Efficient and Effective Similar Subtrajectory Search with Deep Reinforcement Learning

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1. INTRODUCTION

Trajectory data, which corresponds to a type of data for capturing the traces of moving objects, is ubiquitous. It has been used for various types of analysis such as clustering [1, 16, 5], and similarity search [6, 7, 39, 27, 18, 41]. The majority of existing studies take a trajectory as a whole for analysis [6, 7, 39, 27, 18, 41]. Motivated by the phenomenon that two trajectories could be dissimilar to each other if each is considered a whole but similar if only some portion of each is considered, there have been a few studies which take a portion of a trajectory as a basic entity for analysis [1, 16, 5, 31, 32]. Some examples include subtrajectory clustering [1, 16, 5] and subtrajectory join [31, 32]. For example, the subtrajectory clustering method in [16] first partitions raw trajectories into different subtrajectories using some principle and then groups those subtrajectories that are similar to one another into clusters.

In this paper, we study a query with its goal to search for a portion of a trajectory from a database storing many trajectories called data trajectories, which is the most similar to a given trajectory called query trajectory. In this query, a portion of a trajectory, called subtrajectory, is considered as a basic entity and a query trajectory is taken as a whole for analysis. Therefore, it captures trajectory similarity in a finer-grained way than conventional similar trajectory search. For instance, consider a data trajectory and a query trajectory. When considered as a whole, the data trajectory is not similar to the query trajectory based on some trajectory similarity measurement, but some portion of it is very similar to the query trajectory. With the conventional similar trajectory search query, this data trajectory would be ruled out, though a portion of it is very similar to the query trajectory, which is interesting.

Moreover, in quite a few real-life applications, subtrajectories are naturally considered as basic units for analysis, e.g., subtrajectory search [29], subtrajectory join [31], subtrajectory clustering [5], etc. For illustration, consider the subtrajectory search query on sports play data. In sports such as soccer and basketball, a common practice nowadays is to track the movements of players and/or the ball using some special-purpose camera and/or GPS devices [37]. The resulting trajectory data is used to capture the semantics of the plays and for different types of data analyses. One typical task on such sports play data is to search for a portion/segment of play from a database of plays, with its trajectories of players and/or its trajectory of the ball similar to those and/or that of a given query play [29]. This task is essentially one of searching for similar subtrajectories.

A key problem that is involved in answering the query mentioned above is to find a subtrajectory of a data trajectory, which is the most similar to a given query trajectory. We call this problem the similar subtrajectory search (SimSub) problem. Let $T$ be a data trajectory involving $n$ points and $T_q$ be a query trajectory involving $m$ points. Note that there are $\binom{n+m}{2}$ possible subtrajectories of $T$. While there are many existing studies on the similar trajectory search problem with each trajectory considered a whole, there are very few studies on the SimSub problem. An intuitive solution to the SimSub problem is to enumerate all possible subtrajectories of $T$, compute the similarity between each subtrajectory and the query trajectory, and return the one with the greatest similarity. A straightforward implementation of

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this method would incur $O(n^3)$ time for some measurements such as t2vec (for which the cost of similarity computation is $O(n)$) and even $O(n^3m)$ time for some other measurements such as DTW and Frechet (for which the cost of similarity computation is $O(nm)$), and this is prohibitively expensive for large-scale applications. We improve the implementation via an incremental strategy for computing similarities and achieve $O(n^2)$ for some measurements such as t2vec and $O(n^2m)$ for some other measurements such as DTW and Frechet. We also follow some existing studies on subsequence matching and design an algorithm which considers only those subtrajectories with their sizes similar to that of the query trajectory and controlled by a user parameter. This would provide a controllable trade-off between efficiency and effectiveness.

To push the efficiency further up, we propose several algorithms which share the idea of splitting a data trajectory into some subtrajectories to be candidate solutions to the problem and differ in using different methods for splitting the data trajectory. Specifically, the process is to scan the points of a data trajectory one by one sequentially and for each one, it decides whether to split the data trajectory at the point. Some of them use pre-defined heuristics, e.g., a greedy one. Others model the process as a Markov decision process (MDP) and use deep reinforcement learning to learn an optimal policy for the MDP which is then used for splitting the data trajectory. These splitting-based algorithms have time complexities much lower than the exact algorithm, e.g., for measurements such as t2vec, each splitting-based algorithm runs in $O(n)$ time and for measurements such as DTW and Frechet, it runs in $O(nm)$ time. The major contributions of this paper are as follows.

- We propose the SimSub problem, and this to the best of our knowledge, corresponds to the first systematic study on searching subtrajectories that are similar to a query trajectory. The SimSub problem relies on a trajectory similarity measure, and in this paper, we assume an abstract one, which could be instantiated with any existing measurement such as DTW and Frechet.

- We develop a suite of algorithms for the SimSub problem: (1) one exact algorithm, (2) one approximate algorithm which provides a controllable trade-off between efficiency and effectiveness, and (3) several splitting-based algorithms including both heuristics-based ones and deep reinforcement learning-based ones. These algorithms should cover a wide spectrum of application scenarios in terms of efficiency and effectiveness requirements.

- Third, we conducted extensive experiments which verified that splitting-based algorithms in general have good efficiency and among them, the algorithms based on deep reinforcement learning achieve the best effectiveness and efficiency.

Organization. We review the related work in Section 2 and provide the problem definition and some preliminaries in Section 3. Section 4 presents all non-learning based algorithms and Section 5 presents the deep reinforcement learning-based algorithms. We report our experimental results in Section 6 and conclude this paper and discuss some future work in Section 7.

2. RELATED WORK

(1) Trajectory Similarity Measurements. Measuring the similarity between trajectories is a fundamental problem and has been studied extensively. Some classical solutions focus on indexing trajectories and performing similarity computation by the alignment of matching sample points. For example, DTW is the first attempt at solving the local time shift issue for computing trajectory similarity. Frechet distance is a classical similarity measure that treats each trajectory as a spatial curve and takes into account the location and order of the sampling points. Further, ERP and EDR are proposed to improve the ability to capture the spatial semantics in trajectories. However, these point-matching methods are inherently sensitive to noise and suffer from quadratic time complexity. EDS and EDwP are two segment-matching methods which operate on segments for matching two trajectories. In recent years, some learning-based algorithms were proposed to speed up the similarity computation. Li et al. propose to learn representations of trajectories in the form of vectors and then measure the similarity between two trajectories as the Euclidean distance between their corresponding vectors. Yao et al. employ deep metric learning to approximate and accelerate trajectory similarity computation. Different specialized index techniques are developed for these similarity measures, such as DTW distance, LCSS, ERP, EDR, and EDwP. However, these index techniques do not generalize to other similarity measures or subtrajectory similarity search. Following the work by Yao et al., we also consider employing two spatial-based indexes (R-tree and Grid Inverted) for subtrajectory similarity search in our experiments. In this paper, we assume an abstract trajectory similarity measurement which could be instantiated with any of these existing similarity measurements and our techniques still apply.

(2) Subtrajectory Similarity Related Problems. Measuring subtrajectory similarity is also a fundamental functionality in many tasks such as clustering and similarity join. Lee et al. propose a general partition and group framework for subtrajectory clustering. Further, Buchin et al. show the hardness of subtrajectory clustering based on Frechet distance, and Agarwal et al. apply the trajectory simplification technique to approximate discrete Frechet to reduce the time cost of subtrajectory clustering. Recently, Tampakis et al. proposed a distributed solution for subtrajectory join and clustering by utilizing the MapReduce programming model. Although these algorithms need to consider subtrajectory similarity, similarity computation is not their focus and they usually first segment a trajectory into subtrajectories and employ an existing measure, such as Freechet distance.

(3) Subsequence (Substring) Matching. Subsequence matching is a related but different problem. It aims to find a subsequence that has the same length as the query in a given candidate sequence, which usually contains millions or even trillions of elements. Efficient pruning algorithms have been proposed for the matching, and these pruning algorithms are generally designed for a specific similarity measure, such as DTW, and cannot generalize to other measures. On the other hand, substring matching often focuses on approximate...
matching based on the Edit distance. It aims to find a substring in a string to best match the query. Our problem differs from the substring matching problem mainly in two aspects. First, characters in a string have exact match (0 or 1) in the alphabet; however, the points of a trajectory are different. Second, substring matching techniques are usually designed based on the characteristics of strings. e.g., grammar structure patterns, or word concurrence patterns; however, a trajectory does not have such patterns.

