Decisive role of electron-phonon coupling for phonon and electron instabilities in transition metal dichalcogenides

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The origin of the charge density wave (CDW) in transition metal dichalcogenides has been in hot debate and no conclusive agreement has been reached. Here, we propose an ab-initio framework for an accurate description of both Fermi surface nesting and electron-phonon coupling (EPC) and systematically investigate their roles in the formation of CDW. Using monolayer 1H-NbSe2 and 1T-VTe2 as representative examples, we show that it is the momentum-dependent EPC softens the phonon frequencies, which become imaginary (phonon instabilities) at CDW vectors (indicating CDW formation). Besides, the distribution of the CDW gap opening (electron instabilities) can be correctly predicted only if EPC is included in the mean-field model. These results emphasize the decisive role of EPC in the CDW formation. Our analytical process is general and can be applied to other CDW systems.

The formation of charge density wave (CDW) is a spontaneous symmetry breaking process with periodic charge density modulations and lattice distortions below a critical temperature (T_{CDW}) [1, 2]. However, the origin of CDW is a long-standing problem, which attracts broad research interest in condensed matter physics [3–6]. The first mechanism, Fermi surface nesting (FSN), was originated from Peierls’ model of an ideal one-dimensional (1D) metal atomic chain. FSN relates to an elastic electronic scattering at the Fermi surface [7]. The zero-energy electronic excitations screen the phonon vibration at the CDW vector Q, inducing an abrupt phonon softening (known as Kohn anomaly) [8]. However, the extension of FSN to real materials is not ideal. With a few exceptions, FSN is argued to have limited power in inducing CDW distortions in higher-dimensional systems [4–6, 9].

Instead, momentum-dependent electron-phonon coupling (q-EPC), which involves an inelastic electronic scattering mediated by a phonon, is argued to be more prevailing in higher-dimensional systems [4–6, 9]. In an electron-phonon interaction dominant system, the electron field can be integrated out as a perturbation to the free phonon field, softening phonon frequencies from their bare values [see SI-III for details]. Thus, both FSN and q-EPC may soften phonons to imaginary values (phonon instabilities) and induce CDW distortions. However, quantitative studies of q-EPC are rare, and the reported method to directly obtain q-EPC is a tight-binding model by merely using the electronic band structure [10–12]. Besides, such a semi-empirical method is hard to give rigorous results compared to the first-principles calculations. Therefore, it is imperative to explore a general method to accurately describe q-EPC in the CDW materials.

Another important feature of a CDW is the band gap opening (electron instabilities) accompanied by the CDW distortions, which lowers the energy of the system [2, 4, 13]. The location of the CDW gap in the Brillouin zone (BZ) can be identified accurately by a band unfolding scheme [14–16], which however requires the prior knowledge of the corresponding CDW structure and cannot provide insight into the underlying mechanism. Thus, the driving force of the CDW gap is still elusive. A convenient method that can give physical insight into the gap opening is needed.

In this letter, we use monolayer 1H-NbSe2 and 1T-VTe2 (in short NbSe2 and VTe2) as representative examples for the most common high-symmetry phases of the transition metal dichalcogenides (TMDs). Another reason for such a choice is the mechanisms of their CDWs are considered to be different. The electron-phonon coupling (EPC) has been shown to be dominant in NbSe2 [15, 17, 18], while FSN in VTe2 seems to be substantial, which leads to a peak in the static Lindhard susceptibility [19, 20]. Thus, we conduct a comprehensive study of their CDW properties from both phononic and electronic perspectives. In the phononic part, we obtain the accurate q-EPC from the fully first-principles calculations, which allows us to make a straightforward comparison between the contributions from FSN and q-EPC in the CDW formation. Interestingly, the q-EPC is shown to play the dominant role in designating the CDW vectors for both NbSe2 and VTe2. While in the electronic part, we find the rigorous EPC matrix elements are the key to predict the distribution of the CDW gaps by our mean-field model. In addition, we argue that our analysis process, besides its conciseness and accurateness, should be
a general framework, which can be easily exploited in other CDW systems.

