Magnetic skyrmion spectrum under voltage excitation and its non-linear modulation

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Magnetic skyrmions are topological quasiparticles with great potential for applications in future information storage and processing devices because of their nanoscale size, high stability, and large velocity. Recently, the high-frequency properties of skyrmions have been explored for magnon nanodevices. Here we systematically study the dynamics of an isolated skyrmion under voltage excitation through the voltage-controlled magnetic anisotropy effect in a circular thin film. A theoretical model considering the demagnetization energy for the skyrmion breathing behavior is developed. With our model, the periodic oscillation of the skyrmion radius can be solved numerically with similar precision compared to micromagnetic simulations, and the characteristic frequency ($f_c$) of the skyrmion breathing can be determined analytically with greater precision than previous studies. Furthermore, we find that the breathing skyrmion can be analogized as a frequency modulator by investigating its nonlinear modulation functionality under sinusoidal-form voltage excitation. Our findings can provide useful guidance for both theoretical and experimental skyrmions research as well as the development of skyrmion-based magnonic devices.
Magnetic skyrmions are topologically nontrivial, particle-like spin textures, which have been experimentally observed in B20-type bulk materials or ultrathin films exhibiting the Dzyaloshinskii–Moriya interaction (DMI)\textsuperscript{1-3}. They have been extensively studied recently for potential applications in future information magnetic storage\textsuperscript{4-6} and processing devices\textsuperscript{7-9} because of their intrinsic properties of nanoscale size, extremely low depinning current density, and high motion velocity\textsuperscript{10-12}. Detailed fundamental research is necessary for the development of devices that can exploit all these beneficial properties of skyrmions. An important component is the understanding of their dynamical excitations to utilize their characteristic properties and to manipulate them more efficiently\textsuperscript{13-19}. Skyrmion breathing modes, in which the core of the swirling spin structure expands and compresses periodically over time, were first studied by micromagnetic simulations (MS) in skyrmion lattices\textsuperscript{20} and then investigated experimentally in helimagnetic insulators\textsuperscript{21}. However, theoretical study of skyrmion breathing is rare. For example, the properties of magnon modes localized on a ferromagnetic skyrmion were studied in Ref. 22, but the analytical result for the breathing-mode frequency of an isolated skyrmion was not validated. Ref. 16 derived and identified the precession frequency of a skyrmion but the DMI energy contribution was excluded. These studies neglected or imprecisely treated one of the most important energy contributions, the demagnetization energy (DE), or the stray field energy\textsuperscript{16,22,23}, as this energy term is difficult to treat analytically because of its nonlocal nature.

Recently, pure electric field or voltage application has been proposed as an energy-efficient method to manipulate magnetism; it is very promising for the development of skyrmion-based devices\textsuperscript{24-26}. Extensive research on the voltage-controlled magnetic anisotropy (VCMA) effect has led to the rapid advances of both the understanding of the underlying physical mechanisms and the realization of
skyrmion-hosting systems suitable for applications\textsuperscript{27-30}.

In this work, we study the dynamics of an isolated skyrmion under voltage excitation in a thin film with a large radius by MS and develop a theoretical model for determining the periodic oscillation of the skyrmion radius and the characteristic frequency ($f_r$) of the skyrmion breathing mode. We compare the corresponding results in the absence and presence of the DE, which is demonstrated to have importance in determining the dynamic state of a skyrmion. Our analytical result demonstrates greater accuracy and robustness compared to those from previous studies\textsuperscript{16,22}. Moreover, we find that the oscillatory skyrmion embraces the properties of the modulation function, similar to a conventional frequency modulator. Our results could offer evidence and guidance for skyrmion-related theories and experiments as well as for the development of skyrmionic devices.

Results

Modeling. We consider the case of a chiral magnetic skyrmion in the center of a circular ferromagnetic (FM) thin film with a large radius and perpendicular magnetic anisotropy (PMA). The skyrmion can be excited to the radially symmetrical magnon mode, or the so-called breathing mode, via the VCMA effect under an applied time-varying voltage, e.g., a sinc pulse voltage on the electrode gate, as illustrated in Fig. 1(a). In our configuration, we consider four contributions to the total energy of the system $E$,

$$E = t_I \int \int \left[ A \varepsilon_{\text{ex}} + D \varepsilon_{\text{IM}} + K_a (1-m_z^2) + \varepsilon_d \right] dS,$$

