NEW MECHANISM OF ACCELERATION OF PARTICLES BY STELLAR BLACK HOLES

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In this paper we study efficiency of particle acceleration in the magnetospheres of stellar mass black holes. For this purpose we consider the linearized set of the Euler equation, continuity equation and Poisson equation respectively. After introducing the varying relativistic centrifugal force, we show that the charge separation undergoes the parametric instability, leading to generation of centrifugally excited Langmuir waves. It is shown that these waves, via the Langmuir collapse damp by means of the Landau damping, as a result energy transfers to particles accelerating them to energies of the order of $10^{16}$eV.

Keywords: acceleration of particles; cosmic rays; black holes

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1. Introduction

Recently we have published an article, where we have developed a novel model of particle acceleration via the damping of centrifugally excited Langmuir waves (Ref. [1]). In particular, we have considered magnetospheric particles close to active galactic nuclei (AGNs) and found that the particle acceleration is composed of several steps: in the first stage the electrostatic waves are efficiently excited pumping rotational energy of the magnetosphere, on the next stage the already amplified Langmuir waves transfer their energy to plasma particles, accelerating them by means of the "Langmuir collapse" (henceforth: Langmuir-Landau-Centrifugal Drive (LLCD)). The mentioned mechanism strongly depends on the relativistic effects of rotating magnetospheres, which are present not only in AGNs but also in the galactic sources. In particular, in (Refs. [2] [3] we have considered the Crab-like and newly born millisecond pulsars and it has been shown that the LLCD can lead to extremely high energy electrons.

The centrifugally excited electrostatic waves has been considered for pulsars (Ref. [4] and AGN (Ref. [5]) where it has been found that by means of the varying centrifugal force charge separation becomes parametrically unstable, creating amplified Langmuir waves.
In general, it is worth noting that this double stage mechanism requires pre-acceleration of particles, that can be provided by a direct centrifugal acceleration (Refs. [6]). It is worth noting that relativistic centrifugal force is a crucial physical element in astrophysical objects endowed with rotating magnetospheres, energy density of magnetic field exceeds that of the plasma. The plasma particles are, thus, forced to follow the field lines, which in turn are corotating. Under these circumstances the particles moving along the magnetic field lines, undergo powerful centrifugal acceleration.

This process is extremely efficient close to the light cylinder surface (a hypothetical zone, where the linear velocity of rotation exactly equals the speed of light), where energy losses become strong as well. In particular, the accelerated particles encounter soft photons always present in the magnetospheric media. These photons via the inverse Compton scattering (ICS) might saturate energy gain driven by the centrifugal effects. Another mechanism that could potentially limit the maximum attainable energies is the so-called breakdown of the bead on the wire (BBW) approximation. During the motion along the field lines, charged particles also gyrate around them and the dynamics can be studied in the framework of the bead on the wire approximation. Particles are bound by the Lorentz force, which is not enough to keep particles close to the field lines, when particles approach the light cylinder. This process leads to deviation of the particles from the field lines terminating the subsequent acceleration.

Although, by means of the direct centrifugal acceleration the particles gain relatively low relativistic energies, they are high enough for subsequent acceleration to ultra high energies via the LLCD.

Apart from AGNs and pulsars, there is another class of astrophysical objects, which are characterized by effects of rotation: stellar black holes (SBH). It is strongly believed that SBHs are formed by the gravitational collapse of a massive star and are members of X-ray binary systems (Refs. [7]). The rotating magnetosphere is formed by the falling accreting matter, which transfers from a companion star. On the other hand, taking into account the number of stars, which potentially might produce such black holes, it is estimated that there are as many as ten million to a billion SBHs in the Milky Way alone. Therefore, it is interesting to apply the new mechanism of acceleration to the SBHs and study efficiency of a corresponding process and the role of these objects in producing the high energy cosmic rays.

In this article we apply the mechanism of LLCD to the SBHs, analyzing the results versus physical parameters. The paper is organized as follows: In Section 2 we introduce the mechanism of LLCD applying it to SBHs and obtain results and in Sect. 3 we summarize them.

