THE HISTORY OF GRB OUTFLOWS: EJECTION LORENTZ FACTOR AND RADIATION LOCATION OF X-RAY FLARES

HUI-JUN MU1,2,6, DA-BIN LIN1,2, SHAO-QIANG XU1,3, TING-TING LIN1,2, YUAN-ZHU WANG1,2, YUN-FENG LIANG4, LIAN-ZHONG LO1,2, JIN ZHANG5, and EN-WEI LIANG1,2,5

1 GJU-NAOC Center for Astrophysics and Space Sciences, Department of Physics, Guangxi University, Nanning 530004, China; lindabin@gxu.edu.cn
2 Guangxi Key Laboratory for the Relativistic Astrophysics, Nanning 530004, China
3 Department of Mathematics and Physics, Officers College of CAPF, Chengdu, 610213, China
4 Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
5 National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
6 Department of Astronomy and Institute of Theoretical Physics and Astrophysics, Xiamen University, Xiamen, Fujian 361005, China

Received 2015 February 9; revised 2016 July 28; accepted 2016 August 17; published 2016 October 31

ABSTRACT

We present time-resolved spectral analysis of the steep decay segments of 29 bright X-ray flares of gamma-ray bursts (GRBs) observed with the Swift/X-ray telescope, and model their light curves and spectral index evolution behaviors with the curvature effect model. Our results show that the observed rapid flux decay and strong spectral index evolution with time can be well fitted with this model, and the derived characteristic timescales (τc) are in the range of 23 ~ 264 s. Using an empirical relation between the peak luminosity and the Lorentz factor derived from the prompt gamma-rays, we estimate the Lorentz factors of the flares (ΓX). We obtain ΓX = 17 ~ 87 with a median value of 52, which is smaller than the initial Lorentz factors of prompt gamma-ray fireballs. With the derived τc and ΓX, we constrain the radiating regions of 13 X-ray flares, yielding RX ≈ (0.2 ~ 1.1) × 1017 cm, which are smaller than the radii of the afterglow fireballs at the peak times of the flares. A long evolution feature from prompt gamma-ray phase to the X-ray epoch is found by incorporating our results with a sample of GRBs whose initial Lorentz factors are available in the literature, i.e., Γ ≈ [τp/(1 + z)]1.69±0.06. These results may shed light on the long-term evolution of GRB central engines.

Key words: gamma-ray burst: general

1. INTRODUCTION

Gamma-ray bursts (GRBs) are the most extreme explosive events in the universe. The duration of the prompt gamma-rays ranges from milliseconds to thousands of seconds (Kouveliotou et al. 1993), and afterglows following the prompt gamma-ray phase were found in the X-ray, optical, and radio bands. It is generally believed that the prompt gamma-rays are from internal shocks of collisions among fireball shells and the afterglows are from external shocks when the fireball shells propagate into the circumburst medium (e.g., Mészáros & Rees 1992; Rees & Mészáros 1994; Mészáros & Rees 1997; Piran 2004; Zhang & Mészáros 2004). With promptly slewing capacity, the X-ray telescope (XRT) on board the Swift mission observed erratic flares during the prompt gamma-ray phase and even up to several days post the GRB trigger (e.g., Burrows et al. 2005; Falcone et al. 2006, 2007; Nousek et al. 2006; O’Brien et al. 2006; Zhang et al. 2006; Chincarini et al. 2007, 2010). These flares are found to be of internal origin and they signal the restart of the GRB central engine after the prompt gamma-rays (e.g., Burrows et al. 2005; Fan & Wei 2005; Dai et al. 2006; Liang et al. 2006; Perna et al. 2006; Proga & Zhang 2006; Romano et al. 2006; Wu et al. 2005; Zhang et al. 2006; Maxham & Zhang 2009; Margutti et al. 2010; Guidorzi et al. 2015; Mu et al. 2016).7 Taking these flares into account, the duration of the GRB central engines is much longer than the duration of the prompt gamma-rays (Qin et al. 2013; Virgili et al. 2013; Levan et al. 2014; Zhang et al. 2014).

X-ray flares are one of the most powerful diagnostic tools for the GRB central engines, especially their long-term evolution behavior (e.g., Dai et al. 2006). The radiation region and the bulk Lorentz factor of fireballs for early prompt gamma-rays and late X-ray flares are of great theoretical interest. Although it is still quite uncertain, the radiation region of the prompt gamma-rays is generally believed to be around 1013 ~ 1015 cm and the fireballs are ultra-relativistic with a Lorentz factor (Γγ) being greater than 100 (e.g., Mészáros & Rees 1993; Rees & Mészáros 1994; Piran 2004; Zhang & Mészáros 2004). With the deceleration timescale observed in the afterglow light curves, the derived Γγ values are usually in the range from 100 ~ 1000 (Sari & Piran 1999; Kobayashi & Zhang 2007; Molinari et al. 2007), and they are tightly correlated with the isotropic gamma-ray energy (Eiso) and luminosity (Liso) of the prompt gamma-rays (Γγ−E−iso relation; Liang et al. 2010, 2013; Lu et al. 2012). Furthermore, Liang et al. (2015) discovered a tight relation with a high-energy cutoff in the prompt gamma-ray spectrum based on the “compactness” argument (Fenimore et al. 1993; Woods & Loeb 1995; Baring & Harding 1997; Lithwick & Sari 2001; Gupta & Zhang 2008). More recently, Tang et al. (2014) systematically searched for such a high-energy spectral cutoff/break in GRBs observed with Fermi/LAT. They estimated the Γγ values for nine GRBs with the observed cutoffs by assuming that these cutoffs are caused by pair-production absorption within the sources. They found Γγ values are also larger than 100, and
confirmed the $\Gamma - E_{\gamma, \text{iso}}$ relation. For late X-ray flares, the Lorentz factor ($\Gamma_X$) was suggested to be smaller than that of the fireballs producing the prompt gamma-rays (e.g., Fan & Wei 2005; c.f. Burrows et al. 2005). With the thermal emission observed in X-ray flares, Peng et al. (2014) obtained $\Gamma_X$ of around 60 $\sim$ 150. The curvature effect, which is due to the observer receiving the progressively delayed emission from higher latitudes (Fenimore et al. 1996; Kumar & Panaitescu 2000; Qin 2002; Dermer 2004; Uhrl & Zhang 2015), may present tight constraint on the emission region and its Lorentz factor of X-ray flares (e.g., Lazzati & Begelman 2006; Zhang et al. 2006). Jin et al. (2010) estimated $\Gamma_X$ with both the thermal emission in the flares and the curvature effect on the decay phases of the flares and found that $\Gamma_X$ ranges around tens. The radiation regions of the X-ray flares ($R_X$) are even more poorly known. Troja et al. (2015) analyzed the flares with peaking time at 100 $\sim$ 300 s post the GRB trigger. They showed that $R_X = 10^{13}$ $\sim$ $10^{14}$ cm with $\Gamma_X$ > 50 in the framework of an internal shock model if the variability timescale is significantly shorter than the observed flare duration.

As mentioned above, the origin of the flares may be the same as that of prompt gamma-rays. One may estimate $\Gamma_X$ by assuming that the flares follow the same $\Gamma - L_{\gamma, \text{iso}}$ relation. Furthermore, it is believed that the steep decay observed in the X-ray flares is due to the curvature effect (Dyks et al. 2005; Fan & Wei 2005; Liang et al. 2006; Panaitescu et al. 2006; Wu et al. 2005; Zhang et al. 2006, 2007, 2009; Qin 2008; c.f., Hascoet et al. 2015). This paper is dedicated to the study of $\Gamma_X$ and $R_X$ of X-ray flares, based on the $\Gamma - L_{\gamma, \text{iso}}$ relation and the curvature effect on the X-ray flare tails. We select a sample of 29 bright X-ray flares (Section 2) and fit the light curve and the evolving spectral index during the steep decay phases of these X-ray flares, based on our curvature effect model (Section 3). We constrain $R_X$ and $\Gamma_X$ values based on our fitting results in Section 4. Conclusions and discussions are presented in Section 5. Notation $Q_n = Q/10^n$ in cgs units are adopted.

