Nuclear $\mu^- - e^-$ conversion in strange quark sea

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Abstract

We study nuclear $\mu^- - e^-$ conversion in the general framework of effective Lagrangian approach without referring to any specific realization of the physics beyond the standard model (SM) responsible for lepton flavor violation ($\mathcal{U}_f$). All the possible types of short range interactions (non-photonic mechanisms), i.e. (pseudo-)scalar, (axial-)vector and tensor, are included in our formalism. We show that the $\mu^- - e^-$ conversion in the scalar interactions is comparable with that in the valence quarks. This provides an insight into the strange quark couplings beyond the SM. From the available experimental data on $\mu^- - e^-$ conversion and expected sensitivities of planned experiments we derived upper bounds on the generic $\mathcal{U}_f$ - parameters of $\mu^- - e^-$ conversion sensitive to the relevant u-,d- and s-quark couplings.

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The muon-flavor violating processes

\[ \mu^{-} + (A, Z) \rightarrow e^{-} + (A, Z)^{\ast}, \quad (1) \]

i.e. muon-to-electron \((\mu^{-} - e^{-})\) conversion in nuclei, is known as a very sensitive probe of lepton flavor violation \((L_{f})\) and related physics beyond the standard model (SM) \([1]-[6]\). This fact has been recently strengthened by the evidence for the muon-neutrino oscillations, drawn by the Superkamiokande experiment, which is the first convincing signal of the non-standard physics connected to the lepton flavor non-conservation. The distinct feature of coherent enhancement in nuclear \(\mu^{-} - e^{-}\) conversion makes it more promising probe of \(L_{f}\) than other lepton flavor violating processes \((\mu^{-} \rightarrow e^{-}\gamma, \text{etc.})\). In general the structure of a participating nucleus brings some uncertainties into the theoretical predictions for \(\mu^{-} - e^{-}\) conversion. However, in the most interesting case of coherent \(\mu^{-} - e^{-}\) conversion \([7]\) these uncertainties can be significantly reduced by the possibility of using the available experimental data on nuclear densities.

On the experimental side, at present, there is one running \(\mu^{-} - e^{-}\) conversion experiment, SINDRUM II \([8]\), and two planned experiments, MECO \([9, 10]\) and PRIME \([11]\). The SINDRUM II experiment at PSI \([8]\) with \(^{48}\text{Ti}\) as stopping target has established the best upper bound on the branching ratio

\[ R_{\mu e}^{T} = \frac{\Gamma(\mu^{-} + ^{48}\text{Ti} \rightarrow e^{-} + ^{48}\text{Ti})}{\Gamma(\mu^{-} + ^{48}\text{Ti} \rightarrow \nu_{\mu} + ^{48}\text{Sc})} \leq 6.1 \times 10^{-13}, \quad (90\% \text{ C.L.}) \quad [8]. \quad (2) \]

The MECO experiment with \(^{27}\text{Al}\) is going to start soon at Brookhaven \([10]\). The sensitivity of this experiment is expected to reach the limit

\[ R_{\mu e}^{Al} = \frac{\Gamma(\mu^{-} + ^{27}\text{Al} \rightarrow e^{-} + ^{27}\text{Al})}{\Gamma(\mu^{-} + ^{27}\text{Al} \rightarrow \nu_{\mu} + ^{27}\text{Mg})} \leq 2 \times 10^{-17} \quad [10] \quad (3) \]

This year the PSI experiment is running with the very heavy nucleus \(^{197}\text{Au}\) aiming to improve by a factor of about 20-30 over the previous limit, \(R_{\mu e}^{Au} \leq 2.0 \times 10^{-11}\), set on \(\mu^{-} - e^{-}\) in \(^{197}\text{Au}\) by the same experiment some years ago \([8, 12]\). Now the expected limit is

\[ R_{\mu e}^{Au} = \frac{\Gamma(\mu^{-} + ^{197}\text{Au} \rightarrow e^{-} + ^{197}\text{Au})}{\Gamma(\mu^{-} + ^{197}\text{Au} \rightarrow \nu_{\mu} + ^{197}\text{Pt})} \leq 6 \times 10^{-13} \quad [8, 12] \quad (4) \]

