CP asymmetries in the supersymmetric trilepton signal at the LHC

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Abstract

In the CP-violating Minimal Supersymmetric Standard Model, we study the production of a neutralino-chargino pair at the LHC. For their decays into three leptons, we analyze CP asymmetries which are sensitive to the CP phases of the neutralino and chargino sector. We present analytical formulas for the entire production and decay process, and identify the CP-violating contributions in the spin correlation terms. This allows us to define the optimal CP asymmetries. We present a detailed numerical analysis of the cross sections, branching ratios, and the CP observables. For light neutralinos, charginos, and squarks, the asymmetries can reach several 10%. We estimate the discovery potential for the LHC to observe CP violation in the trilepton channel.
1 Introduction

The Minimal Supersymmetric extension of the Standard Model (MSSM) [1] features many potential sources for CP violation [2] beyond the Standard Model (SM) of particle physics. Most of these additional phases are associated with flavor mixing. In the flavor conserving sector, the $SU(2)$ gaugino mass parameter $M_2$ is conventionally chosen to be real and positive. The CP-violating, complex parameters are then the higgsino mass parameter $\mu$, the $U(1)$ and $SU(3)$ gaugino mass parameters $M_1$ and $M_3$, respectively, and the trilinear scalar coupling parameters $A_f$,

$$\mu = |\mu| e^{i\phi_\mu}, \quad M_1 = |M_1| e^{i\phi_1}, \quad M_3 = |M_3| e^{i\phi_3}, \quad A_f = |A_f| e^{i\phi_A_f}. \quad (1)$$

These phases in general contribute to electric dipole moments (EDMs), in particular to those of the neutron, and the Thallium and Mercury atoms. In fact, scenarios where (at least) one of the phases appearing in Eq. (1) is large are often in conflict with current experimental upper bounds on these EDMs [3]. However, large phases are not excluded by these constraints, even if first generation sleptons are rather light. For example, a certain degree of fine-tuning allows for cancellations among the different terms contributing to the EDMs [4,5]. The EDM bounds can also be fulfilled by including lepton flavor violating couplings in the slepton sector [6]. See also Refs. [7].

It is clear that CP observables outside the low energy EDM sector have to be measured to independently determine or constrain possible SUSY phases at colliders. In that respect, T-odd and CP-odd asymmetries based on triple or epsilon products have been proposed [8]. For the ILC [9], triple product asymmetries have been intensively studied in the production and decay of neutralinos [10,11] and charginos [16,20], also using transversely polarized beams [21,22]. At the LHC [23,24], triple product asymmetries have been studied for the decays of stops [5,26,29], sbottoms [30], staus [31], and neutralinos which originate from squark decays [32,33]. The triple-product asymmetries can be of the order of 60%, since they already appear at tree level due to spin correlations. In particular, it was shown that the CP-violating effects in stop decays [28] can be measured at the $3\sigma$ confidence level at the LHC for a luminosity of $\mathcal{L} = 300$ fb$^{-1}$, using the technique of momentum reconstruction for on-shell decay chains [33,34].

We are thus motivated to explore the discovery reach at the LHC for CP violation in the production of a neutralino-chargino pair

$$p + p \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_j^\pm, \quad i = 2, 3, 4; \quad j = 1, 2. \quad (2)$$

The contributing tree-level diagrams involving $\tilde{u}_L$, $\tilde{d}_L$ squark and $W$ exchange are shown in Fig. 1. The phases $\phi_1$ and $\phi_\mu$ in the neutralino and chargino sector will trigger CP violation. CP-sensitive contributions to the differential cross section will originate at tree level from the spin and spin-spin correlations among the neutralinos and charginos. These can be analyzed with the help of the visible decay products of the neutralino and chargino. Leptonic decays are especially useful [35]. On the one hand, hadronic decays lead to final states with very large SM backgrounds, and therefore do not lead to viable signals at the LHC. On the other hand, the construction of T-odd observables requires the measurement of the charges of final state particles, which is difficult, if not impossible, for jets.
We therefore use the leptonic two-body decays of the chargino and neutralino as “spin analyzers”:

\[
\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_R^\pm \ell_1^\pm, \\
\tilde{\ell}_R^\pm \rightarrow \tilde{\chi}_1^0 \ell_2^\pm; \\
\tilde{\chi}_j^\pm \rightarrow \tilde{\nu}_\ell + \ell_3^\pm, \\
\tilde{\nu}_\ell \rightarrow \tilde{\chi}_1^0 + \nu_\ell. 
\]

(3) 

(4) 

See Fig. 2 for a schematic picture of the production and decay process. The LHC signature is three isolated leptons (two of them with same flavor and opposite charge) and missing energy, carried away by the two neutralinos $\tilde{\chi}_1^0$ and the neutrino $\nu_\ell$. This process, known as the trilepton signal [35–37], has low QCD and SM background, and thus has been studied in detail as a SUSY discovery channel at the Tevatron [38–40], and also at the LHC [23, 24].

In this paper we first calculate the amplitude squared for the entire process of neutralino-chargino pair production and decay in the spin density matrix formalism [25]. The explicit formulas allow us to identify the CP-sensitive parts in the spin and spin-spin correlations, see Section 2. From those we define T-odd asymmetries of triple- and epsilon products. In Section 3, we numerically analyze these asymmetries, the production cross sections, and the neutralino and chargino branching ratios in general MSSM scenarios with complex $\mu$ and $M_1$. We discuss several production modes $\tilde{\chi}_i^0, \tilde{\chi}_j^\pm$, and two-body decays via gauge bosons and three-body decays. The asymmetries analyzed in Sections 2 and 3 depend on the 4-momenta of the quarks in the initial state, and of the chargino and/or neutralino in the final state. These can in general not be determined exactly. In Section 4, we therefore discuss approximations for the initial quark and intermediate gaugino momenta, which work quite efficiently to enhance the asymmetries. In Section 5, we comment on the minimal luminosity required at the LHC to observe CP-violating effects in the trilepton mode, before we summarize and conclude in Section 6. The Appendices contain a review of chargino and neutralino mixing (Appendix A), of the relevant parts of the MSSM Lagrangian with complex couplings (B), definitions of momenta and spin vectors as well as details of phase space integration (C), analytical expressions for the amplitude squared in the spin-density matrix formalism (D), and a discussion of how our asymmetries depend on the lepton charges (E).
2 CP asymmetries

In this Section, we first identify the CP-sensitive parts in the amplitude squared of the entire process of neutralino-chargino pair production and their subsequent leptonic two-body decays, see Eqs. (2)-(4) and Fig. 2. In order to probe these parts, we then define several T-odd products of 4-momenta and the corresponding asymmetries. Explicit expressions for the squared amplitude, relevant parts of the MSSM Lagrangian with complex couplings, and details of phase-space integration, are summarized in Appendices A to D. In this and the following Section we consider all (products of) 4-momenta to be observables. Since not all of these momenta can be measured in actual LHC experiments, we will address solutions to this complication later in Section 4.

2.1 T-odd observables

The amplitude squared $|T|^2$ for neutralino-chargino pair production and decay, see Fig. 2 can be decomposed into contributions from the neutralino spin correlations, the chargino spin correlations, the neutralino-chargino spin-spin correlations, and a spin-independent part, see Eq. (D.97). At tree level, CP-sensitive parts can only originate from the spin and the spin-spin correlations. They receive contributions from the exchange of the $W$ and the squarks $\tilde{u}_L, \tilde{d}_L$.

The CP-sensitive contributions to the neutralino spin correlations are

\begin{align}
\Sigma_{PD_1}(WW) & \propto \text{Im}\{O_{ij}^L O_{ij}^{R*}\} \chi^0_i, \\
\Sigma_{PD_1}(W\tilde{u}_L) & \propto \text{Im}\{f_{ui}^{L*} l_{ij} O_{ij}^{R*}\} \chi^0_i, \\
\Sigma_{PD_1}(W\tilde{d}_L) & \propto \text{Im}\{f_{di}^{L*} l_{ij} O_{ij}^{L*}\} \chi^0_i, \\
\Sigma_{PD_1}(\tilde{u}_L\tilde{d}_L) & \propto \text{Im}\{f_{ui}^{L*} f_{d_j}^{L*} l_{ij} l_{ij}\} \chi^0_i. 
\end{align}

1The squares of the $\tilde{u}_L$ and $\tilde{d}_L$ contributions, given by $\Sigma_{PD_1}(\tilde{u}_L\tilde{u}_L)$ and $\Sigma_{PD_1}(\tilde{d}_L\tilde{d}_L)$, do not contain sufficiently many different couplings to be sensitive to any CP-violating phase. Note that we have assumed vanishing $L-R$ mixing for first generation squarks, making their couplings chiral, whereas the $W-\tilde{\chi}^0_i-\tilde{\chi}^\pm_j$ couplings have no fixed chirality.
These spin correlation terms explicitly depend on the imaginary parts of the products of the W-\(\tilde{\chi}^0_i - \tilde{\chi}^\pm_j\) couplings \(O^{L,R}_{ij}\), the \(q\bar{q}_L - \tilde{\chi}^0_i\) couplings \(f^L_{\bar{q} i}\), and the \(q\bar{q}_L - \tilde{\chi}^\pm_j\) couplings \(l^L_{\bar{q} j}\). The imaginary parts are manifestly CP-sensitive, i.e. sensitive to the phases \(\phi_\nu, \phi_1\) in the neutralino-chargino sector, and are each multiplied by the T-odd epsilon product

\[
\mathcal{E}_{\chi^0_i} \equiv [p_u, p_d, p_{\tilde{\chi}^0_i}, p_{\ell_1}] \equiv \varepsilon_{\mu\nu\alpha\beta} p^\mu_u p^\nu_d p^\alpha_{\tilde{\chi}^0_i} p^\beta_{\ell_1}.
\]  

(9)

We use the convention \(\varepsilon_{0123} = 1\), and here and in the following we not put a bar on any of the \(u\) or \(d\) quark indices, as long as the statements made apply for both \(u\bar{d} \rightarrow \tilde{\chi}^0_i \tilde{\chi}^0_j\) or \(\bar{u}d \rightarrow \tilde{\chi}^0_i \tilde{\chi}^0_j\) production processes.

Similarly, the CP-sensitive contributions to the chargino spin correlations are

\[
\Sigma_{PD_3}(WW) \propto \text{Im}\{O^L_{ij} O^{R*}_{ij}\} \mathcal{E}_{\chi^\pm_j},
\]

(10)

\[
\Sigma_{PD_3}(W\tilde{u}_L) \propto \text{Im}\{f^L_{\bar{u} i} f^L_{\bar{d} j} O^{R*}_{ij}\} \mathcal{E}_{\chi^\pm_j},
\]

(11)

\[
\Sigma_{PD_3}(W\tilde{d}_L) \propto \text{Im}\{f^L_{\bar{d} i} f^L_{\bar{d} j} O^{L*}_{ij}\} \mathcal{E}_{\chi^\pm_j},
\]

(12)

\[
\Sigma_{PD_3}(\tilde{u}_L\tilde{d}_L) \propto \text{Im}\{f^L_{\bar{u} i} f^L_{\bar{d} j} l^L_{\bar{d} j}\} \mathcal{E}_{\chi^\pm_j},
\]

(13)

with the short-hand notation for the epsilon product

\[
\mathcal{E}_{\chi^\pm_j} \equiv [p_u, p_d, p_{\chi^\pm_j}, p_{\ell_1}] \equiv \varepsilon_{\mu\nu\alpha\beta} p^\mu_u p^\nu_d p^\alpha_{\chi^\pm_j} p^\beta_{\ell_1}.
\]

(14)

Note that the chargino spin correlations probe the same coupling combinations as the neutralino spin correlation terms.

Finally, the CP-sensitive contributions to the spin-spin correlations are

\[
\Sigma_{PD_1,D_3}(WW) \propto \text{Im}\{O^L_{ij} O^{R*}_{ij}\} f,
\]

(15)

\[
\Sigma_{PD_1,D_3}(W\tilde{u}_L) \propto \text{Im}\{f^L_{\bar{u} i} f^L_{\bar{d} j} O^{R*}_{ij}\} f,
\]

(16)

\[
\Sigma_{PD_1,D_3}(W\tilde{d}_L) \propto \text{Im}\{f^L_{\bar{d} i} f^L_{\bar{d} j} O^{L*}_{ij}\} f,
\]

(17)

\[
\Sigma_{PD_1,D_3}(\tilde{u}_L\tilde{d}_L) \propto \text{Im}\{f^L_{\bar{u} i} f^L_{\bar{d} j} l^L_{\bar{d} j}\} f,
\]

(18)

with the short-hand notation for the kinematical function

\[
f \equiv (p_u \cdot p_{\ell_1})[p_d, p_{\tilde{\chi}^0_i}, p_{\chi^\pm_j}, p_{\ell_1}] + (p_u \cdot p_{\tilde{\chi}^0_i})[p_d, p_{\chi^\pm_j}, p_{\ell_1}, p_{\ell_1}] + (p_d \cdot p_{\ell_1})[p_u, p_{\tilde{\chi}^0_i}, p_{\chi^\pm_j}, p_{\ell_1}] + (p_d \cdot p_{\chi^\pm_j})[p_u, p_{\tilde{\chi}^0_i}, p_{\chi^\pm_j}, p_{\ell_1}],
\]

(19)

see Eq. \([D.127]\) in Appendix \[D.3]\ Again the spin-spin correlations contain the same dynamics, i.e. the same coupling combinations, as the spin correlations. Note that the neutralino and chargino decays will not yield additional CP-sensitive contributions, since these are two-body decays via scalar particles. The T-odd spin-spin correlation term \(f\) is analogous to the corresponding term in neutralino \([11]\) or chargino \([18]\) pair production at the ILC, since all these processes have the same kinematical structure.

Each of the epsilon products \(\mathcal{E}_{\chi^0_i}\), Eq. \([9]\), and \(\mathcal{E}_{\chi^\pm_j}\), Eq. \([14]\), or the tensor product \(f\), Eq. \([19]\), is T-odd. That is since each of the spatial components of the four-momenta changes sign under a naive time reversal, \(t \rightarrow -t\). Due to CPT invariance, these T-odd products are multiplied with the CP-sensitive, imaginary parts of products of couplings, \(\text{Im}\{O^L_{ij} O^{R*}_{ij}\}\), \(\text{Im}\{f^L_{\bar{u} i} f^L_{\bar{d} j} O^{R*}_{ij}\}\), \(\text{Im}\{f^L_{\bar{d} i} f^L_{\bar{d} j} O^{L*}_{ij}\}\), or \(\text{Im}\{f^L_{\bar{u} i} f^L_{\bar{d} j} l^L_{\bar{d} j}\}\).
2.2 T-odd asymmetries

The task is to define observables that project out the CP-sensitive parts in the spin and the spin-spin correlation terms in the amplitude squared. This can be achieved by defining for the several possible T-odd products $T = \bar{E} \chi_0^i, \bar{E} \chi_{\pm}^j,$ or $f$ the corresponding T-odd asymmetries of the cross section $\sigma$ for neutralino-chargino production and decay

$$A = \frac{\sigma(T > 0) - \sigma(T < 0)}{\sigma(T > 0) + \sigma(T < 0)} = \frac{\int \text{Sign}[T]|T|^2d\text{Lips}d\text{PDF}}{\int |T|^2d\text{Lips}d\text{PDF}},$$

(20)

with the amplitude squared $|T|^2$, Eq. (D.97), the Lorentz invariant phase-space $d\text{Lips}$ (C.42), and the short-hand notation $d\text{PDF} \equiv dx_1 dx_2 f_u(x_1, \mu^2) f_d(x_2, \mu^2)$, see Eq. (D.100).

We obtain explicit expressions for the asymmetries by inserting the amplitude squared $|T|^2$ (D.97) into Eq. (20):

$$A(\bar{E} \chi_0^i) = \frac{\int \text{Sign}(\bar{E} \chi_0^i) \Sigma_{PD} D_1 D_3 d\text{Lips}d\text{PDF}}{\int P D_1 D_3 d\text{Lips}d\text{PDF}},$$

(21)

$$A(\bar{E} \chi_{\pm}^j) = \frac{\int \text{Sign}(\bar{E} \chi_{\pm}^j) \Sigma_{PD} D_1 d\text{Lips}d\text{PDF}}{\int P D_1 D_3 d\text{Lips}d\text{PDF}},$$

(22)

$$A(f) = \frac{\int \text{Sign}(f) \Sigma_{PD} D_1 D_3 d\text{Lips}d\text{PDF}}{\int P D_1 D_3 d\text{Lips}d\text{PDF}},$$

(23)

using Eqs. (D.102) and (D.110), and the phase space element $d\text{Lips}$ as given in Eq. (C.42), where we have already used the narrow width approximation of the propagators, see Eq. (D.98). In the numerators of the asymmetries $A$, Eqs. (21)-(23), only those spin or spin-spin correlations remain, which contain the corresponding T-odd products $T = \bar{E} \chi_0^i, \bar{E} \chi_{\pm}^j,$ or $f$. The other terms of the amplitude squared vanish due to the phase space integration over the sign of the T-odd product, $\text{Sign}(T)$. In the denominator, all spin and spin-spin correlation terms vanish, and only the spin-independent parts contribute. This last statement remains true even after applying acceptance cuts on the final state momenta, as long as these cuts are CP symmetric.

