BOUNDING THE Higgs Boson Mass in the Next–to–Minimal Supersymmetric Standard Model

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ABSTRACT

We discuss the upper bound on the lightest CP-even Higgs boson mass in the next–to–minimal supersymmetric standard model within the framework of a low energy renormalisation group analysis. We find $m_h < 146$ GeV for $m_t = 90$ GeV, decreasing to $m_h < 123$ GeV for $m_t = 180$ GeV.

The minimal supersymmetric standard model (MSSM) is the simplest supersymmetric (SUSY) generalisation of the standard model which has many attractive features. Because of the non-renormalisation theorem and phenomenological SUSY, the technical hierarchy problem is solved, namely, why is $M_Z \ll M_{\text{planck}}$ stable against perturbation theory? Moreover, the realisation that the electroweak group, $SU(2)_L \otimes U(1)_Y$, may be broken radiatively due to the top quark’s large Yukawa coupling provides an elegant and perhaps compelling explanation for the smallness of $M_Z$ compared to $M_{\text{planck}}$. Furthermore, it is easy to bound the lightest CP-even Higgs boson mass at tree–level, yielding the classic result $m_h < M_Z$, though it is well–known that the tree–level bound is subject to large radiative corrections, the size of which may be estimated using triviality limits on the top quark Yukawa coupling. Radiative corrections to the Higgs boson masses in the MSSM have been the subject of much recent discussion. However, one aspect of the MSSM which is unsatisfactory is the so–called $\mu$–problem, the occurrence of a mass–scale $\mu \sim O(M_Z)$ in the superpotential. One would naively expect, in a SUSY theory, that either $\mu = 0$ or $\mu \sim O(M_{\text{planck}})$.

The MSSM is not, however, the only generalisation of the standard model compatible with grand unification. It is possible that SUSY grand unified theories (GUTs) give rise to a low energy theory containing an additional gauge singlet field, the so–called next–to–minimal supersymmetric standard model (NMSSM). In this talk we shall be concerned with bounding the lightest CP-even Higgs mass in this more general model by using the triviality limits of Yukawa couplings in the theory.

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The NMSSM contains two Higgs doublets and one Higgs gauge singlet field, leading to three CP-even neutral states, two CP-odd neutral states and two charged bosons in the physical spectrum, and is defined by the superpotential

$$W = h_U Q H_2 U^c + h_D Q H_1 D^c + \lambda N H_1 H_2 - \frac{1}{3} k N^3,$$

where gauge and family indices are suppressed, $H_{1,2}$ contain the standard Higgs doublets and $N$ contains the Higgs gauge singlet. The only Yukawa couplings which we retain are $h_t$, the top quark Yukawa coupling, $\lambda$ and $k$, since these may all be potentially large (we assume $h_b \ll h_t$). Thus, the superpotential becomes

$$W = h_t Q H_2 t^c + \lambda N H_1 H_2 - \frac{1}{3} k N^3,$$

where $Q^T = (t_L, b_L)$ contains the left–handed top and bottom quarks, and $t^c$ contains the charge conjugate of the right–handed top quark.

The $\mu$–problem is eliminated in the NMSSM by replacing $\mu$ by the gauge singlet $N$ which develops a vacuum expectation value (vev), $\langle N \rangle = x$. The remaining vevs are $\langle H_{1,2} \rangle = \nu_{1,2}$, where $\nu = \sqrt{\nu_1^2 + \nu_2^2} = 174$ GeV. The trilinear term $\frac{1}{3} k N^3$ removes a global $U(1)$ symmetry which would result in an unwanted axion when the fields acquire their vevs.

An upper bound on the lightest CP-even scalar $h^0$ in the NMSSM may be obtained from the real, symmetric $3 \times 3$ CP-even scalar mass–squared matrix, by using the fact that its minimum eigenvalue is bounded by the minimum eigenvalue of its upper $2 \times 2$ submatrix. In this manner, we may obtain the tree–level bound

$$m_h^2 \leq M_Z^2 + (\lambda^2 \nu^2 - M_Z^2) \sin^2 2\beta,$$

where $\tan \beta = \nu_2/\nu_1$, and $\lambda = \lambda(M_{\text{SUSY}})$. This is maximised by taking $\lambda$ as large as possible; the triviality limit, $\lambda = \lambda_{\text{max}}$, provides the most reasonable maximum value, for a given top quark mass. $\lambda_{\text{max}}$ is determined by solving the SUSY renormalisation group (RG) equations for the Yukawa couplings $h_t$, $\lambda$, and $k$ in the region $M_{\text{SUSY}} = 1$ TeV to $M_{\text{GUT}} = 10^{16}$ GeV.

