Stronger gravity in the early universe

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Introduction

It is natural to expect that both inflation at earliest epoch of cosmological evolution and dark energy of late-time acceleration [1] have a common origin in a scalar degree of freedom called inflaton [2]. Since a temporarily stationary inflaton field needed in both cases resembles a cosmological constant, it would be ideal that the problem of fine-tuned cosmological constant [3] is solved together with inflation and dark energy.

Scalar-tensor theory of gravity has been a focal point in the advent of small, but finite dark energy. Among them a class of conformal gravity theories have appealing features, and many popular cosmology models [4], [5], [6] of inflation and late-time acceleration are interpreted as this class of theories.

In a recent publication [7] it was shown that a special class of conformal gravity models has a potential of solving the long-standing fine-tuning problem of cosmological constant. It was also pointed out that in these models the cosmological mass variation of standard model particles occurs. Historically, the problem of varying mass in cosmology is traced back to the Jordan-Brans-Dicke (JBD) theory [8]. A partial motivation of constructing this class of theory is the Dirac’s large number hypothesis [9] proposed to explain the unnaturally large mass ratio found in nature. When a cosmological constant [10] is added to the JBD theory, the mass variation becomes inevitable [10]. The Dirac’s original idea is untenable due to the nucleo-synthesis constraint explained below, but the question remains on variation of the proton to the Planck mass ratio \( \approx 10^{-18} \).

The crucial question to be addressed to is how one can regulate the pattern of mass variation in cosmology. There exists a constraint on the mass ratio of proton to weak W-boson from nucleo-synthesis [11], [12], which should be respected. We shall formulate in the present work the method of modification in accordance with the general principles of gauge invariant quantum field theory, a basis of standard particle theory.

We use the natural unit of \( \hbar = c = 1 \) and the Boltzmann constant \( k_B = 1 \) throughout the present work unless otherwise stated. The flat Friedman-Robertson-Walker (FRW) metric we use is \( ds^2 = dt^2 - a^2(t)(d\vec{x})^2 \) with \( a(t) \) the cosmic scale factor.

A class of conformal gravity under consideration

We start from formulation of local quantum field theory in four space-time dimensions. The lagrangian density we consider is given in what is called the Jordan metric frame [8],

\[
\mathcal{L} = \mathcal{L}_{\text{gy}} + \mathcal{L}_{\text{SM}}, \quad \mathcal{L}_{\text{SM}} = \sqrt{-g}L_{\text{SM}}(\psi, g_{\mu\nu}),
\]

\[
\mathcal{L}_{\text{gy}} = \left(-M_P^2 F(\chi)R(g_{\mu\nu}) - 2M_P^2 \Lambda + \frac{1}{2} (\partial \chi)^2 - V(\chi)\right),
\]

\[
M_P^2 = \frac{1}{16\pi G_N} \approx (1.72 \times 10^{18} \text{GeV})^2, \quad \Lambda > 0.
\]

The spinless field \( \chi \) plays the role of inflaton that mediates inflation and late-time acceleration, while \( L_{\text{SM}}(\psi, g_{\mu\nu}) \) is the standard model lagrangian density, \( \psi \) generically representing fields of standard particle theory: gauge bosons, Higgs boson, and fermions (leptons and quarks). The conformal function \( F(\chi) \) is assumed to be positive definite. The cosmological constant \( \Lambda \) is assumed to be positive. The Jordan metric frame is suitable when four dimensional theory descends from higher dimensional theories such as Kaluza-Klein unification [13], [14], and superstring theories.

There are various possibilities for the potential function \( V(\chi) \). Many popular models [4], [5], [6] of non-minimal kinetic terms \( K(\chi)(\partial \chi)^2 \) with a non-trivial function \( K \) in front are transformed to conformal gravity of the type [2] by a metric rescaling called Weyl transformation later elaborated. Many remarks in the present work are applied to these models, as well.

