Effect of Ratio of Visco-Elastic Material Viscosity to Fluid Viscosity on Stability of Flexible Pipe Flow

S. ANBUKUMAR¹ AND MUNENDRA KUMAR²
¹ and ² Assistant Professor, Civil Engineering Department, Delhi Technological University (DTU), Delhi -110042.
Email: anbukumars@yahoo.com

Abstract: In the present study, a flexible pipe has been considered to study the effect of ratio of visco-elastic material viscosity to fluid viscosity on the stability of flexible laminar pipe flow with axi-symmetric disturbances. The effect of thickness of visco-elastic material on the stability of flexible pipe flow with outer rigid shroud has also been studied. The stability curves are drawn for various values of the ratio of visco-elastic material viscosity to fluid viscosity. It is observed that stability of flow is increasing by decreasing the ratio of visco-elastic material viscosity to fluid viscosity.

Keywords: Viscosity, stability, flexible pipe flow, laminar flow, rigid shroud.

I. INTRODUCTION

Flexible pipe flow is normally seen in nature such as human body, industrial applications for reactors and membranes. Two types of flexible pipe flow are found in nature. First type is one in which shape and size of pipe is changed due to internal pressure of the fluid. Second type is the one in which shape and size are not deforming, which means pressure gradient is constant along the direction of flow.

Kramer’s [1] - [4] studied the flow over flexible surface on a flat plate and it was found that flexibility of flat surface delay the transition and drag is reduced that means flow remains laminar for a longer period. Carpenter [5], [6], Davies, Carpenter and Lucey [7], [8] also obtained the same result.

Reynolds [9] performed the experiment in rigid circular pipe and classified the flow as laminar, transition and turbulent based on the Reynolds number. Davey and Drazin [10] shown by numerical analysis that flow is stable to very small disturbances at all Reynolds number R and axial wavenumbers \( \alpha \). Rouleau and Garg [11] and Grosch and Salwen [12] have also confirmed this result by numerical studies. Salwen and Grosch [12] showed that center line modes (waves) are more unstable as compared to wall modes. Venkateswarlu et.al [13] have studied that non-linear terms increased the instability of pipe flow.

Hamadiche and Gad-el-Hak [14] also studied the flexible tube flow with 2-Dimension and 3-Dimension disturbances problems. They found that flexible tube is unstable at all Reynolds number R and all axial wavenumbers \( \alpha \), which indicates flexible tube is unstable at very small, medium and higher Reynolds number R and all axial wavenumbers \( \alpha \). In the present study...
considered a non-collapsible pipe flow problem. The flow have been chosen in such a way that the interface of the visco-elastic material and the fluid is getting deformed in the normal (N) as well as on the tangential (T) direction to flow and hence such a flow is defined by (N+T).

\[ \text{Reynolds number } \frac{\pi V}{\alpha} \]

where \( V \) is maximum center line velocity and \( \alpha \) is the kinematic fluid viscosity.

The \( \eta \) and \( \xi \) are the deformation of flexible material along \( r \)-direction and \( x \)-direction and \( \hat{p} \) is the pressure in the visco-elastic material. All details of mathematical terms are given in figure-1.

In this paper, the stability is studied for flexible laminar pipe flow and outer surface of pipe is considered as rigid. The Fourier waves (modes) are inserted into pipe as a disturbance as given below:

\[ (\hat{u}, \hat{v}, \hat{p}) = [u(r), v(r), p(r)] e^{i(r-r)} \]

Here the disturbance velocity is given on the fluid-side disturbance equations

Here, the laminar flow through circular flexible pipe for linear stability is considered as given in figure-1. The outer side of the pipe is considered as rigid surface. Figure 1 shows the two dimensional flexible pipe flow. The \( u \) and \( v \) are the velocity along axial \( x \)-axis and radial \( r \)-axis. The laminar average velocity can be written as \( \pi = 1 - r^2 \).

