Buckling strength of thin-walled steel cylindrical shells with stiffened plates

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Abstract. Thin-walled steel shell members are structurally superior as structural members. However, the thin-walled steel shell with stiffeners has been only studied in a part by reason of excelling dynamically on ordinary thin-walled shell without stiffeners. In this paper an investigation in regard of the increase of the buckling capacity on steel cylindrical shells with out- and inside stiffened plates is generated under pure axial compression. Then, so as to clarify the effect of the buckling capacity on thin-walled steel cylindrical shells with out- and inside stiffened plates, exact buckling strength and buckling mode shapes are generated by using the extended transfer matrix method. The buckling strength of cylindrical shell with out- and inside stiffened plates are obtained during the combination of half wave number (n) for perpendicular direction to axis and half wave number (m) for axis direction. Depending on circumstances, the local buckling strength of thin-walled shells with stiffened plates is sometimes failing by local buckling of stiffened plate to differ from local buckling of shell-panel in comparison with that of ordinary thin-walled shells without stiffener. Therefore, this study aims to plan increase of the local buckling capacity on the thin-walled steel cylindrical shells with out- and inside stiffened plates due to pure axial compression.

1. Introduction
Thin-walled steel shell members are structurally superior as structural members. The stiffened thin-walled members are important components of many industrial complexes. Most of these components are used as stiffened thin-walled plates for deck-plate and channel sections, etc. in the field of construction. However, the thin-walled steel shell with stiffeners has been only studied in a part by reason of excelling dynamically on ordinary thin-walled shell without stiffeners. The buckling capacity of thin-walled shell with plate-stiffeners sometimes reduce by local buckling of stiffened plate to differ from local buckling of shell-panel in comparison with that of ordinary thin-walled shell without stiffeners.

Then in the case of cylindrical shell with stiffened plates, the increase of the buckling strength owing to the influence to the plate-height ratio of stiffened plate is partially observed in comparison with that of member without stiffener in small local range.

Then, this study aims to plan increase of the local buckling capacity on the thin-walled steel cylindrical shells with out- and inside stiffened plates due to pure axial compression.

And then in order to clarify the effect of the local buckling capacity on steel thin-walled cylindrical shell with stiffened plates, exact buckling strength and the buckling mode shapes are generated by
using the extended transfer matrix method.

In analysis the transfer equations are proposed by considering the compatibility and the equilibrium conditions between shell panel and stiffened plate on cylindrical shell with stiffened plates. The local buckling mode shapes of two types are in existence during the local buckling mode shapes like showing the deformation of stiffened plate and the deformation of wave-shaped form for cross section of member are in existence on the thin-walled cylindrical shell with stiffened plates.

The buckling strength and the buckling mode shape of cylindrical shell with out- and inside stiffened plates are obtained during the combination of half wave number (n) for perpendicular direction to axis and half wave number (m) for axis direction.

2. Analytical theory

The increase of the buckling capacity on steel cylindrical shells with outside and inside stiffened plates due to pure axial compression is researched by use of the extended transfer matrix method.

![Figure 1](image_url)

(a) outside stiffened plates  (b) inside stiffened plates

Figure 1. (a) and (b) Thin-walled steel cylindrical shells with out- and inside stiffened plates.

Stability equation for thin-walled cylindrical shell with outside and inside stiffened plates [1-3]. The original transfer matrix method is not directly applied to thin-walled steel cylindrical shell with outside stiffened plates. Therefore, in order to apply this method to thin-walled steel cylindrical shell with stiffened plates, the extended transfer matrix method for stiffened plates is presented. The compatibility and equilibrium conditions between the state vectors of the main shell-panel and those of stiffened plates (branched panel) of the half cylindrical shell section I are described as follows (Figure 2a,b);

\[
\begin{bmatrix}
Z^L \\
0
\end{bmatrix}_{M1} = \begin{bmatrix}
F_{S1}^L \\
F_{S1}^R
\end{bmatrix}^T - \begin{bmatrix}
P_{B1}^L \\
P_{B1}^R
\end{bmatrix}^T \begin{bmatrix}
Z_M \\
Z_{B1}
\end{bmatrix}_0
\]

