Vector space as an area of the operation risks characteristics for asynchronous electric machines

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Abstract. The article is written in terms of topological theory for the asynchronous electric machine (AEM). During operation the vector space properties of AEM are changed under the influence of operational aging and operational damage. The information about its current properties is used to assess the AEM routine operating condition and the risks of the operational integrity loss. The authors analyze the vector spaces of the stator symmetric three-phase winding and the AEM rotor 9-phase cage. Their common working area is discovered, which serves as a communication channel for a stator and a rotor. A Working area operational testing methodology is described. Absolute and relative measures of risk for AEM operational integrity loss are introduced.

1. Introduction

The article, offered to the attention of readers, is written in terms of topological theory for the asynchronous electric machine (AEM). According to the topological concept the properties of the mathematical model of the electric machine are the simultaneous formal properties of the electric machine itself. In the nature of operating conditions implementation, in technical and energy efficiency characteristics, in operational reliability indicators of AEM there are contributions from both its physical nature and topological nature. The first one is determined by the physical properties of metals, their volumes as well as electromagnetic loads realized in AEM. The second is determined by the number of phases of AEM, the nature of their interaction, location and movement, constrains imposed on possible states.

The theory of AEM appeared and existed as a physical and technical concept for a long time. The accumulated experience of a mathematical modeling allows to supplement it with a topological concept. The main question is which electromagnetic states of an object are allowed and which are forbidden by its topology. It can be stated that electromagnetic and power states analysis of AEM represents an analysis of the AEM vector space properties and the sets of those vectors coordinates which are possible in it. The topological concept of AEM, unlike its classical physical and technical concept, proposes the evident form of such analysis.

The topological properties of AEM find their concentrated expression in the vector space properties of a mathematical model: in the dimension of the space, in its parametric properties, in the number of
characteristic subspaces and in the structure of eigenvectors belonging to these subspaces. In terms of the topological theory the possible states of AEM, that is the sets of current vectors, flux linkages, electromotive force and the windings voltage, powers corresponding to them, energies and electromagnetic moments, are identified as the properties of the vector space; in particular, by restrictions imposed in it on the vectors of electromagnetic values by the topology and the properties of parametric windings matrixes.

During the operation under the influence of operational aging and possible operational damages the properties of parametric matrix of windings are changed, and as a result the properties of the AEM vector space are also changed. If these changes are periodically recorded during the operational diagnostic, the received information can be used to assess both the current operating condition of an object and the risk of its operational integrity loss. Hence, the study of the vector space gives the scientific and methodological basis for the operational audit of the AEM stock.

In modern scientific works, dealing with the problems of electromechanical and electric power equipment, [1–6] the topological issues are addressed as an applied aspect in connection with the development or operation efficiency in a concrete electrotechnical system. It usually occurs in the areas of operation for heterogeneous electric equipment groups, the development of electric equipment energy efficiency control, when developing the measures to limit the equalize currents or to increase the electrical equipment safety. In the most concentrated form the topology issues also appear while developing the mathematical models for electromechanical objects, AEM, in particular [7]. The work originality of the authors of this article [8–13] is in the system approach to the study of the vector space of AEM and the application of these results to the topical problems of the development and operation of AEM. One of these problem is the development of the theoretical bases, methods and tools for operating diagnostic of AEM [14–16].

The authors consider the problem of this article to present the AEM vector space as the area containing information about the current AEM operation properties and to form the information characteristics of these properties.

2. Study results and their discussion

With the reference to AEM main indicators for the absence of the internal damages caused by the poor quality manufacturing, violation of operating rules and operation aging is the parametric homogeneity of the vector space working areas.

As the necessary elements for the further description of the vector space concept the results of the study for vector spaces of a symmetric three-phase stator winding and a parametric homogeneous 9-phase* rotor short-circuited cage of AEM are given below.

* Note. Relatively small numbers of cage phases are chosen with the reason to make the calculation results more compact within the article and in general.

2.1 Vector space of a three-phase winding

One of the stator three-phase winding topology is presented in figure 1.

