Neutrino-nucleus interactions at low energies within Fermi-liquid theory

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Cross sections are calculated for neutrino scattering off heavy nuclei at energies below 50 MeV. The theory of Fermi liquid is applied to estimate the rate of neutrino-nucleon elastic and inelastic scattering in a nuclear medium in terms of dynamic form factors. The cross sections, obtained here in a rather simple way, are in agreement with the results of the other much more sophisticated nuclear models. A background rate from the solar neutrino interactions within a large Ge detector is estimated in the above-mentioned approach. The knowledge of the rate is in particular rather important for new-generation large-scale neutrino experiments.

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I. INTRODUCTION

Neutrino-nucleus reactions play an essential role in various fields of particle, nuclear and neutrino physics like, for example, neutrino oscillation, beta and double beta decay experiments, search for the dark matter, investigation of the nucleon and nuclear structure, etc. A precise knowledge of the neutrino-nucleus cross sections is necessary first of all to correct interpretation of the results of above-mentioned experiments. This is due to the fact that the neutrino-nucleus interactions in one experiment provide us with a signal (when we, for example, look for neutrino oscillations) and in another experiment constitute an inevitable background (when one, for example, looks for a dark matter signal). Therefore it is also obvious that the knowledge of neutrino-nucleus cross sections is very important when one studies the feasibility of a new neutrino project.

At rather low neutrino energies, comparable to the nuclear excitation energy, the neutrino-nucleus (νA) reactions are very sensitive to the nuclear structure and the form of strong nucleon correlations in a nucleus. At these energies (several MeV) the nuclear shell model is usually applied to calculate the νA cross sections (see for example [11,2,3,4,5]). This model reproduces the Gamow-Teller (GT) response for nuclei [6] and quite satisfactorily describes the available experimental data [7,8,9]. At higher energies the Random Phase Approximation (RPA) is used to analyze the νA processes corresponding to the neutral and charged weak currents. It describes the nuclear collective excitation one-hole excitations of the correlated ground state. This model and its modification, the continuum version CRPA, are appropriate at neutrino energies above the nuclear excitation energy [10]. All these models are applied to investigate various astrophysical aspects of nucleosynthesis, supernova collapse, observation of solar and supernova neutrinos as well. The most important ingredient of neutrino transport calculations in such applications the neutrino opacity at supra-nuclear densities.

Besides the above-mentioned nuclear models there are a lot of models also used to analyze neutrino propagation in dense matter (see for example [11,12,13,14,15] and references therein). They are very successfully applied to calculate the neutrino cross sections and neutrino mean free path in dense infinite nuclear matter and neutron matter. One of them is the model [12] based on the Fermi liquid theory suggested by L.D.Landau [16] and developed by A.B.Migdal [17], G.Baym and C.J.Pethick [18] and others. The strong nucleon correlation and the collective excitation mode are included in this theory [18,19].

In this paper we apply the Fermi liquid theory (FLT) to calculate the cross sections of neutrino scattering off heavy nuclei due to the neutral and charged weak currents at rather low neutrino energies (below the Fermi energy). In Sec. II we briefly present the general formalism for neutrino scattering in dense matter in terms of dynamic form factors (FF). In Sec. III we calculate the vector and axial vector FF for symmetric nuclear matter within the FLT for neutral and charged current neutrino processes. The procedure for the calculation of neutrino-nucleus cross sections is described in Sec. IV. To illustrate the available application of our approach we estimate the expected event rate induced by the interaction of solar neutrinos with the $^{71}$Ge isotope in a 1-kg target detector. In Sec. V we compare our results with the results of other different nuclear models. The conclusion is presented in Sec. VI.

II. GENERAL FORMALISM

The elastic or quasi-elastic neutrino-nucleon scattering is due to the weak neutral current. According to the Weinberg-Salam model, the Lagrangian of such interaction has the form [12,20,21,22]

$$\mathcal{L}_i(x) = \frac{G_F}{\sqrt{2}} j_\mu^\nu(x) j^\mu_\nu(x),$$

(1)
where $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ = 1.436 $\times$ 10$^{-10}$ erg cm$^3$ is the Fermi weak coupling constant,

$$ l^\mu_\nu(x) = \bar{\psi}_\nu \gamma_\mu (1 - \gamma_5) \psi_\nu \quad (2) $$

is the lepton weak neutral current,

$$ j^\mu_\nu(x) = \frac{1}{2} \bar{\psi}_N \gamma_\mu (C_V - C_A \gamma_5) \psi_N \quad (3) $$

is the third component of the isospin current, $C_V$, $C_A$ are the vector and axial coupling constants for the neutral current reactions.

