New Upper Limit of Terrestrial Equivalence Principle Test for Rotating Extended Bodies

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Abstract

Improved terrestrial experiment to test the equivalence principle for rotating extended bodies is presented, and a new upper limit for the violation of the equivalence principle is obtained at the level of \( 1.6 \times 10^{-7} \), which is limited by the friction of the rotating gyroscope. It means the spin-gravity interaction between the extended bodies has not been observed at this level.
I. INTRODUCTION

The equivalence principle (EP), as one of the fundamental hypotheses of Einstein’s general relativity, has been tested by many experiments, including torsion balance scheme [1–4] and free-fall one [5–7]. Lunar laser ranging from the Earth to the Moon has provided up to now the most accurate test of the EP to $5 \times 10^{-13}$ [8]. Recently, some different tests of EP for gravitational self-energy and spin-polarized macroscopic objects have been reported [9–11]. However, in all of the experiments including the Satellite Test of the Equivalence Principle (STEP) and the Galileo Galilei (GG) space projects as well as the MICROSCOPE space mission [12–14], the test masses are all non-rotating.

It is well known that spin-interactions of elementary particles, spin-orbit coupling and spin-spin coupling, have been studied in both theory and experiment. Furthermore, the spin-gravitational couplings, i.e. the spin-gravitoelectric coupling and the spin-gravitomagnetic coupling, and the spin-rotation coupling between intrinsic spins have been also investigated for a long time [15–20].

Over the last few years there has been a growing interest in effects of gravitational fields on gyroscopes [21–24]. From the experimental point of view, the NASA/Stanford Relativity Mission Gravity Probe B (GP-B) experiment will provide two extremely precise tests of general relativity based on observations of four identical superconducting gyroscopes in a satellite in a 400 miles polar orbit around the Earth [25]. These gyroscopes are carefully isolated from nearly all sources of Newtonian torques, and their residual drift is presented less than 0.020 marc-second/year for a gyroscope in a fully inertial orbit [26]. General relativity predicts that, though isolated from external torques, the spin axes of these gyroscopes will precess with respect to a distant inertial reference frame at a rate of 6.6 arc-second/year for the geodetic effect, and 0.042 arc-second/year due to frame dragging. Cerdonio et al. have proposed a novel detector (gyromagnetic electron gyroscope) to locally detect the frame dragging due to the terrestrial rotation [27]. Recently, Zhang et al. also developed a phenomenological model for the spin-spin interaction between rotating extended bodies,
which predicts the effect of the spin-spin coupling on the orbital acceleration of the gyroscope free-falling in gravitational field rather than the spin procession of the gyroscope [28]. In the mode, a dimensionless parameter representing the strength of violation of EP can be defined as follows

$$\eta_s = \frac{\Delta g}{g} = \kappa \left( \frac{\hat{S}_1 \cdot \hat{S}_c}{G m_1 M_e R_1} - \frac{\hat{S}_2 \cdot \hat{S}_c}{G m_2 M_e R_2} \right),$$

where $G$ is the Newtonian gravitational constant, $m_1$, $m_2$ and $M_e$ are the masses of the two gyroscopes and the Earth, respectively, and $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_c$ are their spin angular momentums, $R_1$ and $R_2$ are the distances between the centers of the two gyroscopes and the Earth, respectively, and the parameter $\kappa$ represents the universal coupling factor for the spin-spin interaction for rotating extended bodies. As pointed out in Refs. 28 and 29, the phenomenological model developed by Zhang et al. is to investigate the effect of the spin-spin coupling on the orbital acceleration of the rotating gyroscope free falling in gravitational field, which is different from the aim of the GP-B.