![Figure 1. Three-phase winding, “star” topology.](image-url)
Let \( z \) be the resistance of the mutual induction for the winding phases, and \( Z \) be the full intrinsic resistance of a winding phase. The parametric matrix of the winding presents the symmetric matrix in the following form

\[
Z_k = \begin{pmatrix}
  Z & -z & -z \\
  -z & Z & -z \\
  -z & -z & Z
\end{pmatrix}.
\]

The study of the vector space of matrix \( Z_k \) indicates the following. The vector space of the winding is three-dimensional. The matrix has eigenvectors presented in the form of the matrix

\[
V = \begin{pmatrix}
  1 & -1 & -1 \\
  1 & 0 & 1 \\
  1 & 1 & 0
\end{pmatrix}
\]

and eigenvalues (numbers)

\[
\lambda_1 = Z - 2z, \quad \lambda_2 = Z + z, \quad \lambda_3 = Z + z.
\]

The eigenvectors \( V_i, i=1,2,3 \) present the characteristic systems of windings phase currents. Any realizing current distribution in the winding phase according to figure 1 can be expressed as a linear combination of eigenvectors \( V_i, i=2,3 \). It should be noted, that the systems for currents of \( V_1 \) type cannot be realized because of the restrictions imposed by the circuit in figure 1, for which there is

\[
i_A + i_B + i_C \equiv 0.
\]

Thus, the space 0, that is one–dimensional space for the values of zero sequence given by vector \( V_1 \) and having the characteristics of the resistance \( \lambda_1 \), does not contain vectors of the phase current. But it can contain the components of phase voltage vectors as for them we have

\[
u_A + u_B + u_C \neq 0.
\]

Two-dimensional space –plane \( \alpha\beta \), given by vectors \( V_i, i=2,3 \) and orthogonal to the subspace 0, is parametrically homogeneous, which is shown by the equality \( \lambda_2 = \lambda_3 \). This subspace contains phase currents vectors and phase voltage vectors without the component of a zero sequence. The vectors \( V_i, i=2,3 \) are shown in figures 2 and 3 in the form of coordinates sets along the axes of the basis of orthogonal phase axes 1, 2, 3. The basis \( V_2, V_3 \) of the plane \( \alpha\beta \) is not orthonormal, but, if necessary, it can be changed into the orthonormal basis. As such an orthonormal basis of the plane \( \alpha\beta \) a pair of vectors are usually used

\[
n_\alpha = \sqrt{\frac{2}{3}} \begin{pmatrix}
  -1 \\
  2 \\
  1
\end{pmatrix},
n_\beta = \sqrt{\frac{2}{3}} \begin{pmatrix}
  -\sqrt{3} \\
  2 \\
  0
\end{pmatrix},
\]

which together with the vector

\[
n_0 = \frac{1}{\sqrt{3}} \begin{pmatrix}
  1 \\
  1 \\
  1
\end{pmatrix},
\]

form the orthonormal basis of the vector space. In all other relations vectors \( V_i, i=1,2,3 \) and \( n_0, n_\alpha, n_\beta \) are equivalent.
2.2 Vector space of a short-circuited rotor cage

The \( z_2 \)-phase topology of the cage is presented in figure 4 in the form of planar circuit. The circuit shows the positive directions of the cage phase currents, which are taken as currents in the short-circuiting rings

\[ i_{y_i}, \quad i = 1, 2, \ldots, z_2. \]

**Figure 2.** Stator three-phase winding, the vector of current \( V_2 \).

**Figure 3.** Stator three-phase winding, the vector of current \( V_3 \).

The subspace \( \alpha \beta \) presents a working area of the vector space for the stator three-phase winding. The currents in the cage rods are the differences of the neighboring phase currents

\[ i_i = i_{y_{i+1}} - i_{y_i}, \quad i = 1, 2, \ldots, z_2. \]

In this case, due to the periodicity, the number \( z_2+1 \) is assumed to be equal to 1.

In accordance to the expression given above, the currents in the rods of the cage do not contain zero sequence components even if these components are present in phase currents.

**Figure 4.** Planar circuit of a rotor cage.

They also do not contain zero sequence voltage on the elements of the cage rings. The sum of these voltages with the absence of the cage unipolar excitation is equal to zero

\[ \sum_{j=1}^{z_2} 2 \cdot z_{y_i} i_{y_i} = 0, \]

where \( z_{y_i} \) is the resistance of the element for the short-circuiting ring with number \( i \).

For the cages with parametrically homogeneous short-circuiting rings the following expression is true

\[ z_{y_1} = z_{y_2} = \ldots = z_y, \]
in this connection we have

\[ 2 \cdot z_y \sum_{i=1}^{z_x} \frac{z_i}{z_x} = 0 \]

and the appearance of zero sequence components in the phase currents is excluded. Thus, it can be stated, that the subspace 0 of parametrically homogeneous cages is empty.

Let \( z \) be the resistance for the rod of the cage, and \( Z \) be the full resistance of the cage frame. In this case the following relation is true

\[ Z = 2z + 2z_y. \]

Since the frame of a homogenous cage contains two rods and two elements of the ring with resistances \( z_y \).

