Effective attraction between oscillating electrons in a plasmoid via acoustic wave exchange

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We consider the effective interaction between electrons owing to the exchange of virtual acoustic waves in a low-temperature plasma. Electrons are supposed to participate in rapid radial oscillations, forming a spherically symmetric plasma structure. We show that under certain conditions, this effective interaction can result in the attraction between oscillating electrons and can be important for the dynamics of a plasmoid. Some possible applications of the obtained results to the theory of natural long-lived plasma structures are also discussed.

Keywords: acoustic waves; effective attraction between electrons; long-lived atmospheric plasma structures

1. Introduction

The interaction between a test charged particle, embedded in a warm plasma, and the collective response of a plasma results in the concept of a dressed particle. Accounting for the collective plasma interaction, for a particle at rest, one obtains screening of the vacuum Coulomb interaction, leading to the Debye–Hückel potential (Landau & Lifshitz 1980). Nambu & Akama (1985) showed that a test particle moving in plasma is a source of ion acoustic waves that form in its wake. If the speed of a test particle is close to the speed of the ion acoustic wave propagation, the resulting wake potential prevails the Debye–Hückel interaction, leading to the effective attraction of charged particles of the same polarity (Nambu & Akama 1985). This process may be responsible for the formation of complex structures in dusty plasmas (Shukla & Mamun 2002; Tsytovich 2007).

There is, however, another possibility for charged particles of equal polarities to experience an effective attraction in a plasma. It happens when charged particles interact via an acoustic wave appearing in the neutral component of plasma (Vlasov & Yakovlev 1978). This process is analogous to the phonon exchange between two electrons in metal (Madelung 1978). For the implementation of this mechanism, the temperature of the plasma should be at least less than the ionization potential of an atom—otherwise, neutral atoms can
exist only in a small fraction. The results of Vlasov & Yakovlev (1978) may have various applications. In particular, they can be used for the explanation of the stability of atmospheric plasma structures (Stenhoff 1999).

In the present work, using the formalism of Vlasov & Yakovlev (1978), we will study the effective attraction between charged particles participating in spherically symmetric oscillations of a low-temperature plasma. Recently, radial plasma pulsations were considered by Dvornikov & Dvornikov (2007) and Dvornikov (2010, 2011), in frames of both quantum and classical approaches, as a theoretical model of natural plasmoids (Stenhoff 1999). Unlike Vlasov & Yakovlev (1978), in the present work, we will suggest that the stability of a plasma structure is provided by some other mechanisms such as various nonlinear (Škorić & ter Haar 1980; Laedke & Spatschen 1984; Dvornikov 2011) or quantum (Haas & Shukla 2009) effects.

This work is organized as follows. In §2, we develop the general theory of the effective interaction between two charged particles via the exchange of an acoustic wave and apply it to electrons performing spherically symmetric oscillations in plasma. Then, in §3, we evaluate the parameters of the effective potential and analyse the conditions that result in the attraction between oscillating electrons. In §4, we consider the application of the described effective interaction to the dynamics of natural long-lived plasmoids. Finally, in §5, we summarize our results.

2. Effective interaction in a spherically symmetric plasma structure

In this section, we formulate the general dynamics of a spherically symmetric oscillation of a low-temperature plasma. The electron temperature should be less than the ionization potential of an atom. In this case, both electron, ion and neutral components can coexist in a plasma. For a hydrogen plasma, this critical temperature is approximately $10^5$ K. Then, we consider the effective interaction between two oscillating electrons owing to the exchange of a virtual acoustic wave and establish conditions when this interaction corresponds to a repulsion and an attraction between particles.

Suppose that one has excited a plasma oscillation in a low-temperature plasma. In this situation, both electrons and ions will oscillate with respect to neutral atoms. Note that electrons will oscillate with a higher frequency than that of ions since their mobility is much greater. If the density of neutral atoms is sufficiently high, charged particles will collide with neutral atoms, generating perturbations of their density or even waves in the neutral component of the plasma. Typically, this process will result in acoustic wave emission and finally in energy dissipation in the system. However, if an acoustic wave is emitted coherently to be absorbed by another charged particle, one can expect the appearance of an effective interaction. The analogous phenomenon is well known in solid-state physics (Madelung 1978). It was suggested by Vlasov & Yakovlev (1978) that such an effective interaction may well happen in a low-temperature plasma.

