Residual interaction effects on deeply bound pionic states in Sn and Pb isotopes

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We have studied the residual interaction effects theoretically on the deeply bound pionic states in Pb and Sn isotopes. We need to evaluate the residual interaction effects carefully in order to deduce the nuclear medium effects for pion properties, which are believed to provide valuable information on nuclear chiral dynamics. The s- and p-wave \( \pi N \) interactions are used for the pion-nucleon residual interactions. We show that the complex energy shifts are around \((10-20)+i(2-7)\) keV for 1s states in Sn, which should be taken into account in the analyses of the high precision data of deeply bound pionic 1s states in Sn isotopes.

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I. INTRODUCTION

Deeply bound pionic states in heavy nuclei were predicted to be quasistable by Friedman and Soff 1, and Toki and Yamazaki 2 independently. According to the theoretical predictions for the formation reactions 3,4, the deeply bound pionic 2p states are observed in 207Pb nucleus experimentally in \((d,^3He)\) missing mass spectra in 1996 5,6. After this discovery, precise data of the observed states in Sn isotopes 7,8. Furthermore, Umemoto et al. predicted that the Sn isotopes are ideal target nuclei to observe 1s pionic states and to deduce the isotope shifts of the pionic atoms 9. Recently, K. Suzuki et al. performed the experiments of the \((d,^3He)\) reactions on the Sn targets and succeeded to observe deeply bound 1s pionic states in Sn isotopes quite precisely 10. Experimental errors for the binding energies of the 1s states are around \(\Delta E \sim 20\)keV. 

From these experiments, we can study the s-wave part of the pion-nucleus interaction, which is very interesting since the s-wave strength is expected to provide information on the pion mass excess and pion decay constant \(f_\pi\) in the nuclear medium through the Tomozawa-Weinberg theorem 11,12. The \(f_\pi^2\) is the order parameter of chiral symmetry breaking of QCD, and is connected to the quark condensate through the Gell-Mann-Oakes-Renner relation 13. Thus, it is very interesting to determine the s-wave potential parameters from deeply bound pionic atoms. For this purpose, it is required to observe the pionic 1s-states because these states depend predominantly on the s-wave potential 14 and, furthermore, the 1s states in heavy nuclei \((N > Z)\) provide key information on the isovector part of the s-wave potential. As described above, K. Suzuki et al. performed the experiment and obtained excellent new data of the deeply bound pionic 1s states in Sn isotopes 9, which are very suitable for the purpose.

However, since we make use of the single neutron pickup \((d,^3He)\) reactions, the final pionic states are the pion plus neutron-hole state \([\pi \otimes n^{-1}]_J\) with respect to the target nuclei 14,2,9. So far all theoretical calculations and analyses of the data, except for Ref. 14, postulate that the residual interaction effects are small enough and can be neglected. This is actually true for pionic atoms in 207Pb case since experimental errors are significantly larger than the estimated residual interaction effects 14. However, in the present cases for the 1s states in Sn isotopes, it is not obvious whether the effects are negligible or not since the experimental errors for Sn cases are comparable to the calculated residual interaction effects for 207Pb 14. Thus, it is very important to evaluate the residual-interaction effects for 1s states in Sn isotopes to deduce physical quantities related to pion behaviors in the nuclear medium from the observed spectra. In this report we evaluate the residual-interaction effects on pionic states in 207Pb, 205Pb and Sn isotopes by taking into account both s-wave and p-wave \(\pi N\) interactions.

In section 2, we describe the theoretical formula to evaluate the residual interaction effects. In section 3, we show the numerical results and section 4 is devoted to the conclusions.

II. FORMULATION

We consider the pionic states whose Hamiltonian is expressed as follows:

\[
H = \sum_i \omega_i c_i^+ c_i + \sum_i \varepsilon_i a_i^+ a_i + \sum_{ijkl} V_{ijkl} c_i^+ a_j^+ a_k c_l ,
\]

(1)

where the \(c^+ (c)\) and \(a^+ (a)\) are creation (annihilation) operators of the pion and the nucleon respectively. The indices characterize their quantum numbers. In Eq. 1, \(\omega_i\) is the pion binding energy , \(\varepsilon_i\) the single-nucleon energy and \(V_{ijkl}\) indicate the matrix elements of the pion-nucleon residual interaction.

