I review the theoretical and experimental status of proton decay theory and experiment. Regarding theory, I focus mostly, but not only, on grand unification. I discuss only the minimal, well established SU(5) and SO(10) models, both ordinary and supersymmetric. I show how the minimal realistic extensions of the original Georgi - Glashow model can lead to interesting LHC physics, and I demonstrate that the minimal supersymmetric SU(5) theory is in perfect accord with experiment. Since no universally accepted model has of yet emerged, I discuss the effective operator analysis of proton decay and some related predictions from a high scale underlying theory. A strong case is made for the improvement of experimental limits, or better the search of, two body neutron decay modes into charged kaons and charged leptons. Their discovery would necessarily imply a low energy physics since they practically vanish in any theory with a desert in energies between $M_W$ and $M_{GUT}$.

1 Introduction

As Maurice Goldhaber had put nicely more than half a century ago, we feel it in our bones that proton is long lived, for otherwise the radiation from the decay would kill you. All you need to know is how much radiation is hazardous for you, and you get a lower limit on proton lifetime, on the order of $10^{18}$ years. It was, as it is today, puzzling that the proton should live so much longer than the neutron, and it was attributed to the conservation of the baryon number, first by Weyl [1] already in 1929, and the again by Stuckelberg [2] and Wigner [3] every ten years afterwards. And then it was becoming a dogma, as usually happens when a cause and a consequence are confused, and the questioning became heretic. It is amusing to see how almost apologetic Reines et al [4] were in justifying their pioneering experimental probe of nucleon decay:

"It has often been surmised that there exists a conservation law of nucleons, i.e., that they neither decay spontaneously nor are destroyed or created singly in nuclear collisions. In view of the fundamental nature of such an assumption, it seemed of interest to investigate the extent to which the stability of nucleons could be experimentally demonstrated.”

They established a limit of $10^{21}$ yr, which they would improve to $10^{26}$ yr later on. And while the baryon number was becoming sacred, lepton number violation was seriously discussed already in 1937 by Majorana in his classic paper [5] on majorana spinors. In turn Racah [6] and Furry [7] were to discuss at depth neutrino-less double decay still desperately searched for as a probe of lepton number violation. This shows how crucial for experimentalists it is to have a theory behind.

Since there is nothing sacred about global symmetries we will follow a belief that: the only good global symmetry is a broken global symmetry. The symmetry point is not a special point in the parameter space of a theory, for an almost exact global symmetry is just as protected and useful as a fully exact one. Furthermore a zero is useless for experimentalists.

And baryon number is not an exact gauge symmetry unless you accept a corresponding gauge coupling to be ridiculously small: $g_B \leq (10^{-20} - 10^{-19})$ (a repulsive competition with gravity).

While Goldhaber et al had to justify their search for proton decay, today the atmosphere has changed completely, as illustrated by the fact that one is asked to give plenary talks on it at major conferences. The point is that there is a theory behind: grand unification of strong, weak and electro-magnetic interactions. It grew out of pioneering ideas of Pati and Salam [8] on the unification of quarks and leptons and is exemplified perfectly on the minimal SU(5) theory of Georgi and Glashow [9]. When Georgi, Quinn and Weinberg [10] computed the unification scale and predicted proton lifetime $\tau_p \sim 10^{30}$ yr, many experimentalists rushed underground [11], all over the world, from India to Japan to US to Europe. Here is a list of experiments

Calorimeter detector

• Kolar Gold Field - Kolar district (Kamataka, India)
• NUSEX - Mont Blanc (Alps, France)
• FREJUS - Frejus tunnel (Alps, France)
• SOUDAN- Soudan underground mine (Minnesota, US)

Cherenkov detector

• IMB - Morton salt mine (Ohio, US)
• Kamiokande - Mozumi mine (Hida, Japan) \rightarrow Super-Kamiokande \rightarrow atmospheric neutrino oscillations

The search resulted in great improvements on proton decay limits, culminating with SK that recently pushed the pionic channel all the way to about $10^{34}$ yr. Here are limits on some proton decay channels.
For realistic neutrino Dirac Yukawa couplings it predicts a light fermion triplet \((1_C, 3_W)\), with a mass below TeV so that the running of the SU(2) gauge coupling is slowed down and meets U(1) above \(10^{15}\) GeV. It’s phenomenology is quite interesting for it leads to lepton number violation at colliders in the form of same sign di-leptons as suggested originally in seesaw a long time ago [19]. For the relevant studies in the context of the type III seesaw see [20].

