Alpha Cluster Structures and Isoscalar Monopole Excitations in Light Nuclei

T Yamada
Lab. of Physics, Kanto Gakuin University, Yokohama 236-8501, Japan
E-mail: yamada@kanto-gakuin.ac.jp

Abstract. Isoscalar monopole excitations to cluster states in light nuclei are in general strong as to be comparable with the single particle strength. Their experimental strengths share about 20 % of the sum rule value in the case of $^4$He, $^{11}$B, $^{12}$C, and $^{16}$O etc. We discuss the isoscalar monopole strength function in $^{16}$O, which is typical in light nuclei, is discussed up to $E_x \approx 40$ MeV. It is found that 1) two different types of monopole excitations exist in $^{16}$O; one is the monopole excitation to cluster states which is dominant in the lower energy part, and the other is the monopole excitation of the mean-field type such as one-particle one-hole (1p1h) which is attributed mainly to the higher energy part, and 2) this character of the monopole excitations originates from the fact that the ground state of $^{16}$O with the dominant doubly closed shell structure has a duality of the mean-field-type as well as alpha-clustering character.

1. Introduction
Isoscalar monopole excitation is related to a density fluctuation in nuclei. The typical example is the isoscalar giant monopole resonance (ISGMR) observed as a single bump in medium and heavy nuclei, which exhausts almost 100 % of the energy weighted sum rule value. It is interesting to study what happens for the ISGMR in lighter nuclei. When the nuclear masses decrease from medium nuclei to light nuclei, the isoscalar monopole strengths are in general fragmented. In this paper, we concentrate on the case of $^{16}$O as a typical example. The histogram in Fig. 1 shows the experimental isoscalar monopole strength function in $^{16}$O [1] (the date below $E_x \sim 10$ MeV is absent due to the experimental condition). The monopole strengths split over wide energy region, and one can see discrete peaks in low energy ($E_x \lesssim 16$ MeV) and gross three-bump structure in higher energy region ($16 \lesssim E_x \lesssim 40$ MeV). The experimental data is compared with the relativistic RPA (RRPA) calculation [2]. In order to match their calculation to the experimental centroid, the calculated strength function was shifted down in energy by 4.2 MeV and furthermore they normalized it by multiplying the RRPA curve by a factor of 0.25 [1], although their calculation failed to reproduce the $0^+$ states found in the low energy region. In the nonrelativistic RPA and second-order RPA (SRPA) calculations for $^{16}$O [3, 4] a significant discrepancy is also revealed as compared with the experimental data, in particular, in the low energy region, although the gross structures at the higher energy region in the RPA calculations are in rather good agreement with the data.

Recently the structure study of $^{16}$O has made a great advance up to $E_x \simeq 16$ MeV around the $4\alpha$ disintegration threshold. The six lowest $0^+$ states of $^{16}$O, up to $E_x \simeq 16$ MeV, including the ground state, have for first time been reproduced very well within the framework of the $4\alpha$
orthogonality condition model (OCM) [5] (see Fig. 2) combined with the Gaussian expansion method (GEM) [6]. The OCM is a semi-microscopic cluster model, which is an approximation of RGM (resonating group method) and is extensively described in Ref. [7]. Many successful applications of OCM to ordinary nuclei as well as hypernuclei are reported in Refs. [8, 9]. The six \(0^+\) states have the following characteristic structures [5]: 1) the ground state \((0^+_1)\) has dominantly a doubly-closed-shell structure, 2) the \(0^+_2\) state at \(E_x = 6.05\) MeV and the \(0^+_3\) state at \(E_x = 12.05\) MeV have mainly \(\alpha + ^{12}\text{C}\) structures where the \(\alpha\)-particle orbits around the \(^{12}\text{C}(0^+_1)\) core in an \(S\)-wave and around the \(^{12}\text{C}(2^+_1)\) core in a \(D\)-wave, respectively, 3) the \(0^+_4\) \((E_x = 13.6\) MeV) and \(0^+_5\) \((E_x = 14.1\) MeV) states mainly have \(\alpha + ^{12}\text{C}(0^+_1)\) structure with higher nodal behavior and \(\alpha + ^{12}\text{C}(1^-)\) structure, respectively, and 4) the \(0^+_6\) state at 15.1 MeV is a strong candidate of the \(4\alpha\) condensate [11], \((0S)^4_{16}\), with the probability of 61 %. The monopole strengths and decay widths are well reproduced, together with the r.m.s. radius of the ground state (see Table 1). We see that the monopole strengths in Table 1 are comparable with the single particle strength \((\sim 4\) fm\(^2\)) [12].

