Scale-invariant scalar spectrum from the nonminimal derivative coupling with fourth-order term

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Abstract

An exactly scale-invariant spectrum of scalar perturbation generated during de Sitter spacetime is found from the gravity model of the nonminimal derivative coupling with fourth-order term. The nonminimal derivative coupling term generates a healthy (ghost-free) fourth-order derivative term, while the fourth-order term provides an unhealthy (ghost) fourth-order derivative term. The Harrison-Zel'dovich spectrum obtained from Fourier transforming the fourth-order propagator in de Sitter space is recovered by computing the power spectrum in its momentum space directly. It shows that this model provides a truly scale-invariant spectrum, in addition to the Lee-Wick scalar theory.

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1 Introduction

In the evolutionary phase of density inhomogeneities, one often makes the simplifying assumption that the primordial power spectrum has the simple power-law expression like \( P(k) = A_s(k/k_\star)^{n_s-1 + \alpha_s \ln(k/k_\star)} \) with \( k_\star \) a pivot scale. The case of \( n_s = 1 \) and \( \alpha_s = 0 \) corresponds to the Harrison-Zeldovich (HZ) spectrum \([1, 2, 3]\) and this has been ruled out by different datasets \([4]\). In the picture of slow-roll inflation (quasi-de Sitter expansion), the momentum dependence at any given time arises as a consequence of the time dependence of \( H \) and \( \dot{\phi} \) compared to the de Sitter (dS) expansion of constant \( H \) and \( \dot{\phi} = 0 \). Recent data from Planck has shown that the scalar spectrum is a nearly scale-invariant one with the amplitude \( A_s = \frac{1}{2\epsilon M_p^2} \left( \frac{H^2}{2\pi} \right)^2 = (2.441 \pm 0.092) \times 10^{-9} \), implying that it is approximately 1 (more precisely, \( n_s = 0.9603 \pm 0.0073 \)) \([5]\).

On the other hand, it is worth noting that the power spectrum of a massless minimally coupled (mmc) scalar in dS expansion takes the form of \( \left( \frac{H}{2\pi} \right)^2 \left[ 1 + \left( \frac{k}{aH} \right)^2 \right] \). It reduces to the HZ scale-invariant spectrum of \( \left( \frac{H}{2\pi} \right)^2 \) in the superhorizon region of \( k \ll aH \), whereas it leads to \( \left( \frac{k}{2\pi a} \right)^2 \) in the subhorizon region of \( k \gg aH \) \([6]\). We note here that the latter is just the spectrum of a massless conformally coupled (mcc) scalar generated during dS expansion. This happens to the tensor spectrum too. Clearly, the HZ scale-invariant spectrum is related to the two-point function which is logarithmically growing for largely separated points in dS space. However, it was shown that the IR growing of the two-point function is physical because Fourier transforming logarithmic two-point function can lead to the HZ spectrum when one tames logarithmic divergence by using Cesàro-summability technique \([7]\). This implies that one could obtain the HZ spectrum through an IR regularization procedure.

More recently, the authors have obtained the HZ spectrum \( \left( \frac{H}{2\pi} \right)^2 \) from the Lee-Wick model of a fourth-order derivative scalar theory in dS spacetime \([8]\). Here we have obtained the fourth-order propagator as the inverse of the Lee-Wick operator \( \Delta_{\text{LW}} = -\frac{1}{M^2} (\nabla^4 - M^2 \nabla^2) \) with \( M^2 \) mass parameter. The operator of “\(-M^2 \Delta_{\text{LW}}\)” becomes the Weyl operator \( \Delta_4 = \nabla^4 - 2H^2 \nabla^2 \) in dS spacetime \([9]\) which is a conformally covariant operator only when choosing \( M^2 = 2H^2 \). The HZ spectrum obtained by Fourier transforming the propagator in dS spacetime was confirmed by computing the power spectrum directly in momentum space. Also, the scale-invariant tensor spectrum generated during dS inflation could be found from the conformal gravity of \( \sqrt{-g} \mathcal{C}^2 \) \([10]\). This suggests strongly that a fourth-order derivative
theory related to conformal symmetry provides the HZ scale-invariant spectrum in whole dS spacetime.

