Exact Langevin equation for electrons in thermostat, Fokker-Planck kinetics and flicker noise

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Abstract

Generalized Langevin equation for characteristic functional of many-electron system dynamically interacting with a thermostat and besides subjected to external perturbation and observation is derived and formulated in terms of one-particle but stochastic density matrix. The scheme naturally includes also direct electron-electron interactions. Corresponding approximate Fokker-Planck kinetic equation is discussed. We point out that it possesses continuous degeneracy of stationary solution and therefore can hide long-correlated fluctuations of electron transport even if thermostat noise is short-correlated.

A. Introduction

The low-frequency flicker noise (1/f noise) is of growing interest among scientists. Usually, investigators interpret it as manifestation of some slow kinetics, for instance, of thermally activated switchings with a broad variety of memory-times (life-times, relaxation times, etc.). From this point of view, 1/f-noise says that present events in a system depend on its old history. Opposite principal possibility is that 1/f-noise is due to forgetting history, concretely, forgetting how many events took place in the past, which means that a system has no reasons to keep well certain "number of events per unit time". Then 1/f spectrum reflects only statistical freedom of flow of events but not real cause-consequence connections between them. From this point of view, 1/f noise in electronics and, say, 1/f fluctuations of flowing of the river Niel are similar phenomena. General ideology of this approach and its concrete realizations were published elsewhere [1-11].

The two different ideas imply different programs for future theory. In our approach, one should reconsider derivations of kinetic models of dissipation (irreversibility) and noise from underlying Hamiltonian dynamics, avoiding the ansatz that "numbers of events per unit time" (i.e. collision integrals, etc.) have quite definite (and well achievable) limits as being averaged over time. The suspicion that such quantities may be uncertain was claimed and argued by Krylov [12] although with no relation to 1/f-noise. It must be emphasized that violation of ergodicity with respect to events, i.e. transitions between phase space regions, does not mean violation of ergodicity with respect to visiting these regions, therefore, usual ergodic theorems are not under doubts. Also we have no doubts of importance of slow kinetics (thermally activated or other).
In particular, it is interesting to elucidate whether many-electron system interacting with a thermostat can possess 1/f type or other flicker fluctuations in electron transport if autonomous thermostat dynamics itself is free of such fluctuations. Any definite answer for this question would be equally useful. In this work we make several steps in this direction and argue that the answer is perhaps positive.

The object under our interest is characteristic functional of electron current. First, we derive formally exact "Langevin equation" for fermions subjected to Hamiltonian interaction with a thermostat, to external dynamical perturbation (bias electric field) and besides to external observation. Secondly, we show that operator-valued thermostat variables in this equation can be effectively replaced by doubled c-number-valued variables ("noises"). As the consequence, the equation allows to perform evaluation of fermionic statistical sums before averaging with respect to thermostat. Therefore, the problem reduces to analysing statistical properties of one-electron but instead stochastic ("twice randomized") density matrix. Next, supposing that electron-thermostat interaction is weak and short-correlated, we derive approximate Fokker-Planck equation for probability distribution of the density matrix, and then consider corresponding kinetic equation for its eigenvalues which just determine the characteristic functional. Finally, we attract attention to that stationary solution of the Fokker-Planck equation is continuously degenerated, and that this fact can signify existence of long-correlated fluctuations in electron transport (flicker type excess current noise). Mathematically rigorous proof (or refutation) of this statement needs in analysis of spectra of complicated non-self-adjoint second-order differential operators. This will be next stage of the work.

B. Model

Let a quantum system consists of an essential "dynamical subsystem", with Hamiltonian $H_e(t)$, and a "thermal bath" ("thermostat"), with Hamiltonian $H_b$. We assume that (i) quantum states of these two parts belong to different independent Hilbert spaces, and (ii) their interaction is described by Hamiltonian

$$H_{int} = \sum_j B_j \ast D_j$$

(1)

where $D_j$ and $B_j$ act in dynamical space and bath space, respectively. This is the case in most of physically meaningful models. For concreteness, we may take in mind that dynamical subsystem (to be abbreviated "DS") is formed by electrons obeying secondary quantization, while bath by bosons (for instance, phonons). Thus the total Hamiltonian is

$$H(t) = H_e(t) + H_b + H_{int}$$

(2)

Besides the interaction, DS may be subjected to an external quasi-classical (c-number) perturbation, $u(t)$. In many cases it also can be described by bilinear form,

$$H_e(t) = H_e - u(t)Q$$

(3)

with $H_e$ being unperturbed Hamiltonian and $Q$ a conjugated quantum variable. For example, if $u(t)$ is an external voltage then $Q$ represents amount of charge transported by free
(conducting) electrons, or if \( u(t) \) is applied electric field then \( Q \) is polarization. Generally speaking, operator \( Q \) does not have definite expression, but the corresponding flow (current), \( i[H(t), Q] \), is well defined (square brackets will serve for operator commutation, \([X, Y] = XY - YX\)). We assume that (iii) \( Q \) belongs to the dynamical subspace (or to some its extension) and (iv) \( Q \) commutes with all the operators \( D_j \), and thus \([H_{int}, Q] = 0\). This means that thermal bath producing dissipation and decoherence in DS at the same time do not contribute to the current in a direct way, as usually in models of electron transport. Then the current operator

\[
J = i[H(t), Q] = i[H_e, Q]
\]

also purely belongs to the DS subspace, and

\[
[H_b, J] = 0
\]

Of course, in any non-trivial case operators \( D_j \) do not commute with free DS Hamiltonian, \([H_e, D_j] \neq 0\), and correspondingly \([J, D_j] \neq 0\), although commutation \([H_e, J] = 0\) is possible.

### C. Quantum characteristic functional

The things under our interest will be fluctuations in DS, firstly in the current, \( J \), and its response to external perturbation. To find these, one should consider time-dependent operators in the Heisenberg representation,

\[
J(t) = S^+(t)JS(t), \quad S(t) \equiv \exp \left[ -i \int_0^t H(t') dt' \right],
\]

where superscript "\(^+\)" denotes Hermitian conjugation, and symbols \( \exp \) and \( \exp \) serve for chronological and anti-chronological time-ordering exponents, respectively. In the Heisenberg picture, fluctuations are described by statistical moments \( \langle J(t_1)...J(t_n) \rangle \), where angle brackets is averaging via some statistical ensemble of initial states,

\[
\langle ... \rangle \equiv \text{Tr} \{ ... \} \rho_{in},
\]

with \( \rho_{in} \) being an initial density matrix.

To have real-valued moments, it is necessary to choose a certain symmetrization rule, in other words, some quantum generalization of classical characteristic functional

\[
\Xi(v) = \left\langle \exp \left[ \int_0^t v(t')J(t')dt' \right] \right\rangle,
\]

where \( v(t) \) is arbitrary test function. The first and second moments are indifferent to the choice. One of possible definitions is

\[
\Xi(v) \equiv \left\langle \exp \left[ \frac{1}{2} \int_0^t v(t')J(t')dt' \right] \exp \left[ \frac{1}{2} \int_0^t v(t')J(t')dt' \right] \right\rangle
\]

It corresponds to following time-ordered symmetric product:
where the circle means Jordan symmetric product, $X \circ Y \equiv \frac{1}{2}(XY + YX)$.

The convenience of this variant of symmetrization is that it allows to reduce calculation of the integral quantity $\Xi(v)$ to solution of a differential equation. With the help of well known general recipes for disentangling operator exponents we can rewrite expression (8) as

$$
\Xi(v) = \langle \overrightarrow{\exp} \left( \int_{t_0}^{t'} \left[ \frac{1}{2} v(t') J + iH(t') \right] dt' \right) \overleftarrow{\exp} \left( \int_{t_0}^{t'} \left[ \frac{1}{2} v(t') J - iH(t') \right] dt' \right) \rangle
$$

(9)

Because of general properties of the Trace operation this means that

$$
\Xi(v) = \text{Tr} \, \rho(t),
$$

where operator $\rho(t)$ obeys the equation

$$
\frac{d}{dt} \rho(t) = v(t) J \circ \rho(t) + i[\rho(t), H(t)] \equiv v(t) J \circ \rho(t) + L(t) \rho(t),
$$

(10)

with initial condition $\rho(0) = \rho_{in}$. Equation (10) remains valid also in classical limit, if replace $L(t)$ by Liouville operator and $J$ by phase function of canonical variables, respectively. At $v(t) = 0$ it turns into mere von Neumann (Liouville) evolution equation for density matrix (probability distribution). Hence, the testing is equivalent to non-Hamiltonian perturbation which can not be introduced by means of Poisson brackets.

