Higgs Boson Production and Decay:  
Dalitz Sector∗

Giampiero Passarino†

Dipartimento di Fisica Teorica, Università di Torino, Italy  
INFN, Sezione di Torino, Italy

The processes $H \to \bar{t}f\gamma(g)$, $pp \to \bar{q}+q \to H+\gamma(g)$ and $pp \to q(\bar{q})+g \to H+q(\bar{q})$ pose severe challenges to the experimental analysis. They represent rare decays and production mechanisms of the Higgs boson at LHC. However, they are not Yukawa suppressed at next-to-leading order opening a window for the correct definition of pseudo-observables, e.g. a definition of $\Gamma(H \to Z\gamma)$ with universal inherent meaning, that are currently used in extracting information for the couplings of the newly discovered resonance at LHC. The impact of genuinely electroweak NLO corrections is discussed, as well as the comparison of $\sigma(pp \to ggX \to e^+e^-\gamma)$ to its zero-width approximation.

Keywords: Feynman diagrams, loop calculations, radiative corrections, Higgs physics

PACS: 12.15.Lk 11.15.Bt, 12.38.Bx, 13.85.Lg, 14.80.Bn, 14.80.Cp

∗Work supported by MIUR under contract 2001023713_006 and by Compagnia di San Paolo under contract ORTO11TPXK.
†EMAIL: giampiero@to.infn.it
1 Introduction

The original motivation for this paper is on interpretation of the pseudo-observable (PO) \( H \to Z\gamma \) which is one of the key ingredients, with \( H \to \gamma\gamma \), in studying Higgs boson couplings at LHC, see Refs. [12,13]. For recent and past developments on the experimental side we quote Refs. [4,5,6].

The Z boson is an unstable particle, predominantly decaying into a \( ff \)-pair, so that the Higgs Dalitz decay, \( H \to ff\gamma \), is the process to be compared with the data; original work along these lines can be found in Refs. [7,8,9,10] (see also Ref. [11]). There are important points to keep in mind when discussing Dalitz decay of the Higgs boson, in particular that the next-to-leading (NLO) electroweak (EW) and QCD corrections are not Yukawa suppressed \[12\], contrary to what happens in lowest order (LO). Therefore, we have extended the analysis to cover all related processes that share this property, \( H \to qqg \) and Higgs - photon(gluon) associated production at hadron colliders. For the original work on NLO EW corrections to Higgs - gluon associated production we quote Ref. [13] (see also Ref. [14]). For the inclusion of b quarks, see Ref. [15].

Returning to the original question of how to link the pseudo-observable \( \Gamma(H \to Z\gamma) \) to a specific set of experimental data, we observe the following: it came dangerously close to realizing a nightmare, physics done by sub-sets of diagrams (e.g. \( H \to Z\gamma \)) instead of kinematical cuts (e.g. on \( H \to \tilde{\tau}\gamma \)). Several years ago we avoided that fate \[16,17\], may be the history will repeat itself?

Why Dalitz decay? For a Standard Model (SM) Higgs boson of \( 125.5 \) GeV we find \( \text{BR}(H \to e^+e^-) = 5.1 \times 10^{-9} \), while a naive estimate gives \( \text{BR}(H \to Z\gamma) \times \text{BR}(Z \to e^+e^-) = 5.31 \times 10^{-5} \), which is 4 orders of magnitude larger. However, how much of this number will be reflected into the corresponding PO, consistently extracted from full Dalitz decay? Once again, a fully inclusive estimate is given by \( \Gamma(H \to e^+e^-\gamma) = 5.7\% \Gamma(H \to \gamma\gamma) \[18\] but the question cannot be answered before discussing photon isolation\[1\]. In the following, we introduce categories: the name “Dalitz decay” must be reserved for the full process \( H \to \tilde{\tau}\gamma \) and subcategories are defined by:

\[
\begin{align*}
H \to Z^* \rightarrow & \tilde{\tau} + \gamma & \text{unphysical} \\
H \to \gamma^* \rightarrow & \tilde{\tau} + \gamma & \text{unphysical} \\
H \to Z_c \rightarrow & \tilde{\tau} + \gamma & \text{PO}
\end{align*}
\]

where \( Z^* \) is the off-shell Z boson and \( Z_c \) is the Z boson at its complex pole. More generally, for a given massive particle, we define its “Dalitz sector” as the one containing all four-body processes involving the particle, a massless gauge boson and two massless fermions. Understanding the problem of POs means understanding the difference between \( H \to \tilde{\tau}\) and \( H \to \tilde{\tau} + n \gamma \); this is most easily done using an argument based on the cuts of the three-loop H self-energy: only the sum over all cuts is infrared and collinear finite so that we must isolate photons, otherwise we will be mixing different processes, \( H \to \tilde{\tau} + n \gamma \) at NNLO and \( H \to \tilde{\tau}\gamma \) at NLO.

