Log-Poisson non-Gaussianity of Lyα transmitted flux fluctuations at high redshift

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ABSTRACT

We investigate the non-Gaussian features of the intergalactic medium (IGM) at redshift $z \sim 5–6$ using the Lyα transmitted flux of quasar absorption spectra and a cosmological hydrodynamic simulation of the concordance $\Lambda$CDM universe. We show that the neutral hydrogen mass density field and Lyα transmitted flux fluctuations possess all the non-Gaussian features predicted by the log-Poisson hierarchy. This depends only on two dimensionless parameters $\beta$ and $\gamma$, describing, respectively, the intermittence and singularity of the random fields. We find that the non-Gaussianity of the Lyα transmitted flux of quasars from $z = 4.9$ to $z = 6.3$ can be well reconstructed by the hydrodynamical simulation samples. Although the Gunn–Peterson optical depth and its variance undergoes a significant evolution in the redshift range of 5–6, the intermittency measured by $\beta$ is almost redshift-independent in this range. More interestingly, the intermittency of the quasar’s absorption spectra on physical scales $0.1–1\,h^{-1}\,\text{Mpc}$ in redshift 5–6 is found to be about the same as that on physical scales $1–10\,h^{-1}\,\text{Mpc}$ at redshifts 2–4. Considering the Jeans length is less than $0.1\,h^{-1}\,\text{Mpc}$ at $z \sim 5$, and $1\,h^{-1}\,\text{Mpc}$ at $z \sim 2$, these results imply that the non-linear evolution in high and low redshifts will lead the cosmic baryon fluid to a state similar to fully developed turbulence. The log-Poisson high-order behaviour of the current high-redshift data of a quasar’s spectrum can be explained by the uniform ultraviolet background in the redshift range considered. We have also studied the log-Poisson non-Gaussianity by considering an inhomogeneous background. With several simplified models of the inhomogeneous background, we have found that the effect of the inhomogeneous background on the log-Poisson non-Gaussianity is no larger than $1\sigma$.

Key words: cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

The cosmic density and velocity fields of the intergalactic medium (IGM) at low redshifts are highly non-Gaussian, while the temperature fluctuations of the cosmic microwave background radiation are basically Gaussian or, at most, weakly non-Gaussian. Thus, the non-Gaussian features of the IGM at low redshifts should mainly result from non-linear dynamical processes. A good knowledge of these non-Gaussian features and the history of their development can fundamentally improve our understanding of the formation and evolution of the structures in the Universe. For instance, the lognormal clustering model of the IGM (Bi 1993; Bi & Davidsen 1997; Liu, Bi & Fang 2007; Feng et al. 2008) can explain well all the basic properties of the Lyα forests of a quasar’s absorption spectrum at various redshifts. However, high-order statistics do reveal the deviation of observations from the lognormal model (Jamkhedkar et al. 2003; Lu, Chu & Fang 2009).

In this paper, our aim is to study the non-Gaussian features of Lyα transmission flux of a quasar’s spectrum at high redshift. The non-Gaussianities of the high-redshift Lyα transmission flux have been studied using various statistics, including the probability distribution function (PDF) of the flux (Fan et al. 2002; Becker, Rauch & Sargent 2007), the distribution of the size of dark gaps (Songaila & Cowie 2002; Fan et al. 2006), the largest peak width distribution (Gallerani et al. 2007) and the distributions of the width of leaks (Liu et al. 2007; Feng et al. 2008). Most of these statistics are designed based on the following observational facts: the high-redshift quasar absorption spectra consist of complete absorption troughs separated by the spikes of the transmitted flux. Obviously, these statistics are not suitable for comparing the non-Gaussian features at high redshift and low redshift, as the absorption spectra of low-redshift quasars...
consistent Lyα forests and do not contain Gunn–Peterson troughs. Our focus is on the log-Poisson non-Gaussianity, which is characterized by two dimensionless parameters that are available for both high- and low-redshift samples. Therefore, it can be a useful tool to study the redshift evolution of non-Gaussianity.

More important, the log-Poisson hierarchy is directly related to the dynamics of the cosmic baryon fluid. With the cosmological hydrodynamical simulation of the concordance ΛCDM model, it has been shown that, in the scale range from the onset of non-linear evolution to the scale of dissipation, the velocity fields of the cosmic baryon fluid at low redshift are extremely well described by the She–Leveque (SL) scaling formula (He et al. 2006), which was inferred from the log-Poisson hierarchical cascade (Dubrulle 1994; She & Waymire 1995; Benzi, Biferale & Trovatore 1996). The non-Gaussian behaviour of the mass density field of the baryon fluid can also be well described by log-Poisson processes (Liu & Fang 2008). Recently, the high-order statistics of observed high-resolution and high signal-to-noise (S/N) Lyα absorption spectra of quasars at redshift $z \sim 2-3$ have been found to be very consistent with the non-Gaussian features predicted by the log-Poisson hierarchy (Lu et al. 2009). It would be worth studying whether the scenario of the self-similar log-Poisson hierarchical cascade still holds at high redshifts. Does the log-Poisson non-Gaussianity experience a strong evolution in the redshift range 5–6? Can we explain the log-Poisson non-Gaussianity and its redshift dependence with the concordance ΛCDM model?

A possible application of the non-Gaussianity of the neutral hydrogen distribution is to constrain the fluctuations of the hydrogen-ionizing radiation background field. Considering the inhomogeneity of the ultraviolet (UV) background might be a source of the non-Gaussianity of ionized and neutral hydrogen; the non-Gaussianity of the Lyα transmitted flux would shed light on the evolution of the UV background. The inhomogeneity of the UV background at lower redshifts is negligible and a uniform ionizing background is a reasonable approximation. The situation becomes more complex and debatable when $z > 5$ (Fan et al. 2002; Wyithe & Loeb 2005; Fan et al. 2006; Liu et al. 2006, 2007; Furlanetto & Mesinger 2009; Mesinger & Furlanetto 2009). Therefore, it is also worth studying the effect of the non-uniform UV background on the log-Poisson high-order behaviour at high redshifts.

