The $\rho \gamma^* \rightarrow \pi(\rho)$ transition form factors in the Perturbative QCD factorization approach

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In this paper, we studied the $\rho \gamma^* \rightarrow \pi$ and $\rho \gamma^* \rightarrow \rho$ transition processes and made the calculations for the $\rho \pi$ transition form factor $Q^4 F_{\rho \pi}(Q^2)$ and the $\rho$ meson electromagnetic form factors, $F_{LL,LT,TT}(Q^2)$ and $F_{1,2,3}(Q^2)$, by employing the perturbative QCD (PQCD) factorization approach. For the $\rho \gamma^* \rightarrow \pi$ transition, we found that the contribution to form factor $Q^4 F_{\rho \pi}(Q)$ from the term proportional to the distribution amplitude combination $\phi_\rho^T(x_1)\phi_\pi^P(x_2)$ is absolutely dominant, and the PQCD predictions for both the size and the $Q^2$-dependence of this form factor $Q^4 F_{\rho \pi}(Q^2)$ agree well with those from the extended ADS/QCD models or the light-cone QCD sum rule. For the $\rho \gamma^* \rightarrow \rho$ transition and in the region of $Q^2 \geq 3$ GeV$^2$, further more, we found that the PQCD predictions for the magnitude and their $Q^2$-dependence of the $F_1(Q^2)$ and $F_2(Q^2)$ form factors agree well with those from the QCD sum rule, while the PQCD prediction for $F_3(Q^2)$ is much larger than the one from the QCD sum rule.

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I. INTRODUCTION

During the past years, due to its very important role in understanding the hadron structure, the various hadron form factors have been studied intensively for example in Refs. [1–3]. The transition and the electromagnetic form factors of the pseudoscalar mesons, especially the pion meson as the lightest QCD bound state, attracted the most attentions theoretically [4–16] and experimentally[17–19]. The transition form factors between the pseudoscalar and vector mesons are also investigated by employing rather different approaches [20–25], the resulted theoretical predictions however are self-consistent and comparable from each other. The radiative form factors of vector meson, such as the $\rho$ meson, also draw some interests [26, 27].

The $k_T$ factorization theorem[28, 29] is one of the major factorization approaches based on the factorization hypothesis[30–32] and the resummation image in the end-point region[33, 34]. Due to it’s clear advantages, such as no end-point singularity and can provide a large strong phase to generate the sizable CP violation for B meson decays, the PQCD factorization approach based

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on the $k_T$ factorization theorem has been widely used to study the two-body hadronic decays of $B/B_s/B_C$ mesons for example in Refs. [35–44]. Recently, the next-to-leading-order (NLO) corrections to some important hadron form factors have been calculated [15, 45–47]. With the inclusion of these NLO corrections, the PQCD predictions for the heavy-to-light pseudoscalar form factors in $B \to \pi$ transition, for example, become well consistent with those from the lightcone QCD sum rule [48–51]. In this paper, we will study the $\rho\pi$ transition from factor and the $\rho$ meson electromagnetic form factor by employing the pQCD factorization approach.

This paper is organized as follows. The relevant kinetics and the meson wave functions are introduced in Sec. II. In Sec. III, the pseudoscalar and vector transition form factors corresponding to the process $\rho\gamma^* \to \pi$ are studied with the use of the two-parton meson wave functions up to sub-leading twist. In Sec. IV, the $\rho$ meson electromagnetic form factors in $\rho\gamma^* \to \rho$ process are calculated. Sec. V contains the summary of this paper.

II. KINETICS AND INPUT WAVE FUNCTIONS

In this section, we first consider the relevant kinetics and the input meson wave functions to be used in our calculation. The leading-order (LO) topological diagrams for the meson transition or electromagnetic form factors are presented in Fig. 1. Of course, the fully diagrams should also include the other two sub-diagrams with the photon vertex being on the lower anti-quark lines. Because of the isospin symmetry, these extra two sub-diagrams have same structure as Fig. 1(a,b) with exchanging quark and anti-quark in the initial and final mesons, the only difference may be appeared is the quark charge.

