Simulation of disruptions triggered by Vertical Displacement Events (VDE) in tokamak and leading edge effect in plasma energy deposition to material surfaces

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Abstract.

The paper describes two major non-linear properties of the vertical instability of a tokamak plasma, which has a vertical elongation: (a) inductive excitation of surface (edge) currents stabilizing the instability and converting it into fast equilibrium evolution, and (b) creation of a wetting zone without the normal component of the magnetic field when the plasma has contact with material surfaces. Two major disruption effects for both mitigated and non-mitigated disruptions, important for JET and ITER, were considered: (a) excitation of vertical disruption during the current quench (i.e., abnormal plasma current ramp down) and (b) related to the wetting zone, potential leading edge effect in plasma energy deposition to the in-vessel tiles during disruptions. Our considerations together with a 2-D version of the VDE (Vertical Disruption Event) code are based on a new mathematical model, called Tokamak MHD (TMHD), as a replacement for the conventional model, a model that cannot solve numerical problems related to extreme plasma anisotropy and negligible mass. The code includes a 3-D model of surface currents on a thin conductive wall and has a well-specified algorithm for extension to vertical disruptions that excite asymmetric kink modes.

1. Introduction

Disruptions on JET were capable of melting the plasma facing beryllium tiles. Similar wall damage in ITER should be assessed. Accordingly, in this paper we will address two important disruption effects in both mitigated and non-mitigated disruptions in JET and ITER: (a) excitation of vertical disruption during the current quench (i.e., abnormal plasma current ramp down) and (b) calculation of the wetting zone of plasma and plasma facing surfaces and of potential leading edge effect in power deposition to the in-vessel tiles during disruptions.

The plasma current quench deteriorates vertical stability of plasma and has a tendency to low stability and excite VDE which can go into fast inductive phase, causing big forces to the vessel. One of the goals of this paper is to determine the limitations on current decay rate given the geometry of vessel structure and characteristics of feedback stabilization systems.
During fast VDE disruptions, the plasma acquires a contact with the plasma facing material surfaces (e.g., protective plates in ITER). The instability generates the edge Hiro currents which make plasma surface to follows the profile of the plate shape in the touching area. The Hiro currents from the free plasma surface enter the wall surface mostly through the contour of the wetting zone. This creates the situation of the leading edge effect with concentrated power deposition at the edges of the plates and potential destruction of the edges. This effect, overlooked so far, but considered by the authors as a cause of wall melting in JET, is as important as, e.g., formation of runaway electrons.

The determination of the geometry of the wetting zone as well as the power deposition to edges of the protective plates in ITER is an essential step in disruptions simulation.

In 2007 Leonid Zakharov created the theory of current sharing between plasma and the wall, revealing Hiro currents as the main leading inductive effect in disruptions [1], which explained the unexpected direction of currents in the wall in Asymmetric Vertical Displacement Events (AVDE) on JET and justified the scaling of sideways forces from JET to ITER [2, 3]. In 2012, the Hiro currents, predicted by the same theory, have been measured in symmetric VDEs on EAST [4].

In 2015, based on the following time scale relations, typical for tokamaks [5]

\[
\tau_{MHD} \simeq \frac{R}{V_A} \ll \tau_{TMHD} \ll \tau_{\text{transport}} \ll \tau_{\text{resistive}},
\]

where \( R \) is the the major plasma radius, \( V_A \) is the Alfvén velocity, \( \tau_{TMHD} \) is the low limit of events in the TMHD and \( \tau_{\text{transport}} \) is the characteristic time of evolution of plasma parameters, a new mathematical model, called Tokamak MHD (TMHD) [5] was formulated for disruptions simulation as a replacement of conventional MHD. By considering for MHD simulations adaptive grids which are aligned with the 3-D ergodic magnetic field, new, so-called Reference Magnetic Coordinates (RMC) have been introduced [5]. TMHD considers disruption as fast equilibrium evolution underlying that the plasma inertia plays no role, being for example, 8-9 orders of magnitude lower than the electromagnetic forces in a JET VDE.

In 2013-2015, a 2-D version of TMHD/RMC model VDE-code, was created [6, 7] and applied for EAST VDE disruptions simulations [4], being able to reproduce the Hiro currents, predicted by the theory and measured directly for the first time in axisymmetric disruptions. It is to note that even if, at this stage, VDE code is a free boundary non-linear stability code for simulation of 2D vertical disruptions in tokamaks, it considers a 3-D wall describing both eddy and source/sink currents.

