A Survey Length for AGN Variability Studies

S. Kozłowski

Astronomical Observatory, University of Warsaw, Al. Ujazdowskie 4, 00-478 Warszawa, Poland

Received June 25, 2021

ABSTRACT

The damped random walk (DRW) process is one of the most commonly used and simplest stochastic models to describe variability of active galactic nuclei (AGN). An AGN light curve can be converted to just two DRW model parameters – the signal decorrelation timescale $\tau$ and the asymptotic amplitude $S_F^\infty$. In principle, these two model parameters may be correlated with the physical parameters of AGN. By simulation means, we have recently shown that in order to measure the decorrelation timescale accurately, the experiment or the light curve length must be at least 10 times the underlying decorrelation timescale. In this paper, we investigate the origin of this requirement and find that typical AGN light curves do not sufficiently represent the intrinsic stationary process. We simulated extremely long ($10^4 \tau$) AGN light curves using DRW, and then measured the variance and the mean of short light curves spanning $1-10^4 \tau$. We modeled these light curves with DRW to obtain both the signal decorrelation timescale $\tau$ and the asymptotic amplitude $S_F^\infty$. The variance in light curves shorter than $\approx 30 \tau$ is smaller than that of the input process, as estimated by both a simple calculation from the light curve and by DRW modeling. This means that while the simulated stochastic process is intrinsically stationary, short light curves do not adequately represent the stationary process. Since the variance and timescale are correlated, underestimated variances in short light curves lead to underestimated timescales as compared to the input process. It seems, that a simulated AGN light curve does not fully represent the underlying DRW process until its length reaches even $\approx 30$ decorrelation timescales. Modeling short AGN light curves with DRW leads to biases in measured parameters of the model – the amplitude being too small and the timescale being too short.

Key words: Accretion, accretion disks – Galaxies: active – Methods: data analysis – quasars: general

1. Introduction

Variability of active galactic nuclei (AGN) is of high interest to astronomers at least for two distinct reasons. The first one is related to our desire of understanding physics leading to variability (e.g., Kawaguchi et al. 1998, Kelly, Bechtold, and Siemiginowska 2009, Ross et al. 2018). It seems that the amplitude of light fluctuations and timescales involved must somehow be related to physics of accretion
disks, to their size, to the accretion flow, and to the central black hole mass. This
is why it is critically important to accurately characterize an measure the variabil-
ity of AGN. The second reason, where our understanding of the variability itself
is of less importance, is to use it as a tool to measure, for example, time lags in
the reverberation mapping method (e.g., Peterson 1993). Then AGN light curves
can be modeled as either a stochastic or a deterministic process – a Fourier time
series, consisting of typically a large number of basic functions (sines or cosines,
e.g., Starkey et al. 2016). In such a case the model parameters are generally not the
prime information desired, but a good model describing the light curve.

We are interested here in accurately measuring variability of AGN light curves
that could be linked to the physical parameters of AGNs. One of the most widely
used models of the last decade has been the damped random walk (DRW) stochastic
process, characterized by just two model parameters – the signal decorrelation
timescale $\tau$ and the asymptotic amplitude $S_F^{\infty}$ (e.g., Kelly, Bechtold, and Siemigi-
nowska 2009, Kozłowski et al. 2010, MacLeod et al. 2010). While this model
is computationally fast and reproduces AGN light curves very well (in terms of $\chi^2$),
in Kozłowski (2016a) and Kozłowski (2017b), we presented and explored
a number of issues that arise when modeling AGN light curves with DRW. In
Kozłowski (2016b), we showed that DRW model is able to describe non-DRW
stochastic processes well. This means that we may obtain reasonable DRW fits
to data, while in fact we may not be dealing with the DRW process after all, and
hence the estimated model parameters may be simply meaningless. In Kozłowski
(2017b), we found that a light curve length must be at least 10 times longer than
the decorrelation timescale, otherwise the measured timescales are underestimated
and correlated with the survey length (the result confirmed by Suberlak, Ivezic, and
MacLeod 2021). We also showed that measured timescales from currently existing
($\approx$ decade-long) surveys are unlikely to be correct when using DRW, in particu-
lar for AGNs with massive black holes and/or at high redshifts as the timescale is
stretched by $(1 + z)$.

