**N-BODY SIMULATIONS OF SMALL GALAXY GROUPS**

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**RESUMEN**

Se presenta una serie de simulaciones de $N$-cuerpos destinadas a estudiar la dinámica de grupos pequeños de galaxias. En particular, se comparan los resultados obtenidos con las propiedades dinámicas de los grupos de Hickson. Se consideran condiciones iniciales partiendo de una ‘expansión máxima’ y de equilibrio virial, y no se considera un halo oscuro común primigenio. Se encuentra muy buena concordancia con las propiedades de grupos pequeños de galaxias, y las propiedades de los grupos compactos de Hickson son reproducidas adecuadamente por aquellos sistemas que se encuentran en un estado avanzado de un colapso gravitacional. No se encuentra un problema de ‘sobre-fusión’ de las galaxias. Para grupos que comienzan en virial, se encuentra que una fracción importante ($\sim 40\%$) puede durar por $\sim 10$ Gyr sin colapsarse completamente. Los resultados encontrados proporcionan una solución alternativa al problema de sobre-fusión para los grupos de Hickson. Asimismo, se encuentra que la razón masa-luminosidad de los grupos de Hickson podría ser similar a la de cúmulos de galaxias, sugiriendo que ambos tienen aproximadamente la misma fracción de materia bariónica a masa total.

**ABSTRACT**

A series of $N$-body simulations aimed at studying the dynamics of small groups of galaxies is presented. In particular, our results are compared with the dynamical properties of Hickson’s compact groups (HCG). ‘Maximum expansion’ and virial initial conditions are tested, and no primordial common dark halo is considered. The properties of small galaxy groups are very well reproduced, and those of Hickson’s groups are well reproduced for the most advanced stage of collapsing groups. We find no overmerging problem in our simulations. An important fraction of groups ($\sim 40\%$) initially in virial equilibrium can last for $\sim 10$ Gyr without complete merging. These results provide an alternative solution to the overmerging expected in Hickson’s compact groups. Also, the mass-to-light ratio of HCG are probably similar to those found in clusters, suggesting that both kinds of systems have about the same fraction of baryonic to total mass.

**Key Words:** GALAXIES: INTERACTIONS — GALAXIES: KINEMATICS AND DYNAMICS — METHODS: NUMERICAL

**1. INTRODUCTION**

Small groups are the most common galaxy associations and contain about 50% of all galaxies in the universe (Huchra & Geller 1982; Nolthenius & White 1987). Important work has been done to compile catalogs with relevant kinematical data. In this way Nolthenius & White (1987) and Nolthenius (1993) found that small groups from the CFA catalog (hereafter NCFA) have the following median values: a one-dimensional velocity dispersion of $\sigma \approx 116$ km s$^{-1}$, a mean harmonic radius of $R_H = 480$ kpc, a deprojected median radius of $R_S = 720$ kpc and a crossing time of $H_0 \tau_c = 0.44$.\(^1\) Hereafter a Hubble constant.

\(^1\)In these expressions,

\[
R_S \equiv \frac{4R}{\pi}, \quad R_H^{-1} \equiv 4 \sum_{i<j} \frac{R^{-1}_{ij}}{\pi N_k(N_k - 1)}, \quad \tau_c \equiv \frac{2R_H}{\sqrt{3}\sigma},
\]

where $R$, $R_{ij}$ and $N_k$ are the average projected separation, the projected separation between galaxy pairs and the number of galaxies, respectively.
of $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is assumed. Gourgoulhon, Chamaraux, & Fouqué (1992, hereafter GCF) found for nearby small groups (< 80 Mpc) a median $\sigma \approx 75 \text{ km s}^{-1}$, a virial radius of about 770 kpc, and a crossing time-scale of $\sim 4 \text{ Gyr}$, these quantities corrected for the effect of the Hubble flow. More recently, Makarov & Karachentsev (2000, hereafter MK) derived from a sample of 839 groups, in particular for those consisting of five galaxies, the following mean values, with quartiles: $\sigma = 57^{+26}_{-13} \text{ km s}^{-1}$, a non-deprojected harmonic radius of $230^{+25}_{-23} \text{ kpc}$ and a dimensionless crossing time of about 0.1.

A special and very important case is that of Hickson's compact groups (HCG), characterized by being composed of at least four galaxies with a median projected separation of about three times the diameter of their luminous component, and densities similar to the central region of rich clusters (Hickson 1982, 1997). Hickson's initial catalog contained 100 groups with a median projected separation $R = 52^{+180}_{-21}$ kpc, a velocity dispersion $\sigma = 200^{+157}_{-127} \text{ km s}^{-1}$ and a dimensionless crossing time $H_0 t_c = 0.016^{+0.007}_{-0.007}$; where $t_c \equiv 4R/(\pi\sqrt{\sigma})$. However, Sulentic (1997) pointed out that only 61 of them show according redshifts.

It is noteworthy that HCG provide an ideal site to study the evolution of galaxies, via interactions and/or mergers, in a high density environment. One of the most striking challenges of the compact groups is how to reconcile their apparent longevity with their quite small crossing time of about 1% of the Hubble time. This short time-scale suggests a rapid evolution of these systems toward a complete merging: the so-called overmerging problem (Hickson, Richstone, & Turner 1977; Hickson 1982; White 1990; Hickson 1997). However, Hickson et al. (1984) and Rubin, Hunter, & Ford (1991) have suggested that compact groups may have formed relatively recently.

