Electromagnetic Signatures of the Color Glass Condensate: Dileptons

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Abstract

We evaluate the invariant cross section for production of dileptons in forward rapidities at RHIC and LHC, using the Color Glass Condensate formalism and present results for the nuclear modification factor $R_{d(p)A}$ as a function of dilepton invariant mass for the most central deuteron (proton)-nucleus collisions.
1 Introduction

The recent results on suppression of hadron spectra in deuterongold (dA) collisions in the forward rapidity region [1] of the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Lab (BNL) has generated a lot of interest in the applications of semi-classical QCD and the Color Glass Condensate [2] to RHIC. Even though the Color Glass Condensate is the prediction of QCD for the wave function of a hadron or nucleus at high energies [3, 4, 5], it is not a priori clear at what energy this happens. There is some experimental evidence [6] that RHIC may be at just high enough energy to see glimpses of the Color Glass Condensate.

The applications of the Color Glass Condensate formalism to the heavy ion collisions at RHIC have been most successful at low $p_t$ [6] which probe the kinematic region where $x_{bj} \sim 0.01$ at mid rapidity. However, the mid rapidity region in heavy ion collisions is not the best place to look for the Color Glass Condensate because of the dominance of the final state effects, such as the energy loss of energetic partons [7] from the possibly formed Quark Gluon Plasma.

The forward rapidity region in dA collisions (the deuteron fragmentation region) is the best place in a hadronic/nuclear collision to probe the Color Glass Condensate [8]. First, there is presumably no Quark Gluon Plasma formed in a deuteron gold collision so that the dominant final state effects such the jet energy loss from the plasma are absent. Second, the forward rapidity region probes the small $x_{bj}$ part of the nuclear wave function and the large $x_{bj}$ part of the deuteron wave function. This is the ideal situation for the Color Glass Condensate probes since the high gluon density effects in the nucleus which give rise to the Color Glass Condensate are the strongest in this kinematics.

RHIC is a unique experiment in the sense that it has almost a continuous rapidity coverage, $0 < y < 4$, among its various detectors where it can detect various particles. The STAR detector can measure hadrons and photons in mid rapidity as well as at $y = 4$. The PHENIX collaboration can measure hadrons at mid rapidity as well as dimuons in the rapidity region between 1.2 – 2.2 while BRAHMS has measured hadrons at mid rapidity as well as rapidities of 1, 2.2 and 3.2. Therefore, one has the chance to map out the rapidity ($x_{bj}$) dependence of particle production and confront it with the predictions of the Color Glass Condensate formalism. Already, the qualitative agreement between the predictions of the Color Glass Condensate formalism [9, 10, 11] and the data from BRAHMS, both the suppression of $R_{dA}$ and its centrality dependence, are quite remarkable specially since all the available models in the market missed this suppression despite their many free parameters [12].

Electromagnetic probes such as photons and dileptons provide another tool, in addition to hadrons, with which to investigate the properties of the Color Glass Condensate [13]. They are cleaner in the sense that they do not undergo strong interactions with the other partons produced after the collision. Furthermore, in the case of hadrons, one needs to convolute the produced parton with the desired hadron fragmentation function. This means that a hadron measured at a given transverse momentum $p_t$ comes from the fragmentation of a parton at a yet higher transverse momentum $k_t = p_t/z$. Since the collision energy is
fixed, this takes one to higher momenta which correspond to higher $x_{bj}$ where the Color Glass Condensate effects become less dominant. Dileptons and photons do not suffer from this and are therefore, a better signature of the high gluon density effects.

In this short paper, we provide a numerical analysis of the dilepton production cross section in deuteron-gold collisions \cite{13} at RHIC and proton-lead collisions at LHC in the forward rapidity region. We consider $y = 2.2$ at RHIC where the PHENIX detector is located and $y = 5$ at LHC where there are plans to measure dileptons. We show our results for the absolute cross sections as well as the nuclear modification factor $R_{d(p)A}$ and show that, similar to hadrons, dilepton production is also suppressed in the forward rapidity region.

