Non-linear absorption and motional effects in Rydberg EIT-media

Martin Gärtnert\textsuperscript{1,2} and Jörg Evers\textsuperscript{1}

\textsuperscript{1}Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
\textsuperscript{2}Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany

(Dated: May 8, 2013)

Light propagation through an ensemble of ultra-cold Rydberg atoms in electromagnetically induced transparency (EIT) configuration is studied. In strongly interacting Rydberg EIT media, non-linear optical effects lead to a non-trivial dependence of the degree of probe beam attenuation on the medium density and on its initial intensity. We develop a Monte Carlo rate equation model that self-consistently includes the effect of the probe beam attenuation to investigate the steady state of the Rydberg medium driven by two laser fields. We compare or results to recent experimental data and to results of other state-of-the-art models for light propagation in Rydberg EIT-media. We find that for low probe field intensities, our results match the experimental data best if a density-dependent dephasing rate is included in the model. At higher probe intensities, our model deviates from other theoretical approaches, as it predicts a spectral asymmetry together with line broadening. These are due to off-resonant excitation channels, which however have not been observed in recent experiments. We interpret these results as signatures for atomic motion and effects beyond the frozen gas approximation. At resonant driving with low probe intensity, the motion is consistent with the required additional density-dependent dephasing. At higher probe intensities and off-resonant driving, the motion renders off-resonant excitation channels ineffective, suppressing the spectral asymmetry.

I. INTRODUCTION

Electromagnetically induced transparency (EIT) in Rydberg gases has been the subject of intense studies both theoretically \cite{1,2,3,4} and experimentally \cite{5,6,7,8,9,10,11,12,13} in the recent years. One motivation is to achieve strong interactions between photons by interfacing them with interacting states of matter. In particular, based on the excitation blockade, non-classical states of light can be prepared out of an initially classical driving field \cite{16,17}. Possible applications include deterministic single photon sources, storage and retrieval of photons, as well as quantum gates based on photon-photon interactions. However, already the simulation of classical light propagating through a strongly interacting medium is a substantial theoretical challenge due to the high complexity of the underlying many-body physics. At the heart of this is the exponential complexity of the quantum many-body problem of interacting 3-level atoms and the non-linearity and non-locality of the propagation equations of the light related to the long range interactions.

Various approaches using different approximations have been pursued to tackle light propagation through Rydberg EIT media. Sevinçli \textit{et al.} \cite{5} derived an analytical expression for the third order optical non-linearity based on the cluster expansion approach \cite{14}. This approach yields interesting results for moderate atomic densities but the cluster expansion is expected to break down at high densities \cite{15}. In the weak probe regime, where the probe field consists only of a few photons significant progress has been made recently \cite{16,17}. However, for more than two photons in the probe field and imperfect EIT, numerical calculations become very demanding. Petrosyan \textit{et al.} \cite{17} developed a model including correlations in the light field. This model is based on coarse graining the atomic medium by introducing super-atoms.

All these approaches treat the atomic cloud as a continuous medium. Alternatively, the atoms can be treated individually as discrete objects. This has the advantage that the simulation can realistically model non-homogeneous trap geometries, atom numbers and densities. However, models focusing on the atomic properties, such as inter-atomic correlations and other many-body effects, generally have the problem that the dimension of the Hilbert space grows exponentially with the number of atoms. This problem can be overcome by excluding states that are never populated due to the Rydberg blockade effect \cite{20,21}. But this state reductions is not possible for non-Rydberg excited states, since they are not affected by strong interaction-induced level shifts. At off-resonant two-photon driving of Rydberg excited states, non-Rydberg excited states are never populated. But in an EIT configuration, where both lasers are near-resonant with a low-lying intermediate excited state, this intermediate level cannot be eliminated adiabatically. As a consequence, the state space truncation becomes ineffective. A further restriction arises because incoherent processes such as the spontaneous decay of the intermediate level are important in the EIT setting, which make a full Master equation treatment necessary.

