Transverse $\tau$ polarization in $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay

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Abstract

The possible transverse polarization of the $\tau$-lepton, which is $T$-odd and can be used to measure $CP$-violation, is estimated precisely in the radiative decay mode $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ with possible parameters for the multi Higgs doublet model and the R-parity violating supersymmetric model. We find that the up-bound of the transverse polarization with possible parameters in the models is at very different levels i.e. it can be $\leq 15 \sim 23\%$ for the former but $\leq 0.1\%$ for the latter, and it is accessible experimentally soon.
I. INTRODUCTION

The $B_c$ meson has attracted special attention as it contains two heavy quarks and its decay channels are very rich compared to those of $B_q$ ($q = u, d, s$) mesons. Its production, spectroscopy and different decay channels are widely discussed in the current literature. Recently, it has been observed in 1.8 TeV $p\bar{p}$ collisions using the CDF detector on the Fermilab Tevatron and the values: mass $M_{B_c} = 6.40 \pm 0.39 \pm 0.13$ GeV and lifetime $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03$ ps have been obtained. Further detailed experimental studies will be performed on Tevatron Run II and CERN LHC, which is the motivation for extensive theoretical studies of this system. In particular, on LHC with luminosity $\mathcal{L} = 10^{34} cm^{-2}s^{-1}$ and $\sqrt{s} = 14$ TeV, the number of $B_c^\pm$ events is expected to be about $10^8 \sim 10^{10}$ (or even more), so some interesting rare decays could be studied experimentally in the foreseen future.

The comparatively ‘long’ lifetime of the $B_c$ meson is related to the fact that it can decay only weakly, thus providing the rather unique opportunity of investigating weak decays in a heavy quarkonium-like system. Its weak decay may be due to three ‘components’:

i. The $b$ quark decays with the spectator $c$ quark (e.g. in the decay $B_c \rightarrow J/\psi \ell \bar{\nu}_\ell$ etc);

ii. The $c$ quark decays with the spectator $b$ quark (e.g. in the decay $B_c \rightarrow B_s \ell \bar{\nu}_\ell$ etc);

iii. The annihilation of $b$ and $c$ (e.g. the decay $B_c \rightarrow l^+ \nu_l, (l = e, \mu, \tau)$ etc).

Sometimes there is interference of two or three of the above processes in certain nonleptonic decays.

As far as the annihilation processes are concerned, the pure leptonic decay to $e^+$ or $\mu^+$, due to helicity suppression, represents a minor fraction of the $B_c$ full width but the decay to $\tau$ does not. Whereas all the pure leptonic decays are the simplest and can be used in determining the decay constant $f_{B_c}$. Thus in fact it is very difficult to use the $B_c \rightarrow l^+ \nu_l, (l = e, \mu)$ decay modes to determine $f_{B_c}$, but it is possible in principle to use the channel $B_c \rightarrow \tau^+ \nu_\tau$ if we have good efficiency in the detection of $\tau$-leptons.

The helicity suppression can be avoided, if there is a third particle in the final state. In particular, ‘adding a photon’ to the corresponding pure leptonic decay does not appreciably change the fact that the decay ratio is proportional to the decay constant $f_{B_c}$, but the decay is changed to a ‘radiative one’. The radiative leptonic decay modes $B_c \rightarrow \ell^+ \nu_\ell \gamma$ have already been studied using the quark models, light cone QCD and effective field theory.

In this paper we intend to study the $B_c \rightarrow \tau^+ \nu_\tau \gamma$ decay mode. This mode is of particular interest because it can provide quite a sensitive test of certain theories of $T$ or $CP$ violation. Since the transverse $\tau$ polarization is proportional to $(\vec{p}_\tau \times \vec{k}) \cdot \vec{s}_\tau$, where $\vec{p}_\tau$ and $\vec{k}$ are the momenta of the $\tau$ and photon particles, respectively, and $\vec{s}_\tau$ is the spin of $\tau$, it is an odd quantity under time reversal. The standard model (SM) has only negligible contribution to the polarization in the decay mode, therefore measurement of the transverse $\tau$ polarization will reveal to us possible new sources of $T$ or $CP$ violation beyond SM (assuming $CPT$ symmetry). Namely, many models (beyond SM) can contain certain new physical phases which induce $CP$ violation, and measurement of the transverse $\tau$ polarization in this decay mode is possible ($\tau$’s lifetime in the order of 0.3 ps and weak interaction decay). If a nonzero value of the polarization is observed, it will be a clear indication of the existence of new $CP$ phases beyond SM. Here, as two examples to see the possibility or advantages in observing certain new sources of $CP$ violation, we consider the multi Higgs doublet model (MHDM) and R-parity violation models of the supersymmetric extensions of SM, because in these

\[1\] In SM, at tree level the polarization is null.

