Choice of mathematical models for technological process of glass rod drawing

L B Alekseeva

St. Petersburg Mining University, 21 line, 2, St. Petersburg, 199106, Russia

E-mail: lbalek@rambler.ru

Abstract. The technological process of drawing glass rods (light guides) is considered. Automated control of the drawing process is reduced to the process of making decisions to ensure a given quality. The drawing process is considered as a control object, including the drawing device (control device) and the optical fiber forming zone (control object). To study the processes occurring in the formation zone, mathematical models are proposed, based on the continuum mechanics basics. To assess the influence of disturbances, a transfer function is obtained from the basis of the wave equation. Obtaining the regression equation also adequately describes the drawing process.

1. Introduction

The structure of any fiber-optic element includes single-core and multi-core rods (light guides) with certain geometric and optical characteristics. From the very beginning of the development of fiber optics, the dominant problems of optics were the stability of the original rods diameter and the small light-attenuation in them. If the second problem is successfully solved, then the problem of stability of the fiber diameter continues to exist. To solve it, it is very necessary to maintain constant control and management of the technological process for manufacturing light guides. The complexity of tasks arising in the management consists in the fact that since the quality of the products obtained depends on a variety of factors, accounting for which is a task with a high level of uncertainty [1].

Each type of a fiber component (single-core, multi-core rigid light guides, flexible fibers, bundles for transmitting light or for image transmission, phocons, etc.) requires its own specific production technology. In all technological processes, the operation of manufacturing single-core rods (light guides) is invariably present. This can be hollow tubes or thin bars with a cross-sectional diameter of 0.5 ... 2 mm. When they are manufactured, a method of drawing is used from the Spinneret or the method of Pulling from the workpiece, and their ends are softened by heat.

Fiber parts with high resolution can only be obtained by using light guides with the same geometry. The stability of the fiber geometry depends on many factors. The kinematic and dynamic parameters of the drawing devices are of great importance: uniformity of feeding and drawing speeds; vibratory activity level for moving parts of the installation; inertia; the inertia of the transients caused by random disturbances and the need to adjust the parameters of the process.
2. Automated control of the drawing process
Automated control of the drawing process is reduced to the decision-making process by ensuring products of a given quality. The drawing process is considered as a control object, including a drawing device (control device) and an optical fiber forming zone (control object) [2]. The dynamics of the formation of fiber from the melt is considered as a transient process. The reaction of the molten mass in the Annex to the formative force is seen as a function of time, passed on from the moment when molten mass exits the spinneret hole to the time of the formation of the light guide.

The process of formation of the optical fiber geometry takes place in the formation zone, which represents a transition from a preheated glass mass to an optical fiber. For research processes occurring in this zone, mathematical models are proposed, built on the basis of continuum mechanics and including three groups of equations: equations of equilibrium of forces, the equation of continuity and equations that determine the physical state of the environment [3].

3. The formation zone
The formation zone is a body of revolution. Therefore, a cylindrical coordinate system \( rz \) is used, where \( z \) is the axial coordinate and \( r \) - radius of the circle in the section with the coordinate \( z \). Two coordinates are sufficient (since one-dimensional motion is considered), for which the authors have a system of three equations.

\[
\begin{align*}
\frac{\partial \nu}{\partial t} + \frac{\partial (\nu S)}{\partial z} &= 0; \\
\rho \left[ \frac{\partial (\nu S)}{\partial t} + \frac{\partial (\nu^2 S)}{\partial z} \right] &= \rho S g + \frac{\partial (p S)}{\partial z}; \\
p + \lambda \left( \frac{\partial p}{\partial t} + \nu \frac{\partial p}{\partial z} \right) &= 3\mu \frac{\partial \nu}{\partial z},
\end{align*}
\]

where \( t \) is time; \( \nu, p \) - speed and voltage, respectively, in the considered section of the zone in the section of the formation zone under consideration; \( S \) - area of the section under consideration; \( \rho \) - density of the glass mass, which is assumed to be constant; \( \mu \) - dynamic viscosity of the glass mass; \( \lambda = \mu / G \) - relaxation time; \( G \) - modulus of elasticity of the glass mass.

The results obtained make it possible to draw qualitative conclusions. For example, in order to obtain a more accurate geometry of light guides, it is necessary that the glass mass in the formation zone possess the properties of a viscoelastic body. This state can be achieved by lowering the temperature in the formation zone. This predetermines the choice of the method of manufacturing light guides- from the workpiece. When using the spinneret method, it is necessary to create special conditions for the passage of viscous glass mass through the spinnerets.

Static properties are determined by the sensitivity of the process to various kinds of perturbations, including technological parameters. To develop a methodology for estimating this kind of sensitivity, it is necessary to investigate the steady motion of the glass mass, in which the configuration of the formation zone is stable.

In the solution of the problem, real ranges of the parameters of the process under investigation (drawing speed \( \nu = 1 \text{m/ min}...10 \text{m/s} \); viscosity of the glass mass \( \mu = 10^6...10^9 \text{Pa} \cdot \text{s} \); optical fiber radius \( r_c = 0.5...2 \text{mm} \)) are identified.

For the selected parameter ranges, the first equation of the system (1) has the dominant effect on the configuration of the formation zone. For example, the value of the inertial component even at the drawing
speed $1 \text{ m s}^{-1}$ hardly reaches 0.1% of the viscosity term. There is also minimal influence of the second term on the configuration of the formation zone.

As expected, the configuration depends mainly on the viscosity of the glass mass. This allows us to use the simplest solution for the stationary configuration of the optical fiber formation zone when deriving the perturbation equations [4, 5].

The obtained equations allow us to normalize the technological parameters (drawing speed, viscosity, pulling force), depending on the requirements for the cross-sectional dimension accuracy of the fiber. Rationing is based on sensitivity assessment of the formation zone to a change in viscosity.

