A Convex Formulation of a Bilevel Optimization Problem for Energy Markets

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Abstract—This paper studies the static economic optimization problem of a system with a single aggregator and multiple prosumers associated with the aggregator in a real-time balancing market. The aggregator, as the agent responsible for portfolio balancing, needs to minimize the cost for imbalance satisfaction in real-time by proposing a set of optimal incentivizing prices for the prosumers. On the other hand, the prosumers, as price taker and self-interest agents, want to maximize their profit by changing their supplies or demands and providing flexibility based on the proposed incentivizing prices. We model this problem as a bilevel optimization problem. The state-of-the-art approach to solve a bilevel optimization problem is to reformulate it as a Mixed-Integer Programming (MIP) problem. Despite recent developments in the solvers for MIP problems, the computation time for a problem with a large number of decision variables may not be appropriate for real-time applications. We propose a convex equivalent optimization problem for the original bilevel one and prove that the global optimum of the prosumers/aggregator bilevel problem can be found by solving a convex problem. Also, we demonstrate the efficiency of our convex equivalent with respect to an MIP formulation in terms of computation time and optimality.

Index Terms—Real-time balancing market, convex optimization, bilevel optimization, flexibility management.

ACRONYMS

- ADS: Active Demand and Supply
- BRP: Balance Responsible Party
- DER: Distributed Energy Resources
- HP: Heat Pump
- MIP: Mixed-Integer Programming
- MPEC: Mathematical Programming with Equilibrium Constraints
- mCHP: Micro Combined Heat and Power
- RTBM: Real-Time Balancing Market
- TSO: Transmission System Operator

I. INTRODUCTION

In recent years, the increase in the penetration of DERs at the demand side has drastically changed the structure of our power system. As a result, the old passive households, which only consumed energy, found a more active role with the help of the demand side generation. The new term prosumer was introduced in the energy community to represent this transition for households [1].

The emergence of prosumers calls for a new real-time market structure in contrast to the existing day ahead and intraday markets. Since output power of many DERs is volatile due to their intrinsic environmental dependency, planning for supply and demand matching needs to be done as close as possible to real-time to keep the system stable and economically efficient. Therefore, an RTBM [2] that incorporates available unused capacity of prosumers’ controllable DERs and flexible loads, which together we denote here as controllable ADS units, should be developed to address the supply volatility by incentivizing prosumers.

Currently, there is only an ex-post financial settlement procedure in the Netherlands and most of Europe, and no actual or physical real-time balancing occurs [3]. Communication infrastructure in the new paradigm of smart grid [4] facilitates the participation of the prosumers with controllable ADS units in an RTBM. Moreover, to prevent direct interaction of the prosumers with higher level agents in the market and aggregate them, a market participant, the aggregator, has been introduced [5]. The aggregators have different roles in different market structures.

The goal of an aggregator in an RTBM is to optimize its operational costs for balancing by incentivizing the prosumers to utilize their unused assets. There are many approaches which an aggregator can employ to steer its associated prosumers to an optimal operation point [6]. One of the most popular approaches is to consider the aggregator as a leader, who can anticipate the reaction of the prosumers, proposes some prices to the following prosumers such that their reactions would be optimal for the aggregator. This price incentive oriented setup falls into the category of bilevel optimization problems [7] and Stackelberg games [8], where the lower level problems and the upper level problem are the problems related to the prosumers and the aggregator, respectively.

Bilevel optimization problems have already been employed in the literature for electricity markets. The well-known paper [9] models strategic offering of a dominant generating firm as a two level optimization problem, where at the upper level a generator firm maximizes its profit and at the lower level a system operator maximizes social welfare or minimizes total system cost. It is assumed that the dominant firm knows about the other non-dominant firms bids and offers. More recent papers [10] and [11] propose a similar approach for...
participation of microgrids and aggregators in an RTBM. In these types of models, the price for the electricity is equal to the dual variable corresponding to the clearing constraint in the lower level problem.

