Phenomenology of axial-vector and pseudovector mesons: decays and mixing in the kaonic sector

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Abstract

We study the decays of the lightest axial-vector and pseudovector mesons, interpreted as quark-antiquark states, into a vector and a pseudoscalar meson. We show that the quarkonium assignment delivers a good description of the decays and allows also to make further testable predictions.

In the kaonic sector, the physical resonances $K_1(1270)$ and $K_1(1400)$ emerge as mixed objects of an axial vector state $K_1,A$ and a pseudovector state $K_1,B$. We determine the mixing angle as $|\theta_K| = (33.6 \pm 4.3)^\circ$, a value compatible with previous studies but with a smaller uncertainty. This result may be helpful for testing models beyond the Standard Model of particle physics in which decays into the kaonic resonances $K_1(1270)$ and $K_1(1400)$ are investigated.

1 Introduction

The understanding of hadron masses and decays using hadronic models, which embody some of the symmetries of the underlying theory of quarks and gluons (Quantum Chromodynamics, QCD), is an important task of modern particle physics. The question about the nature of hadrons, if it is described in terms of the ‘old quark model’ or if new exotic hadrons, such as glueballs, hybrids, but also tetraquark states are necessary, is in the center of a vivid theoretical and (ongoing as well as planned) experimental efforts [1]. In addition, the emergence of dynamically generated resonances due to meson interactions has attracted much theoretical attention: bumps, which do not correspond to preformed quark substructure, can emerge due to unitarity (loop) corrections on top of preexisting ‘seed states’, see e. g. Ref. [2] and refs. therein.

In particular, $p$-wave quark-antiquark states represent an important subject of hadron spectroscopy [5]. The properties of $p$-wave quark-antiquark states are rather different according to which nonet is taken into consideration and at the two extremes there are the scalar and the tensor mesons. Namely, on the one hand scalar $p$-wave quarkonia (total angular momentum $L = 1$ and total spin $S = 1$ coupled to $J^{PC} = 0^{++}$) offer long-standing puzzles in hadron spectroscopy. They have been widely investigated both with theoretical models, see for instance Refs. [1] and refs. therein. Being the chiral partners of the pseudoscalar mesons, they are extremely important in QCD also in relation to chiral symmetry breaking and its restoration at nonzero temperature and density. Moreover, the emergence of further scalar fields, such as the scalar glueball and scalar tetraquark states, has rendered the whole scalar sector of QCD at the same time rich and difficult. On the other hand, the lowest nonet of tensor mesons ($L = 1$, $S = 1$ coupled to $J^{PC} = 2^{++}$) represents one of the best established ground state quark-antiquark nonets with a very good agreement between the quark-antiquark assignment and the measured masses and decay widths [5] and refs. therein. (The situation is, however, less clear when going to excited tensor states, see the discussion in Ref. [1].)
In this work we concentrate our attention to the two remaining nonets of \( p \)-wave states, the axial-vector and pseudosvector quarkonia mesons. They are not as enigmatic as the scalar ones but also not unambiguous as the tensor states; in addition, interesting mixing effects between them are present.

We study the quarkonia axial-vector and pseudoscalar mesons by using a phenomenological flavor symmetric relativistic Lagrangian in which the axial-vector and the pseudovector mesons are introduced as standard quark-antiquark fields. The Lagrangian is built in agreement with parity, charge conjugation, flavor symmetry, and to the explicit breaking of the latter due to different quark masses \((m_s > m_u = m_d)\). In particular, we shall study the three subjects that we present in the following.

(i) Phenomenology of the ground state axial-vector quark-antiquark fields: these quarkonia mesons correspond to the quantum numbers \( L = 1 \) and \( S = 1 \) coupled to \( J^{PC} = 1^{+} \). They are the chiral partners of the vector mesons, and are therefore part of chiral models in which the vector d.o.f. are included, see Refs. \[15, 16\] and the recently developed extended Linear Sigma Model \[8, 9\] (in particular, in Ref. \[9\] for the first time a linear chiral model with three flavors and (axial-)vector quarkonia d.o.f. was studied). Still, a debate on the nature of the lightest axial-vector states is ongoing: in our approach the isovector \((I = 1)\) axial-vector state \( a_1 \equiv a_1(1260) \) is described by the quark-antiquark isotriplet states \( ud, \bar{u}d, \sqrt{1/2}(uu - \bar{u}d) \); however, the very same resonance has been interpreted as a \( \rho \pi \) quasi-bound state in the works of Ref. \[17\], see also the general discussion of Ref. \[3\] about dynamically generated and/or reconstructed states. In the present manuscript we test to which extent the decays of \( a_1 \) are in agreement with the measured decay widths by utilizing the quark-antiquark assignment in a relativistic approach. The very same question can be extended to the two isoscalar members of the nonet, \( f_1(1285) \) and \( f_1(1420) \), and to the axial-vector kaonic state \( K_{1,A} \) (present in both resonances \( K_1(1270) \) and \( K_1(1400) \), see below). Our findings confirm the quark-antiquark nature of these axial-vector resonances.

