An Existence Condition for Power-Flow of DC Microgrids with CPLs Considering Voltage Disturbance and Distributed Generations under MPPT Control

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Abstract. For the power-flow equation of a power network, whether a feasible power-flow solution exists is necessarily important for the correct operation of the system. Besides, it is directly related to the system’s voltage stability. Therefore, it is of great importance to investigate the existence of the feasible power-flow solution of DC microgrids, especially for those with CPLs and a large amount of distributed generations under MPPT control (MPPPT-DGs). In this paper, a mathematical power-flow equation of the DC microgrid with CPLs is built firstly. Next, a solvability condition for the power-flow equation of the DC networks considering voltage disturbance and MPPT-DGs is derived based on Brouwer’s fixed-point theorem. Moreover, simulations are performed using MATLAB to verify the effectiveness of the proposed theorem.

1. Introduction
In recent decades, due to renewable energy resources’ DC property as well as the increase in DC demand, DC microgrids have attracted widespread attention. DC microgrids have highlighted advantages such as high transmission efficiency and high reliability\textsuperscript{[1]}, suitable for applications in commercial sector, aviation sector and electric vehicles\textsuperscript{[2-3]}. In DC microgrid, the distributed generations (DGs) can be divided into two kinds of DGs, that is the renewable energy DGs under MPPT control (MPPT-DGs) and the traditional DGs under droop control (Droop-DGs). MPPT-DGs are applied to enhance the efficiency of energy, while Droop-DGs are used to support system’s voltage.

It is noted that the existence of the equilibrium is a necessary condition for the system’s stability. Through DC-DC/DC-AC converters, loads in DC microgrid normally behave as constant power loads (CPLs). Then, the nonlinear characteristics of CPLs would bring a challenge to DC microgrids\textsuperscript{[2-3]}. The negative impedance of CPLs may cause instability of the system\textsuperscript{[4]}. Therefore, it is of prime importance to derive the existence condition of power-flow equation of DC microgrid containing CPLs.

The presence of MPPT-DGs and CPLs makes the power-flow equation a nonlinear equation, which is difficult to solve. In addition, the power-flow equation of a meshed DC network is a multidimensional quadratic equation (MDQE) containing multiple unknowns\textsuperscript{[5]}. Among the methods to solve MDQE, most methods such as “Tarski’s fixed-point theorem” and “nested interval theorem” only applied for DC microgrids with Droop-DG. However, the existence of MPPT-DGs also affects the power-flow equation. Moreover, there are limitations on the joint consideration of voltage...
disturbance and MPPT-DGs. Therefore, when considering voltage disturbance and distributed generations under MPPT Control, it is necessary to derive the existence condition for the power-flow equation of the DC microgrid containing CPLs.

2. Definitions and Lemmas

Definition 1. In this paper, Droop-DGs are defined as the traditional renewable DGs under droop control. While MPPT-DGs are renewable DGs under MPPT control.

Definition 2. Let $I = [l_1, l_2, ..., l_m]^T$ ($l_i \neq 0$) and define $\|I\| = \text{diag} \{l_i\}$, $I^m = [l_{m1}, l_{m2}, ..., l_{mn}]^T$. Define $1_m = [11...1]^T$, $0_m = [00...0]^T$ and $L = [l_{ij}]$.

Definition 3. Define the infinite norm of matrix $A$ as $\|A\|_\infty = \max_{1 \leq i \leq m} \left\{ \sum_{j=1}^{m} |a_{ij}| \right\}$.

Lemma 1. Brouwer’s fixed-point theorem: Given a compact and convex set $S \subseteq \mathbb{R}^n$ and a continuous function $f(x): S \rightarrow S$, then there has some $x \in S$ satisfying $f(x) = x$, i.e., $x$ is a fixed point [6].

3. Problem Formulation

3.1. The Power-Flow Equation of DC Microgrids Containing CPLs

In this article, a meshed DC microgrid which has $m$ Droop-DGs, $n$ MPPT-DGs and $l$ loads is studied. Figure 1 depict the topology of this DC microgrid. According to figure 1, the system is made up of power sources, loads and cables. For a low voltage system, the cables are purely resistive and the loads are all assumed as CPLs.