(1)

Where superscripts δ and F indicate the displacement and force components of the state vectors respectively. Superscripts L and R indicate the left and right side of the section, and subscripts M and B indicate the variables of the main (shell-panel) and branched (stiffener) panels.
Further considering the compatibility and equilibrium conditions between the state vectors of the main shell-panel and those of stiffened plates of half cylindrical shell at section 2, the relation between the final state vector \( (Z_{M3}) \) and the initial state vector of the main shell-panel and branched panels \( (Z_{M0}) \) and the state vectors of branched panels \( (Z_{B10}) \) and \( (Z_{B20}) \) can be obtained as follows:

\[
\begin{bmatrix}
Z_L \\
0 \\
0_{M3}
\end{bmatrix}
= L
\begin{bmatrix}
F_{S3}F_{S2}F_{S1} & F_{S3}F_{S2}P_{B1}F_{B1} & F_{S3}P_{B2}F_{B2} \\
[F_{S1}]^\delta & 0 & 0 \\
[F_{S2}F_{S1}]^\delta & [F_{S2}P_{B1}F_{B1}]^\delta & 0
\end{bmatrix}
\begin{bmatrix}
Z_M \\
Z_{B1} \\
Z_{B20}
\end{bmatrix}
\]

(2)

Here, stability equation of cylindrical shell with inside stiffened plates is same to that with outside stiffened plates. However, in numerical analysis positive and negative of angle for rotation matrix is only changing.

3. Elastic buckling strength and buckling mode shapes of thin-walled steel cylindrical shells with stiffened plates

3.1. Analytical models

Thin-walled steel cylindrical shell with stiffened plates are analyzed for the buckling strength and buckling mode shapes under the uniform axial load. The parameters of analytical models are shown in Table 1 and Figure 3 (a), (b) and (c). In Figure 3 thin-walled steel cylindrical shells without stiffener (Mode A) and with outside stiffened plates (Mode B) and with inside stiffened plates (Model C) are shown. Here, in the cross-sectional dimensions of steel cylindrical shell a radius is \( r=500 \text{mm} \), and panel-thickness is \( t=2\text{mm} \). The number of stiffened plate is 4 plates. Further the number of groove-height ratios \( (R_g / R) \) are 0.05, 0.10, 0.15 and 0.20. And then thin-walled steel cylindrical shell without stiffener (Model A) is preparing for comparison at the same time.
3.2. Buckling strength of thin-walled steel cylindrical shells with stiffened plates

Figure 4 shows the buckling coefficients of thin-walled steel cylindrical shells with outside stiffened plates for member aspect ratios $a/R=1\sim 70$ under each plate-height ratio $h_s/R=0.05, 0.10, 0.15$ and 0.20 subjected to uniform axial load. Here the vertical axis is shown for the buckling coefficients $k = \frac{\sigma_a b^2 t}{\pi^2 D}$, (D: bending stiffness of shell panel), and the horizontal axis is shown for common logarithms of the member aspect ratio. As compared with member with plate shape–ratio $h_s/R=0.50$ and member without stiffener (Model A), the local buckling strength of cylindrical shell with $h_s/R=0.50$ is a little increasing by the stiffened effect of the plates in local aspect ratio range (about $a/R=2\sim 10$) and after that the local buckling strength is slightly depreciating in local large aspect ratio. In the case of cylindrical shell with $h_s/R=0.10$ the fall of the local buckling strength is observed by local buckling of stiffened plate in local short aspect ratio (about $a/R=1\sim 2$) in comparison with that of Model A and then the increase of local buckling strength is seen in local short aspect ratio (about $a/R=2\sim 9$), and afterwards the local buckling strength is slightly depreciating by the the interaction of the local buckling of stiffened plate and the local buckling of shell-panel till overall buckling range. Then the local buckling strength of cylindrical shell with $h_s/R=0.15$ and 0.20 are failing by local stiffened plate-buckling in short local range (about $a/R=1\sim 7$) in comparison with that of Model A, and then the fall of local buckling strength are observed before overall range. And the local buckling strength of cylindrical shell with $h_s/R=0.20$ is failing as much as about 35% in remarkable range. It is observed that the local buckling strength are partially increasing during the stiffened effect by cosidering the plate-height of stiffrener on the cylindrical shell with $h_s/R=0.05$ and 0.10. The buckling strength in overall buckling range roughly agree in any cases of Model A, B and C.
Figure 4. Buckling coefficients of cylindrical shells with outside stiffened plates.

