Roton minimum at $\nu = 1/2$ filled fractional quantum Hall effect of Bose particles

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We have studied the collective excitation of fractional quantum Hall effect (FQHE) in the rotating Bose-Einstein condensate (BEC) using CF theory at the filling fraction $\nu = 1/2$. The roton type of excitation in the FQHE of electron system is established over the years for all the filling fraction, whereas the collective excitation at $\nu = 1/2$ filling fraction in the rotating BEC shows no roton minimum. We have investigated this using composite fermion theory with the Pöschl-Teller interaction potential between the particles. We have seen that the long range interaction will give the roton, whereas short range interaction gives no roton minimum.

Fractional quantum Hall effect (FQHE) \[1\] occurs due to the strong Coulomb interaction between electrons in two-dimensional system in presence of a perpendicular magnetic field. The natural quasiparticle of FQHE is the composite fermion (CF) \[2\], each electron in the lowest Landau level (LLL) captures an even number of flux quanta and forms CF. So the CF experience reduce amount of magnetic field, in this magnetic field CF forms new kind of Landau levels called $\Lambda$ levels, and most of the observed FQHE of strongly interacting electron can be mapped into the IQHE of non-interacting CFs. FQH states have reach collective phenomenon which has been studied over the past three and half decades.

If we rotate two dimensional harmonic trapped Bose-Einstein condensate (BEC)\[3\], above some critical angular velocity the motion of the superfluid forms vortex, if we increase the angular velocity we will have large number of vortices in triangular lattice arrangement\[4, 5\], which is also established by solving GP equation\[6\]. As we increase the rotation more and more vertices will generate and due to the increase of centrifugal force atoms will fly away of centre and the density of the fluid will reduce. In this low density, there is a theoretical proposal of the FQHE in rapidly rotating BEC of charge neutral dilute Bose gas in low temperature\[7–9\]. The neutral atoms do not interact with the magnetic field, but the rotation in the confinement potential plays a similar mathematical role of magnetic field in the two-dimensional electron system, sometimes this field refers as synthetic magnetic field\[10\]. Atomic density of the system in the high rotation is very low, so we can assume that all the atoms will confine in the lowest LL.

The CF realization of Bose atomic system is simply attaching odd number (say $p = 1, 3, 5, \cdots$) of vortices with each atom\[11–13\]. The magnetic field experience by the CF of Bose particle is

$$B^* = B - p\rho\phi_0$$

where $B$ is the actual fictitious magnetic field, $\rho$ is the number density of the Bose particle, $\phi_0$ is the magnetic flux quantum. In this reduced magnetic field CF forms Landau level, called $\Lambda$ levels. The lowest LL filling fraction of boson and filling fraction of CF ($n$, an integer number of filled $\Lambda$ level) is given by

$$\nu = \frac{n}{np + 1}$$

In FQHE of electron, $\nu$ is the ratio of the number density of electrons to the degeneracy per unit area of LL, here in rotating trap $\nu$ is the ratio of the number of bosons to the average number of vortices. The ground state properties of the Bosonic counterpart of FQHE have been studied but little attention has been given to the collective excitation in this system in the thermodynamic limit. There are some exact diagonalization calculation and CF calculation for small number of particles. The roton type of excitation in the FQHE of electronic system is a natural phenomenon, which is established theoretically as well as experimentally\[14, 15\], roton minimum has been predicted theoretically in the collective excitation of filled LL of dipolar interacting fermions\[16\]. It has been observed that the excitation spectrum FQHE of Bosonic system at filling fraction $\nu = 1/2$ is not associated with the roton type of excitation\[11, 13, 17, 18\], whereas some other filling fractions contain roton-minimum in their energy spectrum. In this article, we have tried to explain the absence of roton minimum in the energy spectrum at the filling fraction $\nu = 1/2$. In contrast of considering delta function potential we have considered short-range Poschl-Teller(PT) interaction between atoms. In addition of PT interaction, we have considered coulomb interaction too\[19\] to see the nature of excitations in the long-range interaction.