### D. Free bath representation

We will suppose that the only significant role of thermal bath is destroying coherence of DS evolution, i.e. production of noise, dephasing and dissipation. Otherwise (for instance, if the bath changes ground state of electronic subsystem), we must redefine division of the whole system into two parts. Then, it is reasonable to speak the free bath language in which bath behaves independently on DS, while the latter is disturbed by bath and governed by the Hamiltonian

$$
H(B, t) \equiv H_e(t) + \sum_j B_j(t) D_j
$$

(11)

Here the operators

$$
B_j(t) \equiv \exp(iH_b t) B_j \exp(-iH_b t)
$$

describe free bath evolution. Taking into account the equality (5), it is easy to verify that expressions (6) and (9) are indeed identical to

$$
J(t) = S_B^+(t) J S_B(t) , \quad S_B(t) \equiv \exp \left[ -i \int_0^t H(B, t') dt' \right],
$$

(12)

$$
\Xi(v) = \left\langle \overrightarrow{\exp} \left( \int_0^t \left[ \frac{1}{2} v(t') J + iH(B, t') \right] dt' \right) \overleftarrow{\exp} \left( \int_0^t \left[ \frac{1}{2} v(t') J - iH(B, t') \right] dt' \right) \right\rangle
$$

(13)
E. Quantum thermostat to classical noise conversion

The ”free bath representation” (12)-(13) allows to replace quantum bath by an effective classical disturbance (noise), or to be precise, by pseudo-classical one. In perfect sense this can be made under assumption that (v) initial statistical ensemble has the factored form

$$\rho_{in} = \rho_e \ast \rho_b$$

with no initial correlations between two subsystems, and therefore the complete averaging exactly divides into two stages:

$$\langle \ldots \rangle = \langle \langle \ldots \rangle_e \rangle_b \equiv \text{Tr}_e \{ \ldots \} \rho_e \ , \ \langle \ldots \rangle_e \equiv \text{Tr}_e \{ \ldots \} \rho_e \ , \ \langle \ldots \rangle_b \equiv \text{Tr}_b \{ \ldots \} \rho_b \ ,$$

where Tr$_e$ and Tr$_b$ are traces in dynamical and bath subspaces.

Consider the construction

$$\Xi(v) = \langle e^{\overline{\mathcal{P}}} \left( \int_0^t \left[ \frac{1}{2} v(t')J + iH(\eta, t') \right] dt' \right) e^{\overline{\mathcal{P}}} \left( \int_0^t \left[ \frac{1}{2} v(t')J - iH(\xi, t') \right] dt' \right) \rangle$$

with different left-hand and right-hand Hamiltonians defined by replacing $B(t)$ in (11) by $\eta(t)$ and $\xi(t)$ , and $\eta_j(t)$ and $\xi_j(t)$ being formally treated as classical (c-number valued) noises. Obviously, this expression coincides with (13) if interpret $\langle \ldots \rangle_b$ as averaging with respect to noises $\eta(t)$ and $\xi(t)$ under an appropriate choice for their statistics. To get the exact identity, we should introduce their characteristic functional by

$$\langle \exp \left( \int \sum_j [a_j(t)\xi_j(t) + b_j(t)\eta_j(t)]dt \right) \rangle$$

$$\equiv \text{Tr}_b \left( \exp \left( \int \sum_j b_j(t)B_j(t)dt \right) \text{exp} \left( \int \sum_j a_j(t)B_j(t)dt \right) \rho_b \right),$$

where $a_j(t)$ and $b_j(t)$ are test functions. As consequence, instead of (10), we can write

$$\Xi(v) = \langle \text{Tr}_e \rho(t) \rangle_b,$$

where now ”density matrix” $\rho(t)$ obeys the generalized von Neumann equation

$$\frac{d}{dt} \rho(t) = v(t)J \circ \rho(t) + L(t)\rho(t) \ , \ \ L(t)\rho = \{\rho H(\eta, t) - H(\xi, t)\rho\},$$

with initial condition $\rho(0) = \rho_e$. It concerns DS only and evolves purely in the dynamical subspace, while $\eta(t)$ and $\xi(t)$ play a role of noise perturbation externally applied to DS.

The ansatz (14) is natural approximation if the interaction $H_{int}$ is sufficiently weak. But in general case we also may start from the ensemble (14), if relating $\Xi(v)$ to later times than characteristic time of mutual relaxation of subsystems, and thus considering their thermodynamical interaction as result of dynamical time evolution.
F. Generalized Langevin equation

The convenience of the pseudo-classic representation introduced by Eqs.16-18 is that one can deal with \( \eta(t) \) and \( \xi(t) \) as objects which commute with both themselves and arbitrary other objects. The pay for this simplification is that DS evolution as determined by Eqs.16 and 18 becomes non-unitary, because of \( \eta(t) \neq \xi(t) \). It is useful to introduce new noises,

\[
\xi \equiv x + iy/2, \quad \eta \equiv x - iy/2, \quad x = (\eta + \xi)/2, \quad y = i(\eta - \xi)
\]

In their terms Eqs.17 and 18 read

\[
\left\langle \exp \left\{ \int \sum_j [g_j(t)x_j(t) + f_j(t)y_j(t)] dt \right\} \right\rangle_b \equiv \exp \left\{ \int \sum_j \left[ \frac{g_j(t)}{2} + if_j(t) \right] B_j(t) dt \right\}
\]

with \( g_j(t) \) and \( f_j(t) \) being test functions, and Eq.18 takes the form

\[
\frac{d}{dt} \rho(t) = v(t)J \circ \rho + L(t)\rho(t), \quad L(t)\rho = \sum_j y_j(t) D_j \circ \rho + i[\rho, H(x, t)],
\]

where \( H(x, t) \) is defined again by replacing \( B(t) \) in (11), \( H(x, t) = H_e(t) + \sum_j x_j(t)D_j \). According to (19), in bath phase space \( x(t) \) is associated with time-ordered symmetric multiplying, \( B(t) \circ ... \), while \( y(t) \) with time-ordered commutation, \( i[B(t), ...] \).

At \( v(t) = 0 \), Eq.20 (or equivalently Eq.18) turns into what can be named generalized Langevin stochastic equation for DS interacting with a thermostat. It differs from frequently used quantum Langevin equations by the terms containing \( y_j(t) \). Clearly, just these terms destroy unitarity of marginal DS evolution. Like the testing they represent non-Hamiltonian perturbation of DS. Their physical meaning is the phase volume exchange between DS and bath. Of course, the unitarity effectively restores after the complete averaging is performed.

Notice that evidently dissipative terms appear in Eq.20 if divide products \( x_j(t)\rho \) into the averages \( \langle x_j(t)\rho \rangle_b \) and normalized forms : \( x_j(t)\rho \). From the other hand, comparison of Eqs.10 and 20 shows that \( \text{Tr}_x \rho(t) \) may be interpreted as mutual characteristic functional of observables \( J(t) \) and \( D_j(t) \) as if governed by Hamiltonian \( H(x, t) \) and tested by \( v(t) \) and \( y_j(t) \), respectively.

It should be emphasized that Eq.20 also directly extends to classical mechanics if replace commutators by classical Poisson brackets. From the point of view of classical intrinsic bath dynamics, multiplying by \( y(t) \) looks as chronologically ordered action of the brackets,

\[
y(t) \leftrightarrow \sum \frac{\partial B(t, q, p)}{\partial p} \frac{\partial}{\partial q} - \sum \frac{\partial B(t, q, p)}{\partial q} \frac{\partial}{\partial p},
\]

where \( q, p \) are bath canonical coordinates and momenta, \( B(t, q, p) \) are \( B(t) \) as expressed via initial values of \( q, p \), and differentiations \( \partial/\partial q, \partial/\partial p \) are oriented in the direction of initial bath distribution \( \rho_b = \rho_b(q, p) \).
G. Effective bath noise statistics

Thus $y_j(t)$ substitute superoperators even in classical case. Nevertheless, even in quantum case we may treat them as well as $x(t)$ like $c$-number variables if only remember the definition (19) and carefully keep in mind their time assigning. The evident profit of such a symbolic calculus is that now a "ready-made" statistics of bath noises $x(t)$ and $y(t)$ may serve as a model fit of the interaction. But underlying operator nature of $y(t)$ results in their unusual partly "ghost" effective statistical properties. For instance, at $a(t) = 0$ Eq.19 turns into

$$\left\langle \exp \left\{ \int \sum_j b_j(t)y_j(t)dt \right\} \right\rangle_b = 1 ,$$

that is all statistical moments of $y_j(t)$ are zero. In spite of this, $y_j(t)$ are non-zero, since they have non-zero cross correlations with $x_j(t)$.

Of course, $x(t)$ and $y(t)$ are closely related being two manifestations of the same intrinsic bath life. General connections between them follow from the properties of the "trace" procedure. In the stationary case, when $[H_b, \rho_b] = 0$, for two-point correlators (which are translationally invariant in this case as well as higher correlators) we have

$$K_{jm}(\tau) \equiv \text{Tr}_b B_j(\tau)B_m(0)\rho_b = K^*_m(-\tau) \quad (21)$$

and correspondingly from Eqs.17 and 19 obtain

$$\langle \eta_j(\tau)\xi_m(0) \rangle = K_{jm}(\tau) ; \quad \langle \xi_j(\tau)\xi_m(0) \rangle = \langle \eta_j(\tau)\eta_m(0) \rangle^* = K^*_{jm}(\tau), \quad \tau > 0$$

$$K_{jm}^{(xx)}(\tau) \equiv \langle x_j(\tau)x_m(0) \rangle = \text{Re} K_{jm}(\tau) ; \quad K_{jm}^{(xy)}(\tau) \equiv \langle x_j(\tau)y_m(0) \rangle = \frac{2\text{ Im} K_{jm}(\tau)}{\vartheta(\tau)}, \quad \tau > 0$$

Under the canonical bath distribution,

$$\rho_b \propto \exp(-H_b/\Theta) ,$$

where $\Theta$ is bath temperature, additional relation takes place,

$$K_{jm}(\tau - i/\Theta) = K_{mj}(-\tau) \quad (23)$$

The latter if combined with (21) means that

$$K_{jm}^{(xx)}(\tau) = \int_0^\infty \cos(\omega\tau)\sigma_{jm}(\omega)d\omega , \quad K_{jm}^{(xy)}(\tau) = 2\vartheta(\tau) \int_0^\infty \sin(\omega\tau) \tanh \left( \frac{\omega}{2\Theta} \right) \sigma_{jm}(\omega)d\omega$$

with some real non-negative spectral measure $\sigma_{jm}(\omega)$ and $\vartheta(\tau)$ being step function.

