An efficient controlled elitism non-dominated sorting genetic algorithm for multi-objective supplier selection under fuzziness

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Abstract

Supplier selection and order allocation constitute vital strategic decisions that must be made by managers within supply chain management environments. In this paper, we propose a multi-objective fuzzy model for supplier selection and order allocation in a two-level supply chain with multi-period, multi-source, and multi-product characteristics. The supplier evaluation objectives considered in this model include cost, delay, and electronic-waste (e-waste) minimization, as well as coverage and weight maximization. A signal function is used to model the price discount offered by the suppliers. Triangular fuzzy numbers are used to deal with the uncertainty of delay and e-waste parameters while the fuzzy Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is used to obtain the weights of the suppliers. The resulting NP-hard problem, a Pareto-based meta-heuristic algorithm called controlled elitism non-dominated sorting genetic algorithm (CENSGA), is developed. The Non-dominated Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Particle Swarm Optimization (MOPSO) are used to validate the applicability of the CENSGA algorithm and the Taguchi technique to tune the parameters of the algorithms. The results are analysed using graphical and statistical comparisons illustrating how the proposed CENSGA dominates NSGA-II and MOPSO in terms of mean ideal solution distance (MID) and spacing metrics.

Keywords: multi-objective supplier selection; fuzzy logic; controlled elitism; PSO; NSGA-II
1. Introduction

Supply Chain Management (SCM) is a complex and integrated logistics system where raw materials are efficiently converted into finished goods (Ghiani, Laporte, & Musmanno, 2004). An integrated supply chain can reduce total cost much more effectively than one whose parts are assumed to work independently of each other. Hugos (2005) defines SCM as the coordination between location, inventory, transportation, and production for a network of facilities and distribution options to reach the best combination of efficiency and responsiveness for the market being served. SCM ensures the efficient flow of raw materials, work-in-process inventory, and finished goods among suppliers, manufacturers, distribution centres, and retailers (Simchi, Kaminsky, & Simchi levi, 2002). In this sense, supplier evaluation and selection are a key activity in SCM that involves multiple and often conflicting criteria and metrics (Dulmin & Mininnno, 2003), while the primary goal is generally represented by cost minimization.

Supplier selection models are designed to answer questions such as ‘How many suppliers are needed?’, ‘Which suppliers should be selected?’, ‘What is the optimal ordering policy to each supplier?’, and so forth. Numerous deterministic models have been developed in the literature to answer these questions. The main disadvantage of deterministic models is that they fail to be accountable when dealing with the uncertain nature of real-world systems. To handle the effect of uncertainty of the input parameters, either probabilistic or fuzzy approaches have been applied. However, in many real cases, stochastic programming cannot be applied due to the lack of enough historical data to fit appropriate probability distributions to the uncertain parameters. Furthermore, the computational complexity of stochastic programming models is another reason to limit its application (Tseng, Chiang, & Lan, 2009).

In this paper, a non-linear multi-objective programming model is developed where the objective functions account for the cost, delays, and wastes imposed by the suppliers, the coverage provided by the suppliers, and the suppliers’ weights. Fuzzy variables represent delays and wastes due to the suppliers. At the same time, the weights of the suppliers are defined on the basis of the closeness coefficients produced by the fuzzy Technique for Order Performance by Similarity to Ideal Solution (TOPSIS). The way the weights of the suppliers are obtained is one of the novelties of the proposed model. Besides, the proposed supplier selection problem takes into consideration the distance of the customers from the suppliers as well as the extensiveness of the coverage (the coverage can be partial or complete). This is an aspect of supplier selection and allocation problems usually overlooked in the related literature. The model also assumes a discount constraint for simplification purposes. The method proposed to solve the model is based on a multi-objective meta-heuristic known as controlled elitism non-dominated sorting genetic algorithm (CENSGA). The well-known Non-dominated Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Particle Swarm Optimization (MOPSO) algorithm are used to validate the applicability of CENSGA. The parameters of both the algorithms are tuned using the Taguchi method.

The remainder of this paper is organized as follows: Section 2 reviews some of the related relevant research. Section 3 provides some basics on fuzzy numbers and Maximal Covering Location Problems (MCLPs). Section 4 formulates the proposed multi-objective programming problem. Section 5 outlines fuzzy TOPSIS and illustrates how fuzzy and non-fuzzy parameters are handled in our model. Section 6 explains the three Pareto-based meta-heuristics used to solve the problem. Section 7 illustrates the calibration process of the algorithms by the Taguchi approach. Section 8 compares the algorithms using three different metrics and performs a statistical analysis. Finally, Section 9 concludes.

2. Literature Review

Karasakiak and Karasakal (2004) suggested considering the partial coverage problem as an instance of the maximum coverage problem. In their model, the customer’s demand coverage rate by a distribution centre depended on the inverse of the customer distance from that centre. Liang (2008) developed a fuzzy multi-objective model in a multi-product, multi-period case in two levels. In his model, he considered delivery costs and time as two objective functions and solved his model using a dynamic approach. Tseng et al. (2009) proposed a hierarchical evaluation framework to assist groups of experts in selecting the optimal supplier. They performed a multi-criteria decision-making analysis under multiple conflicting criteria using the analytic network process (ANP) technique and the Choquet integral to reduce the interactivity of subjective judgments. Torabi and Hassini (2009) developed a three-dimensional model in a multi-objective fuzzy case assuming multi-products with a fixed demand. Their objective functions minimized the deviation variables for a store constraint, a future coverage constraint, and cost. Fatih, Serkan, Mustafa, and Diyar (2009) developed a multi-item system to select the suppliers using fuzzy and TOPSIS techniques in a group decision-making problem.

Onot, Selin, and Isik (2009) ranked the suppliers utilizing fuzzy TOPSIS and fuzzy ANP. They implemented a practical application of their technique to communication systems. Amid, Ghodspour, and O’Brien (2009) developed a linear multi-objective model that assumed the objective functions and demand to be indefinite and fuzzy; then, they solved their model using the weighted sum technique. Kokangol and Susuz (2009) considered capacity, budget, and discount conditions to formulate and solve the supplier selection problem. They developed an integrated model combining hierarchical analysis techniques, a non-linear mathematical programming model, and a multi-objective programming model. For more details on the problem development, one can refer to Arumugam and Rao (2008) and Fatih et al. (2009). Tsai and Wang (2010) applied a mixed integer programming procedure to solve the problem and allocate the orders for a multi-source and multi-product case in the supply chain. Their objective functions included cost and minimizing the delay and wastes from the supplier side. Chu and Varma (2012) suggested a multi-level multi-criteria decision-making model for supplier selection in a fuzzy setting. They developed a hierarchical structure for criteria and sub-criteria and rated the alternatives respect to qualitative criteria in linguistic terms represented by triangular fuzzy numbers.

