An Investigation of Poorly Studied Open Cluster NGC 4337 Using Multicolor Photometric and Gaia DR2 Astrometric Data

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Abstract

We present a comprehensive analysis (photometric and kinematical) of the poorly studied open cluster NGC 4337 using 2MASS, WISE, APASS, and Gaia DR2 databases. By determining the membership probabilities of stars, we identified the 624 most probable members with membership probability higher than 50% by using proper motion and parallax data taken from Gaia DR2. The mean proper motion of the cluster is obtained as \( \mu_x = -8.83 \pm 0.01 \) and \( \mu_y = 1.49 \pm 0.006 \) mas yr\(^{-1}\). We find the normal interstellar extinction toward the cluster region. The radial distribution of members provides a cluster radius of \( 7.75 \) pc. The estimated age of 1600 \( \pm 180 \) Myr indicates that NGC 4337 is an old open cluster with a bunch of red giant stars. The overall mass function slope for main-sequence stars is found as \( 1.49 \pm 0.06 \) within the mass range \( 0.75-2.0 \) \( M_\odot \), which is in fair agreement with Salpeter’s value \((x = 1.35)\) within uncertainty. The present study demonstrates that NGC 4337 is a dynamically relaxed open cluster. Using the Galactic potential model, Galactic orbits are obtained for NGC 4337. We found that this object follows a circular path around the Galactic center. Under the kinematical analysis, we compute the apex coordinates \((A, D)\) by using two methods: (i) the classical convergent point method and (ii) the \( AD \)-diagram method. The obtained coordinates are \((A_{\text{conv}}, D_{\text{conv}}) = (96.2^{\circ}27 \pm 0^{\circ}10, 13^{\circ}14 \pm 0^{\circ}27)\) and \((A_\odot, D_\odot) = (100^{\circ}282 \pm 0^{\circ}10, 9^{\circ}5777 \pm 0^{\circ}323)\) respectively. We also computed the Velocity Ellipsoid Parameters, matrix elements \((\mu_{ij})\), direction cosines \((\xi, \eta, \zeta)\), and the Galactic longitude of the vertex \((\ell)\).

Unified Astronomy Thesaurus concepts: Open star clusters (1160); Proper motions (1295); Hertzsprung Russell diagram (725); Stellar mass functions (1612); Apex (54); Luminosity function (942)

1. Introduction

To understand the stellar evolution and dynamics of stellar systems, open clusters (OCs) are considered to be ideal tools. Before the Gaia era, the studies of open star clusters were mostly hampered by the presence of field stars. Proper motions (PMs) and parallax information provided by Gaia’s mission in the clusters are very valuable tools to identify the probable members of the clusters. These identified cluster members are very crucial in deriving the fundamental parameters, mass function (MF), and kinematical parameters of the clusters.

NGC 4337 \((\alpha_{2000} = 12^{h}24^{m}04^{s}, \delta_{2000} = -58^{\circ}74724^{\prime}, l = 299^{\circ}31, b = 4^{\circ}55)\) was previously investigated by Carraro et al. (2014) and Seleznev et al. (2017) using \(UBVI\) photometric and spectroscopic data. Apart from these studies, no other detailed analysis is available in the literature for this object. Because most OCs are affected by the field star contamination, the information of cluster membership is necessary to understand cluster properties (Kharchenko et al. 2013; Cantat-Gaudin et al. 2018). Our main aim of the present study is to perform a deep investigation of this object using a photometric and kinematical database. The Gaia DR2 was made public on 2018 April 25 (Gaia Collaboration et al. 2016a, 2016b). The data include astrometric five-parameter solutions of central coordinates, PM in R.A. and decl., and parallaxes \((\alpha, \delta, \mu_x \cos \delta, \mu_y, \pi)\) for more than 1.3 billion sources (Gaia Collaboration et al. 2018b).

OCs are considered ideal objects for the determination of initial mass function (IMF; Bisht et al. 2017, 2018, 2019). In the current paper, one of our motives is to provide useful information about IMF using the most probable cluster members, which will be helpful to understand the star formation history in the region of NGC 4337. In the solar neighborhood, the characteristic feature of the stellar motion is often represented by the peculiar velocities. The peculiar velocities have an axis of greatest mobility and this characteristic is represented based on ellipsoidal law of velocity distribution (Ogorodnikov 1965). If we consider the ellipsoidal law to be valid at all points within the steady-state of a stellar system, the function \( f \) can be expressed in the form:

\[
 f = F(x, y, z; ax^2 + bx^2 + cy^2 + 2fxy + 2gxy + 2huvb),
\]

where \(a, b, c, ..., h\) are the functions of \(x, y,\) and \(z\). In the above generalized formula, the length and distributions of the principal axes of the velocity ellipsoid varies from point to point in the system. The analysis can be simplified by exploiting the empirical fact that one axis of the velocity ellipsoid is always found to be oriented perpendicular to the Galactic plane, while the other two axes lie in the plane. It is also found empirically that the longest axis of the ellipsoid (i.e., the direction of maximum velocity dispersion) points approximately toward the direction of the Galactic center. Therefore, to specify the orientation of the velocity ellipsoid, we need to determine only the Galactic longitude along which the principal
axis lies. This longitude is called the longitude of the vertex (l2). The importance of the VEPs lies in their connection to the phase density function, which is one of the most important mathematical functions of stellar astronomy.

The outline of the paper is as follows. The brief descriptions of the data used are described in Section 2. Section 3 deals with the study of PM and determination of membership probability of stars. The structural properties and derivation of fundamental parameters using the most probable cluster members have been carried out in Section 4. Luminosity and MF are discussed in Section 5. The dynamics and kinematics of the cluster are devoted to Section 6. The conclusions are presented in Section 7.

2. Data Used

We collected photometric and astrometric data from Gaia DR2 along with the broadband photometric data sets from APASS, 2MASS, and WISE for NGC 4337. We cross-matched each catalog for the present study. The description of the data sets used is as follows.

