Exploring and mining attributed sequences of interactions

Tiphaine Viard  
LTCI, Institut Polytechnique de Paris  
Paris, France

Henry Soldano  
NukkAI  
Paris, France

Guillaume Santini  
LIPN  
Villetaneuse, France

ABSTRACT

We are faced with data comprised of entities interacting over time: this can be individuals meeting, customers buying products, machines exchanging packets on the IP network, among others. Capturing the dynamics as well as the structure of these interactions is of crucial importance for analysis. These interactions can almost always be labeled with content: group belonging, reviews of products, abstracts, etc. We model these stream interactions as stream graphs, a recent framework to model interactions over time. Formal Concept Analysis provides a framework for analyzing concepts evolving within a context. Considering graphs as the context, it has recently been applied to perform closed pattern mining on social graphs. In this paper, we are interested in pattern mining in sequences of interactions. After recalling and extending notions from formal concept analysis on graphs to stream graphs, we introduce algorithms to enumerate closed patterns on a labeled stream graph, and introduce a way to select relevant closed patterns. We run experiments on two real-world datasets of interactions among students and citations between authors, and show both the feasibility and the relevance of our method.

ACM Reference Format:
Tiphaine Viard, Henry Soldano, and Guillaume Santini. 2021. Exploring and mining attributed sequences of interactions. In Proceedings of ACM Conference (Conference’17). ACM, New York, NY, USA, 8 pages. https://doi.org/10.1145/nmmmm.nmmmm

1 INTRODUCTION

We consider mining connected data with the following view: part of the data consists in attributes values reporting information about objects, while the remaining part of the data reports information about how objects are related. We search then for attribute patterns i.e. sentences expressing constraints on the attributes values and that may be valid, i.e. occur, in some objects. Various previous work on graphs (see Section 2.1) confront such attribute patterns to the connected structure, i.e. consider poorly connected objects as poorly relevant to the knowledge to extract. As a result the mining process enumerates and selects both attribute patterns and the dense subgraphs associated with them. The purpose of this article is to extend one of such methodology, namely the core closed pattern methodology, in order to mine temporal interaction data.

Modelling data that has a structural component over time has been done in multiple ways, and in particular recently, by considering interaction data: the connected data is then designed as a sequence of triplets \((t, u, v)\) indicating that nodes \(u\) and \(v\) interacted at time \(t\) (see Section 2.2). They may represent, for instance, the interactions between scientists attending a conference, social networks exchanges between high school students, or interactions on the web, among others. Enriching such connection data with attributes describing individuals allows to extract knowledge relating individuals descriptions, to the way these individuals are connected at some moment. Note that the individuals descriptions may themselves depend on time: while, for instance, the background of a scientist may be considered as unrelated to the interaction time, their state of mind may depend on the time of the interaction.

The main characteristic of the stream graph formalism is to represent interaction data is that it is based on the extensions of static graph notions in a natural way. As a consequence we may transfer conveniently results and methods from graph analysis and mining. The present work focus on extending core closed pattern methodology to attributed stream graphs, a process which is facilitated by the fact that the notion of graph cores, which core closed pattern mining heavily relies on, has a natural counterpart in stream graphs.

We develop our contributions as follows: after discussing related work in Section 2, we present the core closed pattern formalism to mine connected data in Section 3. In Section 4, we present the stream graph formalism to model interactions over time, and show how to adapt the mining methodology to stream graphs. We then present algorithms, in Section 5, and apply them to closed pattern mining on two real-world datasets, in Section 6. Finally, we conclude and present some tracks for future work in Section 7.

2 RELATED WORK

2.1 FCA and closed pattern mining on graphs

A recent review on mining and finding dense subgroups within attributed graphs [2] discusses a variety of approaches, algorithms and programs addressing this task. Among them, various works such as [17], [22] and [23] define the subgraph properties that are suitable both from formal and application viewpoints. The latter introduced core closed pattern mining whose various definitions and results necessary for our purpose to mine attributed interaction data are presented in Section 3.

Closed pattern mining is strongly related to Formal concept analysis [30] which focuses on describing formally concepts associated to a context, i.e. an object-attribute table, and ordered in a concept
lattice according to a general-to-specific ordering. A FCA process results in producing a lattice of concepts each made of a closed pattern (the concept intent), together with its support set (the concept extent) i.e. the set of objects in which the pattern occurs. A closed pattern is then the most specific pattern among all those sharing the same support set. While FCA is a formal methodology strongly interested in the ordering of such concepts, the closed pattern mining framework focuses on the efficient enumeration of closed patterns in large datasets (see for instance [32]).

Core closed pattern mining is a variant of closed pattern mining in which the support set of a pattern is reduced to its core support set i.e. the core of the subgraph induced by the original support set. The first core notion is the k-core proposed by Seidman [21] that reduces a simple and undirected graph to the unique maximal subgraph whose nodes (forming the k-core) all have degree at least k. Core definitions, as generalized in [3] always rely on some topological property that have to be shared by its elements and has proved to be a key notion for real-world network analysis. In [23] it is shown that the core of a graph is obtained by applying an interior operator to its vertex set, so ensuring that closed patterns exists when reducing support sets to core support sets (see Section 3).

