Inertial force, Hawking Temperature and Quantum Statistics

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To explore the mechanism for the entropic force proposal in Entropic Gravity, we propose a specific thermodynamic process for states thermalized in local Hawking Temperature. We find when Casini’s version of the Bekenstein bound is saturated, the thermodynamic force derived in the entanglement first law matches the local inertial force for the Schwarzschild solution, except for a negligible statistics-dependent factor. We argue the gravity viewed by static observers may have observable effects emerged from quantum statistics. The successful detailed calculation in this simple model inspires and is in support of the further development in our following research arXiv:2004.14059.

I. INTRODUCTION

Three different aspects are combined to consider the inertial force in gravitational field as a thermodynamic dual effect. They are Entropic Gravity [1,2], Hawking Temperature [3], and research [4,5] on the Bekenstein bound [7,8].

Entropic Gravity is to understand the connection between gravity and thermodynamics involving entropy change. Several major approaches and proposals have been done [9,12], while [13] examined the major assumptions for the origin of entropy in [9,10].

Verlinde’s theory [1,2] suggested the gravity as a macroscopic entropic force, emphasizing the existence of an entropy gradient in spacetime responsible for the gravity as the inertial force. Thus the information structure of spacetime influences the gravitational behavior. The open questions to answer are what causes the entropy change and can we form a more specific mechanism to generic situations?

We find clues from historical Hawking Temperature and Casini’s work on the Bekenstein bound. Noticing Hawking Temperature is position-dependent for the thermal situation seen by different local static observers in the spacetime of asymptotic flat Schwarzschild black holes, we use it to replace the role of Unruh Temperature in common approaches of Entropic Gravity theories. Thus the resulting temperature gradient can vary the entropy and energy of thermal states.

Meanwhile, Casini [6] proved a precise version of the Bekenstein bound in quantum field theory which avoided species problem from an argument of non-negativity of relative entropy based on formal research [4,5]. The saturation of the bound was then regarded as a condition of holography [14]. These developments inspired us to relate the subtracted entanglement entropy and energy and bring in the first law of entanglement [15] to the improvement of the foundation of Verlinde’s theory.

In this paper, we find the existence of the external force measured by accelerating observers as the thermodynamic force turns the situation to be entropic. Thus the entropic mechanism is compatible with Susskind’s complexity approach [16-18] to explain the gravitational attraction tendency because they are viewed in different processes. We will show the thermodynamic force is produced exactly from the difference between the modular Hamiltonian $K_1$ of an excited particle state and $K_0$ of the vacuum state

$$F_\mu = T \nabla_\mu \langle K - K_1 \rangle_1.$$  \hfill (1)

where $T = \frac{k}{3\pi\nu(r)}$ represents the local temperature with the surface gravity $k$ and the redshift factor $V(r)$.

When Casini’s version of the Bekenstein bound $\Delta S \leq \frac{\Delta(H)}{T} = S_\infty$ is saturated, the thermodynamic expression for $F_\mu$ to compare with the local inertial force $F_{\text{inertial}} = -m\mu$ is

$$F_\mu = S_\infty \nabla_\mu T = \frac{\nabla_\mu T}{T} \Delta \langle H \rangle,$$  \hfill (2)

where the subtracted energy $\Delta \langle H \rangle$ is statistics-dependent and the expression approximates to the local inertial force when applying local Hawking Temperature to $T$.

This paper is rather an unperfected story organized to show the coincident derivation of those results in a short cut and then look back to see the conditions and implications of the derivation. In the whole context, we adopt the Natural Unit $c = k = \hbar = 1$.

II. THERMAL PARTICLE STATES

According to the Unruh Effect [19], an accelerating observer will see the thermal spectrum of particle states. Even the Minkowski vacuum contains thermal particles to the observer.

In [3,6], to prove $\log M$ increasing in species problem will not ruin the Bekenstein bound, 1-particle mixed states of single frequency $\omega$ with $M$ species of fields are considered. In Minkowski spacetime, the mixed states are taken to be

$$\frac{1}{\sqrt{M}} \sum_j |\psi_j\rangle = \frac{1}{\sqrt{M}} \sum_j a_j^\dagger |0\rangle |a_j\rangle$$

with the vacuum state $|0\rangle = \otimes_j |0\rangle_j$ and excited particle added.
to the right Rindler wedge, where $j$ labels species and $a_j^\dagger$ creates one particle of frequency $\omega$.

