Iterative learning fault-tolerant control for networked batch processes with event-triggered transmission strategy and data dropouts

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ABSTRACT
This paper studies the iterative learning fault-tolerant control (ILFTC) problem for networked batch processes with event-triggered transmission strategy and data dropouts. During the transmission of input signal, the event-triggered mechanism is adopted to reduce the number of updated data items. The data dropouts are assumed to obey the Bernoulli random binary distribution. The objective of this paper is to design a state feedback controller such that the system is fault-tolerant and satisfies the robust $H_{\infty}$ performance requirement. By combining 2D stochastic system theory and linear matrix inequality (LMI) technique, some sufficient conditions are given to ensure the existence of the designed controller. Finally, an example of nozzle pressure control is utilized to verify the availability of the proposed method.

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1. Introduction
Over the past several decades, as a class of production modes with low-volume and high-value, batch processes have been widely applied in manufacturing and chemical industries (Korovesi & Linninger, 2006; Reklaitis & Sunol, 1996). Due to the fact that the repetitive nature of batch processes can be exploited by iterative learning control (ILC), the ILC strategy has been successfully introduced to batch processes. Since iterative learning control was originally proposed in 1984 by Arimoto (Arimoto, Kawamura, & Miyazaki, 1984), a lot of results have been presented (Gao, Yang, & Shao, 2001; Liu, Wang, & Chen, 2014; Shao, Gao, & Yang, 2003; Shi, Gao, & Wu, 2005a, 2005b; Xiong & Zhang, 2003). In Gao et al. (2001), a robust iterative learning control algorithm has been studied for the injection molding process with uncertain initial resetting and external disturbance. By using the designed ILC scheme, the batch process can be modelled as a 2D system in Shi et al. (2005a), meanwhile, the iterative learning controller design, which is the main work in this paper, has been transformed into a robust stabilization problem for 2D system. A robust indirect-type iterative learning control strategy has been investigated to ensure the robust stability and tracking performance of the batch process with time-varying uncertainties in Liu et al. (2014).

It is well known that faults are inevitable in batch processes on account of complex work environments, which greatly affect the steady performance of the system. Hence, it is significant to design a fault-tolerant controller such that the relevant system performance of batch processes can be ameliorated after faults occur. So far, various papers about fault-tolerant control scheme have been reported (Gao, Sheng, Zhou, & Gao, 2017; Tao, Zou, & Yang, 2016; Wang, Mo, Zhou, Gao, & Chen, 2013; Wang, Sun, Yu, Zhang, & Gao, 2017; Wang, Shi, Zhou, & Gao, 2006; Wang, Zhou, & Gao, 2008). Based on the traditional reliable control (TRC) theory, the iterative learning reliable control (ILRC) scheme has been presented to deal with actuator/sensor faults in Wang et al. (2006, 2008). In Tao et al. (2016), combining the lifting technology, an iterative learning fault tolerant control (ILFTC) method has been established for multi-rate sampling batch process with actuator faults such that the robust dissipative performance of the system can be satisfied. The multistage batch process with uncertainties has been converted to a 2D switched system in Wang et al. (2017), and a hybrid iterative learning control scheme has been presented by using the average dwell time method to ensure the 2D robust stability.

With the improvement of electronic technology, the networked control systems have received extensive attention in engineering practice due to the outstanding economy and applicability in the past several years (Bu & Hou, 2011; Bu, Wang, Zheng, & Qian, 2014; Ding, Wang, Shen, & Wei, 2015; Gao et al., 2017; Liu, Xu, & Wu, 2009;
Shen & Wang, 2015; Shen, Zhang, & Xu, 2017; Tao et al., 2016; Wang et al., 2017; Xiong, Yu, Patel, & Yu, 2016; Xuhui, Zhongsheng, Shangtai, & Ronghu, 2016; Yuan, Wang, & Guo, 2018; Yuan, Wang, Zhang, & Dong, 2018; Yuan, Yuan, Wang, Guo, & Yang, 2017; Zhou, Pan, Xiao, & Sun, 2016). However, in the signal transmission of the system, there exist a series of networked phenomena, which may reduce the product quality and production efficiency. In most existing literature, the communication between the components of the networked control system is perfect. Nevertheless, the packet losses cannot be neglected in the actual transmission in consequence of network congestion, which seriously damage the tracking performance of networked batch processes. Hence, the problem of data dropouts has been studied extensively in the past years (Bu & Hou, 2011; Bue et al., 2014; Liu et al., 2009; Shen & Wang, 2015; Shen et al., 2017; Xuhui et al., 2016). Some Bernoulli random variables have been introduced to convert the original discrete system into a 2D stochastic system in Xuhui et al. (2016), and a state feedback iterative learning controller has been given to guarantee the mean square asymptotic performance for the stochastic system. Moreover, different from the past, an arbitrary stochastic sequence which satisfies suitable conditions has been presented to model the data dropouts in Shen and Wang (2015).