\[ \chi_q = \sum_k g_{k,k+q}^2 \frac{f(\varepsilon_k) - f(\varepsilon_{k+q})}{\varepsilon_{k+q} - \varepsilon_k} \]  

which is a pure electron effect, and its peak reflects the electronic instability by FSN [4]. Similarly, \( \chi_q \) can reduce to the q-EPC \( \bar{g}_q \) under the “constant fraction” approximation \( (\frac{f(\varepsilon_k) - f(\varepsilon_{k+q})}{\varepsilon_{k+q} - \varepsilon_k}) = 1 \):

\[ \bar{g}_q = \sum_k |g_{k,k+q}|^2 \]  

which reflects a pure EPC effect. Focusing on the low energy interaction around the Fermi level, we only consider the coupling between the lowest phonon branch and the single electronic band which crosses the Fermi level [see the pink bands in Figs. S1(b) and S1(e)]. Due to the complexity in describing the EPC matrix elements, only static Lindhard susceptibility has been widely used to understand the CDW formation, while the q-EPC which however may play a more important role is ignored [4, 6].

Here, to remedy this blemish, we applied density functional perturbation theory (DFPT) to obtain the accurate element \( g \), which is given by [24]:

\[ g_{k,k+q} = \left( \frac{\hbar}{2M\omega_q} \right)^{1/2} \langle \varphi_{k+q} | \partial_q V | \varphi_k \rangle \]  

where \( \partial_q V \) is the derivative of the electron-ion potential with phonon frequency \( \Omega_q \). \( \varphi_k \) is the electronic wavefunction with wavevector \( k \). For a direct comparison, we also calculated the matrix element \( g \) between electronic states \( k \) and \( k+q \) by the tight-binding (TB) model [10–12], which is:

\[ g_{k,k+q} \propto (\nu_k - \nu_{k+q}) \cdot \frac{q}{|q|} \]  

where \( \nu \) is the electron velocity at \( k \) point in the coupled band. \( \frac{q}{|q|} \) is the longitudinal projection as only LA phonons soften to zero. And this method succeeded in describing EPC properties in bulk 2H-NbSe2 [11, 12] and monolayer 1T-VSe2 [25]. To distinguish the two different sources of EPC matrix elements in the following discussions, the related quantities obtained by the TB method are denoted as \( \bar{g}_q \) and \( \chi_q \), while the quantities obtained by DFPT are used bare notation \( g_q \) and \( \chi_q \).

The calculated \( \chi_q, g_q, \chi_q \), and \( \omega_q \) for NbSe2 (upper panels) and VTe2 (lower panels) are shown in Fig. 2. For NbSe2, the static Lindhard susceptibility \( \chi_q \) has a broad plateau from 2/5\( \Gamma \)M to 4/5\( \Gamma \)M [see Fig. 2(a) and the blue line in Fig. 3(a)], which indicates the weakness of FSN [18]. Nonetheless, \( \bar{g}_q \) shows a strong electron-phonon interaction near 2/3\( \Gamma \)M for NbSe2 [see Fig. 2(b)].
and the red line in Fig. 3(a)]. In addition, the topology of \( \chi_q \) is very similar to \( \tilde{g}_q \) [Figs. 2(b) and 2(c)] in the NbSe\(_2\) first BZ, indicating the dominated role played by the \( \mathbf{q} \)-EPC in \( \chi_q \). Fig. 2(d) displays the softened phonon modes of NbSe\(_2\), which are concentrated in the arch-like area [see the purple dashed line in Fig. 2(d)], in agreement with the highland in \( \chi_q \) [see the purple dashed line in Fig. 2(c)]. More importantly, the peaks in \( \tilde{g}_q \) and \( \chi_q \) [see the red areas in Figs. 2(b) and 2(c)] are at 2/3\( \Gamma \), which is consistent with the dip in \( \omega_q \) [see the blue areas in Fig. 2(d)].

