Observations of powerful radio waves from neutron star magnetospheres raise the question of how strong waves interact with particles in a strong background magnetic field \( B_{bg} \). This problem is examined by solving the particle motion in the wave. Remarkably, waves with amplitudes \( E_0 > B_{bg} \) pump particle energy via repeating resonance events, quickly reaching the radiation reaction limit. As a result, the wave is scattered with a huge cross section. This fact has implications for models of fast radio bursts and magnetars. Particles accelerated in the wave emit gamma-rays, which can trigger an \( e^\pm \) avalanche and, instead of silent escape, the wave will produce X-ray fireworks.

**Introduction.** — Magnetized compact objects are capable of generating strong electromagnetic waves of low frequencies \( \omega \). In particular, pulsars and magnetars produce bright radio emission, and magnetars are thought to be the sources of fast radio bursts (FRBs) [1, 2]. A strong background magnetic field \( B_{bg} \) is believed to suppress the plasma response to the wave electric field \( E \) when \( E \perp B_{bg} \). This allows the linearly polarized radio waves to freely propagate through the magnetospheric plasma even when \( \omega \) is far below the plasma frequency. A standard calculation of the cross section for wave scattering by a magnetized electron gives \( \sigma_{sc} \approx (\omega/\omega_B)^2 \sigma_T \ll \sigma_T \) [3], where \( \omega_B \) is the gyrofrequency and \( \sigma_T \) is the Thomson cross section. However, the standard analysis fails for waves with amplitudes \( E_0 > B_{bg} \), and this regime is inevitably encountered as a strong wave packet propagates away from the magnetized star, in the decreasing \( B_{bg} \). Note that both conditions \( B_{bg} < E_0 \) and \( \omega_B \gg \omega \) may be expressed as

\[
1 < \frac{\omega_B}{\omega} < a_0, \quad a_0 = \frac{eE_0}{mc^2} \sqrt{\frac{\gamma}{\gamma - 1}},
\]

where \( e \) and \( m \) are the electron charge and mass. Low-frequency waves can have enormous \( a_0 \). For instance, FRB models picturing a \( \text{GHz} \) source of luminosity \( L \sim 10^{42} \text{erg/s} \) at radii \( R < 10^8 \text{cm} \) have \( E_0^2 = 2L/cR^2 \) and \( a_0 \sim 10^5 R^{-1} \). The wave encounters regime (1) in the outer magnetosphere \( R \gtrsim 3 \times 10^8 \text{cm} \) where \( B_{bg} \propto R^{-3} \) drops below \( E_0 \) [4]. We find below that regime (1) triggers quick stochastic acceleration of particles in the wave, and \( \sigma_{sc} \) becomes huge.

**Method.** — Let us consider a wave packet propagating along \( \hat{z} \) in an initially static magnetized plasma of low density, and calculate the particle motion in the packet. It obeys the dynamical equation for velocity \( \mathbf{v} = \beta c \) or four-velocity \( \mathbf{u}^\mu = (\gamma, \mathbf{u}) \) (where \( \mathbf{u} = \gamma \mathbf{\beta} \)):

\[
mc \frac{d\mathbf{u}}{dt} = e \{ \mathbf{E} + \beta \times (\mathbf{B} + \mathbf{B}_{bg}) \} + \mathbf{f}, \quad (2)
\]

where \( f \) is the radiation reaction force. Relevant scales in this dynamical problem are microscopic compared with the scale \( R \) of the background field variation, so \( B_{bg} \) can be approximated as uniform. The simple case of \( B_{bg} = 0 \) has been extensively studied in laser plasma physics [5]. The particle motion was also solved for circularly polarized waves propagating along \( B_{bg} \neq 0 \) [6]. In these cases, the stochastic pump effect described below disappears.