A preliminary double free-fall (DFF) experiment to test the EP for rotating extended bodies, in which two gyroscopes with differing rotating senses drop freely, has been performed, and the results show that the EP is still valid for rotating extended bodies at the level of $2 \times 10^{-6}$ [29]. A main limit of preliminary experimental precision has been proved to come from the pump outgassing effect due to the asymmetrical outgassing for the two tubes. In the initial experimental setup, the vacuum pump system is set in the top part of the tube. In this case, when the test masses fall through the tee part of the tube ($\sim 0.3$ s free-fall), a force due to the pump outgassing will deflect the test masses or lead them to a more complex motion [30]. An abrupt acceleration change of about 20 mGal is observed at this height. To avoid it, the pump system is moved down to the bottom part of the tube, and meantime the vacuum level is improved from initial 50 mPa to 2 mPa. At the same time, the vibration excited by the operating pump is effectively isolated by a rubber-gas-steel isolator, and the isolation ratio is measured about -30 dB to -60 dB in the range of above 3 Hz. And then, the effective falling height is prolonged from initial 20 cm (0.2 s) to 9 m (1.0 s). In this
article, error sources of our DFF experiment will be carefully discussed and a new upper limit of the EP for rotating extended bodies will be presented.

II. EXPERIMENTAL DESCRIPTION AND ERROR ANALYSIS

A Michelson-type interferometer including a frequency-stabilized He-Ne laser beam with a relative length standard of $1.3 \times 10^{-8}$ is used to monitor the differential vertical displacement between two gyroscopes, in which one is rotating and another is non-rotating, and then the interference fringes are sampled by means of a 10 MHz 12-bit AD card combined with an external rubidium atomic clock, and then stored in a computer. The diameter of the laser beam is collimated about 3.0 mm so that the beam wavefront effect can be neglected. An aligned verticality is kept within 50" for each laser beam, and the maximum uncertain differential acceleration due to the aligned verticality is below 20 $\mu$Gal.

Each of the two test masses consists of a steel gyroscope with a mass of 420.0±2.5 g, a diameter of about 55 mm and a height of about 32 mm, which together with a corner-cube-retroreflector (CCR) of 76.4±0.4 g is sealed in an aluminum frame of 159.4±0.9 g. Tinned copper wires with a diameter of 0.25 mm are used to suspend the test masses, and the initial suspending differential height between them is kept within 1 mm, which implies the vertical gravity gradient correction is about 0.3 $\mu$Gal. The test mass with a non-rotation rotor is released about 3 ms before the rotating one, which sets a systematic error of about 0.3 $\mu$Gal due to the finite speed of light. The rotating gyroscope is driven by a DC motor and its rotating speed is kept at (17000±200) rpm.

An uncertain acceleration due to the residual gas drag effect is less than 0.2 $\mu$Gal at $p = 2$ mPa and $T = 300$ K [29]. In addition, the outgassing effect on the dropped objects should be carefully considered because of the continuous operation of a turbo-molecular pump with a full rated pumping speed $v_p$ of 1500 L/s. The acceleration contribution for a single dropped object can be estimated as follows [31]

$$a \leq R\rho v_p V/m$$

(2)
where $R$ represents the ratio of the surface of the dropped object and the inner surface of the vacuum tube, and is about 0.04 here, $\rho$ and $V$ are the residual gas density and mean gas particle speed, respectively. Then, the acceleration for a single dropped object is about 100 $\mu$Gal at $p = 2$ mPa. Fortunately, the DFF scheme can reduce the common mode effect, and a differential acceleration for the two test masses due to the outgassing effect depends on both the outgassing difference and the gas density difference between the two tubes. It is very difficult to calculate the real difference due to the complex flux motion exactly. However, it can be roughly estimated based on the pressure difference between the two tubes. When the turbo-molecular pump runs normally, the pressure of the bottom part of the tube (close to the pump) is measured about 1.7 mPa, and at the same time, the top parts of the two tubes are measured about 8.7 mPa and 5.6 mPa, respectively. This means that the outgassing speeds in the two tubes are about $0.61 \, v_p$ and $0.39 \, v_p$ if the pressure distribution in both tubes is the same. So in this assumption, the acceleration difference for the two test masses due to the outgassing effect is estimated less than 22 $\mu$Gal.

