Difficulties in Obtaining Weak Phases from $b \to d$ Penguin Decays

Alakabha Datta$^a$, C.S. Kim$^b$ and David London$^a$

$^a$ Laboratoire René J.-A. Lévesque, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

$^b$ Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea

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Abstract

We re-examine a recent proposal for obtaining $\beta$ using the measurements of the $b \to d$ penguin decays $B_t^0(t) \to K^0\bar{K}^0$ and $B_s^0(t) \to \phi K_s$, along with a theoretical assumption. We show that there are in fact three assumptions one can make, so that the method can in principle be used to extract $\alpha$, $\beta$ or $\gamma$. We also show that it is the assumption which yields $\gamma$ which is the best. However, the theoretical error on this assumption is still 25–30%, which leads to an error on $\gamma$ of at least 20–25%. Given our current understanding of hadronic physics, it does not seem possible to reduce this error.
Measurements of CP-violating rate asymmetries in the neutral $B$ system will allow one to obtain the CP angles $\alpha$, $\beta$ and $\gamma$ \cite{1}. From these the unitarity triangle \cite{2} can be constructed, and one can test the predictions of the standard model (SM). If we are lucky, these measurements will reveal the presence of physics beyond the SM.

One of the best ways of searching for new physics is to consider two different decay modes whose CP asymmetries probe the same CP phase within the SM. Any discrepancy between the measured values of these asymmetries will point unequivocally to the presence of new physics. One possibility is to consider pure $b \to d$ penguin decays such as $B_0^0(t) \to K^0\bar{K}^0$ or $B^0(t) \to \phi K_s$. At the quark level, these decay amplitudes take the form $b \to d\bar{s}s$. If such decays are dominated by internal $t$-quark exchange, the amplitude is proportional to $V_{tb}V_{td}^*$, where the $V_{ij}$ are elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. In the (approximate) Wolfenstein parametrization of the CKM matrix \cite{3}, the only two (approximate) matrix elements which have a nonzero weak phase are $V_{td}$ [$\sim \exp(-i\beta)$] and $V_{ub}$ [$\sim \exp(-i\gamma)$]. Thus, CP asymmetries in pure $b \to d$ penguin decays probe the weak phase $\beta$. By comparing this value of $\beta$ with that extracted via the conventional mode $B_d^0(t) \to J/\psi K_s$, one can search for new physics.

Unfortunately, the $b \to d$ penguin amplitude is not dominated by $t$-quark exchange. For example, the $u\bar{u}$ quark pair of the tree-level decay $b \to u\bar{u}d$ can rescatter strongly into an $s\bar{s}$ quark pair, giving an effective $V_{ub}V_{ud}^*$ contribution to the $b \to d$ penguin amplitude, and similarly for the $b \to c\bar{c}d$ tree-level decay. Buras and Fleischer \cite{4} have noted that the $u$- and $c$-quark contributions can be between 20% and 50% of the leading $t$-quark contribution to the $b \to d$ penguin amplitude. And since $V_{ub}V_{ud}^*$ and $V_{cb}V_{cd}^*$ have different weak phases as compared to $V_{tb}V_{td}^*$, this implies that CP asymmetries in pure $b \to d$ penguin decays do not cleanly probe the weak phase $\beta$.

But this then begs the question: is it possible to isolate the $t$-quark contribution to the $b \to d$ penguin amplitude? If so, one could then obtain $\beta$ from this piece of the amplitude, and compare it with the value found in $B_d^0(t) \to J/\psi K_s$. Unfortunately, as was shown in Ref. \cite{5}, this is not possible. In a nutshell, the argument is as follows. The $b \to d$ penguin amplitude can be written generally as

$$P = P_u V_{ub}V_{ud}^* + P_c V_{cb}V_{cd}^* + P_t V_{tb}V_{td}^* .$$

(1)

Now, the unitarity of the CKM matrix implies that $V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$. Thus, any of the three pieces in the above amplitude can always be eliminated in terms of the remaining two. For example, if we eliminate the $V_{ub}V_{ud}^*$ piece, we obtain

$$P = (P_c - P_u) V_{cb}V_{cd}^* + (P_t - P_u) V_{tb}V_{td}^* \equiv P_{cu} e^{i\delta_{cu}} + P_{tu} e^{i\delta_{tu}} e^{i\beta} ,$$

(2)

where we have explicitly separated out the weak and strong phases and absorbed the magnitudes $|V_{cb}V_{cd}|$ and $|V_{tb}V_{td}|$ into the definitions of $P_{cu}$ and $P_{tu}$, respectively. Similarly, eliminating $V_{tb}V_{td}^*$ gives

$$P = P_{ct} e^{i\delta_{ct}} + P_{ut} e^{i\delta_{ut}} e^{-i\gamma} .$$

(3)

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Now, suppose that a method existed which would permit one to extract the CP phase \( \beta \), with no hadronic uncertainties, using the parametrization of Eq. (3). If so, then \( \beta \) could be expressed entirely in terms of experimentally measured quantities. However, Eq. (3) has the same form as Eq. (2). Thus, this same method would allow us to cleanly obtain \(-\gamma\) using Eq. (3). In particular, \(-\gamma\) would be expressed as the same function of observables as \( \beta \), leading to the conclusion that \( \beta = -\gamma \). Since this is clearly not true in general, one concludes that it is not possible to cleanly measure the weak phase of the \( t \)-quark piece (or indeed any other piece) of the \( b \to d \) penguin amplitude. In Ref. [5], this is referred to as the “CKM ambiguity.”

