Self-stabilizing Byzantine-tolerant Broadcast
(preliminary version)

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We study a well-known communication abstraction called Byzantine Reliable Broadcast (BRB). This abstraction is central in the design and implementation of fault-tolerant distributed systems, as many fault-tolerant distributed applications require communication with provable guarantees on message deliveries. Our study focuses on fault-tolerant implementations for message-passing systems that are prone to process-failures, such as crashes and malicious behavior.

At PODC 1983, Bracha and Toueg, in short, BT, solved the BRB problem. BT has optimal resilience since it can deal with $t < n/3$ Byzantine processes, where $n$ is the number of processes. The present work aims at the design of an even more robust solution than BT by expanding its fault-model with self-stabilization, a vigorous notion of fault-tolerance. In addition to tolerating Byzantine and communication failures, self-stabilizing systems can recover after the occurrence of arbitrary transient-faults. These faults represent any violation of the assumptions according to which the system was designed to operate (provided that the algorithm code remains intact).

We propose, to the best of our knowledge, the first self-stabilizing Byzantine-tolerant BRB solution for signature-free message-passing systems. Our contribution includes a self-stabilizing variation on a BT that solves a single-round BRB for asynchronous systems. We also consider the problem of recycling instances of single-round BRB. Our self-stabilizing Byzantine-tolerant recycling for time-free systems facilitates the concurrent handling of a predefined number of BRB invocations. Our proposal can serve as the basis for self-stabilizing Byzantine-tolerant consensus.

1 Introduction

Fault-tolerant distributed systems are known to be hard to design and verify. High-level communication primitives can facilitate such complex challenges. These high-level primitives can be based on low-level ones, such as the one that allows processes to send a message to only one other process at a time. When an algorithm wishes to broadcast message $m$ to all processes, it can send $m$ individually to every other process. Note that if the sender fails during this broadcast, it can be the case that only some of the processes have received $m$. Even in the

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presence of network-level support for broadcasting or multicasting, failures can cause similar inconsistencies. In order to simplify the design of fault-tolerant distributed algorithms, such inconsistencies need to be avoided. Many examples show how fault-tolerant broadcasts can significantly simplify the development of fault-tolerant distributed systems, e.g., State Machine Replication [31] and Set-Constrained Delivery Broadcast [2]. The weakest variance, named Reliable Broadcast (RB), lets all non-failing processes agree on the set of delivered messages. This set includes all the messages broadcast by the non-failing processes. Stronger RB variants specify additional requirements on the delivery order. Such requirements can simplify the design of fault-tolerant distributed consensus, which allows reaching, despite failures, a common decision based on distributed inputs. RB and consensus (as well as message-passing emulation of read/write registers [31]) are closely related to distributed computing problems. This work aims to design an RB solution that is more fault-tolerant than the state of the art.

1.1 The problem

Lamport, Shostak, and Pease [26] said that a process commits a Byzantine failure if it deviates from the algorithm instructions, say, by deferring (or omitting) messages that were sent by the algorithm or sending fake messages. Such malicious behavior can be the result of hardware malfunctions or software errors as well as coordinated malware attacks. Bracha and Toueg [13], BT from now on, proposed the communication abstraction of Byzantine Reliable Broadcast (BRB), which allows every process to invoke the \texttt{brbBroadcast\texttt{}(v)} operation and raise the \texttt{brbDeliver()} event upon message arrival according to Definition 1.1. Following Raynal [31, Ch. 4], we consider the (single instance) BRB problem.

**Definition 1.1** The operations \texttt{brbBroadcast\texttt{}(v)} and \texttt{brbDeliver()} should satisfy the following properties.

- **BRB-validity.** Suppose a correct process BRB-delivers message \texttt{m} from a correct process \texttt{p}. Then, \texttt{p} had BRB-broadcast \texttt{m}.
- **BRB-integrity.** No correct process BRB-delivers more than once.
- **BRB-no-duplicity.** No two correct processes BRB-deliver different messages from \texttt{p} (who might be faulty).
- **BRB-Completion-1.** Suppose \texttt{p} is a correct sender. All correct processes BRB-deliver from \texttt{p} eventually.
- **BRB-Completion-2.** Suppose a correct process BRB-delivers a message from \texttt{p} (who might be faulty). All correct processes BRB-deliver \texttt{p}’s message eventually.

For the sake of completeness, we also consider the problem of recycling an unbounded sequence of BRB invocations using bounded memory. We require the (single-instance) BRB object, \texttt{O}, to have an operation, called \texttt{recycle()}, that allows the recycling mechanism locally reset \texttt{O}, after all non-faulty processes had completed the delivery of \texttt{O}’s message. Also, we require the mechanism to inform (the possibly recycled) \texttt{O} regarding its availability to take new missions. Specifically, the \texttt{txAvailable()} operation returns \texttt{True} when the sender can use \texttt{O} for broadcasting and \texttt{rxAvailable()} returns \texttt{True} when \texttt{O}’s new transmission has arrived at the receiver.

1.2 Fault models

Recall that our BRB solution may be a component in a system that solves consensus. Thus, we safeguard against Byzantine failures by following the same assumptions that are often used when solving consensus. Specifically, for the sake of deterministic and signature-free solvability [30].
we assume there are at most $t < n/3$ crashed or Byzantine processes, where $n$ is the total number of processes. The studied problems focus on asynchronous message-passing systems that have no guarantees on the communication delay and the algorithm cannot explicitly access the clock. The fault model includes (i) undetectable Byzantine or crash failures, as well as (ii) communication failures, e.g., packet omission, duplication, and reordering, as long as fair communication holds. Following Raynal [31], we denote the model by $\text{BAMP}_{n,t}[\text{FC}, t < n/3]$ which stands for Byzantine asynchronous message-passing with at most $t$ (out of $n$) faulty nodes, fair communications (FC), and $t < n/3$. We use $\text{BAMP}_{n,t}[\text{FC}, t < n/3]$ for studying the problem of single-instance BRB.

By Doudou et al. [21], processes commit muteness failures when they stop sending specific messages, but they may continue to send “I-am-alive” messages. For studying the problem of BRB instance recycling, we enrich $\text{BAMP}_{n,t}[\text{FC}, t < n/3]$ with a muteness detector of class $\Diamond P_{\text{mute}}$ and assume bounded message lifetime (BML). That is, in any unbounded sequence of BRB invocations, at the time that immediately follows the $x$-th invocation, the messages associated with the $(x-\lambda)$-th invocation (or earlier) are either delivered or lost, where $\lambda$ is a known upper-bound. (Note that BML does not imply bounded communication delay since an unbounded number of messages can be lost between any two successful transmissions.) We denote this time-free model by $\text{BAMP}_{n,t}[\text{FC}, \Diamond P_{\text{mute}}, \text{BML}]$.

We clarify that our muteness detector implementation follows an assumption about the number, $\Theta$, of messages that some non-faulty processes can exchange without hearing from all non-faulty processes.

1.2.1 Self-stabilization

In addition to the failures above, we also aim to recover after the occurrence of the last arbitrary transient-fault [1, 15]. A transient-fault can model any temporary violation of assumptions according to which the system was designed to operate. This includes the corruption of control variables, such as the program counter and packet payloads, as well as operational assumptions, such as that at most $t < n/3$ processes are not faulty. Since the occurrence of these failures can be arbitrarily combined, we assume that these transient-faults can alter the system state in unpredictable ways. In particular, when modeling the system, Dijkstra [14] assumes that these violations bring the system to an arbitrary state from which a self-stabilizing system should recover, see [1, 15] for details. I.e., Dijkstra requires recovery after the last occurrence of a transient-fault and once the system has recovered, it must never violate the task requirements.

1.3 Related work

In the context of reliable broadcast, there are (non-self-stabilizing) Byzantine fault-tolerant solutions [31] and (non-Byzantine-tolerant) self-stabilizing solutions [28]. Our study focuses on the BT [13] solution to which we propose a self-stabilizing variation. BT is the basis for advanced Byzantine fault-tolerant algorithms for solving consensus, such as the one by Mostéfaoui and Raynal [29]. BT is based on a simpler communication abstraction called no-duplicity broadcast (ND-broadcast) by Toueg [32, 31]. It includes all of the above requirements except BRB-Completion-2.

In the broader context of self-stabilizing Byzantine-tolerant solutions for message-passing systems, we find solutions for topology discovery [18], storage [11, 10, 9, 8, 7], clock synchronization [20, 27, 25], approximate agreement [12], asynchronous unison [24] to name a few. Also, Byzantine-tolerant state-machine replication by Binun et al. [4, 5] for synchronous systems and Dolev et al. [16] for practically-self-stabilizing partially-synchronous systems.
This work also considers a self-stabilizing Byzantine-tolerant mechanism for recycling single-instance BRB objects. This mechanism uses a muteness detector inspired by Doudou et al. \[21, 22\]. Doudou et al. consider the problem of consensus, whereas we consider BRB.