In the important specific case, when $B(t)$ represent a free bosonic field and therefore self-commutator of $B(t)$ is $c$-number,
\[i[B_j(t), B_m(t')] = 2 \text{Im } K_{mj}(t' - t),\]

for any functional \(\Phi(x, y)\) the Eq.19 implies

\[\langle y_j(t)\Phi(x, y) \rangle = \left\langle \int_{t>t'} dt'[2 \text{Im } K_{mj}(t' - t)] \frac{\delta \Phi(x, y)}{\delta x_m(t')}dt' \right\rangle\]

In this sense, in case of free bosonic thermostat

\[y_j(t) = \sum_m \int_{t'>t} dt'[2 \text{Im } K_{mj}(t' - t)] \frac{\delta}{\delta x_m(t')} \tag{24}\]

\section*{H. Ghost fields and dissipation}

According to relations (22) and (24), all ghost noises \(y(t)\) are correlated with more late \(x(t)\) only. The meaning of this property becomes trivial if rewrite Eq.19 in the form

\[\left\langle \exp\int\sum_j[g_j(t)x_j(t) + f_j(t)y_j(t)]dt\right\rangle_b = \text{Tr}_b \exp\left\{\frac{1}{2} \int\sum_j g_j(t)B_j(t, f)dt\right\} \exp\left\{\frac{1}{2} \int\sum_j g_j(t)B_j(t, f)dt\right\} \rho_b,\]

where \(B_j(t, f)\) correspond to non-free bath evolution perturbed by classical forces \(f(t)\),

\[B_j(t, f) = \exp\left[i \int_0^t H_b(t')dt'\right] B_j \exp\left[-i \int_0^t H_b(t')dt'\right],\]

\(H_b(t) \equiv H_b + \sum_j f_j(t)B_j\)

Differentiation of this identity by \(g(t)\) and \(f(t)\) shows that cross correlators are nothing but bath response functions to the perturbation:

\[\left\langle \prod_j x(t_j) \prod_m y(t'_m) \right\rangle = \left[\prod_m \frac{\delta}{\delta f(t'_m)} \left\langle \prod_j B(t_j, f) \right\rangle \right]_{f=0} \tag{25}\]

Consequently, for all higher-order correlators

\[\left\langle \prod_j x(t_j) \prod_m y(t'_m) \right\rangle = 0 \text{ if } \text{max}\{t'_m\} > \text{max}\{t_j\}, \tag{26}\]

in accordance with causality principle. This property just guarantees eventual safety of the unitarity and of summary phase volume of two subsystems. Due to (26), as one can see from Eq.20, at \(v(t) = 0\) always \(\text{Tr}_b \langle \rho(t) \rangle_b = 1\).

Because of obvious dual symmetry of the subsystems, with reference to bath \(f(t)\) and \(g(t)\) play in Eq.25 the same roles of exciting forces and test functions, respectively, as \(x(t)\) and \(y(t)\) in Eq.20 with reference to DS. It is not hard to guess that the intention of variables \(y(t)\) in the symbolic bath noise calculus is involving a back action of DS onto bath, i.e. reverse dissipative energy flow which must ensure conservation of total energy. If \(y(t)\) were absent then only non-reciprocal pumping of DS by forces \(x(t)\) would be present.
I. Fermionic dynamical subsystem

Let DS be purely fermionic representing free conducting electrons (plus some electron depository if necessary). Our main ansatz will be that (vi) all the operators related to this subsystem have the quadratic form

\[ A = \sum_{\alpha \beta} A_{\alpha \beta} C_{\alpha}^+ C_{\beta} \]  

where \( C_{\alpha}^\dagger (C_{\alpha}) \) are fermionic creation (destruction) operators, indices \( \alpha, \beta \) enumerate some complete set of one-fermion states, and \( A \) stands for \( H_e, D_j, Q \) and consequently \( J \).

Thus the Hamiltonian (2) does not contain direct electron-electron interactions like the Coulombian one. But this simplification implies no principal losses, since it is known that such interactions can be introduced by means of auxiliary actual or ghost bosonic degrees of freedom included to both \( H_b \) and the set \( B_j \).

Indeed, let \( H_e \) is quadratic and we add to it "Coulombian" or any other pair interaction described by the Hamiltonian

\[ H_c = \sum_{\alpha \beta} V_{\alpha \beta} N_\alpha N_\beta, \quad N_\alpha \equiv C_{\alpha}^+ C_{\alpha} \]  

(of course, here \( \alpha, \beta \) may relate to different set of electron states than in (27)). Then Eq.20 turns into

\[ \frac{d}{dt} \rho(t) = v(t) \circ \rho(t) + L(t) \rho(t) + i[\rho(t), H_c] \]

It is easy verifiable that its solution can be represented in the form

\[ \rho(t) = \langle \tilde{\rho}(t) \rangle, \quad \frac{d}{dt} \tilde{\rho}(t) = v(t) \circ \tilde{\rho}(t) + L(t) \tilde{\rho}(t) + L'(t) \tilde{\rho}(t), \]  

\[ L'(t) \tilde{\rho} \equiv \sum_{\alpha} y'_\alpha(t) N_\alpha \circ \tilde{\rho} + i x'_\alpha(t) [\tilde{\rho}, N_\alpha], \]

where \( x'(t) \) and \( y'(t) \) are Gaussian white noises with correlators

\[ \langle x'(t)x'(t_0) \rangle = \langle y'(t)y'(t_0) \rangle = 0, \quad \langle x'_\alpha(t)y'_\beta(t_0) \rangle = 2V_{\alpha \beta} \delta(t-t_0), \]  

and angle brackets in (28) notate averaging with respect to these white noises if mentioned in the sense of Stratonovich (not of Ito).

This is merely the variant of so called Stratonovich transformation. Now all fermionic operators in the evolution equation (28) become quadratic. Hence, after including \( x'(t) \) and \( y'(t) \) to the set of bath noises (with operators \( N_\alpha \) in place of \( D_j \) ) electron-electron interaction reduces to the above general stochastic scheme. Because of \( \langle x' x' \rangle = 0 \), one may say that \( x'(t) \) and \( y'(t) \) represent interaction with zero-energy bath.
J. Fermions to matrices conversion

To complete our simplifications, in addition to (14) we assume (vii) an initial distribution of the electron DS be canonically produced by quadratic-type operators. For instance,

$$\rho_e = \frac{\exp(-H_0/T)\exp(\mu \sum_{\alpha} N_{\alpha}/T)}{\text{Tr}_e \{ \exp(-H_0/T)\exp(\mu \sum_{\alpha} N_{\alpha}/T) \}}$$

(30)

where $H_0$ is some quadratic operator, and sum $\sum_{\alpha} N_{\alpha}$ is the operator of total number of electrons (it commutes with any other operators). In particular case, $H_0$ may coincide with $H_e$. If wanting to fix a number of electrons, $N_{el}$, instead of chemical potential, $\mu$, we may consider the composition of grand canonical distributions,

$$\rho_e = \frac{\delta(N_{el} - \sum_{\alpha} N_{\alpha})\exp(-H_0/T)}{\text{Tr}_e \{ \delta(N_{el} - \sum_{\alpha} N_{\alpha})\exp(-H_0/T) \}}, \quad \delta(n) = \int_{-\pi}^{\pi} e^{i\varphi T} \frac{d\varphi}{2\pi}$$

(31)

with variety of imaginary chemical potentials, $i\varphi T$. In this approach, thermodynamical effects of Coulombian interaction, if included into dynamics as prescribed by Eqs.28 and 29 (but with the averaging put off to a later stage), will come into play after a time of evolution.

Then, clearly, in Eq.16 we can confine ourselves by calculation of only specific fermionic spurs $\text{Tr}_e \{ \Pi_{\alpha} M_n \}$, where each multiplier is an exponent of one of three sorts, $\exp(A)$, $\exp\{ \int A(t)dt \}$, or $\exp\{ \int A(t)dt \}$, with $A$ being quadratic fermionic operator in the sense of (27). This calculation is easy performable if take into account three remarkable facts, namely, (1) any exponent of the second or third sort can be written as an exponent of the first sort, (2) product of two such "quadratic exponents" can be represented as a single quadratic exponent and (3) there is one-to-one correspondence between such operator expressions and analogous matrix expressions. In summary,

$$\exp(\sum X_{\alpha\beta} C^+_{\alpha} C_{\beta}) \exp(\sum Y_{\alpha\beta} C^+_{\alpha} C_{\beta}) = \exp(\sum Z_{\alpha\beta} C^+_{\alpha} C_{\beta}) \Leftrightarrow e^{X} e^{Y} = e^{Z}$$

(32)

$$\delta\exp \left[ \int X_{\alpha\beta}(t) C^+_{\alpha} C_{\beta} dt \right] = \exp(\sum Z_{\alpha\beta} C^+_{\alpha} C_{\beta}) \Leftrightarrow \delta\exp \left[ \int X(t) dt \right] = e^{Z}$$

(33)

Here $X_{\alpha\beta} \equiv (X)_{\alpha\beta}$, and so on, and the right-hand sides are meant in the sense of matrix functions. These relations can be proved if use the commutator identity

$$\left[ \sum X_{\alpha\beta} C^+_{\alpha} C_{\beta}, \sum Y_{\alpha\beta} C^+_{\alpha} C_{\beta} \right] = \sum [X, Y]_{\alpha\beta} C^+_{\alpha} C_{\beta}$$

