Pioneer acceleration and variation of light speed: experimental situation

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Abstract

The situation with respect to the experiments is presented of a recently proposed model that gives an explanation of the Pioneer anomalous acceleration $a_P$. The model is based on an idea already discovered by Einstein in 1907: the light speed depends on the gravitational potential $\Phi$, so that it is larger the higher $\Phi$. The potential due to all the mass and energy in the universe increases in time because of its expansion, which has the consequence that light must be slowly accelerating. Moreover it turns out that the observational effects of a universal adiabatic acceleration of light $a_\ell = a_P$ and of an extra acceleration towards the Sun $a_P$ of a spaceship would be the same: a blue shift increasing linearly in time, precisely what was observed. The phenomenon would be due to a cosmological acceleration of the proper time of bodies with respect to the coordinate time. It is shown that it agrees with the experimental tests of special relativity and the weak equivalence principle if the cosmological variation of the fine structure constant is zero or very small, as it seems now.

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1 Introduction

The purpose of this paper is to discuss the situation with respect to the observations of the recent proposal that there could be an adiabatic universal acceleration of light of the order of \( a_\ell \approx 10^{-9} \text{m/s}^2 \) [1, 2, 3]. Such an acceleration would give an explanation for one of the most intriguing riddles in today’s cosmology: the anomalous and unmodelled acceleration observed in the Pioneer 10/11, Galileo and Ulysses spacecrafts. It would be explained then as an effect of the adiabatic attenuation of the quantum vacuum, which would be cause the proper time of bodies to accelerate with respect to the coordinate time. Strange as it might seem, this value is so small that it is plausible that it could have not been detected, in spite of the enormous number of optical phenomena which are studied and analyzed daily.

In October 1998, Anderson et al reported a tiny but significant anomaly in the motion of the Pioneer 10/11 spaceships: the solar attraction seems to be slightly larger than what predicted by standard gravity [4]. Their analysis of the data from the two missions found in the motion of both spacecrafts an extra unmodelled constant acceleration towards the Sun, equal to \( a_P \approx 8.5 \times 10^{-10} \text{m/s}^2 \). The data of the Galileo and Ulysses spaceships showed the same effect. Surprisingly, no similar extra acceleration was found in the case of the planets, as it would be required by the equivalence principle if the effect were due to gravitational forces (of dark matter for instance). Anderson et al stated “it is interesting to speculate on the possibility that the origin of the anomalous signal is new physics.” In spite of a thorough search, they could not find as yet any reason for that extra acceleration (see [5] for a detailed review of the problem and of the observational techniques involved).

It was suggested in 2003 by this author that this effect could be explained if the light is increasing its speed with an adiabatic acceleration \( a_\ell \) equal to \( a_P \) [1, 2, 3]. More precisely, a mechanism was proposed in these papers which must produce such an acceleration as a consequence of the combined effect on the quantum vacuum of (i) the fourth Heisenberg relation and (ii) the expansion of the universe, the treatment being Newtonian and phenomenological. It follows from this mechanism that the electric and magnetic constants of empty space, \( \varepsilon_0 \) and \( \mu_0 \), are decreasing adiabatically (and its optical density as well), the light speed increasing consequently. Curiously enough, as shown in [6], it turns out that a general relativistic argument predicts the existence of such an acceleration if the expansion of the universe is taken into
account, its value being close, at least, to $a_t \simeq H_0c \approx 0.8a_P$, $H_0$ being the present value of the Hubble parameter. Furthermore, an adiabatic acceleration $a_t$ would imply a blue shift in the radio signal of the spacecrafts, which increases linearly in time such that $\dot{\nu} = \nu a_P/c$, precisely what was observed by Anderson et al. The possibility must be studied that the ships did not accelerate but followed instead the laws of standard gravity, the blue shift not being due to their motion but to the acceleration of light. The model based on that mechanism will be called here “this model” or “the attenuating vacuum model”.

However and understandably, the first reaction of many physicists would probably be that the light speed is too much well known to admit such an acceleration without violating some of the experimental bounds imposed by the tests of special relativity and the equivalence principle, or that it would have been detected in interferometry, optical communication and similar effects.

## 2 The mechanism

In this section a terse summary of the attenuating vacuum model is given, in the frame of a Newtonian approximation, although with the inclusion of the rest energy of the particles. As is explained in previous work [1, 2, 3], it can be argued that the quantum vacuum can be compared to an inhomogeneous optical medium such that its optical density depends on the gravitational potential. This dependance would be important since the quantum vacuum fixes some important quantities through renormalization processes, as is the case of the electron charge $e$ (and eventually of the monopole charge), the light speed and the fine structure constant. A coupling between its density and the gravitational field would entail a space and time inhomogeneity across the universe of such quantities as $e$, $c$ and $\alpha$. To understand why, note that the empty space can be considered as a sea of virtual pairs and other particles which pop up and disappear continuously. On the average, a pair created with energy $E$ (including rest energy, kinetic energy and electromagnetic energy) lives during a time $\tau \approx h/E$ according to the fourth Heisenberg relation. If there is a Newtonian gravitational potential $\Phi$, the pair pops up with an extra energy $E\Phi/c^2$, so that its lifetime must

$$
\tau_\Phi = h/(E + E\Phi/c^2) = \tau_0/(1 + \Phi c^2).
$$

(1)
As a clear if unexpected consequence, the number density of pairs $N$ would depend on the gravitational potential $\Phi$ as

$$N_\Phi = N_0/(1 + \Phi/c^2).$$

(2)

In other words, the optical density of the quantum vacuum would vary in space and time, because it must have there an extra number density of pairs depending on $\Phi$. In this model, the lower (more negative) is $\Phi$, the denser is the quantum vacuum, and conversely the higher (less negative) is $\Phi$, the thinner is the quantum vacuum. Note that this variation of the virtual pairs lifetime in a gravitational field is not an ad hoc hypothesis, but an compelling consequence of the fourth Heisenberg relation. Note also that the virtual particles considered here are not created by gravity: they are just the usual zero point particles that fill the space everywhere, according to elementary quantum physics: they live a bit longer (or shorter), that is all.

It is unavoidable here to take $\Phi$ as the gravitational potential created by all the universe, so that this model has something in common with the Mach principle, since some important properties of the bodies are determined by their interaction with all the matter and energy in the visible universe. Indeed the gravitational potential energy of a body is defined as the energy required to bring it from the infinity to its actual position, without changing its kinetic energy or any other nongravitational energy. It could be argued that a body can not be brought actually from the infinity, since this is farther away than the horizon of the visible universe. However, it is clear that less energy is needed to create a virtual pair if $\Phi$ is negative than if it is zero. Therefore, the variation of the lifetime of the virtual pairs is due in this model to the gravitational interaction of the virtual pairs of the quantum vacuum with all the matter and radiation in the universe, and also with all the quantum vacuum itself.

As a result of the previous arguments some physical constants must vary in space and time because they are renormalized in a way that depends on $\Phi$. Any one, say $f$, can be expressed, at first order in the potential, either as $f(r, t) = f_\infty[1 + \sigma \Phi/c^2]$ or as $f(r, t) = f_R[1 + \sigma'(\Phi - \Phi_R)/c^2]$, where $\Phi = \Phi(r, t)$, the subindexes $\infty$ and $R$ indicate value at zero potential and at a reference terrestrial laboratory $R$, respectively, and $\sigma, \sigma'$ are two coefficients. We will consider only cases in which $(\Phi - \Phi_R)/c^2$ is small.

In particular, the relative permittivity and permeability of empty space must depend on the gravitational potential, their expressions at first order
being

$$
\epsilon_{r}(r, t) = 1 - \beta[\Phi(r, t) - \Phi_R]/c^2, \quad \mu_{r}(r, t) = 1 - \gamma[\Phi(r, t) - \Phi_R]/c^2, \quad (3)
$$

where $\beta$ and $\gamma$ are certain coefficients, which must be positive since the quantum vacuum is dielectric but paramagnetic (its effect on the magnetic field is due to the magnetic moments of the virtual pairs). For later convenience, we will used instead the combinations $\chi = (\beta + \gamma)/2, \xi = (3\beta - \gamma)/2 \text{ [so that } \beta = (\chi + \xi)/2, \gamma = (3\chi - \xi)/2].$ Clearly, $\epsilon_{r} = 1, \mu_{r} = 1$ at the reference laboratory at Earth.

This implies that the electron charge, the light velocity and the fine structure constant do vary in the universe, their values at a generic spacetime point $(r, t)$ being expressed, at first order, as

$$
e(r, t) = e[1 + \frac{\chi + \xi}{2} \frac{\Phi - \Phi_R}{c^2}], \quad (4)
$$

$$
c(r, t) = c[1 + \chi \frac{\Phi - \Phi_R}{c^2}], \quad (5)
$$

$$
\alpha(r, t) = \alpha[1 + \xi \frac{\Phi - \Phi_R}{c^2}], \quad (6)
$$

Here and along this paper $e, c, \alpha$ in an equation (without spacetime dependence) will be the values now at Earth (i.e. the constant in the tables). When this need to be emphasized, they will be denoted $e_0, c_0, \alpha_0$, unless otherwise specified. When they are considered to be variables, they will be written with space and/or time arguments. It was shown in [1] that eq. (6) agrees with the observed time evolution of the fine structure constant [7] if $\xi = (3\beta - \gamma)/2 \approx 1.3 \times 10^{-5}$. If the variation of $\alpha$ has been overestimated, as indicated by some recent data [8], then $\xi$ would be smaller. If $\alpha$ does not vary, then $\xi = 0$ (and $\gamma = 3\beta$). On the other the best value for $\chi$ turns out to be $\chi = 1$, as was shown in [3] and will be discussed later.