This paper is organized as follows. In Section 2 we give the theoretical background. In Section 3 we present the log-Poisson hierarchy of the Lyα transmitted flux fluctuations of observed samples of high-redshift quasar absorption spectra. In Section 4, we describe the method used for producing simulation samples, and we show the log-Poisson non-Gaussianity of the neutral hydrogen component of the cosmic baryon fluid. In Section 5 we compare the log-Poisson non-Gaussianities of observed data with simulation samples of the Lyα transmitted flux. The effect of the non-uniform UV background on the non-Gaussianity is also studied in Section 5. Finally, we give a conclusion and discussion in Section 6.

2 THEORETICAL BACKGROUND

2.1 Hierarchical clustering

It has been recognized for a long time that the clustering process of cosmic matter is probably hierarchical (Peebles 1980). That is, the non-linear dynamics of large-scale structure formation can be characterized by the merging of holes from smaller to larger scales. An early model of hierarchical clustering assumes that the correlation functions of the density field satisfy a linked-pair relation, $\zeta_\alpha = Q_\alpha \zeta^{\alpha-1} (\alpha = 1, 2, \ldots)$ (White 1979), where $\zeta_\alpha$ is the $\alpha$th irreducible correlation function with variable $\delta(x) = \rho(x) - \bar{\rho}$, where $\rho(x)$ and $\bar{\rho}$ are the cosmic mass density field and its mean, respectively. However, observation samples of the transmitted flux of the Lyα forest do not support the linked-pair relation if the coefficients $Q_\alpha$ are assumed to be constants (e.g. Feng, Pando & Fang 2001).

The hierarchical merging has also been modelled by an additive cascade rule (Cole & Kaiser 1988). The basic step of the cascade rule is to assume that a cell of mass $M$ and spatial scale $x$ will evolve into two cells, 1 and 2, with mass $M_1 = M + \delta m$ and $M_2 = M - \delta m$, on scale $x/2$, where $\delta m$ is a Gaussian random variable. However, the central limit theorem shows that the field produced by an additive cascade process should be Gaussian. It cannot explain the non-Gaussian features of the Lyα transmitted flux (Pando et al. 1998). Moreover, this cascade needs ‘very small initial units’ (Peacock 1999) as the first generation halo of the hierarchy, of which the mass is non-zero. These ‘initial units’ are not compatible with hydrodynamic equations with continuous variables.

A proper model of the hierarchical clustering should be randomly multiplicative, and infinitely divisible. For a randomly multiplicative cascade, the mass $m_n$ in a cell at step $n$ is related to step $n - 1$ by $m_n = (1 \pm \delta)m_{n-1}$, where $\delta$ is a random variable. Non-Gaussian features can be formed through randomly multiplicative cascade processes, even if the original field is Gaussian. The infinite divisibility means that there are no finite ‘initial units’ in the hierarchical merging. The ‘initial units’ of the merging process can be infinitesimal.

The log-Poisson hierarchy has all these desired properties. More importantly, the log-Poisson hierarchy was actually inferred from the invariance and symmetry of the Navier–Stokes equations. It works well in explaining the high-order behaviour of fully developed turbulence (Dubrulle 1994; She & Waymire 1995; Leveque & She 1997).

2.2 Log-Poisson hierarchy

To measure the non-Gaussianity caused by the log-Poisson hierarchy, it is better to use the variable $\delta \rho_i = \rho(x + r) - \rho(x)$, but not $\rho(x + r) - \bar{\rho}$. For a statistically isotropic and homogeneous random field, we can just consider $|\delta \rho_i|$, as the distribution of positive and negative $\delta \rho_i$ is statistically symmetric. The basic statistical quantity is the structure function defined by

$$S_p(r) = \langle |\delta \rho_i|^p \rangle,$$

where $p$ is the order of statistics, and the average $\langle \ldots \rangle$ is taken over the ensemble of density fields. The second-order structure function $S_2 = \langle |\delta \rho_i|^2 \rangle$ as a function of $r$ (scale) is actually the power spectrum of the mass density field (Feng & Fang 2000).

In the scale-free range, the structure function should be a function of the power law of $r$ as

$$S_p(r) \propto r^{\xi(p)},$$

where $\xi(p)$ refers to an intermittent exponent. For a Gaussian field, $\xi(p)$ is a linear function of $p$, but for an intermittent field, the function $\xi(p)$ is non-linear.

The log-Poisson hierarchy assumes that, in the scale-free range, the variables $\delta \rho_i$ on different scales $r$ are related to each other by a statistically hierarchical relation as (Dubrulle 1994; She & Waymire 1995)

$$\delta \rho_{i2} = W_{i1} \delta \rho_{i1},$$

where $W_{i1}$ is a multiplicative random variable.
The factor $W_{\gamma,\tau}$ is a function of the ratio $r_1/r_2$ given by

$$W_{\gamma,\tau} = \beta^\gamma(r_1/r_2)^\gamma,$$  
(4)

which describes how the fluctuation $\delta \rho_\gamma$ on the larger scale $r_1$ relates to fluctuations $\delta \rho_\tau$ on the smaller scale $r_2$. In equation (4), $m$ is a Poisson random variable with the PDF

$$P(m) = \exp(-\lambda_{\gamma,\tau})\lambda_{\gamma,\tau}^m/m!.$$  
(5)

The random variable $m$ can be considered as the step of the evolution from $\delta \rho_\gamma$ to $\delta \rho_\tau$. To ensure the normalization ($W_{\gamma,\tau} = 1$, where the average $\langle ... \rangle$ is over $m$, the mean $\lambda_{\gamma,\tau}$ of the Poisson distribution is then

$$\lambda_{\gamma,\tau} = \gamma [\ln(r_1/r_2)](1 - \beta).$$  
(6)

The log-Poisson hierarchy contains both the spatial size and amplitude of the density fluctuations. This point is different from other hierarchical models, which consider only the size of hierarchical units.

The log-Poisson hierarchy given by equation (3) depends only on the ratio $r_1/r_2$, which is obviously scale-invariant. The hierarchy is determined by two dimensionless positive parameters, $\beta$ and $\gamma$, describing the intermittence and singularity of the random fields, respectively. Equation (3) relates $\delta \rho_\tau$ on different scales by multiplying a random factor $W$, which generally yields a non-Gaussian field even if the field originally is Gaussian (Pando et al. 1998).