Note that in general the largest asymmetries are obtained by using T-odd products of 4-momenta that match exactly the kinematical dependence of the CP-sensitive terms in the amplitude squared. In the literature, these are therefore sometimes referred to as optimal observables [41]. Other combinations of momenta lead in general to smaller asymmetries.

By construction, the asymmetries $A$, Eqs. (21)-(23), are manifestly Lorentz invariant. Due to the boost between the partonic center-of-mass frame and the lab frame, the use of triple products, which are not Lorentz invariant, generally leads to smaller asymmetries. One famous example is the triple product of the three outgoing leptons [38]

$$T = p_{\ell_1} \cdot (p_{\ell_2} \times p_{\ell_3}) \equiv (p_{\ell_1}, p_{\ell_2}, p_{\ell_3}).$$

(24)

Note that strictly speaking the largest observables are the expectation values of the T-odd products $\langle T \rangle = \int |T|^2 d\text{Lips}/\int |T|^2 d\text{Lips}$, as used e.g. in Ref. [11, 21]. Typically their statistical significance is increased by $\sim 20\%$ [11, 18], compared to the asymmetries given in Eq. (20). However, we show in Section 4 that these optimal observables are more difficult to measure experimentally than the asymmetries we analyze.

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Table 1: MSSM benchmark scenario.

| $M_2$ | $|\mu|$ | $\phi_\mu$ | $\phi_1$ | $\tan\beta$ | $M_E^\tilde{\ell}$ | $M_L^\tilde{\ell}$ | $m_{\tilde{q}}$ |
|-------|----------|-----------|---------|----------|----------------|----------------|--------|
| 240 GeV | 150 GeV | 0 | $\frac{\pi}{9}$ | 5 | 110 GeV | 150 GeV | 400 GeV |

That triple product probes the CP-sensitive terms in the spin-spin correlations and in the neutralino spin correlations, but not in the chargino spin correlations. Since it does not directly match the kinematical dependence of the CP-sensitive terms, and since it is not Lorentz invariant, the corresponding asymmetry will be reduced by a factor of about 2 to 4 compared to the maximal asymmetries as given in Eqs. (21)-(23). These however contain the initial quark and the intermediate neutralino/chargino momenta, which cannot be measured directly at the LHC and thus need to be approximated, whereas the lepton momenta appearing in Eq. (24) are measurable with high accuracy. In Section 4 we will compare numerically the sizes of different experimentally measurable CP-asymmetries around a benchmark scenario.

Another complication arises because the initial state at the LHC, which contains two protons but no antiprotons, is not CP self-conjugate. Thus LHC experiments will strictly speaking not be able to measure true CP-odd asymmetries. However, it is sufficient for our purpose that our asymmetries are odd under naive time reversal. This ensures that at tree level they are non-zero only in the presence of non-trivial CP phases. Finally, one needs to be careful when summing over events with different lepton charges, since some of the asymmetries change sign when the sign of a lepton charge is flipped. This is discussed in Appendix E.

3 Numerical analysis

We perform a quantitative study of the CP asymmetries $A$, Eqs. (21)-(24), the $pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm$ production cross sections, and branching ratios of the neutralinos and charginos around a benchmark scenario defined in Table 1. We fix all relevant parameters directly at the weak scale. First we choose rather small values of $|\mu| = 150$ GeV and $M_2 = 240$ GeV, which results in small chargino and neutralino masses, but still distant from the current experimental bounds [12]. Larger chargino and neutralino masses considerably reduce the production cross sections. In order to reduce the number of parameters, we use the GUT inspired relation for the modulus of the $U(1)$ gaugino mass parameter $|M_1| = 5/3 \tan^2 \theta_w M_2 \approx 0.5 M_2$, but leave $\mu$ and the phase $\phi_1$ as independent parameters. In order to enable the neutralino and chargino two-body decays, $\tilde{\chi}_2^0 \to \tilde{\ell}_R \ell_1$ and $\tilde{\chi}_2^\pm \to \ell_3^\mp \tilde{\nu}_i^{(s)}$, we fix the soft-breaking parameters in the slepton sector to $M_{\tilde{\ell}_E} = 110$ GeV and $M_{\tilde{\ell}_L} = 150$ GeV for $\ell = e, \mu, \tau$. The CP asymmetries, as well as the chargino and neutralino branching ratios are rather independent of this choice, as long as the desired neutralino and chargino two-body decays are kinematically allowed. We fix $\tan \beta = 5$ since we observe a mild dependence of the cross sections and asymmetries on $\tan \beta$. We take stau mixing into account, and fix the trilinear scalar coupling parameter $A_{\tau} = 250$ GeV. This choice only has a small impact on the neutralino and chargino branching ratios. Since its phase does not contribute to the CP asymmetry, we set $\phi_{A_{\tau}} = 0$. Finally, we

\[3\text{Note that this choice significantly constrains the neutralino sector [13].}\]
chose a large mass for the charged Higgs boson, i.e. we work in the “decoupling limit” of the MSSM Higgs sector. The only Higgs state of relevance to us is then the lightest neutral Higgs boson, which has a mass of about 115 GeV.$^4$

Finally, we fix the squark masses $m_{\tilde{q}_L} = m_{\tilde{d}_L} = 400$ GeV to relatively low values, to enhance squark exchange in the production. For mixed or bino-like gauginos, the squark exchange channels will give the dominant contributions to the asymmetries and the $\tilde{\chi}_0^i \tilde{\chi}_j^\pm$ production cross sections. Note that in the context of the constrained MSSM/mSUGRA, the experimental lower bounds for squark masses of the first generation at the 95% confidence level have recently increased from some 450 GeV (in specific $m_{\tilde{q}} = m_{\tilde{g}}$ scenarios) for data-sets corresponding to an integrated LHC luminosity of about 35 pb$^{-1}$ for the year 2010 [44–46], to now up to some 1.1 TeV, based on data samples corresponding to up to 1.45 fb$^{-1}$, collected in the first half of the year 2011 [47, 48]. Although we think that the reported special and CP-conserving cMSSM/mSUGRA model based bounds can also be transferred to some extent to our general SUSY models considered, we still take our benchmark scenario as a starting point for parameter scans, which we will perform in the following. In this sense our benchmark scenario has to be seen as the most optimistic, since it provides the largest CP asymmetries possible. We discuss and comment on the asymmetries and the cross sections with heavier squark masses in detail at the end of the next Subsection.

The relevant resulting SUSY masses, branching ratios and production cross sections for the benchmark scenario are summarized in Tables 2 and 3 respectively. Note that the production cross sections for the charge conjugated gaugino pairs $pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^-\bar{\nu}_\ell \ell^+$ are about half as big as for $pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^+\tilde{\nu}_\ell \ell^-$ production, see Table 3. This is due to the PDFs, approximately reflecting the valence quark ratio $u : d \approx 2 : 1$, as the partonic cross sections are identical for the two charge conjugated pairs at tree level.

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Table 2: Superparticle masses and branching ratios for the benchmark scenario as given in Table 1. The branching ratios are summed over $\ell = e, \mu$, and for the neutralino also summed over both slepton charges.

| $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_R}$ | $m_{\tilde{\chi}_1^\pm}$ |
|-------------------------|----------------------|-------------------------|
| 89 GeV                  | 118 GeV              | 119 GeV                 |
| 146 GeV                 | 157 GeV              | 281 GeV                 |
| 160 GeV                 | 117 GeV              | 119 GeV                 |
| 281 GeV                 | 137 GeV              | 281 GeV                 |

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$^4$ As mentioned in the Introduction, the combination of small sparticle masses and rather large phases tends to give too large electric dipole moments. This requires some finetuning of parameters not relevant for our analysis. For example, our benchmark point satisfies the 95% c.l. bound on the EDM of the electron [42] if the phase of $A_e$ lies between $-1.543$ and $-1.445$ ($-1.0254$ and $-0.9118$) for $|A_e| = 150$ (300) GeV, indicating a finetuning at the 1% level. The EDM of the neutron, and of atoms, in addition depends on the trilinear soft breaking parameters $A_u$ and $A_d$, as well as on the gluino mass. Since the experimental bound on the EDM of the neutron is about one order of magnitude weaker than for the electron, and since squarks are significantly heavier in our benchmark scenario, somewhat less finetuning is required in the squark sector.

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individual contributions can be as large as \( \pm \sqrt{W} \), with opposite sign, and thus cancel each other partly. See the sum of all their contributions to the asymmetry \( A_\sigma \) the corresponding neutralino-chargino cross section the neutralino matrix \([49]\), are driven apart by the complex off-diagonal terms. reaching from \((\tilde{\chi}_2^0)\rightarrow(\tilde{\nu}_\tau \tau^+)\), \( p\tilde{\nu}_\tau \), \( p\tilde{\tau}^+ \) \( \tilde{\tau}_2^+ \nu_\tau \approx \text{BR}(\tilde{\tau}_2^+ \rightarrow \tilde{\tau}_2^+ \nu_\tau) = 12\% \), \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \tilde{\nu}_\tau \tau^+) = 12\% \), \( \text{BR}(\tilde{\tau}_i^+ \rightarrow \tilde{\nu}_i^0 W^+) = 17\% \), summed over \( i = 1, 2, 3 \), and \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^+ Z) = 10\% \), \( \text{BR}(\tilde{\tau}_2^+ \rightarrow \tilde{\tau}_2^+ h) = 2\% \). 

In Fig. [5](b), we show the \(|\mu|-M_2\) dependence of the cross section \(\sigma(pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^+ )\) for neutralino-chargino pair production, which reaches several 10 fb for light neutralinos and charginos. Our benchmark scenario is indicated by a cross in the \(|\mu|-M_2\) plane. The neutralino and chargino branching ratios \(\text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\nu}_\tau \tau^+)\) and \(\text{BR}(\tilde{\tau}_i^+ \rightarrow \tilde{\nu}_i^0 W^+)\) are shown in Fig. 5(c), (d). The (Lorentz invariant) asymmetry for the T-odd product \(\mathcal{E}_{\tilde{\chi}_2^+} = [p_\mu, p_{d\tilde{\nu}}, p_{\tilde{\chi}_2^0}, p_{\tilde{\tau}_i^+}]\), which appears in the neutralino spin correlations, is shown in Fig. 5(a). For the asymmetry we use the short-hand notation \(\mathcal{A}[p_\mu, p_{d\tilde{\nu}}, p_{\tilde{\chi}_2^0}, p_{\tilde{\tau}_i^+}]\). That asymmetry will probe the CP-sensitive parts of the neutralino spin correlations, as discussed in Section 2.

Looking at Fig. 5(a), we see that our benchmark point lies on a line, approximately reaching from \((|\mu|, M_2) = (100, 150) \text{ GeV}\) to \((250, 400) \text{ GeV}\), where the asymmetry obtains its maximum. In the vicinity of that line there is a level-crossing of the neutralino states \(\tilde{\chi}_2^0\) and \(\tilde{\chi}_3^0\) for \(\phi_1 = 0\), and between \(\tilde{\chi}_1^0\) and \(\tilde{\chi}_2^0\) for \(\phi_1 = \pi\). The level crossing of the neutralinos is also reflected by the abrupt change in the cross section, see Fig. 5(b). Note that for non-vanishing phases there is no true level-crossing since the neutralino masses, \emph{i.e.} the singular values of the neutralino matrix \([49]\), are driven apart by the complex off-diagonal terms.

In Fig. 4 we show the phase dependence of the asymmetry \(\mathcal{A}[p_\mu, p_{d\tilde{\nu}}, p_{\tilde{\chi}_2^0}, p_{\tilde{\tau}_i^+}]\) (left), and the corresponding neutralino-chargino cross section \(\sigma(pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^+ )\). The asymmetry receives contributions from the different production channels of \(W, \tilde{u}_L\), and \(\tilde{d}_L\) exchange. The contribution from \(WW\) exchange is shown in Fig. 5(a), that of \(W\tilde{u}_L\) exchange in Fig. 5(b), the \(W\tilde{d}_L\) exchange in Fig. 5(c), and finally the \(\tilde{u}_L\tilde{d}_L\) exchange in Fig. 5(d). We can see that the individual contributions can be as large as \(\pm 60\%\), however the different contributions enter with opposite sign, and thus cancel each other partly. See the sum of all their contributions to the asymmetry \(\mathcal{A}[p_\mu, p_{d\tilde{\nu}}, p_{\tilde{\chi}_2^0}, p_{\tilde{\tau}_i^+}]\) in Fig. 4 (left).

In Fig. 4 (left), we show the \(|\mu|-M_2\) dependence of the asymmetry of the T-odd product \(\mathcal{E}_{\tilde{\chi}_2^+} = [p_\mu, p_{d\tilde{\nu}}, p_{\tilde{\chi}_2^0}, p_{\tilde{\tau}_i^+}]\), which appears in the chargino spin correlations. For the asymmetry we use the short-hand notation \(\mathcal{A}[p_\mu, p_{d\tilde{\nu}}, p_{\tilde{\chi}_2^0}, p_{\tilde{\tau}_i^+}]\), to indicate the different momenta used. In Fig. 4 (left), our benchmark scenario is indicated by a cross in the \(|\mu|-M_2\) plane. For that

| \(\sigma(pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^+)\) | \(\sigma(pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^+)\) | \(\sigma(pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^+)\) |
|---|---|---|
| 600 (332) fb | 574 (313) fb | 2 (1) fb |
| 11 (6) fb | 27 (13) fb | 34 (16) fb |
Figure 3: Contour lines in the $|\mu|-M_2$ plane for neutralino-chargino pair production $pp \to \tilde{\chi}_2^0\tilde{\chi}_2^\pm$ and subsequent leptonic two-body decays $\tilde{\chi}_2^0 \to \tilde{\ell}_R^- \ell^+_R$ and $\tilde{\chi}_2^\pm \to \tilde{\nu}_e \ell^+_R$ at the LHC at 14 TeV: (a) CP asymmetry $A[p_u, p_d, p_{\tilde{\chi}_1^0}, p_{\tilde{\ell}_1}]$ in %, see Eq. (20), (b) the production cross section $\sigma(pp \to \tilde{\chi}_2^0\tilde{\chi}_2^\pm)$ in fb, (c) the neutralino branching ratio $\text{BR} (\tilde{\chi}_2^0 \to \tilde{\ell}_R^- \ell^+_R)$ in %, and (d) the chargino branching ratio $\text{BR} (\tilde{\chi}_2^\pm \to \tilde{\nu}_e \ell^+_R)$ in %. The cross indicates the position of our SUSY benchmark scenario, see Table 1. The area above the black dashed line is excluded by $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0}$. Below the gray dashed line we have $m_{\tilde{\chi}_1^\pm} < 100$ GeV.

scenario we show the phase dependence of the asymmetry in the right panel. By comparing its size with the corresponding asymmetry which probes the neutralino spin correlations, see Figs. (a) and (c) for the $|\mu|-M_2$ dependence and the phase dependence, respectively, we see that the asymmetry which probes the chargino spin-correlations is about half as large. This is only
asymmetries almost vanish in this limit. Four different asymmetries (right) on the squark masses, 
\[ m_q = m_{\tilde{q}} = m_{\tilde{d}_L}. \] Since the different interference contribution depend sensitively on the squark masses, also the cross section and the asymmetries are very sensitive. For increasing squark masses, the \( W \) boson contribution dominates, which results in the asymptotic values \( \sigma_p(pp \to \tilde{\chi}_2^0 \tilde{\chi}_2^+ \to 35 \text{ fb}, \) whereas the asymmetries almost vanish in this limit.

