Interacting modified holographic Ricci dark energy model and statefinder diagnosis in flat universe

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Abstract

In this work we have considered the modified holographic Ricci dark energy interacting with dark matter through a non-gravitational coupling. We took three phenomenological forms for the interaction term $Q$ in the model, where in general $Q$ is proportional to the Hubble parameter and densities of the dark sectors, $\rho_{de} + \rho_m$, $\rho_m$ and $\rho_{de}$ respectively. We have obtained analytical solutions for the three interacting models, and studied the evolutions of equations of state parameter, deceleration parameter. The results are compared with the observationally constrained values for the best parameters of the model. We have also done the statefinder analysis of the model to discriminate the model from other standard models. In general we have shown that the model is showing a de Sitter type behavior in the far future of the evolution of the universe.

1 Introduction

The recent observational advance in cosmology have shown that, the expansion rate of the universe is accelerating [1, 2]. This discovery was announced in 1998 based on the data on Supernovae Type IA. A series of subsequent experiments regarding the Cosmic Microwave Background (CMB) radiations, Large Scale Structure (LSS) and other measurements have all confirmed the above claim on accelerating expansion of the universe. Theoretical analysis which explaining this recent acceleration, predicts that, the major component of the universe is dark energy, an exotic cosmic component with negative pressure. However the nature of dark energy is still enigmatic, so that it’s nature is one of the biggest challenges in cosmology. Cosmological constant $\Lambda$ is an important candidate for dark energy and it provides a good explanation for the current acceleration. But cosmological constant faces the severe draw backs such that, the theoretical value of $\Lambda$ is many orders of magnitude larger than the current observational value [3, 4, 5, 6] and it is not able to explain, why densities of dark energy and dark matter are of same order at present while they evolve in rather different ways. So as an alternative, dynamical dark energy models have been proposed and analyzed in recent literature. Among these holographic dark energy models [7, 8, 9, 10] have got much recent attention, because it originates from the holographic principle of quantum gravity [11]. According to this principle, the vacuum energy density can be bounded as $\rho_{vac} L^3 \leq M_{pl}^2 L$, [12], where $\rho_{vac}$ is the vacuum energy density and $M_{pl}$ is the reduced plank mass. This bound implies that, the total energy inside a region of size $L$, should
not exceed the mass of a black hole of the same size. From effective quantum field theory, an effective IR cut of can saturate the length scale, so that the dark energy density (the vacuum energy density) can be written as \( \rho_{de} = 3c^2 M_{pl} L^{-2} \) [13], where \( c \) is a dimensionless numerical factor. In literature, the IR cut-off has been taken as the Hubble horizon [13, 14], particle horizon, event horizon [13] and some generalized IR cut-off [15, 16, 17]. The holographic dark energy models with Hubble horizon and particle horizon as the IR cut-off, cannot lead to the current accelerated expansion of the universe. When event horizon is taken as the cut-off, the model is suffered from the following problem. Future event horizon is global concept of space-time, while dark energy density is a local quantity. So the relation between them will pose challenges to causality. These leads to the introduction of the new holographic dark energy, the holographic Ricci dark energy, where the IR cut-off is taken as proportional to the Ricci scalar curvature, \( R^{1/2} \). The holographic Ricci dark energy introduced by Granda and Oliveros [18] is in fairly good agreement with the observational data. This model have the following advantages. The fine tuning problem can be avoided in this model of dark energy. Moreover the presence of event horizon is not presumed in this model, so that the causality problem can be avoided and the coincidence problem can also be avoided in this model. Recently a modified form of holographic Ricci dark energy in interaction with the dark matter was analyzed [19].

The current observational evidence indicates that around 95% of matter-energy in the universe is in the dark sector, composed of dark matter and dark energy [1, 2, 20]. Dark matter is substantially non-baryonic in nature and would responsible for the structure formation in the universe. There is an unavoidable degeneracy between dark matter and dark energy existing within the Einstein’s gravity. So there could be a hidden non-gravitational coupling between them. This interestingly leads to develop various ways of testing different kinds of interaction in the dark sector. In the present paper we consider the interaction between dark energy and dark matter, by considering dark energy density as the modified holographic dark energy. Owing to the lack of mechanism for the microscopic origin of the interaction, one has to assume various forms for the interaction phenomenologically. Several forms for the interaction have been put forward [21, 22, 23, 24, 25]. The commonly used interaction forms are those in which the interaction is depend linearly on the Hubble parameter and the density of dark matter, dark energy or sum of both the densities. In all these works the dark energy taken as the holographic dark energy. As we have mentioned, in this work we have considered the modified holographic Ricci dark energy in interaction with the dark matter.

Statefinder parameters, introduced by Sahni et al. [26] is a sensitive diagnostic tool used to discriminate various dark energy models, because the Hubble parameter and the deceleration parameter alone cannot effectively discriminate various dark energy models. These parameters are defined as

\[
\begin{align*}
  r &= \frac{1}{a H^3} \frac{d^3a}{dt^3}, \\
  s &= \frac{r - \Omega_{total}}{3(q - \Omega_{total}/2)}
\end{align*}
\]

where \( a \) is the scale factor and \( \Omega_{total} \) is the total energy density parameter containing the matter and dark energy. The \( r - s \) plot can discriminate the various dark energy models, for example, the well known ΛCDM model is one with, \( r = 1 \) and \( s = 0 \). The cosmological behavior of various dark energy models were differentiated using the statefinder parameters [27, 28].

2 Interacting MHRDE model

The Friedmann equation for the flat universe with FRW metric is,

\[
3H^2 = \rho_m + \rho_{de}
\]
where $H$ is the Hubble parameter, $\rho_m$ is the dark matter density and $\rho_{de}$ is the dark energy density. We have considered the flat universe because the inflationary model of the universe predicted a flat universe which has been confirmed by observations that the current density parameter corresponds to curvature is $\Omega_k \sim 10^{-3}$ [29]. The modified Holographic Ricci dark energy (MHRDE), taking Ricci scalar as the IR cut-off is given as

$$\rho_{de} = \frac{2}{\alpha - \beta} \left( 2\dot{H} + \frac{3}{2} \alpha H^2 \right)$$

(3)

where $\dot{H}$ is the derivative of the Hubble parameter with respect to cosmic time, $\alpha$ and $\beta$ are constants, the model parameters. This model was studied in the non-interacting case in reference [30], and Chimento et. al. have analyzed this this type of dark energy in interaction with dark matter as Chaplygin gas [19, 31]. The interaction between MHRDE and dark matter can be included through the continuity equations,

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q$$

(4)

$$\dot{\rho}_m + 3H\rho_m = Q$$

(5)

Where $p_{de}$ is the pressure density of dark energy, $Q$ is the interaction term, over-dot representing derivative with respect to time and dark matter is assumed to be pressure less. Since there is no conclusive theory for the microscopic origin of the interaction, one has to assume the form of $Q$ phenomenologically. The interaction term must be a function of a quantity with dimension inverse of time and thus $Q$ can take forms [38, 39] such as $Q = 3bH(\rho_{de} + \rho_m)$, $Q = 3bH\rho_m$ and $Q = 3bH\rho_{de}$. By convention, $b > 0$ means energy is transferring from dark energy to cold dark matter. For convenience we will abbreviate the three interacting models as: IMHRDE1 corresponds to $Q = 3bH(\rho_{de} + \rho_m)$, IMHRDE2 corresponds to $Q = 3bH\rho_m$ and IMHRDE3 corresponds to $Q = 3bH\rho_{de}$.

