Influence of complex configurations on properties of pygmy dipole resonances

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Abstract. Starting from the quasiparticle random phase approximation based on the Skyrme interaction SLy5, we study the effects of phonon-phonon coupling (PPC) on the low-energy electric dipole responses in some spherical nuclei. The inclusion of the PPC results in the formation of low-energy $1^-$ states. There is an impact of the PPC effect on low-energy $E1$ strength. The PPC effect on the electric dipole polarizability is discussed. We predict a strong increase of the summed $E1$ strength below 10 MeV, with increasing neutron number from $^{48}$Ca till $^{58}$Ca.

1. Introduction

It is well known that collective dipole excitations in atomic nuclei can arise from out-of-phase oscillations of the proton and neutron “fluids” giving the giant dipole resonance (GDR) [1]. Systematic investigations established it to be a global feature of nuclei from the very light to the heaviest nuclei [2, 3]. In recent years, the interest is more focused on the low-lying dipole strength, that is located below the GDR energies. The concentration of the $E1$ strength around the particle separation energy is commonly called the pygmy dipole resonance (PDR) because of its weak strength in comparison with the GDR, which dominates the $E1$ strength distribution in nuclei [4]. In analogy to the GDR, the PDR has been interpreted as a collective oscillation of the neutron skin with respect to a $\Lambda=Z$ inert core (see Ref. [5] and references therein). The total sum of the measured energy-weighted sum rule of such $E1$ distributions is less than 1-2% of the Thomas-Reiche-Kuhn (TRK) sum rule value for stable nuclei and less than 5-6% for unstable neutron-rich nuclei [4]. Recent theoretical calculations indicate that such a low-energy peak is a common property of neutron-rich nuclei lying in different mass regions [6, 7]. The occurrence of non-negligible low-lying $E1$-strength can influence the radiative neutron capture cross section by orders of magnitude and, consequently, also the rate of the astrophysical $r$-process nucleosynthesis [8].

The strong proton shell closure at $Z = 20$ and the already good experimental knowledge of the chain of calcium isotopes makes [9, 10, 11, 12] calcium an attractive element for a PDR study. Indeed, indications of a trend for increasing low-energy dipole strength with increasing mass can be observed in the dipole excitation functions (above neutron separation energy) in the stable Ca isotopes [9, 10]. The results were generally consistent with the theoretical prediction regarding the shifting of dipole strength to lower energies, see, e.g., Refs. [6, 13, 14, 15]. Moreover, recent
experimental studies indicate $N = 32$ as a new magic number in Ca isotopes due to the high energy of the first $2^+$ state in $^{52}$Ca [16] and the trend obtained for the two-neutron separation energies [17]. The first experimental spectroscopic study of low-lying states in $^{54}$Ca have been performed at RIKEN [18]: the $2^+_1$ energy in $^{54}$Ca was found to be only $\sim 500$ keV below that in $^{52}$Ca, suggesting a $N = 34$ new shell closure. Finally, we note that new progress in the production of neutron-rich Ca isotopes can be expected at the NSCL at Michigan State University [19]. Future measurements of excited states and masses for the neighboring Ca isotopes will further enhance our understanding of nuclear states in very neutron-rich systems. Thus, the spectroscopy of neutron-rich calcium isotopes provides a valuable information, with important tests of theoretical calculations.

A powerful microscopic approach is the quasiparticle-phonon model (QPM) [20, 21]. A special emphasis on a reliable description of the mean-field part, reproducing as close as possible the ground-state properties of nuclei along an isotopic chain has been done in [22, 23]. This is achieved by solving the ground-state problem in a semimicroscopic approach based on a Skyrme energy density functional (EDF) [22, 23]. The EDF+QPM calculations have been applied for the low-energy dipole strength [22, 23]. The QRPA with a self-consistent mean-field derived from Skyrme EDF is one of the most successful methods for studying the low-energy dipole strength, see e.g., [24, 14, 25, 26, 27]. Such an approach describes the properties of the low-lying states less accurately than more phenomenological ones, but the results are in a reasonable agreement with experimental data. On the other hand, due to the anharmonicity of vibrations there is a coupling between one-phonon and more complex states [20, 21]. The main difficulty is that the complexity of calculations beyond standard QRPA increases rapidly with the size of the configuration space, so that one has to work within limited spaces. Using a finite-rank separable approximation (FRSA) [28, 29, 30, 31] for the residual interaction one can overcome this difficulty. The FRSA was thus used to study the electric low-energy excitations and giant resonances within and beyond the QRPA [30, 31, 32, 33]. In this paper our approach applied for PDR features of neutron-rich nuclei. We will give an illustration of our approach for $^{48}$Ca with closed neutron shell in comparison to the $N = 30$ isotope $^{50}$Ca. Preliminary results of our studies for neutron-rich Sn isotopes are reported in Refs. [26, 34, 35].

