Secret Broadcasting of W-type state

I.Chakrabarty $^{1,2,*}$, B.S.Choudhury $^2$

$^1$ Heritage Institute of Technology, Kolkata-107, West Bengal, India
$^2$ Bengal Engineering and Science University, Howrah, West Bengal, India

Abstract

In this work we describe a protocol by which one can secretly broadcast W-type state among three distant partners. This work is interesting in the sense that we introduce a new kind of local cloning operation to generate two W-type states between these partners from a W-type state initially shared by them.

1 Introduction:

The no-cloning theorem [1], as modified in [4], states that there is no method to blindly copy a pair of non orthogonal pure states. More importantly, for any pair of non orthogonal pure states $\rho_i$, $i \in \{1, 2\}$, there is no trace-preserving completely positive map $\epsilon$ such that $\epsilon(\rho_i) = \rho_i \otimes \rho_i \forall i$. Although nature prevents us from amplifying an unknown quantum state but nevertheless one can construct a quantum cloning machine that duplicates an unknown quantum state with a fidelity less than unity [1,2,3,4,5,6].

Beyond the no-cloning theorem, one can clone an arbitrary quantum state with some non-zero probability [7]. In the past years, much progress has been made in designing quantum cloning machine. Buzek-Hillery took the first step towards the construction of approximate quantum cloning machine [2]. They showed that the quality of the copies produced by their machine remain same for all input state. This machine is known as

*Corresponding author: E-Mail-indranilc@indiainfo.com
universal quantum cloning machine (UQCM). Later D. Bruss et al. showed this universal quantum cloning machine to be optimal [5]. After that different sets of quantum cloning machines like the set of universal quantum cloning machines, state dependent quantum cloning machines (i.e. the quality of the copies depend on the input state) and the probabilistic quantum cloning machines were proposed.

Entanglement [8], the heart of quantum information theory, plays a crucial role in computational and communicational purposes. Therefore, as a valuable resource in quantum information processing, quantum entanglement has been widely used in quantum cryptography [9,10], quantum super dense coding [11] and quantum teleportation [12]. An astonishing feature of quantum information processing is that information can be encoded in non-local correlations between two separated particles. A lot of work have been done to extract pure quantum entanglement from partially entangled state [10]. Now at this point one can ask an question: whether the opposite is true or not i.e. can quantum correlations be ”decompressed”? The probable answer to this question is ”Broadcasting of quantum entanglement”. Broadcasting is nothing but local copying of non-local quantum correlations. That is the entanglement originally shared by a single pair is transferred into two less entangled pairs using only local operations.

Suppose two distant parties A and B share two qubit-entangled state

$$|\psi\rangle_{AB} = \alpha|00\rangle_{AB} + \beta|11\rangle_{AB} \tag{1}$$

Let us assume that the first qubit belongs to A and the second qubit belongs to B. Each of these two parties A and B now perform local cloning operation on their own qubit. It turns out that for some values of $\alpha$

(1) non-local output states are inseparable, and

(2) local output states are separable.

V.Buzek et.al. [25] were the first who proved that the decompression of initial quantum entanglement is possible, i.e. from a pair of entangled particles, two less entangled pairs can be obtained by local operations. That means inseparability of quantum states can be partially broadcasted (cloned) with the help of local operations. They used optimal universal quantum cloners for local copying of the subsystems and showed that the non-local outputs are inseparable if $\alpha^2$ lies in the interval $\left(\frac{1}{2} - \frac{\sqrt{39}}{16}, \frac{1}{2} + \frac{\sqrt{39}}{16}\right)$.
Further S.Bandyopadhyay et.al. [13] showed that only those universal quantum cloners whose fidelity is greater than \( \frac{1}{2}(1 + \sqrt{\frac{4}{3}}) \) are suitable because then the non-local output states become inseparable for some values of the input parameter \( \alpha \). They proved that an entanglement can be optimally broadcasted only when optimal quantum cloners are used for local copying and also showed that broadcasting of entanglement into more than two entangled pairs is not possible using only local operations. I.Ghiu investigated the broadcasting of entanglement by using local 1 → 2 optimal universal asymmetric Pauli machines and showed that the inseparability is optimally broadcast when symmetric cloners are applied [14].

Few years back we studied broadcasting of entanglement using state dependent quantum cloning machine as a local copier. We showed that the length of the interval for probability-amplitude-squared (\( \alpha^2 \)) for broadcasting of entanglement using state dependent cloner can be made larger than the length of the interval for probability-amplitude-squared for broadcasting entanglement using state independent cloner [15]. In that work we showed that there exists local state dependent cloner which gives better quality copy (in terms of average fidelity) of an entangled pair than the local universal cloner [15]. In recent past Adhikari et.al in their paper [16] showed that secretly broadcasting of three-qubit entangled state between two distant partners with universal quantum cloning machine is possible. They generalized the result to generate secret entanglement among three parties. Recently Adhikari et.al proposed a scheme for broadcasting of continuous variable entanglement [17]. In another work [18] we presented a protocol by which one can broadcast five qubit entangled state between three different parties.