Under the light-cone (LC) coordinate, the momentum for initial and final mesons in Fig. 1 are defined as,

$$P_1 = \frac{Q}{\sqrt{2}}(1, r_{\pi/\rho}^2, 0_T), \quad P_2 = \frac{Q}{\sqrt{2}}(r_{\pi/\rho}^2, 1, 0_T);$$

$$k_1 = (x_1 \frac{Q}{\sqrt{2}}, 0, k_{1T}), \quad k_2 = (0, x_2 \frac{Q}{\sqrt{2}}, k_{2T}),$$

(1)

where $k_i$, $i = 1, 2$ is the momentum carried by the anti-quark for initial or final meson with the momentum fraction $x_i$, and $k_{iT}$ represent the corresponding transversal momentum. The dimensionless parameter $\gamma_{\pi}^2 \equiv M_{\pi}^2/Q^2$, $\gamma_{\rho}^2 \equiv M_{\rho}^2/Q^2$. Then the momentum transfer squared is $q^2 = (P_1 - P_2)^2 = -Q^2$. The polarization vectors of the initial and final $\rho$ mesons are also defined.
in the LC coordinate:

\[ \epsilon_{1\mu}(L) = \frac{1}{\sqrt{2}2\gamma_\rho}(1, -\gamma_\rho^2, 0_T), \quad \epsilon_{1\mu}(T) = (0, 0, 1_T); \]

\[ \epsilon_{2\mu}(L) = \frac{1}{\sqrt{2}2\gamma_\rho}(-\gamma_\rho^2, 1, 0_T), \quad \epsilon_{2\mu}(T) = (0, 0, 1_T). \]  \( \tag{2} \)

with \( \rho \) meson mass \( M_\rho = 0.77 \) GeV. The LC definitions in Eq. (1,2) satisfy the relations,

\[ P_1 \cdot \epsilon_1 = 0, \quad P_2 \cdot \epsilon_2 = 0; \quad \epsilon_1^2 = \epsilon_2^2 = -1. \]  \( \tag{3} \)

As elaborated in Ref. [37], the power-suppressed three-parton contribution to the pion electromagnetic form factor in the \( k_T \) factorization theorem can only provide \( \sim 5\% \) correction to the LO form factor in the whole range of experimentally accessible momentum transfer squared. This sub-leading piece amount only up to few percents of the \( B \to \pi \) transition form factor at large recoil of the pion in \( k_T \) factorization theorem. So in our calculation for the \( \rho \pi \) transition form factor and \( \rho \) meson electromagnetic form factors, we can just consider the dominant contributions from the two-parton meson wave functions and neglect the very small three-parton part safely. The initial wave functions for transversal and longitudinal polarized \( \rho \) meson can be written as,

\[ \Phi_\rho(P_1, \epsilon_1(L)) = \frac{1}{\sqrt{6}} \left[ \ell_1(L) M_\rho \phi_\rho(x_1) + \ell_1(L) P_1 \phi_\rho^T(x_1) + M_\rho \phi_\rho^b(x_1) \right], \]  \( \tag{4} \)

\[ \Phi_\rho(P_1, \epsilon_1(T)) = \frac{1}{\sqrt{6}} \left[ \ell_1(T) P_1 \phi_\rho^T(x_1) + \ell_1(T) M_\rho \phi_\rho^b(x_1) + i M_\rho \epsilon_{\mu
u\rho\sigma} \gamma^\nu \gamma^\rho \epsilon_1^\nu(T) n^\rho \sigma \phi_\rho^b(x_1) \right], \]  \( \tag{5} \)

The final pion or rho meson wave function are written as,

\[ \Phi_\pi(P_1) = \frac{1}{\sqrt{6}} \left[ \gamma_5 P_2 \phi_\pi^A(x_2) + \gamma_5 m_0^5 \phi_\pi^P(x_2) - \gamma_5 m_0^5 (\phi_\pi^a - 1) \phi_\pi^T(x_2) \right]; \]  \( \tag{6} \)

\[ \Phi_\rho(P_2, \epsilon_2(L)) = \frac{1}{\sqrt{6}} \left[ \ell_2(L) M_\rho \phi_\rho(x_2) + \ell_2(L) P_2 \phi_\rho^T(x_2) + M_\rho \phi_\rho^b(x_2) \right], \]  \( \tag{7} \)

\[ \Phi_\rho(P_2, \epsilon_2(T)) = \frac{1}{\sqrt{6}} \left[ \ell_2(T) P_2 \phi_\rho^T(x_2) + \ell_2(T) M_\rho \phi_\rho^b(x_2) + i M_\rho \epsilon_{\mu
u\rho\sigma} \gamma^\nu \gamma^\rho \epsilon_2^\nu(T) n^\rho \sigma \phi_\rho^b(x_2) \right]. \]  \( \tag{8} \)

Here \( \phi_\pi^A, \phi_\rho^T \) and \( \phi_\rho \) are the leading twist-2 (T2) distribution amplitudes(DAs), while \( \phi_\pi^{P,T}, \phi_\rho^{\nu,a} \) and \( \phi_\rho^{T,s} \) are the sub-leading twist-3 (T3) DAs which are power suppressed by \( m_0/\rho \) and \( M_\rho/Q \).

