Spin-dependent transport of carriers in semiconductors

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This article reviews spin-dependent transport of carriers in homogeneous three-dimensional and two-dimensional semiconductors. We begin with a discussion of optical orientation of electron spins, which allows both the creation and detection of spin-polarized carriers in semiconductors. Then we review non-equilibrium spin flow including spin drift and diffusion caused by electric fields and concentration gradients. A controlled spin precession is possible both in external magnetic fields and in effective magnetic fields due to a broken inversion symmetry. Although the Coulomb interaction does not couple to the spin degree of freedom, it affects the spin-dependent transport via the spin Coulomb drag. In gyrotropic media, the optical creation of spin-oriented electrons gives rise to spin photocurrents, which reverse their direction when the radiation helicity is changed from left-handed to right-handed. The reverse process is possible, too, i.e., an electric current in a gyrotropic medium gives rise to a spin polarization in the bulk of the sample.

1 Introduction

Broadly speaking, spin-dependent transport phenomena in semiconductors can be divided into two categories. On the one hand, we have those effects such as spin drift, spin diffusion, and spin precession that refer to the transport of spin-polarized carriers. These effects are of central importance for spintronics device concepts where the generation of spin-polarized distributions of carriers are spatially separated from those elements that manipulate and detect the spins. On the other hand, we also have spin-dependent phenomena such as the spin Coulomb blockade or weak localization and spin-split Shubnikov-de Haas oscillations visible in magneto-transport of two-dimensional (2D) electron systems. These effects provide important insights into the nature of the spin-dependent interactions, such as exchange and spin-orbit coupling, that can be exploited for the manipulation of spin systems. In this review, we will focus mostly on the former class of phenomena. Also, we will focus mostly on homogenous systems and touch only briefly on the properties of structured devices that are discussed [elsewhere].

We begin with a discussion of optical orientation of electron spins in semiconductors (Section 2). Then we review non-equilibrium spin flow including spin drift and diffusion...
(Section 3.1), and spin precession (Section 3.2). Coulomb effects in spin transport are discussed in Section 3.3. Finally, we review in Section 4 spin photocurrents and the reverse effect, the electrical generation of a spin polarization.

2 Optical orientation of electron spins

Various schemes have been developed to generate spin-polarized carrier distributions in non-magnetic semiconductors. Broadly speaking, these fall into three categories. First, optical excitation allows creation of spin-polarized electrons inside the semiconductor. Second, magnetic layers can be used to inject spin-polarized carriers into the semiconductors. These magnetic layers can be either ferromagnetic or semimagnetic semiconductors (see [elsewhere]) or ferromagnetic metal electrodes attached to the semiconductor (see [elsewhere]). Finally, dynamic phenomena based on electric fields and charge currents can give rise to spin polarization inside the semiconductor or spin accumulation at the edges of the sample (see Section 4.2 and [elsewhere]).

Here we will focus on the optical orientation of electrons that has proven to be a powerful technique since some of the earliest studies of spin-related phenomena in semiconductors were performed [Lampel 1968, Meier and Zakharchenya 1984]. In direct semiconductors like GaAs, the electron states in the conduction band have spin $S = 1/2$, whereas the hole states in the valence band have an effective spin $S = 3/2$. The hole states with spin $z$ component $S_z = \pm 3/2$ are denoted heavy-hole (HH) states, whereas the light-hole (LH) states have $S_z = \pm 1/2$. Left (right) circularly polarized photons carry a $z$ component of angular momentum of $-1 (+1)$ so that conservation of angular momentum results in the selection rules for optical transitions depicted in Figure 1. A more detailed analysis shows that the probabilities for transitions from the HH states to the conduction band are three times larger than the probability for optical transitions from the LH states. In bulk semiconductors, the maximum attainable degree of spin polarization is thus $P = 50\%$, where $P$ is defined as

$$P = \frac{N_+ - N_-}{N_+ + N_-},$$

and $N_+$ ($N_-$) is the number of electrons with spin up (down), respectively. In 2D systems the degeneracy of the HH and LH states is lifted as sketched in Figure 1. For resonant excitation at the HH energy we thus expect a rise of the maximum attainable degree of polarization up to $P = 100\%$.

A particular advantage of the optical orientation scheme lies in the fact that it holds both for absorption and emission, so that it can be used for creating and for detecting spin-polarized carrier distributions. However, the holes lose their spin orientation significantly faster than the electrons, and the oriented electrons can recombine with any hole. Therefore, Figure 1 implies that the polarization of the recombination photoluminescence (PL) in bulk semiconductors does not exceed $\sim 25\%$. (This does not apply to 2D systems where the recombination PL is due to a transition from the lowest electron to the lowest HH state.)
Even in a single-particle picture for the optical excitation, the 3 : 1 ratio of HH and LH transitions is obtained only if HH-LH coupling of the hole states at nonzero wave vectors $k$ is neglected. Due to this HH-LH coupling, the hole states with $k > 0$ are not spin eigenstates. Furthermore, a realistic picture must take into account that optical absorption gives rise to the formation of excitons, i.e., Coulomb correlated electron-hole pairs. Thus even for excitations close to the absorption edge we get substantial HH-LH coupling because the exciton states consist of electron and hole states with $k$ of the order of $1/a_B^*$, where $a_B^*$ is the effective Bohr radius. The Coulomb coupling between different electron and hole states yields a second contribution to the mixing of single-particle states with different values of $S_z$.

