PN Modified Differential Game Strategy for Two-on-one Cooperative Interception

Jiaqiang Zhang*, Yuanwei Lou, Xiaolong Liang and Duo Qi

National Key Laboratory of Air Traffic Collision Prevention, Air Traffic Control and Navigation College, Air Force Engineering University, Xi’an, China

*jiaqiang-z@163.com

Abstract. In order to deal with the Chaff, Infrared Decoy, Electronic Jamming and the escaping maneuvering of the target, two-on-one cooperative interception strategy is discussed in this paper. The planar engagement of two air-to-air missiles cooperative intercepting one target by the alliance of differential game (DG) strategy and proportional navigation (PN) guidance law is investigated. The equations of motion for two-on-one cooperative intercepting are derived first, and zero-effort-miss (ZEM) representation technique is employed. Then a linear quadratic differential game cooperative zero-sum cost function is phrased, and the cooperative control variables of two-on-one cooperative guidance law under pure DG strategy are derive. Finally, the proportional navigation guidance law is introduced to alleviate the acceleration command oscillation. The simulation results in the ideal dynamic scenarios show that, the combination of DG and PN guidance law is effective to the two-on-one cooperative pursuit-evasion problem, especially for the high maneuvering target at the cost of much more control effort.

1. Introduction

For a missile-target pursuit-evasion (PE) problem, no one knows exactly what maneuvering approaches the target will take in the future, so differential game (DG) based guidance laws are developed and exploited, which have no assumptions on the target’s maneuvering or trajectory, and are insensitive to a specific evasion strategy. As the Chaff, Infrared Decoy, Electronic Jamming and the escaping maneuvering of the target can lead the failure of one missile’s attack on one target, two-on-one cooperative interception seems to be a feasible and affordable strategy. Considering the worst evasive maneuver of the target is different for each missile, the target cannot best evade both missiles simultaneously; therein raise the strength of cooperation. However merely multiplying the quantity of the missiles may not satisfy the goal, a well-designed cooperative guidance law can exploit the advantages to the full.

To answer the question that when two cooperative missiles preferred over a single missile and what are are the preferred cooperation strategies between two missiles, many researchers have been carried out by predecessors [1-6]. Most studies are formulated with constraints on terminal impact angle and impact time, the approaches include the traditional guidance laws, such as proportional navigation in [7-8], and modern missile guidance law, such as the optimal guidance law in [9-10], differential game guidance law in [11-15] and model predictive control in [16].
Inspired by the predecessors’ work, this paper attempts to combine the DG and PN guidance law to suggest a solution to the two-on-one pursuit-evasion problem, under the assumptions of planar kinematic linearization and full information. The introduced method can take advantage of DG in dealing with high maneuvering target and the advantage of PN to overcome the acceleration oscillation generated from DG guidance law.

The remainder of this paper is organized as follows: the equations of motion for two-on-one cooperative intercepting are derived first, and zero-effort-miss (ZEM) representation technique is employed in Sec. II. Then a linear quadratic differential game cooperative zero-sum cost function is phrased, the cooperative control variables of two-on-one cooperative guidance law under pure DG strategy are derived, and finally a PN based two-on-one DG strategy is developed in Sec. III. A performance analysis of the proposed algorithm is presented in Sec. IV. followed by concluding remarks.

2. Problem formulation

To simplify the equations of motion for the missile-target engagement, we assume the following conditions to facilitate the capturability analysis of the problem:

- The missiles and target are considered as point masses moving in the planar plane;
- Both the missiles and target have ideal dynamics, and the seeker and autopilot dynamics of the missiles are fast enough in comparison with the guidance loop;
- The state vector of the missiles and target are common knowledge.

The two dimensional planar pursuit-evasion engagement geometry between a single missile and a single target is given in Fig. 1. The $X$ axis is along the initial line of sight (LOS) between the missile and target, the $Y$ axis is perpendicular to it, and The $X$-$Y$ plane is fixed with regard to the initial engagement geometry. The speed, normal acceleration, and flight-path angles are denoted by $V$, $a$, and $\gamma$, respectively; variables associated with the missile and target are denoted by additional subscripts $M$ and $T$ respectively. The range between missile and target is denoted as $r$. The angle between missile-to-target LOS and the Cartesian inertial reference frame $X$ axis is $\gamma_{LOS}$. And $x$ is denoted as the relative displacement projected over $Y$ axis and $y$ is the relative displacement between missile and target, relative to $X$ axis, thus:

\[
\begin{align*}
  x(t) &= x_T(t) - x_M(t) \\
  y(t) &= y_T(t) - y_M(t)
\end{align*}
\]
The zero lag (ZL) dynamic model is the simplest pursuit-evasion game model, describing ideal vehicles. Within this model, the lateral accelerations equal to the corresponding lateral acceleration commands. Using the linearization approximation, the state vector of the missile-target couplet can be written as follows:

\[
X = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}
\]  

