Direct Estimates of the Solar Coronal Magnetic Field Using Contemporaneous Extreme-ultraviolet, Radio, and White-light Observations

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Abstract

We report a solar coronal split-band type II radio burst that was observed on 2016 March 16 with the Gauribidanur Radio Spectro-Polarimeter in the frequency range ≈90–50 MHz, and the Gauribidanur RadioheliograPH at two discrete frequencies, viz. 80 and 53.3 MHz. Observations around the same epoch in extreme ultraviolet (EUV) and white light show that the above burst was associated with a flux-rope structure and a coronal mass ejection (CME), respectively. The combined height-time plot generated using EUV, radio, and white-light data suggests that the different observed features (i.e., the flux rope, type II burst, and the CME) are all closely associated. We constructed an empirical model for the coronal electron density distribution \( N_e(r) \), where \( r \) is the heliocentric distance) from the above set of observations themselves and used it to estimate the coronal magnetic field strength \( B \) over the range of \( r \) values in which the respective events were observed. The \( B \) values are consistent with each other. They vary as \( B(r) = 2.61 \times r^{-2.21} \) G in the range \( r \approx 1.1–2.2 \) R\(_e\). As far as we know, similar direct estimates of \( B \) in the near-Sun corona without assuming a model for \( N_e(r) \), and by combining cotemporal set of observations in two different regions (radio and white-light) of the electromagnetic spectrum, have rarely been reported. Further, the present work is a novel attempt where the characteristics of a propagating EUV flux-rope structure, considered to be the signature of a CME close to the Sun, have been used to estimate \( B(r) \) in the corresponding distance range.

Key words: Sun: activity – Sun: corona – Sun: coronal mass ejections (CMEs) – Sun: magnetic fields – Sun: radio radiation

1. Introduction

The formation, evolution, and characteristics of coronal mass ejections (CMEs), coronal streamers, coronal holes, and coronal loops in the solar atmosphere are primarily determined by the coronal magnetic field. But measurements of the solar coronal magnetic field are presently limited due to practical difficulties (see, e.g., Lin et al. 2000; Tomczyk et al. 2008). It is inferred by extrapolating the observed solar surface magnetic field distribution using the potential or force-free field approximations (see, e.g., Wiegelmann et al. 2017 for a recent review on the subject). Estimates of the coronal magnetic field strengths, particularly in the middle corona \( (r \approx 1.1–3.0 \) R\(_e\))., are largely obtained using observations of either the circularly polarized radio emission (i.e., the Stokes \( V \) emission) from the transient low-frequency \( (\lesssim 150 \) MHz) radio events like the type I, II, III, IV, and V bursts, or the split-band feature exhibited by some of the radio type II bursts (Smerd et al. 1975; Dulk & McLean 1978; Dulk & Suzuki 1980; Gopalswamy et al. 1986; Bastian et al. 2001; Vršnak et al. 2002; Mancuso et al. 2003; Ramesh et al. 2003, 2004, 2011, 2013; Cho et al. 2007; Zimovets et al. 2012; Mancuso & Garzelli 2013; Sasikumar Raja & Ramesh 2013b; Tun & Vourlidas 2013; Harirharan et al. 2014; Sasikumar Raja et al. 2014; Zucca et al. 2014b; Kishore et al. 2016, 2017). Weak circularly polarized components in the thermal radio emission from discrete sources at low frequencies (Sastry 2009; Ramesh et al. 2010b) and geometrical properties of the propagating disturbances observed in extreme-ultraviolet (EUV) images of the solar atmosphere (Gopalswamy et al. 2012) have also been used to estimate coronal magnetic strength. Kwon et al. (2013) carried out global coronal seismology from the propagation speed of a fast magnetosonic wave to determine \( B(r) \) in the extended corona. Despite all these different measurements, a combined estimate of \( B(r) \) using observations in the different regions of the electromagnetic spectrum and particularly close to the Sun are very limited (Dulk & McLean 1978; Vršnak et al. 2002; Mancuso et al. 2003, 2019; Cho et al. 2007; Zimovets et al. 2012; Zucca et al. 2014b; Kumari et al. 2017b, 2017c). Equally rare are reports where the same set of observations are used to independently derive the coronal electron density \( N_e(r) \) required to estimate \( B(r) \). This is important since \( B(r) \) will be otherwise sensitive to the density model used (see, e.g., Vršnak et al. 2002).

