Nuclear matter distributions in the neutron-rich carbon isotopes $^{14-17}\text{C}$ from intermediate-energy proton elastic scattering in inverse kinematics

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Abstract

The absolute differential cross sections for small-angle proton elastic scattering off the nuclei $^{12,14-17}\text{C}$ have been measured in inverse kinematics at energies near 700 MeV/u at GSI Darmstadt. The hydrogen-filled ionization chamber IKAR served simultaneously as a gas target and a detector for the recoil protons. The projectile scattering angles were measured with multi-wire tracking detectors. The radial nuclear matter density distributions and the root-mean-square nuclear matter radii were deduced from the measured cross sections using the Glauber multiple-scattering theory. A possible neutron halo structure in $^{15}\text{C}$, $^{16}\text{C}$ and $^{17}\text{C}$ is discussed. The obtained data show evidence for a halo structure in the $^{15}\text{C}$ nucleus.

Keywords: $^{12}\text{C}$, $^{14}\text{C}$, $^{15}\text{C}$, $^{16}\text{C}$, $^{17}\text{C}$, nuclear matter distribution, nuclear matter radii, proton-nucleus elastic scattering

1. Introduction

The study of nuclei far from stability is a topic of great current interest. A number of experiments have shown that these nuclei may have exotic structures such as a neutron skin or a halo $^{[1,2]}$. The neutron skin describes an excess of neutrons on the nuclear surface whereas the neutron halo corresponds to such an excess along with an extended tail of the neutron density distribution. The necessary conditions for the halo formation in nuclei are a small binding energy and a low angular momentum of the valence nucleon(s). It has been found that a halo structure manifests itself by large interaction (reaction) cross sections, by enhanced removal cross sections and by narrow momentum distributions of reaction products in the processes of nuclear break-up and Coulomb dissociation $^{[1,2,5]}$.

A long isotopic chain of carbon nuclei was extensively studied both experimentally and theoretically with the aim to understand the evolution of the nuclear structure as one approaches the drip line. Among other
topics, the variation of the nuclear shape with the neutron excess \[ \delta \delta \] , the development of a halo \[ \delta \delta \delta \] - \[ \delta \delta \delta \delta \] , and the change of the shell structure \[ \delta \delta \delta \delta \] are important subjects in the study of the nuclei of carbon isotopes. Recently, an experimental evidence for a prevalent \( Z = 6 \) magic number in neutron rich carbon isotopes was presented \[ \delta \delta \delta \] based on a systematic study of proton radii, electromagnetic transition rates and atomic masses of light nuclei. Small neutron separation energies are known in \( \delta^{15} \text{C} \), \( \delta^{17} \text{C} \), \( \delta^{19} \text{C} \) and \( \delta^{22} \text{C} \) \[ \delta \delta \] , so these nuclei are suggested to be candidates to exhibit a neutron halo. Large enhancements in the values of the root-mean-square (rms) nuclear matter radius \( R_m \) evaluated from the measured interaction cross sections were found for \( \delta^{15} \text{C} \), \( \delta^{19} \text{C} \) \[ \delta \delta \delta \delta \] and \( \delta^{22} \text{C} \) \[ \delta \delta \delta \delta \] . These results also signal the formation of a neutron halo. Narrow fragment momentum distributions of the reaction products in the nuclear break-up of \( \delta^{15} \text{C} \) \[ \delta \delta \delta \delta \] \[ \delta \delta \delta \delta \delta \delta \delta \] \[ \delta \delta \delta \delta \delta \delta \delta \] \[ \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \delta \δ
structure in $^{15-17}$C. The $^{14}$C nucleus was chosen as a presumable core for the $^{15}$C and $^{16}$C nuclei. The measurement of the differential cross section for elastic $p^{12}$C scattering was used as a consistency check of the experimental method, including the data analysis procedure.

2. Experimental set-up and the measurement procedure

The measurements were performed at GSI, Darmstadt, at the exit of the fragment separator FRS [41] using the experimental set-up shown in Fig. 1. The carbon isotopes were produced through fragmentation of the $^{22}$Ne primary beam interacting with a 8 g/cm$^2$ thick Be target. The produced secondary beams with an energy of $\sim 700$ MeV/u and an energy spread of $\sim 1.3\%$ were focused at the centre of the active target IKAR, the mean energies of the beam particles being determined with an accuracy of about 0.1%. The intensity of the secondary carbon beams was at the level of 3000 ions/s with a duty cycle in the range of 50–70%.

The experimental set-up was the same as in the previous experiment [37]. It includes the active target IKAR [40, 42, 43], a tracking system based on multi-wire proportional chambers PC1–PC4, scintillator detectors S1–S3 and VETO, the ALADIN magnet with a drift chamber and a scintillator wall. The active target IKAR is the hydrogen-filled ionization chamber which serves as a hydrogen target and a proton recoil detector. IKAR consists of six identical cells, one of which is shown in Fig. 1. It permits to measure the energy $T_R$ of the recoil proton (or its energy loss in case it leaves the active volume), the scattering angle $\Theta_S$ of the scattered proton, and the coordinate $Z_V$ of the interaction point along the chamber axis in the grid-cathode space [33].

The scattered beam particles were registered in coincidence with the recoil protons. The scattering angle $\Theta_S$ of the projectiles was determined with a set of two-dimensional multi-wire proportional chambers PC1–PC4. The $\Theta_S$ angular resolution was estimated to be in the range from $\sigma_\Theta = 0.6$ mrad for the case of $^{17}$C to $\sigma_\Theta = 0.85$ mrad for $^{12}$C.

