Dense Pruning of Pointwise Convolutions in the Frequency Domain

Mark Buckler, Neil Adit, Yuwei Hu, Zhiru Zhang, and Adrian Sampson

Computer Systems Lab, Cornell University, Ithaca, NY, USA
markabuckler@gmail.com, {na469, yh457, zhiruz, asampson}@cornell.edu

Abstract

Depthwise separable convolutions and frequency-domain convolutions are two recent ideas for building efficient convolutional neural networks. They are seemingly incompatible: the vast majority of operations in depthwise separable CNNs are in pointwise convolutional layers, but pointwise layers use $1 \times 1$ kernels, which do not benefit from frequency transformation. This paper unifies these two ideas by transforming the activations, not the kernels. Our key insights are that 1) pointwise convolutions commute with frequency transformation and thus can be computed in the frequency domain without modification, 2) each channel within a given layer has a different level of sensitivity to frequency domain pruning, and 3) each channel’s sensitivity to frequency pruning is approximately monotonic with respect to frequency. We leverage this knowledge by proposing a new technique which wraps each pointwise layer in a discrete cosine transform (DCT) which is truncated to selectively prune coefficients above a given threshold as per the needs of each channel. To learn which frequencies should be pruned from which channels, we introduce a novel learned parameter which specifies each channel’s pruning threshold. We add a new regularization term which incentivizes the model to decrease the number of retained frequencies while still maintaining task accuracy. Unlike weight pruning techniques which rely on sparse operators, our contiguous frequency band pruning results in fully dense computation. We apply our technique to MobileNetV2 and in the process reduce computation time by 22% and incur <1% accuracy degradation.

1. Introduction

Since the advent of convolutional neural networks for vision, computational efficiency has been a primary concern. A broad swath of techniques have successfully decreased the time and space required for CNN inference, such as model compression and pruning [6, 10], frequency-domain computation [14], and depthwise separable convolutions [4]. The resulting highly efficient networks are particularly relevant to industrial deployments of vision, especially in embedded and mobile settings where energy is a scarce resource.

High-efficiency CNNs have recently trended toward pointwise $1 \times 1$ convolutions because of the high cost of larger kernels. SqueezeNet [10], replaced many $3 \times 3$ kernels in a traditional architecture with $1 \times 1$ kernels, and depthwise separable CNNs replace traditional $3 \times 3$ convolutions with a combination of pointwise convolutions and depthwise $3 \times 3$ convolutions. The recent proliferation of depthwise separable CNNs [4, 9, 18, 26, 16] has demonstrated the effectiveness of this approach.

A concurrent but independent research direction has focused on weight pruning as a mechanism for compressing CNNs, based on absolute magnitude of parameters [6] and frequency-domain based compression [1, 14]. However, weight pruning typically results in a sparse model which can be counterintuitively slower than the dense computations found in unpruned models. A recent study found, that even pruning 89% of AlexNet’s [12] weights resulted in a 25% slowdown [25].

While depthwise separable networks and frequency-based weight pruning both reduce parameter volume and computational cost, they are not trivially compatible. The $1 \times 1$ filters that dominate computation in these architectures are not amenable to a frequency-domain transform because they contain no spatial context, and the $3 \times 3$ filters account for an order of magnitude less computation—so pruning them will have a negligible impact on overall performance. The central goal in this paper is to properly combine these two trends by using frequency-domain computation and pruning to speed up the costly pointwise convolutions. In doing so, we make the following observations:

- Pointwise convolutions, the primary computational bottleneck in modern efficient CNNs, can be computed in the frequency-domain without modification.
• Certain frequency coefficients for certain channels can be pruned, resulting in fewer necessary operations.
• Any given channel’s sensitivity to frequency pruning is approximately monotonic with respect to frequency.

These observations inspired our contributions:
• Channel–frequency band pruning: A new technique which prunes contiguous frequency coefficients above a given frequency threshold by wrapping pointwise convolutions with Discrete Cosine Transforms (DCT).
• FCMask: A new parameter which can be used to learn the level of channel–frequency band pruning per-layer and per-channel during training time, short for Frequency Contiguous Mask.

2. Related Work

Our work builds upon and combines three lines of work on making CNN inference efficient: one that develops CNN architectures that can be trained from scratch for small model sizes and fast inference; one that lowers model sizes of an already trained network by pruning weights; and one that uses frequency-domain methods to compute CNNs more efficiently, mostly by leveraging the convolution theorem. In this section, we describe these three lines of work and explain how they relate to our contributions.