2. Theoretical model

In this section we outline the theoretical model of LLCD examining the centrifugal excitation of electrostatic waves, the subsequent development of the Langmuir
collapse, leading to the Landau damping of the generated waves.

By taking into account the accretion rate onto the black hole (Ref. [8])
\[
\dot{M} \approx 1.2 \times 10^{12} \times M_{10}^2 \times \left( \frac{T_\infty}{10^4 K} \right)^{-3/2} \, g \, s^{-1},
\]
where \( M_{10} \equiv M/10M_\odot \) is the normalized mass of the SBH, \( M_\odot \approx 2 \times 10^{33} g \) is the Solar mass and \( T_\infty \) is the temperature of the accreting matter far from the black hole, one can show that the bolometric luminosity, \( L = \eta \dot{M} c^2 \), is given by
\[
L \approx 1.1 \times 10^{32} \times M_{10}^2 \times \frac{\eta}{0.1} \times \left( \frac{T_\infty}{10^4 K} \right)^{-3/2} \, \text{erg s}^{-1}.
\]

The proposed mechanism of LLCD strongly depends on relativistic effects of rotation, dynamically provided by strong magnetic fields, therefore, it is important to estimate the angular velocity of the black hole
\[
\Omega \approx \frac{a c^3}{G M} \approx 10^3 \frac{a}{M_{10}} \text{ rad s}^{-2},
\]
and the equipartition magnetic field (when the magnetic energy density is of the order of the emission energy density) in the light cylinder area (Ref. [6])
\[
B \approx \sqrt{\frac{2 L}{R_{lc}^2 c}} \approx 2.9 \times 10^3 \times \frac{a}{0.1} \times \left( \frac{\eta}{0.1} \right)^{1/2} \times \left( \frac{T_\infty}{10^4 K} \right)^{-3/4} \, G,
\]
where \( 0 < a \leq 1 \) and \( R_{lc} = c/\Omega \) is the light cylinder radius. One can straightforwardly check that the gyroradius of a proton with the Lorentz factor, 10 is much less than the kinematic lengthscale, \( R_{lc} \), which in turn means that the particles will co-rotate with the magnetic field lines.

The magnetospheric plasma consists of protons and electrons and therefore, their dynamics is crucial for studying the LLCD. For describing the centrifugally excited Langmuir waves we apply the 1+1 approach (Ref. [9]) following the model developed in (Ref. [1]) and linearize the system of equations, composed of the Euler equation
\[
\frac{\partial p_\beta}{\partial t} + i k v_{\beta 0} p_\beta = v_{\beta 0} \Omega^2 r_\beta p_\beta + \frac{e_\beta}{m_\beta} E,
\]
the continuity equation
\[
\frac{\partial n_\beta}{\partial t} + i k v_{\beta 0} n_\beta, + i k n_{\beta 0} v_\beta = 0
\]
and the Poisson equation
\[
i k E = 4\pi \sum_\beta n_{\beta 0} e_\beta,
\]
where by \( \beta \) we denote the species index of stream particles: electrons and protons, \( k \) is the wave number, \( v_{\beta 0}(t) \approx c \cos(\Omega t + \phi_\beta) \) is the zeroth order velocity and \( r_\beta(t) \approx \frac{c}{\Omega} \sin(\Omega t + \phi_\beta) \) is the radial coordinate (Ref. [1]), \( c \) is the speed of light, \( \phi_\beta \) is the initial phase of particles, \( p_\beta \) is the first order dimensionless momentum
\( p_\beta \rightarrow p_\beta/m_\beta \), \( m_\beta \) and \( e_\beta \) are the particle’s mass and charge respectively, \( E \) is the induced electrostatic field and \( n_\beta \) and \( n_{\beta 0} \) are the perturbed and unperturbed Fourier components of the density. It is worth noting that the aforementioned behaviour of the radial coordinate (and velocity as well), is physically applicable until the particle reaches the light cylinder surface and the dynamics cannot be extended for timescales exceeding \( P/4 \).