\section{Dilepton Production in $d(p)A$ Collisions}

Our starting point is the dilepton production in quark-nucleus scattering using the Color Glass Condensate formalism as derived in \cite{13} (see also \cite{14} for an equivalent approach). The diagrams corresponding to this process are shown in Fig. 1 where the virtual photon is emitted before, after or during the multiple scatterings of the quark from the target nucleus.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diagram1.png}
\caption{Dilepton production in quark-nucleus scattering.}
\end{figure}

It can be shown that the diagram where the virtual photon is emitted during the scattering of the quark from the nucleus is suppressed by the Lorentz factor $\gamma$ and does not contribute. Assuming the incoming quark is on shell and ignoring all the quark masses, one gets

\[
\frac{d\sigma^{A \rightarrow q\bar{q} \ell^+ \ell^-}}{dz \, d^2b_t \, d^2k_t \, d\log M^2} = \frac{2\alpha_{em}^2}{3\pi} \int \frac{d^2l_t}{(2\pi)^4} \hat{\sigma}_{\text{dipole}}(x, b_t, l_t) \left\{ \frac{1 + (1 - z)^2}{z} \frac{z^2 l_t^2}{[k_t^2 + M^2(1 - z)][(k_t - z l_t)^2 + M^2(1 - z)]} \right\}
\]
\[-z(1 - z) M^2 \left[ \frac{1}{k_t^2 + M^2 (1 - z)} - \frac{1}{(k_t - z t)^2 + M^2 (1 - z)} \right]^2 \]  

(1)

with \( l_t = q_t + k_t \) and \( q_t \) and \( k_t \) are the transverse momenta of the outgoing quark and dilepton pair respectively while \( z \) is the fraction of the energy of the incoming quark carried away by the virtual photon and \( M \) is the dilepton pair invariant mass. \( \hat{\sigma} \) is the Fourier transform of the dipole cross section given in (5).

To proceed further, we will integrate over the transverse momentum of the dilepton pair. This can be done analytically and simplifies the expression for the cross section considerably. The transverse momentum integrated cross section is also more useful experimentally since the production rate is quite small, at least at RHIC. Rewriting the dipole cross section in the coordinate space, we get

\[
\frac{d\sigma}{d^2 b_t dM^2} = \frac{\alpha^2_{em}}{2\pi^2} \int dz \frac{1 - z}{z^2} \int dr_t^2 \sigma_{dipole}(x_g, b_t, r_t) 
\left[ [1 + (1 - z)^2] K^2_1[\sqrt{1 - \frac{z}{z M r_t}}] + 2(1 - z) K^2_0[\sqrt{1 - \frac{z}{z M r_t}}] \right]
\]  

(2)

To relate this to deuteron (proton)-nucleus scattering, we need to convolute (2) with the quark (and anti-quark) distributions \( q(x, M^2) \) in a deuteron (proton). As shown in [14], this can be written in terms of the deuteron (proton) structure function \( F_2 \)

\[
\frac{d\sigma}{d^2 b_t dM^2 dx_F} = \frac{\alpha^2_{em}}{6\pi^2} \frac{1}{x_q + x_g} \int_{x_q}^{1} dz \int dr_t^2 \frac{1 - z}{z^2} F^d_{2}(x_q/z) \sigma_{dipole}(x_g, b_t, r_t) 
\left[ [1 + (1 - z)^2] K^2_1[\sqrt{1 - \frac{z}{z M r_t}}] + 2(1 - z) K^2_0[\sqrt{1 - \frac{z}{z M r_t}}] \right]
\]  

(3)

where

\[
x_q = \frac{1}{2} \left[ \sqrt{x_F^2 + 4 \frac{M^2}{s}} + x_F \right]
\]

\[
x_g = \frac{1}{2} \left[ \sqrt{x_F^2 + 4 \frac{M^2}{s}} - x_F \right]
\]

(4)

and

\[
F^d_{2}(x) = \sum_f x \left[q_f(x, M^2) + \bar{q}_f(x, M^2)\right]
\]

is the deuteron (proton) structure function with \( x_F \equiv \frac{M}{\sqrt{s}}[e^y - e^{-y}] \). Note that the sum over quark and anti-quark flavors is different for protons and deuterons. Here, we will use eq. (3) to calculate the dilepton production cross section in deuteron (proton) nucleus collisions. In case of a proton projectile, we use the GRV98 parameterization [15] of the structure
function $F_2^p$. We note that, in this kinematic region, the incoming quark or anti-quark distributions in a proton are well known and there is very little difference between different parameterizations. For a deuteron projectile, we use the HKM parameterization [16] of the $F_2^d$ structure function which includes shadowing. Again, since the projectile quarks and anti-quarks are in the large $x_{bj}$ region, the nuclear effects in the projectile deuteron are not large except at very large $x_{bj} \sim 0.7 - 0.9$ where shadowing can be a $20\% - 30\%$ effect.

The forward rapidity region in a $d(p)A$ collision probes small $x_{bj}$ gluons in the nucleus. Therefore, the target nucleus is treated as a Color Glass Condensate. To proceed further, we need to know the dipole cross section $\sigma_{\text{dipole}}(x_g, b_t, r_t)$. The dipole cross section satisfies the JIMWLK equation [17]. This equation has recently been solved on a lattice in [18]. Alternatively, one can solve the large $N_c$ limit of the JIMWLK equation, known as the BK equation [19]. This has been done numerically by several authors [20].

