Selected Topics in Astro-Hadron Physics:
Going from a Proton to Nuclei to Neutron Stars

Mannque Rho

Service de Physique Théorique, CE Saclay, 91191 Gif-sur-Yvette, France
E-mail: rho@spht.saclay.cea.fr
and
School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

Abstract

The effort currently in vogue in some small circle of physicists to go from a proton to nuclei to compact stars involves various aspects of particle and nuclear physics that require input from laboratory experiments, transcending narrow specialization in diverse sub-fields. Several topics on this matter are discussed in this note. The notion of Cheshire Cat Principle is introduced for nucleons, nuclei and dense hadronic matter and is confronted with experimental data on the proton, two-nucleon systems and heavy-ion experiments, with a leaping extrapolation to the structure of neutron stars. The matter discussed here illustrates that a close contact with experiments, indispensable in the present case, is essential for significant progress in any field of physics. I suggest that this is a field that has a tremendous potential for breakthrough in the Asia Pacific countries, particularly in Korea, where some of the seminal works in this area have been done by young theorists, both pre- and post-graduate.
1 Introduction

A relatively new branch of physics combining hadronic physics and astrophysics is called “astro-hadron physics.” I would like to describe some recent efforts to understand such extremely compact astrophysical objects as neutron stars starting from data obtained in laboratories, in particular from heavy-ion experiments that have been performed in such laboratories as GSI of Germany, CERN (SPS) of Switzerland and elsewhere. The central thesis that I will develop is that there are what may look like “alternative” descriptions for the same process in different languages ranging from bare hadrons to quasiparticle hadrons to quasiparticle quarks characterized by what I would call “Cheshire Cat mechanism.” This “duality” nature seems to be operative from elementary hadrons to superdense matter perhaps existing in compact stars, suggesting a unity of particle, nuclear and astro physics.

As in all works at an embryonic stage, there are false starts, pitfalls, wrong tracks and controversies. What I shall describe therefore are not necessarily well-established facts. Much work will be needed to validate or invalidate some or all of them. Even so, what we have at this stage is so exciting that it deserves much more attention than presently paid, particularly from the Asia Pacific physics community where some of the early significant works have been done by young theorists working on PhD theses #1.

I will start the discussion with the proton, the constituent of the nucleus, go to the simplest nucleus, i.e., two-nucleon system including the deuteron, then to nuclear matter and finally to dense hadronic matter relevant to the interior of neutron stars. The topics are selected to highlight work done in Korea by thesis students or young theorists. Clearly part of the connections are incomplete or faulty, needing more solid structure but the logic appears sound and worth pursuing for possible breakthrough.

2 “Proton Spin Problem”

I start with the nucleon, specifically the proton which is the lowest baryonic state. In the fundamental theory of strong interactions, QCD, this state is a bound state of three quarks in the color-singlet state confined in as yet poorly understood way within a region wherein the gluons play a crucial role. To describe this from the QCD Lagrangian using its microscopic variables is at present practically impossible but the most tantalizing fact is that it can be given various different (albeit approximate, yet qualitatively correct) descriptions indicating some sort of “dualities” are in action. One apt way of seeing this is to use “Cheshire Cat principle” (for a general introduction, see [2]).

#1 Some of the earlier developments were already discussed in the 1997 APCTP-sponsored workshop on astro-hadron physics held in Seoul [1]. As a follow-up and for more updated developments, a workshop is planned for the year 2000 at Korea Institute for Advanced Study (KIAS) in Seoul devoted to such phenomena as supernovae, neutron stars, black holes, gamma-ray bursts etc.
2.1 Cheshire Cat Principle

Consider three colored quarks \( uud \) of the quantum numbers of a proton confined in a spherical “bag” of radius \( R \) surrounded by a cloud of Goldstone pions, the latter being indispensable to the system in order to be consistent with the spontaneously broken chiral symmetry. With a suitable set of boundary conditions consistent with the symmetries of QCD that mediate the communication between the inside microscopic QCD variables and the outside macroscopic hadronic variables, nearly all low-energy observables of the proton can be understood independently of the size of the confining bag [2, 3]. Indeed for a large bag with \( R \sim 1 \) fm, the quark-gluon degrees of freedom dominate while for a small bag with \( R \lesssim 1/2 \) fm, it is the Goldstone degrees of freedom that govern the dynamics of the system. This feature is known as the Cheshire Cat phenomenon [4]. When the “bag” is shrunk to a point \( R \to 0 \), which is allowed by Cheshire Cat principle, one gets the celebrated skyrmion [3, 4, 5], a description of the baryon that is the more accurate the larger the number of colors \( N_c \). It seems that in nature, \( N_c = 3 \) is already quite large, so the skyrmion picture is qualitatively correct.