2.1. The 2MASS Data Sets

For photometric analysis, we used 2MASS data within the 10′ radius of the cluster. The Two Micron All-Sky (2MASS; Skrutskie et al. 2006) survey uses two highly automated 1.3 m telescopes, one at Mt. Hopkins, Arizona (AZ), USA, and the other at CTIO, Chile, with a three-channel camera (256 × 256 array of HgCdTe detectors in each channel). 2MASS catalog provides J (1.25 μm), H (1.65 μm), and Ks (2.17 μm) band photometry for millions of galaxies and nearly a half-billion stars (Carpenter 2001). The sensitivity of this catalog is 15.8 mag for J, 15.1 mag for H, and 14.3 mag for Ks band at S/N = 10. The identification map of NGC 4337 taken from the Leicester database and archive service (LEDAS) is shown in Figure 1. The errors in J, H, and Ks bands are plotted against J magnitude in the upper left panel of Figure 2. This figure shows that the mean error in J, H, and Ks bands is ≤0.04 at J ~ 13 mag. The error becomes ~0.1 at J ~ 16 mag.

2.2. WISE Data

The effective wavelengths of the Wide-field Infrared Survey Explorer (WISE; Wright et al. 2010) are 3.35 μm (W1), 4.60 μm (W2), 11.56 μm (W3), and 22.09 μm (W4) in the mid-IR bands. We used the ALLWISE source catalog to extract data for NGC 4337. This catalog has achieved 5σ point-source sensitivities better than 0.08, 0.11, 1, and 6 mJy at 3.35, 4.60, 11.56, and 22.09 μm, which is expected to be more than 99% of the sky. These sensitivities are 16.5, 15.5, 11.2, and 7.9 mag for W1, W2, W3, and W4 bands correspond to vega magnitudes. The photometric errors for W1, W2, and W3 bands are plotted against J magnitude in the upper left panel of Figure 2.

2.3. Gaia DR2 Data Sets

Gaia DR2 (Gaia Collaboration et al. 2018a) database within a 10′ radius of the cluster is used here for the astrometric study of NGC 4337. This data consist of positions on the sky (α, δ), parallaxes, and PMs (μα, cos δ, μδ) with a limiting magnitude of G = 21 mag. The photometric errors in Gaia passbands (G, Grp, and Gbp) versus G magnitudes are plotted in the lower left panel of Figure 2. The uncertainties in parallax values are ~0.04 milliarcsecond (mas) for sources at G ≤ 15 mag and ~0.1 mas for sources with G ~ 17 mag. The PMs with their respective errors are plotted against G magnitude in the right panel of Figure 2. The uncertainties in the corresponding PM components are ~0.06 mas yr⁻¹ (for G ≤ 15 mag), ~0.2 mas yr⁻¹ (for G ~ 17 mag), and ~1.2 mas yr⁻¹ (for G ~ 20 mag).

2.4. APASS Data

The American Association of Variable Star Observers (AAVSO) Photometric All-Sky Survey (APASS) is organized in five filters: B, V (Landolt) and g′, r′, i′ proving stars with V band magnitude range from 7 to 17 mag (Heden & Munari 2014). Their latest catalog DR9 covers almost 99% of the sky (Henden et al. 2015). For NGC 4337, we extracted APASS data from http://vizier.u-strasbg.fr/viz-bin/VizieR?-source=II/336.

3. Mean PM and Field Star Separation

PMs play a vital role in order to separate field stars from the main sequence and to derive authentic fundamental parameters as well (Yadav et al. 2013; Sariya et al. 2018a; Bisht et al. 2019). PM and parallax are very reliable parameters to extract field stars from the cluster zone, because cluster stars share the same kinematical properties and distances (Rangwal et al. 2019). To detect the distribution of cluster and field stars, PM components (μα, cos δ, μδ) are plotted as VPD in the bottom panels of Figure 3. The panels of the top rows show the corresponding J versus J − H and the middle row of panels shows G versus (Gbp − Grp) color–magnitude diagrams (CMDs). The left panel shows all the stars, while the middle and right panels show the probable cluster members and field region stars. A circle of 0.5 mas yr⁻¹ around the distribution of cluster stars in the VPD characterize our membership criteria. The chosen radius in such VPD is a compromise between losing cluster members with poor PMs and including field

![Figure 1. Finding chart of stars in NGC 4337, taken from LEDAS.](image)
Figure 2. (Left upper panels) Photometric errors in 2MASS $J$, $H$, and $K_s$ bands against $J$ magnitude. (Left lower panels) Photometric errors in Gaia bands ($G$, $G_{BP}$, and $G_{RP}$) with $G$ magnitudes. (Right panels) Plot of proper motion and their respective errors vs. $G$ magnitude.

Figure 3. (Bottom panels) Proper-motion vector point diagram (VPD) for NGC 4337 based on Gaia DR2. (Top panels) $J$ vs. $J - H$ color–magnitude diagrams. (Middle panels) $G$ vs. ($G_{BP} - G_{RP}$) color–magnitude diagrams. (Left) The entire sample. (Center) Stars in VPDs within a circle of 0.5 mas yr$^{-1}$ of the cluster mean. (Right) Probable background/foreground field stars in the direction of these clusters. We have used only stars with PM $\sigma$ smaller than 0.5 mas yr$^{-1}$ in each coordinate.
region stars (Sariya et al. 2017, 2018b). We have also used mean parallax for the cluster member selection. We estimated the weighted mean of parallax for stars inside the circle of VPD having G mag brighter than 20 mag. We considered a star as the most probable members if it lies within 0.5 mas yr$^{-1}$ radius in VPD and has a parallax within 3σ from the mean parallax of the cluster. The CMD of the most probable cluster members is shown in the upper-middle panel in Figure 3. In this figure, the main sequence of the cluster is identified. These stars have a PM error of $\leq$0.5 mas yr$^{-1}$.

To estimate the mean PM, we consider probable cluster members selected from VPD and CMD as shown in Figure 3. We constructed the histograms for $\mu_\alpha \cos \delta$ and $\mu_\delta$ as shown in the left panel of Figure 4. The fitting of a Gaussian function to the histograms provides mean PM in both directions. In this way, we found the mean-PM of NGC 4337 as $-8.83 \pm 0.01$ and $1.49 \pm 0.006$ mas yr$^{-1}$ in $\mu_\alpha \cos \delta$ and $\mu_\delta$ respectively. The estimated values of mean PMs for this object are in very good agreement with Cantat-Gaudin et al. (2018).