Alternatively, one can model the dipole cross section based on the known properties of the solution to the non-linear evolution equation in various limits. This has been done in [21] and a comparison to the HERA data on structure functions has been performed. It is shown in [21] that one can fit the HERA data in the kinematic region $Q^2 < 50\text{GeV}^2$ and $x_{bj} < 0.01$. Furthermore, this ansatz has a simple form which includes the right anomalous dimension in the extended scaling region [22, 23], unlike some previous parameterizations of the dipole cross section [24]. Having the right anomalous dimension is extremely important specially since the recent data from BRAHMS at RHIC may indicate that we are in the region corresponding to the linear evolution and the BFKL anomalous dimension.

In [21] parameterization, the dipole cross section has the following simple form

$$\int d^2b_t \sigma_{\text{dipole}}(x_g, b_t, r_t) \equiv 2\pi R^2 N(x_g, r_tQ_s)$$

(5)

where

$$N(x_g, r_tQ_s) = 1 - e^{-a \ln^2 b_tQ_s} \quad r_tQ_s > 2$$

and

$$N(x_g, r_tQ_s) = N_0 \exp \left\{ 2\ln \left( \frac{r_tQ_s}{2} \right) \left[ \gamma_s + \frac{\ln 2}{\kappa \lambda \ln 1/x_g} \right] \right\} \quad r_tQ_s < 2$$

(6)

The constants $a, b$ are determined by matching the solutions at $r_tQ_s = 2$ while $\gamma_s = 0.63$ and $\kappa = 9.9$ are determined from LO BFKL. The form of the saturation scale $Q^2_s$ is taken to be $Q^2_s \equiv (x_0/x)^\lambda \text{GeV}^2$ with $x_0, \lambda, N_0$ determined from fitting the HERA data on proton structure function $F_2$. We refer the reader to [21] for details of the fit. In case of a nucleus, we make the assumption that the saturation scale of the nucleus is $Q^2_{sA} \equiv A^{1/3}Q^2_{sp}$. We now have all the ingredients necessary to evaluate the dilepton production cross section as given by eq. (3).

It is instructive to consider the kinematic regions where a parameterization of the form used in [21] is appropriate. This parameterization is valid in the saturation region, defined by scales $M < Q_s$, and in the extended scaling region defined by $M < Q_{es}$ where
\( Q_{cs} \equiv Q_s^2/Q_0 \). In the case of the proton, the initial scale \( Q_0 \) is of the order of \( \Lambda_{QCD} \) while in the case of nuclei, it is \( Q_0 = Q_s(x_0) \) where \( \ln 1/x_0 \) is the initial rapidity where high gluon density effects become important. This is usually taken to be \( x_0 \sim 0.05 - 0.01 \). It is important to notice that the actual value of the extended scaling scale does not enter anywhere in our results, but that it just defines how high in dilepton invariant mass \( M \) we can go in this formalism.

To get a feeling for the saturation and extended scaling scales, we note that at \( y = 2.2 \), the extended scaling scale of a proton is \( \sim 3 \) GeV for \( M \sim 3 \) GeV. This is why, at RHIC, we show results for dilepton invariant masses up to \( M \sim 3 \) GeV. For rapidity \( y \sim 5 \) at LHC, the extended scaling scale goes up to 13 GeV.

We show our results for the absolute cross sections, as given in eq. (3), in Fig. (2). For comparison, we show the cross sections for both proton-proton and deuteron-gold scattering for different dilepton invariant masses at rapidity \( y = 2.2 \), appropriate for the PHENIX detector at RHIC.

![Graph showing dilepton production at RHIC: \( y = 2.2 \) and \( b_t = 0 \).](image)

**Figure 2:** Dilepton production at RHIC: \( y = 2.2 \) and \( b_t = 0 \).

It should be kept in mind that \( M \sim 3 \) GeV is quite likely at the edge of the extended scaling region for this kinematics while our formalism is best suited for the saturation and extended scaling region and could very well begin to break down as we go to higher dilepton masses. Also, since the saturation and extended scaling scales are larger for a nucleus than a proton, our results are probably more reliable for deuteron-gold collisions than proton-proton collisions for this particular kinematics. This will improve as we go to higher energies and/or higher rapidities such as those covered by LHC.

The nuclear modification factor \( R_{dA} \) defined as

\[
R_{dA} \equiv \frac{d\sigma^{dA \rightarrow l^+l^-X}/dy 
\dM^2 d^2b_t}{A^{1/3} d\sigma^{pp \rightarrow l^+l^-X}/dy 
\dM^2 d^2b_t}
\]
is shown in Fig. (3) for the most central deuteron-gold collisions. We also show the nuclear modification factor for the proton-gold collisions for sake of comparison. It should be noted that nuclear shadowing of the incoming deuteron wave function is taken into account in the HKM parameterization [16]. As is clear from the figure, there is a large difference between dilepton production in proton-gold and deuteron-gold collisions as there must be since the flavor and as a result, electric charge, decomposition of proton and deuteron are different.

