OBSERVATIONAL CONSTRAINTS ON COSMOLOGY FROM THE MODIFIED FRIEDMANN EQUATION

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Received 2003 August 28; accepted 2003 November 24

ABSTRACT

Recent measurements of Type Ia supernovae, as well as other concordant observations, suggest that the expansion of our universe is accelerating. A dark energy component has usually been invoked as the most feasible mechanism for the acceleration. However, effects arising from possible extra dimensions can mimic dark energy through a modified Friedmann equation. In this work, we investigate some observational constraints on a scenario in which this modification is given by \( H^2 = (8\pi G/3)(\rho + C\rho^n) \). We mainly focus our attention on the constraints from recent measurements of the dimensionless coordinate distances to Type Ia supernovae and Fanaroff-Riley Type Iib radio galaxies compiled by Allen et al. We obtain the confidence region on the power index \( n \) and the density parameter \( \Omega_m \) of the universe from a combined analysis of these databases. We find that \( n = 0.06^{+0.22}_{-0.15} \) and \( \Omega_m = 0.30^{+0.02}_{-0.02} \), at the 95.4% confidence level, which is consistent within the errors with the standard ΛCDM model. These parameter ranges give a universe whose expansion switches from deceleration to acceleration at a redshift between 0.52 to 0.73.

Subject headings: cosmological parameters — cosmology: theory — distance scale — supernovae: general — X-rays: galaxies: clusters

1 INTRODUCTION

The Hubble expansion, the cosmic microwave background radiation (CMBR), primordial big bang nucleosynthesis, and structure formation are the four pillars of the standard big bang cosmology. In recent years, it seems that all these cornerstones have combined to point out that the expansion of the universe is speeding up rather than slowing down (for a recent review, see Peebles & Ratra 2003). The main evidence comes from the recent well-known distance measurements of some distant Type Ia supernovae (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2001). Possible explanations for such an acceleration include: a cosmological constant \( \Lambda \) (Weinberg 1988; Carroll, Press, & Turner 1992; Krauss & Turner 1995; Ostriker & Steinhardt 1995; Chiba & Yoshii 1999), a decaying vacuum energy density or a time varying \( \Lambda \) term (Ozer & Taha 1987; Vishwakarma 2001; Alcaniz & Maia 2003; Jain, Dev, & Alcaniz 2003), an evolving scalar field (referred to by some as quintessence; Ratra & Peebles 1988; Caldwell, Dave, & Steinhardt 1998; Wang & Lovelace 2001; Li, Hao, & Liu 2002; Weller & Albrech 2002; Li et al. 2002a, 2002b; Chen & Ratra 2003; Mukherjee et al. 2003), a phantom energy for which the sum of the pressure and energy density is negative (Caldwell 2002; Hao & Li 2003a, 2003b; Dabrowski, Stochowiak, & Szdylofki 2003), the so-called “X-matter” (Turner & White 1997; Zhu 1998, 2000; Waga & Miceli 1999; Podariu & Ratra 2001; Zhu, Fujimoto, & Tatsumi 2001; Sereno 2002; Alcaniz, Lima, & Cunha 2003; Lima, Cunha, & Alcaniz 2003), and the Chaplygin gas (Kamenshchik, Moschella, & Pasquier 2001; Bento, Bertolami, & Sen 2002; Alam et al. 2003; Alcaniz, Jain, & Dev 2003; Dev, Alcaniz, & Jain 2003b; Silva & Bertolami 2003; Makler, de Oliveira, & Waga 2003). All the above mechanisms for accelerating are obtained by introducing a new hypothetical energy component with negative pressure—the dark energy.

