Chiral corrections in hadron spectroscopy

A.W. Thomas\textsuperscript{1} and G. Krein\textsuperscript{2}

\textsuperscript{1} Department of Physics and Mathematical Physics and Special Research Center for
the Subatomic Structure of Matter, University of Adelaide, SA 5005, Australia

\textsuperscript{2} Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona, 145 - 01405-900 São Paulo, SP, Brazil

Abstract

We show that the implementation of chiral symmetry in recent studies of the hadron spectrum in the context of the constituent quark model is inconsistent with chiral perturbation theory. In particular, we show that the leading nonanalytic (LNA) contributions to the hadron masses are incorrect in such approaches. The failure to implement the correct chiral behaviour of QCD results in incorrect systematics for the corrections to the masses.

PACS NUMBERS: 24.85.+p, 11.30.Rd, 12.39.Jh, 12.39.Fe, 12.40.Yx

KEYWORDS: Chiral symmetry, quark model, potential models, hadron spectrum
There is an extremely interesting recent series of papers by Glozman, Riska and collaborators \[1\]- \[6\] who have investigated hadron spectroscopy on the basis of a residual $q - q$ interaction governed by chiral symmetry. Their residual interaction, which is meant to correspond to Goldstone boson (GB) exchange, has the attractive feature, in comparison with one-gluon-exchange (OGE) \[7\], that it does not produce large spin-orbit effects which are certainly not present in the spectrum. While our remarks apply to all GB exchanges, for simplicity we concentrate on the SU(2) sector – i.e. pion exchange. In this sector GB exchange leads to an effective interaction of the form

\[ H_{\text{int}} = \frac{g^2}{4\pi^2} \frac{1}{3} \sum_{i<j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j \left[ m_\pi^2 e^{-m_\pi r_{ij}} - 4\pi \delta(r_{ij}) \right], \quad (1) \]

where $m_i$ and $m_j$ denote the masses of the constituent quarks and $m_\pi$ is the pion mass. In principle there is also a tensor component, which will not be written explicitly since it is not relevant in the context of the present paper. This interaction has also been employed in studies of the hadron properties and hadron-hadron interactions \[8,9\]. In practice, the short-distance behaviour of this interaction is not expected to be reliable \[1\] - unlike the long range Yukawa piece - and in the spectroscopic studies by Glozman and Riska the radial strength is replaced by single fitting parameter in each shell. On the other hand, the spin-isospin structure of Eq. (1) is maintained and the corrections from Eq. (1) to the energy of the nucleon (N) and the $\Delta(1232)$ are given as

\[ M_N = M_0 - 15P_0^\pi, \quad (2) \]
\[ M_\Delta = M_0 - 3P_0^\pi, \quad (3) \]

where $M_0$ is the corresponding unperturbed energy and $P_0^\pi$ is the fitting parameter corresponding to the radial matrix element of Eq. (1), in the lowest-energy unperturbed shell of the 3-quark system.

Because the basis for this approach to hadron spectroscopy is chiral symmetry, we were interested to check that the formalism is consistent with chiral perturbation theory ($\chi$PT) - i.e., that at least the leading nonanalytic (LNA) contribution to hadron masses is correct. It turns out to be very easy to check this and the result is that Eq. (1) is inconsistent with the LNA behaviour of QCD.

The LNA contribution to the mass of the nucleon is proportional to $m_\pi^3 \sim m_q^{3/2}$ \[10\]. In the quark model of Glozman and Riska, such a contribution can only arise from the linear term in the expansion of the Yukawa potential in Eq. (1)
\[ H^{LNA}_{\text{tot}} = \frac{g^2}{4\pi} \frac{1}{3} \sum_{i<j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j m^2 \frac{1}{r_{ij}} + \mathcal{O}(m^2) \]

\[ \sim -m^3 \frac{g^2}{4\pi} \frac{1}{3} \sum_{i<j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j. \]  

(4)