A set of scintillation counters (S1, S2, S3 and VETO) was used for triggering and identification of the beam particles via time-of-flight (ToF) and energy loss ($\Delta E$) measurements. The identification plot for the case of the $^{17}$C secondary beam is shown in Fig. 2. The time-of-flight and energy loss of the projectiles in the scintillators allow for unambiguous discrimination of the different isotopes present in the beam. The contamination with other nuclei for each selected carbon isotope was below the 0.1% level.
The ALADIN magnet with a drift chamber and a scintillator wall behind it was utilized to discriminate against break-up reaction channels using magnetic rigidity and energy loss of the reaction products. Some features of the experimental lay-out and a detailed description of the procedure of the measurements have already been described in earlier publications [33–39].

The differential cross section \( \frac{d\sigma}{dt} \) was determined after the event selection using the relation

\[
\frac{d\sigma}{dt} = \frac{dN_{el}}{dt N_b n \Delta L}.
\]

Here, \( dN_{el} \) is the number of elastic proton-nucleus scattering events in the interval \( dt \) of the four-momentum transfer squared, \( N_b \) is the total number of incident beam particles, \( n \) is the density of protons in the target, and \( \Delta L \) is the total target length. The value of \( t \) was calculated as \( |t| = 2mT_R \) (where \( m \) is the mass of the proton) for the lower momentum transfers, or as \( |t| = 4p^2 \sin^2(\Theta S/2)/(1 + 2E \sin^2(\Theta S/2)/mc^2) \) (where \( p \) and \( E \) denote the projectile initial momentum and total energy, correspondingly) for the higher momentum transfers [39].

The procedure of the selection of elastic events was the same as in the previous experiments with IKAR [33, 36, 37, 39]. The measured differential cross sections are to a large extent cross sections for elastic scattering. However, they may contain some admixture of inelastic scattering. Possible contributions of inelastic scattering to the measured cross sections were estimated by calculations.

The calculations of the inelastic cross sections for proton scattering off the carbon isotopes under study were performed using the eikonal model. In particular, the formalism of Ref. [44] was adopted as a starting point, but it was extended in order to distinguish between scattering on protons and neutrons in the nuclei under investigation. Note that for the case of neutron-rich nuclei, such a distinction is obviously necessary. In the calculations, the basic inputs were the nucleon-nucleon (\( NN \)) scattering amplitudes and the ground-state (transition) densities for the cases of elastic (inelastic) scattering, respectively. The parameters of the \( NN \) amplitudes were taken from Ref. [45]. The ground-state densities were described as Gaussians, while the transition densities were as in the Tassie model [46]. The rms radii of the proton and neutron distributions \( R_p \) and \( R_n \) were taken from Ref. [4]. The total differential inelastic cross sections for the different carbon isotopes were calculated by summing up the contributions of all experimentally known states below the
Figure 3: Absolute differential cross sections $d\sigma/dt$ for $p^{12,14,15,16,17}C$ elastic scattering versus the four-momentum transfer squared $-t$. The indicated energies correspond to the equivalent proton energies for direct kinematics. Solid lines are the results of fits to the experimental cross sections performed within the Glauber theory using the GH parameterization with the fitted parameters. The deformation parameters $\beta_p$ and $\beta_n$ used in the calculations were based on the existing experimental information (proton or other scattering data, Coulomb excitation or electromagnetic decay properties). The details of the calculations will be published elsewhere [48].

The calculated inelastic cross sections are significantly smaller than the measured values of $d\sigma/dt$ and make a noticeable contribution to $d\sigma/dt$ (up to about 10%) only at the highest values of $|t|$ (at $|t| \approx 0.06$ (GeV/c)$^2$). The absolute differential cross sections $d\sigma/dt$ deduced in the present experiment according to Eq. (1) for proton elastic scattering on the $^{12}C$, $^{14}C$, $^{15}C$, $^{16}C$, and $^{17}C$ nuclei in the momentum-transfer range of $0.002 \leq |t| \leq 0.06$ (GeV/c)$^2$ after subtraction of the calculated contributions from the inelastic scattering are displayed in Fig. 3 and listed in a tabular form in the Appendix. The indicated energies $E_p$ correspond to the equivalent proton energies in direct kinematics. A high detection efficiency for the beam particles and the elastic-scattering events provide the 2% accuracy of the absolute normalization of the measured cross sections. The uncertainty in the $t$-scale calibration is estimated to be about 1.5%. Note that the above discussed procedure of subtraction of the estimated contributions of the inelastic scattering had a rather small effect (within the error bars) on the deduced radii.
3. The data analysis and results

The Glauber multiple-scattering theory was used to obtain the nuclear density distributions from the measured cross sections similarly as in the previous experiments with IKAR [34, 35]. The calculations were performed using the basic Glauber formalism for proton-nucleus elastic scattering and taking experimental data on the elementary proton-proton and proton-neutron scattering amplitudes as input (for details see Ref. [34]). In the analysis of the experimental data, the nuclear many-body density \( \rho_\Lambda \) was taken as a product of the one-body densities, which were parameterized with different functions. The parameters of these densities were found by fitting the calculated cross sections to the experimental data. The fitting procedure is described in detail in Ref. [34].