In general, two complementary approaches have been proposed to address this issue—see Athanassoula (2000) for review of this topic. On the one hand, compact groups are assumed to be formed in the early universe and survive somehow until the present epoch (Ishizawa 1986; Diaferio, Geller, & Ramella 1994; Governato, Tozzi & Cavaliere 1996). Diaferio et al. (1994) introduced a scenario where a continuous replenishment of the compact group occurs within a single collapsing rich loose group. Governato et al. (1996) proposed an alternative model that relies on the evolution of a compact group as an ongoing and frequent process through secondary infall of galaxies in a critical universe. However, in these scenarios merging is not suppressed, so one would expect more compact groups and merger remnants than those observed today (Mamon 2000).

On the other hand, there is a dynamical view where some kind of tuning of the initial conditions is imposed in order to guarantee the survival of the compact group. Along these lines, Barnes (1985) was the first to investigate the effects of different kinds of initial conditions on the evolution of compact groups. He found that a compact group immersed in a massive common halo would, in general, delay its merging process if more mass is initially placed in this common halo. This merging delay results from the fact that the halo common mass is obtained from the galaxy haloes, thus reducing their masses and increasing the dynamical friction timescale. This result was also corroborated by Bode, Cohen, and Lugger (1993). Barnes also found that groups whose initial conditions begin in ‘turnaround’ or ‘expansion’ merge more rapidly than their virialized counterparts.

More recently, Athanassoula et al. (1997) carried out a more systematic study of the parameter space for small groups (e.g., concentrations of the common and individual haloes, common-halo-mass to total-group-mass ratios, etc.). Their main findings are: first, that centrally concentrated groups merge faster for high common-halo to total-mass ratios and slower for low common-halo to total-mass ratios; and, second, that the overmerging issue of compact groups may not be a problem if appropriate initial conditions are chosen. To support this last point, they built up a virialized group with a large common-halo to total-group ratio and an almost homogeneous central concentration that was able to survive for as long as $\sim 20 \text{ Gyr}$. According to these results, the merger rates obtained from group simulations without including a common envelope, such as the ones to be considered here, would be an upper limit.

In this work, we test the hypothesis of Barnes (1989) and White (1990): that diffuse groups might be the progenitors of compact ones. This hypothesis has been criticized by Diaferio et al. (1994) for lacking quantitative support. We also test the hypothesis, advanced on observational grounds, that HCG might be relatively young systems (Hickson et al. 1984; Rubin et al. 1991). A dynamical approach is used and, to this end, a series of $N$-body simulations are performed without including a common halo. These simulations allow us to compare our results with the observed kinematical properties of small groups. The rest of this paper has been organized as follows: in § 2 the numerical methods used to set up the galaxy models and the initial conditions.
for the groups are described. In § 3 the merging histories of our simulations are described, and the dynamical properties are given in § 4. In § 5 the mass estimation of our groups is addressed. A constrained representation of the ‘phase-space’ is shown in § 6. A more realistic N-body simulation involving spiral galaxies is given in § 7, and § 8 contains a general discussion of the results. Finally, the main conclusions are summarized in § 9.

2. DESCRIPTION OF THE SIMULATIONS

This section contains a detailed description of the method used to set up our galaxy model and the initial conditions for our small galaxy groups. Also, the computational tools and the parameters used to evolve our simulations are described.

2.1. Galaxy Model

Each individual galaxy is represented by a self-consistent Plummer model. In constructing the galaxy model no explicit distinction between dark and luminous matter is made. However, to gain some feeling about the evolution of a luminous component, we classify as ‘luminous’ a fraction (10%) of the particles that are initially the most tightly bound.

By using the procedure described by Aarseth, Hénon, & Wielen (1974), positions and velocities of the particles are randomly generated from the following mass and phase-space distributions $f(E)$:

$$M(r) = M_e \frac{(r/R_0)^3}{[1 + (r/R_0)^2]^{3/2}$$

and

$$f(E) = \frac{24\sqrt{2}R_0^2}{7\pi^3 G^3 M_e^4} |E|^{7/2},$$

where $M_e$ is the total galaxy mass, $R_0$ its scale-length, and $E$ the energy per unit mass, respectively.

A system of model units such that $G = 1, M_e = 1$ and $R_0 = 1$ is chosen. Since at large radii the Plummer density goes as $\rho \propto r^{-5}$ a cut-off radius is introduced, for numerical purposes, at about $r_{\text{cut}} \approx 10 R_0$ which contains about 99% of the total galaxy mass. In this system of units, the dynamical time-scale, the half-mass and the cut-off radii are given by $t_d = \sqrt{8R_0^2/(GM_e)} = \sqrt{\pi}, R_h \approx 1.3$ and $r_{\text{cut}} = 10$, respectively. In order to compare with observational data the values $r_{\text{cut}} \approx 135$ kpc and $M_e = 5.5 \times 10^{11} M_\odot$ (Model B of Kuijken & Dubinski 1995) are adopted. For these values the time and velocity units are 31.5 Myr and 419 km s$^{-1}$. In general, the simulations can be easily scaled through the following expressions for the velocity and time:

$$v = 2.1 \times 10^{-3} \sqrt{M_e/R_0} \text{ km s}^{-1} \text{ and } t = 4.7 \times 10^5 \sqrt{R_0^3/M_e} \text{ Myr},$$

where $R_0$ and $M_e$ must be given in kpc and $M_\odot$, respectively. With these definitions our initial ‘luminous’ core is contained within a radius of 0.74 model units, $\approx 10$ kpc. Finally, each galaxy is represented by 3000 equal-mass particles.

