THE STAR-FORMATION RELATION FOR REGIONS IN THE GALACTIC PLANE:
THE EFFECT OF SPATIAL RESOLUTION

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ABSTRACT

We examined the relations between molecular gas surface density and star-formation rate surface density in an 11 deg2 region of the Galactic plane. Dust continua at 1.1 mm from the Bolocam Galactic Plane Survey and 22 μm emission from the Wide-field Infrared Survey Explorer (WISE) all-sky survey were used as tracers of molecular gas and the star-formation rate, respectively, across the Galactic longitude of 31.5 deg l deg 20.5 deg and Galactic latitude of 0.5 deg b deg 20 deg. The relation was studied over a range of resolutions from 33 arcsec to 20 arcmin by convolving images to larger scales. The pixel-by-pixel correlation between 1.1 mm and 22 μm increases rapidly at small scales and levels off at the scale of 5 arcmin–8 arcmin. We studied the star-formation relation based on a pixel-by-pixel analysis and on an analysis of the 1.1 mm and 22 μm peaks. The star-formation relation was found to be nearly linear with no significant changes in the form of the relation across all spatial scales, and it lies above the extragalactic relation from Kennicutt. The average gas-depletion time is ≈200 Myr and does not change significantly at different scales, but the scatter in the depletion time decreases as the scale increases.

Key words: ISM: clouds – local interstellar matter – stars: formation

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1. INTRODUCTION

Because star formation is an important process in the formation and evolution of galaxies, understanding what controls the conversion of gas into stars is essential. One of the key parameters in determining the star-formation rate (SFR) in any region should be the amount and nature of the gas available. Our goal is to use surveys of the Galactic plane to study the relation between gas and star formation on scales ranging from 1 to 200 pc, smaller than the scales accessible in studies of other galaxies.

The study of the relation between gas mass and the SFR goes back to the study of Schmidt (1959), who proposed a power-law relation between SFR and the gas density. Kennicutt (1998) studied a set of galaxies and found a relation between the disk-averaged SFR surface density (ΣSFR) and gas surface density (Σg) to be ΣSFR ∝ Σg1.4. Since then, there have been many studies on the star-formation relation in various types of galaxies, as reviewed by Kennicutt & Evans (2012).

With improvements in instrumental resolution and sensitivity, it became possible to study the relation on smaller scales within individual galaxies (Wong & Blitz 2002). Many studies used high-resolution data to study the relation down to the subkiloparsec scale (Kennicutt et al. 2007; Bigiel et al. 2008; Blanc et al. 2009; Verley et al. 2010; Liu et al. 2011; Schruba et al. 2011; Leroy et al. 2013). Star formation has been shown to be associated with molecular gas rather than total gas (Schruba et al. 2011), and the relations between ΣH2 and ΣSFR tend to be linear (ΣSFR ∝ ΣH21⋅0) (Bigiel et al. 2008; Leroy et al. 2013). While there is a strong correlation between gas mass and SFR surface density at disk-averaged scales, the relation showed a much larger scatter at smaller scales, suggesting that the star-formation relation breaks down below a certain scale (Onodera et al. 2010; Liu et al. 2011; Schruba et al. 2010). The large scatter of the star-formation relation on small scales implies that the relation does not arise simply from propagating the same relationship up from the scale of individual star-forming regions (see also Evans et al. 2014). Currently, the extragalactic studies can resolve star-forming regions down to the subkiloparsec scale, with a few reaching the scale of about 100 pc (Schruba et al. 2010; Onodera et al. 2010).

The relations seen in the extragalactic studies are measured on scales that are larger than the individual molecular cloud. To understand the physics that underly the relation, one should look at an individual star-forming region. To study star formation down to the scale of an individual molecular cloud, we need to look at closer regions. Studies of nearby molecular clouds (distance <1 kpc) have found that star formation was highly concentrated in dense gas (Heiderman et al. 2010; Lada et al. 2010, 2013; Evans et al. 2014). Where star formation happens, the star-formation rate lies well above the relation for extragalactic regions, indicating that the extragalactic relation emerges from averaging over both star-forming and non-star-forming gas. Regions of high-mass star formation, more representative of what is probed in studies of other galaxies, are generally more distant and require techniques different from those used in the nearby clouds. Studies of massive, dense regions indicated a linear relation between SFR and dense gas (Wu et al. 2005, 2010), similar to those found in other galaxies when tracers of dense gas like HCN emission were used (Gao & Solomon 2004a, 2004b). The relation using tracers of dense gas also shows larger ΣSFR at the same ΣH2 than found in extragalactic studies using all of the molecular gas.

Extragalactic studies obtain star-formation relations by looking at regions covering large areas of the galactic disk, and star-formation studies in the Milky Way have focused on individual star-forming regions that were selected based on certain criteria. The regions studied by Wu et al. (2010) trace small, very dense regions, and they were selected to have signposts of massive star formation and thus may be biased. To bridge the gap between the Galactic and extragalactic scales and to understand some of the differences in the extragalactic results, we need to study Galactic star formation on larger scales and without the biases of previous studies.
There have been many new large-scale observations of the Milky Way in various wavelength bands. The Spitzer legacy projects include the MIPS GAL (Carey et al. 2009) and GLIMPSE (Churchwell et al. 2009) surveys, giving a view of the Galactic plane in the infrared from 3.6 μm to 70 μm. All-sky surveys also provide information on star-formation rates in the Galactic plane (e.g., Wide-field Infrared Survey Explorer (WISE); Wright et al. 2010). The Bolocam Galactic Plane Survey (BGPS) observed the northern part of the Galactic plane in a 1.1 mm dust continuum (Aguirre et al. 2011; Ginsburg et al. 2013 for version 2 of the data). The molecular gas distribution of the Milky Way was studied by Dame et al. (2001) in 12CO and by the Galactic Ring Survey in 13CO (Jackson et al. 2006), with both studies using the J = 1 → 0 line. With these data available, we can study the relation in a larger area of the Milky Way and perform a similar analysis on the Galactic plane as in the extragalactic data and study how the change in resolution and the change in the region-selection method affect the result. Understanding the effect of the change in resolution and selection methods will be useful in comparing between Galactic and extragalactic studies.

In this paper, we used the 1.1 mm dust continuum data from the BGPS survey as a molecular gas tracer and the 22 μm emission by first looking at the pixel-by-pixel relation (Section 4) and then by identifying separate sources inside the regions (Section 5). The discussion of the results is in Section 6, and the results are summarized in Section 7.

2. DATA

2.1. Emission at 1.1 mm

Dust emission has been used as a total gas tracer in previous studies by assuming that dust and gas are well mixed (Leroy et al. 2007; Bolatto et al. 2011). The correlation between gas and dust emission has also been studied by the CO survey (Dame et al. 2001) and the recent PLANCK survey (Planck Collaboration et al. 2011). Planck Collaboration et al. (2011) argued that dust emission is optically thin up to a column density of N_H ≈ 10^{25} cm^{-2} at 1 mm. It is also less sensitive to temperature than the far-infrared (FIR) emission. Using dust opacities, a dust-to-gas ratio, and a typical dust temperature, we can estimate the gas mass.

We used the 1.1 mm dust continuum from the BGPS version 2 as a molecular gas tracer. The BGPS covers about 170 deg^2 in a 1.1 mm continuum at an effective resolution of 33" (Aguirre et al 2011). The 1.1 mm data span the range of Galactic longitude of −10.5 ≤ l ≤ 90.5 and Galactic latitude of |b| ≤ 0.5 with additional coverage in some selected regions (Ginsburg et al. 2013). Extended sources were extracted from the 1.1 mm images to a catalog (Bolocat) using a watershed decomposition algorithm (Rosolowsky et al. 2010; Ginsburg et al. 2013). Follow-up molecular line observations of the sources include HCO+ J = 3−2, N2H+ J = 3−2 (Schlingman et al. 2011; Shirley et al. 2013), and NH3(J, K) = (1, 1), (2, 2), and (3, 3) inversion transitions (Dunham et al. 2011). Distances to a subset of the bolocat sources are also available (Ellsworth-Bowers et al. 2013, 2014). Bolocat sources are relatively dense (∼10^{1.5} cm^{-3}) structures in molecular clouds with angular sizes of ≈0.5−2′ (Ginsburg et al. 2013).

