Effect of Closure Coefficients of $K$-$\Omega$ SST Turbulence Model on Predicting Stall Characteristics of The Airfoil

Changping Liang$^{1, 2, *}$

$^1$School of Mechanical Engineering, Changshu Institute of Technology, Chang’shu 215500, China

$^2$Jiangsu Key Laboratory of Recycling and Reuse Technology for Mechanical and Electronic Products, Changshu, Jiangsu, China

*Corresponding author e-mail: liangcp@cslg.edu.cn

Abstract. The two-dimensional numerical simulation for NACA0015 airfoil and S325 airfoil were conducted with $k$-$\omega$ SST turbulence model. For the $k$-$\omega$ SST turbulence model could not predict the airfoil stall characteristics accurately, the non-equilibrium turbulence transport nature was systematically analyzed. The study showed that: the turbulent viscosity scope simulated by the original $k$-$\omega$ SST turbulence model was smaller which resulting in the model could not accurately simulate the turbulence transport properties in the separation zone; By increasing $\beta^*$ or decreasing $\alpha_1$ can improve the capabilities for predicting the transport characteristics of non-equilibrium turbulent, and then improve the accuracy of numerical simulation for stall characteristics of the airfoil, it also helps to improve the accuracy of numerical simulation of wind turbine blades.

1. Introduction

The prediction accuracy of aerodynamic characteristics of the wind turbine airfoil has an important impact on the design results. The accurate and efficient analysis tool for aerodynamic characteristics of the wind turbine airfoil is the premise and foundation to develop a new generation of wind turbine airfoil. With the rapid development of modern computer technology and computational fluid dynamics (CFD), CFD has been widely used to predict the aerodynamic performance of wind turbine airfoils. In CFD, the pros and cons of turbulence models will have a significant impact on the calculation results. Eleni and Wolfe [1-2] reported that using CFD method to calculate the aerodynamic performance of the airfoil requires special attention to the transition point prediction and turbulence model. Langtry and Shelton et al. [3-4] showed that turbulence models can not accurately predict the stall characteristics of the airfoil when the flow is in stall condition. In the widely used turbulence models, there are some empirical parameters, which are based on the equilibrium turbulence with the simple boundary, or be calibrated by the experimental results with some special conditions. Therefore, in practical applications, these parameters need to be properly adjusted [5]. Zhou et al. [6] analyzed the influence discipline of the eight parameters in $k$-$\omega$ SST model on the calculation of the airfoil aerodynamic characteristics. The result showed that the values of $\sigma_{\omega_2}, \beta_2, \beta', \alpha_1$ in $k$-$\omega$ SST model had great impacts on the calculation results, but the literature did not do further research. In 1994, Yang et al. [7-8] used a two-dimensional incompressible solver to simulate the aerodynamic
characteristics of the S805 [9] and S809 [10] airfoil, the turbulence model was the Baldwin-Lomax algebraic model. The result showed that at low angle of attack, the lift coefficient and pressure distribution obtained by numerical simulation agreed well with the experimental results; however, at high angle of attack, the lift coefficient of numerical simulation was significantly higher than the experimental data. Therefore, researchers believe that to get better simulation when the airfoil under stall state, it may be necessary to rewrite the generation items of \( k \) and \( \omega \) in the transport equation of the turbulence model.

Currently, the turbulence models based on the Boussinesq assumption can not give the accurate simulation results for the stall characteristics when the airfoil at high angle of attack [11-12]. Zhong Wei [13] used the \( \gamma-\theta \) transition mode to simulate the aerodynamic forces of the S809 airfoil under the angle of attack from \( 0^\circ \) to \( 30^\circ \), involving the flow conditions of attached flow, light stall, and deep stall. Therefore, it is important to accurately simulate the aerodynamic performance after the airfoil stall.