A possible lifting force for a rotating rotor due to the residual gas flow’s circulation can be calculated based on the Zhukovskii’s theorem, and this effect can be neglected here [29]. A possible horizontal velocity difference $\Delta v_h$ is estimated smaller than 4.2 mm/s according to the change of the interference pattern intensity during free-fall of test masses, and an acceleration difference due to the Coriolis’s effect is less than 54 $\mu$Gal [29]. It means that the horizontal velocity difference would have to be monitored in the further experiment with a higher precision.

The silent amplitude spectrum of the seismic noise in our laboratory contributes an uncertainty of about 1 $\mu$Gal to the final experiment result [32]. But the mechanical vibration modes of the optical measurement system are dominant due to excitation of the vacuum pumps. Figure 1 is a typical residual differential displacement of the DFF experiment between both non-rotating test masses, in which the linear term has been subtracted. Three main modes have been observed about 16.8 Hz, 36.6 Hz, and 96.1 Hz, and their amplitudes comes to about 0.05 $\mu$m, which contribute an uncertain acceleration of 8 $\mu$Gal based on the
following equation [30]

\[
\Delta g_t = \frac{120x_n}{\omega_nT^3} \sqrt{1 + \frac{12}{\omega_n^2T^2} + \left(\frac{12}{\omega_n^2T^2}\right)^2 \cos\left(\frac{\omega_nT}{2} + \phi_n\right) \cos\left(\frac{\omega_nT}{2} + \tan^{-1}\frac{12 - \omega_n^2T^2}{6\omega_nT}\right)},
\]

(3)

where \(x_n\), \(\omega_n\), and \(\phi_n\) represent the amplitude, the angular frequency, and the phase of the high-frequency vibration, respectively, and \(T\) is the effective time length. Parabolic curve fitting result shows that the differential acceleration \(\Delta g_{N-N}\) is -58 \(\mu\)Gal, which is consistent with the total systematic uncertainty, as listed in Table 1, of 64 \(\mu\)Gal.

### III. EXPERIMENTAL RESULTS

Figure 2(a) and 2(b) are, respectively, typical residual differential displacements of the DFF experiment with non- and left-rotating (spin vector pointing upward) gyroscopes, and non- and right-rotating (spin vector pointing downward) ones. From both figures, it is very clear that there are a dominant slow frequency motion but not a parabolic term. This slow frequency (1.6±0.2 Hz) motion is confirmed to be resulted from the friction coupling between the rotating rotor and the aluminum frame by observing the motion of the reflected beam. Since effective free-fall duration in our DFF experiment is about 1 s and the correlative coefficient between a parabolic term (\(\Delta g \sim 100 \mu\)Gal) and a harmonic term of 1.6 Hz (\(x_n \sim 0.06 \mu\)m) comes to 0.3 \(\sim\) 0.4, the slow frequency fluctuation is very difficult to be subtracted by fitting. Nevertheless, a maximum uncertain acceleration due to the slow motion can be estimated based on Eq. (3), and its effect comes to about 130 \(\mu\)Gal (\(x_n \sim 0.06 \mu\)m) for the non- and left-rotating gyroscopes and 150 \(\mu\)Gal (\(x_n \sim 0.07 \mu\)m) for the non- and right-rotating gyroscopes, respectively. The fitting results of the both residual curves show that the differential acceleration between the non- and left-rotating gyroscopes \(\Delta g_{N-L}\) is 0.48 \(\mu\)Gal, and that between the non- and right-rotating gyroscopes \(\Delta g_{N-R}\) is -110 \(\mu\)Gal, which are consisted with the systematic uncertainty of 160 \(\mu\)Gal as listed in Table 1. In addition, a high-frequency mechanical vibration of the CCR at the frequency of the rotating rotor, caused by the friction coupling, has also been observed and its modulation amplitude is
in the order of 0.1 $\mu$m, which contributes an uncertainty of about 5 $\mu$Gal. Fortunately, the limit can be suppressed by a factor of $\sin(\omega_n t_s/2)/(\omega_n t_s/2)$ by means of a time-domain data-smoothing-process, here $t_s$ is the time length of the smoothing-process.