Surveys by ground-based instruments at λ ≈ 1 mm lose sensitivity to emission beyond some angular scale because it cannot be separated from the atmospheric emission variation. The BGPS maps completely recover emission up to 80′′ and partially recover emission to ≈5′′ (Ginsburg et al. 2013). These surveys thus pick out regions of characteristic volume densities, which depend on distance. Dunham et al. (2011) calculated sizes and other properties of a subset of sources and characterized the bolocat sources as cores, clumps, and clouds, with size scales on the order of 0.1 pc, 1 pc, and 10 pc, respectively, depending on the sources’ distances. As discussed later, the majority of the structures in the regions we study here correspond to relatively dense (n ≈ 10^{3.5} cm^{-3}) clumps within larger molecular clouds. Battisti & Heyer (2014) extracted 13CO clouds associated with selected BGPS sources with dense gas observations and compared the dust mass with the mass of the parent 13CO clouds. Comparing a total mass of BGPS sources inside giant molecular clouds (GMCs) with the mass of the GMC gave a median mass ratio of 0.11^{+0.09}_{−0.06}. If the mass was restricted to regions with a mass surface density higher than 200 M⊙ pc^{-2}, the ratio decreased to 0.07^{+0.05}_{−0.03} (Battisti & Heyer 2014). The dense gas mass fraction does not appear to depend on cloud mass or cloud mass surface density. This result shows that the 1.1 mm sources occupy only a small fraction of the mass and volume within the clouds. The maps at 1.1 mm also have a smaller chance of source confusion along the line of sight than does CO, which traces more extended emission from the less dense parts of molecular clouds.

2.2. Emission at 22 μm

The mid-infrared continuum emission has been used as a tracer of SFR. The MIPS GAL survey covers our target region in the 24 μm MIPS band at a resolution of 6′. However, the saturation level is too low for the purpose of our large-scale study. Instead, we used the WISE (Wright et al. 2010) all-sky release images at 22 μm as a tracer of star formation. The WISE observed the entire sky in multiple exposures in four IR bands at 3.4, 4.6, 12, and 22 μm with a resolution of 12′0 at 22 μm. The star-formation rate (24 μm) has been calibrated using the Spitzer MIPS 24 μm band. The WISE 22 μm band overlaps the MIPS 24 μm band with a slightly bluer response curve. The comparison between the two bands shows that they are comparable (Jarrett et al. 2011; Anderson et al. 2014). We considered the difference between measurements centered at 22 μm and those centered at 24 μm to be negligible, so we use the relations derived for 24 μm to calculate star-formation rates.

The 22 μm emission comes from dust heated by strong stellar radiation or from transiently heated small dust grains (Draine et al. 2007; Calzetti et al. 2007). The bright emission peaks are expected to indicate dust concentrations that are heated by high-mass stars. The 22 μm emission can then be used to trace the presence of high-mass stars and so traces the current star-formation activities. We used the 22 μm maps to study the relation between star-formation activities and the gas distribution.

To convert the 22 μm emission to the SFR, however, a conversion factor or a calibration is needed. Several studies calibrated a relation between MIR and SFR using extragalactic data (Calzetti et al. 2007, 2010, and references therein). These calibrations were done on extragalactic scales assuming a fully sampled IMF and a long timescale of constant star...
formation. These assumptions are not always valid when applied to smaller scales or to regions with low mass or low SFR (Vutisalchavakul & Evans 2013; Kennicutt & Evans 2012). The effect of stochastically sampling the initial mass function (IMF) and the star-formation history has been studied quantitatively by Fumagalli et al. (2011) and da Silva et al. (2012, 2014). They did not study the SFR measured from the 24 μm emission explicitly, but they did study the SFR measured from the bolometric luminosity, which is the closest in sensitivities to the mid-infrared emission. These two tracers correlate closely (Vutisalchavakul & Evans 2013). The SFR determined from the bolometric luminosity, assuming continuous star formation and an IMF, has a scatter of 0.4 to 0.6 dex for SFR < 10^{-4} M_⊙ yr$^{-1}$ and begins to systematically underestimate the actual SFR below 10^{-5} M_⊙ yr$^{-1}$ (da Silva et al. 2014). Vutisalchavakul & Evans (2013) found that total infrared luminosity and 24 μm emission can underestimate SFR by more than an order of magnitude for molecular clouds with SFR < 10^{-6} M_⊙ yr$^{-1}$. The effects of stochasticity get smaller as the total SFR of the region increases. For this work, 22 μm can underestimate the SFR on small scales and especially at low surface density, but it should trace SFR better when we look at larger scales, where we average over multiple star-forming regions and toward bright regions with high surface density.

2.3. The Regions Studied

The part of the Galactic plane covered in this study includes a Galactic longitude range of 20.5 ≤ l ≤ 31.5 and Galactic latitude range of |b| ≤ 0.5, a total of 11 deg$^2$, divided into two equal-size regions. Both the 1.1 mm images from BGPS version 2 and WISE 22 μm images were combined into two separate mosaics using Montage (Jacob et al. 2010) for the purpose of data analysis: region 1 at 20.5 ≤ l ≤ 26.0 and region 2 at 26.0 ≤ l ≤ 31.5. Figure 1 shows 1.1 mm and 22 μm images for region 1 in the top two panels and region 2 in the third and fourth panels. All of the analysis described in the next section was performed similarly on images for the two regions. The results were then combined for further discussion. The two regions show sources with strong emission at 22 μm, including well-studied high-mass star-forming regions W41 and W42 in region 1 and W43 for region 2 (Benjamin et al. 2005). The end of the Galactic bar should be near the end of region 2 at around 1 ≈ 28–31° (Benjamin et al. 2005).

3. DATA PROCESSING

3.1. Diffuse IR Background Estimation

The 22 μm images show diffuse background emission that is not necessarily associated with recent star formation. The diffuse (cirrus) infrared emission within our Galaxy has long been observed and studied (Low et al. 1984; Miville-Deschênes et al. 2007; Compiègne et al. 2010). We constructed diffuse emission maps to facilitate removal of the diffuse background in the 22 μm images for photometry and for better comparison with the 1.1 mm images where large-scale emission was automatically removed.

The WISE 22 μm images were convolved to a resolution of 33′ and aligned to the BGPS images, which were then used for estimating the diffuse emission maps. To capture the variations in the background, we adopted the following method. First, source areas were determined by selecting a 22 μm flux contour level that matched bright 22 μm emission areas when inspected by eye. All pixels inside the source areas were masked as source pixels. Second, the images were divided into smaller rectangular subgrids at a size of 200 by 200 pixels (≈10/8). Iterative applications of Chauvenet’s rejection criterion were performed on each subgrid by iteratively applying a 3σ cut until all of the remaining pixels are within 3σ of the average pixel value. The average of the remaining pixel values was then taken as the background value for the subgrid. Finally, subgrids with fractions of the source area above a certain clipping threshold were omitted. The rest of the subgrid’s background values were interpolated with a thin-plate spline interpolation to create a final background image.

This method gave a reasonable representation of the diffuse emission as seen in the original 22 μm images. We chose a source contour level, a grid size, and a clipping threshold that resulted in the closest approximation of the diffuse emission when inspected by eye. Our method gives a diffuse background image that is similar to the method of Battersby et al. (2011). More detailed descriptions on the parameters chosen and the associated uncertainties are provided in Appendix B, and the comparison between two background subtraction methods is provided in Appendix C. The background-subtracted 22 μm images were used for the image convolution.

The result of removing diffuse emission is shown in Figure 2 by comparing the fraction of the estimated diffuse emission to total emission as a function of surface brightness. The diffuse emission dominates the 22 μm flux at low surface brightness and contributes a considerable fraction up to a surface brightness of ≈700 MJy sr$^{-1}$, where the diffuse emission accounts for 50% of the total emission, showing that removing diffuse emission is crucial.