In order to reduce the model dependence of the obtained results, four parameterizations of phenomenological nuclear density distributions were applied in the present analysis, labeled as SF (Symmetrized Fermi), GH (Gaussian-Halo), GG (Gaussian-Gaussian) and GO (Gaussian-Oscillator). A detailed description of the SF, GH, GG and GO parameterizations is given in Ref. [34]. Within the GH and SF density parameterizations, the many-body density is the product of the one-body densities, assuming that all nucleons have the same density distribution, while within the GG and GO parameterizations, the nuclear density is subdivided into the core and valence (“halo”) nucleon components. The free parameters in the GG and GO parameterizations are the rms radii \( R_C \) and \( R_v \) (\( R_h \)) of the core and valence (“halo”) nucleon distributions. The matter radius \( R_m \) is connected with \( R_C \) and \( R_v \) by the following relation:

\[
R_m = \left( \frac{(A_c R_C^2 + A_v R_v^2)}{A} \right)^{1/2},
\]

where \( A \) is the nuclear mass number, \( A_c \) is the number of nucleons in the core, and \( A_v \) is the number of valence nucleons.

The results of the fits to the measured experimental cross sections with the phenomenological density distributions SF, GH, GG and GO for the carbon isotopes under investigation are presented in Table I. For each density parameterization, the deduced rms nuclear matter radius \( R_m \), the \( \chi^2 \) value of the fitting procedure, the values of the fit parameters, and the normalization coefficient \( A_v \) with which the calculated cross section \( d\sigma/dt \) was multiplied to obtain the same absolute normalization as the experimental one are presented. Note that the errors in Table I are statistical only.

The solid lines in Fig. 3 represent the results for the cross sections \( d\sigma/dt \) calculated using the GH parameterization with the fitted parameters. At \( |t| < 0.005 \) (GeV/\( c \))^2, a steep rise of the cross section with decreasing \( |t| \) is caused by Coulomb scattering. It is seen that the fits describe the experimental cross sections fairly well with the reduced \( \chi^2 \) values close to 1.0. The calculations of the cross sections with the nuclear matter density parameterizations SF, GG, and GO with the fitted parameters give practically the same results.

For the description of the cross sections in the case of the \(^{12}\text{C} \) and \(^{14}\text{C} \) nuclei, only the SF and GH density parameterizations were used. The weighted mean values of \( R_m \) averaged over the results obtained with these density parameterizations are:

\[
\begin{align*}
R_m &= (2.34 \pm 0.05) \text{ fm} \quad \text{for } ^{12}\text{C}, \\
R_m &= (2.42 \pm 0.05) \text{ fm} \quad \text{for } ^{14}\text{C}.
\end{align*}
\]

The errors indicated here and below for the deduced values of the radii include statistical and systematic uncertainties [34]. The systematic errors appear as the result of uncertainties in the absolute normalization of the experimental cross sections, as an error in the \( t \)-scale and errors introduced to the analysis from uncertainties in the parameters of the free \( pp \) and \( pn \) scattering amplitudes. Also, the contributions to the systematic errors due to corrections for the inelastic scattering and due to different model density parameterizations used are taken into account.

In the analysis it was assumed that the nuclei \(^{15}\text{C} \) and \(^{17}\text{C} \) consist of the \(^{14}\text{C} \) and \(^{16}\text{C} \) cores, respectively, and a loosely bound valence neutron. For these nuclei good descriptions of the cross sections have been achieved with all the density parameterizations used. The corresponding values of the rms matter radii \( R_m \) deduced with all four parameterizations for \(^{15}\text{C} \) and \(^{17}\text{C} \) are close to each other within rather small errors. The values of \( R_m \) averaged over the results obtained with all the density parameterizations are:
Table 1: Parameters obtained by fitting the calculated proton elastic scattering cross sections for the carbon isotopes under investigation to the measured ones for the parameterizations SF, GH, GG and GO of the nuclear matter density distributions. The presented parameters refer to point-nucleon density distributions. The parameters are as follows:

- $R_m$ – rms nuclear matter radius;
- $R_c$ – rms nuclear core radius;
- $R_v$ – rms radius of the valence ("halo") nucleon(s) distribution;
- $R_0$ – "half density radius" and $a$ – diffuseness parameter of the SF distribution;
- $\alpha$ – the parameter of the GH distribution which influences the shape of the distribution (see [34]);
- $A_n$ – normalization parameter of the calculated cross section.

$\chi^2/N_{df}$ and $A_n$ are dimensionless, all other fit parameters are given in fm. The radii $R_c$ and $R_v$ are in the c.m. system of the nucleus. All errors given are statistical only.

