A NOVEL METHODOLOGY FOR PORTFOLIO SELECTION IN FUZZY MULTI CRITERIA ENVIRONMENT USING RISK-BENEFIT ANALYSIS AND FRACTIONAL STOCHASTIC

YAHIA ZARE MEHRJERDI
Department of Industrial Engineering, Yazd University, Yazd Iran

(Communicated by Gerhard-Wilhelm Weber)

ABSTRACT. This article proposes an efficient approach for solving portfolio type problems. It is highly suitable to help fund allocators and decision makers to set up appropriate portfolios for investors. Stock selection is based upon the risk benefits analysis using MADM approach in fuzzy environment. This sort of analysis allows decision makers to identify the list of acceptable portfolios where they can assign some portions of their asset to them. The purpose of this article is two folds; first, to introduce a methodology to select the list of stocks for investment purpose, and second, to employ a stochastic fractional programming model to assign money into selected stocks. This article proposes a hybrid methodology for finding an optimal or new optimal solution of the problem. This hybrid approach considers risks and benefits at the time of stocks prioritization. This is followed by solving a fractional programming to determine the percentages of the budget to be allocated to stocks while dealing with two sets of suitable and non-suitable stocks. For clarification purposes, a sample example problem is solved.

1. Introduction. Markowitz is the first researcher developed a bi-criterion mathematical formulation for portfolio problem taking return maximization function and variance minimization function into account. Consideration of variance as risk has caused some researchers to criticize this kind of modeling of the problem. There are other researchers who followed this type of modeling and expansion of the problem taking all possible restrictions of real world problems into consideration. Others have defined new objective functions to have new measuring tools for evaluation purposes. Researchers Jia and Dyer (1996) shown that mean-variance approach can lead to an optimal solution when investment returns are symmetrically distributed. As Jia and Dyer claimed (1996), it seemed that Markowitz assumptions (1952, 1956, and 1959) are unrealistic due to the facts that it rejects possible asymmetry in return distributions. Indeed, the asymmetry investment return is more common, especially in the stock and bond markets.

Researchers have used two, three, and sometimes four objective functions to model a portfolio type problem. The objective functions used are: (1) maximizing return on portfolio, (2) minimizing portfolio risks, (3) maximizing short term return, and (4) maximizing the dividend. To deal with uncertainty, fuzzy approach is often used in portfolio modeling. Stochastic modeling of the problem using chance
constrained programming (Charnes and Cooper, 1962), scenario based approaches (Guastaroba, 2012, Liesio and Salo, 2012), bi-level stochastic programming (Ma, 2016), and fuzzy chance constrained programming (Liu, 2001a, 2001b) are also used for portfolio modeling. Models using crisp and fuzzy chance constraints for dealing with double inequality constraints have also been employed in the formulation of portfolio type problem (2009). To solve such problems, authors converted each chance constraint into its equivalent deterministic form constraint (EDF) (Charnes and Cooper, 1962, 1963) and then solved the created deterministic model instead. For fuzzy situations, similar conversion to crisp equivalent is also conducted just before starting to solve the equivalent deterministic form of the problem.

Mizgier, K. et al (2017) proposed a multi-objective model for capital allocation for supplier development under risk. Researchers have used an example of a global car manufacturer and support the decision-making process with data downloaded from the Bloomberg database. They used stock market returns and cost of capital of suppliers to assess their performance. In 2016, Mizgier et al. conducted a study on multi-objective Optimization of Credit Capital Allocation in Financial Institutions. Authors introduced a methodology for optimal credit capital allocation by focusing on the efficient allocation of capital to business lines characterized by credit risk losses and cost of capital. Qu et al. (2017) studied on large scale portfolio optimization using multi objective evolutionary algorithms and pre selection methods. Soleymani, F. and Paquet (2020) studied on financial portfolio optimization with online deep reinforcement learning and restricted stacked auto-encoder-Deep-Breath. Ivanova, M., Dospatleiv, L. (2017) concentrated on the application of Markovitz portfolio optimization on Bulgarian stock market from 2013 to 2016. Doumpos, M., and Zopounidis (2020) studied multi-objective optimization models in finance and investment while Simo-Kengne et al. (2018) studied behavioral portfolio selection and optimization with its application to international stocks. De Prado, et al. (2019) conducted a research by proposing an optimal risk budgeting under a finite investment horizon.

The remaining of article is organized as follow. Background of study is discussed in section 2. Research contribution and gap analysis is discussed in section 3 while research methodology is discussed in section 4. Criteria selection for stock selection is the topic of section 5. Section 6 is devoted to the case study. Simple Additive Weighting (SAW) Method is discussed in section 7. A fractional programming model for fund allocation is the topic of section 8. Goal programming formulation of the model is presented in section 9 while authors conclusion is given in section 10.

2. Background.

2.1. MADM. Multi criterion decision making is comprised of two broad fields of decision making known as Multiple objective decision making (MODM) and multi-attribute decision making (MADM). By using MADM, it is possible to obtain the most attractive solution considering all alternatives and criterions into consideration. In TOPSIS, the logic is based upon two solution points namely, positive ideal solution point (PISP) and negative ideal solution point (NISP). Alternatives to be ranked are evaluated based upon the relative similarity to these ideal solution points in such a way that alternative should have largest distance from the NISP and smallest distance to PISP. Kahrman and his co-researchers (2007) introduced a hierarchical fuzzy TOPSIS method with the ability of considering the hierarchy
among the attributes and alternatives. Baykasoglu and Golcuk (2017) proposed a novel multiple-attribute decision making model via fuzzy cognitive maps and hierarchical fuzzy TOPSIS. In another study, Baykasoglu, Kaplanoglu, Durmusoglu, and Sahin (2013) developed an Integrating fuzzy DEMATEL and fuzzy hierarchical TOPSIS methods for truck selection. Researches Can and Demirok (2019) proposed universal usability evaluation using an integrated fuzzy multi criteria decision making. Özceylan, Eren, Kabak, Mehmet, and Dagdeviren, Metin (2016) employed Fuzzy ANP and PROMETHEE approaches for machine selection purposes. Zare Mehrjerdi (2013) utilized hierarchical fuzzy TOPSIS approach to study risk-benefit analysis in library systems. Amiri and Zare Mehrjerdi (2020) used fuzzy TOPSIS and food system security and sustainability in Iran. Babae and his co-researchers (2019) employed fuzzy decision making and multi-objective programming for selecting a sustainable-reliable supplier. To show the applications of MADM methods in portfolio study, this author searched the literature and found that there are a large number of articles using the crisp and fuzzy variants of MADM approaches as such as TOPSIS, AHP, ANP, ELECTRE, PROMETHEE, compromise programming, and others. Obviously, there are many more articles relating to some of the variants of MADM approaches that are not presented in Table 1.

![Table 1. Sample of variants of MADM approaches](image)

| Variants of Fuzzy TOPSIS and MADM | Authors and years of publications |
|-----------------------------------|----------------------------------|
| 1 Fuzzy TOPSIS                   | Mirabi et al. (2012), Baykasoglu and Golcuk (2015) |
| 2 Hierarchical fuzzy TOPSIS      | Zare Mehrjerdi (2020), Baykasoglu (2013) |
| 3 Type-2 fuzzy TOPSIS            | Baykasoglu and Golcuk (2017) |
| 4 Interval value fuzzy TOPSIS    | Zare Mehrjerdi (2013) |
| 5 Group DM Fuzzy TOPSIS          | Wange (2008), Zare Mehrjerdi (2013) |
| 6 AHP and Fuzzy AHP              | Aydein Celen (2014) |
| 7 ANP and Fuzzy ANP              | Fahlavan et al. (2012), Amiri, M. (2010) |
| 8 ELECTRE                        | Chen-Tung-Chen (2009), Thien Phue Ho Quang (2014) |
| 9 Compromise Programming          | Bilbao-Terol et al. (2006) |
| 10 PROMETHEE                     | Fallahpour et al. (2014) |

2.2. Chance Constrained Programming. A subject matter originally developed by Charnes and Cooper in 1962 and then employed by many researches over times and applied to many industrial areas. The originators of this methodology have applied this procedure into different fields of study, suggesting that others do so in spite of some critics that were made to this approach matter. This approach allows constraints to be flexible and have a tolerance for not always to be satisfied. The facts that such constraints, sometimes violates, it can be regarded as a risk issue that is acceptable by the decision makers. The degree of constraint violation
is referred to as risk level and usually is shown by $\alpha$ when its associated constraint is shown as below:

$$P(AX \leq B) \geq (1 - \alpha) \quad (1)$$

When $\alpha$ is set to be 0.05 then $(1 - \alpha) = 0.95$ means that the above constraint holds for 95 percent of the time. Table 2 shows CCP and variants of that and their applications in portfolio management and other areas. CCP is widely used with fuzzy numbers for dealing with uncertainty in portfolio modeling. For this purpose, readers can refer to the articles of Elahi, Yousefi and Zare Mehrjerdi (2012), Moradian, Fareidouni, and Zare Mehrjerdi (2010), Shahmohammadi, Emami, and Zare Mehrjerdi (2011), and Zare B.F. and Zare Mehrjerdi (2013).