3.2. Image Convolution

The original resolutions of the 1.1 mm and 22 μm images were 33′ and 12′ respectively. Because we intended to study the relations between 1.1 mm and 22 μm at different scales, we first created a set of images at different spatial resolutions. This was done by convolving the images with a two-dimensional (2D) Gaussian profile of varying FWHM. Both the 22 μm and 1.1 mm images at 33′ resolution were convolved with a 2D Gaussian kernel to resolutions of ≈1′, 2′, 3′, 4′, 5′, 8′, 10′, 15′, and 20′. The scale of 20′ was the largest scale we could achieve because of the limited coverage of Galactic latitude in the BGPS data. After the convolutions, the convolved images were binned to oversampling rates of ≈10 pixels per beam’s FWHM. Figure 1 shows the 1.1 mm and 22 μm images at three resolutions: 33′, 10′, and 15′. The whole set of the convolved images was then used for further analysis.

3.3. Detection Limit

We identified regions with unreliable detections by estimating the sensitivity levels of both the 1.1 mm and 22 μm maps. The sensitivity of the WISE 22 μm images is very low compared to the average flux, and the uncertainty in source emission is dominated by the small-scale variations in the diffuse background emission. We estimated the noise in the diffuse emission based on several sky regions in the image at 1′ resolution. The average of the standard deviation of all of the sky regions was taken as the 1σ value of the noise per pixel. The detection limits at other resolutions were estimated by assuming that the noise drops as 1/√N_{pixels}, where N_{pixels} is the number of pixels at the 1′ scale inside a resolution element. Tests on the convolved images supported this assumption.
Figure 1. BGPS 1.1 mm and WISE 22 μm images for the entire region at three different resolutions of 33\textquoteleft (top four panels), 5\textquoteleft (middle four panels), and 10\textquoteleft (bottom four panels).

(A color version of this figure is available in the online journal.)
covering the entire image, each with a corresponding 1.1 mm and 22 μm flux surface density.

The mass surface density can be calculated from

$$\Sigma_{\text{H}_2} = \left( \frac{\Omega}{\Omega_{\text{dust}}(1.1)} \right) \left( \frac{\rho_g}{\rho_d} \right),$$

(1)

where $$\rho_g$$ is the gas density, $$\rho_d$$ is the dust density, $$\Omega$$ is the solid angle of a pixel, and $$\kappa_{\text{dust},1.1} = 1.14 \text{ cm}^2 \text{ g}^{-1}$$ is the dust opacity at 1.1 mm per dust mass (Ossenkopf & Henning 1994), and $$\rho_g/\rho_d$$ is the gas-to-dust mass ratio, taken to be 100 (Hildebrand 1983). Assuming standard values and a dust temperature of 20 K for all sources yields

$$\Sigma_{\text{H}_2} = \frac{37.2 \times S_{1.1 \text{ mm}}}{\theta_{\text{arcsec}}^2}(\text{g cm}^{-2}),$$

(2)

where $$S_{1.1 \text{ mm}}$$ is the 1.1 mm flux density in Jansky, and $$\theta_{\text{arcsec}}$$ is the size of the region in arcseconds (Schlingman et al. 2011).

The SFR surface density was calculated from the extragalactic relation (Calzetti et al. 2007):

$$\Sigma_{\text{SFR}}(M_{\odot} \text{ yr}^{-1} \text{ kpc}^{-2}) = 1.56 \times 10^{-35} \times [S_{24}(\text{erg s}^{-1} \text{ kpc}^{-2})]^{0.8104}.$$  

(3)

The 24 μm luminosity surface density is described by

$$S_{24}(\text{erg s}^{-1} \text{ kpc}^{-2}) = \frac{\nu(H_2)L_{24}(\nu)}{A(kpc^2)},$$

where $$L_{24}(\nu)$$ is the 24 μm luminosity per unit frequency, $$\nu$$ is the frequency, and $$A$$ is the projected physical area of the region. We substituted the 22 μm flux for the 24 μm flux for the calculations.

The resulting $$\Sigma_{\text{H}_2}$$ and $$\Sigma_{\text{SFR}}$$ for various spatial scales are shown in Figure 4. The figure shows the star-formation relations as contour plots at each resolution. The contours represent number densities of data points of 1, 3, 5, 10, 20, 40, 60, and
Figure 4. Pixel-by-pixel star-formation relations of \( \Sigma_{\text{H}_2} \) and \( \Sigma_{\text{SFR}} \) at different resolutions show large scatters at small scales and tighter relations at larger scales. The contours represent source number densities of 1, 3, 5, 10, 20, 40, 60, and 80 data points at the binning of 0.2 in \( \Sigma_{\text{H}_2} \) and \( \Sigma_{\text{SFR}} \). The same color represents the same number density in all of the plots. The three dotted lines show lines of constant depletion time with \( t_{\text{dep}} = 10^8, 10^9, 10^{10} \) yr from top to bottom. The horizontal and vertical dashed lines show the 3\( \sigma \) detection limit for \( \Sigma_{\text{SFR}} \) and \( \Sigma_{\text{H}_2} \) respectively.

(A color version of this figure is available in the online journal.)
The molecular depletion time is defined to be

\[ t_{\text{dep}} = \frac{\Sigma_{\text{H}_2}}{\Sigma_{\text{SFR}}} \]

The variable \( t_{\text{dep}} \) can be thought of as the timescale for all of the molecular gas to be converted into stars at the current rate of star formation. The three dotted lines in Figure 4 show lines of constant depletion time with \( t_{\text{dep}} = 10^8, 10^9, \) and \( 10^{10} \) yr from top to bottom. The distribution of \( t_{\text{dep}} \) provides a good measure of the scatter in the star-formation relations.

Figure 5 shows the distributions of the \( \log(t_{\text{dep}}) \) at various spatial scales, which include all of the data points with positive fluxes. The dashed blue line in each plot represents a Gaussian fit to the distribution. The result shows that the average \( \log(t_{\text{dep}}) \) is about 8.3, corresponding to 200 Myr, independent of smoothing length, and the scatter, \( \sigma(\log(t_{\text{dep}})) \), decreases as the smoothing length increases.

The distributions in Figure 5 include many points below the detection limit. To account for the sensitivity issue, we assigned

80 data points at the binning of 0.2 in \( \log(\Sigma_{\text{H}_2}) \) and \( \log(\Sigma_{\text{SFR}}) \). The same color represents the same number density in all of the plots. The plot includes all data points with positive fluxes with the vertical and horizontal dashed red lines representing detection (3\( \sigma \)) limits for \( \Sigma_{\text{H}_2} \) and \( \Sigma_{\text{SFR}} \) respectively. Naturally, the number of data points drops as the scale gets larger, and there are fewer pixels with values <3\( \sigma \).

The mean \( \mu \) and the standard deviation \( \sigma \) from the Gaussian fit are shown in red for each plot. The mean \( t_{\text{dep}} \) from the fit does not change much at different scales, and the standard deviation decreases as the scale increases.

(A color version of this figure is available in the online journal.)
The three methods give a comparable fit for data sets where large fractions of the data points are censored. The average resolution for pixel-by-pixel analysis is shown in Table 1. The pixel-by-pixel analysis provides a way to study the relation between $\Sigma_H$ and $\Sigma_{SFR}$ for all of the data points with values below the detection limits. Data points with upper limits on both $\Sigma_H$ and $\Sigma_{SFR}$ were omitted from further analysis because most of them should be regions that are not involved in current star formation. Table 1 shows $\langle \log(t_{dep}) \rangle$ and $\sigma(\log(t_{dep}))$ estimated by three different methods. The first method (fit) is the Gaussian fit to the distributions of all of the data points with positive fluxes before assigning upper limits, as shown in Figure 5. The second method (limit) used the data, including upper-limit values of $\Sigma_H$ and $\Sigma_{SFR}$, to calculate the median and standard deviation of $\log(t_{dep})$. The third method (EM) used an expectation-maximization algorithm as described in Wolynetz (1979) to estimate the mean and standard deviation of a censored normal distribution. The EM method is based on calculating the maximum likelihood estimates to a distribution assumed to be normally distributed. The data includes left-censored (upper limit in $\Sigma_H$) and right-censored (upper limit in $\Sigma_{SFR}$) data points. The uncertainties in the EM method increase for data sets where large fractions of the data points are censored. The three methods give a comparable $\langle \log(t_{dep}) \rangle$ of about 8.3 regardless of the resolution scale (Table 1). They differ more regarding $\sigma(\log(t_{dep}))$, but these differences disappear at larger smoothing scales.

