Trade-offs analysis of sustainability dimensions using integer-valued data envelopment analysis

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Abstract. Conducting an in-depth exploration of trade-offs between sustainability aspects is a notable matter of taking decisions. Furthermore, there are many real world investigations that trade-offs and sustainability should be dealt with in the presence of desirable and undesirable materials while some of them accept integer amounts. Therefore, this study addresses trade-offs of sustainability dimensions when undesirable and integer-valued measures are presented. For this purpose, approaches based upon data envelopment analysis (DEA) are proposed. To explain, DEA models are introduced to calculate individual and group marginal rates of substitution and also directional marginal rates of substitution when integer and undesirable variables are observed. These procedures are applied to calculate trade-offs between different sustainability dimensions, including economic, environmental and social ones. The applications of ports and industrial parks are provided to clarify the approaches appeared in this study. The results derived from the proposed strategies show the usefulness and validity of them.

Keywords: data envelopment analysis, integer, marginal rates of substitution, sustainability, undesirable outputs

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1. Introduction

The consideration of sustainability performance and trade-offs of sustainability dimensions is a significant aspect for making decisions. In many real world conditions, there are also integer-valued variables and undesirable products. A valuable approach to benchmark decision making units (DMUs) and to assess the performance is the non-parametric data envelopment analysis (DEA) technique, initially developed by Charnes et al. [5].

In the DEA literature, some studies have been presented to deal with the sustainability performance of processes. Tajbakhsh and Hassini [22] provided centralized and non-cooperative DEA models to analyze sustainable multi-stage structures while integer and undesirable performance measures were not investigated. Amirteimoori et al. [3] introduced a DEA-based approach to evaluate the sustainability of proceedings over multiple periods when weakly disposable undesirable outputs are appeared in the operation. Modeling with weakly disposable undesirable outputs and non-uniform abatement factor initially developed by Kuosmanen [13] in the DEA studies. Tajbakhsh and Shamsi [23] rendered efficiency and super-efficiency DEA models to assess the sustainability of countries matters when undesirable integer measures are presented. Undesirable outputs (inputs) are supposed as desirable inputs (outputs) and trade-offs of sustainability dimensions are not addressed in that research. Jahani Sayyad Noveiri and Kordrostami [10] determined the sustainability performance of systems with discrete and

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bounded factors in several periods of time while trade-offs between sustainability areas have not been tackled. Also, Zhou et al. [25] conducted a review of the literature on DEA applications in sustainability issues.

Furthermore, from one hand, there are DEA–based studies without including sustainability issues to measure the efficiency in the presence of undesirable and integer variables. Chen et al. [6] introduced the integer additive DEA model with undesirable factors and provided its associated additive super–efficiency model to discriminate the efficient entities. Undesirable inputs (outputs) managed as desirable outputs (inputs) in that consideration. Amirteimoori and Maghbouli [2] assessed the efficiency of DMUs in the presence of integer measures and weakly disposable undesirable outputs. In the DEA literature, there are different approaches to address undesirable outputs. Halkos and Petrou [9] classified approaches applied to handle undesirable outputs into four categories, including ignoring them, treating them as inputs, regarding them as ordinary outputs and applying transformations. For instance, the assumptions of weak and managerial disposability are some of plans to concentrate undesirable outputs with their benefits and drawbacks. Also, following [14], researchers can take each approach in order to deal with undesirable outputs as a result of the applications that they conduct. Due to the main purpose of this study that is rendering DEA approaches to analyze trade–offs of sustainability areas in the presence of integer values, we consider the weak disposability assumption for undesirable outputs. However, approaches proposed in this study can be extended for other existing techniques to consider undesirable outputs such as modelling with managerial disposability proposed by Sueyoshi and Goto [21].

And from the other hand, some DEA investigations have been concentrated on trade-offs between indicators. Asmild et al. [4] expanded Rosen et al. approach [19] to examine non-marginal trade-offs between measures. Khoshandam et al. [12] estimated marginal rates of substitutions in the presence of undesirable outputs. Actually, they calculated the directional marginal rates of substitution in a weak disposability production technology while all measures considered as real values. Mirzaei et al. [15] applied binding supporting surfaces in an efficient point to compute different marginal rates. Mirzaei et al. [16] planed a method in accordance with the free disposal hull model to estimate marginal rates of substitution where the convexity principle is not satisfied. Podinovski et al. [18] calculated different directional differentials for some technologies organized as polyhedral sets such as the ones with weakly disposable undesirable products using a unified linear method. Podinovski [17] discussed more about the application of the technique presented by Podinovski et al. [18] to technologies with weakly disposable undesirable outputs and dealt with different marginal rates and elasticity measures.

However, to our knowledge, marginal rates of substitutions in the presence of integer measures have not been investigated in the DEA context whilst it is important to find optimal integer values for integer performance measures in trade-offs. On the other hand, trade-off analysis as the interrelation between sustainability indicators is vital for comparison and decision-making among various choices. Undesirable products and integer-valued indicators are also present in many occasions of sustainability assessment. Therefore, procedures founded on DEA are proposed in this research to analyze marginal rates of substitution of sustainability areas when undesirable and integer-valued indicators are present in the system. To illustrate in more details, individual and group marginal rates of substitution and also directional marginal rates of substitution between sustainability components, including economic, environmental and social ones are estimated whilst there are integer and undesirable indicators. Integer variables are, actually, dealt with in trade-offs and integer results are found for integer-valued performance measures. The study areas of ports and industrial parks are used to examine the validity of the designed approach herein. In summary, the main purpose of this study is the response to the following:

What-if questions related to sustainability areas and in the presence of integer values and undesirable measures.
The structure of this research is arranged as follows. Some preliminaries and concepts which are prerequisite to clarify the proposed method are provided in Section 2. The suggested methodology is disclosed in Section 3. Two applications of ports and industrial parks are given in Section 4 to illustrate and verify the introduced technique. Finally, conclusions and some recommendations are presented in Section 5.

2. Preliminaries

In this section, some terminologies, concepts and procedures associated with the proposed methodology are given that are essential for better insight of the issue under investigation.

2.1. Weakly disposable undesirable outputs

Assume there are $n$ DMUs ($DMU_j; j = 1, \ldots, n$) with $m$ inputs $x_j = (x_{1j}, \ldots, x_{mj})$, $s$ desirable outputs $y_j = (y_{1j}, \ldots, y_{sj})$ and $K$ undesirable outputs $w_j = (w_{1j}, \ldots, w_{Kj})$.

