Photometric and Kinematic Study of the Open Clusters SAI 44 and SAI 45

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Abstract

We carry out a detailed photometric and kinematic study of the poorly studied sparse open clusters SAI 44 and SAI 45 using ground-based $BVR_J$ data supplemented by archival data from Gaia eDR3 and Pan-STARRS. The stellar memberships are determined using a statistical method based on Gaia eDR3 kinematic data, and we found 204 members in SAI 44 while only 74 members are identified in SAI 45. The average distances to SAI 44 and SAI 45 are calculated to be 3670 ± 184 and 1668 ± 47 pc. The logarithmic age of the clusters are determined to be 8.82 ± 0.10 and 9.07 ± 0.10 yr for SAI 44 and SAI 45, respectively. The color–magnitude diagram of SAI 45 hosts an extended main-sequence turnoff (eMSTO). The apparent age spread is found to be similar to the apparent age spread predicted on the basis of the age spread and cluster age relation predicted by rotation models. This indicates that eMSTO is a stellar evolution rather than star formation phenomenon in SAI 45. We conclude that eMSTO in SAI 45 is mainly caused by the different rotation rates of stars as the SYCLIST synthetic population with different rotation rates was able to reproduce the observed eMSTO, and stars in the red part of the eMSTO were preferentially concentrated in the inner region, which again hints at different rotations being the reason for the extension in the upper MS. This finding supports the theory attributing the origin of eMSTO to the different rotations of eMSTO stars. The mass function slopes are obtained as $−2.24 ± 0.66$ and $−2.58 ± 3.20$ in the mass ranges $2.426–0.990 \ M_\odot$ and $2.167–1.202 \ M_\odot$, for SAI 44 and SAI 45, respectively. SAI 44 exhibits the signature of mass segregation while we found weak evidence of mass segregation in SAI 45 possibly due to tidal stripping. The dynamical relaxation times of these clusters indicate that both clusters are in a dynamically relaxed state. Using the AD-diagram method, the apex coordinates are found to be $(69.79 ± 0.11, −30.82 ± 0.15)$ for SAI 44 and $(−56.22 ± 0.13, −56.62 ± 0.13)$ for SAI 45. The average space velocity components of the clusters SAI 44 and SAI 45 are calculated in units of km s$^−1$ as $−15.14 ± 3.90, −19.43 ± 4.41, −20.85 ± 4.57$ and $28.13 ± 5.30, −9.78 ± 3.13, −19.59 ± 4.43$, respectively.

Unified Astronomy Thesaurus concepts: Open star clusters (1160); CCD photometry (208); Stellar mass functions (1612)

1. Introduction

Most stars, if not all, are born in clusters in our Galaxy (Lada & Lada 2003; Portegies Zwart et al. 2010). Therefore, the study of star clusters is important in order to understand star formation and stellar evolution. Open star clusters are relatively younger systems and witness recent star formation events. These clusters are mostly found in the spiral arms of the Milky Way galaxy; therefore, they are suitable tracers in the studies of Galactic disk formation and structures (Carraro et al. 1998; Chen et al. 2003; Joshi et al. 2016). Open clusters also provide an ideal testing environment for star formation and evolution theories as stars belonging to clusters are born from almost similar initial conditions and have similar age, distance, and chemical composition (Lada & Lada 2003).

The intermediate-age open clusters in the Milky Way are considered as clusters hosting single main-sequence (MS) i.e., coeval, populations. However, a few recent studies claim the presence of an extended main-sequence turnoff (eMSTO) caused by the spread in age due to the existence of multiple populations in Galactic open clusters (Gossage et al. 2019; Piatti & Bonatto 2019; Li et al. 2021). The age spread inferred from eMSTO was thought to be present due to extended star formation in clusters (Goudfrooij et al. 2014) but detection of extended star formation responsible for eMSTO has been elusive (Bastian et al. 2013). There have been other theories like variability, rotation, and binary systems explaining the origin of eMSTO (de Grijs 2017; Li et al. 2019; Sun et al. 2021). The most discussed reason behind the presence of eMSTO is the rotation of stars affecting the effective temperature of stars, thus causing the spread in the color of stars in the upper MS (Georgy et al. 2019). Fast-rotating stars have been found to be located in the red part of eMSTO while slow-rotating stars are located in the blue part of eMSTO (Sun et al. 2019). The origin of eMSTO in Galactic open clusters is currently intensely debated, and this makes these clusters even more interesting for investigators. The dynamical evolution of stars in clusters are important in the study of the initial mass distribution and dynamics of the Galactic disk. Mass functions and the dynamical evolution of clusters can be estimated once the physical parameters like age and distance modulus are accurately determined. The various aspects of the initial mass function (IMF) like spatial variation and universality with time are still open questions to answer (Bastian et al. 2010; Dib & Basu 2018). Sometimes, spatial variation in mass function slopes indicates mass segregation in open clusters (Sollima 2019). There are two theories used to explain the mass segregation phenomenon: primordial mass segregation and segregation caused by the dynamical evolution of clusters. According to primordial theory, massive stars are formed preferentially in the central part of the cluster (Dib et al. 2008).
The other theory suggests that mass segregation in a cluster is caused by two-body relaxation (Allison et al. 2009). It is still debated which one or if both mechanisms are responsible for the segregation of massive stars in clusters (Dib et al. 2018). Thus, a comprehensive photometric study of open clusters is useful in understanding the physics of stars from star formation events to the stellar and dynamical evolution of stars.

In this study, we present a photometric study of the unstudied open clusters SAI 44 and SAI 45. These two clusters are listed in the catalog compiled by a research group at Sternberg Astronomical Institute (SAI), Russia (Koposov et al. 2008; Glushkova et al. 2010). The catalog has the initial parameters of the newly discovered open clusters using 2MASS data (Skrutskie et al. 2006). These two clusters are not photometrically studied in a great detail, and our photometric study based on precise membership analysis using kinematic data of Gaia eDR3 (Gaia Collaboration et al. 2021; Lindegren et al. 2021b) will enrich the study of these clusters.

In this paper, the data analysis is given in Section 2. The spatial distribution of the stellar density and radii of these clusters is determined in Section 3. Membership determination using Gaia eDR3 kinematic data and the estimation of physical parameters are described in Section 4. The analysis of the eMSTO present in the upper MS of SAI 45 is given in Section 5. The MF and dynamic evolution are discussed in Section 6. The kinematic study of the clusters is given in Section 7. Section 8 is dedicated to a discussion and conclusion of the study.

2. Data

We used archived data from the Gaia eDR3, 2MASS, and Pan-STARRS surveys to complement our observed data in the $BVRcIc$ bands. The kinematic data from Gaia eDR3 have been particularly useful in the determination of membership, distance, and kinematic structure parameters. The 2MASS data in the near-IR $J$, $H$, and $K_s$ bands were useful in the calculation of total-to-selective extinction $R_v$. We converted the 2MASS $K_s$-band magnitude to $K$ magnitude using the relation given by Carpenter (2001).

