Kaluza-Klein Picture and Mass Spectrum of Two-Pion System

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Abstract

In this note we present additional arguments in favour of Kaluza and Klein picture of the world. In fact, we have shown that formula (10) provided by Kaluza-Klein approach with the fundamental scale early calculated (Eq. 8 [1]) gives an excellent description for the mass spectrum of two-pion system.

1 Introduction

In our previous papers [1, 2] we have presented the arguments in favour of that the Kaluza-Klein picture of the world has been observed in the experiments at very low energies where the nucleon-nucleon dynamics has been studied. In particular we have found that geniusly simple formula for KK excitations provided by Kaluza-Klein approach gives an excellent description for the mass spectrum of two-nucleon system. Surely, this was quite an event and, certainly, this very nice fact inspired us to study the other two-particle hadronic systems in this respect.

Recently there is a large amount of data concerning resonance states of two-pion system [3, 4]. The modern strong interaction theory formulated in terms of known QCD Lagrangian do not allow us at present time to make an appreciable breakthrough in the problem of calculating the masses of compound systems mainly because this problem is a significantly non-perturbative one. Therefore, great efforts have been undertaken to develop various kinds (semi)phenomenological models for systematics of experimentally observed resonance states.

We have performed an analysis of experimental data on mass spectrum of the resonance states of two-pion system and compared them with Kaluza-Klein picture. The results of this analysis have been presented here.

2 Kaluza-Klein picture and KK excitations in two-pion system

It is well known that the basic idea of the Kaluza-Klein scenario may be applied to any model in Quantum Field Theory (see for the details e.g. the excellent review articles...
and many references therein). As example, let us consider the simplest case of (4+d)-dimensional model of scalar field with the action

\[ S = \int d^{4+d}z \sqrt{-G} \left[ \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{m^2}{2} \Phi^2 + \frac{G_{(4+d)}}{4!} \Phi^4 \right], \]

(1)

where \( G = \text{det} |G_{MN}| \), \( G_{MN} \) is the metric on \( \mathcal{M}_{(4+d)} = M_4 \times K_d \), \( M_4 \) is pseudo-Euclidean Minkowski space-time, \( K_d \) is a compact internal \( d \)-dimensional space with the characteristic size \( R \). Let \( \Delta_{K_d} \) be the Laplace operator on the internal space \( K_d \), and \( Y_n(y) \) are orthonormalized eigenfunctions of the Laplace operator

\[ \Delta_{K_d} Y_n(y) = -\frac{\lambda_n}{R^2} Y_n(y), \]

(2)

and \( n \) is a (multi)index labeling the eigenvalue \( \lambda_n \) of the eigenfunction \( Y_n(y) \). \( d \)-dimensional torus \( T^d \) with equal radii \( R \) is an especially simple example of the compact internal space of extra dimensions \( K_d \). The eigenfunctions and eigenvalues in this special case look like

\[ Y_n(y) = \frac{1}{\sqrt{V_d}} \exp \left( i \sum_{m=1}^{d} n_m y^m / R \right), \]

(3)

\[ \lambda_n = |n|^2, \quad |n|^2 = n_1^2 + n_2^2 + \ldots + n_d^2, \quad n = (n_1, n_2, \ldots, n_d), \quad -\infty \leq n_m \leq \infty, \]

where \( n_m \) are integer numbers, \( V_d = (2\pi R)^d \) is the volume of the torus.

To reduce the multidimensional theory to the effective four-dimensional one we write a harmonic expansion for the multidimensional field \( \Phi(z) \)

\[ \Phi(z) = \Phi(x,y) = \sum_n \phi^{(n)}(x)Y_n(y). \]

(4)

The coefficients \( \phi^{(n)}(x) \) of the harmonic expansion (4) are called Kaluza-Klein (KK) excitations or KK modes, and they usually include the zero-mode \( \phi^{(0)}(x) \), corresponding to \( n = 0 \) and the eigenvalue \( \lambda_0 = 0 \). Substitution of the KK mode expansion into action (1) and integration over the internal space \( K_d \) gives