In Fig. 4 (left), we show the phase dependence of the CP asymmetry \( A[p_u, p_d, p_{\tilde{\chi}_2^0}, p_{\ell_1}] \) in \( \% \), see Eq. (20), and (right) the production cross section \( \sigma(pp \to \tilde{\chi}_2^0 \tilde{\chi}_2^+) \) in fb, for neutralino-chargino pair production \( pp \to \tilde{\chi}_2^0 \tilde{\chi}_2^+ \) and subsequent leptonic two-body decays \( \tilde{\chi}_2^0 \to \ell_R \ell_1^\pm ; \ell_R \to \tilde{\chi}_1^0 \ell_2^\pm \) and \( \tilde{\chi}_2^+ \to \ell_R \ell_3^\pm \) at the LHC at 14 TeV. The SUSY parameters are given in Table 1.
Figure 5: Phase dependence of the interference contributions (in %) to the CP asymmetry $A[p_u,p_d,p_{\tilde{\chi}_0^2},p_{\ell_1^+}]$, Eq. (20), from (a) $WW$ exchange, (b) $W\tilde{u}_L$ exchange, (c) $W\tilde{d}_L$ exchange, and (d) $\tilde{u}_L\tilde{d}_L$ exchange, for neutralino-chargino pair production $pp \to \tilde{\chi}_2^0\tilde{\chi}_2^\pm$ and subsequent leptonic two-body decays $\tilde{\chi}_2^0 \to \tilde{\ell}_R^{-}\ell_1^+$; $\tilde{\ell}_R^- \to \tilde{\chi}_1^0\ell_2^-$ and $\tilde{\chi}_2^\pm \to \tilde{\nu}_{\ell}\ell_3^\pm$ at the LHC at 14 TeV. The SUSY parameters are given in Table 1.
Figure 6: Dependence of the CP asymmetry $A[p_u, p_d, p_{\tilde{\chi}_2^0}, p_{\tilde{\chi}_2^+}]$ in %, see Eq. (20), on $|\mu|-M_2$ (left) and on the phases (right) for neutralino-chargino pair production $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^+$ and subsequent leptonic two-body decays $\tilde{\chi}_2^0 \rightarrow \tilde{\nu}_R \ell_1^+$ and $\tilde{\chi}_2^+ \rightarrow \tilde{\nu}_R \ell_3^+$ at the LHC at 14 TeV. The SUSY parameters are given in Table 1. Compare with Figs. 3 (a) and 4 (left), where the asymmetry $A[p_u, p_d, p_{\tilde{\chi}_2^0}, p_{\ell_3^+}]$ of the neutralino spin correlations is shown.

Figure 7: Phase dependence of (left) the CP asymmetry $A[f]$ in %, and (right) the triple product asymmetry $A(p_{\ell^+}, p_{\ell^-}, p_{\ell_3^+})$ in %, see Eq. (20), for neutralino-chargino pair production $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^+$ and subsequent leptonic two-body decays $\tilde{\chi}_2^0 \rightarrow \tilde{\nu}_R \ell_1^+$ and $\tilde{\chi}_2^+ \rightarrow \tilde{\nu}_R \ell_3^+$ at the LHC at 14 TeV. The SUSY parameters are given in Table 1. Compare their sizes with the other asymmetries in Figs. 4 (left) and 6 (right).
3.2 Remarks on \( \tilde{\chi}_2^0 \tilde{\chi}_1^\pm \) and \( \tilde{\chi}_3^0 \tilde{\chi}_1^\pm \) production

As we can see from Table 2, the cross sections for \( pp \to \tilde{\chi}_2^0 \tilde{\chi}_1^\pm \) production are much larger than the \( pp \to \tilde{\chi}_{2,3}^0 \tilde{\chi}_2^\pm \) cross sections. However, in scenarios with \( M_L^t < M_L^\ell \), it is difficult to simultaneously satisfy the inequalities

\[
m_{\ell_R} < m_{\tilde{\chi}_2^0} < m_{\tilde{\ell}_L} \quad \text{and} \quad m_{\tilde{\ell}_R} < m_{\tilde{\chi}_1^\pm}, \quad (\ell = e, \mu);
\]

these are needed to enable the neutralino and chargino two-body decays, Eqs. (3), (4), respectively, while suppressing undesirable \( \tilde{\chi}_2^0 \to \tilde{\ell}_R \ell \) decays. These decays, if allowed, reduce the branching ratio \( \text{BR}(\tilde{\chi}_2^0 \to \tilde{\ell}_R \ell) \) we are interested in. Moreover, the CP asymmetry for the neutralino \( \mathcal{A}[p_u, p_d, p_{\tilde{\chi}_2^0}, p_{\tilde{\ell}_1}] \) changes sign with an intermediate \( \tilde{\ell}_L \) compared to an intermediate \( \tilde{\ell}_R \) in the neutralino decay chain \[1011\]. Thus the two contributions, from intermediate \( \tilde{\ell}_R \) and \( \tilde{\ell}_L \), will tend to cancel, if they cannot be distinguished experimentally on an event-by-event basis.

\[5^\text{Note that the} pp \to \tilde{\chi}_{2,3}^0 \tilde{\chi}_2^\pm \text{channels can wash out the strong CP signal from} \tilde{\chi}_2^0 \tilde{\chi}_2^\pm \text{production, if the different} \tilde{\chi}_2^\pm \text{modes are not separated. However, for the present benchmark scenario, the pollution from the} \tilde{\chi}_2^\pm \text{three-body decays can probably be made quite small, in particular if final states are excluded where all three leptons have the same flavor. Since} \tilde{\chi}_2^\pm \text{decays produce rather soft leptons, they can be suppressed by putting a cut on the energy or} p_T \text{of the unpaired lepton} \ell_3. \text{Similarly, one may be able to suppress contributions from} \tilde{\chi}_{3,4}^0 \tilde{\chi}_2^\pm \text{contributions through cuts on the paired leptons} \ell_1 \ell_2. \text{In particular, the neutralino two-body decay chain has a well-defined upper edge of the} \ell_1 \ell_2 \text{invariant mass distributions, with most events not too far from this edge. A cut on the} \ell_1 \ell_2 \text{invariant mass should therefore suppress unwanted contributions. Depending on the masses, this could also suppress} \tilde{\chi}_2^0 \to \tilde{\ell}_R \ell, \text{if open.}\]
Note that in the MSSM one generally has $m_{\tilde{\chi}_i^0} \gtrsim m_{\tilde{\chi}_i^\pm}$, while $SU(2)$ gauge invariance implies $m_{\tilde{\chi}_L} \sim m_\nu$. Thus some fine-tuning of parameters is needed to achieve $m_{\tilde{\chi}_i^+} > m_\nu$ without getting also automatically $m_{\tilde{\chi}_2^0} > m_{\tilde{\ell}_L}$.

As an example of a somewhat fine-tuned scenario, we keep the hierarchy $M_L > M_E$ as in our benchmark scenario, Table I, but choose a slightly smaller $M_L = 120$ GeV (and close to $M_R = 110$ GeV), which enables the two body decay $\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_L \ell^\pm$. We also adopt the phase $\phi_1 = 0.9\pi$, which gives maximal asymmetries for this scenario. We then scan the parameter space around that modified benchmark point. Although the production cross section $pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^+$ reaches up to 2 pb for small $M_2$, $|\mu| \lesssim 200$ GeV, the asymmetry $\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_2^0}, p_{\tilde{\ell}_R^+}]$ does not exceed 0.5%. The neutralino branching ratio $BR(\tilde{\chi}_2^0 \rightarrow \tilde{\ell}_R^- e^+)$ gets slightly reduced since now the $\tilde{\nu}_L$ are lighter than $\tilde{\ell}_R$, and reaches not more than 10%. The chargino branching ratio can reach $BR(\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_e e^+) = 33\%$, since only the chargino decay channels into leptons are open. Similarly, the $pp \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_1^+$ production cross section reaches 1 pb, with $BR(\tilde{\chi}_3^0 \rightarrow \tilde{\ell}_R^- e^+) < 10\%$, but the asymmetries are again small, $\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_3^0}, p_{\tilde{\ell}_R^+}] \lesssim 1\%$.

In a second approach, we invert the hierarchy between the left and right slepton states, taking $M_L < M_E$. Then we choose $M_E$ sufficiently heavy to close the channel $\tilde{\chi}_2^0 \rightarrow \tilde{\ell}_R \ell$, but still allow for $\tilde{\chi}_1^\pm \rightarrow \tilde{\ell}_L \ell^\mp$ and $\tilde{\chi}_3^0 \rightarrow \tilde{\ell}_L \ell$, such that there is only the contribution from the left slepton to the neutralino asymmetry, and no cancellations appear. However we cannot find asymmetries larger than $\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_2^0}, p_{\tilde{\ell}_R^+}] > 0.5\%$.

### 3.3 Note on other chargino and neutralino decays

So far we have assumed rather large CP phases and small sfermion masses. This will generate too large electric dipole moments of SM fermions, unless the first and second generation $A$-parameters are fine-tuned accurately in both size and phase. This could be ameliorated by choosing sufficiently large slepton masses. The phase sensitivity of spin and spin-spin correlations could then be studied via chargino and neutralino decays into real $Z^0$ and $W^\pm$ gauge bosons; background reduction and charge determination would again force one to focus on purely leptonic decays of the gauge bosons.

However, the asymmetries for the neutralino decay $\tilde{\chi}_i^0 \rightarrow Z \tilde{\chi}_1^0$ with the subsequent leptonic decay $Z \rightarrow \ell \ell$ are reduced by a factor $f_Z = (|L|^2 - |R|^2)/(|L|^2 + |R|^2) \approx 0.15$, where $L$ ($R$) is the left (right) SM coupling of the $Z$ to charged leptons. Thus one loses almost an order of magnitude in all neutralino spin asymmetries. In addition, the overall statistical significance is reduced, since $BR(Z \rightarrow \ell \ell) \approx 6.7\%$ summed over $\ell = e, \mu$. This reduces the total cross section for the purely leptonic final state, see the discussion at the end of Section 3 in Ref. [12]. Due to the Majorana properties of the neutralinos, the neutralino spin asymmetries identically vanish for decays into a Higgs bosons, $\tilde{\chi}_i^0 \rightarrow H \tilde{\chi}_1^1$, since the couplings obey $|c_L|^2 = |c_R|^2$.

A similar argument holds for the chargino decay into a $W$ boson, $\tilde{\chi}_i^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$. If the $W$ momentum is reconstructed, which is in principle possible for hadronic decays, the reduction factor $f_W = (|l|^2 - |r|^2)/(|l|^2 + |r|^2)$ to the chargino spin asymmetries due to the left ($l$) and right ($r$) $\tilde{\chi}_i^\pm - \tilde{\chi}_1^0 W$ couplings is typically of the order of 0.2 to 0.4. For leptonic decays the

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Note 8: Clearly the size of the branching ratios, and hence the degree to which these two contributions cancel, also depend also on the wino/bino admixtures of $\tilde{\chi}_i^0$. 

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overall statistical significance is reduced due to the branching ratio \( \text{BR}(W \rightarrow \ell \nu) \approx 21\% \), for \( \ell = e, \mu \). Lastly, for the chargino decay via a \( W \), and the neutralino decay via an on-shell \( Z \) boson, one would have to fight the large SM background from \( pp \rightarrow ZW^\pm \) production, of order 16 pb \(^{[23]} \). Thus maximally P-violating chargino and neutralino decays, which can be realized only if sleptons are light, are ideal for analyzing the CP-violating effects in chargino-neutralino production.

There are also regions of parameter space where some of the charginos and neutralinos only have three-body decays. This happens if the sleptons (and squarks) are heavier than the charginos and neutralinos in question; in addition, the mass difference between these states and the lightest neutralino has to be smaller than the masses of the \( W \) and \( Z \) boson. Three-body decays, which proceed via the exchange of virtual sfermions, gauge and Higgs bosons, could then provide additional CP-violating contributions to the asymmetries \(^{[15, 20]} \). (This also happens for two-body decays into spin-1 \( Z \) \(^{[12]} \) and \( W \) \(^{[17]} \) bosons.) Although that could be interesting, as those CP-violating contributions can be of the order of 10\% \(^{[12, 15, 17, 20, 32, 33]} \), it could be more difficult to disentangle the two different CP-violating contributions from production and decay. Since the three-body decay scenario would require quite different calculations and phenomenology, we defer its analysis to another work.

4 Constructing accessible asymmetries at the LHC

So far we have discussed those CP asymmetries, based on the optimal epsilon products, which exactly match the kinematical dependence of the CP-sensitive terms in the amplitude squared. For example, in the neutralino spin correlations this is the epsilon product \( \mathcal{E}_{\tilde{\chi}_0^0} = \{p_u, p_d, p_{\tilde{\chi}_1^0}, p_{\ell_1}\} \), see Eq. (9). This epsilon product contains the quark and neutralino momenta, which are not directly accessible at the LHC. Even if we knew all particle masses, we would not be able to completely reconstruct the momenta in the event. There are 8 unknowns: the 3-momenta of \( \tilde{\chi}_1^0 \) from the neutralino decay, the invisible sneutrino \( (\tilde{\nu}_\ell \rightarrow \nu_\ell \tilde{\chi}_1^0) \) from the chargino decay, as well as the z-components of the \( u \) and \( \bar{d} \) momenta, but only 7 kinematical constraints: 4 from energy-momentum conservation, and 3 invariant mass constraints for the decays of the chargino, the neutralino, and the slepton. A complete reconstruction is thus impossible, since the decay chain on the chargino side is too short due to the invisible sneutrino.

In this Section we thus discuss different methods to approximate the intermediate neutralino and the initial partonic momenta. We have checked that all our results for the asymmetries and cross sections are in agreement with the public code \texttt{MadGraph} \(^{[51]} \). Since the estimates we present in the following cannot be perfect, we quantify the dilution of the asymmetries \( A \) due to the momenta replacements, and the dilution of the cross section \( \sigma \) due to selection cuts. The figure of merit will be the statistical significance, which roughly scales like

\[
S = |A| \sqrt{\mathcal{L} \sigma},
\]

for a given luminosity \( \mathcal{L} \), assuming 100\% acceptance. For our benchmark point, with \( pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0 \) production and decay, see Table 1 we have \( \sigma(pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0) = 11 \, \text{fb}, \) \( \text{BR}(\tilde{\chi}_2^0 \rightarrow \ell_R \ell) = 66\%, \) \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \tilde{\nu}_\ell \ell^+) = 23\% \). The task is to obtain larger significances than that of the triple product asymmetry \( A(p_{\ell^+}, p_{\ell^-}, p_{\ell^0}) = -4.7\% \), for which we have \( S = 0.6 \) for \( \mathcal{L} = 100 \, \text{fb}^{-1} \).
4.1 Lepton preselection

As a preselection, we apply cuts on the transverse momentum and rapidity of each lepton

\[ p_T > 10 \text{ GeV}; \quad |\eta| < 2.5; \quad \Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} > 0.4, \]  

where \( \Delta \phi \) is the difference of the azimuthal angles of a lepton pair in radiant, and \( \Delta \eta \) their rapidity difference. The preselection are standard cuts to isolate leptons, which are for example included as basic cuts in MadGraph [51]. About 50% of our signal events pass these cuts.

4.2 Replacing the neutralino momentum

We replace the neutralino momentum \( p_{\tilde{\chi}_i^0} \) by the lepton momenta \( \ell_{1,2} \) from its decay, such that the epsilon product becomes \([p_u, p_d, p_{\tilde{\chi}_i^0}, p_{\ell_1}] \rightarrow [p_u, p_d, p_{\ell_1}, p_{\ell_2}] = [p_u, p_d, p_{\ell_2}, p_{\ell_1}]\). The corresponding asymmetry is reduced from 21% to 14%. We can improve the approximation of the neutralino momentum by focusing on events where the lepton pair \( \ell_1 \ell_2 \) from the neutralino decay has a large invariant mass, \( m_{\ell_1\ell_2} > b \max(m_{\ell_1\ell_2}) \), which optimizes the significance for \( b \approx 0.5 \). This cut ensures that the sum of the 3-momenta of the leptons is small in the \( \tilde{\chi}_2^0 \) rest-frame. Note that a cut on this quantity might be needed to reduce SM backgrounds where the lepton pair originates from a virtual photon. In addition we only keep those lepton pairs with a large transverse momentum sum, \((p_{T\ell_1} + p_{T\ell_2}) > c m_{\tilde{\chi}_2^0}\), which optimizes the significance for \( c \approx 0.2 \). About two thirds of the preselected events pass both these cuts, but the asymmetry \( A[p_u, p_d, p_{\ell_2}, p_{\ell_1}] \) is enhanced from 14% to 18%.

