On the formation of planetary systems in photoevaporating transition discs

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ABSTRACT

In protoplanetary discs, planetary cores must be at least $0.1 \, M_\oplus$ at 1 au for migration to be significant; this mass rises to $1 \, M_\oplus$ at 5 au. Planet formation models indicate that these cores form on million year time-scales. We report here a study of the evolution of $0.1$ and $1 \, M_\oplus$ cores, migrating from about 2 and 5 au, respectively, in million year old photoevaporating discs. In such a disc, a gap opens up at around 2 au after a few million years. The inner region subsequently accretes on to the star on a smaller time-scale. We find that, typically, the smallest cores form systems of non-resonant planets beyond 0.5 au with masses up to about $1.5 \, M_\oplus$. In low-mass discs, the same cores may evolve in situ. More massive cores form systems of a few Earth-mass planets. They migrate within the inner edge of the disc gap only in the most massive discs. Delivery of material to the inner parts of the disc ceases with opening of the gap. Interestingly, when the heavy cores do not migrate significantly, the type of systems that are produced resembles our Solar system. This study suggests that low-mm flux transition discs may not form systems of planets on short orbits but may instead harbour Earth-mass planets in the habitable zone.

Key words: accretion, accretion discs – methods: numerical – planets and satellites: formation – planet-disc interactions – planetary systems.

1 INTRODUCTION

Before the discovery of extrasolar planets, explaining the formation of terrestrial planets in our Solar system was already a challenging task. The detection of a large variety of planetary systems containing Earth-mass or super-Earth planets by the Kepler satellite has made the task even more arduous. Terrestrial mass planets are usually considered to form either in situ or through inward migration. In situ formation was the early scenario proposed for forming planets in our Solar system (Wetherill 1988; Lissauer 1993 and references therein), and has been put forward as a way to explain the Kepler candidates (Hansen & Murray 2013). However, it has been argued that this model is not consistent with the distribution of solids in discs (Raymond & Cossou 2014; Schlichting 2014). Also, in situ formation models have difficulties forming cores of giant planets before the gas dissipates (Thommes, Duncan & Levison 2003; Chambers 2016). Formation through inward migration is an efficient way of obtaining tight systems of super-Earths with short periods (Terquem & Papaloizou 2007; Haghighipour 2013 and references therein), but fails to explain terrestrial planets similar to those in our Solar system. However, migration of planets in dissipating discs has recently been studied (Coleman & Nelson 2014, 2016; Cossou et al. 2014) and seems to offer a way of combining the advantages of both models for forming terrestrial planets.

Coleman & Nelson (2014, 2016) studied the evolution of systems containing initially 36 or 52 cores with masses of 0.3 or 0.1 $M_\oplus$, respectively, spread between 1 and 20 au. In addition, there were thousands of planetesimals with masses 10, 20 or 50 times smaller than that of the embryos and distributed in between the planets. The disc had a surface gas density at least equal to that of the minimum solar mass nebula, i.e. $\Sigma \propto r^{-3/2}$ and $\Sigma = 1.7 \times 10^2 \, g \, cm^{-2}$ at 1 au, and was subject to photoevaporation and viscous evolution. They found that terrestrial-mass planets and super-Earths formed in the discs with the lowest masses. In the models with small abundance of solids and large planetesimals, growth was found to be limited so that migration was inefficient. Such models resulted in systems of low-mass planets spread out through the disc and in which mean motion resonances (MMR) were destroyed after the disc dissipated.

Cossou et al. (2014) started with cores with masses between 0.1 and 2 $M_\oplus$, spread between 1 and 20 au. The total mass in the cores was between 21 and 84 $M_\oplus$. They considered a disc with a surface gas density of $300 \, g \, cm^{-2}$ at 1 au and $\Sigma \propto r^{-1/2}$. In some of their simulations, the disc’s mass was decreased exponentially to mimic disc’s viscous evolution and photoevaporation. Systems of hot super-Earths in MMR were produced in discs which were not
dissipating, whereas the systems were more spread out and not in resonances when dissipation was included. In these studies, all types of planets formed from the same parent population and, in general, the planetary systems that were produced at around 1 au contained planets more massive than the terrestrial planets in our Solar system. In this paper, we consider a model where different parent populations exist at different locations and the disc is photoevaporating.

In typical protoplanetary discs, the type-I migration time-scale becomes comparable to the disc lifetime for cores at least as massive as 0.1 and 1 $M_\oplus$ at 1 and 5 au, respectively. Planet formation models suggest that it takes at least 1 Myr for 0.1 $M_\oplus$ cores to form at $\sim$1 au. More massive cores, with a mass $\sim$1 $M_\oplus$, may form on this time-scale beyond the snow line, at around 5 au. This prompts us to consider a model where the initial conditions are a population of 0.1 $M_\oplus$ cores between 1 and 5 au and a population of 1 $M_\oplus$ cores beyond 5 au, which start migrating when the disc is $\sim$1 Myr old. The total mass in these populations of cores is set by the initial surface density in the disc.