2.1. Observed $BVRcIc$ Data

The observations of SAI 44 and SAI 45 in the $BVRcIc$ bands were captured using the 1.3 m Devasthal Fast Optical Telescope (DFOT) situated at a Himalayan site known as Devasthal in India. The observations of SAI 44 were taken on 2017 March 24 while SAI 45 was observed on 2017 March 25. The observations of SAI 44 were taken in the $VRcIc$ bands, and SAI 45 was observed in the $BVRcIc$ bands. The exposure time for the $B$, $V$, $R_c$, and $I_c$ bands were 300, 200, 100, and 60 s, respectively. The DFOT has a field of view of $18' \times 18'$ for the 2k $\times$ 2k CCD camera used for the observations. To standardize instrument magnitudes and find standard magnitudes, observations of Landolt’s standard field SA 98 were taken on both nights. Observations of astronomical twilight flats and bias images were also acquired on each night. We used the Image Reduction and Analysis Facility (IRAF) to clean our images. The instrumental magnitudes were estimated through PSF techniques using the DAOPHOT II package. We calibrated the instrumental magnitudes to find the standard magnitudes through the process given in Stetson (1992). The transformation equations used for the standardization are given in our previous paper (Maurya & Joshi 2020). We calculated the $VRcIc$ magnitudes from Gaia DR2 $G$, $G_{BP}$, and $G_{RP}$ magnitudes using the Carrasco conversion formula. We have shown a comparison in Figure 1 between the observed $VRcIc$ magnitudes and the calculated $VRcIc$ magnitudes. We found that the two magnitudes are in good agreement. We also estimated the completeness of observed data in the $V$ band using the method described in Maurya & Joshi (2020). The $V$-band data were found to be complete up to 19 mag for both clusters SAI 44 and SAI 45.

2.2. Gaia eDR3 Data

The Gaia eDR3 (Gaia Collaboration et al. 2021) data are used in this study for membership determination and distance estimation. The Gaia eDR3 provides celestial positions and magnitude in $G$ bands up to 21 mag for 1.8 billion sources. It provides parallaxes, proper motions, and colors ($G_{BP} - G_{RP}$) for 1.5 billion of those sources. In Gaia eDR3 data, the uncertainties in parallaxes are up to 0.02–0.03 mas for sources with $G < 15$ while there is an uncertainty of 0.07 mas for objects of 17 mag brightness in the $G$ band. The uncertainties are significantly higher for fainter stars as uncertainties go up to 0.5 mas for sources having 20 mag in the $G$ band while the uncertainty rises up to 1.3 mas for objects with $G = 21$ mag. The uncertainty in the Gaia eDR3 proper motion components has also improved to 0.02–0.03 mas yr$^{-1}$ for $G$ magnitudes less than 15 mag. The uncertainty becomes 0.07 yr$^{-1}$ for 17 mag in the $G$ band while it is 0.5 mas yr$^{-1}$ for objects having a $G$-band magnitude equal to 20 mag. The uncertainty becomes 1.4 mas yr$^{-1}$ for objects having 21 mag in the $G$ band.

2.3. Pan-STARRS Data

Pan-STARRS used the 1.8 m telescope in the first part of its survey (PS1) to image the northern sky in the $g$, $r$, $i$, $z$, and $y$ filters. The PS1 survey data have seesings of 1"–31, 1"–19, and 1"–11 in $g$, $r$, and $i$, respectively (Magnier et al. 2020). The survey includes faint stars up to $\sim 23.2$, 23.2, and 23.1 mag for stacked images in the $g$, $r$, and $i$ bands, respectively (Magnier et al. 2020). We calculated Johnson $B$-band magnitudes for
stars in SAI 44 from $g$ and $r$ magnitudes using the conversion formula given by Kostov & Bonev (2018). These $B$-band magnitudes for SAI 44 are used throughout the present study.

3. Spatial Stellar Distribution

The study of the spatial structure of an open cluster sheds light on the stellar distribution in the cluster according to the mass (or brightness) of stars (Joshi et al. 2014). We calculated radial stellar density distributions in selected clusters to obtain the center and radii of the clusters. We used stars detected in the $V$ band to calculate the stellar density. The cluster centers were estimated using the maximum density method by assuming the cluster center to be the point where stellar density is maximum. Our estimated cluster centers are (05:11:10.51; $+45:42:10.25$) and (05:16:29.45; $+45:35:35.89$) for SAI 44 and SAI 45, respectively. Glushkova et al. (2010) calculated the cluster centers using 2MASS data as (05:11:07.4; $+45:43:09$) and (05:16:35.0; $+45:34:56$) for SAI 44 and SAI 45, respectively. The R.A. of the cluster centers is in agreement with those given by Glushkova et al. (2010), while the decl. of the cluster centers are slightly different from the values reported by Glushkova et al. (2010).

The cluster radii were calculated by fitting the radial density profile (RDP) provided by King (1962) to the radial stellar distribution of stars in the observed regions of the selected clusters. We calculated the radial distributions in these clusters using concentric annular regions of $\sim$0′5 width. The RDP profile given by King (1962) is given below:

$$
\rho(r) = \rho_b + \frac{\rho_0}{1 + \left(\frac{r}{r_c}\right)^2},
$$

where $\rho_0$ and $\rho_b$ are the maximum stellar density and background stellar density, respectively. $r_c$ is the core radius defined by the distance from the center to where the stellar density falls to half of its maximum value. The plots of the RDPs with the fitted King (1962) radial density model are shown in Figure 2 for SAI 44 and SAI 45. We found the core radii to be $2/5 \pm 0/6$ and $2/1 \pm 0/8$ for SAI 44 and SAI 45, respectively. We considered the cluster boundary to be the points where the radial density is $1\rho_b$ above the background density $\rho_b$. The radius of the cluster, $r_{\text{cluster}}$, is calculated using the following formula:

$$
r_{\text{cluster}} = r_c \sqrt{\frac{\rho_0}{\rho_b} - 1}.
$$

We obtained the cluster radii to be $11/4 \pm 0/5$, and $8/9 \pm 0/5$ for the clusters SAI 44 and SAI 45, respectively. The estimated radii are approximate in nature due to the irregular shape of the clusters, as also visible in Figure 3 showing the contour maps of the spatial density distribution for these clusters. The estimate of the cluster radius based on the boundary condition depends on the chosen cutoff; therefore, the resulting radius is the lower limit of the possible radius, and some stars belonging to the cluster may be found beyond the calculated cluster radius. The stellar density contrast of these clusters against the background population can be estimated through the density contrast parameter, $\delta_c$, which is mathematically defined as follows:

$$
\delta_c = 1 + \frac{\rho_0}{\rho_b}.
$$

We calculated $\delta_c$ to be 2.0 and 1.7 for SAI 44 and SAI 45, respectively. These clusters are sparse clusters relative to their background population density as the $\delta_c$ for the two studied clusters are below the range ($7 \leq \delta_c \leq 23$) suggested by Bonatto & Bica (2009) for the compact clusters. As the ages of open clusters increase, stellar dynamics within the clusters influences the central part of the cluster to become circular while the overall cluster extent becomes larger and the stellar density of the clusters becomes sparser (Chen et al. 2004). The clusters SAI 44 and SAI 45 having ages near $\sim$1 Gyr are expected to be sparser, which is also confirmed from the $\delta_c$ obtained for these clusters. All of these estimated structural parameters are included in Table 1 for both clusters.
4. Member Stars and Cluster Parameters

