The Power of M Theory

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Abstract

A proposed duality between type IIB superstring theory on $\mathbb{R}^9 \times S^1$ and a conjectured 11D fundamental theory ("M theory") on $\mathbb{R}^9 \times T^2$ is investigated. Simple heuristic reasoning leads to a consistent picture relating the various $p$-branes and their tensions in each theory. Identifying the M theory on $\mathbb{R}^{10} \times S^1$ with type IIA superstring theory on $\mathbb{R}^{10}$, in a similar fashion, leads to various relations among the $p$-branes of the IIA theory.

1Permanent address.
1 Introduction

Recent results indicate that if one assumes the existence of a fundamental theory in eleven dimensions (let’s call it the ‘M theory’\(^2\)), this provides a powerful heuristic basis for understanding various results in string theory. For example, type II superstrings can be understood as arising from a supermembrane in eleven dimensions \([\text{1}]\) by wrapping one dimension of a toroidal supermembrane on a circle of the spatial geometry \([\text{2, 3, 4, 5}]\). Similarly, when the spatial geometry contains a \(K3\), one can obtain a heterotic string by wrapping a five-brane with the topology of \(K3 \times S^1\) on the \(K3\) \([\text{6, 7}]\). This provides a very simple heuristic for understanding ‘string-string duality’ between type IIA and heterotic strings in six dimensions \([\text{8, 9, 10, 6, 11, 12}]\). One simply considers the M theory on \(\mathbb{R}^6 \times S^1 \times K3\). This obviously contains both type II strings and heterotic strings, arising by the two wrappings just described.

Moreover, since the membrane and 5-brane are electric-magnetic duals in 11 dimensions, the two strings are dual in six dimensions, and so it is natural that the strong-coupling expansion of one corresponds to the weak-coupling expansion of the other. The remarkable thing about this kind of reasoning is that it works even though we don’t understand how to formulate the M theory as a quantum theory. It is tempting to say that the success of the heuristic arguments that have been given previously, and those that will be given here, suggest that there really is a well-defined quantum M theory even when perturbative analysis is not applicable. The only thing that now appears to be special about strings is the possibility of defining a perturbation expansion. In other respects, all \(p\)-branes seem to be more or less equal \([\text{13, 14}]\).

Recently, I have analyzed heuristic relationships between Type II strings and the M theory \([\text{15}]\). The approach was to compare the 9D spectrum of the M theory on \(\mathbb{R}^9 \times T^2\) with the IIB theory on \(\mathbb{R}^9 \times S^1\). A nice correspondence was obtained between states arising from the supermembrane of the M theory and the strings of the IIB theory. The purpose of this paper is to extend the analysis to include higher \(p\)-branes of both theories, and to see what can be learned from imposing the natural identifications.

Let us begin by briefly recalling the results obtained in \([\text{15}]\). We compared the M theory compactified on a torus of area \(A_M\) in the canonical 11D metric \(g^{(M)}\) with the IIB theory compactified on a circle of radius \(R_B\) (and circumference \(L_B = 2\pi R_B\)) in the canonical 10D IIB metric \(g^{(B)}\). The canonical IIB metric is the convenient choice, because it is invariant

\(^2\)This name was suggested by E. Witten.
under the $SL(2,\mathbb{R})$ group of IIB supergravity. By matching the 9D spectra of the two models (especially for BPS saturated states), the modular parameter $\tau$ of the torus was identified with the modulus $\lambda_0 = \chi_0 + ie^{-\phi_0}$, which is the vev of the complex scalar field of the IIB theory. This identification supports the conjectured non-perturbative $SL(2,\mathbb{Z})$ duality symmetry of the IIB theory. (This was also noted by Aspinwall [19].)