The wave fields \( E \) and \( B \) depend on \( \xi = t - z/c \) and described by the dimensionless potential \( a = eA/mc^2 = (a(\xi), 0, 0) \):

\[
\frac{eE}{mc} = \left( \frac{da}{d\xi}, 0, 0 \right), \quad \frac{eB}{mc} = \left( 0, -\frac{da}{d\xi}, 0 \right). \tag{3}
\]

We choose \( \xi = 0 \) at the leading edge of the wave packet, so it propagates at \( \xi > 0 \) (and \( a = 0 \) at \( \xi \leq 0 \)). In numerical examples we will use a modulated sine wave with amplitude rising linearly at \( 0 < \xi < \xi_{\text{rise}} \) and then staying constant in the packet: \( E(\xi) = E_0 \sin(\omega \xi) \).

Consider a particle initially at rest before the wave \( (\beta = 0 \text{ at } \xi \leq 0) \). The wave overtakes the particle with relative speed \( d\xi/dt = 1 - \beta_z \), and we define

\[
u_\xi \equiv \frac{d\xi}{dt} = \gamma (1 - \beta_z) = \gamma - u_z, \quad d\tau \equiv \frac{dt}{\gamma}. \tag{4}
\]

Note that the wave potential \( a \) varies along the particle worldline with \( da/d\tau = -(eE/mc)u_\xi \). We also define

\[
\omega_B = \frac{eB_{bg}}{mc} = (0, \omega_B \sin \theta, \omega_B \cos \theta), \quad \theta \neq 0. \tag{5}
\]

The case of \( \theta = \pi/2 \) is particularly simple — then the wave does not excite \( u_y \), i.e. \( \mathbf{u} = (u_x, 0, u_z) \).

We first examine particle motion without radiation reaction, \( f \approx 0 \). Then, Eq. (2) gives

\[
\frac{dU_x}{d\tau} = \omega_B^2 u_y - \omega_B u_{yz}, \quad \frac{du_\xi}{d\tau} = -\omega_B^2 u_x, \quad \frac{du_y}{d\tau} = -\omega_B u_x, \tag{6}
\]

where \( U_x = u_x + a \). Variables \( (U_x, u_\xi, u_y) \) determine all components of \( \mathbf{u}^\mu \) (using \( u^\mu u_\mu = -1 \)). We solve Eqs. (6) with initial conditions \( \mathbf{u} = 0, U_x = 0, u_\xi = 1, u_z = 0 \) at \( \xi = 0 \).

When \( B_{bg} = 0 \), the solution is trivial: \( U_x \) and \( u_\xi \) keep their initial values, which yields

\[
u_x = -a, \quad u_z = \frac{a^2}{2}, \quad \gamma = 1 + \frac{a^2}{2}. \tag{7}
\]
This motion is well known, although it is usually viewed in the center-of-momentum frame $K'$ where the average $\bar{z}' = 0$ and the particle executes an $S$-shaped orbit [7].

For waves with $E_0 \ll B_{bg}$, the particle motion is also known: it oscillates in the wave with small $|\mathbf{u}| \sim E_0/B_{bg}$.

Hereafter we focus on waves with $E_0 > B_{bg} \neq 0$. Then, we find that the particle motion is a superposition of fast $\omega$-oscillations in the wave with amplitude $|\Delta \mathbf{u}| \sim a_0$ and a slower Larmor rotation of $\bar{u}$ (averaged over the $\omega$-oscillations) in the average field $\bar{B} + B_{bg} = B_{bg}$. In particular, $\Delta u_x = a_0$ and $\bar{u}_z = U_x$.

Fig. 1 shows sample solutions demonstrating two types of motion: $|\bar{u}| \gg |\Delta \mathbf{u}|$ (found when $\omega < \omega_B$) and $|\bar{u}| \ll |\Delta \mathbf{u}|$. Both solutions were calculated for the same wave $(a_0 = 30, \xi_{rise} = 150(2\pi/\omega))$, but with different $\omega_B = (0, 10\omega, 0)$ and $\omega_B = (0, 0/5, 0)$.