4.1. Membership

Membership assessment of stars in a cluster region is of central importance in a detailed analysis of the cluster. The stars belonging to a cluster are gravitationally bound together; therefore, they are expected to have proper motions tightly distributed around the mean proper motion value. Thus, the distribution of stars in the proper motion plane has been very useful in membership determination after the advent of precise astrometric data of Gaia DR2 and eDR3 (Cantat-Gaudin et al. 2018; Monteiro et al. 2020). We utilize this property of the distribution of stars in a proper motion plane in vector-point diagrams (VPDs) as an initial step to separate cluster stars from field stars. The VPDs of these clusters are shown in Figure 4. It is clear from Figure 4 that cluster stars are conspicuously separated from field stars in the VPDs of the selected open clusters. The stars lying in the red circle in the VPDs are probable cluster members. We derived the center of the red circle using the maximum density method for the stars in the proper motion plane. We also calculated the radial distribution of the number density of probable member stars in the proper motion plane, which is shown in Figure 4. The fitted red curves on the radial density distributions of the probable members in the proper motion plane are similar to the curve fitted on the RDP in the spatial plane in Section 3. The radii of the red circles in the VPDs of the clusters are also estimated from the radial density distribution in the proper motion plane. The radii were taken as the radial distance up to the point where the cluster stellar density starts merging with the field star density. We found the centers of the circles in VPDs to be \((-0.21, -1.62)\) and \((-1.68, -1.33)\) in units of mas yr\(^{-1}\) for SAI 44 and SAI 45, respectively. There are 420 and 101 probable member stars encircled in the VPDs of clusters SAI 44 and SAI 45, respectively.

We calculated the membership probabilities for the stars present in the observed regions of the selected clusters through a statistical method using the proper motions (PMs) of stars as given in previous studies (Sanders 1971; Joshi et al. 2020). The equations used in the membership calculation using this method are given in our previous study (Maurya & Joshi 2020).

4. Member Stars and Cluster Parameters

Table 1

| Cluster | Central Coordinates | \(r_{\text{cluster}}\) | \(\delta\) |
|---------|---------------------|----------------------|--------|
| SAI 44  | R.A. (J2000) = 05:11:10.51, Decl. (J2000) = +45:42:10.25 | 11/4 ± 0/5 | 2.0 |
| SAI 45  | R.A. (J2000) = 05:16:29.45, Decl. (J2000) = +45:35:35.89 | 8/9 ± 0/5 | 1.7 |

Figure 4. The plots of the vector-point diagram (left panel), stellar density distribution as a function of radial distance in the proper motion plane (middle panel), and color–magnitude diagrams (right panel) for SAI 44 and SAI 45. In the vector-point diagrams, the probable member stars are encircled in red. The red circular points and blue “+” marks denote the member stars and field stars, respectively, in the color–magnitude diagrams.
be different as the method of membership probability calculation is based on the distribution of stars in the proper motion plane (Sanders 1971). The cluster stars are known to have very similar proper motions; therefore, they would be concentrated in only a few bins in the histogram of the probability distribution. We plotted histograms of the probability distribution as shown in Figure 5. The number of stars started showing a clear upward trend from 70% and 90% for SAI 44 and SAI 45, respectively. Therefore, we took the probability cutoff for SAI 44 and SAI 45 to be 70% and 90%, respectively. However, there is still a possibility that stars satisfying the probability cutoff may belong to a field population as a few field stars may have proper motion similar to the cluster stars. The most reliable way to remove these field stars from the cluster star sample is the application of a parallax cutoff because stars belonging to the cluster are expected to have a very tight parallax distribution (Maurya et al. 2020). We took the parallax cutoff to be within 1 σ of the mean parallax value (\( \mu_p \)) calculated from probable members for SAI 44, while within 3 σ of the mean parallax was used as the selection criterion for SAI 45. The cluster SAI 44 has a smaller \( \sigma_p \) and larger \( \mu_p \) than SAI 45 so we applied a narrow selection criterion in parallax for SAI 44 compared to SAI 45. We identified 204 and 74 member stars in SAI 44 and SAI 45, respectively, after applying both the probability cutoff and the parallax cutoff.

In order to evaluate the cluster parameters with a high confidence level, all further analyses in this study have been carried out considering only these member stars unless explicitly mentioned otherwise. The average PM values (\( \mu_{p,*,*} \), \( \mu_0 \)) are calculated to be \((-0.20 \pm 0.25, -1.63 \pm 0.22)\) and \((-1.68 \pm 0.08, -1.33 \pm 0.08)\) in mas yr\(^{-1}\) units for SAI 44 and SAI 45, respectively. Cantat-Gaudin et al. (2020) found 80 and 27 member stars having magnitude only up to \( G = 17 \) mag in SAI 44 and SAI 45, respectively. The mean PM values (\( \mu_{p,*,*} \), \( \mu_0 \)) of \((-0.265; -1.628)\) and \((-1.609; -1.288)\) for SAI 44 and SAI 45, respectively, calculated by Cantat-Gaudin et al. (2020) are in good agreement with our estimated values of mean PMs.

### 4.2. Physical Parameters

The accurate determination of the physical parameters of the clusters plays an important role in the study of the stellar and dynamical evolution in clusters as well as the evolution and structure of the Galaxy disk (Frinchaboy & Majewski 2008). The determination of the distance of a cluster is important to know its location, which is in turn related to properties like cluster morphology and reddening (Green et al. 2019; Hu et al. 2021). The distance calculated through parallax is independent of the intrinsic properties of the objects unlike in the case of isochrone fitting. The accuracy of kinematic data including parallax has improved very significantly in Gaia eDR3 in comparison to Gaia DR2 (Lindegren et al. 2021b) so we used the Gaia eDR3 parallax to determine the distance to the clusters SAI 44 and SAI 45. The distances to these clusters were calculated using the method suggested by Bailer-Jones et al. (2018), which uses a Bayesian approach using a prior of exponentially decreasing space density. We applied the systematic offset of \(-0.017\) mas reported by Lindegren et al. (2021a) to find offset-corrected mean parallax of \(0.272\) and \(0.600\) mas for SAI 44, and SAI 45, respectively. The distances to the clusters SAI 44 and SAI 45 are found to be \(3670 \pm 184\) and \(1668 \pm 47\) pc, respectively. Cantat-Gaudin et al. (2020) reported the mean parallax and distance as \(0.252\) mas and \(3575\) pc, respectively, for SAI 44. The mean parallax and distance of the cluster SAI 45 are given to be \(0.561\) mas and \(1694\) pc, respectively, in Cantat-Gaudin et al. (2018).