Finally, we must keep in mind that for higher excitation energies we get a superposition of exciton continua that are predominantly HH- or LH-like. These different excitons contribute oppositely to the spin orientation of electrons. We note that these arguments are valid for the optical excitation of bulk semiconductors and quasi-2D systems (Pfalz et al., 2005).

Optical orientation in bulk systems was reviewed by Dyakonov and Perel (1984). Early works on 2D systems were published by Weisbuch et al. (1981) and Masselink et al. (1984) who reported on polarization-resolved transmission and PL experiments on GaAs/AlGaAs quantum wells (QWs) under cw excitation. In later works, the electron spin polarization in quasi-2D systems was studied using time-resolved photoluminescence excitation spectroscopy. For excitation energies even slightly above the HH resonance, several authors (Freeman et al., 1990; Dareys et al., 1993; Muñoz et al., 1995) observed a polarization that was significantly smaller than one. These measurements were carried out on fairly narrow GaAs/AlGaAs QWs. A first well-width dependent study of optical orientation was performed experimentally by Roussignol et al. (1992). For energies near the HH resonance, they found initial spin polarizations in the range of 60 – 80%.

Twardowski and Hermann (1987) as well as Uenojama and Sham (1990) studied the polarization of QW PL theoretically, taking into account HH-LH coupling in the valence band. However, these authors neglected the Coulomb interaction between electron and hole states. On the other hand, Maiialle et al. (1993) investigated the spin dynamics of excitons taking into account the exchange coupling between electrons and holes, but disregarded the HH-
3 Non-equilibrium spin flow in semiconductors

3.1 Spin drift and diffusion

Similar to electric charge distributions in semiconductors, a non-equilibrium spin distribution can spread out diffusively or it can drift in the presence of an electric field. However, these phenomena behave qualitatively different in $p$- and $n$-type semiconductors (D'yakonov and Perel', 1971a). In $p$-type semiconductors, only the spins of the non-equilibrium electrons become oriented. Their number is proportional to the intensity of the light, but the degree of orientation does not depend on the intensity (Figure 1). As drift and diffusion of the spin orientation must preserve charge neutrality, the kinetics of the spin orientation follows the kinetics of the charge distribution. Charge diffusion in doped semiconductors is characterized by the diffusion coefficient of the minority carriers (Smith, 1978). Therefore, electron spin diffusion
in $p$-type semiconductors is essentially characterized by the charge diffusion coefficient for electrons.\(^1\)

In $n$-type semiconductors the situation is qualitatively different due to the fact that the optically excited electrons augment the equilibrium electrons \(\text{(D'yakonov and Perel', 1971a)}\). Therefore, significant optical orientation of electron spins is possible at moderate degrees of excitation when the excess photoelectron density is still much less than the equilibrium electron density. The mechanism underlying this effect is the following. Absorption of circularly polarized light creates electrons with mainly a single spin orientation. The spin relaxation time $\tau_s$ of these electrons is typically much greater than the excess carrier lifetime. Holes, on the other hand, have a short spin relaxation time so that the spin orientation of the optically created holes is quickly lost. Therefore any electron can recombine with these holes with a recombination rate that is usually independent of the sign of the spin. Thus, optical excitation is a source for spin-polarized electrons whereas recombination represents a drain for electrons with the “wrong” spin orientation. Under stationary excitation, the oriented electrons are the equilibrium ones.

In a bulk sample, the light is usually absorbed in a narrow layer near the surface of the crystal. In this case the excess carriers penetrate a distance of the order of the diffusion length $L = \sqrt{D_p \tau}$, where $D_p$ is the hole diffusion coefficient in the $n$-type sample (which is usually small), and $\tau$ is the lifetime of the non-equilibrium carriers. On the other hand, the orientation penetrates a depth of the order of the spin diffusion length $L_s = \sqrt{D_s \tau_s}$, where $D_s$ is the spin diffusion coefficient of the electrons (which is usually large; it is approximately equal to the electron diffusion coefficient $D_e$). Under typical experimental conditions we thus have $L_s \gg L$ in an $n$-type sample \(\text{(D'yakonov and Perel', 1971a)}\). Beyond a layer of thickness $\sim L$, recombination cannot change the degree of polarization $P$ that falls off like $P(z) = P(0) \exp(-z/L_s)$, i.e., a spin orientation of the order of $P(0)$ penetrates into a layer of depth $\sim L_s$, where there are no excess carriers. [A small number of photoexcited carriers can be present within this layer because of reabsorption \(\text{(Dzhioev et al., 1997)}\).]

In general, the motion of the spin density $S$ is characterized by a drift-diffusion equation \(\text{(D'yakonov and Perel', 1976; Garbuzov et al., 1976; Dyakonov and Perel, 1984)}\)

\[
\frac{\partial S}{\partial t} = D_s \nabla^2 S + \frac{eE \cdot \nabla S}{k_B T} + \Omega \times S - \frac{S}{\tau_s} - \frac{S - S_0}{\tau},
\]

(2)

similar to the drift and diffusion of charge. Here $E$ is a built-in or external electric field; $T$ is the temperature; and $\Omega$ is the spin precession frequency, which can be due to an external magnetic field $B$, i.e., $\Omega = g^* \mu_B B/\hbar$, or due to spin-orbit coupling at $B = 0$, see Section 3.2 below. The last two terms reflect two reasons for the spin orientation to vanish, spin

\(^1\)When a semiconductor has a large absorption coefficient near the band edge, an emitted photon is usually reabsorbed before it can escape the crystal \(\text{(Dumke, 1957)}\). The detailed analysis of spin diffusion in $p$-type semiconductors performed by \(\text{Garbuzov et al., 1976} \) and \(\text{Gioev et al., 1994}\) showed that allowance for diffusion and reabsorption was essential for the proper interpretation of their experimental data. Even in $n$-type GaAs it was found that reabsorption can be important for spin diffusion \(\text{(Dzhioev et al., 1997)}\). Please note that the first authors of the latter two publications are, in fact, the same.


relaxation and recombination, where $S_0$ is the average spin orientation at the moment of photocreation. Recently, the drift-diffusion equation \cite{Flatté2000} was reconsidered by Flatté and Byers \cite{Flatté2000} and Yu and Flatté \cite{Yu2002}.