And \( m_0 \) is the the chiral mass of pion meson.

The pion meson DAs with the inclusion of the high order effects as given in Ref. [52] are adopted in our numerical calculation:

\[ \phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}} x(1 - x) \left[ 1 + a_2^\pi C_2^3(t) + a_4^\pi C_4^3(t) \right], \]

\[ \phi_\pi^{P,T}(x) = \frac{f_\pi}{2\sqrt{6}} \left[ 1 + \left( 30 \eta_3 - \frac{5}{2} \rho_\pi^2 \right) C_2^1(t) - 3 \left( \eta_3 \omega_3 + \frac{9}{20} \rho_\pi^2 (1 + 6 a_2^\pi) \right) C_4^3(t) \right], \]

\[ \phi_\pi^{T,s}(x) = \frac{f_\pi}{2\sqrt{6}} (1 - 2x) \left[ 1 + 6 \left( 5 \eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho_\pi^2 - \frac{3}{5} \rho_\pi^2 a_2^\pi \right) (1 - 10x + 10x^2) \right]. \]  \( \tag{9} \)
where the new Gegenbauer moments $a_i^2$, the parameters $\eta_3, \omega_3$ and $\rho_\pi$ are adopted as[51]:

$$a_2^\pi = 0.16, \quad a_4^\pi = 0.04, \quad \rho_\pi = m_\pi/m_0, \quad \eta_3 = 0.015, \quad \omega_3 = -3.0,$$

(10)

with $f_\pi = 0.13$ GeV, $m_\pi = 0.13$ GeV, $m^0_\pi = 1.74$ GeV.

For the rho meson, the following twist-2 DAs ($\phi_\rho$ and $\phi_\rho^T$) and twist-3 DAs ($\phi_\rho^{v,a,t,s}$) will be used in our numerical calculation:

$$\phi_\rho(x) = \frac{3f_\rho}{\sqrt{6}}x(1-x)[1 + a_{2\rho}^\parallel C_2^{3/2}(t)],$$

$$\phi_\rho^T(x) = \frac{3f_\rho^T}{\sqrt{6}}x(1-x)[1 + a_{2\rho}^\perp C_2^{3/2}(t)],$$

(11)

$$\phi_\rho^{v}(x) = \frac{f_\rho}{2\sqrt{6}}[0.75(1+t^2) + 0.24(3t^2 - 1) + 0.12(3 - 30t^2 + 35t^4)],$$

$$\phi_\rho^{a}(x) = \frac{4f_\rho}{4\sqrt{6}}(1 - 2x)[1 + 0.93(10x^2 - 10x + 1)];$$

$$\phi_\rho^{t}(x) = \frac{f_\rho^T}{2\sqrt{6}} [3t^2 + 0.3t^2(5t^2 - 3) + 0.21(3 - 30t^2 + 35t^4)],$$

$$\phi_\rho^{s}(x) = \frac{3f_\rho^T}{2\sqrt{6}}(1 - 2x) \left[1 + 0.76 \left(10x^2 - 10x + 1\right)\right],$$

(12)

where $t = 2x - 1$, the Gegenbauer moments $a_{2\rho}^\parallel = 0.18, a_{2\rho}^\perp = 0.2$ and the decay constants $f_\rho = 0.209, f_\rho^T = 0.165$. The Gegenbauer polynomials in Eqs. (11,12) are of the following form:

$$C_2^{1/2}(t) = \frac{1}{2} [3t^2 - 1], \quad C_2^{3/2}(t) = \frac{3}{2} [5t^2 - 1],$$

$$C_4^{1/2}(t) = \frac{1}{8} [3 - 30t^2 + 35t^4], \quad C_4^{3/2}(t) = \frac{15}{8} [1 - 14t^2 + 21t^4].$$

