Guidance Law Design with Attack Angle Control and Maneuvering Avoidance

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Abstract. In order to realize the tactical task of anti-ship missile attacking target in the predetermined direction, and improve its penetration effect under the condition of enemy ship-to-air missile interception, a guidance law with attack angle control and maneuvering avoidance is designed. Using the terminal sliding mode theory, based on the relative motion model between missile and target, the guidance law defined a sliding surface, which includes miss distance requirement, terminal angle constraint and terminal maneuvering information. The guidance law can attack the target in the predetermined direction and realize its own trajectory maneuvering penetration. The simulation results verify the validity of the designed method.

Introduction

With the continuous development of modern science and technology, anti-ship missiles are under the guard of enemy satellites, early warning aircraft, long-range warning radar and other sensor early warning systems from launch, and are always facing the interception of enemy anti-missile weapons. Therefore, how to successfully penetrate under the interception of anti-missile missiles and strike the target accurately is an important problem to be solved. A large number of research documents show that [1,2], in order to break through the interception of anti-missile missiles, maneuvering in the course of missile guidance is a feasible technical way to miss the target of anti-missile missiles. At present, the related literature mainly focus on the trajectory transfer technology, terminal maneuvering penetration guidance law designing, penetration effect caused by maneuvering transfer, and miss distance analysis of ship-to-air missile caused by maneuvering transfer, and so on [3-5].

According to guidance theory, the guideline for designing missile guidance law without considering the maneuver of missile terminal is to make the line-of-sight angular rate reach zero as soon as possible in the course of missile attacking target. That is, the line-of-sight angle between the missile and the target does not change and attacks the target with a constant line-of-sight angle, which can realize the smooth trajectory of the missile [6-7]. Considering the terminal maneuver, the angular rate of the line of sight (LOS) of the missile changes with the change of its trajectory in the course of hitting the target. That is to say, the line-of-sight angular rate between missile and target is no longer a constant value in the course of tracking target. It converges to zero with a certain law, so as to ensure that the final target is hit. However, there is an additional uncertainty factor affecting the miss distance at the end of the missile, that is, the uncertainty caused by the variable line-of-sight angular rate.

Therefore, how to realize the organic combination of Maneuvering Penetration trajectory and self-guided trajectory, so that the anti-ship missile under the guidance of guidance law can not only hit the target accurately, but also achieve its own trajectory maneuverability, thus improving its penetration probability, is the problem to be solved in this paper.

Relative Motion Model of Missile and Target

The relative motion relationship between the anti-ship missile and the ship target is shown in Fig. 1. The reference coordinate system is the ground coordinate system (x, y, z). M, T, and MT
respectively represent missiles, targets and line of sight between missile and target (LOS), r is the distance between missiles and targets, \( q_\theta \) is the inclination angle of MT, \( q_\phi \) is the azimuth angle of MT, \( v_t \) is the velocity of target and \( \phi_t \) is the azimuth angle of \( v_t \). The inclination angle of \( v_t \) is zero, this is because the enemy ship always moves in the course plane, and there is no movement in the longitudinal plane. \( v_m \) is the velocity of the missile. The inclination and azimuth angles of \( v_m \) are \( \theta_m \) and \( \phi_m \), respectively. \( \theta_m \) and \( \phi_m \) are defined in the same way as \( q_\theta \) and \( q_\phi \).

![Figure 1. Relative motion of missile and target.](image)

References [8], the three-dimensional relative motion equation between anti-ship missile and ship target can be found in Eq.1 and Eq.2, and the detailed derivation process can be found in references.

\[
\begin{align*}
    r\ddot{q}_\theta &= -2r\dot{q}_\theta + g \cos \theta_m \cos(q_\theta - \theta_m) - u_t \cos(q_\theta - \theta_m) + v_t \sin(q_\theta \sin(q_\theta - \phi_t) - \omega_t \sin q_\theta) \\
    r\ddot{q}_\phi &= \dot{q}_\theta [r\dot{q}_\theta \sin q_\theta - \dot{r} \cos q_\theta + v_m \cos \theta_m \cos(q_\theta - \phi_m) \\
                 &\quad - v_t \cos(q_\theta - \phi_t)] + (g \cos \theta_m - u_t) \sin \theta_m \sin(q_\theta - \phi_m) + u_z \cos(q_\theta - \phi_m) - \omega_2
\end{align*}
\]

(1)

(2)

Among them, \( \omega_1 \) and \( \omega_2 \) are functions related to target maneuvering acceleration, which can be regarded as bounded interference.