2. The FRSA model

The starting point of the method is the Hartree-Fock (HF)-BCS calculation [36] of the ground state based on Skyrme interactions. Spherical symmetry is imposed on the quasiparticle wave functions. The continuous part of the single-particle spectrum is discretized by diagonalizing the Skyrme HF Hamiltonian on a harmonic oscillator basis. In the particle-hole (p-h) channel we use the Skyrme interaction with the tensor components and their inclusion leads to the modification of the spin-orbit potential [37, 38]. The pairing correlations are generated by a density-dependent zero-range force

$$V_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left[ 1 - \eta \left( \frac{\rho(\mathbf{r}_1)}{\rho_0} \right)^\gamma \right] \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (1)$$

where $\rho(\mathbf{r}_1)$ is the particle density in coordinate space, $\rho_0$ being the nuclear matter saturation density; $\gamma$, $\eta$, and $V_0$ are adjusted parameters. The parameters are determined by adjusting the empirical odd-even mass differences of the nuclei in the region under study.

To build the QRPA equations on the basis of HF-BCS quasiparticle states, the residual interaction is consistently derived from the Skyrme force in the p-h channel and from the zero-range pairing force in the particle-particle (p-p) channel [39]. The FRSA for the residual interaction enables us to find QRPA eigenvalues as the roots of a relatively simple secular equation [28, 31]. The cut-off of the discretized continuous part of the single-particle spectra is at the energy of 100 MeV. This is sufficient to exhaust practically all the sum rules.
To take into account the effects of the phonon-phonon coupling (PPC) we follow the basic QPM ideas [20, 21]. We construct the wave functions from a linear combination of one- and two-phonon configurations

\[ \Psi_\nu (\lambda \mu) = \left( \sum_i R_i(\lambda \nu) Q^+_i \right) + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^i_{\lambda_1 i_1 \lambda_2 i_2}(\lambda \nu) \left[ Q^+_i Q^+_i \right]_{\lambda_1 i_1 \lambda_2 i_2} \mid 0 \rangle, \]

where \( \lambda \) denotes the total angular momentum and \( \mu \) its z-projection in the laboratory system. The ground state is the QRPA phonon vacuum \( \mid 0 \rangle \). The unknown amplitudes \( R_i(\lambda \nu) \) and \( P^i_{\lambda_1 i_1 \lambda_2 i_2}(\lambda \nu) \) are determined from the variational principle, which leads to a set of linear equations [30, 33]

\[ (\omega_{\lambda i} - E_\nu) R_i(\lambda \nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^i_{\lambda_1 i_1 \lambda_2 i_2}(\lambda i) P^i_{\lambda_1 i_1 \lambda_2 i_2}(\lambda \nu) = 0, \]

\[ 2(\omega_{\lambda i} + \omega_{\lambda_2 i_2} - E_\nu) P^i_{\lambda_2 i_2}(\lambda \nu) + \sum_i U^i_{\lambda_1 i_1 \lambda_2 i_2}(\lambda i) R_i(\lambda \nu) = 0. \]

The rank of the set of linear equations (3) and (4) is equal to the number of one- and two-phonon configurations included in the wave function (2). Its solution requires to compute the Hamiltonian matrix elements coupling one- and two-phonon configurations:

\[ U^i_{\lambda_1 i_1 \lambda_2 i_2}(\lambda i) = \langle 0 \mid Q_{\lambda i} H \left[ Q^+_1 Q^+_2 \right]_{\lambda_1 i_1 \lambda_2 i_2} \mid 0 \rangle. \]

Equations (3) and (4) have the same form as the QPM equations [20, 21], where the single-particle spectrum and the residual interaction are derived from the same Skyrme EDF.

