Environment-induced uncertainties on moving mirrors in quantum critical theories via holography

Da-Shin Lee and Chen-Pin Yeh

Department of Physics, National Dong-Hwa University, Hualien, Taiwan, R.O.C.

Abstract

Environment effects on a $n$-dimensional mirror from the strongly coupled $d$-dimensional quantum critical fields with a dynamic exponent $z$ in weakly squeezed states are studied by the holographic approach. The dual description is a $n+1$-dimensional probe brane moving in the $d+1$-dimensional asymptotic Lifshitz geometry with gravitational wave perturbations. Using the holographic influence functional method, we find that the large coupling constant of the fields reduces the position uncertainty of the mirror, but enhances the momentum uncertainty. As such, the product of the position and momentum uncertainties is independent of the coupling constant. The proper choices of the phase of the squeezing parameter might reduce the uncertainties, nevertheless large values of its amplitude always lead to the larger uncertainties due to the fact that more quanta are excited as compared with the corresponding normal vacuum and thermal states. In the squeezed vacuum state, the position and momentum of the mirror gain maximum uncertainties from the field at the dynamic exponent $z = n + 2$ when the same squeezed mode is considered. As for the squeezed thermal state, the contributions of thermal fluctuations to the uncertainties decrease as the temperature increases in the case $1 < z < n + 2$, whereas for $z > n + 2$ the contributions increase as the temperature increases. These results are in sharp contrast with those in the environments of the relativistic free field. Some possible observable effects are discussed.

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*Electronic address: dslee@mail.ndhu.edu.tw
†Electronic address: chenpinyeh@mail.ndhu.edu.tw
I. INTRODUCTION

Macroscopic quantum phenomena often refer to collective quantum behavior in objects, consisting of a large number of particles in atomic scales \([1, 2]\). The best known examples are superconductivity and superfluidity. Additionally, experimental realizations of Bose-Einstein condensation in dilute gases certainly provide a more fruitful venue, in which various macroscopic quantum phenomena are explored under experimental controls. Moreover the progress in electro- and opto-mechanical techniques makes it possible to prepare macroscopic or mesoscopic mechanical objects in nearly pure quantum states (See \([3–6]\)) where the center of mass of an object obeys a quantum mechanical equation of motion. Recently experiments to demonstrate quantum interference between the macroscopic objects have been proposed in \([7, 8]\). In those experiments it is essential that a macroscopic system like the mirror is prepared in the quantum superposition state.

Because of a large number of the degrees of freedom in macromechanical systems, the observability of the quantum behavior will be strongly influenced by interactions with the environment and the experimentally accessible quantum region will also depend on the decoherence dynamics due to the presence of the environment \([5]\). A viable microscopic approach to investigate the environmental effects on the system would start with a specific system-environment model. Then the environmental degrees of freedom are integrated out by the method of Feynman-Vernon influence functional. This approach consistently and systematically accounts for the influence of the environment on the system of interest \([9–11]\). The influence functional can be exactly derived if the environment variables are Gaussian and their coupling with the system is linear \([12, 13]\). In particular, the effects from the quantized electromagnetic fields on a point charge in the dipole approximation have been studied extensively by \([14–17]\).

In the work \([18]\), the environment is modeled by a free massless scalar field in vacuum and thermal states, and its coupling to the system of the particle, which is a harmonic oscillator, is linear in particle’s position. They focused on the evolutions of particle’s reduced density matrix which initially is in vacuum and squeezed states, and explored the uncertainties of particle’s position and momentum due to the interaction with the environment. What they found is that if the system is prepared in a pure state, the loss of quantum coherence can happen as a result of the coupling to the environment. In particular, when the environ-
ment field is in zero temperature, the off-diagonal terms of the reduced density matrix in
the position representation decrease more rapidly than in the momentum representation,
resulting in relatively small position uncertainty. This comes from the fact that the system
is coupled to the environment by its position variable. They also discussed the changes in
these uncertainties by varying the squeeze parameters of the system and the temperature
of the environments. Here we would like to explore these effects from the environments of
strongly coupled fields and also allow the dimensions of probe objects and environments to
be arbitrary. The purpose is to make possible comparisons with various cases in weakly
coupled environments.

In quantum field theory, the correlators of weakly interacting quantum fields are nor-
manly computed perturbatively in terms of the small coupling constant. As for strongly
coupled fields in high dimensions, the holographic correspondence is among very few known
nonperturbative ways to calculate their correlators. Thus in this paper, we will extend the
results in [18] by considering the strongly coupled environment that admits a holographic
description. The idea of holographic duality is originally proposed as the correspondence
between 4-dimensional conformal field theory (CFT) and gravity theory in 5-dimensional
anti-de Sitter (AdS) space [19]. Other backgrounds and field theories are soon to be general-
ized with the possibility to study the strong coupling problems in the condensed matter
systems (see [20] for a review). Considerable efforts also have been focused on using the
holography idea to explore the Brownian motion of a particle moving in a strongly coupled
environment [21–41]. A review on the holographic Brownian motion can be found in [28].

Here we will apply a bottom-up holographic method, proposed in our earlier work [38],
to find the uncertainties of a \( n \)-dimensional mirror in the environment of \( d \)-dimensional
quantum critical theories at zero and finite temperature. The holographic dual for such
quantum critical theories has been proposed in [42] where the gravity theory is in the Lifshitz
background (See [29, 30] for details). Several physical phenomena have been studied in this
theory, including linear DC conductivity, power-law AC conductivity, and strange fermion
behaviors \[30, 43–46\]. In our set-up, the bulk counterpart of the mirror is a \((n + 1)\)-brane
in the Lifshitz geometry in \( d + 1 \) dimensions. The motion of the mirror can be realized from
the dynamics of the brane at the boundary of the bulk. As explained in [38] and will also
be reviewed in the appendix, this holographic identification is based upon the fact that the
coupling of the brane to the boundary field shares similar feature as the coupling between
the mirror and the environment quantum field where the mirror is of perfect reflection for the field \[38, 47\]. The force on the mirror is given by the position change of the mirror \[38\]. It has been discussed in \[38\] that, for \(z = 1\) (the relativistic environmental field) and for a 2-dimensional mirror, the ohmic dynamics of the mirror with the \(T^4\) dependence of the damping constant due to the strongly coupled environment field at finite temperature \(T\), is in agreement with the finding in \[47\] where the relativistic thermal free field is considered. It is also expected that the proportionality constant in the damping constant is different between the strongly coupling environment field and the free field. In this paper, we consider the environment of the squeezed vacuum (thermal) baths, whose holographic duals arise from gravitational wave perturbations in Lifshitz (black hole) background, as suggested in \[40, 41, 48\]. Using the method of holographic influence functional, developed in \[39\], we then study the uncertainties of the position and momentum of the mirror.

Our presentation is organized as follows. In next section, we briefly review the method of holographic influence functional for the squeezed states and explain the duality between the squeezed state and the gravitational wave perturbed Lifshitz black hole. The reviews on the method of influence functional in field theory and the construction of holographic influence functional for pure Lifshitz geometry and Lifshitz black hole are in the appendices. In Sec. \[III\], the evolution of the uncertainties of a mirror, influenced by the environment of strongly coupled fields, are computed. The comparisons with the results from the environment given by the relativistic free-field are also discussed. Summary and outlook are in Sec. \[IV\].

The sign convention \((-,-,\ldots,+)\) is adopted in the \(d+1\)-dimension metric in dual gravity theory with indices \(\mu, \nu, \ldots\) Indices \(a, b, c, \ldots\) denote all spacetime coordinates in the boundary field theory while \(i, j, k, \ldots\) denote only spatial dimensions.

**II. HOLOGRAPHIC INFLUENCE FUNCTIONAL FOR THE ENVIRONMENT FIELD IN SQUEEZED VACUUM/ THERMAL STATES**

The environment fields we consider in this paper is the theory of quantum critical points with the following scaling symmetry:

\[
t \to \mu^z t, \quad x \to \mu x
\]  \hspace{1cm} (1)
where \( z \) is called the dynamical exponent. The holographic dual for such quantum critical theories in 2+1-dimension has been proposed in [42], where the corresponding gravity theory is in the 3+1-dimensional Lifshitz background. Here we consider the \( d+1 \)-dimensional background, which is asymptotic to the Lifshitz metric,

\[
ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -\frac{r^{2z}}{L^2} dt^2 + \frac{1}{r^2} dr^2 + \frac{r^2}{L^2} dx_i dx_i, \tag{2}
\]

where the scaling symmetry (1) is realized as an isometry of this metric. This \( d+1 \)-dimensional Lifshitz metric can be constructed by coupling gravity with negative cosmological constant to massive Abelian vector fields [49]. The corresponding action is given by:

\[
S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-g} (R + 2\Lambda - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} m^2 A_\mu A_\mu). \tag{3}
\]

