Compton scattering from the proton at NLO in the chiral expansion

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Abstract

We present calculations of differential cross sections for Compton scattering from the proton, using amplitudes calculated to fourth order in heavy baryon chiral perturbation theory. We compare with available data up to 200 MeV. We find that the agreement for angles below 90° is acceptable over the whole energy range, but that at more backward angles the agreement decreases above about 100 MeV, and fails completely above the photoproduction threshold.

12.39Fe 13.60Fz 11.30Rd

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I. INTRODUCTION

Heavy baryon chiral perturbation theory (HBCPT) is a systematic framework in which to describe the interactions of nucleons with pions and photons. Respecting gauge and Lorentz invariance, it reproduces low-energy theorems, which have been known since the 50’s and 60’s, as the first terms in chiral expansions, and in a number of cases also makes further parameter-free predictions. One of the simplest and cleanest processes to which it can be applied, theoretically at least, is Compton scattering, and indeed this was first studied in the very early days of the theory. First spin-independent scattering was considered. At lowest order (technically second order in HBCPT) the scattering amplitude is simply the energy-independent Thomson term, which is entirely independent of all details of the proton’s structure. This arises simply as a seagull term in the Lagrangian, one of the terms that arise when antinucleons are integrated out of the lowest-order relativistic Lagrangian. Energy and angle dependence in the amplitude enter at third order, where the only diagrams which contribute are one-pion loop graphs. The next lowest terms in an energy expansion of the amplitude (of order $\omega^2$) have coefficient which are called the polarisabilities, $\alpha$ and $\beta$, and at this order in HBCPT they are simply predicted in terms of well-known nucleon and pion properties. The numerical values, $\alpha = 12.5 \times 10^{-4}$ fm$^3$ and $\beta = \alpha/10$, are in remarkably good agreement with the current world averages for the proton, $\alpha = 12.1\pm0.8\pm0.5\times10^{-4}$ fm$^3$ and $\alpha + \beta = 14.1 \pm 0.5 \times 10^{-4}$ fm$^3$. This was an early triumph for HBCPT.

The calculation of the forward spin-independent scattering amplitude to fourth order followed quickly. However at that order there is no further predictive power for the polarisabilities within the strict framework of the theory, as there are four unknown low-energy constants in the fourth-order Lagrangian which give contributions to the isoscalar and isovector $\alpha$ and $\beta$. Also, the amplitude is not easily compared with experiment on its own.

The next step was the calculation of the full spin-dependent amplitude to third order. There are four independent amplitudes, each with a leading term (of order $\omega$) which is given by a low-energy theorem. The next terms, of $\mathcal{O}(\omega^3)$, have coefficients, denoted $\gamma_i$, which are called spin polarisabilities. Unfortunately these are not well known experimentally; indeed till recently only the contribution which enters forward scattering, called $\gamma_0$, was known at all, and that only from pion photoproduction data rather than direct Compton scattering experiments. More recently $\gamma_\pi$ (for backward scattering) has been extracted from unpolarised Compton scattering experiments, with conflicting results. The first direct measurements of polarised photon-proton scattering at MAMI yield a more reliable value $\gamma_0$ of $-0.8 \pm 0.1 \times 10^{-4}$ fm$^4$ which is within the range of previous estimates. In addition pion photoproduction data has been used as input into fixed-$t$ dispersion relations to obtain estimates of the $\gamma_i$ individually. Unfortunately the lowest order HBCPT values are in total disagreement with the experimental numbers, the only saving point of agreement being that the isovector quantities, which vanish in LO HBCPT, do seem to be smaller in general than the isoscalar ones.

Most recently, three groups have calculated the spin polarisabilities to fourth order (NLO). Unlike the spin-independent case, this order contains no LEC’s, and so there are again genuine predictions. Surprisingly the NLO contributions are as large as the LO ones. The agreement with the experimental numbers however is variable.
Of course, classic HBCPT has no explicit $\Delta$. This is assumed to have been integrated out, and its legacy is in the LEC's of the nucleon-only theory. It is seen first at different orders in different processes: second order for pion scattering, fourth order for spin-independent and fifth order for spin dependent Compton scattering. It is expected to be important; it may improve the agreement of the spin polarisabilities, but risks spoiling the spin-independent ones.