3. Details of calculations

We apply the above approach to study the influence of the PPC on the strength \( E1 \) distributions of the neutron-rich Ca isotopes. We use the Skyrme interactions SLy5 [40] and SLy5+T [41] in the p-h channel. The parameters of the force SLy5 have been adjusted to reproduce nuclear matter properties, as well as nuclear charge radii, binding energies of doubly magic nuclei. The force SLy5+T involves the tensor terms added without refitting of the parameters of the central interaction (the tensor interaction parameters are \( c_T = -170 \text{ MeVfm}^5 \) and \( \beta_T = 100 \text{ MeVfm}^5 \)). These parametrizations describe correctly the binding energies of even-even Ca isotopes [42]. It is shown [42] that the proton-neutron rms differences become larger when the neutron number increases. The same evolution is obtained with other Skyrme EDF’s [6]. In the case of \(^{48}\text{Ca}\), the experimental neutron skin thickness (0.14 – 0.20 fm) has been determined from the \( E1 \) strength distribution which is extracted from proton inelastic scattering [43]. HF-BCS analysis gives the neutron skin \( \Delta R_{np} \) of \(^{48}\text{Ca}\) to be 0.16 fm and 0.14 fm with the SLy5 and SLy5+T EDF’s, respectively. The theoretical “model-averaged” estimate for \( \Delta R_{np} \) is 0.176 ± 0.018 fm [44]. In addition, the \( ab\ initio \) calculations for the neutron skin in \(^{48}\text{Ca}\) is 0.12 ± 0.15 fm [45].

For the interaction in the p-p channel, we use a zero-range volume force, i.e., \( \eta = 0 \) in Eq. (1). The strength \( V_0 \) is taken equal to −270 MeVfm\(^3\). This value of the pairing strength is
Figure 1. Low-energy $E1$ strength distribution of $^{40}$Ca (resp. $^{48}$Ca) is shown in the left (resp. right) panels. The dashed and solid lines correspond to the SLy5 calculations within the RPA and taking into account the PPC effects, respectively. Panels (b) and (d): experimental data are from Ref. [12].

fitted to reproduce the experimental neutron pairing energies of $^{50,52,54}$Ca obtained from binding energies of neighboring isotopes. This choice of the pairing interaction has also been used for a satisfactory description of the experimental data of $^{70,72,74,76}$Ni [46], $^{90,92}$Zr and $^{92,94}$Mo [33]. Thus, hereafter we use the Skyrme interaction SLy5 with and without tensor components in the particle-hole channel together with the volume zero-range force acting in the particle-particle channel.

In order to construct the wave functions (2) of the $1^−$ states, in the present study we take into account all two-phonon terms that are constructed from the phonons with multipolarities $\lambda \leq 5$ [26, 34, 35]. As it was shown in [42]the overall agreement of the calculated energies and $B(E1)$ values with the experimental data looks reasonable.

All dipole excitations with energies below 35 MeV and 15 most collective phonons of the other multipolarities are included in the wave function (2). We have checked that extending the configuration space plays a minor role in our calculations.

4. Results and discussion
As a first step in the present analysis, we examine the PPC effects on the $E1$ strength distributions for doubly-magic $^{40,48}$Ca isotopes. A comparison of such calculations with recent experimental data [12] demonstrates that the RPA approach cannot reproduce correctly the low-energy $E1$ strength distributions, see Fig. 1. For $^{40}$Ca the PPC calculation predicts the first $1^−$ state significantly higher than the experimental two-phonon candidate [47] (see Fig. 1). It is worth pointing out that our results for $^{40,48}$Ca are in good agreement with the RQTBA calculations taking into account the effects of coupling between quasiparticles and phonons [15]. In addition, we discuss the GDR energy region. For $^{48}$Ca, the photo-absorption process is well studied experimentally. The photo-absorption cross section up to 27 MeV is displayed in Fig. 2(a). The cross section is computed by using a Lorentzian smearing with an averaging parameter $\Delta = 1.0$ MeV. The PPC effects yield a noticeable redistribution of the GDR strength in comparison with the RPA results. It is worth mentioning that the coupling increases the GDR width from 6.9 MeV to 7.3 MeV in the energy region 10 – 26 MeV. The experimental GDR width and energy are 6.98 MeV and 19.5 MeV [48], respectively. The calculated characteristics
Figure 2. (a) The estimated photo-absorption cross section for $^{48}$Ca (filled circles) are taken from Ref. [48]. The dotted and solid lines correspond to the calculations within the RPA with the SLy5 EDF and taking into account the PPC, respectively. (b) Running sum of the electric dipole polarizability for $^{48}$Ca calculated within the RPA with the SLy5 EDF (dotted line) and the RPA plus PPC (solid line) in comparison to experimental determination of $\alpha_D$ (the two dashed lines indicate upper and lower limits) [43].

of the GDR are in agreement with the observed values. We conclude that the main mechanisms of the GDR formation in $^{48}$Ca can be taken into account correctly and consistently in the PPC approach.

In order to perform further investigations on the $^{48}$Ca nucleus we have extracted the electric dipole polarizability [1, 49, 50], which represents a handle to constrain the equation of state of neutron matter and the physics of neutron stars.