The action yields the equations of motion for the metric and the vector fields,

\[
R_{\mu\nu} = -\frac{2\Lambda}{d-1} g_{\mu\nu} + \frac{1}{2} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} + \frac{1}{2} m^2 A_\mu A_\nu - \frac{1}{4(d-1)} F_{\beta\alpha} F^{\beta\alpha} g_{\mu\nu}, \tag{4}
\]

\[
D_\mu F^{\mu\nu} = m^2 A^\nu, \tag{5}
\]

where \( D_\mu \) is the covariant derivative. The solutions of the vector fields are assumed to be

\[
A_\mu = A \frac{r^z}{L^z} \delta_\mu^0. \tag{5}
\]

Then the Lifshitz background in (2) is the solution of (1) with

\[
A = \sqrt{\frac{2(z-1)}{z}}, \quad m^2 = \frac{(d-1)z}{L^2}, \quad \Lambda = \frac{(d-1)^2 + (d-2)z - z^2}{2L^2}. \tag{6}
\]

In particular, we consider the Lifshitz black brane perturbed by the gravitational wave with metric (we set the radius of curvature \( L \) to one),

\[
d\tilde{s}^2 = -r^{2z} f(r) dt^2 + \frac{dr^2}{f(r)r^2} + r^2 dx_i dx_i + r^2 \phi(t,r) \xi_{\mu\nu} dx^\mu dx^\nu, \tag{7}
\]

where \( \xi_{\mu\nu} \), the polarization tensor, has non-zero components only in the spatial directions \((i,j)\) of the boundary, and is assumed transverse and traceless. Here \( \phi(t,r) \) is assumed to be small, and its equation of motion will be determined later. We also have \( f(r) \to 1 \) for \( r \to \infty \) and \( f(r) \approx c(r - r_h) \) near the black brane horizon \( r_h \) with \( c = (d + z - 1)/r_h \). For example, the model in (3), for the case \( d = 3 \) and \( z = 2 \), has the exact black hole solution [50],

\[
f(r) = \sqrt{1 + \frac{1}{10r^2} - \frac{3}{400r^4}} \tag{8}
\]
Also, in the case of $d = 3$ and $z = 1$, the AdS black brane solution is found with $f(r) = 1 - \frac{r^3}{r_h}$. However, for general values of $z$ and $d$, only numerical and perturbative solutions are available. Nevertheless, the detailed form of $f(r)$ is not very relevant when considering the perturbations in low frequency limits, as will be seen in our subsequent discussions. The black hole temperature, which is also the temperature in the boundary field theory, is

$$T = \frac{4\pi}{d + z - 1} r_h^z.$$  

As suggested in [48] and also in our earlier works [40, 41], a possible holographic realization of the weakly squeezed vacuum (thermal) state of the boundary field is given by gravitational wave perturbed Lifshitz (black brane) background in (7). To justify this identification, we consider two ways of deriving correlators for the position of mirror. One is by the holographic influence functional method [39], and the other is through the Bogoliubov transformations of the excitations on the probe brane. We first describe the holographic influence functional in the following.

In quantum field theory, the influence functional is a way to summarize the effect of the quantum field to a mirror’s position. We give a review in Appendix A. In the holographic setup, the dual description of the mirror is a $n+1$-dimensional probe brane in the asymptotic Lifshitz background (7). In accordance with the closed-time-path formalism [9, 10, 12] that we have discussed in Appendix A, we introduce $Q^+(t, r_1)$ and $Q^-(t, r_2)$, which correspond to the branes living in two regions with different asymptotic boundaries in the maximally extended Lifshitz black hole geometry [51, 52]. $Q^+(t, r_1)$ and $Q^-(t, r_2)$ are determined by their analytical properties at $r = r_h$ [52] or equivalently by the unitarity arguments of the boundary theory [39]. We also impose the following boundary conditions

$$X^\pm(t) = Q^\pm(t, r_b),$$

where the variable $X(t)$ can be identified as the displacement of the moving mirror. Then the classical on-shell action of the brane is identified as the influence functional for the mirror [51]:

$$\mathcal{F}[q^+, q^-] = S_{\text{gravity}}(Q^+(t, r_h), Q^-(t, r_h)) = S_{\text{on-shell}}^{\text{DBI}}(Q^+) - S_{\text{on-shell}}^{\text{DBI}}(Q^-)$$

where $S_{\text{on-shell}}^{\text{DBI}}$ is the on-shell DBI action for the probe brane. To quadratic order with the background (7), we can write

$$S_{\text{DBI}} = -\frac{T_{n+1} S_n}{2} \int dr dt \left( r^{z+n+3} f(r) (1+\phi(r,t)) X'^2 X'^2 - (1+\phi(r,t)) \frac{\dot{X}^I \dot{X}^I}{f(r)r^{z-n-1}} \right),$$

(12)
where $T_{n+1}$ and $S_n$ are the tension and area of the brane respectively and $X^I(t, r)$ parameterizes the brane’s position with $I = n + 1, \ldots, d$ denoting the transverse directions to the brane. Also, $X^I = \partial_r X^I$, $\dot{X}^I = \partial_t X^I$. We assume the mirror does not deform when moving in its transverse directions so $X^I$‘s depend only on $t$ and $r$. Then, up to the first order in $\phi$, the equation of motion of the perturbation of the brane derived from (12) becomes

$$\frac{\partial}{\partial r} \left[ r^{z+n+3} f(r) \frac{\partial}{\partial r} X^I(r, t) \right] - \frac{\partial}{\partial t} \left[ \frac{1}{f(r) r^{z-n-1}} \frac{\partial}{\partial t} X^I(r, t) \right] = -\frac{\partial}{\partial r} \left[ f(r) r^{z+n+3} \phi(r, t) \frac{\partial}{\partial r} X^I(r, t) \right] + \frac{\partial}{\partial t} \left[ \frac{1}{f(r) r^{z-n-1}} \phi(r, t) \frac{\partial}{\partial t} X^I(r, t) \right].$$ (13)

Using the equation of motion above, the classical on-shell action with the boundary terms is given by

$$S_{DBI}^{\text{on-shell}}(1+\phi) \simeq -\frac{T_{n+1} S_n}{2} \int dt \left( 1 + \phi(t, r_b) \right) \left( X^I(t, r_b) \partial_r X^I(t, r_b) \right).$$ (14)

The solution in frequency space can be expressed perturbatively as

$$X^I_\omega(r) = X^I_\omega(0)(r) + X^I_\omega(\phi)(r),$$ (15)

where the zeroth-order solutions $X^I_\omega(0)(r)$ at zero temperature and finite temperature will be reviewed in Appendix B and are given respectively by (B10) for all $\omega > 0$ and (B6) for small $\omega$. Then the equation of motion for $X^I_\omega(\phi)(r)$ to leading order is given by

$$\frac{\partial}{\partial r} \left[ r^{z+n+3} f(r) \partial_r X^I_\omega(\phi)(r) \right] + (r^{-z+n+1} / f(r)) \omega^2 X^I_\omega(\phi)(r)$$

$$= - \int d\omega' \left[ r^{z+n+3} f(r) \partial_r \phi(\omega, r) \partial_r X^I_{\omega-\omega'}(r) + \omega' (\omega - \omega') (r^{-z+n+1} / f(r)) \phi(\omega', r) X^I_{\omega-\omega'}(r) \right].$$ (16)

Now we would like to find the perturbed on-shell action. To this end, we only need the asymptotical forms of the solutions $\phi(t, r)$ and $X^I_\omega(\phi)(r)$ in the large $r$ limit by taking the $f(r) \to 1$ limit in (16). In addition, the dynamics of $\phi(t, r)$ in the limit of large $r$ can also be obtained by considering the gravitational waves on the Lifshitz metric $g^{(0)}_{\mu \nu}$ in (2). Thus, the equation of motion for $\phi(t, r)$ in this limit is

$$r^{-2z} \frac{\partial^2}{\partial t^2} \phi(t, r) + (d - 1 + z) r \frac{\partial}{\partial r} \phi(t, r) + r^2 \frac{\partial^2}{\partial r^2} \phi(t, r) = 0,$$ (17)

which can be derived by linearizing (4) about the background solutions (2) and (5). The Fourier transform of the $\phi(t, r)$ field in frequency space is defined as

$$\phi(t, r) = \int_0^\infty \frac{d\omega}{2\pi} \phi(\omega, r) e^{-i\omega t}.$$ (18)
and the normalizable solution of \((17)\) is
\[
\phi(\omega, r) = r^{-\frac{d+z-2}{2}} \varphi(\omega) J_{\frac{d+z-2}{2}} \left( \frac{\omega}{2r^2} \right) + c.c. .
\] (19)

The function \(\varphi(\omega)\) is determined by the boundary conditions of the gravitation waves at \(r = r_b\). Note that we impose the normalizable boundary condition for \(\phi(t, r)\) at \(r_b\) rather than the infalling boundary condition in the black brane horizon, normally adopted to construct the retarded correlators. This boundary condition is obtained by adding the correct boundary counterterms in the gravity theory so that the dual boundary stress tensor obtained satisfies the trace Ward identity and is independent of \(r_b\). This is also necessary for us to identify \(\varphi(\omega)\) as a squeezed parameter in the boundary theory.

