Cryptocurrency Equilibria Through Game Theoretic Optimization

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Optimization methods are used to determine equilibria of investment in cryptocurrencies. The basic assumptions involve existence of a core group (the "wealthy") that fears the loss of substantial assets through government seizure. Speculators constitute another group that tends to introduce volatility and risk for the wealthy. The wealthy group exhibits risk aversion through a utility function, we establish the existence and uniqueness of Nash equilibrium. Also examined is the more realistic optimization problem in which the government policy cannot be reversed, while the wealthy can adjust their allocation in reaction to the government’s designation of probability. The methodology leads to an understanding of the equilibrium market capitalization of cryptocurrencies.

Cryptocurrencies have evolved into a new speculative asset form that differs from others in that most represent no intrinsic value; they cannot be redeemed by a financial institution for any amount. The roller-coaster ride of Bitcoin prices from $6,000 to $20,000 back to $6,000 and subsequently near $10,000, all during the period October 2017 to May 2018, was shadowed by other major cryptocurrencies. This has been accompanied by the general feeling in government, business and academia that the speculative fever is of concern only to those who own the cryptocurrencies. There is some justification for this perspective as the total market capitalization of all cryptocurrencies is about $300 billion, so that large moves in the cryptocurrency price are not likely to have a significant impact on the world’s stock and bond markets. However, this impact will present a significant risk to the world’s markets if the market capitalization of the cryptocurrencies increases significantly. During the dramatic round trip of Bitcoin between $6,000 to $20,000, the market capitalization of all cryptocurrencies nearly doubled in six months. Moreover, 10,000 Bitcoins were used to purchase two pizzas in the first transaction in 2010 (3). If people gradually become more comfortable with cryptocurrencies, as they did with internet shopping, it is likely that the market capitalization would grow to a few percent of the $75 trillion Gross World Product (GWP). At this point, large price changes in cryptocurrencies would likely have an impact on the broader markets.

Hence, it is important to understand the factors behind the market capitalization and price of cryptocurrencies. With all other assets there is some theoretical methodology to estimate the value, which is at least a first step estimating the trading price. For example, the value of a stock is estimated by measures such as the expected dividend stream (see Graham (4), (5), Luenberger (6), Bodie et al. (7), and Jost (8)). The absence of an intrinsic value of a cryptocurrency means that classical finance methods are inapplicable.

Our analysis begins with a game theoretic examination of the motivations of three groups that are the key players. For many people, the basic need for a cryptocurrency arises from the inadequacy of the ‘home’ currency and banking system (9). There are also a significant number of people who are not able to obtain a credit card or even open a bank account in the US, for example (10). In many countries, owning large amounts of the currency can present a significant risk. There is the possibility of expropriation by the government, sometimes in the guise of a corruption probe. The government could institute policies in which inflation is very high, e.g., the extreme example in Venezuela (11) where 4000% (and rising) inflation quickly decimates any individual savings. Onerous taxes can be placed by the government on the wealthy. Thus a group of people in the world have rational reasons to replace their country’s currency with one that is outside the control of their government or financial institutions, even if it presents some risk. Subsequently they would have the option of buying a more reliable currency or asset in another country. We denote this group by \( W \) (the "wealthy").

The second group, \( D \), represents a government that is totalitarian, at least with respect to monetary policy, so that its citizens are not free to transfer their wealth into other currencies. There is a probability, \( p \), that the government can initiate policies that will deprive citizens of a fraction \( k \) of their wealth, e.g., by printing money. This possibility is noted

Significance Statement

The core capital for cryptocurrencies arises from the desire of citizens (denoted \( W \)) to protect their wealth from seizure by various entities (denoted \( D \)). Thus, \( W \) must make a choice of allocation of its assets between the home currency and the cryptocurrency. While the home currency faces risk of seizure, the cryptocurrency faces risk due to volatility induced by speculators. We then determine the Nash equilibrium, defined to be an allocation fraction \( x^* \) and probability of seizure, \( p^* \), such that neither party can unilaterally improve its position. We introduce a second method in which group \( D \) cannot make rapid changes, unlike \( W \). Our work has implications for understanding market capitalization and the impact on the world’s economy, and economic freedom.