The cascade from scale $r_2$ to $r_1$ and then to $r_2$ is identical to the cascade from $r_1$ to $r_2$. It is because $W_{\gamma,\tau} = W_{\gamma,\tau} W_{\gamma,\tau} = \beta^N(r_1/r_2)^\gamma$, where $N$ is again a Poisson random variable with $\lambda_{\gamma,\tau} = \lambda_{\gamma,\tau} + \lambda_{\gamma,\tau} = \gamma [\ln(r_1/r_2)](1 - \beta)$. The log-Poisson hierarchy removes an arbitrariness in defining the cascade from $r_1$ to $r_2$ or $r_2$ to $r_1$. Therefore, the log-Poisson hierarchy, suggested by equation (3), is discrete in terms of the discrete random number $m$. However, the scale $r$ is infinitely divisible. Namely, there is no lower limit on the difference $r_1 - r_2$. It can be infinitesimal, and the hierarchical process is of infinite divisibility. With the log-Poisson model equations (3)-(6), we can show that the intermittent exponent $\xi(p)$ is given by (Liu & Fang 2008)

$$\xi(p) = -\gamma[p - (1 - \beta^p)/(1 - \beta)].$$  
(7)

This is actually the SL scaling formula (She & Leoneveque 1994).

From equation (7), the power spectrum $S_2(r) = \text{const}$ is flat, or called white. However, the power spectrum of the initial Gaussian field of the cosmic matter generally is not white, but coloured with the power law $S_2(r) \propto r^{-2\alpha}$. In this case, we should adjust the log-Poisson hierarchy equation (3) by replacing $\delta \rho_\tau_1$ and $\delta \rho_\tau_2$ with $r_1^2 \delta \rho_\tau_1$ and $r_2^2 \delta \rho_\tau_2$, respectively. The intermittent exponent $\xi(p)$ is

$$\xi(p) = -\alpha p - \gamma[p - (1 - \beta^p)/(1 - \beta)].$$  
(8)

When parameter $\alpha = 0$, equation (8) will simplify to equation (7). The non-Gaussian features of the field described by the log-Poisson hierarchical clustering have been given in Liu & Fang (2008) and Lu et al. (2009).

3 Log-Poisson Non-Gaussianity of Observed Samples

3.1 Observed Data

The observational data used here consist of the spectra of 19 quasi-stellar objects (QSOs) with redshifts from $z = 5.74$ to 6.42 as compiled in Fan et al. (2006). The data have a resolution of $R \sim 3000-4000$ and are rebinned to a resolution $R = 2600$. The observed flux, $F_{\text{obs}}$, is normalized with a power-law continuum $F_{\text{con}} \propto \nu^{-0.5}$. The noise and continuum uncertainty of the transmitted flux $F = F_{\text{obs}}/F_{\text{con}}$ is at the level of $0.018 \pm 0.012$ in the range $z \leq 5.7$, and $0.014 \pm 0.008$ in the range $z > 5.7$ (Liu et al. 2007). For more details, we refer to Fan et al. (2006). The wavelength of the data covers roughly the rest-frame wavelength from 900 to 1350 Å. To avoid the mixing of Ly\(\beta\) absorption and the proximity effect, only pixels having the rest-frame wavelength $>1040$ Å and below the maximum Ly\(\alpha\) are used.

In our analysis below, we use only the Ly\(\alpha\) transmitted flux in the redshift range from 4.7 to 6.3. There actually are 12 quasars available in the range $z > 5.9$, reduced further to 4 in the range $6.1 < z < 6.3$. We divide the redshift range from 4.9 to 6.3 into seven bins by $4.9 + n \times 0.20 < z < 4.9 + (n + 1) \times 0.20$ where $n = 0, 1, \ldots, 6$. More specifically, the redshift size of each bin is $\Delta z = 0.20$. As a comparison, we also use the sample in the redshift range from 4.7 to 6.2, and divide it into 10 bins by $4.7 + n \times 0.15 < z < 4.7 + (n + 1) \times 0.15$ where $n = 0, 1, \ldots, 9$; each bin has a size of $\Delta z = 0.15$. All the transmission flux pixels in a given redshift bin form an ensemble. The numbers of pixels in the different ensemble are not uniform.

In order to compare with observations at moderate redshift, we also analyse the high-resolution and high S/N ratio Ly\(\alpha\) absorption spectra of 28 Keck High Resolution (HIRES) QSOs (Kirkman & Tytler 1997). This is the same as used in Lu et al. (2009). The details of the data set and its reduction have been described in Jamkhedkar (2002) and Jamkhedkar et al. (2003).

With these samples we calculate the optical depth $\tau(z) = -\ln F(z)$, and the fluctuation of the optical depth $\delta \tau_r = \tau(x + r) - \tau(x)$, where the spatial coordinates $x$ and $r$ are in the physical scale. Because the variable $\delta \tau_r$ is given by the difference between $\tau(x + r)$ and $\tau(x)$, $\delta \tau_r$ is independent of fluctuations of $\tau(x)$ on scales larger than $r$. Therefore, the variable $\delta \tau_r$ is actually insensitive to the continuum used in the data reduction (Jamkhedkar, Bi & Fang 2001).

As has been shown in Lu et al. (2009), the variable $\delta \tau_r$ is approximately a measurement of the density fluctuation $\rho_r$. We can then study the log-Poisson non-Gaussianity with variable $\delta \tau_r$. For instance, the structure function with the variable $\delta \tau_r$ is defined now by

$$S_p(r) = \langle |\delta \tau_r| \rangle.$$  
(9)

To treat the unwanted data, including low S/N and bad pixels, we use the algorithm of wavelet denoising by threshold (Donoho 1995; Jamkhedkar et al. 2003). This method is effective for pixelated data. First, we calculate the wavelet scaling function coefficients (SFCs) of both the transmission flux field $F(x)$ and the noise field $n(x)$ with

$$\epsilon_p^F = \int F(x) \phi_{jl}(x) \, dx,$$  
$$\epsilon_p^n = \int n(x) \phi_{jl}(x) \, dx,$$  
(10)

where $\phi_{jl}(x)$ is the scaling function of the wavelet on scale $j$ and position $l$. We then identify unwanted mode $(j, l)$ by using the threshold condition

$$|\epsilon_p^F/\epsilon_p^n| < f.$$  
(11)

This condition flags all modes with S/N less than $f$. We skip all the flagged modes when performing statistics. To reduce the boundary effect of unwanted chunks, we also flag two models around an unwanted model. With this method, no rejoining and smoothing of the data are needed. The threshold $f$ is given by the same way as Jamkhedkar et al. (2003) and Lu et al. (2009).