Along with Einstein-Podolsky-Rosen (EPR)state and Greenberger-Horne-Zeilinger (GHZ) state, there exist other entangled states such as W-class states and zero sum amplitude (ZSA) states [19] which have substantial importance in quantum information theory.

In this work we introduce a new cloning transformation. Each of three friends Alice, Bob and Carol is supplied with this cloning machine so that they can approximately clone their respective qubits. We start with a W type state of the form

\[
|X\rangle_{123} = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123}
\]

(2)

shared by three distant parties Alice,Bob and Carol. Then each party apply local approx-
imate cloning machine on their respective qubits. After that they perform measurements on their respective machine vectors. Not only that, each party informs others about their measurement results using Goldenberg and Vaidmans quantum cryptographic scheme [20] based on orthogonal state. Since the measurement results are interchanged secretly among them, so Alice, Bob and Carol share secretly six qubit state. Among six qubit state, we interestingly find that there exists two three qubit W-type states shared by Alice, Bob and Carol.

The advantage of this protocol from the previous broadcasting protocols is that here we secretly generate two states: (1) One between Alice’s original qubit and cloned qubits of Bob and Carol, (2) Another between original qubits of Bob and Carol with the cloned qubit of Alice, independent of the input parameters \( \alpha, \beta, \gamma \). Now to have a knowledge about the quantum information, evesdroppers have to do two things: First, they have to gather knowledge about the initially shared entangled state and secondly, they have to collect information about the measurement result performed by three distant partners. Therefore, the quantum channel generated by our protocol is more secured and hence can be used in various protocols viz. quantum key distribution protocols [23,24].

2 Secretly Broadcasting W-type state among three different partners

In this section we describe our whole protocol below step by step.

Step1: A new Cloning Transformation:

First of all we introduce a new cloning operation of the form

\[
|0\rangle \longrightarrow \frac{1}{\sqrt{x^2+y^2}}(x|00\rangle|\uparrow\rangle + y|10\rangle|\downarrow\rangle)
\]

\[
|1\rangle \longrightarrow \frac{1}{\sqrt{x^2+y^2}}(x|11\rangle|\uparrow\rangle + y|01\rangle|\downarrow\rangle)
\]

(3)
where \{ |\uparrow\rangle, |\downarrow\rangle \} are post operation orthogonal quantum cloning machine state vectors. Without loss of generality, \( x \) and \( y \) can always be considered to be real parameters. Now each of the three parties are supplied with identical cloning machines (defined by equation (3)), so that they can approximately clone their respective qubits.

**Step 2: Local Cloning and Measurement**

Let us consider a scenario, where three friends Alice, Bob and Carol, who are far away from each other, are sharing an entangled state (W-type) of the form

\[
|X\rangle_{123} = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123}
\]

(4)

where \( \alpha, \beta, \gamma \) are all real with \( \alpha^2 + \beta^2 + \gamma^2 = 1 \). The qubits 1,2,3 are with Alice, Bob and Carol respectively.

Alice, Bob and Carol then operate quantum cloning machine defined in equation (3) locally to copy the state of their respective particles. Therefore, after operating quantum cloning machine, Alice, Bob and Carol are able to approximately clone the state of the particle and consequently the combined system of six qubits is given by

\[
|X^C\rangle_{142536} = \frac{1}{(x^2 + y^2)^3} \left\{ \alpha(x|00\rangle|\uparrow\rangle^A + y|10\rangle|\downarrow\rangle^A)(x|00\rangle|\uparrow\rangle^B + y|10\rangle|\downarrow\rangle^B)(x|11\rangle|\uparrow\rangle^C + y|01\rangle|\downarrow\rangle^C) + \beta(x|00\rangle|\uparrow\rangle^A + y|10\rangle|\downarrow\rangle^A)(x|11\rangle|\uparrow\rangle^B + y|01\rangle|\downarrow\rangle^B)(x|00\rangle|\uparrow\rangle^C + y|10\rangle|\downarrow\rangle^C) + \gamma(x|11\rangle|\uparrow\rangle^A + y|01\rangle|\downarrow\rangle^A)(x|00\rangle|\uparrow\rangle^B + y|10\rangle|\downarrow\rangle^B)(x|00\rangle|\uparrow\rangle^C + y|10\rangle|\downarrow\rangle^C) \right\}
\]

(5)

The subscripts 4,5,6 refer approximate copies of qubits 1,2,3 which are with Alice, Bob and Carol respectively. Also \(|\rangle^A\), \(|\rangle^B\) and \(|\rangle^C\) denotes quantum cloning machine state vectors in Alice's, Bob's and Carol's side respectively.

