From DAMA/LIBRA To Super-Kamiokande

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Abstract

We consider the prospects for probing low-mass dark matter with the Super-Kamiokande experiment. We show that upcoming analyses including fully-contained events with sensitivity to dark matter masses from 5 to 10 GeV can test the dark matter interpretation of the DAMA/LIBRA signal. We consider prospects of this analysis for two light dark matter candidates: neutralinos and WIMPless dark matter.

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I. INTRODUCTION

The DAMA/LIBRA experiment has seen, with 8.2σ significance \cite{1}, an annual modulation \cite{2} in the rate of scattering events, which could be consistent with dark matter-nucleon scattering. Much of the region of dark matter parameter space that is favored by DAMA is excluded by null results from other direct detection experiments, including CRESST \cite{3}, CDMS \cite{4}, XENON10 \cite{5}, TEXONO \cite{6, 7}, and CoGeNT \cite{8}. On the other hand, astrophysical uncertainties \cite{9, 10} and detector effects \cite{11} may open up regions that can simultaneously accommodate the results from DAMA and these other experiments.

A. A New Window for Light Dark Matter

The sensitivity of direct detection experiments is largely determined by the kinematics of non-relativistic elastic nuclear scattering. If the nucleus recoil energy is measured, the momentum transfer can then determined, which in turn determines the dark matter mass (for a standard dark matter velocity distribution). This connection allows a direct detection experiment to convert an observed event rate into a detection of dark matter with a particular mass, or alternatively, convert a lack of a signal into an exclusion of a region of \((m_{DM}, \sigma_{SI})\) parameter space.

But the procedure described above implicitly contains the uncertainties which could potentially lead to consistency between the dark matter interpretation of the DAMA/LIBRA signal and the lack of signal at other detection experiments. All direct detection experiments have a recoil energy threshold; nuclei recoiling with energies below this threshold cannot be detected. Low nuclei recoil energies imply a low dark matter mass. Thus, the sensitivity of direct detection experiments tends to suffer at low mass, as seen if Fig. \text{1}. DAMA has a recoil energy threshold which is lower than many other direct detection experiments, thus raising the possibility that its signal is a result of elastic scatter with a light dark matter candidate.

However, there are some direct detection experiments, such as TEXONO \cite{6, 7} and CoGeNT \cite{8}, which also have very low energy thresholds, and whose negative results would seem to exclude the light dark matter region which would be preferred by the DAMA signal. But there are more uncertainties at issue. One example is the measurement of the recoil energy. DAMA uses a NaI crystalline scintillator which converts the recoil energy of the nuclei into light, which is measured. But, a fraction of the recoil energy is transferred to phonons in the lattice and lost. So the measured energy must be scaled by a quenching factor to determine the actual nucleus recoil energy. But the DAMA experiment has recently noted the existence of the “channeling effect,” a property of crystalline scintillators wherein an ion moving through certain channels in the lattice will lose no energy to phonons. In this case, scaling by the old quenching factor would overestimate the recoil energy, and thus the dark matter mass. The DAMA collaboration has adjusted its analysis to account for this effect, and their resulting preferred parameter region is not excluded by any other experiment \cite{11, 12}.

On the other hand, any uncertainty in the dark matter velocity distribution would alter the dark matter mass which should be inferred from any particular measured recoil energy distribution. If local streams of dark matter alter the velocity distribution from that expected in our local neighborhood, then this could also potentially shift the mass region preferred by the DAMA recoil energy signal \cite{9, 10}. 

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These effects are detailed in Fig. 1. The application of both these effects to the DAMA signal is somewhat controversial. It is not at all clear that a dark matter stream with sufficient velocity to shift the DAMA preferred region appreciably is reasonable from the point of view of astrophysics. And the effect of channeling on a scintillator like DAMA for the $1 - 10$ GeV energy range is currently being studied and cross-checked by various groups.

As if this controversy were not enough, three groups [13, 14, 15] have analyzed the spectrum of modulations within the recoil energy bins which DAMA reports. Naturally, they all reach different conclusions, ranging from ruling out the dark matter interpretation of DAMA to declaring it completely consistent with the spectrum of the DAMA signal.