On the other hand, many models have appeared that make use of branes and extra dimensions to obtain an accelerating universe (Randall & Sundrum 1999a, 1999b; Deffayet, Dvali, & Gabadadze 2002; Avelino & Martins 2002; Alcaniz, Jain, & Dev 2002; Jain, Dev, & Alcaniz 2002). The basic idea behind these braneworld cosmologies is that our observable universe might be a surface or a brane embedded in a higher dimensional bulk spacetime in which gravity could spread (Randall 2002). The bulk gravity sees its own curvature term on the brane, which accelerates the universe without dark energy. Here we are concerned with the cosmological model from the modified Friedmann equation

\[
H^2 = \frac{8\pi G}{3} (\rho + C\rho^n). \tag{1}
\]

Freese & Lewis (2002) showed the above term proportional to \( \rho^n \) (dubbed the Cardassian term by the authors) may arise as a consequence of embedding our observable universe as a (3+1)-dimensional brane in extra dimensions, although an elegant and unique five-dimensional energy-momentum tensor \( T_{\mu\nu} \) that gives rise to equation (1) has not yet been found. If \( n < 1 \), the new term dominates at late times, which implies a modification of gravity at very low-energy scales. Particularly, if \( n < \frac{2}{3} \), it gives rise to a positive acceleration. Note that in this scenario, although the universe is flat and accelerating, it contains matter (and radiation) only, without any dark energy contribution.

The main goal of this paper is to set observational constraints on this Cardassian expansion model and check whether it is consistent with current cosmological data. We perform a combined analysis of data including the dimensionless coordinate distances to type Ia supernovae (SNe Ia) and Fanaroff-Riley type Iib (FR IIb) radio galaxies compiled by Daly & Djorgovski (2003) and the X-ray gas mass fraction

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in clusters of galaxies published by Allen, Schmidt, & Fabian (2002) and Allen et al. (2003). These results are timely and complementary to the previous constraints from the angular size of high-z compact radio sources (Zhu & Fujimoto 2002), CMBR (Sen & Sen 2003a, 2003b), the SNe Ia database (Zhu & Fujimoto 2003; Wang et al. 2003; Cao 2003; Szydlowski & Czaja 2003; Godlowski & Szydlowski 2003), large-scale structures (Multamaki, Gaztanaga, & Manera 2003), and from optical gravitational lensing surveys (Dev, Alcaniz, & Jain 2003a).

The plan of the paper is as follows. In § 2, we provide a brief summary of the Cardassian expansion scenario relevant to our work. Constraints from dimensionless coordinate distance data of SNe Ia and FR IIb radio galaxies are discussed in § 3. In § 4 we discuss the bounds imposed by the X-ray gas mass fraction in galaxy clusters. Finally, we present a combined analysis, our concluding remarks, and discussion in § 5.

2. THE CARDASSIAN MODEL: BASIC EQUATIONS

In the modified Friedmann equation (1), there are two model parameters: the power index $n$ and the coefficient $C$ of the Cardassian term. Instead of $H(z)$, we have $H(z) = \frac{dz}{dz} = \frac{1}{1 + z}$. If we further ignore the radiation components in the universe that are not important for the cosmological tests considered in this work, we have $\rho = \rho_0(1 + z)^3$, and

$$H^2 = \frac{8\pi G}{3} \rho_0(1 + z)^3 \left[ 1 + \left( \frac{1 + z}{1 + z_c} \right)^{3(n-1)} \right],$$

where $\rho_0$ is the current matter density of the universe. Hence, evaluating the Hubble parameter today gives $H_0^2 = \left(\frac{8\pi G}{3}\right)\rho_0 \left[ 1 + (1 + z_c)^3(1-n) \right]$. Conventionally, the critical density of the universe is $\rho_c = 3H_0^2/8\pi G = 1.8791 \times 10^{-29} \text{ g cm}^{-3}$, where $h$ is the present-day Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the present matter density of the universe is written in terms of $\rho_c$ as $\rho_0 = \Omega_m \rho_c$, where $\Omega_m$ is the standard matter density parameter. Now in the Cardassian model, matter alone makes the universe flat, which means that $\rho_0 = \rho_{c,\text{flat}}$, $\rho_{c,\text{flat}}$ is the critical density of the universe in the Cardassian expansion model (Freese & Lewis 2002),