A typical statistical quantity of log-Poisson non-Gaussianity is

$$F_p(r) = S_p + (r)/S_p(r).$$  
(12)
We test the effect of noise on \( F_4(r) \equiv S_4(r)/S_4(r) \). We calculate \( F_4(r) \) for data sets given by different threshold \( r \). The \( r \)-dependence of \( F_4(r) \) is shown in Fig. 1. For the data set at \( 4.9 < z < 5.1 \), the values of \( F_4(r) \) are \( r \)-independent when \( \log r \geq 0.3 \). In other words, the statistical results are stable with respect to threshold \( r \approx 2 \). We use only data with S/N larger than 2. This threshold is larger than the error level given by Fan et al. (2006). For a sample set of \( 5.7 < z < 5.9 \) (right panel of Fig. 1), \( F_4(r) \) are very weakly dependent on \( \log r \). This is because, in the redshift range \( z > 5.7 \), the high-order statistics \( F_4(r) \) are dominated by modes with high \( \delta \tau_\alpha \), and therefore it is insensitive to dropping modes with low \( \delta \tau_\alpha \). However, the fewer the modes, the larger the variance of the Poisson process. Therefore, we should consider the variance of the Poisson process in our statistics.

3.2 Redshift dependence of parameter \( \beta \)

We first study the \( \beta \)-hierarchy predicted by the log-Poisson hierarchy. It reads (Liu & Fang 2008)

\[
\ln F_{p+1}(r)/F_p(r) = \beta \ln F_p(r)/F_2(r). \tag{13}
\]

Equation (13) requires that for all \( r \) and \( p \), \( \ln [F_{p+1}(r)/F_p(r)] \) versus \( \ln [F_p(r)/F_2(r)] \) should be on a straight line with slope \( \beta \), which is called the \( \beta \)-hierarchy. Equation (13) does not contain the parameters \( \gamma \) and \( \alpha \). Fig. 2 presents the \( \beta \)-hierarchy of the observed transmitted flux in four redshift ranges \( z = 5.0, 5.4, 5.8 \) and 6.2. The statistical quantity \( F_{p+1}(r) \) is given by all available observed data, of which the physical scale \( r \) covers the range from \( \sim 0.1 \) to \( 1.5 \) h\(^{-1}\) Mpc, and the order parameter \( p \) increases from 1 to 2.5. All the distributions of \( F_{p+1}(r)/F_p(r) \) versus \( F_p(r)/F_2(r) \) in Fig. 2 can be well fitted with a straight line. This shows that the Ly\(\alpha \) transmitted flux of real observations at high redshifts still satisfies the \( \beta \)-hierarchy and indicates that the log-Poisson non-Gaussian features are significant in the considered redshift range.

From the \( \beta \)-hierarchy straight line, we calculate the parameter \( \beta \) in redshift ranges from \( z = 5.0 \) to 6.2. Fig. 3(a) presents the mean of \( \beta \) in each redshift bin. The error bars are given by the variance of the Poisson process, which generally are larger than the variance of \( \beta \) in the given redshift bin. Fig. 3(a) shows that the redshift evolution of \( \beta \) is weak in the range from \( z = 5 \) to 6. We also calculate the redshift dependence of \( \beta \) with the same data, but the size of the redshift bin is taken to be \( \Delta z = 0.15 \). The result is also plotted in Fig. 3(a), which shows the same redshift dependence as that of \( \Delta z = 0.20 \). Therefore, the weak redshift evolution of parameter \( \beta \) is not affected by \( \Delta z \). From now on, we only give results with \( \Delta z = 0.20 \).

Fig. 3(b) plots the redshift dependence of \( \beta \) for the Keck HIRES quasar spectra sample. The error bars are given by the maximum and minimum in each redshift interval. Comparing Figs 3(a) and (b), we can conclude that the non-Gaussian parameter \( \beta \) evolves weakly from redshift 2 to 6. It should be pointed out that the physical scale \( r \) in Fig. 3(a) (high-redshift sample) is actually smaller than that of Fig. 3(b) (low-redshift sample). Fig. 3 reveals that the \( \beta \) non-Gaussianity on a small scale at high redshift is about equal to that on a large scale at low redshift. This result indicates that the turbulence state on scales \( 0.1–1 \) h\(^{-1}\) Mpc at redshift \( z \approx 5 \) would be the same as that on scales \( 1–10 \) h\(^{-1}\) Mpc at redshift \( z \approx 2 \). This is consistent with the fact that the Jeans length at \( z \approx 5 \) is less than \( 0.1 \) h\(^{-1}\) Mpc and increases to \( \sim 1 \) h\(^{-1}\) Mpc at \( z \approx 2 \).

The quasar Ly\(\alpha \) absorption spectra at high redshift \( z \approx 5–6 \) are significantly different from that at low redshift \( z \approx 2–3 \). The latter show Ly\(\alpha \) forests, while the former consist of complete absorption troughs (Gunn–Peterson troughs) separated by tiny transparent regions. In other words, the Ly\(\alpha \) transmitted flux experiences a strong evolution with redshift rising from \( z \approx 2–3 \) to \( 5–6 \). The low-order statistics of the Ly\(\alpha \) transmitted flux, such as the mean optical depth and its variance, also shows strong redshift evolution. Therefore, the weak redshift evolution of \( \beta \) is very interesting. It implies that the \( \beta \) non-Gaussian feature is mainly dependent on the non-linear state of the fluid, but weakly dependent on the optical depth and its variance.

