A Possible Late Time ΛCDM-like Background Cosmology in Relativistic MOND Theory

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Received Day Month Year
Revised Day Month Year
Communicated by Managing Editor

In the framework of Relativistic MOND theory (TeVeS), we show that a late time background ΛCDM cosmology can be attained by choosing a specific $F(\mu)$ that also meets the requirement for the existence of Newtonian and MOND limits. We investigate the dynamics of the scalar field $\phi$ under our chosen $F(\mu)$ and show that the "slow roll" regime of $\phi$ corresponds to a dynamical attractor, where the whole system reduces to ΛCDM cosmology.

Keywords: Relativistic MOND; TeVeS; Dark Energy

1. Introduction

Current astronomical observations on cosmological scales (CMB anisotropy $^{12}$, supernovae $^{345}$ and SDSS$^{6}$) reveal that our universe is spatially flat, with about two thirds of the energy content resulting from what is referred to as dark energy. This energy has a negative pressure to account for the accelerated expansion of the universe. While, on smaller, galactic scales, the rotational curves of galaxies strongly indicate that the biggest contributions to their mass density arises from non-luminous matter, which has given rise to speculations on the existence of dark matter$^{7}$.

On the other hand, it is also interesting to inquire if we can solve the current observational puzzles, both dark matter and dark energy, by modifying Einstein gravity in spite of its many successes, in particular, the solar system tests. MOND theory is a striking modification of the Newtonian laws of motion and explains the rotational curves with amazing accuracy$^{8}$ without introducing any dark matter. After some initial problems, a consistent relativistic extension of the MOND theory has recently been proposed by Bekenstein$^{9}$, which has spurred extensive studies on
its successes and pitfalls\cite{10,11,12,13,14,15,16,17,18}. In that extension, general relativity has been modified by including two scalar fields (one dynamical while the other is non-dynamical), one vector field. In addition, the theory contains one arbitrary function whose form is dictated in part by its primary purpose, i.e., to address the data on the rotational curves of galaxies, or equivalently, the dark matter problem. Consistency with data from the Bullet clusters\cite{19} indicates that eV scale hot neutrinos must also be included as explicit dark matter candidate\cite{20}. The remarkable features of this theory are that it can explain the galaxy rotational curves without introducing dark matter (just as its non-relativistic counterpart, MOND theory) and at the same time reduce to general relativity in the appropriate limits. The price one pays is the additional complication of the whole system: two scalar fields, one vector field and gravitational field.

A natural question to ask is if the new framework also works for cosmology? Can it help to tackle the dark energy problem? We will show in this paper that under appropriate conditions, the scalar field could mimic the behavior of the dark energy models\cite{21,22,23,24,25}. Specifically, in the "slow roll" regime, it will be seen to reduce to the standard ΛCDM model. Although the current formulation of TeVeS is still not as mature as the dark matter models, it has the potential, to naturally solve the cosmic puzzles currently explained by dark energy and dark matter without invoking either of them explicitly\cite{26}.

2. Basic setup

We begin with a brief outline of Gravity in the TeVeS framework. Following Bekenstein\cite{9}, the action for the whole system could be written as

\[ S = S_g + S_s + S_v + S_m \]  

where

\[ S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta} \sqrt{-g} d^4x \]  

\[ S_s = -\frac{1}{2} \int \left[ \sigma^2 h^{\alpha\beta} \phi,\alpha \phi,\beta + \frac{1}{2} G \ell^{-2} \sigma^4 F(kG\sigma^2) \right] \sqrt{-g} d^4x \]  

\[ S_v = -\frac{K}{32\pi G} \int \left[ g^{\alpha\beta} g^{\mu\nu} \mathfrak{U}_{[\alpha,\mu]} \mathfrak{U}_{[\beta,\nu]} - 2(\lambda/K)(g^{\mu\nu} \mathfrak{U}_\mu \mathfrak{U}_\nu + 1) \right] \sqrt{-g} d^4x, \]  

\[ S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^a, f^a|_\mu, \cdots) \sqrt{-\tilde{g}} d^4x \]