In this work, we will mainly discuss spherically symmetric plasma oscillations. The possibility of the existence of such a plasma structure is predicted in both classical (Škorić & ter Haar 1980; Laedke & Spatschen 1984; Dvornikov...
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2011) and quantum (Dvornikov & Dvornikov 2007; Haas & Shukla 2009) cases. Stable classical plasmoids can exist owing to the various plasma nonlinearities, which arrest the collapse of oscillations (Goldman 1984), whereas the stability of quantum plasmoids is provided by the additional quantum pressure. In our study, we do not specify physical processes underlying the stability of the plasma structure in question.

Omitting the motion of ions we can represent the plasma characteristics in terms of the electron density $n_e$, the electron velocity $v_e$ and the electric field $E$, in the following way:

$$
\begin{align*}
n_e &= n_0 + n_1 e^{-i\omega t} + n_1^* e^{i\omega t} + \cdots, \\
v_e &= v_1 e^{-i\omega t} + v_1^* e^{i\omega t} + \cdots, \\
E &= E_1 e^{-i\omega t} + E_1^* e^{i\omega t} + \cdots,
\end{align*}
$$

(2.1)

and

where $n_0$ is the background electron density and the subscript ‘1’ denotes the oscillating parts of the electron density $n_1$, the electron velocity $v_1$ and the electric field $E_1$. As we study a spherically symmetric plasma oscillation, all the functions in equation (2.1) depend only on the radial coordinate $r$, and the vectors $v_1$ and $E_1$ only have a radial component.

Note that the frequency of the plasma oscillation $\omega$ in equation (2.1) is typically less than the Langmuir frequency for electrons $\omega_p = \sqrt{4\pi e^2n_0/m}$, where $e > 0$ is the proton charge and $m$ is the mass of the electron. The discrepancy between $\omega$ and $\omega_p$ depends on the physical processes proving the plasma structure stability. For example, for a classical plasmoid, which is treated by Škorić & ter Haar (1980), Laedke & Spatschen (1984) and Dvornikov (2011) with the help of perturbation theory, the value of $|\omega - \omega_p|/\omega_p$ can be about 10 per cent. However, if a system possesses a high degree of nonlinearity, the difference between $\omega$ and $\omega_p$ can be significant. We will suppose that $\omega = \xi \omega_p$, with $\xi < 1$.

The quantities $n_1$, $v_1$ and $E_1$ are related by the plasma hydrodynamic equations derived by Škorić & ter Haar (1980),

$$
\begin{align*}
v_1 &= -i e/m \omega_p E_1 \
(\nabla \cdot E_1) &= -4\pi e n_1.
\end{align*}
$$

(2.2)

Therefore, we can describe the plasma oscillation in terms of only one function, e.g. $E_1 = |E_1|$. The form of this function depends on the peculiar type of a plasmoid, and it is rather difficult to find it analytically. Thus, one should rely only on the numerical simulations of the plasmoid dynamics. Nevertheless, we can use the following ansatz proposed by Anderson (1983),

$$
E_1(r) = Ar \exp\left(-\frac{r^2}{2\sigma^2}\right),
$$

(2.3)

which quite accurately describes the dynamics of a plasmoid (Bang et al. 2002). Now a plasmoid is described in terms of the amplitude $A$ and the width $\sigma$. In figure 1, we show a schematic of the perturbations of the electric field and the normalized number density of electrons $f_n = (3 - r^2/\sigma^2) \exp(-r^2/2\sigma^2)$.

As we mentioned above, rapidly oscillating electrons, described by equations (2.1) and (2.3), can interact with neutral atoms generating oscillations in the neutral component of plasma. Under certain conditions, these acoustic
waves can be absorbed by other electrons, producing an effective interaction between charged particles. The process of the exchange of a virtual acoustic wave between two electrons is schematically shown in figure 2.