Since we make use of the single-neutron pickup \((d,^3He)\) reaction, each final state is the pion plus single neutron-hole state with respect to the target nucleus. To calculate the residual-interaction effects between pion and the neutron-hole, we introduce a neutron-hole creation
(annihilation) operator $b^\dagger(b)$, which are defined as:

$$a^+_j = (-1)^j - m_j b_j - m.$$  

Here, we show the angular momentum quantum numbers explicitly while the isospin indices are abbreviated. The third term in Eq.(1) can be rewritten as

$$\sum_{ijkl} V_{i,jk} c^+_i a^+_k a_k c_l \rightarrow \hat{V} = \sum_{ijkl} \bar{V}_{jk,\ell i} b^+_k b_i c_l,$$  

where we discarded the core contribution which is already included in the second term of Eq. (1) and $\bar{V}_{jk,\ell i}$ are the interaction matrix elements between pion and the nucleon hole which correspond to the Pandya transformation of the pion-nucleon interaction.

The state of pionic atom with a single neutron-hole state can be expressed as

$$|\pi, N_\alpha; J\rangle = (\epsilon^\pi_\alpha \otimes b^\dagger_{N_\alpha})J(0),$$  

where the suffixes $\pi$ and $N_\alpha$ specify the quantum numbers of the pion and the neutron-hole respectively, and $J$ is the total angular momentum of the pion-nucleus system. The matrix elements of the Hamiltonian with respect to these states are expressed as

$$\langle \pi', N_\beta; J | \hat{V} | \pi, N_\alpha; J \rangle = (\epsilon^\pi_\alpha - \epsilon^\pi_\beta)\delta_{\pi,\pi'} \delta_{\alpha,\beta} + \langle \pi', N_\beta; J | \hat{V} | \pi, N_\alpha; J \rangle,$$

where $\epsilon^\pi_\alpha$ is the eigenenergy of the pionic state specified as $\pi$, and $\epsilon_\alpha$ is the separation energy of neutron from the target nucleus.

As the residual interaction, we consider the pion-nucleon interaction,

$$V = -\frac{2\pi}{m_\pi} [b_0 + b_1 \tau \cdot I + (c_0 + c_1 \tau \cdot I) \nabla \cdot \nabla |\delta(r)|].$$

Here, we have taken into account both the s- and the p-wave pion-nucleon interaction. The gradient operators act on the right and the left-hand-side pion wave functions, respectively. In Ref. [14], we reported the results with the s-wave contribution only. We fix the parameters as $b_0 = -0.0283m^{-1}_\pi$, $b_1 = -0.12m^{-1}_\pi$, $c_0 = 0.223m^{-3}_\pi$, $c_1 = 0.25m^{-3}_\pi$, which are taken from Ref. [14]. The pion-nucleon interaction adopted here is consistent with the pion-nucleus optical potential used to calculate the pion wavefunctions. By folding the pion-nucleon interaction with the nuclear density, we obtain exactly the same real part of the pion-nucleus optical potential except for the small corrections coming from the transformation of the center of mass coordinates. As for the imaginary part, we simply assume the pion-nucleon residual interaction is real and has no absorptive effects since two nucleon degrees of freedom are necessary at least in absorptive processes. The effects of the pion absorption by the core nucleus are incorporated phenomenologically as the density quadratic term in the imaginary parts of the pion-nucleus optical potential as usual. In this theoretical framework, we do not evaluate the absorptive effects due to processes including both nucleon-hole and nucleon-particle degrees of freedom simultaneously, which we expect to be small.

