Chemical Impact on Heat and Mass Transfer Flow in Presence of Radiation and Rotation with Variable Temperature and Concentration

Sujan Sinha, Manoj Kr. Sarma

Abstract: A parametric study to investigate the effect of chemical reaction parameter on an MHD mixed convective mass transfer flow of an incompressible viscous electrically conducting fluid past an infinite vertical porous plate. The equations of motion are worked out by assuming Laplace Transform approach. The velocity profile, temperature, concentration, viscous drag, Nusselt number and the rate of mass transfer are discussed graphically by assuming some arbitrary criterion given in the present paper and physical descriptions are made. It is emphasized from the graphical portion that chemical species retards the fluid flow.

Keywords: MHD, mass transfer, Sherwood number, chemical reaction parameter.

I. INTRODUCTION

The science of activities through which all the magnetic and chemical assets and behavior of fluids which are electrically conducted is termed as MHD. For the last several decades, MHD has attracted numerous Scientists, Biotechnologists, engineers because of its fascination and importance in various technological devices, application in drugs and biology and understanding the diverse cosmic phenomena. Besides it provides a simplified and mathematically distinguishable picture of the more complicated domain of Plasma Physics. There are various types of research papers in which the field of MHD is concerned with heat and mass transfer. In this regard Ahmed [1], and Singh and Singh [2] have presented MHD effects with heat and mass transfer. Chemical reaction is a route which transforms chemical properties between two substances. The impact of chemical reaction has got exceedingly sensible importance in the practical field of different areas of science and technology. The effect of chemical reaction under the fluid characteristics with or without MHD have studied by Anderson et.al [3] and Apelblat [4].

The geophysical importance of the flows in rotating frame of reference has involved the interest of many researchers. Mazumder et al. [5], Soundalgekar and Pop [6] and Gupta [7] have studied various consequences of magnetic intensity with or without suction on different fluid properties. Radiative heat and mass transfer play a significant position in modern industries. For the higher temperature of the encircling field, the radiation impact plays a crucial role in space technology.

Many researchers such as Ahmed et.al. [8], Rajesh and Varma [9], Hussain and Takhar [10] have made an investigation to study the prospects of radiation in various convective flows. To the best of the knowledge of present authors till now no attempt has been made to study the effect of mass transfer and chemical reaction on an MHD mixed convective flow past an infinite vertical porous plate following the technique adopted by Rajput and Kumar [11]. The present work is an extension to the work done by U.S. Rajput and Surendra Kumar [11] with the inclusion of mass transfer and chemical reaction.

II. MATHEMATICAL FORMULATION

In this paper a three dimensional flow with variable temperature and concentration under the action of applied magnetic field in a rotating frame is incorporated. With the standard assumptions made by Ahmed et.al. [8], the physical situations are described by the following governing equations:

\[
\frac{\partial u'}{\partial t'} - 2\Omega u' = g\beta(T' - T_0') + \nu \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^2 u'}{\rho} + g\beta(C' - C_a)
\]

\[
\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial z'^2} \quad \text{with} \quad \frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + K\nu' (C'_a - C')
\]

Parameters used above have their usual meaning explained by N. Sandeep et.al. [12], with the following boundary conditions:

For \( t' \leq 0 \)
\[ u' = 0, \quad T' = T'_a, \quad C' = C'_a \quad \forall \ z' \]

For \( t' > 0 \)
\[ u' = \frac{m_1}{\nu'}, \quad T' = T'_a + (T'_a' - T'_a) \frac{v}{v'}, \quad C' = C'_a + (C'_a' - C'_a) \frac{v}{v'} \quad \forall \ z' = 0 \]
and \( u' \rightarrow 0, \quad T' \rightarrow T'_a, \quad C' \rightarrow C'_a \quad \text{as} \ z' \rightarrow \infty \)