For the inelastic $\nu N$-scattering the Lagrangian is related to the charged lepton and baryon weak currents

$$ \mathcal{L}_i(x) = \frac{G_F}{\sqrt{2}} l^\mu_\nu(x) j^\mu_W(x), \quad (4) $$

where

$$ j^\mu_W(x) = \bar{\psi}_N \gamma_\mu (V - g_A \gamma_5) \psi_N. \quad (5) $$

Here $g_V$, $g_A$ are the vector and axial-vector weak coupling constants for the charged current reactions. The lepton weak charged current has the following form

$$ l^\mu_\nu(x) = \bar{\psi}_\nu \gamma_\mu (1 - \gamma_5) \psi_\nu. \quad (6) $$

At the low neutrino energies we may use the following approximation for the neutral hadronic current $^{12}$

$$ \bar{\psi}_N \gamma_\mu (C_V - C_A \gamma_5) \psi_N \rightarrow C_V \bar{\psi}_N^\dagger \psi_N^\dagger \delta^\mu_0 - C_A \bar{\psi}_N^\dagger \sigma_\mu \psi_N \delta^\mu_t. \quad (7) $$

The similar approximation can also be used for the charge hadronic current. The rate of neutrino-nucleon scattering in a medium at low energies can be presented in the following form (see, for example, $^{12}$, $^{14}$, $^{15}$)

$$ W_{fi} = \frac{G_F^2 \alpha}{4V} [C_V^2 (1 + \cos \theta) S_V(q, \omega) + C_A^2 (3 - \cos \theta) S_A(q, \omega)], \quad (8) $$

where $\theta$ is the scattering angle, $V$ is the normalized volume, $\alpha$ is the nuclear density, $S_V$ and $S_A$ are the vector and axial-vector dynamic form factors (FF), which depend on the transferred 3-momentum $q$ and the energy transfer $\omega$.

In principle, the hadronic vector current $\bar{\psi}_N^\dagger \psi_N \delta^\mu_0$ and the axial vector current $\bar{\psi}_N^\dagger \sigma_\mu \psi_N \delta^\mu_t$ can be modified in a medium. The vector and axial coupling constants $C_V$ and $C_A$ undergo renormalization; however, their values can be changed in a medium within 10–15% $^{23}$ $^{22}$. Therefore we neglect this effect. We also neglect the antisymmetric part $\mathcal{L}^{\eta}(q, \omega)$ of the spin-spin dynamic form factor because it is proportional to $q^2/m^2$ $^{12}$ and use only its symmetric part $\mathcal{L}_{ij}(q, \omega) = \delta_{ij} S_{\eta}(q, \omega)$.

### III. Dynamic Form Factors in Neutrino Processes

#### A. Neutral currents

First, we consider the FF for the neutral current neutrino processes, e.g., the elastic and quasi-elastic neutrino-nucleon scattering in a medium. The FF for a one-component infinite nuclear system were constructed in Ref. $^{12}$ to compute the neutrino mean free path in pure neutron matter for the densities of relevance for neutron stars. In Ref. $^{23}$ the FF were obtained for two-component nuclear matter consisting of protons and neutrons also within the FLT. Within the RPA the FF in isospin-symmetric nuclear matter (SNM) were calculated in Ref. $^{14}$ by applying the Dyson type equation for the vector and axial vector polarization operators. Here we will demonstrate how the procedure suggested in Ref. $^{14}$ can be modified by applying the FLT $^{23}$ to get the FF for SNM.