1.4 Our contribution

We present a fundamental module for dependable distributed systems: SSBRB, a self-stabilizing Byzantine-tolerant reliable broadcast for asynchronous message-passing systems, \(i.e.,\) for the model of \(\text{BAMP}_{n,t}[\text{FC},t < n/3]\). We obtain this new self-stabilizing solution via a transformation of the non-self-stabilizing BT algorithm \[13\] while preserving BT’s resilience optimality of \(t < n/3\).

In the absence of transient-faults, our asynchronous solution for single-instance BRB achieves operation completion within a constant number of communication rounds. After the occurrence of the last transient-fault, the system recovers eventually (while assuming execution fairness among the non-faulty processes). The amount of memory used by the proposed algorithm is bounded and the communication costs of the studied and proposed algorithms are similar, \(i.e.,\) \(O(n^2)\) messages per BRB instance. The main difference is that our solution unifies all the types of messages sent by BT into a single message that is repeatedly sent. This repetition is imperative since self-stabilizing systems cannot stop sending messages [15, Chapter 2.3].

Our contribution also includes a self-stabilizing Byzantine-tolerant recycling mechanism for time-free systems that are enriched with muteness detectors, \(i.e.,\) \(\text{BAMP}_{n,t}[\text{FC},\hat{P}_{\text{mute}},\text{BML}]\). The mechanism is based on an algorithm that counts communication rounds. Since individual BRB-broadcasters increment the counter independently, the algorithm is named the independent round counter (IRC) algorithm. Implementing a self-stabilizing (and Byzantine-tolerant) IRC is a non-trivial challenge since this counter should facilitate an unbounded number of increments, yet it has to use only a constant amount of memory. The proposed solution recovers from transient faults eventually, uses a bounded amount of memory, and has communication costs of \(O(n)\) messages per BRB instance.

To the best of our knowledge, we propose the first self-stabilizing Byzantine-tolerant BRB and IRC solutions. As said, BRB and IRC consider different fault models. Section 2 defines \(\text{BAMP}_{n,t}[\text{FC},t < n/3]\) and self-stabilization. The non-self-stabilizing BT algorithm for \(\text{BAMP}_{n,t}[\text{FC},t < n/3]\) is studied in Section 3. Our self-stabilization Byzantine-tolerant variation on BT for \(\text{BAMP}_{n,t}[\text{FC},t < n/3]\) is proposed in Section 4. IRC is presented in two steps. A self-stabilizing IRC for time-free node-failure-free message-passing systems appears in Section 6. In Section 7, we revise these time-free settings into \(\text{BAMP}_{n,t}[\text{FC},\hat{P}_{\text{mute}},\text{BML}]\) and propose a self-stabilizing Byzantine-tolerant IRC.

2 System Settings for \(\text{BAMP}_{n,t}[\text{FC},t < n/3]\)

This work focuses on asynchronous message-passing systems that have no guarantees on the communication delay. Also, the algorithm cannot explicitly access the (local) clock (or use timeout mechanisms). The system consists of a set, \(\mathcal{P}\), of \(n\) fail-prone nodes (or processes) with unique identifiers. Any pair of nodes \(p_i, p_j \in \mathcal{P}\) has access to a bidirectional communication channel, \(\text{channel}_{j,i}\), that, at any time, has at most channelCapacity \(\in \mathbb{Z}^+\) messages on transit from \(p_j\) to \(p_i\) (this assumption is due to a known impossibility [15, Chapter 3.2]).

In the interleaving model [15], the node’s program is a sequence of (atomic) steps. Each step starts with an internal computation and finishes with a single communication operation, \(i.e.,\) a message send or receive. The state, \(s_i\), of node \(p_i \in \mathcal{P}\) includes all of \(p_i\)’s variables and
channel. The term system state (or configuration) refers to the tuple \( c = (s_1, s_2, \ldots, s_n) \). We define an execution (or run) \( R = c[0], a[0], c[1], a[1], \ldots \) as an alternating sequence of system states \( c[x] \) and steps \( a[x] \), such that each \( c[x+1] \), except for the starting one, \( c[0] \), is obtained from \( c[x] \) by \( a[x] \)'s execution.

2.1 The fault model and self-stabilization

The legal executions (LE) set refers to all the executions in which the requirements of task \( T \) hold. In this work, \( T_{BRB} \) denotes the task of Byzantine-tolerant Reliable Broadcast, which Section 1 specifies, and the executions in the set \( LE_{BRB} \) fulfill \( T_{BRB} \)'s requirements.

2.1.1 Benign failures

A failure occurrence is a step that the environment takes rather than the algorithm. When the occurrence of a failure cannot cause the system execution to lose legality, i.e., to leave \( LE \), we refer to that failure as a benign one.

- Communication failures and fairness. We focus on solutions that are oriented towards asynchronous message-passing systems and thus they are oblivious to the time at which the packets depart and arrive. We assume that any message can reside in a communication channel only for a finite period. Also, the communication channels are prone to packet failures, such as loss, duplication, and reordering. However, if \( p_i \) sends a message infinitely often to \( p_j \), node \( p_j \) receives that message infinitely often. We refer to the latter as the fair communication assumption. As in [24], we assume that the communication channel from a correct node eventually includes only messages that were transmitted by the sender.

The studied algorithm assumes reliable communication channels whereas the proposed solution does not make any assumption regarding reliable communications. Section 4.1.2 provides further details regarding the reasons why the proposed solution cannot make this assumption.

- Arbitrary node failures. Byzantine faults model any fault in a node including crashes, and arbitrary malicious behaviors. Here the adversary lets each node receive the arriving messages and calculate their state according to the algorithm. However, once a node (that is captured by the adversary) sends a message, the adversary can modify the message in any way, delay it for an arbitrarily long period or even omit it from the communication channel. The adversary can also send fake messages, i.e., not according to the algorithm. Note that the adversary has the power to coordinate such actions without any computational (or communication) limitation. For the sake of solvability [26, 30, 32], the fault model that we consider limits only the number of nodes that can be captured by the adversary. That is, the number, \( t \), of Byzantine failures needs to be less than one-third of the number, \( n \), of nodes in the system, i.e., \( 3t + 1 \leq 3n \). The set of non-faulty indexes is denoted by \( Correct \), so that \( i \in Correct \) when \( p_i \) is a correct node.

2.1.2 Arbitrary transient-faults

We consider any temporary violation of the assumptions according to which the system was designed to operate. We refer to these violations and deviations as arbitrary transient-faults and assume that they can corrupt the system state arbitrarily (while keeping the program code intact). The occurrence of a transient fault is rare. Thus, we assume that the last arbitrary transient fault occurs before the system execution starts [15]. Also, it leaves the system to start in an arbitrary state.
Algorithm 1: ND and BRB; code for $p_i$.

1. operation $ndBroadcast(m)$ \[ $brbBroadcast(m)$ \] do broadcast INIT($m$);
2. upon INIT($mJ$) first arrival from $p_j$ do broadcast ECHO($j, mJ$);
3. upon ECHO($k, mJ$) arrival from $p_j$ begin
   4. if ECHO($k, mJ$) received from at least $(n+1)/2$ nodes \[ \land \text{READY}(k, mJ) \text{ not yet broadcast} \] then $ndDeliver(j, mJ)$;
4. broadcast READY($k, mJ$);
5. upon READY($k, mJ$) arrival from $p_j$ begin
   6. if READY($k, mJ$) received from $(t+1)$ nodes \[ \land \text{READY}(k, mJ) \text{ not yet broadcast} \] then broadcast READY($k, mJ$);
7. if READY($k, mJ$) received from at least $(2t+1)$ nodes \[ \land (k, mJ) \text{ not yet BRB-Delivered} \] then $brbDeliver(k, mJ)$;

2.2 Dijkstra’s self-stabilization

An algorithm is self-stabilizing with respect to LE, when every execution $R$ of the algorithm reaches within a finite period a suffix $R_{\text{legal}} \in LE$ that is legal. Namely, Dijkstra \[ [14] \] requires $\forall R : \exists R' : R = R' \circ R_{\text{legal}} \land R_{\text{legal}} \in LE \land |R'| \in \mathbb{Z}^+$, where the operator $\circ$ denotes that $R = R' \circ R''$ is the concatenation of $R'$ with $R''$.

2.3 Execution fairness

This work assumes execution fairness only during the period in which the system recovers from the occurrence of the last arbitrary transient fault. Given a step $a$, we say that $a$ is applicable to system state $c$ if there exists system state $c'$, such that $a$ leads to $c'$ from $c$. We say that a system execution is fair when every step of a correct node that is applicable infinitely often is executed infinitely often, and fair communication is kept.

3 Non-self-stabilizing Byzantine-tolerant Reliable Broadcast

Recall that the studied algorithm, BT \[ [13] \], is a BRB solution for BAMP$_{n,t}[\text{FC}, t < n/3]$. BT is based on a simpler communication abstraction called no-duplicity broadcast (ND-broadcast) by Toueg \[ [32, 31] \]. The ND-broadcast task includes all of the BRB requirements (Section 1.1) except BRB-Completion-2. We review BT after studying Toueg’s ND-broadcast algorithm.