2.1 Interacting model with $Q = 3bH(\rho_{de} + \rho_m)$-IMHRDE1

In this section we are analyzing the interaction of the MHRDE with cold dark matter, with interaction given as $Q = 3bH(\rho_{de} + \rho_m)$. Substituting the MHRDE density equation (3) in the Friedmann equation (2), we get

$$h^2 = \frac{\rho_m}{3H_0^2} + \frac{2}{3\Delta} \left( \frac{1}{2} \frac{dh^2}{dx} + \frac{3\alpha}{2} h^2 \right)$$

(6)

where $h = H/H_0$, $H_0$ is the present value of the Hubble parameter, $\Delta = \alpha - \beta$ and the variable $x = \log a$ with $a$ as the scale factor of the universe. Differentiate this equation once more and substituting $\dot{\rho}_m$ form the continuity equation, leads to

$$\frac{d^2 h^2}{dx^2} + 3(1 + \beta) \frac{dh^2}{dx} + 9(\beta + b\Delta) h^2 = 0$$

(7)

The solution of the above second order differential equation is obtained as,

$$h^2 = c_1 e^{\frac{3}{2}m_1 x} + c_2 e^{\frac{3}{2}m_2 x}$$

(8)

where

$$m_{1,2} = -1 - \beta \pm \sqrt{1 - 4b\alpha - 2\beta + 4b\beta + \beta^2}.$$  

(9)

The coefficients $c_1$ and $c_2$ are determined using the initial conditions,

$$h^2|_{x=0} = 1, \quad \frac{dh^2}{dx}|_{x=0} = 3\Omega_{de0}\Delta - 3\alpha$$

(10)
where $\Omega_{de0}$ is the current value of dark energy density and is related to matter density as $\Omega_{de0} = 1 - \Omega_{m0}$ for the flat universe and $\Omega_{m0}$ is present value of cold dark matter density. From these the coefficients $c_1$ and $c_2$ are found to be,

$$c_1 = \frac{2(\Omega_{de0}\Delta - \alpha) - m_2}{m_1 - m_2}, \quad c_2 = 1 - c_1 \quad (11)$$

Comparing equation (8) with the standard Friedmann equation, the dark energy density can be identified as,

$$\Omega_{de} = c_1 e^{\frac{3}{2}m_1 x} + c_2 e^{\frac{3}{2}m_2 x} - \Omega_{m0} e^{-3x} \quad (12)$$

The pressure of the dark energy can then be obtained as

$$p_{de} = -\Omega_{de} - \frac{1}{3} \frac{d\Omega_{de}}{dx} = -\left[ c_1 \left( 1 + \frac{m_1}{2} \right) e^{\frac{3}{2}m_1 x} + c_2 \left( 1 + \frac{m_2}{2} \right) e^{\frac{3}{2}m_2 x} \right] \quad (13)$$

The corresponding equation of state can be obtained using the standard relation,

$$\omega_{de} = -1 - \frac{1}{3} \frac{d \ln \Omega_{de}}{dx} \quad (14)$$

which after using the equation (12), gives,

$$\omega_{de} = -1 - \frac{1}{2} \left( \frac{c_1 m_1 e^{\frac{3}{2}m_1 x} + c_2 m_2 e^{\frac{3}{2}m_2 x} + 2\Omega_{m0} e^{-3x}}{c_1 e^{\frac{3}{2}m_1 x} + c_2 e^{\frac{3}{2}m_2 x} - \Omega_{m0} e^{-3x}} \right) \quad (15)$$

If there is no interaction between the dark sectors, i.e. $b = 0$, and the contribution form non-relativistic cold dark matter behavior ($\sim \Omega_{m0}$) term is negligible in the dark energy density, the constants takes the values $m_1 = -2, m_2 = -2\beta, c_1 = 0$ and $c_2 = 1$. Consequently the equation of state parameter become

$$\omega_{de} = -1 + \beta. \quad (16)$$

This is in agreement with the earlier results in the non-interacting case [30]. So in the non-interacting case the equation of state can be greater than or less than -1, depending on the value of the the parameter $\beta$.

We have analyzed the equation of state parameter for different parameter values. The interaction parameter $b$, is chosen to be $b = 0.001$, because for values greater than this IMHRDE1 does not satisfy the coincidence of matter and dark energy (the coincidence problem), for the ($\alpha, \beta$) parameter sets using by us. The evolution of the IMHRDE1 along with the dark matter is shown in the figure 1. In studying this co-evolution of dark energy and dark matter, we have neglected the phase transitions, transitions form non-relativistic to relativistic particles at high temperature and also new degrees of freedom thus arises. However it is expected that these will not make much changes in the result. The plot shows that, the interacting dark energy and the dark matter were comparable with each other in the past universe and MHRDE is dominating at low redshift, which in effect solves the coincidence problem. We have found that the IMHRDE1 is compatible with the coincidence of matter and dark energy for all the parameter values of ($\alpha, \beta$) we used through out our analysis.

We have plotted the equation of state with redshift for the parameters ($\alpha, \beta$) = (1.15, 0.15), for the standard values of $\Omega_{de0} = 0.7,$ and $\Omega_{m0} = 0.3$. The plot is shown in the figure 2. The evolution of the equation of state of IMHRDE1 shows that, in the remote past, at large redshift, the equation of state parameter $\omega_{de}$ of the dark energy is very near to zero, in which it behaves like cold dark matter. But in the far future, as redshift $z \to -1$, equation of state approaches a negative saturation value. From the figure 2 the current value of the equation of state is found to be around $\omega_{de} \sim -0.82$ for parameters ($\alpha, \beta$)=(1.15,0.15). From figure 3 it is clear that for the parameter set (1.2,0.1),
Figure 1: Evolution of IMHRDE1 in comparison with the cold dark matter, with parameters $(\alpha, \beta) = (1.15, 0.15)$ and $b=0.001$. Blue line representing MHRDE and violet line is for cold dark matter.

Figure 2: Variation of the equation of state parameter $\omega_{de}$ with redshift $z$ for the model parameters $\alpha = 1.15$, $\beta = 0.15$ and $b = 0.001$

$\omega_{de}=-0.78$ and as $z \to -1$, the equation of state parameter shows behavior $\omega_{de}(z \to -1) > -1$. For $(4/3, 0.05)$, $\omega_{de0}=-0.78$ and $\omega_{de}(z \to -1) > -1$. When $\beta$ takes negative values, the equation of state parameter have following values. For $(1.2, -0.1)$, $\omega_{de0}=-0.96$ and $\omega_{de}(z \to -1) < -1$. For $(4/3, -0.05)$, $\omega_{de0}=-0.88$ and $\omega_{de}(z \to -1) < -1$. For $(1.01, -0.01)$, $\omega_{de0}=-0.96$ and $\omega_{de}(z \to -1) = -1$. The WMAP-7 data predicts $\omega_{de0} \sim -0.93$, when joint analysis of the WMAP+BAO+H$_0$+SN data [29, 19] for constraining the present value of the equation of state parameter for the dark energy is made. From our analysis, the present values of $\omega_{de}$ for the parameter sets $(1.2, -0.1)$ and $(1.01, -0.01)$ are very close to the observationally deducted values. However the parameters $(1.2, -0.1)$ have the behavior that in the far future, as $z \to -1$, the $\omega_{de}$ crosses the phantom divide -1, so the model posses phantom behavior in the future evolution. While for the parameter set $(1.01, -0.01)$, the $\omega_{de}$ approaches -1, which corresponds the ΛCDM model where the energy density is fully dominated with the cosmological constant. So the parameter set $(\alpha, \beta)=(1.01, -0.01)$ can be considered as preferable values. The plots also shows that irrespective of the values of the parameters, $\omega_{de} \to 0$, at very large redshift. That is in the remote past the IMHRDE1 is behaving almost like pressureless cold dark matter for all values of the parameter set.

