Anisotropic stress as signature of non-standard propagation of gravitational waves

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We make precise the heretofore ambiguous statement that anisotropic stress is a sign of a modification of gravity. We show that in cosmological solutions of very general classes of models extending gravity — all scalar-tensor theories (Horndeski), Einstein-Aether models and bimetric massive gravity — a direct correspondence exists between perfect fluids apparently carrying anisotropic stress and a modification in the propagation of gravitational waves. Since the anisotropic stress can be measured in a model-independent manner, a comparison of the behavior of gravitational waves from cosmological sources with large-scale-structure formation could in principle lead to new constraints on the theory of gravity.

Over the last decade, we have established beyond reasonable doubt that, in its recent past, the expansion of the universe has been accelerating. This has challenged our beliefs about the theory of gravity: the only possibility available in general relativity with non-exotic matter is a cosmological constant, which would suffer from severe fine-tuning issues. Alternatively, the mechanism could be dynamical, i.e. feature at least one new degree of freedom. These dynamics would modify the predictions of concordance cosmology and give us a means to carry out precision tests of gravity at extremely large scales.

Frequently in extended models of gravity perfect fluids apparently carry anisotropic stress: there is gravitational slip, i.e. the values of the two scalar gravitational potentials sourced by matter are not equal. This affects structure formation and weak lensing. Recently, it was shown that the ratio of the two potentials is actually a model-independent observable [1, 2], which Euclid should be able to measure to a precision of a few percent, depending on the precise assumptions [3]. This begs the question as to what detecting or not detecting anisotropic stress actually means.

In this Letter we show that the propagation of tensor modes (gravitational waves, GWs) is also modified whenever the anisotropic stress is present at first order in perturbations sourced by perfect-fluid matter. We demonstrate this relationship in the context of three very large classes of extensions of the gravitational sector: general scalar-tensor theories (Horndeski [4, 5]), Einstein-Aether models [6–8] and bimetric massive gravity [9, 10]. GWs are the only propagating degrees of freedom in General Relativity, and it is natural to define modified gravity models as those where the gravitational waves are modified in such a non-trivial manner. Since imperfect fluids with anisotropic stress also split the two gravitational potentials but do not modify the propagation of tensor modes, this definition allows us to separate modifications of gravity from imperfect fluids.

The emphasis of this paper is not on new calculations (see e.g. the review [11]), but rather on new relations which are very general, were not noted before in the literature and could have a significant impact on tests of gravity on cosmological scales.

ASSUMPTIONS

We assume that the universe is well-described by small linear perturbations living on top of a spatially flat Friedmann metric. We take the line element for the metric on which matter and light propagate as

\[ ds^2 = a^2(\tau) \left[ -1 + 2\Phi d\tau^2 + (1 - 2\Phi) [\delta_{ij} + h_{ij}] dx^i dx^j \right] \]

where \( \tau \) is the conformal time, \( a \) the scale factor, \( \Phi \) and \( \Psi \) are the scalar gravitational potentials and \( h_{ij} \) is the traceless spatial metric (tensor) perturbation, i.e. the gravitational wave. We assume that the matter sector can be described as a fluid arising from the averaging of the motion of particles. We comment on the effect of this fluid’s being imperfect. We use the prime to denote a derivative with respect to conformal time.

The presence of anisotropic stress results in a difference in values between the two scalar potentials and can be described through the gravitational slip,

\[ \eta \equiv \frac{\Phi}{\Psi}. \tag{1} \]

In concordance cosmology, \( \eta = 1 \), with small corrections appearing from neutrino free-streaming. At second order in perturbations, anisotropic stress also always appears even when the matter consists completely of dust [12], but in the late universe should be smaller than \( |\eta - 1| \lesssim 10^{-3} \) [12, 13].

On the other hand, various modifications of gravity (such as \( f(R) \) [14], \( f(G) \) [15] or DGP [16]) do feature an \( O(1) \) correction to the slip parameter at linear order in perturbations, at least at some scales and even in the presence of just a perfect-fluid matter. It is, however, well
known that the value of $\eta$ can be modified by a change of frame, e.g. a conformal rescaling of the metric, making its value seemingly ambiguous.