### 2. SAMPLE AND DATA ANALYSIS

We present an extensive temporal and spectral analysis for the X-ray flares observed with Swift/XRT during ten observation years (from 2005 to 2014). The XRT light curve data are downloaded from the website [http://www.swift.ac.uk](http://www.swift.ac.uk) (Evans et al. 2009) and fitted with a multi-component model comprising a single power law and broken power-law functions. This analysis focuses on the steep decay segments of the flares only. We obtain a sample of 29 X-ray flares which satisfy the following criteria. First, they are bright with $F_X > F_{3 \gamma} > 10$, where $F_X$ and $F_{3 \gamma}$ are the peak flux of the flares and the flux of the underlying afterglow component. Second, the decay segments of the flares clearly decline without superimposing other flares or significant fluctuations. The time-resolved spectral analysis for the steep decay segments of the flares are made with an absorbed single power-law model, i.e., $N(E) = N_0 \times \text{wabs} \times \text{zabs} \times E^{-\beta} \times 1\text{keV}$ and $N(E) = N_0 \times \text{wabs} \times \text{zabs} \times E^{-\beta} \times 1\text{keV}$ with known redshift and unknown redshift, where “wabs” and “zabs” are the photo-electric absorption of both our Galaxy and GRB host galaxies, respectively. Since the gas-to-dust ratio of GRB host galaxies is very uncertain (Starling et al. 2007; Li et al. 2008; Schady et al. 2012), we ignore the dust-scattering effect in the calculation of $N_0$ values and adopt the same absorption model as that of our Galaxy for the GRB host galaxies at redshift $z$. We derive the $N_0$ value of the host galaxy from the X-ray afterglow data for a given burst and make the time-resolved spectral analysis by keeping this value as a constant. The spectral analysis results are reported in Table 1. The evolution of the spectral index with time is also shown in Figure 1, where the peak time of the X-ray flare is set as the beginning time ($t = 0$) of the steep decay segment for our analysis.

### 3. MODELING THE STEEP DECAY SEGMENTS IN THE CURVATURE EFFECT SCENARIO

As mentioned in Section 1, the steep decay segment of the flares with a slope $\alpha = 2 + \beta$ would be due to the curvature effect, where $\beta$ is the power-law index of the radiation spectrum (e.g., Fenimore et al. 1996; Kumar & Panaitescu 2000; Dermer 2004; Liang et al. 2006; Zhang et al. 2007). The curvature effect is a combination of the time delay and the Doppler shifting of the intrinsic spectrum for high latitude emission with respect to that in the light of sight. The time delay of photons from radius $R_X$ and latitude angle $\theta$ with respect to those from $R_X$ and $\theta = 0$ is given by

$$t = (1 + z)(R_X/c)(1 - \cos \theta),$$

where $z$ is the redshift of the studied source and $c$ is the speed of light. For a relativistic moving jet with a Lorentz factor $\Gamma_X$, the comoving emission frequency $\nu'$ is boosted to $\nu = D\nu'$ in the observer’s frame, where $D$ is the Doppler factor described as

$$D = [\Gamma_X(1 - \beta_{\text{jet}} \cos \theta)]^{-1} \approx \left[ \frac{1}{2\Gamma_X} + \Gamma_X(1 - \cos \theta) \right]^{-1} = \frac{2\Gamma_X}{(1 + t/t_c)},$$

where $\beta_{\text{jet}} c$ is the jet velocity, and $t_c$ is a characteristic timescale of the curvature effect, which is

$$t_c = \frac{R_X(1 + z)}{2\Gamma_X^2 c}.$$  

In the case of a single power-law radiation spectrum, the observed spectral index would not evolve with time (e.g., Fenimore et al. 1996; Dermer 2004; Liang et al. 2006). The observed significant spectral softening, as shown in Figure 1, would be due to the curvature effect on a curved radiation spectrum (e.g., Zhang et al. 2007, 2009). Following Zhang et al. (2009), we fit the light curves and the spectral evolution features of the steep decay segments in our sample in the curvature effect scenario. We take the intrinsic radiation spectrum as a cutoff power law parameterized as (Zhang et al. 2009)\footnote{The choice of this function was also due to the fact that the spectral evolution of some GRB tails can be fitted by such an empirical model (Campana et al. 2006; Yonetoku et al. 2008; Zhang et al. 2007).},

$$N'(E') = N_0 \left( \frac{E'}{1\text{keV}} \right)^{\beta} \exp \left[ -\left( \frac{E'}{E_c'} \right)^\kappa \right],$$

