Numerical modeling of the controlled lifting of the structure

Alexandra Bestuzheva and Ivan Chubatov
Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, Russia
E-mail: alex_bestu@mail.ru, chubatovz@gmail.com

Abstract. The numerical simulation of the stress-strain state of the soil mass at the base of the settled foundations of structures with the aim of predicting the possibility of their rise by the method of compensating directed discharge is based on the developed finite element method (FEM) program, verified on a number of test problems. The verification results are compared with the decisions of O. Zenkevich [p. 3], with analytic solutions of V.G. Rekacha [p. 5] and G. Lame [p. 6]. Comparisons of the results of a numerical solution of the inverse Lame problem are presented. The error in the results of the comparisons does not exceed 0.5%. In the framework of the linear theory of elasticity, inverse problems have been solved for obtaining displacements on the day surface of the soil at the base of the foundation of the structure with targeted injection of self-expanding mixtures in the thickness of the soil base. The process of targeted injection of self-expanding mixtures using the FEM is modeled by point displacements of the nodes of the computational domain and a change in the boundary conditions. The results obtained on the displacements of the day surface, depending on the local increase in volume in the soil stratum, allow us to judge about the adequate operation of the program and the possibilities of numerical modeling of the processes of work during compensatory injection.

1. Introduction

Due to accidents that may be caused by various factors, the foundations of structures may be subjected to out-of-tolerance settlements incompatible with normal operation. There are various methods to correct or prevent situations created. One of these methods is compensation grouting [1, 2].

The essence of the compensation grouting method is that the development of structure settlements is prevented by introducing special mortars in the soil mass of the structure base. When grouting an injection composition into a soil mass, the stress-strain state of this mass changes. This stops both the settlement development and subsequent rise of the daylight surface, and at the same time, of the structure [3, 4, 5].

World practice contains numerous examples of implementing the compensation grouting technology for stabilizing the settlements or leveling the position of structures. Best known examples include leveling of the Hessigheim hydroelectric installation building on the navigable Neckar River near Hessigheim (Germany), compensation for the settlement and tilt of the Elizabeth Tower (Big Ben) in the Palace of Westminster (London, UK) [6], building leveling in New Orleans (USA) [7], compensation of the settlement of the Bertelsmann AG office building in Berlin (Germany).

On September 17, 2013, an accident occurred during the construction of the Zagorskaya PSPP-2. Seepage distortion of foundation soils resulted in the differential settlement of the PSPP building, which made its further operation impossible. PSPP building dimensions are 106x75 m in plan. A proposal was put forward and approved for the use of controlled compensation grouting to restore the
design position of the station. A special self-expanding mortar was developed that is suitable for field sands lying at the structure base. An experiment was carried out to raise the building in field soils on a test site 50 meters from the station. A detailed description of the process is given in the paper [8].

When raising a real structure, it is necessary to solve a number of problems such as minimizing the total number of injection wells arranged under the structure, reducing the time of work, and limiting stresses that occur in the structure in case of uneven raising in order to avoid new cracking. These problems can be solved by feeding the mortar through injectors in a certain sequence, i.e. acting on the foundation of the structure along a certain loading path.

To find such a path, it is necessary to develop a special technique for numerical simulation of geotechnical processes that occur in the foundation soil when using compensation grouting. When setting the task, we relied on the papers [9,10]. To implement this task, an author program is developed based on the finite element method. Similar numerical studies as part of “The Soil” universal software package were carried out by Ph.D. A.N. Simutin and Ph.D. A.V. Alexandrov [11, 12].

2. Main points of the finite element method

In the program under development, the problem of a flat deformed state is solved when the calculations are divided by triangular elements with a linear approximation of displacements [13, 14, 15].

In matrix form is written as:

$$[K] \{U\} = \{F\}$$  \hspace{1cm} (1)

where: $[K]$ is a stiffness matrix, $\{U\}$ is a displacement vector, $\{F\}$ is a force vector.

The stiffness matrix has the entry:

$$[K] = \int [B]^T [D] [B] dV$$

where: $[D]$ is an elasticity matrix, $[B]$ is a form matrix, $V$ is a volume of the considered area.

