Perspectives of Using Oscillators for Computing and Signal Processing

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Abstract
It is an intriguing concept to use oscillators as fundamental building blocks of electronic computers. The idea is not new, but is currently subject to intense research as a part of the quest for ‘beyond Moore’ electronic devices. In this paper we give an engineering-minded survey of oscillator-based computing architectures, with the goal of understanding their promise and limitations for next-generation computing. We will mostly discuss non-Boolean, neurally-inspired computing concepts and put the emphasis on hardware and on circuits where the oscillators are realized from emerging, nanoscale building blocks. Despite all the promise that oscillatory computing holds, existing literature gives very few clear-cut arguments about the possible benefits of using oscillators in place of other analog nonlinear circuit elements. In this survey we will argue for finding the rationale of using oscillatory building blocks and call for benchmarking studies that compare oscillatory computing circuits to level-based (analog) implementations.

Keywords: Oscillator-based computing, oscillatory neural networks, non-Boolean computing, nanoelectronic devices

1. Introduction
The goal of this paper is to give a survey of oscillatory computing architectures (OCAs) and help the reader to understand the value and utility of this concept in engineering applications, and in the design of beyond Moore computing and signal processing devices.

The vast majority of signal processing / computing devices uses a level-based representation of signals - in most cases, the levels are discrete or binary voltage values. In these models of computation the dynamic properties of the signals play a secondary role – the information that is carried by the timing of the signals or by their exact waveform, is largely neglected. Wasting the information carried by the timing of the signal inevitably wastes energy - this suggests that any purely level-based computing scheme is suboptimal in terms of power consumption, which is arguably the most important figure of merit in todays computing devices.

Information is physical and in any hardware realization, the signal must be represented by a physical quantity (such as current, voltage, or an other emerging variable). One could almost certainly do better and use the carriers of information more efficiently. One way of accomplishing this is to use the phase and /or frequency of oscillatory signals to carry information, besides the signal amplitude. This is one of the key ideas behind OCAs.

The other key characteristics of an OCA is that the signals have to be processed by the physical interaction of nonlinear oscillators. There are many emerging computing concepts that use the interaction of nonlinear elements for computation, but, as it turns out, it is not at all easy to find physically realizable low-power, robust, nonlinear elements that could serve as building blocks in such computers. Oscillators are good candidates for a practical nonlinear devices - one reason for that is that oscillatory systems are ubiquitous in the physical world, so one may hope to find an attractive (low power, compact, fast, etc) device realizations. Most physical oscillators show a suitable nonlinearity in their phase or frequency response and may act as the nonlinear, active circuit element at the heart of the computing architecture.

Figure 1 gives a schematic high-level overview of analog, dynamic computing concepts, that are relevant for OCAs. The field of OCAs overlaps with many nonconventional computing paradigms and many analog, nonconventional computing model can be implemented using oscillatory building blocks.

Von Neumann’s oscillatory computer, is an early and still very relevant example of OCAs - this scheme uses phases of oscillatory signals to realize Boolean, digital computation. Von Neumann’s oscillatory computer serves as a perfect example how one can redesign a level-based computing scheme (in this case a standard Boolean computer) to phase / frequency / based realization and also illustrates most of the challenges inherent to this approach. For this reason we will review this concept in Section 2.

Many different kinds of oscillators and oscillator coupling schemes can be implemented in transistor-based (CMOS-
Power figures will likely be advantages of ‘standard’ CMOS circuitry - in particular, overlap with other analog computing concepts - some of those related to neurally-inspired circuit architectures. We mentioned above that oscillators may be attractive, power hungry operations in ANNs. Prime example here are image-preprocessing operations in an image processing pipeline, associative memory units, reservoirs of an echo state network or units handling computationally hard problems. Our review will devote Sec. 6 and Sec. 7 to these new application areas.