In order to overcome these difficulties, here, we use a model based on the rate equation (RE) ansatz developed by Ates \textit{et al.} \cite{8,22,23} and extended by Heeg \textit{et al.} \cite{24} to calculate the steady state of a cloud of three-level atoms subject to coherent laser driving. In this model, interactions are included as level shifts only, making a classical Monte Carlo treatment possible. In the regime of weak probe laser fields, the probe beam is attenuated while traveling through the atomic cloud. For this, we extend the existing RE models to include absorp-
tion based on the propagation equations of classical light fields. These lead to spatially varying local probe fields experienced by the different atoms, and we solve the combined propagation equations and RE self-consistently. The strength of the RE model lies in its great flexibility and applicability in arbitrary density regimes and geometries. This allows us to compare our results to recent experimental data for resonant probe fields of low intensity [19]. We find that best agreement is achieved if in addition to the constant dephasing induced by the finite laser linewidth also a density-dependent dephasing is introduced. This additional dephasing could arise from motion-induced dephasing, and we find that the density-dependent rate extracted from comparison to the experimental data is compatible in magnitude to a naive estimate based on kinetic gas theory. We further study the model with off-resonant probe field, and compare our results to that of other models [5,7]. We find that the models disagree at higher probe intensities, as the RE include resonant excitation channels at off-resonant laser driving which are not captured in the other super-atom based models. The resulting asymmetry in the spectra predicted by the RE, however, were not observed in recent experiments [13,14]. We attribute this to the fact that atomic motion renders the resonant excitation channels ineffective.

II. MODEL DESCRIPTION

II.1. Monte Carlo rate equation model

The RE model provides a way to calculate the steady state of a strongly interacting many-body system subject to lasers in EIT configuration that scales almost linear with the atom number as long as the Rydberg excited fraction is small. The setup we mainly refer to in this work is as in Ref. [19]. The ground state $|g\rangle = |5S_{1/2}\rangle$ of $^{87}$Rb is coupled to an intermediate state $|e\rangle = |5P_{3/2}\rangle$ by the (weak) probe laser with Rabi frequency $\Omega_p$. The state $|e\rangle$ is coupled to the Rydberg state $|R\rangle = |5S_{1/2}\rangle$ by the (strong) coupling laser with Rabi frequency $\Omega_c$. The intermediate state $|e\rangle$ can spontaneously decay to the ground state with rate $\Gamma$, while the Rydberg state is long lived. The additional dephasings caused by the finite laser bandwidths lead to the total line widths $\gamma_{eg}$ and $\gamma_{gR}$ of the probe transition and the two photon transition, respectively. Two atoms that are in the Rydberg state show repulsive Van der Waals interaction with $C_6/2\pi = 50\text{GHz}/\mu\text{m}^6$.

The Hamiltonian of an ensemble of $N$ such atoms, in rotating wave approximation, reads ($\hbar = 1$)

$$
H = \sum_{i=1}^N [H_L^{(i)} + H_\Delta^{(i)}] + \sum_{i<j} C_6 |\langle R_i R_j \rangle| |\langle R_i R_j \rangle| [R_i - R_j]^6 \tag{1}
$$

where

$$
H_L^{(i)} = \Omega_p/2 |g_i\rangle \langle e_i| + \Omega_c/2 |e_i\rangle \langle R_i| + h.c. \tag{2}
$$

describes the coupling of the atoms to the laser fields and

$$
H_\Delta^{(i)} = -\Delta_1 |e_i\rangle \langle e_i| - (\Delta_1 + \Delta) |R_i\rangle \langle R_i| \tag{3}
$$

accounts for the detuning from the one and two photon resonance. Incoherent processes can be included as Lindblad terms $\mathcal{L}[\rho]$ [20–29] leading to the Master equation for the density matrix

$$
\dot{\rho} = -i[H, \rho] + \mathcal{L}[\rho]. \tag{4}
$$

For a single atom ($N = 1$) one can transform the master equation into a set of RE for the populations of the atomic levels by adiabatically eliminating the coherences ($\dot{\rho}_{ij} = 0$ for $i \neq j$) [23]. For the many-body case one can straightforwardly generalize this to a RE for the populations of the product states $|\sigma\rangle = |\sigma_1, \sigma_2, \ldots, \sigma_N\rangle$, where $\sigma_i \in \{g, e, R\}$. We employ a Monte Carlo technique for the solution of the many body RE, that is, starting in the global ground state $|g, g, \ldots, g\rangle$ we perform a random walk through the configuration space of states $|\sigma\rangle$ [25] and average over many such trajectories, ensuring the convergence to a global steady state. The Hamiltonian $H$ couples two such many body states only if they differ in exactly the state of one atom. Therefore, it is sufficient to randomly pick one atom in each Monte Carlo step and determine the probability (jump rate) with which its state is changed. In order to calculate these rates, a mean-field-like approximation is required: The interaction between atoms in the Rydberg state is incorporated merely as a shift of the Rydberg energy.