The asymptotical form of the solution \(X_\omega^{f(\phi)}(r)\) in the large \(r\) limit has been discussed in \([40, 41]\), where we found that, to the leading order in \(\varphi(\omega)\), we can ignore the contribution of \(X_\omega^{f(\phi)}(r)\) in the zero temperature on-shell action by taking the large \(r_b\) limit. Here the same conclusion can be reached for the finite-temperature on-shell action \((14)\) due to the similar arguments as follows. Using \((16)\), the dependence of \(X_\omega^{f(\phi)}(r)\) on \(r\) for large \(r\) is mainly determined by \(\partial_r [r^{z+n+1} \partial_r X_\omega^{f(\phi)}(r)] \approx r^{-z+n+1} \phi(\omega, r)\) as \(f(r_b) \to 1\) and \(X_\omega^{f(0)}(r_b) \to 1\) in \((B10)\). From the asymptotic behavior of \(\phi(\omega, r) \approx r^{-d-z+2}\) as given in \((19)\), it leads to \(X_\omega^{f(\phi)}(r) \approx r^{-d-3z+2}\). Thus we conclude that \(\phi(\omega, r_b) \gg X_\omega^{f(\phi)}(r_b)/X_\omega^{f(0)}(r_b)\) for large \(r_b\), so the contributions from \(X_\omega^{f(\phi)}\) to the above perturbed action \((12)\) can be ignored when we keep terms up to linear order in \(\varphi(\omega)\). Thus, using the holographic influence functional prescription \((11)\) where \(Q\) denotes one of the directions \(X_I\), the on-shell perturbed action \(S_{DBI, \phi}^{\text{on-shell}}\) obtained from \((14)\) is expressed as

\[
S_{DBI, \phi}^{\text{on-shell}}(Q^+) - S_{DBI, \phi}^{\text{on-shell}}(Q^-) =
- \frac{T_{n+1} S_{n}}{2} r_b^{z+n+3} \int \frac{dt}{r_b} \phi(t, r_b) \left( Q_+^+(t, r_b) \partial_r Q_+^+(t, r_b) - Q_-(t, r_b) \partial_r Q_-^-(t, r_b) \right)
= - \frac{T_{n+1} S_{n}}{2} r_b^{z+n+3} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \phi(\omega + \omega', r_b) \left( Q_+^+(r_b) \partial_r Q_+^+(r_b) - Q_-^-(r_b) \partial_r Q_-^-(r_b) \right).
\] (20)

Substituting the expression \((B4)\) with \(\mathcal{X}_\omega(r)\) constructed by the zeroth-order solutions \(X_\omega^{f(0)}(r)\) in Appendix \([B]\) for zero and finite temperature into the above expression, the respective perturbed holographic influence functionals are obtained. These nonequilibrium Green’s functions constructed from the perturbed influence functionals can be compared.
with the form of the Green’s functions in the squeezed vacuum and thermal states, obtained
by means of the corresponding squeeze operator as we will describe in the following. Then
the function $\varphi$ in \[19\], determined by the boundary condition of gravitation waves, can be
identified as the squeezing parameters of the squeezed vacuum and thermal states.

Now we turn to the second way of deriving correlators for the mirror’s position. According
to the holographic correspondence \[28, 29\], the correlation functions of the boundary fields
can also be found from the correlation functions of the probe brane’s position by taking
the near boundary limit. From the boundary point of view, this link between two ways of
deriving correlation functions is established through the Langevin equation for mirror’s posi-
tion, which can also be derived from the corresponding holographic influence functional \[40\].
Using this method, we first consider the mode expansion of the brane’s position operator
$X(t, r)$. As in previous discussions, we assume that the brane’s position is independent of
$x_1, x_2, \ldots, x_n$. In the small displacement limit, the dynamics of the brane in different direc-
tions along $X^I$ with $I = n + 1, \ldots, d$, are decoupled. Thus we can just consider the brane’s
motion in one of those directions, which is denoted by $X(t, r)$ . Accordingly, the mode
expansion of the position operator evaluated at $r = r_b$, which is identified as the position of
the mirror is given as

$$X(t) \equiv X(t, r_b) = \int_0^{\infty} d\omega U_\omega(r_b) \left( a_\omega e^{-i\omega t} + a_\omega^\dagger e^{i\omega t} \right),$$

where $a_\omega$ and $a_\omega^\dagger$ are the annihilation and creation operators, and they obey canonical
commutation relations

$$[a(\omega), a^\dagger(\omega')] = \delta(\omega, \omega'), \quad [a(\omega), a(\omega')] = [a^\dagger(\omega), a^\dagger(\omega')] = 0 \, .$$

In the background of Lifshitz black hole, the brane’s perturbations are in thermal states
where $\langle a_\omega^\dagger a_\omega \rangle_T = (e^{\frac{\pi}{T} \omega} - 1)^{-1}$ with black hole temperature $T$. The mode functions $U_\omega(r)$
are found from \[133\] with the Neumann boundary condition and the Wronskian condition
(See \[38\] for details). Since the mode function near the horizon $r = r_h$ exhibits logarithmic
divergence, an infrared energy cutoff scale as $r \to r_h$ is introduced for regularization. We
may absorb this infrared divergence by carefully defining the density of states \[28\]. The
square of the divergence-free mode function in the low frequency limit is obtained in our
previous work \[38\] as

$$U_{\omega}^2(r_b) = \frac{1}{\pi^2 T_{n+1} S_n r_h^{n-2}} + O(\omega^0) \, .$$

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In the zero-$T$ limit, the mode function squared evaluated at $r_b$ can be found exactly as

$$U^{(0)}_\omega(r_b) = \frac{2\pi r_b^{2-n}}{\omega^2 T^{n+1} S_n J^{2\frac{n+2}{2}} - \frac{1}{2} \left( \frac{\omega}{\pi T} \right)} + \frac{Y^{2\frac{n+2}{2}}}{\pi T} \left( \frac{\omega}{\pi T} \right).$$

(24)

The squeezed vacuum and thermal states can be constructed from the Bogoliubov transformations of the creation and annihilation operators of the normal vacuum and thermal states. Here we assume the general two-mode squeezed thermal states. We will see later that the corresponding Green’s functions of boundary fields, in the small squeeze parameter limit, have the same form as those of the Green’s function constructed from the perturbed influence functional in gravitational wave perturbed Lifshitz black hole. The squeezed vacuum state can be obtained by taking the zero temperature limit. Using the squeeze operator, the two-mode squeezed thermal states can be defined as

$$\rho^T_{\xi_{\omega\omega'}} = S(\xi_{\omega\omega'}) \rho_T S^\dagger(\xi_{\omega\omega'}) \quad S(\xi_{\omega\omega'}) = \exp \left[ \frac{1}{2} \left( \xi_{\omega\omega'}^* a_\omega a_{\omega'} - \xi_{\omega\omega'} a_\omega^\dagger a_{\omega'}^\dagger \right) \right]$$

(25)

with the thermal density matrix $\rho_T$ in (A3) and the squeeze parameter $\xi_{\omega\omega'} = r_{\omega\omega'} e^{i\theta_{\omega\omega'}}$.

With the help of the Baker-Campbell-Hausdorff formula, we readily find the Bogoliubov transformations of the creation and annihilation operators due to the squeeze operator $S(\xi_{\omega\omega'})$,

$$S^\dagger(\xi_{\omega\omega'}) a_\omega S(\xi_{\omega\omega'}) = \mu_{\omega\omega'} a_\omega - \nu_{\omega\omega'} a_{\omega'}^\dagger,$$

$$S^\dagger(\xi_{\omega\omega'}) a_{\omega'} S(\xi_{\omega\omega'}) = \mu_{\omega\omega'} a_{\omega'} - \nu_{\omega\omega'} a_\omega^\dagger,$$

$$S^\dagger(\xi_{\omega\omega'}) a_\omega^\dagger S(\xi_{\omega\omega'}) = \mu_{\omega\omega'} a_\omega^\dagger - \nu_{\omega\omega'} a_{\omega'}^\dagger,$$

$$S^\dagger(\xi_{\omega\omega'}) a_{\omega'}^\dagger S(\xi_{\omega\omega'}) = \mu_{\omega\omega'} a_{\omega'}^\dagger - \nu_{\omega\omega'}^* a_\omega,$$

(26)

and we have

$$\langle a_\omega \rangle_{T \xi} = 0, \quad \langle a_\omega a_{\omega'} \rangle_{T \xi} = -\mu_{\omega\omega'} \nu_{\omega\omega'} (1 + n_\omega + n_{\omega'}) \quad \langle a_\omega^\dagger a_{\omega'} \rangle_{T \xi} = n_\omega + \eta_{\omega\omega'}^2 (1 + 2n_{\omega'}) \delta(\omega - \omega'),$$

