Pair Production and Vacuum Polarization of Vector Particles with Electric Dipole Moments and Anomalous Magnetic Moments

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Abstract

The matrix, 8-component Dirac-like form of $P$-odd equations for boson fields of spins 1 and 0 are obtained and the $GL(2, c)$ symmetry group of equations is derived. We found exact solutions of the field equation for vector particles with arbitrary electric and magnetic moments in external constant and uniform electromagnetic fields. The differential probability of pair production of vector particles with the EDM and AMM by an external constant and uniform electromagnetic field has been found using the exact solutions. We have calculated the imaginary and real parts of the electromagnetic field Lagrangian that takes into account the vacuum polarization of vector particles.

1 Introduction

Vector particles $W^\pm$, $Z^0$ play very important role as carriers of the weak interactions. The standard model of electroweak interactions (SM) which implies the Higgs mechanism of acquiring mass of vector particles is renormalizable. Renormalizable electrodynamics for massive charged vector bosons is based in the framework of the SM on the spontaneous breaking of the local $SU(2)_L \otimes U(1)$ - symmetry. The $U(1)$ subgroup is unbroken and the corresponding gauge electromagnetic field remains massless. At the same time the gauge fields which are identified with vector intermediate bosons ($W^\pm$, $Z^0$) corresponding to broken $SU(2)_L$ - subgroup acquire masses. There is a certain symmetry of the vector electromagnetic vertices in the renormalizable SM and as a result the gyromagnetic ratio for vector particles is equal to two. It should be noticed that for the nonrenormalizable case of the Proca Lagrangian the gyromagnetic ratio $g = 1$. So, if an anomalous magnetic moment (AMM) of vector particles is observed, which corresponds to $g \neq 2$, that would signal physics beyond the SM.

The $CP$ violation observed in the decays of the $K^0$ mesons and in $B^0_d/\bar{B}^0_d \to J/\psi K^0_s$ decays remains mysterious. In the SM $CP$ - violating interactions can be explained by the Kobayashi-Maskawa mechanism which supposes single phase for three quark generations. In this scheme the predicted electric dipole moments (EDM's) of elementary particles are extremely small. In some supersymmetric and multi-Higgs models which are extensions of the SM, $CP$ - violating effects are much stronger [1]. The EDM of particles violates the
time-reversal (T) symmetry and the CP invariance which are equivalent due to the CPT theorem [2]. Some aspects of the CP-violating effects which follow from the EDM of the neutron, electron and atoms are discussed in [3]. The EDM bounds of the neutron and the electron can be established in low energy experiments.

There are some investigations of the EDM of vector W bosons in the framework of the SM and beyond in [4]. But for W bosons it is necessary to analyze the high energy processes for extracting CP-odd asymmetries. The EDM of W bosons can give the large contribution to the EDM of fermions (in particular to electrons). The EDM of particles may be also induced by Higgs-boson exchange [5]. The prediction of the EDM of the W-boson in the SM is $d_W \sim 10^{-29} \, \text{e cm}$ [6] but beyond the SM it can be (for example in the Two-Higgs-doublet model) about $10^{-21} - 10^{-20} \, \text{e cm}$ (see last reference in [4]). The experimental constraint on the EDM of the W-boson which follows from the experimental upper bound for the neutron EDM $d_n = (-3 \pm 5) \times 10^{-26} \, \text{e cm}$ [7] is $d_W \leq 10^{-19} \, \text{e cm}$. So, the presence of the EDM can indicate physics beyond the SM.

Strong interacting composite hadrons $\rho$, $\omega$ (and others) possess spin one. The theory of strong interactions of quarks and gluons - quantum chromodynamics (QCD) is renormalizable. However properties of hadrons are described by the infrared region of QCD where perturbation theory in small parameter $\alpha_s$ is not acceptable. In this region some phenomenological models are used. The nonperturbative theory of strong interactions of hadrons has not being developed yet but there is a progress in describing hadrons in the framework of QCD string theory [8]. In this approach the EDM of mesons [9] and baryons [10] appears naturally. It should be noticed that the EDM of the neutron may be induced by the $\vartheta$-term of the QCD vacuum. The QCD vacuum angle $\vartheta$ violates $P$ and $CP$ symmetries and gives $CP$-odd electromagnetic observables. As the EDM of the neutron is small the $\vartheta$-parameter of the QCD vacuum is also small. It is possible to explore the axion mechanism [11] to solve the strong $CP$ problem for having the parameter $\vartheta = 0$. Vector mesons may possess the EDM due to the $\vartheta$-term [12] and the $CP$-odd electromagnetic form-factors of $\rho$-mesons can be introduced.

In view of the great interest to physics in framework and beyond the SM it is very important to study various processes involving massive vector bosons with the EDM and AMM. The important and interesting vacuum quantum effects are pair production of particles and antiparticles and the vacuum polarization [13]. In particular, there is the vacuum instability of the vector particles in a magnetic field [14]. This is due to large contribution of the tachyon mode to the negative part of the Callan-Symanzik $\beta$-function, and as a result the vacuum is reconstructed in a magnetic field. Some studies were performed to investigate the vacuum quantum effects for vector fields. The pair production and the vacuum polarization of vector fields with gyromagnetic ratio $g = 2$ by a constant uniform electric field were investigated in [15]. The semiclassical imaginary-time method was used in [16] to find the probability of pair production by a constant electromagnetic field for arbitrary spin $s$ and gyromagnetic ratio $g$. In [17] we found the pair production probability and the vacuum polarization of fields for arbitrary $s$ and
on the basis of the exact solutions of the wave equation for particles in a constant and uniform electromagnetic field and with the help of the Fock-Schwinger proper-time method. In this approach fields realize \((s, 0) \oplus (0, s)\)-representation of the Lorentz group. Nonlinear corrections to the constant uniform electromagnetic field due to the vacuum polarization of a charged vector field in the framework of the renormalizable gauge theory were studied in [18]. The pair production probability of charged vector bosons with \(g = 1\) by a non-stationary electric field was derived in [19].

In this work we study the pair production probability and the vacuum polarization of the charged vector particles with arbitrary EDM and AMM. The paper is organized as follows. In Sec. II we proceed from the Dirac-Kähler equations for boson fields of spins one and zero. We show that this system of wave equations can be represented as two subsystems of \(P\)-odd equations for self-dual and antself-dual antisymmetric tensors of second rank. The matrix form and the symmetry group of equations is investigated in Sec. III. In Sec. IV the \(P\)-odd system of first order equations for vector fields with the EDM and AMM is introduced. We found exact solutions of the second order field equation in external constant and uniform electromagnetic fields. The pair production probability of vector particles with the EDM and AMM is calculated in Sec. V with the help of found solutions. Sec. VI devoted to finding the vacuum polarization of vector particles. Section VII contains the conclusion.

