NONLOCAL PROPERTIES OF ENTANGLED TWO-PHOTON GENERALIZED BINOMIAL STATES IN TWO SEPARATE CAVITIES

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Abstract—We consider entangled two-photon generalized binomial states of the electromagnetic field in two separate cavities. The nonlocal properties of this entangled field state are analyzed by studying the electric field correlations between the two cavities. A Bell’s inequality violation is obtained using an appropriate dichotomic cavity operator, that is in principle measurable.

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I. INTRODUCTION

Entanglement of spatially separate quantum systems holds an important role both in the investigation of quantum theory foundations \[ \text{[1]} \] and in quantum information and computation processing \[ \text{[2]} \]. Manifestations of quantum nonlocal properties of entangled distant systems can be characterized by Bell’s inequality violations \[ \text{[3–4]} \]. Therefore, for their striking quantum nonlocal properties and their possible applications, entangled quantum systems are subject of intense study in several contexts.

In particular, in cavity quantum electrodynamics (CQED) several schemes have been proposed to generate entangled number states in two separate single-mode high-\( Q \) cavities, the cavities having zero or one photon \[ \text{[5–6, 7–8, 9, 10]} \]. There are some proposals to prove Bell’s inequality violations for such states \[ \text{[11, 12]} \] but an experimental test has not yet been made. It appears also of interest to obtain entanglement between electromagnetic field states with mesoscopic characteristics, so that the classical-quantum border may be approached. This is possible, for example, if the electromagnetic field states present non-zero mean fields. This fact seems to exclude the number states, and some schemes to generate entangled coherent states in two separate cavities have been then proposed \[ \text{[13, 14]} \]. However, since two different coherent states are never orthogonal, entangled states of this kind cannot be made totally distinguishable. Therefore, it may be useful to have entangled states in separate cavities formed by field states that present non-zero mean fields, and can be obtained by standard resonant interactions of two-level atoms with the cavities.

Nonclassical states of electromagnetic field suited for this goal are, for example, the generalized binomial states \[ \text{[15, 16, 17]} \]. These states, characterized by a finite maximum number of photons \( N \), interpolate between the coherent state and the number state and present non-zero mean electric fields. Moreover, for each \( N \)-photon generalized binomial state (NGBS) it is always possible to find an orthogonal one \[ \text{[18]} \]. For large \( N \), these states thus present mesoscopic properties. The point remains on how to generate entangled orthogonal couples of these binomial states and characterize their nonlocal properties. It has been recently indicated how both entangled 1GBSs and 2GBSs \[ \text{[18, 19]} \] can be generated in two separate cavities by resonant atom-cavity interactions. In the one-photon case, the electric field correlations of the two cavities have been analyzed and Bell’s inequality violations are shown to be observable \[ \text{[18]} \].

Since entangled 2GBSs may be generated, it appears of interest to analyze their nonlocal properties and compare them with the ones of the one-photon case. This constitutes the aim of this paper. In particular, we shall examine the correlations of electric field for entangled 2GBSs and, by introducing an appropriate dichotomic cavity operator, we also construct a Bell’s inequality that it is shown to be violated for a wide range of the degree of entanglement and is amenable to an experimental verification.

The paper is organized as follows. In Sec. \[ \text{II} \] we recall the definition of generalized binomial state and some of their useful properties. In Sec. \[ \text{III} \] we define the entangled two-photon generalized binomial states, briefly describing their possible generation scheme \[ \text{[19]} \]. In Sec. \[ \text{IV} \] we study the electric field correlations, while in Sec. \[ \text{V} \] we show Bell’s inequality violations for these entangled states. In Sec. \[ \text{VI} \] we summarize our conclusions.

