Model on Selection of Shells for Gun to Fire on Ground Target Based on Probability Ranking Decision

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Abstract. During gun firing on ground target, because the ammunition consumption is large, it is necessary to select shells in the light of effectiveness-cost. In order to solve the problem of selecting shells, this paper puts forward analysis method of firing effectiveness-cost firstly. Then by using the calculation model of probability ranking decision, we can select appropriate shells by comparing average value of effectiveness-cost expectation extremum. This provides reference for user of gun to make decision during firing on ground target.

1. Introduction
During gun firing on ground target, because the ammunition consumption is large, it is important for us to consider the economical factor. In order to analyze the economical factor, it is necessary to compare the effectiveness-cost between different shells, then we may select correct shells to undertake the combat mission.

The firing observation is affected by many factors when gun firing, such as remote distance, terrain masking and meteorological condition and so on. Even if remote observation means such as UAV (unmanned aerial vehicle) are used to execute firing observation, it is difficult to determine the type of target exactly [1]. But if the type of target is different, the effectiveness-cost for gun using different shells to fire on ground target is very different.

In order to solving the problem of selecting shells, this paper puts forward analysis method of firing effectiveness-cost firstly. Then by using the calculation model of probability ranking decision, we can select appropriate shells by comparing average value of effectiveness-cost expectation extremum. This provides reference for user of gun to make decision during firing on ground target.

2. The analysis method of firing effectiveness-cost
Effectiveness-cost is normally the ratio between combat effectiveness and consumption cost of weapon equipment. In the situation of gun firing on ground target, it is refer to the ratio between combat effectiveness which gained by gun and consumption cost of shells.

When we determine the effectiveness-cost for gun to fire on ground target, the combat effectiveness $E$ is equal to the product of target’s damage probability $R$ and target’s value $Q_d$. The formula to calculate $E$ is as follows [2]:

$$E = RQ_d$$ (1)
According to some factors, such as repeated errors, non-repeated errors, target’s area and shells’ damage ability, damage probability $R$ can be calculated by statistics simulation method. The simulation and calculation process is as Figure 1.

![Simulation and Calculation Process](image)

**Figure 1.** The simulation and calculation process of damage probability

Supposed $B$ is effectiveness-cost, then $B$ is the ratio between combat effectiveness $E$ and material consumption $Q_c$. The formula to calculate $B$ is as follows:

$$B = \frac{E}{Q_c} = \frac{RQ_d}{Q_c}$$  \tag{2}

The consumption cost $Q_c$ of gun is as below:

$$Q_c = N \times p + V$$  \tag{3}

In above formula, $N$ is ammunition consumption, $p$ is one shell’s price, and $V$ is the total expenses of service support, such as checking, maintenance and so on.

Usually, the value of $V$ should be analyzed according to shells’ life-cycle cost. But the service support methods for different shells are basically the same, and the difference of support cost is small, so we can ignore the support cost in the process of comparing different shells’ effectiveness-cost.

If combat effectiveness $E$ is unchanged during gun firing on ground target, when the effectiveness-cost of weapon system is larger, the combat benefit becomes better accordingly[3].

### 3. The model of probability ranking decision

If possible types of ground targets are as follows: $L_1, L_2, \ldots, L_{n-1}, L_n$, and occurrence possibilities of different targets are $P_1, P_2, \ldots, P_{n-1}, P_n$. At the same time, we can’t get to know the certain value of $P_1, P_2, \ldots, P_{n-1}, P_n$, but only know their relationships are: $P_n \geq P_{n-1} \geq \cdots \geq P_2 \geq P_1$. Then the decision which is made under the above mentioned conditions is called probability ranking decision[4].
3.1. Calculate the expectation extremum of effectiveness-cost

When shooting at different targets, we may suppose that effectiveness-cost of shells \( Z \) is \( B_i \) \((i=1, 2, \ldots, n-1, n)\). For shells \( Z \), it’s expectation maximum value(or minimum value) of effectiveness-cost can be solved by the following linear programming equation:

\[
\begin{align*}
\max (\text{or min}) E(Z) &= \sum_{i=1}^{n} P_i B_i \\
\text{s.t.} \quad &\sum_{i=1}^{n} P_i = 1 \\
&\quad P_{i+1} - P_i \geq 0 \quad (i = 1, 2, \cdots, n-1) \\
&\quad P_i \geq 0 \quad (i = 1, 2, \cdots, n)
\end{align*}
\] (4)

In order to solve equation (4), we may suppose:

\[
\begin{align*}
s_i &= \sum_{k=1}^{i} B_k \quad (i = 1, 2, \cdots, n) \\
r_i &= \frac{P_i - P_{i+1}}{P_n} \quad (i = 1, 2, \cdots, n-1) \quad r_n = 0
\end{align*}
\] (5)

Then we can get the following equation:

\[
\begin{align*}
\sum_{i=1}^{n} P_i B_i &= \sum_{i=1}^{n} r_i s_i \\
\sum_{i=1}^{n} P_i &= \sum_{i=1}^{n} i r_i
\end{align*}
\] (6)

Because \( r_i \geq 0 \), we can get the conclusion: \( P_i \geq 0 \).