are the actions for gravity, scalar field, vector field and luminous matter respectively. In the framework of TeVeS, there are two different frames, physical frame corresponding to the "real universe" and Einstein frame. The metric in physical frame \((\tilde{g}_{\mu\nu})\) relates that in Einstein frame \((g_{\mu\nu})\) by \(\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta} + \mathfrak{U}_\alpha \mathfrak{U}_\beta) - e^{2\phi} \mathfrak{U}_\alpha \mathfrak{U}_\beta.\)
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Here $U_\alpha$ is a timelike 4-vector field, $\phi$ and $\sigma$ are respectively the dynamical and the non-dynamical scalar fields, $F$ is a free function to be specified by dynamics while $k$ and $K$ are two positive dimensionless parameters. Varying the action, one can arrive at the following equations of motion for the vector, scalar and gravitational fields:

$$K \left( g^{\mu_1 \nu_1} \left( \frac{\partial}{\partial \tau} U_{\mu_2} + \dot{U}_{\mu_2} \right) - \frac{g^{\tau \mu_1} \phi_{\tau \mu_1}}{2} \right) + 8\pi G \sigma^2 \left[ g^{\mu \nu} \phi_{\nu} g^{\alpha \beta} \phi_{\alpha \beta} + \frac{1}{2} \phi_{\nu} g^{\alpha \beta} \right]$$

$$= 8\pi G (1 - e^{-4\phi}) \left[ g^{\alpha \beta} \dot{T}_{\alpha \beta} + \frac{1}{2} \phi_{\alpha \beta} \dot{T} \right]$$

$$- \mu F(\phi) - \frac{1}{2} \mu^2 F'(\phi) = k\ell^2 h^{\alpha \beta} \phi_{\alpha \beta}$$

$$[\mu (k\ell^2 h^{\mu \nu} \phi_{\mu} \phi_{\nu})]_{\alpha \beta} = kG \left[ g^{\alpha \beta} + (1 + e^{-4\phi}) \epsilon^{\alpha \beta} \right] \dot{T}.$$  

$$G_{\alpha \beta} = 8\pi G \left[ \dot{T}_{\alpha \beta} + (1 - e^{-4\phi}) \epsilon^{\alpha \beta} + \tau_{\alpha \beta} \right] + \Theta_{\alpha \beta}$$

where

$$\tau_{\alpha \beta} \equiv \sigma^2 \left[ \phi_{\alpha \beta} - \frac{1}{2} g^{\mu \nu} \phi_{\mu \nu} g_{\alpha \beta} - \phi_{\mu} \phi_{\nu} \left( g_{\mu \alpha} \phi_{\beta} - \frac{1}{2} g_{\alpha \beta} \phi_{\mu} \right) \right]$$

and

$$\Theta_{\alpha \beta} \equiv K \left( g^{\mu \nu} \dot{U}_{(\mu \nu)} - \frac{1}{4} g^{\sigma \tau} g^{\alpha \beta} \dot{U}_{(\sigma \tau)} \right) - \lambda \dot{U}_{\alpha} U_{\beta}$$

$\dot{T}_{\mu \nu}$ is the energy momentum tensor of ordinary matter in the physical coordinate system. $h^{\alpha \beta} = g^{\alpha \beta} - U^{\alpha} U^{\beta}$ and $\mu = kG\sigma^2$.

3. Cosmological dynamics in Einstein frame

In a cosmological setting, the symmetries of the FRW universe will simplify the above equations considerably. It is worth recalling here an underlying assumption in such applications. One interprets the fields in the problem, say the scalar field, as consisting of both a spatially homogenous (time dependent) part and an inhomogeneous one. On cosmological scales the spatially inhomogeneous part may be neglected in the first approximation while at galactic scales, the inhomogeneous part plays a prominent role and the homogenous part is negligible (quasi-static approximation). Henceforth, we will restrict ourselves to cosmological scales, more specifically to the flat universe case, and consider the line element:

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$
where $d\tilde{t} = e^{\phi}dt$ and $\tilde{a} = e^{-\phi}a$. It is clear that the physical frame and Einstein frame become the same once the $e^{\phi}$ is constant 1. The vector field in this case could be chosen as $\Omega^\mu = \delta_t^\mu$ and the scalar field depends only on time $t$. In Einstein frame, the equations of motion and the field equations have simpler forms though their dynamical evolution are essentially the same as those in physical frame (they are related by a non-singular transformation). So, in this section, we will analyze the dynamics of the system in Einstein frame.