Waves with $\omega < \omega_B$.— In this case, the gyration of $\bar{u}$ in $B_{bg}$ develops a huge amplitude $|\bar{u}| \approx \gamma \gg |\Delta \mathbf{u}| \sim a_0$, and the particle’s motion becomes dominated by Larmor rotation with frequency $\omega_L = \omega_B/\gamma \ll \omega$ (Fig. 1, left). We observe that $|\bar{u}|$ is pumped in nearly impulsive events that occur every Larmor rotation. These events are resonances where the particle exchanges energy with one wave oscillation $\delta \xi \sim \omega^{-1}$.

The resonance may be described as follows. The wave oscillation along the particle’s worldline resembles an oscillator with a changing frequency $\omega'_w = (1 - \beta_z) \omega$. It becomes slowest near moment $t_0$ where $\beta_z \approx 1$ is maximum and $u_x = 0$ is minimum (then the particle moves almost together with the wave). Gyration of $u \approx \bar{u}$ gives $\beta_x \approx \gamma/\gamma_\star$, and $\bar{u}_x = \gamma_\star t_0$. The resonance occurs when

$$\omega_w \delta \xi \approx 1 \Rightarrow \delta \psi_{res} = \omega L \delta t_{res} \sim (\omega_L/\omega)^{1/3}. \tag{8}$$

The obtained $\delta \psi_{res}$ determines the characteristic $u_\xi = \gamma(1 - \beta_x)$ and $u_x$ during the resonance,

$$u_{\xi res} \approx \left(\frac{\gamma_\star^2 \omega_B}{\omega^2}\right)^{1/3} \frac{u_{\xi res}}{a_0} \approx \left(\frac{\gamma}{\gamma_\star}\right)^{2/3} \gamma_\star \approx \left(\frac{a_0^2 \omega_B}{\omega^2}\right) \tag{9}$$

These expressions assume $u_{\xi res} > a_0$ which requires $\gamma > \gamma_\star$ (then $u_{\xi res} \approx U_{\xi res} \approx -\gamma_\star \sin \delta \psi_{res}$). One can verify that $\omega_L/\omega = (\gamma_\star/\gamma)(B_{bg}/E_0)^{3/2} \ll 1$ and $\delta \psi_{res} \ll 1$.

Gain $\gamma$ from the resonance event may be found from $d\gamma/d\tau = eE_{ux}/mc$ and $du_x/\delta \tau = -\omega_B u_{\tau}$, which gives

$$d\gamma = -\frac{E}{B_{bg}}, \quad \delta \tau = -\frac{E_0}{B_{bg}} \int \sin(\omega \delta \xi + \phi) \, du_x. \tag{10}$$

Here $\phi$ is the (practically random) phase of the wave at the particle’s location at time $t_0$. Note that $u_\xi$ is even in $\delta \xi$ during the resonance, near the minimum $u_\xi$. Hence, the odd part of $\sin(\omega \delta \xi + \phi)$, i.e. $\sin(\omega \delta \xi) \cos \phi$, determines the integral and $\delta \gamma \sim -u_{\xi res} (E_0/B_{bg}) \cos \phi$. This result may be stated as

$$\delta \gamma \approx -H \left(\gamma \gamma_\star^2\right)^{1/3} \cos \phi \quad (\gamma > \gamma_\star). \tag{11}$$

Exact integration gives the coefficient $H \approx 2.6$ (Fig. 2). If the particle approaches the resonance with $\gamma < \gamma_\star$, it gains $\delta \gamma \sim \gamma_\star$. As the resonances repeat every $\xi_L = t_L = 2\pi/\omega_L$, the particle performs random walk in $\gamma$ to $\gamma \gg \gamma_\star$. The wave acts as a stochastic “pump” that can accelerate the particle to arbitrary high $\gamma$ (limited only by radiative losses discussed below). The deterministic particle motion gives the chaotic walk in $\gamma$ because $\delta \gamma$ is sensitive to $\phi$. It remains regular in repeating resonances every gyration.