(4) Reinforcement Learning. The goal of reinforcement learning is to guide agents on how to take actions to maximize a cumulative reward in an environment, and the environment is generally modeled as a Markov decision process (MDP) [24]. Recently, RL models have been utilized successfully to solve some database related problems. For example, Zhang et al. [42] and Li et al. [17] use RL model for automatic DBMS tuning. Trummer et al. [33] use RL to learn optimal join orders in the SkinnerDB system. Wang et al. [36] design an effective RL-based algorithm for bipartite graph matching. Overall, there are two types of popular reinforcement learning methods: (1) model-based methods [4, 12] that require to understand the environment and learn the parameters of the MDP in advance, and (2) model-free methods [48, 20] that make no efforts to learn a model and get feedback from the environment step by step. In this paper, we follow the model-free methods because they are more efficient. Specifically, we make use of a popular reinforcement learning method, namely Deep Q Network (DQN) [20], for splitting a trajectory into subtrajectories to be candidate solutions to the similar subtrajectory search problem.

3. PROBLEM DEFINITION AND PRELIMINARIES

The trace of a moving object such as a vehicle and a mobile user is usually captured by a trajectory. Specifically, a trajectory T has its form as a sequence of time-stamped locations (called points), i.e., $T=< p_1, p_2, ..., p_n >$, where point $p_i = (x_i, y_i, t_i)$ means that the location is $(x_i, y_i)$ at time $t_i$. The size of trajectory $T$, denoted by $|T|$, corresponds to the number of points of $T$.

Given a trajectory $T=< p_1, p_2, ..., p_n >$ and $1 \leq i \leq j \leq n$, we denote by $T[i, j]$ the portion of $T$ that starts from the $i^{th}$ point and ends at the $j^{th}$ point, i.e., $T[i, j] = < p_i, p_{i+1}, ..., p_j >$. Besides, we say that $T[i, j]$ for any $1 \leq i \leq j \leq n$ is subtrajectory of $T$. There are in total $\binom{n}{2}$ subtrajectories of $T$. Note that any subtrajectory of a trajectory $T$ belongs to a trajectory itself.

3.1 Problem Definition

Suppose we have a database of many trajectories which we call data trajectories. As discussed in Section 1, one common application scenario would be that a user has a trajectory at hand which we call a query trajectory and would like to check what is the portion of the data trajectories that is the most similar to the one at his/her hand. Note that in some cases, by looking each data trajectory as whole, none is similar enough to the query trajectory, e.g., all data trajectories are relatively long while the query trajectory is relatively short.

We note that a more general query is to find the top-k similar subtrajectories to a query trajectory, which reduces to the user’s query as described above when $k = 1$. In this paper, we stick to the setting of $k = 1$ since extending the techniques for the setting of $k = 1$ to general settings of $k$ is straightforward. Specifically, the techniques for the setting $k = 1$ in this paper are all based on a search process which maintains the most similar subtrajectory found so far and updates it when a more similar subtrajectory is found during the process. These techniques could be adapted to general settings of $k$ by simply maintaining the $k$ most similar subtrajectories and updating them when a subtrajectory that is more similar than the $k^{th}$ most similar subtrajectory.

An intuitive solution to answer the user’s query is to scan the data trajectories, and for each one, compute its subtrajectory that is the most similar to the query one based on some similarity measurement and update the most similar subtrajectory found so far if necessary. This solution could be further enhanced by employing indexing techniques such as the R-tree based index and the inverted-file based index for pruning [40, 35], e.g., the data trajectories that do not have any overlap with the query trajectory could usually be pruned. The key component of this solution (no matter whether indexing structures are used or not) is to compute for a given data trajectory, its subtrajectory that is the most similar to a query trajectory. We formally define the problem corresponding to this procedure as follows.

Problem 1 (Similar Subtrajectory Search (SimSub)).

Given a data trajectory $T=< p_1, p_2, ..., p_n >$ and a query trajectory $q=< q_1, q_2, ..., q_m >$, the similar subtrajectory search (SimSub) problem is to find a subtrajectory of $T$, denoted by $T[i, j]$ ($1 \leq i \leq j \leq n$), which is the most similar to $T_q$ according to a trajectory similarity measurement $\Theta(\cdot, \cdot)$, i.e., $[i^*, j^*] = \arg \max_{1 \leq i \leq j \leq n} \Theta(T[i, j], T_q)$.

The SimSub problem relies on a similarity measurement $\Theta(T, T')$, which captures the extent to which two trajectories $T$ and $T'$ are similar to each other. The larger the similarity $\Theta(T, T')$ is, the more similar $T$ and $T'$ are. In the literature, several “dissimilarity measurements” have been proposed for $\Theta(\cdot, \cdot)$ such as DTW [41], Frechet [2], LCS [34], ERP [6], EDR [7], EDS [39], EDW [27], and t2vec [18]. Different measurements have different merits and suit for different application scenarios. In this paper, we assume an abstract similarity measurement $\Theta(T, T')$, which could be instantiated with any of these existing measurements by applying some inverse operation such as taking the ratio between 1 and a distance.

3.2 Trajectory Similarity Measurements

The SimSub problem assumes an abstract similarity measurement and the techniques developed could be applied to any existing measurements. Since the time complexity analysis of the algorithms proposed in this paper relies on the time complexities of computing a specific measurement in several different cases, in this part, we review three existing measurements, namely DTW [41], Frechet [2] and t2vec [18], and discuss their time complexities in different cases as background knowledge. The first two are the most widely used measurements and the last one is the most recently proposed one which is a data-driven measurement.

We denote by $\Phi$ the time complexity of computing the similarity between a general subtrajectory of $T$ and $T_q$ from scratch, $\Phi_{inc}$ be the time complexity of computing $\Theta(T[i, j], T_q)$ ($1 \leq i < j \leq n$) incrementally assuming that
\( \Theta(T[i, j - 1], T_q) \) has been computed already, and \( \Phi_{ini} \) the time complexity of computing \( \Theta(T[i, i], T_q) \) \((1 \leq i \leq n)\) from scratch since it cannot be computed incrementally. As will be discussed later, \( \Phi_{inc} \) and \( \Phi_{ini} \) are usually much smaller than \( \Phi \) across different similarity measurements.

**DTW** [41]. Given a data trajectory \( T = \langle p_1, p_2, \ldots, p_n \rangle \) and a query trajectory \( T_q = \langle q_1, q_2, \ldots, q_m \rangle \), the DTW distance is defined as below

\[
D_{i,j} = \begin{cases} 
\sum_{h=1}^{i} d(p_h, q_1) & \text{if } j = 1 \\
\sum_{k=1}^{j} d(p_k, q_j) & \text{if } i = 1 \\
\min(D_{i-1,j-1}, D_{i-1,j}, D_{i,j-1}) & \text{otherwise}
\end{cases}
\]

(1)

where \( D_{i,j} \) denotes the DTW distance between \( T[i,i] \) and \( T_q[1,j] \) and \( d(p_k, q_j) \) is the distance between \( p_k \) and \( q_j \) (typically the Euclidean distance which could be computed in \( O(1) \)).