Hence, any product of quadratic fermionic exponents can be transformed into a single such exponent, for which it is known that

$$\text{Tr}_e \exp(\sum Z_{\alpha\beta} C^+_{\alpha} C_{\beta}) = \text{det} (1 + e^{Z}) = \exp \text{Sp} \ln(1 + e^{Z})$$

(34)

with $\text{det} (...)$ being matrix determinant, and $\text{Sp} M \equiv \sum_{\alpha} M_{\alpha\alpha}$ for any matrix $M$. After that, according to (32) and (33), the exponent $e^{Z}$ may be back unfolded as the exact matrix copy of initial operator product. Therefore, if denote quadratic operators and their exponents by the same letters as their matrix copies then

$$\text{Tr}_e \{ \prod_n M_n \} = \det \{ 1 + \prod_n M_n \}$$

(35)

Although $x(t)$ and especially $y(t)$ are to some extent ghost variables, we can pass them through all these transformations like scalar $c$-number variables, since they by their definition commute with arbitrary objects and are uniquely marked by their time arguments.
K. Secondary randomization instead of secondary quantization

Let $\rho_0$ be matrix copy of quadratic fermionic exponent $\exp(-H_0/T)$, with $H_0$ now defined in the space of one-electron states. Then, as it follows from previous section, in case of grand canonical ensemble (30) the characteristic function (16) can be represented by

$$\Xi(v) = \frac{\langle \det[1 + z\rho(t)] \rangle_b}{\det(1 + z\rho_0)} = \frac{\langle \exp \text{Sp} \ln[1 + z\rho(t)] \rangle_b}{\exp \text{Sp} \ln(1 + z\rho_0)} , \quad z \equiv e^{\mu/T} , \quad (36)$$

where time-dependent one-particle density matrix

$$\rho(t) = \exp\left(\int_0^t \left[\frac{1}{2}v(t')J - iH(\xi, t')\right] dt'\right) \rho_0 \exp\left(\int_0^t \left[\frac{1}{2}v(t')J + iH(\eta, t')\right] dt'\right) \quad (37)$$

is matrix copy of the above considered $\rho(t)$. It obeys exact matrix analogies of Langevin equations (18) and (20), but now under initial condition $\rho(0) = \rho_0$. The case of canonical ensemble (31) can be reduced to calculating the same quantities as in (36):

$$\Xi(v) = \left[\left(\frac{\partial}{\partial z}\right)^{N_{el}} \langle \exp \text{Sp} \ln\{1 + z\rho(t)\} \rangle_b \right]_{z=0}^{-1} \left[\left(\frac{\partial}{\partial z}\right)^{N_{el}} \exp \text{Sp} \ln(1 + z\rho_0) \right]_{z=0} \quad (38)$$

Now, when Fermi statistics is explicitly took into account, we may pay our attention to one-electron version of Langevin equations (18) and (20) and analyse statistical properties of one-electron density matrix (37) (or corresponding wave functions) as evolving under influence by random processes $x(t)$ and $y(t)$. If we first performed bath averaging we would deal with evolution of many secondary quantized fermions. Instead we will deal with twice random (quantumly and excitedly) motion of single particle.

The above consideration clearly demonstrated that we have all rights to deal with $x(t)$ and $y(t)$ as real-valued variables and correspondingly with $\eta(t)$ and $\xi(t)$ as complex conjugated variables, $\eta(t) = (\xi(t))^*$. Hence, if suppose test function $v(t)$ be also real-valued then we can formally treat $\rho(t)$ as Hermitian matrix and write

$$\rho(t) = S(t, t_0)\rho(t_0)S^+(t, t_0) , \quad S(t, t_0) \equiv \exp\left(\int_{t_0}^t \left[\frac{1}{2}v(t')J - iH(\xi, t')\right] dt'\right) \quad (39)$$

In general, $S(t, t_0)$ is not unitary transformation, $SS^+ \neq 1$.

L. Grand canonical ensemble

Consider general relations what follow from one-particle matrix versions of Langevin equations (18) and (20). It is easy to see from Eq.20 that

$$\frac{d}{dt} \text{Sp} F(\rho) = v(t) \text{Sp} J\rho \frac{dF(\rho)}{d\rho} + \sum_j y_j(t) \text{Sp} D_j \rho \frac{dF(\rho)}{d\rho}$$

for any function $F(\rho) = F(\rho(t))$. Therefore, if putting on $F(\rho) = \ln(1 + z\rho)$, we may rewrite Eq.36 as
\[ \Xi(v) = \left\langle \exp \left[ \int v(t) \text{Sp} J f(t) \, dt + \sum_j \int y_j(t) \text{Sp} D_j f(t) \, dt \right] \right\rangle_b, \quad f(t) \equiv \frac{z \rho(t)}{1 + z \rho(t)} \] (40)

Clearly, matrix \( f(t) \) reflects occupancy of one-electron states in factual many-electron system. According to Eq.20, \( f(t) \) is solution to the non-linear stochastic equation

\[ \frac{df}{dt} = \frac{v(t)}{2} [f J (1 - f) + (1 - f) J f] + \sum_j \frac{y_j(t)}{2} [f D_j (1 - f) + (1 - f) D_j f] + i[f, H(x, t)] \] (41)

To find statistical moments of the current we have to differentiate \( \Xi(v) \) by test function \( v(t) \) and then turn it into zero. But the terms in (40) containing \( y_j(t) \) remain and work even at \( v(t) = 0 \). Hence, it is comfortable to introduce special notation for bath averaging under \( v(t) = 0 \):

\[ \langle \Phi(x, y) \rangle_0 \equiv \left\langle \Phi(x, y) \exp \sum_j \int y_j(t) \text{Sp} D_j f(t) \, dt \right\rangle_{v=0} \]

with \( \Phi(x, y) \) being arbitrary functional of bath influence. Due to the property (26), \( \langle 1 \rangle_0 = 1 \). Because of (26), here as well as in (40) in the exponent we may omit limits of the integration. Then the mean current is

\[ \langle J(t) \rangle = \left[ \frac{\delta \Xi(v)}{\delta v(t)} \right]_{v=0} = \langle \text{Sp} J f(t) \rangle_0 \] (42)

The current correlation function produced by second-order derivative of \( \Xi(v) \) generally looks more complicated:

\[ \langle J(t) J(t_0) \rangle = \left[ \frac{\delta^2 \Xi(v)}{\delta v(t) \delta v(t_0)} \right]_{v=0} = \]

\[ = \langle \text{Sp} \{ f(t) J \} \text{Sp} S(t, t_0) [J \circ z \rho(t_0)] S^+(t, t_0) \{1 + z \rho(t)\}^{-1} \rangle_0 + \]

\[ + \langle \text{Sp} J \{1 + z \rho(t)\}^{-1} S(t, t_0) [J \circ z \rho(t_0)] S^+(t, t_0) \{1 + z \rho(t)\}^{-1} \rangle_0, \]

where all matrices correspond to \( v(t) = 0 \), and it is assumed that \( t > t_0 \).

M. Non-degenerated electrons

Let number of electrons is small as compared with total number of one-electron states, \( N_{el} \ll N \), \( N \equiv \text{Sp} 1 \), to the extent that all mean occupancies are significantly smaller than unit. Then, in the frame of canonical ensemble, Eq.38 approximately transforms into

\[ \Xi(v) \approx \left\langle [\text{Sp} \tilde{\rho}(t)]^{N_{el}} \right\rangle_b, \quad \tilde{\rho}(t) \equiv \frac{\rho(t)}{\text{Sp} \rho_0} \] (44)

Here the normalized density matrix \( \tilde{\rho}(t) \), of course, also obeys Eqs.18 and 20. In its tems,
\[ \langle J(t) \rangle = N_{el} \langle \text{Sp} \, J\bar{\rho}(t) \rangle_{N_{el} - 1} \]  \hspace{1cm} (45)

\[ \langle J(t)J(t_0) \rangle = N_{el}(N_{el} - 1) \left\langle \text{Sp} \{ J\bar{\rho}(t) \} \text{Sp} \{ S(t, t_0) [J \circ \bar{\rho}(t_0)] S^+(t, t_0) \} \right\rangle_{N_{el} - 2} + \]  \hspace{1cm} (46)

\[ + N_{el} \left\langle \text{Sp} J S(t, t_0) [J \circ \bar{\rho}(t_0)] S^+(t, t_0) \right\rangle_{N_{el} - 1} , \]

where \( t > t_0 \), and now the bath averaging at \( v(t) = 0 \) is defined by

\[ \langle \Phi(x, y) \rangle_n \equiv \langle \Phi(x, y) \left[ \text{Sp} \, \bar{\rho}(t) \right]_n \rangle_b \approx \left\langle \Phi(x, y) \exp n \sum_j \int y_j(t) \text{Sp} D_j \bar{\rho}(t) dt \right\rangle_b \]

(the latter expression relates to large \( N \) and \( n \) under small ratio \( n/N \ll 1 \)).

Since generally \( \text{Sp} \, \bar{\rho}(t) \neq 1 \), we see that even in dilute electron gas contribution of any separate electron to total current and its fluctuations statistically depends on all other electrons. The matter is that all they feel the same thermostat and thus the latter becomes mediator of electron-electron correlations if not interactions. Since non-degenerated electrons obey Boltzmannian statistics, formulas (44)-(46) relate also to classical limit.

