Finite nilpotent symmetry in Batalin-Vilkovisky formalism

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Abstract – We consider the Batalin-Vilkovisky formulation of both 1-form and 2-form gauge theories in the context of generalized BRST transformations with a finite field-dependent parameter. In the usual Faddeev-Popov (FP) formulation of gauge theories such finite field-dependent BRST (FFBRST) transformations do not leave the generating functionals invariant as the path integral measure changes in a non-trivial way for finite transformations. Here we show that FFBRST transformation, with the appropriate choice of a finite field-dependent parameter, is the symmetry of the generating functionals in the Batalin-Vilkovisky formalism. The finite parameter is chosen in such a way that the contribution from the Jacobian of the path integral measure is adjusted with gauge fixed fermions which do not change the generating functionals. Several examples for such finite parameters are constructed.

The field/antifield formulation, alternatively known as Batalin-Vilkovisky (BV) formalism [1–4], is one of the most powerful techniques to study the gauge field theories. This formulation is developed in a Lagrangian framework and is extremely useful as it allows us to deal with very general gauge theories including those with open or reducible gauge symmetry algebras. The essential aspects of the BV formalism were originally developed by Zinn-Justin [5] in order to prove the renormalizability of gauge theories. This method provides a convenient way of analyzing the possible violations of symmetries of the action by quantum effects. This formulation is based on the BRST symmetry [6–9] which plays a crucial role in the discussion of quantization, renormalization, unitarity and other aspects of the gauge theory. The nilpotent BRST transformation is characterized by an infinitesimal, anticommuting and space-time independent parameter which leaves the FP effective action as well as the path integral measure in the generating functional invariant.

In this present work, we show that FFBRST transformations [10] with appropriate choices of finite parameter are the symmetry of the generating functional in the BV formulation. In the usual FP formulation, FFBRST transformations do not leave the generating functionals invariant as the path integral measure in the expression of generating functionals changes non-trivially under finite transformations. We choose the finite parameter in such a way that the contribution from the Jacobian for the path integral measure is adjusted with gauge fixed fermions which do not affect the generating functional in the BV formulation. We construct few finite parameters and show the results in the context of both 1-form and 2-form gauge theories. Generalized anti-BRST transformations are also constructed and are shown to be the symmetry of the generating functional in the BV formulation of 1-form and 2-form gauge theories with the appropriate choice of finite parameters. In principle, one can construct infinitely many such finite parameters for which FFBRST transformations are the symmetry of the generating functionals in BV formulation.

The main idea of the BV formalism is to construct an extended action $W_\Psi(\Phi, \Phi^*)$ by introducing antifields $\Phi^*$ corresponding to each field $\Phi$ with opposite statistic. The sum of ghost number associated to a field and its antifield is equal to $-1$. Generically, $\Phi$ denotes all the fields involved in the theory. The generating functional can be written as

$$Z[\Phi^*] = \int D\Phi e^{iW_\Psi[\Phi, \Phi^*]} \left[\Phi^* = \frac{i}{\Psi}\right].$$

$\Psi$ is the gauge fixed fermion with Grassman parity 1 and ghost number $-1$. The generating functional $Z[\Phi^*]$ is proved to be independent of the choice of $\Psi$ [11]. This extended quantum action satisfies certain rich mathematical relation called quantum master equation [12] and is
given by
\[ \Delta e^{iW_{\Phi,\Phi^*}} = 0, \quad \text{with} \quad \Delta \equiv \frac{\partial_x}{\partial \Phi} \frac{\partial_x}{\partial \Phi^*} (-1)^{x+1}. \] (2)

The master equation reflects the gauge symmetry in the zeroth order of antifields and in the first order of antifields it reflects the nilpotency of the BRST transformation. This equation can also be written in terms of antibrackets as
\[ (W_\Phi, W_\Phi) = 2i \Delta W_\Phi, \]
where the antibracket is defined as
\[ (X, Y) \equiv \frac{\partial_x X}{\partial \Phi} \frac{\partial_x Y}{\partial \Phi^*} - \frac{\partial_x Y}{\partial \Phi} \frac{\partial_x X}{\partial \Phi^*}. \] (4)

The invariance of the FP effective action does not depend on whether the BRST parameter is finite or field dependent as long as it is anticommuting and space-time independent. Keeping this in mind, Joglekar and Mandal have generalized the BRST symmetry by considering the anticommuting BRST parameter finite and field dependent [10]. Such a finite transformation relates the generating functionals corresponding to different effective theories in FP formulation [10]. The path integral measure is not invariant under such transformations as the parameter is finite and field dependent. It has been shown that the Jacobian can be exponentiated to modify the effective action. Because of these interesting properties, the FFBRST has found many applications [13–20].