In the present work we take advantage of the simultaneous imaging and spectropolarimetric observations of a type II radio burst with the ground-based facilities and EUV, white-light observations of the solar corona with instruments on board space platforms to estimate \( B(r) \) in the distance range \( r \approx 1.1–2.2 \) R\(_e\). The paper is arranged as follows: In Section 2, we have reported the observations and the related instruments. The data analysis and results are discussed in Section 3 with a summary given in Section 4.

2. Observations

2.1. Radio Observations

The radio observations reported in the present work were carried out using the different facilities operated by the Indian Institute of Astrophysics (IIA) in the Gauribidanur
Observatory (Ramesh 2011). The Gauribidanur Radio Spectro-Polarimeter (GRASP; Hariharan et al. 2015; Kishore et al. 2015) observed a split-band type II radio burst from the Sun on 2016 March 16 during the period 06:45–07:00 UT. The frequency range of the burst was ≈90–50 MHz. Figure 1 shows the dynamic spectra of the burst observed with the GRASP in Stokes I and V. Radio frequency interference in the observations is minimal (Monstein et al. 2007). The estimated peak degree of circular polarization (dcp) is in the range ≈8%–11%. The durations of the lower (L) and upper (U) bands of the split-band burst at a typical frequency like 88 MHz are ≈2.3 minutes and ≈2.5 minutes, respectively (see Figure 2). The half-power width of the response pattern of GRASP is ≈90° × 60° (R.A. × decl.) and is nearly independent of frequency. The primary receiving element used in GRASP is a Crossed Log-Periodic Dipole (Sasikumar Raja et al. 2013a). The integration time is ≈250 ms, and the observing bandwidth is ≈1 MHz at each frequency. The antenna and the receiver systems were calibrated by carrying out observations in the direction of the Galactic center as described in Kishore et al. (2015). The burst was observed elsewhere also including the Gauribidanur Radio Interferometer Polarimeter (Ramesh et al. 2008), the Gauribidanur Low-frequency Solar Spectrograph (GLOSS; Ebenezer et al. 2001, 2007; Kishore et al. 2014), and e-Callisto (Benz et al. 2009) in Gauribidanur and Ooty. It was associated with a C2.2 class soft X-ray (SXR) flare observed with the Geostationary Operational Environmental Satellite (GOES-15) from the NOAA sunspot active region AR12522 located at the heliographic coordinates N12W83. The above flare was present in the time interval 06:34–06:57 UT, with peak at 06:46 UT. The location of the split-band burst in the solar atmosphere was inferred from observations with the Gauribidanur Radioheliograph (GRAPH; Ramesh et al. 1998, 1999a, 2006b at 80 and 53.3 MHz (see Figure 5). The GRAPH is a T-shaped radio interferometer array that produces two-dimensional images of the solar corona with an angular resolution of ≈5′ × 7′ (R.A. × decl.) at a typical frequency like 80 MHz. The integration time is ≈250 ms and the observing bandwidth is ≈2 MHz. We would like to add here that both the type II bursts shown in Figure 5 correspond to the lower (L) band of the split-band type II burst in Figure 1.

