Constraints on ultralight axions, vector gauge bosons, and unparticles from geodetic and frame-dragging effects

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Abstract

The geodetic and frame-dragging effects are the direct consequences of the spacetime curvature near earth which can be probed from the Gravity probe B satellite. The satellite result matches quite well with Einstein’s general relativistic result. However, there is uncertainty between the results of general relativity and the gravity probe satellite. The gyroscope of the satellite which measures the spacetime curvature near earth contains lots of electrons and nucleons. Ultralight axions, vector gauge bosons, and unparticles can interact with those SM particles through different operators and change the drift rate of the gyroscope. Some of these ultralight particles can either behave as a long range force between some dark sector or earth and the gyroscope or they can behave as a background oscillating dark matter fields or both. These ultralight particles can contribute to the uncertainties in the measurement of drift rate of the gyroscope obtained from the GR and GP-B results and we obtain bounds on different operator couplings. The bounds on the couplings obtained in this paper are stronger than any other bounds available in the literature. These ultralight particles can be promising candidates for dark matter which can be probed from the measurements of geodetic and frame-dragging effects.

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I. INTRODUCTION

The gravity probe B (GP-B) is a satellite-based experiment that was launched in April, 2004 to test the general relativity (GR) phenomena such as the geodetic effect or the de-Sitter effect [1] and the Lense-Thirring precession [2] or the gravitomagnetic frame-dragging effect predicted by Einstein’s GR theory. This earth satellite contained four gyroscopes and it was orbiting at 650km altitude. The spacetime near the earth is changed due to the presence of the earth and its rotation which modifies the stress-energy tensor near the earth [3]. When the gyroscope’s axis of the satellite is parallel transported around the earth then after one complete revolution the tip does not end up pointing exactly in the same direction as before. The geodetic effect measures the drift rate of the gyroscope due to the presence of the earth which wraps the spacetime near it and the frame-dragging effect measures the drift rate due to the rotation of the earth which drags the spacetime around it. The GR predicts the geodetic drift rate as $-6606.1 \text{ mas/yr}$ whereas the frame-dragging drift rate is $-39.2 \text{ mas/yr}$. In 2011, the GP-B experiment published the data for the geodetic drift rate as $-6601.8 \pm 18.3 \text{ mas/yr}$ and the frame-dragging drift rate as $-37.2 \pm 7.2 \text{ mas/yr}$, where “mas” stands for milliarcsecond [4]. The GP-B result matches quite well with the GR predicted value, however, there is uncertainty between the GR and the GP-B results. To resolve these uncertainties in both geodetic and frame-dragging effects, we assume interactions of nucleons and electrons in the gyroscope with the time-dependent and time-independent axions, vector gauge boson fields, and unparticles which can change the drift rate of the gyroscope. By considering the uncertainties, we can obtain the constraints on axions, gauge bosons, and unparticles mediated forces. In the following, we have briefly discussed what are axions and axion like particles, the gauge bosons and the unparticles.

Axion is a pseudo-Nambu Goldstone Boson (NGB) which was first postulated by Peccei and Quinn (1977) to solve the strong CP problem [5–8]. The most stringent probe of the strong CP problem is the measurement of neutron electric dipole moment (nEDM). From chiral perturbation theory, one can obtain the nEDM as $d_n \simeq 10^{-16} \bar{\theta} e \text{cm}$, where $\bar{\theta}$ is related with the QCD $\theta$ angle by $\bar{\theta} = \theta + \arg \det(M)$ [9, 10], where $M$ is the general complex quark mass matrix. A natural choice of $\bar{\theta} \sim O(1)$ violates the current experimental bound on nEDM is $d_n^{\text{expt}} < 10^{-26} e \text{cm}$. Hence, $\bar{\theta}$ is as small as $\bar{\theta} < 10^{-10}$ [11]. The smallness of $\bar{\theta}$ is called the strong CP problem. Quantum Chromodynamics (QCD) is the theory which can
calculate the nEDM and its Lagrangian is

\[ \mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{j=1}^{n} [i \bar{q}_j D \gamma_j (m_j q^i_j q^j + h.c.)] + \theta \frac{g_s^2}{32 \pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}, \]  

(1)

where \( q \) denotes the quark field, \( D \) denotes the covariant derivative, \( \tilde{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta} \) is the dual gluon field tensor, \( \theta \) is the QCD theta angle and \( h.c. \) denotes the Hermitian conjugate. The last term in Eq. 1 is a CP violating term. The last term comes from the symmetry of the Lagrangian and it must be present since all the quark masses are non-zero. However, QCD is good CP symmetric. To solve this strong CP problem, Peccei and Quinn came up with an idea that \( \bar{\theta} \) is not a parameter but a dynamical field that goes to zero by its classical potential. They postulated a global \( U(1)_{PQ} \) symmetry which spontaneously breaks at a symmetry breaking scale called the axion decay constant \( f_a \) and explicitly breaks due to non-perturbative QCD effects at a scale \( \Lambda_{QCD} \), where the pseudo-Nambu Goldstone bosons called the QCD axions to get mass. The mass of the QCD axion is related to the axion decay constant as \( m_a \simeq 5.7 \times 10^{-12} \text{eV} \left( \frac{10^{18} \text{GeV}}{f_a} \right) \) [12]. Also, there exist other pseudoscalar particles which are not exactly the QCD axions but have similar kinds of interactions to the QCD axions. These are called the axion like particles or ALPs. These particles are motivated by string/M theory [13–16]. We assume the ALPs do not get any instanton induced mass and remain naturally light. For ALPs, \( m_a \) and \( f_a \) are independent of each other whereas, for QCD axions, \( m_\pi f_\pi \sim m_a f_a \), where \( m_\pi \) is the pion mass and \( f_\pi \) is the pion decay constant. The Lagrangian which describes the interaction between the ALPs and the standard model (SM) particles is [17]

\[ \mathcal{L} \supset \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{\alpha_s}{8\pi} g_{a0} a \int_a G^a_{\mu\nu} \tilde{G}^{a\mu\nu} - \frac{\alpha}{8\pi} g_{a\gamma} a \int F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{2} \frac{g_{af}}{f_a} \partial_\mu a \tilde{f} \gamma^\mu \gamma_5 f, \]  

