Strong CP violation in spin-1/2 singly charmed baryons

Y. Ünal1,2,a, Ulf-G. Meißner1,3,4,b
1 Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics Universität Bonn, D-53115 Bonn, Germany
2 Physics Department, Çanakkale Onsekiz Mart University 17100 Çanakkale, Turkey
3 Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany
4 Tbilisi State University, 0186 Tbilisi, Georgia
a unal@hiskp.uni-bonn.de, b meissner@hiskp.uni-bonn.de

Abstract We report on the calculation of the CP-violating form factor $F_3$ and the corresponding electric dipole moment for charmed baryons in the spin-1/2 sector generated by the QCD $\theta$-term. We work in the framework of covariant baryon chiral perturbation theory within the extended-on-mass-shell renormalization scheme up to next-to-leading order in the chiral expansion.

Keywords Baryon chiral perturbation theory · CP violation · charmed baryons

1 Introduction

CP violation has recently been established in the charm sector, more precisely in the meson decays $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ [1], and LHCb has also measured the difference of CP-asymmetry of the three-body singly Cabibbo-suppressed $\Lambda_c^+$ decays [2]. There have also been quite a number of studies predicting CP asymmetries in charmed baryon decays, see e.g. [3] and references therein. It is therefore of interest to investigate other possible effects of CP violation in singly-charmed baryons. Indeed, a first measurement of CP violation in $\Xi_c \to pK^-\pi^+$ decays has been performed by LHCb [4]. However, these data are consistent with the hypothesis of no CP violation.

Here, we concentrate on the effects generated by the strong CP-violating $\theta$-term of QCD, that also induces electric dipole moments in light baryons, as pioneered in Refs. [5, 6]. The proper framework to address such questions is baryon chiral perturbation theory, see [7] for a review. In fact, the masses, axial charges, and electromagnetic decays of the charmed and bottomed baryons have already been calculated in the framework of the heavy baryon approach [8, 9]. More recently, the magnetic moments of the spin-1/2 singly charmed baryons were analyzed in covariant baryon chiral perturbation theory [10]. Here, we extend these studies and work out the CP-violating effects induced by the QCD $\theta$-term.

The manuscript is organized as follows. In Section 2, we briefly discuss the underlying chiral Lagrangian. The CP-violating electromagnetic form factor of the singly-charmed baryons is worked out in Section 3 followed by the display of our numerical results in Section 4. Section 5 contains the summary and outlook. The appendices contain some technicalities as well as more detailed tables of results.

2 Chiral Lagrangian including CP-violating terms

The QCD Lagrangian of the strong interactions including the $\theta$ term reads

$$L_{\text{QCD}} = \bar{q}(i\not\!D - M)q - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{g^2 \theta}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} G^{\mu\nu}_{a} G_{a}^{\rho\sigma}, \quad a = 1, ..., 8,$$

(1)

where $G^{\mu\nu}_{a}$ is the gluon field-strength tensor and $M$ is the quark mass matrix. Strong CP violation arising from the $U(1)$ anomaly in QCD is specified via the vacuum angle $\theta$. Here, to describe the phenomena related to the $\theta$-term, we seek a description in a properly tailored effective field theory, see e.g. Refs. [11, 12] for the detailed construction of the corresponding effective Lagrangian to one loop accuracy.

The Goldstone bosons together with the flavor singlet $\eta_0$, resulting from the spontaneous symmetry breaking of $U(3)_R \times U(3)_L$ into $U(1)_V$, are represented by the matrix-valued field $\tilde{U}$. Treating the vacuum angle $\theta(x)$ as an external field, it transforms as $\theta(x) \to \theta'(x) = \theta(x) - 2N_{f}\alpha$, where $N_{f}$ is the number of flavors, and $\alpha$ is the rotation angle. Following the spontaneous chiral symmetry breaking, under the axial $U(1)$ transformation, $\tilde{U}$ changes but the combination of $\theta_0(x) = \theta(x) - i \ln(\det(\tilde{U}(x))$ stays invariant. Using this invariant combination of $\theta_0(x)$, one can construct the most general mesonic chiral effective Lagrangian up-to-and-including second chiral order

$$L = -V_0 + V_1(\nabla_\mu \tilde{U}^\dagger \nabla^\mu \tilde{U}) + V_2(\bar{\psi} \tilde{U} + \bar{\tilde{\psi}} \tilde{U}^\dagger) + iV_3(\bar{\tilde{\psi}} \tilde{U} - \bar{\psi} \tilde{U}^\dagger) + V_4(\bar{\psi} \nabla_\mu \tilde{U}^\dagger \tilde{U}^\dagger \nabla^\mu \tilde{U}).$$

(2)
We now turn to the baryon sector of the effective Lagrangian. In the SU(3) flavor representation the spin-1 \( V \) multiplets. Here, we only present the terms pertinent to the calculation. In the quark mass and momentum expansion, where \( v_\mu \) and \( a_\mu \) are the conventional vector and axial-vector external sources. The \( V_i \) coefficients are functions of \( \theta_0 \). One needs to determine the vacuum expectation value of \( \bar{U} \) so that the Lagrangian (2) can be used. Writing \( \bar{U} = \sqrt{U_0 U} \) with the choice of

\[
U = \exp \left(i \sqrt{\frac{2}{3} \frac{\eta_0}{F_0}} + i \frac{\sqrt{2}}{F_\pi} \phi \right),
\]

the chiral effective Lagrangian in terms of the Goldstone boson fields composed in \( \bar{U} \) reads [13]

\[
\mathcal{L}_\phi = - V_0 + V_4 \langle \nabla_\mu U \nabla^\mu U \rangle + (V_2 + B V_3) \langle \chi (U + U^\dagger) \rangle - i A V_2 \langle U - U^\dagger \rangle + A V_3 \langle U + U^\dagger \rangle + V_4 \langle U \nabla_\mu U \rangle \langle U^\dagger \nabla^\mu U \rangle.
\]

