Signatures of the Unruh effect from electrons accelerated by ultra-strong laser fields

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We calculate the radiation resulting from the Unruh effect for strongly accelerated electrons and show that the photons are created in pairs whose polarizations are maximally entangled. Apart from the photon statistics, this quantum radiation can further be discriminated from the classical (Larmor) radiation via the different spectral and angular distributions. The signatures of the Unruh effect become significant if the external electromagnetic field accelerating the electrons is not too far below the Schwinger limit and might be observable with future facilities. Finally, the corrections due to the birefringent nature of the QED vacuum at such ultra-high fields are discussed.

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Introduction One of the most fascinating phenomena of non-inertial quantum field theory is the Unruh effect: An observer or detector undergoing a uniform acceleration $a$ experiences the Minkowski vacuum as a thermal bath with the Unruh temperature

$$T_{\text{Unruh}} = \frac{\hbar}{2\pi k_B} a.$$  \hspace{1cm} (1)

As one might expect from the principle of equivalence, the Unruh effect is closely related to Hawking radiation, i.e., black hole evaporation: The uniformly accelerated observer (detecting the Unruh effect) corresponds to an observer at a fixed distance to the horizon (feeling the gravitational pull and measuring the Hawking radiation); whereas the inertial observer in flat space-time is analogous to an unfortunate astronaut freely falling into the black hole. However, there is also a crucial difference between the two phenomena: In contrast to the case of uniform acceleration, the free fall into a black hole is (per definition of a black hole) not invariant under time-reversal. Hence, while Hawking radiation generates a real out-flow of energy (black hole evaporation), the Unruh effect corresponds to an equilibrium thermal bath and does not create any energy flux per se.

The most direct way of observing this striking effect would be to accelerate a detector and to measure its excitations. However, this is extremely difficult since moderate accelerations correspond to extremely low temperatures and thus, the Unruh effect has not been directly observed so far (see, however, [3, 4]). Therefore, we focus on a somewhat indirect signature in the following: Since the uniformly accelerated detector acts as if it was immersed in a thermal bath, there is a finite probability that it absorbs a virtual particle from this bath and passes to an excited state. Translated back into the inertial frame, this process corresponds to the emission of a real particle and goes back to its ground state, also corresponds to the emission of a real particle in the inertial frame.

In the limiting case that the time between absorption and re-emission becomes arbitrarily small, the detector transforms into a scatterer which scatters (virtual) particles from one mode into another mode of the thermal bath in the accelerated frame. Translated back into the inertial frame, this process corresponds to the emission of two real particles by the accelerated scatterer. This effect is analogous to moving-mirror radiation and can be interpreted as a signature of the Unruh effect. In the following, we calculate this quantum radiation given off by point-like non-inertial scatterers (as a model for electrons accelerated in ultra-intense electromagnetic fields) and compare it to the classical (Larmor) radiation. An analogous idea has already been pursued in [7] but in the derivation presented therein did not take into account crucial features of the radiation (such as the fact that the photons are always created in correlated pairs).

The Model Assuming that the electric field $E$ is much stronger than the magnetic field $B$ in the region of interest $E^2 \gg c^2 B^2$, we neglect the spin of the electron since the related energy $\mu_e \cdot B$ is much smaller than the other relevant energies (such as the Unruh temperature). For electromagnetic waves whose wavelength is much larger than the Compton wavelength, the electron acts as a classical point-like scatterer with an energy-independent cross section (Thomson scattering). Since we are mostly interested in backward scattering (reflection, see below), we further neglect the angular dependence of the scattering amplitude ($s$-wave scattering approximation). Under these assumptions, the electromagnetic field under the influence of such a point-like scatterer (i.e., electron) with the trajectory $r_e[t]$ is governed by the Lagrangian density ($\hbar = \varepsilon_0 = \mu_0 = c = 1$ throughout)

$$\mathcal{L} = \frac{1}{2} \left( E^2 - B^2 \right) - g A^\mu A_\mu (r_e[t] - r) \sqrt{1 - \dot{r}_e^2[t]} , \hspace{1cm} (2)$$
where the coupling $g$ determines the $s$-wave scattering length and $A$ is the vector potential in radiation gauge. The last factor ensures the relativistic invariance of the action $A = g \int ds \mu_\mu A^\mu - \int d^4x F_{\mu\nu}F^{\mu\nu}/4$. 

