Emergent Universe Scenario and the Low CMB Multipoles

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Abstract. In this paper, based on Phys. Rev. D 91, no. 8, 083534 (2015), we study super inflation in the context of the Emergent Universe (EU) scenario. The existence of a super inflating phase before the onset of slow roll inflation arises in any Emergent Universe model. We found that the super inflationary period in the EU scenario produces a suppression of the CMB anisotropies at large scale which could be responsible of the observed lack of power at large angular scales of the CMB.

1. Introduction
Cosmological inflation has become an integral part of the standard model of the Universe. Apart from being capable of removing the shortcomings of the standard cosmology, it gives important clues for large scale structure formation [2, 3, 4, 5] (see [6] for a review).

The standard slow-roll inflation predicts a slightly red-tilted power spectrum of the primordial perturbation. This means that large scale (small $k$) perturbation has more power than small scale perturbation. This red tilt arises from the fact that the Hubble parameter slowly decreases with time as the scalar field rolls downwards to its potential.

The recent cosmological observations are entirely consistent with the simplest slow-roll inflationary models [8, 9, 10]. Probing that the observed universe is almost flat, the bispectal non-Gaussianity parameter $f_{\text{NL}}$ measured is consistent with zero. The scalar perturbation spectral index $n_s$ is less than one, which is a measure of the tilt discussed above ($n_s < 1$ red tilt, $n_s > 1$ blue tilt and $n_s = 1$ scale invariant spectrum). All these results are predicted by the simplest slow-roll inflationary models.

However, there are intriguing observations on cosmic microwave background radiation, suggesting a lack of power at large angular scales (very low multipoles, $l < 40$). These results were first obtained by COBE [11] and WMAP [8] and now are confirmed by Planck [9]. Although these results are well within our cosmic variance and statistically their significance is still low, the power deficit is not insignificant. The Planck Collaboration reported a power deficit in the low multipoles CMB power spectrum of order $5\%-10\%$ (with respect to the Planck best-fit $\Lambda$CDM model [12, 13]) with statistical significance $2.5 \sim 3 \sigma$.

There are different approaches developed in order to explain this problem. One possibility is to introduce a cutoff in the primordial power spectrum [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. This cutoff is normally introduced by hand but linked to the spatial curvature scale [25], string physics [20, 22, 23], the bouncing universe scheme [23, 24], or a fast-rolling stage in the evolution of the inflaton field [17]. This approach is interesting since it has been claimed that from the
Figure 1. Schematic representation of a potential for a standard emergent universe scenario.

observed angular power spectrum, it is possible to deconvolve the primordial power spectrum for a given set of cosmological parameters. The most prominent feature of the recovered primordial power spectrum is a sharp, infrared cutoff on the horizon scale [26, 27].

In this respect, it has been suggested that the low-$l$ power could be related to a period of superinflation, previous to the standard slow-roll inflationary regimen [29, 28, 30], where the superinflationary period is characterized by the condition $H > 0$. In particular, in Ref. [28], this possibility was studied in the context of bouncing universes and was discussed regarding the emergent universe scheme.

The emergent universe (EU) refers to models in which the universe emerges from an Einstein static state, inflates, and is then submitted into a hot big bang era. Such models are appealing since they provide specific examples of nonsingular (geodesically complete) inflationary universes. These models have been studied in different contexts during recent years; see [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 45, 47, 48, 49, 50, 51, 52].

In this work, we study the period of superinflation in the context of the EU scenario. The existence of a superinflating phase before the onset of slow-roll inflation arises in any EU scenario [28] and has not been studied thus far. As an arena to explore the superinflation phase, we consider the EU model developed in Ref. [31, 32]. This EU model is based on general relativity and considered a closed universe dominated by a scalar field minimally coupled to gravity. We found that the superinflationary period in the EU scenario produces a suppression of the CMB anisotropies at large scale which could be responsible of the observed lack of power at large angular scales of the CMB [11, 8, 9].

2. The Emergent Universe scenario

In the emergent universe scenario, the Universe is initially in a past-eternal classical Einstein static state which eventually evolves into a subsequent inflationary phase; see [31, 32, 33, 34, 35, 36, 37, 38]. During the past-eternal static regime, it is assumed that the scalar field is rolling on the asymptotically flat part of the scalar potential with a constant velocity, providing the conditions for a static universe. But once the scalar field exceeds some value, the scalar potential slowly drops from its original value. The overall effect of this is to distort the equilibrium behavior breaking the static solution. If the potential has a suitable form in this region, slow-roll inflation will occur.
In the EU scenario, the evolution of the scale factor could be modeled by the following expression (see [31, 32]):

\[ a(t) \simeq a_0 + A e^{H_0 t} \quad (1) \]

where \( a_0, A, H_0 \) are positive constants. This universe is past asymptotic to an Einstein static state, since \( a(t) \to a_0 \) as \( t \to -\infty \). Thus, \( a_0 \) is identified with the radius of the Einstein static universe. At late times, on the other hand, \( a(t) \to A e^{H_0 t} \) and the model approach a de Sitter expansion phase.

