Parametric dependence of two-plasmon decay in homogeneous plasma

Dejan R. Dimitrijević
Department of physics, Faculty of sciences and mathematics, University of Niš, Višegradska 33, 18001 Niš, Serbia
dimke@junis.ni.ac.rs

Abstract. A hydrodynamic model of two-plasmon decay in a homogeneous plasma slab near the quarter-critical density is constructed in order to improve our understanding of the spatio-temporal evolution of the daughter electron plasma waves in plasma in the course of the instability. The scaling of the amplitudes of the participating waves with laser and plasma parameters is investigated. The secondary coupling of two daughter electron plasma waves with an ion-acoustic wave is assumed to be the principal mechanism of saturation of the instability. The impact of the inherently nonresonant nature of this secondary coupling on the development of two-plasmon decay is researched and it is shown to significantly influence the electron plasma wave dynamics. Its inclusion leads to nonuniformity of the spatial profile of the instability and causes the burst-like pattern of the instability development, which should result in the burst-like hot-electron production in homogeneous plasma.

1. Introduction
Two-plasmon decay (TPD) is a nonlinear parametric instability during which a large amplitude electromagnetic pump wave decays into two electron plasma waves near the quarter-critical plasma density \( n_c/4 \). TPD has primarily been known for its detrimental aspect, as a source of suprathermal (hot) electrons capable of preheating the fusion targets in the inertial confinement fusion experiments [3-5]. The electrostatic nature of the daughter electron plasma waves renders the experimental verification of various aspects of the theory of TPD rather difficult, and the comparison of theoretical and numerical predictions with the experimental evidence is mainly qualitative.

Here we report the results of our investigation of the spatio-temporal evolution of the amplitudes of the waves participating in the instability, especially the daughter electron plasma waves, and its dependence on the pump laser intensity and plasma parameters. Since the secondary coupling of the daughter electron plasma waves with ion-acoustic waves is widely recognized as the principal mechanism leading to the saturation of the instability, we are particularly interested in investigating the influence of the nonresonant nature of this secondary coupling on the wave dynamics and the spatio-temporal development of TPD. A transparent hydrodynamic model of TPD in spatially homogeneous plasma slab near the quarter-critical density is constructed, taking into account numerous nonlinear features of the instability. Similar model has been utilized earlier [6, 7] in order to search for the time-dependent local solutions for the slowly varying wave amplitudes, by neglecting all the convective terms and, consequently, group velocities of the coupled waves in the corresponding coupled-wave equations, which was justified by the fact that the nature of the instability is strictly
resonant. This approach has been generalized [8, 9] by simulating the complete set of coupled-wave equations numerically, in order to gain a more complete insight into the spatio-temporal behavior of the wave amplitudes inside the homogeneous plasma slab.

In the next section, we will introduce the theoretical model that describes the evolution of the slowly varying amplitudes of the waves taking part in the development and the saturation of the instability in the homogeneous plasma slab. The model hydrodynamic equations are simulated numerically and the corresponding results are presented and discussed in the final section.

2. Theoretical model

We consider the propagation of a linearly polarized high intensity laser radiation (pump wave), with frequency \( \omega_0 \) and wavelength \( \lambda_0 \), in a homogeneous underdense plasma slab whose width is \( L = 100\lambda_0 \). The pump wave enters the slab perpendicularly through its left boundary. If plasma density is in the vicinity of \( n_e/4 \), the pump electromagnetic wave decays nonlinearly into two daughter electron plasma waves, whose frequencies and wave vectors are \( \omega_{1,2} \) and \( \vec{k}_{1,2} \), respectively. Resonant nature of TPD implies that the well-known matching conditions for the frequencies and wave vectors, \( \omega_0 = \omega_1 + \omega_2 \) and \( \vec{k}_0 = \vec{k}_1 + \vec{k}_2 \), must be satisfied strictly. Bearing in mind that the electron plasma wave frequencies are temperature-dependent through the Bohm-Gross dispersion relations \( \omega_{1,2}^2 = \omega_{pe}^2 + 3k_e^2v_e^2 \), where \( \omega_{pe} \) is the electron plasma frequency and \( v_e \) the thermal velocity, it is clear that the plasma slab density must be somewhat lower than \( n_e/4 \) in order to provide conditions necessary for the onset of TPD.