The purpose of the present paper is to study whether the \(4\alpha\) OCM can reproduce the experimental isoscalar monopole strength function in the low energy region up to \(E_x \simeq 16\) MeV in \(^{16}\text{O}\). It is noted that this energy region is difficult to be treated in the mean-field theory as mentioned above.

2. Isoscalar monopole strength function of \(^{16}\text{O}\) with \(4\alpha\) OCM

Figure 3 shows the calculated isoscalar monopole strength function of \(^{16}\text{O}\) with the \(4\alpha\) OCM, where we use the calculated monopole matrix elements and the calculated decay widths for the six \(0^+\) states up to \(E_x \simeq 16\) MeV obtained by the \(4\alpha\) OCM calculation, also the
Figure 2. (Color online) Comparison of energy spectra among experiment, the 4\(\alpha\) OCM calculation [5], and the \(\alpha^{+12}\text{C}\) model calculation [10], where the \(\alpha^{+12}\text{C}\) and 4\(\alpha\) thresholds are shown. Experimental data are taken from Ref. [14] and from Ref. [15] for the 0\(^{+}\) state.

The experimental excitation energies for the six 0\(^{+}\) states are employed [13]. We can see a rather good correspondence with the experimental data. The fine structures in the calculated strength function, i.e. one peak at \(E_{x} = 12.1\) MeV (corresponding to the 0\(^{+}\) state), one shoulder-like peak at \(E_{x} = 13.8\) MeV (0\(^{+}\))\(^{1}\)), two peaks at \(E_{x} = 14.1\) MeV (0\(^{+}\))\(^{2}\)) and 15.1 MeV (0\(^{+}\))\(^{5}\)), are well reproduced. It is noted that the data below \(E_{x} \sim 10\) MeV in Figs. 1 and 3 is absent due to the experimental condition. Thus, the signal at 6.05 MeV is not seen by the experiment.

From the present results, one concludes that there exist two features of the isoscalar monopole excitations of \(^{16}\text{O}\), i.e. the monopole excitation to cluster states dominates the low energy region \((E_{x} \lesssim 16\) MeV\)), sharing about 20 % of the EWSR, while that to the 1p1h-type states looks likely to be predominant at the higher energy region.

3. Why are alpha cluster states excited by the isoscalar monopole transitions?

It is instructive to discuss the mechanism of why the five \(\alpha\) cluster states (0\(^{+}\))\(^{2}\), 0\(^{+}\))\(^{3}\), 0\(^{+}\))\(^{4}\), 0\(^{+}\))\(^{5}\), and 0\(^{+}\))\(^{6}\) of \(^{16}\text{O}\) are excited relatively strongly from the shell-model-like ground state by the isoscalar monopole transition. First we discuss a dual nature in the ground state wave function of \(^{16}\text{O}\), and then demonstrate an interesting role of the monopole operator.

The wave function of the \(^{16}\text{O}\) ground state has the dominant \((0\alpha)^{4}(0\rho)^{12}\) configuration, corresponding to the SU(3) \((\lambda,\mu) = (0,0)\) wave function. According to the Bayman-Bohr theorem [17], this doubly closed shell model wave function is mathematically equivalent to a single \(\alpha^{+12}\text{C}\) cluster model wave function as well as a single 4\(\alpha\) cluster wave function with the
Table 1. Excitation energies ($E_x$), charge r.m.s. radii ($R_c$), E0 transition matrix elements [$M(E0)$], and particle decay widths ($\Gamma$) of the $0^+$ states in $^{16}$O obtained by the 4$\alpha$ OCM calculation [5, 13], together with the experimental data [14, 15]. They are given in the unit of MeV, fm, fm$^2$, and MeV, respectively. The experimental monopole matrix elements are obtained by the $^{16}$O($e, e'$) reaction [14]. $P^\text{e.w.}$ represents the percentage of the energy weight strength to the isoscalar-monopole energy weighted sum rule value. The finite size effects of $^\text{12}$C are taken into account in estimating $R_c$ with the 4$\alpha$ OCM (see Ref. [16] for details).

