To the issue of the control law influence on the active dynamic vibration damper efficiency

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Abstract. The work is devoted to the investigation of the influence of control laws on the tuning conditions of the active vibration damper. Active dynamic vibration damper is an additional mass with elasticity and power drive (electrodynamic, electromagnetic) and mounted on the main oscillating mass. Control of the power drive (actuator) is carried out according to information on the movement of the main mass and mass of the absorber. To define the fundamental capabilities of the vibration isolation system with an active dynamic vibration damper, the work of the actuator was assumed to be ideal. The basis of the dynamic model is the differential equations written relative to the equilibrium position of the two-mass system and taking into account the adopted control law of the actuator. It is shown that the use of an active vibration damper makes it possible to significantly expand the frequency range of the minimum value of the force transfer coefficient to the base in comparison with passive dynamic vibration dampers. Based on the results of the study, recommendations were received for the design of vibration isolation systems with an active dynamic vibration damper.

1. Introduction

Widely used in various branches of technology, passive vibration isolation systems weaken well the relatively high-frequency components of the vibratory forces transmitted to the hull. But reducing the load on the base for low frequencies is still quite an actual problem. Note that for the purpose of vibration protection, active vibration protection systems (AVPS) are widely used, in which hydraulic, electrodynamic, piezoelectric, etc. are used as a power device (actuator) with an active suppression range of 5 - 20 Hz. However, for the purpose of vibration isolation, i.e. to reduce the force transfer to the base, which is especially important, for example, for shipbuilding, active systems are practically not used, although the creation of an effective vibration isolation system at frequencies of 2 to 10 Hz and below is an unresolved problem to date. Schematic diagrams and the work of active systems of vibration isolation are considered in [1 - 5]. A detailed analysis is given in the review papers [1, 2] and the limiting possibilities of active systems with actuators of various types (electrodynamic, magnetoelectric, piezoelectric, etc.) mounted between the oscillating mass and the body, whose operation is determined by the control system by the signals of the accelerometer and (or) the force sensor are analyzed in detail and presented. Active vibration isolation systems increase the efficiency of attenuation of force transfer to the base compared to passive systems in a rather narrow frequency range behind the resonance of the oscillatory system, can have a tuning frequency in this region with a minimum value of the vibration damper coefficient $K_p$ [6] and lower the value of the resonant frequency [7]. A decrease in the vibration isolation coefficient in the near-resonance region by means of an actuator mounted between the oscillating mass and the shell is fundamentally impossible, since at these frequencies the decrease in the amplitude of the mass oscillations is compensated by the increase in the actuator force on the shell. To solve the problem of reducing the force transfer on the shell in the low frequency region, it is possible to use active dynamic vibration dampers mounted on an oscillating...
mass [2]. The control law of the motion of the mass of the vibration damper largely determines the effectiveness of vibration isolation. In this paper, the effectiveness of the use of an active vibration damper for a linear control law of the movement of the mass of the damper proportional to the displacements of the main mass and the mass of the damper is considered.

Figure 1. Vibration damper circuit

2. Formulation of the problem
The circuit of the simplest vibration damper, as a two-mass mechanical system, is shown in figure 1. The mathematical model of this vibration isolation system with a dynamic vibration damper can be written in the form [8].

\[
m_0 \ddot{x}_0 + c_0 x_0 + c_1 (x_0 - x_1) = F(t) \\
m_1 \ddot{x}_1 + c_1 (x_1 - x_0) = 0.
\]  

(1)

Equations (1) are compiled without allowance for viscous friction, which at the initial stage of the study allows us to determine the damper settings, which ensure complete damping of the oscillations at a given frequency.

The allowance of viscous friction does not allow to obtain such representative results on tuning the damper and requires a separate consideration.

If the perturbing force has a harmonic form, i.e. \( F(t) = F_0 \cdot e^{i\omega t} \), then the solution of equations (1) in the steady mode can be represented in the form:

\[
x_0 = \bar{x}_0 \cdot e^{i\omega t}, \quad x_1 = \bar{x}_1 \cdot e^{i\omega t}
\]

where \( \bar{x}_0 \) and \( \bar{x}_1 \) – oscillation amplitudes.