Now after local cloning, each of them perform measurement on the quantum cloning machine state vectors in the basis \{ |\uparrow\rangle, |\downarrow\rangle \} and exchange their measurement results with each other using Goldenberg and Vaidmans quantum cryptographic scheme [20]. In this
way Alice, Bob and Carol interchange their measurement results secretly.
The tensor product of machine state vectors of three friends after the measurement is given by the following table.

| Serial Number | Measurement Results |
|---------------|---------------------|
| 1             | | ↑⟩^A| | ↑⟩^B| | ↑⟩^C |
| 2             | | | ↑⟩^A| | ↑⟩^B| | ↓⟩^C |
| 3             | | | ↑⟩^A| | ↓⟩^B| | ↓⟩^C |
| 4             | | | ↑⟩^A| | ↓⟩^B| | ↑⟩^C |
| 5             | | | ↓⟩^A| | ↑⟩^B| | ↑⟩^C |
| 6             | | | ↓⟩^A| | ↑⟩^B| | ↓⟩^C |
| 7             | | | ↓⟩^A| | ↓⟩^B| | ↑⟩^C |
| 8             | | | ↓⟩^A| | ↓⟩^B| | ↓⟩^C |

**Step 3: Analysis of a Particular Measurement Result**

Now let us consider the case when the measurement outcome is | ↑⟩^A| | ↑⟩^B| | ↑⟩^C, then the six qubit entangled state shared by Alice, Bob and Carol is given by

$$|Y^C⟩_{142536} = \frac{x^3}{(x^2 + y^2)^{\frac{3}{2}}} \{\alpha|000011⟩_{142536} + \beta|001100⟩_{142536} + \gamma|110000⟩_{142536}\} \quad (6)$$

Now it remains to be seen whether one can generate two 3-qubit W-type state from above six qubit entangled state or not.

$$\rho_{156} = \rho_{234} = \frac{x^4y^2}{(x^2 + y^2)^{\frac{3}{2}}} \{\alpha^2|001⟩⟨001| + \beta^2|010⟩⟨010| + \gamma^2|100⟩⟨100| + \alpha\beta|001⟩⟨010| + \alpha\gamma|001⟩⟨100| + \beta\alpha|010⟩⟨001| + \beta\gamma|010⟩⟨010| + \alpha\gamma|010⟩⟨100| + \gamma\alpha|100⟩⟨100| + \gamma\beta|100⟩⟨010|\} \quad (7)$$

It is evident from the outer products of equation(7), that the density operators $\rho_{156}$ and
\( \rho_{234} \) represent the density matrix of W-type of states.

One can investigate the problem of inseparability of the states obtained as a consequence of other possible measurement results as shown in the table 1.

**Step 4: Inseparability of Local Output states**

In broadcasting of inseparability, we generally use Peres-Horodecki criteria [21,22] to show the inseparability of non-local outputs and separability of local outputs.

**Peres-Horodecki Theorem** : The necessary and sufficient condition for the state \( \rho \) of two spins \( \frac{1}{2} \) to be inseparable is that at least one of the eigen values of the partially transposed operator defined as \( \rho_{m\mu,n\nu}^T = \rho_{m\mu,n\nu} \), is negative. This is equivalent to the condition that at least one of the two determinants

\[
W_3 = \begin{vmatrix} 
\rho_{00,00} & \rho_{01,00} & \rho_{00,10} \\
\rho_{00,01} & \rho_{01,01} & \rho_{00,11} \\
\rho_{10,00} & \rho_{11,00} & \rho_{10,10} 
\end{vmatrix}
\quad \text{and} \quad
W_4 = \begin{vmatrix} 
\rho_{00,00} & \rho_{01,00} & \rho_{00,10} & \rho_{01,10} \\
\rho_{00,01} & \rho_{01,01} & \rho_{00,11} & \rho_{01,11} \\
\rho_{10,00} & \rho_{11,00} & \rho_{10,11} & \rho_{11,11} \\
\rho_{10,01} & \rho_{11,01} & \rho_{10,11} & \rho_{11,11} 
\end{vmatrix}
\]

is negative.

