External-Feedback Laser Cooling of Gases

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We analyze the laser cooling of polarizable particles by continuous dispersive position detection and active feedback. The magnitude of the dissipative force is proportional to the particles’ photon scattering rate into the detector, while its velocity dependence is determined by the programmable frequency dependence of the loop gain. The method combines final temperatures near the recoil limit with large velocity capture range, and is applicable to multilevel atoms or molecules.

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Low-temperature, high-brightness sources of particles are of interest in many areas of physics. In particle accelerators, stochastic cooling has been a key technology to increase collision rates [1]. In a single-particle picture, an electrical signal that depends on a particle’s motion generated in one part of the accelerator ring is used to apply a correction force further downstream. In view of Liouville’s theorem, which forbids compression of phase space volume by conservative forces, the cooling of a finite-number sample has been explained as an expulsion of the empty phase space between occupied points [1]. Feedback cooling has been applied to other systems including a resonant gravity gradiometer [2], a single mirror vibration mode via radiation pressure [3, 4], and a single axially oscillating electron in a Penning trap [5]. Feedback concepts have been further extended to the quantum domain [6, 7, 8, 9, 10]. In order to increase the cooling bandwidth, an optical version of stochastic cooling has been proposed for accelerators [10].

For atomic samples, Raizen et al. have proposed to optically measure fluctuations of the center-of-mass momentum, and to apply momentum kicks with laser-induced dipole forces [11]. Balykin and Letokhov have suggested a cooling scheme based on velocity measurement and optical feedback [12]. In cavity experiments, single atoms have been trapped by applying feedback [14, 15], while in an optical lattice, a sample’s center-of-mass oscillation has been damped [10].

In this Letter, we analyze the laser cooling of a gas of polarizable particles by continuous dispersive position measurement and optical feedback. In particular, the sign and velocity dependence of the dissipative force are determined exclusively by the sign and frequency dependence of the loop gain. The force is proportional to the radiation pressure associated with photon scattering into the detector, but is otherwise independent of the target level structure. We derive closed-loop heating rates due to light and photodetector quantum noise, and due to the thermal motion of the sample. Using an optical resonator for signal enhancement in combination with nearly shot-noise-limited photodetection, a large variety of atomic or molecular samples can be cooled to temperatures near the photon recoil energy.

In the following we adopt the single-particle viewpoint of stochastic cooling [1], where the signal generated by a single atom is propagated through the linear feedback loop, and the thermal motion of the other atoms in the sample constitutes a source of heating. Consider an atom of mass $m$ moving as $x_a = vt$ at constant velocity $v$ in a weak periodic potential $U(x, t) = U(t) \cos 2kx$ whose depth $U(t) \ll m v^2 / 2$ is influenced by the atom’s motion. Since the time variation of the force on the atom $f(t) = 2k U(t) \sin 2kv t$ arises both from the spatial variation $\cos 2kx$ of the potential, and from the dependence of the depth $U(t)$ on the atom’s trajectory, a frequency component of $U(t)$ in phase with the atomic-motion-induced force variation $\sin 2kv t$ produces a non-zero average force that can heat or cool the atom. To establish this type of dissipative system, we propose to use a standing light wave to both monitor the position $x_a(t)$ of the polarizable atom by means of its index of refraction [12, 13, 15], and to generate a periodic potential of adjustable depth $U(t)$ via the light shift.

Viewed in the time domain, the atom is cooled because it finds itself moving uphill on a deeper potential than experienced when moving downhill (see also Refs. [17, 18]). Of course, this is only true for an appropriately chosen transfer function mapping $x_a(t)$ onto $U(t)$. In the frequency domain picture, both the moving atom and the feedback actuator modulate the light wave at the Doppler frequency $\nu k$. Under appropriate conditions, amplitude and phase modulation from both processes interfere to produce asymmetric sidebands in the light exiting the system. If the blue Doppler sideband is stronger, the atom will be cooled at the rate at which energy is carried away by the frequency-shifted light. The atom-light interaction, and consequently the cooling force, can be increased by means of an optical resonator (Fig. 1).