(1)

where $\varepsilon_{\text{ex}} = |\nabla m|^2$ with an exchange constant $A$, and $\varepsilon_{\text{IM}} = m_z \nabla \cdot m - m \cdot \nabla m_z$ with an interfacial DMI coefficient $D$. The third term of the integrand is the PMA energy.
with constant $K_u$. The last term $\varepsilon_d$ is the DE density. Here, $\mathbf{m}$ is the unit magnetization vector, $m_z = \mathbf{m} \cdot \mathbf{\hat{z}}$ is the magnetization component normal to the surface of the film, $t_f$ is the thickness of the FM layer, and the integration is performed over the whole area of the FM layer. In the spherical angular parametrization, where $\mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, the magnetization dynamics are well described by the Landau–Lifshitz equation,

$$\sin \theta \theta' = -\frac{\gamma \partial \varepsilon}{M_s} \theta - \alpha \sin^2 \theta \phi', \quad \sin \theta \phi' = \frac{\gamma \partial \varepsilon}{M_s} \phi + \alpha \theta', \quad (2)$$

where prime denotes the derivation of the indicated variable with respect to time. $\varepsilon$, $\gamma$, and $\alpha$ represent the total energy density, the gyromagnetic constant, and the Gilbert damping parameter, respectively.

In our theoretical model, the breathing mode of a skyrmion is described by considering the time-dependent skyrmion radius $R(t)$ and angle $\phi(t)$, which is the azimuthal angle of the magnetization $\mathbf{m}$ relative to the radial direction [see Fig. 1(b)], near their equilibrium positions $R(t) = R_e$ and $\phi(t) = 0$. Here, $\phi(t)$ is assumed to remain consistent along the radial direction of the skyrmion. Combining the ansatz of the skyrmion profile$^{31-34}$ [see Fig. 1(d)] with equation (2), we derive the following coupled differential equations [see Supplementary Note 1],

$$\phi' = \alpha \Delta R' + \frac{\gamma \Delta}{M_s} G_1(\phi, R),$$

$$R' = -\alpha \Delta \phi' - \frac{\gamma \Delta}{M_s} G_2(\phi, R), \quad (3)$$

where the functions $G_1(\phi, R)$ and $G_2(\phi, R)$ are related to the total energy of the system [see Supplementary Note 2]. Equation (3) captures the properties of skyrmion breathing behavior more concisely and clearly than micromagnetic modeling can.
In the following parts, the breathing dynamics of a skyrmion is studied both by MS and by using our theoretical model.

**Figure 1. Schematic of the model.** (a) Schematic structure of the model. A voltage is applied on the electrode gate through an insulating layer (omitted here) to change the PMA of the FM layer. (b) The coordinates used in the theory. $R_s$, $\varphi$, and $\chi$ denote the equilibrium skyrmion radius, the azimuthal angle of the magnetization $m$ relative to the radial direction, and the (real space) polar angle, respectively. Here, $\phi = \chi + \varphi$. (c) Variation of the PMA in the form of the sinc function in the FM layer. (d) Illustration of the circular domain wall ansatz. The plot shows the normalized perpendicular magnetization $m_z$ as a function of position $x$ along the diameter of the skyrmion and defines the skyrmion radius $R_s$.  

**Characteristics of breathing dynamics of a skyrmion.** Based on the modeling above, MS are performed in a circular thin film with thickness $t_f = 1\text{ nm}$ and radius $R_d = 500\text{ nm}$, which is sufficiently large to avoid boundary effects. By applying a sinc
pulse voltage to the electrode gate with radius \( R_s = 40 \text{ nm} \), the increment of the PMA of the FM layer beneath the electrode gate varies accordingly, as, \( \Delta K_u = K_0 \sin[2\pi f_0(t-t_0)]/[2\pi(t-t_0)] \) with \( f_0 = 100 \text{ GHz} \), \( t_0 = 1 \text{ ns} \), and \( K_0 \) being the amplitude of the excitation [see Fig. 1(c)], because of the VCMA effect\(^{35-38} \) [see Methods for the simulation details]. The breathing mode is clearly observed under the excitation [see Fig. 2(a)]. Thus, we can determine \( f_c \) of the breathing mode by performing the fast Fourier transform (FFT) on the skyrmion radius \( R(t) \) [see Methods for FFT calculations]. Normally, a greater strength causes more prominent oscillation of \( R(t) \), which also suppresses \( f_c \) slightly [see Fig. 2(b)] because of the increased relaxation distance \( (a_\text{max} = |R(t)_{\text{max}} - R_s|) \) defined as the maximum derivation from the equilibrium radius \( R_s \). In the MS study, we focus on the case of small oscillations, where \( a_\text{max} \) is within 4 nm.