Adapting enumeration algorithms from closed pattern mining [18], that has a polynomial delay between outputting two patterns, has also been a necessary result for real-world applications. The core closed pattern mining framework has since then been applied to bipartite [26] and directed [23] networks, and the methodology has been extended in various ways [1, 24, 27].

2.2 Stream graphs and modelling of interactions over time

Modelling data that has a structural component over time has been done in multiple ways, typically through different variants of dynamic graphs. In this setting, one typically has a sequence of graphs \( \{G_t\} \) and a time frame \( \Delta \), and for all \( i, E_i \) contains all the interactions that happened between times \( i\Delta \) and \( (i+1)\Delta \). There are multiple variants, for example in which the graph only grows in time [10], or in which multiple concurrent values of \( \Delta \) are considered [15], but the principle remains similar. The main limit of these approaches is linked to the loss of temporal information induced by this aggregation. The choice of \( \Delta \) is non trivial: a value too small will yield small, empty graphs, while a value too large will destroy the temporal information and the interaction causalities [7].

Recently, a few models take a different perspective, where aggregating is not necessary and one considers the sequence of interactions for itself. The sequences of interactions are then modelled as temporal networks [12], time-varying graphs [8] or stream graphs [14], depending on the research goals and the scientific community. In all cases, the base object is identical: a sequence of \( (t,u,v) \) indicating that nodes \( u \) and \( v \) interacted at time \( t \). From this object, different communities have researched with different goals in mind: temporal networks has large bodies of work around diffusion and temporal causality [11]; time-varying graphs focuses on reachability and elaborating algorithmic complexity classes [5]; stream graphs focus on extending the notions used for large-graph analysis [28] and applying them to real-world scenarios such as traffic analysis [31], or financial network analysis [9], among others.

3 CORE CLOSED PATTERN MINING

In this section we report the needed definitions and results to introduce our attributed stream graph mining methodology. Except regarding Proposition 3.4, they are extracted from [26]. To be self-contained, let us first recall closure and interior operator definitions: Let \( S \) be an ordered set and \( f : S \to S \) a self map such that for any \( x, y \in S, f \) is monotone, i.e. \( x \leq y \) implies \( f(x) \leq f(y) \) and idempotent, i.e. \( f(f(x)) = f(x) \). Then If \( f(x) \geq x \), \( f \) is called a closure operator while if \( f(x) \leq x \), i.e. \( f \) is intensive, \( f \) is called an interior operator.

3.1 Abstract closed pattern mining

In closed pattern mining, a pattern \( q \) belongs to a pattern language \( L \) which is ordered through a partial order where \( q \geq q' \) means that \( q \) is more specific than \( q' \). Consider then a set of objects \( V \), each object \( v \) has a description \( d(v) \) in \( L \) representing the most specific pattern in which it occurs, i.e. \( d(v) \) occurs in \( v \) and also occurs in any pattern less specific than \( d(v) \). Pattern \( q \) extension, also called its support set, \( X = \text{ext}(q) \) is then the set of its occurrences in \( V \). Applying then an interior operator \( p \) to \( \text{ext}(q) \) results in reducing the support set of \( q \) into its so-called abstract support set. The most specific pattern with abstract support set \( X \) is then unique, as far as the pattern language is a lattice, and is called an abstract closed pattern. Computing the abstract closed pattern \( f(q) \) with same support set as some pattern \( q \) relies on an intersection operator \( \text{int} \) such that \( \text{int}(X) \) returns the most specific pattern which is less specific than any object description \( d(o) \) in \( X \). We obtain then the abstract closed pattern \( f(q) \) with same abstract support set as pattern \( q \), where \( f \) is a closure operator, as \( f(q) = \text{int} \circ p \circ \text{ext}(q) \).

In the closed itemset mining setting objects are described as itemsets i.e. subsets of a set of items \( I \). In this case the intersection operator simply is the set theoretic intersection operator \( \cap \).

Example 3.1. Let us consider \( L = \{1, 2\}, I = \{abc\}, V = \{1, 2, 3\}, d(1) = abd, d(2) = acd, d(3) = abc \). Pattern \( \emptyset \) has support set 123 and \( \text{int}(123) = abd \cap acd = abc \). Now consider the interior operator \( p \) such that \( vX \subseteq V, p(X) = X \setminus X \). We obtain then \( p(123) = 12 \) and Following Equation 7?, the abstract closed pattern \( f(q) = \text{int}(12) = \emptyset \).

3.2 Core closed pattern mining

The following result allows us to define an interior operator on the object powerset \( 2^V \) from a logical property \( P \) regarding an object \( v \) in the context of an object subset \( X \) to which it belongs:

Proposition 3.2. Whenever a property \( P \) is monotone, i.e. for any \( X \subseteq V \) and \( v \in X \), we have that \( P(v,X) \) and \( X' \supseteq X \) implies \( P(v,X') \), then there is a unique greatest subset \( C \subseteq X \) such that \( P(v,C) \) holds for all \( v \in C \) and \( p \) defined as \( p(X) = C \) is an interior operator.