(13)

### III. THE $\rho\pi$ TRANSITION FORM FACTOR $Q^4F_{\rho\pi}(Q)$

We firstly consider the $\rho \gamma^* \rightarrow \pi$ transition, here the two sub-diagrams Fig. 1(a,b) will contribute to the $\rho\pi$ transition form factor. The final state is a pseudoscalar pion meson, which can’t be generated by a scalar current, so only the transversal polarized initial $\rho$ meson with the vector current $J_{\mu,|\lambda|=1}$ contributes to $\rho\pi$ transition form factor, which can be written as,

$$< \pi(P_2)|J_\mu|\rho(P_1,\epsilon_1)> = < \pi(P_2)|J_{\mu,|\lambda|=1}|\rho(P_1,\epsilon_1(T))> = F_{\rho\pi}(Q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{\nu}(T)n^\rho v^\sigma p_1^+ p_2^-.$$

(14)

For the case of the large momentum transfer, the asymptotic behavior of the hadron form factors is the form of [31]

$$<P_2,\lambda_2|J_\lambda(0)|P_1,\lambda_1> \sim \left(\frac{1}{\sqrt{|q^2|}}\right)^{|\lambda_1-\lambda_2|+2n-3}.$$  

(15)

The $\rho\pi$ transition amplitude is suppressed by $1/Q^2$ because of the helicity flipping at the vector vertex for the quark lines ($|\lambda| = 1$). In Eq. (15), $\lambda_1$ and $\lambda_2$ denote the helicity on the z-axis and $n$ is the parton number of hadron: when the hadron is a meson, $n = 2$. 


From Eqs. (14,15) one can see that the \( \rho \pi \) transition form factor \( F_{\rho \pi}(Q^2) \) has the asymptotic behaviour \( [Q]^{-4} \) at the limit of large transfer momenta, so one should study the dimensionless form factor \( Q^4 F_{\rho \pi}(Q^2) \) rather than \( F_{\rho \pi}(Q^2) \) itself. After the inclusion of the contributions from all sub-diagrams, Fig. (1)(a,b) and their partner diagrams with the vertexes on the lower anti-quark lines, the vector and pseudoscalar \( \rho \pi \) transition hard kernel can be written in the following form:

\[
Q^4 H_{\rho \pi}(Q; x_1, x_2; k_1 T, k_2 T) = \frac{16\pi\alpha_s}{3} \left\{ \frac{M_\rho \left[ \frac{\phi_0^a(x_1) - \phi_0^\rho(x_1)}{P_1 - k_2} \right] \phi_\pi^A(x_1)}{(P_1 - k_2)^2(k_1 - k_2)^2} + \frac{x_1 M_\rho [\phi_0^a(x_1) - \phi_0^\rho(x_1)] \phi_\pi^A(x_1)}{(P_2 - k_1)^2(k_1 - k_2)^2} \right\}.
\]

(16)

By integrating over the longitudinal momentum fractions \( (x_1, x_2) \) and the transversal momentum in it’s conjugate coordinate spaces \( (b_1, b_2) \), we can obtain the \( \rho \pi \) transition form factor:

\[
Q^4 F_{\rho \pi}(Q) = \frac{16\pi}{3} Q^4 \cdot \alpha_s(\mu) \cdot \int_{0}^{1} dx_1 dx_2 \int_{0}^{\infty} b_1 db_1 b_2 db_2 \cdot \exp[-S(x_1; b_1; Q; \mu)] \cdot \left\{ M_\rho \left[ \frac{\phi_0^a(x_1) - \phi_0^\rho(x_1)}{x_1 M_\rho} \right] \cdot \phi_\pi^A(x_1) \cdot h(x_2, x_1, b_1, b_2) \right\}
\]

(17)

where the \( k_T \) resummation Sudakov factor \( S(x_1; b_1; Q; \mu) \) and the threshold resummation function \( S_t(x) \) are the same ones as being used in Refs. [15, 38, 47]. In numerical calculation we choose \( c = 0.4 \) in the function \( S_t(x) \). The hard function \( h(x_1, x_2, b_1, b_2) \) in Eq. (17) can be written as the following form:

\[
h(x_1, x_2, b_1, b_2) = K_0 \left( \sqrt{x_1 x_2 Q} b_2 \right) \left[ \theta(b_1 - b_2) I_0 \left( \sqrt{x_1 Q} b_1 \right) K_0 \left( \sqrt{x_1 Q} b_2 \right) + (b_1 \leftrightarrow b_2) \right].
\]

(18)

where the function \( K_0 \) and \( I_0 \) are the modified Bessel function. Following Refs. [15, 38, 47], we here also choose \( \mu = \mu_f = t = \max (\sqrt{x_1 Q}, \sqrt{x_2 Q}, 1/b_1, 1/b_2) \).