According to [20], outputs $(y, w)$ are stated to be weakly disposable if and only if $(x, y, w) \in V = \{x \text{ can produce } (y, w)\}$ reveals that $(x, \theta y, \theta w) \in V$ for all $0 \leq \theta \leq 1$, $x \in R^+_m, y \in R^+_s$ and $w \in R^+_K$. $\theta$ is refereed as the abatement factor. It means that proportional decreases of attainable outputs are feasible. Fare and Grosskopf [8] considered a single abatement factor $\theta$ and presented the following technology:

$$V = \{(x, y, w) | \sum_{j=1}^n \lambda_j x_{ij} \leq x_i, i = 1, \ldots, m, \theta \sum_{j=1}^n \lambda_j y_{rj} \geq y_r, r = 1, \ldots, s, \theta \sum_{j=1}^n \lambda_j w_{kj} = w_k, k = 1, \ldots, K, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, 0 \leq \theta \leq 1\}$$

(1)

that the technology is based on variable returns to scale assumption and $\lambda_j$ is intensity variables.

Kuosmanen [13] took different abatement factors $\theta_j$ into account in technology (1) and rendered the following linear form using a change of variables:

$$V = \{(x, y, w) | \sum_{j=1}^n (\lambda_j + \mu_j) x_{ij} \leq x_i, i = 1, \ldots, m, \sum_{j=1}^n \lambda_j y_{rj} \geq y_r, r = 1, \ldots, s, \sum_{j=1}^n \lambda_j w_{kj} = w_k, k = 1, \ldots, K, \sum_{j=1}^n (\lambda_j + \mu_j) = 1, \lambda_j \geq 0, \mu_j \geq 0\}$$

(2)

in which $\lambda_j$ and $\mu_j$ are weight variables.

2.2. Technology with integer and real input-output factors and undesirable outputs

Suppose that $(x, y, w)$ can be partitioned into $(x^I, x^{NI}, y^I, y^{NI}, w^I, w^{NI})$. In other words, each set of inputs, desirable outputs and undesirable outputs are divided into sets of integer and real measures denoted by superscript $I$ and $NI$ that $x^I (I \in I^I)$, $y^I (I \in O^I)$, $w^I (I \in K^I)$, $x^{NI} (NI \in I^{NI})$, $y^{NI} (NI \in O^{NI})$ and $w^{NI} (NI \in K^{NI})$. Notice that $I^I \cup I^{NI} = m, O^I \cup O^{NI} = s$ and $K^I \cup K^{NI} = K$. Amirteimoori and Maghbouli [2] advanced the following integer technology including the weak disposability assumption of undesirable outputs.

$$V^{IW} = \{(x^I, x^{NI}, y^I, y^{NI}, w^I, w^{NI}) | \sum_{j=1}^n (\lambda_j + \mu_j) x_{ij} \leq x_i, i \in I^I, \sum_{j=1}^n (\lambda_j + \mu_j) x_{ij} \leq x_i, i \in I^{NI}, \sum_{h=1}^n \lambda_h y_{rj} \geq y_r, r \in O^I, \sum_{h=1}^n \lambda_h y_{rj} \geq y_r, r \in O^{NI}, \sum_{j=1}^n \lambda_j w_{kj} = w_k, k \in K^I, \sum_{j=1}^n \lambda_j w_{kj} = w_k, k \in K^{NI}, \sum_{j=1}^n (\lambda_j + \mu_j) = 1, \lambda_j \geq 0, \mu_j \geq 0\}$$

(3)
2.3. Trade-offs and marginal rates of substitution

The information of trade-offs between indicators in processes is significant for making appropriate decisions. Trade-offs such as marginal rates of substitution show partial differential or slopes of the performance frontier for certain items. To give further details, the marginal rates of substitution are the rates wherein one or multiple variables of units can change for the variations of one or more other indicators. At this point, the procedure described by Asmild et al. [4] to calculate marginal rate of substitution is given briefly.

Consider each entity as a throughput vector \( z_j = (-x_j, y_j) \) that \( x_j \in R^+_{m} \) and \( y_j \in R^+_{s} \). As mentioned in [4], at the point \( z_o \) on the frontier, the marginal rates of substitution of throughput \( j \) to throughput \( b \) can be computed through the following four steps:

a. Specify a slight expansion \( h \) for the throughput \( b \).

b. Solve the following model to attain the optimal value \( z^*_{oj} \):

\[
\begin{align*}
\text{Max} & \quad z^*_{oj} \\
\text{s.t.} & \quad \{ Z\lambda \geq z^*_o, 1^T \lambda = 1, z^*_o = z_{ol}, l \neq j, b; z^*_ob = z_{ob} + h, \lambda \geq 0 \}
\end{align*}
\]

in which the \( n \)-dimensional vectors \( \lambda \) and \( 1 \) are the intensity variable and unity, respectively.

c. Apply the following formula and calculate the marginal rates of substitution from the right:

\[
MRS^+_{jb}(z_o) = \frac{z^*_{oj} - z_{oj}}{h}
\]

d. Estimate the marginal rates of substitution from the left by iterating stages \( b \) and \( c \) for \( -h \) instead of \( h \).

In the next section, plans based on DEA are proposed to assess marginal rates of substitution for sustainability dimensions with encountering integer and real indicators and undesirable outputs.

3. Methodology

Suppose there are \( n \) entities, \( DMU_j(j = 1, ..., n) \) with \( m \) inputs, \( x_{ij}^d(i = 1, ..., m), s \) desirable outputs \( y_{irj}^d(r = 1, ..., s) \) and \( K \) undesirable outputs, \( w_{kj}^d(k = 1, ..., K) \). Performance measures are taken integer and non-integer values into account and they are incorporated into three dimensions, including economic, social and environmental areas, that is \( i \in I^d \cup I^N^d = I = \{1, ..., m^d \}, r \in O^d \cup O^N^d = O = \{1, ..., s^d \}, k \in U^d \cup U^N^d = U = \{1, ..., K^d \} \), in which \( d \in D = \{ \text{economic, social, environmental} \} \).