| State | $E_x$ ($\text{MeV}$) | $R_c$ (fm) | $M(E0)$ (fm$^2$) | $\Gamma$ (MeV) | $E_x$ ($\text{MeV}$) | $R_c$ (fm) | $M(E0)$ (fm$^2$) | $P^\text{e.w.}$ (%) |
|-------|---------------------|------------|-----------------|----------------|---------------------|------------|-----------------|-------------------|
| $0_1^+$ | 0.00 | 2.7 | 0.00 | 2.70 | 6.05 | 3.55 ± 0.21 | 3.5% |
| $0_2^+$ | 6.37 | 3.0 | 3.9 | 6.05 | 4.03 ± 0.09 | 8.9% |
| $0_3^+$ | 9.96 | 3.1 | 2.4 | 12.05 | no data | 0.6 |
| $0_4^+$ | 12.56 | 4.0 | 2.4 | 0.60 | 13.60 | no data | 0.6 |
| $0_5^+$ | 14.12 | 3.1 | 2.6 | 0.20 | 14.01 | 3.3 ± 0.7 | 6.9% | 0.185 |
| $0_6^+$ | 16.45 | 5.6 | 1.0 | 0.14 | 15.10 | no data | 0.166 |

Figure 3. (Color online) Calculated isoscalar monopole strength functions of $^{16}$O with the 4$\alpha$ OCM (bold line) [13] and experimental data (thin line: see Fig. 1 [1]). The date below $E_x \sim 10$ MeV is absent due to the experimental condition. Thus, the signal at 6.05 MeV is not seen by the experiment.

total harmonic oscillator quanta $Q = 12$ [12, 13],

\[
\det[|0s^4(0p)^{12}|] = N_0 \times \mathcal{A} \left\{ \left[ u_{40}(\xi_3, 3\nu) \phi_{L=0}^{(12)C} \right]_{J=0} \phi(\alpha) \right\} \phi_{\text{cm}}(R_{\text{cm}}),
\]

(1)

\[
N_2 \times \mathcal{A} \left\{ \left[ u_{42}(\xi_3, 3\nu) \phi_{L=2}^{(12)C} \right]_{J=0} \phi(\alpha) \right\} \phi_{\text{cm}}(R_{\text{cm}}),
\]

(2)

\[
\tilde{N}_0 \times \mathcal{A} \left\{ \left[ u_{40}(\xi_3, 3\nu) \left[ u_{40}(\xi_2, 8\nu/3) u_{40}(\xi_1, 2\nu) \right] \right]_{L=0} \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3) \phi(\alpha_4) \right\} \phi_{\text{cm}}(R_{\text{cm}}),
\]

(3)
where $\phi_{\text{cm}}$ denotes the wave function of the center-of-mass motion of $^{16}$O, and $\xi_k$ ($k = 1 \sim 3$) stand for the Jacobi coordinates between the clusters. $\phi(\alpha)$ and $\phi(1^{12}\text{C})$ represent, respectively, the internal wave function of the $\alpha$ cluster and that of $^{12}\text{C}$ with the angular momentum of $L$. $N_L(N_0)$ is the normalization factor. The relative wave function between the $\alpha$ and $^{12}\text{C}$ clusters in Eqs. (1) and (2) is described by the harmonic oscillator wave function $u_{QL}(\xi, \beta)$ with $Q = 4$. Eqs. (1) and (2) mean that the doubly closed shell model wave function has an $\alpha+^{12}\text{C}$ cluster degree of freedom. In addition, Eq. (3) demonstrates that the doubly closed shell model wave function also possesses a 4$\alpha$ cluster degree of freedom. Thus the ground state of $^{16}$O has the mean-field of degree of freedom as well as the cluster degree of freedom. We call this the dual nature.

On the other hand, the isoscalar monopole operator of $^{16}$O can be decomposed into the internal part (for the constituent clusters) and relative part (with respect to the clusters) as follows [12, 13]:

$$O_{E0}^{\text{IS}}(16\text{O}) = \sum_{i=1}^{16} (r_i - R_{\text{cm}})^2 = O_{E0}^{\text{IS}}(\alpha) + O_{E0}^{\text{IS}}(^{12}\text{C}) + 3\xi_3^2 \quad (4)$$

$$= \sum_{k=1}^{4} O_{E0}^{\text{IS}}(\alpha_k) + \sum_{k=1}^{4} 4(R_{\alpha_k} - R_{\text{cm}})^2, \quad (5)$$