Consider \( \Phi \). It is clear that \( \Phi = O(n^2 \cdot m) \) since it needs to compute all pairwise distances between a point in a subtrajectory of \( T \) and a point in \( T_q \) and in general, the subtrajectory has its size of \( O(n) \) and \( T_q \) has its size of \( m \). Consider \( \Phi_{inc} \). This should be the same as the time complexity of computing \( D_{i,j} \) given that \( D_{i-1,1}, D_{i,1}, \ldots, D_{i-1,m} \) have been computed. Since \( D_{i-1,1} \) has been computed, we can safely assume that \( D_{i-1,1}, D_{i-1,2}, \ldots, D_{i-1,m} \) have been computed also according to Equation (1) (note that we can always make this hold by enforcing that we compute \( D_{i,j} \) for any other DTW distance in this way). Therefore, in order to compute \( D_{i,j} \), we compute \( D_{i,1}, D_{i,2}, \ldots, D_{i,m} \) sequentially, each of which would take \( O(1) \) time with the information of \( D_{i-1,k} \) \((1 \leq k \leq m)\) all available. That is, it takes \( O(m) \) to compute \( D_{i,j} \), and thus we know \( \Phi_{inc} = O(m) \). Consider \( \Phi_{ini} \). We know \( \Phi_{ini} = O(m) \) since \( T[i,i] \) \((1 \leq i \leq n)\) has its size always equal to 1 and \( T_q \) has its size of \( m \).

**Frechet** [2]. Given a data trajectory \( T = \langle p_1, p_2, \ldots, p_n \rangle \) and a query trajectory \( T_q = \langle q_1, q_2, \ldots, q_m \rangle \), the Frechet distance is defined as below

\[
F_{i,j} = \begin{cases} 
\max_{h=1}^{i} d(p_h, q_1) & \text{if } j = 1 \\
\max_{k=1}^{j} d(p_1, q_k) & \text{if } i = 1 \\
\max(d(p_1, q_j), d(p_i, q_1), d(p_i, q_j)) & \text{otherwise}
\end{cases}
\]

(2)

where \( F_{i,j} \) denotes the Frechet distance between \( T[i,i] \) and \( T_q[1,j] \) and \( d(p_i, q_j) \) is the distance between \( p_i \) and \( q_j \) (typically the Euclidean distance which could be computed in \( O(1) \)). When the Frechet distance is used, we have \( \Phi = O(n \cdot m) \), \( \Phi_{inc} = O(m) \), and \( \Phi_{ini} = O(m) \), based on similar analysis as for the DTW distance.

**t2vec** [18]. \( t2vec \) is a data-driven similarity measure based on deep representation learning. It adapts a sequence-to-sequence framework based on RNN [3] and takes the final hidden vector of the encoder [28] to represent a trajectory. It computes the similarity between two trajectories based on the Euclidean distance between their representations as vectors.

Given \( T \) and \( T_q \), it takes \( O(n) \) and \( O(m) \) time to compute their hidden vectors, respectively and \( O(1) \) to compute the Euclidean distance between two vectors [18]. Therefore, we know \( \Phi = O(n^2 \cdot m + 1) = O(n^2 \cdot m) \). Since in the context studied in this paper, we need to compute the similarities between many subtrajectories and a query trajectory \( T_q \), we assume that the representation of \( T_q \) under \( t2vec \) is computed once and re-used many times, i.e., the cost of computing the representation of \( T_q \), which is \( O(m) \), could be amortized among all computations of similarity and then that for each one could be neglected. Because of the sequence-to-sequence nature of \( t2vec \), given the representation of \( T[i, j - 1] \), it would take \( O(1) \) to compute that of \( T[i, j] \) \((1 \leq i < j \leq n)\). Therefore, we know \( \Phi_{inc} = O(1) \). Besides, we know \( \Phi_{ini} = O(1) \) since the subtrajectory involved in the computation of similarity, i.e., \( T[i,i] \) \((1 \leq i \leq n)\), has its size equal to 1.

The summary of \( \Phi \), \( \Phi_{inc} \) and \( \Phi_{ini} \) for the similarity measurements corresponding to the distance measurements DTW, Frechet and \( t2vec \) is presented in Table 1.

**4. NON-LEARNING BASED ALGORITHMS**

In this part, we introduce three types of algorithms, namely an exact algorithm ExactS, an approximate algorithm SizeS, and splitting-based algorithms including PSS, POS and POS-D. The ExactS algorithm is based on an exhaustive search with some careful implementation and has the highest complexity, the SizeS algorithm is inspired by existing studies on subsequence matching [15] [43] and provides a tunable parameter for controlling the trade-off between efficiency and effectiveness, and the splitting-based algorithms are based on the idea of splitting the data trajectory for constructing subtrajectories as candidates of the solution and run the fastest. A summary of the time complexities of these algorithms is presented in Table 2.

### 4.1 The ExactS Algorithm

A straightforward algorithm for the SimSub problem on \( T \) and \( T_q \) is to enumerate all possible subtrajectories \( T[i,j] \) \((1 \leq i \leq j \leq n)\) of the data trajectory \( T \), and for each of them, compute the similarity between \( T[i,j] \) and \( T_q \), i.e., \( \Theta(T[i,j], T_q) \), and then return the one with the greatest similarity. We call this algorithm *ExactS*. A straightforward implementation of ExactS would incur a time cost of \( O(n^2 \cdot \Phi) \) since \( T \) has \( \sum_{i=1}^{n} \left( \begin{array}{c} n+1 \\ i \end{array} \right) \) subtrajectories and the cost of computing the similarity between each subtrajectory and \( T_q \) is \( \Phi \) in general. For example, when DTW or Frechet is used, the time complexity is \( O(n^2 \cdot nm) = O(n^3m) \) and when \( t2vec \) is used, it is \( O(n^2 \cdot (n+m)) \).

A better implementation of ExactS is to compute the similarities between the subtrajectories and \( T_q \) incrementally as much as possible as follows. It involves \( n \) iterations, and in the \( i \)-th iteration, it computes the similarity between each subtrajectory starting from the \( i \)-th point and the query trajectory in an ascending order of the ending points, i.e., it computes \( \Theta(T[i,i], T_q) \) (from scratch) first and then computes \( \Theta(T[i,i+1], T_q) \), ..., \( \Theta(T[i,n], T_q) \) sequentially and

| **Time complexities** | **DTW** | **Frechet** | **t2vec** |
|------------------------|---------|------------|-----------|
| \( \Phi \) (general)   | \( O(n \cdot m) \) | \( O(n \cdot m) \) | \( O(n \cdot m) \) |
| \( \Phi_{inc} \) (incremental) | \( O(m) \) | \( O(m) \) | \( O(1) \) |
| \( \Phi_{ini} \) (initial) | \( O(m) \) | \( O(m) \) | \( O(1) \) |
incrementally. During the process, it maintains the sub-trajectory that is the most similar to the query one, which has been traversed so far. As could be verified, it would traverse all possible sub-trajectories after \( n \) iterations. The ExactS algorithm with this implementation is presented in Algorithm 1.

Consider the time complexity of this implementation. Since there are \( n \) iterations and in each iteration, the time complexity of computing \( \Theta(T[i,i], T_q) \) is \( O(n) \) and the time complexity of computing \( \Theta(T[i,i+1], T_q) \), ..., and \( \Theta(T[i,n], T_q) \) is \( O(n \cdot \Phi_{inc}) \), we know that the overall time complexity is \( O(n \cdot \Phi_{inc}) \). For example, when DTW or Frechet is used, the time complexity is \( O(n \cdot (m + n + m)) = O(n^2m) \) and when \( t2vec \) is used, the time complexity is \( O(n \cdot (1 + n + 1)) = O(n^2) \). Compared with the straightforward implementation, this implementation based on incremental computations is faster by a factor of \( O(n) \) roughly. Besides, it is hard to improve the implementation of ExactS to see this, consider the case when \( t2vec \) is used, where ExactS has its time complexity of \( O(n^2) \) which is the best we can achieve since ExactS needs to traverse \( O(n^2) \) sub-trajectories.

