Type IIB Theory on Half-flat Manifolds

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ABSTRACT

In this note we derive the low-energy effective action of type IIB theory compactified on half-flat manifolds and we show that this precisely coincides with the low-energy effective action of type IIA theory compactified on a Calabi–Yau manifold in the presence of NS three-form fluxes. We provide in this way a further check of the recently formulated conjecture that half-flat manifolds appear as mirror partners of Calabi–Yau manifolds when NS fluxes are turned on.
1 Introduction

Calabi–Yau compactification is one of the most common procedures to obtain four-dimensional models from ten-dimensional string theories. However, the physics obtained in this way generically feature a large number of scalar fields (moduli) which are flat directions of the potential and moreover, there exist no viable mechanism to further break supersymmetry. It was recently realized [1]–[14] that if one allows a non-zero background value (flux) for some of the field strengths, a potential is generated in the lower-dimensional effective theory and supersymmetry can be spontaneously broken [1].

Beside the phenomenologically interesting features of such compactifications it is also attractive to study flux compactifications in the context of string dualities [3]–[8], [10, 11, 14, 27], [29]–[35] and in particular, in this note we will concentrate on mirror symmetry which is supposed to relate the two type II theories when compactified on mirror Calabi–Yau three-folds.

The issue of mirror symmetry when fluxes are turned on was addressed in several works [3, 4, 6, 10, 11, 14, 27, 29]. In type IIA theory the RR fluxes lie in the even cohomologies of the Calabi–Yau manifold as the RR sector of this theory contains even form field strengths. For type IIB on the other hand one encounters odd form field strengths and thus the RR fluxes are parameterized in this case by elements of the odd cohomologies of the Calabi–Yau space. As mirror symmetry precisely exchanges the odd and even cohomologies it is not surprising that it still holds when RR fluxes are turned on. For the NS fluxes the situation was until recently less clear as none of the two type II theories contain even form field strengths in the NS-NS sector. It was in turn proposed [6] that the mirror of the NS fluxes should now come from the geometry of the internal manifold. This proposal was made more concrete in [14] where it was conjectured that when NS fluxes are turned on in type IIB theory, mirror symmetry requires the presence of a new class of manifolds, known as half-flat manifolds with SU(3) structure on type IIA side. The main argument supporting this proposal was provided by showing that the low-energy effective actions for the type IIB compactified on a Calabi–Yau three-fold in the presence of electric NS three-form flux and type IIA compactified on a half-flat space are equivalent. The purpose of this note is to test the conjecture formulated in [14] in the reversed situation, namely we want to show that compactifying type IIB theory on half-flat manifolds produces an effective action which is mirror equivalent to type IIA theory compactified on Calabi–Yau three-folds with NS three-form flux turned on.

The paper is structured as follows. In section 2 we briefly recall some of the results obtained in [14] and mainly we are interested in those features which are relevant for the KK reduction. In section 3 we compute the low-energy effective action of type IIB supergravity compactified on such a manifold and show that it indeed reproduces the action obtained in the type IIA case when NS fluxes are turned on. In section 4 we present our conclusions while in the appendix we record the main steps of the compactification of type IIA theory on Calabi–Yau three-folds with NS fluxes [11]. Throughout the paper

\[3\]Strictly speaking this idea appeared for the first time in [15]. In the context of finding supersymmetric ground states this was initially addressed in [16, 17, 18]. More recently, orientifolds and Calabi–Yau fourfolds with fluxes have been discussed in [12, 13, 19–29].

\[4\]Manifolds with SU(3) structure also appeared recently in the heterotic string compactifications [30], though from a slightly different perspective.
we use the conventions of [11] (see appendix A of this paper).

2 Preliminaries

Let us start by recording the main results obtained in [14]. Turning on NS three-form fluxes in type IIB compactification on a Calabi–Yau manifold $\tilde{Y}$ introduces $2(h^{(1,2)} + 1)$ flux parameters via

$$H_3 = p^A \alpha_A + q_A \beta^A ,$$

(2.1)

where $(\alpha_A, \beta^B), \ A, B = 0, \ldots, h^{(1,2)}$ form a basis for $H^3(\tilde{Y})$ and is normalized as in (A.9). These fluxes will appear in the four dimensional theory as charges which couple to electric or magnetic fields and it is just pure convention to call them electric or magnetic fluxes depending on how we choose to describe the gauge sector. However, in the setup of [11] which we also adopt here, $p^A$ appear as magnetic charges, while $q_A$ as electric ones. Thus, from now on we will refer to the fluxes $p^A$ and $q_A$ in (2.1) as magnetic and electric fluxes respectively.

