Performance of MIMO FBMC-OQAM and MIMO OFDM-QAM under time Synchronization Errors in Perfect CSI and Imperfect CSI

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Abstract: The requirement of flexible allocation of available limited frequency resources led to rethink the possibilities to find an substitute to Orthogonal Frequency Division Multiplexing (OFDM) [1], which can also meet the necessities of future wireless communication. In this paper, we have compared one such alternative to OFDM, which is Filter Bank Multi Carrier (FBMC) [2] with Offset Quadrature Amplitude Modulator (OQAM). The BER performance of OQAM based MIMO FBMC compared with QAM based MIMO OFDM for various channels conditions and using different prototype filters [3] for FBMC and under the influence of Timing Offset errors with perfect CSI and imperfect CSI. Implementation of MIMO into OQAM-FBMC has a computational complexity compared to MIMO OFDM and we have used block coded MIMO OQAM-FBMC for simulation. Three prototype filters RRC, Hermite and PHYDAS are used to analyze the performance of OQAM-FBMC in relations of Signal to Interference Ratio under time offset and found that performance of PHYDAS prototype filter outperforms RRC and Hermite prototype filters.

Keywords: CP-OFDM,CSI,MIMO,OQAM-FBMC

I. INTRODUCTION

Introduction: Multi carrier modulation systems are considered as the key technique for greater data rates with better bandwidth efficiency for future wireless communication. Typically Orthogonal Frequency Division Multiplexing (OFDM) based transceivers uses MCM enabling higher data rates with bandwidth efficiency. OFDM uses Cyclic Prefix (CP) to reduce ISI and ICI at the cost of bandwidth. Alternatively, Filter Bank based multi carrier (FBMC) techniques has been well thought-out as potential contender for future wireless communication. An FBMC system, because of its lower spectral side lobes does not require Cyclic Prefix (CP) and in comparison to OFDM systems, FBMC has an edge of improved bandwidth efficiency with the help of well-designed prototype filters. Further to upturn channel capability Multiple Input Multiple Output (MIMO) systems are introduced which has multiple transmit and receive antenna.

II. MULTI CARRIER MODULATION & MIMO

A. Orthogonal Frequency Division Multiplexing (OFDM)

OFDM is a type of Multi Carrier Modulation technique, which consists of conventional modulators and demodulators, each with dissimilar orthogonal carrier frequencies. The transmitter output will be a combination of modulator outputs with orthogonal subcarriers. Let us consider if N data to be transmitted which are represented as \( A_k \), \( k = 0,1,\ldots,N-1 \) where \( A_k \) is a complex number of modulators such as QAM or QPSK. Also suppose that the \( k \)th carrier frequency for \( X_k \) is \( f_k \). Then, the transmitter output is given by,

\[
a(t) = \sum_{k=0}^{N-1} A_k e^{j2\pi f_k t}
\]  

A digital source will produce its output in a sampled-data fashion. By letting \( t = nT_s \), where \( T_s \) is the sample interval, the digital MCM transmitter output will be as given in equation 2,

\[
a(nT_s) = \sum_{k=0}^{N-1} A_k e^{j2\pi f_k nT_s}
\]  

The MCM transmitter output in equation 2 take the form with frequency spacing of \( f_s \), that is \( f_k = kf_s \), \( k = 0,1,\ldots,N-1 \), of,

\[
a(nT_s) = \sum_{k=0}^{N-1} A_k e^{j2\pi k f_s nT_s}
\]  

Letting \( f_s = 1/NT_s \) which is the minimum parting to maintain right angularity among signals of different modulators. The corresponding OFDM output signal is given by,

\[
a_n = a(nT_s) = \sum_{k=0}^{N-1} A_k e^{j2\pi nk/N}
\]  

B. MIMO–OFDM

The combination OFDM with MIMO is beneficial as OFDM supports multiple antennas with larger bandwidth because of its simplicity in equalization with MIMO systems. The performance of OFDM increases dramatically with the MIMO even under frequency selective fading channels and channel fading can be mitigated with one tap equalizers.
However, synchronization plays an important role in correctly demodulating signals at the receiver. Let us now characterize the MIMO-OFDM with transmitting and receiving antennas. Figure 1 is the MIMO system model, which consists of M Transmit antennas and N Receive antennas.