Although the value of total to selective extinction \( R_v \) is taken to be \(3.1\) for diffuse interstellar medium (Cardelli et al. 1989), the value of \( R_v \) has been found to vary in the Galaxy for different lines of sight (Valencic et al. 2004). The total to selective extinction is very helpful in the photometric study of clusters as it allows the calculation of extinction \( A_v \) directly from the reddening \( E(B-V) \), which is an easily measured parameter of the clusters. We obtained \( R_v \) to be \(3.1\) and \(2.8\) for SAI 44 and SAI 45, respectively, from the slopes of the color–color diagrams using optical and near-IR \( J, H, K \) and \( B \)-band data. The method to calculate \( R_v \) is described in Maurya & Joshi (2020). The \( R_v \) values have been used to infer the size of dust grains (Schlafly et al. 2016). The lower value of the total-to-selective extinction for SAI 45 may indicate a dust grain size for the cluster region smaller than normal for the diffuse interstellar medium. Schlafly et al. (2017) found a correlation between the \( R_v \) values and distances, which states nearby dusts are associated with lower \( R_v \) values than distant dust. Although SAI 44 and SAI 45 are situated in approximately the same part of the sky, the lower \( R_v \) for SAI 45 is consistent with Schlafly et al. (2017) as SAI 45 is situated at a significantly smaller distance compared to SAI 44. We calculated the reddening \( E(B-V) \) to be \(0.34^{+0.01}_{-0.07}\) and \(0.34^{+0.03}_{-0.02}\) mag for SAI 44 and SAI 45, respectively, utilizing the 3D reddening map provided by Green et al. (2019), which gives a very good resolution to the north of \(-30^\circ\) decl. using the Gaia parallax and multiband photometric data from Pan-STARRS and 2MASS. The \( E(B-V) \) values were obtained using R.A., decl., distance, and \( R_v \) values found in this study through the extinction ratio relations given by Wang & Chen (2019). The extinction \( A_v \) for these clusters was calculated using \( A_v = R_v \times E(B-V) \). We found the values to be \(1.05\) and \(0.95\) in the direction of the line of sight of the clusters SAI 44 and SAI 45, respectively. The \( A_v \) value we calculated is larger than the \( A_v \) value of \(0.86\) for SAI 44 given by Cantat-Gaudin et al. (2020). However, our estimated \( A_v \) value for SAI 44 is in good agreement with \( A_v = 1.065 \) obtained by Bossini et al. (2019) for SAI 44.

We used the \( A_v \) values calculated in the present study to find the apparent distance modulus and extinction-corrected isochrone of Marigo et al. (2017). We used \( V(B-V) \) and \( G/(G_{BP}-G_{RP}) \) color–magnitude diagrams (CMDs) of the

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Figure 5. Histograms of the probability distribution in SAI 44 and SAI 45.
clusters SAI 44 and SAI 45 to estimate the ages of the clusters. The plots of the CMDs for SAI 44 and SAI 45 are shown in Figure 6. We fitted the Marigo et al. (2017) extinction-corrected isochrones of the solar metallicity on the CMDs of these clusters for our estimated extinction $A_v$. The best fits were achieved for isochrones having logarithmic-scaled ages of $8.82 \pm 0.10$ and $9.07 \pm 0.10$ yr for distance moduli of $12.82 \pm 0.11$ and $11.11 \pm 0.06$ mag for the clusters SAI 44 and SAI 45, respectively. However, the isochrone fitted on the $V/(B-V)$ CMD seems to be slightly redward shifted, which might have been introduced during the conversion of the $g$- and $r$-band magnitudes from Pan-STARRS to the $B$-band magnitude. The log(Age) values reported by Glushkova et al. (2010) are 8.95 and 9.20 yr for SAI 44 and SAI 45, respectively. The ages we estimated are slightly younger than the same found by Glushkova et al. (2010) for the clusters SAI 44 and SAI 45.

4.3. Half-mass Radius, Tidal Radius, and Cluster Structure

The half-mass radius is defined as the radial distance containing half of the total mass belonging to a cluster. The masses of stars for the $R_h$ calculation were found using the Marigo et al. (2017) isochrones. We found the half-mass to be $132.935$ and $49.530 \, M_\odot$ while corresponding $R_h$ values were obtained to be $4.1 \, (4.4 \, pc)$ and $4.5 \, (2.2 \, pc)$ for SAI 44 and SAI 45, respectively. The $R_h$ value combined with the tidal radius, $R_t$, is very useful in the study of dynamics like the extent of cluster disruption due to tidal forces (Angelo et al. 2021). Open cluster structure and dynamical evolution are influenced by tidal interactions (Chumak et al. 2010). Open clusters lose stars continuously due to tidal interactions. The tidal radius is calculated using the relation given by Kim et al. (2000) as follows:

$$R_t = \left(\frac{M_C}{2M_G}\right)^{1/3} \times R_{gc}.$$  

In the above equation, $M_C$ is the total mass of the cluster and $R_{gc}$ is the distance to the cluster from the Galactic center. $R_{gc}$ is calculated in Section 7 as $11.742$ and $9.802$ kpc for SAI 44 and SAI 45, respectively. $M_G$ denotes the mass of the Galaxy contained within $R_{gc}$. $R_t$ is the radial distance from the center of a cluster where the gravitational field of the cluster is equal to the tidal field of the Galaxy. The value of $M_G$ is calculated using the Genzel & Townes (1987) relation given as

$$M_G = 2 \times 10^8 \left(\frac{R_{gc}}{30 \, pc}\right)^{1.2}.$$  

Using the above two equations, we determined the tidal radii of SAI 44 and SAI 45 to be $10.1$ and $6.5$ pc, respectively. We obtained $R_h/R_t$ ratios to be $0.4$ and $0.3$ for SAI 44 and SAI 45, respectively. SAI 45 has a smaller value of the $R_h/R_t$ ratio, implying that it has a more compact structure and survival possibility (Angelo et al. 2021). The clusters located at larger Galactocentric distances are subject to weaker tidal field, and hence these clusters show a larger $R_h/R_t$ ratio and have less compact structures. Because SAI 44 is at a large $R_{gc}$ value, it is expected to be less compact and have a larger core radius compared to SAI 45, which is found to be true as it has a larger core radius (see Section 3). It has been found that the core radius and dynamical time ratio of a cluster are negatively
correlated (Angelo et al. 2021). The dynamical time ratio is calculated as \( \tau_{\text{dyn}} = \frac{\text{age}}{\tau_{\text{cross}}} \), where the crossing time, \( \tau_{\text{cross}} \), is the timescale required for a star belonging to the cluster to complete an orbit across the system. \( \tau_{\text{cross}} \) was calculated using the formula \( \tau_{\text{cross}} = \frac{R_h}{\sigma_v} \), where \( R_h \) is the half-mass radius and \( \sigma_v \) is the velocity dispersion. We obtained the velocity dispersion from the space velocities calculated in Section 7 to be 8.47 and 1.39 km s\(^{-1}\) for SAI 44 and SAI 45, respectively. The values of \( \tau_{\text{cross}} \) were found to be 0.52 and 1.58 Myr for SAI 44 and SAI 45, respectively. \( \tau_{\text{dyn}} \) values were found to be 1270 and 743 for SAI 44 and SAI 45, respectively. The obtained values of the core radius, 2.7 and 1.0 pc for SAI 44 and SAI 45, are comparable to the values shown in the core radius–log (\( \tau_{\text{dyn}} \)) correlation plot of Angelo et al. (2021). Therefore, we can conclude that the structure and dynamics of a cluster is influenced by the tidal field interactions.

