Modified Ratio-Cum-Product Estimators of Population Mean Using Two Auxiliary Variables

J. O. Muili1*, E. N. Agwamba1, Y. A. Erinola1, M. A. Yunusa2, A. Audu2 and M. A. Hamzat2

1Department of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria.
2Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.

Authors’ contributions

This work was carried out in collaboration among all authors. Author JOM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors ENA, YAE, MAY and AA managed the analyses of the study. Author MAH managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJRCOS/2020/v6i130152
(1) Dr. G. Sudheer, Gayatri Vidya Parishad College of Engineering for Women - [GVPW], India.
(2) Heiber Cardenas Henao, University of Valle, Colombia.
(3) Muamber Yılmaz, Bartın University, Turkey.
Complete Peer review History: http://www.sdiarticle4.com/review-history/59248

ABSTRACT

A percentile is one of the measures of location used by statisticians showing the value below which a given percentage of observations in a group of observations fall. A family of ratio-cum-product estimators for estimating the finite population mean of the study variable when the finite population mean of two auxiliary variables are known in simple random sampling without replacement (SRSWOR) have been proposed. The main purpose of this study is to develop new ratio-cum-product estimators in order to improve the precision of estimation of population mean in sample random sampling without replacement using information of percentiles with two auxiliary variables. The expressions of the bias and mean square error (MSE) of the proposed estimators were derived by Taylor series method up to first degree of approximation. The efficiency conditions under which the proposed ratio-cum-product estimators are better than sample mean, ratio estimator, product estimator and other estimators considered in this study have been established. The numerical and empirical results show that the proposed estimators are more efficient than the sample mean, ratio estimator, product estimator and other existing estimators.

*Corresponding author: E-mail: jamiunice@yahoo.com;
1. INTRODUCTION

A percentile is one of the measures of location used by statisticians showing the value below which a given percentage of observations in a group of observations fall. Percentiles play an important part in descriptive statistics and their use is well recommended. Percentiles divide a set of ordered data into hundredths. Median (Mₚ) is the 50th percentile. In a situation where auxiliary information is available, it is possible to devise suitable ways of using it in obtaining the sample strategies which are better than those in which no such information is used. When the information on an auxiliary variable X is known, a ratio, product or linear regression estimator could be employed for the estimation of finite population mean or variance. Cochran [1] made an important contribution to the modern sampling theory by suggesting methods of using the auxiliary information for the purpose of estimation of population mean so as to increase the precision of the estimates. Cochran [1] developed the ratio estimator to estimate population mean or variance. Cochran [1] made an important contribution to the modern sampling theory by suggesting methods of using the auxiliary information for the purpose of estimation of population mean so as to increase the precision of the estimates. Cochran [1] developed the ratio estimator to estimate population mean or variance.

Many authors have developed ratio and product type estimators for estimating population mean of study variable like Upadhayaya and Singh [2], Abu-Dayyeh [3], Singh et al. [4], Kadilar and Cingi [5], Tailor et al. [6], Jeelani et al. [7], Gupta and Yadav [8], Muili et al. [9], Muili and Audu [10], Muili et al. [11], etc. None of the above authors have used percentiles as population parameters for estimating population mean of study variable.

The purpose of this study is to develop new ratio-cum-product estimators to improve the precision of estimation of population mean in sample random sampling without replacement using information of percentiles with two auxiliary variables.

Consider \( U = \{U_1, U_2, U_3, \ldots, U_N\} \) be a finite population having \( N \) units and each \( U_i = (X_i, Y_i) \), \( i = 1, 2, 3, \ldots, N \) has a pair of values. \( Y \) is the study variable and \( X_1 \) and \( X_2 \) are the two auxiliary variables which are correlated with \( Y \). Let \( y = \{y_1, y_2, \ldots, y_n\} \), \( x_1 = \{x_{11}, x_{12}, \ldots, x_{1n}\} \), and \( x_2 = \{x_{21}, x_{22}, \ldots, x_{2n}\} \) be \( n \) sample values. \( \bar{Y}, \bar{X}_1, \bar{X}_2 \) are the sample means of the study and auxiliary variables respectively. \( s_y^2 \) and \( s_{x_i}^2 \) be the mean square population of \( Y \) and \( X_i \) respectively and \( s_y^2 \) and \( s_{x_i}^2 \) be respective sample mean square based on the random sample of size \( n \) drawn without replacement.

\( N \) : Population size, \( n \) : Sample size, \( \bar{Y}, \bar{X}_1, \bar{X}_2 \) : Population means of study and auxiliary variables \( \rho_{xy} \) : Coefficient of correlation, \( C_y, C_{x_i} \) : Coefficient of variations of study and auxiliary variables, \( \beta_{2(x,y)} \) : Coefficient of Kurtosis of auxiliary variables, \( M_d(x_i) \) : Median of the auxiliary variables. Sampling fraction \( \left( f \right) \) is the ratio of sample size to population size. Percentage Relative Efficiency (PRE) is a statistical tool used to measure and ascertain the efficiency of one estimator over another.