In this work, we wish to look for another model which may provide a HZ scale-invariant spectrum. This would be the nonminimal derivative coupling with fourth-order term because this coupling provides $-2H^2\nabla^2$-term naturally which is necessary to obtain a mcc scalar operator. In this sense, this model is more attractive than the Lee-Wick model. Even the computation of the slow-roll inflation (quasi-dS expansion) is promising to compare with observation data, we wish to compute power spectrum generated during dS inflation because the computation is more simple and intuitive than those in quasi-dS spacetime.

2 Gravity model

We start with the gravity model whose action is given by

$$S_{\text{ENF}} = S_E + S_{\text{NF}} = \int d^4x \sqrt{-g} \left[ \left( \frac{R}{2\kappa} - \Lambda \right) + \frac{1}{2M^2} \left( \xi G_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - (\nabla^2 \phi)^2 \right) \right],$$  \hspace{1cm} (1)

where $\kappa = 8\pi G = 1/M_P^2$, $M_P$ being the reduced Planck mass, $M^2$ is a coupling parameter with dimension of mass squared, and $\xi$ is a coefficient to be adjusted as $\frac{2}{3}$. $\Lambda$ is introduced as a positive cosmological constant to obtain dS spacetime, instead of potential $V$. Here, the scalar $\phi$ has dimension of mass. The former term in the last parenthesis is the nonminimal derivative coupling (NDC) term and the latter denotes the fourth-order kinetic (FK) term. We note that the former generates no more degrees of freedom (DOF) than general relativity canonically coupled to a scalar field, while the latter generates a new scalar DOF because it is a fourth-order derivative term. This implies that the former generates healthy (ghost-free) higher derivative terms, while the latter is unhealthy (ghost) higher derivative terms. We have combined these to be $S_{\text{NF}}$. For reference, the action of the Lee-Wick scalar model is given by

$$S_{\text{LW}} = -\frac{1}{2} \int d^4x \sqrt{-g} \left[ g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + \frac{1}{M^2} (\nabla^2 \phi)^2 \right],$$  \hspace{1cm} (2)

where the first term is the canonically coupled kinetic term.

Varying the action (1) with respect to the metric tensor $g_{\mu\nu}$ leads to the Einstein equation

$$G_{\mu\nu} + \kappa \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}^{\text{NF}} = \kappa (T_{\mu\nu}^{\text{NDC}} + T_{\mu\nu}^{\text{FK}}),$$  \hspace{1cm} (3)
where two energy-momentum tensors are given by

\[
M^2 T^\text{NDC}_{\mu\nu} = \xi \left[ \frac{1}{2} R \nabla_\mu \phi \nabla_\nu \phi - 2 R_{(\nu} \nabla_{[\mu} \phi \nabla_{\mu]} \phi + \frac{1}{2} G_{\mu\nu} (\nabla \phi)^2 - R_{\mu\rho\nu\sigma} \nabla^\rho \phi \nabla^\sigma \phi 
- \nabla_\mu \nabla^\rho \phi \nabla_\nu \nabla_\rho \phi + (\nabla_\mu \nabla_\nu \phi) \nabla^2 \phi 
+ g_{\mu\nu} \left( R^{\rho\sigma} \nabla_\rho \phi \nabla_\sigma \phi - \frac{1}{2} (\nabla^2 \phi)^2 + \frac{1}{2} (\nabla^\rho \nabla^\sigma \phi) \nabla_\rho \nabla_\sigma \phi \right) \right] \tag{4}
\]

and

\[
M^2 T^\text{FK}_{\mu\nu} = - \nabla_\mu (\nabla^2 \phi) \nabla_\nu \phi - \nabla_\nu (\nabla^2 \phi) \nabla_\mu \phi + g_{\mu\nu} \left[ \nabla_\rho (\nabla^2 \phi) \nabla^\rho \phi + \frac{1}{2} (\nabla^2 \phi)^2 \right] \tag{5}
\]

It seems that \( T^\text{NDC} \sim T^\text{FK} \) when one disregards curvature coupled derivative terms in Eq. \(4\). However, we note that even though fourth-order derivative terms are generated in \( T^\text{NDC} \), there is no ghost state which means that any dangerous higher time-derivative is not generated. This is why one favors the NDC term to enhance friction effects in the slow-roll inflation \([11, 12]\). On the contrary, \( T^\text{FK} \) generates ghost states because of fourth-order derivative term. This is a basic difference between NDC and FK.

