Abstract: We study $\mathcal{T}\mathcal{T}$ deformations of chiral bosons using the formalism due to Sen. For arbitrary numbers of left- and right-chiral bosons, we find that the $\mathcal{T}\mathcal{T}$-deformed Lagrangian truncates at linear order in the $\mathcal{T}\mathcal{T}$ coupling constant, a behaviour exhibited by free fermions as well. Furthermore, the stress tensor of both these theories receives no corrections due to the deformation. We conclude by discussing the interplay between $\mathcal{T}\mathcal{T}$ deformations and bosonisation.
1 Introduction

The subject of soluble irrelevant deformations of two-dimensional quantum field theories has, in recent year, enjoyed significant attention. Various aspects of the so-called $TT$ deformation — triggered at each step of the “flow” by the determinant of the stress tensor [1, 2] — such as the finite-volume spectrum [3] and the S-matrix of the deformed theory [4–6] are known exactly. These deformations have been studied from a various points of view, for example holography [7–11], supersymmetric quantum field theories [12–17], and string theory [18, 19]. An excellent introduction to this field can be found in [20].

The goal of this paper is to study $TT$ deformations of chiral bosons. In this introduction, we will motivate our analyses, situating our own study in the context of recent developments in this and adjacent fields.

Chiral Decoupling

In [21], we explored a curious aspect of the $TT$ deformation: the limit in which the $TT$ coupling constant is sent to infinity. In order to make sense of the above limit, it was imperative that we studied the deformed theory within the Hamiltonian formalism. We argued that this limit generically effects a decoupling of left- and right-chiral degrees of freedom. Schematically, we may represent our findings as follows:

Free Bosons $\xrightarrow{TT}$ Nambu-Goto String $\xrightarrow{\lambda \to \infty}$ Chiral Bosons

A similar progression was observed for the case of free fermions:

Free Fermions $\xrightarrow{TT}$ Interacting Fermions $\xrightarrow{\lambda \to \infty}$ Free Fermions
The curious nature of these progressions, from free to interacting to free theory, naturally motivates us to ask: what next? That is, what happens to a theory of free chiral bosons under a $TT$ deformation?

We will argue in this section that the choice to focus on this question places us at the intersection of various interesting aspects of the $TT$ deformation that merit further investigation.

A Choice of Formalism

Now that we are decided on the study of chiral bosons, we must first decide how to describe them. In the study of chiral fields, it is well-known that the constraint of chirality precludes the possibility of their study via the usual methods of free field theory. This is true more broadly of all self-dual $(2n+1)$-form field strengths in $4n+2$ spacetime dimensions. (The case $n=0$ is a theory of chiral bosons.)

This subject of self-dual field strengths has an interesting and tortured history, with theorists caught for decades between Scylla and Charybdis. The earliest investigations [22–24] used actions that were not manifestly Lorentz invariant. More recently, too, [25] studied $TT$ deformations of chiral bosons utilizing the formalism of Floreanini and Jackiw [26, 27].

The lack of manifest Lorentz invariance was subsequently overcome [28, 29] at the cost of the introduction of auxiliary fields: either an infinite number of them, or a finite number of auxiliary field with non-polynomial actions. For the better part of the last two decades, this was the most widely accepted framework for the discussion of chiral $p$-form fields despite the lack of off-shell self-duality; this permitted a classical analysis but complicates the discussion of the corresponding quantum theory. Another recent proposal for a Lorentz invariant action for chiral bosons was put forward in [30].

Following the first successful construction of the quantum Batalin–Vilkovisky master action for closed superstring field theory [31], the aforementioned difficulties were overcome in the context of Type IIB SUGRA by Sen in [32]. The construction was readily extended to theory of self-dual forms with arbitrary rank in [33]. Sen’s formalism, in addition to the $(2n+1)$-form field strength, also contains an additional $(2n)$-form field with the wrong sign for the kinetic term. Despite this, it was shown that the (free) dynamics of this additional field completely decouples — even from the background metric, and in the quantum theory as well — thereby forming an autonomous sector inert to the dynamics of other physical interacting fields.

We have elected to use Sen’s formalism in our study of chiral bosons. This choice was made in part by the demands of the problem at hand: various aspects of $TT$ deformations are quantum mechanically exact, and it is desirable to work with a formalism that is capable, at least in principle, of eventually accessing the quantum theory. Chirality is baked into the theory, i.e. the field variables are self-dual off-shell, and not imposed as an equation.
of motion. Additionally, manifest Lorentz invariance and polynomial nature of the action makes the case for adoption of Sen’s formalism very strong indeed.¹

To work with a formalism where the self-duality of the chiral boson is imposed at the level of an equation of motion (i.e. on-shell) is to foreclose on the possibility of studying the quantum aspects of this theory within this formalism. With this perspective, while the demonstration of [25] is elegant, it is not surprising, since at the level of the classical Lagrangian (an off-shell quantity), we are dealing with a decomposition of the scalar field that is only permissible on-shell. This is evident in the relation between the Hamiltonian densities of the deformed free scalar on one hand and the deformed left-chiral and right-chiral scalar on the other, see [25, eq. (20)].

Another way of seeing this is that for [25], the seed theory described by the Floreanini-Jackiw Lagrangian is a theory of chiral bosons on-shell, and that this is no longer the case once the TT flow begins. This is because in the Floreanini-Jackiw formalism, chirality is imposed on-shell by solving an equation of motion. Once deformations are turned on, the equation of motion that imposes chirality changes.

Our set-up is quite different. While the on-shell dynamics of the formalisms of Floreanini-Jackiw and Sen agree, in the latter formalism one works with fields that are self-dual off-shell, i.e. they continue to be off-shell once deformations are turned on. This off-shell self-duality imposes severe constraints on the form of the interactions one can introduce in Sen’s formalism. We will see that this implies an interesting, even familiar form of the TT-deformed Lagrangian.