The resonator of finesse $F$ and field decay rate constant $\gamma_c$ supports a standing-wave Gaussian $TEM_{00}$ mode of wave number $k$ with waist $w$. Incident light produces an electric field of amplitude $2E_c$ at an antinode ($x=0$) inside the resonator. An atom with complex polarizability $\alpha$ moving on the resonator axis coherently scatters photons into free space at an average rate $\Gamma_{sc} = k^3 |\text{Re}(\alpha)| E_c^2 / (6\pi \epsilon_0 \hbar)$, and experiences an op-
tional potential \( U_0 \cos 2kx \) of depth \( U_0 = -|E_\infty|^2 R e(\alpha)/2 \). Conversely, the forward-scattered radiation by the spatially varying induced atomic dipole, or equivalently, the atom’s index of refraction, results in an atomic-position-dependent detuning \( \delta_{at} = \zeta \gamma c \cos kvt_s \) of the resonator from its average resonance frequency \( \omega_c \). Here \( \zeta = h \eta \Gamma_{sc}/U_0 \) is a dimensionless parameter characterizing the atom-cavity coupling, and the quantity \( \eta = 6F/(\pi k^2 w^2) \) can be interpreted as the fraction of photons scattered into one direction of the resonant cavity.

For incident light detuned by an amount \( \delta_1 = c k - \omega_c \) relative to the resonator, the time-varying detuning \( \delta_{at}(t) \) of the resonator by the moving atom changes the resonator transmission by a fraction \( \varepsilon(t) \). The transmitted power is measured and used as an error signal in a feedback loop to adjust the incident power by a fraction \( -\varepsilon(t) \). In order to ignore cavity-induced forces \[17, 18\], we take the resonator linewidth \( 2\gamma_c \) to be much larger than both the feedback bandwidth and the Doppler frequency \( 2kv \), such that the intracavity power \( P_c(t) \) adjusts instantaneously to a value determined by the total light-resonator detuning \( \delta_1(t) = \delta_1 - \delta_{at}(t) \). Then the fractional deviation \( \varepsilon(t) = P_c(t)/P_0 - 1 \) of the intracavity power from its unperturbed value \( P_0 \) (for \( \delta_{at} = \varepsilon(t) = 0 \) ) is given by

\[
\varepsilon(t) = \frac{\gamma^2 + \delta_1^2}{\gamma^2 + \delta_1^2}(1 - \varepsilon(t)) = 1. \tag{1}
\]

Assuming \( |\delta_{at}|/\gamma_c, \varepsilon(t) \ll 1 \), we can linearize the fractional change \( \varepsilon(t) \) in resonator transmission,

\[
\varepsilon(t) = r \gamma^{-1}\delta_{at}(t) - \varepsilon(t). \tag{2}
\]

In this approximation, the moving atom modulates the intracavity power by an amount \( r \delta_{at}/\gamma_c \) proportional to the normalized resonator slope \( r = 2\delta_1/\gamma_c (\gamma^2 + \delta_1^2) \), while the feedback loop adjusts the incident power by a fraction \( -\varepsilon(t) \). If the atom’s kinetic energy far exceeds the light shift \( U_0 \), then to lowest order \( \delta_{at}(t) \) is determined by the atom’s unperturbed motion \( x_a = vt \). We introduce the open-loop feedback gain \( H(s) \) for the Laplace transformed quantities \( \varepsilon(t), \bar{S}^q(s) \) via \( \varepsilon(t) = H(s) \bar{S}^q(s) \). Then \( H(\omega) = H_1(\omega) + iH_2(\omega) \), with real and imaginary parts \( H_1 \) and \( H_2 \), respectively, is the complex gain in the frequency domain. If the loop is stable, the steady-state solution in the time domain is given by

\[
\varepsilon(t) = r \zeta (1 + H_1(2k\nu) \cos 2kvt + H_2(2k\nu) \sin 2kvt) / [1 + H(2k\nu)^2]. \tag{3}
\]