where $\beta$ is the photon index and $E_c'$ is the cutoff energy. Parameter $\kappa$ measures the steepness of the spectrum at $E' > E_c'$, as shown in Figure 2. We normally take $\kappa = 1$ in this analysis. Zhang et al. (2009) showed that such an intrinsic radiation spectrum can present the observed spectral index and...
| GRB       | Interval (s) | Mean Time (s) | $\beta$     | $N^{\text{best}}$ (10$^{32}$ cm$^{-2}$) | C$_{\text{stat/bin}}$ |
|-----------|-------------|--------------|------------|----------------------------------------|--------------------|
| 050502B   | 714–767     | 760          | 1.14$^{+0.09}_{-0.09}$ | 0.03                        | 878/859           |
| ...       | 768–792     | 780          | 1.17$^{+0.08}_{-0.07}$ | ...                        | ...               |
| ...       | 793–825     | 808          | 1.32$^{+0.08}_{-0.08}$ | ...                        | ...               |
| ...       | 827–887     | 853          | 1.53$^{+0.08}_{-0.08}$ | ...                        | ...               |
| ...       | 892–1021    | 945          | 1.66$^{+0.10}_{-0.09}$ | ...                        | ...               |
| ...       | 1031–1130   | 1062         | 1.96$^{+0.22}_{-0.21}$ | ...                        | ...               |
| 060111A   | 283–294     | 288          | 0.89$^{+0.09}_{-0.08}$ | 0.13                       | 810/743           |
| ...       | 295–319     | 308          | 1.12$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 320–331     | 326          | 0.83$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 332–347     | 339          | 1.06$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 347–370     | 358          | 1.16$^{+0.11}_{-0.11}$ | ...                        | ...               |
| ...       | 371–410     | 387          | 1.05$^{+0.11}_{-0.11}$ | ...                        | ...               |
| ...       | 413–483     | 443          | 1.19$^{+0.12}_{-0.12}$ | ...                        | ...               |
| 060124    | 761–771     | 766          | 2.00$^{+0.08}_{-0.08}$ | 0.13                       | 630/623           |
| ...       | 772–782     | 777          | 2.12$^{+0.10}_{-0.09}$ | ...                        | ...               |
| ...       | 782–797     | 789          | 2.33$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 798–813     | 805          | 2.38$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 814–836     | 824          | 2.43$^{+0.11}_{-0.10}$ | ...                        | ...               |
| 060904B   | 172–184     | 178          | 0.93$^{+0.09}_{-0.09}$ | 0.49                       | 692/645           |
| ...       | 187–220     | 196          | 1.33$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 207–252     | 212          | 1.66$^{+0.17}_{-0.16}$ | ...                        | ...               |
| ...       | 219–278     | 225          | 1.86$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 233–308     | 240          | 2.22$^{+0.12}_{-0.12}$ | ...                        | ...               |
| ...       | 249–340     | 256          | 2.53$^{+0.15}_{-0.14}$ | ...                        | ...               |
| ...       | 268–396     | 284          | 2.98$^{+0.18}_{-0.17}$ | ...                        | ...               |
| 060929    | 527–542     | 535          | 0.46$^{+0.10}_{-0.10}$ | 0.11                       | 500/470           |
| ...       | 543–557     | 550          | 0.77$^{+0.16}_{-0.15}$ | ...                        | ...               |
| ...       | 557–574     | 565          | 0.80$^{+0.11}_{-0.11}$ | ...                        | ...               |
| ...       | 575–600     | 587          | 0.97$^{+0.11}_{-0.11}$ | ...                        | ...               |
| ...       | 602–640     | 620          | 1.24$^{+0.12}_{-0.12}$ | ...                        | ...               |
| ...       | 642–709     | 668          | 1.40$^{+0.13}_{-0.13}$ | ...                        | ...               |
| 070520B   | 180–184     | 182          | 1.01$^{+0.15}_{-0.14}$ | 0.20                       | 563/453           |
| ...       | 185–197     | 190          | 1.47$^{+0.17}_{-0.17}$ | ...                        | ...               |
| ...       | 197–214     | 205          | 1.50$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 216–239     | 227          | 1.77$^{+0.11}_{-0.11}$ | ...                        | ...               |
| ...       | 239–269     | 253          | 2.00$^{+0.12}_{-0.12}$ | ...                        | ...               |
| ...       | 270–340     | 298          | 2.05$^{+0.14}_{-0.14}$ | ...                        | ...               |
| 070704    | 338–350     | 344          | 0.93$^{+0.13}_{-0.13}$ | 0.32                       | 977/924           |
| ...       | 351–365     | 358          | 0.96$^{+0.13}_{-0.13}$ | ...                        | ...               |
| ...       | 366–386     | 376          | 1.04$^{+0.13}_{-0.13}$ | ...                        | ...               |
| ...       | 387–420     | 401          | 1.26$^{+0.13}_{-0.13}$ | ...                        | ...               |
| ...       | 428–523     | 468          | 1.55$^{+0.19}_{-0.18}$ | ...                        | ...               |
| 080928    | 355–361     | 358          | 0.39$^{+0.14}_{-0.13}$ | 0.38                       | 259/257           |
| ...       | 361–368     | 364          | 0.69$^{+0.15}_{-0.15}$ | ...                        | ...               |
| ...       | 369–377     | 373          | 0.78$^{+0.15}_{-0.15}$ | ...                        | ...               |
| ...       | 378–391     | 384          | 0.86$^{+0.17}_{-0.16}$ | ...                        | ...               |
| ...       | 392–412     | 402          | 0.97$^{+0.17}_{-0.17}$ | ...                        | ...               |
| 091130B   | 103–112     | 107          | 0.91$^{+0.12}_{-0.12}$ | 0.31                       | 657/665           |
| ...       | 112–122     | 117          | 1.46$^{+0.20}_{-0.19}$ | ...                        | ...               |
| ...       | 123–132     | 128          | 1.61$^{+0.17}_{-0.16}$ | ...                        | ...               |
| ...       | 134–146     | 139          | 1.70$^{+0.17}_{-0.17}$ | ...                        | ...               |
| ...       | 147–161     | 154          | 1.85$^{+0.21}_{-0.20}$ | ...                        | ...               |
| ...       | 163–177     | 169          | 2.38$^{+0.26}_{-0.25}$ | ...                        | ...               |
| 100619A   | 949–968     | 959          | 0.73$^{+0.10}_{-0.10}$ | 0.47                       | 818/772           |
| ...       | 969–991     | 980          | 1.01$^{+0.16}_{-0.15}$ | ...                        | ...               |
| ...       | 992–1019    | 1005         | 1.12$^{+0.10}_{-0.10}$ | ...                        | ...               |
| ...       | 1020–1052   | 1036         | 1.43$^{+0.11}_{-0.11}$ | ...                        | ...               |
| GRB     | Interval (s) | Mean Time (s) | $\beta$     | $N_{\text{H}}$ (10$^{25}$ cm$^{-2}$) | $C_{\text{stat/bin}}$ |
|---------|-------------|--------------|-------------|---------------------------------|----------------------|
| ...     | 1053–1097   | 1073         | 1.44$^{+0.11}_{-0.11}$ | ...                             | ...                  |
| ...     | 1100–1191   | 1137         | 1.64$^{+0.12}_{-0.12}$ | ...                             | ...                  |
| ...     | 1195–1286   | 1234         | 2.06$^{+0.18}_{-0.18}$ | ...                             | ...                  |
| 100704A | 196–228     | 208          | 1.37$^{+0.07}_{-0.08}$ | 0.31                            | 1064/820             |
| ...     | 228–242     | 234          | 2.10$^{+0.13}_{-0.12}$ | ...                             | ...                  |
| ...     | 242–254     | 248          | 2.43$^{+0.17}_{-0.16}$ | ...                             | ...                  |
| ...     | 255–264     | 259          | 2.60$^{+0.20}_{-0.19}$ | ...                             | ...                  |
| ...     | 265–275     | 270          | 2.96$^{+0.23}_{-0.23}$ | ...                             | ...                  |
| ...     | 270–330     | 300          | 3.24$^{+0.23}_{-0.13}$ | ...                             | ...                  |
| 100802A | 507–538     | 522          | 0.81$^{+0.07}_{-0.07}$ | 0.03                            | 995/826              |
| ...     | 541–564     | 552          | 1.04$^{+0.13}_{-0.13}$ | ...                             | ...                  |
| ...     | 567–594     | 581          | 1.03$^{+0.09}_{-0.09}$ | ...                             | ...                  |
| ...     | 598–635     | 615          | 1.16$^{+0.09}_{-0.09}$ | ...                             | ...                  |
| ...     | 640–707     | 668          | 1.25$^{+0.10}_{-0.09}$ | ...                             | ...                  |
| ...     | 719–818     | 784          | 1.41$^{+0.12}_{-0.12}$ | ...                             | ...                  |
| 100902A | 418–431     | 424          | 0.95$^{+0.05}_{-0.07}$ | ...                             | ...                  |
| ...     | 432–454     | 442          | 1.86$^{+0.10}_{-0.10}$ | ...                             | ...                  |
| ...     | 457–473     | 464          | 2.31$^{+0.13}_{-0.14}$ | ...                             | ...                  |
| ...     | 473–482     | 477          | 2.49$^{+0.14}_{-0.15}$ | ...                             | ...                  |
| ...     | 482–492     | 487          | 2.62$^{+0.16}_{-0.16}$ | ...                             | ...                  |
| ...     | 493–511     | 501          | 2.84$^{+0.15}_{-0.15}$ | ...                             | ...                  |
| 110801A | 453–472     | 462          | 1.53$^{+0.09}_{-0.09}$ | ...                             | 1103/833             |
| ...     | 473–486     | 479          | 1.71$^{+0.13}_{-0.13}$ | ...                             | ...                  |
| ...     | 487–507     | 496          | 1.81$^{+0.13}_{-0.13}$ | ...                             | ...                  |
| 110820A | 258–262     | 260          | −0.08$^{+0.31}_{-0.21}$ | ...                             | 181/157              |
