THINKING ABOUT TOP
within the Standard Model

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I present an overview of standard-model top-quark physics at the Fermilab Tevatron. Topics discussed include the top-quark mass, weak interaction, strong interaction, and rare decays.

1 Why think about top?

Before we begin thinking about the top quark, let’s clarify why we should think about it. The main reason is that the near future holds the promise of a large number of top quarks at the Fermilab Tevatron.

For example, let’s consider some of the cleanest top-quark events, $t\bar{t} \rightarrow W + 4j$, with at least one $b$ tag. These events are fully reconstructable and have very little background. In the Run I data ($\sqrt{S} = 1.8$ TeV, 100 pb$^{-1}$), each experiment had about 25 such events. There are expected to be about 1000 events per experiment in Run II ($\sqrt{S} = 2$ TeV, 2 fb$^{-1}$), due mostly to the factor of 20 increase in integrated luminosity, but also due to the 37% increase in production cross section at $\sqrt{S} = 2$ TeV and the increased acceptance for top-quark events. Further running beyond Run II could deliver as much as 30 fb$^{-1}$ (“Run III”), which corresponds to about 15,000 events. The large number of events produced in Runs II and III will allow a detailed scrutiny of the properties of the top quark.

What are the chances that a close inspection of the properties of the top quark will yield surprises? One way to address this question is to consider the top-quark’s SU(2) partner, the $b$ quark. The $b$ quark was discovered in 1977, and in 1983 it yielded its first surprise: its lifetime was found to be much longer than expected. This is a consequence of the fact that $V_{cb} \ll V_{us}$, something which could not have been anticipated. The large $b$ lifetime has a number of very desirable consequences, such as large $B^0 - \bar{B}^0$ mixing, large $CP$ violation, enhanced rare decays, and the ability to tag $b$ jets via a secondary vertex using a silicon vertex detector.

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*a* Presented at the Thinkshop on Top-Quark Physics for the Tevatron Run II.

*b* In this talk I restrict my attention to top-quark physics at the Tevatron. I do not consider top-quark physics at the LHC or future lepton colliders.

*c* The $W$ is identified via its leptonic decay.

*d* For example, CDF had 34 such events, of which about 8 are thought to be background.
The top quark was discovered in 1995 and has already yielded its first surprise: the large value of its mass, approximately 174 GeV. Fifteen years ago, there were few who would have guessed its mass would be so large. A detailed scrutiny of the top-quark’s properties will reveal whether there are more surprises in top-quark physics.

## 2 Top mass

The top-quark mass has been measured by the CDF and D0 collaborations to be

\[
m_t = 176.0 \pm 6.5 \text{ GeV (CDF)} \tag{1}
\]

\[
= 172.1 \pm 7.1 \text{ GeV (D0).} \tag{2}
\]

This yields a world-average mass of

\[
m_t = 174.3 \pm 5.1 \text{ GeV (CDF + D0).} \tag{3}
\]

\[\text{This is the top-quark pole mass. The corresponding } \overline{\text{MS}} \text{ mass is } m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}}) = 165.2 \pm 5.1 \text{ GeV.}\]
To put this into context, I plot all the quark masses in Fig. 1, on a logarithmic scale. The width of each band is proportional to the fractional uncertainty in the quark mass. We see that, at present, the top-quark mass is the best-known quark mass, with the $b$-quark mass a close second ($m_b^{\text{MS}}(m_b) = 4.25 \pm 0.15 \text{ GeV}$).

An important question for the future is what precision we desire for the top-quark mass. There are at least two avenues along which to address this question. One is in the context of precision electroweak data. Fig. 2 summarizes the world’s precision electroweak data on a plot of $M_W$ vs. $m_t$. The solid ellipse is the 1σ contour. If the standard electroweak model is correct, the measured top-quark mass should lie within this contour. Since the contour spans about $\pm 8 \text{ GeV}$ along the $m_t$ axis, we conclude that the present uncertainty of $\pm 5 \text{ GeV}$ in the top-quark mass is more than sufficient for the purpose of precision electroweak physics at this time.