All this emphasis on the polarisabilities obscures the fact the HBCPT does in fact give complete amplitudes for scattering, which one might hope would be believable up to energies of the order of the pion mass, since the chiral expansion is both in powers of $\omega/\Lambda$ and $M_\pi/\Lambda$ (where $\Lambda$ is either $m_N$ or the chiral scale $4\pi f_\pi$). When the full third-order amplitudes were published, it was claimed that they gave good agreement with cross section data for unpolarised scattering up to the photoproduction threshold. However a more wide-ranging study was done by Babusci et al. \[20\], who showed that the agreement started to breakdown for backward angles above about 120 MeV. They stopped their comparisons at the threshold.

We have now calculated the full scattering amplitude to fourth order in HBCPT. Since the fourth-order contributions to the spin polarisabilities are so large, it is clearly of interest to see whether the full amplitudes and cross sections are also changed dramatically, and this is the subject of this paper. We have compared with the available data up to 200 MeV \[21,23-26\], and find that in fact the difference between third- and fourth-order cross sections is small. Above the threshold the good agreement at forward angles continues, while the lack of agreement at backward angles is dramatically worse.
II. THE CROSS SECTION IN HBCPT

The usual notation for the scattering amplitude in the Breit frame is, for incoming real photons of energy $\omega$ and momentum $\vec{q}$ and outgoing real photons of the same energy and momentum $\vec{q}'$,

$$T = \epsilon'^\mu \Theta_{\mu\nu} \epsilon'^\nu = \epsilon' \cdot \epsilon A_1(\omega, \theta) + \epsilon' \cdot \hat{\epsilon} \epsilon \cdot \hat{\epsilon}' A_2(\omega, \theta) + i \sigma \cdot (\epsilon' \times \epsilon) A_3(\omega, \theta) + i \sigma \cdot (\hat{\epsilon}' \times \hat{\epsilon}) \epsilon' \cdot \epsilon A_4(\omega, \theta) + \left(i \sigma \cdot (\epsilon' \times \hat{\epsilon}) \epsilon \cdot \hat{\epsilon}' - i \sigma \cdot (\epsilon \times \hat{\epsilon}) \epsilon' \cdot \hat{\epsilon}ight) A_5(\omega, \theta) + \left(i \sigma \cdot (\epsilon' \times \hat{\epsilon}') \epsilon \cdot \hat{\epsilon}' - i \sigma \cdot (\epsilon \times \hat{\epsilon}) \epsilon' \cdot \hat{\epsilon}ight) A_6(\omega, \theta), \tag{1}$$

where hats indicate unit vectors. By crossing symmetry the functions $A_i$ are even in $\omega$ for $i = 1, 2$ and odd for $i = 3 - 6$. The amplitudes may be decomposed into pole and non-pole pieces. The pole, which occurs at unphysical values of $\omega$ and $t (= 2\omega^2(\cos \theta - 1))$, arises from Born diagrams in which the intermediate nucleon is on shell, and the contribution containing
this pole can be calculated using Dirac nucleons and on-shell photon-nucleon couplings. The leading terms are given by low-energy theorems [6]. With the pole terms truncated at fourth order (that is, at $\mathcal{O}(1/m_N^3)$), the amplitudes read

$$A_1(\omega, \theta) = -\frac{Q^2 e^2}{m_N} + \frac{e^2}{4m_N^3}((Q + \kappa)^2(1 + \cos \theta) - Q^2)(1 - \cos \theta)\omega^2 + 4\pi(\alpha + \cos \theta \beta)\omega^2 + \mathcal{O}(\omega^4)$$

$$A_2(\omega, \theta) = \frac{e^2}{4m_N^3}\kappa(2Q + \kappa)\cos \theta\omega^2 - 4\pi\beta\omega^2 + \mathcal{O}(\omega^4)$$

$$A_3(\omega, \theta) = \frac{e^2}{2m_N^2}\omega\left(Q(Q + 2\kappa) - (Q + \kappa)^2\cos \theta\right) + A_3^{\pi^0} + 4\pi\omega^3(\gamma_1 + \gamma_5\cos \theta) + \mathcal{O}(\omega^5)$$

$$A_4(\omega, \theta) = \frac{e^2}{2m_N^2}\omega(\kappa^2 + A_4^{\pi^0} + 4\pi\omega^3\gamma_2 + \mathcal{O}(\omega^5)$$

$$A_5(\omega, \theta) = \frac{e^2}{2m_N^2}\omega(\kappa^2 + A_5^{\pi^0} + 4\pi\omega^3\gamma_4 + \mathcal{O}(\omega^5)$$

$$A_6(\omega, \theta) = -\frac{e^2}{2m_N^2}\omega\kappa(Q + \kappa) + A_6^{\pi^0} + 4\pi\omega^3\gamma_3 + \mathcal{O}(\omega^5)$$