Efficient CNN architectures. This paper builds on a line of work on designing CNN architectures that support efficient inference. One of the earliest of these architectures was SqueezeNet [10], which achieved accuracy comparable to a state-of-the-art CNN (AlexNet [12]) with substantially fewer weights. MobileNet achieved an even greater reduction in computation by leveraging depthwise separable convolutions (which were first introduced in another parameter-efficient architecture, Xception [4]). According to the authors, “Our model structure puts nearly all of the computation into dense 1 × 1 convolutions... MobileNet spends 95% of its computation time in 1 × 1 convolutions which also has 75% of the parameters” [9]. Since then, several other architectures have been built with similar structure, including MobileNetV2 [18], ShuffleNet [26], ShuffleNetV2 [16], and ResNeXt [24]. EfficientNet [19], EfficientDet [21], and EfficientNet2 [20] expanded on these architectures by proposing unified scaling methodologies for increasing network scale and demonstrating effectiveness when used for object detection and semantic segmentation.

Weight pruning. A second approach to efficient inference is pruning in which some of the signals in a neural network are removed [3]. This can be done at the channel granularity [15, 8, 29, 7, 23] or the weight granularity [6]. While channel pruning results in dense computation, weight pruning results in sparse computation. Pruning at either level of granularity results in a smaller model size, but the sparse nature of weight pruning methods typically results in a significant increase in computation time unless extremely high levels of pruning are possible [25].

Frequency Domain CNNs. One well-known method to perform convolutions in the frequency domain is to leverage the Fourier transform’s convolution theorem [17]. This method applies the DFT to a padded convolutional kernel and the input image, element-wise multiplies each frequency component, and then applies the inverse DFT to the result. While this method can significantly reduce the total number of operations required, its benefit relies on large kernel sizes to amortize the cost of the DFT. A closely related approach is to apply weight pruning in the frequency domain [14]. This technique suffers from the same sparsity problem seen in typical weight pruning and is also inapplicable to the pointwise convolutions which dominate depthwise separable CNNs. Another related technique in the literature is OctConv [2]. Similar to this work, OctConv observes that channels within a layer experience heterogeneous frequency sensitivity. Rather than applying a frequency transformation, however, OctConv subsamples the subset of channels which are less sensitive to high frequencies. Our DCT-based approach offers a finer level of granularity in the frequency domain than OctConv’s subsampling approach.

3. Methodology

This section describes our technique for computing pointwise convolutions in a DCT-based frequency space, how pruning can be applied in the frequency domain, and then how this pruning can be learned with standard back-propagation.

3.1. Pointwise Frequency Domain Convolution

Our approach to frequency-domain computation is to compress activation data using a similar approach as JPEG image compression [22]: we divide each channel’s activations into small, fixed-size, square tiles called macroblocks and apply the discrete cosine transform (DCT) to macroblock. The key observation is that running a 1 × 1 convolution on this transformed data is equivalent to running it on the original activation:

\[ \text{Conv}_{1 \times 1}(X) = \text{IDCT}(\text{Conv}_{1 \times 1}(\text{DCT}(X))) \]
where \(X\) is an activation tensor, DCT is the macroblocked discrete cosine transform, and IDCT is the inverse transformation. Therefore, we can insert DCT and IDCT “layers” before and after every pointwise convolution and obtain an equivalent network.

Just like JPEG, our technique is sensitive to macroblock size. Where JPEG uses macroblocks to localize frequency information to specific locations within the image, in our setting macroblock size represents a trade-off between the precision of frequency-domain pruning and the computational overhead from frequency transformation. Larger macroblocks result in more coefficients per macroblock which offers more precision in the frequency space and thus leads to better pruning potential. On the other hand, the DCT forms the primary overhead cost for our technique and the computational cost scales quadratically with the size of the macroblock. In our experiments, we find that \(3 \times 3\) macroblocks work well across both networks we evaluate.