A second result was that the IIB theory has an infinite spectrum of strings, which forms an $SL(2,\mathbb{Z})$ multiplet. The strings, labelled by a pair of relatively prime integers $(q_1, q_2)$, were constructed as solutions of the low-energy 10D IIB supergravity theory using results in Refs. [7, 18]. They have an $SL(2,\mathbb{Z})$ covariant spectrum of tensions given by

$$T_1^{(B)} = \Delta_q^{1/2} T_1^{(B)},$$  \hspace{1cm} (1)

where $T_1^{(B)}$ is a constant with dimensions of mass-squared, which defines the scale of the theory, and

$$\Delta_q = e^{\phi_0} (q_1 - q_2 \chi_0)^2 + e^{-\phi_0} q_2^2.$$  \hspace{1cm} (2)

Note that strings with $q_2 \neq 0$, those carrying RR charge, have tensions that, for small string coupling $g_B = e^{\phi_0}$, scale as $g_B^{-1/2}$. The usual $(1,0)$ string, on the other hand, has $T \sim g_B^{1/2}$. In the string metric, these become $g_B^{\frac{1}{2}}$ and 1, respectively.

The mass spectrum of point particles (zero-branes) in nine dimensions obtained from the two different viewpoints were brought into agreement (for BPS saturated states, in particular) by identifying winding modes of the family of type IIB strings on the circle with Kaluza–Klein modes of the torus and by identifying Kaluza-Klein modes of the circle with wrappings of the supermembrane (2-brane) on the torus. The 2-brane of the M theory has a tension (mass per unit area) in the 11D metric denoted $T_2^{(M)}$. If one introduces a parameter $\beta$ to relate the two metrics ($g^{(B)} = \beta^2 g^{(M)}$), then one finds the following relations [15]

$$(T_1^{(B)} L_2^2)^{-1} = \frac{1}{(2\pi)^2} T_2^{(M)} A_M^{3/2},$$  \hspace{1cm} (3)

$$\beta^2 = A_M^{1/2} T_2^{(M)} / T_1^{(B)}.$$  \hspace{1cm} (4)

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3Equation (3) was given incorrectly in the original versions of my previous papers [13]. Also, $T_1^{(B)}$ and $T_2^{(M)}$ were called $T$ and $T_{11}$, and $A_M$ was called $A_{11}$. A more systematic notation is now desirable.

4The rule that gave sensible results was to allow the membrane to cover the torus any number of times (counting orientation), and to identify all the different ways of doing as equivalent. For other problems (such as Strominger’s conifold transitions [19]) a different rule is required. As yet, a single principle that gives the correct rule for all such problems is not known. I am grateful to A. Strominger and A. Sen for correspondence concerning this issue.
Both sides of eq. (3) are dimensionless numbers, which are metric independent, characterizing the size of the compact spaces. Note that, since $T_{1}^{(B)}$ and $T_{2}^{(M)}$ are fixed constants, eq. (3) implies that $L_B \sim A_M^{-3/4}$.

Strings (1-branes) in nine dimensions were also matched. A toroidal 2-brane with one of its cycles wrapped on the spatial two-torus was identified with a type IIB string. When the wrapped cycle of the 2-brane is mapped to the $(q_1, q_2)$ homology class of the spatial torus and taken to have minimal length $L_q = (A_M/\tau_2)^{1/2}|q_2\tau - q_1| = (A_M\Delta_q)^{1/2}$, this gives a spectrum of string tensions in the 11D metric $T_{1q}^{(M)} = L_q T_{1q}^{(M)}$. Converting to the IIB metric by $T_{1q}^{(B)} = \beta^{-2}T_{1q}^{(M)}$ precisely reproduces the previous formula for $T_{1q}^{(B)}$ in eq. (1), which therefore supports the proposed interpretation.

2 More Consequences of M/IIB Duality

Having matched 9D point particles (0-branes) and strings (1-branes) obtained from the IIB and M theory pictures, let us now explore what additional information can be obtained by also matching $p$-branes with $p = 2, 3, 4, 5$ in nine dimensions.

It should be emphasized that even though we use extremely simple classical reasoning, it ought to be precise (assuming the existence of an M theory), because we only consider $p$-branes whose tensions are related to their charges by saturation of a BPS bound. This means that the relations that are obtained should not receive perturbative or non-perturbative quantum corrections. This assumes that the supersymmetry remains unbroken, which is certainly believed to be the case.