The interaction matrix elements between the pion and the nucleon hole are expressed as,

$$\langle \pi', N_\beta; J | \hat{V} | \pi, N_\alpha; J \rangle$$

$$= -\frac{1}{2m_\pi}(1)^{j_\pi+j_\alpha+j_\beta+1/2} \sqrt{(2j_\alpha+1)(2j_\beta+1)(2\ell_\alpha+1)(2\ell_\beta+1)(2\ell_\pi+1)}$$

$$\times \sum_{L}(-1)^L \left\{ \begin{array}{ccc} \ell_\alpha & j_\beta & \ell_\pi \\ j_\alpha & \ell_\beta & L \end{array} \right\} \left\{ \begin{array}{ccc} \ell_\beta & 0 & \ell_\alpha \\ 0 & 0 & L \end{array} \right\}$$

$$\times \left[ (b_0 + b_1) \int_0^\infty dr r^2 R^\ell_\beta(r) R^{\ell_\alpha}(r) R^{\ell_\alpha}(r) + (c_0 + c_1) \int_0^\infty dr r^2 R^\ell_\beta(r) R^{\ell_\alpha}(r) \right]$$

$$\times \left\{ \left( \frac{dR^{\ell_\alpha}}{dr} \right) \left( \frac{dR^{\ell_\alpha}}{dr} \right) + \ell_\pi(\ell_\pi + 1) + \ell_\alpha(\ell_\alpha + 1) - L(L+1) \right\}$$

Where $R^{\ell_\alpha}(r)$ and $R^{\ell_\beta}(r)$ are the radial wave function of the pion and the neutron-hole, respectively. We consider the pionic orbits of 1s, 2s, 2p, 3s, 3p and 3d states which are obtained by solving the Klein-Gordon equation numerically. Because the Klein-Gordon equation includes the complex optical potential which makes the Hamiltonian non-Hermite and makes the eigenenergies complex, and hence we normalize the pionic wave function on the proper orthonormal condition according to the prescription in Ref. [16].
For the proton and the neutron, we use the two-parameter Fermi type density distributions as, 
\[
\rho_{p(n)}(r) = \frac{\rho_0}{1 + \exp\left[\left(\frac{r - r_{p(n)}}{\alpha_{p(n)}}\right)\right]},
\]
and assume the same radius parameters of the proton and the neutron. These radius parameters and the proton diffuseness parameter are taken from the experimental values in Ref. [17]. For the diffuseness parameter of the neutron we adopt the values in Ref. [18]. These density parameters are compiled in Table I. These density parameters are taken from the experimental calculations.

For the neutron-hole states, we have taken into account the orbits \(p_{1/2}^{1}, f_{5/2}^{1}, p_{3/2}^{1}, d_{3/2}^{-1}, s_{1/2}^{-1}, h_{11/2}^{-1}, g_{9/2}^{-1}, d_{5/2}^{-1}\) for \(^{115,119,123}\text{Sn}\). These states are calculated using the potential of Woods-Saxon form in Ref. [19]. The neutron separation energies \(\varepsilon_{\alpha}\) are determined from experimental data as far as possible. We can disregard the spreading widths of the neutron hole states which are considerably narrower than the width of the pionic states and little affect the results here. For \(^{205}\text{Pb}\), the separation energies are obtained from the isotope table as the excited energies of the levels coupled to the neutron pick-up reactions. For open shell nuclei \(^{205}\text{Pb}\) and \(^{115,119,123}\text{Sn}\), we adopt the excited states observed in a single neutron pick-up reactions and use the observed excitation energies to deduce the neutron separation energies. In the case that there exist plural states assigned to the same spin and parity, we choose the level which has a larger spectroscopic factor. As for \(^{131}\text{Sn}\), we use the separation energies deduced from the systematics in Ref. [21] since no data of the neutron pick-up reactions are available. We diagonalize the matrix elements of the whole Hamiltonian expressed in \(H\). Then, we can calculate the complex energy shifts defined as, 
\[
\Delta E \equiv E(\pi, N_{\alpha}; J) - (\omega_{\pi} - \epsilon_{\alpha}),
\]
where \(E(\pi, N_{\alpha}; J)\) are the corresponding eigenenergies of the pion-nucleus system.