\[2\] Sometimes people call the two Higgs doublet model as a MHDM too, but here we do not. Namely here we precisely mean that there are more than two Higgs doublets in the model.
models new \(CP\)-violation phases are rich \cite{[11,12]} and the transverse polarization of \(\tau\)-leptons in the \(B_c \rightarrow \tau^+ \nu_{\tau} \gamma\) decay mode may be at tree level induced by the \(CP\) phases. Therefore throughout the paper, we restrict ourselves to estimate the effects in the two models just at tree level.

The paper is organized as follows. In section II we obtain the transition amplitude for the \(B_c \rightarrow \tau^+ \nu_{\tau} \gamma\) decay using the nonrelativistic quark model. The standard model contribution to the amplitude corresponding to \(W\)-exchange, the MHDM additional contribution (charged Higgs exchange) and the RPV contribution (slepton exchange) are explicitly given in subsections A, B and C, respectively. The formalism for \(\tau\) polarization is presented in section III. Section IV contains numerical results and discussions.

II. TRANSITION AMPLITUDE FOR THE \(B_C \rightarrow \tau^+ \nu_{\tau} \gamma\) DECAY PROCESS

According to the Feynman rules for the SM, MHDM and RPV-SUSY models, the relevant Feynman diagrams for the decay \(B_c \rightarrow \tau^+ \nu_{\tau} \gamma\) are obtained to be just those with a photon attached in turn to each of the charged fermion lines of the diagrams for the pure leptonic decay \(B_c \rightarrow \tau^+ \nu_{\tau}\), as shown in figures 1, 2, 3-a, b, c, d, respectively. The matrix element can thus be written down immediately. Based on the experimental facts, in the models charged gauge bosons \(W^\pm\) and the concerned Higgs particles or sleptons are heavy compared to \(B_c\), so we do not take into account the contributions arising from the diagrams where the photon is attached to the mediating charged bosons (figures 1, 2, 3-d). This is because they are suppressed at least by a factor of \(M^2/m_w^2\), where \(M\) and \(m_w\) are the masses of the \(B_c\) meson and the charged boson \(W^\pm\) respectively. The precise contributions of the SM, MHDM and RPV-SUSY models to the transition amplitude are given in the following subsections.

A. Standard model contributions

The amplitudes arising from the Feynman diagrams figures 1-a,b,c are given as

\[
A_{a+b} = -\frac{i e G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu \left[ Q_c \frac{k - p_c + m_c}{(p_c \cdot k)} (1 - \gamma_5) + Q_b (1 + \gamma_5) \frac{p_b - k + m_b}{p_b \cdot k} \right] b \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau, \tag{1}
\]

\[
A_c = -\frac{i e G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{\tau} \left[ \frac{k + p_l + m_l}{(p_l \cdot k)} \gamma_\mu (1 - \gamma_5) \right] \nu_\tau, \tag{2}
\]

where, respectively, \(Q_c\) and \(p_c\) (\(Q_b\) and \(p_b\)) are the charge (in units of \(e\)) and momentum of the constituent \(c\) (\(b\)) quark, \(\epsilon\) and \(k\) denote the polarization and momentum of the photon and \(p_l\) is the momentum of the \(\tau\) lepton.

Now we use the non-relativistic quark model to convert these quark level amplitudes into the hadronic level. This model was previously used by us \cite{[8]} to study the process \(B_c \rightarrow l \bar{\nu} \gamma\). Since in the \(B_c\) meson both quarks (\(b\) and \(c\)) are heavy, the relative momentum and their binding energy to their masses are small, so the quark and anti-quark inside the meson may be treated as approximately moving with small velocity. This means that the following equations are valid to quite good accuracy:

\[
M \simeq m_c + m_b, \quad p_c \simeq \frac{m_c}{M} P \quad \text{and} \quad p_b \simeq \frac{m_b}{M} P \tag{3}
\]

where \(P\) is the momentum of the \(B_c\) meson. Using these approximations and the interpolating field technique \cite{[10]} to relate the hadronic matrix element to the decay constant of the meson as
\(\langle 0|\bar{c}\gamma^\mu\gamma_5b|B_c(P)\rangle = if_{Bc}P^\mu\)