5. Presence of eMSTO in the CMD

The eMSTO refers to a wider upper MS than is usually expected from a single stellar population in a cluster. The cluster SAI 45 shows a broad MS at the brighter end in both \( V/(B - V) \) and \( G/(G_H - G_R) \) CMDs, which suggests the presence of eMSTO in this cluster. The presence of eMSTO in Galactic open clusters is a relatively recent phenomenon and has been reported in recent studies of open clusters (Cordoni et al. 2018; Maurya & Joshi 2020; Li et al. 2021). We fitted the Marigo et al. (2017) isochrone of various ages in Figure 7 but using the same metallicity, \( E(B - V) \), and distance modulus as used in Figure 6. The isochrones of different ages are almost linear in the lower part of the MS while they are extended in the upper part of the MS. The rectangular region illustrated in Figure 7(a) is the extended MS region used for the calculation of the apparent age spread through isochrone fitting. We have drawn a histogram of the apparent age spread for the stars within the rectangular region of the CMD. The plot of the histogram is shown in Figure 7(b) where a bimodal distribution is clearly evident, suggesting an age spread as has also been reported in some previous studies (e.g., Goudfrooij et al. 2017; Piatti & Bonatto 2019). The mean logarithmic ages corresponding to the peaks are 9.15 and 9.28 yr for small and large peaks, respectively. The apparent age separation between these peaks is 493 Myr. The photometric uncertainty in the magnitudes can also introduce a very small apparent age spread as bright stars belonging to the eMSTO have a relatively smaller uncertainty in magnitudes than the fainter stars in the lower MS. We calculated the apparent age spread caused by the photometric uncertainty in the \( (B - V) \) color through the ratio of the spread in age to the spread in \( (B - V) \) color for eMSTO stars. The minimum and maximum apparent logarithmic ages of stars in the eMSTO region are 9.00 and 9.35 yr, respectively. The total spread in \( (B - V) \) color and log(Age) is 0.237 mag and 0.35 yr, respectively. The average photometric uncertainty in the \( (B - V) \) values is found to be 0.006 mag, which can produce an apparent age spread of only \( \sim 54 \) Myr. The spread in the upper MS can also be caused by the spread in the metallicity of stars. We have fitted isochrones with different Z values while keeping other parameters constant as shown in Figure 8. The leftmost isochrone corresponds to \( Z = 0.01 \) while the rightmost isochrone has \( Z = 0.055 \). We obtained the [Fe/H] values from Anders et al. (2019) and converted them into the Z abundance value using the relation log(Z) = 0.977 \times [Fe/H] - 1.699 given by Bertelli et al. (1994). The histogram of the metallicity of stars in the eMSTO region with a Gaussian fit is given in Figure 8. We obtained the mean and standard deviation of the metallicity Z of eMSTO stars to be 0.0152 and 0.001, respectively. The corresponding FWHM was

![Figure 7](image-url)  
5. Presence of eMSTO in the CMD

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![Figure 8](image-url)
calculated to be 0.002. The apparent age spread due to the spread in the metallicity of eMSTO stars was calculated from the ratio of the spread in $Z$ required for the production of eMSTO to the FWHM of the actual distribution of the metallicity $Z$ of eMSTO stars. We obtained the apparent spread possible due to metallicity spread to be $\sim$43 Myr. Thus, this large apparent spread of 493 Myr has been caused by reasons other than photometric uncertainty in magnitudes and the spread in the metallicity of eMSTO stars. The existence of eMSTO in the CMD of open clusters primarily seems to suggest prolonged star formation in the clusters. However, Niederhofer et al. (2015) found that the width of eMSTO increases with the age of the clusters, which suggests eMSTO is related to stellar evolution instead of stellar formation. After subtracting the apparent age spread due to photometric uncertainty and metallicity spread, the apparent age spread possibly caused by rotation would be 396 Myr. The apparent age spread of 396 Myr is comparable to the previous 300–700 Myr apparent age spread prediction due to rotation for a cluster with age similar to SAI 45 (Niederhofer et al. 2015; Goudrooij et al. 2017). To further investigate the reason behind the eMSTO, we generated a synthetic cluster population using the SYCLIST7 interface (Ekström et al. 2012; Georgy et al. 2014). The synthetic data are overplotted on the observed CMD in Figure 9. In Figure 9(a) the nonrotational synthetic CMD is overplotted over the observed CMD, and it fairly reproduces the spread in the lower part of the MS possibly caused by binaries. However, the nonrotational synthetic CMD is not able to reproduce the eMSTO. We then used the SYCLIST interface to retrieve only the MS turnoff with a large number of rotations and overplotted this synthetic CMD over the observed CMD as shown in Figure 9(b). The eMSTO is nicely reproduced by the synthetic CMD with rotating populations. The fast-rotating stars are preferentially located in the red part of the CMD while slow-rotating stars are in the blue part. The gravity darkening in the fast-rotating stars causes an apparently lower effective temperature (Li et al. 2019) while rotational mixing in the interior of stars causes higher luminosity and cooler temperature (Bastian & de Mink 2009). This makes fast-rotating stars appear redder on the CMD. There are direct spectroscopic studies that also found that, generally, fast-rotating stars are redder than slow-rotating or nonrotating stars (Dupree et al. 2017; Marino et al. 2018; Sun et al. 2021). We also plotted histograms using initial rotation rates $\Omega/\Omega_c$ of the synthetic population as shown in Figure 9(c). The critical rotation rate $\Omega_c$ is the rotation rate at which centrifugal force becomes equal to the surface gravity of stars. The histogram of initial rotation rates exhibits a bimodal distribution as obtained from the histogram of the apparent age spread shown in Figure 7(b). This suggests that the bimodal apparent age distribution of stars in the cluster is mainly caused by the different rotation rates of stars. The bimodal distribution of rotation rates is suggested to be due to the slowing down of fast-rotating stars caused by tidal braking and binary interaction (D’Antona et al. 2017; Sun et al. 2021). The synthetic data were generated for solar metallicity $Z = 0.014$, log (Age) = 9.1 yr, rotation distribution according to Huang et al. (2010), and random rotation axis distribution. The limb darkening (Claret 2000) and gravity darkening (Espinosa Lara & Rieutord 2011) were also accounted for in the synthetic cluster population. The binary fraction was chosen to be 0.3 for the synthetic cluster. As stellar rotation mimics the extended region of the MS turnoff, eMSTO seems to be mainly a stellar evolution phenomenon in SAI 45. Fast-rotating stars are expected to be more centrally concentrated (Bastian & de Mink 2009). We also noticed that the stars in the red part of the CMD in the eMSTO region i.e., fast-rotating stars, are mostly inner region stars while the blue part of the CMD, i.e., slow-rotating stars, are predominantly outer region stars (refer to Figure 12). This again indicates that the spread of the eMSTO region is caused mainly by the different rotation rates of stars in SAI 45. We conclude that the stellar rotations are the main reason behind the presence of eMSTO in the cluster SAI 45.

