A POISSON-POISSON MODEL TO ANALYZE CONGESTION DATA

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Abstract

Congestion in internet network often worsens the performance of the network. An analysis of congestion data plays a significant role in congestion detection and control. To detect and control congestion, analysis of number of packet arrival, packet inter arrival times, throughput, queuing delay, etc. are some important phenomena. Fitting probability distributions to packet arrival, throughput, inter arrival time, etc. are essential in order to control congestion in future, setting appropriate values of parameters beforehand (buffer size, number of packet arrival, inter arrival time, average queuing delay, etc.). Number of packet arrival within a certain time interval is a count random variable that follows a Poisson distribution. Similarly, throughput/number of packet departure within a certain time interval is
also a Poisson random variable but it is correlated with the number of packet arrival. In most of the previous studies, univariate distributions have been used to identify the best fit probability distributions for congestion data. In this study, an attempt is made to use a Poisson-Poisson model for packet arrival and departure (throughput) from the network where correlation between the packet arrival and departure is considered. Using simulation, the performance of models in each situation (small/large sample size, low/heavy congestion) is evaluated. This technique is expected to simplify the analysis of big data stemming from the congestion in TCP/IP network.

1. Introduction

At present, the world cannot survive a single moment without internet. Internet has made the world smaller. The majority of data transfer on the internet relating to the transfer jobs (web pages, audio/video downloads, file transfers, etc.) are being coordinated by TCP/IP (Lassila et al. [10]). TCP is a two-layer program. The higher layer, transmission control protocol, manages the assembling of a message or a file into smaller packets that are transmitted over the internet and received by a TCP layer that reassembles the packets into the original message. The lower layer, internet protocol, handles the address part of each packet so that it gets to the right destination. Each gateway computer on the network checks this address to see where to forward the message. Even though some packets from the same message are routed differently than others, they will be reassembled at the destination.

The performance of internet usage is worsened due to congestion in TCP/IP network. Network congestion occurs when a network node is carrying more data/traffic than it can handle. When the router buffer cannot handle the arriving packets, then the congestion occurs. That is, if rate of arrival and rate of departure of packets to their destination are not the same, then congestion arises. Congestion worsens the network’s performance by increasing packet dropping probability. In addition, it increases the mean waiting time of packets in the queue and the mean queue length. As a result, the amount of packets passing through the buffer of the routers that is throughput will be degraded (Lin and Morris [12]).
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An analysis of congestion data plays a significant role in congestion detection and control. To detect and control congestion, some important phenomena are: number of packet arrival, packet inter arrival time, throughput, queuing delay, etc. To control the congestion in network, many detection algorithms are suggested such as: random early detection (RED) (Floyd and Jacobson [4]), adaptive random early detection (ARED) (Floyd et al. [3]), dynamic stochastic early discovery (DGRED) (Balziki et al. [1]), stochastically adaptive RED (SARED) (Suthaharan [14]), etc. These methods have one goal in common, which is to detect the congestion early to reduce the packet dropping, mean queue length, mean waiting time as well as to improve networks’ performance.

Uses of probability distributions are also seen in several literatures of network performance. Deng [2] constructed an empirical model of WWW document arrivals at access link. He used actual traffic data with over 20000 data points to fit distributions. He applied heavy tailed Weibull and Pareto distributions to model on and off periods. He also used another Weibull distribution to model the inter arrival times of requests coming within the on periods. Deng [2] suggested this empirical model to work as a basis for future planning of access network capacity. Padhye et al. [13] modeled TCP’s congestion avoidance behavior in terms of rounds. The size of the congestion window in a round ($W_i$) is considered as a Markov process where stationarity can be obtained numerically and the number of packets in the last round is assumed to be uniformly distributed between 1 and $W_i$.

Garsva et al. [6] presented results of statistical analysis of network packet inter arrival time distribution in TCP and UDP, most popular transport protocols in academic computer network. Packet inter arrival time distributions were found by dividing network traffic into sections according to its direction and usage trends. Garsva et al. [6] used Kolmogorov-Smirnov test to evaluate the goodness of fit of packet inter-arrival time distribution. Among the distributions (Weibull, Pareto second kind, gamma, exponential and lognormal), it appears that Pareto second kind is fitted well in both TCP and UDP protocols. Garetto and Towsley [5] described an analytical approach for estimating the queuing delay distribution on an internet link.
carrying realistic TCP traffic, such as that produced by a large number of finite-size connections transferring files whose sizes are taken from a long-tail distribution. Their modelling approach contained two components: the first one was a stochastic model of TCP where amount of data transferred by a TCP connection is assumed to follow a geometric distribution. The second component was concerned in obtaining a queue length distribution in which batch size is assumed to have a beta distribution. The arrival of batches to a connection is assumed to be Poisson arrival in Garetto and Towsley [5]. Karp et al. [9] made an attempt to solve several optimization problems regarding internet congestion control. They derived several algorithms for optimization under static and dynamic cases using different cost functions. In their study, use of uniform distribution is made to model maximum number of packets that can transmit without experiencing any packet loss. Hong et al. [7] proposed a queuing model which describes a packet encapsulation and aggregation process assuming Poisson arrivals phase-type service time distributions for the transmitted packets. The proposed model was useful in packet aggregation which would result in increased throughput and would minimize the average total delay of a packet for various system loads.

