SUPPLIER FINANCING SERVICE DECISIONS FOR A CAPITAL-CONSTRAINED SUPPLY CHAIN: TRADE CREDIT VS. COMBINED CREDIT FINANCING

QIANG LIN, YING PENG AND YING HU*

College of Management and Economics
Tianjin University
Tianjin, 300072, China

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ABSTRACT. In practice, suppliers and third-party logistics providers sometimes both offer credit in supply chain financing. To examine the supplier’s financing decision, we firstly design a multiple-participant supply chain finance system comprising a supplier, a capital-constrained retailer, and a 3PL firm. Secondly, we compare combined credit financing (CCF), which includes both the supplier’s partial trade credit and the 3PL firm’s credit with trade credit financing (TCF), to analyze the supplier’s optimal decision given the retailer’s initial capital level and immediate payment coefficient. Thirdly, we consider the operational and financial parameters to obtain the optimal decisions of supply chain participants under both TCF and CCF. Finally, we perform a numerical analysis of the retailer’s initial capital level and immediate payment coefficient. The results show that: when the retailer’s initial capital level is low or the retailer’s capital constraint is insignificant, the supplier will choose TCF; otherwise, the supplier would better choose CCF. It is more profitable for the supplier to cooperate with a retailer with limited assets under both TCF and CCF. Moreover, we obtained the threshold level of the retailer’s initial capital to ensure the retailer’s participation and the immediate payment coefficient that ensures the 3PL firm’s participation under CCF.

1. Introduction. As a type of short-term loan, the aim of the supply chain financing is to finance capital-constrained enterprises and to substantially enhance the entire supply chain’s financial efficiency. There are two widely used supply chain finance (SCF) systems: external bank credit financing (BCF) and internal trade credit financing (TCF) where the enterprise that has more power in the supply chain extends credit to its partners by providing accounts payable or accounts receivable [17]. According to Petersen and Rajan (1997)[13], trade credit is the most important way of short-term financing for enterprises in the US. Information from the less-developed world indicates that it may be even more important for firms in such countries to cooperate on trade credit. (Berlin, 2003)[11].

In a financing crisis, the suppliers, the dominant firms in the supply chain, also experience a shortage of funds and cannot provide fully deferred payments to downstream retailers. The automobile manufacturer FAW-Xiali Automobile Co. Ltd.

* Corresponding author: Ying Hu.
has shifted from the full trade credit system to a partial trade credit system in which it asks retailers to make larger (60%–90%) partial payments immediately after production. Numerical analysis reveals that a similar partial guarantee based system of financing can achieve supply chain coordination when the guarantee coefficient is as large as 0.92 (Yan et al., 2016)[17].

3PL firms occupy a cardinal position in supply chain management. 3PL firms can have considerable effects on the profits of upstream and downstream enterprises. Many successful logistics service providers, such as UPS, FedEx and DHL, offer financial services that are integrated with their traditional logistics services to obtain higher profits (Chen and Cai, 2011)[23].

This paper is inspired by a real-world case in which a Chinese 3PL firm called Eternal Asia Supply Chain Management Ltd. (EASCML) offers SCF (credit financing) to help small and medium enterprises (SMEs). These SMEs achieved 191 million yuan in SCF income (poundage income and interest), and their profit margins increased by as much as 77.36% according to China Business News, 2015.

Adopted by the practices described above, we derive the question of the supplier’s optimal decision. As the retailer may exhibit bankruptcy risk, the supplier and the retailer may face capital constraints. The supply chain participants need to answer the following questions:

1. When working with capital-constrained retailers, what kind of SCF should a supplier choose (TCF or CCF)? Under what conditions should the supplier cooperate with the 3PL firm and the retailer under CCF?

2. When should 3PL firms participate in CCF given the retailer’s initial capital and immediate payment coefficient?

3. What effects do the retailer’s initial capital and immediate payment coefficient have on the decisions and profits of supply chain participants?

In order to answer these questions, we propose a three-stage game involving the supplier, retailer and 3PL. By solving this game, we find that the optimal mode decision is greatly affected by different retailer’s initial capital. Our results show that the SCF mode decision of supplier varies with the retailer’s different initial capital level. Moreover, when the retailer’s initial capital level is too low, almost close to 0, the risk of SCF is too high for both supplier and 3PL. Therefore, 3PL should consider carefully whether to participate in CCF. In general, compared to the traditional TCF mode, the supplier in the CCF mode can set higher wholesale prices and get higher profits. Moreover, due to the financial constraints of retailer, deferred payment and combined financing models can promote retailer to increase order quantity, both modes are always beneficial to capital-constraint retailer.

In spite of the important practical meaning of this topic, there are limited theoretical analysis regards to the 3PL’s credit financing with supplier’s trade credit. Our study contributes to the literature by combining a supplier’s trade credit with credit from a 3PL firm. Most previous researchers have not focused on a 3PL firm that subsidizes a retailer. Instead, they focus primarily on the initial capital level and bankruptcy risk under TCF. This paper incorporates a 3PL firm’s behavior to help suppliers choose a suitable financing mechanism. Moreover, we find that when the retailer’s capital level is moderate, if a 3PL firm participates in the credit system, the supplier can earn higher profits than under TCF. The 3PL company should cooperate with the supply chain when the supplier charges a higher immediate payment coefficient and the retailer has limited assets.
This paper is organized as following. Section 2 reviews the related literature. Section 3 presents TCF as a benchmark. Section 4 proposes a three-stage mode of CCF. In Section 5, we perform a series of numerical experiments to highlight the key features of our model. Finally, Section 6 concludes this work.

2. Literature review. Various scholars have investigated trade credit theory between supply chain in different insights. Trade credit originally played a non-financial role and was intended to reduce transaction costs (Ferris, 1981)[5]. Because customers have different levels of credit-worthiness, price discrimination can occur (Zechner, Brennan and Maksimovic, 1988)[16]. Trade credit helps creditors to enhance their long-term relationships with partners (C. Jézabel, and H. Jérôme 2002)[4]. Unlike earlier theoretical work on trade credit, more recent research considers factors such as the distinguishing features of trade credit (Daripa and Nilsen, 2010[1]; Chen, Kök and Tong, 2013[3]) and compares bank credit and trade credit financing (Miwa and Ramseyer, 2005[26]; Daripa and Nilsen, 2010[1]). Rather than examine trade credit from a finance perspective, some papers treat trade credit as a decision variable. Chen (2015)[22] compares financing costs between trade credit and bank credit, and he finds that trade credit allows for lower costs than bank credit. Our paper obtains an analogous result when we compare the interest rates under the two modes (TCF and CCF) and find that the interest rate is lower in TCF than CCF. Kouvelis and Zhao (2012) [10] formulate a two-player Stackelberg game for a supplier’s choice between credit and bank credit. The key contribution of this paper is to introduce a new credit mode which combines trade credit with credit provided by 3PL and expand the formal study of the game theory between the retailer and supplier.

Our study also relates to the retailer’s initial capital level. A firm’s initial capital level affects the performance and decisions of the supply chain. When a supply chain member’s initial capital is insufficient, this increases bankruptcy risk at the end of the repayment period. Boyabatlı and Toktay (2006)[19] analyze the effect of the amount of a transaction on the operating and financial decisions of a capital-constrained enterprise under imperfect capital markets and emphasize that external financing liquidity could reduce the risk of bankruptcy. H.L. Chang and B.D. Rhee (2011)[2] evaluate the risk coefficient of enterprise bankruptcy using an expert scoring method.