3.1. Membership Probabilities

It is essential to identify the most probable cluster members toward the cluster zone for the reliable determination of its fundamental parameters. Vasilevskis et al. (1958) built up a mathematical technique to estimate the membership probabilities using PMs. Sanders (1971) introduced the maximum likelihood principle for the evaluation of membership of stars in the cluster zone. In this paper, we adopted the approach given by Balaguer-Núñez et al. (1998) by using the Gaia PM database. In this method, two frequency distribution functions are set up for a particular ith star. The frequency distributions of cluster members ($\phi_i^c$) and field stars ($\phi_i^f$) are presented by the equations given below:

$$\phi_i^c = \frac{1}{2\pi\sqrt{(\sigma_c^2 + \epsilon_{\mu_c}^2)(\sigma_c^2 + \epsilon_{\mu_c}^2)}} \times \exp \left\{ -\frac{1}{2} \left[ \frac{(\mu_{\alpha_i} - \mu_{\alpha_c})^2}{\sigma_c^2 + \epsilon_{\mu_c}^2} + \frac{(\mu_{\delta_i} - \mu_{\delta_c})^2}{\sigma_c^2 + \epsilon_{\mu_c}^2} \right] \right\}$$ (1)

and

$$\phi_i^f = \frac{1}{2\pi\sqrt{(1 - \gamma^2)(\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2)(\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2)}} \times \exp \left\{ -\frac{1}{2(1 - \gamma^2)} \frac{(\mu_{\alpha_i} - \mu_{\alpha_c})^2}{\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2} \right\} - \frac{2\gamma(\mu_{\alpha_i} - \mu_{\alpha_c})(\mu_{\delta_i} - \mu_{\delta_c})}{\sqrt{(\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2)(\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2)}} \times \exp \left\{ -\frac{1}{2(1 - \gamma^2)} \left[ \frac{(\mu_{\alpha_i} - \mu_{\alpha_c})^2}{\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2} + \frac{(\mu_{\delta_i} - \mu_{\delta_c})^2}{\sigma_{\mu_c}^2 + \epsilon_{\mu_c}^2} \right] \right\}$$ (2)

where ($\mu_{\alpha_i}, \mu_{\delta_i}$) are the PMs of the ith star, although ($\epsilon_{\mu_c}, \epsilon_{\mu_c}$) are corresponding errors in PMs. ($\mu_{\alpha_c}, \mu_{\delta_c}$) represent the cluster’s PM center and ($\mu_{\alpha_f}, \mu_{\delta_f}$) are PM center coordinates for field stars. The intrinsic PM dispersion is denoted by $\sigma_{\mu_c}$ for members, whereas $\sigma_{\mu_c}$ and $\sigma_{\mu_f}$ show the field intrinsic PM dispersions. The correlation coefficient $\gamma$ is calculated as:

$$\gamma = \frac{(\mu_{\alpha_i} - \mu_{\alpha_c})(\mu_{\delta_i} - \mu_{\delta_c})}{\sigma_{\mu_f}\sigma_{\mu_f}}.$$ (3)

To figure out $\phi_i^c$ and $\phi_i^f$, we considered only those stars that have PM errors better than $\sim$0.5 mas yr$^{-1}$. A tight bunch of stars is found at $\mu_{\alpha_c} = -8.83$ mas yr$^{-1}$, $\mu_{\delta_c} = 1.49$ mas yr$^{-1}$ and in the circular region having radii of 0.5 mas yr$^{-1}$.
Assuming a distance of 2.5 kpc and the radial velocity dispersion of 1 km s⁻¹ for open star clusters (Girard et al. 1989), the expected dispersion ($\sigma_v$) in PMs would be 0.08 mas yr⁻¹. For field region stars, we have estimated ($\mu_x^f$, $\mu_y^f$) = (−6.5, 0.5) mas yr⁻¹ and ($\sigma_x^f$, $\sigma_y^f$) = (4.5, 3.8) mas yr⁻¹.

Considering $n_c$ and $n_f$ are the normalized number of cluster and field stars, respectively, (i.e., $n_c + n_f = 1$), the absolute distribution function can be computed as:

$$\phi = (n_c \times \phi_c^i) + (n_f \times \phi_f^i).$$  \hspace{1cm} (4)

Finally, the membership probability of the $i$th star is given by:

$$P_m(i) = \frac{\phi_c(i)}{\phi(i)}. \hspace{1cm} (5)$$

From this analysis, we identified 541 stars as cluster members with membership probability higher than 50% and $G \leq 20$ mag. In the lower right panel of Figure 4, we plotted membership probability versus $G$ magnitude. In this figure, cluster members and field stars are separated. In the upper right panel of this figure, we plotted $G$ magnitude versus parallax of stars. In Figure 5, we plotted an identification chart in the left panel, PM distribution in the middle panel and $G$ versus $B_P - R_P$ CMD in the right panel using the most probable cluster members. The most probable cluster members with high membership probability (≥50%) are shown by red dots in Figures 4 and 5.

4. Structural Properties of NGC 4337

4.1. Spatial Structure: Radial Density Profile (RDP)

Accurate information of the central coordinates of a cluster is very necessary for a reliable estimation of the cluster’s fundamental parameters, such as age, distance, reddening, etc. For calculating the center, we applied the count-number method to the whole area of NGC 4337. The resulting histograms in both the R.A. and decl. directions are shown in the left panel of Figure 6. The Gaussian curve-fitting is applied at the central regions in the histograms. The fitting provides the central coordinates as $\alpha = 186.01 \pm 0.01$ deg (12h24m23.3) and $\delta = -58.12 \pm 0.003$ deg ($-58^\circ9'12")$. These values of the cluster center are in agreement with the values given by Dias et al. (2002). The central coordinates are also very close to the values given by Cantat-Gaudin et al. (2018) for NGC 4337.