### N. Diagonalization of stochastic density matrix

According to Eqs.36, 38 and 44, in principle, to find the characteristic functional we need in eigenvalues, \( \lambda_k \) (\( k = 1..N \)), of the one-electron density matrix only. In view of Eq.39, we can treat \( \rho(t) \) as Hermitian operator and write \( \rho(t) = \Psi(t) \lambda(t) \Psi^+(t) \), where \( \lambda = \text{diag} \{ \lambda_k \} \) with real elements and \( \Psi(t) \) is unitary transformation. Then in grand canonical ensemble

\[ \Xi(v) = \frac{\int N \prod_{k=1}^{N} [1 + z \lambda_k] W_{\lambda}(t, \lambda) d\lambda}{\prod_{k=1}^{N} [1 + z \lambda_{0k}]} , \]  \hspace{1cm} (47)

where \( W_{\lambda}(t, \lambda) \equiv W_{\lambda}(t, \lambda_1, \ldots, \lambda_N) \) is probability density distribution of the eigenvalues, \( d\lambda = \prod_{k=1}^{N} d\lambda_k \), and \( \lambda_{0k} \) are their initial values. In ensemble with a fixed number of particles

\[ \Xi(v) = \int \left( \sum_{k_1 < k_2 < \ldots} \lambda_{k_1} \lambda_{k_2} \ldots \lambda_{k_{N_{el}}} \right) W_{\lambda}(t, \lambda) d\lambda \left( \sum_{k_1 < k_2 < \ldots} \lambda_{0k_1} \lambda_{0k_2} \ldots \lambda_{0k_{N_{el}}} \right)^{-1} \approx \]  \hspace{1cm} (48)

\[ \approx \int \left( \sum_{k=1}^{N} \lambda_k \right)^{N_{el}} W_{\lambda}(t, \lambda) d\lambda \left( \sum_{k=1}^{N} \lambda_{0k} \right)^{-N_{el}} , \]

where the latter expression relates to dilute gas.

To realize this most adequate approach to the problem, we should combine evolution equation (20) and the first-order perturbation theory to describe infinitesimal time steps of the diagonalization. After standard manipulations we obtain coupled evolution equations for eigenvalues and eigenfunctions of density matrix:
\[
\frac{d\lambda_k}{dt} = \left[ v(t)(\Psi_k^*, J\Psi_k) + \sum_j y_j(t)(\Psi_k^*, D_j \Psi_k) \right] \lambda_k
\]  
(49)

\[
\frac{d\Psi_k}{dt} = \frac{1}{2} \sum_{m \neq k} \Psi_m \frac{\lambda_k + \lambda_m}{\lambda_k - \lambda_m} \left[ v(t)(\Psi_m^*, J\Psi_k) + \sum_j y_j(t)(\Psi_m^*, D_j \Psi_k) \right] - iH(x, t)\Psi_k
\]  
(50)

Here \((a, b) = \sum a_\alpha b_\alpha\) means pseudo-scalar product of two complex vectors (thus \((a^*, b)\) is usual scalar product), and we suppose that eigenvalues are non-degenerated. The latter assumption is not too restrictive since \(\lambda_k\) will acquire tendency to mutual repulsion.

O. Weak noise and Fokker-Planck equation

Let us emphasize that in fact the Langevin stochastic equations (49) and (50), as well as Eqs.18 and 20, are still exact equations, in the sense that they represent explicit mould of underlying detail Hamiltonian dynamics of the electron-bath interaction. The natural next stage of the investigation is derivation of corresponding kinetic equation for probability distribution of \(\lambda\) and \(\Psi\) (and thus for probability functional of \(\rho(t)\)).

If \(x(t)\) and \(y(t)\) were white noises the result would be time-local Fokker-Planck (or Kolmogorov) equation again exactly reflecting the Hamiltonian dynamics. As Eqs.28 and 29 demonstrate, this is just the case when the noises represent time-local electron-electron interaction only. We put off this interesting example for the future. Another such case is \(y(t) = 0\) while \(x(t)\) being white noise which may be interpreted as the infinite bath temperature limit. We will touch it below.

In general, exact kinetic equation is inevitably non-local since at any finite bath temperature the relation (23) forbids to at once turn all the correlators (22) into delta-functions. But we believe that a time-local kinetic equation will be as usually good approximation if the bath noise is sufficiently weak and short correlated. Moreover, under such assumptions we can use simplest one-loop approximation resulting in a Fokker-Planck equation. For any set of stochastic equations

\[
\frac{dZ_k}{dt} = A_k(t, Z) + \sum_\alpha x_\alpha(t)B_{\alpha k}(Z) + \sum_\alpha y_\alpha(t)C_{\alpha k}(Z),
\]

with the same structure as Eqs.49 and 50, this approximation yields

\[
\frac{\partial W}{\partial t} = \Lambda(t)W = \Lambda_f(t)W - \sum_k \frac{\partial}{\partial Z_k}F_k(t)W, \quad \Lambda_f(t) \equiv -\sum_k \frac{\partial}{\partial Z_k}A_k(t, Z),
\]

where \(W = W(t, Z)\) is probability density function, operator \(\Lambda_f(t)\) corresponds to free evolution and operators \(F_k(t)\) describe the bath noise contribution to the probability flow,

\[
F_k(t) = -\sum B_{\alpha k}(Z) \int_0^\infty \langle x_\alpha(\tau)x_\beta(0) \rangle e^{\tau \Lambda_f(t)} \frac{\partial}{\partial Z_\beta}B_{\beta m}(Z) e^{-\tau \Lambda_f(t)} d\tau -
\]

\[
-\sum B_{\alpha k}(Z) \int_0^\infty \langle x_\alpha(\tau)y_\beta(0) \rangle e^{\tau \Lambda_f(t)} \frac{\partial}{\partial Z_\beta}C_{\beta m}(Z) e^{-\tau \Lambda_f(t)} d\tau
\]

(supposing noises are stationary and have zero mean values).
P. Kinetics of probability measure of density matrix

When applying this scheme to our Eqs.49 and 50 we may neglect a contribution by $v(t)$ to the exponents like $\exp[\pm \tau \Lambda_f(t)]$ in the above integrals, since under weak and short correlated bath noise this contribution might result in relatively small corrections only (physically, this means that we neglect displacements of an electron during its short collisions with bath excitations).

Then Fokker-Planck equation for the mutual probability density distribution of the eigenvalues and eigenvectors, $W(t, \lambda, \Psi)$, looks as

$$ \frac{\partial W}{\partial t} = \Lambda(t)W = [v(t)\Lambda_{test} + \Lambda_{free}(t) + \Lambda_{xx}(t) + \Lambda_{xy}(t)]W, $$

(51)

where kinetic operators $\Lambda_{test}$, $\Lambda_{free}$, $\Lambda_{xx}$ and $\Lambda_{xy}$ describe testing of electron DS, its free evolution as if in absense of noise, its stochastic pumping by $x(t)$ which itself influences $\Psi$ only, and common action of $y(t)$ and $z(t)$ which leads to dissipation and besides to fluctuations of $\lambda_k$, respectively. These four parts of $\Lambda(t)$ directly correspond to four different type terms in stochastic equations (49)-(50). The first is

$$ \Lambda_{test} = - \sum_k \frac{\partial}{\partial \lambda_k} \lambda_k (\Psi_k^*, J\Psi_k) - \frac{1}{2} \sum_{k \neq m} \left( \frac{\partial}{\partial \Psi_k^{*m}}, \Psi_m \right) \left( \frac{\partial}{\partial \Psi_k^{*}}, \Psi_m \right) + C.C. $$

(52)

with $\partial/\partial \Psi_k$ being gradients and $(\partial/\partial \Psi_k)^* \equiv \partial/\partial \Psi_k^*$. To express other parts of the kinetic equation (51), for any Hermitian matrix $M$ let us define first-order differential operator

$$ \Omega(M) \equiv i \sum_k \left[ \left( \frac{\partial}{\partial \Psi_k}, MP_k \right) - \left( \frac{\partial}{\partial \Psi_k}, MP_k^* \right) \right] $$

Obviously, $\Lambda_{free}(t)$ is such operator, $\Lambda_{free}(t) = \Omega\{H_e(t)\}$, while effect of noise $x(t)$ is described by the second-order operator

$$ \Lambda_{xx}(t) = \sum_{\alpha\beta} \Omega(D_{\alpha}) \Omega \left\{ \int_0^\infty K_{\alpha\beta}^{(xx)}(\tau) \exp[-iH_e(t)\tau] D_{\beta} \exp[iH_e(t)\tau] d\tau \right\} $$

(53)

The second-order operator $\Lambda_{xy}(t)$ contains $x-y$-correlators instead of $x-x$ ones,

$$ \Lambda_{xy}(t) = - \Omega(D_{\alpha}) \frac{\partial}{\partial \lambda_k} \lambda_k \left( \Psi_k^*, \int_0^\infty K_{\alpha\beta}^{(xy)}(\tau) e^{-iH_e(t)\tau} D_{\beta} e^{iH_e(t)\tau} d\tau \Psi_k \right) - $$

$$ - \frac{1}{2} \Omega(D_{\alpha}) \left( \frac{\partial}{\partial \Psi_k}, \Psi_m \right) \left( \frac{\partial}{\partial \Psi_k}, \Psi_m^* \right) \left[ \int_0^\infty K_{\alpha\beta}^{(xy)}(\tau) e^{-iH_e(t)\tau} D_{\beta} e^{iH_e(t)\tau} d\tau \Psi_k \right] + C.C., $$

(54)

with summation over all repeated indices and $k \neq m$. Its structure resembles that of $\Lambda_{test}$ and it also mixes kinetics of $\Psi$ and $\lambda$.

