Role of deformed Coulomb potential in the fusion process at different orientations of the colliding nuclei

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Abstract. We discuss the effect of adding deformation in the Coulomb potential in the fusion process using well tested proximity based nuclear potentials. This effect is analyzed at different orientations of the two colliding nuclei. Our study reveals that different (oblate/prolate) quadruple deformations in the Coulomb potential affect the total interaction potential in the inner region in different manner. In addition to nature of deformation (oblate/prolate), the depth of the fusion pocket also varies with change in orientations of colliding nuclei. A totally opposite trend of variation is observed in case of spherical-oblate and oblate-oblate systems as compared to spherical-prolate and prolate-prolate systems.

1. Introduction
The heavy-ion collisions produce a variety of phenomena that depend on the mass/charge of the colliding nuclei as well as on the incident energy and impact parameter of a reaction. At low incident energies, colliding matter leads to complete (incomplete) fusion [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Whereas, with increase in the incident energy, rare phenomena such as multifragmentation [15, 16, 17, 18, 19] as well as associated phenomena such as collective flow and elliptic flow [20, 21, 22] can also emerge. Due to large excitation energy, the fine structures such as shell effects, shape of a nucleus as well as quantum features don’t play significant role at higher incident energies where multifragmentation is a dominant channel. These effects, however, can play a role at low excited matter where fusion has a large share.

In recent years, one has seen renewed interest in the field of fusion. This renewed interest has been due to availability of neutron rich nuclei and synthesis of super heavy elements. In this direction, Puri and coworker [4, 5, 6, 7] studied the fusion barrier heights and positions by using various proximity potentials. Their studies revealed that out of different proximity potentials, Akyüz Winther (AW) 95, Bass 80 and Denisov DP reproduce the experimental data better than others. These potentials nicely explain the fusion probabilities at above barrier energies. In addition, many other proximity based potentials are also available in the literature [8, 9, 10, 11, 12, 13, 14]. In all above calculations, nuclei were assumed to be spherical, therefore, deformation was neglected in the nuclear as well as Coulomb potential. Though many calculations have been reported for deformed nuclear potentials, only few address the question of deformed Coulomb potential [23]. Our present aim is to use the best adjusted proximity based potentials along with deformed Coulomb potential to see if deformed Coulomb potential plays any role at different orientations of the colliding nuclei or not. In present study, the nuclear potential is assumed to be independent of deformation effects.
In present work, we employ AW 95, Bass 80 and Denisov DP to study the effect of oblate and prolate quadruple deformations (in the Coulomb potential) on the fusion process.

2. The Model
The nuclear part of the interaction potential $V_N(R)$ between two colliding surfaces, taken for present analysis, is derived using proximity approach and is mentioned in Refs. [4, 5, 6, 7]. The expression for the Coulomb interaction $V_C(R)$ between two deformed colliding nuclei is given by Wong [24] as

$$
V_C(R, \theta) = \frac{Z_1 Z_2 e^2}{R} + \sqrt{\frac{9}{20\pi}} \frac{Z_1 Z_2 e^2}{R^3} \sum_{i=1}^{2} [R_i(\theta_i)]^2 \beta_{2i} P_2(\cos\theta_i)
$$

$$
+ \frac{3}{7\pi} \frac{Z_1 Z_2 e^2}{R^3} \sum_{i=1}^{2} [R_i(\theta_i)]^2 [\beta_{2i} P_2(\cos\theta_i)]^2.
$$

(1)

where $Z_1$ and $Z_2$ are their respective charges, $R$ is the internuclear distance, $\theta_i$ is the angle of orientation (taken anticlockwise) and is defined as the angle between the axis of collision and symmetry axis as shown in figure 1. $R_i(\theta_i)$ is the nuclear radius for a deformed nucleus which is given by

$$
R_i(\theta_i) = R_i[1 + \sqrt{\frac{5}{4\pi}} \beta_{2i} P_2(\cos\theta_i)],
$$

(2)

Figure 1. Collision of a spherical and prolate-deformed nucleus in the same plane (i.e. $\phi=0$).

$R_i$ being the effective sharp radius of $ith$ nucleus. The different forms of effective sharp radius used in case of AW 95, Bass 80 and Denisov DP are given in Ref. [7]. The parameter $\beta_{2i}$ corresponds to quadruple deformation for $ith$ nucleus. In general, the quadrupole moment for a deformed nucleus is given by

$$
\beta_2 = \frac{2}{5} Z(b^2 - a^2),
$$

(3)