One should not get trapped by intuition when dealing with data: the infrared/collinear component of the decay will not survive in the limit \( M_f \to 0 \) while there are genuinely non-radiative (QED and QCD) terms surviving the zero-Yukawa limit. Therefore, only the Dalitz decay has a meaning and it can be differentiated through kinematical cuts; the most important one is the definition of “visible photons” to distinguish between different final states, \( \tilde{\tau}\) and \( \tilde{\tau}\gamma \). Other cuts can be applied on the invariant mass \( M_{\tilde{\tau}f} \) to isolate pseudo-observables and one has to distinguish: a) \( H \to \tilde{\tau} + \text{soft(collinear)} \) photon(s) which is part of the real corrections to be added to the virtual ones in order to obtain \( H \to \tilde{\tau} \) at (N)NLO; b) a visible photon

\[1\] LHCHXSWG BR Subgroup Meeting: focus on Dalitz decay, https://indico.cern.ch/conferenceDisplay.py?confId=250520.
and a soft $\tilde{t}f$-pair where one probes the Coulomb pole and get large (logarithmic) corrections that should be exponentiated.

Once again, $H \rightarrow Z^* \gamma \rightarrow \tilde{t}f\gamma$ and $H \rightarrow \gamma^* \gamma \rightarrow \tilde{t}f\gamma$ are unphysical: none of these contributions exists by itself, each of them is not even gauge invariant. However, one can put kinematical cuts: with a small window around the $Z$-peak the pseudo-observable $H \rightarrow Z_{\gamma}$ can be enhanced (but there is a contamination due to many non-resonant backgrounds). One should also beware of generic statements about box contamination in $H \rightarrow Z\gamma$ being known to be small and of ad-hoc definitions of gauge-invariant splittings. Of course, at small di-lepton invariant masses $\gamma^*$ dominates.

Our summary is as follows: $H \rightarrow \tilde{t}f$ is well defined and $H \rightarrow \tilde{t}f + \gamma$ (\(\gamma^*\) soft+collinear) is part of the corresponding NLO corrections while $H \rightarrow Z\gamma$ is ill-defined, being a gauge-variant part of $H \rightarrow \tilde{t}f + \gamma$ (\(\gamma^*\) visible) and can be extracted (in a PO framework) by imposing cuts on the di-lepton invariant mass.

The outline of the paper is as follows: in Section 2 we describe the salient features of the calculation, in Section 3 we present results for the Higgs boson decay while the associated production is discussed in Section 4. Theoretical uncertainties are discussed in Section 5.

2 Computational setup

We compute helicity amplitudes for $H + t + f + \gamma(g) \rightarrow 0$ according to Refs. [19,20] and express them in terms of Mandelstam invariants.

Loop integrals are treated with a) standard reduction to scalar integrals to be evaluated analytically, b) BST functional relations [21,22,23] and numerical evaluation. Comparison of the two approaches provides a powerful check on the results. Furthermore, for the EW NLO corrections we use the Complex-Pole scheme (CPS) of Refs. [24,25,26,27]; as input parameters for the numerical evaluation we have used the following values:

\[
\begin{aligned}
M_W &= 80.398 \text{ GeV} \\
M_Z &= 91.1876 \text{ GeV} \\
M_t &= 172.5 \text{ GeV} \\
\Gamma_W &= 2.0887 \text{ GeV} \\
G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2} \\
\alpha(0) &= 1/137.0359911 \\
\alpha_s(M_Z) &= 0.12018 \\
\Gamma_Z &= 2.4952 \text{ GeV}
\end{aligned}
\]

For the PDF we use MSTW2008 at NLO [28]. At LO we use $M_b = 4.69 \text{ GeV}$ and derive $\Gamma_t = 1.480 \text{ GeV}$.

Once the helicity amplitudes are computed we use an optimization scheme based on the notion of “abbreviations”. We are dealing with multivariate polynomials in the Mandelstam variables; we require their evaluation to be performed with the least number of arithmetic operations and each polynomial will receive a name (abbreviation). The strategy, also known as “subexpression elimination”, represents a code transformation in which variables are introduced for each subexpression such that it is calculated only once and can be used at any later point in the calculation.

Schematically, invariants are collected (bracketed) and brackets are factored out; the procedure is repeated until the innermost brackets contain only monomials or polynomials that are irreducible over $R$. The innermost brackets are “abbreviated”, the next level of brackets is again “abbreviated” etc. All abbreviations are then pre-computed (once and only once) in the numerical code. For an alternative approach we refer to the work in Ref. [29]; we mention that for multivariate polynomials there is no a priori knowledge of the scheme that leads to the smallest number of operations.

