Over half a century of studying carbon-12

Tzany Kokalova Wheldon
School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham
B15 2TT, UK
E-mail: t.wheldon@bham.ac.uk

Abstract. Carbon-12 is one of the most studied light nuclei yet it continues to surprise and provide a rigorous testing ground for a wide range of physics, from nucleosynthesis models to theories of symmetries. This paper discusses the background motivating the investigations of $^{12}$C and summarises the recent results, with an emphasis on collective excitations and the high-energy structure together with possible future directions for this most intriguing of nuclei.

1. Introduction
The earliest measurements in the field of nuclei involved using alpha-particles as a probe of the nucleus through Rutherford (Coulomb) scattering [1]. The observations made at the beginning of the 20th century led to the formation of the most basic ideas about the structure of matter, and the presence of the heavy nucleus at the heart of the atom. During the first half of the 20th century, alpha particles continued to play a central role in nuclear physics, with the Liquid Drop Model [2] providing evidence for the tightly bound nature of alpha-conjugate nuclei, e.g. $^{12}$C, $^{16}$O, $^{20}$Ne etc. This led to the development of a geometric model of nuclei by Halstad and Teller [3], in which the number of bonds between pairs of alpha particles was considered. As the 20th century progressed, predictions for a special state in $^{12}$C were made by the Yorkshire astronomer Fred Hoyle [4]. This prediction was soon verified [5], corresponding to a $0^+$ state at 7.654 MeV – now known as the Hoyle state – and which has a special three-alpha structure [6]. The importance of this state to the field of nuclear clustering can not be overstated. Since the mid 1950s, physicists have striven to uncover its precise configuration and its very existence has paved the way for countless theoretical calculations using a wide range of models.

The essential underlying concept behind the emergence of clustering as nuclei are excited, is elegantly captured in the Ikeda diagram [7]. Figure 1 shows threshold energies for a variety of cluster configurations. Nuclear matter is predicted to crystallise and clump as the internal energy approaches that of the mass-difference needed to form the cluster sub-units. The evidence in favour of this picture is most easily seen by observing the close proximity of the Hoyle state to the $^{12}$C $\rightarrow ^8$Be-$\alpha$ threshold at 7.367 MeV.

2. Development of models
It became clear from mean-field shell- and Nilsson-models that these approaches were inadequate to reproduce cluster structure in nuclei and new ideas were needed, to unravel on the structure of the Hoyle state. A modern illustration of this inability of shell-model calculations to reproduce
the excitation energy of the Hoyle state is shown in figure 2 [8] and demonstrates, in a striking way, the absence of any candidate in the correct energy range despite including a model space of $4\hbar\Omega$.

In the 1960s, building on the original idea of Margenau [9] and developed by Brink and Block [10] among others [11], cluster models were developed in which the nucleons were combined into spin-zero quartets. In these models, scale parameters determine the size of the alpha-particle and the relative position of each quartet is described by a vector. The $N$-alpha wave-function is constructed using a Slater determinant and the anti-symmetrisation operator incorporates the essential qualities of the Pauli Exclusion Principle [12]. Finally, the optimal arrangement and size of alpha-particles is obtained through the variation approach. Such microscopic cluster models could be applied to calculations of excited nuclear (cluster) states, but ultimately ignore the internal structure of the clusters.

Despite the significant strides made by microscopic alpha-cluster models, the development of theoretical frameworks has continued unabated to the present day. Several of the most notable advancements during the last half century were the moves to address clustering in neutron-rich matter. This involved the adaptation of ideas from atomic physics in which valence neutrons are
covalently bonded between large sub-clusters, providing additional binding [13, 14, 15, 16] and, in some cases, predicted to stabilise linear structures that in the corresponding alpha-conjugate nucleus would be unbound due to bending or breathing modes [17]. These ideas have led to Antisymmetrised Molecular Dynamics (AMD) in which no preformed alpha particles or relative coordinates between nucleons are assumed a priori [18]. The energy of the system is calculated variationally using an effective nucleon-nucleon potential and $N$ degrees-of-freedom included. Other features are nucleon position described by Gaussian wave packets. This framework and a variant known as Fermi-Molecular Dynamics (FMD) [19], that additionally allows the widths of the Gaussian wave packets to be varied, have been highly successful at predicting density distributions across isotopic chains and have become increasingly relevant with the advent of more exotic radioactive beams able to probe nuclear structure farther from stability.