At the second order of the perturbation theory, the matrix element of the electrons interaction owing to the exchange of the ‘quantum’ of an acoustic wave has the form (Madelung 1978)

\[ V_{kk'q} = \frac{2\hbar\omega_q|M_{k,k-q}|^2}{[E(k) - E(k - q)]^2 - (\hbar\omega_q)^2}, \]  

(2.4)

where \( \omega_q \) is the frequency of a virtual acoustic wave, \( E(k) \) are the energy levels of an electron participating in rapid oscillations and \( M_{k,k-q} \) is the matrix element of the electron’s interaction with the ‘quantum’ of an acoustic wave taken in the first order of the perturbation theory. Note that an electron state in equation (2.4) is specified with help of the single quantum number \( k \) rather than of a vector \( k \), since here we consider a radial motion of charged particles.
To determine the type of the effective interaction (2.4), one should examine the sign of the denominator in equation (2.4). The situation when \(|E(k) - E(k - q)| > \hbar \omega_q\) corresponds to repulsion between electrons and \(|E(k) - E(k - q)| < \hbar \omega_q\) to effective attraction. In §3, we will evaluate the energy change \(\Delta E = |E(k) - E(k - q)|\) in a collision with a neutral atom as well as the typical frequency of an acoustic wave \(\omega_q\).

3. Parameters of the effective potential

In this section, we evaluate the parameters of the effective interaction (2.4) and show that under certain conditions, it can be attractive. Then, we estimate the magnitude of the effective interaction and compare it with the typical kinetic energy of oscillating electrons.

To evaluate the energy change \(\Delta E\) of an electron while it emits an acoustic wave, we should notice that this acoustic wave is generated when an electron collides with neutral atoms. Thus, knowing the total power of acoustic waves emitted and the mean number of collisions with neutral atoms per unit time, we can evaluate the energy change in a collision.

We describe the dynamics of a neutral gas using a set of inhomogeneous hydrodynamic equations proposed by Ingard (1966),

\[
\begin{align*}
\frac{\partial \rho_a}{\partial t} + \nabla \cdot (\rho_a \mathbf{u}) &= 0 \quad \text{and} \quad \rho_a \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} + \nabla p &= \mathbf{F}, \\
\end{align*}
\]

(3.1)

where \(\rho_a\) is the mass density of the neutral gas, \(\mathbf{u}\) is its velocity, \(p\) is the neutral gas pressure and \(\mathbf{F}\) is the rate of the momentum transfer per unit volume from an external source, which is the electron subsystem in our case, to the neutral gas. In equation (3.1), we suppose that the generation of acoustic waves occurs at constant entropy. We should also supply equation (3.1) with the equation of state of the neutral gas, \(p = p(\rho_a)\).

Taking into account the oscillatory character of the electron number density (2.1), we get that the external force \(\mathbf{F}\) should also have similar behaviour, \(\mathbf{F} = \mathbf{F}_1 e^{-i\omega t} + \mathbf{F}_1^* e^{i\omega t}\). Linearizing equation (3.1), we obtain that the pressure, the density and the velocity of the neutral gas also oscillate on the same frequency \(\omega\)

\[
\begin{align*}
p &= p_0 + p_1 e^{-i\omega t} + p_1^* e^{i\omega t} , \\
\rho_a &= \rho_0 + \rho_1 e^{-i\omega t} + \rho_1^* e^{i\omega t} , \\
\mathbf{u} &= \mathbf{u}_1 e^{-i\omega t} + \mathbf{u}_1^* e^{i\omega t} ,
\end{align*}
\]

(3.2)

and

where \(p_0\) and \(\rho_0\) are the equilibrium values. As in equation (2.1), the subscript ‘1’ denotes the perturbed quantities, and higher harmonics are omitted.

Using equation (3.2), we can represent the linearized equation (3.1) for the neutral gas pressure as the inhomogeneous Helmholtz equation,

\[
\nabla^2 p_1 + k^2 p_1 = (\nabla \cdot \mathbf{F}_1),
\]

(3.3)

where \(k = \omega/c_a\) is the wavevector of the acoustic wave and \(c_a = \sqrt{(\partial p/\partial \rho_a)_{S}}\) is the sound velocity in the neutral gas, with the derivative being taken at constant
entropy. Note that acoustic waves will be emitted with the same frequency as the plasma oscillation, $\omega_q = \omega$.