Physically more transparent is the Einstein metric frame obtained by a Weyl rescaling of the metric tensor, \( \tilde{g}_{\mu\nu} = F(\chi)g_{\mu\nu} \) that eliminates \( F \)-factor in front of the Ricci curvature in the Jordan frame. To simplify our notation, we replace the transformed metric \( g_{\mu\nu} \) in the Einstein frame by \( g_{\mu\nu} \), to derive the scalar-tensor gravity.
part;
\[
\frac{\mathcal{L}_{g\chi}}{\sqrt{-g}} = (-M_p^2 R(g_{\mu\nu}) + \frac{5}{2F(\chi)}(\partial\chi)^2 - \frac{1}{F^2(\chi)}(2M_p^2 \Lambda + V(\chi))). \tag{4}
\]

In this frame the gravitational constant or the Planck energy scale \(M_p\) is kept as an invariant constant with cosmological evolution. Different powers, \(1/F\) and \(1/F^2\), that appear in kinetic and potential terms are determined by the presence or the absence of inverse metric tensor \(g^{\mu\nu}\) in the Jordan frame. The original cosmological constant in the Jordan frame becomes a variable function \(\Lambda/\chi\) that is allowed to cosmologically evolve with inflaton field \(\chi\).

**Time evolution of inflaton field and Higgs boson mass**

The variational principle gives the inflaton field equation under the background of the Friedman-Robertson-Walker metric,

\[
\ddot{\chi} + \frac{3}{a} \dot{\chi} - \frac{1}{F'} \frac{\partial V_{\text{eff}}^{(E)}}{\partial \chi} = -\partial_{\chi} V_{\text{eff}}^{(E)}(\chi), \tag{5}
\]

\[
V_{\text{eff}}^{(E)}(\chi) = \frac{1}{5} \int_{-\infty}^{\chi} d\chi' F(\chi') \partial_{\chi'} V_{\chi g}^{(E)}(\chi'), \tag{6}
\]

\[
F \frac{\partial}{\partial \chi} V_{\chi g}^{(E)}(\chi) = \frac{1}{F} \left( \partial_{\chi} V - 2(V + 2M_p^2 \Lambda) \frac{\partial F}{\partial \chi} \right). \tag{7}
\]

The dot here means time derivative. We considered spatially homogeneous mode, hence time-independent equation. The effective potential \(V_{\chi g}^{(E)}(\chi)\) was derived from the force term of \(\ddot{\chi}\), and the original potential \(V\) may include the centrifugal repulsive potential when the multiple scalar field \(\chi\) exhibits spontaneous symmetry breaking as discussed in [7].

A choice of quartic polynomials for two functions, \(F(\chi), V(\chi)\), resolves in the simplest way the cosmological constant problem along with realization of slow-roll inflation and late-time acceleration [7].\footnote{We shall not restrict \(F(\chi), V(\chi)\) to these quartic forms in the present work in order to treat a wider class of models. But in this section we consider quartic polynomials as an example.}

The leading behavior of potential at large field values is of a logarithmic type,

\[
V_{\text{eff}}^{(E)}(\chi) \approx \frac{M_p^4}{5} \left( \text{constant} - \frac{g}{\xi_4} \ln \frac{\chi}{\chi_*} \right), \tag{8}
\]

with \(\chi_*\), a value of order the Planck energy \(M_p\) and \(g, \xi_4\) coupling constants that appear in the potential \(V\) and the conformal function \(F = \xi_4 (\chi/M_p)^4 \cdots\). After inflation a spontaneously broken symmetry is restored and the field \(\chi\) rolls down very slowly to the field infinity, giving a resolution of cosmological constant problem. During this monotonic inflaton motion the effective inflaton mass \(M_\chi\) changes from of order the Hubble energy \(O(10^{27})\) eV to of order the present Hubble constant \(O(10^{-33})\) eV. The product \(M_\chi \chi\) is kept constant in the radiation-dominated epoch, with a value \(M_\chi H_0 \sim (\text{a few meV})^2\), to give the present dark energy density of order (a few meV)\(^4\) as observed. Derivation of these and other results for quartic polynomial model is given in [7].

