On the Problem of Optimal Path Encoding for Software-Defined Networks

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Abstract

Packet networks need to maintain state in the form of forwarding tables at each switch. The cost of this state increases as networks support ever more sophisticated per-flow routing, traffic engineering, and service chaining. Per-flow or per-path state at the switches can be eliminated by encoding each packet’s desired path in its header. A key component of such a method is an efficient encoding of paths through the network. We introduce a mathematical formulation of this optimal path-encoding problem. We prove that the problem is APX-hard, by showing that approximating it to within a factor less than $\frac{8}{7}$ is NP-hard. Thus, at best we can hope for a constant-factor approximation algorithm. We then present such an algorithm, approximating the optimal path-encoding problem to within a factor 2. Finally, we provide empirical results illustrating the effectiveness of the proposed algorithm.

I. INTRODUCTION

New networking technologies such as network virtualization [1], [2], policy-based routing [3], [4], per-flow routing [5], and service chaining [6] are leading to an explosion of state maintained at switches in the network core. Current efforts to control this state rely on restricting the per-flow state to the network edges and using tunnels in the network core, which only maintains the traditional per-destination forwarding state. While solving the state problem, this approach results in suboptimal routing, because multiple flows, each with potentially different delay, jitter, and bandwidth requirements, are aggregated into a single tunnel.

One way of extending per-flow state from the network edge to the core is for the header of each packet to contain an encoding of the packet’s required path. One such approach, termed source routing, encodes the path as a sequence of identifiers (such as the IP addresses) of the intermediate hops along the path [7]. This approach results in large packet headers and still requires each switch along the path to perform a table lookup to translate the identifier to an interface (i.e., an output port) from a table that grows as the size of the network grows.

Fig. 1. Packet traversal using the path-encoding architecture.

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To reduce the size of state, i.e., the size of the lookup table at each core switch, one can instead encode the path as a sequence of interface identifiers ([8], [9]). For instance, if a switch has \( k \) interfaces, then each of them could be assigned a distinct label of length \( \lceil \log k \rceil \) and a path encoding is a sequence of such labels. We refer to this approach as path switching.

For example, consider the scenario depicted in Fig. 1 in which an incoming packet to the network from endpoint \( E_4 \) to endpoint \( E_5 \) has its destination encoded at the ingress switch \( S_1 \) as 3133. This encoded path represents a sequence of labels, each uniquely identifying a switch interface on the path between the source and the destination. In our example, the first switch \( S_1 \) forwards the packet through its interface 3 to switch \( S_2 \); \( S_2 \) forwards the packet through its interface 1 to switch \( S_3 \); \( S_3 \) forwards the packet through its interface 3 to switch \( S_4 \); and so on.

Encoding a packet’s path in this manner eliminates expensive lookup tables at the switches in the network core, since each switch can identify the next hop from the interface label within the encoded path. Thus, path encoding enables arbitrary routing without the need to maintain per-path or per-flow lookup tables in the core switches. In particular, paths through the network can be arbitrarily complicated, making this approach ideally suited for service chaining and sophisticated traffic engineering with per-flow granularity. With such an encoding, the size of a switch’s lookup table to translate a label to an interface depends only on the number of interfaces at that switch and therefore remains constant despite network growth.

In this paper, we propose a path encoding that, rather than constraining the interface labels at a given switch to have the same length, allows the interface labels at the same switch to have variable lengths. The flexibility of variable-length interface encoding advocated in this paper has the advantage of resulting in shorter encoded paths. In particular, our contribution is a method for path encoding that minimizes the maximum length of any encoded path in the network. Minimizing this performance measure is appropriate when each encoded path is to be placed inside existing source/destination address header fields. As with any reasonable encoding using a sequence of interface labels, the proposed encoding allows the state kept at each switch to remain of constant size independent of the growth of the network in terms of topology as well as in terms of the number of distinct flows. Finally, the proposed method allows each switch along a path to easily and unambiguously determine the correct outgoing interface.

We begin by introducing a mathematical formulation of the problem of computing interface labels that minimize the longest encoded path for a given set of paths. We call this the optimal path-encoding problem. We prove that this problem is APX-hard, by showing that it is NP-hard to approximate to within any factor better than \( 8/7 \) of optimal. We next describe a 2-approximation algorithm for the optimal path-encoding problem. Finally, we apply the proposed algorithm to several real-world networks (the AT&T MPLS backbone network [10] and 11 networks from the RocketFuel topology set [11]). The proposed variable-length encoding results in a reduction of up to 30% in the maximum encoded path length.

The remainder of this paper is organized as follows. Section II introduces the optimal path-encoding problem. Algorithmic solutions to this problem are presented in Sections III and IV. Finally, Section V contains concluding remarks.

II. Problem Formulation

We start with a more detailed description of the network architecture in Section II-A. A key component of this architecture is the optimal encoding of paths described in Section II-B. We introduce a mathematical formulation of this problem in Section II-C.

A. Network Architecture

Consider the conceptual model of a network shown in Fig. 1. Endpoints are connected to the network via an edge switch. The network is assumed to be software-defined networking (SDN) enabled, i.e., there is a centralized SDN controller that configures each switch in the network. The SDN controller installs an interface-label table in each switch, assigning to each (outgoing) switch interface a unique binary...
string called the interface label. In addition, the SDN controller installs a flow-table in each edge switch, assigning to each incoming flow an encoded path. Each such encoded path is a binary string, consisting of the concatenation of the interface labels in the path.

Edge switches use the flow-table entries to modify the packet headers of incoming and outgoing flows. How this mapping between flows and encoded paths is performed depends on the nature of the packet endpoints. If the network ingress and egress interfaces in the path uniquely identify the packet endpoints, then the encoded path replaces the source and destination fields. Otherwise, the encoded path is placed in a tunnel header, leaving the existing packet header unchanged.

To facilitate the forwarding operations inside the network, the path label also contains a pointer field indicating the current position in the encoded path. In order to forward a packet, the switch reads its path label starting from the position of the pointer. It then searches its interface-label table for the unique interface that could result in this encoded path (starting from the current position). It increments the pointer by the length of the label, and forwards the packet over the corresponding interface.

From the above description, we see that a key component of this architecture is the assignment by the SDN controller of binary labels to switch interfaces. This assignment needs to be such that switches are able to make correct forwarding decisions. At the same time, the resulting encoded paths cannot be too large so that they can fit inside existing packet source/destination address fields, thereby leaving packet formats invariant. We next describe this path-encoding problem in detail.

**B. The Optimal Path-Encoding Problem**

As mentioned above, the goal of path encoding is the assignment of labels to switch interfaces such that the following two objectives are satisfied. First, to ensure that packets can be properly routed, the encoded paths need to be uniquely decodable. Second, because the encoding is appended to every packet sent through the network, usually in a header of fixed size, the longest encoded path needs to be small.