As computing power has increased, so has the ability to to utilise this for increasingly complex nuclear systems. An example from the turn of the century is the calculation of $^8$Be using Green’s function Monte-Carlo ab initio calculations [20] for which the two-alpha density distribution is clearly manifest. The current state-of-the-art in ab initio models is lattice QCD calculations using effective field theory as performed by Epelbaum and coworkers [21]. Though immensely challenging, these have become sufficiently sophisticated that not only has the $^{12}$C ground-state configuration been calculated, but also the excited Holye state [21, 22].
In parallel with the above recent developments, over the preceding decade, an alternative interpretation of the nature of nuclear matter in the Hoyle-state and its analogues has emerged; that is one of an alpha condensate. Tohsaki, Horiuchi and others [23, 24] have modelled this exotic state of matter by implementing the THSR wave function (name after the authors of Ref. [23]) in which, taking $^{12}\text{C}$ as an example, the three alpha particles are trapped in the $0S$ state of a wide harmonic oscillator potential (of width $B$) as shown in Figure 3(a). The four nucleons of each alpha are confined in the $0S$-state of a narrow potential (of width $b$) and all the nucleons are anti-symmetrised. The right-hand panel of figure 3 shows the mechanism for excitation in the condensate picture, with an alpha-particle being excited to a $d$-state to form a collective resonance.

![Figure 3](image)

**Figure 3:** Panel (a): A pictorial representation of a three-alpha condensate in a shallow $0S$ of a harmonic oscillator potential defined by width $B$. Each quartet of nucleons is confined in a smaller, local potential governed by width $b$. Based on Ref. [24]. Panel (b): Representation of how a $2^+$ excitation can be generated in an alpha condensate, via promotion of an alpha particle to a $d$-orbit [25, 26].

Overall much progress has been made in modelling clustered nuclear matter, but as is evident from the above discussion, precision measurements are needed in order to discriminate between the different predictions. Some background and recent progress on such investigations is given below.

3. Experimental progress

Experimentally, the properties of $^{12}\text{C}$ have been scrutinised, not only to probe the Hoyle state and its decay with evermore precision, but also to search for excited collective modes that would also reveal the exact three-alpha nature. Meanwhile, other investigations have concentrated on searching for so called Hoyle-state analogue states in other alpha-conjugate nuclei. The searches for collective modes of the Hoyle state have been driven by the rotational model of Bohr and Mottelson [27] and such structure can be seen in the lightest alpha-conjugate nucleus – $^8\text{Be}$ – forming the well known two-alpha dimer with excitations built on the ground-state configurations enabling the moment-of-inertia and, hence, the size, to be elucidated. As mentioned in section 2 the eight-nucleon $^8\text{Be}$ ground-state structure has been calculated beginning from the nucleon-nucleon force [20]. On the experimental side, Datar et al., [28] have successfully measured the very weak ($1.5 \times 10^{-7}$) $\gamma$-decay branch between the $4^+ \rightarrow 2^+$ excitations, despite the initial $4^+$ state lying at an excitation energy of 10.8 MeV. Though this branching ratio is small, its relative intensity represents a strong collective enhancement indicative of a rotational cluster-band member. Such measurements represent the cutting edge and are the ultimate goal for precision experiments of cluster bands; necessary to definitively show that a series of states...
has the same underlying structure.

In $^{12}$C, curiosity to understand the structure in detail has led to recent renewed experimental activity across the globe. This is motivated largely by improvements in detector apparatus that make the required precision achievable – higher excitation energies and better background rejection coupled to the high excitation energies accessible compared to the early measurements, first performed over half a century ago.