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} (\partial_\mu \phi^{(0)})^2 - \frac{m^2}{2} (\phi^{(0)})^2 + \frac{g}{4!} (\phi^{(0)})^4 + \sum_{n \neq 0} \left[ \frac{1}{2} (\partial_\mu \phi^{(n)}) (\partial^\mu \phi^{(n)})^* - \frac{m_n^2}{2} \phi^{(n)}(x)^2 \phi^{(n)*}(x)^2 \right] + \frac{g}{4!} (\phi^{(0)})^2 \sum_{n \neq 0} \phi^{(n)}(x)^2 \phi^{(n)*}(x)^2 \right\} + \ldots. \]

(5)

For the masses of the KK modes one obtains

\[ m_n^2 = m^2 + \frac{\lambda_n}{R^2}, \]

(6)

and the coupling constant \( g \) of the four-dimensional theory is related to the coupling constant \( G_{(4+d)} \) of the initial multidimensional theory by the equation

\[ g = \frac{G_{(4+d)}}{V_d}, \]

(7)

where \( V_d \) is the volume of the compact internal space of extra dimensions \( K_d \). The fundamental coupling constant \( G_{(4+d)} \) has dimension \([\text{mass}]^{-d}\). So, the four-dimensional
coupling constant $g$ is dimensionless one as it should be. Eqs. (6,7) represent the basic relations of Kaluza-Klein scenario. Similar relations take place for other types of multidimensional quantum field theoretical models. From four-dimensional point of view we can interpret each KK mode as a particle with the mass $m_n$ given by Eq. (9). We see that in according with Kaluza-Klein scenario any multidimensional field contains an infinite set of KK modes, i.e. an infinite set of four-dimensional particles with increasing masses, which is called the Kaluza-Klein tower. Therefore, an experimental observation of series KK excitations with a characteristic spectrum of the form (6) would be an evidence of the existence of extra dimensions.

We have applied the main issues of Kaluza-Klein approach to our analysis of the structure of proton-proton total cross section at very low energies and calculated the fundamental scale (size) $R$ of the compact internal extra space. One obtained by this way

$$\frac{1}{R} = 41.481 \text{ MeV} \quad \text{or} \quad R = 24.1 \text{ GeV}^{-1} = 4.75 \times 10^{-13} \text{ cm}. \quad (8)$$

It turned out the fundamental scale has a clear physical meaning: This scale corresponds to the scale of distances where the strong Yukawa forces in strength come down to the electromagnetic forces [1]. After that we have built the Kaluza-Klein tower of KK excitations by the formula

$$M_n = 2\sqrt{m^2_p + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots) \quad (9)$$

and compared it with the observed irregularities in the mass spectrum of diproton system. The result of the comparison has been presented in extended Table 1 of ref. [2]. It was established that the Kaluza-Klein tower of KK excitations built by the calculated fundamental scale was in a good correspondence with the experimentally observed picture of irregularities in the mass spectrum of two-nucleon system.

Now, let us build the Kaluza-Klein tower of KK excitations for two-pion system by the formula

$$M_n^{\pi^1 \pi^2} = \sqrt{m^2_{\pi^1} + \frac{n^2}{R^2}} + \sqrt{m^2_{\pi^2} + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots), \quad (10)$$

where $\pi^i (i = 0, +, -) = \pi^0, \pi^+, \pi^-$ and $R$ is the same fundamental scale (Eq. 8) calculated early from the analysis of nucleon-nucleon dynamics at low energies. Kaluza-Klein tower such built is shown in Table 1 where the comparison with experimentally observed mass spectrum of two-pion system is also presented.

We have used Review of Particle Physics [3] and recent review article of Crystal Barrel Collaboration [4] (see also many references therein) where the experimental data on mass spectrum of the resonance states of two-pion system have been extracted from. As it is seen from Table 1, there is a quite remarkable correspondence of the calculated KK excitations for two-pion system with the experimentally observed mass spectrum of the resonance states of two-pion system, which we consider as an additional strong evidence of Kaluza-Klein picture of the world.

