Continuous-variable quantum key distribution (CVQKD) has potential advantages of high secret key rate, which is very suitable for high-speed metropolitan network application. However, the secret key rates of the reported CVQKD systems are only a few Mbps over typical transmission distance so far. Here, we address the fundamental experimental problems and demonstrate a single-carrier four-state CVQKD with sub-Gbps key rate within metropolitan area. In the demonstrated four-state CVQKD using local local oscillator, an ultra-low level of excess noise is obtained and a high efficient post-processing setup is designed for practically extracting the final secure keys. Thus, the achieved secret key rates are 190.54 Mbps, 137.76 Mbps and 52.48 Mbps using linear channel assuming security analysis method and 233.87 Mbps, 133.6 Mbps and 21.53 Mbps using semidefinite programming security analysis method over transmission distances of 5 km, 10 km and 25 km, respectively. This result increases the asymptotic secret key rate to sub-Gbps level, which is sufficient to achieve the one-time pad cryptographic task. Moreover, our work shows the road for future high-rate and large-scale CVQKD deployment in secure broadband metropolitan and access networks.
Continuous-variable quantum key distribution (CVQKD) provides a secret key shared between the sender (Alice) and the receiver (Bob) with information-theoretical security\(^1\), which is very suitable for broadband metropolitan and access networks due to its inherent advantages of high key rate and good compatibility with commercial off-the-shelf components\(^2\). However, the reported CVQKD systems with several Mbps secret key rate (SKR)\(^3\) are still not up to the requirements of one-time-pad encryption (e.g., high-speed secure access networks). Therefore, the development of ultra-high SKR CVQKD is of great importance for its practical application\(^4\).

According to the modulation method of the coherent state, two practical CVQKD schemes have been proposed. One is based on Gaussian modulation coherent state (GMCS)\(^2\), and the other is based on discrete modulation coherent state (DMCS)\(^12\). The GMCS CVQKD has made great progress in both theory and experiment in recent years\(^13\) (e.g., see sec. VII and sec. VIII in ref. 18 for an overview). However, the high-rate GMCS modulation/detection practically requires high-speed digital-to-analog converter (DAC) and analog-to-digital converter (ADC) with higher linearity to quantify large amplitude range following Gaussian distribution, which potentially limits the SKR. As a comparison, the DMCS CVQKD, such as four-state protocol, has more practical advantages of working at low signal to noise ratio (SNR) and low linearity in large operating bandwidth, which can improve the SKR significantly\(^20,22\). Currently, high-speed DMCS CVQKD has been extensively researched by combining the local oscillator (LLO) scheme, which is free from the security loopholes and the intensity bottleneck of the transmitting LO\(^23\)–\(^27\). However, to improve the SKR in practice, the DMCS LLO-CVQKD system faces the following issues: (1) A precise phase noise compensation (PNC) scheme is required to achieve good coherence between two independent laser sources in LLO-CVQKD system\(^28\)–\(^30\). Meanwhile, the DMCS CVQKD with large operating bandwidth needs robust approaches to eliminate other excess noises, such as photon-leakage noise, modulation and detection noise and quantization noise in quantum state preparation, the photoleakage noise in co-fiber transmission, the detection noise and ADC quantization noise in simultaneous detection. Moreover, a precise fast-slow PNC scheme to eliminate the dominate phase noise, including the pilot-tone-assisted fast-drift phase recovery and the least mean square (LMS) adaptive slow-drift phase recovery, is innovatively proposed and efficiently realized for achieving an ultra-low level of excess noise in experiment. Besides, a high-efficient post-processing setup is designed to achieve rate-adaptive reconciliation efficiency better than 95% and practically extract the final secure keys in experiment. Based on the above key technological breakthroughs, the SKRs of the demonstrated CVQKD setup are 190.54 Mbps@5 km, 133.6 Mbps@10 km, and 52.48 Mbps@25 km with the LCA security analysis method and 233.87 Mbps@5 km, 137.76 Mbps@10 km, and 21.53 Mbps@25 km with the SDP security analysis method, achieving a single-carrier CVQKD with sub-Gbps key rate within metropolitan area.