we obtain the total amplitude for the \(B_c \rightarrow \tau\bar{\nu}_\tau\gamma\) process in the SM from equations (1) and (4) to be:

\[
A^{SM} = \frac{eG_F}{\sqrt{2}}V_{cb}f_{B_c}\left[m_{\tau}\bar{\tau}\left\{\frac{P \cdot \epsilon}{P \cdot k} - \frac{\epsilon \cdot k + 2p_{\tau} \cdot \epsilon}{2p_{\tau} \cdot k}\right\}(1 - \gamma_5)\nu_{\tau}\right.
\]

\[
+ \frac{1}{6P \cdot k}\left\{\left(\frac{M}{m_b} - \frac{2M}{m_c}\right)i\epsilon_{\mu\nu\alpha\beta}P^\nu k^\alpha\epsilon^\beta\right\}
\]

\[
+ \left(6 - \frac{M}{m_b} - \frac{2M}{m_c}\right)(P \cdot k)e^\mu - (P \cdot \epsilon)k^\mu\bar{\tau}\gamma^\mu(1 - \gamma_5)\nu_{\tau}\right].
\]

(5)

It should be noted that, to ensure gauge invariance of the transition amplitude exactly, we need to add a contact term which causes the diagram figure 1-d with the propagators of the charged W-boson to shrink into a ‘point’. Figure 1-d has one more propagator of the charged gauge boson \(W\), thus the gauge violation, which originates from the ignoring of figure 1-d as stated above, is suppressed by a factor of \(m_{B_c}^2/m_W^2\).

For convenience, we call the first term in equation (5) as internal bremsstrahlung part \((A_{IB})\), whereas the second term as the structure dependent part \((A_{SD})\).

B. Additional contributions from the multi Higgs doublet model

In MHDM, the effective scalar-pseudoscalar four-Fermi interaction can be induced by exchanges of additional charged scalar Higgs particles with \(CP\) violating complex couplings at the tree level. To be more specific, here we consider the general MHDM [12] with natural flavour conservation. In order to have observable \(CP\) violation, the lightest charged scalar particle has to be much lighter than the heavier ones [13]. Here we assume that all the charged scalar Higgs particles, apart from the lightest one, decouple from the fermions effectively due to suppression from their heavy masses in propagators. The Yukawa couplings of the lightest charged scalar to up-type quarks, down-type quarks and charged leptons are determined by the parameters \(X\), \(Y\) and \(Z\), respectively.

The fresh terms of the effective Lagrangian corresponding to the \(B_c \rightarrow \tau\bar{\nu}_\tau\) decay is

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}m_H^2}\bar{c}\left[m_bX(1 + \gamma_5) + m_cY(1 - \gamma_5)\right]b\left\{m_{\tau}\bar{\tau}Z^*(1 - \gamma_5)\nu_{\tau}\right]\}
\]

(6)

where \(m_H\) denotes the mass of the lightest charged scalar particle. In the above equation the term proportional to \(m_cY\) can be safely neglected: first, it is suppressed by the mass ratio \(m_c/m_b\); second, \(Y\) is bounded to be of \(\mathcal{O}(1)\), while \(X\) can be large [12].

Now attaching the photon to the charged fermion lines of figure 2-a,b,c, as in the SM case we obtain the quark level amplitudes to be

\[
A'_{\alpha} + A'_{\beta} = \frac{ieG_F R}{2\sqrt{2}}V_{cb}\bar{c}\left[Q_c\left(\gamma^\mu p_{\alpha} + m_{\alpha}(1 + \gamma_5)\right)\right.
\]

\[
+ Q_b(1 + \gamma_5)\left(p_{\beta} - \frac{k + m_b}{p_{\beta} \cdot k}\right)\sigma^\mu\bar{\tau}(1 - \gamma_5)\nu_{\tau},
\]

(7)

\[
A'_c = \frac{ieG_F R}{2\sqrt{2}}V_{cb}\bar{c}(1 + \gamma_5)b\bar{\tau}\left[\gamma^\mu \frac{p_{\tau} + m_{\tau}}{p_{\tau} \cdot k}(1 - \gamma_5)\right]\nu_{\tau}
\]

(8)
where $R = X Z^* m_b m_\tau / m_H^2$.

