General Relativity As an Æther Theory

Maurice J. Dupré and Frank J. Tipler
Department of Mathematics, Tulane University, New Orleans, LA 70118
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Most early twentieth century relativists — Lorentz, Einstein, Eddington, for examples — claimed that general relativity was merely a theory of the æther. We shall confirm this claim by deriving the Einstein equations using æther theory. We shall use a combination of Lorentz’s and Kelvin’s conception of the æther. Our derivation of the Einstein equations will not use the vanishing of the covariant divergence of the stress-energy tensor, but instead equate the Ricci tensor to the sum of the usual stress-energy tensor and a stress-energy tensor for the æther, a tensor based on Kelvin’s æther theory. A crucial first step is generalizing the Cartan formalism of Newtonian gravity to allow spatial curvature, as conjectured by Gauss and Riemann.

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I. INTRODUCTION

Richard Feynman often emphasized the importance of having many mathematically equivalent ways of expressing the same physical theory. In the lecture which he gave on the occasion of receiving the 1965 Nobel Prize for physics, Feynman said: “Theories of the known, which are described by different physical ideas may be equivalent in all their predictions and are hence scientifically indistinguishable. However, they are not psychologically identical when trying to move from that base into the unknown. For different views suggest different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in one’s attempt to understand what is not yet understood. I, therefore, think that a good theoretical physicist today should as a class have this. . . . If my own experience is any guide, . . . if the peculiar viewpoint taken is truly experimentally equivalent to the usual in the realm of the known, there is always a range of applications and problems in this realm for which the special viewpoint gives one a special power and clarity of thought, which is valuable in itself” [1].

We shall follow Feynman and give a derivation of the Einstein field equations from æther theory. Most of the leading relativists in the early twentieth century, for examples Eddington [18] and even Einstein himself [19], claimed that general relativity was an æther theory, but they gave no mathematical demonstration of their claim.

We shall provide the demonstration in this paper. A huge number of æther theories were proposed over the nineteenth century, and one could write a book describing them. In fact, Edmund Whittaker wrote a two volume book [20, 21] describing them. All we shall need is two of these æther theories, namely the theory of Lorentz, and the theory of Kelvin.

The first step is to generalize the Cartan theory of Newtonian gravity to allow spatial curvature. To this curved space Cartan theory, we add Lorentzian æther, which says the Maxwell equations are the theory of the æther ([11], p. 13). We show that this implies that the curved space Poisson equation, $R_{tt} = 4\pi G\rho$ must become $R_{\mu\nu} = 4\pi G S_{\mu\nu}$, where all components of the Ricci tensor $R_{\mu\nu}$ must be present.

According to Einstein, in his Autobiography [12], the most natural choice for the tensor $S_{\mu\nu}$ is the stress-energy tensor. Einstein was uncomfortable with adding the term $\frac{4}{3}g_{\mu\nu}R$ to the Ricci tensor, saying it was only introduced for “technical reasons,” required by the vanishing of the covariant divergence of the stress-energy tensor.

Einstein was wise in being uncomfortable with this justification for adding this term. The modern view follows Noether and sees conservation laws as an expression of symmetries. Energy conservation is a consequence of a timelike Killing vector, momentum conservation a consequence of an appropriate spacelike Killing vector, corresponding to invariance under spatial translation, and so on. So the total energy need not be conserved in a spacetime with no timelike Killing field. But $T^\mu_{\mu\nu} = 0$ follows from $T^\mu_{\mu\nu} = 0$ only by assuming the comma goes to semicolon rule. The Noether theorems point out just how powerful an assumption this is. The vanishing of the divergence of the stress energy tensor is derived in Minkowski space using all the symmetries of Minkowski space. But leaving Minkowski space for a general spacetime means losing the symmetries that allowed the derivation of $T^\nu_{\mu\nu} = 0$ to start with!