(2)

where \( g \)'s are the coupling constants. The first term denotes the kinetic term of ALP, the second term denotes the coupling of ALP with the gluon field \( G_{\mu\nu} \), the third term denotes the coupling of ALP with the electromagnetic photon field \( F_{\mu\nu} \) and the fourth term denotes the derivative coupling of ALP with the fermion field \( f \). The couplings of ALPs with SM particles are small since all the coupling are proportional to \( \frac{1}{f_a} \) and \( f_a \) generally takes larger value. The axions or ALPs can also couple with the nucleons or quarks through the electric and magnetic dipole moment operators described by the terms \( g_{EDM} a \bar{N} \gamma_5 NF^{\mu\nu} \) and \( g_{MDM} a \bar{N} \sigma_{\mu\nu} NF^{\mu\nu} \) respectively.
The axion is a promising non thermal dark matter candidate which can solve some of the small scale structure problems in the universe [18–21]. The axion field can oscillate with time as \( a(t) \sim \frac{\sqrt{\rho_{DM}}}{m_a} \sin(m_a t) \), where \( \rho_{DM} \) is the dark matter energy density. Axion can also form topological defects like cosmic strings and domain walls [22–24]. They can also behave as dark radiation [25–30]. If the mass of the axion is very small then it can also mediate long range forces and the corresponding potential is Yukawa type \( \frac{1}{r} e^{-m_a r} \). The axions can also contribute to the monopole-monopole, monopole-dipole, and dipole-dipole interactions between the visible sector particles [31–34].

There is no direct evidence of axions so far. However, there are bounds on the axion mass and decay constant from the laboratory, astrophysical and other experiments. SN1987A puts the bound on axion decay constant as \( f_a \gtrsim 10^9 \text{GeV} \), which is in tension with the solar axion experiments with sub eV axion mass and the axion decay constant \( f_a \sim 10^7 \text{GeV} \) [35, 36]. Axions can be a component of hot dark matter if \( f_a \lesssim 10^8 \text{GeV} \) [37–39]. Cold axions can be produced from vacuum realignment mechanism which can give cold dark matter relic density [40, 41]. Some laboratory and cosmological bounds on axions are discussed in [42–49]. ALP dark matter can rotate the cosmic microwave background (CMB) modes which constrain the mass of the axions and axion photon couplings [50]. Axions can also be probed from the superradiance phenomena for a black hole [51, 52]. Ultralight axion like particles can be a candidate of fuzzy dark matter (FDM) if \( m_a \sim 10^{-22} \text{eV} \) and \( f_a \sim 10^{17} \text{GeV} \) [53, 54]. The de Broglie wavelength of the FDM particles is of the order of the size of a dwarf galaxy (1–2)kpc. However, Lyman-\( \alpha \) bounds disfavor ALPs as FDM [55]. The Shapiro time delay also puts constraints on the ALPs as \( m_a \lesssim 10^{-18} \text{eV} \) and \( f_a \lesssim 10^7 \text{GeV} \) [56]. The neutron star-white dwarf binary systems put bounds on ALPs as \( m_a \lesssim 10^{-19} \text{eV} \) and \( f_a \lesssim 10^{11} \text{GeV} \) [57]. The axions can also be constrained from the birefringence phenomena due to the interactions of axions and electromagnetic photons [58]. There are few experiments like ABRACADABRA [59, 60], CASPEr [61–64], GNOME [65–67] which are looking for axions using magnetometers [68]. Storage rings can also be used for the detection of axions [69]. Constraints on axion mediated force from torsion pendulum experiment is discussed in [70].

The light gauge bosons can also mediate long range forces or it can also serve as a background oscillating dark matter fields. The SM of particle physics is a \( SU(3)_c \times SU(2)_L \times U(1)_Y \) gauge theory. However, in the leptonic sector one can construct three symmetries, \( L_e - L_\mu, L_e - L_\tau \) and \( L_\mu - L_\tau \) in an anomaly free way and they can be gauged. The
$L_e - L_{\mu,\tau}$ type of gauge force can be constrained from neutrino oscillation experiments [71] and perihelion precession of planets [72]. The $L_{\mu} - L_{\tau}$ type of gauge force can be constrained from the orbital period loss of the binary systems [73]. The other bounds on $L_i - L_j$ force are discussed in [74, 75]. $B - L$ symmetry can also be gauged in an anomaly free way and it can mediate long range force. The bounds on ultralight $B - L$ gauge bosons is discussed in [76]. Constraints on long and short range forces mediated by scalars and vectors are discussed in [77, 78]. The dark photon which is another standard dark matter candidate, can have a kinetic mixing with the visible photons or couple with the SM particles if they are charged under the new $U(1)$ gauge group. Its mass can be light and mediate long range force as usual [79–82].

Unparticles are also good candidates which can mediate long range forces. In 2007, Georgi proposed a new type of massless particles which can arise in effective low energy theory due to its non canonical dimensions [83, 84]. Due to its non canonical scaling dimensions, the force mediates by unparticle can deviate from inverse square law and can give rise to long range forces. Suppose, an ultraviolet (UV) theory has an infrared (IR) fixed point at some energy scale $\Lambda_u$. The fields become conformally invariant at this scale. If the operator of the UV theory is denoted by $O_{UV}$ with a canonical dimension $d_{UV}$ and if any SM operator is denoted by $O_{SM}$ with dimension $d_{SM}$, then the effective coupling is suppressed by some heavy mass scale $M_u$ and is denoted by $\frac{1}{M_u^{d_{UV} + d_{SM} - 4}}$. The fields of the UV theory become scale invariant below a scale $\Lambda_u$ (typically $\Lambda_u \sim 1$ TeV) and it acquires a dimension $d_u$ different from the canonical one by the dimensional transmutation. Thus, the unparticle operator $O_u$ coupled with the SM operator as $\left(\frac{\Lambda_u}{M_u}\right)^{d_{UV} + d_{SM} - 4} \frac{1}{\Lambda_u^{d_u + d_{SM} - 4}} O_u O_{SM}$.