### III. NUMERICAL RESULTS

In this section, we show the numerical results of the residual-interaction effects for the pionic atoms. As we explained above, we include 6 pionic states and 4(5) neutron states for Pb(Sn) isotopes in the present calculation to evaluate the matrix elements. Since the residual-interaction effects are larger for deeper bound pionic states, we will show the numerical results for pionic 1s and 2p states.

In Table II, we show the complex energy shifts of the pionic states on \(^{207}\text{Pb}\). In order to see the contributions from \(\pi N p\)-wave interactions, which are newly included in present work, we show both results with only s-wave interaction, and with s- and p-wave residual interactions. Here, since we have used more realistic neutron wave functions and nuclear density distributions than those used in Ref. [14], the present results are slightly different from those in the previous work. In Table III the results only with the s-wave residual interaction are written in the parentheses and have the same negative sign for all configurations which means that the s-wave residual interaction effects makes the bound states deeper and the level widths wider. This fact can be understood intuitively as the result of the lack of the repulsive s-wave interaction from the removed one neutron. The calculated results with both s- and p-wave residual interactions are also shown in the same Table. We find that the p-wave interaction has the opposite effects to the s-wave interaction in general and the complex energy shifts become less attractive and absorptive in almost all configurations except for a few cases. This tendency also can be understood as the result of the missing attractive \(p\)-wave interaction from one picked-up neutron. In this case with \(^{207}\text{Pb}\) nuclei, the calculated shifts are reasonably smaller than the experimental errors and we think we can safely neglect the residual interaction effects as concluded in Ref. [14].

We show the calculated results in Table III for pionic atoms in \(^{205}\text{Pb}\) with the experimental errors reported in Ref. [3]. In this case, the largest shifts appears for \([(2p)_{s} \otimes f_{5/2}^{-1}]_{3/2}\) configuration and is around half of the corresponding experimental error for the real part. However, this configuration only has minor contribution to the formation cross section \([7,8]\). The dominant contribution to the formation process of pionic 2p state is from \([(2p)_{s} \otimes p_{3/2}^{-1}]\) configurations and the residual-interaction shifts for this configuration are evaluated to be around 1/3 or less of the experimental error in real part. The each level corresponding to different total angular momentum \(J\) has different energy shifts and splits by a few keV, which will be seen as a broadening of the resonance peak since each level overlaps due to their large natural widths. As for the pionic 1s state, the residual-interaction effects are around 1/4 ∼ 1/5 of experimental error for real part and smaller by about 10 for imaginary part. Therefore, we can also conclude that the residual interaction effects can be neglected safely for pionic atoms in \(^{205}\text{Pb}\) case.

For Sn isotopes, we have made similar calculation for \(^{115}\text{Sn}, 119\text{Sn}\) and \(^{123}\text{Sn}\). In these cases together with \(^{205}\text{Pb}\) case, the target nuclei are not closed and thus the

### Table I: Nuclear density parameters used in the present calculations.

| Nucleus | \(r_{p}(= r_{n})[fm]\) | \(\alpha_{p}[fm]\) | \(\alpha_{n}[fm]\) |
|---------|-----------------|----------------|----------------|
| \(^{120}\text{Sn}\) | 5.417 | 0.5234 | 0.5837 |
| \(^{124}\text{Sn}\) | 5.459 | 0.5234 | 0.6014 |
| \(^{132}\text{Sn}\) | 5.491 | 0.5234 | 0.6175 |
| \(^{206}\text{Pb}\) | 5.548 | 0.5234 | 0.6487 |
| \(^{208}\text{Pb}\) | 6.631 | 0.5234 | 0.6389 |
| \(^{208}\text{Pb}\) | 6.647 | 0.5234 | 0.6439 |
description of the nuclear structure is much complicated. The purpose of the present calculation is, however, not to make detailed comparison with the experiment but to estimate the size of the correction coming from the effects of the residual interaction between pion and the residual nucleus. Then, we simply assumed that the residual nucleus \( \Psi_r \) consists of a single-hole state with respect to the target nucleus \( \Psi_i \) as,