Running sums of $\alpha_D$ values for $^{48}$Ca in the energy region below 27 MeV are given in Fig. 2(b). It is shown that the PPC does not affect the description of the electric dipole polarizability. The results differ insignificantly. Moreover, we have checked that inclusion of the tensor components does not change the value of $\alpha_D$ obtained by integrating the $E1$ strength up to 60 MeV: $\alpha_D = 2.28$ fm$^3$ in the case of the SLy5 EDF and $\alpha_D = 2.20$ fm$^3$ in the SLy5+T. Both effective interactions reproduce the experimental data $\alpha_D = 2.07 \pm 0.22$ fm$^3$ [43] and they are in good agreement with the “model-averaged” value of $2.306 \pm 0.089$ fm$^3$, which is predicted in Ref. [44]. Although the GDR strength dominates, contributions to $\alpha_D$ value at lower and higher excitation energies must be taken into account.
To complete the discussion we consider $\alpha_D$ as a function of the neutron number for Ca isotopes. The result of the SLy5 calculation with the PPC predicts a monotonic increase of the $\alpha_D$ value with neutron number, and only a small kink in the calculated excitation energies is found at the $N = 28$ shell closure. The calculated polarizabilities $\alpha_D$ of $^{40}$Ca and $^{48}$Ca are in excellent agreement with the experimental data [43]. We find that the correlation between the value of $\alpha_D$ and neutron skin $\Delta R_{np}$ is discerned, see [42]. With the increase of the neutron skin one observes a smooth increase of $\alpha_D$. Thus, the $\alpha_D$ value and the neutron skin $\Delta R_{np}$ are correlated as predicted in Ref. [50]. Besides this, the $\alpha_D$ value can be measured in finite nuclei and, as a result, the $\Delta R_{np}$ value can be extracted.

Let us now discuss the strong increase of the summed $E1$ strength below 10 MeV ($\sum B(E1)$), with increasing the neutron number. In Fig. 3(a) the calculated running sum for $^{48}$Ca is plotted as a function of the excitation energy. In the same plot the calculated $S_{1n}$ values are shown. In the case of the RPA, there is no $1^-$ state below 10 MeV (see Fig. 1). The RPA calculations predict the first dipole state around 10.5 MeV. In contrast to the RPA case, the PPC results in the formation of low-lying $1^-$ states in this energy region. The dominant contribution in the wave function of the $1^-$ states comes from the two-phonon configurations ($> 60\%$). These states originate from the fragmentation of the RPA states above 10 MeV. As one can see in Fig. 3(a),

![Figure 3](image-url)
the calculated running sum of the $\sum B(E1)$ value is close to the experimental $\sum B(E1)$ value. The PPC calculations give a total dipole strength of 0.063 e²fm². The experimental value of $\sum B(E1)$ is 0.0687±0.0075 e²fm² in the same interval [10]. The PPC effects produce a sizable impact on the low-energy $E1$ strength of $^{48}$Ca. It is remarkable that the contributions of the low-lying $1^+$ states to the value of $\alpha_D$ is small (0.033 fm²), three times as much as the value deduced experimentally [43]. It is shown that the $\alpha_D$ value is more sensitive to the fine structure of the $E1$ strength distribution. The PPC calculations reproduce the observed trend in light Ca isotopes, although the theoretical value of $\sum B(E1)$ for $^{44}$Ca underestimates the experimental value by a factor of 2 [42].

In contrast to the case of $^{48}$Ca, the PPC effects on the low-energy dipole spectrum of $^{50}$Ca is weak (see Fig. 3(b)). Thus, the $\sum B(E1)$ values for $^{50}$Ca results predominantly from the QRPA distribution of $E1$ strength. A similar result is observed in the case of $^{52,54,56,58}$Ca isotopes. The separation energies decrease much faster than the value of $\sum B(E1)$. This means, of course, that the observation of the PDR in $(\gamma, \gamma')$ experiments will be strongly hindered.

Our investigation [42] demonstrates that there is a correlation between the PDR properties and the neutron skin $\Delta R_{np}$.

5. Conclusions

The electric dipole polarizability is a particularly important observable, as it can be measured in finite nuclei and it provides important information on the neutron skin thickness that can be extracted. In this study, Skyrme QRPA calculations including the phonon-phonon coupling have been performed for the $E1$ response in neutron-rich Ca isotopes, some of which should become experimentally accessible in the near future. Our calculations reproduce the data for the neutron skin thickness and neutron separation energies. It is shown that the phonon-phonon coupling has small influence on the dipole polarizability.