**Results**

**Experimental setup.** The experimental setup of the proposed four-state discretely modulated LLO-CVQKD scheme is demonstrated in Fig. 1. At Alice’s site, a continuous optical carrier is divided into two optical paths by a beam splitter (BS). The upper optical carrier is modulated by the quadrature phase-shift keying (QPSK) digital signal with \(R_{\text{sym}} = 5\) Gbaud symbol rate in an in-phase/quadrature (IQ) modulator (FUJITSU FTIM7962EP), where the digital signals \(I(t) = \text{real}[[R(t) + jQ(t)]\exp(j2\pi f_{\text{c}} t)]\) and \(Q(t) = \text{imag}[[R(t) + jQ(t)]\exp(j2\pi f_{\text{c}} t)]\) are generated from a high-speed arbitrary waveform generator (AWG, Keysight M8195A) with a single channel sampling rate of 30 GSa s\(^{-1}\). The security of

![Fig. 1 Schematic setup of the proposed four-state LLO-CVQKD scheme.](image-url)

LLO-CVQKD continuous-variable quantum key distribution with local local oscillator, BS beam splitter, PC polarization controller, AWG arbitrary waveform generator, MA microwave amplifier, PSA polarization synthesis analyzer, VOA variable optical attenuator, PBC polarization beam combiner, SMF single mode fiber, PBS polarization beam splitter, OC optical coupler, BHD balanced homodyne detector, IPC industrial personal computer.
QPSK modulation with a carrier $f_s$ is theoretically equivalent to that of QPSK with baseband modulation. Moreover, the bandwidth and amplitude of the QPSK signal from the AWG should be carefully controlled for well matching linear gain of broadband amplifier and ensuring the IQ modulation without distortion. In our experiment, the DAC amplitude is properly set to 320 mV. The QPSK bandwidth is further reduced by the root-raised cosine filter with a roll-off factor $a_{ro} = 0.3$ without the influence on the phase recovery accuracy in digital signal processing (DSP). The QPSK modulated signal is then attenuated by a variable optical attenuator (VOA) to be four-state discretely modulated quantum signal. The corresponding average number of photons per pulse is 0.47 with the quantum optical power of $-65.2$ dBm and frequency of 193.5 THz. From Fig. 1, the polarization controller 1 (PC1) is used to maintain the polarization direction of the quantum signal and ensure the optimal modulation in the IQ modulator. Meanwhile, PC2 and PC3 are used to align the polarization direction and optical power of the pilot tone and the LO signal, respectively. The lower optical carrier is directly attenuated to be a pilot tone with reasonable amplitude. The prepared quantum signal and pilot tone with different frequency bands and orthogonal polarization are transmitted through the quantum channel (single mode fiber with a wavelength of 1550 nm) and separated by a polarization beam splitter (PBS) at Bob's site. In order to separate the quantum signal and pilot tone efficiently, a polarization synthesis analyzer (PSA, General Photonics PSY-201) is used for correcting the polarization deterioration resulted from the fiber channel disturbance. Subsequently, the quantum signal and pilot tone are respectively detected with LLO signals by two commercial BHDs (Optilab BPR-23-M). In our experiment, the optical carrier at Alice's site and the LLO signal at Bob's site are independently generated from two free-running lasers (NKT Photonic Basik E15). Moreover, two BHDs' output signals are collected and digitized by a high-speed oscilloscope (Keysight DSOV084A) for the subsequent DSP and post-processing.

In the proposed four-state LLO-CVQKD system, the intense pilot tone and the weak quantum signal are independently generated in different optical paths, which is beneficial to improve the preparation accuracy of the quantum state in the case of finite DAC quantization bits and modulation extinction ratio, so that the DAC quantization noise and modulation noise can be well reduced compared with the conventional RF-subcarrier-assisted LLO-CVQKD scheme as in Eqs. (17) and (18). Moreover, the upper optical carrier is shifted by frequency $f_s = 3.5$ GHz in 5 GBAud QPSK modulation relative to the lower optical carrier at Alice's site, where the photo-leakage noise from intense pilot tone to weak quantum signal can be eliminated in co-bias transmission due to their complete isolation in frequency domain as in Eq. (19). In experiment, the shifting frequency $f_s$ is mainly determined by the quantum operating bandwidth $\Delta f_s = R_{sym}*(1 + a_{ro})$ and the laser frequency difference $\Delta f_{AB}$. At Bob's site, the intense pilot tone and weak quantum signal are separated in orthogonal polarization state for fully guaranteeing low-noise coherent detection of broadband quantum signal and high-saturation limit detection of intense pilot tone. Moreover, referenced to Eqs. (20) and (21), the detection noise and ADC quantization noise can be further reduced by separately detecting the intense pilot tone and weak quantum signal in the case of the limited detection dynamic and ADC quantization bits. As shown in Fig. 2a, b, the designed QPSK quantum frequency component and the designed pilot component are in different frequency bands, verifying no crosstalk between quantum signal and pilot tone. Meanwhile, in Fig. 2a, the pilot tone is not completely suppressed due to the PBS with finite polarization isolation ratio.