As a part of the model, we have developed the SSEC code [8] for calculation of 3-D source/sink and eddy currents in the thin wall approximation. The SSEC code is a component of the disruption code VDE and can be coupled with any other calculation of plasma dynamics achieved in the EUROfusion community. Our code received the status of open source license and can be accessed now by the EUROfusion community. Our code has been checked on analytical cases, for both surface currents components (eddy currents and source-sink currents) [9, 10, 11, 14]. Recently, our SSEC interface of wall with disruption simulation codes [12] has been implemented (in FORTRAN) at IPP Garching into the JOREK-STARWALL code [13].

2. Vertical instability in tokamaks

In tokamak devices the high temperature plasma is confined by poloidal magnetic field, representing a superposition of magnetic field of the toroidal plasma current and currents in external equilibrium coils. In the 1970s, it was established that the best confinement can be obtained in a vertically elongated plasma cross-section. At the same time, the elongated plasma
is prone to vertical instability. In practice, this instability is suppressed by stabilizing effect of electrically conducted structures of the vacuum vessel as well as by the feedback positional control system.

The simplified sketch in Fig. 1a explains the origin of the vertical instability when stabilization structure are absent. The plasma cross section is made elongated by two currents in the Poloidal Field Coils (PFCoils) with the same direction as the plasma current.

If plasma is shifted, e.g., downward as in Fig. 1, a the plasma current exhibits larger attraction to the lower PF Coil than to the upper coil. This accelerates the plasma in the direction of the initial shift signifying the vertical instability. The same figure shows also the direction of surface currents generated by electromotive force ($\mathbf{V} \times \mathbf{B}_{PFC}$), where $\mathbf{V}, \mathbf{B}_{PFC}$ are plasma velocity and magnetic field of PF Coils, respectively. Note that on the leading side of the plasma surface the surface current is opposite in direction to the plasma current. This is a general property of the free boundary plasma instabilities in tokamaks.

![Figure 1.](image-url)

Figure 1. Straight cylinder model of a tokamak plasma for illustration of vertical instability. (a) Plasma column with elongated cross-section is shifted downward from equilibrium position. (b) 2-dimensional cross-section of magnetic configuration with a plasma slightly shifted downward in the center.

The right frame Fig. 1b shows the cross-section of magnetic configuration for slightly shifted elongated plasma as calculated by the VDE code. The color contours represent the levels of the normalized poloidal magnetic flux $\bar{\Psi}(r,z) = \text{const}$, related to the poloidal magnetic field $\mathbf{B}_{pol}$ ($\bar{\Psi} \equiv \Psi/2\pi$, with $\Psi$ the poloidal magnetic flux) as

$$\mathbf{B} \equiv \mathbf{B}_{pol} + \mathbf{B}_{\phi} = (\nabla \bar{\Psi} \times \nabla \phi) + F \nabla \phi$$

with $F = r B_{\phi}$ representing the toroidal field and $r, \phi, z$ being cylindrical coordinates.

Due to the conservation of magnetic fluxes (poloidal and toroidal), conservation valid for high temperature plasmas, currents are excited right at the plasma edge and can be represented by surface currents. They play a significant role in plasma dynamics. Regarding the nonlinear evolution, if we were to follow a naive thinking, the plasma column should approach the lower coil which attracts the plasma current. In fact, because of flux conservation and generation...
of surface currents, the downward motion of the plasma stops. The negative surface currents creates a force which balances the vertical force.

The plasma cross-section is deformed like a triangular shape and creates two Y-points in magnetic configuration in its corners as shown in Fig. 2. The following evolution represents a plasma reconnection at the Y-points rather than a positional instability. Plasma cross-section should shrink while the plasma moves along the separatrix to the open magnetic field lines. The reconnection phase is beyond the scope of the VDE code and is not illustrated.

![Figure 2. Late non-linear phase of vertical instability. Plasma stops moving in vertical direction. Plasma cross-section becomes triangle-like. (a) Strong negative surface current in blue at the leading plasma side. The shape represents its profile across the flat area. (b) Strong positive current in red is generated at the opposite, free plasma surface.](image)

### 3. TMHD model of Vertical Disruptions.

The numerical code VDE, used in the above simulations, has implemented a specific for tokamak MHD 2-dimensional model version, called TMHD.

