Gauge Unification and Dynamical Supersymmetry Breaking

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Abstract

Under the assumption that all the gauge groups in supersymmetric theories unify at the fundamental scale, the numbers and the mass scales of messenger quarks and leptons, as well as the beta-function coefficient of the sector for dynamical supersymmetry breaking are constrained depending on various gauge mediation mechanisms. For this, we use one-loop renormalization group equations and draw constraints on the scales in each gauge mediation model.

PACS numbers: 12.10.Kt, 12.60.Iv, 12.90.+b
Dynamical supersymmetry breaking (DSB) has been known to occur in some supersymmetric gauge theories. Recently, following the new understanding of the quantum behavior of supersymmetric gauge theories, the number of theories for DSB increased rapidly. Concurrently, various new mechanisms for transmitting supersymmetry breaking through gauge mediation have been proposed. The supersymmetric standard model (SSM) has to be implemented with the sector for supersymmetry breaking and a way of its mediation to the SSM sector. It will be a future task to find a right DSB sector and messenger mechanism which can yield phenomenologically acceptable soft supersymmetry breaking in the SSM sector.

As is well-known, the minimal supersymmetric standard model supports the idea of grand unification. It remains however to be explained why the grand unification scale \( \sim 10^{16} \) GeV differs from the fundamental scale which could be the string scale \( \sim 10^{17-18} \) GeV or the Planck scale \( \sim 10^{19} \) GeV. This question has been addressed in SSM-like string theories where the masses of extra fermiones can reside anywhere between the electroweak scale and the string scale. It can be expected that this problem is resolved in gauge mediation models of supersymmetry breaking where extra heavy quarks and leptons are necessary and their mass scales are determined from the successful prediction of ordinary superparticle masses. The existence of such messenger fermions may remove the discordance between the conventional unification scale and the fundamental scale. Then the fundamental theory should be such that not only the standard model gauge interactions but also DSB gauge interactions unify at the fundamental scale of the theory. In this case, the supersymmetry breaking scale can be also determined dynamically in terms of the fundamental scale. An attempt to find a realistic string model with such a property was made in Ref. The aim of this paper is to investigate the general consequences of the ultimate unification of the SSM sector and the DSB sector. Specifically, we will draw restrictions on the various scales in the theory and the number of messenger quarks and leptons depending on theories for DSB and mechanisms for gauge mediation.

We begin with considering the renormalization group equations of the SSM and DSB gauge coupling constants. Our analysis will rely on a rough order-of-magnitude calculation which is enough for our purpose. Gauge mediation models contain additional vector-like quarks or leptons (denoted by \( f, \bar{f} \)) whose masses (denoted by \( M_m \)) can be generated by the following
schematic form of superpotential:
\[ W = \lambda S f \bar{f}. \]  (1)

Here the field \( S \) can be a fundamental or a higher dimensional composite field. The nonzero vacuum expectation values (VEVs) of \( S \) and its F-term \( F_S \) result from the mediation of supersymmetry breaking in the DSB sector. Then the ordinary superparticles obtain the soft masses of the order \( \frac{\alpha}{4\pi} \Lambda_S \) where

\[ \Lambda_S \approx \frac{F_S}{S} \approx (10 \sim 100) \text{ TeV} \]  (2)

and \( \alpha \) is a standard model fine structure constant. The messenger quarks and leptons at the messenger scale \( M_m \) participate in the renormalization group evolution up to the fundamental scale \( M_X \) at which all gauge groups unify. The assumption of the gauge unification allows us to compute the dynamical scale \( \Lambda_D \) once the gauge structure of the DSB sector (more precisely the coefficient of the \( \beta \)-function) is fixed. For simplicity, we assume that there is only one dynamical scale in the DSB sector. This assumption is not so restrictive since the largest dynamical scale can be taken when the DSB sector has a product group.