5. RESULTS: SOURCE-BASED RELATIONS

The pixel-by-pixel analysis provides a way to study the relation between $\Sigma_H$ and $\Sigma_{SFR}$ for an entire region without the need to extract sources from the images. This method was used in some extragalactic studies (Bigiel et al. 2008; Liu et al. 2011; Leroy et al. 2013). However, the pixel-by-pixel analysis includes regions with low 22 $\mu$m and 1.1 mm surface densities that do not necessarily represent star formation. In addition, no information on distance is available.

Another approach to studying the relation is to choose regions with strong emission, as was done in some extragalactic studies (e.g., Kennicutt et al. 2007). We started by looking at extracted sources from the BGPS source catalog. These sources also have the advantage of having additional properties determined for a subset of them in previous studies, such as distances, sizes, temperature, and densities.

We started with the sources in the BGPS catalog, describing the properties of sources in this sample (Section 5.1) and studying the star-formation relation (Section 5.2). Then we considered what happens when we smoothed the images to larger scales and extracted data centered on the 1.1 mm peaks (Section 5.3). Finally, we compared these results to those obtained when we extracted data centered on the 22 $\mu$m peaks (Section 5.4).

5.1. The BGPS Source Properties

The BGPS source catalog (Bolocat) provides sources extracted from the 1.1 mm images along with the integrated source flux and the flux inside aperture diameters of 40', 80', and 120'. These sources are possible sites of star formation, and they correspond to cores, clumps, or clouds, depending on the distance (Dunham et al. 2011). Because the 1.1 mm emission provides an unbiased tracer for dense molecular gas without a preselected criteria, Bolocat gives a good set of sources for studying the properties of potential star-forming regions.

Distances are not available to all of the Bolocat sources, but kinematic distances to a subset of sources have been determined by Ellsworth-Bowers et al. (2013, 2014) using a Bayesian distance probability distribution function to resolve the kinematic distance ambiguity in the inner Galaxy. The distance catalog provides distances along with the uncertainties. In our targeted regions, 33% of the sources have distances. Figure 6 shows the distance distribution for sources inside our targeted region, where the distances plotted are the maximum likelihood distances from Ellsworth-Bowers et al. (2014). The shaded gray area represents a distance range inside which 90% of the total number of sources reside (see Section 6.1). The subset of the sources with distances (referred to hereafter as the distance catalog) are generally representative of the entire Bolocat source catalog. Their distribution in Galactic latitude...
is comparable to that of sources in the full catalog. However, the distance catalog is slightly biased toward sources with larger surface brightness, with a median of 6.8 and 8.0 MJy sr$^{-1}$ for full catalog and distance catalog, respectively (see the full discussion in Ellsworth-Bowers et al. 2014).

Seventy-three percent of the sources with distances are located in the 5 kpc peak (between 0 and 7.5 kpc), 25% of the 10 kpc peaks (between 8 and 14 kpc), and a very small fraction of sources are at distances around 16 kpc. The mean of the distance is 5.9 kpc, the median is 5.1 kpc, and the standard deviation is 2.8 kpc. We will use the median value to calculate characteristic properties. At 5 kpc, 1 arcmin corresponds to 1.45 pc, and BGPS sources correspond to clumps with typical densities of $10^3$ cm$^{-3}$ (Ginsburg et al. 2013). The X-ray, IR, and optical bands of the source were determined from the calibration and photometric uncertainties (Ginsburg et al. 2013).

The result is shown in Figure 7. The contours show the source number density, as in Figure 4. The data have a large scatter, with a rank correlation coefficient between log($\Sigma_{H_2}$) and log($\Sigma_{SFR}$) of 0.40. We assumed a power-law relation of the form

$$\Sigma_{SFR} \propto \Sigma_{H_2}^n,$$

or equivalently

$$\log(\Sigma_{SFR}) = n \log(\Sigma_{H_2}) + a.$$  

A linear curve fit to the log data including the uncertainties in both axes using MPFITEXY (Markwardt 2009) gave fitting parameters of $n = 0.84 \pm 0.03$ and $a = -2.68 \pm 0.06$, as shown by the solid black line. The dashed red line represents the extragalactic star-formation relation from Kennicutt et al. (1998) of

$$\Sigma_{SFR} = (2.5 \pm 0.7) \times 10^{-4} \Sigma_{H_2}^{1.4 \pm 0.15}.$$  

The vertical and horizontal dashed lines represent the 3$\sigma$ detection limit, and the dot-dashed blue line represents the relation observed for dense gas as traced by HCN from Wu et al. (2005):

$$(SFR)(M_\odot \text{yr}^{-1}) \approx 1.2 \times 10^{-8} M_{\text{dense}}(M_\odot).$$  

5.2. Analysis Based on Bolocat Sources

We started the investigation at the scale of individual Bolocat sources by looking at the 22 $\mu$m emission from these sources. We performed aperture photometry on the 22 $\mu$m images of 33$''$ resolution at each of the Bolocat source positions with an aperture radius of 40$''$ to compare to the 1.1 mm Bolocat flux of the same aperture size.

The photometry resulted in a total of 1,981 Bolocat sources in the whole 11 deg$^2$ region; 738 of them have a 22 $\mu$m background-subtracted flux below the detection limit, or about 37%. The photometric uncertainties were determined by combining the observational and Poisson errors from the WISE uncertainty maps with the estimated random uncertainties in our method of background subtraction, the details of which can be found in Appendix B.

We next calculated the SFR surface density from the 22 $\mu$m flux (Equation (3)) and the molecular gas surface density from the 1.1 mm flux (Equation (2)) within an 80$''$ aperture. The typical uncertainties are $\approx$28% and 7% for $\Sigma_{H_2}$ and $\Sigma_{SFR}$ respectively. The uncertainties for $\Sigma_{H_2}$ were determined from the calibration and photometric uncertainties (Ginsburg et al. 2013).

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A linear curve fit to the log data including the uncertainties in both axes using MPFITEXY (Markwardt 2009) gave fitting parameters of $n = 0.84 \pm 0.03$ and $a = -2.68 \pm 0.06$, as shown by the solid black line. The dashed red line represents the extragalactic star-formation relation from Kennicutt et al. (1998) of

$$\Sigma_{SFR} = (2.5 \pm 0.7) \times 10^{-4} \Sigma_{H_2}^{1.4 \pm 0.15}.$$  

The vertical and horizontal dashed lines represent the 3$\sigma$ detection limit, and the dot-dashed blue line represents the relation observed for dense gas as traced by HCN from Wu et al. (2005):

$$(SFR)(M_\odot \text{yr}^{-1}) \approx 1.2 \times 10^{-8} M_{\text{dense}}(M_\odot).$$  

5.3. Analysis Based on 1.1 mm Peaks

On the scale of BGPS sources, the 22 $\mu$m emission shows a weak correlation with the 1.1 mm flux with a large scatter. From visual inspections, the 22 $\mu$m emission in the whole region is more diffuse than the 1.1 mm emission. A lot of the 22 $\mu$m extended emission also does not coincide with the Bolocat source contours. In this section, we studied how the correlation between SFR tracers and gas tracers changes when we look at the regions on larger scales.