Spin drift and diffusion have been studied experimentally by several groups. Dzhioev \textit{et al.} \cite{Dzhioev1997} estimated that the spin diffusion length in their $n$-type GaAs sample was $L_s = 10 \mu m$. Hägele \textit{et al.} \cite{Hägele1998} found that the spin orientation in intrinsic GaAs was almost completely preserved over a distance of $4 \mu m$. Kikkawa and Awschalom \cite{Kikkawa1999} performed a detailed study of spin transport in intrinsic and $n$-type GaAs samples in which gates allowed one to stir the drift of the spin-polarized electrons. Using non-local Faraday rotation, they found that the drift distance of the spin-oriented electrons was linear in the electric field, and it could exceed a distance of $100 \mu m$ for electric fields of $16 V \text{ cm}^{-1}$. Fiederling \textit{et al.} \cite{Fiederling1999} used semimagnetic $\text{Be}_x\text{Mn}_y\text{Zn}_{1-x-y}\text{Se}$ to inject spin-polarized electrons into a $0.1-\mu m$-thick layer of $n$-type AlGaAs followed by a $15$-nm-wide GaAs, where the spin-polarized electrons recombined with holes that were injected from the other side of the QW (a spin light-emitting diode). In a similar experiment, Ohno \textit{et al.} \cite{Ohno1999} used ferromagnetic GaMnAs to inject spin-polarized electrons into an intrinsic layer of GaAs, followed by an InGaAs QW.

It has been found that interfaces between different semiconductors do not affect spin transport. This was first noticed by Garbuzov \textit{et al.} \cite{Garbuzov1976}, who studied spin orientation for a sample that contained a GaAs QW embedded in thick graded layers of $p$-type $\text{Al}_x\text{Ga}_{1-x}\text{As}$. Malajovich \textit{et al.} \cite{Malajovich2000} found that even the interface between ZnSe and GaAs, a II-VI and a III-V semiconductor, did not suppress spin transport.

Lateral spin diffusion was studied by Cameron \textit{et al.} \cite{Cameron1996}. When two laser beams with crossed polarizations interfere, the light intensity on the sample is uniform, but the polarization alternates between left polarized, linear, and right polarized. Therefore, a spin grating is generated in the sample where the optical orientation of the electrons alternates across the excitation region. By analyzing the orientation decay as a function of time, these authors could determine the spin diffusion coefficient $D_s$ and the spin relaxation time $\tau_s$. The spin diffusion length $L_s = \sqrt{D_s \tau_s}$ appeared to be approximately $8 \mu m$ \cite{Kavokin2002}.

### 3.2 Spin precession

The magnetic-field-dependent term $\Omega \times S$ in equation (2) describes the precessional motion of the oriented spins in an external field $B$ or an effective field due to spin-orbit coupling. For a transverse external field $B$, it gives rise to the Hanle effect, a depolarization of luminescence induced by the field $B$ \cite{Dyakonov1984}. In a homogenous system [i.e., $\nabla S = 0$ in equation (2)], we get the expression for the Hanle curve

$$S_z(B) = \frac{S_z(0)}{1 + (\Omega T_s)^2},$$

where $S_z(0) = \frac{S_0}{1 + \tau/\tau_s}$. \hspace{1cm} (3)

Here we have assumed that the $z$ direction is the direction of the exciting radiation with $S_0 \perp B$, and $T_s$ is the “spin lifetime” defined by $T_s^{-1} = \tau^{-1} + \tau_s^{-1}$. From the Hanle curve as a function of field $B$, one can thus extract the lifetime $\tau$ and the spin relaxation time $\tau_s$ of the
carriers (provided the effective Landé factor $g^*$ is known). However, a particular situation arises in $n$-type semiconductors where recombination is not possible past the surface layer of thickness $\sim L$. The depolarization induced by the magnetic field thus changes the gradient of the degree of polarization within the layer where electrons are oriented. Therefore, the spin diffusion rate becomes magnetic-field-dependent, which results in a distinct change of the functional form of the Hanle curve as a function of magnetic field (D'yakonov and Perel', 1976).

