Notes on Theories with 16 Supercharges*

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We survey the various field theories with 16 real supercharges. The most widely known theory in this class is the $N = 4$ theory in four dimensions. The moduli space of vacua of these theories are described and the physics at the singularities of the moduli spaces are studied.

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1. Introduction

The magic of supersymmetry makes supersymmetric theories amenable to exact treatment. With more supercharges, the theory is more constrained and more observables can be analyzed exactly. The largest number of supercharges, which is possible in free field theory, is sixteen. With more supercharges the free multiplet includes fields whose spin is larger than one and no consistent theory (without gravity) exists. There are three motivations for studying these theories. First, as the most supersymmetric theories they are the most constrained theories, and therefore they exhibit interesting features like exact electric/magnetic duality in the \( N = 4 \) theory in four dimensions [1,2]. Second, these theories appear in string compactifications as the theory of the collective coordinates of various branes. Finally, the \( N_c \to \infty \) limit of these \( U(N_c) \) gauge theories have been proposed as exact descriptions of toroidally compactified M-theory [3].

In section 2 we survey the various theories in this class and examine when they have non-trivial infrared dynamics. In section 3 we review briefly the \( N = 4 \) theory in four dimensions. In section 4 we focus on the \( N = 8 \) theory in three dimensions and study its moduli space of vacua and its singularities. Section 5 is devoted to the compactification of the four dimensional \( N = 4 \) theory on a circle to three dimensions. We discuss the image of the famous electric/magnetic duality of the four dimensional theory in three dimensions. In section 6 we make some comments on the \((8,8)\) theory in two dimensions. In section 7 we present a few observations on the special theory with \((0,2)\) supersymmetry in six dimensions.

2. The zoology of theories with 16 supersymmetries

The most symmetric classical field theory with 16 supersymmetries is supersymmetric Yang-Mills theory in ten dimensions. The simplest such theory is an Abelian gauge theory. The supermultiplet includes a massless photon and a massless fermions. The non-Abelian extension of this theory exists as a classical field theory but its quantum version is anomalous and therefore inconsistent.

The theory in \( d \) dimensions, which is obtained by dimensional reduction of the classical theory in ten dimensions is anomaly free. Its Lorentz symmetry is \( Spin(d − 1, 1) \). The dimensionally reduced theory has an R-symmetry \( Spin(10 − d) \), which originates from the
ten dimensional Lorentz group. The sixteen generators transform as

| \(d\) | \(Spin(d-1,1) \times Spin(10-d)\) | Automorphism \(\supseteq Spin(10-d)\) |
|-------|----------------------------------|-------------------------------------|
| 9     | \(16_r\)                         | \(U(1) = Spin(2)\)                 |
| 8     | \((8_s, 1) + (\bar{8}_s = 8_c, -1)\) | \(SP(1) = Spin(3)\)                |
| 7     | \((8_p, 2_p)\)                   | \(SP(1) \times SP(1) = Spin(4)\)  |
| 6     | \((4_p, 2_p) + (4'_p, 2'_p)\)   | \(SP(2) = Spin(5)\)                |
| 5     | \((4_p, 4_p)\)                   | \(U(4) \supset Spin(6)\)           |
| 4     | \((2, 4) + (\bar{2}, \bar{4})\) | \(Spin(8) \supset Spin(7)\)        |
| 3     | \((2_r, 8_r)\)                   | \(Spin(8) \times Spin(8) \supset Spin(8)\) |
| 2     | \((1_r, 8_s) + (-1_r, 8_c)\)    | \(Spin(16) \supset Spin(9)\)       |
| 1     | \(16_r\)                         |                                     |

where the subscripts \(p\) and \(r\) label pseudoreal and real representations respectively and we label the three eight dimensional representations of \(spin(8)\) (or its non-compact versions) as \(8_s, 8_c, 8_v\). The automorphism group of the supersymmetry algebra can be larger than the R-symmetry of the Lagrangian. It is determined by the anticommutation relations of the supercharges and their reality properties (\(SP\) for pseudoreal, \(Spin\) for real and \(U\) for complex). The automorphism group is also included in the table along with its \(Spin(10-d)\) subgroup.

The only irreducible massless representation, with spin less than one, of the superalgebra is obtained by dimensional reduction from ten dimensions. It includes a photon \(A_\mu\), \(10 - d\) scalars \(\phi\), and some fermions.

One characteristic of all these theories is the existence of a moduli space of vacua. The large amount of supersymmetry constrains the moduli space to be locally flat but there can be singularities. The moduli space of a gauge group \(G\) of rank \(r\) in \(d \geq 4\) dimensions is

\[
\mathcal{M} = \frac{\mathbb{R}^{r(10-d)}}{W},
\]

where \(W\) is the Weyl group of \(G\). For \(SU(2)\), \(r = 1\) and \(W = \mathbb{Z}_2\). The effective Lagrangian along the flat directions is constrained to be free

\[
\mathcal{L} = \frac{1}{g^2} (F_{\mu\nu}^2 + (\partial \phi^i)^2 + \text{fermions})
\]

where \(g\) is the gauge coupling constant. By supersymmetry it is independent of \(\phi\).

The coupling constant \(g\) in (2.3) seems unphysical because the theory is free and one might attempt to absorb it by rescaling the dynamical variables \(\phi\) and \(A_\mu\). However, such a redefinition puts \(g\) in the gauge transformation laws. Furthermore, when the theory (2.3)
is studied on nontrivial manifolds, there exist nontrivial bundles with magnetic fluxes of the photon. Their action depends on $g$ and hence $g$ is physical.

The most interesting aspect of the dynamics of these theories is their behavior at the singularities of the moduli space. In a classical theory based on a non-Abelian gauge theory there are new massless particles at the singularities. What happens in the quantum theory? The standard criterion for nontrivial dynamics in the theory of the renormalization group is the dimension of the coupling constant. We always assign dimension one to gauge fields, $A_\mu$, since their Wilson line is dimensionless. By supersymmetry we should also assign dimension one to the scalars $\phi$. The gauge coupling $g$ is then of dimension $\frac{4-d}{2}$. For $d > 4$ its dimension is negative, and the corresponding interaction is irrelevant at long distance. Therefore, we do not expect any interesting infrared dynamics in the gauge theory above four dimensions.

