A study is made of what SUSY signals would be observable for mSUGRA models in a 500-GeV linear collider. All current experimental bounds on the mSUGRA parameter space are imposed. For $m_0 < 1 \text{ TeV}$ (or alternately if the current $g_\mu - 2$ anomaly maintains) the only observable signals that remain are slepton pair production and neutralino production of $\tilde{\chi}_0^2 + \tilde{\chi}_1^0$. Slepton pair production can occur for masses $< 250 \text{ GeV}$ which for the selectron and smuon pairs require $\tan \beta < 40$. In this domain very accurate selectron and smuon masses could be measured. Light staus, $\tilde{\tau}_1$, with mass $< 250 \text{ GeV}$ can be pair produced for any $\tan \beta$ and the neutralino signal can be seen provided $m_{1/2} \approx 400 \text{ GeV}$. However, the detection of these requires a much more complicated analysis due to the fact that the dark matter co-annihilation constraint requires that the $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$ mass difference be $\approx 15 \text{ GeV}$. The point $m_{1/2} = 360 \text{ GeV}$, $A_0 = 0$, $\mu > 0$ is analyzed in detail, and it is shown that the stau and neutralino signals can be detected provided an active mask down to $2^o$ is used. However, large parts of the mSUGRA parameter space exists where a 500-GeV machine would not be able to see any SUSY signal.

1 Introduction

There is a growing consensus that the next high energy machine to be built after the Large Hadron Collider (LHC) should be an electron-positron linear collider (LC), and initial designs call for a 500-GeV machine. While the technology to be chosen (TESLA or NLC/JLC) and where the siting should be is still under discussion, the proponents believe that such a machine is technically feasible. We consider here what aspects of mSUGRA might be tested at a 500-GeV LC.

There has been in the past a huge amount of analysis on methods of detecting SUSY at LCs. However, the minimal supergravity model, mSUGRA has several special aspects that make its predictions clearer and hence more directly accessible to experimental study. Hence it is worthwhile to examine this particular model. Thus:

(1) mSUGRA depends on only four additional parameters and one sign beyond those of the Standard Model (SM). These are $m_0$ (the universal soft breaking
mass at the GUT scale $M_G$; $m_{1/2}$ (the universal gaugino soft breaking mass at $M_G$); $A_0$ (the universal cubic soft breaking mass at $M_G$); $\tan \beta = < H_2 > / < H_1 >$ at the electroweak scale (where $H_2$ gives rise to $u$ quark masses and $H_1$ to $d$ quark and lepton masses); and the sign of $\mu$, the Higgs mixing parameter in the superpotential ($W_\mu = \mu H_1 H_2$). Note that the lightest neutralino $\tilde{\chi}_1^0$ and the gluino $\tilde{g}$ are approximately related to $m_{1/2}$ by $m_{\tilde{\chi}_1^0} \approx 0.4 m_{1/2}$ and $m_{\tilde{g}} \approx 2.8 m_{1/2}$. We will examine here the following parameter range: $0 < m_{1/2} < 1$ TeV, $|A_0| < 4 m_{1/2}$ and $1 < \tan \beta < 55$. The $m_{1/2}$ range thus covers the limit of gluino discovery at the LHC.

(2) mSUGRA makes predictions for a wide range of phenomena, and thus the model is already significantly constrained by experiment. Most important for limiting the parameter space are:

(i) The light Higgs mass bound from LEP $m_h > 114$ GeV. Since theoretical calculations of $m_h$ still have a 2-3 GeV error, we will conservatively assume this to mean that $(m_h)^{\text{theory}} > 111$ GeV.

(ii) The $b \rightarrow s + \gamma$ branching ratio $\delta$. We assume here a relatively broad range (since there are theoretical errors in extracting the branching ratio from the data):

$$1.8 \times 10^{-4} < B(B \rightarrow X_s \gamma) < 4.5 \times 10^{-4}$$

(iii) In mSUGRA the lightest neutralino, $\tilde{\chi}_1^0$, is the candidate for dark matter (DM). Previous bounds from balloon flights (Boomerang, Maxima, Dasi, etc.) gave a relic density bound for DM of $0.07 < \Omega_{DM} h^2 < 0.21$ (where $\Omega_{DM}$ is the density of dark matter relative to the critical density to close the universe, and $h = H/100 km/secMpc$ where $H$ is the Hubble constant). However, the new data from WMAP greatly tightens this (by a factor of four) and the 2$\sigma$ bound is now:

$$0.095 < \Omega_{DM} h^2 < 0.129$$

(iv) The bound on the lightest chargino mass $\tilde{\chi}_1^{\pm} > 104$ GeV

(v) The muon magnetic moment anomaly, $\delta a_\mu$. Here the calculation of the leading order of the hadronic SM contribution is still in doubt. Using the $e^+e^-$ data to calculate this, one gets a 3$\sigma$ deviation of the SM from the experimental result, while using tau decay and CVC analysis with CVC breaking included one get a 1$\sigma$ deviation. However, comparison between the $e^+e^-$ analysis and the tau decay analysis exhibits more than a 4$\sigma$ disagreement in one channel making it difficult to argue one should average the two results. Most recently CMD-2 has done a reanalysis of their $e^+e^-$ data correcting their treatment of vacuum polarization diagrams. This reduces the $e^+e^-$ disagreement with the SM to perhaps 2$\sigma$. Thus the situation is still much up in the air, and future data from KLOE, BaBar and Belle as well as additional BNL E821 data for the $\mu^-$ may help to clarify matters in the future.

(3) One can now qualitatively state the constraints on the parameter space produced by the above experimental bounds:
(i) The relic density constraint produces a narrow rising band of allowed parameter space in the $m_0 - m_{1/2}$ plane.

(ii) In this band, the $m_h$ and $b \to s + \gamma$ constraints produce a lower bound on $m_{1/2}$ for all $\tan \beta$:

\[ m_{1/2} \gtrsim 300 \text{GeV} \tag{3} \]

which implies $m_{\tilde{\chi}_1^0} > 120 \text{ GeV}$ and $m_{\tilde{\tau}} > 250 \text{ GeV}$.

(iii) If the $g_\mu - 2$ effect is real, then $\mu > 0$, and the combined effects of $g_\mu - 2$ and the $M_{\tilde{\chi}_1^\pm} > 104 \text{ GeV}$ eliminates all other possible domains satisfying the relic density constraint.

In the following, we will analyze the case where $\mu > 0$, but leave open the question of the validity of the $g_\mu - 2$ effect. (See also 13 for $\mu < 0$.) Figs. 1, 2 and 3 illustrate the above constraints on the mSUGRA parameter space for $\tan \beta = 10, 40$ and 50 with $A_0 = 0$. The red area is the parameter space allowed by the earlier balloon CMB experiments, while the (reduced) blue area is the region now allowed by WMAP, Eq. (2). The dotted red lines are for different Higgs masses, and the light blue region would be excluded if $\delta a_\mu > 11 \times 10^{-10}$. It is important to note that the narrowness of the allowed dark matter band is not a fine tuning. The lower limit of the band comes from the rapid annihilation of neutralinos in the early universe due to co-annihilation effects as the light stau mass, $m_{\tilde{\tau}_1}$, approaches the neutralino mass as one lowers $m_0$. Thus the lower edge of the band corresponds to the lower bound of Eq.(2), and the band is cut off sharply due to the Boltzmann exponential behavior. The upper limit of the band [corresponding to the upper bound of Eq.(2)] arises due to insufficient annihilation as $m_0$ is raised.

As the WMAP data becomes more accurate, the the allowed band will narrow even more. (Note that the slope and position of the band changes, however as $A_0$ is changed.) Thus the astronomical determination of the amount of dark matter effectively determines one of the four parameters of mSUGRA.

(4) In order to carry out the calculations it is necessary to include a number of corrections to obtain results of sufficient accuracy, and we list some of these here:

(i) Two loop gauge and one loop Yukawa RGE equations are used from $M_C$ to the electroweak scale , and QCD RGE below for the light quarks. Two loop and pole mass corrections are included in the calculation of $m_h$.

(ii) One loop corrections to $m_b$ and $m_t$ are included.

(iii) Large $\tan \beta$ SUSY corrections to $b \to s + \gamma$ are included.

(iv) All $\tilde{\tau}_1 - \tilde{\chi}_1^0$ co-annihilation channels are included in the relic density calculation.

