Homogeneity trend on social networks changes evolutionary advantage in competitive information diffusion

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Abstract

Competitive information diffusion on large-scale social networks reveals fundamental characteristics of rumor contagions and has profound influence on public opinion formation. There has been growing interest in exploring dynamical mechanisms of the competing evolutions recently. Nevertheless, the impacts of homogeneity trend, which determines powerful collective human behaviors, remains unclear. In this paper, we incorporate homogeneity trend into a modified competitive ignorant-spreader-ignorant rumor diffusion model with generalized population preference. Using microscopic Markov chain approach, we first derive the phase diagram of competing diffusion results on Erdös–Rényi graph and examine how competitive information spreads and evolves on social networks. We then explore the detailed effects of homogeneity trend, which is modeled by a rewiring mechanism. Results show that larger homogeneity trend promotes the formation of polarized ‘echo chambers’ and protects the disadvantaged information from extinction, which further changes or even reverses the evolutionary advantage, namely, the difference of stable proportions of the competitive information. However, the reversals may happen only when the initially disadvantaged information has stronger transmission ability, owning diffusion advantage over the other one. Our framework provides profound insight into competing dynamics with homogeneity trend, which may pave ways for further controlling misinformation and guiding public belief systems. Moreover, the reversing condition sheds light on designing effective competing strategies in many real scenarios.

1. Introduction

Owing to the rapid development of Internet technologies, the dynamical evolutions of information spreading on large-scale social networks have been widely concerned and studied in recent years [1–9]. How to calculate the final scope of information delivery? How to predict the threshold of information dissemination? What’s the coupled effects of information diffusion and other evolution processes on multiple networks? Exploring the underlying dynamical mechanisms of information diffusion is of vital significance for understanding these important questions, which can further instruct powerful applications in many different fields, such as controlling rumors, promoting innovations, designing efficient marketing strategies and etc [10–14].

Of particular interest, competitive information diffusion on social networks, which reproduces the ubiquitous situations where individuals are exposed to multiple polarized information related to the same social event, has attracted great attention recently [15–17]. Revealing the diffusion patterns of competitive information helps to deal with some great challenges nowadays. A direct application scenario is to understand the simultaneous spreading of the truth and the rumors [18, 19]. This is of great importance not only in physics, but
also in economics and social science, considering the fact that misinformation has been recorded as one of the main threats to human society by World Economic Forum [20, 21]. Moreover, the final competing diffusion results have profound influence on the formation of public opinions [22–27]. A typical example is the US presidential election, where the outcome of competitive information diffusion on large-scale social networks, like Facebook and Twitter, would directly affect the election result [28, 29].

The competing dynamics was first studied on epidemic spreading where two competitive diseases infect the same population [30–32]. Newman utilized the generalized susceptible-infective-removed model to describe the transmission of two pathogens on networks [33]. Karrer et al further studied the dynamics of competing diseases with cross immunity and derived the theoretical phase diagram of the system [34]. Leventhal et al applied competitive susceptible-infected-susceptible (SIS) model to describe the evolution of infectious diseases and highlighted the significant role of network heterogeneity [35].

Inspired by the idea that the dynamical process of information diffusion is analogous to epidemic spreading to some extent, many literatures attempted to use epidemic-like model to describe the competitive information diffusion [36–38]. Trpevski et al proposed a competitive SIS model with stubbornness and completely asymmetry preference, where the competing rumors satisfy cross immunity and individuals always select rumor 1 when they are informed of two rumors simultaneously [39]. Wang et al incorporated the neighborhood influences into theoretical framework and found rich dynamics of two competing ideas [40]. Further, the easier availability of online social network data and the development of computational ability make it possible to find real-world dynamical characteristics of the diffusion process. Studies then tried to explore the unique consumption patterns as well as the psychological origins of information diffusion, which cannot be described by the epidemiological model, leading to the complex contagion models that are closer to reality [8, 41–43].

More generally, the competing dynamics of behavioral and cultural spreading are also developed by model-data integration [44]. Gleeson et al introduced the competition-induced criticality into the meme popularity and successfully predicted the power-law distributions of popularity observed in empirical data [45].

Despite the efforts and progress, it remains largely unknown that how complicated human behaviors, especially those collective evolutions caused by the group psychology, influence the competitive information diffusion. For example, it has been long understood that people are more likely and more frequently to interact and exchange their ideas with those who have similar traits in real world [46–48]. Similar motivation called homogeneity trend in information dynamics is also observed on online social networks, which means individuals prefer to seek for like-minded people while avoid further communications with those who hold opposite opinions [49–51]. In particular, homogeneity trend was verified as the main psychological origin for the formation of echo chambers, which further leads to social differentiation and polarization [52]. Hence, homogeneity trend is an essential factor in social diffusion process, which not only influences the efficiency of information spreading, but also changes the potential diffusion paths, i.e. affects the underlying network topology. However, there still lacks a proper understanding of the detailed impacts of homogeneity trend on diffusion results of competitive information.