2.2 Flavor-singlet axial charge

One conspicuous exception to the success of the Cheshire Cat phenomenon has, up to date, been the flavor singlet axial charge (which will be referred in short to as FSAC) of the proton which is often associated with the so-called “proton-spin problem.” It turns out that there is no obstacle to the Cheshire Cat manifesting in this quantity, that is, there is no “proton-spin crisis.” The resolution lies in subtle quantum anomalies which are now understood. We do not yet have a complete answer to the issue which is the PhD thesis subject of Hee-Jung Lee at Seoul National University but I will briefly describe how this Cheshire Cat property can be recovered in the FSAC when chiral symmetry and chiral anomaly are judiciously taken into account [8]. The interplay between the boundary conditions and Casimir effects is found to play a crucial role.

Since the flavor-singlet axial current is not conserved because of the anomaly, the color cannot be confined inside the bag unless a suitable boundary condition is put at the surface that cancels the outflow of the color as discovered by H.B. Nielsen et al [9]. The boundary term that does this is proportional to the Chern-Simons current on the surface, i.e., the Chern-Simons flux (which is invariant under neither small nor large gauge transformation). This influences nontrivially the FSAC of the proton. In a nut-shell, what happens is that the FSAC contributed by the matter fields (quarks inside the bag and \( \eta' \) outside the bag) and the FSAC contributed by the gauge field (gluons inside the bag) more or less (or possibly exactly if treated rigorously) cancel, leaving behind only the small contribution from the (gauge field) vacuum fluctuation which is effectively a Casimir effect caused by the boundary with its color-anomalous boundary condition. The cancellation and the remnant small FSAC are shown in Fig. [8].
Figure 1: The flavor singlet axial charge of the proton as a function of the bag radius compared with the experiment; it consists of three contributions: (a) matter field contribution: quark plus $\eta$ ($a_{BQ}^0 + a_{\eta}^0$), (b) gauge field contribution: the static gluons coupled to the quark source ($a_{G,\text{stat}}^0$), (c) Casimir contribution: the gluon vacuum fluctuation ($a_{G,\text{vac}}^0$), and (d) the sum total ($a_{\text{total}}^0$). The shaded area represents the range admitted by experiments.

The small FSAC of the proton that is left over – which is independent of the size $R$ – provides another evidence that the proton could be equivalently understood both in terms of quarks/gluons and in terms of macroscopic hadronic variables. When $R$ is taken to be big, it is the QCD variables that figure significantly, e.g., the MIT bag. When the size $R$ is shrunk to a point, the proton is a skyrmion. Thus we have the equivalence of the skyrmionic proton and the quark-gluonic (QCD) proton, that is, the Cheshire Cat. Since there is no way that one can exactly bosonize four-dimensional QCD, the equivalence can be only approximate. Thus we simply have an approximate equivalence which can be made more precise by doing more work.

In what follows, I will develop the thesis that it is this feature observed in the proton that underlies many-body dynamics going all the way to neutron star matter.

3 Nuclei in Effective Field Theory

Let me now turn to nuclei. For this, consider two nucleon interactions at very low energy. The two-nucleon system is the nuclear system that is the simplest – and the only – nuclear system that can be treated accurately and systematically. At present it is only at the probed energy-momentum much less than the pion mass $m_\pi \sim 140$ MeV that is amenable to an accurate computation. At very low energy, according to Weinberg’s “theorem,” the content of QCD can be phrased in terms of the nucleons and pions treated as effective
fields. In fact if we are probing a scale much less than the pion mass, we can even ignore the pions and work with the nucleons only. The corresponding framework is an effective field theory (EFT). What this means is that we can shrink – following the notion of the Cheshire Cat – the bag of the nucleon and work in the skyrmion limit, namely, zero-radius bag. Thus a nucleus will be a collection of point-like objects interacting strongly subject to the constraint of QCD symmetries.

