A fuzzy goal programming model for biodiesel production

D S Lutero\textsuperscript{1}, E M U Pangue\textsuperscript{1}, J M Tubay\textsuperscript{1} and S P Lubag\textsuperscript{1,2}

\textsuperscript{1} Mathematics Division, Institute of Mathematical Sciences and Physics, University of the Philippines Los Baños, Laguna, Philippines 4031
E-mail: dslutero@up.edu.ph

Abstract. A fuzzy goal programming (FGP) model for biodiesel production in the Philippines was formulated with Coconut (Cocos nucifera) and Jatropha (Jatropha curcas) as sources of biodiesel. Objectives were maximization of feedstock production and overall revenue and, minimization of energy used in production and working capital for farming subject to biodiesel and non-biodiesel requirements, and availability of land, labor, water and machine time. All these objectives and constraints were assumed to be fuzzy. Model was tested for different sets of weights. Results for all sets of weights showed the same optimal allocation. Coconut alone can satisfy the biodiesel requirement of 2% per volume.

1. INTRODUCTION AND LITERATURE REVIEW

Biodiesel production has been in place since 1991 to address the pressing need for a renewable source of diesel and diminished carbon emission. To sustain biodiesel production however requires external incentives such as tax exemptions or subsidies provided by the government [1]. Moreover, biodiesel are mostly harvested from food crop feedstocks [2]. Consequently, it has been seen as a cause of increase in food prices between 2003 and 2008. Extraction of biodiesel from non-food feedstock such as jatropha and algae is seen as a solution to the food problem [3].

Republic Act 9367 (Biofuels Act of 2006) mandates the 2% blend of biodiesel per volume of diesel in the country. The current feedstock source of biodiesel in the country is coconut (Cocos nucifera). Jatropha (jatropha curcas) was also identified as a non-food feedstock source for biodiesel. Although Philippines is one of the top coconut oil producers in the world, RA 9367 mandates an increase in biodiesel blend of up to 10% per volume by 2020. The price and supply of coconut oil in the local and international market may be affected by the increase in blend.

Biodiesel production requires intensive production planning that involves steady supply of raw materials and use of efficient and cost effective production processes. Moreover, feedstocks have food or non-food uses other than for biodiesel. Optimal allocation of feedstock is essential to maintain a desirable trade-off among its uses.

Several authors have used multiple objective decision making (MODM) approaches in production planning. Goal programming is often used for problems that involve multiple objectives such as that in production planning. Originally introduced by Charnes and Cooper...
GP can arrive at an optimal solution to multiple objectives [4]. The goal programming approach transforms an originally multiple objective problem into a single goal [4]. Moreover, most real world problems function under an uncertain environment that involves imprecise and inexact data. Fuzzy set theory of Zadeh enables the inclusion of vagueness, imprecision and inexactness in models through the use of membership functions [5]. It was in 1980 that Narasimhan first integrated fuzzy set theory in goal programming [6]. Since then, fuzzy goal programming has been used in production planning and renewable energy management [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

In this paper, a fuzzy goal programming model was formulated to determine an optimal allocation of feedstocks used in biodiesel production given food and non-food requirements. Coconut and jatropha were the biodiesel feedstock sources. Objectives and constraints were assumed to be fuzzy. Models were tested using different sets of weights.

2. FUZZY GOAL PROGRAMMING

2.1. Fuzzy Set Theory
Let $U$ be a universal set and a set $A \subseteq U$. Unlike an ordinary mathematical set, a fuzzy set is defined by a membership function, often denoted by $\mu$, that takes on function values ranging within the interval $[0, 1]$. If $A$ is considered a fuzzy set, then the degree of membership of $x$ to $A$, denoted by $\mu_A(x)$ can be any value between 0 and 1 inclusive. A higher degree of membership of $x$ in $A$ implies a closer value of $\mu_A(x)$ to 1. With the definition of a membership function, fuzzy sets allow partial membership of elements in a (fuzzy) set.

2.2. Fuzzy Set Theory in Goal Programming
Fuzzy set theory was integrated in goal programming formulations by using membership functions that allow inexactness, ambiguity and imprecision of parameters in the model.

The conventional goal programming approach converts a multiple (linear) objective problem to a conventional (linear) single objective problem by assigning goals to each of the objectives and combining all these goals in a single objective function [4]. Both objectives and constraints can be treated as fuzzy and will take the form of one of the fuzzy goal types.

2.3. Fuzzy Goal Types
Fuzzy goals can either be "$\lesssim$" (at most), "$\geq$" (at least) or "$\approx$" (almost equal to). In the discussion, we will only consider the first two types: $AX \lesssim b$ or $AX \geq b$.

