Analyses of Concentration and Wavelength Dependent Refractive Index of Sugar Solution Using Sellmeier Equation

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Abstract. The concentration and wavelength-dependent refractive index of sugar solution have been determined using Sellmeier Equation. The equations describe the refractive index as a function of wavelength parameter. They could be generalized as a function of material concentration by investigating the characteristic of their Sellmeier constants A and B. The three wavelengths used to identify the refractive index of sugar solutions were 455 nm, 525 nm, and 633 nm, while the concentration of the sugar solution ranged from 0 to 40%. This paper reported in this research performed the empirical expression of concentration-dependent of the sugar solution. The A and B Sellmeier constants were the main subjects to be concerned. A constant has a linear relationship with the sugar solution at 15% concentration to 40%. Under a concentration of 15%, the refractive index is quadratic towards engagement. The sellmeier B constant has a quadratic relation characteristic below the attention of 15%. Above 15%, the constant B and concentration of sugar solution were associated with the 4th order polynomial equation.

1. Introduction

The refractive index has been widely applied in the industrial sector. One of the uses is the measurement of the refractive index to determine the composition of a solution. One example is the determination of the characteristics of glass and plastic materials. Therefore, the measurement of the refractive index of a material is widely developed in various studies and industries to determine the solution's concentration. Some of the methods commonly used in measuring the refractive index are very diverse. Each technique is adapted to the application applied. Some refractive index measurement methods include the prism spectrometer in the phenomenon of polychromatic light deviation angle [1, 2], Fraunhofer diffraction method [3], and a measurement method based on Michelson Interferometry [4]. For more effective and efficient purposes, the refractive index measurement uses a refractometer at variations in the liquid measured concentration.

Several factors that influence the refractive index value include the concentration [5], temperature, pressure, and wavelength of the light beam used [6-9]. The dependence of the refractive index of water on temperature and wavelength had observed with a simple approach [7]. The effect of concentration and temperature on the refractive index of electrolyte solutions, polar solutions, nonpolar solutions, and protein solutions had also been reported [9]. Various measurement techniques have also been developed to determine sugar concentration [10-13]. Improved retroreflection methods have also
applicable to measuring the refractive index [14]. Determination of sugar cane sucrose has been described using multidimensional statistical analysis, based on their mid-infrared attenuated total reflectance spectra [15]. Each method of measurement should be followed by its application [16]. The characteristics of refractive index dependence on the concentration and wavelength of sugar solution have also been well described using the prism spectrometer method [17].

The measured refractive index value is affected by the concentration of solution [18] and also the wavelength of the utilized light source [2, 8, 11, 12]. Researchers develop several methods to measure sugar solution concentration, such as using a capacitive sensor [14, 19, 20], using density, polarimetry, refractometry, and ultraviolet and infrared spectroscopy [16, 21, 22]. In addition, the measurement of the refractive index is related to the temperature variations of the sugar solution [2, 8, 14, 16, 23]. Concentration mapping of sugar, which represents the refractive index, has been studied using ultrasonic waves [24]. Moreover, the index refractive of the liquid has also been related to its impurities [25].

The measurement of sugar solution concentration described above is still using expensive devices which are not provided in the local area. Thus this research aims to develop a sugar concentration measurement instrument at a low cost. The light source, i.e., red and green laser pointer, and the existing laser He-Ne in the physics laboratory. The applied method is refractometry. The measurement result is then used to explain the relationship between the refractive index of sugar solution and laser source wavelength by using Sellmeier equation.

On an industrial scale, for the purposes of measuring ease, effectiveness, and accuracy, a refractometer is more widely applied. Therefore, in this study will be conveyed the use of these methods in describing the concentration and wavelength-dependent the refractive index of the sugar solution. All characteristics of the refractive index towards the two parameters are then mathematically modeled using the Sellmeier equation [26]. The Sellmeier equation is an equation in the form of a mathematical relationship between wavelengths and the refractive index of a material. The constants of this equation have determined the purity of olive oil [21]. Thus, the entire dependence of the refractive index of the sugar solution on the concentration of the material and the wavelength of the light beam used is sufficiently analyzed using the Sellmeier equation.

2. Research Method
The parameters measured in this study are the refractive index value for every concentration of sugar solutions. The refraction index is measured by a digital refractometer with a laser beam in three different wavelengths as a light source. The setup of equipment for the measurement is shown in Figure 1 below.

![Figure 1. The set up of equipment for refractive index measurement](image)

There are two major steps conducted in this research that are refractive index measurement and calculation of two constant values in the Sellmeier equations.

