What is Surrealistic about Bohm Trajectories?

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Abstract

We discuss interferometers in Bohmian quantum mechanics. It is shown that, with the correct configuration space, Bohm trajectories in a which way interferometer are not surrealistic, but behaves exactly as common sense suggests. Some remarks about a way to generalize Bohmian mechanics to treat density matrix are also made.

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1 Introduction

Orthodox quantum mechanics, in the sense of Copenhagen interpretation, has succeed very well in its role of taking account experimental results. It is a kind of common sense that orthodox quantum mechanics is fapp, i.e. it is good for all practical purposes.

But it is also almost common sense that in the scope of Copenhagen interpretation it is not possible to understand quantum mechanics[1]. Just in the green years of quantum theory, one of his fathers, de Broglie, proposed[2] the pilot wave interpretation. One great problem of this attempt was that de Broglie pilot waves live in ordinary physical space, what made the generalization to many body systems a little fuzzy.

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Another attempt to insert trajectories in quantum mechanics was made by Bohm[3], in 1952. In his brilliant interpretation, the wave field actually creates a quantum potential, and particle trajectories are obtained as rays of a Hamilton-Jabobi like equation, where the quantum potential is added to the classical one. Two elements play decisive roles in Bohmian mechanics: the wave field $\Psi$ and configuration space, where Bohm trajectories live.

Up to now (and to author’s knowledge), Bohmian mechanics has survived to all critics, except perhaps for the one which will be treated in this letter. In the year of 1992, when Bohmian mechanics completes 40 years and his father passed away, Englert, Scully, Süssmann, and Walther gave to the community the striking article[4] entitled Surrealistic Bohm Trajectories. In this work, authors considered one bit which way (WW, also for welcher Weg) interferometers’ thought experiments (but almost realizable with modern quantum optics technology) and Bohm trajectories of simple (in opposition to WW) interferometers[5] to construct an incongruence between common thought and Bohm trajectories. With this in hands, referred authors claimed Bohm trajectories could not be realistic, and should be “surrealistic”.

This example of “surrealistic” Bohm trajectories was criticized in many ways, some of them claimed that between Bohmian mechanics and common sense, the first is stronger[6], other suggested there should be mathematical mistakes[7], and other even appealed to nonlocal instantaneous teleportation of energy to save the appearances[8].

The intention of this letter is to reanalyze ESSW experiment (in fact a pedagogical modification suggested by Dewdney, Hardy, and Squires[8]) but taking account of the role played by configuration space in Bohmian mechanics. In this way we show there is nothing “surrealistic” about Bohm trajectories in the example worked, and even that they agree with common sense when worked in the right way. Once again, it can be viewed as a feature of Bohr’s insistence on the essential wholeness of quantum mechanics.

This letter is organized as follows: we first review interferometers in Bohmian mechanics and sketch ESSW argument, then we review WW interferometers in the orthodox approach, and finally we revise the Bohmian approach to these systems. We then close the letter with brief conclusions.

2 Interferometers in Bohmian Mechanics and ESSW Argument

In this section we discuss how to understand interferometers in Bohmian mechanics. For simplicity we work with an incomplete (i.e. without the last beam
splitter) Mach-Zender interferometer (fig 1). As is the essence of any interferometer, two ways are permitted and an interference “region” takes place. Then, we sketch ESSW argument on “surrealistic” Bohm trajectories.

In Bohmian mechanics we write the wave field $\Psi (x)$, where $x$ denotes a configuration space parametrization, in polar representation

$$\Psi = R \exp \left( \frac{i}{\hbar} S \right),$$

and Schrödinger equation implies \cite{3} $P = R^2$, and $S$ should obey a continuity equation

$$\frac{\partial P}{\partial t} + \nabla \cdot \left( P \frac{\nabla S}{m} \right) = 0,$$

