Game among interdependent networks: The impact of rationality on system robustness

Yuhang Fan\textsuperscript{1,2}, Gongze Cao\textsuperscript{2}, Shibo He\textsuperscript{1}, Jiming Chen\textsuperscript{1} and Youxian Sun\textsuperscript{1}

\textsuperscript{1} State Key Lab. of Industrial Control Technology, Zhejiang University - Hangzhou, 310027, China
\textsuperscript{2} School of Mathematics, Zhejiang University - Hangzhou, 310027, China

received 10 October 2016; accepted in final form 16 January 2017
published online 8 February 2017

PACS 89.75.Fb - Structures and organization in complex systems
PACS 89.75.Hc - Networks and genealogical trees
PACS 87.23.Ge - Dynamics of social systems

Abstract — Many real-world systems are composed of interdependent networks that rely on one another. Such networks are typically designed and operated by different entities, who aim at maximizing their own payoffs. There exists a game among these entities when designing their own networks. In this paper, we study the game investigating how the rational behaviors of entities impact the system robustness. We first introduce a mathematical model to quantify the interacting payoffs among varying entities. Then we study the Nash equilibrium of the game and compare it with the optimal social welfare. We reveal that the cooperation among different entities can be reached to maximize the social welfare in continuous game only when the average degree of each network is constant. Therefore, the huge gap between Nash equilibrium and optimal social welfare generally exists. The rationality of entities makes the system inherently deficient and even renders it extremely vulnerable in some cases. We analyze our model for two concrete systems with continuous strategy space and discrete strategy space, respectively. Furthermore, we uncover some factors (such as weakening coupled strength of interdependent networks, designing a suitable topology dependence of the system) that help reduce the gap and the system vulnerability.

In interdependent networks, nodes from different networks rely on one another. The failure of a node in one network causes its dependent nodes to also fail, leading to an iterative cascade of failures. They are, consequently, much more fragile than independent networks [1–3]. Much attention has been paid on how interdependence results in the catastrophic cascade of failures and how to improve the robustness of such systems [4–9].

The game, especially the evolutionary game, on interdependent networks has been well studied before [10–12]. Some methods, such as optimizing the interdependence topology [13] and designing proper cooperative mechanisms [14], have been proposed to promote the cooperation on interdependent networks. Further relevant research has been done under the elevated levels of cooperation in interdependent networks more precisely, particularly in relation to information transfer [15,16].

However, these studies, mainly based on conventional networked evolutionary game [17,18], regarded nodes in networks as the players of the game and represented the multilayer relation between them via interdependent networks. So far, little is known about the game among interdependent networks in which the players are networks themselves. In practice, different networks are typically designed and operated by varying entities [19–21]. For example, the power and communication networks are owned by different companies in China. Each entity aims at maximizing its own payoff when building the network, without considering the overall system performance. Clearly, there exists a game during the formation of the interdependent networks, where a network is taken as a player. Studying such a game helps understand how the topology of the practical interdependent networks is formed, and provides an insight into their inherent performance degradation, which has not been studied yet.

We introduce a mathematical framework based on random graph theory and percolation theory [22] for studying this game. The system is composed of $n$ interdependent networks $N_i, i \in \{1,2,\ldots,n\}$, and the dependence is fixed. After a fraction $1 - p_i$ of nodes being randomly removed from network $N_i$, there is an iterative cascade of failures. We denote the fraction of nodes in the giant component of network $N_i$ by $P_{\text{gc},i}$ (when the number of nodes approaches infinity, it represents the probability
of the existence of the giant component) which is a function of \( p_j, j \in \{1, 2, \ldots, n \} \). Let \( \langle k \rangle \) be the average degree of network \( N_i \). The payoff of \( N_i \) is the difference of the income \( I_i \) and the cost \( O_i \) associated with building and operating the network \( N_i \). Clearly, the income of \( N_i \), denoted by \( I_i(P_{\infty,i}) \), is positively correlated to the ratio of functional nodes (i.e., the fraction of nodes in the mutually connected giant component \( P_{\infty,i} \)) which is positively correlated to the robustness of the network) in practice. As \( P_{\infty,i} \) is a function of \( p_j \), we can regard \( I_i \) as a function of \( p_j \).