The FF $S_{V,A}$ are related to the corresponding response function $\chi_{V,A}$ (see, for example, $^{12}$ $^{26}$)

$$ C_{V,A}^2 S_{V,A}(\omega, q) = \frac{2}{n} \frac{\text{Im} \chi_{V,A}(\omega, q)}{1 - \exp(-\omega/T)}. \quad (9) $$

As was shown in Ref. $^{14}$, the response functions $\chi_V$ and $\chi_A$ for the two-component system consisting of protons and neutrons can be found as the solutions of the following Dyson perturbation equations for the spin-independent $\mathcal{F}$ and spin dependent $\mathcal{G}$ interactions of quasiparticles presented in the matrix form:

$$ \chi_V = \chi_0 - \chi_V \mathcal{F} \chi_0, \quad \chi_A = \chi_0 - \chi_A \mathcal{G} \chi_0 \quad (10) $$

Here $\chi_0 = \begin{pmatrix} \chi_0^0 & 0 \\ 0 & \chi_0^n \end{pmatrix}$ is the diagonal $2 \times 2$ matrix consisting of $\chi_0^0$ and $\chi_0^n$ which are zero approximations of the proton and neutron response functions in the interaction, $\mathcal{F}$ and $\mathcal{G}$ are the spin-independent and spin-dependent interaction amplitudes. For symmetric nuclear matter these amplitudes become the matrices $^{14}$:

$$ \mathcal{F} = \begin{pmatrix} f_{pp} & f_{pn} \\ f_{pn} & f_{nn} \end{pmatrix}, \quad \mathcal{G} = \begin{pmatrix} g_{pp} & g_{pn} \\ g_{pn} & g_{nn} \end{pmatrix} $$

where $f_{pp}$, $f_{nn}$, $f_{pn}$ and $g_{pp}$, $g_{nn}$, $g_{pn}$ are the spin-independent and spin-dependent amplitudes of $pp$, $nn$ and $pn$ interactions, respectively.

Note that the amplitude of the interaction between two quasi-particles $q$ and $q'$ with the three-momenta $\mathbf{p}$ and $\mathbf{p}'$ when the tensor forces are neglected has the following form $^{14}$ $^{17}$ $^{27}$:

$$ f_{qq'}(\mathbf{p}, \mathbf{p}') = f + f' (\tau \cdot \tau') + g (\sigma \cdot \sigma') + g' (\sigma' \cdot \sigma')(\tau \cdot \tau'), \quad (11) $$

where $f$, $f'$, $g$, $g'$ are the functions of only the angle $\theta$ between $\mathbf{p}$ and $\mathbf{p}'$, $\sigma$, $\sigma'$ and $\tau$, $\tau'$ are the spin and
isospin Pauli matrices. The amplitudes associated with the proton-proton, neutron-neutron and proton-neutron interactions have the following forms \[ T_{pp} = T_{nn} = f_{pp} + g_{pp} \sigma \cdot \sigma ', \]

\[ T_{pn} = T_{np} = f_{pn} + g_{pn} \sigma \cdot \sigma '. \] (12)

From eqs. (11,12) we have \[ f_{pp} = f_{nn} = f + f', \]

\[ g_{pp} = g_{nn} = g + g', \] (13)

\[ f_{pn} = f_{np} = f - f', \]

\[ g_{pn} = g_{np} = g - g'. \] Inserting the matrices \( F, G \) and \( \chi^0 \) in (10) we get a couple of equations for \( \chi^0 \), and \( \chi_A^0, \chi_A^0 \).

\[ \chi_V^{n,m} = \frac{(C_V^p)^2(1 + f_{nn} \chi_n^0) \chi_p^0 + (C_V^n)^2(1 + f_{pp} \chi_n^0) \chi_p^0 - 2C_V^n C_F F_{pm} \chi_n^0 \chi_p^0}{1 + 2 f_{nn} \chi_n^0 + f_{pp} \chi_n^0 + f_{pp} \chi_n^0 + f_{pp} \chi_n^0 - f_{pm} \chi_n^0 f_{pm} \chi_n^0}, \]

(16)

\[ \chi_A^{n,m} = \frac{(C_A^p)^2(1 + g_{nn} \chi_n^0) \chi_p^0 + (C_A^n)^2(1 + g_{pp} \chi_n^0) \chi_p^0 - 2C_A^n C_F F_{pm} \chi_n^0 \chi_p^0}{1 + 2 g_{nn} \chi_n^0 + g_{pp} \chi_n^0 + g_{pp} \chi_n^0 + g_{pp} \chi_n^0 - g_{pm} \chi_n^0 g_{pn} \chi_p^0} \]. (17)

Note that eq. (16) and eq. (17) for the response functions are similar in form to the equations for the polarization operators obtained in Ref. [14].