2.1. Refractive Index as a Function of Sugar Solution Concentration
The measurement of the refractive index of sugar solution was carried out at various concentrations (m / m%) of 0%, 5%, 10%, 15%, 20%, 25%, 30%, 35% and 40%. The sugar solution is obtained by mixing some sugar into the aquadest. For example, a 5% sugar solution is resulted by dissolving 5 grams of sugar into100 grams of aquadest. The Refractive index measurement was carried out at a temperature of 28°C. The results obtained are in the form of a graph of the refractive index and concentration for three kinds of laser beams. In this research, the refractive index of the sugar solution...
was analyzed using three laser beams, which had three specific wavelengths i.e., 455 nm, 525 nm, and 633 nm, respectively. The refractive index measurement is carried out using a digital refractometer.

2.2. Sellmeier Equation of Sugar Solution
For each concentration of sugar solution, three data sets of a refractive index will be obtained according to three different laser beam wavelengths ($\lambda$, $n$). The three data sets are then written in the Sellmeier equation shown by equation (1).

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - B}$$  \hspace{1cm} (1)

The combination of the three Sellmeier equations is then used to determine Sellmeier constants $A$ and $B$. Both of these constants are obtained by solving those three linear equations simultaneously. The analysis had conducted in order to determine the relationship between the refractive index and sugar concentration and also between incident wavelength and material concentration with the refractive index.

3. RESULTS AND DISCUSSION
By using three different wavelengths as a monochromatic light source, the refractive index was obtained as a function of the concentration of sugar solution. The characteristics of the refractive index curve for each wavelength are shown in Figure 2.

![Figure 2. Refractive index as a function of the concentration of sugar solution for laser beams 455 nm, 525 nm, and 633 nm](image)

It is shown that the refractive index of the sugar solution changes linearly to the concentration of sugar for three types of light, i.e, the laser 455 nm, 525 nm, and 633 nm. This characteristic seems similar to other research shows that the refractive index of sucrose performed a linear relationship with concentration [18].

The linear equation produced for each wavelength is shown in equation (2-4).

$$n_{\lambda=455\text{nm}} = 0.0014c + 1.3270 \hspace{1cm} (2)$$
$$n_{\lambda=525\text{nm}} = 0.0013c + 1.3312 \hspace{1cm} (3)$$
$$n_{\lambda=633\text{nm}} = 0.0013c + 1.3310 \hspace{1cm} (4)$$
Based on Figure 2 and supported by equations (3) and (4), the line slope has the same number. The red laser wavelength in the area does not provide a significant difference in the refractive index. The difference starts when the laser used has a smaller wavelength. Equation (2) shows a shift in the graph slope. The slope shift in the graph, and the resulting equation shows the characteristics of the refractive index dependence on the concentration of sugar solution. Increasing the wavelength will cause the slope of the graph to shift smaller [17]. Hence, the refractive index data produced in this study has good suitability with the results of the Belay and Assefa [17], as shown in Table 1.

Table 1. Table of Refractive Index Sugar Solutions in Various Concentrations for wavelengths of 455 nm, 525 nm and 633 nm at Room Temperature (28°C)

| Concentration (%) | n_λ=455 nm | n_λ=525 nm | n_λ=633 nm | n^*_λ=633 nm |
|-------------------|------------|------------|------------|--------------|
| 0                 | 1.3322     | 1.3320     | 1.3318     | 1.3282       |
| 5                 | 1.3379     | 1.3377     | 1.3375     | 1.3380       |
| 10                | 1.3424     | 1.3421     | 1.3419     | 1.3465       |
| 15                | 1.3521     | 1.3516     | 1.3512     | 1.3558       |
| 20                | 1.3583     | 1.3579     | 1.3576     | 1.3602       |
| 25                | 1.3645     | 1.3642     | 1.3639     | 1.3688       |
| 30                | 1.3715     | 1.3712     | 1.3709     | 1.3770       |
| 35                | 1.3753     | 1.3748     | 1.3745     | 1.3848       |
| 40                | 1.3849     | 1.3844     | 1.3838     | 1.3948       |

n^* (Belay dan Assefa, 2018)

The nature of dispersion in light when passing through material causes a change in the refractive index of the material. Therefore, the refractive index changes with the nature of the incident light, according to their wavelength. In electromagnetic theory, when a beam of light, which is an electromagnetic wave, concerning the atoms or molecules of matter, the bond between the charge in the material vibrates. The vibration produced has the same frequency as the incident wave due to a resonance event in the material [5]. The different concentration of sugar solution defines changes in the material that stimulate changes in the vibrational frequency of the atomic bonds in the material. Empirically, the characteristic changes in the refractive index to the coming wavelength are indicated by the Sellmeier equation.

