A novel experimental method for determining the eigen-axes angle of MEMS ring gyroscope

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Abstract. MEMS ring gyroscope with sensitive structural errors has frequency splitting in the working modes and the eigen-axes are no longer located at any arbitrary position. Among the imperfection, the fourth harmonic distribution components of the mass and Young’s modulus have the most significant influence on the frequency split. In order to determine the position angle of the defects, the frequency response characteristics at the sense electrodes of the ring structure were analysed, and a calculate method to ascertain the eigen-axis position was found by using the frequency response parameters. To verify the validity of this method, a finite element model was established, and the harmonic response was analysed. The simulation results show that the relative error is about 8%. This method was applied in a multi-ring eigen-axes experiment, the harmonic response under different drive and sense electrodes combinations were tested. The average of the calculated angle is 43.3050°, and the standard deviation is 0.8940° which shows the consistency of the calculated values. This method provides a support for the modification of ring structure errors.

1. Introduction
MEMS ring vibration gyroscope, as a MEMS device, can measure the angular rate by Coriolis Effect. With the help of the micromachining process, it has many advantages such as small size, easy to integrate with the circuit and low cost. Compared with the gyroscopes with sensitive beams or proof mass, the structure of ring, especially the multi-ring gyroscope has a high degree of symmetry and larger probability of high performance. Therefore, MEMS ring gyroscope is more widely used in fields with high requirements such as guidance of missiles and platform stabilization. [1-2].

The properties of ring MEMS gyroscope are closely related to the structure symmetry. In ideal condition, the resonant mass, stiffness and damping of the rings are consistency, therefore, it is easy to achieve the tuning of the vibration frequency between the drive and sense modes. With a good structure design to reduce the thermal elastic damping, and a perfect machining process to achieve a high quality factor under the condition of vacuum encapsulation, the ring gyroscope can achieve high angular rate sensitivity [3-5].

However, the actual MEMS processing technology cannot guarantee the perfect symmetry of the ring structure. The main sources of the structure errors are the inhomogeneity of structural thickness and beam width and so on. The existence of these defects break the perfect symmetry of the structure, leading to frequency splitting. And the eigen-axes are no longer the arbitrary position in the circumference but related to the position of the defects. Among the structural parameter inhomogeneity, the fourth harmonic of Fourier expansion has the most significant influence on the harmonic oscillator [6-8]. This paper proposed a method to calculate the angular position by using the
frequency response information, which can provide a support for the follow-up mechanical or electrostatic modification of the structure.

2. Theoretical analysis

MEMS ring gyroscope works with two identical elliptically-shaped flexural modes that 45° apart from each other (Figure 1) [6, 9-10]. The ring gyro analyzed in this paper is composed of multi-ring, center supporting anchor and isolator rings, the rings are connected by 8 evenly distributed connecting beams, and the connecting beams between different rings are staggered. Outside of the rings are 16 electrodes evenly distributed around the circumference, (Figure 2). These electrodes can be used as drive or sense electrodes and the corresponding modes are called drive mode and sense mode respectively. Capacitance is formed between those electrodes and the ring structure. When the angular rate is employed in the plane perpendicular to the vibration, the sense mode is excited by Coriolis force. The change of capacitance can be detected by sense electrode, and the angular velocity information can be obtained by the post signal processing.

Figure 1. Working modes and define of eigen-axis angle $\varphi_0$.

Figure 2. The schematic structure of the multi-ring gyroscope.

The displacement in the vibration work modes can be expressed as [6]:

$$u = 2q_1(t)\cos 2\varphi + 2q_2(t)\sin 2\varphi$$
$$v = -q_1(t)\sin 2\varphi + q_2(t)\cos 2\varphi$$

(1)

where $q_1(t)$, $q_2(t)$ are the displacement in time domain of two work modes respectively, and $\varphi$ is the angle in the space domain. Equation (1) implies that the mode equation in space domain is the combination of $\sin 2\varphi$ and $\cos 2\varphi$.

Both the drive and the sense mode of ring vibration gyro are mass-spring-damper systems, and the transfer functions of the two modes are [6, 9]:

$$G_1(s) = \frac{1}{m_1 s^2 + 2\zeta_1 \omega_1 s + \omega_1^2}$$
$$G_2(s) = \frac{1}{m_2 s^2 + 2\zeta_2 \omega_2 s + \omega_2^2}$$

(2)

Where $m_1$, $m_2$ are the resonant mass; $\omega_1$, $\omega_2$ are the natural angular frequency and $\zeta_1$, $\zeta_2$ are the damping ratio, respectively.
In reality, the ring gyroscope has frequency split, and the eigen-axes are no longer located at arbitrary position but some fixed angle. The primary eigen-axis angle is defined as $\varphi_0$ in Figure 1, and the secondary eigen-axis is $\varphi_0 \pm 45^\circ$. The mode equation modified to the combination of $\sin(-2\varphi_0 + 2\varphi)$, $\cos(-2\varphi_0 + 2\varphi)$. Suppose the amplitude of the harmonic force applied on the drive mode is unit 1, and the force can be decomposed into the distributed force $N_1$ and $N_2$ acting on the two vibration modes [6].