(27)

where $\mu_{\omega\omega'} = \cosh r_{\omega\omega'}$, $\nu_{\omega\omega'} = e^{i\theta_{\omega\omega'}} \sinh r_{\omega\omega'}$ and $\eta_{\omega\omega'} = |\nu_{\omega\omega'}|$. Notice that the retarded Green’s function (A9), given by the expectation value of the commutator of the field $F$, remains the same in the two-mode squeezed thermal state because the involved Bogoliubov transformations are the canonical transformations that preserve the commutation relations between the creation and annihilation operators. Moreover the position correlator $\langle X(t) X(t') \rangle$ in the squeezed vacuum and thermal states can be calculated straightforwardly.
Now we are in the stage to find the corresponding Green’s function of the environment field in squeezed thermal state, which can be obtained by the associated Langevin equation. The Langevin equation of the mirror with the effects from the environment can be straightforwardly derived from the influence functional \[40\] as

\[
\int dt' G_R(t, t') X(t') = \eta(t).
\] \hspace{1cm} (28)

The noise force correlation function is given by

\[
\langle \eta(t) \eta(t') \rangle = G_H(t, t') .
\] \hspace{1cm} (29)

Then, according to the Langevin equation in its Fourier transformed form, the fluctuations on the position can be related to the retarded and Hadamard functions with respect to the normal vacuum and thermal states as follows:

\[
\langle X(\omega)X(-\omega) \rangle = \frac{G_H(\omega)}{G_R(\omega)G_R^*(\omega)} .
\] \hspace{1cm} (30)

At finite temperature, the Langevin equation gives the relation

\[
G^T_H(\omega) = \pi \left( \frac{\frac{e^\omega}{T} + 1}{e^\omega - 1} \right) \frac{G^T_R(\omega)G^T_R^*(\omega)}{U_\omega^2(r_b)} .
\] \hspace{1cm} (31)

We can check that our approximate results for Green’s functions in (B5), (B6), (B9) and (23), in the low frequency limit, satisfy this relation. A similar relation at zero-$T$ can be found by taking the $T \to 0$ limit. Using the Langevin equation in (28) and (29), we find the corresponding Hadamard function of the boundary fields in the squeezed thermal states is

\[
G^{(T\xi)}_H(t, t') = \int_0^\infty d\omega \int_0^\infty d\omega' W(\omega)W(\omega') U^T_\omega U^T_{\omega'} G^T_R(\omega) G^T_R^*(\omega') \left[ -\mu_{\omega\omega'} \nu_{\omega\omega'} (1 + n_\omega + n_{\omega'}) \frac{G^T_R(\omega')}{G^T_R^*(\omega')} e^{-i\omega t - i\omega' t'} + \delta(\omega - \omega')(n^2_{\omega\omega'} + \frac{1}{2} (1 + 2n_\omega) e^{-i\omega t + i\omega' t'} \right] + \text{c.c.} ,
\] \hspace{1cm} (32)

where $G^T_R(\omega)$ is the Fourier transform of the retarded Green’s function in the normal thermal state in (B7). In the above expression, we have introduced the simplest window function $W(\omega)$ given by the unit-step function

\[
W(\omega) = 1 , \quad \text{if } \omega_0 - \Delta \leq \omega \leq \omega_0 + \Delta .
\] \hspace{1cm} (33)
Thus only modes within the frequency band $\omega_0 - \Delta \leq \omega \leq \omega_0 + \Delta$ are excited to the squeezed thermal states. The other modes remain in normal thermal states. This result can be compared to the perturbed holographic influence functional derived in (20). From (20) with the expression (B4) and the influence functional (A7), the corrections to the Hadamard function of boundary fields in thermal states, denoted by $G_H^{(\phi)}(t, t')$, can be obtained as

$$G_H^{(\phi)}(t, t') = \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{d\omega'}{2\pi} G^T_H(\omega) \{ \phi(\omega + \omega', r_b) e^{-i\omega t - i\omega' t'} + \phi(\omega - \omega', r_b) e^{-i\omega t + i\omega' t'} + \text{c.c.} \}. \quad (34)$$

In the limits of small squeeze parameters and the narrow bandwidth ($\Delta/\omega_0 < 1$ in (33)), we can approximate $\omega \approx \omega'$ since $\omega$ and $\omega'$ lie within the frequency band. Compared (34) with (32) and using (31), the field $\varphi(2\omega)$ obtained from $\phi(\omega, r_b)$ in (19) can be related to the squeeze parameters up to a constant phase by

$$r_b^{-2-z} \varphi(\omega + \omega' \approx 2\omega) = -4\pi r_\omega \Gamma\left(\frac{3}{2} + \frac{1}{z}\right) \left(\frac{\omega}{2z}\right)^{-2\frac{1+z}{2z}}. \quad (35)$$

The large $r_b$ limit is taken to obtain (35). Notice that this identification is held for any temperature, and thus, as expected, the same identification is found at zero temperature in [40]. Also, the squeeze parameters are expected to be small, since the holographic dual of squeezing vacuum (thermal) states is considered in gravitational wave perturbed background. Later, we will express some of the results in terms of general squeezing parameters, however it should keep in mind that small squeeze parameters are considered.

### III. UNCERTAINTIES ON THE POSITION AND MOMENTUM OF THE MIRROR

The presence of the environment will give additional uncertainties to the observables associated with the mirror. The effects from the environment on the uncertainties of the position and momentum of the mirror are contained in the two-point functions of the environment field. In the following we consider the environment field to be strongly coupled and use the holographic influence functional discussed in the previous section to study its effect on the mirror’s uncertainties in the position and momentum. For finding correlators of mirror’s position, we have proven the equivalence between influence functional method and the method of the mode expansion via the identification in (35). It is then quite straightforward to compute two-point correlation of $X(t)$ from the mode expansion in (21) with the
squeezed thermal state. We will mainly study the late time behavior of the uncertainties, resulting from the small frequency limit of the mode function.

A. The uncertainties in squeezed vacuum states

Let us now discuss the squeezed vacuum state by taking zero temperature limit. Here we choose the squeeze parameter as \( \xi_{\omega'} = \xi_{\omega} \delta (\omega - \omega') \). We also consider that the interaction between the mirror and fields is turned on at \( t_i = -\infty \). Then the difference of the mirror’s position uncertainty at time \( t \) and the result at time \( t = 0 \) can be expressed as

\[
\langle (X(t) - X(0))^2 \rangle_{\xi} = \int_0^\infty d\omega W(\omega) U_{\omega}^{(0)2} \left[ -\mu_\omega \nu_\omega (e^{-i\omega t} - 1)^2 - \mu_\omega \nu_\omega^* (e^{+i\omega t} - 1)^2 \right. \\
\left. - (2 \eta_\omega^2 + 1) (e^{-i\omega t} - 1)(e^{+i\omega t} - 1) \right] \int_{\omega_0-\Delta}^{\omega_0+\Delta} d\omega 8 U_{\omega}^{(0)2} \left[ \mu \eta \cos[\omega t - \theta] + (\eta^2 + 1/2) \right] \sin^2 \frac{\omega t}{2} \right. ,
\]

where the mode function \( U_{\omega}^{(0)} \) of the fields at vacuum is given in (24). The window function \( W(\omega) \) with a finite bandwidth is also included for squeezed modes, in which the squeeze parameters are assumed to be independent of frequency within the frequency band. The saturated value of the uncertainty can be found from the late-time behavior of (36) in the limit \( (\omega_0 \pm \Delta) t \gg 1 \). In this limit, the main contribution to the integration comes from the regions of small \( \omega \). The small \( \omega \) expansion of \( U_{\omega}^{(0)2} \) takes different forms for \( 1 < z < n + 2 \) and \( z > n + 2 \), and they are respectively given by

\[
U_{\omega}^{(0)2} \sim \begin{cases} \\
\frac{N_{1<z<n+2}}{T_{n+2}S_n} (r_b^z)^{2-(4+2n)/z} \omega^{-3+(n+2)/z} , & 1 < z < n + 2 ; \\
\frac{N_{z>n+2}}{T_{n+2}S_n} \omega^{-1-(n+2)/z} , & z > n + 2 ,
\end{cases}
\]

\[
N_{1<z<n+2} = \frac{(2z)^{2-(n+2)/z}}{\Gamma^2 \left( \frac{n+2}{2z} - \frac{1}{2} \right)} , \\
N_{z>n+2} = \frac{(2z)^{(n+2)/z}}{\Gamma^2 \left( \frac{1}{2} - \frac{n+2}{2z} \right)} ,
\]

Notice that different \( \omega \)-dependence in these two regions of \( z \) is mainly attributed to the fact that the low frequency behavior of the retarded Green’s function in (B12) is dominated respectively by the mass term when \( 1 < z < n + 2 \) and by the \( \gamma \) term when \( z > n + 2 \).