\[ \text{Continuity:} \]

\[ i\alpha u + v' + \frac{v}{r} = 0. \]
The Fourier waves are inserted into the linearized Navier-Stokes and continuity equations (2-4). After doing some mathematical exercise, the 4th order differential equation in terms of \( v \) for axi-symmetric disturbances (2-D waves) in the fluid-side is obtained as follows:

\[
\frac{v'''}{r^2} + \frac{2v''}{r^3} + \frac{3}{r^3} + 2a' + i(\alpha a - c)\frac{v''}{r^2} + \left(\frac{3}{r} - \frac{i}{r} R \left(\frac{a}{r} - c\right)\right) v' + \frac{3}{r} \frac{v'}{r} + i\left(\alpha a - c\right) \frac{v}{r} = 0.
\]

(B) The flexible material equations

The Navier and continuity equations for displacements in the flexible materials may be expressed as follows:

\[
\dot{\xi} = -\frac{\partial \rho}{\partial t} + \frac{1}{R} \left[ \frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r} \frac{\partial \xi}{\partial r} + \frac{3}{r^2} \frac{\partial^2 \xi}{\partial \theta^2} \right],
\]

\[
\dot{\eta} = -\frac{\partial \rho}{\partial t} + \frac{1}{R} \left[ \frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r} \frac{\partial \eta}{\partial r} + \frac{3}{r^2} \frac{\partial^2 \eta}{\partial \theta^2} \right],
\]

Continuity:

\[
\frac{\partial^2 \xi}{\partial r^2} + \frac{\partial^2 \eta}{\partial r^2} = 0
\]

Where, \( K \) is the flexible term and is given as \( K = K_e + iK_c \).

\[
\frac{K}{R} = \frac{K_e}{R} = \frac{\mu}{R} \frac{\partial^2}{\partial r^2},
\]

Kumaran (1995) gave a flexible parameter \( \Gamma \) as \( \Gamma = \sqrt{\rho V^2/G} \), where \( G \) is shear modulus of flexible material. The \( \dot{K} \) is given by Kumaran as

\[
\frac{\dot{K}}{R} = \frac{\mu}{R} \frac{\partial^2}{\partial r^2} \frac{\partial \xi}{\partial t} = \frac{1}{\Gamma} \frac{\partial \xi}{\partial t}.
\]

For the visco-elastic material and fluid sides. Introducing \( \frac{\partial \xi}{\partial t} \to -i\alpha \epsilon \), we have \( \dot{K} \to K \), where

\[
K_e = K_e - iK_c
\]

as follows:

\[
K_e = \mu \frac{\partial^2}{\partial r^2} + a\mu \epsilon, \quad K_c = a\mu \epsilon.
\]

The flexible material deformations may written as follows:

\[
\xi = \xi(r, \theta) = [\xi_0(r, \theta), \eta_0(r, \theta), p_0(r, \theta)] \exp[i(\alpha \epsilon - c\epsilon t)]
\]

The following differential equation for the visco-elastic material can be obtained in terms of \( \eta \) displacement as follows:

\[
\eta'' + \frac{2\eta'}{r} + \left[ \frac{3 - 2\alpha^2 \epsilon - \alpha^2 \epsilon^2 R}{R} \right] \eta' + \left[ \frac{3 - 2\alpha^2 \epsilon - \alpha^2 \epsilon^2 R}{R} \right] \eta = 0.
\]

(C) Boundary conditions for visco-elastic and fluid sides

The boundary conditions for combined fluid-solid problem for axisymmetric case, \( n = 0 \), are given as below:
Pipe center at, $r = 0$:

$$u'(0) = 0, \quad v(0) = 0.$$  \hfill (15)

Surface of the pipe

At the surface of the visco-elastic pipe, viz. at $r = H$:

$$\xi(H) = 0, \quad \eta(H) = 0.$$  \hfill (16)

