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An analysis of the relation between return and beta for portfolios of Turkish equities

Salvatore J. Terregrossa¹* and Veysel Eraslan²

Abstract: The present study investigates the possible existence of a systematic relation between beta and excess-return for portfolios of Turkish equities. In the process, no systematic relation is found between beta and realized portfolio excess-return, in an unconditional sense. However, the study does find a systematic relation between realized portfolio excess-return and beta, conditioned upon the sign of realized market-portfolio excess-return. Moreover, an even stronger systematic relation is found between realized portfolio excess-return and beta, conditioned not only upon the sign, but also the magnitude of realized market-portfolio excess-return, with the estimation of the security market plane (SMP) model. The study has several useful implications for portfolio managers. Firstly, the empirical findings strongly suggest that employment of the SMP model may generate more accurate estimations of expected asset-return, compared with straightforward application of the capital asset pricing model (CAPM). Enhanced accuracy of expected asset-return, in turn, may lead to more accurate appraisals of asset value, resulting in more profitable investment opportunities and decisions. Employment of the SMP model may thus lead to enhanced efficient-portfolio development, by leading to construction of portfolios with greater expected-return, for a given class of quantifiable-risk.

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Salvatore J. Terregrossa holds a PhD in Economics and Finance from Binghamton University, NY, US, and currently teaches Economics at Istanbul Aydin University. Salvatore has published articles in SSCI and Econlit-indexed journals, regarding empirical studies in combination forecasting; CAPM and beta; international finance and asset pricing. The research reported in this paper relates to the wider issue regarding factors that influence or determine expected and realized excess-return of Turkish equities, and portfolios of Turkish equities. Related potential future projects include the identification and modeling of the various factors that influence the excess-return of Turkish equities.

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PUBLIC INTEREST STATEMENT
The present study is concerned with the process of estimating an asset’s expected monetary return. For example, investors are concerned with estimating the expected percentage return of financial assets, such as a share of common stock, which represents part-ownership of a firm. An asset’s true (or intrinsic) value depends in part on its expected monetary return. Therefore, the more accurate the estimation of an asset’s expected percentage return, the more accurate will be the appraisal of the asset’s true value. In turn, more accurate appraisals of asset intrinsic value may lead to more profitable investment opportunities and decisions.

Different models may be employed to estimate an asset’s expected return. The current study presents empirical evidence strongly suggesting that employment of the security market plane (SMP) model may lead to more accurate estimation of an asset’s expected percentage return, compared to straightforward application of the capital asset pricing model (CAPM).
1. Introduction

The present study conducts an empirical analysis regarding the possible existence of a systematic relation between beta and excess-return for portfolios of Turkish equities. In the process, the security market plane (SMP) model of Bollen (2010) and the related models of Pettengill, Sundaram, and Mathur (1995) and Fama and French (1992) are all estimated with Turkish equity market data.

The SMP is a concept introduced and developed by Bollen (2010), with the primary purpose to demonstrate an alternative, more economical method to conduct conditional tests of beta. The SMP is derived from the market model of Sharpe (1964), Lintner (1965), and Black (1972), (or SLB market model), the empirical counterpart of the theoretical capital asset pricing model (CAPM), developed by Markowitz (1959), Sharpe (1964) and Lintner (1965). The SMP embodies a conditional relation among three variables: beta, realized excess market-return, and expected excess portfolio-return. This relation infers that it is an interaction-effect between portfolio-beta and realized excess market-return that largely determines expected excess portfolio-return.

In the course of the study, a significant, positive relation between portfolio excess-return and beta, conditioned upon both the sign and magnitude of market portfolio excess-return, is found for portfolios of Turkish equities. Cross-equity-market confirmation of the SMP empirical model is thereby provided. In contrast, no systematic relation is found between portfolio excess-return and beta, in an unconditional sense, with the estimation of the Fama and French (1992) model.

These findings have several useful implications for portfolio managers and investors. To begin with, the empirical analysis strongly suggests that employment of the SMP model may generate more accurate estimations of expected asset-return, compared with straightforward application of the CAPM, given the finding of no systematic relation between asset-return and beta, in an unconditional sense.

In turn, more accurate estimations of asset expected-return may lead to more accurate appraisals of asset intrinsic-value; and as a result, more profitable investment opportunities and decisions. Employment of the SMP model may thereby lead to enhanced efficient-portfolio development, by leading to the construction of portfolios with greater expected-return, for a given level of quantifiable-risk.