Precise fast-slow phase noise compensation. In order to realize a reasonably low excess noise, a precise fast-slow PNC scheme is designed and realized in DSP to accurately compensate the dominate phase noise. As is illustrated in Fig. 3, the output electrical signals $i_{sig}(t)$ and $i_{pilot}(t)$ of two BHDs are digitized by dual-channel 8 bit ADCs at 40 Gs$^{-1}$, respectively. Firstly, the pilot tone $\Delta f_{AB}$ is precisely estimated to be 7.505 GHz by searching the peak value of the pilot frequency spectrum, and the center frequency $\Delta f_{AB} - f_s$ of the desired quantum frequency spectrum is determined to be 4.005 GHz when the shifting frequency $f_s$ is 3.5 GHz. By using the estimated frequencies, the desired quantum and pilot signals are band-pass filtered for eliminating the out-of-band noise and orthogonally down-converted for extracting the in-phase and quadrature components in baseband, respectively. Next, the baseband components of QPSK quantum signal and pilot tone are obtained by matching root-raised cosine filtering and the narrow band low-pass filtering, respectively. Note that the quantum filtering bandwidths in DSP are selected based on the detected power, the QPSK quantum symbol rate, the roll-off factor of the root-raised cosine filter and the employed laser linewidth, which requires a compromise between noise suppression and phase estimation accuracy. Therefore, the fast-drift laser phase difference $\Delta \varphi_A(t)$ involved in QPSK quantum signal $i_{sig}(t) + j k_{sig}(t)$ can be recovered by sharing the phase of the pilot tone $I_{pilot}(k) + j Q_{pilot}(k)$. Moreover, the slow-drift phase difference $\Delta \varphi_Q(t)$ of the QPSK quantum signal originated from different fiber delay and disturbance is adaptively recovered by the designed LMS algorithm with 51 tap and 1e$^{-3}$ step. Besides, the symbol synchronization between the transmitted and received data is finely corrected for further improving the phase recovery accuracy. Furthermore, the optical-frequency difference of two free-running lasers is fixed as much as possible by precise laser wavelength control in our experiment, while the influence of small optical-frequency deviation can be eliminated by the adaptive filtering in the designed DSP. To verify the proposed DSP, the constellation diagrams of the detected QPSK quantum signal without and with the phase recovery are demonstrated respectively under transmission distance $L = 25$ km, as shown in Fig. 3a, b.
High-efficient post-processing. To extract the final key efficiently, a high-efficient post-processing setup is designed as follows. Since the SNR is very low in our experiment, the raw keys after DSP, which are essentially correlated random data, are firstly reversedly reconciled with the multidimensional reconciliation method. After the reconciliation, the raw keys of Alice and Bob are both transferred into binary sequences, which are unidentical due to inevitable noise and will be further corrected by employing the error correction matrix based on multi-edge-type low-density parity check (MET-LDPC) method. Note that in order to guarantee the extraction of the final key in our experiments and hence validate the practicality of the high SKR CVQKD system proposed in this paper, the reconciliation efficiency should be achieved as high as possible. Thus, three parity check matrices are correspondingly designed for the experiments under the transmission distance of 5, 10, and 25 km with a code rate of 0.07, 0.06, and 0.03, respectively, as shown in Table 1. Specially, for the design of the matrices, a 10 bit quantization based on density evolution algorithm is chosen to obtain the degree distribution functions under such low SNRs, through which the convergence threshold \( \omega_{DE} \) of degree distribution function and the corresponding threshold reconciliation efficiency \( \beta_{r} \) required by the demonstrated CVQKD system is guaranteed. Subsequently, the layered LDPC decoder algorithm and the adaptive decoding algorithm are combined for error correction step. After the error correction, privacy amplification with Toeplitz matrix is employed to extract the final keys. It can be observed from Table 1 that the threshold reconciliation efficiency \( \beta_{r} \) and the efficient rate-adaptive reconciliation efficiency \( \beta_{a} \) are both gained to be better than 95% over the distance of 5, 10, and 25 km. Moreover, the SNRs and reconciliation efficiencies without rate-adaptive versus FERs are computed under the three code rates and shown in Fig. 4, for further verifying our designed post-processing setup. For our post-processing, the high-efficient check matrices are innovatively designed and efficiently realized on graphics processing unit (GPU, NVIDIA TITAN Xp) with low SNR and final secure keys are successfully extracted in off-line situation, which are experimentally achieved in the four-state LLO-CVQKD system compared with the reported literatures according to our knowledge. Note that it is significant for high-rate CVQKD system to distribute the final
secure keys between two legitimate parts by post-processing in real time, which will be deeply researched in our future work.

