Two-Stage Inflation in Supergravity

G. Lazarides\(^{(a)}\) and N. Tetradis\(^{(b)}\)

\(^{(a)}\) Physics Division, School of Technology, University of Thessaloniki, 54006 Thessaloniki, Greece
\(^{(b)}\) Dipartimento di Fisica, Scuola Normale Superiore, 56100 Pisa, Italy

Abstract

We investigate the viability of a two-stage inflationary scenario in the context of supergravity, so as to resolve the problem of initial conditions for hybrid inflation. We allow for non-renormalizable terms in the superpotential and consider the most general form of the Kähler potential and the gauge kinetic function. We construct a model with two stages of inflation, the first driven by $D$-term and the second by $F$-term energy density. The viability of this scenario depends on the non-minimal terms in the Kähler potential, for which we derive the necessary constraints.

PACS number: 98.80.Cq

January 1998
1 Introduction

An attractive realization of the inflationary scenario is obtained when hybrid inflation \[1\] is embedded in the context of supersymmetry \[2\]–\[11\]. Flat directions with non-zero potential energy density appear without fine-tuning and are not lifted by quantum corrections. On the contrary, the quantum corrections generate a small slope of the potential along the flat directions, which results in the slow rolling of the inflaton field \(S\). An important aspect of the hybrid inflationary scenario is that the part of inflation with observable consequences takes place for values of the inflaton field below the Planck scale \[2\] where models can be reliably constructed. (Throughout the paper we use the “reduced” Planck scale \(m_{Pl} = M_{Pl}/\sqrt{8\pi}, M_{Pl} = 1.22 \times 10^{19}\) GeV.)

Recently, the question of initial conditions for hybrid inflation has been addressed \[12\]–\[14\]. In ref. \[14\], it was shown that severe fine-tuning of the initial configuration that will lead to inflation is necessary. This is a consequence of the presence of (one or more) scalar fields orthogonal to the inflaton and the need to satisfy the experimental constraints resulting mainly from the COBE observation of the cosmic microwave background anisotropy. We briefly summarize the arguments below.

For hybrid inflation, the slow rolling of the inflaton occurs along a valley of the potential with a small slope. In the original scenario \[1\], the slow rolling ends when the valley turns into a ridge and fluctuations of fields orthogonal to the inflaton begin to grow. In supersymmetric formulations \[2\]–\[11\], it may end even earlier, if the slope along the valley becomes large. Typically the end of slow rolling corresponds to a value of the inflaton below the Planck scale. The COBE observation of the cosmic microwave background anisotropy constrains the properties of the model along the inflationary trajectory \[15\]–\[17\]. On general grounds, one expects the inflationary energy scale \(V^{1/4}\) (determined by the vacuum energy density during inflation) to be at least two or three orders of magnitude smaller than the Planck scale \[18\]:

\[
V^{1/4}/\epsilon^{1/4} \simeq 7 \times 10^{16}\text{ GeV. (1.1)}
\]

Here \(\epsilon\) is a “slow-roll” parameter \[14\] that must be much smaller than 1 during inflation.

The onset of inflation requires a region of space with a size of a few Hubble lengths where the fields take almost constant values, so that the gradient energy density is negligible compared to the potential energy density \[19\]. The earliest time at which one could start talking about such regions of space is when the Planck era (during which quantum gravitational fluctuations
dominate) ends and classical general relativity starts becoming applicable. The initial energy density is near $m_{Pl}^4$. For a theory with couplings not much smaller than 1, the initial field values within each region are expected to be of order $m_{Pl}$.

Inflation could start at the end of the Planck era, provided that the fields take appropriate values. However, it is very likely that the fields will evolve from some initial values that do not give rise to inflation to different values that do. The difference between the initial energy scale $m_{Pl}$ of the field evolution and the inflationary scale $V^{1/4}$ implies that the fields evolve for a long time before settling down along the inflationary trajectory. The Hubble parameter $H$ sets the scale for the “friction” term in the evolution equations, which determines how fast the energy is dissipated through expansion. When the energy density drops much below $m_{Pl}^4$, the smallness of the “friction” term results in a very long evolution, during which the fields oscillate many times. Some of the trajectories eventually settle down in the valley of the potential that produces inflation. However, the sensitivity to the initial conditions is high because of the long evolution. A slight variation of the initial field values separates inflationary trajectories from trajectories that lead to the minima of the potential, where inflation does not occur.

This has severe implications for the initial configuration that can lead to the onset of inflation. In ref. [14], it was shown that, for the prototype model of hybrid inflation [1] with a scale consistent with the COBE observations, the most favourable area of inflationary initial conditions is a thin strip of width $\sim 10^{-5}m_{Pl}$ around the $S$ axis. Throughout a region of space with a size of the order of the Hubble length (which is $\sim m_{Pl}^{-1}$ initially), the initial values of the fields orthogonal to the inflaton must be zero with an accuracy $\sim 10^{-5}m_{Pl}$. This should be compared to the natural scale of the initial fluctuations of the fields, which is of order $m_{Pl}$. If this condition of extreme homogeneity is not satisfied the fields in different parts of the original space region will evolve towards very different values. In one part they may end up in the valley along the $S$ axis, while in another they may settle at the minima of the potential. Before inflation sets in, the size of space regions shrinks compared to the Hubble distance. As a result, large inhomogeneities are expected at scales smaller than $\sim H^{-1}$ when the evolution of the fields finally slows down. These will prevent the onset of inflation. The fine-tuning of the initial configuration must be increased by several orders of magnitude (to the $10^{-10}$ level) if the initial time derivatives of the fields are non-zero.

The initial condition problem has another serious aspect. The size $L$ of a homogeneous
region evolves proportional to the scale factor and, in terms of the Hubble length, is given by

\[
\frac{LH}{L_0 H_0} \sim \left( \frac{\rho}{\rho_0} \right)^{\frac{1+3w}{6(1+w)}}.
\]  

(1.2)

The parameter \( w \) determines the relation between energy density and the mean value of the pressure \( (p = w \rho) \). For a system of massless oscillating fields, or a radiation-dominated Universe, \( w = 1/3 \). For a system of massive oscillating fields, or a matter-dominated Universe, \( w = 0 \). At the onset of inflation, where \( \rho/m_{Pl}^4 \sim 10^{-12} \), we must have \( LH \gg 1 \). Otherwise the gradient terms may dominate over the vacuum energy density. For \( w = 0 \), this leads to the requirement \( L_0 H_0 \gg 100 \) at the end of the Planck era, where \( \rho_0/m_{Pl}^4 \sim 1 \). This means that the initial extreme homogeneity must extend far beyond the typical size of regions that can be considered causally connected. In ref. [14] the evolution of the scale factor \( R \) relative to the Hubble parameter was studied numerically, starting from an initial value \( R_0 \sim H_0^{-1} \). At the onset of inflation, \( R \) was found to be smaller than \( H^{-1} \) typically by a factor of order \( 10-100 \). This reflects the variation of the effective value of \( w \) between \(-1\) and \( 0 \) during the initial evolution.

In summary, for hybrid inflation, one must assume the presence of a large number (typically \( \sim 10^6 \)) of causally disconnected adjacent initial regions within which the corresponding fields have almost equal values. More specifically, the fields orthogonal to the inflationary trajectory must be zero with an accuracy at least \( \sim 10^{-5} \). Despite the difficulty of calculating the probability of such an initial configuration, we believe that, under normal circumstances, it is highly improbable. In a self-reproducing Universe, however, it may become quite probable and the problem of initial conditions may not exist. Note that the degree of fine-tuning of initial data required in non-inflationary cosmology is incomparably worse than the one necessary for the onset of inflation.

In ref. [20], a simple resolution of the issue of fine-tuning described above was suggested. A scenario with two stages of inflation was proposed within the context of global supersymmetry. The first stage has a typical scale \( \sim m_{Pl} \), which implies that the “friction” term proportional to \( H \) in the evolution equations of the fields is large. As a result, the system settles down quickly along an almost flat direction of the potential and this stage of inflation occurs “naturally”. By generating an exponential expansion of the initial region of space it also provides the homogeneity that is necessary for the second stage of inflation. The latter has a characteristic scale much below \( m_{Pl} \) and generates the density perturbations that result in the cosmic microwave background anisotropy observed by COBE.

