Associated production of vector gauge boson & graviton to NLO QCD

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In this talk, we discuss the next-to-leading order (NLO) QCD corrections to the associated production of the vector gauge boson (Z/W±) and the graviton in the large extra dimension model, namely the ADD model, at the LHC. After a brief review of the ADD model, we present the importance of QCD correction to these preferred processes and the impact of the QCD corrections on the total cross sections as well as the differential distributions of the gauge bosons. The dependence of the cross sections on the arbitrary factorization scale is studied and the reduction in the scale uncertainties at NLO level is shown. The ultraviolet sensitivity of the theoretical prediction is also presented.

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1. Introduction

To address the large hierarchy between the electroweak scale and the Planck scale remains an interesting and challenging task for decades. A variety of models has been proposed to address the hierarchy problem. All these models are subject to verification and with the advent of the Large Hadron Collider (LHC) with its unprecedented energy and luminosity, it is expected to test the TeV scale gravity models, which can give rise to new and interesting signals. One such beyond Standard Model (SM) candidate is the ADD model, proposed by Arkani-Hamed, Dimopoulos and Dvali [1] and one such interesting signal is the production of vector boson in association with a graviton leading to missing energy.

The ADD model was the first extra dimension model in which the compactified dimensions could be of macroscopic size. A viable mechanism to hide the extra spatial dimension, is to introduce a 3-brane with negligible tension and localise the Standard Model (SM) particles on it. Only gravity is allowed to propagate in the full 4 + d dimensional space time. For simplicity, the extra dimensions can be assumed to be flat, of the same size and compactified on a d-dimensional torus of radius \( R/(2\pi) \). After the compactification, the scale \( M_s \) of the extra dimensional theory is related to the Planck scale \( M_p \) as:

\[
M_p^2 = C_d M_s^{2+d} R^d ,
\]

where \( C_d = 2 \left( 4\pi \right)^{-\frac{d}{2}} / \Gamma(d/2) \) and \( R \) is the size of the extra dimensions. This compactification implies that a massless graviton propagating in 4 + d dimensions manifests itself as a tower of massive graviton modes in 4-dimensions, with mass \( m_{\bar{n}}^2 = 4\pi^2 \bar{n}^2 / R^2 \) where \( \bar{n} = \{n_1, n_2, ..., n_d\} \) and \( n_i = \{0, 1, 2, ...\} \). Here, the zero mode corresponds to the 4-dimensional massless graviton. As the inverse square law of gravity has been tested down to only few \( \mu m \) so far [2], the size of the extra spatial dimensions in this model can be taken as large as this limit. For \( M_s \sim O(\text{TeV}) \), the above limit on \( R \) constrains the number of extra dimensions to \( d \geq 2 \).

In the effective theory valid below the scale \( M_s \), these gravitons couple to the SM fields through energy momentum tensor \( T^{\mu\nu} \) of the latter with the coupling \( \kappa = \sqrt{16\pi / M_p} \), as given by [3, 4]

\[
\mathcal{L}_{int} = -\frac{\kappa}{2} \sum_{\bar{n}=0}^{\infty} T^{\mu\nu}(x) h_{\mu\nu}^{(\bar{n})}(x) .
\]

The Feynman rules for the above interaction Lagrangian are given in [3, 4]. To order \( \kappa^2 \), the above action allows scattering processes involving SM fields and virtual gravitons in the intermediate state or real gravitons in the final state. In the context of collider phenomenology, this gives rise to a very rich and interesting signals that can be seen at the present LHC. The virtual exchange of the gravitons can lead to the deviations from the SM predictions whereas the real emission of the gravitons can lead to the missing energy signals. Though the coupling of each graviton mode to the SM fields is \( M_p \) suppressed, the large multiplicity of the available graviton modes can give rise to observable effects. As the size of the extra dimensions could be large in this model, the mass splitting i.e. \( 2\pi / R \) is very small and hence this summation over the graviton modes can be approximated to be an integral in the continuum limit, with the density of the graviton modes given...
by \[\rho(m_{\tilde{n}}) = \frac{R^d m_{\tilde{n}}^{d-2}}{(4\pi)^{d/2} \Gamma(d/2)}\].

For the real graviton production process at the collider experiments, the inclusive cross section is given by the following convolution:

\[d\sigma = \int dm_{\tilde{n}}^2 \rho(m_{\tilde{n}}) d\sigma_{m_{\tilde{n}}},\]

where \(d\sigma_{m_{\tilde{n}}}\) is the cross section for the production of a single graviton of mass \(m_{\tilde{n}}\).