In order to relieve the communication burden and improve the transmission efficiency, the event-triggered transmission mechanism has been introduced to networked control systems. Compared with the traditional clock-triggered transmission strategy, a large of redundant data will be rejected under the new transmission strategy. Recently, the event-triggered scheme has been increasingly investigated and a large number of researches can be obtained (Ding et al., 2015; Xiong et al., 2016; Zhou et al., 2016). The event-triggered scheme has been firstly introduced to iterative learning control systems in Xiong et al. (2016), which decreases the number of iterations to be updated. In Ding et al. (2015), an event-triggered controller has been investigated for stochastic multi-agent system with state dependent noises such that the system can realize consensus performance in probability.

Motivated by the above discussion, we aim to design an event-triggered iterative learning fault-tolerant controller for a class of networked batch processes with data dropouts and sensor faults. The measurement data dropouts can be modelled by a random variable which satisfies Bernoulli distribute in this paper. During the transmission of input signal, the event-triggered transmission strategy is considered to decrease the occupation of network bandwidth and improve the transmission efficiency. The main contributions of this paper can be outlined as follows. (1) Based on the 2D Fornasini-Marchesini model, the event-triggered iterative learning fault-tolerant control strategy is proposed for networked batch processes with sensor faults. (2) The phenomenon of data dropouts is considered in the process of the measurement output transmission.

The rest of the paper is organized as follows. The problem description of fault-tolerant control and some preliminaries are presented in Section 1. In Section 2, the asymptotical stability and the robust $H_{\infty}$ performance of the networked batch process are proved by means of some sufficient conditions. Furthermore, the fault-tolerant controller parameters can be obtained via using the approach of linear matrix inequality. A numerical example of nozzle pressure control is given in Section 3, and we conclude this paper in Section 4.

### Notations

Throughout this paper, $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space and $\mathbb{R}^{n \times m}$ denotes the set of all $n \times m$ real matrices. $I$ and $0$ stand for the identity matrix and the zero matrix, respectively. For a matrix $X$, $X^T$ denotes its transpose. $X > 0$ means $X$ is a positive matrix. $\mathbb{Z} = \{0, 1, 2, \ldots, N\}$, in which $N$ is a given positive integer. $\text{diag}\{\cdot\}$ represents a diagonal matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ stand for the minimum and maximum eigenvalue, respectively. The sign $\ast$ stands for the symmetric term in a matrix. $\| \cdot \|$ refers to the Euclidean norm and $\mathbb{E}\{\cdot\}$ represents the expectation of $x$. For a 2D signal $x(t, k)$, if $\|x(t, k)\|_2 = \sqrt{\sum_{t=0}^{T} \sum_{k=1}^{N} \|x(t, k)\|^2} < \infty$, for any $T > 0$ and $N \to \infty$, then $x(t, k)$ is regarded to be in $l_2$ space. Denote $\|x\|_E = \sqrt{\mathbb{E}\{\sum_{t=0}^{T} \sum_{k=1}^{N} \|x(t, k)\|^2\}}$.

### 2. Definitions and preliminaries

Consider a class of typical discrete batch processes:

$$
\Sigma_{BP} : \begin{cases}
    x(t + 1, k) = Ax(t, k) + Bu(t, k) \\
    y(t, k) = Cx(t, k), & t \in \mathbb{Z}, k = 1, 2, \ldots
\end{cases}
$$

where $t$ and $k$ denote the discrete time and the cycle. $x(t, k) \in \mathbb{R}^p$, $u(t, k) \in \mathbb{R}^p$ and $y(t, k) \in \mathbb{R}^m$ represent the state, input and measurement output of the batch process at time $t$ in the $k$th iteration, respectively. To describe the sensor faults which are common in the batch process, we denote the following failure model:

$$
y^f(t, k) = \Lambda Cx(t, k).
$$

The fault gain matrix $\Lambda$ is a diagonal matrix in the form of

$$
\Lambda = \text{diag} \{\lambda_1, \lambda_2, \ldots, \lambda_p\},
$$
in which
\[ 0 \leq \dot{\alpha}_i \leq \bar{\alpha}_i, \quad i = 1, 2, \ldots, p. \] (4)

Denote \( T_j = \text{diag}(T_{j1}, T_{j2}, \ldots, T_{jp}), \) \( T_{ji} = \dot{\alpha}_i \) or \( \bar{\alpha}_i, j = 1, 2, \ldots, 2p, i = 1, 2, \ldots, p. \) \( T_0 = \text{diag}(T_{01}, T_{02}, \ldots, T_{0p}), \)
\[ T_{0i} = \frac{\dot{\alpha}_i + \bar{\alpha}_i}{2}, \quad i = 1, 2, \ldots, p. \] (5)

Thus, we obtain
\[ \Lambda \in \left\{ \sum_{j=1}^{2p} b_j T_j \middle| 0 \leq b_j \leq 1, \sum_{j=1}^{2p} b_j = 1 \right\}, \] (6)

where \( T_0 \) is the polyhedron centre.