Here, \( \phi \) represents the Goldstone boson octet and \( \chi = 2B_0 \text{diag}(m_u \cos \varphi_u, m_d \cos \varphi_d, m_s \cos \varphi_s) \). To leading order, \( A \) and \( B \) are given as

\[
A = \frac{V_0^{(2)}}{V_2^{(0)}} \tilde{\theta}_0 + O(\delta^4), \quad B = \frac{V_4^{(1)}}{V_2^{(0)}} \tilde{\theta}_0 + O(\delta^6).
\]

After vacuum alignment, the \( V_i \) coefficients are now functions of \( \tilde{\theta}_0 = \sqrt{3} \eta_0 / F_0 \). Further, the normalization of the kinetic terms in the Lagrangian (2) provides

\[
V_1(0) = V_2(0) = \frac{F_\pi^2}{4}, \quad V_4(0) = \frac{1}{12} (F_0^2 - F_\pi^2).
\]

In principle, the coupling of the \( \eta_0 \) singlet is different from \( F_\pi \) because the subgroup \( U(3)_V \) does not present a nonet symmetry. However, in the large \( N_c \)-limit \( F_0 = F_\pi \). Moreover, the quantity of \( \tilde{\theta}_0 \) can be denoted in terms of physical quantities [16]

\[
\tilde{\theta}_0 = \left[ 1 + \frac{4V_0^{(2)}}{F_\pi^2 \left( M_K^2 - M_\pi^2 \right)^2} \right]^{-1} \theta_0.
\]

Here, we note that \( \tilde{\theta}_0 = O(\delta^2) \), and take \( 1/N_c = O(\delta^2) \) as counting rules [14]. More detail and information on the formalism used in the work can be found in e.g. in Refs. [13, 15].

We now turn to the baryon sector of the effective Lagrangian. In the SU(3) flavor representation the spin-1/2 anti-symmetric triplet and symmetric sextet charmed baryon states are denoted as in the following matrices, respectively,

\[
B_3 = \begin{pmatrix}
0 & \Lambda_+^c & \Xi_+^c \\
-\Lambda_+^c & 0 & \Xi_0^c \\
-\Xi_+^c & -\Xi_0^c & 0
\end{pmatrix}, \quad B_6 = \begin{pmatrix}
\Sigma_+^c & \Xi_+^c & \Omega_+^c \\
\Sigma^c & \Xi^c & \Omega^c \\
\Xi^c & \Xi^c & \Omega^c
\end{pmatrix}.
\]

Similarly to the mesonic Lagrangian one can write down the most general effective Lagrangian for the charmed baryon multiplets. Here, we only present the terms pertinent to the calculation. In the quark mass and momentum expansion,
As can be seen from the contributing Lagrangians, there are quite a number of low-energy constants (LECs). The charge matrix for the singly-charmed baryons is

\[ Q = \begin{pmatrix} 0 & \tilde{q} & \tilde{q} \\ q & 0 & \tilde{u} \\ q & \tilde{u} & 0 \end{pmatrix} \]

where the relevant building blocks are

\[ \tilde{\chi}_- = \chi_- - iA(U + U^\dagger) - iB\chi_+ , \]
\[ \tilde{\chi}_+ = \chi_+ + iA(U - U^\dagger) - iB\chi_- , \]
\[ D_\mu B = \partial_\mu B + \Gamma_\mu B + B\Gamma_\mu^T , \]
\[ \Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - i\mu u)u + u(\partial_\mu - i\mu u)u^\dagger] , \]
\[ u = i[u^\dagger(\partial_\mu - i\mu u)u - u(\partial_\mu - i\mu u)u^\dagger] . \]

The charge matrix for the singly-charmed baryons is \( Q_h = \text{diag}(1, 0, 0) \), while for the light quarks the charge matrix is \( Q_l = \text{diag}(2/3, -1/3, -1/3) \). We use \( w_{10/11} + 3w_{12} = w'_{10} \) as in Ref. [16].

As can be seen from the contributing Lagrangians, there are quite number of low-energy constants (LECs). The meson-baryon coupling constants \( g_i (i = 1, \ldots, 6) \), the symmetry-breaking LECs \( b_D \) and \( b_F \) as well as the LECs \( \omega_{16/17}, \omega_{18} \) related to the CP-conserving electromagnetic response can all be taken from earlier studies of different observables, as detailed in Section 4.

This leaves us with the yet undetermined LECs \( w'_{10}, w'_{13/14}, w'_{15} \). As will be shown, we can fix \( w_{13/14}, w_{15} \) from recent lattice results QCD for the neutron and proton electric dipole moments, \( d_n \) and \( d_p \), respectively. The remaining of these LECs will be varied as \( 0.05 \) GeV\(^{-1} \), that is within a natural range. This naive dimensional analysis should be eventually overcome by a more sophisticated modeling of the LECs or invoking further lattice QCD results. Having fixed/estimated all the LECs will then allow to estimate the CP-violating contributions to the singly-charmed baryons induced by the \( \theta \)-term.