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_N
\end{bmatrix} = H_{N \times M} \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_M
\end{bmatrix} + \begin{bmatrix}
n_1 \\
n_2 \\
\vdots \\
n_N
\end{bmatrix}
\]  

(5)

Where \(y_1, y_2, \ldots, y_N\) are the received symbols corresponding to \(x_1, x_2, \ldots, x_M\) symbols. \(n_1, n_2, \ldots, n_N\) are the noise course and \(H_{N \times M}\) is the channel matrix. Therefore MIMO model can be represented as,

\[y = Hx + n\]  

(6)

Where the \(N \times N\) channel matrix is,

\[
H = \begin{bmatrix}
h_{11} & h_{21} & \cdots & h_{1M} \\
h_{21} & h_{22} & \cdots & h_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
h_{1N} & h_{2N} & \cdots & h_{NN}
\end{bmatrix}
\]  

(7)

The above channel model is used with 2 X 2 transmit and receive antenna in this paper.

C. OQAM-FBMC

The main drawback associated with OFDM based Multi carrier modulation is its high spectral sidelobes. The interferences because of high spectral sidelobes can only mitigated at the cost of transmission bandwidth, as a result alternative multi carrier modulation techniques has been a research interests for many researchers and because of employing prototype pulse shaping filter, Filter Bank Multi Carrier(FBMC) System gained interests in futuristic communication. FBMC systems have a flexibility to adjust system parameters like subcarrier spacing and frequency which is very difficult to achieve in OFDM systems. In Literature, three types of FBMC systems are familiarized, namely Offset-QAM based FBMC(OQAM-FBMC), Filtered multitone based FBMC(FMT-FBMC) and Cosine Modulated multitone based FBMC(CMT-FBMC). Because of its high data rates and bandwidth utilization efficiency we have considered OQAM-FBMC in this paper. OQAM/FBMC functions built on QAM codes whose in-phase and quadrature components are flabbergasted by half the symbol period. Right-Angularity between carriers is achieved through time flabbergasting in-phase and quadrature elements of the subcarrier symbols and using well designed pulse shaping filters which has decent property of frequency localization. The system model of the OQAM/FBMC communication system with the complex input symbols are inscribed as,

\[x_k(m) = p_k(m) + jq_k(m)\]  

(8)

Where \(k\) in equation 8 represents subcarrier with \(m^{th}\) symbol associated to it and it consist of \(p_k(m)\) and \(q_k(m)\) which are the real and unreal parts. The in-phase and quadrature components are flabbergasted in time domain by \(T/2\), where \(T\) is the symbol period. The symbols are then distributed through a group of filters which are used to transmission and modulated using \(K\) subcarrier modulators whose carrier frequencies are \(1/T\)-spaced aside. The OQAM/FBMC modulated signal is [8],

\[s(t) = \sum_{k=0}^{K-1} \sum_{m=-\infty}^{\infty} \left[ p_k(m)h(t - mT) + jq_k(m)h(t - mT - T/2) \right] e^{jk\varphi_t}\]  

(9)

Equation 9 is the representation of OQAM/FBMC modulated signal which has prototype filter with filter response \(h(t)\). OQAM/FBMC is up converted to radio frequency (RF) band after modulating \(s(t)\) and transmitted. For an ideal communication system, the received signal at the receiver should be equal to the transmitted signal counterpart at the transmitter. After down conversion from RF band, the received signal \(r(t)\) is demodulated with the help of \(K\) subcarrier demodulators and directed to a bank of matched filters. With sampling filtered signal every \(T\) Period the output symbols can be written as,

\[\hat{x}_k(m) = \hat{p}_k(m) + j\hat{q}_k(m)\]  

(10)

Where,

\[\hat{p}_k(m) = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{K-1} \int h(mT - t) \left[ a_k(m')h(t - m'T - mT) \cos[(k' - k)\varphi_t] - b_k(m')h(t - m'T) \sin[(k' - k)\varphi_t] \right] dt,\]  

(11)
\[ \hat{x}_k(m) = x'_k(m) \]  

(13)