Atakhan and Ali Fuat (2011) formulated a multi-objective model with fuzzy parameters and solved it using a weighted max–min technique. They obtained the weights of the suppliers using TOPSIS and utilized a weighing method to integrate the objectives. Haleb and Hamidi (2011) developed a fuzzy multi-objective model to allocate orders to suppliers. In their model, a hierarchical technique was used to obtain the weights of the suppliers. They set these weights as an objective function to select the suppliers and solved their model using the max–min
technique of the membership function. Liang (2011) developed a fuzzy multi-objective model with parameters and objective functions defined in a fuzzy environment. They converted their model into one with a single-objective function using the maximin technique. Liao, Lin, and Lai (2011) developed a maximum distance constraint for covering the customers’ demand by distribution centres in the inventory location problem. Lin (2012) considered cost, delay, and quality objective functions and developed a model for supplier selection under fuzzy conditions. Shaw, Shankar, Yadav, and Thakur (2012) developed an integer multi-objective model whose objective functions included purchase cost, delay, wastes or returned products, and environmental effect or greenhouse gases. They converted the objective functions into a single-objective function using a weighted method that allows determining the weights of the suppliers by a fuzzy hierarchical technique.

Nazarikhani, Shakouri, Javadi, and Keramati (2013) presented a supplier selection problem for several cost levels and products with three objective functions, including cost, delay, and wastes. Esfandiari and Seifbarghy (2013) developed a multi-objective model aiming at minimizing cost, delay, and waste while maximizing the weights of the suppliers. Their model had a stochastic component, with the demand being modelled by a Poisson probability function. At the same time, the product cost from the supplier side was assumed to follow a linear discount function. A metric LP-technique was used to convert the problem to a single objective model. Arikan (2013) developed an integer multi-objective model for supplier selection considering cost, on-time delivery, and delivered units percentage as objective functions. Subsequently, he converted the objective functions to a single objective using the max-min technique. Meena and Sarmah (2013) developed a non-linear single-objective model for supplier selection using a mixed-integer programming model. Hajipour, Khodakarami, and Tavana (2014a) and Hajipour, Rahmati, Pasandideh, and Niaki (2014b) proposed two Pareto-based meta-heuristic approaches to solve a multi-objective facility location-allocation problem, namely NSGA-II and the non-dominated ranking genetic algorithm.

Jadidi, Zolfaghari, and Cavalieri (2014) modelled the problem of supplier selection as a multi-objective optimization problem where minimization of price, rejects, and lead-time are considered as three objectives. Moghaddam (2015) developed a fuzzy multi-objective mathematical model to identify and rank the candidate suppliers and find the optimal number of new and refurbished parts and final products in a reverse logistics network configuration. Prasannavakatesan and Goh (2016) presented a multi-objective mixed integer linear programming (MILP) model to find the optimal choice of suppliers and their order quantity allocation under disruption risk. Tran and Park (2016) defined eight groups of twenty nine scoring criteria that aimed at helping designers and practitioners to select an appropriate methodology in product-service system design. Hajikhani, Khalilzadeh, and Sadjadi (2018) defined a fuzzy multi-objective model to select and allocate orders to the suppliers in uncertain conditions and performed a case study in the urban agricultural industry. They designed a MOPSO algorithm and NSGA-II to solve a fuzzy multi-objective supplier selection model.

Facility location models and covering problems have a vast number of applications to real-life situations such as determining the number and location of public service facilities (schools, libraries, post offices, etc.) or emergency facilities (hospitals, police stations, fire and rescue service, etc.) (Francis and White, 1974; Schilling, Jayaraman, & Barkhi, 1993; Owen and Daskin, 1998; Fallah, NaimiSadigh, & Aslantzadeh, 2009). A survey of the latest works on covering models and problems is provided in Farahani, Asgarib, Heidar, Hosseiniaiic, and Gohd (2012). Babbbar and Hassanazadeh Amin (2018) developed a novel mathematical model to select a set of suppliers and assign the order quantity with environmental considerations. Sureeytanapas, Sriwattananusart, Niyamosoth, Sessomboon, and Arunyanart (2018) proposed an approach to facilitate practitioners to select suppliers under uncertainty and/or unavailability of information. Khalilzadeh and Derikvand (2018) presented a comprehensive model based on real-world conditions for the supplier selection problem in the green supply chain under uncertainty. In addition to economic issues, environmental features were considered when selecting environment-friendly suppliers and deciding on the resulting purchases. Park, Okudan Kremer, and Ma (2018) defined a two-phase approach for a sustainable global supply chain design and illustrated its performance through a bicycle manufacturing supply chain. Moheb-Alizadeh and Handfield (2019) designed a hybrid three-step solution methodology based on the ε-constraint method and Benders decomposition algorithm to solve sustainable supplier selection and order assignment problems. Yadavalli, Barhi, Bhayana, Jha, and Agarwal (2019) developed an analytical model for a manufacturing firm to select suppliers based on the expectations of customers, together with financial and socio-environmental stability factors. Kirschstein and Meisel (2019) proposed a supplier assessment and storage selection heuristic based on kernel search for multi-period multi-commodity lot-sizing problems. Finally, Luan, Yao, Zhao, and Song (2019) introduced a hybrid model consisting of an Ant Colony Optimization and a Genetic Algorithm to settle the supplier selection problem.

2.1. Contribution

In this paper, we formulate a multi-objective SCM supplier selection problem in a fuzzy environment to maximize the coverage provided by the suppliers. That is, we focus on the following two aspects:

• the coverage provided by the suppliers when selecting and allocating the orders; and
• the uncertainty that characterizes the delays and e-wastes as well as the importance weights to be assigned to the suppliers.

Taking into account the coverage provided by the suppliers constitutes an innovative feature for this kind of problem. At the same time, we use fuzzy variables to represent delays and e-wastes due to the suppliers and fuzzy TOPSIS to determine the weights of the suppliers. In particular, we complete the fuzzy TOPSIS algorithm with an additional step that allows assigning weights to the alternatives based on their closeness coefficients (the closeness coefficient of an alternative represents its distance to the fuzzy positive and the fuzzy negative ideal solution, simultaneously).

The choice of defining the suppliers’ weights using the closeness coefficients is an innovative feature of the proposed model. The weights of the suppliers could be defined by fuzzy variables directly. However, the use of fuzzy TOPSIS guarantees that the suppliers’ weights are assigned to reflect the evaluation criteria and the corresponding importance weights, which characterize the fuzzy multi-criteria decision-making setting where the suppliers are evaluated.
Table 1. Extensions implemented relative to the model of Hajikhani et al. (2018).

| Problem characteristics | Hajikhani et al. (2018) | Current model |
|--------------------------|-------------------------|---------------|
| **Objectives**           | Cost, Delay,           | Cost, Delay,  |
|                          | Maximal covering       | Waste,       |
|                          | Suppliers weight       | Maximal covering, |
|                          |                        | Suppliers weight |
| **Constraints**          | Hajikhani et al. (2018) | Hajikhani et al. (2018) plus acceptable quantity of e-waste |
| **Optimizers**           | NSGA-II, MOPSO         | CENSGA, NSGA-II, MOPSO |

The model of Hajikhani et al. (2018) constitutes the closest formal framework to the supplier selection environment analysed in the current paper. Table 1 summarizes the main differences existing between the model developed by Hajikhani et al. (2018) and the current one. Note that, together with the complexity inherent to the introduction of an additional objective, a novel optimizer has been designed to solve the resulting optimization problem. Thus, after formulating the problem mathematically, CENSGA is used to solve the resulting NP-hard problem while NSGA-II and MOPSO are implemented to validate the results. The Taguchi technique is used to tune the parameters of both MOPSO and NSGA-II. The results are analysed using quantitative criteria and performing parametric and non-parametric statistical analyses.

