Determination of the State of Rock Mass by the Mathematical Modeling Method

N Gorbunova¹,², O Mirkushov¹

¹Department of mining and oil and gas business, Peoples’ Friendship University of Russia, 6, Miklukho-Maklaya str., Moscow, 117198, Russian Federation
²Department problems of managing the development and conservation of the Earth's interior, Institute of Comprehensive Exploitation of Mineral Resources Russian Academy of Sciences, 4, Krijukovskij tupik str., Moscow, 111020, Russian Federation

E-mail: gorbunova-nn@rudn.ru

Abstract. The article discusses a method for finding the intensity of fracturing of a rock mass by mathematical modeling. To determine the frequency of fracturing, the unit circle direction segment method will be used, and for the concept of extrema, a diagram of fracture frequency in directions is constructed, which is divided into sectors by the directions of the fracture systems. When studying the intensity of fracturing for a rock mass of a section of the "Pshada-Bettinskoe" interfluve, a graph was constructed that is divided into sectors by the directions of the fracture systems. It was established that the intensity minima are at the boundaries of the sectors and are Lmin = 0.13, and the maxima are inside the sectors in the direction of the sums of vectors Lmax = 1.87 describing crack systems. As a result of the work, it was found that the rock mass of the section of the "Pshada-Bettinskoe" interfluve can be attributed to the third category of fracturing - medium-fractured (large-block massif). This model of fracturing of the rock mass will allow us to effectively solve the problems of rock mechanics.

1. Introduction

In assessing the stability of the rock massif, one of the decisive factors that must be taken into account is the structure of the rock massif, characterized by rock fracture, anisotropy, the presence of weakening surfaces and other surfaces. The intensity of the cracks determines the strength characteristics of an array of rocks [1]. In general, when studying the fracture of the array, it is necessary to establish elements of the underlying cracks systems, the intensity of the cracking, the size and shape of elementary structural blocks. The intensity of the crack is a value inversely to the average size of an elementary block of rocks limited by adjacent cracks (at least three) of the most intense systems. The shape and size of the elemental block is determined by the relative location (orientation) of the cracks, which are the indicators necessary to move from the strength of the rocks in the sample to the strength of the array [2, 3]. The effect of the fracture character on the properties of the rock massif is taken into account by the structural weakening factor.

2. Relevance

Taking into account the widespread use of computer technology, to reduce the cost of design and survey work in geomechanics, the use of modern numerical methods, in particular the finite element
method, is becoming increasingly widespread. Years of laboratory and in-kind studies of the strength of rock samples and cracked arrays have shown that the main factors influencing the stability of the array are [4]:

- The degree (intensity) of the fractured rocks of the array;
- Clutch of rocks, determined by laboratory tests of cores;
- The size and shape of elementary structural blocks on which an array of rocks is broken down.

Thus, one of the key factors determining the state of the rock massif is the intensity of fracture, which has an impact on the tense - deformation parameters of rock massifs and therefore requires a comprehensive study. Mathematical modeling is one of the most accurate ways to study fracture [5, 6]. The following scientists made a significant contribution to the solution of this problem: G.L. Fisenko, I.I. Popov, V.V. Sokolovsky, S.S. Golushkevich, Yu.I. Turintsev, A.B. Fadeev, E.L. Galustyan, AM Halperin, AM Mochalov, T.K. Pustovoitova, P.S. Shpakov, V.V. Rzhevsky, M.E. Pevzner, G.G. Poklad, F.K. Nizametdinov and many others.

3. Statement of the problem
The object of research is the stability of rocky rocks in fractured rock massifs. The subject of research is the regularities of the formation of functions of the stress-strain state and destructibility of rocks in the vicinity of the underground development. The aim of the work is to increase the reliability and reliability of predicting the stability of rocks in underground workings in a fractured rock mass.