(4)

When representing these equations in the form of velocity lateral acceleration’s command, a vectorial equation of motion of the pursuit-evasion problem can be formulated as follows:

\[
\dot{X} = AX - Bu_m + Cu_r
\]  

(5)

where, \( A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ \beta_m \end{bmatrix} \), \( C = \begin{bmatrix} 0 \\ c_r \end{bmatrix} \), and \( \beta_m = \cos(\gamma_m(t) - \gamma_{LOS}(t)) \), \( c_r = \cos(\gamma_r(t) - \gamma_{LOS}(t)) \).

When linear approximation of missile and target relative motion can be performed around the ideal collision course, the during time for missile to catch the target can be evaluated with respect to the initial distance between the missile and target and their closing speed:

\[
t_f = -x(t) / \dot{x}(t) |_{t_f}
\]  

(6)

The zero-effort-miss (ZEM) variable represents the anticipated miss distance assuming both the missile and target do not apply any controls, from the current time onwards. ZEM can enable the order reduction of linear-form equations of motion. For the missile-target pursuit-evasion problem mentioned before, ZEM can be formulated as follows:

\[
Z_{ZEM}(t) = [1 \ 0] \Phi(t_f - t) X(t), \ 0 \leq t \leq t_f
\]  

(7)

where \( \Phi(t_f - t) = \begin{bmatrix} 1 & t_f - t \\ 0 & 1 \end{bmatrix} \) is the transition matrices of (5), then this equation can be written as

\[
Z_{ZEM}(t) = [1 \ 0] \begin{bmatrix} 1 & t_f - t \\ 0 & 1 \end{bmatrix} X(t) = [1 \ t_f - t] X(t), \ 0 \leq t \leq t_f
\]  

(8)
3. Two-on-one cooperative guidance law

The two dimensional planar engagement geometry of two missiles cooperative interception one target is given in Fig. 2. Within this paper, we assume that both missiles launched simultaneously. Considering that the termination conditions of the missiles are not necessarily restricted to the same timing, the termination time and time-biasing between the scheduled captures are fixed by $t_f = \max(T_1, T_2)$ and $\Delta T = \text{abs}(T_2 - T_1)$ respectively. After the 1st missile ‘impacts’ the target, the target will go on flying until the 2nd missile hits it. Considering the goal of the cooperating missiles is to minimize a linear combination of the miss distances between the target and each of the missiles, while the target is to maximize it. The cost function of the two-on-one cooperative interception problem may be phrased as follows:

$$J_{2on} = \frac{\rho_1}{2} \left[ Z_{ZEM1}(t_f) \right]^2 + \frac{\rho_2}{2} \left[ Z_{ZEM2}(t_f) \right]^2 + \frac{1}{2} \int_0^{t_f} \left[ \kappa_1 u_{t1}^2(t) + \kappa_2 u_{t2}^2(t) - \kappa_3 u_t^2(t) \right] dt$$

(10)

![Figure 2. Planar engagement geometry of two-on-one cooperative interceptions.](image)

Within (10), it is generally assumed $\rho_1 = \rho_2$, $\kappa_1 = \kappa_2$. When applying large positive values to parameters ($\rho_1$, $\rho_2$), small misses will result. However, when these values are too high, the resulting miss will be at the price of increasing the control efforts.
Without restricting the generality, it may be assumed that \( t_j = T_2 \), thus 2\(^{th}\) missile terminates last. When the 1\(^{th}\) missile has already reached its capture terms, the PE scenario degenerates to a classic one-on-one PE scenario, and the miss distance of the 1\(^{th}\) missile doesn’t change anymore. Then (10) can be transform as follows:

\[
J_{\text{final}} = \frac{\rho_2}{2} \left[ Z_{ZEM2} (t_f) \right]^2 + \frac{1}{2} \int_{t_f - \Delta T}^{t_f} \left[ \kappa_i u_{M1}^2 (t) - \kappa_i u_T^2 (t) \right] dt
\]  

(11)