2.2. Group Initial Conditions

In all cases each group consists of five equal-mass galaxies. Galaxies are assumed to be already formed and no secondary infall is considered, which is justifiable in a low density universe (e.g., Bahcall 1999; Hranecky et al. 2000). The positions and velocities for the center of mass of these galaxies are randomly generated from initial conditions starting at ‘turnaround’ and at virial equilibrium. In all simulations no primordial common dark halo is included. Although this assumption seems to contradict current cosmological models of structure formation, some observations of clusters and groups suggest that most of the dark matter is associated with the individual dark haloes of galaxies (e.g., Bahcall, Lubin, & Droman 1995; Puche & Carignan 1991). Hence, the merging activity observed in our simulations could be an upper limit.

2.2.1. Collapsing Groups

For groups initially in ‘turnaround’, the positions for the galaxy centers were selected randomly from a homogeneous sphere of radius $R_{\text{max}}$, as defined by Gunn & Gott (1972), ignoring any contribution from the cosmological constant: $R_{\text{max}} = (8GM_C t^2/\pi^2)^{1/3}$, where $M_C$ represents the mass of the group enclosed within this radius at time $t$; mass conservation will be assumed. Compact groups are assumed to be on the verge of complete collapse at the present epoch (e.g., Hickson et al. 1984; Rubin et al. 1991), and thus their collapse time-scale $t_{\text{clips}}$ turns out to be the age of the universe $t_0$. Hence, $R_{\text{max}}$ was evaluated at $t_0/2$ (Gott & Rees 1975).

For $\Omega_0 = 1$ it is found that $t_0 = 2H_0^{-1}/3$, and for $\Omega_0 \rightarrow 0$ that $t_0 \approx H_0^{-1}(1 + 0.5\Omega_0 \ln \Omega_0)$ (Gott et al. 1974). In particular, for $\Omega_0 = 0.2$ and $H_0 = 75$ km s$^{-1}$ Mpc$^{-1}$ Gyr the ‘turnaround’ epoch was 5 Gyr ago (i.e., at redshift $z \approx 0.6$, Sandage 1961). In Table 1 the ‘turnaround’ radius of the group is listed for different values of $H_0$ and $\Omega_0$. A radius $R_{\text{max}} = 700$ kpc is taken as the fiducial ‘turnaround’ radius for collapsing groups. Note that this value is

This approximation differs by less than 1% from its true value for $fH_0 = 0.2$ (Peacock 1999).
consistent with $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_0 = 1$, and with the observational data of Sulentic (1987), who found that there is not a significant population of normal galaxies within $\sim 1 \text{ Mpc}$ of the central region of HCG.

Finally, the bulk velocity magnitudes for the galaxy centers were obtained randomly from a Gaussian distribution with a one-dimensional velocity dispersion $\sigma = \sqrt{4GMG/(5R_{\text{max}})}$ (Gott 1975) subject to the constraint $2T/|W| = 1/4$ (Barnes 1985), where $T$ and $W$ are the kinetic and potential energy of the group, respectively. These energies were computed assuming that galaxies are equal point-mass particles. Velocity vectors were oriented randomly.

2.2.2. Virialized Groups

Theoretical arguments and $N$-body simulations (Peacock 1999) indicate that by a time $\approx 3t_{\text{clps}}/2$ a system has reached an approximate equilibrium state with a virial radial scale of $R_v \approx R_{\text{max}}/2$. Hence, a radial scale for groups in near virial equilibrium at $t_0$ is $R_v = R(3t_0)/2$ and, in the particular case of our fiducial model, a value $R_v = 267 \text{ kpc}$ is obtained. The positions of the galaxies are randomly chosen from a uniform spherical distribution with radius $R_v$, and their velocities from a Gaussian distribution constrained to satisfy the virial ratio $2T/|W| = 1$ with random orientation vectors.

2.3. Computational Issues

To evolve our galaxy and group models a serial TREECODE was used (Barnes & Hut 1986) with a tolerance parameter $\theta = 0.9$ and with the quadrupole corrections to the potential included. A softening parameter $\epsilon = 0.07 \approx R_0/14$ and a time-step $\Delta t = 0.1$ were adopted. With these parameters, the simulations were evolved for about 321.4 time units ($\approx 10 \text{ Gyr}$) and the energy conservation, in all cases, was better than 0.7%. A total of 30 simulations were performed for each kind of initial conditions considered for the groups.