\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_\phi) \tag{13}
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_M + \rho_\phi + 3p_M + 3p_\phi) \tag{14}
\]

where $\rho_M = \bar{\rho}e^{-2\phi}$, $p_M = \bar{p}e^{-2\phi}$ are respectively the density and pressure of non-relativistic matter. $\rho_\phi$ and $p_\phi$ are the effective energy density and pressure of the scalar field $\phi$, defined as:

\[
\rho_\phi = \frac{\mu^2}{kG} + \frac{\mu^2}{4\ell^2k^2G}F(\mu) = \frac{\mu^2[3F(\mu) + \mu F'(\mu)]}{4\ell^2k^2G} \tag{15}
\]

\[
p_\phi = \frac{\mu^2}{kG} - \frac{\mu^2}{4\ell^2k^2G}F(\mu) = \frac{\mu^2[F(\mu) + \mu F'(\mu)]}{4\ell^2k^2G}
\]

where $\mu$ relates to $\dot{\phi}$ via the $\sigma$ equation (Eq.7):

\[
\mu F(\mu) + \frac{1}{2}\mu^2F'(\mu) = 2k\ell^2\dot{\phi}^2 \tag{16}
\]

Here we need to emphasize, as suggested in [17], that the above definitions are held in the effective sense due to the fact that $\rho_M$ and $p_M$ terms contain $\phi$. But in the "slow roll" regime ($\dot{\phi} \sim 0$ as we will see later), they are well defined as the $\phi$ is merely a constant factor. The equation of motion of the scalar field $\phi$ in a FRW background is given by

\[
\ddot{\phi} + (3H + \frac{\dot{H}}{\mu})\dot{\phi} + \frac{kG}{2\mu}(\rho_M + 3p_M) = 0 \tag{17}
\]

where $H = \dot{a}/a$ and dot denotes the derivative with respect to $t$. It is worth nothing that, under the specific form of $F(\mu)$ we choose in Eq.(21), Eq.(17) is consistent with the "slow roll" approximation $\dot{\phi} \sim 0$ under two different situations: First, as in the usual scalar field dark energy dynamical system, the matter density $\rho_M$ decreases to zero at late times, which leads to a $\rho_\phi$ dominated universe; Second, $\frac{\dot{H}}{\mu} \ll 1$, which lead to a universe with both $\rho_M$ and $\rho_\phi$. In next section, we consider the second possibility when fitting the model to the Supernovae data. On the other hand, if we
assume that \( \dot{\phi} \) varies slowly, then we can consider the two energy components to be approximately adiabatic, and energy momentum conservation leads to

\[
\frac{d\rho_i}{dt} = -3\ddot{a}(\rho_i + \rho_i)
\]

where the subscript \( i \) denotes \( M \) and \( \phi \). To get a closed system of equations, we also need to specify the equation of state of the energy/matter content. This equation of state is specified by \( w = p/\rho \), which is 0 for non-relativistic matter. While for the scalar field, this equation of state is given by its equation of motion, i.e., Eq.(17) which can be expressed as,

\[
w_\phi = \frac{4\ell^2k\mu\dot{\phi}^2 - \mu^2F(\mu)}{4\ell^2k\mu\dot{\phi}^2 + \mu^2F(\mu)} = \frac{F(\mu) + \mu F'(\mu)}{3F(\mu) + \mu F'(\mu)}
\]

In the "slow roll" regime, \( \ddot{\phi} \sim \dot{\phi} \sim 0 \) and \( w_\phi \sim -1 \). It is worth noting that from Eq.(19), the equation of state could approach -1 from either greater than -1 (quintessence case) or less than -1 (phantom case) if we choose the form of \( F(\mu) \) appropriately. In this paper, we focus on the case with \( w_\phi \geq -1 \). Eqs.(13-14) are expressed in terms of the scale factor in the Einstein picture. In the following, we will continue to work in Einstein frame to investigate the dynamics of the scalar field.