3. Basic Notions

In this paper, we formulate a supplier selection problem in a fuzzy environment with the objective of minimizing cost, delays, and wastes imposed by the suppliers, maximizing the coverage provided by the suppliers and minimizing the suppliers’ weights. In order to proceed to the mathematical formulation of the problem, we need to recall a few notions of fuzzy set theory and briefly describe the features of an MCLP.

3.1. Fuzzy set theory

Fuzzy set theory, introduced by Zadeh (1965), deals with the uncertainty and imprecision associated with information.

**Definition 1.** Let $U$ be a universe set and $X \subset U$. The fuzzy set $\hat{X}$ of $U$ is characterized by a membership function $\mu_X$ such that, $\forall x \in U, \mu_X(x) \in [0, 1]$ indicates the grade of membership of $x$ in $X$.

**Definition 2.** A fuzzy number $\hat{x}$ is a fuzzy set that is both normal and convex in the universe set $U$.

**Definition 3.** A triangular fuzzy number $\hat{a} = (l, m, u)$ is a fuzzy number characterized by the following membership function:

$$\mu_a(x) = \begin{cases} 
0 & x \leq l \\
(x - l)/(m - l) & l \leq x < m \\
(u - x)/(u - m) & m \leq x \leq u \\
0 & x > u 
\end{cases}$$

(1)

**Definition 4.** Let $\hat{M} = (m_0, m_2, m_0)$ and $\hat{N} = (n_0, n_2, n_0)$ be two triangular fuzzy numbers. Then, the distance between $\hat{M}$ and $\hat{N}$ is calculated as follows:

$$d(\hat{M}, \hat{N}) = \sqrt{\frac{1}{3} \left( (m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 \right)}.$$  

(2)

This is also known as the ‘vertex method’.

3.2. MCLP

MCLPs optimize the number of demand points covered within a specified distance/time by a fixed number of facilities. In the standard approach, the demand points need not be all covered. However, allowing for partial coverage provides a more practical approach to MCLP than the classical setting (Karasakal & Karasakal, 2004).

Suppose, for instance, that there are two potential facilities, $Y_1$ and $Y_2$, covering a certain number of demand points, and we have to choose the one with maximal coverage. Thus, interpreting facilities as suppliers and demand points as customers to serve, the coverage by a supplier $j$ of a customer $i$ can be represented by the following probability distribution:

$$F_j(i) = \begin{cases} 
1 & o_{ij} \leq S_j \\
L(o_{ij}) & S_j < o_{ij} < R_j \\
0 & o_{ij} \geq R_j 
\end{cases}$$

(3)

$$L(o_{ij}) = \frac{R_j - o_{ij}}{R_j - S_j}, \quad 0 < L < 1$$

(4)

where $S_j$ and $R_j$ stand for the maximum distances for complete and partial coverage by supplier $j$, respectively, and $o_{ij}$ is the distance of supplier $j$ from customer $i$.

The continuous random variable with probability distribution defined by $F_j(i)$ in equation (3) will be denoted by $b_{ij}$.

4. Problem Modelling

4.1. Indices and parameters

$i$ : Index of customers ($i = 1, 2, \ldots, I$)

$j$ : Index of suppliers ($j = 1, 2, \ldots, J$)

$k$ : Index of products ($k = 1, 2, \ldots, K$)

$t$ : Index of periods ($t = 1, 2, \ldots, T$)

$r$ : Index of discount levels ($r = 1, 2, \ldots, R$).

$p_{ijkt}$ : Unitary purchasing cost of product $k$ by customer $i$ in period $t$ from supplier $j$

$b_{ij}$ : Price of the product $k$ offered by supplier $j$ in period $t$

$B_{ij}$ : Price of defective product $k$ offered by supplier $j$ in period $t$

$b_{ij}$ : Coverage rate of supplier $j$ for customer $i$

$D_{ijk}$ : Demand of customer $i$ for product $k$ in period $t$

$W_j$ : Weight of supplier $j$

$f_{ijk}$ : Fixed cost of ordering for supplier $j$ in period $t$ for product $k$

$P_{R_{ijk}}$ : Price of each unit product $k$ offered by supplier $j$ in period $t$ with discount level $r$

$C_{ijk}$ : Capacity of supplier $j$ for product $k$ in period $t$

$n_{ik}$ : Maximum number of suppliers for customer $i$ and product $k$ in period $t$

$Q_{ij}$ : Maximum price of the defective products purchased by buyer $i$ from supplier $j$

$T_i$ : Maximum price of the delay for purchased products by buyer $i$ from supplier $j$

$S_j$ : Maximum distance for complete coverage by supplier $j$
\( R_j \): Maximum distance for partial coverage by supplier \( j \)
\( V_{ij} \): Cost of shipment per each unit product \( k \) from supplier \( j \) to customer \( i \) in distance unit
\( o_{ij} \): Distance of supplier \( j \) from customer \( i \)
\( H_i \): Minimum ordering to supplier \( j \)
\( O_t \): Maximum capital of customer \( i \) in period \( t \)
\( q_{jktr} \): Discount bound of product \( k \) supplied by supplier \( j \) in period \( t \) at discount level \( r \)

The parameters with a tilde cannot be evaluated with certainty by the decision maker and shall be represented by fuzzy variables. We will use triangular fuzzy numbers for the parameter \( \tilde{e}_{jk} \) and \( \tilde{b}_{jk} \). For what concerns the parameters \( W_j \), instead of regarding them as fuzzy variables, we will generate them using an extension of fuzzy TOPSIS. That is, the suppliers’ weights will be crisp values obtained by a fuzzy procedure that suitably accounts for the uncertainty behind a weight assignment.

\[ R_j : \text{Maximum distance for partial coverage by supplier } j \]
\[ V_{ij} : \text{Cost of shipment per each unit product } k \text{ from supplier } j \text{ to customer } i \text{ in distance unit} \]
\[ o_{ij} : \text{Distance of supplier } j \text{ from customer } i \]
\[ H_i : \text{Minimum ordering to supplier } j \]
\[ O_t : \text{Maximum capital of customer } i \text{ in period } t \]
\[ q_{jktr} : \text{Discount bound of product } k \text{ supplied by supplier } j \text{ in period } t \text{ at discount level } r \]

4.2. Decision variables
\( x_{ijk} \): Purchasing quantity of product \( k \) by customer \( i \) from supplier \( j \) in period \( t \)
\( y_{ijk} \): One if customer \( i \) buys product \( k \) in period \( t \) from supplier \( j \); zero, otherwise.

4.3. Assumptions
- Demand is fixed and definitive.
- Shortage is not permitted.
- Discount is universal and determined by a sign function.

