Optimization for First Stage of Multistage Pump Based on Response Surface Methodology

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Abstract. To improve the efficiency of a multistage centrifugal pump at design point, the response surface methodology was applied to construct an approximation function between the design-point efficiency of the first stage in multistage centrifugal pump and the design variables of the impeller. First, important geometric parameters are selected as factors of a one-factor experimental design. Four parameters that had greater influence on the hydraulic efficiency were screened out from the simulation results using parameter sensitivity analysis. The optimal design points for each level of the parameters were then determined by central composite design and the response surface analysis method. The efficiencies of the impeller schemes generated in the Design-Expert software was executed with CFD. The effects of the single parameters were analysed, and the approximation model solved to find the optimal parameter combination that improve the efficiency by 3.62% at design point. And the inner flow was improved in high flow rate condition.

1. Introduction
The global water crisis is becoming increasingly serious. Seawater desalination is one of the recognized methods to solve the freshwater crisis. It is not affected by time, space and climate, and can meet the urgent needs of industries and residents in coastal areas for freshwater resources. The electricity cost of the high-pressure multistage pump accounts for about 35% of the operation cost of the whole desalination system. Therefore, optimization for improving efficiency of the pump is of significance for energy saving and emission reduction.

Different optimization methods were used to improve the performance of multistage centrifugal pumps. Hui W[3] investigated the matching relationship between guide vane outlet and impeller inlet of multi-stage ramjet pump, and studied the influence of different combination of guide vane and impeller parameters on pump performance. Liu et al.[4] assumed that the performance of impellers of multistage multiphase pump was not affected by the guide vanes between stages. Orthogonal experimental design optimization was carried out for the five parameters of impellers of multistage multiphase pump. Wei W et al.[5] used orthogonal experimental design method to optimize the of impeller of multistage centrifugal pump, and analysed the sequence of influence of impeller parameters on efficiency and head. Kawashima and Kanemoto[6] studied the effect of two positive diffuser schemes on the performance of multistage centrifugal pump. It was pointed out that the good matching of the positive diffuser and de-swirl vane were very important to improve the pump performance. Swarm intelligence algorithm were carried out to optimize the centrifugal pump because of the superiority in solving complex problems. Benturki et al.[7] used multi-objective genetic algorithm to optimize the
impeller, guide vane and volute of two-stage centrifugal pump involving 34 parameters, and obtained multi-objective solution set of head and efficiency in the global scope. Based on MATLAB and particle swarm optimization method, an automatic multi-parameter optimization platform has been established by Wenjie W[8] to optimize a centrifugal pump.

The methods to optimize the multi-stage pump can be divided into two categories: design of experiments and intelligent algorithms. For a pump, there are so many parameters can be the factors of experiment designing, which make it difficult to select factors and keep less number of designed schemes; Although the intelligent algorithms have superiority in solving complex problems, the time of optimizing depends largely on the convergence speed of the algorithms. Therefore, in this paper, a parameter sensitive analysis and single factor test were carried out to select factors and decrease the number experiments before response surface method experiment design. And the advantage of response surface method experiment design is less experiments for more accurate prediction.

2. Initial model and mesh

The optimization object is the first stage impeller of a three-stage centrifugal pump (shown in Figure 1(a)). Four parts are contained in the computational domain which is shown in Figure 1(b). The 3D geometry of impeller was generated in BladeGen and the outlet part of impeller are cut off to match with diffuser. The inlet pipe, outlet pipe and diffuser are built by UG NX 12.0. the inlet pipe is added to make the flow in impeller more similar to the actual situation.

The flow rate and head of single stage at design point are 650m$^3$/h and 205m respectively. The specific speed is 84.46 and the rotation speed is 2950 r/min. Other basic parameters of single stage are shown in Table 1.

![Figure 1. The whole pump and the first stage 3D model](image)

| Parameters                        | Value    |
|-----------------------------------|----------|
| Inlet diameter of impeller $D_1$  | 222mm    |
| Outlet diameter of impeller $D_2$ | 400mm    |
| Blade outlet width $b_2$          | 27mm     |
| Number of impeller blades $Z$     | 6        |
| Wrap angle $\phi$                 | 140°     |
| Blade inlet angle $\beta_1$       | 10°      |
| Blade outlet angle $\beta_2$      | 27°      |
The structured hexahedron mesh was generated for impeller employed TurboGrid (1 passage grids shown in Figure 2), and the unstructured Tetrahedron/Mix mesh for diffuser and pipes was generated by ANSYS ICEM. Total number of grids in computational domain, as shown in Figure 2, is approximately \(5.47 \times 10^6\), which is determined by mesh independence analysis. The different head and efficiency with 6 level of grid number at design point condition are shown in Table 2. As is shown in Figure, the head remains stable when element number exceeds \(5.47 \times 10^6\), the head remains stable, which means that the accuracy of numerical simulation is no longer affected by the elements number.