We evolve these initial populations in a disc which undergoes a transition due to X-ray photoevaporation. Transition discs are defined as discs lacking emission in the near-infrared, which means that there is a (large) hole in the dust distribution in their inner parts. Observations strongly suggest two types of transition discs: those with low-mm flux, which have low accretion rates and hole sizes smaller than about 20 au, and those with high-mm flux, which have higher accretion rates and hole sizes larger than 20 au (Owen & Clarke 2012; Owen 2016, and references therein). It is believed that low-mm flux transition discs are in the process of dispersing, whereas high-mm flux transition discs are not. The most commonly accepted interpretation for low-mm flux transition discs is X-ray photoevaporation. In this model, a gap opens up after about 3 Myr (75 per cent of the disc lifetime) at $\sim$2 au, where the accretion rate in the disc matches the photoevaporation rate (Owen, Ercolano & Clarke 2011). The inner parts of the disc then become decoupled from the outer parts and cannot be resupplied in gas and dust. They subsequently accrete on to the central star whereas the outer edge of the gap recedes due to photoevaporation. By contrast, it is believed that, in high-mm flux transition discs, a massive planet (with a mass of a few Jupiter masses) is responsible for creating a gap.

The above discussion suggests that, in low-mm flux transition discs, 0.1 and 1 $M_\oplus$ cores forming between 1 and 5 au and beyond 5 au, respectively, on a time-scale of 1 Myr, would start to migrate a million years or so before a gap opens up at around 2 au. As their migration time-scale itself is of the order of a million years, it can be expected that the dispersion of the disc will prevent migration of cores to very small radii and also that the formation of the gap will prevent massive cores to be delivered to the region of terrestrial planets. This is the model we explore in this paper.

The plan of the paper is as follows. In Section 2, we describe the disc evolution model and justify the parameters that are used in the simulations. We also give expressions for the planet migration and eccentricity damping time-scales and briefly discuss core formation time-scales. In Section 3, we present $N$-body simulations of cores migrating in photoevaporating discs. We first describe the numerical scheme and the initial set up. We then present the results of the simulations. We show that, for reasonable parameters that are consistent with the observations, a population of 0.1 $M_\oplus$ cores originating from between 2 and 4 au migrate down to 0.5–1 au. The final masses of planets are between a fraction of an Earth mass and $\sim$1.5 $M_\oplus$. As for the population of 1 $M_\oplus$ cores originating from $\sim$5 au, it forms a few cores of a few Earth masses which may migrate down below the inner edge of the gap only in the most massive discs considered here. In less massive discs, cores with a mass comparable to that of Jupiter may be left at a few au from the star. Finally, in Section 4, we summarize and discuss our results.

2 DISC EVOLUTION AND PLANET FORMATION

In this section, we review the different time-scales that are used in the simulations.

2.1 Disc evolution and parameters

We adopt an initial surface density profile in the disc:

$$\Sigma(r, t = 0) = \Sigma_1 \left( \frac{r}{1 \text{ au}} \right)^{-1},$$

where $\Sigma_1$ is the initial surface density at 1 au. This density gradient gives the best fit to the thermal continuum emission from $\sim$1 Myr old discs in the Ophiuchus star-forming region, as shown by Andrews et al. (2010). We express $\Sigma_1$ in terms of the disc mass within 50 au, $M_{50}$:

$$\Sigma_1 = 2.8 \times 10^4 \frac{M_{50}}{M_\odot} \text{ g cm}^{-2}. \quad (2)$$

In this paper, we focus on the evolution of planetary systems in transition discs which have low-mm fluxes and are supposed to be the consequence of X-ray photoevaporation (Owen 2016; Andrews et al. 2010). Here, we evolve the surface density profile $\Sigma(r, t)$ in such a way as to reproduce their calculation.

We start with $\Sigma(r, 0)$ given by equation (1). After a time $t = t_{\text{gap}}$ which is between 1 and 3 Myr ($\sim$75 per cent of the disc lifetime, see Owen et al. 2011), a gap opens up between the radii that we fix to be $r_{10} = 2$ au and $r_{20} = 3$ au. Between $t = 0$ and $t_{\text{gap}}$, we assume that $\Sigma$ decreases linearly with time, i.e.:

$$\Sigma(r, t) = \Sigma(r, 0) \left[ 1 - \frac{(1 - \eta) t}{t_{\text{gap}}} \right], \quad \text{for } 0 \leq t \leq t_{\text{gap}}, \quad (3)$$

where $\eta = \Sigma(r, t_{\text{gap}})/\Sigma(r, 0)$ is the fraction of mass left after a time $t_{\text{gap}}$. Subsequently, the inner disc disperses on a time-scale $t_c$ such that

$$\Sigma(r_{\text{can}} \leq r \leq r_{10}, t) = \eta \Sigma(r, 0) \left[ 1 - \frac{(t - t_{\text{gap}})}{t_c} \right], \quad \text{for } t_{\text{gap}} \leq t \leq t_{\text{gap}} + t_c, \quad (4)$$

