Kinetic Analysis of Weakly ionized Plasmas in presence of collecting walls

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Abstract. Description of plasmas in contact with a wall able to collecting or emitting charged particles is a research topic of great importance. This situation arises in a great variety of phenomena such as the characterization of plasmas by means of electric probes, in the surface treatment of materials and in the service-life of coatings in electric thrusters. In particular, in this work we devote attention to the dynamics of an argon weakly ionized plasma in the presence of a collecting wall. It is proposed a kinetic model in a 1D1V planar phase-space geometry. The model accounts for the electric field coupled to the system by solving the associated Poisson’s equation. To solve numerically the resulting non-linear system of equations, the Propagator Integral Method is used in conjunction with a slabbing method. On each interrelating plasma slab the integral advancing scheme operates in velocity space, in such a way that the all the species dynamics dominating the system evolution are kinetically described.

1. Introduction

The description of plasma-wall interactions is an open problem of major importance in experiential plasma physics since the dynamics of light and heavy species close to a wall bears on probe measurement processes, service life of walls and coatings and surface treatment procedures. This interaction appears in experiments carried out involving plasmas [1, 2, 3] as well as in vessels flying in space [4]. Many works have provided fluid models, as the those by T. Gyergyek et al. [5, 6, 7] or kinetic descriptions, as the works by M. D.Campanell et al. [8, 9] or J. P.Sheehan et al. [10, 1], among many other authors [11, 12, 13, 14, 15, 16, 17]. An important case of plasma-wall interaction appears in the characterization of plasmas, usually carried out with emissive [18, 19, 20] or collecting probes [21, 22, 3] to calculate plasma parameters, such as species temperature, density or the plasma potential, responsible of the energy transference. The description of this interaction is usually established by models that may not capture all microscopic processes involved, specially in Weakly Ionized Plasmas (WIP), for which the inclusion of a dense heavy cold neutral species hinders the development of such models [21, 23, 24]. Our research group is actually interested in the use of collecting probes to analyse the low temperature plasma generated in our laboratory [25, 26, 27], making this topic of main interest in the characterisation of, for example, the plasma plume produced by an electric thruster [25].

In this work, some progresses performed to analyse the evolution of Weakly Ionized Plasmas close to collecting walls are presented. In particular, the collection of electrons and ions coming from a quasi–neutral weakly ionized plasma is studied by analysing the different behaviours of
ions and electrons through their time evolving dynamics. A theoretical kinetic model is proposed, giving rise to non-linear drift-diffusion equations. The Propagator Integral Method [13, 28, 29] is used to solve the resulting one-dimensional kinetic problem in the velocity space, meanwhile the spatial dependence is introduced by dividing the plasma in a series of interrelating thin slabs. This slabbing method [30, 31] accounts for the flux of particles as a function of the position.

2. Kinetic model and slabbing method

For a plasma bounded by a wall capable of collecting or emitting charged particles, the species dynamics gives rise to a very complex system, related to the fast charge separation that governs the plasma evolution, at the same time modified by the collection of particles. In this section, we present a kinetic model to describe ions and electron dynamics of a weakly ionized plasma close to a metallic wall, with no previous assumptions about the particle density profiles or electric potential shapes. The kinetic model in the 1D1V planar phase-space employed is mathematically expressed as

\[
\frac{\partial f_\alpha}{\partial t} + v \frac{\partial f_\alpha}{\partial x} + \frac{q_\alpha E}{m_\alpha} \frac{\partial f_\alpha}{\partial v} = \left( \frac{\partial f_\alpha}{\partial t} \right)_{\alpha 0} \tag{1}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = -\frac{e}{\epsilon_0} (n_i - n_e)
\]

\[
\phi(0) = \phi_w \tag{2}
\]

where \(\partial f_\alpha/\partial v\) is a Dougherty collision term for charged particles interactions with neutrals, represented by the drift and diffusion parameters in a Fokker—Planck model approach [32, 33] as

\[
D^\gamma 0 = -\nu^{\gamma \gamma'} (v - u_0); D^\gamma v = \nu^{\gamma \gamma'} (E_\gamma + E_{\alpha 0} - 2u_\gamma u_0), \tag{3}
\]