$$
\Delta^{(i)} = \Delta_{\text{int}} = \sum_{j \neq i} V_{ij}, \quad \text{where the sum only runs over atoms that are currently in the Rydberg state.} \tag{5}
$$

for the density matrix

II.2. Including propagation effects

We now discuss how the attenuation of the probe beam can be included in the RE model. Classical light propagating through an atomic medium with electric susceptibility $\chi = \text{Im}(\chi_{eg})$ and thickness $L$ is damped exponentially

$$
\Omega_p(L) = \Omega_p(0)e^{-\chi k L}, \tag{5}
$$

where $k$ is the wave vector of the light. For resonant probe fields, dispersion and transverse beam dynamics can be neglected [5]. In terms of atomic properties $\chi$ is given as

$$
\chi = \frac{2|\mu_{eg}|^2 n_0}{\epsilon_0 \hbar \Omega_p} \text{Im}(\rho_{eg}) = \frac{3\lambda^2 n_0 \Gamma^2}{2\pi k \Omega_p^2 \rho_{ee}} \tag{6}
$$
where $\mu_{eg}$ is the dipole matrix element of the probe transition, $n_0$ the atomic density, $\lambda = 2\pi/k$ the probe wavelength, and $\Gamma$ the the spontaneous decay rate from $|e\rangle$ to $|g\rangle$.

In order to include the propagation effect in the Monte Carlo simulation, we have to calculate the local probe Rabi frequency that a certain atom $i$ experiences. For this, we define a cylindrical volume (tube) of cross section $A$ located around atom $i$ and extending into the opposite direction of the probe light propagation, see Fig. 1. All atoms inside this tubes contribute to the attenuation of the probe beam before it reaches atom $i$. The attenuation is calculated recursively, starting at the first atom in the tube ($i_1$), that experiences the full probe laser power corresponding to the Rabi frequency $\Omega_p^{(0)}$. Using $\Omega_p^{(0)}$ we calculate the steady state value of $\rho^{(i_1)}_{ee}$ for the current configuration $\sigma$ and use this to determine the Rabi frequency behind atom $i_1$ as

$$\Omega_p^{(i_1)} = \Omega_p^{(0)} \exp \left[ -\frac{3\lambda^2 \Gamma^2 \rho^{(i_1)}_{ee}}{2\pi A (\Omega_p^{(0)})^2} \right]. \quad (7)$$

Using $\Omega_p^{(i_1)}$ this procedure is repeated with the next atom $i_2$ in the tube and so on until atom $i$ is reached. The local Rabi frequency $\Omega_p^{(i)}$ is then used to determine the steady states and thus the jump probabilities for atom $i$ and to update its state. This procedure is repeated until the global observables converge. Additionally we average over many random Monte Carlo samples of atom positions.

In our numerical routines the recursive calculation of $\Omega_p$ is not done newly in every step. Instead the values of the local susceptibility and Rabi frequency are stored and reused. They only have to be updated, when an atom jumps into the Rydberg state, since in this case the interaction shifts of all other atoms change.

The only parameter that we can choose freely is the tube cross section $A$. We found that the results are independent of the exact choice of $A$ as long as two criteria are fulfilled: $A$ must be large enough to obtain $N_{\text{tube}} \gg 1$ atoms per tube on average, and it must be small enough, such that the atomic density does not vary much over the tube diameter. When simulating samples of varying density, we choose $A$ such that the average $N_{\text{tube}}$ is the same for all densities.

### III. RESULTS

#### III.1. Density dependence on resonance

The experimental setup we refer to in this section is the same that is described in Ref. [19]. A small ensemble of $^{87}$Rb atoms is trapped in a cigar shaped dipole trap, see Fig. 1. After releasing the gas from the trap Rydberg excitations are induced by switching on counter-propagating resonant probe and coupling beams for about 2 $\mu$s. The atomic density is varied by letting the cloud expand thermally for a variable time (TOF) before applying the excitation laser pulses. The cloud density as a function of expansion time can be measured by studying the absorption images in absence of the coupling beam, where only ordinary two level absorption takes place. This confirms that the cloud is Gaussian in all spatial directions and expands thermally having a temperature of 5 $\mu$K. The absorption image of the probe beam is recorded on a CCD camera. After the excitation pulses the produced Rydberg atoms are ionized by an electric field ramp and the ions are collected by a micro channel plate detector.