At the hadron colliders like LHC or Tevatron, the QCD radiative corrections are very significant for they can enhance the LO predictions as well as decrease the arbitrary scale uncertainties in theoretical predictions. Further, the presence of a hard jet in the final state, due to these radiative corrections, has the potential to modify the shapes of the transverse momentum distributions of the particles that are under study at LO. Obtaining such a modification to the shapes of the distributions is beyond the scope of the normalization of the corresponding LO distributions by a constant K-factor, and it requires an explicit computation of the cross sections or distributions to next-to-leading order (NLO) in QCD. In the context of missing energy signals in the large extra dimensional model, the NLO QCD corrections are presented for the processes (i) jet plus graviton production \([5]\) and (ii) photon plus graviton production \([6]\). In each of these two cases, it is shown that the K-factors can be as high as 1.5 at the LHC.

2. Importance of graviton plus vector boson production

The gravitons when produced at the collider experiments escape the experimental detection due to their small couplings and negligible decays into SM particles. The production of vector bosons \((V = Z, W^\pm)\) together with such an invisible gravitons \((G)\) can give rise to a very large missing transverse momentum signals at the collider experiments. The study of graviton plus gauge boson production, hence, in general will be a useful one in probing the new physics at the LHC. This process has been studied at leading order (LO) in the context of lepton colliders \([7, 8]\) as well as at the hadron colliders \([3]\), and also has been implemented in Pythia8 \([10]\). The process is an important one and stands complementary to the more conventional ones involving the graviton production, like jet plus graviton or photon plus graviton productions, that are generally useful in the search of the extra dimensions at collider experiments.

It is important to note that there is a Standard Model (SM) background which gives signals similar to those of associated production of \(Z\) and \(G\). This SM background receives a dominant contribution coming from the \(ZZ\) production process, where one of the \(Z\)-bosons in the final state decays into a pair of neutrinos \((Z \rightarrow \nu\bar{\nu})\) leading to \(Z\)-boson plus missing energy signals. The other \(Z\)-boson can be identified via its decays to leptons, mostly electrons and muons, and then constraining the lepton invariant mass close to the mass of the \(Z\)-boson to consider only the on-shell \(Z\)-bosons. A detailed study of the event selection and the minimization of other SM contributions to this process \(ZZ \rightarrow l\bar{l}\nu\bar{\nu}\), using MC@NLO and Pythia, is taken up in the context of ATLAS detector simulation and is presented in \([11]\). Any deviation from this SM prediction will hint some beyond SM scenario and hence a study of this process will be useful in searching the new physics.
In what follows, we describe the computation of NLO cross sections for the process under study. Since our focus is on the QCD part in this work, we will confine our calculation to the production of on-shell \( Z/W^\pm \).

### 3. Calculational details

At the lowest order in the perturbation theory, the associated production of the vector gauge boson and the graviton takes place via the quark anti-quark initiated subprocess, given by

\[
q_a(p_1) + \bar{q}_b(p_2) \rightarrow V(p_3) + G(p_4),
\]

where \( V = Z, W^\pm \) and \( a, b \) are flavor indices. The Feynman rules and the summation of polarization tensor of the graviton are given in [3, 4]. For the vector gauge boson, the propagator in the unitary gauge \((\xi \rightarrow \infty)\) has been used throughout because of some advantages [12, 13].

At the NLO in the perturbation theory, the cross sections receive \( \mathcal{O}(\alpha_s) \) contributions from real emission as well as virtual diagrams. The integration over the phase space of the real emission diagrams will give rise to infra-red (IR) (soft and collinear) divergences in the limit where the additional parton at NLO is either soft and/or collinear to the initial state partons. On the other hand, the integration over the loop momenta in the virtual diagrams will also give rise to infrared divergences, in addition to the ultraviolet (UV) divergences. In our calculation, we regulate all these divergences using dimensional regularization with the number of space-time dimensions \( n = (4 + \epsilon) \). Completely anti-commuting \( \gamma_5 \) prescription [14] is used to handle \( \gamma_5 \) in \( n \) dimensions. Here, it should be noted that as the gravitons couple to the energy momentum tensor of the SM fields, which is a conserved quantity, there won’t be any UV divergences coming from the loop diagrams.