There has recently been an intense activity on this EFT for two-nucleon systems [9, 10, 11, 12, 13]. Some of the developments are summarized in a series of workshops devoted to the issue [14]. Up to date, there are two successful approaches to EFT in nuclear physics. One is the original Weinberg approach [9] where a systematic power counting is made only to the “irreducible graphs,” for which chiral perturbation theory (with pions figuring prominently) becomes applicable in organizing the expansion of the series and the reducible graphs are summed to all orders with the irreducible graphs entering as vertices. This scheme used in Ref. [9, 10, 11, 13] – which in spirit is close to the original Wilsonian EFT but incurs possible errors in the power counting – involves a chiral-symmetry scale $\Lambda \sim m_\pi$ (where $m_\pi \sim 140$ MeV is the pion mass) as the counting is applied only to the irreducible terms. I will call this the $\Lambda$ counting. The other approach [12] motivated to account more explicitly for the anomalously large s-wave scattering lengths in the two-nucleon scattering allows one to do a systematic counting for the S-matrix as a whole. This approach renders a more systematic accounting of the powers of $Q/\Lambda$ where $Q = \sqrt{MB}$ (where $B$ is the deuteron binding energy and $M$ is the proton mass), $p$ (probe momentum) as well as $m_\pi$ (pion mass) but at the expense of certain predictivity. This is referred to as $Q$-counting scheme.

In essence, both approaches, though somewhat different in strategy, are equally consistent with the tenet of EFT and more or less equivalent in their predictive power. In what follows, I will simply focus on the $\Lambda$-counting approach which works stunningly well for two-body systems. Actually in the processes that I will consider, the $\Lambda$ counting is found to be more readily adaptable to — and predictive in treating — nuclear physics problems. One non-trivial advantage of the $\Lambda$ scheme is that it allows one to calculate precisely defined corrections to what can be obtained from so-called realistic potential models (PM in short) that have been developed by nuclear theorists since a long time, thus giving the realistic potential models (PM) a first-principle justification. It allows us to study processes involving not only few-body but also many-body systems. For instance, it is possible to calculate the “hep” process in the Sun $p + ^3$He $\to ^4$He + $e^+ + \nu_e$ (which is currently an exciting issue after the recent Superkamiokande neutrino data) with an accuracy that can be controlled systematically. Such a calculation in the $Q$-counting scheme is most likely to be a hard task.

In a series of recent papers, Tae-Sun Park (currently a post-doc at TRIUMF in
Vancouver) and his collaborators \[13\] have shown in a finite-cutoff regularization that the EFT results of the leading order terms in all two-body observables at low energy \(E \ll m_\pi\) are precisely reproduced by the potential model results. This is the case not only for scattering amplitudes but also all electroweak response functions. What EFT can do that the potential models (PM) cannot is that the corrections to the leading order results given accurately by the PM are calculable systematically. This is the power of EFT as a theory.

For low-energy processes, this privileged role of the PM in EFT can be understood by the fact that the tail of the wave functions is a physical quantity and the realistic potential models which are fit to experiments have the correct asymptotic properties in the wave function \[17\].

Considered to order \(Q^n\) where \(n\) is the order in the \(\Lambda\) counting (which I will consider relative to the leading order term in the expansion of the irreducible graphs), the \(s\)-wave scattering amplitudes are accurately postdicted by T.-S. Park et al \[13\] and further improved by Chang-Ho Hyun (a graduate student of Seoul National University) et al \[18\] up to \(p \leq m_\pi\) for \(n = 2\) and a cutoff appropriate to the number of pions exchanged (one or two) in the irreducible graphs. All deuteron properties are also well understood within the same scheme \[13\]. As an important spin-off, the scheme allowed the calculations to order \(Q^2\) and \(Q^3\) of the proton fusion process in the Sun \[19\]

\[
p + p \rightarrow d + e^+ + \nu_e
\]

and of the threshold np capture \[1, 20\] with polarized projectile and target nucleons

\[
\vec{n} + \vec{p} \rightarrow d + \gamma.
\]

The process \(\text{(1)}\) crucial for the solar neutrino problem is given in the scheme to an accuracy of \(1 \sim 3\) percents (the uncertainty here is due to the exchange current that appears at order \(Q^3\)). The unpolarized cross section for \(\text{(1)}\) has been computed to the accuracy of 1 percent in complete agreement with the experiment. More significantly, the polarization observables \(P\) (circular polarization) and \(\eta\) (anisotropy) have been predicted parameter-free in Ref.\[20, 21\]. Since there are no experimental data available yet, this is a genuine prediction involving matrix elements that are suppressed relative to the allowed term by \(\sim 3\) orders of magnitude. These quantities are currently being measured in several laboratories and will soon be available. The outcome will be an exciting check of the prediction of the theory, perhaps the first of the kind in nuclear physics.