(2)

where the charge of the nucleon is $Q = (1 + \tau_3)/2$ and its anomalous magnetic moment is $\kappa = (\kappa_s + \kappa_v\tau_3)/2$. The polarisability terms are the leading terms in the expansion of the full non-pole amplitudes; the polarisabilities are isospin dependent. The contribution from the $\pi^0$ graph, $A_i^{\pi^0}$, has been separated out, though its contribution is often included in

Fig. 3: Predictions for various CM angles. See Fig. 1 for legend.
the definition of the polarisabilities. Only four of the spin polarisabilities are independent since three are related by $\gamma_5 + \gamma_2 + 2\gamma_4 = 0$. The spin-dependent amplitudes have no pole contributions beyond the leading ones at this order. (At fifth order there are pieces of order $\omega^3/m_N^4$, though these vanish for both forward and backward scattering amplitudes.) The amplitudes are given for a nucleon spinor normalisation of $\bar{u}u = 1$.

To calculate these amplitudes in HBCPT, we need the all the diagrams which appear at fourth order. (The pion-loop diagrams are shown in Fig. 1.) There is no clear separation at a diagramatic level between pole and non-pole; heavy-baryon seagulls give pieces of the Dirac pole but, at fourth order, they also contribute to $\alpha$ and $\beta$. Conversely fourth-order loop diagrams have contributions at $O(\omega)$ which correct the bare anomalous magnetic moment in the LET terms. However, taking all diagrams together HBCPT reproduces the pole terms of Eq. 4, with the physical (as opposed to bare) nucleon mass and magnetic moment, as an expansion in powers of $1/mN$. Everything else is non-pole. (While this separation into pole and non-pole is useful in discussion, nothing in this paper depends on them. We simply calculate the full amplitudes to a given order and use them to produce cross sections. This is not affected by the divergence of opinion that is found in the literature over the definition of the polarisabilities [28].)

For the spin-dependent amplitudes, no LEC’s enter except well-known nucleon and pion properties. We take $g_A = 1.267$, $f_\pi = 92.32$, $\kappa_s = -0.120$, $\kappa_v = 3.706$ and $M_\pi = 139.6$ (as the loops are of charged pions in the main). However in the spin-independent amplitudes two sets of LEC’s enter. One set is the pair of LEC’s which contribute to $\alpha$ and $\beta$; these have been fixed by requiring the full polarisabilities (with third and fourth order contributions) to take the experimental values. The other set are the parameters $c_i$ from $\mathcal{L}^{(2)}$, which enter in the tadpole graphs. These are rather poorly known; furthermore they have been determined from pion-nucleon scattering at second, third and fourth order with quite different results (and indeed from different data sets with different results). Clearly the second-order determinations are the most appropriate to use in these calculations as vertex dressing is absent at this order. We have used $c_1 = -0.81$ GeV, $c_2 = 2.5$ GeV and $c_3 = -3.8$ GeV [24].

**Fig. 4:** Effect of different handling of third-order predictions. Solid: prediction taken as referring to Breit frame and variable change performed to reproduce the physical value of the threshold; dashed: as solid, but no variable change; dotted: predictions taken as referring to the centre of mass frame, as in ref. [20].
Fig. 5: Effect of altering the spin polarisabilities by hand. Solid line: 4th order HBCPT (as in previous figures); long dashes, short dashes and dots: polarisabilities taken from the DR analyses of refs [13,15,14] respectively.

Sensitivity to these choices is explored later.

