The Deformation Structure of the Nuclei $^{32}$S and $^{36}$Ar

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Abstract

We applied two different approaches to investigate the deformation structures of the two nuclei $^{32}$S and $^{36}$Ar. In the first approach, we considered these nuclei as being deformed and have axes of symmetry. Accordingly, we calculated their moments of inertia by using the concept of the single-particle Schrödinger fluid as functions of the deformation parameter $\beta$. In this case, we calculated also the electric quadrupole moments of the two nuclei by applying Nilsson model as functions of $\beta$. In the second approach, we used a strongly deformed nonaxial single-particle potential, depending on $\beta$ and the nonaxiality parameter $\gamma$, to obtain the single-particle energies and wave functions. Accordingly, we calculated the quadrupole moments of $^{32}$S and $^{36}$Ar by filling the single-particle states corresponding to the ground- and the first excited states of these nuclei. The moments of inertia of $^{32}$S and $^{36}$Ar are then calculated by applying the nuclear superfluidity model. The obtained results are in good agreement with the corresponding experimental data.

Keywords

Nuclear structure, deformed nuclei, single-particle Schrödinger fluid, Nilsson model, nuclear superfluidity model, moments of inertia, electric quadrupole moments.

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1. Introduction

Many of the light nuclei are spherical. This is due to the success of the shell model, which is based on states in a field of spherical symmetry. According to the basic ideas of quantum mechanics the concept of rotation in a spherically symmetric system is meaningless. However, in an elongated nucleus the concept of rotation is meaningful, and the nucleus can rotate about an axis perpendicular to the axis of symmetry. The basic ideas concerning non spherical nuclei have been most completely described by A. Bohr [1]. A non-spherical nucleus is characterized by the moment of inertia about the axis perpendicular to the symmetry axis of the nucleus, its magnetic dipole moment and its electric quadrupole moment. The elongation of the nucleus is related to the interaction between the surface and the nucleons outside closed shells.

In treating the internal motion in a deformed nucleus, it is assumed that the individual nucleons move independently in a certain fixed non-spherical field of the nucleus. The Hamiltonian of the internal motion can then be represented, as in the ordinary model, in the form of a sum of one-particle Hamiltonians. One of the most successful models for generating realistic intrinsic single particle states of deformed nuclei is that first proposed by Nilsson [2]. According to Nilsson's model, the nucleons inside the nucleus are moving independently in an averaging field in the form of anisotropic oscillator, with $\omega_x = \omega_y = \omega_z$, added to it a spin-orbit term and a term proportional to the square of the orbital angular momentum of the nucleon. The nucleon energy eigenvalues and eigenfunctions are then obtained by solving the time-independent Schrödinger wave equation in spherical polar coordinates and applying the method of diagonalizing the matrices. This model was limited to nuclei with axially symmetric quadrupole deformations, where the deformation is measured by the deformation parameter $\beta$. Positive values of $\beta$ correspond to prolate deformation and negative values to oblate deformation. The success of the description of many nuclei by means of deformed potential can be taken as an indication that by distorting a spherical potential in this manner we automatically obtain the right combination of spherical eigenfunctions which make the corresponding Slater determinant a better approximation to the real nuclear wave function. From this point of view, the deformed potential is a definite prescription for a convenient mixing of various configurations of the spherical potential. Considerable evidence has accumulated for the rotational structure of nuclei. The absolute values of the rotational energies or equivalently the moments of inertia require a knowledge of the fine details of the intrinsic nuclear structure. Consequently, the investigation of the nuclear moments of inertia is a sensitive check for the validity of the nuclear structure theories [3].

The study of the velocity fields for the rotational motion of the axially symmetric deformed nuclei led to the formulation of the so-called Schrödinger fluid [4,5]. Since the Schrödinger-fluid theory is an independent particle model, the cranking model approximation for the velocity fields and the moments of inertia play the dominant role in this theory. For axially symmetric deformed nuclei, the best description of the moment of inertia can be carried out by applying the concept of the single-particle Schrödinger fluid [6-8]. For these nuclei one can apply Nilsson's model [2] to calculate the nuclear quadrupole moments.

