The rest mass

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A relation connecting the rest mass and separation of events in space-time continuum is suggested and the idea of Compton scattering is used as a method for the determination of rest mass. An experiment involving collision of photons resulting in creation of rest mass is discussed theoretically in order to illustrate the connection formula.

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Historically the rest mass first appears in the special theory of relativity as a mathematical by-product when proper momentum was defined and the conservation of 4-momentum was taken into consideration [1]. P. G. Bergmann [1] in course of discussion on the relation between energy and mass in relativistic mechanics observed that if kinetic energy of a mechanical system decreases, at least some of the rest masses of the constituent system must increase. This so-called mass-energy equivalence accounts for mass defect in atomic nuclei, disintegration energy of decay processes and so on and so forth. Rest masses change appreciably when interaction energies have the order of magnitude of the rest energies. Till to-date only a few attempts are made to determine the rest mass experimentally; Jian Qi Shen [2] performed experiments on photon rest mass. Though an attempt has been made by Donald Chang [3] to investigate the wave properties corresponding to rest mass, no real effort has yet been made to resolve the fundamental question regarding the status of the rest mass and to consider whether any basic principle is involved that inherently restricts its determination.

It is quite obvious that following definition [4] of energy in special theory of relativity and principle of energy and momentum framed therein the loss of rest mass accounts for the production of energy and vice versa. However the transfer of energy ($\Delta E$) is connected to time interval ($\Delta t$) needed for the transfer to take place by

$$\Delta E \Delta t \geq \hbar$$

and ($\Delta E$) can be measured with the available measuring apparatus. But how can one measure rest mass? Obviously it cannot be done directly because any such effort will disturb the system to the extent of changing the rest energy $E$ to some value $E + \Delta E$. One possible way is to consider Compton effect where the theory assumes the particle (scatterer) to be free and at rest before collision with the photon takes place. The Compton shift is given by

$$\lambda - \lambda_0 = (\hbar/mc)(1 - \cos \theta)$$

where $\lambda$ and $\lambda_0$ are the wavelengths respectively of scattered and incident waves, $m$ is the
rest mass of the particle and \( \theta \) is the angle of scattering. At a particular value of \( \theta \), the measurement of the Compton shift can give an estimate of the rest mass. If one employs the expression (2) to find \( m \), then it becomes clear that the limit to measure \( m \) accurately depends on the limit to which \( \lambda \) and \( \lambda_0 \) can be resolved. One can use Rayleigh’s criterion of resolution to get the maximum value of rest mass that can be measured accurately, at least in principle.

Now, the resolving power of an optical system is \( R = \lambda_0/(\lambda - \lambda_0) \), following symbols of expression (2). The determination of rest mass depends on clear identification of the peaks for the two waves (incident and scattered). For this to happen the shift in wavelength must be at least \( \lambda_0/R \), which leads to the inequation

\[
m \leq hR(1 - \cos \theta)/(c\lambda_0)
\]  

In other words, \( m \) must have an upper limit.

Now, let us consider the following process (ref. Fig. 1) viz. collision of two oppositely directed photons resulting in the creation of rest mass \( m \), which in terms of usual symbols of the equation for the conservation of 4-momentum can be written as

\[
(h\nu/c)(1, \vec{k}) + (h\nu/c)(1, -\vec{k}) = (2mc, \vec{0}),
\]

\( \vec{k} \) being the unit vector along the momentum of ‘photon 1’. The Fig. 1 describes the process in the space-time continuum (only one space dimension is shown). Thus,

\[
2h\nu/c = 2mc \quad \text{i.e. } h\nu = mc^2; \quad \text{and } (h\nu/c)\vec{k} - (h\nu/c)\vec{0} = \vec{0}
\]

In this particular example, therefore, change \( \Delta m \) in rest mass is of the order of \( h\nu/c^2 \) which is equal to \( h/(c\lambda_m) \) where \( \lambda_m \) is the wavelength of either of the photons. Let \( \Delta s \) denote an infinitesimally small interval separating events viz. collision of photons and creation of mass at rest, \( s \) being given by

\[
(\Delta s)^2 = c^2(\Delta \tau)^2 - |\Delta \vec{r}|^2
\]

(the terms have their usual significance).
To illustrate the process where two photons projected simultaneously from $x = a$ and $x = -a$, meet at $x = 0$ to form a point mass at rest.

The separation $\Delta s$ in this example should be of the order of $\lambda_m$. Thus,

$$\Delta m \Delta s \sim \hbar/c$$

In order to include the idea expressed in (3), expression (4) should be rewritten as

$$\Delta m \Delta s \leq \hbar/c$$

The very description that mass is at rest, makes one conceive of localization (in spacetime) of the system, and this is what is precisely given in (5). For the sake of an application if we substitute in (5) the Planck mass ($10^{19} \text{ GeV}$) for $\Delta m$, we get for $\Delta s$ a
value which is of the order of $10^{-33} \text{ cm}$, the Planck length.

For time like separation of events it might well be that with respect to an observer $|\Delta \mathbf{r}| = 0$; then $\Delta s = c \Delta \tau$ and in the context of creation of mass (as discussed above in connection with the problem of two photons) the world line of the physical system will obviously be traced in such a way that $\Delta \tau$ becomes positive, otherwise ‘causality’ will be violated. This $\Delta \tau$ signifying the separation in time of the two events, must have a magnitude of the order of uncertainty $\Delta t$ of relation (1) when $\Delta E$ becomes the energy equivalent of $\Delta m$. Now for space-like separation it can be that with respect to an observer $\Delta \tau = 0$, which means that $(\Delta s)^2 < 0 \ i.e. \ \Delta s$ is imaginary. Thus it is evident from (5) that for space like separation of events mass creation is not possible.

Since loss of rest mass makes corresponding $\Delta E$ positive and the gain in rest mass is always associated with loss in energy, $\Delta m$ and $\Delta E$ must have opposite signs; the uncertainty relation expressed by (1) then follows simply from relation (5) for a time like separation of events when mass is being created.

References

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