Figure 2 shows how the probe beam is attenuated while propagating through the atomic cloud. The higher the atomic density, the faster the probe intensity drops.
Therefore the maximum of the Rydberg density does not coincide with the maximum of the atomic density. This is indicated by the dashed vertical lines in Fig. 2. A comparison of the spatial dependence to experiment is not possible here, since the ion detector does not provide spatial resolution. The next quantity of interest is the transmitted probe intensity relative to the respective intensity observed in the two-level medium obtained in the absence of the coupling beam (without EIT). Figure 3 shows the distribution of this relative intensity in a section transverse to the beam propagation direction. In (a), a single Monte Carlo trajectory is shown. The noise is due to fluctuations in the local atomic density. The two dips close to the trap center are signatures of Rydberg excitations reducing the transmission in their vicinity. Such images cannot be obtained easily with current state-of-the-art experiments since the exposition time required to obtain an absorption image of sufficient signal-to-noise ratio is long on the time scale of the excitation dynamics. Thus excitations will vanish and reappear at other positions while the image is acquired making the spatially resolved detection of Rydberg excitations impossible. To overcome this difficulty, alternative imaging schemes have been proposed [30, 31].

Experimentally a time-integrated transmission signal is recorded, which is in addition averages over several repetitions of the experiment. This procedure is mimicked in the Monte Carlo simulation by averaging over several Monte Carlo trajectories and several realizations of randomly chosen atom positions. Such an averaging results in a transmission pattern as shown in Fig. 3(b) which can be compared directly to the experimentally recorded camera image [19].

We simulated the probe intensity behind the cloud (z = 0) in the center of the excitation region (x = y = 0) in order to quantitatively compare to experimental results. The results for the EIT-absorption are divided by the absorption obtained with the coupling laser switched off in order to eliminate trivial density dependences. In the low and high density limit the results agree well (see solid black line in Fig. 3), however there appears to be a shift in density by about a factor of two. This discrepancy can be attributed to additional density dependent dephasing effects due to collisions, ion formation and motional effects. The exposure time for the acquisition of the absorption images was 100 µs, while the ion signal (c.f. Fig. 5) was recorded after an excitation pulse of 2 µs. We note that the simulations of the scaled absorption have no adjustable parameters. All experimental parameters have been determined in independent measurements.

For comparison with our results on the propagated probe light, we employed a model proposed by Petrosyan et al. [7]. This work makes use of a simple super-atom model for the atom dynamics and focuses on the propagated light which is characterized via coupled propagation equations for the intensity and the correlation function of the probe light. This way, correlations in the light field going beyond the classical treatment in our approach can be included. But the model only describes light propagation through a one-dimensional array of super-atoms with diameter 2r_b. A consistent extension to a three-dimensional sample is not straightforward. The blockade radius r_b is defined by equating the EIT-width to C_0/r_b^6. Interactions between super-atoms are included as a small mean field shift appearing in the susceptibility, which will be discussed in more detail in Sec. III.2. We extended the original model by replacing the EIT width w = |Ω_c|^2/γ_0 by γ_R + |Ω_c|^2/γ_0 due to the larger dephasing rates in our setup, such that the contribution γ_R can not be neglected. Furthermore we include spatially varying densities, i.e., the number of atoms per super-atom n_S becomes spatially dependent. With the above extensions, we obtain very good agreement for the properties of the propagated light between the two models. However, we found that for our parameters, the simulation results remain unchanged if the photon statistics is forced to remain classical in the extended model of Petrosyan et al. For this, we set the g^{(2)} of the light field to one. This indicates that for the parameters of this experiment, the non-classical character of the light does not influence the total absorption. Thus, the discrepancy in the absorption between the experiment and our theoretical modeling is likely due to some other effect.

As a cross check for our model, in Fig. 4(a) we show
the scaled absorption obtained including the third order non-linear absorption calculated in [5]. But this model deviates from the experimental data as soon as density is increased. One reason for this could be that for the trap geometry used here, it is probably not justified to neglect the transverse beam dynamics as it was done in [5] since the density varies rapidly perpendicular to the propagation direction. Moreover, this model is based on a truncation in the correlation order at the two particle level, and is thus expected to fail at high densities, since here higher order correlations are crucial.