(ii) Phenomenology of the ground state pseudovector quark-antiquark fields: these quarkonia mesons correspond to the quantum numbers \( L = 1 \) and \( S = 0 \), which implies that \( J^{PC} = 1^{+} \). The quarkonium nature of the lightest isovector states \( b_1(1235) \), the isoscalar states \( h_1(1170), h_1(1380) \) and the isodoublet state \( K_{1,B} \) (present in both resonances \( K_1(1270) \) and \( K_1(1400) \), see below) is established \[5, 11, 18\]. Our approach confirms these results: a good description of the decay widths is obtained by simply using the constraints of flavor symmetry in the channels in which experimental data are available.

(iii) Mixing in the kaonic sector: an interesting property which links the two, otherwise separated, nonets of axial-vector and pseudovector mesons is the fact that the two isodoublet axial-vector and pseudovector states \( K_{1,A} \) and \( K_{1,B} \) mix and generate the two physical resonances \( K_1(1270) \) and \( K_1(1400) \) \[5, 11, 18, 19, 20, 21, 22\]. Within our model this mixing is generated by a flavor-symmetry breaking term, which is proportional to \( m_u^2 - m_s^2 \). This also explains why the mixing only takes place in the \( I = 1/2 \) sector. Namely, a mixing between the charged states \( a_1^{\pm} \) and \( b_1^{\pm} \) is proportional to \( m_u^2 - m_s^2 \); it is then suppressed and neglected in this work. The neutral states \( a_1^0, b_1^0, f_1(1285), f_1(1420), h_1(1170), h_1(1380) \) are eigenstates of the charge conjugation operator \( C \) and do not mix, because \( C \) is a conserved quantity in QCD.

We also calculate the mixing angle by studying the decays of \( K_1(1270) \) and \( K_1(1400) \). In the notation of Refs. \[5, 18, 19, 20, 21, 22\] usually adopted in the literature, the values \( |\theta_K| \sim 35^o \) and \( |\theta_K| \sim 55^o \) have been obtained in a variety of phenomenological models, which have used the masses and strong decays of the quarkonia nonets and also decays of \( \tau \) lepton and, more recently, the decays of heavy \( D \)-type and \( B \)-type mesons. In this work, a unique answer for the absolute value of the mixing angle is obtained from all available strong decays: \( |\theta_K| = (33.6 \pm 4.3) \). This result is in agreement with the recent discussion of Ref. \[22\], in which the solution close to \( 55^o \) has been shown to be not favoured, and also to the result of Ref. \[20\], in which, using the weak decays of \( B \) mesons, the value and also the sign have been determined as \( \theta_K = -(34 \pm 13) \). Quite remarkably, the knowledge of this mixing angle is not only relevant for a better understanding of hadron physics, but is also an important information for testing the existence of new particles beyond the Standard Model and has been for this reason subject of recent investigations \[23, 24, 25\]. Namely, the transitions of heavy \( B \)-mesons to axial- and pseudovector mesons can shed light to details of the
CKM matrix and to the presence of a fourth generation. In this view, our approach represents an 'up to date' and complete hadronic-based way to determine the angle and this result can be of interest for studies of physics beyond the Standard Model as well.

In order to achieve the goals (i)-(iii) described above we first calculate the expression of the tree-level decay widths of axial- and pseudovector states into a vector and a pseudoscalar meson as it follows from our interaction Lagrangian; however, the use of simple tree-level expressions of the decay widths is not enough for our purposes because many decays are affected by threshold effects: the (nominal) masses of the produced particles are just below (or even above) the threshold for their production. For this reason, an integration over their spectral functions [26, 27, 28, 29] is necessary. To this end we use a relativistic version of the Breit-Wigner distribution to which the corresponding energy threshold and the necessary normalization are implemented.

The paper is organized as follows: in Sec. 2 we present the Lagrangian and the employed decay formulae which take finite width effects into account; details of the Lagrangian are presented in an Appendix. In Sec. 3 we present our results for the decays of isoscalar and isovector states, and in Sec. 4 we determine the mixing and the decays in the isodoublet (kaonic) sector. Finally, in Sec. 5 we present our conclusions.

2 The model: Lagrangian and decay widths

The resonances we are interested in are the axial-vector and pseudovector quark-antiquark nonets \( A^\mu \) and \( B^\mu \). These states decay predominantly into a vector and a pseudoscalar meson, which are part of the corresponding quark-antiquark nonets \( V^\mu \) and \( P \). The four nonets are described by the following matrices:

\[
P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_1 + \eta_0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_1 - \eta_0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sqrt{2} \rho_1 + \rho_0}{\sqrt{2}} & \rho_1^\mu+ & 0 \\ \rho_1^- & \frac{\sqrt{2} \rho_1^\mu-}{\sqrt{2}} & K^{\mu+} \\ 0 & \sqrt{2} \omega_1^\mu & K^{\mu0} \end{pmatrix} \quad (1)
\]

\[
A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sum_1 + \sum_0 \sqrt{2} & a_1^\mu+ & K_1^\mu+ \\ a_1^- & \sum_1 - \sum_0 \sqrt{2} & K_1^\mu0 \\ K_1^- & \bar{K}_1^0 & \eta_S \end{pmatrix}, \quad B^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sum_1 + \sum_0 \sqrt{2} & b_1^\mu+ & K_1^\mu+ \\ b_1^- & \sum_1 - \sum_0 \sqrt{2} & K_1^\mu0 \\ K_1^- & \bar{K}_1^0 & \eta_S \end{pmatrix}
\]

The assignment of the fields is as follows: The matrix \( P \) refers to the usual nonet of pseudoscalar states \( \{ \pi, K, \eta, \eta' \} \). The mixing of strange and non-strange isoscalar sector implies that: \( \eta = \eta_N \cos \varphi_P + \eta_S \sin \varphi_P \) and \( \eta' = -\eta_N \sin \varphi_P + \eta_S \cos \varphi_P \), where \( \eta_N \) is the pure nonstrange state \( \sqrt{1/2} (uu + dd) \) and \( \eta_S \) the pure strange state \( ss \). We use the numerical value \( \varphi_P = -41.4^\circ \), which has been evaluated by the KLOE collaboration in Ref. [30]. Varying this mixing angle between the phenomenological range \( (-36^\circ, -45^\circ) \) generates only small numerical changes.

