Soft gravitational effects in Kadanoff-Baym approach

Hiroyuki Kitamoto\textsuperscript{1)}* and Yoshihisa Kitazawa\textsuperscript{2),3)}†

\textsuperscript{1)}Department of Physics and Astronomy
Seoul National University, Seoul 151-747, Korea

\textsuperscript{2)}KEK Theory Center, Tsukuba, Ibaraki 305-0801, Japan

\textsuperscript{3)}Department of Particle and Nuclear Physics
The Graduate University for Advanced Studies (Sokendai)
Tsukuba, Ibaraki 305-0801, Japan

Abstract

In de Sitter space, the gravitational fluctuation at the super-horizon scale may make physical quantities time dependent by breaking the de Sitter symmetry. We adopt the Kadanoff-Baym approach to evaluate soft gravitational effects in a matter system at the sub-horizon scale. This investigation proves that only the local terms contribute to the de Sitter symmetry breaking at the one-loop level. The IR singularities in the non-local terms cancel after summing over degenerate states between real and virtual processes. The corresponding IR cut-off is given by the energy resolution like QED. Since the energy resolution is physical and independent of cosmic evolution, the non-local contributions do not induce the de Sitter symmetry breaking. We can confirm that soft gravitational effects preserve the effective Lorentz invariance.

May 2013

*E-mail address: kitamoto@post.kek.jp
†E-mail address: kitazawa@post.kek.jp
1 Introduction

In quantum field theories in de Sitter (dS) space, it is well known that the propagator for a massless and minimally coupled scalar field does not respect the full dS symmetry \[1,2,3\]. Such a symmetry breaking is caused by the fact that the propagator is sensitive to the increasing degrees of freedom at the super-horizon scale and so we need to introduce an infra-red (IR) cut-off. The IR cut-off is an initial size of universe when the exponential expansion starts and gives the propagator a logarithmic dependence of the scale factor of universe: \(\log a(\tau)\). In some field theoretic models in dS space, physical quantities may acquire growing time dependences through the propagator.

On the dS background, some modes of gravity behave just like massless and minimally coupled scalar fields. Unlike a scalar field, the gravitational field is massless without fine-tuning the action. Thus it is an attractive candidate to make physical quantities time dependent \[4\].

Since we need to introduce all possible counter terms to renormalize ultra-violet (UV) divergences in quantum gravity, there are infinite choices of finite UV contributions. However there is no ambiguity in investigating time dependences of physical quantities. Concerning the internal loop contributions, we can separate IR contributions and UV contributions in comparison to the Hubble scale. The degrees of freedom at the sub-horizon scale are constant with cosmic evolution and so the UV contributions respect the dS symmetry.

In the previous studies \[5\], we investigated soft gravitational effects on matter systems at the sub-horizon scale which are directly observable. As specific examples, we adopted massless and conformally coupled scalar and massless Dirac fields. Since dS space is conformally flat, the matter actions possess the Lorentz invariance at the classical level after the conformal transformation. In a generic non-conformally coupled case, the conformal invariance holds at the sub-horizon scale and the Lorentz invariance appears as an effective symmetry. We found that the effective Lorentz invariance is preserved even if soft gravitational effects are considered.

Here we recall that in massless field theories, IR singularities occur even in flat space when virtual particles approach on-shell. Such singularities originate in the fact that the integration over the infinite past is divergent when the frequency of the integrand is zero. In QED, these IR singularities are known to cancel after summing over degenerate states between real and virtual processes. The corresponding IR cut-off is given by the energy resolution \[6,7\]. We found that the cancellation takes place in \(\varphi^3, \varphi^4\) theories in dS space \[8\]. The energy resolution is physical and independent of cosmic evolution. Therefore these non-local contributions do not induce the dS symmetry breaking. It would be appropriate to postulate that the cancellation takes place in any unitary model as the total spectral weight is preserved.

From the above discussion, we need to distinguish the dS symmetry breaking which is local from the non-local contribution. In the previous studies, we adopted the effective equation of motion \[9\] as a tool to evaluate time dependent quantum corrections. There we have shown that the non-local contribution does not lead to the dS symmetry breaking in the off-shell case.
In order to investigate the on-shell limit, we adopt the Kadanoff-Baym method \[10\] in this paper. The method is valid when the external momentum is at the sub-horizon scale as a particle description is valid. By using it, we can systematically obtain the on-shell term and the off-shell term. So it is clearly visible when they are degenerated. We show that the IR singularities appearing at the on-shell limit cancel out between real and virtual processes also in a matter system with gravity.

The organization of this paper is as follows. In Section 2, we quantize the gravitational field on the dS background. We make a brief review of the Kadanoff-Baym approach in Section 3. The subsequent two sections are the main parts of this paper where we investigate interaction effects through the collision term. In our investigation, we focus on the local contribution in Section 4 and the non-local contribution in Section 5. We find that the local term contributes to the dS symmetry breaking while the non-local term does not. Furthermore we confirm that soft gravitational effects induced by the local terms preserve the effective Lorentz invariance. In Section 6, we show that the results obtained in the previous sections do not depend on the parametrization of the metric. We conclude with discussions in Section 7.

2 Gravitational propagators in dS space

In this section, we review gravitational propagators in dS space. In the Poincaré coordinate, the metric of dS space is written as

\[
ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2),
\]

\[
a = e^{Ht} = -\frac{1}{H\tau}, \quad (2.2)
\]

where the Hubble parameter $H$ is constant and the dimension of dS space is taken as $D = 4$. The conformal time $\tau$ runs in the range: $-\infty < \tau < 0$. After a sufficient exponential expansion, the dS space is well described locally by the above metric irrespective of the spatial topology.

In investigating gravitational fluctuations, we primarily use the following parametrization of the metric:

\[
g_{\mu\nu} = \Omega^2(x)\tilde{g}_{\mu\nu}, \quad \Omega(x) = a(\tau)e^{\kappa x}, \quad (2.3)
\]

\[
det \tilde{g}_{\mu\nu} = -1, \quad \tilde{g}_{\mu\nu} = \eta_{\mu\rho}(e^{\kappa x})^\rho_\nu, \quad (2.4)
\]

where $\kappa$ is defined by the Newton’s constant $G$ as $\kappa^2 = 16\pi G$. To satisfy (2.4), $h_{\mu\nu}$ is taken to be traceless

\[
h_{\mu}^\mu = 0. \quad (2.5)
\]
In this parametrization, the scalar density and the Ricci scalar are written as
\[ \sqrt{-g} = \Omega^4, \quad R = \Omega^{-2} \tilde{R} - 6\Omega^{-3} \tilde{g}^{\mu\nu} \nabla_\mu \partial_\nu \Omega, \] (2.6)
where \( \tilde{R} \) is the Ricci scalar constructed from \( \tilde{g}_{\mu\nu} \)
\[ \tilde{R} = -\partial_\mu \partial_\nu \tilde{g}^{\mu\nu} - \frac{1}{4} \tilde{g}^{\mu\nu} \tilde{g}^{\rho\sigma} \partial_\mu \tilde{g}_{\rho\sigma} \partial_\nu \tilde{g}_{\sigma\beta} + \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{g}^{\rho\sigma} \partial_\mu \tilde{g}_{\sigma\alpha} \partial_\nu \tilde{g}_{\alpha\beta}. \] (2.7)
By using the partial integration, the Lagrangian density for the Einstein gravity is written as
\[ \mathcal{L}_{\text{Gravity}} = \frac{1}{\kappa^2} \sqrt{-g} \left[ R - 2\Lambda \right] = \frac{1}{\kappa^2} \left[ \Omega^2 \tilde{R} + 6\tilde{g}^{\mu\nu} \partial_\mu \Omega \partial_\nu \Omega - 6H^2 \Omega^4 \right], \] (2.8)
where \( \Lambda = 3H^2 \).
In order to fix the gauge with respect to general coordinate invariance, we adopt the following gauge fixing term [4]:
\[ \mathcal{L}_{\text{GF}} = -\frac{1}{2} a^2 F_\mu F^\mu, \] (2.9)
\[ F_\mu = \partial_\mu h^\rho - 2\partial_\mu w + 2h_\rho^\mu \partial_\rho \log a + 4w \partial_\rho \log a. \]
Note that in this paper, the Lagrangian density is defined including \( \sqrt{-g} \) and the Lorentz indexes are raised and lowered by \( \eta_{\mu\nu} \) respectively. The corresponding ghost term at the quadratic level is given by
\[ \mathcal{L}_{\text{ghost}} = -a^2 \partial^\mu \bar{b}^\mu \{ \eta_{\mu\rho} \partial_\rho + \eta_{\nu\rho} \partial_\mu + 2\eta_{\mu\nu} \partial_\rho (\log a) \} b^\rho \] (2.10)
\[ + \partial_\rho (a^2 \bar{b}^\mu) \{ \partial_\mu + 4\partial_\rho (\log a) \} b^\mu, \]
where \( b^\mu \) is the ghost field and \( \bar{b}^\mu \) is the anti-ghost field. From (2.8)-(2.10), the quadratic part of the total gravitational Lagrangian density is
\[ \mathcal{L}_{\text{quadratic}} = a^4 \left[ \frac{1}{2} a^{-2} \partial_\mu X \partial_\mu X - \frac{1}{4} a^{-2} \partial_\mu \tilde{h}^{ij} \partial^\mu \tilde{h}^{ij} - a^{-2} \partial_\mu \tilde{b}^i \partial^\mu \tilde{b}^i \right. \] (2.11)
\[ + \frac{1}{2} a^{-2} \partial_\mu h^{0i} \partial^\mu h^{0i} + H^2 h^{0i} h^{0i} - \frac{1}{2} a^{-2} \partial_\mu Y \partial^\mu Y - H^2 Y^2 \]
\[ + \left. a^{-2} \partial_\mu \bar{b}^0 \partial^\mu \bar{b}^0 + 2H^2 \bar{b}^0 \bar{b}^0 \right]. \]
Here we have decomposed \( h^{ij}, \ i, j = 1, \cdots, 3 \) into the trace and traceless part
\[ h^{ij} = \tilde{h}^{ij} + \frac{1}{3} h^{kk} \delta^{ij} = \bar{h}^{ij} + \frac{1}{3} h^{00} \delta^{ij}. \] (2.12)
The action has been diagonalized by the following linear combination
\[ X = 2\sqrt{3} w - \frac{1}{\sqrt{3}} h^{00}, \quad Y = h^{00} - 2w. \] (2.13)
The quadratic action (2.11) contains two types of fields, massless and minimally coupled fields: \( X, h^{ij}, b^i, \bar{b}^i \) and massless conformally coupled fields: \( h^{0i}, b^0, \bar{b}^0, Y \). We list the corresponding propagators as follows