It is far from clear what DAMA is actually seeing. What is clear, however, is that if DAMA is seeing dark matter, one preferred region of parameter space has dark matter mass in the range $m_X \sim 1 - 10$ GeV and spin-independent proton scattering cross section $\sigma_{SI} \sim 10^{-5} - 10^{-2}$ pb. This is a mass region where several different experimental effects can push in different directions, and potentially create a window where dark matter could be observed at DAMA while not being ruled out by other experiments.

Moreover, there are a variety of theoretical models which attempt to explore this region of parameter space. Although neutralinos have been proposed as an explanation [16], such low masses and high cross sections are not typical of weakly-interacting massive particles (WIMPs), and alternative candidates have been suggested to explain the DAMA signal [17, 18, 19, 20, 21, 22, 23, 24, 25].

B. Cross-checking DAMA

The current state of affairs also makes it abundantly clear that complementary experiments are likely required to sort out the true nature of this result. Other direct detection experiments may play this role. In this work, we note that corroborating evidence may come from a very different source, namely, from the indirect detection of dark matter at Super-Kamiokande (Super-K) [26]. In contrast to direct detection experiments, which rapidly lose sensitivity at low masses, Super-K’s limits remain strong for low masses. But in contrast to other indirect detection experiments, which can only be compared to DAMA after making astrophysical assumptions which are highly uncertain (such as the cuspiness of the dark matter density profile near the galactic center), Super-K offers a way of testing the DAMA result which is largely model independent. Super-K is therefore poised as one of the most promising experiments to either corroborate or exclude many dark matter interpretations of the DAMA/LIBRA data.

In Sec. II, we show the relation between the DAMA and Super-K event rates. In Sec. III we show that there is significant potential for Super-K to extend its reach to dark matter masses from 5 to 20 GeV and provide sensitivity that is competitive with, or possibly much better than, direct detection experiments. In Sec. IV we apply our analysis to two specific dark matter candidates that have been proposed to explain DAMA: neutralinos [16] and WIMPless dark matter [18, 19, 20]. We present our conclusions in Sec. V.

II. BOUNDING $\sigma_{SI}$ WITH SUPER-KAMIOKANDE

Super-K can probe dark matter in the Sun or Earth’s core annihilating to standard model (SM) particles, which subsequently emit neutrinos. Muon neutrinos then interact weakly at
or near the detector to produce muons, which are detected at Super-K. The observed rate of upward-going muon events places an upper bound on the annihilation rate of dark matter in the Sun or the Earth’s core. For low-mass dark matter, the dominant contribution to neutrino production via dark matter annihilation is from the Sun [27], on which we focus.

The total annihilation rate is

$$\Gamma = \frac{1}{2} C \tanh^2[(aC)^{1/2} \tau] ,$$

(1)

where $C$ is the capture rate, $\tau \approx 4.5$ Gyr is the age of the solar system, and $a = \langle \sigma_{\text{ann}} v \rangle / (4\sqrt{2}V)$, with $\sigma_{\text{ann}}$ the total dark matter annihilation cross section and $V$ the effective volume of WIMPs in the Sun ($V = 5.7 \times 10^{30} \text{cm}^3 (1 \text{GeV}/m_X)^{3/2}$) [27, 28, 29]. If $\langle \sigma_{\text{ann}} v \rangle \sim 10^{-26} \text{cm}^3 \text{s}^{-1}$ (to get the observed dark matter relic density), then for the range of parameters considered here, the Sun is in equilibrium [27, 30, 31] and $\Gamma \approx \frac{1}{2} C$. WIMP evaporation is not relevant if $m_X \gtrsim 4 \text{GeV}$ [28, 29, 30].