However, in Ref. [5] it is also argued that it is possible to resolve the CKM ambiguity, and hence isolate a particular piece of the \( b \to d \) penguin amplitude, if one makes an assumption regarding the hadronic parameters involved in this amplitude. This fact has been used by two of the present authors (Kim, London) and Yoshikawa in Ref. [6] to obtain \( \beta \) from the \( t \)-quark piece of the \( b \to d \) penguin amplitude. The idea is to use the two penguin decays \( B^0_d(t) \to K^0\bar{K}^0 \) and \( B^0_s(t) \to \phi K^0 \). If one eliminates the \( V_{ub}V_{ud}^* \) piece from the penguin amplitudes, as in Eq. (2), the amplitudes for these two decays can be written as

\[
A(B^0_d \to K^0\bar{K}^0) = P_{ct} e^{i\delta_{ct}} + P_{ut} e^{i\delta_{ut}} e^{-i\beta},
\]

\[
A(B^0_s \to \phi K^0) = P'_{ct} e^{i\delta'_{ct}} + P'_{ut} e^{i\delta'_{ut}} e^{-i\beta}.
\]

(4)

The following assumption is now made:

\[
\frac{r_u}{r'_u} \equiv \frac{P_{ct}}{P_{ut}} \frac{P'_{ct}}{P'_{ut}} = 1.
\]

(Note that the dependence on the CKM matrix elements cancels in this ratio, so that this really is an assumption about the hadronic parameters of the two amplitudes.) Measurements of \( B^0_d(t) \to K^0\bar{K}^0 \) and \( B^0_s(t) \to \phi K^0 \), combined with the assumption in Eq. (5), will then allow the extraction of \( \beta \), the weak phase of the \( t \)-quark piece of the \( b \to d \) penguin amplitude. By comparing this value of \( \beta \) with that found in \( B^0_d(t) \to J/\psi K^0 \), one may be able to detect the presence of new physics. In what follows, we will refer to this as the “KLY method.”

However, there is a problem with this method which has been overlooked in Ref. [5]. It is similar to the CKM ambiguity: how do we know that \( V_{ub}V_{ud}^* \) is the correct term to eliminate? For example, had we eliminated the \( V_{tb}V_{td}^* \) piece, as in Eq. (3), the amplitudes would take the form

\[
A(B^0_d \to K^0\bar{K}^0) = P_{ct} e^{i\delta_{ct}} + P_{ut} e^{i\delta_{ut}} e^{i\gamma},
\]

\[
A(B^0_s \to \phi K^0) = P'_{ct} e^{i\delta'_{ct}} + P'_{ut} e^{i\delta'_{ut}} e^{i\gamma}.
\]

(6)

Using the assumption that

\[
\frac{r_t}{r'_t} \equiv \frac{P_{ct}}{P_{ut}} \frac{P'_{ct}}{P'_{ut}} = 1,
\]

(7)

the KLY method can be used to allow us to obtain \( \gamma \). Similarly, the elimination of the \( V_{cb}V_{cd}^* \) piece in the amplitudes, combined with the assumption that

\[
\frac{r_c}{r'_c} \equiv \frac{P_{uc}}{P_{tc}} \frac{P'_{uc}}{P'_{tc}} = 1,
\]

(8)
would permit the extraction of $\alpha$. The confusion regarding the validity of the three assumptions in Eqs. (3), (7) and (8) can be thought of as a “second CKM ambiguity.”

It is clear that the three assumptions in Eqs. (3), (7) and (8) cannot be simultaneously true. If they were, then, as was the case in the discussion of the CKM ambiguity, one would obtain results such as $\beta = -\gamma$, which do not hold in general. We therefore deduce that (at least) two of the assumptions are poor. That is, the values of the CP phases obtained using these assumptions will differ enormously from their true values.

In fact, it can be argued that all three assumptions are likely to be poor. After all, even though they are based on the same quark-level process, the hadronic quantities (form factors, strong phases, etc.) describing the decays $B_0^0(t) \rightarrow K^0\bar{K}^0$ and $B_s^0(t) \rightarrow \phi K_s$ should be quite different. Thus, there is no reason to expect any of the ratios in Eqs. (3), (7) and (8) to equal one, in which case it appears that the method is of little practical use.

However, this is not as serious a problem as appears at first glance. Consider again the assumption of Eq. (3). In fact, the KLY method does not require that $r_u/r'_u = 1$. All that is necessary is that we know the value of this ratio. Thus, if we could theoretically calculate the value of $r_u/r'_u$, the KLY method could then be used to obtain $\beta$. Similarly, if we could calculate $r_t/r'_t$ or $r_c/r'_c$, we could extract $\gamma$ or $\alpha$. We therefore see that the second CKM ambiguity is not necessarily a disadvantage: depending on what hadronic information is known, any of the three CP phases could, in principle, be obtained using the KLY method.

This is therefore the key question: how well can we estimate the values of the three ratios $r_u/r'_u$, $r_t/r'_t$ and $r_c/r'_c$? Can we make general statements regarding this question, or is it process-dependent? Also, for a given theoretical uncertainty on a particular ratio, what is error on the corresponding extracted CP phase? These are the issues which we address in this paper.

We argue below that, with our present theoretical understanding of hadronic $B$ decays, neither $r_u/r'_u$ nor $r_c/r'_c$ can be computed with any degree of reliability. Therefore the KLY method cannot be used to obtain $\beta$ or $\alpha$. On the other hand, for pure $b \rightarrow d$ penguin decays such as $B_0^0(t) \rightarrow K^0\bar{K}^0$ and $B_s^0(t) \rightarrow \phi K_s$, $r_t/r'_t$ can be calculated to lie in a much narrower range. Thus, for these decays, the KLY method can be used in principle to extract $\gamma$. Unfortunately, as we will see, the error on $\gamma$ remains fairly large, in the range of 20–25%, so that this method cannot be used to obtain a precise measurement of $\gamma$. We examine possible ways to reduce this error.

We begin our analysis by addressing the question of how to estimate the ratios $r_u/r'_u$, $r_t/r'_t$ and $r_c/r'_c$. The important point to realize is that the $P_i$’s which appear in the ratios [Eqs. (3), (7), (8)] are actually matrix elements of penguin operators. Thus, in order to answer this question, we need some sort of framework in which to evaluate hadronic matrix elements. Factorization is usually employed to calculate nonleptonic amplitudes. Corrections to naive factorization have been calculated within QCD factorization [7] and the perturbative QCD (pQCD) [8] approach. In QCD factorization, naive factorization is recovered as the leading-order term, and one systematically computes corrections to it in an expansion in $\alpha_s(m_b) \sim 0.2$ and $\Lambda_{QCD}/m_b$. At $O(\alpha_s)$ the vertex and hard spectator corrections modify only the top penguin amplitude and do not introduce any additional weak phases. The penguin
or the rescattering corrections generate the up and the charm penguin pieces. The
SM effective hamiltonian for hadronic $B$ decays is [9]:

$$
H_{\text{eff}}^q = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* (c_1 O_{1f}^q + c_2 O_{2f}^q)
- \sum_{i=3}^{10} (V_{ub} V_{uq_i}^{*} c_{i}^u + V_{cb} V_{cq_i}^{*} c_{i}^c + V_{tb} V_{tq_i}^{*} c_{i}^t) O_{i}^q] + \text{h.c.} \tag{9}
$$