Table 2. Sample of chance constrained programming applications in Portfolio selection

| CCP and its Variants | Authors and Years of publications |
|----------------------|----------------------------------|
| 1 CCP with random    | Tavana, M., Khanjani, R., and Di Caprio, D. (2019) |
| 2 CCP with multi period portfolio selection | Hassanlou, K. (2017) |
| 3 Joint CCP and Portfolio selection | Adam, L., Branda, M., Heitsch, H., and Henrion, R. (2018) |
| 4 CCP with Multi objective modeling of Portfolio selection and GA | Miryekemani, S.A., Sadeh, E., Amini Sabegh, Z. (2017) |
| 5 Sparse CCP Portfolio selection | Chen, Z., Peng, S., and Liser, A. (2020) |
| 6 CCP and Robust and reliable portfolio optimization | Sengupta, R.N., and Kumar, R. (2017). |
| 7 CCP and Data driven robust | Chen, Z., Peng, S., Liu, J. (2018) |
| 8 Ambiguous joint CCP | Hanususanto, G.A., Roitch, V., Kuhn, D., Wiesemann, W. (2017) |
| 9 CCP and Type-2 fuzzy fractional integrated modeling | Zhou, C., Huang, G., and Chen, J. (2019). |
| 10 CCP with a sparse model | Xu, F., Wang, M., Dai, Y.H., Xu, D. (2018) |
| 11 CCP and Data envelop analysis | Alinedjad and Zare Mehrjerdi (2013) |
| 12 CCP and Fuzzy computer simulation | Liu (2009), Zare Mehrjerdi et al. (2010) |

2.3. Portfolio. Markowitz is the first researcher developing a mathematical programming model for stocks selection using bi-criterion optimization approach. With the passage of time, this model was criticized and new models, dealing with real world’s realities, were introduced instead. One such model was introduced by Konno and Yamazaki (1991) using absolute deviation risk which substitutes the risk proposed in Markowitz’s model. The Markowitz model was based upon the following assumptions:
A METHODOLOGY FOR PORTFOLIO SELECTION

(1) Investors are risk averse and having a utility function reflecting their expectation
(2) Investors choose their portfolios based upon the mean and variance of the expected return.
(3) Each investing alternative is infinitely divisible
(4) The investing horizon is assumed to be one.

Simaan (1997) compared the mean variance model and the mean absolute deviation model. After introducing the semi variance by Markowitz, other researchers have used the semi-absolute deviation to measure the risk and expressed a new portfolio selection model. Some researchers debated that higher moments should be considered in modeling when we believe that the returns are not symmetrically distributed (Samuelson, 1970), (Kraus and Litzenberger, 1976), to mention a few. Samuelson (1970) pointed out that higher moments can play an important role for investors in making good decisions regarding their portfolio selection. It is also indicated that all investors should choose a portfolio with a third order moment if the first and second moments are equal. Chunhachinda et al. (1997) and Machado-Santos et al. (2005) provided evidence for skewness using stock markets' data. Table 3 shows some research works conducted using portfolio modeling and analysis using different mathematical approaches. Also, this Table shows some applications of portfolio modeling in financial market, product portfolio management, credit capital allocation in financial institutions, and risk budgeting.

Table 3. Sample of portfolio applications

| Portfolio Application areas | Authors and Years of publications |
|----------------------------|-----------------------------------|
| 1 Multi-objective capital allocation for supplier development under risk | Mizgier, Kamil J., Joseph M. Pasia, Srinivas Talluri (2017) |
| 2 Multi-objective Optimization of Credit Capital Allocation in Financial Institutions | Mizgier, Kamil J., Pasia, Joseph (2016), Doumpos, M., and Zopounidis, (2020) |
| 3 Large Scale Portfolio optimization using Multi-objective Programming | Qu, B.Y., Zhou, Q., Xiao, J.M., Liang, J.J., Suganthan, P.N (2017) |
| 4 Financial portfolio optimization, Bank portfolio management monetary policy, economic uncertainty | Özceylan, Eren, Kaba, Mehmet, Dağdeviren, Metin (2016), Soleymani, F., and Paquet, E. (2020), Amirian, S., Amiri, M. (2013), Udomrachtavanich, W. (2005) |
| 5 Application of Markowitz portfolio optimization on Bulgarian stock market | Ivanova, M., Dospatliev, L. (2016) |
| 6 Behavioral portfolio selection and optimization | Simo-Kengne, B.D., Ababio, K.A., Ur Koumba, J.M (2018) |
| 7 Scenario-based portfolio selection | Liesio, J., Salo A. (2012) |
| 8 Nonlinear bi-level programming approach for product portfolio management | Ma, S. (2016) |
| 9 Effectiveness of scenario generation techniques in single-period portfolio optimization | Guastaroba, G., Mansini, R., Speranza, (2009) |
| 10 Optimal risk budgeting under a finite investment horizon | De Prado, M.L., Vince, R., and Zhu, Q.J. (2019) |

2.4. Fractional Programming. There are many cases in which businesses deal with the linear fractional programming problem. In this regard, we can point to the work of Steuer (1986) who says that the mathematical optimization problems with
an objective function that is a ratio of a linear numerator and a linear denominator function. This sort of modeling has many applications in areas as such as waste management, corporate planning, bank balance sheet management, tax planning, marine transportation, water resources, and health care to mention some. Zare Mehrjerdi and Faregh (2017) employed a fractional function modeling for waste management situation in the city of Yazd. A linear fractional programming problem can be formulated as:

\[ \text{Max} f(x) = \frac{f_1(x)}{f_2(x)} \]  \hspace{1cm} (2)

s.t.

\[ S = \{x | Ax = b, x \geq 0\} \]  \hspace{1cm} (3)

where \( S \) is assumed to be a nonempty bounded polyhedron.

To solve this type of problem, many authors starting with Charnes and Cooper (1962), and then Martos (1975), Wolf (1985), Pal (1995), Dutta (1992), and Hezam (2013) have conducted research on this subject and proposed different algorithms for different form and shape of this problem. Later, in 2003, Pal employed goal programming and utilized Particle Swarm optimization (PSO) as a tool to solve the fuzzy linear fractional programming problem. Comparative investigations on such algorithms can be found in Arsham and Kahn (1990), and Bhatt (1989). In their book, Nonlinear Programming, Theory and Algorithms, Bazaar and Shetty (1979) have shown that fractional type objective function presented in (1)-(2) has several important properties - it is (simultaneously): pseudo convex, pseudo concave, quasi-convex, quasi-concave, strict quasi-convex and strict quasi-concave. This means that the point that satisfies the Kuhn-Tucker conditions for the maximization problem gives the global maximum on the feasible set. In addition, each local maximum is also a global maximum. This maximum is obtained at an extreme point of \( S \) (Metev and Gueorguieva, 2000). The general format of the fractional stochastic programming problem is as shown below:

\[ P_1 : \text{Maximize} \frac{H_1}{H_2} \]  \hspace{1cm} (4)

s.t.

\[ \text{Prob}\{ \sum_{j=1}^{n} D_j X_j \geq H_1 \} \geq \alpha_1 \]  \hspace{1cm} (5)

\[ \text{Prob}\{ \sum_{j=1}^{n} C_j X_j \geq H_2 \} \geq \beta_1 \]  \hspace{1cm} (6)

\[ \sum_{j=1}^{n} a_{ij} X_j \leq b_i \text{ for all } i \]  \hspace{1cm} (7)

where, \( i = 1, \cdots, m \) and \( j = 1, \cdots, n \). A major difficulty in using CCP when input-output coefficients and/or cost vectors are random variables, having a known distribution function, is the use of a nonlinear computer program. Table 4 shows a summary of research articles associated with chance constraint programming.

