LENSING EFFECTS OF MISALIGNED DISKS IN DARK MATTER HALOS

RYAN QUADRI, Ole Möller, and Priyamvada Natarajan

Received 2003 January 3; accepted 2003 July 11

ABSTRACT

In this paper we study the observational signatures of the lensing signal produced by dark matter halos with embedded misaligned disks. This issue is of particular interest at present, since several of the observed multiple-lens systems have magnification ratios and image geometries that are not simultaneously well fitted by standard mass models. The presence of substructure exterior to the lens has been invoked by several authors in the context of cold dark matter (CDM) models in order to explain the anomalous magnification ratios. We emphasize that the anomalous magnification ratios may be an artifact of the simple mass models currently in use; the inclusion of a misaligned disk may be able to mimic the effect of substructure. These slight spatial offsets between the dark matter halo and the disk, which are likely to occur as a consequence of interactions or mergers, lead to complex image configurations and nonstandard magnification ratios. We investigate the effects of disk misalignment on two illustrative lenses: a spiral disk embedded within a dark matter halo, and a compact, disklike component within an early-type galaxy. The expected fraction of galaxies with a misaligned disk is estimated to be of the order of the merger fraction, since all mergers are expected to progress via a transiently misaligned phase. We find that there is a significant change in all observed lensing properties due to the added complexity of such mass profiles. In particular cases, the resulting lensing geometries are unusual, with high image multiplicities. The caustic structures—both radial and tangential—are drastically modified, and the magnification ratios differ from expectations from standard lens models. The additional parameters required to specify the relative alignment of multiple mass components in the primary lens introduce yet another source of uncertainty in the mass modeling of gravitational lens systems.

Subject headings: dark matter — galaxies: fundamental parameters — galaxies: halos — gravitational lensing — methods: numerical

1. INTRODUCTION

Lensing by individual galaxies provides a plethora of observational signatures, ranging from multiply imaged, highly magnified background sources to a weakly sheared background (see Schneider, Ehlers, & Falco 1992; Blandford & Narayan 1992). Several authors have attempted to use the observed image geometries, positions, and magnitudes to infer and reconstruct the mass profile of the lens using ground-based optical and radio surveys (Browne et al. 2001 [JVAS/CLASS] and references therein; Lehar et al. 2000 [CASTLES]). Attempts have been made to constrain both the density profiles of the mass distribution within the inner regions of galaxies (Cohen et al. 2001) and to obtain constraints on the Hubble parameter $H_0$ (Kundić et al. 1997; Fassnacht et al. 2002). Both of these enterprises are complicated by the presence and precise configuration of structure within and exterior to the lensing galaxy, leading to degeneracies in the mass profiles. The degeneracies in such modeling attempts arise from the presence of structural complexity in the inner regions of galaxies, the prevalence of disky components, bulges, central black holes (Möller & Blain 1998; Mao, Witt, & Koopmans 2001), as well as the presence of external mass perturbations: nearby galaxies, groups, or clusters in the vicinity of the lens (Keeton, Kochanek, & Seljak 1997; Möller et al. 2002). In this work we explore a further source of uncertainty—the presence of misaligned disks within dark halos.

The motivation for studying galaxy-scale systems that have their baryonic disks misaligned slightly with their dark matter halos is twofold. First, given what is currently accepted regarding the hierarchical assembly of structure in cold dark matter (CDM)–dominated structure formation models (White & Frenk 1991), i.e., that merging of galaxies is frequent, then dynamically induced small misalignments between the disk and halo are expected during the merger process. Since the lensing properties of a system (an individual galaxy in this case) depend on the projected surface mass density, we demonstrate in the following sections that these small misalignments are likely to have very interesting and important observable consequences. Second, it has recently been claimed in the literature (Mao & Schneider 1998; Metcalf & Madau 2001; Dalal & Kochanek 2002) that the observed magnification ratios of several multiply imaged systems (notably B1422+231 and PG 1115+08) can only be explained as arising as a result of the presence of small-scale substructure within galaxy halos. Here, we illustrate that there are in fact other physical scenarios, such as an anisotropic mass distribution within the lensing galaxy: nonzero quadrupole and octopole components induced in the lensing potential by a misaligned disky component can also give rise to magnification ratios and image characteristics that differ from those predicted by the standard isothermal/ pseudo-isothermal mass models currently in use. While the evidence for substructure is compelling, it is useful to explore the degeneracy with primary lens properties.
2. WHY AND WHEN ARE HALOS AND DISKS MISALIGNED?