2.2. Optical Observations

The optical data reported in the present work were obtained in EUV at 211 Å with the Atmospheric Imaging Assembly (AIA; Lemen et al. 2012) on board the Solar Dynamics Observatory (SDO), and in white light with the COR1 coronagraph of the Sun-Earth Connection Coronal and Heliospheric Investigation (Howard et al. 2008) on board the Solar Terrestrial Relationship Observatory (STEREO) and the Large Angle and Spectrometric Coronagraph (LASCO; Brueckner et al. 1995) on board the Solar and Heliospheric Observatory (SOHO). The STEREO-A/COR1 instrument observed a CME around the same time as the type II burst in Figure 1. The CME was first seen in the STEREO-A/COR1 field of view (FOV) at ≈06:50 UT and was noticeable until ≈07:05 UT (see Figure 3). The projected heliocentric distance of the centroid of the CME \( r_{\text{CME}} \) during its first appearance was 1.66\( \odot \). The angular width of the CME was ≈36°.
source region for this CME was the active region AR12522 (N12W83) mentioned in Section 2.1. STEREO-A was at \( \approx E16^\circ \) during the onset of the CME.\(^{10}\) The location of the active region therefore corresponds to \( \approx 24^\circ \) behind the limb for the STEREO-A view.

The deprojected heliocentric distances of the CME were calculated for STEREO-A/COR1 images by assuming that the projection effects vary as \( 1/\cos(\phi) \), where \( \phi \) is the angle from the plane of the sky and is equal to \( \approx 24^\circ \) in the present case (see Table 1 for the deprojected \( r_{\text{CME}} \) at different epochs). Figure 4 shows \( SDO/AIA 211 \) Å observations of activity in the source region of the above CME. The evolution of a flux rope (marked with a blue line) and a diffuse shock ahead of it (marked with a yellow line), as described in Gopalswamy et al. (2012), can be clearly noticed. The leading edge (LE) of the flux rope \( (r_L) \) and the shock \( (r_{sh}) \) are located at \( \approx 1.06R_\odot \) and \( \approx 1.13R_\odot \), respectively, at \( \approx 06:36:36 \) UT. The values of \( r_L, r_{sh} \), and the radius of curvature \( (r_c) \) of the flux rope at different epochs are listed in Table 2. Figure 5 shows the \( SOHO/LASCO-C2 \) observations of the CME at \( \approx 07:00 \) UT, along with the \( SDO/AIA 211 \) Å and GRAPH observations at epochs earlier than the appearance of the CME in the \( SOHO/LASCO-C2 \) FOV. It appears that the flux-rope structure in EUV, the type II radio burst, and the white-light CME are all closely associated. Note that the projection effects are very minimal in all the above three observations since AR12522 is almost at the limb of the Sun. We find that the shock is not noticeable in the STEREO-A/COR1 white-light observations (see Figure 3). It is possible that the shock had become fainter by the time the CME reached the STEREO-A/COR1 FOV.

3. Analysis and Results

3.1. Estimates of Coronal Electron Density (\( N_e \))

3.1.1. Radio Imaging Observations with GRAPH

An inspection of Figure 5 indicates that the centroid of the type II burst \( (r_{\text{radio}}) \) observed with GRAPH at 80 MHz and 53.3 MHz are located at \( \approx 1.6 \pm 0.2R_\odot \) and \( \approx 1.9 \pm 0.2R_\odot \), respectively. Any possible error in the position of the burst due to propagation effects such as scattering by density inhomogeneities in the solar corona and/or refraction in the Earth’s ionosphere is expected to be within the above error limit (Stewart & McLean 1982; Ramesh et al. 1999b, 2006a, 2012b; Kathiravan et al. 2011; Mercier et al. 2015; Mugundhan et al. 2016, 2018). The fact that the Sun is presently in the phase of minimum activity (during which the observations reported in the present work were carried out) also indicates that the scattering will be less (Sasikumar Raja et al. 2016; Mugundhan et al. 2017). We calculated \( N_e \) at the above two heliocentric distances using the relation \( N_e = \left( \frac{f_p}{9 \times 10^{-13}} \right)^2 \), where \( f_p \) is the fundamental plasma frequency in units of MHz and \( N_e \) is in units of \( \text{cm}^{-3} \). We would like to note here that the type II burst in the present case is mostly due to harmonic plasma emission \( (2f_p) \) since the locations of the bursts as observed with GRAPH at 80 and 53.3 MHz are above the limb (see Figure 5). The consistency of the estimated peak dcp of the bursts from the GRASP observations (\( \approx 8\% - 11\% \); see Section 2.1) with those reported in the literature for harmonic plasma emission also indicate the same (see, for example, Dulk & Suzuki 1980). An inspection of the dynamic spectra of the type II burst as observed with GLOSS indicates the presence of a faint fundamental component of the type II burst at