So far the probability models applied to network traffic data are all univariate probability distributions or queuing models looking at different aspects of network congestion individually. If one looks at two important variables of TCP/IP network data, packet arrival and packet departure (throughput), they are not independent. Rather packet departure (passing of packets throughput) depends on the number of packets arriving on a gateway where the two variables are correlated. So if this correlation is taken into account while modeling these two important characteristics, the modeling of network congestion data will be more effective. The present study applies the bivariate Poisson model of Islam and Chowdhury [8] to model network data. The estimation and tests using these models are shown in this study applying simulation.

Following this section, the study is organized as follows. The methodology using models, likelihood, and test statistics are described in Section 2, an application of the methods using simulated data of network
congestion is described in Section 3, Section 4 consists of results and discussion followed by conclusion in Section 5.

2. Methods

2.1. Bivariate Poisson model for an analysis of congestion data

Let us assume a network where number of packets arriving per time point (e.g., second) within a given time interval \([t_0, t]\) (e.g., 0 to 10 seconds; 0 to 100 seconds, etc.) in a gateway with a specific rate \(\lambda_1\). We may denote the number of packets arriving in a gateway in a specific time point during a time interval by \(Y_1\). Then \(Y_1\) has a Poisson distribution with mass function

\[
g_1(y_1) = \frac{e^{-\lambda_1} \lambda_1^{y_1}}{y_1!}, \quad y_1 = 0, 1, \ldots
\]

Again let us assume among the packets arriving in a gateway, the number of packets those reach to their destination constitute the amount of throughput of the specific network within the given time interval. Let the rate of packet departure in any time point within the interval be \(\lambda_2\) and it also has a Poisson distribution. Also, it is evident that the number of packets departed successfully within the interval (throughput) may be denoted by \(Y_2\) depending on the number of packets arrived in the gateway within each time point of the given interval \([t_0, t]\). So the joint distribution of the number of packet arrival and the number of packet departure can be shown as follows (Leiter and Hamdan [11]):

\[
g(y_1, y_2) = g(y_2 | y_1) \cdot g(y_1) = \frac{e^{-\lambda_1} \lambda_1^{y_1} e^{-\lambda_2 y_1} (\lambda_2 y_1)^{y_2}}{(y_1! y_2!)} \cdot y_2 = 0, 1, \ldots, \quad (1)
\]

where \(y_2\)'s are assumed to be mutually independent. The number of packets departed, \(Y_2\), out of \(Y_1 = y_1\) arrivals, in any time interval, is Poisson with parameter \(\lambda_2 y_1\). In a time interval, the possible number of packets to be
departed, $Y_2$, can be shown as the sum of departed packets corresponding to each of $1, 2, ..., y_1$ possible arrivals and the variable $Y_2$ is defined as $Y_2 = Y_{21} + Y_{22} + \cdots + Y_{2y_1}$. Then the conditional probability of the total number of packets departed as throughput among the $y_1$ arrivals occurring in a time interval denoted by $P(Y_2 = y_2 | Y_1 = y_1)$ which is shown as Poisson with parameter $\lambda_{2y_1}$. Then it can be shown that

$$g(y_2 | y_1) = \frac{e^{-\lambda_{2y_1}}(\lambda_{2y_1})^{y_2}}{y_2!}, \quad y_2 = 0, 1, ..., $$

where $Y_{2i}$ is a random variable with the number of packets departed (becoming throughput) resulting from $i$th arrival, and suppose it has a Poisson distribution with parameter $\lambda_{2i}$; that is,

$$g_2(y_{2k}) = \frac{e^{-\lambda^2_{2i}y_{2k}}}{y_{2k}!}, \quad y_{2k} = 0, 1, ...$$

For the above Poisson-Poisson model (1), it can be shown that

$$E(Y_1) = \mu_1 = \lambda_1 \quad \text{and} \quad E(Y_2) = \mu_2 = \lambda_1\lambda_2.$$

### 2.2. Test of goodness of fit and overdispersion

The proposed test for goodness of fit and the test of overdispersion for bivariate Poisson-Poisson model (Islam and Chowdhury [8]) can also be applied in the present study to check the goodness of fit of this model applied to network congestion data and to detect overdispersion or underdispersion in the data.

The suggested test for goodness of fit is shown below, where the test statistic is asymptotically distributed as Chi-square distribution ($x^2_g$), where $g$ is the number of groups of observed values, $y_1, ..., y_g$: 
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\[ T_1 = \sum_{y_1} \left( \frac{y_1 - \hat{\mu}_{y_1}}{\bar{y}_2 | y_1 - \hat{\mu}_{y_2 | y_1}} \right) \left( \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{1i} / n_{y_1} \right) \left( \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{2i} y_1 / n_{y_1} \right)^{-1} \]

\[ \cdot \left( \frac{y_1 - \hat{\mu}_{y_1}}{\bar{y}_2 | y_1 - \hat{\mu}_{y_2 | y_1}} \right) \]

where \( \hat{\mu}_{y_1} = \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{1i} / n_{y_1} \) and \( \hat{\mu}_{y_2 | y_1} = \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{2i} y_1 / n_{y_1} \).