In the standard paradigm for structure formation in CDM-dominated models, galaxies are assembled over time via the merger of smaller subclumps (White & Frenk 1991). The merger rate for halos of a given mass can be computed semianalytically using the extended Press-Schechter formalism (see Sheth, Mo, & Tormen 2001), and can be directly compared with cosmological N-body simulations (Raig et al. 2001). When dark matter halos merge, their baryonic components also merge; detailed numerical simulations of this process have been carried out by Mihos & Hernquist (1994) and Barnes & Hernquist (1998). In these simulations, it is found that the baryonic gas flows to the center of the more massive halo and the inner regions of the halo become baryon dominated. The tidal torquing during the merger process is significant and can easily cause the embedded disk and halo to be misaligned. It is also likely that during the "relaxation" process, the disk wanders transiently within the halo (Nelson misaligned. It is also likely that during the "relaxation" process, the disk wanders transiently within the halo (Nelson & Tremaine 1995; Dubinski & Kuijken 1995).

In fact, more recent N-body simulation of galaxy mergers with baryons suggest that the light (baryonic mass) distribution can easily become misaligned with the dark matter by a distance of a few kiloparsecs. This misalignment may not be evident in the light distribution, as the baryons may appear smoothly distributed and viralized (Gonzales-Garcia 2003; Abel 2002). Photometric detection of disk misalignment in a lensing galaxy is made more difficult by the presence of a bright background quasar (the lensed source) in the image, which must be carefully subtracted to retrieve the light profile of the underlying lens.

Finally, interactions that do not result in mergers can also perturb the disk-halo configuration and induce transient misalignments. These are expected to be damped well within a Hubble time (Dubinski & Kuijken 1995).

2.1. Estimating the Number of Misaligned Disks in Halos

Lens galaxies tend to sample the high-mass end of the galaxy mass function. Since massive galaxies are assembled via mergers, and because such galaxies are particularly efficient as lenses, it is likely that even transitory misalignments could be important for a number of lens systems. We use the observationally determined galaxy merger rate to estimate the expected frequency of misaligned systems. Patton et al. (2002) report a merger rate of \( dN/dz \propto (1+z)^{2.31 \pm 0.7} \) from a study of a sample of bright galaxies, \(-18 < M_B < -21\), in the Canadian Network for Observational Cosmology Field Galaxy Redshift Survey (CNOC2). They find that 3% of galaxies at \( z = 0.3 \) are mergers in an \( h = 0.6, \Omega_m = 0.7, \Omega_m = 0.3 \) cosmology. Note that this merger rate, found by Patton et al. (2002), is consistent with semianalytic estimates from models that are calibrated to observational data, as shown in the top panels of Figure 6 in Somerville et al. (2000). We assume here that all mergers lead to misalignments through tidal torquing during the merger process and that these misalignments persist for the duration of the merger. From detailed N-body simulations of interacting galaxies, the merger timescale is found to be of the order of 1 Gyr (see Fig. 4 of Mihos & Hernquist 1994). Since mergers are more frequent at higher redshifts, and since typical lens galaxies are found at up to \( z = 1 \), we can expect the misaligned fraction to be higher in fact than the estimate of Patton et al. (2002).

At present, there are approximately 70 confirmed cases of individual galaxies lensing high-redshift background quasars or galaxies. We expect that at least a few of these are likely to contain misaligned disks. Note that while the majority of lenses detected thus far are typically early-type galaxies, observationally it is becoming clear that early-type galaxies also do often contain embedded compact disky components (Rest et al. 2001). In fact, we demonstrate below that even a highly compact misaligned disk that lies interior to the Einstein radius can produce the anomalous-image geometries. Therefore, our analysis is generic and applicable to all lensing galaxies that contain a disky mass component.

3. METHODOLOGY: THE RAY-TRACING METHOD

In order to study the strong lensing properties of a misaligned halo and disk, we used the multiple-plane ray-tracing code developed by Möller & Blain (1998). Computing the lensing properties of multicomponent lens systems is complicated and can be very time consuming if this is not done with an optimized algorithm. The code we employ here is based on the "grid search method" (Schneider et al. 1992) and uses an adaptive grid that is optimized to calculate deflection angles for parametric multiple component lens models quickly and accurately. A regular grid of triangle pairs on the image plane is mapped to the source plane. The magnification at each point on the source plane is calculated as the sum of the ratios of the area of the unmapped triangles on the image plane to that of the mapped triangles on the source plane. Similarly, shears are calculated from the distortions of mapped (source plane) and unmapped (image plane) triangles. In order to reduce the computation time, the mapping process is iterated adaptively. The numerical inaccuracies on magnifications and shears are very small, usually less than 0.1%. The numerical uncertainty when calculating the number of images for a given point source is greater; faint images near a caustic line may be missed by this method (Schneider et al. 1992). However, increasing the resolution of the grid can overcome this problem, and it does not affect any of the results discussed in this paper.