The study is organized into four sections: Following the introduction, Section 2 presents a backdrop with a review of the relevant literature, and descriptions of the various models to be estimated. Section 3 presents the empirical analysis, and has two segments. The first explains the data and methodology employed. The second segment presents the regression results of the various estimated models and also discusses the relevant implications regarding the empirical findings. The paper ends with concluding remarks and comments in Section 4.

2. Backdrop

The CAPM equation in its ex ante form states that in equilibrium an individual security’s expected excess-return is a positive, linear function of its relative degree of systematic, market-related risk as measured by beta ($\beta_i$):

$$E(r_i) = \beta_i [E(r_m)]$$

(1)
where \( r_i = R_i - R_m \) is the excess-return on security \( i \), \( r_m = R_m - R_f \) is the excess-return on the market portfolio, \( R_i \) is the return on security \( i \), \( R_m \) is the return on the market portfolio, \( R_f \) is the risk-free rate of return, all at time \( t \). \( E (*) \) is the expectation operator. And where \( \beta_i \) is the security’s beta (an index of systematic, market-related risk), and is equal to: \( \text{cov} (R_i, R_m) / \text{var} (R_m) = \rho(i,M) (\sigma_i) (\sigma_m) \), where \( \sigma_i \) is the SD of return for security \( i \), \( (\sigma_m) \) is the SD of return for the market portfolio, \( (\sigma_m)^2 \) is the variance of return for the market portfolio, \( (\rho(i,M)) \) is the correlation coefficient between the return for security \( i \) and the corresponding return for the market portfolio, all at time \( t \).

The CAPM is stated in terms of expectations. Empirical applications of the CAPM conventionally require the use of historical data to estimate expectations. That is to say, in practice ex ante expectations are formed from ex post observations.

Fama and French (1992) challenge this empirical convention by testing the hypothesis that portfolios with lower levels of systematic, market-related risk (i.e. with lower betas) attract lower realized returns, and vice versa: portfolios with higher betas command higher realized returns. Challenging the validity of the CAPM at the time, Fama and French (1992) find no positive, systematic relation between beta and average realized portfolio-returns, of the kind implied by the SLB market model. Instead, Fama and French (1992) find that “… allowing for variation in beta that is unrelated to size flattens the relation between average return and beta, to the point where it is indistinguishable from no relation at all” (p. 458).

The SLB market model is stated as:

\[
R_i = \alpha_i + \hat{\beta}_i (R_m) + \epsilon_i 
\]  

(2)

where \( r_i = \) the excess-return on portfolio \( i \), \( r_m = \) the excess-return on the market portfolio, \( \hat{\beta}_i = \) the portfolio’s beta, \( \alpha_i = \) a fixed term for a given portfolio \( i \), \( \epsilon_i = \) error term, \( E [\epsilon_i] = 0 \), all at time \( t \).

Consequently, Fama and French (1992) employ the following related model to test (and ultimately refute) their hypothesis stated above:

\[
R_i = \alpha + \gamma \hat{\beta}_i + \epsilon_i 
\]  

(3)

where \( r_i = \) the excess-return on portfolio \( i \), \( \hat{\beta}_i = \) the estimated historical beta of portfolio \( i \), \( \alpha \) and \( \gamma \) are fixed parameters, and \( E [\epsilon_i] = 0 \). After controlling for the size effect anomaly, Fama and French (1992) find that the \( \gamma \) regression coefficient to be not statistically significant, and conclude that “… the relation between beta and average return … is perhaps non-existent” (p. 464).

Subsequently however, Pettengill et al. (1995) perceive a flaw in the Fama and French (1992) study, which is the implicit assumption that expected excess market-returns are positive; when in fact realized excess market-returns are often negative. Pettengill et al. (1995) offer that “… when realized market-returns fall below the risk-free rate [i.e. negative realized excess market-returns], an inverse relationship is predicted between realized returns and beta” (p. 102). Pettengill et al. (1995) present a model that allows tests of beta that are conditioned upon excess market-returns, by investigating separately the effect of beta on portfolio returns, (1) when realized excess market-returns are negative; and (2) when realized excess market-returns are positive. The dual-beta CAPM market-model is stated as:

\[
r_i = a + b_1 D_i \hat{\beta}_i + b_2 (1 - D_i) \hat{\beta}_i + \epsilon_i 
\]  

