Interaction of whole fiber and destroyed neighbour fiber in reinforced plastic

R Turusov¹,², E Bogachev³ and V Andreev¹

¹Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia
²FRC of Chemical Physics them. N.N. Semenova RAS, ul. Kosygin 4, 119991 Moscow, Russia
³Joint-stock company “Composite”, ul. Pioneer, 4, 141070 Korolev, Moscow Region, Russia

Abstract. Composite materials, both on the basis of polymeric binder, and other materials, for example, concrete are widely distributed. The paper considers the case of longitudinal location of elements. The paper deals with the question the intensity of the adhesive interaction, of course, with the true strength of the adhesive bond. To study the interaction of two adjacent fibers, whole and broken in work, it is proposed to use the concept of a contact layer [1–4]. To study the interaction two adjacent fibers, whole and torn, we propose use the concept of contact layer. This anisotropic layer has a certain thickness, equal to the length of the bonds, the shear modulus in the contact plane, and the Young’s modulus along the rod-bonds. In fact, it is an anisotropic continuous medium. When solving one-dimensional problems, the shear modulus and thickness of the contact layer are combined into one parameter, whose influence is decisive on the distribution pattern and stress intensity near the edge. This parameter can be determined quite easily from a macro experiment.

1. Introduction

Quite a number of works by various authors [2–7] are devoted to the edge effect near the rupture (end) of a single fiber in the composite. But the role of adhesive interaction was not practically studied in them, since adhesive interaction in these works was not characterized in any way, and by default it was assumed absolute. The latter means that only the fundamental requirements of the continuity of displacement and stress vectors, i.e. the requirements of maintaining continuity and the implementation of Newton's third law. In this case, inevitably, solutions are obtained in which the tangential stresses at the corner points (at the ends of the fiber) tend to infinity, i.e. the so-called singularity is manifested. The use of physically justified strength criteria in these conditions becomes almost impossible.

Works on the contact layer method in adhesive mechanics (initially called the boundary layer) began in the Soviet Union in the early seventies (for example, [8–15]) and continue now [1-3]. In the West, the first works on the characterization of adhesive contact appeared only in the nineties [16–22]. Basically, it is a shear spring model like the Winkler base.

2. State of the problem

Figure 1 shows the studied here scheme, when only one fiber is broken, while the surrounding matrix and the neighbouring fiber remain solid.
We assume that there is a contact layer between the fiber and the matrix [1-4]. The contact layer is represented as a set of elastic short rods - bonds. These bonds are perpendicular to the surfaces of the adhesive and the substrate. They are rare compared to the density of atoms on the surfaces being joined (approximately \( n = 10^{14} \text{ cm}^{-2} \)). That is they do not touch each other and therefore the stresses perpendicular to the rods are zero. This contact layer is characterized by two parameters: shear modulus, and thickness equal to the length of the bonds. As a result, the contact can be characterized by one parameter - the ratio. It characterizes the contact of the adhesive with the substrate and is called the intensity of the adhesive interaction or the stiffness of the contact layer [3]. Using the contact layer method, the following flat problem was solved in the article (Figure 1).

![Figure 1](image)

**Figure 1.** The central fiber is torn. Matrix and the neighboring fibers remain undivided; \( m \) – matrix, \( f \) – fiber.

Nearby are one broken fiber and whole fibers (Figure 2). The main question of the problem - how are distributed in this situation at the specified loading normal stresses in fibers & matrix and shear stresses at the matrix-fiber boundaries.

![Figure 2](image)

**Figure 2.** Flat one-dimensional model for the calculation of the stress-strain state. 0 - broken fiber, to the right (\( x = 1 \)) there is no load; matrix (1) and fiber (2) remain solid; contact layers are shaded.

We introduce the notation: \( d_0 = E_0 h_0 \), \( d_1 = E_1 h_1 \), \( d_2 = E_2 h_2 \). Here \( E_0, E_1, E_2 \) – Young’s modules, respectively, torn fiber, matrix and solid fiber, \( h_0, h_1, h_2 \) – their thickness.