2.5. **Fuzzy Information.** Fuzzy sets and linguistic variables are widely used as two fundamental concepts in qualitative assessments. A real fuzzy number \( x \) is described as a fuzzy subset of the real line \( R \) with member function \( \mu(x) \) that
Table 4. Sample of research articles associated with fractional programming

| Authors | Purpose of research | Solution methodology |
|---------|---------------------|----------------------|
| 1 Dalman, H. (2016) | bi-level multi-objective fractional programming problem | Interactive fuzzy goal programming algorithm |
| 2 Xiao, L. (2010) | Solving linear fractional programming | Neural network method |
| 3 Farag, T.B. (2012) | integer fractional decision-making problems | Parametric analysis |
| 4 Hezam, I.M., Raouf MMH (2013) | solving fractional programming problems and complex variable fractional programming | Meta-heuristic algorithms, Particle swarm optimization |
| 5 Dür M, Khompatraporn C, Zabinsky ZB (2007) | Solving fractional problems | Dynamic multi-start improving hit-and-run |
| 6 Udhayakumar, et al. (2010) | solving chance constrained fractional programming | Simulation based genetic algorithm |
| 7 Sameeullah et al. (2008) | Linear fractional programming. | Genetic Algorithm |
| 8 Gupta (2009) | chance constrained approach to fractional programming with random numerator | Convex programming |
| 9 Wang C-F, Shen P-P (2008) | linear fractional programming | Global optimization algorithm |
| 10 Biswas and Bose (2011) | quadratic fractional bi-level programming. | Goal programming approach |
| 11 Charnes and Cooper (1962, 1973) | converted the fractional programming (FP) into equivalent linear programming | Linear programming |
| 12 Pal (2003, 2013), Zare Mehrjerdi (2011) | Fractional chance constraint programming | Fuzzy modeling and goal programming approach |
| 13 Zhang, Li, and Guo (2017) | Two stage stochastic chance constrained fractional programming | Decision support system |
| 14 Zare Mehrjerdi and Faregh (2017), | Linear fractional programming Arsham (1990), Dutta (1992) | Optimization |
| 15 Amiri, M., Shariatpanah, M., Benekar, M. (2010) | The effects of using fuzzy multi attributes approaches on selective portfolio returns in Tehran securities exchange market | Multi attribute decision making |
| 16 Phuc Ho Quang, T. (2014) | Applications in Portfolio Selection Problems | Multiple criteria decision making |
| 17 Y. Simaan. (1997) | Estimation risk in portfolio selection | mean variance model and the mean-absolute deviation model |
| 18 Zhu, H., Hung, G. H. (2011) | Stochastic linear fractional programming approach for sustainable waste management | linear fractional programming |

represents uncertainty. A membership function is defined from the universe of discourse to [0, 1]. A triangular fuzzy number can be defined as a triplet $(a, b, c)$. 
Therefore, a membership function of the fuzzy number \( x \) is defined as
\[
\mu(x) = \begin{cases} 
\frac{x - a}{b - a} & \text{if } a \leq x \leq b \\
\frac{x - c}{b - c} & \text{if } b \leq x \leq c \\
0 & \text{Otherwise}
\end{cases}
\] (8)

Using this representation, we can do arithmetic operations on fuzzy numbers simply and quickly. With the notations given above, the arithmetic operations of \((+), (-), (x)\), and \((\div)\) on fuzzy numbers are defined as follows:
\[
(a_1, b_1, c_1)(+)(a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2) 
\] (9)
\[
(a_1, b_1, c_1)(-)(a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 + a_2) 
\] (10)
\[
(a_1, b_1, c_1)(x)(a_2, b_2, c_2) = (a_1 x a_2, b_1 x b_2, c_1 x c_2) 
\] (11)
\[
(a_1, b_1, c_1)(\div)(a_2, b_2, c_2) = (a_1 \div a_2, b_1 \div b_2, c_1 \div c_2) 
\] (12)

The inversion of a fuzzy number and the multiplication of constant times a fuzzy number are done according to following formula:
\[
(a_1, b_1, c_1)^{-1} = \left( \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right) 
\] (13)
\[
kx(a_1, b_1, c_1) = (ka_1, kb_1, kc_1) 
\] (14)

3. Research Contribution and Gap analysis. Using information gathered in Tables 1, 2, 3, and 4 above, we can list the gaps exist in the literature of portfolio. The gaps are:

(1) Identifying the lists of risks and benefits
(2) Providing a proper framework for risk and benefit analysis taking various investing sectors
(3) Prioritizing all funds at the presence of risks and benefits identified in (1) and (2) above using hierarchical fuzzy TOPSIS
(4) Determining the percentage of money to be allocated to each fund for obtaining optimal portfolio using goal programming and fractional programming.

The Gap discussed above is the topic of this article. Table 5 demonstrates the areas that this research will cover to make its significant contributions to the literature of optimal portfolio selection.

### Table 5.
Reported use of decision making approaches with risks and benefits analysis and strategies prioritization associated with Joyful Organization studies

|                       | MADM approaches          | MODM approaches          | Integrating approaches |
|-----------------------|---------------------------|--------------------------|------------------------|
| (1) risks and benefits analysis for portfolio analysis | hierarchical fuzzy topsis (hf topsis) (this study) | gp (this study) | this research |
| (2) assessment of portfolio alternatives and prioritization of them | this study | x | this study |
| portfolio analysis | | optimization approaches | | madm and modm integration approach |
4. **Research Methodology.** This section lists steps to be followed to accomplish this research. The steps are: (1) selecting criteria for portfolio evaluation; (2) determining the list of criterions that are related to model building; (3) identifying the risks and benefits of the portfolio-based systems in relation to its domain of application; (4) advising a group of consultants in listing the most significant strategies for relating portfolio to the needs of individuals and industries; (5) giving and receiving appropriate consultations to/from the team of experts as needed to make the study process possible; (6) consulting stock market experts in weighting, determining attractive score, and process validation; (7) developing a hierarchical fuzzy TOPSIS approach for Risk-benefit analysis of the portfolio; (8) identify the ranking of stocks by SAW technique; (9) comparing the results of SAW and Hierarchical fuzzy TOPSIS for validation purposes; (10) proposing a new fractional fuzzy programming approach for determining the percentages of the fund to be allocated to each stock. Figure 1 schematically shows how this research is done from literature review to scenario development and analysis.

![Diagram](image-url)

**Figure 1.** Steps to develop a hierarchical fuzzy TOPSIS-SAW-GP-Fractional Programming approach for portfolio risk-benefit analysis
5. **Selection criteria for evaluation of Portfolio.** The most important issue for investors is the way of assigning their money to one or more different investment alternatives such that with the least possible risk the maximum return become obtainable. In the economic literature, this is known as the problem of portfolio selection.

5.1. **Risks of Portfolio.** Researchers are interested in identifying risks associate with portfolio in relation to financial market. Table 6 lists some of the most important risks associated with the portfolio selection.

| Portfolio Risks         | Descriptions                                                                                                                                 |
|------------------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| Broker (C1)            | Since investors in the developing countries have little access to brokers on the international markets so the risk can be relatively high for investors. |
| Technical Analysis (C2)| Access to professional technical analysts is not a simple task. This because (1) there are not too many of such analysts and (2) they are far less experienced in this task in general. |
| Capital management (C3)| Capital management techniques are related to three categories of: Tools, Objectives and Costs. More on these terminologies can be studied in the work of Amiri, et al. (2010). |
| Trading System (C4)    | A good on-line trading system with fast access to internet may help capital management to access the main trading board and then buy or sell as required. Most likely in developing countries the internet access is not fast and always available for political and social reasons as it is in many middle eastern countries. |
| Technology (C5)        | Due to the fact that foreign exchange is rapidly growing and will continue to do so and reaching over 3 trillion dollars (Amiri et al. 2010), hence we there is a need to spend more on technology and expect a good level of risk at any level of trading. |
| Trading Psychology (C6)| The most effective factors in trading psychology can be identified as: trading commitment, personal trading style, personal discipline, trading coach, courage, and familiar with the secrets of successful traders. |

5.2. **Benefits of Portfolio.** Table 5.2 lists some of the most significant benefits of portfolios associated with financial markets.