| ...     | 262–267     | 265          | 0.32$^{+0.42}_{-0.36}$ | ...                             | ...                  |
| ...     | 267–271     | 269          | 0.51$^{+0.21}_{-0.20}$ | ...                             | ...                  |
| ...     | 298–328     | 309          | 1.28$^{+0.28}_{-0.20}$ | ...                             | ...                  |
| 121027A | 259–277     | 268          | 1.19$^{+0.14}_{-0.15}$ | 1.33                            | 338/322              |
| ...     | 278–303     | 290          | 1.58$^{+0.14}_{-0.15}$ | ...                             | ...                  |
| ...     | 305–337     | 320          | 1.50$^{+0.15}_{-0.14}$ | ...                             | ...                  |
| ...     | 339–392     | 363          | 1.56$^{+0.15}_{-0.15}$ | ...                             | ...                  |
| ...     | 397–441     | 418          | 1.65$^{+0.19}_{-0.18}$ | ...                             | ...                  |
| ...     | 446–494     | 467          | 1.44$^{+0.23}_{-0.23}$ | ...                             | ...                  |
| 121211A | 182–203     | 192          | 1.44$^{+0.10}_{-0.10}$ | 1.20                            | 1211/818             |
| ...     | 204–228     | 215          | 1.58$^{+0.11}_{-0.10}$ | ...                             | ...                  |
| ...     | 229–241     | 235          | 1.78$^{+0.13}_{-0.13}$ | ...                             | ...                  |
| ...     | 242–253     | 247          | 1.95$^{+0.14}_{-0.13}$ | ...                             | ...                  |
| ...     | 253–265     | 259          | 2.55$^{+0.15}_{-0.15}$ | ...                             | ...                  |
| ...     | 265–277     | 271          | 2.81$^{+0.18}_{-0.17}$ | ...                             | ...                  |
| 121229A | 458–467     | 462          | 1.13$^{+0.17}_{-0.17}$ | 0.50                            | 352/309              |
| ...     | 468–478     | 473          | 1.07$^{+0.17}_{-0.17}$ | ...                             | ...                  |
| ...     | 479–489     | 484          | 1.06$^{+0.16}_{-0.16}$ | ...                             | ...                  |
| ...     | 490–501     | 496          | 1.14$^{+0.17}_{-0.17}$ | ...                             | ...                  |
| ...     | 490–518     | 510          | 1.49$^{+0.18}_{-0.17}$ | ...                             | ...                  |
| ...     | 490–545     | 531          | 1.54$^{+0.18}_{-0.17}$ | ...                             | ...                  |
| ...     | 490–585     | 564          | 1.76$^{+0.21}_{-0.20}$ | ...                             | ...                  |
| ...     | 490–619     | 601          | 2.01$^{+0.27}_{-0.28}$ | ...                             | ...                  |
| 130131A | 293–310     | 301          | 1.45$^{+0.08}_{-0.08}$ | 0.30                            | 281/219              |
| ...     | 311–332     | 320          | 1.94$^{+0.18}_{-0.13}$ | ...                             | ...                  |
| ...     | 335–391     | 355          | 2.25$^{+0.16}_{-0.15}$ | ...                             | ...                  |
| 130615A | 356–368     | 362          | 0.62$^{+0.09}_{-0.09}$ | 0.00                            | 764/659              |
| ...     | 368–383     | 375          | 0.64$^{+0.09}_{-0.09}$ | ...                             | ...                  |
| ...     | 384–400     | 391          | 0.73$^{+0.09}_{-0.09}$ | ...                             | ...                  |
| ...     | 401–421     | 411          | 0.85$^{+0.10}_{-0.09}$ | ...                             | ...                  |
| ...     | 422–451     | 436          | 1.00$^{+0.09}_{-0.09}$ | ...                             | ...                  |
| ...     | 453–501     | 474          | 1.15$^{+0.09}_{-0.09}$ | ...                             | ...                  |
| GRB        | Interval (s) | Mean Time (s) | \( \beta \) | \( N_{\text{H}_{\text{best}}} \) (10^{24} cm^{-2}) | \( C_{\text{stat/bin}} \) |
|------------|--------------|---------------|-------------|--------------------------------|------------------|
| ...        | ...          | ...           | ...         | ...                            | ...              |
| 130925A1   | 505–591      | 540           | 1.37\( ^{+0.12}_{-0.10} \) | ...                            | ...              |
| ...        | ...          | ...           | ...         | ...                            | ...              |
| 130925A2   | 5020–5031    | 5026          | 0.51\( ^{+0.10}_{-0.10} \) | 1.59                           | 1189/1218        |
| ...        | 130925A2     | ...           | ...         | ...                            | ...              |
| 131030A    | 116–125      | 120           | 0.90\( ^{+0.09}_{-0.09} \) | 0.61                           | 907/769          |
| ...        | 126–138      | 132           | 1.23\( ^{+0.10}_{-0.10} \) | ...                            | ...              |
| ...        | 140–160      | 149           | 1.51\( ^{+0.09}_{-0.09} \) | ...                            | ...              |
| ...        | 160–181      | 170           | 1.75\( ^{+0.09}_{-0.08} \) | ...                            | ...              |
| ...        | 182–207      | 193           | 2.03\( ^{+0.09}_{-0.09} \) | ...                            | ...              |
| ...        | 208–218      | 213           | 1.99\( ^{+0.12}_{-0.12} \) | ...                            | ...              |
| ...        | 219–225      | 222           | 2.09\( ^{+0.16}_{-0.16} \) | ...                            | ...              |
| 140430A    | 222–233      | 227           | 1.76\( ^{+0.16}_{-0.16} \) | 0.91                           | 319/298          |
| ...        | 233–239      | 236           | 2.06\( ^{+0.18}_{-0.18} \) | ...                            | ...              |
| ...        | 239–246      | 242           | 2.49\( ^{+0.20}_{-0.20} \) | ...                            | ...              |
| ...        | 247–257      | 251           | 2.78\( ^{+0.20}_{-0.20} \) | ...                            | ...              |
| ...        | 259–274      | 266           | 3.22\( ^{+0.27}_{-0.27} \) | ...                            | ...              |
| 140506A    | 359–372      | 365           | 2.64\( ^{+0.15}_{-0.14} \) | 0.64                           | 772/567          |
| ...        | 373–384      | 378           | 2.38\( ^{+0.11}_{-0.11} \) | ...                            | ...              |
| ...        | 385–394      | 389           | 2.57\( ^{+0.14}_{-0.14} \) | ...                            | ...              |
| ...        | 395–417      | 405           | 2.76\( ^{+0.12}_{-0.12} \) | ...                            | ...              |
| ...        | 417–440      | 428           | 3.11\( ^{+0.15}_{-0.15} \) | ...                            | ...              |
| ...        | 441–483      | 459           | 3.40\( ^{+0.16}_{-0.16} \) | ...                            | ...              |
| 140512A    | 144–150      | 147           | 0.16\( ^{+0.11}_{-0.11} \) | 0.20                           | 346/304          |
| ...        | 151–156      | 153           | 0.51\( ^{+0.14}_{-0.14} \) | ...                            | ...              |
| ...        | 157–163      | 160           | 0.64\( ^{+0.13}_{-0.13} \) | ...                            | ...              |
| ...        | 164–173      | 169           | 0.88\( ^{+0.17}_{-0.17} \) | ...                            | ...              |
| ...        | 175–187      | 180           | 1.06\( ^{+0.19}_{-0.19} \) | ...                            | ...              |
| 140709A    | 189–192      | 190           | 1.97\( ^{+0.33}_{-0.32} \) | 0.28                           | 187/187          |
| ...        | 193–199      | 196           | 1.94\( ^{+0.29}_{-0.28} \) | ...                            | ...              |
| ...        | 200–206      | 203           | 2.02\( ^{+0.24}_{-0.24} \) | ...                            | ...              |
| ...        | 207–216      | 211           | 2.33\( ^{+0.31}_{-0.29} \) | ...                            | ...              |
| ...        | 218–235      | 225           | 2.80\( ^{+0.40}_{-0.36} \) | ...                            | ...              |
| 141031A    | 1104–1109    | 1106          | 1.09\( ^{+0.29}_{-0.28} \) | 0.12                           | 263/220          |
| ...        | 1111–1139    | 1124          | 1.10\( ^{+0.13}_{-0.12} \) | ...                            | ...              |
| ...        | 1141–1164    | 1152          | 1.05\( ^{+0.19}_{-0.19} \) | ...                            | ...              |
| ...        | 1167–1187    | 1177          | 0.95\( ^{+0.20}_{-0.20} \) | ...                            | ...              |
| ...        | 1191–1225    | 1206          | 1.06\( ^{+0.20}_{-0.20} \) | ...                            | ...              |
| ...        | 1229–1256    | 1241          | 0.92\( ^{+0.29}_{-0.29} \) | ...                            | ...              |
| ...        | 1262–1578    | 1264          | 0.51\( ^{+0.44}_{-0.44} \) | ...                            | ...              |
| 141130A    | 329–336      | 332           | 2.22\( ^{+0.39}_{-0.36} \) | 0.20                           | 110/126          |
| ...        | 338–352      | 344           | 2.64\( ^{+0.40}_{-0.39} \) | ...                            | ...              |
| ...        | 355–376      | 364           | 2.77\( ^{+0.37}_{-0.35} \) | ...                            | ...              |
| ...        | 380–404      | 391           | 3.26\( ^{+0.46}_{-0.43} \) | ...                            | ...              |
| ...        | 409–445      | 425           | 3.11\( ^{+0.48}_{-0.45} \) | ...                            | ...              |
Figure 1. Light curves (upper part of each panel) and spectral index $\beta$ evolution (lower part of each panel) of the steep decay segment in our selected 29 flares. The zero time is set to the peak time of the flares to timing the flare tails. Our joint fits are also shown with solid curves. The inset shows the light curve in 0.3–10 keV of XRT observation (red and gray) and extrapolation based on BAT observation (blue), where the red is the data we fitted. The blue dotted line is $\beta = 1$. 