### 4.2 The SizeS Algorithm

While the ExactS algorithm could be implemented with incremental computations as above, it has time complexity still at least \( O(n^2) \), which makes it limited to small and moderate datasets only. This is essentially due to the fact that ExactS explores all possible \( \binom{n+1}{2} \) sub-trajectories, many of which might be quite dissimilar from the query trajectory and could be ignored. For example, by following some existing studies on subsequence matching \[15\] \[23\], we could restrict our attention to only those sub-trajectories which have similar sizes as the query one for better efficiency. Specifically, we enumerate all sub-trajectories that have their sizes

Table 2: Time complexities of algorithms (\( n_1 < < n \))

| Algorithms         | abstract similarity measurement | DTW               | Frechet          | \( t2vec \)         |
|---------------------|----------------------------------|-------------------|------------------|---------------------|
| ExactS              | \( O(n \cdot (\Phi_{ini} + n \cdot \Phi_{inc})) \) | \( O(n^2 \cdot m) \) | \( O(n^2 \cdot m) \) | \( O(n^2) \)     |
| SizeS               | \( O(n \cdot (\Phi_{ini} + (m + \xi) \cdot \Phi_{inc})) \) | \( O((\xi + m) \cdot n \cdot m) \) | \( O((\xi + m) \cdot n \cdot m) \) | \( O((\xi + m) \cdot n) \) |
| PSS, POS, POS-D     | \( O(n_1 \cdot \Phi_{ini} + n \cdot \Phi_{inc}) \) | \( O(n \cdot m) \) | \( O(n \cdot m) \) | \( O(n) \)      |
| RLS, RLS-Skip (learning-based) | \( O(n_1 \cdot \Phi_{ini} + n \cdot \Phi_{inc}) \) | \( O(n \cdot m) \) | \( O(n \cdot m) \) | \( O(n) \)      |

1: \( T_{best} \leftarrow \emptyset \); \( \Theta_{best} \leftarrow 0 \);
2: for all \( 1 \leq i \leq |T| \) do
3: \( \Theta(T[i,i], T_q) \);
4: if \( \Theta(T[i,i], T_q) > \Theta_{best} \) then
5: \( T_{best} \leftarrow T[i,i] \); \( \Theta_{best} \leftarrow \Theta(T[i,i], T_q) \);
6: end if
7: for all \( i + 1 \leq j \leq |T| \) do
8: \( \Theta(T[i,j], T_q) \) based on \( \Theta(T[i,j-1], T_q) \);
9: if \( \Theta(T[i,j], T_q) > \Theta_{best} \) then
10: \( T_{best} \leftarrow T[i,j] \); \( \Theta_{best} \leftarrow \Theta(T[i,j], T_q) \);
11: end if
12: end for
13: end for
14: return \( T_{best} \);

within the range \( [m-\xi, m+\xi] \), where \( \xi \in [0, n-m] \) is a pre-defined parameter that controls the trade-off between the efficiency and effectiveness of the algorithm. Again, we adopt the strategy of incremental computation for the similarities between those sub-trajectories starting from the same point and the query trajectory, i.e., \( \Theta(T[i,i+(m-\xi-1)], T_q) \), \( \Theta(T[i,i+(m-\xi)], T_q) \), ..., \( \Theta(T[i,i+(m+\xi-1)], T_q) \) for some \( i \in [1,n] \) (note that for some \( i \)'s such as \( i = n \), there exist no sub-trajectories starting from the \( i^{th} \) point and having their sizes in \( [m-\xi, m+\xi] \) and in these cases, no sub-trajectories are considered). We call this algorithm SizeS and analyze the time complexity of this algorithm as follows. The time complexity of computing the similarities among all sub-trajectories starting from a specific point and having their sizes within the range \( [m-\xi, m+\xi] \) is \( O(\Phi_{ini} + (m-\xi-1) \cdot \Phi_{inc} + 2\xi \cdot \Phi_{inc}) \), where \( \Phi_{ini} + (m-\xi-1) \cdot \Phi_{inc} \) is cost of computing \( \Theta(T[i,i+(m-\xi-1)], T_q) \) and \( 2\xi \cdot \Phi_{inc} \) is the cost of computing \( \Theta(T[i,i+(m-\xi-1)], T_q) \) for \( j \in [1,i-\xi,i,m+\xi-1] \). It could be further reduced to \( O(\Phi_{ini} + (m+\xi) \cdot \Phi_{inc}) \). Therefore, the overall time complexity of SizeS is \( O(n \cdot (\Phi_{ini} + (m+\xi) \cdot \Phi_{inc})) \). For example, when DTW or Frechet is used, the time complexity of SizeS is \( O(n \cdot \Phi_{inc}) \) and when \( t2vec \) is used, the time complexity is \( O(n \cdot (1 + (\xi + m) \cdot 1)) = O((\xi + m) \cdot n) \) - the cost of its effectiveness. Besides, SizeS still needs to explore \( O(\xi \cdot n) \) sub-trajectories, which restricts its application on small and moderate datasets only.

### 4.3 Splitting-based Algorithms

The ExactS algorithm is costly since it explores \( O(n^2) \) sub-trajectories and the SizeS algorithm runs faster than ExactS since it explores about \( O(\xi \cdot n) \) sub-trajectories \( (\xi < < n) \). Thus, an intuitive idea to push the efficiency further up is to explore fewer sub-trajectories. In the following, we design a series of three approximate algorithms which all share the idea of splitting a data trajectory into several sub-trajectories and among them, returning the one that is the most similar to the query trajectory. These algorithms differ from each other in using different heuristics for deciding where to split the data trajectory. With this splitting strategy, the number of sub-trajectories that would be explored is bounded by \( n \) and in practice, much less than \( n \). We describe these algorithms as follows.

1. **Prefix-Suffix Search (PSS).** The PSS algorithm is a greedy one during which it maintains a variable \( T_{best} \) storing the sub-trajectory that is the most similar to the query trajectory found so far. Specifically, it scans the points of the data trajectory \( T \) in the order of \( p_1, p_2, ..., p_n \). When it scans \( p_i \), it checks whether any of the two sub-trajectories it would form if it splits \( T \) at \( p_i \), i.e., \( T[h,i] \) and \( T[i,n] \) where \( p_h \) is the point following the one where the last split was done if any and \( p_h \) is the first point \( p_1 \) otherwise, is more similar

\[5\]
It is \( \Phi \) corresponding to a prefix of the trajectory that is to be split and \( T[i, n] \) a suffix, and this is why we have the name “Prefix and Suffix Search”. If so, it performs a split operation at \( p_i \) and updates \( T_{best} \) to the subtrajectory which is more similar to the query trajectory among the two formed subtrajectories; and if not, it continues to scan the next point \( p_{i+1} \). At the end, it returns \( T_{best} \). The procedure of PSS is presented in Algorithm 2 where we replace the part of computing the similarity between a suffix \( T[i, n] \) with the part of computing the similarity between their reversed versions, denoted by \( T[i, n]^R \) and \( T_q^R \), respectively, for efficiency consideration (explanations are provided in the time complexity analysis next).

We analyze the time complexity of PSS as follows. When it scans a specific point \( p_i \), the time costs consists of that of computing \( \Theta(T[h, i], T_q) \) and also that of computing \( \Theta(T[i, n], T_q) \). Consider the former part. If \( i = h + 1 \), it is \( (\Phi_{ini} + \Phi_{inc}) \), where \( \Phi_{ini} \) is the cost of computing \( \Theta(T[h, h], T_q) \) and \( \Phi_{inc} \) is the cost of computing \( \Theta(T[h, h + 1], T_q) \) based on \( \Theta(T[h, h], T_q) \). If \( i > h + 1 \), it is \( \Phi_{inc} \) since \( \Theta(T[h, i], T_q) \) could be computed based on \( \Theta(T[h, i - 1], T_q) \) incrementally. With a straightforward implementation, the latter is \( O(\Phi) \), i.e., the cost of computing the similarity from scratch, resulting the time complexity of PSS the same as that of the ExactS algorithm, which is not desirable. To retain the efficiency of PSS, we use a little trick here which is to replace the part of computing the similarity between \( T[i, n] \) and \( T_q \), which cannot be done incrementally, by a part of computing the similarity between their reversed versions, i.e., \( \Theta(T[i, n]^R, T_q^R) \), which could done incrementally (i.e., \( \Theta(T[i, n]^R, T_q^R) \) could be computed incrementally based on \( \Theta(T[i + 1, n]^R, T_q^R) \)). Note that for some similarity measurements such as DTW and Frechet, the similarity between two trajectories does not change when they are reversed, i.e., \( \Theta(T[i, n]^R, T_q^R) = \Theta(T[i, n], T_q) \) while for some other similarity measurements such as t2vec, this does not hold but \( \Theta(T[i, n], T_q) \) and \( \Theta(T[i, n]^R, T_q^R) \) are positively correlated as we found via experiments. With this trick applied, the latter time cost would be \( O(\Phi_{inc}) \). Overall, the time complexity of PSS (with the trick applied) is \( O(n_1 \cdot (\Phi_{ini} + \Phi_{inc} + \Phi_{hinc}) + n_2 \cdot (\Phi_{ini} + \Phi_{inc})) = O(n_1 \cdot \Phi_{ini} + \Phi_{inc} + \Phi_{hinc}) \), where \( n_1 \) is the number of points where splits are done, \( n_2 \) is the number of points where no splits are done and \( n_1 + n_2 = n \). For example, when DTW or Frechet is used, the time complexity of PSS is \( O(n_1 \cdot m + n \cdot m) = O(n \cdot m) \) and when t2vec is used, it is \( O(n_1 \cdot 1 + n_1 \cdot 1) = O(n) \).