Here, $q$ can be either a $d$ or an $s$ quark, depending on whether the decay is a $\Delta S = 0$
or a $\Delta S = -1$ process. In the first terms, $f$ can be a $u$ or a $c$ quark, while in the
last terms, the superscript $u, c$ or $t$ indicates the flavour of the internal quark. Note
that, in the effective Hamiltonian $H_{\text{eff}}^q$ above, we have explicitly included the up
and charm penguin pieces which are generated by rescattering. The values of the
Wilson coefficients $c_i^q$ for the penguin operators evaluated at the scale $\mu = m_b = 5$
GeV, for $m_t = 176$ GeV and $\alpha_s(m_Z) = 0.117$, are [10]:

$$
c_3^q = 0.017, \quad c_4^q = -0.037, \quad c_5^q = 0.010, \quad c_6^q = -0.045,
$$

$$
c_7^q = -1.24 \times 10^{-5}, \quad c_8^q = 3.77 \times 10^{-4}, \quad c_9^q = -0.010, \quad c_{10}^q = 2.06 \times 10^{-3},
$$

$$
c_{3,5}^q = -c_{4,6}^q / N_c = P_s^i / N_c \quad c_{7,9}^q = P_e^i \quad c_{8,10}^q = 0, \quad i = u, c \tag{10}
$$

where $N_c$ is the number of colors. The leading contributions to $P_{s,e}^i$ are given by

$$
P_s^i = (\frac{\alpha_s}{\pi}) c_2 (\frac{10}{9} + G(m_i, \mu, q^2)) \quad P_e^i = (\frac{\alpha_s}{\pi}) (N_c c_1 + c_2) (\frac{10}{9} + G(m_i, \mu, q^2)),
$$

where the function $G(m, \mu, q^2)$ takes the form

$$
G(m, \mu, q^2) = 4 \int_0^1 x(1-x) \ln \frac{m^2 - x(1-x)q^2}{\mu^2} \, dx \tag{11}
$$

where $q$ is the momentum carried by the virtual gluon in the penguin diagram.

Of course, as mentioned above, we are really interested in the matrix elements of
the various operators for the decay $B \to f_1 f_2$. We therefore define new coefficients
$
\tilde{c}_i^{u,c}$ as

$$
\tilde{c}_i^{u,c} = \frac{\langle f_1 f_2 | c_i^{u,c}(q^2) O_i | B \rangle}{\langle f_1 f_2 | O_i | B \rangle} \tag{12}
$$

The values of $\tilde{c}_i^{u,c}$ can be calculated in the approaches of Ref. [7] and Ref. [8]
if the light cone distributions of the various mesons are known. However, the values
for the $\tilde{c}_i^{u,c}$ will, in general, be different in the QCD factorization and the pQCD
calculations. This is because pQCD assumes that the the light quarks forming the
final state light mesons must all be energetic while QCD factorization assumes that
the spectator quark coming from the $B$ mesons remains soft as it combines with
an energetic quark to form one of the final-state light mesons. Without adopting a
particular approach, we will follow the usual practice, which is to simply replace

$$
\tilde{c}_i^{u,c} \rightarrow c_i^{u,c}(q_{av}^2) \tag{13}
$$

and we will conservatively allow $q_{av}^2$ to vary between $m_b^2/4 \rightarrow m_b^2/2$. (Note that this
is the range which has been used in the past to take into account possible process
dependence \([11, 12]\). With this prescription, the relations between the various \(\bar{c}_{u,c}^{4} \) are the same as those between the various \(c_{i}^{u,c}(q^2)\). In particular, we still have \(\bar{c}_{4}^{u,c} = \bar{c}_{6}^{u,c}\). \hspace{1cm} (14)

In order to calculate matrix elements, we need to consider specific final states. Consider first the decay \(B^0_d \rightarrow K^0\bar{K}^0\). Using the naive factorization approximation, along with the fact that \(\bar{c}_{6}^{u,c} = \bar{c}_{4}^{u,c}\) \hspace{1cm} \text{[Eq. (14)]}, one can write

\[
P_{u,c} = \bar{c}_{6}^{u,c}(1 - \frac{1}{N_c}) \langle O_{LL} \rangle - 2 \langle O_{SP} \rangle ,
\]

(15)

where

\[
\langle O_{LL} \rangle = \langle K^0 | \bar{s}\gamma_{\mu}(1 - \gamma_5)b | B^0_d \rangle \langle K^0 | \bar{d}\gamma_{\mu}(1 - \gamma_5)s | 0 \rangle ,
\]

\[
\langle O_{SP} \rangle = \langle K^0 | \bar{s}(1 - \gamma_5)b | B^0_s \rangle \langle K^0 | \bar{d}(1 + \gamma_5)s | 0 \rangle ,
\]

(16)

and we have dropped factors common to \(P_{u,c,t}\). (The operator \(O_{SP}\) appears due to a Fierz transformation: \((V - A) \otimes (V + A) = -2(S - P) \otimes (S + P)\).) The contribution from the top penguin amplitude is given by

\[
P_t = \left[ (c_4^t + \frac{c_3^t}{N_c}) - \frac{1}{2}(c_{10}^t + \frac{c_9^t}{N_c}) \right] \langle O_{LL} \rangle - 2(c_6^t + \frac{c_5^t}{N_c}) \langle O_{SP} \rangle .
\]

(17)

In the above, we have neglected the contributions from \(c_7,8\).

It is convenient to rewrite \(P_{u,c}\) and \(P_t\) as

\[
P_{u,c} = \bar{c}_{6}^{u,c}(1 - \frac{1}{N_c}) X_r \langle O_{LL} \rangle ,
\]

\[
P_t = +(a_4 - a_6 - \frac{1}{2}a_{10}) \langle O_{LL} \rangle + a_6 X_r \langle O_{LL} \rangle ,
\]

(18)

where

\[
a_i = \begin{cases} c_i + \frac{c_i-1}{N_c}, & i = 4, 6, 10 ; \\ c_i + \frac{c_i+1}{N_c}, & i = 3, 5, 9 ; \end{cases}
\]

(19)

and

\[
X_r \equiv \left[ 1 - \frac{2 \langle O_{SP} \rangle}{\langle O_{LL} \rangle} \right] .
\]

(20)

For \(B^0_d \rightarrow K^0\bar{K}^0\),

\[
X_r = \left[ 1 + \frac{2m_K^2}{m_b} \frac{1}{m_s + m_d} \right] .
\]

(21)

Note that, for \(m_K = 500\text{ MeV}, m_b \approx 5\text{ GeV}, m_s \approx 100\text{ MeV}\) and \(m_d \approx 0\), one finds that \(X_r \approx 2\) for this decay.