The NS-NS sector of type IIA theory also contains a two form potential with a three-form field strength. However, the corresponding fluxes will again lie in the third cohomology and they can not be mirror to (2.1) since mirror symmetry exchanges the even and odd cohomologies. In order to find a configuration mirror symmetric to (2.1) one needs to find NS even form field strengths. It was suggested in [3] that the missing fluxes should come from the geometry of the internal manifold which now should be taken to be non-complex and the NS even form field strength should be associated to the lack of integrability of the almost complex structure. It was shown in [14] that one can obtain the mirror electric fluxes by considering the internal space to be a half-flat manifold with $SU(3)$ structure which is indeed non-complex. Such manifolds admit a globally defined nowhere vanishing spinor which is covariantly constant with respect to a connection with torsion $\nabla^{(T)} \eta = 0$. This assures that the low-energy effective action obtained by compactifying either of the type II theories on such manifolds still has $N = 2$ supersymmetries in four dimensions. Equivalently, one can think about these spaces as being endowed with an almost complex structure $J$ and a $(3,0)$ form $\Omega$ which are covariantly constant with respect to the same connection with torsion

$$\nabla^{(T)} J_{np} = 0 ; \quad \nabla^{(T)} \Omega_{npq} = 0 .$$

(2.2)

The Levi-Civita connection fails to preserve $J$ and $\Omega$ thus, unlike the Calabi–Yau case $J$ and $\Omega$ are no longer closed. In [14] it was found that the NS four-form was provided by $d\Omega^+, \Omega^+$ being the real part of the $(3,0)$ form $\Omega$, and the (electric) fluxes were obtained in the expansion of this four-form in some appropriately chosen basis of $(2,2)$ forms $\tilde{\omega}^i$

$$d\Omega^+ = e_i \tilde{\omega}^i .$$

(2.3)

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5By flux we understand a background value for some p-form field strength. This value can not be arbitrary as the equations of motion restrict it to be a harmonic form on the internal manifold. Thus we can write

$$F_{(p)} = m^i \omega_{(p)}^i ,$$

where $m^i$ are the flux parameters and $\omega_{(p)}^i$ represents a basis of harmonic p-forms.

6For a systematic study of such manifolds we refer the reader to [37, 38] and references therein, or for a more physical discussion to [14, 34].
Moreover, it was argued that in order for mirror symmetry to work there should be a precise relation between the half-flat space \( \hat{Y} \) and some Calabi–Yau three-fold \( Y \). In particular, the moduli space of metrics of the half-flat manifold should coincide with the moduli space of the corresponding Calabi–Yau and the metrics on these spaces should be equivalent. This means that one should perform the usual moduli expansion

\[
\Omega = z^A \alpha_A - \mathcal{F}_A \beta^A, \quad J = v^i \omega_i, \quad (2.4)
\]

and the forms \((\alpha_A, \beta^B)\) and \((\omega_i, \tilde{\omega}^j)\) should have the same intersection numbers as on the Calabi–Yau manifold, i.e.

\[
\int_Y \alpha_A \wedge \beta^B = \delta^B_A, \quad \int_Y \omega_i \wedge \tilde{\omega}^j = \delta^j_i, \quad \int_Y \alpha_A \wedge \alpha_B = \int_Y \beta^A \wedge \beta^B = 0. \quad (2.5)
\]

The above relation proved to impose strong constraints on the topology of the half-flat manifold \( \hat{Y} \). In particular it was argued in [14] that in order to have both (2.3) and (2.4) the only solution is to consider the following action of the exterior derivative on the basis \((\alpha_A, \beta^B)\) \(7\)

\[
d\alpha_0 = e_i \tilde{\omega}^i, \quad d\alpha_a = d\beta^B = 0. \quad (2.6)
\]

Consistency with (2.5) further required that

\[
d\omega_i = e_i \beta^0, \quad d\tilde{\omega}^i = 0. \quad (2.7)
\]

Using these relations one can immediately see that the cohomology groups are reduced compared to the corresponding Calabi–Yau manifold and one has

\[
h^{(2)}(\hat{Y}) = h^{(1,1)}(Y) - 1, \quad h^{(3)}(\hat{Y}) = h^{(3)}(Y) - 2. \quad (2.8)
\]

From a physical point of view this can be easily understood as, due to the fluxes, some of the previous moduli of the Calabi–Yau now acquire masses and thus appear no longer as flat directions of the potential. Consequently, in order to obtain the same light spectrum some of the forms considered previously in the zero modes expansion have to become non-harmonic on the mirror side.

Using the above setup it was shown in [14] that the effective action of type IIA supergravity compactified on a half-flat manifold \( \hat{Y} \) precisely reproduces the effective action of type IIB supergravity compactified on a Calabi–Yau manifold \( Y \) in the presence of NS electric fluxes (2.1) and thus provides strong evidence that half-flat manifolds are indeed the mirror configuration of the NS fluxes (2.1).