(4)

where \( r_i = \) the excess-return on portfolio \( i \), \( \hat{\beta}_i = \) the estimated historical beta of portfolio \( i \); a dummy variable \( D_i \) is defined as: \( D_i = 1 \) if \( r_m \geq 0 \), \( D_i = 0 \) if \( r_m < 0 \), where \( r_m = \) the excess-return on the market portfolio, all at time \( t \). Parameters \( a, b_1 \), and \( b_2 \) are fixed and \( E [\epsilon_i] = 0 \). Estimating this model with US data, the Pettengill et al. (1995) study simultaneously generates a highly significant, positive relation
between beta and excess portfolio-returns, when realized excess market-returns are positive; and a highly significant, inverse relation between beta and excess portfolio-returns, when realized excess market-returns are negative. In this way, Pettengill et al. (1995) “… provide strong support for a systematic but conditional relation between beta and realized returns” (p. 109).

Bollen (2010) builds upon the Pettengill et al. (1995) study to offer an alternative approach to condition expected asset excess-return upon realized market excess-return, that not only takes into account the sign but also the magnitude of realized excess market-return, while attempting to demonstrate the validity of beta. To construct this approach, the SLB market model is employed to derive a conditional relation among three variables: portfolio-beta, realized excess market-return, and expected excess portfolio-return. The SMP relation infers that “… it is the interaction effect between [portfolio-beta] and realized, excess market-returns that determines expected excess portfolio-returns” (Bollen, 2010, p. 1233).

The conditional SMP relation, as derived from the SLB market model, is stated by Bollen (2010) as:

\[
E[r_i | r_M] = \gamma_0 + \gamma_1 (\hat{\beta}_i r_M) + \epsilon_i
\]

The conditional SMP relation is tested by Bollen (2010) with Australian equity market data, with a regression of realized excess portfolio-returns against a corresponding product-term of beta and realized excess market-returns \(\hat{\beta}_i r_M\). The empirical SMP is ultimately stated as:

\[
r_i = \gamma_0 + \gamma_1 (\hat{\beta}_i r_M) + \epsilon_i
\]

where \(r_i\) is the observed excess-return on portfolio \(i\), \(r_M\) is the observed excess-return on the market portfolio, and \(\hat{\beta}_i\) is the estimated historical beta of portfolio \(i\), at all time \(t\). \(\gamma_0, \gamma_1, \epsilon_i\) are fixed parameters and \(E[\epsilon_i] = 0\). In this implementation of a derivative of the SLB market model, both the sign and the magnitude of excess market-return are thereby taken into account in the modeling of excess portfolio-returns. The Bollen (2010) study generates a highly significant, and very close-to-one in value, estimated regression coefficient \((\gamma_1)\) of the empirical SMP model’s interactive product-term of beta and excess market-return \(\hat{\beta}_i r_M\); and thus concludes that “… to a very large extent, the interaction term \(\hat{\beta}_i r_M\) models the level of portfolio returns well” (p. 1237).

The framework of the interaction-effect between portfolio-beta \((\hat{\beta}_i)\) and realized excess market-return \((r_M)\) may be summarized as follows, beginning with \(\hat{\beta}_i\), which measures the degree to which changes in general economic conditions influence portfolio-performance: relatively high values for \(\hat{\beta}_i\) (>1) suggest changes in the general economy have a strong effect on portfolio-return. Low values for \(\hat{\beta}_i\) (<1) indicate a weak effect. Next, is a consideration of the excess-return of the market portfolio \((r_M)\) which reflects the state of general economic conditions: Relatively high values of \(r_M\) reflect a strong economy, with low values indicating a weak economy. Therefore, it follows that a portfolio corresponding with: (1) a high \(\hat{\beta}_i\) and a high \(r_M\) will have an expectation of a very strong performance; (2) a high \(\hat{\beta}_i\) and a low \(r_M\) will have an expectation of a very weak performance; and so forth.

3. Empirical analysis

3.1. Data and methodology

The empirical analysis of the present study is divided into two different stages. The first stage includes an estimation of the SLB market model (Equation 2) employing a time-series methodology, as established in previous studies. The second stage of the analysis, following Bollen (2010), utilizes a methodology that pools cross-sectional and time-series data to estimate the empirical SMP model (Equation 6), and, for purposes of comparison and context, the Fama and French (1992) model (Equation 3), and the Pettengill et al. (1995) model (Equation 4).
These various models are tested using monthly data of a sample of Turkish firms trading on the Istanbul Stock Exchange (ISE), or the Borsa Istanbul (Istanbul Stock Exchange Website or Borsa Istanbul Website, n.d.) as it is presently referred to, over the 126-month period from 2003 to June 2013. The Borsa Istanbul is the only stock exchange in Turkey; the exchange facilitates primary market transactions, as well as conducting secondary market transactions.