There are several methods available in the literature to compute NLO QCD corrections. Standard methods based on fully analytical computation deal with the phase space and loop integrals in \( n \)-dimensions and give a finite \( \mathcal{O}(\alpha_s) \) contribution to the cross sections, after the real and the virtual contributions are added together and the initial state collinear singularities are absorbed into the bare parton distribution functions. However, these methods are not useful whenever the particles in the final state are subjected to experimental cuts or some isolation algorithms. In such cases, semi analytical methods like phase space slicing method or dipole subtraction method are extremely useful. In the present work, we have resorted to the former [15] with two cut offs \( (\delta_s, \delta_c) \) to compute the radiative corrections. In this method, the IR divergences appearing in the real diagrams can be handled in a convenient way by slicing the soft and collinear divergent regions from the full three body phase space. The advantage of this method is that the integration over the remaining phase space can be carried out in 4-dimensions, rather than in \( n \)-dimensions, using standard Monte-Carlo techniques. For any further calculational details about the real and virtual parts of these processes, we refer to [12, 13].

### 4. Numerical Results

In this section, we present various kinematic distributions for the associate production of the graviton and the vector gauge boson to NLO in QCD at the LHC. The results are presented for
proton-proton collision energy of $\sqrt{s} = 14$ TeV. The limits on the integral over the graviton mass are set by the kinematics from 0 to $\sqrt{s} - m_V$, where $\sqrt{s}$ is the parton center of mass energy and $m_V = m_Z, m_W$. The masses of the gauge bosons and the weak mixing angle are given by $m_Z = 91.1876$ GeV, $m_W = 80.398$ GeV, $\sin^2\theta_w = 0.2312$. For $W$ boson production cross sections, we will consider the mixing of quarks among different quark generations, as allowed by the CKM-matrix elements $V_{ij}$, with $(i = u, c, t)$ and $(j = d, s, b)$. Since all our calculations are done in the massless limit of the partons, we have not included the top quark contribution in our calculation and set all $V_{ij}$’s to zero. The fine structure constant is taken to be $\alpha = 1/128$. Throughout our study, we have used CTEQ6L1 and CTEQ6.6M parton density sets for LO and NLO cross sections respectively. The strong coupling constant is calculated at two loop order in the $\overline{MS}$ scheme with $\alpha_s(m_Z) = 0.118$ ($\Lambda_{QCD} = 0.226$ GeV). We have also set the number of light flavors $n_f = 5$. The following cuts are used for our numerical study,

$$ p_T^{Z,W} > p_T^{\text{min}}, \quad p_T^{\text{miss}} > p_T^{\text{min}}, \quad |y^{Z,W}| \leq 2.5. \quad (4.1) $$

For the 2-body process, the missing transverse momentum is same as that of the gauge boson. On the other hand, for the 3-body process, it need not be so due to the presence of an observable jet in the final state and hence it amounts purely to the graviton transverse momentum. The observable jet is defined as the one that satisfies the following conditions:

$$ p_T^{\text{jet}} > 20\text{GeV} \quad \text{and} \quad |\eta^{\text{jet}}| \leq 2.5. \quad (4.2) $$

Whenever the jet does not satisfy the above conditions, the missing transverse momentum is approximated to be that of the gauge boson.

We check for the stability of the cross sections against the variation of the slicing parameters, $\delta_s, \delta_c$ for all of the three processes and find that our results are independent of the choice of these slicing parameters that are introduced in the intermediate stages of the calculation. It can be seen from the fig. (1) that both the 2-body and the 3-body contributions vary with $\delta_s$ but their sum is fairly stable against the variation of $\delta_s$ over a wide range. The cross sections are given for both the truncated as well as the un-truncated cases, as the ADD model is an effective theory [3], with the choice of model parameters $M_s = 3$ TeV and $d = 2$. The same is plotted for $W^+$ case [13] also. All of these plots are studied with different value of $d$. In fig. (2), we have shown the variation of the truncated as well as un-truncated total cross sections with respect to the scale $M_s$, for the case $d = 2$ for the $Z/W^+$ and the graviton associated production. The K-factors are shown in fig. (3) for $Z$ (left panel) and $W^+$ (right panel) production with the graviton as a function of $M_s$. A similar study of variation of K-factors with $P_T^{\text{min}}$ for all of these three cases is done [13]. Further, in fig. (4), we present the transverse momentum distribution of $W^-$ (left panel) and $W^+$ (right panel) respectively as a function of the number of extra dimensions $d$ and for $M_s = 3$ TeV. The missing transverse momentum distributions ($P_T^{\text{miss}}$) for the $Z$-boson case is plotted in the left panel of fig. (5) for $d = 2, 4$. In the right panel of fig. (5), the rapidity distribution of the $Z$-boson both at LO and at NLO for two different choices of the factorization scale: $\mu_F = P_T^Z/2$ and $2P_T^Z$ is plotted. This distribution is obtained by integrating over the transverse momentum of the $Z$-boson from 700 GeV to 750 GeV, for $d = 4$. As expected, the inclusion of order $\alpha_s$ corrections reduces the dependence on the arbitrary factorisation scale $\mu_F$. The percentage of uncertainty in the cross
sections at the central rapidity region $Y = 0$, due the variation of the scale from $\mu_F = P_T^Z/2$ to $\mu_F = 2P_T^Z$, is 18.9% at LO and it gets reduced to 8.6% at NLO. Similar considerable amount of scale uncertainty reductions at NLO are observed for the rest of the two cases [13].