LORENTZIAN VS. EUCLIDEAN SPACETIME

The decision to study chiral bosons in two dimensions forces us to work with a Lorentzian signature. It is helpful to quickly review this argument, so let us suppose that we are studying chiral (i.e. self-dual or anti-self-dual) p-forms on M, an n-dimensional manifold whose metric has s time coordinates, and derive this conclusion.

Consider the Hodge star operator \( \star : \Omega^p(M) \to \Omega^{n-p}(M) \), which maps p-forms to \( (n-p) \)-forms. We find that

\[
\star^2 = (-1)^{s+p(n-p)},
\]

where \( s = 0 \) for Euclidean space and \( s = 1 \) for Lorentzian spacetimes. A self-dual (+) or anti-self-dual (−) form A satisfies the property

\[
A = \pm \star A,
\]

which in turn implies \( n = 2p \), i.e. we can only have chiral forms in even dimensions. Further, on Hodge dualising the above equation once more, we find that

\[
\star^2 = (-1)^{s+p(n-p)} = +1,
\]

¹Sen’s formalism has been successfully used in the context of self-dual 3-forms in six-dimensional (2,0) theories, see [34–36].
which in turn implies that for Euclidean signature, $p$ is even, implying $n = 4k$ for $k \in \mathbb{N}$. However, for the Minkowski signature, we find $p^2 + 1$ is even, so $p$ is odd and $n = 4k + 2$. Therefore, in order to study real chiral bosons in two dimensions, we must work with a Lorentzian signature.

In contrast, the $\mathcal{T}\mathcal{T}$ deformation has been most rigorously studied in Euclidean space. For example, one typically defines the composite $\mathcal{T}\mathcal{T}$ operator via point-splitting and the proof of Zamolodchikov’s factorization formula [1] uses Euclidean operator product expansions. In the Lorentzian setting, it is natural to expect additional lightcone singularities. To the best of our knowledge, such an analysis has not yet been carried out.

Despite this, various Lorentzian aspects of the $\mathcal{T}\mathcal{T}$ deformation — defined in the same way, except on Minkowski spacetime — have been studied in the literature. As illustrative examples: [7] builds on [37] and studies signal propagation speeds; [38] interprets the inviscid Burgers equation which determines the spectrum of the $\mathcal{T}\mathcal{T}$-deformed theory as encoding the gauge invariance of the target spacetime energy and momentum of a non-critical string theory quantised in a one-parameter family of deformations of the standard lightcone gauge called the uniform lightcone gauge; finally, [39] adopt a Hamiltonian point of view on $\mathcal{T}\mathcal{T}$ deformations and show that their results in Lorentzian spacetime are consistent with known exact results like the Castillejo-Dalitz-Dyson phase that relates the deformed and undeformed S-matrices.

Our perspective in this note will be similar.

**Bosonisation**

Another interesting aspect of Sen’s formalism is that the classical Euclidean action is not of the general form assumed by [40], i.e. of the form

$$ S = \int d^2 x \sqrt{g} \mathcal{L} . $$

We will therefore construct the $\mathcal{T}\mathcal{T}$-deformed Lagrangian order-by-order in the $\mathcal{T}\mathcal{T}$ coupling constant, with the deformations sourced by the determinant of the stress tensor.

Our central result is that the $\mathcal{T}\mathcal{T}$-deformed Lagrangian for a pair of chiral bosons can in fact be computed in closed form. Strikingly, these deformations truncate at linear order in the $\mathcal{T}\mathcal{T}$ coupling constant. On considering arbitrary numbers of left- and right-chiral bosons, the same result is obtained. This result bears a striking resemblance to the $\mathcal{T}\mathcal{T}$-deformed free fermion Lagrangian, which also truncates at linear order irrespective of the number of left- and right-chiral fermions [17].

Based on this observation, we are tempted to conjecture that the bosonisation duality relating chiral bosons and fermions persists in their $\mathcal{T}\mathcal{T}$-deformed avatars. That is, bosonisation “commutes” with $\mathcal{T}\mathcal{T}$ deformation.

Of course, bosonisation duality (say, between sine Gordon theory and the massive Thirring model [41]) is a quantum mechanically exact statement about correlation functions. This
demonstration is made possible in part by the relevant nature of deformations, which makes the corresponding quantum theories well-defined, i.e. renormalisable. In this paper, we have restricted our attention to the classical aspects of $T\overline{T}$-deformed theories.

We hope to address quantum mechanical features of this problem, leveraging the analyses of [42–45] on correlation functions and [46] on renormalisation, in the near future.

**Outline**

We now summarise the contents of our paper.

In §2, we provide a pedagogical review of Sen’s formalism as it is applied to chiral bosons in two dimensions. Following this, in §3 we explain our strategy for computing the stress tensor. We feel this will help acclimatise our readers to Sen’s formalism, which may be unfamiliar. Our main results are contained in §4, where we study $T\overline{T}$ deformations of theories with left- and right-chiral bosons, and establish that the deformations truncate at linear order in the $T\overline{T}$ coupling. In §5, we point out that $T\overline{T}$-deformed free fermions also possess a similar structure, and discuss the possibility of a bosonisation duality relating the two $T\overline{T}$-deformed theories, i.e. that bosonisation “commutes” with $T\overline{T}$ deformations. We conclude in §6 by summarising our results and listing some further directions for research.

### 2 Sen’s Formalism for Chiral Bosons

A chiral boson in two dimensions is nothing but an 1-form field $A_\eta$ which is self-dual, i.e. $A_\eta = \star_\eta A_\eta$. Typically one thinks of this 1-form as a field strength for a scalar, viz. $A_\eta \sim d\varphi$, where $d$ is the exterior derivative and $\varphi$ is a scalar field. However, in Sen’s formalism, as we will shortly see, we need not invoke a rewriting of the self-dual 1-form as an exact 1-form. Further, it is sufficient for us to confine our analysis to the classical aspects of this action. Nonetheless, we wish to emphasize that the fully quantum mechanical analysis in the Hamiltonian formulation of this formalism is known as well. For more detailed discussions, we refer the reader to the original papers [32, 33] and also [35].

