Constraining the ellipticity of strongly magnetized neutron stars powering superluminous supernovae

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Accepted 2016 April 14. Received 2016 April 14; in original form 2016 March 24

ABSTRACT

Superluminous supernovae (SLSNe) have been suggested to be powered by strongly magnetized, rapidly rotating neutron stars which are often called magnetars. In this process, rotational energy of the magnetar is radiated via magnetic dipole radiation and heats the supernova ejecta. However, if magnetars are highly distorted in their geometric shape, rotational energy is mainly lost as gravitational wave radiation and thus such magnetars cannot power SLSNe. By simply comparing electromagnetic and gravitational wave emission timescales, we constrain upper limits to the ellipticity of magnetars by assuming that they power the observed SLSNe. We find that their ellipticity typically needs to be less than about a few $10^{-3}$. This indicates that the toroidal magnetic field strengths in these magnetars are typically less than a few $10^{16}$ G so that their distortions remain small. Because light-curve modelling of SLSNe shows that their dipole magnetic field strengths are of the order of $10^{14}$ G, the ratio of poloidal to toroidal magnetic field strengths is found to be larger than $\sim 0.01$ in magnetars powering SLSNe.

Key words: supernovae: general – gravitational waves – stars: magnetic fields – stars: neutron

1 INTRODUCTION

Superluminous supernovae (SLSNe) are a newly recognized class of supernovae (SNe) whose peak luminosity is more than about 10 times brighter than that of typical core-collapse SNe (Quimby et al. 2011). These powerful transients are therefore observable at cosmological distances although they are thought to be intrinsically rare – less than $\sim 0.01\%$ of the core-collapse population (Quimby et al. 2013; McCrum et al. 2015). How SLSNe achieve such high luminosities is not well-understood (Gal-Yam 2012). One of the favoured models to explain their peak luminosity is the formation of a rapidly spinning, strongly magnetized neutron star (NS), which is often called a ‘magnetar’ in the literature. If such a magnetar has an initial magnetic dipole field strength of $\sim 10^{14}$ G and an initial spin period of $\sim 1$ ms, a huge rotational energy reservoir can be emitted as magnetic dipole radiation to heat SN ejecta, thus making a SN superluminous (Kasen & Bildsten 2010; Woosley 2010; Dessart et al. 2012; Inoue et al. 2013; Chatzopoulos et al. 2013; Metzger et al. 2015; Sukhbold & Woosley 2016).

On the other hand, magnetars are also suggested to be strong sources of gravitational waves (Cutler 2002; Stella et al. 2005; Dall’Osso, Shore, & Stella 2009; Gualtieri, Cioni, & Ferrari 2011; Dall’Osso et al. 2015). The main reason for this is a geometric distortion of the NS caused by a strong toroidal magnetic field. Rapid rotation of a time-varying quadrupole moment results in efficient emission of gravitational waves. If SLSNe are actually powered by magnetars, however, the released spin-down energy from magnetars must be dominated by electromagnetic radiation which can be thermalized in the SN ejecta. Therefore, as gravitational waves cannot power the SN ejecta, rotational energy loss by gravitational waves must be insignificant. A resulting criterion should be that the spin-down timescale by gravitational waves must be shorter than that by gravitational wave radiation (cf. Kashiyama et al. 2016).

Although magnetars are suggested to be distorted by their strong toroidal magnetic fields, it is not easy to constrain this distortion and its resulting ellipticity. Furthermore, the toroidal magnetic field components themselves are also hard to determine observationally (Makishima et al. 2014; Lasky & Glampedakis 2016). An advantage of using SLSNe to constrain NS ellipticities is that we can use their light curves to estimate the spin period and the dipole mag-
netic field strength of these magnetars. Assuming that magnetars are indeed distorted by toroidal magnetic fields, the derived limits on their ellipticities can be used to constrain the toroidal magnetic field strengths in these NSs as an independent sanity check. Thus, we can constrain both poloidal and toroidal magnetic field strengths in magnetars via modeling of their light curves. In this Letter, we show such constraints obtained from SLSNe for which light-curve modeling by magnetars has been performed.

Constraining both poloidal and toroidal field strengths in SLSNe also serves as an important test of the magnetar model of SLSNe. It is known that purely poloidal magnetic fields in NSs are not stable, and toroidal magnetic fields which are much stronger than the poloidal fields are still required to have stable and significant poloidal magnetic fields (e.g. Brathwaite 2009; Akgün et al. 2013). By constraining both poloidal and toroidal field strengths in magnetars powering SLSNe, we can check if these NSs actually satisfy the stability condition, and thus, if the magnetar model of SLSNe is self-consistent.

