Load Distribution Coefficient of Node of Lattice Beam Based on Two-Parameter Elastic Foundation Model

Taiyue Zhang*
Architectural Engineering Institute, Jiangsu College of Engineering and Technology, Nantong, China

*Corresponding author’s email: taiyue12@126.com

Abstract. According to the force characteristic of lattice beam, used the Two-parameter elastic foundation model to deduce the answer of internal forces of concentrated force on the finite-length beam, and calculate load distribution coefficient of node, then get the internal forces of lattice beam, used the analytical solution combined with practical engineering ,and compared with numerical solution of simulation of ABAQUS finite element software, to verify the correctness of theoretical solution ,and analyze its deficiency.

Keywords-Lattice beam; two-parameter elastic foundation model; distribution coefficient

1. Introduction
Slope support technology is changing with each passing day, with the development of the times also continuous improvement and innovation, in which the anchor lattice beam is widely used because of its simple design, effective support, neat and beautiful .However, in the design of lattice beams, in order to ensure engineering safety, the load distribution of node generally assumes that there is a hinge between the lattice longitudinal beam and the crossbeam, regardless of the influence of the torque and adjacent load caused by the intersection of the two directions at the joint, the longitudinal and cross beams are split directly, and the load on the anchor is evenly distributed to the longitudinal and cross beams, so the design is relatively conservative and causes some waste. Based on Winkler elastic foundation model to deduce the answer of the finite-length beam of the load distribution coefficient of node. Although it can overcome the unreasonable assumption of average distribution, but, the force of anchor lattice beam should be infinite-beam mode, the answer of finite-beam based on the Winkler elastic foundation model is not suitable, fails to consider the continuity of soil [1,2]. In this paper, used the Two-parameter elastic foundation model to deduce answer of based on finite-beam of load distribution coefficient of node of lattice beam ,and combined with an engineering example, comparative analysis the results obtained by this method and the simulation results of ABAQUS finite element software ,to verify the correctness of the load distribution coefficient in this paper.

2. Two-parameter elastic foundation model introduction
In this section, based on the Pasternak model of two-parameter is used to calculate the load distribution coefficient of node of lattice beam, to consider effectively the continuity of soil and overcome the shortcomings of existing methods. Subgrade reaction of Two-parameter elastic foundation model:

\[ p(x) = ky - G_p \frac{d^2y}{dx^2} \]  

(1)

Among them, \( k \) and \( G_p \) are the two elastic parameters of the foundation, and \( k \) is the coefficient of subgrade reaction, \( G_p \) is the shear coefficient of ground.

In this paper, the design model of beam-column load distribution of node of foundation cross beams is applied to the slope support technology of anchor lattice beam. According to the finite beam element, to deduce the load distribution coefficient of node of lattice beam in general element (inner column 'cross') [3,4].

3. The answer of internal forces of finite beam based on two-parameter elastic foundation model

For the finite width (b) of effective beam, which is shelved on the two-parameter elastic foundation, the governing equation is as follows [5]:

\[ EI \frac{d^4y}{dx^4} - G_p b^* \frac{d^2y}{dx^2} + \beta^* y = b q(x) \]  

(2)

Where \( b^* \) is the equivalent width of the lattice beam.

\[ b^* = b \left[ 1 + \sqrt{\frac{G_p}{k b}} \right] \]  

(3)

Definition

\[ \lambda^* = \frac{kb^*}{4EI} \]  

(4)

\[ \alpha^2 = \frac{G_p}{k} \]  

(5)

The governing equation (2) to simplify homogeneous differential equation:

\[ \frac{d^4y}{dx^4} - 4 \lambda^* \alpha^2 \frac{d^2y}{dx^2} + 4 \lambda^4 y = 0 \]  

(6)

In most cases, the lattice beam is satisfied \( \lambda^* \alpha^2 < 1 \), obtain the general solution of homogeneous differential equation (7):

\[ y = (C_1 e^{-\beta_1 x} + C_2 e^{\beta_1 x}) \cos \beta_2 x \]

\[ + (C_3 e^{-\beta_2 x} + C_4 e^{\beta_2 x}) \sin \beta_2 x \]

(7)

To solve the displacement and internal force of the finite beam by using the initial parameter method, and assumed that the finite beam without external load. Substituting \( y = y_0, \theta = \theta_0, M = M_0, Q = Q_0 \) into equation (7)

\[ y_0 = C_1 + C_3 \]

