Numerical Simulation and Analysis of Cave Penetration by Hydraulic Fractures

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Abstract. The cavernous carbonate reservoir in the Tahe oilfield in northwestern China has high development potential. Fluid is distributed in caves due to low matrix permeability, and recombination acid fracturing is a primary development method. The effects of fracturing depend on the number of caves connected by fractures. An estimate of whether a hydraulic fracture can penetrate a cave and continue propagation is therefore important for production safety and efficiency. We developed a numerical model using COMSOL Multiphysics to calculate the fluid-solid coupling stress distribution after fractures connect with caves and determine whether the fractures can continue to expand. We first developed an analysis group based on parameters from fracturing experiments in which caves were placed in man-made samples. A comparison of the calculation and experimental results verifies that the model is reasonable. The penetration ability of hydraulic fractures and influencing factors under different conditions were analyzed in combination with characteristics of the cavernous carbonate reservoir in the Tahe oilfield. The results show that when holding fixed certain working parameters, such as displacement and fluid viscosity during fracturing, the hydraulic fractures more easily penetrate smaller caves, the in-situ stress ratio increases, and the distance between the well and caves that would lead to a higher penetration ability decreases. If the fractures of a given cave are connected but not penetrated, several methods can be applied to effectively increase the hydraulic pressure in the cave, such as increased fracturing injection fluid displacement or replacing low-viscosity fracturing fluid. This can form new fractures in the wall that continue to expand and connect more caves. We calculated the fluid-solid coupling stress distribution after the fractures had connected caves to determine whether hydraulic fractures could also achieve penetration. The main influencing factors and optimization methods affecting the ability of fracture penetration are clarified, which provide an important reference for fracturing design in the Tahe cavernous carbonate reservoir.

1. Introduction
Carbonate reservoirs in the Tahe oilfield are unique because the formation contains numerous caves. Oil and gas resources are typically stored in such dissolution caves due to the compaction, low porosity, and permeability of carbonate rock [1]. Hydraulic fracturing is therefore generally used for cavernous carbonate reservoirs to improve production efficiency.
Previous studies on the fracturing of cavernous carbonate reservoirs mainly included numerical simulations and experiments. Jeffrey and Mills [2] studied the propagation of hydraulic fractures in shallow karst caves with developed micro-fractures using numerical simulations and showed that
increased liquid filtration can inhibit fracture propagation. Jiang [3] performed simulations using the extended discrete element method and found that the dip angle and micro-fracture spacing around a cave are the main factors that affect fracture initiation and propagation. Kaiser [4] considered that the intersection of hydraulic fractures and caves is mainly affected by the maximum and minimum horizontal ground stress differences, as well as the minimum horizontal and vertical ground stress differences. A numerical model based on the extended finite element method for the interactions between fractures and caves was established by Cheng [5] to solve the stress field of hydraulic fracturing caves and simulate the influence of karst caves on fracture propagation. Hui [6] used the FEMM-FracFlow method to analyze the propagation of fractures around a cave. Wang [7] developed the unit splitting method to calculate the propagation track of fractures in a cavernous formation. Zhou [8] performed an experimental physical fracturing model study and found that major fractures form in carbonate rocks under high-level ground stress differences, whereas small differences lead to the formation of dendritic complex fractures. In an experiment study by Wei [9], the primary fracture propagation morphology of hydraulic fracturing in carbonate rock was comprehensively affected by the lithology of natural joints and ground stress differences. Subsequently, Hou and Zhang [10-12] studied the influence on hydraulic fracture expansion in carbonate rocks in fracturing tests by considering the natural fracture development degree, fracture inclination angle, and fracturing fluid properties, and clarified the expansion morphology of hydraulic fractures in fractured carbonate reservoirs. Yang [13] calculated the path of cracks as they approached a hole defect. For cavernous formations, Liu [14] studied the disturbance of hydraulic fractures propagation by caves. Weng [15] studied the interference of eccentric caves and fracturing displacement on fracture deflection. However, the behavior of hydraulic fractures penetrating caves remains scarcely reported. In this study, we established a solid-fluid coupling model in COMSOL to simulate the ability of hydraulic fractures to penetrate a cave that considers the influence of factors such as cave size, in-situ stress, distance, viscosity, and displacement. Based on the results, we propose an optimization method for increasing the penetration ability of hydraulic fractures.

