Z3RO Precoder Canceling Nonlinear Power Amplifier Distortion in Large Array Systems

François Rottenberg, Gilles Callebaut, Liesbet Van der Perre
KU Leuven, ESAT-WaveCore, Ghent Technology Campus, 9000 Ghent, Belgium

Abstract—Large array-based transmission uses the combination of a massive number of antenna elements and clever precoder designs to achieve array gain and spatially multiplex different users. These precoders require linear front-ends, and more specifically linear power amplifiers (PAs). However, this reduces energy efficiency since PAs are most efficient close to saturation, where they generate most nonlinear distortion. Moreover, the use of conventional precoders, such as maximum ratio transmission (MRT), induces a coherent combining of distortion at the user location, degrading the signal quality. In this work, a linear precoder is proposed that allows working close to saturation while canceling the coherent combining of the third order nonlinear PA distortion at the user location. In contrast to other solutions, the zero third-order distortion (Z3RO) precoder does not require prior knowledge of the signal statistics and the PA model. The design consists of saturating a single or a few antennas on purpose together with an opposite phase shift to compensate for the distortion of all other antennas. The resulting array gain penalty becomes negligible as the number of base station antennas grows large.

Index Terms—Large antenna systems, precoder, nonlinear power amplification.

I. INTRODUCTION

A. Problem Statement

The power amplifier (PA) in wireless communications accounts for a major part of the energy consumption [1], and its operation requires a trade-off between linearity and energy efficiency. On the one hand, the linearity of the PA is beneficial for the signal quality. On the other hand, the PA has a maximal efficiency when its operating point is close to saturation, where its characteristic is nonlinear and creates distortion [2]. Making a trade-off between these two aspects has always been a challenging problem [3]. Specifically in large array based transmission, such as massive MIMO in 5G, the combination of multicarrier transmission and precoding over a large number of antennas, leads to a high peak-to-average power ratio, requiring a large linear range of the PA [4].

More specifically, in large array systems, i.e., one of the key technology enabler of 5G, authors have shown that the PA nonlinear distortion is in general not uniformly radiated [5], [6]. It is known that, when the base station (BS) transmits in a dominant beamforming direction, the distortion is affected by the same array gain as the linearly amplified signal. This implies that nonlinear distortion can strongly limit the user performance, more specifically its signal-to-distortion ratio (SDR), when working close to saturation and using conventional precoders.

B. State-of-the-Art

A first solution to improve the SDR is to back-off power, i.e., to reduce the transmit power to let the PAs work further away from saturation and thus in a more linear regime. However, this strongly degrades the PA efficiency and reduces the transmit power, which can be problematic for users experiencing a large attenuation.

Another popular solution is to use digital pre-distortion (DPD) compensation techniques [7]. The idea is to pre-distort the signal that is at the input of the PA to compensate for its nonlinear distortion. These techniques are data dependent and often require a feedback. Applying these techniques requires a certain complexity burden and power consumption, especially in large antenna systems, where it has to be implemented for each PA [4]. Moreover, even a perfect DPD only linearizes input signal with an amplitude lower than the saturation level of the PA (weakly nonlinear effects). Higher fluctuations are clipped due to saturation, resulting in nonlinear distortion (strongly nonlinear effects). This phenomenon is more likely as the back-off power is reduced, especially for high Peak-to-Average Power Ratio (PAPR) signals such as multicarrier and massive multiple input multiple output (MIMO) systems.

As opposed to previous techniques, we propose a novel linear precoder, which compensates nonlinear distortion and can be implemented in addition to or instead of a DPD. A recent work [8] has followed a similar approach. However, no simple closed-form solution for the precoder was presented. The contributions are properly formalized in the next subsection.

C. Contributions

The limitations of the maximum ratio transmission (MRT) precoder are put forward. This precoder is obtained by maximizing the signal-to-noise ratio (SNR), without taking into account nonlinear distortion. As the PAs enter saturation, distortion appears and MRT is not optimal anymore. It even induces a coherent combining of distortion at the user locations.