![Diagram](image)

One way to solve the path-encoding problem is to assign fixed-length labels to each switch interface. More precisely, for a switch with $k$ interfaces, we can assign a binary label of size $\lceil \log k \rceil$ bits to each of its interfaces. As an example of this fixed-length labeling, consider the network and label assignment depicted on the left side of Fig. 2. In this example, we consider all possible paths from switch $S_0$ to any of the edge switches $S_3, S_4, \ldots, S_9$. The longest encoded path is $(S_0, S_1, S_2, S_8)$ with encoding 00000 of length 5. Observe that, for ease of presentation, we do not explicitly distinguish between switches and endpoints in this example and in the following.
In this work, we instead advocate the use of variable-length interface labeling, in which labels for interfaces of the same switch may have different lengths. As an example of this variable-length labeling, consider the same network as before, but with the label assignment depicted on the right side of Fig. 2. Assuming the same set of paths as before, one longest encoded path is again $(S_0, S_1, S_2, S_8)$, but this time, its encoding 000 has only length 3. Thus, we see that in this example the use of variable-length labels has reduced the longest encoded path from 5 to 3 bits.

Variable-length interface labels have to be used with some care to ensure that the encoded paths are properly decodable. Two problems can occur. First, the encoded paths may not be unique, i.e., two different paths with same starting switch may be mapped into the same encoding, thereby preventing proper routing of packets. As an example, consider the network and label assignment depicted on the left side of Fig. 3. In this example, we consider all possible paths from switch $S_0$ to any of the edge switches $S_3$ through $S_7$. Consider the path encoding 01 at switch $S_0$. This encoding could result from either path $(S_0, S_4)$ or path $(S_0, S_1, S_5)$. The switch $S_0$ has therefore no way of deciding whether to forward a packet with encoded path 01 to switch $S_1$ or switch $S_4$.

A second problem is that the encoded paths may not be locally decodable. This occurs when a switch requires global information about the entire network in order to make the correct local forwarding decision. As an example, consider the same network as before, but with the label assignment depicted on the right side of Fig. 3. Assuming the same set of paths between $S_0$ and all possible edge switches as before, this assignment leads to unique encoded paths. However, the encoded paths are not locally decodable. Consider for example the path encoding 01 at switch $S_0$. Given that $(S_0, S_2)$ is not a valid path, this encoded path is uniquely decodable to path $(S_0, S_1, S_5)$. Therefore, switch $S_0$ needs to forward a packet with path 01 to switch $S_1$. However, to make this forwarding decision, $S_0$ needs to be aware of the collection of all possible paths in the network. Local decodability may also be compromised if a switch needs to know the label assignment at other switches in the network.

Both of these problems can be avoided if we impose the additional constraint that, at every switch, the collection of labels assigned to the interfaces of this switch form a prefix-free set, meaning that no label is a prefix of any other label. If this prefix-free condition is satisfied, then each switch can make its forwarding decision using only local information by finding the unique of its interface labels that forms a prefix of the encoded path. Observe that neither of the two label assignments in Fig. 3 are prefix free (since 0 is a prefix of 01 at switch $S_0$). On the other hand, both the label assignment in Fig. 2 are prefix free.

C. Mathematical Problem Description

We are now ready to introduce a mathematical description of the optimal path-encoding problem. We are given a directed graph $G = \{V, A\}$ describing the network and a set of paths $\mathcal{P}$ in $G$. We are tasked
with assigning binary labels \( x_a \in \{0, 1\}^* \), i.e., a binary string of arbitrary finite length, to each arc \( a \in A \). Denote by

\[
\ell_a \triangleq \ell(x_a)
\]

the length of the label string \( x_a \). For a path \( p \in \mathcal{P} \), the size of the path encoding resulting from this assignment of labels is

\[
\sum_{a \in p} \ell_a,
\]

where the summation is over all arcs \( a \) in the path \( p \).

In order to minimize the field size needed to store the path encoding, our goal is to minimize the length of the labels for the longest (with respect to \( \ell_a \)) encoded path in \( \mathcal{P} \). As discussed above, we ensure that messages are correctly routable through the network by imposing that the set of labels \( \{x_a\}_{a \in \text{out}(v)} \) corresponding to arcs \( a \) out of any vertex \( v \in \mathcal{V} \) forms a prefix-free set, meaning that no label \( x_a \) is a prefix of another label \( x_{\tilde{a}} \) in the same set. Clearly, this prefix condition implies that the encodings of (partial) paths with same starting vertex are unique. Moreover, it allows each vertex \( v \) to make routing decisions based on only its local set of labels \( \{x_a\}_{a \in \text{out}(v)} \).

Consider a vertex \( v \) and its outgoing arcs \( \text{out}(v) \), and consider a set of label lengths \( \{\ell_a\}_{a \in \text{out}(v)} \). A necessary and sufficient condition for the existence of a corresponding prefix-free set of labels \( \{x_a\}_{a \in \text{out}(v)} \) with these lengths is that they satisfy Kraft’s inequality

\[
\sum_{a \in \text{out}(v)} 2^{-\ell_a} \leq 1,
\]

see, e.g., [12, Theorem 5.2.1]. Moreover, for a collection of label lengths satisfying Kraft’s inequality, the corresponding set of labels can be found efficiently as follows. Let \( \Lambda \) be the largest label length in the set. Construct a perfect binary tree of depth \( \Lambda \), with each vertex in the tree representing a binary sequence of length up to \( \Lambda \). Next, find the arc \( a \) with shortest label length \( \ell_a \). Assign this arc to the (lexicographically) first available vertex of length \( \ell_a \) in the binary tree, and remove all its descendants from the tree. Continue this procedure with the second shortest label length until all labels are chosen. The vertices of the tree chosen by this procedure correspond to a set of prefix-free labels with the specified label lengths. With this, we can focus our attention in the following on finding the label lengths.

With this necessary and sufficient condition on the label lengths, we can now write the optimal path-encoding problem in the following compact form.

\[
\begin{align*}
\text{min} & \quad L \\
\text{s.t.} & \quad \sum_{a \in p} \ell_a \leq L, \quad \forall p \in \mathcal{P} \\
& \quad \sum_{a \in \text{out}(v)} 2^{-\ell_a} \leq 1, \quad \forall v \in \mathcal{V} \\
& \quad L \in \mathbb{R} \\
& \quad \ell_a \in \mathbb{Z}, \quad \forall a \in A.
\end{align*}
\]

Observe that the constraint \( \sum_{a \in \text{out}(v)} 2^{-\ell_a} \leq 1 \) guarantees that the optimal values of \( \{\ell_a\} \) are nonnegative. Further, since all \( \ell_a \) are integers, the optimal value of \( L \) is also guaranteed to be an integer. The remainder of this paper focuses on this minimization problem.