TMHD represents the replacement of conventional MHD by utilizing (a) neglection of plasma inertia relative to large electromagnetic forces, and (b) the equilibrium plasma evolution during disruptions, which is fast compared to plasma diffusion time scales but slow relative to what would be dynamic scales due to violation of equilibrium. For the 2-dimensional model of vertical instability TMHD suggests three step solutions for each time step (for explanation in details see Ref. [5]).

(i) *Equilibrium in plasma core.* It is described by equation

\[
\nabla p = (\vec{j} \times \vec{B})\quad \text{(GSh equilibrium)}
\]

where \( p \) is the plasma pressure, \( \vec{j}, \vec{B} \) are current density and magnetic field inside the plasma. In 2-D version, this equation is solved as Grad-Shafranov (GSh) equation with specified plasma boundary, pressure profile and given toroidal and poloidal magnetic fluxes inside the plasma. The non-linear Grad-Shafranov equation was solved using its linearization [15].
(ii) **Advancing plasma boundary.** Toroidal flux conservation and incompressible plasma motion are resulting in the equation for plasma deformation $\delta \vec{r}$

$$\left( \nabla \cdot \frac{\nabla \tilde{F}^2}{r^4} \right) = 0, \quad \lambda \delta \vec{r} = -\frac{\nabla \tilde{F}^2}{2r^4},$$

(3)

where $\tilde{F}$ has the physical sense of perturbed poloidal current in the plasma, $r$ is the cylindrical radial coordinate, and $\lambda$ is calibration parameter, determining the step for evolving equilibrium configuration. There is no time step in the basic level of TMHD. The sequence of equilibrium configurations is unique for given initial conditions. The association with the time instances is determined by the physics of the plasma decay. This is out of scope of the most basic level of TMHD.

(iii) **Boundary condition.** The total magnetic and kinetic pressure should be continuous across the plasma boundary

$$p_{\text{core}}^\text{core} + \frac{B_{\text{pol}}^2}{2\mu_0} \bigg|_{\text{core}} + \frac{F^2}{2\mu_0 r^2} \bigg|_{\text{core}} = \frac{B_{\text{pol}}^2}{2\mu_0} \bigg|_{\text{vac}} + \frac{F^2}{2\mu_0 r^2} \bigg|_{\text{vac}}, \quad F \equiv \tilde{F}(\Psi) + \tilde{F}(r, z).$$

(4)

Here $F$ is the poloidal current consisting from equilibrium part $\tilde{F}$ and $\tilde{F}$, the small deviation from equilibrium. $\mu_0 = 0.4\pi \cdot 10^{-6}$ H/m. All quantities are calculated at the plasma boundary of the core and at vacuum sides.

(iv) **Surface currents.** Magnetic flux conservation implies generation of the toroidal surface current $\bar{i}$ which is calculated from the jump of poloidal component of the magnetic field

$$\mu_0 \bar{i} = B_{\text{pol}}^\text{vac} - B_{\text{pol}}^\text{core}.$$  

(5)

Matching the core magnetic field with external structures (eddy and Hiro currents in vessel, coils) leads to surface currents $\bar{i}$ at the plasma boundary and to a jump in edge pressure $p_{\text{edge}}^\text{edge}$.

### 4. Plasma shrinking due to decay of Hiro currents

Generation of the surface currents is the main inductive effect in free boundary instabilities. As it is shown in Fig. 2, these surface currents can stop the development of the instability nonlinearly and provide a macroscopic equilibrium. The plasma will still disappear due to another physics effects like reconnection.

In tokamak devices, plasma is inside the conducting structure of a toroidal vacuum vessel. This structures affect stability conditions of the elongated plasma by reducing the growth rate of instability to the inverse resistive time of induced eddy current decay. This suppression of the growth rate make full stabilization of the vertical instability possible by an active feedback system.

Still due to sharp change in plasma parameters of a failure of feedback algorithm, vertical instability can happen. Moreover, the fast shutdown of the plasma current (so called current quench), necessary in certain situations, is prone to excitation of vertical instability. Indeed, the driving force of instability $F_z$ is proportional to the gradient of external magnetic field near the plasma center

$$F_z \simeq -2\pi RI_{pl} \frac{\partial B_{\text{pol}}^{\text{PFC}}}{\partial z} \Delta_z,$$