### 3 CP-violating electromagnetic form factor

The electromagnetic form factors of a baryon are defined via the matrix element of the electromagnetic current,

\[
\langle B(p_f) | J_{em}^\mu | B(p_i) \rangle = \bar{u}(p_i) \left[ \gamma^\mu F_1(q^2) + \frac{iF_2(q^2)}{2m_B} \gamma^\mu q_\nu \right] u(p_f),
\]

with \( q = (p_f - p_i)^2 \) the invariant momentum transfer squared, \( m_B \) the baryon mass and \( J_{em}^\mu \) the electromagnetic current. Here, \( F_1(q^2) \) and \( F_2(q^2) \) are the P- and CP-conserving Dirac and Pauli form factors, respectively. \( F_A(q^2) \)------
denotes the P-violating anapole form factor, and $F_3(q^2)$, which will be considered throughout this work, the P- and CP-violating electric dipole form factor. The electric dipole moment of the baryon $B$ is then given by

$$d_B = \frac{F_{3,B}(0)}{2m_B}.$$  \hfill (12)

In what follows, we will use the effective Lagrangian to calculate the CP-violating form factor of the singly-charmed baryons at next-to-leading (NLO) order, which includes tree as well as loop diagrams as shown in Figure 1, where we display the corresponding Feynman diagrams. Tree-level diagrams at leading order are presented in (a) and (b). One-loop diagrams at order $O(\delta^2)$ and $O(\delta^3)$ in (c)-(d), and (e)-(h), respectively. Because the diagrams in (g)-(h) with intermediate Goldstone boson fields are canceled each other exactly for each particle within both anti-triplet and sextet multiplets, they are not displayed here.

![Figure 1: CP-violating contributions of the spin-1/2 charmed baryons.](image)

We show different combinations of the charmed baryon states from anti-triplet and sextet multiplets considered throughout the calculation in Figure 2.

![Figure 2: Different combinations of the spin-1/2 anti-triplet and sextet charmed baryons contributing to $F_3(q^2)$.](image)
The results obtained for the form factor $F_3(q^2)$ of the charmed baryons coming from the tree-level diagrams are collected in Table 1 with

$$
\alpha = \frac{576V_0^{(2)}V_0^{(1)}}{(F_0F_\pi M_{\pi 0})^2}.
$$

Table 1: Tree-level contribution to the $F_3(q^2)$ of the charmed baryons.

| States | Contributions |
|--------|---------------|
| $B_3$  | $\Lambda_c^+$  $e\theta_0 \Lambda_c$  $[2\alpha(w_{13/14} + 2w_{15}) + 8(w_{13/14} + 2w_{15})]$ |
|        | $\Xi_c^+$     $e\theta_0 \Xi_c$  $[2\alpha(w_{13/14} + 2w_{15}) + 8(w_{13/14} + 2w_{15})]$ |
|        | $\Xi_c^0$     $e\theta_0 \Xi_c$  $4(aw_{15} + 4w_{15})$ |

As usual in the EOMS scheme, the loop contributions are rather lengthy expression. Let us discuss the case of the $\Lambda_c^+$.

The one-loop contribution can be written as, cf. Fig. 1,

$$
F_{3\Lambda_c^+}(q^2) = \sum_{i=1}^{2} \frac{e\theta_0 V_0^{(2)}}{\pi^2 F_\pi^2} \left[ C_{\Lambda_c}^{i} \left( m_i (\bar{m} + m_i) \right) \left( 2J_i^{cd}(\bar{m}^2, m_i^2, M_i^2) - 2J_i^{cd}(q^2, M_i^2, M_i^2) \right) \right] + \left( 2M_i^2 + 2\bar{m}^2 - 2m_i^2 - q^2 \right) J_i^{cd}(\bar{m}^2, \bar{m}^2, M_i^2, M_i^2) \]

$$

$$
+ \sum_{i=3}^{4} \frac{e\theta_0 V_0^{(2)}}{\pi^2 F_\pi^2} \left[ C_{\Lambda_c}^{i} \left( -J_i^{cf}(M_i^2)(4\bar{m}^2 - q^2) + J_i^{cf}(\bar{m}^2)(4\bar{m}^2 - q^2) \right) \right] - \left( M_i^2(q^2 + 4\bar{m}^2) - 4\bar{m}^2 q^2 \right) J_i^{cf}(\bar{m}^2, \bar{m}^2, M_i^2) + 8\bar{m}^2(M_i^2 - 2\bar{m}^2) J_i^{cf}(q^2, \bar{m}^2, \bar{m}^2) + 4\bar{m}^2 (2M_i^2 - 8\bar{m}^2 + q^2) J_i^{cf}(\bar{m}^2, \bar{m}^2, M_i^2, M_i^2) \]

$$

$$
+ \sum_{i=5}^{7} \frac{e\theta_0 V_0^{(2)}}{\pi^2 F_\pi^2} \left[ C_{\Lambda_c}^{i} \left( -J_i^{cf}(M_i^2)(4\bar{m}^2 - q^2) + J_i^{cf}(m_i^2)(4\bar{m}^2 - q^2) \right) \right] - \left( M_i^2(4\bar{m} m_i + q^2) + (m_i + m_i)(4\bar{m}^2 m_i - m_i q^2 - \bar{m}(4m_i^2 + q^2)) \right) J_i^{cf}(\bar{m}^2, m_i^2, M_i^2) + 4\bar{m}(\bar{m} + m_i) + 2\bar{m}(\bar{m} + m_i) \]

$$

$$
\times \left( M_i^2(4\bar{m} m_i + q^2) + (m_i + m_i)(4\bar{m}^2 m_i - m_i q^2 - \bar{m}(4m_i^2 + q^2)) \right) J_i^{cf}(\bar{m}^2, m_i^2, M_i^2, M_i^2) \]