Local radiation of optically thin gray gas is given by
\[ \frac{\partial q}{\partial z} = -4a \left( T'^{4} - T'^{4} \right) \]

\[ a^* \rightarrow \text{absorption constant} \]

Considering Temperature difference inside the flow adequately small, expanding \( T'^{4} \) using Taylor series about \( T'_{0} \) by neglecting higher powers we obtain,

\[ T'^{4} \equiv 4T'^{4} T' - 3T'^{4} \quad \rightarrow (7) \]

Using (6) and (7) in (3) we have

\[ \rho C_p \frac{\partial T'}{\partial z} = K \frac{\partial^2 T'}{\partial z^2} + 16a^* \sigma T'^{3} (T' - T') \]

To normalize the flow problem, the following non-dimensional terms are utilized.

\[ u' = \frac{u - u_0}{\mu \Omega}, V' = \frac{V}{\mu \Omega}, t' = t \frac{u_0}{\mu \Omega}, z' = \frac{z - u_0 t}{\mu \Omega}, Gr = g \beta_0 (T'_0 - T'_v) \]

Equations (6) – (9) are compiled to make the equations (1) - (4) in the following form:

\[ \frac{\partial \Theta}{\partial \xi} - 2\Theta = \frac{\partial}{\partial \xi} \frac{\partial}{\partial \xi} - Mq + \Phi \frac{\partial \Gamma}{\partial \xi} \quad \rightarrow (10) \]

where \( M = \frac{\sigma_{\text{Ll}} \beta_0 \mu \Omega}{\rho \mu} \)

\[ \frac{\partial \Phi}{\partial \xi} + 2\Omega \frac{\partial \Phi}{\partial \xi} = \frac{\partial^2 \Phi}{\partial \xi^2} - Mq \quad \rightarrow (11) \]

\[ \frac{\partial \Theta}{\partial \xi} = \frac{1}{\rho \frac{\partial \Theta}{\partial \xi}} \frac{1}{\rho \frac{\partial \Theta}{\partial \xi}} \quad \rightarrow (12) \]

\[ \frac{\partial \Phi}{\partial \xi} = \frac{1}{\rho \frac{\partial \Phi}{\partial \xi}} \frac{\partial \Phi}{\partial \xi} \quad \rightarrow (13) \]

From (10) and (11) using \( q = u + iv, \ m = M + 2i\Omega q \)

\[ \frac{\partial^2 \Phi}{\partial \xi^2} + q = \frac{\partial q}{\partial \xi} + \Phi \frac{\partial \Gamma}{\partial \xi} - Gr \Theta - Gm \Phi \]

With\n
\[ \text{For } \xi \leq 0, \ q = 0, \ \Theta = 0, \ \Phi = 0 \quad \forall \ z \]

\[ \text{for } \xi > 0 \ \begin{cases} \ q = 1, \ \Theta = 1, \ \Phi = 0 \text{ when } z = 0, \\ \text{and } q = 0, \ \Theta = 0, \ \Phi = 0 \text{ when } z \rightarrow \infty \end{cases} \quad \rightarrow (15) \]

\[ \frac{d^2 \Phi}{dz^2} - Pr(s + C) \Phi = 0, \text{ where } C \times R \quad \rightarrow (17) \]

\[ \frac{d^2 \Phi}{dz^2} - S \left( s + Kr \right) \Phi = S \Phi \quad \rightarrow (18) \]

The boundary conditions under Laplace Transform are

\[ \Phi = \frac{1}{s} \left( 1 - e^{-s} \right), \Phi = \frac{1}{s^2} \left( 1 - e^{-s} \right) \quad \text{at } z = 0 \]

The solutions of the equations (16) – (18) using (19) are obtained as:

\[ \Phi = \frac{1}{s^2} \left( 1 - e^{-s} \right) e^{-\sqrt{Sc(s + Kr)} \xi} \quad \rightarrow (20) \]

\[ \Phi = \frac{1}{s} \left( 1 - e^{-s} \right) e^{-\sqrt{Sc(s + Kr)} \xi} \quad \rightarrow (21) \]

