Simple two Parameter Description of Lepton Mixing

Werner Rodejohann and He Zhang

Max–Planck–Institut für Kernphysik,
Saupfercheckweg 1, 69117 Heidelberg, Germany

We note that a simple two parameter description of lepton mixing is possible which reproduces the features that apparently emerge from global fits at the 1σ level: if \( U_{e3} \) is non-zero it implies that the solar neutrino mixing parameter \( \sin^2 \theta_{12} \) is less than \( \frac{1}{3} \) by order \( |U_{e3}|^2 \). If the CP phase \( \delta \) is around \( \pi \) it implies that the atmospheric neutrino mixing parameter \( \sin^2 \theta_{23} \) is less than \( \frac{1}{2} \) by order \( |U_{e3}| \). The mixing scheme can be described by a 23-rotation appearing to the right of a tri-bimaximal mixing matrix. We quantify the excellent agreement of the scheme with data statistically, and comment on model building aspects.
I. INTRODUCTION

All three mixing angles of the lepton sector are now known. The last step towards this marvelous achievement came from reactor neutrino experiments Double Chooz [1], Daya Bay [2] and RENO [3]. Combining the reactor data with other experiments ruled out vanishing $U_{e3}$ at more than 7σ C.L. [4–7]. At the Neutrino 2012 conference in June 2012, Double Chooz have presented new data with $3.1\sigma$ evidence for non-zero $U_{e3}$, and also Daya Bay has increased its significance to more than 7σ, see the URL http://kds.kek.jp/conferenceTimeTable.py?confId=9151#20120604.detailed for the slides. Moreover, additional data from T2K and MINOS can be included, and one can fit the overall data to the parameters in the Pontecorvo-Maki-Nagakawa-Sakata (PMNS) lepton mixing matrix (ignoring possible Majorana phases)

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$ (1)

In this short note we wish to give a possible interpretation of the emerging features of global fits. We will focus on the results from Fogli et al. [6], which spotlight the following interesting properties:

a) $|U_{e3}| \simeq 0.16$ is sizable;

b) solar neutrino mixing is described by $\sin^2 \theta_{12} \simeq \frac{1}{3} - \mathcal{O}(|U_{e3}|^2)$, i.e. slightly less than $\frac{1}{3}$;

c) atmospheric neutrino mixing is described by $\sin^2 \theta_{23} \simeq \frac{1}{2} - \mathcal{O}(|U_{e3}|)$, i.e. significantly less than $\frac{1}{2}$;

d) the CP phase $\delta$ is around $\pi$.

The precise fit parameters are quoted in Table I. Not all features are present in the results of the other groups [4–7] (which have not yet updated their results, and have partly different treatment in their atmospheric codes or do not even fit atmospheric data), and at the 2σ level points b) and d) are absent. While there is no doubt about the value of $|U_{e3}|$, less-than-maximal atmospheric mixing and less-than-$\frac{1}{3}$ solar mixing seem to be common features, at least at the 1σ level.

If these properties of lepton mixing survive the test of time, an interpretation in terms of a mixing scheme will without doubt be useful. We note in this work that there is a mixing scheme that can reproduce features a) – d). It has only two free parameters that can be adjusted to the observables, and possesses the following properties:

i) if $|U_{e3}|$ is non-zero, $\sin^2 \theta_{12} \simeq \frac{1}{3} - \mathcal{O}(|U_{e3}|^2)$ is implied. This links features a) and b);

ii) if $\delta$ is around $\pi$, $\sin^2 \theta_{23} \simeq \frac{1}{2} - \mathcal{O}(|U_{e3}|)$ is implied. This links features c) and d).
TABLE I: Best-fit and estimated 1σ values of the neutrino mixing parameters from Ref. [6] for the normal mass ordering ($m_3 > m_2 > m_1$, NH) and the inverted mass ordering ($m_3 < m_1 < m_2$, IH).