The matrix \( V^\mu \) represents the vector states \( \{ \rho, K^*(892), \omega, \phi \} \), where \( \omega \) is regarded as the purely nonstrange state and \( \phi \) as a purely strange state; we thus neglect the (small) mixing angle in the vector sector. Finally, the matrix \( A^\mu \) contains the nonet of resonances of axial-vector states:

\[
\{ a_1(1230), K_{1,A}, f_1(1285), f_1(1420) \},
\]

and the matrix \( B^\mu \) describes the nonet of pseudovector resonances

\[
\{ b_1(1230), K_{1,B}, h_1(1170), h_1(1380) \}.
\]

In both nonets the strange-nonstrange isoscalar mixing is neglected, thus \( f_1(1285) \) and \( h_1(1170) \) are purely nonstrange states, while \( f_1(1420) \) and \( h_1(1380) \) are purely strange states. The transformation properties of these nonets under charge, parity and flavor transformations are summarized in Table 1.
Table 1: Transformation properties of the nonets \( \Pi \) under charge, parity and flavor transformations. The position of the indices is important for parity: for instance, \( V^\mu \) has odd-parity for \( \mu = 1, 2, 3 \) but has even-parity for \( \mu = 0 \).

The Lagrangian describing the decay of axial- and pseudovector states into vector and pseudoscalar mesons and the mixing in the kaonic sector consists of three parts, each one containing one free parameter:

\[
\mathcal{L} = \mathcal{L}_A + \mathcal{L}_B + \mathcal{L}_{mix},
\]

where:

(i)

\[
\mathcal{L}_A = i a \operatorname{Tr} \left\{ A_\mu \left[ V^\mu, P \right]_\text{−} \right\}
\]

describes the coupling of the axial-vector fields to vector and pseudoscalar ones; the unknown coupling constant is the parameter \( a \) with the dimension of energy; \( \mathcal{L}_A \) is invariant under \( P, C, \) and \( U(3)_V \). The symbol \( \left[ \cdot, \cdot \right]_\text{−} \) is the usual commutator. For the explicit form of \( \mathcal{L}_A \) see Appendix A.

(ii)

\[
\mathcal{L}_B = b \operatorname{Tr} \left\{ B_\mu \left[ V^\mu, P \right]_\text{+} \right\}
\]

is the analogous Lagrangian, with coupling constant \( b \), generating the interaction of pseudovector fields with vector and pseudoscalar ones. \( \mathcal{L}_B \) is also invariant under \( P, C, \) and \( U(3)_V \). The symbol \( \left[ \cdot, \cdot \right]_\text{+} \) is the anticommutator. For the explicit form of \( \mathcal{L}_B \) see Appendix A.

(iii)

\[
\mathcal{L}_{mix} = ic \operatorname{Tr} \left\{ \Delta \left[ A_\mu, B^\mu \right]_\text{−} \right\},
\]

in which \( c \) is a dimensionless coupling constant and \( \Delta \) is a diagonal matrix with the bare quark masses \( \Delta = \text{diag}\{m_u^2, m_d^2, m_s^2\} \). \( \mathcal{L}_{mix} \) is still invariant under \( P \) and \( C \) transformations, but breaks the symmetry under \( U(3)_V \) transformations when the bare quark masses are not equal. Notice that, in the limit in which all the quark masses coincide, the mixing Lagrangian vanishes. Here, we work in the isospin symmetric limit \( m_u = m_d \). In this limit the Lagrangian takes the form

\[
\mathcal{L}_{mix} = \frac{ic}{2} \left( m_s^2 - m_u^2 \right) \left\{ K_{1,A}^\mu K_{1,B}^{\mu +} - K_{1,A}^{\mu +} K_{1,B}^\mu + K_{1,A}^0 K_{1,B}^{\mu 0} - K_{1,A}^{\mu 0} K_{1,B}^0 \right\}.
\]

As a consequence, only the kaonic fields \( K_{1,A} \) and \( K_{1,B} \) mix and generate the two physical resonances \( K_1(1270) \) and \( K_1(1400) \):

\[
\begin{pmatrix}
K_1^\mu (1270) \\
K_1^\mu (1400)
\end{pmatrix} =
\begin{pmatrix}
\cos \varphi & -i \sin \varphi \\
-i \sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
K_{1,A}^\mu \\
K_{1,B}^\mu
\end{pmatrix}.
\]