To fill this theoretical gap, in this paper, we propose a theoretical framework which incorporates homogeneity trend into a modified competitive SIS model with generalized population preference [53]. First, we show how competitive information diffuses and evolves on homogenous social networks (ER graph) without homogeneity trend via both theoretical analysis and Monte Carlo simulations. Second, through examining the time evolutions and competing results of both system states and network structures, we reveal and explain the detailed non-monotinous influence of homogeneity trend on competitive information diffusion. Finally, we find that homogeneity trend can significantly change and even reverse the evolutionary advantage, i.e. the difference of final proportions of the two information. However, the reversals only happen when the initially disadvantaged information has stronger transmission ability but loses population preference. This indicates that to win the competing evolutions, the best way for the information is to take the diffusion advantage regardless of population preference. Our work provides important insights into competitive diffusion processes under the influence of homogeneity trend, which promotes the understanding of misinformation spreading as well as public opinion formation on large-scale social networks.

2. Model description

Consider an undirected network with \( N \) nodes, whose adjacent matrix is denoted as \( A = (a_{ij})_{N \times N} \). If there exists an edge between node \( i \) and node \( j \), \( a_{ij} = 1 \). Otherwise, \( a_{ij} = 0 \). In order to provide better analytical insights, here we adopt simple ignorant-spreader-ignorant (SIS) rumor spreading model instead of complex contagion models to describe the diffusion process of a single piece of information, which is at the cost of being less socially realistic [42, 43]. Ignorant (I) represents an individual who has not known the information and spreaders (S) stand for the individuals who are able to spread information to its neighbors, corresponding to the
susceptible and infected populations in classical epidemic model, respectively [54]. An important characteristic of the SIS model is the reinfection mechanism, i.e. the recovered nodes can be infected again, which could effectively describe the multi-round information dissemination. In addition, the reinfection mechanism provides a simple and natural way to characterize the viewpoint changing process in multiple information diffusion environments.

In this work, we focus on the situation where two pieces of competitive information, denoted as information 1 and information 2, spread simultaneously. Complying with previous studies, we assume that information 1 and information 2, such as the truth and the rumors, are competitive with exclusiveness [55]. This means that all individuals could only support one piece of information at any time. Thus, the population could be divided into three classes according to their states: ignorant (I), spreader of information 1 (S1), spreader of information 2 (S2). The ignorant has not learned any information or is confused about which information to support. S1 strongly supports information 1 and has a probability λ1 to spread information 1 to its ignorant neighbors. Meanwhile, S1 forgets the information or becomes confused again because of the self-awareness or the external social influence, i.e. the individual changes its state from S1 to I, with probability μ1 [56]. Similarly, S2 transmits information 2 to its neighbors with probability λ2 and becomes ignorant with probability μ2. In addition, we assume that all individuals are stubborn once they make a choice, which means that the spreaders of one information can not be persuaded by their neighbors to support the other one directly [57].

It is worthy of note that there exists a special and important situation where the ignorant is informed of two competitive information at the same time. Previous works studied the scenario where information 1 always has a higher priority [39]. In addition, the social bias caused by neighborhood influence was also proposed and modeled through reducing the infective probability of the competing ideas [40]. Here we use population preference to directly describe the social bias, which is characterized by a new parameter α (0 ≤ α ≤ 1), representing the probability that the ignorant will choose to support and spread information 1 when receives both information simultaneously.

Moreover, we incorporate rewiring mechanism into our model to describe the wide-existing homogeneity trend on large-scale social networks, which is also well-known as ‘echo chamber’ phenomenon. At each time step, the links between S1 and S2 break with probability p. Meanwhile, new links will generate from one of the broken links’ endpoints to the randomly selected individuals who support the same information or in ignorant states. In this way, we mimic the homogeneity trend that individuals tend to avoid further communications with those who hold opposite opinions while prefer to seek for those who support the same information. The rewiring probability p reflects the strength of homogeneity trend among populations.

In summary, our model is composed of two dynamical processes: diffusion process and rewiring process, as shown in figure 1:

(i) **Diffusion process.** At each time step, S1 and S2 transmit information to ignorant neighbors with probability λ1 and λ2, respectively. Meanwhile, S1 and S2 become ignorant again with probability μ1 and μ2. Specially, if an ignorant receives both competitive information simultaneously, it chooses to become S1 with probability α. Otherwise, with probability 1 − α it becomes S2.

(ii) **Rewiring process.** At each time step, the links between S1 and S2 break with probability p. Meanwhile, new links will generate, which connect one of the endpoints of the broken links to the randomly chosen individuals who are ignorant or spread the same information.