2 Field equations

One of the nonperturbative approaches of strong interactions is lattice QCD [20]. For describing fermions on the lattice the Dirac-Kähler equation [21] can be used (see [22]). Dirac-Kähler’s equation in four dimensional space-time is given by

\[(d - \delta + m) \Phi = 0,\]

where \(d\) is the exterior derivative, \(\delta = -\star^{-1} d \star\) transforms \(n\)-form into \((n - 1)\)-form, \(\Phi\) denotes the inhomogeneous differential form. The star operator \(\star\) connects a \(n\)-form to a \((4 - n)\)-form so, that \(\star^2 = 1, d^2 = \delta^2 = 0\). The Laplacian is given by \((d - \delta)^2 = - (d \delta + \delta d) = \partial \mu \partial_\mu\) where the operator \((d - \delta)\) is the analog of the Dirac operator \(\gamma_\mu \partial_\mu\). The inhomogeneous differential form \(\Phi\) can be represented as

\[
\Phi = \varphi(x) + \varphi_\mu(x) dx^\mu + \frac{1}{2!} \varphi_{\mu\nu}(x) dx^\mu \wedge dx^\nu +
\]

\[
+ \frac{1}{3!} \varphi_{\mu\nu\rho}(x) dx^\mu \wedge dx^\nu \wedge dx^\rho + \frac{1}{4!} \varphi_{\mu\nu\rho\sigma}(x) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma,
\]

where \(\wedge\) is the exterior product; \(\varphi(x), \varphi_\mu(x), \varphi_{\mu\nu}(x), \varphi_{\mu\nu\rho}(x), \varphi_{\mu\nu\rho\sigma}(x)\) are scalar, vector and antisymmetric tensor fields, respectively. The antisymmetric tensors \(\varphi_{\mu\nu\rho}(x)\),
\( \varphi_{\mu\nu\rho\sigma}(x) \) are connected with pseudovector and pseudoscalar fields by the relationships:

\[
\tilde{\varphi}(x) = \frac{1}{3!}\varepsilon_{\mu\nu\rho\sigma}\varphi_{\nu\rho\sigma}(x), \quad \tilde{\varphi}(x) = \frac{1}{4!}\varepsilon_{\mu\nu\rho\sigma}\varphi_{\nu\rho\sigma}(x),
\]

where \( \varepsilon_{\mu\nu\alpha\beta} \) is an antisymmetric tensor Levy-Civita; \( \varepsilon_{1234} = -i \). The Dirac-Kähler equation formulated in the framework of differential forms [21] is equivalent to the following system of tensor fields [23]:

\[
\partial_\nu \psi_{\mu\nu}(x) - \partial_\mu \psi(x) + m^2 B_\mu(x) = 0, \quad \partial_\nu \tilde{\psi}_{\mu\nu}(x) - \partial_\mu \tilde{\psi}(x) + m^2 C_\mu(x) = 0, \quad (1)
\]

\[
\partial_\mu B_\mu(x) - \psi(x) = 0, \quad \partial_\mu C_\mu(x) - \tilde{\psi}(x) = 0, \quad (2)
\]

\[
\psi_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) - \varepsilon_{\mu\nu\alpha\beta}\partial_\alpha C_\beta(x), \quad (3)
\]

where \( \tilde{\psi}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\alpha\beta}\psi_{\alpha\beta} \) is the dual tensor. Expression (3) is the most general representation for the antisymmetric tensor of second rank [24, 25]. Equations (1)-(3) describe the system of the vector \( (B_\mu(x)) \), pseudovector \( (C_\mu(x)) \), scalar \( (\psi(x)) \), and pseudoscalar \( (\tilde{\psi}(x)) \) fields. For the complex values of the vector-potentials \( B_\mu(x) \) and \( C_\mu(x) \), Eqs. (1)-(3) correspond to the charged vector fields. As the system of equations (1)-(3) contains two four-vectors \( B_\mu(x), C_\mu(x) \) which carry spins one and zero (without Lorenz conditions, \( \partial_\mu B_\mu(x) \neq 0, \partial_\mu C_\mu(x) \neq 0 \)) there is the doubling of the spin states of particles. So, Eqs. (1)-(3) describe fields with two spin-one and two spin-zero states. Using the projection operator technique this states may be separated [23]. The field equations (1)-(3) can be derived from the corresponding Lagrangian and represent the Lagrange-Euler equations. The Proca equations [26] are a special case of Eqs. (1)-(3) when the constraints \( C_\mu = 0, \partial_\mu B_\mu = 0 \) are imposed. At the case of \( C_\mu = 0, \partial_\mu B_\mu \neq 0 \) we arrive at Stueckelberg’s equation [27] describing spin one and zero fields without the doubling of the spin states of a particle. The matrix form of equations (1)-(3) is \( 16 \times 16 \)-dimensional Dirac equation [23]. This makes it possible to describe fermions with spin 1/2 with the help of fields \( \psi(x), B_\mu(x), C_\mu(x) \) which do not realize the tensor representation of the Lorentz group in this case and are connected with spinors. In this work we consider the case when the fields \( \psi(x), B_\mu(x), \psi_{\mu\nu}(x), \tilde{\psi}(x), C_\mu(x) \) are bosonic fields carrying spins 0 and 1.

Dirac-Kähler equations (1)-(3) are equivalent to the following systems

\[
\partial_\nu M_{\mu\nu}(x) - \partial_\mu M(x) + m^2 M_\mu(x) = 0, \quad \partial_\mu M_{\mu}(x) = M(x), \quad (4)
\]

\[
M_{\mu\nu}(x) = \partial_\mu M_\nu(x) - \partial_\nu M_\mu(x) - i\varepsilon_{\mu\nu\alpha\beta}\partial_\alpha M_\beta(x),
\]

with the self-dual tensor \( M_{\mu\nu}(x) = -i\tilde{M}_{\mu\nu}(x) \) and

\[
\partial_\nu N_{\mu\nu}(x) - \partial_\mu N(x) + m^2 N_\mu(x) = 0, \quad \partial_\mu N_{\mu}(x) = N(x), \quad (5)
\]
\[ N_{\mu\nu}(x) = \partial_\mu N_\nu(x) - \partial_\nu N_\mu(x) + i\varepsilon_{\mu\nu\alpha\beta}\partial_\alpha N_\beta(x), \]

with the antiself-dual tensor \( N_{\mu\nu}(x) = i\tilde{N}_{\mu\nu}(x) \), where

\[ M(x) = \frac{1}{\sqrt{2}} \left( \psi(x) - i\tilde{\psi}(x) \right), \quad N(x) = \frac{1}{\sqrt{2}} \left( \psi(x) + i\tilde{\psi}(x) \right), \]

\[
\begin{align*}
M_\mu(x) &= \frac{1}{\sqrt{2}} \left( B_\mu(x) - iC_\mu(x) \right), \\
M_{\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left( \psi_{\mu\nu}(x) - i\tilde{\psi}_{\mu\nu}(x) \right), \\
N_\mu(x) &= \frac{1}{\sqrt{2}} \left( B_\mu(x) + iC_\mu(x) \right), \\
N_{\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left( \psi_{\mu\nu}(x) + i\tilde{\psi}_{\mu\nu}(x) \right).
\end{align*}
\]

