J/ψ production in NRQCD: A global analysis of yield and polarization

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J/ψ Production with NRQCD

Factorization theorem:  \( \sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle \)

- **n**: Every possible Fock state, including **color-octet** (CO) states.
- **\( \sigma_{c\bar{c}[n]} \)**: Production rate of \( c\bar{c}[n] \), calculated in perturbative QCD
- **\( \langle O^{J/\psi}[n] \rangle \)**: Long distance matrix elements (LDMEs): describe \( c\bar{c}[n] \rightarrow J/\psi \), universal, extracted from experiment.

Scaling rules: LDMEs scale with definite power of \( v (v^2 \approx 0.2) \):

| scaling | \( v^3 \) | \( v^7 \) ("CO states") | \( v^{11} \) |
|---------|----------|-------------------------|-----------|
| \( n \) | \( 3S_1^{[1]} \) | \( 1S_0^{[8]}, 3S_1^{[8]}, 3P_J^{[8]} \) | ... |

- **Double expansion** in \( v \) and \( \alpha_s \)
- Leading term in \( v \) (\( n = 3S_1^{[1]} \)) equals **color-singlet model**.
Global Fit to Unpolarized Data

\[ <O^{[1]} S_0^{[8]} > = (4.97 \pm 0.44) \cdot 10^{-2} \text{ GeV}^3 \]

\[ <O^{[3]} S_1^{[8]} > = (2.24 \pm 0.59) \cdot 10^{-3} \text{ GeV}^3 \]

\[ <O^{[3]} P_0^{[8]} > = (-1.61 \pm 0.20) \cdot 10^{-2} \text{ GeV}^5 \]
Global Fit to Unpolarized Data

Fit results after subtracting higher charmonia feed-down contributions from prompt data (pp: 36%, γp: 15%, γγ: 9%, ee: 26%):

\[<O^{[1]S_0^{[8]}}>= (3.04 \pm 0.35) \cdot 10^{-2} \text{ GeV}^3\]
\[<O^{[3]S_1^{[8]}}>= (1.68 \pm 0.46) \cdot 10^{-3} \text{ GeV}^3\]
\[<O^{[3]P_0^{[8]}}>= (-9.08 \pm 1.61) \cdot 10^{-3} \text{ GeV}^5\]

\[<O^{[1]S_0^{[8]}}>= (4.97 \pm 0.44) \cdot 10^{-2} \text{ GeV}^3\]
\[<O^{[3]S_1^{[8]}}>= (2.24 \pm 0.59) \cdot 10^{-3} \text{ GeV}^3\]
\[<O^{[3]P_0^{[8]}}>= (-1.61 \pm 0.20) \cdot 10^{-2} \text{ GeV}^5\]
J/ψ Polarization

- **Angular distribution** of decay lepton $l^+$ in J/ψ rest frame

  Polarization observables $\lambda$, $\mu$, $\nu$:

  \[
  \frac{d\Gamma(J/\psi \to l^+l^-)}{d\cos\theta \ d\phi} \propto 1 + \lambda \cos^2\theta + \mu \sin(2\theta) \cos\phi + \frac{\nu}{2} \sin^2\theta \cos(2\phi)
  \]

- Depends on choice of coordinate system:
  - Helicity frame: $z$ axis $\parallel -(\vec{p}_\gamma + \vec{p}_p)$
  - Collins-Soper frame: $z$ axis $\parallel \vec{p}_\gamma/|\vec{p}_\gamma| - \vec{p}_p/|\vec{p}_p|$
  - Target frame: $z$ axis $\parallel -\vec{p}_p$

- **In Calculation**: Plug in explicit expressions for $cc[n]$ spin polarization vectors according to

  \[
  \lambda = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \mu = \frac{\sqrt{2} \text{Re} \ d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \nu = \frac{2 d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}
  \]

- We use the CO LDME set with feed-down contributions subtracted.
**J/ψ Polarization in Photoproduction: p_T Distribution**

- **Bands:** Uncertainties due to scale variation and CO LDMEs.
- **CSM** predicts **longitudinal** J/ψ at high p_T.
- **CS+CO:** largely **unpolarized** J/ψ at high p_T. α_s expansion converges better.
- **H1 and ZEUS data not precise** enough to discriminate CSM / NRQCD.