Another possible decay mode is \(B^0_d \rightarrow K^*\bar{K}^*\). In this case the above analysis is unchanged, except that

\[
X_r = 1 ,
\]

(22)
since here \( \langle O_{SP} \rangle = 0 \). (Note that the KLY method does not apply to the decay \( B_d^0 \rightarrow \bar{K}K^* \) since this final state is not CP self-conjugate.) Finally, one can also consider decays such as \( B_s^0 \rightarrow \phi K_s \). However, over a large region of parameter space, \( P_c \approx P_s \approx 0 \) \[\text{[13]},\] so that the CP asymmetry probes \( \beta \) directly, and the KLY method does not apply. We therefore concentrate only on \( B_d^0 \rightarrow K^{(*)}\bar{K}^{(*)} \) decays in the analysis below.

One can now construct the ratios \( r_u \), \( r_c \) and \( r_t \) defined in the numerators of Eqs. (5), (7) and (8). We use \( r_{u0}, r_{c0} \) and \( r_{t0} \) to denote the values of \( r_u \), \( r_c \) and \( r_t \) in the naive factorization limit. In addition, it is useful to separate the dependence of \( r_{u0}, r_{c0} \) and \( r_{t0} \) on the CKM matrix elements from the hadronic dependence:

\[
\begin{align*}
    r_{u0} &= \left| \frac{V_{cd}^{\ast} V_{ud}}{V_{tb} V_{td}} \right| r_{u0}^{(had)}, \quad r_{c0} = \left| \frac{V_{ub}^{\ast} V_{ud}}{V_{tb} V_{td}} \right| r_{c0}^{(had)}, \quad r_{t0} = \left| \frac{V_{ub}^{\ast} V_{td}}{V_{ub} V_{ud}} \right| r_{t0}^{(had)}. \quad (23)
\end{align*}
\]

Using the above expressions for the \( P_i \), we find

\[
\begin{align*}
    r_{u0}^{(had)} &= \left| \frac{P_c - P_u}{P_t - P_t} \right| = \left| \frac{X_r (1 - N_i^2) (\tilde{c}_6^c - \tilde{c}_6^u)}{a_4 + a_6 (X_r - 1) - \frac{1}{2} a_{10} - X_r (1 - N_i^2) \tilde{c}_6^c} \right|, \\
    r_{c0}^{(had)} &= \left| \frac{P_u - P_c}{P_t - P_t} \right| = \left| \frac{X_r (1 - N_i^2) (\tilde{c}_6^u - \tilde{c}_6^c)}{(a_4 + a_6 (X_r - 1) - \frac{1}{2} a_{10} - X_r (1 - N_i^2) \tilde{c}_6^c)} \right|, \\
    r_{t0}^{(had)} &= \left| \frac{P_c - P_t}{P_u - P_t} \right| = \left| \frac{(a_4 + a_6 (X_r - 1) - \frac{1}{2} a_{10} - X_r (1 - N_i^2) \tilde{c}_6^c)}{(a_4 + a_6 (X_r - 1) - \frac{1}{2} a_{10} - X_r (1 - N_i^2) \tilde{c}_6^c)} \right|. \quad (24)
\end{align*}
\]

One obtains similar expressions for the ratios \( r_{u0}^{(had)}, r_{c0}^{(had)} \) and \( r_{t0}^{(had)} \), where the \( r_{i}^{(had)} (i = u, c, t) \) are defined in the denominators of Eqs. (5), (7) and (8). Note that the various ratios \( r_{u0,c0,t0}^{(had)} \) and \( r_{u0,c0,t0}^{(had)} \) depend only on the Wilson coefficients, on \( q_{av}^2 \) and on the quark masses in the factor \( X_r \). There is no dependence on hadronic quantities such as form factors and decay constants since they cancel in the ratios.

So far we have considered only the rescattering corrections which occur at \( \alpha_s \), and which generate the up and charm penguins. However, as noted earlier, there are additional vertex and the hard spectator corrections, which are also \( O(\alpha_s) \). These can be taken into account by the replacement \( a_i \rightarrow a_i^{eff} = a_i (1 + t_i) \) in Eq. (19), where \( t_i \sim O(\alpha_s) \) are process-dependent corrections to the naive factorization assumption. Since the corrections \( t_i \) depend on several poorly-known nonperturbative quantities, we will treat them as free parameters. The process dependence of the \( r_{u,c,t}^{(had)} \) and \( r_{u,c,t}^{(had)} \) then comes from three sources: (i) the value of the momentum transfer \( q_{av}^2 \) in Eq. (13), (ii) \( t_i \), the \( O(\alpha_s) \) corrections to the \( a_i \), and (iii) the dependence of the quantity \( X_r \) [Eq. (20)] on the quark and hadron masses.

Including now all the corrections to naive factorization, to first order in \( \alpha_s \) and to leading order in \( \Lambda_{QCD}/m_b \), we can make the replacement \( P_i \rightarrow P_i (1 + x) \), where the process-dependent quantity \( x \sim \alpha_s(m_b) \sim 0.2 \) \[\text{[4]}\]. We can then obtain the corrected values of \( r_{u}^{(had)}, r_{c}^{(had)} \) and \( r_{t}^{(had)} \) as

\[
\begin{align*}
    r_{u}^{(had)} &= r_{u0}^{(had)} \frac{1}{\sqrt{1 + x^2 \frac{P_i |^{2}}{|P_i|} + 2x \frac{|P_i|}{|P_i|} \cos(\delta_{tu} - \delta_t)}}, \\
    r_{c}^{(had)} &= r_{c0}^{(had)} \frac{1}{\sqrt{1 + x^2 \frac{P_i |^{2}}{|P_i|} + 2x \frac{|P_i|}{|P_i|} \cos(\delta_{cu} - \delta_c)}}, \\
    r_{t}^{(had)} &= r_{t0}^{(had)} \frac{1}{\sqrt{1 + x^2 \frac{P_i |^{2}}{|P_i|} + 2x \frac{|P_i|}{|P_i|} \cos(\delta_{ts} - \delta_t)}}.
\end{align*}
\]