![Figure 1. The model and structural diagram of DC microgrid.](image-url)
According to the Ohm’s and Kirchoff’s laws, the current injected into the transmission network can be described as

\[
\begin{pmatrix}
i_s \\
i_m \\
i_l
\end{pmatrix}
= \begin{pmatrix}
Y_{SS} & Y_{SM} & Y_{SL} \\
Y_{MS} & Y_{MM} & Y_{ML} \\
Y_{LS} & Y_{LM} & Y_{LL}
\end{pmatrix}
\begin{pmatrix}
u_s \\
u_m \\
u_l
\end{pmatrix} = \begin{pmatrix}
u_s \\
u_m \\
u_l
\end{pmatrix}
\]

where \(Y = [y_{ij}]\) is system’s symmetric admittance matrix, \(y_{ij}\) is the conductance of cable. \(i_s, i_m\) and \(i_l\) represent the vectors of currents of Droop-DGs, MPPT-DGs and CPLs. While the vectors of voltages of Droop-DGs, MPPT-DGs and CPLs are represented by \(u_s, u_m\) and \(u_l\), respectively.

When the system works in a steady state, the voltage dynamics of Droop-DGs can be described as

\[
u_s = (Y_{SS} + K^{-1})^{-1}K^{-1}\nu_{ref}^ih - (Y_{SS} + K^{-1})^{-1}\begin{pmatrix}
Y_{SM} \\
Y_{SL}
\end{pmatrix}u_m^h + \nu_{eq}^l
\]

where \(K = \text{diag}\{k_i\}\), \(k_i\) represents the droop gain of \(i_{th}\) Droop-DG. \(\nu_{ref}\) is the voltage reference.

For MPPT-DGs and CPLs, the output powers are given by

\[
\begin{pmatrix}
P_M \\
P_L
\end{pmatrix} = \begin{pmatrix}
O \\
O
\end{pmatrix}
\begin{pmatrix}
i_m \\
i_l
\end{pmatrix} - \begin{pmatrix}
P_M \\
P_L
\end{pmatrix}
\]

where \(P_M\) and \(P_L\) represent the power vector of MPPT-DGs and CPLs.

Combining (1)-(3), the power-flow equation is obtained as follows

\[
\begin{pmatrix}
P_M \\
P_L
\end{pmatrix}
= \begin{pmatrix}
Y_{eq}^h \\
Y_{eq}^l
\end{pmatrix} \begin{pmatrix}
(\nu_s)^h \\
(\nu_l)^h
\end{pmatrix} = \begin{pmatrix}
P_M \\
P_L
\end{pmatrix}
\]

where \(Y_{eq} = \begin{pmatrix}
Y_{MM} & Y_{ML} \\
Y_{LM} & Y_{LL}
\end{pmatrix} - \begin{pmatrix}
Y_{MM} \\
Y_{LM}
\end{pmatrix}(Y_{SS} + K^{-1})^{-1}\begin{pmatrix}
Y_{SM} \\
Y_{SL}
\end{pmatrix}\) is the Schur complement of matrix \(H\), where

\[
H = \begin{pmatrix}
Y_{SS} + K^{-1} & Y_{SM} & Y_{SL} \\
Y_{MS} & Y_{MM} & Y_{ML} \\
Y_{LS} & Y_{LM} & Y_{LL}
\end{pmatrix}
\]

Left multiplying \(Y_{eq}^{-1}\) \(\begin{pmatrix}
P_M \\
P_L
\end{pmatrix}\) \(^{-1}\), (4) becomes

\[
\begin{pmatrix}
u_M \\
u_L
\end{pmatrix} = -Y_{eq}^{-1}\begin{pmatrix}
Y_{MS} \\
Y_{LS}
\end{pmatrix}(Y_{SS} + K^{-1})^{-1}K^{-1}\nu_{ref}^h - Y_{eq}^{-1}\begin{pmatrix}
-\begin{pmatrix}
P_M \\
P_L
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
u_M \\
u_L
\end{pmatrix}^{-1}
\]

Considering that \(Y\) is a Laplacian matrix, the following is obtained

\[
\begin{cases}
Y_{SS}^h l^m + Y_{SM}^h l^m + Y_{SL}^h l^l = 0^m \\
Y_{MS}^h l^m + Y_{MM}^h l^m + Y_{ML}^h l^l = 0^n \\
Y_{LS}^h l^m + Y_{LM}^h l^m + Y_{LL}^h l^l = 0^l
\end{cases}
\]

