The relativistic self-energy in nuclear dynamics

O. Plohl and C. Fuchs

Institut für Theoretische Physik, Universität Tübingen,
Auf der Morgenstelle 14, D-72076 Tübingen, Germany

Abstract

It is a well known fact that Dirac phenomenology of nuclear forces predicts the existence of large scalar and vector mean fields in matter. To analyse the relativistic self-energy in a model independent way, modern high precision nucleon-nucleon (NN) potentials are mapped on a relativistic operator basis using projection techniques. This allows to compare the various potentials at the level of covariant amplitudes were a remarkable agreement is found. It allows further to calculate the relativistic self-energy in nuclear matter in Hartree-Fock approximation. Independent of the choice of the nucleon-nucleon interaction large scalar and vector mean fields of several hundred MeV magnitude are generated at tree level. In the framework of chiral EFT these fields are dominantly generated by contact terms which occur at next-to-leading order in the chiral expansion. Consistent with Dirac phenomenology the corresponding low energy constants which generate the large fields are closely connected to the spin-orbit interaction in NN scattering. The connection to QCD sum rules is discussed as well.

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I. INTRODUCTION

A fundamental question in nuclear physics is the role which relativity plays in nuclear systems. The ratio of the Fermi momentum over the nucleon mass is about $k_F/M \simeq 0.25$ and nucleons move with maximally about 1/4 of the velocity of light. This implies only moderate corrections from relativistic kinematics in finite nuclei. Non-relativistic approaches such as, e.g., Skyrme-Hartree-Fock and relativistic approaches describe finite nuclei equally well.

However, there exists a fundamental difference between relativistic and non-relativistic dynamics: a genuine feature of relativistic nuclear dynamics is the appearance of large scalar and vector mean fields, each of a magnitude of several hundred MeV. The scalar field $\Sigma_s$ is attractive and the vector field $\Sigma_\mu$ is repulsive. In relativistic mean field (RMF) theory, both, the sign and the size of the large scalar and vector fields are enforced by the nuclear saturation mechanism [1]. At nuclear saturation density $\rho_0 \simeq 0.16 \text{ fm}^{-3}$ the empirical fields deduced from RMF fits to finite nuclei are of the order of $\Sigma_s \simeq -350 \text{ MeV}$ and $\Sigma_0 \simeq +300 \text{ MeV}$ [2] (In mean field theory only the time-like component of $\Sigma_\mu$ contributes in static systems with time-reversal symmetry).

A problem is, however, that these scalar/vector fields are no direct observables as, e.g., the nuclear binding energy or the nucleon potential. The single particle potential in which the nucleons move originates from the cancellation of the two contributions $U_{\text{cent}} \simeq \Sigma_0 + \Sigma_s$ and is of the order of -50 MeV. Therefore one has no direct experimental access to the interpolating scalar/vector fields. There exist, however, several features in nuclear structure which can be explained naturally within Dirac phenomenology while models based on non-relativistic dynamics have difficulties or, at least, one has to introduce additional model parameters. The most well known feature is the large spin-orbit splitting in finite nuclei. In a relativistic framework the strong spin-orbit force appears naturally from the coupling to the lower components of the Dirac equation where the scalar-vector mean fields add up in the spin-orbit potential $U_{S.O.} \propto (\Sigma_0 - \Sigma_s) \simeq 750 \text{ MeV}$. Due to this fact RMF theory is able to reproduce the strong spin-orbit splitting in spherical nuclei quantitatively without the introduction of additional parameters [2]. A second symmetry, observed more than thirty years ago in single-particle levels of spherical nuclei is the so called pseudo-spin symmetry [3]. While all attempts to understand this symmetry within non-relativistic approaches failed, it can naturally be understood within RMF theory as has been shown by Ginocchio [4] a few
years ago. This symmetry, again a consequence of the coupling to the lower components, is exact in the limit $\Sigma_0 = -\Sigma_s$ and is broken in nature by the amount $(\Sigma_0 + \Sigma_s)/(\Sigma_0 - \Sigma_s)$ which is less than 10%. A third example are the moments of inertia in rotating nuclei. Relativistic dynamics implies that in the rotating system a Coriolis term occurs due to the spatial vector currents, however, with all couplings already fixed through the time-like components.

An alternative approach for nuclear matter are ab initio many-body calculations. Based on high precision nucleon-nucleon ($NN$) interactions one treats short-range and many-body correlations explicitly. A typical example for a successful many-body approach is Brueckner theory. In the relativistic Brueckner approach the nucleon inside the medium is dressed by the self-energy $\Sigma$. The in-medium T-matrix is obtained from the relativistic Bethe-Salpeter (BS) equation and plays the role of an effective two-body interaction which contains all short-range and many-body correlations of the ladder approximation. Solving the BS equation the Pauli principle is respected and intermediate scattering states are projected out of the Fermi sea. The summation of the T-matrix over the occupied states inside the Fermi sea yields finally the self-energy in Hartree-Fock approximation. In contrast to relativistic Dirac-Brueckner-Hartree-Fock (DBHF) calculations which came up in the late 80ties non-relativistic BHF theory has already half a century’s history. Despite strong efforts invested in the development of improved solution techniques for the Bethe-Goldstone (BG) equation, the non-relativistic counterpart of the BS equation, it turned out that, although such calculations were able to describe the nuclear saturation mechanism qualitatively, they failed quantitatively. Systematic studies for a large number of nucleon-nucleon interactions showed that saturation points were always allocated on a so-called Coester-line in the $E/A - \rho$ plane which does not meet the empirical region of saturation. In particular modern one-boson-exchange (OBE) potentials lead to strong over-binding and to too large saturation densities where relativistic calculations work much better.

However, in relativistic approaches the nuclear interaction is always described in some sort of a meson exchange picture. The mesons represent effective bosonic degrees of freedom which are either directly adjusted to the properties of nuclear matter and finite nuclei, as in the case of RMF theory, or to vacuum $NN$ scattering. Hence it is a fundamental question to decide whether the large scalar and vector fields enforced by Dirac phenomenology of nuclear systems are an artefact of the meson exchange picture or whether they reflect a deeper characteristics of nature.
A connection to Quantum-Chromo-Dynamics (QCD) as the fundamental theory of strong interactions is established by QCD sum rules. The change of the chiral condensates $\langle \bar{q}q \rangle, \langle q^\dagger q \rangle$ in matter leads to attractive scalar and repulsive vector self-energies which are astonishingly close to the empirical values derived from RMF fits to the nuclear chart.

It is remarkable that relativistic many-body calculations yield again scalar and vector fields which are of the same sign and magnitude as obtained from RMF theory or, alternatively, from QCD sum rules. Such a coincidence could not have been expected a priori. Moreover, DBHF calculations agree even on a quantitative level surprisingly well with the QCD based approach of Ref. where chiral fluctuations from the long and intermediate range pion-nucleon dynamics were considered on top of the chiral condensates.

These facts suggest that preconditions for the existence of large fields in matter or, alternatively, the density dependence of the QCD condensates, must already be inherent in the vacuum $NN$ interaction. The connection of the nucleon-nucleon force to QCD is given by the fact that the interaction is described by the exchange of the low lying mesonic degrees of freedom. The long-range part of the interaction is mediated by the one-pion-exchange (OPE) while the scalar isoscalar intermediate range attraction is mainly due to correlated two-pion-exchange. The short-range part, i.e., the hard core, is dominated by light vector meson exchange, i.e., the vector isoscalar $\omega$ meson and the vector isovector $\rho$. Modern one-boson-exchange potentials (OBEP) as e.g. the Bonn potentials are based on the exchange of these mesons and provide high precision fits to nucleon-nucleon scattering data. Meson-nucleon coupling constants and form factors are empirically fixed from the data. Thus OBEPs are the result of relativistic phenomenology at the level of the elementary $NN$ interaction. There exist, however, also high precision non-relativistic empirical potentials such as the Argonne potential or the Nijmegen potentials.

A more systematic and direct connection to QCD is provided by chiral effective field theory (EFT). Up to now the two-nucleon system has been considered at next-to-next-to-next-to-leading order (N$^3$LO) in chiral perturbation theory. In such approaches the $NN$ potential consists of one-, two- and three-pion exchanges and contact interactions which account for the short-range contributions. The advantage of such approaches is the systematic expansion of the $NN$ interaction in terms of chiral power counting. The expansion is performed in powers of $(Q/\Lambda_\chi)^\nu$ where $Q$ is the generic low momentum scale given by the nucleon three-momentum, or the four-momenta of virtual pions or a pion mass.
\( \Lambda_\chi \simeq 4\pi f_\pi \simeq 1 \text{ GeV} \) is the chiral symmetry breaking scale which coincides roughly with the Borel mass \( \Lambda_B \) of the sum rules. In such an expansion the low-energy constants (LECs) related to pion-nucleon vertices can be fixed from pion-nucleon scattering data \cite{24}.

A better understanding of the common features and the differences of the various approaches is essential in order to arrive at a more model independent understanding of the \( NN \) interaction, in particular since all the well established interactions fit \( NN \) scattering data with approximately the same precision. A direct comparison of relativistic phenomenology based on the meson exchange picture with chiral EFT and non-relativistic phenomenology is, however, difficult since the latter two approaches lack of a clear Lorentz structure. At low momentum scales the different potentials can be mapped on each other using renormalization group methods \cite{25}. This led recently to the construction of a “model independent” low momentum potential \( V_{\text{low } k} \) by integrating out the dynamics for momenta above a cut-off scale of about \( \Lambda \simeq 2 \text{ fm}^{-1} \) \cite{25}. It has been argued that beyond this scale the short-range part of the interaction, mediated by vector meson exchange or point-like counter terms, becomes dominant and leads to the deviations of the various approaches.

Although a breakthrough in some sense, the renormalization group approach does not help to clarify the relativistic structure of the potentials which is essential e.g. in order to generate (or not to generate) large scalar/vector mean fields in nuclear matter.

The present work tries to answer this question. We apply projection techniques to map the various potentials on Dirac phenomenology. The philosophy behind this approach is based on the fact, that any \( NN \) interaction, independent whether relativistic or non-relativistic, contains a certain spin-isospin operator structure. By projection techniques this operator basis is mapped on the operator basis of Dirac phenomenology which is given by the Clifford algebra in Dirac space. This allows to identify the different Lorentz components of the interaction. Starting from the angular-momentum representation of a given \( NN \) potential, one transforms to plane-wave helicity states and finally to Lorentz invariant amplitudes in Dirac space \cite{26, 27}. Such a transformation is well defined in the positive energy sector for on-shell amplitudes and allows to compare the \( NN \) potentials on the basis of Lorentz invariant amplitudes. A remarkable agreement between relativistic and non-relativistic OBE potentials, non-relativistic phenomenological potentials and EFT potentials, respectively, has been found in \cite{46}. This agreement is also reflected in the structure of the relativistic self-energy when we further calculate the mean field in infinite nuclear matter in Hartree-
FIG. 1: (Color online) Diagonal matrix elements $V(q,q)$ in the $^1S_0$ partial wave for different high precision $NN$ potential models.

The Fock approximation at tree-level. The scalar and vector self-energy components are found to be large, i.e., of the order of several hundred MeV. The present work extends the investigations of Ref. [46]. The formalism is outlined in detail and we discuss the density dependence of the fields as well as the implications for the nuclear equation-of-state. The connection between chiral EFT and QCD sum rules is investigated. The present formalism allows a quantitative extraction of the scalar/vector fields which are generated by pion dynamics and contact terms at different orders in the chiral expansion.

