Introducion: A long-standing problem in physics is the hierarchy problem, which has been one of the main driving forces of physics beyond the Standard Model (SM) during the last few decades [1]. The problem can be formulated as the large difference in magnitudes between the Planck and electroweak scales, $M_{\text{pl}}/M_{\text{EW}} \approx 10^{16}$, where $M_{\text{pl}}(\sim 10^{16} \text{ TeV})$ denotes the four-dimensional Planck mass, and $M_{\text{EW}}(\sim \text{TeV})$ the electroweak scale.

To resolve this problem, in 1998 brane world scenarios were proposed and have been extensively studied since then [2]. In particular, Arkani-Hamed et al (ADD) [3] pointed out that the extra dimensions need not necessarily be small and may even be on the scale of millimeters [4]. For a typical size $R$ of the extra dimensions, the $D$-dimensional fundamental Planck mass $M_D$ is related to $M_{\text{pl}}$ by $M_D = (M_{\text{pl}}^2/R^{D-4})^{1/(D-2)}$. Clearly, for any given extra dimensions ($D \geq 6$), if $R$ is large enough, $M_D$ can be as low as the electroweak scale. In a different model, Randall and Sundrum (RS) [5] showed that if the self-gravity of the brane is included, gravitational effects can be localized near the Planck (invisible) brane at low energy and the 4D Newtonian gravity is reproduced. In this model, often referred to as the RS1 model, the extra dimensions are not homogeneous, but warped. The mechanism to solve the hierarchy problem is different [5]. Instead of using large extra dimensions, RS used the warped factor, for which the mass $m_0$ measured on the Planck brane is related to the mass $m$ measured on the visible (TeV) brane by $m = e^{-ky_c}m_0$, where $e^{-ky_c}$ is the warped factor. Clearly, by properly choosing the distance $y_c$ between the two branes, one can lower $m$ to the order of TeV, even $m_0$ is still of the order of $M_{\text{pl}}$. A remarkable feature of the RS1 and ADD models is that, after ten years of its invention, no evidence of tension with current observations or tests of gravitational phenomena has been found [2]. More important, they are experimentally testable at high-energy particle colliders and in particular at the newly-built Large Hadron Collider (LHC) [6].

Another important problem is the late cosmic acceleration of the universe, first observed from Type Ia supernovae measurements [7], and confirmed subsequently by more detailed studies of supernovae and independent evidence from the cosmic microwave background radiation (CMB) and large scale structure. While the late cosmic acceleration of the universe is now well established [8], the underlying physics remains a complete mystery [9]. The nature and origin of the acceleration have profound implications, and understanding them is one of the biggest challenges of modern cosmology [10].

Various models have been proposed [9], which can be divided into two classes: one is constructed within general relativity (GR), such as quintessence [11] and a tiny positive cosmological constant (CC), and the other is from modified theories of gravity, such as the DGP brane model [12] and the $f(R)$ nonlinear gravity [13]. However, it is fair to say that so far no convincing model has been constructed, yet. A tiny CC might be one of the simplest resolutions of the crisis, and is consistent with all observations carried out so far [3,6]. It is exactly because of this triumph that, together with an early inflationary and subsequently radiation and cold dark matter dominated periods, this model has been considered as the current “standard model” of cosmology.

Brane world scenarios have been intensively studied in the last decade or so, and continuously been one of the most active frontiers of physics. As a matter of fact, the field has been so extensively studied that it is very difficult to provide a list of un-biased references, so we shall simply refer readers to the review articles [2]. However, most of the work carried out so far is phenomenological in nature. Therefore, it is very important to consider these models in a “bigger picture.” At the present, string/M theory is our best bet for a consistent quantum theory of gravity, so it is natural to embed such models...
into string/M theory. In fact, the invention of branes \[13\] was originally motivated by string/M theory \[14\]. However, relatively much less efforts have been devoted to such studies \[13\]. One of the main reasons is that such an embedding is non-trivial and frequently hampered by the complexity of string/M theory. In such a setup, two fundamental issues are: (a) the stability of the large number of moduli resulting from the compactification; and (b) the localization of gravity on the branes. In the Horava-Witten (HW) heterotic M theory, the moduli are naturally split into two families, depending on the type of harmonic form, \((1,1)\) and \((2,1)\). Various mechanisms to stabilize these moduli have been proposed. In particular, one may use the internal fluxes, introduced by Kachru et al. (KKLT) originally for the moduli stabilization of type IIB string \[10\], to stabilize the \((2,1)\) moduli. Brane stretching between the two boundaries, on the other hand, can fix the \((1,1)\) moduli \[17\], while gaugino condensation may fix the volume of the 3 Calabi-Yau manifold \[18\]. In addition, to fix the distance (radion) between the two branes, Goldberger-Wise (GW) mechanism \[19\] and Casimir energy contributions \[20\] have also been widely used.

In the past couple of years, we have studied orbifold branes in the framework of the 11D HW heterotic M Theory on \(S^1/Z_2\) \[14\], developed by Lukas et al \[15\] by compactifying the 11D HW theory on a 6D Calabi-Yau space \[21, 22\]. and orbifold branes in the framework of (type II) string theory \[23, 26\], as well as orbifold branes in the RS setup \[27, 28\]. In particular, we investigated in detail the three important issues: (i) the radion stability and radion mass; (ii) the localization of gravity and high-order Yukawa corrections; and (iii) the hierarchy problem.

**Radion Stability and masses**: Using the Goldberger and Wise (GW) mechanism \[19\], we found that the radion is stable. Fig. 1 shows the radion potential in the case of HW heterotic M Theory \[22\]. A similar result was also obtained in the framework of string theory \[24\].

