Higgs inflation and scalar weak gravity conjecture

Yang Liu\textsuperscript{1,2,a}

\textsuperscript{1} School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, UK
\textsuperscript{2} Nottingham Centre of Gravity, University of Nottingham, Nottingham NG7 2RD, UK

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Abstract

In this article, we intend to find a specific model which can satisfy the further refining de Sitter swampland conjecture and scalar weak gravity conjecture (SWGC) simultaneously, in particular, Higgs inflation model and its two extensions: Higgs-dilaton model and Palatini Higgs inflation. We determine the conditions if the three inflation models satisfy scalar weak gravity conjecture (SWGC) and strong scalar weak gravity conjecture (SSWGC).

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1 Introduction

The swampland program is a very interesting development for the phenomenology of quantum gravity theories. In recent years, people have proposed the “de Sitter swampland conjecture” and “refined de Sitter swampland conjecture” considering the derivative of scalar field potentials, such as \cite{1,2}. If we consider a 4d theory of real scalar field $\phi^i$ coupled to gravity, whose dynamics is governed by a scalar potential $V(\phi^i)$, then the action is given by \cite{2,3}

$$S = \int d^4\sqrt{|g_4|} \left( \frac{M_p^2}{2} R_4 - \frac{1}{2} h_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V \right), \quad (1.1)$$

where $M_p$ is the Planck mass, $|g_4|$ is the determinant of the metric matrix of 4d spacetime, $R_4$ is the 4d Riemann curvature of spacetime and $h_{ij}$ is the metric on the target space of the scalar fields.

Firstly, we review the refined de Sitter conjecture. The conjecture states that an effective theory of quantum gravity, which is not in the swampland, should at least meet one of the following two constraints \cite{1,2}:

$$|\nabla V| \geq \frac{c_1}{M_p} \cdot V, \quad (1.2)$$

or

$$\min(\nabla_i \nabla_j V) \leq - \frac{c_2}{M_p^2} \cdot V, \quad (1.3)$$

where $c_1$ and $c_2$ are both positive constants of the order of 1. The first constraint, i.e., Eq. (1.2), corresponds to original “swampland conjecture”.

Furthermore, Andriot and Roupec proposed a further refining de Sitter swampland conjecture, which suggested that a low energy effective theory of a quantum gravity that takes the form (1.1) should satisfy \cite{2,3},

$$\left( M_p \frac{|\nabla V|}{V} \right)^q - a M_p^2 \frac{\min(\nabla_i \nabla_j V)}{V} \geq b \quad \text{with} \quad a + b = 1, \quad a, b > 0 \quad q > 2. \quad (1.4)$$

In our previous work, we have found that Higgs inflation model and its two extensions: Higgs-dilaton model and Palatini Higgs inflation all satisfy the refining de Sitter swampland conjecture \cite{3}. On the other hand, Palti proposed a generalisation of the Weak Gravity Conjecture in the presence of scalar fields (SWGC) \cite{4}. Moreover, Strong Scalar Weak Gravity Conjecture (SSWGC) has been proposed by Gonzalo and Ibáñez \cite{5}. The potential of scalar field $V(\Phi)$, which is coupled to...
quantum gravity, should meet certain conditions. We will review SWGC and SSWGC briefly in Sect. 3. The main goal of this work is to find specific inflation models which satisfy the two versions of SWGC and the further refining de Sitter swampland conjecture simultaneously, in particular, Higgs inflation model and its two extensions: Higgs-dilaton model and Palatini Higgs inflation.

The article is composed as follows: in Sect. 2, we review Higgs inflation model and its two extensions briefly. In Sect. 3, we review the two versions of scalar weak gravity conjecture and examine if these inflation models can satisfy the conjectures. In Sect. 4, the results we have obtained are discussed.

2 Higgs inflation and its extensions

In this section, we will review the Higgs inflation model and its two extensions briefly: Higgs-dilaton model and Palatini Higgs inflation [6].