On the other hand, the scalar equation for the action \((1)\) is given by

\[
\nabla^4 \phi + \xi G^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0. \tag{6}
\]

A solution of dS spacetime to Eqs. \((3)\) and \((6)\) together with \( \bar{T}^\text{NF}_{\mu\nu} = 0 \) can be easily found when one chooses a constant scalar

\[
\bar{R} = 4 \kappa \Lambda, \quad \bar{\phi} = \text{const}. \tag{7}
\]

Here, the curvature, Ricci, and Einstein tensors can be written by

\[
\bar{R}_{\mu\nu\rho\sigma} = H^2 (\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = 3 H^2 \bar{g}_{\mu\nu}, \quad \bar{G}_{\mu\nu} = -3 H^2 \bar{g}_{\mu\nu} \tag{8}
\]

with Hubble constant \( H = \sqrt{\kappa \Lambda / 3} \). Further, the flat-slicing of dS spacetime can be realized by introducing conformal time \( \eta \) as

\[
d s^2_{\text{dS}} = \bar{g}_{\mu\nu} dx^\mu dx^\nu = a(\eta)^2 [-d\eta^2 + \delta_{ij} dx^i dx^j], \tag{9}
\]

where \( a(\eta) \) is conformal scale factor expressed by

\[
a(\eta) = - \frac{1}{H \eta}. \tag{10}
\]
During dS expansion, the scale factor $a$ goes from small to a very large value like $a_f/a_i \simeq 10^{30}$ which implies that the conformal time $\eta = -1/aH(z = -k\eta)$ runs from $-\infty(\infty)$ [the infinite past] to $0^-(0)$ [the infinite future]. The dS SO(1,4)-invariant distance between two spacetime points $x^\mu$ and $x'^\mu$ is defined by

$$Z(x, x') = 1 - \frac{-(\eta - \eta')^2 + |x - x'|^2}{4\eta\eta'}$$

since $Z(x, x')$ has the ten symmetries which leave the metric of dS spacetime invariant.

## 3 Scalar fourth-order propagator

One begins with general perturbed metric with 10 DOF

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi)d\eta^2 + 2B_id\eta dx^i + (\delta_{ij} + \bar{h}_{ij})dx^idx^j \right],$$

where the SO(3)-decomposition is given by

$$B_i = \partial_i B + \Psi_i, \quad \bar{h}_{ij} = 2\Phi \delta_{ij} + 2\partial_{ij}E + \partial_i\bar{E}_j + \partial_j\bar{E}_i + h_{ij}$$

with the transverse vectors $\partial_i\Psi^i = 0, \partial_i\bar{E}^i = 0$, and transverse-traceless tensor $\partial_ih^{ij} = h = 0$. To investigate the cosmological perturbation around the dS spacetime (9), we might choose the Newtonian gauge as $B = E = 0$ and $\bar{E}_i = 0$. Under this gauge, the corresponding perturbed metric with 6 DOF and perturbed scalar can be written as

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi)d\eta^2 + 2\Psi_i d\eta dx^i + \left\{ (1 + 2\Phi)\delta_{ij} + h_{ij} \right\} dx^idx^j \right],$$

$$\phi = \bar{\phi} + \varphi.$$  

Now we linearize the Einstein equation (3) around the dS background to obtain the cosmological perturbed equations. It is known that the tensor perturbation is decoupled from scalars. The tensor equation becomes a tensor form of a massless scalar equation

$$\delta R_{\mu\nu}(h) - 3H^2h_{\mu\nu} = 0 \rightarrow \nabla^2 h_{ij} = 0.$$  

We mention briefly how do two scalars $\Psi$ and $\Phi$, and a vector $\Psi_i$ go on. The linearized Einstein equation requires $\Psi = -\Phi$ which was used to define the comoving curvature perturbation in the slow-roll inflation and thus, they are not physically propagating modes.
in dS spacetime. During the dS inflation, no coupling between \( \{ \Psi, \Phi \} \) and \( \varphi \) occurs since \( \bar{\phi} = 0 \) implies \( \delta T_{\mu\nu}^{\text{NDC}} = \delta T_{\mu\nu}^{\text{FK}} = 0 \). Furthermore, the vector \( \Psi_i \) is not a propagating mode in the ENF theory because it has no kinetic term. Hence, we have the tensor \( h_{ij} \) with 2 DOF propagating in dS spacetime.