We do not include Yukawa unification or proton decay constraints as these depend sensitively on post GUT physics, about which little is known.
Figure 1. Allowed region in the $m_0 - m_{1/2}$ plane from the relic density constraint for $\tan\beta = 10$, $A_0 = 0$ and $\mu > 0$. The red region was allowed by the older balloon data, and the narrow blue band by the new WMAP data. The dotted red vertical lines are different Higgs masses, and the current LEP bound produces the lower bound on $m_{1/2}$. The light blue region is excluded if $\delta a_\mu > 11 \times 10^{-10}$. (Other lines are discussed in text.)

2 Possible mSUGRA Signals At The 500-GeV LC

To pair produce SUSY particles at a 500-GeV linear collider requires the sparticle mass to be less than 250 GeV. The previous constraints discussed, $m_{1/2} > 300$ GeV and the dark matter allowed bands, already excludes the pair production of the charginos, the heavier neutralinos, $\tilde{\chi}_2^0$, squarks, gluons and the heavy stau, $\tilde{\tau}_2$. Thus the remaining possible signals for mSUGRA are the following:

$$e^+ + e^- \rightarrow \tilde{\tau}_1^+ \tilde{\tau}_1^- \rightarrow (e^+ + \tilde{\chi}_1^0) + (e^- + \tilde{\chi}_1^0)$$

(4)

$$e^+ + e^- \rightarrow \tilde{\chi}_2^+ + \tilde{\chi}_2^- \rightarrow (\mu^+ + \tilde{\chi}_1^0) + (\mu^- + \tilde{\chi}_1^0)$$

(5)
Figure 2. Same as Fig. 1 for $\tan \beta = 40$, $A_0 = 0$, $\mu > 0$ except that now that the $b \rightarrow s\gamma$ constraint (green region) produces the lower bound on $m_{1/2}$.

\begin{align}
e^+ + e^- & \rightarrow \tilde{\tau}_1^+ + \tilde{\tau}_1^- \rightarrow (\tau^+ + \tilde{\chi}_1^0) + (\tau^- + \tilde{\chi}_1^0) \quad (6) \\
e^+ + e^- & \rightarrow \tilde{\chi}_2^0 + \tilde{\chi}_1^0 \rightarrow (\tau + \tilde{\tau}_1) + \tilde{\chi}_1^0 \rightarrow (\tau^+ + \tau^- + \tilde{\chi}_1^0) + \tilde{\chi}_1^0 \quad (7)
\end{align}

Processes (4) and (5) are easiest to detect since the $e$ and $\mu$ can be readily observed. Process (6) is more readily produced since L-R mixing in the tau mass matrix makes $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1}$, but hadronic tau identification (ID) is more difficult. In mSUGRA, $m_{\tilde{\chi}_2} \cong 2m_{\tilde{\chi}_1}$ and so process (7) is kinematically feasible for a 500-GeV machine, and since the $\tilde{\tau}_1$ is the lightest slepton, the final state will be mainly taus as indicated. This allows us to divide the mSUGRA parameter space into three parts:

(1) $m_{\tilde{\chi}_1} < 250$ GeV. Here processes (4) and (5) can occur and SUSY can be detected with high precision. The kinematic reach of these processes is
Figure 3. Same as Fig. 2 for $\tan \beta = 50$, $A_0 = 0$, $\mu > 0$. Note that the large bulge at lower $m_{1/2}$ allowed by the older balloon data is now mostly excluded by the WMAP data.

shown by the dashed black curve in the lower left hand corners of Figs. 1 and 2. One sees that when other experimental constraints are taken into account, these processes require $\tan \beta < 40$. Most of the previous analyses have been concentrated on this region of the parameter space. We note that $\tilde{\tau}_1$ pair production can also occur in this parameter region.

(2) $m_{\tilde{\tau}_1} > 250$ GeV, but $m_{\tilde{\tau}_1} < 250$ GeV. Here processes (6) and (7) are the only ones possible, process (7) being possible if $m_{1/2} \lesssim 400$ GeV. The kinematic reach for process (6) is shown by the solid blue lines in Figs. 1-3, the lower one being for a 500-GeV machine, and the upper one for a 800-GeV machine. The kinematic reach of process (7) is shown in Figs. 1-3 as vertical blue dot-dash lines, the left hand one for a 500-GeV machine and the right hand one for an 800-GeV machine. We see that process (6) has a significantly
larger reach than process (7), but neither cover much of the SUSY parameter space for a 500-GeV LC.