For a nucleus which has not an axis of symmetry (usually called an asymmetric rotor), one can use a single-particle Hamiltonian for a nucleon moving in a nonaxial deformed potential and then solves the Schrödinger equation in this case to obtain the single-particle energy eigenvalues and eigenfunctions [9]. It is then possible to fill the ground state (or the excited state) of the considered nucleus by the resulting single-particle wave functions. As a consequence, the quadrupole moment can be obtained by calculating the expectation value of the well-known quadrupole-moment operator with respect to the ground (or the excited) state of the nucleus. The best description of the nuclear moments of inertia for a nucleus which has not an axis of symmetry can be obtained by applying the nuclear superfluidity model of Belyaev [10].
The moments of inertia and the quadrupole moments of some deformed nuclei in the sd-shell have been investigated in frame work of different models. By applying Nilsson model, Bishop [11] calculated the moment of inertia and the quadrupole moment of the nucleus $^{27}$Al. Also, Doma [6] applied the nuclear superfluidity model to calculate the moments of inertia of the nuclei $^{24}$Mg and $^{26}$Mg. Furthermore, Doma [12] applied the single-particle Schrödinger fluid to calculate the moments of inertia of the even-even nuclei in the sd-shell.

In the present paper, we investigated the deformation structure of the nuclei $^{32}$S and $^{36}$Ar. Accordingly, we calculated two characteristics for these nuclei by using different models which depend on the shape of the nucleus. In the case where the nucleus is assumed to be deformed and has an axis of symmetry, we applied the concept of the single-particle Schrödinger fluid for the calculation of the moments of inertia. Accordingly, the cranking-model moment of inertia and the rigid-body moment of inertia of the two nuclei $^{32}$S and $^{36}$Ar are calculated as functions of the deformation parameter $\beta$ and the non-deformed oscillator parameter $\hbar \omega_0$. Furthermore, we calculated also the electric quadrupole moments of $^{32}$S and $^{36}$Ar by applying Nilsson model as function of $\beta$. We finally considered a single-particle deformed potential consisting of an anisotropic oscillator potential added to it a spin-orbit term and a term proportional to the square of the orbital-angular momentum of the nucleon to calculate the single-particle energy eigenvalues and eigenfunctions for a nucleon in a deformed non axial nucleus. As a consequence, the quadrupole moments of $^{32}$S and $^{36}$Ar are calculated by using the single-particle deformed wave functions. The moments of inertia of $^{32}$S and $^{36}$Ar are then calculated by applying the superfluidity nuclear model, as functions of $\beta$, the non-axiality parameter $\gamma$ and the non-deformed oscillator parameter $\hbar \omega_0$.

2. Calculations Based on the Assumption that the Nucleus Is Deformed and Has an Axis of Symmetry

2.1 The Single-Particle Schrödinger Fluid

The detailed formulation of the concept of the single-particle Schrödinger fluid from the time dependent Schrödinger equation, by suitably chosen single-particle wave function, is given by Kane and Griffin [4,5]. The method of the application of this concept to the calculation of the nuclear moments of inertia is given by Doma [6-8]. The following expressions for the cranking-model and the rigid body-model moments of inertia can be easily obtained on the basis of the concept of the single-particle Schrödinger fluid [4,5]

$$J_{cr} = \frac{E}{w_0^2} \left(1 + \frac{q}{6+2\sigma}\right) \frac{1}{1+\sigma} \left(\sigma^2 (1 + q) + \frac{1}{\sigma} (1 - q)\right).$$

$$J_{rig} = \frac{E}{w_0^2} \left(1 + \frac{q}{6+2\sigma}\right) \frac{1}{1+\sigma} \left((1 + q) + \sigma (1 - q)\right).$$

where $q$ is the anisotropy of the configuration, which is defined by

$$q = \frac{\sum_{n} (n_z + 1)}{\sum_{n} (n_z + 1)}.$$

and $E$ is the total energy

$$E = \sum_{n} [\hbar \omega_x (n_x + n_y + 1) + \hbar \omega_z (n_z + 1)].$$

In equations (2.3) and (2.4) $n_x, n_y$ and $n_z$ are the state quantum numbers of the oscillator. The summations in (2.3) and (2.4) are carried over all the occupied single-particle states. The method of filling these states is illustrated in [8]. Also, in (2.1) and (2.2) $\sigma$ is a measure of the deformation of the potential and is defined by

$$\sigma = \frac{\omega_x - \omega_z}{\omega_y + \omega_z}.$$

For the frequencies $\omega_x, \omega_y$ and $\omega_z$, Doma et al. [6-8] used Nilsson’s frequencies [2], defined by

$$\omega_x^2 = \omega_y^2 = \omega_z^2 (1 + \frac{2}{3} \delta),$$

$$\omega_z^2 = \omega_z^2 (1 - \frac{4}{3} \delta).$$


\( \omega_0(\delta) = \omega_0^0 \left(1 - \frac{4}{3} \delta^2 - \frac{16}{27} \delta^3 \right)^{\frac{1}{2}}. \)  

