Wormhole on the Brane with Ordinary Matter: The Broader View

Rikpratik Sengupta,\textsuperscript{a} Shounak Ghosh,\textsuperscript{b} Meheди Kalam\textsuperscript{a} and Saibal Ray\textsuperscript{c}

\textsuperscript{a}Department of Physics, Aliah University, Kolkata 700 160, West Bengal, India
\textsuperscript{b}Directorate of Legal Metrology, Department of Consumer Affairs, Govt. of West Bengal, Alipurduar 736121, West Bengal, India
\textsuperscript{c}Department of Physics, Government College of Engineering and Ceramic Technology, Kolkata 700 010, West Bengal, India

E-mail: rikpratik.sengupta@gmail.com, shounakphysics@gmail.com, kalam@associates.iucaa.in, saibal@associates.iucaa.in

Abstract. In this paper we attempt to examine the possibility of construction of a traversable wormhole on the Randall-Sundrum braneworld with ordinary matter employing the Kuchowicz potential as one of the metric potentials. In this scenario, the wormhole shape function is obtained and studied, along with validity of Null Energy Condition (NEC) and the junction conditions at the surface of the wormhole are used to obtain a few of the model parameters. The investigation, besides giving an estimate for the bulk equation of state parameter, draws important constraints on the brane tension which is a novel attempt in this aspect and very interestingly the constraints imposed by a physically plausible traversable wormhole is in high conformity with those drawn from more general space-times or space-time independent situations involved in fundamental physics. Also, we go on to claim that the possible existence of a wormhole may very well indicate that we live on a three-brane universe.
1 INTRODUCTION

The idea of a wormhole can be achieved from a set of solutions of the Einstein Field Equation (EFE), materializing into a tunnel-like structure connecting two different regions of space-time of same universe or two different universe, can be traced back to the Einstein-Rosen bridge [1]. However, the term ‘wormhole’ was coined by Fuller and Wheeler [2], they showed that such solutions of the EFE were unstable as the throat of the wormhole would pinch-off, thus trapping any traversing signal in an infinite curvature region making the wormhole non-traversable. The modern idea of traversable wormholes was introduced first by Morris and Thorne [3], who were studying wormhole solutions as a tool for a simpler understanding of General Relativity (GR). They prescribed that in order to have stable traversable wormholes, the corresponding energy-momentum tensor for the wormhole must violate the null energy condition (NEC) of GR, such that the pinching-off of the throat can be prevented. Further, if the wormhole can be sustained, it can allow the possibility of time-travel, violating causality [4]. Such ‘exotic’ matter which violates the NEC has not been observed till date, but also finds existence in theories attempting to explain the present cosmic acceleration [5, 6, 7, 8]. Traversable wormholes admitting exotic matter have been studied in [9, 10, 11, 12, 13, 14, 15].

There have been a number of attempts to modify GR at higher energy scales [16, 17] in order to avoid the singularity problems [18] arising in cosmology and black hole physics, and also at energy scales corresponding to the present epoch of cosmic evolution to accommodate naturally the current cosmic acceleration without incorporating exotic matter [19, 20]. The idea of braneworld [21, 22] involving a higher dimensional approach to modify GR was studied in [23] from a purely geometrical approach. Braneworlds have been used very effectively as a platform to investigate cosmological [24, 25, 26, 27, 28, 29], astrophysical [30, 31, 32, 33, 34] and collapse [35, 36, 37, 38] problems. Due to the non-locality and non-closure properties of the modified EFE on the brane, the Minimum Geometric Deformation (MGD) approach introduced in [39, 40] is a key tool for studying collapse problems on the brane.

Wormhole solutions in the context of modified gravity without accounting for extra dimensions have been investigated in [41, 42, 43, 44, 45, 46, 47]. In higher dimensional modified gravity approach, wormhole solutions can be found in [48, 49, 50, 51, 52, 53, 54]. The possibility of existence of wormholes in the central and outer region of galactic halos has been investigated in [55] and [56, 57], respectively. We attempt to investigate the possibility of construction of a traversable wormhole on the braneworld, such that the metric potential in the temporal direction is of the Kuchowicz type [58]. Also, due to lack of experimental evidence in favour of
exotic matter so far, we choose to consider a linear Equation of State (EOS) corresponding to perfect fluid matter on the brane, describing ordinary matter. In the present article we have studied the mathematical model of the wormhole on the brane including the modified EFE on the braneworld, solution for the wormhole shape function, validity of the NEC on the brane for ordinary perfect fluid matter and obtaining the possible numerical values of the constants from the junction conditions for plotting the physical parameters of interest pertaining to the wormhole. We will then discuss the physical relevance of the obtained solutions and plots for the physical parameters and draw conclusion based on it in the following section.