3
It was pointed out, in ref. [20], that the generalization of the two-stage inflationary scenario in the context of supergravity is difficult. The scenario considered there involved two hybrid inflations along two $F$-flat directions. When global supersymmetry is replaced by supergravity, all flat directions are in general lifted and inflation becomes impossible. In the context of canonical supergravity (minimal Kähler potential) and a linear superpotential of the form $W = -\mu^2 S$ (like the one encountered in hybrid inflationary models) a cancellation takes place that prevents the appearance of a mass term for $S$ [2]. Therefore, there is a possibility for $S$ to play the role of the inflaton. However, for a superpotential with two $F$-flat directions, only a linear combination of the possible inflaton fields stays “massless”, with the orthogonal combination acquiring a large mass term [7]. This implies that only one inflationary stage is likely to survive in the context of supergravity. However, this argument does not apply to the case where inflation is driven by a $D$-term energy density [6]. The two-stage inflationary scenario may then be possible along an appropriate combination of $F$-flat and $D$-flat directions.

The above fine-tuning problem may also be solved if a first stage of inflation, at values of the inflaton $> m_{Pl}$, is incorporated into the scheme. This, however, requires tiny coupling constants. There is, in principle, no reason why this possibility cannot be realized in the context of hybrid inflation. Inclusion of supergravity, however, makes the discussion of this option technically difficult. The reason is that, although $D$-term inflation does not contain exponentially growing terms at values of the inflaton $> m_{Pl}$, a reliable expansion scheme is not available in this domain.

In this paper, we discuss a model that allows for two stages of inflation, the first driven by $D$-term and the second by $F$-term energy density. We compute the supergravity corrections to the potential and discuss under what conditions the two stages are realized. In section 2, we introduce our model. We first present the potential in the globally supersymmetric limit and then compute all the relevant supergravity corrections. In sections 3, 4 and 5, we discuss in detail the first stage of inflation, the intermediate stage between the two inflations and the second stage of inflation respectively. We derive constraints for the parameters of the Kähler potential that guarantee the viability of the scenario. Our conclusions are given in section 6.

2 The model
2.1 Global supersymmetry

We consider the renormalizable superpotential

$$W = \lambda X \Phi^+ \Phi^- + \kappa S \Phi \bar{\Phi} - \mu^2 S. \quad (2.1)$$

The superfields $\Phi, \bar{\Phi}$ are the Standard Model singlet components of a conjugate pair of chiral superfields which transform under the GUT gauge symmetry group $G$. Their expectation values break the group, reducing its rank. The superfields $S, X, \Phi^+, \Phi^-$ are singlets under $G$. The parameters $\lambda, \kappa, \mu$ can be taken real and positive by absorbing their possible phases in a redefinition of the fields. The superpotential possesses the following three $U(1)$ symmetries too:

a) $U(1)_R$: a global $R$-symmetry.
b) $U(1)_{\xi}$: an (“anomalous”) local symmetry with gauge coupling $g$.
c) $U(1)_X$: a global symmetry.

The charges of the fields are

| Field | $U(1)_R$ | $U(1)_{\xi}$ | $U(1)_X$ | $U(1) \subset G$ |
|-------|----------|------------|----------|------------------|
| $X$   | 1        | 0          | 1        | 0                |
| $\Phi^+$ | 0       | 1          | $-\frac{1}{2}$ | 0                |
| $\Phi^-$ | 0       | $-1$       | $-\frac{1}{2}$ | 0                |
| $S$   | 1        | 0          | 0        | 0                |
| $\Phi$ | 0        | 0          | 0        | 1                |
| $\bar{\Phi}$ | 0       | 0          | 0        | $-1$             |

Performing appropriate $U(1)_R, U(1)_{\xi}, U(1)_X$ and $G$ transformations, and using the vanishing condition for the $D$-term with respect to $G$ (i.e. $|\Phi| = |\bar{\Phi}|$), the fields can be written in the form

$$X = \frac{\chi}{\sqrt{2}}, \quad \Phi^+ = \frac{\phi^+_1 + i\phi^+_2}{\sqrt{2}}, \quad \Phi^- = \frac{\phi^-}{\sqrt{2}},$$

$$S = \frac{\sigma}{\sqrt{2}}, \quad \Phi = \bar{\Phi} = \frac{\phi_1 + i\phi_2}{2}. \quad (2.2)$$

The real fields $\chi, \phi^+_1, \phi^+_2, \phi^-, \sigma, \phi_1, \phi_2$ have canonically normalized kinetic terms.

In the globally supersymmetric limit, the scalar potential is given by

$$V = \lambda^2 |X|^2 \left(|\Phi^+|^2 + |\Phi^-|^2\right) + \lambda^2 |\Phi^+|^2 |\Phi^-|^2 + \frac{g^2}{2} \left(|\Phi^+|^2 - |\Phi^-|^2 + \xi\right)^2$$

$$+ \kappa^2 |S|^2 \left(|\Phi|^2 + |\bar{\Phi}|^2\right) + |\kappa \Phi \bar{\Phi} - \mu^2|^2. \quad (2.3)$$
It can be expressed in terms of the real fields defined in eq. (2.2) as

\[
V = \frac{\lambda^2}{4} \chi^2 \left( \left[ \phi_1^+ \right]^2 + \left[ \phi_2^+ \right]^2 \right) + \frac{\lambda^2}{4} \left( \left[ \phi_1^- \right]^2 + \left[ \phi_2^- \right]^2 \right)
\]

\[
+ \frac{g^2}{8} \left( \left[ \phi_1^+ \right]^2 + \left[ \phi_2^+ \right]^2 - \left[ \phi^- \right]^2 + 2 \xi \right)^2
\]

\[
+ \left[ \frac{\kappa}{4} \left( \phi_1^2 - \phi_2^2 \right) - \mu^2 \right] + \frac{\kappa^2}{4} \phi_1^2 \phi_2^2 + \frac{\kappa^2}{4} \sigma^2 \left( \phi_1^2 + \phi_2^2 \right).
\]

(2.4)

The minima of the potential are located at \( \chi = 0, \phi_1^+ = \phi_2^+ = 0, [\phi^-]^2 = 2\xi, \sigma = 0, \phi_1^2 = 4\mu^2/\kappa, \phi_2 = 0. \) We choose the Fayet-Iliopoulos term \( \xi \) to be positive and we consider the case \( g\xi \gg \mu^2 \) in the following. A Fayet-Iliopoulos term can be generated at the one-loop level if the symmetry \( U(1)_\xi \) is anomalous [21]. When our model is embedded in the context of a complete theory (such as string theory) the anomaly is expected to be cancelled through the Green-Schwarz mechanism and \( \xi \) is calculable. For example, within the weakly-coupled heterotic string theory it is given by [22]

\[
\xi = \frac{\text{TrQ}}{192\pi^2 g^2 m_{Pl}^2},
\]

(2.5)

where \( \text{TrQ} \) is the total charge under the \( U(1)_\xi \) symmetry.

For \( \phi_1^+ = \phi_2^+ = \phi^- = \phi_1 = \phi_2 = 0 \) the potential of eq. (2.4) is independent of \( \sigma, \chi \) and has the value \( V = g^2 \xi^2/2 + \mu^4 \), with the \( \mu^4 \) contribution being negligible. Therefore, this range of field values can support a first stage of inflation. The masses of the \( \phi_{1,2}^+ \) and \( \phi^- \) fields are

\[
M_{\phi_1^+}^2 = M_{\phi_2^+}^2 = \frac{\lambda^2}{2} \chi^2 + g^2 \xi.
\]

\[
M_{\phi^-}^2 = \frac{\lambda^2}{2} \chi^2 - g^2 \xi.
\]

(2.6)

An instability appears for

\[
\chi^2 < \chi_{\text{ins}}^2 = \frac{2g^2}{\lambda^2} \xi,
\]

(2.7)

which can trigger the growth of the \( \phi^- \) field.

For \( \phi_1 = \phi_2 = 0 \) the potential is independent of \( \sigma \). The masses of the \( \phi_{1,2} \) fields are given by

\[
M_{\phi_1}^2 = \frac{\kappa^2}{2} \sigma^2 - \kappa \mu^2,
\]

\[
M_{\phi_2}^2 = \frac{\kappa^2}{2} \sigma^2 + \kappa \mu^2.
\]

(2.8)

An instability appears for

\[
\sigma^2 < \sigma_{\text{ins}}^2 = \frac{2}{\kappa} \mu^2.
\]

(2.9)
which can trigger the growth of the \( \phi_1 \) field. For \( \chi = \phi_1^+ = \phi_2^+ = 0 \), \( [\phi^-]^2 = 2\xi \) the potential is \( V = \mu^4 \). Therefore, this range of field values can support a second stage of inflation.