3.1 No-Duplicity Broadcast

Algorithm \[ [1] \] brings Toueg’s solution for ND-broadcast \[ [32] \]. The boxed code lines 4 to 7 are irrelevant to the implementation of ND-broadcast but the struck through code (the declaration of the ND-broadcast operation at line \[ [1] \] and line \[ [4] \]‘s consequent clause of the if-statement) is part of the ND-broadcast implementation. Algorithm \[ [1] \] assumes that every correct node invokes ND-broadcast at most once. Node $p_i$ initiates the ND-broadcasts of $m_i$ by sending INIT($m_i$) to all nodes (line \[ [1] \]). Upon this message’s first arrival to node $p_j$, it disseminates the fact that $p_i$
has initiated $m$’s ND-broadcast by sending $\text{ECHO}(i, m)$ to all nodes (line 2). Upon this message arrival to $p_k$ from more than $(n+t)/2$ different nodes, $p_k$ is ready to ND-deliver $(i, m_i)$ (line 3).

### 3.2 Byzantine Reliable Broadcast

As explained, we present the BT solution for BRB as an extension of Toueg’s solution for ND-broadcast. Algorithm 1 satisfies the BRB requirements (Section 1.1) assuming $t < n/3$. The boxed code lines 4 to 7 are part of BT solution and the strike-through code (the consequent clause of the if-statement in line 4) is irrelevant.

The first difference between the ND and BRB algorithms is in the consequent clause of the if-statement in line 4, where ND-delivery of $(j, m_i)$ is replaced with the broadcast of $\text{READY}(j, m_i)$. This broadcast indicates that $p_i$ is ready to BRB-deliver $(j, m_i)$ as soon as it receives sufficient support, i.e., the arrival of $\text{READY}(j, m_i)$, which tells that correct nodes can BRB-deliver $(j, m_i)$. Note that BRB-no-duplicity protects Algorithm 1 from the case in which $p_i$ broadcasts $\text{READY}(j, m_i)$ while $p_j$ broadcasts $\text{READY}(j, m'_i)$, such that $m_i = m'_i$.

The new part of the BRB algorithm (lines 5 to 7) includes two if-statements. The first one (line 6) makes sure that every correct node receives $\text{READY}(j, m_i)$ from at least one correct node before BRB-delivering $(j, m_i)$. This is done via the broadcasting of $\text{READY}(j, m_i)$ as soon as $p_i$ received it from at least $(t+1)$ different nodes (since $t$ of them can be Byzantine).

The second if-statement (line 7) makes sure that no two correct nodes BRB-deliver different pairs (in the presence of plausibly fake $\text{READY}(j, -)$ messages sent by Byzantine nodes, where the symbol ‘-‘ stands for any legal value). That is, the delivery of a BRB-broadcast is done only after the first reception of the pair $(j, m)$ from at least $(2t+1)$ (out of which at most $t$ are Byzantine). The receiver then knows that there are at least $t+1$ correct nodes that can make sure that the condition in line 6 holds eventually for all correct nodes.

### 4 Self-stabilizing Byzantine-tolerant Single-instance BRB

Before proposing our solution (Section 4.2), we review the challenges that we face when transforming the non-self-stabilizing BT algorithm [13] into a self-stabilizing one (Section 4.1).

#### 4.1 Challenges and approaches

We analyze the behavior of the BT algorithm in the presence of transient-faults. We clarify that our analysis is relevant only in the context of self-stabilization since Bracha and Toueg do not consider transient-faults.

1. **Dealing with memory corruption and desynchronized system states**

Recall that transient faults can corrupt the system state in any manner (as long as the program code remains intact). For example, memory corruption can cause the local state to indicate that a certain message has already arrived (line 2) or that a certain broadcast was already performed (line 6). This means that some necessary messages will not be broadcast. This will result in an indefinite blocking. The proposed solution avoids such a situation by unifying all messages into a single $\text{MSG}(mJ)$, where the field $mJ$ includes all the fields of the messages of Algorithm 1.
4.1.2 Datagram-based end-to-end communications

Algorithm 1 assumes reliable communication channels when broadcasting in a quorum-based manner, i.e., sending the same message to all nodes and then waiting for a reply from $n-f$ nodes. Next, we explain why, for the sake of a simpler presentation, we choose not to follow this assumption. Self-stabilizing end-to-end communications require a known bound on the capacity of the communication channels [15, Chapter 3]. In the context of self-stabilization and quorum systems, we must avoid situations in which communicating in a quorum-based manner can lead to a contradiction with the system assumptions. Dolev, Petig, and Schiller [19] explain that there might be a subset of nodes that are able to complete many round-trips with a given sender, while other nodes merely accumulate messages in their communication channels. The channel bounded capacity implies that the system has to either block or omit messages before their delivery. Thus, the proposed solution does not assume access to reliable channels. Instead, communications are simply repeated by the algorithm’s do-forever loop.

4.2 Self-stabilizing Byzantine-tolerant single-instance solution

Algorithm 2 proposes our SSBRB solution for $\text{BAMP}_{n,t}$ if $FC, t < n/3$. Algorithm 2’s line numbers continue the ones of Algorithm 1. The boxed code fragments in lines 32 to 33 are irrelevant to our single-instance BRB implementation.

4.2.1 Types, constants, variables, and message structure

As mentioned, the message $MSG()$ unifies the messages of Algorithm 1. The array $msg[][]$ stores both the information that is sent and arrived by these messages. Specifically, $msg[i][]$ stores the information that node $p_i$ broadcasts (line 32) and for any $j \neq i$ the entry $msg[j][]$ stores the information coming from $p_j$ (lines 14 to 15). Also, we define the type $\text{brbMSG} := \{\text{init, echo, ready}\}$ (line 8) for storing information related to BRB-broadcast messages, e.g., $msg[i][\text{init}]$ stores the information that BRB-broadcast disseminates of INIT() messages and the results of the content of READY() messages appear in $msg[i][\text{ready}]$.

4.2.2 Algorithm details

The $\text{brbBroadcast}(v)$ operation (line 17) allows Algorithm 2 to invoke BRB-broadcast instances with $v$. Such an invocation causes Algorithm 2 to follow the logic of the BRB solution presented by Algorithm 1 in lines 18 and 27 to 31. We note that our solution also includes consistency tests at line 24.

4.2.3 Interfaces

Recall that Algorithm 2 has an interface to a recycling mechanism of BRB instances (Sec. 6). The interface between the proposed BRB and recycling mechanism includes the $\text{recycle}()$, $\text{txAvailable}()$, and $\text{rxAvailable}()$ operations, see Figure 1 (the interface between IRC and $\Diamond P_{mate}$ is irrelevant to Algorithm 2). The function $\text{recycle}_i(k)$ (line 11) lets the recycling mechanism locally reset $msg[i][]$. The notation $f_i()$ denotes that $p_i$ executes the function $f()$. For the single-instance BRB (without recycling), define $\text{txAvailable}()$ and $\text{rxAvailable}(k)$ (line 10) to return True. Note that we further integrate between BRB and IRC is via the piggybacking of their messages.
Algorithm 2: Self-stabilizing Byzantine-tolerant BRB with instance recycling interface; $p_i$’s code

8 types: $brbMSG := \{\text{init, echo, ready}\};$
9 variables: $msg[P][brbMSG] := [[0, \ldots, 0]]$ /* most recently sent/received message */
10 provided interfaces: $txAvailable()$ and $rxAvailable(k)$
11 required interfaces: $recycle(k)$ do \{msg[k] $\leftarrow$ $[0, 0, 0]$\};
12 $mrg(mJ, j)$ begin
13   foreach $s \in brbMSG, p_k \in P$ do
14      if $s \neq \text{init} \vee \exists s = \text{init}, (k, m), (k, m') \in (msg[j][s] \cup mJ[s]) : m \neq m'$ then
15         msg[j][s] $\leftarrow$ msg[j][s] $\cup$ mJ[s]
16 operations:
17   $brbBroadcast(v)$ do \{if $txAvailable()$ then $recycle(i);$ \; $msg[i][\text{init}]$ $\leftarrow$ $\{v\}$ \}
18   $brbDeliver(k)$ begin
19      if $\exists m : (2t+1) \leq |\{p_k \in P : (k, m) \in msg[l][\text{ready}]\}| \land rxAvailable(k)$ then
20         return $m$
21      else return $\bot$;
22 do-forever begin
23      if $\exists (j, m) \in msg[i][\text{echo}]$ : $m \notin msg[j][\text{init}] \vee \exists (j, m) \in msg[i][\text{ready}] : \neg(\frac{n+t}{2} < |\{p_k \in P : (j, m) \in msg[l][\text{ready}]\}| + t+1) \leq |\{p_k \in P : (j, m) \in msg[l][\text{ready}]\}|$ then
24         $recycle(i)$
25      foreach $p_k \in P$ do
26         if $|msg[k][\text{init}]| > 1 \vee \exists s \neq \text{init} : \exists p_j \in P : \exists (j, m), (j, m') \in msg[k][s] : m \neq m'$ then $msg[k][s] \leftarrow \emptyset$;
27         if $\exists m \in msg[k][\text{init}] : msg[i][\text{echo}] = \emptyset$ then $msg[i][\text{echo}] \leftarrow \{(k, m)\}$;
28         if $\exists m : (n+t)/2 < |\{p_k \in P : (k, m) \in msg[l][\text{echo}]\}|$ then
29            msg[i][\text{ready}] $\leftarrow$ msg[i][\text{ready}] $\cup$ \{(k, m)\}
30         if $\exists m : (t+1) \leq |\{p_k \in P : (k, m) \in msg[l][\text{ready}]\}|$ then
31            msg[i][\text{ready}] $\leftarrow$ msg[i][\text{ready}] $\cup$ \{(k, m)\}
32      broadcast $MSG(brbI = msg[i], ircI = txMSG());$
33 upon $MSG(brbJ, ircJ)$ arrival from $p_j$ do \{ $mrg(brbJ, j) \; brbMSG(brbJ, ircJ, j)$ \}