2 Mathematical Model of the Brane Wormhole

In this section we will try to construct a detailed mathematical model of the brane wormhole for linear EOS, where one of the metric potentials is of the Kuchowicz type. We will start with the modified EFE on the Randall-Sundrum (RS) II braneworld.

2.1 Modified EFE on the Brane

We begin with a the general form of the modified EFE on the 3-brane [23] and move on to obtain the set of EFEs for the case of a spherically symmetric, static metric. The RS II braneworld model considers a single 3-brane embedded in higher dimensional bulk having one extra dimension which is the compact space $S_1/Z_2$, where $Z_2$ refers to the orbifold symmetry or the reflection symmetry.

The general form of the modified EFE on the 3-brane is given as

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu}^{mod},$$

(2.1)

which may be expanded as

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + \kappa_5^2 S_{\mu\nu} - E_{\mu\nu},$$

(2.2)

such that the modified stress-energy tensor $T_{\mu\nu}^{mod}$ contains the corrections to the standard EFE, besides the matter stress-energy tensor $T_{\mu\nu}$ on the brane. Here, the 4-dimensional constant $\kappa^2 = \frac{1}{6}\sigma\kappa_5^4$, where $\sigma$ denotes the brane tension. In the high energy regime the brane tension is small and tends to zero for extremely high energies. We will take $\kappa^2 = 8\pi G = 1$ on the brane.

The effective cosmological constant on the brane is given by

$$\Lambda = \frac{1}{2}\kappa_5^2 \left( \Lambda_5 + \frac{1}{6}\kappa_5^2 \sigma^2 \right).$$

(2.3)

In the RS II set-up the bulk is Anti-de Sitter (AdS$_5$), making the bulk cosmological constant $\Lambda_5$ negative in magnitude, and it is fine-tuned with the brane tension in such a way as to give the value of $\Lambda$ to be zero.

The corrections to the EFE are of two types-(i) Local correction ($S_{\mu\nu}$) which comprises of terms quadratic in the stress-energy tensor on the brane and (ii) Non-local correction ($E_{\mu\nu}$) which comprises of the projection of the bulk Weyl tensor on the brane, transferring the gravitational effect from the bulk.

The local correction is given by
are both conserved separately. So, the conservation equation on the brane reads the same as
\[ S_{\mu \nu} = \frac{1}{12} T T_{\mu \nu} - \frac{1}{4} T_{\mu \alpha} T_{\nu}^{\alpha} + \frac{1}{24} g_{\mu \nu} \left[ 3 T_{\alpha \beta} T_{\alpha \beta} - (T_{\alpha}^{\alpha})^2 \right], \tag{2.4} \]
where \( T_{\alpha}^{\alpha} \) and we consider matter on the brane with perfect fluid stress-energy tensor
\[ T_{\mu \nu} = \rho u_{\mu} u_{\nu} + p h_{\mu \nu}. \tag{2.5} \]
Here \( u_{\mu} \) is the 4-velocity, \( h_{\mu \nu} = g_{\mu \nu} + u_{\mu} u_{\nu} \) is the induced metric on the brane, \( \rho \) and \( p \) are the energy density and pressure.

The non-local correction has the form
\[ E_{\mu \nu} = C_{ACBD}^{(5)} n^{C} n^{D} g_{\mu}^{A} g_{\nu}^{B} = - \frac{6}{\sigma} \left[ U u_{\mu} u_{\nu} + Pr_{\mu} r_{\nu} + h_{\mu \nu} \left( \frac{U - P}{3} \right) \right]. \tag{2.6} \]
where \( C_{ACBD}^{(5)} \) is the bulk Weyl tensor, \( n^{C} \) is unit normal spacelike vector, \( r_{\mu} \) denotes projected radial vector, \( U \) and \( P \) denote the bulk energy density and bulk pressure respectively. \( E_{\mu \nu} \) is symmetric and traceless. It has no components orthogonal to the brane.