Very recently, a proposal for a new experiment at Tokyo (PRIME) was announced \([11]\). It intends to utilize the \(^{48}\text{Ti}\) as stopping target with the impressive expected sensitivity of \(R_{\mu e}^{T} \leq 10^{-18}\) \([11]\).
These experimental limits can put severe constraints on mechanisms of $\mu^- - e^-$ conversion. In the literature there have been studied various mechanisms beyond the SM (see [1]-[7] and references therein) classified into two categories: photonic and non-photonic as shown in Fig. 1. Specific mechanisms from both categories significantly differ in many respects and, in particular, in nucleon and nuclear structure treatment. This is attributed to the fact that they operate at different distances and, therefore, involve different details of the nucleon structure. Long-distance photonic mechanisms (Fig. 1(a)) are mediated by virtual photon exchange between the nucleus and the $\mu^- - e^-$ lepton current. They suggest that the $\mu^- - e^-$ conversion occurs in the lepton-flavor non-diagonal electromagnetic vertex which is presumably induced by non-standard model physics at loop level. The hadronic vertex is characterized in this case by ordinary electromagnetic nuclear form factors. Contributions to $\mu^- - e^-$ conversion via virtual photon exchange exist in all models which allow $\mu \to e\gamma$ decay. On the other hand, short-distance non-photonic mechanisms (Fig. 1(b)) are described by the effective $\mathcal{L}_f$ 4-fermion quark-lepton interactions which may appear after integrating out heavy intermediate states ($W, Z$, Higgs bosons, supersymmetric particles etc.).

In this Letter we focus on the non-photonic mechanisms of $\mu^- - e^-$ conversion (Fig. 1(b)). The generic effect of physics beyond the SM in $\mu^- - e^-$ is described in our approach by an effective Lagrangian which includes all the possible 4-fermion quark-lepton interactions. Our special interest is concentrated on the scalar interactions which are sensitive to the heavy quark content of the nucleon. Also, the current $\mu^- - e^-$ experiments at Brookhaven and PSI can efficiently probe the scalar component of the $\mu^- - e^-$ conversion [8]-[12]. We will show that the contribution to the $\mu^- - e^-$ conversion rate which originates from the strange nucleon sea via the scalar interactions is comparable with that coming from the valence quarks of the nucleon.

We derive a general representation of the $\mu^- - e^-$ branching ratio in terms of generic $\mathcal{L}_f$ parameters of the effective 4-fermion quark-lepton $\mu^- - e^-$ transition operators. Transforming these operators, first to the nucleon and then to the nuclear level, we pay special attention to the effects of nucleon and nuclear structure. The nucleon structure is taken into account on the basis of the QCD picture of baryon masses and experimental data on certain hadronic parameters. For nuclear structure calculations we apply the formalism described in Refs. [5]-[7]. Our applications refer to the nuclei of current experimental interest, $^{27}$Al, $^{48}$Ti and $^{197}$Au, with special attention to $^{197}$Au which has not been previously studied in the context of $\mu^- - e^-$ conversion. We start with the 4-fermion effective Lagrangian describing the non-photonic $\mu - e$ conversion at the quark level which
Figure 1: (a) Photonic (long-distance) and (b) non-photonic (short-distance) contributions to the nuclear $\mu^{-} - e^{-}$ conversion.

corresponds to the upper vertex of the diagram in Fig. 1(b). It can be written in a general Lorentz covariant form as

$$\mathcal{L}_{\text{eff}}^q = \frac{G_F}{\sqrt{2}} \sum_{A,B,C,D;q} \left[ \eta_{AB}^{(q)} j_A^{B\mu} + \eta_{AB}^{(q)} j_A^B + \eta_{T}^{(q)} j_{\mu\nu} J_{(q)}^{\mu\nu} \right]. \quad (5)$$

where the summation involves $A, B = \{A, V\}, C, D = \{S, P\}$ and $q = \{u, d, s\}$. The $\mathcal{L}_f$ parameters $\eta_i^q$ depend on a concrete $\mathcal{L}_f$ model. The lepton and quark currents are

$$j^V_{\mu} = \bar{\nu} \gamma_{\mu} \nu, \quad j^A_{\mu} = \bar{\nu} \gamma_{\mu} \gamma_5 \nu, \quad j^S = \bar{\nu} \mu, \quad j^P = \bar{\nu} \gamma_5 \mu, \quad j^V_{\mu} = \bar{\nu} \gamma_{\mu} \nu, \quad j^A_{\mu} = \bar{\nu} \gamma_{\mu} \gamma_5 \nu, \quad j^S = \bar{\nu} \gamma_5 \nu, \quad j^P = \bar{\nu} \gamma_5 \nu, \quad j_{\mu\nu} = \bar{\nu} \sigma_{\mu\nu} \nu.$$ 