If a conformal gravity of this sort is to describe a hot big-bang after inflation, the inflaton field must couple to standard model particles such as Higgs boson, which further produces gauge bosons and light fermions realizing a thermal equilibrium characterized by a single temperature. If one takes the standard form of \(L_{SM}\), one is led in the Einstein frame to different powers, \(1/F\) and \(1/F^2\) for kinetic and potential terms, respectively, exactly in the same way as in eq. (4) for the inflaton. Inflaton coupling to Higgs doublet is given by the lagrangian density,

\[
V_{H}^{(E)} = \frac{\lambda_H}{4} H^2 - \left( |H|^2 - v^2 \right)^2. \tag{9}
\]

This gives both varying Higgs boson mass and Higgs coupling to inflaton. The Higgs mass is modified from the standard result in general relativity \(\sqrt{2\lambda_H v}\) to \(\sqrt{2\lambda_H/\xi_4 (M_p/\chi)^2} v\). Three-point vertex \(\chi H_0 H_0\) is important to realize thermalized hot big-bang after inflation.

**Dispersion and the Einstein relation for freely moving particles**

Before we proceed, let us go back and find out the core part of the problem. One may decompose the free field equation into Fourier modes of the form, \(\propto e^{-i\omega t + \vec{q} \cdot \vec{x}}\), to derive a particle field \(p\) under the FRW metric,

\[
\omega^2 + i \frac{3}{a} \omega - i \frac{F}{F'} \omega - \frac{q^2}{a^2} - \frac{m^2}{F^2} = 0. \tag{10}
\]

Dot in this paper means the time derivative \(d/dt\). Relative weight difference in kinetic and mass terms is reflected in the \(F\)–factor power, \(1/F_p\), of squared mass term. This is a dispersion relation between complex frequency (or energy) \(\omega\) and real 3-momentum \(\vec{q}\). During time span much shorter than variation time scales of \(F\) and \(a\), the analysis is simplest: the imaginary part signifies decaying or frictional behavior, and let us for the moment ignore this decay part, to derive a dispersion relation among real parts of the right-hand side in (10),

\[
\Re \omega_q = \sqrt{k^2 + \frac{m^2}{F_p^2}}, \quad k = \frac{\vec{q}}{a}. \tag{11}
\]

The negative energy solution exists, as well. The presence of cosmic scale factor \(q^2/a^2\) in this relation is well understood as the redshift of length scale associated with cosmic expansion, and indeed by re-defining a physical momentum by \(k = q/a\) we recover the usual dispersion relation.

Both electroweak gauge bosons and Higgs boson (as shown above) have the power \(p = 1\). For fermions one can square the Dirac spinor equation and derive Klein-Gordon equation. The Dirac mass follows the rule \(\propto 1/F^2\), unlike W-boson mass \(\propto 1/F\). Thus, its mass
ratio is proportional to $1/F$. Nucleo-synthesis is sensitive to a combination of weak interaction parameters, $\nu = G_F m_\nu^2 \propto g^2 m^2_\nu/m_W^2$, and concordance of theoretical calculation with observations places a bound on its variation, $|\delta \nu/\nu| < 0.06$. The naive Weyl scaling from the Jordan frame is thus ruled out from concordance of nucleo-synthesis calculation with observations. A special care on the metric frame in which the standard model is introduced must be taken in many popular models such as [4, 5, 6]. Otherwise, a thermalized hot universe after inflation might never occur in these models.

On the other hand, the stringent Oklo bound [15] is sensitive to the fine structure constant variation, $|\delta \alpha/\alpha| \lesssim O(10^{-7})$. This bound is not applicable to models we discuss in the present work.