The Hamiltonian of the problem could be phrased as follows:

\[
H_{2\text{on}} (t) = \lambda_1 \dot{Z}_{ZEM1} (t) + \lambda_2 \dot{Z}_{ZEM2} (t) + \frac{\kappa_1}{2} u_{M1}^2 (t) + \frac{\kappa_2}{2} u_{M2}^2 (t) - \frac{\kappa_T}{2} u_T^2 (t), \quad 0 \leq t \leq t_f - \Delta T
\]  

(12)

Substituting (9) into (12), yields

\[
H_{2\text{on}} (t) = (t_f - \Delta T - t) \left[ \frac{\lambda_1}{2} (-\beta_{M1} u_{M1} + c_1 a_f) + \frac{\lambda_2}{2} (-\beta_{M2} u_{M2} + c_2 a_f) \right] + \frac{\kappa_1}{2} u_{M1}^2 (t) + \frac{\kappa_2}{2} u_{M2}^2 (t) - \frac{\kappa_T}{2} u_T^2 (t)
\]  

\[
= \frac{\kappa_1}{2} \left[ u_{M1}^2 (t_f - \Delta T - t) \right] + \frac{\kappa_2}{2} \left[ u_{M2}^2 (t_f - \Delta T - t) \right]
\]  

\[
= \frac{\kappa_T}{2} \left[ u_T (t_f - \Delta T - t) \right]^2 + \mathcal{E} (t)
\]  

(13)

where \( \mathcal{E} (t) \) is only associated with \( t \).

Therefore, the optimal controllers for the missiles satisfy

\[
\frac{\partial}{\partial a} H_{2\text{on}} (t) = 0
\]  

(14)

Thus

\[
u_{M1}^* (t) = \text{sgn} \left( \frac{\lambda_1 \beta_{M1}}{\kappa_1} (t_f - \Delta T - t) \right) \times \min \left\{ \frac{\lambda_1 \beta_{M1}}{\kappa_1} (t_f - \Delta T - t), a_{\text{max}} \right\}
\]  

(15)

\[
u_{M2}^* (t) = \text{sgn} \left( \frac{\lambda_2 \beta_{M2}}{\kappa_2} (t_f - \Delta T - t) \right) \times \min \left\{ \frac{\lambda_2 \beta_{M2}}{\kappa_2} (t_f - \Delta T - t), a_{\text{max}} \right\}
\]  

(16)
\[ u^*_1(t) = \text{sgn}\left( \frac{\lambda_2 c_{f1} + \lambda_2 c_{f2}}{\kappa_T} (t_f - \Delta T - t) \right) \times \min \left\{ \left| \frac{\lambda_2 c_{f1} + \lambda_2 c_{f2}}{\kappa_T} (t_f - \Delta T - t) \right|, a_{M\max} \right\} \]  

(17)

Considering the general condition, \( \beta_{M1}, \beta_{M2}, \kappa_1, \kappa_2 > 0 \), then

\[ u_{M1}^*(t) = \text{sgn}(\lambda_1) \min \left\{ \frac{\lambda_1 \beta_{M1}}{\kappa_1} (t_f - \Delta T - t), a_{M\max} \right\} \]  

(18)

\[ u_{M2}^*(t) = \text{sgn}(\lambda_2) \min \left\{ \frac{\lambda_2 \beta_{M2}}{\kappa_2} (t_f - \Delta T - t), a_{M\max} \right\} \]  

(19)

\[ u^*_T(t) = \text{sgn}\left( \frac{\lambda_2 c_{f1} + \lambda_2 c_{f2}}{\kappa_T} \right) \min \left\{ \left| \frac{\lambda_2 c_{f1} + \lambda_2 c_{f2}}{\kappa_T} (t_f - \Delta T - t) \right|, a_{T\max} \right\} \]  

(20)

Hence, the Hamilton’s canonical equations is written as:

\[ \dot{\lambda}_z = -\frac{\partial H}{\partial Z_{\text{ZEM}}} = \begin{bmatrix} \frac{\partial H_{2\text{z1}}}{\partial Z_{\text{ZEM}1}} \\ \frac{\partial H_{2\text{z1}}}{\partial Z_{\text{ZEM}2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]  

(21)

The co-state variable vector, which is denoted by \( \lambda_z(t_f - \Delta T) \), is defined as follows:

\[ \lambda_z(t_f - \Delta T) = \begin{bmatrix} \frac{\partial J_{2\text{z1}}}{\partial Z_{\text{ZEM}1}(t_f - \Delta T)} \\ \frac{\partial J_{2\text{z1}}}{\partial Z_{\text{ZEM}2}(t_f - \Delta T)} \end{bmatrix} = \begin{bmatrix} \rho_z Z_{\text{ZEM}1}(t_f - \Delta T) \\ \rho_z Z_{\text{ZEM}2}(t_f - \Delta T) \end{bmatrix} \]  