The reader will see that oscillatory computing devices are a vast field - the 100+ reference we cite in this review represents only a small fraction of the literature and there are books devoted to the topic. Still, it seems that a fundamental ‘why’ question remains unanswered, namely, why would one use oscillatory components instead of other nonlinear circuit building blocks? It is very much possible to realize analog, neuromorphic, non-Boolean computing devices from non-oscillatory nonlinear elements – much researched Cellular Nonlinear Networks (CNNs) do exactly that and memristor-based, non-oscillatory neuromorphic architectures are a hot topic nowadays. We mentioned above that oscillators may be attractive due to the vast number of possible implementations - but this alone is a rather hand-waving argument as oscillators have obvious disadvantages too. For example they must run continually, dissipating power all the time, and it can be energetically costly to power them up / down - there must be strong benefits to offset such disadvantages. We believe that OCAs indeed may have fundamental benefits and in the conclusion of this paper we offer clues for what those killer benefits could be.

Before diving in the details of oscillatory computing we have to make an important distinction between OCAs and spiking neural networks / spike-based signal processors. Spiking (neural) networks are also an emerging field and perhaps the first successful engineering application related to neurally-inspired circuit architectures. The boundary between OCAs and spiking networks is somewhat diffuse and the terminology is often imprecise. Spiking neural networks employ oscillatory signal representation (they count spikes, use their relative phase and frequency to do processing), but they do not rely on oscillator interactions: most architectures count / integrate pulse sequences using digital circuits or analog integrators. Their promise comes from the fact that a single spike can carry extremely low energy (in the order of 10⁻¹⁶ J) and it is acceptable to lose some fraction of the spikes to noise. Spiking neural networks may use oscillators for generating the signals but the nonlinear interaction, pattern formation of these oscillators is not required for their operation. Spiking devices are not subject of this review paper.

2. Phase logic and von Neumann’s oscillatory computer

Von Neumann’s idea is based on interconnected, subharmonic injection locked oscillators (SHILOs). While we cannot be sure of von Neumann’s motivation, it is worthwhile to note that digital computes in the early fifties reached then-breathtaking several MHz speed already in the early fifties. So it seemed a natural idea to look at a logic circuit not as switch between zeros and ones, but rather as an electrical oscillator switching between different phases of oscillation. Goto and others furthered this concept (the basic oscillator elements are called parametron in their works) and fully functional Boolean computers were realized using these building blocks.

Figure 2 - b is a circuit schematics of a logic gate from a parametron-based computer. This is one particular implementation, where the building blocks are inductively coupled nonlinear LC oscillators and the oscillator nonlinearity comes from the nonlinear hysteresis of the inductive cores. If the LC oscillators have a resonance frequency of \( f = f_0 \) and are excited by an external \( f_{\text{pump}} = 2f_0 \) signal. This parametric pumping supplies the oscillators with energy (they need no DC power supply), and the \( f = f_0 \) frequency oscillator signals become phase-locked to \( 2f_0 \) pumping signal. One may see in Figure 2 that there are two distinct phases in which a \( f_0 \) frequency signal may be synchronized to \( 2f_0 \) signal. This observation is the key for using this system for binary logic - the two dis-

![Diagram](image-url)
tinct phases with respect to the pumping signal, represent the binary ‘0’ and ‘1’.

Multiple, interconnected oscillators (which all run at $f_0$ frequency in one or the other possible phase) will pull toward each other’s phase. If a particular oscillator receives multiple inputs with different phases, then it will follow the phase of the majority of input oscillators. Such majority gates serve as universal logic gates in the computing architecture - they can straightforwardly realize NAND / NOR gates and inverters and satisfy the five tenets of Boolean computation [27].

Clearly, LC oscillators have serious practical limitations: inductors remain a major challenge in any planar and miniaturized technology. The idea of phase-based logic however, is resurrected in many proposals. Ring oscillators may be a microelectronic-friendly implementation [28, 29] - Fig. reffig:parametronb - illustrates this possibility. Using nanomechanical oscillators [30, 23] one could make very compact and extremely low-power stand-alone logic gates - so the idea of parametron is very much alive. One can construct all phase logic elements from single electronic devices [31]. It is argued [28] that phase-based representation are more noise tolerant than level-based signal representation, which can be extremely important in low-power, low-voltage circuitry.