where $Z$ represents atomic number of a nucleus, $a$ and $b$ in the equation represent semi-major and semi-minor axis, respectively. For spherically symmetric nucleus, $\beta_2 = 0$ (i.e. $b = a$). In a prolate deformed nucleus, $\beta_2 > 0$ (i.e. $b > a$), and in oblate deformed nucleus $\beta_2 < 0$ (i.e. $b < a$). The quantity $P_2(\cos\theta_i)$ is the legendre polynomial for $n=2$. Since the colliding nuclei can have many possible orientations ($\theta_1, \theta_2$) during the experiment, therefore, we take different cases of orientations of two deformed colliding nuclei while studying the fusion between them. We restrict to the condition that both the deformed colliding nuclei are lying in the same plane.
(i.e. \( \phi = 0 \)). Therefore, the Coulomb potential calculated above is independent of angle \( \phi \). As we know that the quadruple deformation \( (\beta_2) \), higher order deformations \( (\beta_3, \beta_4) \), etc., satisfy the relation \( \beta_2^2 \approx \beta_1^2 \), therefore, only the terms which are linear and quadratic in quadruple deformation of nuclei are considered and the terms including higher order deformations are neglected [25, 26]. The values of quadruple deformation parameter \( (\beta_2) \) for present study are taken from Ref. [27].

Adding Coulomb potential to the nuclear part, we get the total interaction potential which, for zero value of orbital momentum, is given by

\[
V_T(R) = V_N(R) + V_C(R, \theta).
\] (4)

These models are used to calculate fusion barrier heights and positions. For further details of these models, the reader is referred to Refs. [4, 5, 6, 7].

3. Results and Discussion

For the present study, the reactions of \(^{30}\text{Si} + ^{62}\text{Ni}, ^{30}\text{Si} + ^{24}\text{Mg}, ^{27}\text{Al} + ^{70}\text{Ge} \) and \(^{24}\text{Mg} + ^{24}\text{Mg} \) are considered. These reactions are considered because, the value of deformation parameter for the above mentioned colliding nuclei is large enough to see the effect of deformation on the total interaction potential.

![Figure 2](image-url)

**Figure 2.** (Color online) The total interaction potential \( V_T(\text{MeV}) \) is displayed as a function of inter-nuclear distance \( R \) (fm) for the reactions of \(^{30}\text{Si} + ^{62}\text{Ni} \) (see upper panel) and \(^{30}\text{Si} + ^{24}\text{Mg} \) (see lower panel). Only AW 95 based calculations are displayed here. Similar trends are observed with Bass 80 and Denisov DP (not shown here). Further details are given in the text.
In figure 2, we display the total interaction potential $V_T$(MeV) as a function of internuclear distance $R$ (fm) for spherical (projectile)-deformed (target) systems. Here, black solid line corresponds to the total interaction potential calculated using AW 95, assuming both colliding nuclei to be spherical. The red, blue, magenta and olive dashed lines correspond to different orientations of the colliding nuclei (taking deformation into account), i.e. $(\theta_1 = 0^0, \theta_2 = 0^0)$, $(\theta_1 = 0^0, \theta_2 = 30^0)$, $(\theta_1 = 0^0, \theta_2 = 60^0)$ and $(\theta_1 = 0^0, \theta_2 = 90^0)$, respectively.

We find that the effect of adding deformation to Coulomb potential on total interaction potential in the inner region is significant, whereas, the effect in the tail region is negligible. It is clear from the figure 2(a) that the depth as well as width of the fusion pocket for $^{30}\text{Si} + ^{62}\text{Ni}$ i.e. spherical projectile with oblate-deformed target ($R_x = R_y > R_z$) increase with increasing relative orientation of two deformed colliding nuclei. The maximum depth of fusion pocket is observed in case of $(\theta_1 = 0^0, \theta_2 = 90^0)$ orientation. Whereas, depth is minimum in case of $(\theta_1 = 0^0, \theta_2 = 0^0)$. This is because, an oblate-deformed target finds maximum height of fusion barrier while colliding along its longest axis with the spherical projectile and lowest barrier height while colliding along the shortest axis. However, in case of $^{30}\text{Si} + ^{24}\text{Mg}$ (see figure 2(b)), i.e. spherical projectile with prolate-deformed target ($R_x = R_y < R_z$), the depth as well as width of the fusion pocket decrease with increasing relative orientation of two deformed colliding nuclei. The maximum depth of fusion pocket is observed in case of $(\theta_1 = 0^0, \theta_2 = 0^0)$. Whereas, depth is minimum in case of $(\theta_1 = 0^0, \theta_2 = 90^0)$. The reason is that a prolate-deformed target finds maximum height of fusion barrier while colliding along its shortest axis (side collision) with the
spherical projectile and lowest barrier height while colliding along the longest axis (tip collision).