All our amplitudes are calculated in the Breit frame. At third order, the result is the same in any frame (so long as the energies remain low); the amplitudes could be in the Breit frame, or the centre-of-mass frame, or even the lab frame (since $\omega - \omega'$ is of higher chiral order). At fourth order however the result will be different in different frames. Interestingly there is still no dependence on $\omega - \omega'$, but there will be expressions involving the average nucleon three-momentum $\mathbf{p}$. In the Breit frame $\mathbf{p}$ vanishes, so the amplitudes take on by far their simplest form. In the centre-of-mass frame $\mathbf{p}$ can be written in terms of the photon momenta, and the structures which enter are unchanged, but the amplitudes will have terms with the “wrong” symmetry in $\omega$. (These of course can be obtained from a boost of lower order amplitudes, and are not independent.) The differential cross section in any frame is a kinematic prefactor $\Phi^2$ times the invariant $|T|^2$, which in terms of the Breit-frame amplitudes is given by

\begin{equation}
|T|^2 = \frac{1}{2} A_1^2 (1 + \cos^2 \theta) + \frac{1}{2} A_2^2 (3 - \cos^2 \theta) + \omega^2 \sin^2 \theta [4 A_3 A_6 + (A_3 A_4 + 2A_3 A_5 - A_1 A_2) \cos \theta] + \omega^4 \sin^2 \theta [\frac{1}{2} A_2^2 \sin^2 \theta + \frac{1}{2} A_3^2 (1 + \cos^2 \theta) + A_4^2 (1 + 2 \cos^2 \theta) + 3 A_6^2] + 2 A_6 (A_4 + 3 A_5) \cos \theta + 2 A_4 A_5 \cos^2 \theta \tag{3}
\end{equation}

Above the threshold one should replace $A_i^2$ by $|A_i|^2$ and $A_i A_j$ by $\Re(A_i^* A_j)$.

Experimental data, though taken in the lab frame, are sometimes presented as centre-of-mass frame differential cross sections, so both prefactors are needed:

$$\Phi_{\text{lab}} = \frac{1}{4\pi} \frac{\omega'}{\omega}; \quad \Phi_{\text{cm}} = \frac{1}{4\pi} \frac{m_N}{\sqrt{s}}.$$ \tag{4}

There is a further complication which stems from the heavy baryon reduction. Physically, there is a threshold at a lab energy of $\omega_{\text{th}} = M_{\pi}(1 + M_{\pi}/2m_N)$. However the HBCPT amplitudes at third order have cusps at $\omega = M_{\pi}$, and at fourth order they are actually singular at this point. The resolution was given by Bernard et al. when discussing forward scattering [4]: if calculated to all orders, the sum of all the singular terms will be precisely
what is required to shift the threshold to the physical value. Thus at finite order, to have
the threshold at the right point and maintain finite amplitudes, one changes variables from
the Breit frame energy $\omega$ to $\zeta \equiv \omega/\omega_{th}$, then Taylor expands and discards pieces of higher
order. At fourth order, therefore, if a naive loop amplitude is $f^{(3)}(\omega/M_\pi) + f^{(4)}(\omega/M_\pi)$, the
shifted amplitude is

$$f^{(3)}(\zeta) + f^{(4)}(\zeta) + \zeta \left( \frac{\omega_{th}}{M_\pi} - 1 \right)_{LO} \frac{df^{(3)}(\zeta)}{d\zeta}. \tag{5}$$

The subscript “LO” indicates the dropping of terms of $\mathcal{O}(1/m_N^2)$. The shifted amplitude is
finite at $\zeta = 1$.

Since $|T|^2$ is an invariant, it should make no difference in which frame it is calculated,
and we use the Breit frame as the amplitudes are simplest there. Of course in HBCPT
the invariance is only respected up to the order in $1/m_N$ to which we are working, and so
the results obtained by calculating the amplitudes in different frames would not in fact be
identical.

Although we have calculated the amplitudes consistently to fourth order, those ampli-
tudes substituted in Eq. 3 will not give a differential cross section which is of consistent
order. Since $A_1$ starts at second order with the Thomson term, knowledge of the amplitude
to fourth order gives the cross section to sixth order. The fourth order parts of all the
other $A_i$ are not actually needed, and the seventh- and eighth-order parts of $|A_1|^2$ should
be subtracted. We do not do this. The various orders of contributions are actually of very

Fig. 6: Fourth order HBCPT with $\alpha$ and $\beta$ set to their PDG values (solid), and to $\alpha = 8,$
$\beta = 6 \times 10^{-4}$ fm$^3$ (dashed).
Fig. 7: Effect of varying the pion-scattering LEC’s $c_i$. Values are taken from Fettes and Meißner [29]. Solid and dashed lines are the second order fits based on KA85 and SP98; dotted is the fourth order fit to SP98 from Table 1 of that work.

similar magnitude, and if we try to remove higher order pieces we may not even be left with a positive cross section. Thus all the cross sections we present are calculated with fourth-order amplitudes, and our kinematic prefactors and change of variable from $\omega$ to $\zeta$ and from one frame to another are done exactly.