The EPC properties of bulk 2H-NbSe\(_2\) have been well-described by using the TB method \cite{11, 12}, however, we find this method fails in addressing monolayer 1H-NbSe\(_2\). As for the generalized static electronic susceptibility \( \chi^T_B \), the overall topology in the first BZ [Fig. S3(c)] cannot fit the phonon softening [Fig. 2(d)], which is in stark contrast to the good match achieved by DFPT [Fig. 2(c)]. Besides, the peaks of \( \tilde{g}^T_B \) and \( \chi^T_B \) at the \( \Gamma M \) path [see the green line in Fig. 3(a) and the orange line in Fig. 3(b)] are near 1/2\( \Gamma M \), which predict the formation of 4 x 4 CDW instead of 3 x 3 CDW. By comparing to the prominent peaks of \( \tilde{g}_q \) and \( \chi_q \) at 2/3\( \Gamma M \) obtained by the DFPT [see the red line in Fig. 3(a) and the black line in Fig. 3(b)], we conclude that the \textit{ab-initio} based DFPT method is superior in obtaining the \( \mathbf{q} \)-EPC and predicting the CDW vector in monolayer NbSe\(_2\).

For monolayer VTe\(_2\), \( \chi'_q \) has a peak near 2/5\( \Gamma M \) [see Fig. 2(e) and the blue line in Fig. 3(c)], in line with previous works \cite{19, 20}. However, this peak does not correspond to the 4 x 4 CDW structure observed from the experiments \cite{19, 23}. Besides, the profile of \( \chi_q \) in the first BZ is again very close to \( \tilde{g}_q \) [Figs. 2(f) and 2(g)], which both show maxima close to 1/2\( \Gamma M \), providing a powerful clue of phonon softening at \( \mathbf{Q}^T \). As shown in Fig. 2(h), the distribution of the softened phonon frequency \( \omega_q \) of VTe\(_2\) shows hexapetalous flower-like pattern [see the blue area in Fig. 2(h)], which match well with the “hot” area in \( \chi_q \) [Fig. 2(g)]. One should note that the peak of \( \tilde{g}_q \) is very sharp, which overwhelms the fluctuation of \( \chi^T_B \) in VTe\(_2\) [see the red line and the blue line in Fig. 3(c)], leading to the correct 1/2\( \Gamma M \) peak position of \( \chi_q \) [see the black line in Fig. 3(d)]. However, the TB method still cannot explain the phonon softening in VTe\(_2\). Although \( \tilde{g}^T_B \) shows a peak at 1/2\( \Gamma M \), such peak is even broader than the nesting peak at 2/5\( \Gamma M \), leading to the incorrect peak position of \( \chi^T_B \) at 2/5\( \Gamma M \) [see the green line in Fig. 3(c) and the orange line in Fig. 3(d)]. Compared the distribution of \( \tilde{g}^T_B \) [Fig. S3(e)], the “sharpness” of \( \chi'_q \) make the topology of \( \chi^T_B \) closer to that of the \( \chi'_q \) in the first BZ of VTe\(_2\) [Figs. 2(e) and S3(f)], which cannot explain the phonon softening.
shown in Fig. 2(h). Therefore, with the help of the accurate EPC matrix element $g$ obtained by DFPT, our study clearly demonstrates that it is $q$-EPC rather than FSN determines the phonon softening at correct positions and accounts for the CDW formation in both NbSe$_2$ and VTe$_2$.

**Electron instabilities and CDW gaps.**—Having identified the domination of EPC in phonon softening, we now study the momentum-dependent CDW gap (electron instabilities) based on mean-field theory. The Hamiltonian of the CDW phase is minimally described by including one band crossing the Fermi level and electron-phonon interaction with phonon momentum $Q$.

$$H_{mf} = \sum_k \varepsilon_k c_k^\dagger c_k + \sum_{k, Q} 2g_{k,k+Q} \Delta_Q c_k^\dagger c_{k+Q} + h.c. \quad (7)$$

Here, $c_k^\dagger$ ($c_k$) and $\varepsilon_k$ are creation (annihilation) operator and energy for an electron with momentum $k$. $\Delta_Q$ is the order parameter which was approximated to a constant because of symmetry and small pocket size. This Hamiltonian can then be used to calculate the spectral function of the CDW phases [see SI-IV for more details about the theoretical background].