Adding and subtracting equations (1)-(3) we get equations (4), (5). The self-dual tensor \( M_{\mu\nu} \) which obeys the equations (4) is transformed under \((1,0)\)-representation of the Lorentz group and has 3 independent components (see also [28]). Equations (4) are not invariant under the parity transformation and there is no Lagrangian formulation of them. This also applies to equations (5) for the antiself-dual tensor \( N_{\mu\nu} \) which transforms under the \((0,1)\)-representation of the Lorentz group. But if we consider the whole system of equations (4), (5) (which is equivalent to equations (1)-(3)) on the basis of \((0,0) \oplus (1/2, 1/2) \oplus (1,0) \oplus (0,1) \oplus (1/2, 1/2) \oplus (0,0)\)-representation of the Lorentz group, we will have \( P \)-invariant theory within the Lagrangian formulation. Each of the system of equations (4), (5) describes eight independent variables \((M(x), M_\nu(x), M_{ab}(x)), (N(x), N_\nu(x), N_{ab}(x))\).

### 3 Matrix form of equations

Let us introduce four-component columns:

\[
\begin{align*}
\xi(x) &= -im \begin{pmatrix} M_a(x) \\ M_4(x) \end{pmatrix}, \\
\chi(x) &= \begin{pmatrix} \tilde{M}_a(x) \\ M(x) \end{pmatrix}, \\
\xi'(x) &= -im \begin{pmatrix} N_a(x) \\ N_4(x) \end{pmatrix}, \\
\chi'(x) &= \begin{pmatrix} \tilde{N}_a(x) \\ N(x) \end{pmatrix},
\end{align*}
\]

where \( \tilde{M}_a(x) = (1/2)\epsilon_{amn}M_{mn}(x), \tilde{N}_a(x) = (1/2)\epsilon_{amn}N_{mn}(x) \). Taking into account the notations (6), Eqs. (4), (5) can be represented as

\[
\begin{align*}
\alpha_\mu \partial_\mu \xi(x) &= m\chi(x), \\
\overline{\alpha}_\mu \partial_\mu \chi(x) &= m\xi(x), \\
\alpha'_\mu \partial_\mu \xi'(x) &= m\chi'(x), \\
\overline{\alpha}'_\mu \partial_\mu \chi'(x) &= m\xi'(x),
\end{align*}
\]
where

\[
\alpha_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \\
\alpha_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad \alpha'_1 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \\
\alpha'_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \quad \alpha'_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{pmatrix}, \\
\alpha'_4 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & i \end{pmatrix}, \quad \alpha'_4 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \\
\overline{\alpha}_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \quad \overline{\alpha}_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{pmatrix}, \\
\overline{\alpha}_4 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}, \quad \alpha_4 = iI_4, \quad \overline{\alpha}_\mu = (\alpha_k, -iI_4).
\]

Equations (7), (8) can be also cast in the form

\[
\beta_\mu \partial_\mu \varphi(x) + m\varphi(x) = 0, \\
\beta'_\mu \partial_\mu \varphi'(x) + m\varphi'(x) = 0,
\]

where

\[
\varphi(x) = \begin{pmatrix} \chi(x) \\ \xi(x) \end{pmatrix}, \quad \beta_\mu = -\begin{pmatrix} 0 & \alpha_\mu \\ \overline{\alpha}_\mu & 0 \end{pmatrix}, \\
\varphi'(x) = \begin{pmatrix} \chi'(x) \\ \xi'(x) \end{pmatrix}, \quad \beta'_\mu = -\begin{pmatrix} 0 & \alpha'_\mu \\ \overline{\alpha}_\mu & 0 \end{pmatrix},
\]

and the matrices \(\beta_\mu, \beta'_\mu\) obey the Dirac algebra

\[
\beta_\mu \beta_\nu + \beta_\nu \beta_\mu = 2\delta_{\mu\nu}.
\]
We can combine Eqs. (10), (11) in the 16–component Dirac-type wave equation, as follows

$$(\Gamma_{\mu} \partial_{\mu} + m) \Psi(x) = 0,$$  

(14)

where

$$\Psi(x) = \begin{pmatrix} \varphi(x) \\ \varphi'(x) \end{pmatrix}, \quad \Gamma_{\mu} = \begin{pmatrix} \beta_{\mu} & 0 \\ 0 & \beta'_{\mu} \end{pmatrix}.$$  

(15)

The $16 \times 16$ - matrices $\Gamma_{\mu}$ also obey the Dirac algebra (13). This means that the system of equations (7), (8) is equivalent to four Dirac equations. So Dirac-Kähler equations are equivalent to two matrix equations (10), (11) (or two systems of tensor equations (4), (5)). Equation (10) (and (4)) as well as Eq. (11) (and (5)) are parity noninvariant separately and at the same time the system of the two equations (10), (11) (or Dirac-Kähler equations) are P-invariant.

Now we will find the symmetry group of equations (10), (11). As matrices $\beta_{\mu}$, $\beta'_{\mu}$ obey the same algebra, equations (10) and (11) have the same symmetry group. Therefore, we only need to consider equation (10) which is equivalent to Eqs. (4).

The matrices $\beta_{\mu}$ are 8–component Dirac-type matrices, and in a specific basis they take the form $\overline{\beta}_{\mu} = I_2 \otimes \gamma_{\mu}$. It is obvious that the matrices $\overline{\beta}_{m} = \tau_{m} \otimes I_{4}$ ($\tau_{m}$ are the Pauli matrices) form the symmetry algebra of Eq. (10). It should be noted that the internal symmetry under consideration is not violated by introducing the electromagnetic fields by the substitution $\partial_{\mu} \rightarrow \partial_{\mu} - ieA_{\mu}$. In the representation (12), the following matrices

$$\overline{\beta}_{m} = \begin{pmatrix} \rho_{m} & 0 \\ 0 & \rho_{m} \end{pmatrix},$$  

(16)

commute with the matrices $\beta_{\mu}$, if $[\rho_{m}, \alpha_{n}] = 0$, where the matrices $\alpha_{n}$ are given by Eqs. (9) and satisfy the Pauli commutation relations: $\{\alpha_{i}, \alpha_{k}\} = 2\delta_{ik}$, $[\alpha_{i}, \alpha_{k}] = 2i\epsilon_{ikl}\alpha_{l}$. Such matrices $\rho_{m}$ which commute with $\alpha_{n}$ have the form

$$\rho_{1} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \rho_{2} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix},$$  

$$\rho_{3} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix};$$  

(17)

they also obey the Pauli commutation relations. Further we will use also the matrix $\beta_{4} = iI_{4}$.

