Strain-induced quantum spin Hall effect in methyl-substituted germanane GeCH$_3$

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Quantum spin Hall (QSH) insulators exhibit a bulk insulting gap and metallic edge states characterized by nontrivial topology. We investigated the electronic structure of an isolated layer of methyl substituted germanane GeCH$_3$ by density functional calculations (DFT), and its dynamic stability by phonon dispersion calculations. Our results show that an isolated GeCH$_3$ layer has no dynamic instability, and is a QSH insulator under reasonable strain. This QSH insulator has a large enough band gap (up to 108 meV) at 12% strain. The advantageous features of this QSH insulator for practical room-temperature applications are discussed.

Recently, two-dimensional (2D) topological insulators (TIs), namely, QSH insulators$^1$–$^7$, have attracted much attention. These insulators possess a bulk insulting gap but metallic edge states. The edge states exhibit a linear dispersion near the Fermi energy giving rise to a Dirac Fermion behavior, which can lead to an efficient transport of charge carriers. The Dirac Fermions are protected by time-reversal symmetry and hence are robust against nonmagnetic defects, disorder and surface contamination$^4$–$^7$. The QSH effect was first proposed by Kane and Mele for graphene, considering the opening of a band gap at the Dirac point by spin-orbit coupling (SOC)$^1$. However, this proposal was shown to be unrealistic$^8$–$^9$, because the strength of SOC in graphene is extremely small. Subsequently, Beneviget et al. proposed that the quantum well semiconductor HgTe-CdTe would be a QSH insulator$^5$, which was soon confirmed experimentally by observing ballistic edge channels$^3$. Nevertheless, these quantum-well-based QSH insulators have a very small bulk band gap so that the QSH effect is observed only at low temperatures (below 10 K)$^5$. Besides, the HgTe-CdTe quantum wells as well as their analogues$^{10,11}$ have other serious limitations such as toxicity, difficulty of preparation, and incompatibility with current silicon-based electronic technology.

In searching for practical QSH insulators, two key ingredients are essential: (1) the materials must have a large bulk gap to ensure the realization of a QSH effect at room temperature, and (2) they must be compatible with the current silicon-based electronic technology. The latter condition narrows down our attention to the group IV honeycomb lattices. Silicene can be a nontrivial QSH insulator in its native structure, and can be observed in an experimentally accessible low temperature$^{12}$. However, up to now, no free-standing silicene has been prepared experimentally, and its QSH effect can be observed only at low temperature$^5$. Closely related to silicene in terms of compatibility and structure are germanene and stanene, which have been proposed as QSH insulators with a relative large band gap induced by spin-orbit coupling (SOC)$^{12–15}$. However, they have not yet been realized experimentally. As a stable free-standing species that is experimentally accessible, only the germanane, GeH, can be well produced$^{16,17}$. When strained, germanane can display a QSH effect$^{18}$. However, it begins to become amorphous upon annealing at 75°C and becomes completely amorphous above 175°C$^{16}$, which is undesirable because most transistors should operate up to hundreds of degrees °C. Recently, a methyl-substituted germanane, GeCH$_3$, has been synthesized$^{19}$ (Fig. 1a, b). In contrast to the case of germanane GeH, it is found that GeCH$_3$ has considerably enhanced thermal stability$^{19}$. Therefore, it is worthwhile to probe whether GeCH$_3$ has a QSH effect.

In this work, we study the electronic states of a single layer of GeCH$_3$ by density functional calculations to show that reasonable strain could induce the s-p band inversion in the GeCH$_3$ layer and makes it a QSH insulator, and that SOC plays an important role in opening a local band gap at the crossing points near the Fermi level. In the band-inverted GeCH$_3$ layer under 12% strain, the band gap is 108 meV, so that this QSH insulator can be useful in an experimentally accessible temperature regime.

**Results**

**Stability of GeCH$_3$ monolayer.** GeCH$_3$ crystallizes in a hexagonal structure in which GeCH$_3$ layers stack along the c axis$^{19}$. GeCH$_3$ is thermally stable up to 250°C, so it is quite promising as a building block for future...
Due to its large interlayer separation the electronic interaction between adjacent GeCH₃ layers is extremely weak, contrary to the case of graphite, which has a substantial interlayer overlap between the C 2pₓᵧ orbitals, and MoS₂ at the Γ point, which has a substantial interlayer overlap between the S 3pₓᵧ orbitals. The interlayer binding energy is calculated to be 5.7 meV/Å² for GeCH₃, which is even smaller than that of graphite (9.3 meV/Å²). Such weak coupling between GeCH₃ layers provides a natural cleaving plane. Thus, bulk GeCH₃ can be easily exfoliated to produce stable GeCH₃ monolayers. The stability of GeCH₃ monolayer is further confirmed by the phonon dispersion curves, presented in Fig. 1c, which shows that all branches of the phonon dispersion curves have positive frequencies, and no imaginary phonon modes are found. This confirms the stability of a GeCH₃ monolayer.