Using the small \( \omega \) expansion of the mode functions in (37), the time dependence of the momentum and position uncertainties is explored in the following. We first study the
momentum uncertainty, which is obtained directly from (36) by the relation, \( P = mdX/dt \), and the result is:

\[
(\Delta P(t))^2 = \langle (P(t) - P(0))^2 \rangle_{\xi}
\]

where the function \( g_{\pm} \)'s of squeeze parameters are defined as

\[
g_{\pm}(\eta, \theta) = (2\eta^2 + 1) \pm \eta \mu \cos(\theta).
\]

Thus, the momentum uncertainty due to the squeezed environment fields reach a saturated value at late times, following a power-law saturation rate of \( t^{-1} \). The last expression is obtained by taking the narrow bandwidth approximation \( \Delta \ll \omega_0 \) and setting \( r_b^z = 1/L \), which is the length scale characterizing the breakdown of Lorentz invariance in quantum critical theory, introduced by [55]. We also parameterize the mass in (B13) as \( m_n = T_{n+1}S_nL \) where \( 1/L \) is the largest energy scale in this system. The typical wavelength \( \lambda_0 \equiv 1/\omega_0 \) of the squeezed modes is in general greater than \( L \). So the maximum momentum uncertainty can be achieved when \( z = n + 2 \). The similar results on the velocity dispersion were also found in our earlier work [40]. It is worth noticing that the momentum uncertainty of the mirror is proportional to the brane tension \( T_{n+1} \), which is related to the 't Hooft coupling of the boundary field by \( T_{n+1} \propto \lambda^{n/4 + 1/2} \). The enhancement in the momentum uncertainty from the environment agrees with the result in the field theory calculations [18], where they also considered the linear coupling of the environment field to particle’s position although the environment field under study is a free field. Here, we also find that the momentum
uncertainty is proportional to some negative power of the wavelength $\lambda_0$ of the squeezed modes. So the small value of the ratio $L/\lambda_0$ and the narrow bandwidth approximation ($\Delta/\omega_0 \ll 1$) can reduce the momentum uncertainty.

Now we compute the position uncertainty. From the equation (36), the position uncertainty is given by

$$(\Delta X(t))^2 = \langle (X(t) - X(0))^2 \rangle \xi$$

(42)

$$\approx \begin{cases} \frac{N_{1 < z < n+2}}{T_{n+1} S_n} \frac{g_+(r, \theta)}{4(2 - (n + 2)/z)} (r_b^z)^{2-(4+2n)/z} \left[ (\omega_0 - \Delta)^{-2+(n+2)/z} - (\omega_0 + \Delta)^{-2+(n+2)/z} \right] & , \quad 1 < z < n + 2; \\
\frac{N_{z > n+2}}{T_{n+1} S_n} \frac{g_+(r, \theta)}{4(n + 2)/z} \left[ (\omega_0 - \Delta)^{-(n+2)/z} - (\omega_0 + \Delta)^{-(n+2)/z} \right] + O(1/t), & z > n + 2. \end{cases}$$

(43)

$$\approx \begin{cases} \frac{N_{1 < z < n+2}}{T_{n+1} S_n L^2} \frac{g_+(r, \theta)}{2} \left( \frac{L}{\lambda_0} \right)^{-2+(2+n)/z} \left( \frac{\Delta}{\omega_0} \right) L^2 + O(1/t), & 1 < z < n + 2; \\
\frac{N_{z > n+2}}{T_{n+1} S_n L^2} \frac{g_+(r, \theta)}{2} \left( \frac{L}{\lambda_0} \right)^{-(n+2)/z} \left( \frac{\Delta}{\omega_0} \right) L^2 + O(1/t), & z > n + 2. \end{cases}$$

(44)

Similar to the momentum uncertainty, the saturation of the position uncertainty is reached following a power-law behavior like $t^{-1}$. Again, the last expression is obtained by taking the narrow bandwidth approximation and also setting $r_b^z = 1/L$. Also, the maximum position uncertainty occurs at $z = n + 2$. On contrary to the momentum uncertainty, the position uncertainty is inversely proportional to the brane tension $T_{n+1}$, and is suppressed by the large coupling constant. Thus, the environment effect reduces the position uncertainty on the one hand, and enhances the momentum uncertainty on the other hand. Similar environment effects from free fields on the particle are also seen in [18] where the interaction to the environment is via the position of the mirror. Based on the relation $P = mdX/dt$ and the momentum uncertainty (38), the position uncertainty is proportional to some positive power of $\lambda_0$ instead. Although the narrow bandwidth approximation reduces the position uncertainty, the large value of the ratio $\lambda_0/L$ will lead to some enhancement.

As also discussed in our work [40], the saturated value of the position and momentum uncertainties depend on the functions $g_\pm(r, \theta)$ of squeeze parameters in (41). Since

$$\eta^2 - \frac{1}{2} \eta \mu \geq -\frac{2 - \sqrt{3}}{4} > -\frac{1}{2},$$

(45)
the functions $\eta^2 - \frac{1}{2} \eta \mu$ can be negative for small squeezing parameter $r$, leading to the so-called subvacuum phenomenon. It means that the position or momentum uncertainty, arising from the squeezed vacuum of the environment, can be smaller than the value solely due to the normal vacuum fluctuations. However, the sum of the uncertainties given by the normal vacuum and the shifted value due to squeezing vacuum must be positive.

To fully understand environmental effects on the mirror, we study the cross correlation between the position and momentum uncertainties, which can be obtained straightforwardly as

$$\frac{1}{2} \left\{ (X(t) - X(0))_\xi (P(t) - P(0))_\xi + (P(t) - P(0))(X(t) - X(0))_\xi \right\}$$

(46)

$$\approx \begin{cases} 
- \frac{N_{1<z<n+2}}{T_{n+1} S_n} \frac{\eta \mu \sin \theta}{4((n+2)/z - 1)} (r_b^2 - (4+2n)/z) m_n \left[ (\omega_0 + \Delta)^{(n+2)/z - 1} - (\omega_0 - \Delta)^{(n+2)/z - 1} \right] + O(1/t), & 1 < z < n + 2; \\
- \frac{N_{z>n+2}}{T_{n+1} S_n} \frac{\eta \mu \sin \theta}{4(1 - (n+2)/z)} m_n \left[ (\omega_0 + \Delta)^{1-(n+2)/z} - (\omega_0 - \Delta)^{1-(n+2)/z} \right] + O(1/t), & z > n + 2; 
\end{cases}$$

(47)

$$\approx \begin{cases} 
- \frac{N_{1<z<n+2}}{2} \frac{\eta \mu \sin \theta}{\lambda_0}^{(n+2)/z - 1} \left( \frac{\Delta}{\omega_0} \right) + O(1/t), & 1 < z < n + 2; \\
- \frac{N_{z>n+2}}{2} \frac{\eta \mu \sin \theta}{\lambda_0}^{1-(n+2)/z} \left( \frac{\Delta}{\omega_0} \right) + O(1/t), & z > n + 2. 
\end{cases}$$

(48)

The above cross correlations are found to have no dependence on $T_{n+1}$. Since, in the holographic approach, the quadratic DBI action in (12) is proportional to $T_{n+1}$, the proper rescaling of $X$ by absorbing $T_{n+1}$ gives that $\Delta X \propto 1/\sqrt{T_{n+1}}$. Moreover, since the mass of the mirror is proportional to the energy cost to create the brane, $m_n \propto T_{n+1}$ [29]. As a result, $\Delta P = m_n \Delta (dT/dt) \propto \sqrt{T_{n+1}}$. Therefore, the product of the position and momentum uncertainties, and so the cross correlations have no dependence on the coupling constant of strongly coupled fields. This is probably a general consequence from the holographic approach.

In particular, with the position and momentum uncertainties and their correlations in (12), (38), and (16) respectively, we find that when $t \to \infty$,

$$\left( \frac{\Delta P(\infty)}{\xi} \right)^2 \left( \frac{\Delta X(\infty)}{\xi} \right)^2 \propto N^2 \left[ (\eta^2 + \frac{1}{2} \eta \mu)^2 - \frac{\eta^2 \mu^2}{4} \right] \left( \frac{L}{\lambda_0} \right)^{2(1-(n+2)/z)} \left( \frac{\Delta}{\omega_0} \right)^2,$$

(49)
where $\mathcal{N}$ can be $\mathcal{N}_{1<z<n+2}$ or $\mathcal{N}_{z>n+2}$, depending on the value of $z$. The $z$-dependence of (49) shows that, when $z = n + 2$, $(\Delta P(t))^2(\Delta X(t))^2$ is largest for $L < \lambda_0$ as compared with other $z$. Thus, at $z = n + 2$, the environment effects on the mirror is maximal.

Accordingly, for considering the same frequencies of the squeezed modes and squeezing parameters, the quantum critical theories with the dynamical exponent $z = n + 2$ gives maximum uncertainty effects on mirror’s position and momentum. Also, the position uncertainty can be reduced by the large coupling constant of the strongly coupled fields, whereas the momentum uncertainty is enhanced by the coupling constant. It deserves a further study on finding the holographic dual of the system-environment model, where the interaction between them is via system’s momentum, to explore the dependence of the position and momentum uncertainties on the coupling constant of the environment fields.

B. The uncertainties in squeezed thermal states

As for the environment in squeezed thermal state, the mirror can receive the significant finite-$T$ effects. This can be seen from the retarded Green’s function in the small frequency limit (B7), which gives finite-$T$ modification to the mass and dissipation coefficient $\gamma_T$. On top of that, thermal fluctuations of the environment, summarized in the Hadamard function (B9), renders the mirror undergoing stochastic motion. So the uncertainties of the position and momentum of the mirror are modified when the environment is heated up.