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by the wealthy, \( W \), who must make a decision on the fraction of their assets, denoted \( 1 - x \) held in the home currency and the remainder, \( x \), in cryptocurrency, which presents risks of its own due to the volatility. A third group, \( S \), consists of the speculators whose sole reason to trade is to profit from the transaction at the expense of the less knowledgeable group, \( W \). The speculators can effectively control the volatility. While \( D \) can set the probability, \( p \), with which the assets can be seized, group \( W \) can decide what fraction, \( x \), to convert into the cryptocurrency.

We model this situation to find equilibria in two different ways. The first is to find the Nash equilibrium \((p^*, x^*)\) that neither party can improve its fortunes by unilaterally changing (e.g., one day), \( D \) must make a decision that is irrevocable during a longer time (e.g., one year) as policies (e.g., creating inflation, imposing onerous taxes) are implemented. But in doing so, \( D \) must be aware that \( W \) will self-optimize in its choice of \( x \), knowing \( p \). Thus, both parties are aware of the different time scales involved in anticipating the other party’s decision.

**The Utility Functions of the Groups**

The general framework for this section will be to write the utility functions of the groups, modeled on portfolio theory (6), (7) whereby one seeks allocate resources to maximize return while minimizing risk. The general form for a basic utility function is \( U = m - d^2 \sigma^2 \) where \( m \) and \( \sigma^2 \) are the expectation and variance of the outcome, while the parameter \( d^2 \) quantifies the risk aversion.

The speculators, \( S \), are assumed to be able to manipulate the volatility and are likely to profit at the expense of \( W \), who are likely to be novices. Hence, the role of \( S \) is secondary (and discussed in the Appendix) as they create an expected loss and a variance for \( W \).

Focusing now on the groups, \( W \) and \( D \), we assume that \( W \) has a choice between the home currency, F, and a cryptocurrency, Y. Any money held in F faces the risk that a fixed fraction \( k \in [0, 1] \) will be seized by \( D \) with probability \( p \). Thus, the outcome will be \((1 - x)(1 - k) \) with probability \( p \) and \( 1 \) with probability \( 1 - p \). Letting \( m_F \) and \( \sigma_F^2 \) denote the mean and variance of the investment in F, one finds,

\[
m_F = (1-k)p + 1 \cdot (1-p) = 1 - kp, \\
\sigma_F^2 = k^2p(1-p).
\]

For the investment in Y, we let \( m_Y \) and \( \sigma_Y^2 \) denote the mean and variance that will be determined by the speculators (see Appendix).

The utility function for \( W \) with the fraction \( x \in [0, 1] \) of its assets in \( Y \) and the remainder in \( F \) can then be expressed as

\[
U_W = m - d^2 \sigma^2 = mxm_Y + (1-x)m_F - d^2 \left\{ x^2 \sigma_Y^2 + (1-x)^2 \sigma_F^2 + 2x(1-x) \text{Cov}[Y,F] \right\}. \tag{2}
\]

We will assume that the correlation between the two assets, \( Y \) and \( F \), is zero, but the analysis can easily be carried out if there is a correlation.

The utility function for \( D \) can be expressed in terms of the amount that it seizes, i.e.,

\[
U_D = (1-x) kp. \tag{3}
\]

This can be augmented with a term (as in portfolio theory) that expresses the risk aversion. In particular, one has

\[
U_D = (1-x) kp - d_D^2 p^2, \tag{4}
\]

where \( d_D^2 \) represents the risk aversion of \( D \).