$$\Sigma(r \leq r_{10}, t = 0), \quad \text{for } t \geq t_{\text{gap}} + t_c, \quad (5)$$

where $r_{\text{can}} = 0.05$ au if the radius of the inner disc cavity produced by the magnetic interaction between the disc and the star. We will take $t_c \sim 10^4$–$10^5$ yr as this is the viscous time-scale at 2 au in a standard $\alpha$-disc model (Shakura & Sunyaev 1973).
10\(^{-3}\) and 0.01. After the gap opens up, its outer edge \(r_2\) moves out according to
\[
r_2 = r_{20} + (10 \text{ au} - r_{20}) \frac{t - t_{\text{gap}}}{3.2 \times 10^6 \text{ yr} - t_{\text{gap}}}, \quad \text{for } t \geq t_{\text{gap}},
\] (6)
so that after \(3.2 \times 10^6\) yr (~80 per cent of the disc lifetime, see Owen et al. 2011), the outer edge has moved up to 10 au. The surface density does not vary significantly beyond \(r_2\):
\[
\Sigma(r \geq r_2, t) = \Sigma(r \geq r_2, t_{\text{gap}}) = \eta \Sigma(r \geq r_2, 0), \quad \text{for } t \geq t_{\text{gap}},
\] (7)
while it is zero inside the gap:
\[
\Sigma(r_{10} \leq r \leq r_2, t) = 0, \quad \text{for } t \geq t_{\text{gap}}.
\] (8)

In the numerical simulations presented below, we will take \(\Sigma_1 \sim 10^3\) g cm\(^{-2}\) at \(t = 0\), i.e. \(M_{\text{dust}} \lesssim 0.1 \, M_\oplus\) initially. More massive discs would be gravitationally unstable. It has been pointed out that the observed range of accretion rates in T Tauri stars require an initial disc mass between 0.01 and 0.2 \(M_\odot\) (Hartmann et al. 1998).

Masses derived for the ∼1 Myr old discs in the Ophiuchus star-forming region are of the order of 0.01 \(M_\odot\) within 50 au (Andrews et al. 2010). This suggests that \(\eta = 0.1\) in equation (3). A decrease in disc’s mass by a factor 10 during the first Myr of evolution is also consistent with the observations and modelling of discs around T Tauri stars in the Taurus and Chamaeleon I molecular clouds complex (Hartmann et al. 1998).

The surface density at different times is shown in Fig. 1 for \(t_{\text{gap}} = 3\) Myr, \(t_1 = 0.2\) Myr and \(\Sigma_1 = 2.7 \times 10^3\) g cm\(^{-2}\). The different lines correspond to the following times: \(t = 0\) (solid line), \(t = 2.8\) Myr (dotted line), \(t = 3.04\) Myr (short-dashed line), \(t = 3.15\) Myr (long-dashed line), \(t = 3.195\) Myr (dot–short-dashed line) and \(t = 3.5\) Myr (dot–long-dashed line). This figure is similar to fig. 9 of Owen et al. (2011).

2.2 Migration and eccentricity damping time-scales

Tidal interaction between a planet and the disc in which it is embedded leads to a change of the planet’s angular momentum (i.e. migration) and to eccentricity damping on the time-scales \(t_{\text{mig}}\) and \(t_{\text{ecc}}\), respectively. The cores we consider here are small enough that they undergo inwards type I migration. In this regime, we use for \(t_{\text{mig}}\) the time-scale derived by Tanaka, Takeuchi & Ward (2002) and modified by Papaloizou & Larwood (2000) to account for the effect...
of a finite eccentricity:

\[ t_{\text{ mig}}(\text{yr}) = \frac{2}{2.7 + 1.1n} \left( \frac{M_\odot}{m_p} \right) \left( \frac{H}{r} \right)^2 \frac{M_\odot}{2\pi(1\text{ au})^2\Sigma} \left( \frac{1 \text{ au}}{a} \right)^{1/2} \times \left[ 1 + \left( \frac{e}{1.3H/r} \right)^3 \right]^{-1} \left[ 1 - \left( \frac{e}{1.1H/r} \right)^4 \right]^{-1}, \tag{10} \]

where \( m_p, a, e \) are the mass, semimajor axis and eccentricity of the planet, \( \Sigma \) is the disc surface density of gas and \( H/r \) is the disc aspect ratio at the location of the planet, and \( n \equiv -\ln\Sigma/d\ln r \). Here, we have assumed that the central object is a solar mass star. Thereafter, we will take \( n = 1 \), as in Section 2.1 [see equation (1)].