### III. RESULTS

In Figs. 2 and 3 we show differential cross sections at various lab and centre-of-mass angles for which data exist below 200 MeV. In each graph we show the Born (pion and nucleon pole) contribution at fourth order, and the HBCPT results with the amplitudes calculated to third and fourth order. The third as well the fourth order results have had the variable change $\omega/M_\pi \rightarrow \omega/\omega_{th}$ as explained above; otherwise the threshold would be in the wrong place. This, and the fact that we take the basic third-order amplitude to be in the Breit frame, rather than the centre-of-mass frame, accounts for the slight differences between our third-order results and those of Babusci et al. [20]. In Fig. 4 we show three third order curves: amplitudes calculated in the Breit frame and shifted, in the Breit frame and unshifted, and in the CM frame and unshifted as in ref. [20]. The differences between these are higher order.

For scattering angles up to $90^\circ$ the fit at either third or fourth order is very good except at the highest energies, and much better than the Born terms alone. In the cusp region
the fourth order is arguably better than the third order at all angles. The good fit to the data above threshold is a feature which has not been remarked before. However for larger angles both curves start to fall below the data as the threshold is approached, and fall dramatically short above. Of course one does not expect agreement to continue indefinitely, as in experiments the $\Delta$ resonance is clearly seen, and this will not be reproduced by HBCPT. However it is not obvious why backward angles should be so much worse than forward angles. As the data in that region is only from one experiment \cite{23} one might question its accuracy, but in fact it fits well with dispersion analyses which incorporate the more numerous higher energy data.

At fifth order, the new diagrams which enter fall into several groups. First, there are two-loop graphs, which one might well expect to be small. Then, there are one-loop graphs with two insertions from $L^{(2)}$. Some of these give higher $1/m_N$ corrections and these again ought to be small (some indeed have already been incorporated in the change of variables which corrects the threshold) but some, such as two insertions of the anomalous isovector magnetic moment, may be numerically significant. Then there are one-loop graphs with one insertion from $L^{(3)}$, and seagulls contributing directly to the polarisabilities. It is in these that the effects of the $\Delta$ will show up in the spin-dependent amplitudes, and these contributions are expected to be anomalously large. We have not calculated fifth-order loop graphs. However the seagulls may be incorporated effortlessly, since the free LEC’s will just be fit to give the “experimental” values of the $\gamma_i$. In Fig. 5 we show the effect of setting the $\gamma_i$ to the values found in three fixed-$t$ dispersion-relation studies \cite{13-15}. Below threshold the agreement is slightly improved, but above the enhancement at backward angles is not sufficient to give agreement.

Finally, we show the effects of varying the input parameters within a purely fourth-order calculation. These are the places where the effect of the $\Delta$ is already present; for instance the tadpole graphs of Fig. 1n-r could arise from integrating out an intermediate $\Delta$. These graphs vanish for forward scattering but contribute significantly to the spin-independent amplitudes at backward angles. First we show the results of varying $\alpha - \beta$ while keeping the sum fixed. This shows up only at backward angles; to reproduce the Mainz data at $180^\circ$ we need to make a dramatic shift, to $\alpha = 8$ and $\beta = 6 \times 10^{-4}$ fm$^3$. (Rather similar numbers were found by Grießhammer and Rupak in a study of Compton scattering from the deuteron in the framework of effective field theory without dynamical pions \cite{31}.) However even these extreme values do not cure the problem above the threshold. Then we show the effect of using a very different parameter set for the $c_i$ that enter the tadpole graphs, while keeping the polarisabilities fixed. (These graphs do of course contribute to the polarisabilities, as given ref. 8. However the fourth order LEC’s are adjusted to give the measured polarisabilities whatever the other contributions might be.) These parameters are quite uncertain and differ substantially in different fits; we show two second-order fits and one fourth-order \cite{29}. Again, the only effect is at backward angles, and none of the parameter sets gives agreement above the threshold.