If $t_l$ is the allowed tolerance for the $l^{th}$ fuzzy goal of the form $AX_l \lesssim b_l$, then the membership function is

$$\mu(AX_l) = \begin{cases} 
1 & \text{if } AX_l \leq b_l \\
1 - \frac{AX_l - b_l}{t_l} & \text{if } B \leq AX_l \leq b_l + t_l \\
0 & \text{if } AX_l > b_l + t_l
\end{cases}$$

Similarly, if $t_g$ is the tolerance for $g^{th}$ fuzzy goal of the form $AX_g \geq b_g$, the membership function is

$$\mu(AX_g) = \begin{cases} 
1 & \text{if } AX_g \geq b_g \\
1 - \frac{b_g - AX_g}{t_g} & \text{if } b_g - t_g \leq AX_g \leq b_g \\
0 & \text{if } AX_g < b_g - t_g
\end{cases}$$

The general fuzzy goal programming formulation can be defined as
Maximize

\[ \sum_{l=1}^{L} \mu((AX)_l) + \sum_{g=1}^{G} \mu((AX)_g) \]  

subject to

\[ \mu((AX)_l) \leq 1 - \frac{(AX)_l - b_l}{t_l}, \forall l \in \{1, \ldots, L\} \]
\[ \mu((AX)_g) \geq 1 - \frac{b_g - AX}{t_g}, \forall g \in \{1, \ldots, G\} \]
\[ X \geq 0, \mu((AX)_l) \geq 0, \mu((AX)_g) \geq 0 \]

The objective function can be modified such that weights are assigned to each of the fuzzy goals. Then Equation 1 becomes

\[ \sum_{l=1}^{L} \omega_l \mu((AX)_l) + \sum_{g=1}^{G} \omega_g \mu((AX)_g) \]  

where \( \omega_l \) and \( \omega_g \) is the weight assigned to \( l^{th} \) and \( g^{th} \) fuzzy goal, respectively.

3. MODEL FORMULATION

What makes every general model powerful is its flexibility to be modified and tailor-fitted according to the assumptions and data of a particular problem. As many objectives and constraints can be integrated in the model provided the availability of data. More importantly, FGP model can accommodate assumptions on imprecision and uncertainty.

We chose feedstock allocation for the 2% blend requirement of biodiesel in the Philippines as the problem where choice of objectives, constraints and their targets and tolerances were based.

The general modelling process using fuzzy goal programming can be summarized in these steps:
1. Define each objective and constraint
2. Assign target value(goal) and tolerance to each fuzzy objectives and constraint
3. Formulate objective function

3.1. Identify and define objectives and constraints

3.1.1. Identification and definition of variables

The variables in the general model are defined as follows
These were the objectives considered in the model for biodiesel production. All objectives are considered fuzzy.

1. Feedstock production
   Maximize production output of biodiesel feedstock sources such that output should be greater than or equal to the target production output.

2. Overall revenue
   Maximize overall revenue for every feedstock such with its allocation being between biodiesel and non-biodiesel uses.

3. Energy requirement
   Minimize energy used in processing feedstocks to biodiesel to avoid waste of energy resources.

4. Working capital for farming
   Minimize working capital for farming that includes purchasing of land and machinery, establishment of planting area among others.

3.1.3. Fuzzy constraints description and formulation
All these constraints have fuzzy right hand side values.

1. Biodiesel Requirement
   According to RA 9367, a 2% blend is required for all diesel sold in the Philippines before 2015.

2. Non-biodiesel requirement
   The non-biodiesel requirements such as food or non-food demands for every feedstock should be met.
3. Land Availability
Every feedstock uses land during a given period of time. The land occupied by the feedstock should not exceed the total available land provided for the feedstock.

4. Labor Availability
The manpower who will handle the feedstock farming should not exceed total available labor.

5. Machine Time Availability
Total machine time for farming and processing the feedstock should not exceed the machine time available.

6. Water Availability
Every feedstock should be provided the minimum amount of water for it to grow but should not exceed the total amount of water available.

3.1.4. Non-fuzzy Constraint Description and Formulation
The total yield per feedstock should be greater than or equal to the amount of feedstock allocated to biodiesel and non-biodiesel requirements.

\[ P_f \geq DY_f + NDY_f \forall f \] (3)

3.2. Assign target value(goal) and tolerance to each fuzzy objectives and constraint
The fuzzy objectives and constraints are reformulated in terms of their corresponding membership function. Every fuzzy objective and constraint is assigned with a target value and tolerance.