Note that the variation of the potential between two spacetime points can be written as the sum of a space and a time variation $\Delta \Phi = \Delta_s \Phi + \Delta_t \Phi,$ where $\Delta_s \Phi = \Phi(r_2, t_2) - \Phi(r_1, t_2)$ and $\Delta_t \Phi = \Phi(r_1, t_2) - \Phi(r_1, t_1).$ In this paper the point 1 will be the laboratory R at present time $t_0$, unless otherwise specified. The two variations have different effects, as was shown in [2, 3]. This is important since one or the other of the two variations can be neglected in some interesting cases. In the experiments considered here, the time variation can be neglected, while in the observations leading to the discovery of the Pioneer acceleration, it is dominant.
(i) The spatial variation of $\Phi$ causes the light to behave as in an ordinary inhomogeneous optical medium, in such a way that the frequency remains constant during the propagation, while the wavelength and the light velocity change according to the value of a refractive index $n$, which depends on $\Phi$ as

$$n = \left(1 + \chi(\Phi - \Phi_R)/c^2\right)^{-1},$$

This space variation can be either positive or negative according to how much matter is around. It follows from (5) that $c$ is smaller where $\Phi$ is more negative (or less positive), i.e. it decreases when approaching massive objects. In the case of the Sun surface, for instance, $(\Phi_S - \Phi_R)/c^2 \simeq -2.1 \times 10^{-6}$, with $\Phi_S$ being the potential there (and $n = 1 - 2.1 \times 10^{-6}$, so that light speed is about 2 ppm smaller than here). If, however, we take Jupiter instead of the Sun, then $\Delta \Phi > 0$ and $n > 1$.

(ii) The time variation of $\Phi$, as was shown in [2, 3] and will be seen here, is dominated by a secular component due to the expansion of the universe. This is because the progressive separation of the galaxies implies a monotonous increase of $\Phi$. As a consequence, $\epsilon_r, \mu_r$ decrease slowly what must cause an adiabatic acceleration of light. It can be shown and will be seen later that an adiabatic acceleration of the light speed causes a linear increase of the frequency such that $\dot{\nu} = \nu a_P/c$, the wavelength remaining constant. This variation of $c$ is the most important prediction of the model, although it is negligible in short time terrestrial laboratory experiments.

The potential at the terrestrial laboratory $R$ can be written, with good approximation, as $\Phi = \Phi_{\text{inh}}(R) + \Phi_{\text{av}}(t)$. The first term $\Phi_{\text{inh}}(R)$ is the part due to the near local inhomogeneities (the Earth, the Solar System and the Milky Way). It can be taken as constant since these objects are not expanding. The second $\Phi_{\text{av}}(t)$ is the space averaged potential due to all the mass and energy in the universe, assuming that they are uniformly distributed. It depends on time because of the expansion. The former has a non vanishing gradient but is small, the latter is space independent (it has zero gradient) but is time dependent and much larger. The value of $\Phi_{\text{inh}}/c_0^2$ at $R$ is the sum of the effects of the Earth, the Sun and the Milky Way, which are about $-7 \times 10^{-10}$, $-10^{-8}$ and $-6 \times 10^{-7}$, respectively, which have much smaller absolute values than $\Phi_{\text{av}}$, which of the order of $10^{-1}$ as will be seen below. Moreover $\Phi_{\text{inh}}$ appear in the two terms of the difference of potential in eq. (3), so that, being time independent, it cancels out. In fact, that difference is $[\Phi_{\text{av}}(t) + \Phi_{\text{inh}}(R) - (\Phi_{\text{av}}(t_0) + \Phi_{\text{inh}}(R))] = [\Phi_{\text{av}}(t) - \Phi_{\text{av}}(t_0)]$. 


This means that eq. (3) can be written at R as

\[ \epsilon_r = 1 - \beta [\Phi_{av}(t) - \Phi_{av}(t_0)]/c^2, \quad \mu_r = 1 - \gamma [\Phi_{av}(t) - \Phi_{av}(t_0)]/c^2, \]  

(8)

where \( \Phi_{av}(t) \) is the space averaged gravitational potential of all the universe at time \( t \) and \( t_0 \) is the present time (i.e. the age of the universe).

Let \( \Phi_0 \) be the gravitational potential produced by the critical density distributed up to the distance of \( R_U \) \((\Phi_0 = -\int_0^{R_U} G \rho_c 4\pi r dr \simeq -0.3c^2 \) if \( R_U \approx 3,000 \text{ Mpc} \)) and let \( \Omega_M, \Omega_\Lambda \) be the corresponding present time relative densities of matter (ordinary plus dark) and dark energy corresponding to the cosmological constant \( \Lambda \). Because of the expansion of the universe, the gravitational potentials due to matter and dark energy equivalent to the cosmological constant vary in time as the inverse of the scale factor \( R(t) \) and as its square \( R^2(t) \), respectively. It turns out therefore that

\[ \Phi_{av}(t) - \Phi_{av}(t_0) = \Phi_0 F(t), \quad \text{with} \quad F(t) = \Omega_M[1/R(t) - 1] - 2\Omega_\Lambda[R^2(t) - 1], \]  

(9)

where \( F(t_0) = 0, \dot{F}(t_0) = -(1 + 3\Omega_\Lambda)H_0 \). Let us assume a universe with flat sections \( t = \text{constant} \) (i.e. \( k = 0 \)), with \( \Omega_M = 0.27, \Omega_\Lambda = 0.73 \) and Hubble parameter to \( H_0 = 71 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} = 2.3 \times 10^{-18} \text{ s}^{-1} \). To find the evolution of the average light speed, \( \Phi_{av}(t) - \Phi_{av}(t_0) \) must be substituted for \( \Phi(r, t) - \Phi_R \) in (5), what gives

\[ c(t) = c[1 + \chi F(t)\Phi_0/c^2]. \]  

(10)

The effects of the inhomogeneities of the Milky Way, the Sun and the Earth at a terrestrial laboratory at present time \( t_0 \) are in fact included here in the numerical value of \( c = c(t_0) \). Taking now the time derivative of this equation, the present value of the acceleration \( a_\ell = \dot{c}(t_0) \) is found to be

\[ a_\ell = a_t c, \quad \text{with} \quad a_t = -H_0\chi(1 + 3\Omega_\Lambda)\Phi_0/c^2. \]  

(11)

Note that, with \( \chi = 1 \) and the values given before for \( \Phi_0 \) and \( \Omega_\Lambda \), then \( a_t = 2.2 \times 10^{-18} \text{ s}^{-1} = 0.96H_0 \) and \( a_\ell \simeq H_0c = 6.9 \times 10^{-10} \text{ m/s}^2 \approx 0.8a_P \). In other words, \( a_\ell \) is close to the Pioneer acceleration \( a_P \). The quantity \( a_t \) represents an acceleration of time since \( c(t) = c_0(1 + a_t t) \). Note that it plays the role of an acceleration of the clocks; it will be discussed in section 5.1.

The suggestion that \( a_t = H_0 \) is clear and intriguing. (Remember that the best values of these two coefficients were shown to be \( \chi = 1 \) and \( \xi \simeq 10^{-5} \).
in order to keep the agreement of the predicted time variation of $\alpha$ with the observations and to explain the Pioneer anomaly (see paragraph after eq. (6)).

3 The acceleration of light and the blue shift

It will be shown in this section that an adiabatic acceleration of light implies a blue shift, because the frequency $\nu_0$ of a monochromatic light wave with such an adiabatic acceleration $a_\ell$ increases so that its time derivative $\dot{\nu}$ satisfies

$$\frac{\dot{\nu}}{\nu_0} = \frac{a_\ell}{c}.$$  \hspace{1cm} (12)

The consequence is that an adiabatic acceleration of light has the same radio signature as a blue shift of the emitter, although a peculiar blue shift with no change of the wavelength (i.e. all the increase in velocity is used to increase the frequency).