Using such properties is natural when the object set \( V \) is the set of vertices of a graph \( G = (V, E) \). For instance, the k-core [21] of the subgraph \( G_v \) induced by some vertex subset \( X \) is defined as the greatest subset \( C \subseteq X \) such that all vertices in \( C \) have degree at least \( k \) in \( G_v \), which may be rewritten as \( P(v,C) \) holds for all \( v \in C \).
$P$ is then called a core property, called $p$ a core operator. We obtain that way abstract closed patterns, called core closed patterns.

A second way to obtain an interior operator on $2^V$ is to first build an interior operator $p_b$ on a pair of powersets $(2_{X_1}, 2_{X_2})$ from a logical property $P_b$. By considering then $\mathcal{V} = \mathcal{V}_1 = \mathcal{V}_2$, we derive from $p_b$ a new interior operator $p$ on $2^V$. $p_b$ is obtained as follows:

**Proposition 3.3.** Whenever a property $P_b$ is bi-monotone, i.e. for any $(X_1, X_2)$ pair and any $v \in X_1 \cup X_2$, $P_b(u, X_1, X_2)$ and $(X_1^v, X_2^v) \supseteq (X_1, X_2)$ imply $P_b(v, X_1^v, X_2^v)$, then:

- there is a unique greatest subset pair $(C_1, C_2) \subseteq (X_1, X_2)$ such that $P_b(v, C_1, C_2)$ holds for all $v \in C_1 \cup C_2$ and
- $p_b$ defined on $2^{2V} \times 2^{2V}$ as $p_b(X_1, X_2) = (C_1, C_2)$ is an interior operator.

Bi-cores are then pairs of object subsets whose members all satisfy a bi-monotone property, called a bi-core property. A bi-core property $P_b$ is usually designed from a pair of properties, i.e. $P_b(u, X_1, X_2)$ if and only if $v \in X_1$ then $P_1(v, X_1, X_2)$ and if $v \in X_2$ then $P_2(v, X_1, X_2)$ holds. For instance, when $G$ is the directed graph, the $h-a$ BHA bi-core property states that in the subgraph $G(X_1, X_2)$ induced by the directed edges from $X_1$ towards $X_2$, if $v$ is in $X_1$, it has outdegree at least $h$ and if $v$ is in $X_2$ it has indegree at least $a$. Note that vertices in $X_1 \cap X_2$ have to satisfy both constraints. The following Proposition 3.4 leads then to interior operators on $2^V$ and therefore to core closed patterns.

**Proposition 3.4.** Let $P_b$ be a bi-core property on $(V, 2^V, 2^V)$ and $p_b$ its associated interior operator. Then, $p$ defined as $p(X) = X_1 \cup X_2$, with $(X_1, X_2) = p_b(X, X)$ is an interior operator on $2^V$.

Proof. We need to prove three properties. The proofs straightforwardly follows from the truth of the corresponding properties of the interior operator $p_b$. For instance, we need to prove that $p$ is monotone, i.e. $X \subseteq X'$ implies $p(X) \subseteq p(X')$, we remark that $X \subseteq X'$ means $(X, X) \subseteq (X', X')$. As $p_b$ is an interior operator this implies $p_b(X, X) \subseteq p_b(X', X')$ and it follows that $p(X) \subseteq p(X')$. Idempotency and intensity are proved in the very same way. □

The $h-a$ hub-authority (HA) core $p(X)$ for directed graphs was first defined in [27]. It may be obtained as the union of hubs $H$ and authorities $A$ from the $h-a$ BHA bi-core $(H, A)$ of the subgraph $G(X)$.

### 3.3 Exhibiting patterns of interest

In many real-world contexts, enumeration is only an intermediate step towards the mining of patterns of interest. When selecting individual patterns from a pattern set $Q$, according to various interestingness criteria, the resulting pattern subset may still be redundant, i.e. contain patterns very similar to other patterns. There are various pattern set selection ways of reducing size and redundancy of a pattern set [6, 19, 29]. In our experiments we will use the $g\beta$ pattern set selection algorithm first defined and applied to core closed patterns in [25]. It consists in maximizing in the selected pattern set $Q_\mathcal{V}$ the sum of the values of a pattern interestingness measure $g$ under the constraint that two patterns $q$ and $q'$ in $Q_\mathcal{V}$ have to be at distance $\sigma(q, q')$ at least $\beta$.

Figure 1: Two toy stream graphs, modelling interactions over $T = [0, 10]$. Left: A unipartite stream graph involving 4 nodes $V = \{a, b, c, d\}$ and $W = \{(a) \times [0, 10], (b) \times [0, 4] \cup [5, 10], (c) \times [4, 10], (d) \times [1, 3]\}$ and the set of interactions $E = \{(ab) \times [1, 3] \cup [7, 8], (bd) \times [2, 3]\}$. Right: A bipartite stream graph involving 6 nodes, with $T = \{u, v, w\}$ and $\mathcal{V} = \{x, y, z\}$.

returns a greedy approximation for this problem, obtained after ordering the input pattern list $Q$ in decreasing $g$ order. Choosing the interestingness measure $g$, (or equivalently the corresponding pattern ordering), as well as the distance measure $\sigma$, is typically application-dependent.