D. MIMO OQAM-FBMC

The spectral as well as power proficiency of an offset quadrature amplitude modulation (OQAM) based filter bank multicarrier (OQAM-FBMC) can be improved by Multiple-input and multiple-output (MIMO) methods. For the reason that of the imaginary meddling in OQAM-FBMC systems, the application of MIMO with OQAM-FBMC is more complicated compared to MIMO OFDM systems [13]. Spatial multiplexing and Spatial diversity are the two main methods of the MIMO techniques and same can be useful to the OQAM-FBMC. Spatial multiplexing adapts transmitting diverse data streams over dissimilar antennas to increase the throughput of the system. In Spatial Diversity, the time or frequency domain diversity can be implemented at transmitter or receiver. In this paper we have considered a slab wise Space Frequency Block Coding technique is considered for OQAM-FBMC in which a slab of real valued data symbols will be alienated into an upper and lower slab. Each of these slabs has K subcarriers and they remain isolated by the guard band. Through the use of PHYDYAS prototype filter, in which the intervention mostly originates from the adjacent subcarriers, the upper blocks can be well isolated from the lower slabs with single guard subcarrier. The (k, n)th real SFBC code is considered as,

\[ C_{k,n} = \begin{bmatrix} a_k(n) & b_k(n) \\ (-1)^{n+1}b_k(n) & (-1)^n a_k(n) \end{bmatrix} \]  

(14)

Assuming the channel frequency response do not differ over the subcarriers of the code, the corresponding (k, n)th complex code is assumed as,

\[ C_{k,n} = \begin{bmatrix} a_k(n) + j d^{(i)}_{m(k,1),n,1} & b_k(n) + j d^{(i)}_{m(k,1),n,2} \\ (-1)^{n+1}b_k(n) + j d^{(i)}_{m(k,2),n,1} & (-1)^n a_k(n) + j d^{(i)}_{m(k,2),n,2} \end{bmatrix} \]  

(15)

Where the task of m (k, s) springs the subcarrier index of the (k, n)th SFBC slab, with s = 1, 2 demonstrating the upper and lower blocks, and \( d^{(i)}_{m(k,2),n,2} \) is the imaginary meddling suffered by the data on the subcarrier m(k, s) at the time n starting from first antenna.

III. SYNCHRONIZATION ISSUES IN MCM

Multicarrier modulation systems are more exposed to synchronization errors equated to single carrier systems. The Orthogonality among subcarriers is deeply affected due to synchronization offset problems which results in unadorned Inter-Symbol interference (ISI) and Inter-Carrier Interference (ICI). System performance can be significantly improved using cyclic prefix in OFDM in case of timing errors. The biggest advantage of FBMC lies in using prototype filters and not using Cyclic Prefix as use of CP consumes bandwidth and power. However, MIMO OQAM-FBMC will be affected by Synchronization errors like Carrier Frequency Offset (CFO) and Time Offset (TO) in the absence of perfect channel state information.

A. Modeling Time Offset errors

The received samples are shifted by way of time offset by a value \( \Delta t \) to its right or left liable on sign of \( \Delta t \). Let us assume that the transmitted signal is given by \( S_t[n] \), the received signal \( R_t[n] \) in the presence of time synchronization error can be expressed as:

\[ R_t[n + \Delta t] = S_t[n] + v[n] \]  

(16)

With generality, let us assume \( v[n] = 0 \), then,

\[ R_t[n + \Delta t] = S_t[n] \]  

(17)

Time offset degrades the performances of multi-carrier transceivers by presenting inter-symbol interference (ISI).

B. Impact of Time offset on OFDM

Cyclic prefix acts as a low complexity and effective technique to handle over scattered channels and time synchronization equalizers in OFDM transceivers. OFDM is frequently accommodated with cyclic prefix but infrequently with cyclic postfix. This means we have two different situations which can occur under time synchronization inaccuracies, depending on the direction of the time offset. Time synchronization error may occur to the right of cyclic prefix or to the left of the cyclic prefix. An OFDM structure that is affected by timing slips, that is time synchronization error off from cyclic prefix and where samples of adjacent symbol are erroneously selected practices severe deprivation of the performance. The demodulated OFDM signal after FFT which is affected by time offset can be written for the case \( \Delta t > \text{NCP} \) as[12],
\[ \hat{a}_{u',k'} = \frac{N - \Delta_t}{N} a_{u',k'} e^{j2\pi \Delta_t} \]

The first constituent of Eq. (18) represents valuable signal that is weakened and phase shifted by a term relational to sub-carrier index \( k' \). The second term in above equation constituent of Inter Channel Interference and the third constituent represents Inter Symbol Interference with the next symbol. When the OFDM structure that is affected by timing slips that is time synchronization error towards the cyclic prefix for the case when \( \Delta_t < NCP \), The demodulated OFDM signal after FFT which is unaffected by time offset can be inscribed as,

\[ \hat{a}_{u',k'} = \frac{N - \Delta_t}{N} a_{u',k'} e^{j2\pi \Delta_t} \]  

(19)

Because of the cyclic prefix, the Right-Anglearity is unspoiled and ISI and ICI terms do not appear. The timing inaccuracy towards the cyclic prefix results only as a phase shift.