Equations (8) state that the time derivatives of the permittivity $\epsilon = \epsilon_0 \epsilon_0$ and permeability $\mu = \mu_0 \mu_0$ of empty space at present time $t_0$ are equal to

$$\dot{\epsilon} = \epsilon_0 \beta \frac{\Phi_0}{c^2} (1 + 3\Omega_\Lambda) H_0, \quad \dot{\mu} = \mu_0 \gamma \frac{\Phi_0}{c^2} (1 + 3\Omega_\Lambda) H_0.$$  \hspace{1cm} (13)

These two derivatives are negative and very small. To study the propagation of the light in a medium whose permittivity and permeability decrease adiabatically, we must take the Maxwell equations and deduce the wave equations for the electric field $E$ and the magnetic intensity $H$. It is very easy to show that they are

$$\nabla^2 E - \frac{1}{c^2(t)} \frac{\partial}{\partial t} \left( \mu \frac{\partial}{\partial t} (\epsilon E) \right) = 0, \quad \nabla^2 H - \frac{1}{c^2(t)} \frac{\partial}{\partial t} \left( \epsilon \frac{\partial}{\partial t} (\mu H) \right) = 0, \hspace{1cm} (14)$$

or, more explicitly,

$$\nabla^2 E - \frac{1}{c^2(t)} \frac{\partial^2 E}{\partial t^2} - \frac{1}{c^2(t)} \left( \frac{\dot{\mu}}{\mu_0} + \frac{2\dot{\epsilon}}{\epsilon_0} \right) \frac{\partial E}{\partial t} - \frac{1}{c^2(t) \epsilon_0 \mu_0} \dot{\epsilon} \dot{\mu} E = 0, \hspace{1cm} (15)$$

$$\nabla^2 H - \frac{1}{c^2(t)} \frac{\partial^2 H}{\partial t^2} - \frac{1}{c^2(t)} \left( \frac{2\dot{\mu}}{\mu_0} + \frac{\dot{\epsilon}}{\epsilon_0} \right) \frac{\partial H}{\partial t} - \frac{1}{c^2(t) \epsilon_0 \mu_0} \dot{\epsilon} \dot{\mu} H = 0, \hspace{1cm} (16)$$

since at present time $\epsilon_t = 1, \mu_t = 1$. Because of (13), $\dot{\epsilon}/\epsilon_0$ and $\dot{\mu}/\mu_0$ are of order $H_0 = 2.3 \times 10^{-18}$ s$^{-1}$, so that the third terms in the LHS of (15) and
(16) can be neglected for frequencies $\omega \gg H_0$, in other words for any practical purpose. Note, by the way, that the last is a mass term with coefficient of order $10^{-53}$ m$^{-2}$, i.e. corresponding to a mass $m_\gamma \approx 10^{-39}$ MeV or close. This is exceedingly small and can be neglected also. We are left with two classical wave equations with time dependent light velocity $c(t) = c + a_t t$.

$$\nabla^2 E - \frac{1}{c^2(t)} \frac{\partial^2 E}{\partial t^2} = 0, \quad \nabla^2 H - \frac{1}{c^2(t)} \frac{\partial^2 H}{\partial t^2} = 0.$$  

(17) In order to find the behavior of a monochromatic light beam according to these two wave equations, we start with the first one and take $E = E_0 \exp[-i(\kappa z - (\omega_0 + \dot{\omega}/2)t)]$, where the frequency is the time derivative of the phase of $E$, i.e. $\omega_0 + \dot{\omega} t$. Neglecting the second time derivatives and working at first order in $\dot{\omega}$ (with $\dot{\omega} \ll \omega_0$, $\dot{\omega} \ll \omega_0^2$), substitution in (17) gives $\kappa^2 = [(\omega_0 + \dot{\omega} t)^2 - i\dot{\omega}] / c^2(t)$. It follows that $\kappa = k + i\zeta = \pm(\omega_0/c(t))[1 + 4\dot{\omega}t/\omega_0](\cos \varphi + i \sin \varphi)$, with $\varphi = -\omega/2\omega_0$, so that $k = \pm(\omega_0/c) (1+\dot{\omega}t/\omega_0)/(1+a_t t/c)$ what implies that $k = \pm\omega_0/c$ and the validity of eq. (11), $\dot{\omega}/\omega_0 = a_t/c$, as stated before. Also, $\zeta = -\dot{\omega}/2\omega_0 c = a_t/2c^2$.

The wave attenuates in the direction of propagation as $e^{-\zeta z}$, but since $a_t$ is of order $H_0 c$, $\zeta^{-1}$ is of order of 6,000 Mpc, so that the attenuation of the waves can be neglected. It is easy to show that to take $k + \dot{k} t$ for the wave vector leads to $\dot{k} = 0$. These results are valid both for the solutions of (15) and (16).

This shows that the electromagnetic waves verify eq. (12), so that $k$, and the wavelength $\lambda$ therefore, remain constant while the frequency increases with the same relative rate as the light velocity. Note an important point: in a measurement of the frequency (necessarily very precise), a blue shift is found (unrelated to the velocity of the emitter), but observations of the wavelength fail to find any effect. In other words, the observation using radio waves can discover the blue shift but the standard optical observations would see nothing.

The model would give thus a solution to the Pioneer riddle. However, it seems strange and surprising that this acceleration would not have been detected thus far. So, in order to proceed, it is necessary to show that an acceleration of about $10^{-9}$ m/s$^2$ is small enough to have remained undetected. In other words, one must consider the experimental tests of spacial relativity and the equivalence principle as well as other experiments in which the light speed plays a role. But before going to that, let us consider in next section some ideas of general relativity, as they were proposed by Einstein.
4 Einstein and the speed of light

Just after proposing his special relativity in 1905, Einstein realized that his theory was not an end but just a beginning. Indeed he published later but a few papers on that theory, starting instead an effort to transcend it that would lead to the future general relativity of 1916 [9]. Due to their enormous importance, the interest of the physicists concentrated in the two relativity theories in their finished form, his work during the transition period from 1906 to 1915 being much less known. This explains why, although studied by the historians of science, its influence on the mainstream of physics had been scarce. It is however highly interesting, in particular because of Einstein's discussions on the variation of the light speed in a gravitational field.

In the last section of a review paper in 1907 [10], Einstein introduces his principle of equivalence and concludes that the velocity of light must depend on the gravitational potential. According to Pais, “the study of Maxwell equations in accelerated frames had taught him that the velocity of light is no longer a universal constant in the presence of gravitational fields” [9]. It seems that he was not fully satisfied with his 1907 work so that in 1911 he takes again the question in a paper entitled “On the influence of gravitation on the propagation of light” (after what Pais called “three and a half silent years”). In order to apply the principle of equivalence, he uses two reference frames, K in a gravitational field with acceleration of gravity $g$, parallel to the $z$-axis in the negative direction, and $K'$, situated in a space free of gravitation and moving with acceleration $g$ in the positive direction of the $z$-axis. In the third and last section “Time and velocity of light in the gravitational field”, he analyzes the well known formula for the gravitational redshift $(\nu_1 - \nu_2)/\nu_2 = -\Phi/c^2$, where $\Phi = \Phi_1 - \Phi_2$ is the change of the gravitational potential between the points 1 and 2. He says that this seems to assert an absurdity since it states that the number of periods per second arriving in 1 is different that the number emitted in 2.

How could this be? To answer, he states: “we cannot regard $\nu_1$ or $\nu_2$ simply as frequencies since we have not yet determined the time in system K because [even if we have synchronized the clocks], nothing compels us to assume that the clocks U in [points with] different gravitational potentials must be regarded as going at the same rate” (U is a set of clocks synchronized as in special relativity). In a somewhat obscure passage, Einstein argues then that the effect of the gravitational field is that “if we measure the time in 1 with the clock U, then we must measure the time in 2 with a clock which
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goes $1 + \Phi/c^2$ times more slowly than the clock $U$ when compared with $U$ at one and the same place”. He concludes that “if we call the velocity of light at the origin of coordinates $c_0$, where we take $\Phi = 0$, then the velocity of light at a place with gravitational potential $\Phi$ will be given as

$$c = c_0 \left( 1 + \frac{\Phi}{c^2} \right).$$

(18)

This equation, which was already in the 1907 paper, is number (3) in reference [11]. Einstein says that it is a first order approximation. It can be written a bit more explicitly as

$$c(\Phi) = c_0 \left( 1 + \frac{\Phi - \Phi_R}{c_0^2} \right),$$

(19)

where $c_0 = c(t_0)$ is, and will be from now the speed of light at $R$, i.e. the constant that appears in the tables and $\Phi_R$ is a reference potential (at a terrestrial laboratory, for instance, where the light speed is measured with high precision). Equations (18)-(19) state that $c$ must vary in space, so that the deeper (more negative) is the potential, the lower is the light speed and conversely (at the surface of the Sun, $c$ must be about 2 ppm lower than here at Earth). Note that they are contrary to the frequent assumptions that $c$ is a universal constant of nature and that relativity precludes absolutely any variation of the light speed (see also [11]). It must be emphasized, furthermore, that eq. (19) coincides with eq. (5) if $\chi = 1$ and that we are assuming that $\xi$ is small.

In two papers in 1912 [12], he proceeds with his ideas. He begins by stating that the light speed is not constant in the presence of gravitational fields, a result that “excludes the general applicability of Lorentz transformations”. Turning once more the nut, he considers the light speed as a field in space-time $c(r, t)$ which, in a static situation, verifies $\Delta c = kc\rho$, $k$ being a constant (it was precisely in this paper and as a consequence of his thinking on this equation, where he realized that $\rho$ must include “the density of ponderable matter plus the energy density [evaluated locally]”). In the second paper, he reaches the conclusion that the variation of the light speed affects Maxwell’s equations, since $c$ appears in the Lorentz transformations, what establishes a coupling between the electromagnetic and the gravitational fields which is not easy to analyze since the first is generally time dependent. In the second paper, he proposes, as a dynamical equation for the field $c = c(r, t)$, the equation $\Delta c = k[\rho + (\nabla c)^2/2kc]$ [9, 14].
In 1912, in a reply to a critical paper on relativity by M. Abraham, Einstein states clearly “the constancy of the velocity of light can be maintained only insofar as one restricts oneself to spatio-temporal regions with constant gravitational potential. This is where, in my opinion, the limit of the principle of the constancy of the velocity of light — thought not of the principle of relativity — and therewith the limit of the validity of our current theory of relativity, lies” [13]. Note that what Einstein says here is that the principle of relativity is not the same thing as the principle of constancy of light speed: the latter must not be taken as a necessary consequence of the former. Furthermore, he admits that the light velocity can depend on Φ, as was the case with his eq. (19).