Pöschl-Teller Interaction between dilute Bose gas atoms

In the most of the cases the effective interaction between two charge neutral Bose particles at low energies is a constant in the momentum representation, $U_0 = 4\pi\hbar^2a/m$, where $m$ is the mass of each particle and $a$ is s-wave scattering length. In the real space the interaction can be expressed as

$$V = U_0 \sum_{i<j} \delta^{(3)}(\vec{r}_i - \vec{r}_j)$$

This interaction gives the well known GP equation for BEC and superfluid system. In two dimension this expression
The standard spherical geometry is used in our calculations, which considers electrons moving on the surface of a sphere, subjected to a radial magnetic field. The magnetic field can be thought to emanate from a magnetic monopole of strength $Q$ at the centre, which produces a total magnetic flux of $2Q\phi_0$ through the surface of the sphere of radius $R = \sqrt{Ql}$. This maps into a system of composite fermions at an effective flux $2q = 2Q - (N - 1)$, with $Q$ chosen so that the state at $q$ is an integral quantum Hall state at filling $n = 1$ so that we will have $\nu = 1/2$ filling fraction. In spherical geometry, the angular momentum number is a good quantum number and its value of an electron in the $k$-th LL is $k + Q - 1$[21].

The ground state wave function of the $N$ electron system of FQHE at filling fraction $\nu$, which maps with $n$ filled $\Lambda$ levels of CFs is[2][22]

$$\Psi^0 = J^{-1}P_{\Lambda LL}J^2 \Phi_1(\Omega_1, \Omega_2, \cdots \Omega_N)$$

where $\Omega_i$ are the position of electron on the surface of the sphere, $\Phi_1$ is the Slater determinant of completely filled lowest $\Lambda$ level of CFs, $P_{\Lambda LL}$ is the projection operator onto the LLL and the Jastrow factor is given by

$$J = \prod_{i<j} (u_i v_j - u_j v_i)$$

where the spinor variables are $u = \cos(\theta/2) \exp(-i\phi/2)$ and $v = \sin(\theta/2) \exp(i\phi/2)$ with $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. Here $\Phi_1$ and $J$ both are odd under the exchange of particles so the wave function of the system remains symmetric after the fermionic transformation. The ground state wave function of $\nu = 1/2$ is already in the LLL, but the excited states need the projection. The excited state wave function of $N$-particle system at the filling fraction $\nu = 1/2$ corresponding to the transition of a CF from the filled 0th $\Lambda$ level to an empty $\Lambda$ level $l_f$ ($n_f > 0$), in the spherical geometry is given by[23-25]

$$\Psi(L) = J^{-1}P_{\Lambda LL}J^2 \sum_{m_{h}} |m_h > < q, m_h; n_f + q, m_p|L, M >$$

where $|m_h >$ is the Slater determinant of $N - 1$ number of particles in the lowest $\Lambda$ level with a hole at $m_h$ $Z$-component of angular and one particle in the $n_f \Lambda$ level with $Z$-component of angular momentum $m_p$, $< q, m_h; n_f + q, m_p|L, M >$ are the Clebsch-Gordan coefficients, $L$ is the total angular momentum.

Here we have considered sub-Hilbert space with zero $Z$-component of angular momentum ($M = 0$) without any loss of generality to reduce the numerical complication. Actual collective excitation is not a single CF-exciton state rather the superposition of all possible excitons. We have presented the results of three excitons as we have checked that this energy is identical with the calculation considering four excitons. The excitons are not orthogonal, we have

