Identification of Isomorphism in Multistage Planetary Gear Trains

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Abstract. Isomorphism identification of kinematic chains occupies an important place in the field of mechanism structure synthesis. A method to identify the isomorphism of multistage planetary gear trains is proposed in this paper. Firstly, the structure topology model and the corresponding link-link adjacency matrix are presented, to describe the schematic diagram of multistage planetary gear trains. Then, based on the hamming number method, the hamming number matrix is obtained by the link-link adjacency matrix, and the hamming value of link, the hamming number and the hamming strings of multistage planetary gear trains are introduced, which can determine whether the multistage planetary gear trains is isomorphic or not. The study shows that this work is beneficial to the development of new multistage planetary gear trains.

1. Introduction

In structural synthesis of kinematic chains, one of the most important and challenging problem is the detection of the isomorphism among some given kinematic chains. If the non-isomorphic of kinematic chains were mistaken for isomorphism, which would result in losing possible candidates for new mechanisms. If isomorphic kinematic chains mistaken for non-isomorphism, which would cause duplicate solutions and unnecessary effort. Therefore, isomorphic identification of kinematic chains is an essential step in mechanism creative design.

In the past few decades, many researchers have been trying to solve this problem and consequently various methods came into existence. Rao and Raju [1] proposed hamming number method and applied it to six-, eight-, and ten-bar planar simple joint kinematic chains with one degree of freedom, as well as ten-bar planar multiple joints kinematic chains with three-degree of freedom, but the proposed method proved to be complex and cumbersome. Chu and Cao [2] proposed link’s adjacent-chain table (ACT) method to identify isomorphism of planar hinge kinematic chains, but it is not suitable for computerised structural synthesis. Rao and Pathapati [3] proposed two new invariants: “chain loop string” and “link adjacency string” for isomorphism detection of multiple degrees of freedom planar simple joint kinematic chains, however, there is no proof being offered. Chang et al. [4] proposed comparison eigenvalues and eigenvectors of adjacency matrices for isomorphism identification of planar hinge kinematic chains, although this method possesses the advantages of using standard matrix theory and adapting automatic computation techniques, but it has theoretical errors. Cubillo and Wan [5] discovered the errors of original theory about literature [4], and the authors proposed the necessary conditions of eigenvalues and eigenvectors of adjacency matrices for
isomorphic planar hinge kinematic chains. Ding and Huang [6–8] proposed the canonical perimeter topological graph and the characteristic adjacency matrix approach to test isomorphism for planar hinge kinematic chains. Dargar et al. [9] proposed a method to detect isomorphism among planar simple joint kinematic chains, but no mathematical proof was presented. Ge et al. [10] used standardized adjacency matrix to detect isomorphism of planar simple joint kinematic chains, yet this method is based on the strong basis of graph theory. Yang et al. [11] presented the incident matrices method to identify the isomorphism of topological graphs, but the discrimination will be complicated when there are many identical values in rows and arrays. Dargar et al. [12] proposed a method based on the concept of weighed structural matrices to reveal simultaneously link isomorphic and chain is isomorphic, but the effectiveness of this method still needs testing. Lohumi et al. [13] proposed a computerized loop based approach for identification of isomorphism in planar kinematic chains, but there is no mathematical proof has been presented.

It can be seen from the literature, there are a number of methods proposed by many researchers, to detect the isomorphism of simple joint or multiple joints planar kinematic chains, and the isomorphism identification methods for these two kinds of planar hinge kinematic chains are comparatively mature. But when it comes to planetary gear trains, tests are very rare. As we know, isomorphism identification methods used in the planar hinge kinematic chains are always not satisfying the planetary gear trains, especially multistage planetary gear trains. So, according to the researches above, finding a useful method, to solve the problem of isomorphism identification of planetary gear trains is the existing research content and further study is necessary. In this paper, a method to identify isomorphism in planetary gear trains is presented, and which involves three main steps: 1 Topological description of planetary gear trains. 2 Establishment of hamming number matrix and find out the hamming strings. 3 Analysis of isomorphism identification. And the specific analysis process is shown in Figure 1.

Fig. 1 Flow chart for identifying isomorphism of planetary gear trains

2. Topological description of planetary gear trains

2.1 Topology model of planetary gear trains

The topological description of planetary gear trains plays a crucial role in its structural analysis. In generally, planetary gear trains consist of planet gear set and racks, in which planet gear set contains sun gear, planet carrier, planetary gear and ring gear. In other words, planetary gear trains is a collection of links connected by turning pairs, gear pairs and interstage junction. In order to be able to completely describe the structural feature of multistage planetary gear trains, based on the topological description of planetary gear trains in literature [28–29], the new topology model of planetary gear trains is proposed.

1. Use solid circle (”) to denote planetary gear, hollow circle (“○”) to denote sun gear, solid square (“□”) to denote ring gear, hollow square (“□”) to denote planetary carrier and hollow triangle (“△”) to denote rack.

2. Use thin lines to denote turning pairs, single dash line to denote external gear pairs, double dash lines (“—”) to denote internal gear pairs, thick lines (“—”) to denote fixed connection between
components of planet gear set and racks, and the crooked lines ("\(\sim\)") to denote interstage junction between components in planet gear set.

For example, Fig. 2(b) is the new topological model of the multistage planetary gear trains (Fig. 2(a)). According to above definition, it can be seen that the planetary gear trains can be uniquely represented by a topology model.