**Theoretical framework for modifying the standard model lagrangian**

We need firm theoretical principles to formulate modification of the standard model lagrangian in the Jordan frame. Two principles to be respected are (1) the canonical quantization rule, and (2) the gauge invariance.

Before spelling out these principles, it is useful to introduce a fictitious counting rule applied to coordinate, its operation and field operators. We assign counting numbers, $Q_F$, to these, as if they were conserved quantum numbers:

$$Q_F = -1; \quad x_\mu, \partial_\mu = \frac{\partial}{\partial x^\mu}, A_\mu,$$

$$Q_F = 2; \quad g^{\mu \nu}, \quad Q_F = -2; \quad g_{\mu \nu}, \quad Q_F = 0; \quad H, \chi, \psi_f.$$

The rule is constructed such that

$$ds^2 = g_{\mu \nu} dx^\mu dx^\nu, \quad g^{\mu \nu} \partial_\mu \chi \partial_\nu \chi,$$

are invariant with their $Q_F = 0$ among other working hypothesis such as a well-defined $Q_F = -1$ for covariant derivative $\nabla_\mu$. There is some ambiguity of $Q_F$ assignment on fermion field $\psi_f$, but this ambiguity is irrelevant in subsequent discussion.

One can form bosons from fermions as bound fermion-antifermion pairs, hence it is imperative to insert the same $F$-factor for boson commutators and fermion anticommutators. Hence the equal-time quantization rules for canonical conjugate operators should satisfy a common relation for spinless boson $\phi(x)$, any fermion $\psi(x)$, and vector boson fields $A_i^a, i = 1, 2, 3$:

$$\left[ \phi(\vec{x}, t), \nabla_0 \delta^3(\vec{y}, t) \right] = F^3 \delta^3(\vec{x} - \vec{y}), \quad \left[ \psi_a(\vec{x}, t), \nabla_0 \delta^3(\vec{y}, t) \right] = \delta_{ab} F^3 \delta^3(\vec{x} - \vec{y}), \quad \left[ A_i^a(\vec{x}, t), \nabla_0 A_i^b(\vec{y}, t) \right] = \delta_{a b} g_i F^3 \delta^3(\vec{x} - \vec{y}),$$

with $\nabla_\mu$ covariant derivative. The canonical choice $\beta = 0$ is not excluded. The metric tensor component $g_{ij}$ should satisfy the relation $\partial^i g_{ij} = 0$, since we quantize gauge fields in the Coulomb gauge, but we shall keep it this way for simplicity. The common power $\beta$ here is reflected in creation and annihilation operators when the hamiltonian is written as bilinear forms for the free field parts.

We now address the question of gauge invariance. All masses in the standard model are generated by the Higgs mechanism formulated in a gauge invariant manner. There are five gauge invariant operators that appear in the standard model lagrangian, denoted by I $\sim$ V below:

$$I; \quad L_A = -\frac{1}{4} g^{\mu \nu} g^{\rho \sigma} \left[ (\partial_\mu A_\nu - \partial_\nu A_\mu + ig/2[A_\mu, A_\nu]) \times (\partial_\rho A_\sigma - \partial_\sigma A_\rho + ig/2[A_\rho, A_\sigma]) \right]$$

$$\times \left( \partial_\beta A_\gamma - \partial_\gamma A_\beta + ig/2[A_\gamma, A_\beta] \right) \right) \right)^{\dagger} \tilde{H}, \quad (18)$$

for non-Abelian and Abelian squared gauge field strength written in the matrix form, and for the Higgs kinetic term

$$II; \quad L_{dH} = g^{\mu \nu} \left[ (\partial_\mu - i g/2 B_\mu + g/2 \tilde{\tau} \cdot \tilde{A}_\mu) \right] \tilde{H}, \quad (19)$$