$$\rho_0 = \rho_{c,\text{flat}} = \frac{3H_0^2}{8\pi G [1 + (1 + z_c)^3(1-n)]}.$$  

and the standard matter density parameter of the universe is $\Omega_m = [1 + (1 + z_c)^3(1-n)]^{-1}$ (note that $\Omega_m \equiv \rho_0/\rho_{c,\text{flat}} = 1$, as expected for a flat universe). As shown in equation (3), for some combinations of the parameters $n$ and $z_c$, the critical density of the Cardassian model can be much lower than that of the standard Friedmann model. In other words, in the context of the Cardassian model, it is possible to make the dynamical estimates of the quantity of matter that consistently point to $\rho_0 \simeq (0.2 - 0.4)\rho_c$, compatible with the observational evidence for a flat universe from CMB observations and the flatness prediction made by inflationary cosmology without any dark energy component (see Freese & Lewis 2002 for more details).

Now we evaluate the dimensionless coordinate distance, $y(z)$, the angular diameter distance, $D_A(z)$, and the luminosity distance, $D_L(z)$, as a function of redshift $z$ as well as the parameters of the Cardassian model. The three distances are simply related to each other by $D_L = (1 + z)^2 D_A = (c/H_0)(1 + z)y(z)$. Following the notation of Peebles (1993), we define the redshift dependence of the Hubble parameter $H(z) = H_0 E(z)$. Parametrizing the model as $(\Omega_m,n)$, we get $E$ as

$$E^2(z;\Omega_m,n) = \Omega_m(1 + z)^3 + (1 - \Omega_m)(1 + z)^{3n}.$$  

We note that the quintessence models with a constant equation of state ($p = \omega \rho$) for the dark energy component give rise to the same $H(z)$ presented here. One can make the identification, $\omega = n - 1$, such that $n = 1$ corresponds to a $\Lambda$CDM model. As far as any observation that involves only $H(z)$, the two models predict the same effects on the observation (Freese 2003). It is straightforward to show that the distances are given by

$$D_L^2(z; H_0, \Omega_m, n) = (1 + z)^2 D_A^4(z; H_0, \Omega_m, n) = \frac{c}{H_0} (1 + z) \int_0^z dz' \frac{E(z'; \Omega_m, n)}{E(z, \Omega_m, n)}.$$  

3. CONSTRAINTS FROM THE DIMENSIONLESS COORDINATE DISTANCE DATA

Recently, Daly & Djorgovski (2003) compiled a large database of the dimensionless coordinate distance measurements estimated from observations of SNe Ia and FR IIb radio galaxies. The authors used the database to derive the expansion rate of the universe as a function of redshift, $H(z)$, and the acceleration rate of the universe as a function of redshift, $q(z)$. We use this sample to give an observational constraint on the model parameters, $n$ and $\Omega_m$.

The SNe Ia measurements include the 54 supernovae in the “primary fit C” used by Perlmutter et al. (1999), the 37 supernovae published by Riess et al. (1998), and the highest redshift supernova so far, SN 1997ff, presented by Riess et al. (2001). The standard procedures of Perlmutter et al. (1999) and Riess et al. (1998) were used to determine the dimensionless coordinate distances to the supernovae. The apparent bolometric magnitude $m(z)$ of a standard candle with absolute bolometric magnitude $M$ is related to the luminosity distance $D_L$ by $m = M + 5 \log D_L + 25$. Then, using the relation of equation (5), the $B$-band magnitude-redshift relation can be written as

$$m_B = M_B + 5 \log [c(1+z)y(z)],$$

where $M_B \equiv M_B - 5 \log H_0 + 25$ is the “Hubble-constant-free” $B$-band absolute magnitude of a Type Ia supernova (SN Ia) at maximum. Then the above relation is used to determine $y(z)$ for each SN Ia. There are 14 SNe Ia that are present in both the Perlmutter et al. (1999) and Riess et al. (1998) samples, for which we will use the average values of $y$ with appropriate error bars (see Table 4 of Daly and Djorgovski 2003). Therefore we have in total 78 SN Ia data points, which are shown as filled circles in Figures 1 and 2.