This model can be motivated by the following simple (non-relativistic) picture: The charge $q$ of the electron determines its density $\rho(t, r) = q\delta^3(r_e[t] - r)$ and its current $\mathbf{j} = \dot{r}_e$. Omitting the magnetic field, we get $\dot{r}_e = q \mathbf{E}/m = q\hat{A}/m$, and neglecting the spatial dependence (due to $\dot{r}_e \perp k$) of $A$, this can be solved via $\dot{r}_e = q\hat{A}/m$. Insertion of the resulting current $\mathbf{j} = q^2\delta^3(r_e[t] - r)\hat{A}/m$ into the Lagrangian $\mathbf{j} \cdot \mathbf{A}$ reproduces our ansatz (2) with $q = q^2/m$ which yields the correct cross section for planar Thomson scattering. In the natural units used here, the charge $q$ is related to the fine-structure constant via $q = \sqrt{4\pi\alpha_{\text{QED}}} \approx 0.3$.

**Particle Creation**

In order to calculate the photons created by the non-inertial motion $r_e[t]$ of the scatterer, let us split the total Hamiltonian into a perturbation part

$$\hat{H}_1(t) = g\hat{A}^2(t, r_e[t])\sqrt{1 - \dot{r}_e^2[t]} ,$$

supplemented with the usual adiabatic switching on and off $g[t] \uparrow \infty = 0$, and the undisturbed Hamiltonian $H_0 = \frac{1}{2} \int d^3r (\dot{\mathbf{E}}^2 + \dot{\mathbf{B}}^2)$, which leads to the usual normal mode expansion.

Since the coupling $g$ is much smaller than the other relevant length scales (such as the wavelengths of the photons), the evolution of the initial Minkowski vacuum $|0\rangle$ can be derived via time-dependent perturbation theory

$$|\text{out}\rangle = |0\rangle - i \int dt \hat{H}_1(t) |0\rangle + \mathcal{O}(g^2)$$

$$= |0\rangle + \sum_{k,\lambda, k',\lambda'} \mathcal{A}_{k,\lambda, k',\lambda'} |k, \lambda, k', \lambda'\rangle + \mathcal{O}(g^2) ,$$

with the two-photon amplitude

$$\mathcal{A}_{k,\lambda, k',\lambda'} = \frac{e_{k,\lambda} \cdot e_{k',\lambda'}}{2iV\sqrt{k k'}} \int dt \frac{1 - \dot{r}_e^2[t]}{\sqrt{k k'}} \exp\left\{ i(k + k')t - i(k + k') \cdot r_e[t] \right\} .$$

Here $k$ is the wavenumber and $k = |k|$ the frequency of the photon modes; $\lambda$ labels their polarization described by the unit vector $e_{k,\lambda}$; and $V$ is the quantization volume. Since a time-resolved detection of the created photons is probably infeasible, polarization and momentum are the best observables to be measured. As one may infer from the above expression, the photons are always emitted in pairs (squeezed state) and there is a perfect entanglement of the polarizations of the two photons due to the scalar product $e_{k,\lambda} \cdot e_{k',\lambda'}$; e.g., parallel photons $k \parallel k'$ must have the same polarization $\lambda = \lambda'$. Note that this applies to linear polarization, the circular polarizations of the two created photons are opposite (for Thomson scattering) due to angular momentum conservation.

In terms of the new integration variable $\tau = t - r_e[t]$, with $r_e^\parallel = (k + k') \cdot r_e/(k + k')$, the two-photon amplitude,

$$\mathcal{A}_{k,\lambda, k',\lambda'} = \frac{e_{k,\lambda} \cdot e_{k',\lambda'}}{2iV\sqrt{k k'}} \int d\tau \frac{1 - \dot{r}_e^2[\tau]}{1 - \dot{r}_e^2[t]} \exp\left\{ i(k + k')\tau \right\} ,$$

is determined by the Fourier transform of the effective (direction-dependent) Doppler factor in the integral above.