In order to decrease the occupation of network bandwidth, without sacrificing the system performance, there is no need to update each input data when the changes of the input \( u(t, k) \) are small. Then, we introduce the event-triggered transmission strategy in this paper. Denote the following event-triggered function:
\[ h(u(t, k), \bar{u}(t, k), \xi) = (\bar{u}(t, k) - u(t, k))^T (\bar{u}(t, k) - u(t, k)) - \xi u^T(t, k) u(t, k), \] (7)

where the input signal \( \bar{u}(t, k) \) can be obtained as \( \bar{u}(t, k) \triangleq u(t_\xi, k). \) The variable \( \xi \) represents the triggering threshold in this paper. If the following condition
\[ h(u(t, k), \bar{u}(t, k), \xi) > 0, \] (8)

is satisfied, the current information of controller will be transmitted to the plant. The triggering time \( t_\xi (0 \leq t_\xi < t_1 < \cdots < t_\xi < t_{\xi+1} \cdots < T) \) can be generated by
\[ t_{\xi+1} = \min \left\{ t \in \mathbb{N} \middle| t > t_\xi, h(u(t, k), \bar{u}(t, k), \xi) > 0 \right\}. \] (9)

Then, we can obtain
\[ \bar{u}(t, k) = (l + \Theta(t)) u(t, k), \] (10)

where \( \Theta^T(t)\Theta(t) < \xi. \)

Based on the system (1), the following iterative learning control scheme is adopted
\[ \begin{align*}
    u(t, k) &= u(t, k - 1) + r(t, k) \\
    u(t, 0) &= 0, \quad t \in \mathbb{Z}_+, k = 1, 2, \ldots
\end{align*} \] (11)

in which \( r(t, k) \) is the updating law of the controller. Define
\[ \delta(f(t, k)) = f(t, k) - f(t, k - 1), \] (12)

then
\[ \delta(x(t + 1, k)) = A\delta(x(t, k)) + B(l + \Theta(t)) r(t, k). \] (13)

During the transmission of the measurement output in the system, the phenomenon of data dropouts is considered. Therefore, we redefine the measurement output as
\[ \tilde{y}(t, k) = \alpha(t, k) y(t, k) + (1 - \alpha(t, k)) \bar{y}(t, k - 1), \] (14)

where \( \alpha(t, k) \) is a random variable which satisfies Bernoulli distribution with
\[ \text{Prob} \{ \alpha(t, k) = 1 \} = \text{E} \{ \alpha(t, k) \} = \tilde{\alpha}, \] (15)

If \( \alpha(t, k) = 0, \) it means that the packet \( y(t, k) \) is missed, and the information of \( \tilde{y}(t, k - 1) \) would be utilized in the system.

Defining the following tracking error:
\[ e(t, k) = y_d(t) - \tilde{y}(t, k), \] (16)

we have
\[ e(t + 1, k) = e(t + 1, k - 1) - (\tilde{y}(t + 1, k) - \bar{y}(t + 1, k - 1)) \]
\[ = e(t + 1, k - 1) - \alpha(t + 1, k) \Delta C(A\delta(x(t, k))) \]
\[ + B(l + \Theta(t)) r(t, k) + \nu(t, k), \] (17)

where \( \nu(t, k) = \alpha(t + 1, k)(y(t + 1, k - 1) - \bar{y}(t + 1, k - 1)) \) and it is easy to verify that \( \nu(t, k) \in L_2. \) Denoting \( \tilde{x}(t, k) \triangleq [\dot{o}^T(x(t, k)) \varepsilon^T(t, k)]^T, \) and introducing the variable \( \tilde{a}(t, k) \) to handle the stochastic problem, we get the following augmented system
\[ \tilde{x}(t + 1, k) = A_1(\Lambda) \tilde{x}(t, k) + \tilde{a}(t + 1, k) \Lambda_1(\Lambda) \tilde{x}(t, k) \]
\[ + A_2 \tilde{x}(t + 1, k - 1) + B_1(\Lambda) (l + \Theta(t)) r(t, k) \]
\[ + \tilde{a}(t + 1, k) \tilde{B}_1(\Lambda) (l + \Theta(t)) r(t, k) + D \nu(t, k), \] (18)

where
\[ \tilde{a}(t, k) = \alpha(t, k) - \tilde{a}, \]
\[ \text{E} \{ \tilde{a}(t, k) \} = 0, \quad \text{E} \{ \tilde{a}(t, k) \tilde{a}(t, k)^T \} = \tilde{a}(1 - \tilde{a}), \]