Meanwhile the whole Hilbert space can be decomposed as a tensor product $\mathcal{H} = \mathcal{H}_{-V} \otimes \mathcal{H}_V$ with respect to two casual separated Rindler Wedges $-V$ and $V$. Tracing over the left Hilbert space $\mathcal{H}_{-V}$ which is conventional taken to be invisible, the particle states will follow thermal distribution of Unruh Temperature $T = T_U = \frac{\hbar}{2\pi}$ proportional to the acceleration $a$.

The reduced density matrixes in [5][6] of the vacuum and 1-particle thermal states with $M$ species of free scalar fields are

$$\rho^0_V = \left(1 - e^{-\omega/T}\right)^M \sum_{N=0}^\infty e^{-\omega N/T} |N_1,\ldots,N_M\rangle \langle N_1,\ldots,N_M|,$$  

$$\rho^1_V = \frac{1}{M} e^{\omega/T} \sum_{N=0}^\infty \frac{e^{-\omega N/T} |N_1,\ldots,N_M\rangle \langle N_1,\ldots,N_M|}{(1 - e^{-\omega N/T})^{M+1}},$$

where we adopted a vector notation $\vec{N} = (N_1,\ldots,N_M)$ and the total number operator $\mathcal{N}$ satisfying $\mathcal{N}|N_1,\ldots,N_M\rangle = \sum_{k=1}^M N_k |N_1,\ldots,N_M\rangle$. One important insight is that they have different modular Hamiltonian.

The density matrixes now follow the same thermal distribution as states of $M$ thermal harmonic oscillators with temperature $T$ and single frequency mode $\omega$.

This interesting similarity inspired us to consider the thermal ensembles with the same form of density matrixes. 

The expectation value of the number operator $\mathcal{N}$ of the Bosonic thermal vacuum would be

$$\langle \mathcal{N} \rangle_0^b = \frac{M}{e^{\omega/T} - 1},$$

which agrees with Bose-Einstein statistic, while that of the thermal Fermionic vacuum would be

$$\langle \mathcal{N} \rangle_0^f = \frac{M}{e^{\omega/T} + 1},$$

which is what Fermi-Dirac statistic would tell.

Difference from statistics will appear when the states are confined in subsystem as thermal states. In the following context, we make our calculation using the expression of $\rho^1_V$ for the Bosonic ensembles, then generalize the result to Fermionic ensembles.

**II.1 Observer-Dependent Energy and Entropy**

The Hamiltonian of the quantum oscillators is

$$H = \omega \mathcal{N},$$

where $\mathcal{N}$ is the number operator counting the total number of particles with single frequency $\omega$ mode.

The energy of any state $\rho$ is calculated by counting the expectation of the number operator $\mathcal{N}$

$$\langle H \rangle_\rho = \omega \langle \mathcal{N} \rangle_\rho = \omega \text{Tr} \rho \mathcal{N},$$

and the Von Neumann entropy is

$$S(\rho) = -\text{Tr}(\rho \log \rho),$$

which is also the entanglement entropy for reduced density matrixes.

Because of Unruh Effect, we notice $\rho^1_V$ corresponding to the single mode in consideration is connected to Bose-Einstein statistics. The energy is

$$\langle H \rangle_0^b = \frac{M \omega}{e^{\omega/T} - 1},$$

$$\langle H \rangle_1^b = \frac{\omega}{1 - e^{-\omega/T}} + \frac{M \omega}{e^{\omega/T} - 1} = \frac{\omega}{1 - e^{-\omega/T}} + \langle H \rangle_0^b,$$

and the entanglement entropy of the vacuum is

$$S(\rho^0_V) = M \left( \frac{\omega}{e^{\omega/T} - 1} - \log \left(1 - e^{-\omega/T}\right) \right),$$

which is the entropy of a thermal ensemble of $M$ independent oscillators, while the entropy of $\rho^1_V$ is

$$S(\rho^1_V) = S(\rho^0_V) + \log(M) - \log(e^{\omega/T} - 1) + \frac{\omega/T}{1 - e^{-\omega/T}} - \sum_{\mathcal{N}=0}^\infty \rho^1_V \log \mathcal{N},$$

where the last term $\sum_{\mathcal{N}=0}^\infty \rho^1_V \log \mathcal{N}$ is just $\langle \log \mathcal{N} \rangle_1$.