For reduced dimensionless normal stresses (marked with an asterisk), we have equations [23]:

\[
\sigma_0^* = \sigma_{x0} \frac{bh_0}{P} = \frac{d_0}{4} \left( \frac{d_1}{d_0 + d_1} \left( \frac{sb(\omega_0 x)}{sh(\omega_0 l)} + \frac{ch(\omega_0 x)}{ch(\omega_0 l)} \right) + 4 \right) \left( \frac{sh(\omega_2 x)}{sh(\omega_2 l)} + \frac{ch(\omega_2 x)}{ch(\omega_2 l)} \right)
\]

(1)

\[
\sigma_1^* = \sigma_{x1} \frac{bh_1}{P} = \frac{d_1}{2h_1(d_0 + d_1)} \left( 2 + \frac{d_0}{d_0 + d_1} \left( \frac{sb(\omega_0 x)}{sh(\omega_0 l)} + \frac{ch(\omega_0 x)}{ch(\omega_0 l)} \right) \right)
\]

(2)
\[ \sigma_2^* = \sigma_{x2}^* \frac{bh_0}{P} = \frac{d_0}{4} \left[ \frac{1}{(d_0 + d_1)} \left( \frac{sh(\omega_1)}{sh(\omega_0)} + \frac{ch(\omega_1)}{ch(\omega_0)} \right) + 4 \right] + \frac{1}{(d_0 + d_1)} \left[ \frac{sh(\omega_2)}{sh(\omega_0)} + \frac{ch(\omega_2)}{ch(\omega_0)} \right]. \] (3)

For reduced but dimensional [1/m] shear stresses we have:
\[ \tau_1^* = \tau_1 \frac{b}{P} \] (4)
\[ \tau_2^* = \tau_2 \frac{b}{P} \] (5)

The results of calculations of the distribution of all reduced stresses by formulas (1) - (5) in model (Figure 1) with one broken fiber are presented in Figures 3 and 4. The following parameters were used as the baseline calculations: length \( l = 1 \text{mm} \); width \( b = 1 \text{mm} \); temperature differential \( \Delta T = 0 \); the intensity of the adhesive interaction (stiffness of the contact layer) \( G_s/h = 25 \cdot 10^6 \text{MPa/m} \); thickness of the rods: \( h_0 = h_2 = 10 \cdot 10^{-6} \text{mm} \); matrix thickness \( h_1 = 2.9 \cdot 10^{-3} \text{mm} \); Young’s moduli of bars 0 and 2 \( E_0 = E_2 \approx E_{\text{glass}} = 2 \cdot 10^9 \text{MPa} \); Young’s modulus of the matrix \( E_1 \approx E_{\text{pol}} \approx 3 \cdot 10^8 \text{MPa} \).

3. Calculation results and discussion
We write the boundary conditions for the desired reduced functions.

The boundary conditions for normal stresses in the model are:
\[ \sigma_0^*(l) = 0; \quad \sigma_0^*(-l) = \sigma_0 \frac{bh_0}{P} = \frac{E_0 h_0}{2E_0 h_0 + E_1 h_1}; \]
\[ \sigma_1^*(l) = \frac{E_1 h_1}{E_0 h_0 + E_1 h_1} \sigma_1 \frac{bh_1}{P} = \frac{E_1 h_1}{2E_0 h_0 + E_1 h_1}; \]
\[ \sigma_2^*(l) = \frac{E_0 h_0}{E_0 h_0 + E_1 h_1} \sigma_2 \frac{bh_0}{P} = \frac{E_0}{2E_0 h_0 + E_1 h_1}. \] (6)

Boundary values for reduced shear stresses (dimension 1/m) in the model (Figure 1) with \( x = 1 \), i.e. near broken fiber 0:
\[ \tau_{1,1}^*(l) = \frac{2d_0}{4(d_0 + d_1)} \left[ \omega_1 \frac{d_1}{(2d_0 + d_1)} + \omega_2 \right] = \frac{E_0 h_0}{E_0 h_0 + E_1 h_1} \frac{1}{2} \sqrt{\frac{G^*}{h^*}} \frac{1}{E_0 h_0} \left( 1 + \frac{E_1 h_1}{2E_0 h_0 + E_1 h_1} \right); \]
\[ \tau_{2,2}^*(l) = \frac{E_0 h_0}{E_0 h_0 + E_1 h_1} \frac{1}{2} \sqrt{\frac{G^*}{h^*}} \frac{1}{E_0 h_0} \left( 1 - \frac{E_1 h_1}{2E_0 h_0 + E_1 h_1} \right); \] (7)

Let us estimate the boundaries of change of the functions (6) and (7) when the stiffness parameters change.

