The transmission phase of an electron plays a crucial role in various quantum interference phenomena. Full characterization of the coherent transport therefore requires a reliable phase measurement, but this is still challenging. One may envisage a quantum two-path interferometer because the interference is measured as a function of the phase difference between the two paths. For instance the phase shift across a quantum dot (QD), in which one can control the quantum state of single electrons, can be measured using a QD embedded in one of the two arms of the interferometer. The theory predicts a Breit-Wigner type phase shift across a Coulomb peak (CP) and a $\pi/2$ phase shift across a Kondo-singlet state and both were experimentally investigated.

The Breit-Wigner type phase shift was confirmed by a pioneering experiment for a QD embedded in a multi-terminal Aharonov-Bohm (AB) interferometer. The phase shift was derived from a smooth shift of AB oscillations and a universal phase shift across two Coulomb peaks of a spin degenerated level for a Kondo correlated QD. Although several mechanisms have been proposed to account for the universal phase lapse origins of the behavior remain unaccounted. This is also related to the fact that only a few experiments have been reported for the phase measurement due to difficulty in realizing a reliable phase measurement for QDs. In a two-terminal AB interferometer, which is usually considered as a two-path interferometer, the phase of the AB oscillation is fixed to either $0$ or $\pi$ at zero magnetic field due to boundary conditions imposed by the two-terminal geometry, whereas the real transmission phase across the QD is not. The $0-\pi$ rigidity of the observed phase called phase rigidity therefore implies that the two-terminal AB ring is not a true two-path interferometer; because not only direct two paths but also paths of an electron encircling the AB ring multiple times contribute to the interference.

A multi-terminal as well as a multi-channel AB interferometer was employed to avoid the phase rigidity and to measure the transmission phase shift across a gate-defined QD embedded in one of the two arms. In these experiments lifting of the phase rigidity was confirmed by observation of a smooth phase shift with gate voltage at a fixed magnetic field. On the other hand lifting of the phase rigidity does not readily ensure that the observed interference is a pure two-path interference.

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(Dated: 25 June 2015)

A quantum two-path interferometer allows for direct measurement of the transmission phase shift of an electron, providing useful information on coherent scattering problems. In mesoscopic systems, however, the two-path interference is easily smeread by contributions from other paths, and this makes it difficult to observe the true transmission phase shift. To eliminate this problem, multi-terminal Aharonov-Bohm (AB) interferometers have been used to derive the phase shift by assuming that the relative phase shift of the electrons between the two paths is simply obtained when a smooth shift of the AB oscillations is observed. Nevertheless the phase shifts using such a criterion have sometimes been inconsistent with theory. On the other hand, we have used an AB ring contacted to tunnel-coupled wires and acquired the phase shift consistent with theory when the two output currents through the coupled wires oscillate with well-defined anti-phase. Here, we investigate thoroughly these two criteria used to ensure a reliable phase measurement, the anti-phase relation of the two output currents and the smooth phase shift in the AB oscillation. We confirm that the well-defined anti-phase relation ensures a correct phase measurement with a quantum two-path interference. In contrast we find that even in a situation where the anti-phase relation is less well-defined, the smooth phase shift in the AB oscillation can still occur but does not give the correct transmission phase due to contributions from multiple paths. This indicates that the phase relation of the two output currents in our interferometer gives a good criterion for the measurement of the true transmission phase while the smooth phase shift in the AB oscillation itself does not.
There is a possibility that contributions from multi-path interferences still remain. Previously we have developed a new type of interferometer realized in an AB ring contacted to tunnel-coupled wires. It can be tuned into a two-path interferometer in the weak tunnel-coupling regime such that the interferometer works as a two-path interferometer, where the two output currents oscillate with anti-phase as shown in Fig. 2(a). For panels (a) - (c) of Fig. 2 we plot the oscillating components of the currents as a function of magnetic field, which are obtained from raw data by performing a complex fast Fourier transform (FFT) are plotted. Three figures are measured at the different gate voltages of $\Delta V_{M1,2}$, which are indicated in (d). (d) Modulation of geometrical phase as a function of $B$ and $\Delta V_{M1,2}$. The black solid lines are added to highlight the change of the slope.

FIG. 1. SEM picture of device A and measurement setup. Output currents are measured for a constant voltage bias across the resistance $R = 10 \, \Omega$. Dashed lines indicate electron trajectories for the two-path interference.