Actually, we also see that there are three empty cells in the Table: $M_1(282 - 291)$, $M_2(317 - 325)$ and $M_{13}(1112 - 1114)$. We did not find an experimental confirmation of these states. Probably such two-pion resonance states exist but we don’t know these data. That’s why any experimental information in this respect would gratefully be acknowledged.\(^1\)

\(^{1}\)Recently we found the references [9, 10, 11, 12] (some of them is an old enough) which allowed us to fill the cells 1 and 2.
Table 1: Kaluza-Klein tower of KK excitations of two-pion system and experimental data.

| n  | $M_n^{\pi^0\pi^0}$ MeV | $M_n^{\pi^0\pi^0}$ MeV | $M_n^{\pi^0\pi^0}$ MeV | $M_{exp}^\pi\pi$ MeV |
|----|-----------------|-----------------|-----------------|-----------------|
| 1  | 282.41          | 286.80          | 291.21          | $\sim 300$     |
| 2  | 316.87          | 320.80          | 324.73          | 322 ± 8        |
| 3  | 367.18          | 370.58          | 373.98          | 370 – i356     |
| 4  | 427.78          | 430.71          | 433.64          | 430 – i325     |
| 5  | 494.92          | 497.45          | 499.99          | 506 ± 10       |
| 6  | 566.26          | 568.48          | 570.70          | 585 ± 20       |
| 7  | 640.41          | 642.38          | 644.34          | 650 – i370     |
| 8  | 716.50          | 718.26          | 720.01          | 732 – i123     |
| 9  | 793.96          | 795.55          | 797.13          | 780 ± 30       |
| 10 | 872.44          | 873.88          | 875.33          | 870 – i370     |
| 11 | 951.68          | 953.00          | 954.32          | 955 ± 10       |
| 12 | 1031.50         | 1032.72         | 1033.94         | 1015 ± 15      |
| 13 | 1111.78         | 1112.92         | 1114.05         |               |
| 14 | 1192.43         | 1193.49         | 1194.55         | 1165 ± 50      |
| 15 | 1273.38         | 1274.37         | 1275.36         | 1275.4 ± 1.2   |
| 16 | 1354.57         | 1355.50         | 1356.43         | 1359 ± 40      |
| 17 | 1435.96         | 1436.84         | 1437.72         | 1434 ± 18      |
| 18 | 1517.53         | 1518.36         | 1519.19         | 1522 ± 25      |
| 19 | 1599.24         | 1600.02         | 1600.81         | 1593 ± 8 ± 29/47 |
| 20 | 1681.07         | 1681.82         | 1682.57         | 1678 ± 12      |
| 21 | 1763.00         | 1763.72         | 1764.43         | 1768 ± 21      |
| 22 | 1845.03         | 1845.71         | 1846.40         | 1854 ± 20      |
| 23 | 1927.14         | 1927.79         | 1928.45         | 1921 ± 8       |
| 24 | 2009.32         | 2009.94         | 2010.57         | 2010 ± 8       |
| 25 | 2091.56         | 2092.16         | 2092.76         | 2086 ± 15      |
| 26 | 2173.85         | 2174.43         | 2175.01         | 2175 ± 20      |
| 27 | 2256.19         | 2256.75         | 2257.31         | $\sim 2250$   |
| 28 | 2338.58         | 2339.12         | 2339.66         | $\sim 2330$   |
| 29 | 2421.01         | 2421.53         | 2422.05         | 2420 ± 30      |
| 30 | 2503.47         | 2503.97         | 2504.48         | 2510 ± 30      |
Some known experimental information concerning resonance states in two-pion system is collected in separate tables: Table 2 – Table 22. We can learn from these tables that many different two-pion resonances with the different quantum numbers may occupy one and the same storey in KK tower. This is a peculiarity of the systematics provided by Kaluza-Klein picture. Kaluza-Klein scenario in the considered simplest case predict only masses of KK excitations and do not give any information on quantum numbers of corresponding resonance states. The later information on quantum numbers exceptionally depends on the physical process and the details of a dynamics of the given physical process where KK excitations have been observed. In particular, this also concerns the details (geometrical structures and shapes) of generic compact internal extra space\(^2\).