2 CONSTRaining THE ELLIPticy

2.1 Emission timescales

We consider a magnetar with a radius $R$, a moment of inertia $I$, and an initial spin period $P_0$. The magnetar has a total rotational energy of $E_{\text{rot}} = 1/2I (2\pi/P_0)^2$. Furthermore, we idealize the magnetar as a NS with the shape of a slightly deformed, homogeneous ellipsoid and thus having a small ellipticity of $\varepsilon \equiv (I_1 - I_2)/I_3$, where $I_1$, $I_2$, and $I_3$ are the principal moments of inertia of the NS and $I_3$ is assumed to be aligned with the spin axis. Assuming quadrupolar gravitational wave radiation, the gravitational wave luminosity $L_{\text{GW}}$ from the distorted magnetar is ( Shapiro & Teukolsky 1983):

$$L_{\text{GW}}(t) = \frac{E_{\text{rot}}}{\tau_{\text{GW}}} \left(1 + \frac{2t}{\tau_{\text{GW}}}\right)^{-3/2},$$

where $\tau_{\text{GW}} \equiv |E_{\text{rot}}/E_{\text{GW}}|$ is the gravitational wave emission timescale of the magnetar:

$$\tau_{\text{GW}} = \frac{5}{2^{10}\pi^4} \frac{c^3 P_0^4}{G I \varepsilon^2},$$

$c$ is the speed of light, and $G$ is the gravitational constant. Here, we assume that the angle between the spin axis and the principal axis of the NS distortion is $\pi/2$ (Cutler & Jones 2001).

In principle, rapidly spinning NSs may emit gravitational waves from a combination of ‘mountain’ and ‘wobble’ radiation, see e.g. Lasky & Glampedakis (2016) for a recent discussion on CFS instabilities in the form of f modes (bar-modes) and r modes (inertial modes, Andersson & Kokkotas 2001). However, their mode oscillation amplitudes are likely saturated at modest values, resulting in relatively long spin-down timescales of several years (Lasky & Glampedakis 2016).

Spinning magnetars with a misaligned magnetic dipole moment emit magnetic dipole radiation with a luminosity $L_{\text{EM}}$ which is approximately given by ( Shapiro & Teukolsky 1983):

$$L_{\text{EM}}(t) = \frac{E_{\text{rot}}}{\tau_{\text{EM}}} \left(1 + \frac{t}{\tau_{\text{EM}}}\right)^{-2},$$

where $\tau_{\text{EM}} \equiv |E_{\text{rot}}/E_{\text{EM}}| = P/2P$ is the electromagnetic radiation timescale of the magnetar:

$$\tau_{\text{EM}} = \frac{3}{4\pi} \frac{I c^3 P_0^2}{B_{\text{dipole}}^2 R^6 \sin^2 \alpha},$$

$B_{\text{dipole}}$ is the dipole magnetic field strength at the pole, and $\alpha$ is the misalignment angle between the spin axis and the magnetic dipole axis. We assume $\alpha = \pi/2$.

2.2 Constraining ellipticity from SLSN light curves

Long rise times and large luminosities of SLSNe require extraordinary central engines. If magnetars power the observed light curves they must deposit their rotational energy of more than $10^{51}$ erg through electromagnetic radiation with a timescale of more than 10 days ( Kase & Bildsten 2010). This requires $B_{\text{dipole}} \sim 10^{14}$ G and $P_0 \sim 1$ ms (Table 1). However, on the other hand, if the gravitational wave emission timescale is shorter than the electromagnetic emission timescale, the rotational energy of the magnetar is mainly lost by gravitational wave radiation and SLSNe cannot be powered by magnetars. Thus, we simply impose the condition that the gravitational wave emission timescale needs to be larger than the electromagnetic wave emission timescale in SLSNe, i.e., $\tau_{\text{GW}} > \tau_{\text{EM}}$. Using Equations (2) and (4), we obtain the following constraint on the NS ellipticity,

$$|\varepsilon| < \sqrt{\frac{5}{3G}} \frac{c R^3 P_0 B_{\text{dipole}}}{2^2 \pi I^4} \approx 3.0 \times 10^{-4} \left(B_{\text{dipole}} / 10^{14} \text{ G} \right) \left(P_0 / 1 \text{ ms} \right),$$

where we apply fiducial NS properties of $I = 10^{45}$ g cm$^2$ and $R = 10$ km.