The one-loop renormalization group evolution of the gauge couplings is given by

\[ \alpha_X^{-1} = \alpha_i^{-1}(M_Z) + \frac{b_i}{2\pi} \ln \frac{M_X}{M_Z} - \frac{n_i}{2\pi} \ln \frac{M_X}{M_m} \]

\[ = \frac{b}{2\pi} \ln \frac{M_X}{\Lambda_D} \]  (3)

where \( b_i \) is the minimal value of the coefficient of the one-loop \( \beta \)-function \( (b_1 = -33/5, b_2 = -1, b_3 = 3) \), and \(-n_i \) is the contribution from the messenger fermions at the mass scale \( M_m \), and \( b \) is the coefficient of the DSB sector. Note that the extra quarks or leptons do not have to form complete multiplets of a unification group, e.g, \((5 + \bar{5})\) or \((10 + \bar{10})\) of \( SU(5) \) as also discussed in other studies \[14, 15\]. With the simple one-loop renormalization group equation (3), we can draw information on the values of \( n_i \) and \( b \) which are compatible with the unification idea in various gauge mediation models. In our discussion, we ignore the two-loop evolution which involves also information on the messenger fermion Yukawa couplings and two-loop \( \beta \)-function of the DSB sector. We expect that two-loop effects cause no essential change in the prediction of the numbers \( n_i \) and \( b \) and the orders of magnitude of various scales. More precise phenomenological discussions at two-loop order in gauge mediation models with conventional unification group \( SU(5) \) have been performed in Refs. \[10\].
First, we get the relation between the messenger scale and the unification scale $M_X$,

$$M_m = M_X \left( \frac{M_U}{M_X} \right)^{4/n}, \quad M_U \equiv M_Z e^{2\pi(\alpha_2^{-1} - \alpha_3^{-1})/4}$$  \hspace{1cm} (4)$$

where $n \equiv n_3 - n_2$. Here $M_U$ is the usual unification scale $\approx 2.4 \times 10^{16}$ GeV. For the calculation, we use the central values; $\alpha_1 = 1/58.97, \alpha_2 = 1/29.61, \alpha_3 = 0.118$ at the scale $M_Z$ \[17\]. Assuming $M_X > M_U$ to ensure the experimental bounds on the proton lifetime are not violated, $n$ must be positive or zero. When $n = 0$, the messenger scale $M_m$ is not related to the unification scale, and $M_X = M_U$. The messenger scale and the DSB scale are related by the equation,

$$2\pi\alpha_U^{-1} = n_2 \ln \frac{M_U}{M_m} + b \ln \frac{M_U}{\Lambda_D},$$  \hspace{1cm} (5)$$

where $\alpha_U \approx 1/24$ is the usual unification coupling constant. On the contrary to the case with $n = 0$, the unification scale can be pushed up when $n \geq 1$. The number $n$ cannot be arbitrarily large. The upper bound $M_m < 10^{16}$ GeV [see below Eq. (6)] implies $n \leq 3$ as can be seen from Eq. (4). There is also a lower bound on the messenger scale which becomes smaller for a larger $M_X$ and a smaller $n$. The smallest messenger mass can be obtained by taking $n = 1$ and $M_X = M_{pl} \approx 1.2 \times 10^{19}$ GeV: that is, $M_m > 2 \times 10^8$ GeV. Of course, this lower bound on $M_m$ is not applied to the case with $n = 0$ [see Eq. (4)]. Note that the messenger scale would be related to the axion scale or the heavy right-handed neutrino scale when $M_m \approx 10^{10} \sim 10^{12}$ GeV, which can be obtained with $n = 1$ and $M_X \approx 3 \times 10^{18} \sim 7 \times 10^{17}$ GeV. Given $n$ (or $n_3$) and $n_2$, the number $n_1$ is constrained by the well-known relation