We can generalize this standard argument and include theories, which might not come from a Lagrangian but have a moduli space of vacua, where they are free at long distance. The long distance theories at the singularities must be at fixed points of the renormalization group and therefore they are scale invariant. These scale invariant theories can be free (orbifold theories) or interacting. The effective Lagrangian along the flat directions is constrained by supersymmetry to have the form (2.3). Furthermore, since the flat directions emanate from scale invariant theories this Lagrangian must exhibit spontaneous breaking of scale invariance. However, for $d > 4$ (2.3) exhibits explicit breaking of scale invariance. This follows from the fact that with $A_\mu$ of dimension one $g$ is dimensionful. In fact, since the dimension of $g$ is negative, $g$ approaches zero at long distance. (Of course, since the theory along the flat directions is free, we can absorb $g$ in the fields to find a scale invariant theory. Such a scaling will not be compatible with a possible nontrivial theory at the singularities of the moduli space.) We conclude that above four dimensions the theories with sixteen supersymmetries of (2.1), which have a moduli space of vacua cannot exhibit nontrivial dynamics, and are free at long distance.

The previous argument used only scale invariance and the assumption about the moduli space of free vacua which emanate from the interacting point. Another argument supporting this conclusion is based on the assumption that all nontrivial fixed points of the renormalization group with a gap in the spectrum of dimensions of operators are not only scale invariant but are also conformal invariant. In a supersymmetric theory they should also be superconformal invariant. The possible superconformal algebras were analyzed in [4] with the conclusion that the supersymmetry algebras in (2.1) do not admit an extension. 

\[1 \text{ We thank P. Townsend and E. Witten for helpful discussions on this point.}\]
to a superconformal algebra above four dimensions. We should note, though, that the free
theory is scale invariant but not conformal invariant. This comes about by absorbing the
coupling constant $g$ in the photon. Then the photon field $A_\mu$ does not have dimension one
and the gauge invariant field strength $F_{\mu\nu}$ does not have dimension two. It is expected that
such scale invariance without conformal invariance does not extend to interacting theories.

The most widely known theory in this class is the $N = 4$ theory in four dimensions.
The scaling argument shows that it can be scale invariant – in this case the Lagrangian
(2.3) along the flat directions is scale invariant. Even if higher derivative terms are included
[3], the effective Lagrangian exhibit spontaneous breaking of scale invariance. Indeed, it is
known that the Yang-Mills theory is a finite, superconformal field theory. We will analyze
some of its properties in section 3.

Below four dimensions $g$ has positive dimension. Therefore, it corresponds to a rele-
vant operator in the Lagrangian, and the long distance behavior can be different from the
short distance description. In sections 4, 5 and 6 we will analyze the three dimensional
and the two dimensional theories.

There is one more supersymmetry algebra with sixteen supercharges, which is not
included in (2.1). It is in six dimensions and includes four spinors of the same chirality. It is
usually called the $(0, 2)$ algebra, while the one in (2.1) in six dimensions is the $(1, 1)$ algebra.
Its automorphism group is $SP(2)$ and the supercharges are in $(4_p, 4_p)$ of the $Spin(5, 1) \times
SP(2)$. Its irreducible massless representation consists of a two-form $B_{\mu\nu}$, whose field
strength $H = dB$ is selfdual, five real scalars $\Phi$ and fermions. Upon compactification to
lower dimensions this theory becomes one of the theories in (2.1).

Despite recent successes [6], no fully satisfactory Lorentz invariant Lagrangian for a
two form with selfdual field strength is known. Ignoring the self-duality constraint the free
Lagrangian is

$$H^2_{\mu\nu\rho} + (\partial \Phi)^2 + \text{fermions};$$  \hspace{1cm} (2.4)

i.e. the metric on the moduli space is locally flat. Note that for $H$ to be selfdual, there
cannot be an arbitrary coupling constant $g$ in front of this Lagrangian.

Let us repeat our analysis of scale invariance. We should assign dimension two to the
two form $B_{\mu\nu}$ and hence, by supersymmetry we should assign dimension two to $\Phi$. Hence,
(2.4) is scale invariant. Therefore, if there are singularities in the moduli space, the theory
there can be a nontrivial field theory. Indeed, the analysis of [4] shows that the $(0, 2)$
supersymmetry algebra admits an extension to the superconformal algebra. The $SP(2)$
automorphism group of the supersymmetry algebra is included in the superconformal al-
gebra. In section 7 we will make some comments on this theory.
Although in these notes we focus on theories with 16 supercharges, we would like to mention a similar analysis of theories with 8 supercharges above 4 dimensions. In certain examples in 5 dimensions \[7,8\] the effective Lagrangian along the flat directions
\[\phi F_{\mu\nu}^2 + \phi (\partial \phi)^2 + \ldots \quad (2.5)\]
exhibits spontaneous breaking of scale invariance (as before, the dimension of \(A_\mu\) and of \(\phi\) is one) and the strongly coupled theory at the singularity is scale invariant. Similarly, in six dimensions, the tensor multiplet includes a scalar \(\Phi\) and a two form \(B_{\mu\nu}\) both of dimension 2 and hence the Lagrangian
\[\Phi F_{\mu\nu}^2 + B \wedge F \wedge F + (\partial \Phi)^2 + (dB)^2 \ldots \quad (2.6)\]
is scale invariant and the singularities in the moduli space can be strongly coupled scale invariant theories \[9-13\] Again, this is consistent with the existence of a superconformal extension of \(N = 1\) supersymmetry in five and six dimensions \[4\].

3. \(N = 4\) supersymmetry in \(d = 4\)

The superconformal algebra includes an \(SU(4) = Spin(6)\) symmetry \[4\]. The extra \(U(1)\) factor in the automorphism group of the supersymmetry algebra (see \(2.1\)) is not a symmetry. This theory is labeled by a dimensionless coupling constant \(g\). One can also add the theta angle to make it complex
\[\tau = \frac{\theta}{2\pi} + \frac{4\pi}{g^2} i. \quad (3.1)\]

The moduli space of vacua is, as in \(2.2\), \(R^{10r}/W\). Since the theory is scale invariant, the expectation values of the scalars along the flat directions lead to spontaneous scale symmetry breaking. One of the massless scalars is a dilaton – the Goldstone boson of broken scale invariance. At the generic point in the moduli space the low energy theory is \(U(1)^r\) and the global \(Spin(6)\) symmetry is spontaneously broken. However, at long distances this theory is free and a new \(Spin(6)\) symmetry appears. This is consistent with the fact that at long distance we find a conformal field theory and the \(Spin(6)\) R-symmetry is included in the conformal algebra.