Discussion
The performance of the proposed four-state LLO-CVQKD setup is shown as follows. In our work, the SKRs of the demonstrated experimental four-state LLO-CVQKD system are firstly evaluated by the general SDP security analysis method and then verified by the frequently used LCA security analysis method. Note that the SDP method depends on a lot of computational power for realizing the optimal solution of Z. In the latest work, an improved SDP method has been reported for obtaining Z by explicit solution. Nevertheless, the SKR with explicit solution is equal to SKR with SDP. For achieving an optimized SKR, the SKR as a function of the excess noise and modulation variance are simulated theoretically for choosing an modulation variance in applicable for the SDP and LCA method. As shown in Fig. 5a, b, a preferable modulation variance $V_A$ is chosen to be about 0.45 in shot noise unit (SNU) for supporting a level of sub-

Table 2 Estimated main excess noise components.

| Noise source                              | Noise magnitude (SNU) |
|-------------------------------------------|-----------------------|
| Untrusted noise                           | Laser intensity noise $\epsilon_{\text{LIN}}$ | $8.1 \times 10^{-5}$ |
|                                           | DAC quantization noise $\epsilon_{\text{DAC}}$ | $4.64 \times 10^{-4}$ |
|                                           | Modulation noise $\epsilon_{\text{Mod}}$ | $4.7 \times 10^{-4}$ |
|                                           | Rest phase noise $\epsilon_{\text{phase, rest}}$ | 0.0032 |
|                                           | Other noise | 0.0033 |
|                                           | Detection noise $\epsilon_{\text{Det}}$ | 0.2869 |
|                                           | ADC quantization noise $\epsilon_{\text{ADC}}$ | 0.0101 |
|                                           | Total excess noise excluding trusted electronic noise | 0.0075 |

The experimental results are obtained in the case of 5 GBaud repetition rate and 25 km transmission distance. SNU shot noise unit, DAC digital-to-analog converter, ADC analog-to-digital converter.

The simulated thermodynamic-SKR diagrams at different excess noises and modulation variances. The simulated results are obtained in the case of secure transmission distance of 25 km. a SKR is evaluated with SDP method. b SKR is evaluated with LCA method. SNU shot noise unit, SKR secret key rate, SDP semidefinite programming, LCA linear channel assuming.
Gbps SKR in asymptotic regime. Moreover, compared with previous works, the SKR of our work is significantly improved by increasing the repetition rate up to 5 GBaud, optimizing the excess noise to an ultra-low level and realizing the reconciliation efficiency better than 95%.

The proposed four-state LLO-CVQKD scheme can be experimentally demonstrated with high repetition rate and high SKR by mainly relying on low-noise high-speed transceiver, more precise fast-slow PNC scheme and high-efficient post-processing due to the lower SNR in high-rate LLO-CVQKD. Moreover, in contrast to ref. 35, the SKR evaluated by the LCA method is lower than that of the SDP method within transmission distance of 10 km, because the

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**Table 3** Full comparison between the proposed LLO-CVQKD scheme and the existing literatures.

| Literature | Repetition rate | Quantum state preparation | Multiplexing setup | Excess noise (SNU) | Post processing | SKR evaluation method | Distance (km) | SKR (Mbps) |
|------------|----------------|---------------------------|--------------------|-----------------|----------------|-----------------------|---------------|-----------|
| This work  | 5 GBaud        | Four state                | Frequency          | 0.007           | LDPC           | LCA                   | 5             | 190.54    |
|            |                |                           |                    |                 |                |                       | 10            | 137.76    |
|            |                |                           |                    |                 |                |                       | 25            | 52.48     |
| Wang7      | 100 MHz        | Gaussian                  | Frequency          | 0.022           | -              | Optimality of Gaussian attacks | 25            | 7.04      |
| Wang15     | 500 MHz        | Gaussian                  | Time               | 0.062           | -              | Optimality of Gaussian attacks | 20            | 10.37     |
| Ren17      | 500 MBaud      | Gaussian                  | Time               | 0.083           | -              | Optimality of Gaussian attacks | 15            | 3.13      |
| Eriksson20 | 400 MBaud      | Four state                | Frequency          | 0.007           | LDPC           | LCA                   | 25            | 0.89 channel⁻¹ |
| Roumestan23| 600 MBaud      | 1024 QAM                  | Frequency          | 0.037           | -              | Explicit solution     | 9.5           | 67.6      |
| Roumestan24| 64 QAM         | Quadrature amplitude modulation | Frequency | 0.030           | -              |                       | 66.8          |           |

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**Fig. 6** Measured excess noises in SNU. The results are measured by 16 times over 5, 10, and 25 km secure transmission distance, respectively. The black circles represent the measured excess noises on the block of size $4 \times 10^6$, the red solid lines define the mean of the measured excess noises, the blue dash lines and the orange dash dots denote the excess noises of null SKR threshold with SDP and LCA methods, respectively. SNU shot noise unit, SKR secret key rate, SDP semidefinite programming, LCA linear channel assuming.