These amplitudes can be converted to the hadronic level amplitudes using the non-relativistic approximation \footnote{In fact, to be typical we consider one slepton $\tilde{e}^+$ only for estimating the order.} and the equation

$$\langle 0 | \bar{c} \gamma_5 b | B_c(P) \rangle = \frac{if_B M^2}{m_b + m_c} = if_{B_c} M.$$ \hspace{1cm} (9)

Again, by adding a contact term, we obtain the total gauge invariant amplitude in the MHDM as

$$A_{MHDM} = A'_a + A'_b + A'_c = eG_F R \sqrt{2} V_{cb} f_{B_c} M [P \cdot \epsilon - \frac{i (\not{k} + 2 (p_l \cdot \epsilon))}{2 (p_l \cdot k)}] (1 - \gamma_5) \nu_\tau.$$ \hspace{1cm} (10)

It should be noted that the MHDM contributions only come from the internal bremsstrahlung part (IB) and, as in the case of SM, the diagram figure 2-d is necessary to ensure exact gauge invariance, but with one more propagator of the charged Higgs particle it is suppressed by $m_{B_c}^2 / m_H^2$.

C. Additional contribution from the R-parity violating Model

In the supersymmetric models there may be interactions which violate the baryon number $B$ and the lepton number $L$ generically. To prevent the presence of these $B$ and $L$ violating interactions in supersymmetric models, additional global symmetry is required. This leads to the consideration of so-called $R$-parity which is given by the relation $R_p = (-1)^{3B + L + 2S}$, where $S$ is the intrinsic spin of a field. Thus the $R$-parity can be used to distinguish the particle ($R_p = +1$) from its superpartner ($R_p = -1$). Even though the requirement of $R_p$ conservation gives a theory consistent with present experimental investigations, there is no good theoretical justification for this requirement; in particular, there is not very strong constraint whether the lepton number $L$ is conserved or not. Therefore, models with explicit $R_p$ violation ($\tilde{R}_p$) are considered by many authors \footnote{\footnote{In fact, to be typical we consider one slepton $\tilde{e}^+$ only for estimating the order.}}.

The most general lepton number, so $R$-parity, violating super-potential is given by

$$W_L = \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k$$ \hspace{1cm} (11)

where, respectively, $i, j, k$ are generation indices, $L_i$ and $Q_j$ are $SU(2)$ doublet lepton and quark superfields and $E^c_i$, $D^c_k$ are lepton and down type quark singlet superfields. Furthermore, $\lambda_{ijk}$ is antisymmetric under interchange of the first two generation indices.

The relevant Lagrangian for the decay mode $B_c \rightarrow \tau \bar{\nu}_\tau$ is

$$\mathcal{L}_{R} = -\frac{1}{2} \frac{\lambda_{33}^{\prime} \lambda_{23}^{\prime}}{M_{eLi}^2} \bar{c}(1 + \gamma_5) b \bar{\tau}(1 - \gamma_5) \nu_\tau$$ \hspace{1cm} (12)

where the summation over $i = 1, 2$ is implied.

It should be noted that the $RPV$ Lagrangian has the same form as the MHDM Lagrangian except for the couplings. From equation (11) we can thus easily obtain the amplitude for the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ in the $RPV$ model by replacing the MHDM coupling $((G_F / \sqrt{2}) V_{cb} R)$ by the RPV couplings $(-\lambda_{33}^{\prime} \lambda_{23}^{\prime} / 2 M_{eL_i}^2)$, as shown in figure 3a-d. We will henceforth concentrate on the MHDM only and apply the results to the RPV model with the above replacement.
It should be noted that, quite similar to the case of SM and MHDM, to ensure gauge invariance of the transition amplitude exactly, figure 3-d is necessary but with one more propagator of the slepton $\tilde{e}^+$ it is suppressed by $m_{\tilde{e}^+}^2/m_{\tilde{e}^+}^2$.

III. FORMALISM FOR TRANSVERSE $\tau$ POLARIZATION

Having derived the transition amplitude within and beyond the standard model, we now proceed to analyze the transverse polarization of the $\tau$ lepton. Since within SM alone there is no observable $CP$-violation effect at all, so we take into account the additional contribution from the MHDH(RPV) model i.e. consider the amplitude $SM + MHDM(RPV)$ for the $B_c \to \tau \bar{\nu}_\tau \gamma$ decay

$$A^{SM + MHDM(RPV)} = \frac{eG_F}{\sqrt{2}} V_{cb} f_{B_c} \left[ m_\tau \left( 1 + R \frac{M}{m_\tau} \right) \right] \tau \{ P \cdot \epsilon - \frac{Q}{P \cdot k} + 2(p_l \cdot \epsilon) \} (1 - \gamma_5) \nu_\tau \nonumber$$

$$+ \frac{1}{6P \cdot k} \left( \frac{M}{m_b} - \frac{2M}{m_c} \right) \epsilon_{\mu\nu\alpha\beta} P^\nu k^\alpha e^\beta \nonumber$$

$$+ \left( 6 - \frac{M}{m_b} - \frac{2M}{m_c} \right) \{ (P \cdot k)e^\mu - (P \cdot \epsilon)k^\mu \} \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \} . \quad (13)$$

From this amplitude, a partial decay width and the transverse polarization of $\tau$ are calculated.

