On the Dalitz plot analysis of the $B \to K\eta\gamma$ decays

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ABSTRACT

Recently B-factories have published new results on the $B \to K\eta\gamma$ decays being inspired by the theoretical suggestion to search for new physics in $B \to P_1 P_2 \gamma$ decays. Using heavy meson chiral perturbation theory we find mechanism which governs the amplitude in parts of the Dalitz plot where either $K$ or $\eta$ mesons are soft. The dominant contributions in these cases are coming from the nonresonant decay modes. We discuss also $B \to K\eta'\gamma$ Dalitz plot. Our partially integrated rates are in agreement with the experimental findings.

I. INTRODUCTION

In the last decade B physics was one of the tools in the search for new physics. B-factories made extremely important contributions to these expectations with the numerous measurements. An interesting proposal has been made by the authors of Ref. [1] on the possible effects of new physics in $B \to P_1 P_2 \gamma$ decays. Namely, in these decays new physics might affect the right-handed photons. As it is already known in the standard model (SM), photon emitted in $b \to s\gamma$ is dominantly left-handed [2, 3]. The photon polarization can be measured indirectly by means of time-dependent $CP$ asymmetry of decays to $CP$ eigenstate $f$ plus a photon:

$$
\frac{\Gamma(\bar{B}(t) \to f\gamma) - \Gamma(B(t) \to f\gamma)}{\Gamma(B(t) \to f\gamma) + \Gamma(B(t) \to f\gamma)} = S_{f\gamma} \sin(\Delta m t) - C_{f\gamma} \cos(\Delta m t).
$$

Mixing-induced parameters $S_{f\gamma}$ have been studied in radiative decays of both charged and neutral $B$ decays to $K^*\gamma$ [3], $B \to PP\gamma$ [1, 4], and also $B \to PV\gamma$ [5], where $P$ ($V$) is a light pseudoscalar (vector) meson.

In this work we focus on $B \to K\eta\gamma$ in kinematical region with the hard photon (its energy/momentum is of the order $\sim m_b$) and one soft pseudoscalar (its energy/momentum is of
the order \( \sim \Lambda_{QCD} \). Such kinematical conditions call for using chiral symmetry for soft pseudoscalar and heavy quark effective theory (HQET) combined with the large energy effective theory (LEET) for heavy meson and energetic pseudoscalar. We predict differential decay widths in these regions. This channel has been already seen in Belle and BaBar experiments [6, 7, 8], with the branching fractions [8]

\[
\begin{align*}
\mathcal{B}(B^0 \to K^0\eta\gamma) \times 10^6 &= 7.1^{+2.1}_{-2.0} \pm 0.4, \\
\mathcal{B}(B^+ \to K^+\eta\gamma) \times 10^6 &= 7.7 \pm 1.0 \pm 0.4
\end{align*}
\]

Quoted errors are statistical and systematic, respectively. However, \( CP \) asymmetries are still consistent with zero although experimental resolution is about an order of magnitude above the SM expectation. For three-body decay \( \bar{B}^0 \to K_S\pi^0\gamma \) the authors of Ref. [4] used Soft Collinear Effective Theory (SCET) in the region with soft pion. They used the Breit-Wigner ansatz for the resonant channel via intermediate \( K^*\gamma \) and concluded that right-handed photons are mainly due to the resonance and related interference effects.

Looking into PDG [9] one finds only two strange resonances with spin 2 and 3 which potentially contribute to the \( \bar{B}^0 \to K^0\eta\gamma \) decays in the low to intermediate \( M_{K\eta} \) region. Their effects are small, as for the \( K_2^*(1430) \), the product \( \text{Br}(B \to K_2^*(1430) \gamma) \times \text{Br}(K_2^*(1430) \to K\eta) \sim 10^{-6} \) is one order of magnitude below branching fractions (2). Similar contribution from \( K_3^*(1780) \) is \( 10^{-8} \).

One cannot expect any important contribution coming from these resonant states. This has been confirmed by Belle collaboration in Ref. [6]. On the other hand, spectra of BaBar [8] show some excess of events in the \( 1.4 \text{ GeV} < M_{K\eta} < 1.8 \text{ GeV} \) region, but due to large error bars they are still inconclusive. Following this features we do not include any resonant contributions in our approach.