(2) Prefix-Only Search (POS). In the PSS algorithm, when it scans a point \( p_i \), it considers two subtrajectories, namely \( T[h, i] \) and \( T[i, n] \), that could be potentially formed if a split operation is performed at \( p_i \). An alternative is to consider the prefix \( T[h, i] \) only and the intuition is that the suffix \( T[i, n] \) would eventually be split further and thus it could be ignored from being considered. A benefit of adopting this alternative strategy is that the time cost would be reduced to some extent in practice since it helps to eliminate the cost of computing \( \Theta(T[i, n], T_q) \). The algorithm we obtain by modifying the PSS algorithm with this alternative corresponds to the POS algorithm. As could be verified, theoretically, the time complexity of the POS algorithm is the same as the PSS algorithm though the former runs faster in practice.

### Algorithm 2: Prefix-Suffix Search (PSS)

**Input:** A data trajectory \( T \) and a query trajectory \( T_q \)

**Output:** A subtrajectory of \( T \) that is similar to \( T_q \)

1. \( T_{best} \leftarrow \emptyset; \Theta_{best} \leftarrow 0; \)
2. compute \( \Theta(T[n : n]^R, T_q^R); \)
3. compute \( \Theta(T[n - 1 : n]^R, T_q^R), \Theta(T[n - 2 : n]^R, T_q^R), \ldots, \Theta(T[1 : n]^R, T_q^R) \) incrementally;
4. \( h \leftarrow 1; \)
5. for all \( 1 \leq i \leq |T| \) do
6. compute \( \Theta(T[h, i], T_q) \) incrementally if possible;
7. if \( \max\{\Theta(T[h, i], T_q), \Theta(T[i, n]^R, T_q^R)\} > \Theta_{best} \) then
8. \( \Theta_{best} \leftarrow \max\{\Theta(T[h, i], T_q), \Theta(T[i, n]^R, T_q^R)\}; \)
9. if \( \Theta(T[h, i], T_q) > \Theta(T[i, n]^R, T_q^R) \) then
10. \( T_{best} \leftarrow T[h, i]; \)
11. else
12. \( T_{best} \leftarrow T[i, n]; \)
13. end if
14. \( h \leftarrow i + 1; \)
15. end if
16. end for
17. return \( T_{best}; \)

(3) Prefix-Only Search with Delay (POS-D). The POS algorithm performs a split operation whenever a prefix which is better than the best subtrajectory known so far is found. This looks a bit rush and may prevent a better subtrajectory to be formed by extending it with a few more points. Thus, we design a variant of the POS algorithm, called Prefix-Only Search with Delay (POS-D), which would wait for it to scan more points once a prefix is found to be more similar to the query trajectory than the best subtrajectory known so far and then split at one of these \( D + 1 \) points such that the formed prefix is the most similar to the query trajectory among all possible \( D + 1 \) prefixes. It could be verified that with this delay mechanism, the time complexity of the algorithm does not change and keeps the same as that of the POS algorithm though in practice, the time cost would be slightly higher.

### 5. REINFORCEMENT LEARNING BASED ALGORITHM

A splitting-based algorithm has its effectiveness rely on the quality of the process of splitting a data trajectory, i.e., in order to find a solution of high quality, it requires to perform split operations at appropriate points such that some subtrajectories that are similar to a query trajectory are formed and then explored. The three splitting-based algorithms introduced in Section 4, namely PSS, POS and POS-D, mainly use some hand-crafted heuristics for making decisions on whether to perform a split operation at a specific point. For example, the PSS algorithm would choose to perform a split operation if one of the two subtrajectories including a prefix one and a suffix one that would be potentially formed if a split operation is performed is more similar to the query trajectory than the best subtrajectory found so far. This process of splitting a trajectory into subtrajectories is a typical sequential decision making process, i.e., it scans the points sequentially and at each point, it needs to make a decision on whether to perform a split operation at this point or not. In this paper, we propose to model this sequential decision making process as a Markov

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decision process (MDP) \cite{24} (Section 5.1), adopt a deep-Q-network (DQN) \cite{20} for learning an optimal policy for the MDP (Section 5.2) and then develop an algorithm called reinforcement learning based search (RLS) which corresponds to a splitting-based algorithm that uses the learned policy for the process of splitting a data trajectory (Section 5.3) and an augmented version of RLS called RLS-Skip with better efficiency (Section 5.4).

5.1 Trajectory Splitting as a MDP

A MDP consists of four components, namely states, actions, transitions, and rewards, where (1) a state captures the environment that is taken into account for decision making by a decision-maker called an agent; (2) an action is a possible decision that could be made by the agent; (3) a transition means that the state would be changed from one to another once an action is taken; and (4) a reward which is associated with a transition corresponds to some feedback indicating the quality of the action taken which causes the transition - the larger it is, the higher the quality of the action is. Specifically, we model the process of splitting a data trajectory as a MDP as follows.

1. **States.** We denote a state by \( s \). Suppose it is currently scanning the \( t \)th point of the data trajectory \( T_t \), i.e., \( p_t \), and \( p_1 \) denotes the point following the one at which the last split operation happens if any and \( p_1 \) otherwise.

   We define the state of the current environment as a triplet \((\Theta_{\text{best}}, \Theta_{\text{pre}}, \Theta_{\text{suf}})\), where \( \Theta_{\text{best}} \) is the largest similarity between a subtrajectory found so far and the query trajectory \( T_q \), \( \Theta_{\text{pre}} \) is \( \Theta(T[t,h], T_q) \) and \( \Theta_{\text{suf}} \) is \( \Theta(T[t,n], T_q) \). As could be noticed, a state captures the information about the query trajectory, the data trajectory, the point on which the last split happens, and the point it is scanning, etc. Note that the state space is three dimensional continuous one.

2. **Actions.** We denote an action by \( a \). We define two actions, namely \( a = 1 \) and \( a = 0 \). The former means to perform a split operation at the point that is being scanned and the latter means to move on to scan the next point.

3. **Transitions.** In our process of splitting a trajectory, given a current state \( s \) and an action \( a \) to take, the probability that we would observe a specific state \( s' \) is unknown.

   We note that the method that we use for solving the MDP in this paper is a model-free one and could solve the MDP problem even with its transition information unknown.

4. **Rewards.** We denote a reward by \( r \). We define the reward associated with the transition from state \( s \) to state \( s' \) after action \( a \) is taken as \((s', \Theta_{\text{best}} - s, \Theta_{\text{best}})\), where the \( s', \Theta_{\text{best}} \) is the first component of state \( s' \) and \( s, \Theta_{\text{best}} \) is the first component of state \( s \). The intuition is that by defining the reward this way, the goal of the process of splitting a data trajectory, which is to form a subtrajectory with the greatest possible similarity to the query trajectory, is consistent with that of the MDP problem, which is to maximize the accumulated rewards. To see this, consider that the process goes through a sequence of states \( s_1, s_2, ..., s_N \) and ends at \( s_N \) and \( T_1, T_2, ..., T_{N-1} \) denote the rewards received at these states except for the termination state \( s_N \). Then, when the future rewards are not discounted, we have

\[
\Sigma T_t = \Sigma (s_t, \Theta_{\text{best}} - s_{t-1}, \Theta_{\text{best}}) = s_N, \Theta_{\text{best}} - s_1, \Theta_{\text{best}}
\]

where \( s_N, \Theta_{\text{best}} \) corresponds to the similarity between the best subtrajectory found and the query trajectory \( T_q \) and \( s_1, \Theta_{\text{best}} \) corresponds to the smallest similarity possible between two trajectories, e.g., \( 0 \), which is used to initialize best known similarity at the beginning. Therefore, maximizing the accumulative rewards is equivalent to maximizing the similarity between the subtrajectory to be found and \( T_q \) in this case.

