Interaction corrections at intermediate temperatures: magneto-resistance in parallel field

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We consider the correction to conductivity of a 2D electron gas due to electron-electron interaction in the parallel magnetic field at arbitrary relation between temperature and the elastic mean free time. The correction exhibits non-trivial dependence on both temperature and the field. This dependence is determined by the Fermi liquid constant, which accounts for the spin-exchange interaction. In particular, the sign of the slope of the temperature dependence is not universal and can change with the increase of the field.

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Introduction – In a previous publication\textsuperscript{[1]} we have developed a theoretical framework for studying interaction corrections to conductivity of the two dimensional electron gas (2DEG) due to electron-electron interactions for the arbitrary relation between temperature $T$ and the elastic mean free time $\tau$. To describe strong coupling between electrons we used the conventional Fermi liquid constants. In particular, we found\textsuperscript{[2]} that the temperature dependence of the longitudinal conductivity in a 2DEG is determined by a single Fermi liquid constant $F_0^\sigma$, which describes the strength of the spin-exchange interaction. In principle, its value can be found from a measurement of the Pauli spin susceptibility

$$\chi = \frac{g^2 \mu_B^2 \nu}{1 + F_0^\sigma},$$

where $\mu_B$ is the Bohr magneton, $g$ is the bare electron Lande factor and the density of states $\nu$ should be obtained from a measurement of the specific heat (at $T^{-1} \ll T \ll E_F$). Unfortunately, to the best of our knowledge no measurement of the spin susceptibility has been reported for the 2DEG created at the interface of semiconductor heterostructures, since these measurements are performed in a regime where the temperature $T$ is of the same order of magnitude as the inverse scattering time $\tau^{-1}$ (obtained from the Drude conductivity). The opposite, ballistic limit was considered recently in Refs. \[10]. While giving a reasonable description of the magneto-resistance for weak interaction and at small fields, the authors of Refs. \[10] did not realize that both the temperature and the magnetic field dependence arise due to large-distance (as compared with $\lambda_F$) processes and therefore did not account for Fermi liquid renormalizations. The resulting temperature dependence of the conductivity (also see Ref. \[2]) is qualitatively erroneous: in particular for the case of the fully polarized system Ref. \[2] has the incorrect sign and Ref. \[10] finds no temperature dependence at all. (see Ref. \[2] for more details).

Early theoretical efforts focused on calculating the magneto-conductivity within the diffusive approximation.\textsuperscript{[13,14]} While perfectly justified for metallic thin films, this approximation might be inappropriate for understanding of recent experiments in semiconductor heterostructures, since these measurements are performed in a regime where the temperature $T$ is of the same order of magnitude as the inverse scattering time $\tau^{-1}$ (obtained from the Drude conductivity). The opposite, ballistic limit was considered recently in Refs. \[10]. While giving a reasonable description of the magneto-resistance for weak interaction and at small fields, the authors of Refs. \[10] did not realize that both the temperature and the magnetic field dependence arise due to large-distance (as compared with $\lambda_F$) processes and therefore did not account for Fermi liquid renormalizations. The resulting temperature dependence of the conductivity (also see Ref. \[2]) is qualitatively erroneous: in particular for the case of the fully polarized system Ref. \[2] has the incorrect sign and Ref. \[10] finds no temperature dependence at all. (see Ref. \[2] for more details).

In this paper we calculate the magneto-conductivity for an arbitrary relation between $T$, $\tau$, and the Zeeman energy $E_z$ (however, we are limiting ourselves to $E_z \ll E_F$; the case of strong fields where the electron system is close to full polarization will be addressed elsewhere).