To know about the extent of the cluster, we plotted the RDP (log(radius) versus log(density)) as shown in the right panel of Figure 6 using the derived central coordinates in the above paragraph of this section. To do this, we divided the area of NGC 4337 into many concentric rings. The number density, $R_c$ in the $i$th zone is determined by using the formula $R_c = A_i$, where $N_i$ is the number of stars and $A_i$ is the area of the $i$th zone. This RDP flattens at $r \sim 7.75$ arcmin ($\log(\text{radius}) = 0.89$) and begins to merge with the background density as shown in the right panel of Figure 6. Therefore, we consider $7.75$ as the cluster radius. A smooth continuous line represents the fitted King (1962) profile:

$$f(r) = f_{bg} + \frac{f_0}{1 + (r/r_c)^2}, \hspace{1cm} (6)$$

where $r_c$, $f_0$, and $f_{bg}$ are the core radius, central density, and the background density level, respectively. By fitting the King model to the RDP, we estimated the structural parameters for NGC 4337. The obtained values of central density, background density, and core radius are $13.77 \pm 2.0$ stars per arcmin², 0.41 ± 0.11 stars per arcmin², and $1.77 \pm 0.12$ respectively. The background density level with errors is also shown by the dotted lines. The cluster limiting radius, $r_{lim}$, is calculated by comparing $f(r)$ to the border background density level, $f_{bg}$, defined as:

$$f_b = f_{bg} + 3\sigma_{bg}, \hspace{1cm} (7)$$

where $\sigma_{bg}$ is uncertainty of $f_{bg}$. Therefore, $r_{lim}$ is calculated according to the following formula (Bukowiecki et al. 2011):

$$r_{lim} = r_c \left( \frac{f_0}{3\sigma_{bg}} - 1 \right). \hspace{1cm} (8)$$

The estimated value of limiting radius is found to be 9′+66. By combining $r_c$ and $r_{lim}$ both in terms of the concentration parameter, $c = \log(\text{radius})_c$ (Peterson & King 1975), we can characterize the structure of cluster in the Milky Way. In the present study, the concentration parameter is found to be 0.73. Maciejewski & Niedzielski (2007) reported that $R_{lim}$ may vary for individual clusters from 2$R_c$ to 7$R_c$. The estimated value of $R_{lim}$ (~5.5$R_c$) shows a good agreement with Maciejewski & Niedzielski (2007).

4.2. Tidal Radius

Tidal interactions are crucial to understand the initial structure and dynamical evolution of the clusters (Chumak et al. 2010). Tidal radius is the distance from cluster center
where gravitational acceleration caused by the cluster becomes equal to the tidal acceleration due to the parent Galaxy (von Hoerner 1957). The Galactic mass \( M_G \) inside a Galactocentric radius \( R_G \) is given by Genzel & Townes (1987),

\[
M_G = 2 \times 10^5 M_\odot \left( \frac{R_G}{30 \text{ pc}} \right)^{1.2}.
\]  

(9)

Estimated values of Galactic mass inside the Galactocentric radius (see Section 4.5) are found as \( 1.4 \times 10^5 M_\odot \). Kim et al. (2000) has introduced the formula for the tidal radius \( R_t \) of clusters as

\[
R_t = \left( \frac{M_c}{2 M_G} \right)^{1/3} \times R_G,
\]  

(10)

where \( R_t \) and \( M_c \) indicate the cluster’s tidal radius and total mass (see Section 8), respectively. The estimated value of the tidal radius is \( 9.78 \pm 0.62 \text{ pc} \) for NGC 4337.

4.3. Extinction Law toward the Cluster Region

We combined multiwavelength photometric data with Gaia DR2 astrometric data to test the extinction law from the optical to the mid-infrared region. The resultant \( (\lambda - G_{\text{BP}})/(G_{\text{BP}} - G_{\text{RP}}) \) two color diagrams are shown in the right panel of Figure 7. Here, \( \lambda \) stand for the filters other than \( G_{\text{RP}} \). All stars shown in this figure are the most probable cluster members. A linear fit to the data points is performed and slopes are listed in Table 1. The estimated values of slopes are in good agreement with the value given by Bisht et al. (2019). We calculated \( \frac{A_V}{E(B-V)} \) as \( \sim 3.2 \). This indicates that reddening law is normal in the direction of NGC 4337.

4.4. The Reddening Using 2MASS Colors

Interstellar reddening of the clusters is very important for a reliable determination of the distance and age. To determine the color-excess, we plotted \( (J - H) \) versus \( (J - K) \) color–color diagram as shown in Figure 8 for NGC 4337. Stars used in this figure are the most likely members based on VPD. The zero age main sequence (ZAMS) shown by the solid line is taken from Caldwell et al. (1993). The same ZAMS showed by the dotted line is shifted by \( E(J - H) = 0.07 \pm 0.03 \text{ mag} \) and \( E(J - K) = 0.15 \pm 0.05 \text{ mag} \). The ratio \( E(J - H)/E(J - K) \) is found to be 0.46, which is a little bit lower than the normal interstellar extinction value of 0.55 as suggested by Cardelli et al. (1989). We have determined the value of interstellar reddening \( E(B - V) \) using the relationship \( E(J - K) = 0.72 \pm 0.05 \), as given by Morgan & Nandy (1982). The value of \( E(B - V) \) is found as \( 0.21 \pm 0.01 \). The present value of interstellar reddening is in very good agreement with Carraro et al. (2014).

4.5. Age and Distance

Age and distance are very important parameters to trace the structure and chemical evolution of the Galaxy using OCs (Friel & Janes 1993). The main astrophysical parameters (age, distance, and reddening) of NGC 4337 are estimated by fitting the solar metallicity theoretical evolutionary isochrones of Marigo et al. (2017) to the observed CMDs as shown in Figure 9. In Figure 9, we superimposed isochrones of different log(age) values (9.15, 9.20, and 9.25) having \( Z = 0.019 \) in

![Image: Figure 6. (Left panel) Profiles of stellar counts across cluster NGC 4337. The Gaussian fits have been applied. The center of symmetry about the peaks of R.A. and decl. is taken to be the position of the cluster centers. (Right panel) Surface density distribution (log(radius) vs. log(density)) of the cluster NGC 4337. Errors are determined from sampling statistics \( \frac{1}{\sqrt{N}} \), where \( N \) is the number of stars used in the density estimation at that point. The smooth line represents the fitted profile whereas the dotted line shows the background density level. Long and short dashed lines represent the errors in background density.](image-url)
selected from VPD. The continuous lines represent the slope determined through least-squares linear fit.

Figure 7. (λ - G BP)/(G BP - G RP) two color diagrams using the stars selected from VPD. The continuous lines represent the slope determined through least-squares linear fit.