\[\text{[4]}\] We have neglected a possible small complex phase in \( x \).
\[ r_c^{(\text{had})} = \frac{1}{\sqrt{1 + x^2 \frac{|P_i|^2}{|P_c|^2} + 2x \frac{|P_i|}{|P_c|} \cos(\delta_{tc} - \delta_t)}}, \]
\[ r_t^{(\text{had})} = \frac{1}{\sqrt{1 + x^2 \frac{|P_i|^2}{|P_u|^2} + 2x \frac{|P_i|}{|P_u|} \cos(\delta_{tu} - \delta_t)}}, \]

where \( P_t - P_u = |P_i|e^{i\delta_t} - |P_u|e^{i\delta_u}, \) and similarly for \( P_t - P_c. \) As usual, there are similar expressions for \( r_u^{(\text{had})}, r_c^{(\text{had})} \) and \( r_t^{(\text{had})}. \) From these expressions one can see that the nonfactorizable corrections tend to cancel in \( r_t^{(\text{had})} \) and \( r_t^{(\text{had})}, \) and so these ratios are the least affected by such effects. Note also that, as mentioned previously, the dependence on the CKM matrix elements cancels in the ratios \( r_i/r_i', \)

\[ \frac{r_i}{r_i'} = \frac{r_i^{(\text{had})}}{r_i^{(\text{had})}}. \]

We are now in a position to calculate the theoretically-allowed ranges of the three ratios of Eqs. (3), (7) and (8). There are several factors which can contribute to these ranges: since the quantities \( r_i^{(\text{had})} \) and \( r_i^{(\text{had})} \) \((i = u, c, t)\) are calculated for different processes, the momentum transfer \( q^2, \) the parameter \( X, \) [Eq. (20)], and the nonfactorizable correction \( x \) may all be different. We therefore adopt the following procedure. For each of \( r_i^{(\text{had})} \) and \( r_i^{(\text{had})}, \) we allow the values of the quark masses to vary in the following ranges: \( 4.3 \leq m_b \leq 4.9 \text{ GeV}, \) \( 1.20 \leq m_c \leq 1.30 \text{ GeV}, \) and \( 0.080 \leq m_u \leq 0.120 \text{ GeV}. \) We fix \( m_d = 6 \text{ MeV}. \) All masses are taken to be at the \( b \)-quark mass scale. In addition, for a given value of \( m_b, \) we vary \( q^2 \) between \( m_b^2/4 \) and \( m_b^2/2. \) Finally, we take \( x \) to be in the range \(-0.2 \leq x \leq 0.2.\)

In order to calculate the \( r_i^{(\text{had})} \) and \( r_i^{(\text{had})}, \) we must choose two decay processes. As a first example, we consider \( B_d^0(t) \to K^*(892)\bar{K}^*(892) \) and \( B_d^0(t) \to K^*(1410)\bar{K}^*(1410). \) The quantum numbers \((J^{PC})\) of the \( K^*(892) \) and \( K^*(1410) \) are the same, and the latter can be interpreted as a radially excited \( K^+. \) From the analysis of Ref. [14] we can expect the branching ratio of \( B_d^0 \to K^*(1410)\bar{K}^*(1410) \) to be comparable or enhanced relative to \( B_d^0 \to K^*(892)\bar{K}^*(892) \) [14]. (Note that since the final state consists of two vector mesons, an angular analysis will have to be performed to separate the three helicity states [13]. However, that does not affect our analysis here.) We find that \( r_u^{(\text{had})} \) lies in the range \( 0.14 \leq r_u^{(\text{had})} \leq 0.54, \) and similarly for \( r_u^{(\text{had})}. \) Thus, we have \( 0.26 \leq r_u/r_u' \leq 3.86. \) Similarly, the range of \( r_c^{(\text{had})} \) and \( r_c^{(\text{had})} \) is from 0.13 to 0.47, which leads to \( 0.28 \leq r_c/r_c' \leq 3.62. \) The allowed ranges for both \( r_u/r_u' \) and \( r_c/r_c' \) are clearly enormous. Since the KLY method requires a reasonably accurate theoretical prediction of these ratios, we therefore conclude that \( \beta \) and \( \alpha \) cannot be obtained using this method.

On the other hand, the allowed range for \( r_t^{(\text{had})} \) and \( r_t^{(\text{had})} \) is considerably narrower: \( 0.92 \leq r_t^{(\text{had})}, r_t^{(\text{had})} \leq 1.23. \) This is due partly to the fact that \( P_u \) and \( P_c \) are both quite a bit smaller than \( P_t, \) so that the numerator and denominator of

\footnote{It may also be useful to consider the process \( B_d^0(t) \to K^*(1680)\bar{K}^*(1680), \) since the \( K^*(1680) \) has a significantly higher branching ratio to \( K\pi \) \((\sim 39\%\) \) than the \( K^*(1410) \) \((\sim 7\%\). \) This makes reconstructing the \( K^*(1680)\bar{K}^*(1680) \) final state easier than \( K^*(1410)\bar{K}^*(1410). \)
$r_t^{(\text{had})}$ are roughly equal (and similarly for $r_t^{(\text{had})}'$), and partly to the fact that the nonfactorizable corrections approximately cancel in $r_t^{(\text{had})}$ and $r_t^{(\text{had})}'$ [Eq. (25)]. We therefore find that $0.75 \leq r_t/r_t' \leq 1.34$, a considerably tighter range than that found for $r_u/r_u'$ and $r_c/r_c'$. Thus, of the three ratios, $r_t/r_t'$ has by far the narrowest range, so that in fact it is the CP phase $\gamma$ which can be extracted with the smallest error using the KLY method. Note also that the average value of $r_t/r_t'$ in its range is 1.04, which is quite close to unity. Thus, the assumption of Eq. (7) is justified, though of course the key question is the error on the assumption. From now on, we therefore consider only the measurement of $\gamma$ using the assumption of Eq. (7).