The two theories have in common a ‘fast’ proton decay, with \(\tau_p \leq 10^{35}\) yr, to keep in mind in what follows.

### 3 Minimal supersymmetric SU(5)

The underground rush continued, or better to say, got boosted with the success of the minimal supersymmetric SU(5). Low energy supersymmetry, suggested in order to stabilize the Higgs mass hierarchy, predicted correctly \(\sin^2\theta_W = 0.23\) in 1981 [21] [22] [23] [24] ten years before its confirmation at LEP. It actually did even better: the prediction of \(\sin^2\theta_W = 0.23\) was tied to the prediction of the heavy top quark, with \(m_t \approx 200\) GeV. Namely, in 1981 the low indirect measurements gave \(\sin^2\theta_W = 0.21\), with the assumed value \(\rho = 1\). In order to make a case for low energy supersymmetry, Marcano and I [21] had to say that \(\rho\) was bigger, which required loops, which required at least one large coupling, and a natural SM candidate was the top quark, with \(y_t \approx 1\). It is remarkable that both the \(\sin^2\theta_W = 0.23\) and the heavy top would turn out to be true. It should be stressed that heavy top quark played also a crucial role in the radiative Higgs mechanism [25] [26]. Thus heavy top is an integral part of low energy supersymmetry.

The GUT scale was predicted: \(M_{GUT} \approx 10^{16}\) GeV and in turn \(\tau_p(d = 6) \approx 10^{35.1}\) yr, which would have rendered proton decay out of experimental reach. However, supersymmetry leads to a new contribution: \(d = 5\) operators [27] through the exchange of heavy color triplet Higgsino \((T\text{ and }\tilde{T})\). A rough estimate gives

\[
G_T \approx \frac{\alpha}{4\pi y_u y_d m_{\text{gaugino}}^2 m_{\tilde{T}}^2} \approx 10^{-30} \text{ GeV}^{-2}
\]

which for \(y_u \approx y_d \approx 10^{-4}\), \(m_{\text{gaugino}} \approx 100\) GeV, \(m_{\tilde{T}} \approx \text{TeV and } M_T \approx 10^{16}\) GeV gives \(\tau_p(d = 5) \approx 10^{30.31}\) yr. It would seem that today this theory is ruled out. It was actually proclaimed dead in 2001 when the triplet mass was carefully computed to give \(M_{\tilde{T}}^2 = 3 \times 10^{15}\) GeV [28] (for the superscript 0 explanation, see below). Caution must be raised however for two important reasons: i) the uncertainty in sfermion masses and mixings [29] and ii) uncertainty in \(M_T\) [30] due to necessity of higher dimensional operators [31] to correct bad fermion mass relations \(m_d = m_{\ell} [32]\). The \(d = 4\) operators, besides correcting these relations also split the masses \(m_3\) and \(m_8\) of weak triplet and color

| Channel | \(\tau_p(10^{33}\text{ years})\) |
|---------|-------------------------------|
| \(p \to e^+ \pi^0\) | 8.2 |
| \(p \to \mu^+ \pi^0\) | 6.6 |
| \(p \to \mu^+ K^0\) | 1.3 |
| \(p \to e^+ K^0\) | 1.0 |
| \(p \to \nu K^+\) | 2.3 |
| \(n \to e^+ K^-\) | 0.02 |
| \(n \to e^- K^+\) | 0.03 |
octet, respectively, in the adjoint $24_H$ Higgs super multiplet and one gets

$$M_{\text{GUT}} = M_{\text{GUT}}^0 \left( \frac{M_\text{GUT}}{2m_8} \right)^{1/2} \quad M_{\text{GUT}}^0 \simeq 10^{16} \text{ GeV}$$

$$M_T = M_T^0 \left( \frac{M_\text{GUT}}{m_8} \right)^{5/2} \quad M_T^0 = 3 \times 10^{15} \text{ GeV}$$