Sen’s action is of the following form

$$S = \int_{(\mathbb{R}^{(1,1)}, \eta)} \left[ \frac{1}{2} d\phi \wedge \star_\eta d\phi - 2 A \wedge d\phi + A \wedge \mathcal{M}(A) \right].$$  \hspace{1cm} (2.1)

The action describes a theory defined over a Lorentzian 2-manifold $\mathcal{M}$ equipped with a metric $g_{\mu\nu}$. However, note that the integral is defined over the two-dimensional Minkowski spacetime $\mathbb{R}^{(1,1)}$. The field $\phi$ is a scalar field on $\mathbb{R}^{(1,1)}$, and the field $A$ is an 1-form on $\mathbb{R}^{(1,1)}$ which is defined to be self-dual with respect to the flat metric, i.e. $A = \star_\eta A$. These are just some of the several unfamiliar features of this action.

Notice that due to this unusual self-duality constraint on the 1-form $A$, it does not belong to the space of self-dual 1-forms associated to $(\mathcal{M}, g)$. Following [35], we refer to these objects as pseudoforms.
The map $\mathcal{M}(A)$ is a linear map on the space of self-dual pseudoforms with the following properties (all of which are satisfied at the level of action itself, i.e. off-shell).

- $\mathcal{M}(A)$ maps self-dual pseudoforms to anti-self-dual pseudoforms. That is, for all $A = \star_\eta A$, we have
  \[ \mathcal{M}(A) = - \star_\eta \mathcal{M}(A) . \] (2.2)

- $\mathcal{M}$ annihilates anti-self-dual pseudoforms. That is, for all $B = - \star_\eta B$ we have
  \[ \mathcal{M}(B) = 0 . \] (2.3)

- For any two self-dual pseudoforms $A_1$ and $A_2$, the map satisfies
  \[ A_1 \wedge \mathcal{M}(A_2) = A_2 \wedge \mathcal{M}(A_1) . \] (2.4)

- Even though $A$ and $\mathcal{M}(A)$ are pseudoforms, the following specific linear combination belongs to the space of self-dual forms associated to $(\mathcal{M}, g)$.
  \[ A - \mathcal{M}(A) = \star_g (A - \mathcal{M}(A)) . \] (2.5)

The astute reader will conclude from this last property that the map $\mathcal{M}(A)$ is necessarily cognizant of the background metric, even though the pseudoforms themselves are not. This is indeed true. An explicit construction of the map with the above properties was supplied in [32] (also see [33, 35]). We will, however, not require the explicit construction of the map in this paper.

From the action given in eq. (2.1), one can easily derive the equations of motion which are

\[
d \left[ \frac{1}{2} \star_\eta d\phi + A \right] = 0 ,
\] (2.6)

\[
(1 - \star_\eta) \left[ d\phi - \mathcal{M}(A) \right] = 0 .
\] (2.7)

Let us define the following fields

\[ \hat{A} := A + \frac{1}{2} (d\phi + \star_\eta d\phi) , \] (2.8)

\[ A_g := A - \mathcal{M}(A) . \] (2.9)

In terms of these new fields, the equations of motion assume a very simple form:

\[ d\hat{A} = 0 , \] (2.10)

\[ dA_g = 0 . \] (2.11)

Notice that we can rewrite the property in eq. (2.5) as $A_g = \star_g A_g$. This is true off-shell. Therefore, we identify the variable $A_g$ as the desired chiral boson on $(\mathcal{M}, g)$ which satisfies the free massless equation of motion eq. (2.11) as expected.
The field $\hat{A}$ also satisfies the free massless equation of motion and we can see that at the level of equation of motion at least, it is completely decoupled from the physical degree of freedom carried by $A_g$. It was shown in [33] that this decoupling continues to hold even at the quantum level. Since the only term in the action that is cognizant of the background metric is $\mathcal{M}(A)$, we see that the field $\hat{A}$ does not couple to the background metric.

The decoupling of $\hat{A}$ is absolute in this regard; it does not couple to any of the physical fields and is therefore completely inaccessible to any experiment one might wish to perform. For all intents and purposes, the field $\hat{A}$ is just an auxiliary aid which enables us to formulate a description of the dynamics of self-dual form fields which possesses the desirable properties we outlined in the introduction.

There is, however, one minor trade-off we must accept when adopting Sen’s formalism: the action is not manifestly invariant under general coordinate transformations. In fact, under diffeomorphisms, the transformation properties of the field $\phi$ and the pseudoform $A$ are rather unusual (the details can be found in [32, 35]). Despite the lack of manifest general coordinate invariance, one can verify that the action is indeed invariant under general coordinate transformations, as it should be.

Before proceeding, a brief word about our notation and nomenclature is in order. Throughout the rest of the paper the Hodge dual with respect to $\eta$ will be denoted $\ast$ and we suppress the $\eta$ subscript for convenience. For the Hodge dual with respect to the background metric $g$ on $\mathbb{M}$, we will continue to explicitly write $\ast_g$. Furthermore, we suppress the subscripts $\mathbb{R}^{(1,1)}$ and $\eta$ on all integrals over Minkowski spacetime. Wherever we use the integral over $\mathbb{M}$, we mention it in the subscript or write the diffeomorphism invariant measure factor explicitly. Finally, we will use terms left-chiral (respectively, right-chiral) and self-dual (respectively, anti-self-dual) interchangeably.