3.3 Non-Gaussianity related to parameter \( \gamma \)

We now turn to the \( \gamma \)-related non-Gaussianity, which is given by (Liu & Fang 2008)

\[
\ln \left( \frac{\langle \delta \tau^{2p} \rangle}{\langle \delta \tau^2 \rangle^p} \right) = K_p \ln r + \text{const}, \tag{14}
\]

and

\[
K_p = -\gamma \frac{p(1 - \beta^2) - (1 - \beta^{2p})}{1 - \beta}. \tag{15}
\]

This requires that for a given \( p \), the relation of \( \ln (\langle \delta \tau^{2p} \rangle/(\langle \delta \tau^2 \rangle^p) \) and \( \ln r \) has to be a straight line with slope \( K_p \). As the parameter \( \beta \) has already been determined by the \( \beta \)-hierarchy in the previous section, we can thus calculate the parameter \( \gamma \) from \( K_p \).
Figure 2. The $\beta$-hierarchy of the observed sample of the Ly$\alpha$ transmitted flux at redshift $z = 5.0$ (top left), 5.4 (top right), 5.8 (bottom left) and 6.2 (bottom right). The physical scale $r$ is in the range $\sim 0.1 - 1.5 \, h^{-1} \, \text{Mpc}$, and order $p$ is from 1 to 2.5. The error bars are given by the maximum and minimum of bootstrap resampling.

Figure 3. Redshift dependence of the parameter $\beta$ for samples of (left) Fan et al. (2006) and (right) the Keck HIRES quasars (Jamkhedkar et al. 2003). The result is presented in Fig. 4, which shows the relation of $\ln \left( \langle \delta \tau^2_p \rangle / \langle (\delta \tau_r)^2 \rangle \right)$ versus $\ln r$ for observational data at redshift ranges $z = 5.0, 5.4, 5.8$ and 6.2. The order parameter $p$ is set to 2 and 3 (i.e. the statistics of equations 14 and 15 are of the order of 4 and 6, respectively). The error bars are given by the maximum and minimum of each $r$. The $\ln r$-dependency of $\ln \left( \langle \delta \tau^2_p \rangle / \langle (\delta \tau_r)^2 \rangle \right)$ approximately at each range can be given by a straight line.

The slopes $K_p$ of the straight lines of Fig. 4 are listed in Table 1; the parameter $\gamma$, given by equation (15), is also listed. In the log-Poisson hierarchy, the parameter $\gamma$ has to be independent of $p$ and therefore the values of $\gamma$ determined by $K_p$ with different straight lines should be the same. Table 1 indeed confirms this point. We see that for a given redshift, the statistics of $p = 2$ and 3 yield the same $\gamma$ within their errors. Therefore, the high-redshift Ly$\alpha$ transmitted flux fulfils the $\gamma$-related non-Gaussianity well.

A basic feature of the log-Poisson model is to yield the non-linear terms of $p$ (i.e. the term $\beta^p$ in equation 7 and $\beta^{2p}$ in equation 15; Frisch 1995). The terms with linear $p$ can also be given by other models. As $\beta < 1$, the tests with $\beta^p$ and $n > 6$ do not give a new test on the log-Poisson model. Therefore, the statistical order $n$ generally is taken to be less than 6.
4 HYDRODYNAMIC SIMULATION SAMPLES

Although the mean optical depth and its variance of the Ly\(\alpha\) transmitted flux of the quasar’s absorption spectrum underwent a strong evolution at high redshift, the log-Poisson non-Gaussianity, as the previous section reveals, shows only weak dependence on redshift. Thus, an important question is whether the two aspects of the redshift evolutions can be reconciled within the concordance/\Lambda\text{CDM} model. We study this problem using cosmological hydrodynamic simulation samples.

### 4.1 Simulation samples of neutral hydrogen

We first produce the samples of mass density, temperature and velocity fields of cosmic hydrogen with hydrodynamic simulation with the same code as Liu et al. (2007) and Liu & Fang (2008). This code is based on the Eulerian method for hydrodynamics with the fifth-order weighted essentially non-oscillatory (WENO) finite difference scheme and the particle mesh (PM) method for dark matter particles (Feng et al. 2004). We use the cosmological parameters given by the latest result from the Wilkinson Microwave Anisotropy Probe (WMAP; Komatsu et al. 2009). We run simulations in a period box of side \(25h^{-1}\) Mpc with \(512^3\) grids and dark matter particles that have a mass resolution of \(1.04 \times 10^7\) M\(_\odot\). Assuming ionization equilibrium, the atomic processes, including radiative cooling, heating and fraction of species, are modelled using the primordial composition (\(X = 0.76, Y = 0.24\)) and formalism in the appendix of Theuns et al. (1998), under an optically thin approximation. Photoionization and photoheating are switched on after the UV background is added at \(z = 11.0\). Starting at \(z = 99\), a sample output at \(z = 11\) first. Simulations with different UV histories are then performed from this snapshot at \(z = 11\) and produce snapshots at redshifts from \(z = 6.5\) to \(z = 4.9\) at an interval of \(\Delta z = 0.1\). As we focus on the evolution at high redshifts, simulations are stopped at \(z = 4\). From snapshot dumps, we produce the mass density field of hydrogen \(\rho(x)\), the temperature field \(T(x)\) and the velocity field \(\mathbf{v}(x)\). Star formation is not included. However, the contribution of stars to the UV background is considered by fitting the redshift evolution of the UV background with mean optical depth and its variance of the Ly\(\alpha\) transmitted flux field. This code has recently been used to produce samples to show the turbulence behaviour at a redshift as high as \(z \simeq 4\) (Zhu, Feng & Fang 2010). These samples would also be suitable to test the log-Poisson non-Gaussianity of the IGM at high redshifts.

| Table 1. Parameter \(\gamma\) at redshift \(z = 5.0–6.2\) for observation sample. |
|---|---|---|
| \(z\) | \(p\) | \(K_p\) | \(\gamma\) |
| 5.0 | 2 | \(-0.46 \pm 0.20\) | 0.41 \(+0.17\) |
| | 3 | \(-1.10 \pm 0.40\) | 0.43 \(+0.16\) |
| 5.4 | 2 | \(-0.25 \pm 0.10\) | 0.22 \(+0.08\) |
| | 3 | \(-0.52 \pm 0.30\) | 0.20 \(+0.12\) |
| 5.8 | 2 | \(-0.24 \pm 0.07\) | 0.21 \(+0.06\) |
| | 3 | \(-0.52 \pm 0.15\) | 0.21 \(+0.06\) |
| 6.2 | 2 | \(-0.28 \pm 0.16\) | 0.27 \(+0.15\) |
| | 3 | \(-0.78 \pm 0.35\) | 0.31 \(+0.13\) |

