Hawking temperature and stability in BTZ-Sen Black Hole

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Abstract. By using the appropriate Killing vector for BTZ-Sen black hole, we obtained surface gravity and the corresponding Hawking temperature. The stability of a black hole can be seen from specific heat. From the result, we can infer that the Hawking temperature reduces to the well known BTZ when the charge vanishes and there is no phase transition with appropriate fixed parameters.

1. Introduction
String theory so far is the best candidate for quantum gravity. Yet, to be consistent mathematically, it has to live in higher dimensions than four. To be able to make predictions at all, we need to see what string theory can offer us in its low energy limit. From the cosmological point of view, one of exciting test for it is black hole [1]. In this low energy, Sen [2] obtained a rotating charged black hole in four dimensional spacetime. The solution is characterized by mass, charge, and angular momentum. In his literature, he use the method called the twisting procedure [3] that describes solution with nonzero matter field with vacuum solution as an input. Some aspect including event horizon, ergosphere and Hawking temperature are discussed in the literature.

On the other hand, The well known BTZ black hole [4, 5] describe a three dimensional rotating vacuum black hole in Einstein gravity with Anti de Sitter spacetime. One can find the similar case called Kerr black hole [6] which describe a rotating vacuum solution in four dimensional space-time. The only main distinction between these two solutions is the asymptotic behaviour. Despite, these black holes describe a rotating vacuum that has an event horizon and ergosphere. In other case, while there has been study on Einstein gravity, we could find another exact solution in the low energy string theory by employing generating technique on three dimensional (BTZ) black hole. As a result, we have a three dimensional black hole in the low energy string theory [7].

In this paper, we employ the metric solution obtained in [7] to describe Hawking temperature and the corresponding stability. The paper is structured as follows. As a quick review, we present an Einstein frame BTZ-Sen black hole in section 2. In section 3 and 4, we showed novelty aspect such as Hawking temperature and the stability from specific heat. In section 5, we conclude some of our statement.

2. BTZ-Sen Black Hole
This background solution has been studied in [7]. As a review, we shall discussed only the action in Einstein frame and the metric solution. The action in Einstein frame is

\[ S = \int d^3x \sqrt{-g} \left[ R(g) - 2\Lambda e^{2\Phi} - \nabla_\mu \Phi \nabla^\mu \Phi - \frac{e^{-2\Phi}}{8} F_{\mu\nu} F^{\mu\nu} - \frac{e^{-4\Phi}}{12} H_{K\mu\nu} H^{K\mu\nu} \right] \] (1)
where $g$ is a determinant of metric tensor, $R$ is a Ricci scalar, the $F$ squared is a Maxwell strength tensor, and $H$ squared is a 3-form field. Solving this with H-S transformation, we obtained a black hole solution in Einstein frame which can be written as
\[
\begin{align*}
g_{tt} &= \frac{(M - b)(M + r^2\Lambda - b)}{M + b(M - b + r^2\Lambda)} \\
g_{rr} &= \frac{4r^2[M + b(M - b + \Lambda r^2)]}{(M - b)[a^2(M - b)^2 + 4r^2(b - M - r^2\Lambda)]} \\
g_{\phi\phi} &= \frac{-aM(M - b)}{2[M + b(M + \Lambda r^2)]} \\
g_{\phi\phi} &= \frac{a^2b(M - b)[2M + b(-M + b + r^2\Lambda)]}{4[M + b(M - b + r^2\Lambda)]} + \frac{r^2}{(M - b)}[M + b(M - b + r^2\Lambda)]
\end{align*}
\]
(2)

For non-gravitational field in Einstein frame reads
\[
\begin{align*}
A_\mu dx^\mu &= \frac{Q}{2[M + b(M - b + r^2\Lambda)]} \left[(1 + M - b + r^2\Lambda)dt - \frac{a(M - b)}{2}d\phi\right] \\
B_{\epsilon\phi} &= -B_{\phi\epsilon} = \frac{aMb(M - b)}{2[M + b(M - b + r^2\Lambda)]} \\
\Phi &= \frac{1}{2} \ln \left[\frac{M + b(M - b + r^2\Lambda)}{M - b}\right]
\end{align*}
\]
(3)

Equation (2)-(3) are the complete solution for three dimensional black hole in the low energy string theory describe an object with mass, charge and angular momentum. If we make a limit $b = 0$, all of the metric solution (2) will reduce to the well known BTZ solution [4,5] and the nongravitational field will vanish. Moreover, black hole itself are thermodynamic object that could radiate and undergo phase transition. It is tempting to investigate the related Hawking temperature and the stability of black hole.