It would be better to find the scalar power spectrum by making Fourier transform of propagator in dS spacetime. For \( \xi = 2/3 \), the perturbed scalar equation takes the form

\[
\bar{\nabla}^4 \varphi + \xi \bar{G}^{\mu\nu} \bar{\nabla}_\mu \bar{\nabla}_\nu \varphi = 0 \rightarrow \bar{\nabla}^4 \varphi - 2H^2 \bar{\nabla}^2 \varphi \equiv \Delta_4 \varphi = 0,
\]

where \( \Delta_4 \) is just the Weyl operator (conformally covariant fourth-order operator) in dS spacetime [9, 13]. This is the main reason why we have introduced our action of \( S_{\text{ENF}} \) [1].

Actually, the non-degenerate fourth-order equation can be factorized as

\[
\bar{\nabla}^2 (\bar{\nabla}^2 - 2H^2) \varphi = 0,
\]

which implies two second-order equations for mmc and mcc scalars

\[
\begin{align*}
\bar{\nabla}^2 \varphi^{(\text{mmc})} &= 0, \\
(\bar{\nabla}^2 - 2H^2) \varphi^{(\text{mcc})} &= 0,
\end{align*}
\]

where the solution to \([18]\) is given by \( \varphi = \varphi^{(\text{mmc})} + \varphi^{(\text{mcc})} \). For simplicity, \((i)\) denotes two cases: \((i=1)\) for mmc and \((i=2)\) for mcc. We emphasize that a choice of \( \xi = 2/3 \) leads to a mcc scalar. Otherwise, one has a massive scalar propagating on dS spacetime.

Expanding \( \varphi^{(i)} \) in terms of Fourier modes \( \phi^{(i)}_k(\eta) \)

\[
\varphi^{(i)}(\eta, x) = \frac{1}{(2\pi)^3} \int d^3k \ \phi^{(i)}_k(\eta) e^{i k \cdot x},
\]

Eqs.\([19]\) and \([20]\) become

\[
\begin{align*}
\left( \frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 \right) \phi^{(1)}_k &= 0, \\
\left( \frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 + \frac{2}{z^2} \right) \phi^{(2)}_k &= 0,
\end{align*}
\]

with \( z = -\eta k \). Solutions to \([22]\) and \([23]\) are given by

\[
\begin{align*}
\phi^{(1)}_k &= c_1 (i + z) e^{iz}, \\
\phi^{(2)}_k &= c_2 i z e^{iz}.
\end{align*}
\]
where $c_1$ and $c_2$ are constants to be determined. These will be used to compute the power spectrum directly in the next section.

On the other hand, the linearized equation with an external source $J \phi$ takes the form

$$\Delta_4 \phi = - M^2 J \phi \rightarrow \phi(x) = - \frac{M^2}{\Delta_4} J \phi \equiv - D(Z(x, x')) J \phi(x'),$$

(26)

where the propagator is given by the inverse of $\Delta_4$ as

$$D(Z(x, x')) = \frac{M^2}{2H^2} \left[ \frac{1}{-\nabla^2} - \frac{1}{-\nabla^2 + 2H^2} \right] = \frac{M^2}{2H^2} [G_{\text{mmc}}(Z(x, x')) - G_{\text{mcc}}(Z(x, x'))]$$

(27)

with the dS-invariant distance $Z(x, x')$ (11). Here the propagators of mmc scalar (15) and mcc scalar (16) in dS spacetime are given by

$$G_{\text{mmc}}(Z(x, x')) = \frac{H^2}{(4\pi)^2} \left[ \frac{1}{1 - Z} - 2 \ln(1 - Z) + c_0 \right], \quad G_{\text{mcc}}(Z(x, x')) = \frac{H^2}{(4\pi)^2} \frac{1}{1 - Z},$$

(28)

where the former is the dS invariant renormalized two-point function, while the latter is the simplest scalar two-point function on dS spacetime. Substituting (28) into (27), the propagator takes the form

$$D(Z(x, x')) = \frac{M^2}{16\pi^2} \left( - \ln[1 - Z(x, x')] + \frac{c_0}{2} \right)$$

(29)

which is a purely logarithm up to an additive constant $c_0$.