\[
\begin{align*}
\bar{X}_1 &= \frac{1}{N} \sum_{i=1}^{N} X_{1i}, \quad \bar{X}_2 = \frac{1}{N} \sum_{i=1}^{N} X_{2i}, \quad \bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i, \quad \bar{x}_1 = \frac{1}{n} \sum_{i=1}^{n} x_{1i}, \quad \bar{x}_2 = \frac{1}{n} \sum_{i=1}^{n} x_{2i}, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i, \\
S_y^2 &= \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2, \quad S_{x_i}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_{1i} - \bar{x}_1)^2, \quad S_{x_2}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_{2i} - \bar{x}_2)^2, \quad S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \bar{Y})^2, \\
S_{x_1}^2 &= \frac{1}{N-1} \sum_{i=1}^{N} (X_{1i} - \bar{X}_1)^2, \quad S_{x_2}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_{2i} - \bar{X}_2)^2, \quad \gamma = \frac{1-f}{n}, \quad f = \frac{n}{N}, \quad PRE = \frac{V(t_k)}{MSE(t_k)} X 100, \\
C_y^2 &= \frac{S_y^2}{Y}, \quad C_{x_i}^2 = \frac{S_{x_i}^2}{X_i} \text{ and } C_{x_2}^2 = \frac{S_{x_2}^2}{X_2}
\end{align*}
\]
2. MATERIALS AND METHODS

The problem of estimating population mean of the study variable when the population mean of an auxiliary variable(s) is/are known has been discussed among the statisticians in the field of sample survey. Robson [12] developed a product estimator for estimating population mean. Also, Murthy [13] proposed a product type estimator to estimate population mean of study variable by the used of auxiliary information when the coefficient of correlation is negative. Singh and Tailor [14] developed a family of estimators using known values of some parameters to estimate the population mean of the study variable. Abid et al. [15] proposed a class of ratio estimators incorporating non-conventional location parameters for the estimation of population mean. Other researchers are Kadilar and Cingi [16], Koyuncu and Kadilar [17], Yan and Tian [18], Subramani and Kumarapandiyam [19], Yadav et al. [20], Gupta and Yadav [21], Muili et al. [22-25] Audu et al. [26], [27] Muili et al. [28], etc.

Sample mean \( \bar{y} \) in simple random sampling without replacement is given as:

\[
\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \quad (1.0)
\]

\[
V(\bar{y}) = \gamma \bar{y}^2 C_y^2 \quad (1.1)
\]

Cochran [1] ratio estimator for estimating population mean \( \bar{Y} \) of the study variable \( Y \) is given as:

\[
t_R = \frac{\bar{y}}{\bar{x}_1} \quad (1.2)
\]

\[
Bias(t_R) = \gamma \bar{y} \left( C_{x_1}^2 - \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.3)
\]

\[
MSE(t_R) = \gamma \bar{y}^2 \left( C_y^2 + C_{x_1}^2 - 2 \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.4)
\]

Robson [12] developed a product estimator for estimating population mean \( \bar{Y} \) of the study variable \( Y \) given as:

\[
t_p = \frac{\bar{y}}{\bar{x}_2} \quad (1.5)
\]

\[
Bias(t_p) = \gamma \bar{y} \left( C_{x_1}^2 + \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.6)
\]

\[
MSE(t_p) = \gamma \bar{y}^2 \left( C_y^2 + C_{x_1}^2 + 2 \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.7)
\]

Upadhyaya and Singh [2] developed ratio and product estimators for estimation of population mean using known values of coefficient of variation \( C_{x_1} \) and coefficient of kurtosis \( \beta_2(x_1) \) of variable variables with their biases and mean squares errors (MSEs) given as:

\[
t_1 = \frac{\bar{x}_1 C_{x_1} + \beta_1(x_1)}{\bar{x}_1 C_{x_1} + \beta_2(x_1)} \quad (1.8)
\]

\[
t_2 = \frac{\bar{x}_2 C_{x_2} + \beta_1(x_2)}{\bar{x}_2 C_{x_2} + \beta_2(x_2)} \quad (1.9)
\]

\[
t_3 = \frac{\bar{x}_1 \beta_2(x_1) + C_{x_1}}{\bar{x}_1 \beta_2(x_1) + C_{x_1}} \quad (1.11)
\]

\[
t_4 = \frac{\bar{x}_2 \beta_2(x_2) + C_{x_2}}{\bar{x}_2 \beta_2(x_2) + C_{x_2}} \quad (1.12)
\]

\[
Bias(t_1) = \gamma \bar{y} \left( \lambda_{2}^2 C_{x_1} - \lambda_1 \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.13)
\]

\[
Bias(t_2) = \gamma \bar{y} \left( \lambda_{2}^2 C_{x_2} + \lambda_2 \rho_{y,x_2} C_y C_{x_2} \right) \quad (1.14)
\]

\[
Bias(t_3) = \gamma \bar{y} \left( \lambda_{2}^2 C_{x_1} - \lambda_3 \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.15)
\]

\[
Bias(t_4) = \gamma \bar{y} \left( \lambda_{2}^2 C_{x_2} + \lambda_4 \rho_{y,x_2} C_y C_{x_2} \right) \quad (1.16)
\]

\[
MSE(t_1) = \gamma \bar{y}^2 \left( C_y^2 + \lambda_{2}^2 C_{x_1}^2 - 2 \lambda_1 \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.17)
\]

\[
MSE(t_2) = \gamma \bar{y}^2 \left( C_y^2 + \lambda_{2}^2 C_{x_2}^2 + 2 \lambda_2 \rho_{y,x_2} C_y C_{x_2} \right) \quad (1.18)
\]

\[
MSE(t_3) = \gamma \bar{y}^2 \left( C_y^2 + \lambda_{2}^2 C_{x_1}^2 - 2 \lambda_3 \rho_{y,x_1} C_y C_{x_1} \right) \quad (1.19)
\]

\[
MSE(t_4) = \gamma \bar{y}^2 \left( C_y^2 + \lambda_{2}^2 C_{x_2}^2 + 2 \lambda_4 \rho_{y,x_2} C_y C_{x_2} \right) \quad (1.21)
\]
where \( \lambda_1 = \frac{\bar{X}_1 C_{x_1}}{\bar{X}_1 C_{x_1} + \beta_1(x_1)} \), \( \lambda_2 = \frac{\bar{X}_2 C_{x_2}}{\bar{X}_2 C_{x_2} + \beta_2(x_2)} \), \( \lambda_3 = \frac{\bar{X}_1 \beta_2(x_1)}{\bar{X}_1 \beta_2(x_1) + C_{x_1}} \), \( \lambda_4 = \frac{\bar{X}_2 \beta_2(x_2) + C_{x_2}}{\bar{X}_2 \beta_2(x_2) + C_{x_2}} \).

