Attenuating acoustic pulse propagation in multilayered rods

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Abstract. The problem of seismic activity and carrying out anti-seismic measures is extremely urgent at the present time. A large number of regions are more or less earthquake-prone and sometimes new regions, in which seismic activity has not been recorded before, fall into this category. Various methods of seismic load reduction are used to build new structures and support existing structures in these areas. The seismic load is transmitted to the structure through the foundation, therefore one of the effective ways to reduce it is to change the properties of the foundation. If instead of soil, which perfectly transmits impulses of seismic load, fill in a certain order with material that dampens these impulses - the desired effect can be achieved. The article analyzes the way of how one-dimensional pulse becomes attenuated in a multilayer rod. The layers in the rod are arranged in a periodical order and have contrasting acoustic properties. Numerical analysis is carried out using the finite element method. A pulse hitting the interface between the media undergoes multiple reflection and refraction, which causes damping in a purely mechanical system. The analysis shows that the larger the difference in acoustic properties between adjacent layers, the higher the attenuation of acoustical properties becomes.

1. Introduction

The problem of seismic activity and carrying out anti-seismic measures is extremely urgent at the present time. A large number of regions are more or less earthquake-prone and sometimes new regions, in which seismic activity has not been recorded before, fall into this category. When building new structures in seismically active regions, all modern anti-seismic measures are used, including those that are arranged directly in the structure. But the shape of some structures does not allow the use of such seismic protection measures. Or there is a need to carry out anti-seismic measures with existing structures. The seismic load is transmitted to the structure through the foundation, therefore one of the effective ways to reduce it is to change the properties of the foundation. In such cases, the base soil, which is good for the signal from the load, is replaced by material, passing through which the impulse is attenuated. The replacement material is not used by itself due to its properties - alternation of layers with different characteristics is used. The observed attenuation in a purely mechanical system is associated with multiple reflections and refractions at the interfaces between layers. To carry out such an analysis, the theory of elastic wave propagation in complex media is considered.

The theory of elastic wave propagation in complex media containing layers with different physical properties has a long history from Bromwich simplified model for incompressible isotropic layer and contacting also incompressible isotropic substrate [1] to modern approaches suitable for analyzing stratified media containing arbitrary number of layers with arbitrary elastic anisotropy [2 – 9].
The current research is targeted to constructing a solution for waves in a stratified rod containing layers with contrast acoustical properties. Observance of attenuation of one dimensional pulse propagating in a multilayered rod with periodic layout of layers with contrast acoustical properties is analyzed by numerical analysis utilizing FE method. The observed attenuation in a purely mechanical system is associated with multiple reflections and refractions at the interfaces between layers. The performed analysis reveals higher attenuation at the larger difference in acoustical properties between the adjacent layers.

2. Materials and methods

2.1. Governing notations

The hyperbolic equation of motion for the considered multilayered rod can be written in a form

$$\rho(x) \frac{\partial^2}{\partial t^2} u(x,t) = \frac{\partial}{\partial x} E(x) \frac{\partial}{\partial x} u(x,t), \quad (1)$$

where $\rho(x)$ is the (specific) material density, possibly varying between layers; $E(x)$ is the varying Young’s modulus; $u$ is the displacement component in direction of propagation (herein, one dimensional waves are considered); $x$ is the spatial coordinate, and $t$ is the time.

Performing differentiation in Eq. (1), yields the equation of motion in a form

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2(x) \frac{\partial^2}{\partial x^2} u(x,t) + \rho^{-1}(x) E'(x) \frac{\partial}{\partial x} u(x,t), \quad (2)$$

where $c(x)$ is the variable longitudinal wave velocity:

$$c(x) = \sqrt{\frac{E(x)}{\rho(x)}}. \quad (3)$$

Equation (2) is the desired non-homogeneous hyperbolic equation for analyzing pulse propagation in the considered multilayered rod.

Performing integral Fourier transform in both spatial and time variables, and denoting the Fourier transform parameters as $\xi_1, \xi_2$, the he principal symbol of Eq. (2) yields:

$$A(\xi_1; \xi_2) \equiv \xi_1^2 - c^2(x) \xi_2^2. \quad (4)$$

Herein $A(\xi_1; \xi_2)$ denotes the corresponding hyperbolic principal symbol.

Apparently the simplest case of heterogeneity corresponds to a constant Young’s modulus, ensuring Eq. (2) of a simpler form

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2(x) \frac{\partial^2}{\partial x^2} u(x,t), \quad (5)$$

Naturally, the symbol of Eq. (5) coincides with its principal symbol. Despite substantial simplicity of the latter equation, in the following analysis the assumption $E(x) = \text{const}$ will not be used.

2.2. FE model

The numerical experiment was carried out in the Abaqus 6.14 software package. For FE model of an elastic medium the author chose a cylindrical rod 50 m long and 0.5 m wide. It was divided into 20 sections with different alternating elastic properties (fig.1):

a) alternating stiffness 1x1
b) alternating stiffness 2x2
c) alternating stiffness 5x5
The description of the stiffness of the sections is given in the form of a table (Table 1).

| Element characteristics          | Hard section | Soft section |
|----------------------------------|--------------|--------------|
| Density, $\rho$                  | 100          | 0.01         |
| Elastic modulus, $E$, MPa         | 100          | 0.01         |
| Poisson's ratio, $\mu$            | 0.25         | 0.25         |

The rod is hinged at one of the ends (left end on the fig.1). The other end of the rod is loaded with a uniform pressure of 0.001 kN that changes over time, and the change over time follows a triangular law (Table 2).

| $T$, s | $P$, kN |
|--------|---------|
| 0      | 0       |
| 0.25   | 1       |
| 0.5    | 0       |

According to the formula (3), the speed of propagation of the wave along the rod is 1 m/s. Accordingly, the travel time of a wave along a rod 50 m long is 50 sec.