Exchange of scalar, pseudoscalar, vector, pseudovector unparticles can give rise to long range forces which are discussed in [85–88]. Unparticles can couple to energy stress tensor and mediate ungravity [89]. The unparticle coupling with Higgs, gauge bosons, and other SM particles are discussed in [90–98]. Unparticle mediated long range force can also be tested from the perihelion precession of Mercury [99]. Also, dark matter and dark energy can interact with unparticles which are discussed in [100–104].

Axions, vector gauge bosons, unparticles can mediate long range forces and can change the precessional velocity of the gyroscope of GP-B satellite. The axions or the vector gauge bosons can also behave as a background oscillating dark matter field and when it interacts with the gyroscope, it can change the precession of the gyroscope. By comparing the geodetic
and frame dragging drift rate obtained from the GR and GP-B results, we can put bounds on the coupling and mass of those particles.

The paper is organized as follows. In Sec.II, we consider axion mediated long range Yukawa type of potential between a visible sector and a dark sector. In Sec.III, we discuss the interaction of the GP-B gyroscope with the background oscillating axionic dark matter field. In Sec.IV, we consider that the time dependent oscillating axionic field can interact with the nucleons of the gyroscope through the electric dipole moment operator and change its drift rate. In Sec.V, we discuss the mediation of $L_e - L_{\mu,\tau}$ type of gauge bosons which gives rise to long range force and changes the precession rate of the gyroscope. The hidden photon can also behave as a background oscillating dark matter field and can interact with the gyroscope’s nucleons through electric and magnetic dipole moment operators which are discussed in Sec.VI. In Sec.VII, we discuss the unparticle mediated long range force which can be constrained from the GP-B result. Finally, in Sec.VIII, we discuss our results.

We have used the natural system of units throughout the paper.

II. CONSTRAINTS ON AXION MEDIATED LONG RANGE YUKAWA TYPE OF POTENTIAL

An axion is a pseudo-Nambu Goldstone boson which arises due to $U(1)_{PQ}$ symmetry breaking at a scale $f_a$ and it obtains mass due to non perturbative QCD effects for the QCD axions. However, there exists very light mass axion like particles which arise in string/M theory. In the following, we consider generic ultralight ALPs. Under CP and shift symmetry, the axion transforms as $a \rightarrow -a$ and $a \rightarrow a + \theta$ respectively, where $\theta$ is a real arbitrary number. Since, CP is violated in nature, we consider CP violating coupling of axions with the dark sector. However, in the QCD sector, CP is conserved and we consider CP conserving coupling of axions with nucleons. Due to very light mass, the ultralight ALPs can mediate a long range Yukawa type of potential ($\sim \frac{1}{r} e^{-m_a r}$, $m_a$ is the mass of the ALP) between the visible sector which consists of standard model particles and the dark sector which consists of dark matter particles. The dark sector can also be some compact objects formed by axion dark matter, topological defects such as domain walls, cosmic strings or even it can be primordial black holes (PBH) [105–107]. The ALP has a CP violating coupling with the dark sector and a CP conserving coupling with the visible sector. The mass of the ALP
is typically constrained by the distance between the dark sector and the visible sector. The
range of this ALP mediated force is $\lambda \gtrsim \frac{1}{m_a}$ which makes the ALPs ultralight. The equation
of motion of the ALP field ($a$) from the CP violating coupling with the dark sector is
\begin{equation}
\nabla_\mu \nabla^\mu a(t, \vec{x}) = -m_a^2 a(t, \vec{x}) - J(t, \vec{x}),
\end{equation}
where the source term $J(t, \vec{x})$ denotes the current density in the CP violating dark sector.
Since, $J(t, \vec{x})$ is the current density, it should be proportional to energy density of the CP
violating dark sector $\rho_D$. From the dimensional analysis, it is also inversely proportional
to the energy scale in the theory, here it is $f_a$, the axion decay constant. Hence, one can
write the current density as $J(t, \vec{x}) = c_D \kappa(t) \rho_D(\vec{x}) / f_a$, where $c_D$ denotes the strength of the CP
violation in the dark sector and $\kappa(t)$ includes other model dependent parameters. Due to
the long range behaviour of the axion field, the dark current density is only a function of
$r = |\vec{x}|$. In a Schwarzschild spacetime background, the Klein-Gordon equation Eq.3 for the
axion field cannot be exactly solvable for a point source with $J_0(\vec{x}) = \delta^3(\vec{x})$. Hence, to solve
Eq.3, we expand the axion field in a perturbative way with the perturbation parameter $\frac{GM_D}{R_D}$
(where $G$ denotes the Newton’s gravitation constant, $M_D$ denotes the mass of dark object
and $R_D$ denotes its radius) and keeping the leading order term with its Yukawa behaviour.
Hence, we can write the axion field as
\begin{equation}
a_0(r) = a_1(r) + \frac{GM_D}{R_D} a_2(r) + O\left(\left(\frac{GM_D}{R_D}\right)^2\right),
\end{equation}
where the leading order term $a_1(r) = \frac{1}{r} e^{-m_a r}$ has an Yukawa behaviour. Putting Eq.4 in
Eq.3 in the Schwarzschild spacetime background, we obtain the resultant axion field as
\begin{equation}
a(r) = \frac{e^{-m_a r}}{r} \left[1 + \frac{GM_D}{r} \{1 - m_a r \ln(m_a r) + m_a r e^{2m_a r} E_i(-2m_a r)\}\right] + O\left(\left(\frac{GM_D}{R_D}\right)^2\right),
\end{equation}
where $E_i(x) = - \int_{-x}^{\infty} \frac{e^t}{t} dt$ is called the exponential integral function. Since, we are not
considering any particular dark sector object therefore, we do not have any information
about $M_D$ and $R_D$. Hence, we keep ourselves in the regime $\frac{GM_D}{R_D} \ll 1$ and the axion field
solution (basically, the Green’s function) becomes $a_0(r) \approx \frac{1}{r} e^{-m_a r}$. Hence, the axion field for
the stationary non relativistic dark source is
\begin{equation}
a(r) = \int d^3 \vec{x}' \frac{c_D \rho_D(\vec{x}')} {f_a} a_0(|\vec{x} - \vec{x}'|),
\end{equation}
where we have assumed $\kappa(t) = 1$. 