**Remark 1:** Our assumption throughout this paper is that the path set \( \mathcal{P} \) is static. In contrast, assume now that after the label assignment is completed and the corresponding label tables installed in the switches a new path \( p \notin \mathcal{P} \) needs to be added. Even though the label assignments were not optimized for \( p \), this new path can nonetheless be encoded using the current labels. Moreover, since the labels form a prefix free set at each switch, the resulting encoded path is uniquely routable through the network. Thus, enforcing the prefix-free condition has the additional advantage that new paths can always be added without
having to change the label assignment. In other words, the network will continue to operate correctly with dynamically changing path set \( P \). However, since the new path \( p \) was not part of the optimization problem yielding the label assignment, its encoded length may be larger than \( L \).

**Remark 2:** To alleviate the problem of newly added paths having encoded length larger than \( L \) as mentioned in Remark 1, the set \( P \) should not only contain those paths that are currently active, but also any anticipated future paths. Such anticipated future paths may for example be designed to handle congestion and node failures.

**Remark 3:** Recall that once label lengths satisfying Kraft’s inequality have been fixed, finding the actual labels with those lengths is straightforward using the algorithm described above. Hence, the difficult part of the optimal path-encoding problem is the assignment of label lengths. In the remainder of the paper, we will therefore focus on this subproblem of assigning label lengths with the understanding that the actual label assignment is then straightforward.

### III. MAIN RESULTS

Ideally, we would like to solve the optimal path-encoding problem (1) exactly. For special cases, such as out-arborescences (i.e., directed trees in which the root has in-degree zero and every other vertex has in-degree at most one), this is possible using dynamic programming, as is illustrated in the next example.

![Graph](image)

**Fig. 4. Graph \( G \) for Example 1.** The figure shows next to each arc \( a \) the length \( \ell_a \) of the binary label \( x_a \) associated with that arc.

**Example 1.** Consider the graph \( G \) shown in Fig. 4. The graph is an out-arborescence with three internal vertices labeled \( v_1, v_2, v_3 \) as shown in the figure. The set \( P \) consists of all paths from the root vertex \( v_3 \) to the leaves. This is the abstract version of the network depicted in Fig. 2 in Section II-B.

For out-arborescences, the optimal path-encoding problem can be solved exactly using dynamic programming. Consider the vertex \( v_1 \). There are two possible paths through \( v_1 \), one for each of its two children. Since these two paths have the same arcs from the root to \( v_1 \), their label lengths are the same except for the last arc. Since we are trying to minimize the maximum encoded path length, the optimal allocation of label lengths for these two arcs is \( 1 \) for both of them. This in effect “equalizes” the two path lengths.

Consider next vertex \( v_2 \). We again aim to equalize the paths through \( v_2 \). There are four such paths, one for each leaf that is a descendant of \( v_2 \). In order to equalize them, we should allocate a shorter length to the arc \((v_2, v_1)\) than the other two outgoing arcs. The optimal allocation is \( 1 \) for the arc \((v_2, v_1)\) and \( 2 \) for the other two arcs. With this assignment, all four paths through \( v_2 \) have path length of \( 2 \) from \( v_2 \) onward. Note that this assignment satisfies Kraft’s inequality.

Finally, consider the vertex \( v_3 \). We would again like to equalize paths. However, due to the integrality constraint, the best we can do here is to assign a label length of 1 to the arc \((v_3, v_2)\) and 2, 3, and 3 to the other three outgoing arcs. With this, the maximizing path is along the topmost branch of the tree with length \( L = 3 \).

The example illustrates the performance improvement due to allocating shorter label lengths to arcs on long paths. In particular, if we were to assign labels of uniform length to each outgoing arc of a vertex,

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\[ \text{For a larger example of this equalization, consider a vertex with six outgoing arcs of partial maximal encoded path lengths } 5, 4, 4, 3, 2, 1. \]

An optimal label-length assignment is then 2, 3, 3, 4, 5, 6, which satisfies Kraft’s inequality and equals the partial paths to length 7.
the topmost branch of the tree path would have encoded length $L = 2 + 2 + 1 = 5$. This example also shows that the label assignment on the right in Fig. [2] is optimal.

While the optimal path-encoding problem can be solved exactly for special cases as seen above, this is unfortunately not the case for general graphs $G$ as the following theorem shows.

**Theorem 1.** Approximating the optimal path-encoding problem \((1)\) by a factor less than $8/7$ is NP-hard. Thus, the optimal path-encoding problem is APX-hard.

The proof of Theorem 1 is reported in Section IV-A. The theorem shows that at most we should hope for an efficient constant-factor approximation algorithm for the optimal path-encoding problem. We next describe such an algorithm with an approximation guarantee of a factor 2.

Consider the relaxed version

$$
\min \quad L \\
\text{s.t.} \quad \sum_{a \in p} \ell_a \leq L, \quad \forall p \in P \\
2^{-\ell_a} \leq 1, \quad \forall v \in V \\
L \in \mathbb{R} \\
\ell_a \in \mathbb{R}, \quad \forall a \in A
$$

of the integer minimization problem \((1)\). The function $\sum_{a \in \text{out}(v)} 2^{-\ell_a}$ is convex in $\{\ell_a\}$, and hence this relaxed problem is a convex minimization problem. In fact, by rewriting Kraft’s inequality as

$$
\ln \left( \sum_{a \in \text{out}(v)} \exp(-\ln(2)\ell_a) \right) \leq 0,
$$

we see that \((2)\) is a geometric program [13, Section 4.5]. Such problems can be solved efficiently by interior-point methods [13, Chapter 11].

Let $(L^C, \{\ell^C_a\})$ be the minimizer of the relaxed problem \((2)\). Construct an integer solution

$$
\ell^I_a \doteq \lceil \ell^C_a \rceil,
$$

and set

$$
L^I \doteq \max_{p \in P} \sum_{a \in p} \ell^I_a.
$$

Note that $(L^I, \{\ell^I_a\})$ is a valid solution of the integer path-encoding problem \((1)\). Moreover, the next theorem asserts that the value $L^I$ of this solution is within a factor 2 of the optimal value $L^*$ of the path-encoding problem \((1)\).

**Theorem 2.** Let $L^I$ be the value of the rounded solution \((3)\) and let $L^*$ be the value of the minimizer of the optimal path-encoding problem \((1)\). Then

$$
L^* \leq L^I \leq 2L^*.
$$

The proof of Theorem 2 is reported in Section IV-B. We illustrate this approximation algorithm with a toy example.