$$

$$
+ \sum_{i=8}^{10} \frac{e\theta_0 V_0^{(2)}}{\pi^2 F_\pi^2} \left[ C_{\Lambda_c}^{i} \left( -J_i^{cf}(M_i^2)(4\bar{m}^2 - q^2) + J_i^{cf}(m_i^2)(4\bar{m}^2 - q^2) \right) \right] - \left( \bar{m}^2 q^2 - 4\bar{m}^3 m_i + q^2(m_i^2 - M_i^2) + 2\bar{m} m_i(q^2 + 4m_i^2 - 2M_i^2) \right) J_i^{cf}(\bar{m}^2, m_i^2, M_i^2) - 4\bar{m} (\bar{m} + m_i)(\bar{m}^2 + m_i^2 - M_i^2) J_i^{cf}(q^2, m_i^2, m_i^2) + 2\bar{m}(\bar{m} + m_i) \]

$$

$$
\times \left( M_i^2(4\bar{m} m_i + q^2) + (m_i + m_i)(4\bar{m}^2 m_i - m_i q^2 - \bar{m}(4m_i^2 + q^2)) \right) J_i^{cf}(\bar{m}^2, m_i^2, q^2, m_i^2, M_i^2, M_i^2) \]

$$

$$
+ \frac{16e\theta_0 V_0^{(2)}}{\pi^2 F_\pi^2 F_0} \left[ C_{\Lambda_c}^{i} \left( J_i^{gh}(M_i^2) - J_i^{gh}(\bar{m}^2) - (M_i^2 - 2\bar{m}^2) J_i^{gh}(\bar{m}^2, M_i^2) \right) \right],
$$

(13)
with \(m_i, \bar{m}_i\) and \(M_i\) denoting the masses of the corresponding internal and external baryons and meson running in the loop, for notational simplicity. In the case at hand, \(\bar{m} = m_{\Lambda^+}\). The \(J(m_i, M_i, q^2)\) functions can be reduced to the scalar loop functions given in Appendix A, and the labels \(cd, ef\) and \(gh\) refer to the types of diagrams shown in Fig. 1. The corresponding coefficients \(C_{cd}, C_{ef}\) and \(C_{gh}\) for the \(\Lambda^+\) together with the intermediate meson-baryon states are shown in Table 2, the corresponding tables for the other particles can be found in Appendix B. A MATHEMATICA notebook with these loop functions can be obtained from the first author of this paper.

### Table 2: Loop contribution to the \(F_3(q^2)\) of the \(\Lambda^+_c\) baryon with \(\beta = (b_{D/F} + b_0 + 3w'_{10})\).

| Diagram type number | meson-baryon state | Coefficient |
|---------------------|---------------------|-------------|
| (c), (d)            | \(\Xi^0_c, K^\pm\) | \(2g_0b_{D/F}\) |
|                     | \(\Xi^0_c, K^\pm\) | \(g_2b_{D/F}\) |
| (e), (f)            | \(\Lambda^+_c, \eta_8\) | \(\frac{8}{3}g_0b_{D/F}(w'_{16/17} + 2w_{18})\) |
|                     | \(\Lambda^+_c, \eta_0\) | \(\frac{3}{4}g_0b_{D/F}(w'_{16/17} + 2w_{18})\) |
|                     | \(\Xi^+_c, K^0\) | \(4g_0b_{D/F}(w'_{16/17} + 2w_{18})\) |
|                     | \(\Xi^0_c, K^\pm\) | \(2g_2b_{D/F}w_{18}\) |
|                     | \(\Sigma^0_c, \pi^\pm\) | \(4g_2b_{D/F}w_{18}\) |
|                     | \(\Sigma^+_c, \eta^0\) | \(2g_2b_{D/F}(w'_{16/17} + 2w_{18})\) |
|                     | \(\Sigma^+_c, \pi^\pm\) | \(4g_2b_{D/F}(w'_{16/17} + 2w_{18})\) |
| (g), (h)            | \(\Lambda^+_c, \eta_0\) | \(\beta(w'_{13/14} + 2w_{15})\) |

### 4 Results

First, we must fix parameters. The pion decay constant is taken as \(F_\pi = 92.2\) MeV and the two symmetry-breaking LECs in the baryon sector can be obtained from baryon mass splittings. We use \(b_{D/F} = -0.606\) GeV\(^{-1}\) and \(b_F = -0.209\) GeV\(^{-1}\) [18, 19]. The tree-level contributions can be expressed in terms of two independent linear combinations of unknown LECs as \(\alpha(w'_{13/14} + 4w'_{13/14})\) and \(\alpha w_{15} + 4w'_{15}\), cf. Table 1. The loop contributions are also dependent on unknown LECs, viz., \(w'_{10}, w_{13/14}\) and \(w_{15}\). The conventional magnetic moment couplings, \(w_{18}\) is taken equal to \(w_{16/17} = 0.40\), determined from fits to calculations to baryon magnetic moments in [20, 21].

Further, \(V_0^{(2)} = -5 \times 10^{-4}\) GeV\(^4\) and \(V_0^{(1)} = 3.5 \times 10^{-4}\) GeV\(^2\) are the values obtained from an analysis of \(\eta - \eta'\) mixing in U(3) chiral perturbation theory [22]. The various baryon-meson couplings are taken from Refs. [8, 9], \(g_1 = 0.94, g_2 = -0.60, g_3 = 0.85,\) and \(g_4 = 1.04\). Because of the forbidden \(B_3B_3\phi\)-vertex, we have \(g_6 = 0\). We use the physical masses of the pertinent mesons and baryons running in the corresponding loops, cf. Tables 4-11.