$$\frac{n_1 - n_2}{n_3 - n_2} = \frac{b_1 - b_2}{b_3 - b_2}$$

from which one gets,

$$n_1 = n_2 - \frac{7}{5}n.$$  \hspace{1cm} (6)$$

If $n = 0$, it is required that $M_X = M_U$ and $n_1 = n_2 = n_3$, which is the case when the messenger fermions form complete representations of certain unification group. The number $n_1$ depends upon the $U(1)_Y$ charge assignment to the messenger fermions. Later, we will see how the relation (6) restricts the number of SSM-like particles with the standard $U(1)_Y$ charges. We are now ready to discuss implications of the ultimate unification in various types of gauge mediation models which can be classified essentially into two classes: models with indirect, or direct mediation.
Indirect mediation models: In this class of models, supersymmetry breaking in the DSB sector is transmitted first to the messenger quarks and leptons by an intermediate gauge interaction and then to the SSM sector as described above. As a consequence, the messenger fermions get masses of order $M_m \approx \langle S \rangle \approx \alpha' \Lambda_D/4\pi$ where $\alpha'$ is the intermediate gauge coupling constant. If one consider renormalizable interactions only, the supersymmetry breaking scale of the DSB sector is given by $\sqrt{F} \approx \Lambda_D$. In general, the gravitino mass is $m_{3/2} \approx F/M_P$ where $M_P \approx 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Generic supergravity contribution to the soft masses of the superparticles (proportional to $m_{3/2}$) are not favor-blind and thus generate too large FCNC effects. To avoid this, we require a conservative constraint that the gravitino mass is smaller than the typical soft mass: $m_{3/2} < 100$ GeV, that is, $\sqrt{F} < 10^{10}$ GeV. From the fact that $\Lambda_S \approx 10$ to 100 TeV and that $M_m$ is roughly of the same order or larger than $\Lambda_S$ \[5\], one can draw the limit: $10^4$ GeV $< M_m < 10^7$ GeV. Now that the hierarchy between $M_m$ and $\Lambda_D$ is generated by loop effects, the more realistic bound on $\Lambda_D$ is $10^5$ GeV $< \Lambda_D < 10^8$ GeV. Altogether, we get the hierarchy among the scales:

$$\Lambda_S \lesssim M_m < \Lambda_D < 10^8 \text{ GeV}. \tag{7}$$

Note here that the gravitino mass ($m_{3/2} \approx \Lambda_D^2/M_P$) is in the range $4\text{ eV} < m_{3/2} < 4\text{ MeV}$. Such a light gravitino can yield distinctive phenomenological consequences in cosmology \[18\] and collider physics \[19\].

The upper bound on $\Lambda_D$ can be relaxed if the DSB sector contains nonrenormalizable terms. The interplay between two scales $\Lambda_D$ and $M_P$ in the effective DSB superpotential can give rise to a VEV $v$ larger than $\Lambda_D$: $v \approx M_P (\Lambda_D/M_P)^\kappa$ with $0 < \kappa < 1$ \[5\]. Now, $F/v$ in the DSB sector plays a role of $\Lambda_D$ in Eq. (7). The order of the DSB superpotential is

$$W \approx \frac{v^{m+3}}{M_P^m} \tag{8}$$

when nonrenormalizable terms of dimension $m+3$ give the largest contribution. Then the supersymmetry breaking scale is given by $F \approx v^{m+2}/M_P^m$, and hence $F/v \approx v^{m+1}/M_P^m$. Assuming the supersymmetry breaking scale $\sqrt{F}$ is below the DSB scale, we obtain

$$\Lambda_S \lesssim M_m < \frac{F}{v} < \sqrt{F} < \Lambda_D < v < 10^{16} \text{ GeV}. \tag{9}$$

Here the upper bound $v < 10^{16}$ GeV comes from $v\Lambda_S < F < 10^{20}$ GeV\(^2\). In this case, the natural range of the DSB scale can be inferred to be $10^6$ GeV $< \Lambda_D < 10^{16}$ GeV.
As shown, the messenger mass in indirect mediation models has an upper bound \( M_m < 10^7 \) GeV which is below the smallest value \( M_m \approx 2 \times 10^8 \) GeV in case of \( n \neq 0 \). Therefore, indirect mediation models can only employ \( n = 0 \), and thus \( M_X = M_U \). In this case, the condition \( n_1 = n_2 = n_3 \) has to be satisfied. A trivial way to achieve such a condition is to take the charge assignments such that the messengers form complete representations of a unification group as noted before. For renormalizable indirect mediation models, the allowed ranges of the messenger scale and the DSB scale [discussed above Eq. (7)] can be realized for a quite restricted ranges of \( n_2 \) and \( b \). From Eq. (5), one finds,

\[
1 \leq n_2 \leq 5, \quad 1 \leq b \leq 7.
\] (10)

