Elementary charge transfer processes in a superconductor-ferromagnet entangler

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Abstract – We study the production of spatially separated entangled electrons in ferromagnetic leads from Cooper pairs in a superconducting lead. We give a complete description of the elementary charge transfer processes, i) transfer of Cooper pairs out of the superconductor by Andreev reflection and ii) distribution of the entangled quasiparticles among the ferromagnetic leads, in terms of their statistics. The probabilities that entangled electrons flow into spatially separated leads are completely determined by experimentally measurable tunnel conductances and polarizations. Finally, we investigate how currents, noise and cross-correlations are affected by transport of entangled electrons.

A solid-state entangler is an electronic analog of the optical setups used for experimental Bell inequality tests, quantum cryptography and quantum teleportation [1]. Ideally, such a device should produce separated currents of entangled electrons. Superconductors are suitable candidates as sources in solid-state entanglers since Cooper pairs constitute entangled states. This prospect has motivated several papers addressing the properties of hybrid superconductor and normal metal entanglers [2–6].

One of the challenges is to prevent processes where pairs of entangled particles reach the same lead, i.e. are not spatially separated. Electrons from Cooper pairs are entangled in spin and energy space, and separation of pairs into different leads using ferromagnets or quantum dots has been suggested [3]. Upon filtering, only the spin or energy part of the two-particle wave function collapses, depending on whether ferromagnets or quantum dots are used. Respectively, energy or spin entanglement remains [4]. Here we consider separation by ferromagnets.

Solid-state entanglers have been analyzed in refs. [2–6] in terms of currents, noise and cross-correlations. A more direct approach, describing the elementary charge transfer processes in terms of experimentally controllable parameters is certainly desirable. We demonstrate how this is possible through the full distribution of current fluctuations, the full counting statistics (FCS), of the solid-state entangler [7–10]. The FCS provides complete information about currents, noise, cross-correlations and higher cumulants, and even more importantly, allows direct access to the probability for transfer of charge between different parts of the device.

We consider the singlet superconductor-ferromagnet (S-F) device shown in fig. 1. A normal metal cavity (c) is connected to one superconducting terminal at ground...
and several equally biased ferromagnetic terminals at voltage \( V \) via tunnel junctions. The cavity is under the influence of the proximity effect. In this device, charge transport occurs via two processes: i) transfer of Cooper pairs out of the superconductor by Andreev reflection and ii) distribution of the entangled quasiparticles among the ferromagnetic leads. The distribution can occur via direct Andreev (DA) reflection, where an entangled pair is transferred into lead \( F_n \) or crossed Andreev (CA) reflection, where each particle of the entangled pair is transferred into spatially separated leads \( F_m \) and \( F_n \) \((m \neq n)\). CA reflection produces spatially separated entangled electrons. Since the ferromagnetic terminals are at the same voltage and we consider zero temperature, there is no direct electron transport between the ferromagnetic terminals \([11]\).

Our general results for the counting statistics show that the processes i) and ii) are independent and therefore the statistics can be factorized. This novel factorization and the probability distribution for process ii) reveals the precise dependence of the probabilities for CA and DA processes on the experimentally measurable conductances and polarizations of the ferromagnetic leads. We find the probability to detect in terminals \( m \) and \( n \) the electrons of a Cooper pair which has been transferred out of \( S \),

\[
p_{mn} = (g_m g_n - g_m g_n)/(g^2 - g^2),
\]

where \( g = \sum_n g_n \) and \( g = \sum_n g_n \). The probabilities \( p_{mn} \) depend solely on the conductances \( g_n \) and spin polarization conductances \( g_{\sigma} \) of the ferromagnetic leads which can be determined by magnetoresistance measurement in the normal state. Equation (1) shows that the detected two-particle processes originate from a pure spin singlet density matrix subspace \([12]\). We emphasize that (1) in combination with the Cooper pair transfer probability, to be discussed below, allows for an unambiguous identification of all statistical properties of the charge transfer.