To follow up the orbital evolution of each galaxy inside the group its center was identified with its most bound particle, which was set ‘by hand’ to have 1% of the galaxy mass (Aguilar & White 1985). In order to check the stability of the galaxy model due to this change, its evolution was followed, in isolation, for $\sim 200$ dynamical time-scales. No significant change in its inner structure or in the total energy conservation was found. From here on, all kinematical properties of the group simulations (e.g., mean separation, velocity dispersion, etc.) will be obtained from these galaxy centers.

| $H_0$ b | $\Omega_0 = 0.2$ | $\Omega_0 = 1$ |
|---------|----------------|----------------|
| 50      | 0.88           | 0.76           |
| 75      | 0.67           | 0.58           |
| 100     | 0.56           | 0.48           |

*Units: Mpc  bUnits: km s$^{-1}$ Mpc$^{-1}$

3. MERGING HISTORIES

In this section, the merging histories for all our simulations of small groups are computed. These histories provide some insight into the merging rate and the overmerging problem of compact groups posed by their small crossing time.

In Figure 1, the evolution of three particular simulations with cold initial conditions is shown. Note that group $g27c$ does not present any merger during the simulation ($\sim 10 \text{ Gyr}$) while group $g02c$ completely merged. At the present epoch (i.e., $\sim 5 \text{ Gyr}$ from ‘turnaround’), group $g22c$ resembles a chain-like compact group which is better appreciated at the bottom-right of this figure where only ‘luminous’ particles have been displayed. Note also that groups $g27c$ and $g02c$ may be classified as triplets.

In order to characterize this merging activity the following pair-wise merging criteria (Aarseth & Fall 1980) are employed:

$$V_{ij} = |v_j - v_i| < V_{\text{rms}}/2,$$

$$R_{ij} = |r_j - r_i| < R_0/2,$$

where $V_{ij}$ and $R_{ij}$ are the relative velocity and separation between a pair of galaxies $i$ and $j$, respectively, and $V_{\text{rms}}$ and $R_0$ are their characteristic internal velocity and radius.

In Figures 2 and 3 the time evolution of the merging activity in our groups is shown. The dynamical time for the virialised groups is $t_d = \sqrt{8R_0^3/(GMG)} \approx 111$, corresponding to about 3.6 Gyr. It is observed in Fig. 2 that no group has merged completely at the present epoch, $t_0$, although some multiple mergers do exist. About 15 groups (50%) do not present any merger at all, 9 (30%) show a merger of two galaxies and 6 (20%) triple or quadruple mergers. Note that at the end of the simulations just one group does not show any merger and 15 (50%) have merged completely.

In the case of virialized groups, 5 ($\approx 17\%$) do not present any kind of merging within a dynamical time-scale, 11 ($\approx 37\%$) show a merger of two
Fig. 1. This figure shows the XY-projection in kpc for the collapsing groups g27c (upper-left), g02c (upper-right) and g22c (bottom-left). The numbers in each frame indicate the time elapsed since turnaround in Gyr. Lines refer to the trajectories followed by each galaxy inside its corresponding group. Also, small boxes of width 300 kpc are shown at time 4.79 Gyr for each group. These boxes have been amplified and show just ‘luminous’ particles at the bottom-right of this figure which corresponds to the XY, XZ and YZ-projections.
Fig. 2. Number of individual galaxies versus time (merging history) for simulations with cold initial conditions. The free-fall time-scale \( t_f = \pi \sqrt{R_{\text{max}}/(32GM_G)} \) and the present epoch \( t_0 (\sim 5 \text{ Gyr from ‘turnaround’) are indicated by arrows. The merging tracks have been displaced vertically and in time for better appreciation. The displacement in time corresponds to 0, 3, 6, 9, and 12 time units. A scale corresponding to 1 Gyr is indicated.

Fig. 3. As in Fig. 2, but for initially virialised groups. In this case, the arrow corresponds to the dynamical time of the group.

galaxies, 10 (\( \approx 33\% \)) have triple or quadruple mergers, and only 4 (\( \approx 13\% \)) have completely merged (see Fig. 3). Note also that about 12 (40%) have not completely merged after \( \sim 10 \text{ Gyr of evolution. This clearly indicates that some compact groups can survive complete merging for about a Hubble time.}

4. DYNAMICAL PROPERTIES

The dynamical properties of the groups along three orthogonal ‘lines-of-sight’ at different times are determined. The median values for the one-dimensional velocity dispersion, the mean harmonic and deprojected radii, the VME and MME masses (see § 5), and two variants of the dimensionless crossing time \( H_0 \tau_c \) and \( H_0 t_c \), see § 1) are summarized in Table 2. The times indicated in the table correspond to 0.5, 1, 2, ..., 10 Gyr of evolution from their respective initial conditions.

4.1. Collapsing Groups

In Figure 4 we show the dynamical parameters as a function of time for our collapsing groups \( g27c \), \( g02c \), and \( g22c \). For comparison, the medians of MK and HCG are indicated by arrows. This figure shows the general trend in a collapse—a decrease in size followed by an increase in velocity dispersion and a reduction in the crossing time. The size of a group depends on the method adopted; the harmonic radius is always lower than the mean separation radius and, hence, \( \tau_c < t_c \). An agreement between both definitions of crossing times occurs when the systems are still rather diffuse. This is due to the fact that the harmonic radius \( R_H \) is more sensitive to smaller separations than \( R_S \).

The kinematical properties of \( g27c \) and \( g02c \) at the present epoch are in good agreement with the values found in NCfA, GCF, and MK groups. In general, our simulations reproduce very well the kinematical parameters of ‘normal’ small groups of galaxies (see Table 2).