2.1 Higgs inflation model

The total action of Higgs inflation is

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \xi H^2 H R + L_{SM} \right], \tag{2.1} \]

which contains two parameters, namely, the Higgs mass expectation value \( v_{EW} \approx 250 \) GeV and the reduced Planck mass \( M_p = 2.435 \times 10^{18} \) GeV [6]. When the value of field is very large, the Planck mass plays an important role for inflation [6–8]. If we take the unitary gauge, Higgs field can be written as \( H = (0, h)^T / \sqrt{2} \) [6]. Then the action Eq. (2.1) can be rewritten as

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \partial h^2 - U(h) \right], \tag{2.2} \]

where

\[ U(h) = \frac{\lambda}{4} (h^2 - v_{EW}^2)^2 \tag{2.3} \]

is the symmetry breaking potential in the Standard Model [6–8].

The action (2.2) can be reformulated in the Einstein frame by a Weyl transformation \( g_{\mu\nu} \to \Theta g_{\mu\nu} \) with [6]:

\[ \Theta^{-1} = 1 + \frac{h^2}{F_{\infty}^2}, \quad F_{\infty} \equiv M_p / \sqrt{\xi}. \tag{2.4} \]

Then action (2.2) in the Einstein frame, can be rewritten as [3,6–8]:

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} M_p^2 K(\Theta)(\partial \Theta)^2 - V(\Theta) \right], \tag{2.5} \]

which contains a non-canonical kinetic sector [3,6]:

\[ K(\Theta) = \frac{1}{4|a|\Theta^2} \left( 1 - \Theta^2 \right), \tag{2.6} \]

where

\[ a = -\frac{\xi}{1 + 6\xi}, \tag{2.7} \]

and a non-exactly flat potential [3,6]:

\[ V(\Theta) = U(\Theta)\Theta^2 = \frac{\lambda F_{\infty}^2}{4} \left[ 1 - \left( 1 + \frac{v_{EW}^2}{F_{\infty}^2} \right) \Theta^2 \right]. \tag{2.8} \]

2.2 Higgs-dilaton model

The existence of robust predictions in (non-critical) Higgs inflation is intimately related to the emerging dilatation symmetry of its tree-level action at large field values [3]. The uplifting of Higgs inflation to a completely scale-invariant setting was considered in several articles [9–13]. If we take the unitary gauge \( H = (0, h)^T / \sqrt{2} \), the action of the gravitational sector of the Higgs-dilaton model is given by

\[ S = \int d^4x \sqrt{-g} \left[ \frac{\xi}{2} h^2 + \frac{\xi}{2} \chi^2 \right] R - \frac{1}{2} \partial h^2 - \frac{1}{2} \partial \chi^2 - V(h, \chi), \tag{2.9} \]

where

\[ V(h, \chi) = \frac{\lambda}{4} (h^2 - \alpha \chi^2)^2 + \beta \chi^4 \tag{2.10} \]

is a scale-invariant version of the Standard Model symmetry breaking potential and \( \alpha, \beta \) positive dimensionless parameters [3,6,9–13].

The action in the Einstein frame can be obtained by a Weyl rescaling \( g_{\mu\nu} \to M_p^2 / (\ell_\hbar h^2 + \ell_\chi X^2) g_{\mu\nu} \) and a field redefinition, namely,

\[ y^{-2} \Theta = \frac{(1 + 6\ell_\hbar h^2 + (1 + 6\ell_\chi X^2)}{\ell_\hbar h^2 + \ell_\chi X^2}, \tag{2.11} \]

\[ \exp \left[ \frac{2y \Phi}{M_p} \right] \equiv \frac{a}{\bar{a}} \left( 1 + 6\ell_\hbar h^2 + (1 + 6\ell_\chi X^2) \right) \tag{2.12} \]

with

\[ y = \sqrt{\frac{\ell_\hbar h^2}{1 + 6\ell_\hbar h^2}}, \quad a = -\frac{\ell_\hbar h^2}{1 + 6\ell_\hbar h^2}, \quad \bar{a} = a \left( 1 - \frac{\ell_\hbar h}{\ell_\hbar h} \right). \tag{2.13} \]

After some algebra, we obtain an Einstein-frame action:

\[ S = \int d^4x \sqrt{-g} \left[ M_p^2 R - \frac{1}{2} M_p^2 K(\Theta)(\partial \Theta)^2 \right]. \tag{2.14} \]
\[-\frac{1}{2} \Theta (\partial \Phi)^2 - U(\Theta) \] \hspace{1cm} (2.14)