4.4. Proposed mathematical modelling

4.4.1. Objective 1: cost minimization function
The cost objective function is composed of three parts, including purchase and shipment costs, and the fixed cost of ordering. For the first part of the cost objective function, the price of each product is offered by the suppliers based on the sign of the discount function. Customers in each period order their products to the suppliers based on the offered price. The second part of the cost objective function is the shipping cost, which is calculated based on the customer distance from the supplier and the order size (which accounts for the selection of the closer supplier). The third part of the cost objective function is the fixed cost of ordering (which accounts for the selection of a supplier with the lowest cost).

\[ \text{Min } Z_1 = \sum_{i,j,k,t} P_{ijk} x_{ijk} + \sum_{i,j,k,t} o_{ij} V_{ijk} x_{ijk} y_{ijk} + \sum_{i,j,k,t} f_{jkt} y_{ijk} \quad (5) \]

4.4.2. Objective 2: delay minimization function
The second objective of the problem is to minimize the delay from the suppliers’ side. The order size of a product to the suppliers is based on the potential delay of the product. At the same time, delays by suppliers are uncertain. Thus, the parameters of the delay objective function are defined by triangular fuzzy numbers.

\[ \text{Min } Z_2 = \sum_{i,j,k,t} \tilde{\delta}_{jk} x_{ijk} \quad (6) \]

4.4.3. Objective 3: e-waste minimization function
The third objective of the problem is minimizing the wastes from the suppliers’ side. This function considers the fact that the order size of a product depends on the percentage of waste generated by the suppliers. Thus, the parameters of the e-waste objective function are also represented by triangular fuzzy numbers. This objective function constitutes one of the main contributions of the current paper, extending the standard mathematical models presented in the literature.

\[ \text{Min } Z_3 = \sum_{i,j,k,t} \tilde{B}_{jk} x_{ijk} \quad (7) \]

4.4.4. Objective 4: coverage maximization function
The fourth objective of the problem is maximizing customer coverage by the suppliers. The coverage \( b_{ij} \) of customer \( i \) by supplier \( j \) is determined by the distance between the customer and the supplier, and the partial or complete coverage provided by the latter. That is, \( b_{ij} \) is a continuous random variable with probability distribution defined by \( F_j(\cdot) \) in equation (3), which guarantees that the selection of suppliers is performed according to the demand of each customer \( i \) for each product \( k \) in each period.

\[ \text{Max } Z_4 = \sum_{i,j,k,t} b_{ij} D_{ikt} y_{ijk} \quad (8) \]

4.4.5. Objective 5: weight minimization function
For this objective function, the product ordering rate is defined according to the suppliers’ weights.

\[ \text{Max } Z_5 = \sum_{i,j,k,t} W_j x_{ijk} \quad (9) \]

As already stated (Subsection 4.1), the suppliers’ weights \( W_j \) will be obtained using the fuzzy TOPSIS technique to evaluate the suppliers more realistically when selecting the best suppliers (see Section 5).

4.4.6. Formulation of the multi-objective programming model
The final proposed mathematical model for supplier selection and order allocation using the signal function discount and the maximal coverage policy is formulated as follows:

\[ \text{Min } Z_1 = \sum_{i,j,k,t} P_{ijk} x_{ijk} + \sum_{i,j,k,t} o_{ij} V_{ijk} x_{ijk} y_{ijk} + \sum_{i,j,k,t} f_{jkt} y_{ijk} \]

\[ \text{Min } Z_2 = \sum_{i,j,k,t} \tilde{\delta}_{jk} x_{ijk} \]

\[ \text{Max } Z_3 = \sum_{i,j,k,t} \tilde{B}_{jk} x_{ijk} \]

\[ \text{Max } Z_4 = \sum_{i,j,k,t} b_{ij} D_{ikt} y_{ijk} \]

\[ \text{Max } Z_5 = \sum_{i,j,k,t} W_j x_{ijk} \]

Subject to:

\[ \sum_{j} x_{ijk} \geq D_{ikt}; \quad \forall i, k, t \quad (10) \]

\[ \sum_{j} x_{ijk} \leq \sum_{j} b_{ij} D_{ikt}; \quad \forall i, k, t \quad (11) \]

\[ \sum_{j} x_{ijk} \leq c_{ijk}; \quad \forall j, k, t \quad (12) \]

\[ 1 \leq \sum_{j} y_{ijk} \leq n_{ikt}; \quad \forall i, k, t \quad (13) \]

\[ \sum_{i,j,k,t} \tilde{\delta}_{jk} x_{ijk} \leq \sum_{i,j,k,t} Q_{ij} b_{ij} D_{ikt} \quad (14) \]

\[ \sum_{i,j,k,t} \tilde{B}_{jk} x_{ijk} \leq \sum_{i,j,k,t} T_{ij} b_{ij} D_{ikt} \quad (15) \]
Let $A_j$ ($j = 1, 2, \ldots, J$) be $J$ suppliers that must be evaluated against $H$ selection criteria, $C_h$ ($h = 1, 2, \ldots, H$). The fuzzy version of the associated multi-attribute decision-making problem can be concisely expressed by a decision matrix and a priority vector as follows:

$$
\mathbf{D} = \begin{bmatrix}
\tilde{x}_{11} & \cdots & \tilde{x}_{1H} \\
\vdots & \ddots & \vdots \\
\tilde{x}_{H1} & \cdots & \tilde{x}_{HH}
\end{bmatrix} \quad \text{and} \quad \mathbf{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_H].
$$

Step 2: Normalizing the fuzzy decision matrix

Raw data are normalized to avoid possible anomalies due to the use of different measurement units and scales. In particular, the normalization process guarantees that the ranges of all the normalized triangular fuzzy numbers are included in $[0,1]$.

The normalized fuzzy decision matrix $\mathbf{\tilde{R}}$ is obtained from $\mathbf{D}$ as follows.

Let $B$ and $C$ denote the index sets for the benefit and the cost related criteria, respectively. Also, let

$$
u_h^* = \max_j u_{jh} \quad \text{and} \quad l_h^* = \min_j l_{jh}. \quad (23)
$$

Then, the $j$th element of the matrix $\mathbf{\tilde{R}}$ is given by the following normalized triangular fuzzy number:

$$
\tilde{r}_{jh} = \begin{cases}
\frac{l_{jh}}{u_{jh}}, m_{jh}, \frac{u_{jh}}{l_{jh}} & \text{if } h \in B \\
\frac{l_{jh}}{u_{jh}}, \frac{u_{jh}}{l_{jh}}, \frac{u_{jh}}{l_{jh}} & \text{if } h \in C
\end{cases}. \quad (24)
$$

Step 3: Constructing the weighted normalized fuzzy decision matrix

The weighted normalized decision matrix $\mathbf{\tilde{V}}$ is obtained from $\mathbf{\tilde{R}}$ by multiplying each element of $\mathbf{\tilde{R}}$ by the importance weight of its corresponding evaluation criterion. That is,

$$
\mathbf{\tilde{V}} = \begin{bmatrix}
\tilde{v}_{11} & \cdots & \tilde{v}_{1H} \\
\vdots & \ddots & \vdots \\
\tilde{v}_{H1} & \cdots & \tilde{v}_{HH}
\end{bmatrix}. \quad (25)
$$

where $\tilde{v}_{jh} = \tilde{r}_{jh} \cdot \tilde{w}_h$ ($j = 1, 2, \ldots, J$ and $h = 1, 2, \ldots, H$). The $j$th element of the matrix $\mathbf{\tilde{V}}$ is the triangular fuzzy number $\tilde{v}_{jh} = (v_{jh1}, v_{jh2}, v_{jh3}).$

Step 4: Determining FPIRP and FNIRP

The fuzzy positive ideal reference point (FPIRP), denoted by $A^+$, and the fuzzy negative ideal reference point (FNIRP), denoted by $A^-$, can be defined by the positive triangular fuzzy numbers included in the interval $[0,1]$ as follows:

$$
A^+ = \{\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_H^+\} \quad \text{and} \quad A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_H^-\}. \quad (26)
$$

where

$$
v_{jh}^+ = \max_j v_{jh}; \quad h = 1, 2, \ldots, H \quad \text{and} \quad v_{jh}^- = \min_j v_{jh}; \quad h = 1, 2, \ldots, H. \quad (27)
$$

Step 5: Computing the distances of each supplier from FPIRP and FNIRP
The distances of the \( j \)th supplier \( A_j \) from FPIRP and FNIRP are defined, respectively, by

\[
s_j^+ = \sum_{h=1}^{n} d(\tilde{v}_{jh}, \tilde{v}_{jh}^+), \quad j = 1, ..., J \tag{28}
\]

\[
s_j^- = \sum_{h=1}^{n} d(\tilde{v}_{jh}, \tilde{v}_{jh}^-), \quad j = 1, ..., J \tag{29}
\]

where \( d(\tilde{v}_{jh}, \tilde{v}_{jh}^\pm) \) denote the distance between two fuzzy numbers and it is calculated using equation (2).

