Concerning modeling of double-stage water evaporation cooling

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Abstract. The matter of need for setting technical norms for production, as well as acceptable microclimate parameters, such as temperature and humidity, at the work place, remains urgent. Use of certain units should be economically sound and that should be taken into account for construction, assembly, operation, technological, and environmental requirements. Water evaporation coolers are simple to maintain, environmentally friendly, and quite cheap, but the development of the most efficient solutions requires mathematical modeling of the heat and mass transfer processes that take place in them.

1. Introduction
The printed works [1, 2] present the principles of operation of the direct cooling units, demonstrate the development of the ratio defining the energy balance in the air cooler channels. The basics of the indirect cooling are stated in the printed work [3]. Various ways and principles of the indirect cooler operations are presented in [4–8]. The analysis shows that under low humidity of the ambient air, there is the reserve of humidity at the exit of the indirect cooling unit, as the air does not change its moisture concentration. So, one can additionally cool the air by means of the direct cooling unit. To develop the most efficient designs of the coolers, there is the need for mathematical modeling of the heat and mass transfer processes that take place in the air cooler fittings of indirect and direct cooling units in succession.

2. Results and Discussion
The simplest designs of the water evaporation conditioners are based on the direct cooling principle. The cooled air flow is directed into the evaporation fitting channels. A reduction of its temperature takes place due to the water evaporation from the wetted surface of the plates under the direct contact. In case of direct cooling, the mathematical model of the heat and mass transfer in the water evaporation channel with the cross-section of \( H=2h \) and with length \( L \) (Fig. 1) is described by the known differential equations in partial derivatives of parabolic type describing the energy and mass transfer:

\[
\rho \cdot V(x, y) \cdot C \cdot \frac{\partial t}{\partial x} = \lambda \frac{\partial^2 t}{\partial y^2},
\]

\[
V(x, y) \cdot \frac{\partial w}{\partial x} = \frac{\partial}{\partial y} \left( D \frac{\partial w}{\partial y} \right),
\]
\[ w(x, y) = \varphi(x, y) \cdot w_n(t), \]
\[ w_n(t) = (0.0004212r^3 + 0.001831r^2 + 0.4195r + 4.727) \cdot 10^{-3}, \]
where \( V \) – air velocity, m/s, which was calculated from the formula for the streamline flow mode,

\[ V(x, y) = V_{in} \left[ 1.5 - \frac{1.5y^2}{h^2} - 2 \sum_{n=1}^{\infty} \left( \frac{\cos \left( \frac{g_n y}{h} \right)}{g_n} \right) \exp \left( \frac{-4g_nVx}{v_{in}h^2} \right) \right], \]

where \( g_n \) – positive roots of equation \( \tan x = x \); \( v \) – kinematic air viscosity, m\(^2\)/s; \( h \) – half of the channel cross-section, m; \( V_{in} \) – input air flow velocity, m/s. Due to the necessity for definition of the plate surface temperature, let us add the equation for heat spread within the plate to the other ones, which can be classified as a differential equation on second-order partial derivatives of elliptic type:

\[ \frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} = 0, \quad x \in (0, L), \quad y \in (0, H_p). \]

Also the plate end impermeability conditions are added:

\[ \left. \frac{\partial T_p}{\partial x} \right|_{x=0} = 0, \quad y \in (0, H_p), \quad \left. \frac{\partial T_p}{\partial x} \right|_{y=L} = 0, \quad y \in (0, H_p). \]

Evenness conditions on the channel and plate symmetry axis are:
\[ \frac{\partial t}{\partial y} \bigg|_{y=-h} = 0 \quad , \quad \frac{\partial w}{\partial y} \bigg|_{y=-h} = 0 \quad , \quad \frac{\partial T_p}{\partial y} \bigg|_{y=H_p} = 0 \quad , \quad x \in (0,L) . \]