(19)

Based on the formula in Eq. (17), we calculate and show the PQCD predictions for the \( Q^2 \)-dependence of the \( \rho \pi \) transition form factor \( Q^4 F_{\rho \pi}(Q) \) in Fig. 2. In Fig. 2(a), the dashed-curve shows the contribution from the first term of Eq. (17), corresponding to the T3&T2 product term from Fig. 1(a) and its partner with the virtual vertex being on the lower anti-quark line; while the dot-dashed and dots curve shows the contribution from the second and the third term of Eq. (17), coming from the Fig. 1(b) and its partner. The solid curve in Fig. 2(a) refers to the total contribution. In Fig. 2(b), the dashed curve shows the theoretical prediction based on the ADS/QCD theory [24], while the dark-region shows the theoretical predictions from the light-cone QCD sum rules [9, 20, 22]. The PQCD prediction (the solid curve in Fig.2(b)) is drawn here as a comparison.

From the curves in Fig. 2, one can see the following points:

1 The third term in Eq. (17) with the DAs combination \( \phi_T^a(x_1) \phi_T^\rho(x_2) \) provide the absolutely dominant contribution. The first term describe the contribution from Fig. 1(a) and is very small in size.

2 The PQCD predictions for both the magnitude and the \( Q^2 \)-dependence of the \( \rho \pi \) transition form factor \( Q^4 F_{\rho \pi}(Q) \) agree well with the theoretical predictions obtained in the extended ADS/QCD models [24] or in the classical light-cone QCD sum rule [9, 20, 22].
FIG. 2. (a) the PQCD predictions for $Q^2$ dependence of the $\rho\pi$ transition form factor $Q^4 F_{\rho\pi}(Q^2)$; and (b) the theoretical predictions from ADS/QCD (dashed curve) or the light-cone QCD sum rules (dark region), and from the PQCD factorization approach (solid curve).

IV. $\rho$ MESON ELECTROMAGNETIC FORM FACTOR $F_i(Q^2)$

In this section we consider the $\rho\gamma^* \to \rho$ process, and calculate the $\rho$ meson electromagnetic form factors by employing the PQCD factorization approach. Because the initial and final states are the same $\rho$ meson, we only investigate Fig. 1(a) in detail, the contributions from the other three topological diagrams can be obtained from the exchanging symmetry [15, 47] from Fig. 1(a). The Lorentz invariant $\rho$ electromagnetic form factors $F_i(Q^2), i = 1, 2, 3$ in $\rho\gamma^* \to \rho$ process are defined as the following matrix element [2, 27],

$$< \rho(P_2, \epsilon_2^*) | J_{\mu, \lambda} | \rho(P_1, \epsilon_1) > = -\epsilon_{2\beta}\epsilon_{1\alpha}\left\{ \left[ (P_1 + P_2)_\mu g^{\alpha\beta} - P_2^\alpha g^{\beta\mu} - P_1^\beta g^{\alpha\mu} \right] \cdot F_1(Q^2) \right. \\
- \left. \left[ g^{\mu\alpha} P_1^\beta + g^{\mu\beta} P_2^\alpha \right] \cdot F_2(Q^2) + \frac{1}{M_\rho^2} P_2^\alpha P_1^\beta (P_1 + P_2)_\mu \cdot F_3(Q^2) \right\}. \tag{20}$$

From the asymptotic behavior as defined in Eq. (15), one can see that the transversal form factors are suppressed by $1/Q^2$ or $1/Q^3$:

$$< \rho(P_2, \lambda_2 = 0) | J_{\mu, |\lambda|=1} | \rho(P_1, |\lambda|=1) > \sim \frac{1}{Q^2}, \tag{21}$$
$$< \rho(P_2, \lambda_2 = \pm 1) | J_{\mu, |\lambda|=0} | \rho(P_1, \lambda_1 = \mp 1) > \sim \frac{1}{Q^3}, \tag{22}$$

while the asymptotic behavior of the longitudinal form factor is the normal one:

$$< \rho(P_2, \lambda_2 = 0) | J_{\mu, |\lambda|=0} | \rho(P_1, \lambda_1 = 0) > \sim \frac{1}{Q}. \tag{23}$$

