Comment on
“Neutron lifetime and dark decay of the neutron and hydrogen”

Bartosz Fornal and Benjamin Grinstein
Department of Physics, University of California, San Diego,
9500 Gilman Drive, La Jolla, CA 92037, USA

February 26, 2019

Abstract
The manuscript by Berezhiani (arXiv:1812.11089) proposes a model that has the neutron decaying into a mirror neutron with a branching fraction of 1%, alleviating the tension between neutron lifetime measurements in beam vs bottle experiments. We show that the model as proposed is inconsistent with experiment. Variations of the model may work at the expense of extreme breaking of the $Z_2$ symmetry between the Standard Model and its mirror copy.

Slightly over a year ago we proposed that the tension between neutron lifetime measurements in beam vs bottle experiments can be alleviated by an unobserved decay of the neutron into new neutral particles with a branching fraction of 1% \[1\]. To describe one of the proposed putative new decay channels ($\nu \rightarrow \chi\gamma$) in a quantitative way, we considered theories in which a dark Dirac fermion $\chi$ has a small mass mixing with the neutron. Diagonalizing the mass matrix gives rise to an interaction $\chi n\gamma$.

\[ L_{\text{eff}} = \bar{n} \left( \frac{i}{\sigma} - m_n + \frac{g_{n\gamma}}{2m_n} \sigma_{\mu\nu} F_{\mu\nu} \right) \gamma \chi + \bar{\chi} \left( \chi n + \frac{m_n}{m_\chi} \right) \bar{\chi} \gamma, \]

where $\varepsilon$ is the mass-mixing parameter. Transforming to the mass eigenstate basis yields, for $\varepsilon \ll m_n - m_\chi$,

\[ L_{\nu \rightarrow \chi\gamma} = \frac{g_{n\gamma} \varepsilon}{2m_n (m_n - m_\chi)} \bar{\chi} \sigma_{\mu\nu} F_{\mu\nu} \chi, \]

Therefore, the neutron dark decay rate is

\[ \Delta \Gamma_{\nu \rightarrow \chi\gamma} = \frac{g_{n\gamma}^2 \varepsilon^2}{8\pi} \left( 1 - \frac{m_\chi^2}{m_n^2} \right)^3 \frac{m_n \varepsilon^2}{(m_n - m_\chi)^2} \approx \Delta \Gamma_{\text{exp}} \left( \frac{1 + x}{2} \right)^3 \left( \frac{1 - x}{1.8 \times 10^{-3}} \right) \left( \frac{\varepsilon \ [\text{GeV}]}{9.3 \times 10^{-14}} \right)^2, \]

where $x = m_\chi/m_n$. Stability of $^9$Be with respect to the potential decay channel $^9$Be $\rightarrow ^8$Be + $\chi$ requires $m_\chi > 937.900$ MeV. A slightly stronger constraint follows from $^9$Be $\rightarrow 2^4$He + $\chi$ \[2\]:

\[ 937.992 \text{ MeV} < m_\chi < 939.565 \text{ MeV}. \]

A minimal particle physics realization of this idea was also presented in Ref. \[1\]. It requires only two particles beyond the Standard Model (SM): a complex scalar $S \sim (3, 1, -1/3)$, and a Dirac fermion $\chi \sim (1, 1, 0)$, with the Lagrangian

\[ L = (\lambda_q \epsilon^{|k|} u^c_L d_R S_k + \lambda_S S^i \bar{\chi} d_R_i + \text{h.c.}) - M_S^2 |S|^2 - m_\chi \bar{\chi} \chi, \]

where $u^c_L$ is the charge conjugate of $u_R$. With $B_\chi = 1$ and $B_S = -2/3$, baryon number is conserved.
The rate for $n \to \chi \gamma$ is given by Eq. (3) with
\[ \varepsilon = \frac{\beta \lambda_q \lambda_{\chi}}{M_S^2}. \] (6)

Here $\beta$ is defined by $\langle 0 | \epsilon^{ijk}(\bar{u}_{Li}^j d_{Rj}) d_{Rk}|n \rangle = \frac{1}{2} \beta (1 + \gamma_5) u_n$, where $u_n$ is the neutron Dirac spinor. Lattice QCD calculations give $\beta = 0.0144(3)(21)$ GeV$^3$ [3]. Assuming $m_{\chi} \approx 938$ MeV to maximize the rate, the parameter choice explaining the anomaly is $|\lambda_q \lambda_{\chi}|/M_S^2 \approx 7 \times 10^{-6}$ TeV$^{-2}$.

