Strength Degradation and Life Calculation of Parts under Complex Loads

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Abstract. For components, the strength will decrease under the action of multiple loads. In this paper, based on the equivalent relationship of the load, a strength degradation model of the part under multi-stage load is established. This model solves the problem of calculating the remaining life of the part after multiple loads, and can guide the replacement or maintenance of the part during use.

Keywords: Deterioration of component strength; Multi-stage complex load; Remaining life.

1. Introduction
During the working process, the material strength of the component is continuously degraded, and the remaining strength is continuously reduced. When the remaining strength is equal to or less than the cyclic stress level, fatigue fracture occurs. The strength degradation of a part is related to the load it receives. During the work, the part is not only subject to the working load, but also to the impact load caused by the structural characteristics of the part itself. If the transmission wheel is subject to cyclic working load, but also due to the impact load caused by processing or traditional equipment characteristics, how to consider the impact of various loads on the strength of this article.

At present, there are many studies on the strength degradation of parts. References [1-2] considered the degradation of strength. The maximum stress theory was used in the stress analysis to fully consider the effect of load on strength. References [3 4] carried out research on the influence of strength degradation and fatigue structure reliability based on the stress-strength degradation interference failure model. References [5] carried out research on random wave charge, and simplified each Gaussian process for each random wave. References [6] used the Gamma process to describe the strength degradation law of each part in the main shaft system of the mine hoist. References [7] considered both load and strength as a stochastic process, and used sequential statistical theory to obtain the maximum equivalent load. References [8] used random variables to describe the uncertainty of the initial intensity, and called the evolution process of intensity degradation the gamma process. References [9] propose a method for cylinder fatigue analysis and life prediction based on finite element analysis and linear cumulative damage theory. References [10] proposed an embedded interference optical fiber strain sensor A general method for assessing the mechanical strength degradation of a material or component in situ. The above research is mainly based on the degradation or fatigue life research under a single load. However, in the actual working process, the parts are not only subject to the working load but also the random load due to the structural characteristics. In actual work, it is desirable to obtain the strength degradation under various loads and the remaining working time of parts. Based on this requirement, this article considers the joint effects of the stable working load and random load on the part during the work process, and establishes a model of the strength degradation of the part in a multi-stage working environment. Based on this model, a part is built into a multi-stage working environment. After the remaining life model.
the process of establishing this model, the form of random load was described by the maximum strength theory.

2. Determination of Part Load
In addition to the normal working load of the part during the work process, due to accidental factors such as shock and vibration, the part will also bear a random load generated by the outside world. This load often acts less frequently and has high strength. It is assumed that the working stress $S_1$ of the part is a stable load, and the operating frequency is high and stable. $S_2$ is the maximum stress generated by the impact load, and the total load $S = S_1 + S_2$ when two stresses occur simultaneously. Suppose that the number of occurrences of stress $S_1$ within time $t$ is $n_1 = t \times f_1$, and $f_1$ is the number of occurrences per unit time of $S_1$; the number of occurrences of stress $S_2$ is a Poisson process with intensity $\lambda(t)$. It is determined that different external factors correspond to different strength values, and the number of times of occurrence of load $\lambda(t)$ is also determined by external factors. There are corresponding values for different stress values $S_2$, and the number of occurrences of $S_1$ is much greater than the number of occurrences of $S_2$.

$$S_2 = S_1, S_n = S_1 + S_2$$ (1)

By satisfying the Poisson distribution with a random load, the probability that the stress $S_2$ appears once in $t \rightarrow t + \Delta t$ is: $P = \lambda(t)\Delta t$, so that the times $N_a$ and $N_b$ in which stress $S_a$ and $S_b$ appear in $t \rightarrow t + \Delta t$ are:

$$N_a = f_1\Delta t - \lambda(t)\Delta t; N_b = \lambda(t)\Delta t$$ (2)

The working stress $S_1$ is calculated by a mechanical analysis method according to the working condition of the part, and the occurrence frequency $f_1$ is determined according to the working frequency of the part. The stress value of $S_2$ and the intensity of the number of times of action $\lambda(t)$ are all obtained through statistical methods such as rain flow method, peak count method, vibration count method, or penetration count method, which are collected from various impact loads during the working process.