Now we should specify the type of the interaction between electrons and neutral atoms. According to the definition of the external force, it can be represented as $F_1 = -n_a \nabla U$, where $n_a = \rho_0 / M_a$ is the background number density of neutral atoms and $M_a$ is the mass of an atom. We also suggest that only oscillating electrons contribute to the effective potential $U(r)$ of the interaction of an acoustic wave with electrons,

$$U(r) = \int d^3r' n_1(r') K(r - r'), \quad (3.4)$$

where $K(r)$ is the potential of an interaction between an electron and a neutral atom.

Let us choose the charge-dipole potential first proposed by Buckingham (1937), which is a good model of the interaction between electrons and neutral atoms in a low-energy plasma,

$$K(r) = -\frac{\alpha e^2}{(|r|^2 + r_0^2)^2}, \quad (3.5)$$

where $r_0$ is the cut-off radius, which is of the order of the atomic size, and $\alpha \sim r_0^3$ is the electric dipole polarizability of an atom. The corrections to equation (3.5) owing to the collective plasma effects were studied by Redmer et al. (1987). It was found that for a hydrogen plasma, the deviations from equation (3.5) are negligible at the low plasma temperature of approximately $10^3$ K used in this section.

Taking into account the general solution of equation (3.3),

$$p_1(r) = -\frac{1}{4\pi} \int d^3r e^{ik|\mathbf{r} - \mathbf{r}'|} (\mathbf{\nabla} \cdot \mathbf{F}_1), \quad (3.6)$$

we can represent the total power, radiated in the form of acoustic waves, in the following form:

$$\langle \dot{E} \rangle = \int d^3r (\mathbf{F}_1^* \mathbf{u}_1 + \mathbf{F}_1 \mathbf{u}_1^*) = \frac{8\pi n_a k^3}{\omega M_a} \left( \int_0^\infty dr r U(r) \sin kr \right)^2. \quad (3.7)$$

To derive equation (3.7), we suppose that the function $U(r)$ rapidly decreases at large distances, cf. equation (2.3).

Now let us evaluate the number of collisions with neutral atoms per unit time at rapid oscillations of electrons. For this purpose, it is more convenient to use the Lagrange variables: $(r, t) \rightarrow (\rho, \tau)$. Suppose that an oscillating electron is at the distance $\rho$ from the centre of the system and its law of motion has the form $r = \rho + A_0(\rho) \sin \omega \tau$, where $A_0$ is the amplitude of oscillations. Taking into account the continuity equation in Lagrange variables for a spherically symmetric
system found by Dvornikov (2011), \( n_0 \rho^2 = n_0 r^2 (\partial r/\partial \rho) \), and supposing that the amplitude \( A_0 \) is not so large, i.e. \( n_1 (r) \approx n_1 (\rho) \), we obtain the number of collisions per unit time as

\[
\dot{N} = 4 n_a \sigma_s \omega \int_0^{\infty} A_0 (\rho) n_1 (\rho) \rho^2 \, d\rho = 4 \sigma_s \omega \frac{n_a}{n_0} \int_0^{\infty} n_1 (\rho) d\rho \int_0^{\rho} r^2 n_1 (r) \, dr,
\]

where \( \sigma_s \) is the cross section of the electron scattering off a neutral atom. To derive equation (3.8), we suppose that \( A_0 (0) = 0 \), i.e. oscillations vanish at the centre of the system.

Now, we can evaluate the energy change in a collision with a neutral atom as \( \Delta E \sim \langle \dot{E} \rangle / \dot{N} \). As was mentioned above, the situation when \( |\Delta E| \) is less than the typical energy of the ‘quantum’ of an acoustic wave, \( h \omega_q \), corresponds to the effective attraction between electrons. Besides this condition, for the effective interaction to become important for the dynamics of the system, its magnitude should be comparable with the typical kinetic energy of oscillating electrons. That is why we have to evaluate the matrix element of an acoustic wave generation by an oscillating electron \( M_{k,k-q} \) in equation (2.4).

This matrix element can be expressed in the following form:

\[
M_{k,k-q} = \int d^3r \psi^*_k (r) V (r) \psi_{k-q} (r),
\]

where \( V (r) \) is the energy of interaction of an electron and the acoustic field. The wave function of an electron \( \psi_k (r) \) in equation (3.9) can be taken in the form of a normalized spherical wave,

\[
\psi_k (r) = \sqrt{\frac{k \sin (kr)}{\pi r}} \quad \text{and} \quad \int d^3r \psi^*_k (r) \psi_{k'} (r) = 2 \pi k \delta (k - k'),
\]

since we study a localized oscillation. Generally speaking, in a realistic situation, the wave function of an electron can differ from that in equation (3.10). However, to get a rough estimate, we can choose it in such a form.

