Majrana Neutrinos as the Dark Matter
in the Cold plus Hot Dark Matter Model

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Abstract

A simple model of the Majorana neutrino with the see-saw mechanism is studied, assuming that two light neutrinos are the hot dark matter with equal mass of $2.4\,\text{eV}$ in the cold plus hot dark matter model of cosmology. We find that the heavy neutrino, which is the see-saw partner with the remaining one light neutrino, can be the cold dark matter, if the light neutrino is exactly massless. This cold dark matter neutrino is allowed to have the mass of the wide range from $5.9 \times 10^{-6}\,\text{eV}$ to $2.2 \times 10^{-5}\,\text{eV}$.

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Recently Primack et al. [1] pointed out that the cold and hot dark matter model agrees very well with the observations of the matter distribution in the universe with the total density parameter \( \Omega = 1 \) and the Hubble constant \( H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 0.5 \). They assumed that two massive neutrinos which have nearly degenerate masses \( 2.4 \text{ eV} \) play a role of the hot dark matter. The hot dark matter, the cold dark matter, and the Baryon occupy 20\%, 72.5\%, and 7.5\% of the total density parameter, respectively.

On the other hand, there are some current status for the masses and flavor mixings of neutrinos. The solar neutrino deficit [2] and the atmospheric neutrino anomaly [3] seem to give the indirect evidences of the non-vanishing masses and flavor mixings of the neutrinos in the view of neutrino oscillation. In addition, recent LSND experiment [4] seems to have brought the first direct evidence for neutrino masses and flavor mixings in the \( \nu_e \leftrightarrow \nu_x \) oscillation. If some species of neutrinos have masses of order eV, they can be appropriate for the hot components of the dark matter in the cold and hot dark matter model.

In the standard model of the elementary particle physics, three species of neutrinos are exactly massless and there is no particle which can be the cold dark matter. Some extension is needed to include the mass of the neutrinos and the cold dark matter. In this letter, we study the model first introduced by Chikashige, Mohapatra and Peccei [5]. This model is a very simple extension of the standard model, which includes massive Majorana neutrinos.

We introduce three species of the right-handed neutrinos and an electroweak-singlet scalar as new particles to the standard model. In order to make neutrinos massive, two kinds of the Yukawa couplings are considered. The Yukawa interaction is described by

\[
L_{\text{Yukawa}} = g_{ij} \bar{L}_i R_j + g'_{ij} \bar{R}_i R_j + h.c.);
\]

where \( L \) is the electric-charge neutral component of the Higgs field in the standard model, and \( i \) and \( j \) denote flavors (\( i, j = 1, 2, 3 \)). The Dirac and the Majorana mass terms appear by the non-zero vacuum expectation values of these scalar fields \( \langle h \rangle \neq 0 \) and \( \langle h' \rangle \neq 0 \). The mass matrix is given by
where $m_D$ ($M$) is the 3 3 Dirac (Majorana) mass matrix defined by $m_{Dij} = g_{ij}h$ and $m_{Mij} = g_{ij}h$, and $L$ and $R$ are the column vectors of the three flavors. Since the symmetry of the lepton number is spontaneously broken by $h$ is 0, a massless Namib-Goldstone boson called a majoron appears. Generally $m_D$ ($M$) is the 3 3 complex (symmetric) matrix and we should diagonalize the whole 6 6 mass matrix in eq. (2).

As the first approximation, we assume that the off-diagonal elements of the mass matrices $m_D$ and $M$ are very smaller than their diagonal elements, namely, the effect of the flavor mixing can be neglected. By this assumption, we can take $m_D = \text{diag}[m_1; m_2; m_3]$ and $M = \text{diag}[M_1; M_2; M_3]$. Furthermore, we assume the hierarchy between $m_D$ and $M$, namely $m_i = M_1$ 1. Then, the see-saw mechanism [4] separately works on each generation.