A next possible candidate is atomic motion induced dephasings. In the experimental situation under discussion, a thermal could of atoms at $T = 5 \mu K$ is considered. The average speed of an atom is thus $v = \sqrt{8kT/\pi m} = 0.035 \text{ m/s}$. This means that within the excitation time of $100 \mu s$ an atom typically moves across a distance of 3.5 $\mu$m. As a consequence, in a binary picture, an atom that is initially unblocked with respect to second atom, can move towards the second atom within the excitation time and undergo a collision. Restricting the analysis to dominant collisions of Rydberg atoms, and estimating the collision rate from classical kinetic gas theory, we obtain $n_{\text{coll}} = \sigma v n_0 \approx 10 \mu s^{-1}$ at a density of $10^{12} \text{ cm}^{-3}$. Here, the scattering cross section $\sigma = \pi r_I^2$ is determined by estimating the classical turning point from $mv^2/2 = \hbar C_6/r_i^6$. As expected, the additional dephasing is proportional to the atomic density $n_0$, analogously to the collision rate itself. This collision rate can have an impact even for relatively short excitation times. Fitting $\alpha$ in the additional dephasing $\Gamma = \Gamma_{\text{reag}}/2\pi = \alpha n_0$, the dot-dashed curve in Fig. 4 is obtained, which shows good agreement to the experimental data. From the fit we estimate $\alpha = 15 \times 10^{-12} \text{ MHz cm}^3$, which means that the density-dependent dephasing exceeds the constant laser-induced dephasing for densities larger than approximately $1.2 \times 10^{11} \text{ cm}^{-3}$. Density dependent dephasing effects have been studied in hot atomic vapors recently [92]. The setup in this experiment is slightly different as the excitation lasers are far detuned from the intermediate level. Nevertheless, a linear dependence of the dephasing on the atomic density was found in this work as well. Additionally, the motional dephasing was found to be proportional to the Rydberg population $f_R$. We should note, that using a linear dependence on $f_R$ the fitted absorption curve would fit the data in Fig. 4 even better. However, we have not found a way yet to include a dephasing that depends on the Rydberg population itself in the RE model in a self-consistent way.

Our theory also gives access to the Rydberg excitations. The predicted number of excitations agrees well with experimental values, see Fig. 5. Here, we adjusted two experimental parameters to match the simulation data that cannot be determined sufficiently precisely from independent measurements. Namely, the semi-major axis of the coupling laser spot was found to be $65 \mu m$, in accordance with what is found by inspecting individual absorption images, and the detection efficiency of the MCP was found to be $\eta = 0.4$, which is also in accordance with the expectation. We have added the results for the excitation number that we obtain if we exclude attenuation and interaction effects (green dashed line in Fig. 5). The obtained number of excitations is given by $f_0 N$, where $f_0$ is the single atom excitation probability. Additionally we simulated the system excluding interactions but including attenuation effects and vice versa. The strong deviations from the experimental
data at high densities in both cases show that both, attenuation of the probe beam and interaction between the atoms, have a significant impact on the number of produced Rydberg excitations. This means that including the probe beam attenuation self-consistently in the rate equation model is indispensable for the simulation Rydberg EIT in a dense gas.

III.2. Dependence on probe field detuning

So far, we have only considered resonant probe and coupling beams. Next, we study the dependence of the transmission through an elongated cloud of length $L = 1.3\text{ mm}$ and constant density $n_0 = 1.2 \times 10^{10}\text{ cm}^{-3}$ on the probe field detuning. The laser parameters are as in Refs. [7, 14]. Dephasings are smaller compared to Ref. [19] and $C_6$ is larger (a $60\text{s}$ state with $C_6/2\pi = 140\text{ GHz}\mu\text{m}^6$ is used). In the super-atom model of Ref. [7], the correlation function of the light field was included to account for the emergence of non-classical states of light.

Scanning the probe detuning $\Delta_1$ for various initial probe Rabi frequencies $\Omega_p(0)$, we obtain the transmission curves depicted in Fig. 6. For low probe intensity the models agree well. In this case $g^{(2)}$ does not deviate much from unity. As the probe intensity is increased, the transmission on resonance decreases showing the non-linearity of the process. The transmission obtained from the RE model shows a clear shift and broadening of the EIT resonance while the super-atom model does not. If $g^{(2)}$ is set to unity in the super atom model, the resulting shift and asymmetry is still small, while the main effect is a decrease of transmission near resonance. The asymmetry observed in the rate equation results is due to resonant excitation effects. If the interaction shift cancels the detuning, Rydberg excitation is enhanced (anti-blockade) which leads to smaller $\rho_{ee}$ and thus reduces absorption as this only happens for positive detunings, the curve becomes asymmetric. This effect is not present in the super-atom model since here interactions between different super-atoms are only included as a small mean field shift in the EIT-absorption. This shift is indeed negligible for the parameters studied here and does not account for the anti-blockade. This is the technical reason why the super-atom model does not predict significant shifts and broadening.