In the presence of both time-inversion symmetry and space-inversion symmetry, all electron states in a solid with a given wave vector $k$ are twofold degenerate. When the potential through which the carriers move is inversion-asymmetric, however, the spin degeneracy is removed even in the absence of an external magnetic field $B$. We then obtain two branches of the energy dispersion, $E_+(k)$ and $E_-(k)$. This spin splitting can be the consequence of a bulk inversion asymmetry (BIA) of the underlying crystal [e.g., a zinc blende structure (Dresselhaus, 1955)], and of a structure inversion asymmetry (SIA) of the confinement potential (Ohkawa and Uemura, 1974; Bychkov and Rashba, 1984). Strain gives rise to a third contribution to $B = 0$ spin splitting (Seiler et al., 1977; Howlett and Zukotynski, 1977). A fourth contribution can be the low microscopic symmetry of the atoms at an interface (Rössler and Kainz, 2002). $B = 0$ spin splitting has been reviewed, e.g., by Pikus et al. (1988) and Winkler (2003). In the present context it is important that the spin splitting can be ascribed to an effective Zeeman term $H = (\hbar/2) \sigma \cdot \Omega(k)$ with an effective magnetic field $\Omega(k)$. In leading order of $k$, the effective field in a 2D electron system on a (001) surface reads

$$
\Omega(k) = \frac{2\gamma}{\hbar} \begin{pmatrix}
    k_x \left( k_y^2 - \langle k_z^2 \rangle \right) \\
    k_y \left( \langle k_z^2 \rangle - k_x^2 \right) \\
    0
\end{pmatrix} + \frac{2\alpha}{\hbar} \begin{pmatrix}
    k_y \\
    -k_x \\
    0
\end{pmatrix}.
$$

The first term characterizes the BIA spin splitting of the electron states. It is called the Dresselhaus or $k^3$ term (Dresselhaus, 1955; Braun and Rössler, 1985). It exists already in bulk zinc blende semiconductors due to the broken inversion symmetry. In quasi-2D systems only the in-plane wave vector $k_{||} = (k_x, k_y, 0)$ is a continuous variable. In first-order perturbation theory, the wave vector components $k_z$ and powers thereof are replaced by expectation values with respect to the subband wave functions. The field $\Omega(k)$ due to BIA is depicted in Figure 3(a). We note that in 2D systems, the Dresselhaus term depends on the crystallographic orientation of the substrate. For 2D systems on an $[mnm]$ surface with integers $m$ and $n$, the Dresselhaus term was given by Braun and Rössler (1985).

The momentum scattering of electrons off other electrons, impurities, phonons, etc., results in a random walk of oriented electrons in the field $\Omega(k)$, which gives rise to the so-called Dyakonov-Perel spin relaxation (D'yakonov and Perel', 1976) discussed [elsewhere]. A controlled precession of electrons in the Dresselhaus field $\Omega(k)$ was first demonstrated by Riechert et al. (1984), who investigated the polarization of photoemission following optical orientation. After deposition of Cs and O on the (110) surface of their strongly $p$-doped GaAs
sample, a surface inversion layer was formed where the bands were strongly bent downward. Electrons moving through this layer gain a large kinetic energy. Yet if the layer is very narrow, they move ballistically with $k$ normal to the surface so that the direction of $\Omega$ is the same for all escaping electrons. The photoelectron orientation is thus rotated away from the initial direction defined by the pumping light beam, as observed by Riechert et al.

In asymmetric QWs, SIA gives rise to the second term in equation (4), which is frequently called the Rashba term (Rashba, 1960; Bychkov and Rashba, 1984). The coefficients $\gamma$ and $\alpha$ depend on the underlying semiconductor bulk material. But $\alpha$ depends also on the asymmetry of the QW in the growth direction. It can be tuned by means of front and back gates (Nitta et al., 1997). This is exploited in the famous spin field-effect transistor proposed by Datta and Das (1990), which is discussed elsewhere. The field $\Omega(k)$ due to SIA is depicted in Figure 3(b).

A third contribution to $\Omega(k)$ at $B = 0$ is obtained by means of strain. In lowest order of $k$ and of the components $\varepsilon_{ij}$ of the strain tensor we get (Pikus and Titkov, 1984)

$$\Omega(k) = C_3 \frac{\hbar}{\varepsilon_{xy} k_y - \varepsilon_{xz} k_z} + C'_3 \frac{\hbar}{k_x (\varepsilon_{yy} - \varepsilon_{zz}) + k_y (\varepsilon_{zz} - \varepsilon_{xx}) + k_z (\varepsilon_{xx} - \varepsilon_{yy})}.$$ (5)

The first term depends on the off-diagonal components of the strain tensor, i.e., these components describe a shear strain. They are nonzero, e.g., when uniaxial stress is applied in the crystallographic directions [111] or [110] of a bulk crystal (Trebin et al., 1979). The prefactor $C'_3$ of the second term in equation (5) is nonzero only because of coupling to remote bands outside the usual $8 \times 8$ Kane Hamiltonian. Therefore, this term is rather small, so usually it can be neglected (Pikus and Titkov, 1984; D'yakonov et al., 1986).

For bulk InSb, the effect of strain on spin splitting has been studied by measuring Shubnikov-de Haas oscillation (Seiler et al., 1977) and cyclotron resonance (Ranvaud et al., 1979). D'yakonov et al. (1986) analyzed the Hanle effect in the presence of strain in order to obtain $C_3 = 20 \text{ eVÅ}$ for GaSb, $C_3 = 5 \text{ eVÅ}$ for GaAs, and $C_3 = 3 \text{ eVÅ}$ for InP. The decrease

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$^2$The Rashba spin splitting discussed below was not taken into account by Riechert et al. (1984) for the interpretation of their experiment. However, this does not change the qualitative picture.
of these values from GaSb to InP correlates with the decrease in the spin-orbit interaction gap in these crystals from 0.82 eV to 0.11 eV.

