Holographic Inclusive Central Particle Production at the LHC

*Richard Nally[
*Timothy Raben[2]
†Chung-I Tan[3]

#Department of Physics
Stanford University, Stanford, California 94305 USA

*Department of Physics and Astronomy
University of Kansas, Lawrence, KS 66045 USA

†Department of Physics
Brown University, 182 Hope St. Providence, RI 02902 USA

We describe how the holographic, BPST Pomeron can be used to describe inclusive central production at the LHC.

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1 Introduction

The AdS/CFT has provided new avenues for investigating hadron behavior in the non-perturbative regime. It has become a novel tool for understanding how conformal theories can describe scattering experiments. Here we present a summary of a comprehensive treatment [1] of a conformal theory describing central inclusive production at the LHC. Conformal scattering can be described as the flow of infrared safe observables leading to vacuum expectations

$$\sigma_w(p) = \int d^4x e^{-ipx} \langle 0 | O(w)^\dagger(x) D[w] O(0) | 0 \rangle,$$

where $O$ is the source for an initial state. The inclusive event shape distribution $D[w]$, a product of a set of local operators, measures flows of conserved quantities.

In momentum space, generalized optical theorems identify inclusive cross sections as discontinuities of appropriate forward amplitudes. For example, the differential cross section $d\sigma_{ab \rightarrow c+X}/(d^3p_c/E_c)$ of the process $ab \rightarrow c+X$ can be identified as the discontinuity in $M^2$ of the amplitude for the six-point process $abc \rightarrow a'b'c$. This process can be described holographically where a high energy scattering process depends crucially on the conformal data. Symbolically, we have

$$\frac{d\sigma_{ab \rightarrow c+X}}{d^3p_c/E_c} \propto \frac{1}{2ts} \text{Disc}_{\text{AdS/CFT}} M^2 T_{abc \rightarrow a'b'c} \rightarrow \text{Regge Limit} \sim p_{\perp}^{-\delta},$$

where $\delta$ is related to a conformal scaling dimension. Here we review the key results needed to arrive at Eq.(1.2) and apply it to scattering at the LHC. Further details and examples can be found in [1].

2 Inclusive Cross-Sections and Discontinuities

In general, n-point momentum space Wightman correlation functions are related to the forward discontinuity of the associated n-point time-ordered Green’s function[2]. For example, the invariant single particle inclusive differential cross section,
\( d\sigma_{ab \to c+X}/d^3p_c/E_c \), can be expressed as \(^2\)

\[
\begin{align*}
    d\sigma & \propto \sum_X (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_X) \left| \langle p_c, X | p_a, p_b \rangle \right|^2 \propto \langle p_a, p_b | \widetilde{O}_c(p_c) | p_a, p_b \rangle. \\
(2.1)
\end{align*}
\]

Here \( \widetilde{O}_c = \int d^4xe^{-ip_c \cdot x} \varphi_c(x) \varphi_c(0) \) is the Fourier transform of product of two local operators. Since \( \varphi_c(x) \varphi_c(0) \) is not time-ordered, one is dealing with a Wightman function. A similar analysis holds for higher order correlators \( \langle \widetilde{O}_w(1) \widetilde{O}_w(2) \cdots \rangle \), leading to

\[
\sigma_w = \sum_{c_1,c_2,\cdots} \int d^4p_{c_1}d^4p_{c_2} \cdots \frac{1}{2i} w(p_{c_1}, p_{c_2}, \cdots) \text{Disc} M^2_\alpha T_{\gamma' c_1' c_2' \cdots} \rightarrow \gamma' c_1 c_2 \cdots. \\
(2.2)
\]

**Holographic Inclusive Cross Sections** Within the AdS/CFT correspondence, scattering amplitudes can be computed via a perturbative sum of “Witten diagrams” in analogy to conventional field theory. Here a 4D, flat space, boundary conformal field theory amplitude is related to a 10D, curved space, bulk string amplitude. Scattering amplitudes can be written as a bulk amplitude, connected to a boundary CFT function via a convolution, over the curved space, with wavefunctions \( \varphi(z) \). We find that the differential inclusive cross section for \( a + b \to c + X \) becomes

\[
\begin{align*}
    \frac{d\sigma_{ab \to c+X}}{d^3p_c/E_c} & \approx \frac{1}{2i\delta} \int \{ \Pi_{i=1-6} d\mu(z_i) \varphi_n(z_i) \} \text{Disc} M^2>0 \{ T_{abc \to a' b' c'}(p_i, z_i) \}. \\
(2.3)
\end{align*}
\]

As in conventional QCD, high energy scattering can be dominated by the exchange of Reggeons. Bulk amplitudes, in a AdS space of curvature \( R \), depend on redshifted external momenta \( p^\mu : \tilde{p}^\mu \simeq (z/R)p^\mu \). The dominant contribution, the BPST Pomeron \(^3\), can be described via a propagator with a discontinuity in \( \tilde{s} \), with its leading behavior given by \( \text{Disc}_s \tilde{K}_P(\tilde{s}, 0, z, z') \propto \tilde{s}^{j_0} \), with \( j_0 \simeq 2 - 2/\sqrt{\lambda} \). In the particular case of DIS, a moment expansion leads to an anomalous dimensions for \( j \simeq 2 \) at strong coupling given by \( \gamma(j) = \sqrt{2\sqrt{\lambda(j - j_0) - j + O(\lambda^{-3/4})}} \).