If we introduce the notation \( \omega_0^2 = \frac{C_0}{m_0}, \omega_1^2 = \frac{C_1}{m_0}, \mu = \frac{m_1}{m_0} \) then for the reaction of the base, we can write the expression:

\[
R = \frac{F_0 \cdot \omega_0^2 (\omega_0^2 - \mu \omega_1^2)}{(\omega_0^2 - \mu \omega_1^2) \omega_0^2 - (\omega_1^2 - \mu \omega_1^2) \omega_0^2 - \mu \omega_0^2 \omega_1^2}.
\]

(2)

It follows from (2) that the total damping of the oscillations for the \( \omega \) frequency will be satisfied if \( \omega_0^2 - \mu \omega_1^2 = 0 \).

For the vibration isolation coefficient \( K = \left| \frac{R}{F_0} \right| \) after the introduction of notation \( \Omega = \frac{\omega}{\omega_0}, \Omega_1 = \frac{\omega_1}{\omega_0} \) we obtain:
For the values \( \Omega_0^2 = 0.028, \mu = 0.01 \), that corresponds to the parameters \( m_0 = 100 \text{ kg}; m_1 = 1 \text{ kg}; C_0 = 3.56 \times 10^4 \text{ N/m}; C_1 = 10^3 \text{ N/m} \). The amplitude-frequency characteristic of the vibration isolation coefficient is plotted (curve 2 in figure 3).

3. Theory

A layout diagram of the vibration isolation with an active vibration damper without considering the damping is shown in figure 2.

![Figure 2. Layout diagram](image)

1 – displacement sensor \( x_0 \); 2 – relative movement sensor \( x_1 - x_0 \); 3 – actuator

The control law of the actuator is formulated in the form

\[
u = K_0 x_0 + K_1 x_1.
\]

The equations of motion of the system in figure 2 with respect to the equilibrium position will have the form

\[
\begin{align*}
\ddot{x}_0 + \left( \omega_0^2 + \frac{K_0}{m_0} \right) x_0 - \left( \omega_1^2 + \frac{K_1}{m_0} \right) x_1 &= \frac{F(t)}{m_0} \\
\ddot{x}_1 + \left( \omega_1^2 + \frac{K_1}{m_1} \right) x_1 - \left( \omega_0^2 + \frac{K_0}{m_1} \right) x_0 &= 0
\end{align*}
\]

As before, assuming that \( F(t) = F \cdot e^{j\omega t} \) for the amplitudes and we obtain:

\[
\dot{x}_0 = \begin{pmatrix}
\frac{F_0}{m_0} \\
-\omega_0^2 + \omega_1^2 + \frac{K_0}{m_0} \\
-\omega_0^2 + \omega_1^2 + \frac{K_0}{m_0} + \frac{K_0}{m_1} \\
-\omega_0^2 + \omega_1^2 + \frac{K_0}{m_0} + \frac{K_0}{m_1} + \frac{K_0}{m_1}
\end{pmatrix}
\]

\[
\dot{x}_1 = \begin{pmatrix}
\frac{F_0}{m_0} \\
-\omega_1^2 + \frac{K_1}{m_0} \\
-\omega_1^2 + \frac{K_1}{m_0} + \frac{K_1}{m_1} \\
-\omega_1^2 + \frac{K_1}{m_0} + \frac{K_1}{m_1} + \frac{K_1}{m_1}
\end{pmatrix}
\]
For the vibration isolation coefficient \( |K(\imath \omega)| = \frac{|K|}{F_0} \) in accordance with (5), we can write:

\[
|K(\imath \omega)| = \omega_0^2 \cdot \tau_0.
\]

It follows from (5) that in order to set the active vibration damper, it is necessary to satisfy condition

\[
- \omega^2 + \frac{\omega_0^2}{\mu} \cdot \frac{K_0}{m_0} = 0,
\]

i.e. the introduction of control on \(+K_1\) allows to shift the tuning point to the low-frequency region, however, it follows from (4) that the inequality \( K_1(\omega_0^2 \cdot m_0) \) must be satisfied, since otherwise, the basic properties of the vibration damper as a two-mass system are violated.