The effect of strain on spin transport in n-type (001) GaAs was first studied by Kato et al. (2004a) using time and spatially resolved Faraday rotation spectroscopy. However, they did not quantify or tune the strain in their samples. The implications of equation (5) have been confirmed in detail in experiments by Crooker and Smith (2005). Similar experiments have been published also by Beck et al. (2005). Crooker and Smith used a small vise to apply tunable uniaxial strain along the [110] or [1Ì0] direction of their n-GaAs sample, while a circularly polarized 1.58-eV laser focused to a 4-µm spot was used to create locally a spin orientation along [001]. In spatially resolved measurements using Kerr rotation they studied the electron spin precession while the electrons drifted and diffused away from the position of the laser spot, where the spin orientation was created (see Figure 4).

Crooker and Smith found that the spin precession of electrons drifting and diffusing in the strain field (5) is more robust than the precession of electrons moving in an external magnetic field. The reason is that in a transverse magnetic field the ensemble spin orientation dephases quickly when the precession period falls below the spin diffusion length (the Hanle effect discussed above). The strain-induced field (5), on the other hand, is linear in the wave vector \( k \) so that slowly moving electrons experience a smaller field \( \Omega(k) \) than the faster electrons. But the distance the electrons must travel for a complete precession period is the same for slow and fast electrons so that the electrons remain in phase [Figures 4(c) and (d)]. This argument also implies that the precession period should be independent of the magnitude of the external electric field used to push the electrons, as confirmed by the experiments of Crooker and Smith and Beck et al.

The strain-induced field (5) has a pronounced dependence on the wave vector \( k \). If a uniaxial strain is applied along the direction [110], we have \( \Omega = 0 \) for \( k \parallel [001] \). This is analogous to the fact that we have no Dresselhaus spin splitting in symmetric QWs on a (110) surface for \( k \parallel [001] \) (Winkler, 2003). Within the (001) plane the \( k \) dependence of \( \Omega \) is the same as for the Rashba term, see Figure 3(b). If in addition to the strain-induced
field an external magnetic field \( \mathbf{B} \) is applied, the electrons with \( \mathbf{\Omega}(\mathbf{k}) \) approximately parallel to \( \mathbf{B} \) precess faster than the electrons with \( \mathbf{\Omega}(\mathbf{k}) \) approximately antiparallel to \( \mathbf{B} \). This was confirmed by the experiments of Crooker and Smith. To show this they used the fact that the radial diffusion in a pure strain-induced field or an external magnetic field \( \mathbf{B} \) is independent of the direction of \( \mathbf{k} \), which reflects the fact that the magnitude of \( \mathbf{\Omega} \) does not depend on the direction of the \( \mathbf{k} \) vector of the electrons. The superposition of both fields, on the other hand, results in an anisotropic total field \( \mathbf{\Omega} \) the magnitude of which depends on the direction of \( \mathbf{k} \). This is similar to the fact that, to lowest (i.e., linear) order in \( \mathbf{k} \parallel \), the magnitude of both the Dresselhaus and Rashba spin splitting in 2D systems are independent of the direction of \( \mathbf{k} \parallel \) (see Figure 3), yet the superposition of both terms gives rise to anisotropic spin splitting even in linear order of \( \mathbf{k} \parallel \) (de Andrada e Silva, 1992).

The interplay of diffusion, drift in electric fields, and precession in external magnetic fields was studied theoretically by Qi and Zhang (2003) using a semiclassical Boltzmann equation for the \( 2 \times 2 \) spin density matrix in order to cope with the different length scales of this problem, such as the diffusion length \( L \), the spin diffusion length \( L_s \), and the spin precession length. Spin diffusion equations for systems with Rashba spin-orbit interaction in an electric field were studied by Bleibaum (2006). Drift and diffusion were also studied theoretically by Hruška et al. (2006) for an experimental setup similar to the one used by Crooker and Smith (2005) as described above.

3.3 Coulomb effects

So far we have completely neglected the Coulomb interaction between the electrons. Although this interaction does not couple to the spin degree of freedom of the electrons, it has a great influence on spin-dependent transport. This has long been known in the context of spin diffusion in spin-polarized liquid \(^3\)He. Leggett and Rice (1968) and Leggett (1970) have shown that the spin polarization gives rise to a molecular field, and any given spin will then see (and precess around) a total field that is the sum of the molecular field and the external field. This molecular field cannot affect the precession of the total spin density \( \mathbf{S} \), since it is automatically parallel to it. However, it produces a torque on the spin current which is present in the equation of continuity for the latter. Leggett showed that, as a result, the equation for \( \mathbf{S} \) in the hydrodynamic limit no longer has a simple form similar to equation (2) but he derived a significantly more complicated hydrodynamic-type spin diffusion equation. More recently, Takahashi et al. (1999) have applied these ideas to the spin diffusion and drift in 2D electron systems. They solved the quantum kinetic equation derived from the equation of motion for the non-equilibrium real-time Green’s functions in order to show that the spin rotation term known for \(^3\)He is indeed also present in degenerate 2D electron systems at low temperatures.