Electronic structures of GeCH₃ monolayer. As shown in Fig. 1a, b, each sp³-hybridized Ge atom is bonded to three Ge atoms and one CH₃ group such that the buckled sheet of Ge atoms is sandwiched between two sheets of CH₃ groups. Each GeCH₃ monolayer has a hexagonal unit cell, and there are two Ge atoms and two CH₃ groups per unit cell. In the optimized structure, the lattice constant is 3.954 Å with Ge-Ge = 2.415 Å and Ge-C = 1.972 Å.

The band structures calculated for a GeCH₃ monolayer without and with SOC are plotted in Fig. 2. Without SOC, the band structure has a direct band gap (1.07 eV) at the Γ point (Fig. 2a). The Ge-4s and Ge-4p states are located near Fermi level, with the Ge-4p states lying lower than the Ge-4s states. The valence band maximum (VBM) is doubly degenerate at Γ because they are derived from the Ge-4pₓᵧ states. When SOC is included, the degenerate level is split by a gap of 193 meV (Fig. 2e, f). Except for this, no significant change is caused by SOC for the band structure of a GeCH₃ monolayer; the conduction band minimum (CBM) at Γ is still occupied by the Ge-4s states while the two valence bands near the Fermi level at Γ point are still dominated by Ge-4p states. Thus, no inverted band order is observed in this 2D lattice, showing that a GeCH₃ monolayer is a trivial insulator.

Strain effects. Mechanical strain can induce the s-p band inversion and hence lead to a nontrivial topological state. For example, under external strain, the layered compound InSe becomes a three dimensional TI. In addition, strain can be readily exerted on nanosheets. Thus, one might consider if a GeCH₃ monolayer becomes a QSH insulator under mechanical strain. We probe this question by introducing 0–12% strain into a GeCH₃ monolayer maintaining all crystal symmetries and the overall honeycomb-like structure. Here we define the strain as ε = Δa/a₀, where a₀ and a = Δa + a₀ are the lattice constants of the unstrained and strained cell, respectively. The GeCH₃ lattice is expanded uniformly along both zigzag and armchair directions in the ab plane and is relaxed along the c axis. To clarify the stability of strained GeCH₃, we also calculate phonon dispersion curves of GeCH₃ under different strain. The corresponding results are shown in Fig. S1. We find that, the strain can reach to 12% without destroying the feature that imaginary phonon mode is absent in the phonon dispersion curves. This suggests that strained systems maintain their stability within the strain range of 12%.

The evolution of the band gap of a GeCH₃ monolayer as a function of the applied mechanical strain, presented in Fig. 1d, shows that with increasing the mechanical strain, the CBM is driven continuously to shift downward to the Fermi level thus significantly decreasing the energy gap. Throughout the changes under strain, it has a direct band gap with the VBM and CBM both located at the Γ point. The band...
structures of a GeCH₃ monolayer under the strain of 4%, 8% and 12% are presented in Fig. 2b–d. From Fig. 2, we can see that, with 12% strain, the GeCH₃ monolayer becomes a semimetal since the VBM and CBM is degenerate at \( \Gamma \) point, and the s-p band inversion occurs around Fermi level; for the other two cases with strain below \( \epsilon = 12\% \), the lowest-conduction band is represented by the Ge-4s states, and the highest two valence bands by the Ge-4p states. For the GeCH₃ monolayer with \( \epsilon = 12\% \), the lowest-conduction band and the highest-valence band are represented by the Ge-4p orbitals, but the next-highest-valence band by the Ge-4s states. This establishes the existence of band inversion for the GeCH₃ monolayer with \( \epsilon = 12\% \). The inversion of band orders is a strong indication for the formation of nontrivial topological phases. The s-p band order inversion in our case is caused predominantly by strain, but the SOC is still indispensable because it lifts the band degeneracy at \( \Gamma \), as shown in Fig. 2h. With SOC, a finite energy gap of 161 meV opens at \( \Gamma \) (an indirect gap of 108 meV) while the band order stays unchanged. This large SOC-induced nontrivial band gap reflects the fact that the states near the Fermi level are made of \( \sigma \) orbitals rather than \( \pi \) orbitals. The SOC-induced nontrivial energy gap is significantly greater than \( k_B T \) at room temperature (≈26 meV), so the QSH effect can be measured at room temperature. Thus GeCH₃ might have useful applications in fields such as phase-change random access memory, quantum computing and spintronics.