Some of the most notable measurements on $^{12}$C in recent years relate to observation of the second, $2^+$ (Hoyle-state) excitation now established through several independent measurements [29, 30]. These include Freer and Itoh et al. [31] reporting a $2^+$ resonance at $E_x = 9.75(15)$ MeV with $\Gamma = 75(150)$ MeV using high-resolution magnetic spectrometers close to zero degrees and Zimmerman et al. [32], using the $^{12}$C$(\gamma,\alpha_0)$ reaction and observing a $2^+$ state at $E_x = 10.01(11)$ MeV with $\Gamma = 800(30)$ MeV. These consistent results represent a huge step forward and already allow a linear configuration of three-alpha particles to be ruled out based on the relatively low moment-of-inertia. However, the data are not yet sufficiently precise as to discriminate between an alpha condensate or a dilute alpha gas structure.

In parallel to the above studies long established states have been re-measured, again, with higher precision and in some cases resulting in a refinement of their properties. One such notable case is the $3^-$ state at 9.641 MeV. This lies close to the newly observed $2^+$ state and definitive knowledge of the properties of the $3^-$ state have been important to be able to confidently characterise the Hoyle-state excitation. As a result the $3^-$ state was re-measured using the $^{12}(p,p')^{12}$C$^*$ reaction, performed at 66 MeV using the K600 spectrometer at iThemba Labs, South Africa. The results from an $R$-Matrix analysis reveal a width of 48(2) keV [33] in contrast to the previous literature value of 34(5) keV [34]. The newly established width is also consistent with that measured at the Munich Q3D spectrometer via alpha inelastic scattering at 32 MeV [35].

| $E_{level}$ (MeV) | $J^\pi$ | $\Gamma_{tot}$ (keV) | Ref. | Comment |
|------------------|---------|----------------------|------|---------|
| 0                | 0$^+$   | stable [34]          |      | compact ground state |
| 4.439            | 2$^+$   | 10.8(6)$\times 10^{-9}$ [34] | ground-state band member |
| 7.654            | 0$^+$   | 8.5(10)$\times 10^{-6}$ [6, 34] | Hoyle state |
| 9.641            | 3$^-$   | 48(2) [33, 34]      | compact state |
| 9.8              | (2$^+$) 800(30) [6, 31, 32] | Hoyle band member |
| 10.3             | (0$^+$) 1600(200) [34] | |
| 10.844           | 1$^-$   | 315(25) [34]        | vibrational bending mode |
| 11.828           | 2$^-$   | 260(25) [34]        | bending mode band member |
| 12.710           | 1$^+$   | 18.1(28)$\times 10^{-6}$ [34] | $T = 0$ non-cluster state |
| 13.3             | 4$^+$   | 1700(200) [6, 36, 37] | candidate for Hoyle band |
| 13.35            | 4$^-$   | 375(40) [38, 39]   | compact band member |
| 14.083           | 4$^+$   | 258(15) [34]       | ground-state band member |
| 15.111           | 1$^+$   | 43.6(13)$\times 10^{-6}$ [34] | $T = 1$ non-cluster state |