For a static, spherically symmetric matter distribution, in terms of shape function \( b(r) \) of the wormhole, the line element for a static, spherically symmetric wormhole can be written as
\[ ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{2.7} \]

The modified EFE on the brane, given by Eq. (2.2) can be computed to be
\[ e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = \left( \rho \left( 1 + \frac{\rho}{2} \right) + \frac{6 U}{\sigma} \right), \tag{2.8} \]
\[ e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = \left( p + \rho \frac{(p + \frac{\rho}{2})}{\sigma} + \frac{2 U}{\sigma} + \frac{4 P}{\sigma} \right), \tag{2.9} \]
\[ e^{-\lambda} \left( \frac{\nu''}{2} - \frac{\lambda' \nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) = \left( p + \rho \frac{(p + \frac{\rho}{2})}{\sigma} + \frac{2 U}{\sigma} - \frac{2 P}{\sigma} \right). \tag{2.10} \]

The brane energy-momentum tensor and the overall effective energy momentum tensor are both conserved separately. So, the conservation equation on the brane reads the same as in GR
\[ \frac{dp(r)}{dr} = -\frac{1}{2} \frac{dv(r)}{dr} (p(r) + \rho(r)). \tag{2.11} \]

### 2.2 Solution for the Wormhole Shape Function

The line element for a static, spherically symmetric wormhole can be written as
\[ ds^2 = -e^{\nu(r)} dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{2.12} \]
where \( b(r) \) denotes the shape function of the wormhole.

Rewriting the above field equations (2.8)-(2.10) for a wormhole with static and spherically symmetric matter distribution, in terms shape function \( b(r) \) we get eventually
\[ \frac{b'}{r^2} = \rho \left( 1 + \rho \frac{2}{\sigma} \right) + \frac{6 (X \rho + Y)}{\sigma}, \tag{2.13} \]
\[ \left( \frac{1 - b}{r} \right) \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = p + \rho \frac{(2 p + \rho)}{2 \sigma} + \frac{2 (X \rho + Y)}{\sigma} + \frac{4 \omega (X \rho + Y)}{\sigma}, \tag{2.14} \]
\[ \left( 1 - \frac{b}{r} \right) \left( \nu'' + \nu'^2 + \frac{\nu'}{r} \right) - \frac{b' - b}{2r} \left( \nu' + \frac{1}{r} \right) = p + \rho \frac{(2 p + \rho)}{2 \sigma} + \frac{2 (X \rho + Y)}{\sigma} - \frac{2 \omega (X \rho + Y)}{\sigma}. \tag{2.15} \]
Here we have considered the bulk EOS $P = \omega U$ \cite{59} and the energy density of the brane and bulk are related by $U = X\rho + Y$ \cite{60}.

Now, we consider the metric potential $e^{\nu(r)}$ to be of the Kuchowicz type \cite{58} which is given as

$$e^{\nu(r)} = e^{Br^2 + 2\ln C}, \quad (2.16)$$

where $B$ and $C$ are arbitrary constants. The dimension of $B$ is $[L^{-2}]$ and $C$ is a dimensionless constant. This metric potential is singularity free and well behaved.

As argued earlier, due to lack of observational confirmation of exotic matter which is usually preferred for constructing traversable wormholes, we chose to propose an ansatz for the EOS describing ordinary matter on the brane as follows

$$p(r) = \mu \rho(r), \quad (2.17)$$

where $\mu > 0$.

Using Eqs. (2.11), (2.16) and (2.17) we get

$$\rho(r) = C_1 e^{-(\mu+1)Br^2/2\mu}, \quad (2.18)$$

where $C_1$ is the integration constant which can be determined from the matching condition. The variation of the energy density $\rho(r)$ on the brane along the radial distance $r$ is plotted in Fig. 1. The figure clearly indicates that the matter density at the throat is much higher and gradually decreasing with respect to the radial parameter attains a minimum value at the boundary.

![Figure 1](attachment:image.png)

**Figure 1.** Variation of the density with respect to $r$.

The obtained $\rho$ can be plugged in along with Eq. (2.16) in the field Eq. (2.15) to obtain the shape function for the wormhole given by
\[ b(r) = \frac{2r e^{-2Br^2}}{Bk^4 \sigma} \left( -\frac{\mu C_1 (\sigma \mu k^4 - 2X(\omega - 1)) e^{\frac{Br^2(3\mu - 1)}{2\mu}}}{(3\mu - 1)(2Br^2 + 1)} + \frac{k^4}{(\mu - 1)(2Br^2 + 1)} \right) \]

\[ \left( -\frac{C_1^2 (2\mu + 1) \mu e^{\frac{Br^2(\mu - 1)}{\mu}}}{4} + \frac{\sigma(\mu - 1)}{2} (B(2Br^2 + 1) + Y(\omega - 1)) e^{2Br^2} + C_2 B \right) \]  

Figure 2. Variation of the shape function with respect to \( r \).