It contains a kinetic sector for \( \Theta \) field
\[
K(\Theta) = \frac{1}{4|\tilde{a}|^2} \left( \frac{c}{|\tilde{a}|} - c \right) + \frac{L}{\Theta} - \Theta - 1 - \Theta \] \hspace{1cm} (2.15)

which has two “inflationary” poles at \( \Theta = 0 \) and \( \Theta = c/|\tilde{a}| \) and a “Minkowski” pole at \( \Theta = 1 \) [6] (The “Minkowski” pole does not play a significant role during inflation and can be neglected for all practical purposes [6]) and a potential
\[
U(\Theta) = U_0(1 - \Theta)^2, \quad U_0 \equiv \frac{\lambda M^4}{4} \left( \frac{1 + 6\tilde{a}}{\tilde{a}} \right)^2, \] \hspace{1cm} (2.16)

where
\[
a \equiv -\frac{\xi_h}{1 + 6\xi_h}, \quad \tilde{a} \equiv a \left( 1 - \frac{\xi_h}{\xi_h} \right). \] \hspace{1cm} (2.17)

2.3 Palatini Higgs model

The action is minimized with respect to the metric in Higgs inflation model. This procedure implicitly assumes the inclusion of a York–Hawking–Gibbons term ensuring the cancellation of a total derivative term with no-vanishing variation at the boundary and the existence of a Levi–Civita connection depending on the metric tensor [6,14,15]. Then one could consider a Palatini formulation of gravity alternatively. In this formulation, no additional boundary term is required and the metric tensor and the connection are treated independently [6,14,15]. We consider the action (2.2) with
\[
R = g^{\mu \nu} R_{\mu \nu}(\Gamma, \partial \Gamma) \] and \( \Gamma \) a non-Levi–Civita connection in order to see explicitly. At \( \phi \gg v_{EW} \), the action in the Einstein frame can be obtained by a Weyl transformation \( g_{\mu \nu} \to \Theta g_{\mu \nu} \) and a field redefinition [3,6],
\[
S = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right], \] \hspace{1cm} (2.18)

with
\[
V(\phi) = \frac{\lambda}{4} F^4(\phi), \quad F(\phi) \equiv F_\infty \tanh \left( \frac{\sqrt{\tilde{a}} \phi}{M_\rho} \right). \] \hspace{1cm} (2.19)

One can find more details of this model in Ref. [6].

3 Higgs inflation and scalar weak gravity conjecture

In this section, we will check if Higgs inflation model and its two extensions satisfy scalar weak gravity conjecture. Firstly, we will review scalar weak gravity conjecture and its strong version briefly.

Palti formulated a first version of a Scalar Weak Gravity Conjecture (SWGC) [4]. Considering a particle \( H \) with mass \( m \) which is coupled to a light scalar \( \phi \) with a trilinear coupling proportional to \( \mu = \partial_\phi m \). SWGC states that the force mediated by \( \phi \) must be stronger than gravitational force and \( m^2(\phi) \) is considered as a function of \( \phi \) so that \( m^2 = V'' \), then we have
\[
(V(\phi))^2 \geq \frac{(V(\phi))^2}{M^2_p}. \] \hspace{1cm} (3.1)

Furthermore, Gonzalo and Ibáñez proposed a strong version of SWGC, i.e., SSWGC [5]. The conjecture states that the potential of any canonically normalized real scalar \( V(\phi) \) in the theory must satisfy for any value of the field the constraint:
\[
2V(\phi)^2 - V(\phi) V(\phi'') \geq \frac{(V(\phi))^2}{M^2_p}. \] \hspace{1cm} (3.2)

with primes denoting derivation with respect to \( \phi \).

Then, we will check if Higgs inflation model and its two extensions satisfy scalar weak gravity conjecture. In the following section, we take \( M_p = 1 \).