There is one electroweak measurement, $M_W$, whose precision could in-
crease significantly. An uncertainty of ±20 MeV is a realistic goal for Run III at the Tevatron. Let us take this uncertainty and project it onto a line of constant Higgs mass in Fig. 2. This is appropriate, because once a Higgs boson is discovered, even a crude knowledge of its mass will define a narrow line in Fig. 2, since precision electroweak measurements are sensitive only to the logarithm of the Higgs mass. An uncertainty in \( M_W \) of ±20 GeV projected onto a line of constant Higgs mass corresponds to an uncertainty of ±3 GeV in the top-quark mass. Thus we desire a measurement of \( m_t \) to ±3 GeV in order to make maximal use of the precision measurement of \( M_W \).

Another avenue along which to address the desired accuracy of the top-quark mass is to recall that the top-quark mass is a fundamental parameter of the standard model. Actually, it is the top-quark Yukawa coupling which is the fundamental parameter, given by

\[
y_t = \sqrt{2} \frac{m_t}{v} \approx 1.
\]  

(4)

The fact that this coupling is of order unity suggests that it may be a truly fundamental parameter. We hope someday to have a theory that relates the top-quark Yukawa coupling to that of its SU(2) partner, the \( b \) quark. The \( b \)-quark mass is currently known with an accuracy of ±3.5%. Since the uncertainty is entirely theoretical, it is likely that it will be reduced in the future. If we assume that future work cuts the uncertainty in half, the corresponding uncertainty in the top-quark mass would be ±3 GeV.

\footnote{A particularly compelling model which relates the \( b \) and top-quark masses is SO(10) grand unification. This model may be able to account for the masses of all the fourth-generation fermions, including the tau neutrino, whose mass is given by the “see-saw” mechanism as \( m_{\nu_\tau} \approx m_t^2/M_{GUT} \approx 10^{-2} \text{ eV} \).}
We conclude that both precision electroweak experiments and $m_t$ as a fundamental parameter lead us to the desire to measure the top-quark mass with an accuracy of ±3 GeV. This is well matched with future expectations. An uncertainty of ±3 GeV per experiment is anticipated in Run II\cite{23} and additional running could reduce this uncertainty to ±2 GeV\cite{23}.

3 Top weak interaction

The standard model dictates that the top quark has the same $V - A$ charged-current weak interaction as all the other fermions, as shown in Fig. 3. It is easy to see that this implies that the $W$ boson in top decay cannot be right handed, i.e., positive helicity. The argument is sketched in Fig. 4. In the limit of a massless $b$ quark, the $V - A$ current dictates that the $b$ quark in top decay is always left-handed. If the $W$ boson were right-handed, then the component of total angular momentum along the decay axis would be $+3/2$ (there is no component of orbital angular momentum along this axis). But the initial top quark has spin angular momentum $±1/2$ along this axis, so this decay is forbidden by conservation of angular momentum. CDF has measured

$$BR(t \rightarrow W^+_b) = 0.11 \pm 0.15 \pm 0.06$$

which is consistent with zero\cite{24}.

The top quark may decay to a left-handed (negative helicity) or a longitudinal (zero helicity) $W$ boson. Its coupling to a longitudinal $W$ boson is similar to its Yukawa coupling, Eq. (4), which is enhanced with respect to the weak coupling. Therefore the top quark prefers to decay to a longitudinal $W$ boson, with a branching ratio

$$BR(t \rightarrow W^0_b) = \frac{m_t^2}{m_t^2 + 2M_W^2} \approx 70\%.$$  

\footnote{\textsuperscript{9}Being far from massless, the decaying top quark can be left- or right-handed.}
CDF has made a first measurement of this branching ratio:

$$BR(t \to W_0 b) = 0.55 \pm 0.32 \pm 0.12,$$

which is consistent with expectations. The anticipated accuracy of this measurement in Run II and beyond will make it an interesting quantitative test of the top-quark weak interaction.\textsuperscript{11,15}