In a sequence of papers, D’Amico and Vignale (2000, 2001, 2002, 2003) performed a detailed theoretical analysis of how the Coulomb interaction affects spin-polarized transport and diffusion in electron systems [see also Flensberg et al. (2001)]. D’Amico and Vignale showed that the Coulomb interaction gives rise to a spin Coulomb drag between the electrons.
moving with spin up and the electrons moving with spin down, similar to the Coulomb drag that has been observed for electrons in two spatially separated layers \textit{[Gramila et al., 1991; Rojo, 1999]}. The spin Coulomb drag reflects the fact that while, in the absence of impurities, the total momentum $P = \sum_i p_i$ of the electrons is preserved, the “up” and “down” components of the total momentum, $P^\uparrow = \sum_i p_i(1 + \sigma_{zi})/2$ and $P^\downarrow = \sum_i p_i(1 - \sigma_{zi})/2$, are not separately preserved, even in the absence of impurities. Here $p_i$ is the momentum of the $i$th electron, and $\sigma_{zi}$ is the Pauli matrix for the $z$ component of the $i$th electron spin. Coulomb scattering can transfer momentum between spin-up and spin-down electrons, thereby effectively introducing a “friction” for the relative motion of the two spin components, which tends to equalize the net momenta of the spin components (see Figure 5).

In a more rigorous formulation, Ohm’s law can be written in the form

$$
\begin{pmatrix}
\mathbf{E}^\uparrow \\
\mathbf{E}^\downarrow
\end{pmatrix}
= 
\begin{pmatrix}
\rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\
\rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow}
\end{pmatrix}
\begin{pmatrix}
\mathbf{j}^\uparrow \\
\mathbf{j}^\downarrow
\end{pmatrix}.
$$

(6)

Here, the effective electric fields $\mathbf{E}_\sigma$ are the sums of a spin-independent external electric field plus the gradient of the local chemical potential, which can be spin-dependent, and $\mathbf{j}_\sigma$ is the electric current carried by the electrons with spin $\sigma$. The spin Coulomb drag gives rise to a spin trans-resistivity $\rho_{\uparrow\downarrow}$ in equation (6), which is the ratio of the gradient of the spin-down electrochemical potential to the spin-up current density when the spin-down current is zero. \textit{D’Amico and Vignale (2000)} evaluated $\rho_{\uparrow\downarrow}$ in a generalized random-phase approximation. \textit{D’Amico and Vignale (2001)} showed that the Coulomb interaction usually gives rise to a significant reduction of the spin diffusion coefficient $D_s$ in equation (2) as compared to its value $D_{ni}$ in a noninteracting system. They obtained

$$
D_s = \frac{\mu k_B T}{e} \frac{S}{S_c} \frac{1}{1 - \rho_{\uparrow\downarrow}/\rho_D},
$$

(7)

where $\mu k_B T/e$ is the diffusion constant of a noninteracting system in the high-temperature limit (Einstein’s relation), $S$ is the spin stiffness (i.e., the inverse of the spin susceptibility), $S_c = k_B T n/(4n_\uparrow n_\downarrow)$ is the Curie spin stiffness of an ideal classical gas, and $\rho_D = m^*/(ne^2 \tau_D)$ is the Drude resistivity. Figure (c) shows the ratio $D_s/D_{ni}$ as a function of density $n$, assuming a dielectric constant $\varepsilon = 12$ appropriate for GaAs and mobility $\mu = 3 \times 10^3 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. Different line styles correspond to different temperatures as indicated. The curves labeled
Figure 6: Ratio $D_s/D_{ni}$ between the spin diffusion coefficient $D_s$ of an interacting electron system and the spin diffusion coefficient $D_{ni}$ of the corresponding noninteracting system, plotted as a function of density $n$, and assuming a dielectric constant $\varepsilon = 12$ appropriate for GaAs and mobility $\mu = 3 \times 10^5$ cm$^2$ V$^{-1}$ s$^{-1}$. The curves labeled SD correspond to the case in which interactions in $D_s$ are taken into account only through the spin Coulomb drag [i.e., the third factor in equation (7)]. [Adapted from D’Amico and Vignale (2002).]

SD correspond to the case in which interactions in $D_s$ are taken into account only through the spin Coulomb drag [i.e., the third factor in equation (7)].

Figure 6 shows that the interaction correction is quite significant and reduces the value of $D_s$. For large densities or $T \lesssim T_F$, where $T_F$ is the Fermi temperature for density $n$, the dominant effect in the full calculation stems from the softening of the spin stiffness. On the other hand, the spin drag contribution dominates at small densities (the nondegenerate limit $T \gg T_F$). Note that $T_F = 1.6$, 20, and 300 K correspond to $n = 7.4 \times 10^{15}$, $3.2 \times 10^{17}$, and $1.9 \times 10^{19}$ cm$^{-3}$, respectively.

The spin Coulomb drag in a 2D electron gas was studied theoretically by D’Amico and Vignale (2003) and Flensberg et al. (2001), giving results quantitatively similar to 3D electron systems. It was also observed experimentally by Weber et al. (2005). These authors used spin gratings as discussed at the end of Section 3.1 to measure the spin diffusion coefficient $D_s$ in a 2D electron gas in a GaAs/AlGaAs QW (circles in Figure 7). Its value as a function of temperature is significantly smaller than the charge diffusion coefficient $D_c$ obtained from transport measurements for samples from the same wafer (solid lines in Figure 7). Yet good agreement is achieved between the measured $D_s$ and calculations taking into account the spin Coulomb drag effect [i.e., the last factor in equation (7)], see the dashed line in Figure 7.
Figure 7: Measured (circles) and calculated (dashed line) spin diffusion coefficient $D_s$ and charge diffusion coefficient $D_c$ (solid line) in a 2D electron gas in a GaAs/AlGaAs QW. The electron concentration is $n = 4.3 \times 10^{11} \text{ cm}^{-2}$, which corresponds to a Fermi temperature $T_F = 220 \text{ K}$. The inset shows the ratio between the measured $D_s$ and $D_c$. [Adapted from Weber et al. (2005).]