3 The Stress Tensor

Despite the unfamiliar structure of the Sen’s action, one can compute the associated stress tensor in an arbitrary background following a rather simple prescription. For the case of chiral 3-forms in six dimensions this has already been done in [35]; it is evident, however, that this analysis can be readily extended to any chiral $p$-form in $2p$ dimensions. For the sake of completeness, we explicitly present the analysis for the case at hand, i.e. the case of chiral 1-form in two dimensions.

The steps to compute the stress tensor are as follows:

1. Consider the standard action for an arbitrary 1-form field $A$ living on $\mathbb{M}$

\[ S_{\text{aux}} = \frac{1}{2} \int_{\mathbb{M}, g} A \wedge \ast g A = \frac{1}{2} \int d^2 x \sqrt{-g} \ g^{\mu\nu} A_\mu A_\nu. \]  

   (3.1)

2. Evaluate the stress tensor of $S_{\text{aux}}$ the usual way by varying with respect to the metric

\[ T_{\mu\nu} := -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{aux}}}{\delta g^{\mu\nu}}. \] 

   (3.2)
3. Let the desired stress tensor of the chiral 1-form \( A_g \) (i.e. \( A_g = \star_g A_g \)) be denoted by \( T_{\mu\nu} \). Then we can extract \( T_{\mu\nu} \) from \( T_{\mu\nu} \) as
\[
T_{\mu\nu} = T_{\mu\nu} \bigg|_{A=A_g} .
\] (3.3)

In the rest of this section we derive the steps outlined above. The reader uninterested in this derivation can safely skip to the next section once they have taken note of the final expression eq. (3.25).

3.1 Relating Forms and Pseudoforms

Due to the unusual structure of Sen’s action, the only term contributing to the stress tensor in an arbitrary background is
\[
S = \int A \wedge M(A) .
\] (3.4)

Recall that \( A \) is an arbitrary self-dual form with respect to the Minkowski metric, even though the theory is defined on a two dimensional manifold \( M \) with a non-flat metric \( g_{\mu\nu} \). \( M \) is a linear map from the space of self-dual pseudoforms to space of anti-self-dual pseudoforms. In component form, the map acts as \( M(A)_{\nu} = M_{\nu}{}^\mu A_\mu \). The properties of \( M(\cdot) \) can then be translated into equivalent statements about the tensor \( M_\nu{}^\mu \).

Let us now consider a basis of 1-forms in two dimensions: \( \{v_+, v_-\} \). In fact, in a coordinate basis we have
\[
v_+ = dx^+ \quad \text{and} \quad v_- = dx^- \quad \implies \quad v_+ \wedge v_- = 2 \, dx^1 \wedge dx^2 .
\] (3.5)

It follows that any self-dual 1-form can be expanded as \( A = a_\mu(x^+) v_+^\mu \), we will abbreviate this often as \( A = a v_+ \), with the boldface serving to remind us it is a component of a 1-form. We will now try to pin down a precise relation between the basis of chiral forms with respect to \( \star_g \) and the basis \( v_\pm \).

Let us suppose we have a basis form \( \vartheta_+ \) such that \( \vartheta_+ = \star_g \vartheta_+ \). Locally, one can always write down a relation of the form
\[
\vartheta_+^\mu = T_\mu{}^\nu v_+^\nu + U_\mu{}^\nu v_-^\nu ,
\] (3.6)

where in the last line we have made use of eq. (3.6). Comparing the coefficient of \( v_+ \) on both sides, we get
\[
\mathcal{P}_\mu{}^\nu T_\mu{}^\nu = \delta_\nu{}^\alpha = \mathcal{P}_\mu{}^\nu (T^{-1})_\mu{}^\nu ,
\] (3.8)
whereas the coefficient of $v_-$ gives (in matrix notation)

$$
\mathcal{M} = -\mathcal{T}^{-1}U.
$$

(3.9)

Now note that we can write

$$
\mathcal{T}^{-1} \vartheta_+ = v_+ + \mathcal{T}^{-1}U v_-
$$

$$
= v_+,
$$

(3.10)

where we made use of eqs. (2.3) and (3.9).

It is important to note that $A_g$ is not just equal to \( \frac{1}{2} (1 + \star_g)A \) as one might naively expect. Rather, we have from the definition in eq. (2.9)

$$
A_g = \frac{1}{2} (A + \star_gA) - \frac{1}{2} \left( \mathcal{M}(A) + \star_g \mathcal{M}(A) \right).
$$

(3.11)

A consequence of self-duality condition is that under a variation with respect to the metric, the anti-self-dual part of the variation is constrained to be

$$
\frac{1}{2} (1 - \star_g) \delta_g A_g = \frac{1}{2} (\delta_g \star_g) A_g.
$$

(3.12)

On the other hand, the self-dual part of the variation can be set to zero \cite{47}. The eq. (3.12) will play an important role when we evaluate the stress tensor of the $\mathbb{T} \mathbb{T}$ deformed Lagrangian.

Finally, note that for $A = a v_+$, using eq. (3.10) we can express $A_g$ as

$$
A_g = \mathcal{T}^{-1} a \vartheta_+.
$$

(3.13)

### 3.2 Evaluating the stress tensor

We define the stress tensor the usual way for a QFT coupled to a curved background:

$$
T_{\mu \nu} := -\frac{2}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial g^{\mu \nu}}.
$$

(3.14)

In Sen’s action, the only term which will contribute in the action is the term reproduced in eq. (3.4). Recall also that $A = a_{\mu} v_{+}^{\mu}$. We can then write the stress-tensor as

$$
T_{\mu \nu} = -\frac{2}{\sqrt{-g}} a_{\mu} a_{\nu} v_{+}^{\mu} \wedge \frac{\partial \mathcal{M}}{\partial g^{\mu \nu}} (v_{+}^{\nu})
$$

(3.15)