| Nucleus | Parameterization | $\chi^2/N_{df}$ | $A_n$ | $R_0$ | $a$ | $R_m$ | $\alpha$ | $R_v$ | $R_m$, fm |
|---------|------------------|-----------------|-------|-------|-----|-------|---------|-------|-----------|
| $^{12}$C | SF               | 30.0/33         | 1.03(1) | $R_0 = 1.98(13)$ | $a = 0.48(3)$ | 2.35(2) |         |       |           |
|         | GH               | 30.2/33         | 1.03(1) | $R_m = 2.33(1)$ | $\alpha = 0.00(2)$ | 2.33(1) |         |       |           |
| $^{14}$C | SF               | 31.1/31         | 1.01(1) | $R_0 = 0.87(32)$ | $a = 0.63(3)$ | 2.43(2) |         |       |           |
|         | GH               | 31.4/31         | 1.01(1) | $R_m = 2.41(2)$ | $\alpha = 0.11(2)$ | 2.41(2) |         |       |           |
| $^{15}$C | SF               | 32.6/29         | 1.03(1) | $R_0 = 1.56(16)$ | $a = 0.62(2)$ | 2.59(2) |         |       |           |
|         | GH               | 32.6/29         | 1.03(1) | $R_m = 2.57(2)$ | $\alpha = 0.06(2)$ | 2.57(2) |         |       |           |
|         | GG               | 34.4/29         | 1.02(1) | $R_c = 2.43(1)$ | $R_v = 4.45(43)$ | 2.61(5) |         |       |           |
|         | GO               | 33.6/29         | 1.02(1) | $R_c = 2.40(1)$ | $R_v = 4.49(33)$ | 2.60(4) |         |       |           |
| $^{16}$C | SF               | 33.5/37         | 1.04(1) | $R_0 = 1.31(25)$ | $a = 0.67(3)$ | 2.70(3) |         |       |           |
|         | GH               | 36.3/37         | 1.04(1) | $R_m = 2.68(3)$ | $\alpha = 0.09(2)$ | 2.68(3) |         |       |           |
|         | GG               | 35.3/37         | 1.04(1) | $R_c = 2.43(2)$ | $R_v = 4.36(29)$ | 2.75(6) |         |       |           |
|         | GO               | 35.0/37         | 1.04(1) | $R_c = 2.38(2)$ | $R_v = 4.35(22)$ | 2.71(4) |         |       |           |
| $^{17}$C | SF               | 35.0/37         | 1.01(1) | $R_0 = 1.97(13)$ | $a = 0.60(2)$ | 2.69(2) |         |       |           |
|         | GH               | 34.7/37         | 1.02(1) | $R_m = 2.67(2)$ | $\alpha = 0.03(2)$ | 2.67(2) |         |       |           |
|         | GG               | 35.5/37         | 1.02(1) | $R_c = 2.58(2)$ | $R_v = 3.86(54)$ | 2.68(3) |         |       |           |
|         | GO               | 35.3/37         | 1.02(1) | $R_c = 2.56(2)$ | $R_v = 4.06(40)$ | 2.67(3) |         |       |           |

For the core radius and the radius of the valence neutrons distribution, the following mean values were determined: $R_c = 2.41(5)$ fm and $R_v = 4.20(26)$ fm.

The deduced nuclear matter density distributions obtained using different parameterizations of the nuclear matter distributions are plotted in Fig. [4]. The shaded areas represent the envelopes of the density variation within the model parameterizations applied, superimposed by the statistical errors. Figure [4] also shows the obtained core matter distributions. All density distributions refer to point-nucleon distributions.
Figure 4: Total and core matter distributions $\rho(r)$ of the nuclear density in $^{14}\text{C}$ (a), $^{15}\text{C}$ (b), $^{16}\text{C}$ (c) and $^{17}\text{C}$ (d) deduced in the analysis by using model density parameterizations SF (Symmetrized Fermi), GH (Gaussian-Halo), GG (Gaussian-Gaussian), and GO (Gaussian-Oscillator), for details see the text. The shaded areas represent the envelopes of the density variation within the model parameterizations applied, superimposed by the statistical errors. All density distributions are normalized to the number of nucleons.
Using the matter radii $R_m$ deduced in the present work and the radii $R_p$ of proton distributions obtained in Refs. [49] and [9], the radii $R_n$ of neutron distributions and thicknesses of the neutron skins $\delta_{np} = R_n - R_p$ for the nuclei of the studied carbon isotopes were determined (see Table 2) with the help of expression (3):

$$R_n = \left[ (AR_m^2 - ZR_p^2)/N \right]^{1/2}. \quad (3)$$

4. Discussion

Recently, the charge-changing cross sections for the $^{12-19}$C nuclei were measured at GSI at 900 MeV/u with a carbon target by Kanungo et al. [9]. Using a finite-range Glauber model, the authors derived radii $R_p$ of the proton density distributions for the studied carbon isotopes. With these values of $R_p$ fixed, they performed a new analysis of the interaction cross sections from Ref. [16] to obtain more accurate values of the matter radii $R_m$. The authors also performed coupled-cluster computations using chiral nucleon-nucleon and three-nucleon interactions which satisfactorily describe the experimental data on proton and matter radii.

Our results on $R_m$ for the carbon isotopes are compared with the results of Ref. [9] in Table 2 and in Fig. 5. It is seen that the present results on $R_m$ turn out to be within the experimental errors in agreement with the results of Ref. [9]. In Fig. 5 are also shown experimental results of Refs. [4, 17] and two sets of theoretical predictions for the matter radii of the carbon isotopes [50, 51]. The matter radii in [50, 51] were calculated using a simple model under the assumption that the considered nuclei consist of a core plus one or two valence neutrons. Note that the radii calculated in [51] exhibit a pronounced staggering effect – the radii for the odd mass numbers are larger than the average of the radii for the neighbouring even mass numbers.

![Figure 5: Nuclear matter radii of carbon isotopes. Experimental data are: this work (circles), the results of [9] (diamonds), the result of [4] (square), and the result of [17] (triangle). Theoretical predictions are taken from [50] (solid line) and [51] (dashed line).](image)

The method applied in the given work to study the nuclear matter density distributions was previously tested with the data on proton scattering from stable nuclei $^4$He [34] and $^6$Li [36]. The differential cross...
section for $^{12}$C elastic scattering measured in this work was also used to check the method. The $^{12}$C matter radius $R_m = 2.34(5)$ fm derived in the present work is in agreement with the value of $R_m = 2.35(2)$ fm of Ref. [9]. Note that the rms charge radius of $^{12}$C is known with high precision [49] from $e^-$ scattering and muonic $x$-ray measurements: $R_{ch} = 2.470(2)$ fm. Taking into account the finite size effect of the nucleon (see, e.g., Ref. [3]) and the value of the proton charge radius $r_p = 0.8414(19)$ fm [52], the rms radius $R_p$ of the proton distribution in $^{12}$C is obtained to be $R_p = 2.34(1)$ fm. The number of neutrons in $^{12}$C is equal to that of protons, therefore the matter and proton distributions (normalized to one nucleon) are expected to be rather similar. Indeed, the $R_m$ value deduced in the present work has occurred to be equal to the value of $R_p$ extracted from the experimental data on the charge radius of $^{12}$C. This result on $^{12}$C scattering demonstrates a consistency check of the present experimental method, including the procedure of the data analysis.