Using the magnetization dependence of the probabilities (1) one can violate the Bell-Clauser-Horne-Shimony-Holt inequality \([13,14]\) and demonstrate entanglement. Consider a device with four \( F_n \) \((n = 1-4)\) terminals. Drains 1, 2 and 3, 4 have pairwise equal conductances and antiparallel magnetizations, \( g_1 = g_2 \) and \( g_1 = -g_2 \) etc. The terminal pairs are spin detectors with respect to magnetizations \( g_{i(3)} \) so that spins up in drains 1, 2 \((3, 4)\) are measured by the current in \( F_{1(3)} \) and so on. Experiments are performed with each spin detector in two different magnetization directions \( g_{i(3)} \) and \( g_{i(3)} \). We find for the Bell parameter \( \mathcal{E} = |g_1 g_3 + g_1 g_3 + g_1 g_3 - g_1 g_3|/(g_1 g_3) \) the largest possible value of \( \mathcal{E} \) in a local theory is 2. Violation of Bell’s inequality \( \mathcal{E} \leq 2 \) can be observed provided the detectors are efficient enough. The condition on the spin polarizations is \( |g_{i(3)}|/g_{i(3)} \geq 2^{1/4} \) which can be experimentally realized in the devices we consider \([15]\).

Our results for the S-F entangler determine the spectral properties of the cavity, currents, cross-correlations and higher-order cumulants in terms of the transport conductances measured in the normal state. This is possible for the spin-active tunnel barriers we consider because the parameters that determine not only the average transport, but all statistical properties, are the sums of spin-dependent transmission eigenvalues \( \sum_k T_{k,\sigma} \) (conductances) \([16]\) and not the individual values of \( T_{k,\sigma} \) \([10]\). Specifically, our result does not depend on individual scattering matrix elements for the nanostructure \([17]\) or phenomenological dephasing that removes the complication of coherence effects \([5]\). The experimental control parameters are the relative orientations of the magnetizations, which from our calculation determine the fraction of the CA current, and thus the spatially separated entangled pair currents for a given set of conductances and spin polarizations.

Ferromagnet detection of entangled spin singlets from a ballistic normal conductor was considered in ref. \([12]\). In that device, there are also one-particle transfers, which can contribute substantially to the current and the noise. Another important qualitative difference between our device and the system in ref. \([12]\) is the latter’s strong coupling to the detectors which can distort the spin singlets emitted from the source and induce triplet correlations upon detection. Also, in the limit of a weak coupling to the detectors, there are no two-particle processes in the system of ref. \([12]\). In contrast, the electrons in our S-F device are always detected from a pure spin singlet state.

The charge transfer probabilities are obtained by identifying the elementary processes in the many-body charge counting statistics. The statistics is determined by the cumulant generating function \( (\text{CGF}) \)

\[
S(\chi_1, \chi_2, \ldots) = S(\{\chi_n\}) \quad \text{of the probability } P(\{N_n\}) \quad \text{to transfer in a time interval } t_0, \text{ } N_1 \text{ electrons to } F_1, \text{ } N_2 \text{ electrons to } F_2, \text{ and so on. Our main finding is the statistics }
\]

\[
P(\{N_n\}) \equiv \int \frac{dM}{(2\pi)^M} e^{S(\{\chi_n\}) - i \sum_n \chi_n N_n} \quad (2a)
\]

\[
= P_S \left( \sum_n N_n \right) P(\{N_n\}) \sum_n N_n \quad (2b)
\]

for \( \sum_n N_n \) even and positive and \( M \) the total number of terminals in the circuit. The interpretation of this result is that the charge transfer is given by two independent processes. The first factor \( P_S(2N) \) is the probability that \( N = \sum_n N_n/2 \) Cooper pairs are emitted from the superconducting source terminal into any of the detectors. The second factor \( P(\{N_n\})(2N) \) in (2b) is the conditional probability that \( N_0 \) out of the \( 2N \) electrons have been transferred into the ferromagnetic terminal \( F_n \). Below we will explain in detail how our calculation yields concrete expressions for the elementary processes described by \( P(\{N_n\})(2N) \). These results facilitates a unique interpretation of the transfer of spin singlet electron pairs.