Matching conditions are:

\[ t \bigg|_{y=0} = T_p \bigg|_{y=0} , \quad x \in (0,L) . \]

On the porous plate surface:

\[ \varepsilon RD \frac{\partial w}{\partial y} = \lambda_{pl} \frac{\partial T_p}{\partial y} - \lambda \frac{\partial t}{\partial y} \quad , \quad y = 0 \quad , \quad x \in (0,L) , \]

\[ D = 10^{-5} \cdot 2.16 \cdot (1 + t / 273)^{1.8} , \quad w \bigg|_{y=0} = w_n \left( T_p \right) . \]

The system is closed by the initial conditions on the channel input:

\[ t \bigg|_{x=0} = t_{in} , \quad \varphi \bigg|_{x=0} = \varphi_{in} . \]

Here \( C, \rho, \lambda, D \) – correspondingly, average heat capacity at constant pressure, J/(kg \cdot K); density, kg/m\(^3\); heat capacity, W/m, of the steam-gas mixture; diffusion factor for the steam-gas mixture, m\(^2\)/s. \( w \) and \( w_n \) – correspondingly, density and steam saturation density, kg/m\(^3\); \( t \) and \( \varphi \) – correspondingly, temperature and relative cooled air humidity; \( T_p \) and \( \lambda_{pl} \) – correspondingly, temperature and heat conductivity of the plate; \( R \) – specific evaporation heat, J/kg; \( \varepsilon \) – coefficient for the energetic component taking into consideration the additional energy spent for evaporation from the porous plate surfaces.

In the earlier works of one of the authors, the heat and mass transfer model was realized by the sweep method for implicit schemes of finite-difference equations. This was possible due to the fact that the equations being the part of it were of parabolic type. The model presented above includes elliptic equations in addition to the parabolic ones, which eliminates the opportunity of application of the said method.

The authors propose the following method of implementation of the suggested mathematical model. Let us divide half of the evaporation fitting channel cross-section with the length of \( L \) and the width of \( h \), correspondingly, into \( Nx \) and \( Ny \) parts. The obtained grid forms \( 2(Nx+1)(Ny+1) \) junctures, for which the authors enter the difference analogues of the proportions forming the model.

Similarly let us address the half of the evaporation fitting plate cross-section with the length of \( L \) and the width of \( H_p \). Let us attribute the numbers to the junctures as integer index \( j = 0...Nx \) along the channel and the plate, and \( i = 0...Ny \) - across the channel and the plate. Thus, one obtains \( (Nx+1)(Ny+1) \) junctures of the grid in the plate with its unknown temperature and \( (Nx+1)(Ny+1) \) junctures of the grid in the channel with unknown temperature steam density. Taking into account that in the input to the channel, the temperature and steam density are set, one gets \( 3Nx Ny+3Nx Ny+3 \) unknown variables. Accordingly, the same number of linear finite-difference equations will form the system.

The algorithm of this model realization is as follows. At the first stage, the values of the diffusion factor and saturated density are calculated based on the formulae listed above for the temperature of the input air. The set of equations is solved and the temperature and steam density field is found for the grid juncture points. Then, based on the calculated temperature field, let us define the diffusion factor value for each juncture, and the saturated steam density value for the plate surface, and after that the system is solved again. This iteration process is over when the temperature at the cooler output
differs from the temperature of the previous iteration by less than 0.1°C.

The obvious deficiency of the direct action cooler is excessive moistening of the air resulting in the limitation of the operation areas. More complex coolers are based on the indirect cooling principle. Cooling of the main air flow in them takes place due to moisture evaporation into the auxiliary air flow and heat transfer through the plates forming the evaporation fitting. Unlike the direct evaporation, in this case, the capillary properties of the plate material start playing their roles as well as their thickness and heat conductivity.