II. FRAMEWORK

The \( b \to s\gamma \) is induced by the \( \Delta B = 1 \) effective Hamiltonian [10]

\[
\mathcal{H} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{6} C_i \mathcal{O}_i + C_{7\gamma} \mathcal{O}_{7\gamma} + C_{8G} \mathcal{O}_{8G} \] + h.c. \quad (3)
\]

The most important contribution in the SM is due to electroweak penguin operator which couples tensor current between \( b \) and \( s \) quarks to the electromagnetic tensor

\[
\mathcal{O}_{7\gamma} = \frac{e}{8\pi^2} \left[ m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b + m_s \bar{s} \sigma_{\mu\nu} (1 - \gamma_5) b \right] F^{\mu\nu}. \quad (4)
\]

Final state photons it produces are dominantly left-handed, with right-handed ones being suppressed by \( m_s/m_b \). Keeping only \( \mathcal{O}_{7\gamma} \), this suppression is evident also in the asymmetry [11],
however, in multibody decays $\mathcal{O}_2$ can induce charm-loop mediated $b \to s\gamma g$, with equal rates for $\gamma_L$ and $\gamma_R$, and lift the suppression to $\sim 10\%$ \cite{2}. For our purpose of calculating decay width we can neglect the $m_s$ part of \cite{1} as well as the $\mathcal{O}_2$ effects, keeping only left(right)-handed photons from $b(\bar{b})$ quark.

In decay of $B$ meson to three light particles, there are at least two energetic final state particles with momentum $\mathcal{O}(m_b)$. We shall study kinematical region of soft $\eta$ and energetic $K$, or the other way around, while the photon will always be hard, as shown on Fig. \ref{fig:1} where $E_\eta$ and $K\eta$ invariant mass are used as kinematical variables.

![Figure 1: \( \bar{B}^0 \to \bar{K}^0\eta\gamma \) phase space regions where soft pseudoscalars have energy below 1.2 GeV (0.8 GeV) in the light-gray (gray) region. Left corner corresponds to soft $\eta$ and right one to soft $K$.](image1)

![Figure 2: On the left, the leading contribution in the region of soft $\eta$. On the right, $K$ is soft. They govern the decay amplitude in the left and right region in Fig. \ref{fig:1} respectively.](image2)

Feynman graphs in the leading order in $\frac{p_{\text{soft}}}{\Lambda_{\chi}}$, $\frac{\Lambda_{\text{QCD}}}{m_b}$ are shown in Fig. \ref{fig:2} where heavy meson emits a soft pseudoscalar and is excited to a vector state that decays due to $\mathcal{O}_{7\gamma}$ to energetic
photon and meson. We stress that those two diagrams are for two different final states, i.e. with different momenta, and their sum has no physical meaning. Each corresponds to precisely defined kinematical region where light meson, attached to heavy line, has low momentum. This is in contrast to the analogous decay $\bar{B} \to \bar{K}^0 \pi^0 \gamma$, where one cannot apply effective description in the soft $K$ region, due to lack of $s\bar{s}$ component in $\pi^0$.

For strong emission of the soft pseudoscalar off the heavy-meson line, we utilize the low-energy chiral lagrangian combined with the heavy-quark symmetry (see \cite{11} and references therein)

$$\mathcal{L}_{\text{strong}} = ig \text{Tr} \left[ H_a(v) A_{ab}^\mu \gamma_\mu \gamma_5 H_b(v) \right].$$ (5)

The low-energy pion coupling to heavy pseudoscalar and vector has been calculated on the lattice with unquenched quarks \cite{12} and its value is $g = 0.5 \pm 0.1$, in agreement with the value extracted in \cite{13}. Contribution of the effective weak vertex $\mathcal{O}_{\gamma\gamma}$ in the left graph of Fig. 2 is

$$\langle \bar{K}^0(k) \gamma(q, \epsilon) \left| \frac{e}{8\pi^2} m_b \bar{s}(0) \sigma^{\mu\nu} F_{\mu\nu}(0) (1 + \gamma_5) b(0) \right| B^*(p, \eta) \rangle$$

$$= \langle \gamma(q, \epsilon) \left| 2\partial_\mu A_\nu \right| 0 \rangle \times \langle \bar{K}^0(k) \left| \frac{e}{8\pi^2} m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b \right| B^*(p, \eta) \rangle$$

$$= \frac{ie m_b}{4\pi^2} q_\mu \epsilon_\nu^* \langle \bar{K}^0(k) \left| \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b \right| B^*(p, \eta) \rangle.$$

For soft $\bar{K}^0$ (right graph of Fig. 2), the above manipulations are performed on flavor rotated states $(B^*_s, \eta) \leftrightarrow (B^*, \bar{K}^0)$. Virtuality of intermediate $B^*$ is zero up to $1/m_b$ corrections, so use of the heavy-quark spin-symmetry is justified up to hard spectator effects \cite{14}. In this picture, we assume heavy-quark interacts with light degrees of freedom solely through soft gluon exchanges and thus we use only upper-components field $h_\nu$ for the $b$-quark. This is similar to approaches in \cite{14, 15}.