The six mass eigenstates are described by the weak eigenstates, $L$ and $R$, as

$$L = \begin{pmatrix} 0 & 1 & 2 & 3 & 0 & 1 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 0 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 2 \\ \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 1 \end{pmatrix};$$

where $L = \text{diag}[1; M_1; M_2; M_3]$ and $r = \begin{pmatrix} h \\ 1 \\ 2 \\ 3 \end{pmatrix}^T$. The masses for $i$ and $h$ are $m_i$ and $m_i^h = M_1$, respectively.

The couplings of the light and heavy neutrinos with the weak bosons are given by

$$L_W = \begin{pmatrix} e \\ 2e \end{pmatrix} \begin{pmatrix} h \\ 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} h + h \gamma;$$

$$L_Z = \begin{pmatrix} e/2 \end{pmatrix} \begin{pmatrix} h \\ 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} h + h \gamma + \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix}^2 h;$$

where $\gamma$ denotes the column vector of three charged leptons, $e = (e, -\bar{e})$. Here we neglect the flavor mixing also in the charged leptons. The couplings of the heavy neutrinos with the weak bosons are suppressed by the small factor $e$. These eikons and $h$ also couple with the majoron (imaginary part of the eikon) [4]. According to eq. (2), these couplings are

$^1$ These eikons and $h$ also couple with the majoron-Higgs (real part of the eikon) and the usual
given by

\[ L = \frac{1}{2} \left( \bar{L} \left( i \gamma^\mu \partial_\mu - m_L \right) L + \sum_i \bar{\nu}_i \left( i \gamma^\mu \partial_\mu - m_{\nu_i} \right) \nu_i + h_c + \sum_i \bar{\nu}_i H \right); \]  

(5)

where the \( \nu_i \) is the majoron field defined by \( \nu_i = \Im \phi_i \), and \( g^D_M \) is a diagonal matrix,

\[ g^D_M = \text{diag}\left[g^D_{M1} ; g^D_{M2} ; g^D_{M3}\right] = M = h_i. \]

In contrast with eq. (4), the couplings of the light neutrinos with the majoron are suppressed by the factor \( h_i \).

Some parameters are fixed according to the cosmological model considered by Primack et al. In their model, the mass spectrum of the light neutrinos is constrained as \( m_1 \), \( m_2' \), \( m_3' \) \( \sim 2 \text{MeV} \). Then, the five parameters are left free in our model, \( g^D_{M1} \), \( g^D_{M2} \), \( g^D_{M3} \), \( m^h_1 \), and \( m^h_1 \) \( \sim (2 \text{MeV}) \). Since it is very complicated to analyze leaving all of these parameters free, we consider the simplest case in this letter. We set \( g^D_{M1} = g^D_{M2} = g^D_{M3} \), therefore \( m^h_1 = m^h_2 = m^h_3 = m_h \). Now we have only three free parameters, \( g^D \), \( m_h \), and \( m_1' \). The value of \( m_1' \) is fixed as the following discussion.

There is a fundamental constraint that the particle has to be stable to be the cold dark matter, namely, its lifetime must be longer than the age of the universe (\( t_u \approx 10^{17} \) s). The heavy neutrinos decay into the light neutrinos and the majoron through the coupling in eq. (5). The lifetime of the \( i \)-th heavy neutrino is given by

\[ \tau_i = \frac{32}{(g^D_{M1})^2 m_i}; \]

(6)

Unless \( g^D \) is very tiny (\( g^D < 5 \times 10^3 \)), two neutrinos, \( h_2 \) and \( h_3 \), are unstable because of \( m_2' \) \( \sim 2 \text{MeV} \). Since such extremely small coupling constant is unnatural, these two neutrinos cannot be naturally the cold dark matter. Then, the heavy neutrino \( h_1 \) is the only remaining candidate for the cold dark matter. If we adopt the natural value (not so small value) for the Yukawa coupling \( g^D \), the mass of the corresponding light neutrino \( \nu_i \) must be extremely small, for instance, \( m_i' < 6 \times 10^{-27} \text{eV} \) for \( g^D \approx 10^2 \). Since we cannot