\[
\langle X(x)X(x') \rangle = -\langle \varphi(x)\varphi(x') \rangle, \tag{2.14}
\]

\[
\langle \tilde{h}^{ij}(x)\tilde{h}^{kl}(x') \rangle = (\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{3}\delta^{ij}\delta^{kl})\langle \varphi(x)\varphi(x') \rangle, \tag{2.15}
\]

\[
\langle b^i(x)\bar{b}^j(x') \rangle = \delta^{ij}\langle \varphi(x)\varphi(x') \rangle, \langle h^{0i}(x)h^{0j}(x') \rangle = -\delta^{ij}\langle \phi(x)\phi(x') \rangle, \langle Y(x)Y(x') \rangle = \langle \phi(x)\phi(x') \rangle, \langle b^0(x)\bar{b}^0(x') \rangle = -\langle \phi(x)\phi(x') \rangle.
\]

Here \( \varphi \) denotes a massless and minimally coupled scalar field and \( \phi \) denotes a massless conformally coupled scalar field. The corresponding wave functions \( \varphi_p(x), \phi_p(x) \) are given by

\[
\varphi_p(x) = \frac{H\tau}{\sqrt{2}p}(1 - i\frac{1}{p\tau})e^{-ip\tau + ip\cdot x}, \tag{2.16}
\]

\[
\phi_p(x) = \frac{H\tau}{\sqrt{2}p}e^{-ip\tau + ip\cdot x}. \tag{2.17}
\]

The massless and conformally coupled field is locally equal to that in Minkowski space up to the scale factor

\[
\langle \phi(x)\phi(x') \rangle = \frac{H^2}{4\pi^2} \frac{1}{y}, \tag{2.18}
\]

where \( y \) is the square of distance which preserves the dS symmetry

\[
y = \Delta x^2/\tau \tau', \quad \Delta x^2 = -(\tau - \tau')^2 + (x - x')^2. \tag{2.19}
\]

On the other hand, the massless and minimally coupled field has a specific property to dS space. At the super-horizon scale as physical momentum: \( P \equiv p/a(\tau) \ll H \), the wave function (2.16) behaves as

\[
\varphi_p(x) \sim \frac{H}{\sqrt{2p^3}}e^{ip\cdot x}. \tag{2.20}
\]

The IR behavior indicates that the corresponding propagator has a logarithmic divergence from the IR contributions in the infinite volume limit. To regularize the IR divergence, we introduce an IR cut-off \( 1/L_i \) which fixes the minimum value of the comoving momentum. Physically speaking, \( L_i \) is recognized as an initial size of universe when the exponential expansion starts. Due to the commutation relation, it is equivalent to set the initial time as

\[
a_i = -1/H\tau_i = 1/HL_i. \tag{2.21}
\]
With this prescription, the propagator for a massless and minimally coupled field is given by

$$
\langle \phi(x)\phi(x') \rangle = \frac{H^2}{4\pi^2} \left\{ \frac{1}{y} - \frac{1}{2}\log y + \frac{1}{2}\log \left( \frac{a(\tau)a(\tau')}{a_t^2} \right) + 1 - \gamma \right\},
$$

where $\gamma$ is Euler’s constant. The existence of the logarithmic term: $\log \left( \frac{a(\tau)a(\tau')}{a_t^2} \right)$ implies the breakdown of the dS symmetry. In particular, it breaks the scale invariance

$$
\tau \rightarrow C\tau, \quad x^i \rightarrow Cx^i.
$$

To explain what causes the dS symmetry breaking, we recall that the minimum value of the physical momentum is $1/a(\tau)L_i$ as the wavelength is stretched by cosmic expansion. That is, more degrees of freedom accumulate at the super-horizon scale with cosmic evolution. Due to this increase, the propagator acquires the growing time dependence which spoils the dS symmetry.

As there is explicit time dependence in the propagator, physical quantities can acquire time dependence through the quantum loop corrections. We call them the quantum IR effects in dS space. In order to clearly separate the minimally coupled modes and the conformally coupled modes, the gravitational propagator is written as

$$
\langle h^{\mu\nu}(x)h^{\rho\sigma}(x') \rangle = P^{\mu\nu\rho\sigma} \langle \phi(x)\phi(x') \rangle + Q^{\mu\nu\rho\sigma} \langle \phi(x)\phi(x') \rangle,
$$

where

$$
P^{\mu\nu\rho\sigma} = -\frac{3}{4}\delta^\mu_0\delta^\nu_0\delta^\rho_0\delta^\sigma_0 - \frac{1}{4}(\delta^\mu_0\delta^\nu_0\tilde{\eta}^{\rho\sigma} + \delta^\rho_0\delta^\sigma_0\tilde{\eta}^{\mu\nu})
\begin{align*}
+ (\tilde{\eta}^{\mu\rho}\tilde{\eta}^{\nu\sigma} + \tilde{\eta}^{\mu\sigma}\tilde{\eta}^{\nu\rho} - \frac{3}{4}\tilde{\eta}^{\mu\nu}\tilde{\eta}^{\rho\sigma}),
\end{align*}
$$

$$
Q^{\mu\nu\rho\sigma} = +\frac{9}{4}\delta^\mu_0\delta^\nu_0\delta^\rho_0\delta^\sigma_0 + \frac{3}{4}(\delta^\mu_0\delta^\nu_0\tilde{\eta}^{\rho\sigma} + \delta^\rho_0\delta^\sigma_0\tilde{\eta}^{\mu\nu})
\begin{align*}
- (\delta^\mu_0\delta^\nu_0\tilde{\eta}^{\rho\sigma} + \delta^\rho_0\delta^\sigma_0\tilde{\eta}^{\mu\nu} + \delta^\nu_0\delta^\mu_0\tilde{\eta}^{\rho\sigma} + \delta^\mu_0\delta^\sigma_0\tilde{\eta}^{\rho\nu})
\end{align*}
\begin{align*}
+ \frac{1}{4}\tilde{\eta}^{\mu\nu}\tilde{\eta}^{\rho\sigma}.
\end{align*}
$$

Here $\tilde{\eta}^{\mu\nu}$ denotes the projection to the spatial subspace: $\tilde{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^\mu_0\delta^\nu_0$.

Before starting the next section, we refer to the parametrization dependence of the metric. There are other choices than the parametrization (2.3)-(2.5). For example, the following parametrization is adopted in [4, 11, 12]:

$$
g_{\mu\nu} = a^2(\tau)(\eta_{\mu\nu} + 2\kappa\Phi(x)\eta_{\mu\nu} + \kappa\Psi_{\mu\nu}(x)), \quad \Psi^\mu_\mu = 0.
$$

Here we have divided the fluctuation into the trace and traceless part to facilitate the comparison with the parametrization (2.3)-(2.5). The relation of them is given by

$$
\kappa w = \kappa\Phi - \kappa^2\Phi^2 - \frac{1}{16}\kappa^2\Psi_{\rho\rho}\Psi^{\rho\rho} + \cdots,
$$

$$
\kappa h_{\mu\nu} = \kappa\Psi_{\mu\nu} - 2\kappa^2\Phi\Psi_{\mu\nu} - \frac{1}{2}\kappa^2\Psi_\mu^\rho\Psi_\rho^\nu + \frac{1}{8}\kappa^2\Psi_{\rho\sigma}\Psi^{\rho\sigma}\eta_{\mu\nu} + \cdots.
$$
We should note that $w, h_{\mu\nu}$ is equal to $\Phi, \Psi_{\mu\nu}$ up to the linear order. As far as we adopt the same gauge:

$$F_\mu = \partial_\rho \Psi_\mu^\rho - 2\partial_\mu \Phi + 2\Psi_\mu^\rho \partial_\rho \log a + 4\Phi \partial_\mu \log a,$$

we have only to identify the field components to obtain the propagator in the parametrization (2.27):

$$w \rightarrow \Phi, \ h_{\mu\nu} \rightarrow \Psi_{\mu\nu}. \quad (2.30)$$

The deference between these two parametrizations emerges in the non-linear order.