For \( t_{\text{ ecc}} \), we adopt the time-scale given by Tanaka & Ward (2004) and again modified by Papaloizou & Larwood (2000) to account for the effect of the eccentricity:

\[ t_{\text{ ecc}}(\text{yr}) = 0.1 \left( \frac{M_\odot}{m_p} \right) \left( \frac{H}{r} \right)^4 \frac{M_\odot}{2\pi(1\text{ au})^2\Sigma} \left( \frac{1 \text{ au}}{a} \right)^{1/2} \left[ 1 + 0.25 \left( \frac{e}{H/r} \right)^3 \right]. \tag{11} \]

where the factor of 0.1 is taken from Cresswell & Nelson (2006) as it gives good agreement between equation (11) and the damping time-scale obtained in hydrodynamical simulations.

For a planet with \( e = 0 \) at \( a = 1 \text{ au} \) in a disc with \( n = 1 \) and \( H/r = 0.05 \), equations (10) and (11) can be written as

\[ t_{\text{ mig}}(\text{yr}) \simeq 6 \times 10^5 \left( \frac{\Sigma[1 \text{ au}]}{10^3 \text{ g cm}^{-2}} \right)^{-1} \left( \frac{m_p}{M_\odot} \right)^{-1} \tag{12} \]

\[ t_{\text{ ecc}}(\text{yr}) \simeq 300 \left( \frac{\Sigma[1 \text{ au}]}{10^3 \text{ g cm}^{-2}} \right)^{-1} \left( \frac{m_p}{M_\odot} \right)^{-1}, \tag{13} \]

where \( \Sigma[1 \text{ au}] \) means that \( \Sigma \) is evaluated at 1 au.

Note that radiation-hydrodynamical simulations of disc/planet interactions have shown that the corotation torque could lead to outward migration, but that does only affect cores with masses between about 4 and 30 \( M_\oplus \) (and eccentricities below \(~0.015\)) larger than the cores we consider here (Paardekooper & Mellema 2006; Kley, Bitsch & Klahr 2009; Bitsch & Kley 2010).

### 2.3 Planet formation time-scale

Assuming that the surface density of gas at 1 au is smaller than about \( 10^3 \text{ g cm}^{-2} \) (see Section 2.1), equation (12) indicates that migration at \( \sim 1 \text{ au} \) only affects cores that have a mass larger than about 0.1 \( M_\oplus \) (Mars’s mass). Smaller mass objects would migrate on a time-scale longer than the disc’s lifetime, which is of a few million years.

Cores with masses \(~0.1 \text{M}_\oplus\) form through oligarchic growth on a time-scale which, at \( \sim 1 \text{ au} \), is of the order of at least 1 Myr (see Chambers 2016 and references therein and Kobayashi & Dauphas 2013 for the formation of Mars). Therefore, migration starts to be significant when the disc’s mass is about one-tenth of its initial value, i.e. \( M_\text{disc} \approx 0.01 \text{M}_\odot \) or \( \Sigma[1 \text{ au}] \sim 10^2 \text{ g cm}^{-2} \).

Beyond the snow line, it is expected that growth is faster, although there is still much uncertainty about building massive cores on a short time-scale there (Chambers 2016). In the simulations below, we will consider cases where 1 \( M_\oplus \) cores have formed on a time-scale of 1 Myr at around 5 au.

### 3 Numerical simulations

In this section, we study the evolution of a population of small cores originating from the region of the terrestrial planets, that of a population of Earth-mass cores originating from beyond the snow line, and finally the evolution of a mixture of both small and large cores.

#### 3.1 Numerical integration

To compute the evolution of a population of cores migrating through a disc, we use the \( N \)-body code described in Papaloizou & Terquem (2001) in which we have added the effect of the disc torques (see also Terquem & Papaloizou 2007). The equations of motion are integrated using the Bulirsch–Stoer method with a timestep which is adjusted to match a prescribed accuracy (e.g. Press et al. 1992).

The equations of motion for each core are

\[ \frac{d^2r_i}{dt^2} = -G \frac{M_\odot r_i}{|r_i|^3} - \sum_{j=1}^{N} \frac{Gm_j (r_i - r_j)}{|r_i - r_j|^3} - \sum_{j=1}^{N} \frac{Gm_j r_j + \Gamma_i}{|r_j|^3}, \tag{14} \]

where \( G \) is the gravitational constant and \( m_i \) and \( r_i \) denote the mass and position vector of core \( i \), respectively. The third term on the right-hand side is the acceleration of the coordinate system based on the central star (indirect term).

Acceleration due to tidal interaction with the disc is dealt with through the addition of extra forces as in Papaloizou & Larwood (2000, see also Terquem & Papaloizou 2007):

\[ \Gamma_i = -\frac{1}{t_{\text{ mig}}} \frac{dr_i}{dt} - \frac{2}{|r_i|^3 t_{\text{ ecc}}} \left( \frac{dr_i}{dr} \times r_i \right) r_i, \tag{15} \]

where \( t_{\text{ mig}} \) and \( t_{\text{ ecc}} \) are given by equations (10) and (11) in which \( m_p \) is replaced by \( m_i \). Note that the time-scale on which the semimajor axis decreases is \( t_{\text{ mig}}/2 \) (e.g. Teysssandier & Terquem 2014). As here cores never approach the star very closely, we do not include contribution from the tides raised by the star nor from relativistic effects.