In Refs [1, 2], it was shown that comparing the evolution of redshift-space distortions of the galaxy power spectrum with weak-lensing tomography allows us to reconstruct $\eta$ as a function of time and scale in a model-independent manner. Such an operational definition removes the frame ambiguity, since the measurement picks out the particular metric on the geodesics of which the galaxies and light move. It is the gravitational slip in that metric that is being measured by such cosmological probes. With the ambiguity of frame removed, the gravitational slip is a *bona fide* observable, rather than just a phenomenological parameter. Fixing the metric also determines what is considered a gravitational wave: we call these the propagating spin-2 perturbations of the metric on which matter moves.$^1$

In this Letter, we assume that the gravitational sector is extended by one of three classes of models featuring a single extra degree of freedom: (1) a very general scalar-tensor theory belonging to the Horndeski class [4]; (2) Einstein-Aether theory featuring an extra vector and spontaneous violation of Lorentz invariance; or (3) bimetric massive gravity. We will discuss each of these in turn and show that similar conclusions hold.

**MODIFIED GRAVITY DEFINED**

Dynamical models of late-time acceleration can feature interactions between the new degree of freedom and curvature/metric (scalar-tensor/Einstein-Aether) and the two metrics (bimetric). On a cosmological background, these interactions can alter the speed of propagation of gravitational waves ($c_T$), make the effective Planck mass ($M_\text{P}$) evolve in time [17] or add a mass $\mu$, giving

$$h_{ij}'' + (2 + \nu)Hh_{ij}' + \beta^2 k^2 h_{ij} + a^2 \mu^2 h_{ij} = a^2 \Gamma \gamma_{ij},$$

where $h_{ij}$ is the tensor wave amplitude in either of the two polarizations, $H \equiv a'/a$ is the Hubble rate in conformal time. The deviations away from standard behavior are contained in $\nu \equiv H^{-1} \dd \ln M_\text{P}^2$, the Planck mass run rate, and $c_T$, the speed of tensor waves, with both of these quantities defined in the Jordan frame of the matter.$^2$ We will show that scalar-tensor and Einstein-Aether models can change $\nu$ and $c_T$. On the other hand, in massive bigravity, the equation is modified by the mass of the graviton $\mu$. The transverse-traceless tensor $\gamma_{ij}$ is a source term for the gravitational waves. In the case of bimetric massive gravity, $\gamma_{ij}$ is the gravitational wave in the second metric and the two tensor modes mix as they propagate. When the matter fluid has anisotropic stress, this appears as the source term $\gamma_{ij}$, but it never modifies the homogeneous part of Eq. (2). However, this anisotropic stress is itself coupled to the gravitational waves and can lead to dissipation for horizon-scale GW modes [18, 19].

As we stressed above, Eq. (2) describes the evolution of the gravitational waves of the Jordan-frame metric. This choice is unique if our observations (e.g. redshifts, time delays) are taken to result from the geometry of the Universe. We should also note that, for bimetric massive gravity, the Einstein frame with standard gravitons does not exist even on a perturbative level. On the other hand, the issue of which of the two metrics matter couples to is an important one, which has to be fixed to define the model properly.

As is frequently said, anisotropic stress is a feature of modified gravity. For any gravity theory at the linear level, the anisotropy constraint in the Newtonian gauge takes the form

$$\Phi - \Psi = \sigma(t)\Pi + \pi_m,$$

with $\Pi$ a function of a particular combination of background and linear perturbation variables, depending on the theory. The quantity $\sigma(t)$ is a background function only, depending on the parameters of the Lagrangian. The $\pi_m$ is the scalar anisotropic stress sourced by the matter fluid. This appears whenever the perfect-fluid approximation breaks down and the particle distribution contains higher moments than those described by a perfect fluid. For example, free-streaming in neutrinos gives such a term even in concordance cosmology, but such contributions are very small in the late universe.

The aim of this Letter is to provide an unambiguous definition of modified gravity as one where the propagation of gravitational waves (2) is affected. The gravitational slip and gravitational waves are connected since both the anisotropy constraint (3) and the GW evolution equation (2) arise from the spatial–traceless part of the linearized Einstein equations. In the remainder of this Letter, we will demonstrate that the coupling $\sigma(t)$ appearing in the anisotropy equation (3) consists of the quantities that also control the modification of the tensor propagator. This means that modified gravity models popular in the literature are included in our definition.