4. Theoretical part
For mathematical modeling of fracturing, it is enough to consider free planes, which can be moved by parallel transfer to any part of the subsoil space from the region (focus) of the crack development [7]. In this case, the orientation of the plane in space does not change and is uniquely determined by the elements of its occurrence: the azimuth of the strike line (A); the angle of the incidence line (δ), which vary in the intervals $0 \leq A \leq 2\pi$, $0 \leq \delta \leq \pi(2)^{-1}$ [8]. The method of segments of the direction of the single circle will be used to determine the frequency of cracking [9, 10]. To determine the extremes, a diagram of the frequency of cracking is built in directions, which is divided into sectors by the directions of crack systems [11-13]. The Pshada-Bettinskoe inter-river section, located on the Black Sea coast between the villages of Krinita and Archipo-Osipovka (Gelendzky district) was used as a model object. The data for the task was by measuring cracks within randomly selected arrays. On each of the two characteristic outcrops selected retinue were measured [14]:
1) the orientation of the cracks (azimuth of the fall and the angle of the fall of the $\beta$);
2) The true distance between the cracks l;
3) The length of the cracks a.

According we will build a polar diagram of the orientation of ruptured disturbances, which will show an approximate picture of the spread of cracks to get a more complete picture of the cracking use the method of mathematical modeling ‘Figure 1’.

Figure 1. Polar diagram of the ripping disturbances of the studied mass of rocks of the Pshada-Bettinskoe inter-river 2D.
Let n crack systems are developed within the randomly selected array of rocks of the Pchadsky-Bettinsky inter-river area, in each of which is made by measurements, then the vector of the i-system of cracks is determined by the expression of the [14]:

\[
\overrightarrow{\omega}_i = \frac{[\omega]}{N_i} \quad n_i = \frac{[\omega]}{N_i} \quad (([a]_i ; [b]_i ; [c]_i) \quad \overrightarrow{\omega}_i = \frac{([a]_i^2 + [b]_i^2 + [c]_i^2)^{1/2}}, i = 1,2,...,n
\]

(1)

In order to assess the validity of the results, each of the meanings of the expression (1) determined the error of Gauss designation:

\[
[a]_i = \sum_{j=1}^{N_i} \omega_i^{(j)} [a]_i = -\sum_{j=1}^{N_i} \sin \alpha_i^{(j)} \sin \beta_i^{(j)}
\]

(2)

\[
[b] = \sum_{j=1}^{N_i} \cos \alpha_i^{(j)} \sin \beta_i^{(j)} [c]_i = \sum_{j=1}^{N_i} \cos \beta_i^{(j)}
\]

(3)

where, \(\alpha_i^{(j)}, \beta_i^{(j)}, \omega_i^{(j)}\), j-measurement respectively the azimuth of the stretch, the angle of the fall and the frequency of the i-system of cracks; Ni is the number of measurements in the i.e. crack system. In the mass of rocks of the Pshada-Bettinskoe section there are three main crack systems for which Gauss's errors on the azimuth of the stretch and the angle of the fall (table 1) are calculated.

| Strike azimuth | Angle of incidence | Frequency of the i-th fracture system | Average crack spacing |
|----------------|-------------------|--------------------------------------|----------------------|
| [a] 1          | 1.89              | 4.34                                 | [\omega] 1          |
| [b] 1          | 1.89              |                                      | 3.56                |
| [c] 1          |                   |                                      |                     |
| [a] 2          | -1.14             | 2.44                                 | [\omega] 2          |
| [b] 2          | -3.42             |                                      | 1.57                |
| [c] 2          |                   |                                      |                     |
| [a] 3          | 1.51              | 4.49                                 | [\omega] 3          |
| [b] 3          | -2.75             |                                      | 1.29                |
| [c] 3          |                   |                                      |                     |

Table 1. Gaussian errors in strike azimuth and dip mm/m.

Within the block model of the cracked array of rocks, the orientation in the plan chosen at the field coordinate system of the OXY will depend on the anisotropy of the fractured mass of rocks, such direction is the direction of the projection of the vectors of crack systems (1) on the plane of OXY. The direction in the Cartesian system of coordinates OXY express a single vector of guide cosinus, which is clearly determined by the angle \(\alpha = (\varepsilon, \Omega)\) between it and the axis Ox.

\[
\varepsilon(\alpha) = (\cos(\alpha); \cos(\pi(2)^{-1} - \alpha) = (\cos(\alpha); \sin(\alpha))
\]

(4)