(6)

where $R$ is the major radius of a toroidal plasma, $I_{pl}$ is the plasma current, and $\Delta_z$ is plasma vertical displacement. This displacement excites eddy currents $\vec{i}^{\text{wall}}$ in the wall, which creates
a horizontal field $B_{wall}^r$, stabilizing the plasma. In addition, the feedback system reacts on perturbation of the magnetic field and creates a stabilizing field $B_{f-b}^r$

$$B_{wall}^r = c_{wall} I_{pl} \Delta z, \quad B_{f-b}^r = c_{f-b} I_{pl} \Delta z, \quad (7)$$

where $c_{wall}, c_{f-b}$ are coefficients specific for the device. In a stabilized phase

$$2 \pi R I_{pl} (B_{wall}^r + B_{f-b}^r) > |F_z| \quad (8)$$

or

$$I_{pl}^2 (c_{wall} + c_{f-b}) \Delta z > I_{pl} \frac{\partial B_{PFC}^{r}}{\partial z} \Delta z, \quad (9)$$

thus, giving the following condition of stabilization

$$I_{pl} (c_{wall} + c_{f-b}) > \frac{\partial B_{PFC}^{r}}{\partial z}. \quad (10)$$

During the current quench, the plasma current decays and the stabilizing left hand side of the stability criterion becomes finally smaller than the stabilizing right hand side stability criterion. This excites the fast vertical instability.

The situation could be better if a feedback system would react directly to the plasma displacement rather than to a magnetic perturbation

$$B_{f-b}^r = b_{f-b} \Delta z, \quad (11)$$

where the coefficient $b_{f-b}$ is related to the amplification of feedback system. The sensitivity of feedback to plasma displacement reduces the driving force effectively and leads to the following stabilization criterion

$$I_{pl} (c_{wall} + c_{f-b}) > \frac{\partial B_{PFC}^{r}}{\partial z} - b_{f-b}. \quad (12)$$

The feedback still cannot completely compensate the driving force, and during the fast current quenches plasma becomes most likely vertically unstable.

In tokamaks plasma is prevented from the direct contact with the vessel wall by protective tiles, either graphite or metal. During vertical instability plasma is moving toward these tiles and makes direct contact with the tiles. The surface currents, which in the absence of the contact would be generated on the plasma surface, are partially now transferred to the tile surface. This situation is illustrated in Fig. 3

Starting from an initial unstable equilibrium, a non-realistic case for a tokamak plasma but used as initial state for generating the non-linear phase of vertical instability, plasma moves toward tile surface. The instability excites now negative surface currents in tiles along the toroidal direction. They are called Hiro currents. The compensating force-free (F-F) positive current is excited at the free plasma surface.

Finally, Hiro currents stop fast plasma motion and create the equilibrium configuration. This one can represent already a realistic case of tokamak plasma during the vertical displacement events (VDE).

The Hiro currents are decaying resistively. As a result, plasma moves to the tile surface, its ions are neutralized and the plasma core shrinks. The final evolution is illustrated by Fig. 4.

During the entire final evolution, Hiro currents maintain plasma equilibrium. Their amplitude goes down together with reduction in plasma cross-section and the core plasma current. At the same time the compensating surface current of the same direction as the plasma current is excited as a force free current. It some stage it becomes a dominant surface current as it is indicated by a darkness of the shrinking plasma surface in Fig. 4.
Figure 3. Vertical instability of toroidal plasma in a tokamak environment. The vessel wall is taking into account in simulations but shown only partially. The tile surface is situated at the bottom of the vessel. The instability excites the Hiro currents along the tile surface and a force free current on the free surface of the plasma. (a) Initially unstable elongated plasma. (b), (c) two phases of plasma deformation still only in partial equilibrium (d) the equilibrium configuration maintained by Hiro currents excited on the tile surface.

Figure 4. Final phase of a shrinking plasma configuration due the decay of the Hiro currents. The core plasma current going down together with plasma cross-section. A compensating current in the same direction is excited by the loop voltage. (a) First phase of shrinkage of plasma cross-section. (b) Dark color on the free plasma surface indicates generation of the surface current compensating the reduction in the core plasma current. (c) The compensating current becomes dominant. (d) Final stage of almost disappeared plasma.

5. Wetting zone, concentration of power deposition and leading edge effect

The plasma configuration maintained by the Hiro currents has the fundamental property of having a well defined wetting zone, where the normal component of the magnetic field is absent.
and the plasma surface repeats the shape of the material surface in the wetting zone.

At the edges of the wetting zone the magnetic configuration has 2-Y points where free plasma surface joins the wetting zone. Plasma cross-section shrinks through the process of magnetic reconnection which is happening at two Y-points.

At the moment, there is no physical model of the reconnection process. But is reliable to conclude that the scraping of the plasma edge together with the surface current are happening locally near the Y-points. Accordingly a big fraction of plasma kinetic and magnetic energy is released along the edges of the wetting zone in localized manner.