**Example 2.** Consider the graph $G$ shown in Fig. 5. The graph is an out-arborescence consisting of $2K+1$ vertices. Consider the $K$ internal vertices $v_1, v_2, \ldots, v_K$ forming the “spine” of the graph. Denote the label lengths of the arcs in this spine by $\ell_1, \ell_2, \ldots, \ell_K$ as shown in Fig. 5. The set $P$ is given by all the paths from the root to the leaves.

The optimal solution for the path-encoding problem is trivial in this case: simply assign a value of 1 to each arc. The resulting value of $L^*$ is $K$. 

\(\diamondsuit\)
Let us next evaluate the relaxed problem (2). Consider an internal vertex \( v_k \) and its two outgoing arcs. Assume the first one has length \( \ell_k \). Then the other outgoing arc has to have length \(-\log(1 - 2^{-\ell_k})\) in order to satisfy Kraft’s inequality with equality. Clearly, the optimal choice of \( \ell_1 \) is 1. The optimal value of \( \ell_2 \) is given by the equation

\[
\ell_2 + \log(1 - 2^{-\ell_2}) = -1,
\]

since this equalizes the lengths of all possible paths going through \( v_2 \) as discussed in Example 1. Using the same argument, we obtain the general recursion

\[
\ell_k + \log(1 - 2^{-\ell_k}) = -\sum_{i=1}^{k-1} \ell_i,
\]

which can be solved to yield the solution

\[
\ell_k = \log\left(1 + \frac{1}{k}\right)
\]

of the relaxed problem. The resulting value of \( L_C \) is

\[
L_C = \sum_{k=1}^{K} \ell_k = \log(K + 1).
\]

Comparing this to the value \( L^* = K \) of the optimal path-encoding problem, we see that the integrality gap is at least \( K / \log(K + 1) \), which is unbounded as \( K \) increases.

Nevertheless, perhaps surprisingly, the rounded solution

\[
\lceil \ell_k \rceil = \lceil \log(1 + 1/k) \rceil = 1
\]

of the relaxed problem is in fact equal to the optimal solution of the path-encoding problem for all arcs on the spine of the graph. Hence, \( L^1 = K = L^* \) in this case. Thus, despite an unbounded integrality gap, the rounded solution is optimal in this case and in general yields a constant factor-2 approximation for the optimal path-encoding problem.

Note that

\[
-\log(1 - 2^{-\ell_k}) = \log(k + 1),
\]

so the rounded value \( \lceil \log(k + 1) \rceil \) on the arcs outside the spine of the graph is in fact considerably larger than the optimal solution of 1 of the path-encoding problem. Since these arcs are not on the longest path, this does not affect the value of \( L^1 \). Nevertheless, it does indicate that, in a practical setting, the solution found by the rounding procedure could be further improved by running a local search optimization procedure.

To obtain further intuition for the solution of the relaxed problem (2), it is instructive to consider its dual given by

\[
\begin{align*}
\max & \quad -\sum_{a \in \mathcal{A}} \left( \sum_{p \ni a} \alpha_p \log \frac{\sum_{p \ni a} \alpha_p}{\sum_{p \ni \text{tail}(a)} \alpha_p} \right) \\
\text{s.t.} & \quad \sum_{p \in \mathcal{P}} \alpha_p = 1 \\
& \quad \alpha_p \geq 0, \quad \forall p \in \mathcal{P}.
\end{align*}
\]

(4)
Here, for arc \( a = (v, u) \), \( \text{tail}(a) \) denotes the vertex \( v \). Moreover, we have used the shorthand notation
\[
\sum_{p \ni a} \alpha_p \triangleq \sum_{p \in P, a \in p} \alpha_p,
\]
with the nonstandard convention that a vertex \( v \) is in the path \( p \) if any of its outgoing arcs are in \( p \). It is easily seen that the relaxed primal problem (2) has a strictly feasible solution, which implies that strong duality holds [13, Section 5.2.3], i.e., the two convex programs (2) and (4) have the same value. The derivation of the dual (4) is somewhat lengthy and can be found in Section IV-C.

Let \( \{\alpha_p\} \) be a solution to the dual (4). Define now a random variable \( A \) taking values in \( A \) with
\[
\mathbb{P}(A = a) \triangleq \frac{\sum_{p \ni a} \alpha_p}{\sum_{p \in P} |p| \alpha_p}
\]
for any \( a \in A \), where \( |p| \) denotes the number of arcs in the path \( p \). Furthermore, define the random variable
\[
V \triangleq \text{tail}(A).
\]
Observe that \( V \) takes values in \( V \) with
\[
\mathbb{P}(V = v) = \sum_{a \in \text{out}(v)} \mathbb{P}(A = a) = \frac{\sum_{p \ni v} \alpha_p}{\sum_{p \in P} |p| \alpha_p}
\]
for any \( v \in V \). Finally, let \( P \) be an independent random variable taking values in \( P \) with
\[
\mathbb{P}(P = p) \triangleq \alpha_p
\]
for any \( p \in P \).

With these definitions in place, we can rewrite the dual problem (4) as
\[
\begin{align*}
\max_{\{\alpha_p\}} & \quad \mathbb{E}(|P(\alpha)|) H(A(\alpha) \mid V(\alpha)) \\
\text{s.t.} & \quad \sum_{p \in P} \alpha_p = 1 \\
& \quad \alpha_p \geq 0, \quad \forall p \in P,
\end{align*}
\]
where we have made the dependence of the random variables on \( \alpha \triangleq \{\alpha_p\} \) explicit. Here, \( H(A \mid V) \) denotes the conditional entropy of \( A \) given \( V \),
\[
H(A \mid V) \triangleq \sum_{v \in V} \mathbb{P}(V = v) H(A \mid V = v)
\]
\[
H(A \mid V = v) \triangleq -\sum_{a \in \text{out}(v)} \mathbb{P}(A = a \mid V = v) \log \mathbb{P}(A = a \mid V = v).
\]
The derivation of this entropy form of the dual problem is reported in Section IV-D.

This reformulation of the dual has an intuitive, informal, information-theoretic interpretation. The quantity \( H(A \mid V = v) \) is approximately (up to an additive gap of 1) the expected length of the optimal binary prefix-free source code for the random variable with distribution \( \{\mathbb{P}(A = a \mid V = v)\}_{a \in A} \) [12, Theorem 5.4.1]. This distribution describes the probability that, at vertex \( v \), a path takes arc \( a \in \text{out}(v) \) under distribution \( \{\alpha_p\} \) on the paths in \( P \). Averaged over all \( v \), the quantity \( H(A \mid V) \) is then the average
expected label length. Since the average path contains $E(|P|)$ arcs, the product $E(|P|)H(A | V)$ can be informally understood as a proxy for the expected size of the path encoding under this path distribution. The dual is this quantity for the worst-case distribution \{α_p\} over the paths \mathcal{P}.