$$\left\{ \begin{array}{l}
N_1 = \cos 2\varphi_0 \\
N_2 = \sin 2\varphi_0
\end{array} \right. \quad (3)$$

At arbitrary detection position on the circumference, the detection amplitude is the result of the superposition of two vibration components [6].

$$y(s) = N_1 \cos(-2\varphi_0 + 2\varphi)G_1(s) + N_2 \sin(-2\varphi_0 + 2\varphi)G_2(s) \quad (4)$$

If we motivate the ring structure in $0^\circ$ position with the first angular frequency $\omega_1$, in theory, the two modes will be both motivated if the motivate position and the eigen-axes are misaligned. But with a high Q value and a higher frequency split, the displacement caused by the other mode is small enough to be ignored compared to the first one. Then the amplitude of the sense mode can be simplified as follows.

At $0^\circ$ sense electrode, the frequency response $y_{11}(j\omega)$ can be expressed as:

$$y_{11}(j\omega) = \cos(2\varphi_0) \frac{1}{m_1} \frac{1}{-\omega^2 + 2\zeta_1\omega_1\omega j + \omega_1^2} \cos(-2\varphi_0) \quad (5)$$

$L_{11}$ represent the amplitude at $\omega_1$: 

$$L_{11} = \left| y_{11}(j\omega) \right| = \left| \cos(2\varphi_0) \frac{1}{m_1} \frac{1}{2\zeta_1\omega_1} \cos(-2\varphi_0) \right| \quad (6)$$

At $22.5^\circ$ sense electrode: 

$$y_{12}(j\omega) = \cos(2\varphi_0) \frac{1}{m_1} \frac{1}{-\omega^2 + 2\zeta_1\omega_1\omega j + \omega_1^2} \cos(-2\varphi_0 + 2 \times 22.5^\circ) \quad (7)$$

$L_{12}$ represent the amplitude at $\omega_1$: 

$$L_{12} = \left| y_{12}(j\omega) \right| = \left| \cos(2\varphi_0) \frac{1}{m_1} \frac{1}{2\zeta_1\omega_1} \cos(-2\varphi_0 + 2 \times 22.5^\circ) \right| \quad (8)$$

Then use formula (6) divides (8): 

$$\frac{L_{11}}{L_{12}} = \frac{\cos(2\varphi_0)}{\cos(-2\varphi_0 + 2 \times 22.5^\circ)} \quad (9)$$

Simplify the result as

$$\tan 2\varphi_0 = \pm \sqrt{2} \frac{L_{12}}{L_{11}} \quad (10)$$

This ratio method means that the drive force has no effect on the calculation of $\varphi_0$ results. Similarly, motivate the ring with angular frequency $\omega_1$, the displacement caused by the first working mode can be ignored.

At $0^\circ$ sense electrode: 

$$y_{21}(j\omega) = \sin(-2\varphi_0) \frac{1}{m_2} \frac{1}{-\omega^2 + 2\zeta_2\omega_2\omega j + \omega_2^2} \sin(-2\varphi_0) \quad (11)$$

$L_{21}$ represent the amplitude at $\omega_1$: 

$$L_{21} = \left| y_{21}(j\omega) \right| = \left| \sin(-2\varphi_0) \frac{1}{m_2} \frac{1}{2\zeta_2\omega_2} \sin(-2\varphi_0) \right| \quad (12)$$

In $22.5^\circ$ sense electrode: 

$$y_{22}(j\omega) = \sin(-2\varphi_0) \frac{1}{m_2} \frac{1}{-\omega^2 + 2\zeta_2\omega_2\omega j + \omega_2^2} \sin(-2\varphi_0 + 2 \times 22.5^\circ) \quad (13)$$

$L_{22}$ is the amplitude at $\omega_1$: 

$$L_{22} = \left| y_{22}(j\omega) \right| = \left| \sin(-2\varphi_0) \frac{1}{m_2} \frac{1}{2\zeta_2\omega_2} \sin(-2\varphi_0 + 2 \times 22.5^\circ) \right| \quad (14)$$
Similarly,
\[
\frac{L_{21}}{L_{22}} = \frac{\sin(-2\varphi_0)}{\sin(-2\varphi_0 + 2\times22.5^\circ)}
\]  
(15)
Simplified as
\[
\cot 2\varphi_0 = \pm \sqrt{2} \frac{L_{22}}{L_{21}} + 1
\]  
(16)

From formula (10) and (16), we can obtain two values of \(\varphi_0\) since we used the absolute values of the amplitude. With the aid of the phase information, we can identify the real motion direction of the detection district at the certain frequency. Furtherly, we can describe the vibration mode and weed out the fake one. Detailed analysis is available in 4.2.