Because the fraction \( p_i \) of nodes that remain in the network \( N_i \) after the initial attack is not constant in the real world and is affected by many factors (such as the weather condition for a power network [23]), we suppose that it has a probability distribution with density \( \psi_i \), that is, \( P(p_i < \alpha) = \int_0^\alpha \psi_i(x)dx \). With this, we can compute the mathematical expectation \( E(I_i) \) of the income \( I_i \). The payoff \( Y_i \) of the network \( N_i \) is thus

\[
Y_i = E(I_i(P_{\infty,i})) - O_i(\langle k \rangle_i),
\]

and the payoff \( Y \) of the whole system (also known as social welfare) is

\[
Y = \sum_{i=1}^n Y_i.
\]

Note that increasing \( \langle k \rangle \) may not always improve the social welfare since both the income and the cost increase with \( \langle k \rangle \). In a cooperative system, an optimal \( \langle k \rangle \) and the corresponding strategy can be calculated to maximize the social welfare. To be more comprehensible, we provide an example of two totally interdependent ER networks with the same average degree \( \langle k \rangle \). According to [25], it is equivalent to removing a \( 1 - p \) (which equals \( 1 - p_1 p_2 \)) fraction of nodes from one network. There is a first-order percolation transition with the threshold \( p_c \). It has been shown that the threshold \( p_c = 2.4554/\langle k \rangle \) [1]. We assume, for simplicity, that the income \( I = 0 \) when \( p < p_c \), and \( I = 1 \) when \( p > p_c \), and that \( p_i \), the fraction of functional nodes in \( N_i \) after the initial attack, follows a uniform distribution. The cost function \( O \) is set to be a linear function with coefficient 0.08. Then, the social welfare \( Y = (1 - p_c) - 0.08(\langle k \rangle - 1) = (2.4554/\langle k \rangle - 0.08) \). Clearly, a higher \( \langle k \rangle \) leads to a higher income as well as a higher cost. It is easy to compute the social optimum at \( \langle k \rangle = 5.54 \) where the optimal social welfare \( Y = 0.11 \) [26, 27] (see fig. 1).

Unfortunately, the average degree is typically implicit in \( p_i \) and \( P_{\infty,i} \), which makes our analysis complicated. Here, we introduce real variables \( \{q_i\}, i \in \{1, \ldots, n\} \), as strategy profiles to quantify the construction pattern of the network \( N_i \) by the entity \( i \). We suppose that the strategy set of entity \( i \) is \( Q_i \), i.e., the range of the variable \( q_i \), and \( Q = Q_1 \times \cdots \times Q_n \) is the set of strategy profiles in this game. Let \( q_{-i} \) be the strategy profile of all entities except for player \( i \). According to [28], the degree distribution of a node in network \( N_i \) can be characterized by \( P_i(k, q_i) \). For instance, \( q_i \) could be the average degree of an ER network decided by entity \( i \). We introduce the generating function of network \( N_i \) whose arguments are \( x \) and \( q_i \) as

\[
G_i(x, q_i) = \sum_{k=0}^{\infty} P_i(k, q_i) x^k.
\]

Different from previous studies [28, 29], the strategy parameters \( q_i \) are included in the generating function to assist the analysis of the game. The average degree \( \langle k \rangle_i \) of network \( N_i \) can be calculated as

\[
\langle k \rangle_i = \frac{\partial}{\partial x} G_i(1, q_i) = \sum_{k=0}^{\infty} k P_i(k, q_i),
\]

which is a continuous function of \( q_i \). Analogously, we introduce the generating function of the underlying branching process [2]

\[
H_i(x, q_i) = \frac{G_i'(x, q_i)}{G_i'(1, q_i)},
\]

where \( G_i'(x, q_i) \) is the derivative of \( G \) with respect to \( x \). The probability that a randomly chosen surviving node belongs to the giant component [1, 7] is given by

\[
g_i(p, q_i) = 1 - G_i[p f_i(p, q_i) + 1 - p, q_i],
\]

where \( f_i \) satisfies

\[
f_i(p, q_i) = H_i[p f_i(p, q_i) + 1 - p, q_i].
\]