2. Analysis model for hydraulic fracture penetration of a cave

In the hydraulic fracture penetration cave analysis model under plane strain conditions, we assume that for a straight well, a hydraulic fracture starts at the wellbore along the direction of maximum horizontal stress and a cave is present in the direction of expansion. The extension of the hydraulic fracture then intersects with the cave. The model is axisymmetric, as shown in Fig. 1. The subsequent analysis is based on the premise that the fracture intersects the cave and focuses on whether the fracture can break through the cave and continue to expand. The interference of the cave to the fracture path is therefore not the key to this study. COMSOL was used for modeling and analysis. The model includes the three parts, which are introduced in the following sections.

![Figure 1. Analysis model of a penetration cave](image)

2.1 Solid mechanics module
The formation is set as a solid and the overall forces of the formation are in balance, as shown by:

\[ 0 = \nabla \cdot S_{\text{ad}} + F_Y \]  

(1)

The total stress \( S \) is expressed as:

\[ S = S_{\text{ad}} + C : \varepsilon_{\text{el}} \]  

(2)

Strain \( \varepsilon \) is given as:

\[ \varepsilon_{\text{el}} = \varepsilon - \varepsilon_{\text{inert}} \]  

(3)

\[ \varepsilon_{\text{inert}} = \varepsilon_0 + \varepsilon_{\text{ext}} \]  

(4)

\[ \varepsilon = \frac{1}{2} [(\nabla u^2)^T + \nabla u^2] \]  

(5)

The additional stress \( S_{\text{ad}} \) is given as:

\[ S_{\text{ad}} = S_0 + S_{\text{ext}} \]  

(6)

The external stress \( S_{\text{ext}} \) is caused by pore pressure \( p_1 \) in the formation:

\[ S_{\text{ext}} = -a_p p_1 I \]  

(7)

where \( S_{\text{ad}} \) is the additional stress, \( F_Y \) is the physical force, \( \varepsilon_{\text{el}} \) is the elastic strain, \( \varepsilon_{\text{inert}} \) is the inelastic strain, \( \varepsilon_{\text{ext}} \) is the external strain, \( \varepsilon_0 \) is the initial strain, \( C \) is the elastic parameter, \( u^2 \) is the displacement field, \( S_0 \) is the initial stress, \( S_{\text{ext}} \) is the external stress, and \( a_p \) is the Biot coefficient.

2.2 Seepage module

We set the formation as a pore elastomer and the fracturing fluid is injected from the wellbore fracture at a fixed rate. Due to increased fracture pressure, the fluid will leak into the formation. The fluid in the cave will also exchange with the formation fluid under the pressure difference. Darcy flow is used to describe the fluid flow in the formation:

\[ u_1 = -\frac{k_2}{\mu} \nabla p_1 \]  

(8)

In COMSOL, the water storage model is used to describe transfer in the formation:

\[ \frac{\partial}{\partial t} (\varepsilon_{p_2} p) = \rho S \frac{\partial p_1}{\partial x} \]  

(9)

The water storage capacity of the formation depends on the integrated compression coefficient matrix:

\[ S = \varepsilon_{p_2} \chi_f + (1 - \varepsilon_{p_2}) \chi_{p_1} \]  

(10)

The length of the hydraulic fracture is substantially larger than its width, thus the flow in the fracture is simplified to fracture flow, which includes flow along the direction of the fracture length and filtration perpendicular to the fracture direction:

\[ d_f \frac{\partial}{\partial t} (\varepsilon_{p_2} p) + \nabla \cdot (d_f \rho u_1) = d_f Q_m \]  

(11)

\[ u_2 = -\frac{k_2}{\mu} \nabla p_1 \]  

(12)

The filtration loss occurring on the fracture wall surface is controlled by:

\[ u_2 n_{f+} = -q_{f+}, \text{ on } G_f^+ \]  

(13)

\[ u_2 n_{f-} = -q_{f-}, \text{ on } G_f^- \]  

(14)

where \( d_f \) is pore diameter, \( \varepsilon_{p_2} \) is fracture porosity and equal to 1, and \( k_2 \) is fracture permeability.

2.3 Flow module

If complex scenarios are ignored, fluid flow in caves can be simplified as a steady flow, which satisfies the classical flow equation. The permeable layer model is adopted to describe the exchange between the fluid in the cave and in the formation:

\[ -n \cdot \rho u_1 = \rho \gamma_b \begin{pmatrix} \frac{p_2 - p_3}{\rho g} \end{pmatrix} \]  

(15)

where \( p_2 \) is the fluid pressure in the cave and \( \gamma_b \) is the fluid conductivity.

Due to the model symmetry setting, fracture expansion occurs along the direction of the maximum
ground stress and no turning behavior occurs. Hence, it is only necessary to consider whether the fracture meets the expansion conditions. The stress intensity factor is used to analyze whether the crack expands:

\[ K \geq K_{IC} \]  

(16)

where \( K \) is the stress intensity factor for the hydraulic fracture and \( K_{IC} \) is the critical value.