The radial matrix element is therefore a normalization integral and hence model independent, as it must be. The overall strength (in hadron \(|H\rangle\)) is given by the spin-isospin matrix element

\[ \langle SI \rangle_H = \langle H | \sum_{i<j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j | H \rangle. \]

(5)

For the N and the Δ this gives

\[ \langle SI \rangle^\text{Eq. (1)}_N = 30 \]

(6)

\[ \langle SI \rangle^\text{Eq. (1)}_\Delta = 6. \]  

(7)

On the other hand, the corresponding matrix elements from the LNA contribution required by \(\chi^\text{PT}\) are given by [11]

\[ \langle SI \rangle^\chi_N = 25 \]

(8)

\[ \langle SI \rangle^\chi_\Delta = 25. \]  

(9)

The formulation of \(\chi^\text{PT}\) including the Δ(1232) as an explicit degree of freedom was originally proposed in Ref. [12]. These contributions arise from the processes shown in Figs. (1a) and (b), respectively. We stress that this requires, as usually assumed in \(\chi^\text{PT}\), that the \(N\) and \(\Delta\) are not degenerate in the chiral limit. For a critical discussion on this subject, we refer the reader to Ref. [13]. The LNA chiral contributions to the octet and decuplet baryons has also been calculated within the framework which combines the \(1/N_c\) expansion with \(\chi^\text{PT}\), where \(N_c\) is the number of colors. Large \(N_c\) \(\chi^\text{PT}\) was originally proposed by Dashen and Manohar [14], and has been further developed by many authors (for a list of references, see Ref. [15]).

\[ \text{FIGURE 1. One-loop pion self-energy of (a) the nucleon (N) and (b) the delta (\Delta).} \]
A comparison of these results shows that Eq. (1) yields the wrong LNA contribution for both the N and the Δ. For the N, the error is not large (30 compared to 25). However, because the error is much larger for the Δ the crucial point is that the systematics are wrong. For example, with the correct coefficients this mechanism provides no Δ-N mass difference at all! Of course, our arguments concern the systematics of the LNA behaviour implied by Eq. (1). Even though the Yukawa term is not actually used in the spectral studies, the coefficient of the short-range piece, which is used, is the same and hence our arguments are directly relevant to the actual calculations.

In ChPT the LNA contribution to the nucleon mass is given by

\[
M_{N}^{LNA} = -\frac{3}{32\pi f_{\pi}^2} g_{A}^2 m_{\pi}^3, \tag{10}
\]

where \( f_{\pi} \sim 93 \text{ MeV} \) is the pion decay constant and \( g_{A} = 1.26 \) is the weak decay constant. In a quark model, the crucial step in ensuing this LNA behaviour is to project the quark states onto bare baryon states [16]. Specifically, in a constituent quark model of the Glozman-Riska type, the bare states would correspond to the three quark states confined by a phenomenological potential. The effective hadronic Hamiltonian is obtained by projecting the quark-model Hamiltonian, which now includes the quark-pion vertices, on the basis of the bare three-quark states. Chiral corrections to hadronic properties, such as masses and magnetic moments, are then calculated in time-ordered perturbation theory with the effective hadronic Hamiltonian. For a constituent quark model of the Glozman-Riska type, such a procedure leads to corrections to the N and the Δ masses of the form

\[
M_{N} = M_{N}^{(0)} - \frac{3}{16\pi^2 f_{\pi}^2} g_{A}^2 \int_{0}^{\infty} dk \frac{k^4 u_{NN}^2(k)}{w^2(k)} - \frac{3}{16\pi^2 f_{\pi}^2} \frac{32}{25} g_{A}^2 \int_{0}^{\infty} dk \frac{k^4 u_{N\Delta}^2(k)}{w(k)(\Delta M + w(k))}, \tag{11}
\]

\[
M_{\Delta} = M_{\Delta}^{(0)} + \frac{3}{16\pi^2 f_{\pi}^2} \frac{8}{25} g_{A}^2 \int_{0}^{\infty} dk \frac{k^4 u_{N\Delta}^2(k)}{w(k)(\Delta M - w(k))} - \frac{3}{16\pi^2 f_{\pi}^2} g_{A}^2 \int_{0}^{\infty} dk \frac{k^4 u_{\Delta\Delta}^2(k)}{w^2(k)}. \tag{12}
\]