(3) $m_{\tilde{e}, \tilde{\mu}, \tilde{\tau}_1} > 250$ GeV and $m_{1/2} \gtrsim 400$ GeV. All final sparticle states are inaccessible and SUSY cannot be seen at a 500-GeV machine. (Unfortunately, this is not a small part of the parameter space allowed by mSUGRA.)

We consider first the region where $m_{\tilde{e}, \tilde{\mu}} < 250$ GeV and $m_{1/2} \sim 400$ GeV. All final sparticle states are inaccessible and SUSY cannot be seen at a 500-GeV machine. (Unfortunately, this is not a small part of the parameter space allowed by mSUGRA.)

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This region of parameter space, where one can pair produce selectrons and smuons at a 500-GeV LC, occurs for low and intermediate tanbeta. As can be seen from Fig. 2, one requires $\tan \beta < 40$. Most of the $\tan \beta = 30$ parameter space is also inaccessible, and one can see from Fig. 1, that the reach is not large even for $\tan \beta = 10$. There has been much study of this region (e.g. 171819) as it allows very accurate determination of SUSY masses. The basic reaction is

$$e^+ + e^- \rightarrow \tilde{e}^+_R + \tilde{e}^-_R \rightarrow (e^+ + \tilde{\chi}_1^0) + (e^- + \tilde{\chi}_1^0)$$

and the signal is thus $e^+ + e^- + \not{p}_T$ (with a similar signal in the $\mu$ channel).

Since the processes are all two body, the kinematics is simple: there is a maximum and minimum lepton energy given by

$$E_{\text{min, max}} = \frac{m_{\tilde{e}_R}}{2} \left[ 1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{e}_R}^2} \right] \gamma (1 \mp \beta)$$

where $\gamma = \sqrt{s}/2m_{\tilde{e}_R}$ and $\beta = \sqrt{1 - 4m_{\tilde{\chi}_1^0}^2/s}$. A measurement of $E_{\text{min}}$ and $E_{\text{max}}$ then determines $m_{\tilde{e}_R}$ and $m_{\tilde{\chi}_1^0}$ at the 1/10% level. In mSUGRA then, this would determine $m_0$ and $m_{1/2}$ very accurately.

Since the $\tilde{\tau}_1$ is the lightest slepton, pair production of these can also occur in this part of the parameter space. The corresponding tau analysis, however, is more complicated for two reasons:

1. There is L-R mixing in the $\tilde{\tau}$ mass matrix:

$$m_{\tilde{\tau}}^2 = \begin{pmatrix} m_{\tilde{\tau}}^2 & m_{\tau}(A_\tau - \mu \tan \beta) \\ m_{\tau}(A_\tau - \mu \tan \beta) & m_{\tilde{\tau}_R}^2 \end{pmatrix}$$

with two eigenstates $\tilde{\tau}_1$ and $\tilde{\tau}_2$

$$\left( \begin{array}{c} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{array} \right) = \left( \begin{array}{cc} \cos \theta_\tau & \sin \theta_\tau \\ -\sin \theta_\tau & \cos \theta_\tau \end{array} \right) \left( \begin{array}{c} \tilde{\tau}_L \\ \tilde{\tau}_R \end{array} \right)$$

In mSUGRA the lightest stau, $\tilde{\tau}_1$, is the next to lightest SUSY particle (NLSP), and is mostly $\tilde{\tau}_R$.

2. The decay pattern is now multiparticle i.e.:

$$e^+ + e^- \rightarrow \tilde{\tau}_1^+ + \tilde{\tau}_1^- \rightarrow (\tau^+ + \tilde{\chi}_1^0) + (\tau^- + \tilde{\chi}_1^0) \rightarrow [jets + \nu_\tau + X] + [jets + \nu_\tau + X]$$

where the jets are mainly $\pi$, $\rho$ and $a_1$ mesons i.e. the hadronic taus give rise to multipion states plus $\not{p}_T$. 

