X-ray detection with arbitrarily low radiation - A stationary implementation of semi-counterfactual interaction-free imaging

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Interaction-free measurement makes it possible to detect objects without interacting with them. Applying the approach to X-ray imaging will have the benefit that the object being imaged will not be exposed to radiation. However, existing counterfactual and semi-counterfactual interaction-free measurement schemes are all designed for visible light. They require the use of (1) high efficiency reflectors for normal incidence, (2) fast responding switchable mirrors and polarization rotators, and (3) optical delay. For X-ray, all these devices are still technical challenges today. Here we manage to evade these three difficulties by proposing a stationary implementation scheme, in which all devices are currently available. Thus it paves the way towards practical application of interaction-free X-ray imaging.

I. INTRODUCTION

X-ray plays an important role in everyday life. Besides medical examinations, it is also widely used in security checks in airports, railway stations, and other public venues. However, it is well-known that an excessive amount of X-ray radiation is harmful to health.

In 1993, Elitzur and Vaidman found that quantum mechanics enables a fascinating quantum effect called interaction-free measurement (IFM) [1]. For example, when putting a light-triggered bomb in one of the two arms of a Mach-Zehnder interferometer, there will be a non-trivial probability for detecting the existence of the bomb via the interference pattern without activating the bomb. This result highlights yet another counterfactual phenomenon that absents in the classical world. But from a practical point of view there is still much room to improve, because in the original setup in Ref. [1], the maximum probability for detecting the bomb without activating it is merely 1/2. With the rest probability 1/2 the bomb will explode, which is too danger for real-life security checks. Later, Kwiat et al. [2] proposed a method to raise the fraction of interaction-free measurement arbitrarily close to 1, but did not provide much details on the experimental implementation.

In 2009, basing on similar ideas, Noh proposed a more ingenious setup of the interferometer and applied it to quantum key distribution (QKD) [3]. It immediately caught great interests, most of which focused on whether there can be fully counterfactual protocols for transferring classical [4-22] or quantum information [23-27].

While there is no doubt that the counterfactuality of the transferred information has deep philosophical significance, here we are more interested on its practical application on interaction-free imaging. The apparatuses in Refs. [2, 4-6] seem to fit this purpose well, because in these proposals, the probability for photons to be actually detected at Bob’s side can all be made arbitrarily close to 1. The proposal in Ref. [6] is also quite feasible, that a proof-of-principle experiment was already performed [28]. Very recently, an interaction-free ghost-imaging experiment was reported [29], showing that it is indeed advantageous to develop the imaging technology using the counterfactual effect. However, all the above works were based on visible light. It is not straightforward to adapt them for X-ray, because many critical experimental devices in these works, e.g., fast responding switchable mirrors and switchable polarization rotators, optical delay, and even regular mirrors for normal incidence light, are all currently unavailable for X-ray.

In this paper, we develop methods to evade these problems, and propose an apparatus for X-ray imaging in which the radiation received by the object being imaged can be made arbitrarily close to zero. All devices in this apparatus are based on the X-ray technology available today. What we need is very precise adjustments. While such adjustments may still be somehow challenging in practice, we believe that the hardest parts were already solved so that radiation-free X-ray imaging is no longer out-of-reach.

In the next section, we will review briefly the apparatuses in Refs. [4-6], which form the base of our proposal. Then in section III, we elaborate in details why they and other existing schemes cannot be applied straightforward to X-ray. Our approach will be presented in section IV. Further adjustment and development issues will be discussed in sections V and VI. Finally, the applications of our result will be outlined in section VII.

II. AN EXISTING SCHEME FOR VISIBLE LIGHT

A. Mechanism of the imaging

To understand the mechanism of our proposal, let us start from the apparatus illustrated in Fig.1. It is much the same as those in Refs. [4, 5] which were designed for counterfactual communications, except that we discarded some of Bob’s devices which are irrelevant to the purpose.
of imaging. This apparatus can accomplish counterfac-
tual interaction-free imaging using visible light, as elab-
orated below.