Of course, we would like to especially emphasize with a pleasure that \(\sigma\)-meson \((M_\sigma \simeq 430 \, \text{MeV})\), \(f_2(0^{++})\)-mesons \((M_{f_2} = 1272 \pm 8 \, \text{MeV} \, [7] \, \text{and} \, M_{f_2} = \text{2175} \pm 20 \, \text{MeV} \, [8])\) investigated by IHEP group under direction of Yu.D. Prokoshkin accurately agree with the calculated values and excellently incorporated in the scheme of systematics provided by Kaluza-Klein picture.

### 3 Conclusion

The central point of Kaluza-Klein approach is related to the existence of a new fundamental scale characterizing a size of compact internal extra space. In previous article \([1]\) we have calculated this fundamental scale and shown that geniusly simple formula \([9]\) provided by Kaluza-Klein approach gives an excellent description for the mass spectrum of two-nucleon system \([2]\). It has also been established that the experimental data obtained at low energies where the nucleon-nucleon dynamics has been studied reveal a special sort of (super)symmetry between fermionic(dibaryonic) and bosonic states predicted by Kaluza-Klein scenario.

In this note we have presented additional arguments in favour of Kaluza and Klein picture of the world. In fact, we have shown that formula \([10]\) provided by Kaluza-Klein approach with the fundamental scale early calculated \((\text{Eq. 8})\) gives an excellent description for the mass spectrum of two-pion system.

Of course, it would be very desirable to state new experiments to search a further justification of the systematics provided by Kaluza and Klein picture of the world. We believe that this is a quite promising subject of the investigations in particle and nuclear physics.

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Table 2: $M_1(282 - 291)$–Storey.

| \( R(I^GJ^P) \) | \( M_R \) MeV | \( \Gamma_R \) MeV | Reaction | Collab.       |
|-------------|--------------|----------------|----------|--------------|
|             | \( \sim 300 \) | \( \sim 300 \) | \( \Upsilon(3S) \to \pi^+\pi^-\Upsilon(2(1)S) \) | CLEO 91 |

Table 3: $M_2(317 - 325)$–Storey.

| \( R(I^GJ^P) \) | \( M_R \) MeV | \( \Gamma_R \) MeV | Reaction | Collab. |
|-------------|--------------|----------------|----------|----------|
| "ABC"(0+0++) | 310 ± 10     | \( \sim 25 \)  | \( pd \to He^3(\pi\pi)^0 \) | 10  60 |
|             | 322 ± 8      | \( \leq 20 \)   | \( \gamma p \to p"ABC" \) | 11  62 |
|             | 300–365      | 45–75           | \( dp \to He^3(\pi\pi)^0 \) | 12  73 |

Table 4: $M_{11}(952 - 954)$–Storey.

| \( R(I^GJ^P) \) | \( M_R \) MeV | \( \Gamma_R \) MeV | Reaction | Collab.  |
|-------------|--------------|----------------|----------|----------|
| \( f_0(0+0++) \) | 955 ± 10     | 240 ± 60       | \( pp \to pp\pi^0\pi^0 \) | GAM2 97 |
Table 5: $M_{12}(1032 - 1034)$–Storey.

| $R(I^G J^{PC})$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.          |
|----------------|-----------|----------------|----------|------------------|
| $f_0(0^+0^{++})$ | 1015 ± 15 | 86 ± 16    | COMPILATION $\pi\pi \to \pi\pi, K\bar{K}$ | RVUE 98 |
|                | 1015     | ~ 30       |          | RVUE 95          |

Table 6: $M_{14}(1192 - 1195)$–Storey.

| $R(I^G J^{PC})$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.          |
|----------------|-----------|----------------|----------|------------------|
| $f_0(0^+0^{++})$ | 1165 ± 50 | 460 ± 40      | $\pi^-p \to \pi^0\pi^0n$ | RVUE 95 |

Table 7: $M_{15}(1273 - 1275)$–Storey.