![Figure 2](image-url)  
**Figure 2.** Grids of whole computational domain and 1 passage domain of impeller

| Number of elements | Efficiency(%) | Head(m)  |
|-------------------|---------------|----------|
| \(3.86 \times 10^6\) | 67.72         | 162.43   |
| \(4.43 \times 10^6\) | 69.41         | 169.16   |
| \(5.08 \times 10^6\) | 70.58         | 172.46   |
| \(5.47 \times 10^6\) | 71.92         | 173.79   |
| \(5.94 \times 10^6\) | 71.86         | 173.84   |
| \(6.42 \times 10^6\) | 71.97         | 173.81   |

![Figure 3](image-url)  
**Figure 3.** Head comparison under different element number

3. Parameter sensitive analysis and single factor test

More values mean more experiments in response surface methodology (RSM). In order to reduce the number of experiment and save computing resources, parameters that have a greater impact on efficiency at design point should be selected. In this section, four geometric parameters of impeller are selected as the values in RSM, and the impact of them on efficiency and head are analyzed, which will be a reference to determine the level of factor in RSM.
3.1. Impeller parameters control

As is shown in Figure 4, the 3D model of impeller are generated in BladeGen, while many parameters can be changed. First of all, meridional plane of impeller can be generated by bgd file. Grid the meridional plane of impeller and get enough coordinates. Then wright the coordinates to bgi file which can be converted to bgd file. Finally, Blade inlet angle, blade outlet angle and blade wrap angle can be controlled by “Blade Angle Dialog”, and the thickness of blade can be adjusted at “Normal Layer Thickness” viewer[1].

![Figure 4. The method of impeller parameters control](image)

3.2. Single factor test design and parameter sensitivity analysis

Impeller inlet angle, impeller outlet angle, blade wrap angle, impeller outlet diameter, thickness of blade and blade outlet width are chosen to design the single factor test, which provide information for sensitive analysis and response surface method experiment design.

For each factor, if we need to understand how much it affect the efficiency of the first stage at design point, a generally accepted criteria are needed. Sensitivity analysis is an analytical technique which provide a way to indicate the response changing degree caused by the change of factor employed a certain key indicator or a group of key indicators from the perspective of quantitative analysis. The essence is to explain the key indicators by these factors by changing the relevant variables one by one. The calculation formula of sensitivity is as equation (1)[9].

$$S_i = \sum_{i=1}^{n} \frac{\Delta y_i}{y_0} \frac{\Delta x_i}{x_0}$$  \hspace{1cm} (1)

In this paper, $\Delta y_i$ is the change in efficiency or head; $y_0$ is the efficiency or head of initial model; $\Delta x_i$ indicate the change in geometry parameters; $x_0$ indicate the initial geometry parameters.

Based on the 6 geometry parameters of initial impeller, decrease impeller outlet width by 3% and increase other parameters by 3% to get the experimental scheme of single factor test show in Table 3. Then generated 3D model and grids by above methods. The solution of 3D Reynolds-average Navier-Stokes equations in the calculation domain was carried out using CFD code CFX 19.2. Sheer Stress Transport (SST) model based on k-ω model, a turbulence model, was adopted to implement numerical analysis of the First stage impeller and diffuser[2]. Pertaining to the boundary condition for numerical simulation, the reference press was set as 1 atm, and a 0 atm pressure condition and flowrate condition are assigned to inlet and outlet respectively. Constant computation are carried out to get efficiency and head of each impeller shown in Table 3. Based on efficiency and head of the impeller, the efficiency sensitivity index and head to different parameters can be calculated by equation (1), and the final sensitivity index $S_\eta$ (Efficiency sensitivity index) and $S_H$ (Head sensitivity index) are shown in Table 3. The parameters that corresponding top 4 sensitivity index are impeller outlet diameter, blade outlet width, impeller outlet angle and impeller inlet angle.
Table 3. Single factor test experimental scheme and sensitivity data

