Heavy Flavors on The Lattice

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ABSTRACT

The lattice QCD results on the hadron spectrum and weak transitions between hadrons are briefly reviewed. Hadrons containing heavy quarks $c$ or $b$ are considered. The focus is on the recent simulations and some older results which are particularly relevant in view of the recent experimental discoveries.

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1 Introduction

The study of hadron properties requires a non-perturbative method since the strong coupling constant at hadronic energy is not small. Lattice QCD is an ab-initio non-perturbative method based directly on the QCD Lagrangian with parameters $m_{q_i}$ and $g_s$. An expectation value of a desired quantity $C$ is obtained via numerical path integration $\langle C \rangle \propto \int DUDq_i Dq_i e^{-S_{QCD}} C$ formulated on a discretized and finite Euclidean space-time.

Here I focus on the lattice results reported between the summer 2016 and the LHCP conference held in mid-May 2017. Some older lattice results are also reported which are particularly relevant in view of the very recent experimental discoveries.

2 Spectroscopy and two-hadron scattering

Excited charmonia, charmed and charmed-strange mesons: The most extensive spectra of the excited charmonia, $D$ and $D_s$ have been calculated Hadron Spectrum Collaboration. Several complete quark-antiquark multiplets $nL$ were found in a recent simulation with $m_π \simeq 240$ MeV: see Figs. 3, 4, and 5 in [1]. Multiplets of hybrid states were also found and some of them carry exotic $J^{PC}$. Light-quark mass dependence of charmonia in comparison to earlier results at $m_π \simeq 400$ MeV is found to be mild. Most of these states, particularly those with $J = 3, 4$ or exotic $J^{PC} = 1^{--}, 0^{++}, 2^{++}, \ldots$ have yet to be discovered experimentally. The main caveat of this calculation is that it disregards strong decays of resonances and threshold effects, which is remedied for some states in what follows.

Strongly decaying hadron resonances and hadron-hadron scattering: The hadronic resonances with masses above threshold strongly decay to a pair of hadrons $H_1 H_2$. In the recent years, the main effort was to simulate $H_1 H_2$ scattering channels on the lattice and extract the underlying scattering matrix $T(E)$ as a function of energy. This is possible thanks to the rigorous Lüscher’s formalism [2] (see [3] for introduction on the topic). The scattering matrix $T(E)$ renders the $H_1 H_2$ cross-section $\sigma(E) \propto |T(E)|^2$, which in principle allows lattice QCD determination of the resonance masses and decay widths from the peaks in the cross-section.

Charmed resonances from $Dπ - Dη - D_s K$ coupled channel scattering: Hadron Spectrum Collaboration extracted $3 \times 3$ scattering matrix for three coupled channels at $m_π \simeq 400$ MeV and searched for poles in the scattering matrix [4]. The resonance pole was found in $d$–wave scattering, which closely resembles experimental $D_2$ with $J^P = 2^+$. The scalar $D_0^*$ state was found as a bound-state pole on the real-axes almost on $Dπ$ threshold due to the heavy $m_π$ employed in the simulation [4]. Experimentally scalar $D_0^*(2400)$ meson is a wide resonance above $Dπ$ threshold; it emerged as a wide resonance in a less detailed lattice simulation of $Dπ$ scattering in the one-channel approximation at $m_π \simeq 266$ MeV [5].

$Z_c(3900)$ from coupled channel $J/ψπ - D ¯D^*$ - $η, ρ$ scattering: In order to search for the exotic $Z_c(3900)$ state with flavor $\bar{c}d\bar{u}$, the HALQCD collaboration extracted $3 \times 3$ scattering matrix for three coupled channels with $J^P = 1^+ [6]$. Less rigorous HALQCD approach was applied, which has not been verified on conventional resonances. The resulting differential ratios as a function of $J/ψπ$ and $D ¯D^*$ invariant masses indeed show a peak around 3.9 GeV, resembling experimental peak. If the coupling between $J/ψπ - D ¯D^*$ channels is set to zero by hand, the peak disappears. The HALQCD results therefore indicate that $Z_c(3900)$ peak is possibly a coupled channel effect rather than a genuine resonance.