Using the mode functions at finite-$T$ and the expectation values of creation and annihilation operators for squeezed thermal states (27), we find the position uncertainty as,

$$\langle (X(t) - X(0))^2 \rangle_{T\xi} = \int_0^\infty d\omega \omega W(\omega) U_{T\omega}^2 \left[ -\mu_\omega \nu_\omega (1 + 2n_\omega) (e^{-i\omega t} - 1)^2 
- \mu_\omega \nu_\omega^* (1 + 2n_\omega) (e^{+i\omega t} - 1)^2 + (2\eta_\omega^2 + 1) (1 + 2n_\omega) (e^{-i\omega t} - 1)(e^{+i\omega t} - 1) \right]$$

$$= \int_{\omega_0 - \Delta}^{\omega_0 + \Delta} d\omega \frac{8 U_{T\omega}^2}{\omega^2} \left[ \mu \eta (1 + 2n_\omega) \cos(\omega t - \theta) + (\eta^2 + 1)(1 + 2n_\omega) \right] \sin^2 \frac{\omega t}{2},$$

(50)

where $U_{T\omega}^2$ is the square of mode functions in (23). The squeeze parameters are assumed to be $\xi_{\omega'} = \xi_{\omega} \delta(\omega - \omega')$ and frequency-independent in the frequency band specified by the window function $W$. However the mode functions in (23) can be found only in the low-frequency limit (or equivalently high temperature limit), so we will restrict our study in this
limit. Thus, as long as the squeezed modes under consideration have frequency \( \omega \ll T \), the number density \( n_\omega \) can be approximated by \( n_\omega \approx T/\omega \).

As a result, the momentum uncertainty given by the squeezed thermal environment at high-\( T \) limit (\( T >> \omega_0 \)) becomes

\[
(\Delta P(t))^2 \approx \frac{8}{\pi T_{n+1} S_n} m_{nT}^2 \omega_0^2 t_h^{-n+2} \left( \frac{T}{\omega_0} \right) g_+(r, \theta) \left( \frac{\Delta}{\omega_0} \right) + O(1/t)
\]

\[
\propto (T_{n+1} S_n L^2) \left( \frac{L}{\lambda_0} \right)^2 \frac{T^{-(n+2)/z}}{T^{(n+2)/z}} \left( \frac{T}{\omega_0} \right) g_+(r, \theta) \left( \frac{\Delta}{\omega_0} \right) \frac{1}{L^2} + O(1/t).
\] (51)

In particular, the last expression is obtained by assuming that \( T \) is larger than the frequency \( \omega_0 \) of the squeezed modes, but is still smaller than \( 1/L \), the largest energy scale in this system. If so, the mass \( m_{nT} \) in (B8) can be approximated by \( m_{nT} \approx T_{n+1} S_n/L \) with no temperature dependence. Similar to the results in the zero temperature case, the momentum uncertainty reaches its saturated value following a power-law \( 1/t \), and is enhanced by the factor of brane’s tension \( T_{n+1} \).

It may be quite instructive if the above dependence of the momentum uncertainty on temperature can be reconstructed by dimensional analysis using the energy scales \( T \) and \( \omega_0 \). Here we take a Brownian particle as an example. The same arguments will also be applied to a \( n \)-dimensional mirror by shifting the value of \( z \) from \( z = 2 \) to \( z = n + 2 \). The mean free path of a particle can be argued to be \( \ell_{mfp} \propto 1/T^{1/z} \) due to the scaling symmetry of quantum critical theories in [1]. Moreover, the relaxation time is inversely proportional to \( T \) as \( \tau \propto 1/T \). For the typical measuring time \( t = 1/\omega_0 \), the relevant time scale given by the squeezed modes, the number of collisions is approximated by \( N_c = t/\tau \propto T/\omega_0 \). Therefore, \( \Delta X^2 = N_c \ell_{mfp}^2 \propto T^{1-2/z}/\omega_0 \), as will also be seen later by direct calculations. The corresponding momentum uncertainty, from the relation \( P = m dX/dt \) and taking the relevant time scale of the mirror \( t = 1/\omega_0 \), becomes \( \Delta P^2 \propto T^{1-2/z} \omega_0 \), which for a \( n \)-dimensional mirror is modified to \( \Delta P^2 \propto T^{1-(n+2)/z} \omega_0 \) as above. However it is peculiar that for \( 1 < z < n + 2 \), the momentum uncertainty is inversely proportional to \( T \). This unanticipated result can be tested experimentally in the future. [28–30].
Now we turn to the corresponding position uncertainty, which is obtained as

\[(\Delta X(t))^2_{\xi} = \langle (X(t) - X(0))^2 \rangle_{T\xi} \]

\[\simeq \frac{4}{\pi T_{n+1} S_n} r_h^{-n-2} \omega_0 \left( \frac{T}{\omega_0} \right) g_- (r, \theta) [(\omega_0 - \Delta)^{-1} - (\omega_0 + \Delta)^{-1}] + O(1/t)\]

\[\propto \frac{1}{T_{n+1} S_n L^2} T^{-(n+2)/z} \left( \frac{T}{\omega_0} \right) g_- (r, \theta) \left( \frac{\Delta}{\omega_0} \right) L^2 + O(1/t). \quad (52)\]

As anticipated, the position uncertainty is inversely proportional to the brane’s tension \(T_{n+1}\) and its temperature dependence shares the same behavior as in the momentum uncertainty.

Finally, the cross correlation between the momentum and position uncertainties is

\[\frac{1}{2} \left\{ \langle (X(t) - X(0))(P(t) - P(0)) \rangle_{T\xi} + \langle (P(t) - P(0))(X(t) - X(0)) \rangle_{T\xi} \right\} \]

\[\simeq -\frac{4}{\pi T_{n+1} S_n} m_n T \omega_0 r_h^{-n-2} \left( \frac{T}{\omega_0} \right) \mu \eta \sin \theta \ln \left[ \frac{\omega_0 + \Delta}{\omega_0 - \Delta} \right] + O(1/t)\]

\[\propto -\left( \frac{L}{\lambda_0} \right) T^{-(n+2)/z} \left( \frac{T}{\omega_0} \right) \mu \eta \sin \theta \left( \frac{\Delta}{\omega_0} \right) + O(1/t). \quad (53)\]

As can be seen, the cross correlation has the same temperature dependence as in the position and momentum uncertainties, and also it has no \(T_{n+1}\) dependence. To sum up, the corresponding product of the position and momentum uncertainties when \(t \to \infty\) is

\[(\Delta P(\infty))^2_{T\xi} (\Delta X(\infty))^2_{T\xi} \propto \frac{256}{\pi^2} \left( \eta^2 + \frac{1}{2} \right)^2 - \frac{\eta^2 \mu^2}{4} \left( T/L^{-1} \right)^{2(1-(n+2)/z)} \left( \frac{\Delta}{\omega_0} \right)^2, \quad (54)\]

in the high-\(T\) \((T >> \omega_0)\) and narrow bandwidth limits. Thus, for \(1 < z < n + 2\), the contribution from the fluctuation of the squeezed thermal states to the product of the position and momentum uncertainties decreases as \(T\) increases whereas for \(z > n + 2\), the contribution increases instead as \(T\) increases. The peculiar temperature dependence seems to be the consequence from the scaling symmetry of quantum critical theories under study, and deserve an experimental check.

**C. Comparison with the case in the environment of the relativistic free-field**

The effects on the system from the strongly coupled environment field we obtain can be compared with the results from free field theories. We concentrate on the cases with the dynamical exponent \(z = 1\), and consider the system consisting of a particle, thus \(n = 0\), since these cases are what have been studied in field theories. In [56], they consider a bilinear
coupling between the particle position and the relativistic free field with a coupling constant $\epsilon$. The uncertainties of particle’s momentum and position, which arise from quantum and thermal fluctuations of the free relativistic fields with all frequency modes, are calculated. In particular, the saturated value of the momentum uncertainty for a free particle affected by the thermal bath in high-temperature $T$ limit is found to be \( \langle \Delta P(t \to \infty) \rangle_T^2 \propto m \epsilon^2 T \), where $m$ here is the mass of a particle. Thus, the increase in $\epsilon$ gives rise to larger momentum uncertainty \[56\]. On the contrary, the position uncertainty in the late time limit does not saturate and they found, \( \langle \Delta X(t) \rangle_T^2 \propto (\epsilon^2 T/m \gamma) t \), where $\gamma$ is the damping constant. So the position uncertainty increases with the square root of time just as in the case of classical Brownian motion. However, the damping constant $\gamma$ is obtained from the retarded Green’s function of the field, which is proportional to the coupling constant $\epsilon^2$, and is independent of temperature $T$ \[56\]. As a result, the position uncertainty is independent of the coupling constant $\epsilon$, and both position and momentum uncertainties increases linearly in temperature $T$ of the environment. As for the system interacting with the strong coupled field under consideration, there is an additional large coupling constant of the quantum critical fields, relative to the interaction strength between the system and the environment, the effects from the environment to the system are mainly determined that large coupling constant of the fields instead. New features we find here is that the large coupling constant of the field reduces the position uncertainty of the particle, but enhances the momentum uncertainty. Moreover, in the case $z = 1$, both position and momentum uncertainties of the particle decrease as the temperature of the heat bath increase. This is in a sharp contrast to the behavior in the free field heat bath. We also find that the coupling of the system to the squeezed state of the environment leads to squeezing the quantum state of the system itself through their bilinear coupling. Also, when squeezed modes are restricted to some finite range of frequency, both position and momentum uncertainties of the particle at late times reach their respective saturated value by following the relaxation behavior as $1/t$. Similar saturation behavior is also found when the particle interacts with relativistic free fields in their squeezed vacuum states with a finite frequency bandwidth in \[17\]. Additionally, it has been emphasized in previous sections that in \[18\], the bilinear coupling between particle’s position and the relativistic free fields leads to the saturation of position and momentum uncertainties at late times for the particle of an oscillator, and also find the reduction in the position uncertainty in comparison with the momentum uncertainty. With the same type
of the coupling, the same reduction behavior is found in our holographic setup to consider the strongly coupled quantum critical fields. Thus, there exist some dramatic differences in the effects on the system from the environments of a free field and a strongly coupled field, and they can be experimentally compared. These results also show that the previous studies on open quantum systems based on perturbative methods to deal with the weakly coupled environment fields are not as robust as asserted even qualitatively in the case of the strongly coupled fields.