Since by construction $v_+ - \mathcal{M}(v_+)$ is self dual with respect to $\star_g$ (recall eq. (2.5)), we can vary the self-duality condition with respect to the metric and obtain

$$
(1 - \star_g) \delta_g \mathcal{M}(v_+) = -(\delta_g \star_g) \left( v_+ - \mathcal{M}(v_+) \right).
$$

(3.16)
Hence, for $\vartheta_+ = *_g \vartheta_+$, we get

$$2 \vartheta_+ \wedge \delta_g \mathcal{M}(v_+) = - \vartheta_+ \wedge (\delta_g *_g) \left( v_+ - \mathcal{M}(v_+) \right). \quad (3.17)$$

Writing the indices explicitly and using eqs. (3.7) and (3.8) to rewrite the right-hand side, we obtain

$$2 \delta_g \mathcal{M}_\alpha \nu L^\alpha \nu \wedge v'_+ = -(\mathcal{T}^{-1})^\alpha \beta \vartheta_+ \wedge (\delta_g *_g) \vartheta_+. \quad (3.18)$$

It follows from eq. (3.6) that $\vartheta_+ \wedge v'_+ = \mathcal{T}_\alpha L^\alpha \nu \wedge v'_+$. Substituting this into the left-hand side of eq. (3.18), we get

$$2 \delta_g \mathcal{M}_\alpha \nu L^\alpha \nu v'_+ \wedge v'_+ = - \frac{1}{2} \left( v_+ - \mathcal{M}(v_+) \right) \vartheta_+ \wedge (\delta_g *_g) \left( v_+ - \mathcal{M}(v_+) \right). \quad (3.19)$$

Contracting the right-hand side with $a_\sigma$ and $a_\alpha$, the stress tensor simplifies to

$$\mathcal{T}_{\mu \nu} = \frac{1}{\sqrt{-g}} \left( A - \mathcal{M}(A) \right) \wedge \frac{\partial *_g}{\partial g^{\mu \nu}} \left( A - \mathcal{M}(A) \right) = \frac{1}{\sqrt{-g}} A_g \wedge \frac{\partial *_g}{\partial g^{\mu \nu}} A_g. \quad (3.20)$$

This stress tensor has an alternative interpretation, one that is better suited for carrying out the explicit computation, which we now explain. Consider the action

$$S_{\text{aux}} = \frac{1}{2} \int \mathcal{M}_{\mu \nu} \wedge *_g \mathcal{M}_{\mu \nu} + \frac{1}{2} \int d^2 x \sqrt{-g} \ g^{\mu \nu} A_\mu A_\nu. \quad (3.21)$$

where $A$ is an arbitrary 1-form field defined on $\mathbb{M}$. The stress tensor corresponding to this action is readily seen to be

$$\mathcal{T}_{\mu \nu} = - \frac{1}{\sqrt{-g}} A \wedge \frac{\partial *_g}{\partial g^{\mu \nu}} A \quad (3.22)$$

$$= \frac{1}{4} g_{\mu \nu} A_\alpha A^\alpha - \frac{1}{2} A_\mu A_\nu. \quad (3.23)$$

Comparing eq. (3.22) with eq. (3.20), we see our intended stress tensor can be read off as

$$\mathcal{T}_{\mu \nu} = \mathcal{T}_{\mu \nu} \bigg|_{A=A_g}. \quad (3.24)$$

This completes the derivation of the prescription outlined in the beginning of this section.

All that remains is to make use of this prescription to obtain the stress tensor for the chiral boson. Using the explicit expression eq. (3.23) and substituting $A \rightarrow A_g$ we get the desired answer

$$\mathcal{T}_{\mu \nu} = - \frac{1}{2} A_g^\mu A_g^\nu. \quad (3.25)$$
4 **$\mathcal{T}\mathcal{T}\mathcal{T}$** **Deformations of Chiral Bosons**

We now have all the ingredients required to compute the $\mathcal{T}\mathcal{T}\mathcal{T}$ deformations of chiral bosons. Of course, to ensure that the deformation does not trivially vanish, we must have both left- and right-chiral bosons in our undeformed action. We start with simplest case, when we have one left- and one right-chiral boson, and subsequently generalise this to an arbitrary number of left- and right-chiral fields.

Let us recall some basics. The $\mathcal{T}\mathcal{T}$ “flow” deforms the classical Lagrangian of the seed theory according to the following linear differential equation

$$\frac{dL_\lambda}{d\lambda} = \det T^{(\lambda)}_{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} T^{(\lambda)}_{\rho\sigma} T^{(\lambda)}_{\mu\nu},$$

and the determinant operator that triggers the flow is defined via point-splitting, which we assume implicitly. We interpret all objects in the above equation as power series in $\lambda$, the $\mathcal{T}\mathcal{T}$ coupling constant. This means:

$$L_\lambda = L_0 + \lambda L_1 + \lambda^2 L_2 + \cdots,$$

and

$$T^{(\lambda)}_{\mu\nu} = T^{(0)}_{\mu\nu} + \lambda T^{(1)}_{\mu\nu} + \lambda^2 T^{(2)}_{\mu\nu} + \cdots.$$

The goal of the following two sections will be to determine the corrections $L_{k \geq 1}$ to the classical Lagrangian, using the methods discussed in the previous sections.