### C. Time Offset in OQAM/FBMC

After passing baseband signal through the channel, the received and demodulated signal can be separated into the signal part and the noise part and is given as[11],

\[ y_{k0}(m_0) = a_{k0}(m_0) a_{k0}(m_0) + j_{k0}(m_0) \]  

(20)

With,

\[ a_{k0}(m_0) = \sum_{l=0}^{K0-1} C_l e^{j\pi (m_0 + l) \frac{T}{K0}} A_g \left[ -l - \frac{1}{T} \right] e^{-j2\pi k0 \frac{l}{K0}} \]  

(21)

And

\[ j_{k0}(m_0) = \sum_{(p,q) \neq (0,0)} a_{k0+p}(m_0 + q) e^{j\pi (p + q + pq) \frac{T}{K0}} e^{j\pi pm_0} \]

\[ \sum_{l=0}^{K0-1} C_l e^{\pi (2m0 + q) \frac{T}{K0}} X A_g \left[ -\frac{qK}{2} - l - \left( p + \frac{1}{T} \right) \right] e^{j\pi (2k0 + p) \frac{L}{K0}} \]  

(22)

Where \( k = k0 + p \), \( m = m0 + q \), \( Ag[ q, p ] = A(qT_s, pF_s) \), \( r = 1/f_sT_0 \) is linked to the Doppler shift and \( F_0 = 1/KT_s \).

### D. General Impact of TO in Communication Channel

For ease, assuming an simple single-path delayed model, which can be represented as[8],

\[ c(\tau) = \delta(\tau - d_0) \]  

(23)

\[ a_{k0}(m_0) = A_c[-l_d,0]e^{-j2\pi k0 \frac{L}{T}} \]  

(24)

\[ j_{k0}(m_0) = \sum_{(p,q) \neq (0,0)} a_{k0+p}(m_0 + q) e^{j\pi (p + q + pq) \frac{T}{K0}} e^{j\pi pm_0} \]

\[ X A_c \left[ -\frac{qK}{2} - l - \left( p + \frac{1}{T} \right) \right] e^{j\pi (2k0 + p) \frac{L}{K0}} \]  

(25)

From above equation it is clear that channel is now, time invariant and the channel coefficients \( a_{k0} \) and \( \tau_{k0}(q,p) = AcqK - l - p + 1re - jn(2k0 + p)K \) are now independent of time. Thus the Signal to interference ratio (SIR) is given by,

\[ \text{SIR}(l_d) = \sum_{(p,q) \neq (0,0)} \cos^2 \left( \frac{\pi}{K0} (p + q + pq) - \pi p \frac{L}{K0} \right) A_g^2 \left[ -\frac{qK}{2} - l_d - p \right] \]  

(26)

### IV SYSTEM PARAMETERS FOR SIMULATION

Performance analysis has been conducted using following systems parameters, To determine the suitability of various filters, frequency response of the filters are considered for PHYDYAS, Hermite and RRC and same has been evaluated using following system parameters as shown in Table I.

| Table-I. Frequency Response Parameters of Prototype Filters |
|----------------------------------------------------------|
| **System Parameter**                                   | **Filters** |
|                                                          | PHYDYAS     | RRC        | Hermite    |
| SubCarrier Spacing                                      | 15KHz       | 15KHz      | 15KHz      |
| Sampling Rate                                           | 15MSPS      | 15MSPS     | 15MSPS     |
| Overlapping Factor                                      | 8           | -          | -          |

Table-II shows the system parameters considered for the simulation of SIR performance in which the performance of OQAM/FBMC is evaluated with three different prototype filters (PHYDYAS, RRC and Hermite) under time offset carrier drift of 10%.
Finally, the BER performance of OQAM/FBMC and CP-OFDM has been evaluated considering the PHYDYAS prototype filters which have better SIR performance than RRC and Hermite prototype filters under Time Offset conditions. System parameters in Table-III are considered for the BER performance evaluation with various time delays spreads of 30ns, 60ns and 90ns. 2 x 2 MIMO system and Alamouti code with perfect and imperfect channel state information has been considered for this performance evaluation.

