Wood fiber modelling using stochastic models

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Abstract. Various kinds of materials such as iron, wood, and concrete are very useful in human life. For example, the wood fiber can be used in building construction such as wall, roof, floor, as well as automotive interior. The design process of such materials is therefore still continuously researched and developed in order to create high quality products. To avoid excessive costs, it is essential to establish a non-destructive design technique of the macroscopic material. In this paper, the design technique of fibrous material using computer based modelling (virtual material design) as the realization of stochastic models will be investigated. In this model, the fiber can be stated as either a dilated curved lines, a chain of balls or cylinders with different cross-section. In this research, we will focus to investigate the image of Medium Density Fiber (MDF). It will be modelled as cylinders using the Poisson cylinder process with a rectangular cross-section. We also use Euler rotation and β-distribution to determine the position and orientation of the fibers in the system. Simulation software was conducted to verify the model. The result showed that the realization of fibers model is similar to the image of the wood fibers sample.

1. Introduction
The application of the fiber has been spread to many fields. For example, the wood fiber can be used in building construction such as wall, roof, floor, and acoustic room. Furthermore, wall of automotive interior also made by fibrous structure. Therefore, the design process of such materials is therefore still continuously researched and developed in order to create high quality products. To avoid excessive costs, it is essential to establish a non-destructive design technique of the macroscopic material. The virtual design of fibrous material can be created as a stochastic model realization. Varying model parameters create virtual fiber systems with different microstructures. The modelling of the fiber has been continuously researched and developed. There are several fiber models, for example, the model with or without interaction. The model with interaction can give us the non-overlapping system, such as fibers packing, sedimentation, and Random Sequential Adsorption. On the other hand, an overlapping system can be obtained by the model without interaction, i.e. the Boolean model and the Poisson cylinder process (PCP). Moreover, the fibers can be modelled either as dilated curved lines [1,2], as chains of balls [3-5] or as cylinders [6-8], with different cross-section, such as circle, square, and triangular. Faesel et al [2] proposed the model for a bending fibers as dilated curves line. The realization of fiber model is shown in Figure 1(a), whereas fibers as chains of balls with the force-biased packing approach
introduced by Altendorf [4]. At the beginning in Altendorf, the fibers are modelled using random walks [4]. Then, for controlling the bending problem, they used the multivariate von Mises-Fisher orientation with two parameters. This research focused to explain non-overlapping stochastic model of fiber with a circular cross section. The model applied to the glass fibers reinforces composite as shown in Figure 1(b). Easwaran also developed fibers model as a chain of the balls, but this research focused to explain overlapping fibers, especially for bundle fibers as depicted in Figure 1(c) [5]. Determination interaction between bundle fibers and the representative domain size also studied in this research. This model was applied for non-woven fibrous material. Fibers can also be modelled as cylinders. Schladitz et al [7] modelled the fibers as cylinder with a circular cross-section. They made a model for non-woven fiber by Poisson lines process, dilated by a sphere with circular cross-section to optimize the acoustic properties. The realization of this model can be found in Figure 1(d). Similar to a model in Schladitz [7] and Vecchio [8] introduced the fibers model using the PCP with circular, square and triangle cross sections. This model is applied for cellulose pulp composites as paper. The fibers system realization of this model displayed in figure 2.

Figure 1. The realization of fiber model respectively as dilates curve lines (a), chain of the balls (b), chain of the balls with bundle fiber (c), cylinder with circular cross-section (d).

Figure 2. The realization of infinite fiber model as a cylinder with triangle, circular, and square cross-section [8].

In this research, we focus to develop the model for (MDF) based on their image sample. Their images can be obtained by the imaging-techniques such as Stereo, Scanning Electron Microscopy (SEM), synchrotron radiation, Second Harmonic Generation (SHG), Scanning Acoustic Microscopy (SAM), FIB SEM and micro Computed Tomography (µCT). Here, we will use Stereo to obtain the image of the wood fibers sample. The Stereo will give the 2D images, and we should prepare the sample with small size to put in Stereo, therefore we cut them and made their surface smoothly. Figure 3(a) described the raw of MDF, figure 3(b) The sample of MDF will used in stereo after some preparation, and the image of its as displayed in Figure 3(c). In this paper, we will develop the model with fiber as a cylinder using...
PCP, this model similar as the model in Vecchio, however so that our model can be more explain the real data of MDF, we will propose a fiber model with rectangular cross-section [8].

Then, because the physical properties of composite materials highly correlation with the fibers orientation [9], therefore the orientation distribution is then fitted to the model. In this part, the Euler rotation and β-distribution are employed to solve their orientation distribution problem. The derived model is finally verified by using simulation software.

2. Methods
A fibers (ρ) is a subset of $\mathbb{R}^3$ with the image of a curve $\rho(t) = (\rho_a(t), \rho_b(t), \rho_c(t))$ so that $\|\rho'(t)\|^2 = \|\rho'_a(t)\|^2 + \|\rho'_b(t)\|^2 + \|\rho'_c(t)\|^2 > 0 \ \forall \ t, \ \rho: [0,1] \to \mathbb{R}^3$ is continuously differentiable and the mapping $\rho$ is injective [10]. In this part, we will describe how the wood fiber was modelled using Poisson Cylinder Process, Euler rotation and $\beta$-distribution as their orientation distribution.