Unlike the direct evaporation, in modeling of the indirect cooling processes, one should take into consideration the transverse thermosensitive resistance of the evaporation plates and the fact that the plate’s symmetry axis is not its temperature field’s symmetry axis. The evaporation fitting channels during the indirect cooling are divided into two essentially different groups (Fig. 2). The first group includes the ‘wet’ channels, through which the auxiliary air flow of temperature \( t \) passes and comes into contact with the wet surfaces of capillary porous plates. This flow becomes saturated with evaporated water steam and then is driven outside the cooled volume with temperature \( t_{out} \) and relative humidity \( \varphi_{out} \). The second group is the ‘dry’ channels through which the main air flow with temperature \( T \) passes. These channels are protected against capillary porous plates with water resistant

![Figure 2. A segment of the evaporation fitting](image)

film (shown as the dark line) and do not come into contact with water. The main air flow passes through the channels without any change of its moisture content and gets into the cooled volume having temperature \( T_{out} \).

The mathematical model of the heat and mass transfer process is similar to the direct evaporation model, but also includes the energy equation in the ‘dry’ channels:

\[
\rho_c \cdot V_c(x,y) \cdot C_c \cdot \frac{\partial T}{\partial x} = \frac{\partial}{\partial y} \left( \lambda_c(T) \frac{\partial T}{\partial y} \right), \quad x \in (0, L), \ y \in (Hp, Hp+H).
\]

Input conditions for the ‘dry’ air are:

\[
T \big|_{x=0} = T_{in}, \ y \in (Hp, Hp+H) \quad \text{for the direct flow case.}
\]
Evenness conditions for the ‘dry’ channel symmetry axis are:

\[
\frac{\partial T}{\partial y} \bigg|_{y=H_p+H} = 0 , \quad x \in (0,L) .
\]

Plate end impermeability conditions are:

\[
\frac{\partial T_p}{\partial x} \bigg|_{x=0} = 0 , \quad y \in (0,H_p) , \quad \frac{\partial T_p}{\partial x} \bigg|_{x=L} = 0 , \quad y \in (0,H_p) .
\]

Matching conditions are:

\[
T_y \bigg|_{y=H_p} = T_p \bigg|_{y=H_p} , \quad x \in (0,L) , \quad \lambda_c(T) \frac{\partial T}{\partial y} = \lambda_{pl} \frac{\partial T_p}{\partial y} , \quad y = H_p , \quad x \in (0,L) .
\]

Here the index ‘\(c\)’ means corresponding factors for dry air. The algorithm of this model implementation is generally the same as the algorithm used for direct cooling.

Implementation of the described mathematical model allows calculation of the temperature and humidity change in the gas and steam mixture in the heat transfer fittings of the water evaporation coolers under the influence of a number of factors, including the physical properties of the plates, ambient air parameters, and dimensions of the direct and indirect blocks. The variation of the unit dimensions for a certain type of ventilator block is the key way for improvement of the cooler unit operation efficiency, as it is impossible to affect the ambient air temperature and humidity. In the course of calculations, the authors obtained the dynamics of the air flow temperature change along the evaporation fitting channels, which is shown in fig. 3. One can see from the figure that under the certain length of the indirect block auxiliary channels, the maximum depth of air flow cooling is achieved. The further increase of the channel length is useless, as it does not result in the increase of the cooling depth. So, the length of the auxiliary block can be reasonably limited to that value. Similarly, it is possible to define the channel length for the direct block, which produces the maximum cooling depth. The authors also managed to evaluate the efficiencies of operation of the water evaporation coolers based on the
different principles. Fig. 4 illustrates the obtained functions of the cooling depths in the ambient air temperature for the direct, indirect, and double-stage coolers. As one can see from the figure, double-stage coolers produce the maximum cooling depth compared to the direct and indirect units, which proves the preferable choice of such cooling system [10].

3. Conclusion
So, the use of the suggested mathematical model allows assessing the efficiency of operation of the water evaporation coolers of various types and selection of the dimensions of the evaporation fittings for the direct and indirect blocks for the double-stage water evaporation air cooling units.

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