Analogously to equation (3.4), we suggest that an electron scatters off the density perturbation caused by an acoustic wave, i.e. at the first order of the perturbation theory, the effective potential \( V (r) \) has the following form proposed by Vlasov & Yakovlev (1978):

\[
V (r) = \frac{1}{M_a} \int d^3r' \rho_1 (r') K (r - r'),
\]

where the energy of interaction between an electron and a neutral atom \( K (r) \) is given in equation (3.5). Note that in equation (3.11), we take into account only the oscillating part of the neutral gas density \( \rho_1 \), cf. equation (3.2). Accounting for equations (3.5) and (3.9)–(3.11), we obtain the matrix element \( M_{k,k-q} \) as

\[
M_{k,k-q} = \text{sign} (k - q) \sqrt{1 - \frac{q}{k}} \frac{2 \alpha e^2}{\hbar} \frac{k^2 \pi^3 n_a}{c^2 M_a} e^{-k r_0} \int_0^{\infty} d\tau \, r U (r) \sin kr.
\]

Here, we take into account that both \( \rho_1 \) and \( \rho^*_1 \) contribute to the matrix element.
It is convenient to compare the potential of the effective interaction between oscillating electrons (2.4) with the typical value of the kinetic energy of an oscillating electron. Using equation (2.3), one gets the kinetic energy

\[
\langle E_k \rangle \sim \left( \int d^3 r \langle n_1 \rangle \right)^{-1} \int d^3 r \langle n_1 \rangle \frac{m|\mathbf{v}_1|^2}{2} \approx 0.08 \frac{e^2 A^2 \sigma^2}{m \omega_0^p}, \tag{3.13}
\]

where we take into account the relation between the main harmonic amplitudes of the electron gas presented in equation (2.2). Note that in equation (3.13), we take into account only the part of the kinetic energy owing to the oscillatory motion rather than the total energy, which also includes the thermal contribution.

To analyse the behaviour of the effective interaction, we mention that one can explicitly calculate the common integral in equations (3.7) and (3.12), rather than the total energy, which also includes the thermal contribution.

\[
\int_0^\infty dr \ r U(r) \sin kr = -\frac{\alpha \pi^2 e^2}{r_0} e^{-k r_0} \int_0^\infty dr \ r (\sin kr - kr \cos kr) E_1(r) \approx \frac{\alpha e A \sigma^5 k^3}{4 r_0} \sqrt{\frac{\pi}{2}} e^{-k r_0} \exp \left( -\frac{k^2 \sigma^2}{2} \right), \tag{3.14}
\]

where we use equations (2.2), (2.3) and (3.4). Then, we rewrite equation (2.4) as

\[
V_{kk'/q}/\langle E_k \rangle = W/(R - 1),
\]

where we introduce the new functions

\[
R = \left( \frac{\Delta E}{\hbar \omega_q} \right)^2 \approx 0.5 \times 10^{-2} \left( \frac{\sigma}{10^{-7} \text{cm}} \right)^{-4} f_R(k \sigma),
\]

and

\[
W = 2 |M_{kk'/q}|^2 \approx 3.1 \times 10^{-2} \left( \frac{n_a}{10^{21} \text{cm}^{-3}} \right)^2 \left( \frac{\sigma}{10^{-7} \text{cm}} \right)^{-3} f_W(k \sigma). \tag{3.15}
\]

Here, \(f_R(x) = x^{16} e^{-2x^2}\) and \(f_W(x) = x^{11} e^{-x^2}\). To derive equation (3.15), we use equation (3.14) and the following plasma parameters: \(\alpha \sim 10^{-24} \text{cm}^3, n_0 \sim 10^{-8} \text{cm}, \sigma_s \sim 10^{-15} \text{cm}^2, M_a \sim 10^{-24} \text{g}\) and \(c_s \sim 10^9 \text{cm s}^{-1}\). The effective interaction turns out to be attractive when \(R < 1\). The function \(W\) is the ‘magnitude’ of the interaction expressed in terms of the mean kinetic energy of oscillating electrons, cf. equation (3.13).

