The functional form of open Wilson lines in gauge theories coupled to matter

F. Gliozzi

Dipartimento di Fisica Teorica, Università di Torino, INFN, Sezione di Torino, via P. Giuria 1, 10125 Torino, Italy

The open Wilson lines are gauge-invariant operators made with a gauge transporter along an open path saturated at the end-points with matter fields. Here it is shown that numerical experiments on 3D $\mathbb{Z}_2$ Higgs model provide useful guidance in addressing the problem of the functional form of their vacuum expectation values. It turns out they satisfy, as long as their size does not exceed the string breaking scale, a remarkable factorization, related to finite-size scaling of the Fredenhagen-Marcus order parameter. This leads to conjecture a decay area law, like ordinary Wilson loops, with the difference that the boundary conditions of the confining string world-sheet are fixed along the path of the gauge transporter whereas are free on the straight line joining the two end-points. A consistency condition fixes uniquely the contribution of the free boundary, which turns out to be proportional to the string-breaking scale. Universal shape effects produced by quantum string fluctuations are also studied.

1. INTRODUCTION

The most basic observables of gauge theories are the Wilson loops. These are operators $W_\gamma$ associated to any closed path $\gamma$ of the space-time. Their importance stems from the fact that they serve as order parameters of confinement. The confining phase shows up in an area law for the vacuum expectation value of large Wilson loops: if $\gamma$ is scaled up keeping its shape fixed and increasing the encircled minimal area $A$, then $\langle W_\gamma \rangle$ vanishes exponentially with $A$: $\langle W_\gamma \rangle \propto e^{-\sigma A}$, where the physical quantity $\sigma$ can be interpreted as the tension of the confining string where the chromoelectric flux is concentrated. When dynamical matter is added to such a system this string breaks, because pairs of matter particles form and act as new end points of the confining string. The broken string state describes a bound state of a static colour source -the fixed end of the string- and a dynamical matter field -the free end of the string.

Contrary to earlier expectations, the string breaks only when its length exceeds a certain threshold value $R_b$. The reason, as we shall see shortly, is very simple. At intermediate distances the behaviour of the Wilson loop is indistinguishable from that of the confining phase of the pure gauge theory. Nonetheless, when the Wilson loop is very large it should decay with a perimeter law, hence one can consequently assume that

$$\langle W_\gamma \rangle \simeq c_u e^{-\sigma A_\gamma - \lambda |\gamma|} + c_b e^{-\mu |\gamma|}$$

where $|\gamma|$ is the length of $\gamma$. The first term, corresponding to the unbroken string regime, should be implemented with a shape-dependent factor due to quantum fluctuations of the confining string $[1]$. Such a universal shape effect has been observed also in a gauge theory coupled to matter $[2]$ as a further support to the conjecture that the confining string in this more general medium behaves exactly like the string in the pure gauge vacuum as long as it is shorter than $R_b$. Eq. (1) can be used to describe the main features of the string breaking mechanism as a level crossing phenomenon. In particular the string breaking scale turns out to be

$$R_b \simeq 2 \left( \frac{\mu - \lambda}{\sigma} \right),$$

as expected $[3]$. Notice that $\mu$ and $\lambda$, being self-energy contributions of the static sources, are UV divergent, however these divergences cancel in their difference, hence $R_b$ is a meaningful physical scale even in the continuum limit.

From a computational point of view it is very hard to observe string breaking directly through
the estimate of large Wilson loops, because Eq. (1) implies [4,7] that in the loops of finite size such a phenomenon is visible at distances much larger than \( R_b \), where the signal is drowned in the noise. As a matter of fact direct observation of string breaking by fitting the numerical data to Eq. (1) as been reported only in two 3d systems: the SU(2) gauge theory with adjoint sources [6] and the \( \mathbb{Z}_2 \) Higgs model [4] (see Fig. 1).

In gauge theories coupled to matter one can construct gauge invariant operators that are more general than the Wilson loops. They are formed by a gauge transporter along an open path \( \gamma \), saturated at the end points by the matter fields. We call these operators open Wilson lines. An important example is the U-shaped operator \( U(r, t) \equiv \) where the black dots represent the matter fields. It can be considered as an operator connecting the string state \( - \) of length \( r \) at the bottom \( t = 0 \) with the broken string state \( \bullet \bullet \) at the top, hence it is expected to have a large overlap with the broken string state. This suggested enlarging the basis of operators to extract the static potential by considering the correlation matrix

\[
C(r, t) = \begin{pmatrix} 1 & 1 & 1 \\ \end{pmatrix},
\]

In this way a rather abrupt crossover between string-like and broken string states has been clearly seen at the expected distance \( R_b \). So far string breaking has been verified in various theories coupled to a fundamental scalar field in 2+1 [5] and 3+1 dimensions [9], in QCD with two flavours [10], as well as breaking of the adjoint string in 2+1 [12] and 3+1 dimensions [13].