When each photon is emitted from the source, the opti-
cal circulator $C$ is set correspondingly so that the photon
from the left can pass straight to the right. Then it is split
by the beam splitters $BS_i$ ($i = 1, \ldots, N$) into wave
packets that travel along different paths $x_1y_1$ ($i = 1, \ldots, N$),
$x_iy_{i+1}$ ($i = 1, \ldots, N - 1$) and $x_Nz$, and are reflected back
by the Faraday mirrors $FM_i$ ($i = 1, \ldots, N$) and $FM_{Bob}$, re-
spectively. Fig.1 showed the case for $N = 4$. When
$N = 1$ it reduced to the counterfactual QKD apparatus
in Ref. [3] (except that Bob’s devices in Ref. [3] were
simplified into a single Faraday mirror $FM_{Bob}$).

The last optical delay $OD_N$ is carefully adjusted to en-
sure that the optical path length $x_{N^2}$ equals to $x_Nz$, so
that they form a balanced Michelson-type interferometry.
Consequently, when no object presented before $FM_{Bob}$,
a photon reached the last beam splitter $BS_N$ from the
left will be split into two wave packets that travel along
the paths $x_Ny_N$ and $x_Nz$, respectively. After being re-
lected by $FM_N$ and $FM_{Bob}$, the two wave packets
will recombine at $BS_N$ and interfere. Since $x_{N^2} = x_Nz$,
constructive interference will occur so that the combined
wave packet will always travel along the path $x_Nx_{N-1}$
and will not be detected by $D_1^{(N)}$. Similarly, for any
$i = 1, \ldots, N - 1$, $OD_i$ is adjusted so that the optical path
lengths satisfy $x_iy_i = x_Nz$, which ensures that con-
structive interference occurs at every $BS_i$. Therefore, after
being split and recombined at all the beam splitters, the
photon will finally return to the path from $BS_1$ to $C$ with
certainty. If at this time the optical circulator $C$ is set
to redirect the path to $D_0$, then the photon will always be
detected by $D_0$ when no object presented before $FM_{Bob}$.

On the contrary, if there is a non-reflective object
blocks the path $x_Nz$, any wave packet enters $x_Nz$ will
not return from $FM_{Bob}$, so that the constructive inter-
ference at $BS_N$ will no longer be guaranteed. As a result,
$D_1^{(N)}$ could click with a non-vanishing probability. For
the same reason, the constructive interference at other
$BS_i$ ($i = 1, \ldots, N - 1$) will also be destroyed, so that the
photon may be detected by other $D_1^{(i)}$ too.

Consequently, once any of $D_1^{(i)}$ ($i = 1, \ldots, N$) clicks, we
know for sure that there is an object blocking the path
$FM_{Bob}$. By moving the object in the plane perpen-
dicular to the path $x_Nz$ while constantly sending pho-
tons from the source and recording whether any of $D_1^{(i)}$
($i = 1, \ldots, N$) clicks, the apparatus can obtain an image
of the object.

**B. Counterfactuality**

The most intriguing feature of this imaging process is
that we can learn the presence of the object even when
it does not interact with the photons. This is because
in the above analysis, we merely require the object to
be non-reflective. Other than that, it can be anything,
even if it is another detector or something that absorbs
photons. We know that the object exists when we finds
any of $D_1^{(i)}$ clicks. In this case, it is clear that the photon
is not detected or absorbed by the object, or has any
energy-related interaction with the object. Otherwise,
the same photon should no longer be able to make $D_1^{(i)}$
click. This is why the process is called interaction-free
[1] or counterfactual [3].

Note that when no object presented and the photon is
detected by $D_0$, the process is no longer counterfactual
[3]. Also, if a photon left a weak trace in the path $x_Nz$
(e.g., in the apparatus proposed in Ref. [6]), it is arguable
whether we can still call it counterfactual even when the

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**FIG. 1:** The apparatus for counterfactual interaction-free imaging using visible light. Photons emitted from the source will pass the optical circulator $C$ to the right, then split by the beam splitters $BS_i$ ($i = 1, \ldots, N$), and reflected back by the Faraday mirrors $FM_i$ ($i = 1, \ldots, N$) and $FM_{Bob}$. Calibration of the optical path length is accomplished by adjusting the optical delays $OD_i$ ($i = 1, \ldots, N$). The photons are finally detected by the detectors $D_0$ and $D_1^{(i)}$ ($i = 1, \ldots, N$).
photon is finally found by \( D^{(i)} \) \([7, 8, 11, 12, 14–16, 22]\). Thus it may be safer to call the whole process “semi-counterfactual”. Fortunately, these downsides are not important for our propose of imaging, because as long as the energy of the photon is not absorbed by the object, we can be sure that it is harmless to the object.