In figure 3, we show the behaviour of the functions \(f_R\) and \(f_W\) versus the dimensionless variable \(k \sigma = \omega \sigma / c_s\). As we can see, these functions have maxima owing to the enhanced emissivity of acoustic waves with a wavelength of the order of the size of a plasmoid, \(2\pi/k \sim \sigma\). Using equation (3.15) and figure 3, we get the lower bound on the width of a plasmoid at which the effective attraction between electrons takes place, \(\sigma > 0.3 \times 10^{-7} \text{cm} \times \xi^{-1}\). To obtain this constraint, we account for the maximum of the function \((f_R)_{\text{max}} \approx 22\).

In our analysis, we take into account only the interaction of oscillating electrons with neutral atoms. It is, however, known that oscillations of plasma are possible if the total number of collisions of background electrons per unit time \(\nu\) is less than the oscillations frequency. Note that one should account for collisions only inside the plasmoid. Thus, we get for this quantity, \(\nu = v_T \sigma_s n_a V_{\text{eff}} n_0\), where \(v_T = \sqrt{T/m}\) is the thermal electron velocity, \(V_{\text{eff}} = (4\pi/3) R_{\text{eff}}^3 = 17 \sigma^3\) is the effective plasmoid volume and \(R_{\text{eff}} = 1.6 \sigma\) is the effective plasmoid radius calculated as \(R_{\text{eff}}^2 = \int E^2 r^2 d^3 r / \int E^2 d^3 r\), cf. equation (2.3).
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Figure 3. The functions $f_R$ (dashed line) and $f_W$ (solid line) versus $k\sigma$.

Taking $T = 10^3$ K and using equation (3.15) at the maximal value of the function $W$, corresponding to the strongest effective attraction, we get that plasma oscillations are possible (i.e. the condition $v < \omega$ is satisfied) if $\sigma < 1.4 \times 10^{-6} \text{cm} \times \xi^{2/7}$. Combining this result with the previously obtained lower bound for $\sigma$, one gets that the width of a plasma structure should be in the range $0.3 \times 10^{-7} \text{cm} \times \xi^{-1} < \sigma < 1.4 \times 10^{-6} \text{cm} \times \xi^{2/7}$. Therefore, for the effective attraction between oscillating electrons to take place, the frequency of oscillations should lie in the following interval: $0.05\omega_p < \omega < \omega_p$. Note that for the chosen parameters, the Debye length for electrons $\lambda_D = v_T/\omega_p$ is of the order of $R_{\text{eff}}$.

4. Applications

In §3, we showed that the exchange of a virtual acoustic wave between two electrons participating in spherically symmetric oscillations may result in their effective attraction. The typical size of the plasma structure when the effective attraction becomes important, i.e. when it is comparable with the mean kinetic energy of oscillating particles, is quite small, approximately $10^{-6} \text{cm}$. Although in the present work we do not analyse the physical processes that underlie the plasmoid existence, such a small size is the indication to the quantum nature of this plasma structure. For example, quantum plasmoids of a similar size were described by Dvornikov & Dvornikov (2007) on the basis of the solution of the nonlinear Schrödinger equation.

If the effective attraction between oscillating electrons is sufficiently strong, we may suggest that these particles can form a bound state. Previously, Nambu & Akama (1985) and Nambu et al. (1995) suggested that bound states of charged particles in plasma can be formed owing to the exchange of ion acoustic waves. Another mechanism based on magnetic interaction was proposed by Meierovich (1984). Note that in our case, the presence of a strong nonlinear effect is required to significantly diminish the frequency of oscillations, $\omega < \omega_p$. We suggest that the phenomenon of the bound state formation of electrons in plasma can be implemented inside a stable atmospheric plasmoid (Stenhoff 1999), called a ball...
lightning (BL). These kinds of plasma structures appear during a thunderstorm and have a lifetime up to several minutes. Despite a great variety of models of BL, summarized by Bychkov et al. (2010, pp. 270–296), these objects are likely to be plasma-based phenomena.