A final comment to close this section. Because the universe is expanding the potential Φ produced by all the matter and energy must increase so that Einstein eq. (19) implies that the speed of light must increase.

5 The potential of all the universe and the acceleration of light

Einstein’s variable light speed approach to General Relativity fell into oblivion, superseded as it was by general relativity that gives a deeper insight on Gravitation. However his arguments leading to eq. (18) are still right and can be used in the study of non local effects in weak gravitational fields, when going beyond special relativity which is only valid locally in spacetime. Indeed, special relativity is an approximation to general relativity which holds good only locally, more precisely in all the Local Inertial Frames (LIF), i. e. free-falling frames in each tangent space. However there is no reason whatsoever to require that the light speed will be the same for all the LIF’s: on the contrary it can be expected that it will depend on the particular LIF $L$, so that $c = c(L)$. For the sake of the argument, let us take a team of physicists in a spaceship during a travel through regions at which the potential is weak, through the Solar System or the Galaxy for instance. If they measure locally the light speed they could find that it varies adiabatically along the trip. At any moment, special relativity would still be valid in a small region around any spacetime point as a very good approximation. The local value of $c$, however, could be different from one region to the other.
This can understood by considering the element of interval in weak gravity

\[ ds^2 = e^{2\Phi/c^2} c^2 dt^2 - d\ell^2 \approx (1 + 2\Phi/c^2) c^2 dt^2 - d\ell^2. \]  

(20)

Note that \( \Phi \) is here the potential of near bodies, those that produce a non-vanishing acceleration \( g(r,t) \) at the observation point. Along a null geodesic one has thus for the light speed at a generic point \( P \),

\[ c(P) = c(R)[1 + (\Phi(P) - \Phi(R))/c^2], \]  

(21)

\( R \) being here a reference spacetime point in the geodesic. Assume for a moment that the light speed increases adiabatically as \( c(t) = \tilde{c} f(t) \), where \( \tilde{c} = c(\tilde{t}) \) is the light speed at some fixed time \( \tilde{t} \) in the past, \( f = 1 + \eta(t) \) (so that \( \eta(\tilde{t}) = 0 \)), \( \eta(t) \) being a small and monotonously increasing function of time. Defining the function \( \Pi(t) = \tilde{c}^2 \log f = \tilde{c}^2 \log[c(t)/\tilde{c}] \approx \tilde{c}^2 \eta(t) \), the interval (20) can be written as

\[ ds^2 = e^{2(\Phi+\Pi)/\tilde{c}^2} \tilde{c}^2 dt^2 - d\ell^2, \]  

(22)

at first order. It is clear then that \( \Pi \), which is space independent but time dependent, can be interpreted as a uniform gravitational potential of global character. Since (20) and (22) give equivalent descriptions, it turns out that an adiabatic acceleration of light is equivalent to the gravitational potential of all the universe in the following sense. At any time, we can use either (i) eq. (20) with a time dependent light speed \( c(t) \) and no potential \( \Pi \), or (ii) eq. (22) with a constant value of the light speed \( \tilde{c} \) and with the potential \( \Pi(t) \). This shows that \( \Pi(t) \) is the change of the gravitational potential of all the universe since the time \( \tilde{t} \). If in the second option we take \( \tilde{t} = t_0 \) and, consequently \( \tilde{c} = c_0 \) (i.e. the value in the tables), then \( \Pi(t_0) = 0 \) but \( \dot{\Pi}(t_0) > 0 \) because the galaxies are separating in the universal expansion, this implying that light must be accelerating.

5.1 Is time accelerating?

As shown in [6], there is a third (and intriguing) interpretation. In section 2, the quantity \( a_t \), such that \( a_t = a_t c \), was introduced and calculated to be

\[ a_t = 2.2 \times 10^{-18} \approx H_0. \]

Anderson et al mentioned in [4] that the Pioneer acceleration could be related to a “clock acceleration \( a_t \) of \( 2.8 \times 10^{-18} \text{ s/s}^2 \)”, such that “\( a_P = a_t c \)” (compare with eq (11) of this work), that “would appear
as a nonuniformity of time; i.e. all clocks would be changing with constant acceleration”. However, although the authors say “We have not yet been able to rule out this possibility.”, they do not further elaborate this idea, at least there, perhaps because they say that some results “rule out the universality of the time-acceleration model”.

It is encouraging that the calculation in section 2 of this paper gives for $a_t$ a value close to the Hubble parameter and to the one deduced in [4] from

the value of the blue shift which was interpreted as the acceleration $a_P$.

Note that this can be formulated by defining an accelerated time $t_{\text{cosm}}$ (measured by atomic clocks) so that $t_{\text{cosm}} = t + H_0 t^2 / 2$ (since $a_t = H_0$) [6]. As is easy to see, although the light speed would increase with respect to time $t$, it would be constant with respect time $t_{\text{cosm}}$. What could this mean? A possibility would be that $t$ is a parametric time, coinciding with the usual time in Newtonian and even in special and general relativity, while $t_{\text{cosm}}$ could be the cosmological time (note that it would reverse its arrow in a contraction of the universe in which the time derivative $\dot{\Phi}_{av}/c^2 = H$ would be negative.) The difference between these two kinds of time is important and complex, a theory embodying the dynamics of time would be then necessary [15, 16]. The Pioneers anomaly would show this duality of times and its understanding would need a good theory for the dynamics of time. All this will be considered in a forthcoming paper.

Another important point must be emphasized. As shown in [1, 2, 3] the potential $\Pi$ due to all the universe is much bigger that the potentials due to the local inhomogeneities, as the Galaxy for instance, so that one could fear that the a linearization or a Newtonian approximation would not valid. But we see here that, a large value of $\Pi$ can be eliminated just by changing accordingly the light speed. We can do away with the potential $\Pi$ in the expression of the interval, by the simple procedure of using the current value of $c$. The linear approximation is thus thus quite acceptable. However as $\Pi = \Pi(t)$, its time derivative, although very small, does not vanish and, consequently, the value of $c$ must vary.
6 First summary: spacetime variations of $e$, $c$, $\alpha$ and $e$

In the following, the predictions of this model will be compared with the experimental tests of special relativity and the equivalence principle (see the classic books by Dicke [18], Misner, Thorne and Wheeler [19] and Will [17]). Taking into account the previous considerations, it follows from eqs. (4)-(6) that the difference between the values of $e$, $c$, $\alpha$ and $m$ at two points are, at first order,

$$\frac{\Delta e}{e} = \frac{\chi + \xi}{2} \frac{\Delta \Phi}{c^2}, \quad \frac{\Delta c}{c} = \frac{\chi}{c} \frac{\Delta \Phi}{c^2}, \quad \frac{\Delta \alpha}{\alpha} = \frac{\xi}{c} \frac{\Delta \Phi}{c^2}, \quad \frac{\Delta m}{m} = \left(-\chi + \xi\right) \frac{\Delta \Phi}{c^2},$$

the last equality coming from the assumption that the electron mass verifies $mc^2 \propto e^2$ [20]. Note that the Planck constant $\hbar$ is assumed to be invariant, not depending therefore on the potential. Remember that we assume that $\chi = 1$, $\xi \lesssim 10^{-5}$.

In the following the predictions based in this model, more precisely in equations (23) will be compared with the experimental tests of the equivalence principle and special relativity.

7 Experiments on the gravitational redshift

General Relativity predicts a shift, the gravitational redshift, of a light ray as it travels from an emitter 2 to a receiver 1. It is emitted with frequency $\nu_2$ and received with frequency $\nu_1$. The shift is given, at first order in the potential, as

$$\nu_1 = \nu_2 \left[1 - \frac{\Phi_1 - \Phi_2}{c^2}\right],$$

or, equivalently

$$\frac{\nu_1 - \nu_2}{\nu_2} = -\frac{\Phi_1 - \Phi_2}{c^2}$$

(if the light goes upwards at Earth surface (or it goes away from the Sun) its frequency decreases and conversely). Note that (24) is a consequence of the weak equivalence principle. When looking for eventual violations of eq. (25) it is customary to write it as

$$\frac{\Delta \nu}{\nu} = -(1 + a) \frac{\Delta \Phi}{c^2}.$$
The prediction of general relativity is \( a = 0 \), so that a nonvanishing \( a \) would indicate a failure of the equivalence principle or of the presence of another effect. The best bounds for \( |a| \) are \( |a| \leq 10^{-2} \) for the nuclear gamma ray spectrum by Pound and coworkers [22, 23] and \( |a| \leq 2 \times 10^{-4} \) obtained by Vessot, Levine and coworkers in the hyperfine spectrum of Hydrogen [24, 25].