For $^{48}$Ca, the PPC effect have a damping and smoothing action which yields a GDR cross section close to the experimental one in shape and magnitude. We find the impact of the shell closure $N = 28$ on the summed $E1$ strength below 10 MeV. The dipole response for $^{52-58}$Ca is characterized by the fragmentation of the strength distribution and its spreading into the low-energy region.

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6. References

[1] Migdal A B 1944 J. Phys. Acad. Sci. USSR 8 331; 1945 J. Exp. Theor. Phys. USSR 15 81
[2] Berman and B L Fultz S C 1975 Rev. Mod. Phys. 47 713
[3] Dietrich S S and Berman B L 1988 At. Data Nucl. Data Tables 38 199
[4] Savran D et al. 2013 Prog. Part. Nucl. Phys. 70 210
[5] Paar Net al. 2007 Rep. Prog. Phys. 70 691
[6] Inakura T et al. 2011 Phys. Rev. C 84 021302(R)
[7] Ebata S et al. 2014 Phys. Rev. C 90, 024303 2015 92, 049902(E)
[8] Arnould M et al. 2007 Phys. Rep. 450 97
[9] Hartmann T et al. 2000 Phys. Rev. Lett. 85 274
[10] Hartmann T et al. 2004 Phys. Rev. Lett. 93 192501
[11] Isaak J et al. 2011 Phys. Rev. C 83 034304
[12] Derya Vet al. 2014 Phys. Lett. B 730288
[13] Soloviev V G et al. 1978 Nucl. Phys. A 304 503
[14] Terasaki J and Engel J 2006 Phys. Rev. C 74 044301
[15] Egorova I A and Litvinova E 2016 Phys. Rev. C 94 034322
[16] Gade A et al. 2006 Phys. Rev. C 74 021302(R)
[17] Wienholtz F et al. 2013 Nature 498 346
[18] Steppenbeck D et al. 2013 Nature 502 207
[19] Tarasov O B et al. 2009 Phys. Rev. Lett. 102 142501
[20] Soloviev V G 1992 Theory of Atomic Nuclei: Quasiparticles and Phonons(Institute of Physics, Bristol and Philadelphia)
[21] Lo Iudice N et al. 2012 J. Phys. G 39 043101
[22] Tsonveva N et al. 2014 Phys. Lett. B 586 213
[23] Tsonveva N and Lenske H 2008 Phys. Rev. C 77 024321
[24] Sarchi et al. 2004 Phys. Lett. B 601 27
[25] Avdeenkov A et al. 2011 Phys. Rev. C 83 064316
[26] Arsenyev N N et al. 2012 EPJ Web of Conf. 38 17002
[27] Repko A et al. 2013 Phys. Rev. C 87 024305
[28] Nguyen Van Giai et al. 1998 Phys. Rev. C 57 1204
[29] Severyukhin A P et al. 2002 Phys. Rev. C 66 034304
[30] Severyukhin A P et al. 2004 Eur. Phys. J. A 22 397
[31] Severyukhin A P et al. 2008 Phys. Rev. C 77 024322
[32] Severyukhin A P et al. 2009 Phys. At. Nucl. 72 1149
[33] Severyukhin A P et al. 2012 Phys. Rev. C 86 024311
[34] Arsenyev N N et al. 2015 Acta Phys. Pol. B 46 517
[35] Arsenyev N N et al. 2016 EPJ Web of Conf. 107 05006
[36] Ring P and Schuck P 1980 The Nuclear Many Body Problem (Berlin: Springer)
[37] Stancu et al. 1977 Phys. Lett.B 68 108
[38] Lesinski T et al. 2007 Phys. Rev. C 76 014312
[39] Terasaki J et al. 2005 Phys. Rev. C 71 034310
[40] Chabanat E et al. 1998 Nucl. Phys. A 635 231
[41] Colò G et al. 2007 Phys. Lett. B 646 227 ; 2008 668 457(E)
[42] Arsenyev N N et al. 2017 Phys. Rev. C 95 054312
[43] Birkhan J et al. arXiv:1611.07072v1 [nucl-ex].
[44] Piekarewicz J et al. 2012 Phys. Rev. C 85 041302(R)
[45] Hagen G et al. 2016 Nature Physics 12 186
[46] Severyukhin A P 2014 Phys. Rev. C 90 044320
[47] Derya V et al.] 2016 Phys. Rev. C 93 034311
[48] O’Keefe G J et al. 1987 Nucl. Phys. A 469 239
[49] Bohigas O et al. 1981 Phys. Lett. B 102
[50] Satula W et al. 2006 Phys. Rev. C 74 011301(R)