Here the lower limits are trivial and the upper limit on \( n_2 \) comes from the perturbative unification condition \( \alpha_X < 1/3 \). In fact, two numbers \( n_2, b \) are correlated and roughly speaking, \( n_2 + b = 6, 7, 8 \) has to be satisfied. This result is consistent with Dubovsky et.al. [20]. A difference results from the fact that the unification of all gauge sectors at once is assumed and the change of \( \alpha_X \) due to non-zero \( n_2 = n_3 \) is taken into account in our case. Indeed, in the case of \( n = 0 \), identification of the unification scale \( M_X \) with, e.g., \( M_{\text{pl}} \) instead of \( M_U \) is possible if there is two step unification; the SSM gauge sector unifies at \( M_U \) and then the ultimate unification including the DSB sector occurs somewhere between \( M_U \) and \( M_{\text{pl}} \) as considered in Ref. [20]. This scheme then requires an explanation how the scales different from, e.g., \( M_{\text{pl}} \) can be generated.

To summarize, renormalizable indirect mediation models require a DSB sector with a small \( b \leq 7 \) to achieve unification. The number of DSB models with such a small \( b \) in the literature is very limited. To our knowledge, there are only a few models with \( b < 10 \). They are \( SU(4) \times SU(3) \times U(1) \) model with \( b = 8 \) [21], and the models listed in Ref. [20]. In order to construct more DSB models with a small \( b \), one would need to find a way to use higher dimensional representations of a given DSB gauge group. For nonrenormalizable class models where \( \Lambda_D < 10^{16} \) GeV, essentially no upper limit on \( b (b < 175) \) can be drawn.

(II) **Direct mediation models** : The above conclusion can change a lot in this type of models where the value of \( M_m \) (or \( \Lambda_D \)) can be larger. In this scheme, the field \( S \) belongs directly to the DSB sector and thus \( F_S = F \). As in the case of indirect mediation, there could be renormalizable and nonrenormalizable classes of models. In nonrenormalizable class
of models, a higher dimensional DSB superpotential as in Eq. (8) generates a large VEV of $S$. On the other hand, since the dimension of the field $S$ can be one or bigger, the messenger mass is in general given by

$$M_m \approx \langle S \rangle \approx M_P(v/M_P)^d \quad (11)$$

where $d$ is the dimension of the operator $S$. The case with $d = 1$ is explored in Ref. [3], and a model with a composite field $S$ ($d > 1$) is presented in Ref. [8]. In the former case, there could be other messenger quarks or leptons whose masses can be much smaller than $\Lambda_S$. These light messenger fields are known to drive the soft mass squared of the ordinary squarks to a negative value [8, 9]. Therefore, we assume the models without such a light messenger fermion. In these cases, the mass scale $\Lambda_S$ is given by $\Lambda_S \approx F/v$.

Recalling $F/v \approx M_P(v/M_P)^{m+1}$ [see below Eq. (8)], the bound $v < 10^{16}$ GeV puts a limit: $m \leq 5$. Furthermore, the fact that the messenger mass ($M_m \approx M_P(\Lambda_S/M_P)^{d/m+1}$) must be heavier than around 100 GeV restricts $d$: $d \leq m + 1$.

A large VEV $v$ can be obtained dynamically also in the renormalizable class of models. In this case, one consider an one-loop effective scalar potential of the form $V = f(v)\Lambda_D^4$ where $f(v)$ is a function of a field with a VEV $v$. Then, the function $f(v)$ may be minimized at a large $v > \Lambda_D$ [10]. In these models, we obtain

$$\Lambda_S \approx \frac{F}{v} < \sqrt{F} < \Lambda_D < v < 10^{16} \text{ GeV}. \quad (12)$$

Here the messenger scale is given by $M_m \approx v$ and $\Lambda_D < 10^{10}$ GeV as above.

