Are non-equilibrium Bose-Einstein condensates superfluid?

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We theoretically study the superfluidity properties of a non-equilibrium Bose-Einstein condensate of exciton-polaritons in a semiconductor microcavity. The dynamics of the condensate is described at mean-field level in terms of a modified Complex Ginzburg Landau equation. A generalized Landau criterion is formulated which estimates the onset of the drag force on a small moving defect. Metastability of supercurrents in multiply connected geometries persists up to higher flow speeds.

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Superfluidity is among the most remarkable consequences of macroscopic quantum coherence in condensed matter systems and manifests itself in a number of fascinating effects \cite{1,2}. A unified description of these phenomena is obtained in the framework of the so-called two-fluid hydrodynamics, in which the macroscopic condensate wavefunction adds up to the standard hydrodynamic variables \cite{3}. The phenomenon of macroscopic coherence is not restricted to systems at (or close to) thermodynamical equilibrium such as liquid Helium, ultracold atomic gases, or superconducting materials, but has been observed also in systems far from thermodynamical equilibrium, whose state is determined by a dynamical balance of driving and losses. Most remarkable examples are lasers and, more recently, Bose-Einstein condensates of magnons in magnetic solids \cite{4} and exciton-polaritons in semiconductor microcavities \cite{5,6}. In particular, the issue of superfluidity in this latter system has attracted a significant interest from both the theoretical \cite{7,8} and experimental \cite{9,10} points of view.

Recent experiments with resonantly pumped polariton condensates \cite{9} have demonstrated superfluidity as a dramatic reduction in the intensity of resonant Rayleigh scattering as originally predicted in \cite{7}. The situation is less clear in the case of non-resonant \cite{6} or parametrical (OPO) \cite{10} pumping schemes: recent experiments \cite{10} have observed propagation of polariton bullets without apparent friction, which is in contrast with the predictions of a naive Landau criterion based on the elementary excitation spectrum predicted in \cite{12}. Another aspect of superfluidity, namely metastability of supercurrent in multiply-connected geometries was investigated theoretically in \cite{8} and experimentally confirmed in \cite{11}. The present paper reports a comprehensive theoretical investigation of the meaning of superfluidity for polariton condensates under a non-resonant pumping. Emphasis will be given to the novel features that originate from their non-equilibrium character.

At the mean-field level, the condensate dynamics can be described in terms of the so-called Gross-Pitaevskii equation (GPE) \cite{1}, which was recently generalized to non-equilibrium condensates by including the effect of pumping and losses \cite{13,14}. This mean-field description has been able to explain a number of experimental observations on polariton condensates, e.g. the ring-shaped momentum distribution of spatially narrow condensates \cite{6,15}, the synchronization/desynchronization transition \cite{16}, the spontaneous appearance of vortices \cite{17}. Nonetheless, the implicit assumption that the pumping mechanism is not frequency-selective can lead to unphysical predictions, e.g. that in a spatially homogeneous or ring-like geometry condensation is equally likely to occur in any momentum state. Kinetic calculations \cite{18} have pointed out the significant energy dependence of the polariton-polariton scattering processes that are responsible for replenishing the condensate. Including this feature as an energy-dependent amplification mechanism turned out to be crucial in order to extract physically meaningful predictions for the condensate fluctuations in the Wigner Monte Carlo simulations of \cite{19}.

A simplest generalization of the GPE to include frequency-dependent pumping has the form:

\begin{equation}
\frac{d\psi}{dt} = \left\{ -\frac{\hbar}{2m} \nabla^2 + V_{ext} - \frac{i}{2} \left[ P \left( 1 - \frac{i}{\Omega_K} \frac{d}{dt} \right) - r |\psi|^2 - \frac{\gamma}{P} + g |\psi|^2 \right] \right\} \psi. \tag{1}
\end{equation}

The energy zero has been set for convenience at the bottom of the lower polariton branch; the efficiency of amplification (proportional to the pumping strength \(P\)) decreases to zero a frequency interval \(\Omega_K\) above it. Assuming a linear form of the frequency dependence of the amplification, the generalized GPE \cite{11} maintains a temporally local form. The others terms describing gain saturation \(r\), losses \(\gamma\), polariton mass \(m\), polariton-polariton interactions \(g\), external potential \(V_{ext}\) have the same meaning as in previous works \cite{13,14}.