However, imperfect-fluid matter while acting as a source to both the anisotropy constraint (3) and the GW equation (2), cannot directly modify the homogeneous part of the GW equation. Our definition of modified gravity therefore breaks the ambiguity that arises in the presence of such a source and points to an approach for differentiating modified gravity from imperfect fluids.

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$^1$ In the case of massive gravity, we are referring to the helicity-2 modes of the metric coupled to matter.

$^2$ Note that no observable quantity depends on $M_\text{P}$ itself, since a changed Planck mass can always be reabsorbed into the definition of masses if it is constant.
SCALAR-TENSOR THEORIES

In this section, we consider the most general class of theories featuring one extra scalar degree of freedom which has Einstein equations with no more than second derivatives on any background and are universally coupled to matter: the Horndeski class of models.\(^3\) This class includes the majority of the popular models of late-time acceleration such as quintessence, perfect fluids, \(f(R)\) gravity, \(f(G)\) gravity, kinetic gravity braiding and galileons (see e.g. the reviews \([23, 24]\)). The Horndeski Lagrangian is defined as the sum of four terms that are fully specified by a non-canonical kinetic term \(K(\phi, X)\) and three arbitrary coupling functions \(G_{3, 4, 5}(\phi, X)\), where \(X = -g_{\mu\nu}\phi^\mu\phi^\nu/2\) is the canonical kinetic energy term and where the comma denotes a partial derivative.

We make extensive use of the formulation for linear structure formation in scalar-tensor theories introduced in Ref. \([25]\). It was shown there that the form of linear perturbation equations for all Horndeski models can be completely described in terms of the background expansion history, density fraction of matter today \(\Omega_{\text{m0}}\), and four independent and arbitrary functions of time only, \(\alpha_K, \alpha_B, \alpha_M\) and \(\alpha_T\), which mix the four functional degrees of freedom of the action, \(K\) and \(G\). The Planck mass run rate \(\alpha_M\) and the tensor speed excess \(\alpha_T\) control the existence of anisotropic stress. Unrelated to the anisotropic stress, if the braiding \(\alpha_B \neq 0\), then the dark energy will cluster at small scales, with the \textit{kineticity} \(\alpha_K\) controlling at what scales this happens.

The anisotropy constraint in the notation of Eq. \((3)\) is \([26]\)

\[
\begin{align*}
\sigma &= \alpha_M - \alpha_T \\
\Pi &= H\delta\phi/\dot{\phi} + \alpha_T/(\alpha_M - \alpha_T)\Phi.
\end{align*}
\]

where \(\delta\phi\) is a perturbation of the scalar field. Note that the split between \(\sigma\) and \(\Pi\) above is arbitrary. The gravitational wave equation \((2)\) is modified through

\[
\begin{align*}
\nu &= \alpha_M, \\
c^2_T &= 1 + \alpha_T, \\
\mu^2 &= 0, \\
\Gamma &= 0.
\end{align*}
\]

It is clear from eq. \((4)\) that when both \(\alpha_M = \alpha_T = 0\) there is no new contribution to either to anisotropic stress or tensor propagation. In the context of scalar-tensor models and the late universe with \(\pi_m \approx 0\), a detection of anisotropic stress therefore is direct evidence that one or both of the parameters \(\alpha_T\) and \(\alpha_M\) are different from their concordance values of zero and that gravity is modified in the sense of this work.

In principle, one could imagine that there may exist models defined by a choice of the functions \(\alpha_i\) in which the scalar perturbation arranges itself dynamically in such a configuration that no gravitational slip appears, even though one of \(\alpha_M, \alpha_T\) is not zero. This would be a very particular situation or one requiring a very tuned choice of model parameters. For example, it happens at the asymptotic future — and static — pure de-Sitter limit. It can be shown that it is in fact impossible to have such a cancellation in a model where the scalar has real dynamics. We defer the proof to a more technical follow-up study.