The shape function is found to be dependent on all the brane and bulk parameters including the brane tension. The variation of the shape function \( b(r) \) along the radial distance \( r \) has been plotted in Fig. 2. The properties of the shape function that can be inferred from the plot and its feasibility in construction of wormhole has been discussed later in the concluding section.

2.3 Validity of NEC

One of the most important energy conditions appearing in GR is the null energy condition given by \( T_{\mu\nu} k^\mu k^{\nu} \geq 0 \), where \( k^\mu \) denotes a null vector. This reduces to \( \rho + p \geq 0 \) for a perfect fluid. So, for the ordinary matter we have considered on the brane having EOS of form given by Eq. (2.17), the NEC holds good. However, as we are concerned with the effective stress-energy tensor on the brane due to the local and non-local correction terms, so it is the effective energy density and pressure that we must consider for constructing the null energy condition on the brane. Hence, the NEC becomes \( \rho^{\text{eff}}(r) + p^{\text{eff}}(r) \geq 0 \), where it is to be noted from Eqs. (2.9) and (2.10) that the effective pressures in the radial and transverse directions differ due to difference in contribution from the bulk pressure term.

Here,
\[
\rho_{\text{eff}}(r) = \left[ \rho(r) \left( 1 + \frac{\rho(r)}{2\sigma} \right) + \frac{6U}{\sigma} \right],
\]
and
\[
p_{\text{eff}}^r(r) = \left( p(r) + \frac{\rho(r) \left( p(r) + \frac{\rho(r)}{2} \right)}{\sigma} + \frac{2U}{\sigma} + \frac{4P}{\sigma} \right).
\]

The sum of the effective energy density and effective radial pressure can be computed to be
\[
\rho_{\text{eff}} + p_{\text{eff}}^r(r) = \frac{1}{k^4\sigma} \left( k^4C_1^2(\mu + 1)e^{-\frac{\mu(\mu+1)}{2}} + 4C_1 \left( \frac{1}{4}\sigma(\mu + 1)k^4 + X(\omega + 2) \right) e^{-\frac{(\mu+1)Br^2}{2\mu}} + 4Y(\omega + 2) \right). \quad (2.22)
\]

Figure 3. Variation of the NEC with respect to \( r \).

The variation of \( \rho_{\text{eff}}(r) + p_{\text{eff}}^r(r) \) along \( r \) has been plotted in Fig. 3. We find that the NEC is violated for effective matter distribution on the brane.

2.4 The Junction Conditions

The exterior of the wormhole is completely vacuum. So the solution for the exterior spacetime line metric of wormhole must be Schwarzschild type can be written as
\[
ds^2 = \left( 1 - \frac{2M}{r} \right) dt^2 - \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).
\]
where \( M \) is the total mass of the wormhole.

Presence of matter on the surface of wormhole must produces an extrinsic discontinuity which generates an intrinsic surface energy density and surface pressure. The surface (i.e.,
the shape function) behaves as the junction between the space-time of wormhole and exterior which makes the wormhole geodesically complete manifold with the ordinary matter and characterizes the configuration of wormhole. Thus according to the fundamental junction condition there should be a smooth matching between the two spacetimes at the junction. Again the metric coefficients are continuous at the junction surface do not confirm the continuity of derivatives of these coefficients there. So to determine the surface stress-energy $S_{ij}$ one can use formalism suggested by Darmois-Israel [61, 62].

Now intrinsic surface stress energy tensor $S_i^j$ can be obtain from Lanczos equation [63, 64, 65, 66] in the following form

$$S_i^j = -\frac{1}{8\pi}(\kappa_i^j - \delta_i^j k^k), \quad (2.24)$$

the discontinuity in the second fundamental form can be written as

$$\kappa_{ij} = \kappa_{ij}^+ - \kappa_{ij}^-, \quad (2.25)$$

whereas the second fundamental form is given by

$${\kappa}_{ij}^\pm = -n^\mp \left[ \frac{\partial^2 X_\nu}{\partial \xi^i \partial \xi^j} + \Gamma^\nu_{ij} \frac{\partial X^\alpha}{\partial \xi^i} \frac{\partial X^\beta}{\partial \xi^j} \right] \mid_{S}, \quad (2.26)$$