To address the shortcomings of the MRT precoder, the zero third-order distortion (Z3RO) precoder is proposed. It is obtained by maximizing the SNR under the constraint that the third-order distortion is null at the user location. The precoder has, as MRT, a low complexity. Moreover, it does not depend on the PA parameters and the transmit signal statistics, which makes it simple to compute and to implement. It is obtained by saturating a given subset of antennas and applying an opposite phase shift to them. While using a single saturated antenna is optimal in terms of SNR, using a larger number of saturated antennas results in a more spatially focused distortion pattern, which is useful regarding unintended locations and total radiated distortion. Moreover, the SNR penalty of this precoder as compared to MRT becomes negligible in the large antenna
case. All code used to generate figures are in open-source at github.com/DRAMCO/3ro-precoder-single-user.

Notations: Superscript $(.)^*$ stands for conjugate operator. $j$ is the imaginary unit. The symbols $E(.)$, $\mathcal{E}(.)$, $\mathcal{R}(.)$ and $\mathcal{I}(.)$ denote the expectation, phase, imaginary and real parts, respectively.

II. SYSTEM MODEL

A. Signal Model

We consider a large array-based system with a single user and single BS equipped with $M$ antennas. The complex symbol intended for the user is denoted by $s$, with variance $p$. The signal $s$ is precoded at transmit antenna $m$ using a precoder coefficient $w_m$. The complex baseband representation of the signal before the PA of the corresponding antenna is denoted by $x_m$ and is given by $x_m = w_m s$.

B. Power Amplifier Model

In the following, all PAs are assumed memoryless and to have the same transfer function. For the sake of clarity and without loss of generality, the linear gain of the PA is set to one. We only consider the third order nonlinear distortion of the PA. This approximation regime is valid as the PA enters saturation regime, which creates nonlinear distortion but not enough for higher order terms to provide a significant contribution. Under these assumptions, the PA output of antenna $m$ can be written as

$$y_m = x_m + a_3 x_m |x_m|^2,$$ \hspace{1cm} (1)

where the coefficient $a_3$ characterizes the nonlinear characteristic of the PA, including both amplitude-to-amplitude modulation (AM/AM) and amplitude-to-phase modulation (AM/PM).

C. Channel Model

The complex channel gain from antenna $m$ to the user is denoted by $h_m$. The received signal is given by

$$r = \sum_{m=0}^{M-1} h_m y_m + v,$$ \hspace{1cm} (2)

where $v$ is zero mean circularly symmetric complex Gaussian noise with variance $\sigma_v^2$. In the following, at some places, a pure line-of-sight (LoS) channel to each user will be considered. In this particular case, $h_m$ can be written as

$$h_m = \sqrt{\beta} e^{-j\phi_m},$$ \hspace{1cm} (3)

The real positive coefficient $\beta$ models the path loss. For a narrowband system, the difference of propagation distance between each of the antennas and the user results in an antenna-dependent phase shift $\phi_m$, which can be directly related to the antenna location and the angular direction of the user. For a uniform linear array (ULA), the phase shift is given by $\phi_m = m \frac{2\pi}{\lambda_c} d \cos(\theta)$, where $\lambda_c$ is the carrier wavelength, $d$ the inter-antenna spacing and $\theta$ is the user angle.

The radiation pattern in an arbitrary direction $\hat{\theta}$ can be computed as

$$P(\hat{\theta}) = E \left( \sum_{m=0}^{M-1} y_m e^{-j\hat{\theta} \phi_m} \right)^2,$$ \hspace{1cm} (4)

where $\hat{\phi}_m = m \frac{2\pi}{\lambda_c} d \cos(\hat{\theta})$. Defining the total transmit power $P_T = \int_{-\pi}^{\pi} P(\hat{\theta}) d\hat{\theta}$, the array directivity is $D(\hat{\theta}) = \frac{P(\hat{\theta})}{P_T}$, i.e., $P(\hat{\theta})$ is normalized with respect to an isotropic radiator.