EINSTEIN-AETHER THEORIES

Einstein-aether models \([27, 28]\) are a class of theories which feature an extra vector degree of freedom (the \textit{aether}) \(u^\mu\). They are a subclass of general vector theories requiring that \(u^\mu\) be given a constant and timelike vacuum expectation value \(u_\mu u^\mu = -1\) and that it be minimally coupled. This chooses a preferred frame, violating Lorentz symmetry. The infrared limit of Hořava-Lifshitz (HL) models \([29-31]\) — relevant for late-time cosmology — is closely related, with the vector field forced to be hypersurface orthogonal and thus providing a natural slicing for the space-time \([32]\).

The Lagrangian can be written in a basis of four operators, through a kinematic decomposition of \(\nabla_\mu u_\nu\) \([32]\): the squares of acceleration, expansion, twist and shear, and their associated dimensionless coefficients \(c_\alpha, c_\theta, c_\omega\), and \(c_\phi\), respectively.\(^4\)

The extra dynamical degree of freedom at the linear level is the perturbation of the spatial components \(u^i\) of the vector \(u^\mu\), which can be decomposed into longitudinal and transverse parts as \(u^i = \theta^i u + \hat{u}^i\). The longitudinal part modifies the anisotropy constraint \([33]\), which in the notation of Eq. \((3)\) is

\[
\Pi = \left(\frac{u}{u^2}\right)\sigma - c_\sigma.
\]

At the same time, the parameters of the tensor equation \((2)\) are given by

\[
\begin{align*}
\nu &= 0, & c^2_T &= (1 + c_\sigma)^{-1}, \\
\mu^2 &= 0, & \Gamma &= 0.
\end{align*}
\]

In conclusion, the modifications of both the anisotropy constraint and the tensor wave equation are driven by the}

\(^3\) We have not considered in detail the extension discussed in \([20-22]\), where higher derivatives appear in the Einstein equations, but can be eliminated by solving the constraints.

\(^4\) In the language of Ref. \([32]\), these correspond to \(c_\phi \equiv -c_1 + c_4, c_\theta \equiv \frac{1}{3}(c_1 + c_3) + c_2, c_\omega = c_1 - c_3, c_\phi \equiv c_1 + c_3\).
same coupling \( c_\sigma \) of the shear. If \( c_\sigma \) appears in the action, it will modify both the anisotropic stress and the gravitational wave propagation. Thus a detection of anisotropic stress in the late Universe with \( \sigma_m \approx 0 \) in the context of these models also implies that gravity is modified in the sense of this work.

**BIMETRIC MASSIVE GRAVITY**

The bimetric massive gravity model features two dynamical metrics, \( g_1 \) and \( g_2 \), each with its own Einstein–Hilbert term in the action. In addition, a potential term describes non–derivative interactions between the two metrics, \( U(g_1, g_2; a_i) \). The five constants \( a_i \) parametrize these interactions and are the theory’s free parameters. The interactions inevitably give mass to one of the two metrics [34], and the theory in general propagates a massless and a massive spin-two field [35], and it provides a non–linear extension of the Fierz-Pauli theory [36], which is free of the so-called Boulware-Deser ghost [9, 37–40]. One usually considers the matter fields to be coupled to one of the metrics, which we shall call \( g_1 \).

Bimetric gravity provides a natural extension of the so-called dRGT massive gravity, with the latter being a subcase of the former, in the limit where the second metric becomes non-dynamical. Cosmological solutions for dRGT and bimetric theories have been studied in, for instance, [10, 41–45] and [46–51] respectively, with the aim of explaining the current acceleration of the universe without the need of an explicit cosmological constant in the action. It has been shown however that in dRGT, homogeneous and isotropic backgrounds are not solutions of the background equations of motion [52], or when these solutions exist, they suffer from strong coupling [42], ghost [53, 54] or non-linear instabilities [54, 55], and we will therefore concentrate on the bimetric version only.\(^5\)

We use the setup and notation of Ref. [56] \(^6\), choosing both the background metrics to be homogeneous and isotropic. At the linear level, the theory predicts the existence of anisotropic stress for the scalar Newtonian potentials of the matter metric \( g_1 \), giving the anisotropy constraint the form

\[
\sigma = a^2m^2f_1 \quad \Pi = E_2 \tag{8}
\]

in the notation of Eq. (3). \( E_2 \) is the scalar coming from the tensor perturbation of the second metric \( g_2 \). The function \( f_1 \) is a background-dependent function that depends on the ratio between the scale factors of the two metrics and the constant parameters \( a_i \).