In contrast to dealing with the interaction between an aggregator and a system operator as in [9], [11], [12] investigates the interaction between prosumers and an aggregator. In this setup, the aggregator minimizes its cost at the upper level and the prosumers minimize their electricity consumption cost based on the price proposed by the aggregator. Also, the papers [13] and [14] investigate the prosumers/aggregator interaction albeit with a different approach. Indeed, these papers deal with personalized price for individual prosumers based on their past consumption behavior.

The state-of-the-art approach to solve these types of bilevel optimization problems is to solve them as MIPs by using commercial off-the-shelf software packages. However, implementing the mentioned setup in real-time requires very fast computations. The time intervals for a real-time balancing market can often be as low as 5 minutes [15]. Therefore, the solution for each interval has to be computed and executed within seconds or even less. While papers like [16] have studied the computational efficiency for generating firms strategic offering setup with personalized prices, this problem, in general, is non-convex and non-convex problem can be obtained as a solution of a certain convex optimization problem. This convex equivalent formulation has two main advantages. On the one hand, it guarantees global optimality. On the other hand, a convex formulation is attractive in real-time applications since the computation time is linear in the number of variables whereas it is exponential for MIPs [18]. In addition, this significant reduction in computation time has the potential to help the aggregator to participate in the ancillary service markets such as primary and secondary reserve markets which have very short time intervals [3].

The paper is organized as follows. Section II explains the prosumers/aggregator interaction model in a real-time balancing market and introduces the bilevel problem. In Section III, we show that the bilevel optimization problem is equivalent to a certain convex problem. The efficiency of the proposed method is illustrated by means of simulations in Section IV. Finally, the paper closes with the conclusions in Section V.

II. PROBLEM FORMULATION

In this section, we formulate the static bilevel economic optimization problem of an aggregator and its portfolio for participation in an RTBM. Each aggregator has a set of prosumers under contract and each prosumer is on a contract with only one aggregator. There are many types of aggregators in an electricity market. In this paper, we consider a commercial aggregator which also acts as a BRP [19]. Therefore, the aggregator here is also responsible for balancing its portfolio. To do so, the aggregator receives a real-time price from the TSO, who usually has the highest role in the market hierarchy, and incentivizes the prosumers to supply or consume more or less based on that. The change in each prosumer electrical energy supply or demand in a time interval is referred as flexibility. Next, we explain the problem setting and market structure in detail.

Prosumers are equipped with various kinds of ADS units. They consist of two prominent categories, namely controllable and uncontrollable units. mCHP units and HP units are examples of controllable active supply and demand units of electricity, respectively. Output generation of units such as solar cells and wind turbines is dependent on environmental conditions. Thus these are uncontrollable supply units. Throughout this paper, we assume that each prosumer has either a modular mCHP or HP as a controllable ADS unit and it might have a solar panel or a wind turbine as an uncontrollable one. Each prosumer heat demand is also assumed to be flexible by considering a loss of comfort factor, that is, it is willing to consume more or less heat if its loss of comfort is compensated by the aggregator. Since heat is an output for both mCHP and HP, prosumers are able to alter their controllable ADS units output level to participate in the balancing market. In this paper, we focus on a static and one time-step optimization problem without considering storage devices.

Due to the uncertain nature and volatility of both the uncontrollable DS units and the prosumers demand, there could be a mismatch between the pre-planned supply and demand schedules in the real-time. To balance this mismatch and to participate in the RTBM, the aggregator incentivizes the prosumers with personalized prices [20] in a centralized way to consume or supply more energy using their controllable ADS units. Before providing a precise mathematical formulation, we elaborate on some technical notions.