The Astrophysical Journal, 831:111 (14pp), 2016 November 1 Mu et al.
Figure 1. (Continued.)
Figure 1. (Continued.)
flux evolution behavior of GRB 050814 with the curvature effect model. The observed flux at photon energy $E$ then can be calculated with $F_E \propto D^2 E'N'(E')$, i.e.,

$$F_E(t) = F_{E,0}(1 + t/t_c)^2 \beta^{-1}\exp\left[-\frac{E}{E_c,0}\left(1 + \frac{t}{t_c}\right)\right](E/1\text{keV})^{-\beta+1},$$

where $F_{E,0}$ and $E_{c,0} \equiv 2\Gamma X E_c'$ are the observed on-axis flux and cutoff photon energy (corresponding to $t = 0$). The observed flux in the XRT band can then be given by

$$F_{\text{XRT}}(t) = \int_{0.3\text{keV}}^{10\text{ keV}} F_E dE.$$  

We simply calculate the observed spectral index in the XRT band at $t$ with

$$\beta(t) = -\frac{\log(F_{10\text{keV}}(t)) - \log(F_{0.3\text{keV}}(t))}{\log(10\text{ keV}) - \log(0.3\text{ keV})}.$$  

We make jointed fits to the light curves and $\beta$ evolution with Equations (6) and (7). The peak time of flares is set as the zero time $t = 0$ in these equations. The goodness of our fits is evaluated with the total reduced $\chi^2$ by weighting the data points between those in the light curves and in the spectral

9 Note that our fits by setting $t_0$ as a free parameter may lead to unreasonable results due to the degeneracy of $t_c$ and $t_0$ in our model. Liang et al. (2006) showed that the $t_0$ values are in the rising segment of the corresponding pulses or flares. Since the flux of flares usually rapidly increases in the rising segment, we simply set the zero time of the flares at the peak times in this analysis.
evolution. The total \( \chi^2 \) value is given by \( \chi^2 = \chi^2_{\text{LC}} + \chi^2_{\beta} \), where subscripts “LC” and “\( \beta \)” indicate the light curves and the \( \beta \) evolution curves. We minimize \( \chi^2 \) in our fits.

Only 13 GRBs in our sample have redshift measurement. Our fit curves are displayed in Figure 1, and the model parameters, including \( F_{\text{E90}}, \beta, E_{\gamma,0} \), and \( t_c \), are reported in Table 2. One can observed that our model can represent the observed flux decay and spectral index evolution in these flares. The \( t_c \) values range from 23–264 s, as shown in Figure 3.

4. THE LORENTZ FACTOR AND RADIATION LOCATION OF X-RAY FLARES

As shown in Equation (3), one may estimate \( R_X \) or \( \Gamma_X \) with the derived \( t_c \) if one of them is available by another way. By deriving \( R_X \) with a tentative jet break, Jin et al. (2010) got \( \Gamma_X = 22, 13 \) for GRBs 050502B and 050724, respectively. Note that a tight \( \Gamma_X-E_{\gamma,\text{iso}} \) or \( \Gamma_X-L_{\gamma,\text{iso}} \) relation were found by Liang et al. (2010) and Lü et al. (2012). Assuming that the flares follow the \( \Gamma_X-L_{\gamma,\text{iso}} \) relation, one can use this relation to estimate \( \Gamma_X \), then derive \( R_X \) with Equation (3).

Lü et al. (2012) derived the \( \Gamma_X-L_{\gamma,\text{iso}} \) relation by using the average luminosity of the prompt gamma-rays. Since burst durations depend on the energy band selected (e.g., Qin et al. 2013), it is difficult to accurately measure the duration of flares. Therefore, we use the peak luminosity of the flares for our analysis. In addition, Lü et al. (2012) adopted the ordinary least-squares regression method to obtain the regression line. For regression analysis, the fitting results depend on the specification of dependent and independent variables (Isidebe et al. 1990). One may find discrepancy in the relations among variables by specifying different dependent variables for a given data set, especially when the data have large error bars or scatters. To avoid specifying independent and dependent variables in the best linear fits, we adopt the algorithm of the bisector of two ordinary least-squares to re-do the regression analysis. The derived \( \Gamma_X-L_{\gamma,p} \) relation is

\[
\log \Gamma_X = (2.27 \pm 0.04) + (0.34 \pm 0.03) \log L_{\gamma,p,52}
\]

It is roughly consistent with that reported in Lü et al. (2012). We check if X-ray flares follow this relation with a flare observed in GRB 050724. With its \( L_{\gamma,p} = 9.2 \times 10^{47} \text{ erg s}^{-1} \), we get \( \Gamma = 7.9 \), which is comparable to that reported by Jin et al. (2010), i.e., \( \Gamma_X = 13 \). We, thus, use this correlation to estimate the \( \Gamma_X \) with the peak luminosity of the flares for the 13 GRBs whose redshifts are available. Our results are reported in Table 3. One can find that \( \Gamma_X = 17 \sim 87 \), with a median value of \( \Gamma_X = 52 \). The derived \( \Gamma_X \) values are generally consistent with those reported in previous papers (e.g., Fan & Wei 2005; Falcone et al. 2006; Panaitescu 2006).

It is generally believed that X-ray flares are powered by the late activities of the GRB central engine. The derived \( \Gamma_X \) values in this analysis are systematically smaller than the \( \Gamma_0 \) values of the prompt gamma-rays as reported in Liang et al. (2010, 2013) and Tang et al. (2014). In addition, with the thermal emission observed in the joint BAT and XRT spectra of 13 early flares, which are usually observed at the end of the prompt gamma-ray phase, Peng et al. (2014) derived the \( \Gamma_X \) values for these flares by assuming that the thermal emission is the photosphere emission of the GRB fireballs. They found that the Lorentz factors range between 50 and 150. They are also systematically larger than the \( \Gamma_X \) values of late flares in our analysis. To explore possible the evolution feature of the Lorentz factor, we plot \( \Gamma \) as a function of \( t_c \) in the burst frame for prompt gamma-rays and X-ray flares in Figure 4 with samples from Liang et al. (2013), Peng et al. (2014), Troja et al. (2015), Tang et al. (2014), and Fan & Wei (2005), where the \( t_c \) of prompt gamma-rays is taken as the middle of the burst duration. One can observe a trend that \( \Gamma \) values (both \( \Gamma_0 \) and \( \Gamma_X \)) decay with time. The Spearman correlation analysis yields a correlation coefficient \( r = 0.70 \) and a chance probability \( p < 10^{-4} \). This may illustrate a long-term evolution feature of the GRB central engines (e.g., Lazzati et al. 2008). We derive the relation of \( \Gamma \) to \( t_c/(1+z) \) with the algorithm of the bisector of two ordinary least-squares, which yields \( \Gamma = 10^{3.05 \pm 0.11} \times (t_c/(1+z))^{-0.69 \pm 0.06} \).