Actually, equations (16) and (17) can be solved within the Landau theory of the Fermi liquid (LFLT) by relating the response function to the density fluctuation \( \delta n^p \) and \( \delta n^n \) for protons and neutrons respectively and presenting (16) and (17) as the linearized quantum kinetic equations for these fluctuations. It was done in Ref. [23] by expanding the quasiparticle interaction in Legendre polynomials and keeping only the \( l = 0 \) and \( l = 1 \) terms. For symmetric nuclear matter \( \chi^0 = \chi_0^0 = \chi_0 \) calculated within the LFLT becomes \[ \chi_V^{s,n,m} = \chi_V^0 + \chi_V^0 \] and \( \chi_A^{s,n,m} = \chi_A^0 + \chi_A^0 \).

\[ \chi_0 = N(0) g(\lambda) = N(0) \left[ 1 - \frac{1}{2} \int_{-1}^{1} \frac{\mu d\mu}{\mu - \lambda} \right] = N(0) \left[ 1 - \frac{\lambda}{2} \ln \left| \frac{\lambda + 1}{\lambda - 1} \right| + \frac{i\pi \lambda}{2} g(1 - |\lambda|) \right], \] (18)

where \( \lambda = \omega/qv_F \) and \( v_F = p_F/m^* \), \( p_F \) is Fermi momentum of a quasiparticle, \( m^* \) is the effective nucleon mass in a medium, \( N(0) \equiv d\rho/dE_F \), \( \epsilon_F = p_F^2/(2m) \) is the quasiparticle energy on the Fermi surface, \( p_F \) depends on the density \( n \). For SNM \( N^s(0) = 2m^*p_F(n)/\pi^2 \) and for the pure neutron matter (PNM) \( N^p(0) = m^*p_F(n)/\pi^2 \).

Neglecting the terms of second order in the interaction of type \( [f_{pp}f_{nn} - (f_{pp})^2]/3 \) and other similar terms one can get the following approximate forms for the response functions:

\[ \chi_V^{s,n,m} \approx \frac{N^s(0) C_V^2 g(\lambda) + (C_V^p)^2(F \cdot g(\lambda))g(\lambda)}{V + (F + F') \cdot g(\lambda) + (F_0g(\lambda))(F_0'g(\lambda))}, \] (19)

\[ \chi_A^{s,n,m} \approx \frac{N^s(0) C_A^2 g(\lambda) + (C_A^p)^2(G \cdot g(\lambda))g(\lambda) + (C_A^p)^2(G' \cdot g(\lambda))g(\lambda)}{V + (G + G') \cdot g(\lambda) + (G_0g(\lambda))(G_0'g(\lambda))}, \] (20)

where \( C_{V,A}^p = C_{V,A}^p + C_{V,A}^n, C_{V,A}^2 = (C_{V,A}^p)^2 + (C_{V,A}^n)^2 \).
Here

\[
(F \cdot g(\lambda)) = \left[ F_0 + \frac{\lambda^2 F_1}{1 + (F_1 + F_1')/3} \right] g(\lambda),
\]

\[
(F' \cdot g(\lambda)) = \left[ F'_0 + \frac{\lambda^2 F'_1}{1 + (F_1 + F_1')/3} \right] g(\lambda),
\]

\[
(F + F') \cdot g(\lambda) = \left[ F_0 + F'_0 + \frac{\lambda^2 (F_1 + F_1')}{1 + (F_1 + F_1')/3} \right] g(\lambda),
\]

(21)

where \( F_1 = N^*(0)f_1, F'_1 = N^*(0)f'_1, G_1 = N^*(0)g_1, G'_1 = N^*(0)g'_1(l = 0, 1) \) are the Landau parameters; in fact, \( f_1, f'_1, g_1, g'_1 \) are the coefficients in the expansion of the interaction amplitude in Legendre polynomials. Convolutions \((G \cdot g(\lambda)), (G' \cdot g(\lambda))\) and \((G + G') \cdot g(\lambda)\) have the same analytical forms as (21), but with \( G_0, G'_0, G_1, G'_1 \) instead of \( F_0, F'_0, F_1, F'_1 \), respectively. More exact forms of these response functions are presented in Ref. [25].

