Global Vlasov simulation on magnetospheres of astronomical objects

Takayuki Umeda¹, Yosuke Ito¹ and Keiichiro Fukazawa²
¹Solar-Terrestrial Environment Laboratory, Nagoya University, Nagoya 464-8601, Japan
²Research Institute for Information Technology, Kyushu University, Fukuoka 812-8581, Japan
E-mail: umeda@stelab.nagoya-u.ac.jp

Abstract. Space plasma is a collisionless, multi-scale, and highly nonlinear medium. There are various types of self-consistent computer simulations that treat space plasma according to various approximations. We develop numerical schemes for solving the Vlasov (collisionless Boltzmann) equation, which is the first-principle kinetic equation for collisionless plasma. The weak-scaling benchmark test shows that our parallel Vlasov code achieves a high performance and a high scalability. Currently, we use more than 1000 cores for parallel computations and apply the present parallel Vlasov code to various cross-scale processes in space plasma, such as a global simulation on the interaction between solar/stellar wind and magnetospheres of astronomical objects.

1. Introduction

No less than 99.9% of the matter in the visible Universe is in the plasma state. The plasma is a gas in which a certain portion of the particles are ionized, which is considered to be the “fourth state” of the matter. The Universe is filled with plasma particles ejected from the upper atmosphere of stars. The stream of plasma is called the stellar wind, which also carries the intrinsic magnetic field of the stars. Our solar system is also filled with plasma particles from the Sun. Neutral gases in the upper atmosphere of the Earth are also ionized by a photoelectric effect due to absorption of energy from sunlight.

Computer simulation has now become an essential approach in studies of space plasma. Since the number density of plasma particles in space is low and the mean-free path (average distance between collisions of plasma particles) is large, the word “space plasma” is generally equivalent to collisionless plasma. For an example, the number density far above the Earth’s ionosphere is \(\sim 100\text{cm}^{-3}\) or much less, and a typical mean-free path of solar-wind plasma is about 1AU (Astronomical Unit: the distance from the Sun to the Earth; 1AU\(\sim 150,000,000\text{km}\)). In the case of long collision length, the scale of plasma variability is set by the Debye length, which varies from a few millimeters at the low-Earth orbit to a few kilometer or so in tenuous plasma. The plasma behaves as a dielectric medium with strong nonlinear interactions between plasma particles and electromagnetic fields. That is, the motion of plasma results in an electric current, electric currents modify the surrounding electromagnetic fields, and electromagnetic fields modify the motion of plasma, which results in a new electric current.

There are numerous types of self-consistent computer simulations that treat space plasma according to various approximations. The “global”-scale processes, such as structures and
dynamics of solar/stellar winds, solar/stellar flares, and magnetospheres of stars, planets and satellites, are commonly described by magneto-hydro-dynamic (MHD), Hall-MHD and multi-fluid models. On the other hand, electron-scale processes, such as acceleration and heating of electrons, are described by the full kinetic model, i.e., the Maxwell equations and either the Newton-Lorentz equation or the Vlasov (collisionless Boltzmann) equation for both electron and ion particles. Hybrid methods treat ions as particles and electrons as a fluid for ion-scale processes.

Conventionally, MHD simulations have been widely used for numerical modeling of global-scale problems such as magnetospheres of stars and planets. The MHD or fluid equations are derived by taking the zeroth, first, and second moments of the kinetic Vlasov equations, with the zeroth, first, and second moments being the conservation laws of the density, momentum, and energy, respectively. Thus, the MHD simulations need diffusion coefficients, which are essentially due to kinetic processes that are eliminated in the framework of the MHD approximation. These coefficients are essentially due to first-principle kinetic processes that are eliminated in the framework of the fluid approximations. Recent high-resolution in-situ observations have also suggested that fluid scale and kinetic scale in space plasma are strongly coupled with each other, which is called cross-scale coupling. To understand the cross-scale coupling in space plasma, it is important to include full kinetics in global-scale simulations, which is a final goal of space plasma physics.