At the singularities some of the gauge symmetry is restored. More precisely, the theory at the singularities is an interacting conformal field theory. In such a theory the notion of particles is ill defined, and in particular, we cannot say that the gauge symmetry is restored because the gauge bosons are meaningless. It is standard to use the superconformal
symmetry to analyze the theory there. Primary operators are defined to be the operators
which are annihilated by all the superconformal generators. The (anti)commutation relations lead to a bound on their dimensions $D(\mathcal{O})$ in terms of their $Spin(6)$ representations. Polynomials in the microscopic fields, which are scalars of the Lorentz group, must be in representations of $SO(6)$. Some examples of the inequality for their dimensions are

\[ D(6) \geq 1 \]
\[ D(10) \geq 3 \]
\[ D(15) \geq 2 \]
\[ D(20') \geq 2 \] (3.2)

The inequality (3.2) is saturated for chiral fields. Indeed, along the flat directions, where we find a free field representation of the algebra the dimension of the free scalar field in the 6 of $Spin(6)$ is one. Two bosons are in $6 \times 6 = 1_s + 15_a + 20'_s$. The operator in the singlet can mix with the identity operator and is not chiral. The 15 occurs only when we have more than one scalar. Then there is no short distance singularity in forming the composite field in 15, and its dimension is clearly 2, which is consistent with (3.2). Finally, the fact that the dimension of the composite field $20'_s$ is 2 follows from examining an $N = 1$ superconformal subalgebra and noticing that it includes a field with $R = \frac{4}{\tau}$.

At the origin the theory is interacting but we can still use (3.2) to determine the dimensions of gauge invariant chiral operators. For example, the dimension of the scalar bilinear $\text{Tr } \phi^i \phi^j$ in $20'_s$ of $Spin(6)$ is 2. Note that the dimensions of chiral operators are independent of $\tau$ – they are given by their value in free field theory. This is not the case for more general operators. The theory at the origin must have a truly marginal operator corresponding to changing the value of $\tau$. It seems to be the $Spin(6)$ invariant $Q^2 \text{Tr } (\phi^i \phi^j \phi^k)_{10} \sim Q^4 \text{Tr } (\phi^i \phi^j)_{20'}$ (the equality of these expressions seems to follow from the equation of motion).

This theory is expected to exhibit electric/magnetic duality – the theory characterized by the gauge group $G$, whose weight lattice is $\Gamma_w(G)$ and coupling constant $\tau$ is the same as the theory based on the dual gauge group $^*G$, whose weight lattice is

\[ \Gamma_w(\text{^*G}) = \text{^*\Gamma}_w(G) \] (3.3)

and coupling $-1/\tau$. The spectrum includes BPS particles with electric and magnetic charges. Since this duality and its action on the spectrum are well known, we will not review it here.
Some of the higher dimension operators along the flat directions, which correct the leading order terms (2.3), were analyzed in [3]. The leading irrelevant operator is of the form
\[
\frac{1}{\phi^4} F^4 + \frac{1}{\phi^4} (\partial \phi)^4 + \text{eight fermion terms.} \tag{3.4}
\]
Supersymmetry leads to a non-renormalization theorem guaranteeing that these terms are generated only at one loop and are not corrected by higher order perturbative or nonperturbative effects. We will not repeat the argument here and refer the reader to [3].

4. \( N = 8 \) supersymmetry in \( d = 3 \)

Here we study field theories with \( N = 8 \) supersymmetry in \( d = 3 \). The super generators are in the real two dimensional representation of the Lorentz group. The automorphism group of the algebra (R-symmetry) is \( \text{Spin}(8) \) and the supergenerators transform as an eight dimensional representation, which we take to be the spinor \( \mathbf{8}_s \).

Since for massless particles the little group is trivial, there is only one massless representation of the superalgebra. It consists of 8 bosons in \( \mathbf{8}_v \) and 8 fermions in \( \mathbf{8}_c \) of \( \text{Spin}(8) \). Starting with a higher dimensional field theory with the same number of supersymmetries (e.g. \( N = 4 \) in \( d = 4 \)) we find a vector field, 7 scalars and 8 fermions. The R-symmetry, which is manifest in this description, is \( \text{Spin}(7) \subset \text{Spin}(8) \). The vector is a singlet of \( \text{Spin}(7) \), the scalars are in \( \mathbf{7} \) and the fermions in \( \mathbf{8} \). After performing a duality transformation on the vector it becomes a scalar and the \( \text{Spin}(8) \) symmetry becomes manifest.

Interacting Lagrangians with \( N = 8 \) supersymmetry do not necessarily exhibit the maximal possible R-symmetry. In particular, the Yang-Mills Lagrangian is invariant only under the \( \text{Spin}(7) \) subgroup. At long distance, the theory must flow to a scale invariant theory, which we assume to be also superconformal invariant. The conformal algebra in 3 dimensions is \( \text{Spin}(3, 2) \). The sixteen supersymmetry generators combine with sixteen superconformal generators to eight spinors of \( \text{Spin}(3, 2) \). For the closure of the algebra we must include the the \( \text{Spin}(8) \) symmetry [1]. Hence, the long distance theory is invariant under the full R-symmetry \( \text{Spin}(8) \). More generally, with \( N \) supercharges the superconformal algebra includes a \( \text{Spin}(N) \) R-symmetry under which the supercharges transform as a vector. For \( N = 2 \) supersymmetry the R-symmetry is \( U(1) \), and normalizing the charge of the supercharge to be one, we have for scalar operators \( D \geq R \). For \( N = 4 \) supersymmetry the R-symmetry is \( SU(2)_1 \times SU(2)_2 \) with the supercharges in the vector \( (I_1 = \frac{1}{2}, I_2 = \frac{1}{2}) \)
and for scalar fields $D \geq I_1 + I_2$. For $N = 8$ we use the triality of $\text{Spin}(8)$ to put the supercharges in $8_s$ rather than in a vector. This leads to the bounds on the dimensions

$$
D(8_v) \geq \frac{1}{2}
$$
$$
D(8_s) \geq 1
$$
$$
D(28) \geq 1
$$
$$
D(56_v) \geq \frac{3}{2}
$$
$$
D(35_s) \geq 2
$$
$$
D(35_v) \geq 1
$$

and the bound on $D(r_c)$ the same as for $D(r_v)$.