From the above discussion, it is clear now that we will get the evolution of the whole dynamical system if the form of \( F(\mu) \) is specified. However, at present there are no theoretical arguments in favor of any specific choice, thereby providing a lot of freedom in this regard. The constraints on the form of \( F(\mu) \) are phenomenological in nature motivated by the condition that the correct physics is obtained. Besides the toy model \( F(\mu) \) suggested by Bekenstein, there are some others forms suggested to account for the better fit to data \( ^{14,16} \). In this section, we will consider the choice of the form of \( F(\mu) \) that will lead to an acceptable cosmology in addition to accommodating the MOND theory and Newton’s Law. From Eq.(15), the positive energy condition, \( \rho_\phi \geq 0 \) and the non-vanishing of \( \rho_\phi \) as \( \dot{\phi} \to 0 \) may restrict the freedom in the choice of the form of \( F(\mu) \). On the other hand, the existence of MOND and Newtonian limits also give constraints on the form of \( F(\mu) \) as \(^{18} \)

\[
\frac{d(\mu^2F(\mu))}{d\mu} = -\frac{k^2\mu^2}{64\pi^2(1-\mu)m}f(\mu)
\]

with \( f(\mu) \) an arbitrary function of \( \mu \) with the requirement of non-vanishing \( f(0) \) and \( m = 1 \) is the case proposed by Bekenstein\(^{9} \). Therefore, we suggest a form of \( F(\mu) \) that meets the above requirements as

\[
F(\mu) = \frac{3}{8}\frac{\mu(4 + 2\mu - 4\mu^2 + \mu^3) + 2\ln[(1 - \mu)^2]}{\mu^2} + \frac{\alpha}{\mu^2}
\]
where $\alpha$ is a dimensionless constant. When $\alpha = 0$, the above reduces to the form in\textsuperscript{9}. Then, from Eq. (16), we can obtain the relation between $\mu$ and $\dot{\phi}$ as

$$3/2(1 + \mu - 3\mu^2 + \mu^3 - 1/\mu) = 4k\ell^2\dot{\phi}^2$$

(22)

For a consistent cosmology, we require $2 \leq \mu < \infty$. Note also that $\dot{\phi} = 0$ when $\mu = 2$, for which the energy density Eq. (15) becomes $\rho_\phi = \alpha^4/3k\ell^2$ and the equation of state reduces to, $w_\phi = -1$ at late times, which corresponds to a de Sitter phase. This not only provides an acceptable cosmological state but also ensures the consistency of our previous assumption $\dot{\phi} \sim 0$. Next, we show that such a phase corresponds to a dynamical de Sitter attractor\textsuperscript{27}. To do this, we introduce the dimensionless variables $x = \phi$, $z = t_0\dot{\phi}$ and $N = \ln a$ with $t_0$ a scaling constant of dimension $t$. We can rewrite the Eqs. (13-18) in terms of $x$, $z$ and $N$ and linearize them around the critical point $(x, z) = (x, 0)$, then, we arrive at the following system of equations

$$\frac{dx}{dN} = \frac{z}{t_0\sqrt{2\pi\alpha/3k^2\ell^2}}$$

(23)

$$\frac{dz}{dN} = -3z$$

It is easy to see that the eigenvalues of the coefficients matrix of Eq. (23) are $(-3, 0)$ which indicates that the critical point is stable, i.e. a dynamical attractor. One comes to the same conclusion by noting that $\dot{\phi} = 0$ leads to a minimum of the energy density $\rho_\phi$. In Figs. (1) and (2), we plot the numerical results for the dynamical system defined above. Note that our intention here is merely to illustrate the possibility of a consistent cosmology and not to fit the exact observational data. So, for convenience, we have set the parameter $\alpha = 0.01$ and the rest to unity. Since the attractor corresponds to all $\phi$ with $\dot{\phi} = 0$, we can choose $\phi \sim 0$ so that $e^{-2\phi} \sim 1$, which is indicated by the previous discussion. In our numerical analysis, we chose $\phi = 0.00001$ and the initial $\dot{\phi}$ from 0.001 to 0.005 with an increment of 0.001. We should point out that the above conclusion won’t change even if we choose other values for $\phi$. Then the physical frame and Einstein frame are different by a factor of $e^{-2\phi}$. In Fig. (3), we plot the corresponding evolution of the $\rho_\phi, p_\phi$ and $w_\phi$ with respect to $\mu$ given the form of $F(\mu)$ in Eq. (21).

