A conceptual rainfall-runoff mathematical model to simulate runoff using daily amount of rainfall for arid and semi-arid region

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Abstract. The structure of water resources needed long-term runoff. Because of the huge operating and maintenance costs of sites collecting large quantities of data, it is always a problem for developing countries. Data collection apart from these estimates of runoff from the river basin, irrigation schedules, flood control and dam design and other engineering structures have been very important to plan and manage. In hydrology, various models are available for understanding and predicting the Runoff process. The tank model is often favored for its simplicity among several conceptual precipitation runoff models. However, due to the calibration of too many parameters of the model, it requires a lot of time and effort to get better results. In fact, the price has gone up for product calibration. In this case, a new mathematical model is being developed to analyze the amount of runoff, which is rainfall. The paper illustrates the application of Ordinary Differential Equation the field of Mathematics which helps to relate the value of function itself for an unknown function of one or several variables.

1. Introduction
Water is a key element for the livelihood. It is a key to economic development and growth, especially in view of rapid population growth of agriculture and industry. The Asian economies are mainly agricultural in most Asian countries. The success of agriculture is dependent entirely on a continuous supply of water from rivers, water reservoirs and canals due to arid and semi-arid conditions. Rainwater, as well as people and cattle falling on the surface of the earth, is extremely important to agriculture. In most parts of the world, the principal source of water for agriculture is rainfall. The rainfall characteristics vary between places, every day, month by month and year by year [4,5,6].

Agricultural land in India was stated at 60.45 % in 2016, as per the records of the World Bank collection of development indicators. But this agricultural land more and more replace with industries because of the fast growth of economy and society [12]. Farmers in India are dependent on rivers, reservoirs, tanks and wells for water at the time of irrigation. So in India it is very important to have technique to predict Runoff. In rainfall runoff modelling, it is almost impossible to measure exact amount of runoff, and that is mainly due to the high catchment heterogeneity and the limitation of measurement techniques [13]. This limitation and the need to extrapolate information from the available measurements initiated the application of mathematical modelling. Mathematical models are used to predict how the hydrologic system will respond to precipitation events, and are applied widely in water resources [15]. However, the multiple variables involved in calibration of the rainfall-runoff patterns with regards to local observation data can be complicated and time consuming in order to enhance model predictability. The assessment of the effects of soil use and land cover change in water sources is a factor in agricultural and civil water management modeling.
Surface runoff is an important consideration when assessing water resources monitoring and water quality resolution and the issue of water quantity such as flood predictions, and environmental and biological water environments [1]. Rainwater also contributes an excessive amount of excess nutrients and pesticides because it washes into the water. Erosion is also caused by intense torrential downpours and flooding that destroys vegetation and structures [2].

Recently, there have been proposed many conceptual models, including but not limited to: models of the Sacramento (Brazil and Hudlow 1981), of Clark (Clark 1945), of Nash (Nash 1957), of HBV (Bergstrom 1992, 1995), of HYMOD (Moore 1985) and of the watershed model (Crawford and Linsley, 1966). (Luquen & Mary, 1974, 1983). (see ref. [3,8,11,14,16,17,20,21,23])

There are various rainfall models that are used to simulate floods and daily rushing events in India. The models are conceptual representations of runoff precipitation. The Sugwara device was developed for flood hydrology from Japan and included three tanks which were vertically stacked. It was created with Japanese environmental conditions in mind [7,10,22].

Hairul Basyar of Aceh (Indonesia) invented the Tank model following.

Phien et al. [24] have developed two watershed models that can be used to simulate water pollution in Thailand. The model tank in India also applies to the various basins [25]. Ramasastri K.S. Airport. Using the Tanzanian model for simulating the Malaprabha basin runoff from southern Indian Western Ghats, the Model for the Malaprabha Basin has been found to be very efficient [19]. The tank model was used in 1984 to collect information about daily streamflows in two central Indian sub-basins. To solve this problem, the model was developed [18].

The use of the model tank for hydrological studies has been quite limited in India and only a few studies have so far been carried out.

2. Methodology and Model Structure

Sugawara, a representative of the Japan Science and Technology Agency, suggested the tank prototype in 1951. This model integrates physical processes into the hydrological model. The idea is to measure the flow rate of the fuel in the tank by dividing it into smaller samples. Factors such as infiltration, filtration, storage, surface runoff, sub-base flow, and base flow are all relevant to the outcome. Runoff in the top tank provides water for collection in the bottom tank. How do the inlets on the second, third, and fourth tank work? When, finally, water flows across the surface area and rushes off the surface, the season of Monsoon will arrive. In order to supply an abundant amount of water to the rinse solution, the rinsing surface was increased.