Singh [29] proposed a ratio-cum-product estimator of population mean using the two auxiliary variables as:

\[
t_5 = \bar{y} \left( \frac{\bar{x}_1}{\bar{x}_1} \right) \left( \frac{\bar{x}_2}{\bar{x}_2} \right)
\]

\[
\text{Bias} \left( t_5 \right) = \gamma \bar{y} \left( C^2_{x_1} \left( 1 - \kappa_{x_{y_1}} \right) + C^2_{x_2} \left( \kappa_{x_{y_2}} - \kappa_{x_{y_2}} \right) \right)
\]

\[
\text{MSE} \left( t_5 \right) = \gamma \bar{y}^2 \left( C^2_{x_1} + C^2_{x_2} \left( 1 - 2 \kappa_{x_{y_1}} \right) + C^2_{x_2} \left( 1 + 2 \left( \kappa_{x_{y_2}} - \kappa_{x_{y_2}} \right) \right) \right)
\]

Singh and Tailor [30] also developed a ratio-cum-product estimator for estimation of population mean incorporated coefficient of variation between auxiliary variables into the work of Singh (1967) as:

\[
t_6 = \bar{y} \left( \frac{\bar{x}_1 + \rho_{x_{y_1}}}{\bar{x}_1 + \rho_{x_{y_1}}} \right) \left( \frac{\bar{x}_2 + \rho_{x_{y_2}}}{\bar{x}_2 + \rho_{x_{y_2}}} \right)
\]

\[
\text{Bias} \left( t_6 \right) = \gamma \bar{y} \left( \mu_1 C^2_{x_1} \left( \mu_1 - \kappa_{x_{y_1}} \right) + \mu_2 C^2_{x_2} \left( \mu_2 \kappa_{x_{y_2}} - \mu_2 \kappa_{x_{y_2}} \right) \right)
\]

\[
\text{MSE} \left( t_6 \right) = \gamma \bar{y}^2 \left( C^2_{x_1} + \mu_1 C^2_{x_1} \left( \mu_1 - 2 \kappa_{x_{y_1}} \right) + \mu_2 C^2_{x_2} \left( \mu_2 + 2 \left( \kappa_{x_{y_2}} - \kappa_{x_{y_2}} \right) \right) \right)
\]

Where \( \mu_1 = \frac{\bar{x}_1}{\bar{x}_1 + \rho_{x_{y_1}}} \), and \( \mu_2 = \frac{\bar{x}_2}{\bar{x}_2 + \rho_{x_{y_2}}} \).

Tailor et al. [31] proposed two ratio-cum-product estimators of population mean using both coefficient of variation and coefficient of kurtosis of auxiliary variables as:

\[
t_7 = \bar{y} \left( \frac{\bar{x}_1 C_{x_1} + \beta_1(x_1)}{\bar{x}_1 C_{x_1} + \beta_1(x_1)} \right) \left( \frac{\bar{x}_2 C_{x_2} + \beta_2(x_2)}{\bar{x}_2 C_{x_2} + \beta_2(x_2)} \right)
\]

\[
t_8 = \bar{y} \left( \frac{\bar{x}_1 \beta_1(x_1) + C_{x_1}}{\bar{x}_1 \beta_1(x_1) + C_{x_1}} \right) \left( \frac{\bar{x}_2 \beta_2(x_2) + C_{x_2}}{\bar{x}_2 \beta_2(x_2) + C_{x_2}} \right)
\]

\[
\text{Bias} \left( t_7 \right) = \gamma \bar{y} \left( \lambda_1 C^2_{x_1} \left( \lambda_1 - \kappa_{x_{y_1}} \right) + \lambda_2 C^2_{x_2} \left( \lambda_2 \kappa_{x_{y_2}} - \lambda_2 \kappa_{x_{y_2}} \right) \right)
\]

\[
\text{MSE} \left( t_7 \right) = \gamma \bar{y}^2 \left( C^2_{x_1} + \lambda_1 C^2_{x_1} \left( \lambda_1 - 2 \kappa_{x_{y_1}} \right) + \lambda_2 C^2_{x_2} \left( \lambda_2 + 2 \left( \kappa_{x_{y_2}} - \kappa_{x_{y_2}} \right) \right) \right)
\]

\[
\text{Bias} \left( t_8 \right) = \gamma \bar{y} \left( \lambda_3 C^2_{x_1} \left( \lambda_3 - \kappa_{x_{y_1}} \right) + \lambda_4 C^2_{x_2} \left( \lambda_4 \kappa_{x_{y_2}} - \lambda_4 \kappa_{x_{y_2}} \right) \right)
\]

\[
\text{MSE} \left( t_8 \right) = \gamma \bar{y}^2 \left( C^2_{x_1} + \lambda_3 C^2_{x_1} \left( \lambda_3 - 2 \kappa_{x_{y_1}} \right) + \lambda_4 C^2_{x_2} \left( \lambda_4 \kappa_{x_{y_2}} - \lambda_4 \kappa_{x_{y_2}} \right) \right)
\]
\[
MSE (t_y) = \gamma \bar{Y}^2 \left( C_Y^2 + \lambda_3 C_{x_1}^2 \left( \lambda_3 - 2 \kappa_{x_1} \right) + \lambda_4 C_{x_1}^2 \left( \lambda_4 + 2 \left( \kappa_{x_2} - \lambda_3 \kappa_{x_2} \right) \right) \right) \tag{1.34}
\]