3. Theoretical framework

3.1. Modified competitive SIS model with population preference

We first explore how the modified competitive SIS model without homogeneity trend behaves on networks. We adopt the microscopic Markov chain approach to construct the basic theoretical framework [58, 59]. Let \( \tilde{q}_i^S(t) \) denotes the probability that node i changes its state from I to S1 at time t. Based on the description of diffusion process in section 2, \( \tilde{q}_i^S(t) \) can be calculated by

\[
\tilde{q}_i^S(t) = (1 - q_i^{S1}(t))q_i^{S2}(t) + \alpha(1 - q_i^{S1}(t))(1 - q_i^{S2}(t)),
\]

(1)

where \( q_i^{S1}(t) \) and \( q_i^{S2}(t) \) indicate the probabilities that node i is not informed of information 1, not informed of information 2 if i is an ignorant at time t, respectively. Thus, the first term of equation (1) represents the probability that node i is informed of information 1 but not informed of information 2, while the second term expresses the probability that node i receives both information concurrently and choose to spread information 1.
Let $p_i^X(t)$ denotes the probability that node $i$ is in $X$ state at time $t$, $X \in \{S_i, S_2\}$. Then we have
\[
q_i^{S_1}(t) = \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)
\]
\[
q_i^{S_2}(t) = \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right).
\]
(2)

Similarly, the probability that node $i$ in ignorant state becomes a spreader of information 2 at time $t$, denoted as $\tilde{q}_i^{S_2}(t)$, is
\[
\tilde{q}_i^{S_2}(t) = q_i^{S_1}(t)(1 - q_i^{S_1}(t)) + (1 - \alpha)(1 - q_i^{S_1}(t))(1 - q_i^{S_1}(t)).
\]
(3)

Therefore, the evolutionary equations of the competitive SIS model with population preference can be derived by microscopic Markov chain approach, which read as
\[
\begin{align*}
p_i^I(t + 1) &= p_i^I(t)\prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)\prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right) + p_i^{S_1}(t)\mu_1 + p_i^{S_2}(t)\mu_2 \\
p_i^{S_1}(t + 1) &= p_i^I(t)\left(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)\right)\prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right) + \alpha(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right))(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right)) \\
&\quad + p_i^{S_2}(t)(1 - \mu_1) \\
p_i^{S_2}(t + 1) &= p_i^I(t)\left(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)\right) + (1 - \alpha)(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right))(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right)) \\
&\quad + p_i^{S_1}(t)(1 - \mu_2).
\end{align*}
\]
(4)

Finally, substituting equations (1)–(3) into equation (4), the complete evolutionary equations can be written as
\[
\begin{align*}
p_i^I(t + 1) &= p_i^I(t)\prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)\prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right) + p_i^{S_1}(t)\mu_1 + p_i^{S_2}(t)\mu_2 \\
p_i^{S_1}(t + 1) &= p_i^I(t)\left(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)\right)\prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right) + \alpha(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right))(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right)) \\
&\quad + p_i^{S_2}(t)(1 - \mu_1) \\
p_i^{S_2}(t + 1) &= p_i^I(t)\left(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right)\right) + (1 - \alpha)(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_1}(t)\right))(1 - \prod_{j} \left(1 - \lambda_{ij} a_{ij} p_j^{S_2}(t)\right)) \\
&\quad + p_i^{S_1}(t)(1 - \mu_2).
\end{align*}
\]
(5)
Note that the sum of equations (5) satisfies
\[ p_i^1(t + 1) + p_i^6(t + 1) + p_i^5(t + 1) = p_i^1(t) + p_i^5(t) + p_i^5(t), \]
which is constantly equal to 1.

3.2. Competitive information diffusion with homogeneity trend

We then examine the dynamics of competitive information diffusion with homogeneity trend, described by the rewiring process in our model. Note that under this condition, the structure of underlying network evolves over time and is dependent on the current distribution of the population states. On the other hand, the diffusion process also relies on the changing topology of the network. Therefore, we have to consider the coupled evolutions of the nodes’ states and the network structure. Denote \( f_i(A, x) \) as the probability distribution that \( A(t) = A \) and \( x(t) = x \) in the dynamical system. Here \( A(t) = (a_{ij}(t))_{N \times N} \) is the adjacent matrix and \( x(t) = (x_1(t), x_2(t), \ldots, x_N(t)) \) is the state vector of all nodes at time \( t \).

In this section, we give mathematical description of evolution process from time \( t \) to \( t + 1 \), i.e. calculating \( f_{t+1}(B, y) \) for any possible \( A(t + 1) = B = (b_{ij})_{N \times N} \) and \( x(t + 1) = y = (y_1, y_2, \ldots, y_N) \).