3. Intensity Degradation Model
According to the characteristics of the fatigue damage process, the strength degradation model can be as follows:

$$d\sigma(n)/dn = -S^p \sigma^{-q}(n)$$ (3)

In the formula, $\sigma(n)$ is the residual strength of the material for the $n$th cycle; $n$ is the number of cycles; $S$ is the cyclic stress level; $p$ and $q$ are material constants. By integrating equation (3), the strength becomes $\sigma_n$ when the part is applied $N$ times. Because $\sigma(n)$ should satisfy the boundary conditions, $\sigma_f$ is the true static tensile fracture strength of the material in the initial undamaged state, and $N$ is the fatigue life under a given stress. When $n=N$, we get:

$$S^{1+q} = \sigma_f^{1+q} - (1+q)S^p N$$ (4)

It can be verified that the strength degradation model meets the thermodynamic irreversible and physical conditions of damage. In formula (4), $S^{1+q}$ is much smaller than $\sigma_f^{1+q}$, and the difference can reach several orders of magnitude\[11\]. Therefore, $S^{1+q}$ can be ignored, so that:

$$S^p N = \sigma_f^{1+q}/(1+q)$$ (5)

Equation (5) is an $S-N$ curve in the sense of mean value, thus verifying that the equation is correct. Because the traditional $S-N$ curve is: $\sigma^n, n = C$, the parameter $m$ can be selected according to the different parts, and the parameter $C$ can be calculated from the ultimate strength under infinite life. However, there are two unknowns of the intensity degradation formula, $p$ and $q$. First of all, it is thought that the parameters in the intensity degradation model can be determined according to the $S-N$ curve. However, the $S-N$ curve of the part is obtained through a large number of $P-N$ tests, which can’t be
checked in the existing data. In order to reduce the cost of the test, this paper uses symmetrical cyclic stress fatigue limit $\sigma_r$ in the fatigue limit diagram of a part under infinite life (that is, the number of stress actions is $10^7$) and Strength limit of parts under static stress $\sigma_f$ to calculate the parameters $p$ and $q$. It can be calculated by looking up the table to reduce the cost of the test. The fatigue curve obtained from the fatigue test of the actual part includes the strength of the material, dimensional factors, surface shape, stress concentration, and the influence of changes in working conditions, which basically reflects the actual state of the part and the characteristics of the strength and stress. However, it is impossible to directly perform the fatigue test for ordinary parts. Not only is the cost high, but also the test often encounters great difficulties and even causes errors in the test data. Therefore, under normal circumstances, non-standard parts are tested by using the standard sample of the material, and then the fatigue limit diagram of the standard sample is converted into the fatigue limit diagram of the specific part.

3.1. Strength Analysis under First-level Working Load

Due to the combined effect of working load and impact load in the same working environment, the failure of strength is also the combined effect of these two loads. Since the number of times of impact load is much smaller than the working load, this paper converts the working stress under the impact load into a stress analysis in the case of only the working load. This takes into account the impact of the impact load on the strength and makes the calculation simple To facilitate engineering applications. After the working stress $S_b$ is applied $n_b$ times, it is equivalent to the stress $S_a$ is applied $n_a$ times. It is obtained that when the stress $S_b$ is applied once, it is equivalent to the number of times that the stress $S_a$ is applied $N_1$.

$$\Delta N = \Delta N_a + \Delta N_b \times N_1 = f_1 \cdot \Delta t - \lambda(t) \Delta t + S_a^p / S_a^p \cdot \lambda(t) \Delta t = \left[ f_1 + \left( S_a^p / S_a^p - 1 \right) \lambda(t) \right] \Delta t$$

(6)

From equation (6), we can get the residual strength degradation function under the load in $N \rightarrow N + \Delta N$, and then we can establish the residual strength in time as:

$$\sigma^{1+q}(N + \Delta N) - \sigma^{1+q}(N) = -(1 + q) S_a^p \left[ f_1 + \left( S_a^p / S_a^p - 1 \right) \lambda(t) \right] \Delta t$$

(7)

Since the relationship between the number of times of load and time is $\Delta N = f_1 \Delta t$, $\sigma(n)$ should satisfy the boundary condition: $\sigma(0) = \sigma_f$, and the integral can be obtained:

$$\sigma^{1+q}(n) = -(1 + q) S_a^p \left[ n + \left( S_1 + S_2 \right)^p / S_a^p - 1 \right] \int_0^{n/f_1} \lambda(t) \lambda(t) dt + \sigma^{1+q}$$

(8)