This breathing mode of the skyrmion closely relies on the material properties. For instance, MS results of \( f_c \) and \( R_s \) under different \( D \) and \( K_u \) are respectively shown in Fig. 2(c) and Fig. 2(d). Significantly, \( f_c \) is negatively correlated with \( R_s \). For quantitative analysis, we numerically solved our theoretical model by endowing the initial values of \( \phi(t) \) and \( R(t) \) for equation (3), the results of which validate the accuracy of our model [see Fig. 2(c) and Fig. 2(d)]. Further, the simplified forms of the functions \( G_1(\phi, R) \) and \( G_2(\phi, R) \) [see Supplementary Note 2] can be expressed through rough approximations\(^{39-41} \) for energy terms in equation (1) in the case of \( R \gg \Delta \). Then, after the first order of approximation in equation (3), the corresponding static equilibrium solution for \( R_s \) and the linear dynamics near the equilibrium position for \( f_c \) when \( a_\text{max} \ll R_s \) can be characterized by [see Supplementary Note 4],
\[ R_s = \sqrt{\frac{C_1}{C_2}}, \quad f_c = \sqrt{\frac{2C_1C_2}{(2\pi R_s)}}, \] (4)

where \( C_1 = \pi \gamma D/(2M_s) \), \( C_2 = \gamma/M_s (A/\Delta + K_u \Delta - \pi D/2 - k_1 \mu_0 M_s^2 \Delta) \), and
\[ C_3 = \gamma A/\Delta + k_2 \mu_0 M_s^2 \Delta^2 \] with \( \Delta = \sqrt{A/K_{\text{eff}}} \), \( K_{\text{eff}} = K_u - \mu_0 M_s^2 / 2 \), \( k_1 = 0.576 \), and \( k_2 = 0.158 \). Here, the coefficients \( k_1 \) and \( k_2 \) are derived from the modified DE term\(^{41,42}\) and the values depend on the film thickness (\( t_f = 1 \text{ nm} \) in our case) [see Supplementary Note 2]. The formula results [equation (4)] in relation with \( D \) and \( K_u \) are also shown in Fig. 2(c) and Fig. 2(d) and demonstrate good agreement with the MS results, especially for a large \( R_s \). More importantly, our results are more accurate in comparison with formulas from some previous studies [see Discussion].

As noted in the introduction, DE has often been excluded or crudely approximated by correcting the anisotropy constant \( K_u \) as an effective anisotropy \( K_{\text{eff}} \), which is satisfactory in cases of ultrathin films, especially for \( t_f \to 0 \). In practical situations, we find that this treatment induces some errors, especially in characterizing the dynamic properties of a skyrmion. To clarify, we compare the numerical results of \( R_s \) and \( f_c \) obtained by substituting the anisotropy constant with \( K_{\text{eff}} \) to solve equation (3) with those obtained by separately calculating the DE contributions. Fig. 2 reveals that approximating the DE term by using \( K_{\text{eff}} \) is sufficiently precise to evaluate the static state of a skyrmion, e.g., its \( R_s \), which may be why this method has been extensively adopted in previous studies. In contrast, the approximation is not as suitable in determining the dynamical characteristics of the skyrmion, e.g., \( f_c \), using this modification.
Our model is also verified when DE is excluded [see Supplementary Note 3]. Fig. 2(e) and Fig. 2(f) display the results of $f_c$ and $R_s$ as a function of $D$ and $K_u$ in the absence of the DE. In this case, formula approximations for $f_c$ and $R_s$ can be readily expressed by setting the coefficients $k_1$ and $k_2$ equal to zero in equation (4). The numerical results of equation (3) using simplified form of functions $G_1(\phi, R)$ and $G_2(\phi, R)$ are also obtained, which show very good agreement with the formula results. However, the errors of the formula approximations for $f_c$ compared to the MS results are caused by the imprecise estimation of energy contributions related to $G_1(\phi, R)$ and $G_2(\phi, R)$, as opposed to the first order of approximation. The ansatz to describe the skyrmion profile, i.e., the circular domain wall ansatz, is well represented for large skyrmion sizes ($R \gg \Delta$), but exists some errors for a skyrmion with a size comparable to $\Delta$, thus yielding imprecise approximations for $G_1(\phi, R)$ and $G_2(\phi, R)$. Nevertheless, our formula expressions still show higher accuracy than previous studies [see Discussion].