Table-II. SIR System Parameters for Simulation

| System Parameter       | Filters          |
|------------------------|------------------|
|                        | PHYDYAS | RRC      | Hermite |
| No. of Subcarriers     | 48      | 48       | 48       |
| No. of FBMC Symbols    | 16      | 16       | 16       |
| Sub-Carrier Spacing    | 15KHz   | 15KHz    | 15KHz    |
| Sampling Rate          | 15MSPS  | 15MSPS   | 15MSPS   |
| Initial Phase Shift    | 0       | 0        | 0        |
| Overlapping Factor     | 8       | -        | -        |

Table-III. BER System Parameters for Simulation

| System Parameter       | CP-OFDM | OQAM/FBMC |
|------------------------|---------|-----------|
| No. of Subcarrier      | 12      | 12        |
| No. of Symbols         | 32      | 32        |
| Sub-Carrier Spacing    | 15KHz   | 15KHz     |
| Sampling Rate          | 1.4MSPS | 1.4MSPS   |
| Filter                 | -       | PHYDYAS   |
| Overlapping Factor     | -       | 8         |
| CP Length              | 2µs     | -         |
| Channel                | Fast Fading | Fast Fading |

V. SIMULATION RESULTS

SIR and BER are the performance metrics used to analyze QAM-OFDM and OQAM-FBMC multi carrier modulation technique under the MIMO in various time offset channel conditions. Before performing SIR and BER analysis of CP-OFDM and OQAM/FBMC, OQAM/FBMC MCM has been analyzed with three prototype filters and system parameters shown in Table-II are used for the simulation. Out of PHYDYAS, RRC and Hermite prototype filters, PHYDYAS prototype filter with Overlapping Factor of 8 has good frequency response with minimum sidelobes and same has been considered for BER and SIR performance evaluation with perfect and Imperfect CSI.

**Fig. 2. Frequency Response of various Prototype Filters**

Figure 2 shows the simulation results of three prototype filters out of which PHYDYAS filter with 8 overlapping factor has minimum sidelobes. The Prototype filter is maintained Right-Angularity has been maintained for $T = T_0$ and $F = 2/T_0$.

**Fig. 3. SIR vs. Time Offset in FBMC with PHYDYAS Filter**

**Fig. 4. SIR vs. Time Offset in FBMC with Hermite Filter**
Performance of MIMO FBMC-OQAM and MIMO OFDM-QAM under time Synchronization Errors in Perfect CSI and Imperfect CSI

Fig. 5. SIR vs. Time Offset in FBMC with RRC Filter

Fig. 6. BER vs. SNR in FBMC with 90ns Delay Spread

Fig. 7. BER vs. SNR in FBMC with 60ns Delay Spread

Fig. 8. BER vs. SNR in FBMC with 30ns Delay Spread

Figure 3, 4 and 5 shows the SIR performance of OQAM-FBMC with PHYDYAS, Hermite and RRC prototype filters respectively. The performance of PHYDYAS filter is significant under time offset and hence BER performance simulation is done with PHYDYAS prototype filters with Doppler spread in time is considered for simulation with perfect and imperfect channel state information.

Figure 6, 7 and 8 shows the performance of CP-OFDM and FBMC with 2 X 2 MIMO and Alamouti coding is used. Figure 6 is the simulated result with 90ns delay spread, figure 7 and figure 8 is the simulated result with 60ns and 30ns respectively.

V. CONCLUSION

In this paper first we have evaluated the performance of FBMC under various prototype filters and found the performance of PHYDYAS filter with 4 overlapping factor is better than RRC and Hermite filters by 5dB under 2 X 2 MIMO systems. It is evident from the simulation results that the performance of OFDM with Cyclic Prefix is slightly better than FBMC under time offset. Also, we have to consider the payload capacity and computational hardware requirements of FBMC which is higher than CP-OFDM because of the absence of overhead like Cyclic Prefix. Further same work can be extended for Massive MIMO systems.

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