where \(\nu^{\gamma \gamma'}\) is the collision frequency, \(f_\alpha = f_\alpha(x, v, t)\) is the distribution function for the species \(\alpha = e, i\) and \(\phi_w\) is the potential of the collecting wall, located at \(x = 0\). To solve this system with an integral advancing stable scheme (PIM), a slabbing method is employed in order to reduce the computational cost that would imply the use of a propagator working in the whole phase-space. This approach allows a better representation of the kinetic processes in the \(v\) direction and reduces the number of points required to represent the \(x\) dependence. This slabbing method is appropriate since the dynamics in the \(x\) direction is dominated by the convective flux of particles, and slightly affected by the diffusive \(x - x\) and \(x - v\) processes. This slabbing method consists on the numerical evaluation of the flux of particles \(v \partial f_\alpha/\partial x\) to replace its corresponding contribution, in (1), by a non–homogeneous term which represents the particle balance through the slab boundaries. Similar approach has been employed in [31], where this effective source term, replacing the so–called Vlasov term, was used to compute the heat flux for transport coefficients calculations of a two-dimensional plasma in the velocity space. Furthermore, this method is based on the pioneer work by J. P. Matte and J. Virmont [30] also used in fully ionized plasma local transport coefficients characterization.

To analyse the behaviour in each different slab, the flux term in the \(x\) direction is evaluated as

\[
-v \frac{\partial f_\alpha}{\partial x} \simeq s_{f_{\text{flux}}}(x, v, t) = \begin{cases} -v f_\alpha(x + \Delta x, v, t) - f_\alpha(x, v, t) \frac{\Delta x}{\Delta x} & \text{if } v < 0 \\ -v f_\alpha(x, v, t) - f_\alpha(x, v, t) \frac{\Delta x}{\Delta x} & \text{if } v > 0 \end{cases}
\]

coming from the physical meaning of the divergence of \(v f\) as the net flux of a vector field, across the closed smooth surface value of a vanishingly small volume, divided by this volume, both
neutral elastic exchange are effects or to the action of the self–consistent electric field. The collision frequencies for the charge–preserving an amount of returning ones to the quasi–neutral plasma, a fact due to collisional and σ selection ensures that the upwind discretisation of the flux in the must be employed, which constrains the evolution to a reduced time step for each slab becomes

$$\frac{\partial f_\alpha}{\partial t} + \frac{q_\alpha E}{m_\alpha} \frac{\partial f_\alpha}{\partial v} = \left( \frac{\partial f_\alpha}{\partial v} \right)_{\alpha\theta} + s_{\text{flux}}. \quad (5)$$

As initial condition we assume drift Maxwellian distributions, for each species of a quasi–neutral argon weakly ionized plasma, characterized by the parameters set on table 1. This initial condition ascertains that a large number of particles reaches the wall at x = 0, but still preserving an amount of returning ones to the quasi–neutral plasma, a fact due to collisional effects or to the action of the self–consistent electric field. The collision frequencies for the charge–neutral elastic exchange are \( \nu^{\alpha\theta} = \sigma^{\alpha\theta} V_{th_\alpha} \), for constant cross–sections such as \( \sigma^{\alpha\theta} = 10^{-18} \text{m}^2 \) and \( \sigma^{\alpha\theta} = 10^{-20} \text{m}^2 \), where \( V_{th_\alpha} = \sqrt{\frac{kT}{m_\alpha}} \) is the species thermal velocity. A total density of particles \( n_T = 10^{10} \text{m}^{-3} \) and the ionization ratio \( r_i = 10^{-6} \) are used. In this approach, charge–charge collisions are neglected due to their weak influence in the species dynamics for this low ionization ratio, as we argued in [28, Section 5.1.1]. Due to the upwind slabling, a small \( \Delta x \) must be employed, which constrains the evolution to a reduced time step \( \tau = 2 \cdot 10^{-5} \). This selection ensures that the upwind discretisation of the flux in the x direction is stable from a numerical point of view. This slightly perturbs the mesh–free basis of the PIM, but it is a requirement to ensure that the transversal flux is properly represented. A total of 200 slabs is considered to properly cover the space gap from \( x = 0 \) up to \( x = 40 \text{ cm} \).