where the unit normal vector $n_\nu^\pm$ are defined as

$$n_\nu^\pm = \left| g_{\alpha\beta} \frac{\partial f}{\partial X^\alpha} \frac{\partial f}{\partial X^\beta} \right|^{-\frac{1}{2}} \frac{\partial f}{\partial X^\nu}, \quad (2.27)$$

with $n^\nu n_\nu = 1$. Here $\xi^i$ is the intrinsic coordinate of the surface of the wormhole and $f(x^\alpha(\xi^i)) = 0$ is the parametric equation of it. Here + and − corresponds to exterior and interior spacetime respectively of the wormhole.

Now surface stress energy tensor, for the spherically symmetric space time, can be written as $S_i^j = \text{diag}(-\Sigma, \mathcal{P})$. Where $\Sigma$ and $\mathcal{P}$ is the surface energy density and surface pressure respectively, which can be obtained at the surface of the wormhole (i.e at $r = R$) by the following equations

$$\Sigma = -\frac{1}{4\pi R} \left[ \sqrt{e^\lambda} \right]^+$$

$$= \frac{1}{4\pi R} \left[ \sqrt{1 - \frac{2M}{R}} - \sqrt{1 - \frac{2e^{-2BR^2}}{Bk^4\sigma}} \left( -\mu C_1 (\sigma \mu k^4 - 2X(\omega - 1)) e^{BR^2/(3\mu - 1)} \right) \right] \left( \frac{3}{3\mu - 1} + M \right), \quad (2.28)$$

$$\mathcal{P} = \frac{1}{16\pi R} \left[ \left( \frac{2f - f'R}{\sqrt{f}} \right) \right]^+ = \frac{1}{16\pi R} \left[ \frac{2(1 - M)}{F1} - F2 \right], \quad (2.29)$$

- 7 -
where \( H_1 = -\frac{C_1^2(2\mu+1)k^4\mu e^{BR^2(\mu-1)}}{4(\mu-1)(2BR^2+1)} + \frac{(4B^2k^4\mu e + Y(\omega-1)\mu e^{2BR^2} + C_2k^4\mu e^{2BR^2})}{4(2BR^2+1)} \), and

\[
F_1 = \frac{1}{k^4\sigma(3\mu - 1)(\mu - 1)(2B^2r^2 + 1)B} \left( -4(\sigma \mu k^4 - 2X(\omega - 1))(\mu - 1) \left( (B^2r^4 + B^2r^2 - 1)\mu + 2B^2r^4 + B^2r^2 \right) e^{-\frac{BR^2(\mu+1)}{\mu}} + 2(\mu - 1) \left( C_2k^4\sigma(2B^2r^4 + 2B^2r^2 - 1)e^{-2BR^2} - Y(\omega - 1) \right) \right) (3\mu - 1),
\]

(2.30)

and

\[
F_2 = \left[ 1 - \frac{2e^{-2BR^2}}{Bk^4\sigma(2BR^2+1)} \frac{\mu C_1(\sigma \mu k^4 - 2X(\omega - 1))e^{\frac{BR^2(\mu-1)}{2\mu}}}{(3\mu - 1)} + H_1 \right]^{(2.31)}
\]

Now for the static wormhole the surface pressure as well as the surface energy must vanishes at the boundary surface. So one must i.e., we have \( \sigma = \mathcal{P} = 0 \) at boundary which eventually provides the condition

\[ b(R) = 2M. \] (2.32)

In order to calculate the different unknown constants we have used the following boundary conditions,

(i) From the junction condition at the boundary (from Eq.(2.32)) \( b(R) = 2M \).

(ii) Continuity of the metric potential \( g_{tt} \) and its derivative \( \frac{\delta g_{tt}}{\delta r} \) at the surface boundary i.e., at \( r = R \).

Now for best fit results we choose \( \mu = 0.501 \), \( \sigma = 10^3 \), \( M = 1.7 \), \( r_0 = 0.75 \), \( R = 4 \), where \( r_0 \) is the throat radius. Making use of the above conditions, we obtain the values different parameters associated with the wormhole as: \( X = -1943.448951, Y = 1684.121721, C_1 = 64.85766573, C_2 = -61.40896649, \omega = 0.9157485630 \). These values of the parameters have been used to plot Figs. (2.2)-(2.3). All the plots have been started from the throat (i.e. \( r = r_0 \)) up to the surface boundary (i.e at \( r = R \)).