and
\[ A_1(\Lambda) = \begin{bmatrix} A & 0 \\ -\tilde{a} \Lambda C A & 0 \end{bmatrix}, \]
\[ \Lambda_1(\Lambda) = \begin{bmatrix} 0 & 0 \\ -\tilde{a} \Lambda C A & 0 \end{bmatrix}, \]
\[ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \]
\[ B_1(\Lambda) = \begin{bmatrix} B \\ -\tilde{a} \Lambda C B \end{bmatrix}, \]
\[ \tilde{B}_1(\Lambda) = \begin{bmatrix} 0 \\ -\tilde{a} \Lambda C B \end{bmatrix}, \]
\[ D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \tilde{C} = [0 \ 1]. \]

In this paper, we design the ILFTC updating law as follows:
\[ r(t, k) = K_1 \tilde{x}(t, k) + K_2 \tilde{x}(t + 1, k - 1), \] (19)
then the system (18) can be rewritten as
\[ \dot{x}(t+1,k) = \bar{A}_1(\Lambda)x(t,k) + \bar{A}_2(\Lambda)\dot{x}(t+1,k-1) + \bar{v}(t+1,k)\bar{A}_1(\Lambda)x(t,k) + \bar{v}(t+1,k)\bar{A}_2(\Lambda)\dot{x}(t+1,k) + Du(t,k) \]
\[ z(t,k) \triangleq e(t,k) = C\dot{x}(t,k), \quad (20) \]

where
\[ \bar{A}_1(\Lambda) = A_1(\Lambda) + B_1(\Lambda)(I + \Theta(t))K_1, \]
\[ \bar{A}_2(\Lambda) = A_2 + B_1(\Lambda)(I + \Theta(t))K_2, \]
\[ \bar{A}_1(\Lambda) = \hat{A}_1(\Lambda) + \hat{B}_1(\Lambda)(I + \Theta(t))K_1, \]
\[ \bar{A}_2(\Lambda) = \hat{B}_1(\Lambda)(I + \Theta(t))K_2. \quad (21) \]

The boundary conditions in the system (20) can be given by
\[ \begin{cases} \dot{x}(t,0) = \bar{x}_0, & t = 0, 1, \ldots, T \\ \bar{x}(0,k) = \bar{x}_{0,k}, & k = 1, 2, \ldots, K \end{cases} \]

Remark 2.1: In the actual industrial process, the transmission of input signal is relatively stable. It means that the phenomenon of input data dropouts rarely occurs. Conversely, due to the sensor faults, bandwidth limitations and other reasons, the probability of output signal loss will greatly increase. Hence, we consider different networked phenomena in the different transmissions.

Some definitions and lemmas are needed to facilitate the development of our main results, which are common in the past several works. Meanwhile, the following assumption is necessary to be given for the boundary condition.

Assumption 2.1: For the closed-system (20), we assume the boundary condition satisfies
\[ \lim_{N \to \infty} \sum_{t=0}^{T} \sum_{k=1}^{N} E \{ \| \bar{x}(0,k) \|^2 + \| \bar{x}(t,0) \|^2 \} < \infty. \quad (22) \]

Definition 2.1 (Bu et al., 2014): For any initial boundary conditions \( \{ \bar{x}(0,k), \bar{x}(0,0) \} \) \( \in \mathbb{R} \), if
\[ \lim_{t \to k \to \infty} E \{ \| \bar{x}(t,k) \|^2 \} = 0 \quad (23) \]
holds, the closed system (20) with Assumption 2.1 is mean square asymptotically stable in the absence of unknown external disturbance.

Definition 2.2 (Bu et al., 2014): For the zero boundary conditions and unknown disturbance \( v(t,k) \in l_2 \), the augment system (20) is fault-tolerant and have robust \( H_\infty \) performance index \( \gamma \), if the system is mean square asymptotically stable and the controlled output satisfies
\[ \| z(t,k) \|_E < \gamma \| v(t,k) \|_E. \quad (24) \]

Lemma 2.1 (Boyd, Balakrishnan, Feron, & Ghaoui, 1994): Let constant matrices \( W, L, V \) with appropriate dimensions, and \( W = W^T, V = V^T > 0 \). Then \( L^TVL - W < 0 \), if and only if
\[ \begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0, \quad (25) \]
or
\[ \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \]

Lemma 2.2 (Xie, 1996): Given \( E \) and \( F \) are real matrices with appropriate dimensions. For any scalar \( \epsilon > 0 \), the following inequality holds:
\[ EF + F^TE^T \leq \epsilon EE^T + \epsilon^{-1}F^TF. \quad (26) \]

Remark 2.2: Event-triggered transmission strategy and data dropouts are common in practical engineering, which may damage the relevant system performance. In most batch processes, high-precision is an important indicator to guarantee the quality of the product. It means that we need to minimize the impact of networked phenomena in batch processes. Hence, the investigation on ILFTC problem for batch processes with event-triggered transmission strategy and data dropouts is significant.

