Flexible Continuous-Time Modeling for Multi-Objective Day-Ahead Scheduling of CHP Units

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Abstract: Increasing applications of CHP units have turned the problem of finding the best optimization model into a significant subject for scholars. In this respect, this paper is aimed at driving a novel formulation to the multi-objective day-ahead scheduling of CHP units using Bernstein polynomials, which more optimally schedules power and heat generations as well as ramping trajectories. This procedure includes yielding an affine function that closely approximates real-time net-load and generation trajectories, which is demonstrated to have a superior performance to the conventional hourly day-ahead scheduling of CHP units based on discrete-time approximation. The problem of how to handle various objective functions by function space method is also addressed. The simulations conducted on the sample test systems, which consist of CHP systems, thermal and heat-only units, as well as thermal and electrical loads, show that the suggested multi-objective model can perfectly cover the total heat and electrical loads in terms of economic and environmental criteria. More importantly, the results indicate that the accuracy of the proposed approach renders cost saving of 1.67% and emission saving of 1.46% in comparison with the conventional hourly-based model, apart from leading to fewer ramping scarcities in real-time operations.

Keywords: continuous-time optimization model; combined heat and power (CHP) system; multi-objective optimization; $\varepsilon$-constraint; Bernstein polynomial

1. Introduction

Nowadays, the ever-increasing need for electric energy power, especially in developing countries, has led to an increase in the consumption of fossil fuels. The growing tendency for fossil fuel together with the environmental concerns they induce by air pollution in power generation have compelled operators to use distributed energy resources [1,2], such as wind power, solar energy, etc. Among all these resources, thermal power plants have been widely regarded. Flexibility and reliable performance of combined heat and power (CHP) [3] units with great efficiency and high security are among the main reasons these units have been holding so much appeal among all available thermal power units [4,5]. The goal of the CHP system is to simultaneously provide electrical and thermal energies for loads. Hence, while thermal units operate, the CHP system captures squandered heat and utilizes this energy to satisfy thermal loads [6]. Furthermore, the extra heat from an electric power plant can be utilized for industrial aims or other goals, which boosts the efficiency of the system [7]. For these reasons and more, CHP units are widely used in the planning and operation of power systems. Given that biomass is being promoted to substitute coal for CHP plants, the importance of large-scale CHP plants in a 100% renewable energy context will be fully recognized in the near future [8].

Accordingly, in order to gain an acceptable power system operation and planning, disparate features such as minimizing fuel costs, losses, and environmental pollution, must be considered, to name but a few. In this respect, some studies have been conducted to co-optimize the CHP units. For instance, a multi-objective stochastic model is proposed...
for scheduling CHP units based on microgrid (MG) considering uncertainties in load and wind power in [5]. Another multi-objective electric model is developed in [9] based on the integration of thermal units, heat-only, and CHP units, where the objective functions include minimization of total cost and maximization of heat generation, and the normal boundary intersection method is applied to solve the model. Moreover, a novel optimal operation of a CHP-based MG is presented in [10] regarding energy storage system and demand response in the form of a multi-objective self-scheduling optimization problem. Both convex and nonconvex models, together with the multi-objective issues, need suitable approaches to cope with the multi-objective issue and both rely on finding a well-organized set of Pareto optimal solutions [11]. Therefore, various methods are expounded to solve multi-objective optimization problems (MOOPs), among all of which, the $\varepsilon$-constrained method is highly practical in handling multi-objective problems [12].

The $\varepsilon$-constrained method is an extensive problem-solving technique to cope with multi-objective issues in which the Pareto front can be generated exactly by changing the $\varepsilon$-vector, and all but one of the objective functions are transformed into constraints. By way of example, the $\varepsilon$-constraint method is utilized to solve the problem of self-scheduling home energy management systems, which is defined as a multi-objective optimization problem in [13].

On the other side, some studies have been conducted on the scheduling of CHP units. For example, the problem of optimal day-ahead scheduling of CHP units with electric and thermal energy storages regarding security constraints is discussed in [14], which includes the deterministic optimal day-ahead unit commitment of generation resources relying on system loads. In another study, a whale optimization algorithm is presented as a new method for solving CHP economic dispatch issues [15]. Moreover, a new perspective is proposed based on microgrid in [16] to resolve the economic dispatch of CHP units, thermal units, heat-only units, wind turbines, photovoltaic systems, and battery storage, where AC optimal power flow is utilized. In addition, the information gap theory is applied to handle the uncertainties of generating units, market price and load. In [17], optimal operation of CHP units in a microgrid is discussed based on minimizing the total costs and contenting heat demand. Likewise, it is indicated that taking the feasible operation region (FOR) of CHPs into consideration, heat and power outputs are inseparable and rest on each other in a manner that one output affects the other one. The stochastic fractal search, which is a novel optimization solver, is utilized to solve CHP economic dispatch in [18]. On the other hand, some studies have been carried out on the linearized formulation of thermal units [19] and CHP units either in the economic dispatch issue or unit commitment [20,21]. A number of state-of-the-art optimal dispatch reviews of CHP in integrated energy systems and CHP operation can be found in [22–24].

Continuous-time modeling, rather than discrete-time modeling, is a new subfield of power systems modeling in which the problem is solved in a continuous-time framework with linear objective functions and linear constraints. A trajectory of the system’s state over time is smoothly given everywhere in the phase space by integrating a continuous-time model over time, leading to the fact that instantaneous abrupt changes, which are inevitable in discrete-time models, would not be seen in continuous-time models.