### 4 STREAM GRAPHS

Stream graphs are a recent formalism [14] to model interactions over time by generalizing many useful notions from complex and social networks analysis. We denote a stream graph by the tuple $S = (T, V, W, E)$, where $T$ is a time interval, $V$ a set of nodes, $W \subseteq T \times V$ denotes the presence times of nodes, such that $(t, v) \in W$ means that node $v$ is "active" at time $t$, and finally, $E \subseteq T \times V \times V$ denotes interactions, such that $(t, v, u) \in E$ means that nodes $v$ and $u$ interacted at time $t$. If we consider that interactions are undirected $(t, v, u) = (t, u, v))$ and without loop $(u \neq v)$ and we denote by $V \circ V$ the set of such pairs of nodes. In the directed case, we denote edges as $(t, u, v) \in E$, and $E \subseteq T \times V \times V$. Figure 1 depicts toy stream graphs.

Furthermore, we say that $S' = (T', V', W', E')$ is a substream of $S$ if and only if $T' \subseteq T$, $V' \subseteq V$, $W' \subseteq W$ and $E' \subseteq E$. We denote this by $S' \subseteq S$. We denote by $S(W')$ the substream graph induced by a time-node vertex subset $W' \subseteq W$, and whose interaction subset $E_{W'}$ contains interaction between time-nodes of $W'$. Finally, let us define $G_S = (V_S, E_S)$ the graph induced by $S$, with $V_S = \{u : \exists (t, u) \in E, t \in T\}$ and $E_S = \{uv : \exists (t, u) \in E, t \in T\}$. In other words, nodes and edges belong to $V_S$ and $E_S$ if and only if there exist some time $t$ such that $(t, u) \in E$. The adaptation to the directed case is straightforward.

For any node $v \in V$, we denote its neighbourhood at time $t$ by $N_t(v) = \{(t, u) : \exists (t, u) \in E, u \in V\}$ the set of $(t, u)$ that interact with node $v$ at time $t$. We further denote the degree of $v$ at time $t$ by $d_t(v) = \left| N_t(v) \right|$. For example, in Figure 1 (left), node $b$ at time 2 interacts with nodes $a$ and $d$, and so $N_2(b) = \{a, d\}$, and $d_2(b) = 2$.

We can extend the stream graph definition to directed case, in which all interactions in $E$ are directed. In that case, the outneighbourhood at time $t$ of node $v$, $N^+_t(v)$, contains time-nodes such that there exists a directed edge $(t, uv)$ in $E$ and its outdegree at time $t$ $d^+_t(v)$ is the size of its outneighbourhood. The inneighbourhood at time $t$ and indegree at time $t$ of a node are defined in the same way.
5 PATTERN ENUMERATION IN STREAM GRAPHS

In this section we define cores and present algorithms to compute them and to enumerate patterns from (real-world) attributed stream graphs.

5.1 Core operators

Let us first define two core operators that will be used in our experiments in core closed pattern mining in streams. We will consider as object set the set of time-nodes \( W \) of a stream graph

\[ S = (T, V, W, E). \]

The \( k \)-Star-Satellite core operator selects in an induced substream graph \( S(W') \) high degree time-nodes together with their neighbours and is defined through the following core property:

**Definition 5.1 (\( k \)-Star-Satellite).** Let \( S \) be an undirected stream graph and \( k \in \mathbb{N} \), the \( k \)-star-satellite property \( P((t, v), W') \) holds if and only if in the induced substream graph \( S(W') \) either \( d_t(v) \geq k \) or there exists \((t, v') \in N_t(v)\) such that \( d_t(v') \geq k \).

The \( h-a \) HA core operator is a counterpart in directed stream graphs of the \( h-a \) HA core operator in directed graphs defined in Sections 3.2. It is designed through the following bi-core property:

**Definition 5.2 (\( h-a \) BHA).** Let \( S \) be a directed stream graph and \( h, a \in \mathbb{N} \), the \( h-a \) BHA property \( P_h((t, v), W_1, W_2) \) holds if and only if in the induced substream graph \( S(W_1, W_2) \) if \((v, t) \) is in \( W_1 \) then \( d_t^-(v) \geq h \) and if \((v, t) \) is in \( W_2 \) then \( d_t^+(v) \geq a \).

The \( h-a \) HA core of \( G(X) \) is then obtained as \( p(X) = H \cup A \) where \((H,A)\) is the \( h-a \) BHA bi-core of the induced substream graph \( G(X) = G(X, X) \). To define these core operators we need to prove that the associate properties are, respectively, monotone and bi-monotone properties (see Section 3.2):

**Theorem 5.3.** Definitions 5.1 and 5.2 are respectively core and bi-core properties.

**Proof.** Let us start with the \( k \)-Star-Satellite property 5.1. We are interesting in proving that this property is monotonous. Suppose that there exists a substream \( S' = (T', V', W', E') \), \( S' \subseteq S \) such that for all elements \((t, v) \in W'\), property 5.1 holds. In other words, there are enough interactions in \( E' \) such that node \( v \) at time \( t \) either has at least \( k \) neighbours (and is a star), or is a neighbour of such a node (and is a satellite).