Oscillator phases could represent analog variables, but parametric pumping reduces the of stationary phase states to two, turning the SHILOs into a digital system. This makes the scheme realization friendly - it is not necessary to use high-quality oscillators with stable frequency and the phase characteristics, the external stimulus stabilizes phases / frequencies. In the original scheme the analog computing power of oscillators is not harnessed - albeit one may extend the idea to non-Boolean, analog computation [32], as illustrated in Fig. 2).

Parametron-type devices offered equivalent functionality to level-based logic gates, but they never became mainstream due to the lack of scalable, fast, low-power, on-chip oscillators. In these years, Moore’s law [33], in its original 'scale everything' form is ending, due to power constraints. Oscillator-based logic (phase logic) may have a chance for a comeback, if it can reach significantly better power figures, either with electrical [28] or with alternate state variables [3].

3. Hardware realizations for oscillatory circuits

Just like as a digital computer is built from billions of transistors, an envisioned OCA will contain millions or billions of interconnected oscillators. The demands for the elementary oscillators are high and the success of OCAs will eventually depend on whether one can find oscillatory building blocks that are (1) compact (2) low lower (3) high-frequency (4) low noise (5) can be efficiently interconnected to each other (6) and easily interfaces with electronic circuitry.

Satisfying all of the above requirements is a tall order - and, depending on the chosen architecture, some of the requirements may just be unimportant. Table 2 below gives an overview of possible physical oscillators, and we also give quick estimation of some important parameters. Some of these oscillators are electrical [34], some use another state variable in between their electrical contacts, others may do every function with non-electrical state variables [3].

3.1. Figures of merits for various oscillators

An ideal oscillator in terms of energy would be an energy-recycling oscillator, i.e. where during each oscillation cycle energy is converted (largely reversibly) between two forms, instead of being dissipated. An LC oscillator converts between electrostatic and magnetic energies, a mechanical (NEMS) oscillator converts between kinetic and potential energy [40] [46] [30] [47] [48]. Such oscillator with a quality factor $Q$ dissipates only 1/Qth of its stored energy. Unfortunately, both LC and NEMS oscillators, which have this property [49], fare quite badly in the other figures of merits [50]. The LC oscillators require inductors, which are very hard to integrate on chip, they are bulky and resistive. So on-chip LC oscillators have low Q factors [37] and little can be gained by the energy recycling. Mechanical oscillators (MEMS / NEMS) can have very high Qs - in their case the transduction efficiency is the main problem: no matter how efficient, high Q oscillators they are, interconversion between electric and mechanical signals can be rarely done better than with a few-percent efficiency. Many NEMS oscillators that were proposed for computation [47] occupy a large chip area - albeit there are a lot of new developments toward practical nanomechanical oscillators [51].

In terms of sheer numbers, superconducting devices may come closest to the a perfect oscillators: they consume ultra low energies, capable of high frequencies, they do not necessarily occupy extreme chip areas, and to some degree they are energy-recycling, just as the LC circuits.
#### Table 1: Possible building blocks of an oscillatory computing architecture. Ring oscillators can be viewed as good baseline for any emerging device concept.

| Oscillator name                  | State variable | Frequency (GHz) | Power dissipation / cycle | Coupling mechanism |
|----------------------------------|----------------|-----------------|---------------------------|--------------------|
| Relaxation oscillator based on phase-transitions | Electrical | up to 10 GHz | 10−12 | Electrical only |
| LC oscillator                    | Electrical | up to 100 GHz | 10−16 | Electrical only |
| Mechanical (NEMS) oscillator / RBO | Mechanical | up to 10 GHz | 10−17 | Electrical or mechatronic |
| Spin torque oscillator (STO)      | Magnetic     | up to 1 GHz   | 10−15 | Electrical or magnetic, or spin wave |
| Magneto-electromechanical          | Magnetic     | up to 20 GHz | 10−17 | Electrical |
| Spin-exchange oscillator          | Magnetic     | up to 20 GHz | 10−15 | Electrical or mechatronic |

3.2. Power considerations

A good baseline for comparing the power dissipation of various oscillators is to calculate the power figures for ultra-low power ring oscillators, which are used in, for example RFID transponders [59] [55]. The voltage-controlled oscillator described in [55] consumes 24 nW at 5.24 MHz, that is: $E_{\text{diss}} = 4.7 \times 10^{-15}$ J per oscillation cycle. With vanadium oxide relaxation oscillator one can go to an order of magnitude better: [60] projects 0.5 μW at 1.6 GHz, giving $E_{\text{diss}} \approx 10^{-16}$ J per cycle.