**Fig. 7** SKRs as a function of the secure transmission distance. The blue and black lines represent the simulated SKRs at different secure transmission distances with the SDP and LCA method. The red star and the olive square correspond experimental SKRs with the SDP and LCA method, respectively. The numbers in the square brackets represent the corresponding refs. [7,15,17,22–24]. SKR secret key rate, SDP semidefinite programming, LCA linear channel assuming.
LCA method considers the trusted receiver (BHD and ADC) in our work. Significantly, the additional side channels in IQ modulation are susceptible to leakage of secret information\(^7\), so the sideband modulation should be filtered out in the practical CVQKD system. Finally, the obtained ultra-low level of excess noise in the proposed four-state LLO-CVQKD also can support the SKR evaluation under finite-size effect when four-state CVQKD with tight finite-size security is reported in future. At the same time, our future work will expand four-state modulation to larger constellations or Gaussian modulation for increasing the SKR and transmission distance of LLO-CVQKD. More importantly, the ultra-low level of excess noise, the high-efficient reconciliation efficiency better than 95%, and the more general secure analysis by SDP method are experimentally demonstrated in this paper, achieving high-rate and more secure four-state LLO-CVQKD system for high-speed metropolitan area network application.

Conclusion
We have experimentally demonstrated a sub-Gbps key rate four-state discretely modulated LLO-CVQKD scheme within metropolitan area. In the proposed scheme, the quantum signal and pilot tone are independently generated, co-propagated and separately detected based on frequency- and polarization-multiplexing method, which effectively reduces the modulation noise, ADC/DAC quantization noise, detection noise and photon-leakage noise. Moreover, the dominate phase noise can be precisely eliminated by the designed fast-slow PNC scheme based on pilot-tone-assisted fast-drift phase recovery and LMS adaptive slow-drift phase recovery, achieving a 5 Gbaud symbol rate four-state LLO-CVQKD with an ultra-low excess noise. Furthermore, a high-rate four-state CVQKD protocol for network can be designed to extract the final secure key experimentally (off-line), i.e., 233.87 Mbps, 137.76 Mbps and 21.53 Mbps by the SDP method and 190.54 Mbps, 133.6 Mbps and 52.48 Mbps by the LCA method over transmission distance of 5, 10, and 25 km, respectively, which allows the sub-Gbps SKR single-carrier CVQKD within metropolitan area. In our work, the SDP method, which is resistant against general collective attack, is firstly used to evaluate the SKR of the experimental DMCS CVQKD setup. Moreover, the high-rate metropolitan QKD will be implemented in practice by further increasing the post-processing rate and the coherent stability of quantum key transceiver with LLO in the future. More importantly, the LLO-CVQKD with ultra-high SKR is realized to pave the way for the one-time pad with LLO in the future. More importantly, the LLO-CVQKD with ultra-high SKR is realized to pave the way for the one-time pad with LLO in the future.

Security analysis with LCA method. For the LCA method, the \( I_{AB} \) in Eq. (3) can be expressed as\(^13,34\)

\[
I_{AB} = \log_2 \left( \frac{V + \chi_{line} + \chi_{h} + T}{1 + \chi_{line} + \chi_{h} + T} \right)
\]

with

\[
V = V_A + 1 \tag{5a}
\]

\[
\chi_{line} = 1/T - 1 + \epsilon \tag{5b}
\]

\[
\chi_{h} = [(2 - \eta) + 2\nu_1]/\eta \tag{5c}
\]

where the transmittance efficiency \( T \), the modulation variance \( V_A = 2a^2 \) and the excess noise \( \epsilon \). At the same time, \( S_{BBE} \) can be calculated as

\[
S_{BBE} = G \left( \frac{(\lambda_2 - 1)}{2} + G \left( \frac{(\lambda_4 - 1)}{2} \right) - G \left( \frac{(\lambda_5 - 1)}{2} \right) \right) \tag{6}
\]

with the Von Neuman entropy \( G(x) = (x + 1)\log_2(x + 1) - x\log_2(x) \), and symplectic eigenvalues \( \lambda_i \) can be derived from the covariance matrix between Alice and Bob, which are expressed as

\[
\lambda_{1,2,4} = \frac{1}{2} \left( A \pm \sqrt{A^2 - 4B} \right) \tag{7a}
\]

\[
\lambda_{3,4} = \frac{1}{2} \left( C \pm \sqrt{C^2 - 4D} \right) \tag{7b}
\]

\[
A = V^2 + T^2 \left( V + \chi_{line} \right)^2 - 2TV_2 \tag{8a}
\]

\[
B = \left( V^2 + TV_2 \right)^2 - 4T^2 \chi_2 \tag{8b}
\]

\[
C = A^4_{line} + B + 2A_{line}V\sqrt{B} + T(V + \chi_{line}) + 2T^2 \tag{8c}
\]