The scalar power spectrum is defined by Fourier transforming the propagator at equal time $\eta = \eta'$ as given by

$$P = \frac{1}{(2\pi)^3} \int d^3r \frac{4\pi k^3}{M^2} D(Z(\mathbf{x}, \eta; \mathbf{x}', \eta')) e^{-ik \cdot r}, \quad \mathbf{r} = \mathbf{x} - \mathbf{x}'$$

(30)

$$= \frac{1}{(2\pi)^3} \frac{k^3 M^2}{4\pi} \int d^3r \left( - \ln \left[ \frac{r^2}{4\eta^2} \right] + \frac{c_0}{2} \right) e^{-ik \cdot r}$$

(31)

$$= - \frac{1}{(2\pi)^3} \frac{k^3 M^2}{4\pi} \int d^3r \ln[r^2] e^{-ik \cdot r} + \frac{k^3 M^2}{4\pi} \left( \ln[4\eta^2] + \frac{c_0}{2} \right) \delta^3(k)$$

(32)

$$= \frac{M^2 k^2}{8\pi^3} \int_0^\infty dr \left\{ r \sin[kr] \ln[r^2] \right\},$$

(33)

where the last (time-dependent) term in (32) disappeared, thanks to

$$k^3 \delta^3(k) = \frac{k \delta(k) \delta(\theta) \delta(\phi)}{\sin \theta} = 0.$$

(34)
We may use Cesàro-summation method to compute a logarithmically divergent integral \( (33) \). For this purpose, we note that the integral of
\[
\int_0^\infty f(x)\,dx
\]
is Cesàro summable, if
\[
(C, \alpha) = \lim_{\lambda \to \infty} \int_0^\lambda dx \left( 1 - \frac{x}{\lambda} \right)^\alpha f(x)
\]
exists and is finite for integer \( \alpha \geq 0 \). Then, \((C, \beta)\) is also Cesàro summable for any integer \( \beta > \alpha \).

To investigate Cesàro-summability of the integral explicitly, we focus on \( f(r) = r \sin[kr] \ln[r^2] \) in \( (33) \). In this case, \((C, \alpha)\) is
\[
(C, \alpha) = \lim_{\lambda \to \infty} \int_0^\lambda dr \left( 1 - \frac{r}{\lambda} \right)^\alpha \left( r \sin[kr] \ln[r^2] \right).
\]
We note that the integral \((C, 0)\) takes the same form as \((33)\). After manipulations, we have
\[
(C, 0) = -\frac{1}{k^2} \lim_{\lambda \to \infty} \left[ (k \lambda \cos[k\lambda] - \sin[k\lambda]) \ln[\lambda^2/2] + \text{Si}[k\lambda] \right],
\]
where \( \text{Si}[x] \) denotes the sine-integral function defined by \( \text{Si}[x] = \int_0^x (\sin[t]/t)\,dt \). The first two terms in \( (37) \) diverge in the \( \lambda \to \infty \) limit and thus, \((C, 0)\) is not a convergent integral. However, the last term in \( (37) \) is finite as it is shown in
\[
\lim_{\lambda \to \infty} \int_0^{k\lambda} \frac{\sin[t]}{t}\,dt = \frac{\pi}{2}.
\]
Further, \((C, 1)\) is also not convergent since it becomes
\[
(C, 1) = -\frac{1}{k^2} \lim_{\lambda \to \infty} \left( \sin[k\lambda] \ln[\lambda^2/2] + \text{Si}[k\lambda] \right),
\]
where the first term diverges in the limit of \( \lambda \to \infty \). On the other hand, for \( \alpha = 2 \), the corresponding integral \( (36) \) has a finite value
\[
(C, 2) = -\frac{2}{k^2} \lim_{\lambda \to \infty} \left( \text{Si}[k\lambda] \right) = -\frac{\pi}{k^2},
\]
where we used \( (38) \). Finally, we have checked that in the limit of \( \lambda \to \infty \),
\[
(C, 2) = (C, 3) = (C, 4) = (C, 5) = \cdots.
\]
This implies that \((C, \beta)\) is also Cesàro summable for any integer \( \beta > \alpha = 2 \).
Finally, considering (33) together with (40), the scalar spectrum takes the form