Beginning with the monthly return for each firm in a given sample obtained from the Borsa Istanbul database, the current study directly estimates beta for each firm over the 126-month test period. Monthly firm-returns are regressed against the corresponding monthly-returns of the market portfolio, employing the preceding 36–60 months of historical data. Thus, a firm is required to have at least 36 months of prior historical return data in order to be included in a given sample for a given month. In this way, 281 firms qualify for sample inclusion, generating 33,202 monthly firm-return observations over the test period. The firm-beta estimation equation is stated as:

\[ R_i = \beta_i(R_M) \]

where \( R_i \) = the observed monthly return on firm \( i \), \( R_M \) = the observed monthly return on the market portfolio, all in month \( t \). The regression coefficient, \( \beta_i \), is the estimated firm beta. In this estimation (and also in the various other model-estimations), the market portfolio is represented by the Borsa Istanbul All Index (BIST-All), a weighted index of all stocks trading on the Borsa Istanbul (except the ones listed on Emerging Companies Market and Investment Trusts). The monthly return of the BIST-All is obtained from the Borsa Istanbul database.

Ten portfolios, based on the estimated beta calculations, are then constructed for each month of the test period, following a similar methodology employed by Black, Jenson, and Scholes (1972), Fama and French (1992), Black (1993), Pettengill et al. (1995), and Bollen (2010). In each of the 126 months under analysis, firms are placed in one of ten portfolios based on their estimated beta. Portfolio 1 contains the 10 percent of firms with the lowest betas; Portfolio 2 contains the 10 percent of firms with the next highest betas; the categorization continuing in this manner until Portfolio 10, which contains the 10 percent of firms with the highest betas. Then 10 time-series regressions are run employing the SLB market model as stated in Equation (2), similar to the analyses of Black et al. (1972), Black (1993) and Bollen (2010): Average excess portfolio-returns are regressed against excess market-portfolio returns for each of the 10 sets of beta-classified portfolios, over the test period. The results of the 10 regressions are presented in Table 1.

Table 1 indicates that all of the portfolio-\( \beta \) estimates are positive and statistically significant, ranging in value from 0.5061 for Portfolio 2 to 1.4337 for Portfolio 10. The portfolio-\( \beta \) estimates generally increase in value from the lower to the higher-beta portfolios (true in eight out of the nine possible cases). This finding helps to confirm the idea that estimated betas (generated using historical data) “... may be useful proxies for contemporaneous betas” (Bollen, 2010, p. 1234).

However, Table 1 also indicates that there appears to be no systematic relation between estimated portfolio-beta and average excess portfolio-return. Figure 1 illustrates this apparent lack of a systematic relation. Thus, the current study provides evidence in support of a failure-to-reject of the null hypothesis that excess portfolio-returns are not systematically related to beta alone.

Next, similar to the Pettengill et al. (1995) and Bollen (2010) studies, the current study categorizes the average excess portfolio-return monthly data for the 10 sets of beta-classified portfolios, into two segments: one in which corresponding, monthly excess market-portfolio returns are positive, and the other in which they are negative. For each of the 10 portfolios in the two categories, equally-weighted average excess portfolio-returns are calculated. Two graphs are then constructed: Figures 2 and 3 reflect the conditional relation between portfolio beta and portfolio excess-return when the market excess-return is positive, and when it is negative, respectively.
There is a direct relation between portfolio-beta and realized portfolio excess-return, during months when realized market excess-return is positive. However, portfolio-betas and realized portfolio excess-returns are inversely related during months when realized market excess-return is negative. The present thereby lends cross-equity-market support to the notion that tests of beta need to be conditioned upon realized market-return.

In the second stage of the empirical analysis, the current study attempts to condition tests of beta not only by the sign but also the magnitude of corresponding, realized market-return, by applying the SMP model to Turkish equity market data. As a preliminary step, an empirical version of the theoretical SMP is constructed to verify that Turkish equity market data can be modeled satisfactorily with the SMP. Figure 4 is in the form of a “3-d” diagram which displays: (1) on the vertical axis, the average excess portfolio-return for each of the 10 beta-portfolios, calculated within each of the 10 monthly excess market-return deciles (see Table 2); (2) on the horizontal axis, the beta decile (1 through 10); (3) on the third axis, the excess market-return decile (1 through 10).

