Nucleon semimagic numbers and low-energy neutron scattering

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It is shown that experimental values of the cross sections of inelastic low-energy neutron scattering on even-even nuclei together with the description of these cross sections in the framework of the coupled channel optical model may be considered as a reliable method for finding nuclei with a semimagic number (or numbers) of nucleons. Some examples of the application of this method are considered.

I. INTRODUCTION

During last two decades the problem of existence of so-called semimagic numbers of nucleons is widely discussed (see e.g. [1], where one can find a bibliography on this problem). The existence of semimagic numbers is considered as a result of the appearance of a new gap in the single-nucleon level scheme by adding a pair of neutrons (or protons) and its disappearance by adding another pair. Such an effect is believed to be caused by the interaction between valence protons and neutrons. For the first time the surmise about importance of the np-interaction was made by A.De Shalit and M.Goldhaber [2] in the shell model in connection with the appearance of nuclear deformation. P.Federman and S.Pittel [3] showed that the interaction between neutrons and protons with strongly overlapped orbits may lead to the appearance of nuclear deformation. At the same time they have shown that this interaction may essentially change the single-nucleon level scheme. This interaction becomes important when interacting neutron and proton have big radial or big and close orbital quantum numbers. Therefore semimagic quantum numbers are less steady than "classical" ones.

As to methods of finding semimagic quantum numbers of nucleons it seems that shell model calculations taking into account the np-interaction of valence nucleons can not be regarded as reliable because the form and intensity of this interaction seem to be rather uncertain. Therefore semimagic numbers can be found by indirect ways, namely using a comparative analysis of some properties of neighboring nuclei, such as binding energies of nucleons, spectroscopic data (e.g. g-factor values, change of the sign for the coefficient of E2-M1 mixture in electromagnetic transitions $2_2^+ \rightarrow 2_1^+$, $3_1^- \rightarrow 2_1^+$, $3_1^+ \rightarrow 2_2^+$ of even-even nuclei etc).

In the next section we show that the analysis of inelastic scattering cross sections of low-energy neutrons on even-even nuclei gives a reliable method for finding semimagic numbers of nucleons.

Section 3 presents some examples of the concrete application of this method. And section 4 contains some conclusive remarks.

II. NEUTRON INELASTIC SCATTERING

Low-energy neutron data for even-even nuclei with $A \geq 56$ were successfully described [4, 5] by the authors of this paper in terms of the coupled channel optical model (CCOM). These data, taken for neutron energies $E_n \leq 3$ MeV, included total and elastic scattering cross sections, cross sections of inelastic scattering corresponding to the excitation of $2_1^+$ level, angular distributions for elastic and inelastic scattering. We used two-phonon (five-channel) approximation of CCOM for spherical nuclei, and three-channel rotational approximation for nonspherical ones. Details of CCOM are described by E.S.Konobeevsky, I.V.Surkova et al [6].

In our calculation we used nonspherical optical potential with the real part of Woods-Saxon’s form. The real part included the spin-orbital term and the symmetry potential. Radial dependance of the absorptive part was taken as the derivative of the real part of the form-factor since namely the nuclear surface is responsible for absorption of low-energy neutrons. Geometrical parameters of the potential were the same for real and imaginary parts: the potential radius was fixed as $R = r_0A^{1/3}$ with $r_0 = 1.22$ fm, the nuclear diffuseness parameter was initially chosen to be equal to 0.65 fm, but for some nuclei (see below) it was somewhat changed. The spin-orbit interaction parameter $V_{so}$ was equal to 8 MeV. The real part of our potential included the isototical term proportional to $(N-Z)/A$, so that the depth of the real part of the potential had the following form:

$$V = V_0 - V_1 \frac{N-Z}{A}, \quad (1)$$

where $V_1 = 22$ MeV. The depths $V_0$ (of real part) and $W$ (of imaginary part) were two free parameters to fit. The values of the quadrupole deformation parameter $\beta_2$ were taken from compilation [7].

The optimal description of all the totality of low-energy neutron data for even-even nuclei under consideration was achieved using practically the same values of model parameters $V_0$ and $W$, namely, $V_0 = 52.5 \pm 1.5$ MeV and $W = 2.5 \pm 0.5$ MeV.