Similarly to Kirchhoff equations, for fixed \( q_i \), we can arrive at a system of iterative equations of unknowns \( x_i \).
where $r_{ji} \geq 0$ is the fraction of the nodes in network $N_i$ that directly depends on the nodes of the network $N_j$ and $r_{ii} = 0$, $x_i$ represents the fraction of the nodes that survive in the network $N_i$ after removing all the nodes affected by the initial attack and the nodes depending on the failed nodes in other networks, $y_{ij}$ is the fraction of the survived nodes in the network $N_i$ after the damage from all the networks connected to the network $N_i$ except for the network $N_j$. We can analytically compute that the fraction $P_{\infty,k} = x_k g_k(x_k)$ of the nodes in the giant component of the network $N_k$ as a function of $p_i$ and $q_i$. Specially, if $n = 2$, eq. (9) yields $y_{12} = p_1$, $y_{21} = p_2$. Equation (8) can be simplified as

$$x_i = p_i \prod_{j=1}^{n} \frac{r_{ji} y_{ji} g_j(x_j) - r_{ji} + 1}{r_{ji} y_{ji} g_j(x_j) - r_{ji} + 1}$$

$$y_{ij} = \frac{x_i}{r_{ji} y_{ji} g_j(x_j) - r_{ji} + 1}.$$  

(8)

If the process of cascade is a first-order percolation transition, there is a single-step discontinuity at the threshold $p_c$, which is also a function of $q_i$ [5]; otherwise, we let $p_c = 0$. According to eq. (1), we can calculate the expectation of network $N_i$’s payoff $Y_i$ by

$$Y_i = \int_{p_c}^{1} \cdots \int_{p_c}^{1} I_i(P_{\infty,i}(q_i, \ldots, q_n, p_1, \ldots, p_n)) \prod_{j=1}^{n} \psi_j(p_j) dp_1 \cdots dp_n - O_i((k)_i)).$$

(12)

and $y_{ij}$ [2,30],

$$x_i = p_i \prod_{j=1}^{n} [r_{ji} y_{ji} g_j(x_j) - r_{ji} + 1],$$

$$y_{ij} = \frac{x_i}{r_{ji} y_{ji} g_j(x_j) - r_{ji} + 1}.$$  

(9)

entities’ payoffs when one entity changes his strategy profile to improve his own payoff. It is easy to see that at Nash equilibrium the system reaches the optimal social welfare. Therefore, in such a scenario, the individual payoff of each entity is maximized at the same point as the optimal social welfare. The cooperation among different networks can be reached.

For example, for the game among interdependent scale-free (SF) networks whose strategy space is the power of degree distribution, a higher power leads to an improvement of the system’s robustness [1]. Due to the above conclusion, when the average degree is fixed, different networks can cooperate to improve the power of distribution as much as possible and reach the optimal social welfare.

However, if each $O_i$ is not fixed, the cooperation among different networks may be unattainable. In fact, we will prove in the following that the cooperation is unreachable if the real variables $q_i$ can range in some interval continuously (that is, this is a continuous game). Without loss of generality, we can assume that the payoff functions are differential. Then the necessary condition for a pure strategy Nash equilibrium is $\partial Y_i / \partial q_i = 0$. The necessary condition for the optimal social welfare in this game is $\partial Y / \partial q_i = 0$.

