Update of the equations of the limit state of the structural material with the realization of their deformation

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Abstract. Two methods are given in the article by considering the type of stressed-deformed state (SDS) based on equations limit condition and analyzing the results of laboratory tests of special specimens for mechanical testing, focus having destruction thereof in the same view of SDS as in focus possible destruction of the structural member. The considered limited use of these methods in terms of considering physically consistent strength criterion type Pisarenko-Lebedev. A revised design-experimental procedure for determining the strength of the material of the structure, combining therein the elements of these two methods, consisting in determining the strength parameters of construction material, entering criterion equation Pisarenko-Lebedev, considering the actual appearance of the region-of-interest SDS structure. The implementation of the procedure is performed on the basis of the selection of the respective experimental laboratory specimens for mechanical testing, plan SDS in working zone coinciding with a SDS: structure whose strength is evaluated. The refinement process limit state equations demonstrated in determining 50CrV4 steel strength parameters, being in a state of biaxial stretching. Design-experimentally determined by, that steel for a given voltage limit value is almost a quarter of its value is reduced compared to the conventional tensile strength.

1. Introduction
Evaluation of the structural strength of parts of machines and mechanisms must in some cases be carried out taking into account the kind stressed-deformed state (SDS) in a possible fire destruction [1-5]. Such account is important, for example, when evaluating the static strength of choke assemblies of pressure vessels, elements railway car-side frame, the whole rolled wheel strength analysis of various parts in the form of plates and shells, bendable in two different directions by the action of temperature differences and in other cases. A number of techniques for strength calculation [6-10] relies on the results of laboratory tests of specimens, focus having destruction thereof in the same view of SDS in the working area. For this purpose, the testing equipment typically used with several actuators creating multidirectional action on the test sample.

Accounting type of SDS can be carried out in two ways. The first of these is represented by the so-called combined strength criteria (hereinafter, the limit state equations) of the type Pisarenko-Lebedev, Yagna-Buzhinsky, Drucker-Prager, etc. [3], and is based on a preliminary determination of the strength characteristics of the material of the design in question, determined by quasistatic failure laboratory samples under the conditions of typical types of loading of this material - uniaxial stretching, compression and shearing (respectively determining the values of \(\sigma_t, \sigma_c\) and \(\tau_s\)). These criteria are combined strength criteria, since they combine with the different weight factors two terms corresponding to the destruction by a cut (the first term) and separation. From this point of view, the
factor limiting the accuracy of this method is the difference between the real type of SDS of the design and the type of SDS of the samples tested before failure in determining the values of $\sigma_t$, $\sigma_c$, and $\tau_s$. In addition, the need for a preliminary determination of these quantities and a corresponding variety of laboratory testing techniques complicates the implementation of this method, but does not exclude it. The second way of accounting for the type of SDS relies on the results of laboratory tests of special samples having the same type of SDS in the center of their destruction as in the center of possible destruction of the structural element in question. When implementing the second method, a special testing technique with several power drives is used, creating multidirectional effects on the test sample, which also makes it difficult to habitually use this method and restrains its use. In this paper, we consider a more precise calculation and experimental calculation technique that combines the elements of these two methods, allowing for the use of standard single-drive test machines to take into account the actual type of SDS of the structural element in the source of destruction. At the same time, refinement of the calculation is achieved through the use of experimental data on the destruction of a laboratory sample, whose stressed state simulates in the working zone the stress state of a real design. These guidelines, written in the style of a submission to J. Phys.: Conf. Ser., show the best layout for your paper using Microsoft Word. If you don’t wish to use the Word template provided, please use the following page setup measurements.

2. A technique for solving the problems

Consider the equation of the limiting state of the Pisarenko-Lebedev type [1, 3], used in evaluating the static strength of structures made of isotropic materials. The corresponding condition at which the quasistatic material destruction occurs, accompanied by the appearance of cracks, has the form

$$a\sigma_{lim}^i + (1 - a)\sigma_{lim}^2 + P = \sigma_1,$$  \hspace{1cm} (1)

where $\sigma_{lim}^i$ is the intensity of stresses in a possible failure point

$$\sigma_{lim}^i = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{lim}^{1i} - \sigma_{lim}^{2i}\right)^2 + \left(\sigma_{lim}^{2i} - \sigma_{lim}^{3i}\right)^2 + \left(\sigma_{lim}^{3i} - \sigma_{lim}^{1i}\right)^2},$$  \hspace{1cm} (2)

$\sigma_{lim}^{1i}$, $\sigma_{lim}^{2i}$, $\sigma_{lim}^{3i}$ – the main stresses arising in this source; $P$ is the Smirnov-Alyaev coefficient [4, 5], which characterizes the form of SDS at the point under consideration, defined by

$$P = \frac{\sigma_{lim}^{1i} + \sigma_{lim}^{2i} + \sigma_{lim}^{3i}}{\sigma_{lim}^i},$$  \hspace{1cm} (3)