In the high energy limit, the appropriate 6-point amplitude is dominated by double Pomeron exchange. The amplitude can be written in a factorized form, \( T_{abc \to a' b' c'} = \Phi_{13} * \tilde{K}_P * V_{ab} * \tilde{K}_P * \Phi_{24} \). With this factorized form, the kernel and initial wave-function dependence can be integrated out leading to an inclusive particle density \( \rho \) for central

\(^\dagger\)This is the leading AdS Pomeron intercept. Higher order corrections can be found in \(^4\)
production given by
\[ \rho(\vec{p}_T, y, s) \equiv \frac{1}{\sigma_{\text{total}}} \frac{d^3\sigma_{ab \to c+X}}{dp_T^3/E_c} = \beta \int_{0}^{z_{\text{max}}} \frac{dz_3}{z_3} \tilde{\kappa}^{j_0} \left[ \varphi_c(z_3) \right]^2 \left[ \text{Im} \mathcal{V}_{c}(\tilde{\kappa}, 0, 0) \right], \tag{2.4} \]
where \( \beta \) is an overall constant and \( \kappa = (-t)(-u)/M^2 \). \( z_s \) can be determined via the string constant by demanding \( 2\alpha'\tilde{\kappa} = O(1) \), so that \( z_s \sim \frac{R}{\sqrt{2\alpha'\kappa}} = \frac{\lambda^{1/4}}{\sqrt{2}\kappa} \).

We can thus approximate Eq. (2.4) by integrating only up to \( z_3 = z_s << z_{\text{max}} \), where the exponential factor is of order one and can be neglected. In the appropriate kinematic regime, \( \varphi(z) \simeq z^\tau \), where \( \tau \) is the twist. Thus Eq. (2.4) becomes
\[ \frac{1}{\sigma_{\text{total}}} \frac{d^3\sigma_{ab \to c+X}}{dp_T^3/E_c} = \beta \int_{0}^{z_s} \frac{dz}{z} z^{2\tau_c} (\kappa z^2/R^2)^j_0 e^{-\left(2\kappa/\lambda^{1/2}\right) z^2} \simeq \beta' \kappa^{-\tau_c}, \tag{2.5} \]
where we have introduced a new normalization constant \( \beta' \). This is the form of Eq.(1.2). For scalar glueballs, \( \tau_c = \Delta_c = 4 \) and we have \( \rho(p_\perp, y, s) \sim p_\perp^{-8} \). This result follows essentially from conformality and does not depend on the details of a confinement deformation. It serves as a generalized scaling law for inclusive distribution, as was worked out for exclusive fixed-angle scattering \[5\].

3 Conformal Central Production at the LHC

Phenomenologically we can look for the behavior of Eq.(1.2) via an ansatz:
\[ (1/2\pi p_T)(d^2\sigma/dp_T d\eta) = A/(p_T + C)^B. \]
We consider three data sets: ALICE collab. p-pb collisions at \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \) \[8\], and ATLAS Collab. p-p collisions at \( \sqrt{s} = 8 \) & 13 TeV \[9, 10\]. The results of fits to our model are shown in Table 1 and Figures 1(a) and 1(b). The ALICE datasets have been run at various pseudorapidity, \( \eta \), ranges and demonstrates virtually no variation in kinematics under changes in pseudorapidity. Overall there is excellent agreement between the fit model and the data.

The fits are compatible at the two-\( \sigma \) level with the power law exponent being independent of both the pseudorapidity and the center of mass measurement. This agrees well with Eq.1.2 applied to the AdS/CFT. There are two important caveats, however. First, the overall normalization of the distributions varies sharply between the two types of measurements, with the proton-lead collisions seeming to have a cross
| Dataset                | $A/10$ (GeV$^{-2}$) | $B$     | $C/(1$ GeV) |
|------------------------|---------------------|---------|-------------|
| ALICE $|\eta| < 0.3$ [8]  | 38.48 ± 8.26 | 7.23 ± 0.09 | 1.32 ± 0.04 |
| ALICE $-0.8 < \eta < -0.3$ [8] | 37.60 ± 7.97 | 7.22 ± 0.08 | 1.30 ± 0.04 |
| ALICE $-1.3 < \eta < -0.8$ [8] | 43.00 ± 9.29 | 7.30 ± 0.09 | 1.31 ± 0.04 |
| ATLAS 8 TeV [9]        | 4.46 ± 2.60        | 7.03 ± 0.264 | 1.07 ± 0.123 |
| ATLAS 13 TeV [10]      | 5.77 ± 3.38        | 6.96 ± 0.265 | 1.12 ± 0.126 |

Table 1: Fitted values of $A/(p_T + C)^B$ for three data sets. Both central values and statistical errors are quoted.

Figure 3.1: Fits of inclusive double-differential cross sections. Datasest have been rescaled for visual clarity.

The holographic argument presented here does not offer an easy way to compute this prefactor, so we have no real prediction for it. Certainly we expect higher-order corrections, which are unaccounted for in our tree-level calculation, to importantly influence the normalization. Moreover, from considerations of the mechanisms for proton-lead and proton-proton scattering, it is clear that the difference between these two can have a physical interpretation, rather than being interpreted as an artifact of our calculation. It is of note that the exact scaling behavior deviates from the expected $\delta = 8$ prediction. However this can be explained via the addition of tensor glueball exchange, finite coupling effects, or eikonalization as discussed in [1].
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