This technique was firstly presented in [25,26]. In this way, to solve the continuous-time infinite-dimensional problem, developing a numerically stable way based on decreasing the dimensionality of the continuous-time problem is necessary. The suggested method in these references is a function space-based method with Bernstein polynomials which has been suggested for modeling the other upcoming problems. For example, an optimization scheduling of thermal generating units integrated with energy storage in a continuous-time framework is discussed in [27], where a continuous-time Gaussian technique is applied to cope with load uncertainty. The outcomes show the proficiency of their presented technique, which reduces total cost. Additionally, a novel formulation is presented in [28] for the locational marginal price of electricity, which is the precisely combined fluctuation of load demand and operation constraints of power network in evaluating market price. Their
suggested method converts the problem into a continuous-time optimal control problem, where the numerical results depict the flexibility and accuracy of the proposed method. Furthermore, a new optimization model is represented in [29] for balancing energy and flexible ramp products.

Despite the differences in formulation and solution techniques, current CHP scheduling models have deployed the discrete-time load and generation trajectories which lead to an obvious contrast between real-time operation and day-ahead operation. Therefore, in order to attain any degree of flexibility or economical operation of CHP units, accurate day-ahead scheduling modeling is demanded.

To bridge the above research gaps, a novel and flexible scheduling model is presented in this study for CHP units, which contains a linear formulation of thermal heat-only units and CHP units, while a continuous-time optimization model is applied to schedule CHP units for the first time in literature. To be more specific, a novel technique has been applied as a continuous-time model for the CHP systems in day-ahead power systems operation. As a matter of fact, the multi-objective scheduling of CHP integrated with thermal and heat-only units is solved by means of the epsilon constraint method in a continuous-time framework, where the objective functions comprise cost and emission. In fact, a flexible optimal model is found to schedule the generating units in comparison with day-ahead scheduling. It is worth noting that although solution time of the represented model is a bit more compared to the hourly model, it engenders a higher accuracy, not to mention that the total cost is considerably lower compared to the hourly model. The presented technique produces a single-objective function by transforming all but one of the objectives into constraints. A function space method is then suggested to handle the problem.

The rest of the paper is organized as follows: The general framework of the discrete-time and continuous-time multi-objective CHP planning are presented in Section 2, while Section 3 presents the use of Bernstein polynomials to model the problem in detail. The numerical results of applying the suggested model on a test system are presented in Section 4, followed by summarizing the conclusions in Section 5.

2. Problem Modeling

2.1. Discrete-Time Multi-Objective CHP Scheduling

The goal here is to optimize the multi-objective operation problem of a CHP-based system to obtain an optimal resource scheduling of the system over the day-ahead scheduling horizon $T = [0, T]$. Discrete-time scheduling subdivides the day-ahead time horizon into $M$ intervals, $T = \bigcup_{m=0}^{M-1} T_m, T_m = [t_m, t_{m+1})$ with the same length $T_m = t_{m+1} - t_m$ where $t_0 = 0$ and $t_M = T$. The objective of this model is to minimize the overall generation cost for energy as well as the atmospheric pollutants, supplying both thermal and electrical load demands with respect to several constraints. In this study, it is assumed that spare heat is wasted if the produced heat is more than the heat demand, and it is provided with auxiliary boilers if it is less than the heat demand. Accordingly, three types of units are considered: power-only units, CHP-based units, and heat-only units.

2.1.1. Multi-Objective Programming Based on $\epsilon$-Constraint Method

Decision-making problems mostly rely on multiple criteria and can infrequently be defined based on a well-defined single objective. In fact, although, multiple facets of a decision process can be put into a single objective, this simplification hardly captures the complexity of real-world decision applications. Given that solving multi-criteria decision issues is not as simple as single-criterion issues, finding a single solution that satisfies all constraints and optimizes different objective functions is not simply possible. As a result, providing a set of non-dominated solutions and finding the “most preferred” alternative among the obtained solutions can be the best bet. As a matter of fact, the $\epsilon$-constraint method is one of the best intuitive approaches to produce the Pareto optimal solutions,
which takes into account the user’s preferences. This method keeps one of the objectives only (the most important one) and restricts the rest of the objectives within user-specific values ($f_i(x, u) \leq \varepsilon_i$). A generalized hourly trigeneration planning framework to arrange the optimal operating strategy is formulated as follows.

2.1.2. Objective Functions

1. Minimizing the total cost of operation ($F_{\text{total}}^{\text{Cost}}$): The primary objective function aims at minimizing the operational cost of the system including the production cost of thermal units, CHP units, and heat-only units for a definite time horizon. Such an objective function can be described as:

$$\min \sum_{t_m \in \mathcal{T}} \left[ F_{\text{Cost}}^{\text{Total}}(P(t_m), H(t_m), c, x(t_m)) + F_{\text{Emission}}^{\text{Total}}(P(t_m), H(t_m)) \right]$$ (1)

where

$$F_{\text{Cost}}^{\text{Total}}(P(t_m), H(t_m), c, x(t_m)) = f_{\text{TH}}^{\text{Cost}}(P_{\text{TH}}(t_m)) + \sum_{i \in I} f_{\text{Cost}}^{\text{CHP}}(c, x(t_m)) + f_{\text{Cost}}^{\text{H}}(H_{\text{H}}(t_m)), \quad t_m \in \mathcal{T}$$ (2)

and

$$F_{\text{Emission}}^{\text{Total}}(P(t_m), H(t_m)) = f_{\text{Emission}}^{\text{TH}}(P_{\text{TH}}(t_m)) + f_{\text{Emission}}^{\text{CHP}}(P_{\text{CHP}}(t_m)) + f_{\text{Emission}}^{\text{H}}(H_{\text{H}}(t_m)), \quad t_m \in \mathcal{T}$$ (3)