Yadav et al. [32] developed a ratio-cum-product for estimation of population mean using known values of coefficient of kurtosis and median of auxiliary variables as:

\[
t_y = \bar{Y} \left( \frac{\bar{X}_1 \beta_2 (x_1) + M_d (x_1)}{\bar{X}_1 \beta_2 (x_1) + M_d (x_2)} \right) \tag{2.1}
\]

\[
Bias (t_y) = \gamma \bar{Y} \left( \eta_1 C_{x_1}^2 \left( \eta_1 - \kappa_{x_1} \right) + \eta_2 C_{x_1}^2 \left( \eta_2 \kappa_{x_2} - \eta_1 \kappa_{x_2} \right) \right) \tag{1.36}
\]

\[
MSE (t_y) = \gamma \bar{Y}^2 \left( C_Y^2 + \eta_1 C_{x_1}^2 \left( \eta_1 - 2 \kappa_{x_1} \right) + \eta_2 C_{x_1}^2 \left( \eta_2 + 2 \left( \kappa_{x_2} - \eta_1 \kappa_{x_2} \right) \right) \right) \tag{1.37}
\]

Where \( \eta_1 = \frac{\bar{X}_1 \beta_1 (x_1)}{\bar{X}_1 \beta_2 (x_1) + M_d (x_1)}, \eta_2 = \frac{\bar{X}_1 \beta_2 (x_1)}{\bar{X}_1 \beta_2 (x_1) + M_d (x_2)} \)

### 2.1 Proposed Estimator

Having studied the works of Singh and Tailor [30], Tailor et al. [31] and Yadav et al. [32], we proposed a family of ratio-cum-product estimators for estimating population mean using information of percentiles of auxiliary variables as:

\[
t_{pi} = \bar{Y} \left( \frac{\bar{X}_1 \beta_2 (x_1) + P_{i5} (x_1)}{\bar{X}_1 \beta_2 (x_1) + P_{i5} (x_2)} \right) \tag{2.11}
\]

\[
t_{pi} = \bar{Y} \left( \frac{\bar{X}_1 \beta_2 (x_1) + P_{i6} (x_1)}{\bar{X}_1 \beta_2 (x_1) + P_{i6} (x_2)} \right) \tag{2.12}
\]

where

\[
i = 1, 2, \ldots, 10 \quad k = 55, 60, \ldots, 99
\]

#### 2.1.1 Properties (bias and MSE) of the proposed estimators

To obtain the bias and MSE, we define

\[
\epsilon_0 = \frac{\bar{Y} - \bar{Y}}{\bar{Y}}, \quad \epsilon_1 = \frac{\bar{X}_1 - \bar{X}}{\bar{X}_1} \text{ and } \epsilon_2 = \frac{\bar{X}_2 - \bar{X}}{\bar{X}_2}
\]

such that

\[
\bar{Y} = \bar{Y} (1 + \epsilon_0), \quad \bar{X}_1 = \bar{X}_1 (1 + \epsilon_1) \text{ and } \bar{X}_2 = \bar{X}_2 (1 + \epsilon_2)
\]

from the definitions of \( \epsilon_0, \epsilon_1 \) and \( \epsilon_2 \), we obtain

\[
such that
\]

\[
\bar{Y} = \bar{Y} (1 + \epsilon_0), \quad \bar{X}_1 = \bar{X}_1 (1 + \epsilon_1) \text{ and } \bar{X}_2 = \bar{X}_2 (1 + \epsilon_2)
\]

from the definitions of \( \epsilon_0, \epsilon_1 \) and \( \epsilon_2 \), we obtain

\[
\bar{Y} = \bar{Y} (1 + \epsilon_0), \quad \bar{X}_1 = \bar{X}_1 (1 + \epsilon_1) \text{ and } \bar{X}_2 = \bar{X}_2 (1 + \epsilon_2)
\]
Expressing (2.12) in terms of $e_0$, $e_1$ and $e_2$, we have

$$\begin{align*}
t_{pl} &= \bar{Y} \left(1 + e_0 \right) \left( 1 + \beta_1(x_1) + P_1(x_1) \right) \\
&= \bar{Y} \left(1 + e_0 \right) \left( 1 + \phi_1 e_1 \right) \left(1 + \phi_2 e_2 \right)
\end{align*}$$

(2.14)

where $\phi_1 = \frac{\bar{X}_1 \beta_1(x_1)}{\bar{X}_1 \beta_1(x_1) + P_1(x_1)}$, $\phi_2 = \frac{\bar{X}_2 \beta_2(x_2)}{\bar{X}_2 \beta_2(x_2) + P_2(x_2)}$

Simplifying (2.15) up to first order approximation, it reduces to (2.16) as:

$$\begin{align*}
t_{pl} &= \bar{Y} \left(1 + e_0 - \phi_1 e_1 - \phi_1 e_0 e_1 + \phi_1^2 e_1^2 + \phi_2 e_2 + \phi_2 e_0 e_2 - \phi_1 \phi_2 e_1 e_2 \right)
\end{align*}$$