1. The stiffness of the matrix is much less than the stiffness of the fibers \( E_0 h_0 >> E_1 h_1 \), then:
\[ \tau_{1,1}^*(l) = \frac{\omega_2}{2} = \frac{1}{2} \sqrt{\frac{G^*}{h^*}} \frac{1}{E_0 h_0}; \quad \tau_{2,1}^*(l) = \frac{\omega_2}{2}. \]

2. If \( E_1 h_1 > E_1 h_1 \), then:
\[ \tau_{1,1}^*(l) = \frac{\omega_2}{4} \left( 1 + \sqrt{\frac{E_1 h_1}{E_0 h_0}} \right); \quad \tau_{2,1}^*(l) = \frac{\omega_2}{4} \left( 1 - \sqrt{\frac{E_1 h_1}{E_0 h_0}} \right). \]

3. If \( E_1 h_1 << E_1 h_1 \), then:
\[ \tau_{1,1}^*(l) = \frac{1}{E_1 h_1} \sqrt{\frac{G^*}{h^*}} \frac{1}{E_0 h_0}; \quad \tau_{2,1}^*(l) = \left( \frac{E_0 h_0}{E_1 h_1} \right)^2 \frac{1}{2} \sqrt{\frac{G^*}{h^*}} \frac{1}{E_0 h_0}. \]
Figures 3 and 4 also show curves in which the comparison with the base is reduced by one order of intensity adhesive interaction (hardness of the contact layer) \( G^*/h^* = 25 \times 10^5 \text{ MPa/m} \).

Calculations on the basic parameters of the normal stress given are shown in Figure 3. There they are represented by solid curves 0, 1, 2. Curve 2 - reflects the change in stresses in the whole fiber, i.e. their decrease with distance from the point of broken fiber "0". The solid curve 0 represents how effectively the broken fiber “0” is included in the full-fledged work. It can be seen from the figure that this process of transferring forces from whole fiber to broken fiber can be considered practically complete at a distance of 0.00025m (20-25\( h_0 \) – thickness of the rods or nominal diameters \( D_f = h_0 \) of fibers). The solid curve 1 at the bottom of the graph corresponds to the unloading of the matrix. Its unloading, as can be seen, is completed almost within five to seven diameters.

![Figure 3. Fiber 0 is broken. Fiber 2 and matrix 1 are solid. The distribution along the length of the normal stresses in the elements of the system near the point of broken fiber 0. The solid curve is the basic variant, where the intensity of adhesive interaction \( G^*/h^* = 25 \times 10^5 \text{ MPa/m} \) and the Young’s modulus of the fibers \( E_0 = E_2 \cong E_{\text{glass}} = 2 \times 10^4 \text{ MPa} \). Dotted curve - option 1, in which \( G^*/h^* = 25 \times 10^5 \text{ MPa/m} \).

The next series of dashed curves 0-2 in Fig. 3 reflects the change in normal stresses in the same elements of the model, but differs from the base series only in lower stiffness of the contact layer \( G^*/h^* = 25 \times 10^5 \text{ MPa/m} \) which here represents the intensity of adhesive interaction of the adhesive material (i.e. matrix) with substrate material (fiber). And here it can be seen that the equality of the stresses in the whole and broken fibers is not achieved even at a distance of 0.0011m from the edge of the broken fibers. This means that “equality” comes much later than in the basic version. According to the calculations it is achieved at a distance of 250 fiber diameters from place of rupture fiber 0. As a result, whole fiber is congested at a much greater length compared to the base case.
Figure 4. Distribution of reduced shear stresses along the length of the boundary near the place of rupture fiber 0. The solid curve - basic version, where the adhesive strength of interaction $G^* / h^* = 25 \cdot 10^6 \text{ MPa/m}$ and Young’s modulus of the fibers $E_0 = E_2 \equiv E_{\text{glass}} = 2 \cdot 10^4 \text{ MPa}$. Dotted curve - option 1, in which $G^* / h^* = 25 \cdot 10^5 \text{ MPa/m}$.

Figure 4 shows the curves of the reduced but dimensional (1/m) shear stresses on the distance from the point of fiber break “0”. Solid curves 1 and 2 correspond to the desired tangential stresses of the base case. Curves numbers correspond: 1 are the tangential stresses $\tau_1^*$ between the broken fiber 0 and the matrix 1; 2 are the tangential stresses $\tau_2^*$ between the solid fiber 2 and the whole matrix 1. The curves indicate a high concentration of tangential stresses in the model.