Spin torque oscillators are current-driven devices and typically run at sub-millamperes current and at GHz frequency - their voltage (and the dissipated power) depends on how low the resistance of the stack can be made. Assuming $V_{\text{STO}} = 0.1$ V, $i_{\text{STO}} = 0.1$ mA and $f = 10$ GHz, the energy consumed per cycle is $E_{\text{diss}} = 4.7 \times 10^{-15}$ J per cycle, not very far from the ring oscillator figure - and in case if STOs one has to deal with the losses to / from conversion to electrical signals. Other magnetic resonators (spin-hall effect based, voltage-controlled) may fare better.

Superconducting devices stand out with a $E_{\text{diss}} \approx 10^{-17}$ J energy per spike.

The energy of thermal fluctuations at room temperature is $kT = 26$ meV = $4.143 \times 10^{-21}$ J. The energy involved in each oscillation cycle should be at least a few times this value to avoid the oscillator signal getting completely lost in noise. The above presented oscillators are still several orders of magnitude away from this value - so there is certainly room for more efficient hardware.

3.3. The role of oscillator noise

It is in general true that low-energy, nanoscale oscillators have noisy waveforms, with relatively poor frequency and phase stability [61] [62] [63]. In non energy-recycling oscillators one has to keep the energy of the signal as low as possible, in order to minimize dissipation. For example, the energy contained in a small-size STO is in the order of hundreds of room temperature $kT$s - significantly higher than the characteristic energy of thermal fluctuations, but not as high as to completely avoid thermal noise to influence oscillator waveforms. In relaxation oscillators, the magnetic energy stored in the device may get fairly close to $kT$ - unfortunately, this does not translate to low power dissipation, due to the poor efficiency of magneto-electric interconversion.
tors or ring oscillators, low values of the charge-storing $C$ capacitor will lead to such regime.

In general, the phase of nanoscale oscillators will juggle, while the frequency remains more stable. Computing schemes relying on the average frequency or average phase of oscillators in a network are more experimentally viable than schemes that require long-term phase coherence \[64\]. Noise itself may be useful for the computation - it may strengthen the interaction of weakly coupled oscillators \[65\] and may eliminate metastable, local energy minima.

An extreme case of noisy oscillators are stochastic oscillators \[66\]. In case of magnetic oscillators stochastic behavior is achieved by reducing the volume of the magnetic free layer and as a result reducing the energy of the oscillator to the $\Delta E \approx 10 \, kT$ range. In this case, the oscillator spontaneously jumps between states, requiring no energy for the switching (albeit energy still will be required to interconnect or read out the oscillators). Some computational schemes may exploit such fairly random oscillations.

### 3.4. Physical coupling of oscillators

In order to turn them into computing networks, oscillators must be interconnected, which can be done electrically or using an emerging state variable, such as the electrical or magnetic degree of freedom. In a high-interconnection architecture (such as a most neuromorphic schemes) the number of interconnections greatly outweighs the number of oscillators. So one may argue that the figure of merits for oscillators as given in Table 3 are not at all that relevant, and 'good' oscillators are the ones that can be coupled by compact, low-power, high-fanout interconnections. Even in standard digital circuits, interconnections often account for most of the circuit complexity and moving data between far-lying points of the circuit accounts for most of the power consumption. It is not hard to see, that in highly interconnected analog (oscillatory) circuitry interconnections will be the realization bottleneck.

Electrical interconnection is a straightforward choice for coupling electrical oscillators \[62\] \[63\]. The strength of electrical interconnections may be tunable or hard wired and often one needs both positive of negative coupling coefficients (i.e. ones that push or pull the phases against / toward each other). Figure 3(a) \- b) gives two examples for electrical oscillator interconnection, for VO$_2$-based relaxation oscillators.