To describe the decay $B_c \to \tau \bar{\nu}_\tau \gamma$ we need two variables

$$x = \frac{2P \cdot k}{M^2} \quad y = \frac{2P \cdot p_l}{M^2} . \quad (14)$$

In the centre mass frame of $B_c$, the variable $x(y)$ is proportional to the photon (lepton) energy

$$x = \frac{2E_k}{M} \quad y = \frac{2E_\ell}{M} . \quad (15)$$

The physical regions for $x$ and $y$ are given as follows

$$0 \leq x \leq 1 - r \nonumber$$

$$1 - x + \frac{r}{1 + x} \leq y \leq 1 + r \quad (16)$$

where $r = m_\tau^2/M^2$. The Dalitz plot density is given by

$$\rho(x, y) \equiv \frac{d^2\Gamma}{dx dy} = \frac{M}{256\pi^3} \sum_{spins} |A|^2 . \quad (17)$$

Here $\rho(x, y)$ has the form

$$\rho(x, y) = \rho_{IB}(x, y) + \rho_{SD}(x, y) + \rho_{INT}(x, y) \quad (18)$$

$$\rho_{IB}(x, y) = |1 + R \frac{M}{m_\tau}|^2 A_{IB} f_{IB} \nonumber$$

$$\rho_{SD}(x, y) = \frac{2}{9M^2 x^2} A_{SD} f_{B_c}^2 \left[ \left( 3 - 2 \frac{M}{m_c} \right)^2 f_{SD} + \left( \frac{M}{m_b} - 3 \right)^2 f_{SD} \right] \nonumber$$

$$\rho_{INT}(x, y) = \frac{2}{3M x} A_{INT} f_{B_c} \{ 1 + \frac{M}{m_\tau} \text{Re} \left( R \right) \} \left[ \left( 3 - 2 \frac{M}{m_c} \right) f_{INT}^+ + \left( \frac{M}{m_b} - 3 \right) f_{INT}^- \right] . \quad (19)$$
where

\[ A_{SD} = \frac{M^5}{32\pi^2} G_F^2 |V_{cb}|^2 \]

\[ A_{IB} = 2r \left( \frac{f_{B_c}}{M} \right)^2 A_{SD} \]

\[ A_{INT} = 2r \left( \frac{f_{B_c}}{M} \right) A_{SD} \]

(20)

and

\[ f_{IB}(x, y) = \left( \frac{1 - y + r}{x(x + y - 1 - r)} \right) \left[ x^2 + 2(1 - x)(1 - r) - \frac{2xr(1 - r)}{x + y - 1 - r} \right] \]

\[ f_{SD}^{\pm}(x, y) = (x + y - 1 - r) \left[ (x + y - 1)(1 - x) - r \right] \]

\[ f_{SD}^{-}(x, y) = (1 - y + r) \left[ (1 - x)(1 - y) + r \right] \]

\[ f_{INT}^{\pm}(x, y) = \left( \frac{1 - y + r}{x(x + y - 1 - r)} \right) \left[ (1 - x)(1 - x - y) + r \right] \]

\[ f_{INT}^{-}(x, y) = \left( \frac{1 - y + r}{x(x + y - 1 - r)} \right) \left[ x^2 - (1 - x)(1 - x - y) - r \right]. \]

(21)

If the transverse polarization \( P_T \) of the \( \tau \)-lepton in the decay \( B_c \to \tau \bar{\nu}_\tau \gamma \) is defined along the normal direction of the plane of the vectors \( \vec{k} \) and \( \vec{p}_{\tau} \), then

\[ P_T \propto s_{\tau}^* \cdot (\vec{k} \times \vec{p}_{\tau}) \]

which is a \( T \)-odd or \( CP \)-odd quantity. If we introduce the unit vector \( \vec{n}_T = \frac{\vec{k} \times \vec{p}_{\tau}}{|k \times p_{\tau}|} \), the transverse polarization of the \( \tau \)-lepton at a fixed point in the phase space for the final state is given as [16]

\[ P_T = \frac{d\Gamma(\vec{n}_T) - d\Gamma(-\vec{n}_T)}{d\Gamma(\vec{n}_T) + d\Gamma(-\vec{n}_T)} \]