Here, the \( M^{(0)} \)’s are the masses in the chiral limit, \( \Delta M = M_{\Delta} - M_{N} \), \( g_{A} = 5/3 \) is the bare axial coupling given by the constituent quark model, \( w(k) = \sqrt{k^2 + m_{\pi}^2} \) is the pion energy and \( u_{NN}(k), u_{N\Delta}(k), \ldots \) are the \( NN\pi, N\Delta\pi, \ldots \) form factors. The LNA contribution to \( M_{N} \) is easily seen to arise from the first integral in Eq. (11) (c.f. Fig. 1(a)), while the LNA contribution to \( M_{\Delta} \) comes from the second integral in Eq. (12) - c.f. Fig. 1(b).

In order to understand why the use of Eq. (1) is wrong, we consider the limit, generally considered physically unlikely [12] [13], that \( \Delta M = 0 \). Then all integrals in Eqs. (11) and (12) have the same LNA behaviour and the contributions are in the ratio
\[
25 \,(N \rightarrow N\pi \rightarrow N) \quad : \quad 32 \,(N \rightarrow \Delta\pi \rightarrow N) \quad \quad \quad \quad (13)
\]
\[
8 \, (\Delta \rightarrow N\pi \rightarrow \Delta) \quad : \quad 25 \,(\Delta \rightarrow \Delta\pi \rightarrow \Delta). \quad \quad \quad \quad (14)
\]

In this case the ratio of the total \(N\) and \(\Delta\) self-energies is \(57 : 33\) and the difference is identical to that given by Eqs. (3) and (7). This recalls the well known result from the early work on chiral bag models \[17–19,16\] that the calculation of the self-energy integrals through projection on all baryon states in which the orbital quantum numbers are unchanged, in the limit where these are degenerate, is equivalent to calculating pion emission and absorption between all quarks. In particular one must include those diagrams where the pion is emitted and absorbed by the same quark. In this case the spin-isospin structure of the pion interaction is

\[
\langle SI \rangle_H = \frac{1}{2} \sum_{ij} \langle H|\sigma_i \cdot \sigma_j \tau_i \cdot \tau_j|H \rangle,
\]

so that \(\langle SI \rangle_N = 57\) and \(\langle SI \rangle_\Delta = 33\) – which agree with the results based on Eqs. (11) and (12), quoted in Eqs. (13) and (14). Precisely this form of the pion self-energy was suggested in the early spectroscopic study of Mulders and Thomas \[20\] – see also Refs. \[18,19,21\], and Ref. \[22\] for more recent work.

For completeness, we remark that the ratios given in Eqs. (13) and (14) are precisely the leading order corrections given by large \(N_c\) \(\chi\)PT, as can be easily checked making use of Eqs. (5.6), (C1) and C(5) of Ref. \[15\].

In practice, the \(\Delta-N\) mass difference is quite large and the contribution from the process \(N \rightarrow \Delta\pi \rightarrow N\) is consequently suppressed. One would still expect to obtain a sizeable fraction of the \(\Delta-N\) splitting from pion exchange. Indeed, at the price of increasing the size of the pion-quark effective coupling one could refit the whole mass difference in terms of pion exchange. This would have the consequence that the total nucleon self-energy associated with pion exchange would need to be of the order of 700 MeV. Whether one is able to live with such large self-energies remains to be seen. The alternative is to add some additional hyperfine interaction, such as gluon exchange or residual instanton effects.