(2.8)

For the non-deformed frequency \( \omega_0^0 \) we used the one which is given in terms of the mass number \( A \), the number of neutrons \( N \) and the number of protons \( Z \) by [13]

\[ h\omega_0^0 = 38.6 A^{-\frac{1}{2}} - 127.0 A^{-\frac{1}{2}} + 14.75 A^{-\frac{1}{2}} (N - Z). \]  

(2.9)

The well-known deformation parameter \( \beta \) is related to the parameter \( \delta \) in equations (2.6), (2.7) and (2.8) by the following relation [2]

\[ \beta = \frac{2}{3} \sqrt{\frac{10}{3}} \delta. \]  

(2.10)

We note that the cranking-model and the rigid body-model moments of inertia are equal only when the harmonic oscillator is at the equilibrium deformation.

### 2.2 The Electric Quadrupole Moment

Assuming a charge distribution in accordance with the Thomas-Fermi statistical model applied to the oscillator potential one obtains, for the case of the axially symmetric nuclei, the intrinsic quadrupole moment, to the second-order in the deformation parameter \( \delta \) [1]

\[ Q_0 = 0.0Z e R^2 \delta \left(1 + \frac{2\delta}{3}\right). \]  

(2.11)

where \( Z \) is the number of protons and \( R \) is to be taken equal to the radius of charge of the nucleus. The relation between the measured quadrupole moment, denoted by \( Q_\nu \), and \( Q_0 \) is given by

\[ Q_\nu = \frac{3K^2 - (l+1)(l+2)}{(l+1)(2l+3)} Q_0. \]  

(2.12)

where \( I \) is the total spin-quantum number of the specified nuclear state and \( K \) is its component along the body-fixed \( z \)-axis. Calculating the charge radius of the nucleus, the measured quadrupole moment for a nucleus with an axis of symmetry is then obtained as function of the deformation parameter \( \delta \).

### 3. Calculations Based on the Assumption that the Nucleus Is Deformed and Has Not an Axis of Symmetry

In the case where the nucleus is assumed to be deformed and has not, in principle, an axis of symmetry we proceed as illustrated in the next sections.

#### 3.1 The Single-Particle Potential and the Method of Solution

Consider a nucleon which is moving in a deformed nuclear field whose Hamiltonian operator is given by [9]

\[
H = -\frac{\hbar^2}{2m} \nabla^2 + \frac{m}{2} \omega_0^2 r^2 + C l^2 + D \ell^2 - m\omega_0^2 r^2 \cos \gamma Y_{2,0}(\theta, \phi) \\
- \frac{\hbar^2}{2} m \omega_0^2 r^2 \sin \gamma \left[Y_{2,2}(\theta, \phi) + Y_{2,-2}(\theta, \phi)\right],
\]  

(3.1)

where \( Y_{l,m}(\theta, \phi) \) are the spherical harmonic functions, \( \beta \) is the deformation parameter and \( \gamma \) is the non-axiality parameter. The constants \( C \) and \( D \) in equation (3.1) are given by [2]

\[
C = -2\chi \hbar \omega_0^0, \quad D = -\mu \chi\hbar \omega_0^0,
\]  

(3.2)
where $\chi$ takes values in the interval $0.05 \leq \chi \leq 0.08$ and $\mu$ depends on the number of quanta of excitation $N$ as given by Nilsson [2].

The Hamiltonian $H$, equation (3.1), can be rewritten in the form:

$$H = H^{(0)} + H^{(1)} + H^{(2)},$$

(3.3)

where

$$H^{(0)} = \frac{-h^2}{2m} \nabla^2 + \frac{m}{2} \omega_0^2 \tau^2 + C \Lambda s + D l^2,$$

(3.4)

$$H^{(1)} = -\mu \omega_0^2 \tau^2 \cos \gamma Y_{2,0}(\theta, \varphi),$$

(3.5)

$$H^{(2)} = -\frac{\sqrt{\pi}}{2} \mu \omega_0^2 \tau^2 \sin \gamma \{Y_{2,2}(\theta, \varphi) + Y_{2,-2}(\theta, \varphi)\}.$$  

(3.6)

The solutions of the Schrödinger equation corresponding to the Hamiltonian (3.4) are straightforward [9] with eigenfunctions denoted by $|N/\Lambda \Sigma\rangle$. Also, the solutions of the Schrödinger equation corresponding to the Hamiltonian $H^{(0)} + H^{(1)}$ are straightforward [9] by applying the variational method and as a result we obtain the eigenfunctions $|N/\Omega^\Lambda\rangle$.