As an example, consider the $N = 8$ supersymmetric $SU(2)$ gauge theory. The gauge coupling $g$ has dimension $\frac{1}{2}$, and therefore the theory is superrenormalizable. To analyze its long distance behavior we start by considering the moduli space of vacua. Along the flat directions the $SU(2)$ gauge symmetry is broken to $U(1)$. The low energy degrees of freedom are in a single $N = 8$ multiplet, which includes seven scalars $\phi^i$ ($i = 1, \ldots, 7$) and a photon. Their Lagrangian is as in (2.3)

$$
\frac{1}{g^2}(F^2_{\mu\nu} + (\partial \phi^i)^2).
$$

The dual of the photon is a compact scalar $\phi^0$ of radius one with the Lagrangian

$$
\frac{1}{g^2}(\partial \phi^i)^2 + g^2(\partial \phi^0)^2.
$$

Because of $N = 8$ supersymmetry, the leading terms in the Lagrangian (4.3) are not corrected in the quantum theory. Therefore, the moduli space of vacua $\mathcal{M}$ is eight real dimensional. The $\phi^i$ label $\mathbb{R}^7$ and $\phi^0$ labels $S^1$. The Weyl group of $SU(2)$ changes the sign of $(\phi^i, \phi^0)$ and therefore

$$
\mathcal{M} = \mathbb{R}^7 \times S^1 / \mathbb{Z}_2.
$$

It has two singularities at $\phi^i = \phi^0 = 0$ and at $\phi^i = 0, \phi^0 = \pi$. The metric around them is an orbifold metric.

At long distance the gauge coupling $g$ goes to infinity. The radius of the circle in (4.4) goes to infinity and we can focus on a neighborhood in the moduli space. At the generic point we find a free field theory. The theory at the two orbifold singularities is more interesting. The moduli space around each of them looks like $\mathbb{R}^8/\mathbb{Z}_2$. We will soon
argue that the singularity at $\phi^i = 0, \phi^0 = \pi$ is simply an orbifold singularity – the theory at this point is a free field theory with a gauged $\mathbb{Z}_2$ symmetry. The other singularity, at $\phi^i = 0, \phi^0 = 0$ is likely to be an interacting superconformal field theory.

Along the flat directions we get from the eight bosons, which are the fluctuations around the expectation values of $\phi^i$ and $\phi^0$, an $8_v$ of the Spin(8) R-symmetry. Equation (4.1) shows that the dimension of the bosons is $\frac{1}{2}$. This is exactly the result in a free field theory. We will show that at the singularity at $\phi^i = 0, \phi^0 = \pi$ the theory is free. Since it is an orbifold theory, the fluctuations $\phi^i$ and $\phi^0$ are not gauge invariant. Only bilinears in them are gauge invariant operators. Their dimensions are determined easily using (4.1). The interacting theory at $\phi^i = \phi^0 = 0$ is more interesting. It will be interesting to find the leading irrelevant operator there (it seems to be $Q^4O_{35}$). The Spin(8) invariance of this theory was crucial in a recent discussion of the Matrix model applications of this theory [14,15].

More generally consider a gauge group $G$ with rank $r$. Along the flat directions $G$ is broken to its Cartan torus $T(G) = \mathbb{R}^r/\Gamma_w(G)$, where $\Gamma_w(G)$ is the weight lattice of the group $G$ and $\Gamma_w(G)$ is its dual. The $r$ photons can be dualized to $r$ scalars taking values in $\mathbb{R}^r/\Gamma_w(G) = \mathbb{R}^r/\Gamma_w(*G) = T(*G)$, where $*G$ is the dual group, whose weight lattice is dual to $\Gamma_w(G)$. Therefore, the moduli space is

$$
\mathcal{M}(G) = \frac{\mathbb{R}^{7r} \times T(*G)}{W},
$$

where $W$ is the Weyl group of $G$.

For example, let us compare the $SU(2)$ gauge theory with its dual group $SO(3) = SU(2)/\mathbb{Z}_2$. Since $T(SU(2))/\mathbb{Z}_2 = T(SO(3))$,

$$
\mathcal{M}(SU(2)) = \frac{\mathcal{M}(SO(3))}{\mathbb{Z}_2}.
$$

The $SO(3)$ gauge theory has a global $\mathbb{Z}_2$ symmetry, which shifts $\phi^0$ by half its periodicity. In the $SU(2)$ theory this $\mathbb{Z}_2$ symmetry becomes a gauge symmetry and the moduli space is modded out by it.

There is a similar symmetry in the analogous $N = 4$ theory in three dimensions. The moduli space of the $SU(2)$ gauge theory was determined in [16] to be the Atiyah-Hitchin space. Its fundamental group is $\mathbb{Z}_2$. If we instead consider the $SO(3)$ theory, the moduli space becomes the double cover of the Atiyah-Hitchin space, and the $\mathbb{Z}_2$ is a global symmetry. This fact is in accord with the discussion of confinement in [16]. In the $SU(2)$ theory there is (with a suitable perturbation) confinement of electric charge modulo 2 – the massive W bosons can screen external sources. This is reflected in the fundamental group of the moduli space being $\mathbb{Z}_2$. In the $SO(3)$ theory there are no integer external sources and therefore there is no confinement. Correspondingly, the fundamental group of the moduli space is trivial.
Such three dimensional gauge theories are realized in the study of D2-branes [17] in the IIA theory in ten dimensions. The collective coordinates of every D2-brane form a single vector multiplet [18]. The 7 scalars correspond to the 7 transverse directions of the brane. The dual of the vector multiplet corresponds to the position of the brane in the eleventh compact dimension [19]. Hence the moduli space of vacua of the D2-brane is $\mathbb{R}^7 \times S^1$. The coupling constant of this three dimensional field theory determines the circumference of the $S^1$ factor, such that in the strong coupling limit the radius goes to infinity and the membrane propagates in flat eleven dimensional space.