Based on the studies [15], it has been established that the intensity of fissability in the OXY plane, caused by n crack systems, is the sum of vector projection modules \(\overrightarrow{\omega}\) (1) per direction \(\varepsilon(\alpha)\) (4) [16]:

\[
L(\alpha) = \sum_{i=1}^{n} |\overrightarrow{\omega}_i| \varepsilon(\alpha)| = \sum_{i=1}^{n} |[\omega]_i |[a]_i \cos \alpha + [b]_i \sin \alpha | N_i([a]_i^2 + [b]_i^2 + [c]_i^2)^{1/2}
\]

(5)

We will find extremes of the intensity of fission for the mass of rocks of the Pshada-Bettinskoe inter-river area (6, 7) (table 3).

\[
a_i = \frac{[\omega]_i |[a]_i |}{N_i([a]_i^2 + [b]_i^2 + [c]_i^2)^{1/2}}
\]

(6)

\[
b_i = \frac{[\omega]_i |[b]_i |}{N_i([a]_i^2 + [b]_i^2 + [c]_i^2)^{1/2}}
\]

(7)
were \( x = \cos \alpha \), \( y = \sin \alpha \), \( i = 1, 2, \ldots, n \).

For the range of rocks under study, we use equality (1) and (5) get the cracking intensity for the three crack systems in the plane OXY (table 2).

\[
L(\alpha) = L(x, y) = \sum_{i=1}^{n} |a_i x + b_i y|
\]  

(8)

**Table 2.** Fracture intensity for three fracture systems in the OXY plane.

| Intensity of cracking mm/m | Extremes of the intensity of fracture | The coordinate of crack systems |
|---------------------------|--------------------------------------|--------------------------------|
| L[a] 0,66                | a1 1,32 a2 -0,41 a3 0,36            | X1 -0,912 Y1 0,411 |
| L[a] 0,13                | a2 -0,41 b2 -1,24                   | X2 -0,912 Y2 0,411 |
| L[a] 0,59                | a3 0,36 b3 -0,65                    | X3 -0,912 Y3 0,411 |

Minimum values are the intensity of fracture (5), (8) are taken at the boundaries of the \( L(\alpha) \) sectors. The intensity of \( L(x, y) \) (8) the cracking is the same in symmetrical-opposite directions of the \( i \) and \( n+i \) sectors, \( i = 1, 2, \ldots, n \) this is due to the fact that the coordinates of the point O of the arcs of the single circle are symmetrical relative to the beginning. Therefore, it is enough to investigate the \( i \)-th sectors of the directions, in which we introduce a function that determines the sign of the expression \( a_i x + b_i y \):

\[
\text{sign}(a_i x + b_i y) = \begin{cases} 
1, & \text{if } a_i x + b_i y > 0 \\
-1, & \text{if } a_i x + b_i y < 0 \\
0, & \text{if } a_i x + b_i y = 0 
\end{cases}
\]  

(9)

Let us transform the fracture intensity function \( L(x, y) \) (8) omitting the modulus signs, we obtain expression (10), which can also be represented as a linear function (11):

\[
L_i(x, y) = \sum_{i=1}^{n} (a_i x + b_i y) \text{sign}(a_i x + b_i y)
\]  

(10)

\[
L_i(x, y) = [\beta] x + [\gamma] y \Rightarrow [\beta] = \sum_{i=1}^{n} a_i \text{sign}(a_i x + b_i y), [\gamma] = \sum_{i=1}^{n} b_i \text{sign}(a_i x + b_i y)
\]  

(11)

Using linear function (11) we will determine the variables from the "a" and the "b" intensity of the fissures of the studied array of rocks of the Pshada-Bettinskoe region (table 3).