We shall avoid using $T^\nu_{\mu\nu} = 0$ by assuming instead that the tensor $S_{\mu\nu}$ is a sum of two stress-energy tensors, the usual stress-energy tensor, and the stress-energy tensor for the æther. We shall show that the Lorentz theory, when combined with the Kelvin theory of the æther gives a form for $S_{\mu\nu}$ such that the resulting theory is the familiar Einstein equations of general relativity.

In Section 2, we shall generalized the Cartan-Newton gravity theory to curved space, by generalizing the Misher axioms for Cartan-Newton gravity theory. In Section 3, we then apply Lorentz-Kelvin æther theory to obtain the Einstein field equations. Finally in Section 4, we shall point out that all the basic ideas to construct the
Einstein-æther field equations existed in the nineteenth century. Using the PPN formalism, we shall show that the experimental evidence already existed in the nineteenth century to confirm this theory.

II. Generalizing the Misner Axioms for Newtonian Gravity

What we shall do in this section is generalize Cartan’s formulation to allow space to be curved. To accomplish this we shall proceed by starting with a set of rigorous axioms for Newtonian gravity as curvature. There are two such systems, one developed by Trautman and the other presented later by Misner. These two systems are mathematically equivalent, but we shall use the Misner axioms, because Misner’s system is much easier to generalize to the non-flat spatial case.

The eight Misner axioms are given in Box 12.4 of MTW.

Axiom 1: There exists a function \( t \) called “universal time,” and a symmetric (i.e., torsion free) covariant derivative \( \nabla \) (with associated geodesics, parallel transport, curvature operator, etc.).

Axiom 2: The 1-form \( dt \) is covariantly constant:

\[

\nabla_u dt = 0
\]

for all vectors \( u \).

Axiom 3: Spatial vectors are unchanged by parallel transport around infinitesimal, closed curves, i.e.,

\[

\mathcal{R}(\mathbf{u}, \mathbf{n}) \cdot \mathbf{w} = 0
\]

if \( \mathbf{w} \) is spatial for all vectors \( \mathbf{u} \) and \( \mathbf{n} \), and \( \mathcal{R}(\mathbf{u}, \mathbf{n}) \) is the curvature operator.

Axiom 4: All vectors are unchanged by parallel transport around infinitesimal, spatial, closed curves, i.e.,

\[

\mathcal{R}(\mathbf{v}, \mathbf{w}) = 0
\]

for every spatial \( \mathbf{v} \) and \( \mathbf{w} \).

Axiom 5: The Ricci curvature tensor \( R_{\alpha\beta} \equiv R^\mu_{\alpha\mu\beta} \) has the form

\[

\text{Ricci} = 4\pi\rho dt \otimes dt
\]

where \( \rho \) is the density of mass.

Axiom 6: There exists a metric “\( \cdot \)” defined on spatial vectors only, which is compatible with the covariant derivative in the following sense:

\[

\nabla_u (\mathbf{w} \cdot \mathbf{v}) = (\nabla_u \mathbf{w}) \cdot \mathbf{v} + (\nabla_u \mathbf{v}) \cdot \mathbf{w}
\]

for any spatial vectors \( \mathbf{w} \) and \( \mathbf{v} \), and for any \( u \) whatsoever.

Axiom 7: The Jacobi curvature operator \( \mathcal{J}(u, \mathbf{n}) \), defined for any vectors \( u, \mathbf{n}, \) and \( p \) by

\[

\mathcal{J}(u, \mathbf{n}) p = \frac{1}{2} \left[ \mathcal{R}(p, \mathbf{n}) u + \mathcal{R}(p, u) \mathbf{n} \right]
\]

is “self-adjoint” when operating on spatial vectors, i.e.,

\[

\mathbf{v} \cdot [\mathcal{J}(u, \mathbf{n}) \mathbf{w}] = \mathbf{w} \cdot [\mathcal{J}(u, \mathbf{n}) \mathbf{v}]
\]

for all spatial vectors \( \mathbf{v} \) and \( \mathbf{w} \), and for any vectors \( u \) and \( \mathbf{n} \) whatsoever.