3 Kadanoff-Baym approach

As the main subject of this paper, we investigate soft gravitational effects on a matter system by introducing a Kadanoff-Baym method [10]. The investigation is up to the one-loop level. Compared to the previous studies with the effective equation of motion, we can systematically take into account the process with a soft or collinear particle in the approach. For simplicity, we adopt a massless and conformally coupled scalar field as a matter field

$$S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \tilde{R} \phi^2 \right]. \quad (3.1)$$

After the field redefinition,

$$\Omega \phi \rightarrow \phi, \quad (3.2)$$

the action is written as

$$S = \int d^4x \left[ -\frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \tilde{R} \phi^2 \right]. \quad (3.3)$$

At the tree level, the corresponding propagator is equal to that in Minkowski space

$$\langle \phi(x_1) \phi(x_2) \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p} e^{-ip(t_1 - t_2) + i\mathbf{p} \cdot (\mathbf{x}_1 - \mathbf{x}_2)}. \quad (3.4)$$

To investigate interaction effects in a time dependent background like dS space, we need to adopt the Schwinger-Keldysh formalism [13, 14]. We introduce the Schwinger-Keldysh indices as

$$G^{++}(x_1, x_2) \equiv \langle \phi(x_1) \phi(x_2) \rangle, \quad (3.5)$$

$$G^{+-}(x_1, x_2) \equiv \langle \phi(x_2) \phi(x_1) \rangle, \quad G^{++}(x_1, x_2) \equiv \theta(\tau_1 - \tau_2) \langle \phi(x_1) \phi(x_2) \rangle + \theta(\tau_2 - \tau_1) \langle \phi(x_2) \phi(x_1) \rangle, \quad$$

$$G^{--}(x_1, x_2) \equiv \theta(\tau_1 - \tau_2) \langle \phi(x_1) \phi(x_2) \rangle + \theta(\tau_1 - \tau_2) \langle \phi(x_1) \phi(x_2) \rangle.$$
Here the propagator $G$ includes interaction effects in general. When we specify the free propagator (3.4), it is denoted by $G_0$. We recall that as for the free propagator, the following identities hold

\[
G_0^{-1} = i(\partial_0^2 - \partial_i^2), \quad (3.6)
\]
\[
G_0^{-1}|_{x_1} G_0^{ab}(x_1, x_2) = c^{ab} \delta(4)(x_1 - x_2),
\]

where $c^{ab}$ is defined as

\[
c^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad a, b = +, -.
\] \hspace{1cm} (3.7)

Up to the one-loop level, the Schwinger-Dyson equation is given by

\[
G^+(x_1, x_2) = G_0^+(x_1, x_2) + \int d^4x'd^4x'' c_{ab} G_0^{-a}(x_1, x') \Sigma^{4\text{-pt}}(x', x'') G_0^{b+}(x'', x_2) + \int d^4x'd^4x'' c_{ab} c_{cd} G_0^{-a}(x_1, x') \Sigma^{3\text{-pt}}_{bc}(x', x'') G_0^{d+}(x'', x_2),
\] \hspace{1cm} (3.8)

where the self-energy $\Sigma$ due to the four-point vertices and the three-point vertices are given by

\[
\Sigma^{4\text{-pt}}(x, x') = i\delta(4)(x - x') \times \left[ \frac{1}{2} \kappa^2 \partial_{\mu} \left\{ \langle h^{\mu}(x) h^{\nu}(x') \rangle \partial_{\nu} \right\} + \frac{1}{12} \kappa^4 \partial_{\mu} \partial_{\nu} \left\{ \langle h^{\mu}(x) h^{\nu}(x') h^{\rho}(x) h^{\sigma}(x') \rangle \right\}
\]
\[
+ \frac{1}{24} \kappa^2 \partial_{\mu} \partial_{\nu} \left\{ \langle h^{\mu}(x) h^{\rho}(x') \partial_{\nu} h^{\nu}(x') \rangle - \frac{1}{12} \kappa^2 \partial_{\mu} h^{\rho}(x') \partial_{\nu} h^{\nu}(x') \right\} \right],
\] \hspace{1cm} (3.9)

\[
\Sigma^{3\text{-pt}}(x, x') = - \kappa^2 \partial_{\mu} \partial_{\nu} \left\{ \langle h^{\mu}(x) h^{\nu}(x') \rangle \partial_{\nu} \phi(x) \partial_{\nu} \phi(x') \right\}
\]
\[
- \frac{1}{6} \kappa^2 \partial_{\mu} \left\{ \langle h^{\mu}(x) \partial_{\nu} h^{\nu}(x') \rangle \partial_{\nu} \phi(x) \phi(x') \right\}
\]
\[
- \frac{1}{6} \kappa^2 \partial_{\nu} \left\{ \langle \partial_{\mu} h^{\mu}(x) h^{\nu}(x') \rangle \phi(x) \partial_{\nu} \phi(x') \right\}
\]
\[
- \frac{1}{36} \kappa^2 \partial_{\nu} \partial_{\nu} h^{\mu}(x) \partial_{\nu} \partial_{\nu} h^{\mu}(x') \phi(x) \phi(x').
\] \hspace{1cm} (3.10)

As for the other indices, $\Sigma^{3\text{-pt}}$ is defined in a similar way to the propagator (3.5).

By introducing the retarded and the advanced functions:

\[
F^R(x_1, x_2) \equiv \theta(\tau_1 - \tau_2) \left[ F^+(x_1, x_2) - F^-(x_1, x_2) \right], \quad F = G, \Sigma_{3\text{-pt}},
\]
\[
F^A(x_1, x_2) \equiv -\theta(\tau_2 - \tau_1) \left[ F^+(x_1, x_2) - F^-(x_1, x_2) \right],
\] \hspace{1cm} (3.11)
the Schwinger-Dyson equation is written as the following form
\[ \begin{align*}
G^{-+}(x_1, x_2) &= G_0^{-+}(x_1, x_2) \\
&+ \int d^4x_1 d^4x'' G_0^{R}(x_1, x') \Sigma_{4\text{-pt}}(x', x'') G_0^{-+}(x'', x_2) \\
&+ \int d^4x' d^4x'' G_0^{-+}(x_1, x') \Sigma_{4\text{-pt}}(x', x'') G_0^{A}(x'', x_2) \\
&+ \int d^4x' d^4x'' G_0^{R}(x_1, x') \Sigma_{3\text{-pt}}^{R}(x', x'') G_0^{-+}(x'', x_2) \\
&+ \int d^4x' d^4x'' G_0^{R}(x_1, x') \Sigma_{3\text{-pt}}^{-+}(x', x'') G_0^{A}(x'', x_2) \\
&+ \int d^4x' d^4x'' G_0^{-+}(x_1, x') \Sigma_{3\text{-pt}}^{3\text{-pt}}(x', x'') G_0^{A}(x'', x_2). 
\end{align*} \] (3.12)

As seen in (3.10), \( \Sigma_{3\text{-pt}} \) contains differential operators. They are applied after the step functions are assigned. The prescription corresponds with the \( T^* \) product.

We introduce the assumption that the full propagator in dS space is written as the following form:
\[ \begin{align*}
G^{-+}(x_1, x_2) &= \int \frac{d^3p}{(2\pi)^3} Z(p, \tau_c) \frac{1}{2p} e^{-ip(\tau_1-\tau_2)+ip(x_1-x_2)} \\
&+ \int_{\epsilon>p} \frac{d\epsilon d^3p}{(2\pi)^3} N(p, \epsilon, \tau_c) \frac{1}{2\epsilon} e^{-i\epsilon(\tau_1-\tau_2)+ip(x_1-x_2)}. 
\end{align*} \] (3.13)

The full propagator depends on the average and the relative time
\[ \tau_c \equiv \frac{\tau_1 + \tau_2}{2}, \quad \Delta \tau \equiv \tau_1 - \tau_2. \] (3.14)

Due to the spatial translational symmetry, we can expand it by spatial plane waves. The existence of interactions do not allow the full propagator to consist of on-shell term alone. Therefore we have introduced the on-shell part and the off-shell part of the spectral function: \( Z, N \). The on-shell part \( Z \) is frequently called the wave function renormalization factor. Focusing on the region \(|\tau_c| \gg |\Delta \tau|\), we assume that they evolve with the average time.

In (3.13), we set the initial state to be the vacuum state. We need to introduce a distribution function when we start with an excited state \([15, 16]\). We have found that in \( \varphi^3, \varphi^4 \) theories, the non-local IR effects contribute to the spectrum function prior to the distribution function. Furthermore, the non-local IR singularities are canceled between \( Z \) and \( N \) \([8]\). The main motivation of this paper is to confirm that such a cancellation takes place in the matter system with gravity.