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Table 1. Masses (in GeV) of SUSY particles for Points 1, 2 and 3. These points satisfy all the existing experimental bounds on mSUGRA.

| Point | $m_{\tilde{\chi}^0_1}$ | $m_{\tilde{\tau}_1}$ | $m_{\tilde{\chi}^0_2}$ | $\delta m$ | $m_{\tilde{\tau}_1} - m_{\tilde{\chi}^0_1}$ |
|-------|----------------|----------------|----------------|-----------|-------------------------------------|
| 1.    | 266           | 149.9          | 144.2          | 5.7       | 266                                 |
| 2.    | 266           | 154.8          | 144.2          | 10.6      | 266                                 |
| 3.    | 266           | 164.4          | 144.2          | 20.2      | 266                                 |

The analysis now is quite complicated [17] but using the known value of $m_{\tilde{\chi}^0_1}$ from the electron and smuon analysis, the mass of the $\tau_1$ can be gotten at the $1/2\%$ level [20, 21]. However, these analyses assume that $\delta m = m_{\tilde{\tau}_1} - m_{\tilde{\chi}^0_1}$ is $(40 - 50)$ GeV. Such situations don’t apply to mSUGRA, where due to the narrow co-annihilation band (see Figs. 1, 2, 3) one has $\delta m \cong (5 - 15)$ GeV. The final state taus are thus much softer and so harder to identify and we discuss this case next.

In region (2) where $m_{\tilde{e}_R, \tilde{\mu}_R} > 250$ GeV the only slepton that can be pair produced is the $\tilde{\tau}_1$.

This situation occurs for $\tan \beta > 40$, and also for large parts of the parameter space with $\tan \beta < 40$. For $\tan \beta > 40$, the remaining possible signals of mSUGRA are then Eq. (6) and also Eq. (7) can occur for $m_{1/2} \sim 400$ GeV. Thus the signal in both cases are two $\tau^+ \tau^- + p_T$ with acoplanar jets in the final state. As discussed in Sec. 2, the $\tilde{\tau}_1$ pair production has more reach as it extends to higher values of $m_{1/2}$, as can be seen in Figs. 1, 2, 3. The narrowness of the co-annihilation band means that $\delta m$ is now quite small, and the techniques used earlier to detect SUSY signal with taus [17, 18, 19] no longer are applicable. To investigate what mass measurements might be made in this most difficult region of the parameter space, we have examined the three points of Table 1 which span the allowed dark matter band at $m_{1/2} = 360$ GeV, $\tan \beta = 40$, $A_0 = 0$, $\mu > 0$. [a]

We use RH polarization $P(e^-) = -0.9$ to enhance the $\tilde{\tau}_1 \tilde{\tau}_1$ signal, and LH polarization, $P(e^-) = +0.9$ to see the $\tilde{\chi}^0_1 \tilde{\chi}^0_2$ signal. The production cross sections are given in Table 2. One sees that a significant signal exits provided the SM four fermion background can be suppressed. These SM backgrounds fall into two classes: (1) $\bar{\nu} \nu \tau^+ \tau^-$ sates arising from WW, ZZ etc. production, and (2) two photon processes $e^+ e^- \rightarrow \gamma^* \gamma^* + e^+ e^- \rightarrow \tau^+ \tau^-$ (or $q \bar{q}$) + $e^+ + e^-$ where the final state $e^+ e^-$ pair are at a small angle to the beam pipe and the $q \bar{q}$ jets fake a $\tau^+ \tau^-$ pair. The LH polarization cuts are optimized to enhance the $\tilde{\chi}^0_1 \tilde{\chi}^0_2$ signal and the RH to optimize the the $\tilde{\tau}_1 \tilde{\tau}_1$ signal. The cuts chosen were the following:

[a] preliminary analysis of this case has been given in ref. [22].
Table 2. SUSY and SM production cross sections (in fb) for polarizations $P(e^-) = -0.9$ (RH), 0, and +0.9.

| $P(e^-)$ | -0.9 (RH) | 0     | 0.9 (LH) |
|----------|----------|-------|----------|
| SM       | 7.84     | 48.9  | 89.8     |
| SUSY     | 0.53, 26.4 | 3.39, 19.6 | 7.10, 12.8 |
| 1.       | 0.52, 24.4 | 3.31, 18.4 | 6.91, 11.8 |
| 2.       | 0.50, 21.1 | 3.15, 15.8 | 6.62, 10.3 |

(1) 2 $τ$'s: $N_{jet} = 2$ with $E_{jet} > 3$ GeV for $Y_{cut} \geq 0.002523$, each passing the $τ_1$ ID of 1 or 3 tracks and the two jets are oppositely charged ($q_1 q_2 = -1$).