The illustrative single-lens systems studied here consist of a dark halo and an embedded disk. The halo model is a cored isothermal sphere (CIS). We have also investigated pseudo-isothermal elliptical mass distributions (PIEMDs) with an ellipticity of \( \epsilon = 0.3 \) (Kassiola & Kovner 1993) and find that finite ellipticity in the halo does not significantly alter our results. The two parameters required to specify the CIS are the core radius and the mass within a radius \( R_c \). The former is not well determined by observations of real galaxies. In order to test the effects of our choice of \( R_c \) on the results, we explore a wide range of values, including \( R_c = 0 \). The total halo mass contained within a 20 kpc radius was held constant for all the models considered here to enable a sensible comparison. The primary effect of varying \( R_c \) is a change in the relative significance of the halo in the lensing system, which affects the size of the elliptical caustic (see § 4.1). This is due to the fact that a small core radius implies that the convergence \( \kappa \) in the central region is very high. In this case, the mass profile is centrally concentrated, and thus the cross section for strong lensing is significantly increased.
4. LENS PROPERTIES OF SIMULATED SPIRAL SYSTEMS

We take the Milky Way as the template for our spiral lens, and hence model the halo as a CIS with a small core radius $r_c = 0.5$ kpc (corresponding to $0.13$ at the lens redshift and using the cosmological model as described below) and mass $M_h = 3.64 \times 10^{11} M_{\odot}$ within a radius of 20 kpc ($5^\circ$). This corresponds roughly to a velocity dispersion of 160 km s$^{-1}$. The Einstein radius of the halo is $0.44$, or $1.5$ kpc. The disk has mass $M_d = 6.4 \times 10^{10} M_{\odot}$ and scale length $r_d = 3.2$ kpc ($0.83$). Note that models with less massive and more compact disks are also studied, and the results are presented in a subsequent section. An inclination angle of $70^\circ$ was assumed for the disk, which is roughly the minimal inclination needed to significantly increase the cross section for multiple imaging (see Fig. 2 in Möller & Blain 1998). The lens is located at $z = 0.2$ and the source plane at $z = 1.5$. The values assumed for cosmological parameters are $H_0 = 60$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$. We have also experimented with alternate cosmologies, such as $\Omega_m = 1.0$, $\Omega_\Lambda = 0.0$ Einstein-de Sitter (EdS) and $\Omega_m = 0.3$, $\Omega_\Lambda = 0.0$ open cold dark matter (OCDM), and find that our results are not significantly affected by the particular choice of these parameters.

In this paper we study simple spatial offsets of the disk relative to the halo. We assume that the disk retains an exponential profile and that the halo retains an isothermal profile. Offsets in the plane of the disk are denoted by $\Delta x$, and offsets outside the plane of the disk are denoted $\Delta y$. The following sections assume offsets of $\Delta x = 0^\circ 6$ (2.31 kpc at the redshift of the lens), $\Delta y = 0^\circ 6$, and $\Delta x = \Delta y = 0^\circ 6$. We consider this to be a reasonable approximation to the disturbed and asymmetric mass distribution that will result from mergers or strong interactions. Note that in all toy models explored in this paper, we have neglected the mass of the bulge. Including a bulge that is aligned with either the disk or the halo will not alter the lensing effects of the misaligned disk significantly, but will introduce two more free parameters into our model.

4.1. Magnification Maps, Cross Sections, and Image Geometries

In this section we present magnification maps in the source plane for several lens configurations, varying the

![Magnification Maps](image)

**Fig. 1.**—Magnification maps in the source plane for the spiral lens system. Spatial offsets of the disk are seen to strongly distort both the diamond-shaped "tangential" caustic and the elliptical "radial" caustic. The location of the disk with respect to the center of halo ($\Delta x$, $\Delta y$) in the different panels is (a) $(0^\circ, 0^\circ)$; (b) $(0^\circ 6, 0^\circ)$; (c) $(0^\circ, 0^\circ 6)$; and (d) $(0^\circ 6, 0^\circ 6)$. For our choice of cosmological parameters, an offset of $0^\circ 6$ corresponds to 2.3 kpc.
position of the disk relative to the halo in each case. We also generate the corresponding magnification maps in the image plane, where the value of each pixel is the magnification of the observed image at that location. The caustics (points of infinite magnification) in the source plane map onto critical curves in the image plane. Magnification cross sections and representative image geometries are also presented.