4.1 **A Pair of Chiral Bosons**

In this section we study a pair of chiral bosons, i.e. one left- and one right-chiral boson. The stress energy tensor for anti-self-dual field $B_\mu$ which takes the same form as self dual form field.

$$T_{\gamma\delta} = -\frac{1}{2} B^g_\gamma B^g_\delta,$$

in which case the total stress tensor for a single self-dual and anti-self-dual field is

$$T^{(0)}_{\mu\nu} = -\frac{1}{2} [A^g_\mu A^g_\nu + B^g_\mu B^g_\nu].$$

The first deformation of Lagrangian is then given by

$$L_1 = \det \left( T^{(0)}_{\mu\nu} \right) = \frac{g}{4} (A_g \cdot B_g)^2.$$

We now proceed to the computation of the $O(\lambda^2)$ deformation of the Lagrangian. Recall from eq. (3.12), since $A_g$ is self-dual, the variation $\delta_g A_g$ is anti-self-dual with respect to $\star_g$. Similarly, we conclude that $\delta_g B_g$ is self-dual with respect to $\star_g$. Therefore, we have

$$\frac{\partial L_1}{\partial g^{\mu\nu}} = \frac{\partial g}{\partial g^{\mu\nu}} (A_g \cdot B_g)^2 + 2 (A_g \cdot B_g) A^g_\alpha B^g_\beta \frac{\partial g^{\alpha\beta}}{\partial g^{\mu\nu}},$$

While we have only discussed the case of self-dual forms so far in Sen’s formalism, the case of anti-self-dual forms are readily obtained by suitably replacing self-duality conditions with anti-self-duality conditions.
where we have used
\[
\frac{\partial A^g_\alpha}{\partial g^{\mu \nu}} B^g_\alpha = 0 \quad \text{and} \quad A^g_\alpha \frac{\partial B^g_\alpha}{\partial g^{\mu \nu}} = 0. \tag{4.8}
\]
This implies
\[
\frac{\partial \mathcal{L}_1}{\partial g^{\mu \nu}} = -g g^{\mu \nu} (A^g \cdot B^g)^2 + g (A^g \cdot B^g) A^g_{(\mu} B^g_{\nu)},
\]
\[
= -g (A^g \cdot B^g) \left[ A^g_{(\mu} B^g_{\nu)} - g_{\mu \nu} (A^g \cdot B^g) \right]. \tag{4.9}
\]
The first deformation of the stress tensor can now be calculated as
\[
T^{(1)}_{\mu \nu} := -2 \sqrt{-g} \frac{\partial \mathcal{L}_1}{\partial g^{\mu \nu}},
\]
\[
= -2\sqrt{-g} (A^g \cdot B^g) \left[ A^g_{(\mu} B^g_{\nu)} - g_{\mu \nu} (A^g \cdot B^g) \right]. \tag{4.10}
\]
One can check that the first order deformation to the stress tensor is identically zero — we demonstrate this explicitly in Appendix A. From this, it follows that higher order deformations vanish identically. That is, the $T\overline{T}$-deformed stress tensor is identical to the stress tensor of a pair of free chiral bosons. We will have more to say about this curious fact in the subsequent section.

For now, returning to our primary interest in this section — the $T\overline{T}$-deformed chiral boson action in flat spacetime — we set $g \to \eta$ and drop the subscripts from the fields. The complete deformed Lagrangian for a pair of chiral bosons is therefore exact at $\mathcal{O}(\lambda)$ and is given by
\[
\mathcal{L}_\lambda = \mathcal{L}_0 - \frac{\lambda}{4} (A \cdot B)^2. \tag{4.11}
\]

### 4.2 Arbitrary Number of Chiral Bosons

A similar calculation can be carried out for an arbitrary (but non-zero) number of self-dual and anti-self-dual bosons. The stress tensor of the undeformed Lagrangian can be written as
\[
T^{(0)}_{\mu \nu} = -\frac{1}{2} \sum_{i=1}^{N_c} A^i_{(g) \mu} A^i_{(g) \nu} - \frac{1}{2} \sum_{j=1}^{N_a} B^j_{(g) \mu} B^j_{(g) \nu}, \tag{4.12}
\]
where $A^i_{(g)}$ and $B^j_{(g)}$ are left- and right-chiral bosons, and $N_c$ and $N_a$ denote the number of such bosons respectively. The first deformation is given by
\[
\mathcal{L}_1 = \det \left( T^{(0)}_{\mu \nu} \right) = \frac{g}{4} \sum_{i=1}^{N_c} \sum_{j=1}^{N_a} \left( A^i_{(g)} \cdot B^j_{(g)} \right)^2, \tag{4.13}
\]
since $A^i_{(g)} \cdot A^j_{(g)} = 0$ and $B^i_{(g)} \cdot B^j_{(g)} = 0$.

The next order of the stress tensor is calculated by varying $\mathcal{L}_1$ with respect to the metric, as was done previously. This gives us
\[
T^{(1)}_{\mu \nu} = 2\sqrt{-g} \sum_{i=1}^{N_c} \sum_{j=1}^{N_a} \left( A^i_{(g)} \cdot B^j_{(g)} \right) \left[ A^i_{(g) \mu} B^j_{(g) \nu} - g_{\mu \nu} \left( A^i_{(g)} \cdot B^j_{(g)} \right) \right]. \tag{4.14}
\]
Similar to the case of a pair of chiral bosons in the previous subsection, we find that in the case of an arbitrary number of chiral bosons, the first order deformation to the stress tensor vanishes identically. This in turn implies that higher order deformations to the Lagrangian vanish identically. We will comment on this in the following section.

For now, we focus on the deformed Lagrangian in flat space by making the following substitutions: $g \rightarrow \eta$, $A_{(g)}^i = A^i$, and $B_{(g)}^i = B^i$ in the above equations, which in turn ensures that a drastic simplification occurs. The core advantage comes from the fact that $(A^i \cdot B^j)$ can be written in terms of components of the fields as

$$ (A^i \cdot B^j) = 2 a_i^b j^b , \quad (4.15) $$

where $A_{\mu}^i = (-a^i, a^i)$ and $B_{\mu}^j = (b^j, b^j)$. We conclude, then, that even for arbitrary number of left- and right-chiral bosons in flat background the deformation is exact at first order in the $T \bar{T}$ coupling, and is given by

$$ \mathcal{L}_\lambda = \mathcal{L}_0 - \frac{\lambda}{4} \sum_{i=1}^{N_a} \sum_{j=1}^{N_a} (A^i \cdot B^j)^2 . \quad (4.16) $$

Before proceeding, let us comment on the question of curved spacetime, since our choice of formalism is naturally developed for a 2-manifold $M$ with an arbitrary metric $g$. There is at present no consensus regarding status of $T \bar{T}$ deformations in an arbitrary curved background, although some progress has been made for spacetimes with constant curvature in [48]. It was found that the paradigmatic properties of the $T \bar{T}$ deformation such as Zamolodchikov’s factorisation formula no longer continue to hold, and one finds curvature-induced corrections.