Note that the sign of the asymmetry depends on the charge of the near \( \ell_1 \) and far lepton \( \ell_2 \) from the neutralino decay in the following way, see also the discussion in Appendix E:

\[
A[p_u, p_d, p_{\ell_1\ell_2}] = -A[p_u, p_d, p_{\ell_1\ell_2}] = +A[p_u, p_d, p_{\ell_1\ell_2}] = -A[p_u, p_d, p_{\ell_1\ell_2}].
\]

The sign change in the first step originates from Eq. (28), and the sign change in the second and third steps is due to the interchange of the momenta of \( \ell_1 \) and \( \ell_2 \) in the antisymmetric epsilon product. These relations are important, since we need not determine from which vertex, near or far, the leptons \( \ell_1 \) and \( \ell_2 \) originate. Instead in the epsilon product one just groups them according to their charge, and uses for example the asymmetry \( A[p_u, p_d, p_{\ell^+\ell^-}] \), see also the discussion in Appendix G of Ref. [5]. One can ensure that the two leptons stem from the neutralino decay, if one requires that the lepton \( \ell_3 \) from the chargino decay has different flavor than the opposite-sign same-flavor lepton pair \( \ell_1^\pm \ell_2^\mp \) from the neutralino decay.

4.3 Approximating the quark momenta

The sign of \( \mathcal{E}_{\tilde{\chi}_i^0} = [p_u, p_d, p_{\tilde{\chi}_i^0}, p_{\ell_1}] \), Eq. (9), depends on the direction of the incoming \( u \)-quark. (The \( d \) antiquark then obviously comes from the opposite direction.) In most events \( pp \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \) the \( u \)-quark will be more energetic than the antiquark \( \bar{d} \), due to the characteristic momentum distributions of valence \( u \) and sea-quarks \( d \) in the PDFs. So the entire event will be mostly boosted in the direction of the incoming \( u \)-quark. The z-component of the sum of
all three lepton momenta, $p_\ell^z = p_{\tilde{\chi}_1}^z + p_{\tilde{\chi}_2}^z + p_{\tilde{\chi}_3}^z$, coincides in 75% of the cases with the direction of the incoming $u$-quark momentum. However, guessing wrong will immediately reduce the asymmetry, since then the event is included with the wrong sign. The efficiency of the guess can be considerably enhanced to over 90%, if we instead require that only the lepton pair from the neutralino decay has a large component in the direction of the beam, $|p_{\tilde{\chi}_1}^z + p_{\tilde{\chi}_2}^z| > d m_{\tilde{\chi}_2}$. About half of the preselected events pass the cut for $d \approx 0.6$, which optimizes the significance.

With these approximated quark momenta, $p_{u}^{\text{aprx}} = (1, 0, 0, \eta)$, $p_{\bar{d}}^{\text{aprx}} = (1, 0, 0, -\eta)$, with $\eta = \text{Sign}[p_{\tilde{\chi}_1}^z + p_{\tilde{\chi}_2}^z] = \pm 1$, we obtain for the asymmetry $A[p_{u}^{\text{aprx}}, p_{\bar{d}}^{\text{aprx}}, p_{\tilde{\chi}_2}^z, p_{\ell_1}^z, p_{\ell_2}^z] = -20\%$.

If we now also replace the neutralino momentum and include the cuts on the invariant mass and the transverse momentum of the lepton pair, we obtain an asymmetry of $A[p_{u}^{\text{aprx}}, p_{d}^{\text{aprx}}, p_{\tilde{\chi}_2}^z, p_{\ell_1}^z] = 17\%$, with a cut efficiency of 35% after preselection cuts, such that the corresponding significance is $S = 0.95$, for $\mathcal{L} = 100$ fb$^{-1}$. This value has to be compared with the significance $S = 0.45$ for the triple product asymmetry $A(p_{\ell_1}^z, p_{\ell_2}^z, p_{\ell_3}^z) = -4.7\%$, with the lepton preselection cuts only. Although our cuts and momenta approximations can more than double the significance, it will be difficult to measure the asymmetries at the LHC due to the low production cross section. We will comment on this issue in the next Section.

5 Discovery reach at the LHC

The trilepton signal for CP-conserving neutralino-chargino pair production at the LHC has been studied by the ATLAS [23] and CMS [24] collaborations. They have focused on the benchmark point SU2, given by $m_0 = 3.55$ TeV, $m_{1/2} = 350$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu > 0$, which lies within the focus point region of the mSUGRA parameter space. Thus it is characterized by heavy squarks and sleptons of order 3 TeV, but relatively light charginos and neutralinos of order 100 GeV to 300 GeV. While the heavy squarks and sleptons only provide rather small production cross sections, the light gauginos provide good SUSY discovery potential in multi-lepton events.

In general, the most important SM backgrounds to the trilepton signal are from $t\bar{t}$, $bZ$, and $WZ$ production [23][24]. The fully leptonic $ZW$ events can be reduced by rejecting lepton pairs (with opposite signs and same flavor) which have an invariant mass $\approx \pm 10$ GeV around $m_Z$. The leptonic decays of $tt$, $Zb$, can generate a third, but rather soft, lepton from the leptonic $b$ decay. Stringent isolation cuts on the lepton tracks can reduce those backgrounds, and also soft leptons from bremsstrahlung, other hadron decays, and photon conversions, which otherwise have large contributions. The dominant source of low momentum trileptons are heavy flavor SM processes, like $pp \rightarrow qZ(\gamma^*)$, $q\bar{q}Z(\gamma^*)$, for $q = b, c$, as pointed out in Ref [36], and a cut on $E_T^{\text{miss}}$ around 30 GeV is proposed.

For the CP-conserving SU2 scenario, the ATLAS collaboration [23] has shown that with appropriate cuts, the SUSY trilepton signal gets reduced from about 33 fb to 3 fb, with a remaining SM background of 21 fb [23]. Thus ATLAS expects a 5$\sigma$ discovery over background in the CP-conserving SU2 scenario for a luminosity of 80 fb$^{-1}$. If we assume a similar loss in the
signal events, due to the cuts, of about an order of magnitude, we can estimate the discovery potential for CP violation in the trilepton signal. For our scenario for $\tilde{\chi}_0^0 \tilde{\chi}_\pm^+$ production as given in Table 1, we have shown that the asymmetry $\mathcal{A}[p_u^{\text{appr}}, p_d^{\text{appr}}, p_{\ell_+}, p_{\ell_-}]$ can reach up to 17%. The necessary cuts for the asymmetry will be in some sense equivalent to those which isolate the signal. We would then need at least a luminosity of $L = n^2 / (A^2 \sigma \text{BR}) = n^2 \times 200 \text{ fb}^{-1}$ for $n$ standard deviations. Clearly the exact answer can only be given after performing a detailed experimental study. However the trilepton signal is probably not best suited to study SUSY CP violation at the LHC, and thus we also defer a detailed Monte Carlo study taking into account the above mentioned backgrounds and cuts.

6 Summary and conclusions

In the complex MSSM, we have analyzed the potential to observe CP violation from the gaugino and higgsino phases $\phi_1$ and $\phi_\mu$ in neutralino-chargino pair production at the LHC, $pp \to \tilde{\chi}_0^0 \tilde{\chi}_\pm^\pm$. Their subsequent leptonic two-body decays give rise to a trilepton signal, with low QCD and SM backgrounds. The trilepton signal is well known as a clean SUSY discovery channel at the Tevatron, and also at the LHC.

In order to find the optimal CP asymmetries in the trilepton signal at the LHC, we have calculated the amplitude squared in the spin density matrix formalism. From the explicit formulas we have identified the CP-sensitive contributions, which already appear at tree level in the neutralino and chargino spin and spin-spin correlations. We have then defined optimal CP asymmetries, which base on epsilon products that exactly match the kinematic dependence of the CP-violating spin correlations. After performing a systematic scan in the MSSM parameter space, we have found that these asymmetries can reach up to 20% for scenarios with light squarks, neutralinos and charginos, in particular for parameter points near level crossings where the neutralinos strongly mix. Only the cancellations between the different exchange contributions from squark $\tilde{u}_L, \tilde{d}_L$ and $W$ boson exchange in the production, prevent the asymmetries from attaining larger values.

These optimal asymmetries would however require a reconstruction of the initial quark and intermediate neutralino/chargino momenta, which is only possible to a certain degree at the LHC. We thus have discussed different replacement and approximation strategies, which are best realized by the analysis of triple products of the outgoing three lepton momenta, and by efficient methods to estimate the initial partonic systems. For example, we have shown that, by using appropriate cuts on the final lepton momenta, the direction of the initial quark momenta can be guessed right in over 90% of the events, such that the resulting asymmetries can still reach values up to 17%. We have checked that all our results are in agreement with the public code MadGraph 5.

These washout effects, compared to the optimal asymmetries, are caused by the strong partonic boosts at the LHC, and by cancellations between the different contributions from the spin and spin-spin correlations. Due to these effects, we conclude that SUSY CP violation in the trilepton signal will be difficult to observe, in particular when squarks and gauginos are much heavier than 400 GeV. We have estimated that one would need at least a luminosity of $L = n^2 \times 200 \text{ fb}^{-1}$, for $n$ standard deviations to observe CP-violating effects at the LHC.
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Appendix

A Chargino and neutralino mixings

The masses and mixing angles of the charginos follow from their mass matrix [1]

\[
M_{\tilde{\chi}^\pm} = \begin{pmatrix}
M_2 & m_W \sqrt{2} \sin \beta \\
m_W \sqrt{2} \cos \beta & \mu
\end{pmatrix},
\]

(A.1)

with the \(SU(2)\) gaugino mass parameter \(M_2\), the higgsino mass parameter \(\mu\), the ratio \(\tan \beta = v_2/v_1\) of the vacuum expectation values of the two neutral Higgs fields, and the mass \(m_W\) of the \(W\) boson. The chargino mass matrix can be diagonalized by two complex unitary \(2 \times 2\) matrices [1],

\[
U^* M_{\tilde{\chi}^\pm} V^{-1} = \text{diag}(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}),
\]

(A.2)

such that the chargino masses satisfy \(m_{\tilde{\chi}_j^\pm} \geq 0\).

The complex symmetric mass matrix of the neutralinos in the photino, zino, higgsino basis \((\tilde{\gamma}, \tilde{Z}, \tilde{H}_a^0, \tilde{H}_b^0)\), is given by [52]

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_2 s_w^2 + M_1 c_w^2 & (M_2 - M_1) s_w c_w & 0 & 0 \\
(M_2 - M_1) s_w c_w & M_2 c_w^2 + M_1 s_w^2 & m_Z & 0 \\
0 & m_Z & \mu s_{2\beta} & -\mu c_{2\beta} \\
0 & 0 & -\mu c_{2\beta} & -\mu s_{2\beta}
\end{pmatrix}.
\]

(A.3)

Here \(s_w = \sin \theta_w, c_w = \cos \theta_w, \theta_w\) being the weak mixing angle, \(s_{2\beta} = \sin(2\beta), c_{2\beta} = \cos(2\beta), M_1\) is the \(U(1)\) gaugino mass parameter, and \(m_Z\) is the mass of the \(Z\) boson. We diagonalize the neutralino mass matrix by a complex, unitary \(4 \times 4\) matrix [1]

\[
N^* M_{\tilde{\chi}^0} N^\dagger = \text{diag}(m_{\tilde{\chi}_i^0}, \ldots, m_{\tilde{\chi}_4^0}),
\]

(A.4)

such that the neutralino masses satisfy \(m_{\tilde{\chi}_i^0} \geq 0\). The \(SU(2)\) gaugino mass parameter \(M_2\) has been chosen real and positive by absorbing its possible phase via field redefinitions. We then parametrize CP-violation in the neutralino and chargino sector by the phases of the complex parameters \(M_1 = |M_1| e^{i\phi_1}\) and \(\mu = |\mu| e^{i\delta_\mu}\).

\footnote{The mass can be made real and positive by a phase transformation of the \(SU(2)\) gaugino fields. We then also need to redefine the fermion and/or sfermion fields in order to keep the \(SU(2)\) gaugino fermion sfermion couplings real and positive. Keeping the fermion sfermion couplings to higgsinos as well as \(U(1)\gamma\) and \(SU(3)\) gauginos real then requires phase transformations of these fields as well. This illustrates that only the relative phases between \(M_2\) and the other gaugino masses, and between \(M_2\) and \(\mu\), are physical.}
B  Lagrangians and couplings

The interaction Lagrangians for $W$ boson exchange in the production are

$$\mathcal{L}_{W-ud} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{d} \gamma^\mu P_L u + \text{h.c.},$$  \hfill (B.5)

$$\mathcal{L}_{W-\tilde{\chi}^0+} = g W_\mu^- \tilde{\chi}_i^0 \gamma^\mu [O^{L}_{ij} P_L + O^{R}_{ij} P_R] \tilde{\chi}_j^+ + \text{h.c.},$$  \hfill (B.6)

with the weak coupling constant $g = e/\sin\theta_w$, $e > 0$, $P_{L,R} = (1 \mp \gamma_5)/2$. In the photino, zino, higgsino basis ($\tilde{\gamma}$, $\tilde{Z}$, $\tilde{H}_u^0$, $\tilde{H}_d^0$), the couplings are

$$O^L_{ij} = -1/\sqrt{2} \left( \cos \beta N_{i4} - \sin \beta N_{i3} \right) V_{j2}^* + \left( \sin \theta_w N_{i1} + \cos \theta_w N_{i2} \right) V_{j1}^*, \hfill (B.7)$$

$$O^R_{ij} = +1/\sqrt{2} \left( \sin \beta N_{i4} + \cos \beta N_{i3} \right) U_{j2} + \left( \sin \theta_w N_{i1} + \cos \theta_w N_{i2} \right) U_{j1}. \hfill (B.8)$$

The interaction Lagrangians for $u$-squark exchange are

$$\mathcal{L}_{u\bar{u}\tilde{\chi}^0} = g\bar{u} f^L_{ui} P_R \tilde{\chi}_i^0 \tilde{u}_L + \text{h.c.},$$  \hfill (B.9)

$$\mathcal{L}_{d\bar{u}\tilde{\chi}^+} = g\bar{d} t^L_{ij} P_R \tilde{\chi}_j^+ \tilde{u}_L + \text{h.c.}, \hfill (B.10)$$

and those for $d$-squark exchange are

$$\mathcal{L}_{d\bar{d}\tilde{\chi}^0} = g\bar{d} f^L_{di} P_R \tilde{\chi}_i^0 \tilde{d}_L + \text{h.c.},$$

$$\mathcal{L}_{u\bar{d}\tilde{\chi}^+} = g\bar{u} t^L_{ij} P_R \tilde{\chi}_j^+ \tilde{d}_L + \text{h.c.}. \hfill (B.12)$$

In the photino, zino, higgsino basis, the couplings are

$$f^L_{ui} = -\sqrt{2} \left[ \frac{1}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) N_{i2} + \frac{2}{3} \sin \theta_w N_{i1} \right],$$  \hfill (B.13)

$$t^L_{ij} = -V_{j1}, \hfill (B.14)$$

and

$$f^L_{di} = -\sqrt{2} \left[ \frac{1}{\cos \theta_w} \left( \frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) N_{i2} - \frac{1}{3} \sin \theta_w N_{i1} \right],$$  \hfill (B.15)

$$t^L_{ij} = -U_{j1}. \hfill (B.16)$$

We neglect mixings in the quark and squark generations. The interaction Lagrangians for neutralino decay $\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_{R,L}^\pm \ell^\mp$, followed by $\tilde{\ell}_{R,L}^\pm \rightarrow \tilde{\chi}_1^0 \ell^\pm$ with $\ell = e, \mu$ are

$$\mathcal{L}_{i\ell\tilde{\chi}^0} = g\tilde{\ell} f^L_{i\ell} P_R \tilde{\chi}_i^0 \tilde{\ell}_L + g\ell f^R_{i\ell} P_L \tilde{\chi}_i^0 \ell_R + \text{h.c.},$$  \hfill (B.17)

with the couplings

$$f^L_{i\ell} = \sqrt{2} \left[ \frac{1}{\cos \theta_w} \left( \frac{1}{2} - \sin^2 \theta_w \right) N_{i2} + \sin \theta_w N_{i1} \right],$$  \hfill (B.18)

$$f^R_{i\ell} = \sqrt{2} \sin \theta_w (\tan \theta_w N_{i2}^* - N_{i1}^*). \hfill (B.19)$$
The interaction Lagrangians for chargino decay $\tilde{\chi}_j^\pm \rightarrow \ell^\mp \tilde{\nu}_\ell$, followed by $\tilde{\nu}_\ell \rightarrow \chi_1^0 \nu_\ell$ are [52]

\[ L_{\tilde{\nu}_\ell \tilde{\chi}_j^+} = -gU_{j1}^* \tilde{\nu}_L \tilde{\ell}_L - gV_{j1}^* \tilde{\chi}_j^+ C P_L \ell \tilde{\nu}_L^* + \text{h.c.}, \quad \ell = e, \mu, \]  
\[ L_{\tilde{\nu}_\ell \tilde{\chi}_j^0} = g f^L_{\nu k} P_R \tilde{\chi}_k^0 \tilde{\nu}_L + \text{h.c.}, \]  
with

\[ f^L_{\nu k} = -\frac{\sqrt{2}}{2} \frac{1}{\cos \theta_w} N_{k2}. \]  