5 Correctness of Algorithm 2

Definition 5.1 defines the terms active nodes and consistent executions. Theorem 5.1 shows that consistency is regained eventually. Then, we provide a proof of completion (Theorem 5.2) before demonstrating the closure properties (Theorem 5.3). The closure proof (Section 5.3) shows that the proposed solution satisfies BRB task requirements (Definition 1.1). It is based on the assumption that BRB objects are eventually recycled after their task was completed (Section 1.1).
 definitions of consistency and active nodes of Algorithm 2. These definitions are used to prove the convergence of Algorithm 2.

**Definition 5.1 (Active nodes and consistent executions of Algorithm 2)** We use the term active for node \( p_i \in P \) when referring to the case of \( msg_i[i][\text{init}] \neq \emptyset \). Let \( R \) be an Algorithm 2’s execution, \( p_i, p_j \in P : i \in \text{Correct} \), and \( c \in R \). Suppose in \( c \):

- **(brb.i)** \( |msg_i[j][\text{init}]| \leq 1 \) and \( \exists t \in \{\text{echo, ready} \} \exists p_k \in P \exists (k,m'), (k,m') \in msg_i[j][t] \ m \neq m' \).
- **(brb.ii)** \( \forall (k,m) \in msg_i[i][\text{ready}] \left( n(t)/2 < |\{p_l \in P : (k,m) \in msg_i[l][\text{echo}]\}| \vee (t+1) \leq |\{p_l \in P : (k,m) \in msg_i[l][\text{ready}]\}| \right) \).
- **(brb.iii)** for any message \( MSG(brbJ = mJ, -) \) in transient from \( p_i \) to \( p_j \), it holds that for any \( p_k \in P \) and \( t \neq \text{init} \) there are no \( (k,m), (k,m') \in msg_i[j][t] \cup mJ[t] : m \neq m' \).

In this case, we say that \( c \) is consistent w.r.t. \( p_i \). Suppose every system state in \( R \) is consistent w.r.t. \( p_i \). In this case, we say that \( R \) is consistent w.r.t. \( p_i \) and Algorithm 2.

Note the term active (Definition 5.1) does not distinguish between the cases in which a node is active due to the occurrence of a transient fault and the invocation of \( \text{brbBroadcast}(v) \).

### 5.1 Consistency regaining for Algorithm 2

**Theorem 5.1 (Algorithm 2’s Convergence)** Let \( R \) be a fair execution of Algorithm 2 in which \( p_i \in P : i \in \text{Correct} \) is active eventually. The system eventually reaches a state \( c \in R \) that Starts a consistent execution w.r.t. \( p_i \) (Definition 5.1).

**Proof of Theorem 5.1** Suppose that \( R \)’s starting state is not consistent w.r.t. \( p_i \). Specifically, suppose that either invariant (brb.i) or (brb.ii) does not hold. I.e., at least one of the if-statement conditions in lines 24 and 26 holds. Since \( R \) is fair, eventually \( p_i \) takes a step that includes the execution of lines 24 to 26 which assures that \( p_i \) becomes consistent with respect to (brb.i) and (brb.ii). Observe that once invariant (brb.i) and (brb.ii) hold w.r.t. \( p_i \) in \( c \), they hold in any state \( c' \in R \) that follows \( c \), cf. lines 12 to 15 and 27 to 33.

Due to the above, the rest of the proof assumes, w.l.o.g., that all correct nodes are consistent w.r.t. \( p_i \), (brb.i), and (brb.ii) in any state of \( R \). Let \( m \) be a message that in \( R \)’s starting state
resides in a channel between a pair of correct nodes. Recall that $m$ can reside in that channel only for a finite time (Section 2.1.1). Thus, by the definition of complete iterations, the system reaches a state in which $m$ does not appear in the communication channels eventually. Thus, (brb.iii) holds eventually, since it is sufficient to consider only messages that were sent during $R$ from nodes in which (brb.i) and (brb.ii) hold.

$\Box$ Theorem 5.3

5.2 Completion of BRB-broadcast

Theorem 5.2 (BRB-Completion-1) Let typ $\in$ brbMSG and $R$ be a consistent execution of Algorithm 2 in which $p_i \in P$ is active. Eventually, $\forall i,j \in \text{Correct} : \text{brbDeliver}_j(i) \neq \bot$.

Proof of Theorem 5.2 Since $p_i$ is correct, it broadcasts $\text{MSG} (\text{brbJ} = \text{msg}_i[j],-) \text{ infinitely often. By fair communication, every correct } p_j \in P \text{ receives } \text{MSG} (\text{brbJ} = m,-) \text{ eventually. Thus, } \forall j \in \text{Correct} : \text{msg}_j[i][\text{init}] = \{ m \} \text{ due to line } 14 \text{. Also, } \forall j \in \text{Correct} : \text{msg}_j[j][\text{echo}] \supseteq \{(i,m)\} \text{ since node } p_j \text{ observes that the if-statement condition in line } 27 \text{ holds (for the case of } k_j = i \). Thus, $p_j$ broadcasts $\text{MSG} (\text{brbJ} = \text{msg}_j[j],-) \text{ infinitely often. By fair communication, every correct node } p_t \in P \text{ receives } \text{MSG} (\text{brbJ},-) \text{ eventually. Thus, } \forall j, \ell \in \text{Correct} : \text{msg}_j[j][\text{echo}] \supseteq \{(i,m)\} \text{ (line } 14 \text{). Since } n-t > n+t, \text{ node } p_t \text{ observes that } (n+t)/2 < \{|p_x \in P : (i,m) \in \text{msg}_x[x][\text{echo}]\} \text{ holds, i.e., the if-statement condition in line } 28 \text{ holds, and thus, } \text{msg}_t[\ell][\text{ready}] \supseteq \{(i,m)\} \text{ holds. Note that, since } t < (n+t)/2, \text{ faulty nodes cannot prevent a correct node from broadcasting } \text{MSG} (\text{brbJ} = \text{msg}_j[j],-) : mJ[\text{ready}] \supseteq \{(i,m)\} \text{ infinitely often, say, by colluding and sending } \text{MSG} (\text{brbJ} = \text{msg}_j[j],-) : mJ[\text{ready}] \supseteq \{(i,m')\} \wedge m' \neq m \text{. By fair communication, every correct } p_y \in P \text{ receives } \text{MSG} (\text{brbJ} = mJ,-) \text{ eventually. Thus, } \forall j, y \in \text{Correct} : \text{msg}_y[j][\text{ready}] \supseteq \{(i,m)\} \text{ holds (line } 14 \text{). Therefore, whenever } p_y \text{ invokes } \text{brbDeliver}_y(i) \text{ (line } 18 \text{), the condition } \exists m(2t+1) \leq \{|p_t \in P : (k_y = i,m) \in \text{msg}_y[\ell][\text{ready}]\} \text{ holds, and thus, } m \text{ is returned.}

$\Box$ Theorem 5.2

5.3 Closure of BRB-broadcast

The main difference between the completion and the closure proofs is that the latter considers post-recycled starting system states and complete invocation of operations (Definition 5.2).

Definition 5.2 (Post-recycle system states and complete invocation of operations)

We say that system state $c$ is post-recycle w.r.t. $p_i \in P : i \in \text{Correct}$ if $\forall j \in \text{Correct} : \text{msg}_j[i] = [\emptyset, \ldots, \emptyset] \text{ holds and no communication channel from } p_i \text{ to } p_j \text{ includes } \text{MSG} (\text{brbJ} \neq [\emptyset, \ldots, -]). \text{ Suppose that execution } R \text{ starts in the post-recycled system state } c \text{ and } p_i \text{ invokes } \text{brbBroadcast}_i(v) \text{ exactly once. In this case, we say that } R \text{ includes a complete BRB invocation w.r.t. } p_i.$

Note that a post-recycled system state (Definition 5.2) is also a consistent one (Definition 5.1).