The paper is organised as follows: in sec. II we discuss the operator structure of the various potentials. The transformation onto the covariant basis is outlined in sec. III where also the results of this analysis namely the Lorentz invariant amplitudes are shown. Sec. IV contains the determination of the relativistic mean fields in nuclear matter. In sec. V the structure of the relativistic self-energy fields from chiral EFT is discussed, as well as the connection to QCD sum rule predictions. Finally the self-consistent Hartree-Fock results for the equation of state calculated with three different potentials (Bonn A, Nijm93 and Nijm I) are discussed in sec. VI.
II. OPERATOR STRUCTURE OF THE NN POTENTIALS

A. OBE potentials

As typical examples for modern high precision OBEPs we consider the Bonn A [37] and the high-precision, charge-dependent Bonn potential (CD-Bonn) [28]. The Bonn potentials are based on the exchange of the six non-strange bosons (π, η, ρ, ω, δ, σ) with masses below 1 GeV. These are the two scalar mesons σ (isoscalar) and δ (isovector), the two pseudo-scalar mesons π (isovector) and η (isoscalar), and the two vector mesons ω (isoscalar) and ρ (isovector). The potentials are derived in the no sea approximation which neglects the coupling to anti-particles.

The Born scattering matrix is given by the sum over the corresponding scalar, pseudo-scalar and vector mesons and has the following structure

\[ \hat{V}(q', q) = \sum_{\alpha = s, ps, v} \mathcal{F}_\alpha^2(q', q) \kappa_\alpha^{(2)} D_\alpha(q' - q) \kappa_\alpha^{(1)}. \] (1)

In the two-nucleon centre-of-mass frame (c.m.) the four-momenta of the incoming nucleons are \( q^{(1)}_\mu = (E(q), \pm q) \) and correspondingly, the four-momenta of the outgoing nucleons are \( q^{(2)}_\mu = (E(q'), \pm q') \). The initial and final relative c.m. momenta are \( q^{(1)}_\mu = \frac{1}{2}(q^{(1)} - q^{(2)}) \) and \( q^{(2)}_\mu = \frac{1}{2}(q^{(1)} - q^{(2)}) \), respectively. For on-shell scattering \( |q| = |q'| \) with \( E(q) = E(q') = \sqrt{M^2 + q^2} \) the energy-transfer is zero, i.e., \( q^{(2)}_\mu - q^{(1)}_\mu = (0, q' - q) \). The matrices (1) factorise for each meson \( \alpha \) into the form factors \( \mathcal{F}_\alpha \) at each meson-nucleon vertex, the meson propagator \( D_\alpha \) and the meson-nucleon vertices \( \kappa_\alpha \) themselves. In the standard Bonn potentials [37] the phenomenological form factors have the form

\[ \mathcal{F}_\alpha(q', q) = \left( \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + (q' - q)^2} \right)^{n_\alpha} \] (2)

where \( m_\alpha \) is the corresponding meson mass and \( \Lambda_\alpha \) is a cut-off in order to avoid divergences at short distances. The meson propagators read

\[ D_{s,ps}(q' - q) = i \frac{1}{(q' - q)^2 - m_{s,ps}^2}, \quad D_{v}^{\mu\nu}(q' - q) = i \frac{-g^{\mu\nu} + (q' - q)^\mu(q' - q)^\nu/m_v^2}{(q' - q)^2 - m_v^2} \] (3)

for scalar and pseudo-scalar mesons \( s, ps \) and vector mesons \( v \). The Dirac structure of the potential is contained in the meson-nucleon vertices

\[ \kappa_s = \frac{g_s}{(2\pi)^2} 1, \quad \kappa_{ps} = \frac{g_{ps}}{(2\pi)^2} \frac{q' - q}{2M} i\gamma^5, \quad \kappa_v = \frac{1}{(2\pi)^2} \left( g_v \gamma^\mu + \frac{f_v}{2M} i\sigma^{\mu\nu} \right). \] (4)
For the pseudo-scalar mesons $\pi$ and $\eta$ a pseudo-vector coupling is used in order to fulfil soft pion theorems. The vertices of the isovector bosons $\pi, \delta, \rho$ obtain additional $\tau_2 \cdot \tau_1$ isospin matrices which are suppressed in Eqs. (4). The $\omega$ meson has no tensor coupling, i.e., $f_v^{(\omega)} = 0$.

The potential, i.e., the OBE Feynman amplitudes are obtained by sandwiching $\hat{V}$ between the incoming and outgoing Dirac spinors

$$V(q', q) = \sum_{\alpha = s, ps, v} F^2_\alpha(q', q) \ D_\alpha(q' - q) \ \bar{u}_2(-q') \kappa^{(2)}_\alpha u_2(-q) \ \bar{u}_1(q') \kappa^{(1)}_\alpha u_1(q) \ .$$

(5)

The relativistic operator structure is thus completely determined by the matrix elements of the vertices $\kappa_\alpha$. In helicity representation the Dirac spinor basis is given by

$$u_\lambda(q) = \sqrt{E + M \over 2M} \ \left( \begin{array}{c} 1 \\ 2 \lambda |q| \\ \frac{E + M}{2M} \end{array} \right) \chi_\lambda \ ,$$

(6)

where $\chi_\lambda$ denotes a two-component Pauli spinor with $\lambda = \pm \frac{1}{2}$. The normalisation of the Dirac spinor is chosen such that $\bar{u}_\lambda u_\lambda = 1$.

A consequence of the Feynman amplitudes (5) is their general non-local structure which distinguishes the field theoretical relativistic OBE approach from local non-relativistic potentials. This is even true for the relativistic OPE compared to the local, non-relativistic OPE (see e.g. the discussion in [29]). However, for on-shell scattering the relativistic amplitudes acquire a local structure in the sense that they are functions of $q'^2$ and $q' - q$. In particular for forward and backward scattering, i.e., $\theta = 0, \pi$, the amplitudes are “local” functions of $q'^2$ and $q$. The non-local structure of the relativistic amplitudes becomes evident when going off-shell, e.g. in the intermediate states in the Bethe-Salpeter equation [29, 30].

The standard Bonn (A,B,C) potentials [37] contain 13 free parameters for coupling constants and cut-off masses and two additional parameters if one considers the masses of the scalar mesons as effective parameters. The matrix elements are calculated with the OBNNS code of R. Machleidt [31] when Bonn A is used and the corresponding CDBONN package of R. Machleidt when CD-Bonn is used.

In contrast to the standard Bonn potentials [37] the OPE part of the CD-Bonn potential [28] accounts for charge symmetry breaking in $nn$, $pp$ and $np$ scattering due to the different pion masses $m_{\pi^0}$ and $m_{\pi^\pm}$. The CD-Bonn potential can be referred to as a phenomenological $NN$ potential, since by fine-tuning of the partial wave fits $\chi^2$ per datum is minimised to 1.02, adding up to a total of 43 free parameters.
1. **Non-relativistic reduction**

The OBE potentials as e.g. the Bonn potentials can be reduced to a non-relativistic representation by expanding the full field-theoretical OBE Feynman amplitudes into a set of spin and isospin operators

\[ V = \sum_i [V_i + V'_i \tau_1 \cdot \tau_2] O_i. \]  

The operators \( O_i \) obtained in this low energy expansion, assuming identical particle scattering and charge independence, are defined as

\[ O_1 = 1, \]
\[ O_2 = \sigma_1 \cdot \sigma_2, \]
\[ O_3 = (\sigma_1 \cdot k)(\sigma_2 \cdot k) \]
\[ O_4 = \frac{1}{2} (\sigma_1 + \sigma_2) \cdot n, \]
\[ O_5 = (\sigma_1 \cdot n)(\sigma_2 \cdot n), \]

where \( k = q' - q, \) \( n = q \times q' \equiv P \times k \) and \( P = \frac{1}{2}(q + q') \) is the average momentum. The potential forms \( V_i \) are then functions of \( k, P, n \) and the energy. In order to perform a non-relativistic reduction, usually the energy \( E \) is expanded in \( k^2 \) and \( P^2 \)

\[ E(q) = \left( \frac{k^2}{4} + P^2 + M^2 \right)^{\frac{1}{2}} \simeq M + \frac{k^2}{8M} + \frac{P^2}{2M}. \]  

and terms to leading order in \( k^2/M^2 \) and \( P^2/M^2 \) are taken into account. The meson propagators \( D_\alpha(k^2) \) given in Eq. (3) are approximated by their static form \((-1)/(k^2 + m^2)\). The equivalent to Eq. (7) in configuration space is given by

\[ O_1 = 1, \]
\[ O_2 = \sigma_1 \cdot \sigma_2, \]
\[ O_3 = S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2, \]
\[ O_4 = L \cdot S, \]
\[ O_5 = Q_{12} = \frac{1}{2} [(\sigma_1 \cdot L)(\sigma_2 \cdot L) + (\sigma_2 \cdot L)(\sigma_1 \cdot L)]. \]

These operators are the well known central, spin-spin, tensor, spin-orbit and quadratic spin-orbit operators, respectively. The total angular momentum is denoted by \( L = r \times P \) and the total spin \( S = \frac{1}{2}(\sigma_1 + \sigma_2) \).
B. Non-relativistic potentials

1. Meson-theoretical potentials

We consider the modern Nijmegen soft-core potential Nijm93 [19] as the first example of a non-relativistic meson-theoretical potential. It is an updated version of the Nijm78 [21] potential, where the low energy $NN$ interaction is based on Regge-pole theory leading to the well known OBE forces. The contributions considered in this model are the pseudo-scalar mesons $\pi$, $\eta$, $\eta'$, the vector mesons $\rho$, $\phi$, $\omega$ and the scalar mesons $\delta$, $S^*$, $\epsilon$ and the Pomeron $P$ and the $J = 0$ tensor contributions, leading all in all to a number of 13 free parameters. Since it is constructed from approximate OBE amplitudes it is based on the operator structure given in Eq. (8) plus an additional operator $O_6 = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L}$ accounting for charge independence breaking which is new compared to the older version Nijm78. Exponential form factors are used. This potential gives a $\chi^2$ per datum of 1.87, which is comparable to similar OBE potentials like the standard Bonn potentials.

2. Phenomenological potentials

Another class of non-relativistic $NN$ potentials are the so called high quality potentials where $\chi^2/N_{data} \approx 1.0$. Here we study the Nijmegen potentials Nijm I, Nijm II and Reid93 [19]. The Nijm I and Nijm II potentials are both based on the Nijm78 potential. In the Nijm I potential some nonlocal terms in the central force are kept whereas in the Nijm II potential all nonlocal terms are removed. Although based on the meson-theoretical Nijm78 potential these potentials are often referred to as purely phenomenological models, since the parameters are adjusted separately in each partial wave leading to a total of 41 parameters. At very short distances, both potentials are regularised by an exponential form factor.

The Nijmegen soft-core Reid93 [19] potential is a phenomenological potential and is therefore based on a completely different approach. In the meson-theoretical Nijmegen potential Nijm93 the potential forms $V_i$ are the same for all partial waves, whereas in the Reid93 potentials every partial wave is parametrised separately by a convenient choice of combinations of central, tensor and spin-orbit functions (local Yukawas of multiples of the pion mass) and the related operators, i.e., the operators $O_1$ to $O_4$ from Eq. (10). It is regularised by a dipole form factor and has 50 phenomenological parameters giving all in all
a $\chi^2/N_{\text{data}} = 1.03$. All the Nijmegen potentials contain the proper charge dependent OPE accounting for charge symmetry breaking in $nn$, $pp$ and $np$ scattering due to different pion masses $m_{\pi^0}, m_{\pi^\pm}$.

The same holds for the Argonne potential $v_{18}$, also an example for a widely used modern high precision phenomenological $NN$ potential. It is given by the sum of an electromagnetic (EM) part, the proper OPE, and a phenomenological intermediate- and short-range part unrestricted by a meson-theoretical picture:

$$V = V^{EM} + V^{\pi} + V^{R}.$$  \hspace{1cm} (11)

The EM interaction is the same as that used in the Nijmegen partial-wave analysis. Short-range terms and finite-size effects are taken into account as well.

The strong interaction part $V^{\pi} + V^{R}$ can be written in a form like given in Eq. (7) in configuration space, where the Argonne $v_{18}$ potential is not constructed by approximating the field-theoretical OBE amplitudes (except for the OPE), but by assuming a very general two-body potential constrained by certain symmetries. The potential forms $V_i$ parametrising the intermediate and short-range part are mostly local Woods-Saxon functions.