We begin with $p = 2$. In the M theory the 2-brane in 9D is the same one as in 11D. In the IIB description it is obtained by wrapping an $S^1$ factor in the topology of a self-dual 3-brane once around the spatial circle. Denoting the 3-brane tension by $T_{3}^{(B)}$, its wrapping gives a 2-brane with tension $L_B T_{3}^{(B)}$. Converting to the 11D metric and identifying the two 2-branes gives the relation

$$T_{2}^{(M)} = \beta^3 L_B T_{3}^{(B)}. \quad (5)$$

Using eqs. (3) and (4) to eliminate $L_B$ and $\beta$ leaves the relation

$$T_{3}^{(B)} = \frac{1}{2\pi} \left( T_{1}^{(B)} \right)^2. \quad (6)$$

The remarkable thing about this result is that it is a relation that pertains entirely to the IIB theory, even though it was deduced from a comparison of the IIB theory and the M

\[ \text{For useful background on } p\text{-branes see Refs. [20, 21, 22].} \]
theory. It should also be noted that the tension $T_3^{(B)}$ is independent of the string coupling constant, which implies that in the string metric it scales as $g_B^{-1}$.

Next we consider 3-branes in nine dimensions. The only way they can arise in the M theory is from wrapping a 5-brane of suitable topology (once) on the spatial torus. In the IIB theory the only 3-brane is the one already present in ten dimensions. Identifying the tensions of these two 3-branes gives the relation

$$T_5^{(M)} A_M = \beta^4 T_3^{(B)}.$$  \hspace{1cm} (7)$$

Eliminating $\beta$ and substituting eq. (8) gives

$$T_5^{(M)} = \frac{1}{2\pi} \left( T_2^{(M)} \right)^2.$$  \hspace{1cm} (8)$$

This result pertains entirely to the M theory. Section 3 of ref. \cite{12} analyzed the implication of the Dirac quantization rule \cite{23} for the charges of the 2-brane and 5-brane in the M theory. It was concluded that (in my notation) $\pi T_5^{(M)}/(T_2^{(M)})^2$ should be an integer. The present analysis says that it is $1/2$. Indeed, I believe that eq. (8) corresponds to the minimum product of electric and magnetic charges allowed by the quantization condition. It is amusing that simple classical reasoning leads to a non-trivial quantum result.

Next we compare 4-branes in nine dimensions. The IIB theory has an infinite $SL(2, \mathbb{Z})$ family of 5-branes. These are labeled by a pair of relatively prime integers $(q_1, q_2)$, just as the IIB strings are. The reason is that they carry a pair of magnetic charges that are dual to the pair of electric charges carried by the strings. Let us denote the tensions of these 5-branes in the IIB metric by $T_5^{(B)q}$. Wrapping each of them once around the spatial circle gives a family of 4-branes in nine dimensions with tensions $L_B T_5^{(B)q}$. In the M theory we can obtain 4-branes in nine dimensions by considering 5-branes with an $S^1$ factor in their topology and mapping the $S^1$ to a $(q_1, q_2)$ cycle of the spatial torus. Just as when we wrapped the 2-brane this way, we assume that the cycle is as short as possible, i.e., its length is $L_q$. Identifying the two families of 4-branes obtained in this way gives the relation

$$L_q T_5^{(M)} = \beta^5 L_B T_5^{(B)q}.$$  \hspace{1cm} (9)$$

Substituting the relations \cite{15}

$$L_q = \Delta_q^{1/2} T_1^{(B)} \beta^2 / T_2^{(M)}$$  \hspace{1cm} (10)$$

4
and

\[ L_B \beta^3 = 2\pi T_2^{(M)} / \left(T_1^{(B)}\right)^2 \]  \hspace{1cm} (11)

and using eq. (8) gives

\[ T_{5q}^{(B)} = \frac{1}{(2\pi)^2} \Delta_q^{1/2} \left(T_1^{(B)}\right)^3. \] \hspace{1cm} (12)