The next step is the reformulation of the quark level Lagrangian (5) in terms of the nucleon effective fields. First we write down the nucleon level Lagrangian in a general Lorentz covariant form with the isospin structure of the $\mu^{-} - e^{-}$ transition operator

$$\mathcal{L}_{\text{eff}}^N = \frac{G_F}{\sqrt{2}} \sum_{A,B,C,D} \left[ j_{\mu}^A (\alpha_{AB}^{(0)} J_{(0)}^{B\mu} + \alpha_{AB}^{(3)} J_{(3)}^{B\mu}) + j_{C}^{(0)} (\alpha_{CD}^{(0)} J_{(0)}^{D} + \alpha_{CD}^{(3)} J_{(3)}^{D}) + j_{D}^{(0)} (\alpha_{T}^{(0)} J_{(0)}^{T^{\mu\nu}} + \alpha_{T}^{(3)} J_{(3)}^{T^{\mu\nu}}) \right]. \quad (6)$$

The isoscalar $J_{(0)}$ and isovector $J_{(3)}$ nucleon currents are defined as $J_{(k)}^{V_{\mu}} = \tilde{N} \gamma_{\mu} \tau_k N, \quad J_{(k)}^{A_{\mu}} = \tilde{N} \gamma_{\mu} \gamma_5 \tau_k N, \quad J_{(k)}^{S} = \tilde{N} \tau_k N, \quad J_{(k)}^{P} = \tilde{N} \gamma_5 \tau_k N, \quad J_{(k)}^{T^{\mu\nu}} = \tilde{N} \sigma^{\mu\nu} \tau_k N$, where $k = 0, 3$ and $\tau_0 \equiv \hat{I}$. In Eq. (6) we neglected derivative terms. In the matrix elements of $\mu^{-} - e^{-}$ conversion they produce a small contribution proportional to $q/m_{\mu} \lesssim m_{\mu}/m_{\mu} \sim 0.1$ where $q = |q|$ is the momentum transfer to the nucleon while $m_{\mu}$ and $m_{\mu}$ are the muon and the proton masses respectively.
Now we relate the coefficients $\alpha$ in Eq. (6) with the “fundamental” $L_f$ parameters $\eta$ of the quark level Lagrangian (5). Towards this end we apply the on-mass-shell matching condition [13]
\[
\langle \Psi_F | L^q_{\text{eff}} | \Psi_I \rangle \approx \langle \Psi_F | L^N_{\text{eff}} | \Psi_I \rangle,
\]
(7)
where $| \Psi_I \rangle$ and $\langle \Psi_F |$ are the initial and final nucleon states.

In order to solve this equation we use various relations for the matrix elements of the quark operators between the nucleon states
\[
\langle N| \bar{q} \Gamma_K q|N \rangle = G^{(q,N)}_K \bar{\Psi}_N \Gamma_K \Psi_N,
\]
(8)
with $q = \{u, d, s\}$, $N = \{p, n\}$ and $K = \{V, A, S, P, T\}$, $\Gamma_K = \{\gamma_\mu, \gamma_\mu \gamma_5, 1, \gamma_5, \sigma_{\mu\nu}\}$. Since the maximum momentum transfer in $\mu^-e^-$ conversion is much smaller than the typical scale of nucleon structure we can safely neglect the $q^2$-dependence of the nucleon form factors $G^{(q,N)}_K$ and drop the weak magnetism as well as the induced pseudoscalar terms proportional to the small momentum transfer.

Isospin symmetry requires that
\[
G^{(u,n)}_K = G^{(d,p)}_K \equiv G^d_K, \quad G^{(d,n)}_K = G^{(u,p)}_K \equiv G^u_K, \quad G^{(s,n)}_K = G^{(s,p)}_K \equiv G^s_K.
\]
(9)
Note that the axial, pseudoscalar and tensor nucleon currents couple to the nuclear spin leading, therefore, to incoherent contributions. Thus, only the vector and scalar nucleon form factors are needed in the case of the coherent nuclear $\mu^-e^-$ conversion which we are studying in the present paper.