Then, it yields
\[
\begin{bmatrix}
Y_{MM} & Y_{ML} \\
Y_{LM} & Y_{LL}
\end{bmatrix}
\begin{bmatrix}
Y_{MS} \\
Y_{LS}
\end{bmatrix}
(Y_{SS} + K^{-1})^{-1}
\begin{bmatrix}
Y_{SM} & Y_{SL}
\end{bmatrix}
I_{n+1} + \begin{bmatrix}
Y_{MS} \\
Y_{LS}
\end{bmatrix}
(Y_{SS} + K^{-1})^{-1}
\begin{bmatrix}
Y_{SS} \\
Y_{SS}
\end{bmatrix}
I_{n}\]

\[= \begin{bmatrix}
Y_{MM} & Y_{ML} \\
Y_{LM} & Y_{LL}
\end{bmatrix}
I_{n+1} - \begin{bmatrix}
Y_{MS} \\
Y_{LS}
\end{bmatrix}
I_{m} - \begin{bmatrix}
Y_{MS} \\
Y_{LS}
\end{bmatrix}
(Y_{SS} + K^{-1})^{-1}
\begin{bmatrix}
Y_{SM} & Y_{SL}
\end{bmatrix}
I_{n+1} + Y_{SS}I_{m} = 0_{n+1}
\]  

According to (8), we obtain

\[Y_{eq}I_{n+1} + \begin{bmatrix}
Y_{MS} \\
Y_{LS}
\end{bmatrix}
(Y_{SS} + K^{-1})^{-1}
\begin{bmatrix}
Y_{SS} \\
Y_{SS}
\end{bmatrix}
= 0_{n+1}, \text{i.e.,}
\]

\[u_{eq}I_{n+1} = -Y_{eq}^{-1}
\begin{bmatrix}
Y_{MS} \\
Y_{LS}
\end{bmatrix}
(Y_{SS} + K^{-1})^{-1}K^{-1}u_{eq}I_{m}
\]  

Left multiplying \([u_{eq}]^{-1}\), equation (6) is rewritten as

\[G(\tilde{u}) = I_{n+1} - J\tilde{u}^{-1}\tilde{u}
\]

where \(\tilde{u} = u_{eq}^{-1}\begin{bmatrix} u_{d} \\ u_{L} \end{bmatrix}, J = u_{eq}^{-2}Y_{eq}^{-1}egin{bmatrix} -[P_{M}] \\ [P_{T}] \end{bmatrix}\).

**Remark 1:** The system has a feasible power-flow solution, if and only if, for given \(J\) and \(u_{eq}\), the nonlinear equation (10) has a real solution. Therefore, our next goal is to derive the solvability condition for (10).

3.2. The Proposed Existence Condition for the Power-Flow Equation

According to (10), the following is derived

\[\tilde{u}^{-1} = J^{-1}I_{n+1} - J^{-1}\tilde{u}
\]

Multiplied by \([\tilde{u}]\), equation (11) becomes

\[I_{n+1} = \tilde{u}^{-1}J^{-1}I_{n+1} - \tilde{u}^{-1}J^{-1}\tilde{u}
\]

Considering the voltage disturbance of MPPT-DGs and CPLs, let \(\tilde{u} = I_{n+1} + \Delta \tilde{u}\), the following holds

\[\Delta \tilde{u} = -JI_{n+1} - J\Delta \tilde{u} \quad J^{-1}\Delta \tilde{u}
\]

Then, constructing \(f(\tilde{u}) = -JI_{n+1} - J\Delta \tilde{u} \quad J^{-1}\Delta \tilde{u}\). According to Lemma 1, if there exists a convex set \(s\) that satisfies \(\forall \tilde{u} \in S, f(\tilde{u}) \in S\), then \(f(\tilde{u}) = \tilde{u}\) has a solution.