Figure 5. Buckling coefficients of cylindrical shells with inside stiffened plates.
Figure 5 shows the buckling coefficients of thin-walled steel cylindrical shells with inside stiffened plates for member aspect ratios \( a/R = 2-150 \) under each plate-height ratio (\( h_s/R = 0.05, 0.10, 0.15 \) and 0.20) subjected to uniform axial load. In numerical analysis, positive of outside and negative of inside angle for rotation matrix is only changing. In the case of cylindrical shell with inside stiffened plates, the same buckling behavior to the case of cylindrical shell with outside stiffened plates is observed.

\[
\begin{align*}
(a) \quad & a/R=3, \quad k=717.3.3 \quad (m, n) = (1, 10) \\
(b) \quad & a/R=8, \quad k=663.4, \quad (m, n) = (2, 6) \\
(c) \quad & a/R=13, \quad k=452.6, \quad (m, n) = (1, 4)
\end{align*}
\]

Figure 5. (a), (b) and (c) Buckling mode shapes of cylindrical shell with outside stiffened plates.

In Figure 6, the local buckling mode shapes of cylindrical shell with outside plate-height ratio (\( h_s/R=0.10 \)) are shown. As regards the local buckling behavior, a very complicated buckling mode shape is obtained during the combination of half wave number (\( m \)) for axis direction and half wave
number (n) for perpendicular direction to axis. In Figure 6 (a), the increase of the local buckling strength owing to the influence of the area of stiffened plates under m=1, n=10 in a/b=3 is partially observed in comparison with that of cylindrical shell without stiffener. In accordance with increase of member aspect ratio (a/R), the local buckling mode shape (Figure 6 (b)) is shifting over from n=6 under m=2 in a/R=8, and the local buckling mode shape (Figure 6 (c)) is reached to n=4 under m=1 in a/R=13. And after that the fall of local buckling strength is observed in comparison with that of member without stiffener in local range. As this cause, the decrease of the local buckling strength is considered to be affected by the local buckling of stiffened plate. Further the overall buckling strength in global member aspect ratio range coincide in any case of Model A, B and C, and then the effect of overall buckling strength is not almost subjected to the influence of the stiffeners.

4. Conclusions

An analytical procedure for the elastic buckling behavior of thin-walled steel cylindrical shell with outside and inside stiffened plates by the extended transfer matrix methods is presented. The buckling strength of cylindrical shell with outside and inside stiffened plates are obtained during the combination of half wave number (n) for perpendicular direction to axis and half wave number (m) for axis direction.

On the thin-walled steel cylindrical shell with outside and inside stiffened plates the local buckling strength are not almost creasing in comparison with that of member without stiffener.

However, in the case of cylindrical shell with stiffened plates, the increase of the local buckling strength owing to the influence of the plate-height ratio (hs/R) of stiffened plate is partially observed in comparison with that of member without stiffener in small local range. In the case of cylindrical shell with inside stiffened plates the almost same buckling behavior to the case of cylindrical shell with outside stiffened plates is observed.

References

[1] Hoshide K, Ohga M and Shigematsu T 2011 Ultimate strength of thin-walled channel columns with shell profiled sections, The 2011 World Congress on Advances in Structural Engineering and Mechanics, (CD-ROM)
[2] Ohga M, Shigematsu T and Kawaguchi K 1995 Buckling Analysis of Thin-Walled Members with Closed Cross Section, Thin-Walled Structures, 22, p 51-70
[3] Shigematsu T, Ohga M and Hara T 2008 Beulverhalten gedrückter Kastenprofile mit verschiedenen Steifen, Stahlbau, 77, Jahrgang, März, Heft 3, p 213-221