In addition to the $V-A$ structure of the top weak interaction, there is also its strength, i.e., the value of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{tb}$. CDF has measured

$$\frac{BR(t \to W b)}{BR(t \to W q)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.99 \pm 0.29$$

and it is interesting to ask what this tells us about $V_{tb}$. If we assume that there are just three generations of quarks, then unitarity of the CKM matrix implies that the denominator of Eq. (8) is unity, and we can immediately extract

$$|V_{tb}| = 0.99 \pm 0.15 (> 0.76 95\% \text{ CL}) \text{ (3 generations).}$$

However, to put this into perspective, recall that three-generation unitarity also implies that $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$, and since $|V_{ub}|$ and $|V_{cb}|$ have been measured to be small, one finds

$$|V_{tb}| = 0.9991 - 0.9994 \text{ (3 generations)}$$

which is far more accurate than the CDF result.
Figure 6: Top-quark pair production via the strong interaction: (a) quark-antiquark annihilation; (b) gluon fusion.

If we assume more than three generations, then unitarity implies almost nothing about $|V_{tb}|$:

$$|V_{tb}| = 0 - 0.9994 \ (> 3 \text{ generations}) \ .$$

(11)

At the same time, we also lose the constraint that the denominator of the middle expression in Eq. (8) is unity. All we can conclude from Eq. (8) is that $|V_{tb}| > |V_{ts}|, |V_{td}|$; we learn nothing about its absolute magnitude.

Fortunately, there is a direct way to measure $|V_{tb}|$ at the Tevatron, which makes no assumptions about the number of generations. One uses the weak interaction to produce the top quark; the two relevant processes are shown in Fig. 5. The cross sections for these two “single top” processes are proportional to $|V_{tb}|^2$. The first process involves an $s$-channel $W$ boson, while the second process involves a $t$-channel $W$ boson (and is often called $W$-gluon fusion, because the initial $b$ quark actually comes from a gluon splitting to $b\bar{b}$). $W$-gluon fusion has the advantage of greater statistics than the $s$-channel process, but the disadvantage of greater theoretical uncertainty. Thus far there is only a bound on single-top-quark production via $W$-gluon fusion from CDF:

$$\sigma(Wg \rightarrow t\bar{b}) < 15.4 \text{ pb} \ (12)$$

which is an order of magnitude away from the theoretical expectation of $1.70 \pm 0.09 \text{ pb}$. Both single-top processes should be observed in Run II.

Single-top-quark production can also be used to test the $V-A$ structure of the top-quark charged-current weak interaction. This structure implies that the top-quark spin is 100% polarized along the direction (in the top-quark
Figure 7: Cross section for $t\bar{t}$ production at the Tevatron vs. the top-quark mass. Dashed band is from next-to-leading-order QCD; solid band includes soft-gluon resummation at next-to-leading-logarithm. Figure adapted from Ref. 23.

rest frame) of the $d$ or $\bar{d}$ quark in the event, in both $W$-gluon fusion and the $s$-channel process.

4 Top strong interaction

The strong interaction of the top quark is best tested in its production. There are two subprocesses by which $t\bar{t}$ pairs are produced by the strong interaction at a hadron collider, shown in Fig. 6. At the Tevatron, the quark-antiquark annihilation process is dominant, accounting for 90% of the cross section at $\sqrt{S} = 1.8$ TeV. When the machine energy is increased to $\sqrt{S} = 2$ TeV in Run II, this fraction decreases to 85%. The cross section increases considerably, by about 37%, when the machine energy is increased from 1.8 to 2 TeV.