To describe the long lifetime of a natural plasmoid, Dijkhuis (1980) suggested that BL is a spherical vortex composed of a dense superconducting plasma. Recently, Zelikin (2008) revisited the idea that plasma superconductivity may be implemented in BL. It was suggested that a natural plasmoid can have a positively charged kernel and a superconducting electron envelop. Nevertheless, those models were based on the phenomenological assumption of the plasma superconductivity without pointing out a physical mechanism that underlies it.

As we mentioned in §2, the incoherent emission of acoustic waves results in energy dissipation in the plasmoid. On the contrary, the coherent exchange by an acoustic wave leads to attractive interaction between electrons. As we showed in §3, this attraction can result in the formation of bound states, analogous to Cooper pairs in metals (Madelung 1978). Thus, one may expect that the electron component of a plasma will experience a phase transition followed by the formation of a condensate of electrons. The collective oscillations of the electron condensate can be responsible for reducing the resistance of the plasma and thus can provide the observed lifetime of a natural plasma structure. Although this hypothesis requires the additional analysis that would carefully account for all thermal effects, the estimates given in §3 show that the described process is quite possible.

For the existence of plasma oscillations, the number of collisions of oscillating particles per unit time should be much less than the frequency of the oscillations, \( n \ll \omega \). Otherwise oscillations will decay. In §3, we checked that the weaker inequality, \( n < \omega \), is satisfied. However, if a plasma is in a reduced resistance state and supposing that rather strong nonlinearity is present, i.e. a significant fraction of electrons participate in oscillations, we may expect that the condition \( n \ll \omega \) is also satisfied. In §3, we also obtained that for the chosen parameters, the Debye length was of the order of the plasmoid size. If \( R_{\text{eff}} \ll \lambda_D \), oscillating electrons can sometimes pass through the plasma structure, which results in energy dissipation (Goldman 1984). Although we do not violate the condition \( R_{\text{eff}} \gtrsim \lambda_D \), the excessive energy dissipation can be avoided in the reduced resistance plasma state.

In the present work, we have considered a plasmoid with a very small core, approximately \( 10^{-6} \text{ cm} \), whereas according to observations, the visible diameter of a natural plasma structure is of the order of several centimetres (Bychkov et al. 2010). Nevertheless, if one analyses the available photographs of this phenomenon published by Burt (2007) and Stenhoff (1999, pp. 129–161), one can conclude that, while moving in the atmosphere, a plasmoid leaves a trace that is much smaller than its visible size. It can be possible if a small and hot kernel exists inside a plasmoid. This core ionizes the air and produces the observed trace. The visible dimensions of a plasmoid are likely to be attributed to auxiliary effects.

One more indication that the actual size of a natural plasma structure is much smaller than the visible one, is in the fact that sometimes it can pass through tiny holes and cracks, with the structure of the plasmoid being unchanged (Bychkov et al. 2010, pp. 203–246). In many cases, the materials that a plasmoid passes through are not damaged either. The most natural explanation of this unusual
behaviour is the suggestion of the small-scale internal structure of the object. Although it is not directly related to our description of natural plasmoids, we should mention that recently Muldrew (2010) discussed the model of BL in which the visible size of the object was bigger than its core, having a radius of approximately 0.1–2 cm.

More evidence of the small-sized core of long-lived plasma structures can be obtained from laboratory experiments (Bychkov et al. 2010, pp. 263–265). Moreover, recently, Klimov & Kutlaliev (2010) reported that plasmoids with a nanoscale kernel were generated in the studies of high-frequency discharges in dusty plasmas. The experiments with silicon discharges carried out by Abrahamson (2002), Lazarouk et al. (2006), Dikhtyar & Jerby (2006), Paiva et al. (2007) and Mitchell et al. (2008), where glowing structures resembling natural plasmoids were generated, should also be mentioned. Although the interpretation of the results of those experiments involves another plasmoid model, the generated objects also have nano-sized cores.

The reports of the energy content of natural plasma structures are rather different: along the objects with relatively small internal energy, plasmoids possessing huge energy were observed (Bychkov et al. 2010, pp. 203–246). The analysis of the present work is applicable for a low-energy plasma structure, which does not seem to have an internal energy source. That is why in §3, while making numerical estimates, we took the temperature of electrons to be approximately $10^3$ K. We remind ourselves that the maximal electron temperature is approximately $10^5$ K. Note that the existence of low-temperature plasmoids is not excluded by observations (Bychkov et al. 2010, pp. 203–246). It is worth mentioning that, if the plasma of such low-energy BL is in the superconducting state, one can avoid the energy losses and plasma recombination, and provide plasma structure stability.