Finally, the Schrödinger equation representing the motion of a single nucleon in the non-axially deformed nuclear field, whose Hamiltonian operator is given by equation (3.1), can be solved by applying the stationary non-degenerate perturbation method, for the Hamiltonian $H^{(2)}$ as a perturbed term to $H^{(0)} + H^{(1)}$, with respect to the eigenfunctions $|N/\Omega^\Lambda\rangle$. As a result, the single-particle energy eigenvalues and eigenfunctions, $|\Omega^\Lambda\rangle$, of a nucleon in a deformed nuclear field can be calculated for every level, with given value of the $z$-component of the total angular momentum $\Omega$ and parity $\pi$ as functions of the potential parameters $\chi$, and $\mu$, the deformation parameter $\beta$, and the non-axisiality parameter $\gamma$. In the above mentioned functions, $N$ is the number of quanta of excitation, $l$ and $\Lambda$ are the nucleon orbital angular momentum quantum number and its $z$-component and $\Sigma$ is the $z$-component of the nucleon spin $(\pm \frac{1}{2})$.

### 3.2 The Nuclear Superfluidity Model and the Moment of Inertia

The moment of inertia of a deformed nucleus which has not an axis of symmetry is then given by applying the nuclear superfluidity model [10], and as a result we obtain

$$\mathcal{J}_{z,f} = \hbar^2 \sum_{l,k} \frac{j_{lj}j_k}{E_{l} + E_k} \left(1 - \frac{(\zeta_l - \lambda)(\zeta_k - \lambda) + \Delta^2}{E_l E_k}\right),$$

(3.7)

where $\zeta_l$ are the eigenvalues of the self-consistent field, the eigenvalues of the Hamiltonian operator (3.1), $\lambda$ is the chemical potential and the energy of elementary excitations of the nucleus, $E_l$, is given by

$$E_l = \sqrt{(\zeta_l - \lambda)^2 + \Delta^2},$$

(3.8)

with $\Delta$ being the energy gap. The summation in equation (3.7) is taken over all states of the self-consistent field. The chemical potential $\lambda$ is given by [10]

$$\sum_{l} \left(1 - \frac{\zeta_l - \lambda}{\sqrt{(\zeta_l - \lambda)^2 + \Delta^2}}\right) = N_{p,n},$$

(3.9)

where the summation, here, runs over all distinct neutron (or proton) energies and $N_{p,n}$ is the number of protons or neutrons inside the nucleus.

### 3.3 The Quadrupole Moment

For the non-axial case, the intrinsic quadrupole moment, of a nucleus consisting of $Z$ protons, is given by [1]

$$Q_0 = \sum_{i=1}^Z Q_i,$$

(3.10)

where the single-particle operator $Q_i$ is given by
Carrying out the integration in equation (3.11) with respect to the wave functions $|\Omega^\nu\rangle$ which is evaluated in terms of the functions $|N\Lambda\Sigma\rangle$, one then obtains

$$Q_i = e^{i\pi} \frac{1}{\xi} \int |\Psi_i^\nu\rangle^2 r_i^2 Y_{2,0}(\theta_i, \phi_i) \, dr.$$  \hspace{1cm} (3.11)

Filling the single-particle wave functions $|\Omega^\nu\rangle$ for the given nucleus in its ground- and excited- state ($^+_1$), it is then possible to calculate the quadrupole moment by calculating the necessary matrix elements of equation (3.12) and evaluating the expansion coefficients of the functions $|\Omega^\nu\rangle$ in terms of the functions $|N\Lambda\Sigma\rangle$ as obtained from the variational and the perturbation methods.

4. Results for the Case of the Axial Symmetry

According to previous works [9], the parameters $\chi, \mu$ and $\beta$ are allowed to take on the values $\chi = 0.05, 0.06, 0.07, \text{and} 0.08, \mu = 0, \text{for} \ N = 0, 1 \text{and} 2; \text{and} \ \mu = 0.35 \text{for} \ N = 3, \ \beta \text{takes values in the interval} -0.50 \leq \beta \leq 0.50 \text{with a step} 0.01$.