The aggregator is in up-regulation if its prosumers’ demand is lower than its supply. Similarly, the aggregator is in down-regulation if the demand is higher than the supply for its prosumers. Likewise, the TSO is in up-regulation if the total system demand is lower than the total system generation. Otherwise, it is in down-regulation. Based on these definitions, we distinguish the following four cases:

**Case 1.** The aggregator and the TSO both are in up-regulation: The aggregator needs to pay the TSO to take care of its excess supply or it can incentivize the prosumers with mCHP to generate less and the prosumers with HP to consume more.

**Case 2.** The aggregator is in up-regulation and the TSO is in down-regulation: The TSO pays the aggregator for its excess supply.

**Case 3.** The aggregator and the TSO both are in down-regulation: The aggregator needs to pay the TSO to provide supply or it can incentivize the prosumers with mCHP to generate more and the prosumers with HP to consume less.
**Case 4.** The aggregator is in down-regulation and the TSO is in up-regulation: The TSO pays the aggregator to consume more.

In both Case 2 and Case 3 the solution for the optimal strategy of the aggregator is trivial: sell the requested flexibility to the TSO. However, in Case 1 and Case 3 the aggregator needs to find a trade-off between the possible options for the optimal strategy. In the following subsection, we focus on modeling Case 1 and Case 3 as a bilevel optimization problem.

### A. The prosumers/aggregator model

We consider both the aggregator and the prosumer as self-interest agents. The aggregator tries to minimize its cost to settle the imbalance and the prosumer’s goal is to maximize its revenue and minimize its cost and discomfort by altering its demand or supply given the personalized price proposed by the aggregator.

We consider one aggregator and $n$ prosumers each has an HP or mCHP. We denote the proposed personalize price by $\pi_i$ for aggregator to the $i$th prosumer by $x_i$ and the prosumer $i$’s optimal flexibility response by $y_i$ for $i \in N = \{1, \ldots, n\}$. To model both Case 1 and Case 3, we employ the following optimization problem for each prosumer:

\[
\begin{align*}
\max_{y_i} & \quad x_i y_i - \frac{1}{2} a_i y_i^2 + b_i y_i \\
\text{subject to} & \quad 0 \leq y_i \leq m_i,
\end{align*}
\]

where $m_i$ is the maximum available flexibility, $b_i$ is the price of providing flexibility and $a_i \geq 0$ models the discomfort for the prosumer $i$.

Next, we elaborate further on the model and parameters.

In (1a), the first term corresponds to the received payment by the prosumer $i$ from the aggregator. The second term models the discomfort of the prosumer $i$ for providing flexibility $y_i$. Finally, the last term captures the payment the prosumer $i$ should receive or the cost it should pay with respect to the intraday market plannings for providing flexibility $y_i$.

The parameter $b_i$ for a prosumer with HP in both the aggregator up-regulation (Case 1) and down-regulation (Case 3) is as follows:

\[
b_i = \begin{cases} 
\pi_e & \text{if aggregator in up-regulation,} \\
-\pi_g & \text{if aggregator in down-regulation},
\end{cases}
\]

where $\pi_e \geq 0$ and $\pi_g \geq 0$ are fixed electricity and gas prices charged by electricity and gas suppliers, respectively. Likewise, for a prosumer with mCHP this parameter is defined as follows:

\[
b_i = \begin{cases} 
-c_i \pi_e & \text{if aggregator in up-regulation,} \\
c_i \pi_g & \text{if aggregator in down-regulation},
\end{cases}
\]

where $c_i$ is dependent on the mCHP technology of the prosumer $i$ and is given by

\[
c_i = \frac{\text{nominal input power}}{\text{nominal electricity output power}}.
\]