It is natural to ask what is the functional form of open Wilson lines which generalises Eq. (1) can be applied to any large planar open path \( \gamma \) joining two points placed at a distance \( r \)

\[
\langle G_r(\gamma) \rangle \simeq c_l e^{-\sigma A_{\tilde{\gamma}} - \lambda |\tilde{\gamma}|-R_b \sigma r/2} + \rho c_l e^{-\sigma A_{\tilde{\gamma}} - \lambda |\tilde{\gamma}|+R_b \sigma r/2} + \ldots
\]

where \( \tilde{\gamma} = \gamma + \ell \) is the closed path obtained by joining the two end points of \( \gamma \) with the straight line \( \ell \), \( A_{\tilde{\gamma}} \) is the area of the planar surface it encircles, \( c \) is (a generalisation of) the Fredenhagen-Marco order parameter [7], and the ellipses denote sub-dominant terms at intermediate distances which become visible only at large scale, where the asymptotic behaviour is dominated by the perimeter law, like in Eq. (1). The first term of Eq. (1) is similar to the first term of an older proposal [3] with the important specification that the coefficient of \( r \) in the exponent, which can be regarded as the self-energy contribution of the constituent Higgs particle, is here directly related to the string breaking scale. When \( \gamma = \ell \) then \( A_{\tilde{\gamma}} = 0 \) and we get the straight line operator \( \bullet \longrightarrow \bullet = G_r(\ell) \equiv L_r \). In such a case Eq. (1) is replaced by the following asymptotic ex-
Figure 2. Open Wilson line. The solid dots represent the matter fields; the shaded region denotes the world-sheet of the underlying confining string.

\[ \langle L_r \rangle \approx c_l e^{-\mu r} + \cdots \]  \hspace{1cm} (5)

where \( \mu \) is, like in Eq. (1), the mass of the ground state of the so called static-light meson. Eq. (4) can be still interpreted in terms of the underlying confining string (see Fig. 2). Clearly this has fixed boundary conditions only along \( \gamma \), while it is free to vibrate along \( \ell \). Also in this case we expect that the area term should be modified by universal shape effects due to quantum fluctuations of the string. An explicit form of such a contribution in the case of the U-shaped operator is calculated in §4.

2. ALMOST CLOSED WILSON LINES

The \( \mathbb{Z}_2 \) Higgs model which has been analysed in detail by a number of Monte Carlo simulations is defined through the action

\[ S = -\beta_G \sum_P U_P - \beta_I \sum_x \sum_{\mu=1}^3 \phi_x U_{x,\mu} \phi_{x+\mu} . \]  \hspace{1cm} (6)

Here \( U_{x,\mu} \) and \( U_P \) are link and plaquette variables respectively, which describe the gauge degrees of freedom; the site variables \( \phi_x \) represent the matter fields. Both \( U_{x,\mu} \) and \( \phi_x \) take values on the \( \mathbb{Z}_2 \) gauge group. This model is self-dual and has a confinement/Higgs phase and a Coulomb-like phase [14]. All the MC simulations are performed in the region \( 0 < \beta_G < 0.755, \ 0 < \beta_I < 0.248 \) which is called confinement “phase”, because the Wilson loops of intermediate size decay with an area law.

Figure 3. A comparison between square Wilson loops and U-shaped operators of the same size at \( (\beta_G, \beta_I) = (0.75245, 0.16683) \) where \( R_b \sqrt{\sigma} \sim 27 \).

Measuring \( \langle W(r, t) \rangle \) and \( \langle L_r \rangle \) yields an estimate of the string breaking scale \( R_b \). While \( \langle W \rangle \), as it turns out, does not depend on \( R_b \) at the probed scales, the U-shaped operator is very sensitive to it: \( \langle U(r, t) \rangle \) drops off rapidly as \( R_b \) increases (see Fig. 3). This is not unexpected: when \( R_b \to \infty \) the theory becomes a pure gauge theory, therefore \( \langle U \rangle \equiv 0 \).