Conservation of vector current implies that the vector charge is equal to the number of the valence quarks of the nucleon and, therefore,
\[
G^u_V = 2, \quad G^d_V = 1, \quad G^s_V = 0.
\]
(10)

The scalar form factors can be extracted from the baryon octet $B$ mass spectrum $M_B$ in combination with the data on the pion-nucleon sigma term $\sigma_{\pi N} = (1/2)(m_u + m_d)\langle p|\bar{u}u + \bar{d}d|p \rangle$. We follow the QCD picture of the baryon masses based on the relation $\langle B|\theta^\mu_B|B \rangle = M_B \bar{B}B$ and on the well known representation [14] for the trace of the energy-momentum tensor $\theta^\mu_B = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s - (\hat{b} \alpha_s / 8\pi) G^a_{\mu\nu} G^{a\mu\nu}_0$, where $G^a_{\mu\nu}$ is the gluon field strength, $\alpha_s$ is the QCD coupling constant and $\hat{b}$ is the reduced Gell-Mann-Low function with the heavy quark contribution subtracted. Using these relations in combination with $SU(3)$ relations [13] for the matrix elements $\langle B|\theta^\mu_B|B \rangle$ as well as the experimental data on $M_B$ and $\sigma_{\pi N}$ we derive
\[
G^u_S \approx 5.1, \quad G^d_S \approx 4.3, \quad G^s_S \approx 2.5
\]
(11)
for the conventional values of the current quark masses: \( m_u = 4.2 \text{ MeV} \), \( m_d = 7.5 \text{ MeV} \), \( m_s = 150 \text{ MeV} \). These masses, however, are not yet well determined and uncertainty factor \( \lesssim 2 \) may affect the results in Eq. (11). For our further numerical estimates these uncertainties are not critical and we take the central values Eq. (11) for the form factors \( G_{u,d,s}^S \). The approach of Refs. [17], relying on the results of the lattice simulations, gives for these form factors similar values with the same level of uncertainties. What remains true is that the strange quarks of the nucleon sea significantly contribute to the scalar nucleon form factor \( G_S \). This result dramatically differs from the naive quark model and the MIT bag model where \( G_{A,S,P}^S = 0 \).

Now the above Eq. (7) can be solved and the coefficients \( \alpha \) of the nucleon level Lagrangian (6) can be expressed in terms of the generic \( \eta \) parameters of the quark level effective Lagrangian Eq. (5) as

\[
\begin{align*}
\alpha^{(0)}_{IV} &= \frac{1}{2}(\eta^{(u)}_{IV} + \eta^{(d)}_{IV})(G^u_{IV} + G^d_{IV}), \\
\alpha^{(3)}_{IV} &= \frac{1}{2}(\eta^{(u)}_{IV} - \eta^{(d)}_{IV})(G^u_{IV} - G^d_{IV}), \\
\alpha^{(0)}_{JS} &= \frac{1}{2}(\eta^{(u)}_{JS} + \eta^{(d)}_{JS})(G^u_{JS} + G^d_{JS}) + \eta^{(s)}_{JS}G^s_{JS}, \\
\alpha^{(3)}_{JS} &= \frac{1}{2}(\eta^{(u)}_{JS} - \eta^{(d)}_{JS})(G^u_{JS} - G^d_{JS}),
\end{align*}
\]

where \( I = V, A \) and \( J = S, P \).

From the Lagrangian (6), following the standard procedure, we can derive the formula for the total \( \mu - e \) conversion branching ratio. In this paper we confine ourselves to the coherent process which is the dominant channel of \( \mu - e \) conversion exhausting, for the majority of experimentally interesting nuclei, more than 90% of the total \( \mu^- - e^- \)-branching ratio [7]. To leading order of the non-relativistic reduction the coherent \( \mu - e \) conversion branching ratio takes the form

\[
R_{\mu e}^{coh} = \frac{G_F^2}{4\pi} \frac{p_eE_e}{\Gamma(\mu^- \rightarrow \text{capture})} \left( \mathcal{M}_p + \mathcal{M}_n \right)^2,
\]

where \( p_e, E_e \) are the outgoing electron 3-momentum and energy. We define the quantity

\[
\mathcal{Q} = |\alpha^{(0)}_{VV} + \alpha^{(3)}_{VV} \phi|^2 + |\alpha^{(0)}_{AV} + \alpha^{(3)}_{AV} \phi|^2 + |\alpha^{(0)}_{SS} + \alpha^{(3)}_{SS} \phi|^2 + |\alpha^{(0)}_{PS} + \alpha^{(3)}_{PS} \phi|^2 \\
+ 2 \Re\{(\alpha^{(0)}_{VV} + \alpha^{(3)}_{VV} \phi)(\alpha^{(0)}_{SS} + \alpha^{(3)}_{SS} \phi)^* + (\alpha^{(0)}_{AV} + \alpha^{(3)}_{AV} \phi)(\alpha^{(0)}_{PS} + \alpha^{(3)}_{PS} \phi)^*\}.
\]