Further, we define the maximum available flexibility $m_i$ as follows. For prosumer $i$, let $P_i \geq 0$ denote the electrical power of its ADS device. As such, $P_i$ is the input electrical power to an HP device or the output electrical power of an mCHP device. Also, let $P_i^{\max}$ denote the maximum electrical power for prosumer $i$. Then, the maximum available flexibility of the prosumer $i$ with an HP is given by

\[
m_i = \begin{cases} 
(P_i^{\max} - P_i) \Delta t & \text{if aggregator in up-regulation,} \\
P_i \Delta t & \text{if aggregator in down-regulation},
\end{cases}
\]

where $\Delta t$ is the duration of each time step for the RTBM and assumed to be equal to 300 seconds in this paper. Similarly, we define $m_i$ for a prosumers which owns an mCHP as follows:

\[
m_i = \begin{cases} 
P_i \Delta t & \text{if aggregator in up-regulation,} \\
(P_i^{\max} - P_i) \Delta t & \text{if aggregator in down-regulation}.
\end{cases}
\]

As the agent responsible for supply and demand balancing in the RTBM, the aggregator has two options to accomplish its goal, namely, to incentivize the prosumers for flexibility provision with the associated cost of $x_i y_i$ or to buy flexibility from the TSO with the price $p \geq 0$. The aggregator’s problem is to find the best strategy given these two options.

Considering the above model, $x_i$ being nonnegative and the prosumers’ optimality conditions, we obtain the bilevel optimization problem (2) which has the problem (1) as a constraint for each prosumer:

\[
\begin{align*}
\min_{x,y} & \quad \sum_{i} x_i y_i + p(f - \sum_{i} y_i) \\
\text{subject to} & \quad x_i \geq 0, \quad \forall i \in N, \\
& \quad \sum_{i} y_i \leq f, \\
& \quad \begin{cases} 
\max_{y_i} & \quad x_i y_i - \frac{1}{2} a_i y_i^2 - b_i y_i \\
\text{subject to} & \quad 0 \leq y_i \leq m_i, 
\end{cases} \quad \forall i \in N,
\end{align*}
\]

\[
\begin{align*}
& \quad \text{subject to} \quad x_i \geq 0, \quad \forall i \in N, \\
& \quad \sum_{i} y_i \leq f,
\end{align*}
\]

Fig. 1: A general overview of interactions for the aggregator, the prosumers and the TSO in the RTBM.
where $x$ and $y$ are vectors with components $x_i$ and $y_i$, respectively. Also, $f \geq 0$ denotes the mismatch between supply and demand in both up- and down-regulation. If the flexibility provided by the prosumers is $\sum_i y_i$. Then, the aggregator needs to trade $(f - \sum_i y_i)$ with the TSO. Figure[1] shows these interactions. We consider an ex-ante pricing scheme, that is, the TSO informs the aggregator about the price $p$ prior to the start of each 5-minute interval.

These types of bilevel problems are very similar to Stackelberg games [8], where a leader announces a policy to its followers and then the followers, who are unaware of the outside world, react by their best response strategy. In other words, the leader has the advantage of anticipating the followers reactions.

In the setup we consider in this paper, the aggregator’s goal is to satisfy its internal imbalance in real-time. However, in other possible settings beyond the scope of this paper, helping the TSO to satisfy the total system imbalance can also be a goal for the aggregator. Therefore, in that setting the problem formulation for Case [1] and Case [3] is given by (2) without considering (25). In this situation, if $\sum_i y_i - f \leq 0$, the aggregator pays $p(f - \sum_i y_i)$ to the TSO and if $\sum_i y_i - f > 0$, then the aggregator receives $p(f - \sum_i y_i)$ from the TSO for providing flexibility.

**B. The bilevel market optimization problem and its solution**

The model above for the aggregator and the prosumers interactions is very close to the bilevel electricity market models in [12]–[14], where different market technicalities have been considered. Furthermore, we restrict our model to a static case. Despite these differences, our model captures the basic properties of a bilevel market.

