Spectrogram and PQ-Plot Based Chaotic Signals Analyses

Jinwei Cai, Yaotian Li and Wenshi Li
Department of Microelectronics, Soochow University
Shizi Road, No.1, Suzhou City, China
Email: lwshi@suda.edu.cn

Abstract. Chaotic signals’ complexities feature the disorder state between the order and stochastic states. The time-frequency analyses will increase the representation power with the help of pq-plot in 0-1 test for chaos. This simulation work tested three equations of Logistic, Li and Chua under two methods of spectrogram and pq-plot. Three window-bands of spectrogram are selected in wide band of 128, trade-off of 512 and narrow of 2048. Three values of the chaos-sensing-factor c in pq-plots are used with 1.0456, 1 and 0.75. The simulation results show that (1) narrow band was fitted for three equations in spectrogram, (2) the evolutions of Li chaos equation met the raised power of frequency bands, and (3) compared with three cases under test, chaos-sensing-factor c in pq-plots was slightly dropped. The conclusions grasped the chaos recognition tools of both spectrogram and pq-plot of 0-1 test for chaos in digging out the visualization of time-frequency-energy-dynamics analyses.

1. Introduction
Pattern recognition loves features in time-frequency-energy-dynamics domains. The strong reasons are co-analyses for the complexity of raw data in the nonlinear world [1]. Brief reviewing remind us that computing-wasted Wigner and Wavelet transforms were kept more attention than short-time Fourier transform during chaotic signals analyses [2-6]. To enhance the spectrogram representation power keeping concise, we utilize pq-plot in 0-1 test for chaos diagnosis as contrast experiment [7-10].

2. Methods and Equations

2.1. Methods
Instruction format of spectrogram in MATLAB R2016a and key parameters selections are as [S,F,T,P]=spectrogram(x, window, noverlap, nfft, fs)%((a, 2048, 1024, 2048, 10000) narrow-band; (a,512,256,512,10000) trade-off-band; (a,128,64,128,10000) wide-band. Illustrations are in Figure 1.
Figure 1. Two examples of spectrogram in narrow-band.

On $pq$-plot algorithms for chaos test of signals $\{x(n) y(n) z(n)\}$, the known formulae are as [7]

\begin{align}
\phi(n) &= x(n) + y(n) + z(n) \\
\theta(n+1) &= c + \theta(n) + \phi(n) \\
p(n+1) &= p(n) + \phi(n) \cos(\theta(n)) \\
q(n+1) &= q(n) + \phi(n) \sin(\theta(n))
\end{align}

Wherein parameter $c$ can be denoted chaos-sensing-factor in the range of 0 to $2\pi$, which is involved in the orthogonal modulation and integration in $pq$-plot. There are features in $pq$-plot like cycle shape signing the period state of the data under test versus random walking spray marking chaotic state.

2.2. Equations
Test case I is Logistic equation in formula (5) [11]. Its bifurcation characteristic depicts in Figure 2.

\[ x_{n+1} = k x_n (1 - x_n) \]
Test case II is Li chaos equation in formula (6), reported firstly in the meeting of ICST 2016.

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= 0.3x + 0.05y + 0.6z^2 \\
\dot{z} &= -0.6z + xy
\end{align*}
\] (6)

Test case III is known Chua chaos equation shown as deduced formula (7) [5].

\[
\begin{align*}
\dot{x} &= -2.564x + 10y + 0.5a_3(x+1|-x-1|) \\
\dot{y} &= x - y + z \\
\dot{z} &= -14.706y
\end{align*}
\] (7)

In next simulation works, the key parameter-scanning concerns firstly the intrinsic growth rate, then the band width, the power in frequency domain, and at last, the chaos-sensing-factor.

3. Results and Discussions
For the results of test case I, please see Figure 3 including 8 sub-pictures. After scanning the wide, trade-off and narrow bands, the fitting recognition shows are selected with narrow band.
Figure 3. Logistic equation chaos evolutions in different intrinsic growth rate $k$ values corresponding to the comparison of the spectrograms marking a and $pq$-plots signing b. Wherein $k$ equals to (i) 3.4495, (ii) 3.5441, (iii) 3.5699, or (iv) 3.6721. Herein the chaos-sensing-factor $c$ values 1.0456. @ data length of 98,000.

Following the changes of intrinsic growth rate $k$ in sub-figures from a(i) to a(iv) in Figure 3, the powers in frequency domain of spikes decrease and shape planer.

In contrast, the relative $pq$-plots show from simple cycle to compound cycle (another bifurcation show), and further into Brown walking just at $k$ equals to 3.6721 (the chaos show in 0-1 test for chaos), while in selected $c$ values 1.0456.