The studies most closely related to ours are Chen and Cai (2011)[23], Zhou Y W et al. (2015)[27] and Yan et al. (2016)[17]. Chen and Cai (2011)[23] analyze supplier credit, bank financing and 3PL financing in the presence of a budget-constrained retailer. They conclude that 3PL financing is better than bank financing because the former is more information symmetrical. We combine supplier trade credit and 3PL credit to design a new mode of financing rather than compare the two modes. We then extend the analysis by compared the new financing mode with that of full trade credit under different initial capital levels for the retailer. Y.W. Zhou et al. (2015)[27] conclude that under partial deferred payment, the cash flow and inventory levels will affect the order quantity. Our study emphasizes that a supplier should consider the retailer’s initial capital level when TCF or the new mechanism (combined credit financing) is optimal to reduce the lost from the retailer’s bankruptcy risk. Moreover, we further analyze how the immediate payment coefficient affects the 3PL firm’s decision to join the supply chain. Yan et al. (2016)[17] analyze a supply chain coordination problem with a combination of partial trade credit and bank financing and describe the effect of different credit coefficients on
the order quantity. Based on this research, Jin et al. (2018) [22] further study the optimal credit contract design considering trade credit and bank credit in the case of suppliers with risk aversion.

There are many other researches on trade credit and bank credit, and the combined credit model, such as Yan et al. (2017) [18], Bi et al. (2018) [6] and Zhan et al. (2018) [7]. Zhan et al. (2018) study an innovative trade credit with rebate contract (TCRC) mode, they find that equilibrium strategies exist between the manufacturer and the retailer under the TCRC and traditional trade credit financing (TTCF) mode. However, for the upstream and downstream of the supply chain, the TCRC model is better than the TTCF mode. Scholars are still seeking breakthroughs in research on trade credit. This research problem is still crucial, so we have also tried to expand theoretical research in this field. We find that in practice, suppliers always work with 3PL firms rather than banks. 3PL companies have considerable liquidity and can transfer goods among different retailers, which reduces risk. However, this financing mode is relatively new to the academic literature. Therefore, we design a new CCF mechanism that integrates partial trade credit (provided by the supplier) and credit financing (provided by the 3PL firm) and evaluate it by obtaining the threshold level of the retailer’s initial capital at which the retailer is willing to participate in CCF.

Our contributions to the literature are as follows: (1) we design a CCF mechanism and analyze a multi-level Stackelberg game comprising a supplier (leader), capital-constrained retailer (follower) and a 3PL firm (sub-leader). This new mode is a practical mechanism to reduce the credit risk since 3PL is the core of the flow of traded goods, and controls the goods in the loan. (2) We present an analysis of the initial capital level and immediate payment coefficient in various scenarios to obtain the threshold for the retailer’s and 3PL firm’s decision to participate in CCF. The key distinction from the abovementioned literature is that we consider different initial capital levels for the retailer under different SCF mechanisms in various scenarios and how the supplier should choose the optimal strategy (TCF or CCF) to maximize profits. (3) We provide a series of numerical examples to illustrate the influence of the initial capital level and immediate payment coefficient on the outcomes of the game. We demonstrate that the supplier charges a higher wholesale price and obtains a lower order quantity under CCF than under TCF.

3. Trade credit financing (a benchmark). Based on a two-echelon SCF system, we derive the corresponding market equilibrium for a SCF system shown in Fig. 1, which consists of two participants (the supplier and the retailer). We use a Stackelberg game to analyze the optimal strategies; the supplier acts as the leader, and the retailer acts as the follower. Knowing the retailer’s optimal response, the supplier first sets the interest rate and wholesale price. After observing the supplier’s decision, the retailer determines its optimal order quantity. In this mechanism, the supplier permits deferred payment when credit terms are offered, the retailer orders a quantity of the product from the upstream supplier, and the retailer encounters a capital constraint. The financial deficit should be repaid before the credit term terminates (at the sales season’s end). The supplier provides TCF for the retailer and bears the retailer’s bankruptcy risk.

3.1. Notations and assumptions. Let \( w \) and \( q \) be the supplier’s wholesale price and retailer’s order quantity. The uncertain demand is determined by the non-negative random variable \( x \). The probability density function is \( f(x) \), the cumulative distribution function is \( F(x) \), and the complementary cumulative distribution function.
Table 1. List of notations

| Parameters | Description |
|------------|-------------|
| $p$        | Retail price |
| $c$        | Manufacturing cost |
| $x$        | Random demand, $x \geq 0$ |
| $B$        | Retailer’s initial capital level |
| $\lambda$ | The immediate payment coefficient |
| $r_f$      | The risk-free interest rate |
| $r_t$      | The interest rate offered by the supplier |
| $P(a_1)$  | The success rate of financing |
| $w$        | The supplier’s wholesale price |
| $q$        | The retailer’s order quantity |
| $r$        | The 3PL firm’s financing interest rate |

function is $\bar{F}(x) = 1 - F(x)$. $F(x)$ is assumed to be differentiable, increasing, and strictly and absolutely continuous with $f(x) > 0$, and $g(x) = f(x)/\bar{F}(x)$ denotes the hazard rate. Therefore, $G(x) = xg(x)$ is assumed to be the generalized failure rate and to increase in $x$. We assume that the demand distribution consists of an increasing failure rate (IFR), which can take the form of many commonly used distributions, such as the uniform, exponential, normal and gamma distributions (Lariviere and Porteus, 2001)[15]. Every supply chain participant face a participation constraint described by $p > w(1 + r_t) > c(1 + r_t)$.

Figure 1. The operation of the trade credit financing mechanism

Figure 1 indicates that the supplier first offers an interest rate of $r_t$, and second, the supplier sets the wholesale price $w$ based on the trade credit interest rate. Third, the retailer sets the order quantity $q_1$. At time 1, when the manufacturing process ends, the retailer pays the initial capital to the supplier to guarantee and obtain the goods. At time 2, the retailer sells the goods according to demand. Before time 2 ends, the retailer pays off the rest of payment and interest. If the retailer’s revenue cannot cover the remaining product payment, the retailer will go bankrupt and must repay all revenue to the supplier to compensate for the loss. The notations used in the model are summarized in Table 1 for ease of reference.
3.2. The optimal decisions of the two SCF participants. 1. Analysis of the capital level

The retailer’s initial capital level affects the supplier’s financing mode and the chosen order quantity and wholesale price. This analysis can be divided into the following two situations: When retailers have sufficient funds to pay for their order, there is no need to use TCF, and the supply chain follows the traditional newsvendor model of capital adequacy. However, when the retailer lacks sufficient funds to pay for order, it is necessary to consider the retailer’s initial capital level, and the threshold level of the retailer’s initial capital level under TCF is then determined.

If the retailer can make the payment without reaching its capital constraint, there is no need for the participants to cooperate on trade credit, and the retailer’s profit function is
\[
\max_q \pi_r(q; w, x) = E[p \min(q, x) - wq].
\]
It can then be determined that the retailer’s optimal quantity is
\[
q^*_0 = \bar{F}^{-1}(\frac{w}{p}).
\]
On the other hand, the supplier’s profit maximization function is
\[
\max_w \pi_s(w; q, x) = E[(w - c)q]
\]
given sufficient capital. The supplier will set an optimal wholesale price \(w^*_0\) to maximize its profit. Consider \(\pi_q = (w - c)q^*_0\); there is an optimal wholesale price \(w^*_0\) that maximizes profits. Then, based on the supplier’s optimal wholesale price, the retailer’s optimal order quantity can be determined as
\[
q^*_0 = \bar{F}^{-1}(\frac{w^*_0}{p}),
\]
and the required capital is 
\[
B_0 = w^*_0 \cdot \bar{F}^{-1}(\frac{w^*_0}{p}).
\]

The following section, in which we analyze each participant’s strategy under TCF, begins by determining the retailer’s optimal order quantity. Next, we solve the Stackelberg game through backward induction to determine the equilibrium TCF. 2. Retailer’s optimal order quantity

A capital-constrained retailer should determine its optimal order quantity while accounting for bankruptcy risk when the supplier offers the trade credit contract. When the sales season ends, the supplier will bear the bankrupt risk if there is low realized demand, when the retailer’s revenue cannot cover the credit payment, and it will face a loss of \(wq - B - pS(q)\), where \(B\) denotes the retailer’s initial capital level. If demand is very low, the retailer must declare bankruptcy, and the supplier can only obtain revenue \(pS(q)\). We formulate the retailer’s expected profit as follows:
\[
\max_q \pi_r(q; w, x) = E[p \min(q, x) - (wq - B)^+ (1 + r)]^+ - B.
\]

\(y^+ = \max(y, 0)\) indicates that if the retailer’s revenue cannot cover the trade credit payment and interest, the expected profit is zero.