What is for the case of sufficiently strong DS-bath coupling (intensive bath noise), one can make the substitution

$$ \Psi(t) \Rightarrow S_x(t)\Psi(t), \ S_x(t) = \exp\left[-i \int_0^t H(t', x) dt'\right] $$

in Eqs.49-50, which allows to treat $(\Psi_m^*, S_x^+(t) D_j S_x(t) \Psi_k)$ and similar quantities as fast noises and again apply Fokker-Plank ideology.
Q. Informational vacuum

Let \( W_{\text{unif}}(\Psi) \) be uniform distribution of \( \Psi \) which is invariant with respect to arbitrary rotations and thus independent on \( \Psi \) at all. Of course, this uniform measure is concentrated on the unit sphere, \( (\Psi^*_m, \Psi_k) = \delta_{mk} \), as well as \( W(t, \lambda, \Psi) \) in general, as Eq.50 does imply.

It is not hard to verify two facts. First, multiplication by \( W_{\text{unif}} \) commutes with complete kinetic operator \( \Lambda(t) \) and with any of its four parts separately,

\[
\Lambda_s W_{\text{unif}}(\Psi) - W_{\text{unif}}(\Psi) \Lambda_s = 0, \quad s \in \{\text{test; free; } xx; xy\} \tag{55}
\]

Second, kinetic equation (51) always has formal stationary solution

\[
W_{\text{stat}}(\lambda, \Psi) = W_{\text{unif}}(\Psi) Y(\lambda), \quad \Lambda(t) W_{\text{stat}}(\lambda, \Psi) = 0, \tag{56}
\]

where function \( Y(\lambda) \) is common solution to the equations

\[
\left( \frac{\partial}{\partial \lambda_k} \lambda_k + \sum_{m \neq k} \frac{\lambda_m + \lambda_k}{\lambda_m - \lambda_k} \right) Y(\lambda) = 0, \quad k = 1..N \tag{57}
\]

The latter reads

\[
Y(\lambda) = \frac{\prod_{m<k}(\lambda_k - \lambda_m)^2}{\prod_{j=1}^{N}\lambda_j^{N}} \tag{58}
\]

Moreover, each of the parts of \( \Lambda(t) \) separately turns \( W_{\text{stat}}(\lambda, \Psi) \) into zero,

\[
\Lambda_s W_{\text{stat}}(\lambda, \Psi) = 0, \quad s \in \{\text{test; free; } xx; xy\}
\]

The immovable point \( W_{\text{stat}}(\lambda, \Psi) \) can be named informational vacuum since it says nothing about the system.

R. Much more stationary states

However, this stationary solution can be attracting point for solution of Eq.51 at non-zero test parameter, \( v(t) \neq 0 \), i.e. in absence of testing only. To see this let us integrate both sides of Eq.51 over \( \Psi \). Since the only part of \( \Lambda(t) \) what has terms free of gradients \( \partial/\partial \Psi_k \) and \( \partial/\partial \Psi^*_k \) is \( \Lambda_{\text{test}} \), only its contribution survives after this integration:

\[
\frac{\partial}{\partial t} W_\lambda(t, \lambda) = -v(t) \sum_k \frac{\partial}{\partial \lambda_k} \lambda_k \int (\Psi^*_k, J\Psi_k) W(t, \lambda, \Psi) d\Psi, \tag{59}
\]

\[
W_\lambda(t, \lambda) \equiv \int W(t, \lambda, \Psi) d\Psi
\]

Here \( W_\lambda(t, \lambda) \) is marginal probability distribution of eigenvalues. At \( v(t) = 0 \) it does not change at all and remains atomic, \( W_\lambda(t, \lambda)|_{v=0} = \delta(\lambda - \lambda_0) \), with \( \lambda_0 \) being initial values.
Consequently, under a constant perturbation, \( u(t) = \text{const} \), \( W(t, \lambda, \Psi) \) tends to a stationary distribution,

\[
W_\infty(\lambda, \Psi|\lambda_0) \equiv \lim_{t \to \infty} W(t, \lambda, \Psi)|_{v=0} ,
\]

which differs from (56) and remembers exact start eigenvalues. Hence, at \( v(t) = 0 \) there is continuous infinite set of stationary solutions to Eq.51.

Interestingly, in general these degenerated solutions are not atomic with respect to \( \lambda \), i.e. \( W_\infty(\lambda, \Psi|\lambda_0) \neq \delta(\lambda - \lambda_0) \cdot \int W_\infty(\lambda, \Psi|\lambda_0) d\Psi \). This unusual property as well as the degeneracy itself is reflection of ghost properties of \( y(t) \). The latters manifest themselves in absence of second derivatives \( \partial^2/\partial \lambda^2 \) in Eq.51, in spite of presence of joint derivatives \( \partial^2/\partial \lambda \partial \Psi \), \( \partial^2/\partial \lambda \partial \Psi^* \). Therefore, strictly speaking, \( \Lambda(t) \) is not purely elliptic operator.

**S. Exclusion of eigenvectors**

In order to analyse the characteristic functional as presented by Eqs.47 or 48, we should consider marginal distribution \( W_\lambda(t, \lambda) \) just at \( v(t) \neq 0 \), when it can change to arbitrary extent. Since then the vacuum solution (56) is attractive, it is natural to use it as ground state in the projection method. All the more, bath noise also pushes \( W(t, \lambda, \Psi) \) towards \( W_{\text{stat}}(\lambda, \Psi) \) making eigenvectors as much random and statistically independent on eigenvalues as possible. At the same time, since the factor \( Y(\lambda) \) is non-normalizable, this factored solution never can be achieved explicitly. Always more or less correlations between \( \lambda \) and \( \Psi \) do exist, and we must write

\[
W(t, \lambda, \Psi) = W_\lambda(t, \lambda) W_{\text{unif}}(\Psi) + \tilde{W}(t, \lambda, \Psi) , \quad \int \tilde{W}(t, \lambda, \Psi) d\Psi = 0 ,
\]

where \( \tilde{W}(t, \lambda, \Psi) \) does not contribute to \( W_\lambda(t, \lambda) \).

Thus we merely introduce the projection operator:

\[
\Pi W(\lambda, \Psi) \equiv W_{\text{unif}}(\Psi) \int W(\lambda, \Psi) d\Psi , \quad \tilde{W}(t, \lambda, \Psi) = (1 - \Pi) W(t, \lambda, \Psi)
\]

With using Eqs.55-56, Eq.51 yields

\[
\frac{\partial}{\partial t} \tilde{W} = (1 - \Pi) \Lambda(t) \tilde{W} + W_{\text{unif}} I(t) , \quad I(t) \equiv \left[ \Lambda(t) W_\lambda - \int W_{\text{unif}} \Lambda(t) W_\lambda d\Psi \right] \quad (60)
\]

Here factor \( I(t) \) in the source term is easily calculable:

\[
I(t) = -\sum_k \left( \frac{\partial}{\partial \lambda_k} \lambda_k + \sum_{m \neq k} \frac{\lambda_m + \lambda_k}{\lambda_m - \lambda_k} \right) W_\lambda(t, \lambda) [v(t)(\Psi_k^*, J \Psi_k) + (\Psi_k^*, V(t) \Psi_k)] , \quad (61)
\]

with the traceless matrix \( V(t) \) defined by the sum of commutators

\[
V(t) = i \sum_{\alpha\beta} \left[ \int_0^\infty K^{xy}_{\alpha\beta}(\tau) \exp{-iH_\alpha(t)\tau} D_\beta \exp{iH_\alpha(t)\tau} d\tau , D_\alpha \right] \quad (62)
\]

Only \( \Lambda_{\text{test}} \) and \( \Lambda_{xy} \) contribute to the source giving two similar terms in (61). We also took into account that

\[
\int (\Psi_k^*, M \Psi_k) W_{\text{unif}}(\Psi) d\Psi = (\text{Sp} M)/N
\]

for any matrix \( M \). Because of this equality and of that the current matrix \( J \) by its definition is traceless, only \( \tilde{W} \) contributes to right-hand side in Eq.59.
T. Kinetic equation for eigenvectors

As usually in projection techniques, Eqs. 59-62 ensure exclusion of a part of variables, producing formally closed although non-local separate evolution equation for eigenvalues. If introduce notations

\[ A_{kq} \equiv (\Psi^*_k, A\Psi_q), \quad \langle (...) \rangle_{\text{unif}} \equiv \int (...) W_{\text{unif}}(\Psi)d\Psi, \]

\[ \hat{U}_v(t, t_0) = \exp \left\{ \int_{t_0}^t [v(t') (1 - \Pi) \Lambda_{\text{test}} + \Lambda_{\text{free}}(t') + \Lambda_{xx}(t') + \Lambda_{xy}(t')] dt' \right\}, \quad (63) \]

this equation reads

\[ \frac{\partial}{\partial t} W_{\lambda}(t, \lambda) = v(t) \sum_{kq} \frac{\partial}{\partial \lambda_k} \lambda_k \int_{t_0}^t \langle J_{kk} \hat{U}_v(t, t') [v(t') J_{qq} + V_{qq}(t')] \rangle_{\text{unif}} \times \]

\[ \times \left( \frac{\partial}{\partial \lambda_q} \lambda_q + \sum_{m \neq q} \frac{\lambda_m + \lambda_q}{\lambda_m - \lambda_q} \right) W_{\lambda}(t', \lambda)dt' - v(t) \sum_{k} \frac{\partial}{\partial \lambda_k} \lambda_k \int J_{kk} \hat{U}_v(t, 0) \hat{W}(0, \lambda, \Psi)d\Psi \]

We take in mind Eqs. 55-56 and the identities \((1 - \Pi) \Lambda_s = \Lambda_s\) for \(s \in \{\text{free}; xx; xy\}\).