Brief discussions are given below for the environment consisting of free Lifshitz field theories with the Lorentz symmetry breaking dispersion relation in the case of a general \( z \). Although such system-environment problems have not been studied yet, we may still expect some differences. The same type of the bilinear coupling between the system and free Lifshitz fields might lead to rather different feature of the retarded Green’s function and the Hadamard function defined by (A9). In particular, the retarded Green’s function is constructed from the expectation value of the commutator of the field variable. As for the free field, based upon the canonical quantization, the retarded Green’s function is found to be independent of the state of the field. For example, for the thermal state the retarded Green’s function has no temperature dependence. This is in sharp contrary to the case of the strongly coupled field with the temperature dependent retarded Green’s function (B7). As a result, the Hadamard function of the free field, as can be seen from the fluctuation-dissipation theorem, also has different temperature dependence, as compared with the one in the strongly coupled field (B9). Accordingly, we also anticipate the dramatically different temperature dependence of the momentum and the position uncertainties, influenced by the thermal state of the free Lifshitz field, and it deserves further study.

IV. SUMMARY AND OUTLOOK

The main goal of this work is to understand the effects of the strongly coupled quantum critical fields on the dynamics of a \( n \)-dimensional mirror using the method of holography. The dual description is a \( n+1 \)-dimensional probe brane moving in \( d+1 \)-dimensional Lifshitz geometry. The dynamics of the mirror can be realized from the motion of the brane at the boundary of the bulk. The correlators of strongly coupled environment fields at squeezed vacuum and thermal states can be obtained via holographic influence functional, constructed
from the probe brane action in the gravitational wave perturbed Lifshitz (black-hole) geometry. The interaction between the mirror and the environment is a bilinear coupling through the mirror’s position. We find that the position uncertainty of the mirror due to the presence of the environment is suppressed by the large coupling constant of the fields but the momentum uncertainty is enhanced by the coupling constant instead. As a result, the product of the position and momentum uncertainties is independent of the coupling constant. This finding can be one of the general consequences in the holographic description of Brownian motion. The amplitude of squeeze parameter $\eta$, counting the number of the quanta in squeezed modes, gives additional enhancement to the uncertainties whereas its phase factor $\theta$ may reduce the uncertainties by some proper tuning. In the squeezed vacuum state, the mirror gains maximum effects on its position and momentum uncertainties from the environment when the dynamic exponent $z = n + 2$. For the squeezed thermal state, the contribution to the uncertainties from the thermal fluctuation decreases as $T$ increases for $1 < z < n + 2$, whereas for $z > n + 2$ the contribution increases as $T$ increases instead.

All results deserve experimental tests in physical systems to justify success in employing holographic ideas for the study of environmental effects of strongly coupled fields on the system.

The interaction between the system and environment may result in the loss of quantum coherence of the system. A quantitative way to characterize the decoherence is by the entanglement entropy, defined as $S = -\text{Tr}\rho_r \ln \rho_r$, where $\rho_r$ is the reduced density matrix of the system. A direct extension of the current work is to calculate the time-dependent entanglement entropy of the system via the holographic influence functional approach when turning on the interaction between the system and environment at some initial time, and also imposing a suitable initial density matrix of the system. Another possible extension is to consider two quantum systems, coupled to one strongly coupled quantum field. In particular, we may explore the development of their quantum entanglement through the interaction with the strongly coupled field. To do so, one needs to extend the current holographic setup to include two objects moving in the asymptotic Lifshitz background. In this case, it will be interesting to compare the time-dependent entanglement entropy of two sub-systems derived from the holographic influence functional approach, which is valid in the linear response region, with the entanglement entropy obtained by the Ryu-Takayanagi Conjecture [58].
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Appendix

Appendix A: Review of the method of influence functional

In this appendix, we give a brief review of the method of influence functional in field theory [9–11]. We begin with the total density matrix $\rho(t)$ of the system-plus-environment, which unitarily evolves according to

$$\rho(t_f) = U(t_f, t_i) \rho(t_i) U^{-1}(t_f, t_i) \quad (A1)$$

where $U(t_f, t_i)$ is the time evolution operator. The effects from the environment to the system can be summarized in the reduced density matrix $\rho_r(t)$, obtained by tracing out the environmental degrees of freedom in $\rho(t)$. Here the initial total density matrix at time $t_i$ is assumed to be factorized as

$$\rho(t_i) = \rho_q(t_i) \otimes \rho_F(t_i) \quad (A2)$$

where $q$ and $F$ generically represent the system and the environment variables respectively. The environment field is assumed initially in thermal equilibrium at temperature $T = 1/\beta$, and the corresponding density matrix $\rho_F(t_i)$ is given by

$$\rho_F(t_i) = \rho_T \equiv e^{-\beta H_F} / Tr_F\{e^{-\beta H_F}\} \quad (A3)$$

where $H_F$ is the Hamiltonian for the $F$ field. The system and environment start to couple at an initial time $t_i$. The vacuum state of the environment field can be achieved by taking the zero-$T$ limit.

In the spirit of linear response, the system is considered to be linearly coupled to the environment. Thus, the full Lagrangian takes the form

$$L(q, F) = L_q[q] + L_F[F] + qF \quad (A4)$$

One can then express the reduced density matrix as [9, 10, 12]

$$\rho_r(q_f, \tilde{q}_f, t_f) = \int dq_1 dq_2 J(q_f, \tilde{q}_f, t_f; q_1, q_2, t_i) \rho_q(q_1, q_2, t_i) \quad (A5)$$
Green's function can be defined as

\[ \mathcal{J}(q_f, \bar{q}_f, t_f; q_1, t_1) = \int_{q_1}^{q_f} Dq^+ \int_{q_2}^{\bar{q}_f} Dq^- \exp \left[ i \int_{t_1}^{t_f} dt \left( L_q[q^+] - L_q[q^-] \right) \right] \mathcal{F}[q^+, q^-]. \]  

(A6)

Up to the quadratic order in particle position \( q \), the influence functional in terms of real-time Green's functions for the environment field can be written as

\[ \mathcal{F}[q^+, q^-] = \exp \left\{ -\frac{i}{2} \int_{t_1}^{t_f} dt \int_{t_1}^{t_f} dt' \left[ q^+(t) G^{++}(t, t') q^+(t') - q^+(t) G^{+-}(t, t') q^-(t') - q^-(t) G^{-+}(t, t') q^+(t') + q^-(t) G^{--}(t, t') q^-(t') \right] \right\}. \]  

(A7)

The time-ordered, anti-time-ordered Green's functions and Wightman functions are defined respectively by

\[ i G^{+-}(t, t') = \langle F(t') F(t) \rangle, \]
\[ i G^{--}(t, t') = \langle F(t) F(t') \rangle, \]
\[ i G^{++}(t, t') = \langle F(t) F(t') \rangle \theta(t - t') + \langle F(t') F(t) \rangle \theta(t' - t), \]
\[ i G^{--}(t, t') = \langle F(t') F(t) \rangle \theta(t - t') + \langle F(t) F(t') \rangle \theta(t' - t). \]  

(A8)

The retarded Green’s function and Hadamard function, which account for dissipative and stochastic effects on the dynamics of the system can be constructed out of the above Green’s functions by

\[ G_R(t - t') \equiv -i \theta(t - t') \langle [F(t), F(t')] \rangle = \left\{ G^{++}(t, t') - G^{--}(t, t') \right\}, \]
\[ G_H(t - t') \equiv \frac{1}{2} \langle [F(t), F(t')] \rangle = \frac{i}{4} \left\{ G^{++}(t, t') + G^{+-}(t, t') + G^{--}(t, t') + G^{-+}(t, t') \right\}. \]  

(A9)

When the environment respects time-translation invariance, the Fourier transform of its Green’s function can be defined as

\[ G(\omega) = \int d\tau \, G(\tau) e^{i\omega \tau}, \]  