The flatness of the potential is lifted by radiative corrections. For \( \chi \gg \chi_{\text{ins}} \), the one-loop \( \chi \)-dependent radiative correction at \( \phi_1^+ = \phi_2^+ = \phi^- = 0 \) is

\[
\Delta V_r(\chi) = \frac{g^4}{16\pi^2} \xi^2 \ln \left( \frac{\chi^2}{2\Lambda_1^2} \right). \tag{2.10}
\]

For \( \sigma \gg \sigma_{\text{ins}} \), the one-loop \( \sigma \)-dependent radiative correction at \( \phi_1 = \phi_2 = 0 \) is

\[
\Delta V_r(\sigma) = \frac{\kappa^2}{16\pi^2} \mu^4 \ln \left( \frac{\kappa^2 \sigma^2}{2\Lambda_2^2} \right). \tag{2.11}
\]

The precise values of the normalization scales \( \Lambda_{1,2} \) are not important for our discussion. Also, away from the flat directions \( (\phi_{1,2}^+, \phi^-, \phi_1,2 \neq 0) \) the radiative corrections are not significant and we neglect them.

### 2.2 Supergravity

The scalar potential in supergravity has the form

\[
V = \exp \left( \frac{K}{m_P^2} \right) \left[ (K^{-1})^i_j F^i F_j - 3 \frac{|W|^2}{m_P^2} \right] + \frac{g^2}{2} \text{Re} f_{AB} D^A D^B, \tag{2.12}
\]

where

\[
F^i = W^i + K^i_j \frac{W}{m_P^2}, \tag{2.13}
\]

\[
D^A = K^i_j \left( T^A \right)_i^j \Phi_j + \xi^A. \tag{2.14}
\]

Here \( K, W \) and \( f \) are respectively the Kähler potential, superpotential and gauge kinetic function. Upper (lower) indices \( (i,j) \) denote differentiation with respect to \( \Phi, \bar{\Phi} \) and \( T^A \) are the generators of the gauge group in the appropriate representation. The \( \xi^A \) are Fayet-Iliopoulos \( D \)-terms, which can only exist for \( U(1) \) gauge groups.

Allowing for non-renormalizable terms, the most general form of the superpotential permitted by the symmetries discussed in the beginning of the previous subsection is

\[
W = S \sum_{n=0}^{\infty} A_n (\Phi \bar{\Phi})^n + X \Phi^+ \Phi^- \sum_{n=0}^{\infty} B_n (\Phi \bar{\Phi})^n, \tag{2.15}
\]

---

1 The radiative correction depends on the dimensionality \( N \) of the representation of the GUT group \( G \) under which the chiral superfields \( \Phi, \bar{\Phi} \) transform. More specifically, the right-hand side of the one-loop contribution of eq. (2.11) must be multiplied by \( N \). As we are not specifying the group \( G \) in this work, we have set \( N = 1 \).
with $A_0 = -\mu^2$, $A_1 = \kappa$, $B_0 = \lambda$. We are interested in the effect of the supergravity corrections on the evolution of the fields during the two stages of inflation and the intermediate stage. These corrections can generate large mass terms for the inflatons and destroy the slow-roll solutions. For this reason, we concentrate on the field space near the flat directions of the potential. Therefore, the corrections that are relevant for our discussion have $\Phi = \bar{\Phi} = \Phi^+ = 0$.

The terms in the Kähler potential that can contribute to $K^i$, $(K^{-1})^i_j$ in eq. (2.12) for $\Phi = \bar{\Phi} = \Phi^+ = 0$ can be parametrized as

$$K = \sum_{n_1, n_2, n_3=0}^{\infty} \frac{a_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3}$$

$$+ \left( \frac{1}{m_p} \right) X^{n_1, n_2, n_3=0} \frac{b_{n_1 n_2 n_3}}{2(\Sigma - 1)} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3} + h.c. \right)$$

$$+ |\Phi|^2 \sum_{n_1, n_2, n_3=0}^{\infty} \frac{c_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3}$$

$$+ |\Phi|^2 \sum_{n_1, n_2, n_3=0}^{\infty} \frac{d_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3}$$

$$+ |\Phi|^2 \sum_{n_1, n_2, n_3=0}^{\infty} \frac{e_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3},$$

with $a_{000} = 0$, $a_{100} = a_{010} = a_{001} = 1$, $c_{000} = d_{000} = e_{000} = 1$. The coefficients $a$, $c$, $d$ and $e$ are real, while the coefficients $b$ may be complex.

The form of the gauge kinetic function $f$ is constrained by its holomorphicity and the symmetries of the model. The $R$-symmetry and the holomorphicity prevent the appearance of $S$ and $X$ in $f$. The $U(1)_\xi$ symmetry permits the presence of the combination $\Phi^+ \Phi^-$. However this combination has $X$-charge $-1$, which cannot be compensated, as $X$ cannot appear in $f$. The only combination that is allowed is $\Phi \bar{\Phi}$. As we are interested in the field space with $\Phi = \bar{\Phi} = \Phi^+ = 0$ and the derivatives of $f$ are not relevant for the potential, we conclude that we can take $f^{-1} = 1$ for our study.

The potential is given by eq. (2.13). For $\Phi = \bar{\Phi} = \Phi^+ = 0$ it can be parametrized as

$$V = \mu^4 \sum_{n_1, n_2, n_3=0}^{\infty} \frac{p_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3}$$

$$+ \lambda^2 |X|^2 |\Phi^-|^2 \sum_{n_1, n_2, n_3=0}^{\infty} \frac{q_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3}$$

$$+ \lambda |X|^2 |\Phi^-|^2 \frac{\mu^2}{m_p} \sum_{n_1, n_2, n_3=0}^{\infty} \frac{r_{n_1 n_2 n_3}}{m_p^{2(\Sigma - 1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^-|^{2n_3}.$$
\[ + |X|^2 |\Phi|^{-2} \left( \frac{\mu^2}{m_{Pl}^2} \right)^2 \sum_{n_1=2, n_2, n_3=0}^{\infty} \frac{s_{n_1 n_2 n_3}}{m_{Pl}^{2(2^n-1)}} |S|^{2n} |X|^{2n_2} |\Phi^{-2n_3} \]
\[ + \frac{g^2}{2} D^2, \]  
(2.17)

where the generalized $D$-term is given by

\[ D = - \sum_{n_1, n_2, n_3=0}^{\infty} n_3 \frac{a_{n_1 n_2 n_3}}{m_{Pl}^{2(2^n-1)}} |S|^{2n_1} |X|^{2n_2} |\Phi^{-2n_3} + \zeta. \]  
(2.18)

Notice that the non-renormalizable terms in the superpotential do not appear in the expression for the potential, even for a non-minimal Kähler potential. Also, the potential depends only on the magnitudes of the fields despite the contribution of terms such as the one in the second line of eq. (2.10). The coefficients $p_{n_1 n_2 n_3}$, $q_{n_1 n_2 n_3}$, $r_{n_1 n_2 n_3}$, $s_{n_1 n_2 n_3}$ can be expressed in terms of the coefficients $a_{n_1 n_2 n_3}$, $b_{n_1 n_2 n_3}$, $c_{n_1 n_2 n_3}$ appearing in the expansion of the Kähler potential. A lengthy calculation for the terms suppressed by up to two powers of $m_{Pl}^2$ gives

\[ p_{000} = 1, \quad p_{100} = -4 a_{200}, \quad p_{010} = 1 - a_{110}, \quad p_{001} = 1 - a_{101}, \]
\[ p_{200} = \frac{1}{2} + 16 a_{200}^2 - 9 a_{300} - 7 a_{200}, \]
\[ p_{020} = \frac{1}{2} + a_{110}^2 - a_{120} - a_{110} + a_{020}, \]
\[ p_{002} = \frac{1}{2} + a_{101}^2 - a_{102} - a_{101} + a_{002}, \]
\[ p_{110} = 1 + 8 a_{200} a_{110} + a_{110}^2 - 4 a_{210} + 2 a_{110} - 4 a_{200}, \]
\[ p_{101} = 1 + 8 a_{200} a_{101} + a_{101}^2 - 4 a_{201} + 2 a_{101} - 4 a_{200}, \]
\[ p_{011} = 1 + 2 a_{110} a_{101} + |b_{000}|^2 - a_{111} - a_{101} - a_{110} + a_{011}, \]  
(2.19)

\[ q_{000} = 1, \quad q_{100} = 1 - c_{100}, \quad q_{010} = 1 - c_{010}, \quad q_{001} = 1 - c_{001}, \]
\[ q_{200} = \frac{1}{2} + a_{200} + c_{100} + c_{200} - c_{100}, \]
\[ q_{020} = \frac{1}{2} + a_{020} + c_{010} + c_{020} - c_{010}, \]
\[ q_{002} = \frac{1}{2} + a_{002} + c_{001} + c_{002} - c_{001}, \]
\[ q_{110} = 1 + a_{110} + 2 a_{100} a_{010} - c_{110} - c_{010} - c_{100}, \]
\[ q_{101} = 1 + a_{101} + 2 a_{100} a_{101} - c_{101} - c_{001} - c_{100}, \]
\[ q_{011} = 1 + a_{011} + 2 a_{010} a_{001} - c_{011} - c_{001} - c_{001} + |b_{000}|^2, \]  
(2.20)
\[ r_{000} = - (b_{000} + \text{c.c.}), \]
\[ r_{100} = 4 b_{000} a_{200} + b_{000} c_{100} - 2 b_{100} - 3 b_{000} + \text{c.c.}, \]
\[ r_{010} = b_{000} a_{110} + b_{000} c_{010} - b_{010} - b_{000} + \text{c.c.}, \]
\[ r_{001} = b_{000} a_{101} + b_{000} c_{001} - b_{001} - b_{000} + \text{c.c.}. \] 

