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Abstract. The electronic states in the GaAs biconical (superellipsoidal) quantum dot (QD) in a strong and weak size quantization (SQ) regimes are theoretically investigated within the framework of the geometrical adiabatic approximation. An atypical linear term in the effective confining potential forms a family of non-equidistant sublevels in the electron energy spectrum, whereas the quadratic term leads to the appearance of the equidistant energy levels as a consequence of the symmetry of the QD. In the weak size quantization regime the motion of the exciton's center-of-gravity is quantized, which leads to the appearance of the additional Coulomb sub-levels in the energy spectrum of the QD. The direct interband absorption of light in the biconical QD is also considered in both strong and weak SQ regimes. The dependences of the absorption edge on QD geometrical parameters are revealed in the linear and quadratic approximations of the effective potential adiabatic terms. Corresponding selection rules for quantum transitions are obtained as well.

1. Introduction
Modern technologies allow one to grow QDs of various shapes and sizes [1,2]. The distinctive feature of these quasi-zero dimensional structures is a linear energy spectrum of the charge carriers (CC) in them similar to atoms, which is a consequence of the full quantization of CCs motion. In recent years, there are many theoretical and experimental works, where QDs having ellipsoidal symmetry are considered [3-12]. In particular, the physical properties of the CCs in both QDs having spherical shape (ellipsoid of revolution which differs little from a shere) and in the strongly flattened and elongated ellipsoidal QDs are studied. For the first time a quantum-mechanical problem of finding the CCs energy states in the slightly flattened (elongated) ellipsoidal quantum well was formulated and solved in the first-order perturbation theory by Migdal for the droplet model of the atomic nuclei [13]. Now the solution of this problem is already used for real QDs having spherical shape.

An exceptional result of the CCs energy spectrum in the QD with weakly expressed ellipsoidality is that the correction to the energy for different values of the quantum numbers (QNs) have different signs, which reconstruct a spectrum in a way that does not shift the center of gravity of the multiplet. It is known that even a small change in the external shape or size of the QD leads to the noticeable reconstruction of the CCs energy spectrum. Obviously, significant change in the geometric dimensions of the nanostructure leads to a fundamental change in the CCs spectrum. Thus, the effective confining parabolic potential may be formed due to the peculiarities of the QD's external form, e.g. QDs having the shape of strongly flattened (elongated) ellipsoid of revolution [14,15]. A similar situation is observed in thin lens shaped QDs [16-18]. It should be noted that the efficient parabolic potential forms due to the
geometrical adiabatic approximation application. For instance, in a thin lens shaped QD having a form of a segment of a sphere or an ellipsoid, the CC motion in the direction normal to the segment is much faster than in the plane of the base; this type of separation of the CC motion into the "fast" and "slow" parts is the essence of the geometric adiabatic approximation. As a result of the strong differences in size of QD along different geometrical directions, the family of equidistant sublevels appear above the levels of the "fast" subsystem in the energy spectrum of a QD. It should be noted also that in contrast to the Born-Oppenheimer approximation, in the case of the geometric adiabatic approximation the mass of a particle does not play a significant role. However, the symmetry of a nanosructure is also important. Thus, in the case of a strongly flattened (elongated) ellipsoidal QD, the energy dependence on the geometric parameters is significantly different than in the case of the thin lens shaped QD, although the geometrical adiabatic approximation is applicable in both cases. From this point of view, reducing the degree of symmetry of the external shape of the object leads to more complex dependence on the QD sizes of the particle energy. Therefore, the investigation of strongly elongated QD having superellipsoidal symmetry is important, in particular, QDs having biconical shape. On the other hand, the formation of new size quantized (SQ) sublevels significantly affect the character of the optical absorption in semiconductor structures, which is a powerful tool for determining many characteristics of these systems: bandgaps, effective masses of electrons and holes, their mobility, dielectric constants, etc. It is expected that even at the absence of external quantizing fields, due to the geometrical reconstruction of the CC spectrum, there should appear new quantum transitions, which are absent in spherical QDs. In turn, manipulating the sizes of QDs during their growth it is possible to achieve the implementation of two SQ regimes: strong and weak. The appearance of the excitonic effects should be also taken into account in the optical absorption spectra.