In order to investigate in more detail such a dependence on the string breaking scale it is convenient to study a new family of gauge-invariant operators, that we call almost closed Wilson lines \( G_r(l, t) \), which somehow interpolate between the U-shaped ones and the Wilson loops (see Fig. 4). It turns out that the vacuum expectation value (vev) of these operators does not depend very
Figure 4. The almost closed Wilson lines interpolate between the Wilson loops and the U-shaped operators.

much on the position of the pair of matter sources \( \bullet \bullet \) along \( \gamma \). This can be expressed with the graphical equation

\[
\frac{\langle W(l,l) \rangle}{\langle G_r(l,l) \rangle} \simeq f(r) ,
\]

which, according to the results of Fig.5, seems to depend only on \( r \). The behaviour of such a function is drawn in Fig.6. More insight into \( f(r) \) will be gained in the next section.

Figure 6. The function \( f(r) \) defined in Eq.8 evaluated at \( (\beta_G, \beta_I) = (0.735, 0.224) \). The continuous line is a fit to \( \frac{c}{r}e^{-mr} \). The parameters \( c \) and \( m \) will be related to other physical quantities of the system.

3. FACTORIZATION

There is no reason to believe that the almost closed Wilson lines play some special role in gauge

depend, within the errors, on the size \( l \), as Fig.4 demonstrates. Although this numerical fact cannot directly promoted to the rank of a physical property of the continuum theory because of unbalanced self-energy divergences between numerator and denominator, by combining the almost closed Wilson line \( G_r(l,l) \) with the straight line operator \( L_r \) one is led to consider the following gauge-invariant, UV finite quantity

\[
\frac{\langle L_r \rangle \langle G_r(l,l) \rangle}{\langle W(l,l) \rangle} \simeq f(r) ,
\]

Evaluating the ratio \( \langle W(l,l) \rangle/\langle G_r(l,l) \rangle \) one discovers the surprising result that it does not depend on the size \( l \), as Fig.4 demonstrates. Although this numerical fact cannot directly be promoted to the rank of a physical property of the continuum theory because of unbalanced self-energy divergences between numerator and denominator, by combining the almost closed Wilson line \( G_r(l,l) \) with the straight line operator \( L_r \) one is led to consider the following gauge-invariant, UV finite quantity

\[
\frac{\langle L_r \rangle \langle G_r(l,l) \rangle}{\langle W(l,l) \rangle} \simeq f(r) ,
\]

which, according to the results of Fig.5, seems to depend only on \( r \). The behaviour of such a function is drawn in Fig.6. More insight into \( f(r) \) will be gained in the next section.

Figure 6. The function \( f(r) \) defined in Eq.8 evaluated at \( (\beta_G, \beta_I) = (0.735, 0.224) \). The continuous line is a fit to \( \frac{c}{r}e^{-mr} \). The parameters \( c \) and \( m \) will be related to other physical quantities of the system.

3. FACTORIZATION

There is no reason to believe that the almost closed Wilson lines play some special role in gauge
Figure 7. Splitting a Wilson loop into two open Wilson lines.

In order to lend some support to such a proposal we measured the ratios \( \langle \mathcal{L}_r \rangle \langle U(r,r) \rangle / \langle W(r,r) \rangle \), \( \langle U(r,r) \rangle^2 / \langle W(2r,r) \rangle \), and \( \langle G_{\gamma}^4(0) \rangle^2 / \langle W(r,r) \rangle \), where \( \mathcal{L}_r \) denotes the path obtained by cutting a square in half along its diagonal. Some results are presented in Fig. 8.

Though these numerical tests of Eq. 8 are manifestly very good, there are regions of the parameter space where the agreement is less good. The general rule seems to be that Eq. 9 is fulfilled, at least approximately, whenever the size of the involved loop is less than the string breaking scale. But even in this region Eq. 8 cannot reflect an exact property of the gauge theories. The reason is very simple: the vev of the Wilson loop \( W(\gamma) \) in the denominator depends explicitly on the shape of \( \gamma \) and this dependence cannot be counterbalanced by the shape effects of the numerator, being the splitting \( \gamma \to \alpha + \beta \) completely arbitrary. Such a fact will be strengthened in the next section by an explicit calculation of the universal shape effects generated by the quantum fluctuations of the underlying confining string in the U-shaped open Wilson line.