In all these postdictions and predictions, there is very little \(\Lambda\) dependence as required by the tenet of EFT, assuring that the scheme is fully consistent.

One can go up in the momentum range by doing higher order calculations. Phillips and Cohen \[17\] discuss how the two-body EM form factors can be described in the \(\Lambda\) scheme. Pushing somewhat the validity of the scheme, one can calculate even the process

\[
e + d \rightarrow e + n + p
\]

\#2The cutoff is not to be sent to infinity in EFT contrary to renormalizable field theories \[16\].
involving large momentum transfers \( q > 1 \) GeV. In fact this process measured in 1980’s at ALS of Saclay and elsewhere is considered to be the unambiguous confirmation of meson-exchange currents in nuclei (see [22, 23]).

In sum, the dilute nuclear systems such as the two-nucleon bound and scattering states can be quite accurately described in terms of certain effective degrees of freedom connected to QCD via a Cheshire Cat mechanism. I shall now jump directly to the infinite-body problem, namely, nuclear matter, although the extension of EFT to three- or more-body problems is not fully worked out yet.

4 Dense Hadronic Matter: BR Scaling

For more than a few nucleons and many-nucleon systems like nuclear matter and denser matter, the EFT described above cannot be straightforwardly applied. In fact a systematic approach of the type does not yet exist. The reason is rather simple to understand. First of all when several nucleons are involved the relevant kinematics is not always one to which the low-energy/momentum expansion with a manageable number of terms is applicable and further a systematic expansion would involve many terms whose coefficients are not fully determined from either theory or experiments. This means that strictly speaking, no parameter-free calculation that does not invoke some ad hoc assumptions can be done. Thus some clever intuition is needed to overcome this technical difficulty. One economic way to short-cut the formidable-looking obstacles is the BR scaling introduced by Brown and Rho [24]. Clearly this cannot be the only way but it has not yet met with contradictions while having a variety of success.

The basic idea is that nuclear matter at its equilibrium density represents the ground state of the matter and that at that point, the nucleons and mesons are quasiparticles as they are at zero density. We know from Migdal’s work that nucleons in nuclear matter are quasiparticles in the sense of Landau Fermi liquid theory [25]. In modern language, this means that the nucleon mass and quasiparticle interactions are fixed-point quantities. We go one step further and assume that mesons are also quasiparticles at the equilibrium point of nuclear matter. This may sound absurd as one knows that mesons interact strongly with many inelastic channels open, so the notion of quasiparticles with well-defined effective mass for them does not sound right. But then before the advent of shell model one used to say the same thing about the nucleons and yet nucleons in nuclear matter are bona-fide quasiparticles. Skeptics will then argue that the quasiparticle notion works for the nucleons because of Pauli exclusion principle but for bosons, there is no such thing. This is a valid objection to which no one can at present offer satisfactory answers. So assuming that bosons in medium can be treated in the tree approximation with a point-like structure is an assumption yet to be tested. I shall simply proceed to use the notion in a variety of processes until it meets contradictions. So far there is no evidence that this notion conflicts with nature.
The next step then is to construct an effective Lagrangian theory which preserves the known symmetries of QCD. In principle, this Lagrangian should be usable for a systematic calculation of the type described above for two-nucleon interactions and indeed for particles near “on-shell” in medium, such higher-order calculations must be done to describe the needed amplitudes. I shall focus, however, on processes which are off-shell and hence can be treated in the tree order.

What are then the relevant degrees of freedom? We assume that Goldstone theorem is applicable in medium, that is, there are zero-mass Goldstone (in the chiral limit) or light-mass pseudo-Goldstone (in nature) particles \( \pi^a \), massive nucleons \( N \) appearing as matter fields, vector bosons \( V_\mu \) and possibly scalars \( \sigma \) etc. We assume that such classic low-energy “theorems” as Goldberger-Treiman, Adler-Weisberger, Gell-Mann-Oakes-Renner, Kawarabayashi-Suzuki-Ryazuddin-Fyyazuddin ... relations hold in the medium but with masses given by

\[
\frac{m_N^*}{m_N} \approx \frac{m_V^*}{m_V} \approx \cdots \approx \frac{f_\pi^*}{f_\pi} \approx \left( \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \right)^n.
\]  

Here the star denotes in-medium quantity and \( \langle \bar{q}q \rangle \) stands for the quark condensate in the vacuum (the starred quantity being the same in the in-medium “vacuum.”) The index \( n \) depends on models. Empirically it is close to \( 1/2 \). The quark condensate is believed to vanish (in the chiral limit) at some critical density \( \rho_c \) (this is more or less supported by models but lattice calculations are not yet available) corresponding to chiral phase transition, so one may think of the mass as an indicator for chiral properties of dense matter. This is currently a hot topic with considerable controversy.