$$P = \frac{M^2}{8\pi^2}. \quad (42)$$

For $M^2 = 2H^2$, it leads to an exactly scale-invariant spectrum

$$P = \left(\frac{H}{2\pi}\right)^2. \quad (43)$$

This can be easily checked by noting that for $M^2 = 2H^2$ and $\xi = 2/3$, $S_{\text{NF}}$ in (11) becomes the Lee-Wick scalar model in dS spacetime [8].

### 4 Scalar spectrum

In order to compute scalar power spectrum directly, we have to obtain the second-order bilinear action. Making use of the Ostrogradski’s formalism, one may rewrite the fourth-order bilinear action $\delta S_{\text{NF}}$ obtained by bilinearizing (11) as the second-order bilinear action with $\xi = 2/3$

$$\delta S_{\text{NF}}^{(2)} = \frac{1}{2M^2} \int d^4x \left[ -2a^2H^2 \left( \alpha^2 + \partial_i \varphi \partial^i \varphi \right) - \left( (\alpha')^2 - 2\partial_i \alpha \partial^i \alpha + \partial^2 \varphi \partial^2 \varphi \right. 
\left. + 4aH \alpha \alpha' - 4aH \alpha \partial^2 \varphi \right) + 2M^2 \lambda (\alpha - \varphi') \right], \quad (44)$$

where $\alpha \equiv \varphi'$ is a new variable to lower fourth-order derivative down and $\lambda$ is a Lagrange multiplier. In (44), the prime (‘) denotes the differentiation with respect to $\eta$.

Let us define the conjugate momenta as

$$\pi_{\varphi} = \frac{1}{M^2} \left( \varphi'' - 2\partial^2 \varphi' + 2aH \partial^2 \varphi \right), \quad \pi_{\alpha} = -\frac{1}{M^2} (\varphi'' + 2aH \varphi'). \quad (45)$$

The canonical quantization is accomplished by imposing two commutation relations

$$[\hat{\varphi}(\eta, \mathbf{x}), \hat{\pi}_{\varphi}(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'), \quad [\hat{\alpha}(\eta, \mathbf{x}), \hat{\pi}_{\alpha}(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'). \quad (46)$$

The field operator $\hat{\varphi}$ can be expanded in Fourier modes as

$$\hat{\varphi}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} \left[ \left( \hat{a}_k \phi_k^{(1)}(\eta) + \hat{b}_k \phi_k^{(2)}(\eta) \right) e^{i\mathbf{k} \cdot \mathbf{x}} + \text{h.c.} \right], \quad (47)$$
where $\phi_k^{(1)}$ and $\phi_k^{(2)}$ were given by (24) and (25). When one substitutes (47) into the operator of $\hat{\pi}_\phi$, $\hat{\alpha}(\equiv \hat{\phi}')$, and $\hat{\pi}_\alpha$, one obtains the corresponding expressions. Plugging these all into (46), two commutation relations take the forms

$$[\hat{a}_k, \hat{a}^\dagger_{k'}] = \delta(k - k'), \quad [\hat{b}_k, \hat{b}^\dagger_{k'}] = -\delta(k - k').$$

(48)