**Group \( g27c \).** This group has properties at the present epoch that resemble those of a ‘normal’ diffuse group: \( R_H \approx R_S \approx 700 \text{ kpc, } \sigma \approx 70 \text{ km s}^{-1} \), and \( H_0 \tau_c \approx H_0 t_c \approx 0.5 \). These values are large with respect to the observed ones in HCG, but in better agreement with the medians of NCfA, GCF, and MK groups.

**Group \( g02c \).** This system is similar to a Hickson’s group, showing \( \sigma \approx 200 \text{ km s}^{-1} \) and \( H_0 t_c \approx 0.09 \) when observed along the \( Y \)-axis (see Fig. 4). However, its mean separation is quite large \( R_S \approx \)}
TABLE 2
DYNAMICAL PROPERTIES OF GROUPS IN MODEL UNITS

| $t^a$ | $\sigma^b$ | $R_h^c$ | $H_0\tau_c^d$ | $R_s^e$ | $H_0t_c^f$ | $M_v^g$ | $M_{\text{med}}^h$ | $\sigma^b$ | $R_h^c$ | $H_0t_c^f$ | $R_s^e$ | $M_v^g$ | $M_{\text{med}}^h$ |
|-------|-----------|--------|----------------|--------|-----------|--------|----------------|--------|--------|-----------|--------|--------|----------------|
| 15.0  | 0.071     | 45.76  | 1.91           | 51.27  | 1.09      | 1.28   | 0.65           | 0.221  | 15.14  | 0.197     | 19.17  | 0.126  | 3.41   | 1.79 |
| 36.0  | 0.077     | 43.25  | 1.57           | 49.74  | 0.99      | 1.50   | 0.67           | 0.223  | 10.79  | 0.126     | 17.18  | 0.111  | 2.71   | 2.06 |
| 63.0  | 0.090     | 36.09  | 1.01           | 46.78  | 0.72      | 1.46   | 0.79           | 0.189  | 2.72   | 0.043     | 18.65  | 0.134  | 0.99   | 1.49 |
| 96.0  | 0.112     | 25.65  | 0.43           | 41.19  | 0.49      | 1.33   | 0.62           | 0.175  | 1.44   | 0.033     | 20.10  | 0.179  | 0.27   | 0.66 |
| 126.0 | 0.132     | 8.41   | 0.11           | 34.04  | 0.33      | 0.88   | 0.92           | 0.141  | 0.55   | 0.015     | 21.23  | 0.231  | 0.06   | 1.22 |
| 156.0 | 0.149     | 3.70   | 0.06           | 27.86  | 0.28      | 0.76   | 1.13           | 0.137  | 0.45   | 0.012     | 20.31  | 0.212  | 0.06   | 0.60 |
| 186.0 | 0.153     | 0.87   | 0.02           | 22.31  | 0.21      | 0.16   | 1.74           | 0.111  | 0.24   | 0.008     | 21.47  | 0.195  | 0.02   | 0.08 |
| 219.0 | 0.158     | 0.53   | 0.02           | 17.21  | 0.19      | 0.11   | 1.10           | 0.111  | 0.21   | 0.008     | 21.10  | 0.241  | 0.01   | 0.04 |
| 252.0 | 0.152     | 0.28   | 0.01           | 15.07  | 0.17      | 0.04   | 0.95           | 0.082  | 0.15   | 0.007     | 20.66  | 0.239  | 0.01   | 0.02 |
| 282.0 | 0.131     | 0.23   | 0.01           | 15.59  | 0.20      | 0.03   | 0.57           | 0.077  | 0.15   | 0.007     | 18.39  | 0.242  | 0.01   | 0.01 |
| 315.0 | 0.100     | 0.19   | 0.01           | 17.20  | 0.21      | 0.02   | 0.14           | 0.083  | 0.11   | 0.005     | 15.41  | 0.197  | 0.01   | 0.00 |

$^a$Time evolved from the initial conditions.  $^b$One-dimensional velocity dispersion.  $^c$Harmonic radius.
$^d$Crossing-time scale where $\tau_c \equiv 2R_h/\sqrt{3\sigma}$.  $^e$Mean separation radius.  $^f$Crossing-time scale where $t_c \equiv R_s/(\sqrt{3\sigma})$.
$^g$Virial mass.  $^h$Median mass.
Fig. 4. Kinematical parameters as a function of time for collapsing groups $g^{27}c$, $g^{02}c$, and $g^{22}c$. Astronomical units are used. The left-arrow (→) indicates the median of the 5-galaxy MK groups, while the right-arrow (→), HCG. The different lines indicate the line-of-sight used to compute the quantities: along X-axis (solid), along Y-axis (dotted), and along Z-axis (broken). For the dimensionless crossing times, thicker lines for $H_0c$ than for $H_0t_c$ have been used.

400 kpc. The average values over the three lines of sight obtained at the present epoch are: $R_H \approx 10$ kpc, $R_S \approx 250$ kpc, $\sigma \approx 90$ km s$^{-1}$, $H_0\tau_c \approx 0.01$, and $H_0t_c \approx 0.2$.