We first consider the evolution of the system starting from an initial state with no condensate \(\psi = 0\). If the strength of pumping is enough to overcome losses \(P > \gamma\), the \(\psi = 0\) state is dynamically unstable against the creation of a finite condensate amplitude in all the low-momentum modes for which \(\hbar k^2/2m < \Omega_K (1 - \gamma/P)\). The rate of this instability is maximum at \(k = 0\) and
decreases for increasing \( k \). This fact is in agreement with the experimental observation of condensation naturally occurring around \( k = 0 \) as soon as the sample is sufficiently large and free from disorder [5].

In spite of this natural preference, condensation can be forced to occur in finite momentum state by seeding the system with a short resonant pulse at the desired \( k_c \) as proposed and numerically assessed in Ref. [8]. A related configuration was experimentally demonstrated for the OPO pump scheme in [11]. The effect of the frequency-dependent pumping is then visible in the state equation relating the pumping strength to the condensate density \( n_c = |\psi|^2 \) in the stationary state:

\[
|\psi|^2 = \left[ 1 - \frac{1}{\Omega_K} \left( \frac{\hbar k_c^2}{2m} + g|\psi|^2 \right) \right] \frac{P}{r}. \tag{2}
\]

As a consequence of the frequency dependent pumping, the interaction-induced blue-shift of the condensate frequency \( \omega_c = k_c^2/2m + g|\psi|^2 \) is responsible for a slower increase of density with pumping strength \( P \).

The dynamical stability of a moving condensate is to be assessed by linearization of the CGLE around the stationary solution. Stability requires that the imaginary part of the frequency is negative for all Bogoliubov modes. Examples of the Bogoliubov dispersion \( \omega_{\text{Bog}}(q) \) for moving condensates are shown in Fig.1. In the leftmost panels (a,b), the limit \( \Omega_K \rightarrow \infty \) of a negligible frequency-dependence of pumping is considered. Apart from the global Doppler tilting of the real part due to the finite condensate velocity, the plotted dispersion fully recovers the diffusive character first discussed in [12–14]: a flat dispersion around the condensate wavevector and an imaginary part quadratically growing in \( q \). In this limit, stable condensates exist for any value of \( k_c \).

The effect of a frequency-selective pumping is addressed in the other panels. The central panels (c,d) refer to the case of a pumping intensity just above the threshold. In this regime, the condensate density is very small and the real part of the Bogoliubov mode frequency reduces to the single-particle one. On the other hand, the characteristic damping rate of density fluctuations (the gapped mode at \( q = 0 \)) and of the high-momentum modes is suppressed by a critical slowing down phenomenon [13]. As a result, the low energy modes around \( q \approx -k_c \) become dynamically unstable. Eventually, this mechanism leads to the disappearance of the original condensate and the formation of another -stable- condensate in a lower momentum state.

The right panels (e,f) refer to the case of a stronger pumping well above the threshold. In this case, the damping rate of all modes other than the Goldstone one is comparable to the polariton lifetime \( \gamma \). In this case, the frequency-dependence of pumping is not able to destabilize the moving condensate. The stability domain as a function of the momentum \( k_c \) and the density \( n_c \) of the condensate is summarized in Fig.2 as the density is increased, stable condensates survive up to larger momenta.

It is of crucial importance to note that this instability mechanism has no direct counterpart in standard equilibrium superfluids and is effective even in the absence of any defect potential. Of course, the present theory is directly applicable only to infinite, homogeneous condensates or in multiply connected ring-shaped geometries. In finite systems, assessing the stability of moving condensates would be made harder by the flow of condensate polaritons outside the pump regions [15].