Taking inverse Laplace Transforms of equations (20), (21) and (22) we have:

\[ \theta = \psi_1 - \psi_2 \]

\[ \phi = \psi_3 - \psi_4 \]

\[ q = \psi_3 + Pe^{-\gamma} \left[ \psi_4 - e^{\gamma} \psi_3 \right] - U \psi_3 + T \psi_3 + R e^{-\gamma} \left[ \psi_4 - e^{\gamma} \psi_3 \right] \quad \rightarrow (25) \]

Now skin friction (\( \tau \)), Nusselt number (\( Nu \)) and the Sherwood number (\( Sh \)) in non dimensional form are expressed as:

\[ \tau = \left( \frac{\partial \Phi}{\partial \xi} \right)_{\xi = 0} = \Phi_2 - \Phi_1 \]

\[ Nu = - \left( \frac{\partial \Theta}{\partial \xi} \right)_{\xi = 0} = \Phi_4 - \Phi_3 \]

IV. DISCUSSION AND CONCLUSION

In the present discussion, all the calculations of fluid characteristics are compiled to get different kinds of graphical representation with their physical interpritations by taking some arbitrary values of different parameters. Figure 1 and 2 demonstrate the effect of magnetic intensity and chemical species on primary fluid velocity. It is observed that due to Lorentz force and buoyancy effects, the fluid motion is retarded. The effects of rotation and mass diffusivity accelerated the velocity profile and reduce the same which is inferred in figure 3 and 4. The asymptotically falling characters of temperature under thermal conductivity and radiation are shown in figure 5 and 6.
It is seen from figure 7 and 8 that the concentration level of fluid drops for higher values of chemical reaction parameter \( Kr \) and Schmidt number \( Sc \).

The viscous drag is minimized under the action of applied magnetic field and consumption of chemical species which is observed from figure 9 and 10.

It is explained from figure 11 – 14 that the coefficient of rate of heat and mass transfer gets enhanced due to thermal conductivity, thermal radiation, mass transport coefficient and chemical reaction.

**APPENDIX**

\[
\psi_r = \beta(Pr, C, z, t)
\]

\[
= \left( \frac{1}{2} \right) \left( \frac{1}{2} \sqrt{\frac{2\xi}{4\eta}} \right) e^{-\frac{\xi^2}{2\eta}} \text{erfc} \left( \frac{\xi}{\sqrt{2\eta}} \right)
\]

\[
\psi_2 = \beta(Gr, M, z, t - 1) H(t - 1)
\]

\[
\psi_r = \gamma(Sc, Kr, z, t - 1) H(t - 1)
\]

\[
\psi_0 = f(m, z, t) = \frac{1}{2} e^{\frac{\xi^2}{2\eta}} \text{erfc} \left( \frac{\xi}{\sqrt{2\eta}} + \sqrt{\eta t} \right) + e^{\frac{\xi^2}{2\eta}} \text{erfc} \left( \frac{\xi}{\sqrt{2\eta}} - \sqrt{\eta t} \right)
\]

\[
\psi_6 = f(\xi, z, t) \quad \xi = m - O
\]

\[
\psi_7 = f(m, z, t) = \left[ \left( \frac{1}{2} \right) \left( \frac{1}{2} \sqrt{\frac{2m}{4\eta}} \right) e^{-\frac{m^2}{2\eta}} \text{erfc} \left( \frac{m}{\sqrt{2\eta}} - \frac{\sqrt{\eta t}}{2\eta} \right) \right]
\]

\[
\psi_0 = f(\xi, z, t - 1) H(t - 1)
\]

\[
\psi_10 = f(m, z, t - 1) H(t - 1)
\]