(8)

\[ \theta_0 = \left[ \frac{dy}{dx} \right]_{x = 0} = -\beta_1 C_1 + \beta_2 C_2 + \beta_2 C_3 + \beta_2 C_4 \]

(9)

\[ M_0 = \left[ -\frac{EI d^2y}{dx^2} \right]_{x = 0} = -EI \left[ \beta_1^2 - \beta_2^2 \right] (C_1 + C_3) \]

(10)
The internal forces of lattice beams are obtained by simultaneous equations (8)-(11).

\[
Q_0 = \left[-EI \frac{d^3 y}{dx^3} + G_p b^2 \frac{dy}{dx}\right]_{x=0} = 2\lambda^2 EI (-\beta C_1 + \beta_1 C_1 - \beta C_2 - \beta_1 C_2) \tag{11}
\]

\[
y(x) = y_0 \left[ F_2(x) - \beta_2^2 \frac{F_1(x)}{\beta_2^2} \right] + \frac{\theta_0}{2} \left( \frac{F_1(x)}{\beta_2} + \frac{F_3(x)}{\beta_1} \right) \frac{M_0}{EI} \left( \frac{F_4(x)}{2\beta_2^2} \right) + \frac{Q_0}{4\lambda^2 EI} \left( \frac{F_1(x)}{\beta_2} - \frac{F_3(x)}{\beta_1} \right) - \frac{P}{4\lambda^2 EI} \left[ \frac{1}{\beta_2} F_1(x-x_p) - \frac{1}{\beta_1} F_3(x-x_p) \right] \tag{12}
\]

\[
\theta(x) = y_0 \left[ F_2(x) - \beta_2^2 \frac{F_1(x)}{\beta_2^2} \right] + \frac{\theta_0}{2} \left( \frac{F_1(x)}{\beta_2} + \frac{F_3(x)}{\beta_1} \right) \frac{M_0}{EI} \left( \frac{F_4(x)}{2\beta_2} \right) - \frac{Q_0}{EI} \left( \frac{F_1(x)}{2\beta_2} \right) + \frac{P}{EI} \left[ \frac{F_3(x)}{2\beta_1} \right]_0 \tag{13}
\]

\[
M(x) = y_0 2EI \lambda \left( \frac{F_1(x)}{\beta_2^2} \right) - \theta_0 \left[ \left( \frac{3\beta_2^2 - \beta_1^2}{\beta_2^2} \right) F_1(x) + \left( \frac{2\beta_2^2 - 3\beta_1^2}{\beta_1^2} \right) F_3(x) \right] + M_0 \left[ F_1(x) + \left( \frac{2\beta_2^2 - 3\beta_1^2}{2\beta_2 \beta_1} \right) F_3(x) \right] + \frac{Q_0}{2} \left( \frac{F_1(x)}{\beta_2} + \frac{F_3(x)}{\beta_1} \right) - \frac{P}{2} \left[ \frac{F_1(x)}{\beta_2} (x-x_p) + \frac{F_3(x)}{\beta_1} (x-x_p) \right] \tag{14}
\]
\[ Q(x) = y_0 2 \lambda^2 EI \left( \frac{2 \beta_i^2 - \lambda^2}{\beta_i^2} F_i(x) \right) + \theta_0 2 EI \lambda^2 \left( \frac{F_i(x)}{\beta_i} \right) + M_0 \lambda^2 \left( \frac{F_i(x)}{\beta_i} - \frac{F_i(x)}{\beta_i} \right) + Q_0 \left[ F_i - \left( \frac{\beta_i^2 - \beta_i^2}{2 \beta_i \beta_i} F_i(x) \right) \right] - P \left[ F_i(x) \left(x - x_p\right) - \left( \frac{\beta_i^2 - \beta_i^2}{2 \beta_i \beta_i} F_i \left(x - x_p\right) \right) \right] \tag{15} \]

Krylov function \( F_i(x), F_j(x), F_k(x), F_l(x) \) [6]:

\[
F_i(x) = \cos (\beta_i x) \sinh (\beta_i x) \\
F_j(x) = \cos (\beta_j x) \cosh (\beta_j x) \\
F_k(x) = \sin (\beta_k x) \cosh (\beta_k x) \\
F_l(x) = \sin (\beta_l x) \sinh (\beta_l x) 
\]