The width of the zone of concentration of tangential stresses, as can be seen from the comparison, is substantially greater than the width of the zone of concentration of normal stresses in fibers 0 and 2 (Fig. 4, solid curves 0 and 2). The narrow zone of concentration is compensated by high values of the tangential stresses at the maximum. And then the question arises about the strength of the adhesive bond of the fiber with the matrix, and after that, the question of the compromise between the strength of the fiber, the intensity of the adhesive interaction and the strength of the adhesive bond. Indeed, it is known that the strength of the fiber depends on its length - the shorter the fiber, the higher its tensile strength. This means, for example, that the lower the intensity of adhesive interaction, the longer the whole fiber next to the broken one will be overloaded compared to the rest of the working long-haul fibers. There is a danger that this overloaded state will coincide with the strength of the fiber at a certain length. Then the destruction of the fiber into short sections may begin [1, 2].

The tangential stress distribution curves look completely different when the intensity of the adhesive interaction $G^* / h^*$ (dashed curves) is substantially less than the baseline - by an order of magnitude. The maximum stresses are almost an order of magnitude smaller than the maximum stresses of the base case. And the width of the zone of concentration is more than an order of magnitude greater than that in the base case. But it should be noted that the shear stresses $\tau_1^*$ between the broken fiber 0 and the matrix are the greatest, and the maximum in the distribution of shear stresses along the fiber length is near the point of break.

The maxima for reduced shear stresses (1/m) in the model with $x=l$, i.e. near the point of fiber break 0.

$$
\tau_1^*(l) = \frac{E_{h_0} - E_{h_1}}{E_{h_0} + E_{h_1}} \left[ \frac{\alpha_0}{(2d_0 + d_1)} + \frac{d_1}{(2d_0 + d_1)} \right] = \frac{E_{h_0} - E_{h_1}}{E_{h_0} + E_{h_1}} \left[ \frac{1}{\sqrt{2\frac{E_{h_0}}{E_{h_1}}}} \left( 1 + \frac{E_{h_1}}{E_{h_0}} \right) \right] \left( \frac{G^*}{h^*} \right); \\
\tau_2^*(l) = \frac{E_{h_0} - E_{h_1}}{E_{h_0} + E_{h_1}} \left[ \frac{1}{\sqrt{2\frac{E_{h_0}}{E_{h_1}}}} \left( 1 - \frac{E_{h_1}}{E_{h_0}} \right) \right] \left( \frac{G^*}{h^*} \right).$$

If in the formulas (8) to maximum shear stress at the boundary of the fiber-matrix to require that the maximum shear stresses are less or equal to the strength of the adhesive bond in shear ($\tau_{ad}$): $\tau_{\text{max}} \leq \tau_{ad}$ then after simple transformations you can get
Here $v_m$ and $v_f$ are the relative volume contents of the matrix and fiber; $D_f$ is the fiber diameter. After substituting the average fiber strength $\sigma_b$ instead of $\sigma_f$ into (9), we obtain from it the relation connecting the fiber strength, adhesive bond strength, stiffness of fiber and matrix, as well as the intensity of adhesive interaction. In essence, this is a solidity condition for reinforced materials.

$$\tau_{ad} \geq 2\sigma_b \left(1 + \frac{v_m}{v_f}\right) \left(1 + \frac{E_m}{E_f}\right) \left(1 + \frac{G^* f}{\alpha^* E_f}\right) \sqrt{\frac{D_f}{h^*}} \sqrt{\frac{1 - v_f^*}{1 - v_f^* + 2v_m^*}}.$$  \hspace{1cm} (10)

Relation (10) can be simplified for different ratios of composite parameters. For example, for today's high-strength fibers and their contents in the composite, a simple ratio is obtained:

$$\tau_{ad} \geq 2\sigma_b \frac{G^* f}{h^*} \frac{D_f}{E_f}.$$  \hspace{1cm} (11)

As a result, the approximate formula lacks the rigidity and strength of the matrix, which is possibly a consequence of that in the approximation, the rigidity of the matrix is small, and its strength in the model was not taken into account.

4. Conclusion

Based on the results obtained in the work, it can be stated that the contact (adhesive) interaction of the adhesive with the substrate (here - matrices with fiber) must be characterized. To characterize the interaction, the concept of a contact layer is used. Its characteristics are described at the beginning of the article. As a result of mathematical modeling of the force interaction of a torn fiber with a single adjacent fiber, a formula is obtained that reflects the conditions of monolithicity for a reinforced material.