The use of FR IIb radio galaxies to determine the angular size distance, or dimensionless coordinate distance, at different redshifts was first proposed by Daly (1994) (see also Guerra, Daly, & Wan 2000; Daly & Guerra 2002; Podariu et al. 2003; Daly & Djorgovski 2003). This method consists in a comparison of two independent measurements of the average size of the lobe-lobe separation of FR IIb sources, namely, the mean size $\langle D \rangle$ of the full population of radio galaxies at
similar redshift and the source average (over its entire life) size $D_{s}$, which is determined via a physical model describing the evolution of the sources. The basic idea is that $D_{s}$ must track the value of $D_{C3}$, such that the ratio $R_{C3} = h D_{s} / D_{C3}$ is assumed to be a constant:

$$R_{C3} = \kappa \left( \frac{d}{C^{12}}(z) \right)$$

where $\kappa$ is a parameter entering into the ratio $R_{C3}$. The variable $\kappa(z)$ can be determined using an iterative technique, as described in detail by Guerra et al. (2000) and Daly & Guerra (2002). We use the values of $\kappa(z)$ for 20 FR IIb radio galaxies obtained using the best fit to both the radio galaxy and supernova data (see Table 1 of Daly & Djorgovski (2003), which are shown as open squares in Figures 1 and 2. The best-fit values of $\kappa$ and $\beta$ and their error bars are included in Table 2 of Daly and Djorgovski (2003), i.e., $\kappa = 8.81 \pm 0.05$ and $\beta = 1.75 \pm 0.04$. In determining the error bar on $\kappa(z)$, the uncertainties in $\kappa$ and $\beta$ have been included (Daly & Djorgovski 2003). It is very important to consider whether significant covariance exists between the different parameters determined by the fit (Daly & Guerra 2002). In the quintessence model framework, Daly & Guerra (2002) have estimated the likelihood contours in the $C_{12}$ - $C_{10}$ plane, respectively (see Figs. 3 and 4 of their paper). As pointed out in $\S$ 2, the Cardassian scenario is equivalent to the quintessence model by identifying $C_{0} = n$. Therefore their results are exactly appropriate to our parameters $n$ and $\Omega_{m}$, i.e., there is no covariance between $\beta$ and $n (\Omega_{m})$. We determine the model parameters $n$ and $\Omega_{m}$ through a $\chi^{2}$ minimization method. The range of $n$ spans the interval $[-1, 1]$ in steps of 0.01, while the range of $\Omega_{m}$ spans the interval $[0, 1]$ also in steps of 0.01. The parameter $\chi^{2}$ is

$$\chi^{2}(n, \Omega_{m}) = \sum_{i=1}^{98} \frac{(y(z_i; n, \Omega_{m}) - y_{oi})^2}{\sigma_i^2},$$

where $y(z_i; n, \Omega_{m})$ refers to the theoretical prediction from equation (5), $y_{oi}$ is the observed dimensionless coordinate distances of SNe Ia and FR IIb radio galaxies, and $\sigma_i$ is the uncertainty ($i$ refers to the $i$th data point, with a total of 98 data points). The summation is over all of the observational data points.

The results of our analysis for the Cardassian expansion model are displayed in Figure 3. We show 68.3% and 95.4%
confidence level contours in the \((\Omega_m, n)\) plane using the shaded and the darker shaded areas, respectively. The best fit happens at \(\Omega_m = 0.38\) and \(n = -0.20\). It is clear from the figure that the dimensionless coordinate distance test alone weakly constrains the Cardassian expansion model. Only models with \(\Omega_m > 0.60\) and \(n > 0.54\) are excluded at the 95.4% confidence level. However, this already strongly suggests an accelerating universe (because \(n < 0.54 < \frac{1}{2}\)). Moreover, as we shall see in § 5, when we combine this test with the X-ray gas mass fraction test, we get very stringent constraints on the Cardassian scenario.