Electrical connections are also the usually the easiest, most flexible option for oscillators operating on different state variables. One example of a relatively simple coupling scheme $y$ is shown in Fig. 3(c)-d). In this case, high-frequency spin-torque oscillators (STOs) are coupled to a field line - which is simple a wire, providing magnetic field for the STOs. The field line current may be controlled by either an external current source (providing a control signal to the wire) or a signal that is provided by the STOs itself and brings them into mutual interaction.

The structure of Fig. 3(b), in fact already illustrates many of the challenges associated with electrical interconnections of emerging oscillators. STO signals are relatively weak, so one must pick up and amplify their signal by a relatively heavy electrical circuitry (this is not shown in Fig. 3). The big advantage of STOs is that they operate at several GHz frequencies, creates many challenges in routing the signals, one has to design appropriate microwave circuitry for routing the signals to the STOs and often account for most of the circuit complexity and moving data between far-lying points of the circuit accounts for most of the power consumption. It is not hard to see, that in highly interconnected analog (oscillatory) circuitry interconnections will be the realization bottleneck.

Stronger electrical signals between the STOs make the feedback amplifiers redundant and allow passive interconnection schemes for mutual coupling, this was was demonstrated in \[68\].

Since the interconversion between non-electric and electric degrees of freedom will inevitable result in some overhead, it seems highly desirable to use emerging state variables for interconnections as well. In the case of NEMS oscillators, this may be done by mechanical (acoustic) coupling \[69\]. In case of spin oscillators, dipole (magnetic field) or spin-wave coupling may work \[72\] (see Fig. 3) - so far, up to nine oscillators were brought to interaction this way \[75\]. It is also possible to sync spin oscillators via the current distributions they create, without the need for additional circuitry. Physical coupling of emerging oscillators is probably the one that enables to fully utilize the potential of emerging oscillators, but the geometry and the damping of acoustic / spin waves strongly limits the...
realizable coupling schemes.

4. Computing by oscillator dynamics, interconnection schemes

Oscillatory computing schemes are realized by interconnecting a number of oscillators and exploit their collective dynamic behavior for doing computation. The input of the network could be the initial phase of the oscillators or it could be encoded in the frequency of (variable frequency) oscillators, or the inputs can be external modulating signals. In most cases the output is a stationary phase, frequency or amplitude pattern. This pattern may be the result of a Boolean computation or the solution of an image processing, optimization, pattern matching problem.

The functionality of the network is defined by the oscillator interconnections and / or the parameters of the oscillators or by external stimuli. Mathematically, oscillator interaction are interpreted in terms of oscillator synchronization, i.e. the emergence of phase / frequency patterns in the oscillator cluster [70] [71] [72]. Theoretically, the simplest model to describe the formation of these states is the celebrated Kuramoto model [73] [74]. A comprehensive overview of different neuron and coupling models is given in [75]. In the engineering / physics community 'oscillatory ground state' is often used to describe the convergence of the dynamics system toward a useful, stationary pattern, and this pattern most often minimizes an energy function.

In practice, one often needs time-demanding numerical simulations or numerical approximations to determine phase dynamics [76] - analytical models are not always useful to describe irregularly connected networks or oscillators that describe highly nonlinear physical systems.

In many cases, one looks at the emergence of a phase pattern, i.e. clusters of phase-correlated oscillators. An important point, which is often lost in the mathematical definitions of synchronization that it does not necessarily mean perfectly in-phase or anti-phase running oscillators - any (phase) correlation between two oscillators could have computational value and such phase correlation may be possible to access via an output circuitry. An illustration for quasi synchronization (correlated phase states) is given in Fig. 4 [77] [78]. This pattern was generated by a 3-oscillator all-to-all coupled oscillator array, which is controlled by externally applied sweeping oscillatory signals. In the time domain, the behavior of such system seems almost like noise - extracting the pairwise phase correlations and after a threshold criteria, one gets the pattern of Fig. 4.

On the implementation level one may follow two distinct routes to define the functionality of the network. It is possible to rely on physically defined, often random couplings oscillators - in these case the oscillator parameters (frequency) or externally injected signals may be used to engineer the function of the network and / or a fully programmable outer layer can define the network function. This latter route is followed, for example in reservoir computing [80] [9].