Collisions between cores are dealt with in the following way: if the distance between cores \( i \) and \( j \) becomes less than \( R_i + R_j \), where \( R_i \) and \( R_j \) are the radii of the cores, a collision occurs and the cores are assumed to merge. They are subsequently replaced by a single core of mass \( M_i + M_j \) with the position and the velocity of the centre of mass of cores \( i \) and \( j \).

#### 3.2 Initial set up

We consider a disc which has either \( \Sigma_1 = 2.7 \times 10^3 \) or 900 g cm\(^{-2}\), i.e. \( M_{\text{50}} = 0.1 \) or 0.03 \text{M}_\odot \) initially. Note that this latter value is about twice as large as the minimum mass solar nebula (Hayashi 1981). We start with a population of \( N \) cores on circular orbits in the disc mid-plane spread between an inner radius \( r_{\text{in}} \) and an outer radius \( r_{\text{out}} \). All the cores are supposed to have an identical mass density \( \rho = 1 \text{ g cm}^{-3} \) so that \( R_i = [3m_i/(4\pi\rho)]^{1/3} \). We vary \( r_{\text{in}} \) and \( r_{\text{out}} \), keeping \( r_{\text{out}} - r_{\text{in}} = 1.5-2 \text{ au} \) in most of the simulations.

From equation (9), the mass of dust in between those two radii is \( \sim 5-10 \text{M}_\oplus \). Therefore, we will take \( N \leq 100 \) or \( N \leq 10 \) when starting with a population of cores with masses \( m_p = 0.1 \) or 1 \text{M}_\oplus \), respectively. The upper values of \( N \) correspond to all the dust mass being accreted into cores, which probably leads to overestimating the number of cores or/and their masses.
As mentioned above, the disc is assumed to be truncated at an inner radius $r_{\text{in}} = 0.05$ au. At time $t = t_{\text{gap}}$, a gap opens up between $r_{20} = 2$ au and $r_{20} = 3$ au. We take $t_{\text{gap}} = 3$ Myr, at which time the disc mass has decreased by a factor of 10 [equation (3) with $\eta = 0.1$]. The cores start to migrate at a time $t_{\text{start}}$ which we take to be either 1.5, 2 or 2.5 Myr. For $\Sigma_1 = 2.7 \times 10^3$ g cm$^{-2}$, this corresponds to the surface density at 1 au being reduced to $1.5 \times 10^3$, 10$^3$ or 680 g cm$^{-2}$, respectively [see equation (3)], while for $\Sigma_1 = 900$ g cm$^{-2}$, it corresponds to 495, 360 or 225 g cm$^{-2}$, respectively. Note that, for $t$ between 0 and $t_{\text{gap}}$, the important parameters are not $t_{\text{gap}}$ and $t_{\text{start}}$ taken separately, but $t_{\text{start}}/t_{\text{gap}}$, as this is what determines the surface density in the disc when the cores start to migrate. Therefore, the same results would be obtained with $t_{\text{gap}} = 1$ Myr and $t_{\text{start}} = 0.5$, 0.7 or 0.8 Myr.

For $t_{\nu}$, we adopt values between $2 \times 10^4$ and $6 \times 10^5$ yr.

The simulations are run at least until the time when there is no gas left in the disc in the regions were the planets are. In some cases, the simulations are run for much longer, to check the stability of the systems over million years time-scales.

3.3 Results

3.3.1 Starting with small cores in the inner disc

We start by calculating the evolution of a population of $N = 50$ cores with initial masses $m_0 = 0.1 M_\oplus$ and initially spread between 2 and 3.5 au. Fig. 2 shows the evolution of the semimajor axes of these cores for $\Sigma_1 = 2.7 \times 10^3$ g cm$^{-2}$, $t_{\text{start}} = 2$ Myr and $t_{\nu} = 0.2$ Myr. At $t = t_{\text{gap}} = 3$ Myr, a gap opens up between 2 and 3 au. At that point, the planet with $a > 3$ au continues to migrate in. As it is in MMR with the planets in the gap, it pushes them inwards. Migration of planets in MMR is accompanied by an increase of the eccentricities. As there is no gas left in the gap to damp the eccentricities, collisions among planets occur and resonances are destroyed. Similarly, the planets which are within 2 au continue to migrate in, and as the gas dissipates collisions occur. By the end of the simulations, only the two outermost planets are still in MMR and there is no gas left around the planets, which therefore do not migrate anymore. The three innermost planets, labelled A, B and C, have $a = 0.8, 0.9$ and 1 au and a mass of 0.5, 0.7 and 1.5 $M_\oplus$, respectively. The other cores left at the end of the simulation have a mass between 0.1 and 0.9 $M_\oplus$.