The antifields \( \Phi^* \) corresponding to the generic field \( \Phi \) are obtained from the gauge fixed fermion as
\[ \Psi = \int d\Phi \exp \left[ i W_\Phi (\Phi, \Phi^*) \right], \] (10)
where
\[ W_\Phi = S_0(\Phi) + \delta_{BRST} \Psi. \] (11)

The antifields \( \Phi^* \) are related to the generic field \( \Phi \) through the relation
\[ \Phi^* = \frac{\delta \Psi}{\delta \Phi}. \] (13)

Now, we apply the FFBRST transformation given in eq. (6) with the finite field-dependent parameter \( \Theta(A, c, \bar{c}, B) \) obtainable from
\[ \Theta'(A, c, \bar{c}, B) = i \int d^4 y \bar{c}^a \left[ \gamma_1 A^\alpha + (\partial \cdot A^\alpha - \eta \cdot A^\alpha) \right], \] (14)
using eq. (7) to the extended generating functional given in eq. (10). Note that even though the parameter \( \Theta \) is finite and field-dependent, it is anticommuting in nature. The path integral measure in the expression of \( Z[\Phi^*] \) is not invariant under such finite field-dependent transformations and the generating functional change to
\[ Z[\Phi^*] = \int D\Phi \exp \left[ i W_{\Psi_1}(\Phi, \Phi^*) \right], \] (15)
where
\[ W_{\Psi_1} = S_0(\Phi) + \delta_{BRST} \Psi_1, \] (16)
with \( \Phi^* \equiv \frac{\delta \Psi_1}{\delta \Phi} \). The gauge fixed fermion is changed from \( \Psi \rightarrow \Psi_1 \) as
\[ \Psi_1 = \int d^4 x \bar{c}^a \left[ \frac{\gamma_2}{2} B^\alpha - \eta \cdot A^\alpha \right]. \] (17)
with $\xi = \lambda(1 + 2\gamma)$. However $Z[\Phi^*]$ is independent of the choice of $\Psi$ as proved in ref. [11]. Thus, we see that nilpotent and finite BRST transformations

$$\Phi'(x) = \Phi(x) + \delta_{BRST} \Phi(x) \Theta[\Phi(x)],$$

with the finite parameter given in eq. (14) along with $\Phi^* \rightarrow \Phi^*$ is a formal symmetry of the generating functional $Z[\Phi^*]$ in BV formulation. We can construct many different choices of the finite field-dependent parameter $\Theta$ which changes only the gauge fixed fermion through a non-trivial Jacobian in the path integral measure and hence are the formal symmetry of the generating functional. For example, we make another choice of the finite field-dependent BRST parameter as

$$\Theta'(A, c, \tilde{c}, B) = i \int d^4y \tilde{c}^\nu [\gamma^1 A^\nu + \partial_0 A_0^\nu],$$

the generating functional $Z[\Phi^*]$ changes to

$$Z[\Phi^*] = \int D\Phi \exp[iW_{\Psi_2}(\Phi, \Phi^*)],$$

where $\Psi_2$ is the gauge fixed fermion given as

$$\Psi_2 = \int d^4x \tilde{c}^\nu \left[ \frac{1}{2} B^\nu - \partial^\nu A_0^\nu \right],$$

Thus, the FFBRST transformation with the parameter given in eq. (19) is also the formal nilpotent symmetry of the generating functional in BV formulation.

In 2-form gauge theory. – In this section, we consider the BV formulation of Abelian 2-form gauge theories which is extremely useful in the study of string theories, dual formulation of Abelian Higgs model, supergravity theories with higher curvature term [21–36]. We show that the FFBRST transformations with a careful choice of finite parameters are the symmetry of the generating functional of the 2-form gauge theory in BV formulation. We start with the effective action for the Abelian gauge theory for the rank-2 antisymmetric tensor field $B_{\mu\nu}$ defined as [35]

$$S = \int d^4x \left[ \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - i \bar{\rho}_\mu \partial_\nu (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) + \partial_\mu \bar{\sigma} \partial^\mu \sigma + \beta_\nu (\partial_\mu B^\nu + \lambda_1 \beta^\mu - \partial^\nu \varphi) - i \bar{\chi} \partial_\mu \rho^\mu - i \chi (\partial_\mu \rho^\mu - \lambda_2 \chi) \right],$$

where $F_{\mu\nu} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$. $B_{\mu\nu}$ is the antisymmetric tensor field of rank-2, $(\rho_\mu, \bar{\rho}_\mu)$ are anticommuting vector fields (ghost), $(\sigma_\mu, \bar{\sigma}_\mu)$ are commuting scalar fields, $(\chi, \bar{\chi})$ are anticommuting scalar fields, and $(\beta_\mu, \varphi)$ are the commuting vector and the scalar field, respectively.