In the usual interpretation of this experiment, \( \nu_1 \) is the frequency of the radiation of a particular line that reaches the laboratory and \( \nu_2 \) is the frequency of the same line when emitted at the source 2, which is assumed to be equal to the frequency of the same radiation when emitted at the laboratory 1. It is accepted that the photons lose energy when going to higher potentials and conversely, just as a particle that loses kinetic energy when gaining potential energy. On the other hand, what happens in this model is that any particular line is emitted with lower frequency if the potential is deeper (and conversely), travelling afterwards with constant frequency from the emitted to the receiver. Indeed, since the frequency and wavelength of an emitted line are functions of the electron charge \( e \) and mass \( m \) and the light speed \( c \), that frequency and that wavelength depend on space and time (because these quantities are slightly variable across the spacetime, see eqs. (4)-(6) and note that \( mc^2 \propto e^2 \)). As a consequence, in this Newtonian model, the frequencies of the spectra are indeed space and time dependant. In the case of the redshift experiments, the time variation of the potential can be neglected. The space variation is then \( \Delta \Phi = \Phi_2 - \Phi_1 \).

Let us find now which is the frequency and wavelength predicted by this model in the main experiments performed until now to test the gravitational redshift. We will consider four possibilities depending on whether the lines are produced in (i) ordinary atomic spectrum, (ii) fine structure atomic transitions, (iii) gamma nuclear transitions and (iv) hyperfine transitions. In each case, the production of a beam of light at space-point \( S \) and its reception at point \( R \) will be considered.

It is convenient to use the following notation. The indices \( R \) and \( S \), the initials of “reference” and “source”, refer to the laboratory at Earth and to the emitter. \( \nu_S \) and \( \lambda_S \) (resp. \( \nu_R \) and \( \lambda_R \)) will denote the frequency and wavelength of the radiation as is emitted at \( S \) (resp. at \( R \)); \( \nu'_R \) and \( \lambda'_R \) are the same quantities of the radiation emitted at \( S \) when it arrives to \( R \); \( c_S \) and \( c_R \) are the value of the light speed at the source and at the laboratory.
With this notation, eqs. (24)-(25) are written

\[ \nu'_R = \nu_R \left[ 1 + (1 + a) \frac{\Phi_S - \Phi_R}{c^2} \right], \quad \nu'_R - \nu_R = (1 + a) \frac{\Phi_S - \Phi_R}{c^2} \]

Remember that we are using \( \chi = 1 \) and \( \xi = O(10^{-5}) \) or smaller and that no variation of \( \hbar \) is considered here.

### 7.1 Ordinary spectrum

The frequencies of this type of spectrum verify the proportionality

\[ \nu \propto mc^2 h^{-1} \alpha^2 \]

For instance, they are equal to \( \nu_{nm} = mc^2 \alpha^2 h^{-1}(1/2n^2 - 1/2m^2) \) in the Hydrogen spectrum.

Taking into account eqs. (23), it turns out that the frequency of a particular line emitted at a spacetime point \( S \) verifies \( \nu_R \propto me^4 \), \( m \) and \( e \) being the values of the electron charge and mass at \( S \). Therefore, under a change of the potential, it suffers the relative change \( \Delta \nu/\nu = (\chi + 3\xi)\Delta \Phi/c^2 \) so that one has

\[ \nu_S = \nu_R \left[ 1 + (\chi + 3\xi) \Delta \Phi/c^2 \right], \quad \nu'_R = \nu_S \]

with \( \Delta \Phi = \Phi(S) - \Phi(R) \). Note that the frequency of the radiation does not change during its flight from \( S \) to \( R \) if the effect of the time variation of \( \Phi \) can be neglected, as it happens in the redshift experiments performed up to now (remember that the space is here an optical medium with refractive index \( n = n(\Phi) \) given by eq. (7)). The frequency of the radiation received at \( R \) is thus \( \nu'_R = \nu_S \). Note also that \( \nu'_R < \nu_R \) (resp. \( \nu'_R > \nu_R \)) if the wave travels in the direction of increasing (resp. decreasing) potential, so that there is a red shift (resp. a blue shift). Concerning the wavelength and since \( c_S = c_R[1 + \chi \Delta \Phi/c^2] \), one has for the wavelength of the radiation emitted at \( S \) and received at \( R \), respectively

\[ \lambda_S = \frac{c_S}{\nu_S} = \lambda_R[1 - 3\xi \Delta \Phi/c^2], \quad \lambda'_R = \frac{c_R}{\nu'_R} = \lambda_R[1 - (\chi + 3\xi) \Delta \Phi/c^2]. \]

The consequence is that this model predicts that the variation is given by eq. (26) with \( |a| = 3|\xi| \leq 4 \times 10^{-5} \). The best bound in ordinary spectrum, obtained by Braoult by observing solar spectrum lines is \( |a| \leq 5 \times 10^{-2} \). So the prediction of this Newtonian model is not in conflict with the observations.
In conclusion the prediction of the model is exactly the same as predicted by the weak equivalence principle and by General Relativity if $\xi = 0$. It is equal within the experimental margin if $|\xi| \leq 10^{-5}$.

### 7.2 Fine structure spectrum

The fine structure must be considered also. The difference between the frequencies within the same multiplet verifies $\nu \propto mc^2 \alpha^4 \propto me^4 \alpha^2$. It is very easy to see, following the same analysis as in the preceding subsection, the only difference being that instead of (27) and (28) one has

$$ (\nu'_{R}) = \nu_S = \nu_R [1 + (\chi + 5\xi)\Delta \Phi / c^2], \quad \lambda_{R}' = \frac{cR}{\nu'_{R}} = \lambda_R [1 - (\chi + 5\xi)\Delta \Phi / c^2]. $$

which corresponds to $|a| \leq 8 \times 10^{-5}$.

### 7.3 Nuclear gamma ray spectrum

A very important, well known and precise experiment was performed by Pound, Rebka and Snyder [22, 23], by using the Mssbauer effect to measure the redshift of a line of 14.4 keV gamma rays of Fe$^{57}$. The source and the receiver were at the the bottom and the top of a tower, respectively, in the Jefferson Laboratory of Harvard University, at the distance of 22.5 m. To measure the shift, They observed the absorption of photons in a target containing Fe$^{57}$, by setting the source in motion to compensate the gravitational redshift with a Doppler effect and measuring the velocity at which the photons were absorbed. Their result was that $a \leq 10^{-2}$. From the semiempirical mass formula of von Weizsäcker the energy of the transition is proportional to $e^2$. Following then the same arguments as before and using (23), it is easy to show that, instead of (27) and (28) one has

$$ (\nu'_{R}) = \nu_S = \nu_R [1 + (\chi + \xi)\Delta \Phi / c^2], \quad \lambda_{R}' = \frac{cR}{\nu'_{R}} = \lambda_R [1 - (\chi + \xi)\Delta \Phi / c^2]. $$

which corresponds to $|a| \leq 2 \times 10^{-5}$, well below the bound $|a| \leq 10^{-2}$ implied by this experiment.

Note that, according to the interpretation of this model, the photons were emitted with a lower frequency at the bottom that they would have been at the top, this frequency being enhanced by the motion of the source.
to produce the resonant absorption. We see that the model agrees with the Pound- Rebka-Snider experiment.

### 7.4 Hyperfine spectrum

The best experimental bound obtained up to now for the gravitational redshift was obtained by Vessot and Levine [24, 25] measuring the frequency variation of the 1,420 MHz line of the hyperfine spectrum of Hydrogen between a spaceship in vertical motion up to a height of 10,000 km and a laboratory at Earth surface. Their result is

\[ |a| \leq 2 \times 10^4. \]

The energy levels of that hyperfine structure have the expressions

\[
E = \frac{\mu_0}{4\pi} g_p \mu_N \mu_B K \left( \frac{L(L+1)}{J(J+1)} \frac{1}{r^3} + \frac{2}{3} R^2(0) \right),
\]

where \( K = F(F+1) - I(I+1) - J(J+1), \mu_N = e\hbar/2M \) and \( \mu_B = e\hbar/2m \) are the nuclear and the Bohr magnetons, \( g_p = 2.79, J, I, F \) are the electron, nucleus and total angular momentum and the average value of \( 1/r^3 \) and the value of the radial function at the origin appear in the last factor [26]. From (23) we have \( \Delta \mu_B = (3\chi - \xi)\Delta \Phi/c^2 \). Let us assume that \( \Delta g_p/g_p = (\chi + \xi)\Delta \Phi/c^2 \), what means that \( g_p \) varies approximately as \( e^2 \), as can be expected. All this means that the frequencies in the hyperfine spectrum are proportional to

\[
\frac{\mu_0}{4\pi} g_p \mu_N \mu_B h^2 \left( \frac{me^2}{4\pi \epsilon_0 \hbar^2} \right)^3 = \frac{g_p}{4\pi \ M} \frac{m^2}{\hbar} \left( \frac{e^2}{4\pi \epsilon_0 \hbar c} \right)^4 c^2 \propto g_p \frac{m^2}{M} \alpha^4 c^2,
\]

where the relation \( \mu_0 = 1/c^2 \epsilon_0 \) has been used. It follows that

\[
\nu_S = \nu_R \left( 1 + [(\chi + \xi) + 2(-\chi + \xi) + 4\xi + 2\chi] \frac{\Delta \Phi}{c^2} \right) = \nu_R \left( 1 + (\chi + 7\xi) \frac{\Delta \Phi}{c^2} \right).
\]

Using the same arguments as for the ordinary spectrum, it is found that in this case \( a = 7\xi \simeq 9 \times 10^{-5} \), which is close from below to the Vessot-Levine bound. However, remember that there are arguments to suggest that \( \xi \) is probably smaller, perhaps even zero.
7.5 Frequency from distant sources