Higgs potential term is

$$III; \quad L_H = -\frac{\lambda}{4} (|\tilde{H}|^2 - v^2)^2. \quad (20)$$

The fermion kinetic term is

$$IV; \quad L_{df} = \bar{\psi}_f i \gamma_\mu \nabla^\mu \psi_f,$$

$$\nabla^\mu = g^{\mu \nu} (\partial_\nu - i (g/2 B_\nu + g/2 \tilde{\tau} \cdot \tilde{A}_\nu)). \quad (22)$$

Finally, fermion masses are generated from gauge invariant Yukawa coupling of Higgs field to fermion $\psi_f$:

$$V; \quad L_f = y_f \left( \bar{\psi}_f \frac{1 + \gamma_5}{2} \psi_f \cdot \tilde{H} + \text{h.c.} \right).$$

All gauge invariant operators are arranged to a common $Q_F = 0$ charge; namely they are $Q_F = 0$ singlets.

Gauge invariance and canonical quantization rule of quantum field theory do not preclude introducing arbitrary powers of $F$ in front of each gauge invariant operator. Anticipating the Weyl rescaling, we count the number of the inverse metric $g^{\mu \nu}$ and multiply the same number of $F$-factor to gauge invariant operators that belong to the same number of inverse metric. Our general proposal to modify the standard model lagrangian in the Jordan frame is to use for $L_{SM}/\sqrt{-g}$

$$F^{p_1} L_A + F^{p_2+1} (L_{dH} + L_{df}) + F^{p_3+2} (L_H + L_f). \quad (24)$$

The Weyl rescaling to the Einstein metric frame changes these powers to $p_1, p_2, p_3$. For particle energies and masses the factor $F$ arising from proper normalization of creation and annihilation operators from $|\phi^a| \sim |\phi^a|$ should also be taken into account. Nucleo-synthesis places a constraint: $(\beta + p_2)/2 = \beta - p_3$. A parametrization that takes this into account is $(\beta + p_1)/2 = -\epsilon_1, (\beta + p_2)/2 = \beta + p_3 = -\epsilon_2$. 


In the following table we list $F$–powers of particle energies, masses and the energy to mass ratio in the Einstein frame, when the Hamiltonian is written as bilinear forms of creation and annihilation operators.

| particles energy power mass power power of $E/m$ |
|-----------------------------------------------|
| $W,Z$ $F^{-\epsilon_1}$ $F^{-\epsilon_2}$ $F^{-\epsilon_1(1)}$ $F^{-\epsilon_2(1)}$ |
| $H_0$ $F^{-\epsilon_2}$ $F^{-\epsilon_2}$ |
| fermions $F^{-\epsilon_2}$ $F^{-\epsilon_2}$ |
| inflaton $F^{-1/2}$ $F^{-1/2}$ |

For reference we also listed in the parenthesis powers given by the naive Weyl rescaling, which however contradicts nucleo-synthesis constraint.

A further restriction to maintain the common Einstein relation $E = mc^2$ ($c$ being the light velocity) to all particles is desirable, and it leads to $\epsilon_1 = \epsilon_2 = \epsilon$. The Jordan-frame lagrangian is then

$$\frac{L_{SM}}{\sqrt{-g}} = F^{-2\epsilon} \left[ F^2 L_A + F(L_{dH} + L_{d\chi} + L_H + L_f) \right],$$

(25)

taking into account the change of quantization rule. When $\epsilon = 0$, the Einstein frame lagrangian density is $\sqrt{-g}$ times the standard model quantity, $L_A + L_{dH} + L_{d\chi} + L_H + L_f$. In this case particle energies and masses are exactly the same as those of GR, except the inflaton. But this solution is not favored, because it is impossible to solve the cosmological constant problem in the standard model lagrangian. A finite positive or negative $\epsilon$ value, however small it is, solves the dynamical cosmological constant towards zero in a class of conformal gravity models [7].