| Parameter | $\sin^2 \theta_{12}$ | $\sin^2 \theta_{23}$ | $\sin^2 \theta_{13}$ | $\delta/\pi$ |
|-----------|-----------------------|-----------------------|-----------------------|--------------|
| NH        | $0.307^{+0.018}_{-0.016}$ | $0.386^{+0.024}_{-0.021}$ | $0.0241 \pm 0.0025$ | $1.08^{+0.28}_{-0.31}$ |
| IH        | $0.392^{+0.039}_{-0.022}$ | $0.0244^{+0.0023}_{-0.0025}$ | $1.09^{+0.38}_{-0.26}$ | 

The defining property for the PMNS matrix $U$ is

$$|U| = \begin{pmatrix} \sqrt{\frac{2}{3}} & # & # \\ \sqrt{\frac{1}{3}} & # & # \\ \frac{1}{\sqrt{6}} & # & # \end{pmatrix},$$

(2)

where the elements in the second and third column can be obtained by unitarity. The entries of this matrix in the first column are exactly as in tri-bimaximal mixing (TBM) [8], and hence this mixing matrix can also be obtained by multiplying the tri-bimaximal mixing matrix with a 23-rotation from the right. The phenomenology of the mixing scheme has been analyzed first in [9] (see also [10]), and is a variant of the so-called trimaximal mixing scheme, which is defined through a mixing matrix with only the second row as for TBM [11–13]. This scheme is however disfavored due to its prediction $\sin^2 \theta_{12} \simeq \frac{1}{3}(1 + |U_{e3}|^2)$. The scheme that we propose to describe the current data has been written down first in [13], and in the convention of [9] is called TM$_1$. We revisit this mixing scheme in this short note, emphasizing that it is able to perfectly accommodate the above features a) – d), which seem to emerge from global fits. In Section II we study its phenomenology, and in Section III we discuss aspects of a possible theoretical background.

II. PHENOMENOLOGY

Consider the tri-bimaximal mixing matrix multiplied with a 23-rotation from the right:

$$U = U_{\text{TBM}}R_{\text{23}}(\theta, \psi) = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\sqrt{\frac{1}{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & e^{-i\psi} \sin \theta \\ 0 & -e^{i\psi} \sin \theta & \cos \theta \end{pmatrix}.$$  

(3)

The observables are in this case

$$|U_{e3}|^2 = \frac{1}{3} \sin^2 \theta, \quad \sin^2 \theta_{23} = \frac{1}{2} - \frac{\sqrt{3}}{2} \sin 2\theta \cos \psi,$$

$$\sin^2 \theta_{12} = 1 - \frac{2}{3 - \sin^2 \theta}, \quad J_{CP} = -\frac{1}{6\sqrt{6}} \sin 2\theta \sin \psi,$$

(4)

where the Jarlskog invariant is defined as $J_{CP} = \text{Im} \left( U_{e1}U_{\mu2}U_{\mu3}^*U_{\mu1} \right) = s_{12}s_{23}s_{13}s_{13}s_{13} \sin \delta$. The main feature of the above mixing matrix is that the first column of $U$ keeps the same
form as for TBM:

\[
\begin{pmatrix}
|U_{e1}|^2 \\
|U_{\mu 1}|^2 \\
|U_{\tau 1}|^2
\end{pmatrix} = \begin{pmatrix}
2/3 \\
1/6 \\
1/6
\end{pmatrix}
\]  \quad (5)

From these relations, as well as from Eq. (4), the observable mixing parameters and their correlation can be obtained. In particular, a consequence of Eq. (5) is that \(\sin^2 \theta_{12} \leq \frac{1}{3}\). Indeed, from \(|U_{e1}|^2 = \frac{2}{3}\) one finds

\[
\sin^2 \theta_{12} = \frac{1}{3} \left(1 - \frac{1}{3} \frac{|U_{e3}|^2}{1 - |U_{e3}|^2} \right) \approx \frac{1}{3} \left(1 - \frac{1}{2} |U_{e3}|^2\right).
\]  \quad (6)

The desired feature b), \(\sin^2 \theta_{12} = \frac{1}{3} - \mathcal{O}(|U_{e3}|^2)\), is reproduced. In Fig. I we show the correlation between \(|U_{e3}|\) and \(\sin^2 \theta_{12}\), using the 1\(\sigma\) range of the global fit results [6]. For simplicity, we use the results for the normal ordering, the difference to the results for the inverted ordering is insignificant. One finds that \(\sin^2 \theta_{12}\) lies between 0.315 and 0.318.