In the following, we are going to relate the $B^* \to \bar{K}^0$ tensor form-factors to the vector ones of $B \to \bar{K}^0$. Standard form factors are

$$\langle \bar{K}^0(k) \left| \bar{s}_\mu \sigma^{\mu\nu} \right| B^*(p_B, \eta) \rangle = 2T_1^{BK}(q^2) \epsilon^{\mu\rho\sigma} p_{B, \mu} k_{\rho} \eta_{\sigma},$$ (7a)

$$\langle \bar{K}^0(k) \left| \bar{s}_\mu \sigma^{\mu\nu} \gamma_5 b \right| B^*(p_B, \eta) \rangle = i T_2^{BK}(q^2) \left[ (M^2 - m_K^2) \eta - \eta \cdot q (p_B + k) \right]^{\nu}$$

$$+ iT_3^{BK}(q^2) \eta \cdot q \left[ q - \frac{q^2}{M^2 - m_K^2} (p_B + k) \right]^{\nu},$$ (7b)

$$\langle \bar{K}^0(k) \left| \bar{s} \gamma^\nu b \right| B(p_B) \rangle = f_+^{BK}(q^2) \left[ p_B + k - \frac{M^2 - m_K^2}{q^2} q \right]^{\nu} + f_0^{BK}(q^2) \frac{M^2 - m_K^2}{q^2} q^{\nu},$$ (7c)

where $M$ and $m_K$ are the $B$ and $K$ meson masses, respectively, and $q = p_B - k$. Now we can use underlying heavy quark and large energy symmetries to constrain the number of independent form factors. Following \cite{14}, we express the matrix element between $B$ and energetic $\bar{K}^0$ as Dirac-trace
of their wave functions

$$\langle K^0(En_-) \left| \bar{s}_n \Gamma h_v \right| B^*(Mv) \rangle = \text{Tr} \left[ A(E) \overline{\mathcal{M}}_K \Gamma \mathcal{M}_B \right].$$

(8)

$E = \frac{M^2 + m^2 - q^2}{2M}$ is energy of the $K$ and $n_-$ is four vector almost parallel to $K$ momentum

$$k = En_- + k', \quad n_+^2 = 0.$$  

(9)

Residual momentum $k'$ is of the order $\Lambda_{QCD}/E$. $s_n$ is the effective large-energy field of the $s$ quark

$$s_n(x) = e^{i En_- x} \frac{\not \gamma_5 - \not p}{4} s(x),$$

(10)

and $n_+ = 2v - n_-$. Long distance physics is parameterized by function $A(E)$, which does not depend on $\Gamma$, since Hamiltonians of HQET and LEET commute with quark spin operators. The most general parameterization of $A(E)$ is then in terms of the four energy-dependent functions [14]:

$$A(E) = a_1(E) + a_2(E) \frac{v}{E} + a_3(E) \frac{n_-}{E} + a_4(E) \frac{n_- v}{E}.$$  

(11)

For wavefunctions of mesons, we use

$$\overline{\mathcal{M}}_K = -i \gamma_5 \frac{\not \gamma_5 + \not v}{4}, \quad \mathcal{M}_B = \frac{1 + \gamma_5}{2} \begin{cases} \not \gamma_5; & B = B^*(Mv, \eta) \\ (-\gamma_5); & B = B(Mv) \end{cases}.$$  

(12)

Evaluating the traces on the right-hand side of (8), one can connect form factors with functions $a_1(E), \ldots, a_4(E)$ and find at $q^2 = 0$ the symmetry relation

$$T_1^{BK}(0) = T_2^{BK}(0) = T_3^{BK}(0) = f_+^{BK}(0).$$  

(13)

Consequently, matrix element of $O_{7\gamma}$ for $B^* \to \bar{K}^0$ transition

$$\langle \bar{K}^0(k) \left| \bar{s}_q \sigma^{\mu\nu}(1 + \gamma_5) h_v \right| B^*(v, \eta) \rangle = f_+^{BK}(0) \left[ 2M \epsilon^{\mu\nu\rho\sigma} v_{\mu} k_{\rho} \eta_{\sigma} + i M^2 \eta^\nu - i \eta \cdot q (Mv + k)^\nu \right]$$

(14)

is proportional to $f_+^{BK}(0)$, the value of which has been determined with the light-cone sum rules approach [16]

$$f_+^{BK}(0) = 0.33 \pm 0.04.$$  

(15)

The left diagram in Fig. 2 valid in the soft $\eta$ region is then

$$\mathcal{A}_{\eta \text{ soft}} = -i G_F V_{ts}^* V_{tb} C_7(m_b) \frac{e m_b}{8 \pi^2} f_+^{BK}(0) \frac{g}{f} \left( \frac{\cos \theta}{\sqrt{6}} - \frac{\sin \theta}{\sqrt{3}} \right)$$

$$\times \frac{(p_\sigma - v \cdot p_\nu)}{v \cdot p} \left[ 2M \epsilon^{\nu\lambda\rho\sigma} v_{\lambda} k_{\rho} + i M^2 g^{\nu} - i(Mv - k)^\sigma (Mv + k)^\nu \right] \epsilon_\nu,$$  