Method – The expression for the leading interaction correction to conductivity can be found either by means of the diagrammatic technique\textsuperscript{[14,15]} or using the quantum kinetic equation\textsuperscript{[16]}. Both methods are completely equivalent and result in the following expression for the correction\textsuperscript{[16]}

$$\frac{\delta \sigma_{xx}}{\sigma_D} = \text{Im} \int \frac{d\omega}{\pi} \frac{\partial}{\partial \omega} \left( \omega \coth \frac{\omega}{2T} \right) \int_0^\infty \frac{qdq}{4\pi} \left[ \text{Tr} \tilde{D}_R(\omega, q) \right] B_{xx}(\omega, q),$$

where the retarded interaction propagator $[\tilde{D}_R(\omega, q)]_{\sigma_1\sigma_2;\sigma_3\sigma_4}$ is a matrix in spin space, and the form-factor $B_{xx}$ is defined as

$$B_{xx}(\omega, q) = \left\{ \frac{v_F^2 q^2 / \tau^2}{C^3(C - 1/\tau)^3} + \frac{3v_F^2 q^2}{2\tau C^3(C - 1/\tau)^2} + \frac{2[C - (-i\omega + 1/\tau)]}{C(C - 1/\tau)^2} + \frac{2C - 1/\tau}{Cv_F^2 q^2} \left( C - (-i\omega + 1/\tau) \right)^2 \right\},$$

where

$$\chi = \frac{g^2 \mu_B^2 \nu}{1 + F_0^\sigma},$$

and

$$\chi = \frac{g^2 \mu_B^2 \nu}{1 + F_0^\sigma}.$$
using the notation
\[ C(\omega, q) = \sqrt{(-i\omega + 1/\tau)^2 + \nu^2 q^2}. \] (3)

In the absence of magnetic field one can choose a basis corresponding to the singlet (charge) and triplet channels in which the interaction propagator becomes diagonal \[ \mathcal{D}^R = \text{diag}(\mathcal{D}^R_s, \mathcal{D}^R_t, \mathcal{D}^R_t, \mathcal{D}^R_t). \] The interaction propagator in the singlet channel is taken in the unitary limit (and hence is independent of the corresponding Fermi-liquid parameter \( F^R \)) and becomes proportional to the inverse of the electronic polarization operator
\[ \mathcal{D}^R_s = -\frac{1}{\Pi^R}. \] (4)

\[ \Pi^R(\omega, q) = \nu \left( 1 - \frac{-i\omega}{C(\omega, q) - 1/\tau} \right). \] (5)

On the contrary, the triplet channel propagator depends on the Fermi-liquid constant \( F_0^R \)
\[ \mathcal{D}^R_t = -\frac{F_0^R}{\nu + F_0^R \Pi^R} = -\frac{1}{\nu \imath \omega + \frac{F_0^R + 1}{F_0^R} (C - 1/\tau)}. \] (6)

and describes spin-exchange coupling. For details of the derivation of the propagators and Eqs. (3) we refer the reader to Ref. 1.

Using the explicit form of propagators (4) and (5) we evaluate the integral Eq. (2) and find the temperature dependent correction to conductivity in the absence of external magnetic field:
\[ \sigma = \sigma_D + \delta \sigma_C + \delta \sigma_T. \] (7a)

Here the charge (singlet) channel contribution is given by
\[ \delta \sigma_C = \frac{e^2}{\pi \hbar} \frac{T\tau}{\hbar} \left[ 1 - 3 f(T\tau) - \frac{e^2}{2\pi^2 \hbar} \ln \left( \frac{E_F}{T\tau} \right) \right], \] (7b)

and the triplet channel correction is
\[ \delta \sigma_T = \frac{3F_0^R}{(1 + F_0^R)} \frac{e^2}{\pi \hbar} \frac{T\tau}{\hbar} \left[ 1 + \frac{3}{8} t(T\tau; F_0^R) \right] \]
\[ -3 \left( 1 - \frac{1}{F_0^R} \ln(1 + F_0^R) \right) \frac{e^2}{2\pi^2 \hbar} \ln \left( \frac{E_F}{T\tau} \right). \] (7c)

The factor of 3 in the triplet channel correction Eq. (7c) is due to the fact that all three spin components of the triplet state contribute equally. The function \( f(T\tau) \) smoothly decays from unity to zero and the function \( t(T\tau; F_0^R) \) is non-monotonous only in the narrow region \(-0.25 > F_0^R > -0.5\) where it has a maximum at \( T\tau = 1/(1 + F_0^R) \). For numerical reasons both \( f(T\tau) \) and \( t(T\tau; F_0^R) \) change the result only by a few percent and therefore their explicit form (given in Ref. 1) is inessential for the present discussion.

