Research Article

Effects of Contact Area and Contact Shape on Nonlinear Fluid Flow Properties of Fractures by Solving Navier-Stokes Equations

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The influences of contact shape and contact area on nonlinear fluid flow properties through fractures are investigated by solving Navier-Stokes equations. The evolutions of nonlinear relationships between flow rate and hydraulic pressure drop, Forchheimer coefficients, nonlinear factor, critical hydraulic gradient, distributions of flow streamlines, and tracer flow paths at different times are systematically estimated. The results show that the nonlinear relationships between flow rate and hydraulic pressure drop can be well described by Forchheimer’s law, in which the nonlinear term coefficient $b$ is approximately three orders of magnitude larger than the linear term coefficient $a$. The smaller contact area corresponds to smaller variations in many aspects such as flow rate, critical hydraulic gradient, flow streamlines, and tracer flow paths. The critical hydraulic gradient decreases with the increasing degree of contact shape variations while the contacts have the same mean area. The increase in hydraulic pressure drop can induce significant eddies and decrease the permeability and/or conductivity of fractures. However, the distributions of streamlines and tracer flow paths are not dramatically disturbed under a large hydraulic pressure drop.

1. Introduction

Accurate estimations of hydraulic properties of rock fractures are of very importance in many fields such as geothermal energy development [1, 2] and enhanced oil recovery [3–5]. It is well known that both the contact area ratio of fractures and high hydraulic pressure drop can significantly influence the permeability and/or conductivity of rock masses (Chen et al. 2015; [6, 7]). However, the effects of contact shape and contact area on nonlinear flow properties of fluids through fractures induced by high hydraulic pressure drop by solving Navier-Stokes equations have not yet been systematically investigated.

In early works, the relationships between hydraulic aperture and mechanical aperture have been studied, and mathematical expressions that incorporated the contact area ratio have been proposed. Walsh [8] concluded that $e^3/E^3 = (1 - C)/(1 + C)$, where $e$ is the hydraulic aperture, $E$ is the mechanical aperture, and $C$ is the contact area ratio that is defined as the ratio of contact area to the area of surfaces. Zimmerman et al. [9] reported that $C = 0.25$ in their studies. Zimmerman and Bodvarsson [10] derived that $e^3/E^3 \approx (1 - 1.5\sigma^2/\sigma_{apert}/E^2 + \cdots)(1 - 2C)$. However, the quantifications of the contact area ratio are difficult because the rock mass is invisible. Wang et al. [6] carried out shear-flow tests on rough fractures and calculated the contact area ratio according to the shear displacement and normal displacement using a code developed by themselves. They found that as shear displacement increases from 1 to 3 mm, the contact area ratio dramatically decreases from 0.33 to 0.09. When the shear displacement continuously increases to 9 mm, the contact area ratio fluctuates in a negligibly small range. Liu et al. [11] performed numerical simulations to model fluid flow through rough-walled fractures during shearing by solving Navier-Stokes equations. The results show that the contact area ratio increases linearly...
with boundary stiffness and is in the range of 0.01–0.16 when boundary stiffness = 0 ~ 2 GPa/m, joint compression strength = 100 ~ 150 MPa, and initial normal stress = 0.5 ~ 2.0 MPa. However, the contact area and locations are dependent on shear displacement and fracture surface morphologies [12], and the effect of the contact area ratio on nonlinear fluid flow properties is not estimated.

The previous studies commonly assumed that fluid flow is in the linear flow regime, in which the flow rate is linearly correlated with hydraulic pressure drop and the cubic law is
typically used for modeling fluid flow through fractures [13–17]. However, in the karst systems and/or in the pumping tests [18, 19], the flow rate is very large and the inertial force cannot be negligible, resulting in a nonlinear relationship between flow rate and hydraulic gradient. Thus, the permeability and/or conductivity of fractures are reduced [20–22]. Although the effects of joint surface roughness (JRC), Reynolds number (Re), and shear displacement on nonlinear properties of fluid flow have been systematically investigated [23–26], the influences of the contact shape and contact area have not been studied in an in-depth way.