3.1 Higgs inflation model

Since Eqs. (3.1) and (3.2) are only for canonically-normalized scalar field, we need to redefine the field as
\[
\frac{d \phi}{d \Theta} = \sqrt{K(\Theta)}, \] \hspace{1cm} (3.3)

and
\[
\Phi = \mathcal{F}(\Theta) \quad \Theta = \mathcal{F}^{-1}(\Phi), \] \hspace{1cm} (3.4)

\[
\frac{d \mathcal{F}^{-1}(\Phi)}{d \Phi} = \frac{d \Theta}{d \Phi} = \frac{1}{\sqrt{K(\Theta)}} = \frac{1}{\sqrt{\frac{1}{4|\tilde{a}|^2} \left( 1 - 6|\tilde{a}| \right) - 1/\Theta}}. \] \hspace{1cm} (3.5)

Moreover, the scalar field potential \( V(\Theta) \) can be denoted as \( \tilde{V}(\Phi(\Theta)) \). Then we can obtain that:
\[
\tilde{V}^{(1)}(\Phi(\Theta)) = -\frac{\lambda}{2} F_\infty^2 \left[ 1 + \frac{v_{EW}^2}{F_\infty^2} \left( 1 - \left( 1 + \frac{v_{EW}^2}{F_\infty^2} \right) \phi \right) \right] \] \hspace{1cm} (3.6)

\[
\tilde{V}^{(2)}(\Phi(\Theta)) = \frac{\lambda}{2} F_\infty^2 \left[ 1 + \frac{v_{EW}^2}{F_\infty^2} \right] \left( \phi - \frac{1}{2} \right) \left( 1 + \frac{v_{EW}^2}{F_\infty^2} \right) \] \hspace{1cm} (3.7)

\[
\tilde{V}^{(3)}(\Phi(\Theta)) = \frac{\lambda}{2} F_\infty^2 \left[ 1 + \frac{v_{EW}^2}{F_\infty^2} \right] \left( \phi - \frac{1}{2} \right) \left( 1 + \frac{v_{EW}^2}{F_\infty^2} \right) \] \hspace{1cm} (3.8)
\[
\left( \frac{d\Theta}{d\Phi} \right) \left[ \frac{d\Phi}{d\Psi} \right] = \frac{d\Theta}{d\Psi} \left( \frac{d\Theta}{d\Phi} \right).
\]

(3.8)

\[
\tilde{V}^{(4)}(\Phi(\Theta)) = \frac{5}{2} \lambda F^2_\infty \left( 1 + \frac{v_{EW}^2}{F^2_\infty} \right) \left( \frac{d\Theta}{d\Phi} \right)^2 \left[ \frac{d\Phi}{d\Psi} \right] \left( \frac{d\Phi}{d\Theta} \right)^2 \left( \frac{d\Theta}{d\Phi} \right). 
\]

(3.15)

\[
\tilde{U}^{(4)}(\Psi(\Theta)) = 4U_0 \left( \frac{d\Theta}{d\Psi} \right)^2 \left[ \frac{d\Phi}{d\Psi} \right] \left( \frac{d\Phi}{d\Theta} \right)^2 
\]

(3.16)

Therefore, if Higgs inflation model satisfies scalar weak gravity conjecture, then the potential \( \tilde{V}(\Phi(\Theta)) \) should obey Eq. (3.1), namely, \( (\tilde{V}^{(3)})^2 \geq (\tilde{V}^{(2)})^2 \). If Higgs inflation model satisfies strong scalar weak gravity conjecture, then the potential \( \tilde{V}(\Phi(\Theta)) \) should obey Eq. (3.2), namely, \( 2(\tilde{V}^{(3)})^2 - \tilde{V}^{(2)} \tilde{V}^{(4)} \geq (\tilde{V}^{(2)})^2 \).

3.2 Higgs-dilaton model

Similarly, we need to redefine the field as

\[
\frac{d\Psi}{d\Theta} = \sqrt{K(\Theta)}
\]

(3.10)

and

\[
\Psi = \mathcal{H}(\Theta) \quad \Theta = \mathcal{H}^{-1}(\Psi),
\]

(3.11)

\[
\frac{d\mathcal{H}^{-1}(\Phi)}{d\Phi} = \frac{d\Theta}{d\Phi} = \frac{1}{\sqrt{K(\Theta)}} = \frac{1}{\sqrt{\frac{1}{4}e^{\frac{1}{3}\Theta} - \frac{1}{6}e^{\frac{1}{2}\Theta}}}.
\]

(3.12)

Moreover, the scalar field potential \( U(\Theta) \) can be denoted as \( \tilde{U}(\Psi(\Theta)) \). Thus we can obtain that:

\[
\tilde{U}^{(1)}(\Psi(\Theta)) = 2U_0(\Theta - 1) \left( \frac{d\Theta}{d\Psi} \right)^2,
\]