Wave function & calculation procedures

The interaction strength can be tuned by changing scattering length $a$ in presence of magnetic field. It is very difficult to handle the delta function potential in quantum Monte Carlo calculation and it also requires huge computational resource. Using delta function potential it is not possible to calculate the energy spectra for large number of particles[13]. To avoid this difficulty and to access the system size in the thermodynamic limit we have considered Pöschl-Teller interaction (PT) potential[20]

$$V = g \sum_{i<j} \frac{2\mu}{\cosh^2 (\mu r_{ij})}$$

where $1/\mu$ is the width of the interaction, $\mu$ is the parameter of interaction in unit of inverse of the magnetic length $l = \sqrt{\hbar c/eB} = \sqrt{\hbar/m\omega}$. Here we have considered a range of $\mu$, to investigate the $\mu$ dependence nature of excitation. Small value of $\mu$ gives flat nature of potential as we increase the value of $\mu$ the nature of potential become delta type. Very large value of $\mu$ will give zero energy as the average separation between particles will be large compared to the width of interaction. This PT interaction gives us the opportunity to study the FQHE from long range to short range of interaction.

FIG. 1: Pöschl-Teller interaction potential, $\frac{2\mu}{\cosh^2 (\mu l)}$ as function of separation distance of two particles in unit of magnetic length $l$ for different values of $\mu$ in unit of inverse of $l$. We plotted the potential for both side to see the delta function nature of the interaction, though the value of $r$ is sholly positive. As we increase the value of $\mu$ the potential become more and more like delta-function potential. The out most line is the $1/r$ plot for comparison.
used Gram-Schmidt Orthonormalization procedure to or-
thogonalize low energy exciton states with a fixed angular
momentum. The method of calculation of energy of such
kind of mixed state is called CF-diagonalization[26].

The excited state energy with respect to the ground state
Ψ0 is given by

$$\Delta(L) = \frac{\langle \Psi(L)|H|\Psi(L) \rangle}{\langle \Psi(L)|\Psi(L) \rangle} - \frac{\langle \Psi^0|H|\Psi^0 \rangle}{\langle \Psi^0|\Psi^0 \rangle} \quad (9)$$

$H$ is Hamiltonian of the system. As the kinetic energy be-
come quantized and we assume that the particles are con-
fined in the LLL, the Hamiltonian of the system will be
$H = V$. The multidimensional integration has been car-
ried out using quantum Monte Carlo method.

Results & discussion

We have calculated the ground state energy per particles
in a range of interaction parameter $\mu$ from 1.0 to 3.0 for
different number of particles $N = 130, 111, 100, 91$ and 81
as shown in the FIG 2. The result shows that the number
of particles is sufficient to get the thermodynamic limit of
physical quantities.

The energy spectrum for different values of $\mu$ has been
shown in the FIG 3. We have used the same number of par-
ticles to calculate the excited state energy, the average en-
ergies has been shown in the figure. Small value of $\mu$ gives
sharp roton minimum, as we decrease the range of inter-
action by increasing the interaction parameter $\mu$ the ro-
ton minimum becomes shallow and vanishes at some limit of
interaction.

So the nature of the roton minimum is due to the short
range region the energy is reducing and we believe that the
energy will be identical with the delta function interaction
energy spectra.

The interaction responsible for the FQHE in electronic
system is the Coulomb interaction, which is long range in
nature. We have calculated the energy spectra for Coulomb
interaction FIG 4 and we see that there is a sharp roton
minimum in the energy spectra. So we confirm that the
disappearance of the roton minimum is due to the short

FIG. 2: Ground state energy per particle in unit of $g$ as a function
of $\mu$, for different particle number.

FIG. 3: Energy spectra for different values of $\mu$.
The arrow line represents the ascending order of
$\mu$. At low value of $\mu$, ie at long
range interaction we have a very sharp roton minimum, as we
increase the value of $\mu$ ie decrease the range of interaction the ro-
ton minimum becomes shallow and vanishes after some limitin g
range of interaction. We have calculated the energy spectra for
131, 111 and 91 number of particles. Our result is accurate up to
fourth decimal points.

We see that as we are going to the short
range region the energy is reducing and we believe that the
energy will be identical with the delta function interaction
energy spectra.
range of interaction.

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