3. Iterative learning fault tolerant controller design

In this section, some sufficient conditions are proposed for the existence of the designed fault-tolerant controller, which can guarantee the mean square asymptotic stability and the robust \( H_\infty \) performance of the closed-loop system (20).

3.1. Asymptotic stability and robust \( H_\infty \) performance analysis

Theorem 3.1: Let the positive scalar of disturbance attenuation level \( \gamma > 0 \), the event-triggered threshold \( \xi > 0 \) and \( \epsilon > 0 \) be given. The fault tolerant system (20) is mean square asymptotically stable and satisfies the robust \( H_\infty \)
We first prove the mean square asymptotic stability of the system (20) with \( v(t, k) = 0 \). Define the following Lyapunov function:

\[
V(t, k - 1) = V_1(t, k - 1) + V_2(t, k - 1),
\]

in which

\[
V_1(t, k - 1) = \bar{x}^T(t, k - 1)(P - Q)\bar{x}(t, k - 1),
\]

\[
V_2(t, k - 1) = \bar{x}^T(t, k - 1)Q\bar{x}(t, k - 1).
\]

It is easy to find that

\[
\delta(V(t, k - 1)) = \delta(V_1(t, k - 1)) + \delta(V_2(t, k - 1))
\]

\[
= V_1(t + 1, k) - V_1(t, k) + V_2(t + 1, k) - V_2(t, k - 1)
\]

\[
= \bar{x}^T(t + 1, k)(P - Q)\bar{x}(t + 1, k) - \bar{x}^T(t, k)(P - Q)\bar{x}(t, k)
\]

\[
+ \bar{x}^T(t + 1, k)Q\bar{x}(t + 1, k) - \bar{x}^T(t, k - 1)Q\bar{x}(t, k - 1).
\]

By taking expectation of both sides, we have

\[
\mathbb{E}[\delta(V(t, k - 1))]
\]

\[
= \mathbb{E}[\bar{x}^T(t + 1, k)P\bar{x}(t + 1, k)] - \mathbb{E}[\bar{x}^T(t, k)(P - Q)\bar{x}(t, k)]
\]

\[
- \bar{x}^T(t + 1, k)Q\bar{x}(t + 1, k) - \bar{x}^T(t, k - 1)Q\bar{x}(t, k - 1)
\]

\[
= \Psi^T X(t, k),
\]

where

\[
\Psi = \begin{bmatrix} \bar{A}_1^T(\Lambda) \\ \bar{A}_2^T(\Lambda) \\ \end{bmatrix} \rho \begin{bmatrix} \bar{A}_1(\Lambda) & \bar{A}_2(\Lambda) \\ \end{bmatrix}
\]

\[
+ \theta^2 \begin{bmatrix} \bar{A}_1^T(\Lambda) \\ \bar{A}_2^T(\Lambda) \\ \end{bmatrix} \rho \begin{bmatrix} \bar{A}_1(\Lambda) & \bar{A}_2(\Lambda) \\ \end{bmatrix} - \begin{bmatrix} P & Q \\ Q & 0 \\ \end{bmatrix}.
\]

According to the inequality (27), it is clear that \( \Psi < 0 \). We can conclude that there exists a positive scalar \( \rho \) such that

\[
\mathbb{E}[\delta(V(t, k - 1)) \leq -\rho \| X(t, k) \|^2],
\]

where \( \rho = \lambda_{\text{min}}(-\Psi) \). By taking expectation of both sides, we can obtain

\[
\mathbb{E}[\delta(V(t, k - 1)) \leq -\rho \mathbb{E}[\| X(t, k) \|^2]].
\]

Letting \( t = 0, 1, \ldots, T \) and \( k = 1, 2, \ldots, N \) \( (N \rightarrow \infty) \), it is easy to derive that

\[
\sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}[\delta(V(t, k - 1))]
\]

\[
\geq \sum_{k=1}^{N} \mathbb{E}[\bar{x}^T(t, 0)(P - Q)\bar{x}(t, 0)]
\]

\[
+ \sum_{t=1}^{T+1} \mathbb{E}[\bar{x}^T(t, 0)(P - Q)\bar{x}(t, 0)]
\]

\[
+ \sum_{t=1}^{N-1} \mathbb{E}[\bar{x}^T(t, 1)(P - Q)\bar{x}(t + 1, k)].
\]