(2.16)

Subtracting $\bar{Y}$ from both sides

$$\begin{align*}
\left(t_{pl} - \bar{Y}\right) &= \bar{Y} + \bar{Y} \left( e_0 - \phi_1 e_1 - \phi_1 e_0 e_1 + \phi_1^2 e_1^2 + \phi_2 e_2 + \phi_2 e_0 e_2 - \phi_1 \phi_2 e_1 e_2 \right) - \bar{Y}
\end{align*}$$

(2.17)

Taking expectation of both sides

$$\begin{align*}
E \left(t_{pl} - \bar{Y}\right) &= \bar{Y} E \left( e_0 - \phi_1 e_1 - \phi_1 e_0 e_1 + \phi_1^2 e_1^2 + \phi_2 e_2 + \phi_2 e_0 e_2 - \phi_1 \phi_2 e_1 e_2 \right)
\end{align*}$$

(2.18)

Applying the results of (2.13) to (2.18), gives the bias as:

$$\begin{align*}
Bias(t_{pl}) &= \gamma \bar{Y} \left( \phi_1^2 C_y - \phi_1 \rho_{y,x_1} C_y C_{x_1} + \phi_2 \rho_{y,x_2} C_y C_{x_2} - \phi_1 \phi_2 \rho_{y,x_1} C_{x_1} C_{x_2} \right)
\end{align*}$$

(2.19)

$$\begin{align*}
Bias(t_{pl}) &= \gamma \bar{Y} \left( \phi_1^2 C_y \left( \phi_1 - \kappa_{y,x_1} \right) + \phi_2^2 C_y \left( \phi_2 - \kappa_{y,x_2} \right) \right), \quad i = 1, 2, ..., 10
\end{align*}$$

(2.20)

where $\kappa_{y,x_1} = \rho_{y,x_1} \left( \frac{C_y}{C_{x_1}} \right)$, $\kappa_{y,x_2} = \rho_{y,x_2} \left( \frac{C_y}{C_{x_2}} \right)$

Squaring and taking expectation of (2.18), gives

$$\begin{align*}
MSE \left(t_{pl}\right) &= \left( \bar{Y} E \left( e_0 - \phi_1 e_1 + \phi_2 e_2 \right) \right)^2
\end{align*}$$

(2.22)

Expanding (2.22)

$$\begin{align*}
MSE \left(t_{pl}\right) &= \bar{Y}^2 E \left( e_0^2 + \phi_1^2 e_1^2 + \phi_2^2 e_2^2 - 2 \phi_1 e_0 e_1 + 2 \phi_2 e_0 e_2 - 2 \phi_1 \phi_2 e_1 e_2 \right)
\end{align*}$$

(2.23)

Applying the results of (2.13) to (2.23), gives
\[ \text{MSE}(t_{pi}) = \gamma F^2 \left( C_y^2 + \phi_i^2 C_{x_1}^2 + \phi_i^2 C_{x_2}^2 - 2 \phi_i \rho_{yx} C_y C_{x_1} + 2 \phi_i \rho_{yx_2} C_y C_{x_2} - 2 \phi_i \rho_{yx_1 x_2} C_y C_{x_1} C_{x_2} \right) \] (2.24)

\[ \text{MSE}(t_{pi}) = \gamma F^2 \left( C_y^2 + \phi_i^2 C_{x_1}^2 \left( \rho_{yx_1} - 2 \kappa_{yx_1} \right) + \phi_i^2 C_{x_2}^2 \left( \rho_{yx_2} + 2 \left( \kappa_{yx_2} - \phi_i \kappa_{yx_1} \right) \right) \right), \quad i = 1, 2, \ldots, 10 \] (2.25)

### 3. EFFICIENCY COMPARISON

Efficiencies of the proposed estimators are compared with efficiencies of the existing estimators in the study.

The proposed estimators \( t_{pi} \) of the finite population mean are more efficient than \( \bar{y} \) if,

\[ \text{MSE}(t_{pi}) < V(\bar{y}) \quad i = 1, 2, \ldots, 10 \]

\[ \gamma F^2 \left( C_y^2 + \phi_i^2 C_{x_1}^2 \left( \rho_{yx_1} - 2 \kappa_{yx_1} \right) + \phi_i^2 C_{x_2}^2 \left( \rho_{yx_2} + 2 \left( \kappa_{yx_2} - \phi_i \kappa_{yx_1} \right) \right) \right) < \gamma F^2 C_y^2 \] (3.1)

The proposed estimators \( t_{pi} \) of the finite population mean are more efficient than \( t_p \) if,

\[ \text{MSE}(t_{pi}) < \text{MSE}(t_p) \quad i = 1, 2, \ldots, 10 \]

\[ \gamma F^2 \left( C_y^2 + \phi_i^2 C_{x_1}^2 \left( \rho_{yx_1} - 2 \kappa_{yx_1} \right) + \phi_i^2 C_{x_2}^2 \left( \rho_{yx_2} + 2 \left( \kappa_{yx_2} - \phi_i \kappa_{yx_1} \right) \right) \right) < \gamma F^2 (C_y^2 + C_{x_1}^2 - 2 \rho_{yx_1} C_y C_{x_1}) \] (3.2)