The local two-body operators are the same charge independent ones used in the Argonne $v_{14}$ potential

$$O_i = 1, \sigma_1 \cdot \sigma_2, S_{12}, L \cdot S, L^2, L^2(\sigma_1 \cdot \sigma_2), (L \cdot S)^2.$$  \hspace{1cm} (12)

Due to isovector exchange these operators have to be multiplied by the isospin matrices $\tau_1 \cdot \tau_2$ which than adds up to 14 operators. Additionally, four operators accounting for charge independence breaking are introduced

$$O_{i=15,18} = T_{12}, (\sigma_1 \cdot \sigma_2)T_{12}, S_{12}T_{12}, (\tau_{z1} + \tau_{z2}),$$  \hspace{1cm} (13)

where $T_{12} = 3\tau_{z1}\tau_{z2} - \tau_1 \cdot \tau_2$, is the isotensor operator, defined analogously to the spin tensor $S_{12}$ operator.

Thus the operator structure is more general than that imposed by a non-relativistic, local OBE picture, in particular for the intermediate and short distance part. In total, Argonne $v_{18}$ contains 40 adjustable parameters and gives a $\chi^2$ per datum of 1.09 for 4301 $pp$ and $np$ data in the range 0–350 MeV. The code used to calculate the potential matrix elements of the Argonne $v_{18}$ model in momentum space was provided by H. Muether and T. Frick.
C. Low-Energy potentials

1. EFT potentials

Following the concept originally proposed by Weinberg [32], there has been substantial progress in recent time in order to derive quantitative NN potentials from chiral effective field theory. As already mentioned, the chiral expansion is performed in powers of \((Q/\Lambda_c)^\nu\) where \(\nu = 0\) corresponds to leading order (LO), \(\nu = 2\) to next-to-leading order (NLO), \(\nu = 3\) to next-to-next-to-leading (N^3LO) and finally \(\nu = 4\) to next-to-next-to-next-to-leading order (N^4LO). It turned out that for a quantitative description of NN scattering data one has to go up to N^3LO [22, 23, 24] in the chiral expansion for the two-nucleon problem. N^2LO contributions were still found to be very large compared to NLO. This implies that 2\(\pi\) (and 3\(\pi\)) contributions have to be included up to order four. The effective chiral Lagrangian can be written as

\[
L_{\text{eff}} = L^{(2)}_{\pi\pi} + L^{(1)}_{\pi N} + L^{(2)}_{\pi N} + L^{(3)}_{\pi N} + \ldots,
\]

where the superscript refers to the number of derivatives or pion mass insertions (chiral dimension) and the ellipsis stands for terms of chiral order four or higher. The corresponding chiral NN potential is then defined by

\[
V(q', q) \equiv \left\{ \begin{array}{l}
\text{sum of irreducible } \pi + 2\pi \text{ contributions} \\
+ \text{ contacts}.
\end{array} \right.
\]

The 2\(\pi\) exchange contributions to the NN interaction at order four have been derived by Kaiser [33]. Recently, quantitative NN potentials including contact terms at order four were derived by Entem and Machleidt, the so-called Idaho potential [22, 23], and by Epelbaum, Glöckle and Meissner [24].

For the present comparison we apply the Idaho potential [23]. The operator structure of the momentum-space NN amplitude has the general form given in Eq. (7) with the operators \(O_i\) from Eq. (8). The potential forms \(V_i\) \((i = C, S, T, LS, \sigma L)\) can be expressed as functions of \(|(q' - q)|\) and \(|k|\).

The Idaho potential is regularised by an exponential cut-off

\[
V(q', q) \mapsto V(q', q) e^{-(q' / \Lambda)^{2n}} e^{-(q / \Lambda)^{2n}}
\]

where \(\Lambda = 0.5\) GeV in all partial waves. This does not affect the chiral order of the potential,
but introduces contributions beyond that order. The total number of free model parameters in the N³LO potential is 29 \([23]\).

For the evaluation of the matrix elements we applied the N³LO program package provided by D. Entem and R. Machleidt.

2. Renormalization Group approach to NN interaction

Recently, another approach has been proposed to arrive at a better model independent understanding of the NN interaction \([25]\). In this approach a low-momentum potential \(V_{\text{low } k}\) is derived from a given realistic NN potential by separating the low-momentum part, i.e., by integrating out the high-momentum modes, and using renormalization group (RG) methods to evolve the NN potential models from the full Hilbert space to the low momentum subspace. At a cutoff of \(\Lambda = 2.1\) fm\(^{-1}\) all the various NN potential models were found to collapse to a model-independent effective interaction \(V_{\text{low } k}\).

Since elastic NN scattering data constrains the NN interaction only up to a momentum scale of about 400 MeV, which corresponds to the pion threshold, modern high precision potentials differ essentially in the treatment of the short-range part, as depicted in Fig. 1. The philosophy behind the RG approach is to replace the unresolved short distance structure by something simpler, e.g. contact terms, without distorting low-energy observables.

III. TRANSFORMATION TO A COVARIANT OPERATOR BASIS

A. Covariant operators in Dirac space

Any two-body amplitude can be represented covariantly by Dirac operators and Lorentz invariant amplitudes. A detailed discussion of the general structure of relativistic two-body amplitudes can be found in Refs. \([26, 34]\). However, a relativistic treatment invokes automatically the excitation of anti-nucleons. Nucleon-nucleon scattering, in both, the non-relativistic approaches discussed above but also in the framework of the standard OBE potentials is restricted to the positive energy sector and neglects the coupling to anti-nucleons. As a consequence one has to work in a subspace of the full Dirac space which leads to on-shell ambiguities which require some care.
The inclusion of negative energy excitations with 4 states for each spinor yields altogether $4^4 = 256$ types of two-body matrix elements with respect to their spinor structure. Symmetry arguments reduce these to 44 for on-shell particles \(^{34}\). If one takes only the subspace of positive energy solutions into account this leads to $2^4 = 16$ two-body matrix elements. Considering in addition only on-shell matrix elements the number of independent matrix elements can be further reduced by symmetry arguments down to 5. Thus, all on-shell two-body matrix elements can be expanded into five Lorentz invariants. These five invariants are not unique since the Dirac matrices involve always also negative energy states. Therefore a decomposition of the one-body $NN$ potential into a Lorentz scalar and a Lorentz vector contribution depends to some part on the choice of these five Lorentz invariants.

A natural choice of a set of five linearly independent covariant operators to represent a $4 \times 4$ Dirac matrix are the scalar, vector, tensor, axial-vector and pseudo-scalar Fermi covariants

\[
S = 1 \otimes 1, \quad V = \gamma^\mu \otimes \gamma_\mu, \quad T = \sigma^{\mu\nu} \otimes \sigma_{\mu\nu}, \quad P = \gamma_5 \otimes \gamma_5, \quad A = \gamma_5 \gamma^\mu \otimes \gamma_5 \gamma_\mu. \quad (17)
\]

Since one works with physical, i.e., antisymmetrized matrix elements, one has to realize that the Fierz transformation \(^{26}\) couples direct and exchange covariants which mixes the different Lorentz structures

\[
\begin{pmatrix}
\tilde{S} \\
\tilde{V} \\
\tilde{T} \\
\tilde{A} \\
\tilde{P}
\end{pmatrix} = \frac{1}{4}
\begin{pmatrix}
1 & 1 & \frac{1}{2} & -1 & 1 \\
4 & -2 & 0 & -2 & -4 \\
12 & 0 & -2 & 0 & 12 \\
-4 & -2 & 0 & -2 & 4 \\
1 & -1 & \frac{1}{2} & 1 & 1
\end{pmatrix}
\begin{pmatrix}
S \\
V \\
T \\
A \\
P
\end{pmatrix} \quad (18)
\]

The covariants on the left hand side of Eq. \(^{18}\) are the interchanged Fermi covariants defined in Ref. \(^{26}\) as

\[
\tilde{S} = \tilde{S}S, \quad \tilde{V} = \tilde{S}V, \quad \tilde{T} = \tilde{S}T, \quad \tilde{A} = \tilde{S}A, \quad \tilde{P} = \tilde{S}P,
\]

where the operator $\tilde{S}$ exchanges the Dirac indices of particles 1 and 2, i.e., $\tilde{S}u(1)_{\sigma}u(2)_{\tau} = u(1)_{\tau}u(2)_{\sigma}$. Therefore the direct covariants $\Gamma_m$ with $m = \{S, V, T, P, A\}$ can be expressed in terms of the exchange covariants $\tilde{\Gamma}_m$ with $m = \{\tilde{S}, \tilde{V}, \tilde{T}, \tilde{P}, \tilde{A}\}$.

In contrast to the $NN$ potentials where the pion-nucleon coupling is given by a pseudo-vector vertex, the set \(^{17,19}\) contains the pseudo-scalar covariant $P$. This suggests to replace
P in Eqs. (17,19) by the corresponding pseudo-vector covariant
\[
PV = \frac{q' - q}{2M} \gamma_5 \otimes \frac{q' - q}{2M} \gamma_5 .
\] (20)

This leads to an on-shell equivalence since the matrix elements of the pseudo-vector and the pseudo-scalar matrix operators are identical in the case of on-shell scattering between positive energy states:
\[
\bar{u}(q')\frac{q' - q}{2M} \gamma_5 u(q) = \bar{u}(q') \gamma_5 u(q) .
\] (21)

On the other hand the PV vertex suppresses a coupling to antiparticles since the overlap matrix elements vanish for on-shell scattering
\[
\bar{v}(q')\frac{q' - q}{2M} \gamma_5 u(q) = 0 .
\] (22)

In order to identify the PV contributions clearly in the antisymmetrized amplitudes - note that due to the Fierz transformation (18) all operators are coupled - one can switch to a set of covariants originally proposed by Tjon and Wallace [34]. Based on the following operator identities
\[
\frac{1}{2}(T + \bar{T}) = S + \bar{S} + P + \bar{P}
\] (23)
\[
V + \bar{V} = S + \bar{S} - P - \bar{P}
\] (24)

one finds that the following set of covariants
\[
\Gamma_m = \{S, \bar{S}, (A - \bar{A}), PV, \bar{PV}\}
\] (25)
provides a set of Dirac operators for the positive energy sector [34] which completely separates the direct and exchange pv contributions from the remaining operator structure. This has the advantage that the OPE exchange which is dominant at low energies is decoupled from the remaining amplitudes and gives only a contribution to the \(\bar{PV}\) operator. In the following we will refer to the set of covariants in Eq. (25) as the \textit{pseudo-vector} representation and that of Eq. (17) as the \textit{pseudo-scalar} representation. Note that on-shell matrix elements of PV, \(\bar{PV}\) in (25) are equivalent to those where the pseudo-vector covariants are replaced by P, \(\bar{P}\).

The on-shell equivalence does not affect physical observables which are built on complete matrix elements as e.g. the single particle potential \(U\)
\[
U(k)_{s.p.} \propto \sum_q \langle \bar{u}(k)\bar{u}(q)|\hat{V}(k,q)|u(k)u(q) - u(q)u(k)\rangle
\] (26)
but it leads to uncertainties in operators which are, like the self-energy $\Sigma$, based on traces over only one particle. As discussed in [35], a pseudo-vector $\pi N$ coupling leads to the pseudo-vector representation (25) as the most natural choice of the relativistic operator basis.

B. Projection onto the covariant operators

In this section the technique is described necessary to project the Born amplitudes from an angular-momentum basis onto the covariant basis, given by Eqs. (17) or (25). The procedure is standard and runs over the following steps

$$|LSJ\rangle \rightarrow \text{partial wave helicity states} \rightarrow \text{plane wave helicity states} \rightarrow \text{covariant basis}.$$ 

The first two transformation can be found in Refs. [36, 37]. The last step depends on the choice of the covariant operator basis, see e.g. [14, 27]. Here we sketch the essential steps briefly.