| $R(I^G J^{PC})$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.          |
|----------------|-----------|----------------|----------|------------------|
| $f_2(0^+2^{++})$ | 1275 ± 13 | 173 ± 53      | $\pi^+n \to p\pi^+\pi^-$ | HBC 70 |
|                | 1276 ± 7  |              |          | DLCO 84          |
|                | 1274 ± 5  |              |          | DM 2 87          |
|                | 1272 ± 8  | 192 ± 5      | $e^+e^- \to e^+e^-\pi^+\pi^-$ | GAM2 94 |
|                | 1275.4 ± 1.2 | 156.9$^{+3.8}_{-1.3}$ | AVERAGE | PDG 00          |

Table 8: $M_{16}(1355 - 1356)$–Storey.

| $R(I^G J^{PC})$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.          |
|----------------|-----------|----------------|----------|------------------|
| $f_0(0^+0^{++})$ | 1315 ± 50 | 255 ± 60     | $pp \to pp\pi^0\pi^0$ | GAM4 99 |
| $\rho(1^+1^-)$ | 1348 ± 33 | 275 ± 10     | $\bar{n}p \to \pi^+\pi^+\pi^-$ | OBLX 98 |
| $\rho(1^+1^-)$ | 1359 ± 40 | 310 ± 40     | $\bar{p}p \to \pi^+\pi^-\pi^0$ | OBLX 97 |
| $\rho(1^+1^-)$ | 1370 ± 90 | 130 ± 40     | $e^+e^- \to \pi^+\pi^-$ | RVUE 97 |

Table 9: $M_{17}(1435 - 1438)$–Storey.

| $R(I^G J^{PC})$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.          |
|----------------|-----------|----------------|----------|------------------|
| $f_0(0^+0^{++})$ | 1420 ± 20 | 460 ± 50     | $pp \to pp\pi^+\pi^-$ | SPEC 86 |
|                | 1434±18±9 | 173±32±6    | $D^+_s \to \pi^-\pi^+\pi^+$ | E791 01 |
| $f_2(0^+2^{++})$ | 1421 ± 5  | 30 ± 9       | $J/\psi \to \gamma\pi^+\pi^-$ | DM 2 87 |
| $\rho(1^+1^-)$ | 1480 ± 50 | 150 ± 50     | $pp \to pp\pi^+\pi^-$ | SPEC 86 |
|                | 1424 ± 25 | 269 ± 31     | $e^+e^- \to \pi^+\pi^-$ | DM 2 89 |
Table 10: $M_{18}(1518 - 1520)$ - Storey.

| $R(I^G J^{PC})$ | $M_R \text{ MeV}$ | $\Gamma_R \text{ MeV}$ | Reaction | Collab. |
|------------------|-------------------|-------------------|----------|--------|
| $f_0(0^+0^{++})$ | 1522 ± 25 | 108 ± 33 | $\bar{n}p \to \pi^+\pi^+\pi^-$ | OBLX 98 |
|                  | 1497 ± 30 | 199 ± 30 | $pp \to pp\pi^+\pi^-$ | OMEG 95 |
|                  | 1502 ± 10 | 131 ± 15 | $pp \to pp\pi^+\pi^-$ | OMEG 99 |
|                  | 1530 ± 45 | 160 ± 50 | $pp \to pp\pi^0\pi^0$ | GAM4 99 |
| $f_2(0^+2^{++})$ | 1507 ± 15 | 130 ± 20 | $\bar{p}p \to \pi^+\pi^-\pi^0$ | OBLX 97 |
|                  | 1540 ± 15 | 132 ± 37 | $\bar{n}p \to \pi^+\pi^+\pi^-$ | OBLX 92 |
| $f_2'(0^+2^{++})$ | 1502 ± 25 | 165 ± 42 | $\pi^-p \to \pi^+\pi^-n$ | OMEG 79 |

Table 11: $M_{19}(1599 - 1601)$ - Storey.

| $R(I^G J^{PC})$ | $M_R \text{ MeV}$ | $\Gamma_R \text{ MeV}$ | Reaction | Collab. |
|------------------|-------------------|-------------------|----------|--------|
| $\pi_1(1^-1^{--})$ | 1593 ± 8 $^{29}_{47}$ | 168 ± $^{118}_{12}$ | $\pi^- p \to \pi^+\pi^-\pi^-\pi^-p$ | MPS 98 |
| $f_0(0^+0^{++})$ | 1580 ± 80 | 280 ± 100 | $\pi^- p \to \pi^+\pi^-\pi^-n$ | GAM4 98 |