In Tables-1 and 2 we present the variations of the values of the reciprocal moments of inertia of the nuclei $^{\text{32}}S$ and $^{\text{36}}Ar$, by using the concept of the single-particle Schrödinger fluid for both of the cranking-and the rigid-body models, with respect to the deformation parameter $\beta$ for the cases $\beta \leq 0$ and $\beta > 0$, respectively. The values of the non-deformed oscillator parameter $\hbar \omega_0^\alpha$ are also given in Tables-1 and 2. Also, in Figures-1 and 2 we present the dependence of the reciprocal moments of inertia of $^{\text{32}}S$ and $^{\text{36}}Ar$ on the deformation parameter $\beta$.

![Fig.1 Reciprocal moments of inertia of the nucleus $^{32}$S. Solid line for cranking model and dashed line for rigid body model.](image-url)
Fig. 2 Reciprocal moments of inertia of the nucleus $^{36}\text{Ar}$. Solid line for cranking model and dashed line for rigid body model.

Table 1 Schildinger fluid reciprocal moments of inertia of $^{32}\text{S}$ and $^{36}\text{Ar}$ as functions of $\beta$, $\beta \leq 0$.

| Case | $^{32}\text{S}$ $\hbar\omega_0^S = 10.908$ | $^{36}\text{Ar}$ $\hbar\omega_0^A = 10.622$ |
|------|---------------------------------|---------------------------------|
| $\beta$ | $\frac{\hbar^2}{2I_{\text{crank}}}$ | $\frac{\hbar^2}{2I_{\text{rig}}}$ | $\frac{\hbar^2}{2I_{\text{crank}}}$ | $\frac{\hbar^2}{2I_{\text{rig}}}$ |
| -.500 | 548.020300 | 193.265300 | 544.238000 | 164.409900 |
| -.490 | 538.614000 | 192.287800 | 535.632800 | 163.596600 |
| -.480 | 529.116500 | 191.338600 | 526.897800 | 162.807400 |
| -.470 | 519.527600 | 190.416700 | 518.031900 | 162.041100 |
| -.460 | 509.847100 | 189.521000 | 509.035600 | 161.296800 |
| -.450 | 500.075600 | 188.650300 | 499.908800 | 160.573600 |
| -.440 | 490.213300 | 187.803700 | 490.652600 | 159.870700 |
| -.430 | 480.261500 | 186.980300 | 481.268200 | 159.187200 |
| -.420 | 470.221200 | 186.179100 | 471.757000 | 158.522500 |
| -.410 | 460.093800 | 185.399300 | 462.120700 | 157.875800 |
| -.400 | 449.881400 | 184.640300 | 452.361800 | 157.246600 |
| -.390 | 439.585700 | 183.901200 | 442.482500 | 156.634200 |
| -.380 | 429.208500 | 183.181400 | 432.485300 | 156.037900 |
| -.370 | 418.752900 | 182.480200 | 422.375300 | 155.457400 |
| -.360 | 408.220500 | 181.797100 | 412.150300 | 154.892200 |
| -.350 | 397.614400 | 181.131500 | 401.818600 | 154.341500 |
| -.340 | 386.937000 | 180.482700 | 391.382200 | 153.805100 |
| -.330 | 376.191400 | 179.850400 | 380.847000 | 153.282600 |
| -.320 | 365.380300 | 179.234000 | 370.210100 | 152.773600 |
| -.310 | 354.506700 | 178.633100 | 359.482000 | 152.277500 |
|   |   |   |   |
|---|---|---|---|
| -.300 | 343.573400 | 178.047200 | 348.664400 | 151.794100 |
| -.290 | 332.583700 | 177.476000 | 337.761400 | 151.323100 |
| -.280 | 321.540300 | 176.919000 | 326.777300 | 150.864000 |
| -.270 | 310.446500 | 176.375900 | 315.715900 | 150.416700 |
| -.260 | 299.305000 | 175.846200 | 304.581400 | 149.980700 |
| -.250 | 288.119000 | 175.329900 | 293.377900 | 149.555800 |
| -.240 | 276.891300 | 174.826300 | 282.109600 | 149.141900 |
| -.230 | 265.624600 | 174.335300 | 270.780300 | 148.738500 |
| -.220 | 254.321800 | 173.856600 | 259.394000 | 148.345500 |
| -.210 | 242.985500 | 173.389900 | 247.954300 | 147.962700 |
| -.200 | 231.618100 | 172.934900 | 236.465100 | 147.589700 |
| -.190 | 220.222300 | 172.491400 | 224.929600 | 147.226400 |
| -.180 | 208.799800 | 172.059200 | 213.351700 | 146.872700 |
| -.170 | 197.353200 | 171.638000 | 201.734100 | 146.528300 |
| -.160 | 185.884000 | 171.227600 | 190.079900 | 146.193000 |
| -.150 | 174.394300 | 170.827900 | 178.392000 | 145.866800 |
| -.140 | 162.885500 | 170.438600 | 166.672800 | 145.549300 |
| -.130 | 151.358800 | 170.059400 | 154.924600 | 145.240500 |
| -.120 | 139.815400 | 169.690400 | 143.149300 | 144.940200 |
| -.110 | 128.255900 | 169.331300 | 131.348700 | 144.648300 |
| -.100 | 116.681200 | 168.981900 | 119.524000 | 144.364600 |
| -.090 | 105.091300 | 168.642100 | 107.676500 | 144.089100 |
| -.080 | 93.486510 | 168.311700 | 95.806760 | 143.821600 |
| -.070 | 81.866290 | 167.990600 | 83.915240 | 143.562000 |
| -.060 | 70.230280 | 167.678800 | 72.001590 | 143.310100 |
| -.050 | 58.577240 | 167.375900 | 60.065510 | 143.065900 |
| -.040 | 46.906120 | 167.082000 | 48.106110 | 142.829300 |
| -.030 | 35.215160 | 166.797000 | 36.121970 | 142.600300 |
| -.020 | 23.502260 | 166.520600 | 24.111300 | 142.378600 |
| -.010 | 11.764880 | 166.252900 | 12.071650 | 142.164300 |
| .000 | 0.000000 | 165.993700 | 0.000000 | 141.957300 |
Table 2: Schrödinger fluid reciprocal moments of inertia of $^{32}$S and $^{36}$Ar as functions of $\beta$, $\beta > 0$.