This relation pertains entirely to the IIB theory. Since 5-brane charges are dual to 1-brane charges, they transform contragrediently under \( SL(2, \mathbb{R}) \). This means that in this case \( q_1 \) is a magnetic R-R charge and \( q_2 \) is a magnetic NS-NS charge. Thus 5-branes with pure R-R charge have a tension that scales as \( g_B^{-1/2} \) and ones with any NS-NS charge have tensions that scale as \( g_B^{-1/2} \). Converting to the string metric, these give \( g_B^{-1} \) and \( g_B^{-2} \), respectively. Of course, \( g_B^{-2} \) is the characteristic behavior of ordinary solitons, whereas \( g_B^{-1} \) is the remarkable intermediate behavior that is characteristic of all \( p \)-branes carrying R-R charge. It is gratifying that these expected properties emerge from matching M theory and IIB theory.

We have now related all 1-brane, 3-brane, and 5-brane tensions of the IIB theory in ten dimensions, so that they are determined by a single scale. We have also related the 2-brane and 5-brane tensions of the M theory in eleven dimensions, so they are also given by a single scale. The two sets of scales can only be related to one another after compactification, however, as the only meaningful comparison is provided by eqs. (3) and (4).

All that remains to complete this part of the story, is to compare 5-branes in nine dimensions. Here something a little different happens. As is well-known, compactification on a space \( K \) with isometries (such as we are considering), so that the complete manifold is \( K \times \mathbb{R}^d \), give rise to massless vectors in \( d \) dimensions. Electric charges that couple to these vectors correspond to Kaluza–Klein momenta and are carried by point-like 0-branes. The dual magnetic objects are \((d-4)\)-branes. This mechanism therefore contributes “Kaluza–Klein 5-branes” in nine dimensions. However, which 5-branes are the Kaluza–Klein ones depends on whether we consider the M theory or the IIB theory. The original 5-brane of the M theory corresponds to the unique Kaluza–Klein 5-brane of the IIB theory, and the \( SL(2, \mathbb{Z}) \) family of 5-branes of the IIB theory corresponds to the Kaluza–Klein 5-branes of the M theory. The point is that there are three vector fields in nine dimensions which transform as a singlet and a doublet of the \( SL(2, \mathbb{R}) \) group. The singlet arises à la Kaluza–Klein in the IIB theory and from the three-form gauge field in the M theory. Similarly, the doublet
arises from the doublet of two-form gauge fields in the IIB theory and à la Kaluza–Klein in the M theory.

We can now use the identifications described above to deduce the tensions of Kaluza–Klein 5-branes in nine dimensions. The KK 5-brane of the IIB theory is identified with the fundamental 5-brane of the M theory, which implies that its tension is \( T^{(B)}_5 = \beta^{-6} T^{(M)}_5 \). Combining this with eq. (11) gives

\[
T^{(B)}_5 = \frac{1}{(2\pi)^3} L^2_B \left( T^{(B)}_1 \right)^4.
\]  

Note that this diverges as \( L_B \to \infty \), as is expected for a Kaluza–Klein magnetic \( p \)-brane. Similarly the \( SL(2,\mathbb{Z}) \) multiplet of KK 5-branes obtained from the M theory must have tensions that match the 5-branes of the 10D IIB theory. This implies that \( T^{(M)}_{5q} = \beta^6 T^{(B)}_{5q} \). Substituting eqs. (4) and (12) gives

\[
T^{(M)}_{5q} = \frac{1}{(2\pi)^2} A^3 M \left( T^{(M)}_2 \right)^3 \Delta_{q}^{1/2}.
\]  

This also diverges as \( A_M \to \infty \), as is expected. As a final comment, we note that if all tensions are rescaled by a factor of \( 2\pi \) (in other words, equations are rewritten in terms of \( \tilde{T} = T/2\pi \)), then all the relations we have obtained in eqs. (3) – (14) have a numerical coefficient of unity.