(2.21)

The minimal Kähler potential corresponds to \( a_{n_1n_2n_3} = 0 \) for \( n_1 + n_2 + n_3 \geq 2 \), \( b_{n_1n_2n_3} = 0 \) for all \( n_1, n_2, n_3 \), and \( c_{n_1n_2n_3} = d_{n_1n_2n_3} = e_{n_1n_2n_3} = 0 \) for \( n_1 + n_2 + n_3 \geq 1 \). The potential is determined by the coefficients given in the above equations if only the contributions equal to 1 and \( \frac{1}{2} \) are kept in the right-hand side.

3 The first stage of inflation

As we discussed in the introduction, inflation does not start immediately after the Universe has emerged from the Planck era. An initial evolution of the fields takes place during which they approach an almost flat direction and eventually settle on a slow-roll trajectory. We assume that a Robertson-Walker metric is a good approximation for the regions of space with uniform fields that we are considering. The evolution of the fields is given by the standard equations (overdots denote derivatives with respect to cosmic time)

\[ \ddot{\phi}_i + 3H\dot{\phi}_i = - \frac{\partial V}{\partial \phi_i}, \]
\[ H^2 = \left( \frac{\dot{R}}{R} \right)^2 = \frac{1}{3m_{Pl}^2} \left( \sum_i \frac{1}{2} \phi_i^2 + V \right). \] 

(3.1)

(3.2)

We have integrated numerically the above equations for the potential of eqs. (2.4), (2.5) with \( g = 0.5, \xi/m_{Pl}^2 = 10^{-2}, \lambda = 0.3, \mu/m_{Pl} = 10^{-3}, \kappa = 0.04 \). The initial values of the fields have been chosen as \( \chi/m_{Pl} = 1.1, \phi_1^+/m_{Pl} = 0.04, \phi_2^+/m_{Pl} = 0.03, \phi^-/m_{Pl} = 0.04, \sigma/m_{Pl} = 1.2, \phi_1/m_{Pl} = 0.4, \phi_2/m_{Pl} = 0.2 \). They are near \( m_{Pl} \), with \( \phi_1^+, \phi_2^+, \phi^-, \phi_1, \phi_2 \) smaller than \( \chi \) and \( \sigma \), so that the evolution starts near the flat direction. We have also taken into account the radiative and supergravity corrections that provide a small slope along the flat direction. To this end, we have included the contributions of eqs. (2.10), (2.11) and the corrections arising from eqs. (2.17)–(2.21) with \( \phi_{1,2}^+ = \phi^- = \phi_{1,2} = 0 \). Only the parameters \( p_{n_1n_2n_3} \) are relevant for our discussion. We have used values corresponding to a minimal Kähler potential: \( p_{100} = 0, p_{010} = 1, p_{200} = 1/2, p_{020} = 1/2, p_{110} = 1 \). The contributions to the potential from the non-minimal terms affect mainly the intermediate stage and the second stage.
of inflation and are discussed in sections 4 and 5. We point out that the non-minimal terms also modify the kinetic terms in the Lagrangian and, therefore, the left-hand side of eq. (3.1). We have studied numerically the resulting corrections to the solutions presented below and found that they are small. For this reason, we consider only the corrections to the potential in the following.

In figs. 1 and 2, we present the evolution of the various fields and the Hubble parameter in units of $m_{Pl}$. The evolution starts at some initial time $t \sim H_0^{-1}$, with $H_0/m_{Pl} \simeq 9.0 \times 10^{-3}$. (For illustrative purposes we have indicated a time $t_0/m_{Pl} = 100$ in figs. 1 and 2). We have taken zero initial time derivatives for the fields. The initial energy density scale is approximately one order of magnitude below $m_{Pl}$ ($\rho_0/m_{Pl}^4 \simeq 2.5 \times 10^{-4}$). It is difficult to follow the evolution of the system at larger energy scales, as this typically takes us outside the region of validity of expansions such as the ones in eqs. (2.17), (2.18). The initial field values $\chi/m_{Pl} = 1.1$, $\sigma/m_{Pl} = 1.2$ are already at the border of this region. (Notice, however, that the expansion parameters are $|X|^2/m_{Pl}^2 = \chi^2/2m_{Pl}^2$, $|S|^2/m_{Pl}^2 = \sigma^2/2m_{Pl}^2$ and they are always multiplied by powers of $\mu^2/m_{Pl}^2$, $|\Phi^+|^2/m_{Pl}^2$, $|\Phi^-|^2/m_{Pl}^2$, $|\Phi|^2/m_{Pl}^2$, $|\bar{\Phi}|^2/m_{Pl}^2$, which we have chosen much smaller than 1.)

In fig. 1, we observe that $\phi_1^+, \phi_2^+$ and $\phi^-$ oscillate around zero with their amplitude decaying rapidly. The field $\chi$ quickly settles on the almost flat direction and approaches a slow-roll solution. This behaviour is induced by the large value of the “friction” term $3H\dot{\phi}_i$ in the evolution equations of the fields. At a time $t \simeq 800 m_{Pl}^{-1} \sim H_1^{-1}$ the energy density is dominated by the contribution $g^2\xi^2/2$ in the potential and inflation sets in. The Hubble parameter during the first stage of inflation is $H_1/m_{Pl} \simeq 2.0 \times 10^{-3}$. In fig. 2, we observe that $\sigma$, $\phi_1$ and $\phi_2$ also approach the almost flat direction rapidly. The slope inducing the slow rolling of $\sigma$ is much smaller than the one for $\chi$. As a result the total evolution of $\sigma$ during the first stage of inflation is negligible.

As we discussed in the introduction, the onset of inflation requires a region of space with a size of a few Hubble lengths where the fields take almost constant values, so that the gradient energy density is negligible compared to the potential energy density. It is reasonable to assume that this region first emerges at the end of the Planck era, at a time $t_{Pl} \sim m_{Pl}^{-1}$. During the evolution until inflation sets in, the size of this region shrinks with respect to the Hubble length. For the parameters of our model, the numerical integration of the evolution equations indicates that the ratio $R/H^{-1}$ is reduced by a factor 4–6 between $t_{Pl}$ and the beginning of inflation. This
implies that a region homogeneous over a Hubble length at the onset of inflation evolves from a region homogeneous over 4–6 Planck lengths at \( t_{P_l} \). We assume that it is not “unnatural” for such a region to appear at the end of the Planck era. The necessary range of homogeneity at \( t_{P_l} \) is reduced if the first stage of inflation takes place closer to the Planck scale. In our model, this requires larger values of \( \xi \), which are difficult to reconcile with the expansions in eqs. (2.17), (2.18) when \( \phi^- \) takes the value \( \phi^- = \sqrt{2\xi} \) after the end of the first stage of inflation. In other models, however, this may be possible. The initial homogeneity is expected to be preserved by the short evolution to the almost flat direction where inflation starts. It should be noted that in the one-stage scenario the size of the homogeneous region at \( t_{P_l} \) must be assumed larger by an order of magnitude than in the two-stage scenario (it must extend across 10–100 Planck lengths [14]).

For our choice of parameters, the dominant contribution to the slope along the almost flat direction for \( \chi \) comes from the radiative contribution of eq. (2.10). A slow-roll solution exists for \( \chi \gtrsim \max(\chi_{ins}, \chi_r) \), where \( \chi_r = g m_{Pl}/2\pi \). The total number of e-foldings during this stage is

\[
N_1 = \frac{2\pi^2 \chi_0^2 - \chi_f^2}{g^2 m_{Pl}^2},
\]

where \( \chi_0 \) is the value of the \( \chi \) field when inflation starts and \( \chi_f \) the value when it ends. For \( \chi_0 \simeq 0.52 \ m_{Pl} \) and \( \chi_f = \chi_{ins} \simeq 0.24 \ m_{Pl} \) (with \( \chi_{ins} \) given by eq. (2.7)), we obtain \( N_1 \simeq 17 \), which is confirmed by the numerical solution. We point out that the expression (2.10) that we have employed for the determination of \( N_1 \) is not valid towards the end of the first stage of inflation. However, the corrections are small because \( N_1 \) is dominated by the expansion of the Universe for large values of \( \chi \). As a result of the expansion during the first stage of inflation, the initial homogeneous region extends several orders of magnitude beyond the Hubble length when the intermediate stage starts.