To deform a two-dimensional quantum theory on a curved background, we must first define the operator $T \bar{T}$ as usual, via point-splitting. For curved backgrounds this involves talking about quantities defined at different spacetime points — this is, in essence, the source of the difficulty. It is not our goal to propose a general resolution to this problem. Rather, we observe that for the specific theories under consideration in this paper, Sen’s formalism enables us to express the theory at all points using pseudoforms. The pseudoforms themselves are completely oblivious to the background metric and therefore allow for the comparison of fields at different spacetime points. It seems to us that Sen’s formalism is well positioned to investigate $T \bar{T}$ deformations on curved spacetimes in greater detail. We leave this study for the future.

Before we conclude this section, we point out that a theory with a chiral anomaly, on being coupled to gravity produces a theory with a gravitational anomaly. While at the classical level, these anomalies play no role, a theory with an arbitrary number of left- and right-chiral bosons coupled to background metric is only consistent quantum mechanically when the original theory has chiral symmetry, i.e. when $N_c = N_a$. Since we focus solely on flat spacetime, these considerations have no bearing our analysis.
5 Fermions and Bosonisation

In this section we will solely work in flat space. In this case the pseudoform coincides with the physical form that is self-dual with respect to $\star$, the flat space Hodge dual.

5.1 Free Fermions and their Deformations

We have seen in the previous section that the $T\bar{T}$-deformed Lagrangian for a pair of chiral bosons is

$$L_\lambda = L_0 - \frac{\lambda}{4} (A \cdot B)^2 .$$

The fact that this deformed Lagrangian truncates at linear order in the $T\bar{T}$ coupling $\lambda$ is strongly reminiscent of the $T\bar{T}$-deformed free fermion Lagrangian, which is

$$L_\lambda = \bar{\psi} \partial \psi + \psi \bar{\partial} \psi + \lambda \left[ \left( \bar{\psi} \partial \psi \right) \left( \psi \bar{\partial} \psi \right) - \left( \bar{\psi} \bar{\partial} \psi \right) \left( \psi \partial \psi \right) \right] ,$$

and also truncates at linear order in the $T\bar{T}$ coupling. In the following section, based on this observation, we will discuss the possibility of a bosonisation duality between these theories.

Before turning to this question, however, we comment on another curious observation we made in the previous section: that the first order correction to stress tensor of theories with chiral bosons vanishes identically, i.e.

$$T^{(1)}_{\mu\nu} = 0 .$$

In fact, it is easily verified that the same is true for the deformed free fermion as well. To summarise, we have a seed conformal field theory and an irrelevant deformation thereof, both of which have the same (traceless, of course) stress tensor, the same degrees of freedom, and the same central charge. To the best of our knowledge, this has not been remarked on in the literature.

That the stress tensors of the seed theory and the deformed theory coincide, despite the fact that the deformed theory contains an interaction term, is highly unusual. The interaction term, it would seem, has no impact on the energetics of the system.

A possible resolution to this puzzle that immediately comes to mind is that the free and deformed Lagrangians are related by a nonlinear field redefinition. If this is the case, one diagnostic in the absence of the explicit field redefinition would be to compute the S-matrices of the deformed theory, which should be trivial. This line of investigation might be able to explain a puzzle we raised in [21]: how is it possible for strongly interacting chiral fermions to, in the infinite coupling limit, decouple? If this resolution is correct, we would answer that the ostensibly interacting chiral fermions were actually always free.

We leave the resolution of this puzzle for future work.
5.2 A Bosonisation Map

It is tempting to conclude, based on the observation that both theories have similar behaviour under $\mathbb{T}T$ deformation, that there might exist a bosonisation duality relating the two theories. Indeed, their respective finite-volume spectra are deformed in a universal manner, dictated by the inviscid Burgers equation.\(^3\) The fact that for both theories the truncation of the deformation at linear order continues to hold independent of the number of left- and right-chiral fields further strengthens the hypothesis of a possible bosonisation relation. In this section we speculate on this possibility.

Let us focus on the deformation terms. On-shell, the deformation on the fermion side is

$$\lambda \left( \psi_+ \partial \psi_- \right) \left( \psi_+ \partial \psi_- \right),$$

whereas on the boson side we have

$$\frac{\lambda}{4} (A \cdot B)^2.$$ 

Let us write the latter in terms of components, for which we use $A_\mu = (-a, a)$ and $B_\mu = (b, b)$ as before. We then find the bosonic interaction term is

$$\lambda a^2 b^2.$$ 

If we were to guess a map, then based on considerations of chirality and dimension, we might say

$$a^2 \longleftrightarrow \psi_+ \partial \psi_+,$$

$$b^2 \longleftrightarrow \psi_- \partial \psi_-.$$ 

While these formula do not look like the usual bosonisation correspondence — between chiral fermions and vertex operators of chiral bosons — we should keep in mind that $a$ and $b$ are really components of the field strengths.

It may be tempting to think of $a \sim \partial_+ \varphi$ and $b \sim \partial_- \varphi$ on-shell, which would help relate eq. (5.7) to the usual bosonisation formulas. However, we take the perspective that this is antithetical to the spirit of Sen’s formalism, not to mention severely limiting any attempt to generalise this to the corresponding quantum theory.