We proceed to analyze the game when the average degree can be adjusted by $q_i$ of each network $N_i$. When the payoff of this system achieves its Nash equilibrium, i.e., $Y_i = Y_i^{max}$, we have $\partial Y_i / \partial q_i = 0$. According to eq. (1), we have

$$\frac{\partial Y_i}{\partial q_i} = \frac{\partial E(I_i(P_{\infty,i}))}{\partial q_i} \frac{\partial O_i((k)_i)}{\partial q_i} = 0.$$  

(15)

Note that $\langle k \rangle_i$ does not rely on $q_j$ if $j \neq i$, leading to $\partial O_i / \partial q_j = 0$. At the Nash equilibrium, we have

$$\frac{\partial Y}{\partial q_i} = \sum_{i=1}^{n} \frac{\partial E(I_i(P_{\infty,i}))}{\partial q_i} \sum_{i=1}^{n} \frac{\partial O_i((k)_i)}{\partial q_j} = \sum_{i \neq j} \frac{\partial E(I_i(P_{\infty,i}))}{\partial q_j}.$$  

(16)

Since $\langle k \rangle_i$ is not fixed about $q_i$, $\partial E(I_i(P_{\infty,i})) / \partial q_i = \partial O_i((k)_i) / \partial q_i \neq 0$. Due to the interdependence, the incomes of $n$ networks, which are decided by the robustness of the system, are positively correlated. $\partial E(I_i(P_{\infty,i})) / \partial q_j$ have the same sign and do not equal 0, leading to $\partial Y / \partial q_i \neq 0$. Therefore, Nash equilibrium (since the concrete payoff function is not given here, the pure strategy Nash equilibrium may not exist, however, our model can be easily extended to the mixed strategy game in which Nash equilibrium always exists [26]) deviates from the optimal social welfare. The cooperation among different interdependent networks cannot be
Table 1: Game matrix for two totally interdependent RR networks: the two components of the vectors in the matrix are the payoffs of two entities with corresponding strategy profile for \( I_i(x) = x \) and \( O_i(x) = 0.008x \). The Nash equilibrium and optimal social welfare are underlined in the matrix. It can be seen that there is a notable gap between the Nash equilibrium \( q_1 = q_2 = 7 \) and social optimum \( q_1 = q_2 = 9 \).

| \( (Y_1, Y_2) \) | 5     | 6     | \( \tilde{q} \) | 8     | 9     | 10    |
|----------------|-------|-------|----------------|-------|-------|-------|
| \( q_1 \)      |       |       |                |       |       |       |
| 5              | (0.340,0.340) | (0.356,0.340) | (0.367,0.351) | (0.374,0.350) | (0.379,0.347) | (0.383,0.343) |
| 6              | (0.340,0.356) | (0.366,0.366) | (0.376,0.368) | (0.383,0.367) | (0.388,0.364) | (0.392,0.360) |
| \( \tilde{q} \) | (0.351,0.367) | (0.368,0.376) | (0.379,0.379) | (0.386,0.378) | (0.391,0.375) | (0.395,0.371) |
| 8              | (0.350,0.374) | (0.367,0.383) | (0.378,0.386) | (0.385,0.385) | (0.390,0.382) | (0.394,0.378) |
| 9              | (0.347,0.379) | (0.364,0.388) | (0.375,0.391) | (0.382,0.390) | (0.387,0.387) | (0.391,0.383) |
| 10             | (0.343,0.383) | (0.360,0.392) | (0.395,0.371) | (0.378,0.394) | (0.383,0.391) | (0.386,0.386) |

The game between two interdependent RR networks is discrete. By numerical validation, we have the payoff matrix and obtain the Nash equilibrium and social optimum of this discrete game. Table 1 is the game matrix for the case where two networks are totally interdependent, \( i.e., r_{12} = r_{21} = 1 \) in eqs. (10) and (11). From this matrix, we can calculate that \( q_1 = q_2 = 9 \) are the strategy in the social optimum and \( q_1 = q_2 = 7 \) are those at the Nash equilibrium. Similarly, we can get the Nash equilibrium and social optimum for partially interdependent RR networks.