Radiative corrections to Eq. (3) have recently been considered using either the Effective Potential method, or the RG approach. However, the implementations of the latter have assumed either SUSY RG equations down to near the top mass, or the existence of only one light Higgs particle in the low energy spectrum below $M_{\text{SUSY}}$. Here we shall perform a low energy RG analysis of the Higgs sector of the model between $M_{\text{SUSY}}$ and some lower scale $\mu$, assuming that all Higgs scalars may have masses less than $M_{\text{SUSY}}$, and using non–SUSY RG equations below $M_{\text{SUSY}}$. We make the usual approximation of hard decoupling of superpartners at $M_{\text{SUSY}}$. We then apply this technique to obtain a radiatively corrected upper bound on $m_h$.

The general low energy scalar Higgs potential is

$$V_{\text{Higgs}} = \frac{1}{2} \lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2} \lambda_2 (H_2^\dagger H_2)^2 + (\lambda_3 + \lambda_4) (H_1^\dagger H_1) (H_2^\dagger H_2) - \lambda_4 |H_2^\dagger H_1|^2 + \lambda_5 |N|^2 |H_1|^2 + \lambda_6 |N|^2 |H_2|^2.$$
Since solving these equations, we decouple the top quark at its mass shell, but do not reproduce the RG equations here, but they may be found elsewhere [8]. In the effects of any other soft SUSY breaking parameters. The low energy couplings and remaining parameters may be obtained by solving their RG equations between $M_{\text{SUSY}}$ and $\mu$. We do not reproduce the RG equations here, but they may be found elsewhere [8]. In solving these equations, we decouple the top quark at its mass shell, but do not decouple the Higgs scalars at their mass shells. We do not expect the latter to be a significant approximation provided $\mu \sim m_h$, and the Higgs scalars are light.

The minimisation conditions implied by $\frac{\partial V_{\text{Higgs}}}{\partial v_1} = 0$ and $\frac{\partial V_{\text{Higgs}}}{\partial v_2} = 0$ allow us to eliminate the low energy parameters $m_1$, $m_2$ and $m_3$ in favour of $v_1$, $v_2$ and $x$. The remaining parameters $m_4$ and $m_5$ cannot so be eliminated, but we may remove $m_4$ in favour of the charged Higgs scalar mass, $m_c$, by using the result

$$m_c^2 = \frac{2x}{\sin 2\beta} (m_4 - \lambda x) - \lambda_4 \nu^2.$$  \hfill (6)

Thus, we are then left with the following parameters in our low energy theory: $\frac{\tan \beta}{m} = \tan \beta, \frac{x}{m} = r, \lambda, k, m_5, m_c.$

The upper $2 \times 2$ submatrix of the full $3 \times 3$ CP-even mass-squared matrix (in the basis $\{H_1, H_2, N\}$) is given by

$$M^2 = \begin{pmatrix} 2\lambda_1 \nu_1^2 & 2(\lambda_3 + \lambda_4) \nu_1 \nu_2 \\ 2(\lambda_3 + \lambda_4) \nu_1 \nu_2 & 2\lambda_2 \nu_2^2 \end{pmatrix} \left( \begin{array}{cc} \tan \beta & -1 \\ -1 & \cot \beta \end{array} \right) \frac{1}{2} (m_5^2 + \lambda A_4 \nu^2) \sin 2\beta. \hfill (7)$$

Notice that this is independent of both $m_5$ and $r$. From this we see that

$$m_c^2 \leq \frac{1}{2} (A + m_c^2) - \frac{1}{2} \sqrt{(m_c^2 + B)^2 + C^2 - B^2}, \hfill (8)$$

where

$$A = \nu^2(\lambda_1 + \lambda_2 + \lambda_4) + \nu^2(\lambda_1 - \lambda_2) \cos 2\beta,$$

$$-B = \nu^2 [(\lambda_1 - \lambda_2) + (\lambda_1 + \lambda_2 - \lambda_4) \cos 2\beta] \cos 2\beta + \nu^2(2\lambda_3 + \lambda_4) \sin^2 2\beta,$$

$$C^2 = \nu^4 [(\lambda_1 - \lambda_2) + (\lambda_1 + \lambda_2 - \lambda_4) \cos 2\beta]^2 + \nu^4(2\lambda_3 + \lambda_4)^2 \sin^2 2\beta. \hfill (9)$$