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*J/ψ production in NRQCD: A global analysis of yield and polarization*
J/ψ Polarization in Photoproduction: $z$ Distribution

- Bands: Uncertainties due to scale variation and CO LDMEs.
- **Scale** uncertainties very large.
- **Error bands** of CSM and NRQCD largely overlap.
- $p_T$ distribution better suited to discriminate production mechanisms than $z$.

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J/ψ production in NRQCD: A global analysis of yield and polarization
Helicity frame: NRQCD predicts strong **transverse** polarization at high $p_T$.

Collins-Soper frame: NRQCD predicts slightly longitudinal $J/\psi$.

Disagreement with CDF Run II data, rough agreement with early ALICE data.

Following high precision LHC data: Confirm/rule out LDME universality!
Polarization in Hadroproduction: Ma et al.

- Chao, Ma, Shao, Wang, Zhang (2012)
- **Fit** to CDF Tevatron $J/\psi$ yield and polarization data with $p_T > 7$ GeV:
  \[
  \langle O_8^{J/\psi(1S_0)} \rangle = 0.089 \text{ GeV}^3 \quad \langle O_8^{J/\psi(3S_1)} \rangle = 0.003 \text{ GeV}^3 \quad \langle O_8^{J/\psi(3P_0)} \rangle = 0.0126 \text{ GeV}^5
  \]
- **Describes** CDF Run II polarization data and $J/\psi$ hadroproduction yield up to **highest measured $p_T$ values**, not below 7 GeV.
- **But:** **Disagreement** with photoproduction at HERA and $e^+e^-$ at BELLE:

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**Bands:** Two alternative LDME sets specified in Ma et al.:

\[
\begin{align*}
\langle O_8^{J/\psi(1S_0)} \rangle &= 0 \\
\langle O_8^{J/\psi(1S_0)} \rangle &= 0.11 \text{ GeV}^3 \\
\langle O_8^{J/\psi(3S_1)} \rangle &= 0 \\
\langle O_8^{J/\psi(3S_1)} \rangle &= 0 \\
\langle O_8^{J/\psi(3P_0)} \rangle &= 0.014 \text{ GeV}^3 \\
\langle O_8^{J/\psi(3P_0)} \rangle &= 0.054 \text{ GeV}^5 \\
\langle O_8^{J/\psi(3P_0)} \rangle &= 0
\end{align*}
\]
Polarization in Hadroproduction: Gong et al.

- Gong, Wan, Wang, Zhang (2012)
- **Fit only hadroproduction yield**, but consider also $\psi'$ and $\chi_{cj}$ contributions:
  - Fit $\chi_{c0}$ CO LDME to LHCb data
  - Fit $\psi'$ CO LDMEs to CDF and LHCb data ($p_T > 7$ GeV)
  - Subtract $\psi'$ and $\chi_{cj}$ feeddowns, fit $J/\psi$ LDMEs to CDF and LHCb data ($p_T > 7$ GeV):
    \[
    \begin{align*}
    \langle O^{J/\psi}_{8'}(1S_0) \rangle &= 0.097 \text{ GeV}^3 \\
    \langle O^{J/\psi}_{8'}(3S_1) \rangle &= -0.0046 \text{ GeV}^3 \\
    \langle O^{J/\psi}_{8}(3P_0) \rangle &= -0.0214 \text{ GeV}^5 \\
    \langle O^{\psi'}_{8}(1S_0) \rangle &= -0.0001 \text{ GeV}^3 \\
    \langle O^{\psi'}_{8}(3S_1) \rangle &= 0.0034 \text{ GeV}^3 \\
    \langle O^{\psi'}_{8}(3P_0) \rangle &= 0.0095 \text{ GeV}^5
    \end{align*}
    \]
- **Predict $J/\psi$, $\psi'$ and $\chi_{cj}$ polarization** in prompt hadroproduction (first time!)
- Predicts **moderate** transverse $J/\psi$ polarization, **contrary to** CDF Run II data
- Also: In disagreement with photoproduction at HERA and $e^+e^-$ at BELLE:
Overview: Three J/ψ Production Works