As the unknown LECs cannot be parameterized such a common constant as in [19], since the combinations coming from different particles are different, they have to be considered individually. Using the lattice data from [23] at \(M_\pi = 170\) MeV, we use the neutron dipole moment to fix \(\beta w_{15}\) from the \(\Xi^0_c\) by comparing the loop contributions. With that, we can use the proton electric dipole moment to determine \(\beta w_{13/14}\) from the \(\Lambda^+_c\). We get

\[
\beta w_{13/14} = -0.54 \text{ GeV}^{-1}, \quad \beta w_{15} = 0.13 \text{ GeV}^{-1},
\]

(15)

With these obtained values, we take the variation of \(w'_{10}, w'_{13/14}\) and \(w'_{15}\), and calculate the CP-violating form factor \(F_3(q^2)\) for the singly-charmed baryons in the range \(q^2 \approx 0.05 \ldots 0.3\) GeV\(^2\) as given in Table 12-14. We note that the absolute values for \(d_p\) and \(d_n\) in Ref. [23] are rather large, e.g. Ref. [24] finds a sizeably smaller value for \(|d_n|\). This would reduce the value for \(\beta w_{13/14}\) and in turn for \(\beta w_{15}\), leading to smaller values (in magnitude) for the CP-violating form factor and electric dipole moments of the singly-charmed baryons. Since our study is largely exploratory, we do not explore the whole possible parameter space.

The electric dipole moments for the various baryons are collected in Table 3. As there is a sizeable uncertainty induced by the unknown LECs, we refrain from performing a systematic error analysis accounting e.g. for the effects of higher orders in the chiral expansion. Hopefully, lattice QCD will be able to supply pertinent information on the LECs so that more accurate predictions can be made.
Table 3: Electric dipole moments for the singly-charmed baryons in units of $10^{-14} e\theta_0$ cm. Set 1,2,3 refers to $w_{10}' = w_{13/14}' = w_{15}' = -0.5, 0, +0.5$, in order.

| Set 1 | $\Lambda_c^+$ | $\Xi_c^+$ | $\Xi_c^0$ | $\Sigma_c^+$ | $\Sigma_c^0$ | $\Xi_c^{'+}$ | $\Xi_c^0$ | $\Omega_c^0$ |
|-------|---------------|-----------|-----------|--------------|--------------|--------------|-----------|-------------|
|       | 1.48          | 1.49      | -0.64     | 0.80         | 0.31         | -0.18        | 0.27      | -0.15       | -0.17       |
| Set 2 | 1.04          | 1.05      | -0.98     | 0.70         | 0.22         | -0.26        | 0.18      | -0.24       | -0.26       |
| Set 3 | 0.47          | 0.49      | -1.20     | 0.43         | 0.07         | -0.29        | 0.03      | -0.26       | -0.29       |

5 Conclusion

In this paper, we have performed a one-loop calculation of the CP-violating form factor $F_3(q^2)$ and the corresponding electric dipole moments of the spin-1/2 singly-charmed baryons, where the mechanism of the CP violation is the QCD $\theta$-term. Not all the appearing low-energy constants could be fixed from experimental or lattice QCD data, so the resulting predictions show a spread, cf. Table 3 and the tables in Appendix C. We hope that with more lattice QCD studies on strong CP violations, these LECs can be determined and more accurate predictions can be made, not to mention possible experimental determinations.

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A Scalar loop integrals

The scalar loop integrals of one-, two-, and three-point functions which are used for the calculation of the diagrams are given by

\[ J_0(m) = \frac{(2\pi \mu)^{4-d}}{i\pi^2} \int \frac{d^d k}{k^2 - m^2 + i0^+}, \]

\[ J_0(p^2, m_1^2, m_2^2) = \frac{(2\pi \mu)^{4-d}}{i\pi^2} \int \frac{d^d k}{[k^2 - m_1^2][(k + p)^2 - m_2^2 + i0^+]}, \]

\[ J_0(p_i^2, (p_f - p_i)^2, p_f^2, m_1^2, m_2^2, m_3^2) = \frac{(2\pi \mu)^{4-d}}{i\pi^2} \int \frac{d^d k}{[k^2 - m_1^2 + i0^+][(k - p_i)^2 - m_2^2 + i0^+][(k - p_f)^2 - m_3^2 + i0^+]]. \]

B Loop contributions

All one-loop contributions to the various baryons take the form as given in Eq. 14. In this appendix, we collect the corresponding intermediate meson-baryon states and the values of the coefficients \( C^{cd}, C^{ef}, \) and \( C^{gh} \) for the baryons not given in the main text.