We denote by \(w^*_1\) the supplier’s optimal wholesale price and by \(q^*_1\) retailer’s optimal order quantity. When \(B < w^*_1 \cdot q^*_1\), the retailer’s revenue cannot cover the order payment, and we discuss this situation next.

Lemma 3.1. For a capital-constrained retailer, the sufficient condition for the retailer not to go bankrupt is that minimum realized demand satisfies
\[
x \geq \hat{x}_t = \frac{(wq - B)(1 + r)}{p}.
\]

Proof. When the sales revenue of retailer can meet the financing gap of trade credit, retailer has no bankruptcy risk, expressing in mathematical formula is \(px \geq (wq - B)(1 + r)\), so \(x \geq \frac{(wq - B)(1 + r)}{p}\), the smallest sales will be just equal the financing gap of trade credit, the critical demand is figured out in this way
\[
x \geq \hat{x}_t = \frac{(wq - B)(1 + r)}{p}.
\]
From Lemma 3.1, if demand is lower than the minimum realized demand, the retailer will be unable to repay the supplier and will go bankrupt. The retailer should predict demand as accurately as possible to reduce bankruptcy risk.

**Proposition 1.** For IFR distributions of demand, when the supplier sets a trade credit rate \( r_1 \) and a wholesale price \( w \), then grants TCF to retailer, where this decision is based on the retailer’s initial capital, a capital-constrained retailer’s unique best response in terms of optimal order quantity satisfies:

\[
q_1^* = F^{-1} \left( \frac{w_0^*}{p} \right), \quad \text{when } B \geq wF^{-1} \left( \frac{w(1+r_1)}{p} F(\hat{x}_t) \right)
\]  

(2)

\[
q_1^* = \tilde{F}^{-1} \left( \frac{w(1+r_1)}{p} \tilde{F}(\hat{x}_t) \right), \quad \text{when } B < w\tilde{F}^{-1} \left( \frac{w(1+r_1)}{p} \tilde{F}(\hat{x}_t) \right)
\]  

(3)

**Proof.** According to the profit function, the retailer’s expected profit can be rewritten as

\[
\pi_t(q) = \int_{q}^{\hat{x}_t} [px - (wq - B)(1 + r_t)] f(x)dx
\]

\[+ \int_{\hat{x}_t}^{+\infty} [pq - (wq - B)(1 + r_t)] f(x)dx - B
\]  

(4)

then simplify the formula as \( \pi_t(q) = p \int_{\hat{x}_t}^{q} \tilde{F}(x)dx - B. \) Taking the first-order and second-order derivative of \( \pi_t(q) \) with respect to \( q \), it follows that:

\[
\frac{d\pi_t}{dq} = p\tilde{F}(q) - w(1+r_t)\tilde{F}(\hat{x}_t), \quad \frac{d^2\pi_t}{dq^2} = -pf(q) + \frac{w^2(1+r_t)^2}{p^2} f(\hat{x}_t) = -p\tilde{F}(q) \left[ g(q) - \frac{w^2(1+r_t)^2}{p^2} g(\hat{x}_t) \right],
\]

because of \( q \geq \hat{x}_t \), so \( g(q) \geq g(\hat{x}_t) \), moreover \( \frac{w^2(1+r_t)^2}{p^2} \leq 1 \), then we can get \( \frac{w^2(1+r_t)^2}{p^2} \leq 1 \), where \( \Omega=\frac{w(1+r_t)}{p} \). That is \( f(q_1^*) > \Omega^2 f(\hat{x}_t) \) indicates that \( \frac{d^2\pi_t}{dq^2} = -pf(q) + \frac{w^2(1+r_t)^2}{p^2} f(\hat{x}_t) \leq 0 \). Since the second-order derivable is negative, there must be a solution which let first-order derivable be zero. So the optimal order quantity is \( q_1^* = F^{-1} \left( \frac{w(1+r_t)}{p} F(\hat{x}_t) \right) \), comparing its own fund and trade loan, when \( B \geq w_1^* q_1^* \), there is no need to use trade credit model. The order quantity of supplier is \( \tilde{F}^{-1} \left( \frac{w_0^*}{p} \right) \). It is the optimal order quantity while there is no fund limitation. \( w_0^* \) is the optimal wholesale price, but when \( B < w_1^* q_1^* \), the retailer needs to adopt trade credit model, so the optimal order quantity should be \( q_1^* = \tilde{F}^{-1} \left( \frac{w(1+r_t)}{p} \tilde{F}(\hat{x}_t) \right) \). \( \square \)

From proposition 1, the retailer’s order quantity is divided two cases according to the different ranges of the initial capital level. In the first case, the retailer’s initial capital can cover the payment for its product order, while in the second other case, retailer’s initial capital cannot cover the order payment. This suggests that the retailer’s optimal order quantity is determined by a combination of the financing and operational parameters, since the retailer should consider both the financing cost and bankruptcy risk. Hence, the interactions between operational and financial decisions cannot be decoupled in a capital-constrained supply chain.

**Lemma 3.2.** If the retailer’s optimal order quantity \( q_1^* \) has a functional relationship with the initial capital level \( B \), \( q_1^*(B) \) will be a decreasing function of \( B \).
Proof. By using the method of limit to prove this lemma, by the previous proposition 1, there is a unique optimal order quantity $q^*_1$ existing. Because the optimal decision has the property of uniqueness, its own capital and the optimal order quantity has one to one correspondence relationship. From the equation of $q^*_1 = F^{-1}\left(\frac{w(1+r_t)}{p}F(\hat{x}_t)\right)$, we can get that $\hat{F}(q^*_1) = \frac{w(1+r_t)}{p}F(\hat{x}_t)$, take derivation of $B, -f(q^*_1)\frac{dq^*_1}{dB} = -\frac{w(1+r_t)}{p}f(\hat{x}_t)\frac{\partial \hat{x}_t}{\partial B}$, and $\frac{\partial \hat{x}_t}{\partial B} = \frac{dq^*_1}{dB} = \frac{w(1+r_t)}{p} - 1 + r_t$, then

$$(-f(q^*_1) + \frac{w^2(1+r_t)^2}{p^2}f(\hat{x}_t)) \frac{dq^*_1}{dB} = \frac{w(1+r_t)}{p}f(\hat{x}_t).$$

Since we have already proved that

$$-f(q^*_1) + \frac{w^2(1+r_t)^2}{p^2}f(\hat{x}_t) < 0$$

from Proposition 1, so $\frac{dq^*_1}{dB} < 0$ holds.

As suggested by Lemma 3.2, the retailer’s optimal order quantity decreases in the retailer’s initial capital level. This illustrates that the TCF mechanism offers better incentives when the retailer has a lower initial capital level. However, a retailer with a high initial capital level has little incentive to increase its order quantity. Thus, the supplier should concentrate more on the retailer’s initial capital level to avoid bankruptcy losses and maximize its profits.