Numerous theoretical, experimental and simulation studies [8-14] have indicated that an efficient coupling of the electron plasma wave electrostatic energy to shorter-wavelength ion density fluctuations, or ion-acoustic waves, is the main factor responsible for the saturation of the instability in homogeneous plasma. Our simulations performed in the limit of time-only dependence [6, 7] confirmed that the saturation occurs simultaneously with the ion fluctuation amplitude reaching its maximum value, whose order was only a fraction of a percent of the bulk plasma density. The matching conditions for the secondary coupling are \( \omega_s = \omega_1 - \omega_s \) and \( \vec{k}_s = \vec{k}_1 - \vec{k}_2 \), where \( \omega_s \) and \( \vec{k}_s \) are the frequency and the wave vector of the ion-acoustic wave. It is clear though that the nature of the secondary coupling is nonresonant, so the corresponding frequency mismatch \( \Delta \omega = \omega_2 + \omega_s - \omega_1 \) must be taken into account when writing down the equations of the system evolution. Our earlier work [6, 7] confirms that it leads to shifting and broadening of the frequency lines in the plasmon spectra, as well as an increase of their complexity and the appearance of new spectral lines corresponding to lower-frequency oscillations. Thus the influence of the nonresonance of the secondary coupling on the spatio-temporal evolution of TPD proved to be significant and has been further investigated [8, 9].

We start with the basic set of electron and ion fluid equations combined with the Maxwell equations, to yield the following system of normalized partial differential equations for the slowly varying amplitudes of the coupled waves, namely electromagnetic wave \( (a) \), electron plasma waves \( (N_1, N_2) \), and the ion-acoustic wave \( (N_s) \) participating in the secondary coupling:

\[
\left( \frac{\partial}{\partial t} + V_0 \frac{\partial}{\partial x} \right) a = -B_0 N_1 N_2, \quad (1)
\]

\[
\left( \frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x} \right) N_1 = B_2 a N_1^* + C_1 N_2 N_s \exp(-i\Omega t) - \Gamma_1 N_1, \quad (2)
\]

\[
\left( \frac{\partial}{\partial t} + V_2 \frac{\partial}{\partial x} \right) N_2 = B_2 a N_1^* - C_2 N_1 N_s^* \exp(i\Omega t) - \Gamma_2 N_2, \quad (3)
\]

\[
\left( \frac{\partial}{\partial t} + V_s \frac{\partial}{\partial x} \right) N_s = C_3 N_1 N_s^* \exp(i\Omega t). \quad (4)
\]
Here, only the wave amplitude dependences in the direction of the pump propagation are accounted for, and the group velocities $V_0$, $V_1$, $V_2$ and $V_s$ stand for the longitudinal components of the corresponding vectors. This can be done without loss in generality in the regime of prime interest for laser fusion applications, $(v_0/v_s) \leq 0.1$, where the transverse components of plasma waves are quite small, which is the case when visible and UV lasers are utilized. From the geometry of the instability follows that in such cases one plasma wave is directed forward, almost along the x-axis, while the other is directed backward, and longitudinal components of their wave numbers relate as $k_1 - k_0 \gg k_2$.

Our simulation results refer to this parametric range, although even in the regime $(v_0/v_s) > 10$, typical for CO$_2$ laser driven plasmas, some qualitatively accurate predictions can be made.

In the system of equations (1)-(4) time and space variables are normalized to $1/\omega_0$ and $c/\omega_0$ respectively, the group velocities of all the waves to the speed of light $c$, and the frequency mismatch is normalized as $\Omega = \Delta \omega/\omega_0$. The amplitude of the electromagnetic wave is represented by the dimensionless parameter $a = v_0/c$, where $v_0 = E_0/m_0 \omega_0$ is the electron quiver velocity in the laser electric field whose amplitude is $E_0$. The amplitudes of the electrostatic waves are normalized to the equilibrium (bulk) plasma density $n_0$, and the damping rates for the electron plasma waves, $\Gamma_{1,2}$, which include both collisional and noncollisional (Landau) damping, to $\omega_0$. The coupling coefficients are given as:

$$B_0 = \alpha^3 \kappa_{\perp} \left( \kappa_1^2 - \kappa_2^2 \right)/4 \kappa_1^2 \kappa_2^2,$$

$$B_{1,2} = \kappa_{\perp} \left( \kappa_1^2 - \kappa_2^2 \right)/8 \alpha \kappa_{\perp}^2,$$

$$C_{1,2} = \alpha^3 \left| \kappa_{\perp} \cdot \kappa_{\parallel} \right|^2 /4 \beta_1^2 \kappa_1^2 \kappa_2^2,$$

$$C_3 = \sqrt{Zm/M_1} \beta \kappa_{\perp}^2 \left| \kappa_{\perp} \cdot \kappa_{\parallel} \right|^2 /4 \kappa_1^2 \kappa_2^2,$$

and the dimensionless plasma parameters are $\alpha = \omega_0/\omega_0 = \sqrt{n_0/n_{ew}}$ and $\beta = v_0/c = \sqrt{T_e/(keV)/511}$, where $v_e$ stands for the electron thermal velocity which corresponds to the electron temperature $T_e$. The value for the electron-ion mass ratio is taken to be $Zm/M_1 = 1/3600$ in our simulations, and all the wave vectors and their components are normalized to $\omega_0/c$.

We will limit our discussion to the optimal case of maximum instability increment, since the contribution of the nonresonant coupling is relatively small due to the fact that TPD is instability of a highly resonant nature. The relation between longitudinal and transversal components of the wave vectors of daughter electron plasma waves in the case of maximum instability increment is the well-known equation of hyperbola $k_1^2 = k_0 \left( k_1 - k_0 \right)$. By combining this equation with TPD matching conditions and the dispersion relations for the electron plasma waves, and by subsequently normalizing all the variables as stated above, we easily obtain the expressions for intensities of the normalized wave vectors of both electron plasma waves and their components in the form:

$$\kappa_{1,2} = \sqrt{(1-2\alpha)/6 \beta_e^2 \pm \sqrt{(1-\alpha^2)(1-2\alpha)/12 \beta_e^2}} ,$$

$$\kappa_{1,23} = \sqrt{(1-\alpha^2)/4 \pm \sqrt{(1-2\alpha)/12 \beta_e^2}} ,$$

$$\kappa_{7} = \sqrt{(1-2\alpha)/12 \beta_e^2 - (1-\alpha^2)/4} ,$$

while normalized intensity of the wave number of the ion-acoustic wave is

$$\kappa_s = \left| \kappa_{\perp} - \kappa_{\parallel} \right| .$$

Finally, longitudinal components of the group velocities, calculated from the corresponding dispersion relations and normalized to the speed of light, can be written in our notation as:
Clearly, the wave vectors and the scattering angles for the optimal case of maximum instability increment at given plasma density, as well as the group velocities of the participating waves, depend solely on plasma parameters, $\alpha$ and $\beta$.

3. Results and discussion

We simulate the system (1)-(4) in order to investigate the behavior of the shorter-wavelength, forward propagating electron plasma wave, whose potential detrimental impact in the inertial confinement fusion experiments is most prominent. We will assume that the pump wavelength is $\lambda_p = 1\mu m$ and the time scale, expressed in picoseconds, will be dependent on that assumption.

The electron temperature dependence of the forward-propagating electron plasma wave amplitude temporal evolution is displayed in Figure 1(a). The saturation value of the amplitude $N_s$ clearly scales with the square root of the electron temperature, and our simulations indicate that the same can be said about the ion-acoustic wave amplitude. This is caused by the fact that the group velocities of the electrostatic waves in plasma, calculated through (9), increase linearly with $\beta_e$. It can also be inferred that the electrostatic wave amplitudes saturate much earlier when the electron temperature is higher, as the higher electron thermal energy leads to a more efficient damping of the amplitude oscillations. On the other hand, deeper into the plasma the amplitudes saturate with considerable delay: for the parameters in Figure 1(a) and the electron temperature of 600 eV, the saturation occurs after about 10 ps for $x = 25\lambda_0$, and less than 3 ps for $x = 75\lambda_0$.