For completeness, we also give the couplings in the bino, wino, higgsino $\tilde{H}_{1,2}^0$ basis [1]. In that basis the neutralino mass matrix is diagonalized by the complex, unitary matrix $Z$, similar to Eq. (A.1). The quark-squark gaugino couplings are

\[ f^L_{ui} = \frac{1}{\sqrt{2}} \left( \frac{1}{3} t_w Z_{i1} + Z_{i2} \right), \quad f^L_{di} = \frac{1}{\sqrt{2}} \left( \frac{1}{3} t_w Z_{i1} - Z_{i2} \right), \]  

with $t_w = \tan \theta_w$, and $t^L_{ai}$ and $t^L_{di}$ as given in Eqs. (B.14), (B.15). With these definitions the CP-sensitive imaginary parts of product of couplings become

\[ \text{Im}\{O_{ij}^L O_{ij}^R\} = \text{Im}\left\{ -\frac{1}{2} Z_{i4} Z_{i3} U_{j2}^* V_{j2} - \frac{1}{\sqrt{2}} Z_{i4} Z_{i2} U_{j1}^* V_{j2}^* \right. \]
\[ + \frac{1}{\sqrt{2}} Z_{i2} Z_{i3} U_{j2}^* V_{j1}^* + Z_{i2}^2 U_{j1}^* V_{j1}^* \}, \]  
\[ \text{Im}\{f^L_{ai} t^L_{aj} O_{ij}^R\} = \frac{1}{\sqrt{2}} \text{Im}\left\{ V_{j1}^* \left( \frac{t_w}{3} Z_{i1} Z_{i3} U_{j2}^* + \frac{t_w}{3} Z_{i1} Z_{i2} U_{j1}^* \right) \right. \]
\[ + \frac{1}{\sqrt{2}} Z_{i2} Z_{i3} U_{j2}^* + Z_{i2}^2 U_{j1}^* \}, \]  
\[ \text{Im}\{f^L_{di} t^L_{dj} O_{ij}^R\} = \frac{1}{\sqrt{2}} \text{Im}\left\{ U_{j1}^* \left( \frac{t_w}{3} Z_{i1} Z_{i4} V_{j2}^* - \frac{t_w}{3} Z_{i1} Z_{i2} V_{j1}^* \right) \right. \]
\[ - \frac{1}{\sqrt{2}} Z_{i2} Z_{i4} V_{j2}^* + Z_{i2}^2 V_{j1}^* \}, \]  
\[ \text{Im}\{f^L_{ai} f^L_{di} t^L_{aj} t^L_{dj}\} = \frac{1}{2} \text{Im}\left\{ U_{j1}^* V_{j1}^* \left( Z_{i2}^2 - \frac{t_w^2}{3} Z_{i1}^2 \right) \right\}. \]  

C Kinematics and phase space

In the center-of-mass (cms) system of the incoming quarks, we parametrize the momenta with the scattering angle $\hat{\theta} \chi (p_u, p_{\chi_j^0})$, and the azimuth $\hat{\phi}$ to be chosen zero,

\[ \hat{p}_u^\mu = \hat{E}_u (1, 0, 0, 1), \quad \hat{p}_d^\mu = \hat{E}_d (1, 0, 0, -1), \]
\[ \hat{p}_{\chi_j^0}^\mu = (\hat{E}_i, -\hat{q} \sin \hat{\theta}, 0, \hat{q} \cos \hat{\theta}), \quad \hat{p}_{\chi_j^+}^\mu = (\hat{E}_j, \hat{q} \sin \hat{\theta}, 0, -\hat{q} \cos \hat{\theta}), \]  

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with the cms energy of the partons \( \hat{E}_b = \sqrt{s}/2 \), and
\[
\hat{E}_i = \frac{s + m_i^2 - m_j^2}{2\sqrt{\hat{s}}}, \quad \hat{E}_j = \frac{s + m_j^2 - m_i^2}{2\sqrt{\hat{s}}}, \quad \hat{q} = \frac{\lambda^j_4(s, m_i^2, m_j^2)}{2\sqrt{\hat{s}}},
\]
with \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \). We label variables in the cms system by a hat in our notation. The energies and the \( z \)-components of the momenta of the outgoing neutralino \( \tilde{\chi}_1^0 \) and chargino \( \tilde{\chi}_j^\pm \) in the laboratory (lab) frame
\[
E = \gamma(\hat{E} + \beta \hat{p}_z), \quad p_z = \gamma(\hat{p}_z + \beta \hat{E}),
\]
are obtained by a Lorentz boost with \( \gamma \)
\[
\beta = \frac{x_1 - x_2}{x_1 + x_2}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{x_1 + x_2}{2\sqrt{x_1x_2}}.
\]
The partons have energy fractions \( E_a = x_1 E_p, E_d = x_2 E_p \), of the proton energy \( E_p = \sqrt{s}/2 \) in the laboratory (lab) frame, such that \( \hat{s} = x_1 x_2 \hat{s} \), and
\[
p_\mu^a = E_a(1, 0, 0, 1), \quad p_\mu^d = E_d(1, 0, 0, -1).
\]
In the laboratory frame, the momenta for the subsequent decays of the neutralino \( \tilde{\chi}_1^0 \rightarrow \tilde{\ell}_R \tilde{\ell}_1^\pm \); \( \tilde{\ell}_R \rightarrow \tilde{\chi}_1^0 \tilde{\ell}_2^\pm \) and of the chargino \( \tilde{\chi}_J^J \rightarrow \tilde{\nu}_J \ell_3^\pm \); \( \ell_\ell \rightarrow \tilde{\chi}_1^0 \nu_\ell \), are given by
\[
\begin{align*}
\rho_\ell_1^J &= E_\rho_1 (1, \sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, \cos \theta_1), \\
\rho_\ell_2^J &= E_\rho_2 (1, \sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2, \cos \theta_2), \\
\rho_\ell_3^J &= E_\rho_3 (1, \sin \theta_3 \cos \phi_3, \sin \theta_3 \sin \phi_3, \cos \theta_3),
\end{align*}
\]
with the energies
\[
\begin{align*}
E_\ell_1 &= \frac{m_1^2 - m_\rho^2}{2(E_\rho_1 - |p_\rho_1| \cos \theta_\rho_1)}, & E_\ell_2 &= \frac{m_2^2 - m_\rho^2}{2(E_\rho_1 - |p_\rho_1| \cos \theta_\rho_2)}, \\
E_\ell_3 &= \frac{m_3^2 - m_\rho^2}{2(E_\rho_1 - |p_\rho_1| \cos \theta_\rho_3)}.
\end{align*}
\]
and the decay angles, \( \theta_\rho_1, \chi(p_\rho_1, p_\rho_{\ell_1}), \theta_\rho_2, \chi(p_\rho_2, p_\rho_{\ell_2}), \) and \( \theta_\rho_3, \chi(p_\rho_3, p_\rho_{\ell_3}) \).

For the description of the polarization of the neutralino \( \tilde{\chi}_1^0 \) and chargino \( \tilde{\chi}_J^J \) we choose three spin vectors in the laboratory frame
\[
\begin{align*}
s_1^{1, \mu} &= \left( 0, \frac{s_1^2 \times s_1^3}{|s_1^2 \times s_1^3|}, \right), & s_2^{1, \mu} &= \left( 0, \frac{p_a \times p_i}{|p_a \times p_i|} \right), & s_3^{1, \mu} &= \frac{1}{m_i} \left( |p_i|, \frac{E_i}{|p_i|} p_i \right), \\
s_1^{2, \mu} &= \left( 0, \frac{s_2^2 \times s_2^3}{|s_2^2 \times s_2^3|}, \right), & s_2^{2, \mu} &= \left( 0, \frac{p_a \times p_j}{|p_a \times p_j|} \right), & s_3^{2, \mu} &= \frac{1}{m_j} \left( |p_j|, \frac{E_j}{|p_j|} p_j \right).
\end{align*}
\]
They form an orthonormal set
\[
\begin{align*}
s_i^a \cdot s_i^b &= -\delta^{ab}, & s_i^a \cdot e_i &= 0, & s_i^a \cdot s_j^b &= -\delta^{ab}, & s_i^a \cdot e_j &= 0.
\end{align*}
\]
with the unit momentum vectors \( e^\mu = p^\mu / m \).

The Lorentz invariant phase-space element for neutralino-chargino production and their subsequent two-body decay chain, see Eqs. (2)-(4), can be decomposed into two-body phase-space elements [16]:

\[
d\text{Lips}(s; p_{\ell_1}, p_{\ell_2}, p_{\ell_3}) = \frac{1}{(2\pi)^3} d\text{Lips}(s; p_{i}, p_{j}) \, ds_i \, d\text{Lips}(s; p_{\ell_1}, p_{\ell_2}) \times ds_j \, d\text{Lips}(s; p_{\ell_3}, p_{\ell_4}).
\]

The several parts of the phase space elements are given by

\[
d\text{Lips}(s; p_i, p_j) = \frac{\hat{q}}{8\pi \sqrt{s}} \sin \hat{\theta} \, d\hat{\theta},
\]

\[
d\text{Lips}(s_i; p_{\ell_1}, p_{\ell_2}) = \frac{1}{2(2\pi)^2} \frac{|p_{\ell_1}|^2}{m_{\ell_1}^2 - m_{\ell_2}^2} \, d\Omega_1,
\]

\[
d\text{Lips}(s_{\ell_2}; p_{\ell_1}, p_{\ell_3}) = \frac{1}{2(2\pi)^2} \frac{|p_{\ell_3}|^2}{m_{\ell_3}^2 - m_{\ell_1}^2} \, d\Omega_2,
\]

\[
d\text{Lips}(s_{\ell_3}; p_{\ell_3}, p_{\ell_4}) = \frac{1}{2(2\pi)^2} \frac{|p_{\ell_4}|^2}{m_{\ell_4}^2 - m_{p_{\ell_3}}^2} \, d\Omega_3,
\]

with \( s = x_1 x_2 s_k = p_k^2 \), and \( d\Omega_k = \sin \theta_k \, d\theta_k \, d\phi_k \), \( k = 1, 2, 3 \).

## D Density matrix formalism

For the calculation of the amplitude squared of neutralino-chargino production, Eq. (2), and their two-body decay chains, Eqs. (3)-(4), we use the spin-density matrix formalism of Ref. [25]. The amplitudes squared in the helicity formalism are given, e.g. in Ref. [38].

### D.1 Production matrices

For the production of the neutralino-chargino pair,

\[
u(p_u) + \bar{d}(p_d) \rightarrow \tilde{\chi}_{i}^{0}(p_i, \lambda_i) + \tilde{\chi}_{j}^{+}(p_j, \lambda_j),
\]

with momentum \( p \) and helicity \( \lambda \), the un-normalized spin-density matrix is

\[
\rho_P(\tilde{\chi}_{i}^{0} \tilde{\chi}_{j}^{+})_{\lambda_{i} \lambda_{j} \lambda_{j}} = T^{\lambda_{i} \lambda_{j}} (T^{\lambda_{i} \lambda_{j}})^{*}.
\]

The helicity amplitudes are

\[
T^{\lambda_{i} \lambda_{j}}(s, W) = \frac{g^2}{\sqrt{2}} \Delta_\lambda(s, W) \left[ \bar{v}(p_d) \gamma^\mu P_L u(p_u) \right] \left[ \bar{u}(p_j, \lambda_j) \gamma_\mu (O_{ij}^{L*} P_L + O_{ij}^{R*} P_R) v(p_i, \lambda_i) \right],
\]

\[
T^{\lambda_{i} \lambda_{j}}(t, \bar{u}) = g^2 \Delta_\lambda \left[ \bar{u}(p_u, \lambda_i) f_{u_{\bar{u}}}^{L*} P_L u(p_u) \right] \left[ \bar{v}(p_d) f_{\bar{d}_{\bar{d}}}^{L*} P_R v(p_j, \lambda_j) \right],
\]

\[
T^{\lambda_{i} \lambda_{j}}(u, \bar{d}) = -g^2 \Delta_\lambda \left( \bar{d}(p_u) f_{d_{\bar{d}}}^{L*} P_R v(p_i, \lambda_i) \right) \left[ \bar{u}(p_d, \lambda_j) f_{\bar{u}_{\bar{u}}}^{L*} P_L u(p_u) \right].
\]
with the propagators
\[
\Delta_s(W) = \frac{i}{s - m_W^2}, \quad \Delta_t(\tilde{u}_L) = \frac{i}{t - m_{\tilde{u}_L}^2}, \quad \Delta_u(\tilde{d}_L) = \frac{i}{u - m_{\tilde{d}_L}^2},
\]
and \(s = (p_u + p_d)^2, t = (p_u - p_t)^2, u = (p_u - p_j)^2\). The Feynman diagrams are shown in Fig. 1.

Having introduced a set of spin four-vectors \(s_t^a\) with \(a = 1, 2, 3\), for the neutralino \(\tilde{\chi}_i^0\), see Eq. (C.39), and \(s_j^b\) with \(b = 1, 2, 3\), for the chargino \(\tilde{\chi}_j^+\), see Eq. (C.40), the production matrix (D.48) can be expanded in terms of the Pauli matrices
\[
\rho_P(\tilde{\chi}_i^0, \tilde{\chi}_j^+) = \delta_{\lambda_i\lambda_j} \delta_{\lambda_i\lambda_j} P + \delta_{\lambda_i\lambda_j} \sigma_{\lambda_i\lambda_j}^{a} \Sigma_p^a + \delta_{\lambda_i\lambda_j} \sigma_{\lambda_i\lambda_j}^{b} \Sigma_p^b + \sigma_{\lambda_i\lambda_j}^{a} \sigma_{\lambda_i\lambda_j}^{b} \Sigma_p^{ab},
\]
using the Bouchiat-Michel formulas for massive spin 1/2 particles [51]
\[
u(p, \lambda) \tilde{u}(p, \lambda) = \frac{1}{2} \left[ \delta_{\lambda\lambda} + \gamma_5 \, \not{\sigma} a \sigma_{\lambda\lambda} \right](\not{p} + m), \quad (D.54)
\]
\[
u(p, \lambda) \tilde{v}(p, \lambda) = \frac{1}{2} \left[ \delta_{\lambda\lambda} + \gamma_5 \, \not{\sigma} a \sigma_{\lambda\lambda} \right](\not{p} - m). \quad (D.55)
\]