It can be seen that the introduction of \( u = +K_1 x_1 \) control is analogous to the decrease of the stiffness \( c_1 \) in the passive system, which leads to an increase in the amplitude of the oscillations along the \( x_1 \) coordinate. For example, at the tuning frequency, the amplitude \( \tau_1 \) will look like:

\[
\tau_1 = \frac{F_0}{c_1 - K_1}.
\]

It can be noted that the control \( u = -K_1 x_1 \) leads to an increase in the stiffness of \( c_1 \) in the analogue of the passive system.

Expressions (5) allow, under the assumptions made, to investigate the dynamics of the system for different ratios \( K_0 \) and \( K_1 \) under the control law of the actuator in the form \( u = K_0 x_0 \pm K_1 x_1 \). For the model problem, we assume \( m_0 = 100 \text{ kg}; m_1 = 1 \text{ kg}; c_0 = 3.56 \cdot 10^4 \text{ N/m}; c_1 = 10^3 \text{ N/m}. \)

The frequency characteristics of the dependence of the vibration isolation coefficient \( K(\omega) \) in accordance with the expression (6) under control only with respect to \( x_0 \), i.e. \( u = K_0 x_0 \) \((K_0 = 5 \cdot 10^3)\) and \( K_1 = 0 \) are shown in figure 3. For comparison, the frequency characteristics of the vibration isolation coefficients for systems with a passive dynamic damper and without a dynamic damper are also given.

From the graphs in figure 3 it follows that control of \( x_0 \) increases the frequency range with a minimum value of \( K(\omega) \leq 0.2 \) from 3 to 11 Hz.
Figure 3. Frequency characteristics of the vibration isolation coefficient $K(\omega)$ for the following systems: 1) with an active dynamic damper (DD); 2) with a passive dynamic damper; 3) without a dynamic damper.

Analysis of the combined control $u = K_0x_0 - K_1x_1$ shows that at $K_0 = 5 \times 10^5$, the $K_1$ increase from 0 to $5 \times 10^3$ shifts the tuning point to the high frequency region (from 5 to 12 Hz) while maintaining a sufficiently wide frequency range with $K(\omega) \leq 0.2$ (figure 4).

Figure 4. Investigation of the influence of the regulator $K1$ factor on the vibration isolation coefficient.
In figure 5 shows the results of constructing the frequency characteristics of \( K(\omega) \) under control of the form \( u = K_0x_0 + K_1x_1 \) for different values of \( K_1 \) and \( K_0 = 5 \times 10^5 \) and which implies that this type of control allows to shift the tuning point to the low-frequency region (for \( K_1 = 980 \) the tuning point corresponds to 0.42 Hz).

**Figure 5.** Frequency response of the vibration isolation coefficient for control of the form \( u = K_0x_0 + K_1x_1 \) \((K_1=980)\)

Analysis of the frequency characteristics \( x_1(\omega) \) for various control laws and \( K_1 \) values shows that by decreasing the magnitude of the \( K(\omega) \) amplitude, it is provided by a control of the form \( u = K_0x_0 - K_1x_1 \).

To clarify the evaluation of the effectiveness of vibration isolation with an active dynamic vibration damper, it is necessary in the circuit in figure 2 allow for damping in the suspension of the mass \( m_0 \) and in the active damper with a mass \( m_1 \).

The mass \( m_1 \) damping coefficient can be realized by supplying an additional signal to the actuator from the measured value \( x_1 - x_0 \), or from the relative velocity sensor \( \dot{x}_1 - \dot{x}_0 \).