Apart from the Hubble parameter $H$, deceleration parameter $q$, is another geometrical parameter which describes the expansion history of the universe. The deceleration parameter can be expressed
Figure 3: Variation of the equation of state parameter $\omega_{de}$ with redshift $z$ for the model parameters $(\alpha, \beta)=(4/3, -0.05)$-thick continuous line, $(4/3,0.05)$ large dashed line $(1.15,0.15)$-dot-dashed line,$(1.2,0.1)$small dashed line, $(1.2,-0.1)$- thin continuous line and $(1.01,-0.01)$ dotted line, with $b=0.001$

in terms of $h$ as,

$$q = -\frac{1}{2h^2 \frac{dx}{dz}} - 1 \quad (17)$$

Accelerated expansion is indicated by the condition $q < 0$. From equation 8 $q$-parameter can be expressed as,

$$q = -\frac{3}{4} \left( c_1 m_1 e^{\frac{3}{2} m_1 x} + c_2 m_2 e^{\frac{3}{2} m_2 x} \right)  - 1 \quad (18)$$

In the non-interaction limit and by avoiding the contribution from dark matter, the $q$-parameter becomes $q = (3\beta - 2)/2$, which is in agreement with our earlier work [30] and it shows, as $\beta$ increases from zero, the $q$-parameter increases from -1. In figure 4 we have plotted the evolution of $q$-parameter of the interacting MHRDE with redshift.

Figure 4: Behavior of the deceleration parameter with redshift for parameters $(\alpha, \beta)=(4/3,0.05)$ (dashed line), $(4/3,-0.05)$ (continuous line), with coupling parameter $b=0.001$.

For evolution of the $q$ parameter for different sets $(\alpha, \beta)$ parameters are shown in figures 4 and 5. Both the plots shows the universe enters the accelerated expansion in the recent past, the corresponding to the transition to the accelerating phase are $z_T=0.47$ for parameters $(4/3,-0.05)$, 0.55 for $(1.2,-0.1)$, 0.70 for $(1.01,-0.01)$, 0.44 for $(4/3,0.05)$, 0.50 for $(1.2,0.1)$ and 0.52 for $(1.15,0.15)$. The combined
analysis of SNe+CMB data with ΛCDM model gives the range for the redshift at which universe enters the accelerated phase is $z_T = 0.45 - 0.73$ \cite{35}. The transition redshift given above for the IMHRDE1 is seen to be in the observational range for almost all parameter sets. However in the case of predicting the equation of state, we have found that the parameter set $(1.01,-0.01)$ giving the best suitable prediction. The transition redshift for this parameter set in $z_T = 0.70$ is agreeing with the upper limit region of the observationally constraint range. Form the plots the present value of the deceleration parameter is -0.46 corresponds to $(4/3,-0.05)$, -0.55 for $(1.2,-0.1)$, -0.56 corresponds to $(1.01,-0.01)$, -0.34 corresponds to $(4/3,0.05)$, -0.35 corresponds to $(1.2,0.1)$ and -0.33 corresponds to $(1.15,0.15)$. The plot shows that, the universe enters the accelerating expansion in the recent past, with redshift in the range $z_T = 0.52 - 0.57$ and the present deceleration parameter is in the range $q = 0.33 - 0.58$. So it can be concluded that, compared to the ΛCDM model, the universe entering the accelerating expansion at a relatively later in the IMHRDE1 model. The observationally constraint value of the deceleration parameter form WMAP data is $q_0 = -0.60$. From this we can conclude that it is the parameter set $(1.01,-0.01)$ giving the best value for $q_0$ as 0.56, which very close to the observational value. The parameter set $(1.2,-0.1)$ is also giving a competent value for the present deceleration parameter. But the IMHRDE1 tends towards phantom behavior in the future corresponds this parameter set, so we consider the set $(1.01,-0.01)$ as the best parameter set.

2.1.1 Statefinder analysis

Various dark energy models predicts $H > 0$, $q < 0$ at the present time for the universe. Therefore Hubble parameter and deceleration parameter cannot discriminate dark energy models. In this light, Sahni et. al. \cite{4} and Alam et. al. \cite{35}, introduced statefinder parameters $(r, s)$, by using third order time derivative of the scale factor. These effectively distinguishes various dark energy models by removing the degeneracy between $H$ and $q$. The definitions of these parameters is given in equation \cite{1}. In terms of $h^2$ statefinder parameters can recast as

$$ r = \frac{1}{2h^2} \frac{d^2h^2}{dx^2} + \frac{3}{2h^2} \frac{dh^2}{dx} + 1 \quad (19) $$

and

$$ s = - \left( \frac{1}{2h^2} \frac{d^2h^2}{dx^2} + \frac{3}{2h^2} \frac{dh^2}{dx} + \frac{9}{2} \right) \quad (20) $$

Figure 5: Evolution of $q-$parameter with redshift for parameter sets $(\alpha, \beta)=(1.2,0.1)$ (thick continuous), $(1.2,-0.1)$ (dot-dashed ) and $(1.01, -0.01)$ (dashed line) with coupling constant $b=0.001$. 
On substituting $h^2$ from equation (8), the above equations become,

$$r = 1 + \frac{9 \left( c_1 m_1^2 e^{\frac{3}{2} m_1 x} + c_2 m_2^2 e^{\frac{3}{2} m_2 x} \right) + 2 \left( c_1 m_1 e^{\frac{3}{2} m_1 x} + c_2 m_2 e^{\frac{3}{2} m_2 x} \right)}{8 \left( c_1 e^{\frac{3}{2} m_1 x} + c_2 e^{\frac{3}{2} m_2 x} \right)}$$

(21)

and

$$s = - \left( \frac{\left( c_1 m_1^2 e^{\frac{3}{2} m_1 x} + c_2 m_2^2 e^{\frac{3}{2} m_2 x} \right) + 2 \left( c_1 m_1 e^{\frac{3}{2} m_1 x} + c_2 m_2 e^{\frac{3}{2} m_2 x} \right)}{2 \left( c_1 m_1 e^{\frac{3}{2} m_1 x} + c_2 m_2 e^{\frac{3}{2} m_2 x} \right) + 4 \left( c_1 e^{\frac{3}{2} m_1 x} + c_2 e^{\frac{3}{2} m_2 x} \right)} \right)$$

(22)

In the limiting case of non-interacting MHRDE ($b=0$), and also avoiding the contribution from dark matter ($\Omega_{m_o} \sim 0$), the above equations reduces to $r = 1 + (9\beta(\beta - 1))/2$ and $s = \beta$, which are in agreement with the earlier results for the non-interacting case [30]. So under these specific condition, at $\beta = 0$, the MHRDE model corresponds to the $\Lambda$CDM (LCDM) model with $(r, s) = (1, 0)$ in the $r - s$ plane.