D. Interface between the fluid and the visco-elastic material

The boundary conditions at the interface between the solid and the fluid-sides may be written as below: The continuity of velocities, linearised with respect to $r = 1$, is given as follows:

$$\frac{\partial \xi}{\partial t} = \dot{a} + \eta_{\nu} \tau_{\nu}, \quad \frac{\partial \eta}{\partial t} = \dot{\nu};$$  \hfill (17a, b)

Particularly, eq. (22a) is the tangential no-slip boundary condition. The above interface velocities can also be written as follows:

$$-i\alpha \cdot \xi_{\nu} = u_{\nu} + \eta_{\nu} \tau_{\nu}, \quad -i\alpha \cdot \eta_{\nu} = v_{\nu}.$$  \hfill (18a, b)

The expressions for the stresses from the fluid-side and solid-side are given as below, where generically $\tau$ represents to the fluid-side stresses and $\sigma$ represents to the visco-elastic material-side stresses.

Fluid-side:

$$\tau_{\nu} = -\dot{p} + \frac{2}{R} \left[ \frac{\partial \eta}{\partial \xi} \right] \quad \text{or} \quad \tau_{\nu} = -\dot{p} + \frac{2}{R}[v'], \hfill (19a, b)$$

$$\tau_{\alpha} = -\dot{p} + \frac{1}{R} \left[ \frac{\partial \eta}{\partial \xi} \right] + \frac{\partial \xi}{\partial r} \quad \text{or} \quad \tau_{\alpha} = \frac{1}{R} [i\alpha v + \dot{u'}], \hfill (20a, b)$$

Solid-side:

$$\sigma_{\nu} = -\dot{p} + \frac{2K}{R} \left[ \frac{\partial \eta}{\partial \xi} \right] \quad \text{or} \quad \sigma_{\nu} = -\dot{p} + \frac{2K}{R}[\dot{v'}], \hfill (21a, b)$$

$$\sigma_{\alpha} = \frac{K}{R} \left[ \frac{\partial \eta}{\partial \xi} \right] + \frac{\partial \xi}{\partial r} \quad \text{or} \quad \sigma_{\alpha} = \frac{K}{R} [i\alpha v + \dot{\xi'}], \hfill (22a, b)$$

The stress matching conditions respectively for the normal and axial directions, are given as $\sigma_{\nu} = \tau_{\nu}$ and $\sigma_{\alpha} = \tau_{\alpha}$.

III. Numerical Methods

The above differential equations can be solved by using the basic concept of the finite difference techniques. The differential equations can be written for the Fourier modes in the matrix forms as follows:

$$[ A_{\nu} ] [ F_{J} ] = [ P_{J} ] + I. \quad J = 1, 2, 3, \ldots, (2N + 2),$$

Here, $N$ is the number of intervals for above matrix. The above matrix can be solved for visco-elastic material side and fluid-side by using the finite difference method. The numerical equation for the fluid side and visco-elastic side are solved by using the FORTRAN programming and these programs are compiled by the Fortran Lahey Fitjtsu.
IV. Results

In present paper, stability curves are drawn for the effect of ratio of visco-elastic material viscosity to fluid viscosity ($\mu_r = \mu_r / \mu_f$) for normal plus tangential (N+T) motion of interface of the fluid and visco-elastic material. Here, it has been considered two types of modes (waves) for the analysis of stability problem. One mode is unstable at very low Reynolds numbers and defined it as M1 types modes. Other mode is unstable at intermediate and higher Reynolds numbers and referred it as M2 types of modes. Further discussion, it has been referred these modes as M1 and M2 modes. The effect of ratio of visco-elastic material viscosity to fluid viscosity on the thickness of visco-elastic wall has also observed. Stability of flexible pipe flow depends upon the damping of the modes. i.e. Whether Fourier waves (modes) are damped or amplified. The modes are unstable for $c_i > 0$, stable for $c_i < 0$ and neutral for $c_i = 0$.