**Step 6: Obtaining the closeness coefficient and ranking the suppliers**

The closeness coefficient \( CC_j \) of the \( j \)th supplier \( A_j \) is calculated as follows:

\[
CC_j = s_j^- / (s_j^- + s_j^+); \quad j = 1, 2, ..., J \tag{30}
\]

If a supplier \( A_j \) has a closeness coefficient \( CC_j = 1 \), then \( A_j \) is the closest possible supplier to FPIRP and the farthest possible one from FNIRP. Thus, the best supplier among the available ones is the one presenting the highest closeness coefficient.

**Step 7: Calculating suppliers’ weights**

Finally, the weight of each supplier is computed as

\[
W_j = CC_j / \sum_{j=1}^{J} CC_j; \quad j = 1, 2, ..., J \tag{31}
\]

### 5.2. Generating fuzzy triangular numbers to use as parameters

In order to discuss any solution method for the proposed programming model, we need to arbitrarily fix the values of all the parameters involved in equations (5)-(21). In particular, we need to define the triangular fuzzy numbers that represent the delay and waste parameters as well as the parameters from which the suppliers’ weights are derived using the fuzzy TOPSIS algorithm, that is

- \( \tilde{\delta}_{jk} \) \((j = 1, 2, ..., J; \; k = 1, 2, ..., K; \; t = 1, 2, ..., T) \) in equations (6) and (15);
- \( \tilde{B}_{jkt} \) \((j = 1, 2, ..., J; \; k = 1, 2, ..., K; \; t = 1, 2, ..., T) \) in equations (7) and (14);
- \( \tilde{\xi}_{jh} \) \((j = 1, 2, ..., J; \; h = 1, 2, ..., H) \) and \( \tilde{w}_h \) \((h = 1, 2, ..., H) \) in equation (22).

We do so by randomly generating both the crisp values and the triangular fuzzy numbers corresponding to the different parameters. The main reason why triangular fuzzy numbers are used so often by researchers is that they make calculations simple and easy. One of the main contributions of the current paper is the development of the optimization problem within an uncertain environment. In order to fuzzify the methodological environment, the most common membership functions, such as the triangular fuzzy one, are implemented.

The values of the non-fuzzy parameters are generated using uniform distributions, each of which is defined in the desired range for the corresponding parameter.

The triangular fuzzy numbers representing the fuzzy parameters are obtained as follows. Applying the uniform distribution on a suitable range, 100 numbers are randomly generated for each parameter and arranged in a 10 × 10 matrix. Hence, the endpoints (lower and upper extremes) and the peak point (middle point) of the triangular fuzzy number are defined as follows:

- \( \Lambda = \) randomly generated matrix;
- \( \text{Lower extreme} = \min(a : a \text{ is an element of } \Lambda) \);
- \( \text{Middle point} = \sum(a : a \text{ is an element of } \Lambda) / 100 \);
- \( \text{Upper extreme} = \max(a : a \text{ is an element of } \Lambda) \).

Finally, the Beta mean distribution is applied to defuzzify the triangular fuzzy numbers generated for the delay and e-waste parameters (Ross, 2005). That is,

\[
\begin{align*}
\tilde{B}_{jkt} &= (B_{jkt}^{\text{lower}}, B_{jkt}^{\text{middle}}, B_{jkt}^{\text{upper}}) \\
& \xrightarrow{\text{defuzzify}}
\end{align*}
\]

and, similarly, for \( \delta_{jk} \).

### 6. A Controlled Elitism Non-Dominated Sorting Genetic Algorithm

In order to tackle the complex problem at hand, an intelligent-based approach must be applied to find the set of near-optimal solutions (Gupta, Kumar, Sahoo, Sahu, & Sarangi, 2017). Thus, in the current paper, the proposed mathematical model is solved by applying three Pareto-based meta-heuristic algorithms: CENSGA, NSGA-II, and MOPSO.

#### 6.1. NSGA-II

NSGA (or NSGA-I) has several drawbacks such as computational complexity, non-elitist operation, and the necessity of a sharing parameter, which are often preventable. NSGA-II was proposed by Deb, Agrawal, and Meyarivan (2002) as a robust Pareto-based multi-objective evolutionary algorithm to address these drawbacks. The main idea of NSGA-II is to reproduce a new population from an initial population and to distribute these two populations over the entire Pareto optimal front(s).

In order to find the best solutions and determine the Pareto set(s), the solutions are prioritized using non-domination sorting shown in Fig. 1. Note that there are two main parameters in non-domination sorting: the number of solutions dominating a specific solution (\( N_p \)) and a set of solutions obtained by a specific solution (\( S_p \)). We should note that: (i) this sorting process is an iterative procedure labelling each solution with a ranking that may not be unique; (ii) when considering minimization problems (such as the problem formulated in the current paper), the best solution is ranked first, the second solution is ranked second, and so on. Once the non-domination sorting is completed, the ranking assigned to each solution is used as a measure of fitness (Deb et al., 2002; Ahmadi-Darani, Moslehi, & Reisi-Nafchi, 2018; Bolaños, Escobar, & Echeverri, 2018; Wichapa & Khokha-Jaikiat, 2018).

#### 6.1.1. Solution representation

The solution structure of the problem (chromosome) consists of two parts. The first part of the chromosome indicates the order rate for each product by the customer in each period. The second part of the chromosome is a binary variable that allows selecting the supplier. That is, in the current algorithm, we have a chromosome in the form of a four-dimensional matrix, where the first part corresponds to the order rate and the second to the supplier selection. A chromosome is defined for each product and period, its genes representing the matrix inputs or the number of suppliers and customers.

#### 6.1.2. Crossover operator

The initial population is constructed through \( n \) crossover operations. The subsequent selection is performed randomly. The crossover operator is a function taking the location of two par-
Figure 1. Non-domination sorting process of NSGA-II (Deb et al., 2002).

6.1.3. Mutation operator
The initial population is generated through \( n \) mutations, with the subsequent selection being performed randomly. The mutation operator is implemented through the Gaussian technique in a continuum space. That is, the selected variable, \( x \), initially located between \( x_{\text{min}} \) and \( x_{\text{max}} \), is converted to \( x' \) adding a different element \( \Delta x \) defined by a normal distribution with a mean 0 and variance \( \sigma^2 \):

\[
\Delta x \sim N(0, \sigma^2) \tag{33}
\]

\[
x' = x + \Delta x \sim N(\mu, \sigma^2). \tag{34}
\]

The parameter \( \sigma \) is defined in a way to allow for some percentage of variable diversity that produces the mutation.

\[
\sigma = 0.1 \times (\text{var}_{\text{max}} - \text{var}_{\text{min}}) \tag{35}
\]

The parameter \( \mu \) represents the mutation rate. Once the parameter \( \mu \) is selected, the operation in equation (34) is applied to the population (Coello et al., 2002).

