Evidence for particle–hole excitations in the triaxial strongly-deformed well of $^{163}$Tm

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Abstract

Two interacting, strongly-deformed triaxial (TSD) bands have been identified in the $Z = 69$ nucleus $^{163}$Tm. This is the first time that interacting TSD bands have been observed in an element other than the $Z = 71$ Lu nuclei, where wobbling bands have been previously identified. The observed TSD bands in $^{163}$Tm appear to be associated with particle–hole excitations, rather than wobbling. Tilted-axis cranking (TAC) calculations reproduce all experimental observables of these bands reasonably well and also provide an explanation for the presence of wobbling bands in the Lu nuclei, and their absence in the Tm isotopes.

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Stable asymmetric shapes have been a longstanding prediction of nuclear structure theory [1]. However, experimental evidence for such triaxial nuclei has proven difficult to establish. Still, triaxial shapes have been invoked to interpret a number of experimentally-observed nuclear structure phenomena such as signature inversion [2] and anomalous signature splittings [3], chiral twin bands [4], and, most recently, the wobbling mode [5]. Indeed, it has been generally agreed that the most convincing experimental evidence for stable triaxial shapes is provided by the wobbling mode, recently established in a number of odd-A Lutetium ($Z = 71$) nuclei [6–12]. The nuclear wobbling motion, akin to the motion of an asymmetric top, is indicative of the three-dimensional nature of collective nuclear rotation [1]. In the quantum picture, the low-spin spectrum of such a system corresponds to that of the well-known Davydov asymmetric rotor. However, the low-spin data do not allow a clear distinction between a rigid rotor and a system that is soft with respect to triaxial deformation. At high spins, the sequence of levels that can be associated with the excitation of wobbling phonons can be better distinguished from soft $\gamma$ vibrations. The mode is evidenced in the Lu nuclei by families of strongly deformed (SD) triaxial rotational bands connected to one another and representing different wobbling phonon quantum numbers $n_w$; bands up to $n_w = 2$ have been observed thus far [8]. However, it has been surprising (and, indeed, somewhat frustrating) that in no other element has this mode been observed so far. Indeed, even though a number of SD bands have been reported in several nearby nuclei (up to 8 in case of $^{174}$Hf!), many of which may be grouped into possible families based on similarities of their dynamic moments of inertia, there has been no evidence for connecting transitions between these bands [13,14]. Such connecting transitions are sine qua non for wobbling bands and strong $\Delta J = 1(0)$, $E2$ linking transitions between the $n_w + 1$ ($n_w + 2$) and $n_w$ wobbling partners are expected to occur over a large spin range.
We report the observation of two SD bands in the $Z = 69$ nucleus $^{163}$Tm, an isobar of $^{163}$Lu, the nucleus with the most extensive experimental evidence for wobbling bands [8]. We have identified several transitions connecting the two bands; however, these are unlike the characteristic transitions between wobblor bands and, instead, are akin to a “particle–hole excitation”. Still, this is the first time that two triaxial SD bands with interconnecting transitions have been observed in any element other than Lu. The properties of these bands are well reproduced by calculations in the framework of the tilted-axis cranking (TAC) model. Moreover, the calculations provide an explanation of why one observes particle–hole excitations in the Tm nuclei, but wobbling in the Lu isotopes.

High spin states in $^{163}$Tm were populated via the $^{130}$Te($^{37}$Cl, 4$n$)$^{163}$Tm reaction, at a bombarding energy of 170 MeV. The beam was provided by the 88-inch cyclotron facility at the Lawrence Berkeley National Laboratory. A self-supporting, isotopically enriched target-foil of about 0.5 mg/cm$^2$-thickness was used. To prevent contamination and degradation of the target, it was coated with an Aluminum layer, about 0.04 mg/cm$^2$-thick, on both sides. Quintuple- and higher-fold coincidence events were recorded with the Gammasphere array [15]; at the time of the experiment, the array had 98 active Compton-suppressed HPGe detectors. A total of about one billion events was accumulated and stored onto magnetic tapes for further analysis. The data-analysis procedures for developing the level schemes from Gammasphere data, and for assignments of spins and parities based on DCO ratio measurements, are more-or-less standard by now and only the most pertinent details are provided here. The data were sorted into three-dimensional and four-dimensional histograms [16,17] and analyzed by projecting double- and triple-gated coincidence spectra. The analysis has resulted in extensive development of the level scheme of $^{163}$Tm; a partial level scheme, relevant to the subject matter of this Letter, is presented in Fig. 1. Supporting coincidence spectra are illustrated in Fig. 2: the top and middle panels show, respectively, the $\gamma$-ray transitions in the sequences labeled TSD1 and TSD2, with an energy spacing $\Delta E \sim 60$ keV; the bottom panel displays many of the “connecting” transitions in coincidence with the bands. Each of these bands is of about 2–3% of the total intensity in $^{163}$Tm.