Independent of the various models, the amplitudes are determined normally in the $|LSJM\rangle$-representation and can be denoted as $V^{JS}_{L'S';L}(q', q)$. In case of on-shell scattering ($|q| = |q'|$), due to time-reversal invariance and spin and parity conservation, only five of sixteen possible matrix elements are linearly independent for a fixed total angular momentum $J$ (spin singlet and triplet states). By inversion of Eq. (3.32) in [36] these five partial wave amplitudes are transformed from the $|LSJM\rangle$-representation into the partial wave helicity representation $|JM\lambda_1\lambda_2\rangle$ and are then decoupled via inversion of Eq. (3.28) from Ref. [36]. Since we deal with two-nucleon states which are two-fermion states, we have to evaluate the fully antisymmetrized matrix elements by restoring the total isospin $I = 0, 1$ via the standard selection rule

$$(-1)^{L+S+1} = -1. \quad (27)$$

The five plane wave helicity matrix elements are then obtained by a summation over the total angular momentum $J$

$$\langle \lambda_1'\lambda_2'|V^I|\lambda_1\lambda_2\rangle = \sum_J \left( \frac{2J + 1}{4\pi} \right) d^J_{\lambda\lambda'}(\theta) \langle \lambda_1'\lambda_2'|V^{J1}(q', q)|\lambda_1\lambda_2\rangle. \quad (28)$$

Here $\theta$ denotes the scattering angle between $q'$ and $q$ while $\lambda = \lambda_1 - \lambda_2$ and $\lambda' = \lambda_1' - \lambda_2'$ denote the in- and outgoing helicity states. The reduced rotation matrices $d^J_{\lambda\lambda'}(\theta)$ are those defined by Rose [38].
These plane wave helicity matrix elements can now be projected onto a set of five covariant amplitudes in Dirac space. A set of five linearly independent covariants is sufficient for such a representation since on-shell we deal with five matrix elements independent of the chosen representation. Using the covariants of Eq. (17) (the ’pseudo-scalar choice’) the on-shell potential matrix elements for definite isospin I can be represented covariantly as

\[
\hat{V}^I(|q|, \theta) = F^I_\Sigma(|q|, \theta) S + F^I_\chi(|q|, \theta) V + F^I_T(|q|, \theta) T \\
+ F^I_P(|q|, \theta) P + F^I_A(|q|, \theta) A.
\]  

(29)

The Lorentz invariant amplitudes \(F^I_m(|q|, \theta)\) with \(m = \{S, V, T, P, A\}\) from Eq. (29) depend only on the relative c.m. momentum \(|q|\) and the scattering angle \(\theta\) and are related to the plane wave helicity states defined in Eq. (28) by

\[
\langle \lambda'_1 \lambda'_2 q' | V^I | \lambda_1 \lambda_2 q \rangle = \sum_m \langle \lambda'_1 \lambda'_2 q' | \Gamma_m | \lambda_1 \lambda_2 q \rangle F^I_m(|q|, \theta).
\]  

(30)

The indices (1) and (2) refer to particle one and two. Eq. (30) is a matrix relation between the five independent plane wave helicity amplitudes \(V^I_i\) (where \(i = \{\lambda'_1, \lambda'_2, \lambda_1, \lambda_2\} = 1, \ldots, 5\) denotes five of sixteen possible amplitudes) and the five unknown covariant amplitudes \(F^I_m(|q|, \theta)\). For fixed values of the variables (\(|q| = |q'|, \theta\)) this equation can be written in a more compact form

\[
V^I_i = \frac{1}{M^2} \sum_m C_{im} F^I_m.
\]  

(31)

The covariant amplitudes \(F^I_m\) are obtained by matrix inversion of Eq. (31) which corresponds to Eq. (3.23) of Ref. [27].

Eq. (31) has to be inverted for two scattering angles, i.e., for \(\theta = 0\) for the direct and \(\theta = \pi\) for the exchange part of the interaction. These two scattering angles are required for the Hartree-Fock potential. Details of the inversion of Eq. (31), as well as the treatment of kinematical singularities of the matrix \(C_{im}\) occurring at \(\theta = 0\) and \(\theta = \pi\) are given in appendix C of Ref. [27] where Eq. (31) is explicitly given for \(\theta = 0\) and \(\theta = \pi\) (Eqs. (C10,11)). Following Ref. [27] we calculate the real part of the five Lorentz invariant amplitudes \(F^I_m(|q|, \theta = 0, \pi)\) for the direct and exchange case in both, the isospin singlet and triplet channels. When derived from physical partial wave amplitudes which are already antisymmetrized according to the selection rule (27), the exchange amplitudes \(F_m(|q|, \pi)\) contain redundant information.
Since we are restricted to the subspace of positive energy states, the choice of a set of five linearly independent covariants suffers from on-shell ambiguities, as discussed above. Thus the set of covariants (25) is a more appropriate choice [14]. In this representation the scattering matrix reads [14, 34]

\[
\hat{V}^1(|q|, \theta) = g^1_S(|q|, \theta) S - g^1_S(|q|, \theta) \tilde{S} + g^1_A(|q|, \theta) (A - \tilde{A}) + g^1_{PV}(|q|, \theta) PV - g^1_{PV}(|q|, \theta) \tilde{PV}.
\]  

(32)

The new amplitudes \(g^1_m\) are related to the Lorentz invariant amplitudes \(F^1_m\) from Eq. (29) by the linear transformation

\[
\begin{pmatrix}
g^1_S \\
g^1_S \\
g^1_A \\
g^1_{PV} \\
g^1_{PV}
\end{pmatrix} = \frac{1}{4}
\begin{pmatrix}
4 & -2 & -8 & 0 & -2 \\
0 & -6 & -16 & 0 & 2 \\
0 & -2 & 0 & 0 & -2 \\
0 & 2 & -8 & 4 & 2 \\
0 & 6 & -16 & 0 & -2
\end{pmatrix}
\begin{pmatrix}
F^1_S \\
F^1_S \\
F^1_A \\
F^1_P \\
F^1_A
\end{pmatrix}
\]  

(33)

As mentioned before, the representation of the potential given in Eq. (32) has the advantage that the OPE contribution to the amplitudes is completely decoupled from the rest of the interaction. The OPE contributes only in the pseudo-vector exchange amplitude \(g^\text{OPE}_{PV}\) and vanishes in all other amplitudes \(g^\text{OPE}_S = g^\text{OPE}_S = g^\text{OPE}_A = g^\text{OPE}_{PV} = 0\). Thus one avoids that the low momentum behaviour of these four amplitudes is to large extent dominated by OPE exchange contributions which are present in all five amplitudes \(F^1_m\) from Eq. (29) due to the Fierz transformation. In order to compare the various potentials at the level of covariant amplitudes the pseudo-vector representation is therefore the most efficient and transparent one.

C. Covariant amplitudes

In order to demonstrate the dependence of the relativistic amplitudes on the choice of the operator basis we consider in Fig. 2 first the single OPE. The figure shows the corresponding amplitudes \(F_m\) of the pseudo-scalar representation (17) and the \(g_m\) amplitudes of pseudo-vector representation (25), both for the OPE part of the Bonn A potential. Since we are dealing with antisymmetrized amplitudes it is sufficient to consider the direct Lorentz invariants \(F_m(|q|, \theta = 0)\) and \(g_m(|q|, \theta = 0)\) at scattering angle \(\theta = 0\). As the starting point
the OPE is given in the $|LSJ\rangle$ basis and antisymmetrization is ensured by the selection rule (27). The figure shows the isospin averaged amplitudes defined as
\[
F_m(|q|, 0) := \frac{1}{2} \left[ F_{I=0}^m(|q|, 0) + 3 F_{I=1}^m(|q|, 0) \right]
\]
and correspondingly for $g_m$. It is evident that in the pseudo-scalar representation all amplitudes $F_m$ have large non-vanishing contributions from OPE due to the mixing of direct and exchange contributions described by the Fierz transformation (18). Moreover, as discussed above the on-shell equivalence for the pseudo-scalar covariant P and the pseudo-vector covariant $\tilde{P}V$ in (29) leads to identical Lorentz invariant amplitudes $F_{PS} = F_{PV} \equiv F_P$ [14].

The pseudo-vector representation (25), on the other hand, has the advantage that it decouples the OPE contribution from the remaining amplitudes, i.e., the OPE gives a non-zero contribution only in the $g_{\tilde{P}V}$ amplitude while the others are zero. For the single pion exchange $g_{\tilde{P}V}$ is now easy to interpret: it is just the pion propagator (3) times the pion-nucleon form factor (2).

When the various $NN$ potentials are compared, this is done most efficiently in the pseudo-vector representation. All potentials contain an OPE of similar strength which dominates at small momenta. The pseudo-vector representation decouples the OPE contribution from the remaining amplitudes $g_m \neq g_{\tilde{P}V}$ and allows thus a more transparent investigation of the short and intermediate range parts of the potentials which are actually the interesting ones.

Fig. 3 shows the isospin-averaged amplitudes $g_m^D(|p|, \theta = 0)$ for Bonn A, CD-Bonn, Argonne.
FIG. 3: (Color online) Isospin-averaged Lorentz invariant amplitudes $g_m^D(|q|, \theta = 0)$ for the different NN potentials after projection on the Dirac operator structure. The pseudo-vector representation of the relativistic operator basis is used. As a reference the amplitudes from solely OPE and from $\sigma + \omega$ exchange, both with Bonn A parameters, are shown.
v_{18}, Nijm93, Nijmegen I and II, Reid93, the effective low momentum interaction $V_{\text{low } k}$ and the chiral Idaho potential. The amplitudes are obtained going through the transformation scheme discussed above. Partial waves are taken into account up to $J = 90$ (Bonn A, CD-Bonn, Idaho), $J = 9$ (Argonne $v_{18}$, Nijmegen I/II, Nijm93, Reid93) and $J = 6$ ($V_{\text{low } k}$).

The amplitudes determined from the complete $NN$ potentials are no more as easy to interpret as for a single meson exchange where they represent essentially the propagators times the form factors. This is also true for the full OBE since the contributions from the various mesons are coupled through their exchange parts. Since these amplitudes are not very transparent quantities, Fig. 3 includes as a reference in addition the contributions from only OPE and from only $\sigma$ and $\omega$ exchange, both taken from Bonn A.

Several features can now be seen from Fig. 3. First of all the four amplitudes $g_S$, $g_{S'}$, $g_A$ and $g_{PV'}$ are very close for the OBEPs Bonn A, CD-Bonn and Nijm93 and the phenomenological non-relativistic Argonne $v_{18}$ and Nijmegen I/II potentials. Only at very small $|q|$ Argonne $v_{18}$ shows a deviating structure. The direct pseudo-vector amplitude $g_{PV}$ falls somewhat out of systematics. This amplitude is, however, of minor importance since it does not contribute to the Hartree-Fock self-energy (41-43) and to the single particle potential.

The dominance of the OPE at low $|q|$ is reflected in the pseudo-vector exchange amplitude $g_{PV'}$ which is at small $|q|$ almost two orders of magnitude larger than the other amplitudes. In the OBEPs the high momentum part of the interaction, on the other hand, is dominated by heavy meson exchange and the corresponding amplitudes $g_S$, $g_{S'}$, $g_A$ approach the $\sigma + \omega$ exchange result. Deviations from the $\sigma + \omega$ amplitudes, e.g. due to exchange of isovector mesons $\rho$ and $\delta$ in the OBEPs are moderate at large $|q|$. These deviations are more pronounced at small $|q|$.

The remarkable agreement between the OBE amplitudes and those derived from the non-relativistic Argonne $v_{18}$ potential demonstrates two things: first of all, it means that for on-shell scattering the Argonne $v_{18}$ can be mapped on the relativistic operator structure where the local phenomenological functions $V_i$, Eq. (7), play the same role as the meson propagators plus corresponding form factors in the meson exchange picture. Secondly, the effective treatment of the short-distance physics in Argonne $v_{18}$ is very similar to that in the OBE potentials Bonn A, CD-Bonn and Nijm93. This fact can be estimated from Fig. 11 where the $^1S_0$ partial wave amplitudes are close as well. On the other hand the softer character of the Reid93 and also the Nijmegen I and II potentials is reflected clearly in the
stronger deviation from the $\sigma + \omega$ amplitudes at large $|q|$.