The physical reason why this asymmetry is not observed in experiment [13, 14] is likely to be related to atomic motion and effects beyond the frozen gas approximation. As a pair of atoms is excited resonantly, the atoms start repelling each other as they feel the repulsive force induced by the Rydberg-Rydberg interactions, thereby moving out of the pair excitation resonance. This effect therefore renders resonant excitation processes inefficient for long exposure times. As an example, we look at the case of $\Delta/2\pi = 1\text{ MHz}$. Two atoms can be excited resonantly if they are at a distance $r_{res} = [C_6/(2\Delta)]^{1/6} = 6.4\mu\text{m}$. Assuming that both atoms get excited initially and calculating the classical trajectory on which the partners move apart one obtains that after $10\mu\text{s}$ the interatomic distance has increased by about $1\mu\text{m}$ and the atoms have taken up a relative velocity of $0.13\mu\text{m}/\mu\text{s}$. This means they have moved out of the pair resonance, such that the double excitation probability decreases again, and they have received a momentum kick well above the mean thermal momenta at cryogenic temperatures. Thus the effect of resonant processes is rather a heating of the gas than an enhancement of the Rydberg population if excitation times are too long. Recalling that the data of Ref. [14] was taken by scanning $\Delta/2\pi$ from $-20\text{ MHz}$ to $20\text{ MHz}$ in $500\mu\text{s}$ it becomes clear that such effects should play a role. We note that for the case of attractive interactions it was found that the transmission spectrum strongly depends on the direction of the detuning scan, indicating that mechanical effects come into play [13]. Mechanical effects playing a role in this context have also been mentioned in Ref. [6].

IV. SUMMARY AND DISCUSSION

We have introduced an extended RE model including the attenuation of the probe beam, which is indispensable in the weak probe and high density regime. We applied our model to two different experimental situations: First, we simulate transmission of a weak probe beam through an atomic cloud at EIT resonance as a function of atomic density. Here we find good agreement with experimental results and other models was found for resonant laser driving in a large range of atomic densities. At high density and for experiments with long excitation times, we find that our model underestimates the probe absorption. This deviation is consistent with a motion induced density dependent dephasing. Second, we studied the dependence of the probe transmission on the single pho-
ton detuning and probe intensity at relatively low atomic density. We find that at low probe intensities, our model agrees well to the experimental data. But towards higher probe intensities, our model predicts a shift and broadening of the EIT resonance that is much stronger than observed experimentally. At the low density considered in the experiment, dephasing caused by collisional effects can have another effect. Mechanical forces between resonantly excited pairs of atoms lead to a repulsion between them, which can render resonant excitation processes ineffective at long excitation times. Therefore, the results in both considered experimental settings suggest effects beyond the frozen gas approximation, and motivate further theoretical modeling and experimental studies on the validity of this approximation.

ACKNOWLEDGMENTS

We thank S. Whitlock and K. P. Heeg for fruitful discussions. This work was supported by University of Heidelberg (Center for Quantum Dynamics, LGFG).