where the superscript 0 denotes the predictions for $m_3 = m_8$ at the tree level with $d = 5$ operators neglected. The fact that $M_{\text{GUT}}$ goes up with $m_8$ below $M_{\text{GUT}}$ was noticed quite some time ago [33]. Imagine that $d = 4$ terms dominate for small cubic Yukawa self coupling, in which case one has $m_3 = 4m_8$ and thus $M_T = 32M_\text{GUT}^0 \simeq 10^{17} \text{ GeV} \simeq M_{\text{GUT}}$ ($m_8 \simeq 10^{15}$ GeV). In turn a strong suppression of proton decay with $\tau_p \simeq 10^3 y \simeq 10^{35} - 34$ yr. In principle the ratio of the triplet and octet masses can be as large as one wishes, so at first glance the proton lifetime would seem to not be limited from above at all. However, all this makes sense if the theory remains perturbative and thus predictive. Increasing $M_{\text{GUT}}$ would bring it too close to the Planck scale, so it is fair to conclude that the proton lifetime is below $10^{35}$ yr. This prediction is not hard, though. The d=4 operators not only cure the bad mass relations, but also also split the Yukawa couplings of the SM doublet Higgs and the color triplet. If you allow for cancellation between the d=3 and d=4 couplings, one can in principle make the color triplet couplings as small as one wishes and thus suppress the proton decay. In principle, it is even possible to make a color triplet mass at the electro-weak scale [34]. The cancellation of matrices is rather unnatural, so I will not pursue it here. However, it may emerge in more complex models, such as SO(10) [35], so this should be kept mind as a serious possibility.

In short, the minimal supersymmetric SU(5) is still a perfectly viable theory, and the $d = 5$ proton decay is expected close to the present limit. The theory is crying for a new generation of proton decay experiments. It would seem that B-L remains an accidental global symmetry of nucleon decay and one expects the dominant mode $p \to K^+\bar{\nu}_\mu$ characteristic of $d = 5$ operators. However, this minimal theory must account for neutrino masses and mixings which implies that R-parity is broken. The first important implication of not assuming R-parity is that the lightest neutralino cannot be dark matter, for it decays too fast with the collider signature of lepton number violation. Thus, the only dark matter candidate is an unstable gravitino. It decays into the neutrino and the photon through the neutrino-gaugino mixing

$$\Theta_{\nu \text{ gaugino}} \simeq \sqrt{\frac{m_\nu}{M_{\text{gaugino}}}$$

The decay is suppressed by the Planck scale [36]

$$\Gamma(3/2 \to \gamma \nu) = \frac{1}{32\pi M_{pl}^2} \Theta_{\nu \text{ gaugino}}^2$$

$$\simeq \frac{1}{32\pi \frac{m_3}{2}} \frac{m_\nu}{M_{\text{gaugino}}}$$

$$\leq 10^{-50} \text{ GeV} \quad (\tau \geq 10^{26} \text{ sec}).$$

The lower limit on gravitino lifetime comes from requiring that the flux of produced photons does not exceed the observed diffuse photon background, and it gives an upper limit on gravitino mass, $m_{3/2} \leq 1 - 10 \text{ GeV}$ [36], reasonable for the LSP. If one relaxes the neutrino-gaugino mixing, gravitino can be heavier; see e.g. [37]. For a review and references on gravitino dark matter, see [38].

Once R-parity is not assumed ad hoc, one has a new source of proton decay too. Due to allowed couplings in the superpotential $\lambda_1 u^c d^c + \lambda_2 q^c \ell^c + \lambda_3 \ell \ell e^c$, $d^c$ mediates $d = 6$ proton decay, which implies the limit $\lambda_1 \lambda_2 \lambda_3 \simeq 10^{-25} \text{ (GeV)}$ for $m_{\tilde{d}} \simeq \text{TeV}$. The question mark indicates an interesting possibility of these couplings causing proton decay. It can be shown that the parameter space allows for a B+L violating mode $n \to e + K^+$ [39]. As I show below, this decay mode cannot come from a conventional picture of grand unification with a desert.

4 SO(10)

Although SU(5) is the minimal theory of grand unification and as such deserves maximal attention as a laboratory for studying proton decay, SO(10) has important merits

- it unifies a fermion family in a spinorial 16 representation and as such is a minimal unified theory of matter and interactions
- it automatically contains right-handed neutrinos $N$
- it gives naturally $M_N \gg M_W$ and so neutrino has a tiny mass through the see-saw mechanism
- in supersymmetry R-parity is a gauge symmetry [40]
- in the renormalizable version R-parity remains exact [41] and the lightest supersymmetric partner (LSP) is stable, and thus becomes a natural dark matter candidate.