As shown in Fig. 4(a), the simulated Fermi surface of the NbSe$_2$ CDW structure with constant EPC matrix elements $|g|$ (i.e., $|g| = \sum_{k,k+Q} |g_{k,k+Q}| / N_{k,k+Q}$, where $N_{k,k+Q}$ is the number of $g$ in the calculation.) reflects the FSN effect under the mean-field picture. The norm of $g$ is used to avoid the arbitrary phase factor problem in the band basis of EPC matrix element. It is clearly shown that each K pocket has 3 couples of partial gaps [see the green arrows in Fig. 4(a)], where the spectral intensity becomes blurred as only partial electronic states are left at the Fermi surface. The partial gaps are at both sides of the $\Gamma K$ path, corresponding to the most heavily nested points of NbSe$_2$ [see the red points in white circle in Fig. S6(c)]. The incorporation of anisotropic matrix elements $g$ develop a more extensive gap opening on the Fermi surface [Fig. 4(c)], which considers the synergistic effects of FSN and EPC. Remarkably, the full band-gapped sectors can be found on the K pockets, where the electronic states on the Fermi surface are completely obliterated [see the blue arrow in Fig. 4(c)]. Furthermore, the spectral function was plotted along the $\Gamma K$ path, and we find the pure nesting effect cannot open a band gap along this path [Fig. 4(b)], in contrast with the spectral function derived with the anisotropic $g$, which obviously displays a full band gap close to the K point [Fig. 4(d)]. Considering there is no experimental report on the Fermi surface of the monolayer NbSe$_2$ CDW state, the predicted CDW gap distribution is compared with the unfolded Fermi surface of the simulated NbSe$_2$ CDW ground state, which displays a remarkable agreement with each other [14, 17].

It is known that VTe$_2$ has triangular hole pockets, which has parallel sides to provide good nesting condition [19, 20]. Such nesting will induce a peak in the static Lindhard susceptibility and possibly open a gap at the heavily nested point [19]. In the phononic part discussion, FSN as the mechanism of the phonon softening in VTe$_2$ has been excluded, we now turn to discuss its relation to the CDW gap. Fig. 4(e) suggests that there is no obvious spectral weight depletion on the Fermi surface, and no CDW gap can be opened at the $MK$ path [Figs. 4(e) and 4(f)]. However, after considering the effect of...
the anisotropic EPC matrix elements $g$, there is a strong suppression of the spectral intensity near the M point [see the blue arrow in Fig. 4(g)]. No electronic state can be found on the Fermi surface at the $MK$ path, which indicates a full gap opening [Fig. 4(h)], consistent with the recent experimental results [19]. Moving toward the $\Gamma$ point, the decreasing of the gap size is accompanied by the full to partial gap transition, and finally the gap is closed at the triangular $K$ pocket apex [see Fig. 4(g)]. Such an anisotropic gap distribution on the Fermi surface agrees well with the angle-resolved photoemission spectroscopy (ARPES) measurements [19].

Discussion.—Although the CDW formation has been widely studied for decades, the underlying mechanism is still under debate. Using NbSe$_2$ and VTe$_2$ as examples, we explore the origin of their CDW orders by providing a comprehensive \textit{ab-initio} theoretical study. This study is vital because it is difficult to reconcile FSN and EPC so far. The main features of this work include an accurate description of the $q$-EPC in the whole BZ for the first time, correctly calculating the generalized static $2\times3$ CDW order in the MBE-grown NbSe$_2$ samples is due to the charge transfer from the graphene substrate [26, 28, 29]. Therefore, the intrinsic NbSe$_2$ is expected to have a larger gap size than 4meV, as revealed by the previous unfolded band structure [14, 15] and our mean-field simulation. For monolayer VTe$_2$, previous work reported that there is no observable charge transfer between VTe$_2$ samples and the graphene substrate [20], hence, the graphene based VTe$_2$ samples may be close to the freestanding one, which should have similar properties including the CDW gap opening. Our mean-field calculations successfully reproduce the experimental ARPES results as expected [19]. Besides, the long parallel sides of the triangular hole pockets provide a good condition for FSN, which was thought to be the origin of the anisotropic CDW gap [19]. However, our work excludes this hypothesis and emphasizes the significance of EPC in the CDW gap opening by providing a convincing description.