II. GENERALIZED BINOMIAL STATE

The normalized \( N \)-photon generalized binomial state (NGBS) is defined as \[ \text{[17]} \]

\[
|N, p, \phi\rangle \equiv \sum_{n=0}^{N} \left( \begin{array}{c} N \\ n \end{array} \right) p^n (1-p)^{N-n} \frac{1}{\sqrt{n!}} e^{i n \phi} |n\rangle,
\]

(1)

where \( 0 \leq p \leq 1 \) is the probability of single photon occurrence, \( \phi \) the mean phase \[ \text{[20]} \] and \( \left( \begin{array}{c} N \\ n \end{array} \right) = N!/[N-n)!n!]. \) As said above, the NGBS of Eq. \[ \text{[11]} \] interpolates between the number state and the coherent state. In fact, for \( p = 0, 1 \) it is reduced to the number states \( |0\rangle, |N\rangle \) respectively. On the other hand, when \( N \to \infty \) and \( p \to 0, \)
fixing $N\rho \equiv |\alpha|^2$, the NGBS reduces to the coherent state $|\alpha\rangle$ with $\alpha = |\alpha|e^{i\phi}$. Two NGBSs $|N, p, \phi\rangle$, $|N, p', \phi'\rangle$ are orthogonal if and only if $18$

$$p' = 1 - p, \quad \phi' = (2k + 1)\pi + \phi \quad (k \text{ integer}). \quad (2)$$

### III. ENTANGLED TWO-PHOTON GENERALIZED BINOMIAL STATES

Since it is possible to generate, by opportune resonant interactions between two-level atoms and cavities, entangled 2GBSs in two separate cavities $10$, we assume that two identical separate single-mode cavities, namely $C_1, C_2$, are prepared in the state

$$|\Psi^{(2)}\rangle = \mathcal{N}[2, p_1, \phi_1]\{2, 1 - p_2, \pi + \phi_2\},$$

where $\eta$ is the 1GBS in the cavity $C_j$ as obtained by Eq. (1). Since each couple of 2GBSs of Eq. (3) in the cavity $C_j$ satisfies the orthogonality condition of Eq. (2), the state $|\Psi^{(2)}\rangle$ thus represents entangled orthogonal 2GBSs in two separate cavities.

For the limit values $p_1, p_2 = 0, 1$, the entangled 2GBSs of Eq. (3) are reduced to entangled number states having zero or two photon of the form

$$|\Psi^{(2)}\rangle_{p_1 = p_2 = 1} = \mathcal{N}[2, 0, \phi_1]\{2, 1, p_2, \pi + \phi_2\},$$

$$|\Psi^{(2)}\rangle_{p_1 = 1, p_2 = 0} = \mathcal{N}[2, 0, \phi_1]\{2, 1, 0, \pi + \phi_2\}. \quad (4)$$

This property will be used later on.

### IV. ELECTRIC FIELD CORRELATIONS

In order to evidence the non-local properties of the entangled 2GBSs, in this section we examine the electric field correlations between the two cavities. The quantized electric field inside each single-mode cavity $C_j (j = 1, 2)$ of frequency $\omega$ and volume $V$, can be written, at the time $t_j = 0$ and in the center of the cavity, as $E_j(z_j) = e_j\hat{E}_j$ where $20$

$$\hat{E}_j(z_j) = \sqrt{4\pi\hbar\omega/V}(a_j + a_j^\dagger). \quad (5)$$

In the following, in order to make a comparison between the two-photon and the one-photon cases, we first briefly review the correlations obtained for entangled 1GBSs $18$ and successively give the results for entangled 2GBSs.

#### A. Electric field correlations for entangled 1GBSs

In this case, the two cavities are in the entangled orthogonal 1GBSs $18$

$$|\Psi^{(1)}\rangle = \mathcal{N}[1, p_1, \phi_1]\{1, 1 - p_2, \pi + \phi_2\},$$

where $\eta$ and $\mathcal{N}$ are the same of Eq. (3) and $1, p_j, \phi_j)$, $j$ indicates 1GBS in $C_j (j = 1, 2)$, as obtained by Eq. (1). The expectation value $\langle \Psi^{(1)}|\hat{E}_j|\Psi^{(1)}\rangle = \langle \hat{E}_j \rangle$ of the electric field in $C_j$ is

$$\langle \hat{E}_j \rangle = 4(-1)^{j-1}\sqrt{\frac{\pi\hbar\omega p_j(1 - p_j)}{V}} \frac{1 - |\eta|^2}{1 + |\eta|^2} \cos \phi_j. \quad (7)$$

$\langle \hat{E}_j \rangle$ vanishes when $p_1, p_2 = 0, 1$, that is when the entanglement is between the number states $|0\rangle, |1\rangle$, or when $|\eta| = 1$, that is for maximally entangled states.