Firstly, given a certain system state \( A(t) = A \) and \( x(t) = x \) at time \( t \). The probability that ignorant node \( i \) is not informed of information 1 or information 2 can be written as
\[ q_i^{S_1}(t) = \prod_{i \neq j} (1 - \lambda_i a_{ij}(t) \delta(x_i(t) - 1)), \]
\[ q_i^{S_2}(t) = \prod_{i \neq j} (1 - \lambda_i a_{ij}(t) \delta(x_i(t) - 2)), \]
where \( \delta(x) = 1 \) if \( x = 0 \) and \( \delta(x) = 0 \) if \( x \neq 0 \). Then we can derive the probabilities that node \( i \) changes its state from \( I \) to \( S_1 \) and \( S_2 \), i.e. \( \tilde{q}_i^{S_1}(t) \) and \( \tilde{q}_i^{S_2}(t) \), by substituting equation (6) into equations (1) and (3). Thus, the evolutionary equations of population states read
\[ p_i^1(t + 1) = \delta(x_i(t)) q_i^{S_1}(t) q_i^{S_1}(t) + \delta(x_i(t) - 1) \mu_1 + \delta(x_i(t) - 2) \mu_2, \]
\[ p_i^6(t + 1) = \delta(x_i(t)) \tilde{q}_i^{S_1}(t) + \delta(x_i(t) - 1) (1 - \mu_1), \]
\[ p_i^5(t + 1) = \delta(x_i(t)) \tilde{q}_i^{S_1}(t) + \delta(x_i(t) - 2) (1 - \mu_2). \]

Further, the conditional probability that node \( i \) is in state \( y \) at time \( t + 1 \), which is denoted as \( p^y \), can be written as
\[ p^y = \delta(y) p_i^1(t + 1) + \delta(y - 1) p_i^5(t + 1) + \delta(y - 2) p_i^5(t + 1). \]

Therefore, the probability that nodes’ states evolve from \( x \) to \( y \) is
\[ p_{x \rightarrow y} = \prod_{i=1}^{N} p^y. \]

Let \( p_{A \rightarrow B} \) denote the probability that network structure evolves from \( A \) to \( B \). The number of links between \( S_1 \) and \( S_2 \) at time \( t \) is
\[ L = \frac{1}{2} \sum_{(i,j)} \delta(x_i(t)x_j(t)a_{ij}(t) - 2). \]

At time \( t + 1 \), the number of remaining \( S_1S_2 \) links which is not broken at time \( t \) is
\[ L^b = \frac{1}{2} \sum_{(i,j)} \delta(x_i(t)x_j(t)b_{ij} - 2). \]

According to the rewiring mechanism defined in section 2, we have
\[ p_{A \rightarrow B} = (1 - p)^L \left[ \frac{p}{2(I(t) + S_1(t))} \right]^{L^b} \left[ \frac{p}{2(I(t) + S_1(t))} \right]^{L - L^b - l_1}, \]
where \( l_1 = \sum_{(i,j)} \phi(b_{ij} - a_{ij}(t)) \) for all \( (i,j) \) that satisfies \( \{(i, j) | x_i(t) = 1 \text{ or } x_j(t) = 1\} \), and \( \phi(x) = 1 \) if \( x = 1 \), otherwise \( \phi(x) = 0 \). Here \( l_1 \) calculates the number of new links rewiring from the \( S_1 \) endpoints, i.e. new \( S_1S_2 \) and \( S_2 \) links at time \( t \). Hence, the first formula \( (1 - p)^L \) is the conditional probability that there are \( L^b \) remaining \( S_1S_2 \) links at time \( t \), and the second formula \( \left[ p/(2(I(t) + 2S_1(t)) \right]^{L^b} \times \left[ p/(2(I(t) + 2S_1(t)) \right]^{L - L^b - l_1} \) represents the conditional probability that the other \( L - L^b \) broken \( S_1S_2 \) links rewire, among which \( l_1 \) new links rewire from the \( S_1 \) endpoints. Note that the second formula here is actually an approximation which does not take the multiple edges or loops into consideration, since the real social networks are large and sparse.

Finally, note that the evolution probabilities of nodes’ states and network structure from time \( t \) to \( t + 1 \) are independent. Therefore, summing up all the possible system states at time \( t \), the coupled probability distribution
We start from the Erdős–Rényi (ER) random graphs with $N = 1000$ nodes. The average degree is $\langle k \rangle = 10$ and initially ten $S_1$ and ten $S_2$ are randomly selected. To eliminate the fluctuation, the numerical simulation results in the rest of the paper is the average of 100 times. The stable states of the dynamical system is approximated by the population in the end as the population preference exists is shown in numerical solutions of equations (13).

We set $p = 0$ which excludes the influence of homogeneity trend.