\[
\Psi_r = C b^+_i \Psi_i,
\]

and we simply assumed the constant \( C = 1 \) to estimate the largest possible residual-interaction effects. The calculated results for the \( ^{115-123}\text{Sn} \) are compiled in Table \( \text{IV} \). As can be seen, the residual interaction shifts are comparable to the experimental errors of real part for the pionic 1s states, which are most important states and have dominant contributions for the formation reaction \( \pi N \). Typically, the real energy shifts are around 15 keV and the imaginary shifts are around 5 keV for pionic 1s states. The residual interaction effects slightly decreases for heavier Sn isotopes, since the binding energies of 1s pionic states are smaller and less bound for heavier Sn isotopes \( \text{III} \). We also show the calculated results for the pionic states in \( ^{134}\text{Sn} \) in Table \( \text{V} \) which has the single neutron-hole configuration with respect to the doubly closed-shell structure. The results are close to those of the other Sn isotopes.

In the pionic atom formation spectra of the \((d,^3\text{He})\) reactions on Sn targets, the dominant configuration is \([1s]_\pi \otimes s^{-1}_1\) \(1/2 \text{ II}\). Thus, the residual interaction effects of this configuration should be considered carefully. As we can see in Table \( \text{IV} \) the residual interaction effects on the \([1s]_\pi \otimes s^{-1}_1\) configuration are slightly smaller than experimental error for all isotopes. Hence, we could just manage to neglect the residual interaction effects again. However, the magnitude of the real energy shifts are more or less comparable to those of the experimental error and should be taken into account seriously in analyses of data with higher precision than Ref. \( \text{II} \).

### IV. CONCLUSION

In summary, we have evaluated the complex energy shifts of the deeply bound pionic states due to the residual interaction in \( \text{Pb} \) and \( \text{Sn} \) isotopes. We have shown the numerical results which include both \( s \)-wave and \( p \)-wave \( \pi N \) residual-interaction effects. For the open-shell nuclei, we have assumed one-neutron-hole configuration as described in Eq. \( \text{II} \). The present results show that the sizes of the residual-interaction effect are slightly smaller than the experimental errors in Ref. \( \text{II} \) for 1s pionic states in Sn isotopes. Hence, we could conclude that we can neglect the residual interaction effects in the analyses of data in Ref. \( \text{II} \) for deeply bound pionic 1s states in Sn as in the cases of Pb. However, the magnitude of the residual interaction effects are more or less comparable to the experimental errors in the latest data and the effects should be taken into account seriously in analyses of data with higher accuracy than Ref. \( \text{II} \). We think that it is essentially important to study deeply bound pionic atoms in future to deduce quantitative information on nuclear chiral dynamics.

### V. ACKNOWLEDGMENTS

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| \( P^{-1}_{1/2} \) | \( 1s \) | \( 2p \) |
|----------------|--------|--------|
| \(-13.3 - 2.9i\) | \( J=1/2 \) | \(-7.7 - 2.1i\) |
| \(-14.2 - 3.1i\) | \( J=3/2 \) | \(-9.4 - 2.7i\) |
| \(-12.9 - 2.9i\) | \( J=1/2 \) | \(-15.8 - 4.4i\) |
| \(-13.8 - 3.1i\) | \( J=3/2 \) | \(-0.16 + 0.2i\) |
| \(-13.1 - 3.5i\) | \( J=5/2 \) | \(-8.9 - 2.4i\) |
| \(-14.1 - 3.6i\) | \( J=7/2 \) | \(-10.1 - 3.0i\) |
| \(-14.8 - 6.3i\) | \( J=13/2 \) | \(2.1 + 1.9i\) |
| \(-17.2 - 5.7i\) | \( J=15/2 \) | \(-14.9 - 6.4i\) |
| Exp. error | \(\pm 20\) (stat.) \(\pm 120\) (sys.) | \(\pm 30\) (stat.) \(\pm 30\) (sys.) |

