Quantum measurement process with an ideal detector array

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Abstract
Any observable with finite eigenvalue spectrum can be measured using a multiport apparatus realizing an appropriate unitary transformation and an array of detector instruments, where each detector operates as an indicator of one possible value of the observable. The study of this setup in the frame of von Neumann’s quantum mechanical measurement process has a remarkable result: already after the interaction of the measured system with the detector array without collapse, exactly one detector is indicating a detection. Each single detector indicates either 0 or 1 detection, and no superposition can be attributed to it.

Keywords Measurement process · Detector array · Definite outcome · Preferred base · Wave function collapse

1 Introduction
Reck et al. [1] demonstrated that any finite dimensional unitary transformation can be realized in the laboratory using a lossless multiport apparatus constructed with beam splitters, mirrors, and phase shifters. Furthermore, any finite dimensional observable can be measured realizing an appropriate unitary transformation, attaching a particle source to the input port and particle detectors to the output ports. In this case, each detector operates as an indicator of one possible value of the observable.

Measurement instruments of this type are used in many modern versions of quantum mechanical experiments. The authors list Einstein-Podolsky-Rosen [2, 3] and Bell test [4, 5] experiments, as well as variants of Stern-Gerlach experiments [6], and quantum cryptography devices [7]. A famous example is Aspect’s Bell test experiment [8] with photons, which was repeated in many variants, even with neutrons [9]. Several other experiments with neutrons (in line with their photon counterparts) are described in [10], with many additional references.

In the following, such measurement instruments are considered theoretically, with a focus on the detectors, which are treated quantum mechanically in the frame of von Neumann’s measurement process [11, 12]. Any observable with finite eigenvalue spectrum can
be measured with an array of detector instruments using an appropriate measurement interaction, described by a unitary transformation, which fulfills the conditions for a repeatable ideal measurement. It is well-known, that this interaction of the measured system and the measurement instrument ends with an entangled state, which exhibits the “measurement problem”: how to get a definite outcome. Schrödinger’s cat, waiting in a killing machine triggered by the measurement outcome, is iconic for this situation: is it dead, alive, or in a superposition of these states? For the Copenhagen interpretation [13], the measurement must be completed by some final reduction or collapse [14] to get a definite outcome. However, with the detector array the entangled state already has some special properties, which are usually assumed to arise with the collapse: exactly one detector is indicating a detection; each single detector indicates either 0 or 1 detection, and no superposition can be attributed to it.

2 Von Neumann’s Measurement Process

Von Neumann [12] described the measurement process quantum mechanically as a short time interaction between the measured system $S$ and a measurement instrument $M$, using Hilbert space $\mathcal{H}_S$ and $\mathcal{H}_M$, respectively, as state space, and the tensor product $\mathcal{H}_S \otimes \mathcal{H}_M$ for the composite system $SM$. The instrument has to display the measurement outcome by entering one of the pairwise orthogonal pointer states $\varphi_1, \varphi_2, ..., \in \mathcal{H}_M$, which correspond to the eigenvalues $a_1, a_2, ..., \in \mathbb{R}$ of the measured observable $A \in \mathcal{B}(\mathcal{H}_S)$ (assumed to be bounded and to have a pure, non-degenerate eigenvalue spectrum). The initial state of the composite system $SM$ before the interaction and its final state after that are connected by a unitary transformation $U_{SM} \in \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_M)$, which must fulfill two conditions to constitute a repeatable ideal measurement of the observable $A$: any eigenstate $\alpha_k$ of $A$ stays unchanged, and the instrument must enter the corresponding pointer state $\varphi_k$, indicating the measurement of the eigenvalue $a_k$, i.e.

$$U_{SM} (\alpha_k \otimes \varphi_0) = \alpha_k \otimes \varphi_k,$$

with $\varphi_0$ as the initial state of the instrument. As a consequence, the interaction of the measurement instrument in initial state $\varphi_0$ with a system in state

$$\psi = \sum_k c_k \alpha_k,$$

with $c_k \in \mathbb{C}$, transforms the composite system into the entangled state