We develop numerical schemes for Vlasov simulations for practical use on currently-existing supercomputer systems. The Vlasov model treats “hyper”-dimensional (≥3D) distribution functions in position and velocity spaces. It is still difficult to treat the 3P3V (three dimensions for position and three dimensions for velocity) phase space even with the recent supercomputers, because a computation with 40⁶ grid points requires 160GB memory. With the help of recent development of new Vlasov solvers, however, there are several successful attempts of 2P2V and 2P3V hyper-dimensional Vlasov simulations for studying the Kelvin-Helmholtz instability[1] and the magnetic reconnection [2, 3], which are “local” processes taking place at boundary layers of magnetospheres. In the present study, on the other hand, we aim to perform a “global” Vlasov simulation of the magnetosphere of a small astronomical object.

2. Overview of Numerical Schemes
The Vlasov model solves the kinetics equations of space plasma, i.e., the Maxwell equations (1) and the Vlasov (collisionless Boltzmann) equation (2),

\[
\begin{align*}
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s} [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \frac{\partial f_s}{\partial \mathbf{v}} &= 0
\end{align*}
\]

where \( \mathbf{E}, \mathbf{B}, \mathbf{J}, \rho, \mu_0, \epsilon_0 \) and \( c \) represent electric field, magnetic field, current density, charge density, magnetic permeability, dielectric constant and light speed, respectively. The Vlasov equation (2) describes the development of the distribution functions by the electromagnetic (Lorentz) force, with the collision term in the right hand side set to be zero. The distribution function \( f_s(\mathbf{r}, \mathbf{v}, t) \) is defined in position-velocity phase space with the subscript \( s \) being the species of singly-charged particles (e.g., \( s = i, e \) for ions and electrons, respectively). The Maxwell equations and the Vlasov equation are coupled with each other via the current density \( \mathbf{J} \) that satisfies the continuity equation for charge

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0
\]
These equations are regarded as the “first principle” of the collisionless plasma.

It is not easy to solve hyper-dimensional Vlasov equation numerically, in terms of both computational resources and computational accuracy. The Vlasov equation (2) consists of two advection equations with a constant advection velocity and a rotation equation by a centripetal force without diffusion terms. To simplify the numerical time-integration of the Vlasov equation, we adopt a modified version of the operator splitting [4],

\[ \frac{\partial f_s}{\partial t} + v \frac{\partial f_s}{\partial r} = 0 \] (4)

\[ \frac{\partial f_s}{\partial t} + \frac{q_s}{m_s} E \frac{\partial f_s}{\partial v} = 0 \] (5)

\[ \frac{\partial f_s}{\partial t} + \frac{q_s}{m_s} [v \times B] \frac{\partial f_s}{\partial v} = 0 \] (6)

Equations (4) and (5) are scalar (linear) advection equations in which \( v \) and \( E \) are independent of \( r \) and \( v \), respectively. We adopt a multidimensional conservative semi-Lagrangian scheme [4] for solving the multidimensional advection equations. In the full electromagnetic method, it is essential to use conservative schemes for satisfying the continuity equation for charge. With the multidimensional conservative semi-Lagrangian scheme, the continuity equation for charge (3) is exactly satisfied. In the present study, we compute the numerical flux by using the multidimensional advection scheme [4] with a positive, non-oscillatory and conservative limiter [5, 6] for stable time-integration of advection equations. Equation (6), on the other hand, is a multi-dimensional rotation equation which follows a circular motion of a profile at constant speed by a centripetal force. For stable rotation of the profile on the Cartesian grid system, the “back-substitution” technique [7] is applied. In addition, Maxwell’s equations are solved by the implicit Finite Difference Time Domain (FDTD) method.

The velocity distribution function has both configuration-space and velocity-space dimensions, and defined as a hyper-dimensional (>3D) array, There are some additional communications overhead in parallelizing over the velocity-space dimensions since a reduction operation is required to compute the charge and current densities (the zeroth and first moments) at a given point in configuration space. Thus we adopt the “domain decomposition” only in configuration space, where the distribution functions and electromagnetic fields are decomposed over the configuration-space dimensions. This involves the exchange of ghost values for the distribution function and electromagnetic field data along boundaries of each procession element. The non-oscillatory and conservative scheme [5, 6] uses six grids for numerical interpolation, and three ghost grids are exchanged by using the “Mpi_Sendrecv()” subroutine in the standard message passing interface (MPI) library for simplicity and portability [8]. We also use the “Mpi_Allreduce()” subroutine for the convergence check on each iteration of the implicit FDTD method. Note that the code allows thread parallelization over the velocity-space dimensions via OpenMP.