6. **Case Study.** A company located in a developing country likes to participate in buying and selling the stocks to develop portfolios for the entire company and its employees. This company is facing with a number of risks in addition to the general market and investment risks. Being located in a developing country, there are a number of limitations acting as risks for its long term investing participation. For instance, on time trading systems availability, capital management expertise, knowledgeable financial consultants, good technical analysts, brokers, and trading tools can have significant impacts on the investors. Although, some of these risks
Table 7. Portfolio Benefits

| Portfolio’s Benefits                  | Descriptions                                                                                                                                 |
|---------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| Dividend (C7)                         | Dividend can be defined according to following formula where \(d_{i}\) is the nominal annual revenue and \(P_{h,i}\) is the highest price of asset \(i\) in the year before. Dividend of a portfolio is defined as the weighted sum of the dividends of individual stocks in the portfolio. \(d_{i} = \frac{d_{i}}{T}\) |
| Short term and long term returns (C8) | Some researchers considered short term and long term returns for 12 months performance and 36 months related formulas for these performances are shown below where \(P_{T,i}\), \(P_{T-1,i}\), \(P_{T-3,i}\) are defined as price of asset \(i\) at periods \(T\), \(T-1\), \(T-3\), respectively. The short term performance for asset \(i\) is as shown below: \(r_{12}^{i} = \frac{P_{T,i} - P_{T-1,i}}{P_{T-1,i}}\) The long term performance for asset \(i\) is as shown below: \(r_{36}^{i} = \frac{P_{T,i} - P_{T-3,i}}{P_{T-3,i}}\) |
| Liquidity (C9)                        | For a specific asset, liquidity is set to be a proportion of that asset which is called turnover rate. Often investors prefer to deal with greater liquidity. |
| Standard and Poor’s Star Ranking (C10)| Based upon the Standard & Poor fund services, the performance ranking which is based on an annual basis is shown by star ranking. Taking \(SR_{i}\) as the number of stars assigned to investment fund \(i\), we can define following objective function for the problem under study. \(f(x) = \sum_{i=1}^{M} SR_{i}x_{i}\) where \(SR_{i}\) denotes the number of stars assigned to investment fund \(i\). |
| Financial System reporting such as ERP system (C11) | This means that accounting system is in good health and organization deals with low risk to do stock trading. ERP financial systems are able to report the status of the company in the right time and at the right amount of information needed for making right decisions. |
| Green Vision of the Company (C12)    | This vision of company indicates that risks are low and benefits are high.                                                               |

Factors have deep impacts on other local investors as well, but it would impact the developing countries’ investors much harder and deeper. Besides of all such risks, the companies in developing countries have to invest their money into international markets to have growing portfolios for the sake of the company and for being responsible in responding to the needs of their employees in the long term. Are you interested in having an investing analysis tool that can help you tremendously in selecting a right portfolio among many, by taking both risks and benefits simultaneously into consideration?

6.1. **Alternative Stock Types.** There are ten stocks that need to be taken into consideration for ranking purposes. These ten stocks are: Index funds, Computer, Durable goods, Pharmaceutical, Chip Industry, Real States, Life Insurance, Health Insurance, Tourism industry, and Auto industry. Table 6.1 shows weights of risks and benefits criteria while Table 6.1 shows weights for risks and benefits components.

The sample calculation shown below is from the data collected on the portfolio using a questionnaire with 26 questions. These questionnaires were all passed to
20 financial consultants of highly active in trading in Iran. The results obtained are summarized and then converted to the type of data necessary as input to the model proposed here for the hierarchical fuzzy TOPSIS. Tables 10 shows the original matrix while Tables 11, 12, 13, 14 shows positive and negative distances and Table 15 shows the ranking of alternatives. Using the result of hierarchical multi criterion approach, it can be concluded that

$$P_1 > P_2 > P_3 > P_7 > P_{10} > P_6 > P_4 > P_5 > P_8 > P_6$$

7. Simple Additive Weighting (SAW) Method. Hwang and Yoon have proposed a technique named SAW for multi attribute decision making problems. This
Table 10. the decision Matrix

|                     | Risk1(C1) | Risk2(C2) | Risk3(C3) | Risk4(C4) | Risk5(C5) | Risk6(C6) |
|---------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Index funds         | (48,68,86)| (50,70,87)| (49,69,87)| (43,63,80)| (43,62,78)| (48,68,84)|
| Computer            | (51,71,87)| (50,70,87)| (43,63,82)| (43,63,80)| (31,49,68)| (46,66,83)|
| Durable goods       | (50,70,87)| (50,70,87)| (49,69,87)| (43,63,80)| (24,39,58)| (27,45,63)|
| Pharmaceutical      | (53,73,89)| (50,70,87)| (49,69,87)| (43,63,80)| (31,48,66)| (38,57,74)|
| Chip Industry       | (49,69,86)| (50,70,87)| (34,53,71)| (43,63,80)| (30,48,67)| (28,46,65)|
| Real States         | (46,66,83)| (50,70,87)| (49,69,87)| (43,63,80)| (37,55,72)| (38,56,74)|
| Life Insurance      | (44,64,82)| (50,70,87)| (49,69,87)| (43,63,80)| (29,47,66)| (33,51,69)|
| Health              | (43,63,82)| (50,70,87)| (49,69,87)| (43,63,80)| (35,53,71)| (41,59,76)|
| Insurance           |           |           |           |           |           |           |
| Tourism industry    | (50,70,88)| (50,70,87)| (49,69,87)| (43,63,80)| (34,52,70)| (32,49,68)|
| Auto industry       | (50,70,87)| (50,70,87)| (36,54,72)| (43,63,80)| (33,50,68)| (37,53,70)|

|                     | Benefit1(C7) | Benefit2(C8) | Benefit3(C9) | Benefit4(C10) | Benefit5(C11) | Benefit6(C12) |
|---------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Index funds         | (58,78,94)   | (50,70,87)   | (55,75,93)   | (63,83,96)   | (63,83,97)   | (25,41,61)   |
| Computer            | (48,68,85)   | (47,66,84)   | (51,71,87)   | (45,65,82)   | (48,68,85)   | (40,60,78)   |
| Durable goods       | (25,42,61)   | (32,49,68)   | (44,63,80)   | (32,49,67)   | (53,73,87)   | (19,35,54)   |
| Pharmaceutical      | (39,57,74)   | (43,61,78)   | (42,62,78)   | (44,63,78)   | (52,72,86)   | (19,33,52)   |
| Chip Industry       | (35,53,69)   | (41,59,76)   | (44,63,78)   | (52,71,85)   | (53,73,87)   | (12,26,46)   |
| Real States         | (35,54,72)   | (37,57,74)   | (20,37,56)   | (21,37,56)   | (53,73,88)   | (16,30,49)   |
| Life Insurance      | (46,65,81)   | (47,67,82)   | (21,37,57)   | (33,50,68)   | (53,73,87)   | (42,61,78)   |
| Health              | (43,63,79)   | (46,66,82)   | (30,47,64)   | (26,43,61)   | (54,74,88)   | (22,38,57)   |
| Insurance           |             |             |             |             |             |             |
| Tourism industry    | (47,67,82)   | (47,67,82)   | (22,40,59)   | (40,58,73)   | (53,73,87)   | (27,43,62)   |
| Auto industry       | (49,68,83)   | (50,69,84)   | (38,56,71)   | (31,48,64)   | (53,73,87)   | (22,38,56)   |

Table 11. Positive distance for Risks

|                     | Risk1 | Risk2 | Risk3 | Risk4 | Risk5 | Risk6 | Total |
|---------------------|-------|-------|-------|-------|-------|-------|-------|
| Index funds         | 0.0258| 0.0000| 0.0915| 0.0000| 0.1420| 0.1363| 0.3956|
| Computer            | 0.0405| 0.0000| 0.0573| 0.0000| 0.613  | 0.1249| 0.2840|
| Durable goods       | 0.0339| 0.0000| 0.0915| 0.0000| 0.0000| 0.0000| 0.1254|
| Pharmaceutical      | 0.0471| 0.0000| 0.0915| 0.0000| 0.0533| 0.0723| 0.2642|
| Chip Industry       | 0.0290| 0.0000| 0.0000| 0.0000| 0.0523| 0.0072| 0.0885|
| Real States         | 0.0112| 0.0000| 0.0915| 0.0000| 0.0985| 0.0056| 0.2668|
| Life Insurance      | 0.0032| 0.0000| 0.0915| 0.0000| 0.0461| 0.0367| 0.1775|
| Health              | 0.0000| 0.0000| 0.0915| 0.0000| 0.0834| 0.0891| 0.2640|
| Insurance           |       |       |       |       |       |       |       |
| Tourism industry    | 0.0356| 0.0000| 0.0915| 0.0000| 0.0778| 0.0278| 0.2326|
| Auto industry       | 0.0339| 0.0000| 0.0070| 0.0000| 0.0667| 0.0539| 0.1616|

Technique which is used by many in research areas is comprised of following steps:

**Steps 1:** Identify the decision matrix D and prepare the common measurable matrix as follows where we use the following formula for positive and negative criterion,
Table 12. Positive Distance for Benefits

| Benefit | Benefit 1 | Benefit 2 | Benefit 3 | Benefit 4 | Benefit 5 | Benefit 6 | Total |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|-------|
| Index funds | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1343 | 0.1343 |
| Computer | 0.0559 | 0.0196 | 0.234 | 0.1051 | 0.878 | 0.0037 | 0.2054 |
| Durable goods | 0.2326 | 0.1246 | 0.0664 | 0.2157 | 0.0583 | 0.1876 | 0.8852 |
| Pharmaceutical | 0.1257 | 0.0521 | 0.0767 | 0.1213 | 0.0678 | 0.2041 | 0.6477 |
| Chip Industry | 0.1568 | 0.0635 | 0.0714 | 0.0686 | 0.0618 | 0.2742 | 0.6063 |
| Real States | 0.1442 | 0.0765 | 0.2570 | 0.3141 | 0.0558 | 0.2351 | 1.0827 |
| Life Insurance | 0.0743 | 0.0185 | 0.2518 | 0.2083 | 0.0618 | 0.0000 | 0.6148 |
| Health Insurance | 0.0897 | 0.0218 | 0.1796 | 0.2660 | 0.0523 | 0.1628 | 0.7720 |
| Tourism industry | 0.0647 | 0.0185 | 0.2298 | 0.1562 | 0.0583 | 0.1182 | 0.6458 |
| Auto industry | 0.0540 | 0.0033 | 0.1167 | 0.2304 | 0.0618 | 0.1672 | 0.6334 |

Table 13. Negative distance for Risks

| Risk | Risk1 | Risk2 | Risk3 | Risk4 | Risk5 | Risk6 | Total |
|------|-------|-------|-------|-------|-------|-------|-------|
| Index funds | 0.0200 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0200 |
| Computer | 0.0060 | 0.0000 | 0.0295 | 0.0000 | 0.0683 | 0.0086 | 0.1124 |
| Durable goods | 0.0121 | 0.0000 | 0.0000 | 0.0000 | 0.1420 | 0.1363 | 0.2904 |
| Pharmaceutical | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0785 | 0.0540 | 0.1324 |
| Chip Industry | 0.0169 | 0.0000 | 0.0915 | 0.0000 | 0.0784 | 0.1265 | 0.3133 |
| Real States | 0.0349 | 0.0000 | 0.0000 | 0.0000 | 0.0355 | 0.0612 | 0.1316 |
| Life Insurance | 0.0436 | 0.0000 | 0.0000 | 0.0000 | 0.0847 | 0.0908 | 0.2190 |
| Health Insurance | 0.0471 | 0.0000 | 0.0000 | 0.0000 | 0.0494 | 0.0401 | 0.1366 |
| Tourism industry | 0.0106 | 0.0000 | 0.0000 | 0.0000 | 0.0531 | 0.1014 | 0.1651 |
| Auto industry | 0.0121 | 0.0000 | 0.0831 | 0.0000 | 0.645 | 0.0747 | 0.2344 |

respectively:

\[
r_{ij} = \frac{x_{ij}}{\max_i \{x_{ij}\}}
\]  \hspace{1cm} (15)

and

\[
r_{ij} = \min_i \{x_{ij}\}
\]  \hspace{1cm} (16)

steps 2: Determine the weight vector of \( W = (w_1, w_2, \cdots, w_n) \)

steps 3: Choose the best alternative using following formula

\[
A^* = \{A_i | \max_j \sum_{j=1}^{m} w_j r_{ij} \}
\]  \hspace{1cm} (17)
Table 14. Negative Distance for Benefits

| Benefit 1   | Benefit 2 | Benefit 3 | Benefit 4 | Benefit 5 | Benefit 6 | Total     |
|-------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Index funds | 0.2326    | 0.1246    | 0.2570    | 0.3141    | 0.0878    | 0.1346    | 1.1506    |
| Computer    | 0.1736    | 0.1039    | 0.2323    | 0.2053    | 0.0000    | 0.2697    | 0.9848    |
| Durable goods | 0.0000  | 0.0000    | 0.1881    | 0.0975    | 0.0310    | 0.0827    | 0.3993    |
| Pharmaceutical | 0.1049 | 0.0717    | 0.1780    | 0.1914    | 0.0214    | 0.0694    | 0.6369    |
| Chip Industry | 0.0753 | 0.0605    | 0.1842    | 0.2444    | 0.272     | 0.0000    | 0.5915    |
| Real States | 0.0858    | 0.0472    | 0.0000    | 0.0000    | 0.0329    | 0.0384    | 0.2042    |
| Life Insurance | 0.1565 | 0.1055    | 0.0025    | 0.1045    | 0.0272    | 0.2742    | 0.6705    |
| Health Insurance | 0.1409 | 0.1020    | 0.0771    | 0.0466    | 0.0367    | 0.1067    | 0.5100    |
| Tourism industry | 0.1658 | 0.1055    | 0.0249    | 0.1583    | 0.0310    | 0.1516    | 0.6372    |
| Auto industry | 0.1774 | 0.1213    | 0.1403    | 0.0860    | 0.0272    | 0.1040    | 0.6662    |

Table 15. Ranking of alternatives

| Rows | Stocks Names   | Si*   | Si-   | Si*+Si- | C1   | Rank |
|------|----------------|-------|-------|---------|------|------|
| 1    | Index funds    | 0.5298| 1.1707| 1.7005  | 0.6884| 1    |
| 2    | Computer       | 0.5795| 1.0972| 1.6767  | 0.6544| 2    |
| 3    | Durable goods  | 1.0106| 0.6897| 1.7003  | 0.4056| 8    |
| 4    | Pharmaceutical | 0.9119| 0.7693| 1.6812  | 0.4576| 7    |
| 5    | Chip Industry  | 0.7848| 0.9048| 1.6897  | 0.5355| 3    |
| 6    | Real States    | 1.3496| 0.3359| 1.6854  | 0.1993| 10   |
| 7    | Life Insurance | 0.7922| 0.8895| 1.6817  | 0.5289| 4    |
| 8    | Health Insurance | 1.0359| 0.6466| 1.6825  | 0.3843| 9    |
| 9    | Tourism industry | 0.8784| 0.8023| 1.6808  | 0.4774| 6    |
| 10   | Auto industry  | 0.7950| 0.8906| 1.6856  | 0.5284| 5    |

Table 7 shows the results of calculations for SAW using the decision matrix table given in 10. The ranking by hierarchical TOPSIS is also shown in Table 7. Using the result of SWA approach, we can conclude that

\[ P_1 > P_2 > P_7 > P_8 > P_4 > P_{10} > P_3 > P_9 > P_6 > P_5 \]

8. Fractional Programming Model for Fund Allocation. To further elaborate on this problem, assume that data of Table 8 presents the fuzzy return values for all these ten stocks. For further simplicity, management prefers to use the following optimization model for identifying the values of decision variables as the percentage of amount of money should be assigned to each stocks.
Table 16. Final ranking by HFTOPSIS and SAW

| Rows | Stocks Names     | Rank by Hierarchical TOPSIS | Rank by SWA |
|------|------------------|-------------------------------|-------------|
| 1    | Index funds      | 1                             | 1           |
| 2    | Computer         | 2                             | 2           |
| 3    | Durable goods    | 8                             | 7           |
| 4    | Pharmaceutical   | 7                             | 8           |
| 5    | Chip Industry    | 3                             | 4           |
| 6    | Real States      | 10                            | 10          |
| 7    | Life Insurance   | 4                             | 3           |
| 8    | Health Insurance | 9                             | 9           |
| 9    | Tourism industry | 6                             | 6           |
| 10   | Auto industry    | 5                             | 5           |

Table 17. Fuzzy returns of 10 securities (units per stock)

| Security | 1   | 2   | 3   | 4   | 5   |
|----------|-----|-----|-----|-----|-----|
| Security | 6   | 7   | 8   | 9   | 10  |
| Fuzzy ξ , return | (0.2, 2, 6, 3.7) | (0.4, 2.5, 3.6) | (0.2, 3.4, 4.6) | (0.8, 1.2, 2.8) | (0.3, 2.3, 3.9) |

\[
Max Z(X) = \frac{F_1(X)}{F_2(X)} = \frac{E\{\eta_1X_1 + \eta_2X_2 + \eta_5X_5 + \eta_7X_7 + \eta_9X_9\}}{E\{\eta_3X_3 + \eta_4X_4 + \eta_6X_6 + \eta_8X_8 + \eta_9X_9\}} \tag{18}
\]

S.t

\[
X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} = 1 \quad X_j \geq 0
\]

\[
\mu_1(X) = \begin{cases} 
1 & \text{if } F_1(X) \geq g_1 \\
(F_1(X) - l_1)/(g_1 - l_1) & \text{if } l_1 \leq F_1(X) \leq g_1 \\
0 & \text{if } F_1(X) \leq l_1 \end{cases} \tag{19}
\]

\[
\mu_2(X) = \begin{cases} 
1 & \text{if } F_2(X) \leq g_2 \\
(u_2 - F_2(X))/(u_2 - g_2) & \text{if } g_2 \leq F_2(X) \leq u_2 \\
0 & \text{if } F_2(X) \geq u_2 \end{cases} \tag{20}
\]

where \(u_2\) is the upper tolerance limit.