Let us show that there is no stream \( R = (T_R, V_R, W_R, E_R), R \supset S' \) such that the property is false. Suppose that such a stream \( R \) exists. Then, there exists elements of \( W' \) that are not in \( W_R \). Since the core properties defined both involves degrees, this can only mean that there are interactions in \( E' \) that are not in \( E_R \), which in turns means that \( R \not\supset S' \). This validates our monotonicity claim for the \( k \)-Star-Satellite property. An identical argument can be made for Definition 5.2. \( \square \)

![Figure 2: Illustration of the core definitions on the examples of Figure 1. Left: The 2-star-satellite core, with \{b\} \times [1, 3] \cup [7, 8] being the stars (depicted in blue), and \{\{a\} \times [1, 3] \cup [7, 8], \{c\} \times [7, 8], \{d\} \times [2, 3]\} being the satellites of \( b \) (depicted in green).](image)
We also know that all current nodes $x$ and recursively all frequent core closed patterns greater than $q_i$, our language, we build the pattern $q_i \cup \{x\}$ and compute its core

$$\sigma(l_i, l_j) = 1 - \mathcal{J}(l_i, l_j) = \frac{|W_i \cap W_j|}{|W_i \cup W_j|}$$

$\sigma(l_i, l_j)$ has values between 0 and 1, is equal to 0 whenever $l_i \subseteq l_j$ and to 1 if $l_i$ and $l_j$ have no element in common. As a $g$ interestingness measure we consider the core support set size.

5.4 Exhibiting patterns of interest

Finally, let us define the distance to be used in the $g\beta$ selection process (see Section 3). Given a pair of patterns $l_i, l_j$ and their associated core support sets $W_i, W_j$, we define their temporal Jaccard distance as:

Let us now discuss the pattern enumeration of all frequent core closed patterns, i.e with core support set at least $s$. The algorithm starts with the closure $q_0$ of the empty pattern $\emptyset$ and associated core support set $X$. Then, for all the items $x$ (i.e. the elements of our language), we build the pattern $q_0 \cup \{x\}$ and compute its core support set in the stream, the associated core closed pattern $q_x$ and recursively all frequent core closed patterns greater than $q_x$. Maintaining a list $EL$ of prohibited items results in building a a tree over the pattern lattice, in such a way that each pattern is only enumerated once. The algorithm is similar to the one defined by [23]; indeed, thanks to the formal work presented in the previous sections, once the notions of pattern, support set and core property are properly extended, the algorithm itself runs a similar course of execution.

5.3 Pattern enumeration

Let us now discuss the pattern enumeration of all frequent core closed patterns, i.e with core support set at least $s$. The algorithm starts with the closure $q_0$ of the empty pattern $\emptyset$ and associated core support set $X$. Then, for all the items $x$ (i.e. the elements of our language), we build the pattern $q_0 \cup \{x\}$ and compute its core support set in the stream, the associated core closed pattern $q_x$ and recursively all frequent core closed patterns greater than $q_x$. Maintaining a list $EL$ of prohibited items results in building a a tree over the pattern lattice, in such a way that each pattern is only enumerated once. The algorithm is similar to the one defined by [23]; indeed, thanks to the formal work presented in the previous sections, once the notions of pattern, support set and core property are properly extended, the algorithm itself runs a similar course of execution.

6 EXPERIMENTS

We now detail experiments on two real-world datasets of web and social interactions to highlight the relevance of our proposal.

6.1 Datasets

We performed our experiments using two data sets, one of individual contacts between high school students (HS-327), and another of research paper co-citations extracted from the Association of Computer Linguistics Anthology website. Both datasets are publicly available, and all the code for the following experiments is available online².

6.1.1 Contacts between individuals. HS-327 is a dataset constructed from the results of a study of social interactions of 327 French

²https://github.com/TiphaineV/pattern-mining
Table 1: Summary of the closed patterns enumerated on both datasets, and the number of closed patterns selected by gβ-selection. For the ACL dataset, we only keep closed patterns with at least 4 keywords.

| Dataset | k | β  | Runtime |
|---------|---|----|---------|
| HS-327  | 3 | 0.0 | 620 362 221 125 76 16mns |
| HS-327  | 4 | 0.2 | 99 75 52 40 31 9mns |
| ACL     | 15, 15 | 0.6 | 1030 406 175 56 12 90mns |
|         |   | 0.8 |         |

students conducted in 2013 [16]. The initial dataset provides us with the stream of contacts over 5 days between the students, which amounts to 33806 temporal interactions. The dataset also contains, for each student u, their class, their gender, and three lists of friends: one is the students u has met (self-report), another is the students that u has declared as friends (self-report), and finally, the friends u has on Facebook. We express each temporal interaction between a pair of nodes as a union of consecutive intervals of the form $[t_{i,j} \pm 20\text{sec}, t_{i,j}]$.