Furthermore, we have

\[
\sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}[\delta(V(t, k - 1))]
\]

\[
\geq \sum_{k=1}^{N} \mathbb{E}[\bar{x}^T(t, 0)(P - Q)\bar{x}(t, 0)]
\]

\[
+ \sum_{t=1}^{T+1} \mathbb{E}[\bar{x}^T(t, 0)(P - Q)\bar{x}(t, 0)],
\]

then,

\[
\sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}[\| X(t, k) \|^2]
\]

\[
\leq \frac{1}{\rho} \sum_{k=1}^{N} \mathbb{E}[\bar{x}^T(t, 0)(P - Q)\bar{x}(t, 0)]
\]

\[
+ \sum_{t=1}^{T+1} \mathbb{E}[\bar{x}^T(t, 0)Q\bar{x}(t, 0)]
\]

\[
\leq \kappa \frac{1}{\rho} \sum_{t=0}^{T+1} \sum_{k=1}^{N} [\| \bar{x}(0, k) \|^2 + \| \bar{x}(t, 0) \|^2] < \infty,
\]

where \( \kappa = \lambda_{\text{max}}(P) \). Finally, we can infer that

\[
\lim_{t \rightarrow \infty} \mathbb{E}[\| X(t, k) \|^2] = 0.
\]
that is,
\[
\lim_{t+k \to \infty} \mathbb{E}\{\|\tilde{x}(t,k)\|^2\} = 0. \quad (38)
\]
According to Definition 2.1, we can obtain that the closed-loop system (20) is mean square asymptotically stable.

Next, we discuss the robust \(H_\infty\) performance of the system (20) with \(v(t,k) \neq 0\). The following performance index is introduced
\[
J(t, k) = \delta(V(t, k - 1)) + \|z(t, k)\|^2 - \gamma^2 \|v(t, k)\|^2. \quad (39)
\]
and
\[
\mathbb{E}\{J(t, k)\} = \mathbb{E}\{\tilde{x}^T(t + 1, k)Px(t + 1, k)\} - \tilde{x}^T(t, k)(P - Q - \bar{C}^T\bar{C})\tilde{x}(t, k) - \tilde{x}^T(t + 1, k - 1)Q\tilde{x}(t + 1, k - 1) - \gamma^2v^T(t, k)v(t, k)
\]
\[
= \begin{bmatrix} \tilde{x}(t, k) \\ \tilde{x}(t + 1, k - 1) \end{bmatrix}^T \Phi \begin{bmatrix} \tilde{x}(t, k) \\ v(t, k) \end{bmatrix}
\]
\[
= \tilde{x}^T(t, k)\Phi \tilde{x}(t, k), \quad (40)
\]
in which
\[
\Phi = \begin{bmatrix} \bar{A}_1^T(\Lambda) \\ \bar{A}_2^T(\Lambda) \\ D^T \end{bmatrix} P \begin{bmatrix} \bar{A}_1(\Lambda) & \bar{A}_2(\Lambda) & D \end{bmatrix}
+ \theta^2 \begin{bmatrix} \bar{A}_1^T(\Lambda) \\ \bar{A}_2^T(\Lambda) \\ 0 \end{bmatrix} \bar{P} \begin{bmatrix} \bar{A}_1(\Lambda) & \bar{A}_2(\Lambda) \\ 0 & 0 \end{bmatrix}
- \begin{bmatrix} P - Q - \bar{C}^T\bar{C} & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & \gamma^2I \end{bmatrix}.
\]

From condition (27), it is easy to get that \(\Phi < 0\), and
\[
\sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}\{J(t, k)\} = \sum_{k=1}^{N} \mathbb{E}\{\tilde{x}^T(0, k)(-P + Q)\tilde{x}(0, k)\}
+ \sum_{k=1}^{N} \mathbb{E}\{\tilde{x}^T(T + 1, k)Px(T + 1, k)\}
+ \sum_{t=1}^{T-1} \sum_{k=1}^{N} \mathbb{E}\{\tilde{x}^T(t, 0)(-Q)\tilde{x}(t, 0)\}
+ \sum_{t=1}^{N} \sum_{k=1}^{N} \mathbb{E}\{\tilde{x}^T(T + 1, k)(-Q)\tilde{x}(T + 1, k)\}
+ \gamma^2 \sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}\{v^T(t, k)v(t, k)\} < 0.
\]

Considering the zero boundary initial conditions, we can derive
\[
\sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}\{\tilde{x}^T(t, k)\bar{C}^T\bar{C}\tilde{x}(t, k)\} < \gamma^2 \sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}\{v^T(t, k)v(t, k)\},
\]
which implies
\[
\|z(t, k)\|_E < \gamma \|v(t, k)\|_E. \quad (43)
\]
The proof of the theorem is complete. □