In this note we want to further test this conjecture. In particular, if the half-flat geometries are to reproduce the mirror NS fluxes this should not depend on which of the type II theories is chosen to be compactified on these spaces. Thus, our purpose here is to show that type IIB compactification on half-flat manifolds reproduces the type IIA compactification on Calabi–Yau three-folds in the presence of electric NS fluxes whose effective action was derived in [1].

\(7\)We have implicitly assumed that \( z^A \) can be written as \( z^A = (1, z^a), \quad a = 1, \ldots, h^{(1,2)}. \)
3 Type IIB on half-flat manifolds

Following [14] we will now perform the compactification of type IIB on a manifold \( \hat{Y} \) which obeys (2.6) and (2.7) which again will turn out to be responsible for generating mass terms in the lower-dimensional action.

Let us start by shortly recording the type IIB supergravity in ten dimensions. The NS-NS sector of the bosonic spectrum consists of the metric \( \hat{g}_{MN} \), an antisymmetric tensor field \( \hat{B}_2 \) and the dilaton \( \hat{\phi} \). In the RR sector one finds the 0-, 2-, and 4-form potentials \( l, \hat{C}_2, \hat{A}_4 \). The four-form potential satisfies a further constraint in that its field strength \( \hat{F}_5 \) is self-dual. The interactions of the above fields are described by the ten-dimensional action [39]

\[
S^{(10)}_{IIB} = \int e^{-2\hat{\phi}} \left( \frac{1}{2} R^{*1} - 2d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{4} d\hat{B}_2 \wedge *d\hat{B}_2 \right) - \frac{1}{2} \int \left( dl \wedge *dl + \hat{F}_3 \wedge *\hat{F}_3 + \frac{1}{2} \hat{F}_5 \wedge *\hat{F}_5 \right) - \frac{1}{2} \int \hat{A}_4 \wedge d\hat{B}_2 \wedge d\hat{C}_2 ,
\]

where the field strengths \( \hat{F}_3 \) and \( \hat{F}_5 \) are defined as

\[
\hat{F}_3 = d\hat{C}_2 - ld\hat{B}_2 ,
\]

\[
\hat{F}_5 = d\hat{A}_4 - d\hat{B}_2 \wedge \hat{C}_2 .
\]

As it is well known the action (3.1) does not reproduce the correct dynamics of type IIB supergravity as the self-duality condition of \( \hat{F}_5 \) can not be derived from a variational principle. Rather this should be imposed by hand in order to obtain the correct equations of motion and we will come back to this constraint later as it plays a major role in the following analysis.

In order to compactify the action (3.1) on a half flat manifold we proceed as in [14] and continue to expand the ten dimensional fields in the forms which appear in (2.6) and (2.7) even though they are not harmonic. We do not want to go again here through the argument presented in [14], but we just mention that the Laplace operator acting on these forms produces terms of order \((\text{flux})^2\) and in the supergravity limit, where the fluxes are supposed to be at a scale much smaller than the compactification one, it is consistent to keep the massive modes coming from expansion in these forms and still neglect the massive KK states. Correspondingly we write

\[
\hat{B}_2 = B_2 + b^i \wedge \omega_i , \quad i = 1, \ldots, h^{(1,1)} ,
\]

\[
\hat{C}_2 = C_2 + c^i \wedge \omega_i ,
\]

\[
\hat{A}_4 = D^i_2 \wedge \omega_i + \rho_i \wedge \tilde{\omega}^i + V^A \wedge \alpha_A - U_A \wedge \beta^A , \quad A = 0, \ldots, h^{(1,2)} ,
\]

and thus one finds the two forms \( B_2, C_2, D^i_2 \), the vector fields \( V^A, U_A \), and the scalars \( b^i, c^i, \rho_i \). Additionally, from the metric fluctuations on the internal space one obtains other
scalar fields $z^a$ and $v^i$ \textsuperscript{(2.4)}, which correspond to the Calabi–Yau complex structure and Kähler class deformations respectively. Due to the self-duality condition which one has to impose on $\hat{F}_3$, not all the fields listed above describe physically independent degrees of freedom. Thus as four dimensional gauge fields one only encounters either $V^A$ or $U_A$. In the same way, the scalars $\rho_i$ and the two forms $D_2^i$ are related by Hodge duality and one can eliminate either of the two in the four dimensional action. In the end one obtains an $N = 2$ supersymmetric spectrum consisting of a gravity multiplet $(g_{\mu\nu}, V^0)$, $h^{(2,1)}$ vector multiplets $(V^a, z^a)$ and $4(h^{(1,1)} + 1)$ scalars $\phi, h_1, h_2, l, b^i, c^i, v^i, \rho^i$ which form $h^{(1,1)} + 1$ hypermultiplets.\footnote{We have implicitly assumed that the two-forms $C_2$ and $B_2$ are massless in four dimensions and they can be Hodge dualized to scalars which we have denoted $h_1$ and $h_2$ respectively.}