A comparative analysis will be given in section 3 based on graphs of accelerations, displacements and velocities. The choice of characteristic points on the boundaries of alternating media (hard / soft) for plotting graphs is shown in figure (fig. 2) for alternating stiffness 1x1, but this numbering is applicable to other types of alternating stiffnesses.

3. Results and discussions

In the course of the study, graphs of the three main characteristics of wave propagation in elastic media were plotted: graphs of accelerations, displacements and velocities. These graphs are presented in the following figures (figures 3-5) for three different stiffness assignments (a, b, c).

The developed methodology allowed us to analyze pulse propagation in a composite rod made of acoustically contrast parts at the interfaces undergoing multiple reflections and refractions due to difference of the acoustic impedances at the interfaces.

The developed method relies on the FEA of a one dimensional rod consisting of 20 alternating parts (figures 1 and 2) with substantially different acoustic properties. The constructed axisymmetric model utilizes a 4-node bilinear axisymmetric quadrilateral CAX4R element with reduced integration and hourglass control. The latter allowed avoiding some purely computational effects on the rise of the
specific mechanical energy caused by the local energy (inside of the individual elements) appearing due to flexural element deformations.

To avoid unphysical attenuation at pulse propagation along the rod, the artificial bulk viscosity, both linear and quadratic terms were set to zero. Thus, some oscillations at the back wave fronts caused by numerical (mainly truncation) errors were admitted. However, vanishing artificial viscosity allowed us analyzing attenuation of the signals arriving at the points of observation.

The presented in figures 3 – 5 plots clearly indicate that the resultant signals arriving at a particular point distant from the loaded rod end, have considerably smaller magnitudes of displacements, velocities and accelerations comparing to those at the loaded end.

The observed effect of the signal attenuation can serve for (i) creating new methods of protection of shock waves, associated with propagation of the delta-like pulses; and (ii) constructing the delay lines of a new type in the electronic industry.

Another remark concerns a natural question on the apparent loss of the mechanical energy in a linear mechanical system. That fact has the following explanation, while all the signals arriving at the distant points are much smaller, their duration is much larger, thus taking into account the longer duration time, gives an explanation for the apparent violation if the mechanical energy conservation.

And the last remark should be given on the nature of the propagating wave. It is known [10] that along a cylindrical rod, different modes of Pochhammer – Chree waves may propagate. It is also known [7, 10] that at the uniform pressure loading (figure 1) only longitudinal dispersive modes of Pochhammer – Chree waves are generated, and at some distance of the loaded end the part of the fundamental mode, known as the “rod wave” starts to dominate over other modes. The “rod wave” propagates with the “rod velocity” \( c = \sqrt{E / \rho} \), which is independent of Poisson’s ratio; herein, \( c \) is the (phase) velocity; \( E \) is Young’s modulus; and \( \rho \) is material density. It should also be noted that at the given frequency, the velocities of the other dispersive modes heavily depend upon Poisson’s ratio. Moreover, being highly dispersive these modes would lead to a completely blurred picture of the signal arrivals. A detailed analysis of the plots presented in figures 3 -5 clearly indicates that at the chosen points of observation the wave propagating with the “rod velocity” dominates over other dispersive modes.

Thus, as the presented numerical analysis shows, the constructed rod model ensures relatively fast attenuation of dispersion modes of the generated Pochhammer – Chree waves, and allows making a conclusion on the arrivals of the mainly non-dispersive “rod” pulse at the chosen points of observation. Such an observation may be important for performing future physical experiments with composite rods, as the presented analysis allows one to anticipate that the observed non-dispersive “rod” pulse will dominate over other dispersive modes of the Pochhammer – Chree waves in physical experiments, as well.

It is advisable in future physical or virtual experiments to analyze attenuation of the signals generated by generating (i) longitudinal harmonic waves; (ii) longitudinal waves generated by pulses of other shape; and (iii) modeling signal attenuation at generating flexural modes of Pochhammer – Chree waves. The latter may have importance in vibration reduction of various civil engineering structures.
Figure 3. Graph of acceleration (m/s², in logarithmic coordinates) versus time (s).

Figure 4. Graph of the dependence of displacement (m, in logarithmic coordinates) versus time (s).

Figure 5. Graph of the dependence of velocity (m/s, in logarithmic coordinates) versus time (s).

4. Summary
As a result of the study, the influence of the change and alternation of the stiffness of the sections that make up the rods on the passage of the wave along this rod was analyzed.

Research has shown that changing the properties of a base does indeed change the properties of an impulse passing through that base. The selection of the properties of the replacement layer and their alternation cause pulse decay. Consideration of different types of alternation of layer stiffness was also of great importance. In the course of comparison, it was possible to find out that the frequent alternation of stiffnesses (one layer after one layer) gives a slight attenuation. Rare interception of stiffness (5 layers after 5 layers) gives maximum impulse damping.
The presented analysis clearly indicates that (i) at propagation of the delta-like pulse generated at the loaded rod end, mainly non-dispersive part of the Pochhammer – Chree waves arrives at the chosen points of observation; (ii) the non-dispersive part of the fundamental mode propagates with the ‘rod velocity’, which is independent of Poisson’s ratio; (iii) the arrival pulses have decreasing intensity with increasing distance of the point of observation from the loaded rod end.

These results can be used to protect structures located in earthquake-prone areas, when it is impossible to arrange other anti-seismic measures related to the structure.

5. References
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