The following model is provided to estimate the sustainable performance of entities in the presence of undesirable and integer-valued measures. Notice that undesirable outputs are treated as weakly disposable and the models are based on the variable returns to scale (VRS) assumption.
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\[ e^* = \text{Min} \left( \frac{1}{m+n} \left( \sum_{i \in I} \theta_i + \sum_{k \in U} \varphi_k \right) \right) \]
\[ \text{s.t.} \]
\[ \sum_{j=1}^{n} (\lambda_j + \mu_j) x_{ij}^d \leq \tilde{x}_i^d, \quad i \in I^d, d \in D, \]
\[ x_i^d \leq \theta_i x_{io}^d, \quad i \in I^d, d \in D, \]
\[ \sum_{j=1}^{n} (\lambda_j + \mu_j) x_{ij}^{d'} \leq \tilde{x}_i^{d'}, \quad i \in I^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} \lambda_j y_{ij}^{d'} \geq \tilde{y}_{i}^{d'}, \quad r \in O^{d'}, d \in D, \]
\[ y_{ij}^{d'} \geq y_{ir}^{d'}, \quad r \in O^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} \lambda_j y_{ij}^{d'} \geq y_{ir}^{d'}, \quad r \in O^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} \lambda_j w_{j}^{d'} = \tilde{w}_{k}^{d'}, \quad k \in U^{d'}, d \in D, \]
\[ \tilde{w}_{k}^{d'} = \varphi_k w_{ko}^{d'}, \quad k \in U^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} (\lambda_j + \mu_j) = 1, \]
\[ \theta_i \leq 1, \varphi_k \leq 1, \tilde{x}_i^d \in Z^+, \forall i \in I^d, \forall d \in D, \tilde{y}_{i}^{d'} \in Z^+, \forall r \in O^{d'}, \forall d \in D, \]
\[ \tilde{w}_{k}^{d'} \in Z^+, \forall k \in U^{d'}, \forall d \in D, \lambda_j, \mu_j \geq 0. \]

**Definition 1.** The unit under evaluation is called efficient if and only if \( e^* = 1 \), i.e. the optimal values \( \theta_i^* = 1 \) and \( \varphi_k^* = 1 \).

### 3.1. The marginal rates of substitution between two components

In this place, tradeoffs are addressed for sustainable entities that appear on the frontier. For unsustainable entities, marginal rates of substitution are measured for the projection points of them into the frontier. Thus, the following steps are applied to determine the marginal rates of substitution between any two measures from economic and environmental dimensions. It is clear that this procedure can be extended and analyzed for different sustainability dimensions.

a. A small increment \( h \) for \( p \)th throughput from the environmental aspect is considered.

b. The following problem is computed to find \( y_{so}^{\text{economic}} \); in other words, the integer-valued economic factor \( y_{so}^{\text{economic}} \) results from the change of \( p \)th environmental measure.

\[ es^* = \text{Max} \ y_{so}^{\text{economic}} \]
\[ \text{s.t.} \]
\[ \sum_{j=1}^{n} (\lambda_j + \mu_j) x_{ij}^d \leq \tilde{x}_i^d, \quad i \in I^d, d \in D, \]
\[ x_i^d \leq \theta_i x_{io}^d, \quad i \in I^d, d \in D, \]
\[ \sum_{j=1}^{n} (\lambda_j + \mu_j) x_{ij}^{d'} \leq \tilde{x}_i^{d'}, \quad i \in I^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} \lambda_j y_{ij}^{d'} \geq \tilde{y}_{i}^{d'}, \quad r \in O^{d'}, d \in D, \]
\[ y_{ij}^{d'} \geq y_{ir}^{d'}, \quad r \in O^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} \lambda_j y_{ij}^{d'} \geq y_{ir}^{d'}, \quad r \in O^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} \lambda_j w_{j}^{d'} = \tilde{w}_{k}^{d'}, \quad k \in U^{d'}, d \in D, \]
\[ \tilde{w}_{k}^{d'} = \varphi_k w_{ko}^{d'}, \quad k \in U^{d'}, d \in D, \]
\[ \sum_{j=1}^{n} (\lambda_j + \mu_j) = 1, \]
\[ \theta_i \leq 1, \varphi_k \leq 1, \tilde{x}_i^d \in Z^+, \forall i \in I^d, \forall d \in D, \tilde{y}_{i}^{d'} \in Z^+, \forall r \in O^{d'}, \forall d \in D, \]
\[ \tilde{w}_{k}^{d'} \in Z^+, \forall k \in U^{d'}, \forall d \in D, \lambda_j, \mu_j \geq 0. \]

### c. The marginal rate of substitution from the right is estimated as follows:

\[ MR_{sp}^d = \frac{y_{so}^{\text{economic}} - y_{so}^{\text{economic}}}{h} \]
3.2. The generalization to multi-component scenarios

The influence of the change of one variable into another measure has been dealt with in the proceeding expressions while integer-valued and undesirable measures are presented. In this subsection, changes of a set of variables resulted from changes of another set of variables are examined when there are integer and undesirable factors. Consider two sets \( A = \{a_1, ..., a_c\} \) and \( B = \{b_1, ..., b_d\} \). The purpose is the investigation of the changes of the set \( A \) derived from the changes of the set \( B \). Assume members of the set \( B \) change by the value \( \beta \). Therefore, we follow the below three-stage approach to calculate the changes of the members of the set \( A \).

a. Solve the following multi objective mixed integer linear programming (MOMILP) for changing the set \( B \) by the value \( \beta \) and find \( y^*_so^{economic}, s \in A \).

\[
\begin{align*}
es^* &= \max y^*_so^{economic}, s \in A \\
s.t. & \sum_{j=1}^{n} (\lambda_j + \mu_j) x^d_{ij} \leq \tilde{x}^d_i, i \in I^{Ild}, d \in D, \\
& \tilde{x}^d_i \leq x^d_{io}, i \in I^{Ild}, d \in D, \\
& \sum_{j=1}^{n} (\lambda_j + \mu_j) x^d_{ij} \leq \tilde{x}^d_i, i \in I^{Nld}, d \in D, \\
& \sum_{j=1}^{n} \lambda_j y^d_{ij} \geq \bar{y}^d_i, r \in O^{Id}, d \in D, \\
& \bar{y}^d_i \geq \bar{y}^d_{io}, r \in O^{Id}, r \neq s, r \notin A, d \in D, \\
& y^*_so^{economic} \geq y^*_so^{economic}, s \in A \\
& \sum_{j=1}^{n} \lambda_j y^d_{ij} \geq y^d_{rko}, r \in O^{Id}, d \in D, \\
& \sum_{j=1}^{n} \lambda_j x^d_{kj} = \tilde{x}^d_k, k \in U^{Id}, d \in D, \\
& \tilde{x}^d_k = w^d_{rko}, k \in U^{Id}, k \neq p, p \notin B, d \in D, \\
& w^d_{rko} = w^d_{so environmental} + \beta, \; p \in B \\
& \sum_{j=1}^{n} \lambda_j x^d_{kj} = \tilde{x}^d_k, k \in U^{Id}, d \in D, \\
& \sum_{j=1}^{n} (\lambda_j + \mu_j) = 1, \\
& y^*_so^{economic} \in Z^+, s \in A, \tilde{x}^d_i \in Z^+, i \in I^{Ild}, \forall d \in D, \bar{y}^d_i \in Z^+, r \in O^{Id}, \forall d \in D, \\
& \tilde{x}^d_k \in Z^+, k \in U^{Id}, \forall d \in D, \lambda_j, \mu_j \geq 0.
\end{align*}
\]