III. LIMITATIONS OF THE MAXIMUM RATIO TRANSMISSION PRECODER

The well known MRT precoder is obtained by maximizing the received SNR, disregarding the nonlinear distortion terms at the output of the PA, i.e., approximating $a_3 \approx 0$. Under this approximation, the output of the PA becomes $y_m \approx x_m$ and the received signal becomes

$$r \approx \sum_{m=0}^{M-1} h_m x_m + v = s \sum_{m=0}^{M-1} h_m w_m + v.$$

The received SNR is given by

$$\text{SNR} = \frac{p \sum_{m=0}^{M-1} |h_m w_m|^2}{\sigma_v^2}.$$ \hspace{1cm} (5)

The MRT precoder is found by optimizing the SNR under a total transmit power constraint

$$\max_{w_0,\ldots,w_{M-1}} \text{SNR} \quad \text{s.t.} \quad p \sum_{m=0}^{M-1} |w_m|^2 = pM.$$

The problem can be solved using, e.g., the Lagrangian multipliers technique, giving the MRT precoder

$$w_m^{\text{MRT}} = \alpha h_m^*,$$

$$\max_{\alpha} \text{SNR} \text{MRT} = pM \frac{\sum_{m=0}^{M-1} |h_m|^2}{\sigma_v^2},$$

where $\alpha$ is a normalization constant. Assuming a unit per-antenna channel variance $E(|h_m|^2) = 1$, the MRT achieves an array gain of a factor $M$ with respect to the transmit power $pM$. The MRT precoder is optimal as long as the PA works in its linear regime. As $p$ increases, nonlinear terms will be amplified and distortion becomes non-negligible. The PA output $y_m$ given in (1) can be evaluated for $x_m = w_m^{\text{MRT}} s$

$$y_m = s \alpha h_m^* + a_3 s^2 |s|^2 \alpha^3 |h_m|^2$$

and the received signal (2) becomes

$$r = s \alpha \sum_{m=0}^{M-1} |h_m|^2 + a_3 s^2 |s|^2 \alpha^3 \sum_{m=0}^{M-1} |h_m|^4 + v,$$

$^1$Note that the solution is equivalent up to a constant phasor applied to each antenna element. It is set to one here for simplicity.
where it can be seen that the channel coherently combines both the linear term and the nonlinear term, i.e., the phases are matched. As a result, distortion coherently adds up at the user location and becomes the limiting factor at high power.

To clarify this, let us consider a pure LoS channel (3) giving \( h_m = \sqrt{\beta} e^{-j\phi_m} \) and \( \alpha = 1/\sqrt{\beta} \). Then,
\[
y_m = e^{j\phi_m} (s + a_3 s |s|^2)
\]
\[
r = \sqrt{\beta} M (s + a_3 s |s|^2) + v,
\]
where it is clear that the array gain \( M \) affects both linear and nonlinear terms. Moreover, one can see that both the linear and nonlinear terms are beamformed in the same direction, as they are both affected by the term \( e^{j\phi_m} \). The radiation pattern can be evaluated using (4). An example of the directivity pattern of linear/nonlinear terms for MRT precoding and a ULA is shown in Fig. 1 (a). It illustrates that the directivity pattern of both linear and nonlinear terms is exactly the same.

IV. ZER0 THIRD-ORDER DISTORTION PRECODER

The MRT precoder induces a coherent combining of distortion at the user location, as demonstrated in the previous section. As \( p \) increases, the PAs will be more saturated and the user performance will be limited by its SDR. Since linear and nonlinear terms are affected by the same array gain, increasing the number of antennas does not help to improve the SDR. This section presents the ZER0 precoder which is designed to maximize the linear array gain to boost the SNR at the user location while canceling the third order distortion.