The parametric matrix of a homogenous 9-phase cage presents a symmetric band matrix of the following form

\[
Z_k = \begin{pmatrix}
Z & -z & & & & & & & \\
-z & Z & -z & & & & & & \\
& -z & Z & -z & & & & & \\
& & -z & Z & -z & & & & \\
& & & -z & Z & -z & & & \\
& & & & -z & Z & -z & & \\
& & & & & -z & Z & -z & \\
& & & & & & -z & Z & \\
& & & & & & & -z & Z & \\
\end{pmatrix}
\]

In each row of the matrix there are intrinsic resistances corresponding to the row of the cage frame and their mutual resistance with the neighboring frames. These are the resistances of the cage rods taken with “minus”. The rest elements of the row are equal to zero.

The study of \( Z_k \) matrix vector space indicates the following. The vector space of the cage is three-dimensional. The matrix has eigenvectors \( V_i \), \( i=1,2,\ldots,9 \) presented in the form of the matrix columns

\[
V = \begin{pmatrix}
-1 & -1 & -1.88 & -1 & -1 & 1.53 & -1 & 0.35 & 1 \\
1 & 0 & 2.53 & 1.88 & -1.53 & -1.35 & -0.35 & -0.88 & 1 \\
0 & 1 & -2.88 & -2.53 & -1.35 & 0.53 & 0.88 & -0.65 & 1 \\
-1 & -1 & 2.88 & 2.88 & -0.53 & -0.53 & 0.65 & 0.65 & 1 \\
1 & 0 & -253 & -2.88 & 0.53 & -1.35 & -0.65 & 0.88 & 1 \\
0 & 1 & 1.88 & 2.53 & 1.35 & -1.53 & -0.88 & -0.35 & 1 \\
-1 & -1 & -1 & -1.88 & 1.53 & -1 & 0.35 & -1 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
\end{pmatrix}
\]

The vector space has five characteristic and mutually orthogonal subspaces.

A two-dimensional subspace 1–2 with the eigenvalue \( \lambda_1=\lambda_2=Z+z \), in which (if present) the currents with structures \( V_1, V_2 \) are localized. The subspace is a working area of a cage with its six pole excitation.

The two-dimensional homogeneous subspace 3–4 with the eigenvalue \( \lambda_3=\lambda_4=Z+1.88z \), in which (if present) the currents with structure \( V_3, V_4 \) are localized. The subspace is a working area of the cage with its eight pole excitation.
The two-dimensional subspace 5–6 with eigenvalue $\lambda_5=\lambda_6=Z - 1.53z$, in which the currents with structure $V_5$, $V_6$ are localized. The subspace is a working area of the cage with its two pole excitation.

The two-dimensional homogeneous subspace 7–8 with the eigenvalue $\lambda_7=\lambda_8=Z - 0.35z$, in which (if present) the currents with structures $V_7$, $V_8$ are localized. The subspace is a working area of the cage with its four pole excitation.

The one-dimensional subspace 9 with the eigenvalue $\lambda_9=Z - 2z$ in which (if present) the currents with structure $V_9$ are localized. The subspace realizes the unipolar excitation of a homogeneous cage or the components of the vectors of the zero sequence for the heterogeneous cage.

With two polar stator winding and homogeneous rotor cage, the subspaces 1-2, 3-4, 7-8, 9 are empty, and the subspace 5-6 is a working area for the vector space of the cage. Essentially, the subspace 5-6 is 9-phase implementation for the plane $\alpha\beta$ of the stator winding and realizes the channel connecting these structural elements of AEM.

Vectors $V_5$, $V_6$ are shown in figures 5 and 6 in the form of coordinates sets along the axes for the basis of orthogonal phase axes 1, 2, ..., 9.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Rotor cage, vector $V_5$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Rotor cage, vector $V_6$.}
\end{figure}

2.3 The methodology of the vector space testing and operational risks assessment of AEM

Periodical testing of the AEM vector space allows to obtain reliable data about the machine current operating condition, its changes during operation and about the risks of the AEM operational integrity loss. Unlike the existing operation diagnostic methods the testing of the vector space gives the detailed data allowing to judge about the operational damages and operational aging during the operation.