| Case | $^{32}$S | $^{36}$Ar |
|------|---------|---------|
|      | $h^2$ $\frac{E_{\text{Crank}}}{\hbar^2}$ | $h^2$ $\frac{E_{\text{rig}}}{\hbar^2}$ | $h^2$ $\frac{E_{\text{Crank}}}{\hbar^2}$ | $h^2$ $\frac{E_{\text{rig}}}{\hbar^2}$ |
| 0.10 | 11.795180 | 165.743000 | 12.106490 | 141.564700 |
| 0.20 | 23.625170 | 165.500600 | 24.252580 | 141.200300 |
| 0.30 | 35.494220 | 165.266600 | 36.482530 | 141.379000 |
| 0.40 | 47.407260 | 165.040800 | 48.682530 | 141.200300 |
| 0.50 | 59.369950 | 164.823100 | 60.977860 | 141.028600 |
| 0.60 | 71.384500 | 164.613500 | 73.335590 | 140.863900 |
| 0.70 | 83.469570 | 164.412000 | 85.763710 | 140.706000 |
| 0.80 | 95.621090 | 164.218400 | 98.267066 | 140.554800 |
| 0.90 | 107.851400 | 164.032800 | 110.866100 | 140.410600 |
| 1.00 | 120.169700 | 163.855100 | 123.560800 | 140.273000 |
| 1.10 | 132.586300 | 163.685300 | 136.365590 | 140.142200 |
| 1.20 | 145.112600 | 163.523300 | 149.296300 | 140.018100 |
| 1.30 | 157.760600 | 163.369100 | 162.364700 | 139.900700 |
| 1.40 | 170.544300 | 163.222700 | 175.587500 | 139.789900 |
| 1.50 | 183.478300 | 163.084000 | 188.982400 | 139.685900 |
| 1.60 | 196.578900 | 162.953100 | 202.569000 | 139.588400 |
| 1.70 | 209.864500 | 162.829900 | 216.368600 | 139.497600 |
| 1.80 | 223.354700 | 162.714400 | 230.405000 | 139.413400 |
| 1.90 | 237.071400 | 162.606700 | 244.704700 | 139.335900 |
| 2.00 | 251.038700 | 162.506600 | 259.297000 | 139.265000 |
| 2.10 | 265.283100 | 162.414300 | 274.214100 | 139.200800 |
| 2.20 | 279.834300 | 162.329600 | 289.492300 | 139.143200 |
| 2.30 | 294.725000 | 162.252700 | 305.171900 | 139.092200 |
| 2.40 | 309.991400 | 162.183600 | 321.298400 | 139.048000 |
| 2.50 | 325.674200 | 162.122300 | 337.922300 | 139.010500 |
| 2.60 | 341.818600 | 162.068600 | 355.100400 | 138.979600 |
| 2.70 | 358.475700 | 162.022800 | 372.897600 | 138.955600 |
| 2.80 | 375.702200 | 161.984800 | 391.386700 | 138.983200 |
| 2.90 | 393.563200 | 161.954700 | 410.651300 | 138.927700 |
| 3.00 | 412.131100 | 161.932400 | 430.786100 | 138.924000 |
| 3.10 | 431.489500 | 161.918000 | 451.900700 | 138.927200 |
| 3.20 | 451.732200 | 161.911600 | 474.123100 | 138.937300 |
| 3.30 | 472.971500 | 161.913200 | 497.594200 | 138.954300 |
| 3.40 | 495.330000 | 161.922800 | 522.490700 | 138.978300 |
| 3.50 | 518.954300 | 161.940500 | 549.011800 | 139.009400 |
| 3.60 | 544.014800 | 161.966200 | 577.395400 | 139.047500 |
| 3.70 | 570.711900 | 162.000000 | 607.925700 | 139.092600 |
| 3.80 | 599.281300 | 162.042000 | 640.943400 | 139.144900 |
In Table 3 we present the best values of the calculated reciprocal cranking-model moments of inertia for the two nuclei $^{32}$S and $^{36}$Ar. The values of the corresponding deformation parameter $\beta$ and the experimental moments of inertia of the two nuclei are also given in this table. Concerning the values of the rigid-body moments, they are not presented in this table since they are not in good agreement with the corresponding experimental values, as shown from Tables 1 and 2, as expected.