Figure 4 represents and explains the empirical SMP constructed with Turkish equity market data.6 This diagram shows that higher-beta portfolios do not always correspond to higher portfolio-returns. This characteristic is in line with the theoretical SMP, which holds that higher-beta portfolios do not always attract higher returns, because, according to the theory, it is not beta alone that determines

### Table 1. The regression results of the 10 regressions estimated by the present study with the application of the SLB market model:

| Port | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|------|------|------|------|------|------|------|------|------|------|------|
| α    | -0.2558 | 1.2065 | 0.3669 | 0.4170 | 0.4713 | 0.7367 | 0.4902 | 0.0942 | 0.7883 | 1.1137 |
| (α)  | (-0.521) | (2.852) | (0.822) | (1.023) | (1.255) | (1.657) | (1.178) | (0.209) | (1.731) | (2.140) |
| β    | 0.6376 | 0.5061 | 0.6397 | 0.7358 | 0.8460 | 0.9064 | 0.9958 | 1.0327 | 1.1915 | 1.4337 |
| (β)  | (9.17) | (8.27) | (14.20) | (13.54) | (18.06) | (18.44) | (21.80) | (24.79) | (25.06) | (23.06) |
| mean r' | 0.2172 | 1.5821 | 0.8416 | 0.9630 | 1.0991 | 1.4092 | 1.2290 | 0.8604 | 1.6724 | 2.1775 |
| σ(r') | 7.4723 | 6.7982 | 7.1558 | 8.1067 | 8.7033 | 9.1641 | 9.7360 | 10.1217 | 11.6677 | 13.7128 |
| σ(ε) | 5.0188 | 5.1957 | 4.5077 | 4.9840 | 4.6499 | 4.6714 | 4.4411 | 4.6601 | 5.3528 | 5.6883 |

Notes: The results summarized in Table 1 indicate that there appears to be no systematic relation between estimated portfolio-beta (β) and average excess portfolio-return (mean r), in an unconditional sense. The symbol t stands for t-ratio; β is the estimated portfolio beta; mean r is the average excess portfolio monthly return over the test period, for a given beta-portfolio; σ(r) is the SD of portfolio excess return; σ(ε) is the regression SE.

*Stated in % terms.
portfolio return. Figure 4 therefore helps support the notion that Turkish equity data can be modeled satisfactorily with the SMP.

With this finding in hand, the current study proceeds to estimate the Bollen (2010) empirical SMP model (Equation 6). The Fama and French (1992) model (Equation 3) and the Pettengill et al. (1995) model (Equation 4) are also estimated, to provide context and perspective. Each of these models is run by pooling time-series with cross-sectional data, following Bollen (2010). Specifically, each of the model regressions in the second stage of the empirical analysis is based on 126 months (time-series) of historical data for the 10 beta-classified portfolios (cross section). Thus, each model regression is based on 1,260 observations of monthly portfolio excess-return.7

To estimate each of these models, an estimated beta $\hat{\beta}$ is required for each of the 10 portfolios in each month, over the 126-month test period. The method employed is to calculate the rolling,
historical $\beta$ for each portfolio in each month, with a regression of the preceding 36 months of excess portfolio-return against the corresponding excess market-returns. This portfolio-$\beta$ estimation regression in equation form is:

$$r_{it} = \beta_i (r_{Mt})$$

where $r_{it} =$ the observed monthly excess-return on portfolio $i$, $r_{Mt} =$ the observed monthly excess-return on the market portfolio, both at time $t$, and the regression coefficient is the estimated portfolio-beta, $\beta_i$. With this approach, each monthly portfolio excess-return observation (for each of the 10 portfolios) is assigned the rolling, historical $\beta$ estimated for that particular beta-classified portfolio, for that month.