It is no hard to generalize the results to be Fermionic by changing statistics-dependent factors. Noticing the statistical differences already appear in energy and entropy for different statistics-dependent factors. Notice that a species number $M$, will the negligible effect remain for gravity in an entropic mechanism?
II.2 Entropy Bound in Large Species Number Limit

When $M$ is very large so $Me^{-\omega/T} \gg 1$, we can follow [5] for a mean-field expansion to evaluate the last term in [15]:

$$\langle \log \mathcal{N} \rangle_1 = \log \langle \mathcal{N} \rangle_0 + O\left(\frac{1}{Me^{-\omega/T}}\right),$$  \hspace{1cm} (16)

and we will get

$$S(\rho_V^1) - S(\rho_V^0) \leq \frac{w/T}{1 - e^{-\omega/T}}.$$  \hspace{1cm} (17)

In analogue, for thermal states, we will get

$$S(\rho_1) - S(\rho_0) \leq \frac{w/T}{1 \pm e^{-\omega/T}},$$  \hspace{1cm} (18)

where $+$ for Fermionic states while $-$ for Bosonic states. Generally, the bound is hold because relative entropy $S(\rho_1||\rho_0)$ is non-negative

$$S(\rho_1||\rho_0) \geq 0,$$  \hspace{1cm} (19)

which is equivalent to Casini’s version of the Bekenstein bound [6]

$$S(\rho_1) - S(\rho_0) \leq (\langle H \rangle_1 - \langle H \rangle_0)/T,$$  \hspace{1cm} (20)

where $H/T$ is also the modular Hamiltonian $K_0$ of vacuum. When $M \to \infty$, the bound $S_\infty$ is saturated

$$S_\infty = \lim_{M \to \infty} (S(\rho_1) - S(\rho_0)) = \frac{w/T}{1 \pm e^{-\omega/T}},$$  \hspace{1cm} (21)

and the relative entropy $S(\rho_1||\rho_0)$ becomes zero, which means the 1-particle state is highly mixed and indistinguishable from the vacuum. The condition is closely related to holography [14], so it provides hints for a holographic explanation later developed in [20].

III. THERMODYNAMICS

We consider distributions of states change with infinitesimal variation with respect to the parameters $\omega/T$ first, so the variation of function $g(\omega/T)$ will vary as

$$\delta g(\omega/T) = \frac{dg(\omega/T)}{d(\omega/T)} \delta(\omega/T).$$  \hspace{1cm} (22)

Then we will distinguish the variation of frequency $\omega$ and temperature $T$ separately, they are relevant to different thermodynamic processes.

III.1 Work term from two First Laws

Here we compare the thermodynamic first law and the entanglement first law, to locate the work term from their difference.

First we write down the thermodynamic first law

$$\delta Q + \delta W = \delta E.$$  \hspace{1cm} (23)

We checked the following equalities are satisfied

$$T\delta S(\rho_0) = \delta \langle H \rangle_0,$$  \hspace{1cm} (24)

$$T\delta S(\rho_1) = -T\delta \langle \log \mathcal{N} \rangle_1 + \delta \langle H \rangle_1,$$  \hspace{1cm} (25)

which indeed agrees the entanglement first law $dS = d\langle K \rangle$ where the modular Hamiltonian $K$ is defined from a reduced density matrix $\rho = e^{-K}/\tr e^{-K}$. We notice $K_0 - K_1 = \log \mathcal{N}$ and $K_0 = H/T$.