(3.13)

\[
\tilde{U}^{(2)}(\Psi(\Theta)) = 2U_0 \left( \frac{d\Theta}{d\Psi} \right)^2 + 2U_0(\Theta - 1) \left[ \frac{d\Theta}{d\Psi} \left( \frac{d\Theta}{d\Psi} \right) \right],
\]

(3.14)

\[
\tilde{U}^{(3)}(\Psi(\Theta)) = 4U_0 \left( \frac{d\Theta}{d\Psi} \right)^2 \left[ \frac{d\Theta}{d\Theta} \left( \frac{d\Theta}{d\Psi} \right) \right] + 2U_0 \left( \frac{d\Theta}{d\Psi} \right)^2 \left[ \frac{d\Theta}{d\Theta} \left( \frac{d\Theta}{d\Psi} \right) \right]
\]

(3.15)

Therefore, if Higgs-dilaton model satisfies scalar weak gravity conjecture, then the potential \( \tilde{U}(\Psi(\Theta)) \) should obey Eq. (3.1), namely, \( (\tilde{U}^{(3)})^2 \geq (\tilde{U}^{(2)})^2 \). If Higgs-dilaton model satisfies strong scalar weak gravity conjecture, then the potential \( \tilde{U}(\Psi(\Theta)) \) should obey Eq. (3.2), namely, \( 2(\tilde{U}^{(3)})^2 - \tilde{U}^{(2)} \tilde{U}^{(4)} \geq (\tilde{U}^{(2)})^2 \).

3.3 Palatini Higgs model

From Eq. (2.16), we can obtain

\[
V^{(1)}(\phi) = \lambda \sqrt{\alpha} F^4_{\infty} \frac{\sinh^3(\sqrt{\alpha}\phi)}{\cosh^3(\sqrt{\alpha}\phi)},
\]

(3.17)

\[
V^{(2)}(\phi) = \lambda a F^4_{\infty} \left[ \frac{3 \sinh^2(\sqrt{\alpha}\phi)}{\cosh^4(\sqrt{\alpha}\phi)} - \frac{5 \sinh^4(\sqrt{\alpha}\phi)}{\cosh^6(\sqrt{\alpha}\phi)} \right] \equiv \frac{\lambda a F^4_{\infty}}{\cosh^6(\sqrt{\alpha}\phi)} A,
\]

(3.18)

where

\[
A = 3 \sinh^2(\sqrt{\alpha}\phi) \cosh^2(\sqrt{\alpha}\phi) - 5 \sinh^4(\sqrt{\alpha}\phi).
\]

(3.19)
Then

\begin{equation}
V^{(3)}(\phi) = -\frac{6\lambda a^{3/2}F_\infty^4}{\cosh^3(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) A + \frac{\lambda a F_\infty^4}{\cosh^6(\sqrt{a}\phi)} A', \tag{3.20}
\end{equation}

\begin{equation}
V^{(4)}(\phi) = \frac{42\lambda^2 a^2 F_\infty^4}{\cosh^3(\sqrt{a}\phi)} \sinh^2(\sqrt{a}\phi) A - \frac{6\lambda a^2 F_\infty^4}{\cosh^6(\sqrt{a}\phi)} A + \frac{12\lambda a^{3/2} F_\infty^4}{\cosh^3(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) A' + \frac{\lambda a F_\infty^4}{\cosh^6(\sqrt{a}\phi)} A''. \tag{3.21}
\end{equation}

Thus we have

\begin{equation}
(V^{(2)})^2 = \frac{\lambda^2 a^2 F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} A^2, \tag{3.22}
\end{equation}

\begin{equation}
(V^{(3)})^2 = \frac{36\lambda^2 a^3 F_\infty^8}{\cosh^{14}(\sqrt{a}\phi)} \sinh^2(\sqrt{a}\phi) A^2 + \frac{\lambda^2 a^2 F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} A^2 - \frac{12\lambda a^{3/2} F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) A'' \tag{3.23}
\end{equation}