When conducting operational diagnostic, the primary task is to test the working area of the vector space –subspace $\alpha\beta$. The characteristic of the current operating condition of the AEM is Green’s matrix [14], having Green’s impulse vectors – functions as columns. Each of them represents the response of AEM three phases to the action for the vector of the phase voltage $u_\alpha$ along one of the axes $x,y,z$ of the plane $\alpha\beta$

$$u_{\alpha x} = \delta(t) \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, u_{\alpha y} = \delta(t) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, u_{\alpha z} = \delta(t) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$  

Where $\delta(t)$ – is the Dirac function. Green’s matrix, formed by the results of the testing under the number $k$ and correlated with the testing time $t_k$, counting from the moment of operation $t_0=0$ start, has the form of [15]
\[ G(t, t_k) = \begin{pmatrix} g_{xx}(t, t_k) & g_{xy}(t, t_k) & g_{xz}(t, t_k) \\ g_{yx}(t, t_k) & g_{yy}(t, t_k) & g_{yz}(t, t_k) \\ g_{zx}(t, t_k) & g_{zy}(t, t_k) & g_{zz}(t, t_k) \end{pmatrix}, \]

To assess the current operating condition Green’s matrix is compared with the etalon matrix

\[ G(t, t_0) = \begin{pmatrix} f(t) & \varphi(t) & \varphi(t) \\ \varphi(t) & f(t) & \varphi(t) \\ \varphi(t) & \varphi(t) & f(t) \end{pmatrix}, \]

in which the functions \( f(t) \) and \( \varphi(t) \) are formed by the results of the output testing at the manufacturing plant. According to the results of the step-by-step testing at the time moments \( t_0, t_1, \ldots, t_k \) the deviation matrices are formed

\[ \Delta G(t, t_k) = \text{abs}(G(t, t_k) - G(t, 0)), \]

which are kept in the technical documentation of a product through the service life of AEM.

For practical assessment, instead of functional matrixes \( G(t, t_0), G(t, t_0), \Delta G(t, t_k) \), it is convenient to use numerical matrixes composed from the absolute values of amplitude corresponding the time functions

\[ G(t_0) = \begin{pmatrix} F & \Phi & \Phi \\ \Phi & F & \Phi \\ \Phi & \Phi & F \end{pmatrix}, \]

\[ \Delta G(t_k) = \begin{pmatrix} \Delta G_{xx}(t_k) & \Delta G_{xy}(t_k) & \Delta G_{xz}(t_k) \\ \Delta G_{yx}(t_k) & \Delta G_{yy}(t_k) & \Delta G_{zy}(t_k) \\ \Delta G_{zx}(t_k) & \Delta G_{zy}(t_k) & \Delta G_{zz}(t_k) \end{pmatrix}. \]

From the point of view of the assessment of the current operating condition for AEM and making forecasts, the matrix \( \Delta G(t_k) \) obtains a number of useful properties.

Matrix \( \Delta G(t_k) \) contains the deviation of AEM vectors parameters from the reference values characterizing the product, which operational integrity is reliable. The deviations occur, to a large extent, under the influence of random factors. According to these reasons the elements of matrix \( \Delta G(t_k) \) can be taken as absolute characteristics for the risk of the machine operational integrity loss.

Following the equality

\[ \Delta G(t_k) = n \cdot G(t_0), \]

even if it is approximate, where \( 0 < n < 1 \) is a coefficient of the proportionality, gives the evidence for a parametric homogeneity of the vector space working area and shows the homogeneous operating aging of AEM. Besides, the relation \( n/ t_k \) is a measure of this process rate and allows to assess the residual life of AEM. It is important to note that these estimates are refined with \( t_k \) increase.

The presence of sharp changes in individual elements of the matrix \( \Delta G(t_k) \), taken place during the time \( t_k - t_{k-1} \), indicate the appearance or rapid developments of operating damages in the AEM construction.

Let \( \Delta G \) be the deviation of the matrix \( \Delta G(t_k) \) element corresponding to so called “limiting state”, that is, the state of AEM when its further operation should be stopped due to the drift of parameters from the technical documentation limitations. The value \( \Delta G \) can be defined by the expert estimation or technical specifications document. The matrix elements
can be used as the current estimations for the probability of the event $X$, which is in the loss of the operational integrity of AEM. By the definition, when the operation is stated

$$P(X,t_k) = \frac{1}{\Delta G} \Delta G(t_k)$$

and the event $X$ is impossible. At time $t_k = t_{k_0}$ when at least one deviation of the matrix $\Delta G(t_k)$ reaches the value $\Delta G$, the event $X$ becomes a certain event. It is essential, that risk assessments are given according to three different directions of the AEM vector working area and they are specified during the operating time increase.

3. Study results and their discussion

1. The vector space is an area containing the information about the current operational condition and the loss of operational integrity for AEM.
2. This information presents in the working area of the vector space – subspace $\alpha\beta$ in a more concentrated form.
3. The measures for the technical level loss and the risks of the AEM operational integrity loss are the absolute and relative deviations of Green’s matrix elements from the reference values.
4. The operation of AEM should be accompanied by the matrixes of absolute and relative deviations, including them into the technical specification documents.

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