Angular correlation analyses helped in ascertaining the $\Delta J = 2$ character of the transitions in these bands; they have all been assigned an $E2$ multipolarity. The linking transitions are found to be $\Delta J = 1$ in character, with small possible admixtures, and have been assumed to be of $M1$ multipolarity. The spin and parity quantum numbers of the two bands are established on the basis of the multiplicities of the transitions linking them to the previously-known states in this nucleus [18]. With the established level scheme and the proposed multipolarity assignments, the alignments, $i_{x}$, and the dynamic moment of inertias, $J(2)$, have been calculated and are plotted as a function of rotational frequency in Fig. 3. Although, their SD nature can be confirmed only via measurements of the associated transitional quadrupole moments, these bands are, in general, not dissimilar to other triaxial strongly-deformed (TSD) bands observed in this region and have been assumed to have large triaxial deformation similarly.

Fig. 1. Partial level scheme of $^{163}$Tm, showing the TSD bands, their interaction, and feeding to normal deformed bands. The transition intensities are proportional to the thickness of the arrows.
Fig. 2. Background subtracted coincidence spectra in $^{163}$Tm from the summation of spectra with double gates set on bands TSD1 (top panel) and TSD2 (middle panel), respectively. The arrows indicate the appearance of transitions from TSD2 in the TSD1 spectrum, and vice versa. The bottom panel provides further evidence for the “connecting” transitions. Two gated spectra for interlinking transitions are shown in the inset of the bottom panel. TG refers to triple-gated spectrum; “C1” and “C2” correspond to the associated cascade transitions (C1: $657 + 730 + 804$ keV; C2: $642 + 839 + 902$ keV).

Fig. 4 shows the excitation energies of these TSD bands relative to a rigid rotor reference. The exhibited pattern is quite different from that observed in the case of wobbling bands (see, for example, Fig. 14 in Ref. [7]), and is indicative of the very different nuclear structure associated with these bands. Another major difference from the wobbling bands is that the transitions between TSD1 and TSD2 are “interconnecting”, i.e., there are linking transitions going both ways between the two bands, whereas for the wobbler bands the connecting transitions always proceed only from the band with a higher $n_w$ value to that with a lower $n_w$.

To understand the observed properties of these bands and their distinct differences from the sequences in the Lu nuclei, we have performed calculations in the frameworks of the configuration-dependent cranked Nilsson–Strutinsky (CNS) model [19] and the tilted-axis cranking model (TAC, shell correction version SCTAC) [20]. The CNS model is a special case of the TAC approach assuming that the axis of rotation is one of the principal axes. If this axis turns out to be stable, CNS calculations provide a solution of the TAC problem. If it is unstable, one has to use the TAC code to find the tilted solution. The technically simpler CNS calculations were carried out first, using the parameters advocated in Ref. [19] for the deformed mean field. The same set was subsequently used for the TAC calculations. Pairing was assumed to be zero; CNS calculations without pairing have been successfully applied in numerous cases in the spin range considered here [21].

Fig. 5 presents the CNS energy of a configuration with $(\pi, \alpha) = (−, −1/2)$ (see details below) as function of the deformation $\varepsilon_2$ and the triaxiality parameter $\gamma$ for $I_{\pi} = 63/2^−$. Similar to ultimate cranker calculations in Refs. [6–12,22], the CNS model gives a prolate minimum at normal deformation ($\varepsilon \approx 0.21$), which we refer to as ND, and two triaxial strongly deformed minima, which we refer to as TSD. It is worth pointing out that, in contrast with previous calculations with $Z > 69$ and $N \sim 94$ [6–12], the $i_{13/2}$ proton level is empty in $^{163}$Tm, which means that this level is not essential in forming the TSD minima. Rather, it is the gap in the neutron spectrum...
Fig. 3. Alignments $i_x$ (upper panel) and the experimental dynamic moments of inertia $J^{\text{exp}}$ (lower panel) for the two TSD bands in $^{163}\text{Tm}$ as a function of rotational frequency. The reference for the alignment is $I_{\text{ref}} = \omega_0 + \omega_3 \omega^3$ with $\omega_0 = 30 \hbar^2 \text{MeV}^{-1}$ and $\omega_3 = 40 \hbar^2 \text{MeV}^{-3}$. The calculated alignments and the dynamic moments of inertia for the TSD bands in the TAC model are also shown.