5.2 Deep-Q-Network (DQN) Learning

The core problem of a MDP is to find an optimal policy for the agent, which corresponds to a function \( \pi \) that specifies the action that the agent should choose when in a specific state so as to maximize the accumulative rewards. One type of methods that are commonly used for finding the optimal policy for a given MDP is those value-based methods. The major idea is as follows. First, we define an optimal action-value function \( Q^*(s,a) \) (or \( Q \) function) which represents the maximum amount of expected accumulative rewards we would receive by following any policy after seeing the state \( s \) and taking the action \( a \). Second, we estimate the optimal action-value function \( Q^*(s,a) \) via some methods such as \( Q \)-learning \cite{55} and deep-Q-network (DQN) \cite{20}.

Third, we return the policy which always chooses the action that maximizes the \( Q^* \) \((s,a) \) function for a given state. Different value-based methods differ in using different methods for estimating the \( Q \) function. For example, the \( Q \)-learning method estimates the function by using following Bellman Equation \cite{21} as an iterative update step:

\[
Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [R + \gamma \cdot \max_a Q(s',a') - Q(s,a)],
\]

where \( s \) is the current state, \( a \) is the action that is chosen based on some policy derived from \( Q \) (e.g., using the \( \epsilon \)-Greedy strategy), \( R \) and \( s' \) are the reward and the next state observed, respectively, \( \alpha \) is the learning rate, and \( \gamma \) is a discount factor.

The DQN method parameterizes the \( Q \) function as \( Q(s,a; \theta) \) and then learns the parameters \( \theta \). \( Q \)-learning is suitable for cases where the state space is discrete and of small or moderate size while DQN is capable of handing cases where the state space is large and even continuous. Consider that in our MDP, the state space is a three dimensional continuous one, we adopt a DQN method. Specifically, we use the deep \( Q \) learning with replay memory \cite{21} for learning the \( Q \) functions. This method maintains two neural networks, one called main network \( Q(s,a; \theta) \) which is used to estimate the \( Q \) function and the other called target network \( \hat{Q}(s,a; \theta^-) \) which is used to compute some form of loss for training the main network. Besides, it maintains a fixed-size pool called replay memory containing the latest transitions that are sampled uniformly and used for training the main network, and the intuition is to avoid the correlation among consecutive transitions. An overview of the method is shown in Figure 1.

Specifically, the detailed procedure of deep \( Q \) learning for our MDP is presented in Algorithm 3 which we go through as follows.

1. We maintain a database \( D \) of data trajectories and a set of \( D_q \) of query trajectories. It first initializes the reply memory \( M \) with some capacity, the main network \( Q(s,a; \theta) \) with random weights, and the target network \( \hat{Q}(s,a; \theta^-) \) by copying \( Q(s,a; \theta) \) (Lines 1 - 3). Then, it involves a sequence of many episodes, and for each episode, it samples a data trajectory \( T \) from \( D \) and a query trajectory \( T_q \) from \( D_q \), both uniformly (Lines 4 - 5). It initializes a variable \( h \) such that \( p_h \) maintains the point following
5.3 Reinforcement Learning based Search Algorithm (RLS)

Once we have estimated the Q functions $Q(s,a;\theta)$ via the deep Q learning with experience replay, we use the policy, which always takes the action that maximizes $Q(s,a;\theta)$ for a given state $s$, as one for making decisions on whether to perform a split operation at a specific point in the process of splitting a trajectory, and among all subtrajectories formed as a result of the process, we return the one with the greatest similarity to the query trajectory $T_q$. We call this algorithm reinforcement learning based search (RLS). Essentially, it is the same as PSS except that it uses a policy learned via deep Q learning instead of human-crafted heuristics for making decisions on how to split a data trajectory.

Algorithm 3 Deep-Q-Network (DQN) Learning with Experience Replay

Input: A database $D$ of data trajectories and a set of $D_q$ of query trajectories;

Output: Learned action-value function $Q(s,a;\theta)$;

1: initialize the replay memory $M$;
2: initialize the main network $Q(s,a;\theta)$ with random weights $\theta$;
3: initialize the target network $\hat{Q}(s,a;\theta^-)$ with weights $\theta^- = \theta$;
4: for episode $= 1,2,3,...$ do
5: sample a data trajectory $T$ and a query trajectory $T_q$;
6: $h \leftarrow 1$;
7: $\Theta_{\text{best}} \leftarrow \Theta(T[h,h],T_q)$; $\Theta_{\text{suf}} \leftarrow \Theta(T[h,n]^R,T_q^R)$;
8: observe the first state $s_1 = (\Theta_{\text{best}},\Theta_{\text{suf}})$;
9: for each time step $1 \leq t \leq |T|$ do
10: select a random action $a_t$ with probability $\epsilon$ and otherwise select action $a_t = \arg\max_a Q(s_t,a;\theta)$;
11: if $a_t = 1$ then
12: $h \leftarrow t + 1$;
13: $\Theta_{\text{best}} \leftarrow \max\{s_t,\Theta_{\text{best}},s_t,\Theta_{\text{pre}},s_t,\Theta_{\text{suf}}\}$ (Line 11 - 14).
14: end if
15: $\Theta_{\text{pre}} \leftarrow \Theta(T[h,h],T_q)$; $\Theta_{\text{suf}} \leftarrow \Theta(T[h,n]^R,T_q^R)$;
16: observe the next state $s_{t+1} = (\Theta_{\text{best}},\Theta_{\text{pre}},\Theta_{\text{suf}})$;
17: observe the reward $r_t = s_{t+1},\Theta_{\text{best}} - s_t,\Theta_{\text{best}}$;
18: store the experience $(s_t,a_t,r_t,s_{t+1})$ in the replay memory $M$;
19: uniformly sample a random minibatch of experiences from $M$;
20: perform a gradient descent step on the loss as computed by Equation (4) wrt $\theta$;
21: end for
22: copy the main network $Q(s,a;\theta)$ to $\hat{Q}(s,a;\theta^-)$;
23: end for

RLS has the same time complexity as PSS since (1) RLS differs from PSS only in the way of making decisions on whether or not to split on a point; (2) in RLS, the time cost of making a decision corresponds to that of going through the main network of DQN with some state information, which is $O(1)$ given that the network is small-size (e.g., a few layers); and (3) in PSS, the time cost is that of simply conducting two comparisons between the similarities of the subtrajectories that are being considered and that of the most similar subtrajectory found so far, which is clearly $O(1)$. In terms of effectiveness, RLS provides consistently better solutions than PSS and other splitting-based algorithms POS and POS-D, as will be shown in the empirical studies.

5.4 Reinforcement Learning based Search with Skipping (RLS-Skip)

In the RLS algorithm, each point is considered as a candidate for performing a split operation. While this helps to attain a reasonably large space of subtrajectories for exploration and hence achieving good effectiveness, it is somehow conservative and incurs some cost of decision marking for each point. An alternative is to go a bit more optimistic and skip some points from being considered as places for split operations. The benefit would be immediate, i.e., the cost of making decisions at these points is saved. Motivated by this, we propose to augment the MDP that is used by RLS by introducing $k$ more actions (apart from two existing ones: scanning the next point and performing a split...
operation), namely skipping 1 point, skipping 2 points, ..., skipping \( k \) points. Here, \( k \) is a hyperparameter, and by skipping \( j \) points \((j = 1, 2, ..., k)\), it means to skip points \( p_{i+1}, p_{i+2}, ..., p_{i+j} \) and scan point \( p_{i+j+1} \) next, where \( p_i \) is the point that is being scanned. All other components of the MDP are kept the same as that for RLS. Note that when \( k = 0 \), this MDP reduces to the original one for RLS. We call the algorithm that is based on this augmented MDP RLS-Skip.