The total non-linear cost of traditional thermal power units can be approximated either as a quadratic convex or as piecewise-linear functions. The quadratic form can be formulated as below [30]:

$$f_{\text{Cost}}^{\text{TH}}(P_{\text{TH}}(t_m)) = \sum_{i \in \mathcal{I}} c_{2,\text{TH}}^i P_{i,\text{TH}}^2(t_m) + c_{1,\text{TH}}^i P_{i,\text{TH}}(t_m) + c_{0,\text{TH}}, \quad t_m \in \mathcal{T}$$ (4)

The heat and power produced by CHP plants are highly coupled and are not generated at similar efficiencies and prices. The maximum efficiency of a cogeneration unit can be reached at the full extent of power and proportional heat. The so-called characteristic points of the feasible area comprise these kinds of operating points. A representative operating area is illustrated in Figure 1 with 6 characteristic points in terms of cost ($c$), power ($p$), and heat ($h$). An operating region is convex if, given any two points, it contains the entire line segment that joins them. Accordingly, the feasible area is supposed to be convex and the cost can be also considered a convex function of the generated power and heat [31]. Based on this assumption, the total cost of a CHP unit can be defined as convex combinations of the corner points as below [32].

$$f_{\text{Cost}}^{\text{CHP}}(c, x(t_m)) = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{C}} c_{j}^i x_{j}^i(t_m), \quad t_m \in \mathcal{T}$$ (5)

![Figure 1](image-url). Characteristic points (A–E) and approximate heat-power operating region for a CHP unit.
Here, the production cost of the CHP \( f_{CHP} \) can be defined as a combination of fuel costs, service costs, taxes, etc. This single plant model can be used as a basis for both the hourly multiple plant CHP planning model and the multi-period planning model.

Generally, in order to supply high heat load demands and increase the flexibility of CHP units, heat-only units are added to the system. The quadratic convex cost function of the formulation \([25]\).

As mentioned before, the traditional CHP planning problem includes discrete-time \( T \) intervals to the same \( M \) intervals \( T_{m} = [t_{m}, t_{m+1}) \), \( T = \bigcup_{m=0}^{M-1} T_{m} \) with length \( T_{m} = t_{m+1} - t_{m} \), e.g., hourly. In this way, the upshot power and heat generation schedule to meet discrete-time load samples at minimum cost and emission and sequentially to solve the traditional \( M \) single-period economic dispatch problems \([34]\).

The use of a discrete-time CHP planning model implies that units must track piecewise constant generation trajectories from one schedule to the next, where the units’ ramping are obtained from the finite difference between two successive time steps. Obviously, not only can continuous-time scheduling appropriately model faster variations of the net-load to avoid ramping scarcity events, but it can also allow a greater flexibility for generating units.

Defining the continuous-time generation trajectories \( P(t) = [P_{TH}(t), P_{CHP}(t)] \) and \( H(t) = [H_{CHP}(t), H_{H}(t)] \), the continuous-time multi-objective CHP scheduling problem
over the day-ahead scheduling horizon $\mathcal{T}$ subject to the operating constraints can be formulated as follows:

$$\min \int_{\mathcal{T}} \left( \frac{\text{Total Cost}}{\text{Emission}} (\mathbf{P}(t), \mathbf{H}(t), \mathbf{c}, \mathbf{x}(t)) \right) dt$$

subject to

$$\int_{\mathcal{T}} \left( \frac{\text{Emission}}{\text{Cost}} (\mathbf{P}(t), \mathbf{H}(t)) \right) dt \leq \varepsilon_2, \quad t \in \mathcal{T}, n_2 = 0, 1, \ldots, p_2$$

$$\frac{d}{dt} \mathbf{P}_{\text{TH}}(t) = \mathbf{P}_{\text{TH}}(t), \quad t \in \mathcal{T}$$

$$\frac{d}{dt} \left( \text{times} (\mathbf{p}_{\text{CHP}}^T, \mathbf{x}_{\text{CHP}}(t)) \right) = \mathbf{p}_{\text{CHP}}(t), \quad t \in \mathcal{T}$$

$$\mathbf{P}_{\text{TH}}(t) = \mathbf{1}_{\text{N}_{\text{TH}}}^T \mathbf{P}_{\text{TH}}(t) + \mathbf{1}_{\text{N}_{\text{CHP}}}^T \mathbf{P}_{\text{CHP}}(t), \quad t \in \mathcal{T}$$

$$\mathbf{H}_{\text{heat}}(t) = \mathbf{1}_{\text{N}_{\text{H}}}^T \mathbf{H}(t) + \mathbf{1}_{\text{N}_{\text{CHP}}}^T \mathbf{H}_{\text{CHP}}(t), \quad t \in \mathcal{T}$$

$$\mathbf{P}_{\text{CHP}}(t) = \text{diag} (\mathbf{p}_{\text{CHP}}^T, \mathbf{x}_{\text{CHP}}^T(t)), \quad t \in \mathcal{T}$$

$$\mathbf{H}_{\text{CHP}}(t) = \text{diag} (\mathbf{h}_{\text{CHP}}^T, \mathbf{x}_{\text{CHP}}^T(t)), \quad t \in \mathcal{T}$$

$$\mathbf{x}_{\text{CHP}}(t) \mathbf{1}_L = 1, \quad t \in \mathcal{T}$$

$$0 \leq \mathbf{x}_{\text{CHP}}(t) \leq 1, \quad t \in \mathcal{T}$$

$$\dot{\mathbf{P}}_{\text{TH}}(t) \leq \mathbf{P}_{\text{TH}}(t) \leq \dot{\mathbf{P}}_{\text{TH}}, \quad t \in \mathcal{T}$$

$$\dot{\mathbf{P}}_{\text{CHP}}(t) \leq \mathbf{P}_{\text{CHP}}(t) \leq \dot{\mathbf{P}}_{\text{CHP}}, \quad t \in \mathcal{T}$$