3. Supplier’s optimal wholesale price decision

We assume that the supplier has sufficient capital to offer the retailer TCF. In addition, the supplier should bear any loss from the retailer’s bankruptcy. Based on the above, we formulate the supplier’s expected profit as follows:

$$\max_w \pi_s(w; q, x) = (w - c)q + (wq - B) \cdot r_t - E\left[(wq - B)^+ \cdot (1 + r_t) - p \min(q, x)\right]^+$$

(5)

**Proposition 2.** For IFR distributions of demand under TCF, given the retailer’s best decision $q^*_1$ and initial capital $B$, the optimal wholesale price uniquely set by the supplier satisfies $w^*_1$.

$$w^*_1 = \frac{p^2 \cdot q^*_1 \cdot f(\hat{x}_t) + c \cdot p\hat{F}(\hat{x}_t)}{p(1 + r_t)F^2(\hat{x}_t) + c(1 + r_t)q^*_1 \cdot f(\hat{x}_t)}$$

(6)

Proof. To find the optimal wholesale price of supplier, the expected profit function can be rewritten as

$$\pi_s = \left[\frac{d\pi_s(w)}{d\hat{x}_t}\right] \cdot [-f(q^*_1) + \frac{w^2(1+r_t)^2}{p^2}f(\hat{x}_t)] \cdot \frac{\partial \hat{x}_t}{\partial B}$$

Simplifying the above equation, we can get $\pi_s = \left[\frac{d\pi_s(w)}{d\hat{x}_t}\right] \cdot [-f(q^*_1) + \frac{w^2(1+r_t)^2}{p^2}f(\hat{x}_t)] \cdot \frac{\partial \hat{x}_t}{\partial B}$. Taking the first-order and second-order derivative of $w$:

$$\frac{d\pi_s(w)}{dw} = \frac{\partial q^*_1}{\partial w} \cdot \frac{1}{p} \left[w \cdot \frac{q^*_1}{\partial w} + q^*_1\right].$$

Taking the first-order and second-order derivative of $w$:

$$\frac{d\pi_s(w)}{dw} = \frac{\partial q^*_1}{\partial w} \cdot \frac{1}{p} \left[w \cdot \frac{q^*_1}{\partial w} + q^*_1\right].$$

Taking the first-order and second-order derivative of $w$:

$$\frac{d\pi_s(w)}{dw} = \frac{\partial q^*_1}{\partial w} \cdot \frac{1}{p} \left[w \cdot \frac{q^*_1}{\partial w} + q^*_1\right].$$

From the equation of $\frac{d^2\pi_s(w)}{dw^2}$, if $\frac{d\pi_s(w)}{dw} < 0$ and $\frac{d^2\pi_s(w)}{dw^2} < 0$, then we can get $\frac{d^2\pi_s(w)}{dw^2} < 0$. First, the proof of $\frac{d\pi_s(w)}{dw} < 0$ is followed.

From proposition 1, $-p\hat{f}(q^*_1)\frac{dq^*_1}{dw} = \hat{F}(\hat{x}_t) - w(1 + r_t)f(\hat{x}_t)\frac{\partial \hat{x}_t}{\partial w}$, take $\frac{\partial \hat{x}_t}{\partial w} = \frac{(1 + r_t)}{p}[w \cdot \frac{q^*_1}{\partial w} + q^*_1]\] into above equation, it can be rewritten as $\frac{dq^*_1}{dw} = \frac{1 - 1\cdot \Omega(q^*_1)}{w\Omega(\hat{x}_t) - q^*_1}$. Similar to Chen and Wang (2012)[25], we prove by contradiction to explain that $\frac{dq^*_1}{dw} < 0$. So we assume that inequality of $\frac{dq^*_1}{dw} \geq 0$ holds. Furthermore, we can obtain that:

$$\frac{d\pi_s(w)}{dw} = \frac{\partial \hat{f}(q^*_1)}{\partial q^*_1} \frac{dq^*_1}{dw} \cdot \frac{\partial q^*_1}{\partial w} \cdot \frac{\Omega(q^*_1)}{w} \cdot \frac{1 - \Omega(q^*_1)}{\Omega(\hat{x}_t) - q^*_1}.$$
So, the inequality of \( \frac{d\hat{x}_t(q_1^*)}{dw} \geq 0 \) holds. Let \( d\hat{x}_t(q_1^*) / dw \big|_{w = w_0} = 0 \) We have \( 1 - q_1^* g(q_1^*) = 0 \), i.e., \( G(q_1^*) = 1 \). We have mentioned above the economic rationality hypothesis of \( c \leq w \leq \hat{w} = p/(1 + r) \), then we have three cases to discuss.

1. When \( w_0 \geq \hat{w} \), for any \( w \leq \hat{w} \), because of the assumption, we have \( G(q_1^*(w)) \leq G(q_1^*(\hat{w})) \). Then we get \( dq_1^* / dw \geq 0 \) for IFR distributions of demand. Hence, we can get \( q_1^* g(q_1^*) < 1 \) and \( \Omega < 1 \), so \( 1 - \Omega q_1^* g(\hat{x}_t) < 0 \). Then we get \( \frac{d\hat{x}_t(q_1^*)}{dw} < 0 \), and thus \( dq_1^* / dw < 0 \) holds. It is a contradiction to our assumption.

2. When \( c < w_0 < \hat{w} \), we have \( \frac{d\hat{x}_t(q_1^*)}{dw} < 0 \) over \((c, w_0)\), therefore, the inequality of \( dq_1^* / dw < 0 \) holds. So, it is also a contradiction to our assumption.

3. When \( w_0 = c \), for any \( w \geq c \), and \( \hat{x}_t(q_1^*) \leq q_1^* \), under the condition of \( \Omega < 1 \), thus \( \Omega q_1^* g(\hat{x}_t(q_1^*)) \leq q_1^* g(q_1^*) \). Then we get \( \frac{d\hat{x}_t(q_1^*)}{dw} < 0 \), and \( dq_1^* / dw < 0 \) holds. It is a contradiction to our assumption.

For all of the cases above, we get the contradictions to our assumption, that is, \( dq_1^* / dw < 0 \). Therefore, we conclude that the monotonicity of \( q_1^* \) and \( \hat{x}_t(q_1^*) \) with respect to \( w \) is immediate, and the inequality of \( \Omega q_1^* g(\hat{x}_t) < q_1^* g(\hat{x}_t) < q_1^* g(q_1^*) \leq 1 \) holds.

Secondly, we find \( \frac{d^2 q_1^*}{dw^2} = -\frac{2e^2 f(\hat{x}_t)\left(F(\hat{x}_t) - \Omega q_1^* f(\hat{x}_t)\right)}{w^2(\Omega T f(\hat{x}_t) - f(q_1^*))^2} \), we can conclude that \( d^2 q_1^* / dw^2 < 0 \) for an IFR distribution. Taking the equation of \( dq_1^* / dw < 0 \) and \( d^2 q_1^* / dw^2 < 0 \) into \( d^2 \pi_s(w) / dw^2 \). It can be concluded that \( d^2 \pi_s(w) / dw^2 < 0 \). So, taking the first-order derivates to be zero, then the optimal wholesale price is \( w_1^* = -\frac{p^2 q_1^* f(q_1^*) F(\hat{x}_t) + c - p F(\hat{x}_t)}{p(1 + r) F^2(\hat{x}_t) + c(1 + r) q_1^* f(\hat{x}_t)} \).

From proposition 2, when solving the expression for \( w_1^* \), we find that \( d^2 \pi_s(w) / dw^2 < 0 \), and so it is sufficient to prove that there is a unique pair of an optimal wholesale price and optimal order quantity \( q_1^* \) that represents a first-best outcome for the participants in the TCF system. The supplier’s price decision under TCF is more sophisticated than that in a traditional supply chain. It is contingent not only on operational parameters, such as product cost and order quantity, but also the retailer’s initial capital, bankruptcy risk and the trade credit rate.

4. Combined credit financing.

4.1. The operation of CCF.

4.1.1. The structure of CCF. In this section, we introduce a 3PL firm with financing capacity into the supply chain that also contains a capital-constrained retailer, and a supplier. In this multiple-participant SCF system, the retailer orders products from the supplier and then sells them to the customers; the retailer faces a capital constraint, which the supplier will address by offering financing mode to the retailer. However, as market competition intensifies, the supplier becomes reluctant to provide full financing in the presence of high risk. To decrease market risk, the supplier (leader) is willing to provide partial trade credit, where the immediate payment coefficient is \( \lambda \) for the retailer (follower), while the 3PL firm (sub-leader) provides financing, where the loan is initially \( \lambda w q - B \). The 3PL firm can charge some rate of interest \( r \) on the financing it provides to the retailer and bear the potential loss associated with the retailer’s bankruptcy risk. In this way, the risk can, to some extent, be transferred from supplier to the 3PL firm. In this innovative model of a three-echelon supply chain, the 3PL company, because it manages the
flow of the products, can gain accurate product information and thereby reduce transaction risks. We formulate the problem as a multi-level Stackelberg game in which the supplier acts as the leader by setting the wholesale price and the credit ratio. In response to the supplier’s decision, the 3PL firm acts as the sub-leader and sets the financing interest rate $r$. Finally, the retailer acts as the follower and sets the order quantity $q$. Fig. 2 illustrates the detailed sequence of the business activities of the SCF system.