Axiom 8: Ideal rods measure the lengths that are associated with the spatial metric, and ideal clocks measure universal time \( t \) or some multiple thereof. Furthermore, freely falling particles move along geodesics of \( \nabla \).

Let us remind the reader what these axioms are intended to accomplish on the connection: (1) ensure that the only non-vanishing components are \( \Gamma^i_{tt} \); (2) ensure that the spatial vector \( \Gamma^i_k \) is the gradient of some scalar field, i.e., \( \Gamma^i_k = \phi_i \); (3) \( \Gamma^i_{kl} = 0 \), so that (4) the spatial metric is the metric of flat space.

We want to generalize these axioms so that the following is true on the connection: (1) ensure that the only non-vanishing components of the connection are \( \Gamma^i_{tt} \) and \( \Gamma^i_{kl} \); (2) ensure that the spatial vector \( \Gamma^i_k \) is still the gradient of some scalar field, and (3) ensure that \( \Gamma^i_{kl} \) arises from a spatial Riemannian metric. For simplicity, we shall assume that in what follows, all vectors are written locally in a coordinate frame basis.

We can accomplish this by deleting Axiom 4 (which imposes spatial flatness), and replacing Axiom 3 by

Axiom 3A The basis vector \( e_t \) dual to the 1-form \( dt \) (that is, \( < dt, e_t > = 1 \)), is itself covariantly constant:

\[

\nabla_w e_t = 0
\]

at least for all spatial vectors \( w \).

Axiom 1 implies that \( \Gamma^i_{\alpha\beta} = 0 \) for all \( \alpha \) and \( \beta \). It is easily checked that Axiom 3A implies that \( \Gamma^i_{tj} = \Gamma^i_{jt} = 0 \), leaving us with \( \Gamma^i_{tt} \) and \( \Gamma^i_{kl} \) as the only non-vanishing connection coefficients. In such a case, Misner’s Axiom 6 will force the spatial components of the connection, \( \Gamma^i_{kl} \), to arise from a spatial Riemannian metric.

Axiom 2 and \( < dt, e_t > = 1 \) implies that \( \mathcal{R}(w, e_t) e_t \) is spatial for \( w \) spatial. Then Axiom 7 implies that \( R_{ikt} = R_{kiti} \). Writing \( \Gamma^i_{tt} = v^i \), this gives:

\[

v_{i;k} = R_{ikt} = R_{kiti} = v_{k;i}
\]

But this implies

\[

v_{i;k} - v_{k;i} = v_{i;k} - v_{k;i} = 0
\]

since the covariant curl equals the curl. Thus the vector field \( v^i = \Gamma^i_{tt} = g^{ik} v_j \) is the gradient of a scalar field, since its curl vanishes. We also have
positive, then Poisson’s equation does not allow a threemsphere universe. To see this, integrate $\nabla^2 \Phi = 4 \pi G \rho$ over the entire three-sphere, getting $(1/(4 \pi G)) \int \rho \sqrt{g} d^3 x = \int \nabla^2 \Phi \sqrt{g} d^3 x = \int \text{div} \text{grad} \Phi \sqrt{g} d^3 x = \int_{S} \text{grad} \Phi dS = 0$, the last two steps using the Gauss Divergence theorem and the fact that the last integral is over the boundary $S$ of the three-sphere, and thus this integral is zero since the three-sphere has no boundary. But $\int \rho \sqrt{g} d^3 x = 0$ is impossible if $\rho \geq 0$, unless $\rho = 0$ everywhere. This fact may be the reason why Gauss and Riemann did not develop their idea that physical space was curved.