It is noted that two mode operators ($\hat{a}_k, \hat{b}_k$) are necessary to take into account of fourth-order theory quantum mechanically as the Pais-Uhlenbeck fourth-order oscillator has been shown in [17]. We remind the reader that the unusual commutator for ($\hat{b}_k, \hat{b}^\dagger_{k'}$) reflects that the FK term contains the ghost state scalar [18]. In addition, two Wronskian conditions are found as

$$\phi_k^{(1)} \{ \left[ \phi_k^{*(1)}(\eta)'''' + 2k^2 \phi_k^{*(1)}(\eta)' - 2aH k^2 \phi_k^{*(1)}(\eta) \right] - \text{c.c.} = iM^2, \quad (49)$$

$$\phi_k^{(2)} \{ \left[ \phi_k^{*(2)}(\eta)'''' + 2k^2 \phi_k^{*(2)}(\eta)' - 2aH k^2 \phi_k^{*(2)}(\eta) \right] - \text{c.c.} = -iM^2, \quad (50)$$

which will be exploited to fix $c_1$ and $c_2$. Therefore, $\phi_k^{(1)}$ and $\phi_k^{(2)}$ are determined completely to be

$$\phi_k^{(1)} = \frac{M}{\sqrt{4k^3}} (i + z)e^{iz}, \quad \phi_k^{(2)} = \frac{M}{\sqrt{4k^3}} i ze^{iz}. \quad (51)$$

On the other hand, the power spectrum of the scalar is defined by [6]

$$\langle 0| \hat{\phi}(\eta, x) \hat{\phi}(\eta, x') |0\rangle = \int d^3k \frac{\mathcal{P}_\phi(k, \eta)}{4\pi k^3} e^{i k (x - x')}.$$  

(52)

Considering the Bunch-Davies vacuum state imposed by $\hat{a}_k |0\rangle = 0$ and $\hat{b}_k |0\rangle = 0$, (52) is calculated as

$$\mathcal{P}_\phi(k, \eta) = \frac{k^3}{2\pi^2} \left( |\phi_k^{(1)}|^2 - |\phi_k^{(2)}|^2 \right). \quad (53)$$

$$= \frac{M^2}{8\pi^2} \left[ 1 + z^2 - z'^2 \right] = \frac{M^2}{8\pi^2} \left[ 1 + (\eta)^2 - (\eta')^2 \right] \quad (54)$$

$$= \frac{M^2}{8\pi^2}. \quad (55)$$

Importantly, the minus sign ($-$) in (53) appears because the unusual commutation relation for ($\hat{b}_k, \hat{b}^\dagger_{k'}$) was used. The minus sign is essential to derive a scale-invariant spectrum from the scale-variant spectrum.
Finally, it is shown that for $M^2 = 2H^2$, \cite{53} corresponds to the HZ scale-invariant power spectrum

$$P_{\phi}^{M^2=2H^2} = \left(\frac{H}{2\pi}\right)^2,$$  \hspace{1cm} (56)

which is just the same form as in \cite{13}.

5 Discussions

We have obtained an exactly scale-invariant spectrum of scalar perturbation generated during de Sitter inflation from the gravity model of the nonminimal derivative coupling with fourth-order term. The nonminimal derivative coupling term generates a healthy (ghost-free) second-order term of $-2H^2\bar{\nabla}^2\phi$, while the fourth-order term provides an unhealthy (ghost) fourth-order derivative term. This combination provided the linearized scalar equation of $\Delta_4 \phi = 0$ expressed in term of the Weyl fourth-order operator $\Delta_4$ in dS spacetime. In this sense, our model of nonminimal derivative coupling with fourth-order term is more promising than the Lee-Wick scalar model where the nonminimal derivative coupling term is replaced by canonical kinetic term of $-\bar{\nabla}^2\phi$.

The HZ scale-invariant spectrum was obtained from Fourier transforming the fourth-order propagator (the inverse of Weyl operator) in de Sitter space. Taming a logarithmic IR divergence by making use of the Cesàro summability technique, we arrived at the power spectrum of $(H/2\pi)^2$. This HZ spectrum was also recovered by computing the power spectrum in its momentum space directly. In obtaining the power spectrum, we have used the Ostrogradski’s formalism and quantization of Pais-Uhlenbeck fourth-order oscillator.

Finally, we remark that in the case of $M^2 = 1$, one has a scalar with zero mass dimension in \cite{11}. This corresponds to the case (7.3) in \cite{14}. In this case, one has a constant power spectrum of $P = 1/(8\pi^2)$ which seems to be trivial.

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