**Guidance Law Design**

For the terminal guidance problem of anti-ship missile with attack angle constraint, the guidance target is to obtain both zero miss distance and desired terminal angle. The ultimate miss distance is zero by making the LOS angular rate \( \dot{q}_\theta \) and \( \dot{q}_\phi \) zero. By making the error \( (\theta_m - \theta_d) \) and \( (\phi_m - \phi_d) \) of the terminal attack angle zero, the expected terminal angle can be guaranteed, which \( \theta_d \) and \( \phi_d \) are the expected longitudinal and heading angles, respectively.

Define sliding surface:

\[
    s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} r\dot{q}_\theta + c_1 v_m (\theta_m - \theta_d) \\ r\dot{q}_\phi + c_2 v_m (\phi_m - \phi_d) \end{pmatrix}
\]

(3)

As can be seen from the definition of sliding surface, it contains miss distance and terminal angle constraint information. In order to enable the missile to maneuver its flight trajectory in the course of automatic guidance, Eq.3 is redefined to add information of terminal maneuver, as shown in Eq.4.

\[
    S_1 = s_1 e^{-\mu t} \sin(w_u t) = [r\dot{q}_\theta + c_1 v_m (\theta_m - \theta_d)] e^{-\mu t} \sin(w_u t)
\]

(4)
In the formula, $c_1, \mu > 0$, $\theta$ is the expected inclination angle of the guidance terminal missile in the longitudinal plane, and $\dot{\theta}_o = 0$, $\mu_o$ is the parameter to control the convergence speed of the sliding mode surface, and the value affects the convergence speed of the exponential function. $\sin(w_o t)$ is a sinusoidal function for controlling the convergence trajectory of sliding surface, and $w_o$ is the frequency of this sinusoidal function. Eq.4 shows that the convergence rate of $S_1$ is affected by $e^{-\mu t}$ and its trajectory is affected by $\sin(w_o t)$ in the process of reaching 0. It can be seen from the definition that the convergence trajectory of $S_1$ is a attenuated sinusoidal trajectory. Once $S_1$ reaches zero under the control of the controller, it will remain above zero, then $q_o \rightarrow 0$, $\theta_m \rightarrow \theta_o$. That is, when $t \rightarrow \infty$, the final miss distance and landing angle constraints of the guidance system can be achieved.

It should be noted that the three-dimensional relative motion model of missile and target can be decoupled in both longitudinal and heading directions. Therefore, only the sliding mode variables related to the longitudinal plane are deduced. $S_2$ relating to the course plane will not be described in detail. Only the conclusion is given. Finally, the simulation is carried out in three-dimensional space. Eq. 4 derives the time and get

$$\dot{S}_1 = \dot{s}_1 e^{-\mu t} \sin w_o t + s_1 \frac{d(e^{-\mu t} \sin w_o t)}{dt}$$
$$\dot{S}_1 = \dot{s}_1 e^{-\mu t} \sin w_o t + s_1 (e^{-\mu t} \sin w_o t + w_o e^{-\mu t} \cos w_o t)$$

By combining formula Eq.3 and Eq.5, Eq.6 can be obtained.

$$\dot{S}_1 = \left[ -\dot{q}_o + g \cos \theta_o \cos (q_o - \theta_o) - w_o \cos (q_o - \theta_o) \right] e^{-\mu t} \sin w_o t + s_1 (-\mu e^{-\mu t} \sin w_o t + w_o e^{-\mu t} \cos w_o t)$$

Select Lyapunov function as equation Eq.7:

$$V_1 = \frac{1}{2} S_1^2$$

From the derivation of Eq.7, it can be concluded that:

$$\dot{V}_1 = S_1 \dot{S}_1$$

Set

$$\dot{V}_1 = -\mu V_1^a$$

It can be obtained that the time when $S_1$ converges to zero is:

$$t_r = \frac{(V_1(0))^{1-a}}{\mu (1-a)}$$

Substituting Eq.7 into Eq.9 yields:

$$\dot{S}_1 = -2^{1-a} \mu S_1^{2a-1} = -2^{1-a} \mu \left| S_1 \right|^{2a-1} \text{sign}(S_1)$$

Contact Eq.6 and Eq.11, Eq.12 can be obtained.

$$s_1 (-\mu e^{-\mu t} \sin w_o t + \left[ -\dot{q}_o + g \cos \theta_o \cos (q_o - \theta_o) - w_o \cos (q_o - \theta_o) \right] e^{-\mu t} \sin w_o t = -2^{1-a} \mu \left| S_1 \right|^{2a-1} \text{sign}(S_1)$$