A quantitative indication of the correlations of electric field between the cavities is given by the covariance $C(\hat{E}_1, \hat{E}_2) = \langle \hat{E}_1 \hat{E}_2 \rangle - \langle \hat{E}_1 \rangle \langle \hat{E}_2 \rangle$ that in this case is

$$C(\hat{E}_1, \hat{E}_2) = \frac{8\pi\hbar\omega}{V}\left\{\frac{\eta}{1 + |\eta|^2}\left[f(p_1, p_2) \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 \right] - \left[1 - (1 - |\eta|^2)^2 \right] \times h(p_1, p_2) \cos \phi_1 \cos \phi_2 \right\}, \quad (8)$$

where

$$f(p_1, p_2) = (1 - 2p_1)(1 - 2p_2),$$

$$h(p_1, p_2) = 2\sqrt{p_1p_2(1 - p_1)(1 - p_2)}. \quad (9)$$

$C(\hat{E}_1, \hat{E}_2)$ is in general different from zero, and it vanishes when $\eta = 0, \pm 0$, i.e., when the entangled state $|\Psi^{(1)}\rangle$ of Eq. (3) is reduced to a product of two uncorrelated 1GBSs. In particular, when $\eta = \pm 1$ (maximal entanglement) and $p_1 = 1/2$, it is$c(\hat{E}_1, \hat{E}_2) = - (4\pi\hbar\omega/V) \cos (\phi_1 + \phi_2)$. In this case, if $\phi_1 + \phi_2 = \pi/2$ the covariance vanishes, while, if $\phi_1 + \phi_2 = 0, \pi$, it takes the maximum absolute value $4\pi\hbar\omega/V$. This shows that the electric fields in two separate cavities prepared in entangled 1GBSs are correlated and non-zero.

#### B. Electric field correlations for entangled 2GBSs

Using Eqs. (3) and (1) for $N = 2$, the expectation value of the electric field for the 2GBS $|2, p_j, \phi_j\rangle (j = 1, 2)$ is

$$\langle \hat{E}_j \rangle = \sqrt{\frac{2\pi\hbar\omega p_j(1 - p_j)}{V}} f(p_j) \cos \phi_j,$$

where $f(p_j) = 4(1 - p_j + \sqrt{2}p_j)$, and it is in general different from zero, as expected. The two cavities are now prepared in the entangled 2GBSs of Eq. (3). The mean electric field $\langle \Psi^{(2)}|\hat{E}_j|\Psi^{(2)}\rangle = \langle \hat{E}_j \rangle$ in $C_j$ for the state $|\Psi^{(2)}\rangle$ is now given by

$$\langle \hat{E}_j \rangle = -\sqrt{8\pi\hbar\omega p_j(1 - p_j)} \left[\frac{f(j + 1 - 3j)}{1 + |\eta|^2} - \frac{|\eta|^2 f(2j - 3j)}{1 + |\eta|^2}\right](-1)^j \cos \phi_j \quad (10)$$
This mean electric ($\hat{E}_j$) field in the cavity $C_j$ is in general different from zero when $p_j \neq 0,1$ and $\phi \neq \pm \pi/2$. However, if the entanglement is maximum ($|\eta| = 1$), it vanishes only if also $p_j = 1/2$. This behavior is different from the one of the 1GBSs case, where $\langle \hat{E}_j \rangle$ is always zero if the entanglement is maximum. In particular, if the entangled 2GBSs are reduced to the entangled number states of Eq. (3) ($p_j = 0,1$), the mean electric field is zero in each cavity $\langle \hat{E}_j \rangle$ is zero independently on the value of $|\eta|$, as expected.

