Pre-Equilibrium Differential and Double Differential Cross Sections of Neutrons Induce Nuclear Reactions in $^{64}$Zn Nucleus

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Abstract:
The pre-equilibrium and equilibrium differential and double differential cross sections have been calculated for 2MeV neutrons induced reactions in $^{64}$Zn nucleus using Kalbach Systematic approach in terms of Exciton model with Feshbach, Kerman and Koonin (FKK) statistical theory. The two-component exciton model and some corrections have been implemented in the calculation the particle-hole state density. In this work, Isospin, finite well depth, and shell effects are considered. The obtained results were compared with the available published experimental and theoretical data that in the international libraries such as TENDL, ENDF, and JEFF. The comparisons with these available data showed an acceptable agreement especially for the reactions: $^{64}$Zn(n,n)$^{64}$Zn, $^{64}$Zn(n,p)$^{64}$Cu, $^{64}$Zn(n,D)$^{63}$Cu, $^{64}$Zn(n,T)$^{62}$Cu, $^{64}$Zn(n,He)$^{62}$Ni, and $^{64}$Zn(n,He)$^{61}$Ni at different emission angles.

Keywords: Cross section; exciton; FKK model; Kalbach; pre-equilibrium

Introduction:

Nuclear physics is the physics sector that studies atomic nuclei’s construction blocks and interactions [1], which enable us to have a deeper understanding of how the whole universe was produced and to discover some helpful applications that serve humanity as a whole. Indeed, one of the four fundamental forces that regulate our life is the nuclear force that is accountable for the creation of all nuclei.

Nuclear reaction mechanisms were characterized by Direct and Pre-equilibrium reactions [2,3]. Follow this definition, as a function of excitation energy, different types of reactions and several forms of particulate emissions are studied and formulated in different effective models; to build up the probability of fulfillment of finding the emitted particle at some energy channel and emission angle [4]. Most
models deal with semi-classical approach techniques collected with statistical theory of Feshbach, Kerman, and Koonin (FKK), where the considering reactions are included: stripping, fission and fusion reactions, knockout, pick-up, and elastic inelastic scattering [4]. However, in this paper, cross sections, differential, and double differential cross sections for all output channels of the nuclear reaction \(^{64}\text{Zn}(n,x)X\) have been calculated using "PRECO Code" that invented by C. Kalbach [5].

The pre-equilibrium energy spectrum:

There are many mechanisms have been used in calculating the energy spectrum of nucleons (neutrons and protons) induced nuclear reactions with \(^{64}\text{Zn}\) as a target nuclei. The pre equilibrium mechanism that depends on two components Exciton model, has been depends in this work. However, the primary and secondary nucleon emissions were taken into accounts. Indeed, the particle emission rates of type \((b)\) as a function of excitation energy \((E)\) in spin mixed case given by [6]:

\[
W_b(p, p_\pi, E, \epsilon) = \frac{2S_b + 1}{\pi^2 h^3} \mu_b \epsilon \sigma_b(\epsilon_b) \times \sum_{T_B} \left[ C_b(T, T_B) \right]^2 \frac{W_{eff}(p, p_x - Z_b, h_x, p_v - N_b, h_v, U, T_B)}{\omega(p_{\mu}, h_x, p_v, E, T)} \quad \ldots \quad (1)
\]

where, the total residual state density represented by:

\[
w_{eff}(p, p_x - Z_b, h_x, p_v - N_b, h_v, U) = \sum_{i} \sum_{j} w(p_x - Z_b, i, p_v - N_b, j, U) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)
\]