4. Solution of load distribution coefficient \((k^1, k^1)\) of node

Figure 1 is a schematic diagram of the force acting on a lattice beam. When the concentrated load is distributed at the node of the lattice beam, to consider the interaction between the lattice beam and the slope, at the node, the vertical displacement and rotation angle of the crossbar beam are the same. According to the deformation coordination condition and force balance condition at the node of lattice beam, to distribute the load by ignoring the influence of torsional deformation on the node.

\[ P = P^x + P^y, \quad y^x = y^y \tag{16} \]

\( P^x \) is the load of horizontal beam. \( P^y \) is the load of vertical beam, \( y^x \) is displacement of horizontal beam; \( y^y \) is displacement of vertical beam. Substituting (13) in (17) to obtain:

\[
y^x = y_0 \left[ F_i^x \left(x\right) - \left(\frac{\beta_i^2 - \beta_i^2}{2 \beta_i \beta_i}\right) F_i^x \left(x\right) \right] + \frac{\theta_i^x}{2} \left( F_i^x \left(x\right) + \frac{F_i^x \left(x\right)}{\beta_i} \right) - M_i^x \left(\frac{F_i^x \left(x\right)}{\beta_i} \right) + \frac{Q_i^x}{4(\lambda^2) \lambda^2} \left(\frac{F_i^x \left(x\right)}{\beta_i} - \frac{F_i^x \left(x\right)}{\beta_i} \right) - \frac{P^x}{4(\lambda^2) \lambda^2} \left[ \frac{1}{\beta_i} F_i^x \left(x - x_p\right) - \frac{1}{\beta_i} F_i^x \left(x - x_p\right) \right] \tag{17} \]
The \( k^t \) and \( k^i \) of result obtained in \( P^t = k^t P \), \( P^i = k^i P \), first to simplified equation.

\[
A^t = y_0 \left[ F^t_2 (x) - \frac{(\beta^t_1)^2 - (\beta^t_1)^2}{2 \beta^t_1 \beta^t_2} F^t_i (x) \right] + \frac{\theta^t_0}{2} \left( \frac{F^t_1 (x)}{\beta^t_1} + \frac{F^t_i (x)}{\beta^t_i} \right) - M^t_0 \frac{F^t_i (x)}{2 \beta^t_1 \beta^t_2} \\
+ \frac{Q^t_0}{4(\lambda^t)^2 EI^t} \left( \frac{F^t_1 (x)}{\beta^t_1} - \frac{F^t_i (x)}{\beta^t_i} \right)
\]

\[
A^i = y_0 \left[ F^i_2 (x) - \frac{(\beta^i_1)^2 - (\beta^i_1)^2}{2 \beta^i_1 \beta^i_2} F^i_i (x) \right] + \frac{\theta^i_0}{2} \left( \frac{F^i_1 (x)}{\beta^i_1} + \frac{F^i_i (x)}{\beta^i_i} \right) - M^i_0 \frac{F^i_i (x)}{2 \beta^i_1 \beta^i_2} \\
+ \frac{Q^i_0}{4(\lambda^i)^2 EI^i} \left( \frac{F^i_1 (x)}{\beta^i_1} - \frac{F^i_i (x)}{\beta^i_i} \right)
\]

The \( P^t \), \( P^i \) result obtained that to Simplify the (19) and (20) substituting into (17) and (18):

\[
P^t = \frac{(A^t - y^t) 4(\lambda^t)^2 EI^t}{\beta^t_2 F^t_1 (x-x_p) - \frac{1}{\beta^t_1} F^t_i (x-x_p)}
\]

\[
P^i = \frac{(A^i - y^i) 4(\lambda^i)^2 EI^i}{\beta^i_2 F^i_1 (x-x_p) - \frac{1}{\beta^i_1} F^i_i (x-x_p)}
\]

The result:

\[
k^t = \frac{(A^t - y^t) 4(\lambda^t)^2 EI^t}{\frac{1}{\beta^t_2} F^t_1 (x-x_p) - \frac{1}{\beta^t_1} F^t_i (x-x_p)} P
\]

\[
k^i = \frac{(A^i - y^i) 4(\lambda^i)^2 EI^i}{\frac{1}{\beta^i_2} F^i_1 (x-x_p) - \frac{1}{\beta^i_1} F^i_i (x-x_p)} P
\]

5. Engineering example
In order to verify the correctness of this method, the theoretical value of internal force of lattice beam is calculated according to engineering example, one is without considering load distribution of node, one is considering load distribution of node based on two-parameter foundation model. Then using the ABAQUS finite element simulation of the project to analyze the force of the lattice beam, so as to
verify that the theoretical value of the load distribution coefficient of node based on the two-parameter foundation model is closer to the value of ABAQUS finite element simulation. ABAQUS is one of the most powerful finite element software in the world. It has a constitutive model which can truly reflect the soil properties. It has strong applicability to geotechnical engineering [7,8].