**Larmor Radiation**

In order to discuss the observability of this quantum radiation, it must be compared with the competing classical process. The Larmor radiation can be derived from the relativistic action $q \int ds \mu_\mu A_\mu$ and corresponds to a coherent state

$$|\alpha\rangle = \prod_{k,\lambda} \exp\left\{ \alpha_{k,\lambda} a_{k,\lambda}^\dagger - \alpha_{k,\lambda}^* a_{k,\lambda} \right\} |0\rangle ,$$

with the coefficients (see, e.g., (8))

$$\alpha_{k,\lambda} = q \int d\tau \frac{e_{k,\lambda} \cdot \dot{r}_e[\tau]}{\sqrt{2V k}} \exp\left\{ i(k - k') \cdot r_e[\tau] \right\} .$$

The numerator $e_{k,\lambda} \cdot \dot{r}_e$ displays the well-known blind spot in forward and backward direction $k \parallel r_e$. The introduction of a new integration variable $\tau = t - k_\parallel \cdot r_e[t]/k$ yields a Fourier transform similar to Eq. (10).

For an investigation of the detectability of the quantum radiation in Eq. (6), the two-photon amplitude $|\mathcal{A}_{k,\lambda, k',\lambda'}\rangle$ must be compared with the amplitude for the competing classical process, which is given by $|\alpha_{k,\lambda} \alpha_{k',\lambda'}\rangle$ to lowest order in $\alpha_{k,\lambda}$. In view of the smallness of the coupling $g$ and assuming comparable results of the Fourier integrals (no resonances etc.), there are basically two possibilities for achieving $|\mathcal{A}_{k,\lambda, k',\lambda'}\rangle > |\alpha_{k,\lambda} \alpha_{k',\lambda'}\rangle$: small velocities $\dot{r}_e < 1$ or small angles $\theta$ between $k$ and $\dot{r}_e$ (blind spot). The first alternative is probably impractical since the total effect becomes too small, but the latter option can be realized for a uni-directional acceleration. For typical $k$-values, quantum radiation becomes comparable to classical radiation for

$$\theta_{\text{max}}^2 = \mathcal{O}\left( \frac{g k}{q^2} \right) ,$$

i.e., inside a small forward and backward cone (blind spot). The total probability of emitting a photon pair inside these cones of “quantum domination” scales as

$$\mathcal{P}(\theta_{\text{max}}) = \sum_{k,\lambda, k',\lambda'} \left| \mathcal{A}_{k,\lambda, k',\lambda'} \right|^2 = \mathcal{O}\left( \frac{g^4 k^4}{q^4} \right) .$$

The typical $k$-value is set by the characteristic time-scale of the trajectory. For a smooth (e.g., Gaussian)
electric field pulse $E(t)$ which accelerates the electron to moderately relativistic velocities (say $\gamma = 2$), this would be the pulse length. Assuming a rather short pulse of order $0.1$ attoseconds, i.e., $\Delta t = \mathcal{O}(10^{-19} \text{ s})$, comparison with the coupling $g \approx 3.5 \times 10^{-14} \text{ m}$ yields $g k^2/m = \mathcal{O}(10^{-2})$, i.e., quantum radiation dominates within a forward/backward cone of a few degrees. Unfortunately, the total probability $\Psi(\vartheta_{\max})$ of emitting a photon pair in these cones is suppressed by several orders of magnitude and hence probably very hard to detect.

In order to measure the quantum radiation, far shorter characteristic time-scales are desirable. One option could be a pulse with a sharp front end whose typical raising time $\delta t \ll \Delta t$ is much shorter than the pulse length $\Delta t$. However, even for a rectangular electric field pulse $E(t)$, the Fourier transform of $\dot{r}_e$ decays like $1/(k + k')^2$ for large wavenumbers. Hence, the enhancement of $\vartheta_{\max}$ would be logarithmic $\ln(\Delta t/\delta t)$ for non-relativistic or moderately relativistic velocities -- which is probably not sufficient. Unless further amplification mechanisms such as resonances are present, the only possibility left (apart from generating much shorter pulses, which is extremely hard) is to use long pulses with strong electric fields which accelerate the electrons to ultra-relativistic velocities and to exploit the large Lorentz boost factors $\gamma \gg 1$.