and a Hamilton-Jacobi like one

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(x) + U(x) = 0,$$

where $U(x)$ is the so called quantum potential, given by

$$U(x) = \frac{-\hbar^2 \nabla^2 R}{2m \frac{\nabla^2 R}{R}}.$$

After passage by the beam splitter, the wave field becomes (we shall neglect normalization factors whenever possible)

$$\Psi (x) = \psi_r (x) + \psi_t (x).$$

Except by the region $I$, $\psi_r$ and $\psi_t$ do not overlap. The quantum potential in this region is the same as for free particles (some details about wave packet structure are also interesting, but are beyond the scope of this letter). But for free particles $R$ is constant and quantum potential exerts no effect! In region $I$, however, the interference pattern is important and we actually have

$$R^2 = R_r^2 + R_t^2 + 2R_r R_t \cos \left( \frac{S_r - S_t}{\hbar} \right) \simeq 2R_r^2 (1 + \cos \Phi),$$

and quantum potential becomes very important because $\nabla^2 R$ grows up and $R$ becomes small. That is an interesting point about Bohmian mechanics which should be emphasized: interference becomes interaction through the quantum potential!
Fig. 1. Experimental scheme of an incomplete Mach-Zender interferometer.

Fig. 2. Bohm trajectories for an incomplete Mach-Zender interferometer.

The role played by this interference region in Bohm trajectories is dramatic. In [5] it can be found pictures of the quantum potential and Bohm trajectories for double slit interferometer, which essence also fills in the analysis made above.

In the simple example worked, if we forget details about Bohm trajectories and analyze only the question: in which detector will the trajectory which passes by \( r \) \( (t) \) end? we can understand the (at first time strange) fact that this trajectory should end up in detector 1 (2) (see fig. 2).

Just by symmetry arguments (in a balanced interferometer) it follows \( \psi_r (x) = \pm \psi_t (Rx) \) where \( R \) denotes reflection of configuration space through the beam splitter plane. Though \( \Psi (Rx) = \pm \Psi (x) \), \( R (Rx) = R (x) \), and \( S (Rx) = \text{mod} h/2 \).
As in Bohmian mechanics the velocity field is given by

\[ \mathbf{v} = \frac{\nabla S}{m}, \]

we obtain that Bohm trajectories can not cross the plane of the beam splitter in the free region.

Just as another equivalent argument, Bohm trajectories are like flux lines and so they can not cross one another. This establishes that Bohm trajectories, for the simple interferometer, are \( r \to 1, \ t \to 2 \).

ESSW argument can be viewed as follows: in orthodox quantum mechanics, there is no interaction in region \( I \), so particles just follow free evolution. Free evolution takes the wave packet described by \( \psi_r (\psi_t) \) to detector 2 (1). So, if we can independently record which way (\( r \) or \( t \)) and which detector (1 or 2) a one particle wave packet marks, one can distinguish between Bohm and Copenhagen quantum mechanics. Assuming the second to give the right result, Bohm trajectories should be “surrealistic”.

### 3 WW Interferometers

In this section we discuss the WW interferometers\[^9\] in orthodox quantum mechanical formalism.

We now change slightly our gedanken experiment apparatus, following ESSW, by adding a one bit which way detector. This can be realized, for example, if we consider our beam as a beam of Rydberg circular atoms and in each of the paths we include a microwave cavity resonant with just one mode decay of the atom, and experimentally we made a velocity selection such that, whenever the atom enters a cavity in the excited state it should emit a photon and leave the cavity in the lower state (in quantum optics terminology, this is called a \( \pi \) pulse). This is shown in figure 3, as an example, but for our purposes it is only necessary to include one bit information about which way, independently of how it is realized (in the example this bit would be in which cavity the photon was emitted). Mnemonically, this bit will assume values \( r \) or \( t \).