**Group $g^{22}c$.** This group has dynamical properties, at the present epoch, that very closely resemble the median properties of Hickson’s groups: $R_H \approx 40$ kpc, $R_S \approx 110$ kpc, $\sigma \approx 120$ km s$^{-1}$, $H_0\tau_c \approx 0.1$, and $H_0t_c \approx 0.05$. However, values of $R_S \approx 50$ kpc, $\sigma \approx 180$ km s$^{-1}$ and $H_0t_c \approx 0.01$ are found along the Z-axis. We should mention that the selection of $g^{22}c$ was based solely on its similarity with HCG at the present time when seen in the XY-plane, which is confirmed with the more realistic models of spiral galaxies in section §6. In this group, the three closest galaxies merge in $\approx 1$ Gyr from the present epoch, while the other two form a wide ‘binary’ at the end of the simulation (10 Gyr). This result suggests that although some HCG have small crossing times they will not merge completely over several Gyr from the present epoch.
Furthermore, the majority of the crossing times fall within the values found in HCG (Fig. 6) and ≈ 50% would be classified as virialized at the present epoch according to the criterion of Gott & Turner (1977).

Hickson’s catalog (Hickson 1982; Hickson et al. 1992) has 16 groups with $\sigma \lesssim 100$ km s$^{-1}$ and just five of them show $\sigma \lesssim 5$ km s$^{-1}$. Four of these latter groups (HCG38, HCG47, HCG49, and HCG88) have a dimensionless crossing time of $H_0t_c \gtrsim 1$, suggesting that they are not close to virial equilibrium. However, our results indicate that such an assumption may not be correct.

Since our models roughly agree with those HCG exhibiting a low velocity dispersion, they suggest a probable explanation for that fact (Rubin et al. 1991; Mamon 2000).

### 4.2. Virialized Groups

Figure 7 shows histograms of the dynamical quantities for virialized initial conditions at times $t \approx 0.5, 3, 6$ Gyr. As shown in Table 2, these groups present median values of $\sigma \approx 90$ km s$^{-1}$, $R_H \approx 200$ kpc, $R_S \approx 250$ kpc, $H_0\tau_c \approx 0.20$, and $H_0t_c \approx 0.13$ in their initial stages. After 5 Gyr of evolution the following medians are obtained: $\sigma \approx 58$ km s$^{-1}$, $R_H \approx 6$ kpc, $R_S \approx 274$ kpc, $H_0\tau_c \approx 0.01$, and $H_0t_c \approx 0.21$, which are far from the values found in Hickson’s groups.

In general, the $\sigma$ distribution shows small changes even after ~ 10 Gyr of evolution; however, the $R_H$ distribution presents significant changes which are also observed in the evolution of the $H_0\tau_c$ distribution. The $\sigma$ distribution tends to populate the regions of lower velocity dispersion and to increase somewhat the mean separation. Notice that about 50% of groups have at $t = 0$ crossing times similar to those of normal small groups (NCfA, GCF, and MK) and ~5% close to the median of Hickson’s groups.

The results of this and the previous sections indicate that a large fraction (~40%) of our initially virialized groups have not merged during 10 Gyr and, hence, overmerging (Hickson 1997) may not be a problem. However, it is important to point out that in our simulations a primordial common dark halo has not been included, which may change our conclusions.

### 5. Mass Estimates

The evolution of the virial mass (VME) and median mass (MME) estimators is computed. These estimators are defined, respectively, as (Heisler,
Fig. 6. Evolution of the distribution of the crossing-time $t_c = R_S/(\sqrt{3}\sigma)$. Line styles are the same as in Fig. 5. The interval of crossing-times for HCG and that assumed to be satisfied by virialized groups, according the criterion of Gott & Turner (1977), are indicated. (a) Results for collapsing groups. (b) Results for virialized groups. Note that both kinds of initial conditions lead to ranges in crossing-time within the observed values.

Fig. 7. Same as in Fig. 5, but for small groups with virial initial conditions.

Tremaine, & Bahcall 1985; Aceves & Perea 1999 and references therein):

\[ M_v = \frac{3\pi N_g}{2G} \frac{\sum_i V_i^2}{\sum_{i<j} 1/R_{ij}} \]

\[ M_{\text{med}} = \frac{6.5}{G} \text{MED}_{ij} \left[ \frac{(V_i - V_j)^2 R_{ij}}{} \right] \]

where $N_g$ is the number of galaxies, $V_i$ the velocity along the line-of-sight with respect to the centroid of velocities, $R_{ij}$ the projected separation on the sky, and the constant in $M_{\text{med}}$ is determined from numerical experiments.

In Figure 8 the above quantities are plotted as function of time for our collapsing groups g27c, g02c, and g22c. This figure reveals that both mass estimators underestimate the mass of the group at $t = 0$ as expected (remember that these numerical groups started with a virial coefficient far away from equilibrium $2T/|W| = 1/4$). As the group evolves and reaches some state of virial equilibrium it is observed that these mass estimators give a reliable mass for the groups as long as no mergers occur between their member galaxies. Note that only bulk motions are considered in these definitions, ignoring the self-gravitating nature of the individual galaxies. This last point is important, since self-gravitating galaxies have the ability to absorb energy as the whole
Fig. 8. Time behavior of the two mass estimators considered in § 5 for our collapsing groups g27c, g02c, and g22c. The upper panels show the virial mass, while the lower the median mass. The horizontal broken line corresponds to the total mass of the groups; \( N \)-body units are used for the mass, but physical units for the time. The different types of line denote different lines-of-sight, as in Fig. 4.