6.1.4. Main operators of NSGA-II
The fast non-dominated sorting operator is used to assign a rank to each of them to differentiate the possible solutions. However, some solutions may have the same rank. Thus, the crowding distance (CD) operator is also applied. CD measures the density of all the solutions distributed around a particular solution. The coding process followed by the CD operator is described in Fig. 2.

6.2. CENSGA
A new generation of multi-objective algorithms has been recently introduced in the literature. These algorithms do not convert multi-objective problems into single objective ones but are more oriented to guide the multi-objective processes. NSGA-II, proposed by Deb et al. (2002), is one of the most implemented algorithms within this category. In this paper, we present an extended version of NSGA-II and denote it by CENSGA. The main difference between CENSGA and NSGA-II is defined in the selection strategy, namely, CENSGA allows all fronts to participate in the selection process through a geometric distribution. When compared with NSGA-II and MOPSO, CENSGA minimizes the effects of the different operators on the performance of the algorithms since it is mainly based on their searchability.
The different selection strategies of CENSGA and NSGAII are described in Fig. 3. Note that, in CENSGA, the specific selection operator is defined to allow all fronts to participate in the selection strategy. However, the participation of better fronts is prioritized so that they have a more significant influence when defining the next generation. This process is controlled by the geometric distribution defined in equation (36b)

\[ n_i = r \times n_{i-1}, \]

where \( n_i \) denotes the maximum number of individuals allowed in the \( i \)-th front and \( r \) corresponds to the reduction rate. In a population of size \( N \), the maximum number of individuals allowed in each \( i \)-th \( (i = 1, 2, \ldots, k) \) front is calculated as follows:

\[ n_i = N \times r^{i-1} \times \frac{1 - r}{1 - r^k} \]  

Figure 4 presents a schematic summary of the CENSGA algorithm.

6.3. MOPSO

The particle swarm optimization (PSO) algorithm was introduced by Eberhart and Kennedy (1995). The basic idea in PSO is to create a swarm of particles that move in the problem space searching for their goal (Boyd & Richerson, 1985; Boeringer & Werner, 2004; Hajipour & Pasandideh, 2012). In the original PSO, particle \( i \) is represented by the potential solution \( P_i = (X_{i1}, X_{i2}, ..., X_{id}) \) in a \( D \)-dimensional problem space. Each particle keeps a memory of its previous best position \( (P_{best}) \) and is endowed with a velocity vector \( V_i = (V_{i1}, V_{i2}, ..., V_{id}) \). At each iteration, the position \( g \) of the particle with the best fitness value in the search space and the \( P \) vector of the current particle are combined to adjust the velocity along each dimension. The velocity vector is then used to compute a new position for the particle by using the \( c_{best} \) and \( L_{best} \) approaches. In the \( c_{best} \) version, the particle swarm optimizer keeps track of the overall best value and its location. This solution is called \( c_{best} \) (Gbestid). For the \( L_{best} \) version, the particle swarm optimizer keeps track of the best solution within a local topological neighbourhood of particles. This solution is called \( L_{best} \) (Lbestid). The particle velocity along each dimension is held to a maximum velocity \((v_{max})\) and it is restricted to it (Alaghebandha, Hajipour, & Hemmati, 2017; Hajipour, Tavana, Di Caprio, Jabbari, & Akhgar, 2019). The updating rules are as follows:

\[
V_{\text{new}} = W \times V_{\text{old}} + C_1 \times \text{Rand}_1 \times (\text{Pbest} - X_{\text{old}}) + C_2 \times \text{Rand}_2 \times (\text{Lbest} - X_{\text{old}})
\]

\[
X_{\text{new}} = X_{\text{old}} + V_{\text{new}},
\]

where \( C_1 \) and \( C_2 \) are the relative influence of the social and cognition components, respectively, and \( \text{Rand}_1 \) and \( \text{Rand}_2 \) denote two random numbers uniformly distributed in the interval \([0, 1]\).

6.3.1. The main loop of MOPSO

Leader selection is the first step in the major cycle of MOPSO, where a probability distribution is defined. Using a rolling cycle, sampling is performed from this probability distribution in order to determine what cell will be selected. Hence, a case is selected among the members of this cell, while the members of unfitted particles are placed in a repository. A cell is selected to meet the competency condition, as follows:

\[ n_i < n_j \Rightarrow p_i \geq p_j. \]

The Boltzmann technique is used to define \( p_i \), that is,

\[ p_i \propto \exp(-\beta n_i); \quad p_i = e^{-\beta n_i}/\sum_j e^{-\beta n_j}. \]

6.3.2. Neighbourhood search structure

The uniform distribution is used to define the new particles’ rate as (Coello et al., 2002)

\[
N_m = \left(1 - \frac{\mu t - 1}{\max Ht - 1}\right)^{1/\alpha},
\]

where \( N_m \) is the position of a mutated particle, \( \mu \) is the mutation rate to control the plot slope, and \( t \) is the number of iterations. In addition, a penalty function is used to handle the constraints. If inequality constraints are violated, a penalty amount \( \Psi \) is multiplied by a coefficient denoted by \( \alpha \) and added to the objective
function (Coello et al., 2002). In our case, a penalty is added to the objective function based on the violations considered in Hajipour and Pasandideh (2012).

7. Parameter Tuning and Objective Values

The Taguchi method is implemented to set the algorithm parameters. The parameter calibration test is executed by the Taguchi technique L27 \((3^{*}5)\), that is, 27 tests are designed from five parameters and three levels. Table 2 shows the parameters of CENSGA, NSGA-II, and MOPSO and the levels defined for them. The signal-to-noise (SN) function is defined as in Montgomery (2003).

The objective function values are computed by implementing the algorithms for three test problems. For each test problem, the five objective functions of the model, \(Z_1, \ldots, Z_5\), are considered individually and five objective values obtained. Hence, for each objective function, the mean of the values obtained for the three problems can be calculated. This produces five mean values that are converted to a single objective function value using the weighted-sum approach (Szidarovszky, Gersbon, & Duckstein, 1985; Tavakkoli-Moghaddam, Noshafagh, Taleizadeh, Hajipour, & Mahmoudi, 2017).

\[
\text{Total } Z = \lambda_1 \cdot Z_1 + \lambda_2 \cdot Z_2 + \lambda_3 \cdot Z_3 + \lambda_4 \cdot Z_4 + \lambda_5 \cdot Z_5.
\]

where the \(\lambda\) parameters indicate the weights or significance of the objectives for the decision maker. Figures 5–7 represent the S/N ratio of the Taguchi execution for CENSGA, NSGA-II, and MOPSO, respectively. The best values of the parameters determined by the Taguchi method are reported in Table 3.
Table 2. The levels defined for the parameters of CENSGA, NSGA-II, and MOPSO.