For example, when imaging using X-ray, as long as the X-ray photons are detected by \( D^{(i)}_1 \), it naturally implies that they were not “detected” by the object being imaged. Therefore, even if the object is a film, it will not be exposed. Similarly, if the object is consisted of biological cells, they will not receive the energy of the X-ray radiation in traditional X-ray imaging are avoided.

C. Performance

Now let us calculate the probabilities for this interaction-free imaging to occur. Let \( t_i \) and \( r_i = 1 - t_i \) denote the transmissivity and reflectivity of \( BS_i \) \((i = 1, ..., N)\), respectively. When there is an object blocking the path \( xNz \), detector \( D_0 \) will click with probability

\[
p_0 = r_1r_2 \cdots r_N \left[ 1 + (t_1t_2 \cdots t_N) \right]^2
= r_1^2 + \sum_{j=2}^{N} \left( r_j \prod_{i=1}^{j-1} t_i \right)^2.
\]

The probability for the photon to hit the object so that none of \( D_0 \) and \( D^{(i)}_1 \) \((i = 1, ..., N)\) clicks is

\[
p_2 = \prod_{i=1}^{N} t_i.
\]

Thus the probability for the photon to be detected by any of \( D^{(i)}_1 \) \((i = 1, ..., N)\) is

\[
p_1 = 1 - p_0 - p_2 = 1 - \left( r_1^2 + \sum_{j=2}^{N} \left( r_j \prod_{i=1}^{j-1} t_i \right)^2 \right) - \prod_{i=1}^{N} t_i.
\]

which is the probability that we learn the presence of the object while the photon is not absorbed by the object, i.e., the interaction-free imaging occurs.

For simplicity, assume that all \( BS_i \) \((i = 1, ..., N)\) have the equal transmissivity \( t \) and reflectivity \( r = 1 - t \). Then

\[
p_0 = (1 - t)^2 \sum_{j=1}^{N} t^{2(j-1)},
\]

\[
p_1 = 1 - (1 - t)^2 \sum_{j=1}^{N} t^{2(j-1)} - t^N.
\]

When \( N \to \infty \), there will be \( p_2 = 0 \) while \( p_1 = 2/(1 + 1/t) \) which remains non-trivial. For example, in the case \( t = r = 1/2 \), we have \( p_0 = 1/3 \) and \( p_1 = 2/3 \). That is, if the apparatus can be applied to X-ray imaging, we can detect the presence of the object with a non-trivial probability \( p_1 \), while the radiation that the object receives can be made arbitrarily small by increasing \( N \).

The efficiency of successful imaging can be further improved by sending more than one photon, and we conclude that the object is not presented only if all photons are detected by \( D_0 \). For example, suppose that we send \( m = 10 \) photons. Then the error rate that we mistakenly conclude that the object is not presented while it actually exists is merely

\[
\epsilon = p_0^m = \left( \frac{1}{3} \right)^{10} \approx 1.7 \times 10^{-5}.
\]

Meanwhile, the radiation that the object receives still remains very low even for finite \( N \), e.g., when \( N = 20 \), the probability that at least one of the photon is absorbed by the object is

\[
P_2^{(m,N)} = 1 - (1 - p_2)^m
= 1 - \left( 1 - \left( \frac{1}{2} \right)^{20} \right)^{10} \approx 9.5 \times 10^{-6}.
\]

Though we assumed above that all \( BS_i \) have the equal transmissivity \( t = t \), it is easy to see that \( \epsilon \) and \( P_2^{(m,N)} \) can remain low even if \( t_i \) varies. Therefore, in practice all \( BS_i \) need not to be perfectly identical.