A configuration of two D2-branes in the IIA theory is described by a $U(2)$ gauge theory [18]. At the generic point in the moduli space of vacua the $U(2)$ gauge symmetry is broken to $U(1)^2$ and the light fields are two vector multiplets of $N = 8$. After dualizing the photons and modding out the the Weyl group (which interchanges the two $U(1)$ factors) we find the moduli space

$$\mathcal{M}_2 = \frac{(\mathbb{R}^7 \times S^1) \times (\mathbb{R}^7 \times S^1)}{\mathbb{Z}_2}, \quad (4.7)$$

which is labeled in an obvious way by $\phi^i_I$ and $\phi^0_I$ ($I = 1, 2$) and the $\mathbb{Z}_2$ interchanges $I = 1$ with $I = 2$. The singularities in $\mathcal{M}_2$ are at $\phi^i_1 = \phi^i_2$ and $\phi^0_1 = \phi^0_2$. Physically, they occur when the two membranes are on top of each other.

What is the physics at this singularity? At short distance all the degrees of freedom of the $U(2)$ gauge theory are physical. The interactions between them become strong as we approach the infrared. At long distance the theory flows to a superconformal field theory. This theory may be an interacting or a free field theory. Either way the degrees of freedom at long distance differ from the degrees of freedom at short distance. If the theory is interacting, the notion of the particles at long distance is ill defined. If, however, the theory there is free, it includes only two supermultiplets (rather than the four supermultiplets of the UV $U(2)$ gauge theory). Although we cannot prove it, we find it more likely that the theory there is actually interacting. As we argued above, this interacting theory has $Spin(8)$ enhanced symmetry.

Consider now modding out this theory by the common “center of mass motion” to derive an $SU(2)$ gauge theory. The moduli space of vacua is $\mathcal{M} = (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$, where the $\mathbb{R}^7$ is parametrized by $\phi^i = \phi^i_1 - \phi^i_2$ and the $S^1$ is parametrized by $\phi^0 = \phi^0_1 - \phi^0_2$. The singularity at $\phi^i = \phi^0 = 0$ is the same as in the $U(2)$ problem and is likely to be

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3 The Matrix model description of IIB strings is based on this 2+1 dimensional fixed point field theory [14,15]. Nontrivial string interactions in this framework arise only if this fixed point is interacting.
interacting. What about the other singularity at $\phi^i = 0, \phi^0 = \pi$? It arises because the notion of center of mass in the $S^1$ direction is ill defined. The moduli space of vacua has a new singularity when the two membranes are at $\phi^1 = \phi^2$ and $\phi^0 = \phi^0 + \pi$; i.e. when the two membranes are at the same point in $\mathbb{R}^7$ but at antipodal points in the $S^1$. Clearly, the dynamics at this point is trivial and the singularity is a consequence of the fact that we change $U(2)$ to $SU(2)$ – this is merely an orbifold singularity. Therefore, the singularity at $\phi^i = 0, \phi^0 = \pi$ in the $SU(2)$ theory is an orbifold singularity and the theory there is free.

We remarked above that the leading irrelevant operators along the flat directions of the corresponding four dimensional theory are subject to a non-renormalization theorem and are generated only at one loop. The analogous theorem does not hold in three dimensions [5]. The classical static configuration of the 'tHooft-Polyakov monopole appears as an instanton in the three dimensional theory. One of the effects of these instantons is to explicitly break the shift symmetry of the various magnetic photons $\phi^0$. In fact, we have already mentioned that although the metric on the moduli space (4.5) is flat and hence invariant under the shift, the singularities are not invariant. These instantons were first studied in the theory without supersymmetry by Polyakov [20]. In theories with $N = 2$ supersymmetry they were discussed by Affleck, Harvey and Witten [21] and in $N = 4$ in [22,23]. Their effects in the $N = 8$ theory were studied in [24,25,5]. In particular, they were shown to contribute to terms with four derivatives and to terms with eight fermions (as well as to other terms). More effects of these terms will be discussed in [26].

5. The $N = 4, d = 4$ theory on $\mathbb{R}^3 \times S^1$

Consider now starting with a higher dimensional theory with 16 supercharges and compactifying on a torus to three dimensions. Some of the scalars in the three dimensional Lagrangian originate from components of gauge fields in the higher dimensional theory. Therefore, the corresponding directions in the moduli space of the three dimensional theory must be compact. Let us start by considering the free $U(1) N = 4$ theory in $d = 4$ with gauge coupling $g_4$ and compactify it on a circle of radius $R$ to three dimensions. The three dimensional gauge coupling $g_3$ satisfies

$$\frac{1}{g_3^2} = \frac{R}{g_4^2}. \tag{5.1}$$

The six scalars in the vector multiplet in four dimensions become $\phi^i$ with $i = 1,\ldots,6$. $\phi^7$ arises from a component of the four dimensional gauge field $\phi^7 = A_4$. It corresponds to
a $U(1)$ Wilson line around the circle. A gauge transformation, which winds around this circle, identifies $\phi^7$ with $\phi^7 + \frac{1}{R}$. Therefore, we define the dimensionless field $\phi_e = RA_4$, whose circumference is one. When we dualize the three dimensional photon to a scalar $\phi_m$, we find the Lagrangian [16]

$$\frac{R}{g_4^2}(\partial \phi)^2 + \frac{1}{R g_4^2} (\partial \phi_e)^2 + \frac{g_4^2}{R} (\partial \phi_m)^2. \quad (5.2)$$

The moduli space of vacua is

$$\mathbb{R}^6 \times T^2 \quad (5.3)$$

where the two circles in $T^2$ correspond to the two compact bosons $\phi_e$ and $\phi_m$. They represent a $U(1)$ Wilson line and a $U(1)$ 'tHooft line around the circle we compactified on. In other words, these two scalars are the fourth component of the $d = 4$ photon $A_4$ and the fourth component of the magnetic photon $\tilde{A}_4$. The non-trivial duality transformation in $d = 4$ is translated to

$$\phi_e \rightarrow \phi_m$$

$$\phi_m \rightarrow -\phi_e$$

$$g_4 \rightarrow \frac{1}{g_4}. \quad (5.4)$$

It is easy to add the $\theta$ angle in four dimensions and recover the $SL(2, \mathbb{Z})$ action in four dimension as an action on the $T^2$ in the moduli space (5.3).