The $r - s$ evolutionary trajectory of the interacting MHRDE model for the parameters $(\alpha, \beta) = (1.33, -0.05), (1.2, -0.1)$ is shown in figure 6. The evolution is starts from the right and evolves to the

![Figure 6: $r - s$ evolutionary trajectories for the model for parameters, $(\alpha, \beta)$=(1.2,-0.1), (4/3, -0.05) and (1.01,-0.01) with b=0.001](image)

left in the $r - s$ plane. The plots shows that, the $r$ parameter stays almost constant in the beginning stages of the expansion of the universe. The $\Lambda$CDM phase, corresponds to $(r, s) = (1, 0)$, as denoted in the plots by the point LCDM. The todays position of the universe is also noted in the $r - s$ plane. From figure 6 it is clear that the for negative values of $\beta$, the IMHRDE1 is evolves through the $\Lambda$CDM phase. After that the universe is evolving in such a way that the $r$ value increases very steeply. The present values $(r_0, s_0)$ for different parameters are, $(1.14, -0.048)$ corresponds to $(\alpha, \beta) = (4/3, -0.05), (1.31, -0.098)$ corresponds to parameters $(1.2, -0.1)$ and $(1.03, -0.008)$ corresponds to parameters $(1.01, -0.01)$. The distance of the point $(r_0, s_0)$ from the $LCDM$ fixed point is the least for the parameters $(\alpha, \beta) = (1.01, -0.01)$. From the figure 6 it is seen that as $\beta$ increases the distance between the today’s point and $\Lambda$CDM point is decreasing. For positive $\beta$ values the $r - s$ evolution is shown in figure 7. Unlike in the case for negative $\beta$ values, here it is seen that the $\Lambda$CDM point is being a part of the $r - s$ evolution. Moreover in the later stages of evolution, the $r$ value decreases rather than increasing as
The state finder diagnostic is clearly distinguishing the IMHRDE1 from other models. For quintessence model, the \( r-s \) trajectory is lying in a region with \( s > 0, r < 1 \), for Chaplygin gas model the \( r-s \) trajectory in the region with \( r > 1, s < 0 \). For Holographic dark energy model with event horizon as the IR cut-off, the \( r-s \) evolution starts its evolution with \( s = 2/3, r = 1 \) and ends at the \( \Lambda \)CDM point in the \( r-s \) plane. In the case of IMHRDE1, the distance between \( \Lambda \)CDM point and the today’s point is the least in the \( r-s \) plane for parameters \((\alpha, \beta) = (1.01, -0.01)\), so this parameters can be favored over the other values. For further comparison with our results on IMHRDE1, one can see that in reference [27], the authors have considered a new holographic dark energy model for which they have obtained \((r_0, s_0) = (1.36, -0.59)\).

We have studied the evolution of the IMHRDE1 model in the \( r-q \) plane also. For interaction coupling constant \( b = 0.001 \), the plots are given figures 8 and 9.

For negative values of \( \beta \), the plots in figure 8 shows that both \( \Lambda \)CDM model and IMHRDE1 model are commence evolving from the same point in the past corresponds to \( r = 1, q = 0.5 \), which corresponds to the matter dominated SCDM universe. For \( \Lambda \)CDM model the \( r-q \) trajectory will end the evolution at \( q = -1, r = 1 \), which corresponds to the de Sitter universe. The behavior of the IMHRDE1 is different from the above case. For the usual holographic dark energy model with event horizon as the IR cut-off, the starting and end point are similar to that of the \( \Lambda \)CDM model [36]. In the present model, even though it has the same starting point as the \( \Lambda \)CDM model, in the further evolution it is seemed to be different. For negative values of \( \beta \) the \( r \) value increase as \( q \) decrease but for positive values of \( \beta \) (fig. 9), the \( r \) value decrease as \( q \) decreases. The today’s position of the universe in the \( r-q \) plane is noted. For negative values of \( \beta \), the today’s positions are \((r_0, q_0) = (1.14, -0.45)\) corresponds to \((\alpha, \beta) = (4/3, -0.05)\), \((1.31, -0.57)\) corresponds to \((1.2, -0.1)\) and \((1.03, -0.56)\) corresponds to \((1.01, -0.01)\). For positive \( \beta \) the today’s position in the \( r-q \) plane are \((0.87, -0.35)\) corresponds to parameters \((4/3, 0.05)\), \((0.74, -0.36)\) corresponds to \((1.2, 0.10)\) and \((0.63, -0.33)\) corresponds to parameters \((1.15, 0.15)\). The parameters \((\alpha, \beta) = (1.01, -0.01)\) can be considered as the best fit parameter, since it is giving the equation of state parameter almost same as deducted by the WMAP observations. For the same parameters, we have seen that \( q_0 = -0.56 \), which is very near to the WMAP prediction -0.60.
Figure 8: $r - q$ plots for parameters $(\alpha, \beta) = (1.33, -0.05), (1.2, -0.1), (1.01, -0.01)$ with interaction constant $b=0.001$

Figure 9: $r - q$ plots for parameters $(\alpha, \beta) = (1.33, 0.05), (1.2, 0.1)$, with interaction constant $b=0.001$

29. The $q_0$ value corresponds to the parameters $(1.2, -0.1)$ is also compatible with the corresponding WMAP values, moreover $\omega_{de0}$ corresponds to these parameters is -0.97. So the parameters $(1.2, -0.1)$ is seems to equally good as the parameters $(1.01, -0.01)$, but the problem with these parameters is that the equation of state approaches value less than -1 as $z \rightarrow -1$. As a result the equation of state parameter crosses the phantom divide and model leads to phantom behavior in the future evolution of the universe. In this light the parameters $(1.01, -0.01)$ is finally preferred over other sets.

2.2 Interacting model with $Q = 3bH\rho_m$-IMHRDE2

In this section we are doing the same analysis of IMHRDE2 as in the previous section, but with interaction term given as, $Q = 3bH\rho_m$. From Friedmann equation the second order differential equation for $h^2$ can be obtained,

$$\frac{d^2h^2}{dx^2} + 3(\beta - b + 1)\frac{dh^2}{dx^2} + 9\beta (1 - b) h^2 = 0 \quad (23)$$
A general solution for this can be written as,

\[ h^2 = k_1 e^{-3\beta x} + k_2 e^{3(b-1)x} \]  \hspace{1cm} (24)

where the constants \( k_1 \) and \( k_2 \) are evaluated using the initial conditions as,

\[ c_1 = \frac{\Omega_{de0}(\alpha - \beta) - \alpha - b + 1}{1 - \beta - b}, \quad c_2 = 1 - c_1 \]  \hspace{1cm} (25)

Again using Friedmann equation the dark energy density parameter can be obtained as,

\[ \Omega_{de} = k_1 e^{-3\beta x} + k_2 e^{3(b-1)x} - \Omega_{m0} e^{-3x} \]  \hspace{1cm} (26)

From this the dark energy pressure and equation of state parameter can be calculated as,

\[ p_{de} = - \left[ (1 - \beta)k_1 e^{-3\beta x} + bk_2 e^{-3(1-b)x} \right] \]  \hspace{1cm} (27)

and

\[ \omega_{de} = -1 + \left[ \frac{k_1 \beta e^{-3\beta x} + k_2 (1 - b) e^{-3(1-b)x} - \Omega_{m0} e^{-3x}}{k_1 e^{-3\beta x} + k_2 e^{-3(1-b)x} - \Omega_{m0} e^{-3x}} \right] \]  \hspace{1cm} (28)

In the non-interaction case, with \( b = 0 \) and \( \Omega_{de0} = 1 \), the coefficients become \( k_1 = 1 \) and \( k_2 = 0 \), then the equation of state reduces to the standard form, \( \omega_{de} = -1 + \beta \), confirming the earlier observations [30]. The value of the interaction coupling constant, \( b \) is to be chosen in such a way that, the model must be viable with respect to coincidence between non-relativistic dark matter and dark energy. We found that the \( b \) parameter can be around \( b = 0.003 \) in the present case, at which the model explaining the coincidence problem very well as shown in figure 10. For values \( b > 0.003 \), the IMHRDE2 fails to be compatible with the co-evolution of dark energy and dark matter. So we take, \( b=0.003 \), for our further analysis in this section.