In the presence of weak defects, the frictionless flow of equilibrium condensates is limited to flow speeds below
the Landau critical velocity $v_c = \min_k \{ \text{Re}[\omega_{Bog}(k)]/k \}$, where $\omega_{Bog}(k)$ is the real part of the Bogoliubov dispersion for a condensate at rest. For dilute systems, $v_c$ coincides with the sound velocity $c_s = \sqrt{gm_{\text{eff}}/m}$ \[1\]. As a consequence of the diffusive character of the Goldstone mode, a naive application of this definition to the Bogoliubov dispersion of non-equilibrium condensates anticipated in \[12–14\] gives a vanishing prediction for the critical velocity, $v_c = 0$: Bogoliubov waves are emitted in a moving condensate hitting a (weak) defect at any value of the condensate speed.

A complete picture of non-equilibrium condensates requires taking into account the non-trivial dynamics of the imaginary part of the Bogoliubov dispersion. Numerical plots of the density perturbation induced by a single weak stationary defect in a (dynamically stable) moving condensate can be calculated from the generalized GPE \[1\] in the presence of a suitable defect potential $V_{\text{def}}(\mathbf{r})$ and are shown in Fig. 3 for different values of the condensate velocity. In contrast to the predictions of the naive Landau criterion stated above, the induced perturbation closely resembles the one induced in an equilibrium condensate described by the standard GPE \[22\]: at high speeds [panel (a)], the defect creates a series of parabolic fringes that propagate away from the defect; at low speeds [panel (c)], the propagating fringes are replaced by a localized perturbation in the vicinity of the defect. Remarkably, the characteristic speed at which the fringes disappear is of the order of the equilibrium sound speed $c_s = \sqrt{gm_{\text{eff}}/m}$, but does not correspond to any noticeable feature in the real part of the Bogoliubov dispersion of Fig. 1(a,c,e).

As it was pointed out in a different context in \[20\], the emission pattern by a localized monochromatic source is better understood in terms of the (complex) wavevector of the emitted wave at the given (real) frequency. As one can see in Fig. 3(b), the wavevector $\tilde{k}$ of the Bogoliubov wave emitted at zero frequency by the static defect starts having a finite real part only after a branching point: extended oscillations in the density are observed as soon as the real part of $\tilde{k}$ exceeds the imaginary part.

FIG. 3: Density perturbation created in a moving condensate by a stationary weak defect for three different values of the condensate velocity $v/c_s = 1.5, 1, 0.4$ across the critical value for superfluidity. Parameters: $\nu_c g/\gamma = \nu_c r/\gamma = 1$, $\Omega_K/\gamma = 50$.

This behavior is reflected in the drag force $F$ exerted by the moving condensate onto the defect as a function of the condensate velocity [Fig. 4(a)]. In terms of the perturbed density profile $n(\mathbf{r})$, the drag force is given by $F = -\int d\mathbf{r} n(\mathbf{r}) \nabla_r \bar{V}_{\text{def}}(\mathbf{r})$ \[21\]. As a consequence of the finite lifetime of the Bogoliubov modes, the drag force has a non-vanishing value at all $v$. In addition, the drag force shows a marked threshold at a velocity value that closely corresponds to the onset of fringes in the density profile: the lower the value of the $n_c r/\gamma$ parameter, the sharper the threshold. On the other hand, the role of $\Omega_K$ on this effect is minor. Even though no experiment has so far investigated the superfluid properties of polariton condensates under non-resonant pumping, it is likely that this generalized form of the Landau criterion provides at least a partial explanation of the recent experimental observation of superfluidity in a OPO regime \[10\].