Higgs in the standard model. However, we do not consider the effects of these fields assuming that masses of these couplings assuming that these scalar particles are very heavy.
believe that such small value of the mass is explained by some mechanism (Dirac mass \( m_1 \) must be extremely smaller than the weak scale), we assume that \( m_1 \) is exactly massless by virtue of some symmetries. Note that the vanishing \( m_1 \), or the Dirac mass \( m_1 = 0 \), is stable against the radiative correction, since the \( U(1) \) symmetry of the phase rotation of the fields and \( e \) forbids the generation of the Dirac mass \( m_1 \). Further, it should be stressed that only the mass difference is important for the neutrino oscillation phenomena, but not the absolute value of the mass. There is nothing wrong with the massless \( m_1 \). According to this assumption, the heavy neutrino \( m_1 \) can be stable cold dark matter. In the following discussion, our aim is to investigate the cosmologically allowed region of the mass \( m_1 \) and the coupling constant \( g_h \).

Let us consider the dynamical evolution of the early universe in this model. At the very high temperature, we assume that all the particles are in thermal equilibrium. As the temperature cools down, some particles decouple from the thermal equilibrium at each specific temperature called decoupling temperature. It is convenient for considering the evolution of the universe to separate the matter contents into two parts. One is the 'heavy neutrino-majoron system' which includes three heavy neutrinos and the majoron. The other is the 'electroweak system' in which all the other particles are included. These two systems weakly interact with each other through the couplings suppressed by the see-saw factor in eqs. (4) and (5). We assume that at the temperature \( T_{EW}^{D} \) the heavy neutrino-majoron system 'decouples from the electroweak system', and that the particles in the 'heavy neutrino-majoron system' are still in thermal equilibrium after this decoupling. Therefore, the another decoupling temperature \( T_D \) is lower than \( T_{EW}^{D} \). Here \( T_D \) is the temperature at which the heavy neutrinos and the majoron no longer interact with each other.

Note that the temperatures of these two systems are different after the decoupling at \( T_{EW}^{D} \). The temperature of the heavy neutrino as the cold dark matter in the present universe \( T_{CDM} \) is different from the temperature of photon at present, \( T_e = 2.7K \). The reheating factor \( R \) is estimated by considering the reheating of photon caused by the charged particle and the anti-particle annihilation. In addition to this usual factor \( R \), the cooling factor \( R \)
should be introduced in our model in order to include cooling effect by the decaying unstable neutrinos. As soon as the 'heavy neutrino-majoron system' decouple from the 'electroweak system' at $T_{EW}^{D}$, two unstable neutrinos $\nu_2$ and $\nu_3$ decay into the light neutrinos and the majoron. Then, the 'heavy neutrino-majoron system' cools down and the 'electroweak system' is heated up, because the energy of the 'heavy neutrino-majoron system' flows to the 'electroweak system' by the emission of the light neutrinos. Since the degrees of freedom of the 'electroweak system' is far larger than that of the 'heavy neutrino-majoron system', we can ignore the heating effect of the 'electroweak system'. The cooling factor $R$ is defined by

$$R = \frac{T_D}{T_{EW}};$$

(7)

where $T_{EW}$ is the temperature of the 'electroweak system' when the heavy neutrinos decouple from the majoron. Since $T_D < T_{EW}^{D}$ as assumed above, the factor $R$ is less than unity. The temperature $T_{CDM}$ is written as

$$T_{CDM} = R R T_r;$$

(8)

by using the factors $R$ and $R_r$.