| Diagram type | Number | Meson-baryon state | Coefficient |
|--------------|--------|--------------------|-------------|
| (c), (d)     | 1      | \( \Xi^0_c, \pi^\pm \) | \( g_6 b_{D/F} \) |
|              | 2      | \( \Omega^0_c, K^\pm \) | \( g_2 b_{D/F} \) |
|              | 3      | \( \Sigma^{++}_c, K^\pm \) | \( g_2 b_{D/F} \) |
|              | 4      | \( \Xi^0_c, \pi^\pm \) | \( g_2 b_{D/F} \) |
| (e), (f)     | 5      | \( \Xi^+_c, \eta_8 \) | \( g_6 b_{D/F}(w_{16/17} + 2w_{18}) \) |
|              | 6      | \( \Xi^+_c, \eta_0 \) | \( \beta g_6(w_{16/17} + 2w_{18}) \) |
|              | 7      | \( \Xi^+_c, \pi^0 \) | \( g_6 b_{D/F}(w_{16/17} + 2w_{18}) \) |
|              | 8      | \( \Xi^0_c, \pi^\pm \) | \( g_2 b_{D/F}w_{18} \) |
|              | 9      | \( \Xi^+_c, \eta_8 \) | \( g_2 b_{D/F}(w_{16/17} + 2w_{18}) \) |
|              | 10     | \( \Sigma^+_c, K^0 \) | \( g_2 b_{D/F}(w_{16/17} + 2w_{18}) \) |
|              | 11     | \( \Xi^+_c, \pi^0 \) | \( g_2 b_{D/F}(w_{16/17} + 2w_{18}) \) |
|              | 12     | \( \Sigma^+_c, K^\pm \) | \( g_2 b_{D/F}(w_{16/17} + 2w_{18}) \) |
|              | 13     | \( \Omega^0_c, K^\pm \) | \( g_2 b_{D/F}w_{18} \) |
| (g), (h)     | 14     | \( \Xi^+_c, \eta_0 \) | \( \beta(w_{13/14} + 2w_{15}) \) |
Table 5: Loop contribution to the $F_3(q^2)$ of the $\Xi_c^0$ baryon.

| Diagram type | Number | Meson-baryon state | Coefficient          |
|--------------|--------|--------------------|----------------------|
|              | 1      | $\Lambda_c^+, K^{\pm}$ | $g_6 b_{D/F}$        |
| (c), (d)     | 2      | $\Omega_c^0, K^{\pm}$ | $g_2 b_{D/F}$        |
|              | 3      | $\Xi_c^+, \pi^{\pm}$   | $g_6 b_{D/F}$        |
|              | 4      | $\Sigma_c^+, K^{\pm}$  | $g_2 b_{D/F}$        |
|              | 5      | $\Xi_c^{'+}, \pi^{\pm}$| $g_2 b_{D/F}$        |
|              | 6      | $\Xi_c^0, \eta_8$      | $g_6 b_{D/F} w_{18}$ |
|              | 7      | $\Sigma_c^0, \eta_0$   | $\beta g_6 w_{18}$   |
| (e), (f)     | 8      | $\Sigma_c^0, \pi^0$    | $g_6 b_{D/F} w_{18}$ |
|              | 9      | $\Xi_c^{'+}, \pi^{\pm}$| $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 10     | $\Sigma_c^0, K^0$      | $g_2 b_{D/F} w_{18}$ |
|              | 11     | $\Xi_c^0, \eta_8$      | $g_2 b_{D/F} w_{18}$ |
|              | 12     | $\Xi_c^0, \pi^0$       | $g_2 b_{D/F} w_{18}$ |
|              | 13     | $\Sigma_c^{'+}, K^{\pm}$| $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 14     | $\Omega_c^0, K^0$      | $g_2 b_{D/F} w_{18}$ |
| (g), (h)     | 15     | $\Xi_c^0, \eta_0$      | $\beta w_{15}$       |

Table 6: Loop contribution to the $F_3(q^2)$ of the $\Sigma_c^{++}$ baryon.

| Diagram type | Number | Meson-baryon state | Coefficient                           |
|--------------|--------|--------------------|---------------------------------------|
|              | 1      | $\Xi_c^{'+}, K^{\pm}$ | $g_1 b_{D/F}$                        |
| (c), (d)     | 2      | $\Sigma_c^+, \pi^{\pm}$ | $g_1 b_{D/F}$                        |
|              | 3      | $\Xi_c^+, K^{\pm}$     | $g_2 b_{D/F}$                        |
|              | 4      | $\Lambda_c^+, \pi^{\pm}$| $g_2 b_{D/F}$                        |
|              | 5      | $\Sigma_c^{++}, \eta_8$| $g_1 b_{D/F} (w_{16/17} + w_{18})$  |
|              | 6      | $\Sigma_c^{++}, \eta_0$| $\beta g_1 (w_{16/17} + w_{18})$   |
| (e), (f)     | 7      | $\Sigma_c^{++}, \pi^0$ | $g_1 b_{D/F} (w_{16/17} + w_{18})$  |
|              | 8      | $\Xi_c^{++}, K^{\pm}$  | $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 9      | $\Lambda_c^{++}, \pi^{\pm}$| $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
| (g), (h)     | 10     | $\Sigma_c^{++}, \eta_0$| $\beta (w_{13/14} + w_{15})$       |
### Table 7: Loop contribution to the $F_3(q^2)$ of the $\Sigma_c^+$ baryon.

| Diagram type | Number | Meson-baryon state | Coefficient |
|--------------|--------|--------------------|-------------|
| (c), (d)     | 1      | $\Xi_c^0$, $K^\pm$ | $g_1 b_{D/F}$ |
|              | 2      | $\Xi_c^0$, $K^\pm$ | $g_2 b_{D/F}$ |
| (e), (f)     | 3      | $\Sigma_c^+$, $\eta_8$ | $g_1 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 4      | $\Sigma_c^+$, $\eta_0$ | $\beta g_1 (w_{16/17} + 2w_{18})$ |
|              | 5      | $\Xi_c^+$, $K^0$ | $g_2 b_{D/F} (w_{16/17} + w_{18})$ |
|              | 6      | $\Xi_c^0$, $K^\pm$ | $g_2 b_{D/F} w_{18}$ |
|              | 7      | $\Lambda_c^0$, $\pi^0$ | $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
| (g), (h)     | 8      | $\Sigma_c^+$, $\eta_0$ | $\beta (w_{13/14} + 2w_{15})$ |