The generating functional for this theory in BV formulation can be written as

$$Z[B^{\mu\nu}, \rho^{\mu\nu}, \bar{\rho}^{\mu\nu}, \beta^\nu, \varphi^\nu] = \int [dB \, d\rho \, d\sigma \, d\bar{\sigma} \, d\varphi \, d\bar{\varphi} \, d\bar{\chi} \, d\chi \, d\beta] \times \exp \left\{ \int d^4x \left[ \frac{1}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} - B^{\mu\nu\lambda} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) - i \rho^{\mu\nu} \partial_\mu \rho_\nu + i \bar{\rho}^{\mu\nu} \beta_\nu - \bar{\sigma} \text{d}\bar{\sigma} \beta^\mu - \bar{\chi} \text{d}\bar{\chi} \varphi^\nu \right] \right\},$$

This can be expressed compactly as

$$Z[\Phi^*] = \int D\Phi \exp[iW_{\Psi_3}(\Phi, \Phi^*)],$$

where

$$W_{\Psi_3} = S_0(\Phi) + \delta_{BRST} \Psi_3,$$

and $\Psi_4$ is the gauge fixed fermion given as

$$\Psi_4 = -i \int d^4x |1_{12} F_{\mu\nu\lambda} (\partial_\mu B^{\lambda\nu} - \eta_\lambda B^{\mu\nu} - \partial^\nu \varphi + \eta_\nu \varphi) + \gamma_2 \lambda_1 \rho_\nu \beta^\nu + \gamma_1 \bar{\sigma} (\partial_\mu \rho^\mu - \eta_\mu \rho^\mu) + \gamma_2 \lambda_2 \bar{\sigma} \chi | \mu 

and apply FFBRST transformations to the generating functional given in eq. (25). This takes $Z[\Phi^*]$ to $Z[\Phi^*]$ where

$$Z[\Phi^*] = \int D\Phi \exp[iW_{\Psi_4}(\Phi, \Phi^*)],$$

and $\Psi_4$ is defined as

$$W_{\Psi_4} = S_0(\Phi) + \delta_{BRST} \Psi_4,$$

with the corresponding antifields $\Phi^* = \delta_{\Psi_4}$. Thus, the FFBRST transformation in eq. (6) with the parameter $\Theta'$ given in eq. (28) is a formal symmetry of the generating functional, $Z[\Phi^*]$. In fact, the FFBRST transformations corresponding to any choice of $\Theta'$, which changes one gauge fixed fermion to another gauge fixed fermion through a non-trivial Jacobian of the path integral measure, are also the symmetry in the BV formulation of the Abelian 2-form gauge theory.

**Anti-BRST.** – Anti-BRST transformations are analogous to BRST transformations where the role of ghosts and antighosts fields is interchanged apart from some numerical factors. Formal nilpotent symmetry transformations for the generating functional in BV formulation can also be constructed using FFanti-BRST transformations.
formations will also leave the quantum master equation (eq. (8)). For example, FFanti-BRST transformations with a finite field parameter corresponding to

\[ \Theta' = -i\gamma \int d^4x \, e^\alpha (\partial \cdot A^\alpha - \partial_j A^{i\alpha}) \]  

(32)

changes the gauge fixed fermion only and can be the symmetry of the generating functional for the 1-form theory in BV formulation

\[ Z[\Phi^*] \xrightarrow{\text{FFBRST with } \Theta' \text{ in eq. (32)}} Z[\tilde{\Phi}^*]. \]  

(33)

In the 2-form gauge theory we can construct the finite field anti-BRST transformations parameter corresponding to

\[ \Theta_{ab}' = -\int d^4x [\gamma_1 \rho_\lambda B^{\mu\nu} - \eta_{\mu\nu} B - \partial^\rho \phi - \eta_\nu \phi^\nu] + \gamma_2 \lambda_1 \rho_\nu \beta^\nu - \gamma_1 \lambda_1 \rho_\nu \beta^\nu + \gamma_2 \lambda_2 \sigma_{\nu} \bar{\chi}], \]  

(34)

which is the symmetry generating functional in the BV formulation of the 2-form gauge theory. Thus, FFanti-BRST transformations with appropriate parameters are also the symmetry of the generating functionals BV formulation.

**Conclusion.** – Generalized BRST transformations in which the parameter is finite and field-dependent are also the symmetry of the EP effective action and are nilpotent. In the path integral formulation of different gauge theories, such FFBRST transformations do not leave the path integral measure in the definition of generating functionals invariant and hence the generating functionals are not invariant under such transformations. In fact, with the appropriate choice of the finite field-dependent parameter, these transformations are shown to relate different generating functionals corresponding to different effective gauge theories. Because of this important results, FFBRST transformations found many applications in the study of gauge field theories. In the present work, we showed that such transformations leave the generating functionals defined in the BV formulation of different gauge theories invariant. We chose the finite field-dependent parameter in such a manner that the contribution from the non-trivial Jacobian of the path integral measure is absorbed in the expression of gauge fixed fermion in BV formulation. Recalling the well-known result in BV formulation that the generating functionals are independent of the choice of the gauge fixed fermion, we claim that FFBRST transformations with an appropriate parameter are the formal symmetry of the generating functionals defined in the BV formulation. In principle, one can construct infinitely many such parameters for which FFBRST leaves the generating functionals invariant in the BV formulation. We have constructed parameters both for 1-form and 2-form gauge theories in this work. However, the disadvantage of such transformations is that these transformations are non-local. We believe that such transformations will also leave the quantum master equation invariant in the BV formulation which can further lead to important consequences.

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