The Standard Model from extra dimensions

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Abstract

We present a simple $N = 1$ five-dimensional model where the fifth dimension is compactified on the orbifold $S^1/Z_2$. Non-chiral matter lives in the bulk of the fifth dimension (five dimensions) while chiral matter lives on the fixed points of the orbifold (four-dimensional boundaries). The massless sector constitutes the Minimal Supersymmetric Standard Model while the massive modes rearrange in $N = 2$ supermultiplets. After supersymmetry breaking by the Scherk-Schwarz mechanism the zero modes can be reduced to the non-supersymmetric Standard Model.
The Standard Model (SM) of electroweak and strong interactions has been probed at high-energy colliders for energies $\leq 200$ GeV, and confirmed with an accuracy $\lesssim 1\%$. In spite of this fact, we know that the SM cannot be a fundamental theory describing all interactions because of its inability to give an answer to a number of more fundamental questions, as e.g. the hierarchy problem or how to consistently include gravity at the quantum level. For that reason extensions of the SM have been proposed and are nowadays the object of experimental searches. In particular the minimal supersymmetric extension of the SM (MSSM), aiming to (technically) solve the hierarchy problem, is expected to arise as an effective theory of some underlying fundamental supersymmetric theory valid at scales close to the Planck scale, as e.g. string of M-theory. However the knowledge of the fundamental theory is not enough to make the link with the low energy SM, since it is known that supersymmetry is not an exact symmetry of the nature and the mechanism of supersymmetry breaking is for the moment unclear.

The presence of extra (compact) space dimensions is a common feature of any fundamental theory valid at high scales. If their radii are of the order of the Planck length, the corresponding excitations are superheavy and decouple from the low-energy physics. However, if there is one or more large radii they might have a number of phenomenological and theoretical implications [1]–[5]. In particular there is recently an increasing interest in understanding the role of large extra dimensions to describe the strong coupling limit of string theory [6], to transmit supersymmetry breaking between different four-dimensional boundaries [7, 8], to provide possible alternative solutions to the hierarchy problem [9, 10], to modify the celebrated LEP unification of gauge couplings [11], and even to explain possible modifications of gravitational measurements in the millimiter range [12]. Also large extra dimensions have been proved useful, and here is where we focus our main interest, to provide a consistent and calculable supersymmetry breaking [13, 14].

In this letter we will follow the previous line of investigation and explore the role of extra dimension(s) to provide the source of new physics beyond the TeV scale and, at the same time, to make the connection with the SM at present energies. In particular we will present a simple $N = 1$ supersymmetric model in five dimensions (5D) whose massless sector, upon compactification of the fifth dimension on $S^1/Z_2$, is the MSSM while its massive modes rearrange in $N = 2 D = 4$ supermultiplets. Furthermore, when the remaining $N = 1 D = 4$ supersymmetry of the massless modes is broken by a Scherk-Schwarz [13] mechanism on the fifth dimension, the zero modes can be reduced to the non-supersymmetric Standard Model, while the $N = 2$ structure of the massive modes is in general spoiled.
1. Here we introduce the model, based on a 5D theory compactified on $S^1/\mathbb{Z}_2$. The model has $N = 1$ supersymmetry. Non-chiral matter, as the gauge and Higgs sector of the theory, lives in (5D) the bulk of the fifth dimension, while chiral matter, the three generations of quarks and leptons and their superpartners, live on the 4D boundary (the fixed points of the orbifold $S^1/\mathbb{Z}_2$). This structure can arise from string theories [2] and has been recently considered for theoretical and phenomenological considerations [3, 4, 5, 6, 7].

In five dimensions, the vector supermultiplet $(V_M, \lambda^i_L, \Sigma)$ of an $SU(N)$ gauge theory consists of a vector boson $V_M (M = 0, \ldots, 3, 5)$, a real scalar $\Sigma$ and two bispinors $\lambda^i_L (i = 1, 2)$, all in the adjoint representation of $SU(N)$. The 5D lagrangian is given by

$$\mathcal{L} = \frac{1}{g^2} \text{Tr} \left[ -\frac{1}{2} F^2_{MN} + |D_M \Sigma|^2 + i \gamma^M D_M \lambda^i - \lambda^i [\Sigma, \lambda^i] \right],$$

where $\lambda^i$ is a symplectic-Majorana spinor

$$\lambda^i = \left( \begin{array}{c} \lambda^i_L \\ \epsilon^{ij} \bar{\lambda}_L \end{array} \right).$$