During the first stage of inflation, almost massless fields like \( \sigma \) have a spectrum of quantum-mechanical fluctuations characterized by

\[
(\Delta \sigma)^2_k = \left( \frac{H_1}{2\pi} \right)^2,
\]

where \( H_1 \simeq 2.0 \times 10^{-3} \ m_{Pl} \) is the Hubble parameter. The freezing of fluctuations that cross outside the horizon generates classical perturbations of the fields at superhorizon scales with the same spectrum [24]. At the end of the first stage of inflation, the energy density in spatial gradient terms associated with the perturbations \( \sim H_1/2\pi \) of massless fields like \( \sigma \) is \( \sim H_1^4/4\pi^2 \).
and provides a negligible contribution to the total energy density. It is useful to consider also the mean square fluctuation of the classical field \( \sigma \) \[ (\Delta \sigma)^2 = N_1 \left( \frac{H_1}{2\pi} \right)^2. \] (3.5)

We obtain \( (\Delta \sigma) \simeq 1.3 \times 10^{-3} \ m_{Pl} \), which should be compared to the mean field value \( \sigma \simeq 0.49 \ m_{Pl} \). We conclude that the \( \sigma \) field is well approximated by its classical value at the end of the first stage of inflation. The fluctuations of the massive fields \( \phi_i \), \( i = 1, 2 \) generated by the first stage of inflation are given by

\[ (\Delta \phi_i)^2_k = \left( \frac{c H_i^2}{M_{\phi_i}} \right)^2, \] (3.6)

where \( M_{\phi_i} \) are their masses, given by eqs. (2.8), and \( c = O(10^{-1}) \). In the following sections we shall follow the evolution of these fluctuations during the intermediate stage between the two stages of inflation.

4 The intermediate stage

When the \( \chi \) field rolls beyond its instability point, large domains start appearing in which the value of the \( \phi^- \) field grows exponentially with time. For statistical systems, for which the expansion of the Universe is not relevant, this process is characterized as spinodal decomposition. The expansion of the Universe complicates the above picture, but the details are not important for our discussion. We assume that this initial stage of instability is fast and soon the fields take values away from the \( \chi \) axis and in the vicinity of the minimum at \( \chi = 0, \phi^- \simeq \sqrt{2\xi} \), where the curvature of the potential is positive. Our assumption is reasonable because the \( \sigma \) field rolls to the origin within a time \( \sim H^{-1} \simeq \sqrt{6}(g\xi/m_{Pl})^{-1} \) or slightly larger. On the other hand, the typical time scale for the growth of the \( \phi^- \) field is given by the absolute value of the curvature at the origin and is \( \sim (g\sqrt{\xi})^{-1} \). As a result, we expect that \( \phi^- \) grows to a value near the minimum within a fraction of a Hubble time. Notice that for \( \sigma \neq 0 \) the minimum is not located exactly at \( \chi = 0, \phi^- = \sqrt{2\xi} \) because of the supergravity corrections to the \( D \)-term of eq. (2.18).

After a short complicated evolution, the massive fields \( \chi, \phi^- \) are expected to settle into a regular oscillatory pattern around the minimum, with the Universe characterized by an equation of state \( p = w\rho \) with \( w \simeq 0 \). This can be verified by calculating the quantity \(-\dot{H}/H^2 = 3(1 + w)/2 \). Following ref. [25], we start with random initial values of \( \chi, \phi^- \) in the vicinity of the minimum and examine how fast the dust-dominated era is reached. In agreement with
ref. [25], we find that \( w \) starts from an initial value near \(-1\) and within less than one e-folding approaches zero. During the same time the total energy density of the Universe is reduced by a factor of about 3. We do not expect significant changes of the value of the \( \sigma \) field during this time. As \( w \) is close to \(-1\) initially, we expect that the typical fluctuations of \( \sigma \) are given by eq. (3.4), and are much smaller than its average value \( \sigma \simeq 0.49 \, m_{Pl} \) at the end of the first stage of inflation. We conclude that, when a regular oscillatory pattern of the \( \chi, \phi^- \) fields is established, the energy density is \( \rho_0 \simeq g^2 \xi^2 / 6 \) and the \( \sigma \) field has a value \( \sigma_0 \simeq 0.49 \, m_{Pl} \).

From this point on, the energy density of the oscillating fields is dissipated through the expansion of the Universe or their possible decay into lighter species. When the energy density becomes comparable to \( \mu^4 \), the second stage of inflation can begin. The stability of the \( \sigma \) field is crucial during this intermediate stage. The supergravity corrections of eqs. (2.17)–(2.21), that are proportional to powers of \( \sigma \), can generate a slope along the \( \sigma \) axis that may result in the fast rolling of \( \sigma \) to zero. The details depend sensitively on how efficient the transfer of energy from the fields \( \chi, \phi^- \) to light particles is.

4.1 Absence of decay channels

We consider first the possibility that the fields \( \chi, \phi^- \) do not have any decay channels. As a result, they perform damped oscillations around the minimum at \( \chi = 0, \phi^- \simeq \sqrt{2} \xi \), while their energy is dissipated through expansion. This generates an effective mass for the field \( \sigma \). The largest contributions to this mass come from the term proportional to \( q_{100} \) in the second line of eq. (2.17) and from the one proportional to \( a_{101} \) in the expansion of the \( D \)-term of eq. (2.18) around the minimum. For small values of \( \sigma/m_{Pl} \), like the ones we are considering, the effective \( \sigma \) mass has the general form

\[
\left[ m_{\sigma}^2 \right]_{\text{eff}} = f h \rho / m_{Pl}^2,
\]

where \( f \) is a constant of order 1 and \( h \) is equal to \( q_{100} \) or \( a_{101} \) (or a linear combination of them) depending on which contribution dominates the energy density \( \rho \) (or whether they are comparable). The equation of motion for the \( \sigma \) field can then be written as

\[
\ddot{\sigma} + 3H \dot{\sigma} = -f h \rho / m_{Pl} \sigma.
\]

The Hubble parameter is \( H = \sqrt{\rho / 3m_{Pl}^2} \), while the energy density satisfies the equation

\[
\frac{d\rho}{dt} = -\sqrt{3}(1 + w)\rho^{3/2} / m_{Pl}.
\]
The parameter $w$ determines the relation between energy density and the mean value of the pressure over one oscillation ($p = w \rho$). For a system of massive oscillating fields, such as the one we are considering, or a matter-dominated Universe, $w = 0$. For a system of massless oscillating fields with a quartic potential or a radiation-dominated Universe, $w = 1/3$. The evolution of $\sigma$ as a function of the energy density is determined by the Euler equation

$$\rho^2 \frac{d^2 \sigma}{d \rho^2} + \left( \frac{3}{2} - \frac{1}{1+w} \right) \rho \frac{d \sigma}{d \rho} + \frac{f h}{3(1+w)^2} \sigma = 0. \quad (4.4)$$

We assume that the $\sigma$ field is initially at rest. For $fh > 3(1-w)^2/16$, the solution of the above equation is

$$\sigma = \sigma_0 \left( \frac{\rho}{\rho_0} \right)^r \left\{ \cos \left[ s \ln \left( \frac{\rho}{\rho_0} \right) \right] - \frac{r}{s} \sin \left[ s \ln \left( \frac{\rho}{\rho_0} \right) \right] \right\}, \quad (4.5)$$

with

$$r = \frac{1-w}{4(1+w)}, \quad (4.6)$$

$$s = \sqrt{\frac{fh}{3(1+w)^2} - \frac{1-w}{4(1+w)^2}}. \quad (4.7)$$

For $fh = 3(1-w)^2/16$, the solution is

$$\sigma = \sigma_0 \left[ 1 - r \ln \left( \frac{\rho}{\rho_0} \right) \right] \left( \frac{\rho}{\rho_0} \right)^r, \quad (4.8)$$

with $r$ given by eq. (4.6). Finally, for $fh < 3(1-w)^2/16$, the solution is

$$\sigma = \sigma_0 \left[ - \frac{r_2}{r_1 - r_2} \left( \frac{\rho}{\rho_0} \right)^{r_1} + \frac{r_1}{r_1 - r_2} \left( \frac{\rho}{\rho_0} \right)^{r_2} \right], \quad (4.9)$$

with

$$r_{1,2} = \frac{1-w}{4(1+w)} \pm \sqrt{\left[ \frac{1-w}{4(1+w)} \right]^2 - \frac{fh}{3(1+w)^2}}. \quad (4.10)$$