(2) To reduce the WW, ZZ, $Z-γ^*$, etc. background, Acoplanarity $>40^\circ$ along with LH Polarization: $-q_{jet}cos(θ_{jet}) < 0.7$; $-0.8 < cos[θ(j_2, P_{vis})] < 0.7$.

RH Polarization: $|cos(θ_{jet})| < 0.65$; $-0.6 < cos[θ(j_2, P_{vis})] < 0.6$.

(3) To reduce the 2 photon ($γ^*γ^*$) events: Veto EM cluster in $5^\circ < θ < 28^\circ$ with $E > 2$ GeV; Veto electrons within $θ > 28^\circ$ with $p_T > 1.5$ GeV; Veto EM clusters ($e^+/e^-$) in $2^\circ(1^\circ) < θ_{cluster} < 5.8^\circ$ with $E_{cluster} > 100$ GeV using an active beam mask of $2^\circ(1^\circ) - 5.8^\circ$.

(4) We examine $p_T > 5, 10$ or 20 GeV.

The Monte Carlo analysis was done using the following programs: (1) ISAJET 7.63 to generate SUSY events. (2) WPHACT v2.02 pol for SM backgrounds. (3) Tauola v2.6 for tau decay. (4) Events were simulated and analysed with LCD Root Package v3.5 with LD Mar 01 detector parametrization.

The number of events for each class of final states for the case $p_T > 5, 10, 20$ GeV is given in Table 3, and the significance ($N_s/\sqrt{N_B}$) for $p_T > 5, 10, 20$ GeV is shown in Table 4. (For the RH polarization, the $χ^0_1 χ^0_2$ events are treated as background and for the LH polarization the $τ_1 τ_1$ events are treated as background.) One sees that the RH polarization strongly suppresses the WW etc. SM background and the neutralino events, and combined with a $2^\circ$ mask it leaves a clean signal for the $τ_1 τ_1$ events. The LH polarization then allows for the detection of the $χ^0_2 χ^0_2$ signal. With no mask there would be $\sim 20,000$ additional SM background events swamping the SUSY signal. Thus the mask is essential to detect SUSY in this region of parameter space.

From Table 4 one sees that one has a robust discovery significance for both SUSY signals, the $χ^0_1 χ^0_2$ case for all $δm$ and all minimum $p_T$, and the $τ_1 τ_1$ for all $δm$ with minimum $p_T = 5$ GeV. Using the above acceptances we can find the $5σ$ (discovery) reach for each signal. For $p_T > 5$ GeV we find for $δm = 5$ GeV that $m_{χ^0_2} = 286$ GeV (corresponding to $m_{1/2} = 383$ GeV) and for $δm = 20$ GeV that $m_{χ^0_2} = 295$ GeV ($m_{1/2} = 396$ GeV). The $5σ$ reach is
Table 3. Number of events for luminosity of 500 fb\(^{-1}\) for points 1, 2 and 3 with \(\delta m = 5, 10, 20\) GeV.

| Process          | 0.9(L.H.) | -0.9(R.H.) |
|------------------|-----------|-------------|
| \(\tilde{\chi}^0_1\) Pt.1 | \(\tilde{\chi}^0_2\) Pt.1 | \(\tilde{\chi}^0_3\) Pt.1 |
| \(\tilde{\tau}_1\) Pt.1 | \(\tilde{\tau}_1\) Pt.1 | \(\tilde{\tau}_1\) Pt.1 |
| SM weak          |           |             |
| \(e^+e^-\tau^+\tau^-\)  | 2\(^\circ\) - 5.8\(^\circ\) mask | 1\(^\circ\) - 5.8\(^\circ\) mask |
| \(e^+e^-qq\)      | 2\(^\circ\) - 5.8\(^\circ\) mask | 1\(^\circ\) - 5.8\(^\circ\) mask |
|                  | 1745      | 129 |
|                  | 1626      | 123 |
|                  | 1241      | 100 |
|                  | 449       | 210 |
|                  | 5         | 2   |
|                  | 0         | 0   |
|                  | [4]       | [2] |
|                  | [0]       | [0] |
|                  | [0]       | [0] |
|                  | 79        | 38  |
|                  | 1         | 1   |
|                  | 0         | 0   |
|                  | [0]       | [0] |
|                  | [0]       | [0] |
|                  | 935       | 1244 |
|                  | 781       | 1074 |
|                  | 356       | 526  |