In figure 2(a), we present the effect of diffusion advantage. To eliminate the influence of population preference, $\alpha$ is set to be 0.5. We fix $\lambda_2 = 0.2$, $\mu = 0.2$ and change $\lambda_1$ from 0 to 1. Results show that the competing results experience three stages as $\lambda_1$ increases. Initially when $\lambda_1 \leq 0.16$, $S_1$ becomes extinct in the competition. When $0.16 < \lambda_1 < 0.3$, the proportion of $S_1$ increases sharply and $S_1$, $S_2$ coexist. Finally when $\lambda_1 \geq 0.3$, only $S_1$ survives. Figure 2(b) shows detailed time evolutions of competing results when information 1 has the diffusion advantage and $S_1$, $S_2$ coexist. In figure 2(c), we provide the impact of population preference. We fix $\lambda_1 = \lambda_2 = 0.2$ and change $\alpha$ from 0 to 1. Similarly, $S_1$ first vanishes, then rises rapidly, and dominates the population in the end as $\alpha$ increases. A typical example for the time evolutions of competing results when population preference exists is shown in figure 2(d). Note that in all figures, our theoretical predictions provided by numerical solutions of equations (5) agree well with the simulation results, as shown by dash lines.

Furthermore, in figure 3, we explore the nonlinear joint effects of population preference and diffusion advantage. The phase plane is divided into four regions: (a) only $S_1$ survives, (b) $S_1$ and $S_2$ coexist; $S_1 > S_2$, (c) $S_1 < S_2$, (d) only $S_2$ survives. All separatrix lines are calculated numerically by equations (5). It is noteworthy that when $\lambda_1 < \lambda_2 = 0.2$, i.e. information 2 has the diffusion advantage, the competing results are possible to change from region (d) where only $S_2$ survives to region (a) where only $S_1$ survives as $\alpha$ varies from 0 to 1. This indicates the great power that the newly discussed population preference has on competing diffusion results. On the other hand, when $\alpha > 0.5$, i.e. information 1 has the population preference, the competing diffusion results are also possible to cross the four regions as $\lambda_1$ increases from 0 to 1. It is also worthy of note that there exists a small area where $\alpha < 0.5$ and $\lambda_1 < 0.2$ that belongs to region (c). This indicates that the completely disadvantaged information (information 1), which has neither diffusion advantage nor population preference, can still survive in the competition. In fact, the system has reached to a dynamic equilibrium in this situation, owing to the spontaneous recovery mechanism in SIS model. More detailed analysis can be obtained in figure A1 (appendix A), where we confirm this phenomenon using Monte Carlo simulations and further show that whether the disadvantaged information will survive or become extinct is actually determined by the recovery rates ($\mu$).

4. Results

4.1. The effects of diffusion advantage and population preference

Firstly, we study how diffusion advantage and population preference affect the competing diffusion results. Here the diffusion advantage refers to the difference between transmission probabilities of the competitive information. We set $p = 0$ which excludes the influence of homogeneity trend.

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4.2. The effects of homogeneity trend

In this section, we explore the detailed effects of homogeneity trend on competing diffusion results using Monte Carlo simulations, as stated in the theoretical analysis part. We mainly focus on the following problems: How homogeneity trend influences diffusion advantage as well as population preference? In particular, can homogeneity trend reverse the evolutionary advantage, i.e. completely change the final competing diffusion results when other influencing factors are settled? Specifically, on what conditions the reversing phenomenon may happen?

To begin with, in figure 4, we explore the impacts of different parameters on the relaxation time of system dynamics. In figures 4(a) and (b), we show the impacts of diffusion advantage and population preference under a fixed homogeneity trend ($p = 0.5$), while in figures 4(c) and (d), we change the homogeneity trend under the
existence of diffusion advantage ($\lambda_1 > \lambda_2$) or population preference ($\alpha = 0.55$), respectively. We provide both a detailed view of 30-step evolutions and a global view of 500-step evolutions in each subfigure. Results show that in most situations, the system states become stable within 30−50 steps. While diffusion advantage ($\lambda_1$) and population preference ($\alpha$) have no obvious influence on the relaxation time, the homogeneity trend ($p$) shows non-monotonous effects on both relaxation time and final proportion of $S_2$. In particular, when $p = 0.2$, the system needs much more evolution steps to reach to the stable equilibrium. Finally, we find that all the dynamical evolutions can be steady within 500 time steps, which is exactly the step number we use in simulations to obtain the stable states.
In figure 5, we show how network structure evolves under the influence of homogeneity trend. The rewiring probability \( p \) is set to be 1. We find that the individuals gradually get close to those who spread the same information while leave away from those who support the different information as time goes on. At \( T = 15 \), the whole network is completely divided into two clusters. Within each cluster, only one of the information survives, forming the echo chamber phenomenon which is widespread on social networks. Therefore, the homogeneity trend can significantly affect the network structure and give rise to the formation of echo chambers, which will further influence the information diffusion paths and change the competing results.