Now we have to check that whether in our protocol the local output states are separable or not. The density operators representing the local output states are given by,

\[
\rho_{14} = \frac{x^6}{(x^2 + y^2)^3}(\alpha^2|00\rangle\langle 00| + \beta^2|00\rangle\langle 00| + \gamma^2|11\rangle\langle 11|)
\]

\[
\rho_{25} = \frac{x^6}{(x^2 + y^2)^3}(\alpha^2|00\rangle\langle 00| + \beta^2|11\rangle\langle 11| + \gamma^2|00\rangle\langle 00|)
\]

\[
\rho_{36} = \frac{x^6}{(x^2 + y^2)^3}(\alpha^2|11\rangle\langle 11| + \beta^2|00\rangle\langle 00| + \gamma^2|00\rangle\langle 00|) \quad (8)
\]

Now if one applies the Peres-Horodecki criterion to see whether the states are entangled or not, he will find that for each of these density operators, \( W_4 = W_3 = 0 \) independent of values of \( \alpha, \beta, \gamma \). This clearly indicates the fact that the local output states are separable. Thus with the help of the above protocol one can generate two three qubit W-type states.
from a W-type state:

(1) **One between Alice’s original qubit and cloned qubits of Bob and Carol.**
(2) **Another between original qubits of Bob and Carol with the cloned qubit of Alice.**

One can use these two secretly broadcasted three qubit W-states as secret quantum channels between three partners for various cryptographic schemes.

### 3 Conclusion:

In this work, we present a protocol for the secret broadcasting of three-qubit entangled state (W-type) between three distant partners. Here we should note an important fact that the two copies of three-qubit entangled state is generated from previously shared three-qubit entangled state independent of the input parameters $\alpha, \beta, \gamma$. They send their measurement result secretly using cryptographic scheme so that the produced copies of the three-qubit entangled state shared between three distant parties can serve as a secret quantum channel. Another important thing is that instead of applying (B-H) cloning machine for twice, as in reference [16] here three parties applied a different cloning transformation. Now these three parties can use these newly broadcasted W-type states as quantum channels more securely than any three qubit entangled states.

### 4 Acknowledgement

I.C acknowledges almighty God for being the source of inspiration of all work. He also acknowledges Prof C.G.Chakraborti for being the source of inspiration in research work. I.C also N.Ganguly for having useful discussions.

### 5 References

[1] W.K.Wootters, W.H.Zurek, Nature 299 (1982) 802.
[2] V.Buzek, M.Hillery, Phys.Rev.A 54 (1996) 1844.
[3] N.Gisin, S.Massar, Phys.Rev.Lett. 79 (1997) 2153.
[4] H. P. Yuen, Phys. Lett. A 113, 405 (1986).
[5] D.Bruss, D.P.DiVincenzo, A.Ekert, C.A.Fuchs, C.Macchiavello, J.A.Smolin,Phys.Rev.A 57 (1998) 2368.
[6] V.Buzek, S.L.Braunstein, M.Hillery, D.Bruss, Phys.Rev.A 56 (1997) 3446.
[7] L.M.Duan and G.C.Guo,Phys.Rev.Lett. 80 (1998)4999.
[8] Einstein, Podolsky and Rosen, Phys.Rev. 47 (1935)777.
[9] C.H.Bennett and G.Brassard, Proceedings of IEEE International Conference on Computers, System and Signal Processing, Bangalore, India, 1984, pp.175-179.
[10] P.W.Shor and J.Preskill, Phys.Rev.Lett. 85 (2000)441.
[11] C.H.Bennett and S.J.Weisner, Phys.Rev.Lett.69 (1992)2881.
[12] C.H.Bennett, G.Brassard, C.Crepeau, R.Jozsa, A.Peres and W.K.Wootters,Phys.Rev.Lett. 70 (1993)1895.
[13] S.Bandyopadhyay, G.Kar, Phys.Rev.A 60 (1999)3296.
[14] I.Ghiu, Phys.Rev.A 67 (2003)012323.
[15] S.Adhikari, B.S.Choudhury and I.Chakraborty, J. Phys. A: Math. Gen. 39 No 26 (2006)8439.
[16] S.Adhikari, B.S.Choudhury, Phys. Rev. A74, (2006)032323.
[17] Satyabrata Adhikari, A. S. Majumdar, N. Nayak, arXiv:0708.1869.
[18] I.Chakraborty (in preparation).
[19] P.W.Shor and J.Preskill, Phys.Rev.Lett. 85 (2000)441.
[20] L.Goldenberg and L.Vaidman, Phys.Rev.Lett. 75 (1995)1239.
[21] A.Peres, Phys.Rev.Lett. 77 (1996)1413.
[22] M.Horodecki, P.Horodecki, R.Horodecki, Phys.Lett.A 223. (1996)1.
[23] A.Cabello, Phys.Rev.A 61 (2000)052312.
[24] C.Li, H-S Song and L.Zhou, Journal of Optics B: Quantum semiclass. opt. 5 (2003)155.
[25] V.Buzek, V.Vedral, M.B.Plenio, P.L.Knight, M.Hillery, Phys.Rev.A 55 (1997)3327.