**Butenschön, Kniehl:**

\[
(O_0^{W}(S_0)) = 0.0497 \text{ GeV}^2
\]
\[
(O_0^{W}(S_1)) = 0.0022 \text{ GeV}^2
\]
\[
(O_0^{W}(P_0)) = -0.0161 \text{ GeV}^2
\]

**Gong, Wan, J.-X. Wang, H.-F. Zhang:**

\[
(O_0^{W}(S_0)) = 0.0973 \text{ GeV}^2
\]
\[
(O_0^{W}(S_1)) = -0.0001 \text{ GeV}^2
\]
\[
(O_0^{W}(P_0)) = 0.0046 \text{ GeV}^2
\]
\[
(O_0^{W}(P_1)) = 0.0034 \text{ GeV}^2
\]
\[
(O_0^{W}(P_2)) = -0.0214 \text{ GeV}^2
\]
\[
(O_0^{W}(P_3)) = 0.0095 \text{ GeV}^2
\]
\[
(O_2^{W}(S_1)) = 0.0022 \text{ GeV}^2
\]

**Chao, Ma, Shao, K. Wang, Y.-J. Zhang:**

\[
(O_0^{W}(S_0)) = 0.089 \text{ GeV}^2
\]
\[
(O_0^{W}(S_1)) = 0.003 \text{ GeV}^2
\]
\[
(O_0^{W}(P_0)) = 0.0126 \text{ GeV}^2
\]
Overview: Three J/ψ Production Works

Chao, Ma, Shao, K. Wang, Y.-J. Zhang:

\(O_{J/ψ}^{1S}(S_1) = 0.089 \text{ GeV}^3\)
\(O_{J/ψ}^{1S}(S_1) = 0.003 \text{ GeV}^3\)
\(O_{J/ψ}^{1P_0} = 0.0126 \text{ GeV}^5\)

**AGREEMENT:**

Can NOT describe e⁺e⁻, yp, pp yield and CDF polarization with same LDMEs.
Summary

- NRQCD provides rigorous **factorization theorem** for heavy quarkonium production. But: Need to proof **LDME universality**.
- **Combined NLO fit** of NRQCD LDMEs to inclusive \( J/\psi \) production data from ALICE, ATLAS, BELLE, CDF, CMS, DELPHI, H1, LHCb, PHENIX, ZEUS.
- Good agreement for **CS+CO** with data except perhaps for \( \gamma \gamma \rightarrow J/\psi + X \).
- **CSM** predictions fall short of data everywhere except for \( e^+e^- \rightarrow J/\psi + X \).
- Fit constrained. CO LDMEs in accordance with **velocity scaling rules**.
- NLO calculations of **polarized** \( J/\psi \) cross section including CO states: Direct photoproduction at HERA and hadroproduction at Tevatron and LHC.
- **CDF Tevatron** Run II data in disagreement with our NRQCD prediction, early low-\( p_T \) ALICE data however still in agreement.

- Two later analyses also show that \( e^+e^-, \gamma p, pp \) yield and CDF Run II polarization data can **not** be described with same LDME set.
  - Following LHC measurements: Hopefully **clarify** LDME universality!
J/ψ production in NRQCD: A global analysis of yield and polarization

M. Butenschön
Calculate Inclusive J/ψ Production within NRQCD

Factorization formulas (here hadroproduction):

- Convolute partonic cross section with proton PDFs:
  \[ \sigma_{\text{hadr}} = \sum_{i,j} \int dx \ dy \ f_{i/p}(x) \ f_{j/p}(y) \cdot \sigma_{\text{part},ij} \]

- NRQCD factorization:
  \[ \sigma_{\text{part},ij} = \sum_n \sigma(ij \to c\bar{c}[n]+X) \cdot \langle O^{J/\psi}[n]\rangle \]