The results from all the simulations carried out are summarized in the histograms of Figure 9. The true mass of the groups is indicated by the vertical line. For collapsing groups (Fig. 9a) the VME estimator (upper panel) gives a median virial mass of \( M_V = 1.28 \) at \( t = 15 \), which is consistent with the expected value of \( M_V = 1.25 \), while the MME (lower panel) underestimates the mass by about an order of magnitude. However, once the groups are allowed to evolve to the present epoch (\( \sim 5 \) Gyr), the mass is underestimated by an order of magnitude for the VME method and by a factor of about 5 for the MME method. For the case of virialized groups, we found that at \( t = 0 \) both mass estimators give a similar median mass of about \( M_V \approx 3.5 \). Once the separation between galaxies decreases, the MME provides a better estimate of the mass. From these results, the MME is considered more appropriate for estimating the mass of systems that resemble HCG.

Hickson’s compact groups have a mass-to-light ratio of \( \Upsilon \approx 50h\odot \). If HCG are physically well defined groups, their matter content is probably \( \gtrsim 5 \) times more than is inferred now. This would imply that a closer estimate to the true mass-to-light ratio of HCG would be \( \Upsilon \sim 250h\odot \). It is interesting to note that this value falls within the values found in clusters (200–300\( h\odot \), Bahcall 1999) suggesting that both types of systems would have approximately the same fraction of baryonic to dark matter (see White et al. 1993 for a discussion of the cosmological implications).

6. \( R_S-\sigma \) DIAGRAM

A restricted ‘phase-space’ consisting of an \( R_S-\sigma \) diagram is constructed in order to try to discern some evolutionary track in our simulated groups. In Figure 10 the collapsing (Fig. 10a) and virialized
Fig. 9. Histograms of the behavior of the VME (upper panels) and MME (lower panels), for (a) cold and (b) virial small groups. The bar (j) indicates the true mass of the groups in N-body units. Dotted lines show the distribution at \(t = 0\) Gyr, short-dashed lines at 3 Gyr, and solid at 5 Gyr.

(Fig. 10b) groups are shown at times \(t = 0.5\) and \(t = 5\) Gyr together with the corresponding diagram for HCG where the ones considered bona fide by Sulentic (1997) are indicated.

This diagram clearly shows that initially cold groups reduce their mean separation and increase their velocity dispersion as they evolve, while for virialized groups their velocity dispersions decrease somewhat and their mean separation tends to increase. Furthermore, about 8 collapsing groups have \(R_S \lesssim 100\) kpc and \(\sigma = 60-200\) km s\(^{-1}\) at the present epoch (\(\sim 5\) Gyr), which roughly correspond to the values shown by normal small groups, and are within the range of values found for HCG. Also, two groups have \(R_S \lesssim 50\) kpc and \(\sigma \approx 200\) km s\(^{-1}\) which are very close to the median values of HCG.

This \(R_S-\sigma\) diagram suggests a resemblance between HCG and those of our the systems that once had a diffuse configuration and are on the verge of collapse at the present time, while some of our virialized groups show properties more similar to the ‘normal’ small groups. These and previous results suggest that HCG are the more advanced systems in a process of gravitational collapse among small groups. Thus, compact groups may be rather young configurations (Hickson et al. 1984; Barnes 1989; White 1990; Rubin et al. 1991).

7. A MORE REALISTIC SIMULATION

In the simulations reported above no clear distinction between luminous and dark matter was made, and hence no precise statement could be established whether, for example, the group \(g22c\) could actually be considered a compact group according to Hickson’s criteria.

To address this particular issue, a more realistic galaxy model was set up by replacing the Plummer models by spiral galaxies using model B of Kuijken & Dubinski (1995); see § 2.1 for some numerical values of this model (we refer the reader to Kuijken & Dubinski’s paper for a detailed description of the method to build up the disk galaxy model). It should be mentioned that a detailed study of groups involving disk galaxies is outside the scope of the present work. Table 3 summarizes the parameters that define our galaxy model and Figure 11 shows its corresponding rotation curve.

For the galaxy centers and bulk motions inside the group the values of our previous collapsing model \(g22c\) are used. For illustrative purposes, arbitrary orientations for the galaxy angular momentum vector are chosen. Each numerical galaxy consists of 65,536 particles, 16,384 for the disk, 4096 for the bulge and 45,056 for the halo; i.e., a total of 327,680 particles were used in this simulation.
Fig. 10. $R_\sigma$-$\sigma$ diagram for (a) collapsing, (b) virialized, and (c) Hickson’s groups. Hickson’s compact groups appear to resemble systems that have suffered a gravitational collapse out of a more diffuse system.