\begin{equation}
V^{(2)} + V^{(4)} = \frac{\lambda a F_\infty^4}{\cosh^4(\sqrt{a}\phi)} A + \frac{42\lambda a^2 F_\infty^4}{\cosh^8(\sqrt{a}\phi)} \sinh^2(\sqrt{a}\phi) A - \frac{6\lambda a^2 F_\infty^4}{\cosh^6(\sqrt{a}\phi)} A - \frac{12\lambda a^{3/2} F_\infty^4}{\cosh^3(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) A' + \frac{\lambda a F_\infty^4}{\cosh^6(\sqrt{a}\phi)} A''. \tag{3.24}
\end{equation}

and

\begin{equation}
V^{(2)}(V^{(2)}) + V^{(4)} = \frac{\lambda^2 a^2 F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} A^2 + \frac{42\lambda^2 a^3 F_\infty^8}{\cosh^{14}(\sqrt{a}\phi)} \sinh^2(\sqrt{a}\phi) A^2 - \frac{6\lambda a^2 F_\infty^4}{\cosh^{12}(\sqrt{a}\phi)} A^2 - \frac{12\lambda a^{3/2} F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) AA' + \frac{\lambda^2 a^2 F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} AA'' \tag{3.25}
\end{equation}

If Polyakov-Higgs inflation satisfies SWGC, then based on Eqs. (3.1), (3.20) and (3.21), we have

\begin{equation}
\frac{36\lambda^2 a^3 F_\infty^8}{\cosh^{14}(\sqrt{a}\phi)} \sinh^2(\sqrt{a}\phi) A^2 + \frac{\lambda^2 a^2 F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} A^2 - \frac{12\lambda a^{3/2} F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) AA' \tag{3.26}
\end{equation}

If Polyakov-Higgs inflation satisfies SSWGC, then based on Eqs. (3.2), (3.21) and (3.23), we have

\begin{equation}
\frac{72\lambda^2 a^3 F_\infty^8}{\cosh^{14}(\sqrt{a}\phi)} \sinh^2(\sqrt{a}\phi) A^2 + \frac{2\lambda^2 a^2 F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} A^2 - \frac{24\lambda a^{3/2} F_\infty^8}{\cosh^{12}(\sqrt{a}\phi)} \sinh(\sqrt{a}\phi) AA' \tag{3.27}
\end{equation}

Equations (3.24) and (3.25) are the constraints of Polyakov Higgs inflation which meet the two versions of scalar weak gravity conjecture.

4 Conclusions and discussions

In Ref. [4], Pali proposed a first version of a Scalar Weak Gravity Conjecture (SWGC). SWGC states that the force mediated by a light scalar \(\phi\) must be stronger than gravitational force and \(m^2(\phi)\) is considered as a function of \(\phi\) so that \(m^2 = V''(\phi)\), then the potential \(V(\phi)\) should satisfy

\begin{equation}
(V^{(3)})^2 \geq \frac{(V^{(2)})^2}{M_p^2}. \tag{4.1}
\end{equation}

Furthermore, Gonzalo and Ibáñez proposed a strong version of SWGC, namely, SSWGC [5]. SSWGC states that the potential of any canonically normalized real scalar \(V(\phi)\) in the theory must satisfy for any value of the field the constraint:

\begin{equation}
2(V^{(3)})^2 - V^{(2)2} \geq \frac{(V^{(2)})^2}{M_p^2}, \tag{4.2}
\end{equation}

with primes denoting derivation with respect to \(\phi\).

Higgs inflation model has important phenomenological meaning [6, 7]. In our previous work [3], we have found that Higgs inflation model and its two extensions: Higgs-dilaton
model and Palatini Higgs inflation all could satisfy the refining de Sitter swampland conjecture [3]. Furthermore, in this work we intend to explore if Higgs inflation model and its two variations can satisfy the two versions of scalar weak gravity conjecture or not. Based on the Lagrangian of these models, we have obtained the constraints for Higgs inflation model, Higgs-dilaton model and Palatini Higgs inflation if they could satisfy the further refining dS swampland conjecture and two versions of SWGC simultaneously.

Future work can be directed along at least three lines of further research. Firstly, since the three inflation models could satisfy two versions of SWGC and the refining de Sitter swampland conjecture simultaneously, they have the potential to be a "real" inflation model of the universe. We should focus on how to derive this inflation model from string theory. Secondly, we should find more inflation models which could satisfy these two kinds of conjectures simultaneously. Thirdly, we should find the common properties of the models which have been found in the second step.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This is a theoretical work, so we have no data to be deposited.]

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