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The Lagrangian which denotes the derivative coupling of ALP with the nucleons in the CP conserving SM sector is
\[ L = \frac{c_f}{f_a} \partial_\mu a f \gamma^\mu \gamma_5 f, \] (7)
where \( c_f \) denotes the dimensionless constant for the SM fermions \( (f = p, n) \). In the non-relativistic limit, the Dirac bilinears take the following form
\[ < \bar{f} \gamma^0 f > \sim N_f, \quad < \bar{f} \gamma^5 f > \sim v_f, \quad < \bar{f} \gamma_0 \gamma_5 f > \sim < \frac{\sigma_f p_f}{E_f} >, \quad < \bar{f} \gamma_5 f > \sim < \sigma_f >. \] (8)

Hence, from Eq.7, we can write the interacting Hamiltonian in the non relativistic limit of the fermions as
\[ \mathcal{H} \approx -\frac{c_f}{f_a} \vec{\nabla} a \vec{\sigma}_f. \] (9)

Also, the Hamiltonian for a particle having spin in a magnetic field \( \vec{B} \) has a magnetic moment \( (\mu_f \sigma_f) \) interaction as \( \mathcal{H}' = -\mu_f \vec{\sigma}_f \cdot \vec{B} \). Comparing \( \mathcal{H}' \) with \( \mathcal{H} \) in Eq.9, we obtain the induced magnetic field due to mediation of long range axion field as
\[ \vec{B}_a = \frac{c_f}{\mu_f f_a} \vec{\nabla} a, \] (10)

where \( \mu_n \approx -1.9 \mu_0, \mu_p \approx 2.8 \mu_0 \) and \( \mu_0 = \frac{e}{2m_N} = 1.5 \times 10^{-10} \text{eV}^{-1} \) in natural units. Due to the primordial density fluctuations and the structure formations, the dark matter of the CP violating dark source is not spatially homogeneous and we have a non zero value of \( B_a \). But if the dark matter distribution is spatially homogeneous then \( B_a \) is zero even if \( c_a \neq 0 \). The long range axion induced magnetic field exerts a force on the GP-B gyroscope and contributes to the uncertainty in the measurement of the precession rate from its general relativistic prediction due to geodetic effect and frame dragging. Hence, using GP-B results, we can put bounds on the axion field parameters by using Eq.10 and Eq.6. The energy density of the dark matter \( \rho_D \) in Eq.6 can be decomposed into two parts, the galactic part \( (\rho_{gal}) \) and the extra galactic part \( (\rho_{egal}) \). We take the galactic contribution of dark matter \( \rho_{gal} \) as simply the NFW profile defined as \([108, 109]\)
\[ \rho_{gal}(r) = \rho_{NFW}(r) = \frac{\rho_s r_s}{r} \left(1 + \frac{r}{r_s}\right)^{-2} \] (11)
where \( \rho_s = 0.184 \text{GeV/cm}^3 = 1.428 \times 10^{-6} \text{eV}^4, \) and \( r_s = 24.43 \text{kpc} = 3.807 \times 10^{27} \text{eV}^{-1} \). Hence, in the region \( r \leq r_s \), i.e; around the galactic centre, the contribution from \( \rho_{gal} \) is
important and the axion induced magnetic field is approximately

\begin{equation}
B_a^{gal}(r) \approx -\frac{c\rho_{DM}r_s}{\mu_f f_a^2} \left[ 2 \left( E_i(-m_a r) - e^{r s m_a} E_i(-m_a (r_s + r)) \right) + \frac{r_s e^{-m_a r} \left( r_s + 2r + 2 m_a e^{m_a(r_s+r)}(r_s + r)^2 E_i(-m_a(r_s + r)) \right)}{(r_s + r)^2} \right].
\end{equation}

(12)

The extra galactic contribution becomes dominant for the region \( r > r_s \) or the compton wavelength of the ALP \( \frac{1}{m_a} \) is much larger than the size of the galaxy. The dark matter energy density from the extra galactic part within the horizon is \( \rho_c \times \Omega_{DM} \), where \( \rho_c = 1.1 \times 10^{-5} h^2 \text{GeV/cm}^3 = 3.83 \times 10^{-11} \text{eV}^4 \), is the critical density of the universe and \( \Omega_{DM} h^2 \approx 0.12 \) is the relic density of the dark matter. \( h = 0.67 \) denotes the reduced Hubble parameter. Due to the primordial density fluctuations (\( \mathcal{O}(10^{-5}) \)) from inflation, an inhomogeneity exists in \( \rho_{egal} \) which is parametrize as

\begin{equation}
\tilde{\eta} = \left( \frac{\rho_c \Omega_{DM}}{3 m_a} \right)^{-1} \int d^3 \vec{x}' \rho_{egal}(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} e^{-m_a |\vec{x} - \vec{x}'|}.
\end{equation}

(13)

Hence, the induced magnetic field mediated by long range axion force due to the extra galactic contribution of dark matter is

\begin{equation}
B_a^{egal}(r) \approx \frac{c \rho \eta \Omega_{DM}}{\mu_f f_a^2 m_a} e^{-m_a r} (2 + m_a r).
\end{equation}

(14)

The axion mediated force from the extra galactic contribution increases with decreasing the axion mass while keeping the decay constant \( f_a \) fixed. If the mass of the axion is less than \( 10^{-33} \text{eV} \) which is constrained by the horizon size, then the axion mediated force could not reach us. The total induced magnetic field due to the long range axion mediated force is \( B_a(r) = B_a^{gal}(r) + B_a^{egal}(r) \). The quantity \( |\mu_f B_a| \) contributes to the uncertainty in the measurement of the precessional velocity of the GP-B gyroscope from the GR prediction and GP-B results of the geodetic effect and frame-dragging. In Fig. 1 we have shown the variation of axion decay constant \( (f_a) \) with the axion mass \( (m_a) \) for the axion mediated force between the CP violating dark sector and the CP conserving SM sector from frame-dragging and geodetic efects using the GP-B result. GR predicts the geodetic drift rate of \(-6606.1 \text{mas/yr}\) and frame dragging drift rate of \(-39.2 \text{mas/yr}\) whereas the GP-B results the geodetic drift rate of \(-6601.8 \pm 18.3 \text{mas/yr}\) and the frame dragging drift rate of \(-37.2 \pm 7.2 \text{mas/yr}\) (1 milliarcssecond (mas)= \(4.848 \times 10^{-9} \text{rad}\)). From the uncertainties between the GR and GP-B results, we obtain the bounds on the ALPs by calculating \( B_a \) numerically as \( f_a \lesssim 6.56 \times \)
FIG. 1. Variation of \( f_a \) with \( m_a \) for the axion mediated force between the CP violating dark sector and the CP conserving SM sector from geodetic effect and the frame-dragging using GP-B result