Up to this point everything looks like ordinary Calabi–Yau compactification. The difference comes when one inserts the Ansatz (3.3) back into the action (3.1). Due to (2.6) and (2.7), the exterior derivatives of the fields (3.3) are going to differ from the standard case

$$d\hat{B}_2 = dB_2 + db^i \land \omega_i + \epsilon_i b^i \beta^0 + \epsilon_0 \beta^0,$$
$$d\hat{C}_2 = dC_2 + dc^i \land \omega_i + \epsilon_i c^i \beta^0,$$  \hspace{1cm} (3.4)
$$d\hat{A}_4 = dD_2^i \land \omega_i + \epsilon_i D_2^i \land \beta^0 + dV^A \land \alpha_A - dU_A \land \beta^A + (d\rho_i - \epsilon_i V_0) \land \hat{\omega}^i.$$

As in \cite{14} we have also allowed for a normal $H_3$ flux proportional to $\beta^0$. This naturally combines with the other fluxes parameters $\epsilon_i$ defined in (2.6) to provide all the $h^{(1,1)} + 1$ electric fluxes. With these expressions one can immediately write the field strengths $F_3$ and $F_5$ from (3.2)

$$\hat{F}_3 = (dC_2 - ldB_2) + (dc^i - ldb^i) \land \omega_i + \epsilon_i (c^i - lb^i) \beta^0 - l \epsilon_0 \beta^0,$$
$$\hat{F}_5 = (dD_2^i - db^i \land C_2 - c^i dB_2) \land \omega_i + (d\rho_i - \mathcal{K}_{ijk} c^j db^k) \land \hat{\omega}^i + F^A \land \alpha_A - \tilde{G}_A \land \beta^A,$$

where we have defined

$$D\rho_i = d\rho_i - \epsilon_i V^0,$$
$$F^A = dV^A, \quad G_A = dU_A,$$  \hspace{1cm} (3.6)
$$\tilde{G}_0 = G_0 - \epsilon_i (D_2^i - b^i C_2) + \epsilon_0 C_2; \quad \tilde{G}_a = G_a.$$

In order to derive the lower-dimensional action we adopt the following strategy \cite{9}. In the first stage we are going to ignore the self-duality condition which should be imposed on $\hat{F}_5$ and treat the fields coming from the expansion of $\hat{A}_4$ as independent. Thus, initially we naively insert the expansions (3.3) into (3.1) and perform the integrals over the internal space using (A.8)–(A.14). To obtain the correct action we will further add suitable total derivative terms so that the self-duality conditions appear from a variational principle. At this point one can eliminate the redundant fields and in this way obtain the four-dimensional effective action and no other constraint has to be imposed. It can be checked that the result obtained in this way is compatible with the ten dimensional equations of motion.
Let us apply this procedure step by step. First one inserts the expansions (3.3) and (3.5) into the ten-dimensional action (3.1). The various terms of this action take the form

\[-\frac{1}{4} \int_Y d\hat{B}_2 \wedge *d\hat{B}_2 = -\kappa \frac{K}{4} d\hat{B}_2 \wedge *d\hat{B}_2 - \kappa g_{ij} db^i \wedge *db^j + \frac{1}{4} (c^i b^i + e_0)^2 \kappa_0 * 1 ,\]

\[-\frac{1}{2} \int_Y \hat{F}_3 \wedge *\hat{F}_3 = -\kappa \frac{K}{2} (dC_2 - ldb_2) \wedge *(dC_2 - ldb_2) - 2\kappa g_{ij} (dc^i - ldb^i) \wedge *(dc^j - ldb^j) + \frac{1}{2} [e_i (c^i - l b^i) - le_0]^2 \kappa_0 * 1 ,\]

\[-\frac{1}{4} \int \hat{F}_5 \wedge *\hat{F}_5 = +\frac{1}{4} \text{Im} \mathcal{M}^{-1} \left( \tilde{G} - \mathcal{M} F \right) \wedge * \left( \tilde{G} - \mathcal{M} F \right) \quad (3.7)\]

\[-\kappa g_{ij} (dD_2^i - db^i \wedge C_2 - c^i dB_2) \wedge *(dD_2^j - db^j \wedge C_2 - c^j dB_2) - \frac{1}{16K} g^{ij} (D \rho_i - \kappa_{ilm} c^l db^m) \wedge *(D \rho_j - \kappa_{jnp} c^n db^p) ,\]

\[-\frac{1}{2} \int \hat{A}_4 \wedge d\hat{B}_2 \wedge d\hat{C}_2 = -\frac{1}{2} \kappa_{ijk} D_2^i \wedge db^j \wedge dc^k - \frac{1}{2} \rho_i (dB_2 \wedge dc^i + db^i \wedge dC_2) ,\]

\[+\frac{1}{2} e_i V^0 \wedge (c^i dB_2 - b^i dC_2) - \frac{1}{2} e_0 V^0 \wedge dC_2 .\]