In this paper, the \( k-\omega \) SST turbulence model was used to simulate the aerodynamic performance of the S825 and NACA0015 airfoils, and compared the calculation results with the experimental data. The results showed that the mis-predicting of the stall characteristics was due to the fact that the tradition turbulence model could not accurately predict the strongly non-equilibrium turbulence transport nature, the turbulent viscosity scope simulated by the original \( k-\omega \) SST turbulence model was smaller which resulting in the model could not accurately simulate the turbulence transport properties in the separation zone. Through the modification of the model parameters \( \beta^* \) and \( \alpha_1 \) that have a direct impact on the generation items and the dissipation items, proposing a modification method of the \( k-\omega \) SST model suitable for the separation flow of wind turbine airfoils.

2. CFD numerical simulation method

2.1. Control equations and \( k-\omega \) SST turbulence model

The control equations used in the CFD numerical simulation in this paper are based on the Reynolds-averaged incompressible Navier-Stokes equations, which can be expressed as follows:

\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x_j} - \frac{\partial F^v}{\partial x_i} = 0
\]  

Where \( Q \) is the flow-conservative variable, \( F \) is the non-stick (convection) flux, \( F^v \) is the viscous flux, which can be respectively expressed as:

\[
Q = \begin{bmatrix} \rho \\ \rho u_i \end{bmatrix}, \quad F_i = \begin{bmatrix} \rho u_i \\ \rho u_i u_j + p \delta_{ij} \end{bmatrix}, \quad F^v_i = \begin{bmatrix} 0 \\ \tau_{ij} \end{bmatrix}
\]  

Where \( \rho, u_i, p, \tau_{ij} \) represents the fluid density, velocity component, pressure, shear stress tensor components, respectively.

\( k-\omega \) SST turbulence model consists of turbulent kinetic energy transport equation and turbulent kinetic energy dissipation rate transport equation:

\[
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho ku_j) = \frac{\partial}{\partial x_j} \left[ \Gamma_k \frac{\partial k}{\partial x_j} \right] + G_k - Y_k + S_k
\]  

\[
\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_j} (\rho \omega u_j) = \frac{\partial}{\partial x_j} \left[ \Gamma_\omega \frac{\partial \omega}{\partial x_j} \right] + G_\omega - Y_\omega + D_\omega + S_\omega
\]
Where, $G_k$, $G_\omega$ represent the generation item of $k$, $\omega$; $\Gamma_k$, $\Gamma_\omega$ represent the diffusion item of $k$, $\omega$; $Y_k$, $Y_\omega$ represent the dissipation item of $k$, $\omega$; $S_k$, $S_\omega$ represent the user-defined source item of $k$, $\omega$; $D_\omega$ represent the cross-diffusion term of $\omega$. Where, the dissipation item and the generation item are defined as follows:

$$Y_k = \rho \beta' k \omega$$  \hfill (5)  

$$Y_\omega = \rho \beta \omega^2$$  \hfill (6)  

$$G_k = \mu_t S^2$$  \hfill (7)  

$$G_\omega = \frac{\alpha}{\nu} G_k$$  \hfill (8)  

Where:

$$\mu_t = \frac{\rho k}{\omega} \max \left[ \frac{1}{\alpha^*}, \frac{SF_2}{\alpha_i(\omega)} \right]$$  \hfill (9)  

$$\beta' = \beta_i^* \left[ 1 + \zeta^* F(M_i) \right]$$  \hfill (10)  

$$\beta = \beta_i \left[ 1 - \frac{\beta_i^*}{\beta_i} \zeta^* F(M_i) \right]$$  \hfill (11)  

The other parameters in the model can be defined by reference [13].