In conclusion, using first-principles calculations and the mean-field theory, we report a quantitative study of CDW properties in monolayer 1H-NbSe$_2$ and 1T-VTe$_2$. Our results confirm the validity of the EPC mechanism in a non-1D CDW system, which supplies previous experimental and theoretical works. The combined analysis of FSN and EPC in both phononic and electronic pictures constructs a profound understanding of the CDW formation mechanism. We argue that the same physics, in principle, should be applied to other higher-dimensional CDW systems. Besides, the proposed analytical method can be decorated by more accurate calculations (i.e., GGA+U, GW, hybrid function, etc.) [24, 30–32], which allows further CDW studies in more complex systems. Our work paves a general way to unravel the physical insights of the CDW formation mechanism with phonon and electron instabilities, which can be extended for the understanding of charge ordering in other transition metal compounds [33], kagome metals [34] and high-temperature superconductors [35].

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References

[1] G. Gr"uner, The dynamics of charge-density waves, Reviews of modern physics 60, 1129 (1988).
[2] K. Rossnagel, On the origin of charge-density waves in select layered transition-metal dichalcogenides, Journal of Physics: Condensed Matter 23, 213001 (2011).
[3] X. Zhu, J. Guo, J. Zhang, and E. Plummer, Misconceptions associated with the origin of charge density waves, Advances in Physics: X 2, 622 (2017).
[4] M. D. Johannes and I. I. Mazin, Fermi surface nesting and the origin of charge density waves in metals, Physical Review B 77, 165135 (2008).
[5] F. Weber, S. Rosenkranz, J.-P. Castellan, R. Osborn, R. Hott, R. Heid, K.-P. Bohnen, T. Egami, A. H. Said, and D. Reznik, Extended phonon collapse and the origin of the charge-density wave in h- nbse 2, Physical review letters 107, 107403 (2011).
[6] X. Zhu, Y. Cao, J. Zhang, E. Plummer, and J. Guo, Classification of charge density waves based on their nature, Proceedings of the National Academy of Sciences 112, 2367 (2015).
[7] R. E. Peierls, Quantum theory of solids (Oxford Univer-
C. M. Varma, E. I. Blount, P. Vashishta, and W. Weber, F. Zheng, Z. Zhou, X. Liu, and J. Feng, First-principles Z. Wang, J. Zhou, K. P. Loh, and Y. P. Feng, Controllable F. Flicker and J. van Wezel, Charge order in nbse 2, Physical Review B 19, 6130 (1979).

F. Flicker and J. Van Wezel, Charge order from orbital-dependent coupling evidenced by nbse 2, Nature communications 6, 1 (2015).

F. Flicker and J. van Wezel, Charge order in nbse 2, Physical Review B 94, 235135 (2016).

Z. Wang, J. Zhou, K. P. Loh, and Y. P. Feng, Controllable phase transitions between multiple charge density waves in monolayer 1l-vse2 via charge doping, Applied Physics Letters 119, 163101 (2021).

F. Zheng, Z. Zhou, X. Liu, and J. Feng, First-principles study of charge and magnetic ordering in monolayer nbse 2, Physical Review B 97, 081101(R) (2018).

C.-S. Lian, C. Si, and W. Duan, Unveiling charge-density wave, superconductivity, and their competitive nature in two-dimensional nbse2, Nano letters 18, 2924 (2018).

P. Chen, W. W. Pai, Y.-H. Chan, V. Madhavan, M.-Y. Chou, S.-K. Mo, A.-V. Fedorov, and T.-C. Chiang, Unique gap structure and symmetry of the charge density wave in single-layer vse 2, Physical review letters 121, 196402 (2018).

F. Zheng and J. Feng, Electron-phonon coupling and the coexistence of superconductivity and charge-density wave in monolayer nbse 2, Physical Review B 99, 161119(R) (2019).

M. Calandra, I. I. Mazin, and F. Mauri, Effect of dimensionality on the charge-density wave in few-layer 2h-nbse2, Physical Review B 80, 241108(R) (2009).

Y. Wang, J. Ren, J. Li, Y. Wang, H. Peng, P. Yu, W. Duan, and S. Zhou, Evidence of charge density wave with anisotropic gap in a monolayer vte 2 film, Physical Review B 100, 241404(R) (2019).