The nuclear transition matrix elements \( \mathcal{M}_{p,n} \) entering Eq. (13), for the case of a ground state to ground state \( \mu^- - e^- \)-transition, are defined as

\[
\mathcal{M}_{p,n} = 4\pi \int j_0(p_e r) \Phi_\mu(r) \rho_{p,n}(r) r^2 dr,
\]
\( j_0 \) is the zero-order spherical Bessel function where \( \rho_{p,n}(r) \) are the spherically symmetric proton (p) and neutron (n) nuclear densities normalized to the atomic number \( Z \) and neutron number \( N \), respectively, of the participating nucleus. \( \Phi_\mu(r) \) is the space dependent part of the muon wave function.

The quantity \( Q \) in (14) depends on the nuclear-target parameters through the factor \( \phi = (\mathcal{M}_p - \mathcal{M}_n)/(\mathcal{M}_p + \mathcal{M}_n) \). However for all the experimentally interesting nuclei this parameter is small. Therefore nuclear dependence of \( Q \) can always be neglected except very special narrow domain in the \( L_f \) parameter space where \( \alpha^{(0)} \leq \alpha^{(3)} \phi \). Another important issue of the smallness of the ratio \( \phi \) in Eq. (14) is the dominance of the isoscalar contribution associated with the coefficients \( \alpha^{(0)} \). Given that the relative significance of the strange quark component, which enters only the isoscalar couplings, is enhanced.

The nuclear matrix elements \( \mathcal{M}_{p,n} \), defined in Eq. (15), are numerically calculated using proton densities \( \rho_p \) from Ref. [18] and neutron densities \( \rho_n \) from Ref. [19] whenever possible. The muon wave function \( \Phi_\mu(r) \) was obtained by solving the Schrödinger equation with the Coulomb potential produced by the densities \( \rho_{p,n} \), taking into account the vacuum polarization corrections [6]. In this way the matrix elements \( \mathcal{M}_{p,n} \) for the nuclear targets \( ^{27}\text{Al} \) and \( ^{48}\text{Ti} \) have been calculated in Ref. [4]. Here we apply this approach to \( ^{197}\text{Au} \). The results for \( \mathcal{M}_{p,n} \) corresponding to the nuclei Al, Ti and Au are given in Table 1 where we also show the muon binding energy \( \epsilon_b \) and the experimental total rates (\( \Gamma_{\mu c} \)) of the ordinary muon capture reaction [20].

By inserting in Eq. (13) the values of the nuclear matrix elements \( \mathcal{M}_{p,n} \) and other quantities from Table 1 and Eqs. (2)-(4) we can derive the upper limits on the parameters of the effective Lagrangians (5), (6). These limits correspond to the existing [8] or expected [10, 11] experimental limits on the branching ratio \( R_{\mu e} \). The most straightforward limit can be set on the quantity \( Q \) of Eq. (13) but its physical meaning is rather obscure. In order to obtain physically interesting upper bounds on the \( L_f \) parameters \( \alpha^{(0)}, \eta^{(q)} \) we adopt the usual assumption that different terms in expressions (12) and (14) do not substantially compensate each other or, equivalently, that only one term dominates at a time. In this way we derived the upper limits given in Table 2. The scaling factors \( B_A \) in Table 2 are defined as

\[
B_{Ti} = \left( \frac{R_{\mu e}^{exp}}{6.1 \cdot 10^{-13}} \right)^{1/2}, \quad B_{Al} = \left( \frac{R_{\mu e}^{exp}}{2.0 \cdot 10^{-17}} \right)^{1/2}, \quad B_{Au} = \left( \frac{R_{\mu e}^{exp}}{6.0 \cdot 10^{-13}} \right)^{1/2}.
\]

Multiplying corresponding column in Table 2 by \( B_A \) one can reconstruct upper limits on the listed parameters for the case when experimental upper bounds on
the branching ratio $R_{\mu e}^{\text{exp}}$ differ from those we used in our analysis.

The constraints for $\eta_{JS}^{(s)}$ in Table 2 originate from the contribution of the strange nucleon sea. As can be seen, they are comparable to the other $\mu^- - e^-$ limits derived from the valence quarks contributions.