Let us consider the group of the transformations of the wave function of Eq. (10):

$$\varphi(x) \rightarrow \exp \left( m_{\mu} \overline{\beta}_{\mu} \right) \varphi(x),$$  

(18)
where the $m_\mu$ are four complex parameters. The transformations (18) are defined for the complex fields describing the charge fields; they form the internal symmetry group of Eq. (10) which is isomorphic to the $GL(2, c)$ group.

It is possible to apply Eq. (10) to the description of spinor particles. In this case the wave function $\varphi(x)$ realize the spinor representation of the Lorentz group and Eq. (10) is equivalent to two Dirac equations; it can be obtained by the variation procedure from the corresponding Lagrangian. Thus generators of the Lorentz group are given by

$$J^{(1/2)}_{\mu\nu} = \frac{1}{4} (\beta_\mu \beta_\nu - \beta_\nu \beta_\mu) ,$$

and the Hermitianizing matrix is $\eta = \beta_4$.

In the case of the bosonic fields (see Eqs. (6), (12)), however, there is no Lagrangian formulation of Eq. (10) because it is parity noninvariant equation based on the reducible $(0, 0) \oplus (1/2, 1/2) \oplus (1, 0)$-representation of the Lorentz group.

The requirement that the Lagrangian of spinor fields ($\eta = \beta_4$) be invariant under the transformations (18) yields the restriction on the parameters: $m_k^* = -m_k$, $m^*_4 = m_4$, that corresponds to the extraction of the $U(2)$ subgroup. According to the Noether theorem, this produces the conservation current

$$\theta_{\mu\alpha} = \varphi(x) \beta_\mu \overline{\beta}_\alpha \varphi(x) ,$$

so that $\partial_\mu \theta_{\mu\alpha} = 0$; $\overline{\varphi}(x) = \varphi^+(x) \beta_4$, $\varphi^+(x)$ is the Hermitian-conjugate wave function. It is easy to verify that the quantity (20) is also conserved in the boson case (see also [28]), when the fields are given by Eqs. (6), (12). We notice that the internal symmetry group of Dirac-Kähler equation (14) is $GL(4, c)$ and the corresponding Lagrangian for bosonic fields is invariant under the transformations of $SO(4, 2)$ group (or locally isomorphic group $SU(2, 2)$) [29, 23].

4 **Vector particle with EDM and AMM in uniform electromagnetic field**

We consider here the description of electromagnetic interactions of vector particles possessing the EDM and AMM. Sakata and Taketani added some terms in equations which describe the effects of anomalous moments [30], and Corben and Schwinger [31] included the AMM in the Proca equations [26]. Yang and Bludman considered an anomalous electric quadrupole moment [32]. Introducing in Eqs. (4) the interaction with the electromagnetic field $\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu$, AMM, and EDM, we arrive at the equations (at the substitutions $M \rightarrow \psi$, $M_\mu \rightarrow \psi_\mu$, $M_{\mu\nu} \rightarrow \psi_{\mu\nu}$)

$$D_\mu \psi_\mu = \psi ,$$
\[
\psi_{\mu\nu} = D_\mu \psi_\nu - D_\nu \psi_\mu + \sigma \epsilon_{\mu \alpha \beta} D_\alpha \psi_\beta, \tag{21}
\]
\[
D_\nu \psi_{\mu\nu} - D_\mu \psi + m^2 \psi_\mu + i e \kappa F_{\mu\nu} \psi_\nu = 0.
\]

If setting \( \sigma = 0 \) we get Stueckelberg's equations with the AMM \( e \kappa \) which describe fields of spin 1 and 0 [27]. It should be noted that for field equations (21) the mass of the field with spin of zero coincides with the mass of the vector field. Quantization of the fields (21) leads to the indefinite metric for scalar state. At \( \sigma = i \) and \( \kappa = 0 \) Eqs. (21) have the 8-component matrix formulation (10) (with the replacement \( \partial_\mu \to D_\mu \)) with matrices \( \beta_\mu \) (12) obeying the Dirac algebra. It is easy to obtain the second order equation for the four-vector \( \psi_\mu(x) \) from Eq. (21). As a result one finds

\[
\left(D_\nu^2 - m^2\right) \psi_\mu(x) + ie \left(\sigma \bar{F}_{\mu\nu} - g F_{\mu\nu}\right) \psi_\nu(x) = 0, \tag{22}
\]

where \( \bar{F}_{\mu\nu} = (1/2) \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta} \) is the dual tensor, \( g = 1 + \kappa \) is the gyromagnetic ratio for the quanta of spin 1. Eq. (22) describes a particle with the magnetic moment \( e g/(2m) \) and the EDM \( \sigma/(2m) \). It should be noted that in the case of the Proca equation with the EDM and AMM, we have in Eq. (22) the additional term \( -D_\mu D_\nu \psi_\mu \) due to the absence of a scalar state. Eq. (22) describes a particle with the magnetic moment \( e g/(2m) \) and the EDM \( \sigma/(2m) \). It should be noted that in the case of the Proca equation with the EDM and AMM, we have in Eq. (22) the additional term \( -D_\mu D_\nu \psi_\mu \) due to the absence of a scalar state. Eq. (22) can be treated in the framework of \( \xi \)-formalism [33] as a wave equation for vector field in the gauge \( \xi = 1 \). We notice that the formal counting of the divergences corresponding to Eq. (22) leads to a renormalizable theory due to the form of the field propagator which is proportional to \( 1/p^2 \) but with the presence of indefinite metric.

It is easier to solve Eq. (22) compared to the Proca equation for a particle in the external electromagnetic fields. To estimate the physical quantities for a vector particle one needs to eliminate the contribution of a scalar state. In the following calculations we will use this procedure.