This can be excited by stripping reaction [6,7], and the total residual state density takes into account more compound configurations, Where \(C_b(T, T_B)\) in the escape channel is the Clebsch-Gordan isospin coupling vector, \(T_B\) is the quantity of isospin in the residual nucleus, \(\epsilon\) is the energy of single particles, \(T\) is the isospin quantum number, \(Z_b\) is the emitted particle proton number, \(N_b\) is the emitted particle atomic number and \(\mu_b\) is the reduced mass. The residual excitation energy can be given by \(U = E - \epsilon - B_b\), where \(B_b\) is the binding energy of emitted particles. The cross – section of the pre-equilibrium model in the center of the mass system for the emitted particles \(b\) at emission energy \(\epsilon\) and spin-dependent formulation is obtained by [6, 8, 9]:

\[
\left[ \frac{d\sigma_{\epsilon_b}(\epsilon)}{d\epsilon} \right]_{pre} = \sum_{T} \left[ \frac{d\sigma_{\epsilon_b}(E, T)}{d\epsilon} \right]_{pre} = \sigma_{b}(\epsilon) \sum_{T} \left[ C_a(T, T_A) \right]^2 \prod_{r} \sum_{p_r} S_{pre}(p, p_x, T) W_b(p, p_x, E, \epsilon, T) \quad \ldots \quad (3)
\]

In either case, "the sum over \(p\) begins with \(p = A_a + 1\) and continues until one of the following three conditions is met":

- \(p\) reaches the most probable value at equilibrium,
- \(p\) reaches the maximum value dimensioned for (currently 12), or
- the initial strength has all gone into particle emission.

At that point, any remaining strength is handled using equilibrium model calculations.
Where \( \sigma_{a,pre} \) is the cross section for modeling the complex nucleus that reduced by a cross section with direct reaction and \((P, \pi, \nu, T)\) is the average amount of time spent in each class of configuration [7]. For each spin, \( T \), the product of equation (3) is added and multiplied by the Clebsch-Gordan coefficient of coupling of the entry channel isospin. Since a second particle emission can occur at the pre-equilibrium point, then the emission may influence the energy spectra at high energy excitation [10]. Within the FKK model the multi-step direct (MSD) or pre-equilibrium or forward-peaked component involves the pre-equilibrium exciton process components. As well as knockout and inelastic scattering, cross-sections of nucleon, (IN) involving cluster degrees of freedom, could be represented with:

\[
\left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{IN}} = \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{pre},1} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{pre},2} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{KO}} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{MSC}}
\]

(4)

and for other reaction channel knockout (KO) is,

\[
\left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{KO}} = \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{pre},1} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{pre},2} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{KO}} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{pre},0}
\]

(5)

where KO only makes a contribution for reactions \((N, \alpha), (C, N)\) and \((C, \alpha)\), whereas \(N\) is a nucleon, \(C\) is a complex particle \((2D, 3T, 3\text{He}, \text{or}\ \alpha\)-particle). By contains the corresponding equilibrium or symmetric element only the primary and secondary evaporation cross sections and is given the Multi step compound spectrum,

\[
\left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{MSC}} = \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{q},1} + \left[ \frac{d\sigma_a}{d\Omega} \right]_{\text{q},2}
\]

(6)

However, for \(^{64}\text{Zn}_{30}\) nuclei the binding energies were initially extracted for all light nuclei from the primary emission process when the nucleus is in the automation state through a special program [1]. The binding energies were then used as the input file during the data entry into the Preco6 code and the emission spectra and angular distribution were then obtained for each emitted particle.

In this work, different mechanisms have been used to calculate the total energy spectrum and angular distribution in terms of MSD and MSC models for the emission nucleons and light nuclei from reactions; \(^{64}\text{Zn}\) (n, n), \(^{64}\text{Zn}\) (n, p), \(^{64}\text{C}\) (n, D), \(^{63}\text{C}\), \(^{64}\text{Zn}\) (n, T), \(^{63}\text{Cu}\), \(^{64}\text{Zn}\) (n, \(^{3}\text{He}\)), \(^{62}\text{Ni}\), and \(^{64}\text{Zn}\) (n, \(^{4}\text{He}\)) at incident (excitation) energy 2 MeV.