The time variation of $\Phi$ can not be neglected in the case of the radiation coming from distant sources, contrary to what happens in the experiments referred to in sections 7.1-7.4. It could seem at first sight, therefore, that the increasing in time of the frequencies of the radiation during its travel to Earth must pose a problem, since the radiation coming from distant sources would be blueshifted with $\Delta\nu = \nu a t D/c^2$, where $D$ is the distance from the source, contrary to what is observed. The previous considerations in this section indicate that there is no such a problem. Let us consider a line which has frequency $\nu_0$ here when it is emitted here. It follows from sections 7.1-7.4 that the frequency with which it is emitted at a source $S$, placed at distance $D$, is

$$\nu_S = \nu_0 \left[ 1 + (\chi + b\xi) \left( \frac{\Phi_\text{av}(t) - \Phi_\text{av}(t_0)}{c^2} \right) \right] = \nu_0 \left[ 1 - (\chi + b\xi) a t \frac{D}{c} \right],$$

where $t_0 - t = cD$ and $\Delta\Phi/c^2 = -a t D/c$. As shown before, the numerical coefficient $b$ is equal to 3 (ordinary spectrum), 5 (fine structure spectrum), 1 (gamma rays) and 7 (hyperfine spectrum). It is clear that it is emitted at $S$ with a lower frequency but, in the travel to Earth during the time $D/c$, it increases in a relative amount $\chi a t D/c$, so that it will be received at Earth with the value

$$\nu'_R = \nu_0 \left[ 1 - b\xi a t \frac{D}{c} \right].$$

The cosmological redshift must be added, what decreases further the frequency by a relative amount $H_0 D/c = a t D/c$, i.e. to $\nu_0[1 - (1 + b\xi)H_0 D/c]$. This means that the effect of the quantum vacuum used in this model increases the observed cosmological redshift by the factor $(1 + b\xi)$. Since $|b\xi| \lesssim 10^{-5}$ and could be nil, this is quite unobservable. This property can be expressed by saying that the changes in the frequency of a line due to the time changes in the emission and in the propagation of the radiation until Earth cancel each other, except for a very small and unobservable multiplicative factor.

The conclusion of this section is that, if there is no cosmological variation of the fine structure constant as some papers suggest (for instance [8]) then $\xi = 0$, the predictions of the model being the same as those of the weak equivalence principle and fully compatible therefore with the experiments. If there is a variation of $\alpha$ with the value obtained in [7] then $\xi \simeq 10^{-5}$,
the model being still compatible with the observations. If that variation is smaller than the first estimation by Webb et al [7], the difference between the predictions of the model and of the weak equivalence principle would be smaller also.

8 Bending of a light around the Sun

Let us consider a light beam that grazes the Sun, its mass being $M$ and its radius $R$. Because the refraction index of the quantum vacuum decreases as the distance to the center increases, the trajectory of a light ray is a curve $y = y(x)$ that bends towards the center of the star. The time variation of the light speed can be neglected here. The velocity of light and the refraction index take the form

$$c(r) = c \left(1 - \eta \frac{R}{r}\right), \quad n(r) = \left(1 + \eta \frac{R}{r}\right),$$

where $\eta = GM/c^2R = 2.1 \times 10^{-6}$ and $c(\Phi = 0) = c(1 - \Phi R)$ has been rebaptized as $c$.

The ray trajectories are the solutions to the variational problem

$$\delta T[y(x)] = \delta \int_1^2 \frac{n(r)}{c} (1 + y'^2)^{1/2} dx = 0 \quad (31)$$

or

$$\delta T[y(x)] = \delta \int_1^2 \frac{(1 + y'^2)^{1/2}}{c} \left(1 + \eta \frac{R}{r}\right) dx = 0. \quad (32)$$

The corresponding Euler-Lagrange equation is

$$\frac{d}{dx} \left[ \frac{y'}{(1 + y'^2)^{1/2}} \left(1 + \eta \frac{R}{r}\right) \right] + \eta (1 + y'^2)^{1/2} \frac{yR}{r^3} = 0. \quad (33)$$

We wish to obtain the solution with initial data $y(0) = R, y'(0) = 0$. As $\eta$ is small, the solutions can be expanded in series $y(x) = y_0(x) + \eta y_1(x) + \cdots$.

At order zero the equation (33) is

$$\frac{d}{dx} \left[ \frac{y_0'}{(1 + y_0'^2)^{1/2}} \right] = 0.$$
which says that $y_0'$ is constant. The zero order approximation is thus $y_0(x) = R$, $y_0'(x) = 0$ so that at order 1:

$$
\frac{d}{dx}y_1' = -(1 + y_0'^2)^{1/2} \frac{y_0 R}{(x^2 + y_0^2)^{3/2}} = \frac{-R^2}{(x^2 + R^2)^{3/2}}.
$$

The solution to with the prescribed initial data is

$$
y_1'(x) = -\int_0^x \frac{R^2 dx}{(x^2 + R^2)^{3/2}} = -\frac{x}{(R^2 + x^2)^{1/2}},
$$

so that $y_1(x) = -(R^2 + x^2)^{1/2}$ as can be seen easily by means of the change $x = R \tan \beta$. It follows that $y(x) = R - \eta(R^2 + x^2)^{1/2}$. The bending angle is $\phi = 2|y'(\infty)|$, i.e

$$
\phi = 2\eta = 2 \frac{GM}{c^2 R} = 0.875''.
$$

(34)

The well tested prediction of General Relativity is twice as much, i.e. $\phi = 1.75''$. This means that the prediction of this model is exactly the same as that of the weak equivalence principle.

9 Time delay of radar signals

According to general relativity, light waves take a longer time to traverse a distance in the gravitational field of a massive body, the Sun for instance, than they would in Newtonian physics with constant $c$. It is said that they suffer a time delay. This effect, predicted by Shapiro in 1964 [27, 19, 17], is one of the classic tests of general relativity. Let a beam of radar waves be emitted from Earth and let it be reflected back from a reflector, somewhere in the Solar System. More precisely, let us consider the case of a signal grazing the Sun surface. Let $a_E$ (resp. $a_R$) be the distance from the emitter at Earth (resp. the reflector, say a planet) and the point where the trajectory touches the Sun, to be approximated as the radii of their orbits, and let $M$, $R$ be the mass and radius of the Sun. The well known prediction of general relativity is that the round trip time is

$$
\tau_{GR} = \sqrt{-g_{00}} \frac{1}{2} \int_{-a_E}^{a_R} \left[ 1 + \frac{2GM}{c^2 \sqrt{x^2 + R^2}} \frac{dx}{c} \right]
$$

$$
= \frac{2a_E + a_R}{c} \left( 1 - \frac{GM}{c^2 \sqrt{a_E^2 + R^2}} \right).
$$

(35)
In this model, there is also a delay caused by the smaller speed of light near the Sun. The time variation of that speed can be neglected and the space variation takes the form

\[ c(r) = c_0 \left[ 1 - \frac{GM}{c_0^2} \left( \frac{1}{r} - \frac{1}{\sqrt{a_E^2 + R^2}} \right) \right]. \]

(Note that \( x^2 + R^2 = r^2 \)). It follows that the time delay due to the variation of the speed of light is in this model

\[ \Delta \tau_{\text{model}} = 2 \int_{a_E}^{a_R} (c(r)^{-1} - c_0^{-1}) \, dx = \frac{2GM}{c^3} \log \left[ \frac{(a_R + \sqrt{a_R^2 + R^2})(a_E + \sqrt{a_E^2 + R^2})}{R^2} \right], \quad (36) \]

The Shapiro time delay is \( \Delta \tau_{\text{GR}} = \tau_{\text{GR}} - \tau_{\text{classic}} \). Since \( c \) in (35) is \( c_\infty \), the speed of light at infinite distance (where \( \Phi = 0 \)), if \( c_0 \) is its value at Earth, then \( c = c_0(1 - GM/c_0^2 \sqrt{a_E^2 + R^2}) \). As \( \tau_{\text{classic}} = 2(a_E + a_R)/c_0 \), it is equal to the second line in (35), at first order in the variation of \( c \), so that the time delay in this non-relativistic model is one half the prediction of general relativity. This model predicts again the same result as the weak equivalence principle.

10 Other tests of the weak equivalence principle

10.1 Effect on the electromagnetic mass of bodies

In this model, the electromagnetic mass of a body varies in a way that depends on its chemical composition, what would lead to a violation of the weak equivalence principle (see [31, 32]). However, the effect would be too small to be a matter of concern, as we will see now. Moreover it might be worth to recall the much quoted sentence by J.L. Synge in his classic treaty
"Relativity: the General Theory" [33]: “The principle of Equivalence performed the essential office of midwife at the birth of General Relativity, but, as Einstein remarked, the infant would never have got beyond its long-clothes had it not been for Minkowski’s concept. I suggest that the midwife be now buried with appropriate honors.”