Using the images convolved to larger angular scales (Section 3.2), we identified the local peaks of the 1.1 mm emission. The local peaks were identified by locating pixels whose values are larger than all of the adjacent pixels. Overlapping regions were eliminated by dropping peaks with distances to the nearest peak less than the radius of the aperture. We then performed aperture photometry on the 22 $\mu$m and 1.1 mm images with an aperture centered at the local 1.1 mm peaks and radius equal to the beam’s FWHM of the convolved images (aperture radius = 10$''$ for the images convolved to FWHM of 10$''$ and similarly for others). The aperture size was chosen to contain most of the source emission without applying aperture corrections. The same procedures were performed on the images convolved to resolutions of 10$'$', 15$'$', and 20$'$. Once we go to higher FWHM, the angular source sizes increase, which corresponds to looking at larger physical areas. At a distance of 5 kpc, 20$'$ corresponds to a physical size of about 29 pc in the plane of the sky.

Once we obtained the 1.1 mm and 22 $\mu$m fluxes from the photometry, $\Sigma_{H_2}$ and $\Sigma_{SFR}$ were calculated using Equations (2) and (3), respectively. All of the data points for the 10$'$, 15$'$', and...
Figure 8. $\Sigma_{\text{SFR}}$ vs. $\Sigma_{\text{H}_2}$ from aperture photometry centered at 1.1 mm peaks (a) and 22 $\mu$m peaks (b) for image resolutions of 10', 15', and 20'. The solid black line in each plot represents a robust linear fit to the data, and the dotted gray line represents a least-square fit. The dashed red line represents the extragalactic relation from Kennicutt (1998), and the dot-dashed blue line is the dense gas relation from Wu et al. (2005).

(A color version of this figure is available in the online journal.)

Table 2

| Parameters for the Source-based Analysis |
|-----------------------------------------|
| Resolution  | 1.1 mm | |
| Res. | $\rho$ | $\rho$(rank) | Robust($n, a$) | LS($n, a$) | $\log(t_{\text{dep}})$ | $\sigma(\log(t_{\text{dep}}))$ |
| 10' | 0.66 | 0.63 | 0.97 | $-2.4$ | 0.65 | $-2.1$ | 8.43 | 0.34 |
| 15' | 0.65 | 0.62 | 0.91 | $-2.2$ | 0.59 | $-1.9$ | 8.34 | 0.32 |
| 20' | 0.76 | 0.79 | 0.84 | $-2.1$ | 0.63 | $-1.9$ | 8.32 | 0.24 |

| 22 $\mu$m |
| Res. | $\rho$ | $\rho$(rank) | Robust($n, a$) | LS($n, a$) | $\log(t_{\text{dep}})$ | $\sigma(\log(t_{\text{dep}}))$ |
| 10' | 0.70 | 0.66 | 0.94 | $-2.3$ | 0.66 | $-2.0$ | 8.37 | 0.34 |
| 15' | 0.67 | 0.64 | 1.0 | $-2.5$ | 0.71 | $-2.1$ | 8.41 | 0.31 |
| 20' | 0.84 | 0.83 | 1.0 | $-2.5$ | 0.87 | $-2.3$ | 8.45 | 0.21 |

| Bolocat |
| Res. | $\rho$ | $\rho$(rank) | Robust($n, a$) | LS($n, a$) | $\log(t_{\text{dep}})$ | $\sigma(\log(t_{\text{dep}}))$ |
| 33' | 0.50 | 0.40 | 0.84 | $-2.68^a$ | 9.20$^b$ | 0.59$^b$ |

Notes.

a Linear fit to the data using MPFITEXY (Markwardt, 2009).
b Values correspond to the mean and standard deviation of $\log(t_{\text{dep}})$ from the expectation-maximization (EM) method.

20' resolutions are above the detection limit in both $\Sigma_{\text{H}_2}$ and $\Sigma_{\text{SFR}}$. The correlation between $\log(\Sigma_{\text{H}_2})$ and $\log(\Sigma_{\text{SFR}})$ increases from a linear correlation coefficient of 0.66 at the 10' scale to 0.76 at the 20' scale (Table 2). Two methods of linear curve fitting were performed on the data: an unweighted least-squares fit using MPCURVEFIT (Markwardt 2009) and a robust bisector linear fit (IDL Robust_Linefit). The relations for the convolved scales of 10', 15', and 20' can be seen in Figure 8(a). Each data point corresponds to a region inside an aperture centered on a peak of emission in the 1.1 mm image. The solid black line represents the robust fit to the data, the dotted gray line represents the least-squares fit to the data, the dashed red line represents the extragalactic star-formation relation (Equation (6)), and the dot-dashed blue line represents the dense gas relation from Wu et al. (2005). The coefficients of the curve fits are shown in Table 2.

The star-formation relations are slightly sublinear at all scales, more so for the least-squares fit. Both methods show increases in the intercept $a$ (the effective star-formation rate) as the scale gets larger.
However, the available distances to a subset of the sources can be used to obtain some rough estimations of the physical size that corresponds to each angular resolution. About 33% of the Bolocat sources in our targeted regions have distances measured with a distribution shown in Figure 6 (Ellsworth-Bowers et al. 2014). In estimating the physical scales, we assumed that the distance distribution of these sources is representative of the whole sample (see Section 5.1).

The first estimation of the physical scale used the median distance to the sources to calculate the size in the plane of the sky. With a median distance of 5.1 kpc, physical sizes in the plane of the sky at different resolutions are shown as the dashed red line in Figure 10(a). These sizes range from those of clumps to those of clouds. For angular scales small enough to be comparable to an individual source size, the estimated physical scales should represent the physical sizes of the sources.

For comparison to extragalactic measurements, we also computed the typical averaging scale for each resolution using the distance distribution. The idea behind the method is to use source locations in Galactic latitude and longitude to estimate the solid angle the sources subtend and use the distance distribution to estimate the range of distances for the sources. The result gives the volume subtended by the sources, which can be converted to a length scale. This would represent an upper limit to the relevant scale. To do this, first we binned the distance distribution with a binning size of 0.5 kpc as shown in Figure 6. Three values were chosen as source number density thresholds (per bin) so that the total number of sources above the thresholds account for about 80%, 90%, and 99% of the total sources. This is analogous to drawing number density contours on an image; however, instead of an image, we are drawing one-dimensional (1D) contours on a distribution. The gray-shaded area in Figure 6 is an example of the area inside the contour of 90% of the total sources. The resulting distance ranges (δd) are 4.5, 7, and 14 kpc for 80%, 90%, and 99% of the total sources, respectively. We calculated an averaging volume (V) for the very long, skinny rectangular prism from

\[
\frac{V}{\text{kpc}^3} = \left( \frac{\theta_r \ d}{\text{rad kpc}} \right)^2 \times \frac{\delta d}{\text{kpc}}.
\]

where \( \theta_r \) is the angular size of a resolution element. Reduced to a sphere, the effective radius \( R \) is

\[
R_{\text{kpc}} = \left( \frac{3}{4\pi \ V_{\text{kpc}^3}} \right)^{1/3}.
\]

The effective averaging scale, the diameter \( 2R \), at different resolutions is shown in Figure 10(a). Using this estimation at the median distance \( d \) of 5.1 kpc, the angular resolution of 20' corresponds to an averaging scale of \( \approx 225 \) pc.

A similar method was used to estimate the number of Bolocat sources that each resolution element contains. Bolocat sources were mapped in the Galactic coordinates, and a source number density map was created by counting the number of sources inside bin sizes of 0.2 in both axes. Then 2D contours of source number density were drawn on the source number density map to contain approximately 80%, 90%, and 99% of the total number of sources. The total solid angle inside each contour gave a total area for each completeness value. The average number of sources per resolution element is the solid angle of the resolution element \( (\theta_r^2) \) divided by the total area for the completeness value times the total number of sources. The average number of sources per resolution element is shown in Figure 10(b). For the larger smoothing lengths, we are typically averaging over 20 to 40 sources.