We can note that a generic characteristic of the EU scenario is the existence of a superinflation phase where \( \dot{H} > 0 \) before slow-roll inflation. In this model the evolution described in Eq. (1) corresponds precisely to the superinflationary phase of the evolution of the EU, which asymptotically approaches the de Sitter expansion phase.

The mechanism which generates this superinflationary period depends on the particular EU model under consideration, but it is a generic characteristic of the EU scenario. For example, in the models in [31, 32], it is considered a closed universe. Then, in this case, the spatial curvature is responsible for the superinflationary period.

3. The primordial perturbation in EU

The scalar perturbations to the FRW geometry, in the longitudinal gauge, can be written as follows:

\[ ds^2 = (1 + 2\Phi) dt^2 - (1 - 2\Phi) a(t)^2 d\vec{x}^2, \quad (2) \]

where \( \Phi \) is the Newtonian gravitational potential.

Normally in the emergent universe scenario, the Universe is positively curved; see for example [31, 32]. In this first approach to the problem, we have neglected the contributions of the space curvature to the primordial perturbation, but we have a first approach to this point in Ref. [1], see also [55].

The equation for the perturbations in momentum space is given by

\[ v_k'' + k^2 v_k - \frac{z''}{z} v_k = 0. \quad (3) \]

where we have used the Mukhanov variable [7, 59],

\[ v_k = a \left( \delta \phi_k + \frac{\phi'}{h} \Phi_k \right), \quad (4) \]

where \( \delta \phi_k \) are the perturbations in the inflaton field and \( ' \) is derivative with respect to the conformal time \( \eta = \int dt / a \) and we have used units \( M_p = (8\pi G)^{-1/2} = 1 \). Also, we have defined

\[ z = \frac{a\phi'}{h}, \quad (5) \]
\[ h = \frac{a'}{a}. \quad (6) \]

In order to solve Eq. (3), we consider the evolution of the scale factor \( a(t) \) given in Eq. (1) written in the conformal time,

\[ a(\eta) = \frac{a_0}{1 - e^{a_0 H_0 \eta}}, \quad \eta < 0. \quad (7) \]

From Eq. (3) and using Eq. (7), we obtain
\[ v''_k - (a_0 H_0)^2 e^{a_0 H_0 \eta} \left( \frac{1 + e^{a_0 H_0 \eta}}{-1 + e^{a_0 H_0 \eta}} \right) v_k + k^2 v_k = 0 , \]  
(8)

where we have considered \( z''/z \approx a''/a \). This equation is solved by

\[ v_k(\eta) = \frac{1}{\sqrt{2k}} \left[ \frac{e^{-ik \eta}}{1 - e^{a_0 H_0 \eta}} \right] \times \]

\[ _2F_1 \left( -1 - \frac{ik}{a_0 H_0} - \sqrt{1 - \left( \frac{k}{a_0 H_0} \right)^2}, -1 - \frac{ik}{a_0 H_0} + \sqrt{1 - \left( \frac{k}{a_0 H_0} \right)^2} ; 1 - 2 \frac{ik}{a_0 H_0}; e^{a_0 H_0 \eta} \right) , \]

where \( _2F_1 \) is the hypergeometric function.

In the solution (9) we have considered (and appropriately normalized) the solution of Eq. (8) such that in the short wavelength limit, the normalized positive frequency modes correspond to the minimal quantum fluctuations,

\[ v_k \approx \frac{e^{-ik \eta}}{\sqrt{2k}}, \quad a H << k . \]

(10)

Following [17], we consider the spectrum of \( Q \equiv v/a \) which becomes constant at late time,

\[ P_Q = \frac{k^3}{2\pi^2} |Q|^2 \rightarrow \frac{H_0^2}{\pi^2} \frac{\chi^2 \Gamma[x_1] \Gamma[x_2]}{\Gamma[x_2] \Gamma[x_2^2] \Gamma[x_3] \Gamma[x_3^2]} , \]

(11)

where we have defined

\[ x_1 = 1 - 2i\chi \]

(12)

\[ x_2 = 2 - i\chi - \sqrt{1 - \chi^2} \]

(13)

\[ x_3 = 2 - i\chi + \sqrt{1 - \chi^2} \]

(14)

\[ \chi = \frac{k}{a_0 H_0} \]

(15)

We can note that in the short wavelength limit\( (k >> H_0) \), we recovered the standard result of a nearly scale-invariant spectrum,

\[ P_Q \rightarrow \left( \frac{H_0}{2\pi} \right)^2 . \]

(16)

At superhorizon scales, the two modes \( Q_k \) and \( \Phi_k \) are related by a \( k \)-independent rescaling so that the spectrum given by Eq. (11) directly translates into the spectrum of \( \Phi \). In Fig. (2) we have plotted the spectrum of \( P_Q \) as a function of \( \chi \) obtained from the analytical calculation Eq. (11), solid line. We can note that there is a suppression of the long wave modes as we expected given the superinflationary regimen.