The second independent condition in Eq. (5) involving \(|U_{\mu 1}|^2 = 1/6\) gives

\[
\cos \delta \tan 2\theta_{23} = -\frac{1 - 3 |U_{e3}|^2}{2\sqrt{2} |U_{e3}| \sqrt{1 - 3 |U_{e3}|^2}} \approx \frac{1}{2\sqrt{2} |U_{e3}|} \left(1 - \frac{7}{2} |U_{e3}|^2\right).
\]  \quad (7)

This relation can be written as

\[
\sin^2 \theta_{23} \approx \frac{1}{2} + \frac{1}{\sqrt{2} |U_{e3}| \left(1 + \frac{1}{4} |U_{e3}|^2\right) \cos \delta}.
\]  \quad (8)

Hence, if \(\cos \delta < 0\) and \(|\cos \delta| = \mathcal{O}(1)\), the desired features c) and d) are reproduced: \(\sin^2 \theta_{23} = \frac{1}{2} - \mathcal{O}(|U_{e3}|^2)\). We plot in Fig. I the correlation between \(\sin^2 \theta_{23}\) and \(\delta\). To reproduce the 1\(\sigma\) range of \(\theta_{23}\), \(\delta\) should lie between 1.30\(\pi\) and 1.38\(\pi\). It may be useful to express the Dirac CP phase in terms of the parameters \(\theta\) and \(\psi\) as

\[
\sin \delta = -\frac{\sqrt{2}(5 + \cos 2\theta) \sin \psi}{\sqrt{39 + 20 \cos 2\theta + 13 \cos 4\theta - 24 \cos 2\psi \sin^2 2\theta}}.
\]  \quad (9)

We also display in Fig. I the correlation between \(\sin^2 \theta_{23}\) and \(\sin^2 \theta_{12}\), when \(|U_{e3}|\) is varied in its 1\(\sigma\) range.

For further illustration on how nicely the mixing scheme describes current data, we perform a somewhat naive statistical fit. To this end, we take the two parameters \(\theta\) and \(\psi\) as independent, and compare the TM\(_1\) predictions to the experimental data with a \(\chi^2\) function

\[
\chi^2 = \sum_i \frac{(\rho_i - \rho_i^0)^2}{\sigma_i^2},
\]  \quad (10)

where \(\rho_i^0\) represents the data of the \(i\)-th experimental observable, \(\sigma_i\) the corresponding 1\(\sigma\) absolute error, and \(\rho_i\) the prediction of the model. The experimental values of the neutrino mixing angles are taken from Table I, which as mentioned above include the data presented at Neutrino 2012. Note that the global-fit data slightly differ for normal and inverted neutrino mass orderings.
FIG. 1: Correlation of observables for our mixing scheme: the upper plot shows $\sin^2 \theta_{12}$ vs. $|U_{e3}|$, the middle plot $\sin^2 \theta_{23}$ vs. $\delta$ and the lower plot $\sin^2 \theta_{23}$ vs. $\sin^2 \theta_{12}$ for different values of $\delta$. Except for the lower plot, where the displayed range of $\theta_{23}$ is larger, the plots cover the allowed $1\sigma$ ranges of the parameters in case of a normal mass ordering [6]. The black solid lines are the best-fit points, the black dashed line in the lower plot the $1\sigma$ range.
In Fig. 2 we present the allowed regions of $\theta$ and $\psi$ at 1$\sigma$, 2$\sigma$, and 3$\sigma$ C.L., defined as contours in $\Delta\chi^2$ for two degrees of freedom with respect to the $\chi^2$ minimum ($\chi^2_{\text{min}} \simeq 1.1$ for NH and $\chi^2_{\text{min}} \simeq 0.9$ for IH). Note that these $\chi^2$ minima show that the scheme describes the data excellently. Since $|U_{e3}|$ is firmly connected to $\theta$ in TM$_1$, it is restricted to a narrow range between 0.07$\pi$ and 0.1$\pi$, and the best-fit value of $\theta$ lies close to $\pi/12$. The constraints on $\psi$ are not as strong as for $\theta$ due to the less precise determination of $\delta$ and $\theta_{23}$. The best-fit values of $\theta$ and $\psi$ are ($0.087\pi$, 0.32$\pi$) for the NH case, and ($0.087\pi$, 0.32$\pi$) for the NH case.