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values of $V_0$ and $W$, we had to change the diffuseness parameter for nonspherical nuclei from the initial value of 0.65 fm up to 0.70 — 0.75 fm. For the same reason we slightly diminished the value of $a$ for some spherical nuclei which were found to be magic or semimagic, taking $a = 0.55 – 0.60$ fm for magic and semimagic nuclei and $a = 0.50$ fm for double-magic ones.

These alterations in numerical values of $a$ seem to be consistent (at least, qualitatively) with the experimental data and theoretical view on the thickness of nuclear surface lay; on the other hand, they are important for finding semimagic numbers of nucleons (see below).

Note, that using CCOM with these parameter values we also obtained a good description [4, 5] of the experimental data on $s$-, $p$- and $d$-neutron strength functions and potential scattering lengths for nuclei under consideration.

The analysis of inelastic neutron cross sections led us to so-called $N_pN_n$-systematics [4, 8]: we showed that the inelastic cross section with the $2^+$ level excitation, taken at the energy equal to 300 keV over the threshold and averaged over the energy range of 100 keV, seems to be a smooth function of $N_pN_n$ — the product of valence proton and neutron numbers (or their holes). The curve, presenting this function $\sigma_{inel}(N_pN_n)$ and shown with experimental points in those publications, was obtained using the least square method. The corrected and more precise version of this plot is given by fig. 1 of this paper. Note, that the curve presenting a function $\sigma_{inel}(N_pN_n)$ may also be well described as

$$\sigma_{inel} = A + B \sqrt{N_pN_n}$$

(2)

with $A = 0.5 b$ and $B = 0.14 b$. Emphasize, that this approximation is not good for big ($> 120$) values of $N_pN_n$.

Since the $N_pN_n$-product depends on the beginning and the end of the upper shells of the nucleus, the $N_pN_n$-systematics combined with CCOM-calculations may serve as a method of finding semimagic numbers of nucleons. The main scheme of this method is as follows.

If the cross section of neutron inelastic scattering for an even-even nucleus is essentially less than for neighboring even-even nuclei (usually it is near by 0.5 b) and therefore the corresponding experimental point on the plot of fig. 1 deviates from the curve $\sigma_{inel}(N_pN_n)$, we change the beginning (or the end) of valence shell in order ”to restore the agreement”. As a result of this operation, we obtain a new value of the valence nucleon product and thereby — a new number of nucleons corresponding to the beginning (or the end) of valence shell, i.e. a new magic number. This result may be considered as preliminary one; to confirm it we can see what is the result of the CCOM-description of this nucleus, namely, what is the value of diffuseness parameter: if the value of the diffuseness parameter is essentially less than 0.65 fm ($0.50 – 0.60$ fm) it means that the preliminary result is confirmed and a new semimagic number is found, if not — it is necessary to use another method to confirm or to reject the preliminary result. Such an important role of the diffuseness parameter in our method is based on the high sensitivity of the CCOM-calculation results to the diffusion parameter value.

Thus, our method is expected to be efficient if neutron inelastic cross sections for the considered nucleus and for its ”neighbors” are known with an appropriate accuracy. As we will show, in absence of such data it is also possible to obtain some results, but, may be, not so certain as in the main scheme of the method.

Next section is devoted to practical applications of the method proposed here.

III. SEMIMAGIC NUMBERS OF NUCLEONS

Here, using the method described in the previous section, we prove the existence of neutron semimagic numbers $N_s = 38, 56, 64$, and proton semimagic numbers $Z_s = 40, 58, 64$. We also consider possibilities of existence of some other semimagic numbers.

A. $N_s = 38$ and $Z_s = 40$

The case of $N_s = 38$ seems to be the best illustration of our method application [4, 5] (see also [9]). The neutron inelastic cross sections for neighboring isotopes $^{70}$Ge and $^{72}$Ge differ by factor 1.7. So, following our scheme we have to assume that the $N_pN_n$-values for these isotopes are equal 0 and 40 respectively. But it means that in $^{70}$Ge the neutron state $p_{1/2}$ is shifted up, so that a considerable energy gap appears between it and low-lying state $f_{5/2}$; this gap disappears by adding a pair of neutrons, i.e. in $^{72}$Ge. In other words, $N_s = 38$ is a semimagic number. Calculation of diffuseness parameter $a$ for $^{70}$Ge confirms this conclusion (the value $a = 0.65$ presented in
does not take into account "the correction" of $V_0$ and $W$; taking into account this effect gives $a = 0.58$).