3.2. Residual Strength Analysis under Secondary Working Load

Sometimes the working environment of the parts will change, such as speed and torque. At this time, the stable working stress $S_1$ that the part is subjected to changes, and the random load also changes with the working environment. For example, the accuracy of the products processed by the processing machine at different speeds is different, so the machine tool will choose the corresponding processing speed according to the accuracy requirements of the parts, which results in a diversified working environment of the calculated spindle and other components. It is inaccurate to analyze the degradation of the strength of the part under such a working environment. In this paper, the strength degradation model of a part is first established under a two-stage working load (related to the working environment and the working force of the part, etc.). Assume that the part works $N$ times under the condition of stable working stress $S_1$ and impact stress $S_2$. It is assumed that the working frequency of the stable load is $f_1$, and the number of times of the impact load satisfies the Poisson distribution of strength $\lambda(t)$. The stress generated by the primary load under the common action of the load is $S_b$, and the stress under the primary action of the stable working load is $S_a$. Then after working $M$ times with stable strength $R_1$ and impact strength $R_2$, the remaining strength of the part is a problem. Assume that the working frequency of the stable load is $f_2$, and the number of times of the impact load satisfies the Poisson distribution of strength 3.
The stress generated by the primary load under the combined action of the impact load and the stable load is $R_b$, the stress under a steady working load is $R_a$.

From equations (5) and (8), the number of operations under the first-level workload is equivalent to the number of operations $n_1$ under $S_1$ and the remaining life is:

$$n_1 = N + \left[ \left( S_1 + S_r \right) / S^p_r - 1 \right] \int_0^{N/\lambda_a} \lambda_a(t) dt$$

$$\sigma^{1+q}(n_1) = - (1 + q) S^p_r \cdot \left\{ N + \left[ \left( S_1 + S_r \right) / S^p_r - 1 \right] \int_0^{N/\lambda_a} \lambda_a(t) dt \right\} + \sigma^{1+q}$$

It is based on the equivalent number of operations $n_1$ under the first-level workload, which is equivalent to the number of operations $N_1$ under $R_1$ under the second-level workload. Under the second-stage working load, a single action in $R_b$ is equivalent to $R_a$, and the number of times of operation is $N_1'$. The number of times of work is equivalent to the number of times of work under the load $R_1$ is $N_2$. The residual strength of the part after two-stage working load is established according to formula (10) is:

$$\sigma^{1+q}(n) = - (1 + q) R^p_a \cdot \left\{ M + \left[ \left( R_1 + R_r \right) / R^p_r - 1 \right] \int_0^{N/\lambda_a} \lambda_a(t) dt \right\} + \frac{S^p_a}{R^p_a} \left\{ N + \left[ \left( S_1 + S_r \right) / S^p_r - 1 \right] \int_0^{N/\lambda_a} \lambda_a(t) dt \right\} + \sigma^{1+q}$$

3.3. Analysis of Residual Intensity in Multi-level Working

Assume that the part is subjected to $n$-level working environment. The working stress of this $n$-level working environment is $(S_1, S_2, S_3, \cdots, S_n)$ and the number of actions is $(m_1, m_2, m_3, \cdots, m_n)$. The corresponding impact load stress is respectively $(R_1, R_2, R_3, \cdots, R_n)$, and the number of impact load occurrences respectively satisfies a Poisson distribution with a strength of $(\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_n)$, thereby establishing the residual strength of the part under the action of $n$-level load.

Under the working environment of all levels, the combined action of the stress $X_i$ and impact load $R_i$ and steady load $S_i$ is equivalent to the number of times that the stable load $S_i$ acts $N_i$ is:

$$N_i = X^p_i / S^p_i = (S_i + R_i)^p / S^p_i$$

It can be obtained that after $n$-level load, the number of actions equivalent to the load $S_n$ is:

$$N_n = m_n + \left[ \left( S_n + R_n \right) / S^p_n - 1 \right] \int_0^{\infty/\lambda_n} \lambda_n(t) dt + \sum_{i=1}^{n-1} \frac{S^p_i}{S^p_n} \left\{ m_i + \left[ \left( S_i + R_i \right) / S^p_i - 1 \right] \int_0^{\infty/\lambda_i} \lambda_i(t) dt \right\}$$