2.2. Computational grid and boundary conditions

During the calculation, the NACA0015 airfoil is selected firstly. The airfoil has a chord length of $c=0.4$ m, a long enough domain ($50c$) is chosen to preclude the effect of the outlet boundary condition. Discretization has been preceded using the Finite-Volume method with second-order upwind scheme for all variables, the turbulent intensity and turbulent viscosity ratio are both 0.01. A constant velocity is set at the inlet boundary, while the constant pressure of 1 atm is specified at the outlet, the airfoil’s walls are set as the no-slip wall condition. The airfoil is meshed with the C-type grid with 800 nodes over the airfoil surface, the first layer of the wall thickness is 0.00001C to make sure the value of the $y^+ < 1$. The flow density $\rho = 1.225 \text{ kg/m}^3$, $\mu = 1.789 \times 10^{-5} \text{ kg/(m\cdot s)}$, the chordal Reynolds number is $3 \times 10^5$, the range of attack angle is $0^\circ \sim 20^\circ$.

Figure 1 is the comparison between the calculated lift coefficient by original turbulence model and the experimental results[15], the original turbulence models is $k-\omega$ SST turbulence model (O $k-\omega$ SST).
When the angle of attack is in $0^\circ$ to $10^\circ$, the flow is in the attachment state, the maximum lift coefficient corresponding to the angle of attack of $15^\circ$. As the angle of attack continues to increase, the airfoil trailing edges appear to separate and produce the separation vortex. The breakdown of the separation vortex makes the ring of the airfoil decrease, thus reducing the lift coefficient, and the airfoil into the stall state. In the attachment state, the lift coefficient calculated by the original $k-\omega$ SST turbulence model is in good agreement with the experimental values; after the flow separation, there is a clear discrepancy between the calculated value and the experimental value. In particular, the calculated values of the original $k-\omega$ SST turbulence model are relatively close to the experimental values, but it still can not accurately simulate the flow separation.

It can be seen from figure 2 that when the flow separation occurs, the separation point simulated by the model is closer to the airfoil’s trailing edge and the separation zone is smaller. Therefore, compared with the experimental results, the simulation of the original $k-\omega$ SST turbulence model is still inaccurate when the flow separation occurs. The result shows that the original $k-\omega$ SST turbulence model can not accurately simulate the turbulence transport properties in the separation zone; therefore, it is necessary to correct the closure parameters to improve the capability of prediction stall characteristics of $k-\omega$ SST turbulence model.

3. Correction of the $k-\omega$ SST turbulence model
It can be seen from the equations 5-11 that the $k-\omega$ SST model is based on the Reynolds-averaged incompressible Navier-Stokes equations, and the model makes the Navier-Stokes equations closed by adding turbulent kinetic energy equation ($k$ equation) and dissipation rate equation ($\omega$ equation). These equations describe the transport properties of turbulence, i.e., the generation, diffusion and dissipation of the turbulence; while the generation and dissipation items account for the largest proportion, their different contributions to the transport process determine the level of non-equilibrium turbulence, the generation and dissipation items have a major impact on the accuracy of the simulation results.
In the $k-\omega$ SST turbulence model, $\alpha_1$ and $\beta^*$ are empirical parameters, which are based on the equilibrium turbulence with the simple boundary, the turbulence generation and dissipation terms are almost equal when the flow in this state. In original model, the definition of Reynolds stress uses the JK model that the Reynolds stress is proportional to the turbulent kinetic energy, and the Reynolds stress can be defined as: $\tau = \rho \frac{P_k}{d_k} \alpha_1 k$ [14]. For the equilibrium boundary layer, the ratio of generation item to dissipation item is close to 1, so $-\overline{u'v'} = \alpha_1 k$, $\alpha_1 = -\overline{u'v'}/k$. Simplify the momentum equation, $k$ equation and $\omega$ equation in logarithmic zone of turbulence boundary layer we can get: $-\overline{u'v'} = \sqrt{\beta^* k}$, namely $\beta^* = (-\overline{u'v'}/k)^2$ [17]. The test in 1976 pointed out that the value of $-\overline{u'v'}/k$ is about 0.3[16], so the default values of the $\alpha_1$ and $\beta^*$ are 0.31 and 0.09, respectively. Therefore, the default values of the $\alpha_1$ and $\beta^*$ can not accurately simulate the transport characteristics of turbulence, and the values of $\alpha_1$ and $\beta^*$ must be corrected.