3 DISCUSSIONS AND CONCLUSION

In this work, we have tried to construct a traversable wormhole on the brane without using exotic matter, as such matter has not yet been observed. To obtain physically valid solution for a traversable wormhole one must have the metric potential \( g_{tt} \) finite throughout the wormhole. On considering the Kuchowicz type metric potential, as discussed earlier, it automatically satisfies the desired condition. To obtain the shape function for a static, spherically symmetric wormhole on the brane we have solved Einstein field equations along with the Kuchowicz type metric potential and studied its properties in details to check the viability of our model. We have also estimated values of the model parameters from junction conditions. As we shall see,
we can find a satisfactory estimate of the bulk EOS parameter besides drawing constraints on the brane tension.

In this section we are going to conclude some of the important results that we have obtained from our present study.

1. **Matter density**: By solving the conservation equation along with the EOS we have obtained the matter density of the wormhole and its variation has been shown in Fig. 2.2. From the plot of density with respect to radial distance confirms that the matter near the throat is much denser than the boundary. The variation shows that the density is gradually decreasing from throat to the boundary of the WH.

2. **Properties of Shape function**: For the construction of a traversable wormhole, the shape function plays an important role. In order to obtain a physically acceptable traversable wormhole the shape function must satisfy a number of conditions [3]. Here we discuss those conditions and corresponding behaviour for our model

   (i) First of all at the throat radius \( r = r_0 \), the shape function \( b(r) \) must be equal to the throat radius \( r_0 \) itself. From Fig. 2.2 one can observe that our model satisfies this condition.

   (ii) The metric coefficient \((g_{rr})\) must be regular and well behaved i.e., \( b(r) < r \) for \( r > r_0 \). Again from Fig. 2.2 we have observed this condition is also satisfied for our present study of wormhole.

   (iii) In addition to the above, the nature of the plot for the shape function must be such that if the curve is rotated with respect to the \( b(r) \) axis, the \( b(r) > 0 \) half for both positive and negative \( r \), must approximately resemble the upper half of a wormhole. Fig. 2.2 tells us that it does so.

3. **Flaring-out condition**: To have a stable wormhole, it must obey the flaring-out condition at the throat in order to ensure that there is no pinch-off making the wormhole non-traversable. The flaring-out condition states that the first order derivative of the shape function with respect to \( r \) at the throat must be less than 1. Now, this implies that the null energy condition (NEC) must be violated. We see from Eq. (2.17) that it is not, but on the brane we are interested only about the effective matter distribution which as seen from Eq. (2.22) and Fig. 2.3 violates NEC. This is a huge advantage of considering wormhole on the brane as contrasted to GR, because we find that there is no need to consider exotic matter but the effective matter description due to local and non-local higher dimensional corrections enforce violation of NEC even with ordinary matter on the brane.

4. **Constraints on brane tension and estimate for \( \omega \)**: Further the value of the bulk EOS parameter \( \omega \) which we obtain from the matching condition is 0.9157485630 which is close to +1. This justifies the bulk space to be \( AdS_5 \) (negative cosmological constant).

We find two other new and interesting features from our wormhole model concerned with the brane tension:

   (i) As the value of the brane tension \( \sigma \) is lowered from the value \( 10^3 TeV^4 \) as considered for our analysis, we see that on lowering \( \sigma \) upto \( 1 TeV^4 \), our wormhole solution is valid but on lowering \( \sigma \) further in \( 0.01 - 0.99 TeV^4 \) range, the essential conditions in 2) for wormhole formation are violated, also the condition for violation of the NEC for effective matter description on the brane no longer holding true. Thus, we can claim that from our wormhole model we draw a constraint on the brane tension such that always \( \sigma > 1 \). This is exactly in confirmation with the constraint drawn on the brane tension from the perspective of fundamental investigations [16, 50]. A possible explanation of this may arise from the fact that, as we approach extremely high energy scales \( \sigma \rightarrow 0 \), as discussed before. So, for energy scales
corresponding to $\sigma < 1$, quantum gravity effects must dominate and the effective description on the brane is no longer valid.

(ii) If we vary $\sigma$ further below 0, it turns out that the values of the parameters to be obtained from the matching conditions become imaginary, which strongly rules out the possibility of a negative tension brane. Such a brane is well known to exhibit instabilities from two different fundamental perspectives of black hole [67] and gravitational [68] physics.