\subsection{3.2. ILFTC design under the ETT strategy}

Theorem 3.2: For any \(T_j (j = 1, \ldots, 2^p)\), and the given robust \(H_\infty\) performance index \(\gamma > 0\), the event-triggered threshold \(\xi > 0\) and \(\varepsilon > 0\). Assume that there exist positive definite symmetric matrices \(P > 0, Q > 0\), and matrices \(M_1, M_2\) with appropriate dimensions satisfying the following inequality:
\[
\begin{bmatrix}
-\dot{\bar{P}} + \bar{Q} & * & * & * & * & * & * & * & * \\
0 & -\bar{Q} & * & * & * & * & * & * & * \\
0 & 0 & -\gamma^2I & * & * & * & * & * & * \\
\Upsilon_{41} & \Upsilon_{42} & D\bar{P} & -\bar{P} & * & * & * & * & * \\
\Upsilon_{51} & \Upsilon_{52} & 0 & 0 & -\bar{P} & * & * & * & * \\
\bar{C}\bar{P} & 0 & 0 & 0 & -I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & -\varepsilon \xi^{-1}I \\
M_1 & M_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon \xi^{-1}I
\end{bmatrix} < 0,
\]
where
\[
\Upsilon_{41} = A_1(T_j)\bar{P} + B_1(T_j)M_1, \quad \Upsilon_{42} = A_2\bar{P} + B_1(T_j)M_2,
\]
\[
\Upsilon_{51} = \theta(\hat{A}_1(T_j)\bar{P} + \hat{B}_1(T_j)M_1), \quad \Upsilon_{52} = \theta(\hat{B}_1(T_j)M_2),
\]
and
\[
A_1(T_j) = \begin{bmatrix} A & 0 \\ -\hat{a}T_jCA & 0 \end{bmatrix}, \hat{A}_1(T_j) = \begin{bmatrix} 0 & 0 \\ -T_jCA & 0 \end{bmatrix},
\]
\[
B_1(T_j) = \begin{bmatrix} B \\ -\hat{a}T_jCB \end{bmatrix}, \hat{B}_1(T_j) = \begin{bmatrix} 0 \\ -T_jCB \end{bmatrix}.
\]
Then the 2D fault tolerant system (20) is mean square asymptotically stable and satisfies the robust \(H_\infty\) performance constraint. Moreover, the gain matrices \(K_1, K_2\) in the
ILFTC controller can be given as follows:

\[ K_1 = M_1 \tilde{P}^{-1}, \quad K_2 = M_2 \tilde{P}^{-1}. \tag{45} \]

**Proof:** According to (21), Lemmas 2.1 and 2.2, and combining the part of 4.2 in Wang et al. (2008), we can obtain

\[
\begin{bmatrix}
-P + Q & -I & -I & -I & -I & -I & -I & -I \\
0 & -Q & -I & -I & -I & -I & -I & -I \\
0 & 0 & -\gamma^2I & -I & -I & -I & -I & -I \\
\Pi_{41} & \Pi_{42} & PD & -P & -I & -I & -I & -I \\
\Pi_{51} & \Pi_{52} & 0 & 0 & -I & -I & -I & -I \\
\hat{C} & 0 & 0 & 0 & 0 & 0 & -I & -I \\
0 & 0 & 0 & \varepsilon \hat{B}_{2}(T_{j})P & \varepsilon \hat{B}_{1}(T_{j})P & 0 & 0 & -c_{l}I \\
K_1 & K_2 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{l}^{-1}I
\end{bmatrix} < 0,
\]

where

\[
\Pi_{41} = P(A_1(T_j) + B_1(T_j)K_1), \quad \Pi_{42} = P(A_2 + B_1(T_j)K_2),
\]
\[
\Pi_{51} = \theta P(\hat{A}_1(T_j) + \hat{B}_1(T_j)K_1), \quad \Pi_{52} = \theta P\hat{B}_1(T_j)K_2.
\]

Per- and post- multiplying (46) by \(\text{diag}(P^{-1}, 1, P^{-1}, P^{-1}, 1, 1, 1, 1)\), and defining \(\tilde{P} = P^{-1}, \tilde{Q} = P^{-1}QP^{-1}, M_1 = K_1P^{-1}, M_2 = K_2P^{-1}\), we can get inequality (44). The proof of this theorem is complete. \(\blacksquare\)

**Remark 3.1:** In this paper, we consider both event-triggered transmission strategy and data dropouts during the different signal transmissions, which increase the complexity of derivation. However, compared with the method in Xuhui et al. (2016), it is easy to find that the relevant procedures have been greatly simplified in this paper.