Alternatively, one may engineer the interconnections (the coupling scheme) of the network for a certain functionality. Most oscillator coupling schemes are derived from a particular artificial neural network (ANN). For example, the well-established Hopfield network [81] [82] is the starting point of oscillatory associative memory concepts, such as [79] and [32]. Couplings in such networks can be computed from a Hebbian rule, in the simplest case. The energy function of a well-known, level-based Hopfield network can be rewritten for phase signals and one can construct the oscillatory version of the Hopfield net (see [79]). This is perhaps the most established route [83]. No matter which route is followed, one may use learning schemes developed for neural networks to determine the parameters of the oscillatory neural network [77] [78].

Most neurally inspired networks are highly intercon-
nected or all-to-all connected, which always raises realizability issues and which was already a major bottleneck in traditional neural network architectures. Oscillatory signals will yield to many parasitic couplings so one may speculate that they are well-suited to highly interconnected systems, if these parasitic couplings can be exploited for useful purposes.

5. Biologically inspired network models and deep learning nets by oscillators

The central nervous system is believed to use time-dependent spiking signals to communicate and process information - the dynamic nature of information processing in the brain is what probably distinguishes it most from today's digital computers. There is a diffuse boundary between spiking circuits and oscillators, consequently, many oscillatory networks are inspired by neuromorphic analogies or brain models. Excellent examples of using biological models to develop computational architectures are given by [84] and [85]. A comprehensive overview of circuits dedicated for neural computation is given in [84]. One may look at the problem the other way: it is a intriguing idea to use special-purpose hardware to simulate complex neural processes and perform for example large-scale brain simulation [87], albeit we are not aware of using oscillatory computers in this way.

Any man-made hardware that attempts to imitate neural systems faces an interconnection bottleneck. Neurons have a fan-out on the order of 10^5, i.e. this is the number of direct point-to-point interconnections emanating from a single neuron in the brain. Microelectronic technologies cannot even come close to this number. One way to create a highly interconnected network is to use frequency-division multiplexing (FDM) in artificial neural networks and use a single, high-bandwidth physical link to create a large number of channels between processing units (neurons) [88]. [89] [90] [91] [92] [93]. The possibility of using FDM provides another hand-waving argument in favor of oscillatory signal representation. Of course, there are other ideas to create highly interconnected network, such as to mimic the internet infrastructure on chip [94] - but since this a high-level solution, it is likely to be suboptimal to circuit or device-layer solutions.

The field of neural networks were viewed by many as an academic backwater until the past few years when deep learning networks started to revolutionize machine intelligence [95] [96] [97] [98] [99] [100]. Deep learning nets are now viewed by many as a panacea and a one fits all type of solution, where only the performance of available hardware limits what can be done. True or not, deep learning nets require immense computer power especially in the training phase. Deep learning was a killer application that made many-core graphics-core (GPU) based processing mainstream, ending the 50-year dominance of single (or few) core CPU systems. Clearly, deep-learning would be a strong motivation for special purpose hardware and [7] and oscillator based solution would be welcome - we are not aware of efforts to this end.

6. Special-purpose analog computing with oscillators

In many computing tasks, vast majority of resources (energy, time, hardware) is spent on relative simple, repetitive jobs. This is especially true in areas such as image processing, where large number of convolutions, filtering, image processing steps have to be done on massive amount of input data (i.e. video streams). Co processors for such tasks could be promising testbeds for oscillator-based computers since they target a well-defined task, which could easily be compared with existing, number-crunching solutions to the same problem. Since the OCA is acting as a hardware accelerator or co-processor in these tasks (not as stand-alone computing unit), it is especially important to estimate a net system-level improvements that may come from the OCA.

One of the outstanding problems is efficient preprocessing of video streams, i.e. filtering / convolution operations at the input side of an image processing pipeline (IPP). The focus of a recent DARPA project [99] targeted exactly such task [44] was the demonstration of complete IPP, with analog oscillatory computinf devices at its heart. The underlying idea is that efficient euclidean distance calculation on analog data can be done by exploiting oscillator interaction [100]. The effort included circuit design and algorithm design components and also a nanodevice work package, where nanoscale mechanical and magnetic oscillators were developed as hardware components for the image processing pipeline (IPP).