Thus the conclusion of this fourth-order study is similar to that of the third-order study of Babusci et al. \cite{20}. For angles up to $90^\circ$, HBCPT without an explicit $\Delta$ does an excellent job of fitting the experimental differential Compton scattering cross section up to about 170 MeV. However it fails at larger angles for photon energies above about 120 MeV. Very large fifth-order contributions would be needed to approach the experimental data above
the threshold at these angles.

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**APPENDIX A: FOURTH-ORDER LOOP AMPLITUDE IN THE BREIT FRAME**

The full amplitudes in the Breit frame for the diagrams of Fig. 1 are as follows. The notation \( t_i \) is used for the tensor structures which multiply the amplitudes \( A_i \) of Eq. [1]: for example \( t_1 = e^i \cdot e \).

\[
T_a = \frac{g^2 e^2}{4 m_N f^2} (t_1 + t_3) \left[ (d-1) \frac{\partial J_2[\omega, m^2]}{\partial \omega} - (w^2 + \frac{1}{3} t) \frac{\partial J_0[\omega, m^2]}{\partial \omega} \right] + \text{crossed}
\]

\[
T_b = -\frac{g^2 e^2}{2 m_N f^2} \int_0^1 dx \left[ (t_1 + t_3) (d+1) \frac{\partial J_2[\omega, m^2]}{\partial \omega} \right.
\]

\[
- \left( x \omega (t_2 + \frac{1}{5} t_5) + (x^2 w^2 + \frac{1}{2} x t) (t_1 + t_3) \right) \frac{\partial J_0[\omega, m^2]}{\partial \omega} \right] + \text{crossed}
\]

\[
T_c = -\frac{g^2 e^2}{2 m_N f^2} \tau_3 (t_1 + t_3) \left( \omega J_0[\omega, m^2] + \Delta_\pi[m^2] \right) + \text{crossed}
\]

\[
T_d = \frac{g^2 e^2}{2 m_N f^2} \tau_3 (t_1 + t_3) \int_0^1 dx \left( \omega J_0[\omega, m^2] + \Delta_\pi[m^2] \right) + \text{crossed}
\]

\[
T_e = \frac{g^2 e^2}{2 m_N f^2} (1 - \tau_3) (t_1 + t_3) \frac{1}{\omega} \left( J_2[\omega, m^2] - J_2[0, m^2] \right) + \text{crossed}
\]

\[
T_f = \frac{g^2 e^2}{4 m_N f^2} \int_0^1 dx \left[ (\mu_v - \mu_s \tau_3) ((1 - 2x)(t_3 + t_1) \cos \theta - t_2 - (1 - 2x) t_4) \right.
\]

\[
\left. - (1 - \tau_3) (2(d+1) t_1 \omega^{-1} J_2[\omega, m^2] + 2x(1 - x) t_1 - \frac{1}{2} (1 - 2x) t_6) \omega J_0[\omega, m^2] \right] + \text{crossed}
\]

\[
T_g = -\frac{g^2 e^2}{4 m_N f^2} ((\mu_v + \mu_s \tau_3) ((t_1 + t_3) \cos \theta - t_2 + t_4 - t_5) + \frac{1}{2} (1 + \tau_3) t_6) \omega \int_0^1 dx J_0[\omega, m^2]
\]

\[
T_h = -\frac{g^2 e^2}{4 m_N f^2} \int_0^1 dy \int_0^{1-y} dx \left[ -(d+1)(d+3) t_1 \frac{\partial J_2[\omega, m^2 - xyt]}{\partial \omega} \right.
\]

\[
+ \left( t_1 (2(d+3) V(x, y) - t) + \omega^2 t_2 ((d+3)((d+5)xy - 2x - 2y) + 2) +
\]

\[
((d+3)x-1) \omega \omega^2 (t_6 - t_5) + (d+3)(1-x-y) \omega \omega^2 t_4 \right) \frac{\partial J_0[\omega, m^2 - xyt]}{\partial \omega}
\]

\[
+ \left( -t_1 V(x, y)^2 - (xt_6 - xt_5 + (1-x-y)t_4) V(x, y) \omega \right.
\]