After applying the following anzatz
\[
n_\beta = N_\beta e^{-iV_\beta k \sin(\Omega t + \phi_\beta)}
\]
(8)
to the aforementioned equations they reduce to
\[
\frac{d^2 N_p}{dt^2} + \omega_p^2 N_p = -\omega_p^2 N_e e^{i\chi},
\]
(9)
\[
\frac{d^2 N_e}{dt^2} + \omega_e^2 N_e = -\omega_e^2 N_p e^{-i\chi},
\]
(10)
where \( \omega_{e,p} \equiv \sqrt{4\pi e^2 n_{e,p}/m_{e,p}^3 \gamma_{e,p}^3} \) and \( \gamma_{e,p} \) are the relativistic plasma frequency and the Lorentz factor for the stream particles, \( \chi = b \cos (\Omega t + \phi_\perp) \), \( b = \frac{2ck}{\Omega} \sin \phi_\perp \), \( 2\phi_\pm = \phi_p \pm \phi_e \) and \( J_\mu(x) \) is the Bessel function of the first kind. By applying the Fourier transform, Eqs. (9-10) lead to the dispersion relation (Ref. 1)
\[
\omega^2 - \omega_e^2 - \omega_p^2 J_0^2(b) = \omega_p^2 \sum_\mu J_\mu^2(b) \frac{\omega^2}{(\omega - \mu \Omega)^2}.
\]
(11)

It is clear from Eq. (11) that the system undergoes the parametric instability for the following resonance condition \( \omega_r = \mu_{\text{res}} \Omega \). To define the growth rate we follow the standard method discussed in (Ref. 3) and express the frequency as \( \omega = \omega_r + \Delta \). After substituting it into Eq. (11) it reduces to
\[
\Delta^3 = \frac{\omega_r \omega_p^2 J_{\mu_{\text{res}}}(b)^2}{2},
\]
(12)
with the imaginary part of the solution for \( \Delta \), which in turn gives the increment of the instability (Ref. 1)
\[
\Gamma = \frac{\sqrt{3}}{2} \left( \frac{\omega_e \omega_p^2}{2} \right)^{\frac{1}{2}} J_{\mu_{\text{res}}}(b)^{\frac{3}{2}},
\]
(13)
where \( \mu_{\text{res}} = \omega_e/\Omega \).

3. Discussion
We have already discussed in the introduction that the LLCD requires pre-acceleration, which can be guaranteed by direct centrifugal mechanism, strongly
limited by two major factors: the ICS and the BBW approximation. As it has been shown in (Ref. 6) the corresponding Lorentz factors are given by

$$\gamma_{IC} \approx \left( \frac{6\pi mc^4}{\sigma_T L\Omega} \right)^2, \quad \gamma_{BBW} \approx \frac{1}{c} \left( \frac{e^2 L}{2m} \right)^{1/3}$$

(14)

where $m$ is the particle’s mass and $\sigma_T \approx 6.65 \times 10^{-25} \text{cm}^{-2}$ is the Thomson cross-section. One can straightforwardly check that for both: electrons and protons the BBW approximation is the dominant factor limiting the maximum attainable energies and the expression for the corresponding maximum Lorentz factor writes as

$$\gamma_{max} \approx 800 \times M_{10}^{2/3} \times \left( \frac{m}{m_e} \right)^{-1/3} \times \left( \frac{\eta}{0.1} \right)^{1/3} \times \left( \frac{T_\infty}{10^4 \text{K}} \right)^{-1/2}.$$  

(15)

Therefore, for the reasonable physical parameters the electrons might reach the Lorentz factor of the order of 800 and the protons - respectively 65, which as we will see is quite enough for the LLCD to work efficiently.