\[
D = \left( V + \chi_{line} \right)^2 \left( V + 2\chi_{line} \sqrt{B} \right) + T(V + \chi_{line} + \chi_{h}) \tag{8d}
\]

with the Hermitian matrices 

\[
Z_4 = 2a^2 \left( \xi_0^2 + \xi_1^2 + \xi_2^2 + \xi_3^2 \right) \tag{8e}
\]

\[
Z_5 = 2a^2 \left( \xi_0^2 + \xi_1^2 + \xi_2^2 + \xi_3^2 \right) \tag{8f}
\]

where \( \xi_0 = 2\exp(-a^2) \cos(\alpha^2) \cos(\alpha^2) \) and \( \xi_1 = 1/2\exp(-a^2) \sin(\alpha^2) \sin(\alpha^2) \)

Methods

Four-state LLO-CVQKD protocol. The four-state CVQKD protocol can be described as follows. At Alice’s site, as shown in Fig. 8, a string of random bits \( x = (x_0, ..., x_{2L-1}) \) is encoded as coherent states \( |\psi_i\rangle \) with equal probability\(^35\)

\[
|\psi_i\rangle := |\psi(a)\rangle = e^{-a^2/2} \sum_{n=0}^{\infty} e^{i\lambda n} n! / (\sqrt{n!} |\psi(a)\rangle n) \tag{1}
\]

with \( a > 0 \) and \( k \in \{0, 1, 2, 3\} \). After transmission over an insecure quantum channel, the prepared coherent states are measured by heterodyne detection at Bob’s site with measurement results \( z = (z_0, ..., z_{2L-1}) \in \mathbb{R}^{2L} \), which is converted into a raw key \( y = (y_0, ..., y_{2L-1}) \), given by

\[
(y_{2L-1}, y_{2L}) = \begin{cases} (1, 1) \text{ when } z_{2L-1} \geq 0, z_L \geq 0 \\ (1, -1) \text{ when } z_{2L-1} < 0, z_L \geq 0 \\ (1, 0) \text{ when } z_{2L-1} < 0, z_L < 0 \\ (1, -1) \text{ when } z_{2L-1} > 0, z_L \leq 0 
\end{cases} \tag{2}
\]

Then, the parameter estimation is performed to calculate how much secret key can be achieved from the raw key via post-processing. The post-processing process includes reverse reconciliation, error correction and privacy amplification. In the asymptotic limit, the SKR with reverse reconciliation can be written as

\[
K = R_{sym}(I_{AB} - S_{BB}) \tag{3}
\]

where \( I_{AB} \) is the Shannon mutual information between Alice and Bob, and \( S_{BB} \) is the Holevo bound between Bob and Eve, respectively. Currently, the security proofs of CVQKD with four-state modulation have been established by LCA method\(^13\) and SDP method\(^35,36\). In most reported four-state LLO-CVQKD experiments, the LCA method is used to evaluate the SKR, which limits the attack of Eve. Meanwhile, the SDP method is applicable for general collective attacks. However, the tolerable excess noise with SDP method is very low (e.g., 0.01 SNU), which is challenging in practical CVQKD system. In our work, the SDP method is verified experimentally with reasonably low excess noise.

Fig. 8 Sketch map of the four coherent states protocol. The four quadrants with different colors represent the four encoded coherent states sent from Alice’s site, respectively. The first quadrant (1, 1) denotes the coherent state encoded with phase of \( \alpha/4 \), the second quadrant (−1, 1) denotes the coherent state encoded with phase of \( 3\alpha/4 \), the third quadrant (−1, −1) denotes the coherent state encoded with phase of \( 5\alpha/4 \) and the fourth quadrant (1, −1) denotes the coherent state encoded with phase of \( 7\alpha/4 \).
Security analysis with SDP method. For the SDP method, the $I_{AB}$ in Eq. (3) is expressed as: \[ I_{AB} = \log_2 \left( 1 + \frac{2\Delta q}{2 + T_{\lambda}} \right) \] (9)

where we have defined the quantum efficiency $q = 1$ and the electronic noise $\nu_{el} = 0$. The Holevo bound $\delta_{H}$ can be simplified as

\[ \delta_{H} = \left( \frac{v_1 - 1}{2} \right) + \left( \frac{v_2 - 1}{2} \right) - \left( \frac{v_1 - 1}{2} \right) \] (10)

where $v_1 = 1 + 2a^2 - \left[ Z^2(1+v) \right]$, $v_1$ and $v_2$ are the symmetrical eigenvalues of the optimized covariance matrix between Alice and Bob, given by

\[ \Gamma' = \left( 1 + 2a \right) H_{fi}^s Z_{\delta_i} Z_{\sigma_i} \] (11)