2. Electronic states
Consider a superellipsoidal QD having a biconical shape (figure 1). Then the potential energy of the CC (an electron, hole, exciton) can be written as

\[
U(x,y,z) = \begin{cases} 
0, & \frac{\sqrt{x^2 + y^2}}{a_1} + \frac{z}{c_1} \leq 1, \\
\alpha, & \frac{\sqrt{x^2 + y^2}}{a_1} + \frac{z}{c_1} > 1,
\end{cases}
\]

(1)

where \( a_1 \) and \( c_1 \) are a minor and major semiaxes of the bicone, correspondingly.

2.1 Strong SQ regime
First we solve the problem in the strong SQ regime, when the condition \( a_{es} \gg c_1 \) takes place, \( a_{es} \) is the effective Bohr radius. In this approximation the Coulomb interaction between an electron and hole is much less than the energy caused by the SQ contribution, and the Coulomb interaction between the electron and hole can be neglected. Then the problem reduces to the determination of CCs energy states separately. The system's Hamiltonian in cylindrical coordinates can be written as

\[
\hat{H} = \frac{\hbar^2}{2m_e} \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial Z^2} \right) + U(\rho,\varphi,Z).
\]

(2)

From the elongated geometrical form of the QD \( (a_1 \ll c_1) \) follows that the particle motion in the radial direction is faster than in the \( OZ \) direction. On this basis the system's Hamiltonian in dimensionless units can be represented as a sum of the "fast" \( \hat{H}_1 \) and "slow" \( \hat{H}_2 \) subsystems:

\[
\hat{H} = \hat{H}_1 + \hat{H}_2 + U(r,\varphi,z),
\]

(3)

where
\[ \hat{H}_1 = -\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \right), \quad \hat{H}_2 = -\frac{\partial^2}{\partial z^2}. \] (4)

Here \( \hat{H} = \frac{\hat{H}}{E_{\text{g}}} \), \( r = \frac{P}{a_\text{b}}, \quad z = \frac{Z}{a_\text{b}}, \quad E_{\text{g}} = \frac{\hbar^2}{2m^*_\text{e} a_\text{b}^2} \) is the effective Rydberg energy of an electron,

\( a_\text{b} = \frac{\kappa \hbar^2}{m^*_\text{e}} \) is the effective Bohr radius of an electron, \( e \) and \( m^*_\text{e} \) are the charge and effective mass of an electron, correspondingly, \( \kappa \) is a dielectric permittivity. The wave functions (WFs) of the problem is sought in the form

\[ \Psi(r, \varphi, z) = C e^{i\varphi} R(r; z) \chi(z), \] (5)

where \( C \) is a normalization constant. At a fixed value of the coordinate \( z \) of the "slow" subsystem the particle motion is localized in a two-dimensional potential well with an effective variable width

\[ r(z) = a \left( 1 - \frac{|z|}{c} \right), \] (6)

where \( a = \frac{a_\text{b}}{a_\text{b}}, \) and \( c = \frac{c_\text{b}}{a_\text{b}}. \) First, let us solve the Schrödinger equation of the "fast" subsystem, which after simple transformations reduces to the following Bessel equation:

\[ R''(r; z) + \frac{1}{r} R'(r; z) + \left( \epsilon(z) - \frac{m^2}{r^2} \right) R(r; z) = 0. \] (7)

The solutions of the equation (7) are the first kind Bessel functions

\[ R(r; z) = C J_m \left( \sqrt{\epsilon(z)} r \right), \] (8)

where \( C \) is a normalization constant, and \( m \) is a magnetic QN. From the boundary condition \( J_m \left( \sqrt{\epsilon(z)} r \right) \rvert_{r=0} = 0, \) and taking into account the expression (6), one obtains the energy of the "fast" subsystem:

\[ \epsilon(z) = \frac{\alpha_{n,m}^2}{a^2 \left( 1 - \frac{|z|}{c} \right)^2}, \quad n = 1, 2, ..., \quad m = 0, 1, .... \] (9)