The helicity of the vector current is defined as $\lambda = \lambda_1 + \lambda_2$, then the transition $\lambda_1 = \pm 1 \to \lambda_2 = \pm 1$ is forbidden because the helicity of vector current is $\lambda \leq 1$. The $\lambda_1 = \pm 1 \to \lambda_2 = 0$ transition in Eq. (21) requires the helicity flipping for one quark line at the vector vertex, which give a suppression $k_T/Q$; While the $\lambda_1 = \pm 1 \to \lambda_2 = \mp 1$ transition in Eq. (22) needs the helicity flipping for both quark lines at the vector vertices, which leads to a suppression $k_T^2/Q^2$.

From the radiative matrix element as defined in Eq. (20) and the asymptotic behavior of the hadron form factors as described in Eqs. (21,22,23), we get to know that:
(i) Only the electric form factor $F_1(Q^2)$ contributes to the transversal from factor $F_{TT}(Q^2)$ with $\lambda_1 = -\lambda_2 = \pm 1$;

(ii) Both the electric form factor $F_1(Q^2)$ and the magnetic form factor $F_2(Q^2)$ contribute to the semi-transversal form factor $F_{LT}$ with $\lambda_i = 1$ and $\lambda_j = 0 (i, j = 1, 2, i \neq j)$;

(iii) All $F_1(Q^2)$, $F_2(Q^2)$ and the quadruple form factor $F_3(Q^2)$ give the contribution to the longitudinal radiation form factor $F_{LL}$ with $\lambda_1 = \lambda_2 = 0$.

These form factors do satisfy the following relations by definition:

$$F_{TT}(Q^2) = F_1(Q^2),$$
$$F_{LT}(Q^2) = \frac{Q}{2M_\rho} [F_1(Q^2) + F_2(Q^2)],$$
$$F_{LL}(Q^2) = F_1(Q^2) - \frac{Q^2}{2M_\rho^2} F_2(Q^2) + \frac{Q^2}{M_\rho^2} \left(1 + \frac{Q^2}{4M_\rho^2}\right) F_3(Q^2). \quad (24)$$

By introducing the transversal momentum $k_T$ to cancel the end-point singularity, integrating over the longitudinal momentum fractions and transversal coordinate conjugated to transversal momentum, one finds the PQCD predictions for the $Q^2$ dependence of the $\rho$ meson electromagnetic form factors:

$$F_{LL}(Q^2) = \frac{32\pi C_F}{3} Q^2 \alpha_s(\mu) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \cdot \exp[-S(x_1; b_1; Q; \mu)] \times \left\{ [x_2 - \frac{1}{2} \gamma_\rho^2 (1 + x_2)] \phi_\rho(x_1) \phi_\rho(x_2) + \gamma_\rho^2 \phi_\rho^s(x_1) \phi_\rho^s(x_2) + 2 \gamma_\rho^2 (1 - x_2) \phi_\rho^s(x_1) \phi_\rho^s(x_2) \right\} \cdot h(x_2, x_1, b_1, b_2), \quad (25)$$

$$F_{LT}(Q^2) = \frac{32\pi C_F}{3} Q M_\rho \alpha_s(\mu) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \cdot \exp[-S(x_1; b_1; Q; \mu)] \times \left\{ \frac{1}{2} \left[ \phi_\rho^s(x_1) + \phi_\rho^s(x_1) \right] \phi_\rho(x_2) + \phi_\rho^s(x_1) \phi_\rho^s(x_2) \right\} \cdot h(x_2, x_1, b_1, b_2), \quad (26)$$

$$F_{TT}(Q^2) = \frac{32\pi C_F}{3} M_\rho^2 \alpha_s(\mu) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \cdot \exp[-S(x_1; b_1; Q; \mu)] \times \left\{ (1 - x_2) \left[ \phi_\rho^s(x_1) \phi_\rho^s(x_2) - \phi_\rho^s(x_1) \phi_\rho^s(x_2) \right] \right\} + (1 + x_2) \left[ \phi_\rho^s(x_1) \phi_\rho^s(x_2) - \phi_\rho^s(x_1) \phi_\rho^s(x_2) \right] \cdot h(x_2, x_1, b_1, b_2), \quad (27)$$

where $C_F = 4/3$, the Sudakov factor $S(x_1; b_1; Q; \mu)$ and the hard function $h(x_2, x_1, b_1, b_2, b_3)$ are the same one as those appeared in Eq. (17). With these form factors of different polarized initial and final states and the relations in Eq. (24), one can obtain easily the $Q^2$ dependence of Lorentz-invariant electric, magnetic and quadruple form factors $F_1(Q^2)$, $F_2(Q^2)$ and $F_3(Q^2)$.