Figure 4. \(\ln \left[(\delta \tau_r)^p / (\langle \delta \tau_r \rangle)^p\right]\) versus \(r\) of the observed sample of the Ly\(\alpha\) transmitted flux for redshifts at 5.0 (top left), 5.4 (top right), 5.8 (bottom left) and 6.2 (bottom right). \(p\) is taken to be 2 (bottom line) and 3 (top line). The solid lines are given by the least-squares fitting. The error bars are given by the maximum and minimum of bootstrap resampling.
Although the Gunn–Peterson optical depth shows a dramatic decrease with redshift and an abnormally large scatter at \( z \sim 6 \), it can still be fitted by models of a uniform ionizing background (Liz et al. 2006; Liu et al. 2006, 2009; Furlanetto & Mesinger 2009). As current observations do not give a sufficient knowledge of ionizing sources at high redshift, instead of the Haardt & Madau (2001) model of UV background history, we use a more general uniform UV background in which the ionizing photons have a power-law spectrum with index \(-1.0\) and a normalized coefficient \( J_{21} \). The hydrogen photoionization rate \( \Gamma_{-12} \) is calculated using the fitting formula from Theuns et al. (1998) and is then used to calculate the heating and cooling in the hydrodynamic simulation. The photoionization rate can be given by

\[
\Gamma_{-12} = 3.15 J_{21}.
\]

Based on the above assumption, the history of the UV background in our simulation is given by the evolution of \( J_{21} \). There is no direct observation of this parameter. Considering the two typical reionization histories in the references (one the extended scenario and the other phase transition), we use the following two redshift-dependent models of \( J_{21} \):

\[
J_{21}(z) = 5.0 \times \exp(-0.475z);
\]

\[
J_{21}(z) = \exp(-0.21z^2 + 1.5z - 3.0) + 0.02 \exp(-7.0/z).
\]

Equation (17) can be considered as a model of the extended reionization scenario. Equation (18) gives approximately the same evolution as equation (17) at \( z = 4.0 - 5.0 \), but drops about an order of magnitude from \( z = 5.0 \) to \( z = 7.0 \) and remains at a very low level when \( z > 7 \). It mimics the reionization as a phase transition over redshift range from \( z = 5.0 \) to \( z = 7.0 \). The parameters used in equations (17) and (18) are actually determined by the fitting of simulated samples with observed mean optical depth and its variance of the Ly\( \alpha \) transmitted flux field. Several simulations are performed before these parameters are selected. It is interesting to see that with the parameters of equations (17) and (18), the intensities of \( \Gamma_{-12} \) given by equation (16) in the redshift range \( z = 4 - 9 \) are just in between the values given by (i) Haardt & Madau (2001) and (ii) the proximity effect of quasars (Dall’Aglio, Wisotzki & Worseck 2008; Gilmore et al. 2009 and references therein).

Last but not least, although the intensities of the UV background given by models (17) and (18) are very different at \( z = 5.0 \) to \( z = 7.0 \), the non-Gaussian features of the transmitted field of Ly\( \alpha \) are less affected. This is because the basic variable \( \delta \tau = \tau(x + r) - \tau(x) \) is not sensitive to the change of the uniform ionizing background given by models (17) and (18), especially when \( r \) is small.

### 4.2 Intermittence of neutral hydrogen density field

Before simulating the Ly\( \alpha \) transmitted flux, we analyse the log-Poisson behaviour of the simulation samples of the neutral hydrogen density field, \( n_{HI}(x) \). We first calculate the \( \beta \)-hierarchy of the neutral hydrogen field. The results are presented in Fig. 5, which contains all data points of \( F_{p+1}(r)/F_p(r) \) and \( F_p(r)/F_{2p}(r) \) with \( p = 1, 1.5, 2 \) and 2.5, and available scale \( r \). To estimate the errors, we divide our 160 one-dimensional samples into 80 subsamples, each of which has two lines. The error bars are given by the scattering ranges of the 80 subsamples. Fig. 5 shows that the \( \beta \)-hierarchy also holds well.

![Figure 5](https://academic.oup.com/mnras/article-abstract/408/1/452/1058682)

**Figure 5.** \( \ln [F_{p+1}(r)/F_p(r)] \) versus \( \ln [F_p(r)/F_{2p}(r)] \) for simulation samples of the mass density field of neutral hydrogen at redshifts 5.0, 5.4, 5.8 and 6.2. The statistical order is \( p = 1, 1.5, 2 \) and 2.5.
for the neutral hydrogen density field. The numbers of $\beta$ are also shown in Fig. 5, which is in the range 3.0–3.5.

In the log-Poisson hierarchy, the $r$-dependence of $F_p(r)$ is given by (Liu & Fang 2008; Lu et al. 2009)

$$F_p(r) = A r^{-\alpha - \gamma(1 - \beta^p)}.$$  \hspace{1cm} (19)

That is, the relation between $\ln F_p(r)$ and $\ln r$ should be a straight line with the slope $-\alpha + \gamma(1 - \beta^p)$. For the Gaussian field, we have $\beta = 1$, and therefore the slope is independent of $p$. For the log-Poisson field ($\beta < 1$), when $p$ is large, the slope converges to $-(\alpha - \gamma)$. However, the parameter $\alpha$ can be determined by the power spectrum of $n_{HI}(x)$ (Section 2.2). Thus, the $\ln F_p(r)$–$r$ relation with larger $p$ can be used to determine the parameter $\gamma$.

Fig. 6 presents $\ln F_p(r)$ versus $\ln r$ of the simulation samples of the neutral hydrogen mass density field in the physical length-scale range of 0.1 < $r$ < 1.5 h$^{-1}$ Mpc and orders of $p = 0.5 \times n$ with $n = 1, 2, ..., 8$. For all redshifts and $p \leq 4$, $\ln F_p(r)$–$\ln r$ can indeed be approximately fitted by straight lines. These straight lines have different slopes. The higher $p$ is, the more steep the straight lines are. This implies that $\beta < 1$ and the density fields are non-Gaussian. When $p > 3$, the fitted straight lines are almost independent of $p$. This shows the convergent of the slope to $-(\alpha - \gamma)$ at high $p$. The parameters $\gamma$ found with the slope of $\ln F_p(r)$–$r$ lines are 0.70, 0.65, 0.60 and 0.60, corresponding to redshifts $z = 5.0, 5.4, 5.8$ and 6.2 respectively.