We can also use the dual (4) to derive a simpler projected gradient-ascent algorithm \cite{14} for the path-encoding problem. Recall that strong duality holds, i.e., that the two problems (2) and (4) have the same value. Moreover, the derivation in Section IV-C shows that the optimal primal solution $(L^C, \{\ell_a^C\})$ can easily be derived from the optimal dual solution \{α_p^C\} as

\[
\ell_a^C = \log \frac{\sum_{p \ni \text{tail}(a)} \alpha_p^C}{\sum_{p \ni a} \alpha_p^C},
\]

\[
L^C = \max_{p \in \mathcal{P}} \sum_a \ell_a^C.
\]

The partial derivative of the dual objective function in (4) with respect to $\alpha_p$ is proportional to

$$
\Delta \alpha_p \triangleq \sum_{a : \text{tail}(a) \in p} \sum_{\tilde{p} \ni a} \alpha_{\tilde{p}} - \sum_{a \in p} \ln \frac{\sum_{\tilde{p} \ni \text{tail}(a)} \alpha_{\tilde{p}}}{\sum_{\tilde{p} \ni a} \alpha_{\tilde{p}}} - |p|.
$$

This yields the following projected gradient-ascent algorithm. Start with an initial solution \[ \alpha_p[0] \triangleq |\mathcal{P}|^{-1} \quad \forall p \in \mathcal{P}. \]

In iteration $t + 1$ of the algorithm, set

$$
\hat{\alpha}_p[t + 1] \triangleq \alpha_p[t] + \gamma[t] \Delta \alpha_p[t] \quad \forall p \in \mathcal{P},
$$

$$
\alpha_p[t + 1] \triangleq (\hat{\alpha}_p[t + 1] - \eta[t + 1])^+ \quad \forall p \in \mathcal{P}.
$$

Here, $(x)^+ \triangleq \max\{0, x\}$, and $\gamma[t]$ is a positive parameter depending on $t$ (but not on $p$) that can be chosen using either a line-search procedure or fixed to some small constant (see the discussion in \cite{13} Section 9.3). The parameter $\eta[t + 1]$ needs to be chosen in each iteration such that

$$
\sum_{p \in \mathcal{P}} \alpha_p[t + 1] = 1,
$$

which can be performed in $O(|\mathcal{P}|)$ expected time as described in \cite{14}.

As was pointed out in Example 2, the integral primal solution found by the rounding procedure can be further improved by refining it with a local search optimization procedure as follows. Find a path with longest encoding. Search along this path for any vertex at which Kraft’s inequality is not tight, and consider the arc out of this vertex along the chosen path. Since Kraft’s inequality is not tight, we may be able to reduce the label length of this arc without violating Kraft’s inequality. If this is the case, reduce this label length by one. Repeat these steps with different longest encoded paths until no further reductions are possible.

Once the label lengths are found, the actual labels themselves can then be easily derived using the algorithm described in Section II-C. We illustrate the proposed approximation algorithm with several examples.

**Example 3.** We applied the proposed gradient-ascent algorithm to the simple graph shown in Fig. 4. With a parameter value of $\gamma = 0.1$, the algorithm converges in 6 steps to the optimal solution of the convex dual problem, from which we then recover the optimal solution of the convex primal problem using (6). The rounding of the primal solution to obtain a solution for the integral version needs to be done with some care, since numerical values (say 1.0001, representing the value 1) may be erroneously rounded up. 

\[ \diamond \]
Example 4. For a more realistic scenario, we consider the AT&T MPLS backbone network [10] as depicted in Fig. 6. This is a network with 25 vertices and 224 arcs. There are 600 paths, chosen as the shortest (by hop distance) path between each ordered pair of vertices. A fixed-length encoding yields a maximum encoded path length of 15 bits. Applying the gradient-ascent algorithm for variable-length path encoding proposed in this paper reduces this length to 10 bits. Thus, by optimized variable-length encoding, the encoded path length is reduced by more than 30% in this setting.

Example 5. We also consider several autonomous systems from the RocketFuel topology set [11]. In each case, the paths are chosen as in Example 4. The path lengths for both fixed and variable-length encodings are summarized in Table I. The average reduction in longest encoded path length is more than 25%.

| Network         | Nodes | Edges | Paths | Fixed | Variable |
|-----------------|-------|-------|-------|-------|----------|
| 3549_3549       | 61    | 184   | 3660  | 26    | 16       |
| 4323_4323       | 51    | 142   | 2550  | 25    | 17       |
| Abilene         | 11    | 28    | 110   | 9     | 7        |
| ATMnet          | 21    | 44    | 420   | 12    | 11       |
| BBN Planet      | 27    | 56    | 702   | 14    | 10       |
| BICS            | 33    | 96    | 1056  | 17    | 13       |
| BT Asia Pac.    | 20    | 62    | 380   | 12    | 8        |
| BT Europe       | 24    | 74    | 552   | 11    | 9        |
| BT N. America   | 36    | 152   | 1260  | 16    | 12       |
| China Telecom   | 42    | 132   | 1722  | 13    | 9        |
| Claranet        | 15    | 36    | 210   | 9     | 7        |

TABLE I
Comparison of maximum encoded path length under fixed-length interface labeling and variable-length interface labeling for network topologies from the RocketFuel dataset.

IV. PROOFS

A. Proof of Theorem 7

In this section, we show that it is NP-hard to approximate the path-encoding problem better than 8/7 of optimal. Thus, the problem is APX-hard. We use a reduction from (2,3)-SAT, a variant of 3-SAT that was analyzed in [15] and shown there to be NP-complete.

A Boolean expression is in conjunctive normal form (CNF) if it can be expressed as the conjunction

\[ B = C_1 \land C_2 \land \cdots \land C_M \]
of clauses $C_1, C_2, \ldots, C_M$. Each such clause $C_m$ is the disjunction
\[ C_m = (l_{m,1} \lor l_{m,2} \lor \ldots) \]
of literals $l_{m,1}, l_{m,2}, \ldots$. Each literal $l_{m,s}$, in turn, is either equal to $x_n$ or its negation $\neg x_n$, where $x_1, x_2, \ldots, x_N$ are Boolean variables. In either case, we refer to $n$ as the index $\text{id}(l_{m,s})$ of the literal $l_{m,s}$, and we say that variable $x_n$ is involved in clause $C_m$.

An instance of $(2,3)$-SAT consists of a Boolean expression $B$ in CNF where each clause of $B$ has either 2 or 3 literals (with both types of clauses present) and each variable is involved in at most 3 clauses. Determining if an instance of $(2,3)$-SAT has a satisfying assignment is NP-complete \cite{15}. Notice that in any CNF expression, we can assume without loss of generality that each possible literal appears in at least one clause since otherwise the variable in this literal can easily be removed from the expression. Thus we can assume that in an instance of $(2,3)$-SAT every literal appears in one or two clauses.