The presented calculation assumes $\theta = \pi/2$ (wave propagation perpendicular to $B_{bg}$). Similar integration of Eq. (2) at $\theta < \pi/2$ also gives pumping of $\gamma$, with additional sliding of the particle along the oblique $B_{bg}$. One can boost the reference frame along $B_{bg}$ so that the wave propagates perpendicular to $B_{bg}$ in the new frame $K'$.

Waves with $\omega > \omega_B$.— In this case, the particle moves with $|\bar{u}| \ll |\Delta \mathbf{u}|$ in slowly rising waves, $\xi_{rise} \gg a_0/\omega_B$. The resulting motion at $\xi > \xi_{rise}$ is periodic, with no pumping effect (Fig. 1, right).

$|\bar{u}|$ can be derived analytically. Let us average the energy equation $mcd\gamma/d\tau = eE_{ux}$ over the $\omega$-oscillations:

$$u_{\xi} \frac{d\delta t}{d\xi} \approx \frac{e}{mc} (\overline{Eu_x} - \overline{E\bar{u}}) \approx \frac{d \bar{u}^2}{d\xi} \frac{\omega^2}{2}. \tag{12}$$

Here we used $eE_{a}/mc = -da/d\xi$ and neglected $\overline{Eu_x} = \bar{U}_x E_0 \sin(\omega \xi)$ since $U_x E_0$ varies slowly and $\sin(\omega \xi) = 0$. Dynamical equations for $u_x$ and $u_z$ may be written as an equation for complex $u = u_x + iu_z$. After averaging, it becomes

$$\frac{d\bar{u}}{d\xi} \approx i\omega_B \bar{u} + i F, \quad F \equiv \frac{1}{2} \frac{d \bar{u}^2}{d\xi} \approx \frac{1}{4} \frac{da_0^2}{d\xi}. \tag{13}$$

The Larmor rotation of $\bar{u}$ is excited where the wave rises, $F \neq 0$, and the solution of Eq. (13) during the rise is

$$\bar{u} \approx i \int_0^\tau e^{-\omega_B(\tau - \tau')} F(\tau') \, d\tau' \approx -\frac{F}{\omega_B} \left(\xi < \xi_{rise}\right), \tag{14}$$

where second equality uses the slow-rise approximation (we integrated by parts and neglected $dF/d\tau \ll \omega_B F$). Then, from $du_x/d\xi \approx -\omega_B \bar{u}_x \approx F \approx \bar{u} \gamma/\delta \tau$, we find $u_x \approx \tilde{\gamma}$, and Eq. (12) yields $\tilde{\gamma}^2 \approx 1 + a_0^2/2$. At $\xi > \xi_{rise}$,

$$\tilde{\gamma} \approx \sqrt{1 + \frac{a_0^2}{2}}, \quad \bar{u} \approx \frac{a_0^2 e^{\psi}}{2 \omega_B \xi_{rise}}, \quad u_x \approx \tilde{\gamma} - |\bar{u}| \sin \psi, \tag{15}$$

where $\psi = \omega_B (\tau - \tau_{rise})$. This analytical result agrees with the numerical solution.

One can also evaluate the integral in Eq. (14) when $\xi_{rise} \ll a_0/\omega_B$. Then, we find periodic motion at $\xi > \xi_{rise}$ with $|\bar{u}| \sim (a_0^2/\omega_B \xi_{rise})^{1/2} \gg |\Delta \mathbf{u}| \sim a_0$. 
Radiation reaction limit (RRL).—A relativistic electron in fields $E$ and $B + B_{bg}$ emits momentum with rate [7]

$$\dot{\gamma}_{em}mc = \frac{\sigma_T}{4\pi}\{[\gamma E + u \times (B + B_{bg})]^2 - (u \cdot E)^2\}. \tag{16}$$