10 https://stereo-ssc.nascom.nas.a.gov/cgi-bin/make_where_gif

Table 1: Density Estimates Using STEREO-A/COR1 Data

| Time (UT) | Deprojected \( r_{\text{CME}} \) \((R_\odot)\) | Background Density \((\times10^9 \text{ cm}^{-3})\) | CME Density \((\times10^9 \text{ cm}^{-3})\) |
|----------|------------------|----------------|----------------|
| 06:50    | 1.82             | 7.34 \pm 1.53 | 2.71 \pm 2.46  |
| 06:55    | 2.00             | 4.32 \pm 0.86 | 2.35 \pm 1.56  |
| 07:00    | 2.06             | 3.49 \pm 0.90 | 1.66 \pm 1.17  |
| 07:05    | 2.24             | 2.21 \pm 0.73 | 0.95 \pm 0.91  |

Note

* Centroid of the CME.
frequencies $\lesssim 50$ MHz.\footnote{https://www.iaap.res.in/gauribidanur/GLOSS-dailyimages/Mar-2016/GBD_DSPEC_20160316.jpeg} These confirm that the type II bursts observed with GRASP (Figure 1) and GRAPH (Figure 5) are due to harmonic emission. So we substituted 40 and 26.7 MHz for $f_0$ in the above relation, and obtained the values of $N_e$ as $1.98 \times 10^{17} \text{ cm}^{-3}$ at $\approx 1.6R_\odot$ ($f_0 = 40$ MHz) and $8.77 \times 10^{16} \text{ cm}^{-3}$ at $\approx 1.9R_\odot$ ($f_0 = 26.7$ MHz).

### 3.1.2. White-light Observations with STEREO-A/COR1

The pB measurements with the STEREO-A/COR1 were used to estimate the densities before the occurrence of the CME (i.e., the background corona at the location of the CME) and during the CME, at different heliocentric distances. The difference images used for this purpose were obtained using the observations of the CME at $\approx 06:50$ UT, $06:55$ UT, $07:00$ UT, and $07:05$ UT, and that of the undisturbed background corona at $\approx 06:45$ UT (see Figure 3). Table 1 provides the CME-related details obtained from the aforementioned difference images. The deprojected $r_{\text{CME}}$ at the above epochs are listed in column 2 of Table 1. Note that we had multiplied the measured projected values of $r_{\text{CME}}$ by $1/\cos(24^\circ)$ to remove the projection effects (see Section 2.2). The $N_e$ values of the undisturbed background corona and the CME at the corresponding heliocentric distances are listed in columns 3 and 4 of Table 1. The densities were calculated using the spherically symmetrical inversion technique (Wang & Davila 2014). Note the aforementioned densities correspond to the average density inside the region enclosed by the red box in the lower panels of Figure 3.

Figure 6 shows the plot of the $N_e$ values obtained using GRAPH and STEREO-A/COR1 observations as mentioned above. The error in the density estimates from STEREO-A/COR1 is chiefly due to the errors associated with the instrumental background subtraction and the spherically symmetric approximation (Wang & Davila 2014; Wang 2017). The error in the density estimates from GRAPH is due to variation in $N_e$ over the bandwidth of observations ($\approx 2$ MHz). The power-law fit to the data indicates that $N_e(r) = 2.3 \times 10^8 r^{-3.3}$ in the range $r \approx 1.6-2.2R_\odot$. Note that $N_e(r)$ varies typically as $r^{-6}$ in the range $1.1 \lesssim r \lesssim 2.3R_\odot$ (Baumbach 1937). Considering this, and since we are interested in understanding the characteristics of the CME close to the Sun also in the present case using the SDO/AIA 211Å observations of the associated flux-rope structure (see Figure 4), we assumed that the above empirical relationship should be valid over $r \approx 1.1-2.2R_\odot$. We find that $N_e(r)$ estimated using the above relation for the SDO/AIA 211Å observations in Figure 4 are reasonably consistent with the $N_e(r)$ values reported by Zucca et al. (2014a) in the same distance range ($r \approx 1.1-1.3R_\odot$) utilizing the emission measures derived from SDO/AIA observations for a similar flare associated CME/type II burst event.