Table 2. Displays the minimum and maximum monthly excess market-return (for the BIST All Index over the 126-month test period from 2003 to June 2013), in each of the 10 monthly excess market-return deciles; and also shows the number of monthly excess market-return observations in each decile

| Decile | Min (%) | Max (%) | Number of monthly observations |
|-------|--------|--------|-------------------------------|
| 1     | -24.1233 | -9.4457 | 13                           |
| 2     | -9.0988 | -7.0312 | 13                           |
| 3     | -6.9030 | -4.2350 | 13                           |
| 4     | -4.1346 | -0.1068 | 13                           |
| 5     | -0.0648 | 2.1980  | 13                           |
| 6     | 2.2990  | 3.5923  | 13                           |
| 7     | 3.5992  | 5.5696  | 12                           |
| 8     | 5.8876  | 7.9475  | 12                           |
| 9     | 8.0283  | 9.9483  | 12                           |
| 10    | 10.1818 | 24.5066 | 12                           |

Table 3. Regression results for Equations (3, 4, and 6), estimated in the current study using Turkish equity market data, over the 126-month period from 2003 to June 2013

| Model | Parameters | $R^2$ | $\sigma(\epsilon)$ (%) |
|-------|------------|------|------------------------|
| Equation 3: Fama and French (1992) model: $r_{it} = \alpha + \gamma \beta_i + \epsilon_{it}$ | $\alpha$ | $\gamma$ | 0.000014 | 9.4712 |
|       | 1.0908     | 0.1304 |                         |
|       | t-ratio: (1.2239) | t-ratio: (0.1345) |  |
| Equation 4: Pettengill et al. (1995) model: $r_{it} = \alpha + b_1 D_i \beta_i + b_2 (1 - D_i) \beta_i + \epsilon_{it}$ | $\alpha$ | $b_1$ | $b_2$ | 0.4783 | 6.8436 |
|       | 0.3981     | 7.0441 | -7.3522                |
|       | t-ratio: (0.6179) | t-ratio: (9.6504) | t-ratio: (-10.0058) |
| Equation 6: Bollen (2010) empirical SMP model: $r_{it} = \gamma_0 + \gamma_1 (\hat{\beta}_i r_{Mt}) + \epsilon_{it}$ | $\gamma_0$ | $\gamma_1$ | 0.73220 | 4.90129 |
|       | 0.6234     | 1.0155 |                         |
|       | t-ratio: (4.5036) | t-ratio: (58.6481) |  |

Notes: The results reported in Table 3 indicate that the estimation of the Fama and French (1992) model (Equation 3) suggests that beta by itself is not related in any systematic way to excess portfolio-return. The results also indicate that the estimation of the Pettengill et al. (1995) model (Equation 4) does find a systematic relation between beta and excess portfolio-return, conditioned upon the sign of realized market portfolio excess-return. The reported results also indicate that the estimation of Bollen (2010) SMP model (Equation 6) finds an even stronger systematic relation (given the higher estimated $R^2$ value) between beta and realized portfolio excess-return, conditioned not only upon the sign, but also the magnitude of realized market portfolio return.
3.2. Regression results and tests of hypotheses

Table 3 indicates that the estimation of the Fama and French (1992) model (Equation 3) with Turkish equity market data generates an estimated regression coefficient ($\gamma$) that is positive, but not statistically significant, and an $R^2$ virtually equal to zero. This evidence further supports a failure-to-reject of the null hypothesis, that beta by itself is not related in any systematic way to excess portfolio-return.

Table 3 also indicates that the estimation of the Pettengill et al. (1995) model (Equation 4) with Turkish data generates a positive estimate of $b_1$, and a negative estimate of $b_2$, with both regression coefficients statistically significant. These regression results offer cross-equity-market confirmation of the finding of “... a systematic but conditional relation between beta and realized returns” (Pettengill et al., 1995, p. 109). The $R^2$ statistic is 0.4783, indicating that beta conditioned upon the sign of market portfolio excess-return has a substantial degree of explanatory power regarding portfolio excess-return movement. This finding lends further cross-equity-market support to the premise that tests of beta need to be conditioned upon realized market-portfolio return.

Table 3 also indicates that the estimation of the empirical SMP model (Equation 6) with Turkish data generates a statistically significant, positive estimate of $\gamma_1$, with the regression coefficient very close-to-one in value ($\gamma_1 = 1.0155$). Further, an $F$-test generates a failure-to-reject of the null hypothesis $H_0: \gamma_1 = 1$, at the 1% level (the $F$-test statistic is 0.797, and the critical values at 5% significance and 1% significance, respectively, are 3.849 and 6.655). Thus, strong evidence is provided regarding a failure-to-reject of the null-hypothesis that an interaction effect between beta and excess market returns ($\hat{\beta}_i r_{M_t}$) is related to portfolio excess-returns; indicating that the interactive product-term of portfolio-beta and market portfolio excess-return ($\hat{\beta}_i r_{M_t}$) largely explains excess-return movement for portfolios of Turkish equities.