The expansion coefficient \(P\) of the production density matrix (D.53) is independent of the chargino and neutralino polarizations. It can be composed into contributions from the different production channels
\[
P = P(WW) + P(\tilde{u}_L\tilde{u}_L) + P(\tilde{d}_L\tilde{d}_L) + P(W\tilde{u}_L) + P(W\tilde{d}_L) + P(\tilde{u}_L\tilde{d}_L), \quad (D.56)
\]
with
\[
P(WW) = \frac{g^4}{2} |\Delta_s(W)|^2 \left[ |O_{ij}^L|^2 (p_u \cdot p_i)(p_d \cdot p_j) + |O_{ij}^R|^2 (p_u \cdot p_j)(p_d \cdot p_i) \right] + \text{Re} \{O_{ij}^LO_{ij}^R \} m_i m_j(p_u \cdot p_d), \quad (D.57)
\]
\[
P(\tilde{u}_L\tilde{u}_L) = \frac{g^4}{4} |\Delta_t(\tilde{u}_L)|^2 \left| f_{ai}^L \right|^2 \left| f_{ai}^L \right|^2 (p_u \cdot p_i)(p_d \cdot p_j), \quad (D.58)
\]
\[
P(\tilde{d}_L\tilde{d}_L) = \frac{g^4}{4} |\Delta_u(\tilde{d}_L)|^2 \left| f_{di}^L \right|^2 \left| f_{dj}^L \right|^2 (p_u \cdot p_j)(p_d \cdot p_i), \quad (D.59)
\]
\[
P(W\tilde{u}_L) = \frac{\sqrt{2}}{4} g^4 |\Delta_s(\tilde{u}_L)| \Delta_t^*(\tilde{u}_L) \left[ 2 \text{Re} \{f_{ai}^L f_{ai}^L O_{ij}^L \} (p_u \cdot p_i)(p_d \cdot p_j) \right] + \text{Re} \{f_{ai}^L f_{ai}^L O_{ij}^R \} m_i m_j(p_u \cdot p_d), \quad (D.60)
\]
\[
P(W\tilde{d}_L) = -\frac{\sqrt{2}}{4} g^4 |\Delta_s(\tilde{d}_L)| \Delta_u^*(\tilde{d}_L) \left[ 2 \text{Re} \{f_{ai}^L f_{ai}^L O_{ij}^L \} (p_u \cdot p_j)(p_d \cdot p_i) \right] + \text{Re} \{f_{ai}^L f_{ai}^L O_{ij}^R \} m_i m_j(p_u \cdot p_d), \quad (D.61)
\]
\[
P(\tilde{u}_L\tilde{d}_L) = -\frac{g^4}{4} \Delta_s(\tilde{u}_L) \Delta_u^*(\tilde{d}_L) \text{Re} \{f_{ai}^L f_{ai}^L f_{ai}^L f_{ai}^L \} m_i m_j(p_u \cdot p_d), \quad (D.62)
\]
with the couplings as defined in Appendix B. The terms for \(P\) are the same for the charge conjugated process, \(\bar{u}d \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^-\).
The coefficients \( \Sigma^a_p \), which describe the polarization of the neutralino \( \tilde{\chi}_i^0 \), decompose into

\[
\Sigma^a_p = \Sigma^a_p(WW) + \Sigma^a_p(\tilde{u}_L\tilde{u}_L) + \Sigma^a_p(\tilde{d}_L\tilde{d}_L) + \\
\Sigma^a_p(W\tilde{u}_L) + \Sigma^a_p(W\tilde{d}_L) + \Sigma^a_p(\tilde{u}_L\tilde{d}_L),
\]

with

\[
\Sigma^a_p(WW) = \frac{g^4}{2} |\Delta_s(W)|^2 \left\{ |O_{ij}^L|^2 m_i (p_d \cdot p_j) (p_u \cdot s_i^a) - |O_{ij}^R|^2 m_i (p_u \cdot p_j) (p_d \cdot s_i^a) + \text{Re}\{O_{ij}^L O_{ij}^R\} m_j [(p_d \cdot p_i) (p_u \cdot s_i^a) - (p_u \cdot p_i) (p_d \cdot s_i^a)] - \text{Im}\{O_{ij}^L O_{ij}^R\} m_j [p_u, p_d, p_i, s_i^a] \right\},
\]

\[
\Sigma^a_p(\tilde{u}_L\tilde{u}_L) = \frac{g^4}{4} |\Delta_s(\tilde{u}_L)|^2 |f_{ui}^L|^2 |f_{lj}^L|^2 m_i (p_d \cdot p_j) (p_u \cdot s_i^a),
\]

\[
\Sigma^a_p(\tilde{d}_L\tilde{d}_L) = -\frac{g^4}{4} |\Delta_s(\tilde{d}_L)|^2 |f_{di}^L|^2 |f_{lj}^L|^2 m_i (p_d \cdot p_j) (p_u \cdot s_i^a),
\]

\[
\Sigma^a_p(W\tilde{u}_L) = \frac{\sqrt{2} g^4}{4} |\Delta_s(W)| \Delta_{\tilde{u}_L} (\tilde{u}_L) \left\{ 2 \text{Re}\{f_{ui}^L f_{lj}^L O_{ij}^{L*}\} m_i (p_d \cdot p_j) (p_u \cdot s_i^a) + \text{Re}\{f_{ui}^L f_{lj}^L O_{ij}^{R*}\} m_j [(p_d \cdot p_i) (p_u \cdot s_i^a) - (p_u \cdot p_i) (p_d \cdot s_i^a)] - \text{Im}\{f_{ui}^L f_{lj}^L O_{ij}^{R*}\} m_j [p_u, p_d, p_i, s_i^a] \right\},
\]

\[
\Sigma^a_p(W\tilde{d}_L) = \frac{\sqrt{2} g^4}{4} |\Delta_s(W)| \Delta_{\tilde{d}_L} (\tilde{d}_L) \left\{ 2 \text{Re}\{f_{di}^L f_{lj}^L O_{ij}^{L*}\} m_i (p_d \cdot p_j) (p_u \cdot s_i^a) + \text{Re}\{f_{di}^L f_{lj}^L O_{ij}^{R*}\} m_j [(p_d \cdot p_i) (p_u \cdot s_i^a) - (p_u \cdot p_i) (p_d \cdot s_i^a)] - \text{Im}\{f_{di}^L f_{lj}^L O_{ij}^{R*}\} m_j [p_u, p_d, p_i, s_i^a] \right\},
\]

\[
\Sigma^a_p(\tilde{u}_L\tilde{d}_L) = \frac{g^4}{4} |\Delta_s(\tilde{u}_L)| \Delta_{\tilde{d}_L} (\tilde{d}_L) \left\{ - \text{Im}\{f_{ui}^L f_{di}^L f_{lj}^L f_{lj}^L O_{ij}^{L*}\} m_j [(p_u \cdot p_i) (p_d \cdot s_i^a) - (p_d \cdot p_i) (p_u \cdot s_i^a)] + \text{Re}\{f_{ui}^L f_{di}^L f_{lj}^L f_{lj}^L O_{ij}^{R*}\} m_j [(p_u \cdot p_i) (p_d \cdot s_i^a) - (p_d \cdot p_i) (p_u \cdot s_i^a)] \right\},
\]

with the short-hand notation

\[
[p_a, p_b, p_c, p_d] = \epsilon_{\mu \nu \rho \sigma} p_\mu^a p_\nu^b p_\rho^c p_\sigma^d, \quad \text{and} \quad \epsilon_{0123} = 1.
\]

With our choice of the spin vectors \( s_i^a \) (39) for neutralino \( \tilde{\chi}_i^0 \), \( \Sigma_p^{a=3}/P \) is the longitudinal polarization of neutralino, \( \Sigma_p^{a=1}/P \) is the transverse polarization in the production plane and \( \Sigma_p^{a=2}/P \) is the polarization normal to the production plane. Only if there are non-vanishing CP phases \( \phi_1 \) and/or \( \phi_2 \) in the chargino and neutralino sector, the polarization \( \Sigma_p^{a=2}/P \) normal to the production plane is non-zero. Thus it is a probe for CP violation in the production of a neutralino-chargino pair. To obtain \( \Sigma_p^{a=2} \) for the charge conjugated process, \( \tilde{u} \tilde{d} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^- \), one has to change the signs of Eqs. (D.64), (D.69).
The coefficients $\Sigma^b_P$, which describe the polarization of the chargino $\tilde{\chi}^+_j$, decompose into

$$
\Sigma^b_P = \Sigma^b_P(WW) + \Sigma^b_P(\tilde{u}_L\tilde{u}_L) + \Sigma^b_P(\tilde{d}_L\tilde{d}_L) + \\
\Sigma^b_P(W\tilde{u}_L) + \Sigma^b_P(W\tilde{d}_L) + \Sigma^b_P(\tilde{u}_L\tilde{d}_L),
$$

with

$$
\Sigma^b_P(WW) = \frac{g^4}{2}|\Delta_s(W)|^2 \left\{ |O^R_{ij}|^2 m_j (p_d \cdot p_i)(p_u \cdot s^b_j) - |O^L_{ij}|^2 m_j (p_u \cdot p_i)(p_d \cdot s^b_j) \right. \\
+ \text{Re}\{O^L_{ij}O^{R*}_{ij}\} m_i \left[ (p_d \cdot p_j)(p_u \cdot s^b_j) - (p_u \cdot p_j)(p_d \cdot s^b_j) \right] \\
+ \text{Im}\{O^L_{ij}O^{R*}_{ij}\} m_i [p_u, p_d, p_j, s^b_j],
$$

(D.72)

$$
\Sigma^b_P(\tilde{u}_L\tilde{u}_L) = -\frac{g^4}{4}|\Delta_s(\tilde{u}_L)|^2 |f^{L*}_{u\tilde{u}_j}|^2 |t^L_{u\tilde{u}_j}|^2 m_j (p_u \cdot p_i)(p_d \cdot s^b_j),
$$

(D.73)

$$
\Sigma^b_P(\tilde{d}_L\tilde{d}_L) = \frac{g^4}{4}|\Delta_s(\tilde{d}_L)|^2 |f^{L*}_{d\tilde{d}_j}|^2 |t^L_{d\tilde{d}_j}|^2 m_j (p_d \cdot p_i)(p_u \cdot s^b_j),
$$

(D.74)

$$
\Sigma^b_P(W\tilde{u}_L) = -\sqrt{2} g^4 |\Delta_s(W)| \Delta_s^*(\tilde{u}_L) \left\{ 2\text{Re}\{f^{L*}_{u\tilde{u}_j}O^{L*}_{ij}\} m_j (p_u \cdot p_i)(p_d \cdot s^b_j) \\
+ \text{Re}\{f^{L*}_{u\tilde{u}_j}O^{R*}_{ij}\} m_i \left[ (p_u \cdot p_j)(p_d \cdot s^b_j) - (p_d \cdot p_j)(p_u \cdot s^b_j) \right] \\
- \text{Im}\{f^{L*}_{u\tilde{u}_j}O^{R*}_{ij}\} m_i [p_u, p_d, p_j, s^b_j] \right\},
$$

(D.75)

$$
\Sigma^b_P(W\tilde{d}_L) = -\sqrt{2} g^4 |\Delta_s(W)| \Delta_s^*(\tilde{d}_L) \left\{ 2\text{Re}\{f^{L*}_{d\tilde{d}_j}O^{L*}_{ij}\} m_j (p_d \cdot p_i)(p_u \cdot s^b_j) \\
+ \text{Re}\{f^{L*}_{d\tilde{d}_j}O^{R*}_{ij}\} m_i \left[ (p_d \cdot p_j)(p_u \cdot s^b_j) - (p_u \cdot p_j)(p_d \cdot s^b_j) \right] \\
- \text{Im}\{f^{L*}_{d\tilde{d}_j}O^{R*}_{ij}\} m_i [p_u, p_d, p_j, s^b_j] \right\},
$$

(D.76)

$$
\Sigma^b_P(\tilde{u}_L\tilde{d}_L) = -\frac{g^4}{4}|\Delta_s(\tilde{u}_L)| \Delta_s^*(\tilde{d}_L) m_i \left\{ - \text{Im}\{f^{L*}_{u\tilde{u}_j}f^{L*}_{d\tilde{d}_j}|t^L_{u\tilde{u}_j}|^2 \} [p_u, p_d, p_j, s^b_j] \\
+ \text{Re}\{f^{L*}_{u\tilde{u}_j}f^{L*}_{d\tilde{d}_j}|t^L_{u\tilde{u}_j}|^2 \} \left[ (p_d \cdot p_j)(p_u \cdot s^b_j) - (p_u \cdot p_j)(p_d \cdot s^b_j) \right] \right\}.
$$

(D.77)

With our choice of the chargino spin vectors $s^b_j$ [C.40], $\Sigma^{b=3}_P / P$ is the longitudinal polarization of chargino $\tilde{\chi}^+_j$, $\Sigma^{b=1}_P / P$ is the transverse polarization in the production plane and $\Sigma^{b=2}_P / P$ is the polarization normal to the production plane. To obtain $\Sigma^b_P$ for the charge conjugated process, $\bar{u}d \to \tilde{\chi}_i^0 \tilde{\chi}_j^-$, one has to change the signs of Eqs. (D.72)-(D.77).
The coefficients $\Sigma_{P}^{ab}$, which contain the spin vectors $s_{i}^{a}$ of the neutralino $\tilde{\chi}_{i}^{0}$ and $s_{j}^{b}$ of the chargino $\tilde{\chi}_{j}^{\pm}$, are the spin-spin correlation terms. They decompose into

$$\Sigma_{P}^{ab} = \Sigma_{P}^{ab}(WW) + \Sigma_{P}^{ab}(\tilde{u}_{L}\tilde{u}_{L}) + \Sigma_{P}^{ab}(\tilde{d}_{L}\tilde{d}_{L}) + \Sigma_{P}^{ab}(W\tilde{u}_{L}) + \Sigma_{P}^{ab}(W\tilde{d}_{L}) + \Sigma_{P}^{ab}(\tilde{u}_{L}\tilde{d}_{L}),$$

with

$$\Sigma_{P}^{ab}(WW) = -\frac{g^{4}}{2}|\Delta_{t}(W)|^{2}\left[|O_{ij}^{L}|^{2}m_{i}m_{j}(p_{a} \cdot s_{i}^{a})(p_{b} \cdot s_{j}^{b}) + |O_{ij}^{R}|^{2}m_{i}m_{j}(p_{a} \cdot s_{i}^{a})(p_{b} \cdot s_{j}^{b})
+\text{Re}\{O_{ij}^{L}O_{ij}^{R}\}g^{ab} + \text{Im}\{O_{ij}^{L}O_{ij}^{R}\}f^{ab}\right],$$

$$\Sigma_{P}^{ab}(\tilde{u}_{L}\tilde{u}_{L}) = -\frac{g^{4}}{4}|\Delta_{t}(\tilde{u}_{L})|^{2}|f_{ij}^{L}|^{2}|f_{ij}^{L}|^{2}m_{i}m_{j}(p_{a} \cdot s_{i}^{a})(p_{b} \cdot s_{j}^{b}),$$

$$\Sigma_{P}^{ab}(\tilde{d}_{L}\tilde{d}_{L}) = -\frac{g^{4}}{4}|\Delta_{t}(\tilde{d}_{L})|^{2}|f_{ij}^{L}|^{2}|f_{ij}^{L}|^{2}m_{i}m_{j}(p_{a} \cdot s_{i}^{a})(p_{b} \cdot s_{j}^{b}),$$

$$\Sigma_{P}^{ab}(W\tilde{u}_{L}) = -\frac{\sqrt{2}}{4}g^{4}\Delta_{t}(W)\Delta_{s}^{*}(\tilde{u}_{L})\left[2\text{Re}\{f_{ij}^{L}f_{ij}^{L}O_{ij}^{L}O_{ij}^{L}\}m_{i}m_{j}(p_{a} \cdot s_{i}^{a})(p_{b} \cdot s_{j}^{b})
+\text{Re}\{f_{ij}^{L}f_{ij}^{L}O_{ij}^{L}O_{ij}^{L}\}g^{ab} + \text{Im}\{f_{ij}^{L}f_{ij}^{L}O_{ij}^{L}O_{ij}^{L}\}f^{ab}\right],$$

$$\Sigma_{P}^{ab}(W\tilde{d}_{L}) = \frac{\sqrt{2}}{4}g^{4}\Delta_{t}(W)\Delta_{s}^{*}(\tilde{d}_{L})\left[2\text{Re}\{f_{ij}^{L}f_{ij}^{L}O_{ij}^{L}O_{ij}^{L}\}m_{i}m_{j}(p_{a} \cdot s_{i}^{a})(p_{b} \cdot s_{j}^{b})
+\text{Re}\{f_{ij}^{L}f_{ij}^{L}O_{ij}^{L}O_{ij}^{L}\}g^{ab} - \text{Im}\{f_{ij}^{L}f_{ij}^{L}O_{ij}^{L}O_{ij}^{L}\}f^{ab}\right],$$

and the short-hand notations for the products

$$g^{ab} = (p_{a} \cdot p_{d}) [(p_{j} \cdot s_{i}^{a})(p_{i} \cdot s_{j}^{b}) - (p_{i} \cdot p_{j})(s_{i}^{a} \cdot s_{j}^{b})] + (p_{a} \cdot s_{i}^{a}) [(p_{i} \cdot p_{j})(p_{d} \cdot s_{j}^{b}) - (p_{d} \cdot p_{j})(p_{i} \cdot s_{j}^{b})] + (p_{a} \cdot p_{i}) [(p_{d} \cdot p_{j})(s_{i}^{a} \cdot s_{j}^{b}) - (p_{j} \cdot s_{i}^{a})(p_{d} \cdot s_{j}^{b})] + (p_{a} \cdot s_{j}^{b}) [(p_{i} \cdot p_{j})(p_{d} \cdot s_{i}^{a}) - (p_{d} \cdot p_{i})(p_{j} \cdot s_{i}^{a})] + (p_{a} \cdot p_{j}) [(p_{d} \cdot p_{i})(s_{i}^{a} \cdot s_{j}^{b}) - (p_{d} \cdot s_{i}^{a})(p_{i} \cdot s_{j}^{b})],$$

$$f^{ab} = (p_{a} \cdot s_{i}^{a})[p_{d}, p_{i}, p_{j}, s_{j}^{b}] + (p_{a} \cdot p_{i})[p_{d}, p_{j}, s_{i}^{a}, s_{j}^{b}] + (p_{d} \cdot s_{j}^{b})[p_{a}, p_{i}, p_{j}, s_{i}^{a}] + (p_{d} \cdot p_{j})[p_{a}, p_{i}, s_{i}^{a}, s_{j}^{b}] = (p_{d} \cdot s_{j}^{b})[p_{a}, p_{i}, p_{j}, s_{i}^{a}] + (p_{d} \cdot p_{j})[p_{a}, p_{i}, s_{i}^{a}, s_{j}^{b}] + (p_{a} \cdot s_{j}^{b})[p_{d}, p_{i}, p_{j}, s_{i}^{a}] + (p_{a} \cdot p_{j})[p_{d}, p_{i}, s_{i}^{a}, s_{j}^{b}].$$

The terms for $\Sigma_{P}^{ab}$ are the same for the charge conjugated process, $\bar{u}d \rightarrow \bar{\chi}_{i}^{0} \tilde{\chi}_{j}^{\pm}$.  