### 3.2. Tracing the Path of the CME

Figure 7 shows the height–time ($h-t$) plot of the LE of the CME close to the Sun also in the present case using the SDO/AIA 211Å observations of the associated flux-rope structure (see Figure 4), we assumed that the above empirical relationship should be valid over $r \approx 1.1-2.2R_\odot$. We find that $N_e(r)$ estimated using the above relation for the SDO/AIA 211Å observations in Figure 4 are reasonably consistent with the $N_e(r)$ values reported by Zucca et al. (2014a) in the same distance range ($r \approx 1.1-1.3R_\odot$) utilizing the emission measures derived from SDO/AIA observations for a similar flare associated CME/type II burst event.

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Figure 4. Evolution of the flux rope and shock in SDO/AIA 211Å FOV near the source region of the CME in Figure 3. The white line indicates the solar limb (radius = $1R_\odot$). The blue and yellow markings indicate the flux-rope structure and shock ahead of it, respectively. The red plus marks correspond to the center of the hemispherical structure (assumed) for the flux rope. The cyan crosses represent the LE of the shock.

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The Astrophysical Journal, 881:24 (8pp), 2019 August 10

Kumari et al.
Table 2
Estimates of $B$ and the Related Parameters from $SDO$/AIA 211 Å Observations

| Time (UT) | $r_{sh}$ ($R_\odot$) | $r_q$ ($R_\odot$) | $r_e$ ($R_\odot$) | $\Delta r$ ($R_\odot$) | $\delta$ | $M_a$ | $v_e$ (km s$^{-1}$) | $B$ (G) |
|-----------|---------------------|------------------|-----------------|------------------------|---------|-------|------------------|--------|
| 06:36:34  | 1.12                | 1.04             | 0.025           | 0.083                  | 3.35    | 1.12  | ...              | ...    |
| 06:37:10  | 1.15                | 1.06             | 0.035           | 0.090                  | 2.59    | 1.15  | 401              | 1.93   |
| 06:37:46  | 1.17                | 1.08             | 0.040           | 0.101                  | 2.50    | 1.16  | 400              | 1.83   |
| 06:38:22  | 1.19                | 1.10             | 0.046           | 0.095                  | 2.06    | 1.19  | 390              | 1.74   |

Figure 5. Locations of the type II bursts observed with the GRAPH on 2016 March 16 at 80 MHz ($\approx$06:47:15 UT) and 53.3 MHz ($\approx$06:49:48 UT) superposed on the $SDO$/AIA 211 Å image ($\approx$06:39:36 UT), and the SOHO/LASCO-C2 difference image ($\approx$07:00 UT) obtained on the same day. Solar north is straight up, and east is to the left. The red and cyan color contours represent the GRAPH observations at 53.3 MHz and 80 MHz, respectively. The peak brightness temperatures ($T_b$) of the burst are $\approx$2.66 $\times$ 10$^5$ K (80 MHz) and $\approx$4.46 $\times$ 10$^5$ K (53.3 MHz). The radio contours shown are at 50%, 65%, 80%, and 99% of the peak $T_e$. The black circle indicates the occulting disk of the coronagraph. Its radius is $\approx$2.2$R_\odot$. The bright patch of emission above the coronagraph occulter on its west corresponds to the CME mentioned in the text.