FIG. 2. (a), (b), (c) Quantum oscillations as a function of magnetic field $B$ observed in $I_1$ (black line) and $I_2$ (red line) in the weak tunnel-coupling regime. Only oscillating parts extracted from raw data by performing a complex fast Fourier transform (FFT) are plotted. Three figures are measured at the different gate voltages of $\Delta V_{M1,2}$, which are indicated in (d). (d) Modulation of geometrical phase as a function of $B$ and $\Delta V_{M1,2}$. The black solid lines are added to highlight the change of the slope.

The device was fabricated on a two-dimensional electron gas formed in a GaAs/AlGaAs heterostructure [electron density $n = 3.21 \times 10^{11} \, \text{cm}^{-2}$, electron mobility $\mu = 8.6 \times 10^5 \text{cm}^2/\text{Vs}$ at the temperature of $T = 4.2 \, \text{K}$; see Fig. 2. The interferometer was defined by applying negative voltages on surface Schottky gates and locally depleting electrons underneath the gates. It consists of an AB ring at the center and tunnel-coupled wires on both ends of the ring. The coupling energy of the tunnel-coupled wires can be controlled by the gate voltages $V_{T1}$ and $V_{T2}$. The gate voltages $V_{M1}$, $V_{M2}$ ($V_{M3}$, $V_{M4}$) are used to modulate the wave vector of electrons in the upper (lower) path. A QD can also be formed by applying the gate voltages $V_L$, $V_P$ and $V_R$. We measured two samples with a slightly different size of the AB ring and QD (device A and B). The data shown in Fig. 2 and Fig. 3 was measured in device A and that in Fig. 4 for device B. Electrons are injected from the lower left contact by applying an AC bias ($20 \sim 100 \, \mu\text{V}$, $23.3 \, \text{Hz}$) and currents are measured at the two right contacts by voltage measurements across the resistance ($I_{1(2)} = V_{1(2)}/R$) using a standard lock-in technique.

We first tuned the tunnel-coupled wires into the weak coupling regime such that the interferometer works as a two-path interferometer, where the two output currents oscillate with anti-phase as shown in Fig. 2(a). For panels (a) - (c) of Fig. 2 we plot the oscillating components of the currents as a function of magnetic field, which are
obtained by performing a complex fast Fourier transform (FFT) of the raw data, filtering out the noise outside the oscillation frequency and performing a back transform. The two-path interference is sensitive to the difference of the transmission phase shift between the two paths across the AB ring \( \theta = \int \mathbf{k} \cdot d\mathbf{l} - \frac{\pi}{\hbar} BS + \varphi_{\text{bot}} \). The first term is the geometrical phase depending on the path length \( \mathbf{l} \) and the wave vector of an electron \( \mathbf{k} \), the second term is the AB phase controlled by the magnetic field \( B \) penetrating the surface area \( S \) enclosed by the two paths, and the third term is the transmission phase shift across the QD, respectively. Fig. 2(a) shows the phase shift induced by the modulation of the AB phase.

We then measured the phase shift induced by modulation of the geometrical phase, where the wave vector of electrons passing through the upper path is controlled by the gate voltages \( V_{M1} \) and \( V_{M2} \). Here \( V_{M1} \) and \( V_{M2} \) are shifted simultaneously by the same amount. The result is shown in Fig. 2(d). The oscillating part of \( I = I_1 - I_2 \) as a function of magnetic field, which mainly consists of the anti-phase components, is plotted for the gate voltage shift \( V_{M1,2} \) along the vertical axis around the configuration used for the measurement of Fig. 2(a). Around \( \Delta V_{M1,2} = 0 \), where the anti-phase oscillations of the two output currents are observed, the phase smoothly shifts along the vertical axis with a certain slope. Around the gate voltage shift from \(-5 \) mV to \(-25 \) mV and the magnetic field range from \(-15 \) mT to \(-30 \) mT, the phase smoothly shifts as well but with a slightly different slope as indicated with the black solid lines, where the two output currents do not oscillate with anti-phase as shown in Fig. 2(b). For the more negative voltage shift and the magnetic field range from \(-30 \) mT to \(-45 \) mT, abrupt phase jumps of \( \pi \) along the vertical axis are observed similarly to a two-terminal device that suffers from the phase rigidity. In this region the two output currents oscillate in phase as shown in Fig. 2(c).