For free nucleons the weak coupling constants have the following values: \( C_{nV} = 0.08, C_{nch} = -1.0, C_{nA} = 1.23, C_{n'V} = -1.23 \). In a medium they are renormalized within 10–20% (see Ref. [23]). Therefore for simplicity we neglect this effect in our consideration. Assuming further that \( C_{nV}^2 \simeq C_{n'V}^2 \), we may present the axial response function in the simple form

\[
\chi_{A,n.m.}^s \simeq C_A^2 \frac{N^*(0)}{V} \frac{g(\lambda)[1 + 4(G \cdot g(\lambda))]}{1 + (G + G') \cdot g(\lambda) + (G_0 g(\lambda) G'_0 g(\lambda))/2},
\]

(22)

where \( C_A = 1.23 \). To get the response functions for pure neutron matter \( \chi_{V,n.m.}^s \) and \( \chi_{A,n.m.}^s \), one can simply drop the proton-proton interaction amplitudes; in practice, one should use \( f_{pp} = 0, f_{pn} = 0 \) and \( C_{nV}^p = C_A^p = 0, \lambda_0 = 0, f_1 = 0, g_1 = 0 \) in (21) and (17). As a result one obtains [12]:

\[
\chi_{V,n.m.}^s = (C_{nV}^s)^2 \frac{N^*(0)}{V} \frac{g(\lambda)}{1 + G \cdot g(\lambda)},
\]

\[
\chi_{A,n.m.}^s = (C_{nA}^s)^2 \frac{N^*(0)}{V} \frac{g(\lambda)}{1 + G \cdot g(\lambda)}.
\]

(23)

(24)

One can see that for nuclear matter the axial-vector response function \( \chi_{A,n.m.}^s \) is determined by the Landau parameters \( G_0, G_1, G'_0, G'_1 \) linked to the spin and spin-isospin terms of the interaction amplitude given by eq. (11). Therefore it is called the spin-isospin response function [25]. All necessary Landau parameters for calculation of these response functions can be taken from [27, 28] at the saturation density \( n = 2p^2/3\pi^2 \).

Let us discuss how the collective excitation in a nuclear medium can be included. Within the LFLT the collective mode excitation corresponds to the pole contribution of the response function as a function of \( \lambda \) at \( \lambda = \lambda_0 \equiv c_s/v_F \), where \( c_s \) is the zero sound velocity [12, 16, 18, 19]. If a pole occurs, the contribution of the collective mode is calculated as the residue of the response function at \( \lambda = \lambda_0 \) [12, 18]. One can show that at the saturation density \( n = 0.16 \text{ fm}^{-3} \), when \( F_0 + F'_0 < 0 \) [27], the effective field is attractive and the vector response function \( \chi_{V,n.m.}^s \) given by (19) has no pole at real values of \( \lambda \) [12, 18, 19]. Therefore, only single-pair excitations exist for the density response spectrum at the normal density. On the other hand, the spin-density or the axial-vector response function \( \chi_{A,n.m.}^s \) given by (20) has a pole at \( \lambda = \lambda_0 \) because \( G_0 + G'_0 > 0 \) [27]. We calculated this spin-zero-sound pole contribution to the axial-vector FF as the residue of \( S_A \) at \( \lambda = \lambda_0 \). The same procedure for inclusion of the collective excitation mode in pure neutron matter was proposed in [12].