The equation for gravitational waves (2) is modified through

\[
\nu = 0, \quad c_t^2 = 1, \quad \mu^2 = m^2f_1, \quad \Gamma = m^2f_1. \tag{9}
\]

Massive bigravity models change neither the Planck mass nor the speed of gravitational waves. They do give gravitons a mass and an interaction term. As we can easily see, the coefficients modifying the anisotropy constraint and the graviton equation of motion are all proportional to \( m^2f_1 \). Yet again, if anisotropic stress is observed in the late Universe with \( \sigma_m \approx 0 \) in the context of these models, we must conclude that gravity is modified in the sense of this work.

**CONCLUSIONS AND IMPLICATIONS**

In this Letter, we have shown that a very close relationship exists between two properties of general extensions of gravity which until now have not been considered together: when anisotropic stress is apparently sourced by perfect-fluid matter perturbations at linear level, the propagation of gravitational waves is modified. Such a relationship generally exists in all Horndeski theories with an extra scalar, Einstein-Aether theories featuring an extra vector field and bimetric massive gravity, featuring a second rank-2 tensor field — this covers a very large fraction of all the extensions of gravity with homogeneous backgrounds. We conjecture that this is a feature of all models *in general configurations* and we choose to use this physics as the unambiguous definition of modified gravity.

We note here that the anisotropic stress and clustering of the new degree of freedom — frequently described as a change to the effective Newton’s constant — are both completely independent quantities, the presence of which is not contingent on each other.

The relationship between tensor propagation and gravitational slip is a result of both being part of the spatial–traceless part of the linearised Einstein equations: the same corrections in the action modify the anisotropy constraint and the action for the graviton.

We stress that this relationship would hold whenever gravity is modified, not only at low redshifts where extensions of gravity are frequently utilized as dynamical models of acceleration. For example, during recombination, if models of gravity with apparent anisotropic stress from perfect fluids are introduced, one would then need to adjust the behavior of gravitational waves. At the same time, this new anisotropic stress would change the lensing and the integrated Sachs-Wolfe effect. All these

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\(^5\) In fact, a gradient instability for the new helicity-0 mode in the bimetric setup appears to exist for for some choices of parameters [51, 56] but not others [57]. Whenever healthy solutions exist, the conclusions of this Letter hold.

\(^6\) For a similar analysis see also Ref. [58].
effects would modify the CMB spectrum, in particular the B-mode polarization [33, 59, 60].

This deep relationship between anisotropic stress and tensor modes implies that measurements of large-scale structure and of gravitational waves can give independent information on the properties of each other. For example, a comparison between the time of arrival of neutrinos and gravitational waves from some energetic event is a probe of the speed of tensor modes \(c_T\) and their mass \(\mu\) [61]. A luminosity distance from standard sirens imputed from the decay of the amplitude of the gravitational waves probes \(\nu, \mu\) and \(\Gamma\) [62]. Such observations are clearly extremely challenging and futuristic, but may one day be possible.7 On the other hand, the slip parameter \(\eta\) in some models can be an order-one ratio of small numbers (e.g. in \(f(R)\) gravity, where the permitted parameter values are \(\alpha_M = -\alpha_B \lesssim 10^{-5}\) [64], while \(\eta = 1/2\) inside the Compton scale). Measurements of anisotropic stress can be more informative about tensor modes than direct probes of gravitational waves in such a case. Ultimately, it should be possible to combine them to disambiguate the various properties of the theory of gravity at cosmological scales. We leave the discussion of how feasible this is to future work.

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7 Tests such as the binary pulsar [63] probe the coupling of matter sources to gravitational waves and therefore are not necessarily sensitive to the modification in propagation described here.