\[ 10^{16} \text{GeV from frame-dragging effect (red line) and } f_a \lesssim 4.21 \times 10^{16} \text{GeV from geodetic effect (blue line) for the axion mass } m_a \lesssim 10^{-26} \text{eV}. \] The regions above those lines are excluded. The geodetic effect put stronger bound on \( f_a \). We fixed \( \eta = 10^{-5} \), \( c_f = 1 \), and \( c_D = 1 \) to obtain those bounds. We have checked that the dominant contribution in the drift rate due to long range axion force comes from the galactic contribution.

The ultralight axions with axion mass \( m_a \sim 10^{-22} \text{eV} \) and decay constant \( f_a \sim 10^{17} \text{GeV} \) can be a candidate of fuzzy dark matter (FDM). The FDM relic density is given as

\[
\Omega_{FDM} h^2 \sim 0.12 \left( \frac{a_0}{10^{17} \text{GeV}} \right)^2 \left( \frac{m_a}{10^{-22} \text{eV}} \right)^{\frac{1}{2}},
\]

where \( a_0 = \theta f_a \) and \( \theta \) is called the misalignment angle which can take values from \(-\pi\) to \(\pi\). Comparing our bounds on axions from GP-B with Eq.15, we conclude that the contribution of our axions in DM relic density is negligible. The Lyman-\(\alpha\) bounds for ultralight dark matter are \( m_a \gtrsim 10^{-21} \text{eV} \) and \( f_a \lesssim 10^{16} \text{GeV} \) also disfavors FDM.

III. CONSTRAINTS ON OSCILLATING AXIONIC FIELD FROM GP-B RESULT

The axionic field can also oscillate with time and behave as a background. The oscillating field can interact with the spin of the gyroscope of GP-B satellite and affects the precession
rate. The oscillating axionic dark matter can be defined as

\[ a(t) = \frac{\sqrt{2\rho_{DM}}}{m_a} \sin(m_a t), \tag{16} \]

where \( \rho_{DM} = 0.3\text{GeV/cm}^3 = 2.33 \times 10^{-6}\text{eV}^4 \) is the local dark matter density. The axions can have a derivative coupling with the nucleons as described by the Lagrangian Eq.7. In the rest frame of the gyroscope, the corresponding Hamiltonian is

\[ \mathcal{H} = -g_{aNN} \vec{\nabla} a \cdot \vec{\sigma}, \tag{17} \]

where \( \sigma \) is the spin of the nucleon which precesses in presence of a magnetic field characterised by \( \vec{\nabla} a \) and \( g_{aNN} \propto \frac{1}{f_a} \) is the axion nucleon coupling in the unit of energy\(^{-1}\). Hence, using Eq.16, the time dependent effective magnetic field becomes

\[ B_a(t) = \frac{g_{aNN}}{\mu_N} \sqrt{2\rho_{DM} v} \cos(m_a t), \tag{18} \]

where \( \mu_N = -1.9\mu_0(2.8\mu_0) \) for neutron (proton), \( \mu_0 = \frac{e N}{2m_N} \approx 0.1\text{e.fm} \) is the nuclear magneton and \( v \) is the relative velocity of the gyroscope. This time dependent induced magnetic field can exert a force on the gyroscope as a whole and the precessional velocity of the gyroscope due to the oscillating axionic dark matter becomes

\[ \mu_N B_a(t) = g_{aNN} \sqrt{2\rho_{DM} v} \cos(m_a t) \tag{19} \]

Eq.19 can contribute to the uncertainty in the measurement of the precessional velocity of the gyroscope between the GR and GP-B result and we obtain bounds on axion parameters. In Fig.2 we have shown the variation of \( g_{aNN} \) with \( m_a \) for the geodetic effect (blue) and the frame-dragging (red) using GP-B result for time oscillating axionic field. The regions above those lines the excluded. We obtain the bounds on the axion-nucleon coupling as \( g_{aNN} \lesssim 2.70 \times 10^{-14}\text{GeV}^{-1} \) from the geodetic effect and \( g_{aNN} \lesssim 9.79 \times 10^{-15}\text{GeV}^{-1} \) from the frame dragging effect. The frame dragging gives the stronger bound on \( g_{aNN} \). The effect adds coherently for \( m_a \lesssim \frac{1}{T} \) whereas for \( m_a \gtrsim \frac{1}{T} \) the precessional velocity oscillates with time. \( T = 97.65\text{minutes} \) is the orbital period of the satellite which puts the bound on axion mass as \( m_a \lesssim 1.12 \times 10^{-19}\text{eV} \). Since, \( g_{aNN} \propto \frac{1}{f_a} \) hence, \( f_a \gtrsim 10^{14}\text{GeV} \). The bounds on \( m_a \) and \( f_a \) imply that these oscillating axionic field can be a candidate of fuzzy dark matter as can be seen from Eq.15.
FIG. 2. Variation of $g_{aNN}$ with $m_a$ for the geodetic effect and the frame-dragging using GP-B result for time oscillating background axionic field.