The eccentricities of the planets left at the end of the simulation are below 0.08 but they may increase on a longer time-scale due to gravitational interactions. We note that the mutual spacing between the three innermost planets is very close to 12 times their mutual Hill radii. This has been shown by Pu & Wu (2015) to be the limit for stability on Gyr time-scales, with smaller separations leading to instabilities. Therefore, further collisions cannot be ruled out.

Fig. 3 shows the difference of the longitudes of pericentre $\Delta \sigma$ for planets A and B and for planets B and C as a function of time. Before $t = 3.1$ Myr, these planets are in MMR. For planets A and B, $\Delta \sigma$ librates around 180$^\circ$, i.e. the apsidal lines are anti-aligned and conjunction occurs when one planet is near pericentre and the other near apocentre. For planets B and C, $\Delta \sigma$ librates around 0, i.e. the apsidal lines are aligned and conjunction occurs when the planets...
are near pericentre. At $t \simeq 3.1$ Myr, a collision occurs that results in planet A merging with another core, and this destroys the MMR. At $t \simeq 3.3$ Myr, planet C itself merges with another core. At the end of the simulation, there is no MMR among the three innermost planets (although there is some degree of dynamical coupling).

The case illustrated in Figs 2 and 3 is very typical. All the simulations we have performed starting with $0.1 M_\oplus$ cores and $t_{\text{start}} = 2$ Myr and in which some of the cores were able to migrate below 2 au before the gap opened up, ended up with a few planets between 0.5 and $\sim 1$ au. The same simulation ran with a value of $t_\nu$, 10 times smaller, i.e. $t_\nu = 0.02$ Myr, gives qualitatively the same results. Similar results were also obtained with $t_{\text{start}} = 1.5$ Myr by reducing the disc’s mass (e.g. adopting $\Sigma_1 = 900$ rather than $2.7 \times 10^3 \text{ g cm}^{-2}$).

The final masses are usually between a fraction of an Earth mass and $\sim 1 M_\oplus$. The fact that the planets do not migrate further down is not surprising as the migration time-scale for cores with masses between 0.1 and 1 $M_\oplus$ in a disc with a surface mass density equal to a few $10^2$ g cm$^{-2}$ is between 1 and 10 Myr. For $t_{\text{start}} = 2$ Myr, we obtained planets at the inner edge of the disc (0.05 au) only when starting with cores more massive than 0.3 $M_\oplus$. By reducing $t_{\text{start}}$ to 1.5 Myr, we could obtain planets near the edge when starting with cores more massive than 0.15 $M_\oplus$. In this case, MMR could be maintained.

In the simulation shown in Fig. 2, where the cores were initially spread between $r_{\text{in}} = 2$ au and $r_{\text{out}} = 5$ au, five cores with a total mass of $4 M_\oplus$ reached the inner disc (within 2 au). The same simulation with $r_{\text{in}} = 3$ au and $r_{\text{out}} = 5$ au ends up with only one $1 M_\oplus$ core slightly below 2 au, all the others being left at larger distances from the star. On the other hand, by reducing $r_{\text{in}}$ and $r_{\text{out}}$ to 1.5 and 3 au, respectively, we obtain eight cores with a total mass of 4.6 $M_\oplus$ between 0.5 and 2 au, the most massive core having a mass of 1 $M_\oplus$. Again, the orbits of these cores are expected to evolve and further collisions to occur on a time-scale longer than that of the simulation.

When the cores do not migrate significantly, either because the initial disc is not massive enough (low $\Sigma_1$) or migration starts late, after the disc’s mass has decreased down to low values (large $t_{\text{start}}$), evolution is very much like what is obtained in in situ formation models. There is not enough gas to damp the eccentricities, so collisions occur and a few planets with masses at most between 1 and 2 $M_\oplus$ form without significant inward migration.

### 3.3.2 Starting with heavier cores beyond the snow line

So far, we have ignored in the simulations heavier cores that may be delivered to the inner parts from further away in the disc. Whether these cores will be present or not depends on whether they can form on a time-scale of 1 Myr. Let us assume this the case, and there is a population of 1 $M_\oplus$ cores that form within $\sim 1$ Myr beyond the snow line at around 5 au (see Chambers 2016 for models). According to equation (10), the migration time-scale of these cores at 5 au would be the same as that of 0.1 $M_\oplus$ cores at 1 au, and therefore we choose the same value of $t_{\text{start}} = 2$ Myr as in the simulations with smaller cores presented above (we also keep $t_\nu = 0.2$ Myr). In Fig. 4, we show the evolution of 10 cores with initial mass of $m_p = 1 M_\oplus$ and initially spread between 5 and 6.5 au, for both $\Sigma_1 = 2.7 \times 10^3$.
Figure 4. Same as Fig. 2 but for $N = 10$, $m_p = 1 M_\oplus$, $r_{in} = 5$ au, $r_{out} = 6.5$ au and $\Sigma_1 = 2.7 \times 10^3$ g cm$^{-2}$ (upper plot) or 900 g cm$^{-2}$ (lower plot). Collisions between initial cores occur very early in the simulations so the 10 cores are not visible on the plots. Also heavy lines indicate several cores which oscillate around the same semimajor axis. The numbers above the lines at the end of the simulations indicate the mass of the cores left in $M_\oplus$.