Table 3 Best values of the calculated reciprocal moments of inertia by using the cranking-model for $^{32}$S and $^{36}$Ar

| Nucleus | $\beta$ | $\frac{h^2}{2I_{\text{crank}}}$ (KeV) | $\frac{h^2}{2I_{\text{exp}}}$ (KeV) [14] |
|---------|--------|---------------------------------|-------------------------------------|
| $^{32}$S | -0.335 | 371.533                         | 371.72                              |
|         | 0.278  | 371.621                         |                                     |
| $^{36}$Ar | -0.325 | 374.544                         | 374.55                              |
|         | 0.272  | 374.442                         |                                     |

We considered the equilibrium moment of inertia, for the two nuclei, as the value where both of the cranking model and the rigid-body model moments of inertia are equal. In Table 4 we present the equilibrium reciprocal moments of inertia of the two nuclei together with the values of the deformation parameter $\beta$ and the corresponding experimental reciprocal moments of inertia of the two nuclei.

Table 4 Equilibrium reciprocal moments of inertia of $^{32}$S and $^{36}$Ar

| Nucleus | $\beta$ | $\frac{h^2}{2I_{\text{eqm}}}$ (KeV) | $\frac{h^2}{2I_{\text{exp}}}$ (KeV) [14] |
|---------|--------|---------------------------------|-------------------------------------|
| $^{32}$S | -0.146 | 170.6241                        | 371.72                              |
|         | 0.135  | 163.3611                        |                                     |
| $^{36}$Ar | -0.121 | 144.8934                        | 374.55                              |
|         | 0.114  | 140.0742                        |                                     |

As seen from Table 4, the values of the equilibrium reciprocal moments of inertia of $^{32}$S and $^{36}$Ar are not in good agreement with the corresponding data since they are closely related to the rigid-body values.

In Table 5 we present the values of the electric quadrupole moments of $^{32}$S and $^{36}$Ar for the axially-symmetric case together with the corresponding values of the total spin of the considered nuclear state and its parity, $I^p$, the deformation parameter $\beta$, the root mean-square radius, $R$, and the experimental data.
Table-5 Electric quadrupole moments of $^{32}$S and $^{36}$Ar for the axially-symmetric case

| Nucleus | $I^\pi$ | $\beta$ | $R$ (in fm) | $Q_s$ (efm$^2$) | $Q_{exp}$ (efm$^2$) |
|---------|---------|---------|-------------|----------------|-----------------|
| $^{32}$S | $2^+$   | 0.278   | 4.92        | -14.88         | -14.9           |
| $^{36}$Ar | $2^+$  | -0.11   | 5.12        | 10.96          | 11.0            |

5. Results for the Case where the Nucleus Has Not an Axis of Symmetry

In Table-6 we present the deformation parameters and the parameters of the single-particle potential for the two nuclei $^{32}$S and $^{36}$Ar.