For two coupled ER networks, we set \( q_i \) to be their average degree \( \langle k \rangle_i (i \in \{1, 2\}) \). Then we have \( G_i(x, q_i) = H_i(x, q_i) = \exp[q_i(x - 1)] \). Similarly to the first example, we set \( I_i \) and \( O_i \) to be linear functions whose coefficients are 1 and 0.05, respectively. We also set the distributions of \( p_i \) to be uniform. As fig. 2 shows, we can solve \( Y_1 \) and \( Y_2 \) as functions of \( q_1 \) and \( q_2 \). By numerical simulations, we can obtain the Nash equilibrium and social optimum for this game. For instance, in the case of two totally coupled ER networks, we get that this system reaches its social optimum at \( \langle k \rangle_1 = \langle k \rangle_2 = 4.37 \) with \( Y = 0.54 \) and reaches its Nash equilibrium at \( \langle k \rangle_1 = \langle k \rangle_2 = 3.09 \) with \( Y = 0.49 \). We can see that there is a notable gap between Nash equilibrium and social optimum.

reached. The rationality of different entities makes the system inherently deficient.

If \( Y_1 = Y_1^{\text{max}} \) is the payoff \( Y_i \) of the network \( N_i \) at the Nash equilibrium and \( Y = Y_1^{\text{max}} \) at the social optimum, we define \( \Delta \) as

\[
\Delta = \frac{Y_1^{\text{max}} + Y_2^{\text{max}}}{Y_1^{\text{max}}} \leq 1
\]

(17)

to evaluate this game. The higher \( \Delta \) is, the closer the payoffs at Nash equilibrium and social optimum are.

Next we analyze our model for concrete interdependent networks and strategy space. For the convenience of our numerical validation but without loss of generality, we set \( I_i \) and \( O_i \) to be linear functions in the following experiments. For coupled Random Regular (RR) networks, we set \( q_i \in Z^+ \) to be their average degree \( \langle k \rangle_i (i \in \{1, 2\}) \). Then we have \( G_i(x, q_i) = x^{q_i}, H_i = x^{k_i-1} \) and \( P_{\infty,i} = x_i q_i(x_i) \), where \( q_i \) are given in eq. (6). Set \( I_i(x) = x \) and \( O_i(x) = 0.008x \) in eq. (1) and the distributions of \( p_1 \) and \( p_2 \) to be uniform distributions. Since the average degree of RR networks can only be integer, the system is more profitable and efficient as a result of reducing the coupled strength.

From the above examples of RR networks and ER networks, we can see that the rational behaviors will prevent the system from achieving the optimal robustness and render it more vulnerable in some cases. As fig. 4 shows, we get the \( P_\infty - p \) graph at the Nash equilibrium and social optimum for \( r = 1 \) and \( r = 0.6 \), respectively. The threshold is higher and \( P_\infty \) is lower at Nash equilibrium.
Fig. 3: (Colour online) For two coupled RR and ER networks whose strategy spaces are the average degree, $Y_1^{\text{max}} + Y_2^{\text{max}}$, $Y_{\text{max}}$ and $\Delta$ are shown as functions of the coupled strength $r$ with $I_i(x) = x$, $O_i(x) = cx$ and the distribution of $p_i (i \in \{1, 2\})$ being a uniform distribution. Panel (a) and panel (c) are $Y_{\text{max}} - r$, $(Y_1^{\text{max}} + Y_2^{\text{max}}) - r$ and $\Delta - r$ graphs for coupled RR networks with $c = 0.014$ and $c = 0.008$, respectively. Panel (b) and panel (d) are $Y_{\text{max}} - r$, $(Y_1^{\text{max}} + Y_2^{\text{max}}) - r$ and $\Delta - r$ graphs for coupled ER networks with $c = 0.05$ and $c = 0.03$, respectively. Red-square, red-circle, blue-square, and blue-circle curves correspond to $Y_1^{\text{max}} + Y_2^{\text{max}}$ at $c = 0.014$, $Y_{\text{max}}$ at $c = 0.014$, $Y_1^{\text{max}} + Y_2^{\text{max}}$ at $c = 0.008$, $Y_{\text{max}}$ at $c = 0.008$ in panel (a), and to $Y_1^{\text{max}} + Y_2^{\text{max}}$ at $c = 0.05$, $Y_{\text{max}}$ at $c = 0.05$, $Y_1^{\text{max}} + Y_2^{\text{max}}$ at $c = 0.03$, $Y_{\text{max}}$ at $c = 0.03$ in panel (b). Red-circle and blue-square curves correspond to $\Delta$ at $c = 0.014$ and $c = 0.008$ in panel (c), and to $\Delta$ at $c = 0.05$ and $c = 0.03$ in panel (d). It can be seen from the inset of panel (a) that $Y_1^{\text{max}} + Y_2^{\text{max}}$ is not strictly decreasing in $r$. We can see that reducing the coupled strength leads to a remarkable improvement of $Y_{\text{max}}$, $Y_1^{\text{max}} + Y_2^{\text{max}}$ and $\Delta$.