III. DIFFICULTIES FOR X-RAY

Unfortunately, the above apparatus was designed for visible light, and is hard to be applied straight forward to X-ray due to the following difficulties.

(1) A relatively trivial but unavoidable problem is that mirrors for normal incidence (as shown in Fig.2(a)) are needed. While they are common for visible light, economic X-ray mirror with high reflectivity is not available (especially for hard X-ray, which cannot be efficiently reflected unless at grazing incidence as shown in Fig.2(b)). Multi-layer X-ray mirror can reach a reflectivity of 70% and may be more. But the photons reflected by different layers will have different optical path length and return to the BS at different time. Thus it will be hard for the photons from the other arm of the interferometer to match their arrival time at the BS. Therefore, this kind of X-ray mirrors does not meet the need of our purpose.

(2) The most difficult part is that the apparatus requires \( N \) sets of \( BS_i \) and \( OD_j \), which is impractical and costly when \( N \) is large. In Ref. [6] this is solved by using a nested interferometer. But to control when to
allow the photons to enter or leave the nested interferometer, the proposal requires high-speed switchable mirrors and switchable polarization rotators, whose status needs to be switched within nanoseconds. Such devices are not only unavailable for X-ray, but also a technical challenge for visible light. Therefore, even though Ref. [28] performed a brilliant proof-of-principle implementation of the proposal in Ref. [6], the switchable mirrors and polarization rotators were not faithfully implemented. Instead, half-mirrors were used strategically as replacements. Consequently, the photons stand a considerable probability to be lost every time they meet the half-mirror, making the density of useful signals very weak, so that the implementation cannot work for large \( N \) even for visible light.

(3) Optical delay is required in the apparatuses in Refs. [4] [28]. For visible light, delay line made of optical fiber is very handy. But there is no such fiber for X-ray either.

IV. OUR SOLUTION

To circumvent these difficulties, we propose an apparatus for X-ray interaction-free imaging, as shown in Fig.3. In the supplemental material [30] we also provide a self-executable program, which gives a 3-D view of Fig.3 that can be zoomed and rotated. Our proposal needs only an X-ray source, two detectors, one beam splitter, and some X-ray mirrors. No switchable mirror nor polarization rotator required. To understand how it works, let us explain it part by part.

(1) Reflection Unit:

Finding a substitution for mirrors for normal incidence is relatively easy. As shown in Fig.2(c), we can use a series of X-ray mirrors to turn the direction of the X-ray beam little by little, until it is 180° reversed. Grazing incidence instead of normal incidence occurs at each mirror, thus it is completely within the capability of currently available technology [31]. When viewing along the \( z \) axis, we can see that the entire unit in Fig.2(c) functions very similarly to a mirror for normal incidence like the one in Fig.2(a). Thus the difficulty of lacking X-ray mirrors for normal incidence is solved, as long as we let the outgoing and returning X-ray beams actually travel in two different planes, as they do in Fig.3.

Note that for most materials, optimal reflectivity for X-ray generally requires a rather low grazing angle. For example, when using the device in Ref. [31] as a mirror, optimal reflection occurs at a grazing angle around 8.5°. Thus the actual number of mirrors in the reflection unit needs to be higher. But for simplicity, we only drew three mirrors in Fig.2(c) and Reflection Units Y and Z in Fig.3. It is also important to note that the actual number of mirrors in Reflection Unit Y in Fig.3 has to be odd. This is because when many beams enter Reflection Unit in parallel, an odd number of mirrors can ensure that the upper (lower) beam will still be the upper (lower) one after being reflected by all mirrors.

(2) Beam Splitter (BS) Unit:

This unit contains a single X-ray BS (which is also currently available [31] [32]) and several X-ray mirrors. Note that like Reflection Unit, to turn the direction of the X-ray beams in circles, the actual number of mirrors in BS Unit may also need to be higher, while we only drew four in Figs. 3-5 for simplicity.