Figure 1 shows the magnification maps in the source plane for various representative values of the disk offset. It is evident that a realistic shift in the disk position relative to the halo does indeed distort the caustic shapes severely. A shift in the $x$-direction, or in the plane of the disk, yields a shift of the diamond-shaped curves that form the "tangential" caustic, which is caused by the presence of the disk. The elliptical ("radial") caustic, which is due to the peak in the central mass concentration of the combined halo and disk, reduces in size and is distorted in the direction of the offset. Offsets outside the plane of the disk lead to more complicated caustic geometries. The tangential caustic shifts and the upper cusp of the caustic appears to fold over into the diamond itself. The radial caustic changes its shape drastically, and is pinched inward near the lower cusp. But a comparison of Figures 1c and 1d shows that an offset in the $x$-direction can partially reduce the effect of a shift in the $y$-direction.

The primary effect of decreasing the core radius to $r_c = 0$ is to increase the size of the radial caustic, while the shape of the tangential caustic is unchanged. Increasing the size of the core radius decreases the overall effect of the halo on the caustic structure, as noted in § 3; the radial caustic decreases in size, and for large enough values ($r_c \sim 2$ kpc), the upper and lower cusps of the tangential caustic become smoothed out.

Figure 2 shows the magnification maps in the image plane. The small, inner critical curve corresponds to the radial caustic, and the outer one corresponds to the tangential, diamond-shaped caustic. It is clear from these images that the locations of highly magnified images can be substantially affected by misalignments. As with the magnification maps in the source plane, offsets out of the plane of the disk also create complicated geometries. In Figure 2c, the upper, roughly triangular-shaped critical curve has split.
significant effect than shifts in the $x$-tail, misaligned cases relative to the aligned case. The differences become evident in the spiral lens. The three curves specify the cross section for our three inner critical curves. In Figure 2, the region within this new critical curve has nearly collapsed. If the disk were shifted further in the $x$-direction, then this new critical curve would be absorbed into the outer critical curve. The evolution of caustics and critical curves is continuous, i.e., the curves gradually distort, pinch off, and absorb each other.

To investigate the effects of misalignment on total magnification, we calculate the magnification cross section ratio of the misaligned cases to the aligned case. The magnification cross section for a lens is derived by adding up the total number of pixels with a value greater than some threshold value $\mu$ in a magnification map. The results are shown in Figure 3. An offset in the plane of the disk is seen to have little effect. In contrast, the cross section for high-magnification ($\mu > 30$) events is significantly increased for offsets outside the plane of the disk. This is a manifestation of the large surface area very near the caustic curves in Figure 2c. The offset in the $y$-direction of $0''6$ corresponds approximately to the maximum possible enhancement in high-magnification cross section for our lens model. The observed trend that offsets in the $x$-direction tend to compensate for the more dramatic effects of shifts in the $y$-direction is apparent here also.

This effect on the cross section for high magnifications is likely to have a measurable effect on the statistics of multiple image systems. Due to the preferential selection of high-luminosity images in a survey, any flux-limited sample of lens systems is biased towards systems with a large cross section for high magnifications (“magnification bias,” e.g., Borgeest 1991). Therefore, the fraction of lens systems with misaligned disks as estimated in § 2.1 is conservative, and the actual number of systems in which a misaligned disk is important might be larger.

The number of images of a source will increase or decrease by two if the source crosses a caustic (see Schneider et al. 1992 for a mathematical illustration of the same). An aligned halo and disk are capable of producing at most five separate images, in the configuration of a four-image, possibly asymmetric, Einstein cross with a demagnified central image. However, our distorted caustic geometries for the misaligned cases have regions where even higher image numbers are possible. For instance, the lens configuration of Figure 1c has a region where seven images are formed, and other lens configurations can produce even more images. Such a configuration with a large multiplicity of images has been observed; B1359+154 is a six-image system (Rusin et al. 2001) in which the lens is a compact group of galaxies at $z = 1$. These asymmetries in caustic geometries are expected to yield complicated, asymmetric image geometries. Figure 4 illustrates examples of the kind of image geometries that can be produced with the increased parameter space provided by disk offsets. The inset within each panel shows the magnification maps in the source plane and location of the point source. A representative range of image geometries with a wide range of magnification ratios is shown. Current observations of such systems might not reveal their complexity entirely; for example, the configuration shown in the lower left panel of Figure 4 might easily be confused with an Einstein cross because the separate fainter image is likely to go undetected.