After a long introduction that reviews the work above, Ref. [4] proposes a model in which $\chi$ is identified as the “neutron” ($n'$) of a mirror copy of the SM. The complete model is a double replica of the model in Eq. (5), extending it by mirroring all SM particles and the new particle $S$, and duplicating the Lagrangian, with $\chi$ as a mediator:
\[ \mathcal{L}_2 = \lambda_q \epsilon^{ijk} \bar{u}_{Li}^j d_{Rj} S_k + \lambda_{\chi} S^{\dagger} \bar{\chi} d_{Ri} + \lambda_q \epsilon^{ijk} \bar{u}_{Li}^j d_{Rj} S'_{Rk} + \lambda_{\chi} S^{\dagger} \bar{\chi'} d_{Ri} + \text{h.c.}. \] (7)

Integrating out $S$, $S'$ and $\chi$ produces the effective operator for the $n - n'$ mixing with
\[ \varepsilon = \frac{(\beta \lambda_q \lambda_{\chi})^2}{M_S^2 M_{S'}^2 m_{\chi}}. \] (8)

This is to be compared with Eq. (2). If the $Z_2$ symmetry is explicit, i.e. $M_{S'} = M_S$, then in order to have $\varepsilon \approx 10^{-13}$ GeV, one requires $M_{S'S'} \approx (210$ GeV$) \sqrt{\lambda_q \lambda_{\chi}/(m_{\chi}$[GeV$])}^{1/4}$. To avoid light colored scalars, $Z_2$ must be spontaneously broken. Following Ref. [4], i.e. taking $M_S/\lambda_q \gtrsim 10$ TeV and $m_{\chi} = 10$ GeV, Eq. (5) results in $M_{S'/}\lambda_{\chi} \lesssim 1.4$ GeV. More conservatively, it is sufficient to have $m_{\chi} \gtrsim 1$ GeV. Using the bound from LHC dijet searches for scalar diquarks, $M_{S}/\lambda_q \gtrsim 7.0$ TeV [5,6], results in $M_{S'/}\lambda_{\chi} \lesssim 6.5$ GeV. A complimentary bound on $M_S$ alone comes from investigating the four-jet signature of scalar diquarks [7,8] and gives $M_S \gtrsim 2.5$ TeV.

Even if spontaneously broken, the $Z_2$ symmetry insures that the UV value of the gauge and Yukawa coupling constants in the SM and its mirror copy are the same. The low-energy values of the gauge couplings are determined by the renormalization group with the equality of couplings as a UV boundary condition, i.e. $g_3'(\mu) = g_3(\mu)$ for $\mu \gg M_S$, assuming $S$ is the heaviest field. We then have at one loop (sufficiently accurate for the point we intend to make):
\[ \frac{\Lambda^{(3)}_{QCD'}}{\Lambda^{(3)}_{QCD}} = \left( \frac{M_{S'}}{M_S} \right)^{\frac{1}{16}}. \] (9)

Here $\alpha_s(\mu) = 2\pi/[\beta_0 \ln(\mu/\Lambda_{QCD}^{(n_f)})]$, with $\Lambda_{QCD}^{(n_f)}$ being the one-loop RG-invariant scale for $n_f$ quarks, and similarly for the mirror sector. The ratio in Eq. (9) is independent of the subtraction scheme at one loop. For the most conservative choice, i.e. $M_S \approx 2.5$ TeV, $M_S/\lambda_q \approx 7.0$ TeV and therefore $M_{S'} \approx (6.5$ GeV$) \lambda_{\chi} \lesssim 61$ GeV (for $\lambda_{\chi} \lesssim 3\pi$), then this gives $\Lambda^{(3)}_{QCD'}/\Lambda^{(3)}_{QCD} \lesssim 0.934$, which immediately translates into $m_{n'} \lesssim 0.934 m_n = 878$ MeV. If, following Ref. [4], one instead takes $m_{\chi} \approx 10$ GeV and $M_S/\lambda_q \approx 10$ TeV, which requires (using the correct value for $\beta$) $M_{S'} \lesssim (1.5$ GeV$) \lambda_{\chi} < 14$ GeV (for $\lambda_{\chi} \lesssim 3\pi$), then, given that $M_S \gtrsim 2.5$ TeV, the mirror neutron mass is $m_{n'} \lesssim 854$ MeV.