Finally we are turning to the effective low momentum potentials $V_{\text{low } k}$ and the chiral Idaho $N^3\text{LO}$ potential. $V_{\text{low } k}$ is only shown up to the intrinsic cut-off of 400 MeV. In this momentum range the amplitudes fall practically on top of those from the Idaho $N^3\text{LO}$ potential. At low $|q|$ the amplitudes derived from Idaho $N^3\text{LO}$ and $V_{\text{low } k}$ behave qualitatively and quantitatively like the previous ones, i.e., they are very close to Bonn A, CD-Bonn and Argonne $v_{18}$. We conclude that also the effective low momentum potentials can be mapped on a relativistic operator structure. For the Idaho $N^3\text{LO}$ potential which is also based on the operator structure given in Eq. (8), the functions $V_i$ and $V'_i$ in combination with the corresponding operators, derived from fourth order $2\pi$ exchange plus contact terms, lead to a structure which is similar to that imposed by the OBE picture. However, clear deviations appear in the cut-off region between 400 and 500 MeV. The short-range interactions are strongly suppressed by the exponential cut-off form factors and as a consequence the Idaho approaches rapidly the OPE result for momenta above 400 MeV.

IV. SELF-ENERGY IN NUCLEAR MATTER

With the covariant amplitudes at hand, one is able to determine the relativistic mean field in nuclear matter with its scalar and vector components. To do so, we calculate the relativistic self-energy $\Sigma$ in Hartree-Fock approximation at tree level. We are thereby not aiming for a realistic description of nuclear matter saturation properties which would require a self-consistent scheme. Moreover, short-range correlations require to base such calculations on the in-medium T-matrix rather than the bare potential $V$. This leads to the relativistic Dirac-Brueckner-Hartree-Fock scheme which has been proven to describe nuclear saturation with quantitatively satisfying accuracy\cite{13,14,15,47}. The self-consistent iteration of the self-energy in combination with the Dyson equation for the in-medium nucleon propagator and the Bethe-Salpeter equation for the in-medium T-matrix leads to self-energy components which are qualitatively of similar magnitude than the tree level results, as will be seen later on.

The self-energy is determined by the summation of the interaction of a nucleon with
four-momentum \( k \) with all nucleons inside the Fermi sea in Hartree-Fock approximation

\[
\Sigma_{\alpha\beta}(k, k_F) = -i \int \frac{d^4q}{(2\pi)^4} G^D_{\tau\sigma}(q) \left[ V(|p|, 0)_{\alpha\sigma;\beta\tau} - V(|p|, \pi)_{\alpha\sigma;\beta\tau} \right].
\] (35)

Since we work with fully antisymmetrized matrix elements which contain already the direct (Hartree) and exchange (Fock) contributions, it is sufficient to evaluate the Hartree integral for the self-energy

\[
\Sigma_{\alpha\beta}(k, k_F) = -i \int \frac{d^4q}{(2\pi)^4} G^D_{\tau\sigma}(q) \left[ V^A(|p|, 0)_{\alpha\sigma;\beta\tau} \right].
\] (36)

\( G^D(q) \) is the Dirac propagator describing the on-shell propagation of a nucleon with momentum \( q \) inside the Fermi sea in the nuclear matter rest frame

\[
G^D(q) = (\slashed{q} + M)2\pi i\delta(q^2 - M^2)\Theta(q_0)\Theta(k_F - |q|).
\] (37)

The \( \Theta \) functions account for the fact that only positive energies are considered. Here, \( k \), taken along the z-axis, is the single particle momentum of the incoming nucleon in the nuclear matter rest frame. The relative momentum in the two-nucleon c.m. frame where the matrix elements \( V \) are evaluated, is given by \( |p| = \sqrt{s/4 - M^2} \), where \( s = (E(k) + E(q))^2 - (k + q)^2 \) is the total energy of the two nucleons.

Using the pseudo-vector representation for the on-shell matrix elements \( V \), Eq. (32), the self-energy operator reads

\[
\Sigma_{\alpha\beta}(k, k_F) = \int \frac{d^3q}{(2\pi)^3} \Theta(k_F - |q|) \left\{ \frac{4E(q)}{(\slashed{q} + \slashed{g})^2} \frac{2g_\mu(k^\mu - q^\mu)}{4M^2} g_{\tilde{P}V} \right. \\
+ m1_{\alpha\beta} \left[ 4g_S - g_\Lambda + 4g_\Lambda - \frac{(k^\mu - q^\mu)^2}{4M^2} g_{\tilde{P}V} \right] \\
+ \left. \frac{\gamma_5}{4M^2} \right\}.
\] (38)

Translational and rotational invariance, hermiticity, parity conservation, and time reversal invariance determine the Dirac structure of the self-energy [27]. In the nuclear matter rest frame the self-energy can be written as

\[
\Sigma(k, k_F) = \Sigma_s(k, k_F) - \gamma_0 \Sigma_0(k, k_F) + \gamma \cdot k \Sigma_v(k, k_F).
\] (39)

Note that the sign convention for the vector field \( \Sigma = \Sigma_s - \gamma_\mu \Sigma^\mu \) with \( \Sigma^\mu = (\Sigma_0, k \Sigma_v) \) in Eq. (39) is that used standardly in DBHF [12, 14, 16]. It differs from that used standardly...
in QHD \((\Sigma = \Sigma_s + \gamma_\mu \Sigma^\mu)\) and also that of Eqs. (49) and (51). The self-energy components are Lorentz scalar functions depending on the Lorentz invariants \(k^2, k \cdot j\) and \(j^2\), where \(j_\mu\) denotes the four-vector baryon current. In nuclear matter at rest the time-like component is just the baryon density and spatial components of the current vanish, i.e., \(j_\mu = (\rho_B, 0)\). Hence, the Lorentz invariants can be expressed in terms of \(k_0, |k|\) and \(k_F\), where \(k_F\) denotes the Fermi momentum. The components of the self-energy are computed by taking the respective traces in the Dirac space [27, 39]

\[\Sigma_s = \frac{1}{4} tr [\Sigma], \quad \Sigma_0 = -\frac{1}{4} tr [\gamma_0 \Sigma], \quad \Sigma_v = -\frac{1}{4|k|^2} tr [\gamma \cdot k \Sigma].\] (40)

In doing so, the Lorentz components of the self-energy operator (38) are given by

\[\Sigma_s(k, k_F) = \frac{1}{4} \int \frac{d^3q}{(2\pi)^3} \Theta(k_F - |q|) M(E(q)) \left[ 4g_s - g_S + 4g_A - \frac{(k^\mu - q^\mu)^2}{4M^2} g_{PV} \right],\] (41)

\[\Sigma_0(k, k_F) = \frac{1}{4} \int \frac{d^3q}{(2\pi)^3} \Theta(k_F - |q|) \left[ g_S - 2g_A + \frac{E(k)}{E(q)} \frac{(k^\mu - q^\mu)^2}{4M^2} g_{PV} \right],\] (42)

and

\[\Sigma_v(k, k_F) = \frac{1}{4} \int \frac{d^3q}{(2\pi)^3} \Theta(k_F - |q|) \frac{k \cdot q}{|k|^2 E(q)} \left[ g_S - 2g_A + \frac{k_z (k^\mu - q^\mu)^2}{4M^2} g_{PV} \right].\] (43)

In Fig. 4 the tree level scalar and vector self-energy components in nuclear matter are shown obtained with the various \(NN\) potentials at nuclear saturation density with Fermi momentum \(k_F = 1.35\text{ fm}^{-1}\) which corresponds to a density of \(\rho = 0.166\text{ fm}^{-3}\). As a remarkable result, all potentials yield scalar and vector mean fields \(\Sigma_s\) and \(\Sigma_0\) of comparable strength: a large and attractive scalar field \(\Sigma_s \simeq -(450 \div 400)\text{ MeV}\) and a repulsive vector field of \(-\Sigma_0 \simeq + (350 \div 400)\text{ MeV}\). These values are comparable to those derived from RMF phenomenologically and also from QCD sum rules. Also the explicit momentum dependence of the self-energy is similar for the various potentials. The Idaho mean fields follow the other approaches at low \(k\) but show a stronger decrease above \(k \simeq 2\text{ fm}^{-1}\) which reflects again the influence of the cut-off parameter. Fig. 5 shows the spatial component of the vector self-energy \(k \Sigma_v\), Eq. (43). Also here the various potentials agree quite well. As known from self-consistent DBHF calculations [12, 14], the spatial vector self-energy is a moderate correction to the large scalar and time-like vector components \(\Sigma_s\) and \(\Sigma_0\). This is found to be also the case at tree level where \(k \Sigma_v\) is about one order of magnitude smaller than the other

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FIG. 4: (Color online) Tree level scalar and vector self-energy components in nuclear matter at $k_F = 1.35$ fm$^{-1}$ obtained with different $NN$ interaction models.

FIG. 5: (Color online) Tree level spatial vector self-energy component $k\Sigma_V$ in nuclear matter at $k_F = 1.35$ fm$^{-1}$ for the various potentials.
two components. The spatial self-energy originates exclusively from exchange contributions, i.e., the Fock term, and vanishes e.g. in the mean field approximation of RMF theory.

Fig. 6 displays the density dependence of the fields, evaluated at momentum \( k = k_F \). At moderate densities the different potentials yield scalar and vector fields which are rather close in magnitude. At higher densities the results start to split up which reflects again the different treatment of short distance physics in the various interactions. Only the two low momentum interactions Idaho N^3LO and \( V_{\text{low } k} \) lie practically on top of each other. In this context we want to stress again that these results are obtained in lowest order in density. Hence, the results are only 'realistic' in the low density limit but not at higher densities since short-range correlations are missing.

In order to estimate the influence of short-range correlations and self-consistency, in Fig. 7 the tree level result from Fig. 4 for Bonn A to a corresponding full DBHF calculation are compared at \( k_F = 1.35 \text{ fm}^{-1} \). For DBHF the approach of [14] is used (subtracted T-matrix in \( pv \) representation). The DBHF calculation yields reasonable saturation properties with a binding energy of \( E_{\text{bind}} = -15.72 \text{ MeV} \) and a saturation density of \( \rho = 0.181 \text{ fm}^{-3} \) [14]. It is
no doubt that higher order correlations are essential for saturation of nuclear matter. The correlations lead to a general reduction of the vector self-energy by a shift of about 70 MeV. Self-consistency and correlations also weakens the momentum dependence, in particular for $\Sigma_s$. However, except of the 70 MeV shift of $\Sigma_0$, the absolute magnitude of the self-energies is not strongly modified in the realistic calculation. This means that one can expect that the large attractive scalar and repulsive vector mean fields will also persist for the other interactions when short-range correlations are accounted for in a full relativistic many-body calculation.

Fig. 7 shows finally the single particle potential in nuclear matter at $k_F = 1.35$ fm$^{-1}$, determined from the relativistic self-energy components. The single particle potential is defined as the expectation value of the self-energy

$$U_{s.p.}(k, k_F) = \frac{<u(k)|\gamma^0\Sigma|u(k)>}{<u(k)|u(k)>} = \frac{M}{E(k)} <\bar{u}(k)|\Sigma|u(k)>$$

(44)
and reads
\[
U_{s.p.}(k, k_F) = \frac{M}{E} \Sigma_s - \frac{k \mu \Sigma \mu}{E} = \frac{M \Sigma s}{\sqrt{k^2 + M^2}} - \Sigma_0 + \frac{\Sigma_s k^2}{\sqrt{k^2 + M^2}}.
\]  
\[\text{(45)}\]

Eq. (45) represents the single particle potential at tree level, i.e., the expectation value of \(\Sigma\) with the bare spinor basis. The next step towards a self-consistent treatment would be to use an in-medium spinor basis which includes the scalar and vector self-energy components via effective masses and effective four-momenta
\[
M^*(k, k_F) = M + \Sigma_s(k, k_F), \quad k^*_\mu = k_\mu + \Sigma_\mu(k, k_F).
\]  
\[\text{(46)}\]

This would, however, involve higher order corrections in the baryon density and is not intended in the present investigations which are restricted to leading order.