**Table 3.** Fracturing of the investigated rock mass mm/m.

| Variable from the "a" intensity of fracture | Variable from the "b" intensity of fracture |
|-------------------------------------------|-------------------------------------------|
| \([\beta]\)1 | 1,32 | \([\gamma]\)1 | 1,32 |
| \([\beta]\)2 | -0,41 | \([\gamma]\)2 | -1,23 |
| \([\beta]\)3 | 0,36 | \([\gamma]\)3 | -0,65 |

Geometrically, according to the horizontal section method [17-19, 21], the intensity of \( L(x, y) \) (11) for the \( i \) sector in question can be presented as a system of parallel straights in the plane of OXY, on each of which the function has a permanent value of Q. We accept that \( Q = 0 \), get a general equation straight and normal \( \vec{N} \) to it (12) which passes through the beginning of the coordinates of point O. When increasing \( q \) the straight moves in the direction of the maximum increase in the intensity of the fracture \( L(x, y) \) (11), that is in the direction of its gradient:

\[
[\beta] x + [\gamma] y = 0, \vec{N} = ([\beta], [\gamma])
\]  

(12)
\[
\text{gradL}(x, y) = \left( \frac{\partial L}{\partial x}, \frac{\partial L}{\partial y} \right) = (|\beta|, |\gamma|)
\]

(13)

Comparing expressions (12) and (13), we set a match for the gradient of the intensity of fracture (13) with the normal straight (12). According to the theory of mathematical modeling [20], the point of detachment of the direct level (12) from the arc of the \(i\)-sector will be the point of the maximum, which is the point of crossing the straight, passing through the beginning of the coordinates in the direction of the gradient of the intensity of fissivity (14), (15), with a single circle, move to the parametric equations and get two solutions of the system:

\[
(x_{11}^i; y_{11}^i) = (|\beta|, |\gamma|)(|\beta|, |\gamma|)^{-1} (14)
\]

\[
(x_{22}^i; y_{22}^i) = (-|\beta|, -|\gamma|)(|\beta|, |\gamma|)^{-1} (15)
\]

the first (14) of which may belong to the \(i\)-sector in question, and the second (15) to the opposite \((n + i)\) sector. In this case, substituting the first solution \((x_{11}^i; y_{11}^i)\) into function (11), we obtain the maximum value of fracture intensity (11), (8) in the \(i\)-th sector of directions:

\[
\max L_i(x_{11}^i; y_{11}^i) = (|\beta|^2, |\gamma|^2)^{1/2} = |\text{gradL}(x_{11}^i; y_{11}^i)|
\]

(16)

According to (table 3) and expressions (8) we get a function in the OXY coordinate system:

\[
L(x, y) = L(x, y) = |1.32x + 1.32y - 0.41x - 1.23y| + |0.36x - 0.65y|
\]

(17)

To remove modules in function (17) equate expressions under module signs to zero and get 3 systems of equations corresponding to 3 crack systems:

\[
\begin{align*}
&i = 1, (x_{11}^i; y_{11}^i) \
&i = n, x = a_{i(n)}t \
&y = b_{i(n)}t \
&i = 2, (x_{21}^i; y_{21}^i) \
&i = 3, (x_{31}^i; y_{31}^i)
\end{align*}
\]

(18)

5. Experimental results

Using the values obtained in the system of coordinates OXY of the study array of rocks of the Pshada-Bettinskoe inter-river, we will build a vector on coordinates X and Y, which pass through the beginning of the coordinates in the direction of the gradient of the intensity of fissivity. Continue these vectors in reverse directions to get the system of sectors ‘Figure 2’.

![Figure 2](image)

Figure 2. Graph of the intensity of the fissures of the studied mass of rocks of the Pshada-Bettinskoe inter-river area 2D.

Solutions break down a single circle ‘figure 2’ into three pairs of symmetrical arcs, on which the sectors of the single circle, setting directions in the plane, are based. Since the intensity of cracking in
mutually opposite directions coincides, we will find the directions of three adjacent asymmetrical sectors characterized by border single vectors. Therefore, in the expression (17) we lower the module and get the intensity of the cracking in the direction of the \( i \)-sector \( L(x, y) = 1.32x + 1.32y + 0.41x + 1.23y - 0.36x + 0.65y \). In the same way, we get a single vector of the direction of the maximum value in the \( IV \) sector \( \epsilon_{\text{max}}^{(i)} = (0.706; 0.708) \). According to formula (16), the maximum value in this direction is \( \max L_i(x_i^m; y_i^m) = 1.87 \). Similarly, we define minimum values at sector boundaries (19):

\[
\begin{align*}
\min L_1 &= 0.66 \\
\min L_2 &= 0.13 \\
\min L_3 &= 0.59
\end{align*}
\]