Table 1 summarizes the estimated magnetar properties ($B_{\text{dipole}}$ and $P_0$) for observed SLSNe and their corresponding constraints on the NS ellipticity obtained with Equation (5). The angle $\alpha$ is defined differently depending on the reference in Table 1. We take $B_{\text{dipole}} \sin \alpha$ in the literature and obtain $B_{\text{dipole}}$ by assuming $\alpha = \pi/2$ (see Equation 4). Figure 1 depicts these values graphically. We can see that the absolute NS ellipticity typically needs to be less than $\sim 10^{-3}$ for electromagnetic wave radiation to be more efficient than gravitational wave radiation in the observed SLSNe.

3 DISCUSSION

If we assume that a NS distortion is indeed caused by a strong internal toroidal magnetic field component, $B_{\text{toroidal}}$, we can constrain the average value of this component by using a relation between $\varepsilon$ and $B_{\text{toroidal}}$. If we take the suggested relation by Cutler (2002), $|\varepsilon| \approx 1.6 \times 10^{-3} (B_{\text{toroidal}} / 10^{16} \text{ G})^2$, and combine with Equation (5), we can constrain $B_{\text{toroidal}}$ as:

$$B_{\text{toroidal}} \lesssim 1.4 \times 10^{16} \text{ G} \left(\frac{B_{\text{dipole}}}{10^{14} \text{ G}}\right)^{1/2} \left(\frac{P_0}{1 \text{ ms}}\right)^{1/2}.$$
The resulting constraints on $B_{\text{toroidal}}$ are shown in Table 1 and are typically less than a few $10^{16}$ G. This is close to the maximum toroidal magnetic field strength which can be achieved by the $\alpha - \Omega$ dynamo mechanism (Duncan & Thompson 1992; M¨osta et al. 2015).

To secure a stable magnetic configuration, one must require $B_{\text{dipole}} \ll B_{\text{toroidal}}$ (Braithwaite 2009; Akg¨un et al. 2013). Here, we have constraints on both magnetic field components from fitting a magnetar to the light curves of the SLSNe (Table 1) and the ratio of the two components is typically found to be $B_{\text{dipole}}/B_{\text{toroidal}} \gtrsim 0.01$. Hence, the criterion for a stable magnetic field configuration is validated and our derived magnetar properties, obtained from the applied magnetar model for SLSNe, are self-consistent (but see also Soker 2016). If future observations of SLSNe yield a much larger ratio of $B_{\text{dipole}}/B_{\text{toroidal}}$ when applying the magnetar model it may well indicate that magnetars, at least in those cases, are not related to SLSNe.

Our estimate of the gravitational wave radiation timescale (Equation 2) is based on the assumption that the angle between the spin axis and the principal axis of the NS distortion is $\pi/2$. Furthermore, if this distortion in the NS quadrupole moment is caused by toroidal magnetic fields, then these fields should be orthogonalized to the spin axis (Cutler & Jones 2001). However, when magnetars are formed and their toroidal fields are wound up, their toroidal fields are presumed to be aligned, and not orthogonal, to the spin axis (Duncan & Thompson 1992). Therefore, if orthogonalization does not occur, then the toroidal magnetic fields are not able to distort the NSs sufficiently to become efficient gravitational wave emitters. The timescale for this orthogonalization, or ‘spin flipping’, is quite uncertain (Dall’Osso, Shore, & Stella 2009). Lasky & Glampedakis (2016) showed that a critical ellipticity exists for newborn magnetars such that they will only cause spin flipping if $|e| \lesssim 5 \times 10^{-3}$ ($P_0/1$ ms)$^{-2}$ for typical values of the NS mass density ($10^{15}$ g cm$^{-3}$) and radius (10 km). Applying Equation (5), we obtain the dashed red line in Figure 1, below which the estimated ellipticities are smaller than the critical ellipticity. Hence, in all observed sources above this red dashed line, the magnetars might always be inefficient gravitational wave emitters and all their rotational energy loss can, in principle, be used to power the observed SLSN light curves. Sources below the red dashed

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### Table 1. Constraints on the ellipticity and toroidal magnetic field component in magnetars applied to observed SLSNe. We assume $\alpha = \pi/2$.