3. Performance Evaluation

We conduct the performance measurement test of our parallel Vlasov code with a phase-space grid of \((N_{x}, N_{y}, N_{z}, N_{x}, N_{y}) = (30, 30, 30, 40, 20)\) on one core, which corresponds to a weak-scaling test with 1GB/core.

We use the following three systems for the benchmark test. The Hitachi HA8000 at the University of Tokyo has four AMD Opteron 8356 (four-core 2.3GHz) processors and 64GB memory per node. A total of 512 nodes are connected each other with four links of the Myrinet-10G interconnect. The Fujitsu FX1 at Nagoya University and JAXA has one SPARC64 VII (four-core 2.5GHz) processor and 32GB/16GB memory per node. A total of 768/3008 nodes are
connected each other with one link of the DDR InfiniBand. The Fujitsu FX10 at the University of Tokyo has one SPARC64 FXfx (sixteen-core 1.848GHz) processor and 32GB memory per node. A total of 4800 nodes are connected each other with the Tofu 6D mesh/torus interconnect.

Figure 1 shows the inter-node parallel performance on these systems. Note that the computational performance is measured by using the hardware counter installed on the Fujitsu FX1 and FX10 systems, and the performances on the Hitachi HA8000 are estimated based on the result on the FX1 and FX10. The peak performances (performance efficiencies) of the parallel Vlasov code are 8.19TFlops on the HA8000 with 8192 cores, 4.10TFlops/15.43TFlops on the FX1 with 3072/12,032 cores, and 37.05TFlops on the FX10 with 23,040 cores. We obtained a high scalability of ∼90% on the FX1 and FX10 even with more than 10,000 cores. The Myrinet-10G internode-connection device of the HA8000 system also gives a high scalability of ∼80% up to 2048 cores. However, the scalability becomes worse with more than 4096 cores because of the network bandwidth capacity.

4. Global Vlasov Simulation
4.1. Simulation Setup
We perform a two-and-half-dimensional (2.5D) Cartesian system in which spatial grids are taken in the two dimensional (2D) $x - y$ plane and velocity grids are taken in the three dimensional (3D) $v_x - v_y - v_z$ space (2P3V). There exists a dielectric circle (cylinder) as a small astronomical object at $r \equiv (x, y) = (0, 0)$, on the surface of which the electric charge accumulates by absorption of plasma particles. The radius of the object is set to be $R_S = 2r_i$, where $r_i$ is the gyro radius of solar-wind ions. The simulation box is taken to be $-8R_S \leq L_x \leq 8R_S$ and $-12R_S \leq L_y \leq 12R_S$, and a total of $N_x \times N_y = 160 \times 240$ grid points are used. Thus the grid size is $\Delta \equiv \Delta x = \Delta y = 0.1R_S = 0.05r_i$. At all edges and corners of the simulation box, we applied so-called absorbing boundary conditions to suppress non-physical reflection of electromagnetic waves.

There exists a uniform plasma flow as the solar wind in the simulation box at the initial state. The plasma flow is directed in the $x$ direction with a flow velocity $V_s = 5V_{ti}$. The uniform plasma flow is continuously injected from the left boundary, i.e., $x = -8R_S$, to the right. In order to save computational costs, electrons are assumed to be much heavier than the reality with the
mass ratio \( m_i/m_e = 25 \). Assuming that the ions and electrons have the same temperature \((T_i/T_e = 1.0)\), and thus the thermal velocity of electrons becomes \( V_{te} = 5V_{ti} \). For the velocity space, we use \( N_x \times N_y \times N_z = 60 \times 60 \times 60 \) grid points. The range of the velocity space is taken to be \(-15V_{ti} \leq v_i \leq 15V_{ti}\) and \(-10V_{te} \leq v_e \leq 10V_{te}\) for all \( v_x, v_y \) and \( v_z \) directions, where \( V_t \) denotes the thermal velocity. Thus a total of 400 GB memory is used for the entire computation.