The dark matter capture rate is [27]

$$C = \left[ \left( \frac{8}{3\pi} \right)^{1/2} \frac{\sigma \rho_X/m_X \bar{v} M_B}{m} \right] \left[ \frac{3}{2} \frac{\langle v^2 \rangle}{\bar{v}^2} \right] f_2 f_3 .$$

(2)

The first bracketed factor counts the rate of dark matter-nucleus interactions: $\sigma$ is the dark matter-nucleus scattering cross section, $\rho_X/m_X$ is the local dark matter number density, $m$ is the mass of the nucleus, and $M_B$ is the mass of the capturing object. The velocity dispersion of the dark matter is $\bar{v}$, and $\langle v^2 \rangle$ is the squared escape velocity averaged throughout the Sun. The second bracketed expression is the “focusing” factor that accounts for the likelihood that a scattering event will cause the dark matter particle to be captured. The parameters $f_2$ and $f_3$ are computable $O(1)$ suppression factors that account for the motion of the Sun and the mismatch between $X$ and nucleus masses, respectively. $f_2 \sim 1$ for solar capture [27].

The capture rate is thus a completely computable function of $\sigma/m_X$. Assuming $\rho_X = 0.3 \text{ GeV cm}^{-3}$, $\bar{v} \sim 300 \frac{\text{km}}{\text{s}}$, $\frac{3}{2} \frac{\langle v^2 \rangle}{\bar{v}^2} \sim 20$ [27], and taking $f_2 \sim f_3 \sim 1$, one finds $C \sim 10^{29} (\sigma/m_X) \text{ GeV pb}^{-1} \text{s}^{-1}$.

The major remaining particle physics uncertainty is the neutrino spectrum that arises from dark matter annihilation. Assuming the dark matter annihilates only to SM particles, a conservative estimate for neutrino production may be obtained by assuming that the annihilation to SM particles is dominated by $b\bar{b}$ production for $m_b < m_X < M_W$, by $\tau\bar{\tau}$ production for $m_W < m_X < m_t$, and by $W, Z$ production for $m_X > m_t$ [32].

Super-K bounds the $\nu_\mu$-flux from dark matter annihilation in the Sun. Since the total annihilation rate is equal to the capture rate, this permits Super-K to bound the dark matter-nucleon scattering cross section using Eq. (2). Fig. 1 shows the published bounds from Super-K, limits from other dark matter direct detection experiments and the regions of $(m_X, \sigma_{SI})$ parameter space favored by the DAMA signal given astrophysical and detector uncertainties. As evident from Fig. 1 the published Super-K bounds (solid line) do not test the DAMA-favored regions. But we will see that consideration of the full Super-K event sample provides significant improvement and extends Super-K’s sensitivity to low masses and the DAMA-favored regions.
FIG. 1: Direct detection cross sections for spin-independent X-nucleon scattering as a function of dark matter mass $m_X$. The black solid line is the published Super-K exclusion limit [33], and the black dashed line is our projection of future Super-K sensitivity. The magenta shaded region is DAMA-favored given channeling and no streams [12], and the medium green shaded region is DAMA-favored at $3\sigma$ given streams but no channeling [10]. The light yellow shaded region is excluded by the direct detection experiments indicated. The dark blue cross-hatched region is the prediction for the neutralino models [16] and the light blue cross-hatched region is the parameter space of WIMPless models with connector quark mass $m_Y = 400$ GeV and $0.3 < \lambda_b < 1.0$. Other limits come from the Baksan and MACRO experiments [33, 34, 35], though they are not as sensitive as Super-K.

III. PROJECTION OF SUPER-K SENSITIVITY

As shown in Fig. I, Super-K currently reports dark matter bounds only down to $m_X = 18$ GeV. For heavier dark matter, it is estimated [33] that more than 90% of the upward-going muons will be through-going (i.e., will be created outside the inner detector and will pass all the way through it). However, one can study dark matter at lower masses by using stopping, partially contained, or fully contained muons, that is, upward-going muons that stop within the detector, begin within the detector, or both.\textsuperscript{1}