In this paper, which is the follow-up paper to Kozłowski (2017b), we are inter-
ested in solving the remaining issue of DRW – the origin of the necessity of AGN
light curves being longer than 10 times the decorrelation timescale. We will tackle
this problem by simulation means. In Section 2, we present our simulation setup
and the experiment. In Section 3, we discuss our findings, while in Section 4, we
summarize our results.

2. The Experiment

In this section, we will present our experiment that will lead us to an answer
why the light curves should be 10 (or more) times longer than the decorrelation
timescale. We will describe properties of the signal, the light curve simulator and
the procedure of modeling the data, a stationary process, and finally we will de-
scribe properties of the experiment.

2.1. The Covariance Matrix of the Signal

A light curve is a set of flux measurements taken over a certain time span. A relation between two data points \((i \text{ and } j)\) in the DRW stochastic process is governed by the covariance matrix of the signal

\[
S_{ij} = \sigma^2 \exp\left(-|t_i - t_j|/\tau\right),
\]

for epochs at \(t_i \text{ and } t_j\) (Kelly, Bechtold, and Siemiginowska 2009). The two model parameters are the signal decorrelation timescale \(\tau\) and the asymptotic amplitude \(\sigma\) or \(S_F = \sqrt{2}\sigma\) (MacLeod et al. 2010). Since the timescale and the amplitude are correlated, Kozłowski et al. (2010) introduced a parameter that is less correlated with \(\tau\) – the modified variability amplitude \(\hat{\sigma} = \sigma \sqrt{2/\tau}\).

2.2. The Light Curve Simulator

Generating a DRW light curve requires a starting point for the variable signal that is obtained as \(s_1 = G(\sigma^2)\), where \(G(\sigma^2)\) is a Gaussian deviate of dispersion \(\sigma\). Subsequent signal values are then iteratively calculated as

\[
s_{i+1} = s_i e^{-\Delta t/\tau} + G\left[\sigma^2 \left(1 - e^{-2\Delta t/\tau}\right)\right],
\]

where \(\Delta t = t_{i+1} - t_i\) (as in Kozłowski et al. (2010), Zu, Kochanek, and Peterson 2011, Kozłowski 2016b).

2.3. Modeling the Light Curve

The full explanation of how to model a light curve with DRW is presented in Appendix of Kozłowski et al. (2010), Zu, Kochanek, and Peterson 2011, and Zu et al. 2013, 2016. For completeness, we also present its basic concepts here.

An AGN light curve \(y(t)\) may be considered generally as a sum of the variable signal \(s(t)\) (having the covariance matrix \(S\)), the photometric noise \(n\) (having the covariance matrix \(N\)), and matrix \(L\) multiplied by a set of linear coefficients \(q\) that are used to subtract or add the mean light curve magnitude or to remove trends

\[
y(t) = s(t) + n + Lq.
\]

The likelihood of the data given \(s(t), q, \text{ and model parameters } \tau \text{ and } \hat{\sigma}\) is

\[
L(y|s, q, \tau, \hat{\sigma}) = |C|^{-1/2} |L^T C^{-1} L|^{-1/2} \exp\left(-\frac{y^T C^{-1} y}{2}\right),
\]

where \(C = S + N\) is the total covariance matrix of the data, and \(C^{-1} = C^{-1} - C^{-1} L (L^T C^{-1} L)^{-1} L^T C^{-1}\). To measure the model parameters, the likelihood \(L\) is optimized. To prevent our model from running into unconstrained parameters,
Kozłowski et al. (2010) and MacLeod et al. (2010) used priors on the likelihood of the model parameters $P(\tau) = 1/\tau$ and $P(\hat{\sigma}) = 1/\hat{\sigma}$, and we use the same priors here. Since in this experiment we are not interested in the impact of the photometric noise $n$ on our model parameters, we do not add the photometric noise to the data. Our light curve $y(t)$ is simply $s(t)$.