(In the case of biaxial stretching, the value of $P$ reaches the value $P = 2$, in the case of a simple uniaxial tension $P = 1$, a simple uniaxial contraction $P = -1$); $\alpha$ and $A$ are empirical constants characterizing the strength of the material and independent of the type and level of SDS of the material of the component in the center of its destruction (hereinafter, the strength parameters of the material) and calculated by formulas

$$\alpha = \frac{\sigma_1}{\sigma_c}, \quad A = \frac{\varphi - \sqrt{3}a}{1 - \alpha}, \quad \varphi = \frac{\sigma_1}{\tau_s}.$$  \hspace{1cm} (4)

As already noted earlier, the accuracy of the equation of the limiting state (1) in a number of cases is insufficient for engineering practice due to the difference in the real type of SDS in the possible source of destruction of the structure and the type of SDS of the samples tested before failure in determining the quantities $\sigma_t$, $\sigma_c$, $\tau_s$ and entering into equation (1) by means of the constants $\alpha$ and $A$, which do not depend on the coefficient $P$ and the level of the material's SDS in the source of its destruction. In [11-12], the value of the quantity $\Pi$ determined by the equality (3) significantly affects the location of the source of destruction: with an increase in $P$ (according to Smirnov-Alyaev – with an increase in the "hardness" of the type of SDS), the limiting values of the stress intensity and the first principal voltage decrease.

In order to refine the structure material limit state equations and, accordingly, increase the reliability of its strength calculation, a calculation-experimental approach is proposed that consists in
determining the strength parameters of the material of the structure under study, entering into equation (1), taking into account the real type of SDS of the most loaded area of the structure.

The essence of the proposed approach is as follows. It can be seen from equation (1) that $\alpha$ and $A$ can be regarded as the coefficients of the empirical formula characterizing the structural strength of the material of the design in question with certain calculated values of the quantities $\sigma_{i}^{lim}$, $\sigma_{i}^{lim}$ and $P^{lim}$. The parameters $\alpha$ and $A$ can be calculated directly from equation (1), knowing the remaining parameters $-\sigma_{i}^{lim}$, $\sigma_{i}^{lim}$ and $P^{lim}$. From this it follows that $\alpha$ and $A$ do not in this case have a traditional physical meaning, originally inherent in the Pisarenko-Lebedev test, calculated from (4), but are constant or unimportant parameters from the kind of SDS of the material from which the design is made. Taking into account the foregoing, the strength parameters $\alpha$ and $A$ can be modified, which allows them to be considered as the coefficients of the empirical formula, determined from the test results before the destruction of the samples simulating the estimated construction's SDS in its area of interest. The parameters $\alpha$ and $A$ can be determined in a more complex way and, at least weakly, depend on the value of the coefficient $P$ determined by (3).

In the paper, as in the Pisarenko-Lebedev equation, the assumption of the independence of the parameters $\alpha$ and $A$ entering into equation (1) remains on the intensity values $\sigma_{i}^{lim}$ and the first principal stress $\sigma_{i}^{lim}$ in a possible failure point, but the dependence of these parameters on the quantity $P$ Equation (1) with this approach is an approximation of the true limit state equation constructed for a specific (or relatively narrow range of variation) the value of the coefficient $P$ that characterizes one or another type of SDS. This type is determined by the design features of the samples tested before destruction, and the source of their destruction should be characterized by the value of $P$, which coincides (or is close in value) with the value of $P$ for the area of the structure calculated for strength.

When implementing the proposed approach, after finding the calculated value of the coefficient $P$ to determine the values of the parameters $\alpha$ and $A$ entering into equation (1), it is necessary to select appropriate experimental laboratory samples so that their SDS type coincides (or was close) with the type of construction SDS, strength which is evaluated. In this case, to determine the two parameters $\alpha$ and $A$, it is sufficient to test before the destruction of two laboratory samples differing from each other in geometric dimensions and the design characteristics of the SDS at the time of destruction of their material. In accordance with this, the determination of the strength parameters $\alpha$ and $A$ is carried out on the basis of the following algorithm:

- selection of geometric parameters of two different design variants of special-shaped specimens with values of $P$, close to $P$ for the area of the design being evaluated $-P^{lim}$ and $P^{2lim}$,
- testing before destruction of design variants of samples;
- calculation of $\sigma_{i}^{lim}$ and $\sigma_{i}^{lim}$ – characteristics of the SDS level entering into equation (1) in the working area of the destroyed selected samples;
- determination of the values of $\alpha$ and $A$ from the resolving system of equations.