During the intermediate stage between the two stages of inflation, the energy density drops by a factor $\rho/\rho_0 \simeq 6 \mu^4/g^2 \xi^2 \sim 10^{-7}$ for the parameters of our model. For $fh \gtrsim 3(1-w)^2/16$, the $\sigma$ field rolls close enough to the origin for the second stage of inflation not to take place. For $f |h| \ll 3(1-w)^2/16$, the final value of $\sigma$ is approximately given by the expression

$$\sigma = \sigma_0 \left( \frac{\rho}{\rho_0} \right)^{\frac{2fh}{3(1-w^2)}} \quad (4.11)$$

and is close to the original value $\sigma_0$. Therefore, this parameter range could support a second stage of inflation. The value $q_{100} = 1$, resulting from the minimal Kähler potential, is too large
to satisfy the above requirement. For $\sigma$ not to roll to the origin and at least 60 e-foldings to be produced during the second stage of inflation, one must assume that the non-minimal corrections fall within the range $|a_{101}|, |q_{100}| = |1 - c_{100}| \lesssim 0.1$. It is not easy to motivate a choice of the non-minimal term $c_{100}$ such that the biggest part of the minimal contribution to $q_{100}$ is cancelled. Another possibility is that the non-minimal corrections take values of order 1 and $a_{101}, q_{100} < 0$. This leads to the rapid growth of the $\sigma$ field during the intermediate stage. One must then rely on higher non-minimal terms in order to stabilize the value of $\sigma$ and keep it sufficiently below $m_{Pl}$ for the supergravity corrections to be under control.

4.2 Production of light particles

The most efficient way of keeping $\sigma$ constant during the intermediate stage is through the transformation of the energy of the $\chi, \phi^-$ fields into lighter particles. This is easily achieved if we introduce a light superfield $\Psi$, singlet under the MSSM and with charges $(0,0,-1/2,0)$ under the symmetries of subsection 2.1. In the globally supersymmetric limit, the only allowed new term in the renormalizable superpotential is $\nu X \Psi^2$. We assume that the coupling constant $\nu$ is of order 1. The new term is not expected to modify the first stage of inflation, because the scalar component of $\Psi$ has a zero expectation value.

In our model we have chosen couplings such that the masses of the $\phi_{1,2}^+, \phi^-, \chi$ particles near the minimum at $\chi = 0, \phi^- \simeq \sqrt{2\xi}$ satisfy $M_{\phi^-} = \sqrt{2g\sqrt{\xi}} > 2M_{\chi} = 2M_{\phi_1^+} = 2M_{\phi_2^+} = 2\lambda\sqrt{\xi}$. As a result, several channels exist for the transformation of the energy of the $\chi, \phi^-$ fields into lighter particles. Although this process may be accelerated by a period of preheating [25], we shall not consider this possibility here, as the standard decay rate is very efficient for our purposes. The various decay channels of the fields are mainly determined by the Lagrangian of the globally supersymmetric theory. The supergravity corrections are less efficient, as they are suppressed by powers of $m_{Pl}$. The $\phi^-$ field decays mainly into $\phi_{1,2}^+$ and $\chi$ scalar particles and their fermionic partners. Subsequently, the $\phi_{1,2}^+$ particles decay into light scalar $\Psi$ particles, the $\chi$ particles decay into fermionic $\Psi$, while the fermionic $X, \Phi^+$ decay into fermionic and scalar $\Psi$ particles. The $\chi$ field oscillates around a zero expectation value and decays only into fermionic $\Psi$ directly. The various decay rates have the general form

$$\Gamma = \frac{G}{16\pi}\sqrt{\xi}, \quad (4.12)$$

where $G$ is a function of the couplings $\lambda, g$ and $\nu$. For example, for the decay rate of the $\phi^-$
field into $\phi_1^+$ particles

$$G = \frac{(g^2 - \lambda^2)^2}{\sqrt{2} g} \sqrt{1 - \frac{2\lambda^2}{g^2}}. \quad (4.13)$$

The total energy density converted into light particles at the end of reheating is approximately given by

$$\rho_r \sim \frac{m_{\text{Pl}}^2}{\Gamma_{\text{eff}}} \frac{\sigma}{T^2 m_{\text{Pl}}^2}. \quad (4.14)$$

Here $\Gamma_{\text{eff}}$ is approximately equal to the total decay rate of the $\chi, \phi$ fields if they decay at comparable time scales, or equal to the smallest decay rate if the two fields decay at different time scales without efficient energy exchange during their oscillations. For our model we expect $\rho_r$ to be about one order of magnitude smaller than the energy density at the beginning of the oscillatory stage $\rho_0 \simeq \frac{g^2\xi^2}{6}$. The solution of eq. (4.5) implies that the value of the $\sigma$ field is reduced by an approximate factor of 2–3 during the decay of the oscillating fields.

Within the globally supersymmetric theory, the $\sigma$ field has no interaction with the gas of $\Psi$ particles whose energy density is dominant after the end of the oscillatory phase. However, supergravity introduces additional terms in the potential. The most important supergravity correction for our discussion arises when the term $\sim \nu^2 |\Psi|^4$ coming from the $F$-term is multiplied by a correction $\sim |S|^2 / m_{\text{Pl}}^2$ originating in the Kähler potential. In analogy to the corrections of eq. (2.17), the resulting term has the form $\nu^2 |\Psi|^4 t_{100} |S|^2 / m_{\text{Pl}}^2$, with $t_{100}$ a new parameter of order 1. There are also interaction terms between the $\sigma$-field and the fermionic $\Psi$ particles. However, they involve also the heavy $\chi$ particles or their fermionic partners which are not expected to be abundant. For this reason we conclude that the $\sigma$ field interacts mainly with the gas of the scalar $\Psi$ particles.

The scalar $\Psi$ particles have a large self-interaction rate and their gas is expected to reach thermal equilibrium quickly. However, the interactions of the fermionic $\Psi$ involve additional heavy $\chi$ particles and their superpartners, and thermalization is more difficult for them. The interaction of the $\sigma$ field with the thermal bath of the $\Psi$ particles is established through the term $\nu^2 |\Psi|^4 t_{100} |S|^2 / m_{\text{Pl}}^2$. The largest contribution to the effective mass of $\sigma$ is obtained from this term if the $\Psi$ legs are contracted in pairs. The thermal part of the resulting two-loop graph gives

$$\left[m_\sigma^2\right]_{\text{eff}} \sim \nu^2 t_{100} \left(\frac{T^2}{12}\right)^2 \frac{1}{m_{\text{Pl}}^2}. \quad (4.15)$$

The temperature $T$ can be expressed in terms of the energy density. If both the scalar and
fermionic $\Psi$ particles were in thermal equilibrium the total energy density would be

$$\rho = g_* \pi^2 T^4,$$

with an effective number of degrees of freedom $g_* = 2 + (7/8)2 = 3.75$. If the fermionic $\Psi$ do not thermalize the total energy density is

$$\rho > g_* \pi^2 T^4,$$

with $g_* = 2$. In any case, the effective mass of the $\sigma$ field is given by eq. (4.11), where $k = \nu^2 t_{100}$ is of order 1 and $f \lesssim 10^{-2}$. The evolution of the $\sigma$ field during the radiation-dominated phase is given by eq. (4.11) with $w = 1/3$ and $\rho_0$ equal to the energy density at the end of the oscillatory phase $\rho_r$. The change of $\sigma$ by the end of the intermediate stage when $\rho \sim \mu^4$ is expected to be less than 10% of its value at the end of the oscillatory phase.

We conclude that, within the scenario that permits the decay of the $\chi$, $\phi^-$ fields into light particles, the value of the $\sigma$ field is reduced by an approximate factor of 2–3 during the whole intermediate stage. As the $\sigma$ field does not roll to zero during this stage, there is no need for an “unnatural” choice of the non-minimal terms in the Kähler potential as in the previous subsection. In section 3, we computed a value $\sigma \simeq 0.49 \ m_{Pl}$ at the end of the first stage of inflation. We shall use an initial value $\sigma \simeq 0.2 \ m_{Pl}$ for the study of the second stage of inflation in the following section.