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Table 1

Multiband Color Excess Ratios in the Direction of NGC 4337

| Band (λ) | Effective Wavelength (nm) | (λ - G BP)/(G BP - G RP) |
|----------|---------------------------|---------------------------|
| Johnson B | 445                       | 1.70 ± 0.03               |
| Johnson V | 551                       | 0.93 ± 0.02               |
| 2MASS J   | 1234.5                    | -0.82 ± 0.03              |
| 2MASS H   | 1639.3                    | -1.14 ± 0.03              |
| 2MASS K   | 2175.7                    | -1.27 ± 0.06              |
| WISE W1   | 3317.2                    | -1.31 ± 0.07              |
| WISE W2   | 4550.1                    | -1.50 ± 0.08              |
| WISE W3   | 12082.3                   | -1.46 ± 0.07              |

\(G\), \((G_{BP} - G_{RP})\), \((G_{BP} - G)\), \((G - G_{RP})\), \(J\), \((J - H)\), \(J\), \((J - W1)\), \(J\), \((J - W2)\), and \(K\), \((J - K)\) CMDs. The overall fit is good for \(\log(\text{age}) = 9.20\) (middle isochrone), corresponding to \(1600 \pm 180\) Myr. The estimated distance modulus \((m - M) = 12.40\) mag provides the heliocentric distance \(2.5 \pm 0.06\) kpc.

To estimate the distance of the cluster, we also used the parallax of cluster members available in the Gaia DR2 catalog. We constructed a histogram using parallax in 0.15 mas bins as shown in Figure 10 using the most probable members selected from the cluster’s VPD. By fitting the Gaussian function, mean parallax is estimated as \(0.40 \pm 0.01\) mas, which corresponds to a distance of \(2.5 \pm 0.07\) kpc. The calculated value of parallax is in good agreement with the value obtained by Cantat-Gaudin et al. (2018). We obtained a similar value of distance for NGC 4337 using mean parallax and distance modulus of the cluster.

5. Luminosity and MF

Luminosity function (LF) and MF are associated with each other with the well-known mass–luminosity relationship. To construct the LF for the cluster NGC 4337, we used \(G\) versus \((G_{BP} - G_{RP})\) CMD. Before building the true LF, we converted the observed \(G\) magnitudes of member stars into the absolute \(G\) magnitudes considering the distance modulus of the cluster. The constructed histogram of LF for NGC 4337 is shown in Figure 11.

MF is derived using LF, which is a relative number of stars in certain interval bins of absolute magnitudes. We have used the model given by Marigo et al. (2017) to convert the LF into MF. The resulting MF is shown in Figure 12. The MF slope can be derived by using the following relation

\[
\log \frac{dN}{dM} = -(1 + x) \log(M) + \text{constant},
\]

where \(dN\) represents the number of stars in a mass bin \(dM\) with central mass \(M\), and \(x\) is MF slope. The MF slope for NGC 4337 is found to be \(1.46 \pm 0.18\). The initial MF for massive stars \((\geq 1 M_{\odot})\) has been studied and well established by Salpeter (1955) and he found the value of \(x\) at 1.35.

According to the Salpeter’s power law, the number of stars in each mass range decreases rapidly with increasing mass. It is noted that our investigated value of MF slope is similar to Salpeter’s value. We have estimated the total mass considering the above MF slope within the mass range \(0.75 - 2.0 M_{\odot}\). The total cluster mass and mean mass are estimated as \(~720 M_{\odot}\) and \(~1.15 M_{\odot}\) respectively.

6. The Orbit of the Cluster

We estimated the Galactic orbit of NGC 4337 using the Galactic potential models. We adopted the method given by Allen & Santillan (1991) for Galactic potentials. Recently, Bobylev & Bajkova (2016) and Bobylev et al. (2017) refined the parameters of Galactic potential models with the help of new observational data for the galactocentric distance.
$R \sim 0–200$ kpc. These potentials are given as

$$
\Phi_b(r, z) = -\frac{M_b}{\sqrt{r^2 + b^2}}
$$

$$
\Phi_d(r, z) = -\frac{M_d}{\sqrt{r^2 + (a_d + \sqrt{z^2 + b_d^2})^2}}
$$

$$
\Phi_h(r, z) = -\frac{M_h}{a_h} \ln \left( \frac{\sqrt{r^2 + a_h^2} + a_h}{r} \right),
$$

where $\Phi_b$, $\Phi_d$, and $\Phi_h$ are the potentials of the central bulge, disk, and halo of Galaxy, respectively. $r$ and $z$ are the distances of objects from Galactic center and Galactic disk respectively. The halo region potential is given by Wilkinson & Evans (1999). All three potentials are axis-symmetrical, time independent, and analytical. Also, their spatial derivatives are continuous everywhere.

6.1. Orbit Calculation

To obtain orbit parameters we have used the main fundamental parameters, like central coordinates ($\alpha$ and $\delta$), mean PMs ($\mu_{\alpha} \cos \delta$, $\mu_\delta$), parallax angles, cluster age, and heliocentric distance ($d_h$). Mean radial velocity is estimated using 28 stars taken from the Gaia DR2 catalog.

The right-hand coordinate system is adopted to transform equatorial velocity components into Galactic-space velocity components ($U$, $V$, $W$), where $U$, $V$, and $W$ are radial,
Radius $t_{\text{a}}$ and note. $RZ_UVW$ Cluster 4337 is orbiting in a circular orbit with eccentricity $\sim 0.0$. The orbit is confined in a box of $7.51 < R_{\text{ap}} \leq 7.58$ kpc. This indicates that NGC 4337 is not interacting within the inner region of the Galaxy. The orbit is within the Solar circle.

| Cluster   | $R$ (kpc) | $Z$ (kpc) | $U$ (km s$^{-1}$) | $V$ (km s$^{-1}$) | $W$ (km s$^{-1}$) | $\phi$ (rad) |
|-----------|-----------|-----------|-------------------|-------------------|-------------------|--------------|
| NGC 4337  | 7.40      | 0.21      | $-18.85 \pm 0.47$ | $-235.40 \pm 0.44$ | $-12.70 \pm 0.54$ | 0.29         |

**Note.** Here $R$ is the galactocentric distance, $Z$ is the vertical distance from the galactic disk, $U V W$ are the radial tangential and the vertical components of velocity, respectively, and $\phi$ is the position angle relative to the Sun’s direction.