\[
- (1 - \tau_3) (2(d+1) t_1 \omega^{-1} J_2[\omega, m^2] + 2x(1 - x) t_1 - \frac{1}{2} (1 - 2x) t_6) \omega J_0[\omega, m^2] \right] + \text{crossed}
\]

\[
T_i = \frac{g^2 e^2}{4 m_N f^2} \int_0^1 \left[ (d^2 - 1) \Delta_\pi[m^2 - x(1-x)t] + 2(1 - 2d + 1)x(1 - x)) \right. \Delta_\pi[m^2 - x(1-x)t]
\]
\[ T_j = \frac{g^2 e^2}{m_N f_\pi^2} \int_0^1 dy \int_0^{1-y} dx \left[(d + 1) t_1 \Delta_\pi [m^2 - xyt] + \left(- (2V(x, y) - t) t_1 + 2(x + y - (d + 3)xy) \omega^2 t_2 \right) \Delta_\pi' [m^2 - xyt] + (4V(x, y) - 2t) xy \omega^2 t_2 \Delta_\pi'' [m^2 - xyt] \right] \]

\[ T_k = - \frac{g^2 e^2}{2m_N f_\pi^2} t_1 \int_0^1 dx \left[(d - 1) \Delta_\pi [m^2 - x(1 - x)t] + (1 - 2x(1 - x)) t \Delta_\pi [m^2 - x(1 - x)t] \right] \]

\[ T_i = - \frac{g^2 e^2}{m_N f_\pi^2} t_1 \Delta_\pi [m^2] \]

\[ T_m = - \frac{g^2 e^2}{8m_N f_\pi^2} t_1 (3 - \tau_3)(d - 1) \Delta_\pi [m^2] \]

\[ T_n = - \frac{e^2}{2f_\pi^2} t_1 \left( 8c_3 - \frac{1}{m_N} (1 + \tau_3) \right) \Delta_\pi [m^2] \]

\[ T_p = \frac{e^2}{2f_\pi^2} t_1 \left( 16c_3 - \frac{1}{m_N} (1 + \tau_3) \right) \Delta_\pi [m^2] \]

\[ T_q = \frac{8e^2}{f_\pi^2} \int_0^1 dy \int_0^{1-y} dx \left[ - \frac{1}{2} (\tilde{c}_2 + (d + 2)c_3) t_1 \Delta_\pi [m^2 - xyt] + \left( 2c_1 M_\pi^2 - (\tilde{c}_2 + c_3) \tilde{\omega}^2 + V(x, y)c_3 \right) t_1 + \right. \]

\[ \left. (x y \tilde{c}_2 + ((d + 4)xy - x - y)c_3) \omega^2 t_2 \right) \Delta_\pi' [m^2 - xyt] \]

\[ + xy \omega^2 t_2 \left( - 4c_1 M_\pi^2 + 2 \tilde{\omega}^2 (\tilde{c}_2 + c_3) - 2V(x, y)c_3 \right) \Delta_\pi'' [m^2 - xyt] \]

\[ T_r = \frac{2e^2}{f_\pi^2} t_1 \int_0^1 dx \left[ (d c_3 + \tilde{c}_2) \Delta_\pi [m^2 - x(1 - x)t] - (4c_1 M_\pi^2 + 2x(1 - xt)c_3) \Delta_\pi' [m^2 - x(1 - x)t] \right] \]

\[ T_s = \frac{3g^2 e^2}{8m_N f_\pi^2} (1 + \tau_3)(d - 1)t_1 \Delta_\pi [m^2] \]

\[ T_j, T_k, T_i, T_m, T_n, T_p, T_q, T_r, T_s \]

\[ \text{(A1)} \]

where the \( c_i \) are low energy constants from \( \mathcal{L}^{(2)}_{N\pi} \), and \( \tilde{c}_2 = c_2 - g^2/8m_N \). The integrals \( J_0[\omega, m^2], J_2[\omega, m^2] \) and \( \Delta_\pi [m^2] \) have their usual meanings, prime denotes differentiation with respect to \( m^2 \), \( \tilde{\omega} = (1 - x - y) \omega \) and

\[ V(x, y) = \tilde{\omega}^2 + \frac{1}{2} t(1 - x - y + 2xy). \]

\[ \text{(A2)} \]

We have also introduced an extra tensor structure, \( t_7 = \sigma \cdot (\hat{q}' \times \hat{q}) \epsilon' \cdot \hat{q} \epsilon \cdot \hat{q}' \). This is not independent; \( t_7 = \sin^2 \theta t_3 + \cos \theta t_5 - t_6 \), but as it arises naturally in the calculations—and enters at too high an order in \( \omega \) to affect the polarisabilities—it is a useful notation.