The topological phase transition in a GeCH₃ monolayer is further confirmed by calculating the \( Z_2 \) topological invariant before and after the band inversion. Nontrivial topological insulators can be distinguished from normal insulators by nontrivial \( Z_2 \) topological invariant associated with the “bulk” electronic structure: \( Z_2 = 1 \) characterizes a nontrivial band topology, but \( Z_2 = 0 \) a trivial band topology. All the structures of a GeCH₃ monolayer considered here possess inversion symmetry. According to the method developed by Fu and Kane, the existence of inversion symmetry allows us to calculate the \( Z_2 \) topological invariant directly from knowledge about the parity of the Bloch wave functions for all the filled bands at all the time-reversal-invariant momenta (TRIM). For a GeCH₃ monolayer, there are four TRIM points, namely, one \( \Gamma \) point and three M points in the Brillouin zone. The \( Z_2 \) invariant is determined by the quantity

\[
\delta_i = \prod_{m=1}^{N} \xi_{2m}(k_i), \text{ where } \xi_{2m}(k) = \pm 1 \text{ is the parity eigenvalue of the}
\]

Figure 2 | The band structures calculated without SOC for an isolated GeCH₃ monolayer structures with (a) 0%, (b) 4%, (c) 8% and (d) 12% strain. The band structures calculated with SOC for an isolated GeCH₃ monolayer structures with (e) 0% and (g) 12% strain. (f) and (h) show zoomed-in views of (e) and (g), respectively. The black and olive lines denote the Ge-4s and Ge-4p orbitals, respectively. The horizontal dashed lines indicate the Fermi level.
occupied band at the TRIM points $k_i$. The $Z_2$ invariant is then given by the product of all the $\delta_i$’s, $\Pi \delta_i$. For the GeCH$_3$ monolayer without strain or with strain below 8%, they have trivial band topology with the topological invariant $Z_2 = 0$. When the strain is 12%, the calculated $Z_2$ topological invariants is 1, which confirms that the strain induces a topological phase transition.

The 2D nontrivial insulating states in the GeCH$_3$ monolayer with $\varepsilon = 12\%$ should support an odd number of topologically protected Dirac-like edge states connecting the conduction and valence band edges at certain k-points. To see the nontrivial topological features of the GeCH$_3$ monolayer with $\varepsilon = 12\%$ explicitly, we performed calculations of its edge-state band structure by constructing a zigzag nanoribbon structure with symmetric edges, for which we saturate the dangling bonds of all the edge atoms by hydrogen atoms. The width of the nanoribbon structure adopted here is 8.2 nm, which is enough to separate the interaction between its two edges. The corresponding band structure of the nanoribbon plotted in Fig. 3a shows that the nontrivial topological gapless edge states connecting the band edge states and form a single Dirac cone at the $\Gamma$ point. For this nanoribbon structure, the two edges are the same, leading to two energetically degenerate Dirac cones located at the opposite edges. At a given edge, two counter-propagating edge states exhibit opposite spin-polarizations, a manifestation of the one-dimensional nature of the system. Thus, the surface-state calculation is consistent with the above parity analysis, and confirms conclusively the topologically non-trivial nature of this material.

Let us now consider how strain influences the band gap and the s-p band order. There are two factors affecting the band order; one is SOC, and the other is the strain. The latter factor has been discussed in many previous studies.\textsuperscript{15,18,22,23,27} The previous showed that the band inversion in a GeCH$_3$ monolayer can be driven by mechanical strain instead of SOC. Thus the physics of band inversion can be understood without considering SOC, while the effect of SOC is to create an energy gap afterwards. As discussed above, the valence and conduction bands away from the $\Gamma$ point are well separated, therefore, one needs to focus on the bands near the $\Gamma$ point. The relevant bands are dominated by the Ge-4s and Ge-4p$_{x,y}$ states. Due to the chemical bonding between adjacent Ge-4p$_{x,y}$ states, these states are split into bonding and antibonding states, which we denote as $|s^+\rangle$ and $|p_{x,y}^\pm\rangle$ with the subscript $\pm$ representing the parity (Fig. 3b, c). For the GeCH$_3$ monolayer without strain (Fig. 3b), the bands near the Fermi level are contributed by the $|s^-\rangle$ and $|p_{x,y}^+\rangle$ orbitals, with the $|s^-\rangle$ orbital being above the $|p_{x,y}^+\rangle$ orbital. For the GeCH$_3$ monolayer with strain, increasing the lattice constant weakens the interaction between the Ge atoms. This decreases the splitting between the bonding and antibonding states, which lowers the $|s^-\rangle$ state and raises $|p_{x,y}^+\rangle$ state. Consequently, depending on the strength of strain, the energy gap of GeCH$_3$ monolayer can be continuously tuned. At 12% strain, the $|p_{x,y}^+\rangle$ level shifts above the $|s^-\rangle$ level, which implies the normal band order is inverted under this condition, as depicted in Fig. 3c. Then the SOC then can lift the band degeneracy and create an energy gap at the $\Gamma$ point, signifying that a GeCH$_3$ monolayer with $\varepsilon = 12\%$ is a QSH insulator.