Recently, Knop et al. (2003) investigated in detail the effects of various systematic errors of SNe Ia on the cosmological measurements (§ 5 of their paper). Their main results were summarized in Table 9 (Knop et al. 2003); for example, the differences in light-curve fitting methods can change the flat universe value of \(\Omega_m\) by \(-0.03\) and \(\omega = (n - 1)\) by \(0.02\). Other systematic errors considered include non–Type Ia SN contamination, Malmquist bias, \(K\)-corrections, SN colors, dust extinction, gravitational lensing, etc. All identified systematic errors together give rise to \(\Delta \Omega_m = 0.04\) and \(\Delta \omega = (\Delta n) = 0.09\), which are smaller than the current statistical uncertainties of SNe Ia (Knop et al. 2003).

4. CONSTRAINTS FROM THE GALAXY CLUSTERS X-RAY DATA

Clusters of galaxies are the largest virialized systems in the universe, and their masses can be estimated by X-ray and optical observations, as well as by gravitational lensing measurements. A comparison of the gas mass fraction, \(f_{\text{gas}} = M_{\text{gas}}/M_{\text{tot}}\), as inferred from X-ray observations of clusters of galaxies, to the cosmic baryon fraction can provide a direct constraint on the density parameter of the universe, \(\Omega_m\) (White et al. 1993). Moreover, assuming that the gas mass fraction is constant in cosmic time, Sasaki (1996) show that the \(f_{\text{gas}}\) measurements of clusters of galaxies at different redshifts also provide an efficient way to constrain other cosmological parameters describing the geometry of the universe, because the measured \(f_{\text{gas}}\) values for each cluster of galaxies depend on the assumed angular diameter distances to the sources as \((D_A)^{3/2}\). The true, underlying cosmology should make these measured \(f_{\text{gas}}\) values invariant with redshift (Sasaki 1996; Allen et al. 2003).

Using the Chandra observational data, Allen et al. (2002, 2003) determined the \(f_{\text{gas}}\) profiles for the 10 relaxed clusters. Except for Abell 963, the \(f_{\text{gas}}\) profiles of the other nine clusters appear to have converged or are close to converging on a canonical radius \(r_{2500}\), which is defined as the radius within which the mean mass density is 2500 times the critical density of the universe at the redshift of the cluster (Allen et al. 2002, 2003). The gas mass fraction values of these nine clusters at \(r_{2500}\) (or at the outermost radii studied for PKS 0745−191 and A478) are shown in Figure 4. We use this database to constrain the Cardassian expansion models. Following Allen et al. (2002), the model function is

\[
f_{\text{gas}}^{\text{mod}}(z; n, \Omega_m) = \frac{b \Omega_h}{(1 + 0.19 h^{1/2}) \Omega_m} \left[ \frac{h}{0.5 D_{\text{car}}^{\text{SCDM}}(z_i)} \right]^{3/2},
\]

where \(b \simeq 0.93\) (Bialek, Evrard, & Mohr 2001; Allen et al. 2003) is a parameter motivated by gas dynamical simulations, which suggests that the baryon fraction in clusters is slightly depressed with respect to the universe as a whole (Cen & Ostriker 1994; Eke, Navarro, & Frenk 1998; Frenk et al. 1999; Bialek et al. 2001). The term \((h/0.5)^{3/2}\) represents the change in the Hubble parameter from the default value of \(H_0 = 50\, \text{km s}^{-1}\ \text{Mpc}^{-1}\), and the ratio \(D_{\text{car}}^{\text{SCDM}}(z_i)/D_{\text{car}}^{\text{mod}}(z_i; n, \Omega_m)\) accounts for the deviations of the Cardassian model from the default standard cold dark matter (SCDM) cosmology.