TABLE II: Calculated complex energy shifts due to the residual interaction in \(^{207}\text{Pb} \). The results are shown in units of keV for \([1s]_\pi \otimes s^{-1}_1\) and \([2p]_\pi \otimes j^{-1}_n\) including the \( s \)-wave and the \( p \)-wave parts of pion neutron-hole residual interaction. The values in the parentheses are the results obtained only with the \( s \)-wave residual interaction. Experimental errors are taken from Ref. \( \text{I} \).
|                  | $1s$                  | $2p$                  |
|------------------|-----------------------|-----------------------|
| $p_{1/2}^{-1}$   | $-13.6 - 3.1i$         | $J=1/2$ $-8.3 - 2.5i$|
|                  | $J=3/2$ $0.4 + 0.2i$  |                       |
| $p_{3/2}^{-1}$   | $-13.2 - 3.1i$         | $J=3/2$ $-0.1 + 0.3i$|
|                  | $J=5/2$ $-9.1 - 2.5i$ |                       |
| $f_{5/2}^{-1}$   | $-13.5 - 3.7i$         | $J=5/2$ $0.9 + 0.6i$  |
|                  | $J=7/2$ $-9.8 - 3.3i$ |                       |
| $i_{13/2}^{-1}$  | $-15.4 - 6.6i$         | $J=11/2$ $-13.9 - 8.0i$|
|                  | $J=13/2$ $2.2 + 1.1i$ |                       |
|                  | $J=15/2$ $-11.7 - 6.8i$|                       |
| Exp. error       | $\pm 61^{+36i}_{-77i}$| $\pm 45^{+30i}_{-31i}$|

TABLE III: Calculated complex energy shifts due to the residual interaction in $^{208}$Pb. The results are shown in units of keV for $[(1s)s \otimes j_{n=1}^{-1}]s$ and $[(2p)s \otimes j_{n=1}^{-1}]s$ including the s-wave and the p-wave parts of pion neutron-hole residual interaction. Experimental errors are taken from Ref. [8].