Had they done so, general relativity would have had an easier time being accepted. Recall that one of the main objections to general relativity was based on Occam’s Razor, namely that general relativity depended on ten potentials $g_{\mu \nu}$, rather than the single Newtonian potential $\Phi$. Had the Poisson equation for curved space been considered the Newtonian gravity equation, then physicists would have realized that Newtonian gravity theory required the determination of seven potentials: $\Phi$ and the six components of the spatial metric $g_{ij}$, with the latter six being undetermined by the boundary conditions. Ten potentials is not significantly greater than seven, and further, the Einstein theory provides an additional nine equations, allowing the boundary conditions to yield a unique solution.

III. Proof that the Einstein Gravity Equations are a Special Case of the Newtonian Gravity Equations Coupled to a Luminiferous Ether

A central point of Lorentz’s 1904 paper, in which he derived the Lorentz transformations, was that the Maxwell equations — for Lorentz, the equations of the æther — do not allow an absolute time to be defined. This is of course now obvious since the speed of light in the vacuum is a constant, independent of an inertial observer. So the æther can be thought of as defining a time direction different from what we may have thought of as Newtonian absolute time. Trautman showed that this time direction can be defined as a 4-dimensional vector $u^\mu$, which he called a “rigging,” with

$$u^\mu \equiv (u^t, u^x, u^y, u^z)$$

(13)

where the spatial components are non-zero, but constant over all space for an inertial observer. For a general observer, all components are functions of space and time. We shall impose the constraint that $(u^t)^2 > (u^x)^2 + (u^y)^2 + (u^z)^2$, so that it will be “timelike,” in the sense that the component in the time direction is larger than any space direction. The components $u^j$ can be viewed as the components of the velocity of the æther with respect to Newtonian time in the $j^{th}$ direction, and we will also set $u^t = c$.

The vector field $u^\mu$ defines a 4-dimensional metric:

$$g_{\mu \nu} \equiv g^{ij} - \frac{u^\mu u^\nu}{c^2}$$

(14)
where $g^{ij}$ is the 3-dimensional spatial metric in Section II, written as a 4-metric in which all non-spatial components are zero. The symmetric rank two tensor $g_{\mu\nu}$ defines a 4-D Lorentz metric.

If space is not spatially flat, then the spatial Riemannian metric will define a metric connection, and we might thus have two connections, one from the spatial metric, and one in the time direction only.

But having the 4-metric $g_{\mu\nu}$ means that there is no longer a “natural” division between time and space, and hence there is no natural division between the purely timelike connection and the purely spacelike connection. Since the rigging defines a pseudo-Riemannian metric, it is natural — but not required —to assume that the entire connection arises from the pseudo-Riemannian metric. We emphasize that this is an added constraint on the full aether theory, which would in principle have two connections, one from Newtonian gravity, and yet another from the pseudo-Riemannian geometry. We suspect, but do not attempt to prove, that maintaining the distinction between two such connections would be very difficult.

Essentially, the requirement that the connection arise entirely from the metric is nothing but the “no prior geometry” assumption, which, as we pointed out earlier, is the only assumption that will allow the geometry to be determined by the matter distribution and the boundary conditions. Once again, MTW have emphasized that the “no prior geometry” assumption is the basic assumption of general relativity. It is also an essential assumption of the curved aetherial Newtonian gravity theory we develop here. In effect, we use it to require that the only connection is the metric connection, and assuming, like Cartan-Newton and Einstein, that particles move along geodesics. The geodesics are necessarily those of the metric connection, since there is no other connection.

Since the existence of the aether by definition tells us that Newtonian time cannot be unique, the time index $t$ in the Cartan-Poisson equation must be replaced with a pair of time indices $t$ and $t'$:

$$R_{tt'} = 4\pi G \rho'$$

where $\rho'$ is some density appropriate to this pair of indices. But (15) is really a tensor equation of the form

$$R_{\mu\nu} = 4\pi G S_{\mu\nu}$$

because the LHS of (14) are components of a tensor, and further, if two symmetric tensors agree for all possible $t'$ time coordinates, they are the same tensor in all space and time dimensions. This last statement is Proposition 3.3.4 of Sachs and Wu ([10], p. 72).