4 Spin polarization and charge currents

4.1 Spin photocurrents

The optical creation of spin-oriented electrons can give rise to charge currents, the so-called spin photocurrents, which are characterized by the fact that these currents reverse their direction when the radiation helicity is changed from left-handed to right-handed and vice versa. Spin photocurrents are described by an axial tensor (or pseudotensor) of second rank. Such tensors play an important role in the context of gyrotropy, so that systems permitting nonzero axial second-rank tensors are often denoted gyrotropic systems. We note that gyrotropy is found neither in inversion-symmetric systems nor in systems with a zinc blende structure. The 18 gyrotropic crystal classes are listed, e.g., by Agranovich and Ginzburg (1984).\footnote{As certain aspects of gyrotropy require a symmetric material tensor, the discussion of gyrotropy is often restricted to those 15 crystal classes that permit a symmetric axial tensor of second rank (Nye, 1957; Landau and Lifshitz, 1984), thus excluding the crystal classes $C_{3v}$, $C_{4v}$, and $C_{6v}$ (the latter includes wurtzite materials). Spin photocurrents and the electric generation of spin discussed below do not require that the corresponding material tensors are symmetric. Therefore, these effects can be observed for all 18 crystal classes that permit an axial tensor of second rank. A general discussion of the symmetry of material tensors was given, e.g., by Bir and Pikus (1974).} Semiconductors with a zinc blende (or diamond) structure become gyrotropic when the symmetry is reduced by means of, e.g., quantum confinement or uniaxial strain. We note that gyrotropy is also a required and sufficient condition for the existence of $k$-linear spin splitting of the energy spectrum of spin-1/2 electron systems.
Two mechanisms contribute to spin photocurrents in gyrotropic media, the circular photogalvanic effect and the spin-galvanic effect (Ganichev and Prettl, 2003). The circular photogalvanic effect (CPGE) was independently predicted by Ivchenko and Pikus (1978) and Belinicher (1978). Subsequently, this effect was observed in bulk Te by Asnin et al. (1978).

The mechanism is illustrated in Figure 8. Excitation with \( \sigma_+ \)-polarized light induces direct optical transitions between the valence subband \( hh1 \) and the conduction subband \( e1 \) (vertical arrows in Figure 8). For a given photon energy \( \hbar \omega \), the optical selection rules and spin splitting result in an unbalanced occupation of the positive \( k_x^+ \) and negative \( k_x^- \) states such that the “centre of mass” of these transitions is shifted from \( k_x = 0 \) to some average value \( \langle k_x \rangle \neq 0 \). This wave vector \( \langle k_x \rangle \) translates into an average electron velocity \( v = \hbar \langle k_x \rangle / m^* \) of the optically oriented electrons, which corresponds to a spin-polarized charge current, i.e., the current is carried by electrons with one spin orientation. For interband transitions in 2D systems, as depicted in Figure 8, a detailed theory for the CPGE has been formulated by Golub (2003). Spin photocurrents can also be generated in a similar way by means of inter-subband and intra-subband transitions (Ganichev et al., 2001, Ganichev and Prettl, 2003, Sherman et al., 2005) and Tarasenko and Ivchenko (2005) have shown that pure spin photocurrents not accompanied by charge transfer or spin orientation can be generated by means of absorption of unpolarized light in low-dimensional semiconductor systems.

Up to now, we have discussed spin photocurrents obtained by means of one-photon absorption. These currents can also be generated by means of two-photon excitation (Bhat and Sipe, 2000). In this case, the spin polarization of the resulting charge currents has been confirmed directly by measuring the phase-dependent spatial shift of the circularly polarized photoluminescence (Hübner et al., 2003). Pure spin photocurrents not accompanied by charge transfer have been generated through quantum interference of one- and two-photon absorption by Stevens et al. (2003).

Besides the CPGE, the spin-galvanic effect (SGE) is a second mechanism that contributes to spin photocurrents (Ivchenko et al., 1989, Ganichev et al., 2002). The SGE is caused by asymmetric spin-flip relaxation of spin-polarized electrons. The mechanism is illustrated in...
Figure 9: One-dimensional microscopic picture of the spin-galvanic effect [after Ganichev et al. (2001)]. If one spin subband is preferentially occupied, e.g., by optical excitation, asymmetric spin-flip scattering results in a current in the $x$ direction.

An unbalanced population of spin-up and spin-down subbands is generated, e.g., by optical orientation. The current flow is caused by $k$-dependent spin-flip relaxation processes. Spins oriented in the up direction are scattered along $k_x$ from the more occupied, e.g., spin-up branch, to the less filled spin-down branch. Four quantitatively different spin-flip scattering events exist and are sketched in Figure 9 as bent arrows. The spin-flip scattering rate depends on the values of the wavevectors of the initial and the final states. Therefore, the spin-flip transitions marked by solid arrows in Figure 9 have the same rates. They preserve the distribution of carriers in the branches and, thus, do not yield a current. However, the two scattering processes shown by dashed arrows are inequivalent and generate an asymmetric carrier distribution around the branch minima. This asymmetric population results in a current flow along the $x$-direction. Within this model of elastic scattering the current is not spin polarized, since the same number of spin-up and spin-down electrons move in the same direction with the same velocity (Ganichev et al., 2002).