Another pair of processes that one can consider are $B_d^0(t) \to K^0\bar{K}^0$ and $B_d^0(t) \to K^0(1460)\bar{K}^0(1460)$ ($K^0(1460)$ has the same quantum numbers as the $K^0$). Since these final states consist of two pseudoscalars, one must use the expression for $X_r$ found in Eq. (21). However, the results do not change much: we find $0.94 \leq r_t^{(\text{had})}$, $r_t^{(\text{had})}' \leq 1.22$, leading to $0.77 \leq r_t/r_t' \leq 1.30$.

Note that we have been extremely generous in estimating the allowed range or $r_t/r_t'$. We have allowed each of $r_t^{(\text{had})}$ and $r_t^{(\text{had})}'$ to take any value in their allowed ranges. That is, we have assumed that the momentum transfer $q_{av}^2$ and the nonfactorizable correction $x$ can each take completely different values in the two decay processes. However, in practice, this is not likely to be the case if two similar final states are chosen. For example, we expect a similar value of $q_{av}^2$ in the two decays $B_d^0(t) \to K^*(1410)\bar{K}^*(1410)$ and $B_d^0(t) \to K^0(1460)\bar{K}^0(1460)$, since the masses of the particles in the final states are almost equal. Also, it is reasonable to expect that the effect of nonfactorizable contributions will be similar for the decays $B_d^0(t) \to K^*(892)\bar{K}^*(892)$ and $B_d^0(t) \to K^*(1410)\bar{K}^*(1410)$, since both final states consist of two vector mesons. Thus, it is likely that the range of $r_t/r_t'$ may actually be much smaller than that calculated above, particularly for similar final states. We will come back to this point later.

Of course, it is not enough to have established that it is $\gamma$ which can be extracted with the smallest error using the KLY method. What we really want to know is: what is the size of the theoretical error on $\gamma$ in this method? This is the question we now address.

We first recall how measurements of two processes, $B_d^0(t) \to M_1 M_2$ and $B_d^0(t) \to M_1' M_2'$, along with an assumption about the ratio of penguin amplitudes, can be used to obtain $\gamma$. Using the convention $B_d^0 = bd$, we write the amplitude for $B_d^0 \to M_1 M_2$ using the parametrization of Eq. (21):

$$A_d^{M_1 M_2} = P_{ct} e^{ikct} + P_{ut} e^{i\delta_{ut}} e^{i\gamma}.$$  

(27)

The amplitude for $B_d^0 \to M_1' M_2'$ can be written similarly:

$$A_d^{M_1' M_2'} = P_{ct}' e^{i\delta_{ct}} + P_{ut}' e^{i\delta_{ut}} e^{i\gamma}.$$  

(28)

The amplitudes for $\overline{B_d^0} \to M_1 M_2$ and $\overline{B_d^0} \to M_1' M_2'$, respectively $\overline{A}_d^{M_1 M_2}$ and $\overline{A}_d^{M_1' M_2'}$, can be obtained from the above amplitudes by changing the sign of the weak phase $\gamma$.

From time-dependent measurements of the process $B_d^0(t) \to M_1 M_2$, one can
obtain the following three observables:

\[
X \equiv \frac{1}{2} \left( |A_d^{M_1M_2}|^2 + |\bar{A}_d^{M_1M_2}|^2 \right) = P_{ct}^2 + P_{ut}^2 + 2P_{ct}P_{ut} \cos \Delta \cos \gamma ,
\]

\[
Y \equiv \frac{1}{2} \left( |A_d^{M_1M_2}|^2 - |\bar{A}_d^{M_1M_2}|^2 \right) = 2P_{ct}P_{ut} \sin \Delta \sin \gamma ,
\]

\[
Z_i \equiv -\text{Im} \left( e^{-2i\beta} A_d^{M_1M_2} \bar{A}_d^{M_1M_2} \right) = P_{ct}^2 \sin 2\beta + 2P_{ct}P_{ut} \cos \Delta \sin (2\beta + \gamma)
\]

\[
+ P_{ut}^2 \sin (2\beta + 2\gamma) ,
\]

(29)

where \(\Delta \equiv \delta_{ct} - \delta_{ut}\). One can also define a fourth observable:

\[
Z_R \equiv \text{Re} \left( e^{-2i\beta} A_d^{M_1M_2} \bar{A}_d^{M_1M_2} \right) = P_{ct}^2 \cos 2\beta + 2P_{ct}P_{ut} \cos \Delta \cos (2\beta + \gamma)
\]

\[
+ P_{ut}^2 \cos (2\beta + 2\gamma) .
\]

(30)

Given that the width difference between \(B_d^0\) and \(\bar{B}_d^0\) is very small, it is unlikely that \(Z_R\) can be measured experimentally. However, note that \(Z_R\) is not independent of the other three observables:

\[
Z_R^2 = X^2 - Y^2 - Z_i^2 .
\]

(31)

Thus, one can obtain \(Z_R\) from measurements of \(X, Y\) and \(Z_i\), up to a sign ambiguity. It is also useful to further define “rotated” observables:

\[
\tilde{Z}_i \equiv Z_i \cos 2\beta - Z_R \sin 2\beta 
\]

\[
= P_{ut}^2 \sin 2\gamma + 2P_{ct}P_{ut} \cos \Delta \sin \gamma ,
\]

\[
\tilde{Z}_R \equiv Z_i \sin 2\beta + Z_R \cos 2\beta 
\]

\[
= P_{ut}^2 \cos 2\gamma + P_{ct}^2 + 2P_{ct}P_{ut} \cos \Delta \cos \gamma .
\]

(32)

Assuming that \(\beta\) is known independently (e.g. from the CP asymmetry in \(B \rightarrow J/\psi K_s\)), we can obtain \(\tilde{Z}_i\) and \(\tilde{Z}_R\) from measurements of \(B_d^0(t) \rightarrow M_1 M_2\). The observables \(X', Y', \) etc. for the second process \(B_d^0 \rightarrow M_1 M_2\) can be defined similarly using the “primed” parameters of Eq. (28).