|                   | $^{115}$Sn | $^{119}$Sn | $^{123}$Sn |
|-------------------|-----------|-----------|-----------|
|                   | $1s$      | $2p$      | $1s$      | $2p$      | $1s$      | $2p$      |
| $s_{1/2}^{+1}$    | $-15.4$   | $J=1/2$   | $-4.0 - 1.1i$ | $-13.5$   | $J=1/2$   | $-5.2 - 2.0i$ | $-12.3$   | $J=1/2$   | $-3.2 - 0.7i$ |
|                   | $-4.2i$   | $J=3/2$   | $-4.0 - 1.1i$ | $-3.3i$   | $J=3/2$   | $-3.8 - 1.1i$ | $-2.4i$   | $J=3/2$   | $-3.5 - 0.8i$ |
| $d_{3/2}^{+1}$    | $-15.9$   | $J=1/2$   | $-9.1 - 3.1i$ | $-14.3$   | $J=1/2$   | $-7.0 - 1.6i$ | $-12.8$   | $J=1/2$   | $-8.1 - 2.5i$ |
|                   | $-4.8i$   | $J=3/2$   | $0.3 + 0.3i$  | $-3.7i$   | $J=3/2$   | $0.4 + 0.3i$  | $-2.9i$   | $J=3/2$   | $0.2 + 0.1i$  |
|                   | $J=5/2$   | $-5.2 - 1.8i$ | $-5.2 - 1.8i$ | $J=5/2$   | $-4.6 - 1.4i$ | $J=5/2$   | $-4.3 - 1.2i$ |
| $g_{7/2}^{+1}$    | $-15.4$   | $J=5/2$   | $-6.0 - 3.8i$ | $-13.0$   | $J=5/2$   | $-5.5 - 3.3i$ | $-11.1$   | $J=5/2$   | $-4.9 - 2.8i$ |
|                   | $-7.3i$   | $J=7/2$   | $1.5 + 0.8i$  | $-5.8i$   | $J=7/2$   | $1.3 + 0.7i$  | $-4.6i$   | $J=7/2$   | $1.2 + 0.6i$  |
|                   | $J=9/2$   | $-4.4 - 2.9i$ | $-4.4 - 2.9i$ | $J=9/2$   | $-3.9 - 2.4i$ | $J=9/2$   | $-3.5 - 2.0i$ |
| $h_{13/2}^{+1}$   | $-18.3$   | $J=9/2$   | $-7.7 - 4.0i$ | $-16.0$   | $J=9/2$   | $-6.9 - 3.5i$ | $-14.1$   | $J=9/2$   | $-6.3 - 3.0i$ |
|                   | $-7.2i$   | $J=11/2$  | $1.7 + 0.8i$  | $-6.0i$   | $J=11/2$  | $1.5 + 0.7i$  | $-5.1i$   | $J=11/2$  | $1.4 + 0.6i$  |
|                   | $J=13/2$  | $-6.2 - 3.3i$ | $-6.2 - 3.3i$ | $J=13/2$  | $-5.6 - 2.8i$ | $J=13/2$  | $-5.1 - 2.5i$ |
| $d_{5/2}^{+1}$    | $-15.1$   | $J=3/2$   | $-7.6 - 2.6i$ | $-13.6$   | $J=3/2$   | $-7.1 - 2.2i$ | $-12.2$   | $J=3/2$   | $-6.5 - 1.9i$ |
|                   | $-4.8i$   | $J=5/2$   | $1.0 + 0.6i$  | $-3.7i$   | $J=5/2$   | $0.9 + 0.4i$  | $-2.8i$   | $J=5/2$   | $0.8 + 0.3i$  |
|                   | $J=7/2$   | $-5.0 - 1.7i$ | $-5.0 - 1.7i$ | $J=7/2$   | $-4.6 - 1.4i$ | $J=7/2$   | $-4.3 - 1.2i$ |
| Exp. error        | $\pm 24^{+44i}_{-40i}$ | $\pm 18$         | $\pm 40i$ | $\pm 36i$ |

TABLE IV: Calculated complex energy shifts due to the residual interaction in $^{115,119,123}$Sn. The results are shown in units of keV for $[(1s)s \otimes j_{n=1}^{-1}]s$ and $[(2p)s \otimes j_{n=1}^{-1}]s$ including the s-wave and the p-wave parts of pion neutron-hole residual interaction. Experimental errors are taken from Ref. [10].

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TABLE V: Calculated complex energy shifts due to the residual interaction in $^{131}$Sn. The results are shown in units of keV for $[(1s)_{\pi} \otimes j_n^{-1}]_J$ and $[(2p)_{\pi} \otimes j_n^{-1}]_J$ including the $s$-wave and the $p$-wave parts of pion neutron-hole residual interaction.

|        | 1s   | 2p   |
|--------|------|------|
| $s_{1/2}^{-1}$ | 10.5 $-$ 1.3i$| J=1/2 | $-3.2 - 0.6i$ |
|        |      | J=3/2 | $-3.3 - 0.6i$ |
| $d_{3/2}^{-1}$ | 10.4 $-$ 2.1i$| J=1/2 | $-7.1 - 2.0i$ |
|        |      | J=3/2 | $0.2 + 0.0i$ |
| $g_{7/2}^{-1}$ | -6.5 $-$ 1.6i$| J=5/2 | $-3.8 - 1.1i$ |
|        |      | J=7/2 | $0.9 + 0.4i$ |
| $h_{11/2}^{-1}$ | -9.6 $-$ 2.6i$| J=9/2 | $-4.6 - 1.8i$ |
|        |      | J=11/2 | $1.1 + 0.4i$ |
| $d_{5/2}^{-1}$ | -9.9 $-$ 1.9i$| J=13/2 | $-5.7 - 1.4i$ |
|        |      | J=15/2 | $-5.8 - 1.5i$ |
|        |      | J=17/2 | $0.6 + 0.2i$ |

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