in the same scenario. Note that the system of interdependent networks is more vulnerable at Nash equilibrium as a result of rationality.

It is important to find a way of reducing the gap between optimal social welfare and Nash equilibrium for the game among interdependent networks. Besides reducing the coupled strength, surprisingly, we find that the gap is narrower for the system with suitable dependence topology. Here we test our model for two systems composed of five interdependent networks with different dependence topology. Let all coupled networks be totally interdependent. According to the one-to-one correspondence, randomly removing $1 - p$ fractions of nodes from each network $N_i$ is equivalent to a single attack on one of the networks which removes $1 - p = 1 - \prod_{i=1}^{\infty} p_i$ fraction of nodes. Set the distribution of $p$ to be uniform and $I_i$ and $O_i$ to be linear functions with coefficients $1$ and $3.5 \times 10^{-4}$ in eq. (1) for RR networks and coefficients $1$ and $2.5 \times 10^{-4}$ for ER networks. We validate the cases of chain-like and star-like system formed from five networks (see fig. 5). By numerical simulations, we obtain that the star-like oligarchic dependence topology tends to reduce the gap (see table 2).

Summarizing, in this paper, we study the influence of rational behavior on system robustness. We reveal that, in the continuous game, the cooperation among different interdependent networks is reachable only when the average degree of each network is fixed in strategy space. In general, there is a huge gap between the Nash equilibrium and the optimal social welfare as a result of rationality which makes the system inherent deficient. We reveal

| RR  | Star-like | Chain-like | ER  | Star-like | Chain-like |
|-----|-----------|------------|-----|-----------|------------|
| $\Delta$ | 1.0      | 0.89       | $\Delta$ | 0.56      | 0.52       |

Fig. 4: (Colour online) $P_\infty$-$p$ graph, of two coupled RR networks and ER networks at Nash equilibrium and social optimum with $I_i = x$, $O_i = 0.008x$ (a), $I_i = x$, $O_i = 0.05x$ (b) and $r = 0.6$, $r = 1$, respectively. $P_\infty$, the fraction of nodes in the giant component of the network $N_i$, is shown as a function of the fraction $p_1$ of the remaining nodes in $N_i$. Square, star, circle and triangle curves correspond to Nash equilibrium at $r = 0.6$, social optimum at $r = 0.6$, Nash equilibrium at $r = 1$ and social optimum at $r = 1$, respectively. We can see that the threshold of percolation transition is higher and $P_\infty$ is lower at Nash equilibrium in the same scenario, and that the system is more vulnerable at Nash equilibrium than those at social optimum.

Fig. 5: Two types of essential systems composed of five coupled networks.
and validate some factors (including weakening coupled strength of interdependent networks and designing suitable topology dependence of the system) that help reduce the gap and deficiency.

Interdependent networks exist in all aspects of our life, nature and technology. The game among interdependent networks is more complicated in the real world since besides the fraction of giant component and average degree, the utility function may rely on many factors and need further investigation. It is of great importance to find other efficient ways of reducing the system’s vulnerability and the gap between Nash equilibrium and optimal social welfare.