The link of the here employed mixing angle \( \varphi \) to the mixing angle \( \theta_K \), introduced in the literature \[5, 18, 19, 20, 21, 22\] as

\[
\begin{pmatrix}
K_1^+ (1270) \\
K_1^+ (1400)
\end{pmatrix} =
\begin{pmatrix}
\sin \theta_K & \cos \theta_K \\
\cos \theta_K & -\sin \theta_K
\end{pmatrix}
\begin{pmatrix}
K_{1,A}^+ \\
K_{1,B}^+
\end{pmatrix},
\]

is given by

\[
\theta_K = 90^\circ + \varphi.
\]
In the context of mixing a comment is important: Our mixing Lagrangian describes this mixing at tree-level. However, a mixing between $K_{1,A}$ and $K_{1,B}$ arises when loops are taken into account: namely, as we shall describe later on, both states $K_{1,A}$ and $K_{1,B}$ couple predominantly to the same final state $K^*\pi$ and thus a loop of $K^*$ and $\pi$ can transform $K_{1,A}$ into $K_{1,B}$ (and vice-versa). (Such kind of mixing is a quantum mixing; it appears also in other contexts, see for instance Ref. [31] for the $a_0(980)-f_0(980)$ system, e.g. Ref. [32] for the neutral kaon system, and Ref. [33] for axial- and pseudovector $D_s$ states). In this work we do not evaluate this loop and we effectively describe this mixing by the constant term in Eq. (5). Going beyond this approximation would be an interesting task for the future.

We now turn to the determination of the parameters and to the results of the decays for isoscalar and isovector axial- and pseudovector $(P)$ particle is given by the formula

$$\Gamma_R \rightarrow VP(m_R, m_V, m_P) = \frac{g_{RV}^2 k_f(m_R, m_V, m_P)}{24\pi m_R^2} \left[ 2 + \frac{(m_R^2 + m_V^2 - m_P^2)^2}{4m_R^2m_V^2} \right] \Theta (m_R - m_V - m_P) ,$$

where $m_R, m_V, m_P$ refer to the masses of the initial $(R)$ and final $(P$ and $V$) states of the decay process and $\Theta (x)$ is the step function; the coupling constant $g_{RV}^2$ is a function of the constants $a, b$ and $c$ entering in Eq. (2) and its explicit value can be read off, case by case, in the full expression in Appendix A (by convention, isospin degeneracy factors are also formally included in $g_{RV}^2$). Finally, the momentum function $k_f(m_R, m_V, m_P)$ is the modulus of the three-momentum of an outgoing particle:

$$k_f(m_R, m_V, m_P) = \frac{1}{2m_R} \sqrt{m_R^4 + (m_V^2 - m_P^2)^2 - 2m_R^2(m_V^2 + m_P^2)} .$$

The use of the tree-level decay width is satisfactory when threshold effects are negligible. Being this not the case for some of the decay channels studied in this work, we include the finite width effects by integrating the decay width weighted with the spectral functions of the involved (axial-)vector and pseudovector particles over the particle masses [28]. Pseudoscalar mesons are taken as stable in view of their very small decay width. For what concerns the form of the spectral function of the other fields, we use a relativistic Breit-Wigner form with the proper corresponding energy threshold:

$$d_R (x) = \frac{N_R}{(x^2 - m_R^2)^2 + (m_R \Gamma_R)^2} \Theta (x - m_{\text{threshold},R}) ,$$

$$d_V (x) = \frac{N_V}{(x^2 - m_V^2)^2 + (m_V \Gamma_V)^2} \Theta (x - m_{\text{threshold},V}) .$$

The constants $N_R$ and $N_V$ are chosen in a way so that $\int_0^\infty dx d_{R/V} (x) = 1$. Finally, the decay width reads

$$\Gamma_R \rightarrow VP = \int_0^\infty \int_0^\infty dxdy \frac{\Gamma_R \rightarrow VP (x, y, m_P)}{m_P} d_R (x) d_V (y) .$$

Notice that in this work we do not take into consideration loop corrections to the masses and decay widths. For what concerns the masses we shall use the experimental values quoted by the PDG [34] as inputs and do not attempt a theoretical derivation, while for what concerns the decay widths the formula (13) represents a valid phenomenological description, as long as the ratio of the decay width and the mass of the unstable state is not too large, see Ref. [35] for a comparison of this treatment with the position (in particular, the imaginary part) of the pole.

3 Results for $I = 0$ and $I = 1$ resonances

We now turn to the determination of the parameters and to the results of the decays for isoscalar and isovector axial- and pseudovector states.
We first concentrate on axial-vector mesons as described by the Lagrangian in Eq. (2): we study the decays of \( f_1(1420) \), \( f_1(1285) \), and \( a_1(1260) \). First of all, we use the decay \( f_1(1420) \to K\bar{K}^* (892) \) to determine the axial-vector coupling constant \( a \). The decay width for this channel is given by Eq. (11) and (12) by setting \( g_{RVP} = a/\sqrt{2} \) and by using the corresponding PDG [34] masses and decay widths (for the vector and pseudoscalar kaons the masses of the charged kaon states are used). The average of the total decay width of \( f_1(1420) \) reported by [34] reads \( \Gamma_{f_1(1420)} = (54.9 \pm 2.6) \) MeV. The dominant decay channels of \( f_1(1420) \) are given by the decay channels into \( K\bar{K}^* (892) \) and \( K\bar{K}\pi \). Using the branching ratio between those two channels, we derive the experimental value

\[
\Gamma_{f_1(1420)}^{\text{exp}} \to K\bar{K}^* (892)+c.c. = (44.5 \pm 4.2) \text{ MeV,}
\]

out of which the coupling constant \( a \) is determined:

\[
|a| = (5.43 \pm 0.26) \text{ GeV.}
\]