|  | $\beta_1$ (°) | $\beta_2$ (°) | $\varphi$ (°) | $D_2$(mm) | $\delta$(mm) | $b_2$(mm) | $\eta$ (%) | $H$(m) | $S_\eta$ | $S_H$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Ini | 10 | 27 | 140 | 400 | 4.2-5.4 | 27 | 71.9 | 174 |
| 1 | 10.3 | 27 | 140 | 400 | 4.2-5.4 | 27 | 73.78 | 176.27 | 0.87 | 0.43 |
| 2 | 10 | 27.8 | 140 | 400 | 4.2-5.4 | 27 | 73.80 | 176.96 | 0.88 | 0.57 |
| 3 | 10 | 27 | 140 | 392 | 4.2-5.4 | 27 | 75.08 | 150.18 | 1.47 | 4.56 |
| 4 | 10 | 27 | 140 | 400 | 4.2-5.4 | 27.8 | 74.05 | 176.70 | 1.00 | 0.52 |
| 5 | 10 | 27 | 144.3 | 400 | 4.2-5.4 | 27 | 73.62 | 173.65 | 0.80 | 0.07 |
| 6 | 10 | 27 | 140 | 400 | 4.4-5.6 | 27 | 73.67 | 176.05 | 0.82 | 0.39 |

3.3. Analysis of single factor test
The performance of initial impeller is shown as blue line in Figure 5, and red lines show the performance of different case in Table 3: Figure 5(a), (b), (c) and (d) show the comparison between case 1, 2, 3 and 4 performance and initial impeller especially. When increase $\beta_1$, $\beta_2$ and $b_2$, the trends of efficiency are similar. In case 1, 2 and 4, the efficiency in high flow rate is higher than initial impeller, while the performance is similar to initial impeller in low flowrate condition, and the trends of head is similar with efficiency. But, as is shown in Figure 5(c), comparing with other three parameters, $D_2$ has a more obvious influence on efficiency and head. When $D_2$ is reduced, the head is reduced in both high flow rate and low flow rate conditions; As to efficiency, there is a remarkable promotion in low flow rate condition but a steep decline in high flow rate condition.

![Figure 5. The result of single factor test](image)
4. Optimization based on response surface methodology

There are many experiment design methods in response surface methodology. The more common ones are Box-Behnken Design (BBD) and Central Composite Design (CCD). BBD is suitable for optimization experiments with 2 to 5 factors [10]. The Box-Behnken design method takes 3 levels for each factor and encodes it with (-1, 0, 1). The design table is arranged with as the center point, and +1, -1 are the high and low values corresponding to the cube points respectively.

Based on the Parameter sensitive analysis and single factor test in section 3, 0 level of $\beta_1$, $\beta_2$ and $b_2$ should be define higher than the parameters in initial model, and range of D_2 should be defined narrower because of stronger influence on efficiency and head. Therefore, using Design-Expert software and BBD method, four important factors screened from the previous section are designed for experiment. See Table 4.

**Table 4. Factor levels of BBD**

| Factor                      | Code Level |
|-----------------------------|------------|
| Impeller inlet angle $\beta_1$ (°) | -1 0 1     |
| Impeller outlet angle $\beta_2$ (°) | 22 28 34   |
| Impeller outlet diameter $D_2$ (mm) | 392 396 400 |
| Impeller outlet width $b_2$ (mm) | 25 28 31   |

According to the experiments design in Design-Expert software, 29 experiments were carried out. Based on the initial model, change the parameters in BladeGen following the cases in Table 5. The numerical simulation was set same as single factor test in section 3, and the results is shown in Table 5.