8.1. Goal Programming Formulation. We know that in fuzzy programming approach the highest degree of membership is equal to 1. As discussed by Mohamed (1997), we can write the following goal constraints for the membership functions:

\[
(F_1(X) - l_1)/(g_1 - l_1) + d_1^- - d_1^+ = 1 \tag{21}
\]

\[
(u_2 - F_2(X))/(u_2 - g_2) + d_2^- - d_2^+ = 1 \tag{22}
\]

8.2. Linearization of Membership Goals. By taking following definitions into consideration

\[
L_1 = 1/(g_1 - l_1) \tag{23}
\]
goal constraint (21) can be written as:

\[ L_1 F_1(X) - L_1 l_1 + d^-_1 - d^+_1 = 1 \]  

(24)

Using the second goal membership function of (22) and letting

\[ L_2 = 1/(u_2 - g_2) \]  

(25)

goal constraint (22) can be written as:

\[ L_2 u_2 - L_2 F_2(X) + d^-_2 - d^+_2 = 1 \]  

(26)

Now, the goal programming model of the problem can be stated as below:

\[ \text{(P12)} \quad \text{Minimize: } w^-_1 d^-_1 + w^-_2 d^-_2 \]  

\[ \text{S.t. } \]

\[ L_1 F_1(X) - L_1 l_1 + d^-_1 - d^+_1 = 1 \]  

(28)

\[ L_2 u_2 - L_2 F_2(X) + d^-_2 - d^+_2 = 1 \]  

(29)

\[ \sum_{j=1}^{n} X_j = 1 \]  

(30)

\[ X_j \geq 0 \]  

(31)

\[ w^-_1 = 1/(g_1 - l_1) \]  

(32)

\[ w^-_2 = 1/(u_2 - g_2) \]  

(33)

By substituting the value of \( F_1(X) \) and \( F_2(X) \) into the above GP model, we have the following final model:

\[ \text{Minimize: } w^-_1 d^-_1 + w^-_2 d^-_2 \]  

\[ \text{S.t. } \]

\[ L_1 * E\{\eta_1 X_1 + \eta_2 X_2 + \eta_3 X_3 + \eta_4 X_4 + \eta_5 X_5 + \eta_6 X_6 + \eta_7 X_7 + \eta_8 X_8 + \eta_9 X_9\} - L_1 l_1 + d^-_1 - d^+_1 = 1 \]  

(35)

\[ L_2 u_2 * E\{\eta_3 X_3 + \eta_4 X_4 + \eta_5 X_5 + \eta_6 X_6 + \eta_7 X_7 + \eta_8 X_8 + \eta_9 X_9\} + d^-_2 - d^+_2 = 1 \]  

(36)

\[ \sum_{j=1}^{n} X_j = 1 \]  

(37)

\[ X_j \geq 0 \]  

(38)

\[ w^-_1 = 1/(g_1 - l_1) \]  

(39)

\[ w^-_2 = 1/(u_2 - g_2) \]  

(40)

Due to the fact that the expectation of a triangular fuzzy number \((a_i, b_i, c_i)\) can be shown as

\[ (a_i + 2b_i + c_i)/4 \]  

(41)

Table 18 gives all these expected values. Using management’s help in setting values
for \( g_1 = 10, l_1 = 5, u_2 = 6, \) and \( g_2 = 2, \) we continue solving the problem as shown in details below.

\[
\begin{align*}
\bar{w}_1 &= 1/(g_1 - l_1) = 1/(10 - 5) = 1/5 = 0.2 = L_1 \\
\bar{w}_2 &= 1/(u_2 - g_2) = 1/(6 - 2) = 1/4 = 0.25 = L_2
\end{align*}
\]

Minimize: \( 0.20d_1^- + 0.25d_2^- \)  \hspace{1cm} (42)

\[
\begin{align*}
S.t \\
0.2 \ast \{2.30X_1 + 1.70X_2 + 2.025X_5 + 2.25X_7 + 2.2X_{10}\} - 0.2 \ast 5 + d_1^- - d_1^+ &= 1 \\
0.25 \ast 6 - 0.25 \ast \{2.6X_3 + 1.95X_4 + 2.275X_6 + 2.90X_8 + 1.50X_9\} + d_2^- - d_2^+ &= 1 \\
X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} &= 1 \hspace{1cm} (43)
\end{align*}
\]

After some calculations we have

Minimize: \( 0.20d_1^- + 0.25d_2^- \)  \hspace{1cm} (46)

\[
\begin{align*}
S.t \\
0.46X_1 + 0.34X_2 + 0.405X_5 + 0.45X_7 + 0.44X_{10} + d_1^- - d_1^+ &= 2 \\
0.65X_3 + 0.4875X_4 + 0.56875X_6 + 0.725X_8 + 0.375X_9 - d_2^- + d_2^+ &= 0.5 \\
X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} &= 1 \hspace{1cm} (47)
\end{align*}
\]

9. **Managerial Implications.** This hybrid approach is suitable for fund managers, financial analysts, and engineers for studying portfolios under uncertainty. This approach can be implemented in different areas of: (1) stock selection and fund portfolio management, patent portfolio selection, and goods portfolio selection. The main advantages of this approach are listed below.

1. Due to the fact that risk and benefit components need to be identified in advance, it encourages researchers to gain a better understanding of the problem right before starting the modeling of the problem.
2. Risk averse investors get a good feeling once a portfolio is proposed to them. This is because they know that it is based upon the simultaneous utilization of risks and benefits components in model building.
3. The ranking of stocks by two approaches (hierarchical fuzzy TOPSIS and SAW) along with the further studying of the ratio of suitable stocks to non-suitable stocks, through a fractional programming problem, generates another guiding tool for better management of portfolios.
4. There are many managerial situations that management needs to prioritize a set of alternatives taking risk and benefit components associated with the case under study.
5. The first part of this approach is simple and can be handled by Excel spreadsheet instead of a computer software optimization. However, the second part requires familiarity with the concept of fractional programming and linear goal programming.
10. **Conclusion.** A good literature review on the topics related to the MADM, chance constrained programming and fractional programming are conducted to present the types of research had been completed in the past. Although, there are many algorithms for selecting suitable stocks among the enormous candidates, none have employed fuzzy hierarchical TOPSIS approach within the framework of risk-benefit analysis to generate a suitable list of stocks for proposing to investors. Due to the fact that a goal programming model with two goal constraints and one system constrained is used here, the model of problem is small-structured and can be solved with no hassle at all. It should be noted that, when portfolio manager deals with a large number of stocks and wish to assign limiting amount of money to each stock, then the number of constraints with the upper and lower bounds will increase. However, since linear goal programming technique is used for solving the problem, it can handle medium size to large size problems of this sort with no difficulty. To assess this methodology, index funds are used as alternatives to be ranked for investing. Through the use of hierarchical fuzzy TOPSIS and SAW approaches, funds are ranked and the results of these two approaches are compared. After ranking was done, fuzzy fractional programming approach was used to develop a new model for determining the percentage of fund to be allocated to each stock. A case problem consisting of ten index funds was ranked to show the process of problem solving for the approach proposed here. For future work, one may select a case study with large number of index funds and comparing the resulting solution of this approach with the results of Markowitz approach. The main beneficiary of this methodology is fund managers and financial analysts who wish to propose the most suitable list of stocks to their clients just before starting to invest and generating a portfolio. The first part of this methodology is simple and can be done using spreadsheet instead of a computer software optimization. The second part requires familiarity with the concept of fractional programming and linear goal programming, however. The proposed hybrid methodology is novel in the sense that it is a new hybrid approach for portfolio selection and analysis within the context of risk benefit analysis. An article entitled "Analysis of new approaches used in portfolio optimization: a systemic literature review" by Milhomem and Dantas (2020) does not report the existence of such hybrid methodology in the literature.