We further show in Fig. 3 the predictions for the mixing angles and the Dirac CP phase $\delta$ for the NH case (upper panel) and the IH case (lower panel). Again, the data are very well reproduced. The best-fit values of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ are found to be (0.317, 0.382) for the NH case, and (0.317, 0.385) for the IH case. The right column shows that the TM$_1$ prediction on $\theta_{13}$ is in good agreement with experiments, and that $\delta$ tends to lie at the upper end of its allowed range. We find the best-fit values of $\sin^2\theta_{13}$ and $\delta$ to be (0.0242, 1.34$\pi$) for NH and (0.0245, 1.35$\pi$) for IH. For comparison, we also show in dotted contours the 1$\sigma$ parameter ranges without considering the experimental data on $\delta$. In such a case, the allowed parameter spaces are symmetric with respect to $\delta = \pi$, and $\delta$ is confined to be either around $\delta = 1.3\pi$ or $\delta = 0.7\pi$, implying the predictive power of the scenario. All in all, three mixing angles together with one CP phase are all compatible with experimental data within 1$\sigma$ C.L., though there are only two free parameters in TM$_1$. 

FIG. 2: The allowed region of the TM$_1$ parameters $\theta$ and $\psi$ at 1$\sigma$, 2$\sigma$, and 3$\sigma$ C.L. for the normal mass ordering (left) and inverted mass ordering (right). The black asterisks denote the best-fit values.
FIG. 3: The allowed region of the physical observables at 1σ, 2σ, and 3σ C.L. for the normal mass ordering (upper panel) and inverted mass ordering (lower panel). The shaded areas are not permitted by the model. For comparison, the allowed 1σ ranges of the parameters are also indicated by using green vertical and horizontal bars. The dotted contours are the 1σ ranges without considering the experimental data on δ.

III. THEORY

One may ask a question arising from the previous analysis: what is the underlying symmetry behind TM1? In this section we wish to give some comments on the possible theoretical background and in particular we will show two examples in the framework of $A_4$ and $S_4$ flavor symmetries.

A. Symmetry behind the TM1

The original trimaximal mixing, defined by $(|U_{e2}|^2, |U_{\mu 2}|^2, |U_{\tau 3}|^2)^T = (1/3, 1/3, 1/3)^T$, has been discussed in the framework of flavor symmetry models $[10, 12, 21]$. Models that can
Therefore, the neutrino mass matrix is invariant under the $Z_G$ and $G_F$ groups, including $A_F$ is diagonal and non-degenerate. Therefore, any unitary matrix with $1$ in \([13, 22]\). The mass matrix that reproduces our mixing scheme is:

$$m_\nu = U^* m^{\text{diag}}_\nu U^\dagger = \begin{pmatrix} A & B + C & B - C \\ \cdot & \frac{1}{2} (A + B + D + 4 C) & \frac{1}{2} (A + B - D) \\ \cdot & \cdot & \frac{1}{2} (A + B + D - 4 C) \end{pmatrix}$$

$$= m^{\text{TBM}}_\nu + \begin{pmatrix} 0 & C & -C \\ \cdot & 2 C & 0 \\ \cdot & \cdot & -2 C \end{pmatrix},$$