5.4. Analysis Based on 22 μm Peaks

What changes if regions are identified using 22 μm peaks instead of 1.1 mm peaks? To answer that question, we used procedures similar to those used in the 1.1 mm peaks analysis, but we started by identifying local emission peaks in the 22 μm images at resolutions of 10', 15', and 20' and performed photometry on both the 1.1 mm and 22 μm images at the locations of the 22 μm peaks. Figure 8(b) shows the plot of the star-formation relation at the three resolution scales. The fit parameters are included in Table 2. The fit parameters are comparable to the parameters for the 1.1 mm peaks.

An initial analysis did not show a significant difference when choosing 22 μm peaks versus choosing 1.1 mm peaks, as can be seen from the values of \( t_{\text{dep}} \) in Table 2. When thresholds were applied to the data, however, the result was different. Instead of choosing all of the identified local peaks, we chose only bright local peaks by dropping all of the regions that had fluxes below the threshold were then compared. The result in Figure 9 shows a larger \( t_{\text{dep}} \) for 1.1 mm peaks than for 22 μm peaks. The difference in \( t_{\text{dep}} \) decreases as the scale increases. The difference in \( \log(t_{\text{dep}}) \) is about 0.37 at the 5' scale and goes down to about 0.05 at the 20' scale. Note, however, that the scatter \( \sigma(\log(t_{\text{dep}})) \) is about 0.4 at the 5' scale and about 0.2 at the 20' scale.

6. DISCUSSION

6.1. Relating to Physical Scales

We have been looking at the relations between gas and star formation at various angular scales. Converting the angular scales to physical scales for our Galactic plane data is different than for resolved extragalactic data because we are looking edge-on through the disk instead of face-on or nearly face-on. However, the available distances to a subset of the sources can
6.2. Scatter in the Star-formation Relation

The results on the star-formation relations show that the relations at small scales have large scatter, as seen in the Bolocat source case or at the small scales in the pixel-by-pixel analysis. The rank correlation coefficients between log($\Sigma_{\text{H}_2}$) and log($\Sigma_{\text{SFR}}$) increase from 0.40 at the Bolocat source scale to 0.79 at the 20′ scale for sources based on the 1.1 mm peaks. This result is clearly seen in the pixel-by-pixel star-formation relation and the distribution of the $t_{\text{dep}}$. The scatter of the $t_{\text{dep}}$ in the pixel-by-pixel analysis was estimated by three methods (Table 1), and the results show decreases of $\sigma(\log t_{\text{dep}})$ as the scale gets larger. Figure 11(a) shows the scatter $\sigma(\log t_{\text{dep}})$ over the resolution scale for the three methods. At small scales (1′–3′), the EM method gives much larger values of the scatter than the other two methods. At these scales, over 50% of the data are below detection limits, making the estimates of $\sigma(\log t_{\text{dep}})$ uncertain. The three methods give comparable values for scales over 8′.

Figure 11(b) shows a comparison of our results with some resolved extragalactic star-formation relations. We used the averaging scale with the 90% distance contour as the maximum relevant scale. We caution that we are comparing across different data sets, and the differences in observations, methodologies, and other factors could contribute in the differences in the scatter of $t_{\text{dep}}$. Our result for the Galactic plane covers small scales where only a little comparable extragalactic data exists, to a scale of about 200 pc. The trend from our data suggests a smaller scatter in the depletion time for the Galactic plane than for the extragalactic data.
One important difference in our study is the choice of molecular gas tracers. The BGPS 1.1 mm in general traces denser and smaller parts of molecular clouds than CO or $^{13}$CO (Battisti & Heyer 2014). A single GMC can contain multiple 1.1 mm sources. The smaller scatter in our result is consistent with the fact that star formation is more closely associated with denser regions than with the general molecular cloud.

To examine possible causes of the change in the scatter, we looked at contributions to the scatter in log $t_{\text{dep}}$ ($\sigma(\log t_{\text{dep}})$). Several possible sources of uncertainties in log $t_{\text{dep}}$ include observational and photometric uncertainties, uncertainties in the parameters assumed in calculating $\Sigma_{\text{H}_2}$ and $\Sigma_{\text{H}_\alpha}$, uncertainties in the 22 $\mu$m flux that are due to spatial offsets between the 1.1 mm and 22 $\mu$m emissions, and scatter that is due to variations in intrinsic properties of the sources.

The uncertainties from the observations and the photometry were estimated as the errors associated with the data. The other sources of uncertainties will be discussed below.

### 6.2.1. Parameter Uncertainties

In the calculation of $\Sigma_{\text{H}_\alpha}$, we assumed a dust temperature of 20 K, a gas-to-dust mass ratio of 100, and a dust opacity of 1.14 cm$^2$ g$^{-1}$ for all of the sources, following the study of Schlingman et al. (2011). The variations in the real values would contribute to the scatter in $\Sigma_{\text{H}_\alpha}$. Spectroscopic observations of several molecular lines for Bolocat sources show a median temperature of $\approx$18 K with a temperature range of 10–30 K (Shirley et al. 2013).

In calculating $\Sigma_{\text{SFR}}$, we used the SFR-$24\mu$m relation (Equation (3)) from Calzetti et al. (2007). The calibration was derived assuming a constant SFR on a timescale of 100 Myr and a Kroupa IMF. When applied to an individual molecular cloud or a star-forming region, the assumptions cannot always be valid. The timescale assumed for constant star formation is much longer than the average lifetime of molecular clouds (Murray 2011). Several studies show that infrared tracers underestimate SFR with large uncertainties in clouds with low mass or low SFR (Vutisalchavakul & Evans 2013; da Silva et al. 2012, 2014). The combined effects of stochastically sampling the IMF and the star-formation history causes SFR indicators such as H$\alpha$, far-UV, and bolometric luminosity to underestimate the SFR (da Silva et al. 2014), and the size of the underestimate gets larger as the SFR gets smaller. Da Silva et al. (2014) also showed that when stochasticity is taken into account, SFR indicators do not provide a unique value of the SFR.

The variations in the properties of each source contribute to the uncertainties in how well the calculated SFR agrees with the true SFR. This is especially important for the sources with low $\Sigma_{\text{SFR}}$ in our data. As discussed earlier in Section 2.2, infrared luminosity starts to underestimate SFR below $10^{-5} M_\odot$ yr$^{-1}$ (da Silva et al. 2014). Using the averaging scales with a distance contour of 90%, we estimated the corresponding $\Sigma_{\text{SFR}}$.

### 6.2.2. Spatial Offsets between IR and 1.1 mm

The 1.1 mm emission comes from cold dust from dense molecular gas regions in GMCs, and the 22 $\mu$m emission should be dominated by warmer dust heated by stellar radiation. The two emitting regions might not perfectly coincide spatially with each other. The sizes of the emitting regions could also be different for 1.1 mm and 22 $\mu$m. These two factors will result in spatial offsets, contributing to the scatter in the star-formation relation if the scale size is smaller than a typical offset.

If there is a general offset between the 22 $\mu$m and 1.1 mm sources, the pixel-by-pixel correlation between the two images should get better once the spatial resolution becomes larger than the offset. From Figure 3, the correlation increases rapidly at small scales until they level off at around the 5′–8′ scale. This result suggests that there is an offset of small–scale variations between the 22 $\mu$m and 1.1 mm emissions of about 5′–8′, corresponding to 7–12 pc at the median distance of 5.1 kpc. This offset is typical of cloud sizes, consistent with the idea that the 1.1 mm source may be a remnant clump, and the infrared emission traces star formation in a now-destroyed clump in the same cloud.

For the case of the Bolocat sources, $t_{\text{dep}}$ from the data is greater than the average $t_{\text{dep}}$ at larger scales. The $t_{\text{dep}}$ of $\approx$1 Gyr is close to the average values found in extragalactic studies (Wong & Blitz 2002; Leroy et al. 2013). The 22 $\mu$m flux for each 1.1 mm source was calculated by centering the photometry aperture on the center of the 1.1 mm source. If the infrared emission associated with the 1.1 mm sources does not coincide with the 1.1 mm peak, then the estimated 22 $\mu$m flux would not be representative of the total emission. The infrared emission could also be more extended than the size of the aperture used in the photometry, in which case we would be underestimating the SFR.