The $^{14}$C nucleus is of interest as the presumable core in $^{15}$C and $^{16}$C [14]. This nucleus is supposed to have a spherical shape due to the neutron closed shell effect [45]. The present value of $R_m = 2.42(5)$ fm is in agreement within errors with the result $R_m = 2.33(7)$ fm of Ref. [9]. The charge radius $R_{ch} = 2.503(9)$ fm [49] of $^{14}$C may be used to find the corresponding radius of the proton distribution $R_p = 2.38(2)$ fm. By combining the matter radius $R_m$, deduced in the present work for $^{14}$C with the value of $R_p$, and using expression (3), the rms radius of the neutron distribution $R_n$ in $^{14}$C has been determined to be $R_n = (2.45 \pm 0.09)$ fm. Thus, within the error bars, the $^{14}$C nucleus has the same radius of the neutron distribution $R_n$ as that of the proton distribution $R_p$: $R_n \approx R_p$.

The structure of the odd isotope $^{15}$C has been considered in a $(^{14}$C-core + $n$) model. This nucleus has a small neutron separation energy $S_n = 1.218$ MeV, so it is suggested to be a candidate for a halo nucleus. A special feature of the present method is that it makes possible to determine the sizes of the nuclear core and of the halo. The ratio of the determined valence nucleon to the core nucleon radius, $\kappa = R_v/R_c$, may be used as a gauge for the halo existence [53]. Theory predicts typically values of $\kappa \leq 1.25$ for light nuclei near the valley of beta stability, while for a halo structure this value can be $\kappa \approx 2$, or even larger [2]. In the present analysis, a value of $\kappa = 1.81$ for $^{15}$C is obtained, which confirms the suggestion [3] that this nucleus demonstrates a “moderate halo formation”.

Due to the low binding energy of the halo neutron in $^{15}$C, it is natural to expect that the internal core size $R_v^*$ (size of the core in its own c.m. system) is close to that of the free $^{14}$C nucleus. The motion of the c.m. of the core around the c.m. of the whole nucleus slightly increases the effective core size $R_c$ [34]. Following Tanihata et al. [3], the internal core size $R_v^*$ in the $(\text{core} + n)$ model turns out to be

$$R_v^* = (R_c^2 - \rho_c^2)^{1/2},$$

(4)

where $\rho_c$ is the rms distance between the c.m. of the core and the c.m. of the whole nucleus:

$$\rho_c = R_v/(A - 1).$$

(5)

In the present analysis we obtain $\rho_c = 0.31(3)$ fm and $R_v^* = 2.39(5)$ fm for $^{15}$C. The latter value agrees with $R_m = 2.42(5)$ fm for $^{14}$C. Taking for $^{15}$C the proton radius $R_p = (2.37 \pm 0.03)$ fm [45], and using Eq.

| Isotope | $R_m$, fm | $R_m$, fm | $R_p$, fm | $R_n$, fm | $\delta_{np}$, fm |
|---------|-----------|-----------|-----------|-----------|-----------------|
| $^{12}$C | 2.34 (5)  | 2.35 (2)  | 2.34 (1)  | 2.34 (10) | 0.00 (10)       |
| $^{14}$C | 2.42 (5)  | 2.33 (7)  | 2.38 (2)  | 2.45 (9)  | 0.07 (9)        |
| $^{15}$C | 2.59 (5)  | 2.54 (4)  | 2.37 (3)  | 2.73 (8)  | 0.36 (9)        |
| $^{16}$C | 2.70 (6)  | 2.74 (3)  | 2.40 (4)  | 2.86 (9)  | 0.46 (10)       |
| $^{17}$C | 2.68 (5)  | 2.76 (3)  | 2.42 (4)  | 2.81 (8)  | 0.39 (9)        |
(3), the rms neutron radius for $^{15}$C is determined to be $R_n = (2.73 \pm 0.08)$ fm, and for the thickness of the neutron skin we deduce the value of $\delta_{np} = (0.36 \pm 0.09)$ fm (see Table 2).

There are several theoretical considerations of the structure of $^{16}$C, which is treated as a $(^{14}$C-core + $n+n+n$) three-body system [51]. The experimental value of $R_m = 2.70(6)$ fm, deduced in the present work for $^{16}$C, is in good agreement with existing experimental data as well as with theoretical results (Fig. 5 and Table 2). The core size in $^{16}$C ($R_c = 2.41(5)$ fm) is close to the size of the free $^{14}$C nucleus ($R_m = 2.42(5)$ fm). According to the present analysis, the ratio of the valence nucleon radius $R_v$ to the core radius $R_c$ turns out to be equal in $^{16}$C to $\kappa = 1.74$, which is smaller than the $\kappa$ values of the $2n$ halo nuclei $^{11}$Li ($\kappa = 2.71$ [84]) and $^{14}$Be ($\kappa = 1.91$ [85]) determined earlier with the same method. This observation suggests that the spatial distribution of two valence neutrons in $^{16}$C should be considered rather as a skin, than as a halo. Using the matter radius of the present work $R_m = (2.70 \pm 0.06)$ fm and the radius of the proton distribution $R_p = (2.40 \pm 0.04)$ fm $^{[3]}$, we obtain for the radius of the neutron distribution $R_n = (2.86 \pm 0.09)$ fm, and for the thickness of the neutron skin, the value $\delta_{np} = (0.46 \pm 0.10)$ fm has been deduced (see Table 2). This result is an indication of a noticeable neutron skin in $^{16}$C.