Now we can write down the condition for the heavy neutrino $\nu_1$ to be the cold dark matter, $\nu_1 = CDM$. Here $CDM = 1.9 \times 10^6$ GeV cm$^{-3}$ is the energy density of the cold dark matter in the present universe based on the model considered by Primack et al. The present energy density of the cold dark matter neutrino $\nu_1$ is given by

$$\rho_{\nu_1} = m_{\nu_1} n(T_D) \left( \frac{T_{CDM}}{T_D} \right)^{-1};$$

(9)

where $n(T_D)$ is the number density of the cold dark matter at the decoupling temperature $T_D$. This number density is given by

$$n(T_D) = \frac{1}{2} \int_0^Z \frac{p^2 dp}{\exp \left[ \frac{E}{T_D} \right] + 1} = f(x_D) (T_D)^3;$$

(10)

where $E = \sqrt{p^2 + m_{\nu_1}^2}$, $x_D = T_D = m_{\nu_1}$, and $f(x_D)$ is defined by
f (x_D) = \frac{1}{2} \int_{0}^{\frac{1}{\sqrt{x_D}}} \frac{y^2 dy}{\sqrt{y^2 + x_D}}.

(11)

Considering that the cold dark matter should decouple in the non-relativistic regime, we provide the upper bound of x_D as x_D \approx 1. By using the condition, h_i = c_{DM}, and eqs. (8)-(11), the mass m_h is given by

m_h (x_D, R) = 1 \times 10^9 \frac{3 R}{f (x_D)} 1 R 3 GeV

(12)

as the function of x_D and R.

On the other hand, the coupling constant g, is obtained from the definition of the decoupling temperature T_D. The decoupling temperature is defined by

n (T_D) h \gamma_{\text{rel}} = H;

(13)

where $\gamma_{\text{rel}}$ is the average value of the annihilation cross section of a heavy neutrino times relative velocity, and H is the Hubble parameter. For the non-relativistic heavy neutrino, we obtain

h \gamma_{\text{rel}} = \left( \frac{g_i}{128} \right) \frac{T_D}{(m_h)^3};

(14)

by considering the heavy neutrino annihilation process, $h_i = h_i$. The Hubble parameter H is given by

H = \frac{8}{90} 3 \frac{T^2}{P} \frac{g}{M_P};

(15)

where $M_P \approx 1.2 \times 10^{8} GeV$ is the Planck mass, and g is the total degrees of freedom of the all particles in thermal equilibrium. Note that T is not equal to T_D. Since the degrees of freedom of the electroweak system are far larger than that of the heavy neutrino-adjunction system, the expansion rate of the universe, or the Hubble parameter, is approximately controlled only by the electroweak system. Therefore, we can set T and g the temperature $T_{EW}$ and the degrees of freedom of the electroweak system, respectively. From the definition of R in eq. (4), the Hubble parameter is rewritten as
H = \frac{8}{90} \left(\frac{2^3}{3}\right)^{1/2} p \frac{(T_D)^2}{M_p R^2} \quad \text{(16)}

Substituting eqs. (14), (14) and (16) into eq. (13) and eliminating $m_h$ by using eq. (12), $q_m$ is given by

$$q_m(x_D;R) = 5 \times 10^{-7} g_{18}^{3/4} \frac{1}{R^{3/4}} x_D^{1/2} f(x_D) R^{1/2} \quad \text{(17)}$$

as the function of $x_D$ and $R$. Since we obtain both $m_h$ and $q_m$ as the functions of $x_D$ and $R$, one line is drawn in the $m_h$-$q_m$ plane for one fixed value of $R$ (1) varying $x_D$ from zero to unity. The allowed region of $m_h$ and $q_m$ is very large, if the value of $R$ is absolutely free.

Next we estimate the cooling factor $R$ by using the 'sudden-decay' approximation for two unstable heavy neutrinos $\nu_2$ and $\nu_3$. We approximately consider that all the unstable neutrinos decay and disappear at once when the age of the universe is equal to their lifetime, $(\approx 2 \cdot 3)$ years. In addition to this approximation, we assume that the disappeared $\nu_2$ and $\nu_3$ are quickly supplied by the majoron annihilation, and the thermal equilibrium is recovered. The same situation is expected to occur also at the age $t = 2 \cdot 3$ and so on, until the temperature of the 'heavy neutrino-majoron system' cools down to the decoupling temperature $T_D$. According to these approximations, the ratio $T_r/T$ can be estimated, where $T_r$ is the temperature of the 'heavy neutrino-majoron system' just after quick supplement and $T$ is the one just before the 'sudden-decay'.