5 The second stage of inflation

At the end of the intermediate stage, the fields $\sigma, \phi_{1,2}$ are located on the flat direction of the potential where $V = \mu^4$. When the energy density falls below $\mu^4$, the second stage of inflation can begin. Due to the rapid expansion during the first stage of inflation, the homogeneous regions extend far beyond the typical Hubble length $H_2^{-1}$ of the second stage. However, the size of the fluctuations of the $\sigma, \phi_{1,2}$ fields that were generated during the first stage must be calculated with care, as they can prevent the onset of inflation.

The fluctuations of the massless field $\sigma$ that cross inside the horizon during the intermediate stage start propagating as massless particles and their energy density drops $\sim R^{-4}$. As a result, we do not expect a significant contribution to the total energy density from them.

The fluctuations of the massive $\phi_{1,2}$ fields, that were generated by the first stage of inflation, are given by eq. (3.6). During the intermediate stage, the amplitude of these fluctuations drops
as $R^{-3/2}$. As a result we expect that, when the energy density becomes comparable to $\mu^4$, this amplitude is approximately given by

$$\langle \Delta \phi_i \rangle_k \simeq c \frac{H_i^2}{M_{\phi_i}} \left( \frac{H_2}{H_1} \right)^{-\frac{1}{1+w}},$$  \hspace{1cm} (5.1)$$

with $i = 1, 2$. Here $H_1 \simeq 2.0 \times 10^{-3} \, m_{Pl}$ and $H_2 \simeq 5.8 \times 10^{-7} \, m_{Pl}$ are the values of the Hubble parameter during the first and second stage of inflation respectively, $w$ is determined by the equation of state during the intermediate stage and varies between 0 and 1/3, $M_{\phi_i}$ are given by eqs. (2.8), and $c = \mathcal{O}(10^{-1})$.

It was shown in ref. \cite{1} that for $\mu \lesssim 10^{-1} m_{Pl}$ the most favourable area of inflationary initial conditions is a thin strip around the $\sigma$ axis. If the fields start in this area without initial time derivatives $\sigma$ does not oscillate around zero, but quickly settles along the flat direction. We can obtain a rough estimate of the width of this area if we consider the equation of motion of the $\sigma$ field and replace $\phi_1^2$ and $\phi_2^2$ by their average values $\langle \phi_{1,2}^2 \rangle \sim [\phi_{1,2}]_0^2 / 2$ during the evolution ($[\phi_{1,2}]_0$ are the amplitudes of $\phi_{1,2}$). The equation reads

$$\ddot{\sigma} + 3H \dot{\sigma} = -\frac{\kappa^2}{4} \left( [\phi_1]^2_0 + [\phi_2]^2_0 \right) \sigma. \hspace{1cm} (5.2)$$

Two time scales characterize the solutions of this equation. The first one is related to the “friction” term and is given by $t_H^{-1} = 3H/2$. For $[\phi_{1,2}]_0 \ll 2\mu / \sqrt{\kappa}$ and $\kappa^2 \sigma^2 \left( [\phi_1]^2_0 + [\phi_2]^2_0 \right) / 8 \ll \mu^4$, we have

$$t_H = \frac{2 \, m_{Pl}}{\sqrt{3} \, \mu^2}. \hspace{1cm} (5.3)$$

The other time scale is obtained if we neglect the “friction” term and consider the oscillations of the $\sigma$ field. One-fourth of the period is the typical time for the system to roll to the origin and away from an inflationary solution. It is given by

$$t_{osc} = \frac{\pi}{\kappa} \frac{1}{\sqrt{[\phi_1]^2_0 + [\phi_2]^2_0}}. \hspace{1cm} (5.4)$$

Inflation sets in if $t_{osc} \gtrsim t_H$, which gives

$$\sqrt{[\phi_1]^2_0 + [\phi_2]^2_0} \lesssim \frac{\sqrt{3} \, \pi}{2 \, \kappa} \left( \frac{\mu}{m_{Pl}} \right)^2.$$  \hspace{1cm} (5.5)$$

We have verified numerically that the above relation gives the correct order of magnitude for the size of the strip around the $\sigma$ axis that leads to inflationary solutions. Our assumptions for the derivation of the above bound break down when $\sigma \gtrsim \sqrt{32/3 \pi^2} m_{Pl} \simeq m_{Pl}$.
We can approximate the initial amplitudes $[\phi_i]_0$ by the typical scale-invariant fluctuations at the end of the intermediate stage, given by eq. (5.1). For the parameters of our model, the bound of eq. (5.3) is comfortably satisfied. As a result, the $\sigma$ field is expected to settle quickly on a slow-roll trajectory. We conclude that the necessary conditions for the onset of the second stage of inflation are “naturally” satisfied in our model.

During the second stage of inflation, the main contributions to the slope along the $\sigma$ direction come from the term $\mu^4 p_{100} \sigma^2/2m_{Pl}^2$ in eq. (2.17) and the radiative correction of eq. (2.11). Higher-order corrections are suppressed by powers of $\sigma^2/2m_{Pl}^2 \lesssim 2 \times 10^{-2}$ and $\xi/m_{Pl}^2 = 10^{-2}$. The existence of a slow-roll solution requires $p_{100}$ to be smaller than 1. According to eqs. (2.19) this translates into a constraint for the parameter $a_{200}$ of the non-minimal Kähler potential, which must be negative and $|a_{200}| \lesssim 10^{-1}$. However, if one demands that the slow-roll solution is driven by the radiative contribution of eq. (2.11) one must choose a much smaller value. The values of the other parameters are not constrained for sufficiently small $\sigma/m_{Pl}$. As in our scenario $\sigma/m_{Pl} \lesssim 0.2$ during the second stage of inflation, we assume that they are of order 1 in general. However, for scenarios of inflation with field values closer to $m_{Pl}$ a significant fine-tuning of an infinite number of parameters is required for the flatness condition to be maintained by the supergravity corrections. For example, a small value of $p_{200}$ in eqs. (2.19) can be obtained only if a careful cancellation is arranged involving the factor $1/2$ arising from the minimal Kähler potential and the non-minimal parameters $a_{200}, a_{300}$.

Because of the supergravity corrections to the $D$-term of eq. (2.18), its minimum is a function of the $\sigma$ field. However, no significant contributions to the slope along the inflationary trajectory are generated by this term. The reason is that such contributions are proportional to the value of the $D$-term, which is very rapidly adjusted to zero through a change of the expectation value of $\phi^-$ as $\sigma$ slowly rolls down the $\sigma$ axis. We have verified numerically this very rapid adjustment of the $D$-term to zero.

The second stage of inflation terminates when $\sigma$ rolls below either the instability point of eq. (2.9), or the value $\sigma_r = \kappa m_{Pl}/\sqrt{8\pi^2}$ for which the logarithmic contribution to the potential destroys the slow-roll solution. The number of $e$-foldings during the second stage of inflation is

$$N_2 = \frac{1}{2p_{100}} \ln \left( \frac{\sigma_0^2}{m_{Pl}^2} + \frac{\kappa^2}{8\pi^2} \frac{1}{p_{100}} \right) \left( \frac{\sigma_f^2}{m_{Pl}^2} + \frac{\kappa^2}{8\pi^2} \frac{1}{p_{100}} \right),$$

with $\sigma_0$ the value of $\sigma$ at the beginning of this stage and $\sigma_f$ the largest of $\sigma_{ins}, \sigma_r$. For the
The parameter range of interest to us, the above expression can be approximated by

$$N_2 \simeq \frac{1}{p_{100}} \ln \left( \frac{2 \pi \kappa}{2 \sigma_0 m_{Pl}} \sqrt{2p_{100}} \right). \quad (5.7)$$

The most stringent constraint on $p_{100}$ results from the need to generate at least 60 e-foldings during the second stage of inflation. If this were not the case, the predicted spectrum of adiabatic density perturbations would depend on the parameters of the first stage of inflation and would be incompatible with the COBE measurements of the cosmic microwave background anisotropy. For $\sigma_0$ of order $m_{Pl}$ and taking $\kappa$ an order of magnitude smaller than $\sigma_0/m_{Pl}$, we find $p_{100} \lesssim 0.05$. This results in the constraint $|a_{200}| \lesssim 10^{-2}$, consistently with the results of ref. [7]. (Our parameter $a_{200}$ corresponds to $-\beta/4$ of ref. [7].)