Islam and Chowdhury [8] showed simple tests for overdispersion to be carried out individually for variables \( Y_1 \) and \( Y_2 \). So, in the present study, simple tests for detecting overdispersion are carried out for both the number of packet arrival \( (Y_1) \) and number of packet departure or throughput \( (Y_2) \) within a specific time interval, \([t_0, t]\). For the simple tests, the hypotheses used are:

\[ H_0 : \mu_k = \mu_{k0} \text{ and } H_1 : \mu_k \neq \mu_{k0} \]

Under Poisson assumption, we can assume that \( \mu_k = \mu_{k0} \), where \( \mu_{k0} = \hat{V}_{qk} \), where \( \hat{\mu}_k \) and \( \hat{V}_{qk} \) are the estimates for \( \mu_k \) (population mean) and \( V_{qk} \) (population variance), respectively, \( k = 1, 2 \). For large sample, the test statistic is,

\[ Z_k = \frac{\hat{\mu}_k - \mu_{k0}}{\sqrt{\frac{\mu_{k0}}{n}}} \]

which is asymptotically distributed as \( N(0, 1) \). Rejection of null hypothesis indicates that mean-variance equality may not hold, that is overdispersion may be present in the data set.

The test for underdispersion or overdispersion has been generalized for the bivariate Poisson-Poisson model by Islam and Chowdhury [8] where marginal distribution of \( Y_1 \) (number of packet arrival) and conditional
distribution of $Y_2 | y_1$ (number of packet departure given the number of packet arrival) follow Poisson distribution with parameters $\lambda_1$ and $\lambda_2 y_1$, respectively. Based on the mean-variance equality in both the marginal model for $Y_1$ and the conditional model for $Y_2 | y_1$ the proposed test for underdispersion or overdispersion is shown below where the test statistic $T_2$ is distributed asymptotically as $x^2_{2g}$, where $g$ is the number of groups of observed values, $y_1, ..., y_g$:

$$T_2 = \sum_{y_1} \left( \frac{y_1 - \hat{\mu}_{y_10}}{\hat{\sigma}_{y_1}} \right) \left( \frac{\hat{\phi}_1 \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{1i}/n_{y_1}}{0} \right)^{-1} \left( \frac{\hat{\phi}_2 \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{2i} y_1 / n_{y_1}}{0} \right) \cdot \left( \frac{y_1 - \hat{\mu}_{y_10}}{\hat{\sigma}_{y_1}} \right),$$

(3)

where $\mu_{y_10} = \hat{\phi}_1 \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{1i}/n_{y_1}$ and $\mu_{y_2 | y_1} = \hat{\phi}_2 \sum_{i=1}^{n_{y_1}} \hat{\lambda}_{2i} y_1 / n_{y_1}$, $\hat{\phi}_k = \frac{1}{n - p}$, and $\sum_{i=1}^{n_{y_1}} \left( \frac{y_{ki} - \hat{\mu}_{kl}}{V(\hat{\mu}_{kl})} \right)^2$, $k = 1, 2$. Also, $V(\hat{\mu}_{kl}) = \hat{\mu}_{kl}$.

3. Application

For fitting of bivariate Poisson model and for tests of goodness of fit and overdispersion, data are generated using R programming language. By taking different combinations of $\lambda_1$ (rate of packet arrival) and $\lambda_2$ (rate of packet departure or throughput), considering different values of correlation coefficient ($r = 0.1, 0.5, 0.7, 0.9$) between $Y_1$ and $Y_2$, sample values of $Y_1$ and $Y_2$ are generated for small sample size ($n = 10$) and large sample size ($n = 100$) (Yahav and Shmueli [15]). Here sample sizes are the number of discrete time points within a specific interval. The simulations are carried out under two scenarios: heavy congestion and low congestion. Heavy
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Congestion refers to the network congestion scenario where the difference between the rate of arrival and rate of packet departure is high, resulting in increased number of packets waiting in the queue to be delivered and eventually many of the packets get dropped (fail to reach destination). On the other hand, low congestion refers to the network congestion scenario where the difference between the rate of arrival and departure is not that high which results in a fewer number of packets waiting in the queue to be delivered and also the packet dropping probability becomes lower.

Using the simulated data sets under each congestion scenario, for large and small sample sizes, the rates ($\lambda_1$ and $\lambda_2$) are estimated from the fitted model; bias and MSE of the estimated values are computed to see the appropriateness of the estimation techniques suggested by the employed model of Islam and Chowdhury [8]. The goodness of fit of the proposed model is tested using the test proposed in the paper and also the tests for overdispersion or underdispersion are carried out for each of the samples. The whole process of data generation and computation under each scenario and sample size is repeated 1000 times and the mean of the results obtained from the 1000 repetitions are presented and discussed in the present study.