There have recently been attempts to test the idea of braneworld against observations [69, 70, 71, 72, 73, 74]. There have also been a number of recent investigations on possible ways for detection of wormholes [75, 76, 77, 78, 79, 80]. Our theoretical model of brane wormhole satisfies all the properties required to describe a traversable wormhole without requiring exotic matter. It also imposes two very important constraints on the brane tension, which is one of the most crucial parameters involved in the study of braneworld and more importantly, both the constraints are in confirmation with ones obtained from studies on different aspects of braneworld including fundamental aspects and also aspects related to black holes. Thus we conclude with the notion that the Kuchowicz metric potential provides physically acceptable and theoretical sound solutions and showing consistency with both wormhole and braneworld in presence of ordinary matter. More boldly we can claim from our analysis that if wormholes do exist in nature, then it must be true that our universe has a very high likelihood of being a 3-brane embedded in higher dimensions.

References

[1] A. Einstein and N. Rosen, Phys. Rev. 48 (1935) 73.
[2] R. W. Fuller and J. A. Wheeler, Phys. Rev. 128 (1962) 919.
[3] M. S. Morris and K. S. Thorne, Am. J. Phys. 56 (1988) 395.
[4] M. S. Morris, K. S. Thorne, and U. Yurtsever, Phys. Rev. Lett. 61 (1988) 1446.
[5] T. Padmanabhan, AIP Conf. Proc. 861 (2006) 179.
[6] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D. 15 (2006) 1753.
[7] R. R. Caldwell and M. Kamionkowski, Ann. Rev. Nucl. Part. Sci. 59 (2009) 397.
[8] J. Frieman, M. Turner and D. Huterer, Ann. Rev. Astron. Astrophys. 46 (2008) 385.
[9] C. Barcelo and M. Visser, Phys. Lett. B 466 (1999) 127.
[10] S. A. Hayward, Phys. Rev. D 65 (2002) 124016.
[11] C. Armendariz-Picon, Phys. Rev. D 65 (2002) 104010.
[12] S. Sushkov, Phys. Rev. D 71 (2005) 043520.
[13] F. S. N. Lobo, Phys. Rev. D 71 (2005) 084011.
[14] O. B. Zaslavskii, Phys. Rev. D 72 (2005) 061303.
[15] S. Chakraborty and T. Bandyopadhyay, Int. J. Mod. Phys. D 18 (2009) 463.
[16] R. Maartens and K. Koyama, Living Rev. Rel. 13 (2010) 5.
[17] M. Bojowald, Living Rev. Relativity. **8** (2005) 11.

[18] J. M. M. Senovilla, Einstein and the Changing Worldviews of Physics, **12** (2012) Springer Chapter 15.

[19] T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, Phys. Rev. D **84** (2011) 024020.

[20] S. Nojiri and S. D. Odintsov, Int. J. Geom. Methods Mod. Phys. **4** (2007) 115.

[21] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370.

[22] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370.

[23] T. Shiromizu, K. I. Maeda, and M. Sasaki, Phys. Rev. D **62** (2000) 024012.

[24] P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477** (2000) 285.

[25] K. I. Maeda, D. Wands, Phys. Rev. D **62** (2000) 124009.

[26] D. Langlois, Phys. Rev. Lett. **86** (2001) 2212.

[27] C. M. Chen, T. Harko, M. K. Mak, Phys. Rev. D **64** (2001) 044013.

[28] E. Kiritsis, JCAP **0510** (2005) 014.

[29] A. Campos and C. F. Sopuerta, Phys. Rev. D **63** (2001) 104012.

[30] C. Germani and R. Maartens, Phys. Rev. D **64** (2001) 124010.

[31] N. Deruelle, arXiv:gr-qc/0111065 (2001).

[32] T. Wiseman, Phys. Rev. D **65** (2002) 124007.

[33] M. Visser, D. L. Wiltshire, Phys. Rev. D **67** (2003) 104004.

[34] S. Creek, R. Gregory, P. Kanti, and B. Mistry, Class. Quant. Grav. **23** (2006) 6633.

[35] S. Pal, Phys. Rev. D **74** (2006) 124019.

[36] N. Dadhich, S. G. Ghosh, Phys. Lett. B **518** (2001) 1.

[37] M. Bruni, C. Germani, and R. Maartens, Phys. Rev. Lett. **87** (2001) 231302.