**Discussion**

Our density functional calculations show that a topological phase transition can be induced in a recently synthesized GeCH$_3$ monolayer by strain, and our phonon dispersion analysis shows that this layer is stable. The analysis of the $Z_2$ topological invariant confirms the topological insulating nature of the insulating state of the GeCH$_3$ monolayer with 12% strain. This QSH insulator has a large enough band gap that exceeds the thermal energy at room-temperature, making them suitable for room-temperature spintronics applications. It should be mentioned that there are many different ways to apply strain. For example, the strain can be realized by exerting mechanical forces to suspended samples, while very strong strain can also be introduced in thin films via the lattice mismatch with the substrate. For deposition onto substrates, one main concern is whether interface effect on the topological properties can be ignored.

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**Figure 3** (a) Edge states of a GeCH$_3$ monolayer calculated by using a nanoribbon model (see the text). Schematic diagrams showing the relative positions at $\Gamma$ of the Ge-4s and Ge-4p states of a GeCH$_3$ monolayer near the Fermi level for the case of (b) 0% and (c) 12% strain in the absence of SOC. The horizontal dashed lines indicate the Fermi level.
To this end, lots of efforts are still needed. However, as the first step for its realization, it is necessary to investigate the topological nature of free-standing GeCH₃ as discussed in the present work. In actual conditions, there are many other factors that may affect the topological properties of GeCH₃, so a more comprehensive study concerning these problems should be carried out in further works.

It is of interest to comment on the advantages of this GeCH₃ QSH insulator over other QSH insulators. First, the GeCH₃ monolayer is made up of the commonly used elements and is thermally stable up to 250 °C, offering compatibility with current silicon-based electronic technology. Second, on the experimental side, GeCH₃ has been synthesized whereas other 2D group IV lattice-based QSH insulators with comparable nontrivial gap (such as silicene, germanane, stanene, silicane, and stanane) have not yet been produced in the free-standing form. Third, the CH₃ terminated surface can be resilient against oxidation, whereas the -H terminated surface (such as germanane and silicane) are extremely air sensitive. Therefore, the GeCH₃ monolayer allows one to construct a potential QSH field-effect transistor.

The peculiar judgment of the nontrivial topological nature is the existence of band inversion. Standard DFT calculations tend to underestimate the gap, and this could give an enhanced and not reliable estimate of the strain for band inversion. To overcome this drawback, we repeat the simulations using the Heyd–Scuseria–Ernzerhof (HSE) functional¹⁰, the corresponding results are plotted in Fig. S2. We find that the band inversion occurs at 13% strain based on HSE, which increases only a few percent with respect to the LDA case. From this we can argue that standard DFT is still a good approximation to study the band inversion feature in GeCH₃ monolayer.

Methods

Our density functional calculations for an isolated GeCH₃ layer were performed using the projector augmented wave (PAW) method¹¹–¹² implemented in the Vienna ab initio Simulation Package (VASP)¹³, with the cutoff energy of 450 eV for the plane-wave basis set and the local density approximation (LDA)¹⁴ to describe the exchange-correlation functional. Repeated image interactions were made negligible by including a vacuum layer of 18 Å in the simulations. The Monkhorst–Pack set¹⁵ of 17 × 17 × 1 k-points was used to sample the Brillouin zone for both geometry optimizations and total energy calculations. The lattice constants and the atom coordinates were fully optimized until the forces on each ion are reduced below 0.02 eV/Å. The optimization with the inclusion of van der Waals interactions are also tested, and the results show the main conclusions obtained here are not affected (see Fig. S3). Complementary calculations were performed to determine the phonon dispersion relations using the finite displacement method with the density functional perturbation theory (DFPT) as implemented in the CASTEP code¹²,¹⁴.

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