B. Charged currents

Contrary to the elastic neutrino scattering the isospin changing part of the particle-hole interaction is relevant for the charged current reactions. In terms of the Fermi-liquid parameters, this means that only \( F' \) and \( G' \) contribute to the dynamic form factors [14]. Therefore, to get the density response and the spin-density functions one can use eq. (19) and eq. (20) with \( F_0 = 0, G_0 = 0 \) and with replacement of the coupling constants \( C_{V,V,A}^n, C_{V,A}^n \) by the weak coupling constants corresponding to the charged current vector and axial couplings \( D_V, D_A \). The corresponding vector \( \chi_{V,n.m.}^{s,n} \) and axial-vector \( \chi_{A,n.m.}^{s,n} \) response functions for symmetric nuclear matter have the following forms:
Here \( D_V = 1 \), \( D_A = 1.23 \) and \( F_{0,1} = 2F_{0,1}^c = 2G_{0,1}^c = 2 \) where the factor 2 arises due to isospin consideration \[14\]. In eqs. \((25,26)\) the terms of the second order of interaction of type \((F_{0,1}^c)^2/3\) and \((G_{0,1}^c)^2/3\) are also neglected as it was assumed in eqs. \((19,20)\).

The rate of the neutrino-nucleon scattering in a medium due to the charged currents will have the same form as the rate for neutral current processes given by eq. \((3)\) but multiplied by a factor 4 and with \( S_{V,A} \) replaced by \( S_{V,A}^{\text{ch}} \) related to \( \chi_{V,A}^{\text{ch},s.m.m.} \). The factor 4 is due to the difference between the neutral and charged weak baryon currents (see eqs. \((35)\)).

**IV. NEUTRINO CROSS SECTION**

Using the forms obtained for the FF one can calculate the neutrino mean free path \( l \) in the nuclear matter. According to Ref. \[12\], \( 1/l \) is related to the neutrino rate \( W_{fi} \):

\[
1/l = V \int \frac{d^3q}{(2\pi)^3} W_{fi} .
\]  

With this quantity one can estimate the cross section of the neutrino interaction with a heavy nucleus \( \sigma_{\nu A} \):

\[
\sigma_{\nu A} = \frac{V_A}{l} = \frac{V_A}{V} \int \frac{d^3q}{(2\pi)^3} \tilde{W}_{fi} ,
\]

where \( \tilde{W}_{fi} = V_A \cdot W_{fi} \) and \( V_A = A \cdot v_N \). Here \( A \) is the number of nucleons in a nucleus and \( v_N = 4\pi/3r_N^3 \) is the nucleon volume, \( r_N \) is the nucleon radius about 0.8 fm. To calculate the elastic \( \nu - A \) cross section we substitute into eq. \((28)\) the neutrino rate \( \tilde{W}_{fi} \) due to the neutral weak and baryon currents, whereas for the absorption \( \nu - A \) cross section \( \tilde{W}_{fi} \) is the neutrino rate due to the charge exchange currents corresponding to the absorption processes of type \( \nu + n \rightarrow e^- + p \). We calculated the total neutrino cross section as the sum of the elastic and absorption \( \nu - A \) cross sections. Note that calculating the integral \((28)\) we included the kinematics for the elastic and inelastic neutrino scattering off a nucleus.

We calculated \( S_V \) and \( S_A \) using their relation to the corresponding response functions \( \chi_V \) and \( \chi_A \) given by eq. \((19)\) and putting \( T = 0 \). If the nucleus is isotopically symmetric then the response functions can be calculated using the approximate formulas \((19,20)\) or the exact expressions presented in Ref. \[23\]. Note that, as follows from our calculations the quantitative difference between these two forms for \( \chi_{V,A} \) is negligibly small.

In this paper we computed the cross sections of neutrino interactions with slightly asymmetric nuclei \( ^{71}\text{Ge} \) and \( ^{40}\text{Ar} \). To take into account the isotopic asymmetry for these nuclei, we introduced fractions of protons and neutrons in front of \( p-p \), \( n-n \) and \( p-n \) interactions solving the couple of equations \((10)\). The final forms for \( \chi_{V,A} \) are slightly changed in comparison to eqs. \((19,20)\) when these fractions are included. We took the Landau parameters \( F_0 = -0.4, F_0^c = 0.2, G_0 = 0.19, G_0^c = 0.78, F_1 = -0.66 \) and \( F_1^c = 0.14 \) from Ref. \[27\] and \( F_1 = 0.14 \) from Ref. \[28\] calculated for symmetric nuclear matter at the saturation nuclear density \( n = 2\rho_0/(3\pi^2) = 0.16/fm^3 \). Note that \( F_0, F_0^c, G_0, G_0^c \) are related to the compressibility, symmetry energy \[24, 30\] and spin susceptibility \[31\] respectively, whereas \( F_1 = 3(m^*/m-1)\) \[18, 26\]. The parameters \( G_1 = 0.48, G_1^c = 0.08 \) were taken from Ref. \[28\], using the relation of \( G_0, G_0^c \) to \( G_1, G_1^c \). Unfortunately, there are no calculations of the Landau parameters for asymmetric nuclear matter. The isotopic asymmetry for \( ^{71}\text{Ge} \) and \( ^{40}\text{Ar} \) is small, therefore we assume that one can use the above results for the parameters in question obtained for symmetric nuclear matter.