The proposed estimators \( t_{pi} \) of the finite population mean are more efficient than \( t_p \) if,

\[ \text{MSE}(t_{pi}) < \text{MSE}(t_p) \quad i = 1, 2, \ldots, 10 \]

\[ \gamma F^2 \left( C_y^2 + \phi_i^2 C_{x_1}^2 \left( \rho_{yx_1} - 2 \kappa_{yx_1} \right) + \phi_i^2 C_{x_2}^2 \left( \rho_{yx_2} + 2 \left( \kappa_{yx_2} - \phi_i \kappa_{yx_1} \right) \right) \right) < \gamma F^2 (C_y^2 + C_{x_1}^2 + 2 \rho_{yx_1} C_y C_{x_1}) \] (3.3)

The proposed estimators \( t_{pi} \) of the finite population mean are more efficient than \( t_j \) if,

\[ \text{MSE}(t_{pi}) < \text{MSE}(t_j) \quad i = 1, 2, \ldots, 10 \quad j = 1, 2, 3, 4 \]

\[ \left( C_y^2 + \phi_i^2 C_{x_1}^2 \left( \rho_{yx_1} - 2 \kappa_{yx_1} \right) + \phi_i^2 C_{x_2}^2 \left( \rho_{yx_2} + 2 \left( \kappa_{yx_2} - \phi_i \kappa_{yx_1} \right) \right) \right) < \left( C_y^2 + \lambda_j^2 C_{x_1}^2 - 2 \lambda_j \rho_{yx_1} C_y C_{x_1} \right) \] (3.4)

The proposed estimators \( t_{pi} \) of the finite population mean are more efficient than \( t_5 \) if,

\[ \text{MSE}(t_{pi}) < \text{MSE}(t_5) \quad i = 1, 2, \ldots, 10 \]

\[ \left( C_y^2 + \phi_i^2 C_{x_1}^2 \left( \rho_{yx_1} - 2 \kappa_{yx_1} \right) + \phi_i^2 C_{x_2}^2 \left( \rho_{yx_2} + 2 \left( \kappa_{yx_2} - \phi_i \kappa_{yx_1} \right) \right) \right) < \left( C_y^2 + C_{x_1}^2 \left( 1 - 2 \kappa_{yx_1} \right) + C_{x_1}^2 \left( 1 + 2 \left( \kappa_{yx_2} - \kappa_{yx_1} \right) \right) \right) \] (3.5)
The proposed estimators \( \left( t_p \right) \) of the finite population mean are more efficient than \( t_o \) if,

\[
MSE\left( t_p \right) < MSE\left( t_o \right) \quad i = 1, 2, ..., 10
\]

\[
\left( \phi_1 C_{n1} \left( \phi_1 - 2\kappa_{y1} \right) + \phi_2 C_{n2} \left( \phi_2 + 2\left( \kappa_{y2} - \phi_1 \kappa_{v1} \right) \right) \right) < \left( \mu C_n \left( \mu_x - 2\kappa_{y1} \right) + \mu_2 C_x \left( \mu_2 + 2\left( \kappa_{y2} - \mu \kappa_{v2} \right) \right) \right) \tag{3.6}
\]

The proposed estimators \( \left( t_p \right) \) of the finite population mean are more efficient than \( t_7 \) if,

\[
MSE\left( t_p \right) < MSE\left( t_7 \right) \quad i = 1, 2, ..., 10
\]

\[
\left( \phi_1 C_{n1} \left( \phi_1 - 2\kappa_{y1} \right) + \phi_2 C_{n2} \left( \phi_2 + 2\left( \kappa_{y2} - \phi_1 \kappa_{v1} \right) \right) \right) < \left( \lambda_1 C_n \left( \lambda_1 - 2\kappa_{y1} \right) + \lambda_2 C_x \left( \lambda_2 + 2\left( \kappa_{y2} - \lambda \kappa_{v2} \right) \right) \right) \tag{3.7}
\]

The proposed estimators \( \left( t_p \right) \) of the finite population mean are more efficient than \( t_8 \) if,

\[
MSE\left( t_p \right) < MSE\left( t_8 \right) \quad i = 1, 2, ..., 10
\]

\[
\left( \phi_1 C_{n1} \left( \phi_1 - 2\kappa_{y1} \right) + \phi_2 C_{n2} \left( \phi_2 + 2\left( \kappa_{y2} - \phi_1 \kappa_{v1} \right) \right) \right) < \left( \eta_1 C_n \left( \eta_1 - 2\kappa_{y1} \right) + \eta_2 C_x \left( \eta_2 + 2\left( \kappa_{y2} - \eta \kappa_{v2} \right) \right) \right) \tag{3.8}
\]

The proposed estimators \( \left( t_p \right) \) of the finite population mean are more efficient than \( t_9 \) if,

\[
MSE\left( t_p \right) < MSE\left( t_9 \right) \quad i = 1, 2, ..., 10
\]

\[
\left( \phi_1 C_{n1} \left( \phi_1 - 2\kappa_{y1} \right) + \phi_2 C_{n2} \left( \phi_2 + 2\left( \kappa_{y2} - \phi_1 \kappa_{v1} \right) \right) \right) < \left( \gamma_1 C_n \left( \gamma_1 - 2\kappa_{y1} \right) + \gamma_2 C_x \left( \gamma_2 + 2\left( \kappa_{y2} - \gamma \kappa_{v2} \right) \right) \right) \tag{3.9}
\]

When conditions (3.1), (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9) are satisfied, we can conclude that the proposed estimators are more efficient than sample mean, the ratio estimator, product estimator, and other existing estimators considered in the study.