We apply the DCT to each channel separately. To implement the DCT, each macroblock is multiplied by frequency filters (of size \(k^2\) where \(k\) is the kernel width) for each coefficient (of which there are \(k^2\)). With this implementation of the DCT, the number of required multiply-accumulate operations is:

\[
\text{DCT-MACs} = c_{in} \times \frac{h}{k} \times \frac{k}{w} \times k^4 = c_{in} \times h \times w \times k^2
\]

Processing the same input with a pointwise convolution with a stride of 1 would require the following operations:

\[
\text{Pointwise-MACs} = c_{in} \times h \times w \times c_{out}
\]

To apply the inverse DCT, each coefficient (of which there are \(k^2\)) is multiplied by a kernel (of width \(k^2\)) to transform back to the spatial domain.

\[
\text{IDCT-MACs} = c_{out} \times \frac{h}{k} \times \frac{k}{w} \times k^4 = c_{out} \times h \times w \times k^2
\]

Because our technique transforms to and from the DCT domain for each pointwise convolution that is pruned, our overhead consists of the operations required to do this transformation. The ratio of overhead operations to baseline pointwise operations is the following for each layer:

\[
\frac{\text{DCT-MACs} + \text{IDCT-MACs}}{\text{Pointwise-MACs}} = \frac{(c_{in} + c_{out}) \times k^2}{c_{in} \times c_{out}}
\]

From this overhead equation we can see that the larger \(c_{in}\) and \(c_{out}\) are the smaller the transformation overhead is with respect to number of baseline operations. Typically, the number of channels increases with each subsequent layer within a CNN and thus the overhead for our technique is smaller for later layers than earlier layers.

### 3.2. Frequency Band Pruning

Unlike techniques that use the convolution theorem, transforming activations with the DCT does not save any computation by itself—its purpose is to enable pruning in the frequency domain. We evaluate different pruning techniques shown in Figure 1 building up to our proposed channel–frequency band pruning.

**Channel pruning:** The method we visualize in Figure 1a removes entire channels and then sorts them such that all non-zero channels are contiguous. This technique is equivalent to traditional channel pruning methods. This technique prunes on a coarse level since it does not take frequency coefficients into account. However, this allows for an easy channel reordering to allow dense computation.

**Coefficient pruning:** This approach prunes entire coefficients, independent of the layer’s output channels. Figure 1b depicts a coefficient pruning mask: all channels preserve the same set of “important” frequencies. Coefficient pruning can reduce both the cost of the pointwise convolution and the overhead associated with the DCT and inverse-DCT computations as it is not necessary to produce frequency coefficients that have been pruned. Our technique truncates the DCT to produce only the unpruned coefficients. While this approach results in dense computation, it also expresses the incorrect assumption that all output channels are equally sensitive to each frequency in the activation data. We find that channels are not uniformly sensitive to frequencies in practice, which motivates a more nuanced approach.

**Channel–coefficient pruning:** Channel–coefficient pruning removes the assumption that all channels within a layer are uniformly sensitive to each frequency. Instead, it ac-

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Figure 1: All pruning strategies for frequency-domain 1 \(\times\) 1 convolutions considered in this paper. The frequency axis is according to the output of the 2D DCT, vectorized according to the zigzag pattern commonly associated with JPEG [22]. Each figure depicts a possible pruning mask for each technique with 50\% pruning. Gray areas are preserved (nonzero) and white areas are pruned (zero).
knows that some channels need different frequency information than others. Figure 1c depicts the result: each channel receives a subset of the frequency coefficients.

We find that this per-channel sensitivity significantly increases the amount of pruning that is possible for a given accuracy budget. However, the additional flexibility comes at a cost: the “random” effect of the pruning means that the computation must be sparse. It is not generally possible to reorder channels and coefficients to pack them into a dense tensor for fast computation.

**Channel–frequency band pruning:** Our final approach extends frequency–coefficient pruning to remove contiguous ranges of frequencies (bands), thereby recovering dense computation. The key idea relies on the insight from prior work that CNNs are generally more sensitive to low frequencies than to high frequencies [1, 13, 14]. Put differently, coefficient importance is approximately monotonic with frequency—so if a given coefficient is pruned, all the higher-frequency coefficients are also likely to be pruned. **Channel–frequency band pruning** restricts pruning to a contiguous range of the highest frequencies according to the unique needs of each channel. Figure 1d shows a possible mask: each channel receives a contiguous range of frequencies starting at the lowest frequency.

With this strategy, each channel’s computation in the 1×1 filter is dense. We find that this technique prunes nearly the same set of channels as unrestricted channel–coefficient pruning: significant frequencies naturally tend to be nearly contiguous. So this technique loses only a few pruned coefficients while making computation dense and therefore far more efficient.