Furthermore, the two-component particle-hole state includes many correction parameters was used to calculate the energy spectrum at various incident energies [5]. The matrix element that used in the calculation of energy spectrum can be given by C. Kalbach [5,11].

\[
|M_{ij}|^2 = K_{ij} A_g g^{-3} \left( \frac{E}{3A_g} + 20.9 \right)^{-3}
\]

(7)

The double differential cross section using FKK model can be listed as [11]:

\[
\frac{d^2\sigma}{d\Omega d\varepsilon_h} = \frac{1}{4\pi} \frac{d\sigma}{d\varepsilon_h} \frac{a_{ex}}{\sinh(a_{ex})} \left[ \cosh(a_{ex} \cos\theta) + f_{msd} \sinh(a_{ex} \cos\theta) \right]
\]

(8)
where $a_{ex}$ represents the slope parameter included in the exciton model and its components are related. The angle $\theta$ is adjusted as center-of-mass measured unit [13] and the amount $f_{msd}$ is the fraction of energy in the FKK model at the specified emission cross section. Such cross sections are combined to find the fraction of the cross section $f_{msd}$ which represents the pre-equilibrium part [11]:

$$f_{msd}(\varepsilon) = \frac{\left(\frac{d\sigma}{d\varepsilon}\right)_{msd}}{\left[\frac{d\sigma}{d\varepsilon}\right]_{msd} + \left[\frac{d\sigma}{d\varepsilon}\right]_{msc}}$$

\[
\left[\frac{d\sigma}{d\varepsilon}\right]_{msd}
\]
represents the pre-equilibrium emission spectra for both primary and secondary as well as the cross section from nucleon transfer, knockout and inelastic scattering involving cluster degrees of freedom. The quantity $\left[\frac{d\sigma}{d\varepsilon}\right]_{msc}$ is the equilibrium emission spectra or the symmetric component comprises only the cross-sections of primary and secondary evaporation.

For $^{64}_{30}Zn$ nuclei the binding energies were initially calculated for all light nuclei from the primary emission process when the nucleus is in the exciton state through a special program the connecting energies were then used during the data entry into the Preco06 program and the emission spectra and angular distribution were then obtained each particle is emitted.

In this work, Different mechanisms have been used to calculate the total energy spectrum and angular distribution in terms of MSD and MSC models for the emission nucleons and light nuclei from reactions: $^{64}Zn (n, n)^{64}Zn$, $^{64}Zn (n, p)^{64}Cu$, $^{64}Zn (n, D)^{63}Cu$, $^{64}Zn (n, T)^{62}Cu$, $^{64}Zn (n, 3He)^{62}Ni$, and $^{64}Zn(n,4He)^{61}Ni$, and at different density includes incident energy(2) Mev.

In this work the two-component particle-hole state includes many correction parameters was used to calculate the spectrum at varying incident energies. Its follow-up on transition estimates rates and then demand differentials full spectrum. Not even pre-equilibrium Emissions covered by the calculations, but the particles too emissions in a State of control it Considered the standardization below factors for components in the matrix [11,12].

$$M_{ij} = K_{ij} A_{A} g^{-3} \left(\frac{E}{3A_{A}} + 20.9\right)^{-3}$$

\[(6)\]

Original angular orientation the formalist distribution divides the Section transversal into two components MSD and MSC, The MSD Part is described as having always on Minimum one unbound particle degree in each stage of the reaction, freedom, when in the device portion of the MSC goes by at least one Parameters, where all the Particles are required to include the details on the original position of the projectile is much lost. The cross section on MSD is accordingly, assumed to have forward angular distributions, whilst the cross section of MSC has angularity symmetrical distributions about 90 degrees at mass center [11].