The present study, first, numerically generated a series of fracture models containing contacts with different shapes and different area ratios. Then, the Navier-Stokes equations are solved based on the finite volume method (FVM), and the nonlinear relationships between the flow rate and hydraulic drop, as well as linear and nonlinear coefficients in Forchheimer’s law and critical hydraulic gradient, are quantitatively analyzed. Finally, the streamlines of fluid flow under different shapes and areas of contacts and different hydraulic pressure drops are estimated, and the distributions of tracer flow paths at different times are discussed.

2. Governing Equations of Fluid Flow through Fractures

The fluid flow through fractures is governed by the Navier-Stokes equations, by assuming that the fluid is incompressible and is a kind of Newtonian fluids. The tensor form of Navier-Stokes equations is written as shown in [27, 28]

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}, \]

(1)

where \( \mathbf{u} \) is the flow velocity tensor, \( \rho \) is the fluid density, \( p \) is the hydraulic pressure, \( \mathbf{T} \) is the shear stress tensor, \( t \) is the time, and \( \mathbf{f} \) is the body force tensor. The flow rate \( Q \) can be calculated by multiplying the cross-sectional area \( A \) by \( u \).

The units of \( Q, A, \) and \( u \) are \( \text{m}^3/\text{s}, \text{m}^2, \) and \( \text{m/s} \), respectively.

It is difficult and time-consuming to solve the Navier-Stokes equations, because a series of partial differential equations should be solved by iterations. Therefore, some useful empirical functions are proposed such as Forchheimer’s law [22, 27, 29, 30], which can be written as

\[ -\frac{\Delta P}{\Delta L} = aQ + bQ^2, \]

(2)

where \( \Delta L \) is the length of the model and \( a \) and \( b \) are the linear and nonlinear coefficients, respectively.

When \( Q \) is sufficiently small, the values of \( bQ^2 \) are much smaller than \( aQ \). Thus, equation (2) can be reduced to

\[ -\frac{\Delta P}{\Delta L} = aQ. \]

(3)

Equation (3) is similar to Darcy’s law, in which the flow rate is linearly correlated with hydraulic pressure drop. For fluid flow through fractures, equation (3) can be expressed as the following cubic law [31, 32]:

\[ Q = -\frac{we^3}{12\mu} \frac{\Delta P}{\Delta L}, \]

(4)

where \( w \) is the width of fractures, \( e \) is the hydraulic aperture, and \( \mu \) is the dynamic viscosity.

When \( bQ^2 \) is not sufficiently small with respect to \( aQ \), the terms \( bQ^2 \) cannot be negligible, resulting in a nonlinear relationship between \( Q \) and \( \Delta P/\Delta L \). A nonlinear factor \( E \) is utilized to quantify the nonlinearity of fluid flow, defined as

\[ E = \frac{bQ^2}{aQ + bQ^2}. \]

(5)

\( E \) represents the percentage of hydraulic pressure drop induced by inertial forces to the total hydraulic pressure drop. Although \( E = 0.01 \) and \( E = 0.5 \) have been used for quantifying the onset of nonlinear flow of fluids in some previous works [33, 34], it is commonly adopted that \( E = 0.1 \) is the threshold [20, 21]. In the present study, \( E = 0.1 \) is chosen as the transition of fluids from a linear regime to a nonlinear regime. The hydraulic gradient (\( J \)) that corresponds to \( E = 0.1 \) is the critical hydraulic gradient (\( J_c \)). \( J \) is a non-dimensional parameter, which is defined as the ratio of hydraulic head difference to the length of fractures. Note that the \( J \) can also characterize the fluid flow properties of single fractures yet cannot interpret the flow state of complex fracture networks. However, \( J \) has a known value as long as the hydraulic water heads applied on the opposite boundaries are set. Therefore, \( J \) is suitable to characterize the flow state for both single fractures and fracture networks and is adopted and utilized.

3. Numerical Simulation Results and Analysis

3.1. Numerical Fracture Models Containing Contacts

Ten two-dimensional square numerical fracture models are
established as shown in Figure 1. The side length \((L)\) of fractures is 0.1 m. Within each model, a total of 100 contacts are generated with an interval of the centers to be 0.01 m along both \(x\)- and \(y\)-directions. For simplification, the contacts are assumed to be rectangular-shaped. The mean side length of contacts \((L_{\text{mean}})\) is 3 mm and 5 mm as shown in Figures 1(a)–1(j), respectively. Thus, the contact areas are 900 mm\(^2\) and 2500 m\(^2\), and the contact area ratios are 9%
and 25%, respectively. The side length of each contact ($L_c$) is calculated as follows:

$$L_c = m + nR,$$

where $m$ and $n$ are coefficients and $R$ is the random number that ranges from 0 to 1.