4. Results and Discussion

The results of some simulation are shown in Table 1 to Table 4. In Table 1, the results of simulation are shown where correlation between packet arrival and departure is taken to be 0.1. Under this low correlation, data are generated for heavy congestion and low congestion for both small ($n = 10$) and large ($n = 100$) sample sizes. It is observed that the values of bias for estimated rates of packet arrival and departure are small for small samples and the values are reduced more for large samples (under heavy congestion and low congestion), e.g., under heavy congestion, considering rate of packet arrival as 15 and rate of packet departure as 5, the biases are found to be $-0.4$ and $-1.2$, respectively, for small samples, but for large sample, the values are: $-0.13$ and $0.13$. Also, there is substantial decrease in MSE of the estimated rates of packet arrival and departure ($\lambda_1$ and $\lambda_2$) for the large
samples under both congestion scenarios (for example, the MSE under heavy congestion, considering rate of packet arrival as 10 and rate of packet departure as 4 are 1.060291 and 1.622901 for small sample and 0.10985926 and 0.05150278 for large sample). Similar results for bias and MSE are obtained for other combination of rates ($\lambda_1$ and $\lambda_2$) for small and large samples under heavy and low congestions. These observations indicate that the estimation technique using the proposed model performs as expected. The $p$ values of goodness of fit test are large for all large and small sample cases under heavy and low congestions (Table 1), which indicates that the proposed model is a good fit to the congestion data. The bivariate test for overdispersion shows in every case, $p$ value is large, that means mean variance equality holds for bivariate Poisson distribution for packet arrival and departure. The individual test of overdispersion is also carried out and $p$ values are also shown for the tests in Table 1. In simple tests also, the number of packet arrival and departure does not contain overdispersion. The result is so because we simulated the data from bivariate Poisson distribution, so it is expected that there will be no overdispersion or underdispersion in this simulated dataset. If real life data is used, then the tests of overdispersion might be significant. In those cases, adjustments should be made to the $p$ values of the coefficients of GLM applied (Islam and Chowdhury [8]).

The results of simulation taking correlation values as 0.5, 0.7 and 0.9 between packet arrival and departure are shown in Tables 2, 3 and 4, respectively. Similar results for bias and MSE are observed in Tables 2, 3 and 4. The $p$ values for goodness of fit test exhibit similar behavior, so as the tests for overdispersion. This means, if the correlation between packet departure and packet arrival is changed to 0.5, 0.7 or 0.9 from 0.1, the proposed model performs similarly for both low and heavy congestion scenarios. This leads to the conclusion that the proposed model can perform well in modelling the packet arrival and departure (throughput) in different networks where there could exist different values for correlation between the two variables. It is also evident from the results (Tables 1, 2, 3 and 4) that the model performs well for both large and small sample sizes under both
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heavy and low congestion scenarios as suggested by the small $p$ values in all cases considered in simulations. As in real life, the network congestion data problems are often big data problems. It is worth noting that the model proposed by Islam and Chowdhury [8] can be used satisfactorily for analysis of big data arising from congestion with very large sample sizes. The proposed model is expected to simplify the analysis of network congestion data.

Using real life data of packet arrival and departure in a particular network, one can estimate the rate of arrival and rate of throughput for a given network using the proposed model. The estimation made under the proposed model is quite good in terms of bias and MSE. The bias and MSE decrease with the increase in sample size. So estimations of the rates using the model will help in making congestion control policy more effective for specific networks. In most congestion controlling algorithms, the authors set some threshold or input parameters arbitrarily (Floyd and Jacobson [4] and Suthaharan [14]). But, using this model, the parameters needed can be estimated earlier, then estimated values may be used in the control algorithm, which may provide a more effective control network congestion. Thus, the proposed model is expected to strengthen the development of more effective congestion control algorithm.

To summarize, the findings of fitting the proposed model with simulated data (under low/heavy congestion scenario, for large and small sample sizes) show strong indication that the proposed model is well fitted to the network congestion data. We may conclude that the bivariate Poisson-Poisson regression model proposed by Islam and Chowdhury [8] can be used for analyzing correlated outcomes in network congestion data very well. It has also been shown that the tests for goodness of fit and overdispersion perform very well.

5. Conclusion

The present study is an attempt to apply the newly developed model of Islam and Chowdhury [8] in analysis of count data arising from network
congestion. This is the first ever study where the dependency between number of packet arrival and throughput within a given time interval is taken into account. The application is made using simulated data to validate the use of proposed bivariate Poisson model (Islam and Chowdhury [8]) in analyzing packet arrival and packet departure considering the dependence between the two variables. This application provided new insights of modeling network data TCP/IP network. The proposed model is expected to simplify the analysis of big data arising from network data by modelling two important phenomena as packet arrival and packet departure within a given time interval in a network.

Acknowledgement

This work was supported by the Higher Education Quality Enhancement Project (HEQEP) sub-project (CP-3293) of the Department of Applied Statistics, East West University funded by the World Bank and implemented by University Grants Commission of Bangladesh.