In the process of solving problems, the components of the stress tensor were determined $\{\sigma\}$ using the equation:

$$\{\sigma\} = [D][B]\{U\}$$

Simulating the targeted flow of expanding mortar is assumed by setting the final values of displacements in nodes as boundary conditions [16]. For given displacements in equation (1), the stiffness matrix and the load vector are modified. Figure 1 shows how the stiffness matrix and the load vector change with a given “boundary” displacement $U_1=A$.

$$[K] \{U\} = \{F\} \Rightarrow \begin{bmatrix} k_{11} & k_{12} & \ldots & k_{1n} \\ k_{21} & k_{22} & \ldots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \ldots & k_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & k_{22} & & \ldots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & k_{n2} & \ldots & k_{nn} & k_{nn} \end{bmatrix} \begin{bmatrix} A \\ U_2 \\ U_3 \\ \vdots \\ U_n \end{bmatrix} \Rightarrow \begin{bmatrix} A \\ F_2 - k_{21} \times A \\ \vdots \\ F_n - k_{n1} \times A \\ U_n \end{bmatrix}.$$
In this case, the system stiffness matrix \( [K] \) is formed for all nodes in the area including locked ones since node locking is a special case of specified displacements where the node is set to zero displacement [16].

The author program was verified by comparing it with a number of problems that have accurate analytical solutions, as well as with numerical solutions given using other software units.

### 3. Verification of the program as an example from a monograph by O. Zenkevich

Solving an example from O. Zenkevich's monograph [16] is a problem on the plane stress of a triangular area when a vertical upward force is applied to the vertex (Figure 2). The area is divided into 9 elements with 10 nodes for calculation. The triangle is fixed at Nodes 1 and 4. A comparison of solutions is shown in Tables 1 and 2. Displacements for Nodes 1 and 4 are not shown because they have locking and displacement components are equal to zero in them. The solution using the created program completely coincides with the solution from the monograph.

![Figure 2. Problem on plane stress of a triangular area when loaded by a vertical upward force](image)

#### Table 1. Comparison of results obtained in the solution using the developed software package with the solution from the monograph. Node displacements

| Solution results | Displacement component | 2  | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
|------------------|------------------------|----|----|----|----|----|----|----|----|
| From the monograph | horiz. (X) | 1.0941 | -1.0941 | -1.6412 | 0.0000 | 1.6412 | 0.8206 | -0.8206 | 0.0000 |
| Software package | horiz. (X) | 1.0941 | -1.0941 | -1.6412 | 0.0000 | 1.6412 | 0.8206 | -0.8206 | 0.0000 |
| From the monograph | vert. (Y) | 17.7565 | 17.7565 | 15.6785 | 20.9599 | 15.6785 | 25.3126 | 25.3126 | 44.4729 |
| Software package | vert. (Y) | 17.7565 | 17.7565 | 15.6785 | 20.9599 | 15.6785 | 25.3126 | 25.3126 | 44.4729 |

#### Table 2. Comparison of results obtained in the solution using the developed software package with the solution from the monograph. Stress components

| Solution results | Stress component | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|------------------|------------------|----|----|----|----|----|----|----|----|----|
| From the monograph | \( \sigma_x \) | 1.4902 | -0.7399 | 1.4902 | 0.9503 | 0.9503 | 1.8078 | 1.8078 | -0.2949 | 1.6794 |
4. Verification of the program an example from a tutorial written by Doctor of Engineering, Prof. V.G. Rekach

The second test was solving an example from a tutorial written by Doctor of Engineering, Prof. V.G. Rekach [17] – the problem on plane stress of a parallelepiped when loaded with a vertical distributed load directed downward (Figure 3). The parallelepiped rests on an absolutely rigid and smooth base.

Equations for the analytical solution are written as follows:

\[
U_y = -\frac{(1 - \mu^2)P}{2E} \quad y
\]

\[
U_x = \frac{\mu(1 + \mu)P}{E} \quad x
\]

\[
\sigma_y = -P, \sigma_x = 0
\]

where: \( U_y \) – перемещение is a displacement in the \( Y \)-axis direction at the point with coordinates \( (x,y) \), \( U_x \) – a displacement in the \( X \)-axis direction at the point with coordinates \( (x,y) \), \( \sigma_y \) – is a stress component in the \( Y \)-axis direction, \( \sigma_x \) – is a stress component in the \( X \)-axis direction, \( \mu \) – is a Poisson ratio, \( E \) – is an elasticity modulus, \( P \) – is a uniform distributed load value.