$$U_{SM} (\psi \otimes \varphi_0) = \sum_k c_k \alpha_k \otimes \varphi_k,$$

with probability $p_k = |c_k|^2$ to observe the pointer $\varphi_k$, according to Born’s rule. This superposition of products of eigenstates $\alpha_k$ and corresponding pointer states $\varphi_k$ exhibits the “measurement problem”, how to get a definite outcome, and is a reason for different interpretations of quantum mechanics [15]. According to the Copenhagen interpretation, however, this state terminates only the “premeasurement” [14]. The measurement must be completed by some additional reduction or collapse, giving a definite outcome in a random, irreversible transition

$$\sum_k c_k \alpha_k \otimes \varphi_k \xrightarrow{pd} \alpha_d \otimes \varphi_d$$
where $\varphi_d$, $\alpha_d$, and $p_d = |c_d|^2$ are the pointer state indicating the definite outcome $a_d$, the corresponding eigenstate of the measured system, and the transition probability, respectively. Different authors associate this transition with different circumstances, e.g., the recognition of the measurement outcome by a conscious observer [16] or its registration by a classical device [17]. Some non-Copenhagen interpretations (e.g., the pilot wave interpretation [18, 19], the many-worlds interpretation [20]) deny such an additional transition at all and assume that the final premeasurement state already constitutes the end of the measurement.

The state $\alpha_d$ of the system after reduction is an eigenstate of $A$, so a repetition of the measurement will give the same outcome $a_d$. However, as Pauli already [21] noticed, there are ideal measurements for which an immediate repetition gives not the same result. The measurement scheme of the multiport apparatus with detectors [1] is of this type. Von Neumann’s conditions (1) must be relaxed to describe such non-repeatable ideal measurements [14]: an eigenstate $\alpha_k$ will be transformed into another state $\alpha_k' \in \mathcal{H}_S$, while the instrument enters the corresponding pointer state $U_{SM} (\alpha_k \otimes \varphi_0) = \alpha_k' \otimes \varphi_k$.

For the multiport apparatus we can write $\alpha_k' = U \alpha_k$, with the unitary transformation $U \in \mathcal{B} (\mathcal{H}_S)$ determined by the apparatus. Here, these $\alpha_k'$ are not eigenstates of $A$ but of the observable

$$A' = U A U^{-1}$$

measured repeatably by the detectors. The collapse of the final premeasurement state gives then

$$\sum_k c_k \alpha_k' \otimes \varphi_k \overset{pd}{\rightarrow} \alpha_d' \otimes \varphi_d$$

indicating the measurement outcome $a_d$, with the system in the state $\alpha_d'$.

### 3 Premeasurement With An Ideal Detector

In the following, an ideal detector is considered as an instrument for measuring an indicator observable (binary observable) which is represented by a projection operator $E \in \mathcal{B} (\mathcal{H}_S)$ with $E = E^\dagger = E^2$, eigenvalues $\sigma(A) = \{0, 1\}$, and the complement $\overline{E} = 1 - E$. A suitable detector instrument $D$ has at least 2 orthogonal pointer states $\varphi_0, \varphi_1 \in \mathcal{H}_D$ to display the measurement result. With Hilbert space $\mathcal{H}_D = \mathbb{C}^2$, the detector instrument can be a spin-1/2-system or a qubit.

The mathematical expression of von Neumann’s conditions (1) for the interaction $U_{SD} \in \mathcal{B} (\mathcal{H}_S \otimes \mathcal{H}_D)$ must be generalized to include repeatable ideal measurements of degenerate observables [22]. For an arbitrary vector $\psi \in \mathcal{H}_S$, $E \psi$ and $\overline{E} \psi$ are (unnormalized) eigenvectors of $E$ (or the zero vector) with eigenvalues 1 or 0, respectively. So, the conditions are

$$U_{SD} (E \psi \otimes \varphi_0) = E \psi \otimes \varphi_1$$
$$U_{SD} (\overline{E} \psi \otimes \varphi_0) = \overline{E} \psi \otimes \varphi_0$$

for all $\psi \in \mathcal{H}_S$.