We then study how homogeneity trend affects diffusion advantage in figure 6. We fix \( \alpha = 0.5 \) to exclude the influence of population preference. In figure 6(a), the evolutionary advantage, i.e. the final proportion of \( S_1 - S_2 \), is presented under different combinations of \( \lambda_1 \) and rewiring probability \( p \). Results show that the homogeneity trend can change the evolutionary advantage arisen by diffusion advantage to some extent. When \( \lambda_1 \) is slightly larger than \( \lambda_2 \), which is equal to 0.2 in our simulations, the evolutionary advantage first increases and then decreases as the rewiring probability grows. When \( \lambda_1 \) is much larger than \( \lambda_2 \), the evolutionary advantage reduces as \( p \) increases. Figure 6(b) provides a detailed view of figure 6(a), where \( \lambda_1 \) varies from 0.2 to 0.6. We find the proportion of \( S_1 - S_2 \) is always larger than zero. This indicates that without population preference, the homogeneity trend could not reverse the evolutionary advantage caused by diffusion advantage. In figures 6(c) and (d), we show the global and local phase diagram of the proportion of \( S_2 \). As \( p \) becomes larger, the critical value of \( \lambda_1 \) that makes \( S_2 \) extinct exhibits a slow decrease and then a rapid increase, indicating the fact that higher homogeneity trend helps the information with lower transmission probability survive in the competition while lower homogeneity trend accelerates its extinction process. Note that for a large region of figure 6(c), the proportion of \( S_2 \) is 0. However, the corresponding proportion of \( S_1 - S_2 \) in figure 6(a) still cannot reach to 1, as the system actually reaches to a dynamic equilibrium where the ignorants always occupy a fraction of the population. In figures 6(e) and (f), we give two detailed examples of how homogeneity trend affects the effects of diffusion advantage. The proportion of \( S_1 \) and \( S_2 \) are presented as a function of rewiring probability. We set

**Figure 4.** Time evolutions of competitive information diffusion under the influence of homogeneity trend. We show the impacts of different parameters on the relaxation time of system dynamics. In all subfigures, a detailed 30-step evolution is provided along with a global view of 500-step inset. (a) Impacts of diffusion advantage. We fix \( \lambda_2 = 0.2, \mu = 0.2, \alpha = 0.5, p = 0.5 \) and set \( \lambda_1 = 0.225, 0.3, 0.7 \), respectively. (b) Impacts of population preference. We fix \( \lambda_1 = \lambda_2 = 0.2, \mu = 0.2, p = 0.5 \) and change \( \alpha = 0.55, 0.7, 0.9 \), respectively. (c), (d) Impacts of homogeneity trend. We change \( p = 0, 0.2, 1 \), respectively. Other parameters are as follows: (c) \( \lambda_1 = 0.225, \lambda_2 = 0.2, \mu = 0.2, \alpha = 0.5 \) and (d) \( \lambda_1 = \lambda_2 = 0.2, \mu = 0.2, \alpha = 0.55 \).
λ₁ = 0.225, 0.3, respectively. To sum up, low homogeneity trend enhances diffusion advantage in competitive information diffusion while high homogeneity trend reduces it.

Next, we discuss the impact of homogeneity trend on population preference in figure 7. We set λ₁ = λ₂ = 0.2 to exclude the effects of diffusion advantage. Due to the symmetry, we assume α ∈ [0.5, 1]. In figure 7(a), we present the evolutionary advantage under different combinations of population preference α and rewiring probability p. When α is slightly larger than 0.5, the evolutionary advantage first rises and then declines as p becomes larger. When α is much larger than 0.5, the evolutionary advantage reduces as p increases. While the homogeneity trend does change the evolutionary advantage, the proportion of S₁–S₂ is always larger than zero. This indicates that without the existence of diffusion advantage, the evolutionary advantage solely caused by population preference can not be reversed by effects of homogeneity trend. Figure 7(b) further presents the phase diagram for the proportion of S₂. The critical value of α which results in the extinction of S₂ displays a gradual decrease followed by a sharp increase as the rewiring probability grows, which reveals that lower homogeneity trend accelerates the extinction of information with lower population preference while higher homogeneity trend helps it survive from the competition. In figures 7(c) and (d), two typical examples of how homogeneity trend affects the effect of population preference are given. We set α = 0.55, 0.7, respectively. Results verify that lower homogeneity trend promotes population preference effects while higher homogeneity trend suppresses it.
To give a more intuitive illustration, in figure 8, we further provide snapshots of dynamical networks with different homogeneity trends under the existence of diffusion advantage (figures 8(a)–(c)) or population preference (figures 8(d)–(e)). When $p = 0$, i.e. there is no homogeneity trend, $S_1$ and $S_2$ coexist when the system becomes stable and the network structure is invariant (see figures 8(a) and (d)). When $p = 0.2$, however, the disadvantaged information goes extinct and the advantaged one dominates the population. Meanwhile, the final network structures show no significant difference to the previous situations where $p = 0$ (see figures 8(b) and (d)). Finally, when $p = 1$, the two pieces of competitive information coexist again. Moreover, the network structures are totally changed and show explicit clustering characters. We observe the emergence of two polarized echo chambers, within which only one type of information exists.