Amplitudes for c\bar{c}[n] production by projector application, e.g.:

\[ A_{c\bar{c}\{3S_1^{1/2}\}} = \varepsilon_\alpha(m_s) \ Tr \left[ C \prod_\alpha A_{c\bar{c}} \right] \big|_{q=0} \]
\[ A_{c\bar{c}\{3P_j\}} = \varepsilon_\alpha(m_s) \varepsilon_\beta(m_l) \ \frac{d}{dq_\beta} \ Tr \left[ C \prod_\alpha A_{c\bar{c}} \right] \big|_{q=0} \]

- \( A_{c\bar{c}} \): Amputated pQCD amplitude for open c\bar{c} production.
- \( q \): Relative momentum between c and c. \( \varepsilon \): Polarization vectors.
Overview of IR Singularity Structure
In Detail: Hadroproduction (RHIC, Tevatron)

- Color singlet model **not enough** to describe data (although increase from Born to NLO)
- **CS+CO** can describe data.
- $^{3}P_J^{[8]}$ short distance cross section **negative** at $p_T > 7$ GeV.
- But: Short distance cross sections and LDMEs **unphysical**
  - No problem!
In Detail: Photoproduction (ZEUS HERA1)

- **Distributions:** Transverse momentum ($p_T$), photon-proton c.m. energy ($W$), and $z = \text{Fraction of photon energy going to } J/\psi$.

- Again: Color singlet alone **below** the data, **CS+CO** describes data well.

- Calculation includes **resolved** photon contributions: Important at low $z$.

- **Good description at high $z$:** No increase like in older Born analyses!
Hadroproduction-only Fit

Global fit to hadroproduction data alone, vary low-\(p_T\) cut:

| \(p_T > 1 \text{ GeV}\)  | \(p_T > 2 \text{ GeV}\)  | \(p_T > 3 \text{ GeV}\)  | \(p_T > 5 \text{ GeV}\)  | \(p_T > 7 \text{ GeV}\)  |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| \(<O[1S_0^{[8]}]> [10^{-2} \text{ GeV}^3]\) | 8.54 ± 0.52             | 16.85 ± 1.23            | 11.02 ± 1.67            | 1.68 ± 2.20             | 2.18 ± 2.56             |
| \(<O[3S_1^{[8]}]> [10^{-3} \text{ GeV}^3]\) | -2.66 ± 0.69            | -13.36 ± 1.60           | -5.56 ± 2.19            | 8.75 ± 2.98             | 10.34 ± 3.55            |
| \(<O[3P_0^{[8]}]> [10^{-2} \text{ GeV}^5]\) | -3.63 ± 0.23            | -7.70 ± 0.61            | -4.46 ± 0.87            | 2.20 ± 1.23             | 3.50 ± 1.50             |
| \(M_0 [10^{-2} \text{ GeV}^3]\)            | 2.25 ± 0.12             | 3.51 ± 0.19             | 3.29 ± 0.20             | 5.50 ± 0.29             | 8.24 ± 0.58             |
| \(M_1 [10^{-3} \text{ GeV}^3]\)            | 6.37 ± 0.19             | 5.80 ± 0.19             | 5.54 ± 0.20             | 3.27 ± 0.29             | 1.63 ± 0.43             |

- **Fit underconstrained.** Therefore give two linear combinations of Ma et al.:
  
  \[M_0 = \langle O(1S_0^{[8]}) >= 3.9 \langle O(3P_0^{[8]}) \rangle / m_c^2\]  
  \[M_1 = \langle O(3S_1^{[8]}) > - 0.56 \langle O(3P_0^{[8]}) \rangle / m_c^2\]

- **Fit results depend strongly** on low-\(p_T\) cut.