[1] L. Amendola, M. Kunz, M. Motta, I. D. Saltas, and I. Sawicki, “Observables and unobservables in dark energy cosmologies,” Phys. Rev. D87 (2013) 023501, arXiv:1210.0439 [astro-ph.CO].
[2] M. Motta, I. Sawicki, I. D. Saltas, L. Amendola, and M. Kunz, “Probing Dark Energy through Scale Dependence,” Phys. Rev. D88 (2013) 124035, arXiv:1305.0008 [astro-ph.CO].
[3] L. Amendola, S. Fogli, A. Guarnizo, M. Kunz, and A. Vollmer, “Model-independent constraints on the cosmological anisotropic stress,” arXiv:1311.4765 [astro-ph.CO].
[4] G. W. Horndeski, “Second-order scalar-tensor field equations in a four-dimensional space,” Int. J. Theor. Phys. 10 (1974) 363–384.
[5] C. Deffayet, X. Gao, D. Steer, and G. Zahariade, “From k-essence to generalised Galileons,” Phys. Rev. D84 (2011) 064039, arXiv:1103.3260 [hep-th].
[6] K. J. Nordtvedt and C. M. Will, “Conservation Laws and Preferred Frames in Relativistic Gravity. II. Experimental Evidence to Rule Out Preferred-Frame Theories of Gravity,” Astrophys. J. 177 (1972) 775–792.
[7] C. M. Will and J. Nordtvedt, Kenneth, “Conservation Laws and Preferred Frames in Relativistic Gravity. I. Preferred-Frame Theories and an Extended PPN Formalism,” Astrophys. J. 177 (1972) 757.
[8] R. W. Hellings and K. Nordtvedt, “Vector-Metric Theory of Gravity,” Phys. Rev. D7 (1973) 3593–3602.
[9] S. Hassan, R. A. Rosen, and A. Schmidt-May, “Ghost-free Massive Gravity with a General Reference Metric,” JHEP 1202 (2012) 026, arXiv:1109.3329 [hep-th].
[10] S. Hassan and R. A. Rosen, “On Non-Linear Actions for Massive Gravity,” JHEP 1107 (2011) 009, arXiv:1103.6055 [hep-th].
[11] S. Tsujikawa, “The effective field theory of inflation/dark energy and the Horndeski theory,” arXiv:1404.2684 [gr-qc].
[12] G. Ballesteros, L. Hollenstein, R. K. Jain, and M. Kunz, “Nonlinear cosmological consistency relations and effective matter stresses,” JCAP 1205 (2012) 038, arXiv:1112.4837 [astro-ph.CO].
[13] J. Adamek, D. Daverio, R. Durrer, and M. Kunz, “General Relativistic N-body simulations in the weak field limit,” Phys. Rev. D88 (2013) 103527, arXiv:1308.6524 [astro-ph.CO].
[14] P. Zhang, “Testing \(f(R)\) gravity against the large scale structure of the universe,” Phys. Rev. D73 (2006) 123504, arXiv:astro-ph/0511218 [astro-ph].
[15] I. D. Saltas and M. Kunz, “Anisotropic stress and stability in modified gravity models,” Phys. Rev. D83 (2011) 064042, arXiv:1012.3171 [gr-qc].
[16] K. Koyama and R. Maartens, “Structure formation in the dgp cosmological model,” JCAP 0601 (2006) 016, arXiv:astro-ph/0511634 [astro-ph].
[17] A. Riazuelo and J.-P. Uzan, “Quintessence and gravitational waves,” Phys. Rev. D62 (2000) 083506, arXiv:astro-ph/0004156 [astro-ph].
[18] R. Durrer and T. Kahniashvili, “CMB anisotropies caused by gravitational waves: A Parameter study,” Helv. Phys. Acta 71 (1998) 445–457, arXiv:astro-ph/9702226 [astro-ph].
[19] S. Weinberg, “Damping of tensor modes in cosmology,” Phys. Rev. D69 (2004) 023503, arXiv:astro-ph/0306304 [astro-ph].
[20] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, “Essential Building Blocks of Dark Energy,” JCAP 1308 (2013) 025, arXiv:1304.4840 [hep-th].
[21] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, “Healthy theories beyond Horndeski,” arXiv:1404.6495 [hep-th].
[22] X. Gao, “A unifying framework for scalar-tensor theories,” arXiv:1406.0822 [gr-qc].
[23] L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations*. Cambridge University Press, 2010.

[24] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, “Modified Gravity and Cosmology,” *Phys.Rept.* **513** (2012) 1–189, arXiv:1106.2476 [astro-ph.CO].