The notation “+ crossed” means that to every term, another is added with \( \epsilon \leftrightarrow \epsilon' \), \( \hat{q} \leftrightarrow - \hat{q}' \) and \( \omega \leftrightarrow - \omega \). Since the \( t_i \) are all either symmetric or antisymmetric under this transformation, the net effect is to add a term with \( \omega \leftrightarrow - \omega \) to the coefficients of \( t_1 \) and \( t_2 \), and subtract such a term from the coefficients of \( t_3 \ldots t_7 \).
The spin-dependent terms do not all look identical to the expressions given in ref. [17], but they are equivalent.
REFERENCES

[1] V. Bernard, N. Kaiser, J. Kambor and U.-G. Meißner, Nucl. Phys. B 388 315 (1992).
[2] D. E. Groom et al., European Physical Journal C15 1 (2000).
[3] V. Bernard, N. Kaiser, A. Schmidt and U.-G. Meißner, Phys. Lett. B 319 269 (1993).
[4] V. Bernard, N. Kaiser, U.-G. Meißner and A. Schmidt, Z. Phys. A348 317 (1993)
[5] V. Bernard, N. Kaiser and U.-G. Meißner, Int. J. Mod. Phys. E 4 193 (1995).
[6] F. Low, Phys. Rev. 96 1428 (1954); M. Gell-Mann and M. Goldberger, Phys. Rev. 96 1433 (1954).
[7] I. Karliner, Phys. Rev. D7 2717 (1973).
[8] A. M. Sandorfi, C. S. Whisnant and M. Khandaker, Phys. Rev. D50 R6681 (1994).
[9] D. Drechsel and G. Krein, Phys. Rev. D 58 116009 (1998).
[10] J. Tomnison, A. M. Sandorfi, S. Hoblit and A. M. Nathan, Phys. Rev. Lett. 80 4382 (1998).
[11] F. Wissman, Talk at GDH-2000 symposium, Mainz, June 2000.
[12] P. Pedroni, Talk at GDH-2000 symposium, Mainz, June 2000.
[13] D. Drechsel, G. Krein and O. Hanstein, Phys. Lett. B 420 248 (1998).
[14] D. Babusci, G. Giordano, A. I. L’vov, G. Matone and A. M. Nathan, Phys. Rev. C58 1013 (1998).
[15] D. Drechsel, M. Gorchein, B. Pasquini and M. Vanderhaeghen, Phys. Rev. C61 015204 (2000).
[16] X. Ji, C-W. Kao and J. Osborne, Phys. Rev. D 61 074003 (2000).
[17] K. B. V. Kumar, J. A. McGovern and M. C. Birse, hep-ph/9909442.
[18] G. C. Gellas, T. R. Hemmert and U.-G. Meißner, Phys. Rev. Lett. 85 14 (2000).
[19] K. B. V. Kumar, J. A. McGovern and M. C. Birse, Phys. Lett. B 479 167 (2000).
[20] D. Babusci, G. Giordano and G. Mantone, Phys. Rev. C55 R1645 (1997).
[21] P. S. Baranov et al., Sov. J. Nucl. Phys. 21 355 (1975).
[22] P. S. Baranov, A. L. L’vov, V. A. Petrun’kin and L. N. Shtarkov, Phys. Part. Nucl. 32 376 (2001).
[23] A. Ziegler et al., Phys. Lett. B 278 34 (1992).
[24] F. J. Federspiel et al., Phys. Rev. Lett. 67 1511 (1991).
[25] E. L. Hallin et al., Phys. Rev. C48 1497 (1993).
[26] B. E. MacGibbon et al., Phys. Rev. C52 2097 (1995).
[27] V. Olmos de León et al., Eur. Phys. J. A 10 207 (2001).
[28] M. C. Birse, X. Ji, J. A. McGovern, nucl-th/0011054.
[29] N. Fettes and U.-G. Meißen, Nucl. Phys. A 676 311 (2000).
[30] H. W. Grießhammer and G. Rupak, nucl-th/0012096.