\( H_1(2k\nu) \) and \( H_2(2k\nu) \) are the open-loop gain in phase and in quadrature with the atomic-motion-induced intensity modulation \( \delta_{at} = \zeta \gamma_c \cos 2kvt \), respectively. They determine the closed-loop signal \( \varepsilon(t) \), and thereby the time variation of the optical-potential depth \( U(t) = U_0(1 + \varepsilon(t)) \). In the limit \( U_0 \ll m\nu^2/2 \), work is done on the atom at a rate \( \dot{W} = \varepsilon(t) f_a(t) v \) to lowest order, where \( f_a = 2kU_0 \sin 2kvt \) is the unperturbed force. The component \( 2kve(t) \) of \( \varepsilon(t) \) in phase with \( f_a \) produces a dissipative force, whose spatial average \( f = \langle \dot{W} \rangle / v \) can be written as

\[
f(v) = h\kappa n_\Gamma_{sc} \Gamma v H_2(2k\nu)(1 + H(2k\nu)^2). \tag{4}
\]

This expression, derived without invoking the two-level or rotating-wave approximations, is valid for arbitrary laser detuning from atomic resonances below saturation. The friction force \( f \) depends on target parameters exclusively through the scattering rate \( \Gamma_{sc} \) and is controlled by the average intracavity intensity. \( f \) is proportional to the rate of photon momentum transfer \( 2bh\kappa n_\Gamma_{sc} \) due to backward scattering into the resonator at rate \( \eta \Gamma_{sc} \), multiplied by a dimensionless function of the atomic velocity. In particular, the sign and velocity dependence of \( f \) are determined by the frequency-dependent loop gain \( H(i\omega) \). They are independent of the atom’s level structure, such that atoms in different internal states, or different species, can be cooled simultaneously. Cooling occurs if the product of the resonator line slope \( r \) and the quadrature loop gain \( H_2(2k\nu) \) at the Doppler frequency \( 2k\nu \) is negative. The force is maximized at a resonator-light detuning \( \delta_1 = \pm \gamma_c \) that gives the largest slope \( r = \pm 1 \).

In the following we assume \( \delta_1 = -\gamma_c \) (\( r = -1 \)), such that \( H(s) > 0 \) corresponds to negative feedback.

The cooling force \( f \) is proportional to the quadrature component \( \sin 2kvt \) of the intracavity light modulation in closed loop, given by \( H_2/|1 + H|^2 \). In particular, for very small or very large open-loop gain \( |H|^2 = H_1^2 + H_2^2 \ll 1 \) or \( |H|^2 \gg 1 \), this relevant quadrature of the intensity variation, and hence \( f \), will be small. The velocity-dependent term takes on its maximum value \( 1/(2 + 2H_1) \) when the quadrature gain \( H_2 \) and the in-phase gain \( H_1 \) are related by \( |H_2| = |1 + H_1| \). When \( 1 + H_1 \ll 1 \), the feedback loop regeneratively amplifies the intensity variation caused by the moving atom, so that the force is increased by a factor \( 1 + H_1^{-1} \gg 1 \) compared to the case of small in-phase gain \( |H_1| \ll 1 \). However, the heating of the atom due to noise amplification is then also increased, as we show below.

The simplest stable cooling loop is a differentiator \[15\], \( H_d(\omega) = i\omega/(2k\nu) \), with unity gain frequency \( 2k\nu \). (The
The maximum force $\hbar k \eta \Gamma_{sc}$, attained for the unity-gain velocity $v=\eta$, is the same as in Doppler cooling at the photon scattering rate $\eta \Gamma_{sc}$, but the differentiator force, falling off as $1/v$ rather than $1/v^3$, provides a substantially larger velocity capture range than the Doppler force. Fig. 2 also shows that higher-order loops can substantially further increase the velocity capture range while maintaining the low-velocity friction coefficient $\alpha_0=\partial f/\partial v$ that determines the final temperature. $\alpha_0$ can be increased in real time simply by increasing the loop gain $H$, i.e. by reducing $u$. We also note that a loop $H(s)=(s/\Gamma')(1+(s/2\Gamma'))$ reproduces the velocity dependence of the Doppler force \(^7\) for a (fictional) transition with linewidth $2\Gamma'<2\gamma_c$.