### TABLE 3

| Model | $M_D$ | $R_D$ | $R_t$ | $z_D$ | $\delta R_{\text{out}}$ | $\Psi_c$ | $\sigma_B$ | $\rho_B$ | $\Psi_0$ | $\sigma_0$ | $q$ | $C$ | $R_a$ |
|-------|-------|-------|-------|-------|------------------------|----------|------------|---------|----------|------------|----|----|------|
| MW-B  | 0.87  | 1.0   | 4.0   | 0.15  | 0.4                    | −2.9     | 0.71       | 14.5    | −5.2     | 0.96       | 1.0| 0.1| 0.8  |

**Notes:** $M_D$, $R_D$, $R_t$, $z_D$, and $\delta R_{\text{out}}$ correspond to the mass, radial scale-length, cut-off radius, vertical scale-length of the disk, and disk truncation width, respectively. $\Psi_c$, $\sigma_B$, and $\rho_B$ refer to the cutoff potential, velocity dispersion, and central density of the bulge, respectively. Finally, $\Psi_0$, $\sigma_0$, $q$, $C$, and $R_a$ indicate the central potential, velocity dispersion, potential flattening, concentration, and characteristic radius of the halo, respectively.

To evolve the system a parallel treecode developed by Dubinski (1996) with a tolerance parameter of 0.8 was used. Forces were computed with the quadrupole terms included and with a fixed time-step corresponding to 1 Myr. With these parameters the energy conservation was better than 0.2%. The simulation was run on a Beowulf class cluster consisting of 32 Pentium processors each of 450 MHz (Velázquez & Aguilar 2002). It took a wall time of 16 seconds per time-step.

In Figure 12a the $XY$-projection of the luminous component of the group at time 4.8 Gyr, the same as Fig. 1 of model $g22c$, is shown. Although differences in the evolutionary trend of our more realistic model are appreciated with respect to model $g22c$, it is rather easy to conclude that it satisfies Hickson’s criteria for compact groups. Hickson’s criterion on the difference of magnitudes between galaxies is satisfied immediately, since our galaxies are identical. Therefore, a group is obtained that is compact at the present epoch and will not merge completely for another $\sim 5$ Gyr.

In Figure 12b only the dark particles of the haloes are shown. Notice that a common dark halo has al-
Fig. 12. XY-projection at time 4.8 Gyr of our more realistic model. Luminous particles are shown in (a) and the dark component is indicated in (b). Note that a common dark envelope for the three galaxies close to merging is already present, while the other two in the upper-left will not merge for $\gtrsim 5$ Gyr from the present epoch. Compare with our previous model g22c of Fig. 1.

ready formed for three of the galaxies that will merge in the next few Gyr. This result is consistent with the suggestion by Rubin et al. (1991) that compact groups reside in a common dark halo at the present epoch.

8. GENERAL DISCUSSION

It has been found that $\approx 10\%$ of initially diffuse groups reach the present epoch with dynamical properties consistent with the median of Hickson’s compact groups. Hickson’s groups appear as those systems that have managed to evolve more rapidly toward a compact configuration, due to particular initial conditions in the density perturbation that led to the formation of small groups. The formation of a compact group is, according to our results, consistent with a hierarchical clustering scenario for the formation of structures in the universe, thus supporting the hypothesis advanced by Barnes (1989) and White (1990), and the one suggested on observational grounds by Hickson et al. (1984) and Rubin et al. (1991).

The present model leads to $\sim 50\%$ of originally diffuse groups arriving at the present epoch without any merging. Also, it is found that $\sim 10\%$ (5 simulations) of these groups will not have any merger within the next $\sim 5$ Gyr. In this sense, our model provides an alternative explanation to the properties of compact groups and their existence today, and complements several previous studies (e.g., Athanassoula et al. 1997).

In general, initial conditions from ‘maximum expansion’ reproduce very well the properties of ‘normal’ small groups. Our simple model avoids the requirement for an initial overdensity and/or significant secondary-infall (Diaferio et al. 1994; Governato et al. 1996), and the need for special initial conditions to avoid the complete merging of a group (e.g., Athanassoula et al. 1997).

The present model does not show the merging instability, but many questions remain. One of the most obvious caveats of our simulations is not taking into account a primordial dark halo for virialized initial conditions. However, the merging activity found may still be considered a lower limit if a concentrated dark halo were to be included. Certainly, more realistic simulations of small groups, considering a spectrum of masses and a clear distinction between luminous and dark matter, as well as initial conditions taken from a cosmological simulation, are necessary to address these issues. We plan to study some of these matters in the future.
9. CONCLUSIONS

The main results of this work are as follows:

1. Groups starting from 'maximum expansion' with a diffuse configuration have, at the present epoch, dynamical properties very similar to the medians obtained from several catalogs of small groups. In particular, about 10% of them show values close to the observed medians of Hickson's groups.

2. The suggestion that Hickson's compact groups originated from diffuse systems, and are relatively young systems, finds quantitative support in our numerical experiments.

3. It is found that overmerging is not an important problem in our simulations, either with virialized or cold initial conditions. For the case of virialized groups, about 40% survive merging during ≈ 10 Gyr. This also indicates that the existence of a large, massive, common dark halo does not appear to be a strong requirement to explain the present day properties of small groups.

4. The median mass estimator (MME) appears to be a better estimator of mass than the virial mass estimator (VME) for groups at the present epoch. It is estimated that the mass-to-light ratio of HCG is probably $\sim 250$, suggesting that clusters and compact groups have about the same fraction of baryonic to dark matter.

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