To estimate the number of neutrino interactions \( R \) per second within a detector we use the formula

\[
R = M_T N_A \sigma_{\nu A}^{tot} f_\nu .
\]

Here \( f_\nu \) denotes the initial neutrino flux presented in Fig.3, \( N_A \) is the Avogadro constant, \( M_T \) is the detector mass, \( \sigma_{\nu A}^{tot} \) is the total cross section of neutrino scattering off a heavy nuclei \( A \).

**V. RESULTS AND DISCUSSIONS**

In Fig.1 the total \( \nu-^{71}\text{Ge} \) cross section including both elastic and absorption parts is presented as a function of the neutrino energy \( E_\nu \). The solid line corresponds to the Fermi gas approximation (FG), e.g., when the nucleon-nucleon correlations in the \( ^{71}\text{Ge} \) nucleus are neglected. The long-dashed curve is the calculation within the framework of the FLT when the tensor forces and the realistic total \( \nu-^{71}\text{Ge} \) cross section calculated within the FLT including all these effects can differ from the
Fermi gas approximation by a factor of 1.5–2. The short-dashed line in Fig. 1 is the result of Ref. [2] obtained within the shell model including the short-range part of NN interaction, which is due to the π- and ρ-meson exchange, and the possible isobar excitation, whereas the dotted curve (right panel of Fig. 1) is the result of Ref. [2] without this isobaric effects. Therefore, the result corresponding to the short-dashed curve in Fig. 1 (right panel) can be overestimated at \( E_\nu > 10 \text{MeV} \). One can see from the left panel of Fig. 1 that for solar neutrinos at energies less than 5 MeV the FLT results in the total cross section values close to the shell model calculation presented in Ref. [2]. At larger neutrino energies our calculation within the FG approximation (the solid line in Fig. 1) is similar to the result of Ref. [2] without the isobar excitation effect (the dotted line in right panel of Fig. 1).

In Fig. 2 the total absorption \((\nu, e^-)\) cross section on \(^{40}\text{Ar}\) is presented. The solid line corresponds to the FG approximation, whereas the long-dashed curve is the calculation within the FLT. Crosses correspond to the calculation of Ref. [10] within the RPA. One can see from the left panel of Fig. 2 that at \( E_\nu < 3 \text{ MeV} \) the result from Ref. [10] is similar to our calculations within the FLT, whereas at larger energies (see right panel of Fig. 2) it does not contradict our results obtained within the FG approximation.

Figure 3 (see, for example, Ref. [32]) illustrates the solar neutrino flux continuum as a function of the neutrino energy \( E_\nu \).