The covariance $C(E_1; E_2)$ of the electric fields for entangled 2GBSs is

$$C(E_1; E_2) = \frac{\pi \hbar \omega (p_1, p_2)}{V} \left\{ \left[ \frac{F(p_1) F(1-p_2)}{1 + |\eta|^2} \right. \right.$$

$$\left. - \frac{F'(p_1, p_2) + |\eta|^2 F(p_1, p_1)}{1 + |\eta|^2} \right\} \cos \phi_1 \cos \phi_2$$

$$- \frac{8|\eta| (3 - 2\sqrt{2}) F(p_1, p_2, \phi_1, \phi_2)}{1 + |\eta|^2},$$

(11)

where $F(p_j) = \bar{f}(p_j) - |\eta|^2 \bar{f}(1-p_j)$,

$$F'(p_1, p_2) = \bar{f}(p_1) \bar{f}(1-p_2),$$

$$F(p_1, p_2, \phi_1, \phi_2) = \bar{f}(p_1, p_2) \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2,$

and $f(p_1, p_2), h(p_1, p_2)$ are defined by Eq. (9). The covariance vanishes when $p_1, p_2 = 0,1$. Thus, the electric fields in the two cavities are always uncorrelated for entangled zero and two-photon number states, as given by Eq. (4). The covariance $C(E_1; E_2)$ also vanishes when $|\eta| = 1, p_1 = p_2 = 1/2$ and $\phi_1 = \phi_2 = \phi$, the covariance of Eq. (11) becomes $C(E_1; E_2) = -2\pi \hbar \omega (3 + 2\sqrt{2} \cos 2\phi)/V$, that is different from zero, independently from the value of the mean phase $\phi$ appearing in the entangled 2GBSs of Eq. (3). This behavior is different from the one of entangled 1GBSs having the same values of the parameters, where the covariance becomes constant and equal to $-4\pi \hbar \omega /V$ when $\eta = -1$, while it is $(-4\pi \hbar \omega /V \cos 2\phi$ when $\eta = +1$. Therefore, preparing entangled 1GBSs or entangled 2GBSs with given values of the characteristic parameters $p, \phi$ [18, 19], we can obtain a different behavior of the covariances in the two cases. Thus, we have shown that the electric fields of two cavities, prepared in entangled 2GBSs, are correlated.

V. BELL’S INEQUALITY VIOLATION

In the previous section we have found that the electric fields of two cavities, prepared in entangled 2GBSs, are correlated. In this section, instead, we shall analyze the quantum nonlocal correlations for entangled 2GBSs by using Bell’s inequality in the Clauser-Horne-Shimony-Holt (CHSH) form [4, 21]. To this purpose, we introduce a measurable dichotomic operator expressed in terms of the cavity field states.

The two orthogonal 2GBSs $|2, p, \phi\rangle, |2, 1-p, \pi+\phi\rangle$ constitute the basis vectors of a 2-dimensional subspace, $B = \{ |2, p, \phi\rangle \equiv |+\rangle, |2, 1-p, \pi+\phi\rangle \equiv |−\rangle \}$, of the 3-dimensional Hilbert space [17]. We therefore construct, for each single-mode cavity, a dichotomic operator $\hat{F}$, acting within the 2-dimensional subspace spanned by the basis $B$. Defining a 3-dimensional vector $\vec{F} \equiv (F_x, F_y, F_z)$, we construct the operator $\vec{F}$

$$\hat{F} = \vec{F} \cdot \vec{\sigma} = \left( \begin{array}{ccc} F_z & F_x - iF_y \\ F_x + iF_y & -F_z \end{array} \right) = \left( \begin{array}{ccc} F_{11} & F_{12} \\ F_{12} & -F_{11} \end{array} \right),$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices acting on the $B$ subspace while the parameters $F_x, F_y$ will be shown to be linked to the coefficients of the linear superposition of the basis states. In order that this dichotomic operator $\hat{F}$ has eigenvalues $\pm 1$, it must be

$$|\vec{F}|^2 = F_x^2 + F_y^2 + F_z^2 = |F_{12}|^2 + F_z^2 = 1 \Rightarrow$$

$$|F_{12}| = \sqrt{1 - F_z^2}, \quad F_{12} = |F_{12}| e^{i\theta}.$$

(12)