### 4.1 Nuclear matter

The first question one must answer for the viability of the theory is: Can this theory describe nuclear matter correctly? If the answer were no, then the theory should be abandoned. The answer comes out to be yes: the theory that gives the correct properties of nuclear matter is a Walecka mean-field-type theory with the nucleon \( N \), isoscalar-vector \( \omega_\mu \), and scalar \( \sigma \) fields that are coupled linearly. The Lagrangian is of the form of a linear Walecka model \([26]\) with, however, the masses of the particles scaling a la BR \([4]\). It has been thought for some time that such a theory with the BR scaling would not give a stable system, not to mention a correct binding energy, saturation density \( \rho_0 \) and compression modulus \( K \). But this (thinking) turns out not to be correct. In a Seoul National University PhD thesis work that succeeds to resolve some of the long-standing problems, Chaejun Song (presently a post-doc at SUNY, Stony Brook) – with help from his senior collaborators \([27]\) – has shown that to correctly interpret the theory, it is essential to express the density dependence of the masses in terms of certain chiral-invariant fermion bilinears. The so-called “rearrangement terms” do come out correctly for the resulting equation of state. All properties of nuclear matter are found to be satisfactorily reproduced while maintaining
consistency with all thermodynamic properties. This gives the assurance that one should be able to make small fluctuations around the ground state using the effective Lagrangian, the equilibrium minimum representing the fixed point.

### 4.1.1 Evidence in heavy nuclei

There are some experimental data already available in the literature that we can use to test certain aspect of the BR scaling in fluctuations around the equilibrium density. A number of cases are available but I shall pick two here for illustration.

- The first is the “anomalous” orbital gyromagnetic ratio \( \delta g_l \) in heavy nuclei. The orbital gyromagnetic ratio \( g_l \) is the coefficient figuring in the convection current for a nucleon sitting on the Fermi surface responding to slowly varying electromagnetic field:

\[
\vec{J} = g_l(e\vec{p}/m_N).
\]

(5)

Because of the many-body interactions, \( g_l \) has an anomalous term \( \delta g_l \),

\[
g_l = \frac{1 + \tau_3}{2} + \delta g_l.
\]

(6)

Note that the current (5) carries the “bare” mass \( m_N \), not the effective quasiparticle (or Landau) mass \( m_N^{\text{eff}} \). Thus the first term of (6) correctly describes charge conservation. This is the analog to Kohn’s theorem in electronic systems [28] and makes the calculation of \( \delta g_l \) a highly constrained one providing a stringent consistency condition. Indeed the “mapping” of the BR scaling Lagrangian theory to Landau Fermi-liquid fixed point theory gives a highly non-trivial result as shown in [29]:

\[
\delta g_l = \frac{4}{9}[\Phi^{-1} - 1 - \frac{1}{2}\tilde{F}_1']\tau_3
\]

(7)

where \( \Phi \) is as given in [3] and \( \tilde{F}_1' \) is the pion contribution to the Landau \( F_1 \) parameter which is completely determined by chiral Lagrangian. At the nuclear matter density \( \rho = \rho_0 \), both \( \Phi \) and \( \tilde{F}_1' \) are known numerically,

\[
\Phi(\rho_0) = 0.78, \quad \tilde{F}_1'(\rho_0) = -0.459.
\]

(8)

Since heavy nuclei must have \( \rho \sim \rho_0 \), the prediction in heavy nuclei is

\[
\delta g_l \approx 0.23\tau_3.
\]

(9)

This prediction is consistent with the experimental data \( \delta g_l^{\text{proton}} = 0.23 \pm 0.03 \) extracted from giant resonances in the lead region. It is also consistent with magnetic moments in the lead region.
Another case that provides support for the scheme is the axial charge transitions in heavy nuclei \( A(0^\pm) \rightarrow A'(0^\mp) + e^+(e^-) + \nu(\bar{\nu}) \) with change of one unit of isospin. As shown in the PhD thesis (at Seoul National University) of Tae-Sun Park \[30\], this particular transition is highly enhanced in nuclei (by a factor of \( \sim 2 \) with respect to the single-particle strength) due to one soft pion being exchanged between two nucleons that are involved in the response to the axial charge operator. In heavy nuclei, this is further enhanced because of the scaling \( f_\pi/f_\pi^* = \Phi^{-1} > 1 \) at nuclear matter density \[31\] which can be seen as one goes up in mass number. Experiments are available in medium and heavy nuclei where this enhancement has been seen and confirmed unambiguously \[32, 15\].