A. Effect of $\mu_r$ on visco-elastic wall thickness ($H$)

The figures 2(a) and 2(b) show the effect of different values of $\mu_r$ for the axi-symmetric disturbances of M1 types of modes. From these figures, it is seen that the higher values of $\mu_r$ decreases the imaginary values of wave speed ($c = c_r + ic_i$) because it is seen that the lower positive values of $c_i$ is more stable as compare to higher positive values of $c_i$. So, the lower value of $c_i$ makes the modes more stable. The real part of wave speed ($c_r$) has no effect on the $\mu_r$ values. It is also found that higher value of wall thickness ($H$) makes the modes more unstable as compare to lower $H$.

![Figure 2: (a) Curve for variation of $H$ and $c_i$ for modes M1, $\Gamma = 6$. 'A' is for $\mu_r = 0.0$ 'B' is for $\mu_r = 0.5$. (b) Curve for Variation of $H$ and $c_r$ for modes M1, $\Gamma = 6$. 'A' is for $\mu_r = 0.0$ 'B' is for $\mu_r = 0.5$.](image)

The figures 3(a) and 3(b) show the variation of visco-elastic wall thickness ($H$) versus $c_i$ and $c_i$ for M2 types of modes for different values of $\mu_r$. From these figures, we found the same observations and behavior of effect of $\mu_r$ as discussed above for M1 types of modes on the stability of flexible pipe flow i.e. flexible pipe surface becomes more rigid for higher values of $\mu_r$. So, we can conclude that higher $\mu_r$ value makes the all types of modes more stable.
Figure 3: (a) Curve for variation of H and c, for modes M2, \( \Gamma = 6 \). ‘A’ is for \( \mu_r = 0.0 \) ‘B’ is for \( \mu_r = 0.8 \). (b) Curve for variation of H and c, for modes M2, \( \Gamma = 6 \). ‘A’ is for \( \mu_r = 0.0 \) ‘B’ is for \( \mu_r = 0.8 \).

B. Effect of wall compliance \( \Gamma \)

Figures 4 (a) shows the neutral stability curves in \( \alpha - R \) plane for different values of \( \Gamma \) for M1 types of modes. Here, we have taken the \( \mu_r = 0 \) and \( H = 2 \). From this figure 4, we observed that higher values of wall compliance \( \Gamma \) increases the instability of the modes. i.e. area under curves are increased by higher values \( \Gamma \). It is also seen from figure 4 that M1 types of modes are more unstable for very lower values of Reynolds number R. Modes are unstable inside the curve, neutral on the curve and stable outside the curves. Figure 4.(b) shows variation of the neutral stability curves in \( \alpha - R \) plane for different values of \( \Gamma \) for M2 types of modes and these modes are unstable for medium and higher Reynolds number R. We found that the higher value increases the instability of modes. So, we can conclude that all types of modes increases the instability for higher values of \( \Gamma \).

C. Effect of wall damping \( \mu_r \)

The figures 5 (a) and 5 (b) show the effect of visco-elastic viscosity to fluid viscosity (\( \mu_r \)) for M1 types of modes. Here, we considered \( H = 2 \) and different values of \( \mu_r \). From these figures, it is observed that the higher value of \( \mu_r \) decrease the instability of visco-elastic pipe flow. Higher values of \( \mu_r \) means the more rigidity i.e. less visco-elasticity. It is found that even with a large value of \( \mu_r = 0.2 \), there is little influence of \( \mu_r \) on the upper limb of the neutral curve, and modest influence on the lower limb of the neutral curve. We may conclude that higher values of wall damping \( \mu_r \) increases the stability of visco-elastic pipe flow.
Figure 4: (a) Stability Curves in $\alpha$ - $R$ plane various $\Gamma$ for modes M1, $H=2.0$. ‘A’ is for $\Gamma = 6$, $\mu_r=0$, ‘B’ is for $\Gamma = 8$, $\mu_r = 0$, ‘B’ is for $\Gamma = 9$, $\mu_r = 0$. (b) Stability curves in $\alpha$ - $R$ plane various $\Gamma$ for modes M2, $H=2.0$. ‘A’ is for $\Gamma = 4$, $\mu_r = 0$, ‘B’ is for $\Gamma = 6$, $\mu_r = 0$, ‘B’ is for $\Gamma = 8$, $\mu_r = 0$.