Technical or quantum noise in the light-induced potential $U(t)$ will heat an atom moving at velocity $v$ in proportion to the spectral noise density at the modulation frequency $2kv$ of the unperturbed force. We define the single-sided fractional spectral noise density $S_c(\omega)=(2/\pi)\int_0^\infty dt \langle \hat{\varepsilon}(0)\hat{\varepsilon}(t) \rangle \cos \omega t$, normalized such that $\int_0^\infty d\omega S_c(\omega)=\langle \hat{\varepsilon}^2(0) \rangle$ is the mean-square fractional intensity noise, and calculate the heating power due to momentum diffusion \(^9\),

$$\bar{W}_{\text{fluct}} = \frac{\pi k^2}{m} U_0^2 S_c(2kv).$$

(6)

This general formula describes the heating of an atom moving at velocity $v$ due to classical intracavity intensity noise. For quantum noise, Eq. (6) needs to be multiplied by a factor of two to take into account fluctuations of the induced atomic dipole \(^9\). Assuming negligible technical noise, the shot noise of the light incident on the cavity with power $P_i$ corresponds to a fractional power spectral density given by $S_i^{\text{sn}}=\hbar k/(P_i\pi)$. In closed loop, the intracavity noise density is $S_c^{\text{sn}}(\omega)=S_i^{\text{sn}}/|1+H(i\omega)|^2$ for $\omega << \gamma_c$. In addition, shot noise in the detector photocurrent at limited quantum efficiency $q < 1$ arising from the undetected photons will produce a false error signal, causing the feedback loop to generate (classical) intracavity intensity noise. This photoelectron detection noise corresponds to an uncorrelated fractional noise density of magnitude $S_{\text{esn}}(\omega)=S_i^{\text{sn}}(q^{-1}-1)$ in open loop, and $S_{c\text{sn}}=S_{\text{esn}}(1+H)(1+H')^2$ in closed loop. The total shot noise from these two sources $S_{c\text{sn}}=S_{\text{esn}}+S_{c\text{sn}}$, expressed in terms of the intracavity power $P_c=P_i/(2\pi)$ for the detuned cavity with $|\delta|<<\gamma_c$, is given by

$$S_{c\text{sn}}(\omega) = \frac{F\hbar ck}{2\pi^2 P_c} \frac{1+|H(i\omega)|^2(q^{-1}-1)}{|1+H(i\omega)|^2}.$$ 

(7)

Eq. (6) then yields a closed-loop heating rate $\bar{W}_{\text{sn}}$ due to shot noise

$$\bar{W}_{\text{sn}} = E_r \eta \Gamma_{sc} \frac{2+|H(2ikv)|^2(q^{-1}-1)}{|1+H(2ikv)|^2},$$

(8)

where $E_r=\hbar^2 k^2/(2m)$ is the recoil energy. We see that $\bar{W}_{\text{sn}}$ can be expressed as the recoil heating rate $2E_r \eta \Gamma_{sc}$, multiplied by a dimensionless function that depends on the loop gain $H$ and the detector quantum efficiency $q$. In the absence of feedback ($H=0$), Eq. (8) reproduces the well-known recoil heating \(^9\) due to scattering into the resonator, where each backscattering event, occurring at a rate ($\eta/2\Gamma_{sc}$) for the detuned cavity, heats the atom by an amount $4E_r$. For unity quantum efficiency ($q=1$) the feedback loop reduces (increases) this heating by suppressing (enhancing) the intracavity fluctuations for $|1+H| > 1$ ($|1+H| < 1$). For $q < 1$, the feedback loop also generates intensity noise originating from random photoelectron detection. In addition to the cavity heating $\bar{W}_{\text{sn}}$, scattering into free-space heats the atom at a rate $\bar{W}_{fs}=2E_r \Gamma_{sc} \hbar k/v$ \(^9\).

For the differentiator loop $H_d(i\omega) = i\omega/(2kv)$, a unity-gain velocity $u=(q^{-1}-1+2\eta^{-1})\hbar k/m$ minimizes the temperature $T_d$, yielding

$$k_B T_d = \frac{4E_r(1+\eta)(q^{-1}-1+2\eta^{-1})}{\eta}.$$ 

(9)

where $k_B$ is Boltzmann’s constant. Thus for good atom-cavity coupling, $\eta = 1$, the final temperature is limited.
by the finite detection efficiency to $8(1+q^{-1})E/\kth$. This implies that except for the lightest atoms and molecules, microkelvin temperatures can be reached even for limited detector quantum efficiency $q$.