Here, the ramping trajectories of thermal and CHP units are described in (14) and (15), wherein times($\mathbf{p}_{\text{CHP}}, \mathbf{x}_{\text{CHP}}(t)$) is a matrix obtained by multiplying the $\mathbf{p}_{\text{CHP}}$ and $\mathbf{x}_{\text{CHP}}(t)$ arrays element by element. The total operation cost (12) is equal to (2), representing the real power generation cost of thermal units (4) plus the total cost of the CHP units (5) and heat-only units (6). The $\varepsilon$-constraint is applied to the secondary objective function by (13). (15) and (16) represent the continuous-time real power and heat balance constraints, respectively, wherein $\mathbf{1}_{\text{N}_{\text{TH}}}, \mathbf{1}_{\text{N}_{\text{CHP}}}$, and $\mathbf{1}_{\text{N}_{\text{H}}}$ signify $N_{\text{TH}}, N_{\text{CHP}}$, and $N_{\text{H}}$-dimensional vectors of ones. (18)–(21) are the box constraints of the power and heat trajectories.

Moreover, (22) and (23) embody the relationship between power production and characteristic points as well as heat production and characteristic points of the CHP units, wherein diag() yields a column vector of the main diagonal elements of multiplying $\mathbf{h}_{\text{CHP}}^T$ and $\mathbf{x}_{\text{CHP}}^T(t)$. Convexity and non-negativity constraints for CHPs are also enforced in (24) and (25), wherein $\mathbf{1}_L$ represents the L-dimensional vector of ones. (26) and (27) signify
the ramping trajectories of thermal and CHP units over \( \mathcal{T} \), where \( \dot{P}_{TH}(t) \) and \( \dot{P}_{CHP}(t) \) are obtained as follows,

\[
\frac{d}{dt} P_{TH}(t) = \lim_{m \to 0} \frac{P_{TH}(t_{m+1}) - P_{TH}(t_m)}{T_m}, \quad t \in \mathcal{T}
\]

\[
\frac{d}{dt} P_{CHP}(t) = \lim_{m \to 0} \frac{(\text{times}(p_{CHP}, x_{CHP}(t_{m+1}))) - (\text{times}(p_{CHP}, x_{CHP}(t_m)))}{T_m}, \quad t \in \mathcal{T}
\]

3. Cubic Spline Model for Multi-Objective Scheduling of CHPs

Although modeling in infinite-dimensional spaces is more precise, it is, in general, more computationally intractable. Under suitable assumptions, let us develop an approximation bridge from the infinite-dimensional linear programming to tractable finite convex models in which the performance of the approximation is computed clearly. In this paper, modeling is proposed by means of the cubic spline function space spanned by Bernstein polynomials, which enables making up a function and refining the approximation to any desired precision while also providing additional flexibility to fit the continuous-time load variations [25]. Bernstein polynomials are first defined to provide a foundation for the rest of the paper. The Bernstein-basis polynomials of degree \( Q \) include \( Q + 1 \) polynomials can be described as:

\[
b_{q}^{(Q)}(t) = \binom{Q}{q} t^{q} (1-t)^{(Q-q)}, \quad q = 0, \ldots, Q, \quad t \in [0,1]
\]

where \( \binom{Q}{q} \) is the binomial coefficient.

A convex combination of Bernstein-basis polynomials is called a Bernstein polynomial. The Bernstein polynomial (degree \( Q \)) of a continuous-time function on the interval \([0, 1]\) is described as:

\[
B^{Q}(f)(t) = \sum_{q=0}^{Q} f\left(\frac{q}{Q}\right) b_{q}^{(Q)}(t)
\]

Figure 2a simply displays Bernstein-basis polynomials of degree 3 on the unit square and Figure 2b displays an approximation of a continuous function as a linear combination of Bernstein basis polynomials of degree 3.

![Figure 2](image)

**Figure 2.** (a) Bernstein-basis polynomials of degree 3; (b) approximation of a continuous function using Bernstein basis polynomial of degree 3.

**Notation 1.** The smallest convex set \( L \) (shaded in Figure 2b) that contains the control points (marked by squares) is the so-called “convex hull”, inside which the whole curve is confined.

**Notation 2.** Considering \( M \) subdivisions of the scheduling horizon, a vector of Bernstein-basis functions over the entire \( \mathcal{T} \) can be obtained as:

\[
v_{m(Q+1)+q}^{(Q)}(t) = b_{q}^{(Q)}\left(\frac{t - t_m}{T_m}\right), \quad t \in [t_m, t_{m+1}]
\]
3.1.1. Continuous-Time Modeling of the Generation/Demand Trajectories

The projection of generation/demand trajectory on the Bernstein function space is defined in (34)–(36):

\[
\begin{align*}
P_{TH}(t) &= P_{TH} v^{(Q)}(t), & P_{CHP} &= P_{CHP} v^{(Q)}(t) \quad (34) \\
H_{H}(t) &= H_{H} v^{(Q)}(t), & H_{CHP} &= H_{CHP} v^{(Q)}(t) \quad (35) \\
P_{load}(t) &= P_{load} v^{(Q)}(t), & H_{heat}(t) &= H_{heat} v^{(Q)}(t) \quad (36)
\end{align*}
\]

where \( P_{TH}, P_{CHP}, \) and \( H_{H} \) are the \( N_{TH} \times (M(Q + 1)) \), \( N_{CHP} \times (M(Q + 1)) \), and \( N_{H} \times (M(Q + 1)) \) mapping matrices associated with the thermal units' power, CHP units' power and heat, and heat-only units' heat generation, respectively. Bernstein coefficient row vectors of the electric and heat demand are described by the \((M(Q + 1))\)-dimensional vectors, respectively.