Table 3 shows a comparison of displacement values obtained in the solution of the author program and example from the tutorial for characteristic nodal points. No stress component value is given since \( \sigma_x = 0.000, \sigma_y = 1.000 \) across the area, i.e. correspond to the values of external forces. The solution using the developed program completely coincides with the analytical solution.

| Software package | \( \sigma_x \) | 1.49023 | -0.73993 | 1.49023 | 0.95032 | 0.95032 | 1.80775 | 1.80775 | -0.29492 | 1.67942 |
|------------------|---------------|---------|----------|---------|---------|---------|---------|---------|----------|---------|
| From the monograph | \( \sigma_y \) | 3.7727 | 1.4167 | 3.7727 | 0.5189 | 0.5189 | 3.9487 | 3.9487 | 2.1027 | 10.0000 |
| Software package | \( \sigma_y \) | 3.7727 | 1.4167 | 3.7727 | 0.5189 | 0.5189 | 3.9487 | 3.9487 | 2.1027 | 10.0000 |
| From the monograph | \( \tau_{xy} \) | 3.1137 | 0.0000 | -3.1137 | -0.6733 | 0.6733 | 1.3845 | -1.3845 | 0.0000 | 0.0000 |
| Software package | \( \tau_{xy} \) | 3.1137 | 0.0000 | -3.1137 | -0.6733 | 0.6733 | 1.3845 | -1.3845 | 0.0000 | 0.0000 |
| From the monograph | \( \sigma_1 \) | 5.9477 | 1.4167 | 5.9477 | 1.4417 | 1.4417 | 4.6283 | 4.6283 | 2.1027 | 10.0000 |
| Software package | \( \sigma_1 \) | 5.9477 | 1.4167 | 5.9477 | 1.4417 | 1.4417 | 4.6283 | 4.6283 | 2.1027 | 10.0000 |
| From the monograph | \( \sigma_2 \) | -0.6847 | -0.7399 | -0.6847 | 0.0276 | 0.0276 | 1.1281 | 1.1281 | -0.2949 | 1.6794 |
| Software package | \( \sigma_2 \) | -0.6847 | -0.7399 | -0.6847 | 0.0276 | 0.0276 | 1.1281 | 1.1281 | -0.2949 | 1.6794 |
| From the monograph | \( \alpha \) | 34.935 | 0.0000 | -34.935 | -53.881 | 53.881 | 26.145 | -26.145 | 0.0000 | 0.0000 |
| Software package | \( \alpha \) | 34.9354 | 0.0000 | -34.9354 | -53.8809 | 53.8809 | 26.1449 | -26.1449 | 0.0000 | 0.0000 |
Table 3. Comparison of results obtained in the solution using the developed program with the solution from the monograph. Node displacements

| Solution results | Displacement component | 8   | 13   | 18   | 23   | 11   | 12   | 14   | 15   |
|------------------|------------------------|-----|------|------|------|------|------|------|------|
| From the monograph | horiz. (X)         | 0.0000 | 0.0000 | 0.0000 | 0.0000 | -0.3900 | -0.1950 | 0.1950 | 0.3900 |
| Software package | horiz. (X)          | 0.0000 | 0.0000 | 0.0000 | 0.0000 | -0.3900 | -0.1950 | 0.1950 | 0.3900 |
| From the monograph | vert. (Y)           | -0.4550 | -0.9100 | -1.3650 | -1.8200 | -0.9100 | -0.9100 | -0.9100 | -0.9100 |
| Software package | vert. (Y)           | -0.4550 | -0.9100 | -1.3650 | -1.8200 | -0.9100 | -0.9100 | -0.9100 | -0.9100 |