### 4. Illustration

In order to show the effectiveness of the designed controller, we choose injection molding, a typical batch process, as a simulation object in this section. In the process of packing, which is a main phase of injection molding, the nozzle pressure is necessary to be controlled to ensure the quality of products. Then, the dynamics of nozzle pressure control can be given as follows:

\[
x(t + 1, k) = \begin{bmatrix} 1.607 \ 0 \end{bmatrix} x(t, k) + \begin{bmatrix} 1.2390 \ -0.9282 \end{bmatrix} u(t, k)
\]

\[
y(t, k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t, k), \quad k = 1, 2, \ldots, 100,
\]

\[
t = 0, 1, \ldots, 150.
\]  \tag{47}

The initial conditions are set as \(x(0, k) = [5 \ 0]^T\) in each cycle in the example. For simplicity, we assume that all the signals of the measurement output \(y(t, k)\) can be transmitted at the original iteration. The desired output trajectory \(y_d(t)\) can be given as the following form

\[
y_d(t) = \begin{cases} 15, & 0 \leq t \leq 80 \\ 25, & 80 < t \leq 150 \end{cases}
\]

**Case 1.** In this case, on the basis of the designed ILFTC scheme, the problem of fault-tolerant control is discussed for the batch process (47) with sensor faults. We choose the disturbance attenuation level index \(\gamma = 3.2878\), the triggered threshold \(\xi = 0.15\), and the sensor fault gain \(0.75 \leq \Lambda \leq 1.05\). As for the problem of data dropouts, we assume \(\bar{\alpha} = 0.9\). Then, the controller parameters \((K_1, K_2)\) can be shown as follows by solving the LMI (44)

\[
K_1 = \begin{bmatrix} -1.3881 & -0.9218 & 0.0000 \end{bmatrix},
\]

\[
K_2 = \begin{bmatrix} 0.0000 & 0.0000 & 0.7046 \end{bmatrix}.
\]  \tag{49}

When the sensor is in normal state, the measurement output trajectories \(y(t, k)\) can be plotted by Figure 1, in which we can find the tracking objective can be rapidly realized with the increase of cycle. Then, we assume that the sensor faults occur after Cycle 50, as shown in Figure 2. From this figure, it is clear that the tracking performance is degraded after faults occur, nonetheless, the negative effect can be quickly eliminated under the designed controller.

Meanwhile, we denote the following mean square error index \(H(k)\) to evaluate the tracking performance more clearly.

\[
H(k) = \frac{1}{150} \sqrt{\sum_{i=1}^{150} e^2(t, k)}
\]  \tag{50}

The smaller value of \(H(k)\) means the better tracking performance. Figures 3 and 4 show the changes of tracking performance in cycle direction without and with sensor...
faults, respectively. From Figure 4, it is obvious that the tracking performance can be restored to its original level with the increase of the iteration after the sensor faults occur.

**Case 2.** In this case, we will illustrate how the event-triggered threshold $\xi$ effects the system performance. Choose the different thresholds ($\xi = 0.05, 0.15, 0.3$), then the responses of measurement output can be shown by Figures 5–7, respectively. In Figure 8, the triggering instants of control signal can be depicted for the system (47) with different triggered thresholds.

In Figures 5–7, we can see that the fault tolerance of the system (47) can be reached with different event-triggered thresholds. However, the tracking performance and stability of the system (47) become worse with the increase of $\xi$. From Figure 8, it can be found that the number of the triggering instants are also gradually decreased. Hence, it is necessary to choose a appropriate triggering threshold to guarantee the system performance when we apply the event-triggered scheme in the signal transmission.
Figure 7. Output responses $y(t, k)$ for $\xi = 0.3$ in Case 2.

Figure 8. The triggering instants for different $\xi$.

Figure 9. Tracking performances $H(k)$ for $\bar{\alpha} = 0.9, 0.7, 0.6$ in Case 3.

Case 3. In this case, we consider the problem of data dropouts during the transmission of $y(t, k)$ for the batch process (47). In Figure 9, the tracking performance can be depicted by using the error index $H(k)$, in which the different packet loss probabilities ($\bar{\alpha} = 0.9, 0.7, 0.6$) are chosen in the system (47). From this figure, it is clear to find that the stability of the system will be gradually reduced with the decrease of $\bar{\alpha}$. Then, we can even deduce that the system is going to be instable if too much data is lost during the transmission.

5. Conclusions

In this paper, the problem of iterative learning fault-tolerant control has been studied for the batch processes with sensor faults. The event-triggered transmission mechanism has been adopted for the batch processes to reduce the burden of the signal communication. Meanwhile, a stochastic variable has been introduced to describe the phenomenon of data dropouts. By using the theory of 2D stochastic system and linear matrix inequality approach, the iterative learning fault tolerant controller can be obtained to guarantee the tracking performance of the closed-loop system. Finally, the simulation results on the nozzle pressure control have illustrated the usefulness of the designed scheme. Based on the obtained results, the ILFTC problem will be studied for more complex systems with different networked phenomena in the future.

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