In general, bilevel optimization problems are very difficult to solve. They have been extensively studied in the framework of MPEC. We refer to [17] for a full investigation of MPECs. The simplest case of a bilevel optimization problem is when both the upper and lower level problems are linear. Even in this simplest case, [21] has shown that the problem is strongly NP-hard. Some classes of bilevel optimization problems can be reformulated as MIP problems and solved by commercial software packages [22]. This approach has been extensively used to solve electricity market optimization problems as a state-of-the-art approach [23], [24].

An aggregator can have up to several thousands of prosumers under its contract. To implement an RTBM with 5-minute time intervals, the optimal solution of the problem (2) should be found as fast as possible. The increase in the number of the optimization variables, as a result of the growth in the number of the prosumers, leads to an unacceptable computation time in real-time applications for combinatorial optimization problems such as MIP problems. In the following section, we introduce a convex equivalent for the problem (2).

**III. ON THE CONVEXITY OF THE BILEVEL ELECTRICITY MARKET PROBLEM**

One can parameterize the solution of (1) by using a piece-wise linear map. Indeed, given $x_i$, (1) is a concave quadratic problem in $y_i$. Solving this problem analytically leads to the following piece-wise linear map from $x_i$ to $y_i$:

$$y_i = \begin{cases} 0 & x_i < b_i, \\ \frac{x_i - b_i}{a_i} & b_i \leq x_i \leq a_i m_i + b_i, \\ m_i & x_i > a_i m_i + b_i, \end{cases}$$

which is depicted in Figure[2]. The interpretation for the sign of $b_i$ was given in the previous section. As can be seen in Figure[2b], a prosumer can provide flexibility $y_i = -\frac{b_i}{a_i}$ without any incentive ($x_i = 0$) when $b_i < 0$. The following assumption on the parameters $a_i$, $b_i$ and $f$ guarantees feasibility of the problem (2).

**Assumption 1.** The total flexibility provided by the prosumers with negative $b_i$ without any incentive is less than or equal to the total requested flexibility, i.e.,

$$\sum_{i \in \{ j \in N | b_j < 0 \}} \frac{-b_i}{a_i} \leq f.$$  

A prosumer with an mCHP (HP) in up-regulation (down-regulation) is not able to decrease its supply (demand) drastically in real-time and provide flexibility without any incentive. This is mainly due to decisions the prosumers made in the intraday market. No incentive strategy is designed for the prosumers in the intraday market. Because of this, $\sum_{i \in \{ j \in N | b_j < 0 \}} \frac{-b_i}{a_i}$ is much less than $f$ in practice. Thus, Assumption [1] is satisfied.
Having (5) as the solution to the problem (2), let us rewrite the bilevel optimization problem (2) as the piece-wise quadratic optimization problem:

\[
\underset{x, y}{\text{min}} \quad \phi(x, y) = \sum_i x_i y_i + p(f - \sum_i y_i) \tag{4a}
\]
subject to

\[
x_i \geq 0, \quad \forall i \in N \tag{4b}
\]

\[
\sum_i y_i \leq f, \tag{4c}
\]

\[
y_i = \begin{cases} 
0 & x_i < b_i, \\
\frac{x_i - b_i}{a_i} & b_i \leq x_i \leq a_i m_i + b_i, \\
1 & x_i > a_i m_i + b_i, 
\end{cases} \quad \forall i \in N. \tag{4d}
\]

In general, this problem is non-convex as illustrated by the following example.

**Example 2.** Suppose a two-dimensional case of the problem (4) where \(a_1 = a_2 = 1, b_1 = b_2 = 2, m_1 = m_2 = 6, p = 10, f = 30\). Figure 4 depicts objective function of the problem (4) with these parameters. As can be seen, the objective function is non-convex. Note that its minimum coincides with the minimum of the convex quadratic problem obtained from (12) by taking \(y_i = \frac{x_i - b_i}{a_i}\) with \(b_i \leq x_i \leq a_i m_i + b_i\) for \(i \in \{1, 2\}\).