**Example 1** Premeasurement with a detector. a) A vector of the Hilbert space $\mathcal{H}_S \otimes \mathbb{C}^2$ can be written as 2-component column vector (similar as a spinor). In this notation, the matrix

$$V_{SD} = \begin{pmatrix} \overline{E} & E \\ E & \overline{E} \end{pmatrix} \in \mathcal{B} (\mathcal{H}_S \otimes \mathbb{C}^2),$$
with $V_{SD} = V_{SD}^{-1} = V_{SD}^\dagger$, fulfills the conditions for a repeatable ideal measurement. b) If the measured system $S$ is a spinless particle, with $\mathcal{H}_S = L^2(\mathbb{R}^3)$, an indicator observable for position detection is the indicator function $I_V : \mathbb{R}^3 \to \{0, 1\}$, with $I_V(r) = 1 \iff r \in V$, of a Borel set $V \in B(\mathbb{R}^3)$ representing the detection volume (in position representation) with

$$V_{SD} = \begin{pmatrix} 1 - I_V & I_V \\ I_V & 1 - I_V \end{pmatrix} \in \mathcal{B}(L^2(\mathbb{R}^3) \otimes \mathbb{C}^2).$$

c) If the measured system $S$ is a qubit [23], with $\mathcal{H}_S = \mathbb{C}^2$, a indicator observable for a measurement in the computational base is

$$\tilde{E} = |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \overline{E} = |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

and with its matrix representation one gets a $4 \times 4$ matrix for the measurement transformation

$$\tilde{V}_{SD} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

which is a representation of a CNOT circuit with 2 qubits (Fig. 1).

### 4 Premeasurement With An Ideal Detector Array

Any observable $A \in \mathcal{B}(\mathcal{H}_S)$ with a finite eigenvalue spectrum $\sigma(A) = \{a_1, ..., a_n\} \subset \mathbb{R}$ can be written as a linear combination of projection operators $E_k \in \mathcal{B}(\mathcal{H}_S)$ onto the eigenspaces

$$A = \sum_{k=1}^n a_k E_k,$$

with

$$E_j E_k = E_k E_j = \delta_{j,k} 1$$

for all $j, k \in I_n = \{1, ..., n\}$ and

$$\sum_{k=1}^n E_k = 1 .$$

The projection operators $E_1, ..., E_n$ are indicator observables for the corresponding eigenvalues and can be measured using ideal detector instruments $D_1, ..., D_n$, assuming a bijection

$$E_k \leftrightarrow D_k .$$

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**Fig. 1** Premeasurement with CNOT circuit
An array of these $n$ detectors can be used as a measurement instrument for the observable $A$. With pointer states $\phi_0^{(k)}$, $\phi_1^{(k)}$ of detector $D_k$, the pointer states of this instrument $M$ are
\[ \tilde{\phi}_0 = \phi_0^{(1)} \otimes \ldots \otimes \phi_0^{(n)}, \]
\[ \tilde{\phi}_k = \phi_0^{(1)} \otimes \ldots \otimes \phi_1^{(k)} \otimes \phi_0^{(k+1)} \otimes \ldots \otimes \phi_0^{(n)} \]
for $k \in I_n$, in Hilbert space $\mathcal{H}_M = \mathcal{H}_D^{(1)} \otimes \ldots \otimes \mathcal{H}_D^{(n)} = \mathcal{H}_D^{\otimes n}$. The measurement result is the index $k$ of the detector with the pointer state $\tilde{\phi}_k^{(k)}$, equivalent to the pointer state $\tilde{\phi}_k$ of the whole array, indicating the value $a_k$ of the observable $A$. Here, von Neumann's conditions (1) for an repeatable ideal measurement of observable $A$ are
\[ U_{SM}(E_k\psi \otimes \tilde{\phi}_0) = E_k\psi \otimes \tilde{\phi}_k \] (9)
for all $\psi \in \mathcal{H}_S$ and $k \in I_n$.