With the derived \( \Gamma_X \), we calculate \( R_X \) values for the 13 GRBs with Equation (8). The results are also reported in Table 3. It is found that \( R_X = 2.0 \times 10^{15} \sim 1.1 \times 10^{16} \text{ cm} \), with a median value of \( R_X = 6.5 \times 10^{15} \text{ cm} \). The radiation regions of the flares are within the regions of the prompt gamma-rays and afterglows. It was generally accepted that the locations of the prompt gamma-rays and afterglows are \( \sim 10^{13} \) and \( 10^{17} \text{ cm} \) (e.g., Zhang & Mészáros 2004), respectively. Troja et al. (2015) analyzed the flares with peaking time at 100–300 s post the GRB trigger. They found \( R_X \sim 10^{13} \sim 10^{14} \text{ cm} \) with \( \Gamma_X > 50 \) in the framework of internal shock models, if the variability timescale is significantly shorter than the observed flare duration. Their \( R_X \) values are consistent with those producing early prompt gamma-rays. However, as shown in Peng et al. (2014), the photosphere radii of the flares at the end of the prompt gamma-ray emission phase in their sample are usually \( 10^{13} \text{ cm} \). The internal shock regions of these flares should be beyond the photosphere radius, i.e., \( R_X > 10^{13} \text{ cm} \). The \( R_X \) values for the late flares in this analysis are in the range of \( 2.0 \times 10^{15} \sim 1.1 \times 10^{16} \text{ cm} \), which is much larger than that of the prompt gamma-rays. We estimate the fireball radii of the afterglows at the flare epochs with \( R_{AG} = 1.1 \times 10^{17} \text{ cm} \).
Table 2
Curvature Effect Model Fits to the Evolution Behaviors of Flux and Spectral Index in the Steep Decay Phase of the X-Ray Flares in Our Sample

| GRB   | $E_{k,0}$  | $\beta$  | $E_{c,0}$ (keV) | $t_c$ (s) | $\chi^2$/dof |
|-------|------------|----------|-----------------|-----------|-------------|
| 050502B | 1.77 ± 0.05 | 1.69 ± 0.04 | 5.72 ± 0.24 | 171.71 ± 3.38 | 1.32 |
| 060111A | 2.28 ± 0.07 | 1.51 ± 0.04 | 7.76 ± 0.24 | 121.13 ± 2.17 | 3.03 |
| 060124  | 2.74 ± 0.16 | 1.04 ± 0.06 | 2.90 ± 0.12 | 123.25 ± 4.58 | 1.80 |
| 060904B | 16.46 ± 0.72 | 0.93 ± 0.18 | 2.54 ± 0.24 | 65.39 ± 1.41 | 3.98 |
| 060929  | 1.37 ± 0.07 | 1.10 ± 0.06 | 4.62 ± 0.16 | 121.01 ± 2.00 | 8.31 |
| 070520B | 2.33 ± 0.12 | 1.56 ± 0.06 | 3.28 ± 0.10 | 138.59 ± 2.71 | 3.76 |
| 070704  | 3.58 ± 0.19 | 1.17 ± 0.06 | 3.96 ± 0.12 | 129.90 ± 2.39 | 4.42 |
| 080928  | 1.03 ± 0.08 | 0.68 ± 0.10 | 3.42 ± 0.26 | 67.73 ± 3.13 | 1.36 |
| 091130B | 9.05 ± 0.52 | 0.28 ± 0.53 | 1.64 ± 0.46 | 86.13 ± 2.34 | 2.09 |
| 100619A | 2.73 ± 0.10 | 1.12 ± 0.05 | 3.22 ± 0.06 | 253.92 ± 3.21 | 6.51 |
| 100704A | 24.07 ± 0.99 | 1.11 ± 0.16 | 2.74 ± 0.20 | 49.50 ± 1.13 | 3.32 |
| 100802A | 2.14 ± 0.07 | 1.23 ± 0.05 | 4.62 ± 0.16 | 242.57 ± 4.42 | 2.93 |
| 100902B | 52.50 ± 1.65 | 1.12 ± 0.15 | 2.88 ± 0.24 | 43.86 ± 0.76 | 6.64 |
| 110801A | 2.40 ± 0.41 | 1.46 ± 0.08 | 2.84 ± 0.14 | 113.27 ± 5.56 | 1.24 |
| 110820A | 2.59 ± 0.56 | 0.31 ± 0.28 | 4.32 ± 1.08 | 23.98 ± 1.19 | 6.01 |
| 112027A | 1.06 ± 0.14 | 1.81 ± 0.05 | 5.38 ± 0.14 | 264.63 ± 8.92 | 1.64 |
| 121011A | 5.96 ± 0.23 | 0.10 ± 0.08 | 1.22 ± 0.02 | 179.31 ± 6.44 | 2.22 |
| 121229A | 1.26 ± 0.06 | 1.35 ± 0.07 | 3.74 ± 0.16 | 130.88 ± 3.18 | 3.69 |
| 130113A | 2.02 ± 0.16 | 2.00 ± 0.05 | 9.52 ± 0.20 | 57.87 ± 1.75 | 3.24 |
| 130615A | 1.05 ± 0.04 | 0.88 ± 0.06 | 3.94 ± 0.16 | 156.82 ± 3.20 | 2.34 |
| 130925A1 | 5.18 ± 0.14 | 0.66 ± 0.04 | 3.56 ± 0.04 | 157.63 ± 1.35 | 13.79 |
| 130925A2 | 6.61 ± 0.15 | 0.39 ± 0.04 | 2.38 ± 0.02 | 247.51 ± 1.45 | 18.70 |
| 131030A | 47.79 ± 1.86 | 1.46 ± 0.14 | 4.54 ± 0.54 | 54.61 ± 1.40 | 1.76 |
| 140430A | 7.25 ± 0.64 | 2.25 ± 0.07 | 3.42 ± 0.12 | 51.52 ± 1.36 | 2.48 |
| 140506A | 11.43 ± 1.38 | 1.96 ± 0.97 | 2.04 ± 1.14 | 129.17 ± 2.95 | 1.21 |
| 140512A | 3.11 ± 0.26 | 0.10 ± 0.54 | 2.54 ± 0.12 | 43.60 ± 2.83 | 0.88 |
| 140709A | 10.43 ± 1.76 | 1.82 ± 0.11 | 3.00 ± 0.12 | 33.55 ± 1.16 | 1.90 |
| 141031A | 0.73 ± 0.05 | 1.44 ± 0.07 | 5.86 ± 0.16 | 180.30 ± 6.71 | 1.39 |
| 141130A | 0.66 ± 0.35 | 2.11 ± 0.17 | 2.04 ± 0.14 | 167.36 ± 10.17 | 1.78 |

![Figure 3](image_url)

Figure 3. Distributions of $t_c$ derived from our fits.

The derived $\Gamma$, $t_c$, and $R_X$ from the fits of Table 2 are plotted in Figure 4. The derived $\Gamma$, $t_c$, and $R_X$ are within the range of $23 \sim 264$ s, spreading about one order of magnitude. Using the relation between the peak luminosity and the Lorentz factor derived from the prompt gamma-rays, we estimate the Lorentz factors of the flares and find $\Gamma_X = 17 \sim 87$ with a median value of 52. With the flares in our sample, together with samples collected from the literature for prompt gamma-ray emission and early X-ray flares, we find a tentative correlation between the Lorentz factor and the time of the flares (or the middle time of the prompt gamma-ray duration), i.e., $\Gamma \propto \frac{t_p}{(1 + z)}^{0.99 \pm 0.06}$. With the derived $t_c$ and $\Gamma_X$, we constrain the radiating region as $R_X = 2.0 \times 10^{15} \sim 1.1 \times 10^{16}$ cm, with a median value of $6.5 \times 10^{15}$ cm. The radiation regions of the flares are within the regions of the prompt gamma-rays and afterglows, and the narrow distribution of $R_X$ of the flares in this analysis would be due to the sample selection effect since we select only late flares in order to eliminate the contamination of adjacent flares.