### Table 8: Loop contribution to the $F_3(q^2)$ of the $\Sigma_c^0$ baryon.

| Diagram type | Number | Meson-baryon state | Coefficient |
|--------------|--------|--------------------|-------------|
| (c), (d)     | 1      | $\Sigma_c^+$, $\pi^\pm$ | $g_1 b_{D/F}$ |
|              | 2      | $\Lambda_c^0$, $\pi^\pm$ | $g_2 b_{D/F}$ |
| (e), (f)     | 3      | $\Sigma_c^0$, $\eta_8$ | $g_1 b_{D/F} w_{18}$ |
|              | 4      | $\Sigma_c^0$, $\eta_0$ | $\beta g_1 w_{18}$ |
|              | 5      | $\Sigma_c^0$, $\pi^0$ | $g_1 b_{D/F} w_{18}$ |
|              | 6      | $\Xi_c^0$, $K^0$ | $g_2 b_{D/F} w_{18}$ |
|              | 7      | $\Lambda_c^0$, $\pi^\pm$ | $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
| (g), (h)     | 8      | $\Sigma_c^0$, $\eta_0$ | $\beta w_{15}$ |

### Table 9: Loop contribution to the $F_3(q^2)$ of the $\Xi_c^+$ baryon.

| Diagram type | Number | Meson-baryon state | Coefficient |
|--------------|--------|--------------------|-------------|
| (c), (d)     | 1      | $\Omega_c^0$, $K^\pm$ | $g_1 b_{D/F}$ |
|              | 2      | $\Sigma_c^+$, $K^\pm$ | $g_1 b_{D/F}$ |
|              | 3      | $\Xi_c^0$, $\pi^\pm$ | $g_1 b_{D/F}$ |
|              | 4      | $\Xi_c^0$, $\pi^\pm$ | $g_2 b_{D/F}$ |
| (e), (f)     | 5      | $\Xi_c^+$, $\eta_8$ | $g_1 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 6      | $\Xi_c^+$, $\eta_0$ | $\beta g_1 (w_{16/17} + 2w_{18})$ |
|              | 7      | $\Xi_c^+$, $\pi^0$ | $g_1 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 8      | $\Sigma_c^+$, $K^0$ | $g_1 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 9      | $\Xi_c^+$, $\eta_8$ | $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 10     | $\Lambda_c^+$, $K^0$ | $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 11     | $\Xi_c^+$, $\pi^0$ | $g_2 b_{D/F} (w_{16/17} + 2w_{18})$ |
|              | 12     | $\Xi_c^0$, $\pi^\pm$ | $g_2 b_{D/F} w_{18}$ |
| (g), (h)     | 13     | $\Xi_c^+$, $\eta_0$ | $\beta (w_{13/14} + 2w_{15})$ |
This appendix contains results for the loop contributions to the form factor \( F_3(q^2) \) for the various baryons, for photon virtualities below \( q^2 \approx 0.3 \text{GeV}^2 \).

### Table 10: Loop contribution to the \( F_3(q^2) \) of the \( \Xi_c^0 \) baryon.

| Diagram type | Number | Meson-baryon state | Coefficient |
|--------------|--------|--------------------|-------------|
| (c), (d)     | 1      | \( \Sigma_c^+ \), \( K^\pm \) | \( g_1 b_{D/F} \) |
|              | 2      | \( \Xi_c^0 \), \( \pi^\pm \) | \( g_1 b_{D/F} \) |
|              | 3      | \( \Lambda_c^+ \), \( K^\pm \) | \( g_2 b_{D/F} \) |
|              | 4      | \( \Xi_c^0 \), \( \pi^\pm \) | \( g_2 b_{D/F} \) |
| (e), (f)     | 5      | \( \Xi_c^0 \), \( \eta_8 \) | \( g_1 b_{D/F} w_{18} \) |
|              | 6      | \( \Xi_c^0 \), \( \eta_0 \) | \( \beta g_1 w_{18} \) |
|              | 7      | \( \Xi_c^0 \), \( \pi^0 \) | \( g_1 b_{D/F} w_{18} \) |
|              | 8      | \( \Sigma_c^0 \), \( K^0 \) | \( g_1 b_{D/F} w_{18} \) |
|              | 9      | \( \Omega_c^0 \), \( K^0 \) | \( g_1 b_{D/F} w_{18} \) |
|              | 10     | \( \Lambda_c^0 \), \( K^\pm \) | \( g_2 b_{D/F} (w_{16/17} + 2w_{18}) \) |
|              | 11     | \( \Xi_c^0 \), \( \eta_8 \) | \( g_2 b_{D/F} w_{18} \) |
|              | 12     | \( \Xi_c^0 \), \( \pi^0 \) | \( g_2 b_{D/F} w_{18} \) |
| (g), (h)     | 13     | \( \Xi_c^0 \), \( \eta_0 \) | \( \beta w_{15} \) |

### Table 11: Loop contribution to the \( F_3(q^2) \) of the \( \Omega_c^0 \) baryon.

| Diagram type | Number | Meson-baryon state | Coefficient |
|--------------|--------|--------------------|-------------|
| (c), (d)     | 1      | \( \Sigma_c^+ \), \( K^\pm \) | \( g_1 b_{D/F} \) |
|              | 2      | \( \Xi_c^+ \), \( K^\pm \) | \( g_2 b_{D/F} \) |
| (e), (f)     | 3      | \( \Omega_c^0 \), \( \eta_8 \) | \( g_1 b_{D/F} w_{18} \) |
|              | 4      | \( \Omega_c^0 \), \( \eta_0 \) | \( \beta g_1 w_{18} \) |
|              | 5      | \( \Xi_c^+ \), \( K^\pm \) | \( g_2 b_{D/F} (w_{16/17} + 2w_{18}) \) |
|              | 6      | \( \Xi_c^0 \), \( \pi^0 \) | \( g_2 b_{D/F} w_{18} \) |
| (g), (h)     | 7      | \( \Omega_c^0 \), \( \eta_0 \) | \( \beta w_{15} \) |

### C Results for the CP-violating form factor

This appendix contains results for the loop contributions to the form factor \( F_3(q^2) \) for the various baryons, for photon virtualities below \( q^2 \approx 0.3 \text{GeV}^2 \).