The solar wind carries an Inter-planetary Magnetic Field (IMF), which is the intrinsic magnetic field of the Sun. In the present study, an ambient magnetic field as the IMF is taken in the \( x – z \) plane in order to include both ion gyro motion and magnetic convection in the \( x – y \) plane. The magnitude of the ambient magnetic field is given by \( B_0 = \sqrt{B_{x0}^2 + B_{z0}^2} = m_i/q_0 \omega_{ci} \) with \( \omega_{ci}/\omega_{pi} = 0.02 \), where \( \omega_c \) and \( \omega_p \) are the cyclotron and plasma angular frequency, respectively. Thus the ion inertial length and electron Debye length become \( l_i = 2r_i \) and \( \lambda_D = 0.02r_i \), respectively. The IMF is imposed in the 45\(^\circ\) with respect to the \( x \) axis with \( B_{x0} = B_{z0} = B_0/\sqrt{2} \). To carry the ambient magnetic field in the direction of plasma flow (\( x \) direction), we need a uniform electromotive (Faraday’s inductive) force. A uniform external electric field is applied in the \( y \) direction with the magnitude of \( E_{y0} = V_S B_{z0} \). The ion beta and Alfven Mach number are \( \beta_i = 0.5 \) and \( M_A = 2.5 \), respectively.

There exists an inner boundary at \( r^2 = x^2 + y^2 = R_S \). We assume that the electric conductivity of the object is small, like an insulator. Numerically, the spatial numerical flux is set to be zero for \( x^2 + y^2 < R_S \). Then, spatial advection in the object is not allowed and the electric charge of plasma particles onto the object accumulates on its surface. Note that we do not use any special treatment for electromagnetic waves at the inner boundary. That is, electromagnetic waves propagate freely into the object without conductance.

Suppose a typical velocity of the solar wind and the thermal velocity of ions are \( V_S = 350 \text{km/s} \) and \( V_{ti} = 70 \text{km/s} \) (i.e., about 50 eV), respectively, and a typical ion density is \( N_i = 5 \text{cm}^{-3} \), the ion plasma frequency and Debye length become about \( f_{pi} = 468 \text{Hz} \) and \( \lambda_D = 24 \text{m} \), respectively. Thus the radius of the object becomes about \( R_S = 2.4 \text{km} \), which corresponds to the typical size of an asteroid. The ion cyclotron frequency and the gyro radius become about \( f_{ci} = 9.4 \text{Hz} \) and \( r_i = 1.2 \text{km} \), respectively.

In the present study, we performed two simulation runs without and with an intrinsic magnetic field of the object for Run A and Run B, respectively. In Run B, we assume a 2D dipole magnetic field as an intrinsic magnetic field,

\[
B_d = \frac{1}{2\pi} \left( 2d \frac{M \cdot d}{|d|^2} - \frac{M}{|d|^2} \right) \quad (7)
\]

where \( M \) denotes the dipole moment and \( d \equiv r - d_0 \) denotes the distance from the center of the dipole \( (d_0) \). In the present study, we use \( d_0 = (-0.6R_s, -0.3R_s) \) and \( M_y = -6.4\pi B_0 \) for Run B.

### 4.2. Simulation Result

Figure 2 shows the temporal development of the ion density together with magnetic field lines as a result of the interaction between the solar wind and the small dielectric object. It is found that a low-density region is formed on the nightside of the object in both runs. The result indicates the formation of wake tail by the absorption of solar-wind plasma on the surface of the body [9, 10]. As seen in figure 2, there are closed field lines at the dayside in Run B. A high-density region is formed at the dayside of the body, indicating the formation of bow shock by the interaction between the intrinsic magnetic field of the object and the solar wind with IMF. The structure of the wake tail at the nightside in Run B is modified from that in Run A, although the magnetic field lines at the nightside are mostly open.
Figure 3 shows the reduced velocity distribution functions of ions, $f_i(y, v_x)$, $f_i(y, v_y)$, and ion density $N_i$ at $x/R_S = 1.1$ and $\omega_{ci}t = 9$. In Run A, the ion density in the deep wake tail is $< 0.01$ of that of solar-wind ions. The $f_i(y, v_y)$ velocity distribution function shows that ions enter into the wake tail from both $+y$ and $-y$ directions. The entry of ions into the wake tail from the $+y$ side is due to the ion gyro motion. Since the thermal gyro radius of the solar-wind ions is $r_i = V_{ti}/\omega_{ci} = 0.5 R_S$, only a small part (high-energy thermal tail) of solar-wind ions can enter into the deep wake tail. On the other hand, the entry of ions from the $-y$ side is due to the magnetic convection ($E \times B$ drift) [10]. At the nightside of the body, there appears a strong electric field directed toward the body ($-E_x$) due to the accumulation of electrons on the nightside surface of the body [11, 9]. This electric field is not Debye-shielded because of a low plasma density in the wake tail and becomes a motional (convective) electric field which moves magnetic fields in the $+y$ direction.