The single particle potential reflects the well known fact that phase-shift equivalent two-body potentials which describe \(NN\) scattering data with about the same accuracy \[30\], can be rather different \[30\]. This can already be seen from Fig. 1 where the \(^1S_0\) matrix elements of the various potentials are shown. The differences are mainly due to a different treatment of the short-range part of the nuclear interaction, i.e., the hard core which is not well constraint by scattering data. Thus the various potentials lead to about the same T-matrices when iterated in the Lippmann-Schwinger or Bethe-Salpeter equation. However, at tree-level the hard core contributes fully to \(U_{s.p.}\) which explains the shift of the various results in Fig. 8. Integrating out the high momentum components, e.g. by renormalization group methods, one arrives at equivalent low-momentum potentials \(V_{\text{low } k}\) \[25\]. Since \(V_{\text{low } k}\) contains no significant contributions from the hard core it gives already at tree level a realistic single-particle potential. The situation is similar for the chiral EFT \(N^3\)LO Idaho potential. As can be seen from Fig. 1 Idaho is rather close to \(V_{\text{low } k}\), not only in the \(^1S_0\) partial wave, and correspondingly both lead to comparable potentials. However, the slight shift of about 10 MeV between \(V_{\text{low } k}\) and Idaho reflects again the subtle cancellation effects between the large scalar/vector fields, since at the scale of the fields, Fig. 6, both lie practically on top of each other.

In the present context the single particle potential serves as an important check of the whole procedure. In Fig. 8 the single particle potential \(U_{s.p.}\) is shown, calculated from Eq. (45), i.e., after projecting the \(NN\) potentials from the partial wave basis onto the covariant operator basis, determining then the relativistic self-energy components and finally \(U_{s.p.}\). Fig. 8 includes also the results from a ‘non-relativistic’ calculation of \(U_{s.p.}\) where
FIG. 8: (Color online) Single particle potential in nuclear matter at $k_F = 1.35$ fm$^{-1}$, determined from the tree level Born amplitudes of the various potentials. The single particle potential determined from the relativistic self-energy components after projection onto the covariant operator basis is compared to a non-relativistic calculation (stars) where partial wave amplitudes are summed up directly.

The partial wave amplitudes are directly summed up. To do so we used a non-relativistic Brueckner-Hartree-Fock program [40] and determined the single particle potential in Born approximation. The non-relativistic results are represented by stars in Fig. 8 and shown up to a momentum of 400 MeV. This avoids distortions from non-relativistic kinematics which occur at higher momenta. At moderate momenta the non-relativistic and the relativistic calculations show an excellent agreement which demonstrates the accuracy of the applied projection techniques. One has thereby to keep in mind that $U_{s.p.}$ originates in the relativistic approach from the cancellation of the two scalar and vector fields which are both of the order of about 400 MeV.
V. THE STRUCTURE OF THE SELF-ENERGY FROM CHIRAL EFT

With the projection formalism at hand one is now able to investigate the connection between the appearance of the matter fields and chiral dynamics in more detail. It allows in particular a straightforward and transparent discussion of the contributions which arise at different orders in the chiral expansion of the $NN$ interaction, see Eqs. (14) and (15). Such an investigation allows also to build the bridge to the reduction of the in-medium quark condensates which is usually interpreted as a signature for a partial restoration of chiral symmetry.

A. Role of contact terms

We are now in the situation to calculate the relativistic scalar and vector self-energies from a chiral EFT nucleon-nucleon potential order by order. For this purpose we apply again the chiral Idaho potential [44]. This allows to separate the contributions from different orders in the chiral expansion of the $NN$ interaction and provides a connection to the low energy constants (LECs) which appear at the different orders.

Fig. 9 shows the tree level results for the scalar and vector self-energy components in nuclear matter at $k_F = 1.35$ fm$^{-1}$ obtained in leading order (LO) up to next-to-next-to-next-to-leading order (N$^3$LO).

To leading order the chiral $NN$ interaction does not generate significant mean fields. The scalar self-energy $\Sigma_s$ is of the order of about -70 MeV and the vector self-energy is practically zero. At LO only the static OPE and contact terms without derivatives appear which involve the operators $O_1$ and $O_2$ from the operator basis (8). Hence at LO no pieces from vector exchange occur which would involve all operators $O_i, i = 1..5$. The small scalar field means, on the other hand, that the nucleon mass $M^*$, Eq. (46), does not change significantly in matter to leading order in chiral EFT. The dominant contributions arise at next-to-leading order (NLO). NLO involves leading two-pion-exchange (2PE) and contact terms with two derivatives. The NLO contact terms contain the full operator structure $O_i$. At this level both, scalar and vector self-energy components of about $\mp 400$ MeV magnitude are generated. Also the signs, i.e., the attractive scalar and the repulsive vector mean field, are fixed at NLO. The higher orders, N$^2$LO and N$^3$LO provide corrections which tend to
reduce the NLO result, are, however, moderate. \(N^2\text{LO}\) contains subleading 2PE and no contact terms at all, while \(N^3\text{LO}\) contains sub-subleading 2PE, leading three-pion-exchange, corrections to OPE and 2PE and contact terms with four derivatives \[24\].

In order to investigate the role of pion dynamics and that of contact terms in more detail, Table II contains the contributions which arise from pion dynamics \(\Sigma^{(\pi)}\), i.e., OPE, 2PE, 3PE and corrections, and those from the contact terms \(\Sigma^{(\text{cont})}\) separately. The contributions to the self-energy at a particular order is given by the sum \(\Sigma^{(\pi)} + \Sigma^{(\text{cont})}\), the full self-energy at a certain order \(\nu\) is obtained by adding the contributions from the lower orders \(\Sigma^{(\nu)} = \sum_{\lambda=0}^{\nu} \Sigma^{(\lambda)}\). From Table II it becomes evident that the dominant contributions to the scalar and vector self-energy are generated by the contact terms which arise at next-to-leading order. At \(N^2\text{LO}\) no contact terms occur in the chiral expansion. The \(N^3\text{LO}\) contacts provide sizeable corrections to both, scalar and vector self-energy components and are of opposite sign than the NLO contributions. The contribution from pion dynamics to
TABLE I: Contributions from pion dynamics and contact terms to the scalar and vector self-energy components (in MeV) which appear at different orders in the chiral expansion. The evaluation is performed at nuclear saturation density $k_F = 1.35$ fm$^{-1}$.

The self-energy components are found to be generally moderate. The largest contributions appear at N$^3$LO and are of opposite sign than those from corresponding contact terms.

Hence the reduction of the nucleon mass $M^* = M + \Sigma_s$ is driven by short-distance physics, dominantly by contact terms which occur at NLO. These are four-nucleon contacts with two derivatives. At this order the short-range spin-orbit interaction (proportional to $O_4$ in (8))

$$iC_5(\sigma_1 + \sigma_2) \cdot (q \times q')$$

is generated. The appearance of large scalar/vector fields at NLO is therefore in perfect agreement with Dirac phenomenology where the large spin-orbit force is intimately connected to the appearance of the scalar/vector fields which are generated by short-range isoscalar scalar ($\sigma$) and vector meson ($\omega$) exchange [14, 47]. In EFT the strength of the short-range spin-orbit interaction is determined by the $C_5$ parameter which is given by a linear combination of the $^3P$-wave low energy constants (LECs) [23, 24]

$$C_5 = \frac{1}{16\pi} [2C_{3P0} + 3C_{3P1} - 5C_{3P2}] .$$

Hence the short-range spin-orbit interaction is dictated by $P$-wave $NN$ scattering. As shown by Kaiser [52] the large values of the $C_5$ parameter is in good agreement with corresponding values extracted from high precision OBE type potentials (Bonn, CD-Bonn, Nijm93, Nijmegen I,II) and from Argonne $v_{18}$ which are all in the range of $3C_5/8 \sim 80 \div 90$ MeV fm$^5$. In [52] these values were also compared to purely phenomenological Skyrme type density functionals designed for nuclear structure calculations [53, 54]. The values of the corresponding spin-orbit strength parameter $W_0$ in Skyrme models are also very close in magnitude,
i.e. $3W_0/4 \sim 75 \div 97$ MeV fm$^5$. The contribution from chiral OPE to the spin-orbit terms in the density functional were found to be almost negligible (less than 1%). The lowest order irreducible 2PE which occurs at NLO in the chiral expansion provides moderate corrections to the iso-scalar spin-orbit strength function whereas the iso-vector strength is more strongly affected (2PE contributions lead to a $\sim 30\%$ reduction) \cite{52}. Thus the analysis of Kaiser is fully consistent with the small fields $\Sigma_s^{(\pi)}$ and $\Sigma_0^{(\pi)}$ of $\sim \pm 5$ MeV generated by pion dynamics at NLO, as observed within the framework of the present analysis.

Fig. 10 analyses the dependence of the fields on the value of the $C_5$ low energy constant in more detail. As already mentioned, at LO two contact terms ($C_1$ and $C_2$) appear and at NLO, respectively, 5 contacts ($C_3$ to $C_7$). The figure contains the full NLO result, including contributions from LO and NLO pion dynamics and contacts and compares this to the case where all contacts which appear up to NLO were switched off except of the $C_5$ contribution. It contains in addition results with again all contributions, however, scaling the value of $C_5$
down to 50%, 10% and 0.1%. It becomes evident that the large scalar and vector mean fields are a direct consequence of the large value of $C_5$. Chiral EFT is therefore not only in qualitative but quantitative agreement with the picture known from meson-exchange. In both cases the fields are related to short distance physics and their strength is dictated by $P$-wave $NN$ scattering data where the spin-orbit forces occur.

B. Connection to QCD sum rules

In finite density QCD sum rules scalar and vector fields arise naturally from the structure of the quark propagator which is proportional to the corresponding condensates. As shown by Cohen et al. [6] the quark correlation function can be expressed to leading order in terms of the scalar condensate $\langle \rho | \bar{q} q | \rho \rangle$ already present in vacuum, and the vector condensate $\langle \rho | q^\dagger q | \rho \rangle$ which is introduced by the breaking of Lorentz invariance due to the presence of the medium. The identification of the correlation function with the in-medium nucleon propagator of a dressed quasi-particle leads to scalar and vector self-energies $\Sigma_s$ and $\Sigma_0$ which are of the same order in the condensates $\rho$.

$$\Sigma_s = -\frac{8\pi^2}{\Lambda_B^2} [\langle \rho | \bar{q} q | \rho \rangle - \langle \bar{q} q \rangle] = -\frac{8\pi^2}{\Lambda_B^2 m_u + m_d} \rho S$$  \hspace{1cm} (49)

$$-\Sigma_0 = -\frac{64\pi^2}{3\Lambda_B^2} \langle \rho | q \gamma_0 q | \rho \rangle = -\frac{32\pi^2}{\Lambda_B^2} \rho .$$  \hspace{1cm} (50)

These expressions are of leading order in density. $\rho S$ in (49) is the scalar nucleon density, $f_\pi = 93$ MeV the weak pion decay constant and $m_{u,d}$ are the current quark masses of about 5 \(\div\) 10 MeV. The pion-nucleon sigma term $\sigma_N = \langle N | m_u \bar{u} u + m_d \bar{d} d | N \rangle$ is determined by the $u$ and $d$-quark content of the nucleon and represents the contribution from explicit chiral symmetry breaking to the nucleon mass through the small, but non-vanishing current quark masses. It has an empirical value of about $\sigma_N \simeq 50$ MeV. The Borel mass scale $\Lambda_B \simeq 4\pi f_\pi \simeq 1$ GeV is the generic low energy scale of QCD which separates the non-perturbative from the perturbative regime. It coincides with the chiral symmetry breaking scale $\Lambda_\chi$ of ChPT. Applying Ioffe’s formula [8] for the nucleon mass $M \simeq -\frac{8\pi^2}{\Lambda_B^2} \langle \bar{q} q \rangle$ one finally obtains the fields in the form

$$\Sigma_\rho (\rho) = -\frac{\sigma_N M}{m^2 \pi f_\pi^2} \rho S ,$$  \hspace{1cm} (51)

$$-\Sigma_0 (\rho) = 4(m_u + m_d) M \frac{m^2 \pi f_\pi^2}{\rho} .$$  \hspace{1cm} (52)
However, the dependence of the nucleon mass in matter on the quark condensate is not as straightforward as expression (51) suggests. Concerning the in-medium condensate one has carefully to distinguish between contributions from the pion cloud and those of non-pionic origin [41, 42].