The mechanism of oscillator interaction and how this is used for analog computing is described in [64] in detail. A much simplified sketch of the architecture was sketched in Fig. 4b. The oscillator frequencies should be controllable via their driving current or voltage (VCOs) and their interactions should be sufficiently strong to pull the entire oscillator array in a synchronized state if their natural frequencies are sufficiently close. In fact, the presence or absence of the synchronized state indicates how close the oscillator frequencies are to each other. If the oscillator signals are superposed, rectified and integrated then the value of this output will be higher for synchronized states (where oscillator signals add up coherently) than for unsynchronized states. The network gives a measure of how the oscillator frequencies (i.e. the oscillator driving current or voltages) are clustered. This can directly be used for analog Euclidean distance calculation [8] [17] [101]. The circuit itself requires all oscillators to be connected via a single control node. Distance calculations on n-element long vector require n coupled oscillators.

One may wonder how this all comes together for a useful computing task - Fig. 5 illustrates the ingredients for an IPP. The level of oscillator synchronization is measured is simply measured via a power detector (integrator) Fig.
The output of the integrator is to a good approximation linear with the Euclidean distance of the oscillator frequencies (Fig. 5) - the input is measured in currents, since the used STOs are current-controlled oscillators). Using a cluster of 25 coupled STOs one can scan through an image in $5 \times 5$ patches - Fig. 5) demonstrates this scan for detecting 45-degree lines in the Lena image.

Figure 5: An image-processing pipeline based on oscillators. Euclidean distance calculation calculates dot products, which, in turn, can be used for Gabor filtering. a) integrator circuit b) oscillator output vs. Euclidean distance c) scanning through an image in patches.

The oscillator unit can perform the analog distance calculation with potentially very high energy efficiency and due to the relative simplicity of the coupling scheme, this circuit has recently become possible with nanoscale oscillators, such as electrically coupled STOs [102][103]. The relative simplicity of the computational task, however, turns out to be a disadvantage - as all the other image processing steps are done in the digital domain. The simple analog oscillator network requires some sort of interface circuitry between the surrounding digital circuitry and the analog oscillator components. The high cost of A/D and D/A converters, current sources for driving the oscillators etc. and other input / output components cannot be amortized. If the OCA targets a 'too simple' problem, then the benefits on the system level could be dwarfed by the high cost of interfaces to the conventional digital components. For emerging oscillators this problem is usually more serious, as they often require more non-trivial interfaces to the digital world.

There are a number of other special-purpose computing tasks that had been demonstrated with oscillators, such as associative memory and pattern recognition tasks. Unlike the above described euclidean distance calculation, these problems can be scaled to operate on large, complex inputs [34][104] and therefore the cost of input / output interfaces can be amortized. Nanoscale oscillators (STOs) were also demonstrated as building blocks [9] albeit it seems that the external circuitry and not the nanoscale oscillators do the heavy lifting in the computing process and potential system-level benefits are hard to estimate.

In standard neural implementations of the associative memory / pattern recognition networks the computational task is encoded in the interconnection weights of the neurons and all-to-all interconnections are required. Such complex network of finely tuned connections does not lend itself to practical implementations. There are a few recent ideas for oscillatory neurons that are a lot more (nano)technology friendly. The work of [92] uses the a frequency-domain multiplexing scheme to drastically reduce the number of interconnections. Along a similar line of thought, the work of [105] uses external oscillatory signals to control partial synchronization of oscillators and effectively control their interconnection strengths this way.

### 7. Neural networks for hard problems

As we argued above, a practically useful OCA should target a sufficiently hard problem in order to mitigate the cost of I/O circuitry - ideally, a hard problem meaning that it is entirely intractable using digital circuitry or non-oscillatory networks.