3.1.2. Continuous-Time Modeling of the Power Balance Equations

Considering these coefficients, power balance equations can be reformulated based on Bernstein polynomials as (37) and (38) and then (39) and (40):

\[
\begin{align*}
P_{load} v^{(Q)}(t) &= 1_{N_{TH}}^{T} P_{TH} v^{(Q)}(t) + 1_{N_{CHP}}^{T} P_{CHP} v^{(Q)}(t), & t \in \mathcal{T} \quad (37) \\
H_{heat} v^{(Q)}(t) &= 1_{N_{H}}^{T} H_{H} v^{(Q)}(t) + 1_{N_{CHP}}^{T} H_{CHP} v^{(Q)}(t), & t \in \mathcal{T} \quad (38)
\end{align*}
\]

 Bernstein-basis functions can be removed from both sides of (33) and (34). Thus, the following equations are obtained:

\[
\begin{align*}
P_{load} &= 1_{N_{TH}}^{T} P_{TH} + 1_{N_{CHP}}^{T} P_{CHP}, & t \in \mathcal{T} \quad (39) \\
H_{heat} &= 1_{N_{H}}^{T} H_{H} + 1_{N_{CHP}}^{T} H_{CHP}, & t \in \mathcal{T} \quad (40)
\end{align*}
\]

3.1.3. Continuous-Time Modeling of the Box Constraints

The inequality constraints in the Bernstein function space can be reformulated in the following. Satisfying the convex hull property is one of the strengths of Bernstein polynomials, which keeps the continuous-time trajectories inside the convex hull [35].

\[
\begin{align*}
P_{TH} &\leq P_{TH} v^{(Q)}(t) \leq P_{TH}, & t \in \mathcal{T} \quad (41) \\
P_{CHP} &\leq P_{CHP} v^{(Q)}(t) \leq P_{CHP}, & t \in \mathcal{T} \quad (42) \\
H_{H} &\leq H_{H} v^{(Q)}(t) \leq H_{H}, & t \in \mathcal{T} \quad (43)
\end{align*}
\]
3.1.5. Modeling Ramping Constraints

The ramping power constraints of the units, which restrict the value power production of the thermal or CHP units, in terms of the Bernstein representation over \( T \), are equal to:

\[
\begin{align*}
\dot{P}_{-\text{TH}} & \leq \hat{P}_{\text{TH}}(t) \leq \dot{P}_{\text{TH}}, & t & \in T \\
\dot{P}_{-\text{CHP}} & \leq \hat{P}_{\text{CHP}}(t) \leq \dot{P}_{\text{CHP}}, & t & \in T
\end{align*}
\]  

(49)  

According to Notation 2, the derivative of power trajectories of the Qth degree Bernstein polynomials is polynomials of degree \((Q – 1)\). This significant property helps to define the continuous-time ramping trajectory of the power and generating units in the Bernstein function space as:

\[
\begin{align*}
\dot{P}_{\text{TH}}(t) &= P_{\text{TH}}^v(t) = P_{\text{TH}}P_{\text{TH}}^{(Q-1)}(t) = \hat{P}_{\text{TH}}^v(t) \\
\dot{P}_{\text{CHP}}(t) &= P_{\text{CHP}}^v(t) = P_{\text{CHP}}P_{\text{CHP}}^{(Q-1)}(t) = \hat{P}_{\text{CHP}}^v(t)
\end{align*}
\]  

(51)  

where \( P_{\text{TH}} \) and \( P_{\text{CHP}} \) are the \( M(Q + 1) \times MQ \) mapping matrices, and \( \dot{P}_{\text{TH}} \) and \( \dot{P}_{\text{CHP}} \) are matrices of order \( N_{\text{TH}} \times MQ \) and \( N_{\text{CHP}} \times MQ \), respectively.

3.2. Modeling the Objective Functions

3.2.1. Continuous-time modeling of the cost function as the main objective

In order to benefit from the Bernstein polynomials, linear approximation of the non-linear functions is essential. To accomplish of which, the piecewise linear version of (4) and (6) are first described as (51) and (52):

\[
\begin{align*}
J_{\text{Cost}}(P_{\text{TH}}(t)) &= c_{2,\text{TH}}P_{\text{TH}}^2 + c_{1,\text{TH}}P_{\text{TH}} + c_{0,\text{TH}} + \sum_{s=0}^{N_{\text{TH}}-1} s_{s,\text{TH}}P_{s,\text{TH}}^2(t), & i & \in I, & t & \in T  \\
J_{\text{Cost}}(H_{\text{TH}}(t)) &= c_{2,\text{TH}}H_{\text{TH}}^2 + c_{1,\text{TH}}H_{\text{TH}} + c_{0,\text{TH}} + \sum_{s=0}^{N_{\text{TH}}-1} s_{s,\text{TH}}H_{s,\text{TH}}^2(t), & k & \in K, & t & \in T
\end{align*}
\]  

(52)  

where

\[
\begin{align*}
P_{i,\text{TH}}(t) &= P_{i,\text{TH}} + \sum_{s=0}^{N_{\text{TH}}-1} P_{i,s,\text{TH}}^s(t), & i & \in I, & t & \in T  \\
H_{k,\text{TH}}(t) &= H_{k,\text{TH}} + \sum_{s=0}^{N_{\text{TH}}-1} H_{k,s,\text{TH}}^s(t), & k & \in K, & t & \in T
\end{align*}
\]  

(53)  