For computing the aforementioned MOMILP model, it can be transformed into the following MILP:

\[
\begin{align*}
\max & \varphi \\
s.t. & \varphi \leq y^*_so^{economic}, s \in A, \\
& \text{and all constraints of model (9)}.
\end{align*}
\]

b. The marginal rate of substitution from the right is estimated as follows:

\[
MR^+_{sp} = \frac{y^*_so^{economic} - y^*_so^{economic}}{\beta}, p \in B, s \in A
\]

(11)

c. The marginal rate of substitution from the left is computed by repeating (a) and (b) and substituting \( \beta \) by \(-\beta\).

3.3. Directional trade-offs between sustainability dimensions for multi-component scenarios

Now in this part we deal with reactions of multiple components arisen from changes of several variables in direction \( d \). To estimate the marginal rates of substitution for environmental undesirable outputs \( p \in B \) to economic desirable outputs \( s \in A \), the following approach is propounded by considering the vectors \( d^{UO} \) and \( d^{DO} \). Actually, the changes of desirable outputs in \( A \) in direction \( d^{DO} \) is assessed when undesirable outputs in \( B \) are changed in the direction \( d^{UO} \). Thus, we have:
a. A small increment $\beta$ for throughputs belong to $B$ from the environmental aspect is considered.

b. The following MOMILP is provided to obtain the optimal values $y_{so}^{economic}, s \in A$:

$$
es^* = \max \sum_{i,j} (\lambda_j + \mu_j) x_{ij}^d, i \in I^id, d \in D,$$

$$\text{s.t.} \sum_{i,j} (\lambda_j + \mu_j) x_{ij}^d \leq x_{io}^d, i \in I^id, d \in D,$$

$$\sum_{i,j} (\lambda_j + \mu_j) x_{ij}^d \leq y_{io}^d, i \in I^id, d \in D,$$

$$\sum_{j=1}^n \lambda_j y_{rj}^d \geq \tilde{y}_r^d, r \in O^ld, d \in D,$$

$$y_{ro}^d \geq y_{ro}^d, r \in O^ld, d \in D,$$

$$\sum_{j=1}^n \lambda_j y_{rj}^d \geq \tilde{y}_r^d, r \in O^ld, d \in D,$$

$$\sum_{j=1}^n \lambda_j w_{kj}^d = \tilde{w}_k^d, k \in U^ld, d \in D,$$

$$\sum_{j=1}^n \lambda_j y_{rj}^d \geq \tilde{y}_r^d, r \in O^ld, d \in D,$$

$$y_{so}^{economic} \geq y_{so}^{economic}, s \in A,$$

$$\sum_{j=1}^n \lambda_j y_{rj}^d \geq \tilde{y}_r^d, r \in O^ld, d \in D,$$

$$\sum_{j=1}^n \lambda_j w_{kj}^d = \tilde{w}_k^d, k \in U^ld, d \in D,$$

$$\sum_{j=1}^n (\lambda_j + \mu_j) = 1,$$

$$y_{so}^{economic} \in Z^+, s \in A, x_{ij}^d \in Z^+, i \in I^id, d \in D, y_{io}^d \in Z^+, r \in O^ld, d \in D,$$

$$\tilde{w}_k^d \in Z^+, k \in U^ld, d \in D, \lambda_j, \mu_j \geq 0.$$

To calculate the above mentioned MOMILP model, it can be replaced by the following MILP:

$$
\max \varphi \\
\text{s.t.} \quad \varphi \leq d_s^{(DO)}y_{so}^{economic}, s \in A,
$$

and all constraints of model (12).

c. The marginal rate of substitution from the right is estimated as follows:

$$MR_{sp}^+ = \frac{y_{so}^{economic} - y_{so}^{economic}}{\beta}, \ p \in B, s \in A$$


d. The marginal rate of substitution from the left is computed by repeating (b) and (c) and substituting $\beta$ by $-\beta$.

It should be mentioned that we examined marginal rates of substitution between environmental and economic factors in this section. However, it can be extended and rewritten for other dimensions and also interconnected factors such as economic-social ones.

In the next section, applications are provided to clarify and to show the practicality of the described techniques.

4. Applications

In this section first the data set of 13 ports is used to analyze trade-offs of measures and the performance. Data is initially applied by Adegoke [1] to assess sustainability of ports. Then, a dataset of 31 Iranian industrial parks is considered and the proposed approach is applied to analyze trade-offs of variables. Data of industrial parks that is partially used in this investigation have been provided in [11] originally. Due to this fact that the proposed approach in this research has been based on the assumption of the weak disposability of undesirable outputs and the correlation of desirable and undesirable outputs in applications, the weak disposability assumption has been considered.
4.1. An application to ports

13 ports, containing three inputs, one desirable output and one undesirable output are considered. Data are presented in Table 1. Facilities the loading and unloading of ships (berth length), infrastructure that facilitates storage capacity and drayage transportation (terminal area) and number of gantry cranes are taken as inputs with economic dimension. Notice that number of cranes is an integer-valued factor. Annual cargo handled by a port (economic-social dimension) is treated as a desirable output, which is shown in Table 1 as “container throughput”. Greenhouse gases emissions (environmental dimension) shown by “GHG Emissions” in Table 1 is deemed as an undesirable output [1]. The purpose is to calculate the changes of the amount of cargo handled annually respect to the changes of greenhouse gases emissions. Firstly, model (6) is computed to assess the sustainability of ports when integer-valued measures and weakly disposable undesirable outputs are presented. Column 2 of Table 2 shows the sustainability scores. As can be seen, six ports, Los Angeles, Montreal, Prince Rupert, Tacoma, Vancouver and Virginia are sustainable. Furthermore, the projection points of input and output measures are revealed in columns 3-7 of Table 2. As can be seen in column 5, the projections of the number of cranes are integer values.