**Stronger gravity at early epochs** We formulated the problem of the dispersion relation and the mass variation in the Einstein metric frame in which the gravitational constant is kept invariant with the cosmic evolution. The dimensionless measure of gravity given by $G_N M^2$ or $G_N E^2$ for a body of mass $M$ and energy $E$ ($G_N E^2$ being relevant when the major part of constituent particles move with relativistic velocities) changes with the cosmological evolution. The gravity strength thus defined is different, depending on whether the major component of the clamp mass arises from ordinary baryons or from CDM (cold dark matter) inflatons. As in the model of [7], we assume for definiteness that the inflaton increases monotonically towards the field infinity, its dynamical mass following $\propto \chi^2 \propto (z+1)^4$.

First, when the clamp mass is energetically dominated by CDM inflaton,

$$G_N M^2, \ G_N E^2 \propto F^{-2}(\chi) \propto (z+1)^8, \quad (26)$$

Irrespective of the sign and magnitude of $\epsilon$, the growth rate of the effective gravitational strength towards earlier epochs is enormous for CDM clumps.

We could think of three major epochs where strong gravity manifests itself: (1) stochastic gravitational wave (GW) backgrounds throughout the entire cosmological history [10], (2) the preheating stage of inflationary epoch, [17], (3) the first order electroweak phase transition that may take place in extended Higgs models [18]. In (2) and (3) both GW emission and primordial black hole formation [19] may be copious. It is likely that even a tiny gaussian tail of fluctuation $M_{CDM}(0)$ provides a very strong gravity to CDM clumps. In the model of [6] the total amount of primordial black holes is limited by $O(\text{meV})^4$, hence created black holes do not over-close the universe. We feel it pressing to more seriously study gravitational collapse of the inflaton cold dark matter in the radiation-dominated universe.

Even at modest redshifts a high statistic data of GW emission from black hole mergers may provide a crucial test of strong gravity effect discussed here. In particular, the frequency distribution of events in terms of merger masses and redshifts have characteristic features of dependence, $\propto M_1 M_2$ and $\propto (z+1)^8$. One has to distinguish the primordial origin from astrophysical origins in data analysis.

Gravity strength of clamp mass $M$ made of baryonic matter changes according to

$$G_N M^2 \propto F^{-4\epsilon}(\chi) \propto (z+1)^{16\epsilon}. \quad (27)$$

Only when $\epsilon$ is positive and not too small, one can expect a substantial growth of gravitational strength at early epochs.

We note a promising opportunity in the near future. High statistics observations of GW emission from neutron star mergers [20] at different redshifts may provide an observational hint on the important parameter $\epsilon$, noting the gravity strength $G_N M_{NS}^2 (z+1)^{16\epsilon}, M_{NS} \sim 1.3 \times$ the solar mass.

**Summary and outlook** General relativity has been a remarkable success including recent detections of gravitational wave emission from merging black hole binaries [21]. Nevertheless, there is a deep conundrum related to a small, but finite cosmological dark energy of late-time evolution, which seems to require the scalar degree of freedom, presumably also related to inflation. Resolution of the fine-tuned cosmological constant problem along with other cosmological conundrums may find a solution in a class of conformal gravity theories, as outlined in [7].

We studied in the present work consequences of cosmology models based on, or interpreted as, a class of conformal gravity theories. Severe constraint from nucleosynthesis restricts how the inflaton field couples to standard model particles: the mass ratio of proton to W-boson must not change with cosmological evolution. We formulated how to evade this difficulty by modifying the standard model lagrangian in the Jordan frame and the canonical quantization rule of quantized fields. Our simplest solution is to introduce a slight modification of
the ordinary standard model lagrangian after the Weyl rescaling to the Einstein metric frame, and not in the Jordan frame. Many popular models interpreted as a sort of conformal gravity must obey the same rule if they are to describe a hot big-bang after inflation.

A sacrifice to pay, or an exciting possibility to the future, is the unexpected strong gravity in the early universe. It predicts much stronger gravitational wave emission and much more copious primordial black hole formation than GR predicts, if clumpy parts of fluctuation are primarily made of inflaton cold dark matter.

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