In the next step we use this value for \( a \) to determine the other decay widths of axial-vector mesons and compare them to available experimental data. These results for the \( I = 0 \) and \( I = 1 \) decays are summarized in Table 2.

| Decay process         | Theory (MeV) | Experiment (MeV) |
|-----------------------|--------------|------------------|
| \( f_1(1420) \to K\bar{K}^* (892) + c.c. \) | 44.5 ± 4.2 | 44.5 ± 4.2       |
| \( a_1(1260) \to \rho\pi \) | 396 ± 37     | Dominant; \( \Gamma_{a_1}^{\text{tot}} = 250-600 \) (estimate [34]) |
| \( a_1(1260) \to K\bar{K}^* (892) + c.c. \) | 32.1 ± 3.03 | 6-55             |
| \( f_1(1285) \to K\bar{K}^* (892) + c.c. \) | 2.79 ± 0.26 | not seen         |
| \( f_1(1285) \to \rho\pi \) | 0            | < 0.075          |

Table 2: Decays of \( I = 1 \) and \( I = 0 \) axial-vector states. The first experimental entry has been used to fix the coupling constant \( a \).

The following comments are in order:

a.1) The resonance \( a_1(1260) \) is very broad and the dominant channel, both experimentally and theoretically, is the \( \rho\pi \) one. Our result for this channel confirms this expectation and fits well in the estimated range of the PDG [34] and is in very good agreement with the recent experimental result of the Compass Collaboration in Ref. [36].

a.2) We also have evaluated the decay of \( a_1(1260) \) into \( K\bar{K}^* (892) \). This decay mode has been seen; although no average or fit is performed by the PDG, the listed experimental values obtained by various experiments lie in the range between 5 and 55 MeV [34]. Our theoretical result is in agreement with these results. Notice that this decay mode could not be calculated without using the spectral functions because the sum of the nominal masses of the final state is higher than the nominal mass of the decaying particle.

a.3) In our theoretical framework the resonance \( f_1(1285) \) couples only to the channel \( K\bar{K}^* (892) \). We thus evaluated the decay width, which is about 3 MeV. Experimentally, this decay has not yet been seen [34]. Note, the full decay width of \( f_1(1285) \) amounts to \((24.2 \pm 1.1) \) MeV, thus very narrow. The fact that this resonance is narrow is well explained in our quarkonium framework: the would-be dominant \( K\bar{K}^* (892) \) channel is in this case very small because it is kinematically suppressed. The dominant decay channels of \( f_1(1285) \) are the \( 4\pi \) and the \( \eta\pi\pi \) ones, which are not included in our model. The role of light scalar mesons is relevant for these decays; for instance, for the \( \eta\pi\pi \) channel the contribution of \( a_0(980) \) is sizable [34].

a.4) It is interesting to stress that within our model \( f_1(1285) \) does not couple to \( \rho\pi \); this is a consequence of the symmetries of our approach. Our theoretical prediction is thus zero. This is very well verified by the experiment, for which a small upper limit has been set.
We now turn to the pseudovector sector. To fix the coupling constant $b$ we use the total decay width of $b_1(1235)$: $\Gamma_{b_1(1235)}^{\exp} = (142 \pm 9)$ MeV \[34\]. The relevant channels contributing to the total decay width are $\omega \pi$, $\rho \eta$ and $KK^*(892)$. Thus, out of the equation

$$\Gamma_{b_1(1235)} \rightarrow \omega \pi + \Gamma_{b_1(1235)} \rightarrow \eta \rho + \Gamma_{b_1(1235)} \rightarrow KK^*(892) + c.c. = (142 \pm 9) \text{ MeV},$$

we obtain

$$|b| = (7.0 \pm 0.22) \text{ GeV}.$$  

We use this value of $b$ to determine the other decays of $I = 0$ and $I = 1$ pseudovector mesons. The results are summarized in Table 3.

| Decay process | Theory (MeV) | Experiment (MeV) |
|---------------|--------------|-----------------|
| $b_1(1235) \rightarrow \omega \pi$ | $110.0 \pm 7.0$ | dominant ($\Gamma_{b_1(1235)}^{\exp} = 142 \pm 9$) |
| $b_1(1235) \rightarrow \eta \rho$ | $18.4 \pm 1.2$ | seen |
| $b_1(1235) \rightarrow KK^*(892) + c.c.$ | $16.2 \pm 1.0$ | seen |
| $b_1(1235) \rightarrow \eta \rho$ | $1.07 \pm 0.07$ | not seen |
| $b_1(1235) \rightarrow \phi \pi$ | $0$ | $< 2.1$ |
| $h_1(1170) \rightarrow \rho \pi$ | $364 \pm 23$ | dominant ($\Gamma_{h_1(1170)}^{\exp} = 360 \pm 40$) |
| $h_1(1170) \rightarrow \eta \rho$ | $7.15 \pm 0.46$ | not seen |
| $h_1(1170) \rightarrow KK^*(892) + c.c.$ | $7.99 \pm 0.51$ | not seen |
| $h_1(1380) \rightarrow KK^*(892) + c.c.$ | $54.0 \pm 3.4$ | $91 \pm 30$ |
| $h_1(1380) \rightarrow \phi \eta$ | $2.28 \pm 0.13$ | not seen |
| $h_1(1380) \rightarrow \phi \eta'$ | $0.93 \pm 0.06$ | not seen |

Table 3: Decays of $I = 1$ and $I = 0$ pseudovector states. The sum of the first three experimental entries has been used to fix the parameter $b$.