(22)

The solution to (21) and (22) is:

\[ \lambda_z(t) = \begin{bmatrix} \lambda_1(t_f - \Delta T) \\ \lambda_2(t_f - \Delta T) \end{bmatrix} = \begin{bmatrix} \rho_z Z_{\text{ZEM}1}(t_f - \Delta T) \\ \rho_z Z_{\text{ZEM}2}(t_f - \Delta T) \end{bmatrix}, 0 \leq t \leq t_f - \Delta T \]  

(23)

Substituting (23) into (18) ~ (20), and we assume \( \rho_1 = \rho_2 \), and \( \rho_1, \rho_2 \to \infty \) to minimize the miss distance between the missiles and target, yields

\[ u_{M1}^*(t) = a_{M\max} \text{sgn}\left[ Z_{\text{ZEM}1}(t_f - \Delta T) \right] \]  

(24)
Substituting (24) ~ (26) into (9), integrating the equation form \( t=0 \), and assuming \( Z_{EM1}(0) = 0 \), obtains

\[
Z_{EM1}(t) = t \left( t_f - \frac{1}{2} t \right) \left[ -\beta_{m1} a_{m_{\max}} \text{sgn} \left[ Z_{EM1} \left( t_f - \Delta T \right) \right] \right] + \gamma_{T1} a_{T_{\max}} \text{sgn} \left[ c_{T1} Z_{EM1} \left( t_f - \Delta T \right) \right], \quad 0 \leq t \leq t_f - \Delta T
\]  
(27)

From (27), we have

\[
\text{sgn} \left[ Z_{EM1}(t) \right] = \text{sgn} \left[ Z_{EM1} \left( t_f - \Delta T \right) \right], \quad 0 \leq t \leq t_f - \Delta T
\]  
(28)

Similarly

\[
\text{sgn} \left[ Z_{EM2}(t) \right] = \text{sgn} \left[ Z_{EM2} \left( t_f - \Delta T \right) \right], \quad 0 \leq t \leq t_f - \Delta T
\]  
(29)

\[
\text{sgn} \left[ Z_{EM2} \left( t_f - \Delta T \right) \right] = \text{sgn} \left[ Z_{EM2} \left( t_f \right) \right], \quad t_f - \Delta T \leq t \leq t_f
\]  
(30)

Substituting (28) ~ (30) into (24) ~ (26), obtains

\[
u_{m1}^* \left( t \right) = a_{m_{\max}} \text{sgn} \left[ Z_{EM1} \left( t \right) \right], \quad 0 \leq t \leq t_f - \Delta T
\]  
(31)

\[
u_{m2}^* \left( t \right) = a_{m_{\max}} \text{sgn} \left[ Z_{EM2} \left( t \right) \right], \quad 0 \leq t \leq t_f
\]  
(32)

\[
u_t^* \left( t \right) = \begin{cases} 
\text{sgn} \left[ c_{T1} Z_{EM1} \left( t \right) + c_{T2} Z_{EM2} \left( t \right) \right], & 0 \leq t \leq t_f - \Delta T \\
0, & t_f - \Delta T \leq t \leq t_f
\end{cases}
\]  
(33)

Equation (31) ~ (33) show that the optimized pursuit-evasion strategy for both the missiles and target are ‘bang-bang’ maneuvering. However, if the maximum acceleration of the missiles and target are engaged, the sign of the zero effort miss (ZEM) will switch rapidly, that means the control command is non-executable and useless. Therefore, a linear prediction method is used in this paper to improve the control performance. At current time, if the ZEM sign remains unchanged under the control of (31) ~ (33) in a certain time \( t_p \) in the future, the current control command will be executed. Otherwise, the proportional guidance law will be implemented instead in this time step:

\[
u_{pg} \left( t \right) = k v u \dot{q} \left( t \right)
\]  
(34)

where \( k \) is the navigation gain, and \( \dot{q} \left( t \right) \) is the LOS angular velocity.