While ordinary, non supersymmetric SO(10) was studied at length over the years, no predictive realistic model of fermion masses and mixings ever emerged.

All the fermion masses and mixings can be accounted for with the $10_H$ and $126_H$ representations, the latter providing a large mass to the right-handed neutrinos. In the Pati- Salam $SU(2)_L \times SU(2)_R \times SU(4)_C$ language

$$10_H = (2, 2, 1) + (1, 1, 6)$$
and

$$126_H = (2, 2, 15) + (3, 1, 10) + (1, 3, ar{1}0) + (1, 1, 6)$$

and one can see that in principle no bad mass relations come out. If $10_H$ is taken to be real, a minimal scenario, the predictions turn out to be wrong \[45\], and making it complex kills the predictions. This is why ordinary SO(10) does not do the job. The theory deserves attention for it allows for an intermediate L-R symmetry \[14\]; for recent attempts to revive ordinary SO(10) see \[43\], \[45\].

This situation improves in supersymmetry: the couplings being holomorphic simplifies things. Although a single $10_H$ is necessarily complex, still, analyticity guarantees a single Yukawa, and similarly for $126_H$. This minimal supersymmetric version, coined renormalizable, although suggested already in 1982 \[46\], and revisited ten years later \[17\], has been studied at length only in recent years \[18\] \[19\]. A boost was provided by an observation that $b \rightarrow \tau$ unification can be naturally tied with the large atmospheric mixing angle in the type II seesaw \[50\]. This means that small quark and large lepton weak mixing angles follow naturally from the common Yukawa couplings without any need for flavor symmetries. This is an important result which shows that the much talked about issue of rather different quark and lepton mixings is not a problem as normally argued. In the SO(10) grand unified theory it is simply a product of a broken quark-lepton or Pati-Salam symmetry. In other words, although at the GUT scale quarks and leptons are completely equivalent with the same Yukawa couplings, at the SM energies, their different masses lead to different mixing angles.

After a great initial success, when pinned down, the theory ran into tension between fast proton and neutrino masses. It was revisited recently \[51\] and shown to work with the so called split supersymmetry spectrum of heavy sfermions. It is important to note that the nucleon decay branching ratios are determined. This is an example for a kind of theory of proton decay we are searching for. It remains to be shown that the solution found is unique.

Instead of a large $126_H$ Higgs, one may choose a $16_H$ and then build $126_H$ Higgs effectively as $16_H^2$. This induces a proliferation of couplings and one must introduce extra flavor symmetries, and thus go beyond a simple GUT picture. For a discussion of this approach, see \[52\]. In any case, it is clear that no accepted minimal model has yet emerged. In this sense it is really important to keep in mind that the minimal supersymmetric SU(5) theory is not ruled out. It should be viewed as the laboratory for studying grand unification and related issues, such as proton decay and magnetic monopoles, the way the minimal ordinary SU(5) used to be when it worked.

5 Effective operator analysis of nucleon decay and the desert picture

Since we do not have the theory of grand unification it is worthwhile to study the generic features of high scale theories of proton decay. This is done through the effective operator expansion \[53\]. In this program one expands the baryon violating operators in $M_W/M_B$ or $m_p/M_B$, where $M_B$ is the scale responsible for proton decay. If $M_B$ is very large, one can safely assume the SM gauge symmetry unbroken at that scale. This is surely true in conventional grand unification with a desert where $M_B = M_{GUT}$.

There are only four leading $d = 6$ operators \[54\]

\[
O_1 = (u_R d_R) (q_L \ell_L) \quad O_2 = (q_L q_L) (u_R e_R) \\
O_3 = (q_L q_L) (q_L e_R) \quad O_4 = (u_R d_R) (u_R e_R)
\]

The gauge meson dominance would imply only the first two which can lead to a number of predictions. Even in general one has an immediate prediction of an accidental B - L symmetry in any theory with a high scale $M_B$, which explains why any GUT model was predicting it.