IV. CONSTRAINTS ON AXION EDM COUPLING FROM THE GP-B RESULT

The axions can couple with the nucleons through electric dipole moment operator described by the Lagrangian

$$\mathcal{L} \supset g_d a \bar{N} \sigma_{\mu \nu} N F^{\mu \nu},$$

(20)

where $g_d$ is the coupling constant. Hence, in the non-relativistic limit, the precessional velocity of the gyroscope due to the electric dipole moment operator becomes

$$v_{\text{prec}}^d(t) \sim g_d a(t) E \sim g_d \frac{\sqrt{2 \rho_{DM}}}{m_a} \sin(m_a t) E \tag{21}$$

where $a$ is the time-oscillating axionic field and $E$ is the electric field induced by the earth’s magnetic field in the rest frame of the satellite and it is $\vec{E} \sim \vec{v} \times \vec{B}$. We take the earth’s magnetic field as $B \sim 0.1$ Gauss and we assume that it is attenuated by a factor of $\delta \sim 10^{-12}$. Hence, the value of the electric field becomes $E = 2.51 \times 10^{-18}$ Gauss $= 4.89 \times 10^{-20}$ MeV$^2$, 1 Gauss $= 1.95 \times 10^{-20}$ GeV$^2$. Hence, putting the uncertainty obtained from the GR and GP-B results, Eq.21 becomes

$$5.02 \times 10^9 \text{GeV}^{-2} \left( \frac{m_a}{\text{eV}} \right) = g_d \sin(m_a t),$$

(22)

for frame dragging effect and

$$1.35 \times 10^{10} \text{GeV}^{-2} \left( \frac{m_a}{\text{eV}} \right) = g_d \sin(m_a t)$$

(23)
FIG. 3. Variation of \(g_d\) with \(m_a\) for axion EDM coupling from the geodetic effect and the frame-dragging using GP-B result

for the geodetic effect. The mass of the axion is constrained from the orbital period of the satellite which yields \(m_a \lesssim 1.12 \times 10^{-19}\) eV. In Fig.3, we have shown the variation of the coupling constant \(g_d\) in GeV\(^{-2}\) for electric dipole moment operator with the axion mass \(m_a\) in eV. The red line denotes the variation of \(g_d\) with respect to \(m_a\) for frame dragging effect whereas the blue line denotes the corresponding variation for the geodetic effect. The region \(g_d \gtrsim 10^{-9}\text{GeV}^{-2}\) is excluded from SN 1987A and static EDM experiments. Here the frame dragging gives the stronger bound. If \(m_a \gtrsim 10^{-19}\) eV then \(g_d\) oscillates with time. For the QCD axions, \(m_\pi f_\pi \sim m_a f_a\) and \(g_d^{QCD} \sim 5.9 \times 10^{-10} \left(\frac{m_a}{eV}\right)\) GeV\(^{-2}\). We have checked that the QCD axion cannot contribute to the uncertainty between the GR and GP-B result.

V. CONSTRAINTS ON VECTOR GAUGE BOSONS IN A GAUGED \(L_e - L_{\mu,\tau}\) SCENARIO

Due to the presence of electrons inside the earth and the gyroscope of the GP-B telescope, the exchange of \(L_e - L_{\mu,\tau}\) gauge bosons between the electrons can give rise to a flavor dependent long range force between the satellite and the earth. The long range force can contribute to the precession rate of the gyroscope and from the uncertainty in the measurement between the GR and GP-B result, we can put bounds on gauge boson coupling and its mass.
The electrons inside the earth can generate a potential \( V(r) \) at the quartz sphere surface as

\[
V(r) = \frac{g^2 N_e}{4\pi r} e^{-M_{Z'} r},
\]

(24)

where \( N_e \approx 3.35 \times 10^{51} \) is the number of electrons inside the earth, \( M_{Z'} \) is the mass of the gauge boson and \( a = 7027.4 \text{km} = 3.55 \times 10^{13} \text{eV}^{-1} \) is the distance between the earth and the gyroscope. The mass of the gauge boson is constrained by the inverse of the distance between earth and the gyroscope which demands \( M_{Z'} \lesssim 2.82 \times 10^{-14} \text{eV} \). Hence, the electric field at the gyroscope due to the long range Yukawa potential is

\[
E_{Z'}(r) = -\nabla V(r) = \frac{g^2 N_e}{4\pi r} \left( \frac{1}{r} + M_{Z'} \right) e^{-M_{Z'} r}.
\]

(25)

Now the orbital velocity of the satellite in the polar orbit is \( v \sim 2.51 \times 10^{-5} \). Hence, the corresponding magnetic field is \( \vec{B} \) and from the relation \( \vec{E} = \vec{v} \times \vec{B} \), we obtain the magnitude of the magnetic field as

\[
B_{Z'}(r) = \frac{g^2 N_e}{4\pi r v} \left( \frac{1}{r} + M_{Z'} \right) e^{-M_{Z'} r}.
\]

(26)

The radius of the superconducting quartz sphere gyroscope is \( R = 1.9 \text{cm} = 9.59 \times 10^4 \text{eV}^{-1} \) and the angular velocity is \( \omega = 9000 \text{rpm} = 6.20 \times 10^{-13} \text{rad.eV} \). The mass of the quartz sphere is \( M = 63 \text{grams} = 3.54 \times 10^{25} \text{GeV} \). The numerical value of the charge of the quartz sphere is same as its mass and hence \( q = 3.54 \times 10^{25} \). Since, the gyroscope is rotating, so the magnetic moment of the rotating charged sphere is

\[
\mu = \frac{q}{5} R^2 \omega = 4.037 \times 10^{22} \text{rad.eV}^{-1}.
\]

(27)

Hence, the precessional velocity due to the exchange of \( L_e - L_{\mu,\tau} \) gauge boson is

\[
\mu \times B = 4.289 \times 10^{77} \frac{g^2}{r} e^{-M_{Z'} r} \left( \frac{1}{r} + M_{Z'} \right) \text{rad.eV}.
\]

(28)

The uncertainty in the GR prediction of the frame dragging and geodetic effect from the GP-B result must be greater than the contribution in precession rate due to \( L_e - L_{\mu,\tau} \) gauge bosons from which we obtain bounds on \( g \) as \( g \lesssim 6.49 \times 10^{-41} \) from geodetic effect and \( g \lesssim 4.01 \times 10^{-41} \) from frame-dragging effect. In Fig. 4, we have shown the variation of gauge coupling with the gauge boson mass from geodetic (blue line) and frame-dragging effect (red line). The regions above those lines are excluded. The frame-dragging puts the stronger bound on the gauge coupling for the gauge bosons of mass \( M_{Z'} \lesssim 2.82 \times \)
FIG. 4. Variation of $g$ with $M_{Z'}$ for $L_e - L_{\mu,\tau}$ vector gauge boson mediation from the geodetic effect and the frame-dragging using GP-B result

$10^{-14}$eV. This is the stronger bound on $g$ available in the literature so far. The bound on the $L_e - L_{\mu,\tau}$ gauge coupling from the GP-B result is $10^{16}$ times more stronger than the perihelion precession of planets measurements and $10^{11}$ times more stronger than the neutrino oscillation experiments.