Table 12: $M_{20}(1681 - 1683)$ - Storey.

| $R(I^G J^{PC})$ | $M_R \text{ MeV}$ | $\Gamma_R \text{ MeV}$ | Reaction | Collab. |
|------------------|-------------------|-------------------|----------|--------|
| $\rho_3(1^+3^{--})$ | 1677 ± 14 | 246 ± 37 | $\pi^- p \to 2\pi p$ | OMEG 81 |
|                  | 1679 ± 11 | 116 ± 30 | $\pi^+ p \to \pi^+\pi^-\pi^-n$ | HBC 78 |
|                  | 1678 ± 12 | 162 ± 50 | $\pi^- p \to p3\pi$ | CIBS 77 |
|                  | 1690 ± 7 | 167 ± 40 | $\pi^+ n \to \pi^+\pi^-p$ | DBC 74 |
|                  | 1693 ± 8 | 200 ± 18 | $\pi^- p \to \pi^+\pi^-n$ | ASPK 74 |
|                  | 1678 ± 12 | 156 ± 36 | $\pi^+ N$ | DBC 71 |
|                  | 1692 ± 12 | 240 ± 30 | $\pi^- p \to \pi^+\pi^-n$ | RVUE 75 |
|                  | 1650 ± 35 | 180 ± 30 | $\pi^- p \to N2\pi$ | HBC 70 |
|                  | 1687 ± 21 | 267 $^{72}_{46}$ | $\pi^- p, \pi^+d$ | HDBC 70 |
|                  | 1683 ± 13 | 188 ± 49 | $\pi^+d$ | DBC 68 |
|                  | 1670 ± 30 | 180 ± 40 | $\pi^+d, \pi^-p$ | HBC 65 |
|                  | 1688.8 ± 2.1 | 186 ± 14 | AVERAGE | PDG 00 |
Table 13: $M_{21}(1763 - 1764)$ – Storey.

| $R(I^GJ^P C)$ | $M_R MeV$   | $\Gamma_R MeV$ | Reaction                          | Collab. |
|---------------|-------------|----------------|-----------------------------------|---------|
| $\rho(1^{+1-})$ | 1780 ± 29  | 275 ± 45       | $\bar{p}n \to \pi^-\pi^0\pi^0$  | CBAR 97 |
|               | 1730 ± 30  | 400 ± 100      | $e^+e^- \to \pi^+\pi^-$          | RVUE 94 |
|               | 1768 ± 21  | 224 ± 22       | $e^+e^- \to \pi^+\pi^-$          | DM2 89  |
|               | 1745.7 ± 91.9 | 242.5 ± 163     | $e^+e^- \to \pi^+\pi^-$          | RVUE 89 |
| $f_0(0^{+0++})$ | 1740 ± 30  | 120 ± 50       | $J/\psi \to \gamma(\pi^+\pi^-\pi^+\pi^-)$ | BES 00  |
|               | 1750 ± 20  | 160 ± 30       | $pp \to pp\pi^+\pi^-$            | OMEG 99 |
|               | 1750 ± 15  | 160 ± 40       | $J/\psi \to \gamma(\pi^+\pi^-\pi^+\pi^-)$ | MRK3 95 |
|               | 1750 ± 30  | 250 ± 140      |                                   |         |

Table 14: $M_{22}(1845 - 1846)$ – Storey.

| $R(I^GJ^P C)$ | $M_R MeV$   | $\Gamma_R MeV$ | Reaction                          | Collab. |
|---------------|-------------|----------------|-----------------------------------|---------|
| $\eta_2(0^{+2-})$ | 1840 ± 25 | 200 ± 40       | $pp \to pp2(\pi^+\pi^-)$          | OMEG 97 |
|               | 1881 ± 32 ± 40 | 221 ± 92 ±44     | $e^+e^- \to e^+e^-\eta\pi^0\pi^0$ | CBAL 92 |
|               | 1840 ± 15  | 170 ± 40       | $J/\psi \to \gamma\eta\pi^+\pi^-$ | BES 99  |
|               | 1854 ± 20  | 202 ± 30       | $J/\psi \to \gamma(\pi^+\pi^-\pi^+\pi^-)$ | PDG 00  |