Figure 10: Evolution of interacting MHRDE model with \( Q = 3bH\rho_m \), for parameters, \((\alpha, \beta) = (4/3, -0.05)\) and \( b=0.003 \). Continuous line for interacting MHRDE and dashed line is for dark matter. The plot shows that, dark energy is dominant in the recent past of the universe.

The evolution of the equation of state parameter with redshift is shown in figure 11. The figure shows that, the equation of state parameter, is approaching zero, at very large positive values of redshift. Hence in the remote past, the interacting MHRDE is behaves like cold dark matter. As the universe evolves, the equation of state parameter become more and more negative, and approaches stabilization as \( z \rightarrow -1 \). For parameters \((\alpha, \beta) = (4/3, -0.05), (1.2, -0.1)\), the \( \omega_{de} \) approaches values below -1, at which it behaves as phantom dark energy. But for parameters \((\alpha, \beta) = (4/3, 0.05), (1.2, 0.1)\), the equation of state saturate at values above -1. For the best fit \((1.01, -0.01)\) the equation of
Figure 11: Evolution of the equation of state for different parameters \((\alpha, \beta)\) with \(b=0.001\). Small dashed line is for \((\alpha, \beta)=(1.15,0.15)\), long dashed line for \((1.2, -0.1)\), dot-dashed line is for \((4/3, -0.05)\), thin continuous line is for \((4/3, 0.05)\) and thick continuous line is for \((1.2, 0.1)\), doted line if for \((1.01,-0.01)\). State approaches -1 as \(z \to -1\). The present value of the equation of state parameter, is -0.88 for the parameters \((4/3,0.05)\), -0.98 for \((1.2,-0.1)\), -0.97 for \((1.01,-0.01)\), -0.78 for \((4/3,0.05)\), -0.78 for \((1.2,0.1)\) and -0.76 for \((1.15,0.15)\). It is seen that in contrast to the WMAP value, \(\omega_{de0}\) corresponds to \((1.2,-0.1)\) and \((1.01,-0.01)\) are best values. As we mentioned earlier we will consider \((1.01,-0.01)\) as the best parameters, which give the present equation of state as -0.97.

The deceleration parameter \(q\) in this case is found to be of the form

\[
q = -1 + \frac{3}{2} \left( \frac{c_1 \beta e^{-3\beta x} - c_2 (b - 1) e^{-3(1-b)x}}{c_1 e^{-3\beta x} + c_2 e^{-3(1-b)x}} \right) \tag{29}
\]

In the non-interacting limit with negligible contribution from dark matter sector, the above equation reduces to \(q = (3\beta - 2)/2\). We have studied the the evolution of the \(q\) parameter with redshift, and is shown in the figure 12. The plot reveals that, at large redshift the \(q\)-parameter saturates at around 0.5. As the universe evolves, \(q\) parameter starts decreasing and entering the negative value region corresponds to accelerating universe. The transition to the accelerating phase is occurred at \(z_T=0.44\) corresponds to \((\alpha, \beta)=(4/3,0.05)\), 0.50 corresponds to \((1.2,0.1)\). For comparison the range deduced using observational data is \(z_T=0.45 - 0.73\) [35]. This shows that, the present model IMHRDE2 is agreeing with the observational result for the parameters \((1.2,-0.1)\) and \((1.01,-0.01)\) with interaction coupling constant \(b=0.003\). For the best fit parameters \((1.01,-0.01)\) the the transition occurred at

Figure 12: Evolution of the deceleration parameter \(q\) with redshift for parameters \((\alpha, \beta)=(1.15,0.15)\) (dash-dot line), \((1.33, -0.05)\) (dashed line), \((1.33, 0.05)\) (continuous line), in the left and \((1.2, -0.1)\) (dash-dot line), \((1.2, 0.1)\) (continuous line), \((1.01,-0.01)\) (dashed line) in the right, both for \(b=0.001\).
around \( z_T=0.68 \). It is also seen from the plots that, the present value corresponds to different sets parameters are -0.34 for \((4/3,0.05)\), -0.36 for \((1.2,0.1)\), -0.33 for \((1.15,0.15)\), -0.46 for \((4/3,-0.05)\), -0.57 for \((1.2,-0.1)\) and -0.57 for \((1.01,-0.01)\). For the best fit parameters \((1.01,-0.01)\) the \( q_0=-0.57 \) is very much near to the observational prediction -0.60.

2.2.1 Statefinder analysis

In the following we will analyses the evolution of IMHRDE2 in the \( r - s \) plane. For the present case these parameters, are given as

\[
r = 1 + \frac{9c_1\beta^2 e^{-3\beta x} + 9c_2(1-b)^2 e^{-3(1-b)x} - 9c_1\beta e^{-3\beta x} - 9c_2(1-b)e^{-3(1-b)x}}{2(c_1 e^{-3\beta x} + c_2 e^{-3(1-b)x})}
\] (30)

and

\[
s = -\left[9c_1\beta^2 e^{-3\beta x} + 9c_2(1-b)^2 e^{-3(1-b)x} - 9c_1\beta e^{-3\beta x} - 9c_2(1-b)e^{-3(1-b)x}ight]
-9c_1\beta e^{-3\beta x} - 9c_2(1-b)e^{-3(1-b)x} + 9c_1 e^{-3\beta x} + 9c_2(1-b)e^{-3(1-b)x}
\] (31)

In the non-interacting limit, the above parameters become, \( r = 1 + 9\beta(\beta - 1)/2 \) and \( s = \beta \), which confirms the earlier results. In this limit \( r \) decreases as \( \beta \) (provided it is positive) increases, while for negative \( \beta \) the \( r \) parameter decreases as \( \beta \) increases. In this model with coupling constant \( b=0.003 \), the evolution in the \( r - s \) plane is as shown in the following figures 13 and 14.