The important point is that last term which represents a legacy of initial state,

\[ \hat{W}(0, \lambda, \Psi) = \delta(\lambda - \lambda_0) [\delta(\Psi - \Psi_0) - W_{\text{unif}}(\Psi)], \]

may be insignificant even in spite of the degeneracy. The matter is that all the appointed stationary distributions do not depend on initial \(\Psi\) distribution, therefore \(\hat{U}_v(t, 0) \hat{W}(0, \lambda, \Psi)\) must decay after a fixed finite time determined mainly by the diffusion operator \(\Lambda_{xx}\), i.e. characteristic correlation time of eigenvectors fluctuations. At the same time, as already Eq. 59 shows, the evolution rate of \(W_{\lambda}(t, \lambda)\) is dictated by magnitude of test function \(v(t)\). Since the latter serves eventually for differentiation by it, we may treat it as infinitesimally small parameter and hence \(W_{\lambda}(t, \lambda)\) as slow varying, thus having all rights to neglect the last term.

U. Kinetics of eigenvalues in hot bath

Let the perturbation \(u(t)\) which namely determines time dependence of operators \(\Lambda_{\text{free}}, \Lambda_{xx}\) and \(\Lambda_{xy}\) is not too strong and not too fast varying (for instance, \(u(t)\) is constant). Then we can neglect time variations of \(u(t)\) in the integrand kernel in Eq. 64 while in (62) neglect \(u(t)\) at all. Besides, let us neglect also the kernel dependencies on both \(v(t)\) and \(\lambda\), i.e. throw away \(\Lambda_{\text{test}}\) and \(\Lambda_{xy}\) from the propagator (63). Formally, it is possible because the diffusion operator \(\Lambda_{xx}\) itself quite ensures decay of the kernel, while effects of operator \(\Lambda_{xy}\) is partially took into account by the term containing \((\Psi_k^*, V(t)\Psi_k)\) in the source (61). This term involves dissipation and represents the storage for non-zero dissipative mean current response.
Clearly, in such an approximation the current is modelled as Gaussian random process, and all effects of the degeneracy are asided. Neglecting $\Lambda_{xy}$ as compared with $\Lambda_{xx}$ corresponds to what can be named "infinitely hot bath limit", since according to Eq.23 action of $\Lambda_{xx}$ much overpowers action of $\Lambda_{xy}$ when bath temperature $\Theta$ grows to infinity. Thus we come to maximally simplified kinetic equation for eigenvalues distribution, in the form

$$\frac{\partial W}{\partial t} = \frac{\Gamma(t)}{N} \sum_k \frac{\partial}{\partial \lambda_k} \lambda_k \left( \frac{\partial}{\partial \lambda_k} \lambda_k + \sum_{m \neq k} \frac{\lambda_m + \lambda_k}{\lambda_m - \lambda_k} \right) W_{\lambda}$$

(65)

Here the evolution rate $\Gamma(t)$ is determined by

$$\frac{\Gamma(t)}{N} = v(t) \int_0^\infty \langle (\Psi_0^*, J\Psi_0) \exp\{[\Lambda_{\text{free}}(t) + \Lambda_{xx}]\tau\} (\Psi_0^*, V\Psi_0) \rangle_{\text{unif}} d\tau +$$

$$+ v(t) \int_0^\infty \langle (\Psi_0^*, J\Psi_0) \exp\{[\Lambda_{\text{free}}(t) + \Lambda_{xx}]\tau\} (\Psi_0^*, J\Psi_0) \rangle_{\text{unif}} v(t - \tau) d\tau ,$$

(66)

with $\Lambda_{\text{free}}(t) = \Omega\{H_e\} - u(t)\Omega\{Q\}$, vector $\Psi_0$ is any of columns of $\Psi$, and matrix $V$ is given by (62) but under $H_e(t)$ replaced by $H_e$. We assumed also that total number of one-electron states (dimensionality of matrices) is large, $N >> 1$, which allows for many formal simplifications.

The first integral on right hand of Eq.66 describes mean current. We may suppose that this integral turns into zero at $u(t) = 0$, because of a proper symmetry of the matrix (62), i.e. that there is no spontaneous current breaking time symmetry. We expect also at non-zero low-field conductivity and sufficiently small and slow varying $u(t)$ (and besides slow $v(t)$) expression (66) reduces to

$$\Gamma(t) \approx v(t) \frac{\Delta}{\Theta} u(t) + \Delta v^2(t) = v(t)w(t) + \Delta v^2(t) ,$$

(67)

where $\Delta$ and $w(t)$ represent diffusivity and drift velocity of a separate electron.

Notice that Eq.65 closely resembles kinetic equations for transmission matrix eigenvalues in the theory of statically disordered conductors [16].

V. Current noise in Gaussian approximation

Rather trivial model (65)-(67) is nevertheless useful probe to compare roles of diffusive component of $W_{\lambda}(t, \lambda)$ evolution (described by second-order derivatives $\partial^2/\partial \lambda^2$ in Eq.65), from one hand, and of drift caused by mutual repulsion of eigenvalues (described by ratios $(\lambda_m + \lambda_k)/(\lambda_m - \lambda_k)$), from another hand. Note that at possibly negative $\Gamma(t)$ values Eq.65 corresponds to mathematically incorrect time-reversed diffusion, but in fact this implies nothing incorrect in final results of calculations.

Simple analysis shows that, at $N >> 1$, from the point of view of statistical moments of $\lambda$, all diffusive terms in Eq.65 occur relatively insignificant. Therefore, in the asymptotical sense, for any function $F(\lambda)$ we can write
\[
\frac{d}{dt} \langle F(\lambda) \rangle = \Gamma(t) \left\{ \frac{1}{N} \sum_k \left( \sum_{m \neq k} \frac{\lambda_k + \lambda_m}{\lambda_k - \lambda_m} \right) \lambda_k \frac{\partial}{\partial \lambda_k} F(\lambda) \right\}, \quad \langle (...) \rangle \equiv \int (...) W_\lambda d\lambda \quad (68)
\]

This circumstance is due to that many of ratios \((\lambda_m + \lambda_k) / (\lambda_m - \lambda_k)\) are large numbers \(~ N\), hence the drift overpowers diffusion approximately \(N\) times.

Particularly, in case (48), i.e. at fixed \(N_{el}\) (may be comparable with \(N\)),

\[
\sum_k \left( \sum_{m \neq k} \frac{\lambda_k + \lambda_m}{\lambda_k - \lambda_m} \right) \lambda_k \frac{\partial}{\partial \lambda_k} \sum \lambda_{k_1} \lambda_{k_2} \cdots \lambda_{k_{N_{el}}} = N_{el} (N - N_{el}) \sum \lambda_{k_1} \lambda_{k_2} \cdots \lambda_{k_{N_{el}}} \quad (69)
\]

(primed sum means that all indices are different one from another), and Eq.68 exactly yields

\[
\Xi(v) = \exp \left[ \theta N_{el}(N - N_{el})/N \right], \quad \theta \equiv \int_0^t \Gamma(t')dt'
\]

This result corresponds to Gaussian current noise, with only those electron-electron statistical correlations what come from Pauli principle and Fermi statistics of electrons (or holes if \(N_{el} \approx N\)).

In case of grand canonical ensemble (47), we obtain

\[
\Xi(v) = \exp \left[ \theta \frac{N_{el}(N - N_{el})}{N} + \frac{\theta^2}{2} \left( 1 - \frac{2N_{el}}{N} \right) \sum f_k (1 - f_k) + \ldots \right], \quad (71)
\]

\[f_k \equiv z\lambda_k / (1 + z\lambda_k), \quad N_{el} \equiv \sum f_k,
\]

where \(\lambda_k\) should be replaced by their initial values (i.e. represent diagonal elements of \(\rho(t = 0)\)). The points mean higher degrees of \(\theta\). The second term in the exponent has no analogy in (70) and formally corresponds to non-decaying flicker type current and conductance fluctuations. But in fact this is nothing but artifact of grand canonical ensemble.

**W. Degeneracy and low-frequency noise**

Now let us be so courageous and discuss what will occur beyond the hot bath Gaussian approximation, i.e. if keep \(v\), \(\lambda\) and \(\partial/\partial \lambda\) dependencies of the kernel in Eq.64. Generally speaking, we may expect that above mentioned strong degeneracy of stationary state results in long-lasting statistical correlations when observing DS, i.e. in low-frequency excess fluctuations of the current. Indeed, as Eq.59 shows, measuring of the current transforms initial values \(\lambda_0\) to one or another different values \(\lambda'\) which then remain for arbitrary long time being unchangably encoded in \(W_\infty(\lambda, \Psi|\lambda')\) (or in similar time varying distribution if \(u(t)\) is not constant). Therefore, next measurement will be inevitably correlated with previous one regardless of their time separation (this resembles low-frequency Goldstonian excitations under breaking a continuous degeneracy).