[38] M. Govender and N. Dadhich, Phys. Lett. B **538** (2002) 223.

[39] J. Ovalle, Mod. Phys. Lett. A **23** (2008) 3247.

[40] J. Ovalle, Int. J. Mod. Phys. D **18** (2009) 837.

[41] B. Bhawal and S. Kar, Phys. Rev. D **46** (1992) 2464.

[42] A. Bhadra and K. Sarkar, Mod. Phys. Lett. A **20** (2005) 1831.

[43] E.F. Eiroa and C. Simeone, Phys. Rev. D **71** (2005) 127501.

[44] O. Bertolami and R.Z. Ferreira, Phys. Rev. D **85** (2012) 104050.

[45] P.H.R.S. Moraes, R.A.C. Correa and R.V. Lobato, JCAP **07** (2017) 029.

[46] A.G. Agnese and M. La Camera, Phys. Rev. D **51** (1995) 2011.
[47] F. He and S.-W. Kim, Phys. Rev. D 65 (2002) 084022.

[48] V. Dzhunushaliev and D. Singleton, Phys. Rev. D 59 (1999) 064018.

[49] K.A. Bronnikov and S. W. Kim, Phys. Rev. D 67 (2003) 064027.

[50] F.S.N. Lobo, Phys. Rev. D 75 (2007) 064027.

[51] A. Banerjee, P. H. R. S. Moraes, R. A. C. Correa, and G. Ribeiro, arXiv:gr-qc/1904.10310.

[52] S. Chakraborty and T. Bandyopadhyay, Astrophys. Space Sci. 317 (2008) 209.

[53] F. Rahaman, M. Kalam, K. A. Rahman, S. Chakraborti, Gen. Rel. Grav. 39 (2007) 945.

[54] D. Wang and X.-H. Meng, Front. Phys. 13 (2018) 139801.

[55] F. Rahaman, P. K. F. Kuhfittig, S. Ray and N. Islam, Eur. Phys. J. C (2014) 2750 74 (2014) 2750.

[56] F. Rahaman et al, Ann. Phys. 350 (2014) 561.

[57] P.K.F. Kuhfittig, Eur. Phys. J. C 74 (2014) 2818.

[58] B. Kuchowicz, Acta. Phys. Pol. 33 (1968) 541.

[59] L. B. Castro et al, J. Cosm. Astrop. Phys. 08 (2014) 047.

[60] R. Sengupta et al, Phys. Rev. D (2020).

[61] G. Darmois, Mémorial des sciences mathématiques XXV (1927) Fasticule XXV Chap. V.

[62] W. Israel, Nuo. Cim. 66 (1966) 1.

[63] C. Lanczos, Ann. Phys. (Leipzig) 74 (1924) 518.

[64] N. Sen, Ann. Phys. (Leipzig) 378 (1924) 365.

[65] G.P. Perry, R.B. Mann, Gen. Relativ. Gravit. 24 (1992) 305.

[66] P. Musgrave, K. Lake, Class. Quant. Gravit. 13 (1996) 1885.

[67] D. Marolf and M. Trodden, Phys. Rev. D 64 (2001) 065019.

[68] C. Charmousis and J. F. Dufaux, Phys. Rev. D 70 (2004) 106002.

[69] A. Abdujabbarov and B. Ahmedov, Phys. Rev. D 81 (2010) 044022.

[70] A.R. Liddle and A.J. Smith, Phys. Rev. D 68 (2003) 061301

[71] C.R. Keeton and A.O. Petters, Phys. Rev. D 73 (2006) 104032.

[72] L. Iorio, J. Cosm. Astrop. Phys. 09 (2005) 006.

[73] L. Visinelli, N. Bolis, and S. Vagnozzi, Phys. Rev. D 97 (2018) 064039.

[74] S. Vagnozzi and L. Visinelli, Phys. Rev. D. 100 (2019) 024020.

[75] R. Shaikh and S. Kar, Phys. Rev. D 96 (2017) 044037.

[76] Z. Li and C. Bambi, Phys. Rev. D 90 (2014) 024071.
[77] T. Ohgami and N. Sakai, Phys. Rev. D 91 (2015) 124020.

[78] N. Tsukamoto et al, Phys. Rev. D 86 (2012) 104062.

[79] K.K. Nandi et al, Phys. Rev. D 95 (2017) 104011

[80] R. Shaikh, Phys. Rev. D 98 (2018) 024044.