Another improvement in calculation speed is given by the introduction of collinear-free functions [30]. As it is well known [31], infrared/collinear singular configurations in one-loop n-point functions arise only from three-point sub-diagrams. The best way of introducing collinear-free functions is given by the BST decomposition [21,22,23]; for instance, a box diagram in four dimensions can be written as a linear combination of a box in six dimensions (which is never soft/collinear divergent) plus vertices in four. In this way it
is very simple to check (analytically) for the cancellation of divergent three-point functions, while grouping six-dimensional boxes and finite vertices into a single (finite) function.

If one wants to have a PO definition for the Higgs boson decaying into $Z\gamma$, one must accept that the only completely consistent choice is $H \rightarrow Z\gamma$, i.e. the $Z$ at its complex pole, as discussed in Refs. [26,27].

3 Decay: numerics

We start by considering $H \rightarrow e^+e^-\gamma$ and introduce kinematical cuts as done in Ref. [10]: $M_{ij} > k_{ij}M_H$ with $i,j = e^+, e^-, \gamma$. Furthermore, always following Ref. [10], we require that one fermion has energy greater than 25 GeV, the other greater than 7 GeV, while $E_\tau > 5$ GeV. Note that in Ref. [5] a cut $M_{1^+\tau} > 50$ GeV is required.

A blind comparison (input parameters such as $M_t$ are not given in Ref. [10]) gives a substantial agreement, at the level of few percentages. Our results are given in Table 1, for $M_H = 125$ GeV.

The results of Table 1 should be compared with the SM (on-shell) prediction for $\Gamma(H \rightarrow Z\gamma) \times Br(Z \rightarrow e^+e^-\gamma)$, which is 0.214 keV and with $\Gamma(H \rightarrow Z\gamma) = 9.27$ keV [32]. It may be of interest to observe that $\Gamma(H \rightarrow e^+e^-e^+e^-) = 0.133$ keV.

The process $H \rightarrow \not{T}\not{f}\gamma$ We have extended the calculation to include different fermions in the final state. First, we define cuts, following Ref. [10]:

$$M_{T\gamma} \equiv M(\not{T}\gamma) > 0.1M_H \quad M_{f\gamma} \equiv M(\not{f}\gamma) > 0.1M_H \quad M_{T\gamma} \equiv M(\not{T}\gamma) > 0.1M_H$$

(1)

With the cuts of Eq. (1) we obtain the results shown in Table 2 for different lepton and quark final states. It is worth noting that LO and NLO amplitudes do not interfere, as long as fermion masses are neglected in NLO, since the corresponding amplitudes belong to different helicity sets. For $\tau$ and b the LO result is the leading one.

A noteworthy effect of a finite $M_t$ can be seen in the b-channel; for a b final state there are more Feynman diagrams contributing to the process due to the fact that the H boson has a non-zero coupling with top quarks.

The effect of a cut on $M_{T\gamma}$, designed to enhance the contribution of the Z peak, are given in Table 3, here we fix the cuts such that $k_{\gamma} = k_{\gamma} = 0.1$ and compare $k_{\gamma} = 0.1$ with $k_{\gamma} = 0.6$. The change corresponds to a 19% reduction of the signal for the $e^+e^-\gamma$ final state.

Our calculation shows that $\Gamma(H \rightarrow e^+e^-\gamma)$, with $k_{ij} = 0.1$, is an increasing function of $M_H$. At $M_H = 120$ GeV we find $\Gamma(H \rightarrow e^+e^-\gamma)/\Gamma(H \rightarrow Z\gamma) = 3.9\%$ and $\Gamma(H \rightarrow e^+e^-\gamma)/\Gamma(H \rightarrow \gamma\gamma) = 2.0\%$ while at $M_H = 160$ GeV the ratios become 3.1% and 4.5% respectively.
We now study distributions; in Figures 1-3 we show various distributions for the Dalitz decay of the Higgs boson at 125 GeV. The total $M_{ff}$ distribution is given in Figure 1 (left panel), showing the $Z$-peak as well as the Coulomb peak at small values of the invariant $ff$-mass. The right panel of Figure 1 gives the $M_{e\gamma}$ distribution.

In Figure 2 we compare the total $M_{T\gamma}$ distribution with the unphysical component of the decay, given by the off-shell $Z$ boson. Although the latter is a gauge-dependent quantity the figure gives a qualitative description of the $Z$ non-resonant background.

The $E_{\gamma}$ distribution for the $H \to e^+e^-\gamma$ decay is given in the right panel of Figure 2, showing that the process is dominated by sufficiently hard photons, with a maximum around $E_{\gamma} = 30$ GeV.