The radion mass is given by \[22\]

\[
m_{\phi} = \left(\frac{1}{2} \frac{\partial^2 V^{\phi}}{\partial \phi^2}\right)^{1/2} \approx \left(\frac{M_{11}}{M_{pl}}\right)^{\alpha_1} \left(\frac{R}{l_{pl}}\right)^{\alpha_2} M_{pl}, \tag{1}
\]

where \(\alpha_1 = 3\) and \(\alpha_2 = 2\). For \(M \simeq 1 \text{ TeV}\) and \(R \simeq 10^{-22} \text{ m}\), we find that \(m_{\phi} \simeq 0.1 \text{ GeV}\). In the framework of string, the radion mass is also given by Eq. (1) but now with \(\alpha_1 = 8/3\) and \(\alpha_2 = 5/3\) \[22\].

**Localization of Gravity and 4D Effective Newtonian Potential**: In contrast to the RS1 model in which the gravity is localized on the invisible brane \[3\], we find that the gravity in the framework of both string \[24\] and M theory \[22\] is localized on the visible brane, because in the present setup the warped factor increases as one approaches the visible brane from the invisible one. The spectrum of the gravitational KK modes is discrete and the corresponding masses are given by \[22, 24, 26\]

\[
m_n \simeq n \pi \left(\frac{l_{pl}}{y_c}\right) M_{pl}, \tag{2}
\]

where \(y_c\) is the distance between the two branes. Thus, for \(y_c \simeq 10^{-19} \text{ m}\) we have \(m_1 \simeq 1 \text{ TeV}\).

The 4D Newtonian potential, on the other hand, takes the form,

\[
U(r) = G_4 M_1 M_2 r \left(1 + \frac{M_2^2}{M_1^2} \sum_{n=1}^{\infty} e^{-m_n r} |\psi_n(z_c)|^2 \right), \tag{3}
\]

where \(\psi_n(y_c) \simeq \sqrt{2/y_c}\). Clearly, for \(y_c \simeq 10^{-19} \text{ m}\) and \(r \simeq 10 \mu \text{m}\), the high order corrections to the 4D Newtonian potential are exponentially suppressed, and can be safely neglected.

**The hierarchy problem**: This problem can also be addressed in our current setups, but the mechanism is a combination of the ADD large extra dimension \[3\] and the RS warped factor mechanisms \[8\], together with the brane tension coupling scenario \[29\], and the 4D Newtonian constant is given by \[22, 24\],

\[
G_N = \frac{g_b}{48\pi M^2 R^2}, \tag{4}
\]

where \(g_b\) denotes the tension of the brane, \((\beta_1, \beta_2) = (18, 12)\) for the HW heterotic M-Theory \[22\], and \((\beta_1, \beta_2) = (16, 10)\) for the string theory \[24\].

It is interesting to note that the 4D effective cosmological constant can be cast in the form \[21, 24\]

\[
\rho_{\Lambda} = \frac{\Lambda_4}{8\pi G_4} = 3 \left(\frac{M}{M_{pl}}\right)^{\beta_1} \left(\frac{R}{l_{pl}}\right)^{\beta_2} M_{pl}^4, \tag{5}
\]

For \(R \simeq 10^{-22} \text{ m}\) and \(M_{10} \simeq 1 \text{ TeV}\), in both cases we have \(\rho_{\Lambda} \sim \rho_{\Lambda, ob} \simeq 10^{-47} \text{ GeV}^4\).

**Cosmological Applications**: Applying such setups to cosmology, we found the generalized Friedmann-like equations on each of the two orbifold branes, and showed that the late acceleration of the universe is transient, due
to the interaction of the bulk and the branes. Fig. 2 shows the future evolution of the acceleration of the universe in the framework of string [23]. Similar result was also obtained in M-Theory [21].

Bouncing universe can be also constructed. In particular, in the setup of M-Theory [21,22], the generalized Friedmann equations for moving branes in a five-dimensional bulk with a 4-dimensional Poincare symmetry take the forms,

\[ H^2 = \frac{2\pi G}{3\rho_\Lambda} (\rho + \tau_\phi + 2\rho_\Lambda)^2 - \frac{1}{25L^2a^2}, \quad (6) \]

\[ \dot{\rho} + \dot{\tau}_\phi + 3H(\rho + p) = -6H(2\rho_\Lambda + \rho + \tau_\phi), \quad (7) \]

where \( L \) is a constant, and \( \tau_\phi \equiv 6\alpha\kappa_5^2 e^{-\phi} + V_4(\phi) \), with \( V_4(\phi) \) denoting the potential on the brane. Clearly, at the early time the last term in the right-hand side of Eq. (6) dominates, and there always exists a non-zero minimum \( a_i > 0 \) at which \( H(a_i) = 0 \). For \( a < a_i \) the motion is forbidden, whereby a bouncing universe is resulted. Similar results can be also obtained in the setup of string theory [23,24].

Concluding Remarks: With all these remarkable features, it is very desirable to investigate other aspects of these models. In particular, in our previous studies, we have not addressed the issue of supersymmetry. Working with TeV scale, a distinctive feature is the possibilities of finding observational signals to LHC [6]. Meanwhile, in the framework of brane cosmology, significant deviations come from the early universe [2]. To explain the late cosmic acceleration of the universe, various models have been proposed [9]. While different models can give the same late time accelerated expansion, the growth of matter perturbation they produce usually differ [30]. Recently, the use of the growth rate of matter perturbation in addition to the expansion history of the Universe to differentiate dark energy models and modified gravity attracted much attention. Therefore, it would be very important to study perturbations of the cosmological models in our string/M theory setups.

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