Let us now extract rough constraints on the numbers $n, n_2$ and $b$ for each class of indirect mediation models. To be specific, let us take two canonical candidates of $M_X$: the Planck scale $M_{pl}$ and the string scale $M_{st} \approx 5 \times 10^{17}$ GeV [12]. For $n = 0$, the discussion in part (I) applies here as well. However, we will concentrate on the cases with $n \geq 1$ for which the unification scale $M_X$ can be made close to the Planck scale. As we discussed, given $M_X$ and $n$ determines the messenger scale residing in the range: $M_m \approx 2 \times 10^8 \sim 10^{16}$ GeV. Furthermore, putting $M_m = \Lambda_D < 10^{16}$ GeV in Eq. (3) one finds the upper bound: $n_2 + b < (2\pi \alpha_U^{-1} - \ln(M_X/M_U))/\ln(M_X/10^{16} \text{ GeV}) \approx 20 (38)$ for $M_X = M_{pl} (M_{st})$. The number $n_2$ is constrained individually assuming a perturbative unification: $n_2/n < 2\pi (\alpha_U^{-1} - \alpha_{per}^{-1})/4\ln(M_X/M_U) - 1/4$. Taking $\alpha_{per} = 1/3$, we get $n_2/n < 5.8 (10.8)$ for $M_X = M_{pl} (M_{st})$. 

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Table 1: Various scales in unit of GeV and allowed numbers of $n, n_2$ and $b$ in the non-renormalizable class of models. The messenger mass $M_m$ for each $n$ is the same as in Table 2.

For $M_X = M_{pl}$.

| $n$ | $(d, m)$ | $v$       | $\sqrt{F}$ | $n_2$ | $b$       | $\Lambda_D$            | $\Lambda_S$ |
|-----|----------|-----------|------------|--------|-----------|------------------------|-------------|
| 1   | (3,3)    | $1 \times 10^{15}$ | $9 \times 10^9$ | $5 \sim 0$ | $2 \sim 15$ | $1 \times 10^{10} \sim 1 \times 10^{15}$ | $8 \times 10^4$ |
| 2   | (1,2)    | $5 \times 10^{13}$ | $1 \times 10^9$ | $10 \sim 0$ | $1 \sim 11$ | $2 \times 10^{10} \sim 2 \times 10^{13}$ | $2 \times 10^4$ |
| 3   | (1,4)    | $3 \times 10^{15}$ | $5 \times 10^9$ | $12 \sim 0$ | $3 \sim 17$ | $9 \times 10^{10} \sim 2 \times 10^{15}$ | $7 \times 10^3$ |

For $M_X = M_{st}$.

| $n$ | $(d, m)$ | $v$       | $\sqrt{F}$ | $n_2$ | $b$       | $\Lambda_D$            | $\Lambda_S$ |
|-----|----------|-----------|------------|--------|-----------|------------------------|-------------|
| 2   | (1,3)    | $1 \times 10^{15}$ | $1 \times 10^{10}$ | $12 \sim 0$ | $5 \sim 24$ | $2 \times 10^{10} \sim 1 \times 10^{15}$ | $1 \times 10^5$ |
| 3   | (1,5)    | $9 \times 10^{15}$ | $7 \times 10^9$ | $12 \sim 0$ | $6 \sim 36$ | $8 \times 10^{10} \sim 8 \times 10^{15}$ | $5 \times 10^3$ |

Given specific models, only certain combinations of the numbers $n_2$ and $b$ can be consistent with unification which can be seen from Eq. (3) together with Eqs. (12) and (13). For the nonrenormalizable class of models, the dimensionalities $(d, m)$ are also constrained due to the relations: $M_m \approx M_U (M_X/M_U)^{n/4-n}$, $v \approx M_P (M_m/M_P)^{1/d}$ and $\Lambda_S \approx M_P (v/M_P)^{m+1}$ implying that fixing $n, d$ and $m$ determines the values $v$ and $\Lambda_S$ for each $M_X$. First, the number $d$ is restricted by the bound $v < 10^{16}$ GeV:

$$d < \frac{\ln \frac{M_U}{M_P} + \frac{4-n}{n} \ln \frac{M_U}{M_X}}{\ln \frac{10^{16} \text{GeV}}{M_P}}.$$  \hspace{1cm} (14)