One can compare (24) and (25) with the thermodynamic first law (23) to get

$$\delta W + T\delta S(\rho_1) = \delta \langle H \rangle_1.$$  \hspace{1cm} (26)

We can relate the work term $\delta W$ for 1-particle states to the term $T\delta \langle \log \mathcal{N} \rangle_1$, where $\log \mathcal{N}$ is just the difference of two modular Hamiltonians

$$\delta W = T\delta \langle \log \mathcal{N} \rangle_1 = T\delta (K_0 - K_1)_1.$$  \hspace{1cm} (27)

This is the first result we have got.

III.2 Fixed Frequency Process

![Temperature increases](image)

FIG. 1: A test exited state $\rho(T)$ is hold by an external force $F_\mu$ in a static thermal atmosphere of position-dependent temperature. $F_\mu$ can be calculated by varying the states to be $\rho(T')$ nearby. This picture is in a triumph to relate the change of thermal states, to the phenomenon of gravity.

A thermal reversible progress is an ideal quasi-static process that changes the ensemble while keeping it always in equilibrium with the outside heat bath.

We propose a specific thermodynamic process in this spirit when we have a temperature gradient $\nabla_\mu T$ as shown in FIG.1. We assume exact exterior influence is done to the states so that the frequency $\omega$ won’t change, while the particle state gains no momentum change during the process: $dp = 0$. 
If the temperature of the states depends on position parameter $r$ while its frequency keeps fixed, we have
\[ \nabla_\mu g(\omega/T) = \partial_r g(\omega/T) = \delta_\mu^r \partial_T g(\omega/T). \] (28)
The covariant derivative of a scalar function depends on how the temperature varies with position viewed by the observer moving along with the states.

### III.3 Derivation of Inertial Force

The local inertial force should be opposite to the external force $F_{\text{inertial}} = -ma_\mu$. From (27) we can derive the thermodynamic force as
\[ F_\mu(r) = T \nabla_\mu \langle \log \mathcal{N} \rangle_1. \] (29)
This expression depends on species number $M$. However, when the bound is saturated (21) at $M \to \infty$, the general expression for $F_\mu(r)$ is
\[
F_\mu(r) = -T \frac{\partial T}{\partial r} \partial_T \left( \log \left( \frac{e^{\omega/T}}{T} - 1 \right) \right) \delta_\mu^r \\
= -T \frac{\delta T}{\partial r} \frac{\omega}{T^2} e^{\omega/T} - \frac{\delta_\mu^r}{T} \\
= \frac{\omega}{T} \frac{\partial T}{\partial r} \delta_\mu^r. \] (30)
Since $e^{-\omega/T} \approx 0$ in low temperature limit $T \to 0$, we can get an approximation
\[ F_\mu(r) \approx \frac{\omega}{T} \frac{\partial T}{\partial r} \delta_\mu^r. \] (31)

Now we put the situation in a fixed curved spacetime background. For a asymptotical flat Schwarzschild black hole of mass $M$, with the metric $ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_2$ and $f(r) = g_{tt} = \frac{1}{g_{rr}} = 1 - \frac{2GM}{r}$, the Hawking radiation has the Hawking Temperature $T_H$ at infinity. For a static observer at $r$ with four-velocity $U_\mu = (1, 0, 0, 0)$, he will observe a local temperature blueshift from Hawking Temperature $T_H$ that follows Tolman law (see [21])
\[ T(r) = \frac{T_H}{V(r)} = \frac{\kappa}{2\pi V(r)}, \] (32)
where $\kappa = \frac{1}{4GM}$ is the surface gravity of the event horizon and $V(r) = \sqrt{1 - \frac{2GM}{r}}$ is the redshift factor.

This is a natural candidate for the origin for the temperature gradient. Applying the local Hawking temperature (32) at position $r$ to the right side of (31), we get
\[ \frac{\omega}{T} \frac{\partial T}{\partial r} = \omega V(r) \partial_v \frac{1}{V(r)} = -\frac{G M \omega}{r^2(1 - \frac{2GM}{r})}, \] (33)
which matches the local inertial force for the Schwarzschild solution
\[ F_{\text{inertial}} = \omega \partial_\mu \phi = -\frac{G M \omega}{r^2(1 - \frac{2GM}{r})} \delta_\mu^r, \] (34)
rather than external force $ma_\mu$, where $V(r) = e^{\delta(r)}$ and $\phi(r)$ is the generalized Newton’s potential. At the same time, the entropic gradient during the process is indeed in the opposite direction to that of Verlinde’s original thought, as later we will show in [20].