### 6.1.2 Academic paper citing in the ACL

We also focus on a larger dataset. ACL-papers is built from the ACL anthology, which regroups research papers related to the Association of Computer Linguistics. It is a co-citation temporal network, that we use to track the scientific specialities of scientists that co-author papers together between 1979 and 2008, over the span of 29 years. The dataset contains 250,000 interactions between roughly 8000 authors. The attributes for each author in time are extracted from the abstracts’ content, using the CSO ontology, as described in [4, 20, 33]. We end up with 2500 attributes, and each author keeps all their attributes over time. It would have been interesting to consider attributes on a per-paper basis, which we leave as future work.

### 6.2 Results

Using our implementation of the algorithms presented in Section 5, we mine patterns on our two datasets. Notice that our goal here is to showcase the potential of our method, rather than find an optimal set of parameters that will necessarily be application-dependent.

#### 6.2.1 HS-327

For the HS-327 dataset, we use the $k$-star-satellite property. We present in Table 1 some numerical results depending on the value of $k$ and the selection parameter $\beta$. Notice that rapidly (when $k \geq 5$), there are no more patterns to enumerate other than the empty pattern. This is due to the temporal nature of the data, that spreads out interactions as compared to a static graph.

In the selected patterns, we capture generic patterns, that spread in time (for example, students of a classroom), as well as more specific patterns related to particular time intervals. This allows us to study the interactions at multiple time scales.

As expected, a more specific pattern is correlated with smaller support sets, with the largest support set supporting the empty pattern. However, in particular for smaller patterns, many sizes of supports sets exist. Concerning the patterns, we noticed that many patterns contain the gender of the students (either G_M or G_F), reinforcing claims that students regroup in non-mixed gender groups. In comparison, in the bottom left we display one pattern with no gender information Î = D_894, F_265, D_205, F_170, F_425, F_871, F_1, F_883, C_2BIO3, F_272, F_106, mixing Facebook friendships and self-declared friendships. Notice that this points to strong differences between whom the students declare as friends versus who they are Facebook friends with. For instance, the closed pattern at bottom center is C_2BIO3, D_265, D_272, D_117, this time regrouping only declared friends of the 2BIO3 class (Biology specialty). Notice that student 272 is declared by everyone in the closed pattern as a friend, but this is not mutual. The last pattern (bottom right) F_119, F_425, F_871, F_1, F_883, C_2BIO3, F_181 points to students that are friends on Facebook but did not declare themselves as friends.

Let us compare the patterns resulting from mining the stream to those obtained from the static graph. To enumerate the core closed patterns from the static graph, we implemented the code from [25]. Notice first that when considering the static graph associated to a stream graph, nodes descriptions which do not depend on time, and $k$-star-satellite cores in both cases, the core closed patterns in the stream graph also are core closed patterns in the static graph. Indeed, if node $u$ has $k$ neighbours at a time $t \in T$, then $u$ has also $k$ neighbours in the static graph; however the converse is untrue: it is possible for $u$ to have $k$ neighbours in the static graph, each related to $u$ at different times. This means that the core definition in the static graph is a weaker constraint than the one required by the core definition in the stream graph.

As a consequence of this, when mining close patterns on the graph induced by the stream graph of the HS-327 dataset with the $k = 4$-star-satellite core property, we obtain 11600 closed patterns, to be compared to the 99 closed patterns obtained from the stream graph. Notice however that many of these patterns do not have any grounding in reality, as we show on a toy example in Figure 4. In that sense, we argue that our patterns are fewer but of higher relevance.

#### 6.2.2 ACL

For the ACL dataset, we mine patterns using the $h$-BHA-core property, and report results for $h = a = 15$. We have experimented with different values for both $h$ and $a$, and report these results since they provide enough closed patterns to be interesting without offering an overwhelming number of closed patterns. In total, 1664 closed patterns are enumerated in a bit less than 90 minutes.

As for HS-327, a more specific description is correlated with fewer authors. The intents help us highlight different subfields of the ACL Anthology; typical intents for closed patterns around 1990 involves the keywords syntactics, context-free, while keywords such as learning, natural_language_processing appear much later, around 2005 for most authors.

In the dataset a few (16) researchers are active over more than 14 years. This is particularly interesting, since it allows us to follow their closed patterns over time.

We can see that for most researchers, the terms parsing and natural_language_processing appear late (around 2003), even though one of them, Lynette Hirschman, has keyword natural
Figure 3: Some examples of patterns on the HS-327 dataset, all selected with $\beta = 0.8$. On top, one “long” pattern spread in time (blank spaces represent time periods where nothing happens). On the bottom, three more specific patterns, involving less nodes over a shorter time span. Notice that the long, less specific pattern was selected before the more specific ones.

Figure 4: 2-star-satellites on a toy stream and its induced graph. There are 4 closed patterns on the graph, but 3 in the stream, as the closed pattern $ab$ with core support set $\{u, x, y\}$ cannot exist in time, since $u$ never interacts with $x$ and $y$ at the same time.

Figure 5: The 6 closed patterns selected with $\beta > 0.9$ in the ACL dataset.

Language understanding in her closed patterns since 1991. However, the support sets help paint an even more interesting picture, showing how some researchers change specialty without changing their favourite coauthors, while authors likely change domains.