3. Simulation verification
To verify the calculated method, we established a ring gyroscope model (see Figure 3(a)) in ANSYS [10-11]. The ring has a radius of 2970um, the ring width is 60um with the thickness of 80um. In the ring structure, we dug out four identical defects evenly distributed on the circumference to represent the fourth harmonic simply. The model ring’s two natural frequencies are \(f_1 = 6800.7\text{Hz}\) and \(f_2 = 6896.2\text{Hz}\). Set \(\varphi_0\) to 8°, the harmonic frequency response at 0° sense electrode are showed in Figure 3(b). From the curve, the amplitude and phase at \(f_1\) can be obtained. Similarly, we can get the amplitude and phase curves at 22.5° sense electrode, then we can calculate \(\varphi_0\) at \(f_1\) by using formula (10). Simulated at other different \(\varphi_0\) values such as 10°, 11.25°, 15° and 20°, and dealt with the same method. The comparison between the calculated value and the simulated setting values are shown in Table 1.

![Figure 3](image)

Figure 3. (a) Model structural diagram and (b).frequency response in 0° drive and 0° sense.

Table 1 showed the amplitude in 0° and 22.5° sense electrode with different \(\varphi_0\) and the calculated values compared with the simulation values.

The results in Table 1 imply that the proposed method can be employed to effectively calculate the eigen-axis angle, and the relative error between the calculated value and value in simulation model is within 8%. The relative error contains the errors of ignoring the second mode of vibration and the finite element simulation errors. E.g., the accuracy of the amplitude and frequency will be affected by different meshing settings.
Table 1. The amplitude of the model and calculated values.

| Set defects angle | 0° sense electrode | 22.5° sense electrode | Calculate value | Relative error |
|-------------------|--------------------|-----------------------|-----------------|----------------|
| $\phi_0 / ^\circ$ | $L_1 / \text{um}$  | $L_2 / \text{um}$     | $\phi_0 / ^\circ$ | $\%$           |
| 8°                | 7.1363             | 6.6141                | 8.6305          | 7.88           |
| 10°               | 6.7763             | 6.6548                | 10.627          | 6.25           |
| 11.25°            | 6.5372             | 6.6567                | 11.873          | 5.57           |
| 15°               | 5.6271             | 6.4378                | 15.857          | 5.72           |
| 20°               | 4.2288             | 5.7578                | 21.392          | 6.96           |

4. Experiment

4.1. Frequency response of a ring gyroscope
The device used in experiment is showed in Figure 4. Figure 5 shows the the frequency response curves that drive in 0° and sense in 0°, the natural frequencies of the ring structure are 10640.74 Hz and 10791.82 Hz, the frequency split is 151.12Hz, and the quality factors are 39941 and 47964 respectively.

Figure 4. Diagram of experiment device.

Figure 5. Natural frequency of the ring gyroscope.
We marked 0° and 22.5° electrodes as A and B. And use AA to represent drive in 0° and sense in 0°. Then exchange the drive and sense electrodes, we can get different frequency response curves. Obtaining the amplitude and phase information at natural frequency from the experiment data, we calculated the possible $\phi_0$ values using formula (10) and (16). The results are showed in Table 2.

| Drive /sense | Amplitude /dB | Phase/° | Calculate $\phi_0$/° | Amplitude /dB | Phase/° | Calculate $\phi_0$/° |
|--------------|---------------|---------|-----------------------|---------------|---------|-----------------------|
| AA           | 9.53          | 61.15   | 43.15                 | -43.36        | 35.32   | -103.34               |
| AB           | 11.80         | -103.80 | 43.44                 | -43.25        | 31.53   | -103.97               |
| BA           | 1.03          | 69.09   | 43.56                 | -43.69        | 32.38   | -102.99               |
| BB           | 24.40         | -97.09  | 43.76                 | -43.649       | 28.78   | -106.29               |

4.2. Mutual validation of valid values

Continue with the calculated results above, we use the phase information to figure out the fake angle by the following analysis process. According to the experiment system, the positive phase means that the vibration at the detection position is in the contraction state, and the negative phase means that the vibration structure is in the stretching state. Then, by using phase information in Table 2, we can simply describe the shape of the deformed ring when the driving force is at the phase of 0°, as showed in Figure 5. Take AA and AB at $f_1$ for example (Figure 5 (a)), 61.15° means the vibration is in contraction state as the red arrow A shows and -103.80° means the stretching state shown as the red arrow B. Combined with the elliptic mode, the primary eigen-axis angle should be 43.15°.

**Figure 6.** The vibration modal in $f_1$ (a) and $f_2$ (b). The blue arrows represent the drive force, the red arrows represent the sense displacement.
Figure 6 shows that the primary eigen-axis angle should be greater than 0°, therefore, we can sort out the eligible values from Table 2. The average value of these efficient ones showed in Table 3 is 43.3050°, and the standard deviation is 0.8940° which shows the consistency of the values.

Table 3. The eligible values of $\varphi_0$

| No. | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| $\varphi_0$ /° | 43.15 | 43.44 | 43.56 | 43.76 | 42.55 | 44.95 | 43.13 | 41.90 |

5. Conclusions

By analysing the parameters of the frequency response of the ring gyroscope, the eigen-axes angle can be calculated by using equation (10) and (16). The eigen-axes calculation method proposed in this paper can be used to ascertain the location of the defects in the structure and provide support for the modification of the structural errors by femtosecond laser processing or electro-static method to improve the symmetry of the structure.

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