Acknowledgments

This study was performed with the financial support of the RF Ministry of Education and Science, President Grant #NSh-3492.2018.8.

References

[1] Freydin A and Turusov R 1990 Properties and Calculation of Adhesive Joints (Moscow: Khimiya) p 255
[2] Turusov R 2015 Adhesion Mechanics (Moscow: MGSU Press) p 230
[3] Turusov R and Manevich L 2010 Contact layer method in adhesive mechanics Polymer Science D 31 p 11-9
[4] Andreev V, Turusov R and Tsybin N 2016 Determination of Stress-Strain State of a Three-Layer Beam with Application of Contact Layer Method Mechanics of Composite Materials Apr 212-19
[5] Turusov R 2014 Elastic and temperature behavior of a layered structure Part I Experiment and theory Mechanics of Composite Materials 50 6 pp 1119-30
[6] Rabinovich A 1970 Introduction into the Mechanics of Reinforced Polymers (Moscow: Nauka) p 482
[7] Rosen B 1970 Strength of Uniaxial Fibrous Composites (Pergamon Press) p 694
[8] Manevich L and Pavlenko A 1982 On taking into account the structural heterogeneity of the composite in assessing adhesive strength Appl. mech. and tech. physics. Sib. Department of the USSR Academy of Sciences 3 pp140-5
[9] Salganik R and Malyshev B 1965 Application of crack theory to determine the strength of brittleness of glues Reports of USSR Academy of Sciences 160 1 pp 91-3
[10] Turusov R, Vuba K and Freidin A 1972 Investigation of the influence of temperature and humidity factors on the strength and deformation properties of adhesive joints of wood with steel reinforcement *Proceedings TSNIISK behalf of VA Kucherenko* 24 pp 86-124

[11] Turusov R, Nikishin A and Ivanova-Mumzhieva V 1973 Some tasks associated with determining adhesive strength *International Symposium "Polymers-73" Reports (Bulgaria, Varna)* pp 198-203

[12] Turusov R, Nikishin A and Gorbatkina Yu 1973 The boundary layer method in the problems of mechanics of a solid deformable body *Int. Symposium "Polymers-73" Reports (Bulgaria, Varna)* pp 229-33

[13] Turusov R, Sakvarelidze Zh, Malinsky Yu and Vuba K 1974 Investigation of the mechanism of destruction of reinforced plastics at normal and elevated temperatures "Reinforced plastics - 74", *Reports (Czechoslovakia, Karlovy Vary)* pp 97-103

[14] Turusov R, Sakvarelidze Zh and Malinsky Yu 1975 Stretching of composite rods taking into account bending as a model of adhesive joints *Proceedings TSNIISK behalf V.A. Kucherenko* 53 pp 72-80

[15] Turusov R and Vuba K 1978 The role of heterogeneous stress state in assessing the strength of adhesion joint models. *In the book Strength Physics of Composite Materials* (Leningrad: Institute of Physics) pp 75-84

[16] Turusov R and Vuba K 1979 The stress state and features of the assessment of the strength of adhesive joints in shear *Physics and chemistry of materials processing* 5 pp 87-94

[17] Turusov R and Vuba K 1980 The stress state and features of the assessment of the strength of adhesive joints in separation *Physics and chemistry of materials processing* 2 108-15

[18] Hashin Z 1991 Thermoelastic properties of particulate composites with imperfect interface *J. Mech. Phys. Solids* 39 745–62

[19] Hashin, Z 1991 The spherical inclusion with imperfect interface *J. Appl. Mech.* 58 pp 444–49

[20] Hashin Z 1992 Extremum principles for elastic heterogeneous media with imperfect interface and their application to bounding of effective elastic moduli *J. Mech. Phys. Solids* 40 pp 767–81

[21] Lipton R and Vernescu B 1995 Variational methods, size effects and extremal microgeometries for elastic composites with imperfect interface *Math. Models Meth. Appl. Sci.* 5 pp 1139–73

[22] Benveniste Y and Miloh T 2001 Imperfect soft and stiff interfaces in two-dimensional elasticity *Mech. Mater.* 33 pp 309–24

[23] Turusov R, Bogachev E and Elakov A 2016 The role of the intensity of adhesive interaction and stiffness of the matrix in the transfer of forces from a single fiber to a tear in a fiber composite and in the realization of the strength of reinforcing fibers. Part I. *Mechanics of composite materials and structures* 22 3 pp 430-51