Table 3 also reports that the $R^2$ statistic of Equation 6 is 0.73220, indicating that the interactive product-term ($\hat{\beta}_i r_{M_t}$) has a high degree of explanatory power regarding excess portfolio-return movement. This comparatively high value for the $R^2$ statistic is a substantial improvement on the $R^2$ statistic of the Pettengill et al. (1995) model (Equation 4) estimated in the current study, confirming a similar finding of the Bollen (2010) study.

These joint findings of a significantly positive, close-to-one in value estimate of the regression coefficient $\gamma_1$; and a comparatively high value for the $R^2$ statistic, both in Equation 6, strongly suggest that employment of the SMP model may lead to accurate estimations of asset expected-return.

The above findings of the present study, in conjunction with similar findings in Bollen (2010), suggest several useful implications for portfolio managers and investors. Firstly, the analyses strongly imply that employment of the SMP model may generate more accurate estimations of expected asset-return, compared to a straight utilization of the CAPM. This implication follows from the empirical findings indicating that (1) beta by itself does not appear to be related in any systematic way to asset excess-return; and (2) a significant, positive relation is found to exist between asset excess-return and beta, conditioned upon both the sign and magnitude of market portfolio excess-return.

Consequently, comparatively more accurate SMP-model estimations of expected asset excess-return, in turn, may generate more accurate appraisals of asset intrinsic-value, when employed in discounted-expected-cash-flow asset valuation models. More accurate estimation of asset intrinsic-value may result in better identification of under and over-valued securities, thereby leading to more profitable investment opportunities and decisions.

In this way, utilization of the SMP model may lead to enhanced efficient-portfolio development, by leading to the construction of portfolios with higher expected-return, for a given level of quantifiable-risk.
4. Summary remarks and conclusions

The present study runs the Fama and French (1992) model with Turkish equity data, and no systematic relation is found between portfolio excess-return and beta, in an unconditional sense. However, a systematic relation conditioned upon the sign of realized market-portfolio excess-return, is found with the estimation of the Pettengill et al. (1995) model with Turkish data. Thus, the present analysis lends cross-equity-market support to the premise that tests of beta need to be conditioned upon realized market-portfolio excess-return.

The Bollen (2010) SMP model is estimated with Turkish data and an even stronger systematic relation is found between portfolio excess-return and beta, conditioned not only upon the sign, but also the magnitude of realized market-portfolio excess-return. Cross-equity-market empirical support of the SMP model is thereby provided.

The analysis suggests important implications for portfolio managers, whose main objective is to develop efficient portfolios. As indicated above, the empirical findings strongly suggest that the SMP model may generate more accurate estimations of asset expected-return, compared to direct application of the CAPM. In turn, greater accuracy in estimating asset expected-return may lead to more accurate estimations of asset intrinsic-value, resulting in more profitable investment decisions. Employment of the SMP model may thereby lead to the construction of portfolios with greater expected-return, for a given class of quantifiable-risk.

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Notes
1. See Bollen (2010, p. 1233) for a diagrammatical representation of the theoretical SMP.
2. For the full derivation of the empirical SMP model from the SLB market model, see Bollen (2010, p. 1233).
3. Thus, the overall data-set begins with the year 2000. While there is stock price data for the years before 2000, it is not possible to calculate excess returns for these years without the necessary data for the Turkish treasury-bill rates, which is not available prior to May 1999. Therefore, the necessary data-set required to estimate and test the various models is not available for any test period with a beginning date preceding 2003.
4. This is the broadest possible index, thus ensuring the most robust empirical results.
5. In the estimation of Equation 2, and in the various other model-estimations of the current study, the risk-free rate of return (R_f) denominated in the domestic currency is based on the monthly return of the 3-month or 6-month Turkish Treasury bill.
6. This diagram is very similar to the one constructed by the Bollen (2010) study with Australian data. See Figure 5 in Bollen (2010, p. 1236).
7. The approach employed in the Fama and French (1992) and Pettengill et al. (1995) studies is to run monthly cross-sectional regressions, and then to calculate the average of cross-sectional regression-coefficients over time. The pooling of time-series with cross-sectional data is a well-established, alternative method to determine if a significant relation among variables exists; and is also employed by Bollen (2010).

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