To show that the optimal path-encoding problem is APX-hard, we construct a reduction from $(2,3)$-SAT. Let $I$ be an instance of $(2,3)$-SAT consisting of clauses $C_1, C_2, \ldots, C_M$ over the variables $x_1, x_2, \ldots, x_N$. From $I$ we construct an instance $L$ of the optimal path-encoding problem consisting of a directed graph $G = (\mathcal{V}, \mathcal{A})$ and a set of paths $P$ as follows.

![Fig. 7. Subgraph $G_n$ corresponding to variable $x_n$.](image)

We begin by defining the graph $G$. For each variable $x_n, 1 \leq n \leq N$, we define the subgraphs $G_n$ of $G$ as depicted in Fig. 7. Subgraph $G_n$ consists of 8 vertices labeled $n, x_n, d_n, \neg x_n, t_1^n, t_2^n, f_1^n, f_2^n$ with arcs $(n, x_n), (n, \neg x_n), (n, d_n), (x_n, t_1^n), (x_n, t_2^n), (\neg x_n, f_1^n), (\neg x_n, f_2^n)$. The simple but crucial observation is that at most one of the arcs $(n, x_n)$ or $(n, \neg x_n)$ in $G_n$ can be assigned length 1 if the lengths are to obey Kraft’s inequality at $n$.

For each variable $x_n, 1 \leq n \leq N$ we further define the subgraphs $G_{N+n}$. Subgraph $G_{N+n}$ consists of 5 vertices $N + n, u_{N+n}, v_{N+n}, y_{N+n}, z_{N+n}$ with arcs $(N + n, u_{N+n}), (N + n, v_{N+n}), (u_{N+n}, y_{N+n})$ and $(u_{N+n}, z_{N+n})$. Observe that the total length of the arcs $(N + n, u_{N+n})$ and $(u_{N+n}, y_{N+n})$ is at least 2 if at each node the lengths of the outgoing arcs satisfy Kraft’s inequality.

We now describe how the subgraphs $G_n$ are connected to one another to form the graph $G$. Fig. 8 illustrates the construction for the clause $C_m = (\neg x_1 \lor x_2 \lor \neg x_3)$.

For each clause $C_m = (l_{m,1} \lor l_{m,2} \lor l_{m,3})$ containing 3 literals, we assume without loss of generality that the variable indices are ordered to satisfy $\text{id}(l_{m,1}) < \text{id}(l_{m,2}) < \text{id}(l_{m,3})$. We say that $\text{id}(l_{m,2})$ is a successor index of $l_{m,1}$ and that $\text{id}(l_{m,3})$ is a successor index of $l_{m,2}$. For each clause $C_m = (l_{m,1} \lor l_{m,2})$ containing 2 literals, we again assume without loss of generality that the variable indices are ordered to satisfy $\text{id}(l_{m,1}) < \text{id}(l_{m,2})$. We say that $\text{id}(l_{m,2})$ is a successor index of $l_{m,1}$. We also say that $N + \text{id}(l_{m,2})$ is a successor index of $l_{m,2}$.

Thus, each literal has at most two successor indices since each literal is assumed to be in at least two clauses. We now describe how $G$ is formed by connecting the various subgraphs $G_i, 1 \leq i \leq 2N$. For each literal $x_n, 1 \leq n \leq N$, and for each successor index $i$ of $x_n$, we identify vertex $i$ in $G_i$ with either vertex $t_1^n$ or vertex $t_2^n$ in $G_n$ so that if there are two successor indices of $x_n$ then one is identified with $t_1^n$...
and the other with \( t_2 \). Similarly for each literal \( \neg x_n \) and for each successor index \( i \) of \( \neg x_n \) we identify vertex \( i \) in \( G_i \) uniquely to one of the vertices \( f_1 \) or \( f_2 \) in \( G_n \). This identification of vertices describes how the subgraphs \( G_1, G_2, \ldots, G_{2N} \) are connected to one another. The example shown in Fig. 8 illustrates how node \( f_1 \) and node 2 are identified as a single node and how node \( t_2 \) and node 3 are identified as a single node.

It remains to specify the collection of paths \( \mathcal{P} \) in \( G \). Consider clause \( C_m = (l_{m,1} \lor l_{m,2} \lor l_{m,3}) \) and let \( i_s = \text{id}(l_{m,s}), s \in \{1, 2, 3\} \). For \( r \in \{1, 2\} \), let \( p_r^m \) be the path in \( G \) from \( i_r \) to \( i_{r+1} \). Then define \( p_m \) as the concatenation of the paths \( p_m, p_m^2, \) and \( \text{arc} \ (i_3, l_{m,3}) \). The red/gray arcs in Fig. 8 show \( p_m \) for the example clause \( C_m = (\neg x_1 \lor x_2 \lor \neg x_3) \).

Now consider a clause \( C_m = (l_{m,1} \lor l_{m,2}) \) and let \( i_s = \text{id}(l_{m,s}), s \in \{1, 2\} \). Define \( p_1^m \) to be the path in \( G \) from \( i_1 \) to \( i_2 \). Define \( p_2^m \) to be the path in \( G \) from \( i_2 \) to \( N + i_2 \). Then let \( p_3^m = (N + i_2, u_{N+i_2}, y_{N+i_2}) \). Finally, define \( p_m \) to be the concatenation of \( p_1^m, p_2^m, \) and \( p_3^m \). The set of paths is then chosen as \( \mathcal{P} = \{p_1, p_2, \ldots, p_M\} \).

This completes the construction of the instance \( L \) of the optimal path-encoding problem corresponding to the instance \( I \) of the \((2,3)\)-SAT problem. One can easily verify that this construction can be done in time polynomial in the size of the instance \( I \).

Suppose there is a satisfying assignment \( S \) for \( I \). Then for each \( x_n \) assigned the value True in \( S \), give \( \text{arc} \ (n, x_n) \) length 1 and \( \text{arc} \ (n, \neg x_n) \) length 2. Similarly for every \( x_n \) assigned the value False in \( S \) give \( \text{arc} \ (n, \neg x_n) \) length 1 and \( \text{arc} \ (n, x_n) \) length 2. Assign length 2 to each \( \text{arc} \ (n, d_n) \), and assign length 1 to every other arc. It can easily be verified that the lengths of the arcs out of each vertex satisfy Kraft’s inequality. For path \( p_m \) the length of \( p_m \), is the sum of the lengths of the arcs on \( p_m \). Then the length of \( p_m \) is at most 7 for all \( m \in \{1, 2, \ldots, M\} \) since at least one of the literals in each clause is True, and hence the corresponding arc has length 1. To be more precise, for each clause \( C_m \), the length of \( p_m \) is 5, 6 or 7 depending on whether clause \( C_m \) has 3, 2 or 1 true literals respectively. Of course, if \( C_m \) only contains 2 literals then the length \( p_m \) can only be 6 or 7. To summarize, if a satisfying assignment for \( I \) exists, then \( L \) is at most 7.