As we said above, at long distance in the three dimensional theory only the local structure of the moduli space (5.3) matters. It is $\mathbb{R}^8$. The eight scalars transform as a vector under the enhanced $Spin(8)_R$ symmetry. The duality transformation (5.4) becomes part of the $Spin(8)_R$ symmetry.

We can easily extend this discussion to compactified interacting theories. For example, consider the $SU(2)$ $N = 4$ theory in $d = 4$. Repeating the analysis of the $U(1)$ theory and modding out by the Weyl group, we find the moduli space of vacua

$$\frac{\mathbb{R}^6 \times T^2}{\mathbb{Z}_2}. \quad (5.5)$$

The moduli space has four orbifold singularities. As before, the theory at three of them are orbifold theories (the metric at all of them is an orbifold metric) and the fourth is likely to be an interacting superconformal field theory.

The full theory is invariant only under the $Spin(6)$ symmetry of the four dimensional theory. The $SL(2, \mathbb{Z})$ duality is not a symmetry of the theory. It relates theories with different values of the coupling constant. After the compactification this $SL(2, \mathbb{Z})$ acts on
the $T^2$ factor. Again, it is not a symmetry. However, at long distance its $Z_2$ subgroup becomes a symmetry. Therefore, the symmetry at long distance includes $Spin(6) \times Z_2$. The three dimensional Lagrangian is obtained by shrinking the compactification radius $R$ with $g_3$ fixed. Then, the $Spin(6)$ R-symmetry of the four dimensional theory is enhanced to $Spin(7)$, which is manifest in the three dimensional Lagrangian. Since in this limit $g_4 \to 0$, the $Z_2$ subgroup of $SL(2, \mathbb{Z})$ is not visible. In the long distance limit we should find a symmetry, which includes both this $Spin(7)$ R-symmetry and $Spin(6) \times Z_2$. This must be $Spin(8)$. This leads to an independent derivation of the $Spin(8)$ symmetry of the long distance theory (the other derivation was based on its superconformal invariance). This derivation was also given in [14].

We conclude that the electric-magnetic duality of the four dimensional theory becomes a symmetry of the three dimensional theory. It is included in its $Spin(8)_R$ R-symmetry.

We now generalize to compactification of a $d = 4, N = 4$ gauge theory with gauge group $G$ of rank $r$ on a circle of radius $R$ to three dimensions. Along the flat directions $G$ is broken to its Cartan torus $T(G) = \mathbb{R}^r/\Gamma_w(G)$. The Wilson lines around the circle lead to $r$ scalars in $T(G)$, whose scale is $\frac{1}{\sqrt{R g_4}}$. The 'tHooft loops around the circle (or equivalently the dual of the three dimensional photons) lead to $r$ scalars on $T(\ast G)$, whose scale is $g_4 \sqrt{R}$. The total moduli space is therefore

$$\frac{\mathbb{R}^{6r} \times T(G) \times T(\ast G)}{\mathcal{W}}$$

where again $\mathcal{W}$ is the Weyl group. In this form it is clear that electric-magnetic duality exchanges $g_4$ with its inverse and $G$ with $\ast G$.

6. (8, 8) supersymmetry in $d = 2$

The (8, 8) supersymmetry algebra in two dimensions has a simple massless free field representation consisting of 8 bosons $\phi^i$, 8 left moving fermions $S_-$ and 8 right moving fermions $S_+$. It is interesting to consider the possible action of the $Spin(8) \times Spin(8)$ automorphism group of the algebra (2.1). Without loss of generality, let the 8 right moving supercharges, $Q^\alpha_+$, be in $8_s$, and the 8 right moving fermions, $S^\alpha_+$, in $8_c$ of one of the $Spin(8)$ factors. This implies that the 8 bosons are in $8_v$. Since the bosons are rotated by this symmetry, the corresponding conserved currents $j_{\mu}^{[ij]} = \phi^{[i} \partial_{\mu} \phi^{j]}$ are not conformal fields $- \phi^i$ (and not only their derivatives) appear in the current. Therefore, we do not have separate left moving and right moving currents. This means that the same $Spin(8)$ symmetry must also act on the left moving supercharges $Q_-$ and fermions $S_-$. Here we have
two options: the left moving supercharge, $Q^\alpha$, can be in $8_c$ and the left moving fermion, $S_\dot{\alpha}$, in $8_s$, or the left moving supercharge, $Q^{\dot{\alpha}}$, can be in $8_s$ and the left moving fermion, $S^\alpha$, in $8_c$. These two assignments appear in the Green-Schwarz light cone formalism for IIA and IIB superstrings respectively.

Interacting theories can be constructed by starting with super Yang-Mills theory in higher dimensions. These theories have a global $Spin(8)$ symmetry. The left moving and right moving supercharges have opposite $Spin(8)$ chirality and hence this corresponds to the first option above (as in IIA strings). Classically these theories have a moduli space of vacua (2.2) $\mathbb{R}^{8r}/\mathcal{W}$ ($r$ is the rank of the gauge group and $\mathcal{W}$ is its Weyl group). Along the flat directions the massless spectrum consists of $r$ copies of the free representation discussed above with the global $Spin(8)$ symmetry acting as in the first assignment. However, in two dimensional field theory the notion of moduli space of vacua is ill defined because we should integrate over it. To define it we can integrate out the high energy modes and construct an effective action for the light modes. At short distance the theory is the non-Abelian gauge theory. The long distance theory is a scale invariant theory. Its target space is the orbifold [27]

$$\mathbb{R}^{8r}/\mathcal{W}. \quad (6.1)$$

The main question is how to treat the theory at the singularities.

In order to answer this question we could attempt to extend the scale invariance of the theory to conformal invariance and construct a superconformal field theory with $(8,8)$ supersymmetry. However, there is no superconformal extension of this $(8,8)$ supersymmetry algebra[4]. One way to see that is to recall the fact, demonstrated above in the free representation, that the $Spin(8) \times Spin(8)$ automorphism group cannot be a symmetry. Furthermore, even the currents of the diagonal $Spin(8)$, which can be conserved, are not conformal fields. This suggests that perhaps the only scale invariant theories with $(8,8)$ supersymmetry are free. In this case, the long distance theory is simply the orbifold theory based on the orbifold (6.1).