[1] J. Reslen, Journal of Physics B: Atomic, Molecular and Optical Physics 44, 195505 (2011).
[2] A. V. Gorshkov, J. Otterbach, M. Fleischhauer, T. Pohl, and M. D. Lukin, Phys. Rev. Lett. 107, 133602 (2011).
[3] A. V. Gorshkov, R. Nath, and T. Pohl, arXiv:1211.7060 [quant-ph].
[4] J. D. Pritchard, C. S. Adams, and K. Mølmer, Phys. Rev. Lett. 108, 043601 (2012).
[5] S. Sevinçli, N. Henkel, C. Ates, and T. Pohl, Phys. Rev. Lett. 107, 153001 (2011).
[6] S. Sevinçli, C. Ates, T. Pohl, H. Schempp, C. S. Hofmann, G. Günter, T. Amthor, M. Weidemüller, J. D. Pritchard, D. Maxwell, A. Gauguet, K. J. Weatherill, M. P. A. Jones, and C. S. Adams, J. Phys. B: At. Mol. Opt. Phys. 55, 184018 (2011).
[7] D. Petrosyan, J. Otterbach, and M. Fleischhauer, Phys. Rev. Lett. 107, 213601 (2011).
[8] C. Ates, S. Sevinçli, and T. Pohl, Phys. Rev. A 83, 041802 (2011).
[9] D. Petrosyan and M. Fleischhauer, Journal of Physics: Conference Series 350, 012001 (2012).
[10] D. Yan, Y.-M. Liu, Q.-Q. Bao, C.-B. Fu, and J.-H. Wu, Phys. Rev. A 86, 023828 (2012).
[11] J. Stanojevic, V. Parigi, E. Bimbard, A. Ourjoumtsev, and P. Grangier, arXiv:1303.4927 [quant-ph].
[12] J. Otterbach, M. Moos, D. Muth, and M. Fleischhauer, arXiv:1304.8096 [quant-ph].
[13] H. Schempp, G. Günter, C. S. Hofmann, C. Giese, S. D. Saliba, B. D. DePaola, T. Amthor, M. Weidemüller, S. Sevinçli, and T. Pohl, Phys. Rev. Lett. 104, 173602 (2010).
[14] J. D. Pritchard, D. Maxwell, A. Gauguet, K. J. Weatherill, M. P. A. Jones, and C. S. Adams, Phys. Rev. Lett. 105, 193603 (2010).
[15] J. D. Pritchard, A. Gauguet, K. J. Weatherill, and C. S. Adams, Journal of Physics B: Atomic, Molecular and Optical Physics 44, 184019 (2011).
[16] T. Peyronel, O. Firstenberg, Q.-Y. Liang, S. Hofferberth, A. V. Gorshkov, T. Pohl, M. D. Lukin, and V. Vuletic, Nature 488, 57 (2012).
[17] Y. O. Dudin and A. Kuzmich, Science 336, 887 (2012).
[18] D. Maxwell, D. J. Szwer, D. Paredes-Barato, H. Busche, J. D. Pritchard, A. Gauguet, K. J. Weatherill, M. P. A. Jones, and C. S. Adams, Phys. Rev. Lett. 110, 103001 (2013).
[19] C. S. Hofmann, G. Günter, H. Schempp, M. Robert-de Saint-Vincent, M. Gärttner, J. Evers, S. Whitlock, and M. Weidemüller, arXiv:1211.7265 [physics.atom-ph].
[20] F. Robicheaux and J. V. Hernández, Phys. Rev. A 72, 063403 (2005).
[21] K. C. Younge, A. Reinhard, T. Pohl, P. R. Berman, and G. Raithel, Phys. Rev. A 79, 043420 (2009).
[22] M. Gärttner, K. P. Heeg, T. Gasenzer, and J. Evers, Phys. Rev. A 86, 033422 (2012).
[23] C. Ates, T. Pohl, T. Pattard, and J. M. Rost, Phys. Rev. A 76, 013413 (2007).
[24] C. Ates, T. Pohl, T. Pattard, and J. M. Rost, Phys. Rev. Lett. 98, 023002 (2007).
[25] K. P. Heeg, M. Gärttner, and J. Evers, Phys. Rev. A 86, 063421 (2012).
[26] G. S. Agarwal, in Quantum Optics, Springer Tracts in Modern Physics, Vol. 70 (Springer Berlin/Heidelberg, 1974) pp. 1–128.
[27] Z. Ficek and S. Swain, Quantum interference and coherence: theory and experiments, Springer series in optical sciences (Springer, 2005).
[28] M. Kiffner, M. Macovei, J. Evers, and C. Keitel, in Progress in Optics, Vol. 55 (Elsevier Science, Burlington, 2010) pp. 85–197.
[29] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. 77, 633 (2005).
[30] G. Günter, M. Robert-de Saint-Vincent, H. Schempp, C. S. Hofmann, S. Whitlock, and M. Weidemüller, Phys. Rev. Lett. 108, 013002 (2012).
[31] B. Olmos, W. Li, S. Hofferberth, and I. Lesanovsky, Phys. Rev. A 84, 041607 (2011).
[32] T. Baluktsian, B. Huber, R. Löw, and T. Pfau, Phys. Rev. Lett. 110, 123001 (2013).