**Nash Equilibria**

We assume the utility functions described in Section 3, using the risk aversion form of \( U_D \) above (4). Thus, we need to find \((p^*, x^*)\) such that

\[
\partial_x U_W (p^*, x^*) = 0, \quad \partial_{xx} U_W (p^*, x^*) = 0, \quad \partial_{pp} U_D (p^*, x^*) \leq 0.
\]

i.e., \((p^*, x^*)\) satisfies the definition of a Nash equilibrium (e.g., (12)). Briefly, the definition ensures that at \((p^*, x^*)\) neither party can unilaterally improve its situation. We compute

\[
0 = \partial_p U_D (p, x) = (1-x) k - 2d_D^2 p, \tag{5}
\]

\[
0 = \partial_x U_W (p, x) = m_Y - m_F + 2d^2 \sigma_Y^2 - 2d^2 \left( \sigma_Y^2 + \sigma_F^2 \right) x
\]

\[
= m_Y - 1 + kp + 2d^2 k^2 p (1-p) - 2d^2 \left( \sigma_Y^2 + k^2 p (1-p) \right). \tag{6}
\]

Denote the solution of (5) by \( x_1(p) \) and that of (6) by \( x_2(p) \), so that

\[
x_1(p) = 1 - \frac{2d_D^2 p}{k}, \tag{7}
\]

\[
x_2(p) = \frac{\sigma_F^2 + (2d^2)^{-1} (m_Y - m_F)}{\sigma_Y^2 + \sigma_F^2} = \frac{k^2 p (1-p) + (2d^2)^{-1} (m_Y - 1 + kp)}{k^2 p (1-p) + \sigma_Y^2}. \tag{8}
\]

The intersection of \( x_1(p) \) and \( x_2(p) \) determine a Nash equilibrium. We first establish sufficient conditions for at most one equilibrium, and then prove that under broad conditions, there exists a Nash equilibrium. Some of these curves for sample values of the parameters are illustrated in Figure 1.

**Theorem 1** For \( p \in [0, 1/2] \) one has \( x_2(p) \geq 0 \) for all values of the parameters, so there can be at most one value of \( p \) such that \( x_1(p) = x_2(p) \), and thus at most one Nash equilibrium for \( p \in [0, 1/2] \).

**Proof.** For convenience set \( f(p) = k^2 p (1-p) \), \( c_1 = (2d^2)^{-1} (1-m_Y) \), \( c_2 = (2d^2)^{-1} k \) and \( c_3 = \sigma_Y^2 \) so that

\[
x_2(p) = \frac{f(p) + c_2p - c_1}{f(p) + c_3}
\]

and

\[
x_2(p) = \frac{c_2 k^2 p^2 + c_2 c_3 + (c_1 + c_3) k^2 (1-2p)}{f(p) + c_3^2} \tag{9}.
\]

Clearly, for \( p \in [0, 1/2] \) all terms are positive and the conclusion follows.
Fig. 1. An assortment of curves modelling the fraction $x$ of wealth to be placed into crypotocurrency from the perspectives of groups $D$ and $W$. The linear curves represent the optimization for $D$ and the group for $W$. We plot the curves for values of $a := 2d_D^2 = 8, 4, 2, 1/2$ against fraction under seizure $k$ and expected loss/trading cost on crypotocurrency $b := 1 - m_W$ given by $(k, b) = (.5, .7), (.55, .85), (.6, .9), (.75, .95)$ with volatility $c := \sigma_Y^2 = .01$ and group $W$’s risk tolerance $d = 1$ both held constant in both sets of curves. When there is an intersection in the range $(p^*, x^*) \in (0, 1)^2$, we have a Nash equilibrium. It is possible to have other intersections outside of this box, but such “equilibria” are irrelevant and their behavior will lead to boundary cases such as $p = 0$ or $p = 1$.

**Theorem 2** If the parameters $d$, $k$ and $m_W$ satisfy

$$c_1 + c_2 \leq c_2 \ i.e., \ (1 + 2d^2) (1 - m_W) \leq k$$

then $x'_2 (p) \geq 0$ for all $p \in [0, 1]$. Thus there can be at most one Nash equilibrium under these conditions.

**Proof.** For $p \in [0, 1/2]$ the result has been established. For $p \in [1/2, 1]$ the numerator of (9) is

$$c_2 k^2 p^2 + c_2 c_3 + c_3 k^2 (1 - 2p) = c_2 c_3 + c_3 k^2 (1 - p^2) > 0,$$

and the result follows. 