(upper plot) and 900 g cm$^{-2}$ (lower plot). As pointed out above, if these cores had formed in situ they would have used all the dust initially present in the disc at these locations. Alternatively, they could have formed further away and migrated in, although it takes more time to form cores at larger distances from the star. As seen on the figure, these cores grow through collisions and migrate in while maintaining MMR. When the gap opens up, in the case where $\Sigma_1 = 2.7 \times 10^3$ g cm$^{-2}$, some of the cores have reached the inner parts of the disc below 2 au while the other cores are in the gap. The depletion of gas leads to eccentricity growth and further collisions occur, leaving at the end of the simulation two cores with masses of 3 and 6 $M_\oplus$ between 1 and 2 au and one core with a mass of 1 $M_\oplus$ near 3 au. In the case of a less massive disc, i.e. $\Sigma_1 = 900$ g cm$^{-2}$, none of the cores reach the gap before it opens up. Here, by the end of the simulation, a 1 $M_\oplus$ core has been scattered to large distance (∼11 au) while two cores with masses of 7 and 2 $M_\oplus$ are left at 4 and ∼3 au. In that case, evolution is very much like what is obtained with in situ models.

As can be seen from the figure, the systems are stable after the last collisions occur and until 6 Myr. The eccentricities, although large, vary smoothly and in a periodic way. However, further collisions over much longer time-scales cannot be ruled out.
Cores with a mass of a few $M_\oplus$ could be obtained at smaller distances from the star by starting the migration at earlier times.

3.3.3 Starting with a mixture of small and heavier cores

If we now start with both small cores ($m_p = 0.1 M_\oplus$) at around 2 au and heavier cores ($m_p = 1 M_\oplus$) at around 5 au, the evolution can be predicted on the basis of the results presented above. The small cores evolve as described above, and always end up forming planets with masses at most between 1 and 2 $M_\oplus$, at locations that depend on how fast they have migrated, but usually between 0.5 and 2 au. If the heavier cores are able to catch up, they may sweep some of the smallest ones on their way in and the final system has a few cores of several Earth masses below 2 au, mixed with less massive planets. On the other hand, the heavier cores may not be able to migrate below the inner edge of the gap (e.g. for low values of $\Sigma_1$, or if the heavy cores have large $t_{\text{malloc}}$), in which case they will also evolve mostly in situ to form a few massive cores beyond 4 au or so.

This is exactly what we have found in simulations starting with 50 small cores ($m_p = 0.1 M_\oplus$) between 2 and 5 au together with 5 or 10 heavier cores ($m_p = 1 M_\oplus$) between 5 and 6.5 au. We considered $\Sigma_1 = 2.7 \times 10^3$ or 900 g cm$^{-2}$ and, as above, $t_{\text{malloc}} = 2$ Myr and $t_c = 0.2$ Myr. The case $\Sigma_1 = 2.7 \times 10^3$ g cm$^{-2}$ is illustrated in Fig. 5. The simulation has been stopped after 10 Myr but here of course more collisions are going to occur on longer time-scales. The same simulation with $\Sigma_1 = 900$ g cm$^{-2}$ does not show any significant migration. In that case, the cores evolve mostly in situ, and after the disc dissipates there is no core below 1.8 au.

4 SUMMARY AND DISCUSSION

In this paper, we have studied the outcome of core migration in transition discs produced by photoevaporation. The initial surface mass density in discs at around 1 au is at most of the order $10^3$ g cm$^{-2}$, and it decreases with time. Therefore, planetary cores that form at this distance from the star migrate on a time-scale smaller than the disc lifetime only if their mass is larger than about 0.1 $M_\oplus$. As it takes at least about 1 Myr to form such cores at around 1 au, migration of cores forming below the snow line starts to be relevant only when the disc is at least 1 Myr old. Similarly, if the surface density varies as $r^{-1}$, cores at ~5 au have a migration time-scale shorter than the disc lifetime only when their mass reaches about 1 $M_\oplus$. Assuming these cores form on a time-scale of about 1 Myr, they will also start to migrate significantly when the disc is about 1 Myr old.