Here we will find the solutions of Eq. (22) for a particle in the field of uniform and constant electromagnetic fields. We note [13] that matrices \( F_{\mu\nu}, \bar{F}_{\mu\nu} \) have eigenvalues as follows

\[
F_{\mu\nu} \psi_\mu^{(\lambda)} = F^{(\lambda)} \psi_\mu^{(\lambda)},
\]
\[
\bar{F}_{\mu\nu} \psi_\nu^{(\lambda)} = \frac{1}{F^{(\lambda)}} G \psi_\mu^{(\lambda)} ,
\]
\[
F^{(\lambda)} = \pm F^{(1)}, \pm F^{(2)}, \quad \lambda = 1, 2, 3, 4, \tag{23}
\]
\[
F^{(1)} = \frac{i}{\sqrt{2}} \left[ (\mathcal{F} + i \mathcal{G})^{1/2} + (\mathcal{F} - i \mathcal{G})^{1/2} \right],
\]
\[
F^{(2)} = \frac{i}{\sqrt{2}} \left[ (\mathcal{F} + i \mathcal{G})^{1/2} - (\mathcal{F} - i \mathcal{G})^{1/2} \right],
\]
\[
\mathcal{F} = \frac{1}{4} F_{\mu\nu}^2 = \frac{1}{2} (\mathbf{H}^2 - \mathbf{E}^2), \quad \mathcal{G} = \frac{1}{4} F_{\mu\nu} \bar{F}_{\mu\nu} = \mathbf{E} \cdot \mathbf{H}, \tag{24}
\]
and $\mathbf{E}$, $\mathbf{H}$ are the electric and magnetic fields, respectively. In the diagonal representation (23) Eq. (22) becomes

$$
\left( D^2 - m^2 \right) \psi^{(\lambda)}_\mu(x) + ie \left( \frac{1}{F^{(\lambda)}} G - g F^{(\lambda)} \right) \psi^{(\lambda)}_\mu(x) = 0. 
$$

Equation (25) represents the Klein-Gordon type equation for every component of the eigenfunction $\psi^{(\lambda)}_\mu(x)$. We consider the general case when two Lorentz invariants of the electromagnetic fields $F \neq 0$, $G \neq 0$. It is convenient to use a coordinate system in which the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields are parallel ($\mathbf{E} = n \mathbf{E}$, $\mathbf{H} = n \mathbf{H}$, $n = (0, 0, 1)$) and the 4-vector potential takes the form

$$
A_\mu = (0, x_1 H, -t E, 0). 
$$

After introducing the variables [34] (see also [35])

$$
\eta = \frac{p_2 - e H x_1}{\sqrt{e H}}, \quad \tau = \sqrt{e E} \left( t + \frac{p_3}{e E} \right), 
$$

$$
\psi^{(\lambda)}_\mu(x) = \exp \left[ i \left( p_2 x_2 + p_3 x_3 \right) \right] \Phi^{(\lambda)}_\mu(\eta, \tau) 
$$

Eq. (25) reads

$$
\left[ e H \left( \partial_\eta^2 - \eta^2 \right) - e E \left( \partial_\tau^2 + \tau^2 \right) - m^2 + ie \left( \frac{1}{F^{(\lambda)}} G - g F^{(\lambda)} \right) \right] \Phi^{(\lambda)}_\mu(\eta, \tau) = 0 
$$

where $\partial_\eta = \partial / \partial \eta$, $\partial_\tau = \partial / \partial \tau$. The solution to Eq. (28) exists in the form

$$
\Phi^{(\lambda)}_\mu(\eta, \tau) = \xi_\mu \phi^{(\lambda)}(\eta) \chi^{(\lambda)}(\tau), 
$$

with a constant vector $\xi_\mu$, and the eigenfunctions $\phi^{(\lambda)}(\eta)$, $\chi^{(\lambda)}(\tau)$ obey the following equations

$$
\left[ e H \left( \partial_\eta^2 - \eta^2 \right) - m^2 + ie \left( \frac{1}{F^{(\lambda)}} G - g F^{(\lambda)} \right) + k_\lambda^2 \right] \phi^{(\lambda)}(\eta) = 0, 
$$

$$
\left[ e E \left( \partial_\tau^2 + \tau^2 \right) + k_\lambda^2 \right] \chi^{(\lambda)}(\tau) = 0, 
$$

where $k_\lambda^2$ are the eigenvalues. The finite solution (at $\eta \to \infty$) to Eq. (30) is

$$
\phi^{(\lambda)}(\eta) = N_0 \exp \left( -\frac{\eta^2}{2} \right) H_n(\eta), 
$$

where $N_0$ is the normalization constant, $H_n(\eta)$ are the Hermite polynomials. The requirement that this solution be finite leads to the condition

$$
k_\lambda^2 - m^2 + ie \left( \frac{1}{F^{(\lambda)}} G - g F^{(\lambda)} \right) = e H (2n + 1), \quad n = 1, 2, ..., 
$$
$n$ is the principal quantum number and $k_\lambda$ is spectral parameter. Equation (31) has four solutions with different asymptotics at $t \to \pm \infty$ [34]

\[
\begin{align*}
+\chi^{(\lambda)}(\tau) &= D_\nu[-(1-i)\tau], \quad -\chi^{(\lambda)}(\tau) = D_\nu[(1-i)\tau], \\
+\chi^{(\lambda)}(\tau) &= D_{\nu^*}[(1+i)\tau], \quad -\chi^{(\lambda)}(\tau) = D_{\nu^*}[-(1+i)\tau],
\end{align*}
\]

(34)

where $D_\nu(x)$ are the parabolic-cylinder functions (the Weber-Hermite functions) and

\[
\nu = \frac{ik^2_\lambda}{2eE} - \frac{1}{2}.
\]

The four solutions of equation (25) for the potential (26) with different asymptotic forms are given by

\[
\pm \psi^{(\lambda)}(x) = N_0 \xi_\mu \exp \left\{ i(p_2 x_2 + p_3 x_3) - \frac{\eta^2}{2} \right\} H_n(\eta) \pm \chi^{(\lambda)}(\tau).
\]

(35)

Exact solutions (35) will be used to estimate the pair production probability of vector particles and antiparticles in the external constant and uniform electromagnetic fields.

5 Pair production of vector particles with EDM and AMM

The probability for pair production of vector particles with the EDM and AMM by constant electromagnetic fields can be obtained through the asymptotic form of solutions (35) when the time $t \to \pm \infty$. The functions $\pm \psi^{(\lambda)}(\tau)$ at $t \to \pm \infty$ have positive frequency and $-\psi^{(\lambda)}(\tau)$ have negative frequency. Three quantities $k^2_\lambda$ and the momentum projections $p_2, p_3$ entering solutions (34), (35) are conserved. The functions (34) (see [34]) obey the relations

\[
\begin{align*}
+\chi^{(\lambda)}(\tau) &= c_{1n\lambda}^+ \chi^{(\lambda)}(\tau) + c_{2n\lambda}^- \chi^{(\lambda)}(\tau), \\
+\chi^{(\lambda)}(\tau) &= c_{1n\lambda}^+ \chi^{(\lambda)}(\tau) - c_{2n\lambda}^- \chi^{(\lambda)}(\tau), \\
-\chi^{(\lambda)}(\tau) &= -c_{2n\lambda}^+ \chi^{(\lambda)}(\tau) + c_{1n\lambda}^- \chi^{(\lambda)}(\tau), \\
-\chi^{(\lambda)}(\tau) &= c_{2n\lambda}^+ \chi^{(\lambda)}(\tau) + c_{1n\lambda}^- \chi^{(\lambda)}(\tau),
\end{align*}
\]