The values of $L_{\text{mean}}$, $m$, and $n$ are listed in Table 1. A parameter $K$ is used to represent the deviation of side length of contacts from the mean values, which is calculated as follows:

$$K = L_{\text{mean}} - m.$$  

A larger $K$ represents that the contacts are more scatteredly distributed, while a larger $L_{\text{mean}}$ indicates a larger contact area.

The fracture models as shown in Figure 1 are then imported into ANSYS ICEM for meshing with a maximum side length of 0.2 mm. The meshed files are imported into ANSYS FLUENT to model fluid flow. The hydraulic pressure difference ($\Delta P$) is assigned to be 0.01, 0.05, 0.1, 0.3, and 0.5 Pa, respectively. The water at a room temperature is selected as the fluid, flowing from the left side of the model to the right side. The density is 998.2 kg/m$^3$ and the dynamic viscosity is 0.001003 Pa·s.

### 3.2. Nonlinear Relationships between Flow Rate and Hydraulic Pressure Drop

Figure 2 shows that with the increment of $Q$, $\Delta P$ increases nonlinearly at an increasing rate. The nonlinear relationships between $Q$ and $\Delta P$ follow the Forchheimer’s law as shown in equation (2), in which the intercept of the quadratic functions is zero. As $L_{\text{mean}}$ increases, the contact area increases while the void spaces for fluid flow decrease, resulting in decreases in $Q$ when $\Delta P$ is fixed. Taking $L_{\text{mean}} = 3$ mm and $K = 0$ (Figure 2(e)) and $L_{\text{mean}} = 5$ mm and $K = 0$ (Figure 2(j)) as the examples, when $\Delta P = 0.5$ Pa, $Q$ decreases from 0.00104 m$^3$/s to 0.0004895 m$^3$/s, by a rate of 52.93%. When $L_{\text{mean}} = 3$ mm, $Q$ increases in a limited range with decreasing $K$ from 2 to 0, whereas when $L_{\text{mean}} = 5$ mm, $Q$ increases significantly (from 0.0002557 m$^3$/s to 0.0004895 m$^3$/s by a rate of 91.44%) with decreasing $K$ from 4 to 0. This indicates that when the contact area represented by $L_{\text{mean}}$ is small, the change in contact shape distributions represented by $K$ will not change robustly, because the small area of each contact would not influence the flow paths to a large extent. However, when $L_{\text{mean}}$ is large, the void spaces between contacts may be narrowed and the flow paths change significantly. As a result, energy losses and $Q$ decreases obviously.

The variations in Forchheimer coefficients $a$ and $b$ of functions as shown in Figure 2 are presented in Figure 3 and Table 1. When $L_{\text{mean}} = 3$ mm, $a$ changes slightly in the range of 252–264 Pa·s·m$^{-4}$, whereas $a$ changes significantly from 800 Pa·s·m$^{-4}$ to 1320 Pa·s·m$^{-4}$ when $L_{\text{mean}} = 5$ mm. The variations in $b$ are similar to those of $a$ for $L_{\text{mean}} = 3$ and 5 mm. Therefore, the larger $L_{\text{mean}}$ induces a more significant change in both $a$ and $b$. The values of $b$ are generally 2–3 orders of magnitude larger than those of $a$, which are

![Figure 3: Variations in coefficients a and b for L_mean = 3 and 5.](image-url)
much less than those reported in the literature [6, 35], although the variation trends are consistent, because the fracture models and values of $J$ are different. With increasing $K$ from 0 to 3 when $L_{\text{mean}} = 5 \text{ mm}$, $a$ fluctuates from 800 Pa·s·m$^{-4}$ to 841 Pa·s·m$^{-4}$ by a rate of 5.13% and $b$ increases from $4.52 \times 10^5 \text{ Pa} \cdot \text{s}^2 \cdot \text{m}^{-7}$ to $1.09 \times 10^6 \text{ Pa} \cdot \text{s}^2 \cdot \text{m}^{-7}$ by a rate of 141.15%. However, when $K$ increases from 3 to 4, $a$ increases from 841 Pa·s·m$^{-4}$ to 1320 Pa·s·m$^{-4}$ by a rate of $y = -1.93 \times 10^6 x^3 + 1.80 \times 10^5 x^2 + 4.54 \times 10^4 x + 0.0001$