Having determined sufficient conditions for uniqueness, we now focus on establishing existence of Nash equilibrium. Note first that the $p-$intercept of $x_1 (p)$ can be on either side of $x = 1$ depending on the slope $-2d_D^2 / k$. In particular, we let $p_c := k (2d_D^2)^{-1}$ and consider the two cases separately.

**Theorem 3** (a) If $p_c := k (2d_D^2)^{-1} < 1$ and

$$k^2 p_c (1 - p_c) + (2d^2)^{-1} (m_W - 1 + p_c k) > 0, \ [11]$$

then one has a Nash equilibrium, i.e., there exists $(p^*, x^*) \in [0, 1] \times [0, 1]$ such that $x_1 (p^*) = x_2 (p^*) = x^*$.

(b) If $p_c := k (2d_D^2)^{-1} \geq 1$

$$\frac{(2d^2)^{-1} (m_W - 1 + k)}{\sigma_Y^2} + \frac{2d_D^2}{k} \geq 1$$

then one has again a Nash equilibrium.

If in addition, equation (10) holds, then the Nash equilibrium $(p^*, x^*)$ is unique.

**Remark 4** The Nash equilibrium may not be unique if the condition above, i.e.,

$$(1 + 2d^2) (1 - m_W) \leq k$$

of Theorem 3 is violated. An example for two Nash equilibria can be constructed with the parameters:

$$k = 0.7, \ m_W = 0.8, \ d = 2, \ d_D = 0.355, \ \sigma_Y^2 = 0.1.$$

The two equilibria are given approximately by $(p^*, x^*) = (0.88, 0.68)$ and $(0.96, 0.65)$, as pictured in Figure 2.
Equilibrium with Disparate Time Scales

We consider the situation in which the wealthy, $W$, can decide on an allocation $x$ immediately, (e.g., within one day), and adjust to the probability, $p$, while $D$ must set $p$ that cannot be changed for a long time e.g., one year. Thus, $D$ lacks the opportunity to react to the value of $x$. Both parties are aware of the position of the other group. Hence, $D$ knows that once he sets $p$, group $W$ will set $x = \hat{x}(p)$ in a way that optimizes $U_W$, and that $W$ does not need to be concerned with any readjustment of $p$ in reaction to their choice of $x$. Thus, $D$ must examine $U_W$ (based on the publicly available information on the volatility of $Y$) and decide on a value of $p$ that will optimize $U_D(p, \hat{x}(p))$. Within this setting the utility of $D$ need not be strictly convex in order for an interior maximum (i.e., such that $0 < p < 1$ and $0 < x < 1$). Thus, we consider the case in which $D$ has utility that is proportional to the amount it takes, without any risk aversion, which can be included with a bit more calculation.

We define the quantity

$$A := 2d^2\sigma_Y^2 + 1 - m_Y$$

which arises naturally in the calculations and is a measure of the risk and expected loss from $Y$. Thus a higher value of $A$ means $Y$ is less attractive to the wealthy.

**Theorem 5** Suppose that the utility functions, $U_W$ and $U_D$, given by

- $U_D = (1 - x) kp$
- $U_W = m - d^2\sigma^2$

are known to both parties. Assume that $D$ sets $p$ irrevocably to maximize $U_D$, while $W$ chooses $x$ to maximize $U_W$ based on a knowledge of $p$. For $0 \leq A \leq k$ the optimal choice of $x$ given $p$ is

$$\hat{x}(p) := \frac{m_Y - m_F}{2d^2(\sigma_Y^2 + \sigma_F^2)} + \frac{\sigma_F^2}{\sigma_Y^2 + \sigma_F^2}$$

with $m_Y$ and $\sigma_Y^2$ given by (1), and the optimal value of $p$ is given by

$$p^* := \frac{\sigma_Y^2}{k(k - A)} \left( 1 + \frac{A}{B(k - A)} \right)$$

Thus the optimal point is $(x, p) = (x^*, \hat{x}(p^*))$. The value of maximum, $x^* = \hat{x}(p)$ is 0 if the right hand side of (13) is negative, and 1 if the right hand side exceeds 1.