![Fig. 2 Structure diagram and topology model of two-stage planetary gear trains.](image)

### 2.2 The link-link adjacency matrix

Since topological graph can also be represented by its adjacency matrix, so the topological model information of planetary gear trains is represented digitally by using the link-link adjacency matrix. The row label and column label of the link-link adjacency matrix are the serial number of vertexes about topology model of planetary gear trains, and which is expressed as:

\[
A = \begin{bmatrix}
0 & a_{1,2} & \cdots & a_{1,n} & a_{1,n+1} & \cdots & a_{1,n+k} \\
\vdots & \ddots & \ddots & \ddots & \ddots & & \ddots \\
a_{n,1} & \cdots & 0 & \cdots & \cdots & \cdots & a_{n,n+k} \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
a_{n+1,n} & a_{n,1} & \cdots & a_{n,n} & 0 & \cdots & a_{n+1,n+k} \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
a_{n,k,n} & a_{n,1} & \cdots & a_{n,n} & a_{n,n+1} & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{n+k} \\
\end{bmatrix},
\]

Where \(n\) is the number of vertexes of the topology model. Obviously, the link-link adjacency matrix \(A\) is a square symmetric matrix of order \(n\) with zero diagonal elements, and the value of non-diagonal elements follow the rules as:

\[
A = \begin{bmatrix}
1, & \text{if vertices } i \text{ and } j \text{ are connected by a thin line} \\
2, & \text{if vertices } i \text{ and } j \text{ are connected by a single dash line} \\
3, & \text{if vertices } i \text{ and } j \text{ are connected by a double dash line} \\
-1, & \text{if vertices } i \text{ and } j \text{ are connected by a crooked line} \\
-2, & \text{if vertices } i \text{ and } j \text{ are connected by a thick line} \\
0, & \text{otherwise}
\end{bmatrix}
\]

For the topology model in Fig. 1(b), its link-link adjacency matrix is expressed as:

\[
A = \begin{bmatrix}
0 & 1 & -2 & 0 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & -1 \\
-2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 3 & 0
\end{bmatrix},
\]
3. Establishment of hamming number matrix

3.1 Converting link-link adjacency matrix into hamming number matrix

Once the link-link adjacency matrix $A$ of planetary gear trains is written, the hamming number matrix $C$ can be written from the link-link adjacency matrix $A$, which is also a square symmetric matrix of size $n$. Any hamming number $h_{ij}$ in hamming number matrix $C$ is obtained by comparing the $i$th and $j$th rows of the link-link adjacency matrix $A$ column by column, and performing an exclusive OR operation on them.

For example, comparing first and third rows of the link-link adjacency matrix $A$ of Fig. 2, ones get: $h_{13} = (0-2) + (1+0) + (-2+0) + (0+1) + (1+0) + (1+0) + (0+1) = 2$. And the rules for writing hamming number matrix are:

$$h_{ij} = \sum_{k=1}^{n} S_{k}, \quad S_{k} = \begin{cases} a_{ik} + a_{jk}, & \text{if } a_{ik} \neq a_{jk} \\ 0, & \text{if } a_{ik} = a_{jk} \end{cases}$$

and $h_{ij} = 0$

$n$ is the size of the matrix, i.e., number of links

Similarly, applying the this rules, the hamming number matrix of the Fig. 2 is obtained and showed by Table 1.

| links | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|---|---|---|---|---|---|---|---|---|
| 1     | 0 | 5 | 2 | 9 | 7 | 6 | 5 | 9 | 6 |
| 2     | 5 | 0 | 1 | 8 | 4 | 3 | 2 | 8 | 3 |
| 3     | 2 | 1 | 0 | 5 | 3 | 2 | 1 | 5 | 2 |
| 4     | 9 | 8 | 5 | 0 | 10 | 9 | 8 | 12 | 9 |
| 5     | 7 | 4 | 3 | 10 | 0 | 5 | 4 | 10 | 5 |
| 6     | 6 | 3 | 2 | 9 | 5 | 0 | 3 | 9 | 4 |
| 7     | 5 | 2 | 1 | 8 | 4 | 3 | 0 | 8 | 3 |
| 8     | 9 | 8 | 5 | 12 | 10 | 9 | 8 | 0 | 9 |
| 9     | 6 | 3 | 2 | 9 | 5 | 4 | 3 | 9 | 0 |

3.2 Generation of hamming strings.

The hamming strings of kinematic chains is an invariant, and can be used to detect isomorphism among kinematic chains. Two kinematic chains are considered isomorphic if their hamming strings are identical. For generation of hamming strings of kinematic chains, the first step is to find out the hamming value of links, and the hamming number of kinematic chains. The hamming value of any link $i$ is obtained by adding all the elements in the $i$th row of the hamming number matrix, and the hamming number is the sum of hamming value of all links. Then, the hamming strings is obtained by arranging the hamming number of kinematic chains and the hamming value of links in descending order.

Considering the above hamming number matrix, the hamming value of the nine links in the two-stage planetary gear trains in Fig. 2 in order are 70, 70, 49, 48, 41, 41, 34, 34 and 21, and the hamming number is 408. Therefore, the hamming strings is 408[70-70-49-48-41-41-34-34-21].

4. Conclusions

In this paper, a new graph presentation is introduced to describe the structure diagram of multistage planetary gear trains, and the link-link adjacency matrix can uniquely represent and digitally save the complete structural relationships of the kinematic chains. Then, in association with hamming number method [1], using topological invariants hamming strings to identify isomorphism of multistage planetary gear trains. The result shows that this method is reliable for checking isomorphism in
multistage planetary gear trains. Since the presented method is based on the strong basis of graph theory, and its operation is intuitive. Therefore, it can be easily understood and executed automatically by a computer, the time and calculation of multistage planetary gear trains isomorphism identification will be reduced obviously.

Acknowledgement
The authors would like to thank anonymous reviewers for their helpful comments and suggestions to improve the manuscript. This project is supported by National Natural Science Foundation of China (Grant No.51765020).

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