According to the data received, we are building a graph of the intensity of the fissures of the rock mass of the Pshada-Bettinskoe region 3D ‘Figure 3’.

\[ \text{Figure 3. Graph of the intensity of the fissures of the studied mass of rocks of the Pshada-Bettinskoe region 3D} \]

6. Conclusion

The intensity of fractures found by the method is a necessary element of almost all studies related to determining the stability of the rock massif. In studying the intensity of the cracking for the mass of rocks of the Pshada-Bettinskoe section, a graph was built, which is divided into sectors by the directions of crack systems. It has been established that the minimums of intensity are at the boundaries of the sectors and amount to \( L_{\text{min}} = 0.13 \), and the highs - within the sectors in the directions of the amounts of vectors \( L_{\text{nb}} = 1.87 \) describing the cracks systems. Such images of the mass of rocks of the Pshada-Bettinskoe region can be attributed to the third category of fracture - the medium-cracking (large-scale array). The presented methodology allows the creation of predictive models of cracked arrays, especially with a limited amount of data typical of Russian conditions, which was previously impossible using traditional methods. The use of modern recording and seismic technologies can significantly improve the quality of simulation of cracked arrays. Using this method, it is possible to quantify the fracture and create three-dimensional geological models that can form the basis of a realistic model of the deposit.

7. References

[1] Li W, Sun W, Yan T, Li Y, Ji Z and Tang P 2017 Tianranqi Gongye B 37 22-27
[2] Khalkechev R and Khalkechev K 2017 GIAB B 11 220-226
[3] Holder J, Olsen J and Philip Z 2001 Geophys. Res. Lett. B 28 4 599-602
[4] Kazikaev D, Kozyrev A, Kasparyan E and Iofis M 2016 Management of Geomechanical Processes in Mineral Development: Training Manual (Moscow: Mountain Book Publishing House)
[5] Jun Q, Liang X, Wang G, Xian C, Zhao C and Wang L 2017 Unconventional Resources Technology Conf. vol 33 (Austin: Texas/USA) pp 2626-2637
[6] Kochurov A, Radzhabova L and Borodkin P 2018 GIAB B 4 21-28
[7] Khalkechev K 2016 GIAB B 8 190-194
[8] Redkin G 2005 Izvestia of universities. North Caucasus region B 4 79-82
[9] Sidelnik A 2017 Procedia Structural Integrity B 6 316-321
[10] Downie R, Kronenberger E and Maxwell S 2010 SPE Annual Tech. Conf. and Exhibition (Florence: Italy) p 13
[11] Wang L, Wei J, Di B, Huang P and Zhang F 2018 Applied Geophysics B 15 2 240-252
[12] Khalkechev K 2019 UGOL B 10 1123
[13] Krivosheev I and Shamurina A 2013 Russian Journal of Nondestructive Testing B 9 62-67
[14] Belikov B 1953 About the method of studying crack tectonics deposits of construction and cladding stone (Moscow: Nauka Publishers)
[15] Semenov A 2014 Geology, geography and global energy B 1 60-70
[16] Takranov R 1964 On determining the intensity of fractures of hard-dispersed rocks (Saint Petersburg: State Research Institute of Mining Geomechanics and Mine Surveying)
[17] Barton N 1972 Int. J. Rock Mech. Min. Sci. vol. 9 (Oslo: Norway) pp 579-582
[18] Barton C, Castillo D and Moos D 1998 APPEA Journal B 38 1 466-487
[19] Choi M, Pyrak-Nolte L and Bobet A 2013 47th US Rock Mechanics/Geomechanics Symp. (San Francisco: California) p 405
[20] Brusentsev A, Petrashev V and Ryazanov J Study of Operations and Game Theory: Training (Belgorod: Belgorod State Technological University) p 258
[21] Tkach E, Soloviev V, Temirkhanov R, Solovev D B 2020 The Study of Cement Concrete with Improved Properties Based on the Use of Activated Silica Fume Materials Science Forum 992 228-232. [Online]. Available: https://doi.org/10.4028/www.scientific.net/MSF.992.228