4. Technological system of automated production

The automated production process system consists of a control object and a control device that includes sensing elements, computing units and actuators. The latest directly change the state of the control object in accordance with the control signal. The quality of management is largely determined by the design features of the executive devices. Moreover, these devices can themselves generate uncontrolled disturbances. This must be taken into account when selecting the circuits of actuators.

It is established that the oscillations in the diameter of the optical fiber are characterized by the imposition of a high-frequency band on the low-frequency background. High-frequency oscillations are periodic in nature and are caused (as a rule) by the presence of certain manufacturing errors in the drawing units. In this case, preventive measures should be taken to identify and eliminate these errors.

This can be achieved by means of a harmonic analysis of the oscillations in the diameter of the fiber. Essential role of installation vibrations on the drawing devices during the formation of disturbances. Vibrations of the nodes directly in contact with the pulling fiber can cause oscillations in the drawing speed. But, in addition, vibrations, transmitted through the pulled fiber directly to the formation zone, can influence the process of forming the fiber geometry and, consequently, its quality.

The transmission intensity of such disturbances is determined by the transfer function using the famous wave equation (2)

$$\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial z^2} = 0,$$

$$a = \sqrt{\frac{F}{\rho}}$$

- wave propagation velocity along the filament, $F$ - pulling force; $y(z, t)$ - amplitude of the wave-guide oscillations in the cross section with the $z$ coordinate.

The transfer function is determined by the ratio $w = y(0, t)/y(l, t)$, where $y(0, t)$ - the disturbance transmitted to the formation zone; $y(l, t)$ - kinematic perturbation associated with the operation of the drawing actuator.

For the selected ranges of technological parameters, the value of $w$ does not exceed 1 (one). In this case, it is possible to determine the requirements for admissible vibrations of the grippers $|A| \leq \Delta d$, where $\Delta d$ - the permissible diameter deviation of the fiber section.

5. Regression models

The considered methods presuppose decision-making procedures on the basis of analytical models. They allow developing automated systems of researches and increasing the completeness as well as efficiency of information support of the technological process. Formable control actions can be used not only for object management, but also for its study on the basis of regression models [6].

During the experiments, two factors were identified, i.e. the drawing speed and temperature in the heating zone.
In the overwhelming majority of cases, even very complex systems can be satisfactorily described by a polynomial of no higher second order. In the studied process, the number of varied factors is two. Therefore, the regression equation at the first stage is obtained in the form (3):

\[ y = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2, \]  

where \( y \) - response function; \( x_1, x_2 \) - normalized values of control factors; \( b_0 \) - free term equal to the output at \( x_1 = 0 ; b_1, b_2 \) - coefficients indicating the influence of the factors \( x_1, x_2 \) on the process; \( b_{12} \) - a coefficient indicating the interaction of factors \( x_1, x_2 \); \( x_0 = 0 \) - alternating function.

The regression equation that adequately describes the process under investigation is obtained. The conclusions obtained with the help of analytical models, the significant influence of the interrelation of technological parameters on the geometric quality of optical fibers, are confirmed. On the basis of the regression model, a technique is proposed for the experimental determination of tolerances for deviations of technological parameters, intended for use at the stage of pilot-industrial testing of the technological process under study.

The process of controlled motion of the glass mass in the formation zone is considered. This takes into account the properties of the control object and the control device, to which the drive and the drawing mechanism are assigned [7].

The behavior of the formation zone is determined by the change in its length \( z(t) \), which with the chosen technological parameters corresponds to the required geometry of the drawn product. The differential equation (5) is used in order to determine the formation zone reaction per unit pulse.

\[ m \ddot{z} + \beta \dot{z} + c z = f(t), \]  

where \( m \) - amount of mass in the formation zone; \( \beta, c \) - coefficients of damping and elasticity of the glass mass; \( f(t) = 1(t) \) when \( t \geq 0 \) - unit impulse.

An analysis of the results shows that the parameters of the control device have a major influence on the nature of the transient regimes, and consequently, the choice of control methods. Parameters of the formation zone have a significant influence on the transition time, which is one of the direct indicators of quality management.

6. Conclusion

Mathematical models of the control device and the control object are proposed. The need for an integrated approach to the study of the technological process of drawing optical fibers was identified, in which this process would be viewed like an object of control, and was part of a complete automated system that
allows us to identify the properties and relationships of its elements and give recommendations on the regulation and rationing of the main parameters of the process.

References

[1] Pupkov K A 2007 Time-varying systems of automated control: Analysis, synthesis and optimization. (Moscow, Publishing of MGTU im.N.E. Baumana) p 632
[2] Gayduk A K 2010 Theory of automated control (Moscow, Vysshya shkola) p 415
[3] Alekseeva L B, Uvarov V P 2012 Proceedings of the 2nd Scientific-Practical Conference on Modern Machine Engineering. Science and Education» (Saint-Petersburg, Izd-vo Politekh. un-ta) 108-125
[4] Uvarov V.P. 2003 Standardization of process parameters drawing rods using experimental models. Glass and Ceramics. 11 8-9
[5] Alekseeva L B, Maksarov V V 2008 Izvestiya vuzov. Mashinostroenie. 8 19-24
[6] Myshkis A D 2007 Details of theory of mathematical models (Moscow, Komkniga) p 192
[7] Shtelle M, Bryukner R 1980 Predel'nye parametry protsessa vytyagivaniya steklovolokna. Clastechnische Berichte. 5 130-139
[8] Morton M D 1980 Protsess nepreryvnoy vytyazhki vyazkikh zhidkostey pri pryadenii volokna. Ann. Rev. Fluid Mech. V 12 365-387