The outgoing (from the source to the object) and returning (from the object to the detectors) X-ray beams travel along the upper and lower planes, respectively, as shown in Fig.3. The paths in the two planes are much the same, except for these from BS to \( D_1 \), which are for the returning beam in the lower plane only. For clarity, we gives a top view of BS Unit in Fig.4. In the upper plane, the X-ray photon coming from the source along the path shown in purple meets BS at point \( x_1 \), and was split into two wave packets (shown as the red lines). One packet goes upwards to Delay Unit. The other goes to the right to point \( b_1 \) where it meets an X-ray mirror, then points \( c_1, d_1, a_1 \), until it reaches BS again at point \( x_2 \). Then it was split for the second time into two wave packets (shown as the pink lines). Again, one packet goes upwards to Delay Unit, and the other goes to the right to points \( b_2 \), then points \( c_2, d_2, a_2 \), until it reaches BS again at point \( x_3 \) and was split into two wave packets (shown as the orange lines) which go to Delay Unit or to points \( b_3 \rightarrow c_3 \rightarrow d_3 \rightarrow a_3 \), respectively. The latter hits BS at point \( x_4 \), and was split into two wave packets (shown as the yellow lines). One goes to Delay Unit as before. But the other packet passes Mirror \( B \) by the edge and goes straight to the right to where the object being imaged would be placed. That is, by making the X-ray hits the same BS for 4 times, we manage to implement the \( N = 4 \) case in Fig.1 without actually using four BS. The method for further increasing \( N \) will be discussed later.

The angle of Mirror \( A \) should be precisely adjusted so that the path \( a_1 \rightarrow x_2 \) is parallel to the purple path. Then it automatically guarantees that all the rest paths will always be parallel to those in the previous circle, and the length satisfies

\[
x_1b_1c_1d_1a_1x_2 = x_2b_2c_2d_2a_2x_3 = x_3b_3c_3d_3a_3x_4.
\]

The paths for the returning beam in the lower plane are basically in the reverse direction. The path returns from Reflection Unit Z will rejoin the one returns from
FIG. 3: Our proposed apparatus for X-ray interaction-free imaging. The blue “glasses” in BS Unit, Delay Unit and Reflection Units Y and Z represent X-ray mirrors. The green one in BS Unit is an X-ray beam splitter. $D_0$ and $D_1$ are detectors. The red, pink, orange, and yellow lines represent the beams that split by BS in the 1st, 2nd, 3rd, and 4th round, respectively.

FIG. 4: Top view of BS Unit.

Delay Unit (both are shown in yellow in Fig.4) at point $x_4$. Delay Unit is strategically designed (see point (3) below) so that when no object presented before Reflection Unit Z, the wave packets returned from the above two paths will have constructive interference at BS so that the combined wave packet will always travel along the orange line to point $a_3$. After being reflected to points $d_3$, $c_3$ and $b_3$, it rejoins the wave packet returns from Delay Unit along the orange line at point $x_3$. Constructive interference occurs at BS again and the combined wave packet will always travel along the pink line to point $a_2$ then $d_2$, $c_2$, $b_2$ and $x_2$. Similar process continues, until finally the two red paths rejoin at point $x_1$ and the photon will always travel along the purple path to the left and make detector $D_0$ click. On the other hand, when there is an object blocking the yellow path from Reflection Unit Z, no wave packet can return from this path so that constructive interference can no longer be guaranteed at points $x_4$, $x_3$, $x_2$ and $x_1$. Consequently, detector $D_1$ stands a non-vanishing probability to click, thus revealing the existence of the object.

This idea of reusing a single BS to increase $N$ looks similar to the proposal in Ref. [6]. But there is a critical difference. In our proposal, each time the X-ray photon meets the BS, the split paths do not overlap with the previous ones. Instead, they have a small amount of divergence. After hitting the BS for $N$ times, the divergence sums up to an extent that the outgoing (returning) beam will pass the edge of Mirror B (Mirror A) of BS Unit, and travel towards Reflection Unit Z (detector $D_0$) without being reflected to BS again.

This difference makes our apparatus completely stationary. It no longer requires switchable mirror and switchable polarization rotator, which are currently unavailable for X-ray. Thus it avoids both the complexity problem of Ref. [6] and the efficiency problem of Ref. [28] mentioned in the previous section.