In the rest of this paragraph, a construction for this unitary transformation $U_{SM} \in \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_M)$ is given, using the detector transformation $U_{SD}^{(k)} \in \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_D^{(k)})$ from (5) for each single detector $D_k$. With (6) we can write (5)
\[ U_{SD}^{(k)} \left( E_k\psi \otimes \phi_0^{(j)} \right) = E_k\psi \otimes \phi_0^{(j)} \delta_{j,k} \] (10)
for all $j, k \in I_n$, and $U_{SM}$ can be constructed as product
\[ U_{SM} = U_1^{(1)} \cdot \ldots \cdot U_n^{(n)} \]
where
\[ U_M^{(k)} = 1^{(1)} \otimes \ldots \otimes U_{SD}^{(k)} \otimes 1^{(k+1)} \otimes \ldots \otimes 1^{(n)} \]
is the representation of $U_{SD}^{(k)}$ in the product space $\mathcal{H}_S \otimes \mathcal{H}_M$. It is obvious, that these operators commute
\[ U_M^{(j)} U_M^{(k)} = U_M^{(j)} U_M^{(k)} \]
for all $j, k \in I_n$. To demonstrate that $U_{SM}$ fulfills von Neumann’s conditions for the eigenstates $E_k\psi$ of $A$, we multiply the initial state of the composite system
\[ \Phi_0 = E_k\psi \otimes \phi_0^{(1)} \otimes \ldots \otimes \phi_0^{(n)} = E_k\psi \otimes \tilde{\phi}_0 \]
for each $k \in I_n$ successively with $U_M^{(1)}$, $\ldots$, $U_M^{(n)}$ using (10) 
\[ \Phi_1 = U_M^{(1)} \Phi_0 = E_k\psi \otimes \phi_0^{(1)} \otimes \ldots \otimes \phi_1^{(k)} \otimes \ldots \otimes \phi_0^{(n)} = E_k\psi \otimes \tilde{\phi}_0 \]
\[ \ldots \]
\[ \Phi_k = U_M^{(k)} \Phi_{k-1} = E_k\psi \otimes \phi_0^{(1)} \otimes \ldots \otimes \phi_1^{(k)} \otimes \phi_0^{(k+1)} \otimes \ldots \otimes \phi_0^{(n)} = E_k\psi \otimes \tilde{\phi}_k \]
\[ \ldots \]
\[ U_{SM} \Phi_0 = \Phi_n = U_M^{(n)} \Phi_{n-1} = E_k\psi \otimes \phi_0^{(1)} \otimes \ldots \otimes \phi_1^{(k)} \otimes \phi_0^{(k+1)} \otimes \ldots \otimes \phi_0^{(n)} = E_k\psi \otimes \tilde{\phi}_k, \]
which gives (9).
Example 2 Premeasurement with a 2-detector array. a) With the indicator observables \( E_1, E_2 \in \mathcal{B}(\mathcal{H}_S) \) the matrix representation of \( V_{SD} \) from example 1 gives

\[
V_{SM} = (V^{(1)}_{SD} \otimes 1^{(2)}) (1^{(1)} \otimes V^{(2)}_{SD}) = \begin{pmatrix}
0 & E_1 & E_2 & 0 \\
E_1 & 0 & 0 & E_2 \\
E_2 & 0 & 0 & E_1 \\
0 & E_2 & E_1 & 0
\end{pmatrix},
\]

which fulfills (9). b) If the measured system \( S \) is a particle, the Hilbert space can be restricted to a finite box \( B \in \mathcal{B}(\mathbb{R}^3) \), with \( \mathcal{H}_S = \mathcal{L}^2(B) \). Then, the completeness condition (7) for the partition of the position space \( V_1 \cup V_2 = B \) can be fulfilled with two finite detection volumes and the corresponding indicator functions \( E_1 = I_{V_1}, E_2 = I_{V_2} \) in position representation. Such partitions give a simple model of a tracking chamber. c) If the measured system is a qubit, the measurement interaction for a measurement in the computational base with \( \tilde{E}_1 = |1\rangle\langle 1| \) and \( \tilde{E}_2 = |0\rangle\langle 0| \) can be described with the resulting \( 8 \times 8 \) matrix for \( \tilde{V}_{SM} \) or, equivalently, with an 1-to-2 decoder circuit (Fig. 2).

5 Properties Of The Final Premeasurement State

It is a consequence of (7) and the conditions (9), that for an arbitrary initial state \( \psi \in \mathcal{H}_S \) of the measured system \( S \) the final premeasurement state of the composite system \( SM \) is

\[
U_{SM} (\psi \otimes \tilde{\varphi}_0) = U_{SM} \left( \sum_{k=1}^{n} E_k \psi \otimes \tilde{\varphi}_0 \right) = \sum_{k=1}^{n} E_k \psi \otimes \tilde{\varphi}_k .
\]

To include non-repeatable ideal measurements with the multiport apparatus (2), we discuss the more general final premeasurement state

\[
\Phi = \sum_{k=1}^{n} E'_k \psi \otimes \tilde{\varphi}_k \tag{11}
\]

where \( E'_k = U^{-1} E_k U \) is a projection operator, for all \( k \in I_n \), defined by the inverse transformation of (3).