2.1. Poisson cylinder process (PCP)
Poisson cylinder process is a natural model for systems of long fibers [8]. The fibers are modelled as cylinder using Poisson cylinder process which is developed by Poisson line process. Let $I$ is a line, then $\Phi = \bigcup_{i \in \mathbb{N}} I_i$ is the Poisson line process.

Let all non-oriented $n$-dimensional linear subspaces of $\mathbb{R}^d$ have the Grassmannian manifold denoted $G(n, d)$. If $V \in \mathcal{R}$ (convex ring) with $V \subset I^1$, and $I \in G(n, d)$, then a cylinder with direction space $I$ and cross-section $V$ can be explained by the Minkowski sum $M = I \oplus V$. The set of all cylinders is $\mathcal{M}_n^0$ (in $\mathbb{R}^d$, explained by Eq.1). They have not only cross section position in the origin but also have $n$-dimensional direction space.

$$\mathcal{M}_n = \{M + y: M \in \mathcal{M}_n^0, y \in \mathbb{R}^d\} \subset \mathcal{F}$$  \hspace{1cm} (1)

If $\Xi$ is a poisson point process on $\mathcal{M}_n$ and locally finite, then $\Xi(B)$ has poisson distribution with $\Lambda(B)$ for all Borel sets with $B \subset \mathcal{M}_n$ and $\Xi$ is called Poisson cylinder process [11].

2.2. Euler rotation
Euler rotation commonly used for represent a rotation in 3D with assuming the ZZX-convention. It has 3 Euler angles, namely, phi, theta and gamma ($\phi, \theta, \gamma$). The coordinate system in Euler rotation can be displayed by Figure 4. Euler rotation will be used to determine the position and direction of our fibers but based on the image of wood fibers. Direction is one of important parameter which can be considered in the geometry area research. Sari and Pasaribu also considered direction as a parameter in their model which used to study the difference between the random variable relationship between the sequence of random variables based on the location $s$, and it called semivariogram [12]. Based on the presence or absence of the influence of the angle between pair of locations respectively called anisotropic and isotropic semivariogram [13]. Then, determination of fibers-position using two parameters, $r$ and $\phi$, etc.
whereas fibers-direction using Euler angles \((\theta, \gamma)\). Not only shape, position and direction but also the orientation of fibers system has to be also considered.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{euler_rotation.png}
\caption{The coordinate system in euler rotation.}
\end{figure}

2.3. Orientation distribution
The \(\beta\)-distribution with axis of symmetry mean \(\mu\) corresponding to the \(z\)-axis has the probability density function as in Eq.2:

\[
f(\theta, \phi) = \frac{\beta \sin \theta}{4\pi(1 + (\beta^2 - 1) \cos^2 \theta)^{\frac{3}{2}}}, \quad \theta \in [0, \pi), \phi \in [0, 2\pi)
\]  

(2)

where \(\theta, \phi\) are the polar coordinates representing a direction [8].

3. Results and discussion
The development of the 3D wood fibers model using PCP with rectangular cross section is presented. Let \(K_\delta\) is cross section of cylinder \(M\), then \(S = \bigcup_{i \in \mathbb{N}} L_i \oplus K_\delta\) where \(\delta \in [0, 2\pi]\). Then, the first step is to simulate the single fiber with a rectangular cross section, describe them in different positions and it rotated by \(Z\)-axis rotation. The second step is simulating the fibers system with rectangular cross section using PCP. Euler rotation is also applied to determine their position and direction. Determination of fibers-position using two parameters, \(r\) and \(\phi\), whereas fibers-direction using Euler angles \((\theta, \gamma)\). The fibers system with rectangular cross section can be displayed by figure 5:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fibers_system.png}
\caption{The realization of fibers system with rectangular cross section (a), The realization of fibers system with rectangular cross section with \(\beta > 1\) (b).}
\end{figure}

Now, the \(\beta\)-distribution is used to determine the orientation of our fibers system. The values of \(\beta\) also determine the concentration of the distribution with these criteria; the distribution concentrates on the \(z\)-axis \((\beta < 1\)), isotropic, uniform distribution on the sphere \((\beta = 1)\) and objects oriented around the equator, isotopically in the \(xy\) plane \((\beta > 1)\).

In accordance with the image of the wood fibers sample, all fibers are oriented around the equator, isotopically in the \(xy\) plane, so that, \(\beta > 1\) is appropriate to be used. Figure 5 (b) explains the realization of fibers system with a rectangular cross section with \(\beta > 1\). Finally, we must compare the image of
wood fibers sample (see figure 3b) and the realization of our fibers system (see figure 5b), if they look like similar, so our fiber modelling already successful. Based on their comparison, the direction of each fibers cross section is still arbitrary, whereas the image of wood fibers sample has the same direction (parallel) of each fibers cross section, especially in the red area, therefore it will be the next topic of our research.

4. Conclusion
This research investigates the wood fiber sample image. Based on the natural microstructure of wood fibers, fibers modelled as cylinders with rectangular cross section using the Poisson cylinder process (PCP). Euler rotation and $\beta$-distribution also applied here to solved their orientation distribution problem. Euler rotation used for determination the position and direction of fibers whereas $\beta$-distribution used to reached fibers oriented around the equator and isotropically in the xy plane. The result of this research gave the realization of fibers model which look like similar as the wood fibers sample image. But, the direction of each fibers cross section is still arbitrary, whereas the image of wood fibers sample has the same direction (parallel) of each fibers cross section, therefore it will be the next topic of our research.

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