To optimize the control performance, the predictive period \( t_p \) is associated with the time constant of the missile dynamics \( \tau_M \):
Hence, the control variable can be written as follows:

\[
u^*_M(t) = \begin{cases} 
  a_{\text{max}} \text{sgn}[Z_{\text{ZEM}}(t)] & \text{if } \text{sgn}[Z_{\text{ZEM}}(t)] = \text{sgn}[Z_{\text{ZEM}}(t + t_f)], \\
  k \nu_0 \dot{q}_i(t) & \text{else} 
\end{cases}, \quad 0 \leq t \leq t_f - \Delta T
\]  

(36)

\[
u^*_M(t) = \begin{cases} 
  a_{\text{max}} \text{sgn}[Z_{\text{ZEM2}}(t)] & \text{if } \text{sgn}[Z_{\text{ZEM2}}(t)] = \text{sgn}[Z_{\text{ZEM2}}(t + t_f)], \\
  k \nu_0 \dot{q}_i(t) & \text{else} 
\end{cases}, \quad 0 \leq t \leq t_f
\]  

(37)

\[
u^*_T(t) = \begin{cases} 
  a_{\text{max}} \text{sgn}[Z_{\text{ZEM1,2}}(t)] & \text{if } \text{sgn}[Z_{\text{ZEM1,2}}(t)] = \text{sgn}[Z_{\text{ZEM1,2}}(t + t_f)], \\
  u^*_t(t - \delta t) & \text{else} 
\end{cases} \quad 0 \leq t \leq t_f - \Delta T
\] 

(38)

where \(Z_{\text{ZEM1,2}}(t) = c_1 t Z_{\text{ZEM}}(t) + c_2 t Z_{\text{ZEM2}}(t)\), \(\delta t\) is the time interval of the simulations.

4. Numerical simulation

To analyze the performance of the proposed cooperative guidance algorithms, the test-case scenario is constructed as follows: The target performs the differential game maneuver in (38), its normal acceleration is bounded to 7g, while the missile’s acceleration is bounded to 20g. The simulation inputs are presented in Table 1, the time integration interval is 0.01s, the time constant of the missile dynamics \(\tau_M = 0.5s\), and \(N\) in (35) is set to 2. The initial aiming error \(\xi(t_0)\) between the initial missile heading angle and the ideal collision angle is -\(\pi/12\) and \(\pi/12\) respectively, while the target’s initial heading angle is 0. The terminating condition is the occurrence of the closest approach point between the missile and target, which is corresponding to the miss distance.

|missle | \(x(t_0)\) (m) | \(y(t_0)\) (m) | \(V\) (m/s) | \(\xi(t_0)\) | \(a_{\text{max}}\) (g) | \(k\) |
|-------|----------------|----------------|-------------|-------------|----------------|-----|
|Missile 1 | 0 | 0 | 600 | -\(\pi/12\) | 20 | 3.5 |
|Missile 2 | 5000 | 0 | 600 | \(\pi/12\) | 20 | 3.5 |
|Target | 10000 | 50000 | 300 | | 7 | |

Within the test-case scenarios, the performance of the cooperative guidance law denoted by (38) is compared with one-on-one guidance strategy defined by the idea proportional navigation (IPN) and its navigation gain is set to 3.5. Fig. 3 – Fig. 6 present the trajectories and acceleration curves respectively.
Figure 3. Two-on-one cooperative interception trajectories.

Figure 4. Two-on-one cooperative interception acceleration curves.

Figure 5. Two missiles one-on-one interception trajectories.
It can be seen from the figures that under the control of differential game strategy, the target maneuvers at its maximum normal acceleration all along and the acceleration direction changes only after it’s being captured by 2th missile. The target generates different trajectories when missiles use different engagement strategy, and the IPN guidance law without any cooperation requires considerably less acceleration from the two-on-one cooperative guidance law based on DG guidance law proposed in this paper. But the acceleration required by IPN at the end of the engagement increases significantly, which may lead the increasing of the miss distance. Moreover, Fig. 6 presents an interesting result that, the ‘bang-bang’ maneuvering is an effective strategy to escape from the pursuit guided by proportional navigation.

5. Conclusion
In this paper, the planar engagement of two air-to-air missiles cooperative intercepting one target by the alliance of differential game strategy and proportional navigation guidance law is investigated. The cooperative control variables under pure DG strategy are derivate first, and proportional guidance law is introduced to alleviate the acceleration command oscillation. Simulation results prove the advantage of the cooperative intercepting strategy with respect to the one-on-one pursuit strategy when dealing with the high maneuvering targets. The future work will focus on the nonlinear and time-varying modeling and analyzing of the cooperative pursuit-evasion problem, and proving the algorithm stability and robustness.

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