The following comments are in order:

b.1) The decay $b_1(1235) \rightarrow \omega \pi$ is, according to PDG, the dominant one. This is in well agreement with our result. In addition, we also have predicted the decays of the processes $b_1(1235) \rightarrow \eta \rho$ and $b_1(1235) \rightarrow KK^*(892)$, both of which have been seen, but no branching ratio is reported in \[34\].

b.2) The theoretical ratio $\Gamma_{b_1(1235)} \rightarrow \eta \rho/\Gamma_{b_1(1235)} \rightarrow \omega \pi = 0.17 \pm 0.02$ should be compared to the experimental ratio $\Gamma_{b_1(1235)} \rightarrow \eta \rho/\Gamma_{b_1(1235)} \rightarrow \omega \pi < 0.10$ reported in \[34\]. Our theoretical value is close but slightly above this upper limit. It should be however noticed that only one experiment has reported this ratio \[37\].

b.3) We also predict the ratio $\Gamma_{b_1(1235)} \rightarrow KK^*(892) + c.c./\Gamma_{b_1(1235)} \rightarrow \omega \pi = 0.15 \pm 2$, which has not been measured yet.

b.4) The theoretical prediction for the channel $b_1(1235) \rightarrow \eta' \rho$ has been reported in Table 3. This channel is subleading and has not yet been seen.

b.5) The decay channel $b_1(1235) \rightarrow \phi \pi$ vanishes exactly in our approach and is thus in agreement with the experimental upper limit. Moreover, the very small experimental value for the ratio $\Gamma_{b_1(1235)} \rightarrow \phi \pi/\Gamma_{b_1(1235)} \rightarrow \omega \pi < 0.004$ confirms the absence of this decay channel, in well agreement with the quarkonium assignment.

b.6) The experimental decay $\Gamma_{h_1(1170)}^{\exp} = (360 \pm 40)$ MeV is in very well agreement with our theoretical result of $(373 \pm 24)$ MeV. Moreover, only the channel $h_1(1170) \rightarrow \rho \pi$ has been experimentally seen: this is also reproduced by our results, where the decay modes $h_1(1170) \rightarrow \eta \omega$ and $h_1(1170) \rightarrow KK^*(892)$ are about 7 MeV (and thus much smaller than $h_1(1170) \rightarrow \rho \pi$). Still, these decay modes are testable predictions of our approach.

b.7) The resonance $h_1(1380)$ decays predominantly into $KK^*(892)$; experiment and theory are in well agreement. In addition, we have predicted the decay of the channels $h_1(1380) \rightarrow \phi \eta$ and $h_1(1380) \rightarrow \phi \eta'$, which turn out to be subdominant.
Summarizing, for both nonets the theoretical results are in good agreement with the available measured ones. In addition, we could make predictions for branching ratios of not-yet observed channels, which can be tested in future experiments.

4 Mixing and results in the kaonic sector

In this section we determine the mixing angle \( \varphi \) defined in Eq. (7) in the kaonic sector and the corresponding decays of \( K_1 (1270) \) and \( K_1 (1400) \).

As mentioned in Sec. 2, the non-vanishing difference of the bare quark masses \( m_u^2 - m_d^2 \) induces a mixing of the kaonic fields \( K_{1,A} \) and \( K_{1,B} \). We perform a unitary transformation from the unphysical basis \( \{ K_{1,A}, K_{1,B} \} \) to the physical isodoublets \( \{ K_1 (1270), K_1 (1400) \} \) by using the (inverse form) of Eq. (7). For the determination of \( \varphi \) we use the \( K^* (892) \pi \) decay mode of the resonances \( K_1 (1270) \) and \( K_1 (1400) \). The partial decay widths are given by \( \Gamma_{\text{exp}}^{K_1 (1270) \rightarrow K^* \pi} = (14.4 \pm 5.5) \text{ MeV} \) and \( \Gamma_{\text{exp}}^{K_1 (1400) \rightarrow K^* \pi} = (117 \pm 10) \text{ MeV} \) [35].

We perform a fit by minimizing the \( \chi^2 \)-function,

\[
\chi^2(\varphi) = \left( \frac{\Gamma_{\text{th}}^{K_1 (1270) \rightarrow K^* \pi}(\varphi) - \Gamma_{\text{exp}}^{K_1 (1270) \rightarrow K^* \pi}}{\delta \Gamma^{K_1 (1270) \rightarrow K^* \pi}} \right)^2 + \left( \frac{\Gamma_{\text{th}}^{K_1 (1400) \rightarrow K^* \pi}(\varphi) - \Gamma_{\text{exp}}^{K_1 (1400) \rightarrow K^* \pi}}{\delta \Gamma^{K_1 (1400) \rightarrow K^* \pi}} \right)^2 , \tag{18}
\]

which implies (restricting to the interval \( [-\pi/2, \pi/2] \)):

\[
|\varphi_1| = (56.4 \pm 4.2)^\circ , \quad |\varphi_2| = (19.0 \pm 4.2)^\circ , \tag{19}
\]

with the acceptable \( \chi^2 \)-value \( \chi^2(\varphi_1) = \chi^2(\varphi_2) = 1.54 \).