This shows that the bound on $d$ becomes larger for a larger $M_X$ and a smaller $n$. When $n = 1$, one has $d \leq 4 (2)$ for $M_X = M_{pl} (M_{st})$. On the other hand, only $d = 1$ can be compatible for $n = 2, 3$. The integer $m$ is constrained by the bound on $\Lambda_S \approx M_P (v/M_P)^{m+1} \approx 10^4 \sim 10^5$ GeV:

$$\frac{m+1}{d} \approx \frac{\ln \frac{\Lambda_S}{M_P}}{\ln \frac{M_U}{M_P} + \frac{4-n}{n} \ln \frac{M_U}{M_X}}.$$  \hspace{1cm} (15)

The integer pairs $(d, m)$ most closely satisfying this relation are shown in the second column of Table 1. Now, the restricted ranges of $n_2$ and $b$ can be obtained from the constraints: $\sqrt{F} < \Lambda_D < v$ and $\sqrt{F} \approx M_P (v/M_P)^{1+m/2} < 10^{10}$ GeV where $\Lambda_D$ is obtained by equating the first and the third line of Eq. (3). In selecting $n_2, b$, we impose also the perturbative unification: $\alpha_X < 1/3$. The allowed combinations of $(d, m)$ and $(n_2, b)$ for given $n$ and $M_X$ together with the corresponding supersymmetry breaking scale $\sqrt{F}$ and dynamical scale $\Lambda_D$ are presented in Table 1. As one can see, the allowed values of $b$ can be very large and becomes larger for a
Table 2: Various scales in unit of GeV and allowed numbers of \( n, n_2 \) and \( b \) in the renormalizable class of models.

| \( n \) | \( M_m \) | \( n_2 \) | \( b \) | \( \Lambda_D \) | \( \Lambda_S \) |
|---|---|---|---|---|---|
| 1 | \( 2 \times 10^8 \) | 0 | 5 | \( 2 \times 10^6 \) | \( 3 \times 10^4 \) |
| 2 | \( 5 \times 10^{12} \) | 10 \~ 1 | 1 \~ 6 | \( 7 \times 10^8 \sim 4 \times 10^9 \) | \( 1 \times 10^4 \sim 3 \times 10^5 \) |
| 3 | \( 3 \times 10^{15} \) | 10 \~ 0 | 3 \~ 7 | \( 5 \times 10^9 \sim 1 \times 10^{10} \) | \( 7 \times 10^3 \sim 3 \times 10^4 \) |

For \( M_X = M_{pl} \).

For \( M_X = M_{st} \).

smaller \( M_X \). The number \( n_2 \) is restricted to a small value when \( b \) is large, and vice versa, as in part (I) [see below Eq. (10)]. Therefore, most DSB models using the nonrenormalizable direct mediation mechanism can be consistent with the idea of unification. Note, however, that the supersymmetry breaking scale tends to be large \((\sqrt{F} > 10^9 \text{ GeV})\), and becomes larger for a smaller \( M_X \). Such a large scale gives rise to a heavy gravitino; \( m_{3/2} > 0.4 \text{ GeV} \) which is dangerous cosmologically [22]. However, such a heavy gravitino could be diluted away by a late-time entropy production, e.g., thermal inflation [23].

In the renormalizable class of models, the upper bounds on \( n_2 \) and \( b \) are more restrictive because of \( \Lambda_D < 10^{10} \text{ GeV} \). Following the above process now with the relation, \( \Lambda_S \approx \Lambda_D^2 / M_m \), we can get constraints on \( n_2 \) and \( b \) which are summarized in Table 2. Contrary to the non-renormalizable models, a large \( b \geq 9 \) is not permitted. On the other hand, the supersymmetry breaking scale can be as small as \( 2 \times 10^6 \text{ GeV} \) implying a light gravitino \((m_{3/2} \approx 2 \text{ keV})\) which can form warm dark matter.