The 5D matter supermultiplet, $(H_i, \Psi)$, consists of two scalar fields, $H_i (i = 1, 2)$, and a Dirac fermion $\Psi = (\Psi_L, \Psi_R)^T$. We will consider two matter supermultiplets, $H^a_i$, $\Psi^a$ ($a = 1, 2$), that we will associate with the two Higgs doublets of the MSSM. The $N = 1$ lagrangian for the matter supermultiplets interacting with the vector supermultiplet is given by

$$\mathcal{L} = |D_M H^a|^2 + i \nabla \gamma^M D_M \Psi^a - (i \sqrt{2} H^a_d \bar{H}^a + h.c.) - \nabla \Sigma \Psi^a$$

$$- H^a_i \Sigma^2 H^a_i - \frac{g^2}{2} \sum_{m,\alpha} [H^a_i (\sigma^m)^i_j T^\alpha H^a_j]^2,$$

where $\sigma^m$ are the Pauli matrices. The lagrangians of Eqs. (1) and (3) have an $SU(2)_R \times SU(2)_H$ global symmetry. Under the $SU(2)_R \times SU(2)_H$ symmetry the fermionic fields transform as doublets, $\lambda^i \in (2, 1)$, $\Psi^a \in (1, 2)$, while Higgs bosons transform as bidoublets $H^a_i \in (2, 2)$. The rest of the fields in the vector multiplet are singlets.

Since all interactions in the lagrangians (1) and (3) are gauge interactions the model based on the gauge group $SU(3) \times SU(2) \times U(1)$ is easily constructed from these expressions. It contains 5D vector multiplets in the adjoint representation of $SU(3) \times SU(2) \times U(1)$ $[(8, 1, 0) + (1, 3, 0) + (1, 1, 0)]$ and two Higgs hypermultiplets in the representation $[(1, 2, 1/2) + (1, 2, -1/2)]$. The chiral matter is located on the boundary and contains the usual chiral $N = 1$ 4D multiplets.

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1. The five dimensional $N = 1$ lagrangian can be deduced from the $N = 2$ four dimensional lagrangian of Ref. [10]. We define $V_M = g V_M T^\alpha$, $\Sigma = g \Sigma^\alpha T^\alpha$ and $\lambda = g \lambda^\alpha T^\alpha$ where $T^\alpha$ are the generators of $SU(N)$ with $\text{Tr}[T^\alpha T^\beta] = \delta^{\alpha\beta}/2$. 2
We will reduce the theory from five to four dimensions by compactifying on $S^1/\mathbb{Z}_2$, a circle with the identification $y \rightarrow -y$. The transformation of the fields under the discrete parity $\mathbb{Z}_2$ is determined by the interactions of Eqs. (1) and (3). We find
\[
\Phi(x^\mu, y) \rightarrow \eta \Phi(x^\mu, -y),
\]
where the states with $\eta = \pm 1$ are given in Table 1.

| $\eta = +1$ | $\eta = -1$ |
|-------------|-------------|
| $V_\mu$, $H_2^2$, $H_1^1$ | $V_5, \Sigma$, $H_3^2$, $H_2^1$ |
| $\lambda_L^1$, $\Psi_L^2$, $\Psi_R^1$ | $\lambda_L^2$, $\Psi_R^2$, $\Psi_L^1$ |

Table 1

We will call even the fields with $\eta = +1$ and odd those with $\eta = -1$. Notice that we have rearranged fields in Table 1 in components of $N = 1 \ D = 4$ supersymmetric multiplets that are disposed along the same column.