Conversely, suppose there is an assignment of lengths to the arcs of \( G \) so that they satisfy Kraft’s inequality at every vertex and such that the length of \( p_m \) is at most 7 for \( m \in \{1, 2, \ldots, M\} \). Then for each \( m \in \{1, 2, \ldots, M\} \) it is the case that if \( C_m \) contains 3 literals then for at least one of the literals \( l_{m,1}, l_{m,2} \) or \( l_{m,3} \) (or if \( C_m \) contains only 2 literals then for at least one of \( l_{m,1} \) or \( l_{m,2} \)) the arc \( \text{id}(l_{m,s}), l_{m,s} \) has been assigned length 1. Therefore the truth assignment with \( x_n \) set to False if \( (n, \neg x_n) \) is assigned length 1 and set to True otherwise is a satisfying assignment for \( I \).

Together, this argument shows that there is a solution to \( L \) with maximum path length at most 7 if and only if there is a satisfying assignment for \( I \). Put differently, if \( I \) has no satisfying assignment then any
solution to $L$ will have maximum path length at least 8. By the NP-hardness of (2,3)-SAT, this implies that there cannot be a polynomial-time approximation algorithm for the optimal path-encoding problem with approximation ratio better than $8/7$ unless $P = NP$. \hfill \Box

B. Proof of Theorem \ref{thm:lower-bound}

Since
\[
\sum_{a \in \text{out}(v)} 2^{-\ell_a} \leq \sum_{a \in \text{out}(v)} 2^{-\ell^*_a} \leq 1
\]
for all $v \in V$, the rounded solution $(L^1, \{\ell^I_a\})$ is a feasible point for the integer path-encoding problem (1). The inequality $L^* \leq L^1$ is trivial, since $(L^1, \{\ell^I_a\})$ is a (suboptimal) solution to the integer problem (1).

It remains to show that $L^1 \leq 2L^*$. Observe that the value of a label size $\ell_a^*$ in the optimal solution can be equal to 0 only if $a$ is the only outgoing arc of $\text{tail}(a)$. But then we can without loss of generality assume that $\ell_a^* = 0$ as well. Therefore, for the path $p$ resulting in the largest path encoding according to $\{\ell^I_a\}$,
\[
L^1 = \sum_{a \in p} \ell_a^I = \sum_{a \in p} \lceil \ell_a^C \rceil \\
\leq \sum_{a \in p} \ell_a^C + |\{a \in p : \ell_a^C > 0\}| \\
\overset{(a)}{\leq} L^C + |\{a \in p : \ell_a^* > 0\}| \\
\leq L^* + \sum_{a \in p} \ell_a^* \\
\leq 2L^*,
\]
where $(a)$ follows since $L^C$ is the maximum of $\sum_{a \in p} \ell_a^C$ over all paths, and since $\ell_a^C = 0$ whenever $\ell_a^* = 0$ as argued above. This completes the proof. \hfill \Box

C. Derivation of Dual Problem \ref{eq:dual}

We start with the Lagrangian
\[
f(L, \{\ell_a\}, \{\alpha_p\}, \{\beta_v\}) = L + \sum_{p \in P} \alpha_p \left( \sum_{a \in p} \ell_a - L \right) + \sum_{v \in V} \beta_v \left( \sum_{a \in \text{out}(v)} 2^{-\ell_a} - 1 \right)
\]
\[
= L \left( 1 - \sum_{p \in P} \alpha_p \right) + \sum_{a \in A} \ell_a \sum_{p \ni a} \alpha_p + \sum_{a \in A} 2^{-\ell_a} \beta_{\text{tail}(a)} - \sum_{v \in V} \beta_v.
\]
The dual is given by
\[
\max_{L, \{\ell_a\}} \min_{\{\alpha_p\}, \{\beta_v\}} f(L, \{\ell_a\}, \{\alpha_p\}, \{\beta_v\})
\]
s.t. \quad $\alpha_p \geq 0$, \quad $\forall p \in P$,  \\
$\beta_v \geq 0$, \quad $\forall v \in V$.

We first handle the minimization over $L$. Observe that
\[
\min_L f(L, \{\ell_a\}, \{\alpha_p\}, \{\beta_v\}) = -\infty
\]
unless \(1 - \sum_{p \in P} \alpha_p = 0\). On the other hand, if this equality is satisfied, then the term \(L(1 - \sum_{p \in P} \alpha_p)\) has value 0. Hence, the dual can be simplified to

\[
\max \min_{\{\ell_a\}} \left( \sum_{a \in A} \ell_a \sum_{p \ni a} \alpha_p + \sum_{a \in A} 2^{-\ell_a} \beta_{\text{tail}(a)} - \sum_{v \in V} \beta_v \right)
\]

s.t.

\[
\sum_{p \in P} \alpha_p = 1
\]

\[
\alpha_p \geq 0, \quad \forall p \in P
\]

\[
\beta_v \geq 0, \quad \forall v \in V.
\]

We continue with the minimization over \(\{\ell_a\}\). Taking the derivative of the objective function with respect to \(\ell_a\) and equating to zero yields

\[
\sum_{p \ni a} \alpha_p - 2^{\ell_a} |\beta_{\text{tail}(a)}| \ln 2 = 0,
\]

which has solution

\[
\ell_a = \log \frac{\beta_{\text{tail}(a)} \ln 2}{\sum_{p \ni a} \alpha_p}.
\]

Using this, the dual becomes

\[
\max - \sum_{a \in A} \left( \sum_{p \ni a} \alpha_p \log \frac{\sum_{p \ni a} \alpha_p}{\beta_{\text{tail}(a)} \ln 2} + \sum_{a \in A} \sum_{p \ni a} \alpha_p \ln 2 - \sum_{v \in V} \beta_v \right)
\]

s.t.

\[
\sum_{p \in P} \alpha_p = 1
\]

\[
\alpha_p \geq 0, \quad \forall p \in P
\]

\[
\beta_v \geq 0, \quad \forall v \in V.
\]