Phenomenologically we observe physics at some fixed momentum scales. In the subsequent discussions, we suppress the following integration factor by performing the Fourier transformation with respect to the spatial coordinate \( \Delta \mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2 \)
\[ \int \frac{d^3p}{(2\pi)^3} e^{ip(\mathbf{x}_1-\mathbf{x}_2)}. \] (3.15)
In order to investigate physics which are directly observable, we set the external momentum to be of the sub-horizon scale:

\[ p|\tau_c| \gg 1 \iff P \equiv pH|\tau_c| \gg H. \] (3.16)

In order to derive the differential equation of \( Z, N \), we operate \( G_0^{-1} \) on the Schwinger-Dyson equation from the left and the right respectively. From (3.6) and (3.12), each identity is given by

\[
G_0^{-1}|_{x_1} G^+(x_1, x_2) = \int d^4x' \, \Sigma_{4-pt}(x_1, x')G_0^+(x', x_2) + \int d^4x' \, \Sigma_{3-pt}(x_1, x')G_0^A(x', x_2),
\] (3.17)

\[
G_0^{-1}|_{x_2} G^+(x_1, x_2) = \int d^4x' \, G_0^+(x_1, x')\Sigma_{4-pt}(x', x_2) + \int d^4x' \, G_0^A(x_1, x')\Sigma_{3-pt}(x', x_2).
\] (3.18)

Here we have used the free equation of motion

\[
\partial^2 \phi_p(x) = 0.
\] (3.19)

By substituting (3.13), the left-hand sides of them are written as follows after the Fourier transformation

\[
G_0^{-1}|_{x_1} G^+(x_1, x_2) \overset{\text{F.T.}}{\rightarrow} \left\{ \frac{1}{2} \partial_{\tau_c} Z(p, \tau_c) + \frac{i}{8p} \partial_{\tau_c}^2 Z(p, \tau_c) \right\} e^{-ip\Delta\tau} + \int_p^\infty d\epsilon \, \left\{ \frac{1}{2} \partial_{\tau_c} N(p, \epsilon, \tau_c) + \frac{i}{8\epsilon} \partial_{\tau_c}^2 N(p, \epsilon, \tau_c) - \frac{\epsilon^2 - p^2}{2\epsilon} N(p, \epsilon, \tau_c) \right\} e^{-i\epsilon\Delta\tau},
\] (3.20)

\[
G_0^{-1}|_{x_2} G^+(x_1, x_2) \overset{\text{F.T.}}{\rightarrow} \left\{ -\frac{1}{2} \partial_{\tau_c} Z(p, \tau_c) + \frac{i}{8p} \partial_{\tau_c}^2 Z(p, \tau_c) \right\} e^{-ip\Delta\tau} + \int_p^\infty d\epsilon \, \left\{ -\frac{1}{2} \partial_{\tau_c} N(p, \epsilon, \tau_c) + \frac{i}{8\epsilon} \partial_{\tau_c}^2 N(p, \epsilon, \tau_c) - \frac{\epsilon^2 - p^2}{2\epsilon} N(p, \epsilon, \tau_c) \right\} e^{-i\epsilon\Delta\tau}.
\] (3.21)

To compile them into a simple differential equation, we consider the difference between (3.17)
From (3.20) and (3.21), the left-hand side of (3.22) is given by

\[ G_{0}^{-1}|_{x_{1}}G^{+}(x_{1}, x_{2}) - G_{0}^{-1}|_{x_{2}}G^{+}(x_{1}, x_{2}) \]

and (3.18):

\[ \int d^{4}x' \Sigma_{4-\text{pt}}(x_{1}, x')G^{-+}_{0}(x', x_{2}) \]

\[ - \int d^{4}x' G^{+}_{0}(x_{1}, x')\Sigma_{4-\text{pt}}(x', x_{2}) \]

\[ + \int d^{4}x' \Sigma_{3-\text{pt}}^{R}(x_{1}, x')G^{+}_{0}(x', x_{2}) + \int d^{4}x' \Sigma_{3-\text{pt}}^{-+}(x_{1}, x')G^{4}_{0}(x', x_{2}) \]

\[ - \int d^{4}x' G^{+}_{0}(x_{1}, x')\Sigma_{3-\text{pt}}^{A}(x', x_{2}) - \int d^{4}x' G^{R}_{0}(x_{1}, x')\Sigma_{3-\text{pt}}^{-+}(x', x_{2}). \]

From (3.20) and (3.21), the left-hand side of (3.22) is given by

\[ G_{0}^{-1}|_{x_{1}}G^{+}(x_{1}, x_{2}) - G_{0}^{-1}|_{x_{2}}G^{+}(x_{1}, x_{2}) \]

\[ \text{F.T.} \partial_{\tau_{c}}Z(p, \tau_{c})e^{-ip\Delta\tau} + \int_{p}^{\infty} \text{d} \partial_{\tau_{c}}N(p, \epsilon, \tau_{c})e^{-i\epsilon\Delta\tau}. \]

We can investigate interaction effects through the right-hand side of (3.22). Thus we call it the collision term. Depending on whether the indices \( R, A \) are assigned to \( \Sigma_{3-\text{pt}} \) or \( G \), the latter part of the collision term is separated into the two parts:

\[ \int d^{4}x' \Sigma_{3-\text{pt}}^{R}(x_{1}, x')G^{+}_{0}(x', x_{2}) \text{F.T.} e^{-ip\Delta\tau} \times \ldots, \]

(3.24)

\[ - \int d^{4}x' G^{+}_{0}(x_{1}, x')\Sigma_{3-\text{pt}}^{A}(x', x_{2}) \text{F.T.} e^{-ip\Delta\tau} \times \ldots, \]

(3.25)

\[ \int d^{4}x' \Sigma_{3-\text{pt}}^{-+}(x_{1}, x')G^{A}_{0}(x', x_{2}) \text{F.T.} \int_{p}^{\infty} \text{d} e^{-i\epsilon\Delta\tau} \times \ldots, \]

In terms of the characteristic frequency, we call (3.24) the on-shell terms and (3.25) the off-shell terms. Obviously the integrals including \( \Sigma_{4-\text{pt}} \) are the on-shell terms. Our aim is to evaluate the wave function renormalization factor \( Z \) up to \( \mathcal{O}(\log a(\tau_{c})) \). Since \( Z \) is differentiated in the left-hand side, we need to evaluate the collision term up to \( \mathcal{O}(1/p|\tau_{c}|) \). We expand the collision term by the power series in \( 1/p|\tau_{c}| \) type factors which can be justified well inside the cosmological horizon. It is a kind of the derivative expansion of the Moyal product in the Wigner representation. In the subsequent sections, we investigate the collision term in details.

4 Local contribution in the collision term

In this section, we focus on the local contribution in the collision term. The local terms of the self-energy are identified as they are proportional to \( \delta^{(4)}(x - x') \). As seen in (3.9), the
contribution from the four-point vertices contains only the local terms at the one-loop level. The coefficients of them consist of propagators at the coincident point.

The propagator at the coincident point has an ultra-violet divergence (UV) in general. We are interested in the dS symmetry breaking from the massless and minimally coupled modes of gravity. By adopting the UV regularization which respects the dS symmetry, the propagator for a massless and minimally coupled field is written as

$$
\langle \varphi(x)\varphi(x) \rangle = \frac{H^2}{4\pi^2} \log \left( \frac{a(\tau)}{a_i} \right) + \text{(UV const.)},
\tag{4.1}
$$

$$
\langle \partial_\mu \varphi(x)\varphi(x) \rangle = \frac{H^3}{8\pi^2} a(\tau) \delta^0 \mu,
$$

$$
\langle \partial_\mu \varphi(x)\partial_\nu \varphi(x) \rangle = -\frac{3H^4}{32\pi^2} g_{\mu\nu}.
$$

We should emphasize that the coefficients of the dS symmetry breaking terms (IR logarithms) are UV finite. Concerning the internal loop contributions at the one-loop level, we can clearly separate IR contributions from UV contributions in comparison to Hubble scale. As far as IR logarithms are concerned, we can thus safely ignore UV contributions and UV divergences altogether.