From the parameters $\beta$ and $\gamma$ given above, we plot the intermittent exponent $\xi(p)$ (equation 8) in Fig. 7, in which data points are given by fitting structure functions of simulation samples to equation (2) with $p = 0.5 + 0.5n$, $n = 0, ..., 11$. As mentioned in Section 3.3, the test is limited in the range $p \leq 6$. The error bars of Fig. 7 are given by the maximum and minimum of the 80 subsamples.

We can conclude that the fields of the neutral hydrogen mass density can be well described by the SL scaling formula with a statistical order as high as $p = 6$. It should be noted again that Fig. 7 covers the physical scales from $\sim 0.1$ to $1.5$ h$^{-1}$ Mpc. This indicates that the cosmic neutral hydrogen fluid is turbulent on the scales considered.

5 CONFRONTATION OF OBSERVED DATA WITH SIMULATION SAMPLES

5.1 Ly$\alpha$ spectrum synthesis

Assuming ionization equilibrium under the uniform UV background, we generate the field of the neutral hydrogen fraction $f_{HI}(x) = n_{HI}(x)/n_{H}(x)$ at each cell, where $n_{HI}(x)$ and $n_{H}(x)$ are the number densities of hydrogen and neutral hydrogen at $x$, respectively. We synthesize 160 samples of normalized Ly$\alpha$ transmitted flux $F(z) = \exp[-\tau(z)]$ from $z = 4.9$ to $z = 6.3$ using the same methods as Zhang et al. (1997) and Paschos & Norman (2005). The optical depth $\tau(z)$ is given by

$$\tau(z) = \frac{\sigma_0}{H} \int_{-\infty}^{\infty} n_{HI}(x) V[z - x - v(x), b(x)] \, dx,$$ \hspace{1cm} (20)

in which $\sigma_0$ is the effective cross-section of the resonant absorption and $H$ is the Hubble constant at corresponding redshifts of the samples. The Voigt function is $V[z - x - v(x), b(x)] = 1/(\pi^{1/2} b) \exp[-(z - x - v(x))^2/b^2(x)]$, where $b(x)$ gives the thermal broadening.
Along a randomly selected line of sight, we synthesize an absorption spectrum $F$ from $z = 4.9$ to 6.3 by dividing the spectrum into redshift intervals of $\Delta z = 0.1$. As the corresponding physical length-scale for this redshift interval is larger than our simulation box size, we integrate equation (20) over the simulation dump periodically. Each spectrum is resolved with the same resolution of observation. Gaussian noise is added with the S/N ratio $= 10$.

We calculate the Gunn–Peterson optical depth and its dispersion of the 160 synthesized samples. The results are shown in Fig. 8. The observed results (Fan et al. 2006) are also shown in this figure. The simulation samples are basically consistent with the observations except for the dispersion at redshift $z = 6.2$. The deviation at redshift $z = 6.2$ might be because the available observed samples at the redshift range $z \approx 6.2$ are too few.

We also perform the statistics of the probability distribution function of the transmitted flux $F(z)$ of observation and simulation from $z = 5.3$ to $z = 6.1$. The results are presented in Fig. 9. The PDF of the simulation samples can also fit observations within the 1σ range. Thus, all low-order statistics of the simulated samples are consistent with observation.

5.2 Redshift dependence of $\beta$

Using the method of Section 3.2, we calculate the parameters $\beta$ of the simulated samples of the Ly$\alpha$ spectra. To mimic the observation samples, which contain in total 19 lines, we divide 160 one-dimensional simulation samples into eight subsamples, each of which has 20 one-dimensional samples. The uniform UV background equation (17) and equation (18) are used to produce the simulation samples. Fig. 10 presents the redshift dependence of parameter $\beta$ for observed data and simulation samples. The error bars of the simulation results are given by the maximum and minimum of the eight subsamples.

Although the redshift dependence of the UV background equations (17) and (18) is different at $z > 5$, Fig. 10 shows that the redshift dependence of $\beta$ given by the two models is about the same. This is because the uniform field $J_{21}$ does not change the non-Gaussianity significantly. The simulation results of parameter $\beta$ are basically consistent with the observation; only $z = 5.4$ shows a small deviation. Therefore, the $\Lambda$CDM universe embedded with a uniform UV background history is able to explain the feature of $\beta$ redshift revolution.

Fig. 7 shows that the parameter $\beta$ of the neutral hydrogen density field is in the range 0.30–0.35, which is much less than the results shown in Fig. 10. This indicates that the non-Gaussianity of the neutral hydrogen field is different from that of the Ly$\alpha$ transmitted flux. This is because the non-Gaussianity of the Ly$\alpha$ transmitted flux depends not only on the mass density field of neutral hydrogen, but also on the velocity field via the Voigt convolution equation (20).

5.3 Parameter $\gamma$

Similar to Section 3.3, we use the high-order moment $\langle \delta \tau^p \rangle$ to quantify $K_p$ (equation 15) and then the parameter $\gamma$. The results are listed in Table 2. The values of $\gamma$ for $p = 2$ and 3 in each $z$ range are the same within their errors. Comparing Tables 1 and 2, the parameter $\gamma$ given by the observation data and simulation sample are also consistent with each other within their errors.
Figure 8. Redshift dependence of mean optical depth (left) and its variance (right). The observed results are shown as circles and the simulation samples with UV background equations (17) and (18) are shown by solid and dotted lines, respectively.

Figure 9. Probability distribution functions of the transmitted flux at $z = 5.3–5.5$ (top left), $z = 5.5–5.7$ (top right), $z = 5.7–5.9$ (bottom left) and $z = 5.9–6.1$ (bottom right). Crosses are observation data. The central solid and dashed lines are the results of samples generated with equations (17) and (18), respectively; the sets of lines above and below give the 1σ errors.

The parameter $\gamma$ for the observational samples of the $\text{Ly}\alpha$ transmitted flux at $z = 2.5$ is found to be $0.58 \pm 0.20$ (Lu et al. 2009), which is higher than that listed in Table 2. This might indicate that the parameter $\gamma$ is decreasing with redshift. However, we should keep in mind that the error bars of the observational sample are large at both high and low redshift, and the redshift evolution of parameter $\gamma$ is still not certain.