(54)
where \( H_{k,H}(t) = H_{k,H} + \sum_{s=0}^{N_s-1} h^s_{k,H}(t), \quad k \in \tilde{K}, \quad t \in T \) and \( h^s_{k,H}(t) \) in the Bernstein function-space can be expressed as:

\[
p'_t(T,H) = p'_t(TH)T_{v(Q)}(t)
\]

\[
h^s_{k,H}(t) = h^s_{k,H} T_{v(Q)}(t)
\]

where \( p'_t(T,H) \) and \( h^s_{k,H} \) are the \( M(Q + 1) \)-dimensional vectors of Bernstein coefficients of the thermal and CHP units’ cost function coefficients, respectively. Accordingly, the cost functions obtained from the proposed function space-based method can be reformulated as (57), where \( 1_{M(Q+1)} \) is the \( M(Q+1) \)-dimensional all-ones vector.

\[
\int_T \left( P_{\text{Cost}}(P(t),H(t),c,x(t)) \right) dt = \frac{N_{\text{m}}}{} + \sum_{m=0}^{N_{\text{m}}} \left[ \sum_{i=1}^{N_{\text{TH}}} (c_{2,i,H}^e P_{i,TH} + c_{1,i,H}^e P_{i,TH} + c_{0,i,H}) + \sum_{k=1}^{N_{\text{TH}}} (p_{k,TH}^e + c_{1,k,H}^e P_{k,TH} + c_{0,k,H}) \right] + T_m \left( \text{diag}(p_{\text{CHP}}x_{\text{CHP}}) \right) \leq \varepsilon_{2,n_2}, \quad n_2 = 0, 1, \ldots, p_2
\]

3.2.2. Continuous-Time Modeling of the Emission Function Based on \( \varepsilon \)-Constraint

Let us project the total pollutant emission function converted to an inequality constraint by virtue of the \( \varepsilon \)-constraint in the space spanned by \( v(Q) \) as (58),

\[
\int_T \left( P_{\text{Emission}}(P(t),H(t)) \right) dt = \frac{N_{\text{m}}}{} + \sum_{m=0}^{N_{\text{m}}} \left[ \sum_{i=1}^{N_{\text{TH}}} (c_{2,i,H}^e P_{i,TH} + c_{1,i,H}^e P_{i,TH} + c_{0,i,H}) + \sum_{k=1}^{N_{\text{TH}}} (p_{k,TH}^e + c_{1,k,H}^e P_{k,TH} + c_{0,k,H}) \right] + T_m \left( \text{diag}(p_{\text{CHP}}x_{\text{CHP}}) \right) \leq \varepsilon_{2,n_2}, \quad n_2 = 0, 1, \ldots, p_2
\]

3.2.3. Enforcing the Continuity Constraints

Let us confirm that the generating and ramping trajectories projected on the function space are continuous. Hence, \( C_0 \) continuity is required to be imposed to check the continuity of the generation trajectories at the edge points of the intervals, while \( C_1 \) continuity is crucial to be enforced to guarantee at interval connection points.

\[
P_{i,TH,m} + Q = P_{i,TH,(m+1)} + Q, \quad i \in \mathcal{T}, \quad m = 0, \ldots, M - 1,
\]

\[
P_{j,CHP} + Q = P_{j,CHP,(m+1)} + Q, \quad j \in \mathcal{J}, \quad m = 0, \ldots, M - 1,
\]

\[
\frac{1}{T_m} \left( P_{i,TH,m} + Q - P_{i,TH,(m+1)} + Q - P_{i,TH,(m+1)} + Q - P_{i,TH,m} + Q \right) = \frac{1}{T_m} \left( P_{i,TH,m} + Q - P_{i,TH,m} + Q \right) = \frac{1}{T_m} \left( P_{i,CHP,m} + Q - P_{i,CHP,m} + Q \right)
\]

\[
\frac{1}{T_m} \left( P_{j,CHP,m} + Q - P_{j,CHP,m} + Q \right) = \frac{1}{T_m} \left( P_{j,CHP,m} + Q - P_{j,CHP,m} + Q \right)
\]
4. Numerical Results and Discussion

To assess the proposed multi-objective model, a system [36] including four thermal units, two CHP units, and two heat-only units was considered over a 24 h time interval. The limits of thermal units and heat-only units are given in the Appendix A, Tables A1 and A2, respectively. Further, the characteristic points of the CHP units depending on power and heat generation are described in Table A3 in the Appendix A. Netload data of the California ISO (CAISO) for 22 June 2018 [37] were used for continuous-time power and heat demand. The corresponding data are scaled back to the peak loads of 2048.84 MW and 1024.42 MW for the electric and heat demands, respectively. Bernstein polynomials were used to construct the proposed model, and all simulations were performed in a General Algebraic Modeling System (GAMS) [38] environment with a CPLEX solver on a system with a Core i7-7500U processor, 64 × 2.90-GHz, 12 GB of RAM.

The resultant electrical load profile over the Bernstein function space and hourly-based model are captured in Figure 3. The resultant heat load profile is also similar to the electrical load profile with different peak loads. To compare the results more clearly, simulations were conducted in two different cases, based on the load profiles of Figure 3, categorized as studies I and II:

**Study I:** In this case, the problem is defined as the minimization of total cost and pollutant emission as two competing objectives. Discrete-time modeling is the basis of this study, which aims to evaluate the leading properties of continuous-time modeling. Generally, to solve MO optimization problems, the optimization process is required to [39]:

- Keep the solution points, the decision space, and the non-dominated solutions in the objective space;
- Preserve the algorithmic progress toward the Pareto front;
- Keep the diversity of Pareto front solutions;
- Provide a large enough but limited number of solutions for the Pareto front;
- Search the best compromise solutions (BCSs) among the solution points of the Pareto front.