**Table 5. Scheme and results of Box-Behnken Design**

| $D_2$ (mm) | $b_2$ (mm) | $\beta_2$ (°) | $\beta_1$ (°) | $\eta$ (%) | $H$ (m) | $D_2$ (mm) | $b_2$ (mm) | $\beta_2$ (°) | $\beta_1$ (°) | $\eta$ (%) | $H$ (m) |
|------------|------------|---------------|---------------|------------|--------|------------|------------|---------------|---------------|------------|--------|
| 1          | 396        | 28            | 34            | 8          | 72.49  | 169.87     | 16         | 198           | 31            | 28         | 14      | 73.62  | 178.97 |
| 2          | 396        | 28            | 28            | 11         | 73.77  | 168.37     | 17         | 198           | 28            | 34         | 14      | 72.12  | 171.07 |
| 3          | 198        | 28            | 22            | 8          | 75.12  | 163.05     | 18         | 196           | 28            | 28         | 8       | 73.61  | 157.22 |
| 4          | 198        | 28            | 28            | 11         | 73.77  | 168.37     | 19         | 198           | 31            | 22         | 11      | 74.01  | 172.40 |
| 5          | 198        | 28            | 28            | 14         | 74.51  | 158.57     | 20         | 200           | 31            | 28         | 11      | 71.73  | 185.86 |
| 6          | 200        | 28            | 22            | 11         | 74.37  | 171.31     | 21         | 196           | 25            | 28         | 11      | 74.38  | 149.30 |
| 7          | 196        | 28            | 22            | 11         | 75.34  | 153.80     | 22         | 200           | 28            | 34         | 11      | 70.96  | 178.99 |
| 8          | 198        | 25            | 22            | 11         | 75.40  | 151.90     | 23         | 198           | 25            | 28         | 8       | 74.04  | 157.81 |
| 9          | 200        | 28            | 28            | 14         | 73.08  | 177.08     | 24         | 200           | 28            | 28         | 8       | 72.30  | 177.09 |
| 10         | 198        | 31            | 28            | 8          | 72.82  | 176.83     | 25         | 198           | 28            | 22         | 14      | 75.77  | 164.29 |
| 11         | 198        | 28            | 28            | 11         | 73.77  | 168.37     | 26         | 196           | 28            | 28         | 14      | 73.95  | 158.47 |
| 12         | 196        | 28            | 34            | 11         | 72.38  | 159.12     | 27         | 198           | 31            | 34         | 11      | 71.55  | 180.82 |
| 13         | 198        | 25            | 34            | 11         | 72.70  | 161.91     | 28         | 198           | 28            | 28         | 11      | 73.77  | 168.37 |
| 14         | 200        | 25            | 28            | 11         | 73.56  | 167.63     | 29         | 196           | 31            | 28         | 11      | 73.92  | 167.46 |
| 15         | 198        | 28            | 28            | 11         | 73.68  | 168.13     |            |               |               |            |         |        |        |

The regression equation of efficiency based on actual factor level is obtained by second-order polynomial fitting, which is expressed as follows:
\[ \eta = -4268.49 + 21.29D_2 + 11.36b_2 + 1.81\beta_2 - 3.62\beta_1 - 0.0285D_2b_2 - 0.0047D_2\beta_1 - 0.0035b_2\beta_2 + 0.0091b_1\beta_1 - 0.025\beta_1 - 0.0096b_2 + 0.0071\beta_1^2 \]  

(2)

According to the Analysis of Variance (ANOVA) function in Design-Expert software, \( R^2 \), Adjusted \( R^2 \) (Adj.\( R^2 \)) and Predicted \( R^2 \) (Pred.\( R^2 \)) are used to evaluate the approximate model above. In

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]  

(3)

In the equation (3), \( \bar{y} \) is the mean of the response value; \( \hat{y} \) is the predicted value of response surface. \( R^2 \) indicates the extent to which the response surface fits the experiment data and usually requires an \( R^2 \) value above 0.9. \( R^2 \) in this model is 0.9773. But \( R^2 \) accounts for the number of optimization variables. This means that the R-squared will always increase when a new variable is added to the model whether the new variable has statistical significance. So, it cannot fully indicate that the equation (2) is very close to the relationship between actual optimization objectives and parameters yet, even if \( R^2 \) approach 1. Adj.\( R^2 \) is similar to \( R^2 \), but it will decrease when a non-significant variable has been added.

\[ Adj.\ R^2 = 1 - \frac{n-1}{n-k-1}(1 - R^2) \]  

(4)

In the equation (4), \( n \) is the sample size; \( k \) is the number of optimized variables. The Adj.\( R^2 \) is 0.9547 and Pred.\( R^2 \) is 0.8702 in this model. Both of the \( R^2 \) and Adj.\( R^2 \) are very close to 1, and the difference between Pred.\( R^2 \) and Adj.\( R^2 \) is less than 0.2. It can be considered that the regression equation (2) is significant to predict the efficiency.

Optimization modular in Design-Expert software was employed to predict the best parameters of impeller. Set both efficiency and head optimization condition as ‘maximize’ to get high efficiency impeller with eligible head. The selected solution is shown in Table 6.

| Table 6. Elected solution in Design-Expert software |
|---------------------------------|------|------|------|------|------|
| \( D_2 \) (mm) | \( b_2 \) (mm) | \( \beta_2 \) (˚) | \( \beta_1 \) (˚) | \( H \) (%) | \( H \) (m) |
| 400.000 | 31.000 | 18.298 | 26.000 | 79.679 | 179.328 |

Generate the 3D model as the parameters shown in Table 6 and get the whole computational domain with the optimized impeller. Then the optimized model is calculated in CFX 19.2 to verify the optimization results.