Motivated by this example, consider the following convex quadratic problem:

\[
\underset{x, y}{\text{min}} \quad \psi(x, y) = \sum_i x_i y_i + p(f - \sum_i y_i) \tag{5a}
\]
subject to

\[
x_i \geq 0, \quad \forall i \in N \tag{5b}
\]

\[
\sum_i y_i \leq f, \tag{5c}
\]

\[
y_i = \frac{x_i - b_i}{a_i}, \quad b_i \leq x_i \leq a_i m_i + b_i, \quad \forall i \in N. \tag{5d}
\]

It turns out that the global minimum of the nonconvex problem (4) can be found by solving the convex problem (5).

**Theorem 3.** Let \(x^*\) and \(y^*\) be the minimizers of the convex quadratic minimization problem (5). Also, let \(\psi^*\) be the minimum value, that is \(\psi^* = \psi(x^*, y^*)\). Then, \(x^*\) and \(y^*\) are also the minimizers for the problem (4), i.e.,

\[
\phi(x, y) \geq \psi^*, \quad \text{for all feasible } x, y \text{ of the problem (4),}
\]
or equivalently

\[
\sum_i x_i y_i + p(f - \sum_i y_i) \geq \sum_i x_i^* y_i^* + p(f - \sum_i y_i^*), \quad \text{for all feasible } x, y \text{ of the problem (4),}
\]

**Proof.** Clearly, \(x^*\) and \(y^*\) are feasible for (4). Suppose, for the sake of contradiction, that \(\phi(x, y) < \psi^*\) for some feasible \(x, y\) of the problem (4). This means that

\[
\sum_i x_i y_i + p(f - \sum_i y_i) < \sum_i x_i^* y_i^* + p(f - \sum_i y_i^*), \tag{6}
\]

Define \(\bar{x}\) as follows: For all \(i \in \{j \in N|b_j \geq 0\}\)

\[
\bar{x}_i = \begin{cases} 
b_i & 0 \leq x_i \leq b_i, \\
x_i & b_i \leq x_i \leq a_i m_i + b_i, \\
a_i m_i + b_i & x_i > a_i m_i + b_i,
\end{cases} \tag{7}
\]

and for all \(i \in \{j \in N|b_j < 0\}\)

\[
\bar{x}_i = \begin{cases} 
a_i m_i + b_i & 0 \leq x_i \leq a_i m_i + b_i, \\
x_i & x_i > a_i m_i + b_i.
\end{cases} \tag{8}
\]

It follows from (7) and (8) that

\[
b_i \leq \bar{x}_i \leq a_i m_i + b_i, \quad \forall i \in \{j \in N|b_j \geq 0\}, \tag{9}
\]

\[
0 \leq \bar{x}_i \leq a_i m_i + b_i, \quad \forall i \in \{j \in N|b_j < 0\}. \tag{10}
\]

Furthermore, we define for all \(i\)

\[
y_i = \bar{x}_i - b_i. \tag{11}
\]

Then, (9), (10) and (11) imply that \(\bar{x}\) and \(\bar{y}\) satisfy (5b) and (5d). Since \(y = \bar{y}\) due to (11), (4e) implies (5c). Therefore, \(\bar{x}\) and \(\bar{y}\) are feasible for the problem (5).

Now, suppose that \(b_i \geq 0\) for some \(i\). We consider the following three cases.

Case 1: \(x_i < b_i\). Then, we have

\[
x_i < \bar{x}_i = b_i \quad \Rightarrow \quad x_i y_i = \bar{x}_i y_i = 0. \tag{12}
\]

Case 2: \(b_i \leq x_i \leq a_i m_i + b_i\). In this case, we have

\[
x_i = \bar{x}_i \quad \Rightarrow \quad x_i y_i = \bar{x}_i y_i. \tag{13}
\]

Case 3: \(x_i \geq a_i m_i + b_i\). This leads to

\[
x_i > \bar{x}_i = a_i m_i + b_i \quad \Rightarrow \quad x_i y_i > \bar{x}_i \bar{y}_i. \tag{14}
\]