\[ \text{at } \epsilon_2 \approx 0.39, \quad |\gamma| \approx 17^\circ \text{ which stabilizes the TSD shape (cf. Ref. [22, Fig. 3]).} \]

Fig. 6 presents the single-proton Routhians in both of the TSD minima. The TSD configuration that we assign to the observed band TSD1 is indicated by the large filled circles on the occupied levels. It is the lowest configuration with negative parity and small signature splitting. There are competing configurations with similar energy, which will be discussed below. The relevant single-particle Routhians at the ND minimum can be found in Ref. [22]. Comparing them with the Routhians in the TSD minima (shown in Fig. 6), one finds that the $h_{11/2}$ orbital has a larger splitting between the two signatures, as expected for the associated smaller deformation. Further, the $h_{11/2}$ levels are shifted up by about 2 MeV. We, therefore, interpret bands 1 and 2 after the backbend as the two signatures of the odd proton occupying the $h_{11/2}$ level at the Fermi surface. In contrast to the TSD configuration, the proton pair on the $h_{9/2}$ Routhians is placed on the $[411]_{1/2}$ Routhians in the ND configuration. Fig. 4 compares the calculated and experimental energies. The observed substantial signature splitting is consistent with the calculation.

As seen in Fig. 5, the two TSD minima have nearly the same energy. It is clear from the bottom panel of Fig. 4 that minimum 2 is energetically favored at low spin and minimum 1 at high spin. The two minima have almost the same value of $\epsilon$ and $|\gamma|$, indicating that both are associated with the same shape. The axis of rotation is the short one in minimum 1 (with $\gamma > 0^\circ$), while it is the intermediate one for minimum 2 (with $\gamma < 0^\circ$). Thus, the CNS calculations suggest that at $I = 24 \hbar$, where minimum 1 goes below minimum 2, the orientation of the rotational axis flips from the intermediate to the short axis. This sudden flip is caused by the inherent assumption in the
served for the wobbling bands. Furthermore, as seen in Fig. 7, the dynamic moments of inertia with experimental values. Finally, Fig. 3 demonstrates that the B(γ) for 34° for 34
moment that increase slightly from 8.7 eb at low spins is consistent with the calculated approach of the CNS result. At large frequency, the calculated TSD bands have a lower energy than the ND ones. In the experiment, band 1 crosses TSD1 above spin 20 and have a larger energy above spin 50. However, as can be seen in Fig. 10 of Ref. [18], some residual proton pair correlations in the lower-spin part will disfavor the configurations TSD3 and TSD4 with respect to TSD1 and TSD2. The configurations TSD5 and TSD6, with both signatures of the h_{11/2} orbital occupied and the odd proton on one of the two h_{9/2} Routhians, would correspond to two well-separated \Delta I = 2 sequences with little resemblance to the experiment. The favored signature branch, TSD5, is predicted by the CNS calculations to lie about 500 keV below TSD1 at spin 20 and to have a larger energy than TSD1 above spin 40. We note here that the location of the h_{0/2} orbital has been a longstanding open problem in calculations using the modified oscillator potential (cf. the discussion in Ref. [18]). On the other hand, Ref. [23] demonstrated that the Woods–Saxon potential (universal parameters) reproduces the position of this orbital well for normal deformations. A calculation using the hybrid version of TAC [24], which is a good approximation of the Woods–Saxon potential, with \epsilon_2 = 0.4, \epsilon_4 = 0.04, and \gamma = 17°, all choices close to the self-consistent values, places TSD1 and TSD2 at about the same energy as TSD4 and TSD5 for the low spins, and below them for the higher spins. This points to possible problems with the modified oscillator potential for the TSD shapes which appear to be absent in the Woods–Saxon potential.