$$\alpha\sigma_{i}^{lim} + (1 - \alpha)\sigma_{i}^{lim} A^{-P^{lim}} = \sigma_{i}$$

$$\alpha\sigma_{i}^{2lim} + (1 - \alpha)\sigma_{i}^{2lim} A^{-P^{2lim}} = \sigma_{i},$$

(5)

where the values of the values $\sigma_{i}^{lim}$, $\sigma_{i}^{lim}$ and $P^{lim}$ correspond to the experimentally established moment of destruction of the first of the selected samples, $\sigma_{i}^{2lim}$, $\sigma_{i}^{2lim}$ and $P^{2lim}$ – the second of the destroyed samples. The refinement of the calculation is achieved in the proposed technique due to the use in it of experimental data on the destruction of laboratory samples obtained under real-design SDS conditions. The circumstance that causes difficulties in the implementation of the approach to the refinement of the equation of the limiting state of the construction material is the need to have strength characteristics, taking into account their dependence on the ratio of the principal stresses. The authors of the paper proposed a prismatic specimen for evaluating the strength of a material under a complex stress state [9], which makes it possible to obtain information on the strength of a material, depending on the required coefficient $P$, by loading it on a standard testing machine (see Figure 1).
3. Results of computational and experimental studies

With the aim of realizing the process of refinement of the equation of the limiting state of the material of the structure, taking into account the peculiarities of its deformation, the strength parameters \( \alpha \) and \( A \) for 50CrV4 steel under the conditions of biaxial stretching \((1<P<2)\) were determined. For the study, a SDS was chosen, characterized by a value of \( P = 1.85 \). This condition corresponds to the type of SDS at the time of destruction of the choke unit of the pressure vessel at its outer surface in the joint area of the branch pipe with the housing \([11]\).

A tensile test of standard circular samples of 50CrV4 steel was preliminarily carried out. The results showed that the selected steel is characterized by yield stress 1050 MPa, tensile strength 1300 MPa and a relative elongation \( \delta = 7.5\% \). \([14, 15, 19, 20]\).

According to the method in order to determine the values of the two parameters \( \alpha \) and \( A \) entering into equation (1), it is first necessary to destroy two design variants of the samples, differing from each other in size and corresponding values of the values \( \sigma_{11}^{\text{lim}}, \sigma_{12}^{\text{lim}} \) and \( \sigma_{22}^{\text{lim}}, \sigma_{12}^{\text{lim}} \) at the time of destruction, but coinciding (or close) by the value of the coefficient \( P \) in their working zones. In this case, equations of the form (1) corresponding to the two chosen constructive variants form a system of two nonlinear algebraic equations with respect to the unknown quantities \( \alpha \) and \( A \), which can be solved by the method of successive approximations. The possibility and order of selecting the necessary samples is given, for example, in \([15, 19, 20]\). In accordance with the algorithm, two series of samples were produced - No. 1 and No. 2 - three samples in each series. Their dimensions (see Table 1) were chosen so that in the work area of the samples their SDS was characterized by a value of \( P = 1.85 \) \([19-20]\).

**Table 1.** the main dimensions of the experimental prismatic samples, mm

|   | \( H \) | \( H_1 \) | \( H_2 \) | \( H_3 \) | \( S \) | \( L \) | \( t \) | \( r \) | \( \gamma \) |
|---|---|---|---|---|---|---|---|---|---|
| No. 1 | 33 | 12 | 11 | 21 | 44 | 220 | 4 | 2 | 15 |
| No. 2 | | | | | | | 6 | | |

The experimental destruction of prismatic samples whose geometric dimensions for the series No. 1 and No. 2 are shown in Table 1 was carried out on a typical single-wheeled testing machine Instron 5989. It showed that for the forces \( F_1 \) and \( F_2 \) (in Figure 1 – item 3) at the moment destruction (according to the results of averaging the experimental data) the following equalities were satisfied:

\[
F_1 = 205 \text{ kN}; \quad F_2 = 235 \text{ kN}.
\]
The obtained values of the forces $F_1$ and $F_2$ were used as initial values for the numerical analysis of the design values of the design variants at the moment of their destruction. In this case, pre-designed computational deformation models, supported by computational tests and full-scale experiment, were used. Numerical analysis was carried out using the finite element method, taking into account the possible appearance of elastoplastic deformations in the sample material. In this case, the equations of plastic flow with isotropic hardening (the Prandtl-Reuss equation [17]) are used as a mathematical model of the process of development of elastoplastic deformations. Thus, for the first of the selected prismatic samples ($P_{1 \text{lim}}^{1}=1.9$), the calculated values $\sigma_1^{1 \text{lim}}, \sigma_i^{1 \text{lim}}$ were determined, and for the second ($P_{1 \text{lim}}^{2}=1.8$) of the destroyed specimens $\sigma_1^{2 \text{lim}}, \sigma_i^{2 \text{lim}}$. The results of numerical analysis of prismatic samples of the series No. 1 ($P_{1 \text{lim}}^{2}=1.9$) at the moment of their destruction (the moment of occurrence of the crack) are shown in Fig. 2a is the stress intensity distribution, in Fig. 2b is the distribution of the component of the first principal voltage. The results of a numerical analysis of the deformation of prismatic samples of series No. 2 ($P_{2 \text{lim}}^{2}=1.8$) at the moment of their destruction are shown in Fig. 3a is the stress intensity distribution, in Fig. 2b is the distribution of the component of the first principal voltage.