Differential equations of motion of the vibration isolation system with allowance for damping have the form

\[
\begin{align*}
\ddot{x}_0 &= \frac{b_1}{m_0} (\dot{x}_1 - \dot{x}_0) - \frac{b_0}{m_0} \dot{x}_0 - \frac{c_0}{m_0} x_0 - \frac{c_1}{m_0} (x_0 - x_1) - \frac{K_0}{m_0} x_0 + \frac{K_1}{m_0} x_1 + \frac{F(t)}{m_0}, \\
\ddot{x}_1 &= \frac{b_1}{m_1} (\dot{x}_1 - \dot{x}_0) - \frac{c_1}{m_1} (x_1 - x_0) + \frac{K_0}{m_1} x_0 + \frac{K_1}{m_1} x_1
\end{align*}
\]

(9)

where \( b_0, b_1 \) – are the damping coefficients.

Taking into account the above notation, equation (9) can be written in the form
\begin{align}
\dot{x}_0 &= -2\xi_0\omega_0\mu(x_0 - \dot{x}_0) - 2\xi_0\omega_0\dot{x}_0 - \omega_0^2 x_0 - \omega_1^2\mu(x_0 - x_1) \pm \frac{K_1}{m_0} x_1 - \frac{K_0}{m_0} x_0 + F\sin\omega(t) \\
\dot{x}_1 &= -2\xi_1\omega_1(x_1 - x_0) - \omega_1^2(x_1 - x_0) + \frac{K_0}{m_0} x_0 + \frac{K_1}{m_1} x_1
\end{align}

(10)

where \(2\xi_0\omega_0 = \frac{b_0}{m_0}; \quad 2\xi_1\omega_1 = \frac{b_1}{m_1}\).

From expressions (10), in the case of an ideal actuator for \(K(\omega)\), the amplitude-frequency characteristics are built for the example in figure 6 at \(m_0 = 100 \text{ kg}; \quad m_1 = 1 \text{ kg}; \quad \xi_0 = 0.1; \quad K_0 = 5 \cdot 10^5; \quad K_1 = 0\) for different values of \(\xi_1\).

**Figure 6.** The influence of the coefficient \(\xi_1\) on the frequency characteristic of the vibration isolation coefficient

Analysis of the graphs shows that as the coefficient \(\xi_1\) increases, the increased frequency range due to the influence of \(K_0\) remains, the amplitudes \(\ddot{x}_0\) and \(\ddot{x}_1\) of the oscillations decreases, but the vibration isolation coefficient in the tuning zone increases.

The analysis of the considered vibration isolation system under polyharmonic action within the increased zone of the minimum values of the coefficient of the force transmission \(K\) on the base shows that the small values of \(K\) are remained. For example, for a control of the form \(u = K_0\dot{x}_0 - K_1\dot{x}_1\) (\(K_0 = 5 \cdot 10^4, K_1 = 10^3\)) and a polyharmonic signal \(F(t) = \sum_{i=5}^{10} F_i \cdot \sin 2\pi f_i t\) in the frequency range 5-10 Hz, the vibration isolation coefficient \(\bar{K} = \frac{\overline{R}}{\overline{F}_0}\) does not exceed 0.05.
For example, at $K_0 = 5 \cdot 10^4$ and $K_1 = 10^3$, the tuning point corresponds to 7 Hz, the force transfer coefficient $K(\omega)$ on the base is 0.09, and the amplitude $x_1$ is determined by the expression

$$\bar{x}_1 \approx \frac{F_0}{K_0 + c_1}$$

that at $F_0 = \frac{3H}{m_0}$ equals 0.25 m.

If the actuator (electrodynamic, electromagnetic, etc. drive) ideally fulfills the dynamic signal $K_0 x_0 + K_1 x_1$, there will always be design constraints on the amplitude of the displacement $x_1$ and the maximum drive force.

We assume that the maximum value $\bar{x}_1$ is 0.02 m, and the drive force is not more than 50 N If, during the operation of the drive, the mass $m_1$ part of the oscillation period will be on the stops, then the principle of dynamic compensation. In these moments, the inertial load is not created and in addition, this essentially impact mode can disable the construction of the actuator.