For the lower levels of the energy spectrum, the CC motion is localized mainly in the vicinity of the geometrical center of the QD (interval \( |z| \ll c \)). Expanding \( \epsilon(z) \) in a power series around the point 0 these points one gets the expression

\[ \epsilon(z) = \frac{\alpha_{n,m}^2}{a^2} \left( 1 + \frac{2|z|}{c} + \frac{3|z|^2}{c^2} \right), \] (10)

which serves as an effective potential for the "slow" subsystem. In contrast to the majority of cases of the adiabatic approximation applications (see e.g. [14,17]), in the expansion (10) the linear term is absent. Obviously, this is a consequence of the conical symmetry of the superellipsoidal QD. Therefore, it is important to consider in detail the impact of both linear and quadratic terms of the effective potential on the "slow" subsystem motion.

**2.1.1 Linear approximation.** The effective potential energy (10) of the "slow" subsystem in this case can be written as

\[ \epsilon(z) = \frac{\alpha_{n,m}^2}{a^2} \left( 1 + \frac{2|z|}{c} \right). \] (11)
Since the linear term is the modulus of the coordinate (the triangular potential well), then the positive and negative $z$ coordinates needs to be discussed separately. In case of positive coordinates $|z| = z$, the potential $\varepsilon(z) = \varepsilon_0 + fz$, where $\varepsilon_0 = \frac{\alpha_{n,m}^2}{a^2}$, $f = \frac{2\alpha_{n,m}^2}{a^c}$. The Schrödinger equation of the "slow" subsystem

$$\left(-\frac{\partial^2}{\partial z^2} + \varepsilon_0 + fz\right)\chi(z) = \varepsilon\chi(z), \quad (12)$$

where $\varepsilon$ is the total energy of the electron in the QD. After change of the variable $\xi^+ = f^{\frac{1}{3}}z - f^{\frac{2}{3}}\varepsilon_1$, one gets the Airy equation

$$\chi^{\prime\prime}(\xi) - \xi\chi^{\prime}(\xi) = 0, \quad (13)$$

where $\varepsilon_1 = \varepsilon - \varepsilon_0$. The solutions of (13) satisfying the standard conditions imposed on WFs at $z \to +\infty$, are Airy functions of the first kind:

$$\chi^{\prime}(z) = C_1Ai\left(f^{\frac{1}{3}}z - f^{\frac{2}{3}}\varepsilon_1\right). \quad (14)$$

In the case of negative coordinates $|z| = -z$, the potential has the form $\varepsilon(z) = \varepsilon_0 - fz$. Changing the variable $\xi^- = -f^{\frac{1}{3}}z - f^{\frac{2}{3}}\varepsilon_1$, after the simple transformations the Schrödinger equation takes the form (13), and the solutions satisfying the standard conditions at $z \to -\infty$ can be written as

$$\chi^{-}(z) = C_2Ai\left(-f^{\frac{1}{3}}z - f^{\frac{2}{3}}\varepsilon_1\right), \quad (15)$$

where $C_1$ and $C_2$ are normalization constants. The electron total energy $\varepsilon_{lin}$ in linear approximation can be determined from the condition of continuity of the WFs logarithmic derivatives:

$$\left.\frac{\chi^{\prime}(z)}{\chi^{\prime\prime}(z)}\right|_{z=0} = \left.\frac{\chi^{-\prime}(z)}{\chi^{-\prime\prime}(z)}\right|_{z=0}. \quad (16)$$

### 2.1.2 Quadratic approximation.

Now consider the motion of the "slow" subsystem taking into account the parabolic term in the expansion (10). As in the previous case, consider the positive and negative $z$ coordinates separately. In the case of positive $z$ coordinates, the potential takes the form $\varepsilon(z) = \varepsilon_0 + fz + \gamma^2z^2$, where $\gamma = \frac{\sqrt{3}\alpha_{n,m}}{ac}$. After certain transformations, the Schrödinger equation can be written as

$$\chi^{\prime\prime}(z) + \left(\varepsilon_2 - \gamma^2\left(z + \frac{f}{2\gamma^2}\right)^2\right)\chi(z) = 0, \quad (17)$$