| SN name   | $B_{\text{dipole}} \times 10^{14}$ G | $P_0$ ms | $|e|$ | $B_{\text{toroidal}} \times 10^{16}$ G | Reference            |
|-----------|-----------------------------------|----------|------|---------------------------------|----------------------|
| SN 2005ap | 0.92                              | 3.1      | < 0.85 | < 2.42                           | Chatzopoulos et al. (2013) |
| SCP06F6  | 1.3                               | 2.5      | < 0.98 | < 2.5                            | Chatzopoulos et al. (2013) |
| SNLS 06D-1eu | 1.4                 | 2.0      | < 0.85 | < 2.3                            | Howell et al. (2013)    |
| SN 2007bi | 0.92                              | 2.7      | < 0.75 | < 2.2                            | Chatzopoulos et al. (2013) |
| SN 2010gx | 5.2                               | 2.0      | < 3.1  | < 4.5                            | Inserra et al. (2013)   |
| SN 2010kl | 1.5                               | 2.7      | < 1.2  | < 2.8                            | Chatzopoulos et al. (2013) |
| SN 2010kl | 9.8                               | 3.5      | < 10   | < 8.2                            | Bersten et al. (2016)   |
| PTF10lgi | 2.5                               | 7.2      | < 5.4  | < 5.9                            | Inserra et al. (2013)   |
| SN 2011ke | 4.5                               | 1.7      | < 2.3  | < 3.9                            | Inserra et al. (2013)   |
| SN 2011kl | 3.3                               | 2.0      | < 2.0  | < 3.6                            | Inserra et al. (2013)   |
| PTF11krk | 4.8                               | 7.5      | < 11   | < 8.4                            | Inserra et al. (2013)   |
| SN 2012lf | 2.9                               | 6.1      | < 5.3  | < 5.9                            | Inserra et al. (2013)   |
| PTF12dam | 0.49                              | 2.7      | < 0.39 | < 1.6                            | Chen et al. (2015)      |
| CSS121015 | 1.5                               | 2.0      | < 0.90 | < 2.4                            | Nicholl et al. (2014)   |
| LSQL12lf | 2.6                               | 1.9      | < 1.5  | < 3.1                            | Nicholl et al. (2014)   |
| SSS200810 | 2.8                               | 1.2      | < 1.0  | < 2.6                            | Nicholl et al. (2014)   |
| SN 2013dg | 5.0                               | 2.5      | < 3.7  | < 5.0                            | Nicholl et al. (2014)   |
| iPTF13axg | 1.6                               | 1.1      | < 0.54 | < 1.9                            | Vreeswijk et al. (2014) |
| iPTF13CRY | 0.57                              | 2.55     | < 0.43 | < 1.7                            | Wang et al. (2015)      |
| DES13S2cmn | 1.0                            | 5.3      | < 1.6  | < 3.2                            | Papadopoulos et al. (2015) |
| SN 2015bn | 0.64                              | 2.1      | < 0.40 | < 1.6                            | Nicholl et al. (2016)   |
| ASASSN-15lh | 0.25                         | 1.2      | < 0.090 | < 0.77                           | Bersten et al. (2016)   |

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**Figure 1.** Initial dipole field strengths and spin periods of magnetars in observed SLSNe (blue points, see Table 1). The corresponding constraints on ellipticity (Equation 5) are shown with three solid black lines. Spin flipping in magnetars can occur in magnetars below the dashed red line (Lasky & Glampedakis 2016).
line, can still power SLSNe if their ellipticities are smaller than the derived upper limits shown in Table 1. There are large uncertainties, however, in estimating the spin-flip criterion, and further investigation is needed on this issue.

A slight caveat of concern for our estimated description of $\tau_{\mathrm{EM}}$ is related to the validity of applying the simple magnetic dipole model to a magnetar in a SN. Such a magnetar is surrounded by an expanding envelope and its braking torque depends on the boundary conditions at the wind-envelope interface. If initially the B-field does not penetrate the envelope, then the spin-down torque will be smaller (Lyutikov & Beland 2003); if it penetrates, it will be larger (since the field is twisted more and more). We also note that numerical magnetohydrodynamic simulations indicate that the spin-down luminosity may be larger than that obtained by the classical dipole formula by a factor of $\sim 2$ (e.g. Spikovksy 2006; Tchekhovskoy, Spitkovsky, & Li 2013).