4.2 Heavy-ion collisions and in-medium meson properties

Up to here, I have indicated how the BR scaling works for the nucleon mass and, indirectly, for the pion decay constant. So far the scaling for the meson masses has not been tested although it figures indirectly in both cases discussed above. Heavy-ion collisions could provide a qualitative test of the behavior of mesons in dense medium.

Heavy-ion collisions produce hot and dense matter. The physics of the process must therefore be able to probe the behavior of the relevant degrees of freedom which are mainly mesonic at high temperature and/or high density. This is a big area and much debate has been going on. A recent review can be found in \[33\]. In a recent PhD thesis work, Youngman Kim of Hanyang University (who is currently a post-doc at SUNY, Stony Brook and University of South Carolina, Columbia) has shown that both scalar and vector mesons in hot and dense medium do indeed scale a la BR scaling if trace anomaly and hidden gauge symmetry of QCD are properly taken into account \[34\].

Without going into the details of the dynamics involved – which goes out of scope for this note, it is difficult to be precise about what we are dealing with. So let me be glib about it and just show the result and give some (perhaps biased) remarks.

In fig. \( 2 \) is given the CERN (CERES) experiment for 200 GeV per nucleon central collision of \( S \) on \( Au \) producing dileptons measured at CERN. The differential cross section is plotted vs. invariant mass \( M \). Now if one takes the masses of the particles involved in the collision to be those of free-space particles, workers in the field more or less agree on the predicted cross sections. Despite the large error bars for the experiments, one can see that the free-mass description largely under-estimates the cross section for around \( M \sim 400 \) MeV. As shown by Li, Ko and Brown \[35\], the data can be explained quite economically with the BR-scaling masses, the primary agent for this being the dropping mass of the \( \rho \) meson which plays the principal role in the dilepton process. A similar fit is obtained in the \( Pb-on-Au \) process.

Unfortunately, this simple picture is blurred by a controversy on the precise cause for the shift of the peak in the dilepton data. It appears at present that this BR-scaling mecha-
nism is not the only one that could explain the data. There are various other (alternative?) explanations such as the increased width of the vector meson \[33\] – thereby possibly invalidating the quasiparticle interpretation – or nonperturbative quark-gluon plasma effect \[37\] etc. Whether or not all these alternatives represent different physical phenomena is not known. In any event, it would be premature to conclude that the quasiparticle description a la BR is invalidated. I would say that whether or not a quasiparticle picture for hadrons is applicable in dense medium is an open question that cannot be settled by a few-order calculation in a strong-coupling situation. In fact in condensed matter physics, there are cases where low-order treatments fail to give the correct Fermi liquid structure, the latter resulting only when all-order calculations are performed. For an example, see \[38\].

4.3 Running kaon mass

Let me now turn to fluctuations around the ground state of nuclear matter in other flavor directions than the up- and down-quark flavors, say, in the strangeness flavor direction. For this we need to extend the flavor space to \(SU(3)\). We know how to do this.

A kaon propagating in dense medium interacts with the background with its mass and coupling modified by the medium. The coupling with the matter field (nucleons) is
Figure 3: In-medium kaon ($K^-$) and anti-kaon ($K^+$) masses calculated with the empirically constrained dispersion formula [40].

Figure 4: Kinetic energy spectra of $K^+/K^-$ in Ni + Ni collisions. The circles are experimental data from the KaoS collaboration [42].

given by $\sim f^{*-1}$ where $f^*$ is the in-medium pseudo-Goldstone boson (pion to the leading order) decay constant which scales with density. By (4), the scaling coupling can be related to the scaling in the masses of the mesons that are exchanged between kaon and nucleon in a description where heavy mesons are explicitly accounted for.

The masses for $K^\pm$ so predicted are plotted in fig.(3). Actually what is calculated here is the kaon dispersion relation with inputs from experiments which is equivalent, to leading order, to using a BR-scaled chiral Lagrangian in the tree order. This and related matters are discussed in the PhD thesis work [39] of Chang-Hwan Lee at Seoul National University (presently at SUNY, Stony Brook). This prediction has been beautifully checked by experiments [40] as shown in fig.(4).