The mass of a body includes a part of electromagnetic origin. In fact, the von Weizsäcker semiempirical mass formula tells us that there is a Coulomb contribution to the rest energy of a nucleus given by the expression

$$m_C c^2 = a_C Z (Z-1) A^{-1/3}, \quad \text{with} \quad a_C = \frac{3}{20 \pi \epsilon_0} \frac{e^2}{r_0} \simeq 0.6 \text{ MeV} / c^2 \quad (r_0 \simeq 1.5 \text{ fm}),$$

plus the electromagnetic mass of each of the protons. If the mass of a nucleus is written as $$m = m_0 + m_C$$, then $$u = m_C / m$$ is of the order $$5 \times 10^{-3}$$ for the heavier nuclei and about $$10^{-3}$$ or smaller for the lighter. It follows then form (31) that the difference of the value of the mass between two points due to the variation of $$e$$ and $$c$$ is in the model

$$\Delta m = \Delta m_C = (-\chi - \xi) m_C \Delta \Phi / c^2 = (-\chi - \xi) u m \Delta \Phi / c^2. \quad (38)$$

This implies that the electromagnetic part of the mass of a nucleus changes from one space point to another in a way that depends on $$\Phi$$, on the mass number $$A$$ and on the atomic number $$Z$$. In the case of a macroscopic body, the variation of the mass $$\Delta m$$ depends on its chemical composition and on the direction of the displacement between the two points. This implies that its mass is, in this model, a function of space and time of the form $$m + \Delta m(x, y, z, t)$$. Although $$\Delta m$$ is a scalar, not a tensor, there is an anisotropy in the sense that its derivative does depend on the direction. It must be emphasized, however, that $$\Delta m$$ would be so small that it could not be appreciated at Earth, as shown below.

During its motion the mass change in time, so that its derivative is

$$\frac{dm}{dt} = (-\chi - \xi) m_0 u \nabla \Phi \cdot \mathbf{v} + \partial \Phi / c^2. \quad (39)$$

Newton’s 2nd law $$\dot{\mathbf{p}} = \mathbf{F}$$ takes the form

$$\frac{d(m \mathbf{v})}{dt} = m \mathbf{g}, \quad \text{in other words:} \quad \frac{d\mathbf{v}}{dt} = m \mathbf{g} - \frac{dm}{dt} \mathbf{v} \quad (40)$$
It turns out that there is an extra force \(-\dot{m}v = -m_0u(-\chi+\xi)(\nabla \Phi \cdot v + \partial_t \Phi) \frac{v}{c^2}\). However, it is extremely small at terrestrial scale, quite unobservable as will be seen.

The term in \(\partial_t \Phi/c^2\) can be neglected in laboratory experiments. In fact, as shown before, it is equal to \(a_t \approx 2.3 \times 10^{-18} \text{ s}^{-1}\). Let us see what happens with the gradient term. The value of \(\Phi\) at a terrestrial laboratory is the addition of the contributions of the Earth, the Sun, the Milky Way and the rest of the universe \(\Phi = \Phi_E + \Phi_S + \Phi_{MW} + \Phi_U\), with self-explaining notation. The last one is the larger but has the smallest gradient. We concentrate therefore on the other three. The three accelerations are directed towards the centers of the Earth, the Sun and the Milky Way, respectively, so that the gradients of the three potentials have moduli \(|\nabla \Phi| = GM/R^2\), \(M\) and \(R\) being the corresponding masses and distances. In a terrestrial laboratory, it turns out that (assuming that \(M_{MW} = 10^{11} M_S\))

\[
\frac{|\nabla \Phi_E|}{c^2} \approx 10^{-16} \text{ m}^{-1}, \quad \frac{|\nabla \Phi_S|}{c^2} \approx 10^{-19} \text{ m}^{-1}, \quad \frac{|\nabla \Phi_{MW}|}{c^2} \approx 10^{-27} \text{ m}^{-1}, \quad (41)
\]

If a body is displaced the vector \(h\), the relative variation of its mass is therefore the sum of the corresponding three terms, which verify

\[
\frac{|\Delta m|}{m} \left|_E \right. \leq (\chi - \xi)h \times 10^{-19}, \quad \frac{|\Delta m|}{m} \left|_S \right. \leq (\chi - \xi)h \times 10^{-22},
\]

\[
\frac{|\Delta m|}{m} \left|_{MW} \right. \leq (\chi - \xi)h \times 10^{-30}, \quad (42)
\]

where \(h\) is the number of meters (remember that \(\chi = 1, \xi \leq 10^{-5}\)). The largest change of the mass is the direction to the center of the Earth. Note that this contribution from Earth vanishes if \(h\) is horizontal. In any case the variation of \(m\) along a trajectory can be neglected at Earth.

### 10.2 Free fall

First and in order to understand the problem, let us consider the free fall along the vertical of a small body at the Earth surface, with initial data \(v = \dot{z} = 0, z = z_0\). As explained before

\[
m = m_0 \left\{ 1 + u \left[ 1 + (-\chi + \xi) \frac{\Phi(z) - \Phi(z_0)}{c^2} \right] \right\}, \quad (43)
\]
where \( m = m_0 + m_C \) at the point \( z_0 \). It turns out therefore that the extra force is very small. In fact its ratio to \( mg \) is

\[
\frac{\dot{mv}}{mg} \simeq u(-\chi + \xi) \times 10^{-15} \sqrt{z_0 - z}, \quad (\simeq 10^{-17} \text{ if } z - z_0 = 100 \text{ m}). \tag{44}
\]

This force is extremely small at the terrestrial scale, in fact it is much smaller than the air resistance \( F_S \). To be specific, taking the case of a sphere of aluminium with \( R = 10 \text{ cm} \) and the Stokes expression \( F_S = 6\pi R\eta v \), it turns out that \( \dot{mv}/F_S \simeq \times 10^{-11} \sqrt{z_0 - z} \). The effect is certainly unobservable.

### 10.3 Experiments of Eötvös type.

After some work by Bessel in the 19th century (and by Newton before!), Eötvös and coworkers performed a series of famous experiments with a torsion balance to see if the weak equivalence principle (WEP) is valid (the best one in 1922). To analyze its experiment let us write the passive gravitational mass \( m_P \) as \( m_P = m_0 + m_C + \Delta m_C \), assuming that \( m_I = m_0 + m_C \) is the inertial mass. The term \( \Delta m_C = \Delta m = (-\chi + \xi)um\Delta \Phi/c^2 \) is the one that breaks the Equivalence Principle, since it changes in space in a way that depends on the composition of the body. The time variation of \( \Phi \) can be neglected again here. It follows that

\[
m_P = m_0 + m_C - (\chi - \xi)m_0u \frac{\Phi(r) - \Phi_R}{c^2}. \tag{45}
\]

Note that the equality \( m_P = m_0 + m_C \) holds exactly at \( R \) and that we are working at first order in \( u \). The accelerations of the two bodies are

\[
a_1 = \left[ 1 - u_1(\chi - \xi) \frac{\Phi(r_1) - \Phi_R}{c^2} \right] g, \quad a_2 = \left[ 1 - u_2(\chi - \xi) \frac{\Phi(r_2) - \Phi_R}{c^2} \right] g, \tag{46}
\]

where \( r_1, r_2 \) are the positions of the two arms of the torsion balance and \( g \) the acceleration of gravity. To test WEP one has to measure the Eötvös ratio

\[
\eta \equiv 2 \left| \frac{a_1 - a_2}{a_1 + a_2} \right|. \tag{47}
\]

From (46), its value is in this case

\[
\eta = (-\chi + \xi) \left[ u_1 \frac{\Phi(r_1) - \Phi_R}{c^2} - u_2 \frac{\Phi(r_2) - \Phi_R}{c^2} \right]. \tag{48}
\]
Let $u_1 > u_2$. Taking $\Phi(r_2) = \Phi_R$, we have

$$\eta = (-\chi + \xi)u_1 \frac{\nabla \Phi \cdot h}{c^2},$$

(49)

where $h = r_1 - r_2$. In an Eötvös type experiment $h < 1$ m. It turns out that

$$|\eta| \leq h \times 10^{-18}$$

(50)

where $h$ is in meters. The best experiments by Roll, Krotkov and Dicke [34] and by Braginsky and Panov [35] establish as an upper bound for this quantity $10^{-11}$ and $10^{-12}$. The effect used in this model can not be detected in an Eötvös experiment.

### 11 Experimental tests of the local Lorentz invariance

#### 11.1 The Hughes-Drever experiments

The experiments by Hughes et al [28] and Drever [29], following a suggestion by Cocconi and Salpeter [30], were devised as tests of the Mach principle. They establish bounds to the part of mass of a body $\Delta m$ which is anisotropic. According to that principle and since the matter in our galaxy is not distributed isotropically around the Earth, the mass of a body might depend here on the direction of its acceleration, what would imply in the strict sense that the inertial mass would be a tensor $M_{ij}$ with three different eigenvalues $m + \Delta m_i$. Let $\Delta m$ be the greatest $|\Delta m_i|$. By studying the Zeeman effect in excited states of atoms and nuclei when the magnetic field varies with respect to the direction to the center of the Milky Way as the Earth rotates, Hughes and Drever established that

$$\frac{\Delta m}{m} < 10^{-20}, \quad \text{and} \quad \frac{\Delta m}{m} < 5 \times 10^{-23},$$

(51)

respectively.