The limits in Table 2 represent a general outcome of the $\mu^- - e^-$ conversion experiments for the $\mathcal{L}_f$ physics. These limits can be easily translated into limits on the parameters of specific $\mathcal{L}_f$ model predicting the $\mu^- - e^-$ conversion. This is achieved by adjusting the quark level effective Lagrangian of the model to the form of Eq. (5) and by identifying the effective parameters $\eta_{AB}^{(q)}$ with expressions in terms of model parameters. Then the upper bounds on $\eta_{AB}^{(q)}$ from Table 2 can be translated to certain constraints on the model parameters present in these expressions.

In conclusion, we constructed a general effective Lagrangian describing the non-photonic $\mu^- - e^-$ conversion and specified all the $\mathcal{L}_f$ parameters characterizing this process without referring to any specific model beyond the SM. It includes (axial-)vector, (pseudo-)scalar and tensor interactions. We derived general formula for the coherent $\mu^- - e^-$ conversion branching ratio in terms of the $\mathcal{L}_f$ parameters of the quark level effective Lagrangian. We calculated previously unknown nuclear matrix elements of $^{197}$Au (the current SINDRUM II target). We found that the $\mu^- - e^-$ conversion branching ratio in the strange quark sea of the nucleon is comparable with that in the valence quarks. We have inferred the generic $\mu^- - e^-$ constraints on the $\mathcal{L}_f$ parameters from the existing and expected experimental bounds on the $\mu^- - e^-$ conversion rate. These results are readily used for derivation of the constraints on the parameters of any specific $\mathcal{L}_f$ model without need of the nucleon and nuclear structure calculations. This provides useful nuclear and particle physics inputs for the expected new data from the PSI, MECO, PRIME and other $\mu^- - e^-$ experiments to set bounds on the muon $\mathcal{L}_f$ violating parameters.

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\[
\begin{align*}
\text{Nucleus} & & p_e (fm^{-1}) & & \epsilon_b (MeV) & & \Gamma_{\mu e} \times 10^6 s^{-1} & & \mathcal{M}_p & & \mathcal{M}_n \\
{^{27}}\text{Al} & & 0.531 & & -0.470 & & 0.71 & & 0.047 & & 0.045 \\
{^{48}}\text{Ti} & & 0.529 & & -1.264 & & 2.60 & & 0.104 & & 0.127 \\
{^{197}}\text{Au} & & 0.485 & & -9.938 & & 13.07 & & 0.395 & & 0.516 \\
\end{align*}
\]

Table 1: Transition nuclear matrix elements $\mathcal{M}_{p,n}$ (in $fm^{-3/2}$) of Eq. (15) and other useful quantities (see the text).

| Parameter \( \mathcal{L}_f \) | Present limits \( \text{Present limits} \) | Expected limits \( \text{Present limits} \) |
|---|---|---|
| \( Q \) | \( 2.1 \times 10^{-14} \) | \( 1.2 \times 10^{-18} \) | \( 7.8 \times 10^{-15} \) |
| \( \alpha_{AB}^{(0)} \) | \( 1.5 \times 10^{-7} \) | \( 1.1 \times 10^{-9} \) | \( 8.5 \times 10^{-8} \) |
| \( \eta_{IV}^{(u,d)} \) | \( 1.0 \times 10^{-7} \) | \( 7.3 \times 10^{-10} \) | \( 5.7 \times 10^{-8} \) |
| \( \eta_{JS}^{(u,d)} \) | \( 3.0 \times 10^{-8} \) | \( 2.3 \times 10^{-10} \) | \( 1.8 \times 10^{-8} \) |
| \( \eta_{JS}^{(s)} \) | \( 5.8 \times 10^{-8} \) | \( 4.4 \times 10^{-10} \) | \( 3.4 \times 10^{-8} \) |

Table 2: Upper bounds on the \( \mathcal{L}_f \) parameters (see definitions in the text) inferred from the SINDRUM II data on \(^{48}\text{Ti} \) [Eq. (2)] as well as from the expected sensitivities of the current SINDRUM II [Eq. (4)] and future MECO [Eq. (3)] experiments employing \(^{197}\text{Au} \) and \(^{27}\text{Al} \) as stopping targets respectively. We denoted: \( AB = VV, AV, SS, PS \); \( I = V, A \); \( J = S, P \). The scaling factors \( B_i \) are defined in Eq. (16).