In conclusion, Eq. 9 has no chance to be exact. Rather, it should be viewed as the first term of an expansion in some scale parameter, perhaps \( 1/\sigma A_\gamma \). Note that if \( \gamma \) is scaled up to \( \infty \) keeping its shape fixed one gets (a generalisation of) the Fredenhagen-Marcu order parameter

\[
\lim_{r \to \infty} \frac{\langle G_r(\alpha) \rangle \langle G_r(\beta) \rangle}{\langle W(\gamma) \rangle} = \rho ,
\]

which vanishes in the Coulomb-like phase while it is different from zero in the confinement/Higgs phase.

Eq. 9 suggests the first correction to \( \rho \) should be a shape-independent term. Being only a function of the distance \( r \) of the two end points where are sitting the matter fields, it is conceivable that \( f(r) \) has the asymptotic expansion of a two-point correlation function, namely

\[
f(r) = \rho + \frac{c}{r^{d-2}} e^{-m r} + \frac{c'}{r^{d-2}} e^{-m' r} + \cdots ,
\]
which fits well to the numerical data as Fig. 6 and 8 show. The parameter $\rho$ is too small and can be put safely equal to zero in these fits. A better evaluation of $\rho$ will be obtained shortly.

It has to be noted that the “mass” $m$, though is a physical quantity, needs not to correspond to the mass of any state of the physical spectrum, being associated not to a single correlator, but to a suitable rational function of three of them. A previously unsuspected relation between $m$ and the string breaking scale will be uncovered below.

Neglecting for simplicity the pre-exponential factor $1/x^{d-2}$ and using as input the Ansatz (5) we get

$$ \langle G_r(\gamma) \rangle \simeq \langle W(\tilde{\gamma}) (ce^{(\mu-m)r} + \rho)/c_1 \rangle. \quad (12) $$

The first term shows that open Wilson lines obey a perimeter-area law which differs from that of the ordinary Wilson loops in two respects. First, the proportionality constant is different. Secondly, there is no analogue of the contribution of that part of the boundary where the string is free, encoded in the coefficient $c_f$ of $r$ in the exponential, which clearly is $c_f = \mu - \lambda - m$.

In order to find the link between $m$ and $R_b$ we alluded to above, the clue is provided by the observation that Eq. (9) implies the consistency condition

$$ \langle U(r,t) \rangle^2 \simeq \langle U(r,2t) \rangle \langle L_r \rangle \quad (13) $$

or, diagrammatically,

$$ \langle \Box \rangle \langle \Box \rangle \simeq \langle \Box \Box \rangle \langle \Box \rangle \quad (14) $$

It gives at once

$$ c = \frac{c_1^2}{c_a}; \quad c_f = -\frac{m}{2} ; \quad m = \sigma R_b \quad (15) $$

Combining Eqs. (12) and (15) we get finally Eq. (11), which was the major purpose of this talk.

In a nutshell, we may conclude that the surprising factorization properties of the vev of the open Wilson lines tell us nothing more than their behaviour at intermediate scales is dominated by the exponential law (4).

One could try to look for a more detailed solution of (9) by taking into account the pre-exponential factor $1/x^{d-2}$ of Eq. (11). Note however that such a kind of correction is of the same order of the already mentioned shape effects, which we know do not obey factorization, thus using factorization to find the sub-dominant corrections to the exponential behaviour is no longer justified.

An interesting property of the first two terms of Eq. (4) is that all the parameters but one (the Fredenhagen-Marcu order parameter) can be determined by evaluating the vev of ordinary Wilson loops or straight Wilson lines, hence the fits to this equations are one-parameter fits. As an example a fit to the U-shaped Wilson line is presented in Fig. 9. Note that the dependence on $\rho$ is strongly enhanced by an exponentially increasing factor.

4. UNIVERSAL SHAPE EFFECTS

It is known that the description of the vev of Wilson loops as a simple exponential like in (4) is not sufficiently accurate. Even in gauge theories coupled with matter the confining string, as long as its length does not exceed $R_b$, can vibrate freely exactly like in the absence of dynamical matter. It is then natural to conjecture...
ture that also in the open Wilson line $G_r(\alpha)$ one could detect universal shape effects generated by the quantum fluctuations of the confining string like in the ordinary Wilson loop. The important difference is that now the boundary conditions of the string are fixed only along the open path $\alpha$, whereas are free in the part of the boundary formed by the straight line connecting its endpoints.