### 3.1 Empirical Study

In order to assess the performance of the proposed estimators, we considered a real population as given in Yadav et al [32].

\[N = 30, n = 10, \bar{Y} = 17.5, \bar{X}_1 = 47.1333, \bar{X}_2 = 4.4637, \rho_{y1} = 0.3637, \rho_{y2} = -0.1994, \rho_{v2} = 0.0736,\]

\[\beta_1(x_1) = 0.06206, \beta_2(x_2) = 0.2296, C_y = 0.4758, C_{x1} = 0.6046, C_{x2} = 0.8727, M_d(x_1) = 36,\]

\[M_d(x_2) = 2.21, P_{99x_1} = 125, P_{99x_2} = 13.12, P_{95x_1} = 110.15, P_{95x_2} = 12.89, P_{90x_1} = 96.9, P_{90x_2} = 12.24,\]

\[P_{85x_1} = 95.35, P_{85x_2} = 11.64, P_{80x_1} = 67.8, P_{80x_2} = 8.2, P_{75x_1} = 59.25, P_{75x_2} = 6.88, P_{70x_1} = 53.9, P_{70x_2} = 3.84,\]

\[P_{65x_1} = 49, P_{65x_2} = 2.61, P_{60x_1} = 46.8, P_{60x_2} = 2.49, P_{55x_1} = 40.25, P_{55x_2} = 2.36.\]
Table 1. MSE and PRE of existing and proposed estimators

| Estimator                              | MSE     | PRE     |
|----------------------------------------|---------|---------|
| Sample Mean (\( \bar{y} \))           | 4.6129  | 100     |
| Ratio Estimator (\( t_y \))            | 7.7989  | 59.14808|
| Product Estimator (\( t_p \))          | 16.7563 | 27.52935|
| \( t_1 \) [2]                          | 7.5756  | 60.89155|
| \( t_2 \) [2]                          | 15.2646 | 30.21959|
| \( t_3 \) [2]                          | 7.5865  | 60.80406|
| \( t_4 \) [2]                          | 7.3175  | 63.03929|
| \( t_5 \) [29]                         | 18.3606 | 25.12391|
| \( t_6 \) [30]                         | 17.9278 | 25.73043|
| \( t_7 \) [31]                         | 16.7655 | 27.51424|
| \( t_8 \) [31]                         | 9.4541  | 48.79259|
| Yadav et al. (2016) (\( t_y \))        | 4.4003  | 104.8315|
| Proposed Estimator (\( t_{p1} \))     | 4.3457  | 106.1486|
| Proposed Estimator (\( t_{p2} \))     | 4.2430  | 108.7179|
| Proposed Estimator (\( t_{p3} \))     | 4.1837  | 110.2589|
| Proposed Estimator (\( t_{p4} \))     | 3.9026  | 118.2007|
| Proposed Estimator (\( t_{p5} \))     | 3.7804  | 122.0215|
| Proposed Estimator (\( t_{p6} \))     | 3.7756  | 122.1766|
| Proposed Estimator (\( t_{p7} \))     | 3.8285  | 120.4884|
| Proposed Estimator (\( t_{p8} \))     | 3.8359  | 120.256 |
| Proposed Estimator (\( t_{p9} \))     | 3.8650  | 119.3506|
| Proposed Estimator (\( t_{p10} \))    | 3.8952  | 118.4252|

4. RESULTS AND DISCUSSION

A family of ratio-cum-product estimators for estimation of population mean of the study variable using known population parameters of two auxiliary variables. The bias and mean square error (MSE) of the proposed estimators were derived up to first order of appreciation. Theoretical comparison of the proposed ratio-cum-product estimators of population mean with sample mean (\( \bar{y} \)), ratio estimator (\( t_y \)), product estimator (\( t_p \)) and other existing estimators considered in the study were established. The mean square errors (MSEs) of the proposed estimators are lesser than sample mean, ratio estimator, product estimator and other estimators considered in the study. The performance of the proposed estimators over the sample mean, ratio estimator, product estimator and other selected...
existing estimators using a real population were obtained.

5. CONCLUSION

The study proposed a family of new ratio-cum-product estimators of finite population mean based on the information obtained from the percentiles of auxiliary variables. The results in Table 1 clearly showed that the proposed ratio-cum-product estimators performed better than the sample mean, ratio estimator, product estimator and other existing estimators considered in the study having minimum Mean Square Error (MSE) and the highest Percentage Relative Error (PRE).