VI. CONSTRAINTS ON DARK PHOTON FROM GP-B RESULT

Suppose the background classical dark matter field is an ultralight vector dark photon field ($A'_\mu$) which couples with the nucleons ($N$) of the gyroscope through electric and magnetic dipole moment operators described by the Lagrangian

$$\mathcal{L} \supset g'_{MDM} F'_{\mu\nu} \bar{N} \sigma^{\mu\nu} N + g'_{EDM} F'_{\mu\nu} \bar{N} \gamma^5 \sigma^{\mu\nu} N,$$

where the first term denotes the magnetic dipole moment operator, the second term denotes the electric dipole moment operator and $F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$. In the frame of the gyroscope travelling with a velocity $\vec{v}$, the dark magnetic field is $\vec{B}_{A'_\mu} = \vec{v} \times \vec{E}_{A'_\mu}$, where $E_{A'_\mu} \sim \sqrt{\rho_{DM}}$ is the amplitude of the oscillating dark electric field in the lab frame. Since, in the non relativistic limit $< F'_{\mu\nu} \sigma^{\mu\nu} > \sim \bar{\sigma}_N \cdot \vec{B}_{A'_\mu}$ and $< F'_{\mu\nu} \gamma^5 \sigma^{\mu\nu} > \sim \bar{\sigma}_N \cdot \vec{E}_{A'_\mu}$, the precessional velocity of the gyroscope due to the magnetic dipole moment operator is

$$v^{MDM}_{prec}(t) = g'_{MDM} \sqrt{\rho_{DM}} v \cos \theta \cos (m_A t),$$

(30)
FIG. 5. Variation of dipole moment couplings $g'_{MDM}$ and $g'_{EDM}$ with the dark photon mass $m_{A'}$ from the geodetic effect and the frame-dragging using GP-B result

where $\cos \theta$ picks the normal component of the dark electric field with respect to the gyroscopic plane and $m_{A'}$ is the mass of the dark photon. Similarly, for the electric dipole moment operator, the precessional velocity due to the dark electric field is

$$v_{prec}^{EDM}(t) = g'_{EDM} \sqrt{\rho_{DM}} \sin \theta \cos(m_{A'}t).$$

Eq.30 and Eq.31 can contribute to the uncertainty in the measurement of the precessional velocity of the gyroscope as obtained from GR and GP-B result and from the uncertainty we obtain bounds on dipole moment couplings and the dark photon mass. In Fig.5, we have shown the variation of $g'_{MDM}$ and $g'_{EDM}$ with the ultralight dark photon mass $m_{A'}$ for the geodetic effect (blue line) and the frame-dragging (red line) using GP-B result. Here, we have typically chosen $\cos \theta \sim 1$ and $\sin \theta \sim 1$. The dashed lines correspond to the EDM coupling whereas the solid lines correspond to the MDM coupling. The upper bounds on the MDM coupling from geodetic effect is $g'_{MDM} \lesssim 3.93 \times 10^{-14}$ GeV$^{-1}$ whereas from frame dragging effect, it is $g'_{MDM} \lesssim 1.42 \times 10^{-14}$ GeV$^{-1}$. We also obtain the upper bounds on the EDM coupling from geodetic effect is $g'_{EDM} \lesssim 1.02 \times 10^{-18}$ GeV$^{-1}$ whereas from frame dragging effect, it is $g'_{EDM} \lesssim 3.69 \times 10^{-19}$ GeV$^{-1}$. For both MDM and EDM, we obtain the stronger bounds from frame dragging. However, EDM puts stronger bound than MDM. The mass of the dark photon is constrained from the orbital period of the satellite which yields $m_{A'} \lesssim 1.12 \times 10^{-19}$ eV. Hence, from the GP-B result, one can differentiate the EDM and
MDM couplings. These ultralight dark photons can be a candidate of vector dark matter.

VII. CONSTRAINTS ON UNPARTICLE MEDIATED LONG RANGE FORCE FROM GP-B RESULT

The virtual exchange of axial vector unparticle can give rise to a long range potential at the surface of the quartz sphere due to the presence of SM particles in the earth as

\[ V_u(r, d, \Lambda_u) = -\frac{c^2_u N 4\sqrt{\pi} \Gamma(d + 1/2) \Gamma(2(d - 1))}{(2\pi)^{2d} \Gamma(d - 1) \Gamma(2d)} \left( \frac{1}{\Lambda_u} \right)^{2d-2} \frac{1}{r^{2d-1}}, \quad (32) \]

where \( c_u \) is the axial vector coupling, \( N \) is the number of electrons/nucleons in earth, \( \Lambda_u \) is the scale where the fields become conformally invariant and \( d \) is the scaling dimension. Hence, from Eq.32 the electric field at the surface of the quartz sphere is

\[ E(r, d, \Lambda_u) = -\frac{c^2_u N 4\sqrt{\pi} \Gamma(d + 1/2) \Gamma(2(d - 1))}{(2\pi)^{2d} \Gamma(d - 1) \Gamma(2d)} \left( \frac{1}{\Lambda_u} \right)^{2d-2} \frac{1}{r^{2d-1}}, \quad (33) \]

From Eq.32, we an write the corresponding magnetic field as

\[ B(r, d, \Lambda_u) = -\frac{c^2_u N 4\sqrt{\pi} \Gamma(d + 1/2) \Gamma(2(d - 1))}{(2\pi)^{2d} \Gamma(d - 1) \Gamma(2d)} \left( \frac{1}{\Lambda_u} \right)^{2d-2} \frac{1}{r^{2d-1}}, \quad (34) \]

where \( v \) is the velocity of the satellite at the polar orbit which is \( v \sim 2.51 \times 10^{-5} \). We have previously calculated the magnetic moment of the rotating charged gyroscope as \( \mu = 4.037 \times 10^{22} \text{rad.eV}^{-1} \). Hence, the precessional velocity due to the exchange of unparticles is

\[ \mu \times B = 5.38 \times 10^{-78} \frac{c^2_u N 4\sqrt{\pi} \Gamma(d + 1/2) \Gamma(2(d - 1))}{(2\pi)^{2d} \Gamma(d - 1) \Gamma(2d)} \left( \frac{1}{\Lambda_u} \right)^{2d-2} \frac{1}{r^{2d-1}}. \quad (35) \]