Based on above statement, the EP is still valid at the level of $1.6 \times 10^{-7}$ for rotating extended bodies, which is improved by over one order for preliminary experiment result [23], and the spin-spin interaction between the rotating extended bodies has not been observed at this level. According to the Eq. (1) and the approximately uniform sphere mode of the Earth, it can be concluded that the coupling factor $\kappa \leq 1.6 \times 10^{-19}$ kg$^{-1}$, which sets a new upper limit for the spin-spin interaction between a rotating extended body and the Earth.

**IV. DISCUSSIONS**

Our experiment precision is mainly limited by the mechanical friction of the gyroscope, which could be improved by choosing better gyroscope or extending the free-fall duration. As pointed out in the introduction section, GP-B mission measures the spin axes precession of spinning fused-quartz gyroscopes with respect to an inertial reference frame, partly due to the geodetic effect, and partly due to frame dragging. In general relativity the both effects have small, non-zero values even without violation of the EP. However, based on the model developed by Zhang et al., a perigean precession of the gyroscope in the GP-B experiment is not greater than 100 arc-second/year based on current experimental result $\kappa \leq 1.6 \times 10^{-19}$ kg$^{-1}$. Nevertheless, the GP-B experiment measures the spin axis precession rather than the orbital motion of the gyroscope. As proposed in Ref. 28, a possible scheme is to put a non-spinning shell surrounding a spinning gyroscope in a satellite, and the motion of the spinning gyroscope with respect to the non-spinning reference frame could be monitored using a SQUID or an inertial sensor [33,34]. If the gap between the gyroscope and the reference shell can be measured in the level of 1 nm/year, and the coupling factor $\kappa$ between the spin-spin interaction between the rotating extended bodies could be tested in the level of $10^{-31}$ kg$^{-1}$, which is improved by about 12 orders.
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FIGURES

FIG. 1. Residual differential displacement of the double free-fall experiment between both non-rotating test masses. The fluctuation is due to the mechanical vibration modes of optical measurement system. The parabolic curve fitting shows that the differential acceleration is about 58 $\mu$Gal.

FIG. 2. (a) Residual differential displacement of the double free-fall between non- and left-rotating gyroscopes. The parabolic curve fitting shows that the differential acceleration is about 0.48 $\mu$Gal. (b) Residual differential displacement between non- and right-rotating gyroscopes. The parabolic curve fitting shows that the differential acceleration is about -110 $\mu$Gal.
TABLES

TABLE I. Summary of systematic errors of the equivalence principle test for rotating extended bodies.

| Systematic Error              | Uncertainty (µGal) |
|-------------------------------|--------------------|
| Length standard of laser      | ~ 13               |
| Verticality of laser beam     | ≤ 20               |
| Outgassing effect             | ~ 22               |
| Horizontal motion             | ≤ 54               |
| Mechanical vibrations         | ~ 8                |
| Total for non-rotating        | ≤ 64               |
| Friction coupling             | ≤ 150              |
| Total for rotating            | ≤ 160              |
Parabolic curve fitting

\[ Y = -7.99444E-8 + 3.34986E-7 t - 2.88305E-7 t^2 \]
Residual Displacement (m)

Time (s)

Y = -2.75639E-8 + 8.39751E-8 t + 2.39771E-9 t^2

N-L residual Displacement
Parabolic curve fitting
\[ Y = -9.81562 \times 10^{-8} + 5.56316 \times 10^{-7} t - 5.65752 \times 10^{-7} t^2 \]