When the fracture intersects a cave, the fluid pressure in the cave increases. When the circumferential stress at the maximum point on the wall meets the broken wall condition, a new fracture will start from this point and continue to expand. A pressure threshold is then set in the wellbore. Fracture expansions stops when the pressure reaches the critical value.

2.4 Model verification with experimental results

To verify the accuracy of the model, we compare our results with physical simulation experimental results by Liu [14], in which fracture-cave intersection was considered. In Liu’s experiment, caves of different sizes were set in artificial rock samples located in the path of fracture pre-expansion. It was observed that fractures penetrated smaller caves but could not penetrate larger ones (Fig. 2).

It should be noted that the samples in Liu’s experiment were synthetic rock with mechanical properties (e.g., elasticity modulus, Poisson's ratio) and seepage characteristics (e.g., porosity, permeability) similar to the carbonate rock in the Tahe oilfield. The parameters obtained from the experiments can thus be directly used for simulation.

Based on the experimental parameters shown in Table 1, we used numerical models to simulate the behavior of the fracture-penetrating cave in the experiments. The tensile strength of the carbonate rock was set to 20 MPa. According to the pump pressure curve in the experiments, the wellbore pressure threshold was set to 30 MPa.

| Table 1. Parameters in Liu’s experiments [14] |
| Number | 1# | 2# |
| In-situ stress(\( \sigma_{in} / \sigma_{H} \), MPa) | 16/14/7 |  |
| Cave diameter (cm) | 2/5 | 5/5 |
| Cave pressure (MPa) | 0 |  |
| Distance between cave and wellbore (cm) | 5.5 |  |
| Displacement (ml/min) | 20 |  |
| Viscosity (mPa-s) | 10 |  |
| Elastic parameter (E/\( \nu \)) | 46.3/0.12 |  |
| Results in two sides | penetrate/stop | stop/stop |

Figure 2. Experimental results by Liu [14]

The simulation results based on the above parameters are shown in Fig. 3. Fig. 3a shows the simulation result of sample 1. It can be seen that the hydraulic fracture failed to penetrate the caves...
after making contact. This is because the wellbore pressure had reached the threshold before the wall stress in the cave reached the broken condition, which stopped the injection.

In the simulation result of sample 2 is shown in Fig. 3b. With a continuous injection for 0.7 s after the fracture intersected both sides of caves, a point meeting the broken condition appeared on the wall of the cave on the left; a new fracture then appeared and continued to expand. At this time, the wellbore pressure was 16.57 MPa. As the left fracture touched the boundary, the right cave could not reach the fracture initiation pressure.

The findings show that the rationality model is well verified by the simulation and experimental results of whether the hydraulic fracture can penetrate caves of different characteristics.

3. Analysis of factors affecting the ability of hydraulic fractures to penetrate caves

Based on the established analysis model, this section further addresses the factors that affect the ability of hydraulic fractures to penetrate caves to provide support for improving this effect.

Several factors affect the penetration ability of a hydraulic fracture into a cave. For cavernous formations, the cave size and location are important as well as the in-situ stress, all of which have a notable effect on the stress distribution around the caves. Moreover, the displacement and viscosity of the injected fluid affect the net pressure within the fracture, which can limit the penetration behavior. We mainly study the above influence factors. Additional factors remain (e.g., rock mechanical properties, seepage properties) but generally speaking, for the carbonate rocks in the Tahe oilfield, the rock properties change little due to the matrix density. These parameters are therefore not considered here. The remaining parameters are listed in Table 1 and 30 MPa was used as the wellbore pressure threshold to determine the fracture stop.

In the following simulation, the variable of study is set as the horizontal axis and the wellbore pressure when the fracture penetrates the cave is set in the left y-axis as the red line and legend. The time from the intersection occurrence until penetration is set as the right y-axis in blue. An additional green line is set in each figure as the critical value of 30 MPa.

3.1 Effect of cave size

In this section, we hold the distance between the center of the cave to the wellbore fixed and investigate the influence of cave size on the ability of hydraulic fractures to penetrate a cave. The results are used as the basis for a comparison of subsequent studies.

In this group, five control groups were set: in each sample, the cave sizes were equal with a diameter of 5, 4, 3, 2, or 1 cm. If the maximum circumferential stress of the cave wall reached the crack initiation pressure before the wellbore pressure reached the safe value, the fracture would penetrate the hole; otherwise it would not.
According to the simulation results in Fig. 4, the cave with a 4-cm diameter was the largest size that could be penetrated under a safe pressure of 30 MPa. The time required for the hydraulic fracture to penetrate the cave increased with increasing cave size, and the wellbore pressure at the penetration time also increased. For a large cave, when the fracturing fluid was injected at a fixed rate, the ability to increase the pressure in the cave was lower than in a small cave. A larger contact area between the fluid and wall surface led to a higher leakoff rate, thus it was more difficult to penetrate a large cave.