ACKNOWLEDGEMENTS

The authors appreciate the Journal Editorial Office and the reviewers for their recommendations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Cochran WG. Sampling Techniques, 3rd Edition. Wiley Eastern Limited, India; 1940.
2. Upadhyaya LN, Singh HP. Use of Transformed Auxiliary Variable in Estimating the Finite Population Mean. Biometrical Journal. 1999;41:627–636.
3. Abu-Dayyeh WA, Ahmed MS, Ahmed RA, Muttilk HA. Some Estimators of a Finite Population Mean using Auxiliary Information, Applied Mathematics and Computation. 2003; 139:287–298.
4. Singh HP, Tailor R, Kakran MS. An Improved Estimation of Population Mean using Power Transformation. Journal of the Indian Society of Agricultural Statistics, 2004; 8(2):223–230.
5. Kadilar C, Cingi H. A new estimator using two auxiliary variables. Applied Mathematics and Computation. 2005; 162: 901–908.
6. Tailor R, Jatwa NK, Tailor R, Garg N. Dual to Ratio and Product Type Exponential Estimators in Stratified Random Sampling Using Two Auxiliary Variates, Journal of Reliability and Statistical Studies. 20136; 2: 115-126.
7. Jeelani MI, Maqbool S, Mir SA. Modified Ratio Estimators of Population Mean using Linear Combination of Estimators of Population Mean using Linear Combination of Coefficient of Skewness and Quartile Deviation. International Journal of Modern Mathematical Sciences. 2013; 6:174–183.
8. Gupta RK, Yadav SK. New Efficient Estimators of Population Mean Using Non-Traditional Measures of Dispersion. Open Journal of Statistics, 2017; 7: 394-404.
9. Muili JO, Singh RVK, Audu A. Study of Efficiency of Some Finite Population Variance Estimators in Stratified Random Sampling. Continental Journal Applied Sciences. 2018; 13(2):1 – 17.
10. Muili JO, Audu A. Modification of Ratio Estimator for Population Mean. Annals. Computer Science Series. 2019;17(2):74-78.
11. Muili JO, Audu A, Singh RVK, Odeyale AB, Adebiyi A. Improved Variance Estimator Using Linear Combination of Tri-Mean and Quartile Average, Annals. Computer Science Series. 2019; 17(1):142-147.
12. Robson DS. Application of multivariate polkays to the theory of unbiased ratio-type estimation, Journal of the American Statistical Association, 1957; 52:511–522.
13. Murthy MN. Sampling Theory and Methods; Statistical Publishing Society, Calcutta, India; 1967.
14. Singh HP, Tailor R. Use of known Correlation Coefficient in Estimating the Finite Population Means; Statistics in Transition. 2003; 6(4):555-560.
15. Abid M, Abbas N, Nazir HZ, Lin Z. Enhancing the Mean Ratio Estimators for Estimating Population Mean Using Non-conventional Location Parameters. Revista Colombiana de Estadistica. 2016; 39(1):63–79.
16. Kadilar C, Cingi H. Ratio Estimators in Stratified Random Sampling, Biometrical Journal. 2003; 45(2):218-225.
17. Koyuncu N, Kadilar C. Family of Estimators of Population Mean Using Two Auxiliary Variables in Stratified Random Sampling, Communications in Statistics: Theory and Methods, 2009;38(14):2398-2417.
18. Yan Z, Tian B. Ratio Method to the Mean Estimation using Coefficient of Skewness of Auxiliary Variable. In: Information Computing and Applications. ICICA; 2010.
19. Subramani J, Kumarapandiyam G. Modified Ratio Estimators using known Median and
Co-efficient of Kurtosis. American Journal of Mathematics and Statistics, 2012; 2:95–100. 
DOI:https://doi.org/10.5923/j.ajms.201202 04.05

20. Yadav SK, Mishra SS, Kadilar C, Shukla AK. Improved Dual to Ratio Cum Dual to Product Estimator in the Stratified Random Sampling, American Journal of Operational Research, 2015; 5(3):57-63.

21. Gupta RK, Yadav SK. Improved Estimation of Population Mean Using Information on Size of the Sample. American Journal of Mathematics and Statistics. 2018; 8(2):27-35.

22. Muili JO, Audu A, Singh RVK, Yunusa IA. Improved Estimators of Finite Population Variance Using Unknown Weight of Auxiliary Variable. Annals. Computer Science Series. 2019; 17(1):148-153.

23. Muili JO, Audu A, Singh RVK. Modified Estimator of Finite Population Variance in Simple Random Sampling. Journal of Science and Technology Research. 2019; 1(1):81-90.

24. Muili JO, Audu A, Odeyale AB, Olawoyin IO. Ratio Estimators for Estimating Population Mean Using Trimmean, Median and Quartile Deviation of Auxiliary Variable. Journal of Science and Technology Research 2019; 1(1):91-102.

25. Muili JO, Zakari Y, Audu A. Modified Class of Estimator of Finite Population Variance. FUDMA Journal of Sciences (FJS). 2019; 3(4):67-78.

26. Audu A, Ishaq OO, Zakari Y, Wisdom DD, Muili JO, Ndatsu AM. Regression-cum-Exponential Ratio Imputation Class of Estimators of Population Mean in the Presence of Non-response. Science Forum (Journal of Pure and Applied Sciences) 2020; 20:58–63. Available:http://dx.doi.org/10.5455/sf.71109

27. Audu A, Ishaq OO, Muili JO, Zakari Y, Ndatsu AM, Muhammad S. On the Efficiency of Imputation Estimators using Auxiliary Attribute. Continental J. Applied Sciences. 2020;15(1):1-13 DOI: 10.5281/zenodo.3721046

28. Muili JO, Adebiyi A, Agwamba EN. Improved Ratio Estimators for Estimating Population Mean Using Auxiliary Information. International Journal of Scientific and Research Publications. 2020; 10(5):937-945. Available:http://dx.doi.org/10.29322/IJSRP.10.05.2020.p101111

29. Singh MP. Ratio-cum-product method of estimation. Metrika, 1967; 12:34–42.

30. Singh HP, Tailor R. Estimation of finite population mean using known Correlation co-efficient between auxiliary characters, Statistica. 2005; 65:407–418.

31. Tailor R, Parmar R, Kim JM, Tailor R. Ratio-Cum-Product Estimators of Population Mean using known Population Parameters of Auxiliary Variates, Communications of the Korean Statistical Society. 2011; 18(2):155–164. DOI: 10.5351/CKSS.2011.18.2.155

32. Yadav SK, Subramani J, Mishra SS, Shukla AK. Improved Ratio-Cum-Product Estimators of Population Mean Using Known Population Parameters of Auxiliary Variables. American Journal of Operational Research. 2016;6(2):48-54 DOI: 10.5923/j.ajor.20160602.03