### 3.3. Learned Pruning

Channel, coefficient, channel–coefficient, and channel–frequency band pruning each represent different methods for representing coefficient/channel masks, but how exactly to set the levels for each mask is non-trivial.

A profiling-based selection on a pretrained network was intractable for an exhaustive search. To overcome the shortcomings of the profiling-based approach we developed a new method to learn the best mask for each coefficient using backpropagation. We created a new learnable parameter called the *frequency contiguous mask* (or FCMask) which represents the level of pruning for each channel while also maintaining frequency contiguity. To apply pruning, each coefficient in each channel is multiplied by a coefficient mask value derived from the FCMask value for that channel.

As Figure 2 shows, each coefficient’s mask is a function of the corresponding channel’s FCMask. When FCMask is set to 0.0 all coefficient masks in the channel are equal to 0.0 (removing all data from the channel entirely) and when FCMask is set to 1.0 all frequency masks are equal to 1.0 (no pruning is applied). FCMasks are initialized to 1.0 and are influenced to decrease by our modified loss function. Consider the case where a channel’s FCMask gradient decreases the value of FCMask from 1.0 to 0.9. Coefficient masks 0-2 will remain at 1.0, but frequency mask 3 will slightly decrease in value. Because during inference each coefficient value is multiplied by its corresponding coefficient mask, the decrease in the coefficient mask will decrease the impact that this individual coefficient has on the network as a whole. It is this ability to make small adjustments to the impact of each coefficient that enables this learning technique to succeed, since rounding all masks to 0 or 1 during training would prevent gradients from propagating through the network. After training is completed, all coefficient masks are rounded and fixed to 0 or 1 so that we can save on computation.

The modified loss function which influences FCMask values to be decreased can be seen below:

$$
\text{loss} = \text{cross}\_\text{entropy} + \lambda \times \sum_{l=0}^{\text{num\_layers}} \text{avg}(|\text{FCMask}_l|)
$$

We preserve a cross entropy loss component to maintain the accuracy of the classification task that our model is trained for. We also introduce a new metaparameter, $\lambda$, to control the degree of impact that the new loss component will have on the network as a whole. The primary regularization value in this loss function is a sum taken over all layers of the average absolute value for FCMasks over all channels in each layer. Smaller FCMask values (increased pruning) decrease the loss function and are thus rewarded. On the other hand, masking coefficients may increase the cross entropy if these coefficients are important to the network. We
Figure 3: FCMask values for all channels (sorted from high FCMask to low) in the first pointwise layer of Block 10 of MobileNetV2 using a 3×3 macroblock size. The horizontal lines demarcate the regions of FCMask values which pertain to each of the nine coefficients. The solid horizontal lines indicate a transition between diagonal rows in the DCT, and the dashed horizontal lines indicate a transition within a diagonal row.

Figure 4: Three equivalent pointwise convolutions: typical, frequency domain, separate frequency bands

3.4. Dense Frequency-Domain Computation

The primary motivation for our pruning technique over traditional pruning is that our method enables dense computation while still requiring fewer operations than an unpruned convolution. Figure 4(a) shows a standard pointwise layer, and (b) shows a mathematically equivalent operation which applies the DCT before and IDCT after the pointwise operation. The DCT and IDCT masks in Figure 4(b) shows that not all coefficients are needed for all channels, but (b) would require expensive sparse computation to avoid computing the pruned coefficients.

Figure 4(c) shows our proposal: group coefficients into frequency bands and then compute the pointwise convolution on each frequency band separately. The baseline pointwise convolution would need to multiply all weights by all coefficients in each input channel to produce all coefficients for all output channels. In contrast, dividing the computation into frequency bands offers the ability to only process the channels that pertain to the frequency band of interest. In the example shown in Figure 4(c), frequency band 0 only has non-zero coefficients from channels $c_0 - c_y$ at the input, and only produces non-zero output for channels $\hat{c}_0 - \hat{c}_y$. Similarly, frequency band 1 only has non-zero coefficients from channels $c_0 - c_x$ at the input, and only produces non-zero output for channels $\hat{c}_0 - \hat{c}_x$. By reducing the total volume of input and output data for each pointwise convolution we have saved a significant amount of computation while remaining entirely dense.