We have considered the spatial structure of the $^{17}$C nucleus in a $(^{16}$C-core + $n$) model. The neutron separation energy $S_n$ for $^{17}$C is small: $S_n = 0.728$ MeV. Therefore, one could expect $^{17}$C to be a halo nucleus. However, the ratio of the valence nucleon radius to the core radius, determined in the present work for $^{17}$C, occurs to be relatively small, $\kappa = 1.58$, which does not support the picture that $^{17}$C is a halo nucleus. With the determined value of $R_v$ and Eqs. (4) and (5) in the case of $^{17}$C we obtain $\rho_c = 0.25(3)$ fm and $R^*_c = 2.56(5)$ fm. This value of $R^*_c$ is smaller than $R_m = 2.70(6)$ fm for the free $^{16}$C nucleus. This result demonstrates a noticeable contraction of the $^{16}$C cluster inside $^{17}$C. Obviously, $^{17}$C is a more dense nucleus than $^{16}$C. It was already supposed in Ref. $^{[3]}$ that the configuration of the nucleus $^{17}$C is more complicated than that in the $(\text{core} + n)$ model.

5. Summary

The proton-nucleus elastic scattering at intermediate energies is an efficient method for the investigation of nuclear matter density distributions. In the present work, we have applied this method in inverse kinematics for the investigation of the nuclear radial structure of carbon isotopes. The absolute differential cross sections $d\sigma/dt$ were measured as a function of the four-momentum transfer squared $-t$ in the range $0.001 \leq |t| \leq 0.06$ (GeV/c)$^2$ for proton elastic scattering on the $^{12,14,15,16,17}$C nuclei. The cross sections were determined using secondary beams with energies near 700 MeV/u produced with the fragment separator FRS at GSI. The active target IKAR was used as a recoil-proton detector. The scattered projectiles were registered with a system of multi-wire proportional chambers, scintillation detectors, and a magnetic analysis. The analysis of the experimental data was performed using the Glauber multiple-scattering theory. The nuclear matter radii and the radial nuclear matter distributions for the carbon isotopes were determined from the measured cross sections $d\sigma/dt$. A good description of the experimental cross section is obtained with four phenomenological parameterizations of the nuclear density distributions (SF, GH, GG, and GO). Each of these parameterizations has two free parameters. Our results on the matter radii $R_m$ for the studied carbon isotopes are in agreement within the experimental errors with those of Ref. $^{[8]}$ evaluated from the measured interaction and charge-changing cross sections. The density distribution parameters ($R_m$, $R_p$) for $^{12}$C are well established values from measurements of the interaction cross sections and the charge radii. Therefore, the results on $p^{12}$C scattering were used as a consistency check of the present experimental method, including the procedure of the data analysis.

The measured cross sections are described fairly well within the $(\text{core} + n)$ model for $^{15}$C and $^{17}$C, and the $(\text{core} + 2n)$ model for $^{16}$C. It was shown that the size of the $^{14}$C-core in the $^{15}$C and $^{16}$C nuclei is close to that of the free $^{14}$C nucleus.

A quantitative description of the halo structure for $^{15,16,17}$C was performed in the analysis of the nuclear matter distributions in these nuclei. The ratio of the valence nucleon to the core nucleon radius $\kappa = R_v/R_c$ was used as a gauge for the halo existence, where a value of $\kappa \gtrsim 2$ is expected for a halo nucleus.

The present analysis describes $^{15}$C as a halo nucleus with $\kappa = 1.82$, while $^{16}$C ($\kappa = 1.74$) and $^{17}$C ($\kappa = 1.58$) are considered as nuclei with a noticeable neutron skin. This conclusion is in agreement with
the investigation of fragmentation reactions using radioactive carbon beams. Note that a narrow fragment momentum distribution as a signature of an extended valence nucleon density distribution in a halo nucleus was observed in the considered here carbon isotopes only for $^{15}$C \cite{10, 11, 18, 19}, whereas broad fragment momentum distributions for $^{16}$C \cite{27} and $^{17}$C \cite{10, 11, 19, 20} imply no halo formation in these nuclei.

Besides the determination of the nucleon density distributions and their parameters, the precise data obtained for the differential proton elastic-scattering cross sections allow a sensitive test of theoretical predictions on the structure of the neutron-rich carbon nuclei. For this purpose, the nuclear density distributions obtained from various theoretical approaches may be used as an input to the Glauber multiple-scattering theory. Then the calculated elastic-scattering cross sections should be compared to the experimental data as it was done in Refs. \cite{34}--\cite{36}.

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Appendix

The measured cross sections $d\sigma/dt$ for $^{12,14-17}\text{C}$ elastic scattering as a function of the four-momentum transfer squared $-t$. The indicated errors are statistical only.