4.4 Kaon condensation and neutron stars

An important consequence of the dropping $K^-$ mass is its effect on the equation of state for dense neutron matter and consequently on neutron stars. This is the link between hadron physics and astrophysics that has been emphasized by Gerald E. Brown and Chang-Hwan Lee. The basic idea is quite simple. In dense medium, the $K^-$ mass drops continuously as density increases. But as density increases, the electron chemical potential $\mu_e$ in neutron-star matter increases. When the effective kaon mass crosses the electron chemical potential, the electrons can turn into kaons which by nature of their
The bosonic character can condense as first suggested by Kaplan and Nelson [41]. This is shown schematically in fig.(5).

The onset of kaon condensation is expected to soften the equation of state of the matter. The intricate interplay between this phenomenon, hadronic interactions and gravitational interactions has been worked out by Li, Lee and Brown [40]. Their finding is that the maximum mass of neutron stars can be lowered by about $0.4M_\odot$, once kaon condensation as constrained by the dropping kaon mass – reflected in the empirical dispersion relation – is introduced. The authors point to “the growing interplay between hadron physics, relativistic heavy-ion physics and the physics of compact objects in astrophysics.” This may provide a natural explanation (hopefully without fine-tuning) for the observation that the well-measured neutron-star masses fall within a narrow window $M \lesssim 1.6M_\odot$ (see fig.6 [43]).

5 Superdense Matter and The Cheshire Cat

The last topic I would like to discuss is infinite nuclear matter at large density, a density much greater that considered above. This may be relevant to neutron star cooling although at present it is not clear whether at the density appropriate for neutron star matter other processes cannot compete with it [44, 45].

The old lore that at an asymptotic density, the matter can be described by pertur-
bative QCD with weakly interacting boring quarks is now widely recognized to be simply wrong. What may be happening at super-high density is something a lot more intriguing and exciting than previously thought. This explains a flurry of activity throughout the world, including the present intense activity at Korea Institute for Advanced Study (KIAS) in which I have been participating.

What is most surprising and in some sense unexpected is that at high density the Cheshire Cat picture re-emerges! In fact, at high density, there ceases to be any real distinction between quarks and hadrons. This can be best seen in terms of a quark soliton analogous to the qualiton Kaplan [46] introduced as a model for the constituent quark. The mechanism I will discuss exploits that at high density diquarks condense giving rise to color superconductivity as proposed some years ago [47] and recently revived [48]. Since the resulting qualiton is formed from a color superconducting ground state, it seems proper to call it superqualiton [49]. It has been argued on general symmetry and dynamical grounds [50] that at high density, hadronic matter of flavor $SU(3)$ is characterized by the condensate

$$\langle q_{La} q_{L\beta} \rangle = -\langle q_{Ra} q_{R\beta} \rangle = \kappa \epsilon_{ij} \epsilon_{abc} \epsilon_{\alpha\beta I} q_{Lj}(-\vec{v}_F, x) q_{Li}^+(\vec{v}_F, y)$$

(10)

where $\kappa$ is some constant, $i, j$ are $SL(2,C)$ indices, $a, b$ are color indices, and $\alpha, \beta$ are flavor indices. Equation (10) holds for parity-even states. Such a condensate locks color and flavor so that global color and chiral symmetry are broken to the diagonal subgroup $SU(3)_{C+L+R}$ #3. The consequence of this is that there is an invariant $U(1)$ subgroup that contains a “twisted” photon, measured with which all excitations carry integer charges reminiscent of the Han-Nambu quarks and have quantum numbers that correspond to those of the mesons and baryons present at zero density. There is then a continuity between the excitations at high density in terms of quarks and gluons and hadronic excitations at low density in terms of baryons and mesons. This clearly is a case of Cheshire Cat.

Now to see that this is the Cheshire Cat in the sense formulated in terms of the chiral bag [2], consider the excitation of a quark on top of the diquark-condensed “vacuum.” In [50], such a quark is argued to behave like a baryon. Now I claim that this quark is a quark soliton, i.e., superqualiton [49].