**Agreement with Ma et al.’s fit to Tevatron run II data with \(p_T > 7 \text{ GeV}\):**

| Default: Include feed-downs, directly fit \(M_0\) and \(M_1\): | \(M_0\) = \(7.4 ± 1.9\) \(10^{-2} \text{ GeV}^3\) | \(M_1\) = \(0.5 ± 0.2\) \(10^{-3} \text{ GeV}^3\) |
| Ignore feed-downs, directly fit \(M_0\) and \(M_1\): | \(M_0\) = \(8.92 ± 0.39\) \(10^{-2} \text{ GeV}^3\) | \(M_1\) = \(1.26 ± 0.23\) \(10^{-3} \text{ GeV}^3\) |
| Ignore feed-downs, \(M_0\) and \(M_1\) from 3-parameter fit: | \(M_0\) = \(8.54 ± 1.02\) \(10^{-2} \text{ GeV}^3\) | \(M_1\) = \(1.67 ± 1.05\) \(10^{-3} \text{ GeV}^3\) |

[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001]
Hadroproduction-only Fit

Global fit to hadroproduction data alone, vary low-\(p_T\) cut:

| \(p_T > 1\) GeV | \(p_T > 2\) GeV | \(p_T > 3\) GeV | \(p_T > 5\) GeV | \(p_T > 7\) GeV |
|------------------|------------------|------------------|------------------|------------------|
| \(<O[^1S_0[8]]> [10^{-2} GeV^3]\) | 8.54 ± 0.52 | 16.85 ± 1.23 | 11.02 ± 1.67 | 1.68 ± 2.20 | 2.18 ± 2.56 |
| \(<O[^3S_1[8]]> [10^{-3} GeV^3]\) | -2.66 ± 0.69 | -13.36 ± 1.60 | -5.56 ± 2.19 | 8.75 ± 2.98 | 10.34 ± 3.55 |
| \(<O[^3P_0[8]]> [10^{-2} GeV^5]\) | -3.63 ± 0.23 | -7.70 ± 0.61 | -4.46 ± 0.87 | 2.20 ± 1.23 | 3.50 ± 1.50 |

\(M_0 [10^{-2} GeV^3]\): 2.25 ± 0.12 3.51 ± 0.19 3.29 ± 0.20 5.50 ± 0.29 8.24 ± 0.58

\(M_1 [10^{-3} GeV^3]\): 6.37 ± 0.19 5.80 ± 0.19 5.54 ± 0.20 3.27 ± 0.29 1.63 ± 0.43

- **Fit underconstrained.** Therefore give two linear combinations of Ma et al.:
  \[
  M_0 = \langle O[^1S_0[8]]\rangle + 3.9 \langle O[^3P_0[8]]\rangle / m_c^2 \quad M_1 = \langle O[^3S_1[8]]\rangle - 0.56 \langle O[^3P_0[8]]\rangle / m_c^2
  \]
- **Fit results depend strongly** on low-\(p_T\) cut.

**Agreement with Ma et al.’s fit to Tevatron run II data with \(p_T > 7\) GeV:**

| Default: Include feed-downs, directly fit \(M_0\) and \(M_1\): | \(M_0 = (7.4 ± 1.9) \times 10^{-2} \text{ GeV}^3\) | \(M_1 = (0.5 ± 0.2) \times 10^{-3} \text{ GeV}^3\) |
|---------------------------------------------------------------|--------------------------------------------------|
| Ignore feed-downs, directly fit \(M_0\) and \(M_1\):       | \(M_0 = (8.92 ± 0.39) \times 10^{-2} \text{ GeV}^3\) | \(M_1 = (1.26 ± 0.23) \times 10^{-3} \text{ GeV}^3\) |
| Ignore feed-downs, \(M_0\) and \(M_1\) from 3-parameter fit: | \(M_0 = (8.54 ± 1.02) \times 10^{-2} \text{ GeV}^3\) | \(M_1 = (1.67 ± 1.05) \times 10^{-3} \text{ GeV}^3\) |

[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001]
Global Fit: Dependence on Low-$p_T$ Cuts (1)