Especially note be on the chaos-sensing-factor $c$. Through scanning observation from 0 to $2\pi$, we use $c = 1.0456$ just digging out the chaos show for the Logistic equation evolution while $k = 3.6721$.

For results of test case II, please see Figure 4 including 12 sub-pictures. After our scanning the wide, trade-off and narrow bands, the fitting recognition shows in spectrograms are kept with narrow band.
And following the data lengths growth, from the unstable equilibrium point to single scroll chaos in phase plots in sub-figures of a(i, ii, iii, iv), we just visit that the stochastic walking starts in sub-figure c(iii) coming from the cycles of sub-figures of c(i, ii). Also, the heights of powers in frequency domain go up while the data lengths running from 20,000 points to 70,000 points, please see sub-figures of b(i, ii, iii, iv). The optimal parameter \( c \) is driven by 1.

![Figure 4](image)

**Figure 4.** Li equation chaos evolutions (y series) in different data lengths corresponding to the comparison of the phase plots staring a, spectrograms marking b and \( pq \)-plots signing c. Wherein data length \( l \) equals to (i) 20,000, (ii) 30,000, (iii) 45,000, or (iv) 70,000. Chaos-sensing-factor \( c \) values 1. @ the initial values of [0.1, 0.1, 0.1] in Li chaos equation.

Into the test case III of Chua equation based on spectrogram and \( pq \)-plot, we change the scanning parameters from the intrinsic growth rate \( k \) of Logistic map (case I) and data length \( l \) in Li chaos equation (case II) into the key factor \( a_3 \).

When the factor \( a_3 \) was rising, sub-figures a(i…) show single cycle, double periods, multi-periods and single scroll and double scrolls, respectively. Correspondingly, sub-figures b(i…) descend the power in frequency domain. And most interestingly, while the value 0.75 of chaos-sensing factor \( c \) was used by us then the key three spray immediately appears in the sub-figure c(iv), just like opening the chaos-show window.
Figure 5. Chua equation chaos evolutions (y series) in different key factor $a_3$ values corresponding to the comparison of phase plots staring a, spectrograms marking b and $pq$-plots signing c. Wherein factor $a_3$ equals to (i) 2.70, (ii) 2.95, (iii) 3.02, (iv) 3.11, or (iv) 3.45. Chaos-sensing-factor $c$ weighs 0.75. @ Chua equation initial values of [0.1, 0.1, 0.1], $N = 30,000$.

4. Conclusions
In order to compare the visualized representation advantages based on spectrogram and $pq$-plot in 0-1 test for chaos, we tested three equations during scanning four key parameters. Parameter $k$ and parameter $a_3$ in known chaos equations can lead data scientists searching for optimal chaos-probing factor $c$ values in $pq$-plots in nonlinear dynamic domain. The narrow window spectrogram can depict the time-frequency-energy analyses on chaos evolutions in Li chaos equation, needing the contrast
mirror like $pq$-plot. In summary in time-frequency-energy-dynamics domains we can distinguish intrinsic nonlinear nature of interested chaotic signals.

5. Acknowledgments
This work was supported by Technological Innovation of Key Industries in Suzhou City Prospective Application Study [grant number SYG201701]. This work was supported by Open Project of Ministry-of-Education Key Laboratory of Modern Acoustics, 2017. This work was supported by RIGOL University-Enterprise Cooperative Project of Ministry of Education, 2018.

6. References
[1] Sotoca J M, Sánchez J S, Mollineda R A 2005 Actas del III Taller Nacional de Mineria de Datos y Aprendizaje pp. 77 – 83
[2] Cohen L 1989 Proceedings of the IEEE vol 77 pp. 941– 981
[3] Ping Chen 1994 Proceedings of IEEE-SP International Symposium on Time–Frequency and Timescale Analysis pp. 357–360
[4] Junfeng Sun, Yi Zhao, Nakamura T and Small M 2007 Physical Review E vol 76 p. 016220
[5] Rubezic V, Djurovic I, Dakovic M 2006 Signal Processing vol 86 pp. 2255 – 2270
[6] Frei M G and Osorio I 2007 The Royal Society vol 463 pp. 321–342
[7] Gottwald G A, Melbourne I 2009 SIAM Journal on Applied Dynamical Systems vol 8 pp.129 – 145
[8] Sun K H, Liu X and Zhu C X 2010 Chinese Physics B vol 19 p. 110510
[9] Gopal R, Venkatesan A and Lakshmanan M 2013 Chaos: An Interdisciplinary Journal of Nonlinear Science vol 23 p. 023123
[10] Senlin Wu, Yaotian Li, Wenshi Li and Lei Li 2018 Chinese Journal of Electronics, in publication.
[11] Boeing, G 2016 Systems vol 4 p. 37