We now retain $f = -\dot{\gamma}_{em} mv$ in the dynamical Eq. (2). For waves with $E_0 \gg B_{bg}$, $\dot{\gamma}_{em}mc$ simplifies to $\sigma_T E^2 u^2 / 4\pi$, and averaging over $\omega$-oscillations gives

$$\overline{\dot{\gamma}}_{em} \approx \frac{r_e}{3c} a_0^2 \omega^2 u^2, \quad r_e = \frac{e^2}{mc^2}. \tag{17}$$

In waves with $\omega < \omega_B$, the resonant pumping of $\gamma$ quickly reaches RRL (Fig. 2), and random walk continues with a ceiling $\gamma_{RRL}$. Losses occur with $\langle \dot{u}^2 \rangle = 2\langle \dot{\gamma}^2 \rangle$ (averaged over gyration), and balance the maximum resonant gain $\delta \gamma = H(\gamma^2)^{1/8}$ when $\langle \dot{\gamma}_{em} \rangle t_L = \delta \gamma$. This gives (using Eq. 17)

$$\gamma_{RRL} = \gamma_* \left(\frac{a_*}{a_0}\right)^{15/8} = \left(\frac{3H c}{4\pi r_e a_0}\right)^{3/8} \left(\frac{\omega_B}{\omega}\right)^{1/4} \tag{18}$$

This result holds if $\gamma_{RRL} > \gamma_*$, i.e. if

$$a_0 < a_* \equiv \left(\frac{3H c \omega_B^2}{4\pi r_e \omega^3}\right)^{1/5} \approx 400 \nu^{-1/5} (\frac{\omega_B}{\omega})^{2/5} \tag{19}$$

where $\nu = \omega/2\pi$ is normalized to 1 GHz. (If $a_0 > a_*$, losses completely suppress diffusion in $\gamma$.)

Timescale for reaching RRL is $t_{RRL} \sim (\gamma_{RRL}/\delta \gamma)^2 t_L$, which gives $\omega_{RRL}/2\pi \sim H^{-2}(E_0/B_{bg})^{3/2}(a_*/a_0)^{15/8}$. For a bright FRB, $t_{RRL}$ is shorter than the FRB duration ($\sim 1$ ms), so the wave pushes particles to the RRL.

When $\omega > \omega_B$, we find that radiation reaction is negligible if $q \equiv a_0^3 \tau_{rc}/c \ll 1$; then the particle keeps $\gamma \sim a_0$. If $q > 1$ then $\gamma$ grows (Fig. 3), because $u_\xi$ develops $\omega$-oscillations and radiative losses become asymmetric in phase, inducing a rocket effect.

Initial temperature $T \neq 0$.—A simple way to see the statistics of chaos realizations in a wave packet with $\omega < \omega_B$ is to draw an ensemble of test particles from an initial Maxwellian distribution with some $T \neq 0$. We fol-
Radiative losses offset acceleration at $T = \gamma_{\text{RRL}} \approx 1370$ (horizontal dashed line). The evolution of $\gamma$ consists of stochastic jumps $\delta \gamma$ (resonances) followed by gradual losses. Bottom: $\gamma_\gamma$ vs. wave phase $\phi$ at the resonance. The result confirms Eq. (11) with $H \approx 2.6$ (dotted curve).

![Plot of $\gamma$ vs. $\omega t/2\pi$](image1.png)

**FIG. 3.** Development of radiation reaction with increasing $q$ in waves with $\omega > \omega_B$. Averaging $\langle \ldots \rangle$ is performed over a time longer than the particle gyration in $B_{bg}$: $\langle \gamma \rangle = \langle \gamma \rangle_{\text{ens}}$. The plot was constructed by solving a sequence of models with varying $r_{\gamma}/c$ at fixed $a_0 = 30$ and $\omega_B = 0.1\omega$, however the same result holds for other choices of $a_0 \gg 1$ and $\omega_B < \omega$.