We now supply the microscopic expressions for the two probabilities in (2b). The Cooper pair transfer probability

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is obtained from \( P_S(2N) = \int d\chi_S/(2\pi)\exp(S_S(\chi_S) - i\chi_S) \) with a CGF \( S_S(\chi_S) \) given by
\[
t_0 V \sqrt{2e} \sqrt{g_S^2 + \sqrt{(g_S^2 - g^2 + g^2)^2 + 4g_S^2(g^2 - g^2)e^{2i\chi_S}}} ,
\]
where \( g_S^2 = g_s^2 + g^2 + g^2 \). The contact to the superconducting terminal is characterized by a spin-independent conductance \( g_s \). The \( \pi \)-periodicity of \( S_S(\chi_S) \) on \( \chi_S \) ensures that an even number of charges is transferred. The \( 2N \) electrons are distributed among the \( F_n \) terminals according to the multinomial distribution \( P(\{N_n\}|2N) = \int d^{M-1}\chi/(2\pi)^{M-1}\exp(S_N(\{\chi_n\}) - i\sum_n \chi_n N_n) \) with a CGF
\[
S_N(\{\chi_n\}) = N \ln \left( \sum_{mn} p_{mn} e^{-i\chi_m + i\chi_n} \right) .
\]
The concrete form of the two-particle probabilities \( p_{mn} \) to detect one charge in terminal \( m \) and one in terminal \( n \) is given in (1).

We will explain below how our calculation determines (1), (3), and (4). The interpretation in terms of two independent processes of charge transfer is based on a general result for the calculated counting statistics. We have not made any a priori assumptions on the initial state of the superconducting source or the ferromagnetic terminals, except that they are reservoirs at zero temperature with a voltage bias \( eV \) applied between the source and the terminals. The experimental condition for this approximation is that the energy scale set by temperature \( k_B T \) is smaller than the Thouless energy of the cavity and the superconducting gap. The direct result of our calculation is the CGF \( S(\{\chi_n\}) \) of the S-F entangler of fig. 1 which leads to the factorization (2b) in terms of the CGFs given by (3) and (4). \( S'(\{\chi_n\}) \) is recovered from (3) by replacing the factor \( e^{2i\chi_S} \) with \( \exp(S_N(\{\chi_n\})/N) \) given in (4). The factorization in (2) can be proven straightforwardly from \( S(\{\chi_n\}) \). Actually, such a factorization is valid for any CGF where the \( \chi \)-dependence is \( \exp(S_N(\{\chi_n\})/N) \), irrespectively of its form or the probabilities \( p_{mn} \).

We now discuss some consequences of the charge counting statistics. FCS enables us to express the current and noise correlations in a compact and meaningful form. The currents \( I_n = (ie/t_0)\partial S(\{\chi_n\})/\partial \chi_n|_{\chi_n=0} \) are
\[
I = GV, \quad G = \frac{g_S^2(g^2 - g^2)}{\sqrt{g_s^2 + g^2(g_s^2 + g^2 - g^2)}} , \quad I_n = I p_n ,
\]
where \( p_n = \sum_m p_{mn} \) is the probability to detect one of the electrons in terminal \( n \), irrespective of where the second electron goes. The combined probabilities can be directly accessed in the noise correlators between current fluctuations in terminals \( m \) and \( n \),
\[
C_{mn} = (-2e^2/t_0)\partial^2 S(\{\chi_n\})/\partial \chi_m \partial \chi_n|_{\chi_m, \chi_n=0} = 2eI \left[ p_{mn} + p_n \delta_{mn} - 2(1 - F_2)p_m p_n \right] .
\]