This concentration of the energy deposition in VDE along a contour of the wetting zone represents one of the danger of the disruption events.

In the case of 2-dimensional instability, simulated here by VDE code, the contour of the power deposition is extended by $2 \times 2\pi R$, which is longest possible wetting zone contour. The situation is different in 3-dimensional case as is illustrated by Fig. 5.

The important difference between 2- and 3-dimensional cases is a spot-wise shape of the wetting zone in the case of kink mode, or Asymmetric VDE (AVDE) as is shown in Fig. 5. The localization of the power deposition during plasma shrinkage is even stronger.

There is an additional leading edge effect in the 3-D case of the kink mode. Unlike 2-D VDE, the electric contour of the Hiro currents extends outside the wetting zone to the free plasma surface. A special disruption generated Scrape Off Layer (D-SoL) is generated outside the plasma edge in order to close the circuit of Hiro currents. Now, the electric currents enter the wetting zone locally. Although there the physics of such a plasma-wall contact could be very complicated, it is certain that the most of the energy driving the electric current is released in the area of electric contact with the plasma.

Not only the Hiro currents are carried through this contact. Later, the entire compensation current should also go through this contact. Accordingly, big fraction of the magnetic and kinetic energy is released locally. Moreover, at some position where the wetting zone edge intersects the edge of the protective plates, the energy is release right to the edge of the plates, constituting a very damaging leading edge effect.

At the moment, the 3-D version of VDE code is not operational. But the straightforward extension of 2-D version can be done based on TMHD model and its numerical algorithm.
6. 3-dimensional energy principle for combination of VDE and kink mode

The Tokamak MHD is presented by the following set of equations:

(i) The equation of motion of conventional MHD is split into an equilibrium equation

\[ \nabla p = (\vec{j} \times \vec{B}), \quad \vec{\Psi} = \vec{\Psi}(\vec{\Phi}), \quad (\vec{\Phi} \text{ is the toroidal magnetic flux}) \quad (13) \]

and

(ii) The plasma boundary advancing equation

\[ \lambda \xi = -\vec{\bar{F}} \nabla \bar{F}, \quad \left( \nabla \cdot \frac{\vec{\bar{F}}^2}{r^4} \nabla \bar{F} \right) = 0, \quad (14) \]

(iii) with boundary condition in the form of force balance across the free plasma surface

\[ \left( p + \frac{|\vec{B}|^2}{2\mu_0} + \frac{\vec{\bar{F}} \cdot \vec{\bar{F}}}{r^2 \mu_0} \right)_i = \left( \frac{|\vec{B}|^2}{2\mu_0} \right)_e. \quad (15) \]

Here \( \vec{F} \) is one dimensional function and \( \bar{F} \) is unknown oscillatory part. The subscripts 'i, e' are specifying the inner and outer sides of the plasma surface.

(iv) Faraday’s (Ohm’s) law in plasma and the wall

\[ -\partial \vec{A} \partial t - \nabla \varphi^E + (\vec{V} \times \vec{B}) = \frac{\vec{j}}{\sigma}, \quad \vec{V} \equiv \frac{d\xi}{dt}. \quad (16) \]

Here \( \vec{A} \) is vector potential of magnetic field, \( \varphi^E \) is electric potential, and \( \sigma \) is electric conductivity either of the plasma or of the wall. This is the only equation which determines the time evolution.

(v) Plasma anisotropy

\[ \sigma = \sigma(\vec{\Phi}), \quad (\vec{B} \cdot \nabla) \sigma \simeq 0. \quad (17) \]

(vi) Boundary condition at the wetting zone

\[ (\vec{V} \times \vec{B}) - \nabla \varphi^E = \frac{\vec{j}^{wall}}{\sigma^{wall}}, \quad (\vec{B} \cdot \vec{n}) = 0, \quad (18) \]

which determines \( V_{normal} \equiv (\vec{V} \cdot \vec{n}) \) and \( B_{normal}. \) Here \( \vec{n} \) is unit normal vector to the wall surface.

The outstanding result of TMHD is the existence of energy principles for all TMHD partial differential equations. With finite element or basis function representation they lead to matrix equations with positively defined matrices, thus, guaranteeing stability of numerical schemes.