In the case of the U-shaped operator it is easy to evaluate the contribution of these fluctuations. To be definite, consider the operator $\mathcal{U}(r, t)$, corresponding to the case $r = l$ of Fig.4. The boundary conditions are chosen accordingly. The universal shape effects are encoded in the quantity \[ \text{(11)} \]

\[
[\det(-\partial^2)]^{\frac{q-2}{2}} = | - \partial^2 |^{\frac{q-2}{2}},
\]

where $\partial^2$ is the Laplacian of the 2d bosonic field describing the string displacements in the $d - 2$ transverse directions with respect to the minimal surface. We have

\[
| - \partial^2 | = \prod_{m > 0} \prod_{n > 0} \sqrt{\frac{2}{n^2 + \frac{r^2}{4}(m - \frac{1}{2})^2}},
\]

which clearly needs regularization. A simple method \[ \text{(14)} \]

is based on the assumption that regularised series (or products) fulfill some formal properties of the absolutely convergent series. In particular, we assume the two relationships

\[
\sum_{n > 0} \frac{1}{n} = \ln(2), \quad \sum_{n > 0} \frac{1}{n^2} = \frac{\pi^2}{6},
\]

\[
\sum_{n > 0} \frac{1}{n^4} = \frac{\pi^4}{90}, \quad \sum_{n > 0} \frac{1}{n^6} = \frac{\pi^6}{945},
\]

where the apex indicates the regularised expression and the finite sum $\sum_{n=1}^{n-1} = P_{k+1}(a)$ is a polynomial of degree $k + 1$. For our purpose we need only to know $P_1(a) = a - 1$ and $P_2(a) = a(a - 1)/2$. Assuming they are valid for any real $a$ we get at once

\[
\sum_{n > 0} \frac{1}{n^k} = 2^k P_{k+1}(\frac{1}{2})/(2^{k+1} - 1),
\]

which turns out to coincide with the value of the Riemann zeta function $\zeta(-k)$. In this spirit we get the identities

\[
\prod_{n > 0} \frac{1}{n^k} = \exp[\log c \sum_{n > 0} \frac{1}{n^k}] = \frac{1}{\sqrt{c}}
\]

\[
\prod_{n \in \mathbb{N} + \frac{1}{2}} \frac{1}{n^k} = 1
\]

\[
\prod_{n > 0} \frac{n}{\sqrt{2\pi}} = \prod_{n > 0} \left(1 + \frac{r^2}{4}(m - \frac{1}{2})^2\right).
\]

Using the known identity

\[
\prod_{n > 0} \left(1 + \frac{\alpha^2 + \beta^2}{n^2} \right) = \frac{\sin \pi \alpha \beta}{\pi \alpha \beta} = e^{-\pi \alpha \beta / 2\pi(1 - e^{2\pi \alpha})},
\]

combined with Eqs \[ \text{(15)}, \text{(22)} \text{ and } \text{(23)} \]

leads us finally to the finite result

\[
| - \partial^2 | = \prod_{m > 0} \prod_{n > 0} \left[1 + \frac{r^2}{4}(m - \frac{1}{2})^2\right],
\]

which can be written in a number of equivalent ways:

\[
| - \partial^2 | = \sqrt{\frac{\theta_4(\tau)}{2\theta_4(\tau)}} = \frac{\eta(\tau/2)}{\sqrt{2} \eta(\tau)} = \frac{\eta(-2/\tau)}{\eta(-1/\tau)},
\]

where $\tau = i\frac{\sqrt{2}}{\pi}, \theta_4$ is a Jacobi $\theta$-function and $\eta(\tau) = q^{1/24} \prod_{n > 0} (1 - q^n)$ is the Dedekind’s eta function.

In conclusion, the first term of the asymptotic expansion \[ \text{(7)} \]

applied to the U-shaped Wilson line can be written in the form

\[
\langle \mathcal{U}(r, t) \rangle \propto F(t/r) e^{-r \tau/2(2t+r) \lambda - r R_{\sigma}/2} + \ldots
\]

\[
F(t/r) = \left(\frac{\eta(i\frac{t}{r})}{\eta(i\frac{t}{r})}\right)^{\frac{2n^2}{2}},
\]

As a check, it is easy to see that the static potential defined as $V(r) = -\lim_{l \to \infty} \log(\langle \mathcal{U}(r, l) \rangle)$ contains the $1/r$ Lüscher term with the right coefficient.

Observing these effects is very challenging. Strange though it may seem, boundary terms constitute the major obstruction to detect universal string signals.
Indeed finite size effects in interfaces, where the involved string world-sheet has no boundary, is the first place where these effects were clearly seen [17].