In Figure 3, the left panel shows the angular distribution in terms of $\cos \theta_{T\gamma}$; once again, the dominant

**Table 2:** Partial decay widths for the process $H \to T\gamma$ at $M_H = 125$ GeV and with cuts of Eq.(1). Both LO and NLO ($M_f = 0$) results are shown.

| $f$     | $\Gamma_{LO}$ [keV] | $\Gamma_{NLO}$ [keV] |
|---------|----------------------|----------------------|
| $e$     | $0.29 \times 10^{-6}$| 0.233                |
| $\mu$  | 0.012                | 0.233                |
| $\tau$ | 3.504                | 0.233                |
| $d$     | 0.013                | 0.874                |
| $b$     | 8.139                | 0.866                |
We have extended the calculation including a final state with a pair of light quarks and a gluon. Each helicity amplitude contain a piece proportional to $ggS$ and a piece proportional to $g^3gS$

Table 3: The effect of kinematical cuts on $M_{T\ell}$ for the process $H \rightarrow \ell\ell\gamma$ at $M_H = 125 \text{ GeV}$

| $f$ | $\Gamma_{NLO}[\text{keV}]$ | $\Gamma_{LO}[\text{keV}]$ |
|-----|-----------------------------|-----------------------------|
|     | $M_{T\ell} > 0.1M_H$ | $M_{T\ell} > 0.6M_H$ | $M_{T\ell} > 0.1M_H$ | $M_{T\ell} > 0.6M_H$ |
| $\mu$ | 0.233 | 0.188 | 0.012 | 0.010 |
| $d$ | 0.874 | 0.835 | 0.013 | 0.011 |
| $b$ | 0.866 | 0.831 | 8.139 | 6.745 |

Figure 2: The process $H \rightarrow \ell\ell\gamma$ at $M_H = 125 \text{ GeV}$. Comparing the total $M_{T\ell}$ distribution with the unphysical (off-shell) $Z'$ component (left panel). The $E_{\gamma}$ distribution (right panel).

The ratio $R_c(\xi)$, defined in Table 3, gives the correction factor for extracting the pseudo-observable once a cut $\xi$ is selected around $M_{T\ell} = M_Z$.

**Pseudo-observable**  
As we have described above, the pseudo-observable of interest is $\Gamma_c = \Gamma(H \rightarrow Z_c\gamma \rightarrow e^+e^-\gamma)$ which we compare with $\Gamma_{\text{tot}} = \Gamma(H \rightarrow e^+e^-\gamma)$. Requiring $M_{e^+e^-} > 0.1M_H$ and $M_{e^-\gamma} > 0.1M_H$, we impose an additional cut on $M_{e^+e^-}$ around the $Z$-peak, $M_Z - \xi \Gamma_Z < M_{e^+e^-} < M_Z + \xi \Gamma_Z$ and obtain the results shown in Table 4.

The process $H \rightarrow \ell\ell\gamma$ We have extended the calculation including a final state with a pair of light quarks and a gluon. Each helicity amplitude contain a piece proportional to $ggS$ and a piece proportional to $g^3gS$.
For a d-quark with a cut $k_{ij} = 0.1$, $i,j = \overline{d}, d, g$, we have a partial width (QCD + EW) of \( 7^{+0}_{-0} \) keV where the QCD part is 7.836 keV with an EW contribution of \( -9.58\% \). We present the $M_{\pi d}, M_{d's}$ distributions in Figure 4.

### 4 Production: numerics

To discuss associated Higgs boson production at LHC (8 TeV) we consider the following processes:

\[
\overline{q} + q \rightarrow H + g(\gamma), \; q(\overline{q}) + g \rightarrow H + q(\overline{q}); \;
\]

Here $u$ stands for $u \oplus c$ and $d$ for $d \oplus s$.

We study the total cross-section at 8 TeV as well as the $p_T$ distribution of the parton in the final state.

| $\xi$ | $\Gamma_{\text{tot}}$ [keV] | $\Gamma_c$ [keV] | $R_c = \Gamma_c / \Gamma_{\text{tot}}$ |
|-------|-----------------|-----------------|-----------------|
| 1     | 138.7           | 154.1           | 1.11            |
| 2     | 166.2           | 194.8           | 1.17            |
| 3     | 176.4           | 217.9           | 1.24            |
| 4     | 181.7           | 236.5           | 1.30            |
| 5     | 185.0           | 253.6           | 1.37            |
All processes are computed at NLO accuracy, which is the leading contribution for massless light quarks. Renormalization and factorization QCD scales are fixed at \( \mu_R = \mu_F = M_H \) and their variation is postponed until Section 5.

Cross sections  
In Table 5 we show all the cross sections at \( \sqrt{s} = 8 \text{ TeV} \), \( M_H = 125 \text{ GeV} \), with a cut of \( 30 \text{ GeV} < p_T < 300 \text{ GeV} \), comparing QCD with QCD+EW (no cut on pseudo-rapidity applied); we can split the amplitude into a part proportional to \( gg \) and a part proportional to \( g^2 g_s \); the former is what we define as (hard) QCD component of the \( M_T = 0 \) NLO amplitudes. The effect of NLO EW corrections is parametrized in terms of the relative deviation, \( \delta_{EW} = \sigma_{QCD+EW}/\sigma_{QCD} - 1 \).