5.1 Engineering situation
The project is located in Guantang bus Hub of Zhenjiang City. The road with a total length of 805m and a width of 16m runs from north to south. Slopes with high 6m~9m on both sides. According to different geological conditions and landscape requirements, to adopt various forms of slope support, such as anti-slide pile, counterfort wall, anchor lattice beam and so on.

The rock-soil layer of this section is mainly composed of silty clay, taupe, soft-plastic and medium compressibility, reinforcement with Anchor Lattice Beams. The information of soil layer is shown in Table 1 and Table 2. The length of the slope is 11m, the height is 7.35m, the ratio of slope is 1:0.2, and the anchor is made of HRB400 rebar, which is 12m long. The calculation unit of lattice beam consists of 4 vertical beams and 3 horizontal beams, horizontal distance is 3 m, vertical distance is 2 m, section size of beam is 0.3 × 0.3 m, the C30 concrete is adopted, and the anti-tensile strength of the anchor is 100kN. As shown in Figure 3. Establishment of ABAQUS finite element model according to engineering example based on mohr-coulomb model.

| Table 1. Soil information |
|--------------------------|
| E/MPa | ν | C/kPa | φ/(°) | γ/kN/m³ |
| 20 | 0.3 | 29.7 | 9.12 | 19.8 |

| Table 2. Calculation parameters of lattice beam |
|--------------------------|
| k/kN/m³ | Gp/MPa | E/GPa | I | μ |
| 2.98×10⁴ | 10.78 | 30 | 6.75×10⁻⁴ | 0.3 |

5.2 ABAQUS modeling

Figure 2. ABAQUS three-dimensional model

Figure 3. Mesh of ABAQUS model
Modeling a 3D slope model of ABAQUS. The slope model adopts elastic-plastic stress-strain model, and the constitutive model chooses the Mohr-coulomb. The anchor adopts T3D bar element, which is defined as a truss, and assumed to be a linear elastic material, selected to be implanted into the slope. The lattice beam adopts B32 beam element (quadratic Timoshenko), the section is rectangular, and assumed to be linear elastic material, defined as rigid contact with the slope, as shown in Figure 2. Because the model is three-dimensional, to choose the C3D8 (eight-node hexahedral element) as the element type, and encrypted seed arrangement the of the soil above the ground for accurately calculate. The grid is divided as shown in Figure 3.

When modeling, the force of the slope should be considered to be consistent with the actual situation. First determined the fixed constraint of bottom surface, limited the displacement of XYZ in three directions. And then limited the displacement in the Y direction of the left and right sides. Finally, limited the displacement in the X direction of the front and rear sides. The model can be compressed and deformed under stress, and can slide back and forth, which is in line with the actual situation of the slope.

5.3 Result analysis

Compared with the moment distribution of lattice beam along horizontal direction and vertical direction is calculated by the simulation of ABAQUS, the method of considering distribution coefficient and without considering distribution coefficient. As shown in Figure 4 and 5.

![Figure 4. Compare the moment diagram of horizontal beam](image1)

As can be seen from Figure 4, the maximum bending moment of the lattice beam in the horizontal direction at the 4m section coordinates of crossbeam. The simulated value of ABAQUS bending moment here is -50.89kN.m. The general calculation method without considering the load distribution coefficient, the difference between the analytical solution and the simulation value is 36.34%, while considering the load distribution coefficient of node, the difference between the analytical solution and the simulation value is 14.72%.

![Figure 5. Compare the moment diagram of vertical beam](image2)
As can be seen from Figure 5, the maximum bending moment of the lattice beam in the vertical direction at the 5.7 m section coordinates of vertical beam. The simulated value of ABAQUS bending moment here is -50.89 kN.m. The general calculation method without considering the load distribution coefficient of node, the difference between the analytical solution and the simulation value is 39.59%, while considering the load distribution coefficient of node, the difference between the analytical solution and the simulation value is 26.75%. Therefore, considering the load distribution coefficient of node is closer to the reality, the internal force of the lattice beam is smaller, and the steel consumption can be saved by 11.84% to 21.64%.

