A Stochastic Programming Method for Resources Allocation in Education of Higher Vocational College

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Abstract. A stochastic education resources management (SERM) model has been proposed to solve resource-allocation problems in education under uncertainty. The advantages of SERM are as follows: (1) it is capable for dealing with uncertainty which may exhibit random feature in the right-side of constraints; (2) the optimal education resource-allocation strategies for each higher vocational college under different constraint levels can be generated. The application in education resources sharing systems indicates that the optimal resource-allocation strategies generated by the SERM model can increase the total output of education value, which includes the personnel, technological and social outputs.

1. Introduction
The training objective of higher vocational colleges is professional talents who have a certain quality of technical skills. The sharing of education resources plays a key role in promoting the practice teaching reform and improving the quality of personnel training under the background of coordinated development for the Beijing-Tianjin-Hebei region. There are many interesting themes studied by the researchers about the sharing of resources in education[1,2]. Among them, how to allocate the education resources reasonable have received more concerns[3]. Uncertainties exist in some components in education resources systems which may exhibit random feature due to the data unavailability. Therefore, a stochastic programming method has been applied for tackling such uncertainties in education resources allocation.

2. Methodology
A general stochastic mathematical programming (SMP) can be formulated as follows:

Maximize \( f(X) \) \hspace{1cm} (1a)

Subject to
\[ A(t)X \leq B(t), \] \hspace{1cm} (1b)
\[ x_j \geq 0, x_j \in X, \quad j = 1, 2, \ldots, n. \] \hspace{1cm} (1c)

Where \( f(X) \) is the objective function, \( X \) is a vector of decision variables, \( A(t), B(t) \) and \( C(t) \) are random variables [4]. In model (1), parameters exhibit random feature. For dealing with this type of uncertainty, an ‘equivalent’ deterministic version of constraint (1b) can be defined by using a chance-constrained programming approach. The constraint (1b) can be transformed to [4]:
\[
\Pr\left[\{t|A_i(t)X \leq b_i(t)\}\right] \geq 1 - p_i, A_i(t) \in A(t), \quad i=1,2,\ldots,m.
\]  
(2)

Where \(p_i \in [0,1]\) is a certain level of probability for each constraint \(i\). It is required that the constraints be satisfied with at least a probability of \(1 - p_i\). However, constraint (2) is generally nonlinear. When coefficients in \(A\) is deterministic, constraint (2) becomes linear [4]:

\[
A_iX \leq b_i(t)^{(p_i)}, i=1,2,\ldots,m
\]

(3)

Where \(b_i(t)^{(p_i)} = F_i^{-1}(p_i)\), \(F_i(p_i)\) is the cumulative distribution function of \(b_i(t)\).

3. Application

3.1. Statement of Problems

In an education resources sharing system, the available education resources of higher vocational colleges in a region should be considered. The available education resources are allocated by decision makers in order to obtain the maximum net benefit, i.e. the output of educational value [3]. There are five higher vocational colleges in the study region. They can be represented by HVC1, HVC2, HVC3, HVC4, HVC5, respectively. The history records of each college can be obtained from the Quality Annual Report. Uncertainties exist in the education resources sharing system in a medium and long time period. For example, total material, financial and manpower resources are set with random elements. These uncertainties should be considered by decision makers to generate feasible solutions for allocation of resources in education. The education resource data was scaled to [0,1] for computational convenience [5].

\[
SD = [OD - (1-d)MN]/[(1+d)MX - (1-d)MN]
\]

(4)

Where \(SD\) and \(OD\) are scaled and original values, respectively; \(MX\) and \(MN\) are maximum and minimum values, respectively; \(d\) is the parameter in the original domain. Table 1 shows the scaled original data of resources for the studied higher vocational colleges. The total material, financial and manpower resources are supposed to be normally distributed.