where $R_{\alpha_k}$ denotes the c.o.m coordinate of the $k$-th $\alpha$ cluster in the 4$\alpha$ system. The reasons why the $0^+_2$ and $0^+_3$ states of $^{16}$O with the $\alpha+^{12}\text{C}$ cluster structure are excited by the monopole transition are given as follows: 1) the ground state of $^{16}$O is of the SU(3) $(\lambda, \mu) = (0, 0)$ nature, which has the $\alpha$-clustering degree of freedom as discussed above, 2) the relative part in the monopole operator referring to the $\alpha+^{12}\text{C}$ relative motion, $3\xi_3^2$, in Eq. (4) can activate the $\alpha$-cluster degree of freedom in the ground state, and then, 3) the $\alpha+^{12}\text{C}$ states are excited by the monopole operator. It should be reminded that the contribution from the other parts of the monopole operator becomes significantly smaller for the $\alpha+^{12}\text{C}$ cluster states, and the $\alpha$-cluster-type ground state correlation significantly enhances the monopole strength compared with the case of the $^{16}$O ground state being the pure SU(3) $(0, 0)$ wave function [12].

On the other hand, the reason why the $0^+_5$ state with the 4$\alpha$-gas-like character is excited by the monopole transition can be also understood from the property of the ground state of $^{16}$O. The doubly closed shell-model wave function is mathematically equivalent to the single 4$\alpha$ cluster structure are excited by the monopole operator in Eq. (5) can excite the relative motion among the 4$\alpha$ particles. In other words, the monopole operator has an ability to populate democratically 4$\alpha$ particles by $2\hbar \omega$ with respect to the center-of-mass coordinate of $^{16}$O. The resultant state, thus, has some amount of the overlap with the 4$\alpha$-gas-like state or $\alpha+^{12}\text{C}(0^+_3)$, i.e. $0^+_6$, with the 4$\alpha$-condensate-like structure [5]. It is noted that the mechanism of the 4$\alpha$-gas-like state being populated by the monopole transition is similar to that of the Hoyle state ($0^+_2$) with the $3\alpha$-gas-like structure being excited by the monopole transition, although the ground state of $^{12}\text{C}$ has a shell-model-like compact structure with the main configuration of SU(3) $(\lambda, \mu) = (0, 4)$ [12].

As for the $0^+_5$ state, its main configuration is $\alpha+^{12}\text{C}(1^-_1)$. According to the Bayman-Bohr theorem [17], the SU(3) $(0, 0)$ state of $^{16}$O has no component of the $\alpha+^{12}\text{C}(1^-_1)$ channel. However, the monopole strength to the $0^+_5$ state is as large as 3 fm$^2$ [13]. This is the reason that the $0^+_5$ state has small but important components of the $\alpha+^{12}\text{C}(0^+_1, 2^+_1, 0^+_2)$ configurations. Since these three configurations can be excited from the ground state of $^{16}$O by the monopole operator as discussed above, their respective contributions are coherently added to provide the relatively large monopole strength to the $0^+_5$ state. On the other hand, the situation of the
0+ state is similar to the case of the 0+ state. The 0+ state has also small but non negligible components of the α+12C(01+, 21+, 02+) configurations, which contribute to the monopole strength for the 0+ state.

4. Summary

We have investigated the monopole strength function in the low energy region up to \(E_x \simeq 16\) MeV within the framework of 4OCM. It was found that the fine structures at the low energy region up to \(E_x \simeq 16\) MeV in the experimental monopole strength function obtained by the \(^{16}\text{O}(\alpha, \alpha')\) experiment is rather satisfactorily reproduced within the 4OCM framework. On the contrary, mean-field calculations have encountered difficulties to reproduce the fine structures of the monopole strength function at the low energy region as well as the monopole matrix elements for the 0+ (\(E_x = 6.05\) MeV), 0+ (\(E_x = 12.05\) MeV), and 0+ (\(E_x = 14.01\) MeV) states obtained in the \(^{16}\text{O}(e, e')\) experiment. Our present results indicate that the isoscalar monopole excitation is useful to search for cluster states in light nuclei. Since the existence of the two different types of the monopole excitations and the duality of the ground state discussed in this paper are general in light nuclei [12, 13], it is highly hoped to study systematically the isoscalar monopole excitations in 4n and neutron-rich nuclei as well as hypernuclei.

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