(3) Delay Unit:
Fig.5 shows the top view of Delay Unit. It also features
paths from BS Unit to pass Mirror see that such a shape allows the red/pink/orange/yellow Fig.6. Taking the outgoing beams as an example, we can (for the outgoing/returning X-ray beams to/from Re- to ensure the reflected paths parallel to each other. In this case, it is easy to prove that the length of the paths (3L, 2L, L and 0, respectively. Thus, with a single Delay Unit, we manage to create different amount of delay for the paths that left BS at different time.

Finally, by adjusting the distance between these mirrors, we can have $L = x_1b_1c_1d_1a_1x_2$ (where the latter also includes the extra wave path length caused by BS). Meanwhile, the distance between Delay Unit and Reflection Unit Y should be adjusted so that the sum of the path length between point $x_4$ in Fig.4 and $a'_4$ in Fig.5 (average over the outgoing and returning paths) plus the length between $a'_4$ and Reflection Unit Y equals to the one between $x_4$ and Reflection Unit Z. Then we can see that when no object is presented before Reflection Unit Z, when the wave packets returned from Reflection Unit Z reach BS at points $x_4$, $x_3$, $x_2$ and $x_1$ along the yellow/orange/pink/red paths at different time, respectively, the wave packets returned from Delay Unit will also reach these points at the corresponding times. Thus constructive interference is always guaranteed at these points, so that the apparatus in Fig.3 has exactly the same performance as that in Fig.1, with much less devices while also applicable to X-ray.

Note that in Fig.3 and Fig.4, the grazing angle on BS is drawn as $45^\circ$. This is true when BS is a half-silvered mirror for visible light. But for X-ray BS with equal transmissivity and reflectivity, the actual angle could be different [31, 33]. In this case, the angles of the plane of BS as well as the paths between BS Unit and Delay Unit in Fig.3 and Fig.4 need to be modified. Also, when adjusting the mirrors in Delay Unit, we should count in the extra path length difference that it causes. That is, suppose that the actual grazing angle on BS is $\alpha$, then we should take $L \equiv \alpha_1b_1c_1d_1a_2 = x_1b_1c_1d_1a_1x_2 - s_b (\cot \alpha - \cot 45^\circ)$, where $s_b$ is the spacing between the paths that leave BS.
V. METHODS FOR INCREASING \( N \)

Eqs. (2) and (4) imply that the radiation received by the object drops as \( N \) increases. There are two methods to increase \( N \) so that the wave packets can bounce between the mirrors in BS Unit and Delay Unit for more rounds before they exist. (A) Move Mirror \( A \) (Mirror \( D' \)) along the negative \( y \) axis (the negative \( x \) axis) while keeping its angle unchanged. This can reduce the spacing between the parallel paths. (B) Increasing the size of the whole apparatus (including the area of the mirrors and the distance between them) while keeping the spacing between the paths unchanged.

Method (A) seems harder because it requires the X-ray mirrors in BS Unit and Delay Unit to be very precisely crafted, so that they remain perfectly flat and thin even on the area close to the edge. Otherwise, when the paths have a very narrow spacing, there will inevitably be incident photons that reach the edge area of the mirrors and be reflected or refracted to an unwanted direction. Thus we believe that method (B) is more practical.

Either way, our proposal has the advantage that there is no need to add additional device in our design to increase \( N \). All we need is higher precision in the manufacturing and adjustment of the experimental devices.

VI. FEASIBILITY DISCUSSIONS

In practice, as the X-ray beam passes BS and mirrors so many times, the absorption will be extremely high so that the final signal will be very weak. Fortunately, it is worth noting that our scheme does not require single photon source. Strong X-ray pulse can be used instead. As each photon obeys the analyses of the apparatuses in Fig.1 and Fig.3, the counterfactual character still remains for strong pulse. That is, any X-ray photon survived through all BS and mirrors still has an extremely low probability to be detected by and interact with the object that blocks the path to Reflection Unit Z. Thus, we can feel free to increase the intensity of the input X-ray signal to improve the visibility of the imaging, without needing to worry that the object being imaged will be exposed to strong radiation.