As to $Z = 40$ it seems that in distinction from $N = 38$ the proton state $p_{1/2}$ is not shifted up, and the energy gap appears between this state and the state $g_{9/2}$ (which is higher than $p_{1/2}$). By adding a new pair of proton this gap disappears as a result of the interaction between valence protons and neutrons in states $g_{9/2}$ and $g_{7/2}$. Thus, for protons we obtain $Z_s=40$. The standard (for our method) analysis of the low-energy neutron interaction with isotopes of Sr, Zr and Mo confirms this conclusion.

Note, that the conclusion about $N_s$ and $Z_s$ follows immediately from the investigation by P.Federman and S.Pittel [3]. Later on many authors presented arguments in favor of a possibility of existence of semimagic numbers $N=38$ and $Z=40$ (see e.g. [10]). But in distinction from those arguments our method gives evidence for the existence of these semimagic numbers.

B. $N_s=56$ and $Z_s=58$

To some extent the situation with semimagic numbers 56 and 58 is similar to the case of numbers 38 and 40. The standard analysis of the experimental data on the interaction of low-energy neutrons with even-even isotopes of Sr, Zr and Mo [3] leads to the conclusion that for the isotopes with 56 neutrons the energy level of the neutron state $2d_{5/2}$ is shifted down and the level of $1g_{7/2}$-state is shifted up, so that a gap appears between these states and disappears by adding two more neutrons. In other words, $N = 56$ is a semimagic number. One of the first conclusion about a possibility of the existence of $N_s = 56$ was made by V.A.Morozov [1], who showed that at $N = 56$ the coefficient of mixture of $E2$ and $M1$ in electromagnetic transitions $2^+_2 \rightarrow 2^+_1$, $3^+_1 \rightarrow 2^+_1$, $3^+_2 \rightarrow 2^+_2$ changes the sign. And since such a change normally is treated as the presence of a filled subshell, he assumed that the existence of a semimagic number $N = 56$ is quite possible. Our result (being more definite) confirmed this assumption.

In this connection we should mention recent works by Moscow University group [11] in which it was in particularly shown that the energy gap between neutron subshells in $^{90}$Zr may achieve $\sim 3$ MeV. As a result the authors assumed a possibility of existence of the semimagic neutron number $N_s = 56$. This calculation may be regarded as an additional confirmation for our result.

As to a semimagic number of protons $Z_s=58$ or 56 some authors (see e.g. [1]) assumed that such a number may exist. This assumption was confirmed in the framework of our approach. Trying to describe neutron data for Nd we discovered that neutron inelastic cross sections for some isotopes do not enter the $N_pN_n$-systematics. For instance, from the point of view of this systematics the $N_pN_n$-value for the isotope $^{146}$Nd must be not equal to 40 (if its proton shell would start at $Z=51$), but 8 or 4, and these values correspond to $Z_s=58$ or 56 (see fig.2).

C. $N_s=Z_s=64$

In distinction from the semimagic numbers considered above, the assumption about existence of the semimagic number 64 leads to the conclusion that some nuclei (namely $^{114}$Sn and $^{146}$Gd) must be double magic.

In 1953 J.O.Rasmussen with colaborators [12] payed attention to anomalies in energies of $\alpha$-decay of nuclei with $Z=64$ at $N \approx 82$ and thus assumed that $Z=64$ is a semimagic number. Later on it was assumed [13, 14] that the subshell $Z=64$ exists in the region of rareearth nuclei and disappears in consequence of the $pn$-interaction between $h_{11/2}$- and $h_{9/2}$-states. Many authors (see e.g. [1, 8, 13, 14, 16]), considering the question about $Z_s$, brought different arguments in favor of existence of such a semimagic number. In particular, the description of the neutron inelastic scattering for the isotopes of Sm in the framework of CCOM is possible only under assumption of the disappearance of the shell ($1g_{7/2}$ and $2d_{5/2}$) gap at $Z = 64$ and $N > 80$ [8]. At the same time the value of diffuseness parameter $a$ at the optimal description of the neutron cross sections for $^{146}$Gd is equal to 0.50 fm, while for neighboring nuclei $a=0.55 - 0.60$ fm.
Such values of $a$ give an evidence that $^{146}$Gd is a double magic nucleus, i.e. $Z=64$ is a semimagic number.