Focusing on the most distinct patterns (i.e. the patterns selected with $\beta \geq 0.9$) gives other insights. These patterns are the most mutually dissimilar according to our $q\beta$ measures. We give the intents of these closed patterns in Figure 5. In this case, these closed patterns highlight different sub-areas of research of the Association for Computer Linguistics. The fact that keywords co-occur even in natural languages, semantics, syntactics, syntactic structure

linguistics, machine translations, syntactics, syntactic structure

bilingual, correlation analysis, machine translations, translation process

correlation analysis, learning, parsing algorithm, syntactic analysis, syntactics, syntactic structure

correlation analysis, machine translations, statistical machine translation, syntactic structure

correlation analysis, machine translations, phrase-based statistical machine translation, statistical machine translation, translation models

the 0.9-selected closed patterns likely comes from the fact that the scope of the ACL itself regroups researchers on similar topics of research. As such, even the most dissimilar patterns retain some conceptual similarity.

7 CONCLUSION AND PERSPECTIVES

In this paper, we strengthen the existing bridges between formal concept analysis/closed pattern mining and real-world structural data. We show that beyond graphs, these methods can be adapted to streams of interactions, in order to mine relevant patterns from large real-world such sequences. After recalling the notion of core
of a graph, we define two such cores for stream graphs and show that they exhibit the necessary properties for closed pattern enumeration. A strength of our approach is that we do not challenge the core assumptions made by previous work, allowing for little conceptual modifications algorithms from past work. It opens the way to concurrent mining of structural data of different natures, such as a stream graph and a graph, for example.

We run experiments on two datasets, one of social, online and offline interactions between students and another based on a Web anthology of citations between scientific papers in computational linguistics. In both cases, we mine interesting patterns, and show that post-enumeration pattern set selection allows us to identify dissimilar patterns.

One interesting aspect of this work is the perspectives it opens, some of which we briefly detail now. We have shown that degrees and properties around degrees offer a good trade-off between expressive power and computational efficiency; however, these properties rely on being monotone, which limits our possibilities. Being able to extend the scope of the theoretical framework to convex core properties would be an important progress.

This work relies on the enumeration of closed patterns to do further selection, even though only a fraction of the enumerated patterns is of interest for a typical application. Even if we only compute a spanning tree over the concept lattice, being able to only explore sub-areas of interest is highly sought after. This has recently been done for graphs, using local modularity [1]; there is no consensual definition of modularity for stream graphs and their variants, making this improvement an open problem.

Application-wise, an interesting direction is the use of closed patterns to provide elements of explanation, for example as a complement to recommender systems. One could, given a set of closed patterns and a predicted link (typically, a link between a user and a book), use the set of closed patterns related to the user or the book to provide arguments justifying the prediction. This would allow to tap into the growing number of resources around knowledge representation.