Global fit: Vary low-$p_T$ cut on hadroproduction data:

| hadroproduction data left | $p_T > 1$ GeV (148 points) | $p_T > 2$ GeV (134 points) | $p_T > 3$ GeV (119 points) | $p_T > 5$ GeV (86 points) | $p_T > 7$ GeV (60 points) |
|--------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $<O[^1S_0^{[8]}]> [10^{-2} \text{ GeV}^3]$ | 5.68 ± 0.37 | 4.25 ± 0.43 | 4.97 ± 0.44 | 4.92 ± 0.49 | 3.91 ± 0.51 |
| $<O[^3S_1^{[8]}]> [10^{-3} \text{ GeV}^3]$ | 0.90 ± 0.50 | 2.94 ± 0.58 | 2.24 ± 0.59 | 2.23 ± 0.62 | 2.96 ± 0.64 |
| $<O[^3P_0^{[8]}]> [10^{-2} \text{ GeV}^5]$ | -2.23 ± 0.17 | -1.38 ± 0.20 | -1.61 ± 0.20 | -1.59 ± 0.22 | -1.16 ± 0.23 |
| $M_0 [10^{-2} \text{ GeV}^3]$ | 1.81 ± 0.09 | 1.85 ± 0.09 | 2.18 ± 0.10 | 2.17 ± 0.12 | 1.89 ± 0.12 |
| $M_1 [10^{-3} \text{ GeV}^3]$ | 6.46 ± 0.17 | 6.37 ± 0.17 | 6.25 ± 0.17 | 6.18 ± 0.17 | 5.86 ± 0.18 |

- **Stabilizing** influence of photoproduction data.
- Fit **constrained** enough: Can now extract 3 CO LDMEs.
- Fit results now **almost independent** of low-$p_T$ cut.
- Fit less stable with low-$p_T$ cut below 2 GeV (nonperturbative effects).
Global Fit: Dependence on Low-$p_T$ Cuts (2)

Global fit: Vary low-$p_T$ cut on photoproduction (including γγ-scattering):

- **Fit stable** against varying low-$p_T$ cut in region $1 \text{ GeV} \sim 3 \text{ GeV}$.
- Just 5 or 1 photoproduction against 119 hadroproduction points not enough to stabilize the fit. **Not stable** with low-$p_T$ cut much larger than $3 \text{ GeV}$. (Would need more high-$p_T$ photoproduction data.)

| photoproduction data left | $p_T > 1 \text{ GeV}$ | $p_T > 2 \text{ GeV}$ | $p_T > 3 \text{ GeV}$ | $p_T > 5 \text{ GeV}$ | $p_T > 7 \text{ GeV}$ |
|---------------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| $<O[1S_0^{[8]}]> [10^{-2} \text{ GeV}^3]$ | $4.97 \pm 0.44$ | $5.10 \pm 0.92$ | $4.05 \pm 1.17$ | $5.44 \pm 1.27$ | $9.56 \pm 1.59$ |
| $<O[3S_1^{[8]}]> [10^{-3} \text{ GeV}^3]$ | $2.24 \pm 0.59$ | $2.11 \pm 1.22$ | $3.52 \pm 1.56$ | $1.73 \pm 1.68$ | $-3.66 \pm 2.09$ |
| $<O[3P_0^{[8]}]> [10^{-2} \text{ GeV}^5]$ | $-1.61 \pm 0.20$ | $-1.58 \pm 0.48$ | $-0.97 \pm 0.63$ | $-1.63 \pm 0.68$ | $-3.73 \pm 0.83$ |

$M_0 [10^{-2} \text{ GeV}^3]$ | $2.18 \pm 0.10$ | $2.36 \pm 0.12$ | $2.37 \pm 0.13$ | $2.62 \pm 0.15$ | $3.10 \pm 0.19$ |

$M_1 [10^{-3} \text{ GeV}^3]$ | $6.25 \pm 0.17$ | $6.05 \pm 0.18$ | $5.94 \pm 0.19$ | $5.78 \pm 0.20$ | $5.62 \pm 0.20$ |

Our default fit
Polarization in Hadroproduction: Contributions

- **First**: Sum up contributions of intermediate states:

  \[ \sum \text{ intermediate states } \]

  **Helicity frame:**

  ![Helicity frame graphs]

  **Collins-Soper frame:**

  ![Collins-Soper frame graphs]

- **Then**: \( \lambda_\theta = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}} \), \( \lambda_{\theta\phi} = \frac{\sqrt{2} \text{Re} d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}} \), \( \lambda_{\phi} = \frac{d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}} \)