According to previous analysis, linear programming equation (4) can be changed into the following equation:

\[
\begin{align*}
\max (\text{or min}) E(Z) &= \sum_{i=1}^{n} r_i s_i \\
\text{s.t.} \quad &\sum_{i=1}^{n} i r_i = 1 \\
&\quad r_i \geq 0 \quad (i = 1, 2, \cdots, n)
\end{align*}
\] (7)

Because the above equation has only one constraint restriction, its optimal basic solution include one non-negative numerical value \( r_m \), all the other numerical value is 0. It shows that optimal solution of equation (7) is: \( r_m = 1/m \), \( r_i = \cdots = r_{m-1} = r_{m+1} = \cdots = r_n = 0 \).

Then optimal value can be obtained by following equation:
\[
\max (\text{or min}) E(Z) = \frac{1}{m} s_m = \frac{1}{m} \sum_{k=1}^{m} B_k
\]  
(8)

Supposed that \( B_m = \frac{1}{m} \sum_{k=1}^{m} B_k \), and \( B_m \) is corresponding to the \( m \)th local average value in the sequence \( B_1, B_2, \ldots, B_n \). After calculating all the local average value in the sequence \( B_1, B_2, \ldots, B_n \), we can compare them one by one, then their maximum value (or minimum value) is also maximum value (or minimum value) for \( E(Z) \).

3.2. Select appropriate shells

After we get expectation extremum of effectiveness-cost for different shells, we can select appropriate shells to use via eclectic method.

During gun firing on ground target, we may suppose that there are \( K \) kinds of shells \( Z_1, Z_2, \ldots, Z_K \) which can be chosen. The minimum expectation value of effectiveness-cost is \( m_j \) (\( j=1,2,\ldots, K \)), and the maximum expectation value is \( M_j \). Firstly, the following formula is assumed:

\[
W_j = \frac{1}{2} (M_j + m_j)
\]  
(9)

Then we compare those values of \( W_j \) in turn, and the shells corresponding to maximum value of \( W_j \) are the selected shells to use during firing on ground target.

Notably, when all the expectation extremum of effectiveness-cost for different shells are found out, we can also select the best shells in the light of other principle. For example, conservative decision is to find the largest one among minimum value, in other words, we may select the shells corresponding to maximum value in the sequence of \( m_1, m_3, \ldots, m_k \) as the best shells[5]. And adventurous decision is to find the largest one among maximum value, in other words, we may select the shells corresponding to maximum value in the sequence of \( M_1, M_2, \ldots, M_K \) as the best shells.

4. Application analysis

Supposed that one type of gun is firing on ground target, and UAV is used to execute the remote observation, then we can determine the possible type of target is as following: (1) exposure infantry, (2) armoured group, (3) artillery position. Shells which can be used by gun are kill bomb, explosive cartridge, and armour-piercing projectile.

| Shells                  | Target’s type       | Artillery position | Armoured group | Exposure infantry |
|-------------------------|---------------------|--------------------|----------------|------------------|
| kill bomb               | Artillery position  | 3.6                | 2.2            | 4.3              |
| explosive cartridge     | Artillery position  | 4.9                | 4.5            | 3.0              |
| armour-piercing projectile | Artillery position | 3.3                | 5.7            | 2.5              |

By comprehensive consideration, user of gun can determine the possible sequence of target type is: (1) artillery position; (2) armoured group; (3) exposure infantry. According to the calculation model of
probability ranking decision, we calculate effectiveness-cost for different shells to various target as table 1.

If kill bomb is used to fire on target, we can calculate its three local average value of effectiveness-cost: \( \overline{B_1} = 3.6; \overline{B_2} = \frac{B_1 + B_2}{2} = \frac{3.6 + 2.2}{2} = 2.9; \overline{B_3} = \frac{B_1 + B_2 + B_3}{3} = \frac{3.6 + 2.2 + 4.3}{3} \approx 3.4. \)

After comparing in turn, we can know: \( m_1 = 2.9, M_1 = 3.6. \)

We may consider explosive cartridge and armour-piercing projectile similarly, then calculate local average value and extremum value of effectiveness-cost as table 2.

**Table 2. Local Average Value and Extremum Value of Effectiveness-cost**

| Shells                     | \( \overline{B_1} \) | \( \overline{B_2} \) | \( \overline{B_3} \) | \( m_1 \) | \( M_1 \) |
|----------------------------|-----------------------|-----------------------|-----------------------|----------|----------|
| explosive cartridge        | 4.9                   | 4.7                   | 4.1                   | 4.1      | 4.9      |
| armour-piercing projectile | 3.3                   | 4.5                   | 3.8                   | 3.3      | 4.5      |

According to formula (9), we may get: \( W_1 = \frac{1}{2}(2.9 + 3.6) = 3.25 \), \( W_2 = \frac{1}{2}(4.1 + 4.9) = 4.5 \), \( W_3 = \frac{1}{2}(3.3 + 4.5) = 3.9 \). By comparing with each other, because the value of \( W_2 \) is the maximum value, then we should select explosive cartridge to fire.

5. Conclusion

During gun shooting at ground target such as artillery position, if we want to complete the damage tasks, we have to consume large number of ammunition, so it is necessary to select appropriate shells in the light of effectiveness-cost[6]. This paper puts forward analysis method of firing effectiveness-cost, and the calculation model of probability ranking decision is used to select appropriate shells, it provides reference for user of gun to make decision.

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