where we identify (with $c = \cos \theta$ and $s = \sin \theta$)

$$A = \frac{1}{3} \left(2 m_1 + m_2 c_\theta^2 e^{-2i \alpha} + m_3 s_\theta^2 e^{2i (\psi - \beta)} \right), \quad C = \frac{1}{\sqrt{6}} \left(m_2 e^{-i (\psi + 2 \alpha)} - m_3 e^{i (\psi - 2 \beta)} \right) s_\theta c_\theta,$$

$$B = \frac{1}{3} \left(-m_1 + m_2 c_\theta^2 e^{-2i \alpha} + m_3 s_\theta^2 e^{2i (\psi - \beta)} \right), \quad D = m_2 e^{-2i (\psi + \alpha)} s_\theta^2 + m_3 c_\theta^2 e^{-2i \beta}.$$

Here we have used the form of the PMNS matrix in Eq. (11), including for completeness the two Majorana phases. We see that the $\mu$–$\tau$ symmetry is broken by the extra terms involving $C$. In the limit of $C = 0$ we would have tri-bimaximal mixing. Namely,

$$m_\nu |_{C=0} = m^{\text{TBM}}_\nu = U^{\text{TBM}} \, \text{diag} \, (A - B, A + 2B, D) \, U^{T}_{\text{TBM}}. \tag{12}$$

Note that $C$ is proportional to $\sin \theta$, and thus expected to be somewhat smaller than $A, B, D$.

The eigenvalue $A - B$ has an eigenvector $\frac{1}{\sqrt{6}} (2, -1, -1)^T$, corresponding to the invariant column in TM1. This symmetry of the neutrino mass matrix can be characterized by a unitary matrix $G_1$, satisfying the relation $m_\nu \rightarrow G^T_1 m_\nu G_1$, in which $G_1$ defines a $Z_2$ group, i.e. $G_1^2 = 1$, and it is given by

$$G_1 = \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & 1 \\ -2 & 1 & -2 \end{pmatrix}. \tag{13}$$

The third column of the mixing matrix allows us to define another $Z_2$ symmetry, under which the mass matrix is invariant.

$$G_2 = \frac{1}{3} \begin{pmatrix} 2 + c_\theta & s_\theta (\sqrt{6} e^{i \psi} c_\theta - 2 s_\theta) & -s_\theta (\sqrt{6} e^{i \psi} c_\theta + 2 s_\theta) \\ s_\theta (\sqrt{6} e^{-i \psi} c_\theta - 2 s_\theta) & s_\theta (2 \sqrt{6} c_\theta \cos \psi + s_\theta) & \frac{1}{2} (1 + 5 c_{2\theta} + 2i \sqrt{6} \sin \psi s_{2\theta}) \\ -s_\theta (\sqrt{6} e^{-i \psi} c_\theta + 2 s_\theta) & \frac{1}{2} (1 + 5 c_{2\theta} - 2i \sqrt{6} \sin \psi s_{2\theta}) & -s_\theta (2 \sqrt{6} c_\theta \cos \psi - s_\theta) \end{pmatrix}. \tag{14}$$

Therefore, the neutrino mass matrix is invariant under the $Z_2 \times Z_2$ transformation generated by $G_1 \times G_2$. Note that we are working in the basis in which the charged lepton mass matrix is diagonal and non-degenerate. Therefore, any unitary matrix $F$ that commutes with the charged lepton mass matrix must be diagonal with unit moduli in all its entries. For the three generation case, there are three distinct $F$: diag$(1, \omega, \omega^2)$, diag$(\omega, 1, \omega^2)$ and diag$(\omega^2, 1, \omega)$ with $\omega = \exp(2\pi i/3)$. This allows one to construct a flavor group $G$ generated by $F$, $G_1$ and $G_2$, i.e. $G = \{F, G_1, G_2\}$. The generator $G_1$ is well motivated in several flavor symmetry groups, including $A_4$, $S_4$ and other higher order discrete groups (see Ref. [13] for a detailed
discussion, where in fact $S_4$ was proposed for the scheme under consideration). In contrast, $G_2$ cannot be simply embedded into small discrete groups due to the rotation angle $\theta$. For $\theta = 0$ the matrix $G_2$ becomes the generator of $\mu$–$\tau$ symmetry,