(i) 3-D equilibrium. 3-D Hermite elements, resulted in block three-diagonal matrices

The substitution of magnetic field in terms of the following unknown functions

\[ \vec{\Psi}(a) + \psi(a, \theta, \zeta), \quad \vec{\Phi}(a) + \phi(a, \zeta), \quad \vec{\Psi}(a, \theta, \zeta) \]
and some metric tensor coefficients gives

\[ W^F = \frac{1}{2} \int \left\{ \frac{\rho}{2u_0} - \left( \tilde{A} \cdot \tilde{j} \right) \right\} d^3r \]

\[ = \frac{1}{2u_0} \int \left\{ K(\tilde{\Psi}' + \phi' + \tilde{\Phi}' \eta'_\zeta)^2 - 2N(\tilde{\Psi}' + \phi' + \tilde{\Phi}' \eta'_\zeta)(\psi'_\rho - \phi'_\zeta) + M(\psi'_\rho - \phi'_\zeta)^2 \right. \]

\[ + Q(\tilde{\Phi}' + \phi'_\rho + \tilde{\Phi}' \eta'_\theta)^2 - 2\tilde{N}(\tilde{\Psi}' + \phi' + \tilde{\Phi}' \eta'_\zeta)(\tilde{\Phi}' + \phi'_\rho + \tilde{\Phi}' \eta'_\theta) \]

\[ + 2\tilde{M}(\psi'_\rho - \phi'_\zeta)(\tilde{\Phi}' + \phi'_\rho + \tilde{\Phi}' \eta'_\theta) - (\tilde{\Phi} + \phi)\tilde{F}'_\rho + (\tilde{\Psi} + \psi)(\tilde{J}'_\rho + \nu'_\theta) \left\} d\theta d\zeta. \]

The unknowns are here \( \psi, \phi, \eta \), the others \( K, M, N, Q, \tilde{N}, \tilde{M}, \nu \) represent combination of the metric tensor of toroidal coordinates \( a, \theta, \zeta \). \( \tilde{\Psi}, \tilde{\Phi}, \tilde{F}, \tilde{J} \) are one dimensional profiles in magnetic configuration as it is explained in Ref. [5].

(ii) Advancing the plasma boundary (3-D Hermit elements, block three-diagonal)

\[ W^F = \frac{1}{2} \int \tilde{F}^{a\alpha} \tilde{F}^{a\alpha}' + 2g^{ab} \tilde{F}^{a}_b \tilde{F}^{b}_a + 2g^{ab} \tilde{F}^{a}_b \tilde{F}^{b}_a + 2g^{ab} \tilde{F}^{a}_b \tilde{F}^{b}_a + 2g^{ab} \tilde{F}^{a}_b \tilde{F}^{b}_a + g^{ab} \tilde{F}^{a}_b \tilde{F}^{b}_a \]

\[ = \frac{1}{2} \int \left\{ \frac{\partial (\nabla \phi^S)^2}{2} + j_\perp \phi^S \right\} dS \frac{1}{2} \int \phi^S \sigma_0 [\mathbf{n} \times \nabla \phi^S] \cdot d\mathbf{l}. \]

(iv) Sink/source wall current from the plasma represented as \( \sigma \nabla \phi^S \) (triangle based wall model, small sparse matrix)

\[ W^S = \frac{1}{2} \int \frac{\partial (\nabla \phi^S)^2}{2} + j_\perp \phi^S \right\} dS \frac{1}{2} \int \phi^S \sigma_0 [\mathbf{n} \times \nabla \phi^S] \cdot d\mathbf{l}. \]

Hiro, eddy currents in the wall (triangle based wall model, stationary matrix)

\[ W^I = \frac{1}{2} \int \left\{ \frac{\partial (\tilde{A}^I)}{\partial t} + \frac{1}{\sigma} |\nabla I|^2 + 2 \left( \mathbf{i} \cdot \frac{\partial \tilde{A}^E}{\partial t} \right) \right\} dS \frac{1}{2} \int \phi^E - \phi^S \frac{\partial I}{\partial t} d\mathbf{l}. \]

These energy functionals specify a transition from the present hydro-dynamic MHD schemes to stable numerical schemes, based on plasma physics.

The following notations have been used [5, 8]: \( \mathbf{i} \equiv \nabla I \times \mathbf{n} \) is the divergence free surface current, \( I \) is the stream function if \( \mathbf{i} \), \( \mathbf{n} \) is the unit normal to the wall, \( \tilde{\eta} = 1/(\tilde{d}_w \sigma^w) \) is the surface resistivity of the wall, \( \tilde{d}_w \) is the wall thickness, \( d_w \) is the wall thickness, \( \tilde{d}_w \mathbf{j} = \mathbf{i} - \sigma \nabla \phi^S \) is the surface current density in the wall, with \( \phi^S \) a surface plasma source potential.