**Ultra-relativistic Regime** As motivated above, let us now consider ultra-relativistic velocities $\gamma \gg 1$. Since both, quantum and classical radiation will be boosted forward in this case, we shall focus on a small forward cone $\vartheta \ll 1/\gamma$, see also Eq. (18) below. In this limit, the integral (10) simplifies to (for $\lambda = \lambda'$)

$$
\mathcal{A}_{k,\lambda,k',\lambda'} \approx \int d\tau \frac{g(\tau)}{i V \sqrt{k k'}} e^{i(k + k')\tau},
$$
(12)

with a time-dependent Lorentz factor $\gamma(\tau)$ whose evolution is determined by $d\gamma/d\tau \approx q E(t)/m$ as well as $d\tau \approx 2\gamma^2 d\tau$. The leading contribution of the above integral arises near the maximum Lorentz factor $\gamma_{\max}$, i.e., in the final stage of the acceleration phase, where $d\gamma/d\tau$ drops from $2\gamma q E/m$ to zero on an effective time scale of $\Delta \tau = \mathcal{O}(\Delta t/\gamma_{\max}^2)$, which determines the cutoff wavenumber via $k_{\text{cut}} = \mathcal{O}(\gamma_{\max}^2/\Delta t)$. Since $\gamma_{\max}$ is roughly given by the pulse length $\Delta t$ times the acceleration $q E/m$, the cutoff wavenumber can alternatively be written as $k_{\text{cut}} = \mathcal{O}(\gamma_{\max} q E/m)$. Below this cutoff, the above integral behaves like the Fourier transform of a Heaviside step function of height $\gamma_{\max}$

$$
\mathcal{A}_{k,\lambda,k',\lambda'} = \mathcal{O}\left(\frac{\gamma_{\max}^2}{V \sqrt{k k'} (k + k')}\right).
$$
(13)

A similar estimate for the Larmor radiation yields

$$
\alpha_{k,\lambda} \approx \int d\tau \frac{2q \gamma^2}{\sqrt{2V k}} e^{i k\tau} = \mathcal{O}\left(\frac{q \gamma_{\max}^2}{\sqrt{V k^3}}\right),
$$
(14)

with basically the same wavenumber cutoff. Of course, quantum radiation again dominates for sufficiently small $\vartheta$. For the cutoff wavenumber $k_{\text{cut}}$, the size of the small forward cone of “quantum domination” scales as

$$
\vartheta_{\max} = \mathcal{O}\left(\sqrt{\frac{q E}{m^2 \gamma_{\max}}}\right),
$$
(15)

and is determined by $\gamma_{\max}$ and ratio of the electric field $E$ over the Schwinger limit $E_S = m^2/q$. The probability of (two-photon) emission in this cone is given by

$$
\Psi(\vartheta_{\max}) = \frac{\vartheta_{\max}^2}{\max_{k,\lambda,k',\lambda'} |\mathcal{A}_{k,\lambda,k',\lambda'}|^2} = \mathcal{O}\left(\frac{q E^4}{E_S^2}\right).
$$
(16)

Interestingly, for a given electric field strength $E$, this probability does not depend significantly on the pulse length since $\gamma_{\max}$ cancels -- but the energy $k_{\max}$ and the angular distribution $\vartheta_{\max}$ of the emitted photons does.

**Extensions** Since the observation of the photon pairs requires electromagnetic fields which are not too far below the Schwinger limit, one should also consider the impact of these ultra-high fields on the QED vacuum, which then acts as a medium and displays effects such as bi-refringence. To this end, we consider the first non-linear corrections from the Euler-Heisenberg Lagrangian

$$
\mathcal{L} = \frac{1}{2} \left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{1}{90\pi E_S^2/\alpha_{\text{QED}}} \left(\mathbf{E} \cdot \mathbf{B}\right)^2.
$$
(17)

In media, the dielectric displacement $\mathbf{D} = \partial \mathcal{L}/\partial \mathbf{E}$ may differ from the electric field $\mathbf{E}$ and hence the first Maxwell equation $\nabla \cdot \mathbf{D} = 0$ does not necessarily imply $\nabla \cdot \mathbf{E} = 0$. If we neglect the external magnetic field and linearize around an approximately homogeneous external electric field $\mathbf{E}_0$, we get $\mathbf{D} = \varepsilon \cdot \mathbf{E} \equiv \varepsilon \cdot \mathbf{A}$ with the permittivity tensor $\varepsilon$. Hence, the transversality condition $\mathbf{k} \cdot \mathbf{e}_{k,\lambda} = 0$ deviates from $\mathbf{k} \cdot \mathbf{e}_{k,\lambda} = 0$ and thus the Larmor radiation does not necessarily vanish in forward direction anymore. However, for a trajectory along a field line $\mathbf{r}_e \parallel \mathbf{E}_0$, i.e., along an eigenvector of $\varepsilon$, the Larmor radiation still has the blind spot $\mathbf{k} \perp \mathbf{e}_{k,\lambda}$ in forward direction $\mathbf{k} \parallel \mathbf{E}_0$.