| $^\text{12}\text{C}, E_p=705.2 \text{ MeV}$ | $^\text{12}\text{C}, E_p=705.2 \text{ MeV}$ |
|------------------------------------------|------------------------------------------|
| $-t$, (GeV/c)$^2$ | $d\sigma/dt$, mb/(GeV/c)$^2$ | $-t$, (GeV/c)$^2$ | $d\sigma/dt$, mb/(GeV/c)$^2$ |
| 0.00117 | 14965. ± 297.3 | 0.01300 | 2858.1 ± 67.1 |
| 0.00164 | 9489.3 ± 229.4 | 0.01490 | 2448.0 ± 60.5 |
| 0.00211 | 7942.5 ± 205.8 | 0.01694 | 2061.6 ± 54.3 |
| 0.00258 | 7060.1 ± 195.8 | 0.01910 | 1871.3 ± 50.7 |
| 0.00305 | 6335.2 ± 185.1 | 0.02140 | 1549.8 ± 45.3 |
| 0.00352 | 5742.1 ± 175.8 | 0.02382 | 1358.2 ± 41.7 |
| 0.00399 | 5620.2 ± 173.9 | 0.02636 | 1160.9 ± 38.0 |
| 0.00446 | 5290.9 ± 168.8 | 0.02904 | 924.7 ± 33.5 |
| 0.00493 | 5171.5 ± 166.9 | 0.03185 | 745.1 ± 29.8 |
| 0.00540 | 4517.4 ± 156.2 | 0.03748 | 589.2 ± 26.3 |
| 0.00586 | 4713.5 ± 159.9 | 0.03785 | 495.9 ± 24.0 |
| 0.00633 | 4636.6 ± 160.4 | 0.04104 | 383.7 ± 21.0 |
| 0.00680 | 4250.1 ± 155.3 | 0.04437 | 309.6 ± 18.9 |
| 0.00727 | 4317.7 ± 155.3 | 0.04782 | 223.6 ± 16.1 |
| 0.00774 | 3883.2 ± 147.3 | 0.05141 | 188.6 ± 14.9 |
| 0.00804 | 3793.6 ± 84.1 | 0.05513 | 131.3 ± 12.6 |
| 0.00807 | 3793.6 ± 77.7 | 0.05897 | 85.3 ± 10.4 |
| 0.01122 | 3023.1 ± 71.0 |  |  |