### 3.2 Effect of in-situ stress

In the previous group, the 4-cm diameter cave was the largest size that could be penetrated by the fracture. The cave size in this group and those presented in sections 3.3 and 3.4 was thus set to 4 cm. The influence of the penetrating ability of the hydraulic fracture was studied by changing the stress difference. Two comparison groups were set in which the maximum and minimum in-situ stresses were changed to study the effects and differences.

![Figure 5. Relationship between in-situ stress and penetration ability](image_url)

In Fig. 5, the square symbols represent the results of varying the maximum stress. The minimum ground stress remained unchanged and the increased maximum ground stress made it easier for the wall surface to reach the broken pressure after intersection and reduced the required time. In contrast,
the ability of the fracture to penetrate the cave decreased. The circular symbols in Fig. 5 represent the influence of changing the minimum stress. A decrease of minimum ground stress made it easier to form new fractures on the wall surface, whereas increased minimum ground stress led to a rapid increase of the time and pressure for the cracks to breakthrough. The influence of the minimum and maximum stress show quadratic and primary relationships, respectively. The influence of the minimum stress on the penetration behavior is therefore more apparent than the maximum stress.

3.3 Effect of distance
In this group, the distance from the cave to wellbore was varied, as shown in Fig. 6.

![Figure 6. Relationship between distance and the ability of penetration](image)

Increased distance led to a longer time required for the fracture to break into the cave. Farther distances were also associated with higher filtration and deformation on the fracture wall surface. Compared with the significant change in penetration time, the pressure required for the fracture penetration increased only slightly and all were under a safe pressure range. Due to the small model size, increased fracture wall area had relatively little influence. In an infinite stratum, a cave with a long distance to the wellbore is expected to lose a considerable amount of hydraulic energy due to the filtration and deformation of the wall surface after intersection, thus higher pressure would be required for penetration.

3.4 Effect of viscosity and displacement
Differing from unchangeable geological characteristics, displacement and viscosity as the main fracturing fluid parameters can be actively adjusted to affect fracturing.
Figure 7. Relationship between viscosity and penetration ability

The effect of fracturing fluid viscosity is shown in Fig. 7. With increasing fracturing fluid viscosity, considerably less time is required for a rupture to occur after intersection and follows a quadratic relationship. This was because the amount infiltrated into the formation during the fracturing fluid injection process is less for a high-viscosity liquid, which promotes a stronger increase of pressure in the cave.

Figure 8. Relationship between displacement and penetration ability

The impact of displacement is shown in Fig. 8. Reduction displacement resulted in a larger amount of fracturing fluid filtration over the same time, which thus allowed more time for the pressure to concentrate and ultimately penetrate.

Changes in the fracturing fluid properties strongly influenced the penetration time. However, due to the small model size, low formation permeability, and low fluid penetration rate into the formation, the influence of the fluid performance on the penetration ability was relatively small. In actual formations, however, high seepage zones exist due to geological heterogeneity, thus displacement and viscosity changes could lead to a high filtration rate, which would bring about a large change and indeterminacy in the penetration ability.

4. Discussion and results
(a) In this study, all simulated caves were represented by a circle. However, cave shapes in actual
stratum are more complex. Limited by the diversity of cave morphology and precision of the geological survey, cave morphology can be difficult to accurately describe. A circle was therefore used for simplicity.

(b) During fracturing, even if there is no cave in the formation, the fracture can only extend to a limited extent. Due to the compactness of carbonate rock, the spread distance of unilateral fractures is usually within several tens of meters. For fracturing in cave-containing formations, if the target cave is far from the well location, the fracture cannot intersect it, which demonstrates the importance of the well-location selection.

(c) The fracture penetration capacity is limited owing to the fracturing pump power and safe pressure limitations. Cave size plays a decisive role; when exceeding a certain size, the hydraulic energy is mostly dissipated in the wall surface filtration. Thus regardless of optimization, it will be difficult for the fracture to penetrate the cave. In the case that multiple caves are connected in a formation, it is often impossible to break through and continue to expand after the fracture intersects a given cave.

(d) We established a coupled solid-seepage-flow model and analyzed the main factors that affect the fracture penetration ability after intersecting with a cave. The model accuracy was verified by experiments. The influence of the propagation path of fractures disturbed by caves is the focus point of future research.

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