It is worth noting that the \( \bar{q}q \)-annihilation cross sections are tiny (also due to parton luminosity) while the \( qg \)-annihilation is enhanced, also by the contribution of the gluon exchange in the \( t \)-channel (the vertex diagram). The effect of including the EW part is larger in the annihilation channel where, however, the cross sections are much smaller, 38.8 \( fb \) for light quarks as compared to 2.4 \( pb \) for the quark-gluon channel. For the latter there is a partial cancellation between \( qg \) and \( \bar{q}g \).

Note that we do not discuss the case \( q = b \) at LO + NLO QCD, corresponding to a non-zero value of \( M_b \) (see Ref. [15] for more details). This part of the NLO corrections contains the soft/collinear QCD that can be added incoherently to our result.

If a cut on pseudo-rapidity, \( |\eta| < 2.5 \), is applied we register a reduction of \( \approx 40\% \) on the cross sections.

\( p_T \)-distributions  
We have analyzed the \( p_T \)-distribution for different processes. In Figure 5 (left panel) we show the \( p_T \)-distribution for \( \bar{q} + q \to H + g \) at \( \sqrt{s} = 8 \text{ TeV} \) and \( M_H = 125 \text{ GeV} \).
Table 5: Total cross sections for associate Higgs production at $\sqrt{s} = 8$ TeV and $M_H = 125$ GeV for $30 \text{ GeV} < p_T < 300$ GeV.

| process                  | $\sigma_{\text{QCD+EW}}[fb]$ | $\sigma_{\text{QCD}}[fb]$ | $\delta_{\text{EW}}[\%]$ |
|--------------------------|-------------------------------|-----------------------------|---------------------------|
| $\bar{u} + u \rightarrow H + g$ | 23.24                         | 26.25                       | $-11.5$                  |
| $d + d \rightarrow H + g$      | 15.54                         | 17.71                       | $-12.3$                  |
| Total                     | 38.78                         | 43.96                       | $-11.8$                  |
| $\bar{b} + b \rightarrow H + g$ | 0.221                         | 0.317                       | $-30.3$                  |
| $u + g \rightarrow H + u$     | 1284.5                        | 1312.3                      | $-2.1$                   |
| $\bar{u} + g \rightarrow H + \bar{u}$ | 203.2                        | 192.1                       | $+5.8$                   |
| $d + g \rightarrow H + d$      | 668.0                         | 684.7                       | $-2.4$                   |
| $\bar{d} + g \rightarrow H + \bar{d}$ | 259.5                        | 242.5                       | $+7.0$                   |
| Total                     | 2415.2                        | 2431.6                      | $-0.07$                  |
| $b + g \rightarrow H + b$     | 41.81                         | 33.78                       | $+23.8$                  |
| $\bar{b} + g \rightarrow H + \bar{b}$ | 42.89                        | 33.78                       | $+27.0$                  |

In Figure 5(right panel) we show the $p_T$-distribution for $q(\bar{q}) + g \rightarrow H + q(\bar{q})$ at $\sqrt{s} = 8$ TeV and $M_H = 125$ GeV. To illustrate the effect of QCD scales we have included the band corresponding to $u + g \rightarrow H + u$ for $\mu_R = \mu_F \in [M_H/2, 2M_H]$.

Some of the features of the $p_T$-distributions can be understood by introducing $\rho = p_T^2 = s$, the partonic variable $s = z s$ and the scaled Higgs mass $M_H = \mu_H \sqrt{s}$. For $\rho$ fixed ($0 \leq \rho \leq (1 - \mu_H^2)^2/4$) we have $z_+ \leq z \leq 1$ where $z_+ = \mu_H^2 + 2 \rho + 2 \sqrt{\rho (\rho + \mu_H^2)}$. Cuts of the amplitudes are at $\hat{s} = 4M_t^2$, $4M_z^2$ etc. The value $\delta = 4M_t^2$ corresponds to $p_T = 149.86$ GeV, reflecting the spike in the $p_T$-distribution (crossing a normal threshold in $\hat{s}$). Similarly, the spike in Figure 5 for $b + g \rightarrow H + b$ corresponds to the threshold $\hat{s} = (M_W + M_t)^2$.

The percentage effects of the EW component are summarized in the left panel of Figure 6 where we show $\delta_{\text{EW}}(p_T)$ for the $q(\bar{q})q$-channel and for the $q(\bar{q})g$-channel. For the annihilation channel the effect of including EW components reaches $-25\%$ for $p_T$ around $30$ GeV; note that $\delta_{\text{EW}}$ becomes positive around $p_T = 225$ GeV with a $+10\%$ at $p_T = 300$ GeV. For the process $q(\bar{q}) + g \rightarrow H + q(\bar{q})$ we note the different behavior in the two channels $qg$ and $q\bar{g}$ and the large EW effects in the $bg$-channel where, once again, we have not included LO and soft/collinear NLO.