The expression of the operator $\hat{F}$ in terms of the basis vectors is then

$$\hat{F} = F_z (|+\rangle\langle+| - |−\rangle\langle−|) + \sqrt{1 - F_z^2} (e^{i\theta} |+\rangle\langle−| + e^{-i\theta} |−\rangle\langle+|),$$

(13)

and its eigenvectors are given by

$$|\pm\rangle = [\sqrt{1 + F_z} |+\rangle + \sqrt{1 - F_z} e^{-i\theta} |−\rangle]/\sqrt{2},$$

$$|\mp\rangle = [-\sqrt{1 + F_z} e^{i\theta} |+\rangle + \sqrt{1 + F_z} |−\rangle]/\sqrt{2}.$$

(14)

We shall show that this operator satisfies a CHSH-Bell inequality violations for entangled 2GBSs.

Let us consider the entangled 2GBSs $|\Psi(2)\rangle$ given in Eq. (8) and take $p_1 = p_2 = p$ and $\phi_1 = \phi_2 = \phi$. This choice shall simplify the expressions, making the results more easily readable. The operator of Eq. (14) in the cavity $C_j (j = 1, 2)$ is indicated as $\hat{F}^{(j)} (\theta_j)$, with $F_z$ being equal in the two cavities. The quantum correlation of the operator $\hat{F}$ in the two cavities is defined as

$$\langle \hat{F}^{(1)} (\theta_1) \hat{F}^{(2)} (\theta_2) \rangle = \langle |\Psi(2)\rangle |\Psi(2)\rangle \langle \hat{F}^{(1)} (\theta_1) \hat{F}^{(2)} (\theta_2) \rangle |\Psi(2)\rangle,$$

and it results to be

$$\langle \hat{F}^{(1)} (\theta_1) \hat{F}^{(2)} (\theta_2) \rangle = \frac{2\eta (1 - F_z^2)}{1 + |\eta|^2} \cos (\theta_1 - \theta_2) - F_z^2.$$

(15)

In terms of these correlations it is possible to construct the CHSH-Bell inequality as [4, 21]

$$S_B = |\langle \hat{F}^{(1)} (\theta_1) \hat{F}^{(2)} (\theta_2) \rangle - \langle \hat{F}^{(1)} (\theta_1) \hat{F}^{(2)} (\theta_2') \rangle|$$

$$+ |\langle \hat{F}^{(1)} (\theta_1') \hat{F}^{(2)} (\theta_2) \rangle + \langle \hat{F}^{(1)} (\theta_1') \hat{F}^{(2)} (\theta_2') \rangle| \leq 2. \quad (16)$$

Thus, $S_B$ is formed by correlations of the kind [15], having different phase angles $\theta_j, \theta_j'$ but the same dependence on $F_z$. At this point, we look for opportune values of the
parameters $\eta, F_z, \vartheta_1, \vartheta_1'$ of Eq. (16) such that $S_B > 2$ and thus the Bell’s inequality violation occurs. Setting the partial derivative relating to $F_z$ of the correlation functions appearing in Eq. (16) equal to zero, we readily obtain

$$\frac{\partial S_B}{\partial F_z} = F_z f(\eta, \vartheta_1, \vartheta_2, \vartheta_1', \vartheta_2') = 0 \Rightarrow F_z = 0,$$

(17)

where the function $f(\eta, \vartheta_1, \vartheta_2, \vartheta_1', \vartheta_2')$ is never zero. It is possible to see that $F_z = 0$ corresponds to a maximum of $S_B$. For this value of $F_z$, the CHSH-Bell inequality of Eq. (16) becomes

$$S_B = G^{(E)} \left[ \cos(\vartheta_1 - \vartheta_2) - \cos(\vartheta_1 - \vartheta_2') \right] + \left| \cos(\vartheta_1' - \vartheta_2) - \cos(\vartheta_1' - \vartheta_2') \right| \leq 2,$$

(18)

where $G^{(E)} = 2|\eta|/(1 + |\eta|^2)$ is the degree of entanglement [22], invariant with respect to the substitution $|\eta| \rightarrow 1/|\eta|$, equal to zero for $|\eta| = 0$, $+\infty$ (uncorrelated states) and equal to one for $|\eta| = 1$ (maximally entangled states). Choosing appropriate values of the angles $\vartheta_1, \vartheta_1'$, the $S_B$ function of Eq. (18) can be shown to take values greater than two, so that the CHSH-Bell inequality is violated.