The question is, what should we select for the tensor $S_{\mu\nu}$. According to Einstein in his *Autobiography*, “On the right side [of the Einstein equations] we shall have to place a tensor also in place of [the mass density] $\rho$. Since we know from the special theory of relativity that the (inertial) mass equals energy, we shall have to put on the right side the tensor of energy-density — more precisely the entire energy-density, insofar as it does not belong to the pure gravitational field ([12], p. 75.).

We propose to follow Einstein exactly: the tensor $S_{\mu\nu}$ must be the entire stress-energy tensor “insofar as it does not belong to the pure gravitational field.” Since by hypothesis, we have an aether, we must include the aether stress energy:

$$R_{\mu\nu} = 4\pi G S_{\mu\nu} = 4\pi G (T_{\mu\nu} + T_{\mu\nu}^{\text{aether}})$$

where $T_{\mu\nu}$ is the tensor for the energy density of ordinary ponderable matter, and $T_{\mu\nu}^{\text{aether}}$ is the energy density of the aether. We shall now show that the aether theory of Lorentz and Kelvin gives the form of $T_{\mu\nu}^{\text{aether}}$.

We start with the result, originally derived by Maxwell in 1873 ([13], section 792; p. 391 of volume II) that for an electromagnetic wave traveling in the $i$th direction, where $i$ is either $x$, $y$ or $z$ in the aether, then $\rho = p_i/c^2$ where $\rho$ is the “mass” density of the electromagnetic radiation, and the $p_i$ are the pressures in the $i$th direction. (Lorentz had derived $E = mc^2$ for electromagnetic fields.)

This gives the general relation between the density and the pressure of an electromagnetic wave:

$$T_{tt} = \rho_{\text{EM}} = (p_x^{\text{EM}} + p_y^{\text{EM}} + p_z^{\text{EM}})/c^2 = T_{tt}^i/c^2$$

Notice that we are equating the matter density to the trace of the spatial part of the stress-energy tensor; we need not assume that $T_{\mu\nu}$ can be diagonalized, just that $T_{\mu\nu}^{\text{aether}} = 0$. Since the Lorentz aether theory requires the Maxwell equation to be the equations for the aether, this means that equation (18) must also be the equation for the corresponding quantities for the aether; there is nothing else:

$$\rho_{\text{aether}} = (p_x^{\text{aether}} + p_y^{\text{aether}} + p_z^{\text{aether}})/c^2$$

Now we use the Kelvin theory of the aether, in which all pressures are ultimately due to the aether. Kelvin devoted an entire book ([15]) to various models for how to accomplish this, but for our purposes the details don’t matter. We also note that Lorentz had the same hope, to reduce all phenomena, in particular pressures of all types, to aether phenomena.

So if all pressures are ultimately aether pressures, the most natural way to express this is to simply delete the superscript “aether” in the pressures in equation (19):

$$\rho_{\text{aether}} = (p_x + p_y + p_z)/c^2$$

Equation (20) makes rigorous Kelvin’s belief ([14]) that the aether must generate gravity, but that it cannot do so in the absence of matter.
Equation (20) implies \( T_{\mu\nu}^{\text{aether}} = T_{\mu\nu} - g_{\mu\nu}g^{\alpha\beta}T_{\alpha\beta} \). To see this, choose coordinates locally so that \( g_{\alpha\beta} = \eta_{\alpha\beta} \), so that in particular \( g_{tt} = -1 \), and thus

\[
\rho^{\text{aether}} \equiv T_{tt}^{\text{aether}} = (p_x + p_y + p_z)/c^2 = T_{tt} + [-T_{tt} + (p_x + p_y + p_z)/c^2] = T_{tt} - g_{tt}(T_{tt} - (p_x + p_y + p_z)/c^2) = T_{tt} - g_{tt}g^{\alpha\beta}T_{\alpha\beta} = T_{tt} - g_{tt}T
\]

where we have written the density of ordinary matter as \( T_{tt} \). In other words, if there are no labels to the tensor \( T \), it is the tensor with only non-aether material. This yields

\[
T_{\mu\nu}^{\text{aether}} t^\mu t^\nu = T_{tt} - g_{tt}T = (T_{\mu\nu} - g_{\mu\nu}T)t^\mu t^\nu
\]