6. Dynamical Evolution

The number distribution of member stars of an open cluster according to the magnitude of the stars is called the luminosity...
We calculated the LF for clusters SAI 44 and SAI 45 in the $V$ band down to 19 mag where our photometry is complete; it is given in Table 2. The number distribution of stellar masses during star formation events is known as the IMF. The IMF determines the number of massive stars that were born in a star formation process and determines the fate of the star-forming region. The mass function calculations for open clusters are retrieved from many complex situations like stellar birth rates and life-time correction as stars associated with open clusters are coeval stars. The mass function ($MF$) of a cluster is defined as the number distribution of stars with stellar masses. We calculated the present-day MF as \( N(\log m) \propto G m \). The $G$ is the MF slope calculated as \( G = \frac{d \log N(\log m)}{d \log m} \).

In the above relation, \( N(\log m) \) is the number density on a logarithmic mass scale. The MF obtained for two clusters is given in Table 2. The masses of individual member stars were found by fitting Marigo et al. (2017) isochrones to the observed CMDs of SAI 44 and SAI 45 for their estimated age, apparent distance modulus, and reddening. The slopes were found by least-squares fitting and plots for the MF slopes are shown in Figure 10. The MF slopes of SAI 44 were calculated for the entire region, inner region, and outer regions. The inner region was taken as a circular region of radius 4.3 around the center of the circle. The observed region beyond the inner region was taken as the outer region. The radius of the inner region circle was taken such that the number of stars should be approximately the same in the inner and outer regions. We found different values of the MF slopes in the three regions, hinting at a variation of MF slopes as a function of radial distance from the center. The slopes were found to be $-2.24 \pm 0.66$, $-1.75 \pm 0.72$, and $-2.83 \pm 0.61$ in the entire, inner, and outer regions, respectively, for the same mass range of $2.375 - 0.978 M_\odot$. Here, we excluded one point that corresponds to only one star in the mass range $2.459 - 2.426 M_\odot$ in the calculation of the MF slopes of SAI 44 as shown in Figure 10. The MF slope in the inner region is found to be flatter than the MF slope in the outer region for SAI 44, which suggests the presence of mass segregation (e.g., Maurya & Joshi 2020).

| V Range (mag) | SAI 44 | SAI 45 |
|--------------|-------|-------|
| Mass Range ($M_\odot$) | $\bar{m}$ ($M_\odot$) | $N$ | Mass Range ($M_\odot$) | $\bar{m}$ ($M_\odot$) | $N$ |
| 12–13 | ... | ... | 2.167–1.998 | 2.107 | 2 |
| 13–14 | 2.459–2.426 | 2.452 | 1 | 1.998–1.711 | 1.853 | 10 |
| 14–15 | 2.426–2.086 | 2.278 | 7 | 1.711–1.432 | 1.576 | 19 |
| 15–16 | 2.086–1.716 | 1.910 | 29 | 1.432–1.202 | 1.298 | 16 |
| 16–17 | 1.716–1.415 | 1.568 | 44 | 1.202–1.015 | 1.125 | 12 |
| 17–18 | 1.415–1.149 | 1.300 | 49 | 1.015–0.864 | 0.955 | 11 |
| 18–19 | 1.149–0.990 | 1.080 | 45 | 0.864–0.748 | 0.818 | 3 |

**Figure 10.** Plots of mass function slopes for SAI 44. Plots of entire regions, inner regions, and outer regions are given in (a), (b), and (c), respectively. The open circles in plots denote points excluded in the mass function slope determination of SAI 44.

**Figure 11.** MF for the entire observed region of the cluster SAI 45.

The Astronomical Journal, 162:64 (14pp), 2021 August Maurya et al.
We found a two-step power law for the MF slopes in the case of SAI 45 as can be noticed in Figure 10. The mass at which the turnover of the MF slopes happens is called the turnover mass, $m_t$. The stars having mass above $m_t$ are called high-mass stars while the remaining stars are called low-mass stars. We found MF slopes for the entire observed region of SAI 45 to be $-2.58 \pm 3.20$ in the mass range $2.167-1.202 M_\odot$. The MF slope for the high-mass stars of SAI 45 is steeper than the Salpeter (1955) value of $-1.35$ for the mass range $0.4 M_\odot$–$10.0 M_\odot$. The steeper MF slope indicates the presence of relatively fewer massive stars. The massive stars evolve faster than the low-mass stars so the present-day MF corresponds to a depletion of massive stars compared to the IMF. The extent of the massive-star depletion depends on the initial condition of star formation (Sollima 2019). The turnover mass, $m_t$, for SAI 45 is found to be $1.26 M_\odot$. The $m_t$ value for SAI 45 is higher than the value of $\sim 1.06 M_\odot$ found for NGC 381 in our previous study (Maurya & Joshi 2020). The two-step power law of the MF slope having a turnover around $1 M_\odot$ seems to be commonly present in open clusters as reported in several past studies (e.g., Khalaj & Baumgardt 2013; Sollima 2019). We could not calculate the MF slopes separately for the inner and outer regions of SAI 45 as it has very few member stars (74) in the entire observed region; further division into inner and outer regions could have led to statistically insignificant results.

We noticed a steeper MF slope in the outer region of SAI 44 indicating mass segregation, which can be attributed to the escape of low-mass stars from the cluster aside from the concentration of massive stars in the central region of the cluster caused by the equipartition of energy. The method based on the cumulative distribution of stars with radial distance for various mass bins may give a misleading result due to dependence on the size of mass bins and cumulative radii. We therefore used the method given by Allison et al. (2009) based on the mass segregation ratio (MSR) to analyze the presence of mass segregation in the clusters. This method uses a minimum sampling tree (MST) of the data points to determine the mean edge length $\gamma$. The MST of a sample of points is defined as the shortest path connecting all points without any closed loops (Prim 1957). We first calculated the mean edge length for the $n$ most massive stars $\gamma_{nmm}$ followed by the mean edge length of $n$ random stars of the whole sample $\gamma_{nrand}$. We repeated the same calculation 500 times to get $\langle \gamma_{nrand} \rangle$. Using the relation given by Olczak et al. (2011), we calculated the MSR $\Gamma_{MSR}$ as follows:

$$\Gamma_{MSR} = \frac{\langle \gamma_{nrand} \rangle}{\gamma_{nmm}}.$$ 

The associated standard deviation $\Delta \Gamma_{MSR}$ with the MSR $\Gamma_{MSR}$ was calculated as

$$\Delta \Gamma_{MSR} = \Delta \gamma_{nrand}.$$ 

The method basically assumes that any mass segregation in a cluster will be marked by a spatial distribution of massive stars relatively closer than that of low-mass stars. $\Gamma_{MSR} \sim 1$ will mean that the spatial distribution of the most massive stars would be similar to the spatial distribution of low-mass stars i.e., no mass segregation in the cluster. $\Gamma_{MSR} > 1$ would imply that the most massive stars are relatively closer than the rest of the stars, which means the presence of mass segregation. $\Gamma_{MSR} < 1$ indicates reverse mass segregation (Dib et al. 2018). We found MSR $\Gamma_{MSR}$ values of $1.22 \pm 0.17$ and $1.07 \pm 0.23$ for SAI 44 and SAI 45, respectively. $\Gamma_{MSR} = 1.22 \pm 0.17$ indicates the presence of mass segregation in cluster SAI 44. However, $\Gamma_{MSR} = 1.07 \pm 0.23$ for SAI 45 suggests the presence of weak or no mass segregation in the cluster. Being an old-age cluster, SAI 45 is expected to have evidence of strong mass segregation. The dynamical time of these clusters is useful in understanding the reason behind the discrepancy in mass segregation levels in these clusters. SAI 44 was found to be dynamically older than SAI 45 as discussed in Section 4.3 and hence may have become more mass segregated (Rui et al. 2019). The tidal stripping of outer low-mass stars may be another reason behind the absence of mass segregation in SAI 45 (Rui et al. 2019). As there are only 74 member stars in SAI 45, our result regarding mass segregation is statistically preliminary in nature. We show $G/(G_{BP}-G_{RP})$ plots in Figure 12 for a visual inspection of mass segregation in the clusters. The radius of the inner regions was taken as 4.3 and 4.5 for SAI 44 and SAI 45, respectively. The upper MS is dominated by stars belonging to the inner region while the lower MS is dominated by outer region stars in the case of SAI 44. However, we could not find any conspicuous pre-eminence of stars belonging to either region in the whole MS of SAI 45.

The dynamical relaxation time is a good indicator to determine whether mass segregation is primordial or it happens because of dynamical relaxation (Angelo et al. 2019). The dynamical relaxation time, $T_E$, is physically defined as the time interval in which the velocity distribution of stars in a cluster approaches the velocity distribution of Maxwellian equilibrium through the exchange of energy among member stars. The dynamical relaxation time was calculated using the Spitzer & Hart (1971) formula as described in Maurya et al. (2020). We found $T_E$ values of 48 and 43 Myr for SAI 44 and SAI 45, respectively. We obtained $T_E$ values many times less compared to the observed ages of the clusters, which point toward a dynamically relaxed state of SAI 44 and SAI 45. Thus, mass segregation in SAI 44 is possibly caused by dynamical relaxation.
7. Kinematical Structure

We studied the kinematics of the clusters using a computational algorithm described by Elsanhoury et al. (2018). The line-of-sight velocities, \( V_r \), are given by Zhong et al. (2020) to be \( 2.20 \pm 5.73 \) and \( -33.28 \pm 35.30 \) km s\(^{-1}\) for SAI 44 and SAI 45, respectively. The value of \( V_r \) for SAI 44 is found to be \( 7.33 \pm 0.05 \) km s\(^{-1}\) by Carrera et al. (2019), while Soubiran et al. (2018) reported \( V_r \) values of \( 11.18 \pm 3.21 \) and \( -31.02 \pm 0.35 \) km s\(^{-1}\) for SAI 44 and SAI 45, respectively. We used \( V_r \) values of \( 7.33 \pm 0.05 \) and \( -31.02 \pm 0.35 \) km s\(^{-1}\) in our present analysis from the above-mentioned studies. The estimation of the apex position and spatial velocities of the clusters are described below.

7.1. Apex of the Clusters

The apex position represents the motion of the clusters on the celestial sphere. As open clusters are a gravitationally bound system of stars, member stars of a cluster therefore move with a common velocity vector, and the parallel motions of the common velocity vector on the celestial sphere will point toward a coherent point known as the apex of the cluster. There are two methods to calculate the apex position. The classical method is known as the classical convergent point (CP) method while the other is known as the AD-diagram method. These methods are based on the assumption that a cluster is a nonrotating body without any expansion or contraction.

(i) The CP method: This method assumes that all member stars of a cluster possess the same space velocities. The velocity components along the \( x \), \( y \), and \( z \) axes \( (V_x, V_y, V_z) \) for a cluster having coordinates \( (\alpha, \delta) \) with proper motions \( (\mu_x, \mu_y) \) and radial velocity \( V_r \) at a distance of \( d \) (in parsec) in the heliocentric coordinate system are given by Smart (1968) as described in Postnikova et al. (2020). We found the coordinates of the apex \( (A_{CP}, D_{CP}) \) to be \( (77.79 \pm 0.11, -45.71 \pm 0.15) \) for SAI 44 and \( (-78.55 \pm 0.10, -45.26 \pm 0.10) \) for SAI 45.

(ii) The AD-diagram method: This method uses the radial velocity and parallax of the stars. We can identify the group of stars having common space motions using this method. This method is described in good detail by Chupina et al. (2006). In the AD diagram, the positions of the apexes of the individual stars are calculated in equatorial coordinates. The \( (A, D) \) values of the individual member stars indicate the positions of these stars through space velocity vectors. In this method, the intersection point \( (A_0, D_0) \), also called the apex in equatorial coordinates, can be given as

\[
A_0 = \tan^{-1}\left(\frac{V_x}{V_r}\right)
\]

\[
D_0 = \tan^{-1}\left(\frac{V_x}{\sqrt{V_y^2 + V_z^2}}\right).
\]

Coordinates of the apex \( (A_0, D_0) \) through the AD-diagram method were calculated as \( (69^\circ 79 \pm 0^\circ 11, -30^\circ 82 \pm 0^\circ 15) \) and \( (-56^\circ 22 \pm 0^\circ 13, -56^\circ 62 \pm 0^\circ 13) \) for SAI 44 and SAI 45, respectively. The AD diagrams for the two clusters are shown in Figure 13.