![Figure 2](image1.png)  
**Figure 2.** Estimated stress distribution at the moment of destruction of samples of series No. 1, MPa  
(type of quarter of the sample carved from it by two planes of symmetry)  
a - stress intensities $\sigma_1^{\text{lim}}, \sigma_i^{\text{lim}}$, b - first principal $\sigma_1^{\text{lim}}$  

![Figure 3](image2.png)  
**Figure 3.** Estimated stress distribution at the moment of destruction of samples of series No. 2, MPa  
(type of quarter of the sample carved from it by two planes of symmetry):  
a - intensity of stress $\sigma_1^{\text{lim}}, \sigma_i^{\text{lim}}$, b - first principal $\sigma_1^{\text{lim}}$
In aggregate, the calculated and experimental data obtained are presented in Table 2. The obtained experimental results show that the effect of biaxial stretching in the source of destruction of the structure can be significant - for the tested samples of the series No. 1 the limiting stress intensity $\sigma_{1}^{\text{lim}}$ the moment of their destruction is 895 MPa, which is almost a quarter of the value of the tensile strength in the steel 50CrV4 under study $\sigma_{1}^{\text{lim}} = \sigma_{1} = 1270$ MPa, determined under conditions of uniaxial tension ($P = 1$). The result obtained qualitatively coincides with the result of the experimental studies of Y.A. Wilimok, K.A. Nazarova, A.K. Evdokimova [20], who established a similar effect of a significant reduction (by more than a quarter) of the value $\sigma_{i}^{\text{lim}}$ under conditions of biaxial stretching of steel X10CrNiTi18-10 with the SDS type factor $P \approx 2$ obtained on special testing equipment.

### Table 2. Calculated characteristics of SDS samples at the time of their destruction

| No | working area | $\sigma_{1}^{\text{lim}}$, MPa | $\sigma_{2}^{\text{lim}}$, MPa | $\sigma_{i}^{\text{lim}}$, MPa | $P$ |
|----|--------------|-------------------------------|-------------------------------|-------------------------------|-----|
| 1  |              | 985                           | 743                           | 892                           | 1,9 |
| 2  |              | 1050                          | 615                           | 917                           | 1,8 |

To determine the strength parameters of equation (1), substituting the calculated values of the SDS characteristics into the system of equations (5), obtained from the experiment (see Table 2), we obtain

\[
\begin{align*}
\alpha 892 + (1-\alpha)985A^{1-1,9} &= 1270 \\
\alpha 917 + (1-\alpha)1050A^{1-1,8} &= 1270
\end{align*}
\]  

(7)

Solving the system of equations (7) with respect to the quantities $\alpha$ and $A$ by the method of successive approximations, we obtain the following equalities:

\[
\alpha = 0.73; \quad A = 0.40.
\]  

(8)

These values allow us to use equations (1) for a more accurate calculation of the strength of structural elements made of steel 50CrV4 and characterized by the equality $P \approx 1.85$. If we take into account the fact that the parameters $\alpha$ and $A$ entering into equation (1) have a traditional physical meaning inherent in (1), i.e. Independent of $P$, we can verify that for steel 50CrV4, $\alpha = 0.6$, $A = 0.75$ for any $P$. This circumstance contradicts the well-known assertion of the dependence of the strength of a material on the type of SDS, and the resulting equation (8) confirms it. Taking into account the obtained values of $\alpha = 0.73$ and $A = 0.4$, it can be seen that the influence of the SDS on the strength parameters can be significant.

### 4. Conclusions

The proposed calculation-experimental approach to the refinement of the equation of the limiting state of the construction material, based on a preliminary calculation of the type of SDS in a possible source of structural failure and subsequent quasistatic testing prior to the destruction of the corresponding prismatic samples, made it possible to determine the numerical values of the strength parameters entering into the equation of the limiting state of the Pisarenko-Lebedev, characterized by a value of $P \approx 1.85$ and made of the steel in question.

It has been calculated experimentally that for steel 50CrV4, from which the experimental prismatic samples were made for laboratory tests, the limiting value of the first principal stress under biaxial tension ($P \geq 1.8$) is reduced by almost a quarter in comparison with the value of its strength limit determined Under conditions of uniaxial tension ($P = 1$).
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