![Plot of $\gamma$ vs. $\omega t/2\pi$](image2.png)

**FIG. 4.** Wave pumps $\langle \gamma \rangle_{\text{ens}}$ of a particle ensemble with initial $T \neq 0$ ($a_0 = 30, \omega_\text{rise}/2\pi = 10, \omega_B = 10\omega$). Three models are shown: with no radiative losses for $T = kT/mc^2 = 0.01$ (blue) and 1 (black), and with losses for $T = 1, \gamma_{\text{RRL}} = 1370$ (red). Black dotted line shows the acceleration law $\langle \gamma \rangle_{\text{ens}} \propto \xi^{3/7}$.

**Scattering cross section $\sigma_{sc}$.** — Time-averaged emitted power $\dot{E}_{\text{em}} = \langle \gamma \rangle_{\text{em}}mc^2$ determines the scattering cross section of the particle $\sigma_{sc} = \dot{E}_{\text{em}}/F$, where $F = c\epsilon_0^2/8\pi$ is the wave energy flux. Eq. (17) gives $\sigma_{sc} \approx \langle u^2 \rangle_{\gamma} \sigma_T$. If $B_{bg} = 0$, the particle keeps $u_\xi = 1$, so $\sigma_{sc} = \sigma_T$ (and in frame $K'$, where $u_\xi' = 0, \sigma_{sc}/\sigma_T = 1 + 3u_0^2/8$ [7]).

It is particularly interesting to look at $\sigma_{sc}$ for $\omega < \omega_B$. Then, $\langle u^2 \rangle_{\gamma} \sim \gamma^2_{\text{RRL}}$. A characteristic $\sigma_{sc}^* \equiv \gamma_{\text{RRL}}$ may be defined with $a_0 = a_*$:

$$\frac{\sigma_{sc}^*}{\sigma_T} \approx \left( \frac{c}{r_e\omega} \right)^{3/5} \left( \frac{\omega_B}{\omega} \right)^{1/5} \sim 10^8 \nu^{-3} \left( \frac{\omega_B}{\omega} \right)^{1/5}. \quad (20)$$

Recall that this result holds for $E_0 > B_{bg}$. If $E_0$ is reduced below $B_{bg}$, $\sigma_{sc}$ would drop to $(\omega^2/\omega_B^2)\sigma_T$.

**Energies of emitted photons.** — The emitted power $\dot{E}_{\text{em}} = \langle \gamma \rangle_{\text{em}}mc^2$ is carried by curvature radiation with spectrum extending to a characteristic frequency $\omega_c \approx (3/2)\gamma^{3} r_e$, where $\omega_c^{-1} = (3\dot{E}_{\text{em}}/2cr_e\gamma^4)^{1/2}$ is the curvature of the particle trajectory [7]. Substitution of Eq. (17) gives

$$\frac{\omega_c}{\omega} \approx a_0 \gamma u_\xi. \quad (21)$$

When $\omega > \omega_B$, we find $\langle u_\xi \rangle \sim a_0^2$ (Fig. 3) and $\omega_c \sim a_0^3 \omega$.

For $\omega < \omega_B$, we use $\langle u_\xi \rangle \approx 2\gamma_{\text{RRL}}$ and Eq. (18) to get

$$\frac{\hbar \omega_c}{mc^2} \approx \frac{1}{\alpha} \left( \frac{r_e\omega_B^2a_0}{c\omega} \right)^{1/4} \approx \epsilon_c^* \left( \frac{a_0}{a_*} \right)^{1/4}. \quad (22)$$

$$\epsilon_c^* \approx \frac{1}{\alpha} \left( \frac{r_e\omega_B^3}{c\omega^2} \right)^{1/5} \approx 0.3 \left( \frac{\omega_B}{\omega} \right)^{3/5} \nu^{-1/5}_{\text{GHz}}, \quad (23)$$

where $\alpha = e^2/\hbar c$. Waves with $\omega \ll \omega_B$ generate photons with $\hbar \omega > mc^2$ capable of $e^+ e^-$ creation.