5. CONCLUSIONS AND DISCUSSION

We have fitted the light curves and the spectral evolution during the steep decay segment in 29 late X-ray flares with the curvature effect model. We show that this model may well represent both the spectral and temporal behaviors in the steep decay segments. The derived characteristic timescale $t_c$ are in the range of $23 \sim 264$ s, spreading about one order of magnitude. Using the relation between the peak luminosity and the Lorentz factor derived from the prompt gamma-rays, we estimate the Lorentz factors of the flares and find $\Gamma_X = 17 \sim 87$ with a median value of 52. With the flares in our sample, together with samples collected from the literature for prompt gamma-ray emission and early X-ray flares, we find a tentative correlation between the Lorentz factor and the time of the flares (or the middle time of the prompt gamma-ray duration), i.e., $\Gamma \propto \frac{t_p}{(1 + z)}^{0.99 \pm 0.06}$. With the derived $t_c$ and $\Gamma_X$, we constrain the radiating region as $R_X = 2.0 \times 10^{15} \sim 1.1 \times 10^{16}$ cm, with a median value of $6.5 \times 10^{15}$ cm. The radiation regions of the flares are within the regions of the prompt gamma-rays and afterglows, and the narrow distribution of $R_X$ of the flares in this analysis would be due to the sample selection effect, since we select only late flares in order to eliminate the contamination of adjacent flares.

The derived $\Gamma$, $t_c$, and $R_X$ anti-correlation indicates the decay of the Lorentz factor of ejecta. It may feature the long-term evolution of central engines. With the relation of...
| GRB     | $z$    | $\alpha$ | $\beta$ | $E_p$ (keV) | Model     | $\chi^2$/dof | $F_p$ (10$^{-9}$ erg cm$^{-2}$ s$^{-1}$) | $L_{X,p}$ (10$^{46}$ erg s$^{-1}$) | $\Gamma_X$ | $R_X$ (10$^{15}$ cm) |
|---------|--------|----------|---------|-------------|-----------|-------------|---------------------------------------|----------------------------------|------------|---------------------|
| 060124  | 2.297  | 2$^{+0.08}_{-0.08}$ | ...     | ...         | power law | 1.01        | 6.78 ± 0.16                           | 286 ± 6.74                       | 55.60 ± 3.43 | 6.93 ± 0.89         |
| 060904B | 0.703  | 0.93$^{+0.09}_{-0.09}$ | ...     | ...          | band       | 1.08      | 50.24 ± 1.1                           | 113 ± 2.47                       | 40.54 ± 2.89 | 3.79 ± 0.55         |
| 080928  | 1.692  | 0.39$^{+0.14}_{-0.14}$ | ...     | ...          | power law | 1.01      | 4.17 ± 0.13                           | 82.6 ± 2.58                      | 36.46 ± 2.73 | 2.01 ± 0.32         |
| 110801A | 1.858  | 2.61$^{+0.10}_{-0.11}$ | ...     | ...          | power law | 0.99      | 5.07 ± 0.13                           | 127 ± 3.25                      | 42.15 ± 2.96 | 4.23 ± 0.63         |
| 121027A | 1.773  | 1.19$^{+0.14}_{-0.15}$ | ...     | ...          | power law | 1.05      | 2.72 ± 0.06                           | 60.5 ± 1.33                     | 32.79 ± 2.56 | 6.16 ± 0.98         |
| 12111A  | 1.023  | −0.91$^{+0.38}_{-0.53}$ | −2.49$^{+0.11}_{-0.27}$ | 2.02 ± 1.71 | band       | 1.98      | 14.57 ± 0.22                           | 82.8 ± 1.25                     | 36.49 ± 2.71 | 7.08 ± 1.08         |
| 121229A | 2.707  | 1.12$^{+0.17}_{-0.17}$ | ...     | ...          | power law | 1.14      | 3.19 ± 0.06                           | 200 ± 3.76                      | 49.24 ± 3.21 | 5.14 ± 0.68         |
| 130925A1 | 0.347 | 0.46$^{+0.10}_{-0.10}$ | ...     | ...          | power law | 1.01      | 22 ± 0.11                             | 9.1 ± 0.05                      | 17.19 ± 1.71 | 2.08 ± 0.41         |
| 130925A2 | 0.347 | 0.51$^{+0.10}_{-0.10}$ | ...     | ...          | power law | 0.98      | 23 ± 0.1                             | 9.5 ± 0.04                      | 17.46 ± 1.73 | 3.36 ± 0.67         |
| 131030A | 1.293  | −0.91$^{+0.46}_{-0.34}$ | −3.19$^{+0.34}_{-0.59}$ | 2.13 ± 2.04 | band       | 1.15      | 106.67 ± 3.81                         | 1080 ± 38.8                     | 87.51 ± 4.45 | 10.94 ± 1.15        |
| 140430A | 1.6    | 2.75$^{+0.14}_{-0.14}$ | ...     | ...          | power law | 0.96      | 15.71 ± 0.35                          | 271 ± 6.04                      | 54.61 ± 3.40 | 3.55 ± 0.45         |
| 140506A | 0.889  | 2.64$^{+0.15}_{-0.15}$ | ...     | ...          | power law | 1.36      | 16.1 ± 0.48                           | 64.6 ± 1.93                     | 33.53 ± 2.60 | 4.61 ± 1.06         |
| 140512A | 0.725  | −0.42$^{+0.73}_{-0.50}$ | −1.61$^{+0.05}_{-0.07}$ | 5.14 ± 4.51 | band       | 1.45      | 70.14 ± 4.64                         | 170 ± 11.3                      | 46.60 ± 3.27 | 3.29 ± 0.51         |
GRB spectra are usually well fitted with the Band function in a broad energy band (e.g., Zhang et al. 2011). As shown in Peng et al. (2014), some early X-ray flares are the soft extension of the gamma-ray pulses and the $E_p$ values of their spectra are higher than the XRT band. Their X-ray luminosity values observed in the XRT band are only a small fraction of their bolometric luminosity. Looking at Table 1, one can observe $\beta<1$ for some flares, indicating that their energy fluxes would go up to a higher energy band. For those flares with $\beta>1$, the luminosity observed in the XRT band could be a good representative of the bolometric luminosity, and the $\Gamma_X$ values derived from Equation (8) with $L_{X,p}$ measured in the XRT band are only lower limits of $\Gamma_X$. Then, the true $\Gamma-X_p$ relation would become shallower than that derived in this analysis. In addition, with spectral information of prompt gamma-rays collected from Butler et al. (2007), Ghirlanda et al. (2008), and Heussaff et al. (2013), we also derived the luminosity at 10 keV ($L_{10\text{keV}}$) for the GRBs reported in LV (2012) and checked whether it is still tightly correlated with $\Gamma_0$. We found that it is not. This is reasonable, since $L_{10\text{keV}}$ is only a very small fraction of the radiation luminosity. Although $\Gamma_0$ is tightly correlated with the bolometric luminosity, it is not necessary for it to be correlated with any selected mono-frequency luminosity. Therefore, in the case where the luminosity in the XRT band is not a good representative of the bolometric luminosity, the derived $\Gamma_X$ with Equation (8) may be quite uncertain.

The $R_X$ values derived in this analysis are between the prompt gamma-ray and afterglow radiating regions. Since $\Gamma_X$ may be underestimated as we mentioned above, the inferred $R_X$ with Equation (3) thus may also be underestimated (cf. Beniamini & Kumar 2016), which is somewhat compensated by the assumption that the flare decay timescale $t_c$ is set by the curvature effect. It could be that $t_c$ is dominated by something else, and the $t_c$ reported here may overestimate the true curvature timescale. Recently, Uhm & Zhang (2015) found that the decay slope is steeper than the standard value from the curvature effect model if the jet is undergoing bulk acceleration when the emission ceases. They showed that the decay properties of flares demand that the emission region undergoes significant bulk acceleration (see also Jia et al. 2016). Therefore, the dynamical timescale and the magnetic field decay timescale would dominate the flare decay timescale and the $t_c$ value may be smaller. Hence, the real $R_X$ would be smaller than that reported in this analysis.