| Parameters                  | Level 1 | Level 2 | Level 3 |
|-----------------------------|---------|---------|---------|
| CENSGA Maximum generation (A) | 30      | 50      | 100     |
| Population size (B)         | 25      | 50      | 70      |
| Crossover percentage (C)    | 0.6     | 0.75    | 0.9     |
| Mutation percentage (D)     | 0.1     | 0.25    | 0.4     |
| NSGA-II Maximum generation (A) | 30      | 50      | 100     |
| Population size (B)         | 25      | 50      | 70      |
| Crossover percentage (C)    | 0.6     | 0.75    | 0.9     |
| Mutation percentage (D)     | 0.1     | 0.25    | 0.4     |
| MOPSO Number of iterations (A) | 30      | 50      | 100     |
| Particle population (B)     | 25      | 50      | 70      |
| Repository (C)              | 5       | 10      | 20      |
| Personal best (D)           | 1       | 1.2     | 1.4     |
| Inertia factor (E)          | 0.2     | 0.4     | 0.6     |
| Global best position (F)    | 1       | 1.1     | 1.2     |

8. Results

To evaluate the efficiency of proposed CENSGA in solving the proposed multi-objective model, we applied NSGA-II and MOPSO in the literature. Three well-known performance metrics with 12 test problems are applied for comparisons. The ratios of each measure for each function of each test example are obtained. Hence, the mean value of each measure is defined among the objective function in each test example. We introduce below the performance metrics considered for evaluating and comparing the algorithms; then, we analyse the results from the statistical viewpoint. Table 4 reports the input parameters of the 12 test examples. Note that the input parameters for the test problems are: the number of customers \((i = 1, 2, \ldots, I)\), the number of suppliers \((j = 1, 2, \ldots, J)\), the number of products \((k = 1, 2, \ldots, K)\), and the number of periods \((t = 1, 2, \ldots, T)\). The number of discount levels \((r = 1, 2, \ldots, R)\) does not appear in the table since it did not have a significant impact in generating the test problems.

8.1. Performance measures

We use three measures: the mean ideal solution distance, the spacing metric, and the computational time metric to analyse Pareto solutions in multi-objective optimization.

8.1.1. Mean ideal solution distance (MID)

One of the measures generally used to evaluate an algorithm is the distance from the ideal point. This measure calculates the distance of all points from the best population size. The following equation indicates how to calculate this measure (Hajipour, 2009).

Figure 5. Main effects plot for SN ratios of CENSGA.
MID = \sum_{i=1}^{n} \frac{c_i}{n}, \quad (44)

where \(c_i\) is the distance from the ideal solution \(i\) and \(n\) is the number of Pareto solutions in the final front.

8.1.2. Spacing

By considering the spacing measure, the algorithm covers all the points of the solution space. This measure calculates the relative distance of subsequent solutions. The following equation indicates how to calculate this measure (Boloori Arabani, Zandieh, & Fatemi Ghomi, 2011):

\[
S = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (d_i - \bar{d})^2}, \quad (45)
\]

where \(\bar{d} = \sum_{i=1}^{n} \frac{d_i}{n}\) and \(d_i = \min_{(k: k \neq k_i)} \sum_{m=1}^{2} |f_m^i - f_m^k|\).

8.1.3. Computational time (CPUT)

The final measure is given by the computational time of the algorithms. The algorithms are coded using MATLAB 7.14.0.739 (R2010b) and implemented on a PC using Windows 10, 2.40 GHz, RAM 8 GB.

8.2. Graphical analysis

Table 5 reports the above-mentioned multi-objective metrics after solving the optimization problem via CENSGA, NSGA-II, and MOPSO. Figures 8–10 compare the outputs from executing all algorithms in terms of the MID, spacing, and computational time metrics.

It is clear from the plots that CENSGA performs better than best-developed algorithms in the literature (NSGA-II and MOPSO) with respect to the MID, spacing, and CPUT metrics. The remarkable property of CENSGA is that when the size of the problems grows larger, the difference increases.

8.3. Statistical analysis

A non-parametric test called the Kruskal–Wallis test was applied to analyse the results statistically (Montgomery, 2003). The results for the non-parametric Kruskal–Wallis test for the three metrics are given in Tables 6–9. In this statistical test, the Null hypothesis is defined as 'All medians are equal', while the Alternative hypothesis is given by 'At least one median is different'.

8.3.1. Statistical test on MID metric

According to the statistical test results reported in Tables 6–9, the \(p\)-value of the Kruskal–Wallis non-parametric test is 0.003; thus, \(H_0\) is rejected.

8.3.2. Statistical test on spacing metric

According to the statistical test results reported in Tables 6–9, the \(p\)-value of the Kruskal–Wallis non-parametric test is 0.024; thus, \(H_0\) is rejected.
Table 3. Optimal values for the parameters of CENSGA, NSGA-II, and MOPSO.

| Algorithm  | Parameters                  | Optimal value |
|------------|-----------------------------|---------------|
| CENSGA     | Maximum generation (A)      | 30            |
|            | Population size (B)         | 50            |
|            | Crossover percentage (C)    | 0.6           |
|            | Mutation percentage (D)     | 0.4           |
| NSGA-II    | Maximum generation (A)      | 30            |
|            | Population size (B)         | 50            |
|            | Crossover percentage (C)    | 0.75          |
|            | Mutation percentage (D)     | 0.25          |
| MOPSO      | Number of iterations (A)    | 100           |
|            | Particle population (B)     | 70            |
|            | Repository (C)              | 20            |
|            | Personal best (D)           | 1             |
|            | Inertia factor (E)          | 0.6           |
|            | Global best position (F)    | 1.2           |

8.3.3. Statistical test on CPU time metric

According to the statistical test results reported in Tables 6–9, the p-value of the Kruskal–Wallis non-parametric test is 0.759; thus, $H_0$ is rejected.

Thus, the results showed that in terms of the MID and spacing metrics, the proposed CENSGA performs better and dominates both NSGA-II and MOPSO. Figures 11–13 illustrate the interval plots of all metrics for the different algorithms analysed.

Table 4. Test problems implemented to compare the performance of the algorithms.

| Problem no. | Number of customers | Number of suppliers | Number of products | Number of periods |
|-------------|---------------------|---------------------|--------------------|------------------|
| 1           | 2                   | 3                   | 2                  | 2                |
| 2           | 3                   | 4                   | 3                  | 3                |
| 3           | 5                   | 6                   | 7                  | 3                |
| 4           | 8                   | 8                   | 8                  | 4                |
| 5           | 10                  | 9                   | 10                 | 4                |
| 6           | 12                  | 10                  | 15                 | 5                |
| 7           | 15                  | 12                  | 20                 | 5                |
| 8           | 20                  | 14                  | 25                 | 9                |
| 9           | 30                  | 15                  | 30                 | 9                |
| 10          | 40                  | 18                  | 40                 | 12               |
| 11          | 50                  | 20                  | 50                 | 12               |
| 12          | 70                  | 25                  | 80                 | 12               |

In terms of CPU time, all three algorithms work similarly in statistical terms.

8.4. Model implementation: the case of an information technology company

In order to illustrate the applicability of the model developed, we implement it to analyse a real case study in the Asian Information Technology industry. The name of the company studied has been omitted for privacy reasons. The company analysed focuses on the development of software solutions.
and the selection of infrastructures and suppliers. In particular, it aims at reducing the supplier base of customer firms through the application of lean philosophy. The case study consists of 40 customer types, 18 suppliers, 40 products, two periods of time, and one discount level. The infrastructure procurement activities are carried out traditionally, and purchases take place through methods such as offering, bargaining, or covering.