7. Dynamical and Kinematical Analysis

The dynamical relaxation time is the timescale in which the cluster will lose all traces of its initial dynamic condition (Yadav et al. 2013; Bisht et al. 2019). Because of the internal dynamics among the members, the contraction and destruction forces make the cluster approach a Maxwellian equilibrium. During mass segregation, more massive stars are concentrated toward the cluster core than fainter ones and this phenomenon has been reported recently for many OCs (Piatti 2016; Zeidler et al. 2017; Dib et al. 2018; Rangwal et al. 2019; Bisht et al. 2020; Joshi et al. 2020). To see the mass-segregation effect in the cluster NGC 4337, we have plotted the cumulative radial stellar distribution of member stars for different masses as shown in Figure 14. We divided the main-sequence stars in three mass ranges, i.e., $2.0 \leq \frac{M}{M_{\odot}} \leq 1.8$, $1.8 \leq \frac{M}{M_{\odot}} \leq 1.0$, and $1.0 \leq \frac{M}{M_{\odot}} \leq 0.75$. Figure 14 shows the effect of mass segregation in the sense that bright stars gradually sink toward the core, while fainter ones move away from the core. Furthermore, we checked this signature using the Kolmogrov–Smirnov (K-S) test. This test also indicates the presence of mass-segregation with a confidence level of 80% in this cluster.

During the relaxation time $t_{\text{relax}}$, which depends on both the total number of members $N$ and the cluster diameter $D$. OCs run away to a Maxwellian stability equilibrium due to the mass-segregation phenomenon (Maciejewski & Niedzielski 2007). During mass-segregation in the cluster region, stars with low mass gain the highest random velocities and occupy a larger volume in comparison to massive stars (Mathieu & Latham 1986).

Mathematically $t_{\text{relax}}$ can be expressed according to the relation given by Spitzer & Hart (1971) as,

$$t_{\text{relax}} = \frac{8.9 \times 10^5 \sqrt{R_D} 3/2}{\langle M \rangle 1/2 \log (0.4N)},$$

where $N = 624$ is the number of most probable cluster members, $R_D$ is the radius containing 50% of the cluster mass (i.e., $R_D \approx \frac{1}{2} \times \text{Radius in pc}$), and $\langle M \rangle$ is the average mass of members ($\approx 1.15 M_{\odot}$). Thus, we estimated the value of dynamical relaxation time as $\tau_{\text{dynamical}} \approx 57 \pm 5.55$ Myr. Finally, we described the cluster dynamical state by computing its dynamical evolution parameter (i.e., $\tau = \text{age}/t_{\text{relax}}$). Age of the cluster is found to be higher than its relaxation time with $\tau \sim 28$. This analysis shows that NGC 4337 is a relaxed OC. All the numerical values of dynamical parameters and different times are listed in Table 4.

7.1. Kinematical Structure

The VEPs for the most probable cluster members are computed using a computational algorithm (see Elsanhoury 2015;...
Elsanhoury et al. 2018; Postnikova et al. 2020). We cross-matched cluster members with the catalog given by Soubiran et al. (2018). The mean radial velocity is calculated as 

\[ V_r = \frac{1}{N} \sum_{i=1}^{N} V_{r,i} \]

\[ V_r = -15.58 \pm 0.53 \text{ km s}^{-1} \]

by using the weighted mean method. We estimated the cluster’s position (X, Y, Z in pc, Mihalas & Binney 1981) and its velocity components (V_x, V_y, V_z km s^{-1}) along x-, y-, and z-axes in the coordinate system centered at the Sun using the formulae given by Smart (1968). Elsanhoury et al. (2016) also explained the equations to estimate velocity components.

**Figure 13.** Galactic orbit of the NGC 4337 estimated with the Galactic potential model described in the text in the time interval of the age of each cluster. The left panel shows a side view and the right panel shows a top view of the cluster’s orbit. The filled triangle and filled circle denotes birth and present day position of NGC 4337 in the Galaxy.

**Table 3**

Orbital Parameters for NGC 4337 Obtained Using the Galactic Potential Model

| Cluster     | e   | R_a (kpc) | R_p (kpc) | Z_{max} (kpc) | \(E\) (100 km s^{-1})^2 | J (100 kpc km s^{-1}) | T (Myr) |
|-------------|-----|-----------|-----------|---------------|--------------------------|------------------------|--------|
| NGC 4337    | 0.004 | 7.58      | 7.51      | 0.27          | -11.92                   | -17.43                 | 196    |

**Figure 14.** Cumulative radial distribution of stars in various mass ranges.

**Table 4**

The Kinematical Parameters of NGC 4337 Cluster

| Parameters                                      | NGC 4337 |
|------------------------------------------------|----------|
| No. of members (N)                             | 624      |
| Distance (kpc) (photometric cal.)              | 2.5 \pm 0.06 |
| \(M_{\text{sun}}(M_\odot)\)                   | 720      |
| \(M_\odot)\) (\(M_\odot\))                   | 1.15     |
| \(t_{\text{rel}}\) (Myr)                      | 57.00 \pm 7.55 |
| \(\tau\) (log \(r = 9.20\))                  | 28.00 (relaxed cluster) |
| \(A_{\text{conv}}\)                           | 9627 \pm 0.10 |
| \(D_{\text{conv}}\)                           | 13414 \pm 0.27 |
| \(A_{\odot}\)                                 | 100228 \pm 0.100 |
| \(D_{\odot}\)                                 | 93577 \pm 0.323 |
| \(\vec{V}_r\) (km s^{-1})                     | -26.75 \pm 0.193 |
| \(\vec{V}_\theta\) (km s^{-1})                | 147.46 \pm 12.14 |
| \(\vec{V}_\phi\) (km s^{-1})                  | 25.29 \pm 0.199 |
| \(\vec{L}_r\) (km s^{-1})                     | -149.446 \pm 12.22 |
| \(\vec{L}_\theta\) (km s^{-1})                | 22.831 \pm 0.209 |
| \(\vec{L}_\phi\) (km s^{-1})                  | 255.825 \pm 15.030 |
| \(\lambda_j\) (\(ij = 1, 2, 3\)) (km s^{-1}) | 1061500, 2390.74, 237.264 |
| \(\sigma_j\) (\(ij = 1, 2, 3\)) (km s^{-1}) | 1030.290, 48.895, 15.403 |
| \(l_1, m_1, n_1\)                             | 00°873, 00°480, -00°078 |
| \(l_2, m_2, n_2\)                             | -00°027, -00°113, -00°993 |
| \(l_3, m_3, n_3\)                             | 00°485, -00°869, 00°085 |
| \((x, y, z)_{\odot}\) (pc)                     | -1948.18, -205.177, -3150.79 |
| \(V\) (km s^{-1})                             | 151.985 \pm 12.328 |
| \(B_j\) (\(ij = 1, 2, 3\))                   | -4525.3, -83°314, 4°910 |
| \(L_j\) (\(ij = 1, 2, 3\))                   | -287888, 103°69, -119°178 |
| \(E\) (kpc)^3                                 | 3250.258 |
| \(X_{\odot}\) (kpc)                           | 1.747 \pm 0.004 |
| \(Y_{\odot}\) (kpc)                           | -3.112 \pm 0.005 |
| \(Z_{\odot}\) (kpc)                           | 0.285 \pm 0.002 |
| \(S_{\odot}\) (km s^{-1})                     | 151.985 \pm 12.328 |
| \((u, v, w)_{\odot}\)                        | -23°270, -2°120 |
| \((\alpha, \delta)_{\odot}\)                 | -79°718, -9°578 |
2. The AD-diagram method ($A_0$, $D_0$).