(22)

where \( \vec{k} \) and \( \vec{p}_{\tau} \) are the photon and \( \tau \) momenta in the \( B_c \) rest frame respectively, and \( d\Gamma(\pm \vec{n}_T) \) are the partial decay widths with the \( \tau \) polarization along \( \pm \vec{n}_T \). Thus with the above equations (13-22) it is easy to find

\[ P_T = -\sigma(x, y) \text{Im}(R) \]

(23)

with

\[ \sigma(x, y) = \frac{4f_{B_c}^2}{3M^2} A_{SD} \sqrt{(1 - y + r)(1 - x)(x + y - 1) - r} \rho(x, y) \]

\[ \times \left[ (3 - \frac{2M}{m_c}) \left( \frac{(1 - x)(x + y - 1) - r}{x(x + y - 1 - r)} + \left( \frac{M}{m_b} - 3 \right) \frac{1 + r - y}{x(x + y - 1 - r)} \right) \right]. \]

(24)

The function \( \sigma(x, y) \) is usually referred to as the distribution of the transverse polarization of the charged lepton [16]. However, \( P_T \) as defined above is not a direct observable. What
can be measured in a realistic experiment, for instance, is the average of the polarization over the possible Dalitz plot regions, which is given by

\[
\bar{P}_T = \frac{\int_S dx \; dy \; \rho(x,y) P_T(x,y)}{\int_S dx \; dy \; \rho(x,y)}. \tag{25}
\]

The average is a measure of the difference between the number of \(\tau\)-leptons with their spins pointing above and below the decay plane divided by the total number of \(\tau\)-leptons in the same region of phase space. As final result, the averaged polarization is obtained

\[
\bar{P}_T = -\bar{\sigma}(x,y) \Im(R). \tag{26}
\]

IV. NUMERICAL RESULTS AND DISCUSSION

Here we present the numerical analysis for the branching ratio and transverse \(\tau\) polarization in the \(B_c \to \tau \bar{\nu}_{\tau} \gamma\) decay process with the following values: \(m_c = 1.5\) GeV, \(m_b = 4.9\) GeV, \(M = 6.4\) GeV, \(m_{\tau} = 1.777\) GeV, \(f_{B_c} = 0.360\) GeV, \(\tau_{B_c} = 0.46\) ps, \(|V_{cb}|=0.04\) and \(\alpha = 1/132\).

We first estimate the branching ratio using equations (17 - 21). However, due to the soft photon emission, the total decay width is singular when the photon energy approaches to zero. We therefore impose a cut value for the photon energy, which will set an experimental limit on the minimal detectable photon energy. Since the photon energy \(E_k \geq 100\) MeV, which corresponds to \(x|_{\text{min}} = 3.125 \times 10^{-2}\), we find the branching ratio in the standard model (\(R=0\)) to be

\[
Br(B_c \to \tau \bar{\nu}_{\tau} \gamma) = 3.44 \times 10^{-4}. \tag{27}
\]

This branching ratio may be accessible experimentally. Here we note that the light front model framework yields \[17\]

\[
Br(B_c \to \tau \bar{\nu}_{\tau} \gamma) = 1.1 \times 10^{-4}. \tag{28}
\]

If we compare equations (27) and (28), it is clear that the light cone prediction on the branching ratio is approximately three times smaller than our predicted value. Although right now no one can tell the precise reason what makes the difference, one may see from the definition of \(P_T\) Eq. (22) (being a fraction) that the difference should be canceled quite a lot in \(P_T\).

For MHDM, we use the following parameters from \[12\]. The perturbative constrains on \(|XZ|\) and \(\Im(XZ^*)\) are obtained from the decay \(B \to X\tau\bar{\nu}_{\tau}\). Namely in MHDM, the branching ratio for this decay mode is given as \[18\]

\[
BR^{MHDM}(B \to X\tau\bar{\nu}_{\tau}) = BR^{SM}(B \to X\tau\bar{\nu}_{\tau}) \left(1 + \frac{|R|^2}{4} - D\Re(R)\right), \tag{29}
\]

where the Standard Model result is \[19\]

\[
BR^{SM}(B \to X\tau\bar{\nu}_{\tau}) = (2.30 \pm 0.25)\%
\]

and \(R = m_{\tau}m_bXZ^*/m_H^2\) defined as in Eqs. \([??]\). The expression for \(D\) is given in \[12\]. Comparing Eq. (29) with the experimental value \[20\]
\[ BR(B \to X\tau\bar{\nu}_\tau) < 4\% \quad 95\% \text{ CL.}, \]  

one will obtain a perturbative constraint on \(|XZ|\) when \(m_{H^\pm} > 370 \text{ GeV}\) as follows

\[ |XZ| < \min(0.32 \ m_{H^\pm}^2 \text{ GeV}^{-2}, 44200). \]  

Thus the accordingly up-bound on \(Re(XZ^*)\) is the same as that on \(|XZ|\).