In this respect, a mathematical model of the tank model is attempted. Tank Model assumes four vertical tanks representing the flush surface, intermediate fluid, sub-base fluid, and base fluid as shown in figure 1 respective

The side channel output describes the runoff calculi. The office eliminates the top tank surface runoffs, the second tank’s center runoff, the third tank’s base runoff and the 4th tank base flow. The structure of the zonal underground water typically shown in the figure can be considered as corresponding figure 1 and figure 2.
Let us split the model shown in figure 1 in three different tanks as shown in figure 3(a), figure 3(b) and figure 3(c).

If we move each tank's side outlet(s) to the bottom of the tanks, we convert the model into one of the linear forms displayed in figure 4(a) and figure 4(b).
3. Results
Let us assume below parameters to obtain equation of runoff for each of three distinct tanks
\( p_i(t) \): Precipitation (Input),
\( r_i(t) \): Runoff (Output), where \( i = 1, 2, 3 \)
\( s(t) \): Storage
\( k, k_0, k_1 \) and \( k_2 \) are cross-section area of outlet for different tank.

Case 1: For the tank of type shown in figure the following equations hold:

\[
\frac{d}{dt}[s(t)] = p_1(t) - r_1(t); \quad \text{where } r_1(t) = k s(t)
\]
\[
\therefore \frac{d}{dt}[s(t)] + k s(t) = p_1(t)
\]

Figure 4(a). tank with one outlet
\[
\therefore \frac{d}{dt}[r_1(t)] + k r_1(t) = k p_1(t)
\]

Since rainfall is impulsive in nature at time \( t = 0 \) to an empty linear tank \( p_1(t) = \delta(t) \)
where \( \delta(t) \) is a Dirac Delta function.

Hence by solving above equation using Laplace Transforms output will be exponential function as shown in below equation.

\[ r_1(t) = ke^{-kt} \]

Case 2: For the tank of type shown in figure the following equations hold:

![Figure 5. Tank with two outlets](image)

\[
\frac{d}{dt}[s(t)] = p_2(t) - r_2(t),
\]
where \( r_2(t) = (k_0 + k_1)s(t) \)
\[
\therefore \frac{d}{dt}[s(t)] + (k_0 + k_1)s(t) = p_2(t)
\]
\[
\therefore \frac{d}{dt}[r_2(t)] + (k_0 + k_1)r_2(t) = (k_0 + k_1)p_2(t).
\]

Similarly, by solving above equation using Laplace Transform with initial conditions
\[ p_2(t) = \delta(t) \] Runoff (output) will be an exponential function as shown below:
\[ r_2(t) = (k_0 + k_1)e^{-(k_0+k_1)t} \]

**Case 3:** For the tank of type shown in figure the following equations hold:
\[ \frac{d}{dt}[s(t)] = p_3(t) - r_3(t), \]
where \[ r_3(t) = (k_0 + k_1 + k_2)s(t) \]
\[ \therefore \frac{d}{dt}[s(t)] + (k_0 + k_1 + k_2)s(t) = p_3(t) \]
\[ \therefore \frac{d}{dt}[r_3(t)] + (k_0 + k_1 + k_2)r_3(t) = (k_0 + k_1 + k_2)p_3(t) \]

![Figure 6. Tank with three outlets](image)

Adding Runoff getting by all the three vertical tanks we get total amount of Runoff
\[ R(t) = r_1(t) + r_2(t) + r_3(t) \]
\[ R(t) = ke^{-kt} + 2(k_0 + k_1)e^{-(k_0+k_1)t} + (k_0 + k_1 + k_2)e^{-(k_0+k_1+k_2)t} \]

4. **Conclusion:**
For efficient water resource management, accurate approximation of runoff is needed. It is relatively easy to estimate the runoff of the tank model. Although the tank is divided into zones, there is only one model tank in the entire tank. The rainfall is taken into consideration in each area as rain water. We have developed a differential first order equation and we try to find the rush quantity in each tank at a certain time. Lastly, find Runoff summation for each tank and find the total runoff. By daily rainfall data we can simulate this model. It is widely used by hydrologists and engineers, but not sufficient data can lead to a lack of approximation. The irrigation preparation, crop rotation and cropping pattern can be envisaged with the knowledge of water in the water basin.

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