Meanwhile, when none of \( D_0 \) and \( D_1 \) clicks after a photon was sent, it is hard to tell whether it was absorbed by BS and mirrors or the object being imaged. But note that we only take the clicking of \( D_1 \) as the sign of the presence of the object, while the lost photons should not be counted. Therefore, the absorption from BS and mirrors will not hurt the result of the imaging.

Of course, to make the scheme completely practical, there will still be many technical details to worry, e.g., how to make the whole surface of every X-ray mirror perfectly flat and precisely positioned so that the reflected paths can be rigorously parallel as designed. Our current work is only the first step towards practical application by solving some of the most difficult problems, such as the lack of switchable polarization rotators, mirrors and delay devices for X-ray. It is expected to make proof-of-principle experiment of our scheme using visible light first, before really applying it to X-ray.

It is also worth studying whether the X-ray ghost-imaging method \( \cite{33, 34} \) can be combined with our proposal to further increase the signal-to-noise ratio.

VII. APPLICATIONS AND SIGNIFICANCE

Once our proposed apparatus in Fig.3 can be turned into reality, it can have wide applications on many fields. Besides detecting light-triggered bombs without making them explode \( \cite{2} \), a more practical application with obvious advantage is medical examination for human body, where we can be benefitted from the counterfactual feature of the apparatus that the X-ray radiation absorbed by the object being imaged can be made arbitrarily small with the increase of \( N \). The same feature also makes the apparatus a healthy choice for security gates at public venues. In principle, our proposal also enables stealth radars that are invisible to anti-radar missiles, since they merely emit an arbitrarily small amount of radiation.

Our method can also apply for imaging phase-only objects. These objects are semitransparent, such that a photon passing through it will acquire a phase, even though there is no energy exchange. It was shown that any kind of coincidence imaging technique using a bucket detector in the test arm is incapable of imaging such objects, whether a classical or quantum source is employed \( \cite{27} \). But in our apparatus, once the phase of the wave packet returned from Reflection Unit Z was altered, constructive interference on BS will no longer be guaranteed. Thus \( D_1 \) stands a non-trivial probability to click. Sending many photons for imaging each single position will then reveal the presence of the object. Note that it is arguable whether such a case can still be considered interaction-free due to the phase change \( \cite{2} \). We believe that it is still better than traditional X-ray examination because the energy of X-ray photons are not absorbed by the phase-only object. But it is better to leave it to medical researches to decide whether the phase effect will do harm to the health of living bodies.

The apparatus in Fig.3 also works for visible light. Such counterfactual imaging using visible light can have advantage when photographing ancient arts where shining a light directly may cause damage, as pointed out in Ref. \( \cite{28} \). It can even image unexposed photographic films (maybe a spy will like it).

Also, in practical quantum cryptography, our apparatus makes the conception in Refs. \( \cite{3, 4} \) come true, so that it can serve as a hacking device against specific QKD \( \cite{3} \) and other protocols \( \cite{35} \). In these protocols, the participants encode their secret bits by either measuring the photon they received or reflecting the photon back. Using our apparatus, an eavesdropper can detect whether a participant is placing a reflector on the transmission
channel or not. The counterfactual feature ensures that the probability for detecting this eavesdropping can be made arbitrarily small.

Comparing with previous practical proposal for visible light [6], our scheme has the advantage that all devices are stationary. That is, no need to switch the mirrors and polarization rotators on and off. Moreover, in Ref. [6] the path to the object (like our $x_N$ in Fig. 1) needs to be reused for many times. Thus it has the shortcoming pointed out in the 3rd paragraph of page 5 of Ref. [19], that the method “requires a promise ... that either a particular place is empty at two times or it is blocked at two times. It does not achieve the dual task to the original IFM of finding an object at a particular place at a particular time without the particles being there.” Our proposal does not have this shortcoming because the wave packet of the photon meets the object only once.

If our scheme is expanded for transferring a quantum state following the idea in Ref. [23], then it has the advantage that the transferred state is exact the quantum state at a single time moment (while the method in Ref. [23] is based on Ref. [6] so that it actually transfers the states of different time moments). Details of such an expansion could be left for future research.

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