K. Sugawara, Y. Nakata, K. Fujii, K. Nakayama, S. Souma, T. Takahashi, and T. Sato, Monolayer vte 2: Incommensurate fermi surface nesting and suppression of charge density waves, Physical Review B 99, 241404(R) (2019).

M. M. Ugeda, A. J. Bradley, Y. Zhang, S. Onishi, Y. Chen, W. Ruan, C. Ojeda-Aristizabal, H. Ryu, M. T. Edmonds, H.-Z. Tsai, et al., Characterization of collective ground states in single-layer nbse 2, Nature Physics 12, 92 (2016).

Y. Nakata, K. Sugawara, S. Ichinokura, Y. Okada, T. Hitosugi, T. Koretsune, K. Ueno, S. Hasegawa, T. Takahashi, and T. Sato, Anisotropic band splitting in monolayer nbse 2: Implications for superconductivity and charge density wave, npj 2D Materials and Applications 2, 12 (2018).

P. M. Coelho, K. Lasek, K. Nguyen Cong, J. Li, W. Niu, W. Liu, I. I. Oleynik, and M. Batzill, Monolayer modification of vte2 and its charge density wave, The journal of physical chemistry letters 10, 4987 (2019).

F. Giustino, Electron-phonon interactions from first principles, Reviews of Modern Physics 89, 015003 (2017).

R. Chua, J. Henke, S. Saha, Y. Huang, J. Gou, X. He, T. Das, J. van Wezel, A. Soumyanarayanan, and A. T. Wee, Coexisting charge-ordered states with distinct driving mechanisms in monolayer vse2, ACS nano 16, 783 (2021).

D. Lin, S. Li, J. Wen, H. Berger, L. Forró, H. Zhou, S. Jia, T. Taniguchi, K. Watanabe, X. Xi, et al., Patterns and driving forces of dimensionality-dependent charge density waves in 2 h-type transition metal dichalcogenides, Nature communications 11, 2406 (2020).

X. Xi, L. Zhao, Z. Wang, H. Berger, L. Forró, J. Shan, and K. F. Mak, Strongly enhanced charge-density-wave order in monolayer nbse 2, Nature nanotechnology 10, 765 (2015).

J. Á. Silva-Guillén, P. Ordejón, F. Guinea, and E. Canadell, Electronic structure of 2h-nbse2 single-layers in the cdw state, 2D Materials 3, 035028 (2016).

Y. Chen, L. Wu, H. Xu, C. Cong, S. Li, S. Feng, H. Zhang, C. Zou, J. Shang, S. A. Yang, et al., Visualizing the anomalous charge density wave states in graphene/nbse2 heterostructures, Advanced Materials 32, 2003746 (2020).

J.-J. Zhou, J. Park, I. Timrov, A. Floris, M. Cococcioni, N. Marzari, and M. Bernardi, Ab initio electron-phonon interactions in correlated electron systems, Physical review letters 127, 126404 (2021).

Z. Li, G. Antonius, M. Wu, F. H. daJornada, and S. G. Louie, Electron-phonon coupling from ab initio linear-response theory within the gw method: Correlation-enhanced interactions and superconductivity in ba 1- x k x co 3. Physical review letters 122, 186402 (2019).

Z. P. Yin, A. Kutepov, and G. Kotliar, Correlation-enhanced electron-phonon coupling: Applications of gw and screened hybrid functional to bismuthates, chloronitriles, and other high-t c superconductors, Physical Review X 3, 021011 (2013).

S. Manzeli, D. Ovchinnikov, D. Pasquier, O. V. Yazyev, and A. Kis, 2d transition metal dichalcogenides, Nature Reviews Materials 2, 17033 (2017).

H. Tan, Y. Liu, Z. Wang, and B. Yan, Charge density waves and electronic properties of superconducting kagome metals, Physical review letters 127, 046401 (2021).

X. Wang, Y. Yuan, Q.-K. Xue, and W. Li, Charge ordering in high-temperature superconductors visualized by scanning tunneling microscopy, Journal of Physics: Condensed Matter 32, 013002 (2020).