Table 4. Significance \((N_S/\sqrt{N_B})\) for a luminosity of 500 fb\(^{-1}\) for SUSY discovery for points 1, 2, and 3.

| Min. \(p_T\) | 5 | 10 | 20 |
|---------------|---|----|----|
| Proc. \(\delta m\) | 1\(^{\circ}\) - 5.8\(^{\circ}\) mask | |
| \(\tilde{\chi}^0_2\) (LH) | 5 | 11.2 | 12.2 |
|                      | 10 | 14.5 | 16.1 | 14.6 |
|                      | 20 | 15.6 | 17.0 | 14.5 |
| \(\tilde{\tau}_1\) (RH) | 5 | 12 | 2.5 |
|                      | 10 | 40 | 40 |
|                      | 20 | 61 | 85.6 |

larger for the \(\tilde{\tau}_1\). We find for \(\delta m = 5\) GeV that \(m_{\tilde{\tau}_1} = 232\) GeV (corresponding to \(m_{1/2} = 514\) GeV ) and for \(\delta m = 20\) GeV that \(m_{\tilde{\tau}_1} = 193\) GeV \((m_{1/2} = 463\) GeV).

3 Conclusions

To examine what mSUGRA signals could be observed at a 500-GeV linear collider, it is necessary to take account of all the existing constraints on the parameter space. Particularly important are the light Higgs mass, \(b \rightarrow s\gamma\) and the dark matter constraints. The former two require \(m_{1/2} \gtrsim 300\) GeV, and the latter for the domain \(m_0, m_{1/2} \lesssim 1\) TeV require \(\delta m = m_{\tilde{\tau}_1} - m_{\chi^0_1} \lesssim\)
15 GeV due to the narrowness of the co-annihilation band. Possible signals at a 500-GeV LC then fall into two types:

1. If $m_{\tilde{e}, \tilde{\mu}} < 250$ GeV, then $\tilde{e}$ and $\tilde{\mu}$ pair production is possible, and the selectron and smuon masses can be measured with very high accuracy. This region covers less and less of the parameter space as $\tan \beta$ increases, and has an upper bound of $\tan \beta = 40$.

2. In general the lightest stau is the lightest slepton, and there is a parameter region where pair production of the $\tilde{\tau}_1$ can occur over the full $\tan \beta$ domain. In addition, the production of $\tilde{\chi}_1^0 + \tilde{\chi}_2^0$ is possible for $m_{1/2} < 400$ GeV. The analysis of the stau pair production is greatly complicated by the smallness of $\delta m$. To examine the difficulties, we have considered here the case where $m_{1/2} = 360$ GeV, $\tan \beta = 40$, $\mu > 0$. Signals of both these processes can be detected for $\delta m > 5$ GeV provided an active mask down to $2^o$ can be constructed to eliminate the $\gamma \gamma$ processes (where final states $e^+e^-\tau^+\tau^-$ (or $q\bar{q}$ faking a $\tau$ pair) with the $e^+$ and $e^-$ at very small angles to the beam pipe) occur. Polarized beams are also needed, RH polarization to suppress SM processes and enhance the $\tilde{\tau}_1\tilde{\tau}_1$ process, and LH to see the $\tilde{\chi}_1^0\tilde{\chi}_2^0$ process. We find that the 5$\sigma$ discovery reach for $\delta m > 5$ GeV is $m_{\tilde{\tau}_1} = 232$ GeV (corresponding to $m_{1/2} = 514$ GeV) and $m_{\tilde{\chi}_2^0} = 286$ GeV (corresponds to $m_{1/2} = 383$ GeV). We are analysing how accurately one might determine the $\tilde{\tau}_1$ and neutralino masses. (For a discussion of how this might be done see 22.)

3. The above signals cover perhaps less than one half of the parameter space assuming the current $g_\mu - 2$ bound maintains, and even less if this bound shrinks. Thus over a large amount of the parameter parameter space, a 500-GeV machine would see no signal of SUSY if mSUGRA (or a SUSY theory like it) were valid. This strongly argues for building an 800-GeV linear collider, where the kinematic regions where SUSY can be seen increases significantly. The discussion of SUSY detection reach for an 800-GeV machine would require, however, a separate analysis of the type discussed here, and detection of the $\tilde{\tau}_1$ pair production in the co-annihilation region would likely require an active mask down to $1^o$.

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