In Run B, on the other hand, the ion density in the deep wake tail is $\sim 0.03$ of that of solar-wind ions. The velocity distribution functions show that there exists a nonthermal ion component in a wide area of the downstream (tail) region. The nonthermal ions have a negative velocity in $v_x$ and widely spread over $v_y$. We expect that origins of the nonthermal tail are the dayside bow shock and the dayside magnetopause where solar-wind ions are reflected backward by the magnetic pressure gradient. The reflected ions are picked up and carried downstream by IMF. Note that the pickup ions have a larger gyro radius than solar-wind ions and can enter into the deep wake tail by a trochoidal motion. Detailed analysis on these nonthermal ions is left as a future study. Note that the enhancement of ion flux in the wake tail of an unmagnetized small object is observed by the Kaguya spacecraft at the Moon [12].

5. Conclusion
In this paper, we have made performance measurements of a new Vlasov-Maxwell code on massively-parallel scalar computer systems, which are the Hitachi HA8000, the Fujitsu FX1 and the Fujitsu FX10 supercomputer systems. The weak-scaling benchmark test shows that our parallel Vlasov code achieves a high scalability (>90%) on more than 10,000 cores with recent inter-node connection devices.

We have also performed the “first” global Vlasov simulation of a magnetosphere of a small
Figure 3. Reduced velocity distribution functions of ions, $f_i(y, v_x)$, $f_i(y, v_y)$, and ion density $N_i$ at $x/R_S = 1.1$ and $\omega_{ci}t = 9$.

astronomical object with a spatial scale of ion gyro radius. When the spatial scale of the object and its magnetosphere is the same, the structures of the magnetosphere at the nightside may not be affected by the intrinsic magnetic field because most of magnetic fields at the nightside are open. However, the result indicates that the structure of the nightside wake tail is strongly modified by a weak intrinsic magnetic field at the dayside.

Finally, it should be noted that a 6D simulation (with both three spatial and three velocity dimensions: 3P3V) is essential for full understanding on the global structure and dynamics of magnetospheres. However this requires numerous computing resources with more than 50TB memory, and is left as a (far) future study.
Acknowledgments
The authors are grateful to Yasuhiro Nariyuki and Tatsuki Ogino for discussions. This work was supported by MEXT/JSPS under Grant-in-Aid for Young Scientists (B) No.23740367. The computational resource at Nagoya University is provided as a Nagoya University HPC program and a JHPCN program. The computational resource at JAXA is provided by Iku Shinohara. The computational resources at the University of Tokyo are provided as Large-Scale HPC Challenge programs and a JHPCN program. The global Vlasov simulation was performed on the DELL PowerEdge R815 at Solar-Terrestrial Environment Laboratory (STEL) in Nagoya University as a STEL computational joint program.

References
[1] Umeda T, Miwa J, Matsumoto Y, Nakamura T K M, Togano K, Fukazawa K and Shinohara I 2010 Phys. Plasmas 17 052311
[2] Schmitz H and Grauer R 2006 Phys. Plasmas 13 092309
[3] Umeda T, Togano K and Ogino T 2010 Phys. Plasmas 17 052103
[4] Umeda T, Togano K and Ogino T 2009 Comput. Phys. Commun. 180 365
[5] Umeda T 2008 Earth Planets Space 60 773
[6] Umeda T, Nariyuki Y and Kariya D 2012 Comput. Phys. Commun. 183 1094
[7] Schmitz H and Grauer R 2006 Comput. Phys. Commun. 175 86
[8] Umeda T, Fukazawa K, Nariyuki Y and Ogino T 2012 IEEE Trans. Plasma Sci. 40 1421
[9] Umeda T, Kimura T, Togano K, Fukazawa K, Matsumoto Y, Miyoshi T, Terada N, Nakamura T K M and Ogino T 2011 Phys. Plasmas 18 012908
[10] Umeda T 2012 Earth Planets Space 64 231
[11] Kimura S and Nakagawa T 2008 Earth Planets Space 60 591
[12] Nishino M, Fujimoto M, Maezawa K, Saito Y, Yokota S, Asamura K, Tanaka T, Tsunakawa H, Matsushima M, Takahashi F, Terasawa T, Shibuya H and Shimizu H 2009 Geophys. Res. Lett. 36 L16103