![Graph](image_url)  

**Fig. 1.** Dependence of $t_{ins}/t_{kin}$ on the SBH mass. The set of parameters is $\gamma_e = 800$, $\gamma_p = 1.5$ (solid line), $\gamma_p = 2.5$ (dashed line), $\gamma_p = 4$ (dashed-dotted line), $\eta = 0.1$, $a = 0.1$, $T_\infty = 10^4 \text{K}$.  

We have discussed in the introduction that inside the magnetosphere (inside the light cylinder surface) plasma particles are in the frozen-in condition, and therefore co-rotate with magnetic field lines. In particular, it is straightforward to check that the Larmor radius of protons for the aforementioned Lorentz factors is by four orders of magnitude less than $R_{lc}$. This means that the magnetosphere is rotation powered and therefore, the longitudinal (along the magnetic field lines) electric field is neutralized by charge distribution leading to the density, $n_0$, of the order of $\sim \Omega_B/(2\pi ec)$ also for Black hole magnetospheres (Refs. 1,10). As an example we consider a stream with relativistic electrons, $\gamma_e \sim 800$, examining three different cases of protons $\gamma_p = 1.5; 2.5; 4$. On Fig. 1 we display the behavior of $t_{ins}$ versus $M_{10}$, where $t_{ins} \sim 1/\Gamma$ is the instability timescale and $t_{kin} = 2\pi/\Omega$ is the kinematic timescale. The set of parameters is $\gamma_e = 800$, $\gamma_p = 1.5$ (solid line), $\gamma_p = 2.5$ (dashed line), $\gamma_p = 4$ (dashed-dotted line), $\eta = 0.1$, $a = 0.1$, $T_\infty = 10^4K$. We show the plot versus the black hole mass in the range $(10^{-15}) \times M_\odot$. It is worth noting that up to now the largest known (extragalactic) stellar black hole has mass of the order of $15.6 \times M_\odot$ (Ref. 11). As it is clear from the figure, for the aforementioned parameters $t_{ins} < t_{kin}$, which means that the centrifugally induced electrostatic instability is quite efficient.

An effect of the second stage - the damping of electrostatic waves - can be even stronger if the "Langmuir collapse" precedes it. In particular, by means of the high frequency pressure the particles are pushed out from the perturbation area creating the so-called caverns: relatively small density regions. This in turn, leads to the acceleration of plasmons towards the caverns, leading to the modulation instability (Ref. 12).

Assuming the quasi neutrality of the plasma one can show that in a simplified 1D-case, the process of the "Langmuir collapse" in the rest frame of the fluid is described by the following set of equations (Ref. 12)

\[
\left[ \frac{\partial^2}{\partial t^2} - 3\lambda_D^2 \omega_p^2 \frac{\partial^2}{\partial x^2} - \omega_p^2 \right] E = \frac{\delta n}{n_0} \omega_p^2 E, \tag{16}
\]

\[
\left[ \frac{\partial^2}{\partial t^2} - \lambda_D^2 \omega_p^2 \frac{\partial^2}{\partial x^2} \right] \delta n = \frac{1}{16\pi m_p} \frac{\partial^2 E^2}{\partial x^2}, \tag{17}
\]

where $\lambda_D \equiv \sqrt{k_B T_e/(4\pi n_0 e^2)}$ is the Debye length scale, $k_B$ is the Boltzmann constant, $T_e$ is the electron temperature, $n_0$ is the unperturbed ion number density, $\delta n$ is the electron number density perturbation and $E$ is the induced electric field.

Induced electric field is very significant for studying the development of the Langmuir collapse. For this purpose it is worth noting that perturbation corresponding to density perturbation is small compared with the unperturbed density and therefore frequency of plasmons and correspondingly energy should be constant (Ref. 13)

\[
\int d^3r |E|^2 = \text{const}, \tag{18}
\]
where $q$ denotes spacial dimensions, which might equal 1, 2, 3 depending on a concrete physical system. From the above equation it is evident that the electric field behaves with distance as