Theorem 5.3 (BRB closure) Let $R$ be a post-recycled execution of Algorithm 2 in which all correct nodes are active eventually via the complete invocation of BRB-broadcast. The system demonstrates in $R$ a BRB construction.

Proof of Theorem 5.3 BRB-Completion-1 holds (Theorem 5.2).

Lemma 5.4 (BRB-Completion-2) BRB-Completion-2 holds.
Proof of Lemma 5.4 By line 18, $p_i$ can BRB-deliver $m$ from $p_j$ only once $\exists m(2t+1) \leq \{p \in \mathcal{P} : (k, m) \in \text{msg}_k[t][\text{ready}]\}$ holds. During post-recycled execution, only lines 29 to 31 and 15 can add items to $\text{msg}_k[t][\text{ready}]$ and $\text{msg}_k[t'][\text{ready}]$, respectively. Let $\text{msg}(mJ)$ be such that $mJ[\text{ready}] \supseteq \{(j, m)\}$. Specifically, line 15 adds to $\text{msg}_k[t][\text{ready}]$ items according to information in $\text{msg}(mJ)$ messages coming from $p_k$. This means, that at least $t+1$ distinct and correct nodes broadcast $\text{msg}(mJ)$ infinitely often. By fair communication and line 15, all correct nodes, $p_x$, eventually receive $\text{MSG}(mJ)$ from at least $t+1$ distinct nodes and make sure that $\text{msg}_k[t'][\text{ready}]$ includes $(j, m)$. Also, by line 31, we know that $\text{msg}_k[x][\text{ready}] \supseteq \{(j, m)\}$, i.e., every correct node broadcasts $\text{msg}(mJ)$ infinitely often. By fair communication and line 15 all correct nodes, $p_x$, receive $\text{MSG}(mJ)$ from at least $t+1$ distinct nodes eventually, because there are at least $n-t \geq 2t+1$ correct nodes. This implies that $\exists m(2t+1) \leq \{p \in \mathcal{P} : (k, m) \in \text{msg}_k[t][\text{ready}]\}$ holds (due to line 15). Hence, $\forall i \in \text{Correct} : \text{brbDeliver}_i(j) \notin \{\bot, \Psi\}$. \hfill $\square$ Lemma 5.5

Lemma 5.5 The BRB-integrity property holds.

Proof of Lemma 5.5 Suppose $\text{brbDeliver}(k) = m \neq \bot$ holds in $c \in R$. Also, (towards a contradiction) $\text{brbDeliver}(k) = m' \notin \{\bot, m\}$ holds in $c' \in R$, where $c'$ appears after $c$ in $R$. I.e., $\exists m(2t+1) \leq \{p \in \mathcal{P} : (k, m) \in \text{msg}_k[t][\text{ready}]\}$ in $c$ and $\exists m(2t+1) \leq \{p \in \mathcal{P} : (k, m') \in \text{msg}_k[t'][\text{ready}]\}$ in $c'$. For any $i, j, k \in \text{Correct}$ and any $typ \in \text{brbMSG}$ it holds that $(k, m), (k, m') \in \text{msg}_k[j][\text{ready}]$ (since $R$ is post-recycle, and thus, consistent). Thus, $m = m'$, cf. invariant (brb.iii). Also, observe from the code of Algorithm 2 that no element is removed from any entry $\text{msg}[][\text{ready}]$ during consistent executions. This means that $\text{msg}_k[t'][\text{ready}]$ includes both $(k, m)$ and $(k, m')$ in $c'$. However, this contradicts the fact that $c'$ is consistent. Thus, $c' \in R$ cannot exist and BRB-integrity holds. \hfill $\square$ Lemma 5.6

Lemma 5.6 (BRB-validity) BRB-validity holds.

Proof of Lemma 5.6 Let $p_i, p_j : i, j \in \text{Correct}$. Suppose that $p_j$ BRB-delivers message $m$ from $p_i$. The proof needs to show that $p_i$ BRB-broadcasts $m$. In other words, suppose that the adversary, who can capture up to $t$ (Byzantine) nodes, sends the “fake” messages of $\text{msg}_j[j][\text{echo}] \supseteq \{(i, m)\}$ or $\text{msg}_j[j][\text{ready}] \supseteq \{(i, m)\}$, but $p_i$, who is correct, never invoked $\text{brbBroadcast}(m)$. In this case, our proof shows that no correct node BRB-delivers $(i, m)$. This is because there are at most $t$ nodes that can broadcast “fake” messages. Thus, $\text{brbDeliver}(k)$ (line 18) cannot deliver $(i, m)$ since $t < 2t+1$, which means that the if-statement condition $\exists m(2t+1) \leq \{p \in \mathcal{P} : (k, m) \in \text{msg}_k[t][\text{ready}]\}$ cannot be satisfied. \hfill $\square$ Lemma 5.7

Lemma 5.7 (BRB-no-duplication) Suppose $p_i, p_j : i, j \in \text{Correct}$, $\text{BRB-broadcast MSG}(mJ) : mJ[\text{ready}] \supseteq \{(k, m)\}$, and $\text{resp.}, \text{MSG}(mJ) : mJ[\text{echo}] \supseteq \{(k, m')\}$. We have $m = m'$.

Proof of Lemma 5.7 Since $R$ is post-recycle, there must be a step in $R$ in which the element $(k,-)$ is added to $\text{msg}_x[x][\text{ready}]$ for the first time during $R$, where $p_x \in \{p_i, p_j\}$. The correctness proof considers the following two cases.

- Both $p_i$ and $p_j$ add $(k,-)$ due to line 29. Suppose, towards a contradiction, that $m \neq m'$. Since the if-statement condition in line 29 holds for both $p_i$ and $p_j$, we know that $\exists m(n-t)/2 < \{p \in \mathcal{P} : (k, m) \in \text{msg}_k[t'][\text{echo}]\}$ and $\exists m'(n-t)/2 < \{p \in \mathcal{P} : (k, m') \in \text{msg}_k[t'][\text{echo}]\}$. Since $R$ is post-recycle, this can only happen if $p_i$ and $p_j$ received $\text{MSG}(mJ) : mJ[\text{echo}] \supseteq \{(k, m)\}$, and $\text{resp.}, \text{MSG}(mJ) : mJ[\text{echo}] \supseteq \{(k, m')\}$ from $(n+t)/2$ distinct nodes. Note that $\exists p_x \in Q_1 \cap Q_2 : x \in \text{Correct}$, where $Q_1, Q_2 \subseteq \mathcal{P} : |Q_1|, |Q_2| \geq 2$. This contradicts the fact that $Q_1 = \mathcal{P}$. Hence, $m = m'$. 

- Both $p_i$ and $p_j$ add $(k,-)$ due to line 15. Suppose, towards a contradiction, that $m \neq m'$. Since the if-statement condition in line 15 holds for both $p_i$ and $p_j$, we know that $\exists m(n-t)/2 < \{p \in \mathcal{P} : (k, m) \in \text{msg}_k[t'][\text{ready}]\}$ and $\exists m'(n-t)/2 < \{p \in \mathcal{P} : (k, m') \in \text{msg}_k[t'][\text{ready}]\}$. Since $R$ is post-recycle, this can only happen if $p_i$ and $p_j$ received $\text{MSG}(mJ) : mJ[\text{echo}] \supseteq \{(k, m)\}$, and $\text{resp.}, \text{MSG}(mJ) : mJ[\text{echo}] \supseteq \{(k, m')\}$ from $(n+t)/2$ distinct nodes. Note that $\exists p_x \in Q_1 \cap Q_2 : x \in \text{Correct}$, where $Q_1, Q_2 \subseteq \mathcal{P} : |Q_1|, |Q_2| \geq 2$. This contradicts the fact that $Q_1 = \mathcal{P}$. Hence, $m = m'$. 


1+ (n+t)/2 (as in [31], item (c) of Lemma 3). But, any correct node, $p_x$, has at most one element in $msg_x[f][\text{echo}]$ (line 27) during $R$. Thus, $m = m'$, which contradicts the case assumption.