In order to write the above formulae we have used (A.8)–(A.14) and we have defined \( \kappa_0 = (\text{Im} \mathcal{M}^{-1})^{00} \). In the gravitational sector, beyond the usual part containing the kinetic terms for the moduli of \( \hat{Y} \) there will be a further contribution coming entirely from the internal manifold which is due to the fact that \( \hat{Y} \) is not Ricci flat and which will generate a potential piece in four dimensions. The Ricci scalar for half-flat manifolds was computed in [14] and here we will not present the whole calculation, but just record the effective potential generated in this way

\[V_g = -\frac{\kappa_0}{16K} e^{2\phi} e_i e_j g^{ij} . \quad (3.8)\]

At this point we have to impose the self-duality condition for \( \hat{F}_5 \) which using (A.11), (A.13) and (A.14) translates into the following constraints on the four dimensional fields

\[dD_2^i - db^i \wedge C_2 - c^i dB_2 = \frac{1}{4K} g^{ij} *(D \rho_i - \kappa_{ijk} c^j db^k) ,\]

\[*\tilde{G}_A = \text{Re} \mathcal{M}_{AC} * F^C - \text{Im} \mathcal{M}_{AC} F^C , \quad (3.9)\]

with \( D \rho_i \) and \( \tilde{G}_A \) defined in (3.6). By adding the following total derivative term to the action

\[L_{td} = + \frac{1}{2} dD_2 \wedge d\rho_i + \frac{1}{2} F^A \wedge \tilde{G}_A \]

6
This allows us to eliminate the fields $dD^i$ and the matrix

$$G_A = \frac{1}{2}(e_ib^i + e_0)F^0 \wedge C_2$$

(3.10)

the constraints (3.3) can be found upon variation with respect to $dD^i$ and $G_A$ respectively. This allows us to eliminate the fields $dD^i$ and $G_A$ using their equations of motion and consequently the effective action obtained in this way describes the correct dynamics for the remaining fields which now do not have to satisfy any further constraint.

After the dualization of the 2-forms $C_2$ and $B_2$ to the scalars $h_1$ and $h_2$ one obtains the effective action for type IIB supergravity compactified to four dimensions on a half-flat manifold

$$S_{IIB}^{(4)} = \int -\frac{1}{2} R \ast 1 - g_{ab}dz^a \wedge *d\bar{z}^b - g_{i\bar{j}}dt^i \wedge *d\bar{t}^\bar{j} - d\phi \wedge *d\phi$$

$$- \frac{e^{2\phi}}{8\mathcal{K}}g^{-1}_{ij}(D\rho_i - \mathcal{K}_{ik}c^k db^i) \wedge *(D\rho_j - \mathcal{K}_{jmn}c^m db^n)$$

$$- 2\mathcal{K}e^{2\phi}g_{i\bar{j}}(dc^i - ldb^i) \wedge *(dc^j - ldb^j) - \frac{1}{2}\mathcal{K}e^{2\phi}dl \wedge *dl$$

$$- \frac{1}{2\mathcal{K}}e^{2\phi}(dh_1 - b^i D\rho_i + e_0 V^0) \wedge *(dh_1 - b^j D\rho_j + e_0 V^0) - e^{4\phi}D\bar{h} \wedge *D\bar{h}$$

$$+ \frac{1}{2} \text{Re} \mathcal{M}_{AB} F^A \wedge F^B + \frac{1}{2} \text{Im} \mathcal{M}_{AB} F^A \wedge *F^B - V_{IIB} \ast 1 ,$$

(3.11)

where

$$D\bar{h} = dh_2 + ldh_1 + (c^i - lb^i)D\rho_i + l_{e_0} V^0 - \frac{1}{2}\mathcal{K}_{ijk} c^i c^j db^k .$$

(3.12)

Performing the field redefinitions $a$,

$$a = 2h_2 + h_1 + \rho_i(c^i - lb^i) , \quad \xi^0 = l , \quad \xi^i = lb^i - c^i ,$$

(3.13)

$$\tilde{\xi}_i = \rho_i + \frac{l}{2}\mathcal{K}_{ijk}b^j b^k - \mathcal{K}_{ijk}b^j c^k , \quad \tilde{\xi}_0 = -h_1 + \frac{l}{2}\mathcal{K}_{ijk}b^j b^k + \frac{1}{2}\mathcal{K}_{ijk}b^j c^k ,$$

the metric for the hyperscalars takes the standard quaternionic form of $[11]$ which is now exactly the mirror image of (A.19) with the gauge coupling matrices $\mathcal{N}$ and $\mathcal{M}$ exchanged as prescribed by the mirror map. Introducing the collective notation $q^u = (\phi, a, \xi^i, \tilde{\xi}_i)$ we can write the final form of the four dimensional action

$$S_{IIA} = \int \left[ -\frac{1}{2} R^* \ast 1 - g_{ab}dz^a \wedge *d\bar{z}^b - h_{uv} Dq^u \wedge *Dq^v - V_{IIB} \ast 1 \right.$$

$$\left. + \frac{1}{2} \text{Im} \mathcal{M}_{AB} F^A \wedge *F^B + \frac{1}{2} \text{Re} \mathcal{M}_{AB} F^A \wedge F^B \right] ,$$