### 3. Collecting wall

To simulate the collecting wall, a total absorbing boundary condition is imposed at the origin, therefore we set \( f_\alpha(0, v > 0, t) = 0 \). The distribution function for \( f_\alpha(0, v \leq 0, t) = 0 \) is described as a one-dimensional semi–open problem in the region \((-\infty, 0] \), by the boundary condition \( f_\alpha(0, 0, t) = 0 \). At \( x = 40 \text{ cm} \), the distribution function for negative velocities is kept constant to simulate a sustained plasma, meanwhile for positive velocities values this function is perturbed by the downstream flux and collisional processes. The density current, \( J_\alpha(x, t) = [q_\alpha] \) is plotted against time, at positions \( x = 0 \) and \( x = 40 \text{ cm} \), in figure 1. Clearly, the wall acts as an initial perturbation that disrupts the quasi–neutral plasma. In figure 1b it can be seen how electrons move faster, creating an oscillatory situation that is dumped after a time of \( 1 \mu s \) approximately. After the initial perturbation, plasma adapts itself to the new situation, but each species evolve in its own rate, this is to say, the wall effect is quickly felt by the electrons, due to their high energy and low mass, in fact, it can be seen how the position far from the wall is influenced as positive velocity particles decrease, leading to a more positive electric current. This process leads the electrons to behave in an oscillatory mode to adapt themselves to the fast perturbation experienced by the wall. These oscillations, of unique frequency, are related to the approximated

| Mass (kg) | Velocity (m/s) | Temperature (eV) | Density (m\(^{-3}\)) |
|----------|----------------|-----------------|----------------------|
| e 9.10938356 \times 10^{-31} | \( \sqrt{\frac{kT}{m_e}} \) | 1 | \( r_i \frac{m_T}{m_e} \) |
| i 72819.6 \times m_e | \( \sqrt{\frac{kT}{m_i}} \) | 0.1 | \( n_e \) |
| 0 | \( m_e + m_i \) | 0 | 0.01 | \( n_T(1 - r_i) \) |

Table 1: Argon plasma to analyse the collecting wall by a one-dimensional slabbing method.
cold plasma frequency $f_{pe}$ of order

$$f_{pe} = \sqrt{\frac{n_e e^2}{\pi m_e}} \approx 8980\sqrt{n_e} \text{ Hz},$$

where $n_e$ is the electron density (all magnitudes are in cgs units). For this expression, ions have been assumed as infinitely massive. This expression, ions have been assumed as infinitely massive. Table 2 presents the plasma frequency for three cases. Firstly, two cases corresponding to equation (6), using the electron density at $x = 0$ and far from the wall at time $t \simeq 0.15 \mu s$, are shown for the approximated values $4.2 \cdot 10^6$ and $5 \cdot 10^6 \text{ cm}^{-3}$ respectively. The result obtained after evaluating the mean frequency for the cycles in figure 1b is presented. It can be seen how all frequencies are in the same order of magnitude of $100 \text{ rad/s}$, but the numerical value is slightly lower than the analytical one for $x = 0$. Hence, it is reasonable to conjecture that the electrons evolve in the simulations accomplishing a periodic dynamics close to the electron plasmas characteristic frequency. Moreover, this result arises naturally from the self–consistent resolution of the model, without imposing any constrain. The self–consistent resolution of the electric field is crucial to obtain this result, since small variations of this field are the main reason of the detected electron oscillatory behaviour. Therefore, we can assert that our simulations bring out an evolution that matches realistic physical plasma properties. Moreover, a perceptible evolution of ions is appreciated in figure 1c, meaning that the heavy species reacts to the perturbation produced by the wall, clearly in a larger time scale than the electrons one.

The figure 2 pictures the contours of $f_e$ at four frame times. For short times, the effect of the wall only reaches close positions from it but, as time increases, the influence in the distribution function extends in space and a significant amount of electrons is repelled by the metallic wall. As a result of this dynamics, we can conclude that the wall highly influences the distribution function for positive velocity, a fact that perturbs the quasi–neutral plasma. Although several works, as [34, 9], account for a non–homogeneous term to maintain the quasi–neutral plasma
from a certain position, in the actual model this feature is achieved only by imposing the previously specified boundary conditions at \( x = 40 \text{ cm} \). This means that all the domain, the region even far from the wall, can be affected by the presence of a metallic wall. In practice, it can be found that the use of an external source-sink of particles, to maintain the plasma, could perturb the system evolution [35] forcing a particular electric potential structure able to sustain the dynamics described here. Nevertheless, to properly establish the boundary conditions at 40 cm is still a difficult task that requires further research to describe the quasi-neutral distant plasma. For realistic models, enough space in the \( x \) direction should be provided to ensure that the system recovers a quasi–steady state without external terms, as it would correspond in an experimentally sustained plasma discharge. This would require a large number of plasma slabs and also a large computational time.