Our strategy for projecting dark matter bounds from these event topologies is as follows: we begin by conservatively assuming that the measured neutrino spectrum at low energies matches the predicted atmospheric background. In any given bin with $N$ neutrino events, the $2\sigma$ bound on the number of neutrinos from dark matter annihilation is then $2\sqrt{N}$. This bounds the dark matter annihilation rate to neutrinos, which, for a conservative choice of the neutrino spectrum, implies a bound on the capture rate, and thus on $\sigma/m_X$. To include

\textsuperscript{1} Testing the dark matter interpretation of DAMA has with Super-K through-going muon data has also been considered [29].
experimental acceptances and efficiencies, we scale our results to the Super-K published bound at $m_X = 18$ GeV, assuming these effects do not vary greatly in extrapolating down to the $5 - 10$ GeV range of interest.

The annihilation of dark matter particles $X$ typically produces neutrinos with $E_\nu/m_X \sim 1/3 - 1/2$. The muons produced by weak interactions of $\nu_\mu$ lie in a cone around the direction to the Sun with rms half-angle of approximately $\theta = 20^\circ \sqrt{10\: \text{GeV}/E_\nu}$ [36]. Bounds on dark matter with $m_X = 18$ GeV were set using neutrinos with energies $E_\nu \sim 6 - 9$ GeV [33]. The event sample used consisted of 81 upward through-going muons within a $22^\circ$ angle of the Sun collected from 1679 live days.

For masses $m_X \sim 5 - 10$ GeV, the $\nu_\mu$ are typically produced with energies between $2 - 4$ GeV. At these energies the detected events are dominantly fully-contained events [37], so we use this event topology with only events within the required cone around the Sun. The number of events is

$$N_{\text{solar}} = N \frac{1 - \cos \theta}{2},$$

(3)

where $N$ is the total number of fully-contained muon events expected in the $2 - 4$ GeV energy range and $\theta$ is the cone opening angle. Super-K expects $N_{\text{solar}} = 168$ such fully contained events per 1000 live days [37].

We convert this limit on event rate to a limit on the neutrino flux by dividing by the effective cross section for the Super-K experiment in the relevant energy range. The effective cross-section can be estimated by dividing the estimated rate of events by the predicted atmospheric flux, integrated over the relevant range of energies [37]. For fully contained events with $E_\nu \sim 2 - 4$ GeV, the effective cross section is $\sim 2.1 \times 10^{-8}$ m$^2$. For upward through-going events with $E_\nu \sim 8$ GeV, the effective cross-section is $\sim 1.7 \times 10^{-8}$ m$^2$.

Assuming the neutrino events are detected primarily in either the fully-contained ($2 - 4$ GeV) or through-going sample ($\sim 8$ GeV), one can set $2\sigma$ limits on the time-integrated neutrino flux due to dark matter annihilation:

$$\Phi_{\text{FC}}^{\text{max}} = \frac{2\sqrt{N_{\text{FC}}}}{2.1 \times 10^{-8} \text{ m}^2} \sim 1.6 \times 10^9 \text{ m}^{-2} \sqrt{\frac{N_{\text{days}}}{1679}},$$

$$\Phi_{\text{TG}}^{\text{max}} = \frac{2\sqrt{N_{\text{TG}}}}{1.7 \times 10^{-8} \text{ m}^2} \sim 1.0 \times 10^9 \text{ m}^{-2} \sqrt{\frac{N_{\text{days}}}{1679}},$$

(4)

where $N_{\text{FC}} = 168 (N_{\text{days}}/1000)$ and $N_{\text{TG}} = 81 (N_{\text{days}}/1679)$ are the number of fully-contained and through-going events within the angle and energy ranges, respectively, scaled to $N_{\text{days}}$ live days.