Figure 4: A partial level-scheme for $^{12}$C showing the low-lying states.
The $3^-$ level, together with other states, have played another key role initiated by the recent measurement of a high-lying $5^-$ excitation [40]. Table 1 and figure 4 summarise the low-lying states in $^{12}$C based on current knowledge. Beyond the $2^+$ Hoyle-state excitation, there is evidence for a $4^+$ resonance at 13.3 MeV, first observed by Freer et al., [36] using the $^{12}$C($^4$He, $\alpha\alpha\alpha$)$^4$He$'$ reaction at Notre Dame, USA. This is further supported by Marín-Lambárri [37] and lends further credence to the rotational nature of these collective excitations through their well-behaved $J(J+1)$ dependence. Furthermore, there is a juxtaposed 13.35 $4^-$ excitation [38, 39]. The newly measured higher-lying $5^-$ state lies at $E_x=22.4(2)$ MeV and was observed in an experiment at the Birmingham MC40 cyclotron using an array of four double-sided silicon strip detectors to pick-up the alpha-particles following the $^{12}$C($^4$He, $3\alpha$)$^4$He$'$ reaction at 40 MeV. A full kinematic reconstruction of the detected fragments led to the clear identification of both the state as well as definitive angular distributions. This state has subsequently been independently observed following the inelastic scattering of $^3$He from $^{12}$C at 46 MeV [41]. Taken together, the sequence $J^\pi = 0^+, 2^+, 3^-, 4^\pm$ and $5^-$ is a strong signature of $D_{3h}$ symmetry and this is the first such observation in a nucleus. This equilateral triangle symmetry is represented by a triatomic $U(7)$ mixed-parity structure and the nearly degenerate $4^+$ and $4^-$ states are a particularly strong indicator. An alpha particle lies at each corner of the triangle and the various rotational modes, about an axis perpendicular to the plane of the triangle and about an axis lying in the plane bisecting the triangle (see figure 5), together with the coupling of both of these modes, enables the full band to be generated, again, following a $J(J+1)$ pattern as expected and shown in figure 6.

![Figure 5](image_url)

**Figure 5:** Left: An example of rotation axes to generate the observed sequence of states. Right: The observed energies compared to ACM calculations for the ground-state and Hoyle bands [40]. The ground-state configuration is more compact.

The symmetry and energies of these levels is reproduced well by the Algebraic Cluster Model (ACM) based on three alpha particles [42, 43]. This model includes softness causing fluid-like properties, a near equality of rotational and vibrational energies as well as identical, diffuse alpha constituents. The ACM is widely applicable to cluster nuclei in general and the predictions have
recently been extended to tetrahedral symmetry in $^{16}$O [44].

\[ E_x (\text{MeV}) \]
\[ J(J+1) (\hbar^2) \]

**Figure 6:** Angular momentum versus energy systematics for $^{12}$C adapted from Ref. [41]. The vertical fine dashed lines indicate the integer spins from $J=2$ upwards to guide the eye. Open symbols are used for tentative assignments. The long-dashed line represents the compact ground-state band structure. Squares are positive parity states and triangles negative parity. The short-dashed line is the Hoyle band (circles) and the solid line is the $1^{-}$ band (diamonds). The dotted-dashed horizontal lines indicate the energies of the newly observed states from $^{3}$He scattering [41]. See text for details.

The same study that used $^{3}$He scattering to confirm the presence of the $5^{-}$ state also discovered evidence for a series of resonances at excitation energies of 16.3(0.2), 17.2(0.2), 18.4(0.2), 19.7(0.2), 22.2(0.3) and 25.1(0.3) MeV, all with widths of <600 keV [41]. These are indicated by horizontal lines on figure 6. By unambiguously identifying the outgoing scattered $^{3}$He beam particles, greater precision in the kinematic reconstruction could be achieved, albeit at the expense of a straightforward angular distribution analysis due to the non-zero intrinsic projectile-nucleus angular momentum. As a result, this tantalising observation of structure at higher excitation energies requires spin and parity information from an additional measurement to fully characterise the states.

4. Summary

The seemingly simple nature of the $^{12}$C nucleus, the wealth of structure exhibited and its importance to the formation of the elements, make $^{12}$C a nucleus of increasing contemporary interest. This is in no small part to the presence of the Hoyle state and excitations and the impact on the rate of the triple-alpha process in giant-branch stars. The effect of the Hoyle state and nearby levels is to increase the reaction rate by at least eight orders of magnitude compared to non-resonant contributions. The precise position of the Hoyle excitations – particularly the $2_{+}^{+}$ state – also influence the alpha-burning rate [29, 45, 46].
Despite over 60 years of study the $^{12}$C spectrum still holds surprises and, it seems, that while significant headway has been made in understanding the low-lying band structures, the predictions of strong signatures of symmetry and the elucidation of the higher energy domain look set to keep nuclear spectroscopists occupied for the foreseeable future – perhaps even another half century (or more)!

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