The process $q + q \rightarrow H + \gamma$. This process is highly suppressed, being of purely EW origin; for $d + d \rightarrow H + \gamma$ and $30 \text{ GeV} < p_T < 300 \text{ GeV}$ we find $\sigma = 0.052 \text{ fb}$. The consequences of a central photon requirement make the channel $pp \rightarrow H\gamma$ a process worthwhile to investigate, although the smallness of the signal makes it questionable to discern signal from background in $q + q \rightarrow \bar{b} + b + \gamma$ or $g + g \rightarrow \bar{b} + b + \gamma$.

Higgs boson production in association with a photon via weak boson fusion has received considerable attention in the literature, see Ref. [39]. In Ref. [40] this process has been proposed to probe the $b$-quark parton densities.

In the right panel of Figure 6 we compare the normalized $p_T$-distributions ($30 \text{ GeV} < p_T < 300 \text{ GeV}$) for $d + d \rightarrow H + g$ and $\bar{d} + d \rightarrow H + \gamma$, showing that the $\gamma$-spectrum is softer than the $g$ one. In both cases
we have a spike corresponding to the $\hat{s} = 4M_t^2$ normal threshold.

**Production and decay** Finally, we consider the full (signal) process, $pp \to gg \to H \to e^+e^-\gamma$, going beyond the zero-width approximation. We can distinguish between a DZWA, $\sigma (gg \to H) \times \text{BR} (H \to Z\gamma) \times \text{Br} (Z \to e^+e^-)$ and a ZWA $\sigma (gg \to H) \times \text{BR} (H \to e^+e^-)$.

We require that all final state invariant masses are larger than $0.1M_{e^+e^-}$ and a bin size of $200 MeV$ is used; the result is shown in Figure 7. The calculation is performed within the CPS-scheme [24, 25, 26] as implemented in Ref. [27]. The distribution is asymmetric with a large tail for values of the invariant mass above $M_H$. This is a known effect, described for the first time in Ref. [41]; it has to do with the growth of the partial decay width with growing invariant masses, extending well above the spike due to the WW normal threshold. One has $\Gamma (H \to e^+e^-) = 0.022 keV$ at $M_H = 100 GeV$ and $\Gamma (H \to e^+e^-) = 6.91 keV$ at $M_H = 190 GeV$. Of course, we are not claiming observability of the tail ($\approx 10^{-6} fb$). The main point of this exercise is to compare the full-fledged cross section at $8 TeV$ with the corresponding DZWA and ZWA, although experimentally it will be very hard to construct an hypothesis test which can resolve ZWA versus the shape of Figure 7 especially in this channel. A measure of the effect described in Ref. [41] looks more promising in $H \to 4l$ where the background is orders of magnitude lower.

\[
\begin{align*}
100 GeV < M_{e^+e^-} & < 180 GeV & M_{ij} > 0.1 M_{e^+e^-} & \sigma = 1.13 fb \\
\sigma (gg \to H) & = 19.49 pb & \text{BR} (H \to Z\gamma) \times \text{Br} (Z \to e^+e^-) & = 5.24 \times 10^{-5} \sigma_{\text{DZWA}} = 1.02 fb
\end{align*}
\]
there is a meaning and are used in extracting information for the couplings of the newly discovered resonance. Some

\[ M_{\text{HOO}} \text{ of the same order of those in } H \rightarrow H + g(\gamma) \]

For associated production the estimate is much less precise and usually not discussed in the literature: consider the sub-process with the largest cross section, \( u + g \rightarrow h + u \), the usual strategy of varying the QCD scales gives large effects, as shown in the right panel of Figure 6 where we adopt the standard recipe

\[ \mu_R = \mu_F \in [M_H/2, 2M_H] \]

This is expected since NNLO corrections are missing and NNLO is the first level of precision where one should start discussing uncertainties. According to the point of view expressed in Ref. 43 we are not going to use QCD scale variation as the true estimator of theoretical uncertainty; however, the MHO uncertainty clearly contains QCD scale variation and we must conclude that, at the present level of knowledge, these processes suffer from a large uncertainty, both in the total cross section and in the \( p_T \)-distribution.

6 Conclusions

In this work we provide a general framework for studying production and decay mechanisms of the SM Higgs boson which are Yukawa suppressed at LO but not at NLO. The three-body decay of the Higgs boson, \( H \rightarrow \gamma\gamma(g) \), is naturally framing the extraction of pseudo-observables (e.g. \( HZ\gamma \)) that have universal inherent meaning and are used in extracting information for the couplings of the newly discovered resonance. Some
Figure 7: The invariant mass distribution for $pp \rightarrow gg \rightarrow H \rightarrow e^+e^-\gamma$. All finals state invariant masses are larger than $0.1 M_{e^+e^-\gamma}$ and a bin size of 200 $MeV$ is used.
of these effects have been studied separately in Refs. [7,8,9,10] and in Refs. [13,15]; we have completed all calculations (updating the computational framework), extending previous results to give comprehensive view of the implications, including a comparison of the cross section for $pp \to l^+l^-\gamma$ at 8 TeV with the corresponding zero-width approximation.