The received signal at the user location for a general linear precoder \( w_m \) is
\[
r = \sum_{m=0}^{M-1} h_m x_m + \sum_{m=0}^{M-1} \sum_{m=0}^{M-1} h_m x_m |x_m|^2 + v
\]
\[
= s \sum_{m=0}^{M-1} h_m w_m + \sum_{m=0}^{M-1} \sum_{m=0}^{M-1} h_m w_m |w_m|^2 + v.
\]
The distortion term can be forced to zero by ensuring that
\[
\sum_{m=0}^{M-1} h_m w_m |w_m|^2 = 0. \tag{6}
\]
Note that this constraint does not depend on the statistics of the transmit symbols \( s \) and the PA parameter \( a_3 \), which makes it practical to implement. A similar condition was obtained in [8]. However, the authors made the pessimistic conclusion that considering this constraint leads to a considerable reduction of array gain. Indeed, take the two antenna case \( M = 2 \) and a LoS channel \( h_m = \sqrt{\beta} e^{-j\phi_m} \). If the user angle is coming from broadside, it implies that \( \phi_0 = \phi_1 = 0 \) and the constraint (6) implies that
\[
w_0 |w_0|^2 = -w_1 |w_1|^2
\]
\[
|w_0|^3 e^{j\phi_0} w_0 = -|w_1|^3 e^{j\phi_1} w_1,
\]
which implies that \( w_0 \) and \( w_1 \) should have the same magnitude but opposite phase, e.g., \( w_0 = -w_1 \), which leads to a zero array gain, i.e., \( |w_0 + w_1|^2 = 0 \). The same result occurs for
any user angle, as depicted in Fig. 1(b). However, it is shown further that, with the proposed Z3RO design, as the number of antennas \( M \) grows large, the loss in array gain becomes negligible, as depicted in Fig. 1(c) and (d). In the following, we first derive the expression of the Z3RO precoder in the LoS case, before extending it to general channels.

A. Line-of-Sight Channel

For a LoS channel \( h_m = \sqrt{\beta} e^{-j\phi_m}, \) under constraint \( \sum_{m=0}^{M-1} |g_m|^2 = M \), the precoder optimization problem can be formulated as

\[
\max_{g_0, \ldots, g_{M-1}} \left| \sum_{m=0}^{M-1} g_m e^{-j\phi_m} \right|^2 = \frac{\beta p}{\sigma^2} \left| \sum_{m=0}^{M-1} e^{-j\phi_m} w_m \right|^2,
\]

under the two constraints

1. Transmit power: \( p \sum_{m=0}^{M-1} |w_m|^2 = pM \), \( (8) \)
2. Zero third-order distortion: \( \sum_{m=0}^{M-1} e^{-j\phi_m} w_m |w_m|^2 = 0 \), \( (9) \)

Note that \( (8) \) is not exactly a transmit power constraint, but rather the total power at the input of the PAs. Indeed, the nonlinear transmitted power is disregarded for simplicity and for the sake of comparison with the MRT precoder, which is found under a similar constraint.

The above problem is non convex and not trivial to solve. However, it can be first reformulated in a simpler form using the change of variable \( g_m = w_m e^{-j\phi_m} \). Then, without loss of generality, the optimization problem can be solved as a function of \( g_m \) instead of \( w_m \). The reformulated problem becomes

\[
\max_{g_0, \ldots, g_{M-1}} \left| \sum_{m=0}^{M-1} g_m \right|^2 \quad \text{s.t.} \quad \sum_{m=0}^{M-1} |g_m|^2 = M, \quad \sum_{m=0}^{M-1} g_m |g_m|^2 = 0.
\]

For an optimal solution \( g_m \), the corresponding \( w_m \) is given by \( w_m = g_m e^{j\phi_m} \). Moreover, from the above formulation, a conjecture can be made: an optimal \( g_m \) should be purely real up to a constant phasor. \( ^2 \)

If this phasor is set to one, the problem is converted to an all real problem

\[
\max_{g_0, \ldots, g_{M-1}} \left| \sum_{m=0}^{M-1} g_m \right|^2 \quad \text{s.t.} \quad \sum_{m=0}^{M-1} g_m^2 = M, \quad \sum_{m=0}^{M-1} g_m^3 = 0.
\]

The following theorem solves the problem \( (10) \) and gives the expression for the optimum precoder \( w_m = g_m e^{j\phi_m} \).