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D.2 Decay matrices

Similar to the production matrix, the matrix for neutralino decay $\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_R \ell_1^+$, Eq. (3), can also be expanded in terms of the Pauli matrices

$$\rho_{D_1}(\tilde{\chi}_i^0)_{\chi'_{j\lambda j}} = \delta_{\chi'_{j\lambda j}} D_1 + \sigma^a_{\chi'_{j\lambda j}} \sum^a_{D_1}. \tag{D.88}$$

For the chargino decay $\tilde{\chi}_j^+ \rightarrow \tilde{\nu}_3 \ell_3^+$, Eq. (4), we write

$$\rho_{D_3}(\tilde{\chi}_j^+)_{\chi'_{j\lambda j}} = \delta_{\chi'_{j\lambda j}} D_3 + \sigma^b_{\chi'_{j\lambda j}} \sum^b_{D_3}. \tag{D.89}$$

The expansion coefficients are given by [16]

$$D_1 = \frac{g^2}{2} |f_{\tilde{\ell}1R}|^2 (m_i^2 - m_1^2), \tag{D.90}$$

$$\sum^a_{D_1} = \chi_{\lambda j} \sum^a_{D_1} (s_i \cdot p_\ell), \tag{D.91}$$

$$D_3 = \frac{g^2}{2} |V_{j1}|^2 (m_j^2 - m_{\nu_3}^2), \tag{D.92}$$

$$\sum^b_{D_3} = (s_j \cdot p_\ell), \tag{D.93}$$

where the sign in parenthesis in Eq. (D.91) holds for the charge conjugated process $\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_R \ell_1^-$, and in Eq. (D.93) for the conjugated process $\tilde{\chi}_j^- \rightarrow \chi_3^0 \ell_3^-$. The decay factor for the subsequent slepton decay $\tilde{\ell}_R^\pm \rightarrow \tilde{\chi}_1^0 \ell_2^\pm$, is

$$D_2(\tilde{\ell}) = g^2 |f_{\tilde{\ell}1R}|^2 (m_\ell^2 - m_{\tilde{\chi}_1^0}^2), \tag{D.94}$$

with all couplings as defined in Appendix [3].

D.3 Squared amplitude of production and decay

Having defined all density matrices for production and decay, the amplitude squared of the combined process of neutralino-chargino production, Eqs. (2), and their two-body decay chains, Eqs. (3)-(4), is written in the spin-density matrix formalism as [26]

$$|T|^2 = |\Delta(\tilde{\chi}_i^0)|^2 |\Delta(\tilde{\chi}_j^+)|^2 |\Delta(\tilde{\ell}_R)|^2 \times \sum_{\lambda_i, \chi'_{j\lambda j}, \chi'_{j\lambda j}} \rho_P(\tilde{\chi}_i^0, \tilde{\chi}_j^+) \rho_{D_1}(\tilde{\chi}_i^0)_{\chi'_{j\lambda j}} D_2(\tilde{\ell}_R) \rho_{D_3}(\tilde{\chi}_j^+)_{\chi'_{j\lambda j}}. \tag{D.95}$$

The amplitude squared is composed of the propagators

$$\Delta(k) = \frac{i}{s_k - m_k^2 + im_k \Gamma_k}, \tag{D.96}$$

with mass $m_k$ and width $\Gamma_k$ of the particles. Inserting the density matrices $\rho_P(\tilde{\chi}_i^0, \tilde{\chi}_j^+)$, Eq. (D.53), $\rho_{D_1}(\tilde{\chi}_i^0)$, Eq. (D.88), and $\rho_{D_3}(\tilde{\chi}_j^+)$, Eq. (D.89), into the formula for the amplitude squared, Eq. (D.95), we obtain

$$|T|^2 = 4 |\Delta(\tilde{\chi}_i^0)|^2 |\Delta(\tilde{\chi}_j^+)|^2 |\Delta(\tilde{\ell}_R)|^2 \times \left[ P D_1 D_3 + \sum^a_{D_1} D_3 + \sum^b_{D_1} D_1 + \sum^a_{D_3} D_1 + \sum^b_{D_3} D_1 \right] D_2(\tilde{\ell}_R), \tag{D.97}$$

29
with an implicit sum over $a = 1, 2, 3$ and $b = 1, 2, 3$. The amplitude squared $|T|^2$ is now decomposed into an unpolarized part (first summand), the spin correlations of the neutralino (second summand), those of the chargino (third summand), and the spin-spin correlations of the neutralino and chargino (fourth summand).

We give the amplitude squared $|T|^2$, Eq. (D.97), for the process $\bar{u}d \to \tilde{\chi}_i^0 \tilde{\chi}_j^+$, followed by $\tilde{\chi}_i^0 \to \bar{\ell}_R^+ \ell_1^+$, and $\tilde{\chi}_j^+ \to \tilde{\nu}_R \ell_3^-$. To obtain $|T|^2$ for the charge conjugated process, $\bar{u}d \to \tilde{\chi}_i^0 \tilde{\chi}_j^-$, followed by $\tilde{\chi}_i^0 \to \ell_R^- \ell_1^+$, and $\tilde{\chi}_j^- \to \tilde{\nu}_R \ell_3^+$, one has to reverse the sign of the neutralino spin correlations (second summand), and the spin-spin correlations (fourth summand). The unpolarized part (first summand), and the spin correlations of the chargino (third summand) stay the same. For the neutralino decay into a positively charged selectron, $\tilde{\chi}_i^0 \to \ell_R^+ \ell_1^-$, there is an additional sign change for the neutralino spin correlations (second summand), and the spin-spin correlations (fourth summand), due to the sign change of $\Sigma^a_{\mu\nu}$, see Eq. (D.91).

For the propagators in Eq. (D.97), we use the narrow width approximation

$$\int |\Delta(k)|^2\, ds_k = \frac{\pi}{m_k \Gamma_k},$$

which is justified for $\Gamma_k/m_k \ll 1$, which holds in our case with $\Gamma_k \lesssim \mathcal{O}(1 \text{ GeV})$. Note, however, that the naive $\mathcal{O}(\Gamma/m)$-expectation of the error can easily receive large off-shell corrections of an order of magnitude and more, in particular at threshold, or due to interferences with other resonant or non-resonant processes [55].

The hard partonic cross section in the laboratory frame is then obtained by integrating the squared amplitude, Eq. (D.97), over the Lorentz invariant phase space element, Eq. (C.42),

$$d\hat{\sigma} = \frac{1}{2s} |T|^2 d\text{Lips}. \tag{D.99}$$

Finally the hadronic cross section $\sigma$ in the laboratory frame is obtained by integrating the partonic cross section $\hat{\sigma}$, Eq. (D.99), over the parton distribution functions

$$d\sigma = \frac{2\kappa}{3} \int dx_1 dx_2 f_u(x_1, \mu^2) f_d(x_2, \mu^2) \hat{\sigma}, \tag{D.100}$$

which depend on the factorization scale $\mu$, and the momentum fractions $x_1, x_2$ of the quarks of the proton center-of-mass energy $\sqrt{s}$ in the laboratory frame, such that $s = x_1 x_2 s$. There is a combinatorial color factor of 1/3, since the PDFs typically sum the quark color (3x3), but here only quarks with pairing color-anticolor (3 pairs) contribute. The factor of 2 takes into account the two possible assignments that the quark (antiquark) originates form the left (right) incoming proton beam, and vice versa. The factor $\kappa$ takes efficiently into account the dominant QCD radiative corrections for the production cross section, with typical value $\kappa \approx 1.3$ [24]. For simplicity however, we set $\kappa = 1$ in our numerical calculations.

By using the completeness relations for the neutralino spin vectors

$$s_i^{a,\mu} s_i^{a,\nu} = -g_i^{\mu\nu} + \frac{p_i^\mu p_i^\nu}{m_i^2}, \tag{D.101}$$
the terms with products of the neutralino spin vectors in Eq. (D.97) can be written

\[ \Sigma_{PD} = \Sigma_{PD}(WW) + \Sigma_{PD}(\tilde{u}_L\tilde{u}_L) + \Sigma_{PD}(\tilde{d}_L\tilde{d}_L) + \Sigma_{PD}(W\tilde{u}_L) + \Sigma_{PD}(W\tilde{d}_L) + \Sigma_{PD}(\tilde{u}_L\tilde{d}_L), \]

with

\[
\Sigma_{PD}(WW) = \frac{g^6}{2}|f_{\tilde{\chi}^0}^d|^2 |\Delta_s(W)|^2 \{ |O_{ij}^L|^2 (p_d \cdot p_j) [(p_a \cdot p_i)(p_i \cdot p_t) - m^2 p_a \cdot p_t] - |O_{ij}^R|^2 (p_d \cdot p_j) [(p_a \cdot p_i)(p_i \cdot p_t) - m^2 p_a \cdot p_t] \\
- \text{Re}\{O_{ij}^L O_{ij}^{R*}\} m_i m_j [(p_a \cdot p_i)(p_d \cdot p_t) - (p_d \cdot p_t)(p_a \cdot p_i)] \\
+ \text{Im}\{O_{ij}^L O_{ij}^{R*}\} m_i m_j [p_a, p_d, p_i, p_t],
\]

(103)

\[
\Sigma_{PD}(\tilde{u}_L\tilde{u}_L) = \frac{g^6}{4} |\Delta_t(\tilde{u}_L)|^2 |f_{\tilde{\chi}^0}^d|^2 \{ 2\text{Re}\{f_{u_i u_j}^d L^s O_{ij}^s\} (p_d \cdot p_j) [(p_a \cdot p_i)(p_i \cdot p_t) - m^2 p_a \cdot p_t] \\
+ \text{Re}\{f_{u_i u_j}^d L^s O_{ij}^{R*}\} m_i m_j [(p_a \cdot p_i)(p_d \cdot p_t) - (p_d \cdot p_t)(p_a \cdot p_i)] \\
+ \text{Im}\{f_{u_i u_j}^d L^s O_{ij}^{R*}\} m_i m_j [p_a, p_d, p_i, p_t],
\]

(104)

\[
\Sigma_{PD}(\tilde{d}_L\tilde{d}_L) = \frac{\sqrt{2}}{4} g^6 \Delta_s(W) |\Delta^*_u(\tilde{d}_L)| |f_{\tilde{\chi}^0}^d|^2 \{ 2\text{Re}\{f_{d_i d_j}^d L^s O_{ij}^{R*}\} (p_d \cdot p_j) [(p_a \cdot p_i)(p_i \cdot p_t) - m^2 (p_a \cdot p_t)] \\
+ \text{Re}\{f_{d_i d_j}^d L^s O_{ij}^{L*}\} m_i m_j [(p_a \cdot p_i)(p_d \cdot p_t) - (p_d \cdot p_t)(p_a \cdot p_i)] \\
+ \text{Im}\{f_{d_i d_j}^d L^s O_{ij}^{L*}\} m_i m_j [p_a, p_d, p_i, p_t],
\]

(105)

\[
\Sigma_{PD}(W\tilde{u}_L) = \frac{\sqrt{2}}{4} g^6 \Delta_s(W) |\Delta^*_u(\tilde{u}_L)| |f_{\tilde{\chi}^0}^d|^2 \{ 2\text{Re}\{f_{u_i u_j}^d L^s O_{ij}^{R*}\} (p_d \cdot p_j) [(p_a \cdot p_i)(p_i \cdot p_t) - m^2 (p_a \cdot p_t)] \\
+ \text{Re}\{f_{u_i u_j}^d L^s O_{ij}^{L*}\} m_i m_j [(p_a \cdot p_i)(p_d \cdot p_t) - (p_d \cdot p_t)(p_a \cdot p_i)] \\
+ \text{Im}\{f_{u_i u_j}^d L^s O_{ij}^{L*}\} m_i m_j [p_a, p_d, p_i, p_t],
\]

(106)

\[
\Sigma_{PD}(W\tilde{d}_L) = \frac{\sqrt{2}}{4} g^6 \Delta_s(W) |\Delta^*_u(\tilde{d}_L)| |f_{\tilde{\chi}^0}^d|^2 \{ 2\text{Re}\{f_{d_i d_j}^d L^s O_{ij}^{R*}\} (p_d \cdot p_j) [(p_a \cdot p_i)(p_i \cdot p_t) - m^2 (p_a \cdot p_t)] \\
+ \text{Re}\{f_{d_i d_j}^d L^s O_{ij}^{L*}\} m_i m_j [(p_a \cdot p_i)(p_d \cdot p_t) - (p_d \cdot p_t)(p_a \cdot p_i)] \\
+ \text{Im}\{f_{d_i d_j}^d L^s O_{ij}^{L*}\} m_i m_j [p_a, p_d, p_i, p_t],
\]

(107)

\[
\Sigma_{PD}(\tilde{u}_L\tilde{d}_L) = \frac{g^6}{4} |\Delta_t(\tilde{u}_L)| |\Delta^*_u(\tilde{d}_L)| |f_{\tilde{\chi}^0}^d|^2 m_i m_j \left\{ \text{Im}\{f_{u_i u_j}^d L^s f_{d_i d_j}^d L^s O_{ij}^{R*}\} [p_a, p_d, p_i, p_t] \\
+ \text{Re}\{f_{u_i u_j}^d L^s f_{d_i d_j}^d L^s O_{ij}^{L*}\} [(p_a \cdot p_i)(p_d \cdot p_t) - (p_d \cdot p_t)(p_a \cdot p_i)] \right\},
\]

(108)