4. The effective power spectrum

In the last section we study the spectrum generated during the superinflationary regimen. From this result and by following Ref. [24] we can estimate an effective power spectrum generated by the EU model which takes into account the early period of superinflation and the subsequent period of standard slow roll. In this respect, from the result of previous section, we can note that the scale invariance of the spectrum is the result of inflationary evolution after the
superinflationary stage. Then, if the inflationary period is generated by the standard slow-roll conditions, we can expect the usual slight red-tilted spectrum during this period. From Eq. (11) and by following Ref. [24], we could model this situation by considering the following scalar spectrum:

$$P_\Phi = A \frac{\chi^2 \Gamma[x_1] \Gamma[x_1^*]}{\Gamma[x_2] \Gamma[x_2^*] \Gamma[x_3] \Gamma[x_3^*]}.$$  

(17)

For small $k$, this spectrum reproduces the behavior of spectrum Eq. (11), i.e., suppression of the long wave modes. On the other hand, when $k \gtrsim H_0 a_0$, the spectrum reproduces the usual slightly red spectrum of slow-roll inflation.

Another possibility is to consider a cutoff in the spectrum. This cutoff is related to the transition from the superinflationary regimen to slow-roll inflation which occurs when the mode $k_{max}$ exits the horizon, after which we would get the usual red-tilted power law spectrum. In this case, we can write

$$P_\Phi = \begin{cases} 
\bar{A} \frac{\chi^2 \Gamma[x_1] \Gamma[x_1^*]}{\Gamma[x_2] \Gamma[x_2^*] \Gamma[x_3] \Gamma[x_3^*]}, & k < k_{max} \\
A \left( \frac{k}{k_{max}} \right)^{n_s-1}, & k > k_{max}
\end{cases}$$

(18)

The constants $A$ and $\bar{A}$ are chosen in order to match the superinflationary power spectrum Eq. (11) with the slow-roll inflationary power spectrum at $k = k_{max}$.

5. CMB Anisotropies

We compute the theoretical CMB power spectrum of this model by using CLASS code [53, 54]. In this first approximation, in the context of the EU scenario, for the study of the consequences of the superinflationary phase to the CMB power spectrum, we have simplified the code for computing the $C_l$ by considering an approximation of the spectrums Eqs. (17) and (18). In
particular, we approximate the spectrum of the superinflation phase Eq. (11) with the following expression (see for example [28]),

$$P_Q \sim \frac{H_0^2}{4\pi^2} \frac{\chi^2}{(1 + \chi)^2},$$  \hspace{1cm} (19)

which is obtained by considering the evolution of the scale factor given by Eq. (1) and the Hubble crossing condition,

$$k = a H \Rightarrow k = H_0 A e^{H_0 t}.$$  \hspace{1cm} (20)

In Fig. 2 we have plotted the spectrum Eq. (19), where we can compare it with the analytic calculation Eq. (11). We can note that the approximated spectrum reproduces very well the spectrum obtained by the analytic calculation.

In order to show the suppression at large scales coming from the EU scenario, we have plotted in Fig. 3 the temperature power spectrum obtained from the pure power law (dashed line), from the EU scenario (solid line), and from the EU scenario with a cutoff (dotted line). The points show the Planck data. We can note that compared to the standard power-law model, the $C_l$ spectrum in the emergent universe scenario is suppressed at large scales.

In these examples, we have considered the following values for the parameters in the case of, the EU scenario in Eqs. (17) and (19): $A = 2.07 \times 10^{-9}$, $n_s = 0.9603$, and $a_0 H_0 = 0.0002 \text{ Mpc}^{-1}$. In the case of the EU scenario with the cutoff Eqs. (18) and (19), we consider $A = 2.42 \times 10^{-9}$, $n_s = 0.967$, $a_0 H_0 = 0.0003 \text{ Mpc}^{-1}$, and $k_{\text{max}} = 0.0015 \text{ Mpc}^{-1}$. At this moment we are not doing a best-fit calculation of these parameters, just showing two particular cases and how they produce a suppression of the spectrum at large scales.

6. Conclusions
In recent cosmological observations, there are intriguing results on the cosmic microwave background radiation suggesting a lack of power at large angular scales. This situation is
interesting because it may give us clues towards physics at the very early Universe, perhaps before slow-roll inflation. In this context, the EU models are an interesting arena to explore preinflationary physics and its possible implications for the CMB anomalies.

In this work, based on Ref. [1], we study the primordial perturbations in the context of the EU scenario. In particular, we focus on the primordial perturbations generated during the superinflationary phase. We find that the superinflationary period in the EU scenario produces a suppression of the primordial perturbations at large scale which could be responsible for the observed lack of power at large angular scales of the CMB.

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