While in RLS, the cost of making decisions at those points that are skipped (i.e., that of going through the main network of the DQN) could be saved, the cost of constructing the states at those points that are not skipped would be more or less that of constructing the states at all points since the state at a point consists of some similarities which are computed incrementally based on the similarities computed at those points before the point. Thus, by applying the skipping strategy alone would not help much in reducing the time cost since the cost of maintaining the states dominates that of making decisions. To fully unleash the power of the skipping strategy, we propose to ignore those points that have been skipped when maintaining the states. That is, to maintain the state \((\Theta_{\text{best}}, \Theta_{\text{pre}}, \Theta_{\text{sub}})\) at a point \( p_i \), we compute \( \Theta_{\text{best}} \) and \( \Theta_{\text{pre}} \) in the same way as we do for RLS and \( \Theta_{\text{sub}} \) as the similarity between the query trajectory and the subtrajectory consisting of those points that are before \( p_i \) and have not been skipped. Here, the prefix subtrajectory corresponds to a simplification of that used in RLS [19]. While RLS-Skip has the same worse-case-time complexity as RLS, e.g., it reduces to RLS when no skipping operations happen, the cost of maintaining the states for RLS-Skip would be much smaller. As shown in our empirical studies, RLS-Skip runs significantly faster than RLS and also all non-learning splitting-based algorithms.

6. EXPERIMENTS

6.1 Experimental Setup

Dataset. Our experiments are conducted on two real-world taxi trajectory datasets. The first dataset is collected from the city of Porto, Portugal, which consists around 1.7 million trajectories over 18 months with a sampling interval of 15 seconds and a mean length around 60. The second dataset consists around 1.2 million trajectories collected from 13,000 taxis over 8 months in Harbin, China with non-uniform sampling rates and a mean length around 120.

Parameter Setting. For training t2vec model, we follow the original paper [18] by excluding those trajectories that are short and use their parameter settings. For the neural networks involved in the RL-based algorithms, i.e., RLS and RLS-Skip, we use a feedforward neural network with 2 layers. In the first layer, we use the ReLu function with 20 neurons, and in the second layer, we use the sigmoid function with \( 2+k \) neurons as the output corresponding to different actions, where for RLS we use \( k = 0 \) and for RLS-Skip, we use \( k = 3 \) by default. In the training process, the size of replay memory \( M \) is set at 2000. We train our model on 25k random trajectory pairs, using Adam stochastic gradient descent with an initial learning rate of 0.001. The minimal \( \epsilon \) is set at 0.05 with decay 0.99 for the \( \epsilon \)-greedy strategy, and the reward discount rate \( \gamma \) is set at 0.95.

Compared Methods. We compare RL-based Search (RLS), RL-based Search with skipping (RLS-Skip) and the proposed non-learning based algorithms (Section 4), namely ExactS, SizeS, PSS, POS, and POS-D. For PSS, we use the setting \( \xi = 5 \) (with the results of its effect shown later on). For POS-D, we vary the parameter \( D \) from 4 to 7, and since the results are similar, we use the setting \( D = 5 \). For RLS and RLS-Skip, when t2vec is adopted, we ignore the \( \Theta_{\text{sub}} \) component of a state based on empirical findings.

Evaluation Metrics. We use three metrics to evaluate the effectiveness of the approximate algorithms using the results of the exact algorithm ExactS as the ground truth. (1) Approximate Ratio (AR). AR for the subtrajectory similarity computation is defined as \( \frac{\Psi_{\text{app}}}{\Psi_{\text{ex}}(\Lambda)} \), where \( \Psi_{\text{app}} \) is the dissimilarity value returned by one of the approximate algorithms (e.g., SizeS, PSS, POS, POS-D, RLS, RLS-Skip), and \( \Psi_{\text{ex}}(\Lambda) \) is the dissimilarity value of the ExactS algorithm. A smaller AR value indicates a better algorithm. (2) Mean Rank (MR). We sort all the subtrajectories of the given trajectory \( T \) in ascending order of their dissimilarity values with query \( T_q \). We denote by \( \text{rank}_T(T_q) \) the rank of the returned subtrajectory \( T_s \) by an algorithm for a query trajectory \( T_q \). We report the mean number of \( \text{rank}_T(T_s) \) as MR. (3) Relative Rank (RR). As the number of subtrajectories of a data trajectory \( T \) varies a lot, we normalize the rank by the number of subtrajectories, i.e., \( \frac{\text{rank}_T(T_s)}{\# \text{sub}_T} \), where \( \# \text{sub}_T = |T|(|T|+1) \). We call the metric Relative Rank (RR). Smaller MR or RR value indicates a better algorithm.

Evaluation Platform. All the methods are implemented in Python 3.6. The implementation of RLS is based on Keras 2.2.0. The experiments are conducted on a server with 32-cores of Intel(R) Xeon(R) Gold 6150 CPU @ 2.70GHz 768.00GB RAM and one Nvidia Tesla V100-SXM2 GPU.

6.2 Experimental Results

(1) Effectiveness results. We evaluate the effectiveness of the proposed approximate algorithms, including SizeS, PSS, POS, POS-D, RLS and RLS-Skip, in terms of three evaluation metrics. Specifically, we randomly sample 10,000 trajectory pairs a dataset, and for each pair we use one trajectory as the query trajectory to search the most similar subtrajectory from the other one. Figure 2 shows the results of different algorithms on two datasets, using three trajectory similarity measurements, namely, t2vec, DTW and Frechet. The results clearly show that the proposed learning-based algorithms including RLS and RLS-Skip consistently outperform all other non-learning based approximate algorithms in terms of all three metrics on both datasets and under all three trajectory similarity measurements. For example, RLS outperforms POS-D, the best non-learning algorithm when using t2vec, by 70% (resp. 80%) in terms of RR on Porto (resp. Harbin); RLS outperforms PSS, the best non-learning based algorithm when using DTW, by 25% (resp. 20%) in terms of MR on Porto (resp. Harbin); RLS outperforms PSS, the best non-learning based algorithm when using Frechet, by 25% (resp. 20%) in terms of MR on Porto (resp. Harbin). Among all the non-learning based approximate algorithms, PSS performs the best in terms of all the three metrics for DTW and Frechet; However, for t2vec, PSS

\[1\]https://www.kaggle.com/c/pkdd-15-predict-taxi-service-trajectory-i/data
provides similar accuracy as POS and POS-D on Porto, but
performs much worse than POS and POS-D on Harbin. The
reason is that for DTW and Frechet, PSS computes exact
similarity values for suffix subtrajectories, while for t2vec, it
computes only approximate similarity values for suffix sub-
trajectories. Therefore, PSS for t2vec has a relatively worse
accuracy. The reason that for t2vec, PSS performs relatively
better on Porto than on Harbin could be that the mean

Figure 2: Results of effectiveness evaluation for t2vec (a)-(c), DTW (d)-(f) and Frechet (g)-(i).

Figure 3: Results of efficiency evaluation without index (a)-(f), with R-tree index (g)-(l) and Inverted index
(m)-(r).
length of trajectories of Porto (around 60) is smaller than that of Harbin (around 120), and thus PSS is likely to make more incorrect splitting decisions on Harbin based on the approximate similarity values estimated for the suffix trajectories. As expected, POS-D performs slightly better than POS in terms of accuracy since it essentially checks more subtrajectories for choosing points for splitting. We also observe that SizeS is not competitive compared with other approximate algorithms as it simply increases the search space to check more subtrajectories. Additionally, RLS-Skip, as a variant of RLS for better efficiency, works slightly worse than RLS, but it still has better effectiveness than those non-learning based algorithms.