Note that, apart from the CP phase \(\beta\), the three independent observables \(X, Y\) and \(Z_i\) depend on four unknowns: \(P_{ut}, P_{ct}, \Delta, \gamma\). Thus, one cannot obtain CP-phase information from the process \(B_d^0(t) \rightarrow M_1 M_2\) alone. However, the above equations can be solved to yield \(P_{ut}\) and \(P_{ct}\) as functions of \(\gamma\):

\[
P_{ut}^2 = \frac{\tilde{Z}_R - X}{\cos 2\gamma - 1} ,
\]

\[
P_{ct}^2 = \frac{\tilde{Z}_R \cos 2\gamma + \tilde{Z}_i \sin 2\gamma - X}{\cos 2\gamma - 1} .
\]

(33)

Thus, combining both processes, one can write

\[
r_t \equiv \frac{P_{ct}/P_{ut}}{P_{ct}'/P_{ut}'} = \frac{\sqrt{\tilde{Z}_R \cos 2\gamma + \tilde{Z}_i \sin 2\gamma - X} \tilde{Z}_R - X'}{\tilde{Z}_R' \cos 2\gamma + \tilde{Z}_i' \sin 2\gamma - X' \tilde{Z}_R - X} .
\]

(34)
Table 1: The range for $\gamma$ calculated from Eq. (34) assuming that $r_t/r'_t = 1.0 \pm \Delta r$, for the input parameters of Eq. (35), and for three input values of $\gamma$ ($\gamma_{in} = 30^\circ$, $60^\circ$, $80^\circ$).

| $\gamma_{in}$ | $\Delta r$ | range of $\gamma$    |
|---------------|------------|----------------------|
| $30^\circ$    | 0.01       | $29.8^\circ - 30.3^\circ$ |
|               | 0.05       | $28.8^\circ - 31.4^\circ$ |
|               | 0.1        | $27.6^\circ - 32.9^\circ$ |
|               | 0.2        | $25.4^\circ - 36.3^\circ$ |
|               | 0.25       | $24.4^\circ - 38.3^\circ$ |
| $60^\circ$    | 0.01       | $59.7^\circ - 60.6^\circ$ |
|               | 0.05       | $57.9^\circ - 62.4^\circ$ |
|               | 0.1        | $55.7^\circ - 64.9^\circ$ |
|               | 0.2        | $51.6^\circ - 70.1^\circ$ |
|               | 0.25       | $49.6^\circ - 72.9^\circ$ |
| $80^\circ$    | 0.01       | $79.6^\circ - 80.7^\circ$ |
|               | 0.05       | $77.6^\circ - 82.7^\circ$ |
|               | 0.1        | $75.0^\circ - 85.4^\circ$ |
|               | 0.2        | $69.9^\circ - 89.3^\circ$ |
|               | 0.25       | $67.4^\circ - 93.5^\circ$ |

We therefore see that a prediction for $r_t/r'_t$ will allow us to obtain $\gamma$.

As shown earlier, $r_t/r'_t$ is expected to lie in the range $0.75 \leq r_t/r'_t \leq 1.34$. Although this range is far more narrow than those found for $r_u/r'_u$ and $r_c/r'_c$, it is still very large. How does this translate into an error on the extracted value of $\gamma$? To examine this question, we take the true values of the theoretical parameters to be:

$$P_{ct} = 1.1\ ,\ P_{ut} = 1.0\ ,\ P'_{ct} = 1.5\ ,\ P'_{ut} = 1.36\ ,$$

$$\Delta = 30^\circ\ ,\ \Delta' = 110^\circ\ ,\ \beta = 20^\circ\ .$$ (35)

We also consider three values for $\gamma$: $30^\circ$, $60^\circ$ and $80^\circ$. Given these inputs, we can calculate the values of the experimental quantities in Eq. (34). Then, given a value of $r_t/r'_t$, we can solve for $\gamma$. In all cases, we compute the range for $\gamma$ obtained if one takes $r_t/r'_t = 1.0 \pm \Delta r$, for several values of $\Delta r$: 0.01, 0.05, 0.1, 0.2, 0.25.

The results are shown in Table 1 (We ignore the discrete ambiguities present in the extraction of $\gamma$ from Eq. (34)). For $r_t/r'_t = 1.0 \pm 0.25$, which is almost the full allowed range of $r_t/r'_t$, the theoretical error on the extracted value of $\gamma$ is about 20–25%, which is quite large. Thus, as things stand, this method cannot be used to make a precision measurement of $\gamma$.

Of course, if the theoretical uncertainty on $r_t/r'_t$ could be improved, this would greatly reduce the error on $\gamma$. For example, as shown in Table 1, if the uncertainty on $r_t/r'_t$ were 5%, the error on $\gamma$ would only be 2–3°, which would be quite acceptable. One possibility for reducing this uncertainty might be to use similar final states. Recall that we have assumed that the allowed ranges for $r_t^{(had)}$ and $r_t^{(had)}$
| $\gamma_{in}$ | $\Delta r$ | range of $\gamma$ |
|-------------|-----------|-----------------|
| $30^\circ$  | 0.01      | $27^\circ - 32^\circ$ |
|             | 0.02      | $25^\circ - 39^\circ$ |
|             | 0.03      | $24^\circ - 90^\circ$ |
|             | 0.05      | $22^\circ - 107^\circ$ |
| $60^\circ$  | 0.01      | $54^\circ - 64^\circ$ |
|             | 0.02      | $51^\circ - 74^\circ$ |
|             | 0.03      | $49^\circ - 112^\circ$ |
|             | 0.05      | $45^\circ - 121^\circ$ |
| $80^\circ$  | 0.01      | $73^\circ - 84^\circ$ |
|             | 0.02      | $69^\circ - 95^\circ$ |
|             | 0.03      | $66^\circ - 125^\circ$ |
|             | 0.05      | $62^\circ - 130^\circ$ |

Table 2: The range for $\gamma$ calculated from Eq. (34) assuming that $r_t/r'_t = 1.0 \pm \Delta r$, for the input parameters of Eq. (36), and for three input values of $\gamma$ ($\gamma_{in} = 30^\circ$, $60^\circ$, $80^\circ$).

are completely independent. That is, for each of $r_t^{(had)}$ and $r'_t^{(had)}$, we have assumed that the momentum transfer $q^2_{av}$ and the nonfactorizable correction $x$ may take completely different values in the two decay processes. However, as noted previously, it is quite likely that the momentum transfer $q^2_{av}$ and the nonfactorizable correction $x$ will take comparable values in two similar processes, which will greatly reduce the allowed range of $r_t/r'_t$.