where $\varepsilon_2 = \varepsilon - \varepsilon_0 + \frac{f^2}{4\gamma^2}$. Changing the variable $\eta^\prime = \sqrt{2\gamma}\left(z + \frac{f}{2\gamma^2}\right)$ one gets the equation

$$\chi^{\prime\prime}(\eta) + \left(\frac{\varepsilon_2}{2\gamma} - \frac{\eta^2}{4}\right)\chi(\eta) = 0, \quad (18)$$

which solutions are the parabolic cylinder functions:

$$\chi^{\prime}(z) = C_3D_1\left(\sqrt{2\gamma}\left(z + \frac{f}{2\gamma^2}\right)\right), \quad (19)$$
where \( \lambda = \frac{\varepsilon_2}{2\gamma} - \frac{1}{2} \). For the negative values of the coordinate \( |z| = -z \), and the potential takes the form

\[
e^{-22} \mathcal{E} = \mathcal{E}_0 - \mathcal{E}z + \gamma^2 z^2.
\]

Then, changing the variable \( \eta = \sqrt{2\gamma} \left( z - \frac{f}{2\gamma^2} \right) \), one gets the equation (18), which solutions are the parabolic cylinder functions:

\[
\chi^{-}(z) = C_D z \left[ \sqrt{2\gamma} \left( z - \frac{f}{2\gamma^2} \right) \right].
\] (20)

As in the case of the linear approximation, in the quadratic approximation one gets the total energy of the electron \( e_{\text{quad}} \) from the condition of continuity of logarithmic derivatives (16) of the (19) and (20) WFs.

### 2.2 Weak SQ regime

Let us proceed to the weak SQ, when \( a_{\text{ex}} \ll a_1 \). Then the Coulomb interaction of the electron and the hole prevails over the SQ energy of the biconical QD walls in all geometrical directions. It means that the center-of-gravity motion of the exciton is quantized. The system's WFs can be represented as

\[
\Psi(\tilde{\rho}, \tilde{\varphi}) = \psi(\tilde{\rho}) \Phi_{n, l, m}(\tilde{R}),
\] (21)

where \( \tilde{\rho} = \rho_e - \rho_h \), \( \tilde{R} = m_e \tilde{\rho}_e + m_h \tilde{\rho}_h \), \( m_h \) is the effective mass of the hole. The case of the heavy hole is considered \( m_h \ll m_e \). In the expression (21) the \( \psi(\tilde{\rho}) \) describes the relative motion of the electron and the hole, and \( \Phi_{n, l, m}(\tilde{R}) \) describes the motion of the center-of-gravity of the exciton.

Taking into account the Coulomb interaction, the system's Hamiltonian in \( E_r = \frac{\hbar^2}{2\mu a_{\text{ex}}^2} \) units can be written as

\[
\hat{H} = -\frac{\mu}{M} \nabla^2_F - \frac{2}{\sqrt{r}},
\] (22)

where \( M = m_e^* + m_h^* \) is the mass of the exciton, \( \mu = \frac{m_e^* m_h^*}{m_e^* + m_h^*} \) is the reduced mass of the exciton. After separation of variables and solving the Schrödinger equation for the energy spectrum and the WFs of the relative motion of the exciton one gets known results:

\[
e_{\text{ex}} = -\frac{\mu}{M} \frac{1}{S^2}, \quad S = 1, 2, ...,
\] (23)

\[
\psi(r, \theta, \phi) = r^l e^{-\frac{r}{2}} F_l(-S, 2l + 2; r) Y_{l, m}(\theta, \phi),
\] (24)

where \( S \) and \( l \) are the main and orbital QNs, correspondingly, \( _l F_l(a, b; z) \) is the confluent hypergeometric function of the first kind, \( Y_{l, m}(\theta, \phi) \) are spherical functions [16]. One gets the energy \( e_{\text{ex}} \) of the motion of the exciton center of gravity, by repeating the above calculation procedure of the strong SQ regime for both linear and quadratic approximations, but with \( m_h^* \) being the exciton mass \( M \). It should be noted that formed effective potentials (both linear and parabolic) does not affect the Coulomb interaction between the electron and the hole, and affect only to the motion of the center of gravity of the exciton. In other words, in the weak SQ regime, the quantization of the motion of an exciton in the QD does not affect the internal structure of the exciton. Finally, the total energy of the system in the weak SQ regime can be written as
where $E_{Gr}$ is the energy of the exciton center of gravity quantization, determined from the condition (16).