If \(b_i < 0\), we can follow a similar line of reasoning. Suppose \(0 \leq x_i \leq a_i m_i + b_i\) which results in

\[
x_i = \bar{x}_i \quad \Rightarrow \quad x_i y_i = \bar{x}_i \bar{y}_i, \tag{15}
\]

and now suppose \(x_i > a_i m_i + b_i\). Then we can write

\[
x_i > \bar{x}_i = a_i m_i + b_i \quad \Rightarrow \quad x_i y_i > \bar{x}_i \bar{y}_i. \tag{16}
\]

Based on (12)-(16), we can conclude that

\[
\sum_i \bar{x}_i \bar{y}_i + p(f - \sum_i \bar{y}_i) < \sum_i x_i y_i + p(f - \sum_i y_i). \tag{6}
\]

Using (6), we have

\[
\sum_i \bar{x}_i \bar{y}_i + p(f - \sum_i \bar{y}_i) < \sum_i x_i^* y_i^* + p(f - \sum_i y_i^*),
\]

which is a contradiction since \(x^*\) and \(y^*\) are the minimizers of the problem (5).

**Remark 4.** The piece-wise linear constraint (4d) makes the problem (4) a piece-wise quadratic optimization problem with \(3^n\) quadratic problems where \(n\) is the number of prosumers. Theorem 3 proves that one of these \(3^n\) problems always attains the global optimum.

**Remark 5.** Let \(x_i^* = b_i \geq 0\) and \(y_i^* = \frac{x_i^* - b_i}{a_i} = 0\) for some \(i\). Then we have \(x_i^* y_i^* = 0.\) Based on (6), we can
Kuhn-Tucker optimality conditions as they can be replaced by the necessary and sufficient Karush-Kuhn-Tucker optimality conditions. In our bilevel optimization problem, the aggregator needs to have all the information about the prosumers to solve the problem in a centralized way. This is rather an unrealistic assumption, since the prosumers are not willing to share their information with third parties. Our reformulation, i.e., being convex now allows a distributed solution to find the optimum, see [25].

IV. SIMULATIONS

In this section, we evaluate the performance of our convex equivalent problem for the RTBM in terms of computation time and optimality. We use the state-of-the-art MIP-based approach in [22] as a benchmark for this evaluation. The following subsection briefly explains how to convert the problem to an MIP problem.

A. MIP-based approach

The lower level optimization problems in (2) are concave maximization problems with strong duality [26]. Therefore, they can be replaced by the necessary and sufficient Karush-Kuhn-Tucker optimality conditions as

\[
\min_{x,y,\lambda_1,\lambda_2} \sum_i x_i y_i + p(f - \sum_i y_i) \\
\text{subject to } \quad x_i \geq 0, \quad \forall i \in N, \\
\sum_i y_i \leq f,
\]

\[
\begin{aligned}
& x_i - a_i y_i - b_i + \lambda_{1i} - \lambda_{2i} = 0, \\
& 0 \leq y_i \perp \lambda_{1i} \geq 0, \\
& 0 \leq m_i - y_i \perp \lambda_{2i} \geq 0,
\end{aligned}
\]

where \(\lambda_{1i}\) and \(\lambda_{2i}\) are dual coefficient for the constraints on \(y_i\). By introducing auxiliary binary variables \(z_i\) and \(w_i\) and a sufficiently large constant \(M\), the problem (17) can be turned to an MIP problem as

\[
\begin{align}
\min_{w,y,z,\lambda_1,\lambda_2} & \quad \sum_i (a_i y_i^2 + (b_i - p) y_i + m_i \lambda_{2i}) + pf \\
\text{subject to } & \quad \sum_i y_i \leq f, \\
& \quad a_i y_i + b_i - \lambda_{1i} + \lambda_{2i} \geq 0, \\
& \quad 0 \leq y_i \leq M z_i, \\
& \quad 0 \leq m_i - y_i \leq M w_i, \\
& \quad 0 \leq \lambda_{1i} \leq M(1 - z_i), \\
& \quad 0 \leq \lambda_{2i} \leq M(1 - w_i), \\
\end{align}
\]

Details of this approach can be found in [22].