$$G_2(\theta = 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{14}$$

For $\theta = \pi/12$ (which is close to the best-fit point) and $\phi = \pi$, one has

$$G_2(\theta = \pi/12) = \begin{pmatrix} \frac{1}{12} (4 + \sqrt{3}) & \frac{1}{12} (-4 + 2\sqrt{3} - \sqrt{6}) & \frac{1}{12} (-4 + 2\sqrt{3} + \sqrt{6}) \\ \frac{1}{12} (-4 + 2\sqrt{3} - \sqrt{6}) & \frac{1}{12} (2 - \sqrt{3} - 2\sqrt{6}) & \frac{1}{12} (2 + 5\sqrt{3}) \\ \frac{1}{12} (-4 + 2\sqrt{3} + \sqrt{6}) & \frac{1}{12} (2 - \sqrt{3} + 2\sqrt{6}) & \frac{1}{6} - \frac{1}{4\sqrt{3}} + \frac{1}{\sqrt{6}} \end{pmatrix}, \tag{15}$$

which actually gives $|U_{e3}|^2 \approx 0.025$, $\sin^2 \theta_{12} \approx 0.318$ and $\sin^2 \theta_{23} \approx 0.709$. Both $\theta_{13}$ and $\theta_{12}$ are well within the current $1\sigma$ ranges, while $\theta_{23}$ deviates from its best-fit value at more than $2\sigma$ C.L., which might be improved once an explicit model with perturbations is constructed.

**B. Realization in flavor symmetry models**

As we mentioned above, the TM$_1$ mixing scheme can be viewed as a modification to the TBM mixing pattern by multiplying a 23-rotation from the right to $U_{TBM}$. In this sense, the rotation matrix $R_{23}(\theta, \psi)$ could also be viewed as a perturbation to the exact TBM mixing pattern. Alternatively, and this is what happens in the two short examples we are about to give, one can note that the mass matrix for TM$_1$ is the TBM mass matrix plus a simple additional term, see Eq. (11). It is therefore possible that we modify a successful model leading to TBM, and add additional flavons and particles to it which give precisely the required form for TM$_1$.

In the original Altarelli–Feruglio model [23, 24], the left-handed lepton doublets are assigned to a three dimensional representation 3 under the tetrahedral group $A_4$, whereas the right-handed lepton fields transform as $1$, $1''$ and $1'$, respectively. In addition, three sets of flavon fields $\varphi_T$, $\varphi_S$ and $\xi$, transforming as $3$, $3$ and $1$ under $A_4$, are also introduced together with the vacuum expectation values (VEVs): $\langle \varphi_T \rangle = (v_T, 0, 0)$, $\langle \varphi_S \rangle = (v_S, v_S, v_S)$ and $\langle \xi \rangle = u$, respectively. At leading order, the $A_4$ invariant Lagrangian contains terms like

$$\mathcal{L} = y_e \frac{1}{\Lambda} e^c (\varphi_T \ell) h_d + y_\mu \frac{1}{\Lambda} \mu^c (\varphi_T \ell)' h_d + y_\tau \frac{1}{\Lambda} \tau^c (\varphi_T \ell)'' h_d + \frac{1}{\Lambda} x_a \xi (\ell h_u \ell h_u) + \frac{1}{\Lambda} x_b \varphi_S (\ell h_u \ell h_u) + \text{h.c.}, \tag{16}$$

where $y_\alpha$ and $x_i$ denote the corresponding Yukawa couplings, $\Lambda$ is the cut-off scale of the theory, and two Higgs doublets $h_u$ and $h_d$ with VEVs $v_u$ and $v_d$ are assumed to be invariant.