Assuming a linear relation between the components $S_\beta$ of the electrons’ averaged spin density and the components $j_\alpha$ of the resulting spin photocurrent, we get for the SGE

$$j_\alpha = \sum_\beta T_{\alpha\beta} S_\beta \quad \alpha, \beta = x, y, z,$$

where $T_{\alpha\beta}$ is an axial tensor of second rank.\(^4\) This equation shows clearly that, unlike the case of the CPGE, optical excitation is not required for the SGE. The CPGE, however, is always accompanied by the SGE. Formally, this is due to the fact that both effects are characterized by axial tensors of second rank. Even in a completely optical experiment, CPGE and SGE can be distinguished by their different behaviors when the light source is switched off. Then the circular photogalvanic current decays with the momentum relaxation time whereas the spin-galvanic current decays with the spin relaxation time. If spin relaxation is absent, the spin-galvanic current vanishes (Ganichev and Prettl, 2003).

In recent years, detailed experimental and theoretical investigations of the CPGE and SGE in different systems have been performed by Ganichev et al. This work and related work have been reviewed by Ganichev and Prettl (2003, 2006).

\(^4\)For many crystal classes permitting nonzero axial tensors of second rank it is nonetheless required by symmetry that certain components of these tensors must vanish, see, e.g., the discussion of the experiment of Ganichev et al. (2004, 2006) in Section 4.2.
4.2 Electrical generation of a spin polarization

In general, two possibilities exist for orienting electron spins with electric currents in a semiconductor. The first one is the spin Hall effect. For semiconductor systems, this idea was first discussed by D’yakonov and Perel’ (1971b). It yields a spin accumulation at the edges of the sample in the direction perpendicular to the current. A detailed discussion of the spin Hall effect can be found [elsewhere]. In gyrotropic media, a second mechanism exists that yields a spin polarization in the bulk of the sample (Aronov and Lyanda-Geller, 1989; Edelstein, 1990). We note that equation (8), relating the given spin orientation $S$ with the resulting current $J$, can obviously be inverted, i.e., an electric current $J$ can give rise to a spin density $S$ (Ivchenko and Pikus, 1978). As discussed in detail by Aronov et al. (1991), the different mechanisms contributing to the spin polarization of electrons induced by a current $J$ can be classified analogously to the different spin relaxation mechanisms for $J = 0$: for $J = 0$, these mechanisms drive the system towards its equilibrium configuration characterized by equal occupations of the spin states. For $J \neq 0$, on the other hand, the nonequilibrium configuration is characterized by an unequal filling of the spin states. Apart from a prefactor $Q$ of order one, the details of which depend on the scattering mechanisms present in the electron system, the spin polarization is given by the ratio between the spin splitting $\hbar \Omega(k_E)$ (assumed to be linear in $k$) and the average energy $E$ of the involved electrons (Aronov et al., 1991)

$$S = Qn \frac{\hbar \Omega(k_E)}{E}. \quad (9)$$

Here $k_E = eE \tau_p / \hbar$ is the shift of the Fermi sphere caused by the electric field $E$, and $\tau_p$ is the momentum relaxation time. In degenerate systems, we have $E = \hbar^2 k_F^2 / (2m^*)$. In nondegenerate systems we have $E = (d/2) k_B T$, where $d$ is the dimension. Finally, $n = k_F^d / (d \pi)$ is the number density. The prefactor $Q$ for different scattering mechanisms in $d = 2$ and $d = 3$ dimensions is given in Table I of Aronov et al. (1991).

The electric-field-induced spin orientation inside a semiconductor was also studied theoretically by Magarill et al. (2001) in 2D and Culcer et al. (2005) in 2D and 3D. The effect was first observed experimentally in bulk Te by Vorob’ev et al. (1979). More recently, it was used by Hammar et al. (1999, 2000) to analyze the spin injection from a ferromagnetic film into a 2D electron system, see also Monzon et al. (2000); van Wees (2000) and Silsbee (2001). Moreover, the effect was measured in strained bulk InGaAs by Kato et al. (2004b) and in 2D GaAs systems by Silov et al. (2004) and Ganichev et al. (2004, 2006). As an example, we want to discuss the experiment of Ganichev et al. They used a $p$-type GaAs multi-QW grown on an intentionally miscut (001) surface (tilted by 5° towards the [110] direction). The symmetry of this system is thus fully characterized by one mirror plane ($C_s$) (i.e., point group $C_s$), and electric spin orientation is expected only for a current in the (“active”) direction [110] of the 2D plane, but not for the perpendicular (“passive”) direction. In a transmission measurement using linearly polarized light, it is then possible to identify the current-induced spin orientation via a rotation of the polarization vector of the transmitted light (dichroic absorption and Faraday rotation) in a crossed polarizer setup, see the inset of Figure 10.
the “active” direction [110], Ganichev *et al.* observed a significantly larger signal in the photodetector than for the “passive” direction (Figure 10). The nonzero signal for the “passive” direction was ascribed to imperfections of the infrared polarizers and carrier heating by the current, as confirmed by control experiments.

Finally, we note that Kalevich and Korenev (1990) predicted and observed a current-induced spin precession in the field $\Omega(k_F)$.

### 5 Outlook

We focused here on the fundamental physics underlying the spin-dependent transport of carriers in semiconductors. These phenomena have many important and fascinating applications in the field of spintronics that are discussed [elsewhere]. Particularly important are various laterally structured systems such as the Datta-Das spin transistor (Datta and Das, 1990) and hybrid devices combining nonmagnetic semiconductors with semimagnetic and ferromagnetic materials.

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