Through observing the significant variations of evolutionary advantage and network structures, we have examined the detailed effects of homogeneity trend on diffusion advantage and population preference. For a better understanding of these non-monotonous impacts and based on the insights from figures 6–8, here we provide further explanations from the perspective of the underlying dynamical mechanisms. In general, the homogeneity trend, which is modeled as the rewiring process, plays two main roles in competitive information diffusion processes. One is to advance people to avoid ineffective communications with opposite-minded individuals and to diffuse their information to ignorant or like-minded agents, which improves the efficiency of information diffusion and thus enhances the evolutionary advantage. The other one is to form clusters of like-minded individuals, which separates the competing information into different communities and hence protects the information in disadvantage. Basically, the competing diffusion result is the balance of these two impacts of homogeneity trend. When $p$ is small, the clustering speed is relatively slow compared to the information diffusion. In this situation, the role of improving the efficiency of communication dominates, which promotes the evolutionary advantage (see figures 8(b) and (e)). On the other hand, when $p$ is large, the role of clustering takes the domination which facilitates the formation of two echo chambers, helping the disadvantaged information survives (see figures 8(c) and (f)). Under this circumstance, the evolutionary advantage is reduced. However, for now, the evolutionary advantage can not be reversed by homogeneity trend when it is caused by a single competing factor, regardless of diffusion advantage or population preference. Naturally, we then raise an interesting question which
leads to more complicated situations: if the evolutionary advantage emerges under the circumstances where the initial winning information owns population preference but has no diffusion advantage, or it takes diffusion advantage but loses population preference, can homogeneity trend reverse the competing diffusion results?

In view of this question, in figure 9, we further explore the reversing conditions in competitive information diffusion. First, in figures 9(a)–(c), we examine the situation where information 1 takes diffusion advantage while loses population preference. We fix $\lambda_2 = 0.2, \mu = 0.2, \alpha = 0.3$ and change $\lambda_1$ from 0.2 to 1. Figure 9(a) shows that homogeneity trend can reverse the competing diffusion results, i.e. make $S_1 \rightarrow S_2$ change from negative to positive for a range of $\lambda_1$: $\lambda_1 \in [0.227, 0.33]$. To give a clear sight of this reversing phenomenon, in figures 9(b) and (c), we show the variations of $S_1$ and $S_2$ as a function of rewiring probability $p$ in two detailed examples, where $\lambda_1 = 0.25, 0.3$ respectively. Results show that as $p$ increases, information 1 goes through losing to winning, becoming stably advantaged when $p$ is large. Then in figures 9(d)–(f), we study the circumstance where information 1 owns population preference but has no diffusion advantage. In figure 9(d), when $\alpha > 0.745, S_1 \rightarrow S_2 > 0$ initially which corresponds to the reversing situation discussed in figures 9(a)–(c). Therefore here we focus on $\alpha < 0.745$ in which situation the initially winning information (information 2) has diffusion advantage but loses population preference. Results show that the reversing phenomenon does not happen any more and information 2 always wins as $p$ increases. Two typical examples are shown in figures 9(e) and (f), where $\alpha = 0.745$ and 0.7, respectively. More parameter combinations are verified in appendix B for this circumstance and no reversals are found. To sum up, homogeneity trend is possible to reverse the evolutionary advantage in competitive information diffusion, but only if the initial winning information owns population preference but has no diffusion advantage. In other words, the only chance for the disadvantaged information to win is to occupy the diffusion advantage on large-scale social networks, regardless of the population preference.

5. Conclusions and discussions

Competitive information diffusion is ubiquitous on online social networks, which directly influences the formation of public beliefs. For a better understanding of many real-world challenges such as rumor contagions and political social polarization, great effects have been made in exploring the underlying dynamical mechanisms for competing diffusion processes in recent years [60]. However, there still lacks a proper understanding of how homogeneity trend, which is regarded as one of the most important collective human behaviors on large-scale social networks, affects the diffusion results of competitive information [61].