The PQCD predictions for the $Q^2$-dependence of the $\rho$ meson electromagnetic form factors with different polarizations (i.e. $F_{LL}(Q^2)$, $F_{LT}(Q^2)$ and $F_{TT}(Q^2)$) are presented in Fig. 3, while the $Q^2$-dependence of the $\rho$ meson electric, magnetic and quadruple form factors with different Lorentz structures (i.e. $F_{1,2,3}(Q^2)$) are presented in Fig. 4. From these two figures, one can find the following points:
FIG. 3. The PQCD predictions for $Q^2$ dependence of the $\rho$ meson form factors $F_{\lambda_1\lambda_2}(Q^2)$ with different polarizations for initial and final states. The short dashed, dots and dot-dashed curve represents the form factor $F_{TT}(Q^2)$, $F_{LT}(Q^2)$ and $F_{LL}(Q^2)$, respectively.

(i) For the form factors with different polarizations, there exists an approximate relation: $F_{LT}(Q^2) \gtrsim F_{LL}(Q^2) \gg F_{TT}(Q^2)$. The asymptotic behavior displayed in Eq. (15) is partially violated for $F_{LT}(Q^2)$ and $F_{LL}(Q^2)$. This violation arose because the longitudinal form factor $F_{LL}$ has an additional suppression from $x_2$, although its asymptotic behavior has a $Q/M_\rho$ enhancement when compared to the semi-transversal form factor $F_{LT}$. And this violation is consistent with the light-cone sum rule result [2].

(ii) The value of the electric form factor $F_1(Q^2)$ is rather small, because it happens being equal to the heavily suppressed transversal form factor $F_{TT}(Q^2)$. In the region of $Q^2 \geq 3$ GeV$^2$, there exists also an approximate relation for the values of $F_{1,2,3}(Q^2)$: $F_2(Q^2) \gtrsim F_3(Q^2) \gtrsim F_1(Q^2)$. Our PQCD predictions for $F_1(Q^2)$ and $F_2(Q^2)$ form factors agree well with the QCD sum rule results [27] in the region $Q^2 \geq 3$ GeV$^2$. But the PQCD prediction for the quadruple form factor $F_3(Q^2)$ is much larger than the one from the QCD sum rule, the
hierarchy between $F_1(Q^2)$ and $F_3(Q^2)$ predicted in the QCD sum rule is therefore turn over here.

V. SUMMARY

By employing the pQCD factorization approach, we studied the $\rho\gamma^* \to \pi$ and the $\rho\gamma^* \to \rho$ transition processes and made the analytical and numerical evaluations for the $\rho\pi$ transition form factor $Q^4 F_{\rho\pi}(Q^2)$ and the $\rho$ meson electromagnetic form factors, $F_{LL,LT,TT}(Q^2)$ and $F_{1,2,3}(Q^2)$. We found the following results:

(i) For the $\rho\gamma^* \to \pi$ transition process, the contribution to the $\rho\pi$ transition form factor $Q^4 F_{\rho\pi}(Q)$ from the terms proportional to the DAs combination $\phi^T_\rho(x_1)\phi^P_\pi(x_2)$ is absolutely dominant, and the PQCD predictions for both the magnitude and the $Q^2$-dependence of this form factor agree well with those from the extended ADS/QCD models or from the light-cone QCD sum rule.

(ii) For the $\rho\gamma^* \to \rho$ transition process and in the region of $Q^2 \geq 3$GeV$^2$, we found that the PQCD predictions for the magnitude and their $Q^2$-dependence of the $F_1(Q^2)$ and $F_2(Q^2)$ form factors agree well with those from the QCD sum rule, while the PQCD prediction for the quadruple $F_3(Q^2)$ is much larger than the one from the QCD sum rule.

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