![Figure 13: Evolution of the model in the \( r - s \) plane \((\alpha, \beta)\)=\((1.33,-0.05),(1.2, -0.1),(1.01,-0.01)\) with interaction coupling constant \( b=0.003 \)](image)

and it is seen that, irrespective of the values of parameters, in IMHRDE2, the universe begin with \( r=1 \) in the past. For negative values of \( \beta \), the model is evolving through the \( \Lambda \)CDM model in the past. As \( \beta \) increases the gap between \( \Lambda \)CDM point and the present IMHRDE2 phase is decreasing in the \( r - s \) plane. For negative \( \beta \), the present values of the parameters are \((r_0, s_0)\)=\((1.14, -0.048)\) corresponds to \((4/3,-0.05)\), \((1.31,-0.099)\) corresponds to \((1.2,-0.1)\) and \((1.03,-0.0096)\) corresponds to \((1.01,-0.01)\). For positive \( \beta \) values the distance between the \( \Lambda \)CDM point and the today’s point is increasing as \( \beta \) increases. The present values of the parameters in these cases are \((r_0, s_0)\)=\((0.87, 0.05)\) corresponds
Figure 14: Evolutionary trajectories of $r - s$ plane for parameters $(\alpha, \beta) = (1.33, 0.05), (1.2, 0.1), (1.15, 0.15)$ with $b = 0.003$ to $(4/3, 0.05), (0.74, 0.10)$ corresponds to $(1.2, 0.1)$ and $(0.62, 0.15)$ corresponds to $(1.15, 0.15)$. For the best fit parameters $(1.01, -0.01)$ the present universe is corresponds to $(r_0, s_0) = (1.03, 0.0096)$. So the $r - s$ parameter for the present universe clearly distinguishing IMHRDE2 from the $\Lambda$CDM model with $(r_0, s_0) = (1.0)$ and also from the new holographic dark energy model with $(r_0, s_0) = (1.36, -0.102)$ [27].

In figures 15 and 16 we have plotted the evolution of the IMHRDE2 model in the $r - q$ plane. The time evolution in the $r - q$ plane is from right to left in the plot. For negative values of $\beta$, as seen in figure 15 both IMHRDE2 and $\Lambda$CDM models commence evolving from the same point in the past, corresponds to $r = 1, q = 0.5$, which corresponds to the matter dominates SCDM universe. The $\Lambda$CDM models ends with $r = 1, q = -1$ corresponds to the de Sitter phase, while the IMHRDE2 model, evolves in a different way. For positive values of $\beta$, the $r - q$ behavior is as shown in figure 16, where the behavior is almost of the same characteristics. The present position of the universe in the $r - q$ plane for negative $\beta$ values are $(r_0, q_0) = (1.14, -0.460$ corresponds to parameters $(4/3, -0.05)$, $(1.31, -0.57)$ corresponds to $(1.2, -0.1)$ and $(1.03, -0.55)$ corresponds to $(1.01, -0.01)$. For positive $\beta$ values the present position are $(0.87, -0.35$ corresponds to $(4/3, 0.05)$, $(0.74, -0.36$) corresponds $(1.2, 0.1)$ and
Figure 16: Evolution of the interactive MHRDE model in the \( r - q \) plane for parameters \((\alpha, \beta) = (1.33, 0.05), (1.2, 0.1), (1.15, 0.15)\) with \( b = 0.001 \)

(0.62, -0.3) corresponds to (1.15, 0.15). By comparing the present deceleration parameter corresponds to different parameter sets with the observationally constraint value, \( q_0 = -0.60 \), the parameter sets \((1.01, -0.01)\) and \((1.2, -0.1)\) are found to be good. Out of these two, the parameters \((1.2, -0.1)\) leads to phantom behavior as explained earlier so the present value of the \( q \) parameter by the IMHRDE2 model is taken as \(-0.55\) corresponds to parameters \((1.01, -0.01)\). It is also seen from 15 that corresponds to the parameters \((1.01, -0.01)\) the evolution trajectories of IMHRDE1 and \( \Lambda \)CDM are close to each other compared other parameters.

2.3 Interacting model with \( Q = 3bH\rho_{de} \)-IMHRDE3

In this section we consider IMHRDE model, with \( Q = 3bH\rho_{de} \). The second order differential equation in \( h^2 \) is found to be,

\[
\frac{d^2 h^2}{dx^2} + 3(\beta + b + 1) \frac{dh^2}{dx} + 9(\alpha b + \beta) h^2 = 0 \tag{32}
\]

This can be solved as,

\[
h^2 = f_1 e^{u_1 x} + f_2 e^{u_2 x} \tag{33}
\]

where

\[
f_1 = \frac{3 - 6\alpha + 3b + 3\beta - \sqrt{-36(\alpha b + \beta) + 9(1 + b + \beta)^2} + 6(\alpha - \beta)\Omega_{de0}}{2b - 2\sqrt{-36(\alpha b + \beta) + 9(1 + b + \beta)^2}}, \quad f_2 = 1 - f_1 \tag{34}
\]

and

\[u_1 = -3 - b - 3\beta - \sqrt{9(1 + b + \beta)^2 - 36(\alpha b + \beta)}, \quad u_2 = -3 - b - 3\beta + \sqrt{9(1 + b + \beta)^2 - 36(\alpha b + \beta)}\tag{35}\]

The density parameter and the equation of state in this case is obtained as,

\[
\Omega_{de} = f_1 e^{\frac{u_1}{2} x} + f_2 e^{\frac{u_2}{2} x} - \Omega_{m0} e^{-3x} \tag{36}
\]

and

\[
\omega_{de} = -1 - \left[ \frac{f_1 \frac{u_1}{2} e^{\frac{u_1}{2} x} + f_2 \frac{u_2}{2} e^{\frac{u_2}{2} x} + 3\Omega_{m0} e^{-3x}}{3 \left( f_1 e^{\frac{u_1}{2} x} + f_2 e^{\frac{u_2}{2} x} - \Omega_{m0} e^{-3x} \right)} \right] \tag{37}
\]
For non-interacting case with $b=0$, and by taking $\Omega_{de0}=1$, the constant coefficients become, $f_1=1$, $f_2=0$, $u_1=-6\beta$ and $u_2=-6$, consequently the equation of state parameter reduces to $\omega_{de} = -1 + \beta$ \[30\].

The value of the coupling constant $b$ is chosen to be $b = 0.009$, up to which, this model is compatible with the co-evolution of dark energy and dark matter. For values $b > 0.009$ the model fails to explain the coincidence problem. With this value of $b$, the evolution of the equation of state parameter is as shown figure 17. In this model also, the equation of state parameter of IMHRDE3 is starting form zero in the past stage of universe and evolving to negative values as the universe expands. In the far future, $\omega_{de}$ approaches to value less than -1 for parameters $(4/3,-0.05)$ and $(1.2,-0.1)$. So for these parameters the IMHRDE3 leads to phantom behavior in the far future \[37\]. But for parameters $(1.01,-0.01)$ the equation of state approaches -1 as $z \to -1$. While for positive $\beta$, the equation of state parameter saturates at values greater than -1 in the far future of the universe. The present values of the equation of state parameter for negative values of $\beta$, are -0.86 corresponds to the parameters $(4/3,-0.05)$, -0.97 corresponds to $(1.2,-0.1)$ and -0.97 corresponds to $(1.01,-0.01)$. For positive values of $\beta$ the value $\omega_{de0}$ are -0.78 corresponds to $(4/3,0.05)$, -0.78 corresponds to $(1.2,-0.1)$ and -0.75 corresponds to $(1.15,-0.15)$. The best fit is found to be -0.97 corresponds to the parameter $(\alpha, \beta) = (1.01,-0.01)$, which is in confirmation with the corresponding WMAP value $\omega_{de0}=-0.93$.