| $^\text{14}\text{C}, E_p = 704.4 \text{ MeV}$ | $^\text{14}\text{C}, E_p = 704.4 \text{ MeV}$ |
|------------------------------------------|------------------------------------------|
| $-t$, (GeV/c)$^2$ | $d\sigma/dt$, mb/(GeV/c)$^2$ | $-t$, (GeV/c)$^2$ | $d\sigma/dt$, mb/(GeV/c)$^2$ |
| 0.00117 | 16137. ± 435.8 | 0.00989 | 3859.5 ± 126.6 |
| 0.00164 | 10641. ± 322.6 | 0.01137 | 3242.1 ± 78.9 |
| 0.00211 | 8626.2 ± 284.4 | 0.01350 | 2855.9 ± 71.7 |
| 0.00258 | 7779.0 ± 254.7 | 0.01581 | 2434.2 ± 64.4 |
| 0.00305 | 7125.5 ± 245.8 | 0.01830 | 2024.8 ± 57.9 |
| 0.00352 | 6813.3 ± 241.4 | 0.02096 | 1686.9 ± 50.9 |
| 0.00399 | 6227.4 ± 228.9 | 0.02381 | 1434.8 ± 46.7 |
| 0.00446 | 5802.7 ± 223.0 | 0.02683 | 1131.7 ± 39.6 |
| 0.00493 | 5678.4 ± 218.2 | 0.03004 | 908.8 ± 35.3 |
| 0.00540 | 5115.2 ± 209.8 | 0.03342 | 705.7 ± 30.4 |
| 0.00586 | 4870.5 ± 211.9 | 0.03699 | 614.9 ± 28.6 |
| 0.00633 | 5206.6 ± 224.0 | 0.04074 | 413.7 ± 23.8 |
| 0.00680 | 5071.9 ± 218.3 | 0.04467 | 352.4 ± 23.0 |
| 0.00727 | 4894.1 ± 203.3 | 0.04878 | 253.5 ± 18.2 |
| 0.00774 | 4424.1 ± 180.1 | 0.05307 | 140.5 ± 14.6 |
| 0.00807 | 4368.8 ± 139.0 | 0.05755 | 133.9 ± 12.8 |
| 0.00896 | 4081.9 ± 132.7 |  |  |
\begin{center}
\begin{tabular}{|c|c|c|}
\hline
$p^{15}\text{C}, \ E_p = 702.5 \text{ MeV}$ & $p^{15}\text{C}, \ E_p = 702.5 \text{ MeV}$ \\
\hline
$-t, (\text{GeV}/c)^2$ & $d\sigma/dt, \text{ mb}/(\text{GeV}/c)^2$ & $-t, (\text{GeV}/c)^2$ & $d\sigma/dt, \text{ mb}/(\text{GeV}/c)^2$ \\
\hline
0.00117 & 16475.9 ± 362.3 & 0.01069 & 3769.1 ± 89.2 \\
0.00164 & 12658.6 ± 322.1 & 0.01290 & 3332.1 ± 80.9 \\
0.00211 & 10386.9 ± 291.0 & 0.01532 & 2758.2 ± 71.2 \\
0.00258 & 8421.4 ± 261.6 & 0.01793 & 2253.4 ± 62.6 \\
0.00305 & 7541.3 ± 248.2 & 0.02075 & 1780.6 ± 54.2 \\
0.00352 & 7533.8 ± 248.2 & 0.02377 & 1379.4 ± 46.7 \\
0.00399 & 7071.4 ± 239.6 & 0.02699 & 1125.2 ± 41.3 \\
0.00446 & 6746.6 ± 235.1 & 0.03042 & 850.1 ± 35.3 \\
0.00493 & 6971.2 ± 239.6 & 0.03405 & 607.0 ± 29.4 \\
0.00540 & 6003.9 ± 222.9 & 0.03789 & 410.5 ± 23.9 \\
0.00586 & 6323.3 ± 229.7 & 0.04193 & 325.4 ± 21.0 \\
0.00633 & 5916.3 ± 221.1 & 0.04617 & 206.0 ± 16.6 \\
0.00680 & 5276.6 ± 208.6 & 0.05063 & 165.0 ± 14.7 \\
0.00727 & 5385.0 ± 215.6 & 0.05529 & 94.3 ± 11.2 \\
0.00774 & 4831.4 ± 206.5 & 0.06016 & 62.0 ± 9.1 \\
0.00820 & 4818.7 ± 104.4 & & \\
\hline
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
$p^{16}\text{C}, \ E_p = 700.5 \text{ MeV}$ & $p^{16}\text{C}, \ E_p = 700.5 \text{ MeV}$ \\
\hline
$-t, (\text{GeV}/c)^2$ & $d\sigma/dt, \text{ mb}/(\text{GeV}/c)^2$ & $-t, (\text{GeV}/c)^2$ & $d\sigma/dt, \text{ mb}/(\text{GeV}/c)^2$ \\
\hline
0.00117 & 19706.1 ± 495.4 & 0.01405 & 3295.4 ± 136.0 \\
0.00164 & 12894.0 ± 412.0 & 0.01535 & 2828.7 ± 123.8 \\
0.00211 & 11953.1 ± 394.6 & 0.01671 & 2488.2 ± 114.3 \\
0.00258 & 9308.8 ± 346.6 & 0.01813 & 2115.6 ± 103.8 \\
0.00305 & 9144.8 ± 343.3 & 0.01961 & 2099.7 ± 102.3 \\
0.00352 & 8549.8 ± 331.8 & 0.02114 & 1863.2 ± 95.0 \\
0.00399 & 7662.0 ± 314.4 & 0.02273 & 1496.5 ± 84.1 \\
0.00446 & 7337.3 ± 307.9 & 0.02438 & 1368.8 ± 79.5 \\
0.00493 & 7317.3 ± 308.0 & 0.02609 & 1133.0 ± 71.5 \\
0.00540 & 6907.9 ± 300.3 & 0.02785 & 922.9 ± 63.9 \\
0.00586 & 6707.5 ± 296.5 & 0.02967 & 856.7 ± 61.1 \\
0.00633 & 5881.3 ± 276.2 & 0.03155 & 729.5 ± 55.8 \\
0.00680 & 6069.1 ± 277.1 & 0.03349 & 592.8 ± 49.9 \\
0.00727 & 5263.5 ± 265.4 & 0.03548 & 507.3 ± 45.9 \\
0.00774 & 5162.7 ± 266.9 & 0.03858 & 374.0 ± 27.7 \\
0.00820 & 5235.1 ± 188.7 & 0.04291 & 304.7 ± 24.7 \\
0.00871 & 4564.0 ± 173.9 & 0.04748 & 178.3 ± 18.9 \\
0.01047 & 4302.8 ± 165.1 & 0.05228 & 115.4 ± 15.1 \\
0.01161 & 3954.3 ± 154.7 & 0.05732 & 63.6 ± 11.5 \\
0.01280 & 3543.2 ± 143.6 & & \\
\hline
\end{tabular}
\end{center}
| $-t$ (GeV/c)$^2$ | $\frac{d\sigma}{dt}$, mb/(GeV/c)$^2$ | $-t$ (GeV/c)$^2$ | $\frac{d\sigma}{dt}$, mb/(GeV/c)$^2$ |
|---------------|-----------------|---------------|-----------------|
| 0.00117       | 18437.1 ± 429.9 | 0.01460       | 3160.4 ± 108.3  |
| 0.00164       | 13783.9 ± 361.8 | 0.01601       | 2718.3 ± 98.8   |
| 0.00211       | 12008.3 ± 335.5 | 0.01748       | 2508.4 ± 93.5   |
| 0.00258       | 10474.2 ± 312.1 | 0.01901       | 2210.0 ± 86.5   |
| 0.00305       | 9801.4 ± 301.2  | 0.02062       | 1830.3 ± 77.8   |
| 0.00352       | 9018.8 ± 288.7  | 0.02228       | 1758.0 ± 75.4   |
| 0.00399       | 9179.1 ± 291.5  | 0.02401       | 1443.9 ± 67.6   |
| 0.00446       | 8061.6 ± 273.2  | 0.02581       | 1200.3 ± 61.1   |
| 0.00493       | 7765.3 ± 268.8  | 0.02766       | 1007.4 ± 55.6   |
| 0.00540       | 7172.2 ± 258.9  | 0.02959       | 892.8 ± 51.9    |
| 0.00586       | 7054.6 ± 257.8  | 0.03158       | 689.5 ± 45.5    |
| 0.00633       | 7343.3 ± 266.4  | 0.03363       | 543.5 ± 40.2    |
| 0.00680       | 6387.0 ± 252.1  | 0.03575       | 461.0 ± 37.0    |
| 0.00727       | 6230.8 ± 245.6  | 0.03794       | 394.7 ± 34.1    |
| 0.00774       | 5526.5 ± 231.5  | 0.04019       | 285.1 ± 29.1    |
| 0.00825       | 5705.9 ± 160.4  | 0.04369       | 210.8 ± 17.7    |
| 0.00961       | 4955.8 ± 147.2  | 0.04858       | 125.0 ± 13.8    |
| 0.01076       | 4738.9 ± 140.7  | 0.05374       | 62.9 ± 10.1     |
| 0.01198       | 3829.4 ± 123.7  | 0.05916       | 44.4 ± 8.5      |
| 0.01326       | 3681.4 ± 119.0  |              |                 |