![Figure 1: Temporal evolution of the forward-propagating electron plasma wave amplitude for: (a) $a = 0.03$, $n = 0.248n_{cr}$, and various electron temperatures and distances from the left plasma slab boundary: $T = 0.2$ keV, $x = 25\lambda_0$ (dashed line), $T = 0.6$ keV, $x = 25\lambda_0$ (solid line), $T = 0.6$ keV, $x = 75\lambda_0$ (dotted line); (b) $a = 0.03$, $T = 0.2$ keV, $x = 75\lambda_0$, and for plasma densities: $n = 0.240n_{cr}$ (solid line) and $n = 0.248n_{cr}$ (dotted line).](image-url)

Since the electron plasma wave group velocity decreases with increasing plasma density, as inferred from (9), its saturation amplitude exerts the same dependence, which is illustrated in Figure 1(b). Even though the initial amplitude oscillations are far more rapid for lower plasma densities than for the higher ones, they reach the saturation at approximately the same time, which is of the order of 10 ps. While the saturation amplitude is about 14% for $n = 0.240$, its value is only 1% lower for
Such a weak density dependence, however, does not apply to the ion-acoustic wave amplitude, which plunges from 0.4% to 0.06% in the same interval of densities. Our model thus confirms that even very low values of the ion fluctuations turn out to be sufficient to procure saturation of the instability, especially when the plasma density is close to \( n_{cr}/4 \).

Spatio-temporal evolution of the electron plasma wave amplitude is displayed in Figure 2 in three-dimensional (a) and contour plot (b) representation, for a typical set of laser and plasma parameters, in the case of resonant secondary coupling. The character of the wave dynamics is simply periodic throughout the plasma slab with natural exception of the narrow bands along the boundaries. The saturation level remains mainly constant, approximately 0.14 for the chosen parameters.

![Figure 2: Three-dimensional (a) and contour plot (b) representation of initial spatio-temporal evolution of the forward-propagating electron plasma wave amplitude for \( a = 0.03, \ T = 0.2\text{keV}, \ n = 0.240n_{cr}, \) when the secondary coupling is resonant.](image)

It is rather illustrative to compare these results with those represented in Figure 3, obtained for the same set of parameters when nonresonance of the secondary coupling is taken into account. As before, the results are represented in three-dimensional (a) and contour plot diagram (b) form. The distinct nonuniformity of the electron plasma wave amplitude throughout the plasma slab is revealed for \( \Omega \neq 0 \). Apparently, TPD wave intensity oscillates at the frequency corresponding to that of the ion-acoustic waves and saturates after about 6 ps in the vicinity of the left plasma slab boundary, and up to 10 ps towards the right boundary. As anticipated, the saturation generally occurs later than in the case of resonant secondary coupling, and the saturation amplitude is considerably higher, because the ion-coupling mechanism is less efficient when \( \Omega \neq 0 \). These features of the wave dynamics clearly become more pronounced towards the right plasma slab boundary, as the impact of finite \( \Omega \) causes a cumulative effect on larger space scales. This is in good agreement with benchmark results of Meyer’s group [12] which were obtained by using CO\(_2\) laser radiation with 10.6\,\mu\text{m} wavelength, where the Thomson scattering streak record of the spatial development of TPD wave intensity confirms significant spatial nonuniformity of the electron plasma wave intensity, with closely spaced local maxima of resolution limited size. Apparently, the impact of \( \Omega \) is significant even though its value is of the order \( 10^{-3} \) for most parametric regimes of interest.
Figure 3: Three-dimensional (a) and contour plot (b) representation of initial spatio-temporal evolution of the forward-propagating electron plasma wave amplitude for $a = 0.03$, $T = 0.2\text{keV}$, $n = 0.240n_{cr}$, when nonresonance of the secondary coupling is taken into account.

Some interesting conclusions can be drawn by observing the long-term spatio-temporal evolution of the absolute value of the forward-propagating electron plasma wave amplitude, displayed in Figure 4. After the initial period of rapid oscillations, which unfolds according to the previously described scenario, the electron plasma wave amplitude saturates temporarily to a value of 0.14 to 0.17, depending on $x$, for the chosen laser and plasma parameters, and then enters a period of rapid oscillations. The regime of these oscillations is clearly quasi-periodic, with at least two characteristic frequencies, apparently generated by beating of the ion-acoustic frequency with the frequency mismatch of the secondary coupling. Clearly, introduction of the finite frequency mismatch $\Omega$ contributes to greater richness and complexity of the wave dynamics and causes distinct nonuniformity of the spatial profile of the instability.