From (2.24)-(2.25) and (4.1), the coefficients of the log $a(\tau)$ and $a(\tau)$ terms in (3.10) are evaluated as

$$
\Sigma_{4\text{-pt}}(x, x') \simeq \frac{i\delta^{(4)}(x - x')}{\frac{\kappa^2 H^2}{4\pi^2}} \cdot \log \left( \frac{a(\tau')}{a_i} \right) \left( \frac{3}{8} \partial_0^2 + \frac{13}{8} \partial_i^2 \right) + Ha(\tau') \left( \frac{3}{8} \partial_0^2 \right).
\tag{4.2}
$$

In contrast, the three-point vertices contribute to the local and non-local terms as (3.10). To extract the local terms from (3.10), it is useful to recall that $\delta(\tau - \tau')$ is derived by differentiating $\theta(\tau - \tau')$. As seen in (3.24) and (3.25), the step function is associated with the self-energy in the on-shell terms but not in the off-shell terms. Putting aside differential operators, the retarded self-energy is written as

$$
\Sigma^{R\text{-pt}}(x, x') \propto \frac{1}{2} \langle \{ h^{\mu\nu}(x), h^{\rho\sigma}(x') \} \rangle \times \theta(\tau - \tau') \langle [\phi(x), \phi(x')] \rangle
\tag{4.3}
$$

$$
+ \theta(\tau - \tau') \langle [h^{\mu\nu}(x), h^{\rho\sigma}(x')] \rangle \times \frac{1}{2} \langle \{ \phi(x), \phi(x') \} \rangle.
$$

Here $[,]$ denotes the commutator and $\{ , \}$ denotes the anti-commutator. The advanced self-energy is written in a similar form. The dS breaking logarithms come from the symmetric propagators of gravitational fields. Thus we may focus on $\delta^{(4)}(x - x')$ derived from the retarded propagators of scalar fields

$$
\partial_\mu \partial_\nu (\theta(\tau - \tau') \langle [\phi(x), \phi(x')] \rangle) \big|_{\text{local}} = -i\delta^{(4)}(x - x') \delta^0 \mu \delta^0 \nu,
\tag{4.4}
$$

$$
\partial_\mu \partial_\nu \partial_\rho (\theta(\tau - \tau') \langle [\phi(x), \phi(x')] \rangle) \big|_{\text{local}} = -i\delta^{(4)}(x - x') \left\{ \delta^0 \mu \delta^0 \nu \delta^\rho + \delta^0 \mu \epsilon^0 \nu \partial_\rho + \delta^0 \nu \epsilon^0 \rho \partial_\mu - 2\delta^0 \mu \delta^0 \nu \epsilon^0 \rho \partial_0 \right\}.
\tag{4.5}
$$
\[ \partial_{\mu}\partial_{\nu}\partial_{\rho}\partial_{\sigma}(\theta(\tau - \tau'))([\phi(x), \phi(x')])|_{\text{local}} \]

\[ = -i\delta^{(4)}(x - x') \left\{ \delta_{\rho}^{\mu}\partial_{\rho}'\partial_{\sigma}' + \delta_{\rho}^{\sigma}\partial_{\rho}'\partial_{\sigma}' + \delta_{\rho}^{\sigma}\partial_{\rho}'\partial_{\sigma}' + \delta_{\rho}^{\sigma}\partial_{\rho}'\partial_{\sigma}' \right. \]

\[ + \delta_{\rho}^{\sigma}\partial_{\rho}'\partial_{\sigma}' + \delta_{\rho}^{\sigma}\partial_{\rho}'\partial_{\sigma}' - 2(\delta_{\rho}^{\sigma}\partial_{\rho}^2 + \delta_{\rho}^{\sigma}\partial_{\rho}^2 + \delta_{\rho}^{\sigma}\partial_{\rho}^2 + \delta_{\rho}^{\sigma}\partial_{\rho}^2) \]

\[ + 4\delta_{\rho}^{\sigma}\partial_{\rho}^2 + \delta_{\rho}^{\sigma}\partial_{\rho}^2 \right\}. \]  

Here we have used the fact that the propagator of the conformally coupled scalar field depends only on the relative coordinate \( x - x' \).

From (2.24)-(2.25), (4.1) and (4.3)-(4.6), the following local terms are induced from the three-point vertices

\[ \Sigma_{3\text{-pt}}^R(x, x')|_{\text{local}} \]

\[ = i\delta^{(4)}(x - x') \frac{\kappa^2 H^2}{4\pi^2} \left\{ \log \left( \frac{a(\tau')}{a_i} \right) \left( -\frac{3}{4}\partial_0^2 - \frac{5}{4}\partial_i^2 \right) + Ha(\tau') \right\}. \]  

The sum of (4.2) and (4.7) is evaluated as

\[ \Sigma_{4\text{-pt}}(x, x') + \Sigma_{3\text{-pt}}^R(x, x')|_{\text{local}} \]

\[ = i\delta^{(4)}(x - x') \frac{\kappa^2 H^2}{4\pi^2} \left\{ \log \left( \frac{a(\tau')}{a_i} \right) \cdot \frac{3}{8}\partial^2 + Ha(\tau') \right\}. \]  

In a similar way, we can find the following local terms

\[ -\Sigma_{4\text{-pt}}(x, x') + \Sigma_{3\text{-pt}}^A(x, x')|_{\text{local}} \]

\[ = -i\delta^{(4)}(x - x') \frac{\kappa^2 H^2}{4\pi^2} \left\{ \log \left( \frac{a(\tau')}{a_i} \right) \cdot \frac{3}{8}\partial^2 + Ha(\tau') \right\}. \]  

Here we have assumed that the non-local terms do not contribute to the dS symmetry breaking. In the next section, we show how this assumption is justified. By substituting (3.23) and (4.8)-(4.9) into (3.22), the differential equation of the wave function renormalization factor is written as

\[ \partial_{\tau_c} Z(\tau_c)e^{-ip\Delta\tau} = \frac{3\kappa^2 H^2}{16\pi^2} \left\{ Ha(\tau_1) + Ha(\tau_2) \right\} e^{-ip\Delta\tau} \]

\[ \simeq \frac{3\kappa^2 H^2}{8\pi^2} Ha(\tau_c)e^{-ip\Delta\tau}. \]  

We should emphasize that the log \( a(\tau) \) term vanishes in the first line due to the classical equation of motion (3.19). Here it is crucial that the IR logarithm emerges as an overall factor of \( \partial^2 \) in (4.8)-(4.9). As seen in (3.3), the matter action possesses the Lorentz symmetry at the classical level. Since we can neglect the derivative of the IR logarithm at the sub-horizon scale:

\[ P \gg H \Rightarrow \log a(\tau) \partial_\mu \gg \partial_\mu (\log a(\tau)), \]  

the overall IR logarithm indicates that the Lorentz symmetry is effectively respected even if soft gravitational effects are included.
Furthermore we have extracted the average time dependence as $|\tau_c| \gg |\Delta \tau|$ in the second line. The solution of the differential equation is given by

$$Z(\tau_c) = 1 - \frac{3\kappa^2 H^2}{8} \frac{\log (a(\tau_c)/a_i)}{4\pi^2}, \quad (4.12)$$

where we have set the initial condition as $Z(\tau_i) = 1$. After the field redefinition:

$$\phi(x) \rightarrow Z^{\frac{5}{4}}(\tau)\phi(x), \quad \langle \phi(x_1)\phi(x_2) \rangle \rightarrow Z^{-1}(\tau_c)\langle \phi(x_1)\phi(x_2) \rangle. \quad (4.13)$$

there is no physical effect from soft gravitons in the free field theory (3.3) at the sub-horizon scale. It is consistent with the result obtained in [5].

We also refer to the fact that as far as we consider the dynamics at the sub-horizon scale, the same result (4.12) is derived in non-conformally coupled scalar field theories with one exception. In the minimally coupled case, we need to include the IR logarithm from soft scalar and hard graviton intermediate state.

5 Non-local contribution in the collision term

In this section, we investigate the non-local contribution in the collision term. Specifically we investigate the integrals which do not contain the derivative of $\theta(\tau - \tau')$. To begin with, let us calculate the spatial integration

$$\int d^3x' \Sigma^a_{3-pt}(x_1, x')G^d_0(x', x_2), \quad (5.1)$$

where $(a, b), (c, d) = (\pm, \mp)$. The propagator $G_0(x', x_2)$ with each Schwinger-Keldysh index contains the common spatial plan waves

$$\int \frac{d^3p}{(2\pi)^32p} e^{+ip\cdot(x'-x_2)}. \quad (5.2)$$

From (2.21)-(2.26) and (3.10), a straightforward but cumbersome calculation leads to the following integral

$$\int d^3x' \Sigma^a_{3-pt}(x_1, x') \times \int \frac{d^3p}{(2\pi)^32p} e^{+ip\cdot(x'-x_2)} \quad (5.3)$$

$$\rightarrow \frac{\kappa^2 H^2}{2p} \int \frac{d^3p_1d^3p_2}{(2\pi)^62p_12p_2} (2\pi)^3\delta^{(3)}(p_1 + p_2 - p)e^{-i\epsilon(\tau_1 - \tau')}$$

$$\times \left[ \left\{ -\frac{1}{48}(\epsilon^2 - p^2)(37\epsilon^2 + 11p^2)\frac{1}{p_2} + \frac{1}{12}(37\epsilon^2 - 13\epsilon p^2)\frac{1}{p_2} - \frac{17}{6} \epsilon^2 + \epsilon p_2 \right\} + i(\tau_1 - \tau')(\epsilon^2 - p^2) \left\{ -\frac{1}{48}(37\epsilon^2 + 11p^2)\frac{1}{p_2} + \frac{3}{4} \epsilon - \frac{1}{6} p_2 \right\} + \frac{7}{24}\tau_1\tau'(\epsilon^2 - p^2)^2 \right].$$
Here \( p_1 \) and \( p_2 \) are respectively the comoving momenta of the intermediate scalar and gravitational fields. Furthermore we have introduced the total energy of intermediate particles as

\[
\epsilon \equiv p_1 + p_2. \tag{5.4}
\]

We should emphasize that the integral (5.3) has no IR divergence at \( p_2 = 0 \) (\( \epsilon = p \)). If the non-local terms contribute to the dS symmetry breaking, it appears after performing the remaining time integral.