In low-mm flux transition discs, a gap is believed to open up at around 2–3 au due to X-ray photoevaporation when the disc is a few million years old. The disc’s inner parts are subsequently accreted on to the central star as a result of viscous evolution, while the outer edge of the gap recedes under the effect of photoevaporation. In this context, we find that ~0.1 $M_\oplus$ cores that form within ~1 Myr between 1 and 4 au end up forming a system of a few planets with masses between a fraction of an Earth mass and 1.5 $M_\oplus$ at most. In general, these cores do not migrate down further than ~0.5 au, and may even evolve in situ in low-mass discs. They are not in MMR. Such resonances, which appear during the migration phase, are usually destroyed when the gas dissipates (as previously found by Cossou et al. 2014; Coleman & Nelson 2016). It is difficult to form planets at smaller radii, unless we start with heavier cores or start migrating the cores earlier (as, e.g. in Terquem & Papaloizou 2007). In both cases, it would require a more efficient planet formation process than envisioned here. If ~1 $M_\oplus$ cores can form beyond the snow line also within ~1 Myr, they migrate on the same time-scale as the smaller cores and, depending on the surface density in the disc, they may or may not reach the inner parts of the disc. After the gap has opened up, the inner parts of the disc become isolated from the outer parts, and no more material is delivered there.
We have found that, in a disc with an initial surface mass density of about $10^3 \text{g cm}^{-2}$ at 1 au, it was possible to obtain a system of cores similar to that of the Solar system, with a few planets with masses of a few tenths of an Earth mass to an Earth mass in the terrestrial zone and a more massive core a little bit further away (Fig. 2 and lower panel of Fig. 4). However, as the photoevaporation model predicts that the outer edge of the gap is receding rather fast, it is difficult to envision within this model how the massive core could accrete a gaseous envelope to become a giant planet. Such a system of low-mass planets is similar to those obtained by Cossou et al. (2014) and Coleman & Nelson (2016) in their models with inefficient migration.

The exact parameters that should be used in such simulations are of course not known, and different outcomes could be produced by changing them. However, the point of this paper is to show that, by adopting reasonable parameters that fit the observations and the current understanding of planet formation and migration, systems of cores/planets can be obtained where all the planets do not end up on resonant chains near the disc’s inner edge and delivery of material to the inner parts from the outer parts of the disc can be avoided. To that extent, the type of systems that we obtain resembles more our Solar system than the systems observed by Kepler, which is biased towards detecting planets on very tight orbits. Our results predict that low-mm flux transition disc may harbour terrestrial planets in the habitable zone.

Note that, as collisions between cores occur until rather late in the disc’s evolution, dust may be produced at distances below 1 or 2 au from the central star even after the disc’s inners parts have been accreted. In that case, the disc would not appear as transition disc. This study suggests that low-mm flux transition discs may not be able to form giant planets before X-ray photoevaporation opens up a gap. These discs would then form planetary systems of the type obtained in this study: predominantly low-mass planets at around 1 au with possibly more massive cores further away. High-mm flux transition discs, by contrast, are associated with more massive stars and may form giant planets of a few Jupiter masses, massive enough to open up a gap, before photoevaporation could proceed.

REFERENCES

Andrews S. M., Wilner D. J., Hughes A. M., Qi C., Dullemond C. P., 2010, ApJ, 723, 1241
Bitsch B., Kley W., 2010, A&A, 523, 30
Chambers J. E., 2016, ApJ, 825, 63
Coleman G. A. L., Nelson R. P., 2014, MNRAS, 445, 479
Coleman G. A. L., Nelson R. P., 2016, MNRAS, 457, 2480
Cossou C., Raymond S. N., Hersant F., Pierens A., 2014, A&A, 569, 56
Cresswell P., Nelson R. P., 2006, A&A, 450, 833
Haghighipour N., 2013, Annu. Rev. Earth Planet. Sci., 41, 469
Hansen B. M. S., Murray N., 2013, ApJ, 775, 53
Hartmann L., Calvet N., Gullbring E., D'Alessio P., 1998, ApJ, 495, 385
Hayashi C., 1981, Prog. Theor. Phys. Suppl., 70, 35
Kley W., Bitsch B., Klahr H., 2009, A&A, 506, 971
Kobayashi H., Dauphas N., 2013, Icarus, 225, 122
Lissauer J. J., 1993, ARA&A, 31, 129
Owen J. E., 2016, PASA, 33, 5
Owen J. E., Clarke C. J., 2012, MNRAS, 426, 96
Owen J. E., Ercolano B., Clarke C. J., 2011, MNRAS, 412, 13
Paardekooper S.-J., Mellema G., 2006, A&A, 459, L17
Papaloizou J. C. B., Larwood J. D., 2000, MNRAS, 315, 823
Papaloizou J. C. B., Terquem C., 2001, MNRAS, 325, 221
Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, Numerical Recipes in FORTRAN, Cambridge Univ. Press, New York
Pu B., Wu Y., 2015, ApJ, 807, 44
Raymond S. N., Cossou C., 2014, MNRAS, 440, L11
Schlichting H. E., 2014, ApJ, 795, 15
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Tanaka H., Ward W. R., 2004, ApJ, 602, 388
Tanaka H., Takeuchi T., Ward W. R., 2002, ApJ, 565, 1257
Terquem C., Papaloizou J. C. B., 2007, ApJ, 654, 1110
Teyssandier J., Terquem C., 2014, MNRAS, 443, 568
Thommes E. W., Duncan M. J., Levison H. F., 2003, Icarus, 161, 431
Wetherill G. W., 1988, in Vilas F., Chapman C. R., Matthews M. S., eds, Mercury. Univ. Arizona Press, Tucson, AZ, p. 670

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