In conclusion, we repeat that the use of Goldstone boson exchange interactions of the type given in Eq.(1) is inconsistent with the chiral structure of QCD. In order to reproduce the correct chiral behaviour one must include Goldstone boson exchange between all quarks, including self-interactions, but the intermediate quark states must be projected onto (bare) baryon states – as carried out, for example, within the Cloudy Bag Model \[16\]. While
our analysis of the spectroscopic studies of Glozman and collaborators shows that these are incomplete, the findings are not entirely negative. One can still hope that the major qualitative features of this work will survive in a complete re-analysis. Such a re-analysis must now be an urgent priority.

This work was supported by the Australian Research Council and CNPq (Brazil). One of us (AWT) would like to acknowledge the warm hospitality of the Institute for Theoretical Physics at UNESP, where much of the work was carried out.
REFERENCES

[1] L. Ya. Glozman and D. O. Riska, Phys. Rept. 268 (1996) 263.
[2] L. Ya. Glozman and D. O. Riska, Phys. Lett. B 366 (1996) 305.
[3] L. Ya. Glozman and D. O. Riska, Nucl. Phys. A 603 (1996) 326, Erratum-ibid. A 620 (1997) 510.
[4] L. Ya. Glozman, Z. Papp and W. Plessas, Phys. Lett. B 381 (1996) 311.
[5] L. Ya. Glozman, Z. Papp, W. Plessas, K. Varga and R. F. Wagenbrun, Phys. Rev. C 57 (1998) 3406; ibid. Phys. Rev. D 58 (1998) 094030.
[6] F. Stancu, S. Pepin and L. Ya. Glozman, Phys. Rev. C 56 (1997) 2779; ibid. Phys. Rev. D 57 (1998) 4393.
[7] A. de Rújula, H. Georgi and S.L. Glashow, Phys. Rev. D 12 (1975) 147.
[8] For reviews, see: A. Faessler, A. Buchmann and Y. Yamauchi, Int. J. Mod. Phys. E 2 (1993) 39; K. Yazaki, Prog. Part. Nucl. Phys. 24 (1990) 353; K. Shimizu, Rep. Prog. Phys. 52 (1989) 1.
[9] F. Fernández, A. Valcarce, U. Straub and A. Faessler, J. Phys. G 19 (1993) 2013; A. Valcarce, F. Fernández and P. González, Phys. Rev. C 56 (1997) 3026.
[10] P. Langacker and H. Pagels, Phys. Rev. D 8 (1973) 4595; ibid. Phys. Rev. D 10 (1974) 2904.
[11] E. Jenkins, Nucl. Phys. B 368 (1992) 190.
[12] E. Jenkins and A.V. Manohar, Phys. Lett. B 259 (1991) 353.
[13] B. Borasoy and U.-G. Meissner, Ann. Phys. (N.Y.) 254 (1997) 192; V. Bernard, N. Kaiser and U.-G. Meissner, Z. Phys. C 60 (1993) 111.
[14] R. Dashen and A.V. Manohar, Phys. Lett. B 315 (1993) 425; ibid 438.
[15] Y. Oh and W. Weise, Baryon masses in large $N_c$ chiral perturbation theory, [hep-ph/9901354].
[16] A.W. Thomas, Adv. Nucl. Phys. 13 (1984) 1 ; G.A. Miller, Int. Rev. Nucl. Phys. 2 (1984) 190; S. Théberge, A.W. Thomas and G.A. Miller, Phys. Rev. D 22 (1980) 2838; Erratum-ibid. Phys. Rev. D 23 (1981) 2106.
[17] R.L. Jaffe, Phys. Rev. D 21 (1980) 3215.
[18] W.N. Cottingham, K. Tsu and J.-M. Richard, Nucl. Phys. B 179 (1981) 541.
[19] F. Myhrer, G.E. Brown and Z. Xu, Nucl. Phys. A 362 (1981) 317.
[20] P.J. Mulders and A.W. Thomas, J. Phys. G 9 (1983) 1159.
[21] B.E. Palladino and P. Leal Ferreira, Phys. Rev. D 40 (1989) 3024.
[22] S.K. Ghosh and S.C. Phatak, Phys. Rev. C 58 (1998) 1714.