The Fano factor for Cooper pair transport is defined as the ratio of the full current noise \( C = \sum_{mn} C_{mn} \) to the Poissonian noise of doubled charges, \( F_2 = C/4eI \), and is explicitly found to be \( (1 - F_2) = [5 - g^2/|g_s^2 + g^2|]x(x + 1) \), where \( x = g^2/(g_s^2 + g^2) \). These expressions for the current and the noise provide a transparent interpretation of the transport processes. The current in (5) is proportional to \( g_S^2(g^2 - g^2) \), since two particles have to tunnel through the double junction to transfer a Cooper pair from S. The denominator is due to the proximity effect [18–20] and enhances the current drastically in comparison to calculations based on the tunneling Hamiltonian [11]. The current into each terminal \( I_n \) is then weighted according to the probability \( p_n \). We might also distinguish the contributions to the current originating from crossed and direct Andreev reflection. The probability to detect a DA reflection in terminal \( n \) is given by \( p_n \), and the probability for CA detection in different terminals \( m \neq n \) is given by \( p_{mn} \). We find the ratio of the crossed current to the total current as
\[
\frac{I_{\text{CA}}}{I_n} = \frac{p_n - p_{mn}}{p_n} = \frac{g_n(g - g_n) - g_m(g - g_n)}{g_n g - g_m g} .
\]
This ratio is independent of the coupling to the superconducting terminal. We further observe that the crossed current is enhanced by increasing the polarization of the contact \( n \) and is additionally favored by aligning the magnetization \( g_s \) opposite to the average magnetization \( g \). These results are a direct consequence of the spin singlet nature of the Cooper pairs. Enhancing the magnitude of the polarization \( g_n/g_m \) of one terminal reduces the total current, but enhances the crossed part of the Andreev current, since the tunneling of one spin singlet electron-hole pair through the same contact is strongly suppressed. The sign of cross-correlations in three-terminal beam splitters has been considered for various devices both experimentally [21] and theoretically [5,6,22,23]. Studies of noise [24] and FCS [25] for a beam splitter with entangled electrons show that entanglement gives qualitatively different noise characteristics compared to transport of non-entangled electrons. The physical origin of positive and negative contributions to the cross-correlators can in our case be understood from the dependence on the two-particle probabilities in (6). CA reflection leads to positive cross-correlations since two particles are transferred simultaneously into \( F_m \) and \( F_n \) (bunching behavior) [24]. A negative contribution (anti-bunching) that does not depend on entanglement, is induced by the fermion exclusion principle: The transfer of one electron-hole pair into \( F_m \) by DA reflection, prevents the simultaneous transfer of another pair into \( F_n \). However, if the electron-hole pair transfers are not temporally correlated (Poissonian statistics), the exclusion principle does not affect the cross-correlations. This is the case when there is strong asymmetry in the junction conductances \( g_s \) and \( g_s \) (\( g_s \gg g_s \)) so that the Fano factor \( F_2 = 1 \). In this limit the negative contribution \(-2(1 - F_2)p_m p_n \) in (6) vanishes.
Scattering matrix calculations give similar results for $C_{nn}$ [17].

The strongly asymmetric case is particularly interesting since the cross-correlations $\langle n \neq n \rangle C_{nn} = 2eI_{F_{nn}}$, are a direct measure of the probability that electrons from a Cooper pair are transferred into different terminals.

To illustrate our theory, let us now consider the three-terminal version of fig. 1 with the superconducting source terminal S and two ferromagnetic drains $F_1$ and $F_2$. The ferromagnetic magnetizations can in this device be utilized as filters to produce currents of entangled electrons in separated leads. Let us consider $|g_1| = |g_2|$ in the following and define Fano factors $F_{nn} = C_{nn}/(2eI)$. The autocorrelation noise $F_{11} (22)$ will be reduced in antiparallel alignment $g_1 = -g_2$, as compared to a S-N system, because of $C_{nn} = 0$ due to enhancement of CA reflection. The cross-correlation $F_{12}$, shown in fig. 2 can have both positive and negative sign depending on the conductance asymmetry $g_S/g$ and the spin polarization. The positive contribution to $F_{12}$ is proportional to $g_1 g_2 + (-)|g_1| |g_2|$ in the antiparallel (parallel) alignment demonstrating how spin filtering of entangled pairs enhances (reduces) the correlation between currents in $F_1$ and $F_2$ with respect to an S-N system [5,6]. Note that for sufficiently large spin polarization, $F_{12}$ can be positive for the entire range of $g_S/g$ in the antiparallel alignment (region above the blue online line in the left panel of fig. 2), whereas it remains always negative in the parallel alignment for $g_S/g \simeq 1$ (inset of fig. 2). The change of sign in $F_{12}$ by switching from antiparallel to parallel alignment is due to the enhanced probability of CA events, see (7).