5. Lens Properties of Early-Type Systems

In the previous section we concentrated on misalignment between the disk and halo component in late-type lensing galaxies with Milky Way–type spiral disks. The majority of known lens systems are in fact early-type galaxies, as these tend to be more massive and hence have a larger cross section for lensing. Observations show that disks or disklike structures might well be present in early-type lenses (Kelson et al. 2000). This suggests that early-type galaxies may contain disks that may also be misaligned with respect to the smooth/ellipsoidal mass component. Note here that the bulge+halo are modeled together as a single CIS, as is customary in lensing analysis, and we shall refer to this combination as the smooth/ellipsoidal mass component for early-type systems. In principle, modeling the lens with a de Vaucouleurs bulge plus an isothermal dark matter halo could change the lensing behavior. However, the inclusion of a further component will increase the number of parameters without changing the qualitative effect of misaligned disks in early-type lens galaxies. Whether there is a possible misalignment between a disk and the (baryonic) bulge depends on the exact cause of the misalignment, but a scenario as described in § 2 may cause a misalignment between disk and the bulge, as well as with the dark matter halo.

5.1. Magnification Maps, Cross Sections, and Image Geometries

We investigate here the lensing properties of a misaligned disk component that is similar to that observed in early-type galaxies. In particular, we choose disk parameters that are comparable to those found by fitting the light profile of...
early-type galaxies in CL 1358+62 at $z = 0.33$ with a bulge+disk model (Kelson et al. 2000). For our early-type lens model, we choose a disk-scale length of $r_d = 0.5$ kpc. At our lens redshift of $z = 0.2$, this corresponds to $0\.12$.

The disk mass is taken to be $M_d = 2 \times 10^{10} M_\odot$. We model the smooth/ellipsoidal mass components (bulge+halo) as a CIS with $r_c = 0.5$ kpc and increase the total mass within 20 kpc to $M_h = 7 \times 10^{11} M_\odot$ to reflect the fact that early-type lens galaxies are usually more massive. This choice of parameters gives a velocity dispersion of $220$ km s$^{-1}$ and yields an Einstein radius of $1'' (3.85$ kpc) for the smooth/ellipsoidal mass component.

Figure 5 shows the magnification maps in the source plane for the early-type lens. The values for disk offsets are the same as in the previous section. Although this disk is less massive than the disk in our late-type lens, it is also significantly more compact, meaning that the central $\kappa$ value is in fact high enough for the disk to produce an additional elliptical component to the overall caustic structure. So the magnification maps in Figure 5 represent a superposition of two elliptical caustic components due to the disk and the halo, as well as the tangential caustic due to the disk. Figure 6 shows the corresponding magnification map in the image plane. As seen in Figures 5a and 6a, our model of a compact disk has only a small effect on the structure of the caustics and critical curves when the disk and halo are aligned. The critical curve shown in Figure 6 is a nearly circular Einstein ring. The other panels of Figure 6 show critical curves that are introduced by a misalignment of the compact disk.

The effects of variations in the CIS core radius are similar to those of the spiral lens. Decreasing the core radius causes the central $\kappa$ value of the lens to increase, causing an increase in the size of the radial caustics, but leaving the tangential caustic largely unchanged. Increasing the core radius
lowers the central $\kappa$ value, causing a decrease in the size of the radial caustics. However, in this case there is some change in the shape of the tangential caustic, as it is less distorted by the lensing potential of the halo.

Even though the disk components in the early-type lens galaxy model contain a smaller fraction of the total mass than the disks in the late-type lens models discussed in previous sections, the effect of the misalignment between disk and halo is still very pronounced. For misalignments in both the $x$- and $y$-direction in particular, the effect on the structure of the caustics and critical curves is drastic. Since the majority of observed four-image lens systems are expected to be produced by lensing of a point source that lies close to the high-magnification caustic lines, this indicates that the image properties of these systems are strongly affected by the presence of a misaligned disk, even if it contributes only a small fraction of the total mass and has a scale length of less than a few hundred parsecs, corresponding to a few tenths of an arcsecond at $z = 0.2$.