In this work, the ε-constraint method is suggested for providing the Pareto front, and Fuzzy multi-criteria decision-makers are utilized for selecting the optimal compromise solutions of the two case studies in the presence of two criteria. Detailed information regarding this procedure can be found in [40]. Using a fuzzy decision maker, hourly scheduling obtained the best trade-off between objectives at a point with 4,048,725 (USD) and 385,565 (kg). The generation trajectories of the best compromise solution associated with the thermal, CHP, and heat-only units based on the traditional hourly model are displayed in Figures 4 and 5, and the obtained results are listed in Table 1.
Figure 4. Day-ahead hourly power schedule for the best compromise solution.

Figure 5. Day-ahead hourly heat schedule for the best compromise solution.

Table 1. Day-ahead multi-objective scheduling.

|                          | Hourly Model (Study I) | Continuous-Time Model (Study II) |
|--------------------------|------------------------|----------------------------------|
| Total Day-ahead Cost (USD) | 4,048,725              | 4,051,994                        |
| Real-time Operation Cost (USD) | 113,180               | 40,227                           |
| Total Operation Cost (USD) | 4,161,905              | 4,092,221                        |
| Cost Saving (USD)         | -                     | 69,684 (1.67%)                   |
| Total Day-ahead Emission (kg) | 385,565               | 385,999                          |
| Real-time Emission (kg)   | 10,655                 | 4443                             |
| Total Emission (kg)       | 396,221                | 390,443                          |
| Emission Reduction (kg)   | -                     | 57,778 (1.46%)                   |
| Computed time (min)       | 0.3648                 | 1.9433                           |

Study II: The next set of numerical analyses includes applying the bi-objective Bernstein-polynomial-based modeling. More detailed results are given in Table 1, in which the difference between the real-time operations is clearly seen. A demonstrative set of Pareto fronts for the total day-ahead emission versus the total day-ahead cost provided by hourly-based and continuous-time scheduling is demonstrated in Figure 6.
The horizontal axis represents the cost minimization as the main objective, while the minimization of emission pollutants has been shown on the vertical axis. To better clarify, the location of the best compromises of both models is shown clearly and redrawn inside the figure. Further, according to Figure 7, the presented CHP and thermal units model furnishes a continuous-time schedule for generating units in the electrical part that ably utilizes their capacity to chase the continuous-time net-load, whenever energy loss is reduced. In Figure 8, the continuous-time schedule from CHP and heat-only units in the heat part is captured. Evaluating the above results and the interpretation of related Pareto front approximations, the following significant points are revealed:

(a) Even though the solution points in both Pareto curves in Figure 6 are well distributed, it is clear that the solutions of the continuous-one outperform the non-dominated solutions of the hourly model. However, according to the results of Table 1, the BCS of continuous-time modeling is obtained with a trade-off between 3269 (USD) increase in total day-ahead cost and 434 (kg) increase in total emission with regards to the hourly scheduling. Meanwhile, the real-time operation cost and emission are 113,180 (USD) and 10,655 (kg) for Case I as well as 40,227 (USD) and 4443 (kg) for Case II, respectively. This result highlights cost-saving of 69,684 (USD) (1.67%) and emission saving of 57,778 (kg) (1.46%) in day-ahead scheduling, which is substantial.

(b) Ramping scarcities can increase overall system costs because of the penalty prices due to the ramping capability shortage. Prevention of more ramp scarcities is another fundamental characteristic of the Bernstein expansion, which delivers accuracy and tighter function. For evaluating this attribute, the ramping values of the thermal units are determined by scaling down the maximum value of the units by a ratio of fifteen. In this case, the results represent 6 ramping scarcity events for the hourly based model in comparison with no scarcity events for the proposed model.

(c) The last row in Table 1 shows the computing time of the case studies in a minute, while the upper bound on the duality gap is set to zero. The execution time of the hourly day-ahead operation is 0.3648 min, which is lower than that of the proposed function space-based model, 1.9433 min. It is clear that the proposed method is approximately slower by 5 due to having more variables and constraints in the Bernstein polynomial-based modeling. This shows the main drawback of this expansion, especially when the accuracy of polynomials is increased. However, reducing the accuracy of the polynomials can bring about a significant reduction in the computational burden. Hence, relying on the problem, a tradeoff between accuracy and time should be determined.
Figure 7. Day-ahead continuous-time power schedule for the best compromise solution.

Figure 8. Day-ahead continuous-time heat schedule for the best compromise solution.

5. Conclusions

This paper represents a different way of optimization for continuous-time scheduling of CHP regarding thermal and heat-only units in day-ahead power systems. A flexible optimal model is proposed to optimize cost and emission, simultaneously, in the function space considering the ramping requirements of thermal and CHP units. The paper starts with reformulating the problem as a linear scheduling programming, followed by subsequently evolving a polynomial-based method to solve the continuous-time problem, which includes modeling the parameters in a finite order function space formed by Bernstein polynomials. Numerical results using real load data obtained from the ISO California show that the presented model decreases the total operation cost and emission as well as the number of ramping scarcities in the real-time operations compared with the hourly model. Extending the proposed model to one that includes uncertain parameters is the scope of our future study.

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Nomenclature

A. Objective Functions

| Symbol | Description |
|--------|-------------|
| $F_{Total}$ | Total generation cost of units (USD) |
| $F_{Cost}$ | Total emission of units (kg) |
| $f_{TH}$ | Generation cost of thermal, CHP and heat-only units |
| $f_{Emission}$ | CO2 emission of thermal, CHP and heat-only units |