As pointed out by Birse [41] a naive direct dependence of the nucleon mass on the quark condensate through Eq. (51) leads to contradictions with chiral power counting. The contributions from low momentum virtual pions which enter the in-medium condensate should not contribute by the same amount to the change of the nucleon properties in matter. They can therefore not as easily be associated with a partial restoration of chiral symmetry as the mean field field approximation, Eqs. (51, 52), would suggest. This problem has also been investigated by Chanfray et al. [42] in the framework of the linear sigma model. In their studies the authors were able to reconcile the phenomenology of Quantum Hadron Dynamics with chiral theory, in that case the linear sigma model. Their conclusion was that, in contrast to the scalar condensate $\langle \rho|\bar{q}q|\rho \rangle$ which is driven by the sigma field, i.e., the chiral partner of the pion, the lowering of the nucleon mass $M^*$ is driven by a chiral invariant scalar field which corresponds to fluctuation along the chiral circle. With other words, the condensate is to large extent reduced by the pion cloud surrounding the nucleons while the nucleon mass is not.

To set up the context for the following discussion, we shortly sketch the argumentation of Birse [41]: From Eq. (51) follows that the effective nucleon mass $M^* = M + \Sigma_\sigma(\rho)$ is directly proportional to the nucleon sigma term

$$M^* = M \left(1 - \frac{\sigma_N}{m^2 f^2} \rho_S\right). \quad (53)$$

The chiral expansion of the sigma term leads to [43]

$$\sigma_N = A m^2_\pi - \frac{9}{16\pi} \left(\frac{g_{\pi NN}}{2M}\right)^2 m^3_\pi + \ldots \quad (54)$$

In the chiral limit the pion-nucleon coupling is connected to the axial vector coupling by the Goldberger-Treiman relation $g_{\pi NN} = g_A M/f_\pi$. The coefficient $A$ involves counter terms related to short-distance physics whereas the non-analytic $\mathcal{O}(m^2_\pi)$ term arises purely from long-distance physics of the pion cloud. Inserting (54) into (53) implies a dependence of the effective nucleon mass $M^*$ on the pion mass which is of order $\mathcal{O}(m_\pi)$.

At the mean field level, i.e., in $T - \rho$ approximation, the scalar self-energy [41] is on the other hand given by the scalar forward scattering amplitude $T_s(q = 0)$ ( $T_s(q = 0)$ in (55).
corresponds to the direct amplitudes $F_S$ and $g_S$ in (29) and (32), respectively.)

$$\Sigma_s(k_F) = T_s(q = 0) \rho .$$  \hspace{1cm} (55)

A comparison of Eq. (55) with Eqs. (53) and (54) would imply that the scalar part of the forward scattering amplitude contains a constant and a term of order $m_\pi$. Such a dependence contradicts, however, chiral power counting. In chiral EFT the leading term in the pion mass in the $NN$ interaction originates from the low energy expansion of the OPE and is of order $O(m_\pi^2)$ [22, 23, 24]. Hence the $NN$ interaction cannot contain a term directly proportional to $\sigma_N/f_\pi^2$.

For the comparison of the sum rule predictions we turn to the density dependence of the self-energy. Fig. 11 shows the density dependence of the fields from the various orders. As in Fig. 6 the scalar $\Sigma_s$, time-like vector $\Sigma_0$ and spatial vector $\Sigma_v$ self-energies are determined at momentum $k = k_F$. The density dependence is shown up to $k_F = 1.8$ fm$^{-1}$ which corresponds to about 2.5 times nuclear saturation density. As can be seen from Fig. 11 the relative contributions from the various orders remain the same over the entire density range considered. For comparison the figure contains also the corresponding fields as predicted by leading order QCD sum rules, i.e., Eqs. (51) and (52). For the evaluation of Eqs. (51) the empirical value of $\sigma_N = 50$ MeV has been chosen for the nucleon sigma term, $f_\pi = 93$ MeV and $(m_u + m_d) = 12$ MeV. For the evaluation of the scalar field in (51) we have set the scalar density equal to the vector density, i.e., $\rho_s \simeq \rho$.

Both, the QCD sum rule and the chiral EFT fields are well comparable in terms of a density expansion since both are obtained to leading order in density. In the case of the sum rules this corresponds to a Fermi gas of non-interacting nucleons. To go beyond the Fermi gas approximation would require to include higher order terms in the operator product expansion and the density expansion of the condensates [6, 7, 45]. In the EFT case higher orders in density can be introduced by a self-consistent dressing of the interaction (see discussion in Sec. VI) and of course by higher order terms in perturbation series which would finally end up in a full resummation of the Brueckner ladder diagrams.

At moderate nuclear densities the agreement between the QCD sum rules and $N^3$LO is quite remarkable. At higher densities the results from the sum rules tend to overshoot the $N^3$LO values which is, however, not too astonishing since the relations (51) are valid in the low density limit.
In view of the fact that in chiral $NN$ dynamics the fields are dominantly generated by NLO contact terms, one could be tempted to interpret the present results in the way that the reduction of the quark condensates occurs at NLO in the chiral expansion. However, as discussed above such an interpretation is not straightforward. A closer inspection of the terms which drive the sum rule result reveals the following: the coefficient $A$ in (54) is related to the unknown coupling $C_1$ in the effective ChPT pion-nucleon Lagrangian \cite{48}. Becher and Leutwyler extracted a value of $A = 3.7 \text{ GeV}^{-1}$ fitting the elastic $\pi N$ scattering amplitude at
threshold \[49\]. Inserting this value into the sum rule expression \(53\) corresponds to a scalar self-energy (at \(k_F = 1.35 \text{ fm}^{-1}\) of \(\Sigma_s = -513 \text{ MeV}\) at order \(m_\pi^0\). At order \(m_\pi\), i.e., when the \(\mathcal{O}(m_\pi^2)\) term in the expansion \(54\) is included, the sigma term of 46.7 MeV is already close to its empirical value and a self-energy of \(\Sigma_s = -340 \text{ MeV}\) is obtained. Although this value for \(\Sigma_s\) is astonishingly close to the NLO result from chiral \(NN\) scattering, one has to keep in mind that already the LO result is of order \(m_\pi^2\) in the pion mass. In contrast to the sum-rule approach there appears no significant repulsive contribution from pion dynamics which would correspond to the \(\mathcal{O}(m_\pi^3)\) term in \(54\).

The present results are therefore in qualitative agreement with the findings of Refs. \[41, 42\], namely that long-distance physics related to pion dynamics plays only a minor role for the reduction of the nucleon mass in matter. Relating the in-medium nucleon mass to the in-medium scalar condensate through expression \(53\) one should be very careful. Although the sum rule mean fields, Eqs. \(49\) and \(50\), provide a reasonable approximation to the mean fields from chiral EFT, both approaches do not reflect the same physical concepts. The sum rule approach assumes that the nucleon properties are determined by the interaction with the in-medium condensates while conventional many-body approaches assume that the in-medium properties are determined by the interaction between the nucleons.

VI. EQUATION OF STATE

Until now all calculations in this paper have been performed at tree level. It is, however, a well known fact that a realistic description of nuclear dynamics requires correlations beyond Hartree-Fock. Short-range correlations are known to be essential for nuclear binding whenever realistic interactions are used. This leads in lowest order of the Brueckner hole-line expansion to the ladder approximation of the Bethe-Goldstone equation for the in-medium \(G\)-matrix \[10\], or the Bethe-Salpeter equation in the relativistic case \[11\]. In contrast to non-relativistic BHF where the saturation points of isospin saturated matter are allocated on the so-called \(Coester\) line, the relativistic Dirac-Brueckner-Hartree-Fock approach leads to rather reasonable saturation properties \[12, 13, 14\]. For a review see \[55\].

In Hartree-Fock the matter turns out to be unbound, in particular when high precision potentials with a relatively strong repulsive hard core are applied, e.g. OBE type potentials or Argonne \(v_{18}\). The situation is qualitatively different for low momentum interactions
\(V_{\text{low } k, \text{ Idaho } N^3\text{LO}}\) where the hard core is strongly suppressed by the high momentum cut-offs. For these interactions isospin saturated nuclear matter collapses and Brueckner ladder correlations do not improve on this situation \[50\]. Here the matter has to be stabilised by the inclusion of repulsive three-body-forces \[51\]. Doing so, there appears a strong cut-off dependence at tree-level which can be removed when the second order term of the Brueckner perturbation series is added. \(V_{\text{low } k}\) in combination with three-body-forces does not require a full resummation of the ladder diagrams but can already be treated within second-order perturbation theory \[51\].

In the present work we do not aim for a fully realistic description of the nuclear many-body problem but restrict the investigations to the Hartree-Fock level. The tree-level results discussed up to now are of leading order in density \(\rho\). Higher order corrections in density can be taken into account when the bare potential matrix elements are replaced by in-medium matrix elements \(V \rightarrow V^*\). In the relativistic approach such a treatment is well defined. It means to evaluate the corresponding Feynman amplitudes \[50\] through an in-medium spinor basis \(u^*_\lambda(k)\) where the nucleons are dressed by the self-energy. Such a treatment requires, however, a definite structure of the interaction which allows to evaluate corresponding in-medium amplitudes. It is therefore at present restricted to OBE-type potentials.

The dressing of the interaction through the self-energy leads automatically to a self-consistency problem which is e.g. solved within DBHF. The higher order density dependences which are introduced by such a procedure are considered to be one of the essential reasons for the improved saturation behaviour of relativistic DBHF compared to non-relativistic BHF. In the following we will study the role of self-consistency at the Hartree-Fock level.

As already mentioned, in a relativistic framework one uses an in-medium spinor basis where the scalar and vector self-energy components from Sec. \[\text{IV}\] enter via effective masses and momenta, see Eq. \[16\]. Furthermore the spatial vector self-energy component is usually absorbed introducing reduced effective masses and momenta

\[
\tilde{M}^* = \frac{M^*}{1 + \Sigma_v}, \quad \tilde{k}_{\mu}^* = \frac{k_{\mu}^*}{1 + \Sigma_v}.
\]  

Hence the kinetic energy can be written as

\[
\tilde{k}_0^* = \tilde{E}^* = \frac{E^*}{1 + \Sigma_v} = \sqrt{k^2 + \tilde{M}^{*2}}
\]  

39
and the in-medium spinors of helicity \( \lambda \) are given by

\[ u_\lambda^*(k) = \sqrt{\tilde{E}^* + \tilde{M}^*} \left( \begin{array}{c} 1 \\ \frac{2\lambda|k|}{E^* + M^*} \end{array} \right) \chi_\lambda. \]  

(58)

Thus the effective mass \( \tilde{M}^* \) introduces a density dependence into the interaction. The effective mass is, however, in general not only density but also momentum dependent. Based on the observation that this explicit momentum dependence is moderate, it is usually neglected and \( \tilde{M}^* \) is fixed at the reference point \( |k| = k_F \). In the so-called reference spectrum approximation the reduced effective mass \( \tilde{M}_F^* = \tilde{M}^*(|k| = k_F, k_F) \) serves as an iteration parameter. \( \tilde{M}^* \) is then the solution of the non-linear equation

\[
\tilde{M}^* = M + \Sigma_s(k_F, \tilde{M}^*) - \tilde{M}^* \Sigma_v(k_F, \tilde{M}^*)
\]

(59)

which follows from the formulae above. Self-consistency is now achieved by determining for a given start value of \( \tilde{M}^* \) the in-medium matrix elements \( V^{JS}_{L',L}(q',q) \). Therefore the Lorentz invariant amplitudes \( F^1_m(|q|,\theta) \) and \( g^1_m(|q|,\theta) \), Eqs. (29) and (32), as well as the transformation matrix \( C_{im} \) of Eq. (31) depend on \( \tilde{M}^* \) and the Fermi momentum \( k_F \) since the plane-wave helicity states \( |\lambda_1\lambda_2 q\rangle \) of Eq. (30) are now medium-dependent (58). The next step is to compute the self-energy components \( \Sigma_s, \Sigma_0 \) and \( k \Sigma_v \). Since the Dirac propagator (36) describes dressed quasi-particles now, also in (41), (42) and (43) the mass \( M \) and energy \( E \) have to be replaced by the effective quantities \( \tilde{M}^*, \tilde{E}^* \). Finally the new \( \tilde{M}^* \) is determined. This iteration procedure is repeated until convergence is reached.