4. The calculation results by the modified models

4.1. The calculation results of the NACA0015 airfoil by the modified models

![Comparison of the lift coefficient of NACA0015 airfoil by modified models](image_url)

**Figure. 3** Comparison of the lift coefficient of NACA0015 airfoil by modified models

![Comparison of turbulence viscosity distributions simulated by modified models](image_url)

**Figure. 4** Comparison of turbulence viscosity distributions simulated by modified models
It can be seen from figure 3 that compared with the original turbulence model (O k-ω SST), the lift coefficients of the modified models by increasing $\beta^*$ ($M_{k-\omega\text{SST(}\beta^*)}$) or decreasing $\alpha_i$ ($M_{k-\omega\text{SST(}\alpha_i)}$) are closer to the experimental data. Especially when the airfoil is in stall state, the flow separation occurs in the airfoil’s surface, the accuracy of the lift coefficients calculated by the modified model has been significantly improved. This is because when the flow separation occurs, the generation and dissipation terms are in non-equilibrium state, the original turbulence model can not accurately predict the contributions of the generation and dissipation terms, and thus can not accurately simulate the range of turbulence viscosity distribution. The turbulence viscosity distributions and streamlines are shown in figure 4. It can be seen from the figures that the range of turbulence viscosity distribution and the separation area simulated by the original turbulence model is smaller; the range of turbulence viscosity distribution and the separation area can be increased by increasing $\beta^*$ or decreasing $\alpha_i$. Therefore, the modified model can get more accurate simulation results.

4.2. The calculation results of the S325 airfoil by the modified models

In order to verify the universality of the modified model, the aerodynamic performance of the S325 airfoil is calculated by the modified model. The boundary conditions in the calculation are based on the test conditions of reference [10]. A constant velocity is set at the inlet boundary, while the constant pressure of 1 atm is specified at the outlet, the airfoil’s walls are set as the no-slip wall condition. The flow density $\rho=1.179\text{kg/m}^3$, $\mu=1.789\times10^{-5}\text{kg/(m·s)}$, the static temperature $T=300\text{K}$, $Ma=0.1$, the chordal Reynolds number is $2\times10^6$.

Figure 5 gives the lift coefficients simulated by the original $k-\omega$ SST model and the modified $k-\omega$ SST model. It can be seen from figure 11 that compared with the original turbulence model (O $k-\omega$ SST), the lift coefficients of the modified models by increasing $\beta^*$ ($M_{k-\omega\text{SST(}\beta^*)}$) or decreasing $\alpha_i$ ($M_{k-\omega\text{SST(}\alpha_i)}$) are closer to the experimental data.

Figure. 5 Comparison of the lift coefficients of S325 airfoil by modified models

5. Conclusion

When the flow separation occurs in the airfoil’s surface, the generation and dissipation terms of $k$ and $\omega$ are in non-equilibrium state, the original $k-\omega$ SST turbulence model can not accurately estimate the contributions of the generation and dissipation terms, and thus can not accurately simulate the range of turbulent viscosity, the turbulent viscosity scope and the separation area simulated by the original $k-\omega$ SST turbulence model is smaller. In the main parameters of the $k-\omega$ SST turbulence model, the values of $\alpha_i$ and $\beta^*$ have an important influence on the generation and dissipation of turbulence. The original model has a higher estimate of the magnitude of the generation terms and dissipation terms and their, as a result, the turbulence in the separation area simulated by the original model has been increased. Therefore, the velocity gradient increases, the velocity in the separation zone increases rapidly, and the simulated separation zone is smaller. By increasing $\beta^*$ or decreasing $\alpha_i$, the magnitude of the
generation terms and dissipation terms can be decreased, and the significant range of the generation and dissipation terms of $k$ and $\omega$ can be increased, and thus the capability of prediction stall characteristics and the capability of capturing the non-equilibrium turbulence of $k$-$\omega$ SST turbulence model can be improved obviously.