**Figure 2.** The CCF in a multiple-participant SCF system

The process distribution of CCF operates as follows:

1. First, the supplier determines the wholesale price contract and announces the credit ratio $1 - \lambda$.
2. The retailer then accepts the contract offered by the supplier and determines its order quantity $q$ based on market demand.
3. The retailer faces capital constraints and applies to the 3PL firm for financing, where the financing amount is $L(q) = \lambda wq - B$.
4. The retailer sells the products, and at the end of the sales season, its revenue is $p \cdot S(q)$.
5. and (6) suggest that at the end of the period, if the retailer’s sales income is sufficient to meet its repayment obligation, the retailer will make the first payment to the supplier, and then the 3PL firm will make its payment.
6. (7) and (8) suggest that, at the end of this period, if the retailer’s sales income is not sufficient to meet its repayment obligation, it will declare bankruptcy. Jumping directly from step (4) to step (7), the sales revenue will be paid to the 3PL firm; the 3PL firm will bear retailer’s bankruptcy risk and compensate the supplier by repaying the remaining obligation.

4.1.2. Analysis of the retailer’s initial capital level. From the analysis of the retailer’s initial capital level, we find that it reaches its threshold value when supplier chooses CCF, which can then form the basis for the decisions of the supplier and retailer. We denote by $w_2^*$ the supplier’s optimal wholesale price and by $q_2^*$ the retailer’s optimal order quantity. Different ranges of the retailer’s initial capital level are discussed as follows:
(1) \( B \geq w_2^* q_2^* \) indicates that the retailer can place its order without facing a capital constraint, and the supplier can then secure full payment, which is the same as in a supply chain not subject to capital constraints.

(2) \( \lambda w_2^* q_2^* \leq B < w_2^* q_2^* \) indicates that the retailer’s initial capital level can meet the supplier’s immediate payment requirements, and the supplier will receive the amount \( \lambda w_2^* q_2^* \). The remainder of the payment obligation \((1-\lambda) w_2^* q_2^* \) is usually low because we generally set \( \lambda \) at a high rate following real-world practice (see, for example, the case of FAW-Xiali Automobile Co. Ltd. cited in section one). Therefore, the retailer’s revenue \( p_{\min}(q,x) \) can easily cover the rest of the order payment \((1-\lambda) w_2^* q_2^* \). Thus, we assume that there is little risk for the supplier, and in this case there is no capital constraint.

(3) \( B < \lambda w_2^* q_2^* \) indicates that the retailer’s revenue cannot cover the partial order payment, and this is the situation that we will consider below.

Following the Stackelberg game model, the supplier first determines the wholesale price, the 3PL firm then sets the interest rate, and finally, the retailer determines the product order quantity. The numerical analysis can be divided into five parts.

4.2. The optimal decisions of the multiple SCF participants. 1. Retailer’s optimal order quantity

When the sales season starts, the retailer orders a quantity \( q_2 \) from the supplier. Then the supplier sets the wholesale price at \( w_2^* \). However, the retailer cannot afford the immediate payment and borrows \( L(q) \) from the 3PL firm with interest, where \( L(q) = \lambda wq - B \).

When the sales season ends, the retailer earns revenue of \( p_{\min}(q,x) \). If demand is sufficiently high to cover the rest of product payment, the loan and interest, the retailer will make a net profit and pay the rest of payment obligation \((1-\lambda) wq \) to the supplier and repay the 3PL firm in the amount \( L(q)(1+r) \). If demand is low, the retailer goes bankrupt and must pay all revenue \( p_{\min}(q,x) \) to the 3PL firm. The 3PL firm then helps the retailer to fulfill the remainder of the order payment to the supplier. Therefore, the retailer’s decision problem under CCF is as follows:

\[
\max_q \pi_r(q;w,r,x) = E[p_{\min}(q,x) - (1-\lambda) wq - L(q)(1+r)]^+ - B
\]

(7)

For the situation in which the retailer is constrained by insufficient initial capital, we solve the optimal order quantity equation to maximize profits. The retailer should take its bankruptcy risk into account to make a rational decision when demand is low. Thus, for a capital-constrained retailer, the sufficient condition for the retailer to not go bankrupt is the minimum realized demand that satisfies:

\[
x \geq \hat{x}_c = (1-\lambda) w + L(q)(1+r) - wq(1+\lambda r) - B(1+r)
\]

(8)

Equation (8) suggests the critical point that makes the retailer go bankrupt under CCF; the bankruptcy risk is transferred from the supplier to the 3PL firm due to the interest rate and immediate payment coefficient.

Proposition 3. Based on the CCF contract, under IFR distributions of demand, when the supplier sets a wholesale price \( w_2 \) and the 3PL firm sets a loan rate \( r \), the capital-constrained retailer’s unique best-response optimal order quantity \( q_2^* \) satisfies:

\[
q_2^* = \bar{F}^{-1}(\kappa \bar{F}(\hat{x}_c))
\]

(9)
where $\kappa = \frac{\omega(1+\lambda\cdot r)}{p}$

Proof. According to Lemma 4, simplifying the current profit equation of the retailer, we have $\pi_r = p \int_{\hat{x}_c}^{q} \bar{F}(x) dx - B$. Taking the first-order and second-order derivative of the profit equation: $\frac{d\pi_r}{dq} = p\bar{F}(q) - (1+\lambda \cdot r) w \cdot \bar{F}(\hat{x}_c)$. Firstly, judging whether the second-order derivate is less than zero or not, assuming that $\kappa = \frac{\omega(1+\lambda\cdot r)}{p}$. Due to rational man Proposition, we have $p > w(1+\lambda\cdot r)$, so $\kappa \leq 1$, the second-order derivate can be rewritten as $\frac{d^2\pi_r}{dq^2} = -p\bar{F}(q) \left[ g(q) - \kappa^2 g(\hat{x}_t) \right]$, because of $q \geq \hat{x}_t$, so $g(q) \geq g(\hat{x}_t)$, moreover $\kappa \leq 1$, then we can get $g(q) \geq \kappa^2 g(\hat{x}_t)$ for the market demand distribution of IFR function, so the second-order derivate is negative, which means that there exists equilibrium solution to let the first-order derivate be zero. Hence the first-order derivate of order quantity can be zero, which means $\frac{d\pi_r}{dq} = 0$, so we can have $\frac{d\pi_r}{dq} = p\bar{F}(q) - (1+\lambda \cdot r) w \cdot \bar{F}(\hat{x}_c) = 0$. After simplifying, $\frac{d\pi_r}{dq} = p\bar{F}(q) - (1+\lambda \cdot r) w \cdot \bar{F}(\hat{x}_c) = 0$ □

According to Proposition 3, the expression for the best-response optimal order quantity suggests that it is influenced by the supplier’s wholesale price $w_2$ and the 3PL firm’s loan rate $r$. In addition, the immediate payment coefficient is a key factor influencing the range over which the order quantity varies.

2. 3PL firm’s optimal interest rate choice

The 3PL firm provides financing for retailer based on the order quantity and sales over the past few months. When the sales season ends, the 3PL firm asks the retailer to pay off the loan and interest $L(q) (1+r)$. If demand is low, the retailer will go bankrupt and pay all its revenue to the 3PL firm which is $pS(q)$. This means that the retailer’s revenue cannot cover the loan and interest. Based on the above, we describe this case as follows:

$$\max_r \pi_{3pl} (r; q, w, x)$$

$$= E \min[L(q) (1+r), p \min(q, x) - (1-\lambda) wq] - L(q) (1+r)$$

(10)

We simplify the formula (9) and obtain the following formula

$$\pi_{3pl} = p \left[ \int_{0}^{\hat{x}_c} \bar{F}(x) dx - \hat{x}_c \right] + (\lambda wq - B)(r - rf)$$

(11)

Lemma 4.1. Under IFR distributions of demand, there is a functional relationship between the retailer’s optimal order quantity and the interest rate set by the 3PL firm that satisfies $\frac{dq_2^*}{dr} < 0$ and $\frac{dx(\hat{x}_c)}{dq_2^*} < 0$.