We acknowledge the use of the public data from the Swift data archive. We thank the anonymous referee for helpful suggestions to improve the paper. We also appreciate the helpful discussions with Bing Zhang, Zi-Gao Dai, Xue-Feng Wu, and Shu-Jin Hou. This work is supported by the National Basic Research Program of China (973 Program, grant No. 2014CB845800), the National Natural Science Foundation of China (Grant No. 11533003, 1143005, 11573034, 11373036, 11163001), the Strategic Priority Research Program The Emergence of Cosmicological Structures of the Chinese Academy of Sciences (Grant No. XDB09000000), the Guangxi Science Foundation (Grant No. 2016GXNSFDA380027, 2014GXNSFFA18004, 2013GXNSFFA019001), and the Project Sponsored by the Scientific Research Foundation of Guangxi University (Grant No. XJZ140331).
REFERENCES

Baring, M. G., & Harding, A. K. 1997, ApJ, 491, 663
Beniamini, P., & Kumar, P. 2016, MNRAS, 457, L108
Blandford, R. D., & McKee, C. F. 1976, PhFl, 19, 1130
Burrows, D. N., Romano, P., Falcone, A., et al. 2005, Sci, 309, 1833
Butler, N. R., & Kocevski, D. 2007, ApJ, 668, 400
Butler, N. R., Kocevski, D., Bloom, J. L., & Curtis, J. L. 2007, ApJ, 671, 656
Campana, S., Manganaro, V., Blustin, A. J., et al. 2006, Natur, 442, 1008
Chevalier, R. A., & Li, Z.-Y. 2000, ApJ, 536, 195
Chincarini, G., Mao, J., Margutti, R., et al. 2010, MNRAS, 406, 2113
Chincarini, G., Moretti, A., Romano, P., et al. 2007, ApJ, 671, 1903
Dai, Z. G., Wang, X. Y., Wu, X. F., & Zhang, B. 2006, Sci, 311, 1127
Dermer, C. D. 2004, ApJ, 614, 284
Dyks, J., Zhang, B., & Fan, Y. Z. 2005, ApJ, submitted (arXiv:astro-ph/0511699)
Evans, P. A., Beardmore, A. P., Page, K. L., et al. 2009, MNRAS, 397, 1177
Falcone, A. D., Burrows, D. N., Lazzati, D., et al. 2006, ApJ, 641, 1010
Falcone, A. D., Morris, D., Racusin, J., et al. 2007, ApJ, 671, 1921
Fan, Y. Z., & Wei, D. M. 2005, MNRAS, 364, L42
Fenimore, E. E., Epstein, R. I., & Ho, C. 1993, A&AS, 97, 97
Fenimore, E. E., Madras, C. D., & Nayakshin, S. 1996, ApJ, 473, 998
F Fluid, D. A., Kulkarni, S. R., Sari, R., et al. 2001, ApJL, 562, L55
Ghirlanda, G., Nava, L., Ghiellini, G., et al. 2012, MNRAS, 420, 483
Ghirlanda, G., Nava, L., Ghisellini, G., Firmani, C., & Cabrera, J. I. 2008, MNRAS, 387, 319
Guidorzi, C., Dichiara, S., Frontera, F., et al. 2015, ApJ, 801, 57
Gupta, N., & Zhang, B. 2008, MNRAS, 384, L11
Hascoct, R., Beloborodov, A. M., Daighe, F., & Mochnovitch, R. 2015, arXiv:1503.08333
Heussaff, V., Atteia, J.-L., & Zolnierowski, Y. 2013, A&A, 557, A100
Isooe, T., Feigelson, E. D., Akritas, M. G., & Babu, G. J. 1990, ApJ, 364, 104
Jia, L.-W., Uh, Z. L., & Zhang, B. 2016, ApJ, 225, 17
Jin, Z.-P., Fan, Y.-Z., & Wei, D.-M. 2010, ApJ, 724, 861
Kobayashi, S., & Zhang, B. 2007, ApJ, 655, 973
Kouveliotou, C., Meeegan, C. A., Fishman, G. J., et al. 1993, ApJL, 413, L101
Kumar, P., & Panaiteescu, A. 2000, ApJL, 541, L51
Lazzati, D., & Begelman, M. C. 2006, ApJ, 641, 972
Lazzati, D., Perri, R., & Begelman, M. C. 2008, MNRAS, 388, L15
Levan, A. J., Tanvir, N. R., Starling, R. L. C., et al. 2014, ApJ, 781, 13
Li, Y., Li, A., & Wei, D. M. 2008, ApJ, 678, 1136
Liang, E.-W., Li, L., Gao, H., et al. 2013, ApJ, 774, 13
Liang, E.-W., Lin, T.-T., Li, J., et al. 2015, ApJ, 813, 116
Liang, E.-W., Yi, S.-X., Zhang, J., et al. 2010, ApJ, 725, 2209
Liang, E. W., Zhang, B., O’Brien, P. T., et al. 2006, ApJ, 646, 351
Lithwick, Y., & Sari, R. 2001, ApJ, 555, 540
Liu, J., Zou, Y.-C., Lei, W.-H., et al. 2012, ApJ, 751, 49
Margutti, R., Bernardini, G., Barniol Duran, R., et al. 2011, MNRAS, 410, 1064
Margutti, R., Guidorzi, C., Chincarini, G., et al. 2010, MNRAS, 406, 2149
Maxham, A., & Zhang, B. 2009, ApJ, 707, 1623
Mészáros, P., & Rees, M. J. 1995, ApJ, 405, 278
Mészáros, P., & Rees, M. J. 1997, ApJ, 476, 232
Molinari, E., Vergani, S. D., Malesani, D., et al. 2007, A&A, 469, L13
Mu, H.-J., Gu, W.-M., Hou, S.-J., et al. 2016, arXiv:1609.08290
Nousek, J. A., Kouveliotou, C., Grupe, D., et al. 2006, ApJ, 642, 389
O’Brien, P. T., Willingale, R., Osborne, J., et al. 2006, ApJ, 647, 1213
Panaiteescu, A. 2006, MNRAS, 367, L42
Panaiteescu, A., Mészáros, P., Gehrels, N., Burrows, D., & Nousek, J. 2006, MNRAS, 366, 1357
Peng, F.-K., Liang, E.-W., Wang, X.-Y., et al. 2014, ApJ, 795, 155
Perna, R., Armitage, P. J., & Zhang, B. 2006, ApJ, 636, L29
Piran, T. 2004, REVMP, 76, 1143
Proga, D., & Zhang, B. 2006, MNRAS, 370, L61
Qin, Y., Liang, E.-W., Liang, Y.-F., et al. 2013, ApJ, 763, 15
Qin, Y.-P. 2002, A&A, 396, 705
Qin, Y.-P. 2008, ApJ, 683, 900
Rees, M. J., & Mészáros, P. 1994, ApJL, 430, L93
Romano, P., Moretti, A., Banat, P. L., et al. 2006, A&A, 450, 59
Sari, R., & Piran, T. 1999, ApJ, 520, 641
Schady, P., Dwelly, T., Page, M. J., et al. 2012, A&A, 537, A15
Starling, R. L. C., Wijers, R. A. M. J., Wiersema, K., et al. 2007, ApJ, 661, 787
Tang, Q.-W., Peng, F.-K., Wang, X.-Y., & Tam, P.-H. T. 2014, ApJ, in press (arXiv:1412.3342)
Troja, E., Piro, L., Vasilievou, V., et al. 2015, ApJ, 803, 10
Uhm, Z. L., & Zhang, B. 2015, ApJ, 808, 33
Virgili, F. J., Mundell, C. G., Pal'shin, V., et al. 2013, ApJ, 778, 54
Woods, E., & Loeb, A. 1995, ApJ, 453, 583
Wu, X. F., Dai, G., Wang, X. Y., et al. 2005, arXiv:astro-ph/0512555
Yi, S.-X., Xi, S.-Q., Yu, H., et al. 2016, ApJS, 224, 20
Yonetoku, D., Tanabe, S., Murakami, T., et al. 2008, PASJ, 60, S352
Zhang, B., Fan, Y. Z., Dyks, J., et al. 2006, ApJ, 642, 354
Zhang, B., & Mészáros, P. 2004, DMPA, 19, 2385
Zhang, B.-B., Zhang, B., Murase, K., Connaughton, V., & Briggs, M. S. 2014, ApJL, 730, 141
Zhang, B.-B., Zhang, B., Murase, K., Connaughton, V., & Briggs, M. S. 2014, ApJ, 787, 66