Figure 4: Long-term spatio-temporal evolution in $x = 25\lambda_0$ of the absolute value of the forward-propagating electron plasma wave amplitude for $a = 0.03$, $T = 0.2$ keV and $n = 0.248n_{cr}$, when nonresonance of the secondary coupling is taken into account.
These frequencies are better distinguished in Figure 5, which provides us with an insight into the long-term temporal evolution of the absolute value of the plasmon amplitude for the same laser and plasma parameters as in Figure 4, in $x = 25\lambda_0$ (a), $x = 50\lambda_0$ (b), $x = 75\lambda_0$ (c).

**Figure 5:** Long-term temporal evolution of the absolute value of the forward-propagating electron plasma wave amplitude for $a = 0.03$, $T = 0.2$ keV, $n = 0.248 n_{cr}$, in $x = 25\lambda_0$ (a), $x = 50\lambda_0$ (b), $x = 75\lambda_0$ (c), when nonresonance of the secondary coupling is taken into account.

The maximum amplitude of these rapid oscillations is significantly above the saturation amplitude and can even amount to 25% closer to the right plasma slab boundary. The complexity of the wave dynamics increases simultaneously, barring the narrow band in the vicinity of the right plasma boundary in which the boundary effect prevails, although the oscillations always preserve their quasi-periodic nature. Every maximum is followed by an interval when the amplitude slowly decreases, gradually reaching the saturation value, after which another period of rapid oscillations occurs and so
on. This is somewhat similar to the burst-like behavior of the instability in inhomogeneous plasmas, when the profile modification near the quarter-critical density is taken into account, as the density profile steepens and relaxes under the influence of the ponderomotive force of the pump wave [1, 2]. The frequency of these pulses is better defined in the vicinity of the left plasma boundary, while their amplitude grows toward the right boundary. We conclude that TPD instability should occur in bursts even in homogeneous plasmas, due to the inherently nonresonant nature of the ion saturation mechanism, although the amplitudes of these bursts should be considerably smaller than in the above mentioned case of ponderomotive profile steepening. An increased, burst-like production of hot electrons is also to be expected during periods of instability enhancement.

Acknowledgments
This work has been supported by the Ministry of Science of Serbia, under Project No. 141034.

References
[1] Krue"r W L 1988 The Physics of Laser Plasma Interactions, (Redwood City, CA: Addison-Wesley)
[2] Baldis H A, Campbell E M, and Krue"r W L 1991 Handbook of Plasma Physics: Physics of Laser Plasma, ed A M Rubenchik and S Witkowski (Amsterdam: Elsevier) p. 361
[3] Young F C, Herbst M J, Manka C K, Obenschain S P, and Gardner J H 1985 Phys. Rev. Lett. 54 2509
[4] Yaakobi B, Stoeckl C, Boehly T, Meyerhofer D D, and Seka W 2000 Phys. Plasmas 7 3714
[5] Stoeckl C et al 2003 Phys. Rev. Lett. 90 235002
[6] Dimitrijevi"c D R, and Jovanovi"c M S 2002 Phys. Rev. E 66 056408
[7] Dimitrijevi"c D R, and Jovanovi"c M S 2002 Contrib. Papers 21st Int. Symp. On the Phys. Of Ionized Gases p. 550
[8] Dimitrijevi"c D R, and Maluckov A A 2010 Journal of Plasma Phys. accepted for publishing
[9] Dimitrijevi"c D R and Maluckov A A 2009 Proceedings 7th general Conference of the BPU p. 1456
[10] Karttunen S J 1981 Phys. Rev. A 23 2006
[11] Baldis H A, and Walsh C J 1983 Phys. Fluids 26 1364
[12] Meyer J 1992 Phys. Fluids B 4 2934
[13] DuBois D F, Russell D A, and Rose H A 1995 Phys. Rev. Lett. 74 3983
[14] Labaune C, Baldis H A, Bauer B S, Tikhonchuk V T, and Laval G Phys. Plasmas 5 234