It is therefore likely that the image geometries will also change significantly. In Figure 7 we show some typical image geometries of early-type lens systems with a misaligned disk for two different lens models. The boxes are centered on the image positions and have sizes that are proportional to the logarithm of the magnification. Depending on the source position, both common and unusual image geometries may be produced. The bottom left panel shows a fairly typical 3:1 geometry produced by a source near a cusp. In contrast, the top left panel shows a case with nonstandard magnification ratios; the central image is not the brightest image. These “inverted” magnification ratios are a unique feature of complex multicomponent lens models; as discussed below, single component mass models predict that the central image of the group of three images in 3:1 lens geometries is always the brightest. The top right panel of Figure 7 shows a lensed source near a cusp, but the 3:1 image geometry is not present. Instead,
there are four clustered images, with the two brightest ones nearly overlapping, and a fifth image lying at a distance of a few arcseconds. Finally, the source in the bottom right panel does not lie on a cusp, and the 3:1 image geometry is broken. All images have roughly similar magnification, and there is no clear group of three closely grouped images.

6. INTERACTING HALOS

We point out in this section that even grazing interactions between neighboring halos are likely to produce transient misalignments due to tidal torquing. While these offsets might not persist for significant periods of time, they are worth exploring as a special case of the generic misaligned case. We use the disk+halo model described in § 4 but add a secondary halo with about 30% of the mass of the primary at a distance of 1 to several arcseconds (~4 – 20 kpc) away from the primary lens. Simulations of such interacting halos suggest that the disk misalignment will tend to be along the direction of the secondary. We have explored a variety of halo separations and choose a distance of 1.7 (6.5 kpc) to illustrate the effect. This corresponds to a spatial distance of 2r_d, where r_d = 3.2 kpc is the disk scale length for the Milky Way–type spiral. The magnification maps in the source plane are shown in Figure 8. The main effects are an increase in the area enclosed by the caustics, an overall shift of the caustic structure, and induced distortions in the direction of the secondary halo. Comparing Figures 1c and 8c also shows that the radial caustic is less pinched at the lower edge. Changes in the core radius are very similar to those described in § 4.1 for the single spiral lens. Close pairs of galaxies are in fact observed as lenses; B1608+656 (Koopmans & Fassnacht 1999) is a clear example of this. If one or both of the lens galaxies in this system contain disks, a scenario as discussed above is likely to arise.

7. MAGNIFICATION RATIOS

There has been much interest recently in explaining some of the observed magnification ratios with lens models that include compact substructure with masses that are approximately 10^6 M⊙. Such substructure is ubiquitous in
numerical simulations of structure formation in standard CDM cosmologies, but has not yet been observed. An argument in favor for the possible ubiquity of dark clumps is that there are many viable mechanisms that could prevent star formation in such compact subhalos (Somerville 2002) so that they could indeed exist but be unobservable directly.

Gravitational lensing does in principle provide a method to detect dark matter substructure on these small scales, and the recent arguments presented by Metcalf & Madau (2001) and Dalal & Kochanek (2002) are compelling. The current claim is that in systems like B1422+231, the observed image magnification ratios cannot be obtained without substructure. The most general and convincing case was originally presented by Mao & Schneider (1998) for B1422+231, where there are three highly magnified images on one side of the lens center (images A, B, and C), forming a line. On the other side of the lens lies a single, fourth image, D, which has a radio flux that is a factor of 50 lower than that of the other three images. This means that either image D is demagnified, or the other three images are very strongly magnified. This, however, would imply that the source lies near a cusp. There exists a theorem that holds strictly only for sources on a cusp: the sum of the flux of the outer two images should equal the flux of the innermost image; or, written in terms of the magnifications:

\[ \mu_A + \mu_C = \mu_B \quad \text{for} \quad \mu_{\text{tot}} \to \infty, \]

(1)

where B is the central, brightest image. However, the images in B1422+231 do not obey this relation. Moving the source away from the cusp would mean the theorem need not hold but implies that the total magnification is small, and hence that image D needs to be strongly demagnified. Mao & Schneider (1998) argue that this is unlikely, as the image is far from the lens center and hence in a low \( \kappa \) region, leaving only the possibility that the source does lie close to a cusp but that the magnification of one or more of the images is effected by substructure in the vicinity.

Recently, Keeton, Gaudi, & Petters (2002) investigated the lensing properties of sources lying in cusps in more
detail. Their analysis is based on a generic one-component elliptical power law lens model, including external shear. They find that the image magnification ratios of a number of observed lens systems (B2045+265, RX J0911+0551, and B0712+472) cannot be fitted by such “cusp lenses.” No detailed modeling is done in that paper, and the argument is based on the deviant magnification ratios alone. The magnification ratio
\[ \frac{l_A + l_C}{l_B} \]
of observed systems lies typically between 1.5 and 2 for total magnifications of \( \sim 40 \). Simple lens models of these systems predict a magnification ratio \( \frac{l_A + l_C}{l_B} \) that is within a few percent of unity. The question arises whether the mismatch between observed and simulated image magnification ratios is not due to the specific one-component lens model used; a more complicated lens mass profile may explain the observed magnification ratios.