7 Acknowledgments

Significant discussions with S. Actis, A. David, M. Duhrssen, A. Maier, C. Mariotti, D. Rebuzzi, M. Spira and R. Tanaka are gratefully acknowledged.
References

[1] A. David, A. Denner, M. Duehrssen, M. Grazzini, et al., LHC HXSWG interim recommendations to explore the coupling structure of a Higgs-like particle, arXiv:1209.0040 [hep-ph]

[2] The LHC Higgs Cross Section Working Group Collaboration, S. Heinemeyer et al., Handbook of LHC Higgs Cross Sections: 3. Higgs Properties, arXiv:1307.1347 [hep-ph]

[3] G. Passarino, NLO Inspired Effective Lagrangians for Higgs Physics, Nucl.Phys. B868 (2013) 416–458 arXiv:1209.5538 [hep-ph]

[4] ATLAS Collaboration, Search for the Standard Model Higgs boson in the $H \rightarrow Z\gamma$ decay mode with $pp$ collisions at $\sqrt{s} = 7$ and 8 TeV,

[5] CMS Collaboration Collaboration, S. Chatrchyan et al., Search for a Higgs boson decaying into a $Z$ and a photon in $pp$ collisions at $\sqrt{s} = 7$ and 8 TeV, arXiv:1307.5515 [hep-ex]

[6] D0 Collaboration Collaboration, V. Abazov et al., Search for a scalar or vector particle decaying into $Z\gamma$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, Phys.Lett. B671 (2009) 349–355 arXiv:0806.0611 [hep-ex]

[7] A. Abbasabadi, D. Bowser-Chao, D. A. Dicus, and W. W. Repko, Radiative Higgs boson decays $H \rightarrow$ fermion anti-fermion gamma, Phys.Rev. D55 (1997) 5647–5656 arXiv:hep-ph/9611209 [hep-ph]

[8] A. Abbasabadi and W. W. Repko, Higgs boson decay into $Z$ bosons and a photon, JHEP 0608 (2006) 048 arXiv:hep-ph/0602087 [hep-ph]

[9] A. Abbasabadi and W. W. Repko, A Note on the rare decay of a Higgs boson into photons and a Z boson, Phys.Rev. D71 (2005) 017304 arXiv:hep-ph/0411152 [hep-ph]

[10] D. A. Dicus and W. W. Repko, Calculation of the decay $H \rightarrow e^+e^-\gamma$, arXiv:1302.2159 [hep-ph]

[11] L.-B. Chen, C.-F. Qiao, and R.-L. Zhu, Reconstructing the 125 GeV SM Higgs Boson Through $\ell\bar{\ell}\gamma$, arXiv:1211.6058 [hep-ph]

[12] G. Passarino, Unnaturalness in the Higgs Fermion Sector, Phys.Lett. B195 (1987) 191

[13] W.-Y. Keung and F. J. Petriello, Electroweak and finite quark-mass effects on the Higgs boson transverse momentum distribution, Phys.Rev. D80 (2009) 013007 arXiv:0905.2775 [hep-ph]

[14] C. Anastasiou, S. Buchler, F. Herzog, and A. Lazopoulos, Total cross-section for Higgs boson hadroproduction with anomalous Standard Model interactions, JHEP 1112 (2011) 058, arXiv:1107.0683 [hep-ph]

[15] O. Brein, Electroweak and Bottom Quark Contributions to Higgs Boson plus Jet Production, Phys.Rev. D81 (2010) 093006 arXiv:1003.4438 [hep-ph]

[16] D. Y. Bardin, M. Grunewald, and G. Passarino, Precision calculation project report, arXiv:hep-ph/9902452 [hep-ph]
[17] D. Y. Bardin and G. Passarino, *The standard model in the making: Precision study of the electroweak interactions*, JHEP 1305 (2013) 061, arXiv:1303.2230 [hep-ph]

[18] Y. Sun, H.-R. Chang, and D.-N. Gao, *Higgs decays to γl⁺l⁻ in the standard model*, JHEP 1305 (2013) 061, arXiv:1303.2230 [hep-ph]

[19] G. Passarino, *Helicity Formalism for Transition Amplitudes*, Phys.Rev. D28 (1983) 2867

[20] G. Passarino, *Covariant Polarization Bases for Spin 1/2, Spin 1, Spin 3/2 Particles and their Use*, Nucl.Phys. B237 (1984) 249

[21] G. Passarino, *An Approach toward the numerical evaluation of multiloop Feynman diagrams*, Nucl.Phys. B619 (2001) 257–312, arXiv:hep-ph/0108252 [hep-ph]