3. Direct interband absorption of light. Let us consider direct interband absorption of light in the QDM in the strong SQ regime, when the Coulomb interaction between the electron and a hole is neglected. The case of a heavy hole is discussed, when $m_e' \ll m_h'$. The absorption coefficient [19] is determined by the expression

$$
K = A \sum_{\nu, \nu'} \left| \Psi_\nu^e \Psi_{\nu'}^h \right|^2 \delta \left( \hbar \Omega - E_{nu} - E_{nu'} \right)
$$

(26)

where $\nu$ and $\nu'$ are the QN sets corresponding to the electron and the heavy hole, $E_{\nu}$ is the bandgap of a bulk semiconductor, $\Omega$ is an incident light frequency, $A$ is a quantity proportional to the square of the matrix element taken by the Bloch functions. After simple calculations one gets the following expressions for the absorption edges (AE) of the linear and quadratic approximations $W_{110}$ and $W_{100}$, correspondingly

$$
W_{110} = 1 + \epsilon_{lin} \frac{d^2}{a_b^2},
$$

$$
W_{100} = 1 + \epsilon_{quad} \frac{d^2}{a_b^2},
$$

(27)

where $W_{(0)} = \frac{q_{(1)}}{E_{\nu}}$, and $d = \frac{\hbar}{\sqrt{2 \mu E_{\nu}}}$. The formula (27) describes the dependence of the effective bandgap on the semiaxes $a_1$ and $c_1$. With decreasing both semiaxes, the AE shifts to the short wave region, however, the dependence on minor semiaxis is much stronger.

Consider now the selection rules in quantum transitions. Quantum transitions for the energy levels allowed for the magnetic QNs $m = -m'$, and $n = n'$ for the QNs of the "fast" subsystem. The condition of continuity of logarithmic derivatives of the WFs (16) leads to the complete removal of the "slow" subsystem QNs selection rules. Note, that the analytical expression (27) is given taking into account above mentioned selection rules. Next, consider the interband absorption of light in the weak SQ regime. Since the exciton is localized in a relatively small vicinity of the QD geometric center, the absorption coefficient can be written the following expression [19]:

$$
K = A \sum_{m,n,l,w} \phi(0) \left| \Phi_{m,n,l,w}(\tilde{R}) \right|^2 \delta \left( \hbar \Omega - E_{nu} - E \right),
$$

(28)

where $E$ represents the energy (25) in dimensionless units. Note, that $\phi(0) \neq 0$ only for the ground state, when $l = m = 0$ ($l$ is the orbital QN of the exciton). In this regime one gets the following expressions for the AEs:

$$
W_{1101} = 1 + \epsilon_{lin}^0 \frac{h^2}{a_b^2} - \frac{\hbar^2}{a_{ex} a_{ex}'},
$$

$$
W_{1001} = 1 + \epsilon_{quad}^0 \frac{h^2}{a_b^2} - \frac{\hbar^2}{a_{ex} a_{ex}'},
$$

(29)
where \( W_{\{1\}0}^{(0)} = -\frac{\hbar \Omega_{\{1\}0}}{E_g}, \ h = \frac{\hbar}{\sqrt{2ME_g}}, \ a_{ex} = \frac{\kappa \hbar^2}{\mu e^2}, \) and \( a_{ex}^M = \frac{\kappa \hbar^2}{Me^2}. \) The most important feature of the latter case is that the shift of the exciton level with changes in the semiaxes of the biconical QD is determined by the total mass of the exciton.

4. Discussion of results

As noted above, there is a nonzero linear term in the expansion (10) of the energy of the "fast" subsystem due to the particular symmetry of the biconical QD. In the absence of a linear term, the effective potential turns parabolic for most cases of applications of the geometrical adiabatic approximation [18]. This in turn leads to the appearance of the own equidistant families of sublevels over each energy level of the "fast" subsystem. However, in our case a non-zero linear term in the expansion (10) leads to the appearance of non-equidistant sublevels. Figure 2 represents the dependence of the first two families of the energy levels of the CC in the biconical QD on the minor semiaxis \( a \) at a given value of the major semiaxis \( c \) in the case of the linear approximation.