(36)

where

\[
c_{2n\lambda} = \exp \left[ -\frac{\pi}{2} (\varepsilon + i) \right], \quad \varepsilon = \frac{m^2 - ie \left( \sigma \frac{1}{F^{(\lambda)}} \mathcal{G} - g \mathcal{F}^{(\lambda)} \right) + eH(2n + 1)}{eE},
\]

\[
|c_{1n\lambda}|^2 - |c_{2n\lambda}|^2 = 1.
\]
The value $c_{2n\lambda}$ allows us to calculate the probability of pair production of vector particles in the state with the quantum number $n$ and corresponding to the eigenvalue $F^{(\lambda)}$. The probability for the production of a pair of vector particles in the state with quantum number $n$, components of momentum $p_2, p_3$ and corresponding to the eigenvalue $F^{(\lambda)}$ throughout all space and during all time is

$$|c_{2n\lambda}|^2 = \exp \left\{ -\pi \left[ \frac{m^2}{eE} + \frac{H}{E} (2n + 1) \right] \right\} \exp \left[ i\pi \left( \frac{1}{F^{(\lambda)}} G - gF^{(\lambda)} \right) / E \right] .$$

(38)

The expression (38) gives also the probability of the annihilation of a pair of particles with quantum numbers $n, p_2, p_3$. From Eq. (38) we find the average number of pairs produced from a vacuum

$$\bar{N} = \int \sum_{n,\lambda} |c_{2n\lambda}|^2 dp_2 dp_3 \frac{L^2}{(2\pi)^2},$$

(39)

where $(2\pi)^{-2}dp_2 dp_3 L^2$ means the final state density with the cut-off $L$ along the coordinates ($V = L^3$ is the normalization volume). In accordance with the approach [34] we can use the substitutions

$$\int dp_2 \rightarrow eHL, \quad \int dp_3 \rightarrow eET.$$

(40)

Here $T$ is the time of observation. It is possible to calculate the sum in (39) over the principal quantum number $n$, and eigenvalues $\lambda$ with the help of Eqs. (38), (23). Using Eqs. (40) we obtain the probability of pair production of particles per unit volume and per unit time

$$I(E, H) = \frac{\bar{N}}{VT} = \frac{e^2EH \exp \left[ -\pi m^2 / (eE) \right]}{8\pi^2 \sinh (\pi H / E)} \sum_{\lambda} |\exp \left[ i\pi \left( \frac{1}{F^{(\lambda)}} G - gF^{(\lambda)} \right) / E \right]| .$$

(41)

Evaluating the sum with the help of Eqs. (23)

$$\sum_{\lambda} |\exp \left[ \pi \left( \frac{1}{F^{(\lambda)}} G - igF^{(\lambda)} \right) / E \right]| = 2 \cosh \pi \left( \sigma + \frac{H}{E} \right) + 2,$$

(42)

we arrive at the pair production probability

$$I(E, H) = \frac{e^2EH \cosh \pi \left( \sigma + \frac{H}{E} \right) + 1}{4\pi^2 \sinh (\pi H / E)} \exp \left[ -\pi m^2 / (eE) \right].$$

(43)

So $I(E, H)$ is the intensity of the creation of pairs of particles with spins of 1, 0. Below we extract the pair production probability for particles with the pure spin 1 possessing the gyromagnetic ratio $g$ (and magnetic moment $\mu = eg/(2m)$) and the EDM $\sigma/(2m)$. 

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It follows from (43) that there is a pair production in a purely magnetic field if \( g > 1 \) that indicates about the instability of the vacuum in the magnetic field. For the case \( g > 1, \sigma = 0 \) this property for the higher spin particles was pointed in [16].

It is interesting to compare the probability (43) with those for particles possessing pure spin 1. The probability of pair production per unit volume and per unit time of vector particles on the base of \((0, 1) \oplus (1, 0)\) representation of the Lorentz group at \( \sigma = 0 \) is given by [16, 17]

\[
\begin{align*}
I^{(1)}(E, H) &= \frac{e^2EH \exp \left[-\frac{\pi m^2}{(eE)}\right]}{8\pi^2} \frac{\sinh (\frac{3g\pi H}{2E})}{\sinh (\pi H/E)}
\end{align*}
\]  

(44)

Setting \( \sigma = 0 \) in (43) and using some transformations we arrive at the equality

\[
I(E, H) = I^{(1)}(E, H) + I^{(0)}(E, H),
\]

(45)

where

\[
I^{(0)}(E, H) = \frac{e^2EH \exp \left[-\frac{\pi m^2}{(eE)}\right]}{8\pi^2} \frac{\sinh (\pi H/E)}{\sinh (\pi H/E)}
\]

is the intensity of the creation of pairs of scalar particles [13] (see also the creation of pairs of composite scalar particles in [36]). The physical meaning of equation (45) is clear: the probability of pair production of fields with spins 1, 0 is the sum of production probabilities of vector and scalar particles. By excepting Eq. (45) for arbitrary \( \sigma \) and \( g \) we obtain from Eq. (43) the expression for pair production probability of particles with pure spin one:

\[
I^{(1)}(E, H) = \frac{e^2EH 2 \cosh \pi (\sigma + gH/E) + 1}{8\pi^2} \frac{\sinh (\pi H/E)}{\sinh (\pi H/E)} \exp \left[-\frac{\pi m^2}{(eE)}\right].
\]

(46)

The Eq. (46) obtained is a new result for the intensity of pair production of vector particles with the EDM and AMM. From general formula (46) we find that in the case \( \sigma = g = 0 \), the pair production of vector particles is three times that for scalar pair production. This is due to the three physical degrees of freedom of the vector field.

The imaginary part of the density of the Lagrangian can be obtained using the relationship [34]:

\[
VT \text{Im} \mathcal{L} = \frac{1}{2} \int \sum_{n, \lambda} \ln |c_{1n\lambda}|^2 dp_2 dp_3 \frac{L^2}{(2\pi)^2}.
\]

(47)

From Eq. (47) taking into account Eqs. (37), (40) we arrive at

\[
\text{Im} \mathcal{L} = \frac{e^2EH}{8\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \exp \left(-\frac{\pi km^2}{eE}\right) \cosh \frac{\pi k (\sigma + gH/E) + 1}{\sinh (\pi k H/E)}. \]

(48)
According to the approach [13] the first term in (48) (at \( k = 1 \)) coincides with the intensity of pair production (43) (probability of the pair production per unit volume per unit time) divided by 2. Expression \( \text{Im} \mathcal{L} \) (48) and the pair production probability (46) do not depend on the renormalization scheme because all divergences and the renormalizability are contained in \( \text{Re} \mathcal{L} \) [13].