Additionally, the skyrmion can stabilize with a relatively lower $D$ value in the presence of DE. This is easily observed by the reality that $C_2 > 0$, from which we can determine the upper limit of $D$, i.e., the critical value for $D$,

$$D_c = \frac{2}{\pi} \left( A/\Delta + K_u \Delta - k_1 \mu_0 M_s^2 \Delta \right).$$

Therefore, we can obtain the restriction of $D < 4.41 \text{ mJ/m}^2$ when DE is excluded from the system and $D < 3.78 \text{ mJ/m}^2$ when DE is included in the system for $K_u = 0.8 \text{ MJ/m}^3$ in our simulations.
Figure 2. Comparative analysis between the results from MS and our theoretical model. (a), (b): Comparison between the MS results (blue) and the numerical solutions (yellow) of equation (3) for the normalized skyrmion radius $R(t)/\Delta$ with respect to time in (a) and for $f_c$ as a function of $a_0$ in (b). Here, $D = 3.4 \text{ mJ/m}^2$ and $K_u = 0.8 \text{ MJ/m}^3$. The inset in (b) defines the amplitude of the oscillation $a_0$. (c), (d): MS results (black), numerical solutions of equation (3) including the precise DE term (blue), numerical solutions of equation (3) using $K_{eff}$ (red) and formula results
(dashed lines) of \( f_c \) and \( R_s \) as a function of \( D \) in (c) and \( K_u \) in (d) when DE is present. Here, \( K_u = 0.8 \text{ MJ/m}^3 \) for (c) and \( D = 3.4 \text{ mJ/m}^2 \) for (d). (e), (f): MS results (black), numerical solutions of equation (3) (dashed lines), numerical solutions of equation (3) using simplified \( G_1(\phi,R) \) and \( G_2(\phi,R) \) (orange), and formula results (dashed lines) of \( f_c \) and \( R_s \) as a function of \( D \) in (e) and of \( K_u \) in (f) when DE is absent. Here, \( K_u = 0.8 \text{ MJ/m}^3 \) for (e) and \( D = 3.8 \text{ mJ/m}^2 \) for (f).

**Breathing skyrmion as a modulator.** We have demonstrated that the breathing frequency \( f_c \) of a skyrmion is an intrinsic characteristic closely associated with the material properties. To further investigate and utilize this property, it is also important to explore the skyrmion dynamics under single-frequency excitation. According to the solutions of our theoretical model [see Methods], the skyrmion breathes in hybrid frequencies of the external driving frequency \( f_e \) and \( f_c \) [see Fig. 3]. Moreover, the relative amplitude of \( f_e \) and \( f_c \) in the spectrum depends on the value of \( f_e \).

Specifically, if \( f_e < f_c \), the skyrmion breathing typically synchronizes with \( f_e \), because the external driving frequency \( f_e \) is dominant. Conversely, if \( f_e > f_c \), the skyrmion is more likely to breath at \( f_c \). This phenomenon is very similar to the forced oscillation of a harmonic oscillator: for a periodic force applied to a harmonic oscillator, the solution of the well-known resonance differential equation is a superposition of two sine waves at different frequencies. In addition to the two main peaks in the frequency spectrum, other prominent frequencies can also be discovered, thus revealing the nature of the “resonant skyrmion” as a tunable “modulator.”
Figure 3. Fourier transform spectra of the skyrmion radius under single-frequency excitations. The amplitude of the Fourier transform spectrum under each excitation frequency is normalized by the corresponding amplitude at the position of $f_c$, indicated by the red dashed line. The results are obtained by numerically solving equation (5).