5. Results and discussion
Initial model and optimized model in 11 different flow rate conditions were computed with the same setting as single factor test. External characteristic and internal flow will be compared in this section.

5.1. Hydraulic performance
Figure 6 shows the comparison of hydraulic performance curves between initial and optimized models, which include efficiency and head curves. Compared with initial model, the efficiency of optimized model is higher in high flow rate condition, while they have similar performance in low flow rate condition. At design point, the efficiency increased by 3.62%. As can be seen in Figure 6(a), after response surface method optimization, the point of highest efficiency shifts toward higher flow rate, and a wider high efficiency area are obtained at the same time; As for head, in low flow rate condition, the curves of initial and optimized models almost coincide, as shown in Figure 6(b). In high flow rate, the head of optimized model is higher than initial one.
Figure 6. Hydraulic performance comparison between initial model and optimized model

To sum, after optimization, hydraulic performance gets improved around design point and high flow rate condition. But in low flow rate condition, the optimized modal keeps the similar hydraulic performance as the initial model.

5.2. Internal flow

In order to investigate the causes of different performance in initial and optimized model, an internal flow analysis is executed. The first and second row of Figure 7 shows the velocity contour on the mid-span plane of initial and optimized impeller in different flow rate (Q means flow rate at design point) respectively.

In 0.6Q condition, from whole impeller view, the velocity distribution inside the optimized impeller is more uniform than initial impeller. There is a large low speed area in the outlet of every impeller passage. Except for two passages, uniform velocity distribution are shown in other single passage of optimized impeller, which may lead to a higher hydraulic performance in optimized impeller. But, it is obvious that high speed areas are located around the gap between impeller and diffuser. That may due to the water impact on the guide vanes, which increase impact loss. So, there is no obvious improvement in low flow rate condition.

In design condition, it is similar that low speed areas is distributed in the outlet of impeller. There are two small low speed areas around the suction surface of the blade of optimized impeller on which vortexes exist. But, the change of impeller parameters will change the matching between impeller and diffuser, and so that inner flow in diffuser are changed.
Figure 7. Comparison of velocity contour between initial and optimized model in different flow rate

The streamline in diffuser marching initial impeller is shown in Figure 8 (a), and the streamline in diffuser marching optimized impeller is shown in Figure 8 (b). Although there vortexes in both diffuser, the vortexes in Figure 8 (b) are much smaller than Figure 8 (a) and further away from inlet of the passage of diffuser. That means larger passing area in the diffuser marching optimized impeller.

Figure 8. Comparison of the streamline in diffuser marching initial impeller and optimized impeller

In 1.4Q condition, the velocity distribution in optimized impeller is obviously more uniform than initial impeller. Initial impeller shows high speed flow along the suction side of blade and in the center of some passages, while low speed flow occupies most of the area of the impeller outlet. This velocity distribution lead to non-uniform flow and second flow at the outlet area of impeller.

Figure 9. Comparison of the streamline in initial impeller and optimized impeller

As is shown in Figure 9, streamline in optimized impeller shown in Figure 9(b) fits the surface of blade better than that in initial impeller obviously. In initial impeller, incoming flow impact on the suction surface of the blade, leading to the change of the flow direction at the end of the blade. The
better inner flow in high flow rate condition lead to less impact loss and second flow loss, so that hydraulic performance is improved in optimized model.

6. Conclusion
In this study, response surface method was applied to optimize the first stage of a three-stage multistage pump. To get a reasonable scheme for response surface method experiment design, parameter sensitive analysis and single factor test were carried out. The optimized model was computed in ANSYS CFX 19.2 to compare with initial one and analyze hydraulic performance and inter flow in it. The following conclusion can be drawn:

1) The hydraulic performance at design point can be improved by response surface methodology. Parameter sensitive analysis combining with response surface methodology is an effective method to improve efficiency of centrifugal pump with diffuser by changing geometric parameters of impeller.

2) The changing of impeller parameter can make the matching between impeller and diffuser better, which have leads to better inter flow in whole flow region. So, change the parameters of impeller and diffuser in the same time to improve the matching could be a more efficiency way to improve the inter flow and hydraulic performances.

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