With an external magnetic field $\mathbf{B}_0$, on the other hand, additional terms appear. Assuming $\mathbf{k} \parallel \mathbf{E}_0 \perp \mathbf{B}_0$, one of the (linear) polarizations $\lambda$ has blind spot in forward direction $\mathbf{k} \cdot \mathbf{e}_{k,\lambda} = 0$ but the other one $\lambda'$ has not:

$$
\mathbf{k} \cdot \mathbf{e}_{k,\lambda'} = \frac{4\alpha_{\text{QED}}}{45\pi} \frac{k E_0 B_0}{E_S^2} \neq 0.
$$
(18)

Although the numerical pre-factor is rather small $2 \times 10^{-4}$, this effect should be taken into account when searching for quantum radiation [11]. On the other hand, it might also provide an opportunity for testing the birefringent nature of the QED vacuum in the presence of ultra-high external fields (see also [12]). Similarly, one obtains small corrections to the two-photon correlations.
Summary  We have studied the conversion of (virtual) quantum vacuum fluctuations into (real) particles by non-inertial scattering for the example of strongly accelerated electrons. This quantum radiation can be discriminated from classical (Larmor) radiation via the different angular and spectral distributions and the distinct photon statistics: In the quantum case, the photons are always emitted in pairs (squeezed state) with maximally entangled polarizations – whereas the classical radiation corresponds to a coherent state (independent photons with Poissonian statistics).

The probability of emitting two photons with wave-numbers \( k \ll k_{\text{cut}} = \mathcal{O}(\gamma_{\text{max}} q E/m) \) scales as \( k^4 \) for quantum (Unruh) radiation and as \( 1/k^6 \) for Larmor. For \( k \)-values larger than this cutoff \( k_{\text{cut}} \), the decline is faster than polynomial [smooth \( C^\infty \)-pulse \( E(t) \)]. Hence the typical photon energies are determined by that cutoff \( k_{\text{cut}} = \mathcal{O}(\gamma_{\text{max}} q E/m) \) which corresponds to the Unruh temperature \( \Gamma \) boosted by \( \gamma_{\text{max}} \) and may even exceed the electron mass for large Lorentz factors (thus enabling us to distinguish this quantum radiation from other effects such as annihilation etc.). Note, however, that the spectrum is not Planckian in general. This is caused by the non-trivial translation from the accelerated frame to the inertial frame with a time-dependent Lorentz boost factor \( \gamma(t) \) whose rate of change is of the same order as the frequency corresponding to the Unruh temperature \( \Gamma \) (i.e., non-adiabatic). Of course, a state consisting of pairs of correlated photons can never be exactly thermal.

The Larmor radiation has a blind spot in forward (and backward) direction where the quantum (Unruh) radiation is maximal leading to a cone of “quantum domination” with a small angle \( \theta_{\text{max}} \) in Eq. \( \Gamma \), see also Fig. 1. In this cone, the probability for emitting a photon pair given by Eq. \( \Gamma \) scales with the square of the fine-structure constant times the ratio of the electric field over the Schwinger limit \( E_S = m^2/q \) to the fourth power: \( (4\pi\alpha_{\text{QED}})^2 E^4/\epsilon_S^4 \approx 8 \times 10^{-3}(E/E_S)^4 \). Hence the signatures of the Unruh effect might be detectable in future facilities (cf. [13, 14, 15]) generating electric fields not too far below the Schwinger limit \( E_S \approx 1.3 \times 10^{18} \text{V/m} \) (corresponding to an intensity of order \( 10^{22} \text{W/cm}^2 \)) which accelerates the electrons to ultra-relativistic velocities. (Conversely, one might use the spectral and angular distribution of the radiation to determine the characteristic parameters of the pulse such as \( E, \Delta t, \) and \( \gamma_{\text{max}} \)). The smallness of the above probability could be compensated by accelerating many electrons simultaneously or subsequently. Note that the detection probably also requires coincidence measurements (exploiting the photon statistics) since the probability for a single Larmor photon in this cone scales as \( q^2 E^2/\epsilon_S^2 \).

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