(16)
where $\theta = -15.4^\circ$ is the $\eta_8 - \eta_1$ mixing angle and $f = 93$ MeV is the pion decay constant. Wilson coefficient $C_{7\gamma}$ on energy scale of $b$-quark is $C_{7\gamma}(\mu = 5$ GeV$) = -0.30$. Electromagnetic gauge invariance is restored in the limit of small $E_\eta$. Right diagram of Fig. 2 (with soft $\bar{K}^0$) has amplitude of similar form

$$A_{K\text{ soft}} = iG_F V_{tb} C_{7}(m_b) \frac{e m_b}{8 \pi^2} f_{BK}^{+}(0) \frac{g}{f} \frac{\sqrt{2} \cos \theta + \sin \theta}{\sqrt{3}} \times \left[ 2 M e^{\nu \lambda \rho \sigma} v_\lambda p_\rho + i M^2 g^{\sigma \nu} - i (M v - p)^\sigma (M v + p)^\nu \right] \epsilon^*_\nu, \quad (17)$$

In comparison to the soft $\eta$ amplitude (16), the soft $K$ amplitude (17) has interchanged momenta $p \leftrightarrow k$ and $\eta_8 - \eta_1$ mixing factors now originate from $B_s^* \eta \gamma$ vertex, where we rely on flavor $SU(3)$ symmetry to estimate form factor $f_{BK}^{\eta}$. To get amplitude for $\eta'$ in the final state, one only has to modify $\eta_8 - \eta_1$ mixing coefficients in the amplitudes (16,17) and find for soft $\eta'$

$$A'_{\eta'\text{ soft}} = - iG_F V_{ts} V_{tb} C_{7}(m_b) \frac{e m_b}{8 \pi^2} f_{BK}^{+}(0) \frac{g}{f} \left( \frac{\sin \theta}{\sqrt{6}} + \frac{\cos \theta}{\sqrt{3}} \right) \times \left[ 2 M e^{\nu \lambda \rho \sigma} v_\lambda k_\rho + i M^2 g^{\sigma \nu} - i (M v - k)^\sigma (M v + k)^\nu \right] \epsilon^*_\nu. \quad (18)$$

Momentum of $\eta'$ is denoted by $p$. Amplitude for soft $K$ and energetic $\eta'$ is

$$A'_{K\text{ soft}} = - iG_F V_{ts} V_{tb} C_{7}(m_b) \frac{e m_b}{8 \pi^2} f_{BK}^{+}(0) \frac{g}{f} \frac{\cos \theta - \sqrt{2} \sin \theta}{\sqrt{3}} \times \left[ 2 M e^{\nu \lambda \rho \sigma} v_\lambda p_\rho + i M^2 g^{\sigma \nu} - i (M v - p)^\sigma (M v + p)^\nu \right] \epsilon^*_\nu. \quad (19)$$

Figure 3: $\bar{B}^0 \to \bar{K}^0 \eta \gamma$ spectra. Left: Photon spectrum in the region of $E_\eta < 0.8$ GeV (solid thick line), $E_\eta < 1.0$ GeV (dashed thick), and $E_\eta < 1.2$ GeV (dotted). Right: same for soft $K$, $E_K < 0.8, 1.0, 1.2$ GeV.
III. SUMMARY OF RESULTS

We have investigated Dalitz plots of the $B \rightarrow \eta(\eta')K\gamma$ decays using the combined heavy meson, large energy, and chiral lagrangian theories. The use of this approach is fully justified due to the fact that in the considered areas of the Dalitz plots, the kinematical configuration allows simultaneous expansion in soft momentum and $1/m_b$. Partial branching ratio integrated over both regions in Fig. 1 with upper bound on soft meson energies set to 1.2 GeV accounts for about 10% of the $\bar{B}^0 \rightarrow \bar{K}^0\eta\gamma$ branching ratio [2a]. With increasing statistics, these two corners of the phase space could be studied more thoroughly and bring in complementary information on the scale of $C_7\gamma$. On Figures 3 and 4 we show photon spectra for regions with soft final state mesons. The model we proposed assumes only nonresonant production of the $\bar{K}^0\eta(\eta')$ states. Since $\eta(\eta')$ are isosinglets we do not expect any significant final states effects and therefore strong phase necessary for the observation of the direct $CP$ violation is not likely to be generated. Mixing-induced $CP$ violation in $B \rightarrow K_S\eta\gamma$, on the other hand, should offer cleaner environment (speaking of resonances) to look for right-handed photons, than the analogous decay $B \rightarrow K_S\pi^0\gamma$.

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