The energy density of the 'heavy neutrino-majoron system', $\rho_{hm}$, is described in two different ways. Since the heavy neutrinos and the majoron are in thermal equilibrium just after quick supplement, we obtain

$$\rho_{hm} = \frac{2}{30} (1 + \frac{7}{4} \cdot 3) T^4 \quad \text{(18)}$$

2 The unstable neutrinos $\nu_2$ and $\nu_3$ do not start decaying from $t = 0$, but $t = t_D^{EW}$ at which they decouple from the 'electroweak system'. However, we can approximately set $t_D^{EW} = 0$, because $t_D^{EW}$ is satisfied in our final result.
On the other hand, just after 'sudden-decay' (before quick supplement), $h_m$ is given by

$$h_m = \frac{2}{30} \left(1 + \frac{7}{4} \right) T^4 = \frac{2}{30} \left(\frac{7}{4} + \frac{1}{2}\right) T^4 : \quad (19)$$

The second term denotes the loss of the energy density due to the emission of the light neutrinos. From these two expressions of $h_m$, we can obtain $T = (18=25)^{1-4}$. Since the 'sudden-decay' and the quick supplement repeatedly occur, we obtain $T_{hm} = T_{EW} = (18=25)^{n-4}$ at the age $t = n$, where $T_{hm}$ and $T_{EW}$ are the temperature of the 'heavy neutrino-majoron system' and the 'electroweak system' at the same age, respectively, and $n$ is the positive integer. This ratio is translated to the smooth function of the age of the universe $t$: $T_{hm} = T_{EW} = (18=25)^{n-4}$. Therefore, the cooling factor $R$ is given by

$$R = \frac{18}{25}^{\frac{t}{t_D}} ; \quad (20)$$

where $t_D$ is the age of the universe at which the heavy neutrino decouple from the majoron.

On the other hand, the definition of $R$ in eq. (1) is rewritten as

$$R = \frac{T_D}{T_{EW}} = \frac{m_h x_D}{T_{EW}} : \quad (21)$$

Since $T_{EW}$ is described by $t_D$ by using the relation between the Hubble parameter and the age of the universe:

$$H = \frac{8}{90} \left(\frac{T_{EW}}{M_P}\right)^2 = \frac{1}{2t} ; \quad (22)$$

we obtain

$$R = m_h x_D \left(\frac{T_D}{T_{EW}}\right)^{\frac{1}{2}} \frac{\sqrt{t_D}}{M_P} : \quad (23)$$

By using eqs. (6), (21), and (23), we can obtain a relation among $m_h$, $g$, $x_D$, and $t_D$ as

$$\frac{18}{25}^{\frac{t_D}{t_{ew}}} = \frac{m_h x_D}{T_{EW} (t_D)} : \quad (24)$$

Now we obtain three independent relations, eqs. (12), (17), and (24) for $m_h$, $g$, $x_D$ and $t_D$. Therefore, we can draw a line in the $m_h$-$g$ plane. The result of numerical calculations for these relations is shown in Fig. 1 (upper solid line for various values of $x_D = 1$).
However, note that our approximations underestimate the value of $R$. Because the correct amount of the decaying $\frac{h}{2}$ and $\frac{h}{3}$ is clearly smaller than that estimated by 'sudden-decay' approximation. Furthermore, we provide next decaying neutrinos by quickly supplement approximation, although the disappeared $\frac{h}{2}$ and $\frac{h}{3}$ are not so quickly supplied. Since both $m_h$ and $q_i$ are decreasing functions of $R$ as can be seen in eqs. (12) and (13), the upper solid line in Fig.1 is interpreted as the upper bound of the allowed region.