Notice that we cannot determine the sign of the mixing angle \( \varphi \) because we do not know the sign of \( a \) and \( b \). In particular, \( \varphi < 0 \) implies \( a \cdot b > 0 \) and \( \varphi > 0 \) implies \( a \cdot b < 0 \).

For both angles in Eq. (19) we calculate the decay widths of both resonances \( K_1 (1270) \) and \( K_1 (1400) \) into \( K \rho \) and \( K \omega \). The results are presented in Table 4. Note, while for the resonance \( K_1 (1270) \) we use the branching ratios of the summarizing table and the estimated full width \( \Gamma_{\text{exp}}^{K_1 (1270)} = (90 \pm 20) \text{ MeV} \) reported in Ref. [34], for the resonance \( K_1 (1400) \) we use the results of Ref. [38], in which the partial decay widths of this resonance have been directly determined, see also [34].

| Decay process | Theory (MeV) | Experiment (MeV) |
|---------------|-------------|-----------------|
| \( K_1 (1270) \rightarrow K^* \pi \) | \( 10.8 \pm 0.79 \) | \( 10.8 \pm 0.79 \) | \( 14.4 \pm 5.5 \) [34] |
| \( K_1 (1400) \rightarrow K^* \pi \) | \( 106.4 \pm 7.8 \) | \( 106.4 \pm 7.8 \) | \( 117 \pm 10 \) [38] |
| \( K_1 (1270) \rightarrow K \rho \) | \( 56.2 \pm 4.1 \) | \( 39.6 \pm 2.9 \) | \( 38 \pm 10 \) [34] |
| \( K_1 (1270) \rightarrow K \omega \) | \( 9.5 \pm 0.7 \) | \( 6.74 \pm 0.49 \) | \( 9.9 \pm 2.8 \) [34] |
| \( K_1 (1400) \rightarrow K \omega \) | \( 0.45 \pm 0.03 \) | \( 2.73 \pm 1.2 \) | \( 2 \pm 1 \) [38] |
| \( K_1 (1400) \rightarrow K \rho \) | \( 0.13 \pm 0.01 \) | \( 7.59 \pm 0.56 \) | \( 23 \pm 12 \) [38] |

Table 4: \( K_1 (1270) \) and \( K_1 (1400) \) decay widths for \( K \rho \) and \( K \omega \) decay channel. The first two entries have been used to determine the mixing angle \( \varphi \) via a fit.

Following comments are in order:

(i) The case \( |\varphi_1| = (56.4 \pm 4.3)^\circ \) (in turn: \( |\varphi_2| = (33.6 \pm 4.3)^\circ \)). This large mixing angle implies that \( K_1 (1270) \) is described by the pseudovector state \( K_{1,B} \) with a probability of 69% and by the axial-vector state \( K_{1,A} \) with the remaining probability of 29%; a reversed situation holds for \( K_1 (1400) \). The masses of the bare states \( K_{1,A} \) and \( K_{1,B} \) read

\[
m_{K_{1,A}} = 1.36 \text{ GeV} , \quad m_{K_{1,B}} = 1.31 \text{ GeV} , \tag{20}
\]
thus realizing the bare ordering \( m_{K_{1,b}} < m_{K_{1,a}} \). This bare level ordering is also in agreement with the other members of the nonets: \( h_1(1170) \) and \( h_1(1380) \) are lighter than the corresponding states \( f_1(1285) \) and \( f_1(1420) \). The corresponding value of the parameter \( c \) defined in Eq. (5) and (6) reads

\[
c = \frac{m_{K_{1,a}}^2 - m_{K_{1,b}}^2}{m_s^2 - m_u^2} \tan(2\varphi_1) = \pm \left( 35^{+23}_{-11} \right),
\]

where for the quark masses we have used \( m_s = 95 \text{ MeV} \) and \( m_u = 2.3 \text{ MeV} \) and where a positive sign of \( c \) corresponds to a negative sign of \( \varphi_1 \) and vice-versa.

In Table 4 we have presented the theoretical results of all decay channels, and the first five entries are in good agreement with the experiment. For the last entry \( K_1(1400) \to K\omega \) the theoretical prediction is close to zero in virtue of a destructive interference; this result is not in good agreement with the experimental result of Ref. [38], being off of a factor 2\( \sigma \). However, it should be stressed that the branching ratio reported in the summarizing table of Ref. [34] reads \( (1.0 \pm 1.0)\% \), which is well compatible with our result.