(3.14)

where the scalar potential has the form

$$V_{IIB} = \frac{\kappa_0}{4} e^{+2\phi} e_1 e_j (\text{Im} \mathcal{N}^{-1})^{IJ} - \frac{\kappa_0}{2} e^{4\phi} (e_i \xi^j)^2 ,$$

(3.15)

and the matrix $\mathcal{N}$ is given in (A.20). The non-trivial covariant derivatives have the form

$$D\tilde{\xi}_i = d\tilde{\xi}_i - e_1 V^0 ; \quad Da = da + e_i V^0 \xi^i ,$$

(3.16)
while all the other fields remain neutral.

This ends the derivation of the effective action of type IIB theory compactified to four dimensions on half-flat manifolds. One can immediately notice that the gaugings (3.16) are precisely the same as in the case of type IIA theory (A.16) when all the magnetic fluxes \( p^4 \) are set to zero. It is not difficult to see that in this case also the potentials (3.15) and (A.18) coincide. For this one should just note that under mirror symmetry \( \kappa_0 = \left( \text{Im} \mathcal{M}_B \right)^{00} \) is mapped to \(-\frac{1}{\mathcal{K}_A}\), \( \mathcal{K}_A \) being the volume of the Calabi–Yau manifold on which type IIA is compactified.

4 Conclusions

In this paper we derived the low energy effective action of type IIB supergravity compactified on half-flat manifolds and showed that it is equivalent to the one obtained by compactifying type IIA theory on Calabi–Yau three-folds in the presence of electric NS fluxes. We provided in this way a further check of the conjecture formulated in [14] that half-flat manifolds represent the geometry mirror to Calabi–Yau three-folds with NS three-form fluxes turned on. However, these half-flat manifolds give rise to only \( h^{(1,1)} \) flux parameters and it seems that one still needs an additional parameter in order to recover the mirror partners of all \( h^{(1,2)} + 1 \) electric NS fluxes. Somehow curiously, it was argued in [14] that this extra parameter arises by turning on an ordinary NS flux along some particular element of \( H^3(\hat{Y}), \beta^0 \). Here we again found that this prescription works confirming that this extra flux was not just a coincidence. Moreover, in analogy to [42, 43] and [14], we can write the superpotential

\[
W_B = \int d(B + iJ) \wedge \Omega ,
\]

which naturally incorporates the additional parameter coming from the flux for \( dB_2 \).

We would like to end with an open question which was also posed in [14]: the magnetic fluxes. The subtlety encountered in [14] was that in type IIB when both electric and magnetic NS three-form fluxes were turned on the RR two-form \( C_2 \) became massive and the poor understanding of this issue made it difficult to treat the problem of magnetic fluxes properly. However, in the approach we presented in this note, type IIA with electric and magnetic NS three-form fluxes is well understood and no massive form is present. Thus, it appears that in this picture it would be easier to look for the magnetic fluxes and we hope to report on this subject soon [44].

Appendix

A Type IIA with NS flux

In this appendix we briefly recall the results of [14] for the compactification of type IIA supergravity on Calabi-Yau three-folds \( Y \) when background NS fluxes are turned on.
The bosonic spectrum of type IIA supergravity in ten dimensions features the following fields: the graviton \( \hat{g}_{MN} \), a two-form \( \hat{B}_2 \) and the dilaton \( \hat{\phi} \) in the NS-NS sector and a one form \( \hat{A}_1 \) and a three-form \( \hat{C}_3 \) in the RR sector. The action governing the interactions of these fields can be written as \[ (A.1) \]

\[
S = \int e^{-2\hat{\phi}} \left( -\frac{1}{2} \hat{R} \ast 1 + 2 d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{4} \hat{H}_3 \wedge *\hat{H}_3 \right)
- \frac{1}{2} \int \left( \hat{F}_2 \wedge *\hat{F}_2 + \hat{F}_4 \wedge *\hat{F}_4 \right) + \frac{1}{2} \int \hat{H}_3 \wedge \hat{C}_3 \wedge d\hat{C}_3,
\]

where

\[
\hat{H}_3 = d\hat{B}_2, \quad \hat{F}_2 = d\hat{A}_1, \quad \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3.
\] (A.2)