![Figure 6. Point of horizontal and vertical beam](image)

In order to study the force calculation of lattice beam, considering the load distribution coefficient of node based on the two-parameter foundation model is better than general calculation method, calculated displacement at the point of the vertical and horizontal beam, as shown in Figure 6. Compared their results of displacement with the simulation results of ABAQUS, the simulation results of ABAQUS as shown in Figure 7.

![Figure 7. Displacement of ABAQUS model of lattice beam](image)

| Horizontal beam section point | Numerical solution | Analytical solution of general calculation method | Analytical solution of considering the load distribution coefficient of node |
|------------------------------|-------------------|-----------------------------------------------|-------------------------------------------------|
| J1                           | 4.365             | 6.471                                         | 5.817                                           |
| J2                           | 4.384             | 7.318                                         | 6.693                                           |
| J3                           | 4.372             | 6.515                                         | 5.862                                           |
| J4                           | 4.379             | 7.453                                         | 6.706                                           |
| J5                           | 4.381             | 6.511                                         | 5.869                                           |
Table 4. Compare the displacement of vertical beam (×10-2)

| Vertical beam section point | Numerical solution | Analytical solution of general calculation method | Analytical solution of considering the load distribution coefficient of node |
|-----------------------------|-------------------|-----------------------------------------------|---------------------------------------------------------------------|
| S1                          | 5.512             | 7.168                                         | 6.038                                                              |
| S2                          | 4.372             | 6.419                                         | 5.157                                                              |
| S3                          | 3.501             | 7.226                                         | 6.129                                                              |
| S4                          | 2.991             | 6.506                                         | 5.263                                                              |
| S5                          | 2.625             | 7.308                                         | 6.292                                                              |
| S6                          | 2.133             | 6.586                                         | 5.336                                                              |
| S7                          | 2.122             | 7.562                                         | 6.403                                                              |

Table 3 and Table 4 calculated the displacements of lattice beams. It can be seen from the tables that the displacement of considering the distribution coefficient is smaller than the general calculation method’s, but it is not as good as the variation law of the displacement simulated by ABAQUS. The reason is that ABAQUS simulates the displacement of the lattice beam with the slope, while the displacement calculated by considering the distribution coefficient based on the two-parameter elastic foundation model is a single beam, so there is a big difference.

6. Conclusion

1. According to the mechanical characteristics and form of lattice beam, used the Two-parameter elastic foundation model to deduce the answer of internal forces of concentrated force on the finite-length beam, and calculated load distribution coefficient \( k \) and \( k' \) according to the static equilibrium equation and deformation coordination equation at the node of lattice beam.

2. Compared with bending moment and displacement, the calculation method considering load distribution coefficient of node is closer to ABAQUS simulation.

3. The method is used for the project in this paper to reinforce the lattice beams, the steel consumption can be saved by 11.84% to 21.64%.

4. Since the load distribution coefficient of node derived in this paper still can not take into account the torsional deformation and rotation of node, the mechanical behavior of node needs to be further studied.

Acknowledgment

This work was supported by General Program of Natural science fund for colleges and universities in Jiangsu Province [grant number 18KJD580001]. The financial support is greatly appreciated.

References

[1] Li Qun, and Zhang Guangcheng 2006 Internal force analysis and optimal design of cross lattice beam *Journal of Coal Geology and Exploration* 34 (6) pp 50-53

[2] Dong Jianhua, and Liu Ke 2020 Calculation method of frost heave effect of lattice beam and anchor composite structure in cold region *Chinese journal of rock mechanics and engineering* 34 (05) pp 984-995

[3] Song Jun, Liang Tong 2018 Experimental study on mechanical characteristics of prestressed lattice anchorage structure *Journal of Geomechanics* 24 (03) pp 432-438

[4] Mei Ling, Tian Lu, and Gu Weiwei 2018 Stability analysis of expansive soil slope supported by bolt lattice beam considering expansion force *Journal of Jiangsu University of Science and Technology* 33 (02) pp 298-303

[5] Huang Yi 2005 Beam, plate and shell on elastic foundation *Science Press* Beijing China
[6] FeiKuang, Zhang Jianwei, 2010 Application of ABAQUS in geotechnical engineering, China Water Conservancy and Hydropower Press, Beijing, China.

[7] Chan, S.H, Tuba, I.S (1971). A finite element method for contact problems of solidbodies, Int. J. Mech. Sci., 13 pp 615-639.

[8] F. Hache, N. Challamel, I. Elishakoff, C. M. Wang (2017). Comparison of nonlocal continualization schemes for lattice beams and plates, Archive of Applied Mechanics, 87(7) pp 1105-1138.