In this work, we propose a modified competitive SIS model, which incorporates homogeneity trend and generalized population preference to describe the simultaneous spread of two pieces of competitive information on large-scale social networks. Firstly, we examine how the modified model behaves without homogeneity trend.
Using microscopic Markov Chain approach, we derive the phase diagram of competing diffusion results to show the impacts of diffusion advantage (i.e., the difference of transmission abilities of competitive information) and population preference. Then we explore the detailed effects of homogeneity trend, which is characterized by a rewiring mechanism, by examining both the time evolutions and competing results of the system dynamics. When homogeneity trend is strong, the network structure evolves over time and finally forms two divided clusters, i.e., echo chambers, within which only one of the information survives. Results show that homogeneity trend can significantly change the evolutionary advantage. Lower homogeneity trend accelerates the extinction of disadvantaged information which enhances the evolutionary advantage, while higher homogeneity trend helps the disadvantaged information survive from the competition which reduces the evolutionary advantage. Further, we highlight the conclusion that homogeneity trend can even reverse the evolutionary advantage, but only when the initially disadvantaged information takes diffusion advantage while loses population preference. This indicates that the best chance for the information to win the competing evolution, either to secure the advantage or to reverse the disadvantage, is to occupy the diffusion advantage regardless of population preference.

Our work shows how homogeneity trend makes non-monotonic influence on competitive information diffusion, which provides important insight into misinformation spreading, public opinion formation and many other competing dynamical processes on social networks. Although our theoretical framework is based on the simple epidemiological contagion model, it reproduces and provides underlying dynamical explanations on some interesting social phenomena observed in empirical data, such as the emergence of echo chambers [62, 63], and the reversal of evolutionary advantage [64]. A well-known reversal case would be the shifts in the popularity of different online social networks in the past ten years, such as Facebook’s overtake of Myspace, and the collapse of iWiW [65, 66]. Specifically, iWiW used to be the most popular online social network in Hungary and owed great competitive advantage before 2010, while subsequently Facebook joined the competition and achieved the dramatic reversal [67]. Considering the rapid evolutions of competitive industry or Internet products nowadays, the reversing condition in our work may shed light on designing effective competing strategies in a large range of real situations, such as spreading new ideas, promoting industrial products and doing marketing. Further studies may focus on complex agent-based modeling that can better mimic human behaviors to reveal more precise reversing conditions in competing dynamics based on real data.

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Appendix A. Complementary studies for figure 3

In figure A1, we explore the detailed competing diffusion process under the situation where information 1 has neither diffusion advantage nor population preference using Monte Carlo simulations. Here we fix $\alpha = 0.45$, $\lambda_1 = 0.19$, $\lambda_2 = 0.2$ and $p = 0$. In figure A1(a), we set $\mu = 0.2$ and present the competing results of 500-step evolutions. We find that the completely disadvantaged information (information 1) can still survive and coexist with the advantaged one when the evolutions become stable, which indicates that the system actually reaches a dynamic equilibrium. Considering the spontaneous recovery mechanisms in SIS model, we further examine the effects of recovery rate on final competing results in figure A1(b). We find that the completely disadvantaged information (information 1) successfully survives at $\mu = 0.05$ and $\mu = 0.2$ while goes extinct at $\mu = 0.4$. We conclude that the relative relationship between diffusion rates and recovery rates greatly influences the final position of the dynamical equilibrium in our modified competitive SIS model, which determines the diffusion fate of the disadvantaged information (either survives or becomes extinct).

Appendix B. Complementary studies for reversing conditions

In figure B1, we explore different parameter combinations to check whether reversals may happen under the circumstance that the initial winning information takes diffusion advantage but has no population preference. In figures B1(a) and (b), we fix $\mu = 0.2$ and study the situations where the difference between transmission probabilities of the competitive information (i.e. $\lambda_2 - \lambda_1$) is small, which are necessary complements for the results in figure 9(d). Furthermore, in figure B1(c), we change $\mu$ to 0.1 to check the influence of recovery rate in competing diffusion processes. Results show that no reversal happens when the initial winning information takes diffusion advantage, regardless of the population preference.

Figure A1. Time evolutions of competitive information diffusion where information 1 has neither diffusion advantage nor population preference. We fix $\alpha = 0.45$, $\lambda_1 = 0.19$, $\lambda_2 = 0.2$ and $p = 0$. (a) We set $\mu = 0.2$ and present 500-step evolutions. (b) Effects of recovery rate on final competing results. We set $\mu = 0.05, 0.2, 0.4$ respectively and present proportion of $S_1$ as a function of time.

Figure B1. Exploring whether homogeneity trend can reverse the competing diffusion results when the initial winning information takes diffusion advantage but has no population preference. We present phase graphs of evolutionary advantage under different parameter combinations: (a) $\mu = 0.2, \lambda_1 = 0.36, \lambda_2 = 0.4$; (b) $\mu = 0.2, \lambda_1 = 0.19, \lambda_2 = 0.2$; (c) $\mu = 0.1, \lambda_1 = 0.19, \lambda_2 = 0.2$. 

13
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