The situation with $N_s=64$ is similar to that with $Z_s=64$. Many authors [4, 13, 15] concluded that the existence of this semimagic number is quite possible. The analysis of neutron inelastic scattering on the isotopes of Cd in the framework of the $N_p,N_n$-systematics confirmed that conclusion (just like such an analysis for Sm confirmed the existence of $Z_s=64$). The value of the diffusion parameter for $^{114}$Sn ($a=0.50\text{fm}$) gives an argument in favor of the conclusion that $^{114}$Sn is a double magic nucleus (just like $^{146}$Gd).

Thus, we may conclude that the existence of semimagic numbers $Z_s=N_s=64$ is firmly proved.

### D. Other possible semimagic numbers

Besides the semimagic numbers of nucleons considered above there are some "candidates" for joining the "semimagic community". Unfortunately, the method described here cannot give so far any certain answer about their existence because in most cases the accuracy of necessary experimental data is insufficient. Nevertheless, the use of it can lead to some conclusions (at least, on the level of assumption).

As the first example of such a "candidate" we consider $Z_s=76$ [17]. Values of the neutron inelastic cross section for isotopes of Os ($Z=76$) are essentially bigger than $0.5\text{b}$ and the diffuseness parameter values (corresponding to the best description of low-energy neutron data) are equal to 0.65. All that testifies against the existence of this semimagic number.

It seems to be interesting to investigate the region of heavy ($Z>92$) nuclei in connection with existence of semimagic nucleon numbers. But since for such nuclei the inelastic cross sections of low-energy neutron scattering are not known, it is impossible to use the main scheme of our method. However, one can try to draw some plausible assumptions (or at least some hints) from available data on inelastic neutron scattering on $^{232}$Th. As a matter of fact there exist two sets of the experimental data on neutron inelastic scattering for this nucleus. These cross section values corresponding to $2^+_1$ level excitation for the neutron energy 300 keV over the threshold (after averaging over the energy range of 100 keV) are equal to $1.13\pm0.21\text{b}$ and $1.22\pm0.22\text{b}$ [18]. Both of these values (corresponding to $N_p,N_n=128$) are essentially less than the value required by the $N_p,N_n$-systematics. If we (ignoring the experimental errors) assume that the true value of the cross section considered here is situated between $1.13$ and $1.22\text{b}$, we obtain that the $N_p,N_n$-product must be equal to 48 (instead of 128). Such a change may be correct only if some new semimagic numbers exist. We will try to find them.

Generally speaking, the value of the $N_p,N_n$-product equal to 48 may leads to 6 possible pairs of valence nucleon numbers $N_p$ and $N_n$. But it is necessary that between the new and the old values of these number the following relations were fulfilled:

$$N_p^{\text{new}} \leq N_p^{\text{old}}, \quad N_n^{\text{new}} \leq N_n^{\text{old}}.$$  \hspace{1cm} (3)

The breach of one of these requirements would lead to drastic changes in the nucleon level schemes, in particular, to the removal of classical magic numbers. (In principle, such a reconstruction of the nucleon scheme is possible. But there is no reason for it so far.). Therefore only three pairs $(N_p,N_n)_{\text{kk}}$ (where $k=1,2,3$), namely, $(8,6)$, $(6,8)$, $(4,12)$ have to be taken into consideration. For each pair it is easy to get the nucleon numbers $Z_k$ and $N_k$ from which one has to count the values of relevant valence nucleon numbers:

$$\begin{align*}
Z_1^{1} & = (82 \text{ or } 98), \\
Z_2^{2} & = (84 \text{ or } 96), \\
Z_3^{3} & = (86 \text{ or } 94), \\
N_1^{1} & = (136 \text{ or } 148), \\
N_2^{2} & = (134 \text{ or } 150), \\
N_3^{3} & = (130 \text{ or } 154) \end{align*}$$  \hspace{1cm} (4)

Eqs. (4) contain 12 numbers $Z_k$ and $N_k$. Using these numbers one can form 12 pairs $(Z_k,N_k)$, and if both numbers of a pair are semimagic ones a necessary value of the $N_p,N_n$-product (in our case it is 48) will be guaranteed. It remains to prove that at least there is one such a pair among the pairs considered here. In this connection note that one of the $Z_1$-values is equal to 82 which is a classical magic number, so that in this case it is only necessary to prove that one of 136 or 148 is a neutron semimagic number.