### 2.4. Do Light Curves Represent a Stationary Process?

A stationary process means that all its moments are independent of time. A weakly stationary process of $N$-th order requires all its moments up to $N$ to be time invariant. DRW is a weakly stationary process as it requires its mean, variance, and covariance (and hence the auto-correlation function, ACF) to be independent of time (Brockwell and Davis 2002). On the other hand, the Wold decomposition theorem states that any stationary process can be represented by an autoregressive process, where DRW is simply a first-order autoregressive process.

This has led us to a question if a short (as observed by a survey) light curve was indeed a sufficient representation of the “full” intrinsic process. As we will show this is the basic question to the whole process of light curve modeling with DRW.

### 2.5. Simulated Data

An answer to the above question can be obtained by simulation means. We decided to simulate long light curves (in terms of $\tau$) with the length of $10000\tau$, with the known amplitude $\sigma = 0.2$ mag, and the decorrelation timescale $\tau = 300$ d with two cadences of $3$ d ($100$ points per $\tau$) and $30$ d ($10$ points per $\tau$). The two light curves are presented in Fig. 1.

![Fig. 1. Simulated DRW light curves are shown (with $\sigma = 0.2$ mag, $SF = 0.28$ mag, and $\tau = 300$ d). They are sampled with 100 epochs (top panel) and 10 epochs (bottom panel) per decorrelation timescale $\tau$. While the full simulated light curves span $10000\tau$, here we present only a small fraction (1%) of their lengths.](image-url)
3. Discussion

We randomly draw short light curves from these two long light curves with the length between 1–1000τ. These short light curves will represent observed fractions of the DRW process by astronomical surveys. For each short light curve, we calculate the mean magnitude (shown in top panels of Fig. 2) and dispersions (shown in bottom panels of Fig. 2). From top panels of Fig. 2, we observe that the shorter the light curve the more is the mean magnitude is deviating from the input value. In the bottom panels of Fig. 2, we present the ordinary dispersion as a function of the light curve length. Starting the inspection of the panels from the right side (i.e., the long light curves), we see that the input and measured dispersions are nearly identical. Going in the direction of shorter light curves, we observe that dispersions in light curves becomes increasingly smaller. While it is difficult to pinpoint the exact moment for this transition it is safe to say it happens in the range 30-100τ. Both the mean and dispersion show that light curves shorter than ≈ 30τ no longer adequately represent the stationary process. This means they do not have the properties of the stationary process.

![Fig. 2. The mean magnitude (top panels) and dispersion (bottom panels) for short light curves as compared to the input values (dashed lines). The shorter the light curve, the higher the difference between the mean magnitude in the short light curve and the input value (top panels). The shorter the light curve, the smaller the measured dispersion in the light curve as compared to the input value (bottom panels). The dotted line shows the theoretical variance from Eq.(5) for the continuous DRW process.](image-url)
Fig. 3. Output DRW parameters are shown. Top panels: The amplitudes obtained from DRW as a fraction of the input values are shown against the amplitude (dispersion) derived simply from light curves as a fraction of the input values. DRW delivers the amplitude in accordance with the amplitude present in the data. It is clear (from Fig. 2) that low dispersion fractions represent short light curves, meaning that DRW underestimates amplitudes for short light curves. Bottom panels: The DRW time scales as a fraction of the input values are shown against the amplitude (dispersion) derived simply from light curves as a fraction of the input values. The shorter the light curve (the smaller the amplitude fractions) the more underestimated the DRW time scale as compared to the input value. The golden star marks the input value.

In Kozłowski et al. (2010), we presented the dependence between the variance in the continuous light curve and its length compared to the decorrelation timescale $\tau$:

$$\text{var}(x) = \sigma^2 \left[ 1 - \frac{2}{x} + \frac{2}{x^2} (1 - \exp(-x)) \right].$$  \hspace{1cm} (5)

where $x = \text{length}/\tau$ is the ratio of the survey duration to the timescale $\tau$. This dependence is presented in the bottom panels of Fig. 2 as the curved dotted line.

Next, we model these short light curves with the DRW model and measure the two model parameters. We present the results in Figs. 3 and 4. In top panels of Fig. 3, we show the ratio of the dispersion measured by the DRW model to the input dispersion as a function of the ratio of ordinary dispersion calculated from the light curves to the input dispersion. We can see that DRW generally obtains dispersions that are present in the data (as estimated by a simple dispersion calculation). In the bottom panels of Fig. 3, we show the ratio of the time scale measured with DRW to the input time scale as a function of the ratio of ordinary dispersion calculated...
from the light curves to the input dispersion. It is clear that the two parameters are correlated. The smaller the dispersion the shorter the timescale obtained by DRW.