This entangled state of the system and the detector array has several interesting properties.

![Fig. 2 Premeasurement with 1-to-2 decoder circuit](image-url)
5.1 Exactly One Detection

An observable of the detector array is the number of detectors indicating a detection \( N \in \mathcal{B}(\mathcal{H}_M) \). With the eigenvalue spectrum \( \sigma(N) = \{0, 1, ..., n\} \) it can be written as

\[
N = \sum_{k=0}^{n} k Q_k,
\]

with projection operators \( Q_k \in \mathcal{B}(\mathcal{H}_M) \) onto the closed subspaces spanned by the vectors

\[
V_k = \left\{ \varphi_{i_1}^{(1)} \otimes ... \otimes \varphi_{i_n}^{(n)} \in \mathcal{H}_M \mid i_1, ..., i_n \in \{0, 1\}, i_1 + ... + i_n = k \right\}.
\]

It is obvious that \( \tilde{\varphi}_0 \in V_0 \) is an eigenvector of \( N \) with eigenvalue 0 and \( \tilde{\varphi}_1, ..., \tilde{\varphi}_n \in V_1 \) are eigenvectors with eigenvalue 1. Consequently, the final premeasurement state \( \Phi(\mathbf{11}) \) is an eigenstate of \( 1_S \otimes N \) with eigenvalue 1

\[
(1_S \otimes N) \Phi = \sum_{k=1}^{n} E_k' \psi \otimes N \tilde{\varphi}_k = \sum_{k=1}^{n} E_k' \psi \otimes \tilde{\varphi}_k = \Phi.
\]

So, the observable \( 1_S \otimes N \) has in state \( \Phi \) the dispersion free expectation value 1. This means, exactly one detector is indicating a detection.

5.2 Preferred Pointer Base States

The biorthogonal Schmidt decomposition of an entangled state is in general not unique [24, 25]. Consequently, the same entangled state can arise as a final premeasurement state measuring incompatible observables. However, with the final premeasurement state \( \Phi(\mathbf{11}) \), only eigenstates of the number operator with eigenvalue 1 can be used as factors for the detector array. This can be shown considering the alternative Schmidt decomposition

\[
\Phi = \sum_j d_j \beta_j \otimes \chi_j,
\]

\[
(1_S \otimes N) \Phi = \Phi \Rightarrow \sum_j d_j \beta_j \otimes N \chi_j = \sum d_j \beta_j \otimes \chi_j \Rightarrow \sum_j d_j \beta_j \otimes (N \chi_j - \chi_j) = 0.
\]

Because of the pairwise orthogonality, each summand must be zero. Therefore, each \( \chi_j \) has to be an eigenstate of \( N \) with eigenvalue 1.

There is no factorization of such an eigenstate with other single detector states than the pointer states \( \varphi_0, \varphi_1 \). To see this, let \( \hat{\varphi} \in \mathcal{H}_D^{\otimes n} \) be an eigenvector of the number operator with eigenvalue 1 and

\[
\hat{\varphi} = \chi \otimes \psi
\]
a factorization with \( \psi = a \varphi_0 + b \varphi_1 \in \mathcal{H}_D \). Then, we can write

\[
\chi = c \chi_0 + d \chi_1
\]

with \( N (\chi_0 \otimes \varphi_0) = 0 \) and \( N (\chi_1 \otimes \varphi_0) = \chi_1 \otimes \varphi_0 \) and

\[
\chi \otimes \psi = ac \chi_0 \otimes \varphi_0 + ad \chi_1 \otimes \varphi_0 + bc \chi_0 \otimes \varphi_1 + bd \chi_1 \otimes \varphi_1
\]

\[
N (\chi \otimes \psi) = 0 + ad \chi_1 \otimes \varphi_0 + bc \chi_0 \otimes \varphi_1 + 2bd \chi_1 \otimes \varphi_1.
\]
Non-trivial equality is only possible with $c = 0$ and $b = 0$ or with $d = 0$ and $a = 0$. Since the factorization of the state
\[ \Phi = \sum_j c_j \beta_j \otimes \chi_j = \sum_j c_j \beta_j \otimes \tilde{\chi}_j \otimes \psi \]
is a factorization of all $\chi_j$, it is only possible with $\psi \propto \varphi_0$ or $\psi \propto \varphi_1$.