**Theorem 1.** Critical points of problem \( (10) \), providing a non zero array gain, are obtained by using a number of antennas \( M_s \) with an opposite phase shift and saturated in such a way that they compensate for the distortion due to all other antenna elements. This leads to the Z3RO precoder design:

\[
w_m^{Z3RO, M_s} = \alpha e^{j\phi_m} \begin{cases} -\left( \frac{M - M_s}{M_s} \right)^{1/3} & \text{if } m = 0, \ldots, M_s - 1, \\ 1 & \text{otherwise} \end{cases}
\]

\( ^2 \)Unfortunately, we could not demonstrate this conjecture yet even though it was found to be valid for extensive numerical simulations.

![Fig. 2: As the number of antennas increases, the penalty in array gain of the Z3RO precoder vanishes, as compared to MRT.](image)
to interference for potential observers, especially since PA nonlinearities induce out-of-band emissions. Hence, a user in an adjacent band could suffer from it. On the other hand, as the distortion becomes focused “approximately in the user direction, except for the exact user direction, where it is null by design”. Moreover, the total radiated distortion power is reduced. This comes from the fact that distortion is beamformed and benefits from an array gain. As a result, choosing the value $M_s$ offers a trade-off between array gain and spatial focusing of the distortion.

### B. General Channel

To extend the precoder design to a general channel $h_m$, a heuristic design is proposed, directly inspired by the previous one, and satisfying the zero third-order distortion constraint \((i)\).

#### Proposition 1.

An extension of the Z3RO precoder to a general channel is obtained as

$$w_{m}^{Z3RO,M_s} = \alpha h_m \begin{cases} -\gamma & \text{if } m = 0, \ldots, M_s - 1 \\ 1 & \text{otherwise} \end{cases},$$

where $\gamma$ is the additional gain of the saturated antennas

$$\gamma = \left( \frac{\sum_{m'=M_s}^{M-1} |h_{m'}|^4}{\sum_{m'=0}^{M_s-1} |h_{m'}|^4} \right)^{1/3}$$

and $\alpha$ is a power normalization constraint given by

$$\alpha = \frac{\sqrt{M}}{\sqrt{\sum_{m'=M_s}^{M-1} |h_{m'}|^2 + \gamma^2 \sum_{m'=0}^{M_s-1} |h_{m'}|^2}}.$$  

If the channel gains $h_m$ are assumed to be independent and identically distributed (i.i.d.) and so that $\mathbb{E}(|h_m|^2) = \beta$, as $M$ and $M_s$ grow large, the Z3RO precoder asymptotically achieves the same SNR performance as in Theorem 7.

**Proof.** One can check that the zero third-order distortion constraint \((i)\) is satisfied for the proposed precoder. To show that the precoder achieves the same SNR as given in (11), one can use the law of large number to show that, as $M \rightarrow +\infty$ and $M_s \rightarrow +\infty$, each sample average of i.i.d. elements converges to their expectation.

The Z3RO precoder proposed in Theorem 1 is found as a special case of a LoS channel. Hence, the same notation $w_{m}^{Z3RO,M_s}$ was used. Furthermore, it shows that, for i.i.d. channels and a similar averaged channel power, the same SNR performance as in LoS can be found, provided that the number of antennas $M$ and saturated antennas $M_s$ is large enough. An additional advantage of using a larger $M_s$ here is that it provides some diversity. This helps to prevent the case where saturated antennas would have a small channel gain. Indeed, a pathological case would be to use a single saturated antenna with low channel gain $|h_m|$, so that $\gamma$ would be very large and would consume more power. Using more saturated antennas allows to avoid these fades.