REFERENCES

[1] Martin Atzmueller, Stefan Bloemheuvel, and Benjamin Kloeper. 2019. A Framework for Human-Centered Exploration of Complex Event Log Graphs. In International Conference on Discovery Science. Springer, 335–350.
[2] Martin Atzmueller, Stephan Gunnewink, and Albrecht Zimmermann. 2021. Mining communities and their descriptions on attributed graphs: a survey. Data Mining and Knowledge Discovery (2021). https://doi.org/10.1007/s10618-021-00741-z
[3] Vladimir Batagelj and Matjaz Zaversnik. 2011. Fast algorithms for determining cores in social networks. Adv. Data Analysis and Classification 5, 2 (2011), 129–145.
[4] Steven Bird, Robert Dale, Bonnie J. Dorr, Bryan R. Gibson, Mark Thomas Joseph, Min-Yen Kan, Dongwon Lee, Brett Powley, Dragomir R. Radev, and Yee Fan Tan. 2008. The ACL Anthology Reference Corpus: A Reference Dataset for Bibliographic Research in Computational Linguistics. In LREC: European Language Resources Association.
[5] Nicolas Braud-Santoni, Swan Dubois, Mohamed-Hamza Kaouachi, and Franck Petit. 2016. The next 700 impossibility results in time-varying graphs. Interna- tional Journal of Networking and Computing 6, 1 (2016), 27–41.
[6] Björn Bringmann and Albrecht Zimmermann. 2009. One in a million: picking the right patterns. Knowl. Inf. Syst. 18 (2009). https://doi.org/10.1007/s10115-008-0136-4
[7] Rajmonda Sulo Caceres and Tanya Berger-Wolf. 2013. Temporal scale of dynamic networks. In Temporal networks. Springer, 65–94.
[8] Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, and Nicola Santoro. 2012. Time-varying graphs and dynamic networks. International Journal of Parallel, Emergent and Distributed Systems 27, 5 (2012), 387–408.
[9] Nicolas Gensollen and Matthieu Latapy. 2020. Do you trade with your friends or become friends with your trading partners? A case study in the [Formula omitted] cryptocurrency. Applied Network Science 5, 1 (2020), NA-NA.
[10] Betsy George and Sangho Kim. 2013. Time Aggregated Graph: A Model for Spatio-temporal Networks. In Spatio-temporal Networks. Springer, 7–24.
[11] Petter Holme. 2015. Modern temporal network theory: a colloquium. The European Physical Journal B 88, 9 (2015), 254.
[12] Petter Holme and Jari Saramäki. 2012. Temporal networks. Physics reports 519, 3 (2012), 97–125.
[13] Jon M Kleinberg. 1999. Authoritative sources in a hyperlinked environment. Journal of the ACM (JACM) 46, 5 (1999), 604–632.
[14] Matthieu Latapy, Tiphaine Viard, and Clémence Magnien. 2018. Stream graphs and link streams for the modeling of interactions over time. Social Network Analysis and Mining 8, 1 (2018), 61.
[15] Yannick Léo, Christophe Crespelle, and Eric Fleury. 2019. Non-altering time scales for aggregation of dynamic networks into series of graphs. Computer Networks 148 (2019), 108–119.
[16] Rossana Mastrandrea, Julie Fournet, and Alain Barral. 2015. Contact Patterns in a High School: A Comparison between Data Collected Using Wearable Sensors, Contact Diaries and Friendship Surveys. PLOS ONE (2015).
[17] Pierre-Nicolas Mougel, Christophe Rigotti, and Olivier Gondried. 2012. Finding Collections of k-Clue Percolated Components in Attributed Graphs. In PAKDD 2012, Kuala Lumpur (Lecture Notes in Computer Science). Vol. 7302, 181–192.
[18] Benjamin Negrevergne, Alexandre Terrnier, Marie-Christine Rousset, and Jean-François Méhaut. 2014. Para miner: a generic pattern mining algorithm for multi-core architectures. Data Mining and Knowledge Discovery (2014).
[19] Alfi Abdellah Ouali, Albrecht Zimmermann, Samir Loudahi, Yahia Lebbah, Bruno Crémielle, Patrice Boizumault, and Lakhdar Loukili. 2017. Integer Linear Programming for Pattern Set Mining; with an Application to Tiling. In PAKDD 2017, Jeju, South Korea, May 25-26, 2017.
[20] Angelo Salatino, Thiviyan Thanapalasingam, Andrea Mannocci, Francesco Os- borne, and Enrico Motta. 2018. The Computer Science Ontology: A Large-Scale Taxonomy of Research Areas. In International Semantic Web Conference (2) (Lecture Notes in Computer Science). 187–205.
[21] Stephen B. Seidman. 1983. Network structure and minimum degree. Social Networks 5 (1983), 269–287.
[22] Arlei Silva, Wagner Meira, Jr., and Mohammed J. Zaki. 2012. Mining Attribute-structure Correlated Patterns in Large Attributed Graphs. Proc. VLDB Endow 5, 5 (Jan. 2012), 466–477.
[23] Henry Soldano and Guillaume Santini. 2014. Graph abstraction for closed pattern mining in attributed networks. In ECAI, Vol. 263.
[24] Henry Soldano, Guillaume Santini, and Dominique Bouthinon. 2017. Formal Concept Analysis of Attributed Networks. In Formal Concept Analysis in Social Network Analysis, Rokia Missaoui, Sergei Obiedkov, and Sergei Kuznetsos (Eds.). Springer, 143–170.
[25] Henry Soldano, Guillaume Santini, and Dominique Bouthinon. 2019. Attributed Graph Pattern Set Selection Under a Distance Constraint. In Complex Networks 7th edition, Lisbon, Portugal, December 10-12, 2019 (Studies in Computational Intelligence). Springer, 228–241.
[26] Henry Soldano, Guillaume Santini, Dominique Bouthinon, Sophie Bary, and Emmanuel Lazega. 2019. Bi-pattern mining of attributed networks. Applied Network Science 4, 1 (6 (2019), 37.
[27] Henry Soldano, Guillaume Santini, Dominique Bouthinon, and Emmanuel Lazega. 2017. Hub-Authority Cores and Attributed Directed Network Mining. In International Conference on Tools with Artificial Intelligence (ICTAI). IEEE Computer Society, Boston, MA, USA, 1120–1127.
[28] Tiphaine Viard, Matthieu Latapy, and Clémence Magnien. 2015. Revealing contact patterns among high-school students using maximal cliques in link streams. In ASONAM workshop DyNoIEEE, 1517–1522.
[29] Jilles Vreeken, Matthijs van Leeuwen, and Arno Siebes. 2011. Krimp: mining itemsets that compress. Data Mining and Knowledge Discovery 23 (2011).
[30] Rudolf Wille. 2009. Restructuring lattice theory: an approach based on hierarchies of concepts. In International Conference on Formal Concept Analysis. 77–95.
[31] Audrey Wilmet, Tiphaine Viard, Matthieu Latapy, and Robin Lamarche-Perrin. 2019. Outlier detection in IP traffic modelled as a link stream using the stability of degree distributions over time. Computer Networks 161 (2019), 197–209.
[32] Mohammed Javeed Zaki and Ching-Jui Hsuio. 2002. CHARM: An Efficient Algorithm for Closed Itemset Mining. In SDM. SIAM, 457–473.
[33] Stella Zevio, Guillaume Santini, Henry Soldano, Haïfa Zargayouna, and Thierry Charnois. 2020. A Combination of Semantic Annotation and Graph Mining for Expert Finding in Scholarly Data. In GEW workshop at ECAI PKDD.