(A10)

and the Fourier transforms of the Green’s functions, defined in (A8), are given by

\[ G^{++}(\omega) = \text{Re} G_R(\omega) + (1 + 2n_\omega) i \text{Im} G_R(\omega), \]
\[ G^{--}(\omega) = -\text{Re} G_R(\omega) + (1 + 2n_\omega) i \text{Im} G_R(\omega), \]
\[ G^{+-}(\omega) = 2n_\omega i \text{Im} G_R(\omega), \]
\[ G^{-+}(\omega) = 2(1 + n_\omega) i \text{Im} G_R(\omega) \]  

(A11)
with \(n_\omega = (e^{\frac{\omega}{T}} - 1)^{-1}\). Notice that the fluctuation-dissipation relation is satisfied, and given by

\[
G_H(\omega) = -(1 + 2n_\omega) \text{Im} G_R(\omega). \tag{A12}
\]

**Appendix B: Brief summary of the results in pure Lifshitz geometry and Lifshitz black hole**

Consider the Lifshitz black hole background in (7) without gravitational wave perturbation

\[
ds^2 = -r^{2z} f(r) dt^2 + \frac{dr^2}{f(r) r^2} + r^2 dx_idx_i. \tag{B1}
\]

With the same notations and assumptions as in the main text, the DBI action for the \(n + 1\)-dimensional probe brane in the Lifshitz black hole for small \(X^I\) is given by

\[
S_{DBI}^T \approx \text{constant} - \frac{T_{n+1}}{2} \int dr dt dx_1 dx_2 \ldots dx_n \left( r^{z+n+3} f(r) X'^I X'^I - \frac{\dot{X}^I \dot{X}^I}{f(r) r^{z-n-1}} \right). \tag{B2}
\]

Thus the equation of motion for brane’s position in the Fourier space, \(X^I(\omega) e^{-i\omega t}\) can be derived as follows

\[
\frac{\partial}{\partial r} \left( r^{z+n+3} f(r) \frac{\partial}{\partial r} X^I(\omega) \right) + \frac{\omega^2}{r^{z-n-1} f(r)} X^I(\omega) = 0. \tag{B3}
\]

The solution can be expressed in terms of two linearly independent solutions with the properties \(X^I(\omega) \propto e^{i\omega r^*_s}\) and \(X^I(\omega) \propto e^{-i\omega r^*_s}\), where \(r^*_s = \int dr f(r)^{-1} r^{z-1}\), and the normalization condition \(X^I(\omega) = r^*_s = 1\). Since the different components of \(X^I(\omega)\) are decoupled in the linearized equation of motion, we may just focus on one of the directions \(X^I\) and denote it by \(Q(t, r)\). As described in the main text we introduce \(Q^+(t, r_1)\) and \(Q^-(t, r_2)\), which correspond to the branes living in two regions with different asymptotic boundaries in the maximally extended Lifshitz black hole geometry. Following [39], which is consistent with [51, 52], we find \(Q^\pm(\omega, r)\) to be

\[
Q^+(\omega, r_1) = \frac{1}{1 - e^{-\frac{\omega}{T}}} \left[ (X^- (\omega) - e^{-\frac{\omega}{T}} X^+ (\omega)) X^I(r_1) + (X^+ (\omega) - X^- (\omega)) X^I(r_1) \right],
\]

\[
Q^-(\omega, r_2) = \frac{1}{1 - e^{-\frac{\omega}{T}}} \left[ (X^- (\omega) - e^{-\frac{\omega}{T}} X^+ (\omega)) X^I(r_1) + e^{-\frac{\omega}{T}} (X^+ (\omega) - X^- (\omega)) X^I(r_1) \right]. \tag{B4}
\]

where \(X(\omega)\) is the Fourier transform of \(X(t)\), the displacement of the mirror. This solution is then substituted into the classical on-shell action (14). Using (A7) and (A9), the retarded
Green’s function at finite temperature is obtained as

$$G_R(\omega) = T_{n+1}S_n r_b^{z+n+3} X_\omega(r_b) \partial_r X_\omega(r_b). \quad (B5)$$

In general, the exact solution of $X_\omega(r)$ at finite temperature, denoted as $X^T_\omega(r)$, is not available, and depends on the details of black hole metric ($B1$). However in the small $\omega$ limit, the solution of $X^T_\omega(r)$ can be derived by matching the solution near the black hole horizon at $r = r_h$ to the solution in the large value of $r$ with proper boundary conditions in two regions. We then can obtain the approximate solution as $[38],

$$X^T_\omega(r) = Y_\omega(r)/Y_\omega(r_b), \quad Y_\omega(r) = \frac{i}{z + n + 2} \frac{\omega^{n+2}}{r^{n+2+z}} \left[ 1 - \frac{1}{\frac{n+2}{2z} + \frac{3}{2}} \left( \omega/2zr^z \right)^2 + O(\omega^4) \right] \quad + (1 - i\omega r_h^{n+2}) \left[ 1 + \frac{1}{\frac{n+2}{2z} - \frac{1}{2}} \left( \omega/2zr^z \right)^2 + O(\omega^4) \right]. \quad (B6)$$

Then the retarded Green’s function at finite temperature, $G^T_R(\omega)$, in the small $\omega$ limit, can be found as

$$G^T_R(\omega) = m_{nT}(z)(i\omega)^2 - \gamma_{nT}(z)(i\omega) + O(\omega^3), \quad (B7)$$

where

$$m_{nT}(z) = \frac{T_{n+1}S_n}{r_b^{z-n-2}} \left\{ \frac{1}{n+2-z} + \left( \frac{r_h}{r_b} \right)^{2n+4} \left[ (n + 2 + z) - \kappa r_b^{z+n+2} \right] \right\}, \quad \gamma_{nT}(z) = T_{n+1}S_n r_h^{n+2}, \quad (B8)$$

where $\kappa$ is a constant of integration. The mass $m_{nT}$ and the damping coefficient $\gamma_{nT}$ have the temperature dependence through their dependence on the black hole horizon radius ($B9$). The peculiar dependence of $\gamma_{nT}$ on temperature ($[28, 30]$, will play an important role in determining the temperature effects on the position and momentum uncertainties of the mirror to be explored later. All other correlators can be derived from ($A11$). In particular, through the fluctuation-dissipation relation ($A12$), we find the finite temperature Hadamard function

$$G^T_H(\omega) = \left( \frac{e^{\frac{\omega}{T}} + 1}{e^{\frac{\omega}{T}} - 1} \right) \omega \gamma_{nT}(z). \quad (B9)$$

In the zero temperature limit, there is the exact expression for $X_\omega(r)$ $[38],

$$X_\omega(r) = \frac{z+n+2}{z} \frac{H^{(1)}_{n+2+\frac{1}{2}}(\frac{\omega}{2zr^{z}})}{H^{(1)}_{n+2+\frac{1}{2}}(\frac{\omega}{2zr^{z}_{r_b}})}. \quad (B10)$$
Hence the zero-temperature retarded Green’s function for $\omega > 0$ can be found to be,

$$G_R^{(0)}(\omega) = -T_{n+1} S_n \omega r_b^{n+2} \frac{H_{2+1}^{(1)}(\frac{\omega}{r_b})}{\left(\frac{\omega}{r_b}\right)^{1/2}} - \frac{T_{n+1} S_n}{2} \frac{\omega}{r_b}.$$  \hfill (B11)

Thus, in the small $\omega$ expansion,

$$G_R^{(0)}(\omega) = m_n(z)(i\omega)^2 + \mu_n(\omega, z),$$  \hfill (B12)

where

$$m_n(z) = \frac{T_{n+1} S_n}{(n + 2 - z)r_b^{z-n-2}}, \quad \mu_n(\omega, z) = \gamma_n(z)(-i\omega)^{1+\frac{n+2}{2}} + \delta_n(z)(-i\omega)^4 + \ldots$$  \hfill (B13)

with

$$\gamma_n(z) = \frac{T_{n+1} S_n}{(2z)(n+2)/z} \frac{\Gamma\left(\frac{1}{2} - \frac{n+2}{2z}\right)}{\Gamma\left(\frac{1}{2} + \frac{n+2}{2z}\right)}, \quad \delta_n(\omega, z) = \frac{T_{n+1} S_n}{(n + 2 - 3z)(n + 2 - z)r_b^{3z-n-2}}.$$  \hfill (B14)

The low-frequency expansion is valid as long as $\omega < |[z - (n - 2)][z - (n + 2)/3]|r_b^z$. As mentioned in [29], although both $m$ and $\gamma$ change signs at $z = n + 2$, their ratio $\gamma/m$ still gives sensible results for describing the dynamics of the mirror. The zero-temperature Hadamard function for $\omega > 0$ is derived as

$$G_H^{(0)}(\omega) = \frac{2z}{\pi} r_b^{n+2+z} \frac{T_{n+1} S_n}{2} \frac{J_2^{(1)}(\frac{\omega}{r_b})}{\left(\frac{\omega}{r_b}\right)^{1/2}} + \frac{Y_2^{(1)}(\frac{\omega}{r_b})}{\left(\frac{\omega}{r_b}\right)^{1/2}}.$$  \hfill (B15)

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