There are also a number of immediate isospin relations

\[
\Gamma(p \rightarrow \ell^+_R \pi^0) = \frac{1}{2} \Gamma(n \rightarrow \ell^+_R \pi^-) = \frac{1}{2} \Gamma(p \rightarrow \bar{\nu} \pi^+) = \Gamma(n \rightarrow \bar{\nu} \pi^+)
\]

\[
\Gamma(p \rightarrow \ell^-_R \pi^0) = \frac{1}{2} \Gamma(n \rightarrow \ell^-_R \pi^-)
\]

Also, it is evident that $\bar{s}$ goes out, such as in the allowed decay $p \rightarrow K^+ \bar{\nu}$, characteristic of $d = 5$ operators in supersymmetry.

One conclude in turns that there can be no two body neutron decay into kaons and charged leptons

\[
n \not\rightarrow K^+ \ell^- \quad n \not\rightarrow K^- \ell^+
\]

This is why the R-parity violating mode $n \rightarrow K^+ e$ discussed in section 3. is so important: if discovered, it would point out immediately towards a low scale source of proton decay. It results from the operator

\[
d s d \bar{\ell} \langle H \rangle / \bar{m}^3
\]

where the Higgs vev is needed to break the SM symmetry which otherwise guarantees the conservation of B - L. The low scale $\bar{m}$ of supersymmetry breaking allows for this operator not being suppressed. In GUT $\bar{m}$ becomes $M_{GUT}$ implying the suppression $M_W/M_{GUT}$. Furthermore, the necessary presence of $s$ quark for symmetry reasons implies the absence of the pionic mode: $n \not\rightarrow \pi^+ e$, which makes the it even more predictive \[59\].

This leads to an important message: the decay modes $n \rightarrow K^+ e$ and $n \rightarrow K^- e^+$ imply a low energy source of proton decay. The limits are roughly $10^{19}$ yr and I urge the experimentalists to improve them.
6 Proton decay: matrix elements

Due to the shortage of space, I will be very brief here. There have been many attempts of computing the matrix elements using the non-relativistic quark model, the bag model, the lattice and the chiral Lagrangian techniques. In recent years, the focus has been on the lattice and chiral Langrangians, especially the combination of the two. For the work on lattice see e.g. [55]. A lot of progress was made on the lattice computation of chiral Langrangian coefficients, see e.g. [56]. The trouble is that chiral perturbation theory works great for soft pions, while the pions from the proton decay would have the momentum on the order of proton mass. Thus, one needs to go beyond the leading term of the original classic [57].

7 Summary and Outlook

I would conclude with the following messages:

- there is no universally accepted grand unified theory of nucleon decay
- most models give $\tau_p \lesssim 10^{35}$ yr, especially with low energy supersymmetry
- minimal SU(5) is ruled out and its minimal extension leads to possible LHC physics through the light fermion triplet
- minimal supersymmetric SU(5) still perfectly viable with gravitino being the only possible (unstable) dark matter
- test of GUT desert picture provided by decay modes $n \rightarrow K^+ \ell$ and $n \rightarrow K^- \ell^+$ which would indicate low energy physics as a source of p decay, such as R-parity violating couplings

Thus, a new generation of experiments is badly needed. Here is a list of proposals

**Cherenkov detector** - MEGATON (hopefully): good for the pion modes

- HYPER-Kamiokande
- 3M - (Megaton, Modular, Multipurpose)
  Homestake - DUSEL (Deep Underground Science and Engineering Lab)
- MEMPHYS - MEgaton Mass PHYSics
- Fréjus - LAGUNA (Large Apparatus Grand Unification and Neutrino Astrophysics) project

**Liquid Argon Detector** - 100 kT: good for the kaon modes

- LANNDD (Liquid Argon Neutrino Nucleon Decay Detector)
  Homestake - DUSEL?
- GLACIER (Giant Liquid Argon Charge Imaging ExpeRiment)
  Europe - Laguna
- **Liquid scintillator** - 50 kT: good for the kaon modes
- **LENA** (Low Energy Neutrino Astronomy)
  Europe - Laguna

Hopefully they all will be funded and some will reach $10^{35}$ yr in 10-20 years? One cannot overemphasize the importance of trying to reach this scale, generic of grand unification. Without experiment, a beautiful field of proton decay and grand unification is bound to turn into metaphysics.

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