There exists the upper bound of $m_h$, since we assumed that all particles are in thermal equilibrium at very high temperature. Then there should exist a value of temperature $T = 0$ which can satisfy the condition as follows:

$$\frac{n_h \langle v \rangle}{H} \ll 1.$$  \hspace{1cm} (25)

Here $n$ is the number density of the heavy neutrino and $\langle v \rangle$ is the average value of the annihilation cross section of the heavy neutrino times relative velocity. Considering the annihilation processes $h \rightarrow h$, $\cdots$, or $q^q (q$ denotes quark) according to the weak interaction of eq. (4), we obtain

$$n_h \langle v \rangle = \frac{4N G_F}{32} m_h \frac{2Z}{m_h} + \frac{\rho^4 dp}{e^{2E - T} + 1} \left( \frac{M^4 M_z^4}{M_z^2} \right)^2 + M^2 Z^2 Z^4 Z^2.$$

where $E = p^2 + m^2$, $m \rightarrow 2 \times 10^5$ GeV, $G_F \rightarrow 1$, $10^5$ GeV is the Fermi constant, $M_z$ is the mass of the Z boson, and $Z = 2 \times 10^5$ GeV is the total decay width of the Z boson [4]. The factor $N$ is defined by $N = (I_3, Q \sin^2 \theta) + (I_3)^2$, where $I_3$ and $Q$ are the third component of the weak isospin ($\frac{1}{2}$) and the electric charge of the final state fermion, respectively. Summing $N$ for all the possible final state fermions, we obtain $N \rightarrow 73$. By numerical calculation of eqs. (25) and (26), we obtain the upper bound $m_h \rightarrow 22$ MeV.

Finally, we consider the constraint from the big bang nucleosynthesis (BBN). The number of species of the light neutrinos is constrained as $N = 3 \times 4$ [8]. The contribution of new species of the light neutrinos is constrained as $N = 3 \times 4$ [8]. The contribution of new species of the light neutrinos is constrained as $N = 3 \times 4$ [8].
particles (three heavy neutrinos and the majoron) to the energy density in the BBN era (’ 1M eV) have to be small enough in comparison with

\[ n = \frac{2}{30} \frac{7}{4} N (1M eV)^4; \]

(27)

where \( N = \max(N) \approx 3 = 0.04 \). The energy density of the new particles is given by

\[ n_{\text{new}} = \frac{2}{30} \frac{7}{4} N (1M eV)^4; \]

(28)

where \( \gamma_R \) is the ordinary reheating factor at the BBN era, \( \gamma_R = g (1M eV) = g \left( T_{EW} \right)^{1/3} \). Therefore, we can obtain the upper bound of \( R \):

\[ R \frac{7}{25} N \gamma_R^{\frac{4}{3}} \]

(29)

This provides the lower bound on the allowed region of \( m_h \) and \( g_M \), since both \( m_h \) and \( g_M \) are the decreasing functions of \( R \). The bound \( \square \) is shown in Fig. 1 as the lower solid line for various values of \( x_0 \).

Our result for the mass of the cold dark matter and the coupling to the majoron is shown in Fig. 1. The region among the dashed line, the upper solid line, the lower solid line, and the horizontal line of the upper bound on \( m_h \) is allowed in our analysis. The allowed region of \( m_h \) and \( g_M \) covers over about three and three orders of magnitude, respectively. The mass matrices of the see-saw type are realized in this allowed region of \( m_h \). The right hand side of the dashed line in Fig. 1 satisfies the constraint that the cold dark matter should decouple from the majoron in non-relativistic regime. Requiring the fact that the heavy neutrino has been in thermal equilibrium of the electroweak system once, the upper bound on \( m_h \) (22M eV) is obtained. The ‘sudden-decay’ and ‘quick supplement’ approximations provide the upper bound of the allowed region (upper solid line in Fig. 1). The lower bound of the allowed region is obtained by the BBN constraint (lower solid line in Fig. 1).