To facilitate the subsequent discussions, we adopt \( \epsilon \) and \( p_2 \) as the integral variables:

\[
\frac{1}{2p} \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} \frac{(2\pi)^3 \delta^{(3)}(p_1 + p_2 - p)}{2p_2} = \frac{1}{32\pi^2 p^2} \int_0^\infty d\epsilon \int_0^{\epsilon + p} dp_2. \tag{5.5}
\]

After performing the integral over \( p_2 \), the integral (5.3) is given by

\[
-\frac{\kappa^2 H^2}{32\pi^2 p^2} \int_0^\infty d\epsilon \ A(p, \epsilon, \tau_1, \tau') e^{-i(\tau_1 - \tau')}, \tag{5.6}
\]

\[
A(p, \epsilon, \tau_1, \tau') = \left\{ \frac{1}{12} (37\epsilon^3 - 13p^2) \log \frac{\epsilon + p}{\epsilon - p} - \frac{65}{12} \epsilon^2 p - \frac{11}{12} p^3 \right\}
+ i(\tau_1 - \tau') (\epsilon^2 - p^2) \left\{ -\frac{1}{48} (37\epsilon^2 + 11p^2) \log \frac{\epsilon + p}{\epsilon - p} + \frac{2}{3} \epsilon p \right\}
+ \frac{7}{24} \tau_1 \tau' (\epsilon^2 - p^2)^2 p \right]. \tag{5.7}
\]

We also show the integral which contains \( \Sigma^{+-}_{3-pt}(x_1, x') \)

\[
\int d^4x' \ \Sigma^{+-}_{3-pt}(x_1, x') \times \int \frac{d^3p}{(2\pi)^3} \ e^{ip \cdot (x' - x_2)} \tag{5.8}
\]

\[
\longrightarrow_{\text{F.T.}} -\frac{\kappa^2 H^2}{32\pi^2 p^2} \int_0^\infty d\epsilon \ A^*(p, \epsilon, \tau_1, \tau') e^{+i(\tau_1 - \tau')}.
\]

Let us evaluate the following integral in (3.22)

\[
\int d^4x' \ \Sigma^{R}_{3-pt}(x_1, x')|_{\text{non-local}} \ G^{--}(x', x_2) \tag{5.9}
\]

\[
= \int d^4x' \ \theta(\tau_1 - \tau') \left[ \Sigma^{+-}(x_1, x') - \Sigma^{+-}(x_1, x') \right] G^{--}(x', x_2)
\]

\[
\longrightarrow_{\text{F.T.}} -\frac{\kappa^2 H^2}{32\pi^2 p^2} \int_0^\infty d\epsilon \int_0^{\tau_1} d\tau' \ A(p, \epsilon, \tau_1, \tau') e^{-i(\tau_1 - \tau') - ip(\tau' - \tau_2)}
+ \frac{\kappa^2 H^2}{32\pi^2 p^2} \int_0^\infty d\epsilon \int_0^{\tau_1} d\tau' \ A^*(p, \epsilon, \tau_1, \tau') e^{+i(\tau_1 - \tau') - ip(\tau' - \tau_2)}.
\]
It should be noted that we have retained the integral in the second line which does not contain derivatives of $\theta(\tau_1 - \tau')$. To evaluate the integrals over time, we use the following identities

$$
\int_{-\infty}^{\tau_1} d\tau' e^{\mp i\epsilon(\tau_1 - \tau') - ip(\tau' - \tau_2)} = \frac{1}{i(\pm \epsilon - p)} e^{-ip(\tau_1 - \tau_2)},
$$

$$
\int_{-\infty}^{\tau_1} d\tau' \tau' e^{\mp i\epsilon(\tau_1 - \tau') - ip(\tau' - \tau_2)} = \left\{ \frac{\tau_1}{i(\pm \epsilon - p)} + \frac{1}{(\pm \epsilon - p)^2} \right\} e^{-ip(\tau_1 - \tau_2)},
$$

where the order of double-sign corresponds. These identities indicate that the integral (5.9) contributes to the on-shell terms. After the time integration, the integral (5.9) is given by

$$
+i \frac{\kappa^2 H^2}{32\pi^2 p^2} e^{-ip(\tau_1 - \tau_2)} \int_0^\infty d\epsilon
$$

$$
\times \left[ \frac{1}{\epsilon - p} \left( -\frac{19}{4} \epsilon^2 p + \frac{2}{3} \epsilon p^2 - \frac{11}{12} p^3 \right) + \left( \frac{37}{16} \epsilon^2 + \frac{37}{24} \epsilon p + \frac{11}{48} p^2 \right) \log \frac{\epsilon + p}{\epsilon - p} \right.
$$

$$
+ i \frac{7}{24} \tau_1 (\epsilon + p)^2 p + \frac{7}{24} \tau_1^2 (\epsilon - p)(\epsilon + p)^2 p
$$

$$
+ i \frac{\kappa^2 H^2}{32\pi^2 p^2} e^{-ip(\tau_1 - \tau_2)} \int_0^\infty d\epsilon
$$

$$
\times \left[ \frac{1}{\epsilon + p} \left( -\frac{19}{4} \epsilon^2 p - \frac{2}{3} \epsilon p^2 - \frac{11}{12} p^3 \right) + \left( \frac{37}{16} \epsilon^2 - \frac{37}{24} \epsilon p + \frac{11}{48} p^2 \right) \log \frac{\epsilon + p}{\epsilon - p} \right.
$$

$$
- i \frac{7}{24} \tau_1 (\epsilon - p)^2 p + \frac{7}{24} \tau_1^2 (\epsilon + p)(\epsilon - p)^2 p \right].
$$

There is an IR divergence at $\epsilon = p$ where a soft or collinear particle is present. We should note that the IR divergence originates in the following integrand in (5.9)

$$
\Sigma^{-+}_{3,pt}(x_1, x') G^{+-}(x', x_2).
$$

(5.12)

Our goal is to identify possible IR logarithms in (5.11). We discard UV power divergent terms as they do not induce logarithms. Their IR contribution can be safely neglected. We estimate the logarithmically singular part of (5.11) as

$$
- i p \frac{5\kappa^2 H^2}{32\pi^2} e^{-ip(\tau_1 - \tau_2)} \int_p^\infty d\epsilon \frac{1}{\epsilon - p}.
$$

(5.13)

Since the choice of the initial time corresponds to the IR cut-off of $\epsilon - p$ as

$$
\int_{\tau_1} d\tau' \rightarrow \int_{p+|1/\tau_1|} d\epsilon,
$$

(5.14)

the IR behavior seems to contribute to the dS symmetry breaking. However it is well known that in flat space, the IR singularities in the process with a soft or collinear particle are canceled after summing over degenerate states between real and virtual processes [6, 7]. This is a universal phenomenon in any unitary theory as we have shown to be the case with $\phi^3, \phi^4$ theories in dS space [8]. We can argue that the analogous cancellation holds in the matter field theory interacting with soft gravitons.
To confirm the cancellation, let us evaluate another integral in (3.22)

\[ \int d^4x' \, \Sigma_{\text{pt}}(x_1, x') G^A(x', x_2) \]  
\[ = - \int d^4x' \, \theta(\tau_2 - \tau') \Sigma_{\text{pt}}(x_1, x') \left[ G^{-+}(x', x_2) - G^{+-}(x', x_2) \right] \]

\[ \longrightarrow + \frac{i \kappa^2 H^2}{32 \pi^2 p^2} \int_p^\infty \, d\epsilon \int_{-\infty}^{\tau_2} d\tau' \, A(p, \epsilon, \tau_1, \tau') e^{-i(\tau_1 - \tau') + ip(\tau' - \tau_2)} \]

\[ - \frac{i \kappa^2 H^2}{32 \pi^2 p^2} \int_p^\infty \, d\epsilon \int_{-\infty}^{\tau_2} d\tau' \, A(p, \epsilon, \tau_1, \tau') e^{-i(\tau_1 - \tau') + ip(\tau' - \tau_2)}. \]