**Remark 6** Note that given $p$ the optimal fraction of assets in the cryptocurrency is a sum of the relative variance of the home currency, i.e., $\sigma_Y^2$ as a fraction of $\sigma_Y^2 + \sigma_F^2$ plus the difference in expected loss from the home currency, i.e., $1 - m_Y$ minus the expected loss from the cryptocurrency, $1 - m_Y$ scaled by a risk aversion factor. Thus the fraction invested in the cryptocurrency increases as the expected loss and the variance of the home currency increases, and conversely.

**Remark 7** Note that one obtains an interior maximum with a linear utility function for $U_D$ in this type of optimization, i.e., even though $D$ is interested in pure maximization of its revenue.

**Proof.** Using (2) we determine the maximum of $U_W$ for a fixed $p$, so that

$$0 = \partial_x U_W(p, x) = m_Y - m_F + 2d^2\sigma_Y^2 - 2d^2(\sigma_Y^2 + \sigma_F^2)x.$$

Noting that $\partial_x U_W(p, x) = -2d^2(\sigma_Y^2 + \sigma_F^2) < 0$ we see that $U_D$ is maximized by $\hat{x}(p)$ given by (13) provided $\hat{x}(p) \in [0, 1]$. In the following two cases the maximum is on the boundary:

$$\frac{m_Y - m_F}{2d^2} + \sigma_F^2 < 0 \implies \hat{x}(p) = 0,$$

$$\frac{m_Y - m_F}{2d^2} + \sigma_F^2 > 1 \implies \hat{x}(p) = 1.$$

Thus, $\hat{x}(p)$ interpolates between 0 and 1 by favoring $Y$ if the relative risk of $F$ (measured by $\sigma_F^2(\sigma_Y^2 + \sigma_F^2)^{-1}$) is large in comparison with the relatively greater expected loss in $Y$ (scaled by the sum of the variances).

In anticipation, $D$ optimizes $U_D(p, x(p))$. We thus compute, with $B := \sigma_Y^2$,

$$0 = \frac{2d^2}{k} \partial_p U_D(p, x(p)) = \partial_p \left[ A p - k p^2 \right]$$

$$= \frac{(A - 2kp) [B + k^2p(1 - p)] - (Ap - k^2p) [k^2(1 - 2p)]}{[B + k^2p(1 - p)]^2}.$$

This identity is equivalent to

$$Q(p) := AB - 2Bkp + k^2(A - k)p^2 = 0.$$  \[15\]

Note that $A > 0$ by assumption. The positive root of equation (15) is

$$p^* = \frac{B}{k(k - A)} \left( 1 + \frac{A}{B(k - A)} \right).$$

One can verify that $p^* \in [0, 1]$, and conclude that $(p^*, x^*) = (p^*, \hat{x}(p^*))$ is the optimal point. ■

**Remark 8** Case $A = 0$. By definition (12) we see that $1 - m_Y = 0$. Note that $p^* = 0$ follows from the identity above. Using the definition and the computed values of $m_F = 1 - kp$ and $\sigma_F^2 = k^2p(1 - p)$ we write

$$\hat{x}(p) = \frac{m_Y - 1 + kp}{2d^2(\sigma_Y^2 + k^2p(1 - p))} + \frac{k^2p(1 - p)}{\sigma_Y^2 + k^2p(1 - p)}$$

$$\hat{x}(0) = \frac{m_Y - 1}{2d^2\sigma_Y^2} = 0.$$

In other words, when $A = 0$ there is no risk and no expected loss in the cryptocurrency. Thus, $D$ realizes that any nonzero value of $p$ will result in $W$ investing nothing in the home currency, $F$.

Case $A = k$. The quadratic numerator (15) is then $Q(p) = AB - 2Bkp$ so that one has $p^* = 1/2$.

Case $k < A \leq 2k$. By considering a small positive perturbation, $\delta$, of $A$ we see that $Q \left( \frac{1}{2} \right) > 0$ so that the positive region of $\partial_p U_D$ is extended toward the right as $A$ increases.