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Table 1. Result of simulation taking correlation \((y_1, y_2) = 0.1\) (number of repetitions = 1000, mean value for each output is taken)

| Congestion type | Sample size | λ̂ \(\hat{\lambda}\) | Bias \(\hat{\lambda}\) | MSE \(\hat{\lambda}\) | \(T_1\) | \(p\)-value for \(T_1\) | \(T_2\) | \(p\)-value for \(T_2\) | \(Z_i\) | \(p\)-value for \(Z_i\) |
|-----------------|-------------|---------------------|-----------------------|-------------------|--------|-----------------------------|--------|-----------------------------|--------|-----------------------------|
| Heavy congestion | \(n = 10\)  | 14.6               | -0.4                  | 1.694641          | 13.7352 | 0.561551                    | 12.06788 | 0.6452543                    | 0.08224568 | 0.11967 |
|                 |             | 3.8                | -1.2                  | 1.963330          |         |                             |         |                             | 0.03937126 | 0.18703 |
|                 |             | 9.8                | -0.2                  | 1.060291          | 13.02152 | 0.5493953                   | 11.41045 | 0.6292591                    | 0.09096617 | 0.13945 |
|                 |             | 2.9                | -0.1                  | 1.622901          | 12.67163 | 0.5437807                   | 11.05979 | 0.6195938                    | 0.04799490 | 0.21745 |
|                 |             | 8.7                | -0.2                  | 0.8598068         | 11.97163 | 0.5437807                   | 11.05979 | 0.6195938                    | 0.04799490 | 0.21745 |
|                 |             | 19.8               | -0.2                  | 2.082933          | 14.14843 | 0.5657565                   | 12.47163 | 0.6552187                    | 0.12622739 | 0.09369 |
|                 |             | 8.4                | -1.6                  | 3.604549          | 11.97163 | 0.5437807                   | 11.05979 | 0.6195938                    | 0.04799490 | 0.21745 |
|                 | \(n = 100\) | 14.87              | 0.13                  | 0.16082420        | 52.6922  | 0.1146495                   | 51.71866 | 0.09576928                   | 0.0581393 | 0.11802 |
|                 |             | 5.13               | 0.13                  | 0.06361191        |         |                             |         |                             | 0.0926571 | 0.18488 |
|                 |             | 9.88               | -0.12                 | 0.10985926        | 46.81692 | 0.1025514                   | 45.90287 | 0.08568702                   | -0.0240896 | 0.13477 |
|                 |             | 4.12               | 0.12                  | 0.05150279        | 43.69981 | 0.0994432                   | 42.84441 | 0.08292702                   | -0.0297161 | 0.28141 |
|                 |             | 7.96               | -0.10                 | 0.08620958        | 57.71798 | 0.1204093                   | 56.48851 | 0.09873427                   | -0.0650708 | 0.10149 |
|                 |             | 2.03               | 0.03                  | 0.01967761        | 17.32163 | 0.1146495                   | 16.41866 | 0.09576928                   | -0.0240896 | 0.13477 |
|                 |             | 19.84              | -0.16                 | 0.2185614         | 11.97163 | 0.5437807                   | 11.05979 | 0.6195938                    | 0.04799490 | 0.21745 |
|                 |             | 10.14              | 0.14                  | 0.1129660         | 11.97163 | 0.5437807                   | 11.05979 | 0.6195938                    | 0.04799490 | 0.21745 |
| Low congestion  | \(n = 10\)  | 15.1               | 0.1                   | 1.528726          | 13.20044 | 0.5855379                   | 11.71593 | 0.671688                     | 0.1041118  | 0.11736 |
|                 |             | 11.0               | 0.2                   | 5.356537          |         |                             |         |                             | 0.1602178  | 0.11081 |
|                 |             | 10.1               | 0.1                   | 1.008891          | 12.34442 | 0.5852683                   | 10.92347 | 0.6071209                    | 0.08601472 | 0.14239 |
|                 |             | 9.5                | 0.1                   | 0.8272166         | 12.01838 | 0.5809987                   | 10.56243 | 0.6622211                    | 0.03815898 | 0.16833 |
|                 |             | 5.5                | -0.1                  | 0.7510737         | 13.75957 | 0.5844651                   | 12.1764  | 0.6784057                    | 0.1307715  | 0.10691 |
|                 |             | 20.1               | 0.1                   | 2.036246          | 17.51073 | 0.5844651                   | 12.1764  | 0.6784057                    | 0.1488281  | 0.09855 |
|                 | \(n = 100\) | 14.73              | 0.27                  | 0.2202578         | 45.00255 | 0.23863                     | 44.33852 | 0.2238594                    | -0.0668608 | 0.10769 |
|                 |             | 13.26              | 0.26                  | 0.1903295         |         |                             |         |                             | -0.085386  | 0.11178 |
|                 |             | 9.9                | 0.1                   | 0.10769200        | 12.34442 | 0.5852683                   | 10.92347 | 0.6671209                    | 0.08601472 | 0.14239 |
|                 |             | 8.15               | 0.15                  | 0.09798984        | 12.01838 | 0.5809987                   | 10.56243 | 0.6622211                    | 0.03815898 | 0.16833 |
|                 |             | 7.18               | 0.18                  | 0.09744611        | 13.75957 | 0.5844651                   | 12.1764  | 0.6784057                    | 0.1307715  | 0.10691 |
|                 |             | 19.76              | -0.24                 | 0.2505339         | 17.32163 | 0.5844651                   | 12.1764  | 0.6784057                    | 0.1488281  | 0.09855 |
A Poisson-Poisson Model to Analyze Congestion Data