### Table 12: Loop contribution to the \( F_3(q^2) \) of the \( B_3 \) and \( B_6 \) states for \( w'_{10}, w'_{13/14}, w'_{15} = -0.5 \).

| \( q^2 \text{(GeV}^2) \) | \( \Lambda_c^+ \) | \( \Xi_c^+ \) | \( \Xi_c^0 \) | \( \Sigma_c^{++} \) | \( \Sigma_c'^{+} \) | \( \Sigma_c^0 \) | \( \Xi_c^{'+} \) | \( \Xi_c'^0 \) | \( \Omega_c'^0 \) |
|--------------------------|-----------------|-----------------|-----------------|-------------------|-------------------|-----------------|-----------------|-----------------|-----------------|
| 0.0484                   | 3.4204          | 3.7294          | -1.6089         | 1.7574            | 0.7755            | -0.2062         | 0.7180          | -0.4048         | -0.4764         |
| 0.1024                   | 3.4191          | 3.7286          | -1.6111         | 1.8291            | 0.7774            | -0.2730         | 0.7270          | -0.4103         | -0.4758         |
| 0.1444                   | 3.4180          | 3.7279          | -1.6128         | 1.8549            | 0.7790            | -0.2951         | 0.7318          | -0.4123         | -0.4754         |
| 0.1936                   | 3.4168          | 3.7269          | -1.6146         | 1.8746            | 0.7807            | -0.3104         | 0.7364          | -0.4136         | -0.4748         |
| 0.2500                   | 3.4154          | 3.7258          | -1.6166         | 1.8907            | 0.7828            | -0.3215         | 0.7408          | -0.4143         | -0.4741         |
| 0.3136                   | 3.4138          | 3.7244          | -1.6187         | 1.9044            | 0.7851            | -0.3296         | 0.7451          | -0.414          | -0.4733         |
Table 13: Loop contribution to the $F_3(q^2)$ of the $B_3$ and $B_6$ states for $w_{10}^i$, $w_{13/14}^i$, $w_{15}^i = 0$

| $q^2$(GeV$^2$) | $\Lambda^+_c$ | $\Xi^+_c$ | $\Xi^0_c$ | $\Sigma^+_c$ | $\Sigma^{++}_c$ | $\Sigma^0_c$ | $\Xi^{++}_c$ | $\Xi^{0}_c$ | $\Omega^0_c$ |
|----------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.0484         | 2.4033         | 2.6304      | -2.4473     | 1.5088      | 0.5496      | -0.4093     | 0.4759      | -0.6237     | -0.7059     |
| 0.1024         | 2.4019         | 2.6296      | -2.4495     | 1.5770      | 0.5490      | -0.4778     | 0.4823      | -0.6309     | -0.7071     |
| 0.1444         | 2.4009         | 2.6289      | -2.4512     | 1.6002      | 0.5486      | -0.5012     | 0.4851      | -0.6343     | -0.7077     |
| 0.1936         | 2.3997         | 2.6279      | -2.4530     | 1.6169      | 0.5480      | -0.5181     | 0.4873      | -0.6372     | -0.7091     |
| 0.2500         | 2.3983         | 2.6268      | -2.4550     | 1.6294      | 0.5474      | -0.5309     | 0.4890      | -0.6397     | -0.7103     |
| 0.3136         | 2.3967         | 2.6254      | -2.4571     | 1.6392      | 0.5467      | -0.5410     | 0.4903      | -0.6418     | -0.7116     |

Table 14: Loop contribution to the $F_3(q^2)$ of the $B_3$ and $B_6$ states for $w_{10}^i$, $w_{13/14}^i$, $w_{15}^i = 0.5$

| $q^2$(GeV$^2$) | $\Lambda^+_c$ | $\Xi^+_c$ | $\Xi^0_c$ | $\Sigma^+_c$ | $\Sigma^{++}_c$ | $\Sigma^0_c$ | $\Xi^{++}_c$ | $\Xi^{0}_c$ | $\Omega^0_c$ |
|----------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.0484         | 1.1175         | 1.2411      | -3.0161     | 0.8380      | 0.1796      | -0.4785     | 0.0824      | -0.7020     | -0.7884     |
| 0.1024         | 1.1161         | 1.2403      | -3.0184     | 0.9029      | 0.1765      | -0.5487     | 0.0862      | -0.7109     | -0.7914     |
| 0.1444         | 1.1151         | 1.2396      | -3.0200     | 0.9234      | 0.1741      | -0.5734     | 0.0870      | -0.7157     | -0.7937     |
| 0.1936         | 1.1139         | 1.2386      | -3.0218     | 0.9370      | 0.1712      | -0.5918     | 0.0868      | -0.7202     | -0.7964     |
| 0.2500         | 1.1125         | 1.2375      | -3.0238     | 0.9460      | 0.1679      | -0.6064     | 0.0858      | -0.7244     | -0.7994     |
| 0.3136         | 1.1109         | 1.2361      | -3.0259     | 0.9518      | 0.1643      | -0.6185     | 0.0841      | -0.7286     | -0.8028     |

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