A mathematical model of the vertical dual-mass hydroimpulsive mechanism

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Abstract. In this paper, the model of a hydroimpulsive mechanism for boring machines was presented. Differential equations describing the processes occurring in the mechanism were derived. Keywords: hydroimpulsive mechanism, forced oscillation, natural frequency.

1. Introduction
Papers [3-8] describe the original model of a hydroimpulsive power-actuating mechanism that can be used as a generator of power pulses to intensify drilling of drillholes with a small diameter or to sink piles [1]. One of the features of this mechanism is the mandatory presence of a rigid support, which is replaced by constant compression force $F_{\text{COMP}}$ during the modelling. Creation of such support is problematic and pointless for the practical construction of such mechanism. It is much easier to replace it with another additional mass. At the same time, this mass will not remain stationary and will perform periodic oscillations. This raises the question about studying this mechanism in order to determine its main parameters, under which the system operates in a resonant mode and pressure pulses occurring in the hydraulic cylinder will be maximal, and, hence, the stress on the gear will be maximal as well.

2. Results and discussion
Let us consider the vertical dual-mass hydroimpulsive power-actuating mechanism (Figure 1). The liquid pressure pulses are generated in the following way: when the hydropulsator operates, the plunger performs a reciprocating motion that creates the liquid pressure pulses, which are transferred to the hydraulic cylinder. Since the cylinder is preloaded with the elastic force, it oscillates together with inertia masses $m_1$ and $m_2$. A periodic conversion of kinetic energy of the masses into potential energy of deformed liquid and a hose takes place, hence the system pressure increases, and vice versa. During the system operation mode close to the resonant one the pressure pulses of a significant value appear. They are transferred to the working surface through the piston and the bar.
To derive the equations, we will introduce the following generalized coordinates: \( x_1 \) – displacement coordinate of mass 1 with the hydraulic cylinder housing; \( x_2 \) – displacement coordinate of mass 2; \( x_3 \) – displacement coordinate of the plunger (Figure 2).

The differential equation of the motion of mass 1 with the hydraulic cylinder housing can be represented as:

\[
m_1 \frac{d^2 x_1}{dt^2} + k_{FR} \frac{dx_1}{dt} = F_{EL} + m_1 g - p S_{HC},
\]

where \( m_1 \) – mass 1; \( k_{FR} \) – coefficient of friction between the piston and the hydraulic cylinder; \( p \) – pressure in the hydraulic cylinder; \( F_{EL} \) – elastic force from the spring; \( S_{HC} \) – area of the hydraulic cylinder piston.

The differential equation of the motion of mass 2, excluding the viscous drag from the air and the motion block, can be presented as

\[
m_2 \frac{d^2 x_2}{dt^2} = -F_{EL} + m_2 g.
\]

Spring force is equal to

\[F_{EL} = c(x_0 + x_2 - x_1),\]

where \( c \) – stiffness of spring; \( x_0 \) – the initial spring compression at system equilibrium.

Given that \( x_0 = \frac{m_2 g}{c} \), we obtain the following equation after substitution:

\[
m_1 \frac{d^2 x_1}{dt^2} + k_{FR} \frac{dx_1}{dt} - c(x_2 - x_1) = (m_1 + m_2) g - p S_{HC},
\]
The equation of the actuating rotor motion is as follows:

\[ J_R \frac{d^2 \varphi}{dt^2} = M_{ACT} - \chi \frac{d\varphi}{dt} - p S_{PL} \cos \varphi \cdot r, \]

where \( J_R \) – reduced moment of inertia of the rotating parts of the rotor; \( M_{ACT} \) – actuating torque of the motor; \( \chi \) – damping factor; \( S_{PL} \) – area of the plunger, \( r \) – radius of the bell crank (Figure 3).

The equation of the flowrate is

\[ S_{HC} \frac{dx_1}{dt} + S_{PL} \frac{dx_3}{dt} = C_v \frac{dp}{dt}, \]

where \( C_v \) – coefficient of elasticity of the hydraulic system, which determines the flowrate per the hydraulic system elements deformation and the compressibility of the liquid.

The movement coordinate of the plunger is connected with generalized coordinate \( \varphi \) in the following equation:

\[ \frac{dx_3}{dt} = r \cos \varphi \frac{d\varphi}{dt}. \]

After substituting it in equation (2) we obtain:

\[ S_{HC} \frac{dx_1}{dt} + S_{PL} \frac{dx_3}{dt} = C_v \frac{dp}{dt}. \]

Thus, we obtain the following system of differential equations:

\[
\begin{cases}
    m_1 \frac{d^2 x_1}{dt^2} + k_{pr} \frac{dx_1}{dt} - c (x_2 - x_1) = (m_1 + m_2) g - p S_{HC}, \\
    m_2 \frac{d^2 x_2}{dt^2} + c (x_2 - x_1) = 0, \\
    J_R \frac{d^2 \varphi}{dt^2} = M_{ACT} - \chi \frac{d\varphi}{dt} - p S_{PL} \cos \varphi \cdot r, \\
    S_{HC} \frac{dx_1}{dt} + S_{PL} \frac{dx_3}{dt} = C_v \frac{dp}{dt}. 
\end{cases}
\]
3. Conclusion
With this system of equations, it is possible to perform modelling of the processes occurring in the hydroimpulsive mechanism. This will make it possible to calculate the required parameters for different operational modes.

**Figure 3.** Charts of oscillations of masses 1 and 2 in start-up mode of hydroimpulsive mechanism.

**Figure 4.** Changes of hydraulic pressure.
Figures 3 and 4 show the results of numerical modelling of the resulting system of equations. The following system parameters were taken:

\[ M_1 = 100 \text{kg}, \quad M_2 = 500 \text{kg}, \quad k_{FR} = 0.2, \quad c = 10000 \quad \frac{\text{N}}{\text{m/s}}, \quad C_V = 10^{-7} \quad \frac{\text{m}^3}{\text{Pa}}, \quad S_{HC} = 0.00785 \text{m}^2, \quad S_{HC} = 0.00314 \text{M}^2, \quad \chi = 0.01, \quad M = 500 \text{Hm}, \quad J_R = 0.005 \text{kg} \cdot \text{m}^2, \quad r = 0.25 \text{m} \]

The system is set in motion from the state of rest. Initial conditions are:

\[
\begin{align*}
  x_1(0) &= 0, \quad m_2 \cdot \frac{g}{c}, \quad \varphi(0) = 0, \quad p(0) = \frac{(m_1 + m_2)g}{S_{HC}} \cdot \frac{d x_1(0)}{dt} = 0, \quad \frac{dx_2(0)}{dt} = 0, \quad \frac{d\varphi(0)}{dt} = 0, \\
  \frac{dp(0)}{dt} &= 0.
\end{align*}
\]

The results of numerical modelling clearly show the effect of moving mass 2 on generated hydraulic pressure pulses - its maximum peaks match up with the chart of weight oscillations. Therefore, by controlling the natural frequency of mass 2, i.e. by varying the value of the mass or value of stiffness of spring, it is possible to receive pressure pulses of predetermined value and frequency. In addition, by controlling speed of the plunger, it is possible to ensure resonant mode of the system. In this case the pressure pulses will be maximized.

Numerical modelling of the resulting system of equations and finding its analytical solutions are a matter of the further study.

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