The correction (7c) is written in the approximation of constant (i.e. momentum-independent) \( F_0^R \). For the system close to the Stoner instability such as \( 1/(1 + F_0^R) \ll (1 + F_0^R) \ll 1 \), this limits the applicability of Eq. (7c) by temperatures smaller than \( T^* = \epsilon_F(1 + F_0^R)^2 \), see Ref. 1.

In parallel magnetic field electrons acquire additional Zeeman energy \( E_z = g\mu_B H \), which is proportional to the magnitude \( H \) of the field, the Bohr magneton \( \mu_B \), and the electron g-factor. Consequently, the exact Green’s functions of non-interacting electrons now depend on magnetic field. They are related to the Green’s functions in the absence of the field as
\[ G^{R,A}(\epsilon) = G^{R,A}(\epsilon - \frac{1}{2}E_z \hat{z}), \]

where \( \hat{z} \) is the Pauli matrix in the spin space, and we chose the z-axis along the direction of the magnetic field. Repeating all the considerations of Ref. 1, one finds that two-particle propagators (that depend on the difference of the electron energies) are also modified by the field. This modification depends on the spin state of the two particles. Consider first a system of non-interacting electrons. Identification of the singlet and triplet channels corresponds to the choice of a basis in spin space, namely using the states with the total spin \( L \) and its z-component \( L_z \). The singlet channel is the state with \( L = 0 \) and it is unaffected by the magnetic field, as is the \( L_z = 0 \) component of the triplet. For the remaining two components the Zeeman splitting results in the shift of the frequency \( \omega \) in all diffusions by \( L_z E_z \).

In the presence of electron-electron interaction one takes into account the external magnetic field mostly in the same manner. The only difference is that the g factor is renormalized by the spin-exchange interaction similarly to the Pauli susceptibility Eq. (1). Consequently, the Zeeman splitting is also renormalized:
\[ E_z^* = \frac{g\mu_B H}{1 + F_0^R}. \]

Thus the conductivity correction (7b) is modified as:
\[ \frac{\delta \sigma_{xx}(H)}{\sigma_D} = \text{Im} \int_{-\infty}^{\infty} d\omega \frac{\partial}{\partial \omega} \left( \omega \coth \frac{\omega}{2T} \right) \int dq \frac{q dq}{4\pi} \]
\[ \times \left\{ \mathcal{D}^R_s(\omega, q) + \mathcal{D}^R_t(\omega, q) \right\} B_{xx}(\omega, q) \] (8a)
\[ + \sum_{L_z = \pm 1} \mathcal{D}^R_t(L_z E_z^*; \omega, q) B_{xx}(\omega + L_z E_z^*, q) \} \] (8b)

where the form-factor \( B_{xx}(\omega, q) \) is given by Eq. (2), propagators in the \( L_z = 0 \) channels expression (8a) are still given by Eqs. (3) and (6), while the propagators in expression (8b) are modified by the Zeeman energy as follows:
\[ D_t^R(L_z E_z^*; \omega, q) = -\frac{F_0^\sigma}{\nu + F_0^\sigma \Pi^R(L_z E_z^*; \omega, q)}. \]  
\[ \Pi^R(L_z E_z^*; \omega, q) = \nu \left[ 1 - \frac{-i\omega}{C(\omega + L_z E_z^*, q) - 1/\tau} \right]. \]

Note that the numerator of the polarization operator Eq. (10) is not changed by the Zeeman energy. As a result, the pole of the propagator Eq. (10) at \( q = 0 \) depends only on the bare Zeeman energy \( E_z \) with the bare electron \( g \)-factor. This is a manifestation of the Larmor theorem: the frequency of a homogeneous collective mode (which is the meaning of the pole at \( q = 0 \)) can not be renormalized by electron-electron interaction.