As a consequence, it is necessary to introduce a block into the actuator control system, whose operation would exclude such a mode.

The mass $m_1$ together with the actuator, the spring $c_1$ and the conditional damper $b_1$ is set to the mass $m_0$ and if we take into account that $x_0 \ll x_1$, then the force required for the displacement of the actuator $x_1$ is determined from expression

$$F_u = \bar{x}_1 \cdot \sqrt{(\omega_1^2 - \omega^2)^2 + 4n_1^2\omega^2},$$

(11)

where $n_1 = \frac{b_1}{2m_1}$.

Assuming for the model example, that $\bar{x}_1 = 2 \cdot 10^{-2}$ m, $f_1 = \frac{1}{2\pi} \omega_1 = 5$ Hz, the permissible values of $F_u$ are given in figure 7.

![Figure 7. The dependence of the permissible force of the actuator on the frequency for the displacement amplitude of 2 cm](image)

The basic block diagram of the device in automatic mode limiting the force in the actuator and, accordingly, the displacement $x_1$ for the harmonic mode is shown in figure 8.
To assess the operability of the device in figure 8 in the Matlab / Simulink package the model is compiled (figure 9). In the model, a block diagram of the actuator force limiter for a single-frequency action (frequency and $F_a$ is set) is realized, and in addition, design constraints on the amplitude $x_1 = 0.02$ m and maximum force in the actuator (50 N) are taken into account with the help of Saturation units.

**Figure 9.** Model of vibration isolation system with active vibration damper
In figure 10 is a graph of the development of oscillations along the $x_1$ coordinate for controlling $u = K_0 x_0 - K_1 x_1$ and the following parameter values: $F_0 = 50 \text{ N}$; $f = 5 \text{ Hz}$; $\xi_0 = 0.1$; $K_1 = 10^3$; $K_0 = 5 \cdot 10^5$; $m_0 = 100 \text{ kg}$; $m_1 = 1 \text{ kg}$; $F_a = 19.74\text{N}$.

![Figure 10. Development of oscillations $x_1$ of the dynamic absorber](image)

It follows from the graph that after a transient process for 1.5 s, when the mass $m_1$ reaches the structural stops, the steady-state regime for the amplitude $x_1$ does not exceed the established $2 \cdot 10^{-2} \text{ m}$.

For these parameters, the frequency of 7 Hz is the tuning point (figure 3) and in the system without restrictions on the amplitude $x_1$ and the actuator effort to fully compensate for the external action $F_0 \sin 2\pi ft$, the amplitude $x_1 = 0.032 \text{ m}$.

It should be noted that at $F_0$ values exceeding the capacity of the active vibration damper, the effectiveness of vibration isolation during operation of the force limiter unit is reduced compared to the "ideal" system. When designing an active dynamic vibration damper, its design parameters must be consistent with an oscillating system of mass $m_0$ and possible values of vibration effects in a given frequency range. When the operating modes of vibrating elements exceed the vibration forces taken into account in the design, the force limiter block will ensure reliable operation of the entire system.

4. Conclusions

Investigation of the principles of construction and potential capabilities of a vibration isolation system with an active dynamic vibration damper under the assumption of error-free operation of sensors and an actuator has shown the perspective of the $u = K_0 x_0 \mp K_1 x_1$ control type.

It is shown that by choosing the values of the coefficients $K_0$ and $K_1$, it is possible to increase the frequency range with the minimum value of the force transfer coefficient to the base, while the average value of the tuning frequency can be shifted to a high or a low region due to $\mp K_1$.

Moving the damper mass with the selected control law and the unfavorable operating mode of the system due to increased vibration effects can reach unacceptable values. In the paper, computer simulation of the device in automatic mode limiting the force in the actuator is designed and produced to provide acceptable values of the oscillation amplitude of the damper mass.

Investigation of the vibration isolation system with an active dynamic vibration damper will be continued taking into account the dynamics of the electrodynamic drive as an actuator.

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