where $\delta = v + 1 + 2T_{\lambda}^2 + T_{\tau}$. $H_{fi}$ is $\delta_i$ and $\sigma_i$ are the diagonal matrices. $Z$ is the optimal solution of the following constraint condition

\[ \min \text{tr} \left( \left( \left( \Pi_{\lambda} \right) \otimes \left( 1 + 2d^b \right) \right) \right) = v \]

\[ \text{tr} \left( \left( \left( \Pi_{\lambda} \right) \otimes \left( 1 + 2d^b \right) \right) \right) = \frac{\epsilon}{2 + T_{\lambda}} \] (12)

where $\epsilon_{s}$ is the annihilation and creation operators $a$ ($b$) and $a^\dagger$ ($b^\dagger$) on Fock space at Alice’s site and Bob’s site, respectively. $X$ is positive semidefinite. We have defined $R_{\lambda} \equiv |\psi_{\lambda}\rangle| \langle \psi_{\lambda}|$, $|\epsilon, k = 0, 1, 2, 3 \rangle$ and $|\Pi_{\lambda}\rangle|\langle \Pi_{\lambda}|$. $R_{\lambda}$ and $\Pi_{\lambda}$ correspond to 193.5 THz and 4 dBm, respectively, in our experiment. $\tau$ denotes the pulse duration and $B$ represents the effective detection bandwidth. Note that an additive noise $\epsilon_{el}$ denotes the low-frequency quantum noise, which is mainly determined by the waveform variability of BHD and the linewidth of laser. Since the linewidth of the employed lasers is very low (<0.1 kHz) and the modulation noise is moved to the intermediate frequency (about 4 GHz) in our scheme, the low-frequency quantum noise $\epsilon_{el}$ is extremely low, which can be ignored. Therefore, the heterodyne detection noise $\epsilon_{el}$ can be theoretically calculated to be 0.2714 based on Eq. (20) in the case of the pulse duration $\tau = 0.2$ ns, the effective detection bandwidth $B = 6.5$ GHz and the NEP = 5.84 pw Hz^{-1/2}.

In the heterodyne detection case, the ADC quantization noise $\epsilon_{ADC}$ in Eq. (15) can be expressed as

\[ \epsilon_{ADC} = \frac{\tau}{4B} \frac{1}{\sqrt{P_{LO}}} \frac{1}{2} R_{\lambda} \] (21)

where $P_{LO}$ is the conversion gain of the BHD in (V W^{-1}). $R_{\lambda}$ and $\tau$ are the full voltage range and quantization bits of ADC, respectively. In our experiment, the conversion gain $G_{lo}$ is 1500 V W^{-1} typ. from the specification of the employed BHD. The full voltage range $R_{\lambda}$ and ADC quantization bits $n$ of the oscilloscope are set to 120 mV and 8 bits, respectively. Based on Eq. (21), the ADC quantization noise can be calculated to be 0.0101 ns. Therefore, according to the calculated detection noise $\epsilon_{el}$ and ADC quantization noise $\epsilon_{ADC}$, the electronic noise $\epsilon_{el}$ is theoretically calculated to be 0.2851 close to the measured electronic noise of 0.297 in our experiment. Meanwhile, it is regarded as the trusted noise for the LCA method and ignored for the SDP method in our work. It is obvious that the ADC quantization noise will be alleviated in finite quantization bits if the weak quantum signal and intense pilot tone are independently quantized by two ADCs at Bob’s site. Moreover, we can see from Eqs. (20) and (21) that the separate detection can flexibly provide the sufficient optical power $P_{LO}$ to reduce the detection noise and ADC quantization noise as much as possible in finite detection dynamic and ADC quantization bits.

The last term $\epsilon_{phase}$ in Eq. (15) represents the four-state phase noise in excess noise. In LLO-CVQKD scenario, the phase noise is divided into two parts, given by

\[ \epsilon_{phase} = \epsilon_{fast} + \epsilon_{slow} \] (22)

where the fast-drift phase noise $\epsilon_{phase_{fast}}$ is originated from the fast-drift phase noise. The slow-drift phase noise $\epsilon_{phase_{slow}}$ is well compensated by the proposed precise fast-slow PNC scheme based on pilot-tone-assisted fast-slow phase recovery and LMS adaptive slow-drift phase recovery. After the PNC, the rest phase noise can be expressed as

\[ \epsilon_{phase} = \epsilon_{fast} = \epsilon_{slow} \] (23)

The deviation voltage $\delta V_{DAC}$ is determined by the quantization bits and voltage range of DAC. In this case, the DAC quantization noise is computed to be $4.64 \times 10^{-16}$ based on Eq. (17).

\[ \delta V_{DAC} = \sqrt{\epsilon_{DAC} \frac{\epsilon_{DAC}}{V_{DAC}}} \frac{\epsilon_{DAC}}{V_{DAC}} \] (17)