In a beautiful paper Dijkgraaf, Verlinde and Verlinde [28] analyzed this orbifold conformal field theory and determined the leading irrelevant operator in the long distance orbifold theory. For the simple case of $SU(2)$ it is constructed as follows. The target space is $\mathbb{R}^8/\mathbb{Z}_2$ where the $\mathbb{Z}_2$ acts by changing the sign of the 8 bosons $\phi^i$, the 8 right moving fermions $S_\alpha^r$ and the 8 right moving fermions $S^{\dot{\alpha}}$. The right moving bosonic twist operator

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4 We thank N. Berkovits for a useful discussion on this point.
σ and the right moving fermionic twist fields Σ^i and Σ^α satisfy

\[ \partial_+ \phi^i(z) \sigma(0) \sim \frac{1}{\sqrt{z}} \tau^i(0) \]

\[ S_+^\alpha(z) \Sigma^i(0) \sim \frac{1}{\sqrt{z}} \gamma^i_{\alpha\bar{\alpha}} \Sigma^\alpha(0) \]

\[ S_+^\alpha(z) \Sigma^\alpha(0) \sim \frac{1}{\sqrt{z}} \gamma^\alpha_{\bar{\alpha}i} \Sigma^i(0). \]  

(6.2)

The dimensions of σ, Σ^i and Σ^α are \( \frac{1}{2} \) and the dimension of τ^i is 1. Using these building blocks Dijkgraaf, Verlinde and Verlinde construct the primary field \( O^\dot{\alpha} = \sigma \Sigma^\dot{\alpha} \). Its “descendants” \( Q^\dot{\alpha}_+ O^\beta \) can be in \( 1 \oplus 28 \oplus 35_s \). An explicit computation shows that the field in \( 35_s \) is null

\[ Q^\dot{\alpha}_+ O^\beta + Q^\beta_+ O^\dot{\alpha} = 0. \]  

(6.3)

The field in \( 1 \)

\[ O_+ = \tau^i \Sigma^i \]  

(6.4)

satisfies, by using (6.3) and the anticommutation relations

\[ Q^\dot{\alpha}_+ O_+ = \partial_+ (\sigma \Sigma^\dot{\alpha}). \]  

(6.5)

Note that in establishing (6.5) we use only the supersymmetry algebra and not the non-existing superconformal algebra. We can repeat this analysis for the left movers and construct the operator \( O_- \). Then, because of (6.5) the operator

\[ V = \int O_+ O_- \]  

(6.6)

is supersymmetric. Its dimension is \( (\frac{3}{2}, \frac{3}{2}) \) and it is the leading supersymmetric irrelevant operator.

As in the previous section we can consider the compactification of the four dimensional theory on \( T^2 \) to two dimensions and study the theory as a function of the coupling constant of the four dimensional theory, \( \tau \), and the parameters of the torus. This was done in [27]. The six noncompact scalars from four dimensions lead to a factor of \( \mathbb{R}^{6r} \). The two polarizations of the photon lead to two factors of \( T(G) \) such that the moduli space is

\[ \frac{\mathbb{R}^{6r} \times T(G) \times T(G)}{\mathcal{W}}. \]  

(6.7)
The metric on the two T(G) factors depends on \( \tau \) and the parameters of the compactification. We can use T duality in the two dimensional theory and convert one or both of the T(G) factors to T(*G):

\[
\frac{\mathbb{R}^{6r} \times T(G) \times T(*G)}{\mathcal{W}} = \frac{\mathbb{R}^{6r} \times T(*G) \times T(G)}{\mathcal{W}}.
\] (6.8)

Therefore, S duality, which exchanges \( \tau \rightarrow -\frac{1}{\tau} \) and \( G \rightarrow *G \) in four dimensions, translates to T duality after compactification [27]. Clearly, the physics near the orbifold singularities of (6.7) or (6.8) generalizes in an obvious way the discussion of [28].

7. Theories with (0, 2) supersymmetry in \( d = 6 \) and their compactification

Here we study the theories with (0, 2) supersymmetry in \( d = 6 \) and their compactification. These theories first appeared in the study of K3 compactifications of the Type IIB theory [29] and later in the context of nearby 5-branes in M-theory [30,31]. These theories are expected to be non-trivial fixed points of the renormalization group in six dimensions. Therefore, they have no dimensionful parameter. Furthermore, since these fixed points are isolated, they have no dimensionless parameter.

Along the moduli space of the six dimensional theory there are \( r \) tensor multiplets of (0, 2) supersymmetry. Each of them includes 5 scalars and a two form \( B \), whose field strength \( H = dB \) three form is selfdual. The one form gauge invariance is subject to some global identification corresponding to the allowed non-trivial fluxes of \( H \). If there are \( r \) fields, the fluxes \( \int H_a \) (\( a = 1, \ldots, r \)) through various three cycles are quantized. Since \( H \) is selfdual, the lattice of charges of these fluxes is a selfdual lattice. Therefore, the Abelian one form gauge invariance along the flat directions is characterized by a selfdual lattice \( \Gamma \).

Interesting order parameters in this theory are the generalizations of the Wilson loops, which we can call Wilson surfaces. These are given by \( \exp i \int Q^a B_a \), where the integral is over a two surface and \( Q \in \Gamma \). Since \( H \) is selfdual, these are also the generalizations of the ’tHooft loop. The equality between them is possible only when \( \Gamma \) is selfdual.

Some important subtleties associated with the definition of the theory of such two forms were discussed in [32]. Even without supersymmetry or fermions the theory needs a spin structure for its definition. Since we are studying the supersymmetric theory we need a spin structure anyway. This discussion might interfere with the conclusion above that \( \Gamma \) has to be selfdual.

\footnote{5 We thank E. Witten for a useful discussion on this point.}
The $(0, 2)$ supersymmetry constrains the metric on the moduli space to be locally flat. The only allowed singularities are orbifold singularities in the metric. Hence, the moduli space is

\[ \mathbb{R}^{5r}/W, \tag{7.1} \]

where $W$ is a discrete group. The theory at the singularities is a superconformal field theory. The superconformal algebra includes a $Spin(5)$ R-symmetry \cite{4}, which acts on the 5 scalars.

As in section 2, the scalars on the moduli space $\Phi$ have dimension two. Their expectation values determine the tension of BPS strings, which exist in the theory. The reason for that is that $\Phi$ is in a tensor multiplet, which includes the two form $B$, and $B$ couples canonically to strings. Since the field strength of $B$ is selfdual, these strings are also selfdual.