7.2. Space Velocity of the Clusters

We calculated the space velocity components of the clusters SAI 44 and SAI 45 using space velocity components \( (V_x, V_y, V_z) \) along the \( x \), \( y \), \( z \) axes of the heliocentric coordinate system. We used Liu et al. (2011) relations to calculate the space velocity components \( (U, V, W) \). The Liu et al. (2011) relations are described as follows:

\[
U = -0.0518807421V_x - 0.872226427V_y - 0.486349720V_z
\]

\[
V = +0.4846922369V_x - 0.4477920852V_y + 0.7513692061V_z
\]

\[
W = -0.8731447899V_x - 0.1967483417V_y + 0.4459913295V_z
\]

We calculated the mean space velocity components \( (U, V, W) \) from the values of the space velocity components \( (U, V, W) \) obtained using the above relations. The mean values \( (U, V, W) \) were obtained as \( (-15.14 \pm 3.90, -19.43 \pm 4.41, -20.85 \pm 4.57) \) and \( (28.13 \pm 5.30, -9.78 \pm 3.13, -19.59 \pm 4.43) \) in km s\(^{-1}\) for SAI 44 and SAI 45, respectively. The distribution of these space velocities is shown in Figure 14. It can be inferred from the figure that SAI 44 has more extended velocities compared to the very compact velocity distribution of SAI 45 in \( (UVW) \) velocity space, consistent with our finding that SAI 45 is a compact cluster in Section 4.3.

7.3. Other Kinematic Structure Parameters

The coordinate of the center \( (x_c, y_c, z_c) \) of the cluster is calculated by finding the center of mass of the \( N \) member stars in equatorial coordinates. The relations to calculate \( (x_c, y_c, z_c) \) are given below:

\[
x_c = \frac{\sum_{i=1}^{N} d_i \cos \alpha_i \cos \delta_i}{N}
\]

\[
y_c = \frac{\sum_{i=1}^{N} d_i \sin \alpha_i \cos \delta_i}{N}
\]

\[
z_c = \frac{\sum_{i=1}^{N} d_i \sin \delta_i}{N}.
\]

We obtained \( (x_c, y_c, z_c) \) to be \( (597, 2758, 2893) \) and \( (229, 1188, 1234) \) in parsec for SAI 44 and SAI 45, respectively. The Galactocentric distance of a cluster is calculated using the relation \( R_{gc} = R_{ad} + (d \cos b)^2 - 2R_{gc} d \cos b \cos l \) where the distance of the Sun from the Galactic center \( R_{gc} \) was taken to be \( 8.2 \pm 0.10 \) kpc (Bland-Hawthorn et al. 2019). We obtained \( R_{gc} \).
values of 11.744 ± 0.108 and 9.803 ± 0.099 kpc for SAI 44 and SAI 45, respectively. The projected distances \((X_\odot, \ Y_\odot, Z_\odot)\) in the Galactic plane can be calculated as follows:

\[
X_\odot = d \cos \varphi \cos \lambda; \quad Y_\odot = d \cos \varphi \sin \lambda; \quad Z_\odot = d \sin b
\]

where \(d\) is the heliocentric distance, while \(\varphi\) and \(b\) are the Galactic longitude and latitude. We found \((X_\odot, \ Y_\odot, Z_\odot)\) for SAI 44 and SAI 45 to be \((-3.489 \pm 0.059, 1.116 \pm 0.033, 0.231 \pm 0.015)\) and \((-1.590 \pm 0.039, 0.489 \pm 0.022, 0.125 \pm 0.011)\) in kiloparsec, respectively. The values of all kinematic parameters we calculated are tabulated in Table 3.

### 8. Conclusion

We presented the first deep \(BVRcIc\) photometric and kinematic study of the open clusters SAI 44 and SAI 45. The study is based on the member stars identified taking advantage of kinematic data of Gaia eDR3. We supplemented our photometric data with Gaia eDR3 and Pan-STARRS PS1 DR2. The physical parameters like distance, total-to-selective extinction, reddening, age, half-mass radius, and tidal radius are estimated. The total-to-selective extinction \(R_v\) values are found to follow a correlation between \(R_v\) and distance as \(R_v\) is smaller for cluster SAI 45, which is at a relatively smaller distance. The ratio of the half-mass radius to the tidal radius indicates that SAI 45 is a compact cluster, which was found to be true from structural and kinematic studies. It is found that the structure and dynamics of these clusters are influenced by tidal interaction. The MS of SAI 45 hosts an eMSTO, which corresponds to an apparent age spread of 493 Myr. We calculated the possible apparent spread in age caused by photometric uncertainty and metallicity spread and found that any possible apparent age spread due to these two factors would be much smaller than the actual apparent age spread inferred from the eMSTO. We found (i) an apparent age spread similar to the apparent age spread and cluster age relation predicted by rotation models, (ii) that the SYCLIST synthetic population with different rotation rates was able to reproduce
the observed eMSTO, and (iii) stars in the red part of eMSTO were preferentially concentrated in the inner region. These findings support the theory attributing the origin of eMSTO to the different rotation rates of eMSTO stars. Therefore, we argue that the presence of eMSTO in SAI 45 is a stellar evolution rather than a star formation phenomenon mainly caused by the different rotation rates of stars. We obtained a steeper MF slope in the outer region for SAI 44 indicating the presence of mass segregation in the cluster. We also studied these clusters for mass segregation using a minimum sampling tree method and found that mass segregation is present in SAI 44. However, we could not get any evidence of strong mass segregation in SAI 45, possibly due to tidal stripping of outer low-mass stars. We calculated the kinematic structure parameters of these clusters and the obtained parameters were used in understanding the dynamical evolution of these clusters. The main results of the present study are given below:

1. We calculated the spatial stellar density distribution using the density contrast parameter and contour map of spatial stellar density and found that SAI 44 and SAI 45 are sparse clusters with irregular shapes.

2. We calculated the membership probabilities of stars based on a statistical method using proper motions. We found 204 and 74 member stars in SAI 44 and SAI 45, respectively. We used member stars exclusively to determine the physical and dynamical parameters of these clusters.

3. The ages were determined through isochrone fitting on the CMDs. We obtained log(age) to be $8.82 \pm 0.10$ and $9.07 \pm 0.10$ yr for SAI 44 and SAI 45, respectively.

4. We studied the eMSTO present in the CMD of SAI 45, which is mainly caused by the different rotation rates of member stars.

5. We found relatively steeper MF slopes equal to $-2.24 \pm 0.66$ and $-2.58 \pm 3.20$ for stars in the entire observed region of SAI 44 and SAI 45, respectively. We also found a steeper MF slope in the outer region than in the inner region of SAI 44, which can be a signature of the presence of mass segregation in the cluster.

6. We found that the dynamical relaxation times for SAI 44 and SAI 45 are many times less than the age of these clusters. Thus, these clusters have achieved dynamical relaxation through dynamical evolution.

7. Assuming that the clusters are nonrotating bodies without any contraction or expansion, we calculated the kinematic parameters using radial velocities of the clusters. We found the apex position of SAI 44 and SAI 45 using the AD diagram to be $(69.79 \pm 0.11, -30.82 \pm 0.15)$ and $(-56.22 \pm 0.13, -56.62 \pm 0.13)$, respectively. We obtained space velocity components in km s$^{-1}$ for SAI 44 and SAI 45 to be $(-15.14 \pm 3.90, -19.43 \pm 4.41, -20.85 \pm 4.57)$ and $(28.13 \pm 5.30, -9.78 \pm 3.13, -19.59 \pm 4.43)$, respectively.

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