In Eq. (15), without considering the intense pilot tone, the modulation noise $\epsilon_{mod}$ can be expressed as

\[ \epsilon_{mod} = \frac{|d_s|^2}{\frac{1}{10^{4/10}}} \] (18)

where $d_s$ denotes the intensity of the used IQ modulator and $d_s$ means the amplitude of the quantum signal. Since the weak quantum signal and the intense pilot tone are separately prepared in our scheme, the modulation noise $\epsilon_{mod}$ can be calculated based on Eq. (18), which is $4.7 \times 10^{-4}$ with $d_s = 40$ dB and the quantum optical power of $65.2$ dBm. It is obvious from Eqs. (17) and (18) that the modulation noise $\epsilon_{DAC}$ and modulation noise $\epsilon_{mod}$ are lower relative to the reported RF-subcarrier-assisted LLO-CVQKD scheme due to the weak quantum signal and intense pilot tone generated in different modulation path.

The fourth term $\epsilon_{el}$ in Eq. (15) denotes the photon-leakage noise, which is determined as

\[ \epsilon_{el} = \frac{2|\delta_s|^2}{\frac{R_s}{R_s}} \] (19)

with the pilot amplitude $\delta_s$, $R_s$ denotes the intensity ratio, and it mainly depends on polarization isolation ratio and modulation extinction ratio when the quantum signal and pilot tone are in same time duration or frequency band based on time multiplexing or frequency multiplexing. In the former LLO-CVQKD schemes, the surplus pilot signals cannot be completely suppressed due to finite modulation extinction ratio and polarization isolation ratio, resulting in photon-leakage noise on the quantum signal in practical experiment. Therefore, it is better to completely isolate the quantum signal and pilot tone in frequency or time domain. In our scheme, the photon-leakage noise $\epsilon_{el}$ from the intense pilot tone to the weak quantum signal can be eliminated due to their complete separation in frequency domain.

From Eq. (15), the fifth term $\epsilon_{Det}$ represents the heterodyne detection noise at Bob’s site, given by

\[ \epsilon_{Det} = \frac{2}{h} \frac{B_t}{\sqrt{P_{LO}}} + \epsilon_{el} \] (20)

with the noise-equivalent power NEP and Planck’s constant $h$, and $P_{LO}$ are the LO frequency and power, and they are 193.5 THz and 4 dBm, respectively, in our experiment.
evaluated as follow
\[ \epsilon_{\text{phaseрест}} = \epsilon_{\text{dis}} + \epsilon_{\text{LMS}} + \epsilon_{\text{rest}} \] (23)

where \( \epsilon_{\text{phaseрест}} \) is from the compensation error of the pilot-tone-assisted fast-drift phase recovery scheme, written as
\[ \epsilon_{\text{LMS.error}} = V_A \frac{\epsilon_{\text{dis}} + \epsilon_{\text{phaseрест}}}{|d_{\text{fast}}|} \] (24)

with
\[ \epsilon_{\text{phase resta}} = 2 \frac{Q^T R^{-1}}{Q^T R} + \epsilon_{\text{rest}} + \frac{2\mu_{\text{ref}}}{Q^T R} \] (25)

where \( \epsilon_{\text{dis}} \) is the channel noise in the pilot polarization direction, and it is extremely low and ignored due to the pilot tone with single frequency. The pilot amplitude \( a_p \) is determined by the pilot power of ~26 dBm in the experiment. So, the compensation error \( \epsilon_{\text{phase resta}} \) is computed to be 0.0022 based on Eqs. (24) and (25), where the electronic noise \( v_0 \) is 0.297 measured in experiment. Moreover, the \( \epsilon_{\text{LMS.error}} \) is the compensation error of the LMS adaptive slow-drift phase recovery, which can be reduced as much as possible by properly choosing the tap and step of the designed LMS algorithm. The term \( \epsilon_{\text{LMS.error}} \) is estimated to be about 0.001 by many experimental evaluations. Therefore, the rest phase noise \( \epsilon_{\text{phase resta}} \) is 0.0032 based on Eq. (23) after the PNC in designed DSP.

Data availability
The data used in this study are available from the authors under reasonable request.

Code availability
Code used in the study is available from the authors under reasonable request.

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Author contributions
W.H. proposed the idea and wrote this manuscript. W.H., P.Y.D., and P.Y. carried out the experimental work. S.Y., H.W., and Z.T. carried out the excess noise modeling. M.L., Y.J., and L.Y. carried out the post-processing work. Z.Y.C. and X.B.J. carried out the theoretical analysis on the protocol. All the authors analyzed and discussed the results and contributed to write the manuscript.

Competing interests
The authors declare no competing interests.

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