The AD-diagram is discussed in detail by Chupina et al. (2001, 2006), Vereshchagin et al. (2014), and Elsanhoury et al. (2018), and Postnikova et al. (2020). In stellar apex method, ($A$, $D$) of individual stars give the positions of those stars as a function of space velocity vectors ($V_x$, $V_y$, $V_z$). The intersection point is called the apex ($A_0$, $D_0$) in equatorial coordinates, which is given as follows:

$$A_0 = \tan^{-1}\left(\frac{V_y}{V_z}\right)$$  \hspace{1cm} (18)

$$D_0 = \tan^{-1}\left(\frac{V_x}{\sqrt{V_y^2 + V_z^2}}\right)$$  \hspace{1cm} (19)

Figure 15 shows the AD-diagram for the cluster NGC 4337. The estimated apex position ($[A_{\text{conv}}, D_{\text{conv}}]$ and ($A_0$, $D_0$)) for the cluster NGC 4337 using the above discussed methods are presented in Table 4.

We have derived the velocity ellipsoid parameters (VEPs), matrix elements ($\mu_j$), direction cosines ($l_j$, $m_j$, $n_j$), Galactic longitude of the vertex ($l_2$), and the solar elements for NGC 4337. All these parameters are listed in Table 4. The equations used to derive these kinematical parameters are described in the Appendix. In addition to this, the 3D space distribution of the NGC 4337 cluster has also been described in the Appendix using likely members with membership probability higher than 50% as shown in Figure A1.

8. Conclusions

We presented a comprehensive photometric and kinematical study of the poorly studied OC NGC 4337 using 2MASS, WISE, APASS, and Gaia DR2 data. We have calculated the membership probabilities in the region of cluster NGC 4337 and have found 624 member stars with membership probabilities higher than 50%. We have used those members to derive the fundamental parameters. We also shed some light on the dynamical and kinematical aspects of the cluster. The main points of the current investigation are as follows:

1. The new cluster center is estimated as $\alpha = 186.01 \pm 0.01$ deg ($12^h24^m23.3^s$) and $\delta = -58.12 \pm 0.003$ deg ($-58^\circ 7'12''$) using the most probable cluster members. The cluster radius is estimated as 7.75 using an RDP.

2. Based on the vector point diagram and membership probability estimation of stars, we identified 624 most probable cluster members for this object. The mean PMs of the cluster are estimated as $-8.83 \pm 0.01$ and $1.49 \pm 0.006$ mas yr$^{-1}$ in both the R.A. and decl. directions, respectively.

3. Distances to the cluster NGC 4337 is determined as 2.5 $\pm$ 0.06 kpc. This value is well supported by the distance estimated using the mean parallax of the cluster. Age is determined as 1600 $\pm$ 180 Myr by comparing the cluster CMD with the theoretical isochrones given by Marigo et al. (2017). The isochrones used have a metallicity $Z = 0.01$. 

7.1.1. The Apex of the Cluster

We aim to derive apex coordinates ($A$, $D$) of the cluster on the celestial sphere (Vereshchagin et al. 2014; Postnikova et al. 2020) based on 2MASS and Gaia DR2 catalogs. The apex demonstrates the actual direction in which the object moves. To determine the cluster’s apex, we adopted two different methods: (1) the classical convergent point method and (2) the AD-diagram method. These methods have been discussed below and the obtained results of this analysis are listed in Table 4.

1. The classical convergent point method ($A_{\text{conv}}, D_{\text{conv}}$).

For almost a century, the classical convergent point method has been used to derive the convergent point ($A_{\text{conv}}, D_{\text{conv}}$). In later years, this method has been improved by several authors (e.g., Jones 1971; de Bruijne 1999; Galli et al. 2012). This method assumes the same space velocities for all cluster members. Using the above set of equations and assuming

$$\xi = \frac{V_x}{V_z},$$
$$\eta = \frac{V_y}{V_z},$$  \hspace{1cm} (13)

we get,

$$a_i \xi + b_i \eta = c_i,$$  \hspace{1cm} (14)

where the coefficients $a_i$, $b_i$, and $c_i$ for $N$ cluster members can be given as:

$$a_i = \mu_0^{(i)} \sin \delta_i \cos \alpha_i \cos \delta_i - \mu_0^{(i)} \sin \alpha_i,$$
$$b_i = \mu_0^{(i)} \sin \delta_i \sin \alpha_i \cos \delta_i + \mu_0^{(i)} \cos \alpha_i,$$
$$c_i = \mu_0^{(i)} \cos^2 \delta_i,$$  \hspace{1cm} (15)

then

$$A_{\text{conv}} = \tan^{-1}\left[\frac{\eta}{\xi}\right],$$  \hspace{1cm} (16)

$$D_{\text{conv}} = \tan^{-1}\left[\frac{1}{\sqrt{\eta^2 + \xi^2}}\right].$$  \hspace{1cm} (17)

The coordinates ($A_{\text{conv}}, D_{\text{conv}}$) of the cluster apex are derived from Equations (16) and (17).
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Appendix

The VEPs

To compute VEPs for NGC 4337, we follow a computational algorithm (Elsanhoury et al. 2015, 2018; Elsanhoury 2015; Postnikova et al. 2020). Depending on the matrix that controls the eigenvalue problem for the velocity ellipsoid, we established analytical expressions of some parameters in terms of the matrix elements \(m_{ij}\).