| #  | Port         | Berth Length (m) | Terminal Area (acres) | Number of Cranes | Container Throughput (m TEU) | GHG Emissions (m Tons) |
|----|--------------|------------------|-----------------------|------------------|------------------------------|------------------------|
| 1  | Everglades   | 6928             | 316                   | 8                | 1.06                         | 173623                 |
| 2  | Halifax      | 1860             | 142                   | 12               | 0.48                         | 35292                  |
| 3  | Houston      | 9300             | 550                   | 22               | 2.18                         | 1062509                |
| 4  | Long Beach   | 29676            | 1339                  | 73               | 6.77                         | 776967                 |
| 5  | Los Angeles  | 26812            | 1693                  | 88               | 8.85                         | 881496                 |
| 6  | Montreal     | 4000             | 150                   | 17               | 1.44                         | 66433                  |
| 7  | New York- New Jersey | 27987 | 1518                   | 69               | 6.25                         | 1253001                |
| 8  | Oakland      | 22231            | 780                   | 33               | 2.36                         | 170405                 |
| 9  | Prince Rupert | 360             | 60                    | 4                | 0.73                         | 91000                  |
| 10 | Seattle      | 12340            | 533                   | 21               | 1.4                          | 47797                  |
| 11 | Tacoma       | 10687            | 594                   | 26               | 2.12                         | 48060                  |
| 12 | Vancouver    | 3067             | 425                   | 26               | 2.93                         | 1050593                |
| 13 | Virginia     | 13270            | 1145                  | 28               | 2.65                         | 152308                 |

Table 1: Ports data

To appraise the marginal rates of substitution between two items, including annual cargo handled and greenhouse gases emissions, model (7) and formula (8) measured for \( h = \pm 100 \). The objective optimal value \( y_{so}^{economic-social} \) and the marginal rate of substitution from the right are evaluated and shown in columns 2 and 3 of Table 3, respectively. As can be seen, for \( h = +100 \), infeasible problems raise for Prince Rupert and Vancouver ports. Halifax and Seattle ports have positive tiny marginal rates and Los Angeles has negative tiny marginal rates. Moreover, by considering \( h = +100 \), no marginal changes have been observed for other ports. As the same way, the optimal value of the objective function \( y_{so}^{economic-social} \) and the marginal rate of substitution from the left appear in columns 4 and 5 of Table 3.

Seven ports have positive marginal rates for \( h = -100 \). Other ports have no change from the left. Therefore, trade-offs between economic-social and environmental measures is estimated using the proposed approach. It can be observed in this case that if the changes present, they
will be minor. It should be pointed out that marginal rates of substitution are estimated for sustainable ports and the projection points of unsustainable ports.

| #  | Efficiency | Projection points       |          |          |          |          |          |          |          |          |          |
|----|------------|-------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
|    |            | Berth Length            | Terminal Area | Number of Cranes | Container Throughput | GHG Emissions |
| 1  | 0.57       | 1572.58                 | 120.89      | 8         | 1.06     | 113414.64 |
| 2  | 0.69       | 1200                    | 80.77       | 7         | 0.48     | 33741.65  |
| 3  | 0.57       | 5083.57                 | 351.61      | 19        | 2.18     | 232160    |
| 4  | 0.85       | 20408.63                | 1259.88     | 69        | 6.77     | 652706.39 |
| 5  | 1.00       | 26812                   | 1693        | 88        | 8.85     | 881496    |
| 6  | 1.00       | 4000                    | 150         | 17        | 1.44     | 66433     |
| 7  | 0.70       | 18552.54                | 1161.75     | 62        | 6.25     | 613528.74 |
| 8  | 0.63       | 6832.26                 | 341.57      | 26        | 2.36     | 167628.41 |
| 9  | 1.00       | 360                     | 60          | 4         | 0.73     | 91000     |
| 10 | 0.71       | 5575.83                 | 283.02      | 18        | 1.40     | 47797     |
| 11 | 1.00       | 10687                   | 594         | 26        | 2.12     | 48060     |
| 12 | 1.00       | 3067                    | 425         | 26        | 2.93     | 1050593   |
| 13 | 1.00       | 13270                   | 1145        | 28        | 2.65     | 152308    |

Table 2: Efficiency scores and projection points

| #  | h = +100 | h = −100 |
|----|----------|----------|
|    | Objective function $\gamma^{+}_{y^{econ-social}}$ | $MR^{+}$ | Objective function $\gamma_{y^{econ-social}}$ | $MR^{-}$ |
| 1  | 1.06     | 0.00000  | 1.06     | 0.00000  |
| 2  | 0.4808   | 0.00001  | 0.4792   | 0.00001  |
| 3  | 2.18     | 0.00000  | 2.1796   | 0.00000  |
| 4  | 6.77     | 0.00000  | 6.7694   | 0.00001  |
| 5  | 8.85     | -0.00001 | 8.8492   | 0.00001  |
| 6  | 1.44     | 0.00000  | 1.4381   | 0.00001  |
| 7  | 6.25     | 0.00000  | 6.2496   | 0.00000  |
| 8  | 2.36     | 0.00000  | 2.3594   | 0.00000  |
| 9  | Infeasible | -0.7292  | Infeasible | -0.0000  |
| 10 | 1.4014   | 0.00001  | 1.3081   | 0.00002  |
| 11 | 2.12     | 0.00000  | 2.1156   | 0.00004  |
| 12 | Infeasible | -2.9298  | Infeasible | -0.0000  |
| 13 | 2.65     | 0.00000  | 2.6496   | 0.00000  |

Table 3: Results for marginal rates of substitution

4.2. An application to industrial parks

The application of the proposed approach herein is dealt with by using the data set of industrial parks in 31 provinces of Iran. Data are presented in Table 4. Notice that various approaches are present in the DEA contexts to concern undesirable outputs in industrial parks; see for instance [7, 11, 24]. Because of the relationship between desirable and undesirable outputs in this case, undesirable outputs are deemed as weakly disposable. Each province as the DMU contains three inputs, four desirable outputs and two undesirable outputs. Performance measures are designed as follows:

-Inputs: Number of created companies ($x_1$) (economic integer-valued factor), productions ($x_2$) (economic real-valued factor) and environmental projects ($x_3$) (environmental real-valued factor) are considered as input measures.
-Desirable outputs: Number of active companies ($y_1$) (economic integer-valued factor), revenue ($y_2$) (economic real-valued factor), number of graduates ($y_3$) (social integer-valued factor) and welfare services ($y_4$) (social real-valued factor) are presented as desirable outputs.