2-D version of TMHD, which includes both plasma and thin wall model, is operational in the VDE code and covers axisymmetric VDE. The extension to kink mode requires adding Fourier representation for the \( n = 1 \) mode. The advantage of the energy principles is the possibility of mixing finite elements and basic function representations, what makes the extension of the existing VDE to 3-D version straightforward.

7. Physical and computational basis of a 3D version of AVDE

TMHD/RMC (Reference Magnetic Coordinates) physical model contains intrinsically the physics of inductive effect of disruptions and Hiro currents, thus, is uniquely suitable for
simulation of tokamak disruptions. The model is open for extension to more sophisticated plasma core physics related to initiation of suppressed disruption.

The 3-D wall model of JET and ITER is already in our possession which would allow to perform 2-D studies in short time with our VDE code, including its calibration by JET data. The 3-D wall model is designed to calculate wall eddy currents as well as both Hiro and halo currents due to electric current sharing between plasma and the plasma facing surface.

Simulation of the current quench and transition to fast vertical instability can be done by VDE after implementation to it of some resistive effects of time evolution (e.g., based on Astra transport equations).

Variational principles of TMHD for all its 6 PDEs result in automatically stable numerical schemes with arbitrary combinations of finite elements. This makes the extension from 2-D VDE to 3-D Asymmetric VDE straightforward although labor consuming. In particular, the present 2-D Hermite elements for poloidal plane used in 2-D VDE can be efficiently complemented by a number of Fourier harmonics representing the n=1 kink mode, which create troublesome sideways forces on vacuum vessel.

Advanced description of 3-D magnetic configurations by Reference Magnetic Coordinates will simplify the implementation of core physics transport to the disruption model.

8. Summary and next steps

In this paper the vertical instability with the associated surface current and the resulted waiting zone are described.

For the first time the nonlinear evolution of a vertical instability in a canonical case of a quadrupole external field was presented as being calculated by VDE code. The unexpected termination of the fast evolution and the establishment of an equilibrium were discovered and as a result a new type of plasma reconnection with the magnetic field of vacuum through two Y-points has been predicted.

The calculations of the wetting zones when plasma goes to an equilibrium configuration maintained by the Hiro currents was described. The absence of normal magnetic field in the wetting zone represents the critical property of instability which concentrates the plasma energy deposition along the contour of the wetting zone. In 3-D (when the kink mode is present) the location is expressed even more with the release of the potential energy right at the edge of the protective plates between plasma and vessel.

The paper explains the fast current quench as a special phase of the plasma discharge, which is prone to excitation of vertical instability. This property alarms the disruption mitigation techniques, which provoke a relatively mild disruptions without runaway electrons, but at the same time create the fast current ramp down, which can create a global vertical disruption.

All the above described physics effects require numerical simulations for a specific machine geometry with its conducting structures. The old approached failed even in describing the electromagnetic forces to the structures. VDE code is an example of a new approach. In 2007, the key effect of Hiro currents, resulted from magnetic flux conservation in plasma dynamics, explained the toroidal asymmetry of the plasma current measurements in JET and the current sharing between the plasma and the wall. It became clear that for the next step in solving the disruption problem for ITER it was not possible to move forward based on hydro-dynamic approaches of the present numerical codes and that TMHD has to be used for numerical simulations consistent with theoretical understanding and experimental observations.

Numerical simulations realized via TMHD are consistent with theoretical understanding and experimental observations. The introduced Reference Magnetic Coordinates (RMC) are consistent with high anisotropy of the tokamak plasma. Via the same RMC, the long standing problem of practical coordinates for stochastic and ergodic magnetic fields can be solved.
It is to note that in the equations of motion for plasma confinement configurations TMHD are considering the inertia playing a minor role, while the effect of force balance is dominant. Each of all TMHD equations for description of both inductive effects and direct currents shared with the plasma have her own energy principle leading to a positively defined symmetric matrix, if expressed in terms of finite elements. By using 3-D Hermite finite elements for the plasma core functions, the matrices are simply block three-diagonal.

At the moment, TMHD was implemented into an operational 2-D Vertical Disruption Code (VDE). The finite elements and the corresponding solvers for the plasma equations were created and tested at the level of 2-D. The Hiro and eddy currents were reproduced as predicted by theory, confirming thus the basic consistency of TMHD with theory and observation.