It must be emphasized, however, that the mass of a body does not depend in this model on the direction of the acceleration. In this sense, there is no anisotropy in it. However, this model can be considered as a realization of the Mach principle (since some properties of bodies depend in it of their interaction with all the universe) and embodies another kind of anisotropy.
to which the Hughes-Drever bounds can be applied. As a measure of such anisotropy we must take the value of the changes of $\Delta m$ along different directions around a point. The right thing to do, therefore, is to take the change in the electromagnetic mass for $|\Delta m|$ given in (38) and (41)-(42) and insert them in (51) to see if the inequalities are violated. The main contribution comes from the potential due to Earth. The result is $|\Delta m|/m \leq h \times 10^{-19}$, where $h$ is a length given in meters which is characteristic of the size of the object measured. In the case of an experiment on the Zeeman effect of a nucleus, $h$ must be taken as the diameter of a nucleus, i.e. $h \simeq 10^{-14}$ m, from which we can say that, in any case, the effect of the variation of $\alpha$ verifies in this model,

$$\frac{\Delta m}{m} \leq (-\chi + \xi) \times 10^{-33}. \tag{52}$$

Since $\chi = 1$ and $\xi \leq 10^{-5}$ it is seen that the anisotropy in this model is well below the bounds imposed by the Hughes-Drever experiments. It can be remarked that in experiments of this kind, the direction of the magnetic field is usually fixed with respect to the radius from the center of the Earth. In that case, the main effect would be due to the Sun which is $10^3$ times smaller (41).

## 12 Conclusions

The situation of the attenuating quantum vacuum model [1, 2, 3] with respect to the experiments has been studied. This model gives a unified explanation of the Pioneer anomalous acceleration $a_P \simeq 8.5 \times 10^{-10}$ m/s$^2$ observed by Anderson et al [4] in four different spacecrafts and of the cosmological variation of the fine structure constant detected by Webb et al [7]. Its most important prediction is that there must be an adiabatic universal acceleration of light $a_\ell$, equal to $a_P$, the observational signature of which is the same as that of an extra constant acceleration directed towards the Sun, i.e. a blue shift of the radio signal from the spaceship that increases linearly in time, indeed just what was observed. The model is based on an argument that shows that the quantum vacuum must be decreasing its optical density, what explains why it was dubbed as “attenuating quantum vacuum model”. Accordingly, the empty space (i.e. the quantum vacuum) is treated phenomenologically as an optical medium, its permittivity and permeability $\varepsilon_0$ and $\mu_0$ being changed.
to $\epsilon_t \epsilon_0$ and $\mu_t \mu_0$, where the relative quantities $\epsilon_t$, $\mu_t$ (of course, equal to 1 in a reference terrestrial laboratory) depend on the gravitational potential $\Phi$ through the expressions (3) that include two positive coefficients $\beta$ and $\gamma$. It was argued in [3] that the best values for them are $\beta \simeq 0.5$, $\gamma \simeq 1.5$ (in this paper, the coefficients $\xi = (3\beta - \gamma)/2$, $\chi = (\beta + \gamma)/2$ are used instead).

As a consequence, the electron charge, the light speed and the fine structure constant depend on space and time through the universe as

\[
e(r, t) = e[1 + \frac{\chi + \xi}{2} \frac{\Phi - \Phi_R}{c^2}],
\]

\[
c(r, t) = c[1 + \chi \frac{\Phi - \Phi_R}{c^2}],
\]

\[
\alpha(r, t) = \alpha[1 + \xi \frac{\Phi - \Phi_R}{c^2}],
\]

where $e$, $c$, $\alpha$ are their values at the reference terrestrial laboratory $R$ at present time and $\Phi_R$ is the potential at $R$. These equations fix the values of the coefficients. Einstein formula (19) indicates that $\chi = 1$ [6]. The observations by Webb et al [7] give $\xi \simeq 10^{-5}$. If they have overestimated the variation as some people suggest today, then $\xi$ would be smaller. If there is no cosmological variation of $\alpha$ then $\xi = 0$ (so that $\beta = 0.5$ and $\gamma = 1.5$).

The results of the main experiments concerning special relativity and the equivalence principle have been considered. The conclusions can be summarized as follows.

(i) In the Hughes-Drever experiments, the predicted consequences of the model on the anisotropy of the mass are at least ten orders of magnitude smaller than the experimental bound. The model fully agrees, therefore, with these bounds.

(ii) Since the electron charge would be differently renormalized at different places of the universe, its value would depend on space and time. As a consequence, the electromagnetic mass of a body (due to the Coulomb interaction in their nuclei) would vary when it moves. A new reaction force equal to $-\dot{m} v$ would act on it, that would depend on its chemical composition since different nuclei have different proportions of Coulomb mass. The consequence is that the weak equivalence principle would be violated, since different bodies with the same initial conditions would follow different trajectories in a gravitational field. In principle, this could be observed in free fall or in Eötvös experiments. However, the variation of $\Phi$ being so small, this effect would completely unobservable here at Earth. Indeed, it would
be several orders of magnitude too small, about six orders in the case of the best Eötvös type experiments by Roll, Krotkov and Dicke [34] or Braginsky and Panov [35].

iii) If $\xi = 0$ (i.e. if there is no cosmological variation of $\alpha$), the model predicts exactly the same result as general relativity for the gravitational redshift, while it gives exactly half the value for the bending angle of light passing near the Sun and for the Shapiro time delay. This means that it gives exactly the same result as the weak equivalence principle, which is just a component of general relativity.

iv) If $\xi \neq 0$, the model predicts a cosmological variation of $\alpha$ (remember that, if the observations by Webb et al [7] are correct, $\xi \simeq 10^{-5}$). The predictions concerning the light ray bending around the Sun and the Shapiro time delay would be the same as for $\xi = 0$ (i.e. again the same as predicted by the weak equivalence principle). There would be, however, a difference between the predictions of the model for the gravitational redshift and the observational results, its relative value being a few times $\xi$. The model agrees with the most stringent experimental bound by Vessot and Levine if $\xi \lesssim 3 \times 10^{-5}$, while to explain the observations by Webb et al [7], $\xi \simeq 1.3 \times 10^{-5}$, just borderline. However, there are reasons to believe that Webb et al overestimated the variation, since some researchers have make calculations indicating that either the effect is clearly smaller or it is nil (see [8] for instance). In that case, $\xi$ would be smaller or even zero, and the prediction of the model would be well below any experimental bound on the gravitational redshift.

v) Concerning the light acceleration $a_t \simeq 8.5 \times 10^{-10}$ m/s$^2$ predicted by this model, it must be emphasized that it is very small. During a year, the light speed would increase in about 2.7 cm/s, just about 1 part in $10^{10}$. In 1983, the light velocity was fixed at the measured value of the time, which has an error margin of about 1 m/s. Since that moment the change would have been about 56 cm/s. If such a variation can be detected in an experiment, the change in $c$ since 1983 could be measured. It was stated in section 5.1 that a complete analysis of this acceleration requires to consider a cosmological time $t_{\text{cosm}}$ which is accelerated with respect to the parametric time $t$, the acceleration being the Hubble constant.

vi) The fact that the light propagation depends on the metric indicates that a gravitational field has an effect on the Maxwell equations, a fact first recognized by Einstein in 1907. Since the effect is small in weak gravity, it seems clear that the problem can be treated by introducing a dependence on
the gravitational potential $\Phi$ of the permittivity and permeability of empty space, as done in [1, 2, 3]. It was shown in [6] and in section 4 that the existence of an acceleration of light $a_\ell$ can be deduced from a general relativistic argument based on Einstein formula (19). Furthermore, it was shown in section 2 that an estimation gives a value close to $a_P$. A way to characterize then the situation could be the following: the attenuating vacuum model here studied gives an alternative (and equivalent) description to the relation between gravity and electromagnetism, first recognized by Einstein in 1907, when proposing his equivalence principle. This implies that the Newtonian model proposed in [1, 2, 3] can be integrated in general relativity, provided that $\chi = 1$. If $\xi = 0$, it is certain that there would be no problem for that. In that case, the effect of gravitation on Maxwell equations could be interpreted to be due to its effect on the quantum vacuum, the optical density of which would depend on the potential $\Phi$ as this model proposes. It must be kept in mind that the quantum vacuum fixes the observed values of some important quantities by means of a process of renormalization.

vii) If $\xi$ does not vanish but is small enough, say $|\xi| \leq 10^{-5}$, the integration of the model in general relativity would still be possible. The effect of the attenuation of the quantum vacuum could be observed as a cosmological evolution of $\alpha$, probably smaller than what was reported (with the advantage that it could be compatible with the bound obtained from the Oklo reactor). Indeed, other experiments are needed to determine the right value of the variation of $\alpha$ over cosmological intervals of time, or even if it varies at all.

viii) The fact that the model can be integrated in general relativity, just giving another interpretation of Einstein formula (19) fully compatible with the rest, has an important consequence. If $\xi = 0$ the trajectories of light would be the same as in general relativity, for low curvature at least. This would imply that the Global Positioning System would work just as predicted, so that the variation of the light speed could not be detected from its results. If $\xi < 10^{-5}$ the effect would be still too small to affect the positions given by it. The same would apply to other experiments on light.

To summarize, the attenuating quantum vacuum model must be further studied since, being in agreement with the experimental data, it could give a good insight of some important phenomena and could have interesting cosmological implications. The adiabatic acceleration of light is an example. In any case, the explanation of the Pioneer anomalous acceleration proposed by this model must be studied by the experts, in particular by those who
have the data of the trajectories of the ships.

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