To illustrate this phenomenon, we use a voltage with frequency $f_c = f_c = 7.342$ GHz to excite the skyrmion as a case study, where the resonant oscillation of the skyrmion is expected to be seen. Here, the amplitude of the excitation sine wave is set to a very small value ($K_0 = 0.001$ MJ/m$^3$) to avoid the wild oscillation or destruction of the skyrmion. Both the numerical solutions of equation (3) and MS results are discussed in the following. Interestingly, instead of showing the resonant oscillation phenomenon, the variation of $R(t)/\Delta$ is packaged by a periodic wave with a lower frequency [see Fig. 4(a)]. The fluctuant range of the radius $R(t)$ is $\sim 6$ nm, despite the very low strength of the excitation signal. The fluctuations of the average
The perpendicular component of the magnetization \( \delta m_z(t) = \langle m_z \rangle(t) - \langle m_z \rangle(0) \), where \( \langle m_z \rangle(0) \) is the equilibrium state and the angular bracket denotes the spatial average of the magnetization, are also evaluated [see Fig. 4(b)] because the magnetization variation is readily obtained experimentally. It is apparent that \( \delta m_z(t) \) and \( R(t)/\Delta \) show consistent patterns, which are also identified from the corresponding power spectral density (PSD) [see Fig. 4(c) and Methods]. In the middle of the PSD spectrum, a peak exists very near the position of the frequency \( f_e \), indicating that the exact characteristic frequency \( f_e \) has shifted slightly towards a smaller value because of the relatively large oscillation [see relation between \( f_e \) and \( a_0 \) in Fig. 2(b)]. Moreover, two additional distinct peaks occur on both sides, corresponding to the values of \( f_e - f_e \) and \( f_e + f_e \), respectively. From the time-domain perspective, the appearance of these two peaks can be viewed as a result of modulation, which can be expressed as,

\[
S(t) = S_0 \sin(2\pi f_e t) \times \sin(2\pi f_e t),
\]

with \( S_0 \) being the relative amplitude. The package frequency in Fig. 4(a) and Fig. 4(d) is actually related to \( f_e - f_e \).

The skyrmion also shows a type of “swing” behavior. Specifically, the symmetry axis of the magnetization projecting in the \( xy \) plane rotates back and forth during the breathing process [see Fig. 4(d)]. This phenomenon arises from the magnetization variations along the radial direction, i.e., the angles \( \phi \) are inconsistent because of the drastic changes in the skyrmion radius. The behaviors of the skyrmion under excitation with sine signals at other frequencies are similar, as discussed in detail in Supplementary Note 5. Through the analysis above, the resonant skyrmion shows significant potential applicability for modulation functionality.
Figure 4. Skyrmion dynamics in excitation by sine-wave voltage ($f_c = f_c^* = 7.342$ GHz). (a) MS results (black) and numerical solutions of equation (3) (yellow) for $R(t)/\Delta$. (b) MS results of $\delta m_z(t)$. (c) PSD for $R(t)$ from MS (black) and numerical solutions of equation (3) (orange), as well as for $\delta m_z(t)$. (d) Variation of skyrmion profile (magnetization in $x$ direction) from time $t_0$ to time $t_1$ [indicated in (a)] and from time $t_1$ to time $t_2$ [indicated in (a)] in the breathing process. The red dashed line indicates the symmetry axis of the skyrmion profile, which rotates slightly compared to the equilibrium state.

Discussion

Up to now, we have discussed the formula approximation for the characteristic frequency $f_c^*$ of the skyrmion breathing mode. For greater clarification, we further compare the results from our formula approximations with those from previous research. In Ref. 16, the normalized characteristic frequency $f_c^*$ and the skyrmion radius $R_s^*$ obey the simple relation $2\pi f_c^* R_s^* = 1$, where we obtain $f_c^* \propto 1/R_s$. This
result is obtained when the DMI energy contribution is excluded and the skyrmion radius is tuned by the drive current. In Ref. 22, the normalized frequency $f_\ast$ is characterized by $f_\ast = 1/(2\pi R_s^*\sqrt{1+R_s^*}) \propto 1/R_s^{*2}$ with $R_s$ being normalized. Meanwhile, in our studies, derived from equation (4), the relation between $f_c$ and $R_s$ is expressed as $f_c = \gamma A/(\pi\Delta^2 M_s) \sqrt{1/(R_s/\Delta)^4 - 0.5/(R_s/\Delta)^6}$, from which we determine $f_c \propto \sqrt{1/R_s^{*4} - 1/(2R_s^{*6})}$. If the skyrmion radius is assumed to be sufficiently large, we obtain $f_c \propto 1/R_s^{*2}$, which is the same with that in Ref. 22. Note that the normalizations used in different studies are different. A comparison for the case of DE inclusion is also shown in Fig. 5(b). In brief, the results in Fig. 5 demonstrate the relatively higher accuracy and generality of our formula approximation.