Compactifying on $S^1/\mathbb{Z}_2$ is equivalent to compactifying on a circle and imposing the discrete parity $\mathbb{Z}_2$. In the $S^1$ compactification the fields can be Fourier expanded as
\[
\Phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} e^{iny/R} \Phi^{(n)}(x^\mu),
\]
where $R$ is the $S^1$ radius. After integration with respect to the fifth dimension, the four dimensional theory consists of a tower of KK excitation $\Phi^{(n)}(x^\mu)$ with mass $n/R$. By imposing the $\mathbb{Z}_2$ parity, the Fourier expansion is now given by
\[
\Phi_+(x^\mu, y) = \sum_{n=0}^{\infty} \cos \frac{ny}{R} \Phi^{(n)}(x^\mu), \quad \text{for the even fields},
\]
\[
\Phi_-(x^\mu, y) = \sum_{n=1}^{\infty} \sin \frac{ny}{R} \Phi^{(n)}(x^\mu), \quad \text{for the odd fields}.
\]
Then the $\mathbb{Z}_2$ symmetry projects out half of the tower of KK modes of Eq. (1). Each level of the KK-excitations form massive $N = 2 \ D = 4$ supermultiplets. The $V_5^{(n)}$ field is eaten by the vector $V_\mu^{(n)}$ to become massive while $\lambda_L^{1(n)}$ and $\lambda_L^{2(n)}$ become the components of a massive Dirac fermion. These fields together with $\Sigma^{(n)}$ form an $N = 2$ vector multiplet. $H_i^{a(n)}$ and $\Psi^{a(n)}$ form two $N = 2$ hypermultiplets. The structure of $N = 2$ supermultiplets is displayed in Table 2.

| Vector multiplets | Hypermultiplets |
|------------------|----------------|
| $V_\mu^{(n)}$, $\Sigma^{(n)}$, $\lambda_L^{1(n)}$, $\lambda_L^{2(n)}$ | $H_1^{1(n)}$, $H_2^{1(n)}$, $H_1^{2(n)}$, $H_2^{2(n)}$, $\Psi^{1(n)}$, $\Psi^{2(n)}$ |

Table 2
At the massless level \((n = 0)\), however, only the even fields, see Table 1, are left in the theory and we have an \(N = 1\) supersymmetric theory. Therefore the massless spectrum of this model has just the MSSM content.

Notice that, by the process of compactification over \(S^1/Z_2\), the net number of towers with states \(\Phi^{(n)}\) \((n = -\infty, \ldots, \infty)\) is divided by two. In fact the two towers defined by the 5D fields \(\Phi_+\) and \(\Phi_-\) \((n \geq 0)\) give rise, after compactification, to the single tower \(\Phi\) \((n \in \mathbb{Z})\) defined by \(\Phi = \Phi_+ - i\Phi_-\), i.e.

\[
\Phi^{(\pm n)} = \frac{1}{2}\left\{ \Phi^{(n)}_+ \pm \Phi^{(n)}_- \right\} , \quad (n \geq 0).
\]  

\(2\). In order to get, at the massless level, only the SM fields we have to break supersymmetry. We will use the Scherk-Schwarz (SS) mechanism \([15]\) that consists in imposing to the (superpartner) 5D fields a nontrivial periodic condition under a \(2\pi\) translation of the fifth dimension:

\[
\Phi(x^\mu, y + 2\pi) = e^{2\pi i q T} \Phi(x^\mu, y),
\]

where \(T\) must be a generator of a global symmetry of the five dimensional theory \([15]\) under which the field \(\Phi\) transforms with charge \(q\). The requirement \((8)\) implies a \(y\)-dependence for the fields different from that in eq. \((6)\):

\[
\Phi(x^\mu, y) = e^{iq T y/R} \tilde{\Phi}(x^\mu, y),
\]

where \(\tilde{\Phi}(x^\mu, y)\) are periodic functions \(\tilde{\Phi}(x^\mu, y + 2\pi) = \tilde{\Phi}(x^\mu, y)\) and have the same Fourier expansion as in Eq. \((6)\). Eq. \((9)\) implies a nontrivial \(y\)-dependence for the \(n = 0\) mode that leads to a mass term in the four dimensional theory.

We will consider that \(e^{2\pi i q T}\) belongs to \(SU(2)_R \times SU(2)_H\). Since the theory is compactified on \(S^1/Z_2\), we must require

\[
Z_2 e^{iq T y/R} = e^{iq (-y)/R} Z_2.
\]  

This condition guarantees that \(e^{iq T y/R} \tilde{\Phi}\) has the same \(Z_2\)-transformation as \(\tilde{\Phi}\). The transformation \(Z_2\) on the \(SU(2)_R\) and \(SU(2)_H\) doublets, \(\lambda^i\) and \(\Psi^a\), is respectively given by

\[
Z_2 = \pm \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \gamma_5 ,
\]  