Supersonic flows in equilibrium condensates in ring-shaped geometries are generally strongly sensitive to the presence of defects: as soon as nodes appear in the condensate wavefunction, the topological stability of the supercurrent state is broken, which leads to a rapid slow down of the condensate motion. The finite damping rate of excitations in polariton condensates introduces a substantial modification to this picture: the perturbation created by each defect is not able to propagate on long distances, but rather remains localized in space on a length scale inversely proportional to $\text{Im}[\tilde{k}]$. As a result, we expect it is much harder to break the topological stability of the supercurrents.

This physical picture is confirmed by numerical simulations of the time evolution under the generalized GPE \[1\] starting from an initial condition with a supersonic flow. Periodic boundary conditions are assumed. For a low defect density, the supercurrent state is maintained for very long times with no sign of decay. The characteristic Cerenkov-like density patterns in the vicinity...
of each defect are spatially separated and do not interfere [Fig. 5(a)]. The momentum distribution in $k$-space [Fig. 5(b)] shows the condensate peak right at the initial momentum state and a much fainter resonant Rayleigh scattering ring [7]. The situation is different for a higher momentum state and a much fainter resonant Rayleigh interference [Fig. 5(a)]. The momentum distribution in real space panels indicate the position of the defects. The square in the momentum space panels indicates the position of the defects. Parameters are the same as in Fig. 4 except for $\Omega K / \gamma = 10$.

In conclusion, we have investigated how the non-equilibrium nature of exciton-polariton condensates affects their superfluidity properties. A mean-field model based on a generalized Gross-Pitaevskii equation including a frequency-dependent pumping is developed. Different aspects of superfluidity have been considered. A non-equilibrium version of the Landau critical speed for the onset of drag force is formulated and metastability of supercurrents is shown to persist even at speeds well above the critical speed.

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[1] L.P. Pitaevskii and S. Stringari, Bose-Einstein Condensation, Clarendon Press Oxford (2003).
[2] A. J. Leggett, Rev. Mod. Phys. 71, S318 (1999).
[3] K. Huang, Statistical Mechanics (New York, John Wiley & Sons, 1963).
[4] O. Demokritov, et al., Nature 443, 430 (2006).
[5] M. Richard et al., Phys. Rev. B 72, 201301(R) (2005); J. Kasprzak et al., Nature 443, 409 (2006).
[6] M. Richard et al., Phys. Rev. Lett. 94, 187401 (2005).
[7] I. Carusotto, C. Ciuti, Phys. Rev. Lett. 93, 166401 (2004).
[8] M. Wouters and V. Savona, arXiv:0904.2966.
[9] A. Amo, et al. Nature Phys. 5, 805 (2009).
[10] A. Amo, et al., Nature 457, 291 (2009).
[11] D. Sanvitto, et al. [arXiv:0907.2371].
[12] M. Wouters, I. Carusotto, Phys. Rev. A 76, 043807 (2007).
[13] M. Wouters and I. Carusotto, Phys. Rev. Lett. 99, 140402 (2007).
[14] J. Keeling and N. G. Berloff, Phys. Rev. Lett. 100, 250401 (2008).
[15] M. Wouters, I. Carusotto, and C. Ciuti, Phys. Rev. B 77, 115340 (2008).
[16] A. Baas, et al., Phys. Rev. Lett. 100, 170401 (2008); M. Wouters, Phys. Rev. B 77, 121302 (2008); P. R. Eastham, Phys. Rev. B 78, 035319(R) (2008).
[17] K. G. Lagoudakis et al., Nature Physics 4, 706 (2008).
[18] D. Porras, C. Ciuti, J. J. Baumberg, C. Tejedor, Phys. Rev. B 66, 053303 (2002).
[19] W. C. Tait, Phys. Rev. B 5, 649 (1972).
[20] G. E. Astrakharchik and L. P. Pitaevskii, Phys. Rev. A 70, 013608 (2004).
[21] I. Carusotto, S. X. Hu, L. A. Collins, and A. Smerzi Phys. Rev. Lett. 97, 260403 (2006).