If this precessional velocity Eq.35 can contribute to the uncertainty in the measurement of the precessional velocity between the GR and GP-B results then we can obtain bounds on \( c_u \). In Fig.6, we have shown the variation of axial vector coupling \( c_u \) with the scaling dimension \( d \) for unparticle exchange between the earth and the GP-B satellite for the geodetic (blue lines) and frame-dragging effect (red lines). Here we have chosen the values of the energy scale as \( \Lambda_u = 1 \text{ TeV} \) (solid lines), \( \Lambda_u = 10 \text{ TeV} \) (dot-dashed lines) and \( \Lambda_u = 0.1 \text{ TeV} \) (dashed lines). With increasing \( \Lambda_u, c_u \) increases. The frame-dragging effect gives the stronger bound on \( c_u \) for any value of \( d \). Here, we have chosen \( r \) as the earth-satellite distance. For \( d = 1 \) we obtain \( c_u \lesssim 4.71 \times 10^{-41} \).

In Table I we have summarized the upper bounds on the coupling for the light particles discussed in the paper. To derive the upper bounds on the coupling for spin dependent long
range forces, we assume that all the SM particles in the gyroscope are polarized due to the presence of earth’s magnetic field. Practically, a fraction ($\alpha$) of the spins of the SM particles can be polarized due to earth’s magnetic field and $N_i = \alpha N_j$, where $N_i$ is the number of polarized spins and $N_j$ is the total number of particles. In deriving these bounds on the coupling in the above sections, we take $\alpha = 1$. However, for values of $\alpha < 1$, the coupling in the Table I will be replaced by coupling $\times \alpha$. For $\alpha < 1$, we will obtain weaker bounds on the coupling compared to $\alpha = 1$. 

\[
\begin{array}{l}
\Lambda_u = 0.1 \text{ TeV}, 1 \text{ TeV}, 10 \text{ TeV} \\
\end{array}
\]
VIII. DISCUSSIONS

In this paper, we have obtained the upper bounds on the different operator couplings of ultralight axions, vector gauge bosons, and unparticles from the study of geodetic and frame-dragging effects. The gyroscope of the GP-B satellite which measures the spacetime curvature near earth contains lots of electrons and nucleons. Ultralight axions, vector gauge bosons, and unparticles can interact with those SM particles through different operators and change the drift rate of the gyroscope. Some of these ultralight particles can either behave as a long range force between some dark sector or earth and the gyroscope or they can behave as a background oscillating dark matter fields or both. These ultralight particles can contribute to the uncertainties in the measurement of drift rate of the gyroscope obtained from the GR and GP-B results and we obtain bounds on different operator couplings. If the ultralight axions mediates a long range force between the dark sector and the GP-B gyroscope then the axion induced magnetic field can change the drift rate of the gyroscope and we obtain the upper bound on the axion decay constant as $f \lesssim 4.21 \times 10^{16}$GeV for the axions of mass $m_a \lesssim 10^{-26}$eV. The ultralight axions can also behave as a background time oscillating field and when the gyroscope passes through this background field, the drift rate of the gyroscope changes. Hence, we obtain the axion nucleon coupling as $g_{aNN} \lesssim 9.79 \times 10^{-15}$GeV$^{-1}$ for the axions of mass $m_a \lesssim 1.12 \times 10^{-19}$eV. The time oscillating background axion field can also couple with the nucleons of the gyroscope through electric dipole moment operator. The earth’s magnetic field induces the precession of the gyroscope and we obtain the upper bound on the coupling as $g_d \lesssim 5.51 \times 10^{-10}$GeV$^{-2}$ for axion mass $m_a \lesssim 1.12 \times 10^{-19}$eV. Note, the QCD axions cannot contribute to the drift rate of the GP-B gyroscope within the available uncertainty limit. Due to the presence of electrons in the GP-B gyroscope and the earth, long range force of $L_e - L_{\mu,\tau}$ type can mediate between the earth and the GP-B. The mediation of $L_e - L_{\mu,\tau}$ gauge bosons can change the drift rate of the gyroscope and we obtain the bound on the gauge coupling as $g \lesssim 4.01 \times 10^{-41}$ for ultralight vector gauge boson mass $M_{Z'} \lesssim 2.82 \times 10^{-14}$eV. This bound on flavor changing long range force of $L_e - L_{\mu,\tau}$ type is stronger than any other bound available in the literature. Time oscillating ultralight dark photon field can also interact with the nucleons of the gyroscope through electric and magnetic dipole moment operators and change the drift rate of the GP-B gyroscope. We obtain the upper bounds for EDM operator as $g'_{EDM} \lesssim 3.69 \times 10^{-19}$GeV$^{-1}$ and for MDM operator as
$g^\prime_{MDM} \lesssim 1.42 \times 10^{-14} \text{GeV}^{-1}$ for the dark photon of mass $m_{A^\prime} \lesssim 1.12 \times 10^{-19} \text{eV}$. Note, the EDM coupling puts stronger bound than the MDM coupling. The massless unparticles can also mediate long range force between the earth and the gyroscope and alter the drift rate of the GP-B satellite. We obtain the bound on the unparticle coupling as $c_u \lesssim 4.71 \times 10^{-41}$ for the scaling dimension $d = 1$. To derive the upper bounds on the coupling for spin dependent long range forces, we assume that all the SM particles in the gyroscope are polarized due to the presence of earth’s magnetic field. Practically, a fraction of the spins of the SM particles can be polarized due to earth’s magnetic field and $N_i = \alpha N_j$, where $N_i$ is the number of polarized spins and $N_j$ is the total number of particles. In deriving these bounds on the coupling in the paper, we take $\alpha = 1$. However, for values of $\alpha < 1$, the coupling will be replaced by coupling$\times\alpha$. For $\alpha < 1$, we will obtain weaker bounds on the coupling compared to $\alpha = 1$. Some of these ultralight particles can be a good candidates for fuzzy dark matter and can be probed from the measurements geodetic and frame-dragging effect.

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