To investigate this issue, we looked at the 22 $\mu$m emission for several sources from the images. One of the sources we tested was G23.95+0.16, which is a massive dense clump with an observed water maser. This source had previously been studied by Wu et al. (2010) and Vutisalchavakul & Evans (2013). The source size obtained by fitting a 1D Gaussian is about 3.9′, much larger than our aperture size of 80″. A large fraction of the IR emission lies outside the aperture, resulting in an underestimated SFR. However, this particular source has high $\Sigma_{\text{H}_\alpha}$ and $\Sigma_{\text{SFR}}$ compared to the whole sample. We looked at several other 1.1 mm sources for which there are associated 22 $\mu$m emissions and found that most of the 22 $\mu$m emission is more extended than the aperture size. To see how much this issue affected the star-formation relation result, we performed the photometry again with a larger aperture size of 160″. The result shows a higher average $\Sigma_{\text{SFR}}$, and the relation now lies above the Kennicutt (1998) relation.
6.2.3. Intrinsic Source Properties

Aside from the uncertainties already discussed, the scatter in the star-formation relation can also be contributed by intrinsic variations in the relation itself. If the SFR for each source is not determined only by the amount of gas available, then we would not expect to see a tight correlation between the two. Our results show a much larger scatter at small scales than do the relations found in disk-average studies. What causes the difference? These 1.1 mm sources are expected to be star-forming regions, so these star-forming regions should have variations in their properties. The sources that are in earlier stages of star formation would contain a large amount of gas and little infrared emission from star formation (large \( t_{	ext{dep}} \)), and the sources that are in later stages of star formation would contain less gas because of gas depletions and emit more strongly in the IR because of stars. The difference between the \( t_{	ext{dep}} \) from CO and H\(_2\) peaks decreases as the aperture size increases. They argued that the dependence of \( t_{	ext{dep}} \) on scale was mainly due to the effect of sampling different evolutionary stages. Kruijssen & Longmore (2014) constructed a model to describe the dependence of \( t_{	ext{dep}} \) on scale in the star-formation relation at the scale of individual sources. Sampling a larger number of individual regions at different scales averages out the scatter in \( t_{	ext{dep}} \), resulting in a decrease in the scatter in the relation.

The effect of sampling different stages of star formation on the star-formation relation has been a topic of several recent studies (Onodera et al. 2010; Schruba et al. 2010; Leroy et al. 2013; Kruijssen & Longmore 2014). Schruba et al. (2010) showed, in a study of star formation in M33 at the scale of 75–1200 pc, that the choice of CO or H\(_2\) peaks as centers gave different values of \( t_{	ext{dep}} \). The difference between the \( t_{	ext{dep}} \) from CO and H\(_2\) peaks decreases as the aperture size increases. They argued that the dependence of \( t_{	ext{dep}} \) on scale was mainly due to the effect of sampling different evolutionary stages. Kruijssen & Longmore (2014) constructed a model to describe the dependence of \( t_{	ext{dep}} \) and the scatter in \( t_{	ext{dep}} \) on spatial scales and how the differences between \( t_{	ext{dep}} \) when choosing regions on either gas or star-formation tracer peaks can be used to estimate the timescales involved in star-formation processes.

We found similar results when comparing regions centered on 1.1 mm peaks with regions centered on 22 \( \mu \)m peaks. Regions with strong 1.1 mm emission are more likely to be at an earlier stage of star formation where there is still a large amount of gas, and regions with a strong 22 \( \mu \)m have already formed stars. The differences in the average \( t_{	ext{dep}} \) between choosing different peaks, as shown in Figure 9, support the hypothesis that the differences in the stage of star formation contribute to the large scatter in the star-formation relation at small scales.

All of the mentioned sources of scatter can affect the relations between \( \Sigma_{\text{GFR}} \) and \( \Sigma_{\text{HI}} \). Quantitatively explaining the observed data with these uncertainties requires a careful modeling of how each source of scatter depends on scale, which is beyond the scope of this paper. Future studies of the properties of individual star-forming regions, especially their evolutionary stages, will provide more insights into the problem.

6.3. The Star-formation Relation and Depletion Times

Aside from looking at the scatter in the relation, we can also look for changes in the form of the star-formation relation as the resolution is changed. While the correlation improves with averaging scale, the changes to the fitted values for the slope and intercept are not very significant. The star-formation relations at all scales are slightly sublinear and lie between the extragalactic relation from Kennicutt et al. (1998) and the dense gas relation of Wu et al. (2005).

The typical depletion time, both from the pixel-by-pixel analysis and from the source-based analysis with large averaging scales, is 200 Myr. The Bolocat source-based analysis shows a larger \( t_{	ext{dep}} \) of about 1 Gyr, closer to the typical extragalactic value. However, this value is very likely an overestimation. As discussed in Section 6.2.2, the \( \Sigma_{\text{GFR}} \) for the Bolocat sources are underestimated because not all of the infrared emission was inside the aperture and because there is a bias in using 22 \( \mu \)m to trace SFR for low-mass sources. For this analysis we also centered the apertures on the Bolocat sources, which are bright 1.1 mm regions, so the analysis is biased toward larger \( t_{	ext{dep}} \).

The pixel-by-pixel analysis does not show variations in the average \( t_{	ext{dep}} \) over the resolution scales. When all of the regions are sampled, \( t_{	ext{dep}} \) can be described reasonably well with log-normal distributions centered at \( \approx 10^{8.3} \) yr. Choosing bright 22 \( \mu \)m regions is equivalent to sampling the lower tail of the distribution of \( t_{	ext{dep}} \), and choosing bright 1.1 mm regions is equivalent to sampling the higher tail of the distribution, resulting in the differences in \( t_{	ext{dep}} \) as seen in Figure 9. Evidently, the method of choosing regions affects the result of \( t_{	ext{dep}} \), as does the method of identifying local emission peaks. Before making the cut in the 22 \( \mu \)m and 1.1 mm flux, the data did not show a clear trend in \( t_{	ext{dep}} \) over the spatial scale. This is likely because without the cut all of the identified regions were included. The lower brightness regions tend to sample near the center of the \( t_{	ext{dep}} \) distribution, therefore lowering the distinction between IR or mm peaks. Data for other galaxies could be affected as well because the sensitivity limit varies between data sets.

The constant timescale of 200 Myr seen throughout our data set is similar to the mean value found in the nearby clouds, but it is about five times greater than that found for the dense gas (\( A_V > 8 \) mag) in the nearby clouds (Evans et al. 2014). Because the 1.1 mm emission is mostly tracing clumps with \( n \approx 10^{3.5} \) cm\(^{-3} \), similar to the mean density within the \( A_V > 8 \) mag contours (Evans et al. 2014), this difference may indicate a systematic underestimate of the star-formation rate from the 22 \( \mu \)m emission. Individual young stellar objects could be counted in the nearby clouds, rather than relying on the 22 \( \mu \)m emission. Vutisalchavakul & Evans (2013) showed that the mid-infrared emission does underestimate the star-formation rate in the nearby clouds where high-mass stars are rare. For this reason we believe that the actual value of \( t_{	ext{dep}} \) is likely overestimated.

However, the likely overestimated value we get for \( t_{	ext{dep}} \) is already five times smaller than that found in other galaxies. The BGPS 1.1 mm emission we used in this study traces denser gas than the common tracers used for other galaxies. The 1.1 mm emission only traces about 11% of the gas traced by 13CO, a more typical gas tracer in extragalactic studies (Battisti & Heyer 2014). Therefore, we would expect the average \( t_{	ext{dep}} \) to be smaller than extragalactic values. As a result of these systematic issues, these data tend to lie between the Kennicutt relations for total gas and the Wu relation for even denser (\( n = 10^{4.5} \) cm\(^{-3} \)) gas (Wu et al. 2005).