$$\frac{d^2\sigma}{d\theta d\varepsilon_h} = \frac{1}{4\pi} \frac{d\sigma}{d\varepsilon_h \sinh(a_{ex})} \left[\cosh(a_{ex} \cos \theta) + f_{msd} \sinh(a_{ex} \cos \theta)\right]$$

\[(7)\]
Here $a_{ex}$ represent the slop parameter included in exciton model and its components are related. The angle $\theta$ is adjusted center-of-mass measured unit [13] and the amount $f_{msd}$ is the fraction of energy in the (FKK) model at the specified emit cross section which one is direct multi-step, and replaced her by the pre-equilibrium proportion. Such cross sections are combined to find the fraction of the cross section $f_{msd}$ which represent the preequilibrium part [11]:

$$f_{msd}(\varepsilon) = \frac{\left[\frac{d\sigma}{d\varepsilon}\right]_{msd}}{\left[\frac{d\sigma}{d\varepsilon}\right]_{msd} + \left[\frac{d\sigma}{d\varepsilon}\right]_{msc}}$$  \hspace{1cm} (8)

$\left[\frac{d\sigma}{d\varepsilon}\right]_{msd}$ represent the emission spectra or pre-equilibrium unit peaked includes exciton model pre-equilibrium components, both primary and secondary as well as the cross section from nucleon transfer, knockout and inelastic scattering involving cluster degrees of freedom, as well as the quantity $\left[\frac{d\sigma}{d\varepsilon}\right]_{msc}$ is the equilibrium emission spectra or the symmetric component comprises only the cross-sections of primary and secondary evaporation.

**Results and Discussions:**

Figures (1 – a, b, c, d, e, and f) showed the emission spectra of some neutron reactions incident on $^{64}_{30}Zn$ nucleus, which emitted the particles (n, p, D, T, $^3He$ and $^4He$) at 2 MeV incident neutron energy and the obtained results were compared with other theoretical results of ref. [14]. These figures illustrated that the pre-equilibrium reaction has different probabilities and its predominant in the reactions (b, c, and d) among other interaction mechanisms. The reactions were done by assumed the Isospin has conserved state.

In figures (2 – a, b, and c) the mixed, conserved, and no-conserved isospin states have been studied by registered the energy spectra of the reactions $^{64}_{30}Zn$ (n,n)$^{64}_{30}Zn$, $^{64}_{30}Zn$ (n,p)$^{64}_{31}Cu$, $^{64}_{30}Zn$ (n,D)$^{63}_{29}Cu$, $^{64}_{30}Zn$ (n,T)$^{62}_{30}Cu$, $^{64}_{30}Zn$ (n,$^3He$)$^{63}_{31}Ni$, and $^{64}_{30}Zn$(n,$^4He$)$^{64}_{30}Ni$ at 2 MeV incident neutron energy. We note that isospin values are very close for all interactions involving various light nuclei and the highest values of the differential cross section when the emitted particle are proton, neutron, deuteron, triton, respectively, and almost zero in the cases of $^3He$ and $^4He$.

In order to determine the validity of theoretical treatment of this work, the energy spectrum results obtained for the reactions for reactions: $^{64}_{30}Zn$ (n,n)$^{64}_{30}Zn$, $^{64}_{30}Zn$(n,p)$^{64}_{31}Cu$, $^{64}_{30}Zn$(n,D)$^{63}_{29}Cu$, $^{64}_{30}Zn$(n,T)$^{62}_{30}Cu$, and $^{64}_{30}Zn$(n,$^4He$)$^{64}_{30}Ni$ at incident neutron energy 2 MeV were compared with the available theoretical results of ref. [15] and experimental data of ref. [16], as shown in figures (3 – a, b, and c).