**Results** – Given the expression for the correction Eq. (8) and the explicit expression for the triplet propagator in the presence of the Zeeman field Eq. (3), further calculation consists of evaluating the integral in Eq. (8). The integral is similar to its zero-field counterpart (see Ref. 3). The resulting magneto-conductivity can be written as

\[ \sigma(H, T) - \sigma(0, T) = \frac{e^2}{\pi h} \left[ \frac{2F_0^\sigma}{(1 + F_0^\sigma)} \right] \frac{T \tau}{h} K_b \left( \frac{E_z}{2T}, F_0^\sigma \right) \]

\[ + K_b \left( \frac{E_z}{2\pi T}, F_0^\sigma \right) + m(E_z \tau, T \tau; F_0^\sigma) \]  
\[ \approx \sigma(H, T) - \sigma(0, T) \]  
\[ = \frac{e^2}{\pi h} \left[ \frac{2F_0^\sigma}{(1 + F_0^\sigma)} \right] \frac{T \tau}{h} K_b \left( \frac{E_z}{2T}, F_0^\sigma \right) \]

In the ballistic limit \( T \tau \gg 1 \) the dominating contribution is given by the first term in Eq. (13), where the dimensionless function \( K_b(x, F_0^\sigma) \) contains two contributions:

\[ K_b(x, F_0^\sigma) = K_1(x) + K_2(x, F_0^\sigma), \]  
\[ \text{where} \]

\[ K_1(x) = x \coth x - 1, \]

\[ K_2(x, F_0^\sigma) = \frac{1 + F_0^\sigma}{2F_0^\sigma} \int dx \frac{\partial}{\partial x} (x \coth x) \left[ (x - L_z h) \ln \frac{x - L_z h}{x - L_z h/(1 + F_0^\sigma)} \right] \]

\[ = \left\{ \frac{1}{2\pi F_0^\sigma} \sum_{n=1}^{\infty} \frac{1}{n} \left( \ln \frac{n^2 + h^2}{n^2 + 4h^2/(1 + F_0^\sigma)^2} \right) - \frac{4h}{n^2} \left( \arctan \frac{h}{n} - \frac{\arctan \frac{h}{n}}{\ln(1 + F_0^\sigma)} \right) \right\} - \frac{1}{\pi} \left[ C + \text{Re} \psi \left( 1 - \frac{i\hbar}{1 + F_0^\sigma} \right) \right], \]

where \( C = 0.577\ldots \) is Euler’s constant, and \( \psi(x) \) is the \( \psi \)-function. For weak interaction \( (F_0^\sigma \ll 1) \) Eq. (13) reproduces the result of Ref. 3. At the smallest magnetic field \( h \ll 1 + F_0^\sigma \) Eq. (13) reduces to

\[ K_d(h) \approx \frac{3F_0^\sigma \zeta(3)}{2\pi(1 + F_0^\sigma)^2} h^2, \]  
\[ \text{where} \ z(3) = 1.202\ldots \text{ is the Riemann zeta-function.} \]

In the opposite limit \( h \gg 1 \) we have

\[ K_d(h) \approx \frac{1}{\pi} \left\{ 1 - \frac{1}{F_0^\sigma} \ln(1 + F_0^\sigma) \right\} \ln \frac{h}{1 + F_0^\sigma} \]

and

\[ K_2(x, F_0^\sigma) = \frac{1 + F_0^\sigma}{2F_0^\sigma} \int dx \frac{\partial}{\partial x} (x \coth x) \]

\[ \times \left( y - \frac{x}{1 + F_0^\sigma} \right) \left[ \frac{1}{y} + \frac{2F_0^\sigma}{(1 + 2F_0^\sigma)h - x} \right]. \]