We emphasise that this is merely suggestive of the possibility that $\mathbb{T}T$ deformations “commute” with bosonisation, i.e. that bosonisation duality persists under irrelevant deformations. Much more work would be required to establish that this is in fact the case. In fact, it would be an interesting exercise to just start with a free pair of chiral bosons in Sen’s formalism, and develop a bosonisation dictionary that establishes a correspondence with free chiral fermions. We hope to return to this question in the future.

\(^3\)We thank Shouvik Datta for emphasising this point.
In this paper, we have studied the $T\bar{T}$-deformed Lagrangian corresponding to a theory with an arbitrary number of left- and right-chiral bosons. We have established that the deformations truncate at linear order in the $T\bar{T}$ coupling, a feature also exhibited by $T\bar{T}$-deformed free fermions. The choice to work with Sen’s formalism allowed us to present the first instance of a $T\bar{T}$ deformation of a theory with a non-standard action. In concluding, we highlight a couple of interesting directions for further research we would like to pursue that follow straightforwardly from our analysis.

Firstly, a natural question is to ask what one obtains if one considers the $\lambda \to \infty$ limit of the $T\bar{T}$-deformed Lagrangian in eq. (4.11). This, as we saw in [21], requires that we work in the Hamiltonian formalism. It is natural to expect that the infinite coupling limit will once again result in a decoupling of the chiral sectors of the theory. In the passage to the Hamiltonian formalism presented in [33], it is assumed that a single chiral field is interacting with other physical fields, the latter being described by standard Lagrangians. For the case of the $T\bar{T}$-deformed chiral bosons, the interaction term is not of this form. A small extension of Sen’s formalism to accommodate this case would be necessary.

Secondly, the observation that both the $T\bar{T}$-deformed Lagrangians corresponding to free fermions and chiral bosons truncate at linear order in the coupling raises the intriguing possibility that the deformed theories (like their undeformed parents) are bosonisation duals. In fact, we observed that the stress tensors of the undeformed and deformed theories coincide for both these theories, making the case for bosonisation even stronger. This presents us with a possible answer to the question we posed at the start of the paper. Schematically, we have built a case for the following progression.

\[
\text{Free Bosons} \xrightarrow{T\bar{T}} \text{Nambu-Goto String} \xrightarrow{\lambda \to \infty} \text{Chiral Bosons} \xrightarrow{?} T\bar{T}
\]

In §5, we looked at the interaction terms on-shell and proposed a possible correspondence. However, to shore this correspondence up would require much more work, in particular the computation of deformed correlation functions, which continues to be an active area of research. Indeed, this question is logically prior to any considerations related to deformations, irrelevant or otherwise. As we have discussed, the standard bosonisation map sends chiral fermions to vertex operators of chiral bosons. An understanding of how bosonisation might manifest in Sen’s formalism requires that we first understand how to construct vertex operators in this formalism. It would be an interesting exercise to then check that paradigmatic examples of this duality (for example, the duality between the sine Gordon theory and the massive Thirring model) can be reproduced. We hope to return to these questions in the future.

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A  Vanishing Correction to Stress Tensor

In eq. (4.10), we computed the first order correction to the stress tensor induced by a $T \bar{T}$ deformation. In this appendix we show explicitly that

$$g_{\mu\nu} (A \cdot B) = A^\eta \mathcal{B}^\eta + A_\eta \mathcal{B}_\eta \ ,$$  \hspace{1cm} (A.1)

by explicitly evaluating the components. This is equivalent to the demonstration that $T^{(1)}_{\mu\nu}$ vanishes identically. To avoid clutter, we drop the subscript $g$.

The left-hand side of eq. (A.1) when $(\mu, \nu) = (0, 0)$ can be written as

$$g_{00} (A_0 B^0 + A_1 B^1) = g_{00} (A_0 B^0 + A^0 B_0) \ ,$$  \hspace{1cm} (A.2)

where we have used

$$\epsilon^{\mu\nu} A_\nu = \sqrt{-g} A^\mu \quad \text{and} \quad \epsilon^{\mu\nu} B_\nu = -\sqrt{-g} B^\mu \ ,$$  \hspace{1cm} (A.3)

to rewrite the second term. This equation can be further simplified as

$$A_0 (g_{00} B^0) + (g_{00} A^0) B_0$$
$$= A_0 (B_0 - g_{01} B^1) + (A_0 - g_{01} A^1) B_0$$
$$= 2A_0 B_0 - g_{01} (A_0 B^1 + A^1 B_0)$$
$$= 2A_0 B_0 \ ,$$  \hspace{1cm} (A.4)

which is the R.H.S of (A.1). In the second last line, we make use of (A.6).

Similarly, for $(\mu, \nu) = (1, 1)$,

$$g_{11} (A_0 B^0 + A_1 B^1) = g_{11} (A^1 B_1 + A_1 B^1)$$
$$= (A_1 - g_{01} A^0) B_1 + A_1 (B_1 - g_{01} B^0)$$
$$= 2A_1 B_1 - g_{01} (A^0 B_1 + A_1 B^0)$$
$$= 2A_1 B_1 \ ,$$  \hspace{1cm} (A.5)

and for $(\mu, \nu) = (1, 0)$,

$$g_{10} (A_0 B^0 + A_1 B^1) = g_{10} (A_0 B^0 + A^0 B_0)$$
$$= A_0 (B_1 - g_{11} B^1) + (A_1 - g_{11} A^1) B_0$$
$$= A_0 B_1 + A_1 B_0 - g_{11} (A_0 B^1 + A^1 B_0)$$
$$= A_0 B_1 + A_1 B_0 .$$  \hspace{1cm} (A.6)

This establishes eq. (A.1). Substituting eq. (A.1) into eq. (4.10), one can easily check that indeed $T^{(1)}_{\mu\nu}$ vanishes.

Finally, the above derivation generalizes straightforwardly for the case of arbitrary number of left- and right-chiral boson allowing us to draw the same conclusion for the general case as well.
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