As seen from the maps in Figures 1–8, mergers and misalignments between a halo and disk can cause a strong change in the magnification pattern on the image plane. It is thus not unreasonable to propose that the magnification ratios might be explained by a more complex structure in the primary lens—for example, the presence of a misaligned disk, as discussed in this paper.

In Figures 9 and 10 we show the magnification ratio
\[ \frac{l_A + l_C}{l_B} \]
for our Milky Way–type model and for the early-type lens respectively for different offsets and disk parameters. In all cases, only regions in the source plane that would produce four detectable images are shown. In the case of Figure 9, this is only true for regions in the source plane that form a total of five images. For the early-type lens model, regions that produce a total of seven images in general also produce four bright images, the remaining three images being strongly demagnified (e.g., Fig. 7, top left and bottom left panels). To determine the magnification ratios for these systems, we proceed as follows: for each source position as defined by a grid on the source plane, we identify the four most strongly magnified images. Within this group, the image with the smallest magnification, image D, is identified. Within the remaining group of three magnified images,
we identify the central image as image B. The remaining two images are assigned labels A and C randomly. Having thus identified the images A, B, C, and D for each source position, we determine the magnification ratios from the individual magnifications as described in § 3.

The gray scale in Figures 9 and 10 is such that lightly shaded regions indicate source positions for which the magnification ratios do not obey the cusp relation. The top left panel in Figure 9 shows the case for no halo-disk misalignment for our Milky Way–type lens model; the disk in the remaining three panels has a moderate misalignment in the $x$-, the $y$-, and the $x$-$y$–direction. The caustic shape changes significantly once the disk is misaligned, leading to strong changes in the function of magnification ratio versus source position. Especially for a misalignment in the $y$–direction (Fig. 9, top left panel) and for misalignments of $\Delta x = \Delta y = 0^\circ 6$ (corresponding to 2.3 kpc), the caustic shape becomes very complicated, and in several cusp regions the expected magnification differs drastically from that expected in “normal” cusps. For the early-type lens in Figure 10, we show the magnification ratio only for offsets of $\Delta x = \Delta y = 0^\circ 6$, but for different disk parameters. The top left panel is for the standard early-type lens model described in the previous section. The three other panels are for three different sets of disk masses and sizes. The bottom right panel shows the results for a small, light disk. In all cases, sources lying in the upper right cusp would produce $3:1$ image geometries with $(\mu_A + \mu_C)/\mu_B$ magnification ratios that differ significantly from the expected ratio of 1.

Note that for the Milky Way–type lens, there is a strong effect due the presence of the disk even when it is not misaligned. We show the corresponding result for our standard early-type lens model in Figure 11. The scale is drastically different than that of Figures 9 and 10; the caustic structure is very small. Since we assume a spherical halo, the asymmetry in the potential that causes the inner caustic is solely due to the small disk embedded...
in the halo. However, there is not much qualitative change compared to the Milky Way–type lens: a disk also strongly affects the magnification ratios of four-image lens systems in early-type lens galaxies. This is in concordance with the findings of Möller, Hewett, & Blain (2002), who studied the effects of such aligned disks in early-type lensing galaxies.

Even though there are other explanations for the anomalous magnification ratios, like microlensing, differential extinction, etc., these do not cause drastic changes in image geometries and multiplicities. In contrast, misaligned disks may lead not only to relatively simple image geometries very similar to those expected from simple lens models (except for the magnification ratios), but also to more complicated image geometries with higher image multiplicities. Considering the expected probabilities for significant misalignments, lensing cross sections, and magnifications, simple image geometries will dominate the lensing statistics even when misaligned disks are present. In these cases, a simple lens model may reproduce the image positions but will fail to reproduce the flux ratios.

8. CONCLUSIONS

As has been illustrated in the plots in this paper, the presence of a misaligned disky component embedded in a dark matter halo produces a complex range of strong lensing effects—in the shapes and areas of the caustics, magnification ratios, and image geometries.

Most importantly, we show that for models of high-magnification ($\mu_{\text{tot}} \simeq 20$) systems, the presence of a disky
component can substantially alter the expected magnification ratios and image geometries. This is true for both large, Milky Way–type disks and for compact disks that might be found in early-type galaxies, whether or not the disk is aligned with the halo. However, misalignments of the disk relative to the halo, such as might be caused by mergers or strong interactions, can greatly increase the strength of this effect. The shape of the caustic structure changes dramatically, and depending on the ellipticity of the dark matter halo, the enclosed area is enhanced by up to a factor of 10. Thus, misaligned disks increase the overall cross section for the formation of four magnified images, while at the same time causing strong deviation from the magnification ratios expected from simple one-component lens models. Single-component, elliptical mass models are unable to reproduce such features, which may be the reason why models based on simple elliptical mass models fail to reproduce the observed image magnification ratios, as for B1422+231.