![Graph showing R_{dA} and R_{pA} at RHIC: y = 2.2 and b_t = 0.](image)

Figure 3: $R_{dA}$ and $R_{pA}$ at RHIC: $y = 2.2$ and $b_t = 0$.

In Fig. (4), we show the dilepton production cross section in proton-proton and proton-lead collisions at LHC, at the rapidity of $y = 5$. We now go to dilepton masses as high as $M \sim 13$ GeV because the saturation scale is now much larger so that our formalism is valid for higher dilepton invariant masses.

In Fig. (5), we show our results for the nuclear modification factor $R_{pA}$ at LHC for the most central collisions proton-lead collisions. The suppression of the dilepton spectrum is much stronger than that at RHIC as expected since at LHC smaller $x_{bJ}$ is probed.

3 Discussion

Semi-classical QCD extends the domain of applicability of weak coupling (perturbative) QCD to high energies where the naive perturbative QCD approach fails due to high parton densities. The resulting state of a hadron or nucleus at high energy where parton densities are high is called a Color Glass Condensate. While the existence of such a high parton
Figure 4: Dilepton production at LHC: $y = 5$ and $b_t = 0$

Figure 5: $R_{pA}$ at LHC: $y = 5$ and $b_t = 0$. 
density state follows from QCD, the energy at which this happens cannot be derived, at the moment, from the theory itself and needs to be determined experimentally.

While there is some evidence in favor of Color Glass Condensate from HERA on electron-proton Deep Inelastic Scattering [21, 24], the BRAHMS collaboration at RHIC may have the best signature of the Color Glass Condensate in a nuclear environment so far in their measurement of the negatively charged hadrons in the forward rapidity region. To verify that this indeed the case, it is important to investigate the predictions of the Color Glass Condensate formalism for other processes such as dilepton production.

In this paper, we provide predictions for dilepton production cross sections in proton-proton and deuteron (proton)-nucleus collisions in forward rapidity regions at RHIC and LHC. The universal ingredient in this cross section, as well as cross sections for hadronic observables, is the quark anti-quark dipole- nucleus cross section which is subject to the non-linear evolution equation and which has recently been solved. In this work, we use an ansatz for the dipole cross section which has been successfully used to fit the HERA data on proton structure function. This model includes the physics of the BFKL anomalous dimension as well as scaling properties of the Color Glass Condensate and therefore, is well suited for our purpose.

There are several caveats in this work which need a more careful treatment than considered here. First, the dipole ansatz as given in [22] has been shown to work for a proton target, but has not been used for nuclei. In this work, we assumed that the $A$ dependence comes in through the saturation scale $Q_s^2 A^{1/3}$. Ideally, one would like to have this only in the initial shape of the dipole. The subsequent $A$ dependence will then be determined by the non-linear equations. There are some indications that as long as $x_{bj}$ is not too small, the assumed $A^{1/3}$ dependence may be fine for large nuclei [25].

Another caveat is that the dipole model used here does not have the correct high $p_t$ behavior. In other words, it does not match onto the Double Log DGLAP limit which is contained in the Color Glass Condensate. This may not be very important in the forward rapidity region since the $p_t$ coverage in this region is limited due to kinematics of the experiment.

Most importantly, it remains to be seen whether dileptons at low invariant mass (below $J/\psi$) peak can be measured at RHIC. This is most important since this is where our predictions are most reliable at RHIC. If and when RHIC future upgrades allow measurement of dileptons at higher rapidities, one can go to higher dilepton masses since the saturation scale is larger at more forward rapidities. Therefore, it is highly desirable to have the capability to measure dileptons (as well as photons) at RHIC in as forward rapidity as possible. Also, the STAR collaboration may be able to measure direct photons as well as photon + jets at rapidity $y = 4$ in the next deuteron-gold run at RHIC. The photon + jet process is unique in the sense it directly probes the dipole cross section. Color Glass Condensate predictions for this process will be presented elsewhere [26]. Finally, having another deuteron-gold run at RHIC in the near future will establish conclusively whether the Color Glass Condensate has been observed at RHIC, which seems to be the case, and will allow one to investigate its properties in detail.

Acknowledgments
We would like to thank R. Baier, F. Gelis, B. Kopeliovich, S. Kumano, A. Mueller and J. Raufeisen for useful discussions. This work is supported by DOE under grant number DOE/ER/41132.

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