4. Evaluation

We evaluate the effectiveness of our frequency band activation pruning on two state-of-the-art networks, MobileNetV2 [18] and ResNeXt [24]. We begin by performing experimentation on the CIFAR-10 [11] classification dataset, and then apply the best performing technique to MobileNetV2 when trained on ImageNet [5]. We evaluate computation savings in two ways. First, we measure the number of multiply–accumulate (MAC) operations among all convolutional layers. Second, we measure the wall-clock time to perform all network computation when performing MobileNetV2 on ImageNet.
Table 1: High-level CIFAR-10 results for top-1 accuracy degradation (after refinement training) and computational savings

|          | ResNeXt | MobileNetV2 |
|----------|---------|-------------|
| Acc Degradation | 0.13% | -0.07% |
| MAC Reduction   | 2.4×  | 2×         |

**4.1. CIFAR-10**

This section contains our preliminary experiments to determine the effectiveness of each of the different frequency domain pruning methodologies. Figure 5 and Figure 6 show the accuracy vs pruning level tradeoff, although these sweeps do not include retraining after pruning. Table 1 contains the retraining results after pruning ResNeXt and MobileNetV2 with the most effective technique: learned channel-frequency band pruning.

In the CIFAR-10 baseline, ResNeXt uses 7.7×10^8 MACs and MobileNetV2 uses 8.9×10^7. We add DCT and inverse-DCT transformation layers to each using 4×4 macroblocks, which increases the MAC counts to 8.4×10^8 for ResNeXt and 1.1×10^8 for MobileNetV2. The top-1 CIFAR-10 accuracy for the baseline, unpruned configuration is 94.2% for MobileNetV2 and 95.75% for ResNeXt.

Table 1 summarizes the results from our evaluation for our preferred pruning technique, per-channel frequency band pruning. This technique achieves 2–2.4× computational savings, as measured by the number of multiply–accumulate (MAC) operations, with reductions in top-1 accuracy below one fifth of a percent. These numbers result from first training each network from scratch, learning frequency coefficients to prune while keeping the original network weight constant, and then refining each network with another round of training.

We evaluate each of the pruning techniques in Section 3.2 by sweeping a threshold that masks out a given percentage of operations for each transformation layer. These new pruning techniques are coefficient pruning, channel–coefficient pruning, and per-channel frequency band pruning. We also show the learned technique for per-channel frequency band pruning. We compare against two baselines: channel pruning and reducing the input resolution.

**4.1.1 ResNeXt**

In ResNeXt, the 1×1 convolutions account for 85% of all MACs in the network. The large number of channels per layer (64–1024) means that the DCT overhead is proportionally smaller when compared with MobileNetV2. Figure 5 shows the accuracy for each pruning method at different levels of pruning by relating the resulting MAC reduction with the network’s top-1 CIFAR-10 accuracy. Channel pruning performs very poorly compared with all other techniques, demonstrating that the frequency dimension is important. Also, all frequency-based techniques outperform a simple reduction in input resolution (input_res in the figure), suggesting that it is valuable to remove some but not all of the high-frequency data in the image.

Channel–coefficient pruning, being the most granular pruning technique performs the best compared to other profiling based pruning methods. As Section 3 discusses, this fine pruning granularity comes at the cost of sparse computation.

Fortunately, the final technique, per-channel frequency band pruning, performs nearly as well while yielding dense computation. Below 2× MAC savings, the two techniques are nearly identical. The similarity between unrestricted channel–coefficient pruning and the frequency band equivalent validates our claim that sensitivity is monotonic with frequency. With unrestricted channel–coefficient pruning at a pruning percentile of 30% (1.8× MAC savings), we measure that 98.8% of channel masks in all layers of ResNeXt were already contiguous.

The learning based technique applied to the channel-frequency band pruning technique performs the best. This is achieved by adding our FCMask regularization to the training loss, allowing heterogeneous pruning across layers without significantly impacting the task accuracy after retraining. The learned technique far exceeds the performance of even channel–coefficient pruning when profiling.

**4.1.2 MobileNetV2**

We apply our technique to MobileNetV2 to examine its effects in the context of a network that is already highly efficient. Figure 6 shows our techniques’ performance along-
Figure 6: Comparison of pruning methods for MobileNetV2 on CIFAR-10 (before retraining).

side baseline techniques. The highest degree of per-channel frequency band pruning achieves a 1.8× reduction in total network MACs. With retraining, the network exceeds the baseline top-1 accuracy by 0.05%.