B. Vectors/Matrices

| Symbol | Description |
|--------|-------------|
| $P(t)$ | Power vector including power generation of units |
| $P_{load}(t)$ | Vector of electric load demand (MW) |
| $P_{TH}(t)$ | Vector of thermal units’ power (MW) |
| $P_{CHP}(t)$ | Vector of CHP units’ power (MW) |
| $H(t)$ | Heat vector including heat generation of units |
| $H_{heat}(t)$ | Vector of heat demand (MW) |
| $H_{CHP}(t)$ | Vector of CHP units’ heat (MW) |
| $H_{H}(t)$ | Vector of heat-only units’ heat (MW) |
| $\dot{P}_{TH}(t), \dot{P}_{CHP}(t)$ | Time derivatives of the thermal and CHP units’ power generation |
| $\ddot{P}_{TH}, \ddot{P}_{CHP}$ | Vector of thermal units’ ramp up and ramp down, respectively (MW/h) |
| $\dot{P}_{CHP}, \dot{P}_{CHP}$ | Vector of CHP units’ ramp up and ramp down, respectively (MW/h) |
| $c_{CHP}, p_{CHP}, h_{CHP}, x_{CHP}(t)$ | Cost, power, heat, and coefficient matrices ($N_{CHP}$-by-$L$) of characteristic points for CHP units, respectively |
| $1_R$ | R-dimensional vector of ones |
| $I_R$ | Identity matrix of order R |
| $e_R$ | Standard vector (with a 1 in the $R$th coordinate and zeros elsewhere) |

C. Parameters

| Symbol | Description |
|--------|-------------|
| $N_{TH}$ | The number of thermal units |
| $N_{CHP}$ | The number of CHP units |
| $N_{H}$ | The number of heat-only units |
| $N_{S}$ | The number of piecewise segments |
| $L$ | The number of characteristic points |
| $Q$ | Degree of Bernstein polynomials |
| $c_{B}(t)$ | Bernstein-basis functions of degree Q |
D. Variables

\[ P_{i,\text{TH}}(t) \]
\[ P_{j,\text{CHP}}(t) \]
\[ H_{j,\text{CHP}}(t) \]
\[ H_{k,H}(t) \]

- CHP units' ramp up and ramp down, respectively (MW/h)

\[ x_j,\text{CHP} \]
\[ p_{j,\text{CHP}}, p_{j,\text{CHP}}' \]

Decision variable encoding the convex combination of the operating region of CHP unit \( j \)

E. Symbol, Sets and Indices

\( i \) Index of thermal units
\( j \) Index of CHP units
\( l \) Index of characteristic points of CHP units
\( k \) Index of heat-only units
\( m \) Index of the hourly time interval
\( I \) Index set of thermal units
\( J \) Index set of CHP units
\( L \) Index set of characteristic points of CHP units
\( K \) Index set of heat-only units
\( F \) Index set of linearization segments
\( M \) Index set of hourly time interval
\( T \) Day-ahead scheduling horizon

F. Superscripts and subscripts

\( \cdot_{\text{TH}} \) or \( \cdot_{\text{CHP}} \)

Thermal

Combined heat and power

Heat-only

Minimum/maximum magnitude operator

G. Constants

\( c_{2,\text{TH}}, c_{1,\text{TH}} \) and \( c_{0,\text{TH}} \)
\( c_{2,H}, c_{1,H} \) and \( c_{0,H} \)
\( e_{2,\text{TH}}, e_{1,\text{TH}} \) and \( e_{0,\text{TH}} \)
\( e_{2,H}, e_{1,H} \) and \( e_{0,H} \)
\( c_{1,\text{CHP}} \) and \( c_{0,\text{CHP}} \)

Cost coefficients of thermal units \( i \)
Cost coefficients of heat-only units \( k \)
Emission coefficients of thermal units \( i \)
Emission coefficients of heat-only units \( k \)

H. Operators

\( \cdot^T \)

Transpose operator

\( \text{diag}(\cdot) \)

diag's operator, which returns a column vector of the main diagonal elements of \( \cdot \)

\( \text{times}(\cdot, \cdot') \)

Element-wise multiplication operator which multiplies arrays \( \cdot \) and \( \cdot' \), element by element

Appendix A

Table A1. Required data of thermal units.

| Thermal Unit | \( c_{2,\text{TH}} \) | \( c_{1,\text{TH}} \) | \( c_{0,\text{TH}} \) | \( e_{2,\text{TH}} \) | \( e_{1,\text{TH}} \) | \( e_{0,\text{TH}} \) | \( P_{\text{TH}} \) | \( P_{-\text{TH}} \) |
|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| TH 1         | 0.02069        | 14.83          | 57.11          | 0.00683        | -0.54551       | 40.2669        | 800            | 0              |
| TH 2         | 0.03232        | 18.54          | 57.11          | 0.00683        | -0.54551       | 40.2669        | 500            | 0              |
| TH 3         | 0.01065        | 60.26          | 126.8          | 0.00461        | -0.51116       | 42.8955        | 500            | 0              |
| TH 4         | 0.04222        | 21.19          | 57.11          | 0.00461        | -0.51116       | 42.8955        | 450            | 0              |
Table A2. Required data of heat-only units.

| Heat-Only Unit | $e_{c,CHP}$ | $e_{l,CHP}$ | $e_{o,CHP}$ | $P_H$ | $P_{-H}$ |
|---------------|-------------|-------------|-------------|-------|---------|
| H 1           | 0.0105      | 10.55       | 23.426      | 0.32767 | 13.85932 | 300       | 0       |
| H 2           | 0.0299      | 9.21        | 10.721      | 0.32767 | 13.85932 | 270       | 0       |

Table A3. Required data of CHP units.

| CHP Unit | $P_{CHP}$ | $e_{c,CHP}$ | $e_{l,CHP}$ | $e_{o,CHP}$ |
|----------|-----------|-------------|-------------|-------------|
| CHP 1    | 247, 215, 118, 89 | 0, 240, 104.8, 0 | 1306.85, 1954.2646, 705.6, 982 | 0.5734, 311.57 |
| CHP 2    | 125, 110, 114, 75 | 0, 235, 35, 0 | 1030, 2123, 1120, 1419 | 1.7669, 821.65 |

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