Again, we determine the Cardassian model parameters \(n\) and \(\Omega_m\) through a \(\chi^2\) minimization method. We constrain \(\Omega_m h^2 = 0.205 \pm 0.0018\) (O’Meara et al. 2001) and \(h = 0.72 \pm 0.08\), the final result from the Hubble Key Project by Freedman et al. (2001). The range of \(n\) spans the interval \([-1, 1]\) in steps of 0.01, while the range of \(\Omega_m\) spans the interval \([0, 1]\) also in steps of 0.01. The \(\chi^2\) difference between the model function and SCDM data is then (Allen et al. 2003)

\[
\chi^2(n, \Omega_m) = \sum_{i=1}^{9} \frac{\left[ f_{\text{gas}}^{\text{mod}}(z; n, \Omega_m) - f_{\text{gas, ol}} \right]^2}{\sigma_{f_{\text{gas}}}^2},
\]

\[
+ \left( \frac{\Omega_m h^2 - 0.205}{0.0018} \right)^2 + \left( \frac{h - 0.72}{0.08} \right)^2,
\]

where \(f_{\text{gas}}^{\text{mod}}(z; n, \Omega_m)\) refers to equation (8), \(f_{\text{gas, ol}}\) is the measured \(f_{\text{gas}}\) with the default cosmology (SCDM and \(H_0 = 50\, \text{km s}^{-1}\ \text{Mpc}^{-1}\)), and \(\sigma_{f_{\text{gas}}}\) is the symmetric root-mean-square error (it refers to the \(i\)th data point, with a total of 9 data points). The summation is over all of the observational data points.

The results of our analysis for the Cardassian expansion model are displayed in Figure 5. We show 68.3% and 95.4% confidence level contours in the \((\Omega_m, n)\) plane using the shaded
and the darker shaded areas, respectively. The best fit happens at $\Omega_m = 0.30$ and $n = 0.14$. It is clear from the figure, that although the X-ray gas mass fraction test alone constrains the density parameter $\Omega_m$ very stringently, it weakly limits the Cardassian power index $n$. However, when comparing Figure 4 with Figure 3, we can expect the X-ray gas mass fraction test to break the degeneracy presented in the dimensionless coordinate distance test of §3.

As Figure 5 shows, measurements of $f_{\text{gas}}$ of galaxy clusters provide an efficient way to determine $\Omega_m$. However, the uncertainty of the bias factor $b$ leads to a systematic error in this kind of analysis (Allen et al. 2003). Because $b$ linearly scales the X-ray mass fraction $f_{\text{gas}}$ in equation (8), lowering (raising) it by $\sim 10\%$ would cause the best fitting value of $\Omega_m$ to decrease (increase) by a similar amount. Another systematic uncertainty comes from the $f_{\text{gas}}$ profiles of galaxy clusters: any rise in the $f_{\text{gas}}$ values beyond the measurement radii would cause a corresponding reduction in $\Omega_m$.

5. COMBINED ANALYSIS, CONCLUDING REMARKS, AND DISCUSSION

Now we present our combined analysis of the constraints discussed previously and summarize our results. In Figure 6, we display the 68.3%, 95.4%, and 99.7% confidence level contours in the $(\Omega_m, n)$ plane from a combination of databases of dimensionless coordinate distances to SNe Ia and FR IIb radio galaxies and the X-ray gas mass fraction in clusters of galaxies. The best fit happens at $\Omega_m = 0.30$ and $n = 0.06$. As shown, although the two Cardassian parameters are not very sensitive to the dimensionless coordinate distance data of SNe Ia and FR IIb radio galaxies and the Cardassian power index is not sensitive to the X-ray gas mass fraction data of clusters, a combination of the two data sets gives at the 95.4% confidence level that $\Omega_m = 0.30^{+0.02}_{-0.02}$ and $n = 0.06^{+0.22}_{-0.18}$, a very stringent constraint on the Cardassian expansion scenario.