### V. Simulation Results & Discussion

Let us compare the MRT and the Z3RO precoders. The Bussgang theorem \cite{9, 10} implies that the received signal can be decomposed as $r = Gs + d + v$, where $d$ is the nonlinear distortion, which is uncorrelated with the transmit signal $s$ and noise $v$. The linear gain $G$ can be evaluated as $G = \mathbb{E}(rs^*)/p$. The signal variance is given by $|G|^2p$. Using the fact that $s$, $v$ and $d$ are uncorrelated, the distortion variance is $\mathbb{E}(|d|^2) = \mathbb{E}(|r|^2) - |G|^2p - \sigma_v^2$. The SNR, SDR and signal-to-noise-and-distortion ratio (SNDR) are thus given by

$$\text{SNR} = \frac{|G|^2p}{\sigma^2_v}, \quad \text{SDR} = \frac{|G|^2p}{\mathbb{E}(|d|^2)}, \quad \text{SNDR} = \frac{|G|^2p}{\mathbb{E}(|d|^2) + \sigma^2_v},$$

where the expectations can be evaluated using the statistics of the transmit symbols $s$. In the case where the saturated power of the PA $p_{sat}$ is varied while the other parameters are given in the figure caption. The signal $s$ has a complex Gaussian distribution, which models an OFDM modulated signal. For low values of $p/p_{sat}$, the PA is in the linear regime and the MRT achieves an optimal performance. The Z3RO precoder performs not as well given its

![Signal radiation pattern (dB), $M = 32$.](image1)

![Third-order distortion radiation pattern (dB), $M = 32$.](image2)
Reduced array gain. As the ratio \( p/p_{sat} \) increases, the PA enters the saturation regime and distortion becomes non-negligible. MRT is outperformed by the Z3RO precoder, which is only limited by distortion orders higher than three. In conclusion, the advantages of the Z3RO precoder versus MRT can be seen in two ways, in the saturation regime:

1) For a same SNDR, the Z3RO precoder can work at a larger ratio \( p/p_{sat} \), implying an enhanced energy efficiency. As an example, to achieve a SNDR of 15 dB the Z3RO precoder can work with a ratio \( p/p_{sat} \) about 1.5 dB higher.

2) For a same \( p/p_{sat} \), the Z3RO precoder achieves a larger SNDR, implying an enhanced capacity. As an example, for \( p/p_{sat} = -2 \), the SNDR can be boosted by about 2.5 dB.

VI. CONCLUSION

In this work, the Z3RO precoder is introduced to allow large array systems to work closer to saturation while compensating for nonlinear distortion. The design has a similar implementation complexity as MRT, while it cancels the third-order distortion, without requiring knowledge of the PA model and the channel statistics. Its array gain penalty is negligible in the large antenna case.

Perspectives include a novel precoder which: i) works in a multi-user setting, ii) compensates for higher order distortion terms, iii) takes into account PA differences and iv) relaxes the zero distortion constraint.

VII. APPENDIX

To solve problem (10), the Lagrangian can be formed

\[
L = \left( \sum_{m=0}^{M-1} g_m \right)^2 - \lambda \left( \sum_{m=0}^{M-1} g_m^2 - M \right) - \mu \sum_{m=0}^{M-1} g_m^3.
\]

Setting the derivative with respect to \( g_m \) to zero, a quadratic equation is obtained. Hence, only two values are possible for \( g_m \), that we denote by \( \alpha \) and \( \delta \). Consider that \( M_s \) is the number of coefficients \( g_m \) set to \( \delta \) and \( M - M_s \) are set to \( \alpha \), with \( M_s > 0 \) (otherwise the zero distortion constraint cannot be satisfied). For a fixed value of \( M_s \), applying the two constraints gives

\[
\alpha = \sqrt{M} \sqrt{(M - M_s) + M_s^{1/3}(M - M_s)^{2/3}},
\]

\[
\delta = -\alpha \left( \frac{M}{M_s} \right)^{1/3}.
\]