Let us finally discuss how the relation (6) constrains more the content of messengers. To see this explicitly, we take SSM-like particles for the messengers: that is, \( N_{32} \times [(3, 2) + (3, 2)]_{1/6} \), \( N_3 \times [(3, 1) + (3, 1)]_{2/3} \), \( N'_3 \times [(3, 1) + (3, 1)]_{1/3} \), \( N_2 \times [(1, 2) + (1, 2)]_{1/2} \), and \( N_1 \times [(1, 1) + (1, 1)]_1 \) are added. Here the subscripts denote the absolute values of the \( U(1)_Y \) charges. It is trivial
to calculate the numbers \( n_i \) in this case;

\[
\begin{align*}
    n_1 &= \frac{6}{5}N_{32} + \frac{4}{3}N_3 + \frac{1}{3}N_3' + \frac{1}{2}N_2 + N_1 \\
    n_2 &= 3N_{32} + N_2 \\
    n_3 &= 2N_{32} + N_3 + N_3'.
\end{align*}
\]  

(16)

Then the relation \((16)\) tells us that any set of integers \( N_3, N_1, N_{32} \) and \( n \) satisfying the equation;

\[
N_3 + N_1 - 2N_{32} + \frac{3}{2}n = 0,
\]

(17)

is compatible with the unification. Therefore, in the case of SSM-like messengers, the unification of DSB sector with the SSM sector can be achieved if the messenger contents with \( n = 2 \) and \( N_3 + N_1 - 2N_{32} = -3 \) are taken. It is straightforward to generalize this argument to the cases with any exotic selection of messenger contents.

In conclusion, we investigated the consequences of the assumption that the weak scale and the supersymmetry breaking scale are generated dynamically from the scale of a fundamental theory, \( M_X \), at which gauge couplings of the supersymmetric standard model and the supersymmetry breaking sector unify. Considering one-loop renormalization group evolution, the number and the mass scale of extra vector-like quarks and leptons (messenger fermions), and the structure of the dynamical supersymmetry breaking sector are constrained depending on the ways of mediating supersymmetry breaking.

In indirect mediation models where the messenger fermion masses are smaller than about \( 10^7 \) GeV, the unification scale can not be changed from the usual grand unification value \( \approx 2 \times 10^{16} \) GeV. In its renormalizable class of models the one-loop \( \beta \)-function coefficient of the DSB sector \( b \) has to be less than 8 corresponding to the condition of the DSB scale \( \Lambda_D < 10^8 \) GeV. We notified that there are only a few known examples with such a small \( b \). For the nonrenormalizable class of models, no bound on \( b \) can be found.

In direct mediation models, the messenger mass can be larger than about \( 2 \times 10^8 \) GeV but smaller than about \( 10^{16} \) GeV. Furthermore, the mismatch between the usual unification scale and e.g., the Planck scale can be removed. For this, we need a kind of doublet-triplet splitting for messenger quarks and leptons with a small difference between the numbers of triplets and doublets \( (n = n_3 - n_2 = 1, 2, 3) \). A large value of the messenger mass is obtained when the
fields in the DSB sector get large vacuum expectation values ($< 10^{16}$ GeV) by the presence of nonrenormalizable terms or by a loop-improved effective scalar potential.

In the former case, phenomenologically acceptable models are shown to be compatible with a large $b \leq 36$ or $n_2 \leq 12$ and with a DSB scale $\Lambda_D$ in the range: $2 \times 10^9 \sim 8 \times 10^{15}$ GeV. The supersymmetry breaking scale $\sqrt{F}$ tends to be large indicating a large mass of the gravitino: $m_{3/2} > 0.4$ GeV. This heavy gravitino may cause cosmological troubles unless some dilution mechanism by a late-time entropy production takes place. In the latter case, due to the restriction $\sqrt{F} \approx \Lambda_D < 10^{10}$ GeV, a small $b$ is acceptable: $b \leq 8$, similarly to the indirect renormalizable models. But $n_2$ can be large: $n_2 \leq 10$. The gravitino can be as light as 2 keV to form dark matter. As noted, most supersymmetric gauge theories exhibiting dynamical supersymmetry breaking in the literature have $b$ larger than 10, and thus can be used only in a unified theory with the nonrenormalizable direct mediation mechanism.

**Acknowledgment:** This work is supported by Non Directed Research Fund of Korea Research Foundation, 1996. E.J.C. is a Brain-Pool fellow.

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