5. Verification of the program on the example of calculating a thick-walled pipe (Lame's problem)

The third test was thick-walled pipe calculation (Lame's problem) [18]. The problem on the plane stress state of a thick-walled pipe with its inner and outer surfaces being concentric circular cylinders. The pipe is under internal and external pressure (Figure 4). Equations for the analytical solution are written as follows:

\[ \sigma_\theta = \frac{p_n r_n^2 - p_r r_r^2}{r_n^2 - r_r^2} + \left( \frac{p_n - p_r}{r_n^2 - r_r^2} \right) r_r^2 \]

\[ \sigma_\theta = \frac{p_n r_n^2 - p_r r_r^2}{r_n^2 - r_r^2} - \left( \frac{p_n - p_r}{r_n^2 - r_r^2} \right) r_r^2 \]
where: \( \sigma_r \) – is radial stresses at the point with radius \( r \), \( \sigma_\theta \) – is circumferential stresses at the point with radius \( r \), \( U_r \) – is a displacement of the point with radius \( r \), \( P_i \) – is internal pressure, \( P_e \) – is external pressure, \( r_c \) – is a cylinder internal radius, \( r_a \) – is a cylinder external radius, \( E \) – is an elastic modulus, \( \nu \) – is a Poisson's ratio.

Figure 4. Problem on the plane stress of a parallelepiped when loaded with a distributed vertical load

This problem was used to compare solutions obtained by analytical and numerical methods when setting the internal load using pressure. Solving the Lame's problem involves a search for displacements in an area at a certain distance from the boundaries of the area with specified pressure. Comparison of the calculations of a thick-walled cylinder under external 400 kN/m\(^2\) and internal 4000 kN/m\(^2\) pressures according to the Lame method and using the numerical method are presented in Figure 5.

Figure 5. Comparison of the results of calculating a thick-walled cylinder according to the Lame method using the numerical method: (a) - radial displacement, m; (b) - radial stresses, kN/m\(^2\); (c) - circumferential stresses, kN/m\(^2\)

The inverse Lame's problem was solved to check the software package for adequate operation. Displacements obtained in the direct solution of the Lame's problem on the internal boundary were set as boundary conditions in order to obtain the nature of stress distribution in the computational area and compare them with the previously obtained stresses in the direct Lame's problem. As it can be seen, the error in stress determination is not more than 0.5% (Figure 6).
6. Modeling the "rise" of the soil surface by targeted injection of a self-expanding composition

Next, an attempt was made to model the "rise" of the daylight soil surface by targeted injection of a self-expanding composition at a certain depth in the calculated area. A 10.0 x 10.0 m sand area is considered with a distributed load of 400 kN/m² applied to its surface. There is a conventional “hollow pipe” with a radius of 1 m in the center of the area, at a depth of 5 m from the surface. The conditional pipe expansion to a radius of 1.258 m is simulated by specifying boundary displacements. The problem was solved in a linear formulation with elastic characteristics of the area: E=18.0 MPa, \( \nu =0.3 \), \( \gamma =19.0 \text{ kN/m}^3 \). This problem assumes that the distributed load on the surface is the load from an existing building that has already been erected at the time of calculation. To find daylight surface displacements resulting from pipe expansion, it is necessary to subtract the obtained settlements from the building from the general solution. This action is possible due to a linear statement of the problem. For this, first of all, the problem is solved only with an external distributed load, and then the general problem is solved with an external distributed load and pipe radius expansion. Corresponding actions with the subtraction of settlements provide the resulting daylight surface displacements from pipe expansion, which amounted to 118 mm upwards. Figures 7 and 8 show the final results.
Figure 8. The scheme of displacements of the nodes of the region with increasing internal radius. The displacement scale is increased 5 times for clarity.

The results obtained allow us to judge the adequate operation of the program with linear stress-strain dependence. But this relationship is not linear in the soil and described by a particular soil model. Therefore, for compensation grouting simulation, the next stage of program development will introduce nonlinear stress-strain dependence in accordance with the selected soil model [19, 20]. After that, actions will be taken to find the optimal way to load the structure foundation using the selected optimization method.