7. SUMMARY

We studied the relationship between molecular gas and SFR surface density for 11 deg\(^2\) of the Galactic plane. The 1.1 mm
data from the BGPS, which traces dense gas inside molecular clouds, were used as a tracer of molecular gas, and 22 μm data from the WISE all-sky survey were used to trace the SFR. We studied the relation from the scale of 33′ to the largest scale of 20′ by convolving images with Gaussian beams. We started by looking at the correlations between 22 μm and 1.1 mm images pixel by pixel and found that the rank correlation coefficient increases rapidly as scale size increases, leveling off at the scale of about 5′–8′. The 22 μm and 1.1 mm emission are already well correlated at the scale of 5′ and the correlations do not change much at larger scales, suggesting a spatial offset or small-scale variations around this scale, which corresponds to estimated physical scales of 7–12 pc.

We studied the star-formation relations both by analyzing pixel-by-pixel values and by identifying the 1.1 mm and 22 μm peaks. The star-formation relations from the pixel-by-pixel analysis show close to linear relations. The distribution of $t_{\text{dep}}$ can be closely represented as a log-normal with an average of about 200 Myr regardless of the resolution. The relation on small scales shows large scatter, and the scatter decreases as the scale gets larger. The scatter of the log($t_{\text{dep}}$) decreases from above 0.6 at the 1′ scale to about 0.28 at the 20′ scale. The typical depletion times lie between those for dense clumps and those for total gas or for molecular gas in other galaxies.

For sources centered at 1.1 mm peaks, we found a weak correlation between $\Sigma_{\text{SFR}}$ and $\Sigma_{\text{H}}$ at the 1.1 mm source scale (aperture diameter of 80′′). The correlation gets better at larger scales similarly for the 1.1 mm peaks and 22 μm peaks. There are no significant differences in the form of the relation at different scales or when comparing the 1.1 mm and 22 μm peaks. The star-formation relations at all scales are slightly sublinear and lie above the extragalactic relation from Kennicutt et al. (1998).

The average depletion time of 200 Myr seen across the data is about five times smaller than the typical $t_{\text{dep}}$ of 1 Gyr in extragalactic studies and larger than the $t_{\text{dep}}$ measured for dense clumps. The smaller depletion time for the Galactic plane than the extragalactic value can be explained by the fact that the 1.1 mm emission used as a gas tracer for this study traces denser gas than the usual gas tracer such as 12CO or 13CO. The 22 μm emission could also be systematically underestimating the SFR across all scales.

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## APPENDIX A

### SATURATION

WISE 22 μm images contain some saturated regions with a large number of saturated pixels near peaks of bright, extended emission. The saturated areas are small compared to the total area, but saturation will affect larger portions of the image once we convolve to larger beam sizes. For further analysis, we replaced the saturated pixels with estimated values. Over an extended saturated region, the estimation was done by calculating average values of the surrounding regions and performing a thin plate spline interpolation. Consequently, the values of the flux over saturated regions have large uncertainties. However, the saturated area only covers less than 0.01 percent of the entire image. The uncertainties in the estimations should have minimal effect on the flux calculations.

## APPENDIX B

### PHOTOMETRIC UNCERTAINTY

The uncertainties in the $\Sigma_{\text{SFR}}$ for the Bolocat sources (Section 5.2) came from the estimated uncertainties in the photometry performed on WISE 22 μm images. Since the photometry was performed directly from the WISE all-sky release images, the uncertainties were estimated following the Explanatory Supplement to the WISE All-Sky Data Release Products. The uncertainty of the source flux ($\sigma_{\text{src}}$) was contributed by instrumental and calibration uncertainties, Poisson noise, and uncertainties from background estimations. The $\sigma_{\text{src}}$ was estimated by:

$$\sigma_{\text{src}}^2 = F_{\text{corr}} \left( \sum \sigma_{\text{pix}}^2 + \frac{N_A}{N_B} \sigma_{\text{B/pix}}^2 \right),$$

where

$$F_{\text{corr}} = \text{pixel to pixel correlated noise correction factor}$$

$$\sigma_{\text{pix}} = \text{flux uncertainty for each pixel inside the aperture}$$

$$\sigma_{\text{B/pix}} = \text{uncertainty in the background per pixel}$$

$$N_A = \text{number of pixels inside the aperture}$$

$$N_B = \text{number of pixels used to estimate the background flux}.$$
How the changes in these parameters affect the resulting background estimations was considered in the estimations of \( \sigma_{B/\text{pix}} \).

We calculated \( \sigma_{B/\text{pix}} \) by changing the three parameters around the chosen values and created the background images. We then compared the resulting images to the chosen background images. The differences between two background images were quantified by

\[
\sigma_{\text{diff}}^2 = \frac{1}{N} \sum_{i=1}^{N} (f_i - f'_i)^2,
\]

where \( N \) is the number of pixels in the image, \( f_i \) is the pixel flux of the chosen image, and \( f'_i \) is the pixel flux for the comparison image. \( \sigma_{\text{diff}}^2 \) were calculated for all the comparing background images and the average of the results was adopted as the value for \( \sigma_{B/\text{pix}}^2 \). To see how our estimation of \( \sigma_{B/\text{pix}} \) compared to the spatial variations of the flux, we looked at the pixel flux distribution of the background subtracted 22 \( \mu \)m image of region 1. Figure 12 shows the pixel flux distribution of region 1. To fit the pixel noise variations, we reflected the negative flux distribution about zero and fitted a Gaussian distribution. The resulting fit is shown as the orange dashed line in the figure. The fitting gave a Gaussian distribution parameters of \( \sigma = 0.001 \) Jy/pixel. At the aperture size of 80\', the noise corresponds to \( \sigma \approx 0.02 \) Jy/source. Our estimation of \( \sigma_{B/\text{pix}} \) for region 1 gave \( \sigma \approx 0.11 \) Jy/source.

### APPENDIX C

COMPARING METHODS OF DIFFUSE IR BACKGROUND ESTIMATIONS

Our method of estimating diffuse 22 \( \mu \)m emission is described in Section 3.1. Figure 13 shows the WISE 22 \( \mu \)m images for region 1 before and after diffuse background subtraction. We compared our method to the cirrus removal method from Battersby et al. (2011, hereafter B11). B11 estimated diffuse emission for the Herschel 500 \( \mu \)m emission. The brief summary of the method is as follow. The original image was convolved with a Gaussian beam; then a Gaussian fit was performed in the Galactic latitude at each of the Galactic longitude. The fitted image was subtracted from the original image. The subtracted image was used for estimating a cutoff (4.25 \( \sigma \)) of source flux so that everything above the cutoff was considered as sources. Area above the cutoff was masked out in the original image. The process was then iterated until the source mask cutoff converges. The original image was masked out with the final cutoff value and convolved with a Gaussian kernel to create a background subtracted image.

We compared the background images of region 1 from our method with the method of B11 performed on the same region of the WISE 22 \( \mu \)m image using a Gaussian kernel FWHM = 12\'. The result shows that the background image from B11 method gave a comparable background to our method with slightly stronger background in bright source area. Figure 14 shows a histogram of ((our background image − B11 background image)/(original 22 \( \mu \)m image)). The fractional differences are

![Figure 12](image-url)  
**Figure 12.** Pixel flux distribution for 22 \( \mu \)m image of region 1. The blue dashed line shows the Gaussian fit to the flux distribution of flux \(< 0 \) and a reflected image of the negative flux for flux \( > 0 \). (A color version of this figure is available in the online journal.)  

![Figure 13](image-url)  
**Figure 13.** WISE 22 \( \mu \)m image of region 1. The top image shows the original mosaic, and the bottom image shows the background-subtracted mosaic. The color bar is in units of MJy sr\(^{-1}\). (A color version of this figure is available in the online journal.)
Figure 14. Comparison between two methods of background estimations for the 22 μm image of region 1. The fractional difference is the ratio of the difference between our background image and B11 background image over the original 22 μm image.

small with the highest absolute value of 0.04. This result indicates that the choice of a method of background subtraction does not significantly affect the photometric flux.

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