If the magnetic field is strong, \( x \gg 1 \), the expression Eq. (12) simplifies to

\[ K_b(x \gg 1, F_0^\sigma) = g(F_0^\sigma)x - 1 + O \left( \frac{1}{x} \right), \]

where the dimensionless function \( g(z) \), not to be confused with the Lande \( g \)-factor, is

\[ g(z) = \frac{1}{2z} \ln(1 + z) + \frac{1}{2(1 + 2z)} + \frac{z \ln 2(1 + z)}{(1 + 2z)^2}. \]

For the smallest magnetic field, \( x \ll 1 + F_0^\sigma \), we have

\[ K_b(x \ll 1 + F_0^\sigma, F_0^\sigma) \approx \frac{2}{3} f(F_0^\sigma), \]

where

\[ f(z) = 1 - \frac{z}{1 + z} \left[ \frac{1}{2} + \frac{1}{1 + 2z} - \frac{2}{(1 + 2z)^2} + \frac{2 \ln 2(1 + z)}{(1 + 2z)^3} \right]. \]

The diffusive limit \( T \tau \ll 1 \) is characterized by the function

\[ K_d(h) \approx -\frac{1}{2\pi F_0^\sigma} \ln^2 \frac{h}{1 + F_0^\sigma}. \]

Finally, for intermediate values \( 1 + F_0^\sigma \ll h \ll 1 \) we obtain

\[ K_d(h) \approx -\frac{1}{2\pi} \ln^2 \frac{h}{1 + F_0^\sigma} + O \left( \ln \frac{h}{1 + F_0^\sigma} \right). \]

The cross-over between the ballistic and the diffusive regimes is described by the dimensionless function \( m(E_z \tau, T \tau; F_0^\sigma) \). In the absence of the field \( m(0, T \tau; F_0^\sigma) = 0 \). Similarly to the function \( t(T \tau; F_0^\sigma) \)
Eq. (7c), this function appears to be numerically small and does not modify the sum of the two limiting expressions Eqs. (12) and (13) by more than one per cent.

FIG. 1. Magneto-conductivity in parallel field for different values of $F_0^*$. 

FIG. 2. Temperature dependence of the conductivity corrections in the presence of the parallel field.

Discussion – The resulting temperature dependence of the conductivity correction Eq. 3 is summarized in Figs. 2 and 3. In the ballistic regime $\delta \sigma \propto T$. Remarkably, the value and the sign of the slope depends on the field. At zero field the correction is given by all four (the singlet and three components of the triplet) spin channels so that $\partial \sigma / \partial T \propto 1 + 3F_0^*/(1 + F_0^*)$. For stronger fields $E_z > T$ the $L_z = \pm 1$ channels are gapped and we are left with one singlet and one triplet channel $\partial \sigma / \partial T \propto 1 + F_0^*/(1 + F_0^*)$. The cross-over is described by Eqs. (12) and shown on Fig. 3. This picture is valid up to fields of order $(1 + F_0^*)^2 E_F$. At the strongest fields $E_z^* > E_F$ when the system is fully polarized the spin does not play a role any more and one recovers the universal singlet channel result [see Eq. (7c)]

$$\frac{\partial \sigma}{\partial T} = \frac{e^2 \tau}{\pi \hbar}; \quad \frac{T \tau}{\hbar} \gtrsim 0.1; \quad E_z^* > E_F. \quad (19)$$

This conclusion is in agreement with recently reported measurements of magneto-resistance in GaAs heterostructures.

FIG. 3. Slope of the temperature dependence of the conductivity correction (in the ballistic limit) as a function of the parallel magnetic field.

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