3. Hawking Temperature

Hawking's horizon area theorem in 1970 stated that the total horizon area in a closed system containing black holes never decreases. It can only increase or stay the same. Surprisingly, shortly after the Hawking's theorem, Bekenstein [8, 9] noticed the analogy between the black hole horizon and the second law of thermodynamics therefore black hole have entropy. As is well known, if black hole as a closed system have entropy, then the black hole have temperature. Later, this physical quantity is called Hawking temperature. In his literature [10], he employs quantum mechanics process near black hole horizon and consider an empty space in classical physics into creation-annihilation of particle and anti-particle with the uncertainty. By using surface gravity as fix quantity in the horizon, we get
\[
\kappa = \frac{1}{2} \nabla_\mu \xi^\nu \nabla_\nu \xi_\mu \bigg|_{r \to r_+}
\]
(4)
and the corresponding Hawking temperature becomes
\[
T_H = \frac{\kappa}{2\pi}
\]
(5)

For stationary and axisymmetric black holes, one can obtain the surface gravity using these Killing vector $\xi^t$ and $\xi^\phi$. Therefore, the Hawking temperature reads
$$T_\text{H} = \frac{r^2_+ - r^2_-}{2\pi r_+}$$  \hspace{1cm} (6)

It is shown that the Hawking temperature only depends on the event horizon radius which can be written as

$$r_\pm = l \sqrt{\frac{(M - b)}{2} \left[ 1 \pm \sqrt{1 - \left(\frac{J}{Ml}\right)^2} \right]}$$  \hspace{1cm} (7)

Substituting the radius into equation (6), we get a complete form of Hawking temperature

$$T_\text{H} = \frac{l(M - b) \sqrt{1 - \frac{J^2}{l^2 M^2}}}{\pi \sqrt{2(M - b) \left[ 1 + \sqrt{1 - \frac{J^2}{l^2 M^2}} \right]}}$$  \hspace{1cm} (8)

where \(M\) is a black hole mass and can be obtained by by solving \(g^{rr} = 0\). As charged vanish, we reproduce Hawking temperature in BTZ black hole [4].

4. Stability

After Bekenstein proposed the second law of black hole, the entropy for BTZ black hole is equal to twice the perimeter length of the horizon

$$S_{BH} = 2L = 4\pi r_+$$  \hspace{1cm} (9)

From this information, we could obtain the stability

$$C_q = T_\text{H} \frac{\partial S}{\partial T_\text{H}}$$  \hspace{1cm} (10)

After some algebra, we obtain the specific heat which can be written as

$$C = \frac{4\pi l M^{7/2} \left[ (2M^2 - Q^2) \left( \sqrt{1 - \frac{J^2}{l^2 M^2}} + 1 \right) \right]^{3/2}}{l^2 \left[ 2M^2 - Q^2 \left( 2\sqrt{1 - \frac{J^2}{l^2 M^2}} + 3 \right) \right] + l^2 M^2 (2M^2 + Q^2) \left( 1 - \frac{J^2}{l^2 M^2} + 3 \right)}$$  \hspace{1cm} (11)

Next we analyze this quantity by plot.

\[\text{Figure 1. Specific heat versus radius with } a = 0.5, \text{ and } l = 10\]
In this plot, we present the specific heat versus radius of event horizon with fix parameter shown in the caption. Since it has a positive specific heat \( C > 0 \), we conclude that the black hole is stable. Moreover, specific heat from black hole family in four dimensional space-time always exhibit the infinite value that indicates phase transitions. As we can see, the are no infinite value on this three dimensional black hole in the low energy string theory (as well as the BTZ black hole). From Figure 5, we can infer that there is no phase transition of this fix parameter.

5. Conclusion

Thermodynamics aspect from BTZ-Sen black hole has been studied. Using appropriate Killing vector, we obtained the surface and Hawking temperature. From the temperature and entropy, we find the stability of a black hole from specific heat. In our thermodynamics analysis, it is shown that the simple expression of Hawking temperature reduces to the well known BTZ when charge vanish. The formalism indicates that the Hawking temperature shifts to the higher value as event horizon gets larger. On the other hand, from this condition, we can analyze its stability by obtaining the specific heat. The positivity and linear of specific heat indicates that the black holes are stable and there is no phase transition with this fixed parameters.

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