Due to the numerical simulation of the wind turbine blade is closely related to the numerical simulation of the airfoil, the modified model can be applied to improve the accuracy of numerical simulation of the wind turbine blade.

References
[1] Eleni D C, Athanasios T I, Dionissios M P. Evaluation of The Turbulence Models for The Simulation of The Flow over a National Advisory Committee for Aeronautics (NACA) 0012 Airfoil [J]. Journal of Mechanical Engineering Research, 2012, 4(3): 100-111.
[2] Wolfe W P, Ochs S S. CFD calculations of S809 aerodynamic characteristics [R]. AIAA 1997-0973, 1997.
[3] Langtry R B, Gola J, Mente F R. Predicting 2D Airfoil and 3D Wind Turbine Rotor Performance Using a Transition Model for General CFD Codes [R]. AIAA-2006-395, 2006.
[4] Shelton A, Abras J, Hathaway B E, et al. An investigation of the numerical prediction of static and dynamic stall [C]//American Helicopter Society 61st Forum. Grapevine, TX, United States:[s.n.], 2005.
[5] Ren N X, Ou J P. Numerical Simulation for Pneumatic Characteristics for Two-dimensional Airfoils Large Wind Turbine [J]. ACTA ENERGY SOLARIS SINICA, 2009,30(8):1087-1091.
[6] Zhou Y, Qian W Q, Deng Y Q, et al. Introductory Analysis of The Influence of Menter’s k-$\omega$ SST Turbulence Model’s Parameters [J]. ACTA AERODYNAMICA SINICA, 2010, 28(2): 213-217.
[7] Yang S L, Chang Y L, Arici O. Incompressible Navier-Stokes computation of the NREL airfoils using a symmetric total variational diminishing scheme [J]. Journal of Solar Energy Engineering, 1994, 116 (4): 174-182.
[8] Yang S L, Chang Y L, Arici O. Navier-Stokes Computations of the NREL airfoil using a k-$\omega$ turbulent model at high angles of attack [J]. Journal of Solar Energy Engineering, 1995, 117(4):304-310.
[9] Somers D M. Design and experimental results for the S805 airfoil [R]. Colorado: National Renewable Energy Laboratory, 1997.
[10] Somers D M. Design and experimental results for the S809 airfoil [R]. Colorado: National Renewable Energy Laboratory, 1997.
[11] Shelton A, Abras J, Jurenko R, et al. Improving the CFD predictions of airfoils in stall[R]. AIAA-2005-1227, 2005.
[12] Ma L J, Chen J. Numerical simulation of aerodynamic performance for wind turbine airfoils[J]. ACTA Energiae Solaris Sinica, 2010, 31(2):203-209.
[13] Zhong W, Wang T G. Numerical analysis of transition effect on stall performance of wind turbine airfoils and blades [J]. Acta Aerodynamica Sinica, 2011,29(3): 385–390.
[14] Menter F R. Two-equation Eddy-viscosity Turbulence Models for Engineering Applications [J]. AIAA Journal, 1994, 32(8):269-289.
[15] Costes M, Gleize V, Szydlowski J, et al. Grid sensitivity study for the turbulent viscous flow around a NACA0015 airfoil at stall. 31st European Rotorcraft Forum, Confederation of European Aerospace Societies,2005(105): 1-12.
[16] Menter F R. Performance of popular turbulence models for attached and separated adverse pressure gradient flows [J]. AIAA Journal, 1992, 30:2066-2072.
[17] Wilcox D C. Turbulence model for CFD [M]. California: DCW Industries, Inc, 2007: 155-156.
[18] Townsend A A R. The structure of turbulent flow [M]. England: Cambridge University Press, 1976: 49-53.