The boundary condition at \( x = 0 \) means that no particles are introduced into the numerical domain. However, it can be seen how a population of electrons still holds for \( v > 0 \). This is caused by the amount of electrons that return due to effect of the self-consistent electric field and the collision with neutrals. To analyse the first cause, the electric potential and field are pictured in figure 3. It can be seen how the fast evolution of electrons generates a large electric field due to the charge separation evolving to eliminate it by accelerating electrons positively and ions negatively. The electric field reduces its intensity as time increases and the potential difference between the wall and the end of the numerical domain is reduced drastically, as the plasma self-consistently adapts to the boundary condition. This creates some of the returning trajectories identified in figure 2.
Figure 3: (a) Electric field and (b) potential close to a collecting wall. Monotonic evolution is found for the potential, but interesting variations close to the wall arise for the electric field.

Figure 4: Density current for (a) electrons and (b) ions for a time evolution of $\sim 50 \mu s$. Electron density current at the wall and far from it coincide.

3.1. Long time analysis
In order to determine the influence of the ions on the system dynamics, it is interesting to analyse an extended period of time. The results presented above are analysed here for a period of time of order $50 \mu s$ in figure 4. The electron ensemble shows a fast evolution to a state in which the fluxes far and at the wall are equal at time $\sim 10 \mu s$. In this time, the ions go through a fast change in order to fix the variations induced by the electron dynamics. Consequently, once the current at the wall and far from it become equal, the ions evolve under a slower time scale. However, the ion population seems to continue its evolution towards a situation in which, like the electrons, the current at the wall and far from it are equal.

Two differentiated regions in the system evolution can be distinguished, in a first one, up to a time of $\sim 15 \mu s$, the system dynamics is guided by the fast electron response, which is the main cause of evolution of the self–consistent electric field. Then, electron flux is balanced and the small evolution of the ions is the dominant phenomena. Electrons also response to this variation as the current is slightly reduced for $t > 15 \mu s$, as a response of the changes in the electric potential generated by the ions evolution.
4. Conclusions

The very involved and interesting problem of the plasma–wall interaction and related phenomena has been modelled and numerically solved by a slabbing kinetic model. This problem is of huge importance in the characterization of plasmas and its dynamics in bounded domains. Many disparate and odd processes determine the dynamics of these systems, as the distribution and energy of emitted electrons, the self-consistent electric field and the effects appearing in the vacuum chamber in which the plasma is held. Kinetic descriptions of these phenomena configure, in general, an arduous task, but every approach provides relevant information to understand the processes involved and how they influence the interaction. The aim of this procedures is to capture some behaviours that could be overlapped by some fluid plasma descriptions. For the models presented here, no assumption about the distribution function profile or the electric potential have been assumed, since they are obtained from a pure self–consistent kinetic problem we have stated.

In particular, the description of ions and electrons dynamics, in an argon weakly ionized plasma, in the presence of a collecting wall has been carried out. It has been found that the fluxes of the electrons at the wall and far from it became balanced, meaning that the plasma evolves to self-consistently equilibrate these flows. Moreover, despite the fact of the mass of ions is huge, compared with the electron mass, its dynamics close to a collecting wall modifies the electric field since ions are collected by the wall and its density is being reduced, giving rise to a smaller but important charge-separation. This is also appreciated for a large evolution time. Hence, a fast local change for the ions density occurs when the electrons tend to couple to the new situation, the ions evolve in a slow time scale and their dynamics can dominate the system evolution.

Physical results are obtained as a consequence of the self–consistent resolution of involved models, including the effect of the electric field or charge-neutral collisions. From this preliminary approach it is expected that future works can translate these models into cylindrical and spherical geometries to match the typical probes shapes to analyse the effects of probe geometries. More microscopic kinetic processes should be included to analyse their influence on close and far plasma dynamics. This also will require a more accurate description of the self–consistent electric field in the case of non-uniform wall emission model.

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