• **There is $p_x \in \{p_i, p_j\}$ that adds $(k, -)$ due to line 31**. I.e., $\exists_{n', t+1} \equiv \{ p_x \in P : (k,m') \in msg[f][\text{ready}] \} \land m'' \in \{m, m'\}$. Since there are at most $t$ faulty nodes, $p_x$ received MSG($mJ$) : $mJ[\text{ready}] \geq \{(k,m'')\}$ from at least one correct node, say $p_{x2}$, which received MSG($mJ$) : $mJ[\text{ready}] \geq \{(k,m'')\}$ from $p_{x2}$, and so on. This chain cannot be longer than $n$ and it must be originated by the previous case in which $(k,-)$ is added due to line 29. Thus, $m = m'$.

\[\text{Lemma 57} \quad \Box\]

\[\text{Theorem 63} \quad \Box\]

6 Self-stabilizing Recycling in Time-free Message-passing Systems

Before proposing our self-stabilizing Byzantine-tolerant algorithm for BRB-instance recycling (Section 7), we study a non-crash-tolerant yet self-stabilizing recycling algorithm for time-free systems. Namely, as steppingstones towards a solution for BAMP$_{n,t}$, we present the independent round counter (IRC) task and implement txAvailable() and rxAvailable($k$) (Figure 1 and Algorithm 2).

6.1 Independent Round Counters (IRCs)

We consider $n$ independent counters, such that each counter, $cnt_i$, can be incremented only by a unique node, $p_i \in P$, via the innovation of the increment($i$) operation, which returns the new round number or $\perp$ when the invocation is (temporarily) disabled. Suppose $p_i, p_j \in P$ are correct. Every node $p_j \in P$ can fetch $cnt_i$’s value via the invocation of the fetch($j$) operation, which returns the most recent and non-fetched $cnt_i$’s value or $\perp$ when such value is currently unavailable. We define the Independent Round Counters (IRCs) task using the following requirements.

• **IRC-validity.** Suppose $p_j$ IRC-fetches $s$ from $cnt_i$. Then, $p_i$ had IRC-incremented $cnt_i$ to $s$.

• **IRC-integrity-1.** Let $S_{i,j} = (s_0, \ldots, s_x) : x < B$ be a sequence of $p_i$’s round numbers that $p_j$ fetched—we are only interested in $B$ most recent ones, where $B$ is a predefined constant. It holds that $\forall s_y \in S_{i,j} : y < B - 1 \implies s_y + 1 \mod B = s_{y+1}$. In other words, no correct node IRC-fetches a value more than once from the counter of any other correct node (considering the $B$ most recent IRC-fetches).

• **IRC-integrity-2.** Correct nodes that IRC-fetch numbers from $cnt_i$ do so in the order in which $cnt_i$ was incremented (considering the $B$ most recent IRC-fetches).

• **IRC-preemption.** Suppose $p_i$ IRC-increments $cnt_i$ to $s$. IRC-increment is (temporarily) disabled until all correct nodes have fetched $s$ from $p_i$’s counter.

• **IRC-completion.** Suppose all correct nodes, $p_j$, IRC-fetch $p_i$’s counter infinity often. Node $p_i$’s IRC-increment is enabled infinity often.

Note that any algorithm that solve the IRC task can implement the interface functions txAvailable() and rxAvailable($k$) by returning increment() $\neq$ $\perp$ and fetch($k$) $\neq$ $\perp$, respectively.
6.2 The Studied Time-free Message-passing Systems

Consider a scenario in which, due to a transient fault, \( p_i \)'s copy of its round counter is smaller than \( p_j \)'s copy of \( p_i \)'s counter, say, by \( x \in \mathbb{Z}^+ \). In this case, \( p_i \) will have to complete \( x \) rounds before \( p_j \) could IRC-fetch a non-\( \perp \) value. The proposed IRC algorithm overcomes this challenge by following Assumption 6.1.

Assumption 6.1 (Bounded message lifetime, BML) Let \( R \) be an execution in which there is a correct node \( p_i \in \mathcal{P} \) that repeatedly broadcasts the protocol messages and completes an unbounded number of round-trips with every correct node, \( p_j \in \mathcal{P} \), in the system. Suppose that \( p_j \) receives message \( m(s) \) from \( p_i \) immediately before system state \( c \in R \), where \( s \in \mathbb{Z}^+ \) is the round number. We assume that \( \text{cur}_i[i] - s \leq \lambda \) in \( c \), where \( \lambda \in \mathbb{Z}^+ : \text{channelCapacity} < \lambda < B/6 \) is a known upper-bound, \text{channelCapacity} is defined in Section 3 and \( B \) is defined by line 34.

6.3 Self-stabilizing IRC for Time-free Message-passing Systems

Algorithm 3 presents a non-crash-tolerant self-stabilizing solution for message-passing systems. I.e., it assumes that all nodes are correct. Algorithm 3 makes sure that any node that had IRC-incremented its round counter defers any further IRC-increments until all nodes have acknowledged the latest IRC-increment. Note that the line numbers of Algorithm 3 continue the ones of Algorithm 2. Also, the boxed code in lines 46 and 53 are irrelevant to the IRC solution studied in this section. We remind that the implementation of interface function recycle() (line 39) is provided by Algorithm 2 line 11. Also, for this section, let us assume that trusted\(_i() = \mathcal{P} \).

6.3.1 Constants and variables

All integers used by Algorithm 3 have a maximum value, which we denote by \( B \) (line 34) and require to be large, say, \( 2^{64} - 1 \). The arrays \( \text{cur}[] \) and \( \text{nxt}[] \) (line 36) store a pair of round numbers. The entry \( \text{cur}[i] \) is \( p_i \)'s current round number and \( \text{nxt}[i] \) is the next one. Also, \( \text{cur}[j] \) and \( \text{nxt}[j] \) store the most recently received, and respectively, delivered round numbers from \( p_j \). The array \( \text{lbl}[] \) holds labels that correspond to the number in \( \text{cur}[i] \), where \( \text{lbl}[j] \) is the most recently received label from \( p_j \) (line 37).

6.3.2 The increment() operation

This operation allows the caller to IRC-increment the value of its round number module \( B \). It also returns the new round number. However, if the previous invocation has not finished, the operation is disabled and the \( \perp \) value is returned. Line 45 tests whether the round number is ready to be incremented. In detail, recall that in this section, we assume trusted\(_i() = \mathcal{P} \). Now line 45 checks whether this is the first round, i.e., a round number of \(-1\), or the previous round has finished, i.e., the labels indicate that every node has completed at least \( 2(\text{channelCapacity}+1) \) round trip. By exchanging at least \( 2(\text{channelCapacity}+1) \) labels, the proposed solution overcomes packet loss and duplication over non-FIFO channels, see 17 for a more efficient variation on this technique.

6.3.3 The fetch\(_k() \) operation

This operation returns, exactly once, the most recently received round number. Line 45 tests whether a new round number has arrived. If this is not the case, then \( \perp \) is returned. Otherwise,
the value of the new round number is returned (line 49). In detail, due to Assumption 6.1 immediately after the arrival of message \( m(s) \) to \( p_j \) from \( p_i \), the fact that \( s \notin \{ x \mod B : x \in \{ c - 1, \ldots, c \} \} \) holds implies that \( s \) is new than \( cur_j[i] \). Thus, \( p_i \) can use \( \text{behind}(1, cur_j[i], nxt_j[i]) \) (line 48) for testing the freshness of the round number stored in \( cur_j[i] \) w.r.t. \( nxt_j[i] \). If case the number is indeed fresh, \( \text{fetch}_i() \) updates \( nxt_i[k] \) with the returned round number.
6.3.4 The $txMSG$ and $rxMSG$ operations

The operations $txMSG$ and $rxMSG$ let the sender, and respectively the receiver, process messages. Algorithm 3 sends via the message $MSG()$ two fields: $brb_J$ and $irc_J$. The field $irc_J$ is related to Algorithm 2. Recall that when a message arrives from $p_j$, the receiving-side adds the suffix $J$ to the field name, i.e., $brb_J$ and $irc_J$. The field $irc_J$ is composed of the fields $ack$, which indicates whether acknowledge is required, seq, which is the sender’s round number, and $lbl$, which, during legal executions, is the corresponding label to seq that the sender uses for the receiver.

The operation $txMSG()$ is used when the sender transmits a message (line 50). It specifies that acknowledgment is required, i.e., $ack = True$ as well as includes the sender’s current round number, i.e., $cur[i]$, and the corresponding label that the sender uses for the receiver $p_j \in P$, i.e., $lbl[j]$.

The operation $rxMSG()$ processes messages arriving either to the sender or the receiver. On the sender-side, when an acknowledgment arrives from receiver, $p_j$, the sender checks whether the arriving message has a fresh round number and label (line 52). In this case, the label is incremented in order to indicate that at least one round trip was completed. In detail, $p_i$ uses $behind_i(2, cur[j], sJ)$ for testing whether the arriving round number, $sJ$, is fresh by asking whether $sJ$ is not a member of the set $\{cur[i] - 2\lambda, \ldots, cur[i]\}$, see Assumption 6.1. As we will see in the next paragraph, there is a need to take into account the receiver’s test (line 54), which can cause a non-fresh value to be a member of the set $\{x \mod B : x \in \{c - 2\lambda, \ldots, c\}\}$, but not the set $\{x \mod B : x \in \{c - \lambda, \ldots, c\}\}$.