Table 2. Result of simulation taking correlation \((y_1, y_2) = 0.5\) (number of repetitions = 1000, mean value for each output is taken)

| Congestion type | Sample size | \(x_i\) | \(\hat{\lambda}_i\) | Bias \((\hat{\lambda}_i)\) | MSE | \(T_1\) | \(p\)-value for \(T_1\) | \(T_2\) | \(p\)-value for \(T_2\) | \(Z_i\) | \(p\)-value for \(Z_i\) |
|-----------------|-------------|---------|---------------------|------------------------|-----|--------|------------------------|--------|------------------------|--------|------------------------|
| Heavy congestion | 10          | 15      | 15.1                | 0.1                    | 1.525581 | 13.25127 | 0.582939 | 11.73796 | 0.6705488 | 0.10871441 | 0.11633 |
|                 |             | 5       | 3.7                 | -1.3                   | 2.208217 |          |           |           |           |           | 0.17574 |
|                 |             | 10      | 10.1                | 0.1                    | 1.009911 | 12.36883 | 0.5846338 | 10.93264 | 0.6660564 | 0.08681773 | 0.14317 |
|                 |             | 4       | 2.8                 | -1.2                   | 1.855153 |          |           |           |           |           | 0.20948 |
|                 |             | 8       | 8.1                 | 0.1                    | 0.8268759 | 12.10301 | 0.5775196 | 10.60021 | 0.6595669 | 0.03394564 | 0.16813 |
|                 |             | 2       | 1.2                 | -0.8                   | 0.8425709 |          |           |           |           |           | 0.29802 |
|                 |             | 20      | 20.1                | 0.1                    | 2.034128 | 13.7499 | 0.5852682 | 12.1611 | 0.6789957 | 0.13052893 | 0.10676 |
|                 |             | 10      | 8.3                 | -1.7                   | 3.924910 |          |           |           |           |           | 0.12066 |
| Low congestion  | 100         | 15      | 14.73               | 0.27                   | 0.2201470 | 45.0297 | 0.237625 | 44.36752 | 0.2229847 | -0.0120588 | 0.10771 |
|                 |             | 5       | 5.15                | 0.15                   | 0.06974227 |          |           |           |           |           | 0.18256 |
|                 |             | 10      | 9.9                 | -0.1                   | 0.10766591 | 39.35219 | 0.2203368 | 38.74644 | 0.2060576 | 0.00303438 | 0.13711 |
|                 |             | 4       | 4.1                 | 0.1                    | 0.04772993 |          |           |           |           |           | 0.21961 |
|                 |             | 8       | 7.85                | -0.15                  | 0.10075770 | 36.82378 | 0.2071717 | 36.24271 | 0.1934943 | 0.00626616 | 0.14777 |
|                 |             | 2       | 2.11                | 0.11                   | 0.03076458 |          |           |           |           |           | 0.29432 |
|                 |             | 20      | 19.76               | -0.24                  | 0.2560235 | 49.44603 | 0.2533059 | 48.71052 | 0.2369956 | 0.01571761 | 0.10467 |
|                 |             | 10      | 10.22               | 0.22                   | 0.1429157 |          |           |           |           |           | 0.13237 |
|                 | 100         | 15      | 14.6                | -0.4                   | 1.693527 | 13.71556 | 0.5617646 | 12.07428 | 0.6456624 | 0.08783505 | 0.11996 |
|                 |             | 11.1    | -1.9                | 4.971691               |          |           |           |           |           | 0.14922697 | 0.10995 |
|                 |             | 10      | 9.8                 | -0.2                   | 1.006013 | 13.00038 | 0.5498576 | 11.42012 | 0.6278345 | 0.09712973 | 0.13908 |
|                 |             | 6.4     | -1.6                | 3.395563               |          |           |           |           |           | 0.08357407 | 0.15391 |
|                 |             | 8       | 7.8                 | -0.2                   | 0.8586925 | 12.62826 | 0.5455028 | 11.06024 | 0.6194829 | 0.06541200 | 0.15691 |
|                 |             | 7       | 5.7                 | -1.3                   | 2.4156548 |          |           |           |           | 0.09506805 | 0.15805 |
|                 |             | 20      | 19.8                | -0.2                   | 2.078087 | 14.15732 | 0.5667893 | 12.49559 | 0.654903 | 0.1191602 | 0.09477 |
|                 |             | 17      | 15.0                | -2.0                   | 5.776794 |          |           |           |           | 0.1428620 | 0.10007 |
|                 |             | 15      | 14.87               | 0.15                   | 0.1610611 | 52.51148 | 0.1145316 | 51.57759 | 0.09618123 | -0.0508810 | 0.11806 |
|                 |             | 13.26   | 0.26                | 0.1903850             |          |           |           |           |           | -0.1375371 | 0.11085 |
|                 |             | 10      | 9.88                | 0.12                   | 0.1097407 | 46.66952 | 0.102183 | 45.77923 | 0.08542215 | -0.0235271 | 0.13636 |
|                 |             | 8       | 8.19                | 0.19                   | 0.1110467 |          |           |           |           |           | 0.1466777 | 0.14784 |
|                 |             | 7       | 7.89                | 0.11                   | 0.08868631 | 43.40647 | 0.0988514 | 42.62078 | 0.08302902 | -0.0047596 | 0.16712 |
|                 |             | 7       | 7.12                | 0.12                   | 0.07931130 |          |           |           |           | -0.0922239 | 0.15605 |
|                 |             | 20      | 19.84               | 0.16                   | 0.2185305 | 57.70549 | 0.1205446 | 56.62542 | 0.09896372 | -0.0628897 | 0.10034 |
|                 |             | 17      | 17.25               | 0.25                   | 0.2213135 |          |           |           |           | -0.1888725 | 0.10647 |
### Table 3. Result of simulation taking correlation $(\gamma_1, \gamma_2) = 0.7$ (number of repetitions $= 1000$, mean value for each output is taken)