**Theorem 1:** If the following holds, then the system has solution in \(S = \left\{ \Delta \tilde{u} \|\Delta \tilde{u}\|_{x} \leq r \right\}\).

\[4\|\quad J_{n+1}\|_{x}\|Y_{eq}\|_{x}\|Y_{eq}\|_{x} \leq 1
\]

**Proof:** According to infinite norm definition, it yields

\[\|f(\tilde{u})\|_{x} = \|\quad -JI_{n+1}\|_{x} + \|\quad J\Delta \tilde{u} \quad J^{-1}\Delta \tilde{u}\|_{x} \leq \|\quad J_{n+1}\|_{x} + \|\quad J\Delta \tilde{u} \quad J^{-1}\Delta \tilde{u}\|_{x}
\]

\[= \|\quad J_{n+1}\|_{x} + \|\quad Y_{eq}^{-1}P\Delta \tilde{u} \quad P^{-1}Y_{eq}\Delta \tilde{u}\|_{x} \leq \|\quad J_{n+1}\|_{x} + \|\quad Y_{eq}^{-1}\|_{x}\|Y_{eq}\|_{x}\|\Delta \tilde{u}\|_{x}
\]

Since \(\Delta \tilde{u} \in S\), then (16) is obtained

\[\|f(\tilde{u})\|_{x} \leq \|\quad J_{n+1}\|_{x} + \|\quad Y_{eq}^{-1}\|_{x}\|Y_{eq}\|_{x}\|\Delta \tilde{u}\|_{x}^{2} \leq r
\]
To solve (16), according to the formula of roots of a quadric equation, the following can be obtained

\[ 1 - 4\|Y_{eq}\|_c \|Y_{eq}\|_c \|J_{m,\alpha}\|_c \geq 0 \] (17)

It is noted that when (17) holds, (16) is solvable. Thus, when Theorem 1 holds, \( f(\tilde{u}) \) is as self-mapping. Then according to Lemma 1, when (14) satisfies, the system admits a solution in \( s \).

4. Simulation Verification
In order to confirm the correctness of the derived existence condition, a DC microgrid containing 20 Droop-DGs, 20 MPPT-DGs and 30 CPLs is simulated. (which is shown in figure 2) The Droop-DGs, MPPT-DGs and CPLs are denoted as red, yellow and blue points. Green numbers in figure 2 show the values of cables’ resistances (these values are given in \( \Omega \)), while black numbers present the identifiers of sources and loads. Simulations are performed by using MATLAB. The droop gain of Droop-DGs are designed as \( k_1 = k_2 = \ldots = k_{20} = 2 \).

![Figure 2: The topology of the meshed DC Microgrid.](image)

Let \( \alpha = 4\|J_{m,\alpha}\|_c \|Y_{eq}\|_c \|Y_{eq}\|_c \). According to Theorem 1, if condition (13) holds, i.e., \( \alpha < 1 \), the power-flow equation for DC microgrid considering voltage disturbance has a solution. Next, two cases are designed to verify the proposed result. It is noted that if condition (13) holds, then \( f(\tilde{u}) \) has a solution, otherwise, \( f(\tilde{u}) \) may have no solution.

Case 1: The maximum value of MPPT-DGs and CPL are as \( P_M = [10 \times 1_{10}^T, 15 \times 1_{10}^T, 20 \times 1_{10}^T, 1]\), \( P_L = [10 \times 1_{10}^T, 15 \times 1_{10}^T, 20 \times 1_{10}^T, 1]\). Take \( u_{ref} = 5057V \), then \( \alpha = 0.9997 < 1 \).

Case 2: The maximum value of MPPT-DGs and CPL are as large as the value of case 1. Take \( u_{ref} = 4233V \), then \( \alpha = 1.4267 > 1 \).

By applying MATLAB, we get the corresponding iterative process in figure 3. Figure 3(a) indicates that the solution is convergent when condition (14) holds. Figure 3(b) shows that if condition (14) is not satisfied, the solution may not be convergent. Thus, the correctness of Theorem 1 is verified.
5. Conclusion

In this paper, the existence of the feasible power-flow solution of the DC microgrid considering voltage disturbance and MPPT-DGs is analyzed. Firstly, a mathematical power-flow equation of the DC microgrid with CPLs is investigated. Next, we obtain an existence condition of power-flow equation considering voltage disturbance and MPPT-DGs based on Brouwer's fixed-point theorem. Finally, the effectiveness of the proposed condition is verified by using simulations.

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