According to Eq.12, the guidance law can be obtained by replacing the acceleration and velocity of the target with the upper bound, as follows
Similarly, the guidance law of the course plane can be derived

\[ u_2 = \frac{-2^{-2} \rho_2 \left[ \begin{array}{c} \frac{\sin(S_2)}{c_1 \cos q_0 - \cos(q_0 - \Phi_m)} \\ e^{\frac{\rho_2}{2}} \sin \frac{\Phi_m \cos q_0}{c_1 \cos q_0 - \cos(q_0 - \Phi_m)} \\ e^{\frac{\rho_2}{2}} \sin \frac{\Phi_m \cos q_0}{c_1 \cos q_0 - \cos(q_0 - \Phi_m)} \end{array} \right]}{e^{\frac{\rho_2}{2}} \sin \frac{\Phi_m \cos q_0}{c_1 \cos q_0 - \cos(q_0 - \Phi_m)}} ] \]

(13)

Similarly, the guidance law of the course plane can be derived

\[ u_2 = \frac{q_0 [q_0 \sin q_0 + v_m \cos q_0 - \sin(q_0 - \Phi_m)] + \left( g \cos \theta_q - u_q \right) \sin q_0 - \sin(q_0 - \Phi_m) - a_{sup}}{c_0 \cos q_0 - \cos(q_0 - \Phi_m)} \]

(14)

\[ S_2 \]

is defined as follows.

\[ S_2 = s_2 e^{\frac{\rho_2}{2}} \sin q_0 t = [q_0 \sin q_0 + c_2 \cos \left( \frac{\Phi_m \cos q_0}{c_1 \cos q_0 - \cos(q_0 - \Phi_m)} \right) e^{\frac{\rho_2}{2}} \sin q_0 t], \quad c_2 > 0 \]

(15)

Eq.13 and Eq.14 are terminal sliding mode guidance laws designed in this paper. This guidance law can control attack angle, realize maneuvering avoidance and compensate for target position maneuvering. The guidance law can attack the target in the predetermined direction and realize its own trajectory maneuvering penetration.

**Simulation**

The target ship is located at (5000m, 0, 0), travels forward at speed \( v_i = v_{i, sup} = 20m/s \), and makes snake maneuver to avoid missile attack. Suppose that the terminal guidance of the missile starts at (0,2000 m, -500 m), the velocity is 250 m/s, the initial trajectory inclination angle is - 20°, and the initial trajectory heading angle is - 20°. Expected longitudinal angles is \( \theta_d = -30^0 \), Expected heading angles is \( \phi_d = -30^0 \). The parameters of terminal sliding mode guidance law are selected as follows: \( \rho_1 = 10 \), \( \rho_2 = 400 \), \( c_1 = 0.5 \), \( c_2 = 0.95 \), \( \alpha = 0.1 \), \( \mu = 4.5 \), \( w_a = 2 \). After the switch function in the guidance law is replaced by the saturation function, its parameter is selected as \( \delta = 0.01(s^{-1}) \). In the simulation process, considering that the available overload of the missile is limited in the flight process, the overload instructions on Y axis and Z axis in the trajectory coordinate system are limited in scope, which are 0-10g and 0-8g, respectively.

Under the above simulation conditions, the Terminal sliding mode Maneuvering Penetration guidance law designed in this paper is simulated. The final miss distance of the anti-ship missile is 0.3m, the inclination angle of the ballistic terminal is -30.01°, and the heading angle of the ballistic terminal is -29.95°. The correlation curve is shown in Figure 2-4.

![Figure 2. Z-Y plane trajectory of missile.](image1)

![Figure 3. X-Y plane trajectory of missile.](image2)
Summary

In order to realize the anti-ship missile striking the warship target in the predetermined direction and improve its penetration ability, a guidance law with attack angle control and maneuvering avoidance is designed based on the three-dimensional relative motion model of the missile, by using the terminal sliding mode control theory. In order to verify the validity of the design method in this paper, the mathemetic simulation of a certain type of anti-ship missile against a maneuvering target ship is carried out. The simulation results show that the anti-ship missile can attack the target accurately in the predetermined direction and maneuver the terminal attack trajectory. The combination of the terminal automatic guidance trajectory and the terminal maneuver penetration trajectory is realized, which makes the anti-ship missile not only have no influence on the attack accuracy, but also realize its terminal maneuver penetration.

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