From the identities,

\[ \int_{-\infty}^{\tau_2} d\tau' \, e^{-i(\tau_1 - \tau') + ip(\tau' - \tau_2)} = \frac{1}{i(\epsilon + p)} e^{-i(\tau_1 - \tau_2)}, \]
\[ \int_{-\infty}^{\tau_2} d\tau' \tau' e^{-i(\tau_1 - \tau') + ip(\tau' - \tau_2)} = \left\{ - \frac{\tau_2}{i(\epsilon + p)} + \frac{1}{(\epsilon + p)^2} \right\} e^{-i(\tau_1 - \tau_2)}, \]

the integral contributes to the off-shell terms. After the time integration, the integral (5.15) is given by

\[ -i \frac{\kappa^2 H^2}{32 \pi^2 p^2} \int_p^\infty \, d\epsilon \ e^{-i(\tau_1 - \tau_2)} \]
\[ \times \left[ \frac{1}{\epsilon + p} \left( -\frac{19}{4} \epsilon^2 p + \frac{2}{3} \epsilon p^2 - \frac{11}{12} p^3 \right) + \frac{37}{16} \epsilon^2 + \frac{37}{24} \epsilon p + \frac{11}{48} p^2 \log \frac{\epsilon + p}{\epsilon - p} \right. \]
\[ + i(\tau_1 - \tau_2)(\epsilon + p) \left\{ \frac{2}{3} \epsilon p - \frac{1}{48} (37\epsilon^2 + 11p^2) \log \frac{\epsilon + p}{\epsilon - p} \right\} \]
\[ + i \frac{7}{24} \tau_1(\epsilon + p)^2 p + \frac{7}{24} \tau_1 \tau_2(\epsilon + p)(\epsilon + p)^2 p \left. \right] \]

\[ + i \frac{\kappa^2 H^2}{32 \pi^2 p^2} \int_p^\infty \, d\epsilon \ e^{-i(\tau_1 - \tau_2)} \]
\[ \times \left[ \frac{1}{\epsilon + p} \left( -\frac{19}{4} \epsilon^2 p - \frac{2}{3} \epsilon p^2 - \frac{11}{12} p^3 \right) + \frac{37}{16} \epsilon^2 - \frac{37}{24} \epsilon p + \frac{11}{48} p^2 \log \frac{\epsilon + p}{\epsilon - p} \right. \]
\[ + i(\tau_1 - \tau_2)(\epsilon - p) \left\{ \frac{2}{3} \epsilon p - \frac{1}{48} (37\epsilon^2 + 11p^2) \log \frac{\epsilon + p}{\epsilon - p} \right\} \]
\[ + i \frac{7}{24} \tau_1(\epsilon - p)^2 p + \frac{7}{24} \tau_1 \tau_2(\epsilon + p)(\epsilon - p)^2 p \left. \right] \]

The off-shell term also has an IR divergence at \( \epsilon = p \) which originates in the common integrand (5.12) with the on-shell term (5.9). The logarithmically singular part of (5.17) is evaluated as

\[ + i p \frac{5\kappa^2 H^2}{32 \pi^2} \int_p^\infty \, d\epsilon \ e^{-i(\tau_1 - \tau_2)} \frac{1}{\epsilon - p}. \]

The differences between (5.13) and (5.18) turns out to be the relative opposite sign and their frequencies \( p \) and \( \epsilon \).
Physically speaking, any experiment has a finite energy resolution of observation $\Delta \epsilon$. We may divide the integral region of the off-shell term as

$$\int_p^\infty d\epsilon \ e^{-i\epsilon(\tau_1-\tau_2)} = \int_p^{p+\Delta \epsilon} d\epsilon \ e^{-i\epsilon(\tau_1-\tau_2)} + \int_p^\infty d\epsilon \ e^{-i\epsilon(\tau_1-\tau_2)}. \quad (5.19)$$

Within the energy resolution, we cannot distinguish the off-shell term from the on-shell term

$$\int_p^{p+\Delta \epsilon} d\epsilon \ e^{-i\epsilon(\tau_1-\tau_2)} \sim e^{-ip(\tau_1-\tau_2)} \int_p^\infty d\epsilon. \quad (5.20)$$

Thus we need to redefine the on-shell term by transferring the contribution of the off-shell term within the energy resolution $p < \epsilon < p + \Delta \epsilon$:

$$-ip\frac{5\kappa^2 H^2}{32\pi^2} e^{-ip(\tau_1-\tau_2)} \left\{ \int_p^\infty d\epsilon - \int_p^{p+\Delta \epsilon} d\epsilon \right\} \frac{1}{\epsilon - p} \quad (5.21)$$

The remaining contribution is the well-defined off-shell term:

$$+ip\frac{5\kappa^2 H^2}{32\pi^2} \int_p^{p+\Delta \epsilon} d\epsilon \ e^{-i\epsilon(\tau_1-\tau_2)} \frac{1}{\epsilon - p}. \quad (5.22)$$

We have found that there is no IR divergence after the redefinition. Since the energy resolution of observation is at the sub-horizon scale $\Delta \epsilon \gg |1/\tau_c| > |1/\tau_i|$, the IR cut-off is given by not the inverse of the initial time $|1/\tau_i|$ but the energy resolution $\Delta \epsilon$. As these quantities are pure imaginary, they should be identified as the twice differentiated quantities in (3.20): $i\partial^2_{\tau_c} Z/8p$ and $i\partial^2_{\tau_c} N/8\epsilon$.

We thus integrate twice with respect to $\tau_c$ in order to estimate their contributions to the two point function. The on-shell contribution to the two point function from (5.21) has no time dependence if we fix the physical energy resolution $\Delta E$:

$$\delta Z|_{\text{non-local}} = -\frac{5\kappa^2 P^2}{8\pi^2} \int_{\Delta E}^{\Lambda_{\text{UV}}} d(\epsilon - P) \frac{\epsilon}{E - P}, \quad (5.23)$$

$$P = pH|\tau_c|, \ E = \epsilon H|\tau_c|, \ \Delta E = \Delta \epsilon H|\tau_c|. \quad (5.24)$$

Here $\Lambda_{\text{UV}}$ is the UV cut-off which fixes the maximum value of the physical total energy $E$ and we assume $\Lambda_{\text{UV}} \gg P$. The off-shell contribution to the two point function from (5.22) is evaluated as

$$N = +\frac{5\kappa^2 EP a^{-1}(\tau_c)}{8\pi^2} \frac{a^{-1}(\tau_c)}{E - P}. \quad (5.25)$$

From scale covariance of our definition (3.13), there remains a factor $a^{-1}(\tau_c)$ in this expression. As for the non-local contributions, we can confirm that the total integral of the spectral weight is preserved:

$$\frac{1}{2P} \delta Z|_{\text{non-local}} + \int_{\Delta E}^{\Lambda_{\text{UV}}} d(\epsilon - P) \frac{1}{2E} \tilde{N} = 0, \quad (5.26)$$
where we have redefined the off-shell part of the spectrum function as $\tilde{N} \equiv a(\tau_c)N$. In this regard, the non-local contributions respect unitarity. We should emphasize that the first and second terms in the left-hand side of (5.26) preserve the dS symmetry respectively.

Furthermore, we can show the mechanism how the cancellation takes place. As seen in (5.14), the dS symmetry breaking is expressed as the dependence of the initial time. The contribution from the negatively large conformal time region is dominant only when the frequency vanishes. The zero frequency process is contained in the common integrand (5.12) in (5.9) and (5.15). Then the initial time dependence is canceled as follows:

$$\int d^4x' \Sigma_{3\text{-pt}}(x_1, x')_{\text{non-local}} G^{-+}(x', x_2) + \int d^4x' \Sigma_{3\text{-pt}}^{-+}(x_1, x')G^2(x', x_2)$$

$$\simeq \left\{ \int_{\tau_i}^{\tau_1} d\tau' - \int_{\tau_i}^{\tau_2} d\tau' \right\} \int d^4x' \Sigma_{3\text{-pt}}^{-+}(x_1, x')G^{-+}(x', x_2).$$

The cancellation holds between the remaining two integrals in (3.22):

$$-\int d^4x' G^{-+}(x_1, x')\Sigma_{3\text{-pt}}^{A}(x', x_2)_{\text{non-local}} - \int d^4x' G^R(x_1, x')\Sigma_{3\text{-pt}}^{-+}(x', x_2)$$

$$\simeq \left\{ \int_{\tau_i}^{\tau_2} d\tau' - \int_{\tau_i}^{\tau_1} d\tau' \right\} \int d^4x' G^{-+}(x_1, x')\Sigma_{3\text{-pt}}^{-+}(x', x_2).$$

In a similar way to (5.9)-(5.21), we can confirm the cancellation in terms of the integral over the total energy $\epsilon$.

We summarize this section. When we naively distinguish the off-shell terms from the on-shell terms such as (3.24), (3.25), each term appears to induce the dS symmetry breaking logarithm. Such IR singularities originate in the process with a soft or collinear particle. Since the off-shell terms are not distinguishable from the on-shell terms in the process, we need to sum up them. After the redefinition, the IR cut-off is given by not the initial time but the energy resolution. Once the non-local terms are expressed by physical scales such as $\Delta E, P, A_{UV}$, it is apparent that they do not contribute to the dS symmetry breaking. It justifies the strategy of the previous section that we may investigate the local terms in order to evaluate the dS symmetry breaking.

We also refer to the case that we set the external momentum to be off-shell $p_\mu p^\mu \neq 0$. In this case, the independence from the initial time can be showed more simply. It is because the integral over the total energy is cut-off by the virtuality:

$$\epsilon^2 - (p^0)^2 > p_\mu p^\mu.$$  

The investigation in this section gives a proper interpretation into the on-shell limit of the off-shell effective equation of motion.

### 6 Parametrization dependence

In this section, we clarify how soft gravitational effects depend on the parametrization of the metric. It is the parallel investigation of the previous studies with the effective equation of
motion. In the previous sections, we investigated soft gravitational effects on a matter system by adopting the parametrization (2.3)-(2.5). As another example, we adopt the parametrization (2.27).