We show in Fig. 7 the $t\bar{t}$ cross section vs. the top-quark mass. The dashed band is from a calculation at next-to-leading-order in QCD. The uncertainty in this calculation is about $\pm 10\%$. The solid band includes the effect of soft gluon resummation at next-to-leading logarithm; this increases the cross section by only a few percent, but reduces the uncertainty by almost a factor...
Figure 8: Top-quark and light-quark spins in $q\bar{q} \to t\bar{t}$: (a) near threshold; (b) far above threshold; (c) intermediate energies.

The measurements by CDF and D0,

\begin{align*}
\sigma_{\text{CDF}} &= 7.6^{+1.8}_{-1.5} \text{ pb} \\
\sigma_{\text{D0}} &= 5.9 \pm 1.7 \text{ pb}
\end{align*}

are also shown in the figure, and are seen to agree with theory within one standard deviation.

An interesting aspect of the strong production of $t\bar{t}$ pairs is that the spins of the $t$ and $\bar{t}$ are nearly 100% correlated.\(^h\) The correct basis in which to measure the spins requires some consideration, however. At threshold ($\sqrt{s} \approx 2m_t$), the cross section is entirely $s$ wave, so the spins of the colliding quarks are transferred to the $t$ and $\bar{t}$. Since the quark-antiquark annihilation takes place

\(^h\)These bands reflect the uncertainty in the cross section due to the variation of the renormalization and factorization scales. They do not include the uncertainty from $\alpha_s(M_Z)$ or the parton distribution functions. However, these additional uncertainties are relatively modest.\(^*\)
via a gauge interaction, the quark and antiquark must have opposite helicities, so the spins of the $t$ and $\bar{t}$ are aligned along the beamline as shown in Fig. 8(a). At the other extreme, far above threshold ($\sqrt{s} \gg 2m_t$), the $t$ and $\bar{t}$ behave like massless quarks, and therefore must have opposite helicities, as shown in Fig. 8(b). The question is whether there is a basis which interpolates between the beamline basis near threshold and the helicity basis far above threshold, and the answer is affirmative - it has been dubbed the “off-diagonal” basis.\footnote{26} The $t$ and $\bar{t}$ spins are 100% correlated in this basis, as shown in Fig. 8(c). Since the quark-antiquark annihilation process accounts for most of the cross section at the Tevatron, the spin correlation is nearly 100%. This effect should be observable in Run II.

Another interesting aspect of the strong production of $t\bar{t}$ pairs is an asymmetry in the distribution of the $t$ and $\bar{t}$ quarks.\footnote{27} This effect arises at one loop, and leads to a forward-backward asymmetry of about 5% in $t\bar{t}$ production at the Tevatron.

\section{Rare decays}

Rare top decays in the standard model tend to be very rare, outside the range of the Tevatron. Thus far CDF has placed limits on the rare decays \footnote{28}

\begin{align*}
BR(t \to Zq) &< 33\% \ (95\%CL) \quad (15) \\
BR(t \to \gamma q) &< 3.2\% \ (95\%CL) \quad (16)
\end{align*}

which have tiny branching ratios in the standard model.\footnote{29}

The least rare of the rare decays within the standard model are the CKM suppressed decays $t \to Ws$ and $t \to Wd$, shown in Fig. 9. These decays are interesting because they allow a direct measurement of the CKM matrix.
elements $V_{ts}$ and $V_{td}$. Assuming three generations, the branching ratios are predicted to be

\begin{align}
BR(t \to W s) & \approx 0.1\% \quad (17) \\
BR(t \to W d) & \approx 0.01\% \quad (18)
\end{align}

which are small, but not tiny. Since there will be about 15,000 raw $t \bar{t}$ pairs produced in Run II, and about 200,000 in Run III, events of these type will be present in the data. However, there is no generally-accepted strategy for identifying these events.

Acknowledgments

I am grateful for conversations with and assistance from M. Mangano, K. Paul, R. Roser, S. Snyder, and T. Stelzer. This work was supported in part by Department of Energy grant DE-FG02-91ER40677.

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