Given the two model parameters, $n$ and $\Omega_m$, Zhu & Fujimoto (2004) derived the redshifts $z_{\text{car}}$ and $z_{q=0}$ that satisfy the relation

$$ (1 + z)_{q=0} - (2 - 3n)^{1/3(1-n)}(1 + z_{\text{car}}) = \left(2 - 3n \left(\frac{1}{\Omega_m} - 1\right)\right)^{1/3(1-n)}, $$

where $z_{\text{car}}$ is the redshift at which the two terms inside the bracket of equation (1) are equal, while $z_{q=0}$ is the redshift at which the universe switches from deceleration to acceleration, or, in other words, the redshift at which the deceleration parameter vanishes. It has been shown (Zhu & Fujimoto 2004) that, for every value of $\Omega_m$, there exists a value for the power index of the Cardassian term, $n_{\text{peak}}(\Omega_m)$, that satisfies $(2 - 3n_{\text{peak}})^{1/3(1-n)}\exp\left[3(1 - n_{\text{peak}}) / (2 - 3n_{\text{peak}})\right] = \Omega_m^{-1} - 1$, which makes the turnaround redshift $z_{q=0}$ reach the maximum value, $z_{q=0,\text{max}} = \exp\left[1 / (2 - 3n_{\text{peak}})\right] - 1$.

The lower $\Omega_m$ is, the higher $z_{q=0,\text{max}}$ will be. For the lower bound obtained here, $\Omega_m = 0.28$, we have $n_{\text{peak}} = 0.0576$ and $z_{q=0,\text{max}} = 0.52$. In conclusion, our combined analysis results, $\Omega_m = 0.30_{-0.02}^{+0.02}$ and $n = 0.06^{+0.22}_{-0.18}$ at the 95.4% confidence level, suggest a Cardassian expansion universe that switches from deceleration to acceleration around $z_{q=0} \in (0.52, 0.73)$. However, the modified term of the Friedmann equation would dominate at a redshift around $z_{\text{car}} \in (0.25, 0.55)$, a little bit later than when the expansion turnaround happens (note that, $z_{\text{car}}$ is generally not equal to $z_{q=0}$), simply because the resulting power index $n$ is well below $\frac{1}{3}$ (Zhu & Fujimoto 2004).
Alternative cosmologies from a modified Friedmann equation may provide a possible mechanism for the present acceleration of the universe suggested by various cosmological observations. In this paper we have focused our attention on one of these scenarios, the so-called Cardassian expansion in which the universe is flat, matter (and radiation) dominated, and accelerating, but without any dark energy component. We have shown that stringent constraints on the parameters $\alpha$ and $\Omega_m$ that completely characterize the scenario can be obtained from combining analysis of the dimensionless coordinate distance data of SNe Ia and FR IIb radio galaxies and the X-ray mass fraction data of clusters. We are naturally hopeful that, with a more general analysis such as a joint investigation on various cosmological observations, one could show clearly if this scenario constitutes a feasible alternative to other acceleration mechanisms.

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We would like to thank S. Allen for sending us the compilation of the X-ray mass fraction data and helpful explanation of the data, and J. S. Alcaniz, A. Dev, D. Jain, and D. Tatsami for their helpful discussions. Our thanks go to the anonymous referee for valuable comments and useful suggestions, which improved this work very much. This work was supported by a Grant-in-Aid for Scientific Research on Priority Areas (14047219) from the Ministry of Education, Culture, Sports, Science and Technology. Z.-H. Z. acknowledges support from the National Natural Science Foundation of China and the National Major Basic Research Project of China (G2000077602), and is also grateful to all TAMA 300 members and the staff of NAOJ for their hospitality and help during his stay. X.-T. H. acknowledges support from the National Natural Science Foundation of China.