As next steps we will consider the following tasks: a) developing of a 3-D disruption simulation code based on TMHD, by upgrading of the existing 2-D VDE code to JET and ITER wall geometries, b) including a transport model in VDE and simulate the transition to VDE during the current quench, c) performing calculation of the plasma shape, wetting zone and Hiro currents in asymmetric vertical disruptions (AVDE) (observed on JET and possible in ITER).

**APPENDIX: Illustration of different surface currents appearing during a VDE**

(a) Initial unstable plasma inside an “ideal” wall
(b) Stable equilibrium maintained by eddy currents in the wall and Hiro currents along tiles (JET case)
(c) Reduced plasma maintained by eddy, Hiro, and Evans (halo) currents in the halo zone driven by loop voltage
(d) Shrunken plasma maintained by eddy and Evans currents (C-mod case)

**Figure 6.** Examples of Eddy, Hiro and Evans (Halo) currents in VDE. Eddy and Hiro currents are inductively driven electron. Evans (would be halo) are source-limited ion-currents, with negligible forces on JET.

**Acknowledgments**

This work was partially supported by the US DoE Grant DE-SC0019060 (L.E. Zakharov), and by the China NSF under the grants 11771440 and 11805049 (X. Li).

**References**

[1] Zakharov L E, 2008 *Phys. of Plasmas* 15 062507.
[2] Zakharov L E, S.A. Galkin, S.N. Gerasimov, 2012 Phys. of Plasmas 19 055703.
[3] Gerasimov S N, Hender T C, Morris J, Riccardo V, Zakharov L E and JET EFDA Contributors, 2014 Nucl. Fusion 54 073099.
[4] Xiong H, Xu G, Wang H, Zakharov L E 2015 Phys. of Plasmas 22 060720.
[5] Zakharov L E, Li X 2015 Phys. Plasmas 22 6 062511.
[6] Zakharov L E, Gerasimov S N, Atanasiu C V and Li X 2018 Inductive effects in disruptions (Hiro currents, forces, voltage spikes) and their simulation with VDE code, 2018 European JOREK collaboration meeting, 14.05.2018 18.05.2018, Culham, UK.
[7] Zakharov L E, Xiong H, Hu D, Li X, Atanasiu C V 2013 Hiro currents: physics and a bit of politics, Theory and Simulation of Disruptions Workshop, July 17-19, 2013, PPPL, Princeton, NJ.
[8] Zakharov L E, Atanasiu C V, Lackner K, Hoelzl M, and Strumberger E 2015 J. Plasma Phys. 81 515810610.
[9] Atanasiu C V, Zakharov L E, Lackner K, Hoelzl M, and Strumberger E 2017, Modelling of wall currents excited by plasma wall-touching kink and vertical modes during a tokamak disruption, with application to ITER, 17th European Fusion Theory Conference, Athens Greece, October 9-12, 2017.
[10] Atanasiu C V, Zakharov L E, Lackner K, Hoelzl M, and Strumberger E 2017, Simulation of the electromagnetic wall response to plasma wall-touching kink and vertical modes with application to ITER, 59th Annual Meeting of the APS Division of Plasma Physics, Milwaukee, WI, US, October 23-27, 2017.
[11] Atanasiu C V, Zakharov L E, Lackner K and Hoelzl M 2018, Simulation of the electromagnetic wall response during Vertical Displacement Events (VDE) in ITER tokamak, 7th Int’l Conference on Mathematical Modeling in Physical Sciences, Moscow, Russia, August 27-31, 2018 (oral) arXiv:1810.10277v1.
[12] Zakharov L E, Atanasiu C V, Li X 2017, Interface of wall current modeling with disruption simulation codes, JOREK-STARWALL discussion meeting, IPP, Garching b. M., March 10, 2017.
[13] F.J. Artola, Atanasiu C V, Hoelzl M, Huijsmans G T A, Lackner K, S. Mochalskyy, Oosterwegel G, Strumberger E, Zakharov L E 2018, Second intermediate report for ITER project IO/16/CT/4300001383 on the Implementation and validation of a model for halo-currents in the nonlinear MHD code JOREK and demonstration of 3-D VDEs simulations in ITER, Version 2, March 5th 2018.
[14] Atanasiu C V and Zakharov L E 2013 Phys. Plasmas 20 092506.
[15] Zakharov L E and Pletzer A. 1999, Phys. of Plasmas 6 4693.