The anti-phase oscillations of the two output currents indicate that the total current \( (I_1 + I_2) \) is independent on \( \theta \). This is a clear indication that interferences coming from encircling paths around the AB ring are absent and hence the realization of the pure two-path interference as depicted by the dashed lines in Fig. 1. On the other hand, when the two output currents do not oscillate with anti-phase, paths encircling the AB ring also contribute to the interference even though the magneto oscillations still show a smooth phase shift as a function of gate voltages at a fixed magnetic field. In such a case, however, the observed phase shift is modified from the true transmission phase shift as we will demonstrate in the following.

First we show that the phase relation between the two output currents is a good criteria to exclude the contributions of multi-path interferences and allows for a reliable measurement of the transmission phase shift. For this we carefully tuned the interferometer to observe the anti-phase oscillations as shown in Fig. 3(b). For this condition, we observed a smooth phase shift induced by the modulation of the geometrical phase through \( V_{M3,4} \) with a single constant slope [Fig. 3(a)]. At the same time we also measure the transmission phase shift across a QD, where the experimental results can be compared with theory [11,23] [Fig. 3(c) and (d)]. The QD is formed by tuning the gate voltages \( V_1 \), \( V_p \) and \( V_R \) and the phase shift across a CP is observed by recording quantum oscillations as a function of magnetic field at each value of the plunger gate voltage \( V_p \). This result is presented in Fig. 3(c). The current \( I_2 \) averaged over one oscillation period of the magnetic field mimics the shape of the CP with a finite background current coming from the current through the upper path of the AB ring. The black solid line is a Lorentzian fit of the CP, which is used to calculate the transmission phase shift expected from Friedel’s sum rule and depicted by the red solid lines [11,23].

The numerical values of the observed phase shift are obtained from a complex FFT of \( (I_1 - I_2) \). The observed phase shift is in good agreement with the theoretically expected \( \pi \)-phase shift. This result confirms that the phase evolution obtained under the condition of anti-phase oscillations of the two output currents is the true transmission phase shift observed for the pure two-path interference.

We now turn to the phase shift measurements when the two output currents are not kept anti-phase over the entire gate voltage \( V_p \) scan across a CP. The measured phase shift is shown in Fig. 4(a). The phase smoothly shifts across the CP by \( 1.5\pi \), which is inconsistent with the \( \pi \)-phase shift expected from Friedel’s sum rule (red solid line). In this data the two output currents oscillate with anti-phase for \( V_p \) only around the center of the CP (red circles) as shown in Fig. 4(b). For the en-
In summary, we employed an AB ring with tunnel-coupled wires to demonstrate how to measure the true transmission phase of an electron. We find that lifting the phase rigidity, i.e., the observation of a smooth phase shift at a fixed magnetic field in a multi-terminal AB interferometer does not ensure a correct measurement of the true transmission phase. Our original AB interferometer, on the contrary, allows for the measurement of the true transmission phase shift by simply tuning it into a regime where the two output currents oscillate anti-phase. This interferometer is hence extremely suitable to investigate unsolved problems related to the transmission phase such as the universal phase behavior for large quantum dot\textsuperscript{20,21}.

S. Takada acknowledges financial support from JSPS Research Fellowships for Young Scientists, French Government Scholarship for Scientific Disciplines and the European Unions Horizon 2020 research and innovation program under the Marie Sklodowska-Curie grant agreement No 654603. M.Y. acknowledges financial support by Grant-in-Aid for Young Scientists A (No. 23684019) and Grant-in-Aid for Challenging Exploratory Research (No. 25610070). C. B. acknowledges financial support from the French National Agency (ANR) in the frame of its program BLANC FLYELEC Project No. anr-12BS10-001, as well as from DRECl-CNRS/JSPS (PRC 0677) international collaboration. A.L. and A.D.W. acknowledge gratefully support of Mercur Pr-2013-0001, DFG-TRR160, BMBF - Q.com-H 16KIS0109, and the DFH/UFA CDFA-05-06. S. Tarucha acknowledges financial support by JSPS, Grant-in-Aid for Scientific Re-
search S (No. 26220710), MEXT project for Developing Innovation Systems, and JST Strategic International Cooperative Program.

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