Figure 6. Density estimates from radio (GRAPH) and white-light (STEREO-A/COR1) observations. The solid line is a power-law fit ($N_e(r) = 2.3 \times 10^6 r^{-3.3}$) to the data.

3.3. Estimates of the Coronal Magnetic Field Strength ($B$)

Our aim is to directly estimate $B$ using the observed data and with minimal assumptions. We used the following theoretical relation for this purpose:

$$B = \frac{v_0 \times \sqrt{N_e}}{2.18 \times 10^6},$$

where $B$ is in units of G. We used the empirical relationship in Section 3.1 to obtain $N_e(r)$. The estimated values are in the range $\approx$1.39 $\times$ 10$^8$–3.6 $\times$ 10$^6$ cm$^{-3}$ over $r \approx$ 1.10–2.20$R_\odot$. The combined distance range of the $SDO$/AIA 211 Å and radio observations in the present case. $M_a$ was estimated independently for the aforementioned two observations since they correspond to different heliocentric distance ranges.

3.3.1. $SDO$/AIA 211 Å Observations

Figure 4 shows the initial stages of the CME formation in the $SDO$/AIA 211 Å FOV in the present case. Measuring the locations and characteristics of the corresponding structures, i.e., the flux rope and the shock ahead of it, at different epochs helps to calculate $M_a$ using the relation (see, for example,
Veronig et al. (2010; Gopalswamy et al. 2012)

\[ M_a = \sqrt{1 + \left[ 1.24\delta - (\gamma - 1)/(\gamma + 1) \right]^{-1}}, \]

where \( \delta \) is the relative standoff distance and \( \gamma \) is the adiabatic constant. The heliocentric distance of the shock \( (r_{sh}) \), LE of the CME flux rope \( (r_R) \), thickness of the shock \( \Delta r = r_{sh} - r_R \), and radius of curvature \( (r_c) \) of the CME flux rope are used to calculate \( \delta = \frac{\Delta r}{r_c} \). \( \gamma \) was assumed to be 4/3 for the present calculations (see Kumari et al. 2017b, 2017c for details). The different values estimated using Figure 4 are listed in columns 2–6 of Table 2. We then calculated \( r_c \) for the adjacent time intervals in column 1 using the values of \( r_{sh} \) in column 2. Finally, \( v_x \) values in column 8 were obtained using the relation \( v_x = v_y/M_a \). We find that the location of the active region in the present work and that of the event reported in Gopalswamy et al. (2012) are nearly the same \((\approx W84)\). Furthermore, the \( v_y \) values \((\approx 400–500 \text{ km s}^{-1})\) and the angular width of the CME \((\approx 36^o)\) are also reasonably close in the two cases. So, assuming 06:36:34 UT as the first appearance time \( (t = 0) \) of the flux rope and the shock in Figure 4, we independently calculated the corresponding \( r_{sh} \), \( r_R \), and \( r_c \) values as a function of time using the empirical equations mentioned in Figures 3(a) and (b) of Gopalswamy et al. (2012). The constants in the aforementioned equations were replaced by the values of \( r_{sh} \), \( r_R \), and \( r_c \) at 06:36:34 UT (see Table 2). Interestingly, the empirically calculated values agree well with the direct estimates.

### 3.3.2. Radio Spectral Observations with GRASP

For the radio observations, \( M_a \) was calculated using the following equation (Smerd et al. 1974; Mann et al. 1995; Vršnak et al. 2002):

\[ M_a = \frac{X(X + 5)}{2(4 - X)}, \]

where \( X \) is density jump across the shock during the type II burst. The density jump is calculated from the instantaneous bandwidth \( (BDW) \) of the burst, i.e., \( BDW = \frac{F_H - F_L}{F_L} \) and \( X = (BDW + 1)^2 \). \( F_U \) and \( F_L \) are the upper and lower frequency components of the type II burst in the dynamic spectra. To estimate the \( B \) values, \( F_L \) is used as it corresponds to the undisturbed corona. Table 3 lists the different values estimated from the type II burst observations in Figure 1. The \( v_x \) values in column 8 were obtained in the same manner as the \( SDO/AIA \) 211 Å case described in Section 3.3.1, but Equation (3) was used for the calculations of \( M_a \).