As the figure’s x-axis scale reveals, MobileNetV2 is less robust to pruning overall. The network has already been heavily compressed by design, leaving less opportunity for removing ineffectual computations. However, per channel coefficient pruning and per channel frequency band pruning maintain their record as the best available pruning methods, and all frequency based methods again outperform reducing input resolution.

In Figure 6, coefficient pruning starts with 0.86× MAC savings—i.e., it uses more MACs than the baseline. In this configuration, the 4×4 DCT overhead is significant in the context of a network with fewer channels per layer (16–320). Neither channel pruning nor input resolution changes require DCT transformations, so they both incur no MAC overhead.

In contrast, channel–coefficient pruning and its frequency band variant start with fewer MACs than the baseline, yielding about 1.2× savings in their initial configurations. These initial savings arise because, after applying the DCT, many of the frequency coefficients are zero already and require no thresholding. In fact, the output of the first 1×1 layer and the input to the second 1×1 layer of each residual block exhibit 40–65% sparsity in the frequency domain. This high degree of sparsity arises from the sparsity of the depthwise 3×3 filters themselves when in the frequency domain. Unlike the grouped 3×3 filters of ResNetXt, where any of the input kernels in a group may be sensitive high frequencies and therefore cause the output activations to express high frequencies, each depthwise convolutional layer’s output is only a function of one input channel and one kernel. This increased granularity in the kernel space improves our ability to prune.

The learning based technique outperforms the other profiling based techniques, specifically at speedups greater than 2×. This is consistent with ResNetXt results on CIFAR-10 but the effect is even more significant with MobileNetV2 since it is a much smaller network.

4.2. ImageNet

Table 2 summarizes our results for MobileNetV2 evaluation on ImageNet comparing with other state-of-the-art models which compress the same network resolution to comparable pruning levels. Our method is able to prune the MobileNetV2 model to similar levels of 205M MACs while achieving the best top-1 accuracy.

Figure 7 compares our technique’s ability to reduce MAC operations on varying resolutions of MobileNetV2 when compared with other methods with varied pruning levels. The figure shows the accuracy and operation count
for multiple configurations of MobileNetV2 which forms a Pareto curve. We compare our technique to recent channel pruning models: ThiNet [15], Discrimination-aware Channel Pruning (DCP) [27], Automatic Model Compression (AMC) [7] and Pruning from Scratch (PS) [23]. We also plot OctConv [2] which introduces a convolution operator using a combination of low- and high-resolution feature maps. While OctConv is not strictly a pruning method, it does leverage features’ heterogeneous sensitivity to different frequencies via multi-resolution feature maps. The blue pareto curve is the baseline MobileNetV2 architecture with varying input resolution and network width scaling. The violet curve represent points with different input resolution and varying pruning levels of our learned technique. After learning the frequency pruning, we train the network weights from scratch to allow channels to rearrange weights based on the learned frequency importance. We typically see an improvement of 1–2% in top-1 accuracy compared to a simple refinement strategy. We used a $3 \times 3$ macroblock for the 192 × 192 resolution and a combination of $2 \times 2$ and $7 \times 7$ macroblocks for 224 × 224 resolution for our ImageNet experiments.

Our technique compares favorably with ThiNet, DCP, AMC, PS channel compression, and OctConv as our technique unifies the idea of pruning and tuning channel sensitivity to frequency. Our channel–frequency band pruning can both prune entire channels and reduce spatial redundancy through frequency pruning. The comparison to OctConv is not straightforward since our technique does offer more frequency pruning precision than simple subsampling, but this additional precision comes at the computational cost of frequency transformation. Figure 8 shows the impact our transformation overhead for each layer. The earlier layers have higher resolution input activations and the later layers have much higher number of channels, both of which can be effectively compressed by our learned pruning technique. Since we use a smaller $2 \times 2$ macroblock for the layers up to block 13, we get less pruning than the later layers with $7 \times 7$ macroblocks. However the overhead is higher for the layers with $7 \times 7$ macroblocks, evident by the gap between the two lines. This tension between macroblock sizes and overhead along with the hyperparameter tuning of the loss function allows us to learn fine-grained pruning levels as an independent scaling dimension to input resolution and width multiplier.