For a thermal sample consisting of $N+1$ atoms at temperature $T$, any chosen probe atom will be heated by the intracavity intensity noise induced by the other $N$ randomly moving atoms. The corresponding closed-loop spectral noise density $S_N^\gamma(\omega)$, expressed in terms of the sample’s thermal velocity $\vth=(\kth T/m)^{1/2}$, is

$$S_N^\gamma(\omega) = \frac{N\gamma^2}{8\pi\vth} \exp\left(-\frac{\omega^2/(8\kth \vth^2)}{1 + H(\omega)^2}\right), \quad (10)$$

resulting in a collective heating rate $\dot{W}_N$ for an atom with velocity $v$ given by

$$\dot{W}_N(v) = \frac{E_\gamma \Gamma_{sc}}{1 + H(2\kth v)^2} \frac{\sqrt{2\pi N \eta \Gamma_{sc}}}{2\kth} \exp\left(-\frac{v^2}{2\vth^2}\right). \quad (11)$$

As is typical of stochastic cooling, the heating overwhelms the cooling at too large atom number $N$ or too large cooling speed, i.e. photon scattering rate $\Gamma_{sc}$. For a differentiator loop, we find that the net cooling power averaged over the thermal sample, $\langle \dot{W}_d \rangle = (\dot{W}_d + \dot{W}_N)$ is maximized for $u \approx \vth$. The cooling rate constant, $\gamma_d = -2(\dot{W}_d)/(\kth T)$ can then be written in the form

$$\gamma_d \approx \frac{\kth \vth}{6N} (2\Gamma_{cav} - \Gamma_{cav}^2) \quad (12)$$

where $\Gamma_{cav} = 2N\eta \Gamma_{sc} \kth/(\kth T)$ is the total scattering rate into the cavity normalized to the sample temperature. The cooling rate is maximized for $\Gamma_{cav} = 1$, yielding a rate constant $\gamma_d = \kth \vth/(6N)$. We see that the thermal Doppler broadening $\kth$ takes the role of the stochastic-cooling bandwidth [1]. And that the cooling of smaller subsamples can proceed faster [11]. For a sample of $N=10^8$ magnetically trapped CaH molecules at 0.4 $K$ cooled with light of wavelength 760 nm, the optimum cooling rate is $\gamma_d = 0.1$ s$^{-1}$, attained at a photon scattering rate $\eta \Gamma_{sc} = 3 \times 10^6$ s$^{-1}$. (Here we are assuming that the different degrees of freedom are mixed [1], e.g., by the trapping potential.) An ensemble of $10^8$ molecules at room temperature can be cooled at a rate $\gamma_d = 340$ s$^{-1}$ for a photon scattering rate of $\eta \Gamma_{sc} = 3 \times 10^6$ s$^{-1}$. Eq. 12 implies that the differentiator-induced cooling of a thermal beam containing $N$ atoms with light of wavelength $\lambda$ requires a characteristic length $L = 6N\lambda/\pi$, independent of the initial velocity of the beam, since the cooling bandwidth $\kth$ is larger for a faster beam.

It may seem surprising that an error signal proportional to the atom’s index of refraction generates a cooling force that is proportional to the photon scattering rate into the detector. The reason is that the index of refraction arises from the superposition of the forward-scattered field with the much larger incident field. The dispersive signal can thus be viewed as a homodyne method to measure the small forward-scattered field. Homodyne detection, however, cannot be used to improve over the shot-noise limit resulting from direct detection of the scattered photons. The role of the resonator is to enhance the detection signal-to-noise ratio [12], or equivalently, the photon scattering rate $\eta \Gamma_{sc}$ into the detector. In the absence of the resonator, $\eta$ is replaced by the detection solid angle $\Delta \Omega = 3/(k w^2)$, resulting according to Eq. 10 in much higher temperatures. This may explain why stochastic cooling has not been observed in free-space experiments using laser beams with large waist $w \gg k^{-1}$ for detection [13][16].

In conclusion, we have derived simple analytical expressions for the cooling and heating of a gas interacting with a laser beam whose intensity is adjusted in response to the atoms’ motion. If a resonator is used for signal enhancement in combination with photodetection near the quantum limit, microkelvin temperatures can be reached. By appropriately designing the frequency-dependent loop gain, large friction coefficients can be combined with a very large velocity capture range. The friction force scales with the photon scattering rate by the target atoms, but is otherwise independent of the target level structure. Therefore the proposed method represents a promising route for laser-cooling new atomic or molecular species.

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