Search for $X(5568)$ in $B_c π^+$ scattering: The $X(5568)$ state with exotic content $t s d u$ was recently claimed by D0 collaboration [7]. If this state with $J^P = 0^+$ exists, it can strongly decay only to $B_c π^+$ and lies significantly below all other thresholds, which makes a lattice search for $X(5568)$ cleaner and simpler than for other exotic candidates. The simulation of $B_c π^+$ scattering did not find $X(5568)$ [8], in agreement with the LHCb [9] and CMS [10] results.
**Doubly bottom $BB^*$ and $B_sB^*$ bound states:** Several lattice simulations provide a growing evidence for a strongly-stable state with flavor $bbud$, $J^P = 1^+$, $I = 0$ and mass $m < m_B + m_{B^*}$. The simulation [11 12] was based on the static $b$-quark, while [13] employed NRQCD for $b$-quark. Such a state, if bound by $m_B + m_{B^*} - m = 189 \pm 10$ MeV [13], decays only weakly to $u\overline{u}d\overline{d} \rightarrow B^+\overline{T}^0$, $J/\psi B^+K^0$. The indication for a strongly-state strange partner $B_sB^*$ with $m < m_{B_s} + m_{B^*}$ was also found [13], with expected weak decays to $B^+D_s^-$, $J/\psi B_sK^0$, $B_s\overline{T}^0$, $J/\psi B^0\phi$.

**Excited $\Omega_c^*$:** The extensive excited $\Omega_c^*$ spectrum with $css$ valence content was predicted on lattice in 2013 [14 15], disregarding their strong decays. Five states were found in the energy region $3.0 - 3.2$ GeV, in agreement with LHCb discovery this year. The lattice calculation predicts their quantum numbers: two carry $J^P = 1/2^-$, two carry $3/2^-$ and one $5/2^-$. The new preliminary results for $B_c\rightarrow c\tau\tau$ were presented by S. Hashimoto [25]. The squared amplitude $|M|^2 = |V_{cb}|^2 G_B^2 M_B \bar{b}^\mu\nu W_{\mu\nu}$ based on $H = V_{cb} \frac{G_F^2}{\sqrt{2}} f_{\tau}(1 - \gamma_5)\bar{c}\mu(1 - \gamma_5)\nu J^\mu$ (where $J^\mu \equiv \tau\bar{\tau}(1 - \gamma_5)$) $b = V^\mu - A^\mu$.
contains trivial leptonic part $l^{\mu\nu}$, while the hadronic part

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_x) \frac{1}{2M_B} \langle B(p_B) | J_\mu^A(0) | X \rangle \langle X | J_\nu(0) | B(p_B) \rangle$$

contains a sum over all on-shell final states $X = D, D^*, \ldots$. Instead of summing those explicitly, one can obtain the sum directly by considering forward scattering matrix element

$$T^{\mu\nu} = i \int d^4x \ e^{-ix\cdot\omega} \frac{1}{2M_B} \langle B | T \{ J_\mu^A(x) J_\nu(0) \} | B \rangle$$

which is related to desired $W$ via the optical theorem $2 \text{Im} M(B \to B) = \sum_X \int d\Pi_X |M(B \to X)|^2$ (see for example Section 18.5 of Peskin) as $W = -\frac{1}{\pi} \text{Im} T$. The $T$ can be computed on the lattice from the four-point function

$$T^{\mu\nu}_{\mu\nu}(\omega, \vec{q}) \propto \int_0^\infty dt \ e^{\omega t} \int d\vec{x} \ e^{i\vec{q}\cdot\vec{x}} \langle 0 | B(\vec{p}_B=\vec{0}, t_{snk}) J_\mu^A(\vec{x}, t_2) J_\nu(0, t_1) B^\dagger(\vec{0}, t_{src}) | 0 \rangle$$

where $\omega$ denotes the energy of the final state $X$, $J$ contains $V$ or $A$ parts and various normalizations factors have been omitted for simplicity.