3 The IIA Theory

The analysis given above is easily extended to the IIA theory in ten dimensions. The IIA theory is simply interpreted as the M theory on \( R^{10} \times S^1 \). Let \( L = 2\pi r \) be the circumference of the circle in the 11D metric \( g^{(M)} \). The string metric of the IIA theory is given by \( g^{(A)} = \exp(2\phi_A/3) g^{(M)} \), where \( \phi_A \) is the dilaton of the IIA theory. The IIA string coupling constant \( g_A \) is given by the vev of \( \exp \phi_A \). These facts immediately allow us to deduce the tensions \( T^{(A)}_p \) of IIA \( p \)-branes for \( p = 1, 2, 4, 5 \). The results are

\[
T^{(A)}_1 = g_A^{-2/3} L T^{(M)}_2,
\]

\[
T^{(A)}_2 = g_A^{-1} T^{(M)}_2,
\]

\[
T^{(A)}_4 = g_A^{-5/3} L T^{(M)}_5,
\]
\[ T_5^{(A)} = g_A^{-2} T_5^{(M)}. \]  

Since \( T_1^{(A)} \) and \( T_2^{(M)} \) are constants, eq. (15) gives the scaling rule \( g_A \sim L^{3/2} \). Substituting eqs. (15) and (8) into eqs. (17) and (18) gives

\[ T_4^{(A)} = \frac{1}{2\pi} g_A^{-1} T_{1}^{(M)} T_{2}^{(M)} = \frac{1}{2\pi} T_{1}^{(A)} T_{2}^{(A)}, \]

\[ T_5^{(A)} = \frac{1}{2\pi} \left( T_2^{(A)} \right)^2. \]

Again we have found the expected scaling behaviors: \( g_A^{-1} \) for the 2-brane and 4-brane, which carry R-R charge, and \( g_A^{-2} \) for the NS-NS solitonic 5-brane. Combining eqs. (19) and (20) gives

\[ T_2^{(A)} T_4^{(A)} = T_1^{(A)} T_5^{(A)}. \]

This shows that the quantization condition for the corresponding charges is satisfied with the same (minimal) value in each case.

The IIA theory also contains an infinite spectrum of BPS saturated 0-branes (aka ‘black holes’) and a dual 6-brane, which are of Kaluza–Klein origin like those discussed earlier in nine dimensions. Since the Kaluza–Klein vector field is in the R-R sector, the tensions of these should be proportional to \( g_A^{-1} \), as was demonstrated for the 0-branes in \([5]\).

### 4 P-Branes With \( P \geq 7 \)

The IIB theory has a 7-brane, which carries magnetic \( \chi \) charge. The way to understand this is that \( \chi \) transforms under \( SL(2, \mathbb{R}) \) just like the axion in the 4D N=4 theory. It has a Peccei-Quinn translational symmetry (broken to discrete shifts by quantum effects), which means that it is a 0-form gauge field. As a consequence, the theory can be recast in terms of a dual 8-form potential. Whether or not one does that, the classical supergravity equations have a 7-brane solution, which is covered by the general analysis of \([20]\), though that paper only considered \( p \leq 6 \). Thus the 7-brane in ten dimensions is analogous to a string in four dimensions. Let us call the tension of the IIB 7-brane \( T_7^{(B)} \).

The existence of the 7-brane in the 10D IIB theory suggests that after compactification on a circle, the resulting 9D theory has a 7-brane and a 6-brane. If so, these need to be understood in terms of the M theory. The 6-brane does not raise any new issues, since it
is already present in the 10D IIA theory. It does, however, reinforce our confidence in the existence of the IIB 7-brane. A 9D 7-brane, on the other hand, certainly would require something new in the M theory. What could it be? To get a 7-brane after compactification on a torus requires either a 7-brane, an 8-brane, or a 9-brane in the 11D M theory. However, the cases of \( p = 7 \) and \( p = 8 \) can be ruled out immediately. They require the existence of a massless vector or scalar particle, respectively, in the 11D spectrum, and neither of these is present. The 9-brane, on the other hand, would couple to a 10-form potential with an 11-form field strength, which does not describe a propagating mode and therefore cannot be so easily excluded. Let us therefore consider the possibility that such a 9-brane with tension \( T_9^{(M)} \) really exists and trace through its consequences in the same spirit as the preceding discussions.