As explained in (2.28)-(2.30), the difference between the two parametrizations emerges at the non-linear level. So at the one-loop level, the parametrization difference of the metric (2.28) contributes only to the tadpole diagrams:

$$\Delta(\Sigma_{\text{4-pt}}(x,x')) = i\delta^{(4)}(x-x') \times \left[ -\kappa \partial'_\mu \{ \langle h^{\mu\nu}(x') \rangle|_{\text{NL}} \partial'_\nu \} - \frac{1}{6} \kappa \partial'_\mu \partial'_\nu \langle h^{\mu\nu}(x') \rangle|_{\text{NL}} \right],$$  

where

$$\kappa \langle h_{\mu\nu}(x) \rangle|_{\text{NL}} \text{ is identified as}$$

$$\kappa \langle h_{\mu\nu}(x) \rangle|_{\text{NL}} = -2\kappa^2 \langle \Phi(x) \Psi_{\mu\nu}(x) \rangle - \frac{1}{2} \kappa^2 \langle \Psi_\rho(x) \Psi_{\rho\mu}(x) \rangle + \frac{1}{8} \kappa^2 \langle \Psi_\rho(x) \Psi^{\rho\rho}(x) \rangle \eta_{\mu\nu}. \ (6.2)$$

From (2.24)-(2.25) and (2.30), the coefficients of the $\log a(\tau)$ and $a(\tau)$ terms in (6.1) are evaluated as

$$\Delta(\Sigma_{\text{4-pt}}(x,x')) \simeq i\delta^{(4)}(x-x') \times \left[ -\kappa \partial'_0 \left\{ \langle h^{\mu\nu}(x') \rangle|_{\text{NL}} + v_{\mu\nu} \right\} \partial'_\nu \right],$$  

where

$$\kappa \langle h_{\mu\nu}(x) \rangle|_{\text{NL}} \text{ is identified as}$$

$$\kappa \langle h_{\mu\nu}(x) \rangle|_{\text{NL}} = -2\kappa^2 \langle \Phi(x) \Psi_{\mu\nu}(x) \rangle - \frac{1}{2} \kappa^2 \langle \Psi_\rho(x) \Psi_{\rho\mu}(x) \rangle + \frac{1}{8} \kappa^2 \langle \Psi_\rho(x) \Psi^{\rho\rho}(x) \rangle \eta_{\mu\nu}. \ (6.2)$$

The relative weight of $\partial'_0$ and $\partial'_i$ is not equal to $-1$ in the coefficient of the IR logarithm. That is, the effective Lorentz symmetry is not respected. Since we have confirmed that the effective Lorentz invariance holds in the original parametrization of the metric (2.3)-(2.5), there should be a prescription to retain it in a different parametrization.

We should recall that the parametrization dependence of the metric emerges only in the tadpole diagrams. So we can compensate them by introducing the classical expectation value of the background metric:

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + v_{\mu\nu}, \quad v_{\mu\nu} = -\langle h_{\mu\nu}(x) \rangle|_{\text{NL}}, \quad (6.4)$$

$$\Delta(\Sigma_{\text{4-pt}}(x,x')) \rightarrow i\delta^{(4)}(x-x') \times \left[ -\kappa \partial'_\mu \left\{ \langle h^{\mu\nu}(x') \rangle|_{\text{NL}} + v^{\mu\nu} \right\} \partial'_\nu \right] - \frac{1}{6} \kappa \partial'_\mu \partial'_\nu \langle h^{\mu\nu}(x') \rangle|_{\text{NL}} + v^{\mu\nu} \right] = 0. \ (6.5)$$

Note that the gravitational action is stationary with this shift. At least at the one-loop level, the compensation by shifting the background metric is available not only for the difference between (2.3)-(2.5) and (2.27), but also for an arbitrary difference of the parametrization of the metric. It is because the difference at the non-linear level emerges only in the tadpole diagrams at the one-loop order.

The discussion in this section is summarized as follows. By a judicious choice of the classical expectation value of the background metric, the effective Lorentz invariance is preserved for any choice of the parametrization of the metric. Furthermore the resulting wave function renormalization factor (4.12) does not depend on the choice.
7 Conclusion

Due to the existence of the scale invariant spectrum, we need to introduce an IR cut-off into the propagator for a massless and minimally coupled scalar and gravitational field. The IR cut-off fixes the minimum value of the comoving momentum and it is identified as an inverse of the initial time. As a consequence, the propagator has a logarithmic dependence of the scale factor which breaks the dS symmetry.

It should be noted that there is another IR contribution in massless field theories. In the process with a soft or collinear particle, the frequency of the integrand becomes small and so the integral over the negatively large conformal time is dominant. When we set the external momentum to be off-shell, the time integral is bounded by not the initial time but an inverse of the virtuality.

In the on-shell limit, IR singularities occur since the frequency can vanish. However we cannot distinguish the off-shell term from the on-shell term in the zero frequency process. The IR singularities cancel after summing over degenerate states between real and virtual processes. The corresponding IR cut-off is given by the energy resolution. We observe phenomena in the condition where the physical scale of the virtuality or the energy resolution is fixed. Therefore the non-local contribution respects the dS symmetry.

The above cancellation originates from the fact that the total spectrum weight is preserved. In this regard, it may be a universal phenomenon as far as field theoretic models are consistent with unitarity. By using the Kadanoff-Baym approach, we have specifically shown that the cancellation holds in a matter system with gravity. That is, soft gravitons contribute to the dS symmetry breaking only through the local terms. In other words, if the dS symmetry breaking takes place, it originates only in the increasing degrees of freedom at the super-horizon scale. It has justified the assumption adopted in [5].

We have found that the local contribution appears as a time dependent overall factor of the kinetic term. It indicates that the effective Lorentz symmetry is respected even if soft gravitational effects are considered. After the wave function renormalization, soft gravitons leave no growing physical effect to a free matter system. Of course, the wave function renormalization factor is the same one obtained in [5].

Furthermore we have investigated how soft gravitational effects depend on the parametrization of the metric. The parametrization dependence appears only in the tadpole diagram. Thus it can be compensated by introducing the classical expectation value of the metric. With this prescription, the gravitational action is kept stationary and the results obtained in this paper do not depend on the parametrization of the metric. It is consistent with the previous studies based on the effective equation of motion [17].

The investigation in this paper is on the two-point function. In the previous studies [5], we investigated soft gravitational effects in interacting field theories. There we assumed that only the local terms contribute to the dS symmetry breaking also in multi-point functions. It remains an open problem whether the statement is correct or not. However we believe that the conjecture is reasonable since the cancellation of the non-local IR singularities is intimately connected to the unitarity of the theory.
Acknowledgment

This work is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan, and the National Research Foundation grants NRF-2005-0093843, NRF-2010-220-C0000, NRF-2012KA1A9055.

References

[1] A. Vilenkin and L. H. Ford, Phys. Rev. D 26, 1231 (1982).
[2] A. D. Linde, Phys. Lett. B 116, 335 (1982).
[3] A. A. Starobinsky, Phys. Lett. B117, 175 (1982).
[4] N. C. Tsamis and R. P. Woodard, Commun. Math. Phys. 162, 217 (1994).
[5] H. Kitamoto and Y. Kitazawa, arXiv:1203.0391 [hep-th].
[6] T. Kinoshita, J. Math. Phys. 3, 650 (1962).
[7] T. D. Lee and M. Nauenberg, Phys. Rev. 133, B1549 (1964).
[8] H. Kitamoto and Y. Kitazawa, Nucl. Phys. B 839, 552 (2010) arXiv:1004.2451 [hep-th].
[9] S. A. Ramsey and B. L. Hu, Phys. Rev. D 56, 661 (1997) gr-qc/9706001.
[10] G. Baym and L. P. Kadanoff, Phys. Rev. 124, 287 (1961).
     L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics, Benjamin, New York, 1962.
[11] E. O. Kahya and R. P. Woodard, Phys. Rev. D 76, 124005 (2007) arXiv:0709.0536 [gr-qc].
     E. O. Kahya and R. P. Woodard, Phys. Rev. D 77, 084012 (2008) arXiv:0710.5282 [gr-qc].
[12] S. P. Miao and R. P. Woodard, Class. Quant. Grav. 23, 1721 (2006) gr-qc/0511140.
     S. P. Miao and R. P. Woodard, Phys. Rev. D 74, 024021 (2006) gr-qc/0603135.
     S. P. Miao and R. P. Woodard, Class. Quant. Grav. 25, 145009 (2008) arXiv:0803.2377 [gr-qc].
[13] J. S. Schwinger, J. Math. Phys. 2, 407 (1961).
[14] L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Sov. Phys. JETP 20, 1018 (1965)].
[15] A. M. Polyakov, Nucl. Phys. B 797, 199 (2008) arXiv:0709.2899 [hep-th].
     A. M. Polyakov, arXiv:1209.4135 [hep-th].
[16] E. T. Akhmedov, JHEP 1201, 066 (2012) [arXiv:1110.2257 [hep-th]].
E. T. Akhmedov and P. Burda, Phys. Rev. D 86, 044031 (2012) [arXiv:1202.1202 [hep-th]].
E. T. Akhmedov, F. K. Popov and V. M. Slepukhin, arXiv:1303.1068 [hep-th].

[17] H. Kitamoto and Y. Kitazawa, Nucl. Phys. B 873, 325 (2013) [arXiv:1211.3878 [hep-th]].