| Congestion type | Sample size | $|\hat{\lambda}|$ | $|\hat{\lambda}|$ | Bias $(\hat{\lambda}, \hat{\lambda})$ | MSE $(\hat{\lambda}, \hat{\lambda})$ | $T_1$ | $p$-value for $T_1$ | $T_2$ | $p$-value for $T_2$ | $Z_1$ | $p$-value for $Z_1$ | $Z_2$ | $p$-value for $Z_2$ |
|-----------------|-------------|----------------|----------------|--------------------------------|-------------------------------|-------|-------------------|-------|-------------------|-------|-------------------|-------|-------------------|
| **Heavy congestion** | $n = 10$ | 15 | 14.4 | 1.900575 | 14.14459 | 0.5394574 | 12.43193 | 0.617145 | 0.0967587 | 0.0350773 | 0.10098 | 0.19045 | 0.07751 |
| | | 5 | 3.7 | 2.208576 | | | | | | | | | | |
| | 10 | 9.5 | 2.128282 | 13.60333 | 11.86759 | 0.5961326 | 0.0548877 | 0.0357208 | 0.13466 | 0.22648 | 0.14666 | 0.1078 |
| | 4 | 3.0 | 1.416787 | | | | | | | | | | |
| | 8 | 7.5 | 1.0728967 | 13.10979 | 1.41446 | 0.586233 | 0.0812178 | 0.02003 | 0.15148 | 0.30364 | 0.11623 | 0.07894 |
| | 2 | 1.2 | 1.8439117 | | | | | | | | | | |
| | 20 | 19.4 | 2.430595 | 14.5385 | 12.79725 | 0.6273104 | 0.1372136 | 0.08316 | 0.09585 | 0.13512 | 0.11693 | |
| | 10 | 8.4 | 3.621548 | | | | | | | | | | |
| | 100 | 10 | 15.1 | 0.1605797 | 60.84514 | 0.4612071 | 59.58736 | 0.3152246 | -0.06319 | 0.11380 | -0.13281 | 0.18333 | |
| | | 5 | 5.1 | 0.0564940 | | | | | | | | | | |
| | 10 | 9.9 | 0.0985322 | 54.17574 | 0.4414211 | 53.03326 | 0.364389 | -0.03960 | 0.13749 | -0.10986 | 0.18883 | |
| | 4 | 4.15 | 0.0601443 | | | | | | | | | | |
| | 8 | 7.9 | 0.0832962 | 50.51866 | 0.4725635 | 49.3512 | 0.3454972 | -0.08300 | 0.15712 | -0.05262 | 0.28200 | |
| | 2 | 2.02 | 0.0187624 | | | | | | | | | | |
| | 20 | 19.9 | 0.194501 | 65.53943 | 0.516668 | 64.29379 | 0.3484586 | -0.06401 | 0.09402 | -0.14118 | 0.13161 | |
| | 10 | 10.19 | 0.1290662 | | | | | | | | | | |
| | 100 | 17 | 17.2 | 0.1600579 | 60.69537 | 0.4570256 | 59.51341 | 0.3115006 | -0.0697724 | 0.11136 | -0.18485 | 0.11643 | |
| | | 13 | 13.19 | 0.1564848 | | | | | | | | | | |
| | 10 | 9.94 | 0.0984440 | 54.00638 | 0.437062 | 52.93369 | 0.31457 | -0.03840 | 0.13560 | -0.10104 | 0.13438 | |
| | 8 | 8.19 | 0.1106622 | | | | | | | | | | |
| | 8 | 7.9 | 0.0821553 | 50.06664 | 0.4484626 | 48.98163 | 0.3303733 | -0.08936 | 0.16001 | -0.10454 | 0.15529 | |
| | 7 | 7.11 | 0.0783045 | | | | | | | | | | |
| | 20 | 19.9 | 0.1988194 | 30.0875 | 0.354785 | 15.14952 | 0.48574 | 0.97634 | 0.13429 | 0.81225 | 0.17368 | |
### Table 4. Result of simulation taking correlation \( (y_1, y_2) = 0.9 \) (number of repetitions = 1000, mean value for each output is taken)