In figure 3, we observe the behavior of total interaction potential as a function of internuclear distance for $^{27}\text{Al} + ^{70}\text{Ge}$ (oblate + oblate-deformed system) at different orientations of two colliding nuclei. A careful analysis of the four panels (3(a), (b), (c), and (d)), reveals that when oblate-deformed projectile is oriented at angles $\theta_1 = 0^0$ and $\theta_1 = 30^0$, the depth as well as width of interaction potential for all possible orientations ($\theta_1 = 0^0$ to $\theta_1 = 90^0$) of oblate-deformed target is less as compared to the spherical-spherical case. However, when oblate-deformed projectile is oriented at angles $\theta_1 = 60^0$ and $\theta_1 = 90^0$, the depth and width of fusion pocket start increasing with increasing orientation angles of oblate-deformed target and in most of the cases, these values are more as compared to that in spherical-spherical case. Out of different possibilities of orientation of two colliding nuclei, the lowest fusion barrier for the collision of oblate-deformed projectile with oblate-deformed target is observed at $\theta_1 = 90^0, \theta_2 = 90^0$ i.e. along the polar axis. The maximum repulsion for fusion is found for $\theta_1 = 0^0, \theta_2 = 0^0$ i.e. along the equatorial axis. Moreover, for a fixed orientation of oblate-deformed projectile (say $\theta_1 = 0^0$), the fusion barrier height decreases with increasing orientation angle of oblate-deformed target w.r.t. collision axis.

Similarly, the behavior of total interaction potential as a function of internuclear distance is studied for prolate+ prolate-deformed systems in figure 4. On carefully analyzing the four panels (4(a), (b), (c), and (d)), we find that when prolate-deformed projectile is oriented at angles $\theta_1 = 0^0$ and $\theta_1 = 30^0$, the depth as well as width of interaction potential for all possible

Figure 4. (Color online) Same as figure 2, but for the reaction of $^{24}\text{Mg} + ^{24}\text{Mg}$. Here both colliding nuclei are prolate-deformed.
Figure 5. (Color online) Same as figure 2, but for the reactions of $^{30}$Si + $^{62}$Ni and $^{30}$Si + $^{24}$Mg in the upper panels and for $^{27}$Al + $^{70}$Ge and $^{24}$Mg + $^{24}$Mg in the lower panels.

orientations ($\theta_1 = 0^\circ$ to $\theta_1 = 90^\circ$) of prolate-deformed target is more as compared to the spherical-spherical case and this trend is exactly opposite to that observed in figure 3. Also, when oblate-deformed projectile is oriented at angles $\theta_1 = 60^\circ$ and $\theta_1 = 90^\circ$, the depth and width of fusion pocket start decreasing with increasing orientation angles of oblate-deformed target. Out of different possibilities of orientation of two colliding nuclei, the lowest fusion barrier for the collision of prolate-deformed projectile with prolate-deformed target is observed at $\theta_1 = 0^\circ$, $\theta_2 = 0^\circ$ i.e. tip collision. Maximum repulsion for fusion is found for $\theta_1 = 90^\circ$, $\theta_2 = 90^\circ$ i.e. along the equatorial axis. Moreover, for a fixed orientation angle of prolate-deformed projectile (say $\theta_1 = 0^\circ$), the fusion barrier height increases with increasing orientation angle of prolate-deformed target w.r.t. collision axis. We observe exactly opposite behavior of total interaction potential with internuclear distance for prolate + prolate-deformed systems compared to oblate + oblate-deformed systems.

In figure 5, a comparative study is conducted using AW 95, Bass 80 and Denisov DP. Here, purple, wine and pink solid lines represent the calculations for spherical-spherical cases using AW 95, Bass 80 and Denisov DP, respectively. Whereas, the calculations including deformations using AW 95, Bass 80 and Denisov DP are represented by purple, wine and pink dash-dotted lines, respectively. For different deformed shapes of the two colliding nuclei, the interaction
potential is displayed at only those orientation angles at which fusion barrier are found to be minimum. From last three figures i.e. figures 2, 3 and 4, we find that for spherical + oblate (see figure 2(a)), spherical + prolate (see figure 2(b)), oblate + oblate (see figure 3(d)) and prolate + prolate systems (see figure 4(a)), the minimum fusion barrier is at orientation angle ($\theta_1 = 0^0, \theta_2 = 90^0$), ($\theta_1 = 0^0, \theta_2 = 0^0$), ($\theta_1 = 90^0, \theta_2 = 90^0$) and ($\theta_1 = 0^0, \theta_2 = 0^0$), respectively.

All three proximity based potentials i.e. AW 95, Bass 80 and Denisov DP show similar trend for interaction potential at critical orientations (corresponding to minimum depth of fusion pocket) of the colliding nuclei w.r.t. collision axis. In all the four cases (figs. 5(a), (b), (c) and (d)), the interaction potential becomes less repulsive or more attractive on including the deformed Coulomb potential (at above mentioned orientation angles) in the total interaction potential.

4. Summary
We studied the role of deformed Coulomb potential in fusion process by employing proximity based potentials due to AW 95, Bass 80 and Denisov DP. This detailed study reveals that shapes of the colliding nuclei affect the interaction potential in the inner region in different manner. In addition, the orientations of the nuclei participating in a reaction significantly affect the fusion barrier. The prolate-deformed colliding nuclei find minimum repulsion along tip collision, whereas, the oblate-deformed colliding nuclei find lowest fusion barrier along polar axis. These conclusions can be helpful in deciding the conditions for the fusion of deformed nuclei or synthesis of super-heavy elements during experiments.

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