Case $A \geq 2k$. Since $p \leq 1$ one has

$$Q(p) \geq B(A - 2k) + k^2p^2(A - k) > 0,$$

so $\partial_p U_D > 0$ and the maximum is thus $p^* = 1$.  

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Conclusion

We have examined the optimal strategies for the key parties involved explicitly or implicitly in the formation of an equilibrium for cryptocurrencies. The second method involving different time scales represents a new perspective in determining equilibrium that differs from the standard Nash equilibrium, in which all parties can readjust their positions continuously. This second method we describe in Section 4 can be utilized for many realistic situations in which one entity such as a government optimizes by placing conditions such as taxes, tariffs, fees, etc., or policies that cannot be reversed or adjusted in a short time. In general, optimization in this form favors the group that can make immediate and continuous adjustments.

Each of the methods are based on parameters that can be estimated. For example, the variance of cryptocurrencies can be determined from the trading data. Parameters such as $k$ (the fraction of assets seized) can be estimated from the policies of the government. The capability to estimate these quantities then leads to estimates of the amount of money that is likely to be used to purchase cryptocurrencies in the aggregate. Using the ideas summarized in (18) one can then also estimate average price changes of cryptocurrencies as well as the total market capitalization of cryptocurrencies.

The evolution of the latter is crucial in understanding the implications of instability of cryptocurrencies on other sectors of the world’s economy.

Major governments have often appeared confused and lethargic in their response to cryptocurrency policy, even insofar as deciding whether it is important or not. There is also little understanding of the conditions under which a cryptocurrency could be either beneficial or detrimental to global society. The perspective of our paper suggests that a cryptocurrency that is stable and essentially backed by real assets would serve the cause of economic freedom in the world. In particular, a properly designed cryptocurrency, e.g., linked to the Gross World Product of the world, would be less volatile, so that the equilibrium point $(p^*, x^*)$ would feature $x^*$ that is larger and $p^*$ that is smaller. Thus, the creation of such a cryptocurrency would reduce the volatility and thereby reduce the fraction of savings in the home currency that is under threat by a totalitarian government, whose existence is often contingent on raising money in this manner.

Appendix

In examining the choice faced by $W$ we assume that one option is to remain in the home currency, $F$, and the other to buy the cryptocurrency with the objective of later selling in order to buy other assets such as a more reliable currency, gold, etc.

The group $W$ experiences a loss or gain on these transactions with the speculators, group $S$, which itself has a nonlinear utility function reflecting that fact that high volatility is good for profits up to a point after which it has a negative impact. Thus, one has the following utility function for group $S$:

$$U_S = a_1 V - a_2 V^2$$

where $V$ represents the volatility or variance, for example, and $a_1$, $a_2$ are positive constants. Hence, there will be a maximum value $V = V_m$ that maximizes the utility of the speculators. This can be viewed as a fixed quantity from the perspective of $W$.

The mean, $m_Y$, and variance, $\sigma_Y^2$ of group $W$’s investments in $Y$ can be calculated based on $V_m$ and the other parameters that describe the trading. In particular, we assume that there is a probability $q$ (presumably small) that $W$ will profit, and that their wealth will increase from $1$ to $1 + r_1 V_m$ and a probability $1 - q$ that it will decrease from $1$ to $1 - r_2 V_m$ where $r_2 > r_1 > 0$. In other words, there is a small probability, $q$, that $W$ will benefit by $r_1 V_m$ (as a fraction of their original wealth) and a larger probability, $1 - q$, that they will lose a larger sum $r_2 V_m$. The loss is proportional to the volatility as the professional speculators are able to exploit the ups and downs of the trading at the expense of the inexperienced $W$.

The mean and variance of the outcome are then

$$m_Y = q (1 + r_1 V_m) + (1 - q) (1 - r_2 V_m),$$

$$\sigma_Y^2 = q (1 - q) (r_1 + r_2) V_m.$$  

In other words, there is large probability that $W$ will take a loss on the transaction. One can consider more general probability distributions for $W$’s profits and losses, but ultimately, the two quantities that are relevant for its utility function $U_W$ are given by $m_Y$ and $\sigma_Y^2$ that one can regard as empirical observables.

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