Proof. According to the optimal order quantity of retailer: $q_2^* = \bar{F}^{-1}(\kappa \bar{F}(\hat{x}_c))$, taking the first-order derivative of $r$, then we can get the following formula: $-f(q_2^*) \frac{dq_2^*}{dr} = \frac{d\bar{F}(\hat{x}_c)}{dr} - \kappa f(\hat{x}_c) \frac{dx}{dr}$, next taking $\frac{dx}{dr} = -\frac{\lambda w}{p}$ and $\frac{dx}{dr} = \kappa \frac{dq_2^*}{dr} + \frac{L(q_2^*)}{p}$ into the above formula, we can get

$$\frac{dq_2^*}{dr} = \frac{\lambda w \bar{F}(\hat{x}_c) - \kappa(\lambda w q_2^* - B) f(\hat{x}_c)}{p [\kappa^2 f(\hat{x}_c) - f(q_2^*)]}$$

simplify it as

$$\frac{dq_2^*}{dr} = \frac{\lambda w [1 - \kappa q_2^* g(\hat{x}_c)] + \kappa B g(\hat{x}_c)}{p [\kappa^2 g(\hat{x}_c) - \kappa f(q_2^*)]}$$
and 

\[ \frac{d\hat{x}_c(q_2^*)}{dr} = \frac{\lambda w [1-G(q_2^*)] + B g(\hat{x}_c)}{p [s g(\hat{x}_c) - f(q_2^*)]} \]

Similar to the process of proving \( dq_1^*/dw < 0 \) we use proof by contradiction to imply that \( dq_2^*/dr < 0 \). So we assume that inequality of \( dq_2^*/dr < 0 \) holds. So, the inequality of \( d\hat{x}_c(q_2^*)/dr \geq 0 \) holds. Let \( d\hat{x}_c(q_2^*)/dr|_{r=r_0} = 0 \) We have \( \lambda w - L(q_2^*(r_0))g(q_2^*(r_0)) = 0 \) We have mentioned above the economic rationality hypothesis of \( r_f \leq r \leq 3 \) means retailer’s margin profit rate is no more than \( r \), he won’t borrow money from 3PL. Then we have three cases to discuss.

1. When \( r_0 \geq \hat{r} \), for any \( r \leq \hat{r} \), because of the assumption, we have \( \lambda w - L(q_2^*(r_0))g(q_2^*(r_0)) = 0 \leq \lambda w - L(q_2^*(\hat{r}))g(q_2^*(\hat{r})) \) from the assumption of \( dq_2^*/dr \geq 0 \) for IFR distributions of demand. Hence, we can get \( d\hat{x}_c(q_2^*)/dr < 0 \), and thus \( dq_1^*/dw < 0 \) holds. It is a contradiction to our assumption.

2. When \( r_f < r_0 < \hat{r} \), we have \( d\hat{x}_c(q_2^*)/dr < 0 \) over \( (r_f, r_0) \), therefore, the inequality of \( dq_2^*/dr < 0 \) holds. So, it is also a contradiction to our assumption.

3. When \( r_0 = r_f \), for any \( r \geq r_f \), and \( \hat{x}_c(q_2^*) < q_2^* \), under the condition of \( \kappa < 1 \), thus \( \lambda L(q_2^*(r_f))g(q_2^*(q_2^* (r_f))) \leq \lambda w - L(q_2^*(r_f))g(q_2^*(r_f)) = L(q_2^*(r_0))g(q_2^*(r_0)) = \lambda w \) for IFR distributions of demand. Hence, we can get \( dq_2^*/dr < 0 \). It is inconsistent to our assumption.

For all of the cases above, we get the contradictions to our assumption, that is, \( dq_2^*/dr < 0 \). Therefore, we conclude that the monotonicity of \( q_2^* \) and \( \hat{x}_c(q_2^*) \) with respect to \( r \) is immediate.

According to Lemma 3, the 3PL firm will set a lower interest rate given a larger order quantity to help the retailer decrease its bankruptcy risk.

**Proposition 4.** Considering the retailer’s bankruptcy risk and demand, the 3PL firm provides a rational interest rate to avoid risks and maximize its expected profits; the 3PL firm’s best-response optimal interest rate \( r^* \) uniquely satisfies the following:

\[ r^* = \frac{1 + \lambda \cdot r_f}{\lambda F(\hat{x}_c(q_2^*)) - \frac{1}{\lambda}} \]

**Proof.** Similar to Yan et al. (2016) Taking the first derivative of \( \pi_{3pl} \) with respect to \( r \), we have \( \frac{d\pi_{3pl}}{dr} = \frac{d\pi_{3pl}}{d\hat{x}_c} \cdot \frac{d\hat{x}_c}{dr} \), because we get \( \frac{d\hat{x}_c}{dr} < 0 \) from Lemma 5. The monotonicity guarantees that for any \( r^* \), there is only one corresponding \( \hat{x}_c(r^*) \). Hence, the first-order condition of \( \frac{d\pi_{3pl}}{d\hat{x}_c(r^*)} = 0 \) can be transformed to that of \( \frac{d\pi_{3pl}}{d\hat{x}_c(r^*)} = 0 \).

So, differentiating \( \pi_{3pl} \) with \( \hat{x}_c \), we have \( \frac{d\pi_{3pl}}{d\hat{x}_c} = pF(\hat{x}_c) - w \frac{d\hat{x}_c}{d\hat{x}_c} (1 + \lambda \cdot r_f) \), we know that \( \frac{d\hat{x}_c}{d\hat{x}_c} = w \frac{1 + \lambda \cdot r_f}{p} \), thus \( \frac{d\pi_{3pl}}{d\hat{x}_c} = w \frac{1 + \lambda \cdot r_f}{p} \). Hence, \( \frac{d\pi_{3pl}}{d\hat{x}_c} = p \left( F(\hat{x}_c) - \frac{1 + \lambda \cdot r_f}{1 + \lambda \cdot r_f} \right) \).

Thus we can get that \( \frac{d\pi_{3pl}}{d\hat{x}_c} = -pf(\hat{x}_c) < 0 \). Then from \( \frac{d\pi_{3pl}}{d\hat{x}_c} = \frac{d\pi_{3pl}}{d\hat{x}_c} \cdot \frac{d\hat{x}_c}{dr} \) and \( \frac{d\pi_{3pl}}{d\hat{x}_c} = 0 \), the optimal \( r^* \) can be derived as \( F(\hat{x}_c) = \frac{1 + \lambda \cdot r_f}{1 + \lambda \cdot r_f} \). For every \( \hat{x}_c \), the 3PL’s best response can be solved as \( r^* = \frac{1 + \lambda \cdot r_f}{\lambda F(\hat{x}_c(q_2^*)) - \frac{1}{\lambda}} \).

According to Lemma 3 and Proposition 4, there exists an optimal interest rate solution to demand. Based on an analysis of Proposition 4, the interest rate set by the 3PL firm is higher than the risk-free interest rate. Hence, the 3PL firm needs
to decrease the risk involve and realize higher profits to balance losses and revenue. Under CCF, the 3PL firm is highly reliant on the immediate payment coefficient \( \lambda \). Thus, \( 0 < \lambda < 1 \), which validates the importance of \( r^* \). It is obvious that the optimal rate \( r^* \) increases with \( r_f \); when \( r_f \) is high, the 3PL firm must charge a higher interest rate to guarantee that it does not realize a loss.