The maximization over the dual variables \(\{\beta_v\}\) can be performed analytically. Taking the derivative of the objective function with respect to \(\beta_v\) and equating to zero yields,

\[
\sum_{a \in \text{out}(v)} \left( \sum_{p \ni a} \alpha_p \right) \frac{1}{\beta_v \ln 2} - 1 = 0,
\]

which has solution

\[
\beta_v = \frac{1}{\ln 2} \sum_{a \in \text{out}(v)} \sum_{p \ni a} \alpha_p
\]

\[
= \frac{1}{\ln 2} \sum_{p \ni v} \alpha_p,
\]

where, as before, we use the nonstandard convention that \(v \in p\) if and only if any of its outgoing arcs are in \(p\). Observe that \(\alpha_p \geq 0\) for all \(p \in P\) implies that \(\beta_v \geq 0\) as required. Substituting the optimal value of \(\{\beta_v\}\) and using that

\[
\sum_{a \in A} \sum_{p \ni a} \alpha_p = \sum_{p \in P} \alpha_p |\{a \in p\}| = \sum_{p \in P} \alpha_p |\{v \in p\}| = \sum_{v \in V} \sum_{p \ni v} \alpha_p,
\]
the dual can be simplified to

$$\max - \sum_{a \in A} \left( \sum_{p \ni a} \alpha_p \right) \log \frac{\sum_{p \ni a} \alpha_p}{\sum_{p \ni \text{tail}(a)} \alpha_p}$$

s.t. \[ \sum_{p \in P} \alpha_p = 1 \]
\[ \alpha_p \geq 0, \ \forall p \in P, \]
as claimed.

**D. Derivation of Entropy Form (5) of the Dual**

The dual objective function can be rewritten as

$$- \sum_{a \in A} \left( \sum_{p \ni a} \alpha_p \right) \log \frac{\sum_{p \ni a} \alpha_p}{\sum_{p \ni \text{tail}(a)} \alpha_p} = - \left( \sum_{p \in P} |p| \alpha_p \right) \sum_{a \in A} \mathbb{P}(A = a) \log \frac{\mathbb{P}(A = a)}{\mathbb{P}(V = \text{tail}(a))}.$$

Now,

$$\sum_{a \in A} \mathbb{P}(A = a) \log \frac{\mathbb{P}(A = a)}{\mathbb{P}(V = \text{tail}(a))} = \sum_{v \in V} \mathbb{P}(V = v) \sum_{a \in \text{out}(v)} \mathbb{P}(A = a | V = v) \log \frac{\mathbb{P}(A = a | V = v)}{\mathbb{P}(V = v)}$$
$$= \sum_{v \in V} \mathbb{P}(V = v) \sum_{a \in \text{out}(v)} \mathbb{P}(A = a, V = v) \log \mathbb{P}(A = a, V = v | V = v)$$
$$= \sum_{v \in V} \mathbb{P}(V = v) \sum_{a \in \text{out}(v)} \mathbb{P}(A = a | V = v) \log \mathbb{P}(A = a | V = v)$$
$$= - \mathbb{H}(A | V),$$

so that the dual objective function becomes

$$\mathbb{E}(|P|) \mathbb{H}(A | V),$$
as needed to be shown.

**V. Conclusion**

We presented a mathematical formulation of the problem of minimizing encoded paths and developed a 2-approximation algorithm for this problem. The algorithm allows interface labels of variable length at each switch. Compared to a baseline fixed-length encoding, the flexibility of this variable-length approach allows the algorithm to yield up to a 30% reduction in the length of the maximum encoded paths when tested on real-world ISP topologies. While the problem of path encoding was analyzed in this paper from a theoretical point of view, in follow-up work, our proposed variable-length approach has been implemented in the industry-standard Open vSwitch (OVS) in the Linux kernel [16].

The algorithm presented in this paper assigns labels to switch interfaces, rather than to paths. As a consequence, new paths can always be added at any time. Moreover, these routes can be pre-optimized by adding future projected paths as well as backup paths to the initial list of paths when running the algorithm. An interesting open problem for future research is how to incrementally update interface labels to optimize the path encoding for unanticipated topology or path changes.
REFERENCES

[1] N. M. M. K. Chowdhury and R. Boutaba, “Network virtualization: State of the art and research challenges,” IEEE Commun. Mag., vol. 47, pp. 20–26, July 2009.
[2] GENI Planning Group, “GENI design principles,” Computer, vol. 39, pp. 102–105, Sept. 2006.
[3] P. B. Godfrey, I. Ganichev, S. Shenker, and I. Stoica, “Pathlet routing,” in Proc. ACM SIGCOMM, pp. 111–122, Aug. 2009.
[4] Z. A. Qazi, C.-C. Tu, L. Chiang, R. Miao, V. Sekar, and M. Yu, “SIMPLE-ifying middlebox policy enforcement using SDN,” in Proc. ACM SIGCOMM, pp. 27–38, Aug. 2013.
[5] S. Sen, D. Shue, S. Ihm, and M. J. Freedman, “Scalable, optimal flow routing in datacenters via local link balancing,” in Proc. ACM CoNEXT, pp. 151–162, Dec. 2013.
[6] W. John, K. Pentikousis, G. Agapiou, E. Jacob, M. Kind, A. Manzalini, M. Risso, D. Staessens, R. Steinert, and C. Meirosu, “Research directions in network service chaining,” in IEEE SDN4FNS, pp. 1–7, Nov. 2013.
[7] C. A. Sunshine, “Source routing in computer networks,” ACM SIGCOMM Comput. Commun. Rev., vol. 7, pp. 29–33, Jan. 1977.
[8] S. Saponara, L. Fanucci, M. Tonarelli, and E. Petri, “Radiation tolerant spacewire router for satellite on-board networking,” IEEE Aerosp. Electron. Syst. Mag., vol. 22, pp. 3–12, May 2007.
[9] M. Soliman, B. Nandy, I. Lambadaris, and P. Ashwood-Smith, “Source routed forwarding with software defined control, considerations and implications,” in Proc. ACM CoNEXT Student, pp. 43–44, Dec. 2012.
[10] [http://www.att.com/Common/merger/files/pdf/wired-network/Domestic_0C-768_Network.pdf]
[11] N. Spring, R. Mahajan, and D. Wetherall, “Measuring ISP topologies with rocketfuel,” in Proc. ACM SIGCOMM, pp. 133–145, Aug. 2002.
[12] T. M. Cover and J. A. Thomas, Elements of Information Theory. Wiley, second ed., 2006.
[13] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, 2004.
[14] J. Duchi, S. Shalev-Shwartz, Y. Singer, and T. Chandra, “Efficient projections onto the $\ell_1$-ball for learning in high dimensions,” in Proc. ICML, pp. 272–279, July 2008.
[15] C. A. Tovey, “A simplified NP-complete satisfiability problem,” Discrete Appl. Math., vol. 8, pp. 85–89, Apr. 1984.
[16] A. Hari, T. V. Lakshman, and G. Wilfong, “Path switching: Reduced-state flow handling in SDN using path information,” in Proc. ACM CoNEXT, pp. 1–7, Dec. 2015.