\[ \square \]

\[ \square \]

\[ \text{If we refer the more loose bound, } N \approx 3.3 \text{[9], the upper bound of } R \text{ becomes little larger and the allowed region becomes a little larger.} \]
Here we would like to mention the constraint from the characteristic mass scale of the free streaming of the cold dark matter. The free streaming length is roughly estimated as

\[ L_{FS} \approx \frac{1\text{keV}}{m_h} \frac{T_{CDM}}{T_f} M_{\text{pc}} = \frac{1\text{keV}}{m_h} \frac{R}{R_{\text{Mpc}}}; \quad (30) \]

and we obtain the characteristic mass scale of the free streaming as

\[ M_{FS} = \frac{4}{3} \left( \frac{m_{CDM}}{m_h} \right)^3 2.1 \times 10^6 M_{\text{Mpc}} \frac{R}{m_h \text{ (eV)} }; \]

where \( M \) is the solar mass. This scale means the lower limit of the scale of structure which can be formed by the effect of the cold dark matter. If we consider that the scale of the globular clusters (10^6 Mpc) should be explained by the cold dark matter, more strict lower bound on \( m_h \) is obtained. From the condition of \( M_{FS} = 10^6 M_{\text{Mpc}} \) and using eqs. (12) and (17), we obtain the bound \( m_h > 6.3 \times 10^3 \text{eV} \).

In conclusion, we studied whether the heavy Majorana neutrino can be the cold dark matter or not in the cold plus hot dark matter model considered by Primack et al. The model of the Majorana neutrino was introduced by Chikashige, Mohapatra, and Pecevi, and considered as the simple extension of the standard model. We found that if a light neutrino is exactly massless, the heavy neutrino, which is the see-saw partner of the massless neutrino, can be the cold dark matter, provided that other two light neutrinos play the role of the hot dark matter. Therefore, both the hot and cold dark matters are Majorana neutrinos. We obtained the wide allowed region in the \( m_h - g_\star \) plane by considering the cosmological arguments.
REFERENCES

[1] J.R. Primack, J. Holtzman, A. Klypin and D.O. Caldwell, Phys. Rev. Lett. 74 (1995) 2160.

[2] B.T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38 (1995) 47; K.S. Hirata et al., Phys. Rev. D 44 (1991) 2241; Y. Suzuki, Nucl. Phys. B (Proc. Suppl.) 38 (1995) 54; A. I. Abazov et al., Phys. Rev. Lett. 67 (1991) 3332; J.N. Abdurashitov et al., Nucl. Phys. B (Proc. Suppl.) 38 (1995) 60; P. Anselmann et al., Phys. Lett. B 285 (1992) 376; ibid 327 (1994) 377.

[3] K.S. Hirata et al., Phys. Lett. B 205 (1988) 416; ibid B 280 (1992) 146.

[4] C. Athanassopoulos et al., Phys. Rev. Lett. 75 (1995) 2650; J.E. Hill, ibid 75 (1995) 2654.

[5] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Phys. Lett. B 98 (1981) 265.

[6] T. Yanagida, in: Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK report 79-18, 1979), p. 95; M. Gell-Mann, P. Ramond and R. Slansky, in: Supergravity, eds. P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979), p. 315.

[7] E.W. Kolb and M.S. Turner, The Early Universe (Addison-Wesley Publishing Co., California, 1990).

[8] P. Kerman and L. Krauss, Phys. Rev. Lett. 72 (1994) 3309.

[9] T.P. Walker, G. Steigman, D.N. Olive and H. Kang, Astrophys. J. 376 (1991) 51.
FIGURES

FIG. 1. The allowed region of the mass of the cold dark matter $m_\chi$ and the coupling to the majoron $g_\nu$. The dashed line is the line of $x_0 = 1$ for various values of $R$. The upper solid line is the upper bound for the allowed region. The dots on this line correspond to $x_0 = 1; 0.6; 0.4; 0.2; 0.1$ from below, respectively. The lower solid line is the lower bound for the allowed region. The dots on this line correspond to $x_0 = 1; 0.6; 0.4; 0.2; 0.1$ from below, respectively. The dotted horizontal line is the upper bound of the mass, $m = 22 M_eV$. The region among these four lines is allowed in our analysis.