3. The supplier’s optimal wholesale price

The supplier provides partial trade credit with an immediate payment coefficient \( \lambda \) in the expectation that doing so will increase the order quantity and decrease trade risk. The supplier should set an appropriate wholesale price that increases profits to guarantee liquidity. We describe the supplier’s profit function as follows:

\[
\max_w \pi_s (w; q, r, x) = \left[ w (\lambda (1 + \lambda \cdot r_f) - c) q (\lambda) \right]
\]  

(13)

Lemma 4.2. Under IFR distributions of demand, the functional relationship between the retailer’s optimal order quantity and the wholesale price set by the supplier satisfies \( \frac{dq^*_w}{dw} < 0 \) and \( \frac{d\bar{q}(q^*_s)}{d\bar{w}} < 0 \)

Proof. Taking the first-order derivative of the optimal order quantity, we can have:

\[
-f(q^*_s) \frac{dq^*_w}{dw} = \frac{dx}{d\bar{w}} \hat{F}(\hat{x}_c) - \kappa f(\hat{x}_c) \frac{dx}{dw}
\]

and then taking \( \frac{d\bar{x}}{d\bar{w}} = \frac{1 + \lambda \cdot r_f}{p} \left( w \frac{dq^*_w}{dw} + q^*_w \right) \)

into above equation, we can have the expression about \( \frac{dq^*_w}{dw} \), where

\[
\frac{dq^*_w}{dw} = \frac{\kappa \left( \hat{F}(\hat{x}_c) - \kappa q^*_s \hat{f}(\hat{x}_c) \right)}{w (\kappa^2 \hat{f}(\hat{x}_c) - \hat{f}(q^*_s))} \cdot \frac{1 - \kappa q^*_s \bar{g}(\hat{x}_c(q^*_s))}{w (\kappa^2 \bar{g}(\hat{x}_c(q^*_s))) - \bar{g}(q^*_s))}
\]

Judging the positive or negative of \( \frac{dq^*_w}{dw} \), market demand is distributed as IFR function, as the same proof of \( \frac{d\bar{q}(q^*_s)}{d\bar{w}} \) in Lemma 6, we first assume \( \frac{dq^*_w}{dw} \geq 0 \), thus, \( \frac{d\bar{x}}{d\bar{w}} = \frac{1 + \lambda \cdot r_f}{p} \left( w \frac{dq^*_w}{dw} + q^*_w \right) = \frac{\lambda \cdot \hat{g}(\hat{x}_c(q^*_s))}{\hat{g}(q^*_s)} \), and \( \frac{d\bar{x}}{d\bar{w}} \geq 0 \). We analyse three cases (same as the proof of Lemma 2) and set \( c \leq w \leq w_2 = p/ (1 + \lambda \cdot r) \). We find that all the cases are contradictions to the assumption. We get \( \frac{dq^*_w}{dw} < 0 \) and \( \frac{d\bar{x}(\bar{q}(q^*_s))}{d\bar{w}} < 0 \), also we have the inequation \( \Omega q^*_s \bar{g}(\hat{x}_c) \leq \bar{q}_s g(\hat{x}_c) < \bar{q}_s^2 \bar{g}(q^*_s) \leq 1 \) holds.

Having Lemma 4, we can propose Proposition 5. Under CCF, the supplier can offer a lower price to encourage the retailer to order more to in turn realize its optimal profits.

Proposition 5. Given partial trade credit offered by the supplier, where the immediate payment coefficient is \( \lambda \), and the best responses of the retailer and the 3PL firm, the supplier sets the optimal wholesale price \( w^*_2 \), which satisfies:

\[
w^*_2 = \frac{pc (1 + \lambda \cdot r_f) \hat{F}(\hat{x}_c) + p^2 q^*_s (1 + \lambda \cdot r_f) \hat{f}(\hat{x}_c) + c (1 + \lambda \cdot r^*) q^*_s \hat{f}(\hat{x}_c)}{p (1 + \lambda \cdot r^*) (1 + \lambda \cdot r_f) \hat{F}(\hat{x}_c) + c (1 + \lambda \cdot r^*) q^*_s \hat{f}(\hat{x}_c)}
\]

(14)

Proof. According to \( \frac{dq^*_w}{d\bar{w}} \), taking the second-derivative of \( w \), we can have:

\[
\frac{d^2 \frac{dq^*_w}{dw}}{d\bar{w}^2} = -\frac{2 \kappa \hat{f}(\hat{x}_c) \hat{f}(\hat{x}_c) - \kappa q^*_s \hat{f}(\hat{x}_c)}{w (\kappa^2 \hat{f}(\hat{x}_c) \hat{f}(\hat{x}_c))}
\]

Because of market demand is distributed as IFR function and the proof of Lemma 4, we know that \( q^*_s \bar{g}(\hat{x}_c) \leq 1 \), so \( \hat{F}(\hat{x}_c) - \kappa q^*_s \hat{f}(\hat{x}_c) > 0 \), thus \( 2 \frac{dq^*_w}{dw} < 0 \). According to \( \frac{dq^*_w}{dw} < 0 \) and \( \frac{d^2 \frac{dq^*_w}{dw}}{d\bar{w}^2} < 0 \), we can conclude \( \frac{d^2 \frac{dq^*_w}{dw}}{d\bar{w}^2} = 2 \frac{dq^*_w}{dw} (1 + \lambda \cdot r_f) + \left( w (1 + \lambda \cdot r_f) - c \right) \frac{d^2 \frac{dq^*_w}{dw}}{d\bar{w}^2} < 0 \), we can know the existence of optimal wholesale price that the suppliers can get the maximum profit. Thus, making the first-order derivative of the suppliers profit to wholesale price, that is
\[
\frac{d\pi_s}{dw} = \left[ (w(1 + \lambda \cdot r_f) - c) \cdot \frac{dq_2^*}{dw} + q_2^*(1 + \lambda \cdot r_f) \right] = 0, \text{ applying the properties and equations of the lemma 4, we can get } w_2^* = \frac{p \cdot (1 + \lambda \cdot r_f) \bar{F}(\hat{x}) + p^2 q_2^*(1 + \lambda \cdot r_f) f(q_2^*)}{p (1 + \lambda \cdot r_f)}.
\]

From the Proposition 5, it is obvious that the supplier’s unique optimal wholesale price is influenced by the immediate payment coefficient \( \lambda \), which the supplier can set within a rational range for the retailer to guarantee reduced expected bankruptcy loss. The supplier is heavily reliant on the retailer’s initial capital level \( B \) when making its pricing decision.

5. Numerical analysis.

5.1. Example 1: Suppliers financing decision based on retailers initial capital. This section conducts a numerical simulation analysis based on practice. We now employ numerical examples to further compare TCF and CCF and explore the sensitivity of our results to the decision variables and the change in profit under the influence of different values of the retailer’s initial capital level and the suppliers immediate payment coefficient.

Following Buzacott and Zhang (2004)[9], the supplier’s capital is limited, the cost of production \( c = 0.4 \), and the product price is \( p = 2.5 \). The retailer sells products according to a uniform demand distribution \( x \in [0, 100] \). Similar approaches to modeling the demand function have been used in a number of other papers (e.g., Jing, B., Dewan, R. et al. 2013[8]; Gong, X., Chao, X. et al. 2014[21]). Here, we set the supplier’s immediate payment coefficient \( \lambda \) at 0.5. According to the simulated data above, the comparative analysis is conducted using MATLAB to determine the supplier’s expected profit between under the two financing methods. The results are depicted in Fig.3.

Figure 3 depicts the supplier’s expected profits under CCF and TCF. The dotted line becomes parallel to the horizontal axis when \( \pi_0^s \) (there are no capital constraints) is at 50.89, which suggests that the supplier’s expected profit does not change with the retailer’s initial capital level. When the retailer’s expected profit is negative, it will not participate in CCF, namely, where the threshold of \( B \) is reached, as displayed in Figure 3. Under TCF, the supplier’s expected profit decreases in the initial capital level and is lower than the corresponding profit in the case without capital constrains (\( \pi_0^s \)). This trend indicates a higher optimal wholesale price for the supplier and a larger optimal order quantity for the retailer when the retailer’s initial capital is very low. The supplier thus chooses trade credit and might forgo short-term profits (such as paper profits) and instead consider long-term interests (such as business robustness or market share). Thus, suppliers should select retailers with low initial capital levels under TCF. Similarly, under CCF, the supplier will obtain more profits when the retailer has less initial capital. This suggests that the supplier can encourage a retailer with limited assets to order a larger quantity. Finally, the supplier’s expected profit declines as we move toward the case without capital constraints (\( \pi_0^s \)). The retailer then has sufficient funds to cover the cost of the order without needing any financing.