Figure 5: (a) Neutral stability curves in $\alpha$ - $R$ plane various $\mu_r$ for modes, $M1$, $H=2.0$. ‘A’ is for $\Gamma = 6$, $\mu_r = 0$, ‘B’ is for $\Gamma = 6$, $\mu_r = 0.2$, (b) Stability curves in $\alpha$ - $R$ plane for various $\mu_r$ for modes, $M1$. ‘A’ is for $\Gamma = 9$, $\mu_r = 0$, ‘B’ is for $\Gamma = 9$, $\mu_r = 0.1$.

Figure 6 (a) shows the effect of $\mu_r$ in $\alpha - R$ on the stability of flexible pipe flow for M2 type of modes. Here, we have considered $H = 2$, $\Gamma = 4$ and different values of $\mu_r$. From figure 8, it found that the higher values of $\mu_r$ increases the stability. It is seen that effect of $\mu_r$ is moderate on the neutral stability curve. Figure 6 (b) represent the effect of $\mu_r$ on the neutral stability curve of M2 typed of modes on the flexible pipe flow for axi-symmetric disturbances. In this figure, we considered $H = 2$, $\Gamma = 6$ and different values of $\mu$ . It is observed that there is little influence of $\mu_r$ on the lower limb of the neutral curve, and modest influence on the upper limb of the neutral curve.
Figure 6: (a) Stability curves in $\alpha - R$ plane for various $\mu_r$ for modes, M2. ‘A’ is for $\Gamma = 4$, $\mu_r = 0$, ‘B’ is for $\Gamma = 4$, $\mu_r = 0.2$ and ‘C’ is for $\Gamma = 4$, $\mu_r = 0.4$. (b) Stability curves in $\alpha - R$ plane for different values of $\mu_r$ for modes, M2. ‘A’ is for $\Gamma = 6$, $\mu_r = 0$, ‘B’ is for $\Gamma = 6$, $\mu_r = 0.1$ and ‘C’ is for $\Gamma = 1$, $\mu_r = 0.2$.

Figure 7: Stability curves in $\alpha - R$ plane for different values of $\mu_r$ for modes, M2. ‘A’ is for $\Gamma = 8$, $\mu_r = 0$, ‘B’ is for $\Gamma = 8$, $\mu_r = 0.2$ and ‘C’ is for $\Gamma = 8$, $\mu_r = 0.4$.

Figure 7 shows the variation of neutral stability curve in $\alpha - R$ for different values of $\mu_r$ for M2 typed of modes. It is observed that there is little influence of $\mu_r$ on the higher Reynolds number R on the neutral curve, and large influence on the lower values Reynolds number R on the neutral stability curve. From the above discussions of effect of wall damping $\mu_r$, we may conclude that the higher wall damping damps the modes, i.e higher values of $\mu_r$ increases the stability of flexible pipe flow. The higher values of $\mu_r$ ($\mu_r = \mu_s / \mu_f$) means the higher viscosity of visco-elastic material as per the definition of $\mu_r$. The higher values of $\mu_r$ makes the flexible pipe more rigid. As we know that the rigid pipe flow is more stable than the flexible pipe flow. Hence, final conclusion can be drawn that the more viscosity of visco-elastic material makes the flow more stable.

V. CONCLUSIONS

The following conclusions have been drawn on the basis of the results given above.

1. It is found that the flexible pipe is unstable for all ranges of Reynolds number (R) and azimuthal wave number ($\alpha$).
2. The increase in the visco-elastic material thickness (H) increases the instability of flexible pipe flow, which means higher visco-elastic material thickness makes the pipe more
flexible and hence more flexibility of pipe increases the instability. As it is know that the rigid pipe is more stable as compare to flexible pipe.