1. The \(\sigma_j\) parameters, \(j = 1, 2, 3\).

The velocity dispersion \((\sigma_1, \sigma_2, \sigma_3)\) is given here as:

\[
\sigma_j = \sqrt{\lambda_j}. \tag{A1}
\]

2. The \(l_j, m_j,\) and \(n_j\) parameters, \(j = 1, 2, 3\).

The \(l_j, m_j,\) and \(n_j\) are the direction cosines for the above eigenvalue problem. Thus, we have:

\[
l_j = \frac{\mu_{22} \mu_{33} - \sigma_j^2 (\mu_{22} + \mu_{33} - \sigma_j^2)}{D_j}, \quad j = 1, 2, 3,
\]

\[
m_j = \frac{\mu_{23} \mu_{13} - \mu_{12} \mu_{33} + \sigma_j^2 \mu_{12}}{D_j}, \quad j = 1, 2, 3,
\]

\[
n_j = \frac{\mu_{12} \mu_{23} - \mu_{13} \mu_{22} + \sigma_j^2 \mu_{13}}{D_j}, \quad j = 1, 2, 3,
\]

where \(l_j^2 + m_j^2 + n_j^2 = 1\) and

\[
D_j = (\mu_{22} \mu_{33} - \mu_{23}^2)^2 + (\mu_{23} \mu_{13} - \mu_{12} \mu_{33})^2 + (\mu_{12} \mu_{23} - \mu_{13} \mu_{22})^2 + 2[(\mu_{22} + \mu_{33})(\mu_{23}^2 - \mu_{22} \mu_{33}) + \mu_{12} \mu_{23} - \mu_{13} \mu_{22}] \sigma_j + (\mu_{23}^2 + 4 \mu_{22} \mu_{33} + \mu_{22}^2 - 2 \mu_{23}^2 + \mu_{12}^2 + \mu_{13}^2) \sigma_j^2.
\]

3. The center of the cluster \((x_c, y_c, z_c)\).

The center of the cluster is derived by the simple method of determining the equatorial coordinates of the center of mass of \(N_i\) number of discrete objects,

\[
x_c = \frac{\sum_{i=1}^{N_i} (r_i \cos \alpha_i \cos \delta_i)}{N},
\]

\[
y_c = \frac{\sum_{i=1}^{N_i} (r_i \sin \alpha_i \cos \delta_i)}{N},
\]

\[
z_c = \frac{\sum_{i=1}^{N_i} (r_i \sin \delta_i)}{N}.
\]

4. The velocity \(V\) of the cluster.

As a function of radial velocity \(V_r\), the velocity of the cluster can be written in the following form:

\[
V = \frac{\sum_{i=1}^{N_i} \sqrt{v_i^2} \cos \lambda_i}{\sum_{i=1}^{N_i} \cos^2 \lambda_i} \tag{A3}
\]

where, \(\lambda_i\) is the angular distance of the star from the vertex and

\[
\lambda_i = \cos^{-1} [\sin \delta_i \sin D + \cos \delta_i \cos D \cos (A - \alpha_i)]. \tag{A4}
\]

5. The \(L_j\) and \(B_j\) parameters.

The \(L_j\) and \(B_j\) \((j = 1, 2, 3)\) are Galactic longitude and the Galactic latitude of the directions that correspond to the extreme values of the dispersion. Then,

\[
L_j = \tan^{-1} \left(\frac{-m_j}{l_j}\right), \tag{A5}
\]

\[
B_j = \sin^{-1} [n_j]. \tag{A6}
\]

6. The \(E\) parameter.

The \(E\) parameter represents the volume of the ellipsoid, i.e.,

\[
E = \frac{4}{3} \pi \sigma_1 \sigma_2 \sigma_3. \tag{A7}
\]

7. The Solar elements.

Considering a group with spatial velocities \((\vec{U}, \vec{V}, \vec{W})\), the components of the Sun’s velocities are
8. NGC 4337 in 3D

In the broader terms, our Galaxy comprises two main structural elements: a spheroidal component and a disk. Each of these contains different characteristics of stellar and nonstellar populations, and they have different compositions, kinematics, and dynamical properties as well as evolutionary histories. The spheroidal component can be considered to be an approximately axially symmetric system in which the stars are distributed at random orbits. The disk is an extremely thin (about 200 pc thick), flat system extended in the Galactic plane from the Galactic center to a radius of about 25–30 kpc (Mihalas & Binney 1981). We expect to find one axis of the velocity ellipsoid of stars in the Galactic plane pointing exactly toward the Galactic center. This expectation is compatible here by analysis of VEPs (i.e., $l$, with our series of articles; e.g., Elsanhoury 2015; Elsanhoury et al. 2016, 2018). Thus, we conclude that the longitude of the vertex $l$ differs slightly from zero.

Figure A1 presents a 3D plot of our cluster. Here, we can infer that the deviation into space may be along the Galactic center, which may lead the system to behave in space like a cascaded system. This causes a gradual transformation of the cluster into a stream deviating into ordered ring structures stretched around the Galactic center (Perottoni et al. 2019; Wang et al. 2020).

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Figure A1. The 3D space distribution of NGC 4337 cluster using likely members with membership probability higher than 50%.

$U_0 = -\overline{U}$, $V_0 = -\overline{V}$, and $W_0 = -\overline{W}$. Therefore, the solar elements with spatial velocities considered (w.r.t.) have been estimated using the equations given by Elsanhoury et al. (2016), by considering the position along $x$, $y$, and $z$-axes in the coordinate system which are centered at the Sun. Then the Sun’s velocities with respect to this same group and referred to the same axes are given as: $X_0^s = -\overline{V}_x$, $Y_0^s = -\overline{V}_y$, and $Z_0^s = -\overline{V}_z$. Thereafter, we have obtained the solar elements with radial velocities considered (w.r.v.c.) as:

$$S_0 = \sqrt{(X_0^s)^2 + (Y_0^s)^2 + (Z_0^s)^2}$$  \hspace{0.5cm} \text{(A8)}$$

$$\alpha_0 = \tan^{-1} \left( \frac{Y_0^s}{X_0^s} \right)$$  \hspace{0.5cm} \text{(A9)}$$

$$\delta_0 = \tan^{-1} \left( \frac{Z_0^s}{\sqrt{(X_0^s)^2 + (Y_0^s)^2}} \right)$$  \hspace{0.5cm} \text{(A10)}$$

where $l_0$ and $\alpha_0$ are the Galactic longitude and R.A. while $b_0$ and $\delta_0$ are the Galactic latitude and decl. of the solar apex. $S_0$ is considered the absolute value of the Sun’s velocity to our groups under investigations.
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