Since $C^2 - B^2 \geq 0$, we obtain the $m_c$-independent bound

$$m_c^2 \leq \frac{1}{2} (A - B). \hfill (10)$$
Table 1: Lightest CP-even Higgs mass bound in the NMSSM. In row 1 is the top mass, $m_t$, in GeV; in row 2 the upper bound on the lightest CP-even Higgs mass, in GeV; in rows 3 and 4 those values of $h_t(M_{SUSY})$ and $\lambda_{max}(M_{SUSY})$ which produce the bound, respectively.

| $m_t$ (GeV) | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
|-------------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Bound (GeV) | 146| 143| 140| 137| 134| 131| 128| 126| 124| 123| 126 |
| $h_t$       | 0.61| 0.67| 0.73| 0.78| 0.83| 0.88| 0.92| 0.95| 0.98| 1.00| 1.02|
| $\lambda_{max}$ | 0.87| 0.85| 0.83| 0.81| 0.79| 0.76| 0.73| 0.70| 0.67| 0.63| 0.50|

Inserting $A$ and $B$ from Eq. (9), and using the boundary conditions in Eq. (5) leads to an upper bound of the form

$$m_h^2 \leq M_Z^2 + (\lambda^2 \nu^2 - M_Z^2) \sin^2 2\beta + \frac{\nu^2}{2} (\delta \lambda_1 + \delta \lambda_2 + 2 \delta \lambda_3 + 2 \delta \lambda_4 + 2(\delta \lambda_1 - \delta \lambda_2) \cos 2\beta + (\delta \lambda_1 + \delta \lambda_2 - 2 \delta \lambda_3 - 2 \delta \lambda_4) \cos^2 2\beta),$$

where $\delta \lambda_i = \lambda_i(\mu) - \lambda_i(M_{SUSY})$, and, strictly, $M_Z = M_Z(M_{SUSY})$. By approximating the RG equations (retaining terms quadrilinear in $\lambda, h_t$ and $k$ only, and making a small $\delta \lambda_i$ approximation), we obtain the analytic bound

$$m_h^2 \leq M_Z^2 + (\lambda^2 \nu^2 - M_Z^2) \sin^2 2\beta + \frac{\nu^2}{32\pi^2} \ln \left(\frac{M_{SUSY}}{\mu}\right) [(12h_t^4 - 12\lambda^2 h_t^2 - 8\lambda^2 k^2 - 24\lambda^4) - 24h_t^4 \cos 2\beta + (12h_t^4 + 12\lambda^2 h_t^2 + 8\lambda^2 k^2 + 8\lambda^4) \cos^2 2\beta],$$

where all parameters are evaluated at $M_{SUSY}$. It is easy to show that this is maximised for $\lambda > \lambda_{max}$ and $k = 0$; thus in the full numerical solution of the RG equations yielding $\lambda_i(\mu)$, we set $\lambda = \lambda_{max}$ and $k = 0$. We take $\mu = 150$ GeV, which turns out to be close to the bound on $m_h$.

Our procedure, then, is as follows. For a fixed top mass we determine which values of $h_t(M_{SUSY})$ and $\lambda_{max}(M_{SUSY})$ maximise the right–hand–side of Eq. (11) by solving the RG equations in order to calculate the $\delta \lambda_i$’s. This process is then repeated for all values of the top mass required. In table 1 we present the bound on the lightest CP-even Higgs mass. In row 1 is the top mass; in row 2 the bound on the lightest CP-even Higgs mass; in rows 3 and 4, those values of $h_t(M_{SUSY})$ and $\lambda_{max}(M_{SUSY})$ which give rise to the bound in row 2. Thus, we find that $m_h \leq 146$ GeV, in reasonable agreement will similar analyses [7].

Of course, such a bound is of little experimental interest if it can never be realised physically. To this end, we have studied the spectrum of Higgs particles in various regions of parameter space and determined that, indeed, the bound may become nearly saturated. We refer the reader elsewhere for further discussion of these results [8].
To conclude, then, the lightest CP-even Higgs scalar in the NMSSM must respect the bound $m_h \leq 146$ GeV. This bound is calculated within the framework of a low energy RG approach assuming hard decoupling of superpartners and the existence of a SUSY desert between $M_{SUSY}$ and $M_{GUT}$. The effects of squarks have been neglected. At present, we are in the process of calculating the effects of their contributions to the bound, with preliminary results indicating that they may shift the bound upwards by only a few GeV for small $m_t$, but by perhaps 20 GeV for large $m_t$.

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