Fig. 6 reveals that the parameter $\gamma$ of neutral hydrogen is in the range $0.80–0.89$, which is much larger than that shown in Tables 1 and 2. The parameter $\gamma$ measures the singular structures and higher $\gamma$ represents stronger singularity (Liu & Fang 2008; Lu et al. 2009). Therefore, the field of neutral hydrogen mass density contains many more singular structures than that of the $\text{Ly}\alpha$ transmitted flux.

5.4 Spatially non-uniform UV background

Whether the UV background at redshift $z \sim 5–6$ is spatially uniform is an important problem in the history of reionization. With only
Figure 10. Parameter β versus redshift for the real data of the Lyα transmitted flux (Section 3.2; circles) and the hydrodynamic simulation result samples with UV background equations (17) and (18) (squares and triangles, respectively).

Table 2. Parameter γ at redshift z = 5.0–6.2 for simulation sample.

| z   | p   | K_p  | γ   |
|-----|-----|------|-----|
| 5.0 | 2   | -0.29±0.04 | 0.27±0.04 |
| 5.4 | 2   | -0.29±0.06  | 0.29±0.06  |
| 5.8 | 2   | -0.23±0.03  | 0.20±0.03  |
| 6.2 | 2   | -0.15±0.11  | 0.14±0.10  |

with the density distribution of cosmic baryon matter. In this context, we consider four toy models of δ(x) as follows:

model A, δ(x) = 0.1g(x), where g(x) is a Gaussian random field with variance σ^2 = 1.0;
model B, δ(x) = 0.3g(x);
model C, δ(x) is given by

\[ δ(x) = \begin{cases} \delta_0 \sin((\pi/2) \min[\log(\rho(x))/2.0, 1.0]) & \rho(x) > 1.0 \\ \delta_0 \max[\log(\rho(x))/2.0, -1.0] & \rho(x) < 1.0 \end{cases} \] (22)

where δ_0 = 0.1;
model D, the same as model C, but δ_0 = 0.3.

For models A and B, the fluctuations of the UV background are statistically independent of the IGM density field, while it is correlated to the IGM distribution for models C and D.

Because the radiative transfer has not yet been included in our simulation, we simply add the non-uniform UV background to the outputs of simulations with equation (17) and produce samples of the neutral hydrogen density field and the Lyα transmitted flux at redshift 5.4 and 5.8. As our goal is only to see whether the non-Gaussianity is affected by a fluctuating UV background, the post-processing method would be acceptable as a first try.

The β values of the Lyα transmitted flux samples based on non-uniform UV background equation (22) are listed in Table 3. Although the β values among models A, B, C and D sometimes show 1σ deviation from each other, these simplified models seem to have no effect in general. This may indicate that the effect of the inhomogeneous UV background on the log-Poisson non-Gaussian feature is not yet detectable with the current data.

6 DISCUSSION AND CONCLUSION

The non-linear evolution of baryon fluid at high redshift is a central problem of cosmology. It is well known that in the non-linear regime the dynamical behaviour of the cosmic baryon fluid does not always trace the collisionless dark matter. The non-Gaussianity of the mass and velocity fields of baryonic matter is given by the hydrodynamics of the cosmic flow. A common property of the evolution of a Navier–Stokes fluid is to reach a turbulent state when the Reynolds number is high (e.g. Zhu et al. 2010). In the scale-free range, the fully developed turbulence is statistically quasi-steady characterized by log-Poisson hierarchy. In this regime, the cosmic baryon fluid undergoes the evolution of clustering and finally falls into massive haloes of dark matter to form structures, including light-emitting objects.

The cosmic baryon fluid or the IGM, at both high and low redshifts, exhibits log-Poisson non-Gaussian features in the range from the onset scale of the non-linear evolution to the dissipation scale (i.e. the Jeans length). The log-Poisson non-Gaussianity has been identified in various simulated and observed samples of the IGM, including the mass density and velocity fields of baryonic matter, the density field of neutral hydrogen and the Lyα transmitted flux. Although these fields are different from each other, they show a common behaviour of log-Poisson non-Gaussianity. This scenario is further supported with the high-redshift Lyα transmitted flux.

Table 3. Parameter β and UV background models.

| z   | A   | B   | C   | D   |
|-----|-----|-----|-----|-----|
| 5.4 | 0.48±0.02 | 0.57±0.02 | 0.56±0.02 | 0.55±0.02 |
| 5.8 | 0.52±0.02 | 0.50±0.02 | 0.53±0.02 | 0.52±0.02 |
The log-Poisson non-Gaussian parameter $\beta$ of the mass density field on the same physical scales is found to be increasing with redshift (Feng et al. 2008). This implies that the mass density field of the IGM is less non-Gaussian at high redshifts. The log-Poisson non-Gaussianity at high redshifts should be weaker than that of low redshifts on the same physical scale. However, we have found in this paper that observed data of the Ly$\alpha$ transmitted flux at redshift $z = 5$–6 show about the same level of log-Poisson non-Gaussianity as low-redshift $z \sim 2$ data. This is because the scales covered by the data at $z = 5$–6 are smaller than those covered by the data at $z \sim 2$. The weak evolution of $\beta$ given in this paper implies that the turbulence state of the cosmic baryon fluid at $z \sim 2$ on physical scales of $1$–$10$ $h^{-1}$ Mpc is about the same as that at $z = 5$–6 on physical scales $0.1$–$1$ $h^{-1}$ Mpc. This is reasonable, considering that the Jeans length and the typical scale of the onset of non-linear evolution increase with time. The log-Poisson non-Gaussianity at low and high redshifts is about the same once the non-linear evolution is fully developed.

In a word, on the scales of fully developed turbulence, the parameter $\beta$ of the IGM at high and low redshifts should be about the same. This property is very different from the Gunn–Peterson optical depth and its variance; both show a strong redshift dependence. The Gunn–Peterson optical depth and its variance are sensitive to the intensity of the UV background. In contrast, the parameters of the log-Poisson non-Gaussianity are weakly dependent on the intensity of the UV background. A more self-consistent and delicate handling of the inhomogeneity of the UV background in the cosmic hydrodynamic simulation and much more high-quality observation data would help us to use the log-Poisson non-Gaussianity to investigate the ionizing background at high redshift.

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