To conclude, we have studied skyrmion dynamics under voltage excitation by using the VCMA effect. The skyrmion breathing behaviors were systematically investigated via both MS and our theoretical model. Our model not only provides very exact numerical solutions for the time-dependent skyrmion radius regardless of the form of excitation, but also yields corresponding analytical solutions, which are more accurate and robust than those from previous studies. In addition, the modulation functionality of a skyrmion has also been investigated, which we believe may provide guidance for the design of skyrmion-based high-frequency magnonic devices.
Figure 5. Comparisons of $f_c$ between the results from our formula approximations and those from previous literatures. (a) Calculations of $f_c$ as a function of $R_s$ by using our formula approximations (green) from equation (4), formulas from Ref. 16 (blue), and formulas from Ref. 21 (orange) in the absence of DE in (a) and in the presence of DE in (b), respectively. The black curves show the MS results. The dashed line in (b) indicates the formula results of equation (4), where DE is considered using $K_{off}$. Note that $R_s$ is varied by changing the DMI constant, corresponding to Fig. 2(c) and Fig. 2(e).

Methods

Micromagnetic simulations (MS). MS are performed by using the graphics-processing-unit-based tool MuMax3. The default mesh size of 1.5625 nm×1.5625 nm×1 nm is used in our simulations. We adopted the following material parameters in our simulations: the exchange stiffness $A = 15$ pJ·m$^{-1}$, saturation magnetization $M_s = 580$ kA·m$^{-1}$ and default PMA constant of the FM layer $K_s = 0.8$ MJ·m$^{-3}$. In addition, we set the VCMA coefficient $\xi$ as $100$ fJ·V$^{-1}$·m$^{-1}$ based on recent experiments$^{35-38}$. Here the typical thickness of the insulating layer is 1 nm. Under these conditions, with an applied voltage of 0.1 V (an electric field of 0.1 V/nm), PMA constant in FM layer will change $10$ kJ·m$^{-3}$, which is about 1.25% of the
PMA change. In the case where the DE is not included, the maximum amplitude of the variation of the PMA of the FM layer in excitation with the sinc voltage is $K_0 = 0.1 \text{ MJ/m}^3$, which corresponds to an applied voltage of 1 V. Considering the effect of the demagnetization field in decreasing the degree of varying the magnetization in the $z$ axis, the maximum amplitude of the excitation pulse $K_0 = 0.08 \text{ MJ/m}^3$ (corresponding to 0.8 V) is consequently used to ensure the small oscillations of the skyrmion in the case where DE is included.

In terms of the single frequency excitation case, based on our analytical model, we firstly calculated the variations of the skyrmion radius in excitation with voltages with different $f_c$ by solving the equation (5), where $K_u$ is set as $K_u = K_0 \sin(2 \pi f \epsilon t)$ with $K_0 = 0.01 \text{ MJ/m}^3$ when $f_c = 3 \text{ GHz} (< f_c)$, $12 \text{ GHz} (> f_c)$ and $K_0 = 0.001 \text{ MJ/m}^3$ when $f_c = f_c = 7.342 \text{ GHz}$, then the Fourier transform spectrum is obtained by performing the Fast Fourier transforms.

**Fast Fourier transforms (FFT).** In our simulations, the variation of the skyrmion radius $R(t)$ and the average magnetization $\langle m_z \rangle(t)$ are extracted for over 20 periods and the sampling frequency $f_s$ is 250 GHz, which satisfies the Nyquist sampling theorem ($2f_0 < f_s$). Then, the FFT is performed as, $F(m) = \sum_{n=0}^{N-1} (s(t_n) - s(t_0))W_n^m$, where $m = 0, 1, \cdots, N-1$, $W_n = e^{-i2\pi/N}$ with $N$ being the transformation point and $s(t_n) = R(t_n)$ or $\langle m_z \rangle(t_n)$. Thus, we can obtain the Fourier transform spectrum $F(f)$, from which $f_c$ is determined by the position of the maximum peak. The power spectrum density (PSD) is computed by using the normalized spectrum $F^*(f) = F(f)/F(f_c)$, as $S(f) = \log|F^*(f)|^2$. 
ASSOCIATED CONTENT

Supplementary Information

The Supplementary information contains: Theoretical model derivation, Calculation of the functions $G_1(\varphi, R)$ and $G_2(\varphi, R)$, Numerical solution of equation (3) in the absence of DE, Determination for $R_s$ and $f_c$, Skyrmion dynamics in excitation with sine voltages.

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Notes

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