Hence, we have an equality between two tensors for all timelike unit vectors. But recall that this equality implies the equality of the tensors themselves:

\[
T_{\mu\nu}^{\text{aether}} = T_{\mu\nu} - T g_{\mu\nu}
\]

This completes the derivation of the energy tensor for the aether, and thus derives the Einstein field equations as the Newtonian equations for gravity in which the gravitating aether is included.

Notice that the aether explains, without using the weak field limit, why the the constant \( 4\pi \) in Poisson’s equation is replaced by \( 8\pi \) in the Einstein equations. The aether also enforces \( T_{tt} = 0 \) without using the comma goes to semicolon rule. Thus the aether also implies \( T_{tt} = 0 \) in Minkowski space, a law that would have been grossly violated \(^8\) if we had set \( S_{\mu\nu} = T_{\mu\nu} \).

**IV. Conclusion**

The PPN formalism (MTW, chapter 39) can be used to see the relative effects of time curvature, space curvature, and the fact that the Maxwell aether combines these into a four-metric theory of gravity. Recall that in isotropic coordinates, the PPN metric for a spherically symmetric field is

\[
ds^2 = g_{tt}c^2 dt^2 + g_{ss}[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)]
\]

where

\[
g_{tt} = -\left[1 - \frac{2GM}{rc^2} + 2\beta \left(\frac{GM}{rc^2}\right)^2\right]
\]

and

\[
g_{ss} = \left[1 + 2\gamma \frac{GM}{rc^2}\right]
\]

For general relativity, the PPN parameters have values \( \beta = \gamma = 1 \). For a solar system experiment, \( M \) is the mass of the Sun \( M_\odot \). The angle \( \Delta\alpha \) of deflection by a light ray passing by the Sun is

\[
\Delta\alpha = \frac{1}{2}(1 + \gamma) \frac{(GM_\odot/c^2)}{b}
\]

where \( b \) is the light ray’s impact parameter, and \( GM_\odot/b = 1.75^\circ \) if the light ray just grazes the limb of the Sun.

For a planet in an elliptical orbit around the Sun, with semimajor axis \( a \) and eccentricity \( e \), the perihelion shifts forward by an angle \( \Delta\phi_0 \) with each circuit around the ellipse given by

\[
\Delta\phi_0 = \left[\frac{2 - \beta + 2\gamma}{3}\right] \left[\frac{6\pi}{1 - e^2}\right] \left\{\frac{GM_\odot}{ac^2}\right\}
\]

which for Mercury is \( 42.98 \pm 0.04 \) seconds of arc per century \(^{17}\).

The parameter \( \gamma \) measures the deviation of the spatial metric from flat space, and (27) shows that fully half the value of the deflection of a light ray comes from the spatial curvature; the other half comes from the purely Newtonian curvature in the time direction. So without allowing for spatial curvature, one half of the actual light deflection would be unaccounted for.

In our curved Newtonian spacetime, without the aether, one can obtain the observed Einstein shift by putting in the necessary spatial curvature by hand, basically as a fudge factor since the spatial metric in the curved space Poisson equation is entirely arbitrary. The “no prior geometry” assumption makes the spatial curvature non-arbitrary, and in fact gives exactly the observed value. So from the aether theory point of view, the light deflection experiment is testing the “no prior geometry” assumption.