The angular distribution of the reactions $^{64}_{30}Zn$ (n,n)$^{64}_{30}Zn$, $^{64}_{30}Zn$ (n,p)$^{64}_{31}Cu$, $^{64}_{30}Zn$(n,D)$^{63}_{29}Cu$, $^{64}_{30}Zn$(n,T)$^{62}_{30}Cu$, and $^{64}_{30}Zn$(n,$^4He$)$^{64}_{30}Ni$, have been illustrated in figures (4 – a, b, c, d, e, and f). The results of these figures strongly suggested that the double
differential cross section (DDX) decreases with increasing the incident neutron angle at constant energy, which is 2 MeV. However, one can indicate these reactions the main contribution to the DDX spectra comes from the evaporation reaction that belong to compound (equilibrium) nuclear reaction stage, but when the emission energy increase, the pre-equilibrium stage is predominated. Furthermore, in figures (5 – a, b, c, and d) the DDX spectra at 60° angle of incident neutron and at 2 MeV incident energy for the reactions ⁶⁴Zn(n,p)⁶⁴Cu, ⁶⁴Zn(n,D)⁶³Cu, ⁶⁴Zn(n,T)⁶²Cu, and ⁶⁴Zn(n,³He)⁶¹Ni, respectively, were illustrated. The results have been compared with the experimental and evaluated data of refs. [14, 17, 21-23], and there is clearly a great match of exciton model that used in this work and the results that were compared.
Fig. (1): The energy spectrum of different reaction mechanisms as a function of the particle emission energy in cm-system for emission of neutron and light particles and for different reactions; (a) $^{64}$Zn (n,n)$^{64}$Zn, (b) $^{64}$Zn (n,p)$^{64}$Cu, (c) $^{64}$Zn (n,D)$^{63}$Cu, (d) $^{64}$Zn (n,T)$^{62}$Cu, (e) $^{64}$Zn (n,$^3$He)$^{62}$Ni, and $^{64}$Zn(n,$^4$He)$^{61}$Ni, at incident neutron energy 2MeV.
Fig. (2); the energy spectrum of different Isospin states (mixed, conserved, and no conserved) as a function of the particle emission energy in cm-system for emission neutron and light nuclei and for different reactions; $^{64}\text{Zn} (\text{n},\text{n})^{64}\text{Zn}$, $^{64}\text{Zn} (\text{n},\text{p})^{64}\text{Cu}$, $^{64}\text{Zn} (\text{n},\text{D})^{63}\text{Cu}$, $^{64}\text{Zn} (\text{n},\text{T})^{62}\text{Cu}$, $^{64}\text{Zn} (\text{n},^{3}\text{He})^{62}\text{Ni}$, and $^{64}\text{Zn}(^{4}\text{He})^{61}\text{Ni}$, at incident energy 2MeV.
Fig. (3); A comparison between the calculated energy spectrum with Refs. [19, 20], as a function of particle emission energy, in cm-system for the reactions $^{64}$Zn(n,n)$^{64}$Zn, $^{64}$Zn (n,p)$^{64}$Cu, $^{64}$Zn (n,D)$^{63}$Cu, $^{64}$Zn (n,T)$^{62}$Cu, and $^{64}$Zn(n,4He)$^{61}$Ni, at 2 MeV incident neutron energy.
Fig. (4): The angular distribution at different angles and for different reactions: $^{64}$Zn(n,n)$^{64}$Zn, $^{64}$Zn (n,p)$^{64}$Cu, $^{64}$Zn(n,D)$^{63}$Cu, $^{64}$Zn(n,T)$^{62}$Cu, $^{64}$Zn(n,$^3$He)$^{62}$Ni, and $^{64}$Zn(n,$^4$He)$^{61}$Ni, at 2MeV incident neutron energy.
Fig. (5): The double differential cross sections at 2 MeV incident neutron energy and at 60° angle that calculated using exciton model for the reactions $^{64}$Zn(n,p)$^{64}$Cu, $^{64}$Zn(n,D)$^{63}$Cu, $^{64}$Zn(n,T)$^{62}$Cu, and $^{64}$Zn(n,4He)$^{61}$Ni. Our results have been compared with experimental data [19, 20] and with other theoretical results TENDL2019 [14], ENDF/B-VIII [17], JEFF-3.3 and JEFF-3.2 [18].

Conclusions:
In this paper, the energy spectra and DDX of the reactions of $^{64}$Zn nucleus with a neutron at incident energy 2 MeV were studied using the exciton model with statistical FKK model for pre-equilibrium emission calculations. From the obtained results, one can conclude that the exciton model can be safely used with very good validity in the studying of nuclear reactions, and this claim was proved by several comparisons. For DDX, the results of all reactions showed that when the incident neutron angle increases, the DDX values decreases.

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