First we match the 7-brane obtained by wrapping the hypothetical 9-brane of the M theory on the spatial torus to the 7-brane obtained from the IIB theory. This gives the relation

\[
A_MT_9^{(M)} = \beta^8 T_7^{(B)}. \tag{22}
\]

Substituting eq. \( (4) \) gives

\[
T_7^{(B)} = \left( A_M \right)^{-1} \frac{T_1^{(B)} T_9^{(M)}}{T_2^{(M)}}. \tag{23}
\]

This formula is not consistent with our assumptions. A consistent picture would require \( T_7^{(B)} \) to be independent of \( A_M \) or \( L_B \), but we have found that \( T_7^{(B)} \sim A_M^{-1} \sim L_B^{4/3} \). Also, the 8-brane and 9-brane of the IIA theory implied by a 9-brane in the M theory do not have the expected properties. I’m not certain what to make of all this, but it is tempting to conclude that there is no 9-brane in the M theory. Then, to avoid a paradox for 9D 7-branes, we must argue that they are not actually present. I suspect that the usual methods for obtaining BPS saturated \( p \)-branes in \( d - 1 \) dimensions from periodic arrays of them in \( d \) dimensions break down for \( p = d - 3 \), because the fields are not sufficiently controlled at infinity, and therefore there is no 7-brane in nine dimensions. Another reason to be suspicious of a 9D 7-brane is that a \((d - 2)\)-brane in \( d \) dimensions is generically associated with a cosmological term, but straightforward compactification of the IIB theory on a circle does not give one.

In a recent paper \([24]\), Polchinski has argued for the existence of a 9-brane in the 10D IIB theory and an 8-brane in the 10D IIA theory, both of which carry RR charges. (He also did
a lot of other interesting things.) It ought to be possible to explore whether the existence of these objects is compatible with the reasoning of this paper, but it is unclear to me what the appropriate rules are for handling such objects.

5 Conclusion

We have shown that by assuming the existence of a quantum ‘M theory’ in eleven dimensions one can derive a number of non-trivial relations among various perturbative and non-perturbative structures of string theory. Specifically, we have investigated what can be learned from identifying M theory on $\mathbb{R}^9 \times T^2$ with type IIB superstring theory on $\mathbb{R}^9 \times S^1$ and matching (BPS saturated) $p$-branes in nine dimensions. Similarly, we identified the M theory on $\mathbb{R}^{10} \times S^1$ with type IIA superstring theory on $\mathbb{R}^{10}$ and matched $p$-branes in ten dimensions. Even though quantum M theory surely has no perturbative definition in 11D Minkowski space, these results make it more plausible that a non-perturbative quantum theory does exist. Of course, this viewpoint has been advocated by others – most notably Duff and Townsend – for many years.

Clearly, it would be interesting to explore other identifications like the ones described here. The natural candidate to consider next, which is expected to work in a relatively straightforward way, is a comparison of the M theory on $\mathbb{R}^7 \times K3$ with the heterotic string theory on $\mathbb{R}^7 \times T^3$. There is a rich variety of $p$-branes that need to be matched in seven dimensions. In particular, the M theory 5-brane wrapped on the $K3$ surface should be identified with the heterotic string itself.

The M theory on $\mathbb{R}^4 \times S^1 \times K$, where $K$ is a Calabi–Yau space, should be equivalent to the type IIA superstring theory on $\mathbb{R}^4 \times K$. Kachru and Vafa have discussed examples for which there is a good candidate for a dual description based on the heterotic string theory on $\mathbb{R}^4 \times K3 \times T^2$ \[25\]. A new element, not encountered in the previous examples, is that while there is plausibly a connected moduli space of $N = 2$ models that is probed in this way, only part of it is accessed from the M theory viewpoint and a different (but overlapping) part from the heterotic string theory viewpoint. Perhaps this means that we still need to find a theory that is more fundamental than either the heterotic string theory or the putative M theory.
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