Next we analyses the evolution of the deceleration parameter for this case. With the solution \[33\], the deceleration parameter can be obtained as,

$$q = -1 - \frac{1}{2} \left[ \frac{u_1}{2} f_1 e^{\frac{u_1}{2} x} + \frac{u_2}{2} f_2 e^{\frac{u_2}{2} x} \right]$$

For the non-interacting limit and with $\Omega_{de}=1$, the deceleration parameter reduces to $q = (3\beta - 2)/2$, which is in confirmation with the earlier results \[30\]. The evolution of the $q$ parameter as the universe expands is shown in the figure 18. In this model also it is clear from figure 18 that, the deceleration parameter, is starting with 0.5 in the remote past of the universe. For negative values of $\beta$, the $q$ parameter approaches values less than -1 in the far future for $(4/3,-0.05)$ and $(1.2,-0.1)$. But for parameters $(1.01,-0.01)$ the deceleration parameter $q$ approaches -1 as $z \to -1$. For positive values of $\beta$, the $q$-parameter approaches values less than -1 for the three sets of parameters we have used. The present value for the deceleration parameter for different $(\alpha, \beta)$ are -0.47 corresponds to $(4/3,-0.05)$, -0.57 corresponds to $(1.2,-0.1)$, -0.57 corresponds to $(1.01,-0.01)$, -0.36 corresponds to $(4/3,0.05)$, -0.36 corresponds to $(1.2,0.1)$ and -0.34 corresponds to $(1.15,0.15)$. With the constraints from the observational data and also by the fact that the future universe doesn't show any phantom behavior, we are concluding that the best fit value for $q_0$ is -0.57 corresponds to the parameters $(1.01, -0.01)$. 

![Figure 17](image-url)
From the figure 18 is seen that the redshift at which the universe entering the accelerating phase is \( z_T = 0.49 \) corresponds to \((4/3, 0.05)\), \( 0.60 \) corresponds to \((1.2, -0.1)\), \( 0.70 \) corresponds to \((1.01, -0.01)\), \( 0.45 \) corresponds to \((4/3, 0.05)\), \( 0.50 \) corresponds to \((1.2, 0.1)\) and \( 0.53 \) corresponds to \((1.15, 0.15)\). The observational data prediction for the transition redshift is 0.45 - 0.73 [29]. In the light of this the best fit value from this model is \( z_T = 0.70 \) corresponds to \((\alpha, \beta) = (1.01, -0.01)\).

### 2.3.1 Statefinder analysis

The statefinder parameters \((r, s)\) can be obtained for IMHRDE3, by using the solution (33) as,

\[
r = 1 + \frac{\left[ \frac{u_1^2}{4} f_1 e^{\frac{u_1}{2} x} + \frac{u_2^2}{4} f_2 e^{\frac{u_2}{2} x} + \frac{3}{2} u_1 f_1 e^{u_1 x} + \frac{3}{2} u_2 f_2 e^{u_2 x} \right]}{2(f_1 e^{\frac{u_1}{2} x} + f_2 e^{\frac{u_2}{2} x})}
\]

(39)

and

\[
s = - \frac{\left[ \frac{u_1^2}{4} f_1 e^{\frac{u_1}{2} x} + \frac{u_2^2}{4} f_2 e^{\frac{u_2}{2} x} + \frac{3}{2} u_1 f_1 e^{u_1 x} + \frac{3}{2} u_2 f_2 e^{u_2 x} \right]}{\left[ \frac{3}{2} u_1 f_1 e^{\frac{u_1}{2} x} + \frac{3}{2} u_2 f_2 e^{\frac{u_2}{2} x} + 9 f_1 e^{\frac{u_1}{2} x} + 9 f_2 e^{\frac{u_2}{2} x} \right]}
\]

(40)

In non-interacting limit, \( b = 0 \) with \( \Omega_{de} = 1 \), the statefinder parameters reduces to the standard form \( r = 1 + 9\beta(\beta - 1)/2 \) and \( s = \beta \). The \( r - s \) plots for the present model is shown in the figures below. The figures [19] and [20] shows the behavior of IMHRDE3 in the \( r - s \) plane. The evolution of the \( r - s \) is from right to the left as in two previous cases. The today’s position of the universe and the \( \Lambda \)CDM fixed point (LCDM point) are noted in all the figures. It is clear that the distance between the present position of the universe and the \( \Lambda \)CDM is decreasing as \( \beta \) increases for negative values of \( \beta \). While for positive values of \( \beta \) the separation is increasing the \( r - s \) plane. Also the \( \Lambda \)CDM point is lying on the \( r - s \) plane for negative values of \( \beta \). But for the positive \( \beta \) values, the \( \Lambda \)CDM point is out of the \( r - s \) trajectory. An important point to be noted regarding the \( r - s \) behavior corresponds to the parameters \((1.01, -0.01)\), for which the \( \Lambda \)CDM point is lying on the future phase of the evolution of the universe. This indicating that for the best parameters, \((1.01, -0.01)\) the universe is tending towards the \( \Lambda \)CDM phase in the future. On the contrary for other parameters the \( \Lambda \)CDM point is lying on the past phase of the \( r - s \) plots as it is clear from figure [19].

The present position of the universe in the \( r - s \) plots are \((r_0, s_0) = (1.1, -0.034)\) corresponds to \((4/3, -0.05)\), \((1.27, -0.085)\) corresponds to \((1.2, -0.1)\), \((0.98, 0.0004)\) corresponds to \((1.01, -0.01)\), \((0.83, 0.066)\) corresponds to \((4/3, 0.05)\), \((0.70, 0.115)\) corresponds to \((1.2, 0.1)\) and \((0.60, 0.16)\) corresponds to \((1.15, 0.15)\). For the best fit parameters, \((1.01, -0.01)\) the present position in the \( r - s \) plane is distinguishing the IMHRDE3 from other standard models. For the new HDE model [27], the present position of the
Figure 19: $r - s$ plots for parameters $(\alpha, \beta) = (4/3, -0.05)$ (left panel), $(1.2, -0.1)$ (middle panel), $(1.01, -0.01)$ (the right panel) with $b = 0.009$

Figure 20: $r - s$ plots for parameters $(\alpha, \beta) = (4/3, 0.05)$ (left panel), $(1.2, 0.1)$ (middle panel), and $(1.15, 0.15)$ (right panel) with $b = 0.009$

The universe is corresponds to $(r_0, s_0) = (1.357, -0.102)$. Compared to the new HDE model, the present IMHRDE3 model is comparatively much closer to the $\Lambda$CDM model.

In order to confirm the $r - s$ behavior of IMHRDE3, we analyses the behavior of this model in the $r - q$ plane also. The respective plots are given in figures [21] and [22].

The evolution in the $r - q$ plane is from right to left. For negative values of $\beta$, none of the $r - q$ plots are starting from the SCDM phase, although for parameters $(1.2, -0.1)$, the starting point in the $r - q$ plane is very much close to the SCDM point. For the negative $\beta$ values we have considered, the distance of the starting point of IMHRDE3 and SCDM point in $r - q$ plane is the largest for the parameters $(1.01, -0.01)$, which is our best fit parameters. Also to be noted that the distance between the $\Lambda$CDM point and today’s position in the $r - q$ plane is the smallest for the parameters $(1.01, -0.01)$. For positive values of $\beta$ all the $r - q$ plots commences from the SCDM phase and the distance of the today’s position and $\Lambda$CDM point is increasing as $\beta$ increases. The present value of the $r - q$ parameters are $(1.1, -0.45)$ corresponds to $(4/3, -0.05)$, $(1.27, -0.57)$ corresponds to $(1.2, -0.1)$, $(0.98, -0.57)$ corresponds to $(1.01, -0.01)$, $(0.83, -0.35)$ corresponds to $(4/3, 0.05)$, $(0.70, -0.360$ corresponds to $(1.2, 0.1)$ and $(0.60, -0.33)$ corresponds to $(1.15, 0.15)$. For the best parameters $(1.01, -0.01)$, the present parameters $(r_0, q_0) = (0.98, -0.57)$, for which the $q_0$ value is very close the WMAP value -0.60. So for IMHRDE3 also $(1.01, -0.01)$ is the best fit parameters corresponds to which, the predicted cosmological parameters are $\omega_{de} = -0.97$, $z_T = 0.70$, $q_0 = -0.57$. 