Unfortunately, this does not help to reduce the error on $\gamma$. Suppose that, instead of the values given in Eq. (35), the theoretical parameters take the following values:

$$P_{ct} = 1.1 \, , \, P_{ut} = 1.0 \, , \, P'_{ct} = 1.15 \, , \, P'_{ut} = 1.05 \, ,$$

$$\Delta = 30^\circ \, , \, \Delta' = 40^\circ \, , \, \beta = 20^\circ.$$ (36)

Note that the primed and unprimed parameters are similar to one another, so these could represent $B$ decays to two similar final states. We again consider three values for $\gamma$: $30^\circ$, $60^\circ$ and $80^\circ$. As before, we can calculate the extracted value of $\gamma$ using Eq. (34) for $r_t/r'_t = 1.0 \pm \Delta r$, with $\Delta r = 0.01$, 0.02, 0.03, 0.05. The results are shown in Table 2. Regardless of the true value of $\gamma$, if the theoretical uncertainty on $r_t/r'_t$ is greater than about 2%, the error on the extracted value of $\gamma$ is enormous, particularly as regards the upper limit. In fact, even for an uncertainty of $\Delta r = 2\%$, the corresponding error on $\gamma$ is at least 15%, which is still large.

An examination of Eq. (34) reveals why the use of similar final states does not help to reduce the error on the extracted value of $\gamma$. If the two final states are similar, one expects that the experimental observables will also be similar, i.e. $X \simeq X'$, $Y \simeq Y'$, etc. However, in the limit that these sets of observables are equal to one another, Eq. (34) becomes independent of $\gamma$, and reduces to the tautology $1 = 1$. Thus, although the error on $r_t/r'_t$ may be reduced for similar final states, the KLY method breaks down if the states are too similar. The net effect is that the
error on $\gamma$ for similar final states is actually larger than for final states which are quite different.

From this analysis, we conclude that, for similar final states, we really need a theoretical uncertainty of $\Delta r \lesssim 1\%$ in order to be able to extract $\gamma$ with a reasonable precision. Unfortunately, with our present knowledge of hadron physics, it does not seem possible to definitively establish that $\Delta r \lesssim 1\%$ for a particular pair of $B$ decays. We therefore conclude that the KLY method works best for pairs of final states whose hadronic parameters are quite different. However, in this case $\gamma$ can only be measured with a precision of $\pm 20$–$25\%$, given our current understanding of hadronic physics.

We must emphasize that our calculations have all been done within the framework of factorization, in which there is still a great deal of hadronic uncertainty. However, there is an enormous amount of ongoing work on exclusive hadronic $B$ decays. It may well be that, in a couple of years, we will understand hadronic $B$ decays well enough to theoretically predict the value of $r/t/r'_t$ for exclusive states with reasonable precision, even for very different final states. If this happens, then the KLY method can be used to obtain $\gamma$.

In practice, however, the KLY method will probably be turned around, and will be used to learn about hadronic physics. That is, given an independent measurement of $\gamma$, along with measurements of two $b \to d$ penguin decays, Eq. (34) can be used to obtain $r_t/r'_t$. This information will allow us to test the various models of hadronic $B$ decays.

To summarize, we have re-examined the technique proposed in Ref. [6] for measuring $\beta$ (the KLY method). Their original idea is the following: consider the two $b \to d$ penguin decays $B^0_d(t) \to K^0\overline{K}^0$ and $B^0_s(t) \to \phi K_s$. If one assumes that one knows the value of the ratio of two matrix elements in the first process divided by the ratio of the corresponding matrix elements in the second process, the CP angle $\beta$ can be obtained via time-dependent measurements of these decays.

In this paper we have pointed out that there is an ambiguity inherent in this approach, namely that there are three possible ratios of matrix elements one can use. Depending on which assumption one makes, the KLY method can be used to extract $\alpha$, $\beta$ or $\gamma$. We have used factorization to show that, in fact, it is the assumption which allows $\gamma$ to be obtained which is the most accurate. Thus, it seems that the KLY method can be used to obtain this CP angle.

Unfortunately, $\gamma$ can not be extracted very precisely. There is a theoretical uncertainty in the value of the ratios of matrix elements for the two decays, which we estimate to be as large as 25–30\%. This leads to an error on $\gamma$ of about 20–25\%, which is substantial. Given our current understanding of hadronic physics, it does not appear possible at present to reduce this error.

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References

[1] For a review, see, for example, The BaBar Physics Book, eds. P.F. Harrison and H.R. Quinn, SLAC Report 504, October 1998.

[2] D. E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C 15, 1 (2000).

[3] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).

[4] A. J. Buras and R. Fleischer, Phys. Lett. B 341, 379 (1995) hep-ph/9409244.

[5] D. London, N. Sinha and R. Sinha, Phys. Rev. D 60, 074020 (1999) hep-ph/9905404.

[6] C. S. Kim, D. London and T. Yoshikawa, Phys. Lett. B 458, 361 (1999) hep-ph/9904311.

[7] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) hep-ph/9905312, Nucl. Phys. B 606, 245 (2001) hep-ph/0104110.

[8] Y. Keum, H. Li and A. I. Sanda, Phys. Lett. B 504, 6 (2001) hep-ph/0004004, Phys. Rev. D 63, 054008 (2001) hep-ph/0004173.

[9] A. J. Buras, hep-ph/9806471.

[10] R. Fleischer, Z. Phys. C 58, 483 (1993), Z. Phys. C 62, 81 (1994); G. Kramer, W. F. Palmer and H. Simma, Nucl. Phys. B 428, 77 (1994) hep-ph/9402227, Z. Phys. C 66, 429 (1995) hep-ph/9410403; N. G. Deshpande and X. G. He, Phys. Lett. B 336, 471 (1994) hep-ph/9403260.

[11] N. G. Deshpande and J. Trampetic, Phys. Rev. D 41, 2926 (1990).

[12] H. Simma and D. Wyler, Phys. Lett. B 272, 395 (1991).

[13] A. Datta, C. S. Kim and D. London, Phys. Lett. B 507, 153 (2001) hep-ph/0103321.

[14] A. Datta, H. J. Lipkin and P. J. O'Donnell, hep-ph/0102070.

[15] I. Dunietz, H. R. Quinn, A. Snyder, W. Toki and H. J. Lipkin, Phys. Rev. D 43, 2193 (1991).