Thus, the remaining strength after loading $n$ times is:

$$\sigma^{1+q}(n) = - (1 + q) S^p_n \cdot \left\{ m_n + \left[ \left( S_n + R_n \right) / S^p_n - 1 \right] \int_0^{\infty/\lambda_n} \lambda_n(t) dt + \sum_{i=1}^{n-1} \frac{S^p_i}{S^p_n} \left\{ m_i + \left[ \left( S_i + R_i \right) / S^p_i - 1 \right] \int_0^{\infty/\lambda_i} \lambda_i(t) dt \right\} \right\} + \sigma^{1+q}$$

4. Estimation of Component Life

In general, the safety of parts can be checked based on the calculated residual strength. However, in many cases, people want to know how long (number of times) the part can work after this period of work. This is to ask us to calculate the remaining life of the part based on the remaining strength. The problem of remaining life is mainly divided into the following two categories. One is the number of times the part is left under a single load, and the number of times the second type of part is under a complex load.

4.1. Residual Life under A Single Load

According to formula (5), under this single load $S$, the working life of the part is:

$$N = \left( \sigma^{1+q} - S^{1+q} \right) / [(1 + q) S^p]$$

According to formula (13), the number of times of the previous working load is equivalent to $S$, and the total number of times of work is $N_n$. The remaining life under this load is $M$ as:
4.2. Part Life under Complex Loads

General parts work in multiple environments, each environment corresponds to a working load and impact load. When the life of the part is calculated, the strength degradation formula is used to calculate the remaining strength of the part after each level of load. Calculate the strength of the part after the load is greater than the stress generated by the load, that is, the part is considered safe under the load of this level. If the strength degradation value of the part after the load of this level is calculated is less than the stress generated by the load, it is considered that the part has failed during the action of the load of this level. It is necessary to calculate how many times the part fails after the load is applied. And then the total number of work times is the sum of the load work times at all levels, that is, the number of times the part can work before it fails.

After knowing that the part has experienced $n$-level loads, it is asked whether the part can experience $A$ strengths $x_i$, number of operations $h_i$, and operating frequency $\omega_i$. And there is a maximum random load with a strength of $y_i$, and the number of actions satisfies a Poisson distribution with a strength of $\mu_i(t)$. The calculation process is as follows,

(1) Calculate the degradation value after the $a$-th working cycle: $\sigma(n + \sum_{i=1}^{a} h_i)$

(2) Judging the relationship between the strength degradation value and stress, if $\sigma(n + \sum_{i=1}^{a} h_i) > (x_i + y_a)$, go to (1) $a = a + 1$, continue to calculate, if $\sigma(n + \sum_{i=1}^{a} h_i) \leq (x_i + y_a)$, go to 3)

(3) Calculate how many times the part fails when working under a-level load. Using formula (15) to calculate the working life of the part under a-level load:

$$N = \left( \sigma_{i}^{-1+q} - S_{i}^{1+q} \right) \left[ (1+q)S_{i}^{-p} \right]$$

Calculate the number of times that the part has experienced $n$-level load before, and the $a$-1 level load equivalent to the $a$-level load that it has experienced is

$$N_a = \sum_{i=1}^{a} \frac{S_i^p}{x_i^p} \left[ m_i + \frac{(S_i + R_i)^p}{S_i^p} - 1 \right] \int_{0}^{h_i} \lambda_i(t) dt + \sum_{i=1}^{a} \frac{x_i^p}{x_i^p} h_i + \frac{(x_i + y_i)^p}{x_i^p} - 1 \int_{0}^{h_i} \mu_i(t) dt$$

After experiencing $n$-level loads, the parts can work under complex loads as follows:

$$M = \sum_{i=1}^{n} h_i + N - N_a$$

5. Examples

After 30CrMnSiA steel is used for the parts, the dimensional coefficient, surface processing coefficient, and stress concentration coefficient of the parts are fully considered, and the strength degradation coefficients of the parts are $p=8.9358$ and $q=7.5435$. The initial strength of the part is equal to the strength of the material as $\sigma_f = 491.2143$MPa. The stress magnitude of this part under two levels of working load $S$, working frequency $f$, working time $t$, maximum stress $R$ of random load, and Poisson parameter $\lambda_i$ that the number of occurrences of random load satisfy are shown in Table 1, respectively.