| Congestion type | Sample size | \( \hat{i} \) | \( \hat{j} \) | Bias \( \hat{i} / \hat{j} \) | MSE | \( T_1 \) | \( T_2 \) | \( Z_1 \) | \( Z_2 \) |
|-----------------|-------------|--------------|--------------|-----------------|------|--------|--------|--------|--------|
| Heavy congestion | \( n = 10 \) | 15 | 14.1 | 0.9 | 2.360710 | 14.90278 | 0.5072526 | 13.08821 | 0.0.1114139 | 0.10572 | 0.18792 |
|                 |             | 13 | 3.9 | -1.1 | 1.733602 |            |              |        |          |        |        |
|                 |             | 10 | 9.2 | -0.8 | 1.673316 | 14.32275 | 0.4906741 | 12.55386 | 0.5388246 | 0.11418728 | 0.21449 |
|                 |             | 8  | 3.2 | -0.8 | 1.067277 |            |              |        |          |        |        |
|                 |             | 7  | 7.3 | -0.7 | 0.847987 | 13.95227 | 0.4799417 | 12.08981 | 0.534981 | 0.0336390 | 0.14197 |
|                 |             | 20 | 19.0 | -1.0 | 3.080629 | 15.19984 | 0.5208226 | 13.44499 | 0.585321 | 0.15170322 | 0.08410 |
|                 |             | 17 | 8.7 | -1.3 | 2.746668 |            |              |        |          |        | 0.13983 |
|                 | \( n = 100 \) | 15 | 14.93 | -0.07 | 0.147984 | 71.55171 | 0.118850 | 70.06694 | 0.0540356 | 0.10497 | 0.17880 |
|                 |             | 13 | 5.10 | 0.10 | 0.056198 |            |              |        |          |        |        |
|                 |             | 10 | 9.93 | -0.07 | 0.099686 | 63.78984 | 0.124403 | 62.4258 | 0.0665659 | 0.14386 | 0.19690 |
|                 |             | 8  | 4.05 | 0.05 | 0.039843 |            |              |        |          |        |        |
|                 |             | 7  | 7.98 | -0.02 | 0.076374 | 59.63214 | 0.130854 | 58.2463 | 0.0748594 | 0.15139 | 0.29104 |
|                 |             | 20 | 19.92 | 0.08 | 0.197811 | 76.48961 | 0.13777 | 74.94764 | 0.0615742 | 0.09527 | 0.12670 |
|                 |             | 17 | 8.7 | -1.3 | 2.746668 |            |              |        |          |        |        |
| Low congestion  | \( n = 10 \) | 15 | 14.1 | -0.9 | 2.360284 | 14.80971 | 0.5082268 | 13.06527 | 0.5656887 | 0.1194439 | 0.10290 |
|                 |             | 13 | 11.7 | -1.7 | 4.246604 |            |              |        |          |        | 0.12547 |
|                 |             | 10 | 9.2 | -0.8 | 1.675060 | 14.36784 | 0.4882483 | 12.57262 | 0.5376373 | 0.10691934 | 0.11273 |
|                 |             | 8  | 6.7 | -1.3 | 2.522445 |            |              |        |          |        | 0.15032 |
|                 |             | 7  | 7.3 | -0.7 | 1.314839 | 14.03856 | 0.4752713 | 12.26102 | 0.5204875 | 0.04763608 | 0.14357 |
|                 |             | 20 | 19.0 | -1.0 | 3.078097 | 15.16489 | 0.5222415 | 13.41159 | 0.5875507 | 0.1305752 | 0.08311 |
|                 |             | 17 | 5.2 | -1.8 | 5.040128 |            |              |        |          |        | 0.10943 |
|                 | \( n = 100 \) | 15 | 14.93 | -0.07 | 0.147894 | 71.31807 | 0.122159 | 69.90174 | 0.0569546 | 0.10519 | 0.11137 |
|                 |             | 13 | 13.13 | 0.13 | 0.137839 |            |              |        |          |        |        |
|                 |             | 10 | 9.93 | -0.07 | 0.098966 | 63.48867 | 0.120764 | 62.18414 | 0.0656551 | 0.14021 | 0.13875 |
|                 |             | 8  | 8.09 | 0.09 | 0.083443 |            |              |        |          |        |        |
|                 |             | 7  | 8.02 | 0.02 | 0.076619 | 58.99188 | 0.123111 | 57.82527 | 0.0720977 | 0.0589516 | 0.15307 |
|                 |             | 20 | 19.92 | 0.08 | 0.197871 | 76.43993 | 0.137528 | 74.91629 | 0.0614579 | 0.1365174 | 0.09629 |
|                 |             | 17 | 17.16 | 0.16 | 0.183845 |            |              |        |          |        | 0.09814 |