In Fig. 4, we present the ratio of the time scale measured with DRW to the input time scale as a function of light curve length expressed in time scales $\tau$. We can see that for light curve lengths longer than about $30\tau$ ($\log(\text{length}_{LC}/\tau) = 1.5$) the DRW-measured time scales adequately represent the true value, while for the shorter light curves the time scale seems to be underestimated.

![Fig. 4. The DRW time scales as a fraction of the input values are shown against the light curve length as a fraction of the input time scale. The shorter the light curve, the more biased the measured DRW time scales – typically toward shorter values. The length of a light curve must be at least 10 times the decorrelation time scale $\tau$ to reasonably recover the intrinsic process parameters.](image)

In fact, it now appears that as early as Kozłowski et al. (2010) paper, we had all the ingredients to uncover the issues related to the data length and their impact on the DRW model parameters. From that paper, we knew that DRW is feasible to model both stochastic (DRW) and deterministic data sets (periodic stars), while in Kozłowski (2016b), we found that DRW models stochastic processes with other exponential covariance matrix equally well. From Kozłowski et al. (2010), we knew the variance of a light curve as a function of light curve length and $\tau$ (Eq. 5) and we also commented that $\tau$ and $\sigma$ are correlated. What can we learn from short light curves though? Since getting the correct DRW parameters seems unlikely, we may try other methods to uncover some information about variability. The key disadvantage of DRW is that it has a fixed shape of the covariance matrix of the signal. It is designed in such a way that gives rise to a power spectral distribution with the slope of $-2$ (PSD $\propto \nu^{-2}$), the so called red noise at high frequencies. This is reflected in the fixed slope of the structure function (SF) $\gamma = 0.5$ for $SF(\Delta t) \propto \Delta t^\gamma$, where $\Delta t$ is the time difference between data points.

There exists some evidence that observed PSD and SF slopes for AGN differ from the DRW values (e.g., Mushotzky et al. 2011, Kasliwal, Vogeley, and Richards 2017, Kozłowski 2016a, Caplar, Lilly and Trakhtenbrot 2017). In principle, the slope may depend on the physical parameters of AGN (e.g., Kozłowski 2016a, Simm et al. 2016). Can we measure these slopes from short light curves that do not represent a stationary process?
To test this, we measured structure functions for short light curves from our simulation (see a detailed elaboration on this topic in Kozłowski 2016a) and presented them in Fig. 5. The left column shows 10 individual SFs for very short (1.1τ, top panel), short (4.0τ, middle panel), and medium length (32τ, bottom panel) light curves taken from the original simulated high cadence light curves. The right column shows the corresponding density histograms based on 100 light

Fig. 5. Structure functions for light curve lengths of 1.1τ (top row), 4.0τ (middle row), and 32.2τ (bottom row). The left column shows ten individual SFs, while the right column shows density histograms for 100 SFs. The dashed line is the intrinsic SF or the simulated process (and not a fit).
curves. For the very short light curves, where the light curve length is comparable to the decorrelation time scale, the SFs are very noisy and obviously do not probe the bending SF. Obtaining the SF slopes and drawing conclusions based on individual SFs appears to be fruitless. Once the light curve length grows the situation improves significantly. For light curves spanning $32 \tau$, we may be able to estimate correct slopes (at short time scales) for individual objects. The full shape of SF can be measured from many light curves representing the same process (the bottom-right panel of Fig. 5). This is the so-called “ensemble” variability measurement (e.g., Vanden Berk et al. 2004, MacLeod et al. 2012, Vagnetti et al. 2016, Kozłowski 2017a, Li et al. 2018, Wang and Shi 2019). It only works under assumption that AGNs with the same physical parameters show the same variability properties.