On the receiver-side, $p_i$ uses $behind_i(1, sJ, cur[j])$ to test whether a new round number arrived, i.e., testing whether the arriving number, $sJ$, is a member of $\{cur[j] - 2\lambda, \ldots, cur[j]\}$. In this case, the local round number is updated (line 55) and the interface function $recycle_i(j)$ is called (line 11). Note that whenever the receiver gets a message, it replies (line 56). That acknowledgment specifies that no further replies are required, i.e., $ack = False$, as well as the most recently delivered round number, i.e., $nxt[i]$, and label, $\ell J$.

6.3.5 The do forever loop and message arrival

Note that the processing of messages (for sending and receiving) is along the lines of Algorithm 2. The do forever loop broadcasts the message $MSG()$ to every node in the system (line 57). The operation $txMSG()$ is used for setting the value of the $ircJ$ field. Upon message arrival, the receiver passes the arriving values to $rxMSG()$ for processing (line 58).

6.4 Correctness of Algorithm 3

The proof is implied by Theorem 6.2.

Theorem 6.2 Let $R$ be an Algorithm 3’s execution and $i \in Correct$. Suppose all correct nodes, $p_j$, IRC-fetch $p_i$’s counter infinity often and $p_i$ invokes IRC-increment infinity often. $R$ eventually demonstrates an IRC construction (Sec. 6.1).

Proof of Theorem 6.2

Lemma 6.3 The system demonstrates IRC-completion in $R$ (Section 6.1).

Proof of Lemma 6.3 Recall that $p_i$’s IRC-increment is enabled whenever $increment_i()$ can return a non-$\bot$ value (line 15), where $p_i$ is a correct node. Also, the return of a non-$\bot$ value implies that the value of $cur[i]$ changes (line 16). Thus, towards a contradiction, assume
\(\text{cur}_i[i] = s \geq 0\) holds in every system state of \(R\). The following arguments show the contradiction by demonstrating that, for any correct node \(p_j\), the if-statement condition in line 52 holds eventually for any \(p_j\)'s reply \(\text{MSG}(-,(\text{False}, \bullet))\) arriving to \(p_i\). Note that once \(p_i\) executes line 53 at least once for every \(p_j\), \(\text{increment}_i()\) is enabled since the if-statement in line 45 does not hold. To show that the predicate \(\text{behind}_j(2, \text{cur}_i[i], sJ)\) holds, we note that \(p_i\) is a correct node that broadcasts \(\text{MSG}(-,(\text{True}, \text{cur}_i[i] = s, -))\) infinitely often (line 52). Thus, every correct node \(p_j\) receives \(\text{MSG}(-,(\text{True}, \text{cur}_i[i] = s, -))\) infinitely often (due to the communication fairness assumption). In the system state that immediately follows this message arrival (line 53), the if-statement condition in line 45 holds, \(i.e., \text{behind}_j(1, s, \text{cur}_i[i])\) holds. By the assumption that \(p_j\) invokes \(\text{fetch}_i(j)\) infinitely often, we know that the if-statement condition in line 48 eventually holds. \(\text{i.e., behind}_j(1, \text{cur}_i[i], s')\) and \(\text{behind}_j(2, s, s')\) hold, where \(\text{nxt}_j[i] = s'\) is the value used when \(p_j\) sends \(\text{MSG}(-,(\text{False}, \text{nxt}_j[i] = s', -))\) to \(p_i\). Thus, once \(\text{MSG}(-,(\text{False}, sJ = s', -))\) arrives to \(p_i\) the predicate \(\text{behind}_j(2, \text{cur}_i[i] = s, sJ = s')\) holds. The proof of \(\text{lbl}_i[j] = \ell J\) is by fixing the value of \(\text{lbl}_i[j] = \ell\) and observing that the values of the messages \(\text{MSG}(-,(\text{True}, \text{cur}_i[i] = s, \ell))\) from \(p_i\) to \(p_j\) and \(\text{MSG}(-,(\text{False}, \text{cur}_i[i] = s, \ell))\) from \(p_j\) to \(p_i\) \(\Box\).

**Lemma 6.4** Eventually, the system demonstrates IRC-validity in \(R\) (Section 6.1).

**Proof of Lemma 6.4** W.l.o.g. suppose \(R\) is the suffix of execution \(R' = R'' \circ R\), such that \(\text{increment}_i()\) returns a non-\(\bot\) value more than \(2(\lambda + 1)\) times during \(R''\). We show that IRC-validity holds in \(R\). \(\text{i.e., suppose a correct node} \ p_j \ \text{IRC-fetches} \ s \ \text{in step} \ a_j \in R \ \text{from} \ p_i\)'s counter. We show that \(p_i\) had IRC-incremented \(\text{cnt}_i\) to \(s\) in step \(a_i\) that appears in \(R\) before \(a_j\). Suppose, towards a contradiction, \(\exists a_i \in R'\), yet \(a_j\) returns \(s \neq \bot\) from \(\text{fetch}_i(j)\) when executing line 49.

The starting system state of \(R'\), the \(\text{irc}_i.\ell J\) field of the messages in communication channels between \(p_i\) and \(p_j\) and the variables \(\text{lbl}_i[j]\) and \(\ell J\) include at most \(2(\text{channelCapacity} + 1)\) different labels. Since \(p_i\) does not change \(\text{cnt}_i[i]\) before counting the reception of more than \(2(\text{channelCapacity} + 1)\) labels (line 45), during the period in which \(\text{increment}_i()\) returns non-\(\bot\) values at least twice, the messages in channels between \(p_i\) and \(p_j\) and \(p_j\)'s variables do not include values that have not changed since the starting system state of \(R'\). Thus, \(p_i\) completes an unbounded number of round-trip with \(p_j\) with values that \(p_i\) indeed sent.

Recall that \(a_j\) returns \(s \neq \bot\) from \(\text{fetch}_i(j)\) when executing line 19. This can only happen when \(\text{cur}_j[i] = s \neq \bot\). Node \(p_j\) assigns \(s\) to \(\text{cur}_j[i] : i \neq j\) only in line 53 when it processes a message coming from the sender \(p_i\). However, \(p_i\) can assign \(s\) to \(\text{cur}_j[i]\) only at line 46 \(\text{i.e.,} a_i\) exists. We clarify the last argument: by \(\text{behind}()\)'s definition (line 43), there could be at most \(2\lambda\) consecutive times in which \(\text{behind}(2, \bullet)\) holds in the if-statement condition in line 42 and yet, \(\text{cur}_j[i]\) has not changed while \(\text{cur}_i[i]\) has. \(\Box\)

**Lemma 6.5** Eventually, the system demonstrates in \(R\) an IRC construction (Section 6.1).

**Proof of Lemma 6.5** Recall that lemmas 6.3 and 6.4 demonstrate IRC-completion and IRC-validity. Thus, w.l.o.g. we can assume that IRC-validity holds throughout \(R\).

**IRC-preemption.** Suppose that there is a correct node, \(p_k \in P\), that does not IRC-fetch \(s\) from \(p_i\)'s round counter during \(R\) even after \(a_i\) (in which \(p_i\) IRC-increment \(\text{cnt}_i\) to \(s\)). Also, let \(a'_i\) be a step that appears in \(R\) after \(a_i\) and includes an IRC-increment invocation by \(p_i\). We show that \(a'_i\)'s invocation returns \(\bot\), \(i.e., \ a'_i\)'s invocation is disabled.

By the code of Algorithm 3, the fact that there is no step in \(R\) in which \(\text{fetch}_k(i)\) returns \(s\) implies that \(\text{nxt}_k[i] \neq s\) in any system state during \(R\) (line 49). Therefore, \(p_k\) does not send \(\text{MSG}((\text{False}, \text{nxt}(j) = s), \cdots)\) to \(p_i\). This means that, as long as \(\text{seq}_i[i] = s\), it holds that
Also, as long as \( \text{seq}[i] = s \), whenever \( p_i \) invokes \( \text{increment}_i() \), the if-statement condition in line \( 45 \) holds. Thus, \( \text{increment}_i() \) returns \( \bot \) in step \( a'_i \).

**IRC-integrity-1.** Lines \( 48 \) to \( 49 \) implies that no correct node, \( p_i \), can IRC-fetch the same value twice from the counter of the same node, say, \( p_j \).

**IRC-integrity-2.** Suppose \( p_j \) IRC-fetches \( s' \) from \( p_i \)'s counter in step \( a'_j \) that appears in \( R \) after \( a_j \) (in which \( p_j \) IRC-fetches \( s' \)). We show that \( p_i \) IRC-incremented \( \text{cnt}_i \) to \( s \) and then to \( s' \). Step \( a'_i \) appears in \( R \) after \( a_j \) (IRC-preemption) and \( a_j \) after \( a_i \) (IRC-validity and line \( 46 \)).

Note that \( s \neq s' \), (IRC-integrity-1) i.e., \( a_i \neq a'_i \). By line \( 46 \) \( s \) was IRC-incremented before \( s' \) when considering the \( B \) IRC-increments preceding \( a'_i \).