To obtain \(\Sigma_{PD}\) for the charge conjugated process, \(u \bar{d} \rightarrow \tilde{\chi}^0_1 \tilde{\chi}_j^0\), and \(\tilde{\chi}_j^0 \rightarrow \tilde{\nu}_L \tilde{\ell}_3\), one has to change the signs of Eqs. (103)-(108), due to the sign change of \(\Sigma_{PD}\), see Eq. (103). For the neutralino decay into a positively charged selectron, \(\tilde{\chi}_j^0 \rightarrow \tilde{\ell}_3^+ \tilde{\ell}_1^-\), there is an additional sign change in Eqs. (103)-(108), due to the sign change of \(\Sigma_{PD}\), see Eq. (103).
Analogously, by using the completeness relations for the chargino spin vectors

$$s_j^{b,\mu} s_j^{b,\nu} = -g^{\mu\nu} + \frac{P_j^b P_j^b}{m_j^2},$$  \hspace{1cm} (D.109)

the terms with products of the chargino spin vectors in Eq. (D.97) can be written

$$\Sigma_p^{b} \Sigma_{D_4}^{b} = \Sigma_{P D_3}(W W) + \Sigma_{P D_3}(\tilde{u}_L \bar{u}_L) + \Sigma_{P D_3}(\tilde{d}_L \bar{d}_L) + \Sigma_{P D_3}(W \tilde{u}_L) + \Sigma_{P D_3}(W \tilde{d}_L) + \Sigma_{P D_3}(\tilde{u}_L \bar{d}_L),$$  \hspace{1cm} (D.110)

with

$$\Sigma_{P D_3}(W W) = \frac{g^6}{4} |\Delta_s(\tilde{u}_L)|^2 |V_{1j}|^2 \Bigl\{ |O_{ij}^{R}|^2 (p_d \cdot p_i) [(p_d \cdot p_j)(p_j \cdot p_{\ell_3}) - m_j^2 (p_d \cdot p_{\ell_3})]$$

$$- |O_{ij}^{L}|^2 (p_d \cdot p_i) [(p_d \cdot p_j)(p_j \cdot p_{\ell_3}) - m_j^2 (p_d \cdot p_{\ell_3})]$$

$$+ \text{Re}\{O_{ij}^{L}O_{ij}^{R*}\} m_i m_j [(p_d \cdot p_j)(p_{\ell_3} - (p_d \cdot p_j)(p_{\ell_3})]$$

$$- \text{Im}\{O_{ij}^{L}O_{ij}^{R*}\} m_i m_j [p_d, p_j, p_{\ell_3}]\Bigl\}.$$

$$\Sigma_{P D_3}(\tilde{u}_L \bar{u}_L) = \frac{g^6}{4} |\Delta_s(\tilde{u}_L)|^2 |V_{1j}|^2 \Bigl\{ f_{ij}^{L*} [p_d \cdot p_i] [(p_d \cdot p_j)(p_j \cdot p_{\ell_3}) - m_j^2 (p_d \cdot p_{\ell_3})]$$

$$+ \text{Re}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} m_i m_j [(p_d \cdot p_j)(p_{\ell_3} - (p_d \cdot p_j)(p_{\ell_3})]$$

$$+ \text{Im}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} m_i m_j [p_d, p_j, p_{\ell_3}]\Bigl\}.$$

$$\Sigma_{P D_3}(W \tilde{u}_L) = \frac{\sqrt{3}}{4} g^6 |\Delta_s(W)| D_s^{*}(\tilde{u}_L)|V_{1j}|^2 \Bigl\{$$

$$2 \text{Re}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} [p_d \cdot p_i] [(p_d \cdot p_j)(p_j \cdot p_{\ell_3}) - m_j^2 (p_d \cdot p_{\ell_3})]$$

$$+ \text{Re}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} m_i m_j [(p_d \cdot p_j)(p_{\ell_3} - (p_d \cdot p_j)(p_{\ell_3})]$$

$$+ \text{Im}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} m_i m_j [p_d, p_j, p_{\ell_3}]\Bigl\}.$$

$$\Sigma_{P D_3}(W \tilde{d}_L) = \frac{\sqrt{3}}{4} g^6 |\Delta_s(W)| D_s^{*}(\tilde{d}_L)|V_{1j}|^2 \Bigl\{$$

$$2 \text{Re}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} [p_d \cdot p_i] [(p_d \cdot p_j)(p_j \cdot p_{\ell_3}) - m_j^2 (p_d \cdot p_{\ell_3})]$$

$$+ \text{Re}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} m_i m_j [(p_d \cdot p_j)(p_{\ell_3} - (p_d \cdot p_j)(p_{\ell_3})]$$

$$+ \text{Im}\{f_{ij}^{L*} f_{ij}^{L*} O_{ij}^{R*}\} m_i m_j [p_d, p_j, p_{\ell_3}]\Bigl\}.$$

The terms $\Sigma_{P D_3}$ are the same for the charge conjugated process, $\tilde{u}d \rightarrow \tilde{\chi}_i^0 \tilde{\chi}^-_j$, and $\tilde{\chi}^-_j \rightarrow \tilde{\nu}_\ell \ell^*_3$, due to the sign change of both $\Sigma_p$, see Eq. (D.71), and $\Sigma_{D_3}$, see Eq. (D.93).
Finally, by using the completeness relations for both the chargino and neutralino spin vectors, Eq. (D.101) and Eq. (D.109), respectively, the terms with products of the chargino and neutralino spin vectors in Eq. (D.97) can be written

\[ \Sigma_{D_1} \Sigma_{D_4} \equiv \Sigma_{PD_1D_3}(WW) + \Sigma_{PD_1D_3}(\tilde{u}_L\tilde{u}_L) + \Sigma_{PD_1D_3}(\tilde{d}_L\tilde{d}_L) + \Sigma_{PD_1D_3}(W\tilde{u}_L) + \Sigma_{PD_1D_3}(W\tilde{d}_L) + \Sigma_{PD_1D_3}(\tilde{u}_L\tilde{d}_L), \tag{D.117} \]

with

\[
\Sigma_{PD_1D_3}(WW) = \frac{g^8}{2} |\Delta_u(W)|^2 |f_{\tilde{u}_L}^R|^2 |V_{1j}|^2 \left[ |O_{ij}^L|^2 g_1 + |O_{ij}^R|^2 g_2 + \text{Re}(O_{ij}^L O_{ij}^R) m_m j g_3 + \text{Im}(O_{ij}^L O_{ij}^R) m_m j f \right], \tag{D.118} \]

\[
\Sigma_{PD_1D_3}(\tilde{u}_L\tilde{u}_L) = \frac{g^8}{4} |\Delta_{\tilde{u}}(\tilde{u}_L)|^2 |f_{\tilde{u}_L}^R|^2 |V_{1j}|^2 |f_{\tilde{u}_L}^L|^2 |V_{3j}|^2 g_1, \tag{D.119} \]

\[
\Sigma_{PD_1D_3}(\tilde{d}_L\tilde{d}_L) = \frac{g^8}{4} |\Delta_{\tilde{d}}(\tilde{d}_L)|^2 |f_{\tilde{d}_L}^R|^2 |V_{1j}|^2 |f_{\tilde{d}_L}^L|^2 |V_{3j}|^2 g_2, \tag{D.120} \]

\[
\Sigma_{PD_1D_3}(W\tilde{u}_L) = \frac{\sqrt{2}}{4} g^8 |\Delta_{\tilde{u}}(W)| |f_{\tilde{u}_L}^R|^2 |V_{1j}|^2 \left[ 2\text{Re}(f_{\tilde{u}_L}^R t_{ij}^L O_{ij}^L) g_1 + \text{Re}(f_{\tilde{u}_L}^R t_{ij}^L O_{ij}^R) m_m j g_3 + \text{Im}(f_{\tilde{u}_L}^R t_{ij}^L O_{ij}^R) m_m j f \right], \tag{D.121} \]

\[
\Sigma_{PD_1D_3}(W\tilde{d}_L) = -\frac{\sqrt{2}}{4} g^8 |\Delta_{\tilde{d}}(W)| |f_{\tilde{d}_L}^R|^2 |V_{1j}|^2 \left[ 2\text{Re}(f_{\tilde{d}_L}^R t_{ij}^L O_{ij}^L) g_2 + \text{Re}(f_{\tilde{d}_L}^R t_{ij}^L O_{ij}^R) m_m j g_3 - \text{Im}(f_{\tilde{d}_L}^R f_{\tilde{d}_L}^L t_{ij}^L f_{\tilde{d}_L}^L f_{\tilde{d}_L}^L) f_3 \right], \tag{D.122} \]

and the short-hand notations for the kinematic functions

\[
g_1 = \left[ (p_u \cdot p_i)(p_i \cdot p_{\ell_3}) - m_3^2 (p_u \cdot p_{\ell_3}) \right] \left[ (p_d \cdot p_j)(p_j \cdot p_{\ell_3}) - m_3^2 (p_d \cdot p_{\ell_3}) \right], \tag{D.124} \]

\[
g_2 = \left[ (p_d \cdot p_i)(p_i \cdot p_{\ell_3}) - m_3^2 (p_d \cdot p_{\ell_3}) \right] \left[ (p_u \cdot p_j)(p_j \cdot p_{\ell_3}) - m_3^2 (p_u \cdot p_{\ell_3}) \right], \tag{D.125} \]

\[
g_3 = (p_u \cdot p_d) \left[ (p_j \cdot p_{\ell_3})(p_i \cdot p_{\ell_3}) - (p_i \cdot p_{\ell_3})(p_j \cdot p_{\ell_3}) \right] + (p_u \cdot p_{\ell_3}) \left[ (p_i \cdot p_{\ell_3})(p_d \cdot p_{\ell_3}) - (p_d \cdot p_{\ell_3})(p_i \cdot p_{\ell_3}) \right] + (p_u \cdot p_{\ell_3}) \left[ (p_d \cdot p_{\ell_3})(p_i \cdot p_{\ell_3}) - (p_i \cdot p_{\ell_3})(p_d \cdot p_{\ell_3}) \right] + (p_u \cdot p_{\ell_3}) \left[ (p_d \cdot p_{\ell_3})(p_i \cdot p_{\ell_3}) - (p_i \cdot p_{\ell_3})(p_d \cdot p_{\ell_3}) \right], \tag{D.126} \]

\[
f = (p_u \cdot p_{\ell_3}) [p_d, p_i, p_j, p_{\ell_3}] + (p_u \cdot p_{\ell_3}) [p_d, p_j, p_i, p_{\ell_3}] + (p_d \cdot p_{\ell_3}) [p_u, p_i, p_j, p_{\ell_3}] + (p_d \cdot p_{\ell_3}) [p_u, p_j, p_i, p_{\ell_3}] + (p_u \cdot p_{\ell_3}) [p_d, p_j, p_i, p_{\ell_3}] + (p_u \cdot p_{\ell_3}) [p_d, p_i, p_j, p_{\ell_3}], \tag{D.127} \]

\[
= (p_d \cdot p_{\ell_3}) [p_u, p_i, p_j, p_{\ell_3}] + (p_d \cdot p_{\ell_3}) [p_u, p_j, p_i, p_{\ell_3}] + (p_u \cdot p_{\ell_3}) [p_d, p_i, p_j, p_{\ell_3}] + (p_u \cdot p_{\ell_3}) [p_d, p_j, p_i, p_{\ell_3}]. \tag{D.128} \]
To obtain the terms $\Sigma_{PD_3D_4}$ for the charge conjugated process, $\bar{u}d \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^+$, and $\tilde{\chi}_j^- \rightarrow \nu_\ell \ell_3^-$, one has to change the signs of Eqs. (D.118)-(D.123), due to the sign change of $\Sigma_{D_3}^b$, see Eq. (D.93). For the neutralino decay into a positively charged selectron, $\tilde{\chi}_i^0 \rightarrow \tilde{e}_R^+ \ell_1^-$, there is an additional sign change in Eqs. (D.118)-(D.123), due to the sign change of $\Sigma_{D_4}^a$, see Eq. (D.91).

### E Charge conjugation and asymmetries

In this Appendix we discuss the dependence of our asymmetries on the charges of the near and far leptons, $\ell_1$ and $\ell_2$ respectively, from the neutralino decay, and on the charge of the lepton $\ell_3$ from the chargino decay (which also determines the chargino charge). Note first that changing the chargino charge also requires a charge conjugation of the initial state, i.e. $ud \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^+$, becomes $\bar{u}d \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^-$. Since the total charge of the three leptons is easy to measure, $\bar{u}d \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^+$ production can be distinguished from its charge conjugated production, $ud \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^-$. Due to the Majorana nature of the neutralino, the near lepton produced in its decay chain has equal probability to have positive or negative charge, independent of the charge of the initial state.

The chargino spin correlation asymmetry depends on these charges as follows:

$$\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_j^+}, p_{\ell_3^+}] = \mathcal{A}[p_u, p_d, p_{\tilde{\chi}_j^-}, p_{\ell_3^-}],$$

(E.129)

see the remarks regarding the sign change in the relevant contribution to the amplitude squared for the charge conjugated processes in Appendix D.3 after Eq. (D.116). The asymmetry of the neutralino-chargino spin-spin correlations satisfies

$$\mathcal{A}[f(p_u, p_d, p_{\tilde{\chi}_j^0}, p_{\tilde{\chi}_i^+}, p_{\ell_1^+}, p_{\ell_3^+})] = \mathcal{A}[f(p_u, p_d, p_{\tilde{\chi}_j^0}, p_{\tilde{\chi}_i^-}, p_{\ell_1^-}, p_{\ell_3^-})]$$

$$= -\mathcal{A}[f(p_u, p_d, p_{\tilde{\chi}_j^0}, p_{\tilde{\chi}_i^-}, p_{\ell_1^-}, p_{\ell_3^-})],$$

(E.130)

see the remarks after Eq. (D.128), and the T-odd product $f$ as given in Eq. (19). For the neutralino spin correlation asymmetry we have

$$\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_j^-}, p_{\ell_1^+}] = -\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_j^0}, p_{\ell_1^-}]$$

$$= -\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_j^0}, p_{\ell_1^-}]$$

$$= +\mathcal{A}[p_u, p_d, p_{\tilde{\chi}_j^0}, p_{\ell_1^-}],$$

(E.131)

see the remarks after Eq. (D.108). Although these relations concern the asymmetries which contain the inaccessible quark momenta and neutralino and/or chargino momenta, these relations are essential to understand the properties under charge conjugation of the accessible asymmetries, where these momenta have been replaced and approximated, as discussed in Section 4.

As discussed in Section 4.2, the neutralino momentum cannot be reconstructed and thus is replaced by the momentum of the far lepton $\ell_2$. As shown in Section 4.2 the corresponding asymmetry depends on the charge of the near ($\ell_1$) and far ($\ell_2$) leptons as follows

$$\mathcal{A}[p_u, p_d, p_{\ell_1^+}, p_{\ell_2^-}] = -\mathcal{A}[p_u, p_d, p_{\ell_1^-}, p_{\ell_2^+}]$$

$$= +\mathcal{A}[p_u, p_d, p_{\ell_1^+}, p_{\ell_2^-}]$$

$$= -\mathcal{A}[p_u, p_d, p_{\ell_1^-}, p_{\ell_2^+}].$$

(E.132)
The sign change in the first step originates from Eq. (D.91), and the sign change in the second and third steps is simply due to the interchange of the momenta of $\ell_1$ and $\ell_2$ in the antisymmetric epsilon product. Thus we need not determine from which vertex, near or far, the leptons $\ell_1$ and $\ell_2$ originate. Instead in the epsilon product one would just group the leptons according to their charge, and use the corresponding asymmetry

$$A[p_u, p_d, p_{\ell+}, p_{\ell-}] = -A[p_u, p_d, p_{\ell-}, p_{\ell+}].$$  \hspace{1cm} (E.133)

The triple product asymmetry fulfills similar relations to Eq. (E.132), concerning the interchange of the near and far leptons, and of their charges,

$$A(p_{\ell_1^+}, p_{\ell_2^-}, p_{\ell_3}) = -A(p_{\ell_1^-}, p_{\ell_2^+}, p_{\ell_3})$$
$$= +A(p_{\ell_2^-}, p_{\ell_1^+}, p_{\ell_3})$$
$$= -A(p_{\ell_2^+}, p_{\ell_1^-}, p_{\ell_3}),$$ \hspace{1cm} (E.134)

which each hold for the chargino charge fixed $\ell_3 = \ell_1^+$ or $\ell_2^-$. Thus also for the triple product asymmetry, only the charges of the two leptons from the neutralino decay have to be tagged, and for measurements one would use the triple product asymmetry

$$A(p_{\ell^+}, p_{\ell^-}, p_{\ell_3}) = -A(p_{\ell^-}, p_{\ell^+}, p_{\ell_3}).$$  \hspace{1cm} (E.135)

Concerning now the charged conjugated production, the neutralino spin correlation asymmetry obeys

$$A[p_u, p_d, p_{\ell+}, p_{\ell-}] = -A[p_u, p_d, p_{\ell+}, p_{\ell-}].$$  \hspace{1cm} (E.136)

Note however, that no such simple relations for charge conjugation of the initial state hold neither for the asymmetries where the quark momenta are approximated, see Eqs. (31), (32), nor for the triple product asymmetry,

$$A[p_{\ell_1}^{\text{aprx}}, p_{\ell_2}^{\text{aprx}}, p_{\ell+}, p_{\ell-}] \neq -A[p_{\ell_1}^{\text{aprx}}, p_{\ell_2}^{\text{aprx}}, p_{\ell+}, p_{\ell-}],$$
$$A(p_{\ell_1^+}, p_{\ell_2^-}, p_{\ell_3}) \neq -A(p_{\ell_1^+}, p_{\ell_2^-}, p_{\ell_3}),$$ \hspace{1cm} (E.137, E.138)

The initial partonic systems $\bar{u}d$ and $ud$ have different boost distributions due to the PDFs, thus the efficiencies to guess the initial quark directions are different. Recall that the triple product asymmetry is not Lorentz invariant. Thus also here the different boost distributions of the initial partonic systems, $\bar{u}d$ as opposed to $ud$, will give different values for the triple product asymmetry for $pp \to \tilde{\chi}_1^0 \tilde{\chi}_j^+$ production, and its charge conjugated process $pp \to \tilde{\chi}_1^0 \tilde{\chi}_j^-$, in general. Moreover, as pointed out in Section 4, the asymmetries $A[p_{\ell_1}^{\text{aprx}}, p_{\ell_2}^{\text{aprx}}, p_{\ell+}, p_{\ell-}]$, as well as $A(p_{\ell_1^+}, p_{\ell_2^-}, p_{\ell_3})$, receive contributions both from the neutralino spin correlations and from the spin-spin correlations. Since these two contributions react differently under charge conjugation, also the corresponding asymmetries have a different magnitude.
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