$$|E|^2 \propto \frac{1}{l^q}, \quad (19)$$

As we have discussed in (Ref. 1) inside the light cylinder surface, where the magnetic field dominates plasmas, the particles are forced to move along the field lines, therefore such a physical system is strongly defined by 1D dynamics leading to the following behavior of the high frequency pressure $P_{hf} \propto 1/l$, where we have taken into account the relation $P_{hf} \propto E^2$ (Ref. 12, 13). On the other hand, the plasmons inside the caverns have kinetic and potential energies of the same order of magnitude

$$k^2 \lambda_D^2 \sim \frac{\delta n}{n_0}, \quad (20)$$

leading to the following behaviour of the thermal pressure $P_{th} = k_B T \delta n \propto 1/l^2$. Therefore, for smaller lengthscales the thermal pressure overcomes the high frequency pressure and the collapse cannot develop. This situation drastically changes outside the magnetosphere (light cylinder surface), where particles do not follow the field lines any more and become free. Under such conditions particle dynamics is described by three dimensional geometry and consequently it is evident that unlike the previous case, now, the high frequency pressure dominates leading to the development of the efficient collapse. By applying Eqs. (17,12) and neglecting the term corresponding to the thermal pressure one obtains

$$\frac{\partial^2 \delta n}{\partial t^2} \approx \frac{\delta n \ |E|^2}{16\pi nm_\gamma D}, \quad (21)$$

which after taking into account the following relations $|E|^2 \sim 1/l^3$ and $\delta n \sim 1/l^2$, leads to (Ref. 12)

$$|E|^2 \approx |E_0|^2 \left( \frac{t_0}{t_0 - t} \right)^2, \quad (22)$$

$$l \approx l_0 \left( \frac{t_0}{t_0 - t} \right)^{-2/3}, \quad (23)$$

where $t_0$ denotes the moment of the collapse. As we see, this instability has an explosive character, which is more efficient than standard exponentially amplified processes.

In (Ref. 1) we have found that in the light cylinder area energy gain of a proton is given by

$$\epsilon_p \approx \frac{E^2}{8\pi n}, \quad (24)$$
Fig. 2. Behaviour of $\epsilon_p (\text{eV})$ versus the SBH mass. The set of parameters is $\gamma_e = 800, \gamma_p = 1.5$ (solid line), $\gamma_p = 2.5$ (dashed line), $\gamma_p = 4$ (dashed-dotted line), $\eta = 0.1, a = 0.1, f = 0.01, T_\infty = 10^4 \text{K}$.

The electric field writes as (Ref. [1])

$$E \approx 4\pi e n_0 \Delta r \times \frac{\Delta r^{3/2}}{l_c^{1/2}},$$

(25)

where $\Delta r = R_{lc}/(2\gamma_p)$ is the thickness of a thin layer close to the light cylinder, $l_c \approx c/(\omega_0\sqrt{f})$ is the lengthscale of the cavern, $\omega_0 = \sqrt{4\pi e^2/m}$ and $f = \delta n/n_0$. The number density of protons outside the magnetosphere, $n$, is not governed by rotation any more and can be approximated by assuming the spherical symmetry of accretion

$$n = \frac{L}{4\pi \eta m_p c^2 \nu R_{lc}^2}.$$  

(26)

Here $\nu = \sqrt{GM/R_{lc}}$ is the velocity of an infalling matter.

By considering the beam electrons with $\gamma_e = 800$ and assuming $\eta = a = 0.1$, 

...
$T_\infty = 10^4 \text{K}$ the final energy of protons writes as (see Eq. 24)

$$\epsilon_p (\text{eV}) \approx 7 \times 10^{14} \times M_{10}^3 \times \left( \frac{f}{10^{-2}} \right)^3 \times \left( \frac{2}{\gamma_p} \right)^5 .$$

(27)

On Fig. 2 the dependence of $\epsilon_p (\text{eV})$ on $M_{10}$ is shown. The set of parameters is $\gamma_e = 800$, $\gamma_p = 1.5$ (solid line), $\gamma_p = 2.5$ (dashed line), $\gamma_p = 4$ (dashed-dotted line), $\eta = 0.1$, $a = 0.1$, $f = 0.01$, $T_\infty = 10^4 \text{K}$. As it is clear from the plots, the energy of protons is a continuously increasing function of the SBH mass (see Eq. (27)) and we see that the SBHs might provide ultra high energy protons, $\sim 10^{16} \text{eV}$, via the LLCD. Although, for more massive SBHs the result will be even higher.