**REFERENCES**

[1] M. Abdel-Baset and I. M. Hezam, An improved flower pollination algorithm for ratio optimization problems, *Applied Mathematics and Information Sciences Letters*, 3 (2015), 83–91.

[2] L. Adam, M. Branda, H. Heitsch and R. Henrion, Solving joint chance constrained problems using regulation and benders’ decomposition, *Annals of Operations Research*, 292 (2020), 683–709.

[3] A. Alinedjad and Y. Zare Mehrjerdi, A new approach for portfolio performance evaluation in MVS modeling using data envelopment analysis: (Case study: Iran stock market), Sharif Industrial Journal of Management and Industrial Engineering, 2013.

[4] M. Amiri, An integrated eigenvector-DEA-TOPSIS methodology for portfolio risk evaluation in the FOREX spot market, *Expert Systems with Applications*, 37 (2010), 509–516.

[5] M. Amiri, M. Shariatpanah and M. Benekar, Optimal portfolio selection using multi criterion decision making, *Journal of Securities Exchange*, 11 (2010), 5–24.

[6] S. Amirian and M. Amiri, The effects of using fuzzy multi attributes approaches on selective portfolio returns in Tehran securities exchange market, in *The 10th International Industrial Engineering Conference*, Tehran University, Tehran, Iran, 2013.

[7] H. Arsham and A. B. Kahn, A complete algorithm for linear fractional programs, *Computers & Mathematics with Applications*, 20 (1990), 11–23.
[8] E. Babaee Tirkolaee, A. Mardani, Z. Dashtian, M. Soltani and G. W. Weber, A novel hybrid method using fuzzy decision making and multi-objective programming for sustainable-reliable supplier selection in two-echelon supply chain design, *Journal of Cleaner Production*, 250 (2019).

[9] A. Baykasoglu and I. Golcuk, Development of a novel multiple-attribute decision making model via fuzzy cognitive maps and hierarchical fuzzy TOPSIS, *Information Sciences*, 301 (2015), 75–98.

[10] A. Baykasoglu and I. Golcuk, Development of an interval type-2 fuzzy sets based hierarchical MADM model by combining DEMATEL and TOPSIS, *Expert Systems with Applications*, 70 (2017), 37–57.

[11] A. Baykasoglu, V. Kaplanoglu, Z. D. U. Durmusoglu and C. Sahin, Integrating fuzzy DEMATEL and fuzzy hierarchical TOPSIS methods for truck selection, *Expert Systems with Applications*, 40 (2013), 899-907.

[12] M. S. Bazaraa and C. M. Shetty, *Nonlinear Programming*, Theory and Algorithms, Wiley, New York, 1979.

[13] S. K. Bhatt, Equivalence of various linearization algorithms for linear fractional programming, *ZOR-Methods and Models of Operations Research*, 33 (1989), 39–43.

[14] A. Bilbao-Terol, B. Pérez-Gladish, M. Arenas-Parra, M. Victoria and R. Ura, Fuzzy compromise programming for portfolio selection, *Applied Mathematics and Computations*, 173 (2006), 251–264.

[15] A. Biswas and K. Bose, Fuzzy goal programming approach for quadratic fractional bilevel programming, in *Proceedings of the 2011 International Conference on Scientific Computing (CSC 2011)*, CSREA Press, Las Vegas, (2011), 143–149.

[16] P. Brockett, William W. Abraham Charnes, Cooper Kuyuk Kwon and W. Timothy Ruefli, *Chance Constrained Programming Approach to Empirical Analyses of Mutual Fund Investment Strategies*, National Science Foundation under Grant SES 8722504 and by the IC., 1992.

[17] G. F. Can and S. Demirok, Universal usability evaluation by using an integrated fuzzy multi criteria decision making approach, *International Journal of Intelligent Computing and Cybernetics*, 12 (2019), 194–223.

[18] A. Charnes and W. W. Cooper, Programming with linear fractional functions, *Naval Research Logistics Quarterly*, 9 (1962), 181–186.

[19] A. Charnes and W. W. Cooper, Chance-constrained Programming, *Management Science*, 6 (1962), 73–79.

[20] Z. Chen, S. Peng and A. Lisser, A sparse chance constrained portfolio selection model with multiple constraints, *Journal of Global Optimization*, 77 (2020), 825–852.

[21] Z. Chen, S. Peng and J. Liu, Data-driven robust chance constrained problems: a mixture model approach, *J. Optim. Theory Appl.*, 179 (2018), 1065–1085.

[22] P. Chunhachinda, K. Dandapani, S. Hamid and A. J. Prakash, Portfolio selection and skewness: Evidence from international stock markets, *Journal of Banking and Finance*, 21 (1997), 143–167.

[23] H. Dalman, An interactive fuzzy goal programming algorithm to solve decentralized bi-level multi-objective fractional programming problem, Available at [http://sciencewise.info/media/pdf/1606.00927v1.pdf](http://sciencewise.info/media/pdf/1606.00927v1.pdf).

[24] M. L. De Prado, R. Vince and Q. J. Zhu, Optimal risk budgeting under a finite investment horizon, *Risks*, 7 (2019), 1–15.

[25] W. Dinkelbach, On non-linear fractional programming, *Management Science*, 13 (1967), 492–498.

[26] M. Doumpos and Zapouindis, Multi-objective optimization models in finance and investment, *Journal of Global optimization*, 76 (2020), 243–244.

[27] M. Dür, C. Khompatraporn and Z. B. Zabinsky, Solving fractional problems with dynamic multistart improving hit-and-run, *Ann. Oper. Res.*, 156 (2007), 25–44.

[28] D. Dutta, R. N. Tiwari and J. R. Rao, Multiple objectives linear fractional programming, *A Fuzzy Set Theoretic Approach*, *Fuzzy Sets and Systems*, 52 (1992), 39–45.

[29] M. Elahi, M. Yousefi and Y. Zare Mehrjerdi, Portfolio optimization with mean-variance approach using hunting search meta-heuristic algorithm, *Financial Research Journal*, 16 (2011), 37–56.

[30] S. Fallahpour, H. Safari and N. Omrani, Portfolio selection using fuzzy logarithm modeling and PROMETE approach, *Financial Strategic Management Journal*, 2 (2013), 103–120.
[31] T. B. Farag, A Parametric Analysis on Multicriteria Integer Fractional Decision-Making Problems, PhD Thesis, Faculty of Science, Helwan University, Helwan, Egypt, 2012.

[32] G. Guastaroba, R. Mansini and Speranza, On the effectiveness of scenario generation techniques in single-period portfolio optimization, European Journal of Operational Research, 192 (2009), 500–511.

[33] P. Guo, X. Chen, M. Li and J. Li, Fuzzy chance constrained linear fractional programming approach for optimal water allocation, Stoch. Environ. Res. Risk Assess, (2014), 1601–1612.

[34] S. N. Gupta, A chance constrained approach to fractional programming with random numerator, Journal of Math. Model Algorithm, 24 (2009), 1–5.

[35] G. A. Hanasusanto, V. Roitch, D. Kuhn and W. Wiesemann, Ambiguous joint chance constraints under mean and dispersion information, Oper. Res., 65 (2017), 751–767.

[36] K. Hassanlou, A multi period portfolio selection using chance constrained programming, Decision Science Letters, 6 (2017), 221–232.

[37] I. M. Hezam, O. A. Raouf and Osama Abdel, Solving fractional programming problems using metaheuristic algorithms under uncertainty, Intern. J. Adv. Comput., 46 (2013c), 1261–1270.

[38] I. M. Hezam and O. A. Raouf, Particle swarm optimization programming for solving complex variable fractional programming problems, International Journal of Engineering, 2 (2013), 123–130.

[39] M. Ivanova and L. Dospatleiv, Application of Markowitz portfolio optimization on Bulgarian stock market from 2013 to 2016, International Journal of Pure and Applied Mathematics, 17 (2017), 291–307.

[40] J. Jia and J. S. Dyer, A standard measure of risk and risk-value models, Management Science, 42 (1996), 1691–1705.

[41] H. Jiao, Y. Guo and P. Shen, Global optimization of generalized linear fractional programming with nonlinear constraints, Appl. Math. Comput., 183 (2006), 717–728.