The derivation also works for states following Fermi-Dirac statistics by replacing the factor in (30) to $\frac{1}{1+e^{-\omega/T}}$. We will see how negligible the statistics-dependent factor $\frac{1}{1+e^{-\omega/T}}$ is. The approximation in (33) is indeed very precise for low temperature.

### III.4 Free Falling Process

If we change $\omega$ without changing temperature $T$, the work term will be
\[ \delta W_\omega = T \partial_\omega \langle \log \mathcal{N} \rangle_1 d\omega = \frac{\partial}{\partial \omega/T} \langle \log \mathcal{N} \rangle_1 d\omega. \] (35)
Compare with the thermal reversible process which keeps $\omega$ constant
\[ \delta W_r = T \partial_r \langle \log \mathcal{N} \rangle_1 dr = -\frac{\omega}{T} \frac{\partial T}{\partial r} \partial_\omega/T \langle \log \mathcal{N} \rangle_1 dr, \] (36)
if there is no net external work
\[ \delta W_\omega + \delta W_r = 0, \] (37)
we will get
\[ d\omega = \frac{\omega}{T} \frac{\partial T}{\partial r} dr = -\frac{G M \omega}{r^2(1 - \frac{2GM}{r})} dr. \] (38)

The part $\partial_\omega/T \langle \log \mathcal{N} \rangle_1$ no longer exists! The result agrees with the gravitational redshift effect in General Relativity but we get it from a virtual thermal process, no matter whether the Casini’s version of the Bekenstein bound is saturated or not.

Here we have $F_\mu = 0$ and no entropy change. Actually, it is the external force that maintains the static orbit, and causes the thermodynamics.

### IV. LOOKING BACK TO THE RESULT

The **Principle of Equivalence** The above results lead us to relate the gravitational mass to the frequency
\[ m = \omega. \] (39)

The mechanism inspired from the simple model should work universally as gravity. If each frequency mode $\omega_j$ of a more complex state doesn’t cross into each other during the fixed frequency process, the mode behaves as independent 1-particle single mode. The gravitational effect then follows the superposition principle, so our results from the simple model of 1-particle states of single mode can be generalized into general formula for the external force, regardless of the detail of the states.
However, the new thing is the factor \( f = \frac{1}{1 + e^{-r/r_s}} \). When \( M \to \infty \), \( F_\mu \) in (30) depends on the subtracted energy \( \Delta \langle H \rangle \) measured by the static observer. This is consistent with the Principle of Equivalence where gravity gets response to any form of energy, and origins from the same mechanism viewed by accelerating observers. We will give it a general proof in [20].

**Statistics Behavior** Does gravity seen by the observers in the non-inertial frames as the external force do depend statistics and how much the states are mixed? First we notice that for Unruh Effect, by counting the number of particles, the nearly negligible statistics-dependent factor \( f = \frac{1}{1 + e^{-r/r_s}} \) in inertial force already occurs in energy and entropy. As shown in FIG[2], the statistics-dependent factors is very close to 1 when the ratio \( T/\omega \) is small. Besides, when building up a Supersymmetric mixed state to simulate the classic matter mixed with Bosons and Fermions by replacing the basis \( |N_j\rangle \langle N_j| \to |N^b_j, \ N^f_j, \ N^b_{b,j}\rangle \langle N^b_j, \ N^f_j, \ N^b_{b,j}| \) with \( N_j = N^f_j + N^b_{b,j} \), the difference can be balanced between two kinds of statistics.

![FIG. 2: The error from \( \epsilon = 1 - f \) is less than \( \pm 5\% \) at ratio \( \frac{T}{\omega} \leq 0.3 \), where orange line is for Bosonic states with and green line is for Fermionic states. The difference can be balanced for Supersymmetric mixed states, see the blue line.](image)

We do a numerical calculation for a case of a black hole of the Sun mass \( M = M_\odot \) where Hawking Temperature is \( T_H \approx 6.170 \times 10^{-8} \text{K} \) and using small frequency \( \omega = 10^{14} \text{Hz} \) of a usual photon. The position \( r \) with error \( \epsilon = 5\% \) is still very close to the event horizon at radial \( r_s \): \( r(\epsilon = 5\%) - r_s \approx 0 \) to the accuracy of \( 10^{-23} r_s \). The difference from statistics may be hard to test until the acceleration is very large.