So, in terms of Dirac kets we can schematize the incomplete WW Mach-Zender interferometer as the following unitary transformations:

\[
|\Psi\rangle \xrightarrow{BS} |\Psi\rangle_{bs} \xrightarrow{WW} |\Psi\rangle_{ww}
\]

where the first transformation refers to passage through the beam splitter (BS).
Fig. 3. Experimental scheme of a WW incomplete Mach-Zender interferometer.

and the second through the which way apparatus (WW). We shall consider

\[ |\Psi\rangle_{bs} = |\psi_r\rangle + |\psi_t\rangle, \]

\[ |\Psi\rangle_{ww} = |\psi_r\rangle |r\rangle + |\psi_t\rangle |t\rangle. \]

The central point which reinforces Bohr’s complementarity principle is that

\[ \langle\Psi | \Psi\rangle_{ww} \simeq |\psi_r|^2 + |\psi_t|^2 + 2\text{Re} \{ \langle\psi_r | \psi_t\rangle \langle r | t\rangle \} \]

and so, as we should consider the one bit WW states |r\rangle and |t\rangle orthogonal to each other, with a WW interferometer we see no more interference pattern (unless we record coincidence counts between WW detectors and usual ones).

4 WW Interferometers in Bohmian Mechanics

We now treat the example above in Bohmian mechanics, and show nothing surreal to happen. Our results differ from ESSW ones in reason of the configuration space we utilize includes the WW one bit information.

As a WW interferometer access also the WW bit, we now should consider the wave field \( \Psi (w, x) \), where \( w = r, t \) is exactly this WW bit, and \( x \) has the same meaning as in the simple interferometer. We can now revisit the three arguments implied to justify the \( r \rightarrow 1, t \rightarrow 2 \) rule for Bohm trajectories of the simple interferometer (fig 2):
i) as now we have

\[ \Psi (w, x) = \psi_r (r, x) + \psi_t (t, x), \]

there is no more overlap, even when \( x \) describes points in \( I \) region, as \( w \) assumes different values for each wave packet, there is no overlap. So, as discussed, quantum potential plays no role and particles goes like in free motion.

ii) If we apply the symmetry operation \( R \) now we just have \( \psi_r (r, x) = \pm \psi_t (t, Rx) \). The only conclusion is that there is no net flux across the beam splitter plane, i.e. \( \mathbf{v}_\perp (r, \mathbf{x}) = -\mathbf{v}_\perp (t, \mathbf{x}) \), where \( \mathbf{v}_\perp \) denotes the perpendicular to beam splitter component of velocity vector field, and \( \mathbf{x} \) denotes invariant points with respect to \( R \).

iii) As the configuration space has now two slices, trajectories do not cross, but just passes one “over” the other.

In view of this arguments, specially first of them, one can infer (complete calculations and numerical examples will be given elsewhere) that, for WW interferometers, the correct correspondence is \( r \rightarrow 2, t \rightarrow 1 \), exactly as common sense would say is “the right way”. If we ignore WW variable, then we must project results and the obtained picture shows a cross. We should remember it is not a true cross, but a projected one, which is common result in fluid dynamics (projected flows do not behave like flows!). In figure 4 we show pictorially how this works.

![Figure 4](image-url)

Fig. 4. Bohm trajectories in a WW incomplete Mach-Zender interferometer. The two upper planes refers to \( w = r, t \) and bottom plane is their projection. Configuration space trajectories do not cross, but projected trajectories do.
It should be stressed that this projection property is related to treat impure states, or statistical mixtures. For this case, there is no real Ψ wave field and Bohmian mechanics does not apply. The author is presently working in a way of by pass Bell’s comment[10] “in the de Broglie-Bohm theory a fundamental significance is given to the wave function, and it cannot be transferred to the density matrix”. The idea is to diagonalize the density matrix ρ and then consider each eigenvector independently.

5 Conclusions

We have shown Bohmian mechanics to be, not only self consistent, but also to follow common sense in the example previously considered surrealistic. We have stressed the main points in the confusion: ESSW had used Bohm trajectories of a system to analyze a different one. This letter also made clear how carefully we have to be about configuration space, and also that, up to now, the only way to avoid confusion, is only ascribe Bohm trajectories for pure states with a well determined wave field.

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