After the load, the residual strength value of the part, and the remaining life of the part under the third-level load is calculated. The form of the third-level load shown in Table 1.
Table 1. The role of the multi-level load.

| Load level | Load level | $S$/MPa | frequency $f$/1/h | time $t$/h | $R$/MPa | $\lambda$/1/h |
|------------|------------|---------|------------------|-----------|---------|-------------|
| I          | 80         | 80      | 500              | 30        | 5.8     |
| II         | 70         | 90      | 500              | 25        | 5       |
| III        | 90         | 50      |                  | 28        | 7       |

(1) Calculate the residual strength after two-stage load
The number of times that the working load $S_i$ and the maximum random load $R_i$ in the first and second loads are equivalent to the working load $S_i$ are once:

$$N_1 = \frac{(S_i + R_i)^{\sqrt{\frac{80}{80.9358}}}}{S_i} = (80 + 30)^{0.9358}/80 = 17.2274$$

$$N_2 = \frac{(S_2 + R_2)^{\sqrt{\frac{70}{80.9358}}}}{S_2} = (70 + 25)^{0.9358}/70 = 15.3149$$

The number of actions under the first and second loads is:

$$\bar{m}_1 = m_1 + [N_1 - 1]\int_0^{1/5} \lambda_1(t) dt = 80 \times 500 + [17.2274 - 1] \times 500 \times 5.8 = 8.7059 \times 10^4$$

$$\bar{m}_2 = m_2 + [N_2 - 1]\int_0^{1/5} \lambda_2(t) dt = 90 \times 500 + [15.3149 - 1] \times 500 \times 5 = 8.0799 \times 10^4$$

The equivalent number of times when the first load is equivalent to the second load is:

$$M_1 = S_1^{\sqrt{\frac{80}{80.9358}}} \cdot \bar{m}_1 = 80^{0.9358} \times 8.7059 \times 10^4 = 2.8709 \times 10^5$$

The combined effect of the first two loads is equivalent to the number of the second load:

$$M_2 = M_1 + \bar{m}_2 = 2.8709 \times 10^5 + 8.0799 \times 10^4 = 3.6789 \times 10^5$$

The remaining strength of the part is:

$$\sigma(M_2) = \frac{1}{1 + 7.5435} - \frac{1}{1 + 7.5435} \times 70^{0.9358} \times 3.6789 \times 10^5 + 1088.9^{1 + 7.5435} = 307.8219$$

(2) Calculate the remaining life under the third load after two-stage load
The equivalent number of times of the first two loads to the third load is:

$$N_3 = \frac{(S_i + R_i)^{\sqrt{\frac{80}{80.9358}}}}{S_i} = (90 + 20)^{0.9358}/90 = 11.2513$$

The number of loads of the third load is equivalent to the number of times that the load $S_3$ acts

$$N = f_3 + [N_3 - 1]\int_0^{1/7} \lambda_3(t) dt = 50 + [11.2513 - 1] \times 7 = 121.7591$$

Calculate the total number of times the part is under load $S_1$ and the remaining working time is:

$$N = (\sigma^{\sqrt{\frac{8}{8.5435}} - S_1^{\sqrt{\frac{70}{80.9358}}}) \times [(1 + q)S_1^{\sqrt{\frac{80}{80.9358}}}] \times (491.2134^{1 + 7.5435} - 90^{1 + 7.5435}) / (1 + 7.5435) \times 3.9675 \times 10^4$$

$$t = (N - M_1) / N = (3.9675 \times 10^4 - 3.8943 \times 10^4) / 121.7591 = 6.5046k$$

(3) Verification
After 7.5 hours of work under the III-level load, the parts showed obvious cracks through X-ray inspection, which did not meet the requirements for use. Comparing the actual working time of the parts and the calculation results of the model, the calculation results of this paper are more safe and meet the engineering requirements.

6. Conclusion
(1) This article considers the impact of parts on the combined effects of stable and random loads. A strength degradation model of a part under a multi-stage load environment is established. In the process of establishing the model, since the number of random loads is much smaller than the number of working loads, the maximum strength theory is used to represent the form of random loads. The calculation results are safe and meet the actual engineering requirements.

(2) The analysis of the model shows that under the effect of the strength degradation coefficient $p$, the random load and the working load work together, the random load is more harmful to the part, and the influence of the random load must be considered in the part design process.
(3) Taking a certain part as an example, the calculation shows that the method is simple to calculate, does not require complicated tests and calculations, and can use existing data for analysis to facilitate engineering applications. The calculation of the remaining life of parts provides us with reference and basis for replacement and repair of parts.

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