(2) Efficiency results. We prepare different databases of data trajectories by including different amounts of trajectories from a dataset and vary the total number of points in a database. The results are shown in Figure 4. As expected, ExactS has the longest running time compared with other non-learning based algorithms and the learning-based algorithm due to its high time complexity. ExactS is usually around 7–15 times slower than PSS, POS, POS-D, RLS and 20–30 times slower than RLS-Skip. We observe that PSS, POS, POS-D, RLS and RLS-Skip have similar running time on both datasets for DTW and Frechet. This is expected as they have the same time complexity. We also observe that RLS is slightly slower than PSS, POS, POS-D. This is because RLS makes the splitting decision via a learning model while the other three use a simple similarity comparison. RLS-Skip runs the fastest since on those points that have been skipped, the cost of maintaining the states and making decisions is saved. We also observe that each algorithm takes longer to run on Harbin than on Porto, especially for DTW and Frechet. This is possibly because the average trajectory length of Harbin is around 120 and that of Porto is around 60.

(3) Scalability. We investigate the scalability of all the algorithms based on the results reported in Figure 4(a)-(r). All those splitting-based algorithms including PSS, POS, POS-D, RLS and RLS-Skip scale well. In particular, RLS-Skip runs for less than 100s on all datasets for all similarity measurements with the Inverted Index.

(4) Working with indexes. We further study the effect of indexes for subtrajectory similarity search. Following two recent studies [40, 35] on trajectory similarity search, we employ the same indexing techniques for subtrajectory similarity search: Bounding Box R-tree Index and Grid-based Inverted Index. Note that we do not employ those indexing and pruning methods that are specially designed for a specific trajectory similarity measurement as they cannot be applied to other similarity measurements and/or subtrajectory similarity search. Compared with the results without indexes in Figure 4(a)-(f), the results using the R-tree index as shown in Figure 4(g)-(l) are lower by around 10–20% and the results based on the Inverted Index as shown in Figure 4(m)-(r) are further lower.

Figure 4: Results of grouping evaluation for t2vec (a)-(f), DTW (g)-(l) and Frechet (m)-(r).

Figure 5: Efficiency of varying query lengths.

Table 3: The effect of skipping steps $k$ for RLS-Skip.

| Metrics | $k = 0$ | $k = 1$ | $k = 2$ | $k = 5$ |
|---------|---------|---------|---------|---------|
| AR      | 1.029   | 1.039   | 1.042   | 1.044   | 1.056   | 1.069   |
| MR      | 41.138  | 56.633  | 58.077  | 64.741  | 70.281  | 94.356  |
| RR      | 3.5%    | 5.4%    | 5.6%    | 5.8%    | 6.3%    | 8.9%    |
| Time (s) | 552     | 398     | 385     | 358     | 318     | 229     |

Skipped Pts 0% 3.1% 11.1% 17.7% 28.5% 47.0%
Figure 6: The effect of soft margin $\xi$ for SizeS.

Table 4: Comparison with Trajectory Similarity Computation and Subtrajectory Similarity Computation.

| Similarity measurement | t2vec | DTW | Frechet |
|------------------------|-------|-----|---------|
|                        | AR    | RR  | Time (s) |
| Sizes                   |       |     |          |
| Porto                   |       |     |          |
| SimTra                  | 1.313 | 23.3% | 285     |
| SimSub                  | 1.015 | 3.0%  | 396     |
| SimSub                  | 1.253 | 46.9% | 317     |
| SimSub                  | 1.095 | 1.3%  | 626     |
| RLS                     |       |      |          |
| Porto                   | 2.100 | 70.7% | 181     |
| SimTra                  | 1.026 | 3.5%  | 552     |
| RLS                     | 2.326 | 27%   | 271     |
| RLS                     | 1.084 | 1.8%  | 53.9%   |
| Harbin                  |       |      |          |
| SimTra                  | 752.831 | 1.3% | 317 |
| RLS                     | 1218.908 | 27% | 271 |
| RLS-Skip                | 64.729 | 4.4%  | 53.9%   |

(5) The effect of query trajectory length. To evaluate the effect of query length on the effectiveness of the proposed approximate algorithms, we group the query trajectories according to their lengths into four groups $G1 = [30, 45)$, $G2 = [45, 60)$, $G3 = [60, 75)$ and $G4 = [75, 90)$ and report the result of each group. The results on Porto and Harbin for t2vec, DTW and Frechet are shown in Figure 4. We observe that the effectiveness of PSS, POS, POS-D, RLS and RLS-Skip remains stable when the query length grows. However, the accuracy of SizeS fluctuates significantly with the change of the query length. This is because the length of the most similar subtrajectory may not have similar length as the query trajectory, and thus it may miss better results since its search space is constrained by the parameter $\xi$.

Figure 4(a)-(c) shows the effect of query length on the running time. We can see that for t2vec, the running times of all the algorithms are almost not affected by the query length. This is because we only need to represent the query trajectory as a vector once, which takes $O(m)$ time, and this has little impact on the running time. We observe that the running times of all the algorithms increase linearly or super-linearly with the query length for DTW and Frechet, which is consistent with the time complexity results in Table 4. Compared with the results without indexes in Figure 4(a)-(f), the results using the R-tree index shown in Figure 4(g)-(l) show around 10–20% reduction in running time. The results based on the Inverted Index are shown in Figure 4(m)-(r), and according to these results, the speedups are around 30 times.

(6) The effect of parameter $\xi$. This experiment is to evaluate the effect of the soft margin $\xi$ for SizeS. Intuitively, a big $\xi$ value indicates a larger search space, which approaches the search space of the ExactS algorithm as $\xi$ increases. Therefore, SizeS will check more candidate subtrajectories and its running time will increase. Figure 5 shows the effectiveness results of DTW in terms of AR, MR and RR and the running time on 10,000 trajectory pairs of Porto dataset. In Figure 5, we increase the value of $\xi$ until the result of SizeS is better than that of RLS and RLS-Skip in terms of AR, MR and RR. As expected, as $\xi$ grows the effectiveness of SizeS (as shown in Figure 5(a)-(c)) becomes better (approaches and exceeds RLS gradually); However, its running time increases and approaches to that of the ExactS algorithm while RLS runs much faster (as shown in Figure 5(d)).

(7) The effect of skipping steps $k$. This part is to evaluate the influence of the skipping steps $k$ for RLS-Skip. According to the results, a general trend is that with a larger settings of $k$, the effectiveness in terms of AR, MR and RR drops but the efficiency grows because RLS-Skip tends to skip more points. We present in Table 5 the results for DTW only due to the page limit. For ease of reference, we also report the portion of skipped points in the Porto dataset with 10,000 trajectories. Note that when $k$ is set to 0, the RLS-Skip degrades to RLS with the worst efficiency but the best effectiveness. Here, we choose the setting of $k = 3$ as a reasonable trade-off between effectiveness and efficiency.

(8) Comparison with similar trajectory search (SimTra). The traditional similar trajectory search (SimTra) can be considered as a subproblem of the SimSub problem because a data trajectory as a whole is actually one of subtrajectories of itself. This experiment is to evaluate the accuracy if we use the result of SimTra to approximate the result of SimSub. Similar to earlier experiments, we use the result of ExactS algorithm as the ground truth, and compare the average results of SimTra and SimSub using 10,000 trajectory pairs. We use RLS for SimSub. As shown in Table 4 in terms of effectiveness, SimSub performs much better than SimTra: the MR and RR of SimSub are around 10 times larger than those of SimSub for t2vec and 20 times for DTW and Frechet. This result shows that SimTra is not a good approximation for SimSub, though SimTra runs faster.

7. CONCLUSION

In this paper, we study the similar subtrajectory search (SimSub) problem and develop a suite of algorithms including an exact algorithm, an approximate algorithm providing a controllable trade-off between efficiency and effectiveness, and a few splitting-based algorithms among which some are based on pre-defined heuristics and some are based on deep reinforcement learning called RLS and RLS-Skip. We conducted extensive experiments on real datasets, which verified that among the approximate algorithms, learning-based algorithms achieve the best effectiveness and efficiency. One possible direction for future research is to develop learning-based methods for other subtrajectory related problems such as subtrajectory join.
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