However the CP violating parameter \(Im(XZ^*)\) can be bounded from the transverse polarization of the muon in the decays \(K^+ \to \pi^0\mu^+\bar{\nu}_\mu\) and \(B \to X\tau\bar{\nu}_\tau\). When \(m_{H^\pm} \geq 440 \text{ GeV}\) the perturbative constraint on \(Im(XZ^*)\) is given as

\[ Im(XZ^*) < \min(0.23 \ m_{H^\pm}^2 \text{ GeV}^{-2}, 44200). \]  

With these constraint values of Eq. (32) and Eq. (33), we may evaluate the up-bounds for the branching ratio and averaged transverse polarization of MHDM precisely by integrating Eq. (17) with respect to \(x\) and \(y\) within the limits given by Eq. (16) and Eq. (26). Finally, we obtain quite interesting results and put them in Table I.

Table I. The dependence of the up-bounds for the averaged transverse polarization of \(\tau\) lepton \(P_T\) (in \(10^{-2}\)) and the branching ratio \(Br(B_c \to \tau\nu_\tau\gamma)\) (in \(10^{-3}\)) on the mass of the lightest charged Higgs \(m_H\) (in GeV) in MHDM with the constraint Eq. (32) and Eq. (33) obtained from the decays \(K^+ \to \pi^0\mu^+\bar{\nu}_\mu\) and \(B \to X\tau\bar{\nu}_\tau\).

| \(m_H\) in GeV | \(Br(B_c \to \tau\nu_\tau\gamma)\) in \(10^{-3}\) | \(P_T\) in \(10^{-2}\) | \(m_H\) in GeV | \(Br(B_c \to \tau\nu_\tau\gamma)\) in \(10^{-3}\) | \(P_T\) in \(10^{-2}\) |
|----------------|---------------------------------|----------------|----------------|---------------------------------|----------------|
| 370            | 9.95                           | -              | 400            | 6.81                           | -              |
| 410            | 4.91                           | 14.8           | 450            | 4.55                           | 15.2           |
| 460            | 4.23                           | 15.6           | 470            | 3.94                           | 16.1           |
| 480            | 3.67                           | 16.5           | 500            | 3.22                           | 17.4           |
| 600            | 1.86                           | 20.8           | 700            | 1.245                          | 22.9           |
| 800            | 0.945                          | 23.6           | 900            | 0.743                          | 23.2           |
| 1000           | 0.632                          | 22.1           | 1100           | 0.551                          | 20.7           |

We may see from Table I. that the up-bound of the branching ratio for the radiative leptonic decay

\[ Br(B_c \to \tau\bar{\nu}_\tau\gamma) < 9.05 \times 10^{-3} \]  

which is quite big\(^4\). Similarly from the table the up-bound of the averaged transverse polarization of \(\tau\) lepton obtained from Eq. (26) can be

\[ P_T < 15 \sim 23\% \]  

The \(\tau\) polarization was estimated by Geng et al \(^{[17]}\) \(\text{in MHDM and obtained a up-bound} P_T < 0.67. Since the effect of MHDM in } A_0(x,y) (P_T = A_T(x,y)/A_0(x,y) \text{in their definition}) \text{was not taken into account, so such a big up-bound was obtained.}

As for RPV, when assuming the sfermion masses to be 100 GeV the R-parity violating couplings are obtained by Ref. \(^{[21]}\)

\(^4\)Here the constraint on the parameters \(X, Z\) quoted from Ref. \(^{[12]}\) in MHDM is very weak and the contributions from the lightest charged Higgs particle, being comparatively light, are additive to SM, so a quite big up-bound value of the branching is obtained.
\[ \lambda_{313} = -0.003 \quad \lambda_{323} = -0.06 \]
\[ \lambda'_{123} = 0.20 \quad \lambda'_{223} = 0.18. \quad (36) \]

Assuming maximal CP violation further and with the couplings Eq. (36), we may evaluate the up-bounds on the branching ratio and \( \bar{P}_T \) of RPV precisely, and put the results in Table II.