If the \( o^{th} \) fuzzy objective is of the ”\( \geq \)” form, let

\[ \alpha_o \leq 1 - \frac{b_o - AX_o}{to_o} \Rightarrow AX_o + to_o (1 - \alpha_o) \leq b_o \]

So the fuzzy objectives of this type are transformed as

\[ P_f + to_1 (1 - \alpha_1) \geq TP_f \forall f \in \{1, 2, ..., F\} \] (4)

\[ \sum_{f=1}^{F} DY_f RD_f + \sum_{f=1}^{F} NDY_f RND_f + to_2 (1 - \alpha_2) \geq EP \] (5)

Similarly, if the \( o^{th} \) fuzzy objective is of the ”\( \leq \)” form, let

\[ \alpha_o \leq 1 - \frac{AX_o - b_o}{to_o} \Rightarrow AX_o - to_o (1 - \alpha_o) \leq b_o \]

So the fuzzy objectives of this type are transformed as

\[ \sum_{f=1}^{F} E_f DY_f - to_3 (1 - \alpha_3) \leq TE \] (6)

\[ \sum_{f=1}^{F} I_f P_f - to_4 (1 - \alpha_4) \leq TI \] (7)
Following the same transformation for the fuzzy constraints, the $c^{th}$ fuzzy constraint with membership indicator $\beta_c$ and tolerance $t_c$ is defined as

$$\sum_{f=1}^{F} D_f D_Y f + t_c (1 - \beta_1) \geq DR$$  \hspace{1cm} (8)

$$N D_Y f + t_c (1 - \beta_2) \geq N D R f$$  \hspace{1cm} (9)

$$L_f P_f - t_c (1 - \beta_3) \leq T L f$$  \hspace{1cm} (10)

$$\sum_{f=1}^{F} L R_f P_f - t_c (1 - \beta_4) \leq T L R$$ \hspace{1cm} (11)

$$\sum_{f=1}^{F} M T_f P_f - t_c (1 - \beta_5) \leq T M T$$ \hspace{1cm} (12)

$$\sum_{f=1}^{F} W_f P_f - t_c (1 - \beta_6) \leq T W$$ \hspace{1cm} (13)

3.3. Formulate objective function

The objective of fuzzy goal programming is to find a solution that maximizes membership of all goals and constraints.

Based on the formulation, the weighted sum should be maximized to obtain the maximum membership grade. The objective function is formulated as

$$Z = w_1 \left( \sum_{f=1}^{F} a_f \alpha_{1f} \right) + \sum_{j=2}^{4} w_j \alpha_j + w_5 \left( \omega_1 \beta_1 + \omega_2 \beta_2 + \omega_3 \sum_{f=1}^{F} \rho_f \beta_3 f + \sum_{j=4}^{6} \omega_j \beta_j \right)$$ \hspace{1cm} (14)

where $w_j, j \in 1, 2, 3, 4$ correspond to weights associated with the objectives while $w_5$ is the total weight for all constraints. Note that $\sum_{j=1}^{5} w_j = 1$. $a_f$ is the sub-weight for production of feedstock $f$. $\omega_j, j \in \{1, 2, 3, 4, 5, 6\}$ are the sub-weights for each of the six constraints, and $\rho_f$ is the sub-weight for the land availability constraint of feedstock $f$.

The weights denote the contribution of the goals and constraints to the value of the objective function. In decision-making, unless all weights are equal, the goals or constraints with greater weight are those that are more important to the decision-maker.

3.4. Final FGP model

The general FGP model is

Maximize Equation 14
subject to
Equations 4 to 7
Equations 8 to 13
Equation 3
nonnegativity constraints
4. ILLUSTRATIVE EXAMPLE

4.1. Data for model

Data were collected from different government units in the Philippines, offices from University of the Philippines Los Ba˜nos and Jatropha Project of the same university.

| Feedstock | $RD_f$ | $RND_f$ | $E_f$ | $I_f$ | $D_f$ | $L_f$ | $LR_f$ | $MT_f$ | $W_f$ |
|-----------|--------|---------|-------|-------|-------|-------|--------|--------|-------|
| Coconut ($f = 1$) | 23,940 | 1,000 | 253,4679 | 2,600 | 630 | 0.2 | 17.65 | 5.05 | 0.49 |
| Jatropha ($f = 2$) | 20,000 | 0 | 253,4679 | 6,562 | 250 | 0.17 | 9.22 | 2 | 0.49 |

These are the crisp numerical coefficients of the fuzzy objectives and constraints.

| Fuzzy Objective | Target | Tolerance |
|-----------------|--------|-----------|
| Production (ton) | | |
| Coconut | 12,877,605 | 128,776.05 |
| Jatropha | 2,160 | 21.6 |
| Overall revenue (PHP) | 169,095,045,000 | 1,690,950,450 |
| Energy Requirement (Kw-hr) | 17,031,000,000 | 170,310,000 |
| Working capital for farming (PHP) | 35,796,527,900 | 357,965,279 |