Hence, setting \( M_s \) values of \( g_m \) to \( \delta \) and \( M - M_s \) to \( \alpha \) give critical points of the Lagrangian. We fix \( M/2 > M_s \) to avoid symmetrical/equivalent solutions. Applying the change of variable \( w_m = g_m e^{i\phi_m} \) leads to the first result of Theorem [1]

Let us optimize the resulting array gain with respect to \( M_s \)

\[
\max_x M^2 \left( \frac{2x^2/3 - (1 - x)^2/3}{x^3/3 + (1 - x)^3/3} \right)^2,
\]

where \( x = M_s/M \in [0,1/2] \), with open intervals since \( 0 < M_s < M/2 \). The objective function monotonically decreases on the domain \([0,1/2]\), meaning that one has to choose the minimal \( x \) and thus \( M_s \) possible, i.e., \( M_1 = 1 \).

ACKNOWLEDGMENT

The research reported herein was partly funded by Huawei in the framework of the DASPOP project.

REFERENCES

[1] G. Auer, V. Giannini, C. Desset, I. Godor, P. Skillermark, M. Olsson, M. A. Imran, D. Sabella, M. J. Gonzalez, O. Blume, and A. Fehske, “How much energy is needed to run a wireless network?” IEEE Wireless Communications, vol. 18, no. 5, pp. 40–49, 2011.

[2] H. Enzinger, K. Freiberger, and C. Vogel, “A joint linearity-efficiency model of radio frequency power amplifiers,” in 2016 IEEE International Symposium on Circuits and Systems (ISCAS), 2016, pp. 281–284.

[3] P. M. Lavrador, T. R. Cunha, P. M. Cabral, and J. Pedro, “The linearity-efficiency compromise,” IEEE Microwave Magazine, vol. 11, no. 5, pp. 44–58, 2010.

[4] C. Fager, T. Eriksson, F. Barradas, K. Hausmair, T. Cunha, and J. C. Pedro, “Linearity and Efficiency in 5G Transmitters: New Techniques for Analyzing Efficiency, Linearity, and Linearization in a 5G Active Antenna Transmitter Context,” IEEE Microwave Magazine, vol. 20, no. 5, pp. 35–49, 2019.

[5] E. G. Larsson and L. Van Der Perre, “Out-of-Band Radiation From Antenna Arrays Clarified,” IEEE Wireless Communications Letters, vol. 7, no. 4, pp. 610–613, 2018.

[6] C. Mollen, U. Gustavsson, T. Eriksson, and E. G. Larsson, “Spatial Characteristics of Distortion Radiated From Antenna Arrays With Transceiver Nonlinearities,” IEEE Transactions on Wireless Communications, vol. 17, no. 10, pp. 6663–6679, 2018.

[7] S. C. Cripps, RF power amplifiers for wireless communications. Artech House, 2006, vol. 2.

[8] M. A. Imran, S. Jacobsson, U. Gustavsson, G. Durisi, C. Studer, and T. Eriksson, “Distortion-Aware Linear Precoding for Massive MIMO Downlink Systems with Nonlinear Power Amplifiers,” arXiv preprint arXiv:2012.13337, 2020.

[9] J. J. Bussgang, “Crosscorrelation functions of amplitude-distorted Gaussian signals,” Tech. Rep. 216, Research Lab. Electron, 1952.

[10] O. T. Demir and E. Bjornson, “The Bussgang Decomposition of Nonlinear Systems: Basic Theory and MIMO Extensions [Lecture Notes],” IEEE Signal Processing Magazine, vol. 38, no. 1, pp. 131–136, 2021.

[11] C. Rapp, “Effects of HPA-Nonlinearity on a 4-DPKSF/OFDM-Signal for a Digital Sound Broadcasting System,” in Second European Conf. on Sat. Comm., 22 - 24.10.91, Liege, Belgium., 1991, pp. 179–184, IIDO-Berichtsjahr=1991, pages=6.