The single-particle Routhians in Fig. 6 provide a natural explanation for the presence of collective wobbling excitations in the Lu isotopes with Z = 71 and their absence in ^{163}Tm with Z = 69. The TSD configurations of nuclei with Z > 69 belong typically to minimum 1 with \gamma > 0° [22]. For Z = 71, the Fermi level is the \alpha = 1/2 Routhian of \iota_{13/2} parentage in the frequency range 250 keV < \hbar \omega < 450 keV. The lowest TSD band is observed in this frequency range and has (+, 1/2). The lowest particle–hole (p–h) excitation of the same parity lifts the odd proton on to the other signature, \alpha = -1/2, of this \iota_{13/2} level, which lies at a relatively high energy (~ 1 MeV at
\[ h\omega = 0.4 \text{ MeV} \]. This brings the collective wobbling excitation, which has an excitation energy of about 0.3 MeV, well below the lowest \( p-h \) excitations. For \( Z = 69 \), however, the two signatures of the \( h_{11/2} \) state are quite close together (cf. Fig. 4). Therefore, the wobbling excitation lies above the \( p-h \) excitations, likely too high in excitation energy to be populated with observable strength in the \( (HI, xn) \) reaction employed in the present study. It is also worth mentioning that the relative energy of the collective wobbling mode and of the \( p-h \) excitations in \( ^{163}\text{Lu} \) has been studied by means of the triaxial particle rotor model, where the \( p-h \) excitations have been called the “cranking mode” [12]. These are found to be located well above the one-phonon wobbling excitation. With the level order suggested in Fig. 6, one expects, for \( Z = 69 \), a band structure similar to the one seen in \( Z = 71 \) at somewhat higher energy; it is obtained by lifting the last proton from the \( h_{11/2} \) into the \( i_{13/2} \) orbital. For \( Z = 73 \), several TSD bands of both parities with similar energy are expected.

The possibility to experimentally identify a wobbling band is restricted by the competition of this collective excitation with the \( p-h \) excitations. If the energy of the \( p-h \) excitations is high and the energy of the wobbling band is low, it may become the first excited band above the yrast line. Such a case appears to be realized in the Lu isotopes. The opposite occurs in \( ^{163}\text{Tm} \). The energy for the \( p-h \) excitations between the signature partners of the \( h_{11/2} \) orbital is much smaller than the wobbling energy. In the experiment, only the first excited band, which corresponds to a \( p-h \) excitation, appears to have received sufficient intensity for observation. Moreover, we find that there is a gap at \( N = 94 \) in the neutron diagrams, which prevents the neutron \( p-h \) excitations from competing with the wobbling mode in the Lu isotopes. Around \( N = 102-104 \), the density of neutron orbitals is high in the relevant frequency range. This means that, for these nuclei, many low-lying \( p-h \) excitations are possible, and it would be difficult to disentangle a collective wobbling structure from these many bands. Moreover, the wobbling mode is expected to be fractionated among the \( p-h \) excitations of the same parity. This would account for the presence of many strongly deformed bands in these nuclei [13,14], none of which shows the characteristics of a wobbling mode. Based on this observation, Ref. [13] suggested that these nuclei might be less triaxial than the Lu isotopes. However, as discussed above, the apparent absence of a wobbling band does not necessarily imply a near-axial shape; indeed, that would be in contradiction with our calculations as well as with earlier ones [22].

In summary, two interacting strongly-deformed triaxial (TSD) bands have been observed in the \( Z = 69 \) nucleus \( ^{163}\text{Tm} \). Although confirmation of the strongly-deformed nature of these bands has to wait for measurements of the associated transition quadrupole moments, they appear to be similar, in most respects, to the TSD bands observed in this region. This is the first observation of interacting TDS bands in an element other than Lu where wobbling bands have been identified. The observed TSD bands in \( ^{163}\text{Tm} \) appear to correspond to particle–hole excitations, rather than to wobbling. Tilted axis cranking calculations reproduce all experimental observables for these bands reasonably well. The calculations also provide an explanation for the presence of wobbling bands in the Lu isotopes \( (Z = 71) \) and their absence in the nearby Tm, Hf, and Ta isotopes \( (Z = 69, 72, 73) \).

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