$R^2 = 9.88 \times 10^{-1}$

Figure 4: Nonlinear relationships between $Q$ and $\Delta P$. 

\[
\begin{align*}
(a) & \quad L_{\text{mean}} = 3 \text{ mm and } K = 2.0 \\
(b) & \quad L_{\text{mean}} = 3 \text{ mm and } K = 1.5 \\
(c) & \quad L_{\text{mean}} = 3 \text{ mm and } K = 1.0 \\
(d) & \quad L_{\text{mean}} = 3 \text{ mm and } K = 0.5 \\
(e) & \quad L_{\text{mean}} = 3 \text{ mm and } K = 0 \\
(f) & \quad L_{\text{mean}} = 5 \text{ mm and } K = 4 \\
(g) & \quad L_{\text{mean}} = 5 \text{ mm and } K = 3 \\
(h) & \quad L_{\text{mean}} = 5 \text{ mm and } K = 2 \\
(i) & \quad L_{\text{mean}} = 5 \text{ mm and } K = 1 \\
(j) & \quad L_{\text{mean}} = 5 \text{ mm and } K = 0
\end{align*}
\]
3.4. Streamline Distributions. Figure 6 shows the streamline distributions for different $L_{\text{mean}}$, $K$, and $\Delta P$. Comparing Figure 6(a) with Figure 6(e), it clearly exhibits that the eddies are not formed at $\Delta P = 0.01$ Pa with fluid smoothly flowing by passing the contacts yet are formed at $\Delta P = 0.5$ Pa behind the contacts. These eddies induce energy losses and decrease the permeability and/or conductivity. Figures 6(a) and 6(b) show that although the flow paths are changed between contacts, the distributions of flow paths are not influenced robustly as a whole. However, the flow paths are significantly changed for a relatively large $L_{\text{mean}}$ such as $L_{\text{mean}} = 5$ mm as shown in Figures 6(c) and 6(d). With the increment of $K$ (i.e., comparing Figures 6(h) with Figure 6(g)), the distributions of eddies including size and number of eddies are different, resulting in different energy losses and different permeability and/or conductivities. Figures 6(c) and 6(g) present that the fluid concentrates at the places where the aperture is relatively large. The increasing $\Delta P$ cannot change the streamline distributions to a large extent yet can induce the formations of eddies.

3.5. Tracer Flow Simulations. Figures 7 and 8 show the tracer flow paths under different $t$, $K$, and $\Delta P$ for $L_{\text{mean}} = 3$ mm and 5 mm, respectively. When $L_{\text{mean}} = 3$ mm, the effect of $\Delta P$ plays a negligibly small effect on the distribution of tracers. The fronts of the flow paths generally have the same shape and distance from the inlet. This well interprets that the effect of $K$ slightly influences the values of $Q$, $a$, $b$, and $E$ when $L_{\text{mean}} = 3$ mm as shown in Figures 2–4. For $L_{\text{mean}} = 5$ mm, the effect of $K$ is more important than that for $L_{\text{mean}} = 3$ mm, because the tracer flow paths are significantly changed, which then impact on the variations in $Q$, $a$, $b$, and $E$ as shown in Figures 2–4. Comparing Figure 8(a) with Figure 8(b), the tracers are uniformly distributed at the inlet, where the flow paths are continuous from the inlet to the outlet when $K = 0$, whereas when $K = 2$, the tracers are concentrated on several places where the local apertures are large. With the increment of $t$, when $K = 0$, the tracers gradually flow through the inlet to the outlet. When $K = 2$ or 4, the tracers flow through relatively large local apertures, showing anisotropic flow properties. The dominant flow paths do not change for different $\Delta P$, although a large number of eddies are formed when $\Delta P = 0.5$ Pa, in which the permeability and/or conductivity are significantly decreased.