If this reasonings are true, then the differential operator kernel in Eq.64,

\[
\tilde{R}_{kq}(t, t_0, v) \equiv v(t) \left\langle J_{kk} \tilde{U}_v(t, t_0) [v(t_0) J_{qq} + V_{qq}(t_0)] \right\rangle_{unif}, \quad (72)
\]
has slow decaying non-exponential tail closely related to continuous degeneracy of ground state of the operator \((1-\Pi)\Lambda(t)\) which generates semi-group \(\hat{U}_v(t, t_0)\). Clearly, such a degeneracy takes place (and besides, in opposite to \(\Lambda(t)\), even at arbitrary \(v(t)\), hence, \(\hat{U}_v(t, t_0)\) is not strictly contracting semigroup. Directly, the degeneracy concerns \(\lambda\) dependence of joint probability distribution, but indirectly it expands also to its \(\Psi\) dependence, because of the \(\lambda - \Psi\) coupling produced by operator \(\Lambda_{xy}(t)\). The hot bath Gaussian approximation considered neglects this coupling and therefore retains only rapidly decaying part (66) of the kernel (72) responsible for mean diffusivity and mean drift. The rest of the kernel must contain information on excess contribution to the current correlation function,

\[
K(t, t_0) \equiv \langle J(t)J(t_0) \rangle - \langle J(t) \rangle \langle J(t_0) \rangle ,
\]

including conductance fluctuations, and besides on higher-order cumulants of current. The former, in its turn, consists of two parts whose physical sense is as follows.

When the bath noise \(x(t)\) transforms coherent electron motion into diffusion (plus drift as characteristics of its asymmetry), diffusivity and mobility of any electron depend on random time-varying non-uniformities of density of other electrons, i.e. on random occupancies of one-electron states. As it will be demonstrated elsewhere, this fact results in flicker type conductance fluctuations with spectrum \(\propto f^{-1/2}\), at frequencies higher than inverse time of electron diffusion through overall system. This excess noise arises merely from Pauli principle and has no relation to \(\lambda - \Psi\) coupling. The only role of \(\Lambda_{xy}(t)\) in this mechanism is establishing more or less statistical hierarchy (non-equiparticity) of electron energies, in contrary to \(x(t)\) pumping, and thus translate diffusivity fluctuations into drift ones.

Other mechanism of excess low-frequency noise is due, in principle, just to the \(\lambda - \Psi\) coupling. As we already pointed out, the reverse DS action onto the bath, representing dissipation, makes the latter mediator of indirect electron-electron interaction through bath. This is in rich analogy with action of the \(x'(t)\) - \(y'(t)\) noise in Eqs.28-29 which is equivalent to direct electron-electron interaction. Hence, to keep operator \(\Lambda_{xy}(t)\) in (72) means to take into account such an interaction. But our experience gave evidents [5,8] that particle-particle interaction itself is sufficient natural cause of a true flicker noise, i.e. noise without low-frequency spectrum saturation (1/f fluctuations of self-diffusivity and mobility of particles). In [1,2,4,5] it was explained why this noise can be insensible to spatial and temporal scales of a system.

### X. Current correlation function

To connect \(K(t, t_0)\) and the kernel (72), let us suppose, for simplicity and visuality, that external perturbation \(u(t)\) is time-independent and that after not a long time a mean value of the current, \(\bar{J} = \langle J(t) \rangle\), tends to a constant, while its correlation function becomes depending on time difference only, \(K(t - t_0)\). Then we can put on the test function \(v(t)\) also constant, \(v = \text{const}\), thus testing integrated current, i.e. transported charge, \(\int_0^t J(t')dt'\). This simplifications bring comforts of Laplas transformation.

Applying the latter to the characteristic functional, at sufficiently small \(v\) and \(p\) we have

\[
\Xi(v) \equiv \int_0^\infty \Xi(v)e^{-pt}dt = [p - v\bar{J} - v^2\bar{K}(p) - ... ]^{-1}, \quad \bar{K}(p) \equiv \int_0^\infty K(\tau)e^{-p\tau}d\tau ,
\]  
(73)
where the dots replace higher-order terms of \( v \) expansion. From the other hand, the same quantity can be expressed from Eqs.47-48 and evolution equation (64) (throwing out last term of (64)),

\[
\tilde{\Xi}(v) = F^{-1}(\lambda) \left[ p - v\hat{G}_1(p) - v^2\hat{G}_2(p) - ... \right]^{-1} F(\lambda) \tag{74}
\]

Here differential operators \( \hat{G}_n(p) \) acting on \( \lambda \) are defined by \( v \) expansion of kernel (72), and \( \lambda \) stand for the initial eigenvalues. Direct formal expansion gives

\[
\hat{G}_1(p) = \sum_{kq} [\partial_q + \phi_q(\lambda)] \left\langle J_{kk}[p - \Lambda_0^T]^{-1}V_{qq} \right\rangle_{\text{unif}} \partial_k ,
\]

\[
\hat{G}_2(p) = \sum_{kq} [\partial_q + \phi_q(\lambda)] \left\langle J_{kk}[p - \Lambda_0^T]^{-1}J_{qq} \right\rangle_{\text{unif}} \partial_k +
\]

\[+ \sum_{kq} [\partial_q + \phi_q(\lambda)] \left\langle J_{kk}[p - \Lambda_0^T]^{-1}(1 - \Pi)\Lambda_{\text{test}}^T[p - \Lambda_0^T]^{-1}V_{qq} \right\rangle_{\text{unif}} \partial_k ,
\tag{75}
\]

where the shortened notations are used

\[
\partial_k \equiv \lambda_k \frac{\partial}{\partial \lambda_k} , \quad \phi_k(\lambda) \equiv \sum_{m \neq k} \frac{\lambda_k + \lambda_m}{\lambda_k - \lambda_m} , \quad \Lambda_0 \equiv \Lambda_{\text{free}} + \Lambda_{xx} + \Lambda_{xy}
\]

The superscript "\( T \)" is symbol of conjugation with respect to \( \lambda \) in the Sturm-Liouville sense, that is reversing of order of all differentiations \( \partial/\partial \lambda \) and multiplications by any function of \( \lambda \) and changing \( \partial/\partial \lambda \) by \(-\partial/\partial \lambda\).

Comparing linear and quadratic terms in formulas (73) and (74) we see that

\[
\tilde{\mathcal{J}} = F^{-1}(\lambda)\hat{G}_1(p)F(\lambda) \tag{76}
\]

\[
\tilde{K}(p) = F^{-1}(\lambda)\hat{G}_2(p)F(\lambda) + p^{-1}\{F^{-1}(\lambda)\hat{G}_1(p)F(\lambda) - [F^{-1}(\lambda)\hat{G}_1(p)F(\lambda)]^2\}
\]

Thus, mean current really tends to a constant if \( \hat{G}_1(p \to 0) \) has finite limit. Besides, the natural possibility is if in this long-time limit \( F(\lambda) \) is eigenfunction of operator \( \hat{G}_1(p \to 0) \).

Then Laplas transform of the current correlator reduces to

\[
\tilde{K}(p) \approx F^{-1}(\lambda)\hat{G}_2(p)F(\lambda) \tag{77}
\]

Hence, \( F(\lambda) \) may be also eigenfunction of \( \hat{G}_2(p \to 0) \). Notice that operators (75) always can have polynomial eigenfunctions like presented by (48) since these operators (as well as complete operators \( \Lambda_0 , \Lambda_{\text{test}} \)) are insensible to arbitrary scaling of \( \lambda \). Of course, both \( \tilde{\mathcal{J}} \) and excess part of \( \tilde{K}(p) \) are results of external perturbation, \( u \), hidden in \( \Lambda_{\text{free}} = \Omega\{H_e\} - u\Omega\{Q\} \).
For the meanwhile, our main results are, first, reduction of many-electron problems to investigation of stochastic one-electron density matrix, \( \rho(t) \), which obeys exact Langevin equation (20), and, second, formulation of approximate Fokker-Plank kinetics for eigenvalues and eigenvectors of this matrix.

Equivalently, one could consider statistical moments \( \langle \rho(t) \otimes \ldots \otimes \rho(t) \rangle_b \), i.e. mean strict products of many copies of \( \rho(t) \), and derive from (20) evolution equations for them or, instead, express solution to (20) and then the moments in terms of many-path integrals with counting intersections (raprochements) of paths (like it was considered in [14] with respect to single copy). Under the weak short-correlated bath noise approximation, evolution equations are time-local and then, at least, right-hand side of equation for \( \langle \rho(t) \rangle_b \) can be represented in standard Lindblad form for generators of quantum contracting semigroups (although in our case this is not natural form). But the Langevin equation (20), which in our case realizes stochastic dilation of a dissipative semigroup behaviour, lies rather far from Langevin equations under use in modern ”quantum stochastic calculus” [13] or in well developed bosonic stochastic calculus [15]. The peculiarity of Eq.20 is that it takes into account phase volume exchange between dynamical subsystem (electrons) and thermal bath and, therefore, the dilation it gives is not unitary in momentary sense although eventually (after complete averaging) works as unitary one. To the best of our knowledge, such sort of Langevin equations was still unknown. If it is so then Eq.20 itself also can be qualified as result of this paper.

In the frame of this equation, we came to very intriguing task about spectral and other properties of the Fokker-Planck operator \( \Lambda_0 = \Lambda_{f,ee} + \Lambda_{xx} + \Lambda_{xy} \) whose peculiarities are non-self-adjointness, non-ellipticity (induced by that of \( \Lambda_{xy} \)) and continuous infinitness of Ker \( \Lambda_0 \). Particularly, Fokker-Planck approximation turns into exact theory if throw out thermal bath at all but instead include delta-correlated \( x'(t) - y'(t) \) noise equivalent to pair electron-electron interactions. In this case \( \Lambda_{xx} = 0 \) and ”bad” operator \( \Lambda_{xy} \) becomes responsible for everything (perhaply, in particular, for such ”bad” phenomenon as flicker noise).

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