The exploratory numerical simulation has been done on JLQCD configurations for the spectator $s$ quark $B_s \to X_c \pi\pi$ at zero recoil $\vec{q} = \vec{p}_B - \vec{p}_X = \vec{0}$ and for $b$-quark mass smaller than physical. The resulting $T_{\mu\nu}(\omega, \vec{q} = \vec{0})$ as a function of $\omega$ is shown in Fig. 10 of [25]. The $V_0V_0$ part is found to be dominated by $D_s$ pole (dashed lines) and represents another way to extract of HQET form factor $h_+(1)$ via $T^{VV}_{00} = \frac{|h_+(1)|^2}{M_{D_s}^2 - \omega}$. The $T^{AA}_{11}(\omega, \vec{0}) = \frac{|h_{A1}(1)|^2}{M_{D_s^*}^2 - \omega}$ is dominated by $D_s^*$ pole. The HQET form factors determined in this way are consistent with direct calculation but have currently larger errors. The $V_1V_1$ and $A_0A_0$ contain contribution from $X_c$ final states with other quantum numbers.

**New exclusive $q \to q' \pi\pi$ and $q \to q'\tau^+\tau^-$ baryon decays:** Six $\Lambda_c \to \Lambda \pi\pi$ form factors were determined [26] for the first time in view of the BESIII 2015 measurement of this decay rate. Taking $V_{cs}$ from global CKM fit, these form factors lead to the rate that is consistent, and twice more precise as BESIII rate. This gives confidence in lattice treatment of this and analogous electroweak baryon transitions. Alternatively, the form factors lead to the value of $V_{cs}$, which is currently less precise, but consistent with the one from $D_s \to \pi\pi$.

The $b \to sl^+l^-$ transition and their ratios (relevant for the lepton flavor violation) have attracted large attention recently in view of few tensions between Standard model predictions and experiment. On this front, there is only one new lattice result available since summer 2016. This is a report on the ongoing simulation of $\Lambda_b \to \Lambda^* (1520) l^+l^-$ form factors [24]. The $\Lambda_b \to \Lambda^* l^+l^-$ has namely certain advantages with respect to the more standard $\Lambda_b \to \Lambda l^+l^-$ decay where $\Lambda$ is neutral and long-lived, which is not favorable for the accurate experimental study. The unstable $\Lambda^*$ resonances, one the other hand, immediately decay into charged particles and produce tracks that originate from $b$-decay vertex, which motivates exploring decays also through unstable $\Lambda^*$ in experiment.

### 4 Conclusions

Lattice QCD is a reliable ab-initio non-perturbative method that is based directly on the fundamental theory of strong interactions - QCD.

Spectra of strongly stable hadrons ($B, D, \ldots$) are well under control and in agreement with experiment. Variety of exclusive electro-weak transitions between them are being studied with increasing precision. The formalism and the first exploratory lattice results for the inclusive weak transitions $B \to X_c \pi\pi$ have been presented during the last year.

Experiments provided lots of interesting and puzzling hadrons in the recent years. The five excited $\Omega_c^* \simeq css$ states, discovered by LHCb this year, have been predicted by lattice QCD around 2013. There is a
growing evidence for a strongly-stable state with flavor $\bar{b}ud$, $J^P = 1^+$, $I = 0$ and mass $m < m_B + m_{B^*}$ from the lattice. Most of the experimentally discovered exotic hadrons are actually strongly decaying resonances that decay to a single or to multiple-channels. They have to be inferred from the one-channel or multiple-channel scattering matrix. It is encouraging that scattering matrices for single-channel scattering have been reliably extracted for certain channels in the recent years. Scattering matrices for two-channel and three-channel scattering have also been extracted in some cases. One study along these lines indicates that the experimental $Z_c^+(3900)$ peak arises due to the large coupling between the $D\bar{D}^*$ and $J/\psi\pi^+$ channels. Lots of exciting and challenging problems along these lines still remain to be attacked.

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