For a single detector $D_k$ of the array, with $k \in I_n$, this has the consequence that no superposition of the pointer states $\varphi_0^{(k)}$ and $\varphi_1^{(k)}$ can be attributed to the final premeasurement state $\Phi$ (11) as a factor of a Schmidt decomposition. The pointer base states $\varphi_0^{(k)}, \varphi_1^{(k)}$ are preferred [26] and the reduced state of each detector $D_k$ is a unique mixture of $\varphi_0^{(k)}$ and $\varphi_1^{(k)}$.

5.3 Conditional Expectations Equal Expectations After Collapse

Other studies [27] have shown, that for an ideal repeatable measurement, fulfilling von Neumann’s conditions, the final premeasurement state gives the same conditional probabilities for the measured system as the collapse of the wave function. This is also the case with the collapse of the premeasurement state after a non-repeatable ideal measurement (4).

To see this for state $\Phi$ (11) let $P_k \in \mathcal{H}_M$ be the indicator observable for detection by detector $D_k$, i.e. the projection operator onto $\tilde{\varphi}_k$. It commutes with all system observables. So, a common probability space is available for all these observables and the conditional probability of any event represented by the indicator observable $F \in \mathcal{B}(\mathcal{H}_S)$ of the system, given $P_k \in \mathcal{B}(\mathcal{H}_M)$, is well defined for all $k \in I_n$

\[ p(F \mid P_k) = \frac{p(F \wedge P_k)}{p(P_k)} = \frac{\langle \Phi, F \otimes P_k \Phi \rangle}{\langle \Phi, 1_S \otimes P_k \Phi \rangle} = \frac{\langle \psi, E_k^\prime \psi \rangle}{\langle \psi, E_k^\prime \psi \rangle} \]

if $\langle \psi, E_k^\prime \psi \rangle > 0$. This is a consequence of Born’s rule. The state of the system after the complete measurement with collapse, given the outcome $a_k$, is according to (4)

\[ \psi_{a_k} = \frac{1}{\sqrt{\langle \psi, E_k^\prime \psi \rangle}} E_k^\prime \psi. \]

For this state the probability of $F$ is

\[ \langle \psi_{a_k}, F \psi_{a_k} \rangle = \frac{\langle \psi, E_k^\prime F E_k^\prime \psi \rangle}{\langle \psi, E_k^\prime \psi \rangle} = p(F \mid P_k) \]

and has the same value as the conditional probability of $F$, given $P_k$, for the state $\Phi$. The same is valid for the conditional expectation value of any observable of the system.

Consequently, considering the Heisenberg picture, the isolated evolution of the system $S$ after the measurement interaction, given $P_k$, is the same as for the collapsed wave function after the measurement, given the outcome $a_k$.

6 Conclusion

The entangled state of the system and the detector array after the ideal premeasurement of an observable has the remarkable property that the number of detectors indicating a detection is exactly one. This is a hint at a definite outcome of the measurement and resembles the collapse, since the conditional expectations, given a detection, are the same as for the
collapsed state, given the corresponding outcome. Furthermore, no superposition state can be attributed to a single detector: the pointer base states and their mixtures are preferred.

Using Schrödinger’s cat illustration, one can say: if in a lossless multiport experiment a particle has different exits for all possible measurement outcomes, each one with an ideal detector connected to a killing machine with its own cat, then exactly one cat will be dead, and all others will be alive after a trial; there is no superposition of dead and alive – even after a pure unitary evolution without collapse.

These properties depend on an ideal measurement process where the detector array fulfills the completeness and orthogonality conditions (7), (6) with the bijection (8). Quantum computing circuits (example 2c) demonstrate the possibility of such a setting. In case of position detectors (example 2b), exactly positioned detection volumes without gap and overlap, filling the available space, are required, which seems only possible in a macroscopic quasi-classical limit, in line with Bohr’s Copenhagen [17] interpretation.

Solely, these properties cannot constitute irreversible facts. That, however, could be accomplished through the decoherence of the final premeasurement state [24].

The results of our study may support interpretations of quantum mechanics without collapse. However, if some of the latter’s observable consequences are deducible from other rules, this will not mean a contradiction to the Copenhagen interpretation. Even, if the collapse postulate was rendered unnecessary, it could persist as useful rule of thumb.

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