[25] E. Bellini and I. Sawicki, “Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity,” *JCAP* **1407** (2014) 050, arXiv:1404.3713 [astro-ph.CO].

[26] A. De Felice, T. Kobayashi, and S. Tsujikawa, “Effective gravitational couplings for cosmological perturbations in the most general scalar-tensor theories with second-order field equations,” *Phys.Lett.* **B706** (2011) 123–133, arXiv:1108.4242 [gr-qc].

[27] T. Jacobson and D. Mattingly, “Gravity with a dynamical preferred frame,” *Phys.Rev.* **D64** (2001) 024028, arXiv:gr-qc/0007031 [gr-qc].

[28] T. Jacobson, “Einstein-aether gravity: A Status report,” *PoS* **QG-PH** (2007) 020, arXiv:0801.1547 [gr-qc].

[29] P. Hořava, “Quantum Gravity at a Lifshitz Point,” *Phys.Rev. D79* (2009) 084008, arXiv:0901.3775 [hep-th].

[30] D. Blas, O. Pujolas, and S. Sibiryakov, “Consistent Extension of Horava Gravity,” *Phys.Rev.Lett.* **104** (2010) 181302, arXiv:0909.3525 [hep-th].

[31] D. Blas, O. Pujolas, and S. Sibiryakov, “Models of non-relativistic quantum gravity: The Good, the bad and the healthy,” *JHEP* **1104** (2011) 018, arXiv:1107.3503 [hep-th].

[32] T. Jacobson, “Undoing the twist: the Hořava limit of Einstein-aether,” arXiv:1310.5115 [gr-qc].

[33] E. A. Lim, “Can we see Lorentz-violating vector fields in the CMB?,” *Phys.Rev. D71* (2005) 063504, arXiv:astro-ph/0407437 [astro-ph].

[34] N. Boulanger, T. Damour, L. Gualtieri, and M. Henneaux, “Inconsistency of interacting, multigraviton theories,” *Nucl.Phys.* **B507** (2001) 127–171, arXiv:hep-th/0007220 [hep-th].

[35] S. Hassan and R. A. Rosen, “Bimetric Gravity from Ghost-free Massive Gravity,” *JHEP* **1202** (2012) 126, arXiv:1109.3515 [hep-th].

[36] M. Fierz and W. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field,” *Proc.Roy.Soc.Lond.* **A173** (1939) 211–232.

[37] C. de Rham and G. Gabadadze, “Generalization of the Fierz-Pauli Action,” *Phys.Rev. D82* (2010) 044020, arXiv:1007.0443 [hep-th].

[38] C. de Rham, G. Gabadadze, and A. J. Tolley, “Resummation of Massive Gravity,” *Phys.Rev.Lett.* **106** (2011) 231101, arXiv:1011.1232 [hep-th].

[39] S. Hassan and R. A. Rosen, “Resolving the Ghost Problem in non-Linear Massive Gravity,” *Phys.Rev.Lett.* **108** (2012) 041101, arXiv:1106.3344 [hep-th].

[40] C. de Rham, G. Gabadadze, and A. J. Tolley, “Ghost free Massive Gravity in the Stückelberg language,” *Phys.Lett.* **B711** (2012) 190–195, arXiv:1107.3820 [hep-th].

[41] A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, “Open FRW universes and self-acceleration from nonlinear massive gravity,” *JCAP* **1111** (2011) 030, arXiv:1109.3845 [hep-th].

[42] A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, “Cosmological perturbations of self-accelerating universe in nonlinear massive gravity,” *JCAP* **1203** (2012) 006, arXiv:1111.4107 [hep-th].

[43] M. Fasiello and A. J. Tolley, “Cosmological perturbations in Massive Gravity and the Higuchi bound,” *JCAP* **1211** (2012) 035, arXiv:1206.3852 [hep-th].

[44] D. Langlois and A. Naruko, “Cosmological solutions of massive gravity on de Sitter,” *Class.Quant.Grav.* **29** (2012) 202001, arXiv:1206.6813 [hep-th].

[45] T. Jacobson, “Einstein-aether gravity: A Status report,” *PoS* **QG-PH** (2007) 020, arXiv:0801.1547 [gr-qc].