universe is corresponds to $(r_0, s_0) = (1.357, -0.102)$. Compared to the new HDE model, the present IMHRDE3 model is comparatively much closer to the $\Lambda$CDM model.
3 Conclusions

We have considered the modified holographic Ricci dark energy interacting with the dark matter in a flat universe. The interaction is non-gravitational and linear. We consider three phenomenological form for the interaction term $Q$, which is basically proportional to the Hubble parameter $H$ and the densities of dark energy and dark matter. The three interaction forms are $Q = 3bH(\rho_{de} + \rho_m)$ (IMHRDE1), $3bH\rho_m$ (IMHRDE2), $3bH\rho_{de}$ (IMHRDE3). We have considered these three interaction cases separately, and studied the the evolution of the equation of state, deceleration parameter and also made the statefinder diagnostic analysis to discriminate the models form other standard models.

In the case of IMHRDE1, we have found that the interaction coupling constant can at most be $b = 0.001$, for values higher than this the IMHRDE1 is found to be incompatible with coincidence between dark energy and dark matter. So we took $b = 0.001$ for analyzing IMHRDE1. Further we have found that the best fit parameters are $(\alpha, \beta) = (1.01,-0.01)$. The corresponding the equation of state parameter evolves in such a way that at far future in the evolution of the universe, as $z \rightarrow -1$, $\omega_{de} \rightarrow -1$. This implies that the IMHRDE1 approaching a de Sitter phase in the far future. The present value of the equation of state parameter is found to be as $\omega_{de} = -0.96$ This value is agreeing closely with the value reported by the WMAP project [29], as -0.93. The transition of the universe to accelerating expansion is found to occur at redshift $z_T = 0.70$ This value is in agreement with the observational constraint $z_T=0.45 - 0.73$ obtained form the analysis of SNe+CMB data with $\Lambda$CDM model. We have also obtained the evolution of the deceleration parameter $q$ of IMHRDE1 model. For best parameters $(\alpha, \beta) = (1.01,-0.01)$, we have found that the deceleration parameter approaches -1 as
$z \to -1$, which again shows that the at future evolution the universe tends to the behavior of de Sitter universe. The present value of the deceleration parameter is obtained as $q_0 = -0.56$. The WMAP data constraint the $q_0$ as -0.60. So the IMHRDE1 value of $q_0$ is very close to this observationally constraint value. In a work by Luis et. al.\cite{19} have found that the value of $q_0 = -0.59$, for the IMHRDE with non-linear interaction between dark energy and dark matter for the same $(\alpha, \beta)$ parameters.

The statefinder analysis of the IMHRDE1 model have shown that for the best parameters, the statefinder parameter have the present value as $(r_0, s_0)=1.03,-0.008$. Compared the standard $\Lambda$CDM model, with $(r_0, s_0)=(1,0)$, so it is clear that the distance between the present IMHRDE1 model and $\Lambda$CDM model is very small in the $r-s$ plane. But compared to other standard models, for example, the Chaplygin gas model of dark energy, for which $(r > 1, s < 0)$, the present IMHRDE1 phase is different. However as universe evolves $r-s$ behavior of IMHRDE1 \cite{9} is approaching the Chaplygin gas behavior. But compared to HDE model with event horizon as the IR cut-off, for which $(r, s)=(1, 2/3)$, the IMHRDE1 is shows a different evolution. These are further verified with checking the evolution of the IMHRDE1 in $r-q$ plane.

In IMHRDE2 model the interaction coupling constant if found to be higher as $b = 0.003$, compared to IMHRDE1. For $b$ values above this the model is not compatible with the co-evolution of the dark sectors. The equation state evolution is obtained and found that for the best fit parameters $(\alpha, \beta)=(1.01,-0.01)$, the $\omega_{de} \to -1$ as redshift $z \to -1$. This implies that, in the far future the IMHRDE2 model also tending towards a de Sitter type evolution. The present value of the equation of state parameter is around $\omega_{de0} = -0.96$, which is close to WMAP value -0.93. The evolution of the deceleration parameter is shown in figure \cite{12} Accordingly the transition redshift is found to $z_T = 0.68$ for best fit parameters, which is in close agreement with observationally constraint range \cite{35}. The present value of the $q$-parameter of IMHRDE2 is $q_0 = -0.55$ for the best fit parameters, and is close agreement with the corresponding value predicted WMAP data.

The statefinder evolution is studied for the IMHRDE2, and found that, the evolution is almost similar with that of IMHRDE1. The present values of the statefinder parameters is found to be $(r_0, s_0)=(1.03,-0.0096)$, which implies that the IMHRDE2 will behave as Chaplygin gas in the future. In discriminating the IMHRDE2 from other models, the present position of IMHRDE2 is different compared to the $\Lambda$CDM model with $(1,0)$ and HDE model with event horizon as IR cut-off with $(1,2/3)$. The $r-q$ plane plot seen to be compatible with the above conclusions.

For the analysis of IMHRDE3 we have chosen the coupling constant as $b = 0.009$, because for values higher than this, the model does not predict the co-evolution of the dark sectors. The analysis of the evolution of equation of state parameter for this have clearly shown that $\omega_{de} \to -1$ as $z \to -1$ for the best fit parameters $(1.01,-0.01)$. This implies that as like other two models this model also tend towards a de Sitter evolution in the far future. The present equation of state parameter for the best fit $(\alpha, \beta)$ is found to around $\omega_{de0}=-0.97$, which is close to the observationally constraint value. The deceleration parameter of this model is evolved in such that, the transition to accelerating phase is occurred at around $z_T=0.70$, and is evidently very well in the observational range. The present value of $q_0$ is around -0.57 for best fit parameters, which is quite close to WMAP prediction.

The IMHRDE3 model was discriminated form other standard models using the $r-s$ diagnosis. The behavior of the model in the $r-s$ plane have shown that, for the best fit parameters, the model is evolving to the $\Lambda$CDM phase in the future. This shows that the de Sitter evolution of the IMHRDE3 in future may end on the $\Lambda$CDM phase. The present statefinder parameters are $(r_0, s_0)=(0.98,0.0004)$, which is close to the $\Lambda$CDM point and different from the HDE model with event horizon as the IR cut-off.

In summary we have considered the IMHRDE model with possible interaction between the dark energy and dark matter. We have found that all the three interacting models will approach the de
Sitter phase in the later stages of the evolution of the universe for the best parameters $(\alpha, \beta) = (1.01, -0.01)$. The best fit $(\alpha, \beta)$ parameters have -ve value for $\beta$, which is advisable because the positivity of dark energy density require to take $\beta < 0$ [19]. In particular the IMHRDE3 model evolves to $\Lambda$CDM phase in its future evolution. A similar kind of work was carried out in reference [38], with another form for Ricci dark energy $\rho_{de} = 3\alpha M_{Pl}^2 (H + 2H^2)$ for the same forms for the interaction term $Q$, where the authors mainly concentrated on evaluating the Hubble Parameter and density parameter for dark matter. In our work we have done the evaluation of different parameters and analyses the IMHRDE with statefinder diagnosis.

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