-Undesirable outputs: Two measures, CO$_2$ emission ($w_1$)(environmental real-valued factor) and Effluent ($w_2$) (environmental real-valued factor) are taken as undesirable outputs. As can be seen, there are undesirable measures and also integer-valued factors in the system under evaluation.

| #  | Province           | Inputs | Desirable outputs | Undesirable outputs |
|----|--------------------|--------|-------------------|---------------------|
|    |                    | $x_1$  | $x_2$  | $x_3$  | $y_1$  | $y_2$  | $y_3$  | $y_4$  | $w_1$  | $w_2$  |
| 1  | Azerbaijan Sharghi| 2647   | 1.139 | 112.763 | 1944 | 2.121 | 106 | 212.109 | 74 | 2740 |
| 2  | Azerbaijan Gharbi| 1075   | 1.645 | 133.85  | 1863 | 15.103 | 226 | 1234.006 | 59 | 3170 |
| 3  | Ardabil            | 686    | 0.157 | 11.169  | 394  | 0.305 | 31  | 36.585  | 62 | 2150 |
| 4  | Isfahan            | 3216   | 4.733 | 373.917 | 2521 | 6.93  | 102 | 692.991 | 65 | 3130 |
| 5  | Alborz             | 1686   | 2.223 | 180.061 | 881  | 2.868 | 57  | 200.785 | 47 | 2940 |
| 6  | Ilam               | 312    | 0.133 | 11.43   | 124  | 0.150 | 14  | 12.688  | 39 | 1970 |
| 7  | Bushehr            | 426    | 18.621| 1731.713| 179  | 25.265| 29  | 1515.922| 54 | 2150 |
| 8  | Tehran             | 4810   | 5.776 | 485.213 | 2906 | 10.17 | 243 | 1220.355| 42 | 2410 |
| 9  | Chahar Mahal       | 912    | 0.21  | 20.795  | 582  | 0.372 | 22  | 22.346  | 40 | 3210 |
| 10 | Bakhtiari          | 434    | 0.414 | 35.148  | 200  | 0.575 | 19  | 63.262  | 28 | 2840 |
| 11 | Khorasan Jonoubi   | 2023   | 3.411 | 259.254 | 1351 | 4.744 | 149 | 521.887 | 62 | 2450 |
| 12 | Khorasan Razavi    | 266    | 0.811 | 56.791  | 117  | 1.128 | 24  | 124.122 | 48 | 2120 |
| 13 | Khorasan Shomali   | 1049   | 9.912 | 802.866 | 777  | 13.076| 104 | 1569.171| 31 | 2390 |
| 14 | Khorasan Sharghi   | 849    | 1.07  | 101.632 | 536  | 1.576 | 22  | 94.534  | 56 | 2650 |
| 15 | Khorasan Markazi   | 2114   | 0.236 | 18.161  | 981  | 0.341 | 14  | 34.133  | 51 | 2640 |
| 16 | Sistan             | 1585   | 0.993 | 6.498   | 428  | 0.165 | 55  | 18.203  | 25 | 1990 |
| 17 | Baluchestan        | 1950   | 5.776 | 485.213 | 2906 | 10.17 | 243 | 1220.355| 42 | 2410 |
| 18 | Fars               | 2540   | 0.822 | 77.306  | 1322 | 1.101 | 101 | 66.056  | 59 | 3540 |
| 19 | Qazvin             | 901    | 0.804 | 59.493  | 477  | 0.971 | 36  | 87.387  | 23 | 2380 |
| 20 | Qom                | 1297   | 0.313 | 23.802  | 717  | 0.521 | 27  | 36.477  | 29 | 2790 |
| 21 | Kurdistan          | 298    | 0.179 | 12.731  | 241  | 0.232 | 32  | 27.836  | 18 | 2960 |
| 22 | Kerman             | 1311   | 2.437 | 192.488 | 668  | 3.486 | 67  | 418.293 | 57 | 3610 |
| 23 | Kermanshah         | 557    | 0.44  | 41.821  | 269  | 0.695 | 484 | 62.59   | 32 | 1950 |
| 24 | Kohgiluyeh Boyer-  | 244    | 0.008 | 0.651   | 147  | 0.012 | 14  | 1.412   | 46 | 2100 |
| 25 | Isfahan            | 2540   | 5.776 | 485.213 | 2906 | 10.17 | 243 | 1220.355| 42 | 2410 |
| 26 | Gilan              | 437    | 0.097 | 8.161   | 278  | 0.14  | 71  | 16.75   | 57 | 2750 |
| 27 | Lorestan           | 188    | 0.111 | 7.879   | 102  | 0.154 | 36  | 18.521  | 22 | 2900 |
| 28 | Mazandaran         | 857    | 0.45  | 32.85   | 509  | 0.607 | 64  | 60.729  | 25 | 2770 |
| 29 | Markazi            | 1516   | 0.768 | 53.314  | 969  | 1.441 | 42  | 144.138 | 26 | 2390 |
| 30 | Hormozgan          | 386    | 0.694 | 48.594  | 118  | 1.338 | 41  | 147.133 | 53 | 2790 |
| 31 | Hamadan            | 980    | 1.755 | 165.15  | 2160 | 19.322| 185 | 784.365 | 52 | 2980 |
| 32 | Yazd               | 1424   | 0.948 | 70.153  | 740  | 1.615 | 31  | 161.501 | 65 | 2810 |

Table 4: Industries parks data

At first model (6) is used to assess the sustainability performance in the presence of integer values and weakly disposable undesirable outputs. The results are shown in column 2 of Table 5. As is apparent, 15 provinces are sustainable with the score one. At this stage, we estimate the marginal rates of substitution between two components, CO$_2$ emission ($w_1$) and number of active companies ($y_1$), for sustainable provinces and the projection points of unsustainable
provinces. For this purpose, model (7) and expression (8) is applied for $h = \pm 5$. The findings are provided in Table 5. Columns 3 and 4 show the objective function of model (7) and the marginal rates of substitution from the right, respectively. As shown, the problem is infeasible for 25 industrial parks. Also, there are positive marginal rates for two provinces, Qom and Markazi while negative marginal rates are rendered for four provinces, Isfahan, Semnan, Fars and Golestan.