For example, after choosing $\vartheta_1 = 0, \vartheta_2 = \pi/4, \vartheta_1' = \pi/2$, we plot $S_B = S_B(G^{(E)}), \vartheta_2'$ in Fig. 1, which shows that for some values of $\vartheta_2'$ and $G^{(E)}$, we have $S_B > 2$. The CHSH-Bell inequality is violated also when the degree of entanglement $G^{(E)}$ is not maximum. In particular, when $\vartheta_2' = 3\pi/4$, from Eq. (18) we obtain

$$S_B = 2\sqrt{2}G^{(E)} > 2 \Rightarrow 1/\sqrt{2} \approx 0.707 < G^{(E)} \leq 1.$$  

(19)

When the degree of entanglement is maximum ($G^{(E)} = 1$), the maximum value of $S_B$ ($S_B^{\max} = 2\sqrt{2} \approx 2.828$) is obtained. This value represents the maximal possible violation of the CHSH-Bell inequality [2].

It is important to note that the choice of the operator parameters $F_z, \vartheta$ in each cavity determine the eigenvectors of the operator $\hat{F}$ of Eq. (13), as readily seen from Eq. (14). These eigenvectors represent states of the cavity field expressed as superpositions of two orthogonal 2GBSs. It is possible to show that these field states can be in principle measured by probe two-level atoms that “read” the cavity field [23]. The possibility of measuring the eigenvectors of the cavity operator $\hat{F}$, corresponding to the measurement of its eigenvalues $\pm 1$, opens thus the way to an experimental Bell test for entangled 2GBSs in two separate cavities. The correlations $\langle F^{(1)}(\vartheta_1)F^{(2)}(\vartheta_2) \rangle$ for the desired values of the angles $\vartheta_1, \vartheta_2$ can be obtained by statistical averages on the ensemble of the measurements. The CHSH-Bell inequality of Eq. (16) can be thus finally tested.

VI. CONCLUSION

In this paper we have analyzed the non-local properties of entangled two-photon generalized binomial states (2GBSs) in two spatially separate cavities. In particular, we have investigated the expectation values and the correlations of the electric field for these entangled states. We have also compared these results with the ones for entangled 1GBSs, emphasizing the different behavior of the quantum correlations in the two cases.

We have constructed a Bell’s inequality by using an appropriate dichotomic cavity field operator that is in principle measurable by probe two-level atoms. We have then shown that the CHSH-Bell inequality applied to entangled 2GBSs can be violated for a wide range of the degree of entanglement (Sec. V). We point out here that the atomic state detector efficiency $\alpha$ plays an important role in the experimental realization of a Bell test. Here we have supposed an ideal efficiency ($\alpha = 1$) (see Sec. V), but if we include the detectors efficiencies in the correlation functions, the CHSH-Bell inequality would not be violated for values of $\alpha$ less than a threshold value $\alpha_0 \approx 0.8284$ for maximally entangled states [4,23]. However, this problem can be overcome by the so-called “fair sampling” hypothesis, where the sub-ensemble of detected events (detected probe atoms) represents the whole ensemble. Thus, the results rely only on the detected events, but the detection loophole remains “open” [24,25]. Only for detector efficiencies greater than $\alpha_0$ the detection loophole can be closed. It is worth to note that it could be very difficult to realize experimental loophole free Bell tests, because we would need simultaneous and perfect efficiency measurements of eventual probe atoms. Anyway, recent laboratory developments open promising perspectives for a better and easy control of a well-defined atom numbers sequence [26] and for a high
atomic detection efficiency in microwave CQED experiments [27]. The realization of the Bell test proposed here for entangled 2GBSs would give a direct demonstration of non-local behavior for entangled cavity fields with a photon number $N > 1$. 

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