(3) The higher values of visco-elastic parameter (Γ) increases the instability of flexible pipe flow because more Γ makes the pipe more flexible. So flexible pipes flow becomes more unstable with higher Γ.

(4) Instability of flexible pipe flow decreases with the larger values of μr, i.e. the larger values μr makes the flexible pipe flow more rigid. It means that rigidity of visco-elastic material pipe is increased by increasing the values of μr.

REFERENCES

[1] Kramer, M. O, “Boundary layer stabilization by distributed damping”, J. Aeronaut Science, 24, PP. 459 – 460, 1957.
[2] Kramer, M. O, “Boundary layer stabilization by distributed damping”. J. Aero/Space Science, 27, 69, 1960.
[3] Kramer, M. O, “Boundary layer stabilization by distributed damping”. J. Am. Soc. Naval Engineers, 74, PP. 341 – 348, 1960.
[4] Kramer, M. O, “Hydrodynamics of dolphin, in advance Hydro-science”, ed. V. T. Chow, 2, PP. 110 – 130, Academic press, New York. 1965.
[5] Carpenter, P. W, “ Status of transition delay using compliant walls, in viscous drag reduction in boundary layers”, eds. D. M. Bushell and J. N. Hefner, PP. 79-113, AIAA, Washington, D.C, 1990.
[6] Carpenter, P. W, “Current status of the wall compliance for Laminar flow control”, Exp. Thermal & fluid Science., 16, PP. 133-140, 1998.
[7] Carpenter, P. W., Davies, C. and Lucey, A. D, “Hydrodynamics and compliant walls: Does the Dolphin have a secret?” Current Science 79, PP. 758-765, 2000.
[8] Carpenter, P. W., Lucey, A. D. and Davies C, “Progress on the use of compliant walls for Laminar flow control”, J. Aircraft 38, PP. 504-512, 2001.
[9] Reynolds, O, “An experiment investigation of the circumstances which determines the motion of water in pipe”, Phill. Trans. Roy. Soc. 174, PP. 953-982, 1883.
[10] Davey, A. and Drazin, P. G, “Stability of Poiseuille flow in a pipe”, J. Fluid Mech., 36, PP. 209-218, 1969.
[11] Garg, V. K. and Rouleau, W. T, “Linear spatial problem of the pipe-Poiseuille flow”, J. Fluid Mech., 54, PP. 113-127, 1972.
[12] Salwen, H. and Grosch, C. E, “The stability of Poiseuille flow in a pipe of circular section”, J. Fluid Mech., 54, PP. 93-112, 1972
[13] Sen, P. K., Venkateswarlu, D. and Maji S, “On the stability of plane-Poiseuille flow to finite-amplitude disturbances considering the higher order Landau coefficients”, J. Fluid Mech., 68, PP. 345-351, 1985
[14] Hamadiche, M. and Gad-el-hak, M, “Temporal Stability of flow through a visco-elastic tubes”, J. Fluid Structure, 16, no. 3, PP. 331-359, 2002
S. Anbukumar is an Assistant Professor in the Department of Civil Engineering at Delhi Technological University, since 1999. He has done B.E. in Civil Engineering from Bharathidasan University, Trichy, and M.Tech. in Ocean Engineering from Indian Institute of Technology, Madras. Currently, he is pursuing Ph.D. in Civil Engineering from Delhi Technological University. He has published a number of technical papers in various national and international conferences and reputed journals.

Dr. Munendra Kumar graduated from the Aligarh Muslim University, Aligarh with B.E. and M.E. degrees and from IIT, Delhi with Ph. D degree in the area of Hydrodynamic stability of compliant pipe flow. He has 20 years of teaching experience in technical institutions. Presently, he is working as a senior lecturer in Delhi Technological University, Bawana road, Delhi – 110042. He has published 14 research papers in various National and international journals and conference proceedings.