In spatially flat, non-aether Newtonian theory, \( \beta = \gamma = 0 \), so the PPN formula, valid for all metric theories of gravity, shows us that there would be a perihelion shift of Mercury 2/3 that of general relativity in any metric theory. That is, the number 2 in the numerator of the deflection parameter in (28) is due to a special relativistic effect that would be present even if \( \beta = \gamma = 0 \). To see this, recall that special relativistic effects are of order \( v^2/c^2 \) in the limit \( v \ll c \). Now the perihelion shift is a deviation from an assumed Kepler orbit, for which \( GM_\odot/a = v^2 \) when \( e \ll 1 \). So the last factor in brackets in (28) is just \( v^2/c^2 \), exactly what we would expect for a special relativistic effect. To take the full spatially flat, non-aether limit for the perihelion shift, one must not only set \( \beta = \gamma = 0 \), but also take the limit \( v \rightarrow 0 \), which means taking the limit \( a \rightarrow \infty \) in (28). Thus, just having an aether, which requires a rigging, which in essence is just special relativity, would give 2/3 of the perihelion shift.

Lord Kelvin \(^{16}\) wrote an article in 1859 on LeVerrier’s discovery of Mercury’s perihelion shift, agreeing incorrectly with LeVerrier that it was probably due to a series
of small subMercurian planets. Remarkably, it was actually due to the gravitational effect of Kelvin’s æther!

Equally remarkable is the fact that LeVerrier’s 1859 number for the perihelion shift, $38''$ per century, which is within 12% of the actual value of $43''$ per century, is sufficiently accurate to see not only the spatial curvature correction factor $\gamma$, but also the $\beta$ factor correction to the curvature in the time direction. The $\beta$ factor is to be regarded as an “æther” correction to the gravitational field, since it is a consequence of $R_{\mu\nu} = 0$ rather than the non-æther $R_{tt} = 0$. Setting $\beta = \gamma = 0$ gives $(2/3)43'' = 29''$ which is 24% lower than LeVerrier’s $38''$. Setting only $\beta = 0$, but keeping the spatial curvature at its full general relativistic $\gamma = 1$, gives $(4/3)43'' = 54''$ which is 42% higher than LeVerrier’s $38''$. Since Leverrier was only 12% off the true value, he would have noticed either deviation from the true value. So the nineteenth century physicists were not only conceptually capable of deriving general relativity, but they had by 1859 the data that would confirm the æther theory that is general relativity.

We began with Feynman, let us end with Feynman. In his Nobel Prize lecture, Feynman said he wondered what Dirac meant by saying two mathematical expressions were “analogous” to each other. Feynman calculated that “analogous” meant “equal.” In his Autobiography, Einstein wrote: “In the case of the relativistic theory of the gravitational field, $R_{\mu\nu}$ takes the place of $\nabla^2 \Phi$ ([12], p. 73). Cartan showed that for Newtonian theory in flat space, $R_{tt}$ was actually equal to $\nabla^2 \Phi$. In this paper, we have extended this equality to the case of curved space. Einstein went on to make the remark we quoted earlier, “On the right side [of the Einstein equations] we shall then have to place a tensor also in place of [the mass density] $\rho$. Since we know from the special theory of relativity that the (inertial) mass equals energy, we shall have to put on the right side the tensor of energy-density — more precisely the entire energy-density, insofar as it does not belong to the pure gravitational field.”

In this paper we have shown that Lorentz æther theory requires that all times $t$ must be permitted for $R_{tt}$, and that this implies that the entire tensor $R_{\mu\nu}$ must describe gravity. We showed that the spatial metric and the Trautman vector field $u^\mu$ give a 4-metric, and that “no prior geometry implies that the entire connection must be the 4-metric connection. Finally, we showed that, following Einstein, if the right side is the entire energy density — the usual entire energy density plus the entire energy density of the Lorentz-Kelvin æther — then the full Einstein equations are obtained.

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