These results violate the bound in Eq. (4), giving an unstable $^9$Be and, in addition, an unstable proton. Therefore, the model as presented in Ref. [4] is inconsistent with observation.

\begin{itemize}
  \item \footnote{Without proper accreditation to Refs. [11] and [2].}
  \item \footnote{Our value differs from that of Ref. [4] by a factor of 20, of which a factor of 10 results from the overestimate of $\beta$ used there.}
\end{itemize}
A possible way to avoid this disastrous conclusion is to use the breaking of the $Z_2$ symmetry to invoke large $u’, d’$ and $s’$ masses, that could then increase $m_{n’}$ to within $\sim 1$ MeV of $m_n$. Using

$$\sigma_{\pi N} \equiv \frac{1}{2}(m_u + m_d)\langle N|\bar{u}u + \bar{d}d|N \rangle \quad \text{and} \quad \sigma_{sN} \equiv m_s\langle N|\bar{s}s|N \rangle ,$$

we estimate the additional quark mass contribution to the mirror neutron mass as

$$\Delta m_{n’} \approx \left( \frac{m_{u’} + m_{d’}}{m_u + m_d} - 1 \right) \sigma_{\pi N} + \left( \frac{m_{s’}}{m_s} - 1 \right) \sigma_{sN} ,$$

where $\sigma_{\pi N} \approx 46$ MeV and $\sigma_{sN} \approx 40$ MeV from the lattice calculation in Ref. [9]. Similar values for $\sigma_{\pi N}$ and $\sigma_{sN}$ are found elsewhere [10–14]; the values we use are at the upper end of the range in these determinations, so that our estimate of mirror quark masses required to raise the mirror neutron mass is conservatively low. The $Z_2$ symmetry requires equality of Yukawa couplings that yield quark and mirror quark masses; hence, the mirror quark to quark mass ratios are simply given by the ratio of electroweak vacuum expectation values in the mirror SM to that of the SM, $v'/v$. The masses of heavy mirror quarks also change and the estimate of $\Lambda'$ must be revisited:

$$\frac{\Lambda_{QCD'}}{\Lambda_{QCD}} = \left( \frac{v'}{v} \right)^\frac{2}{3} \left( \frac{M_{S'}}{M_S} \right) ^\frac{1}{3} .$$

In addition, we must distinguish $\beta'$ from $\beta$ in Eq. (5), with $\beta' \approx (\Lambda_{QCD}/\Lambda_{QCD'})^3\beta$. For the values $\varepsilon = 10^{-13}$ GeV, $M_S = 2.5$ TeV, $M_S/\lambda_q = 7.0$ TeV, $m_{\chi} = 1.0$ GeV and $\lambda_{\chi} = 1$, and setting $m_{n'} = 938.8$ MeV (i.e. at the lower bound arising from the search for radiative $n$ dark decay [15]), we find $v'/v = 1.4$ and $M_{S'} = 6.1$ GeV.

A mirror world with $v'/v \sim 2$ is likely to be very different from ours. It has been argued that mirror deuterium is unstable to weak decay for $v'/v \gtrsim 1.2$ and unstable to strong decay for $v'/v \gtrsim 1.4$; these estimates are based on a square well model; when the OBEP model is used instead, the threshold values of $v'/v$ for the weak and strong decays become 1.4 and 2.7, respectively [16,17]. Because our estimates for $v'/v$ are in this range, we cannot draw solid conclusions; while it was not pressing for the work in Refs. [16,17], a refined calculation that can sharply establish the threshold for mirror deuterium stability is needed in our context to determine the fate of mirror deuterium and other mirror nuclei (and the cosmological implications of the mirror world).