[22] A. Ferroglia, M. Passera, G. Passarino, and S. Uccirati, *All purpose numerical evaluation of one loop multileg Feynman diagrams*, Nucl.Phys. B650 (2003) 162–228, arXiv:hep-ph/0209219 [hep-ph]

[23] S. Actis, G. Passarino, C. Sturm, and S. Uccirati, *NNLO Computational Techniques: The Cases H → γγ and H → gg*, Nucl.Phys. B811 (2009) 182–273, arXiv:0809.3667 [hep-ph]

[24] S. Actis and G. Passarino, *Two-Loop Renormalization in the Standard Model Part III: Renormalization Equations and their Solutions*, Nucl.Phys. B777 (2007) 100–156, arXiv:hep-ph/0612124 [hep-ph]

[25] S. Actis, G. Passarino, C. Sturm, and S. Uccirati, *Two-Loop Threshold Singularities, Unstable Particles and Complex Masses*, Phys.Lett. B669 (2008) 62–68, arXiv:0809.1302 [hep-ph]

[26] G. Passarino, C. Sturm, and S. Uccirati, *Higgs Pseudo-Observables, Second Riemann Sheet and All That*, Nucl.Phys. B834 (2010) 77–115, arXiv:1001.3360 [hep-ph]

[27] S. Goria, G. Passarino, and D. Rosco, *The Higgs Boson Lineshape*, Nucl.Phys. B864 (2012) 530–579, arXiv:1112.5517 [hep-ph]

[28] A. Martin, W. Stirling, R. Thorne, and G. Watt, *Parton distributions for the LHC*, Eur.Phys.J. C63 (2009) 189–285, arXiv:0901.0002 [hep-ph]

[29] T. Reiter, *Optimising Code Generation with haggies*, Comput.Phys.Commun. 181 (2010) 1301–1331, arXiv:0907.3714 [hep-ph]

[30] D. Y. Bardin, L. Kalinovskaya, and L. Rumyantsev, *J(A) functions in the Passarino-Veltman reduction*, Phys.Part.Nucl.Lett. 6 (2009) 30–41

[31] S. Dittmaier, *Separation of soft and collinear singularities from one loop N point integrals*, Nucl.Phys. B675 (2003) 447–466, arXiv:hep-ph/0308246 [hep-ph]

[32] LHC Higgs Cross Section Working Group Collaboration, S. Dittmaier et al., *Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables*, arXiv:1101.0593 [hep-ph]

[33] S. Dawson, C. Jackson, L. Reina, and D. Wackeroth, *Higgs boson production with one bottom quark jet at hadron colliders*, Phys.Rev.Lett. 94 (2005) 031802, arXiv:hep-ph/0408077 [hep-ph]
[34] D. Dicus, T. Stelzer, Z. Sullivan, and S. Willenbrock, *Higgs boson production in association with bottom quarks at next-to-leading order*, Phys.Rev. **D59** (1999) 094016, arXiv:hep-ph/9811492 [hep-ph].

[35] F. Maltoni, Z. Sullivan, and S. Willenbrock, *Higgs-boson production via bottom-quark fusion*, Phys.Rev. **D67** (2003) 093005, arXiv:hep-ph/0301033 [hep-ph].

[36] E. Boos and T. Plehn, *Higgs boson production induced by bottom quarks*, Phys.Rev. **D69** (2004) 094005, arXiv:hep-ph/0304034 [hep-ph].

[37] R. V. Harlander and W. B. Kilgore, *Higgs boson production in bottom quark fusion at next-to-next-to leading order*, Phys.Rev. **D68** (2003) 013001, arXiv:hep-ph/0304035 [hep-ph].

[38] A. Abbasabadi, D. Bowser-Chao, D. A. Dicus, and W. W. Repko, *Higgs - photon associated production at hadron colliders*, Phys.Rev. **D58** (1998) 057301, arXiv:hep-ph/9706335 [hep-ph].

[39] K. Arnold, T. Figy, B. Jager, and D. Zeppenfeld, *Next-to-leading order QCD corrections to Higgs boson production in association with a photon via weak-boson fusion at the LHC*, JHEP **1008** (2010) 088, arXiv:1006.4237 [hep-ph].

[40] E. Gabrielli, B. Mele, and J. Rathsman, *Higgs boson plus photon production at the LHC: a clean probe of the b-quark parton densities*, Phys.Rev. **D77** (2008) 015007, arXiv:0707.0797 [hep-ph].

[41] N. Kauer and G. Passarino, *Inadequacy of zero-width approximation for a light Higgs boson signal*, JHEP **1208** (2012) 116, arXiv:1206.4803 [hep-ph].

[42] S. Dittmaier, C. Mariotti, G. Passarino, R. Tanaka, et al., *Handbook of LHC Higgs Cross Sections: 2. Differential Distributions*, arXiv:1201.3084 [hep-ph].

[43] A. David and G. Passarino, *How well can we guess theoretical uncertainties?*, arXiv:1307.1843 [hep-ph].