4. Conclusions

The present work studied the effects of the contact shape and contact area on nonlinear flow properties of fluids through fractures by numerically solving Navier-Stokes equations. A total of ten numerical models with different contact shapes and shapes are generated and utilized for modeling fluid flow. The nonlinear relationships between the flow rate and hydraulic drop, as well as the variations in Forchheimer coefficients and critical hydraulic gradient, are characterized. Finally, the streamline distributions and tracer flow paths are presented and analyzed.
The results show that with the increment of hydraulic pressure drop from 0.01 Pa to 0.5 Pa, the flow rate increases nonlinearly, following Forchheimer’s law. Under a fixed hydraulic pressure drop, the flow rate does not change significantly versus \( K \) when the mean side length of contacts is small (i.e., 3 mm), yet the flow rate decreases dramatically versus \( K \) when the mean side length of contacts is large (i.e., 5 mm). Here, \( K \) represents the maximum deviation of side lengths of contacts from the mean value. As a result, the variations in Forchheimer coefficients \( a \) and \( b \), nonlinear

**Figure 6:** Streamline distributions for different \( L_{\text{mean}} \), \( K \), and \( \Delta P \). Each color of the lines represents a particle.
factor $E$, critical hydraulic gradient, and distributions of flow streamlines and tracer flow paths are in a smaller degree for a smaller mean side length of contacts (or a larger contact area). This is reasonable, because the larger contact area induces the smaller void spaces (or apertures) that are provided for fluid flow, hence the smaller permeability and/or conductivity. The critical hydraulic gradient decreases with the increasing $K$, while the mean side length of contacts is the same. The critical hydraulic gradient varies from $3.36 \times 10^{-5}$ to $4.19 \times 10^{-5}$ for $K = 0 \sim 2$ and mean side length of contacts = 3 mm, while the critical hydraulic gradient varies from $8.59 \times 10^{-5}$ to $1.77 \times 10^{-4}$ for $K = 0 \sim 4$ and mean side length of contacts = 5 mm. The increase in hydraulic pressure drop cannot change the flow streamline distributions and
The distributions of tracer flow paths at different times, although the large hydraulic pressure drop can induce the formations of eddies and reduce the permeability and/or conductivity. The preferential flow paths are more obviously characterized with a larger $K$ and a larger mean side length of contacts.

The present study is aimed at investigating the effects of the contact area and contact shape on nonlinear flow properties of fluids through contacted fractures. Some assumptions are made that the contact is rectangular-shaped and located uniformly along both $x$- and $y$-directions. However, these assumptions greatly deviate from natural cases where the contacts are formed by shearing or coupled thermo-hydro-mechanical-chemical (THMC) processes. Therefore, in the future works, we will focus on the nonlinear fluid flow properties of natural fractures that have rough surface and contacts in a natural state.

**Figure 8:** Tracer flow paths for $L_{\text{mean}} = 5$ mm under different $t$, $K$, and $\Delta P$. 

(a) $t = 42$ s, $K = 0$, $\Delta P = 0.01$ Pa  
(b) $t = 42$ s, $K = 2$, $\Delta P = 0.01$ Pa  
(c) $t = 3$ s, $K = 0$, $\Delta P = 0.5$ Pa  
(d) $t = 3$ s, $K = 4$, $\Delta P = 0.5$ Pa  
(e) $t = 84$ s, $K = 0$, $\Delta P = 0.01$ Pa  
(f) $t = 84$ s, $K = 2$, $\Delta P = 0.01$ Pa  
(g) $t = 6$ s, $K = 0$, $\Delta P = 0.5$ Pa  
(h) $t = 6$ s, $K = 4$, $\Delta P = 0.5$ Pa  
(i) $t = 220$ s, $K = 0$, $\Delta P = 0.01$ Pa  
(j) $t = 220$ s, $K = 2$, $\Delta P = 0.01$ Pa  
(k) $t = 11$ s, $K = 0$, $\Delta P = 0.5$ Pa  
(l) $t = 11$ s, $K = 4$, $\Delta P = 0.5$ Pa  
(m) $t = 530$ s, $K = 0$, $\Delta P = 0.01$ Pa  
(n) $t = 530$ s, $K = 2$, $\Delta P = 0.01$ Pa  
(o) $t = 20$ s, $K = 0$, $\Delta P = 0.5$ Pa  
(p) $t = 20$ s, $K = 4$, $\Delta P = 0.5$ Pa
Data Availability

The data can be available by contacting the corresponding author.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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