We have explored but one consequence of a badly broken $Z_2$ symmetry in the model of Ref. [4]. There may be other effects that as yet have not been accounted for. But given that $Z_2$ is so badly broken, rather than mapping out the consequences of the broken symmetry, it may be more productive to reinterpret our result in a more general context: instead of restricting the dark sector to a $Z_2$ symmetric mirror SM, one may more generally conceive of models with a dark sector involving composite states, bound by a dark-strong interaction, not necessarily based on an SU(3) gauge group nor on “quarks” in the fundamental representation. For example, one may replace the mirror sector in the Lagrangian in Eq. (7) by a simpler model:

$$\mathcal{L}_3 = \lambda_q \epsilon^{ijk} u^a_{Li} d_{Rj} S_k + \lambda_{\chi} \bar{S}^{\ast i} \chi^a d_{Ri} + \lambda_{\phi} \phi \bar{\psi} \chi^a + \text{h.c.} ,$$

where the complex scalar $\phi$ and the Dirac fermion $\psi$ are in the fundamental representation of some strongly interacting gauge group $G$. A confined bound state of $\phi$ and $\psi$ then plays the role of the mirror nucleon in the model of Ref. [4].

\footnote{In addition, the linear relation $m_N = m_q + am_q$ is only a rough approximation. Corrections of order $m_q^3/2$ come in with a negative sign and are not negligible.}
Nota bene
In the “Acknowledgments” of Ref. [4], the author states, in reference to his participation in the INT Workshop “Neutron-Antineutron Oscillations: Appearance, Disappearance, and Baryogenesis” held October 23 – 27, 2017, that:

However, my talk [43] evidently had some subconscious impact on the community, since very similar work of Fornal and Grinstein appeared recently [44], with the difference that the dark particle $n'$ was considered as an elementary fermion with ad hoc chosen mass, which was followed by many other works [45–55]. It is somewhat surprising that none of these authors mentioned about my talk, even those participants of the Seattle INT Workshop which were explicitly present on it – something is rotten in the state of Denmark – and such a solution for the neutron lifetime puzzle was coined as the dark decay solution or Fornal-Grinstein solution, while it remains questionable whether it is a solution at all.

We would like to take this opportunity to clarify that neither Fornal nor Grinstein were present at Berezhiani’s third talk at the Workshop on October 26, 2017 (cited as [43] in Ref. [4]), the talk in which he presented his ideas regarding neutron dark decay in the context of neutron-mirror neutron oscillations: Grinstein did not participate in the Workshop and Fornal had already left the Workshop on October 24, 2017.

We started working on our ideas in May 2017, soon after the article by Geltenbort and Greene “The neutron lifetime puzzle” in the Institut Laue-Langevin 2016 Annual Report [18], based on Ref. [19], was brought to our attention. We have photographic evidence that our ideas leading to the publication [1] were already developed by the time of the aforementioned INT Workshop. The blackboard photo [20] taken two weeks prior to the Workshop (the date can be checked in the file’s properties) shows the diagram for the neutron dark decay that lies at the heart of our proposal.

One of us (BF) was first made aware of the content of Berezhiani’s third talk in an e-mail from Yuri Kamyskho on January 20, 2018, i.e. over two weeks after our paper [1] was posted on the arXiv. Reference [4] seems to suggest that our work was somehow influenced by Berezhiani’s talk. This is obviously not true.

Acknowledgment
The authors were supported in part by the DOE Grant No. DE-SC0009919.

References
[1] B. Fornal and B. Grinstein, “Dark Matter Interpretation of the Neutron Decay Anomaly,”  
Phys. Rev. Lett. 120 no. 19, (2018) 191801, arXiv:1801.01124 [hep-ph].

[2] M. Pfützner and K. Riisager, “Examining the possibility to observe neutron dark decay in nuclei,” Phys. Rev. C97 no. 4, (2018) 042501, arXiv:1803.01334 [nucl-ex].

[3] Y. Aoki, T. Izubuchi, E. Shintani, and A. Soni, “Improved lattice computation of proton decay matrix elements,” Phys. Rev. D96 no. 1, (2017) 014506, arXiv:1705.01338 [hep-lat].