To describe the low-energy dynamics of the color-flavor locking phase, introduce a field $U_L(x)$ which maps space-time to the coset space, $M_L = SU(3)_c \times SU(3)_L/SU(3)_{e+L}$. One can take it to be

$$U_{L\alpha}(x) = \lim_{y \to x} \frac{|x-y|^{\gamma_m}}{\kappa} \epsilon_{ijkl} \epsilon_{\alpha\beta I} q_{Li}^+(\vec{v}_F, x) q_{Lj}^+(\vec{v}_F, y),$$

(11)

where $\gamma_m$ is the anomalous dimension of the diquark field of order $\alpha_s$ and $q(\vec{v}_F, x)$ denotes the quark field with momentum close to a Fermi momentum $\mu \vec{v}_F$. The pairing involves quarks near the opposite edge of the Fermi surface. Similarly, we introduce a right-handed

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#3 I am describing here the situation where the color and flavor lock in such a way that the color symmetry is completely broken. Other cases where color is partially broken with differing flavor patterns could also be addressed in a similar way using a qualiton picture although details remain to be worked out.
field $U_R(x)$, also a map from space-time to $M_R = SU(3)_c \times SU(3)_R/SU(3)_{c+R}$, to describe the excitations of the right-handed diquark condensate. If this field takes a vacuum expectation value as a consequence of the diquark condensation which will, owing to (10), have the form

$$\langle U_{Laa} \rangle = - \langle U_{Raa} \rangle = \kappa \delta_{aa},$$

(12)

then 16 Nambu-Goldstone bosons will get excited \footnote{Actually there are 17 of them, one of which having to do with spontaneous breaking of the baryon number.}. Eight of them will get eaten up by the gluons to give masses to the gluons. The massive gluons then turn into massive vector mesons whose quantum numbers are those of the light-quark vector mesons present at zero density. The remaining eight (pseudoscalar) Nambu-Goldstone bosons are the equivalents of the ones present at zero density and are represented by the interpolating field (11). In analogy to the usual skyrmion at zero density, this field supports a soliton which is a fermion, the quantum numbers of which are identical to those of the usual baryon.

The effective Lagrangian that gives rise to this soliton should in principle be derived from QCD. At the moment such an effective Lagrangian is not known. However one can venture to make a few interesting conjectures. Viewed as a superqualiton whose mass is given by the soliton mass, there is nothing that requires that the soliton mass be equal to or near the superconductivity gap $\Delta$ (which is dictated by the condensate). In fact there is nothing which would prevent the mass from being much less than the gap. Thus one could imagine that light fermions are excited within the gap. Correlations between light superqualitons could rearrange the ground state into a different form from that of the standard superconductivity. There could be other modes of similar nature such as particle-hole excitations from the opposite ends of the Fermi sea (much like the Cooper pair but involving particles and holes) which could give rise to a crystalline structure etc. \footnote{For this and other reasons the phenomenon of color superconductivity in QCD at high density could be completely different from the usual BCS superconductivity. The issue of how this matter could influence the structure of compact stars (e.g., cooling, equation of state etc.) is an open one actively studied presently in KIAS \footnote{\cite{53}}.}

### 6 Conclusion

The most important outcome of the recent development of EFT in nuclear physics is that the highly successful approach to nuclear structure using realistic nuclear potentials (PM) is rendered a first-principle interpretation in that it represents the leading term in the EFT expansion with the corrections thereof systematically calculable. This confers the power of modern field theory techniques to the standard nuclear physics approach that has been practiced with success since a long time. This “bridging” comes about thanks to a possible duality that I refer to as Cheshire Cat Principle between QCD variables and
macroscopic (color-singlet) variables. This also provides a potential link between the physics of the elementary nucleon, nuclei, hadronic matter and compact-star matter. In the case of high density, the picture becomes even more intriguing. There we see emerging the symbolic (approximate) equality

\[ \text{“Quark”} \approx \text{“Qualiton”} \approx \text{“Baryon”}. \]  

(13)

It is amusing that the notion of the Cheshire Cat which was conceived by the need to reconcile the traditional meson-exchange description with the modern QCD description for nuclear processes \[ \text{[2]} \] (i.e., the “little bag” with pion cloud, chiral bag etc) at low density re-emerges at high density where one would have expected the bona-fide QCD to be uniquely applicable.

Most significant of all, the interplay between hadronic physics, relativistic heavy-ion physics and the physics of compact stars in astrophysics highlights the unity of physics, an endeavor that could make a mainstream of physics research in Asia Pacific research centers like APCTP and in Korean institutes like KIAS. Such a potential seems only natural given that some of the most significant contributions have been, and are being, made by young Korean theorists – graduate students and post-docs – actively working in the field and not less significantly that several experimental collaborations between Korean experimenters and the ALICE (CERN) and RHIC (Brookhaven) project teams purporting to probe the hot and dense matter relevant to early Universe and compact stars are in the process of being formed.

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