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1. Note that we assume the solution of the vacuum alignment could be achieved. In fact, several methods (e.g. by using driving fields) have been proposed to explain the different alignments, and these will not be discussed here.
under $A_4$. Note that there is also an additional $Z_3$ symmetry in the model, which decouples the charged lepton and neutrino sectors [23, 24]. By inserting the VEVs of the flavon fields, one finds that the charged lepton mass matrix is diagonal at leading order, i.e.

$$m^{(0)}_{\ell} = \frac{v_x v_T}{\Lambda} \text{diag} (y_e, y_\mu, y_\tau),$$

(17)

whereas the neutrino mass matrix is given by

$$m^{(0)}_\nu = \frac{v^2}{\Lambda} \begin{pmatrix} a + \frac{2b}{3} & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & 2b/3 & a + \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & 2b/3 \end{pmatrix},$$

(18)

with $a = 2x_a \frac{v}{\Lambda}$ and $b = 2x_b \frac{v}{\Lambda}$. The leading order mass matrix $m^{(0)}_\nu$ is then diagonalized by $U_{\text{TBM}}$ as

$$m^{(0)}_\nu = \frac{v^2}{\Lambda} U_{\text{TBM}} \text{diag} (a + b, a, -a + b) U_{\text{TBM}}^T.$$

(19)

Note that $m^{(0)}_\nu$ is consistent with the TBM mass matrix form $m^{\text{TBM}}_\nu$ defined in Eq. (11) since the identification $A = a + \frac{2}{3}b$, $B = -\frac{1}{3}b$ and $D = b - a$ can be made.

In order to modify the TBM mixing pattern, we introduce another flavon field $\phi$, which transforms as $3$ and couples to the lepton doublets via $\frac{1}{\Lambda} x_i c \phi (\ell h_u \ell h_u)$. Similar to $\xi$ and $\varphi_S$, the unwanted couplings between right-handed charged leptons and $\phi$ are forbidden by the additional $Z_3$ symmetry. Different from the flavons $\varphi_T$ and $\varphi_S$, the flavon field $\phi$ is assumed to develop a VEV along the directions $\langle \phi \rangle = (0, -v_\phi, v_\phi)$. This vacuum alignment follows the orthogonality conditions $\langle \phi \rangle \cdot \langle \varphi_T \rangle$ and $\langle \phi \rangle \cdot \langle \varphi_S \rangle$, where the “.” denotes the usual scalar product of 3-vectors. It has been shown in Ref. [22] that such an orthogonality condition can be realized within supersymmetry with “Lagrange multiplier” superfields, which are singlets under the flavor symmetry but couple to the flavon fields in the superpotential. The $F$-term conditions, which are equivalent to the orthogonality conditions, could then yield the desired vacuum alignments.

The residual symmetry in the neutrino sector is now broken by $\langle \varphi_S \rangle = (v_S, v_S, v_S)$ down to a $Z_2$ symmetry $G_1$. The additional term for $m_\nu$ from the extra flavon, after acquiring its VEV alignment $\langle \phi \rangle = (0, -v_\phi, v_\phi)$, gives

$$m^{(1)}_\nu = \frac{v^2}{\Lambda} \begin{pmatrix} 0 & \frac{c}{2} & -\frac{c}{2} \\ \frac{c}{2} & c & 0 \\ -\frac{c}{2} & 0 & -c \end{pmatrix},$$

(20)

where $c = 2x_c \frac{v_\phi}{\Lambda}$. Writing $C = \frac{1}{2}c$, the TM$_1$ matrix structure given in Eq. (11) is then reproduced.