#### 3.3.3. The Radial Variation of the Coronal Magnetic Field Strength

Figure 8 shows the \( B \) values estimated using the \( SDO/AIA \) 211 Å and GRASP observations. The respective estimates are consistent with each other, though they correspond to two different heliocentric distance ranges. A single power-law fit of the form \( B(r) = 2.61 \times r^{-2.21} \) nicely describes the distribution. The only available two-dimensional magnetic field map obtained using coronal Zeeman magnetometry and full-Stokes spectropolarimetric measurements indicates that \( B \approx 3.6 \text{ G} \) at \( r \approx 1.1R_s \) (Lin et al. 2004). Compared to this, the present results predict \( B \approx 2.1 \text{ G} \) at the same distance.

### 4. Summary

We have reported a CME, coronal type II radio burst, and flux-rope structure \( (\text{in EUV}) \) that were observed simultaneously on 2016 March 16. The radio burst was observed in both the imaging and spectral mode. The combined \( h-t \) plot indicates that all the three events are closely associated. We derived an empirical relation for the coronal electron density \( (N_e(r) = 2.3 \times 10^5 r^{-5.3}) \) using EUV observations of the flux-rope structure associated with the CME, spectral and imaging observations of the type II burst associated with the CME, and pB measurements of the corresponding white-light CME. Using the density values thus obtained along with the Alfvén Mach number \( (M_a) \) values from EUV and radio observations, we independently estimated the coronal magnetic field strength \( (B(r)) \). Our results indicate that \( B(r) = 2.61 \times r^{-2.21} \) in the distance range \( r \approx 1.1–2.2R_s \), Mancuso & Garzelli (2013) had derived \( B(r) = 3.76 \times r^{-2.29} \) in the distance range \( r \approx 1.8–14R_s \), by combining split-band type II observations and Faraday rotation measurements of extragalactic radio sources occulted by the solar corona. This is nearly same as the...
The Astrophysical Journal, 881:24 (8pp), 2019 August 10

Kumari et al.

Table 3

| Time (UT) | $F_i$ (MHz) | $F_s$ (MHz) | BDW | $X$ | $M_w$ | $R$ ($R_\odot$) | $v_e$ ($\text{km s}^{-1}$) | $B$ (G) |
|-----------|-------------|-------------|-----|-----|-------|----------------|-----------------|--------|
| 06:47:10  | 102.44      | 81.89       | 0.25| 1.56| 1.45  | 1.58           | 579             | 1.21   |
| 06:48:02  | 91.12       | 72.37       | 0.26| 1.59| 1.47  | 1.65           | 571             | 1.06   |
| 06:49:28  | 79.51       | 57.48       | 0.38| 1.91| 1.78  | 1.86           | 472             | 0.69   |
| 06:51:00  | 65.82       | 47.65       | 0.38| 1.91| 1.77  | 1.99           | 473             | 0.58   |
| 06:53:35  | 54.50       | 40.21       | 0.36| 1.84| 1.70  | 2.12           | 493             | 0.51   |
| 06:57:00  | 45.57       | 35.74       | 0.27| 1.63| 1.51  | 2.15           | 558             | 0.50   |

**Figure 8.** Estimates of $B$ from SDO/AIA 211 Å and radio observations. The solid black line is a power-law fit ($B = 2.61 \times r^{-2.21}$) to the data points.

empirical relation for $B(r)$ in the present case. The present measurements are also in reasonable agreement with that reported by Lin et al. (2004) at $r \approx 1.1R_\odot$ using white-light observations. The consistency between the different measurements, though they correspond to different active regions observed at different epochs, strengthens the robustness of the estimates using radio observations. We expect that the density model-independent direct estimates of $B(r)$ reported in this work would lead to similar attempts in the future for unambiguous estimates of $B(r)$ in the region of the corona where white-light observations are presently difficult.

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