[4] Z. Berezhiani, “Neutron lifetime and dark decay of the neutron and hydrogen,” arXiv:1812.11089 [hep-lat].

[5] CMS Collaboration, A. M. Sirunyan et al., “Search for narrow and broad dijet resonances in proton-proton collisions at $\sqrt{s} = 13$ TeV and constraints on dark matter mediators and other new particles,” JHEP 08 (2018) 130, arXiv:1806.00843 [hep-ex].
[6] ATLAS Collaboration, M. Aaboud et al., “Search for new phenomena in dijet events using 37 fb$^{-1}$ of $pp$ collision data collected at $\sqrt{s}=$13 TeV with the ATLAS detector,” *Phys. Rev. D96* no. 5, (2017) 052004, arXiv:1703.09127 [hep-ex].

[7] P. Richardson and D. Winn, “Simulation of sextet diquark production,” *Eur. Phys. J. C72* (2012) 1862, arXiv:1108.6154 [hep-ph].

[8] N. Assad, B. Fornal, and B. Grinstein, “Baryon number and lepton universality violation in leptoquark and diquark models,” *Phys. Lett. B777* (2018) 324–331, arXiv:1708.06350 [hep-ph].

[9] xQCD Collaboration, Y.-B. Yang, A. Alexandru, T. Draper, J. Liang, and K.-F. Liu, “$\pi N$ and strangeness sigma terms at the physical point with chiral fermions,” *Phys. Rev. D94* no. 5, (2016) 054503, arXiv:1511.09089 [hep-lat].

[10] S. Durr et al., “Sigma term and strangeness content of octet baryons,” *Phys. Rev. D85* (2012) 014509, arXiv:1109.4265 [hep-lat] [Erratum: Phys. Rev. D93, no.3, 039905 (2016)].

[11] QCDSF-UKQCD Collaboration, R. Horsley, Y. Nakamura, H. Perlt, D. Pleiter, P. E. L. Rakow, G. Schierholz, A. Schiller, H. Stuken, F. Winter, and J. M. Zanotti, “Hyperon sigma terms for 2+1 quark flavours,” *Phys. Rev. D85* (2012) 034506, arXiv:1110.4971 [hep-lat].

[12] C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou, “Baryon spectrum with $N_f = 2 + 1 + 1$ twisted mass fermions,” *Phys. Rev. D90* no. 7, (2014) 074501, arXiv:1406.4310 [hep-lat].

[13] G. S. Bali et al., “Nucleon mass and sigma term from lattice QCD with two light fermion flavors,” *Nucl. Phys. B866* (2013) 1–25, arXiv:1206.7034 [hep-lat].

[14] S. Durr et al., “Lattice Computation of the Nucleon Scalar Quark Contents at the Physical Point,” *Phys. Rev. Lett. 116* no. 17, (2016) 172001 arXiv:1510.08013 [hep-lat].

[15] Z. Tang et al., “Search for the Neutron Decay $n \rightarrow X + \gamma$, where $X$ is a Dark Matter Particle,” *Phys. Rev. Lett. 121* no. 2, (2018) 022505, arXiv:1802.01595 [nucl-ex].

[16] V. Agrawal, S. M. Barr, J. F. Donoghue, and D. Seckel, “Anthropic Considerations in Multiple Domain Theories and the Scale of Electroweak Symmetry Breaking,” *Phys. Rev. Lett. 80* (1998) 1822–1825, arXiv:hep-ph/9801253 [hep-ph].

[17] V. Agrawal, S. M. Barr, J. F. Donoghue, and D. Seckel, “Viable range of the mass scale of the standard model,” *Phys. Rev. D57* (1998) 5480–5492, arXiv:hep-ph/9707380 [hep-ph].

[18] G. L. Greene and P. Geltenbort, “The neutron lifetime puzzle,” *ILL Annual Report 2016* (Institut Laue–Langevin, Grenoble, France; May 2017; Editors: G. Cicognani and M. Johnson), pp. 68–69.

[19] G. L. Greene and P. Geltenbort, “The neutron enigma,” *Sci. Am. 314* (2016) 36–41.

[20] Blackboard photo, UC San Diego; October 12, 2017; credit: B. Fornal.