For more illustration, consider the industrial park of Isfahan. Increasing CO$_2$ emission by $h = +5$ leads to the reduction in the number of active companies to 1711. Also, increasing CO$_2$ emission by $h = +5$ for Qom results in the increment of the number of active companies to 728. Similarly, the objective function value of model(7) and the marginal rate of substitution from the left are depicted in columns 5 and 6. In this case, the model (7) is also infeasible for 25 provinces. Furthermore, positive marginal rates are revealed for 6 industrial parks in provinces Azerbaijan Sharghi, Ardabil, Semnan, Fars, Qom and Markazi. Notice that no negative marginal rate is presented in this case. For instance, see the industrial park in Semnan. Decreasing of CO$_2$ emission by $h = -5$ causes the abatement in number of active companies to 895. As evidenced, the integer amounts are obtained for the integer-valued outputs.

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Afterwards, the marginal rates of substitution are calculated between several components that are the environmental factors, CO$_2$ emission and effluent, and the economic and social measures, number of active companies and number of graduates. In this regard, model (10) and the statement (11) is used by considering $\beta = \pm 5$. Results are indicated in Table 6.

### Table 5: The result for two-component case

| # | Eff | $h = +5$ | $h = -5$ |
|---|-----|---------|---------|
|   |     | $y^*_1$ | $MR^+$  | $y^*_1$ | $MR^-$  |
| 1 | 1   | Infeasible | 1883 | 12.20 |
| 2 | 1   | Infeasible | -    | -    |
| 3 | 0.91 | Infeasible | 368  | 5.20  |
| 4 | 0.82 | 1711    | -162 | Infeasible | -    |
| 5 | 0.38 | Infeasible | -    | Infeasible | -    |
| 6 | 0.35 | Infeasible | -    | Infeasible | -    |
| 7 | 1   | Infeasible | -    | Infeasible | -    |
| 8 | 1   | Infeasible | -    | Infeasible | -    |
| 9 | 1   | Infeasible | -    | Infeasible | -    |
| 10 | 0.38 | Infeasible | -    | Infeasible | -    |
| 11 | 0.53 | Infeasible | -    | Infeasible | -    |
| 12 | 0.54 | Infeasible | -    | Infeasible | -    |
| 13 | 1   | Infeasible | -    | Infeasible | -    |
| 14 | 0.37 | Infeasible | -    | Infeasible | -    |
| 15 | 1   | 666    | -63  | 895.00 | 17.20 |
| 16 | 1   | Infeasible | -    | Infeasible | -    |
| 17 | 0.83 | 1209   | -10.60 | 1237 | 17    |
| 18 | 0.47 | Infeasible | -    | Infeasible | -    |
| 19 | 1   | 728    | 2.20  | 551.00 | 33.20 |
| 20 | 1   | Infeasible | -    | Infeasible | -    |
| 21 | 0.31 | Infeasible | -    | Infeasible | -    |
| 22 | 0.6  | Infeasible | -    | Infeasible | -    |
| 23 | 1   | Infeasible | -    | Infeasible | -    |
| 24 | 0.77 | 293    | -10.60 | Infeasible | -    |
| 25 | 1   | Infeasible | -    | Infeasible | -    |
| 26 | 1   | Infeasible | -    | Infeasible | -    |
| 27 | 0.77 | Infeasible | -    | Infeasible | -    |
| 28 | 1   | 1021   | 10.40 | 671.00 | 59.60 |
| 29 | 0.47 | Infeasible | -    | Infeasible | -    |
| 30 | 1   | Infeasible | -    | Infeasible | -    |
| 31 | 0.49 | Infeasible | -    | Infeasible | -    |
Columns 2 and 7 show objective functions for $\beta = \pm 5$. $y_1^*$ and $y_3^*$ for $\beta = +5$ are shown in columns 3 and 4, respectively. As can be seen, they are estimated as integer values. The marginal rates of substitution from the right for two integer-valued desirable outputs are presented in columns 5 and 6. The infeasibility occurs for 24 industrial parks by regarding $\beta = +5$. For more explanation, consider Semnan. The augmentation of the environmental indicators by $\beta = +5$ causes the reduction in the number of active companies $y_1^*$ and the expansion in the number of graduates $y_3^*$. Moreover, the marginal rates of substitution from the left are computed and for two integer-valued desirable outputs are displayed in columns 10 and 11. Findings show that model (10) is infeasible for 19 industrial parks as long as $\beta = -5$.

Integer values $y_1^*$ and $y_3^*$ are revealed in columns 8 and 9. Also, for $\beta = -5$, the number of other active companies decreases, except the one in Hormozgan which increases. The number of graduate also increases in six provinces, Azerbaijan Sharghi, Ardabil, Semnan, Fars, Qom and Markazi, and it decreases for Ilam, Tehran, Sistan Baluchestan, Golestan, Gilan and Hormozgan.

Detections represent that the decrease (increase) of undesirable outputs can cause the reduction or expansion of integer desirable outputs.

It should be noted that by examining the direction $(d_x, d_y, d_w) = (0, 0, 0, 1, 0, 1, 0, 1, 1)$ and computing model (13) and (14), the results of Table 6 are detected. Now, to compare the results