We will finally outline the calculation that yields the FCS of the considered devices. We utilize the circuit theory of mesoscopic superconductivity [8,26] and represent the circuit in terms of terminals, cavities and connectors. Terminals are described by equilibrium quasiclassical Green’s function matrices $\hat{G}_n$ determined by electrochemical potential and temperature. Our notation for matrix subspaces is: $\sim$ for spin, $\sim$ for Nambu, and $\sim$ for Keldysh. Pauli matrices are denoted $\tau_j$. At zero temperature we consider $0 < E \leq eV$ where the Green’s functions for all ferromagnetic terminals $F_n$ are $\hat{G} = \tau_3\hat{\gamma}_n + (\tau_1 + i\tau_2)$, $V$ is the voltage of the ferromagnetic terminals and $E$ the quasiparticle energy. We consider that $eV$ is much smaller than the Thouless energy of the cavity. The superconductor S is at zero voltage and has Green’s function $\hat{G}_S = \tau_1$, where we assume $E \ll \Delta, \Delta$ being the gap of S. The terminals are connected to a cavity c which is under the influence of the proximity effect from S. The cavity is described by an unknown Green’s function $\hat{G}_c$, assumed isotropic due to chaotic or diffusive scattering. We assume that $c$ is large enough so that charging effects can be disregarded, and small enough so that $\hat{G}_c$ is spatially homogeneous. The circuit theory is formulated in terms of generalized matrix currents $I_j$ in spin Nambu Keldysh matrix space and from the matrix current conservation $\sum_j I_j = 0$. This determines the Green’s function on the node together with the normalization condition $\hat{G}_c^2 = 1$. The matrix currents can have arbitrary structure, and allow to derive the FCS by introducing the counting fields $\chi_n$ for each terminal according to [8] $\hat{G}_n(\chi) = e^{\chi \tau_3 \hat{\gamma}_n/2} e^{\chi \tau_1 \hat{\gamma}_n/2}$. Spin-active connectors are taken into account by spin-dependent transmission and reflection amplitudes $t^\sigma_{n,s}$ and $r^\sigma_{n,s}$ for particles incident on the interface $n$ from the cavity side in channel $k$ with spin $\sigma$. The matrix current through a spin-active tunnel barrier between $c$ and $F_n$ evaluated at the cavity side is $[16,27]$ $I_n = |g_n| \hat{G}_n / |\{ g_n, \tau \hat{\gamma}_n, \hat{G}_n \} | / 4, \hat{G}_c$. Here, $g_n = g_0 \sum_{k,\sigma} |t^\sigma_{n,s}|^2$ is the tunnel conductance and $g_0 = e^2/\hbar$ the conductance quantum. The magnetization direction is encoded in the direction of $g_n$, and the conductance polarization in that quantization axis is $|g_n| = gQ \sum_k (|t^\uparrow_{n,s}|^2 - |t^\downarrow_{n,s}|^2)$. We have neglected here an additional term related to spin-dependent phase shifts upon reflection at the interface [16,27], as these are small in some material combinations or can be suppressed by a thin, non-magnetic oxide layer [28]. The matrix current between c and S is $I_S = g_S |\hat{G}_S, \hat{G}_c| / 2$ [26]. We take into account the spin structure of matrix currents $I_n$ and Green’s functions in S-F systems, and derive the CGF in the linear response regime and for $eV \ll \Delta$, generalizing ref. [23]: $S = t_0/(4e^2) \int dE \sum_{\lambda_p} \sqrt{\lambda_p^2}$, where $\{ \lambda_p \}$ is the set of eigenvalues of the matrix $M$ defined by writing matrix current conservation in the cavity $\sum_n I_n = [M, \hat{G}_c] = 0$. The non-trivial spin matrix structure of $I_n$ determines the magnetization dependence of transport processes in the system. Carrying out this procedure yields the FCS for the setup in fig. 1.

In conclusion, we have investigated the elementary charge transfer processes of a S-F entangler. Charge transfers occur via two statistically independent processes, i) Cooper pairs are transferred out of the superconductor by Andreev reflection and ii) entangled quasiparticles are distributed among the different ferromagnetic leads. The
probabilities for entangled electrons to flow into spatially separated leads are completely determined by experimentally measurable conductances and polarizations. This allows complete knowledge of the statistics of charge transfer in the S-F entangler.

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