In Fig. 12 the results for the self-consistently calculated self-energy components \( \Sigma_s \) and \( \Sigma_0 \) for Bonn A, Nijm 93 and Nijmegen I are shown as a function of the Fermi momentum and compared to the tree level results from Fig. 6. For the Bonn A case the result of a full self-consistent DBHF calculation is shown as well [14]. From this figure two features can be observed: the higher order density dependences which are introduced by the dressing of the potential affect mainly the scalar part of the self-energy. The modifications of \( \Sigma_0 \) are moderate while \( \Sigma_s \) is significantly reduced. The short range ladder correlations included in the full DBHF calculation (Bonn A) influence the self-energy in an opposite way. The deviations of \( \Sigma_s \) from the self-consistent HF result are rather small, however, the vector component gets now strongly suppressed. This fact is understandable since the ladder correlations prevent the two-nucleon wave functions from too strong overlap with the hard core. In OBE poten-
tials the hard core is mainly mediated by vector $\omega$-exchange and determines thus the vector self-energy component.

With the self-consistent Hartree-Fock self-energies at hand one can now determine the equation of state (EOS). Like in DBHF the EOS, i.e., the energy per particle is defined as the kinetic plus half of the potential energy

$$E/A = \frac{1}{\rho} \sum_{k,\lambda} \langle \pi^\lambda_{\gamma}(|k|) \mid \gamma \cdot k + M + \frac{1}{2} \Sigma(k) |u^*_\lambda(|k|) \rangle > \frac{M^*}{E^*} - M$$

$$= \frac{1}{\rho} \int_F \frac{d^3k}{2\pi^3} \left[ \left( \sum V(|k|) \right) > \frac{E^*}{E^*} - \Sigma^0(|k|) - \frac{1}{2E^*} \left( \Sigma_s(|k|) M^* - \Sigma^\mu(|k|) k^\mu \right) \right] - M \quad (61)$$

with the self-consistent spinors $u^*_\lambda$ from Eq. (58).

In Fig. 13 we present the self-consistent Hartree-Fock results for the energy per particle in symmetric nuclear matter calculated from the Bonn A, Nijm93, Nijmegen I potentials as a function of the Fermi momentum $k_F$ which is a measure for the density $\rho = 2/(3\pi^2)k_F^3$.

Also a non-self-consistent calculation is shown (dashed line) where the energy per particle is given by

$$E/A = \frac{1}{\rho} \int_F \frac{d^3k}{2\pi^3} \left[ \frac{k^2}{2M} + \frac{1}{2} U_{s.p.}(k, k_F) \right]$$

with $U_{s.p.}(k, k_F)$ as defined in Eq. (55). In this case one obtains the same result as in a non-
FIG. 13: (Color online) Hartree-Fock calculation of the nuclear equation of state, i.e., energy per particle E/A as a function of the Fermi momentum $k_F$ for three different potentials. The dashed line indicates a tree level calculation and the solid line represents a self-consistent Hartree-Fock calculation, i.e., higher order corrections in density are included.

relativistic Hartree-Fock calculation (denoted by stars in Fig. 13). The latter demonstrates again the numerical accuracy of the procedures.

For the Bonn A case again the EOS from the full DBHF calculations is shown as a reference [14]. It is clear that ladder correlations and other in-medium effects such as Pauli-blocking of intermediate states in the Bethe-Salpeter equation are responsible for nuclear saturation. The relatively moderate deviations from self-consistent Hartree-Fock at the scale of the self-energies in Fig. 12 are essential at the scale of the binding energy. Like in relativistic mean field theory of QHD subtle cancellation effects in the large scalar and vector fields are responsible for nuclear binding.

The higher order density dependences introduced via the dressing of the bare interaction $V$ lead to significantly more repulsion at the level of the EOS. This is a direct consequence of the reduced attractive scalar field (see Fig. 12). Thus Fig. 13 serves also as a demonstration for the success of DBHF compared to BHF what concerns the quantitative description of nuclear saturation: In particular for modern high precision potentials such as Bonn, Nijmegen or Argonne $v_{18}$ the BHF approach leads to strong over-binding and too high saturation densities. The additional repulsion introduced by higher order terms in density through
the dressed potentials shifts the corresponding saturation points towards the empirical region \cite{13,16,55}. We want to stress that the density dependence of the dressed potential $V^*$ should not be mixed up with the density dependence of the $G$-matrix. The latter originates from the dressed two-nucleon propagator and the Pauli-operator in the Bethe-Goldstone (or Bethe-Salpeter) equations while $V^*$ enters into the Bethe-Salpeter for iteration. In non-relativistic BHF or variational calculations \cite{56,57} a non-linear density dependence which improves the saturation behaviour is usually introduced through net repulsive three-body-forces. In such a treatment the dependence on the third particle is integrated out such that one is left with an additional effective density dependent two-body force which acts in a similar way as a dressing of the two-body interaction. In this context one should mention that a dressing of the interaction has also more subtle consequences when iterated in the Bethe-Salpeter equation. It leads e.g. to a quenching of the second order OPE exchange \cite{58} which plays an essential role for saturation in non-relativistic approaches.

In summary, one could expect that a dressing of the interaction would allow to comply with weaker three-body forces which may in particular be of interest concerning the application of low momentum EFT potentials to the nuclear many-body problem. As the studies of Bogner et al. \cite{51} have demonstrated, $V_{\text{low } k}$ requires rather strong three-body forces in order to stabilise nuclear matter. There the strength of the three-body contributions has already been pushed to its limits. Although a dressing of the interaction will probably not be possible for $V_{\text{low } k}$ due to the partially non-analytic structure of the potential, it may be a promising perspective for the application of other EFT potentials, e.g. the chiral N$^3$LO.

VII. SUMMARY

The appearance of large scalar and vector fields is a well established feature of relativistic nuclear dynamics. The saturation mechanism of nuclear matter or the single particle potential in finite nuclei are obtained by subtle cancellation effects between large attractive scalar and repulsive vector fields. These fields occur already at tree level and do not change too much when realistic many-body calculations are performed. Full self-consistent Brueckner calculations which account for short-range ladder correlations lead to mean fields of similar size, i.e., of several hundred MeV magnitude. The size of the scalar-vector fields coincides with the values derived from relativistic mean field phenomenology by fits to finite nuclei.
Alternatively, QCD sum rules come to the same results.

The present work addresses the question about the origin of these fields. When the nucleon-nucleon interaction is described within the framework of a meson exchange picture, the situation is rather clear. The Lorentz character of the mesons determines automatically the Lorentz character of the interaction at the corresponding scale: the short-range repulsion is due to vector exchange ($\omega, \rho$) while the intermediate range attraction originates from scalar exchange ($\sigma$). As a direct consequence this leads to the existence of large scalar and vector mean fields in nuclear matter. However, these fields are not observables. It is therefore a fundamental questions of nuclear physics whether the appearance of large scalar/vector fields is intimately connected to the meson exchange picture or if it is a general consequence of the vacuum $NN$ interaction.

To address the question in a model independent way, we based the present study on a broad set of modern high precision $NN$ potentials: Bonn, CD-Bonn, Nijmegen, Argonne $v_{18}$, Reid93, Idaho $N^3LO$ and $V_{\text{low } k}$. Except the fact that all these potentials fit $NN$ scattering data with high accuracy they are based partially on quite different theoretical concepts, i.e., the traditional meson exchange picture (Bonn, CD-Bonn, Nijmegen), a purely phenomenological philosophy (Argonne $v_{18}$, Reid93) or QCD inspired effective field theory approaches (Idaho $N^3LO$, $V_{\text{low } k}$).

For this purpose the potentials were projected on a relativistic operator basis in Dirac space. This was achieved using standard projection techniques which transform from a partial wave basis, i.e., the basis where the potentials are originally given, to the basis of covariant amplitudes in Dirac space. The idea behind this approach is that both, relativistic and non-relativistic descriptions of the $NN$ interaction have common features, i.e., they are based on a certain operator structure in spin-isospin space and invoke certain scales: the long-range part of scale $m_\pi$, essentially given by one-pion exchange, the intermediate range attraction and the short-range repulsion. In the meson exchange picture the various scales are associated with the meson masses which mediate the interaction. The various approaches can now be compared at the level of these covariant amplitudes where we observe a remarkable agreement between the meson exchange potentials (Bonn, CD-Bonn, Nijmegen), the phenomenological non-relativistic potentials (Argonne $v_{18}$, Reid93) and the EFT potentials (Idaho $N^3LO$, $V_{\text{low } k}$).

Moreover, this procedure allows now to calculate the relativistic self-energy operator in
nuclear matter. The key result of the present investigations is the tree level self-energy in nuclear matter. The structure of the nucleon-nucleon interaction enforces the existence of large scalar and vector fields. This is found to be a model independent fact, true for all types of interactions which have been considered. The scale of these fields is set at tree level. Although essential for nuclear binding and saturation, higher order correlations, in particular short-range correlations, change the size of the fields by less than 25%. The magnitude of the tree-level fields is very similar to that predicted by relativistic mean field phenomenology and relativistic many body calculations.

The connection to QCD as the underlying theory of strong interactions is established by chiral effective theory. EFT nucleon-nucleon potentials are derived from a systematic expansion of an effective Lagrangian which respects the basic symmetries of QCD. Chiral EFT is considered as the exact mapping of QCD on effective hadronic degrees of freedom in the non-perturbative regime. Subjecting the chiral N$^3$LO Idaho potential to the present projection scheme we can make the following statements: In nuclear matter scalar and vector mean fields of the same sign and magnitude are generated as by the meson exchange or phenomenological potentials. These fields are generated by contact terms which occur at next-to-leading order in the chiral expansion. These are four-nucleon contact terms with two derivatives which generate the short-range spin-orbit interaction. The strength of the corresponding low energy constants, in particular those connected to the spin-orbit force, is dictated by $P$-wave $NN$ scattering data. Pion dynamics as well as LO and N$^3$LO contacts provide only corrections to the fields generated by the NLO contact terms. EFT is therefore in perfect agreement with Dirac phenomenology where it is known since a long time that the large scalar/vector fields are generated by the short-range vector ($\omega$) and scalar ($\sigma$) mesons which are connected intimately to the large spin-orbit interaction. We conclude that this is a direct consequence of $P$-wave $NN$ scattering.

Like in OBE models and RMF theory, in EFT the reduction of the nucleon mass $M^* = M + \Sigma$ is driven by short-distance physics. Long-distance physics from virtual pions, i.e. the non-analytic term in the expansion of $\sigma_N$ gives a sizable contribution to the modification of the in-medium quark condensate. Such contributions are, however, found to play only a minor role for the reduction of the nucleon mass. Nevertheless, at moderate nuclear densities the N$^3$LO scalar and vector fields agree almost perfectly with the prediction from leading order QCD sum rules. For future perspectives chiral EFT in combination with projection
techniques may allow to determine the relativistic anti-proton potential in matter in a model
independent way. Here the meson-exchange picture predicts a change in sign of the vector
field due to g-parity and hence an extremely deep attractive potential. Such investigations
in particular will be interesting in view of the forthcoming anti-proton facilities, e.g. Panda
at FAIR [59].

Finally we investigated implications of higher order corrections in density on the nuclear
EOS. A dressing of the potential through self-consistently determined self-energies leads
already at the Hartree-Fock level to significantly more repulsion in the EOS. At present
these investigations were restricted to OBE type potentials. But to include such higher
order terms in density might open a promising perspective also for EFT potentials when
applied to the nuclear many-body problem.

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