B. Computation time and optimality comparison

For simulation purposes, we consider one type of HP and two types of mCHP technologies for the prosumers. We assume that half of the prosumers have HP and the other half are equipped with mCHP. We assign to each prosumer a specific technology of HP or mCHP, randomly. Tables I and II show the data regarding these types and also their corresponding \(|b_i|\) parameters. The supplier gas and electricity prices are based on data from [27] for the Netherlands and equal to 0.0861 €/kWh and 0.1707 €/kWh, respectively. The price \(p\) for both up- and down-regulation is set to 0.6 €/kWh based on the settlement price data of TenneT from [28] for a period where the TSO is under high stress. It should be noted that the TSO informs the aggregators about this price ex-ante.

Both the convex formulation and the MIP formulation are implemented in MATLAB r2018b and solved by the Gurobi Optimizer [29]. The simulations were run on four Intel Xeon 2.6 GHz cores and 1024 GB internal memory of the Peregrine high performance computing cluster of the University of Groningen.
C. Discussion on the results

The computation time for the convex optimization problem grows approximately linear with respect to the number of prosumers. This can be seen from the average run time in Table III for the convex formulation. If we consider 30000 as the typical number of prosumers for an aggregator, then the average and the maximum run time are acceptable for a real-time application with 5-minute time interval. However, this is not the case for an MIP formulation. Figure 4 and Table III show that the average and maximum computation time of MIP is not suitable for a real-time market since the computational time grows approximately exponentially. Moreover, there are some cases that the MIP formulation with high number of optimization variables does not converge to the global optimal or even to feasible solution.

V. Conclusions

Currently, prosumers do not participate in real-time balancing. In this paper, we have developed a market with a TSO, an aggregator and prosumers to address real-time balancing. We modeled the corresponding economic optimization problem of a self-interest aggregator and prosumers as a bilevel optimization problem to represent hierarchy in the market. Generally, bilevel optimization problems are non-convex. We have shown that it suffices to solve a specific convex optimization problem to find the global optimum of the original bilevel optimization problem. In contrast to existing approaches (e.g., MIP), the convex equivalent of the bilevel optimization problem has very low computation time and is therefore preferable in real-time. Moreover, in this approach the global optimality of the solution is guaranteed.

Low computation time and global optimality are not the only advantages of having a convex equivalent for the bilevel optimization. Centralized aggregator control over the whole community of prosumers can be a difficult task, especially when the number of prosumers is very high. However, having a convex formulation for the balancing problem opens up new horizons in decentralized and distributed control and optimization.

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| Number of Prosumers | Convex formulation run time | MIP formulation run time | Number of scenarios with infeasible or non-optimal solution for MIP |
|---------------------|-----------------------------|--------------------------|---------------------------------------------------------------|
|                     | Average (sec) | Maximum (sec) | Average (sec) | Maximum (sec) |                          |
| 10                  | 0.0006       | 0.0010       | 0.0016       | 0.0039       | 0                          |
| 100                 | 0.0012       | 0.0017       | 0.0012       | 0.0058       | 0                          |
| 1000                | 0.0038       | 0.0076       | 0.0128       | 0.0371       | 1                          |
| 10000               | 0.0344       | 0.0484       | 0.5548       | 9.0352       | 11                         |
| 20000               | 0.0772       | 0.1231       | 3.9498       | 59.1761      | 27                         |
| 30000               | 0.1161       | 0.1834       | 11.1190      | 161.7937     | 40                         |

**TABLE III: Simulation run time and optimality comparison.**

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