***

This work is supported by NSFC project under grant 61528105 and Zhejiang Provincial Natural Science Foundation of China under grant LR16F020001. We thank Prof. YANG-YU LIU at Harvard University for his valuable suggestion, ZIDONG YANG, YONGTAO ZHANG and HANYUAN LIU at Zhejiang University for their insightful discussions.

REFERENCES

[1] Buldyrev S. V., Parshani R., Paul G., Stanley H. E. and Havlin S., Nature, 464 (2010) 1025.
[2] Gao J., Buldyrev S. V., Stanley H. E. and Havlin S., Nat. Phys., 8 (2012) 40.
[3] Kenett D. Y., Perc M. and Boccaletti S., Chaos, Solitons Fractals, 80 (2015) 1.
[4] Schneider C. M., Yazdani N., Araújo N. A., Havlin S. and Herrmann H. J., Sci. Rep., 3 (2013) 1969.
[5] Parshani R., Buldyrev S. V. and Havlin S., Phys. Rev. Lett., 105 (2010) 048701.
[6] Shao J., Buldyrev S. V., Havlin S. and Stanley H. E., Phys. Rev. E, 83 (2011) 036116.
[7] Gao J., Buldyrev S. V., Havlin S. and Stanley H. E., Phys. Rev. Lett., 107 (2011) 195701.
[8] Di Muro M. A., La Rocca C. E., Stanley H. E., Havlin S. and Braunstein L. A., Sci. Rep., 6 (2016) 22834.
[9] Shekhtman L. M., Danziger M. M. and Havlin S., Chaos, Solitons Fractals, 90 (2016) 28.
[10] Wang Z., Szolnoki A. and Perc M., EPL, 97 (2012) 48001.
[11] Wang Z., Szolnoki A. and Perc M., Sci. Rep., 3 (2013) 1183.
[12] Wang Z., Wang L., Szolnoki A. and Perc M., Eur. Phys. J. B, 88 (2015) 1.
[13] Wang Z., Szolnoki A. and Perc M., Sci. Rep., 3 (2013) 2470.
[14] Wang Z., Szolnoki A. and Perc M., New J. Phys., 16 (2014) 033041.
[15] Jiang L. L. and Perc M., Sci. Rep., 3 (2013) 2483.
[16] Szolnoki A. and Perc M., New J. Phys., 15 (2013) 053010.
[17] Nowak M. A. and May R. M., Nature, 359 (1992) 826.
[18] Hauert C. and Doebeli M., Nature, 428 (2004) 643.
[19] Schweitzer F., Fagiolo G., Sornette D., Vega-Redondo F., Vespignani A. and White D. R., Science, 325 (2009) 422.
[20] Rosato V., Issacharoff L., Tiriticco F., Meloni S., Porcellinis S. and Setola R., Int. J. Crit. Infrastruct., 4 (2008) 63.
[21] Rinaldi S. M., Peerenboom J. P. and Kelly T. K., IEEE Control Syst., 21 (2001) 11.
[22] Callaway D. S., Newman M. E., Strogatz S. H. and Watts D. J., Phys. Rev. Lett., 85 (2000) 5468.
[23] Rosato V., Issacharoff L., Tiriticco F., Meloni S., Porcellinis S. and Setola R., Int. J. Crit. Infrastruct., 4 (2008) 63.
[24] Bollobás B., Modern Graph Theory (Springer) 1998, pp. 215–252.
[25] Gao J., Buldyrev S. V., Havlin S. and Stanley H. E., Phys. Rev. E, 85 (2012) 066134.
[26] Fudenberg D. and Tirole J., Game Theory, Vol. 393 (The MIT Press) 1991.
[27] Gibbons R., A Primer in Game Theory (Harvester Wheatsheaf) 1992.
[28] Shao J., Buldyrev S. V., Cohen R., Kitsak M., Havlin S. and Stanley H. E., EPL, 84 (2008) 48004.
[29] Newman M. E., Phys. Rev. E, 66 (2002) 016128.
[30] Brummitt C. D., D’Souza R. M. and Leicht E. A., Proc. Natl. Acad. Sci. U.S.A., 109 (2012) E680.