The above analysis could also be applied to other groups containing $A_4$ as a subgroup. In the $S_4$ model explored in Ref. [25], the lepton doublets are assigned to a three dimensional representation $3_1$ under $S_4$ as well as two flavons $\psi \sim 2$ and $\Delta \sim 3_1$. The vacuum alignments
are taken to be \( \langle \psi \rangle = (v_\psi, v_\psi) \) and \( \langle \Delta \rangle = (v_\Delta, v_\Delta, v_\Delta) \). The neutrino mass matrix then reads
\[
m^{(0)}_{\nu} = \frac{v_u^2}{\Lambda} \begin{pmatrix}
 2f & d-f & d-f \\
d-f & d+2f & -f \\
d-f & -f & d+2f
\end{pmatrix},
\]
where \( d = 2x_d v_\psi / \Lambda \) and \( f = 2x_f v_\Delta / \Lambda \) stem from the Yukawa coupling terms. This is equivalent to TBM, which can be seen by writing \( A = 2f \), \( B = d - f \) and \( D = d + 3f \). Hence, \( m^{(0)}_{\nu} \) is diagonalized as
\[
m^{(0)}_{\nu} = \frac{v_u^2}{\Lambda} U_{\text{TBM}} \text{diag} (-d + 3f, 2d, d + 3f) U_{\text{TBM}}^T.
\]
Similar to the \( A_4 \) model, here we introduce a new flavon field \( \zeta \), which transforms as \( \mathbf{3}_1 \) but possesses a special vacuum structure, \( \langle \zeta \rangle \sim (0, -v_\zeta, v_\zeta) \), again possible to achieve with an orthogonality condition. The new flavon field leads to the following contribution to the neutrino mass term
\[
m^{(1)}_{\nu} = \frac{v_u^2}{\Lambda} \begin{pmatrix}
 0 & s & -s \\
s & 2s & 0 \\
-s & 0 & -2s
\end{pmatrix},
\]
where \( s = y v_\zeta / \Lambda \) with \( y \) being the Yukawa coupling between \( \zeta \) and the lepton doublets.

In these two examples, the VEVs of the new flavon fields should in principle be smaller than the other flavon VEVs, the reason being that \( C \) is proportional to \( \sin \theta \) and expected to be suppressed with respect to the other entries in Eq. (11). This could be achieved if the scale of new flavon fields is roughly one order of magnitude smaller than the others.

IV. SUMMARY

In this work we have studied an attractive neutrino mixing scheme TM\(_1\), in which the first column of the PMNS matrix has the same form as for tri-bimaximal mixing. The PMNS matrix can be described by using only two parameters: one rotation angle \( \theta \) and one CP phase \( \psi \). The physical observables, i.e. the three mixing angles and the Dirac phase, are therefore correlated via the two parameters, leaving us with rather definite phenomenology. While this was studied before, we noted here that the features that apparently emerge from global fits can be excellently described by this mixing scheme. Namely, if \( |U_{e3}| \) is non-zero, solar neutrino mixing is governed by \( \sin^2 \theta_{12} = \frac{1}{3} - (|U_{e3}|^2) \), i.e. slightly less than \( \frac{1}{3} \). If in addition the CP phase \( \delta \) is such that \( \cos \delta \) is negative, and located around \( \pi \), then atmospheric neutrino mixing is governed by \( \sin^2 \theta_{23} = \frac{1}{4} - (|U_{e3}|) \), i.e. significantly less than \( \frac{1}{2} \). These features, small negative deviations from \( \frac{1}{2} \), large negative deviations from \( \frac{1}{2} \) and \( \delta \) around \( \pi \), are (within 1\( \sigma \)) the outcome of the global fit that we referred to. In fact, the sizable deviation from maximal mixing requires that the CP phase \( \delta \) is around \( \pi \), which can be tested in upcoming long-baseline neutrino facilities. We have also discussed potential flavor
symmetries behind the TM\textsubscript{1} scheme, and showed in particular that rather straightforward additions to existing models leading to tri-bimaximal mixing, be it \(A_4\) or \(S_4\), can lead to the mixing scheme.

It will be interesting to see whether these features to which global fits seem to point survive the test of time. The mixing scheme that we studied here seems to be very well suited to describe the current data, and its rather simple structure adds to its attractiveness.

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