The company has been asked to evaluate and select different infrastructures (provided by a heterogeneous set of suppliers) for a group of customer firms transitioning into the information technology industry. In particular, the company has been asked to reduce the number of suppliers – together with the corresponding set of products – as much as possible to facilitate the successful development of a software system. That is, customers expect a selection of suppliers whose timely performance regarding the provision of basic platforms displaying the specifications and quality required allows them to maintain their project schedules. Indeed, if there were a bug in one of the products, such as a cell, a negative effect would spread through the whole software system, implying that the selection of the products to be provided by the different suppliers constitutes one of the key decisions made by the company being analysed.

The relevant variables have already been defined in Section 4.1. However, the following remarks should be considered
Figure 9. Comparing CENSGA with NSGA-II and MOPSO in terms of the spacing metric.

Figure 10. Comparing CENSGA with NSGA-II and MOPSO in terms of the computational time metric.
within the current study case. The value of the parameters $\hat{b}_{jk}$ and $\hat{b}_{jk}$ has been provided by the different suppliers for the period of two years studied, eliminating their inherent uncertainty and allowing us to incorporate them as crisp values into the analysis. However, fuzziness prevails when considering the subjective evaluation of each supplier via $W_j$. It should be finally noted that the third objective concentrates on e-waste. The solutions obtained regarding the subset of products selected by the company after implementing CENSGA are reported in Table 10. Additional information regarding the selection of suppliers for each of the products is available from the authors upon request.

### 8.5. Managerial implications

Supplier evaluation constitutes a strategic feature for companies with a significant number of suppliers. Rather than dealing with each supplier separately, the buyer can establish a set of rules for interacting with each set of suppliers. Nevertheless, the supplier allocation to a different cluster might not turn into an easy task. The procedures of segmentation, evaluation, and selection of suppliers presented in this paper cover all the scale from simple to very complicated conditions. In some cases, it might turn out that the investment is too high and might not be worth it, as the output will not cover the expenses in terms of business efficiency. As well, many objectives are involved in making decisions more accurate. It is so important to understand that in some companies, the two sets of criterion approach might be enough. Still, in other cases, a more profound analysis must be undertaken, with all the essential variables taken into consideration. Supplier segmentation can incorporate more than two criteria, depending on the market situation of individual companies. Thus, this paper illustrates how the evaluation of suppliers can have strategic managerial implications since these actions involve organizational change, the development of custom-made methods of work, and the appearance of new abilities in the supply chain department, all of them leading to an efficient outsourcing system based on partnership and added value.

### 9. Conclusion

In this paper, we presented a fuzzy multi-objective model to select and allocate the orders to suppliers under uncertainty and for multi-period, multi-source, multi-customer, and multi-product cases at two levels of the supply chain. The objectives of the model consisted of minimizing cost, delays, and e-waste imposed by the suppliers, maximizing the coverage provided by the suppliers, and minimizing the suppliers’ weights.

Delay and e-waste parameters were considered uncertain and represented by triangular fuzzy numbers. The weights of the suppliers could also be interpreted as fuzzy variables. However, in order to make the evaluation of the suppliers more realistic, the suppliers’ weights were determined by implementing the fuzzy TOPSIS technique. Indeed, using fuzzy TOPSIS, the suppliers’ weights were assigned to reflect a given set of evaluation criteria and their relative importance, as is the case in any real-life decision-making situation.

An efficient CENSGA based on controlled elitism mechanism was proposed to solve the multi-objective NP-hard problem resulting from the proposed model. Two best-developed algorithms called MOPSO and NSGA-II were implemented. We applied three performance measures [the MID, spacing, and computational time (CPUT) metrics] to evaluate the efficiency of the algorithms and concluded that CENSGA performs better than both NSGA-II and MOPSO in terms of the MID and spacing metrics, while all three algorithms have a similar performance with respect to the MID metric. The accuracy of this claim is also supported by the Kruskal–Wallis non-parametric test performed on the three algorithms. The results obtained in this study show that the proposed CENSGA is an effective way to determine and manage Pareto solutions to multi-objective supplier selection and order allocation problems.

Finally, it must be noted that the covering constraints were defined as part of the mathematical formulation of the supplier selection and order allocation problem. This kind of constraints is usually overlooked in the SCM literature. Thus, the proposed model allows incorporating MCLPs, which represent a critical component of strategic planning for a broad range of public and private firms and an essential feature of many real-life facility location problems.

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**Table 6.** Kruskal–Wallis non-parametric test for the MID metric.

| Algorithms | Problem no. | Median   | Mean rank | Z-Value |
|------------|-------------|----------|-----------|---------|
| CENSGA     | 12          | 9.62230E + 09 | 10.1      | −3.39   |
| MOPSO      | 12          | 5.83752E + 10 | 23.9      | 2.18    |
| NSGA-II    | 12          | 5.25213E + 10 | 21.5      | 1.21    |
| Overall    | 36          | 18.5     |           |         |

**Table 7.** Kruskal–Wallis non-parametric test for the spacing metric.

| Algorithms | Problem no. | Median   | Mean rank | Z-Value |
|------------|-------------|----------|-----------|---------|
| CENSGA     | 12          | 250880797 | 12.3      | −2.52   |
| MOPSO      | 12          | 768378681 | 23.9      | 2.18    |
| NSGA-II    | 12          | 558586443 | 19.3      | 0.34    |
| Overall    | 36          | 18.5     |           |         |

**Table 8.** Kruskal–Wallis non-parametric test for the CPU time metric.

| Algorithms | Problem no. | Median   | Mean rank | Z-Value |
|------------|-------------|----------|-----------|---------|
| CENSGA     | 12          | 99.885   | 16.7      | −0.74   |
| MOPSO      | 12          | 131.115  | 19.6      | 0.44    |
| NSGA-II    | 12          | 120.315  | 19.3      | 0.30    |
| Overall    | 36          | 18.5     |           |         |

**Table 9.** The p-values of Kruskal–Wallis non-parametric test for the metrics.

| Metric       | Degree of freedom | H-Value | P-Value |
|--------------|-------------------|---------|---------|
| MID          | 2                 | 11.80   | 0.003   |
| Spacing      | 2                 | 7.47    | 0.024   |
| CPU time     | 2                 | 0.55    | 0.759   |
**Figure 11.** Interval plot of CENSGA with NSGA-II and MOPSO in terms of the MID metric.

**Figure 12.** Interval plot of CENSGA with NSGA-II and MOPSO in terms of the spacing metric.

*The pooled standard deviation is used to calculate the intervals.*
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Table 10. Summary of the output obtained in the Information Technology case study.

| Selected products supplied | Number of suppliers selected | Objective 1 | Objective 2 | Objective 3 | Objective 4 | Objective 5 |
|---------------------------|------------------------------|-------------|-------------|-------------|-------------|-------------|
| 5, 8, 10, 12, 15, 16, 17, 19, 22, 28, 32, 33, 37, 39 | 14 | 9.4E + 10 | 1.2E + 02 | 1.3E + 04 | 23.8 E + 04 | 9.3E + 03 |

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Conflict of interest statement

Declarations of interest: none.

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