**Discussion.** — Strong low-frequency waves (Eq. 1) offer a novel mechanism for particle acceleration near astrophysical compact objects. It differs from stochastic...
acceleration by plasma turbulence, where particles gain energy from interactions with many plasma modes [8]. The wave induces a peculiar resonance with the particle motion (without fine-tuning $\omega$) which repeats nearly impulsively at a specific gyration phase $\psi$ with a random wave phase $\phi$, giving random energy boosts to the particle. This behavior is an incidence of chaos development in nonlinear dynamics. Different examples of chaos in plasma waves with a magnetostatic background are found in [9–12]; chaotic motion in electrostatic waves was particularly well studied [13]. Ultrastron waves in regime (1) provide a remarkable new example, which admits simple description presented in this Letter.

Reproducing this acceleration mechanism in the lab is difficult. [14] considered particle acceleration in a laser beam with $a_0 \geq 1$ propagating across a static $B_{bg}$ with a Larmor radius $r_L = c/\omega_L$ exceeding the beam size. Reaching a small $r_L$ and the conditions (1) or (19) is difficult because of limited $B_{bg}$ accessible to experiments. Another experimental setup engineers a slow wave (phase speed $v_{ph} < c$) trapping particles at wave phases $\phi$ where $E > |B - B_{bg}|$ [15, 16]. This surfatron accelerator is not realized in a neutron star magnetosphere (the radio waves have $v_{ph} \geq c$). Instead, stochastic acceleration described in this Letter results from many short resonances with random $\phi$, repeated every Larmor rotation in $B_{bg}$.

Strong waves accelerate protons as well as electrons. The RRL energy scales with the particle mass as $m_{3/2}^3$, however reaching this limit takes time $t_{RRL} \propto m_{7/2}$. Therefore, ion acceleration (to be studied in future work) will be limited by exposure to the wave rather than $t_{RRL}$. Future work should also extend our calculations to non-planar waves with a finite beaming angle $\theta_b$: we expect the plane-wave approximation to hold if $\theta_b < \psi_{res}$.

The quick acceleration of electrons in a strong radio wave has important astrophysical implications, which will be investigated in detail elsewhere. Curvature emission with $\hbar \omega_c > m_e c^2$ will lead to an $e^\pm$ avalanche capable of powering observed X-ray bursts from magnetars. Magnetar quakes first excite low-frequency Alfvén waves, whose nonlinear interactions generate strong radio waves in the magnetosphere [17–19]. Our results suggest that these waves do not silently escape, as usually assumed. Instead, they will generate powerful $e^\pm$ fireworks in the outer magnetosphere where $B_{bg} \lesssim E_0$. Similar waves are expected in a magnetized neutron star binary before its merger, and the resulting $e^\pm$ fireworks may be observed as an X-ray precursor of the merger.

Strong implications are inevitable for FRB models that picture a bright GHz source near a magnetar. The accompanying paper [4] shows that the FRB will experience enormous scattering in the outer magnetosphere, failing to pass through radii $R = 10^9-10^{10}$ cm. This implies that observable FRBs must be emitted by relativistic ejecta from the magnetosphere.

The analysis of particle dynamics in ultrastrong waves in this Letter assumed that the wave propagates with the vacuum speed $v_{ph} = c$ (Eq. 3), neglecting any collective plasma effects on the propagation speed. Collective effects (in particular wave dispersion, $v_{ph} \neq c$) are discussed in [4]. In main applications, dispersion turns out negligible compared to wave damping due to scattering by individual particles, which is an interesting special feature of ultrastron waves. In particular, FRBs are choked by scattering in a plasma of modest density, when deviations of $v_{ph}$ from $c$ are still negligible.

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