References
[1] Rasskazov L. N. Raising and Leveling the Building of the Hydraulic Accumulating Electric Power Station / Rasskazov L. N., Chubatov I. V. // Industrial and civil engineering. 2017. No. 9. pp. 61-65.
[2] Rasskazov L. N. Creation of Injection Massif in Sand Base of Buildings / Rasskazov L. N., Chubatov I. V., Radsinskii A. V. // Industrial and civil engineering. 2017. No. 6. pp. 56-63.
[3] Bellendir E.N. Protection and leveling of buildings and structures using the technology of compensation grouting / E.N. Bellendir, A.V. Alexandrov, M.G. Zertsalov, A.N. Simutin // Hydrotechnical Construction – 2016. – No. 2. – pp. 15-20.
[4] Zertsalov M.G. Compensation grouting technology for protection of buildings and structures / M.G. Zertsalov, A.N. Simutin, A.V. Alexandrov // MGSU Bulletin – 2015. – No. 6. – pp. 32-40.
[5] Bezuijen A. / Compensation Grouting in Sand. Experiments, Field Experiences and Mechanisms / A. Bezuijen. – ISBN 978-90-8570-507-9, 2010. – p. 205.
[6] Ehab Hamed Flexural time–dependent cracking and post–cracking behaviour of FRP strengthened concrete beams / Ehab Hamed, Mark A. Bradford// International journal of Solid and Structures 49 (2012) 1595–1607.
[7] Knitsch, H. Visualization of relevant data for compensation grouting / H. Knitsch // Tunnel.- No. 3, – 2008. – pp. 38-45.
[8] Eliminating settlement consequences of the Zagorskaya PSPP-2 station joint building and restoration works. Development of project documentation for restoration works. Adjustment of
Stage 1. Stage 2. Scientific and engineering support of project works. Execution of scientific and engineering support of Test Site No. 3 when performing tests. Compensation grouting. Test Site No. 3. Report on Stage III results. 1938-40-445. Institute Hydroproject JSC. – M., 2017. – p. 176.

[9] Simutin A.N. Using compensation grouting technology to protect buildings and structures in tunnel construction / Kharchenko I.Y., Merkin V.E., Simutin A.N., Zertsalov M.G. // Transport Construction. 2015. No. 1. pp. 6-9.

[10] Compensation grouting technology to protect buildings and structures / Zertsalov M.G., Simutin A.N., Aleksandrov A.V. // MGSU Bulletin. 2015. No. 6. p. 32-40.

[11] Performing special works for Zagorskaya PSPP-2 building stabilization / V.M. Korolev, O.E. Smirnov, E.S. Argal, A.V. Radzinsky, R.M. Kim, A.V. Alexandrov // Hydrotechnical Construction. 2018. No. 3. p. 2-10

[12] Protection and leveling of buildings and structures using the compensation grouting technology / E.N. Bellendir, A.V. Alexandrov, M.G. Zertsalov, A.N. Simutin // Hydrotechnical Construction. 2016. No. 2. p. 15-19.

[13] E. A. de Souza Neto, D. Peric, D.R.J. Owen. Computational Methods for Plasticity: Theory and Applications — M.: John Wiley & Sons Ltd, 2008, p. 814.

[14] S. Klovanych. The finite element method in nonlinear problems of engineering mechanics— M.: The world of geotechnics, Zaporozhzje, Ukraine, 2009, p. 401.

[15] O.C. Zienkiewicz, R.L. Taylor and J.Z. Zhu. The Finite ElementMethod: Its Basis andFundamentals. Sixth edition — M.: Elsevier Butterworth-Heinemann, 2005, p. 733.

[16] Zenkevich O. Finite element method in engineering — M.: Mir, 1975, p. 541.

[17] V.G. Rekach. Guide to solving problems on the elasticity theory — M.: Vysshaya Shkola, 1966, p. 222

[18] Practical calculations on the strength of structural elements. Part I. / A. G. Dibir, O. V. Makarov, N. I. Pekelny, G. I. Yudin, M. N. Grebennikov. - Textbook. allowance. - Kharkov: Nat. aerospace. University of Kharkiv. Aviation Institute ”, 2007. - 102 p.

[19] An explicit form of the plastic matrix for a Mohr–Coulomb material / D. V. Griffiths, S. M. Willson // Communications in Applied Numerical Methods, 1986, Vol. 2, pp. 523-529.

[20] Fadeev A.B. The finite element method in geomechanics – M.: Nedra, 1987, p. 221.