The two main parameters in the Sellmeier equation, which have a substantial effect in determining the refractive index for the variation in the concentration of sugar solution, are A and B Sellmeier constants. Table 2 shows the Sellmeier constants A and B at various sugar concentrations.

Table 2. Sellmeier constants A and B at various sugar concentration

| Concentration (%) | A          | B          |
|-------------------|------------|------------|
| 0                 | 0.773±0.000 | 588.574±18.057 |
| 5                 | 0.788±0.000 | 578.714±16.255 |
| 10                | 0.000±0.000 | -5.597± 0.000 |
| 15                | 0.800±0.000 | 1252.100±99.767 |
| 20                | 0.827±0.024 | 939.530±80.188 |
| 25                | 0.859±0.000 | 812.354±25.081 |
| 30                | 0.878±0.000 | 798.946±24.465 |
| 35                | 0.896±0.007 | 1049.252±230.161 |
| 40                | 0.912±0.000 | 1249.425±0.000 |
The relationship between Sellmeier constant A and the concentration of sugar had been explained by the graph shown in Figure 3. The constant ranged from 0.773 until 0.912, where the concentration varied from 5 to 40%. The Figure described the two specific areas. The Sellmeier constant A changes linearly when the concentration was varied at (15-40)%, while at concentration (5-15)%, constant A forms a quadratic equation.

![Figure 3](image-url)

**Figure 3.** Sellmeier constant A versus concentrations of sugar solution using three sources of light at wavelength 455 nm, 525 nm, and 633 nm

The best-fitting results for their relationships are shown in equations 5 and 6.

\[
A = 0.0318 c^2 - 0.6339 c + 3.1634, \quad 5\% \leq c \leq 15\% \quad (5)
\]

\[
B = 0.0045 c - 0.7378, \quad 15\% \leq c \leq 40\% \quad (6)
\]

The characteristics of the change in Sellmeier constant B when the sugar concentration varies from 5% to 40% have been described in Figure 4.

![Figure 4](image-url)

**Figure 4.** Sellmeier constant of B versus concentrations of sugar solution using three sources of light at wavelength 455 nm, 525 nm, and 633 nm
At sugar solution concentrations of 5% - 15%, Sellmeier constants B performed quadratic expressions of sugar concentration. In contrast, above 15%, the relationship between the two parameters is expressed in fourth-order polynomial curves. Equations 7 and 8 show the best fittings of the two intervals shown in the graph.

\[
\begin{align*}
B &= 36.84c^2 - 66.46c + 3005, \quad 5\% \leq c \leq 15\% \\
B &= -0.0081c^4 + 0.8342c^3 - 27.909c^2 + 322.52c + 284.36, \quad 15\% \leq c \leq 40\%
\end{align*}
\]  

(7)  

(8)  

The characteristics of the refractive index dependence on concentration and wavelength expressed above will be analyzed in a sugar solution with a concentration of 30%. Figure 4 shows the refractive index of a sugar solution at a concentration of 30%. In the picture, the experimental results are represented by point labels, while the Sellmeier equation is labeled by a solid line.

![Figure 5. Refractive index as wavelength function of sugar solution of 30% concentration at room temperature: a) point label refers to experimental data b) solid line represents Sellmeier equation.](image)

The Sellmeier equation for a sugar solution with a concentration of 30% is mathematically expressed by equation 9.

\[
n^2 = 1 + \frac{0.8776 \lambda^2}{\lambda^2 - 798.9457}
\]  

(9)  

The parameter \(n\) is the refractive index, and \(\lambda\) is the wavelength of light, which experiences dispersion in the material. In this case, the value of \(\lambda\) used is the wavelength of light when in a vacuum [19]. A specific number for the Sellmeier constant obtained in this research could be applied to identify the sucrose content for other solutions. The result obtained in this research could be applied to identify the concentration of the sugar solution. This method could be proposed as a preview report in determining sucrose or sugar content of other commercial beverages, whereas some methods have explained using their density [13].

4. Conclusion
The sugar solution concentration and the wavelength of the incident light affect the refractive index of sugar solution. It is linear to the variation in the concentration of sugar solution in the three observed wavelengths (455 nm, 525 nm, and 633nm). Both of these parameters are well explained in the Sellmeier equation. The Sellmeier constant A changes linearly to the concentration of sugar solution in the interval region (15-40)%. The Sellmeier constant B oscillates in all regions of the concentration of
sugar solution, forming quadratic equations at concentrations (5-15)% and in the form of fourth-order polynomial at 15-40% intervals.

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