| # | $\beta = +5$ | $\beta = -5$ |
|---|---|---|
| | $y_1$ | $y_3$ | $MR_1^+$ | $MR_3^+$ | $y_1$ | $y_3$ | $MR_1^+$ | $MR_3^+$ |
| 1 | Infeasible | - | - | - | 117 | 208 | 117 | 347.20 | -3.4 |
| 2 | Infeasible | - | - | - | Infeasible | - | - | - |
| 3 | Infeasible | - | - | - | 56 | 323 | 56 | 14.20 | -5 |
| 4 | 177 | 235 | 177 | -15 | 457.20 | Infeasible | - | - | - |
| 5 | Infeasible | - | - | - | Infeasible | - | - | - |
| 6 | Infeasible | - | - | - | 11 | 84 | 11 | 8 | 0.60 |
| 7 | Infeasible | - | - | - | Infeasible | - | - | - |
| 8 | Infeasible | - | - | - | 149 | 149 | 149 | 551.4 | 18.8 |
| 9 | Infeasible | - | - | - | Infeasible | - | - | - |
| 10 | Infeasible | - | - | - | Infeasible | - | - | - |
| 11 | Infeasible | - | - | - | Infeasible | - | - | - |
| 12 | Infeasible | - | - | - | Infeasible | - | - | - |
| 13 | Infeasible | - | - | - | Infeasible | - | - | - |
| 14 | Infeasible | - | - | - | Infeasible | - | - | - |
| 15 | 77 | 406 | 77 | -115 | 12.6 | 76 | 421 | 76 | 112 | -12.4 |
| 16 | 53 | 362 | 53 | -13.20 | -0.4 | 40 | 257 | 40 | 34.20 | 3 |
| 17 | 137 | 137 | -237 | 7.20 | 134 | 134 | 134 | 237.60 | -6.60 |
| 18 | Infeasible | - | - | - | Infeasible | - | - | - |
| 19 | 69 | 400 | 69 | -63.40 | 8.4 | 57 | 437 | 57 | 56 | -6 |
| 20 | Infeasible | - | - | - | Infeasible | - | - | - |
| 21 | Infeasible | - | - | - | Infeasible | - | - | - |
| 22 | Infeasible | - | - | - | Infeasible | - | - | - |
| 23 | Infeasible | - | - | - | Infeasible | - | - | - |
| 24 | 36 | 278 | 36 | -13.60 | 0.20 | 33 | 268 | 33 | 15.60 | 0.40 |
| 25 | Infeasible | - | - | - | 65 | 65 | 65 | 42.6 | 1.2 |
| 26 | Infeasible | - | - | - | Infeasible | - | - | - |
| 27 | Infeasible | - | - | - | Infeasible | - | - | - |
| 28 | 101 | 828 | 101 | -28.20 | 11.8 | 62 | 550 | 62 | 83.8 | -4 |
| 29 | Infeasible | - | - | - | 35 | 231 | 35 | -22.60 | 1.20 |
| 30 | Infeasible | - | - | - | Infeasible | - | - | - |
| 31 | Infeasible | - | - | - | Infeasible | - | - | - |

Table 6: The results for multi-component scenarios
of the proposed approach with Khoshandam et al. [12], we consider all measures as real values. In this case, the sustainability scores considering all measures as real values are similar to scores presented in column 2 of Table 5. The results of multi-component scenarios for Khoshandam et al.’s approach [12] are shown in Table 7.

As can be seen, the optimal real values for number of active companies and number of graduates are obtained using Khoshandam et al.’s approach [12] whilst integer values are achieved for these outputs via the approach provided herein. Furthermore, different marginal rates of right and left have been found in two techniques. Nevertheless, as revealed in both approaches, the problem is infeasible for similar industrial parks.

Therefore, to research trade-offs and marginal rates of substitution in the presence of integer measures, the advanced procedure is beneficial and leads to more rational achievements. Thus, economists and decision-makers can utilize it to investigate the changes of some sustainability dimensions caused by the changes of factors from other sustainability areas when they confront with integer and undesirable measures.

| #  | $\beta = +5$ | $\beta = -5$ |
|----|-------------|-------------|
| #  | OF  | $y_1^*$ | $y_3^*$ | $MR_1^*$ | $MR_3^*$ | OF  | $y_1^*$ | $y_3^*$ | $MR_1^*$ | $MR_3^*$ |
| 4  | 177.20 | 1705.91 | 177.20 | -163.02 | -7.17 | 1  | 117.36 | 1769.98 | 117.36 | 34.80 | -3.47 |
| 15 | 77.27  | 406.45  | 77.27  | -114.91 | 12.65 | 3  | 56.19  | 323.39  | 56.19  | 14.12 | -5.04 |
| 16 | 53.63  | 362.01  | 53.63  | -13.20  | 8.12  | 6  | 11.82  | 90.73   | 11.82  | 6.65  | 0.44  |
| 17 | 137.07 | 1012.23 | 137.07 | -61.95  | 7.21  | 8  | 149.57 | 1369.02 | 149.57 | 307.40 | 18.69 |
| 19 | 69.40  | 400.27  | 69.40  | -63.35  | 8.48  | 15 | 76.18  | 421.48  | 76.18  | 111.90 | -12.44 |
| 24 | 36.50  | 278.06  | 36.50  | -13.59  | 0.30  | 16 | 40.30  | 262.41  | 40.30  | 33.12  | 2.94  |
| 28 | 101.83 | 828.97  | 101.83 | -28.01  | 11.97 | 17 | 134.22 | 1048.87 | 134.22 | 54.63  | -6.64 |
| 0  | Others | Infeasible | Infeasible | - | - | 19 | 57.20  | 437.01  | 57.20  | 56.00  | -6.04 |
| 0  |   | Infeasible | Infeasible | - | - | 24 | 33.25  | 268.37  | 33.25  | 15.53  | 0.35  |
| 25 | 65.33  | 255.85  | 65.33  | 4.43   | 1.13  |
| 28 | 62.62  | 560.11  | 62.62  | 81.78  | -4.12 |
| 29 | 35.50  | 256.41  | 35.50  | 3.60   | 1.10  |
| 0  | Others | Infeasible | Infeasible | - | - | 24 | 33.25  | 268.37  | 33.25  | 15.53  | 0.35  |
| 25 | 65.33  | 255.85  | 65.33  | 4.43   | 1.13  |
| 28 | 62.62  | 560.11  | 62.62  | 81.78  | -4.12 |
| 29 | 35.50  | 256.41  | 35.50  | 3.60   | 1.10  |

Table 7: The results for Khoshandam et al.’s approach

5. Conclusions and recommendations

The estimation of trade-offs of sustainability dimensions is the significant aspect for decision makers and managers. Furthermore, there are integer-valued performance measures and undesirable outputs in many real world applications. Therefore, a DEA-based approach has been provided in this research to analyze the sustainability of systems when integer-valued elements and weakly disposable undesirable outputs appear.

Afterwards, procedures on the basis of DEA have been propounded to assess individual and group marginal rates of substitution between sustainability dimensions, including social, economic and environmental ones while undesirable measures and integer factors are disclosed. The technique has also been extended to appraise the directional marginal rates of substitution between sustainability areas. Two applications of ports and industrial parks have been conducted, representing advantages and suitability the designed approaches. The findings show that the approaches provided in this study can estimate marginal rates of substitutions in the rational way.

Moreover, the investigation shows further research can be performed to estimate non-marginal changes in the presence integer-valued and undesirable variables with different assumptions of disposability such as managerial disposability. Also, the calculation of the marginal rates of substitution in sustainable multi-stage processes when there are nondiscretionary measures would be an interesting topic for future study.
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