Upon compactification on a Calabi–Yau three-fold the four-dimensional spectrum can be read from the expansion of the ten-dimensional fields in the Calabi–Yau harmonic forms

\[
\hat{A}_1 = A^0, \quad \hat{C}_3 = C_3 + A^i \wedge \omega_i + \xi^A \alpha_A + \tilde{\xi}_A \beta^A, \quad \hat{B}_2 = B_2 + b^i \omega_i.
\] (A.3)

Correspondingly, in \( D = 4 \) we find a three-form \( C_3 \), a two-form \( B_2 \), the vector fields \( (A^0, A^i) \) and the scalars \( b^i, \xi^A, \tilde{\xi}_A \). Together with the Kähler class and complex structure deformations \( v^i \) and \( z^a \) these fields combine into a gravity multiplet \( (G_{\mu \nu}, A^0), h^{(1,1)} \) vector multiplets \( (A^i, v^i, b^i), \ i = 1, \ldots, h^{(1,1)}, h^{(1,2)} \) hyper-multiplets \( (z^a, \xi^a, \tilde{\xi}_a), \ a = 1, \ldots, h^{(1,2)} \) and a tensor multiplet \( (B_2, \phi, \xi^0, \tilde{\xi}_0) \).

We assume that turning on background fluxes does not change the light spectrum and thus the only modification in the KK Ansatz is a shift in the field strength of \( \hat{B}_2 \)

\[
\hat{H}_3 = H_3 + db^i \wedge \omega_i + p^A \alpha_A - q_A \beta^A.
\] (A.4)

This leads to the following expressions for the different terms appearing in the ten-dimensional action \( (A.1) \)

\[
- \frac{1}{4} \int_Y \hat{H}_3 \wedge *\hat{H}_3 = - \frac{K}{4} H_3 \wedge *H_3 - Kg_{ij} db^i \wedge *db^j - V \ast 1,
\]

\[
- \frac{1}{2} \int_Y \hat{F}_2 \wedge *\hat{F}_2 = - \frac{K}{2} dA^0 \wedge *dA^0,
\]

\[
- \frac{1}{2} \int_Y \hat{F}_4 \wedge *\hat{F}_4 = - \frac{K}{2} (dC_3 - A^0 \wedge H_3) \wedge *(dC_3 - A^0 \wedge H_3)
\]

\[
- 2Kg_{ij} (dA^i - A^0 db^i) \wedge *(dA^j - A^0 db^j)
\]

\[
+ \frac{1}{2} (\text{Im} \, \mathcal{M}^{-1})^{AB} \left[ D\tilde{\xi}_A + \mathcal{M}_{AC} D\xi^C \right] \wedge * \left[ D\tilde{\xi}_B + \mathcal{M}_{BD} D\xi^D \right],
\]

\[
\frac{1}{2} \int_Y \hat{H}_3 \wedge \hat{C}_3 \wedge d\hat{C}_3 = - \frac{1}{2} H_3 \wedge (\xi^A d\tilde{\xi}_A - \tilde{\xi}_A d\xi^A) + \frac{1}{2} db^i \wedge A^i \wedge dA^k \mathcal{K}_{ijk}.
\]
\[ +dC_3 \wedge \left( p^A \xi_A + q_A \xi^A \right). \]

Even from this stage one can notice that some of the fields effectively became charged

\[ D\xi^A = d\xi^A - p^A A^0, \quad D\tilde{\xi}_A = d\tilde{\xi}_A + q_A A^0, \quad (A.6) \]

and a potential term is induced

\[ V = -\frac{1}{4} e^{-\phi} (q - \mathcal{M}p) \text{Im} \mathcal{M}^{-1} (q - \bar{\mathcal{M}}p). \quad (A.7) \]

In order to write the above expressions we have used the following notation for the integrals on the Calabi–Yau manifold. First the harmonic forms are normalized as

\[ \int_Y \omega_i \wedge \tilde{\omega}^j = \delta_i^j, \quad (A.8) \]

while for \( H^3(Y) \) the basis \((\alpha_A, \beta^B)\) obeys

\[ \int_Y \alpha_A \wedge \beta^B = \delta_A^B; \quad \int_Y \alpha_A \wedge \alpha_B = \int_Y \beta^A \wedge \beta^B = 0. \quad (A.9) \]

Furthermore we have denoted

\[ \mathcal{K}_{ijk} = \int_Y \omega^i \wedge \omega^j \wedge \omega^k, \quad \mathcal{K} = \frac{1}{6} \int_Y J \wedge J \wedge J, \quad (A.10) \]

where \( \mathcal{K} \) is the volume and \( J \) is the Kähler form. Finally, the Hodge duals of the harmonic two-forms are given by

\[ *\omega_i = 4\mathcal{K} g_{ij} \tilde{\omega}^j, \quad (A.11) \]

where \( g_{ij} \) denotes the metric on the moduli space of the Kähler deformations which is given by

\[ 4\mathcal{K} g_{ij} = \int_Y \omega_i \wedge *\omega_j. \quad (A.12) \]