Similarly, in the Polyakov loop correlators the boundaries contribution does not depend on their distance; indeed in such a system are concentrated the main theoretical efforts to understand the physics of the confining string, after the introduction of the variance reduction method [18]. For a recent discussion on this argument and a complete list of references see [19].

In the case of Wilson loops the boundary term increases with the loop size, hence it is much more difficult to achieve accurate results. As a matter of fact only in very simple systems, like in 3D $Z_2$ gauge model [20] or even in percolation processes [21] high precision estimates have been reached.

In the case of open Wilson lines the situation dramatically worsen, the reason being that there are two competing requirements: the shape effects would be visible in sufficiently large open Wilson lines provided the string does not break, hence one has to chose a region where $R_b$ is large, however this implies, as Eq. (28) shows, an exponential weakening of the signal, because of the contribution of the free boundary. Studies are under way to gain more insight into these operators.

REFERENCES

1. M. Lüscher, K. Symanzik and P. Weisz, Nucl. Phys. B 173 (1980) 365; M. Lüscher, Nucl. Phys. B 180 (1981) 317.
2. F. Gliozzi, Nucl. Phys. Proc. Suppl. 94 (2001) 550 [arXiv:hep-lat/0010084].
3. W. Bock et al., Z. Phys. C 45, 597 (1990).
4. F. Gliozzi and A. Rago, Nucl. Phys. B 714 (2005) 91 [arXiv:hep-lat/0411004].
5. F. Gliozzi, JHEP 0508 (2005) 063 [arXiv:hep-th/0507016].
6. S. Kratochvila and P. de Forcrand, Nucl. Phys. B 671, 103 (2003) [arXiv:hep-lat/0306011].
7. K. Fredenhagen and M. Marcu, Phys. Rev. Lett. 56 (1986) 223.
8. O. Philipsen and H. Wittig, Phys. Rev. Lett. 81, 4056 (1998) [erratum ibid. 83, 2684 (1999)] [arXiv:hep-lat/9807020].
9. F. Knechtli and R. Sommer [ALPHA collaboration], Phys. Lett. B 440, 345 (1998) [arXiv:hep-lat/9807022]; F. Knechtli and R. Sommer [ALPHA Collaboration], Nucl. Phys. B 590, 309 (2000) [arXiv:hep-lat/0005021].
10. G. S. Ball, H. Neff, T. Duessel, T. Lippert and K. Schilling [SESAM Collaboration], Phys. Rev. D 71 (2005) 114513 [arXiv:hep-lat/0505012].
11. G. I. Poulis and H. D. Trottier, Phys. Lett. B 400, 358 (1997) [arXiv:hep-lat/9504015].
12. P. W. Stephenson, Nucl. Phys. B 550, 427 (1999) [arXiv:hep-lat/9902002]; O. Philipsen and H. Wittig, Phys. Lett. B 451, 146 (1999) [arXiv:hep-lat/9902003].
13. P. de Forcrand and O. Philipsen, Phys. Lett. B 475, 280 (2000) [arXiv:hep-lat/9912050]; K. Kallio and H. D. Trottier, Phys. Rev. D 66 (2002) 034503 [arXiv:hep-lat/0001020].
14. G. A. Jongeward and J. D. Stack, Phys. Rev. D 21 (1980) 3360; L. Genovese, F. Gliozzi, A. Rago and C. Torrero, Nucl. Phys. Proc. Suppl. 119 (2003) 894 [arXiv:hep-lat/0209027].
15. J. Ambjørn, P. Olesen, C. Peterson, Nucl. Phys. B 244 (1980) 365.
16. F. Gliozzi, unpublished, see P. Di Vecchia, NORDITA-77/18, Proceedings of the Bielefeld Summer Inst., 1976.
17. M. Caselle, F. Gliozzi and S. Vinti, Phys. Lett. B 302 (1993) 74 [arXiv:hep-lat/9212013].
18. M. Lüscher and P. Weisz, JHEP 0109, 010 (2001) [arXiv:hep-lat/0108014].
19. M. Caselle, M. Hasenbusch and M. Panero, JHEP 0510, 051 (2005) [arXiv:hep-lat/0510107].
20. M. Caselle, R. Fiore, F. Gliozzi, M. Hasenbusch and P. Provero, Nucl. Phys. B 486 (1997) 245 [arXiv:hep-lat/9609041].
21. F. Gliozzi, S. Lottini, M. Panero and A. Rago, Nucl. Phys. B 719 (2005) 255 [arXiv:cond-mat/0502339].