Table 15: $M_{23}(1927 - 1928)$ – Storey.

| $R(I^GJ^P C)$ | $M_R MeV$   | $\Gamma_R MeV$ | Reaction                          | Collab. |
|---------------|-------------|----------------|-----------------------------------|---------|
| $X(0^{+2++})$ | 1920 ± 10  | 90 ± 20        | $\pi^-p \to \omega\omega n$      | VES 92  |
|               | 1924 ± 14  | 91 ± 50        | $\pi^-p \to \omega\omega n$      | GAM2 90 |
|               | 1921 ± 8   | 90 ± 19        | $\pi^-p \to \omega\omega n$      | PDG 00  |
| $f_2(0^{+2++})$ | 1918 ± 12 | 390 ± 60       | $pp \to pp2(\pi^+\pi^-)$          | OMEG 95 |
|               | 1940 ± 50  | 380 ± 120      | $J/\psi \to \gamma(\pi^+\pi^-\pi^+\pi^-)$ | BES 00  |
|               | 1960 ± 30  | 460 ± 40       | $pp \to pp2(\pi^+\pi^-)$          | OMEG 97 |
| $f_4(0^{+4++})$ | 1935 ± 13 | 263 ± 57       | $\pi^-p \to n2\pi$                | OMEG 79 |
Table 16: $M_{24}(2009 - 2011)$--Storey.

| $R(I^GJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|-------------|--------|--------|----------|--------|
| $f_2(0^+2^{++})$ | $\sim$1996 | $\sim$134 | $pp \rightarrow \pi\pi$ | RVUE 94 |
|             | $\sim$1990 | $\sim$100 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 94 |
|             | 2010 ± 60 | 240 ± 100 | $\pi^-p \rightarrow \pi^0\pi^0n$ | GAM4 98 |
| $f_0(0^+0^{++})$ | 2010 ± 60 | 240 ± 100 | $\pi^-p \rightarrow \pi^0\pi^0n$ | GAM4 98 |
|             | 2020 ± 35 | 410 ± 50 | $pp \rightarrow pp2(\pi^+\pi^-)$ | OMEG 97 |
| $f_4(0^+4^{++})$ | 1998 ± 15 | 395 ± 40 | $\pi^-p \rightarrow \pi^0\pi^0n$ | GAM4 98 |
|             | 2032 ± 30 | 304 ± 60 | $J/\psi \rightarrow \gamma\pi^+\pi^-$ | DM2 87 |
|             | 2020 ± 20 | 240 ± 40 | $\pi^-p \rightarrow n2\pi$ | GAM 2 84 |
|             | 2015 ± 28 | $186^{+103}_{-58}$ | $\pi^+p \rightarrow \Delta^+\pi^0\pi^0$ | STRC 82 |
|             | 2020 ± 30 | 180 ± 60 | $\pi^-p \rightarrow n2\pi^0$ | NICE 75 |
|             | $\sim$2000 | $\sim$170 | $NN \rightarrow \pi\pi$ | RVUE 98 |
|             | $\sim$2010 | $\sim$200 | $NN \rightarrow \pi\pi$ | RVUE 97 |
| $\rho_3(1^+3^{--})$ | 2034 ± 11 | 222 ± 19 | AVERAGE | PDG 00 |
|             | $\sim$2007 | $\sim$287 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 94 |

Table 17: $M_{25}(2091 - 2093)$--Storey.

| $R(I^GJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|-------------|--------|--------|----------|--------|
| $f_4(0^+4^{++})$ | 2086 ± 15 | 210 ± 63 | $J/\psi \rightarrow \gamma\pi^+\pi^-$ | MRK3 87 |
| $\rho_3(1^+3^{--})$ | $\sim$2090 | $\sim$60 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 94 |

Table 18: $M_{26}(2174 - 2175)$--Storey.