Due to the complexity of the lens model investigated here, we have been concerned mainly with investigating the qualitative effect of misaligned disks. Even though detailed modeling of existing systems using a halo in combination with a misaligned disk lens is in principle possible, this will be a complicated and time-consuming task. Because of the large number of parameters and the relatively few observational lensing constraints, it is very unlikely that a definite model can be found; instead, it is to be expected that there will be a considerable degree of degeneracy. However, our results do show clearly that the simple, one-component lens models currently used are likely to be too simplistic and that more complicated models, including a possibly misaligned disk component, can reproduce the observed properties of many lens systems without introducing the need for halo subclumps.

P. N. acknowledges a research fellowship from Trinity College, Cambridge. O. M. acknowledges support from the Marie Curie Fellowship program.

REFERENCES

Abel, T. 2002, in Proc. of the Yale Cosmology Workshop, The Shapes of Galaxies and Their Dark Matter Halos, ed. P. Natarajan (Singapore: World Scientific), 119

Barnes, J., & Hernquist, L. 1998, ApJ, 495, 187

Blandford, R., & Narayan, R. 1992, ARA&A, 30, 311

Borgeest, U, 1991, A&A, 243, 93

Browne, I., et al. 2001, in ASP Conf. Ser. 237, Gravitational Lensing: Recent Progress and Future Goals (San Francisco: ASP), 15

Cohen et al., 2001, ApJ, 554, 1216

Dalal, N., & Kochanek, C. 2002, ApJ, 572, 25

Dubinski, J., & Kuijken, K. 1995, ApJ, 442, 492

Fassnacht, C., Xanhipoulos, E., Koopmans, L. V. E., & Rusin, D. 2002, ApJ, in press (astro-ph/0208420)

González-Garcia, A. C. 2003, Ph. D. thesis, Groningen Univ.

Kassiola, A., & Kovner, I. 1993, ApJ, 417, 450

Keeton, C., Gaudi, B. S., & Petters, A. O. 2002, ApJ, submitted (astro-ph/0210318)

Keeton, C., Kochanek, C., & Seljak, U. 1997, ApJ, 482, 604

Kelson, D. D., Illingworth, G. D., van Dokkum, P. G., & Franx, M. 2000, ApJ, 531, 137

Koopmans, L. V. E., & Fassnacht, C. D. 1999, ApJ, 527, 513

Lehar et al. 2000, ApJ, 536, L584

Mao, S., & Schneider, P. 1998, MNRAS, 295, 587

Mao, S., Witt, H., & Koopmans, L. V. E. 2001, MNRAS, 323, 301

Metcalf, B., & Madau, P. 2001, ApJ, 563, 9

Möller, O., & Blain, A. 1998, MNRAS, 299, 845

Möller, O., Hewett, P., & Blain, A. 2002, MNRAS, submitted (astro-ph/0212467)

Möller, O., Natarajan, P., Kneib, J.-P., & Blain, A. 2002, ApJ, 573, 562

Mihos, C., & Hernquist, L. 1994, ApJ, 425, L13

Nelson, R. W., & Tremaine, S. 1995, MNRAS, 275, 897

Patton, D. R., et al. 2002, ApJ, 565, 208

Raig et al., 2001, MNRAS, 327, 939

Rest, A., et al. 2001, AJ, 121, 2431

Rusin, D., et al. 2001, ApJ, 557, 594

Schneider, P., Ehlers, R., & Falco, E. 1992, Gravitational Lenses (New York: Springer)

Sheth, R., Mo, H. J., & Tormen, G. 2001, MNRAS, 323, 1

Somerville, R. 2002, ApJ, 572, L23

Somerville, R., et al. 2000, MNRAS, 316, 479

White, S. D. M., & Frenk, C. S. 1991, ApJ, 379, 52

Fig. 11.—Magnification ratio \( \mu = (\mu_A + \mu_C)/\mu_B - 1 \) for an early-type galaxy with aligned disk. The parameters are as in Fig. 10 (top left panel), but \( \Delta x = \Delta y = 0 \). The gray scale is as in the previous figures. Note the much smaller scale than in Figs. 9 and 10.