Unfortunately, it seems impossible to find a "semimagic pair" among those 12 ones for lack of necessary experimental data. Nevertheless, we shall try to draw some conclusions from available data (at least on the level of assumptions). Let us consider 12 nuclei consist of $Z_k$ protons and $N_k$ neutrons ($Z_k$ and $N_k$ belong to the same pair). 10 of these nuclei are not interesting for us because some of them are exotic ones (like $^{228}$Pb) and for others nothing is known about their interaction with low-energy neutrons. Remaining two nuclei, namely $^{240}$Cf$_{148}$ and $^{246}$Cm$_{150}$ could be interesting from the point of view of our method. But we do not know anything about interaction of the isotopes of Cf with low-energy neutrons. So, only $^{246}$Cm remains to be considered. For this and some other isotopes of Cm the neutron strength functions are known experimentally. The strength function $S_0$ of $s$-neutrons for $^{246}$Cm is equal to 0.45±0.15 while for neighboring isotopes $^{244}$Cm and $^{248}$Cm $S_0=1.00\pm0.20$ and $1.10\pm0.12$ [14]. Such a big difference between the $S_0$-values for $^{246}$Cm and its neighboring even-even isotopes may be considered as an argument in favor of the assumption that $N=150$ is a semimagic number. The difference between the $S_0$-values for $^{246}$Cm and $^{244}$Pu is of the same order of magnitude, but we cannot say anything about the other "neighbor on the Z-line", i.e. $^{248}$Cf because its interaction with low-energy neutrons is not known. So, we can only say that such a situation may be considered not as an argument, but
only as a "hint in favor" of the assumption that \( Z=96 \) is also a semimagic number.

Generally speaking, almost all the \( S_0 \)-values in the (Pu–Cm)-region are equal to \( \approx 1 \) [19]. The only exception is \( ^{246}\text{Cm} \) \( (S_0 = 0.45 \pm 0.15) \). Such a situation provokes us to assume that \( ^{246}\text{Cm} \) is a double magic (or double semimagic) nucleus. In other words, both \( Z = 96 \) and \( N = 150 \) are semimagic numbers. Emphasize, that this is only a statement about the possibility of existence of two new semimagic numbers and needs more rigorous proof. Unfortunately, the diffusion parameter value taken from our calculation of \( S_0 \) cannot help because such calculations are less sensitive to the diffusion parameter value than calculations of cross sections.

IV. CONCLUSION

The efficiency of the method for finding new semimagic numbers of nucleon, based on the analysis of low-energy neutron data, was demonstrated here by different examples of its application. Using this method we obtained some already known results about existence of some semimagic numbers, confirmed assumptions about other semimags, for the first time proved the existence of the semimagic number \( Z_s = 58 \). Using scarce experimental data we first pointed on the possibility of existence of semimagic \( Z = 96 \) and \( N = 150 \). Thus, this method seems to be rather efficient.

It is necessary to emphasize that the most important part of the method described here is the \( N_pN_n \)-systematics, \textit{i.e.}, smooth dependence of neutron inelastic cross section on the \( N_pN_n \)-product. This dependence itself is out of any doubt: there is no one serious contradiction with it. Therefore this method becomes less efficient in cases when one cannot address the \( N_pN_n \)-systematics, \textit{i.e.} when neutron inelastic cross sections are not known or their accuracy is poor.

Also we mention another essential element of our method having auxiliary character, namely the diffusion parameter value obtained from the requirement of optimal description of neutron data in framework of the coupled channel optical model. But this auxiliary element is efficient only if corresponding calculations are sensitive to this value. For instance, it works well for the cross section calculations, but does not work for calculations of strength functions (as it was mentioned above). Nevertheless, even in such cases our method permits to obtain some results.

Thus, the method described here may be considered as one of the most efficient methods for finding new semimagic (or magic) number of nucleons.

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