Table-6 Model and potential parameters for the $^{32}$S and $^{36}$Ar nuclei

| Nucleus | $\beta$ | $\gamma$ | $\chi$ | $\hbar\omega_0^2$ (MeV) | $\lambda$ (MeV) | $\Delta$ (MeV) |
|---------|---------|----------|--------|--------------------------|----------------|--------------|
| $^{32}$S | 0.395   | 25°      | 0.06   | 10.908                   | 3.24           | 2.07         |
| $^{36}$Ar | -0.376 | 20°      | 0.06   | 10.622                   | 3.25           | 1.93         |

In Table-7 we present the superfluidity reciprocal moments of inertia of $^{32}$S and $^{36}$Ar together with the corresponding moments and the deformation parameters.

Table-7 Superfluidity reciprocal moments of inertia of $^{32}$S and $^{36}$Ar

| Nucleus | $\beta$ | $\gamma$ | $\frac{\hbar^2}{2\hbar}$ (KeV) | $\frac{\hbar^2}{2\hbar_{exp}}$ (KeV) |
|---------|---------|----------|-------------------------------|----------------------------------|
| $^{32}$S | 0.395   | 25°      | 371.782                       | 371.72                           |
| $^{36}$Ar | -0.376 | 20°      | 374.685                       | 374.55                           |

Finally, in Table-8 we present the calculated values of the electric quadrupole moments of $^{32}$S and $^{36}$Ar by using the single-particle wave functions of the nonaxial potential. The values of the deformation parameters, the total spin and parity are also given.

Table-8 The electric quadruple moments of $^{43}$S and $^{36}$Ar in the non-axial case

| nucleus | $I^\pi$ | $\beta$ | $\gamma$ | $Q_{cai}$ (efm$^2$) | $Q_{exp}$ (efm$^2$) |
|---------|---------|---------|----------|----------------|-----------------|
| $^{32}$S | $2^+$   | 0.45    | 25°      | -14.92         | -14.9           |
| $^{36}$Ar | $2^+$  | -0.38   | 20°      | -10.9          | 11.0            |

6. Conclusion

It is seen from Tables-1 and 2 that the calculated values of the moments of inertia of the considered nuclei by using the cranking model of the concept of the single-particle Schrödinger fluid are in good agreement with the corresponding experimental values, a result which shows that the concept of this fluid is reliable and can be applied successfully to deformed nuclei in the $s - d$ shell. It is seen, also, from Tables-1 and 2 that the two nuclei $^{32}$S and $^{36}$Ar have nearly equal values of the deformation parameter $0.27 < \beta < 0.28$ (or $-0.33 < \beta < -0.32$). The disagreement between the value of the rigid-body reciprocal moment of inertia and the corresponding experimental data is due to the fact that the pairing correlation is not taken in concern in this model [3]. Furthermore, according to the results of the moments of inertia by using the concept of the single-particle Schrödinger fluid, the two nuclei $^{32}$S and $^{36}$Ar may have prolate deformation shape (positive value of $\beta$) as well as oblate deformation shape (negative value of $\beta$).

It is well-known that the quantity that characterizes the deviation from spherical symmetry of the electrical charge distribution in a nucleus is its quadrupole moment $Q$. If a nucleus is extended along the axis of symmetry, then $Q$ is a positive quantity, but if the nucleus is flattened along the axis, it is negative. On the other hand, according to the results of the electric quadrupole moments of the two nuclei in the axially-symmetric case, the nucleus $^{32}$S has prolate deformation shape while the nucleus $^{36}$Ar has oblate deformation shape. In the case where the nucleus is assumed to be deformed and has not an axis of symmetry, the results obtained from the calculations of the superfluidity moments of inertia and the electric quadrupole moments of the two nuclei show also that the nucleus $^{32}$S has prolate deformation shape while $^{36}$Ar has oblate deformation shape. Accordingly, the two models, although different in their structures, agree in the assumption that the $^{36}$Ar nucleus has an oblate deformation shape.
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