We showed that the model of a nano-sized plasmoid is able to describe some of the observed properties of BL. Nevertheless, we may also suggest that in a realistic natural plasma structure, there could be multiple tiny kernels where intense electron oscillations happen. Note that an analogous model of a composite BL was discussed by Nikitin (2006). The separate oscillatory centres can be held together by attractive quantum exchange forces (Kulakov & Rumyantsev 1991), which are relevant in our situation since the predicted size of a single core is tiny, approximately $10^{-6}$ cm. However, the detailed description of the coagulation process using the results of Kulakov & Rumyantsev (1991) requires additional special analysis.

The analysis of the observed characteristics of natural plasmoids summarized by Bychkov et al. (2010, pp. 203–246) shows that they are likely to be phenomena of different origin. Therefore, a unique model that would explain all the observations does not seem to exist. For instance, the plasmoid model described in the present work does not explain electromagnetic action of BL, such as the generation of strong electric currents and the emission of radio waves. Nevertheless, our approach could naturally explain some of the BL properties that are hard to account for in alternative models of natural plasmoids.

At the end of this section, we may say a few words about how an atmospheric plasmoid based on spherically symmetric oscillations of electrons appears in natural conditions. As we revealed in §3, for the existence of a plasmoid, the frequency of plasma oscillations should be comparable with $\omega_p$. 

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which is in the gigahertz region. It is very difficult to generate such a high frequency during a thunderstorm since linear lightning is quite a low-frequency phenomenon (Rakov & Uman 2006). Thus, the possible BL generation scenario looks as follows. Suppose that during a linear lightning stroke, a natural capacitor with a small capacity of approximately 10 pF is charged up to a very high voltage. Then, if this capacitor is discharged on a thin point, a gigahertz electromagnetic oscillation can be created, provided the discharge channel inductance is approximately 0.1 mH. However, many other factors, such as the shape of the point, air humidity, etc., should be properly combined for successful BL generation.

5. Conclusion

In this work, we have discussed the effective interaction between electrons in a low-temperature plasma owing to the exchange of virtual acoustic waves. The electron temperature of such a plasma should be low enough to allow the existence of both electrons, ions and neutral atoms. In §2, we have derived the expression for the potential of this effective interaction (2.4) and considered a particular case of the effective interaction between electrons participating in spherically symmetric oscillations. We have supposed that charged particles in plasma perform nonlinear oscillations and form a stable plasma structure. However, in our analysis, we just used the commonly adopted ansatz for electric field distribution (2.3) without going into details about what kinds of physical processes provide plasmoid stability.

Then, in §3, we have evaluated the parameters of the effective interaction. It has been established that under certain conditions, the interaction is attractive and its strength can be comparable with the mean kinetic energy of oscillating electrons. For this situation to happen, the typical size of a plasmoid should be quite small, approximately $10^{-6}$ cm. It is an indication that one should use the concept of quantum plasmas to describe the stability of the plasma structure.

Note that analogous effective interaction will also appear between ions and neutral atoms, as well as between ions and electrons, since equation (2.4) is of a universal type. However, in the former case, the frequency of the ion oscillations is significantly less than $\omega_p$, and the interaction will always be repulsive. In the latter case, we should recall that electrons and ions interact mainly electromagnetically, and the small contribution owing to the collisions will be negligible.

Finally, in §4, we have discussed the possible application of the obtained results to the description of stable natural plasma structures (Stenhoff 1999; Bychkov et al. 2010). According to the model of Dvornikov & Dvornikov (2007) and Dvornikov (2010, 2011) these objects can be implemented as spherically symmetric plasma oscillations. Note that the analogous idea was also discussed by Shmatov (2003). To explain the long lifetime (up to several minutes) of a natural plasmoid, we put forward a hypothesis that plasma could be in a reduced resistance state. The mechanism underlying the existence of such a state could be the exchange of virtual acoustic waves between oscillating electrons, described in §§2 and 3. We have also considered how the characteristics of a plasma structure, predicted in frames of our model, conform to the observed properties of natural plasmoids.
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