\[
\psi_11 = f_1(m, z, t - 1) H(t - 1)
\]
\[ \psi_{12} = f(\xi_1, z, t), \quad \xi_1 = m \cdot O_1 \]
\[ \psi_{13} = f(\xi_2, z, t - 1)H(t - 1) \]
\[ \psi_{14} = \psi_5 - \psi_{10}, \quad \psi_{15} = \psi_7 - \psi_{10}, \quad \psi_{16} = \psi_7 - \psi_{11} \]
\[ K = \frac{Pr - 1}{L}, \quad L = \frac{Pr \cdot C - m}{N} = \frac{Gr}{K}, O = \frac{L}{K} \]
\[ K_1 = \frac{Sc - 1}{L}, \quad L = \frac{Sc \cdot K \cdot m - N_1}{N_1} = \frac{Gr}{K_1}, O = \frac{L_1}{K_1} \]
\[ P = \frac{N}{O^3}, \quad Q = N \cdot O, \quad R = \frac{N}{O^2}, \quad S = N_1 \cdot O_1 \]
\[ \phi = \phi(Pr,C,t) = -\sqrt{Pr} \left[ \frac{1}{\pi} e^{-Ct} - \left( \sqrt{Ct} + \frac{1}{2C} \right) \sqrt{Pr} \text{erf} \left( \sqrt{Ct} \right) \right] \]
\[ \bar{\phi} = \phi(Pr,C,t - 1)H(t - 1) \]
\[ \bar{\phi} = \phi(Sc,Kr,t) \]
\[ \bar{\phi} = \frac{1}{\sqrt{m}} \left( e^{-m} - \frac{m}{2 \sqrt{m}} \right) \text{erf} \left( \frac{m}{\sqrt{m}} \right) \]

**Laplace transform Results**

1. \( L^{-1} \left\{ \frac{1}{s^2} e^{-\sqrt{s} \alpha} \right\} = t \left[ 1 + \frac{\sqrt{y}^2}{2t} \right] \text{erfc} \left( \frac{\sqrt{y}}{2\sqrt{t}} \right) - \frac{\sqrt{y} e^{\sqrt{at}}}{\sqrt{4t}} \]
2. \( L^{-1} \left\{ \frac{e^{-\sqrt{s} \alpha}}{s} \right\} = \frac{1}{2} e^{-\sqrt{at}} \text{erfc} \left( \frac{\sqrt{y}}{2\sqrt{t}} \right) + e^{-\sqrt{s} \alpha} \text{erfc} \left( \frac{\sqrt{y}}{2\sqrt{t}} - \sqrt{at} \right) \]
3. \( L^{-1} \left\{ \frac{e^{-\sqrt{s} \alpha}}{s} \right\} = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} e^{\sqrt{at}} \text{erfc} \left( \frac{\sqrt{y}}{2\sqrt{t}} + \sqrt{at} \right) + e^{-\sqrt{s} \alpha} \text{erfc} \left( \frac{\sqrt{y}}{2\sqrt{t}} - \sqrt{at} \right) \right] \]
4. \( L^{-1} \left\{ \frac{e^{-\sqrt{s} \alpha}}{s} \right\} = \text{erfc} \left( \frac{\sqrt{y}}{2\sqrt{t}} \right) \]
5. \( \text{erfc}(x) - \text{erfc}(-x) = 2 \text{erfc}(x) \]
6. \( \text{erfc} \left( \sqrt{at} \right) - \text{erfc} \left( -\sqrt{at} \right) = -\frac{1}{\sqrt{\pi}} e^{-a^2} \]