In Fig. 4 the total number of \(^8\text{B}-\nu\) interactions \( R \) per month and per MeV in 1 kg of the \(^{71}\text{Ge}\) detector is presented as a function of the neutrino energy \( E_\nu \). The estimation of \( R \) was done using eq. (29). The solid curve corresponds to the calculations when the total neutrino cross section is calculated within the FG approximation, whereas the long-dashed line is the FLT calculation of \( \sigma_{\nu A} \). The short-dashed curve corresponds to \( \sigma_{\nu A}^{\text{tot}} \) taken from Ref. [2]. A small difference between the FLT calculations and the results of Ref. [2] is similar to the one for the cross sections presented in the left panel of Fig. 1. For instance, with the results presented in Fig. 4 one can estimate the total number of neutrino events expected (from \(^8\text{B}-\nu\) flux) in the energy region 1–8 MeV within 1 Tonn (100 kg) of \(^{71}\text{Ge}\) detector during one month. If we also include pep-neutrinos (see Fig. 3), the total number of neutrino events is about 20–30 (2–3) events per month, or 240–360 (24–36) per year. These values do not look negligibly small. Note that the main contribution to these numbers comes from the energy region \( E_\nu \approx 8 \text{MeV} \), where the FG reproduces correctly the cross sections obtained within other models [2, 10] (see the Figs. 1, 2). It is due to the fact that \( R \) falls down very fast when the neutrino energy decreases. Therefore the discussed estimation of this neutrino-induced background in the energy interval from 1 MeV to 8 MeV is mainly determined by the FG calculation and is not sensitive to the calculations done within the FLT. On the other hand, at lower neutrino energies the FLT gives a significantly smaller neutrino background (by a factor of 2) than the FG calculation. It can be seen from Fig. 1 (left panel). The discussed estimation is rather approximate, nevertheless it illustrates the importance of the precise knowledge of solar neutrino-induced background for very accurate neutrino experiments.

VI. CONCLUSION

In this paper we applied the Fermi liquid theory (FLT) to calculate the cross sections of neutrino scattering off heavy nuclei at energies less than 50 MeV. Our results are in agreement with the other more sophisticated calculations based on the nuclear shell model and the RPA. The difference between our results and the other calculations is rather small in the neutrino energy \( E_\nu < 5 \text{ MeV} \). Nevertheless, our approach allows calculations of the cross sections in the most simple way comparing with the other nuclear models. At \( 5 < E_\nu < 50 \text{ MeV} \) the FLT results differ from the RPA [10] and the simple shell model (with neglect of the isobar excitations) [2] by a factor of 2, whereas the use of the Fermi gas approximation (FG) gives the difference much less than a factor 2. It allows us to obtain a quite reliable estimate of the background induced by solar neutrinos with \( E_\nu < 8–10 \text{ MeV} \), for example, a Ge or Ga detector.

The knowledge of such a background is very important for very accurate neutrino experiments like, for example, double \( \beta\)-decay experiments, search for dark matter, etc. One can conclude that for solar neutrinos the FG approximation leads to underestimation of the cross section by a factor of 2, whereas at higher energies the results obtained within the FG are closer to the simple shell model and RPA calculations [10]. Therefore, the FLT can be safely used to estimate the background from solar neutrinos in heavy-nucleus detectors.

In this paper we estimated for the first time \( \nu-A \) cross sections in the FLT, which can be used for present and future neutrino experiments. We showed that this approach did not contradict to other nuclear models. Unfortunately, there are no direct experimental data on total cross sections of solar neutrino interactions with heavy nuclei at \( E_\nu < 10 \text{ MeV} \). The existing data apply to the inverse \( \beta\)-decay on a nucleus integrated over the energy spectrum of the neutrino produced in \( \beta\)-decay of heavy nuclei, for example, \(^{238}\text{U}\). Therefore, in the future we plan to perform a comparison of \( \nu-A \) cross sections obtained within the FLT and other nuclear models with available experimental data. We are also going to improve our approach by including possible multi-pair contributions to the weak response and to calculate collective excitations in a nucleus more carefully.
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FIG. 1: The total $\nu^{-71}\text{Ge}$ cross section as a function of the neutrino energy $E_\nu$. The solid line corresponds to the Fermi-gas approximation (FG), the long-dashed curve is the calculation within the Fermi liquid theory (FLT), the short-dashed line is the calculation within the shell model including the short-range part of the $NN$ interaction which is due to the $\pi$- and $\rho$-meson exchange [2], whereas the dotted line is the results of [2] without these isobaric effects.

FIG. 2: The total absorption $\nu^{-40}\text{Ar}$ cross section as a function of the neutrino energy $E_\nu$. The solid line is the FG approximation, the long-dashed curve is the FLT calculation, the crosses correspond to the calculation of [10] within the RPA.
FIG. 3: The illustration of the energy dependence of the solar neutrino flux continuum from Ref. [32].

FIG. 4: The total number of neutrino interactions per month and per MeV produced from the $^8$B-$\nu$ flux interacting with 1-kg $^{71}$Ge detector as a function of the neutrino energy $E_\nu$. Notation is the same as in Fig. [1].