| $R(I^GJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|-------------|--------|--------|----------|--------|
| $f_2(0^+2^{++})$ | 2175 ± 20 | 150 ± 35 | $pp \rightarrow pp2\eta$ | GAM4 95 |
| $\rho(1^+1^{--})$ | $\sim$2170 | $\sim$250 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 80 |
|             | $\sim$2170 | $\sim$250 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 80 |
|             | $\sim$2191 | $\sim$296 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 94 |
| $\rho_3(1^+3^{--})$ | 2153 ± 37 | 389 ± 79 | $e^+e^- \rightarrow \pi^+\pi^-$ | RVUE 91 |
|             | $\sim$2150 | $\sim$200 | $\bar{p}p \rightarrow \pi\pi$ | CNTR 77 |

Table 19: $M_{27}(2256 - 2257)$--Storey.

| $R(I^GJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|-------------|--------|--------|----------|--------|
| $f_2(0^+2^{(4)}^{++})$ | 2246 ± 36 | $\sim$250 | $J/\psi \rightarrow \gamma\pi^0\pi^0$ | BES 98 |
| $\rho_3(1^+3^{--})$ | $\sim$2250 | $\sim$250 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 80 |
|             | $\sim$2232 | $\sim$220 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 94 |
| $\rho_5(1^+5^{--})$ | $\sim$2250 | $\sim$300 | $\bar{p}p \rightarrow \pi\pi$ | RVUE 80 |
Table 20: \( M_{28}(2339 − 2340) \)–Storey.

| \( R(I^G J^{PC}) \) | \( M_R \text{ MeV} \) | \( \Gamma_R \text{ MeV} \) | Reaction | Collab.  |
|-----------------|-----------------|-----------------|----------|----------|
| \( f_0(0^{+0^{++}}) \) | \( \sim 2321 \) | \( \sim 223 \) | \( \bar{p}p \rightarrow \pi \pi \) | RVUE 94  |
| \( f_4(0^{+4^{++}}) \) | \( \sim 2314 \) | \( \sim 278 \) | \( \bar{p}p \rightarrow \pi \pi \) | RVUE 94  |
|                  | \( \sim 2300 \) | \( \sim 200 \) | \( \bar{p}p \rightarrow \pi \pi \) | RVUE 80  |
| \( \rho_5(1^{+5^{--}}) \) | \( \sim 2330 \) | \( \sim 300 \) | \( \bar{p}p \rightarrow \pi^0 \pi^0 \) | OSPK 78  |
|                  | \( \sim 2300 \) | \( \sim 200 \) | \( \bar{p}p \rightarrow \pi \pi \) | CNTR 77  |
|                  | \( \sim 2303 \) | \( \sim 169 \) | \( \bar{p}p \rightarrow \pi \pi \) | RVUE 94  |
|                  | \( \sim 2300 \) | \( \sim 250 \) | \( \bar{p}p \rightarrow \pi \pi \) | RVUE 80  |

Table 21: \( M_{29}(2421 − 2422) \)–Storey.

| \( R(I^G J^{PC}) \) | \( M_R \text{ MeV} \) | \( \Gamma_R \text{ MeV} \) | Reaction | Collab.  |
|-----------------|-----------------|-----------------|----------|----------|
| \( f_6(0^{+6^{++}}) \) | \( 2420 \pm 30 \) | \( 270 \pm 60 \) | \( \pi^- p \rightarrow \pi^0 \pi^0 n \) | GAM4 98  |

Table 22: \( M_{30}(2503 − 2504) \)–Storey.

| \( R(I^G J^{PC}) \) | \( M_R \text{ MeV} \) | \( \Gamma_R \text{ MeV} \) | Reaction | Collab.  |
|-----------------|-----------------|-----------------|----------|----------|
| \( f_6(0^{+6^{++}}) \) | \( 2510 \pm 30 \) | \( 240 \pm 60 \) | \( \pi^- p \rightarrow \pi^0 \pi^0 n \) | GAM2 84  |
| \( \rho_5(1^{+5^{--}}) \) | \( \sim 2480 \) | \( \sim 210 \) | \( \bar{p}p \rightarrow \pi \pi \) | CNTR 77  |