The ratio of these flux limits obtained from the fully-contained and through-going samples are then equal to the ratio of $\sigma/m_X$ in the $5 - 10$ GeV regime to the same quantity at 18 GeV. We find

$$\frac{1.6 \times 10^9 \text{ m}^{-2}}{1.0 \times 10^9 \text{ m}^{-2}} \sim \left( \frac{\sigma_{5-10}}{m_{5-10}} \right) \left( \frac{\sigma_{18}}{18 \text{ GeV}} \right)^{-1},$$

(5)

where $\sigma_{5-10}$ is the Super-K bound on the dark matter nucleon cross-section for a dark matter particle with mass in the range $5 - 10$ GeV, and $\sigma_{18}$ is the bound for a dark matter particle with mass 18 GeV. In Fig. 1 this projected Super-K bound is plotted, assuming 3000 live days of the SK I-III run. This bound gets better at lower energies and may beat other direct detection experiments.
IV. PROSPECTS FOR VARIOUS DARK MATTER CANDIDATES

We now consider specific examples of theoretical models that have been proposed to explain the DAMA result. We first consider neutralino dark matter. Although neutralinos typically have larger masses and lower cross sections than required to explain the DAMA signal, special choices of supersymmetry parameters may yield values in the DAMA-favored region [16].

The region of the \((m_X, \sigma_{SI})\) plane spanned by these models [16] that do not violate known constraints is given in Fig. 1. We see that if Super-K’s limits can be extended to lower mass, it could find evidence for models in this class. Note, however, that many of these models have \(\rho < 0.3 \text{ GeV cm}^{-3}\), and for these models Super-K’s bound on the cross section will be less sensitive.

WIMPless dark matter provides an alternative explanation of the DAMA/LIBRA signal [19]. These candidates are hidden sector particles that naturally have the correct relic density [18]. In these models, the dark matter particle \(X\) couples to SM quarks via exchange of a particle \(Y\) that is similar to a 4th generation quark. The Lagrangian for this interaction is

\[
\mathcal{L} = \lambda_f X \bar{Y}_L f_L + \lambda_f X \bar{Y}_R f_R .
\]  

(6)

The Yukawa couplings \(\lambda_f\) are model-dependent, and it is assumed that only the coupling to 3rd generation quarks is significant, while the others are Cabibbo-suppressed.\(^2\) One finds that the dominant nuclear coupling of WIMPless dark matter is to gluons via a loop of \(b\)-quarks (\(t\)-quark loops are suppressed by \(m_t\)). The \(X\)-nucleus cross section is given by [19]

\[
\sigma_{SI} = \frac{1}{4\pi} \frac{m_N^2}{(m_N + m_X)^2} \left[ \sum_q \frac{\lambda_b^2}{m_Y - m_X} \left[ Z B^p_b + (A - Z) B^n_b \right] \right]^2 ,
\]  

(7)

where \(Z\) and \(A\) are the atomic number and mass of the target nucleus \(N\), and \(B^p_b, B^n_b = (2/27) m_p f_{p,n}^b / m_b\), where \(f_{p,n}^b \approx 0.8\) [38, 39].

In Fig. 1, we plot the parameter space for WIMPless models with \(m_Y = 400\) GeV and \(0.3 < \lambda_b < 1.0\). These models span a large range in the \((m_X, \sigma_{SI})\) plane, and overlap much of the DAMA-favored region. We see that Super-K’s projected sensitivity may be sufficient to discover a signal that corroborates DAMA’s. But WIMPless models illustrate an important caveat to the analysis above; if there are hidden decay channels, then the annihilation rate to SM particles is only a fraction of \(\Gamma_{tot}\), and Super-K’s sensitivity is reduced accordingly.

V. SUMMARY

The DAMA/LIBRA signal has focussed attention on the possibility of light dark matter, and alternative methods for corroborating or excluding a dark matter interpretation are desired. We have shown that Super-K, through its search for dark matter annihilation to neutrinos, has promising prospects for testing DAMA at low mass.

Using fully contained muon events, we expect that current super-K bounds may be extended down to \(M_{DM} \sim 5 - 10\) GeV, and can test light dark models (such as neutralino models [16] and WIMPless models [19]). We have the intriguing prospect that the

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\(^2\) This is a reasonable assumption and is consistent with small observed flavor-changing neutral currents.
DAMA/LIBRA signal could be sharply tested by an indirect detection experiment in the near future.

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