6 Polarization of vector particle vacuum

In this section we evaluate one-loop corrections to the Lagrange function of a constant and uniform electromagnetic field due to the field interaction with a vacuum of vector particles with the EDM and AMM. This problem has been solved for a number of theories [13, 15-18]. The effect of scattering of light by light is described by the nonlinear corrections to the Lagrangian of the electromagnetic field. Adapting the Schwinger method [13] to the fields described by equation (22), we obtain the nonlinear corrections to Lagrangian of a constant and uniform electromagnetic field

\[
\mathcal{L}^{(1)} = \frac{1}{16\pi^2} \int_0^\infty d\tau \tau^{-3} \exp (-m^2 \tau - l(\tau)) \text{tr} \exp \left[ ie_0 \left( \sigma \bar{F}_{\mu\nu} - g F_{\mu\nu} \right) \tau \right],
\]

with

\[
l(\tau) = \frac{1}{2} \text{tr} \ln \left[ (e_0 F \tau)^{-1} \sin(e_0 F \tau) \right], \quad \exp [-l(\tau)] = \frac{(e_0 \tau)^2 G_0}{\text{Im} \cosh(e_0 \tau X_0)},
\]

where \( X_0 = H_0 + iE_0, X = \sqrt{X^2}, G_0 = E_0 H_0, E_0, H_0 \) are bare (nonrenormalized) electric and magnetic fields, respectively, \( e_0 \) is the bare electric charge (the index 0 refers to the unrenormalized variables). The expression (49) is the effective nonlinear Lagrangian which is represented as an integral over the proper time \( \tau \). Here we consider general case of arbitrary constant vectors \( E_0 \) and \( H_0 \). With the help of Eqs. (23) we calculate the trace (tr) of the matrices

\[
\text{tr} \exp \left[ ie_0 \left( \sigma \bar{F}_{\mu\nu} - g F_{\mu\nu} \right) \tau \right] = 2 \left\{ \cosh e_0 \tau \left( \frac{\sigma G_0}{\text{Re} X_0} + g \text{Re} X_0 \right) + \cos e_0 \tau \left( \frac{\sigma G_0}{\text{Im} X_0} - g \text{Im} X_0 \right) \right\}.
\]

Substituting (51) into (49) and subtracting the additive constant to ensure that the expression \( \mathcal{L}^{(1)} \) vanishes for zero fields, we get

\[
\mathcal{L}^{(1)} = \frac{1}{8\pi^2} \int_0^\infty d\tau \tau^{-3} \exp (-m^2 \tau) \times \left[ (e_0 \tau)^2 G_0 \cosh e_0 \tau \left( \frac{\sigma G_0 / \text{Re} X_0 + g \text{Re} X_0}{\text{Im} \cosh(e_0 \tau X_0)} \right) + \cos e_0 \tau \left( \frac{\sigma G_0 / \text{Im} X_0 - g \text{Im} X_0}{\text{Im} \cosh(e_0 \tau X_0)} \right) - 2 \right],
\]

The integral (52) is the nonlinear correction to Maxwell’s Lagrangian due to the vacuum polarization of vector (with the additional scalar field) fields which possess the EDM.
The Lagrangian (52) contains the term that renormalizes the Lagrangian of the free electromagnetic fields

\[ \mathcal{L}^{(0)} = -\mathcal{F}_0 = \frac{1}{2} \left( E_0^2 - H_0^2 \right). \]

Extracting the divergent constant in Eq. (52) for weak fields, and adding Eq. (52) to the Maxwell Lagrangian (53) we obtain the renormalized Lagrangian of electromagnetic fields

\[ \mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} = -\mathcal{F} + \frac{1}{8\pi^2} \int_0^\infty d\tau \tau^{-3} \exp \left( -m^2\tau \right) \times \]
\[ \times \left[ (e\tau)^2 G \frac{\cosh e\tau (\sigma G/\text{Re}X + g \text{Re}X) + \cos e\tau (\sigma G/\text{Im}X - g \text{Im}X)}{\text{Im} \cosh(e\tau X)} \right. \]
\[ \left. -2 + (e\tau)^2 \left( \frac{2}{3} + \sigma^2 - g^2 \right) \mathcal{F} \right], \]

where the renormalized fields and charges are used:

\[ \mathcal{F} = Z_3^{-1} \mathcal{F}_0, \quad e = Z_3^{1/2} e_0, \]

and the renormalization constant is given by

\[ Z_3^{-1} = 1 + \frac{\epsilon_0^2}{12\pi^2} \left[ 1 + \frac{3}{2} \left( \sigma^2 - g^2 \right) \right] \int_0^\infty d\tau \tau^{-1} \exp \left( -m^2\tau \right). \]

The integral (54) vanishes already if the electromagnetic fields \( E, H \) are absent. We can use the cutoff factor \( \tau_0 \) at the lower limit in the integral (55), and the constant \( Z_3^{-1} \) diverges logarithmically as \( \tau_0 \to 0 \). When the EDM is absent (\( \sigma = 0 \)) and the gyromagnetic ratio \( g = 2 \) that is the linear approximation to the renormalizable gauge theory, we arrive from Eq. (55) to the renormalization constant obtained in [15]. It follows from Eq. (55) that when the inequality

\[ g^2 - \sigma^2 > \frac{2}{3} \]

is valid the renormalization constant of the charge \( Z_3^{1/2} \) becomes larger than one. This case, unlike ordinary electrodynamics, corresponds to the absence of the zero charge situation in the asymptotic region and indicates asymptotic freedom in the field [37, 38]. According to Eq. (56) the asymptotically free behavior in the vector field is due to the AMM but the role of the EDM is opposite. At the case \( \sigma^2 - g^2 > 2/3 \) the situation of the zero charge situation in the asymptotic region, like electrodynamics, is realized. From Eq. (55) we obtain the Callan-Zymanzik \( \beta \)-function that corresponds to the renormalizable theory

\[ \beta = \frac{\epsilon_0^2}{12\pi^2} \left[ 1 + \frac{3}{2} \left( \sigma^2 - g^2 \right) \right]. \]
At the condition (56) the \( \beta \)-function is negative \( (\beta < 0) \) and we arrive at the region of asymptotic freedom. The AMM assures asymptotic freedom and instability of the vacuum in a magnetic field.