![Figure 1](image1.png)

**Figure 1:** Variation of the transverse momentum distribution of $Z$ boson with $\delta_s$, keeping the ratio $\delta_s/\delta_c = 100$ fixed, for $M_s = 3$ TeV and $d = 4$.

![Figure 2](image2.png)

**Figure 2:** Total cross section for the associated production of $ZG$ (left) and $W^+G$ (right) at the LHC, shown as a function of $M_s$ for $d = 2$.

5. Conclusion

We have systematically computed the full NLO QCD corrections to the associated production of the vector gauge boson and the graviton in theories with large extra dimensions at the LHC. The
Figure 3: K-factors of the total cross section for the associated production of the $Z$-boson and the graviton at the LHC, given as a function of the scale $M_s$ for $ZG$ (left) and $W^+G$ (right) production.

Figure 4: Transverse momentum distribution of the $W^-$ (left) and $W^+$ (right) for $M_s = 3$ TeV is shown for different values of the number of extra dimensions $d$.

K-factors for the neutral gauge boson are found to vary from 1.6 to 1.2 depending on the number of extra dimensions $d$, while they vary from 1.8 to 1.3 for the case of charged gauge bosons. At the hadron colliders, the leading order predictions often suffer from large uncertainties resulting from the choice of factorisation scale. Reducing these uncertainties is one of the main motivations for doing NLO computation. We have shown that this is indeed the case for the rapidity distributions of the gauge bosons by varying the factorization scale from $\mu_F = P_T/2$ to $\mu_F = 2P_T$, leading to reduction in the percentage of scale uncertainty to about 9% from 19%. Hence, the results presented in this paper are more suitable for studies on associated production of vector boson and graviton in the context of extra dimension searches at the hadron colliders.
Graviton plus vector boson production to NLO QCD

Satyajit Seth

Figure 5: Missing transverse momentum distribution of the graviton produced in association with Z-boson at the LHC, for $M_s = 3$ TeV (left). The scale uncertainties in the rapidity distribution of Z-boson for $M_s = 3$ TeV and $d = 4$ (right).

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429 (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436 (1998) 257; N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D 59 (1999) 086004.

[2] D. J. Kapner et al. Phys. Rev. Lett. 98 (2007) 021101.

[3] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. B 544 (1999) 3.

[4] T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D 59 (1999) 105006.

[5] S. Karg, M. Karämer, Q. Li, D. Zeppenfeld, Phys. Rev. D 81 (2010) 094036.

[6] X Gao, C S Li, J Gao and J Wang, Phys. Rev. D 81 (2010) 036008.

[7] Kingman Cheung and Wai-Yee Keung, Phys. Rev. D 60 (1999) 112003.

[8] Gian F. Giudice, Tilman Plehn, Alessandro Strumia, Nucl. Phys. B 706 (2005) 455.

[9] Stefan Ask, Eur. Phys. J. C 60 (2009) 509.

[10] S. Ask, I.V. Akin, L. Benucci, A. De Roeck, M. Goebel, J. Haller, Comp. Phys. Comm. 181 (2010) 1593.

[11] G. Aad et al. (ATLAS collaboration), CERN-OPEN-2008-020 [arXiv:0901.0512v1 [hep-ex]].

[12] M.C. Kumar, P. Mathews, V. Ravindran, S. Seth, J. Phys. G 38 (2011) 055001.

[13] M.C. Kumar, P. Mathews, V. Ravindran, S. Seth, Nucl. Phys. B 847 (2011) 54.

[14] M. Chanowitz, M. Furman and I. Hinchliffe, Nucl. Phys. B 159 (1979) 225.

[15] B.W. Harris, J.F. Owens, Phys. Rev. D 65 (2002) 094032.

[16] K. Nakamura et. al. Journal of Physics G 37 (2010) 075021.