**Table II.** The dependence of the up-bounds for the averaged transverse polarization of \( \tau \) lepton \( \bar{P}_T \) (in \( 10^{-4} \)) and the branching ratio \( B_c \rightarrow \tau \nu \gamma \) (in \( 10^{-4} \)) on the mass of the lightest charged slepton \( \tilde{m}_l \) (in GeV) in RPV with the couplings Eq. (36).

| \( \tilde{m}_l \) in GeV | \( Br(B_c \rightarrow \tau \nu \gamma) \) in \( 10^{-4} \) | \( \bar{P}_T \) in \( 10^{-4} \) | \( \tilde{m}_l \) in GeV | \( Br(B_c \rightarrow \tau \nu \gamma) \) in \( 10^{-4} \) | \( \bar{P}_T \) in \( 10^{-4} \) |
|------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 100                    | 3.451           | 9.11            | 200             | 3.447           | 2.59            |
| 300                    | 3.447           | 1.15            | 400             | 3.446           | 0.65            |
| 500                    | 3.446           | 0.04            |                 |                 |                 |

In summary, for RPV the up-bound of the branching ratio is

\[ Br(B_c \rightarrow \tau \bar{\nu} \gamma) < 3.45 \times 10^{-4} \quad (37) \]

and the up-bound of the averaged transverse polarization is

\[ \bar{P}_T > 0.1\% \quad (38) \]

The great differences of MHDM from RPV in the up-bound values for the branching ratio and the averaged transverse polarization are due to the facts that i). The constraint for MHDM Eq. (32) and Eq. (33) is very weak, but the constraint on the couplings Eq. (36) for RPV is quite strained and small. ii). The additional contributions from the charged Higgs in MHDM are constructive (plus) to those of SM, but the additional ones from the charged slepton are destructive (minus) to those of SM.

It is interesting to compare the sensitivity of transverse \( \tau \) polarization in \( B_c \rightarrow \tau \bar{\nu} \gamma \) to that of the muon in \( K^+ \rightarrow \mu \bar{\nu} \gamma \). *A priori* we expect the polarization effects to be larger for \( B_c \rightarrow \tau \bar{\nu} \gamma \) than \( K^+ \rightarrow \mu \bar{\nu} \gamma \), since there are larger quark and lepton masses in the \( B_c \) case. For example, the lepton polarization in these two cases may generically be written in MHDM as \( \bar{P}_T \sim -\sigma_I \text{Im}(R) \). The ratio \( R'/R'' \) is enhanced roughly by the factor \( Mm_\tau/m_K m_\mu \), thus the transverse lepton polarization is enhanced by \( P'/P'' \sim Mm_\tau/m_K m_\mu \sim 2 \times 10^2 \). The rather large enhancement which we generically find means that, to reach a given sensitivity for new physics, we require far fewer \( \bar{B}_c \) decays than \( K \) decays, although it is a great challenge for experiment to produce such a great amount of \( B_c \) mesons and reject possible background.

In conclusion, we have estimated the branching ratio and \( T \)-odd transverse polarization of \( \tau \) leptons in the \( B_c \rightarrow \tau \bar{\nu} \gamma \) decay in the MHDM and RPV models. It is encouraging to note that the polarization effects in these models are found to be quite large. The transverse \( \tau \) polarization observable may provide a means to look for the effects of new physics, since it has received a negligible contribution from the standard model sources. Based on the above estimate for the averaged \( \bar{P}_T \) and branching ratio, we can conclude that at the level, \( n \times \sigma \) (the standard deviation), and with 4% efficiency in detecting the \( \tau \) lepton, the observations of the branching ratio and the averaged \( \tau \) transverse polarization experimentally may make substantial improvements on present constraint obtained by the best observations on suitable observables for MHDM and RPV, if the numbers of \( B_c \) mesons so huge as \( 4.3 \cdot n^2 \times 10^5 \) for MHDM and \( 7.2 \cdot n^2 \times 10^{10} \) for the RPV model are collected. Namely, at LHC one must obtain meaningful results for MHDM and RPV, even at Tevatron with RUN-II data one may be possible to obtain some meaningful result for MHDM if one observes the branching ratio and transverse polarization very carefully.
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FIGURES

FIG. 1. Feynman diagram for the weak boson exchange.

FIG. 2. Feynman diagram for the lightest charged Higgs exchange.
FIG. 3. Feynman diagram for the lightest charged slepton exchange.