In general, the accelerating particles might undergo strong energy losses, therefore, the mechanisms limiting the acceleration process has to be taken into account. We have already discussed in the introduction that by means of the synchrotron mechanism the particles transit to the ground Landau states very soon and therefore it does not influence the acceleration of particles any more.

The ultra-high energy protons interact with soft photons, which are always present in a medium around SBHs, therefore, the corresponding scattering has to be taken into account. In (Ref. 14) it has been found that for extremely energetic particles, the mentioned interaction operates in the so-called Klein-Nishina regime leading to the cooling timescale, which is proportional to the proton energy (Ref. 15). This in turn means that the inverse Compton mechanism does not impose a significant constraint on energy gain.

Apart from the above mentioned processes there is another mechanism - curvature radiation - which potentially might be responsible for limiting the maximum attainable energies of protons. By taking into account the corresponding cooling timescale, $t_{\text{cur}} = \epsilon_p / P_{\text{cur}}$, where $P_{\text{cur}} = 2e^2 \epsilon_p^4 / (3m_p^2 c^3 \rho)$ is the single particle radiation power and $\rho \sim R_{\text{lc}}$ is the curvature radius of a trajectory, one can straightforwardly show that for the typical parameters of a medium surrounding the SBHs (see Fig. 1 Fig. 2) the curvature timescale exceeds that of the Landau collapse instability (Ref. 1, 13)

$$t_{\text{LC}} \approx \gamma_p \left[ \frac{(E^2) e^2 m_p}{4k_B T} \right]^{-1/2} ,$$

(28)

by many orders of magnitude, which means that the curvature radiation is also negligible and does not impose any constraints on achievable energies.

As we have already mentioned in the introduction, the population of SBHs in the Milky Way might be at least ten million, which means that they might be very significant in studying the high energy cosmic rays.

4. Summary

(1) The main novelty of the manuscript is that the recently studied new mechanism of particle acceleration - LLCD process we applied to SBHs. The present mech-
anism is composed of two major stages. In the first stage centrifugal force parametrically excites the unstable electrostatic waves, efficiently extracting energy from rotation. On the second stage the Langmuir collapse develops significantly enhancing the energy gain via the Landau damping.

(2) By linearizing the set of equations composed of the Euler equation, continuity equation and Poisson equation we have shown that by means of the relativistic centrifugal force the very unstable Langmuir waves are generated. It has been shown that the instability timescale is less than the kinematic timescale, indicating high efficiency of the energy pumping process into the waves. As a next step we have considered the possibility of Langmuir collapse and it has been found that energy transfer from the waves into the particles is so efficient that neither the inverse Compton, nor the curvature emission affect particle acceleration. As a result, it is shown that by means of the LLCD the protons in the medium surrounding the SBH with mass exceeding the solar mass 15 times might reach energies of the order of $10^{16}$eV.

The aim of the paper was to show a role of rotation in the jet-like structure of the Crab pulsar. The study was only focused on the dynamic behaviour of particles, moving along prescribed (in the RF) co-rotating channels.

An important restriction in the present model is the consideration of a single particle approach, whereas it is clear that in a general case dynamics of particles is strongly influenced by collective phenomena. Therefore, it would be interesting to explore the dynamics of magnetocentrifugally accelerated particles in this context.

The formalism developed in Section 2 is valid for both Schwarzschild and Kerr black holes even though in the subsequent analysis we completely neglected the gravitational effects and considered only a special-relativistic case. One of the tasks of further study will be to check how magnetocentrifugal acceleration along prescribed trajectories works in fully relativistic situations and physically realistic astrophysical scenarios.

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