Expanding Eq. (54) in the small electromagnetic fields we obtain the Maxwell Lagrangian including the nonlinear corrections (in rational units)

\[
\mathcal{L} = \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{H}^2 \right) + \frac{6\sigma g}{2 + 3(\sigma^2 - g^2)} (\mathcal{G} - \mathcal{G}_0) + \frac{\alpha^2}{m^4} \left[ \frac{14 - 30(g^2 - \sigma^2) + 15(g^4 + \sigma^4)}{45} \right] \mathcal{F}^2
\]

\[
+ \frac{2}{45} \left( 1 + \frac{15(\sigma^4 + 6\sigma^2 g^2 + g^4)}{4} \right) \mathcal{G}^2 + \frac{2}{3} \sigma g \left( g^2 - \sigma^2 - 2 \right) \mathcal{G} \mathcal{F} \right].
\]

where \( \alpha = e^2 / (4\pi) \). The second term in Eq. (58) is induced parity violation anomaly for a vector field with the EDM. This and last terms in Eq. (58) violate parity symmetry due to the EDM of a particle. The effective Lagrangian (58) is like the Heisenberg-Euler Lagrangian of QED [39, 40] but in the case of the polarized vacuum of vector fields with arbitrary EDM and AMM and additional scalar field (with the same mass). The presence of a scalar field is due to the special gauge \( \xi = 1 \) which was chosen to simplify the calculations. Now we will take into consideration the contribution of a scalar (nonphysical) field. It is easy to verify that for the particular case of \( \sigma = 0, g = 0 \), Eq. (58) becomes

\[
\mathcal{L}(\sigma = g = 0) = \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{H}^2 \right) + \frac{\alpha^2}{90m^4} \left[ 7 \left( \mathbf{E}^2 - \mathbf{H}^2 \right)^2 + 4(\mathbf{E}\mathbf{H})^2 \right]
\]

\[
= \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{H}^2 \right) + 4\mathcal{L}_{\text{spin0}},
\]

where

\[
\mathcal{L}_{\text{spin0}} = \frac{\alpha^2}{360m^4} \left[ 7 \left( \mathbf{E}^2 - \mathbf{H}^2 \right)^2 + 4(\mathbf{E}\mathbf{H})^2 \right]
\]

is the correction to the Maxwell Lagrangian due to the vacuum polarization of scalar pointlike particles [13]. As Eq. (22) becomes a Klein-Gordon equation for the field \( \psi_\mu \) at \( \sigma = g = 0 \), there is an equal contribution of four degrees of freedom of fields with spins 1 (three projection \( \pm 1, 0 \)) and 0. To have the contribution from a field of pure spin 1 we should subtract from Eq. (58) the expression (60) corresponding to spin 0 of a field. As a result the Lagrangian of a constant, uniform, electromagnetic field taking into account the vacuum polarization of a charged vector particles with the EDM and AMM is given by

\[
\mathcal{L}_{\text{spin1}} = \mathcal{L} - \mathcal{L}_{\text{spin0}} = \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{H}^2 \right) + \frac{6\sigma g}{2 + 3(\sigma^2 - g^2)} (\mathbf{E}\mathbf{H} - \mathbf{E}_0\mathbf{H}_0)
\]

\[
+ \frac{\alpha^2}{m^4} \left[ \frac{7 - 20(g^2 - \sigma^2) + 10(g^4 + \sigma^4)}{120} \left( \mathbf{E}^2 - \mathbf{H}^2 \right)^2 \right]
\]

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\[
\frac{1}{30} \left[ \sum \left( \sigma^4 + 6\sigma^2g^2 + g^4 \right) \right] (EH)^2 + \frac{1}{3} \sigma g \left( g^2 - \sigma^2 - 2 \right) (EH) \left( E^2 - H^2 \right).
\]

For the particular case \( \sigma = 0, g = 2 \) which corresponds to the linear approximation to the renormalizable SM Eq. (61) leads to the expression

\[
\mathcal{L}_{\text{spin1}} = \frac{1}{2} \left( E^2 - H^2 \right) + \frac{\alpha^2}{10m^4} \left[ \frac{29}{4} \left( E^2 - H^2 \right)^2 + 27(EH)^2 \right]
\]

which coincides with those obtained in [15].

It is possible to obtain the asymptotic form of (54) for super-critical fields at \( eE/m^2 \to \infty \) and \( eH/m^2 \to \infty \). However, for strong electromagnetic fields the AMM and EDM can depend on the external field like the dependence of the electron AMM in QED [41, 42].

## 7 Conclusion

Starting with the Dirac-Kähler equation for tensor fields we arrived at the two \( P \)-odd subsystem for self-dual and antiselfdual antisymmetric tensors of second rank. These equations are based on the \( (0,0) \oplus (1/2,1/2) \oplus (1,0) \) and \( (0,1) \oplus (1/2,1/2) \oplus (0,0) \) representations of the Lorentz group and describe fields with spins of 1 and 0. The 8-component Dirac-like \( P \)-odd matrix wave equation is constructed possessing \( GL(2,c) \) group of symmetry. This symmetry is due to the presence of two spins 1 and 0. The system of tensor equations considered allows us to introduce the EDM and AMM of a particle in the first order equations. The second order equation for a particle with the EDM and AMM is simpler (for solving) compared to the Proca equation. This equation can be treated as an equation for a vector particle with the gauge \( \xi = 1 \) in the framework of the T. D. Lee and C. N. Yang formalism. The contribution of the nonphysical scalar field to physical observables is eliminated at the end of calculations. Such approach allowed us to obtain the pair-production probability, and the effective Lagrangian for electromagnetic fields taking into account the polarization of the vacuum of vector particles with the EDM and AMM. This is the generalization of the Schwinger result on the case of vector particles in the external electric and magnetic fields. The exact formula for the intensity of pair production of fields with spins 1 and 0 is the sum of the intensity of pair production of vector and scalar particles. It is shown that there is a pair production of vector particles by a purely magnetic field in the case of \( g > 1 \) assuring asymptotic freedom and instability of the vacuum in a magnetic field. The role of the EDM of a vector particle is opposite: the EDM of a particle does not lead to instability of the vacuum in a magnetic field and suppresses the phenomena of asymptotic freedom. The pair production probability does not depend on the renormalization scheme because all divergences and the renormalizability are contained in \( \text{Re} \mathcal{L} \). Discussing the procedure of the renormalization we imply that the scheme considered is the linearized version of renormalize gauge theory. This
point of view is due to the smallness of the vector field self-interaction constant (see [14]) and it is possible to ignore processes that allow for the self-interaction of the vector field in vacuum.

The presence of the EDM and the value of the AMM $\kappa \neq 1 \ (g \neq 2)$ of a vector particle lies to physics beyond the SM. Recent experimental muon AMM data [43] have challenged the SM as there is a discrepancy of $2.6\sigma$ deviation between the theory and the averaged experimental value. This can open a window to new physics.

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