Do internal symmetries get restored in hot and dense SUSY?

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I offer some computational details and useful and concise formulae to calculate the effective potential for a general abelian supersymmetric model at high temperature and density. It will be shown that such cases are very good candidates for symmetry nonrestoration at high temperature, providing large densities are present.

1 Introduction
This talk is complementary to the one given by Goran Senjanović at the same conference. The introduction, motivation, physical applications and main references are already given there, so I will try to give some technical details.

2 The general Abelian example
Let me consider a general supersymmetric model with \( N \) chiral superfields, \( M \) global \( U(1) \) symmetries and, for simplicity’s sake, one single gauge \( U(1) \) interaction. For each continuous symmetry a chemical potential corresponding to the 0-component of the conserved current (= charge density) is introduced:

\[
\delta \mathcal{L}^{(1)} = \mu_a \left[ g^a \lambda \lambda + f^a_i \psi_i \bar{\psi}_i + ib^a_i \left[ \phi_i (D_0 \phi_i)^* - \phi_i^* (D_0 \phi_i) \right] \right], \tag{1}
\]

where \( b^a_i, f^a_i \) and \( g^a \) are the \( a \)-th charges of the boson \( \phi_i \), fermion \( \psi_i \) and gaugino \( \lambda \). The summation over repeating indices \( (a = 1,..., M + 1; i = 1,..., N) \) is always assumed. Eq. 1 generates for each boson the term \( \mu_i^2 T^2 / 12 \) for each field, where \( \mu_i = q_i^a \mu_a \) here.

We are interested in the effective potential at temperatures \( T \) much bigger than any mass in the superpotential, and, to simplify the formulae, for chemical potentials not much bigger than the temperature. In this case, the only new relevant terms given by a nonzero charge are \(-a \mu_i^2 T^2 / 12\) for each field, where \( a = 1 \) for fermions and 2 for bosons. To see it, assume for a moment that the chemical potential \( \mu \) corresponding to a field is an independent but nonpropagating field itself. One then calculate the one loop high \( T \) correction to the \( \mu_i^2 \) term in the effective potential: two external “fields” \( \mu_i \) are connected by a loop of the corresponding field via the “interaction” terms in Eqs. 1.
The leading order effective potential at high temperature and density is

$$V_{\text{eff}}(\phi_i, T, n_i) = V_{\text{eff}}(\phi_i, T, 0) - \frac{1}{2} \mu^a \mathcal{M}_{ab} \mu^b + \mu^a n_a ,$$

(3)

where the $(M + 1) \times (M + 1)$ symmetric matrix $\mathcal{M}$ is defined as

$$\mathcal{M}^{ab}(\phi, T) = \frac{T^2}{6}(f^a_i f^b_i + 2b^a_i b^b_i + g^a g^b) + 2b^a_i b^b_i |\phi_i|^2 .$$

(4)

The effective potential does not depend on the chemical potentials, so

$$\frac{\partial V_{\text{eff}}}{\partial \mu^a} = 0 , \quad \mu^a = (\mathcal{M}^{-1})^{ab} n_b ,$$

(5)

$$V_{\text{eff}}(\phi_i, T, n_i) = V_{\text{eff}}(\phi_i, T, 0) + \frac{1}{2} n_a (\mathcal{M}^{-1})^{ab} n_b .$$

(6)

There are some points to be stressed:

1) Nonsupersymmetric models have a very similar matrix as the one defined in Eq. 4 except that the number of complex bosons $\phi_i$ is not necessarily equal to the number of Weyl fermions $\psi_i$ and there are no gauginos $\lambda$.

2) A chemical potential $\mu^a$ can be nonzero even if the corresponding charge density $n^a$ is zero, Eq. 5. So, a large charge density can give a nontrivial vev to fields which are blind to this charge. For example, one can spontaneously break the electromagnetic $U(1)$, even if the universe is electrically neutral.

3) One could work with a nonzero vev $A_0$ instead of the local $U(1)$ chemical potential $\mu^{M+1}$. The equations are completely the same; one must just equate $gA_0 = \mu^{M+1}$, while the charges of the boson field $\phi_i$ and fermion field $\psi_i$ are equal to the “electric charge”, $b_i^{M+1} = f_i^{M+1} = q_i^{\text{gauge}}$.

4) The first part of Eq. 6, i.e. $V_{\text{eff}}(\phi_i, T, 0)$ minimizes for the symmetric vacuum, $\phi_i = 0$. This is a consequence of the constraints of supersymmetry, the result being valid even for nonrenormalizable superpotentials. Ordinary models behave differently and allow also nontrivial minima.

5) Due to the previous remark, it is then the second part of Eq. 6 which decides whether we get symmetry nonrestoration or not. Typically one has $\mathcal{M} \approx T^2 + |\phi|^2$, so that its inverse minimizes for an infinite vev. For small charge densities the first term in Eq. 6 dominates, while for large densities both terms are important, leading to symmetry nonrestoration. The critical value for this to happen is clearly proportional to $T^3$, the temperature being the only remaining scale. Notice that any conserved charge density in the early universe is proportional to $T^3$, so the ratio between the charge density in the universe and the critical density is a constant number as long as possible symmetry breaking terms at low energy are not important.
3 Conclusions

I have given some general formulae and expressions, which can be useful in studying the effective potential at high temperature and density. In all the examples checked, they lead to symmetry nonrestoration for sufficiently large charge densities, the simplest example being the one given in Ref.

The above equations can be generalized for the nonabelian gauge symmetries. One must introduce only the chemical potentials corresponding to the diagonal generators. Nonabelian generators are traceless, so the associated chemical potentials can become nonzero only after the appearance of some nontrivial vev. In order to calculate the first critical charge density (different fields get usually nonzero vevs at different stages), one can then ignore the chemical potentials corresponding to the nonabelian symmetry.

Of course, one is especially interested in realistic models, like for example the MSSM or SUSY GUTs. Typically, at sufficiently large charge densities, all the complex fields get a nonzero vev, so the symmetry seems to be completely broken in this case. Further studies need to be done, however, since the analyses of the effective potential become very involved once many different fields are introduced, as is the case with the MSSM. In any case, the answer to the title seems to be simply no, given a large enough density.

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