### Table 1. Scaled value of resources for five higher vocational colleges

| Higher vocational college | Manpower resource | Material resource | Financial resource |
|---------------------------|-------------------|-------------------|-------------------|
| HVC1                      | 0.5680            | 0.2727            | 0.3358            |
| HVC2                      | 0.1227            | 0.0939            | 0.1447            |
| HVC3                      | 0.8414            | 0.0224            | 0.0161            |
| HVC4                      | 0.8070            | 0.2091            | 0.1581            |
| HVC5                      | 0.3482            | 0.9321            | 0.9378            |

3.2. Stochastic Education Resources Management (SERM) Model

For the study system, a stochastic education resources management (SERM) model will be developed. The decision variables represent manpower, material and financial resources. The objective is to maximum total output of educational value, and the constraints are a number of restrictions. The detailed SERM model is presented as follows:

\[
\text{Maximize } f = \sum_{i=1}^{m} \sum_{j=1}^{n} w_i A_j(\alpha_j, \beta_j, \gamma_j)
\]

(5a)

Subject to
\[
\sum_{j=1}^{n} a_j \leq \alpha_{\text{total}}(t)^{(p)} \\
\sum_{j=1}^{n} \beta_j \leq \beta_{\text{total}}(t)^{(p)} \\
\sum_{j=1}^{n} \gamma_j \leq \gamma_{\text{total}}(t)^{(p)} \\
\alpha_j \geq \alpha_{\min} \quad j = 1,2,\ldots,n \\
\beta_j \geq \beta_{\min} \quad j = 1,2,\ldots,n \\
\gamma_j \geq \gamma_{\min} \quad j = 1,2,\ldots,n
\]

(5b) \hspace{1cm} (5c) \hspace{1cm} (5d) \hspace{1cm} (5e) \hspace{1cm} (5f) \hspace{1cm} (5g)

Where \( f \) is the total output of educational value; \( w_i \) is the weight for output of educational value \( i \); \( i \) is the type of output of educational value. There are three types of output in this study. \( i = 1, i = 2 \) and \( i = 3 \) are the personnel, technological and social outputs of educational value, respectively; \( A_j(\alpha_j, \beta_j, \gamma_j) \) is the function of output of higher vocational college \( j \); \( \alpha_{\text{total}}(t) \), \( \beta_{\text{total}}(t) \) and \( \gamma_{\text{total}}(t) \) are total value of manpower, material and financial resources, respectively; \( \alpha_{\min}, \beta_{\min} \) and \( \gamma_{\min} \) are minimum value of manpower, material and financial resources, respectively; \( p \in [0,1] \) is a certain level of probability.

3.3. Result Analysis

The SERM model has been applied to the problem in education resources sharing system. Figure 1 provides the percentage increase of total output under \( p = 0.1 \). The total output of education value includes the personnel, technological and social outputs. The results show that (1) the colleges in the study region would have an increase in total output during the planning time period; (2) the largest percentage increase would occur in HVC4 with a value of 55.49%; (3) the least percentage increase would occur in HVC1 with a value of 5.86%.

![Figure 1. Percentage increase of total output under \( p = 0.1 \)](image-url)
Figure 2 provides the comparison between original and optimal allocations of education resources for each higher vocational college under $p = 0.1$. For HVC1, all resources should be increased during the planning time period. For HVC2, manpower and material resources should be increased and financial resource should be decreased. For HVC3, manpower and financial resources should be increased and material resource should be decreased. For HVC4, all resources should be decreased. For HVC5, manpower resource should be increased and other resources should be decreased.

Figure 2. Comparison between the original and optimal allocations of education resources under $p = 0.1$

Figure 3 provides the comparison of total social, technological and personnel outputs under $p = 0.1$, $p = 0.05$ and $p = 0.01$. It indicates that the value of probability levels for constraint would influence the solutions of the SERM model.

Figure 3. Comparison of output under various values of constraint levels
4. Concluding Remarks
In this study, a stochastic education resources management (SERM) model has been proposed for resources allocation in education under uncertainty. The proposed model can deal with uncertainty which may exhibit random feature. Moreover, the optimal resource-allocation strategies in education are generated in order to obtain the maximum outputs of education value. The application in education resources sharing systems indicates that (1) the SERM model can handle uncertainty in the right-side of constraints; (2) the higher vocational colleges in the study region would have an increase in total output during the planning time period; (3) the value of probability levels for constraint would influence the solutions of the SERM model.

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