4. Conclusions

In this paper, we identified the origin of problems that one encounters when modeling AGN light curves with DRW. By simulation means, we showed that typical light curves – that are of order of a decade long with cadences of 3–30 d – do not adequately represent the underlying stochastic DRW process, assuming AGNs do indeed generate DRW or DRW-like stochastic variability. Typical AGN light curves do not have the properties of a stationary process, assuming the intrinsic AGN variability is indeed due to such a process. Therefore, it may be difficult, if not impossible, to correctly reproduce the intrinsic process (to measure its parameters with DRW) having the incomplete information about it.

In particular, we showed that the shorter the light curve the smaller its variance as measured by standard procedure (Fig. 2). Then, we identified a strong correlation between that dispersion and the one measured from DRW modeling (Fig. 3). Since both DRW parameters are correlated, increasingly smaller dispersions in increasingly shorter light curves are reflected in increasingly shorter signal decorrelation time scales as measured by DRW.

The DRW stochastic process is mathematically sound concept. As a weakly stationary process, it requires its mean, variance, and covariance (the auto-correlation function) to be time invariant. Once the astronomical time-domain survey reach sufficient lengths to fulfill the stationarity requirements, only then the measured variability parameters will reflect the intrinsic ones.

Acknowledgements. S.K. acknowledges the financial support of the Polish National Science Center through the OPUS grant number 2018/31/B/ST9/00334.
REFERENCES

Brockwell, P.J., and Davis, R.A. 2002, “Introduction to Time Series and Forecasting”, 2nd Ed., New York, NY, Springer.

Caplar, N., Lilly, S.J., and Trakhtenbrot, B. 2017, *ApJ*, 834, 111.

Kasliwal, M.M., Vogt, M.S., and Richards, G.T. 2015, *MNRAS*, 451, 4328.

Kawaguchi, T., Mineshige, S., Umemura, M., and Turner, E.L. 1998, *ApJ*, 504, 671.

Kelly, B.C., Bechtold, J., and Siemiginowska, A. 2009, *ApJ*, 698, 895.

Kozłowski, S., Kochanek, C.S., Udalski, A., et al. 2010, *ApJ*, 708, 927.

Kozłowski, S. 2016a, *ApJ*, 826, 118.

Kozłowski, S. 2016b, *MNRAS*, 459, 2787.

Kozłowski, S. 2017a, *ApJ*, 835, 250.

Kozłowski, S. 2017b, *A&A*, 597, A128.

Li, Z., McGreer, I.D., Wu, X.-B., Fan, X., and Yang, Q. 2018, *ApJ*, 861, 6.

MacLeod, C.L., Ivezić, Ž., Kochanek, C.S., et al. 2010, *ApJ*, 721, 1014.

MacLeod, C.L., Ivezić, Ž., Sesar, B., et al. 2012, *ApJ*, 753, 106.

Mushotzky, R.F., Edelson, R., Baumgartner, W., and Gandhi, P. 2011, *ApJ*, 743, L12.

Peterson, B.M. 1993, *PASP*, 105, 247.

Ross, N.P., Ford, K.E.S., Graham, M., et al. 2018, *MNRAS*, 480, 4468.

Simunovic, M., Maksimovic, D., Poleski, R., et al. 2016, *A&A*, 585, A129.

Simm, T., Salvato, M., Saglia, R., et al. 2016, *A&A*, 585, A129.

Starkey, D.A., Horne, K., and Villforth, C. 2016, *MNRAS*, 456, 1960.

Suberlak, K.L., Ivezic, Z., and MacLeod, C. 2021, *ApJ*, 907, 96.

Vagnozzi, F., Middei, R., Antonucci, M., Paolillo, M., and Serafinelli, R. 2016, *A&A*, 593, A55.

Vanden Berk, D.E., Wilhite, B.C., Kron, R.G., et al. 2004, *ApJ*, 601, 692.

Wang, H., and Shi, Y. 2019, *Astrophysics and Space Science*, 364, 27.

Zu, Y., Kochanek, C.S., and Peterson, B.M. 2011, *ApJ*, 735, 80.

Zu, Y., Kochanek, C.S., Kozłowski, S., and Udalski, A. 2013, *ApJ*, 765, 106.

Zu, Y., Kochanek, C.S., Kozłowski, S., and Peterson, B.M. 2016, *ApJ*, 819, 122.