[42] C. C. Kahraman, S. Evik, N. Y. Ates and M. Gulbay, Fuzzy multi-criteria evaluation of industrial robotic systems, Computers & Industrial Engineering, 52 (2007), 414–433.

[43] A. Kraus and R. Litzenberger, Skewness preference and the valuation of risky assets, Journal of Finance, 31 (1976), 1085–1100.

[44] J. Liesio and A. Salo, Scenario-based portfolio selection of investment projects with incomplete probability and utility information, European Journal of Operational Research, 217 (2012), 162–172.

[45] S. A. Ma, Nonlinear bi-level programming approach for product portfolio management, Springer Plus, 5 (2016), 2–18.

[46] C. A. C. Machado-Santos, Fernandes, Skewness in financial returns: Evidence from the Portuguese stock, Finance, A Uver-Czech Journal of Economics and Finance, 55 (2005), 460–470.

[47] H. Markowitz, Portfolio selection, Journal of Finance, 7 (1952), 77–91.

[48] H. Markowitz, The optimization of a quadratic function subject to linear constraints, Naval Research Logistics Quarterly, 3 (1956), 111–133.

[49] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments, New York, Wiley, 1959.

[50] B. Metev and D. Gueorguieva, Simple method for obtaining weakly efficient points in multi-objective linear fractional programming problems, J. of Operational Research, 126 (2000), 386–390.

[51] V. Mirabi, et al. Identification and prioritization of effective factors on electronic quality services in stock market using fuzzy TOPSIS approach, Financial Engineering and Securities Exchange Journal, 12 (2010), 147–168.

[52] S. A. Miryekemani, E. Sadeh and Z. Amini Sabegh, Using genetic algorithm in solving stochastic programming for multi-objective portfolio selection in Tehran stock exchange, Advances in Mathematical Finance and Applications, 2 (2017), 107–120.

[53] K. M. Mizgier and J. M. Pasia, Multi-objective optimization of credit capital allocation in financial institutions, Central European Journal of Operations Research, 24 (2016), 801–817.

[54] K. M. Mizgier, J. M. Pasia and S. Talluri, Multi-objective capital allocation for supplier development under risk, International Journal of Production Research, 55 (2017), 1–16.

[55] R. H. Mohamed, The relationship between goal programming and Fuzzy programming, Fuzzy Sets and Systems, 89 (1997), 215–222.
[56] P. Moradia, S. Fereidouni and Y. Zare Mehrjerdi, Fuzzy simulation annealing model for solving chance constrained capital budgeting problem, *International Journal of Industrial Engineering and Production Management*, 21 (2010), 94–101.

[57] Özceylan, Eren, Kabak, Mehmet, Dağdeviren and Metin, A fuzzy-based decision making procedure for machine selection problem, *Journal of Intelligent & Fuzzy Systems*, 30 (2016), 1841–1856.

[58] A. Pahlavan, M. Ramazanpour and M. Gholizadeh, Prioritization of effective factors on stock selection in Tehran securities exchange market using Fuzzy ANP approach, in *Financial Mathematics and Applications Conference*, Semnan University, 2012.

[59] A. Pal, S. Singh and K. Deep, Solution of fractional programming problems using PSO algorithm. In Advance Computing conference (IACC), IEEE 3rd International, (2013), 1060–1064.

[60] B. B. Pal, B. N. Moitra and U. Maulik, A goal programming procedure for fuzzy multi-objective linear fractional programming problem, *Fuzzy Sets and Systems*, 139 (2003), 395–405.

[61] M. A. Parra, A. B. Terol and M. V. R. Uria, A fuzzy goal programming approach to portfolio selection, *Journal of Operational Research*, 133 (2001), 287–297.

[62] T. Phuc Ho Quang, *Multiple Criteria Decision Making and Applications in Portfolio Selection Problems*, Ph. D. Thesis, University of New South Wales, Australian School of Business, 2014.

[63] B. Y. Qu, Q. Zhoi, J. M. Xiao, J. J. Liang and P. N. Suganthan, Large scale portfolio optimization using multi-objective evolutionary algorithms and pre selection methods, *Mathematical Problems in Engineering*, (2017), 1–14.

[64] O. A. Raouf and I. M. Hezam, Solving fractional programming problems based on swarm intelligence, *J. Ind. Eng. Int.*, 10 (2014), 56.

[65] A. Sameeullah, S. D. Devi and B. Palaniappan, Genetic algorithm based method to solve linear fractional programming problem, *Asian J. Info. Technol.*, 7 (2008), 83–86.

[66] A. Sameeullah, S. D. Devi and B. Palaniappan, Genetic algorithm based method to solve linear fractional programming problem, *Asian Journal of Information Technology*, 7 (2008), 83–86.

[67] P. Samuelson, The fundamental approximation theorem of portfolio analysis in terms of means, variances, and higher moments, *Review of Economic Studies*, 37 (1970), 537–542.

[68] R. N. Sengupta and R. Kumar, Robust and reliable portfolio optimization formulation of a chance constrained problem, 42 (2017), 83–117.

[69] M. Shahmohammdi, L. Emami and Y. Zare Mehrjerdi, A hybrid intelligent algorithm for portfolio selection using fuzzy mean-variance-skewness, *International Journal of Industrial Engineering*, 23 (2010), 447–458.

[70] Y. Simaan, Estimation risk in portfolio selection: The mean variance model and the mean-absolute deviation model, *Management Science*, 43 (1997), 1437–1446.

[71] B. D. Simo-Kengne, K. A. Ababio and J. M. Ur Koumba, Behavioral portfolio selection and optimization: an application to international stocks, *Financial Markets and Portfolio Management*, 2018.

[72] F. Soleymani and E. Paquet, Financial portfolio optimization with online deep reinforcement learning and restricted stacked auto-encoder-Deep Breath, *Expert Systems with Applications*, 156 (2020), 113456.

[73] R. Steuer, *Multiple Criteria Optimization - Theory, Computation, And Application*, Wiley, New York, Chichester. 1986.

[74] M. Tavana, R. Khanjani and D. Di Caprio, A chance constrained portfolio selection model with random-rough variables, *Neural Computations and Applications*, 31 (2019), 5932–5945.

[75] A. Udhayakumar, V. Charies and V. R. Uthariaraj, Stochastic simulation based genetic approach for solving chance constrained fractional programming problem, *International Journal of Operational Research*, 9 (2010).

[76] W. Udomrachtavanich, *Bank Portfolio Management Monetary Policy, Economic Uncertainty*, Ph.D. Thesis, University of Missouri, Columbia, 2005.

[77] C. F. Wang and P. P. Shen, A global optimization algorithm for linear fractional programming, *Appl. Math. Comput.*, 204 (2008), 281–287.

[78] H. Wolf, A parametric method for solving the linear fractional programming problems, *Operations Research*, 33 (1985), 835–841.

[79] L. Xiao, Neural network method for solving linear fractional programming, in *2010 International Conference On Computational Intelligence and Security (CIS)*, (2010), 37–41.
[80] F. Xu, M. Wang, Y. H. Dai and D. Xu, A sparse enhanced indexation model with chance and cardinality constraints, *J. Glob. Optim.*, 70 (2018), 5–25.

[81] B. F. Zare and Y. Zare Mehrjerdi, Portfolio selection based on risk curve and simulation, *J. of Modern Management and Foresight*, 1 (2013), 1–10.

[82] Y. Zare Mehrjerdi, Solving fractional programming problem through fuzzy goal setting and approximation, *Applied Soft Computing*, 11 (2011), 1735-1742.

[83] Y. Zare Mehrjerdi, Developing a fuzzy TOPSIS method based on interval valued fuzzy set, *International Journal of Computer Applications*, 42 (2012), 7–18.

[84] Y. Zare Mehrjerdi, Group decision making process for RFID-based system selection using fuzzy TOPSIS approach, *Artificial Intelligent Research*, 2 (2013), 1–15.

[85] Y. Zare Mehrjerdi and F. Faregh, Using stochastic linear fractional programming for waste management, *Sharif Journal of Management and Industrial Engineering*, 31 (2017), 3–12.

[86] C. Zhou, G. Huang and J. Chen, A type-2 fuzzy chance constrained fractional integrated modeling method for energy system management of uncertainties and risks, *Energies*, 12 (2019), 2472.

[87] H. Zhu and G. H. Hung, SLFP: A stochastic linear fractional programming approach for sustainable waste management, *Waste Management Journal*, 31 (2011), 1612–1619.

Received November 2020; 1st revision February 2021; Final revision April 2021.

E-mail address: yzare@yazd.ac.ir