A similar dependence of the electron energy on the major semiaxis \( c \) of the biconical QD is shown in the figure 3. As the figure shows, the increase in the major semiaxis leads to a significantly weak dependence due to small contribution of the SQ effects, in contrast to the dependence on the minor semiaxis. In other words, the SQ effects appear brighter in the radial direction as a consequence of the condition \( a_i \ll c_i \), as expected. On the other hand, the effect of the linear term in the effective potential on the CC motion is stronger only for the first two
families of the lower energy levels of the spectrum. Figure 4 shows the dependence of the effective and the real potentials on the coordinate $z$. As it is seen from the figure, the linear part perfectly is converged and approximated with the real potential only at small values of the coordinate $z$. For higher values of the energy, one must take into account the parabolic term, since the linear term strongly diverges the curve of the real effective potential. Obviously, accounting the parabolic term leads to more successful approximation of the real potential. However, it leads to reconstruction of the lower levels of the CC spectrum also. The presence of the parabolic term in (10) leads to a narrowing of the CC localization area and an enhancement of the SQ influence along the $OZ$ axis.

Figure 5 plots the dependence of the first two families of the electron energy on the major semiaxis for the linear approximation. As noted above, the equidistant sublevels of the "slow" subsystem are positioned over each level of the "fast" subsystem. It can be seen that interlevel distances of families increases with transition from the first to the second family, as expected, due to the interlevel distances dependence on the "fast" subsystem QN $n$. Interlevel distances also depend on the value of the semiaxes of biconical QD. With increasing the minor semiaxis the distance between the curves decreases, due to a decrease in the SQ impact in the radial direction. In this case also, the dependence of the CC energy on the small semiaxis appears brighter. However, the interlevel distances of equidistant levels do not depend on the mass of the CC, in contrast to the case of the quantum oscillator. As a consequence, the area of the CC localization is also does not dependent on the mass of the particle. Thus, the characteristic length of a quantum oscillator is $a_o = \sqrt{\frac{\hbar}{m_\omega}}$, where $\omega$ is the frequency of an oscillator. In our case, the parabolic potential arises due to the application of the adiabatic approximation, and the localization area is $a_{oa} = \sqrt{\frac{\hbar}{m_\omega a_{oa}}} = \sqrt{\frac{3\alpha_{oa}}{ac}}$; particle localization condition in the QD geometric center does not on the mass of the particle, but only on the geometrical parameters of the QD.

Weaker expressed energy dependence of the first two families of the equidistant spectrum on the major semiaxis of the biconical QD is shown on the figure 6. As it is seen, with an increase in the semiaxis $c$, the energy curves decrease slowly, due to the SQ weak influence in the perpendicular direction than in the radial direction. Thus, at $a = 1a_b$ and $c = 3.5a_b$, the interlevel distance for the first energy family ($n = 1$) is $\Delta E \approx 3.78E_x$, whereas at $c = 5a_b$ one has $\Delta E \approx 3.17E_x$. For comparison, at $c = 3a_b$ and $a = 0.5a_b$ (see figure 5) one has $\Delta E \approx 10.43E_x$ (the transition frequency is $\omega = 8.352 \cdot 10^{13}$ sec$^{-1}$ and corresponds to the infrared region of the spectrum), but at $a = 0.8a_b$ one has $\Delta E \approx 6.75E_x$ (the transition frequency is $\omega = 5.405 \cdot 10^{13}$ sec$^{-1}$). All numerical calculations are done.

Figure 5. The dependence of the first two families of the electron energy on the minor semiaxis for the linear approximation.

Figure 6. The dependence of the first two families of the electron energy on the major semiaxis for the quadratic approximation.
for the \textit{GaAs} QD with the following parameters: \( m'_e = 0.067 m_e \), \( m_e \) is a free electron mass, \( \kappa = 13.8 \), \( E_R = 5.275 \text{meV} \) and \( a_B = 104 \text{\AA} \).