### 4.3. Wall-Clock Speedup

The purpose of dense pruning in the frequency domain is to offer speedups in actual execution time, not just reduction in total operation count, over the baseline network.

We implement the (1.0-192-0.80) MobileNetV2 configuration from Figure 7 in TVM. The 20% FLOP compression is achieved by only applying pruning to those layers where the pruned $1 \times 1$ operations + transform overhead is smaller than the baseline FLOPs. We measured the time taken with and without our technique on an AWS c5.18xlarge instance with Xeon Platinum 8124M CPU and 140G DRAM, running on single thread. When we apply our technique resulting in the 20% FLOP reduction, the resulting network is 22.1% faster, while incurring <1% accuracy degradation.
5. Implementation Details

Our ResNeXt CIFAR-10 model uses a cardinality of 32 and a bottleneck width of 4, while our MobileNetV2 CIFAR-10 model uses a width multiplier of 1.0. We built and trained our networks using PyTorch and four Nvidia GTX Titan X GPUs. Training hyperparameters are the same as those used in the original papers.

6. Conclusion

This paper does two things that at first seem paradoxical: it prunes CNNs without introducing sparsity, and it applies frequency-based methods to save computation on kernels with no spatial context. Beyond simply saving computation in theory we have achieved real-world speedup.

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A. Appendix

A.1. CIFAR-100 experiments

In the CIFAR100 baseline, MobileNetV2 uses $4.3 \times 10^7$ MACs. We add DCT and inverse-DCT transformation layers using $4 \times 4$ macroblocks, which accounts for 20% overhead and increases the total MAC count to $5.4 \times 10^7$. The top–1 accuracy for MobileNetV2 unpruned baseline configuration on CIFAR100 is 67.92%.

| MobileNetV2          | Top–1 Acc | MAC Reduction |
|----------------------|-----------|---------------|
| (Baseline 67.92%)    | 67.04%    | 3.77×         |

Table 3: High-level CIFAR-100 results for top-1 accuracy (after refinement training) and computational savings

Table 3 summarizes the results from our evaluation for per-channel frequency band pruning. This technique achieves $3.77\times$ computational savings, as measured by the number of multiply–accumulate (MAC) operations, with less than a percent of top–1 accuracy reductions. These numbers result from first training each network from scratch, learning frequency coefficients to prune while keeping the original network weight constant, and then refining each network with another round of training.

We also experimented different learning schemes for MobileNetV2 on CIFAR-100 dataset. CIFAR-100 enabled us to do fast experiments while not overtraining on small datasets like CIFAR-10. The two learning methods we tried are:

- Learn mask followed by weight refinement (baseline): We freeze weights and learn masks for the trained network. Once we achieve the desired pruning levels, we freeze the pruning and refine the weight to improve accuracy.

- Alternate between learning mask and refining weights: This method alternates between learning masks and refining weights in each epoch. We can allow the network to prune more aggressively while maintaining accuracy.

Figure 9 shows the Top–1 accuracy on CIFAR-100 using the two training methods for different pruning levels resulting in the overall speedup. While running experiments we noticed that it was easier to train for higher pruning levels with alternate training method, since it allowed refinement in regular intervals and did not allow the network accuracy to tank. This is evident from the graph as alternate training performs better at higher speedups. The slight decrease in accuracy for alternate training for lower speedups can be attributed to freezing the batchnorm values while training and refining. This was rectified during Imagenet experiments by allowing batchnorm to finetune toward the end.

Figure 10 shows the different hyperparameter tuning results for MobileNetV2 on CIFAR-100. We observe that the developer can choose different learning rates based on the desired speedup/pruning level.

Figure 11 shows different levels of fine-grained frequency pruning existing in the network and the progression of learning with training iterations. We can see that Figure 11b has much more channels which are completely pruned away compared to Figure 11a, which is evident from the fact that the former has a much higher MAC reduction than the latter.
(a) MobileNetV2 trained on CIFAR-100 with 2.27 × MAC reduction  (b) MobileNetV2 trained on CIFAR-100 with 5.83 × MAC reduction.

Figure 11: Learning to prune weights in the frequency domain for MobileNetV2. The coefficients represent pruning starting from that level till the highest frequency coefficient. Since we use 4 × 4 macroblocks, we have 16 coefficients. "coef0" represents the channels which prune away all frequency components and "coef16" represents those which do not prune anything.