However, the effect from different species number \( M \) may be still testable. Since the inertial force for accelerating observers almost matches with Schwarzschild solution only when the Casini’s version of the Bekenstein bound is saturated, observation of gravity as the external force in the view of static observers may differ between quantum pure states (\( M = 1 \)) and mixed states.

**V. IMPLICATIONS AND DISCUSSION**

The coincident derivation has some further possible implications. We discuss three major implications towards our results.

Firstly, we now have a dual thermodynamic explanation of the inertial force indirectly from \( F_{\text{inertial}} = F_\mu \). When Casini’s version of the Bekenstein bound is saturated, we notice the free energy \( U_i = \langle H \rangle_i - TS(\rho_i) \) of the 1-particle state and vacuum are the same. Thus from

\[
\delta U_i = -S(\rho_i)\delta T + \delta W, \tag{40}
\]

where \( \delta W = 0 \) for \( \rho_0 \) and \( \delta W = | F_\mu | | \delta \rho | \) for \( \rho_1 \), \( F_\mu (r) \) can be generalized into the formula (2) made of the entropy bound \( S_\infty \) and temperature gradient because

\[
F_\mu (r) = \lim_{M \to \infty} (S(\rho_1) - S(\rho_0)) \nabla_{\mu} T, \tag{41}
\]

rather than of entropic force expression \( F_\mu = T \nabla_\mu S \) for the inertial force in [1]. That is because when considering statistics, the heat flux \( \delta Q = T\delta S \) not only transforms into external work \( \delta W \), but also into internal energy \( \langle H \rangle \).

We will show that \( F_\mu = T \nabla_\mu S \) can be approximated from (41) when omitting the statistical effect in future research [20].

Secondly, the role of large species number limit resembles the large N limit to form a classical limit. But even if there are not so many different species of fields, the bound may be easily achieved for classical matter distribution as highly mixed states. For example, by referring to the directions of momentum \( \vec{k} \) to replace the role of different species of field, there are infinite eigenstates with the same frequency \( \omega \)

\[
a^\dagger_{j \vec{k}} \mapsto a^\dagger_{j \vec{k}}. \tag{42}
\]

So far without any thing holographic, the mechanism works out fine. But the condition of large species number limit connecting with vanishing of relative entropy promises a holographic origin of gravitational attraction for the entropic mechanism.

Thirdly, we have appointed a new role of Hawking Temperature to replace that of Unruh Temperature in Entropic Gravity theories. The free-field model in the temperature gradient is representative to explain the origin of the entropic gradient for gravity. That is because the gravity should response to any form of energy and obeys a universal mechanisms. The entropic mechanism doesn’t depend on the detail form of states.

We notice that Hawking Temperature is also an observer-dependent effect like Unruh Temperature, and the entropy and energy come from the microscopic degrees of freedom of particle numbers. Thus the above
mechanism relies on those degrees of freedom independently. This kind of temperatures can not be insulated, as a thought experiment FIG. 3 showing the logic.

![Diagram](image)

FIG. 3: A static box in the spacetime a black hole. The triangle inside the box also feels gravity of the black hole. If the temperature gradient is the reason for gravity, can the temperature be prevented by the box?

However, the question comes if general matter distributions such as the Sun and stars should also result in the same temperature as Hawking Effect?

Since in our situation we have argued the mechanism works for black hole and the inertial force is same in the Schwarzschild solution. We have reasons to infer the same mechanism also works for general matter distribution: the general matters also thermalizes the particle states with an equivalent gravitational temperature field $T(r)$. It is possible that the entanglement structure of spacetime for general matter distribution may also exist in the same way as black holes.

In a summary of this paper, thermodynamic processes were built from the point of view of accelerating observers, where we already know the existence of temperature, so degrees of freedoms same in Unruh Effect which are observer-dependent appeared to explain the familiar gravitational results in GR but saw the influence of quantum statistics.

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