The target values for each objective shown in the table were taken from various government agencies namely, Department of Energy, Bureau of Agricultural Statistics and Philippine Coconut Authority. Tolerance for every fuzzy objective was estimated to be 1% of the target.

| Fuzzy Constraint | Target | Tolerance |
|------------------|--------|-----------|
| Biodiesel Requirement | 173,000,000 | 1,730,000 |
| Non-biodiesel requirement | | |
| Coconut | 1,113,912.83 | 11,139.1283 |
| Jatropha | 0 | 0 |
| Land availability | | |
| Coconut | 3,379,740.90 | 337,974.09 |
| Jatropha | 1,100,000 | 11,000 |
| Labor availability | 962,480,000 | 9,624,800 |
| Machine-time availability | 1,320,000,000 | 13,200,000 |
| Water availability | 431,000,000 | 4,310,000 |

The target values for the constraints were taken from the same sources of Table 2. Tolerance for every fuzzy constraint was also estimated to be 1% of the target.

4.2. Results and sensitivity analysis

The model was ran using GAMS with CPLEX 12.1.0 as solver. Results were obtained in less than 0.03 seconds for all sets of weights.
Table 4. Resulting values for variables

| Variable | Value   |
|----------|---------|
| $P_1$    | 12,878,000 |
| $P_2$    | 2,160   |
| $DY_1$   | 274,600 |
| $DY_2$   | 0       |
| $NDY_1$  | 12,603,000 |
| $NDY_2$  | 0       |

The model was tested for several sets of weights (equal and unequal). The results were the same for all sets of weights used. All the goals and constraints were satisfied for both equal and unequal weights. This means that 2% biodiesel blend in all diesel sold in the country is feasible. Coconut alone can satisfy the biodiesel requirement.

Table 5. Ranging Analysis for the Right Hand Side Values

| Equation                  | Lower Limit | Current RHS Value | Upper Limit   |
|---------------------------|-------------|-------------------|---------------|
| Coconut Production        | 8,401,000   | 12,880,000        | 13,760,000    |
| Jatropha Production       | 0           | 2,160             | 352,800       |
| Overall Revenue           | $-\infty$   | 169,100,000,000   | 258,600,000,000 |
| Energy Requirement        | 69,600,000  | 17,030,000,000    | $+\infty$    |
| Working Capital for Farming| 33,500,000,000 | 35,800,000,000   | $+\infty$    |
| Biodiesel Requirement     | 0           | 173,000,000       | 7,411,000,000 |
| Non-biodiesel requirement | $-\infty$   | 1,114,000         | 12,600,000    |
| Land for coconut          | 2,576,000   | 3,380,000         | $+\infty$    |
| Land for jatropha         | 367.2       | 1,100,000         | $+\infty$    |
| Labor availability        | 227,300,000 | 962,500,000       | $+\infty$    |
| Machine-time availability | 65,040,000  | 1,320,000,000     | $+\infty$    |
| Water availability        | 6,311,000   | 431,000,000       | $+\infty$    |

The right-hand side values for each constraint shown above can be adjusted within the specified range one at a time without making the solution infeasible. For example, the current value for the coconut production is 12,880,000 tons of coconut. We can decrease the production to as low as 8,401,000 tons and increase it to as high as 13,760,000 tons without making the solution infeasible.

5. CONCLUSION AND DISCUSSION

This paper proposed a fuzzy goal programming model for biodiesel production in the Philippines. Results showed that all objectives and constraints of the study were satisfied for both equal and unequal weights. This means that the 2% biodiesel blend requirement is possible. Coconut alone can provide for the 2% blend requirement. Jatropha can act as buffer if coconut declines in supply and fails to meet biodiesel requirement or if coconut feedstock will be allocated to non-biodiesel requirements to achieve other benefits, say, higher profit. All membership grades were equal to one which means that the solution can be improved by changing the right hand side values within the specified ranges shown in the sensitivity analysis one at a time.
6. RECOMMENDATIONS
Other feedstocks can be included in the FGP model such as algae. Increased blend requirement can also be considered to assess its feasibility. More biodiesel and non-biodiesel requirements can be incorporated in the model. Land use planning should accommodate the possible changes in land allocation for feedstock when the blend is increased. With this general formulation, FGP approach is envisioned to provide a different view in terms of decision making in the stream of biodiesel production. Math models in biodiesel production can incorporate more ambiguity and inexactness if models are considered to assume fuzziness. Moreover, decision makers have more flexibility in modifying the goals and constraints in a FGP model.

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