[46] M. von Strauss, A. Schmidt-May, J. Enander, E. Mortsell, and S. Hassan, “Cosmological Solutions in Bimetric Gravity and their Observational Tests,” *JCAP* **1203** (2012) 042, arXiv:1111.1655 [gr-qc].

[47] Y. Akrami, T. S. Koivisto, and M. Sandstad, “Accelerated expansion from ghost-free bigravity: a statistical analysis with improved generality,” *Journal of High Energy Physics* **3** (Mar., 2013) 99, arXiv:1209.0457 [astro-ph.CO].

[48] F. Koennig, A. Patil, and L. Pilo, “FRW Cosmology in Ghost Free Massive Gravity,” *JHEP* **1203** (2012) 067, arXiv:1111.1983 [hep-th].

[49] F. Koennig and L. Amendola, “A minimal bimetric gravity model that fits cosmological observations,” arXiv:1402.1988 [astro-ph.CO].

[50] A. De Felice, T. Nakamura, and T. Tanaka, “Possible existence of viable models of bi-gravity with detectable graviton oscillations by gravitational wave detectors,” *PTEP* **2014** no. 4, (2014) 043E01, arXiv:1304.3920 [gr-qc].

[51] A. De Felice, A. E. Gumrukcuoglu, S. Mukohyama, N. Tanahashi, and T. Tanaka, “Viable cosmology in bimetric theory,” arXiv:1404.0008 [hep-th].

[52] G. D’Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, et al., “Massive Cosmologies,” *Phys.Rev.* **D84** (2011) 124046, arXiv:1108.5231 [hep-th].

[53] M. Fasiello and A. J. Tolley, “Cosmological Stability Bound in Massive Gravity and Bigravity,” *JCAP* **1312** (2013) 002, arXiv:1308.1647 [hep-th].

[54] A. De Felice, A. E. Gumrukcuoglu, and S. Mukohyama, “Massive gravity: nonlinear instability of the homogeneous and isotropic universe,” *Phys.Rev.Lett.* **100** (2012) 171101, arXiv:1206.2080 [hep-th].

[55] A. De Felice, A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, “Nonlinear stability of cosmological solutions in massive gravity,” *JCAP* **1305** (2013) 035, arXiv:1303.4154 [hep-th].

[56] D. Comelli, M. Crisostomi, and L. Pilo, “Perturbations in Massive Gravity Cosmology,” *JHEP* **1206** (2012) 085, arXiv:1202.1986 [hep-th].

[57] F. König, Y. Akrami, L. Amendola, M. Motta, and A. R. Solomon, “Stable and unstable cosmological
models in bimetric massive gravity,”

arXiv:1407.4331 [astro-ph.CO].

[58] A. R. Solomon, Y. Akrami, and T. S. Koivisto,
“Cosmological perturbations in massive bigravity: I. Linear growth of structures.”

arXiv:1404.4061 [astro-ph.CO].

[59] L. Amendola, G. Ballesteros, and V. Pettorino, “Effects of modified gravity on B-mode polarization,”
arXiv:1405.7004 [astro-ph.CO].

[60] B. Audren, D. Blas, J. Lesgourgues, and S. Sibiryakov, “Cosmological constraints on Lorentz violating dark energy,” JCAP 1308 (2013) 039,
arXiv:1305.0009 [astro-ph.CO].

[61] A. Nishizawa and T. Nakamura, “Measuring Speed of Gravitational Waves by Observations of Photons and Neutrinos from Compact Binary Mergers and Supernovae,” arXiv:1406.5544 [gr-qc].

[62] C. Cutler and D. E. Holz, “Ultra-high precision cosmology from gravitational waves,”

Phys.Rev. D80 (2009) 104009,
arXiv:0906.3752 [astro-ph.CO].

[63] K. Yagi, D. Blas, E. Barausse, and N. Yunes, “Constraints on Einstein-Æther theory and Horava gravity from binary pulsar observations,”

Phys.Rev. D89 (2014) 084067,
arXiv:1311.7144 [gr-qc].

[64] L. Lombriser, A. Slosar, U. Seljak, and W. Hu, “Constraints on f(R) gravity from probing the large-scale structure,” Phys.Rev. D85 (2012) 124038,
arXiv:1003.3009 [astro-ph.CO].