In addition, there are uncertainties in estimating $B_{\mathrm{dipole}}$ and $P_0$ from SLSN light curves. These magnetar properties are often entangled with SN properties such as ejecta mass, energy and opacity. This degeneracy can be partly solved by using the velocity information from spectra, but still yields $B_{\mathrm{dipole}} \sim 10^{14}$ G and $P_0 \sim 1$ ms (e.g. Nicholl et al. 2015). Thus, we expect that $B_{\mathrm{dipole}}$ and $P_0$ in Table 1 have uncertainties by a factor of a few. We also note that the efficiency in converting magnetar dipole radiation to thermal energy can also affect the estimates of the magnetar properties (Chen et al. 2015).

The ellipticities and spins of NSs at birth are not well constrained, especially not for magnetars. A recent study on the evolution of proto-NSs (Camelio et al. 2016) suggests a minimum proto-NS spin period of about 3 ms (obtained some 10 s after core bounce), and this significantly larger than the mass-shedding limit of $0.6 - 0.7$ ms (Doneva et al. 2013). However, several of the SLSNe investigated here require $P_0 < 3$ ms. Therefore, if proto-NSs (magnetars) are actually hard to spin up efficiently, then these SLSNe cannot be powered by magnetars.

There are several observational constraints on ellipticities of more evolved magnetars. Makishima et al. (2014) estimated that the ellipticity of the magnetar 4U 0142+61 with $B_{\mathrm{dipole}} \sim 10^{15}$ G is $1.6 \times 10^{-4}$. If this deformation is due to a toroidal magnetic field component, the corresponding toroidal magnetic field strength is $\sim 10^{16}$ G.

For NS mergers producing short gamma-ray bursts, the resulting (meta-stable) magnetar is significantly more massive ($> 2.2 M_\odot$) than a typical NS ($1.4 M_\odot$). Hence, these sources can be efficient gravitational wave emitters via f-modes (Doneva, Kokkotas, & Pngouras 2015). Lasky & Glampedakis (2016) estimated that magnetars with $B_{\mathrm{dipole}} \geq 10^{15}$ G powering short gamma-ray bursts, have ellipticities of less than $\sim 10^{-2}$. In comparison, we find that magnetars potentially powering SLSNe have about an order of magnitude smaller values of $B_{\mathrm{dipole}}$ and the magnetar ellipticities that we derived are typically less than a few $\sim 10^{-3}$. Therefore, if magnetars power SLSNe then we do not expect to detect these (extragalactic) NSs by any of the current (aLIGO/VIRGO/KAGRA) nor planned gravitational wave observatories (Einstein Telescope), as their gravitational wave signals must be quite weak. We refer to Kashiya et al. (2016) for further discussion on the gravitational wave detectability from SLSNe.

4 CONCLUSIONS

We have derived constraints on the ellipticity of magnetars powering SLSNe. For magnetars to power SLSNe, electromagnetic radiation should be the dominant channel to extract their large rotational energy reservoir. Thus, the magnetar ellipticity must be small in order to prevent significant loss of rotational energy by gravitational wave radiation from geometrically distorted magnetars. Here, we simply constrained the ellipticity by requiring that the electromagnetic radiation timescale should be shorter than gravitational wave radiation timescale in distorted magnetars.

We find that the magnetar ellipticity, $\varepsilon$ in SLSNe typically needs to satisfy $|\varepsilon| \lesssim 10^{-3}$. Thus, their toroidal magnetic field strengths should be smaller than a few $10^{16}$ G. Combined with the poloidal (dipole) magnetic field strengths constrained by light-curve modelling of SLSNe ($\sim 10^{14}$ G), we find that the ratio of poloidal to toroidal field strengths is larger than $\sim 0.01$ in magnetars powering SLSNe. This ratio is small enough to secure stable magnetic configuration in magnetars powering SLSNe and thus the magnetar model for SLSNe is found to be self-consistent so far.

ACKNOWLEDGEMENTS

This work is based on discussions following a presentation by TJM at the 8th BONN workshop on formation and evolution of neutron stars. We thank Nils Andersson and Kostas Kokkotas for discussions on r-mode and f-mode instabilities of magnetars, and Maxim Lyutikov for insight on the spin-down of a magnetar in a SN. TJM is supported by Japan Society for the Promotion of Science Postdoctoral Fellowships for Research Abroad (2651).

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MNRAS 000, 1–5 (2016)
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