Consider the compactification of these theories to five dimensions on a circle of radius $R_6$. The kinetic terms for the scalars become

\[ \int dx^6 (\partial \Phi)^2 \sim \frac{1}{R_6} (\partial \phi)^2 \tag{7.2} \]

where the scalar $\phi = R_6 \Phi$ is of dimension one. This compactification does not produce more scalars and the moduli space remains as in (7.1), $\mathbb{R}^{5r}/W$.

These theories flow at long distance to super-Yang-Mills theory. To see that, note that the two form $B$ becomes in five dimensions a vector and a two form. The self-duality condition in six dimensions identifies them as dual to each other. Therefore, every $B$ leads to one vector field in five dimensions. As in (7.2), the gauge coupling of the five dimensional theory is

\[ \frac{1}{g_5^2} = \frac{1}{R_6}. \tag{7.3} \]

The scale invariance of the six dimensional theory is explicitly broken by the scale of the compactification. This scale determines the dimensionful gauge coupling of the five dimensional gauge theory (7.3). The five dimensional gauge theory is not renormalizable. It breaks down at a scale of order $\frac{1}{g_5^2}$. Furthermore, as we discussed in section 2, there is no interacting fixed point of the renormalization group in five dimensions. Therefore, we cannot define the five dimensional gauge theory as the low energy limit of a five dimensional fixed point. One way to define it is to use the six dimensional fixed point \cite{33} (see also \cite{34}) as we did above. This definition turns out to be useful in Matrix theory. The fact that the low energy theory is a gauge theory shows that the lattice $\Gamma$ and the discrete
group $W$, which appeared in the data of the six dimensional theory are the weight lattice and the Weyl group of a group $G$.

The five dimensional gauge theory has a conserved current $j = * F \wedge F$. Instantons of the gauge theory are charged BPS particles [7]. Their masses are proportional to $\frac{1}{g_5^2} = \frac{1}{R_6}$. The detailed properties of these instantons depend on the precise way the theory is defined. In the context where the five dimensional theory appears as a compactification of the six dimensional theory, this relation identifies them as Kaluza-Klein momentum modes around the compact circle [34].

Along the flat directions of the five dimensional theory there are massive BPS particles (the “W-bosons” of the gauge theory), whose masses are proportional to $\phi$. These can be interpreted as the strings of the six dimensional theory wrapping the circle, and hence their masses are $\phi = R_6 \Phi$. There are also the BPS strings, which are the the six dimensional strings in the noncompact dimensions. Their tensions are $\Phi = \phi / R_6 = \phi / g_5^2$. This relation identifies them as being ’tHooft-Polyakov monopole solutions of the gauge theory, which are strings in five dimensions.

As we said, the six dimensional field theory has string like excitations. Can it be formulated as a theory of interacting strings? The observation above suggests that if this is the case, it is not simply a string field theory. In the five dimensional theory the W-bosons appear as fundamental particles. The strings are constructed as classical solutions (magnetic monopoles) in the five dimensional field theory. Therefore, they can be interpreted as made out of the W-bosons. We should not include in the five dimensional theory both the W-bosons and the strings as elementary degrees of freedom. However, from a six dimensional point of view these two excitations are very similar; they originate from the six dimensional string when it does or does not wrap the circle. Therefore, it appears that a naive string field theory like description of the six dimensional theory will over-count the elementary degrees of freedom.

When these theories are compactified to four dimensions on a two torus $T_{5,6}$ with radii $R_{5,6}$ they become $N = 4$ theories. Along the flat directions we find the $5r$ scalars of the six dimensional theory and $r$ compact scalars, which arise from the Wilson surface of the two-forms $B$ on $T_{5,6}$. These scalars take values on $\mathbb{R}^r / \Gamma$, and the scale of this torus is $(R_5 R_6)^{-\frac{r}{2}}$. The moduli space is

$$\frac{\mathbb{R}^{5r} \times (\mathbb{R}^r / \Gamma)}{W}.$$  

(7.4)

At the singularities we find an $N = 4$ theory labeled by a gauge group $G$ and the dimensionless coupling constant $\tau$, which is determined as the complex structure of $T_{5,6}$. As in five dimensions, $\Gamma$ and $W$ are the weight lattice and the Weyl group of $G$. The $SL(2, \mathbb{Z})$
freedom in the complex structure of this torus translates to $SL(2, \mathbb{Z})$ duality in the field theory \cite{29}. Since $\Gamma$ is selfdual, $G = *G$. For such theories the $SL(2, \mathbb{Z})$ duality acts without changing the gauge group. As in the compactification to five dimensions, we can identify the W-bosons and the magnetic monopoles as strings wrapping the two different cycles of $T_{56}$ \cite{29}. They are exchanged by $SL(2, \mathbb{Z})$.

We can continue to compactify to three dimensions by adding a circle of radius $R_4$. Combining the previous analysis with the discussion of the compactification from 4 to 3 dimensions above we find the moduli space $\mathbb{R}^5 \times (\mathbb{R}^r/\Gamma) \times (\mathbb{R}^r/\Gamma) \times (\mathbb{R}^r/\Gamma)$, where the scales of the three tori are $\frac{R_{4,5,6}}{\sqrt{R_4 R_5 R_6}}$ (for simplicity we assume that all the angles of the torus are right angles).

The simplest such nontrivial theory is the theory of two 5-branes in eleven dimensions. The group $G$ associated with this theory is $U(2)$, which is selfdual, and $\mathcal{W} = \mathbb{Z}_2$. Before compactification the moduli space is $\mathbb{R}^5 \times \mathbb{R}^5 \mathbb{Z}_2$, where each factors comes from one 5-brane and the $\mathbb{Z}_2$ reflects the fact that they are identical. We now compactify them on a three torus $T_{456}$ with radii $R_{4,5,6}$. We find the moduli space $(\mathbb{R}^5 \times \tilde{T}_{456})^2 \mathbb{Z}_2$, where the radii of the three torus $\tilde{T}_{456}$ are $\frac{R_{4,5,6}}{\sqrt{R_4 R_5 R_6}}$. Note that it is the same as the moduli space of two 2-branes, which move on $T_{456}$. This is consistent with the duality between them in eight dimensions \cite{35}. Furthermore, the theory at the singularity is exactly that of the three dimensional $U(2)$ theory.

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