4. Cosmology in physical frame

The analysis in preceding section is carried out in the Einstein frame. But we need to work in physical frame if we want to compare the theory with observation data. The two frames are mathematically equivalent to each other although the physical interpretations are different. With a simple transformation, one could convert the the equations from Einstein frame to physical frame. Under the ”slow roll” approximation($\dot{\phi} \sim \dot{\phi} \sim 0$), the system can be casted as:
Fig. 1. The evolution of cosmic parameters for matter (indigo curve) and $\phi$ field (pink curve). From the plots, one can find the curves with different initial conditions converges very quickly to a common track corresponding to the attractor.

Fig. 2. The evolution of the equation of state of $\phi$ field for different initial $\dot{\phi}$.

\begin{align}
\left(\frac{\ddot{a}}{a}\right)^2 &= \frac{8\pi G}{3}(\tilde{\rho}e^{-4\phi} + \rho_{\phi}e^{-2\phi}) \quad (24)
\end{align}

\begin{align}
\frac{\dddot{a}}{a} &= -\frac{4\pi G}{3}(\tilde{\rho}e^{-4\phi} - 2\rho_{\phi}e^{-2\phi}) \quad (25)
\end{align}
2.2 \[ \rho \]
2.4 \[ \rho \]
2.6 \[ \rho \]
2.8 \[ \rho \]

\( \Omega_b = \frac{\rho}{\rho_{\text{crit}}} \)
\( \Omega_\phi = \frac{\rho_\phi}{\rho_{\text{crit}}} \)
\( \Omega_M = \frac{\rho_{\text{crit}}}{\rho_{\text{crit}}} \)

\( \phi = -0.46 \pm 0.03 \)

From the discussion in the previous section, we know that \( \dot{\phi} = 0 \) with \( \phi \) at any value is a dynamical attractor. From Eq. (24), we can fix the \( \phi \) value via observation, i.e. the ratio of \( \Omega_b/\Omega_M \). From Big Bang Nucleosynthesis (28), we can give a constraint on the baryon density (\( \dot{\rho} \) and hence \( \Omega_b \)), while from SNIa (29) we can constrain the total density of non-relativistic matter (\( \rho_{\text{crit}} \) and hence \( \Omega_M \)). Combining the two, we can give a constraint on \( \phi \) as \( \phi = -0.46 \pm 0.03 \) and the results are shown in Fig. (4).
Fig. 4. Constraint on $\phi$ from Gold Sample SNIa data. The $\mu$ axis in this plot is the distance modulus.

5. Discussion

In this paper, we show a possible late time background $\Lambda$CDM like cosmology in the framework of TeVeS theory by choosing the $F(\mu)$ as a specific form that also
meets the requirements of Newtonian and MOND limits. We show the dynamical system has a late time de Sitter behavior and the scalar field at the de Sitter attractor should be \(-0.46 \pm 0.03\) so as to be in agreement with current astronomical observations.

On the other hand, the framework of the TeVeS theory involves an arbitrary function in addition to many parameters as well as auxiliary fields. This not only provides much freedom in matching the theory to observations, but also adds uncertainties. Clearly, they must be constrained from further phenomenological investigations of the testable predicions of the theory. It would be interesting to see if the model can be made compatible with all the astronomical data with a unique choice for the arbitrary function. Are there any theoretical constraints on this function?

The analysis in this paper is aimed at suggesting a possibility to reduce the complicated dynamical system of TeVeS into the customary \(\Lambda\)CDM under slow roll approximation and thus get some feedback from the cosmological observational constraints on \(\Omega_b/\Omega_M\). The choice of the function \(F(\mu)\), though without any deep theoretical basis, phenomenologically explains both dark matter and dark energy in the same framework. It remains to be checked in future how this form of the function \(F(\mu)\) can be embedded in the larger context of an appropriate function that explains all the data.

ACKNOWLEDGEMENT: This work was supported in part by the US Department of Energy.

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