where the matrix \(\mp \sigma_3\) in \((11)\) acts on \(SU(2)_R\) or \(SU(2)_H\) indices, while \(\gamma_5\) acts on the spinor indices. Therefore the condition \((10)\) implies \(T = \sin \theta \sigma_1 + \cos \theta \sigma_2\) \([13]\) where \(\sigma_i\) are the Pauli matrices representing the \(SU(2)_{R,H}\) generators. For simplicity we will
only consider $\theta = 0$. Eq. (7) will imply
\[
\begin{pmatrix}
\lambda^1 \\
\lambda^2 \\
\Psi^1 \\
\Psi^2 \\
H_1^1 & H_1^2 \\
H_2^1 & H_2^2
\end{pmatrix}
= e^{i q R \sigma_2 y / R} \begin{pmatrix}
\lambda^1 \\
\lambda^2 \\
\Psi^1 \\
\Psi^2 \\
\tilde{H}_1^1 & \tilde{H}_1^2 \\
\tilde{H}_2^1 & \tilde{H}_2^2
\end{pmatrix},
\]
\[
(12)
\]
Using the $y$-dependence of the fields given in Eq. (12), we obtain, after integrating with respect to the fifth dimension $y$, the following mass spectrum for $n \neq 0$:
\[
\mathcal{L}_m = \frac{1}{R} \left\{ \left( \begin{array}{cc}
\lambda^1_L & \lambda^2_L \\
n & n \\
q_R & q_R
\end{array} \right) \left( \begin{array}{c}
\lambda^1_L \\
n \\
q_R
\end{array} \right)
+ \left( \begin{array}{cc}
\bar{\Psi}^1_L & \bar{\Psi}^2_L \\
n & -q_H \\
q_H & -n
\end{array} \right) \left( \begin{array}{c}
\Psi^1_R \\
n \\
-\Psi^2_R
\end{array} \right) + h.c. \right\}
- \frac{1}{R^2} \left( \begin{array}{c}
H_{0}^{(n)\dagger} & H_{2}^{(n)\dagger} & H_{1}^{(n)\dagger} & H_{3}^{(n)\dagger}
\end{array} \right)
\left( \begin{array}{c}
n^2 + q_-^2 & -2 i q_- & 2 i q_- & n^2 + q_-^2 \\
2 i q_- & -2 n_q^+ & n^2 + q_+^2 & -2 n_q^+
\end{array} \right)
\left( \begin{array}{c}
H_{0}^{(n)} \\
H_{2}^{(n)} \\
H_{1}^{(n)} \\
H_{3}^{(n)}
\end{array} \right),
\]
where we have redefined the scalar fields as $H_i^a = H_{\mu}(\sigma^\mu)^i_\alpha$, $\sigma^\mu \equiv (1, \vec{\sigma})$, and $q_\pm = q_R \pm q_H$. Therefore the $n$-KK modes are now given by two Majorana fermions $(\lambda^1_L \pm \lambda^2_L)$ with masses $|n \pm q_R|$, two Dirac fermions, $(\Psi^1_L \pm \Psi^2_L)$ with masses $|n \pm q_H|$, and four scalars, $(H_0^{(n)} \pm i H_2^{(n)})$ and $(H_1^{(n)} \pm H_3^{(n)})$ with masses $|n \pm (q_R - q_H)|$ and $|n \pm (q_R + q_H)|$ respectively. Of course, the mass spectrum of the fields $V_{\mu}^{(n)}$, $V_{5}^{(n)}$ and $\Sigma^{(n)}$ is not modified by the SS compactification, since they are singlets under $SU(2)_R \times SU(2)_H$.

For $n = 0$, we have
\[
\mathcal{L}_m = \frac{1}{R} \left\{ q_R \lambda^1_L \lambda^1_L + q_H \bar{\Psi}^2_L \Psi^1_R + h.c. \right\}
- \frac{1}{R^2} \left\{ (q_R - q_H)^2 |H_0^{(0)}|^2 + (q_R + q_H)^2 |H_3^{(0)}|^2 \right\}.
\]
(14)
The massless spectrum now consists of only the $n = 0$ mode of the vector fields $V^\mu$. Nevertheless a massless scalar Higgs can be obtained if either of the following conditions are satisfied:

- $q_R - q_H = n,$
- $q_R + q_H = n.$

\[\text{Notice that with this definition $H_0$ and $H_3$ are even, while $H_1$ and $H_2$ are odd fields.}\]
with \( n = 0, \pm 1, \pm 2, \ldots \). Therefore the massless spectrum of the model after the SS compactification can be reduced to the SM with one or two Higgs doublets. For example, for \( q_R - q_H = 0 \) and \( q_R + q_H \neq 0 \) we have a single massless scalar \( H_0^{(0)} \). We would like to associate this scalar with the SM Higgs. Nevertheless we find that its self-interacting quartic coupling, given by the D-term potential

\[
V_D = g^2 \sum_\alpha \varepsilon^{mnp} \left[ H_0^\alpha H_m^\dagger + i H_n^\alpha H_p^\dagger + h.c. \right]^2,
\]

is zero. This is because \( H_0 = H_1^1 + H_2^2 \) is a flat direction of the \( D \)-term contribution, as it is obvious from Eq. (15). Therefore \( H_0 \) has, at the tree level, a flat potential. Of course, quantum effects will lift the flat direction since supersymmetry is broken, and we can expect that they will induce a nontrivial minimum for the scalar Higgs. Notice that the quantum structure of these theories is quite different from ordinary softly broken supersymmetric theories. In our theory, the full tower of massive KK must be included at the one-loop level. We leave this for further investigation. A second alternative is to consider \( q_R - q_H = 0 \) and \( q_R - q_H = 1 \). In this case one has two massless scalars \( H_0^{(0)} \) and \( H_1^{(1)} - H_3^{(1)} \). This scenario corresponds to a two Higgs doublet sector similar to that of the MSSM.

Finally, the family sector of this theory resides in the four dimensional boundary and then do not have KK excitations. Since the theory is \( N = 1 \) supersymmetric, there are massless scalars associated to each fermion (the squarks and sleptons). Nevertheless, supersymmetry is broken in the bulk and those scalar will get masses at the one-loop level. The massless spectrum will only correspond to the fermions of the SM.
References

[1] I. Antoniadis, \textit{Phys. Lett.} \textbf{B246} (1990) 377; Proc. PASCOS-91 Symposium, Boston 1991 (World Scientific, Singapore, 1991), p. 718.

[2] I. Antoniadis, C. Muñoz and M. Quirós, \textit{Nucl. Phys.} \textbf{B397} (1993) 515; I. Antoniadis, K. Benakli and M. Quirós, \textit{Phys. Lett.} \textbf{B331} (1994) 313.

[3] T. Banks and M. Dine, \textit{Nucl. Phys.} \textbf{B479} (1996) 173; \textit{Nucl. Phys.} \textbf{B505} (1997) 445.

[4] E. Cáceres, V. Kaplunovsky and I.M. Mandelberg, \textit{Nucl. Phys.} \textbf{B493} (1997) 73.

[5] I. Antoniadis and M. Quirós, \textit{Phys. Lett.} \textbf{B392} (1997) 61.

[6] P. Horava and E. Witten, \textit{Nucl. Phys.} \textbf{B460} (1996) 506; \textit{Nucl. Phys.} \textbf{B475} (1996) 94.

[7] P. Horava, \textit{Phys. Rev.} \textbf{D54} (1996) 7561.

[8] E.A. Mirabelli and M. Peskin, \texttt{hep-th/9712214}.

[9] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, \texttt{hep-ph/9803313}.

[10] H. Hatanaka, T. Inami and C.S. Lim, \texttt{hep-ph/9805067}.

[11] K.R. Dienes, E. Dudas and T. Gherghetta, \texttt{hep-ph/9803460}.

[12] I. Antoniadis, S. Dimopoulos and G. Dvali, \texttt{hep-ph/9710204}; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, \texttt{hep-ph/9804398}.

[13] I. Antoniadis and M. Quirós, \textit{Nucl. Phys.} \textbf{B505} (1997) 109; \textit{Phys. Lett.} \textbf{B416} (1998) 327; \textit{Nucl. Phys. Proc. Suppl.} \textbf{62A-C} (1998) 312, \texttt{hep-ph/9709023}.

[14] E. Dudas and C. Grojean, \textit{Nucl. Phys.} \textbf{B507} (1997) 553; E. Dudas, \textit{Phys. Lett.} \textbf{B416} (1998) 309.

[15] J. Scherk and J.H. Schwarz, \textit{Nucl. Phys.} \textbf{B153} (1979) 61 and \textit{Phys. Lett.} \textbf{B82} (1979) 60; E. Cremmer, J. Scherk and J.H. Schwarz, \textit{Phys. Lett.} \textbf{B84} (1979) 83; P. Fayet, \textit{Phys. Lett.} \textbf{B159} (1985) 121 and \textit{Nucl. Phys.} \textbf{B263} (1986) 649.

[16] M. Sohnius, \textit{Phys. Rep.} \textbf{128} (1985) 39.