From Figure 4, it is clear that the retailer and 3PL firm obtain higher profits when the retailer has more initial capital under both TCF and CCF. The retailer will obtain higher profits under TCF than under CCF. Furthermore, under CCF, the retailer refuses to cooperate with supplier because its expected profit is negative when facing bankruptcy risk and a high financing interest rate. The 3PL firm will
Figure 3. Effects of retailer’s initial capital on supplier’s expected profit

Figure 4. Effects of retailer’s initial capital on the retailer and 3PL firm’s expected profits
charge a high interest rate to avoid the retailer’s bankruptcy risk and maximize profits, and it is more profitable to provide credit for a retailer with limited assets.

From the comparison of the two funding mechanisms, it is clear that the supplier’s expected profit is more sensitive to the initial capital level under CCF than under TCF. When facing variations in the retailer’s initial capital level, the supplier should consider the operational and financial parameters to make a sensible choice. There are four cases for the supplier to consider:

(1) When the initial capital lies in area (1), the retailer will refuse to cooperate with supplier under CCF, and so the supplier should provide TCF to earn more profits.

(2) When the initial capital lies in area (2), the supplier should choose CCF to avoid risks and boost profits.

(3) When the initial capital lies in area (3), the supplier should provide the retailer with TCF because the retailer has more initial capital and lower bankruptcy risk.

(4) When the initial capital lies in area (4), the retailer’s initial capital can cover the initial payment \( \lambda w q \), and this it does not need to take a loan from the 3PL firm. This is the same as the case without capital constraints.

We can draw the conclusion that retailer’s initial capital level will affect the supplier’s choice of financing mechanism. The less initial capital that retailer holds, the greater the profits earned by the supplier.

![Figure 5. The operation of the trade credit financing mechanism](image)

5.2. Example 2: Sensitivity analysis based on the retailers initial capital level. From Figure 5, we find that under both TCF and CCF, the higher the retailer’s initial capital level is, the lower retailer’s optimal order quantity. TCF is always better in such a case than CCF because the former encourages the retailer to order more. Under TCF, this observation confirms Lemma 2: \( q_1^* (B) \) is a decreasing
function of $B$. When the initial capital level increases and $B \geq w_1^* q_1^*(55.4)$, the retailer will not be capital constrained and will not apply for TCF, and the order quantity $q_0^*$ is equal to that in the newsvendor model. Under CCF, when the initial capital is $B \geq \lambda w_2^* q_2^*(35.2)$, the retailer will not be capital constrained and the order quantity will be $q_0^*$. Obviously, the order quantity under both TCF and CCF is still larger than in the absence of trade credit. We thus conclude that both financing mechanisms can encourage an increase in the retailer’s order quantity.

Figure 6. The operation of the trade credit financing mechanism

Figure 6 indicates that the 3PL firm will set a lower interest rate when the retailer has a higher initial capital level because the retailer has lower bankruptcy risk under CCF. Clearly, the interest rate is considerably higher under CCF than under TCF. It is feasible to extend the supply chain structure to include a 3PL firm, and the transaction costs in a multiple-participant SCF system is higher than that in a two-echelon SCF system.

5.3. Example 3: Sensitivity analysis based on immediate payment coefficient. Figure 7 indicates that as the immediate payment coefficient increases, the supplier and retailer’s profits are decreasing. We suggest that a high immediate payment coefficient exacerbates the retailer’s capital constraint, so that the retailer will order less to ease the capital pressure it faces. Furthermore, the supplier charges a high wholesale price when it cannot obtain much in immediate payment. In this example, the 3PL firm refuses to participate in CCF when the immediate payment coefficient is less than $\lambda_0(0.3)$ in an effort to avoid the retailer’s substantial bankruptcy risk. The 3PL firm gains a positive profit in the feasible region. With a high immediate payment coefficient, the 3PL firm will face lower risk and maximize profits.
According to Figure 8a, the retailer’s optimal order quantity has a concave relationship with $\lambda$, and the optimal immediate payment coefficient $\lambda^*_q$ is the first-best solution for the optimal order quantity, with $\lambda^*_q = 0.35$. Given a higher immediate payment coefficient, the supplier will charge a lower wholesale price, and the 3PL firm will charge a lower interest rate due to the lower bankruptcy risk. As the 3PL firm’s interest rate is always more than $r_t$ under TCF, a multiple-participant SCF system will have higher financing fees.

6. Conclusions. This study investigates the influence of the retailer’s initial capital level and immediate payment coefficient on financing strategies in a SCF system that comprises a capital-constrained retailer, a 3PL firm and a supplier. We design a new CCF mechanism for a multiple-participant supply chain that combines...
trade credit offered by a supplier with credit offered by the 3PL firm. A large body of literature examines trade credit issues, but there are few studies on CCF in multiple-participant SCF systems. We compare TCF and CCF in the presence of bankruptcy risk to help suppliers make decisions when facing different retailers with different initial capital levels.

Through theoretical analysis and numerical examples, our study on retailer’s initial capital level has yielded important insights. First of all, the retailer’s initial capital affects the decision-making of the supplier, 3PL firm and the retailer. When the retailer’s initial capital level is particularly low, the supplier chooses to share the financing risk with 3PL, that is, choose the CCF mode to guarantee its own profit. With the increasing level of retailers’ funds, it’s appropriate for supplier to choose TCF since the risk of bankruptcy of retailers is relatively safe, and the supplier can stimulate retailer to increase the order quantity by adjusting the interest of deferred payments and wholesale price. When the retailer’s capital gap is gradually reduced to a very small level, the incentives for supplier to provide deferred payment are no longer obvious, and the choice of CCF mode can better promote retailer to increase the order quantity, thereby increasing their own profits. To conclude about the optimal financing mode decision, when the retailer’s initial capital level is too low or, on the other hand, when the retailer’s capital constraint is insignificant, the supplier will choose CCF; otherwise the supplier will choose TCF.

Secondly, we further analyze the impact of the initial capital of the retailer on the decision making of the supply chain members. In order to analyze the CCF model and the 3PL’s optimal decisions, we introduce immediate payment coefficient to represent the retailer must pay when announcing an order, which is made by the supplier at first in this mode. This parameter has a great impact on whether 3PL will participate in CCF. Considering the retailer’s initial capital level and immediate payment coefficient, if the supplier wishes to maximize profits under CCF, it must set \( \lambda > \lambda_0 \) and cooperate with the retailer with initial capital above the threshold of \( B \). However, to guarantee that it makes a positive profit, the 3PL firm will participate in CCF only if the supplier’s immediate payment coefficient is more than \( \lambda_0 \). Moreover, the supplier’s profit is decreasing with \( B \) in both mechanisms, and it is thus better for the supplier to choose a retailer with limited assets to maximize profits.

Moreover, our numerical and analytic results have more management insights. First, financing fees are lower in a two-echelon SCF system than in a multiple-participant SCF system due to the interest rate. Thus, the retailer is subject to considerably less financing pressure under TCF than under CCF. Therefore, retailers are reluctant to choose CCF when they have limited capital. Second, the two financing mechanisms present a trade-off between the wholesale price and order quantity. Under different initial capital levels, the supplier charges a lower wholesale price and obtains a higher order quantity under TCF than under CCF. Using our results, managers of the three parties can adjust their decisions to keep pace with the change of different retailer’s initial capital level.

This paper can be extended in at least following directions. First, it would be worthwhile to consider the decisions in the case of asymmetric information. Second, 3PL companies often charge numerous fees for supervision and transportation. Further study on how service fees affecting the supply chain decisions is warranted.
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E-mail address: qianglin@tju.edu.cn
E-mail address: py_cassie@163.com
E-mail address: huying@tju.edu.cn