(ii) The case \( |\varphi_2| = (19.5 \pm 4.5)^\circ \) (in turn: \( |\theta_K| = (70.5 \pm 4.3)^\circ \)). This small mixing angle implies that \( K_1(1270) \) is described by the axial-vector state \( K_{1,A} \) with a probability of 89\% and by the pseudovector state \( K_{1,B} \) with the remaining probability of 11\%; a reversed situation holds for \( K_1(1400) \). The bare masses read

\[
m_{K_{1,A}} = 1.29 \text{ GeV}, \quad m_{K_{1,B}} = 1.39 \text{ GeV}.
\]  

(The parameter \( c \) reads now \( c = \pm (23^{+11}_{-6}) \), where a positive sign of \( c \) corresponds to a negative sign of \( \varphi_2 \) and vice-versa.) However, there are clear reasons why this mixing angle is not favoured: first of all, the decay mode \( K_1(1400) \to K\rho \) reads 27.3 \( \pm \) 1.2 MeV, which is much higher than the experimental value (2 \( \pm \) 1) MeV. Even using the branching ratio 0-6\% reported by the PDG, one finds the upper limit of 10.5 MeV for this decay channel. In addition, the bare level ordering \( m_{K_{1,A}} > m_{K_{1,B}} \) is not expected. In the end, in the literature a solution of the type \( |\theta_K| \sim 70^\circ \) has not been found (typically, the alternative solution found in many works reads, as discussed in the introduction, \( |\theta_K| \sim 55^\circ \)).

In conclusion, a overall good agreement is obtained for the \( |\varphi_1| = (56.3 \pm 4.2)^\circ \), which means \( |\theta_K| = (33.6 \pm 4.3)^\circ \). Using the sign determination of Ref. [20], \( \theta_K = -(34 \pm 13)^\circ \), we are led to choose

\[
\theta_K = (-33.6 \pm 4.3)^\circ.
\]

In turn, this result means that the coupling constants \( a \) and \( b \) have opposite signs: \( a \cdot b < 0 \).

## 5 Conclusions

In this work we have studied the decays of axial-vector and pseudoscalar mesons interpreted, as quark-antiquark states, into a vector and a pseudoscalar meson. To this end, we have used a relativistic quantum field theoretical model which makes use of flavor symmetry and its explicit breaking due to non equal bare quark masses.

Our model contains three unknown parameters. Two of them describe the interaction of axial-vector and pseudovector mesons with pseudoscalar and vector states respectively. The results have been summarized in Table 2 and Table 3, where a good agreement with the experiment is shown and further theoretical predictions for not-yet measured decay channels have been presented. The remaining free parameter describes the mixing of a bare axial-vector kaonic state \( K_{1,A} \) and a bare pseudovector kaonic state \( K_{1,B} \), leading to the resonances \( K_1(1270) \) and \( K_1(1400) \). The results are summarized in Table 4, where it is shown that a good agreement in all decay channels is achieved for the mixing angle \( |\theta_K| = (33.6 \pm 4.3)^\circ \).

In conclusion, our results confirm the predominant quark-antiquark nature of the ground-state axial-vector and pseudovector mesons. In the kaonic sector we have provided an independent determination of the mixing angle, which is potentially useful in the search and/or falsification of physics beyond the Standard Model.
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Appendix A

According to (2) and the discussion of section 2 our Lagrangian consists of three parts:

\[ \mathcal{L} = \mathcal{L}_A + \mathcal{L}_B + \mathcal{L}_{mix}, \]

in which the axial-vector part of the Lagrangian is given by

\[ \mathcal{L}_A = i\alpha \text{Tr} \left\{ A_\mu [V^\mu, P^-] \right\} \]

\[ = \frac{a_1}{2\sqrt{2}} \left\{ f_{1N,A\mu} \left( K^{*\mu+}K^- - K^{*\mu-}K^+ + K^{*\mu}0 \tilde{K}^0 - \tilde{K}^{*\mu}0K^0 \right) + f_{1S,A\mu} \left( K^{*\mu-}K^+ - K^{*\mu+}K^- \right) \right\} \]

and the explicit expression for the pseudovector part is given by

\[ \mathcal{L}_B = b \text{Tr} \left\{ B_\mu [V^\mu, P^-] \right\} \]

\[ = \frac{b}{2\sqrt{2}} \left\{ f_{1N,B} \left( 2\omega_0^\mu N + 2\rho^00\pi^0 + 2\rho^+\pi^- + 2\rho^-\pi^+ + K^{*\mu+}K^- + K^{*\mu-}K^+ + K^{*\mu}0 \tilde{K}^0 \right) + f_{1S,B} \left( 2\omega_0^\mu S + K^{*\mu+}S^- + K^{*\mu-}S^+ + K^{*\mu}0 \tilde{S}^0 - \tilde{S}^{*\mu}0S^0 \right) + \frac{1}{2\sqrt{2}}b_\mu \left( \left( 2\omega_\mu^\mu N \right) \right) \right\} \]

and the explicit expression for the pseudovector part is given by

\[ \mathcal{L}_{mix} = \frac{c}{2\sqrt{2}} \left\{ f_{1N,mix} \left( 2\omega_0^\mu N + 2\rho^00\pi^0 + 2\rho^+\pi^- + 2\rho^-\pi^+ + K^{*\mu+}K^- + K^{*\mu-}K^+ + K^{*\mu}0 \tilde{K}^0 \right) + f_{1S,mix} \left( 2\omega_0^\mu S + K^{*\mu+}S^- + K^{*\mu-}S^+ + K^{*\mu}0 \tilde{S}^0 - \tilde{S}^{*\mu}0S^0 \right) + \frac{1}{2\sqrt{2}}c_\mu \left( \left( 2\omega_\mu^\mu N \right) \right) \right\} \]

The explicit form of the mixing part \( \mathcal{L}_{mix} \) in the isospin symmetric limit is given by Eq. (6).
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