An evident illustration of the difference in the linear and quadratic cases is demonstrated on the figure 7, which plots the dependence of the first two levels of the total energy of the electron on the minor semiaxis \( a \) at the given values of the major semiaxis \( c \). With increase in the minor semiaxis the curves decrease in both cases of the effective potential approximation, and the difference between them reduces. This is due to the SQ impact reduction in radial direction. The curves corresponding to the parabolic approximation are positioned higher, as expected, since the CC localization area is smaller than in the triangular quantum well, corresponding to the linear approximation (see figure 4). For small values of the semiaxis, the SQ effect is stronger, and the particle experiences the difference between the triangular and parabolic wells more stronger, which results in curves discrepancy. For large values of the \( a \), this effect weakens, and the curves corresponding to both approximations merges. A similar pattern is observed in the figure 8, which shows the dependence of the first two levels of the total energy of the electron on the value of the major semiaxis \( c \) at the given \( a \). Again, the energy curves of the CC corresponding to the parabolic approximation are positioned higher due to stronger SQ impact of the QD wells.

Figure 9 shows the dependences of the AE of the of direct interband absorption of light on the biconical QD minor and major semiaxes, respectively, in the strong SQ regime. With a decrease in semiaxes, the AE increases, which is a consequence of the SQ increase (the "effective" bandgap

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure7.png}
\caption{The dependence of the first two levels of the total energy of the electron on the minor semiaxis for the linear and quadratic approximations of the effective potential.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure8.png}
\caption{The dependence of the first two levels of the total energy of the electron on the major semiaxis for the linear and quadratic approximations of the effective potential.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure9a.png}
\caption{The dependences of the AE for the linear and quadratic approximations (a) on the minor semiaxis, (b) on the major semiaxis.}
\end{figure}
increases). As can be seen from the figures, the AE change appears brighter depending on the QD minor semiaxis. In both figures the curve of the parabolic case AE is positioned higher (blueshift) than the AE curve of linear approximation. This blueshift of the edge frequencies is the result of an increase in the effective bandgap due to the stronger confinement in the case of a parabolic approximation. Similar blueshift is observed in the case of a parabolic approximation for weak SQ regime, however, the confinement is not prevalent. In the weak SQ regime the exciton forms, which motion as a whole is influenced by the weak confinement of the QD walls. The accounting of Coulomb interaction leads to a decrease in the "effective" bandgap and a redshift of the AE (the negative excitonic term in (29)). Coulomb interaction between the electron and a hole results in the appearance of energy levels in the bandgap, which leads to the AE shift to the long wavelength region.

Finally, figure 10 illustrates the dependences of the AE on the biconical QD minor and major semiaxes in strong SQ regime at the given values of the major and minor semiaxes, correspondingly. As one can see, even a large variation in the semiaxis \( c \) leads to a small change in the edge frequencies, whereas even a small variation in the minor semiaxis \( a \) results in a sharp leap in the AE value.

5. Conclusion

In the framework of the geometrical adiabatic approximation the electronic states in the GaAs biconical QD have been theoretically investigated in a strong and weak SQ regimes. An atypical linear term in the "slow" subsystem effective confining potential has been revealed as a consequence of the QD's conical symmetry. It has been shown that the linear term in the effective potential forms a family of non-equidistant sublevels on each level of the "fast" subsystem in the electron energy spectrum, whereas the quadratic term leads to the spectrum reconstruction and appearance of the equidistant energy levels. The localization area or the characteristic length of oscillator has been shown to be independent on the particle mass (electron, hole, exciton). The dependences of the AE on the QD semiaxes values and the order of approximation of the effective potential have been studied as well. The appearance of additional Coulomb sublevels in the QD has been shown to result in the edge frequencies redshift, while the quadratic approximation leads to an additional AE blueshift. Partial withdrawal of the selection rules in quantum transitions has been revealed as well.

Acknowledgements

This work is supported by the NSF (HRD-1345219), NASA (NNX09AV07A) and in part by the M. Hildred Blewett Fellowship of the American Physical Society, www.aps.org.

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