**Lemma 6.5**

**Theorem 6.2**

### 7 Self-stabilizing Byzantine-Tolerance via Muteness Detection

Algorithm \ref{alg:byzantine} presents our self-stabilizing Byzantine-tolerant recycling mechanism for the model of BAMP\(_{n,t}^{FC,P_{mute},BML}\). The proposed solution includes the boxed code lines. It uses a muteness detector, which we present in this section. Algorithm \ref{alg:byzantine} lets \( p_i \) to restart the local state of the muteness detector via a call to \( \text{invoc}_i() \) (line \( 46 \)). The algorithm uses \( \text{rtComp}_i(j) \) (line \( 53 \)) for taking into account the completion of a round-trip between \( p_i \) and \( p_j \). The correctness proof shows (Theorem \ref{thm:byzantine}) that this version of the algorithm can consider trusted\(_i() \subseteq \mathcal{P} \) due to the \( P_{mute} \) properties (Section \ref{sec:muteness-detection}).

#### 7.1 Muteness Failures

Let us consider an algorithm, \( \text{Alg} \), that attaches a round number, \( \text{seq} \in \mathbb{Z}^+ \), to every message, \( m(\text{seq}) \) that it sends. Suppose there is a system state \( c_r \in R \) after which \( p_j \), stops forever replying to \( p_i \)'s messages, \( m(\text{seq}) \), where \( p_i, p_j \in \mathcal{P} \). In this case, we say that \( p_j \) is mute to \( p_i \) with respect to message \( m(\text{seq}) \). We clarify that a Byzantine node is not mute if it forever sends all the messages required by \( \text{Alg} \). For the sake of a simple presentation, we assume that the syntax of \( m(\text{seq}) \) corresponds to the syntax of a message generated by \( \text{Alg} \) (since, otherwise, the receiver may simply omit messages with syntax errors). Naturally, the data load of those messages can be wrong. Observe that the set of mute nodes also includes all crashed nodes.

#### 7.2 Muteness Detection: Specifications of \( P_{mute} \)

We deal with mute nodes via the use of the class \( P_{mute} \) of muteness detectors. In the context of self-stabilization, one has to consider the scenario in which the muteness detector suspects a node due to a transient fault. Thus, the muteness detector has to be restarted from time to time. In this work takes the approach in which one restart occurs at the start of a new round.

**Muteness Strong Completeness:** Eventually, every mute node is forever suspected w.r.t. round number \( s \) by every correct node (or the round number changes).

**Eventual Strong Accuracy:** Eventually, the system reaches a state \( c_r \in R \) in which no correct node is suspected.

#### 7.3 Muteness Detection: Related approaches

In the context of self-stabilizing Byzantine-free systems, Beauquier and Kekkonen-Moneta \cite{bamp} and Blanchard et al. \cite{byzantine} implemented perfect failure detectors, i.e., class \( P \), by letting node \( p_i \) to suspect any node \( p_j \in \mathcal{P} \) whenever \( p_i \) was able to complete \( \Theta \) round-trips with other nodes in \( \mathcal{P} \) but not with \( p_j \), where \( \Theta \) is a predefined constant.
Since the studied fault model includes Byzantine failures, we cannot directly borrow earlier proposals, such as the ones in [3, 6]. Consider, for example, a Byzantine node that anticipates the sender’s messages and transmits acknowledgments before the arrival of perceptive messages. Using this attack of speculative acknowledgments, the adversary may accelerate the (false) completion round-trips and let the unreliable failure-detector suspect non-faulty nodes.

7.3.1 Muteness Detection: Implementation

As shown in Figure 1, Algorithm 4 does not send independent messages as it merely provides three interface functions to Algorithm 3. i.e., invoc(), rtComp(j), and trusted(). The algorithm’s state is based on the array rt[i][j][k] (line 62), which stores the number of round trips that node $p_i$ has completed with $p_j$. Note that $rt[i][j]$ lets counting separately the number of round-trips $p_i$ and $p_k$ are able to complete during any period in which $p_i$ and $p_j$ are attempting to complete a single round-trip. The function invoc() (line 64) nullifies the value of $rt[i][j][k]$. We require that, every time $p_i$ has completed with $p_j$, it calls the rtComp(j) (line 65). This function increments, for every $p_k \in \mathcal{P} \setminus \{p_i, p_j\}$, the counter in $rt_i[k][j]$. Then, rtComp(j) assigns zero to every entry in $rt_i[j]$. The function trusted() returns the set of unsuspected nodes. Its implementation relies on Assumption 7.1 which answers to the above challenge (Section 7.3). As a defense against the above attacks that use speculative acknowledgment, $p_i$ ignores the top $t$ round-trip counters when testing whether the $\Theta$ threshold has exceeded. The correctness proof of Algorithm 4 appears in Theorem 7.2.

**Assumption 7.1** Let $R$ be an execution in which there is a correct node $p_i \in \mathcal{P}$ that repeatedly broadcasts the protocol message $m(s) : s \in \mathbb{Z}^+$ and completes an unbounded number of round-trips of message $m(s)$ with every correct node in the system. Let $rt_{i,c} : \mathcal{P} \times \mathcal{P} \rightarrow \mathbb{Z}^+$ be a function that maps any pair of nodes $p_j, p_k \in \mathcal{P}$ with the number of round-trips that $p_i$ has completed with $p_k$ between system states $c' \in R$ and $c \in R$, where $c'$ is the first system state that immediately follows the last time $p_i$ has completed the last round-trip with $p_j$, or the start of $R$ (in case $p_i$ has not completed any round trip with $p_j$ between $R$’s start and $c$). Let $\sum_{x \in \text{withoutTopItems}_{i,c}(t,j)} x$ be the total number of round trips that $p_i$ has completed until $c$ when excluding the top $t$ nodes that have completed with $p_i$ the greatest number of round-trips. We assume that if $\Theta \leq \sum_{x \in \text{withoutTopItems}_{i,c}(t,j)} x$ then $p_j$ is mute to $p_i$, where $\Theta$ is predefined.
Theorem 7.2 Let $R$ be a legal execution of algorithms 3 and 4 that satisfies Assumption 7.1. The system demonstrates in $R$ a construction of class $\Diamond_{\text{P mute}}$ muteness detector (Section 7.2).

Proof of Theorem 7.2 Let us consider the sequence of values of $rt_i[j][k]$ in the different system states $c \in R$. Note that this sequence is defined by the function $rt_{i,c}(k,j)$ (Assumption 7.1). Thus, by line 66 we know that $j \in \text{trusted}_i()$ if, and only if, $\Theta \leq \sum_{x \in \text{withoutTopItems}_i(t,j)} x$.

Let $a_i \in R$ be a step in which $p_i$ invokes $\text{increment}_i()$ and thus calls $\text{invoc}_i()$ (line 46). We demonstrate that the $\Diamond_{\text{P mute}}$ class properties hold (Section 7.2).

Muteness strong completeness: We show that, eventually, every mute node, $p_m \in \mathcal{P}$, is forever suspected w.r.t. round number $s$ by every correct node (or the round number is not $s$). Suppose that the round number is always $s$. By the proof of Lemma 6.3, $p_i$ will call $\text{rtComp}_i(j)$ infinitely often (line 53). I.e., for every correct node $p_k \in \mathcal{P} \setminus \{p_i, p_j\}$, the value of $rt_i[k][j]$ will reach the upper bound $B$ eventually. Since $B(n/3) > \Theta$, eventually $p_j \notin \text{trusted}_i()$ holds.

Eventual Strong Accuracy: We show that eventually, the system reaches a state $c_\tau \in R$ in which every correct node, $p_\ell \in \mathcal{P}$, appears in $\text{trusted}_i()$. Since both $p_i$ and $p_\ell$ are correct, we know that $p_i$ completes round-trips with $p_\ell$ infinitely often. Whenever a round trip is completed, $p_i$ assigns $[0,\ldots,0]$ to $rt_i[\ell]$ (due to lines 53 and 65) and the condition $\Theta > \sum_{x \in \text{withoutTopItems}_i(t,\ell)} x$ (line 66) hold until the next round trip completion (Assumption 7.1).

8 Conclusion

To the best of our knowledge, this paper presents the first self-stabilizing Byzantine-tolerant BRB and IRC algorithms for hybrid asynchronous/time-free message-passing systems. The SS-BRB algorithm takes several communication rounds of $O(n^2)$ messages per instance whereas the IRC algorithm takes $O(n)$ messages but requires synchrony assumptions. The two algorithms are integrated via specified interfaces and message piggybacking (Fig. 1). The integrated solution can run an unbounded number of (concurrent and independent) execution of BRB instances. The advantage is that the more communication-intensive component, i.e., SSBRB, is not associated with any synchrony assumption. We hope that the proposed solutions and studied techniques can be used for the design of new self-stabilizing Byzantine-tolerant building blocks.

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