For the three-forms we assume the following relations

\[ *\alpha_A = A_A^B \alpha_B + B_{AB} \beta^B, \quad *\beta^A = C^{AB} \alpha_B - A_B^A \beta^B, \quad (A.13) \]

where \( A, B, C \), are given in terms of a matrix \( \mathcal{M} \) which represents the gauge coupling functions in the case of type IIB compactification \[47, 48\]

\[ A = (\text{Re} \mathcal{M}) (\text{Im} \mathcal{M})^{-1}, \]

\[ B = - (\text{Im} \mathcal{M}) - (\text{Re} \mathcal{M}) (\text{Im} \mathcal{M})^{-1} (\text{Re} \mathcal{M}), \]

\[ C = (\text{Im} \mathcal{M})^{-1}. \quad (A.14) \]

Next, the compactification proceeds as usually by dualizing the fields \( C_3 \) and \( B_2 \) to a constant and to a scalar respectively. We do not perform these steps here, but we just

\[ ^9 \text{For a systematic study of the Calabi–Yau moduli space we refer the reader to the literature [45, 46].} \]
recall the final results. (for more details see [11, 49]). First the dualization of $C_3$ to a constant $e$ results in

$$L_e = L_{C_3} = -\frac{e^{4\phi}}{2K} \left( p^A \tilde{\xi}_A + q_A \xi^A + e \right)^2 \ast 1 + \left( p^A \tilde{\xi}_A + q_A \xi^A + e \right) A^0 \wedge H_3 \,.$$  (A.15)

It was shown in [11] that the constant $e$ plays a special role in the case of RR fluxes. however, it is irrelevant for the analysis in this paper and thus we will set it to zero. Dualizing now the two-form $B_2$, one obtains an axion, which due to the Green-Schwarz term in (A.15) becomes charged and its covariant derivative reads

$$D a = d a - \left( p^A \tilde{\xi}_A + q_A \xi^A \right) A^0.$$  (A.16)

Collecting all terms one can write the final form of the action

$$S_{IIA} = \int \left[ -\frac{1}{2} R^* \ast 1 - g_{ij} dt^i \wedge \ast d\bar{t}^j - h_{uv} Dq^u \wedge \ast Dq^v - V_{IIA} \ast 1 \right. $$

$$\left. + \frac{1}{2} Im N_{ij} F^i \wedge \ast F^j + \frac{1}{2} Re N_{ij} F^i \wedge F^j \right],$$  (A.17)

where the potential can be read from (A.7) and (A.15)

$$V_{IIA} = -\frac{1}{4K} e^{2\phi} (q - \mathcal{M} p) \mathcal{M}^{-1} (q - \bar{\mathcal{M}} p) + \frac{1}{2K} e^{4\phi} \left( p^A \tilde{\xi}_A + q_A \xi^A \right)^2,$$  (A.18)

while the metric for the hyper-scalars $h_{uv}$ has the standard form of [11]

$$h_{uv} Dq^u \wedge \ast Dq^v = d \phi \wedge \ast d \phi + g_{ab} dz^a \wedge \ast d \bar{z}^b$$

$$+ \frac{e^{4\phi}}{4} \left[ Da + (\tilde{\xi}_A D \xi^A - \xi^A D \tilde{\xi}_A) \right] \wedge \ast \left[ Da + (\tilde{\xi}_A D \xi^A - \xi^A D \tilde{\xi}_A) \right]$$

$$- \frac{e^{2\phi}}{2} \left( \mathcal{M}^{-1} \right)^{AB} \left[ D \tilde{\xi}_A + \mathcal{M}_{AC} D \xi^C \right] \wedge \ast \left[ D \tilde{\xi}_B + \bar{\mathcal{M}}_{BD} D \xi^D \right].$$

Furthermore by $\mathcal{N}$ we have denoted the gauge couplings matrix which can be immediately seen from (A.3) that it has the usual form [45]

$$\text{Re } N_{00} = -\frac{1}{3} \mathcal{K}_{ijk} b^i b^j b^k \,,$$

$$\text{Re } N_{i0} = \frac{1}{2} \mathcal{K}_{ijk} b^i b^k \,,$$

$$\text{Re } N_{ij} = -\mathcal{K}_{ijk} b^k \,,$$

$$\text{Im } N_{00} = -\mathcal{K} - 4\mathcal{K} g_{ij} b^i \,,$$

$$\text{Im } N_{i0} = 4\mathcal{K} g_{ij} b^j \,,$$

$$\text{Im } N_{ij} = -4\mathcal{K} g_{ij} \,.\,$$  (A.20)

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10 We have further redefined the gauge fields as $A^i \rightarrow A^i - b^i A^0$ and also appropriately rescaled the metric in order to go to the Einstein frame.
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