Computationally hard problems (such as NP-hard problems) [106][107] are usually discussed in the context of quantum information theory (quantum computation and quantum simulators). A quantum system in a fully entangled state can be described by exponentially growing number of superposition coefficients - i.e. the time evolution of $N$ coupled 2-state systems generally requires $2^N$ number of internal variables. A quantum computer or quantum simulator could store and operate simultaneously on exponentially large ($2^{30}$ sized) data - so a relatively small-sized hardware could, in principle, process vast-sized problems. Currently, large-scale industrial and academic efforts are ongoing to experimentally verify this claim [108].

It is widely believed that no classical system, only quantum processors could do the feat of storing / processing information that is exponentially growing with the size of the system. Contrary to this common belief, it is quite possible that there is no fundamental difference between quantum and classical systems in this respect. This can be argued for on three grounds: (1) mathematically the relations between complexity classes (P, NP) are unknown (2) the information content in collective states (excitations) of a classical system may grow exponentially with system size (3) eventually, classical and quantum systems will be subjected to the same type of noisy environments and may be limited by the same type of physical constraints. One should appreciate the significance and difficulty level of NP-hard problem (for an excellent insight, see [109]) and it is quite possible that NP hard problems, in their general form, are beyond reach for both classical and quantum computers.

In the light of the above, however, it makes sense to think about oscillator-based accelerators for NP-hard problems, i.e. to design OCAs that compete with realizable quantum computers. The first attempt we are aware of addresses the graph coloring problem [110][111]. It is not hard to see that the dynamics of in-phase and out-of-phase
synchronizing oscillators map to a solution of a 2-color graph coloring problem, and even the oscillator interconnections directly correspond to the graph edges. A more general approach is shown in \cite{Parihar2015}, together with an implementation using vanadium-oxide-based relaxation oscillators. In the approach of \cite{Parihar2015} the problem is first reformulated to finding a circular ordering of the nodes such that the same colored nodes appear together in the ordering - this preserves the hardness of the problem but turns it into an energy-minimization (optimization) task that oscillators can handle.

Another approach for using the collective states for exponentially hard problems is memcomputing \cite{Maass2014} \cite{Paugam-Moisy2006}. This can be implemented in various physical systems, among them oscillators \cite{Parihar2015}, the number of collective excitations in a physical system grows quickly, enabling the solution of hard problems. The arguments of \cite{Parihar2015} are especially important, as they study the feasibility of the exponentially growing state space in the presence of noise.

It must be noted that it is not exclusively OCAs that have been proposed to handle NP-hard problems, but other complex, non-oscillatory analog systems \cite{Deng2015} \cite{Wigington1959} \cite{LeCun2015} \cite{Kuzmina1997} or memristors \cite{Memristors2014}. The prospect that complex analog dynamics systems may attack NP-hard problems could become the most important argument for their research and if this is proven true, all issues with analog / digital interfaces would become non-issue.

8. Conclusions and Outlook

In this paper we gave a hardware-oriented review of the flourishing research field of oscillatory computing architectures (OCAs). Most works on the field are motivated by biological analogies (i.e. neuromorphic computing), or finding new applications for emerging hardware (such as a spin-oscillator-based computing systems). It would be highly useful and stimulating to substantiate fundamental rationale for the superiority of OCAs for certain problems types.

Finding a convincing and somewhat general argument for oscillatory computing system is likely not an easy task. A much older and still unsettled question is whether in general digital or analog solutions are superior for neuromorphic (bio-inspired) computational tasks \cite{Krukowski2013} - but there are many benchmarks and case studies out there. On the other hand, oscillator-based analog vs. level-based analog benchmarks are almost nonexistent.

We conclude with a few, somewhat hand waving arguments to corroborate the usefulness of OCAs. One of their benefits could be that they use of narrow-bandwidth device to device communication channels (i.e. oscillators running at a given frequency) allows efficient intra-circuit communication in the presence of noise \cite{Hull2015} \cite{Jacob2017} and consequently, ultimately low-power operation could be accomplished \cite{Faugham2006}. A second possible argument is that coupled oscillators are ubiquitous in the physical world, so one may find a 'perfect' device for a given computational task. A third, very encouraging fact is that interacting oscillators now appear in circuits proposed for the solution of NP-hard problems. It is very much possible that they will steal the show from quantum computing and yield to hardware that could handle problems that seem intractable with today's resources.

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