**REFERENCES**

1. Ahmed N (2010): MHD free and forced convection with mass transfer from an infinite vertical porous plate. *Journal of Energy, Heat and Mass Transfer*, 32, pp. 55-70.
2. Singh, N.P. and Singh, Atul Kr. (2000): MHD effects on heat and mass transfer in a flow of viscous fluid with induced magnetic field. *Indian Journal of Pure and Applied Physics*, 38, pp. 182-189.
3. Anderson, H.I., Hansen, O.R., and Holmedal, B. (1994): Diffusion of a chemically reactive species from a stretching sheet. *International Journal of Heat and Mass Transfer*, 37, 659-664.
4. Apelblat, A. (1982): Mass transfer with a chemical reaction of the first order effect of axial diffusion, *The Chemical Engineering Journal*, 23, 193-203.
5. Mazumdar B. S., Gupta A. S. and Datta N. (1976): Flow and heat transfer in a hydromagnetic Ekman layer on a porous plate with wall effects. *Int. J. Heat Mass Transfer*, 19, pp. 523-27.
6. Soundalgekar V. M. and Pop I. (1973): On hydromagnetic flow in a rotating fluid past an infinite porous wall, *ZAMM*, 53, pp. 718-19.
7. Gupta A. S. (1972): Magnetohydrodynamics Ekman layer, *Acta Mechanica*, 13, pp. 155-160.
8. Ahmed, N. and Sharma, H.K. (2009): The radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate, *Int. J. of Appl. Math. And Mech.*, 5(5), pp. 87-98.
9. Rajesh, V. and Varma, S.V.K. (2010): Radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion, *Int. J. of Appl. Math. And Mech.*, 6(11), pp. 39-57.
10. Hussain MA and Takhir HS (1996): Radiation effect on mixed convection along a vertical plate with uniform surface temperature. *Heat Mass Transfer*, 31(4), pp. 243-248.
11. Raptis, U.S. and Kumar, S. (2011): Rotation and Radiation Effects on MHD flow past an impulsively Started Vertical Plate with variable temperature, *Int.J. of Math. Analysis*, 5(24), pp. 1155-1163.
12. Sandeep N., Reddy A. V. B. and Sugnamma V., (2012): Effect of Radiation and Chemical Reaction on Transient MHD free convective flow over a vertical plate through Porous media. “Chemical and Process Engineering Research” Vol. 2.

**AUTHORS PROFILE**

Dr. Sujuan Sinha, a resident of the city of Assam, earned his M.Sc. from the prestigious Gauhati University in 2010 and Ph.D. in the area of Fluid Dynamics under the supervision of Dr. G.C. Das. He has contributed his expertise to the field of Mathematics in Assam Down Town University, Panikhaiti, Guwahati-781026. His research work has been acknowledged in various national and international conferences/seminars. Furthermore, Dr. Sinha has been involved in the workshop “KIC-TEQIP WORKSHOP” on Mathematical Analysis and its Application to Engineering, conducted by the Department of Mathematics, IIT Guwahati, and the workshop “North-East Summer workshop in analysis and probability,” conducted by the Department of Mathematics, Rajiv Gandhi University, Arunachal Pradesh, India. Moreover, he was appointed as an external examiner for Ph.D. Progress Report presentation in University of Science & Technology, Meghalaya. Dr. Sinha has more than 8 years of experience in carrying higher level of research in Mathematics. By date, 4 (four) students have obtained Ph.D. and 2 (two) students are pursuing their research work under his supervision.

Mr. Manoj Kumar Sarma, a resident of the state of Assam, currently pursuing his research work under the supervision of Dr. G.C. Das and Dr. C. Uberoi (Deptt. of Mathematical Sciences, IIT Guwahati). He has contributed to various reputed international journals. He has presented (Oral/Poster) 13 papers in various international and national conferences/seminars. Furthermore, Mr. Sarma has been involved in the workshop “ISTEP project under supervision of Dr. G.C. Das” and Dr. C. Uberoi (Deptt. of Mathematical Sciences, IIT Guwahati). He has published a paper with Dr. G.C. Das on “Evolution of ion-acoustic solitary waves in an inhomogeneous discharge plasma” published in the proceedings of the Plasmas (USA) vol. 7, 3964 (2000). He worked as a faculty member of Career Launchers, Guwahati branch, for MBA (CAT) and IIT (JEE) coaching from 2001 to 2008.
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