The spectrum of the adiabatic density perturbations generated by this stage of inflation is

$$\delta_H^2 \simeq \frac{1}{75 \pi^2} \frac{\mu^4}{(p_{100})^2 m_{Pl}^2 \sigma_{60}^2}, \quad (5.8)$$

where $\sigma_{60}$ is the value of the $\sigma$ field at which our present horizon scale crossed outside the inflationary horizon. This value corresponds to $N_Q \simeq 60$ e-foldings. Comparison with the value $\delta_H = 1.94 \times 10^{-5}$, deduced from the COBE observation of the cosmic microwave background anisotropy, leads to the constraint

$$\frac{\mu}{m_{Pl}} \simeq 7.7 \times 10^{-3} \kappa^{1/2} (p_{100})^{1/4} e^{30p_{100}}. \quad (5.9)$$

In our model, we have taken $\kappa = 0.04$, $\mu/m_{Pl} = 10^{-3}$ and obtained $\sigma_0/m_{Pl} \simeq 0.2$ at the beginning of the second stage of inflation. For the choice $p_{100} = 0.03$, $a_{200} = -7.5 \times 10^{-3}$, we find $N_2 = 68$ e-foldings and obtain approximate agreement with the COBE observations.

6 Conclusions

In this paper, we addressed the problem of fine-tuning of the initial conditions for hybrid inflation in the context of supergravity. This problem is generated by the difference between the energy scale at which the Universe emerges from the Planck era (near $m_{Pl}$) and the inflationary scale implied by the COBE observations ($V^{1/4} \sim 10^{-3} m_{Pl}$). A simple resolution of this issue of fine-tuning can be obtained in a scenario with two stages of inflation at two different energy scales [21]. The first stage has a typical energy scale not far from $m_{Pl}$. As a result, the Hubble parameter stays large until the fields settle down along the direction that produces inflation. Due to the large “friction” term proportional to $H$ in the equations of motion, the initial part of
the evolution towards the inflationary trajectory is short and the first stage of inflation occurs “naturally”. After the end of this stage a subset of the fields moves towards the minimum of the potential around which it performs damped oscillations. During this intermediate stage, the energy density is reduced through expansion. A second stage of inflation begins when the energy density falls below the false vacuum energy density associated with a second-order phase transition involving the remaining fields. The homogeneity far beyond the Hubble length, that was produced during the first inflationary stage, makes the onset of the second stage “natural”, despite the fact that its Hubble length is much larger than the one of the first stage. The second stage of inflation generates the density perturbations that result in the cosmic microwave background anisotropy observed by COBE.

The realization of the above scenario in the context of supergravity must overcome two main obstacles. Firstly, it is difficult to preserve flat directions of the potential in supergravity. Typically, fields that are massless at the globally supersymmetric level develop masses of the order of the Hubble parameter that prevent the onset of inflation. A notable exception is a $D$-flat direction, which is typically preserved in supergravity [1]. Another exception is an $F$-flat direction in models with a linear superpotential of the form $W = -\mu^2 S$. For a minimal Kähler potential, this flat direction is not lifted in the context of supergravity [3]. In this paper, we considered a scenario with two stages of inflation, the first driven by $D$-term energy density and the second by $F$-term energy density resulting from a superpotential of the form $W = -\mu^2 S$. One would expect that a scenario with two stages of $D$-term inflation could pose fewer technical problems. However, the Fayet-Iliopoulos term that provides the $D$-term energy density can be naturally generated by invoking an “anomalous” $U(1)$ symmetry in the effective theory resulting from string theory [21, 22]. Such a term is expected to have a scale not far from $m_{Pl}$.

We calculated the potential in our model, allowing for all possible non-renormalizable terms in the superpotential and considering the most general form of the Kähler potential and the gauge kinetic function. We found that the potential depends only on the non-minimal terms of the Kähler potential. We also checked that these terms generate corrections to the kinetic part of the Lagrangian that do not modify our scenario and can be neglected. For the supergravity corrections not to destroy the flat directions of the globally supersymmetric model and inflation to set in, the parameters of the Kähler potential must be chosen appropriately. If inflation driven by $F$-term energy density takes place for field values near $m_{Pl}$, an infinitely large number of such parameters must be tuned in order for the flatness conditions to be satisfied. However,
in our scenario we arranged for inflation to take place for sufficiently small field values, so that only one condition is necessary: The coefficient $a_{200}$ of the $|S|^4$ term in the Kähler potential, where $S$ is the inflaton of the second stage, must be chosen negative and $|a_{200}| \lesssim 10^{-1}$.

The second difficulty faced by two-stage inflation is to keep the inflaton $S$ of the second stage essentially constant at a value with non-zero $F$-term energy density during the intermediate stage. An effective mass for $S$ is generated through its couplings in the Kähler potential with the set of fields that after the first stage of inflation move to a minimum of the potential and oscillate around it. We found that, if these fields have no decay channels into lighter particles, some couplings must be carefully adjusted to values that are difficult to justify. However, if the set of oscillating fields can decay into light species, these constraints are not necessary. The reason is that $S$ has a weak coupling to the generated gas of particles, and its effective mass is small enough for it to remain on the flat direction that leads to the second stage of inflation.

The most stringent constraint on the parameters of the model results from the need to generate at least 60 e-foldings during the second stage of inflation. If this were not the case, the predicted spectrum of adiabatic density perturbations would depend on the parameters of the first stage of inflation and would be incompatible with the COBE measurements of the cosmic microwave background anisotropy. As we arranged for inflation to take place for sufficiently small field values, only one condition is necessary. The coefficient $a_{200}$ of the non-minimal $|S|^4$ term in the Kähler potential that contributes to the slope along the inflationary trajectory of the second stage must be negative with $|a_{200}| \lesssim 10^{-2}$. This range of values is narrower by an order of magnitude than the range that satisfies the slow-roll conditions for inflation. It can be justified within an effective field theory, if we view the non-minimal terms as higher-order corrections to the minimal form of the Kähler potential.

We conclude that a two-stage inflationary scenario can be realized for carefully chosen models in the context of supergravity, without the need to fine-tune an infinite number of parameters. In the model we presented, only one parameter must be mildly tuned. Within a two-stage inflationary scenario the problem of initial conditions of hybrid inflation can be resolved.

**Acknowledgements:** We would like to thank G. Dvali for suggesting the possibility of a $D$- and $F$-term inflation and for many useful discussions. We would also like to thank Z. Berezhiani and D. Comelli for useful discussions. This research was supported by the E.C. under TMR contract No. ERBFMRX–CT96–0090.
References

[1] A.D. Linde, Phys. Lett. B 259, 38 (1991); Phys. Rev. D 49, 748 (1994).

[2] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart and D. Wands, Phys. Rev. D 49, 6410 (1994).

[3] G. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. 73, 1886 (1994).

[4] E.D. Stewart, Phys. Rev. D 51, 6847 (1995).

[5] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D 52, 559 (1995).

[6] J.A. Casas and C. Munoz, Phys. Lett. B 216, 37 (1989); J.A. Casas, J.M. Moreno, C. Munoz and M. Quiros, Nucl. Phys. B 328, 272 (1989); P. Binétruy and G. Dvali, Phys. Lett. B 388, 241 (1996); E. Halyo, Phys. Lett. B 387, 43 (1996).

[7] C. Panagiotakopoulos, Phys. Rev. D 55, 7335 (1997); Phys. Lett. B 402, 257 (1997).

[8] A.D. Linde and A. Riotto, Phys. Rev. D 56, 1841 (1997).

[9] J.A. Casas and G.B. Gelmini, Phys. Lett. B 410, 36 (1997).

[10] J.A. Adams, G.G. Ross and S. Sarkar, Phys. Lett. B 391, 271 (1997); Nucl. Phys. B 503, 405 (1997).

[11] S. Dimopoulos, G. Dvali and R. Rattazzi, Phys. Lett. B 410, 119 (1997).

[12] G. Lazarides, C. Panagiotakopoulos and N.D. Vlachos, Phys. Rev. D 54, 1369 (1996).

[13] G. Lazarides and N.D. Vlachos, Phys. Rev. D 56, 4562 (1997).

[14] N. Tetradis, preprint CERN-TH/97-163, astro-ph/9707214.

[15] A.A. Starobinsky, Phys. Lett. B 117, 175 (1982).

[16] S.W. Hawking, Phys. Lett. B 115, 295 (1982); A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49, 1110 (1982); D.H. Lyth, Phys. Lett. B 147, 403 (1984); ibid. 150, 465 (1985); Phys. Rev. D 31, 1792 (1985).

[17] A.R. Liddle and D.H. Lyth, Phys. Rep. 231, 1 (1993).

[18] D.H. Lyth, preprint LANCS-TH/9614, hep-ph/9609431.

[19] D.S. Goldwirth and T. Piran, Phys. Rep. 214, 223 (1992).

[20] C. Panagiotakopoulos and N. Tetradis, preprint CERN-TH/97-301, hep-ph/9710526.

[21] E. Witten, Nucl. Phys. B 188, 513 (1981).
[22] J. Atick, L. Dixon and A. Sen, Nucl. Phys. 292, 109 (1987); M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. 293, 253 (1987).

[23] E. Cremmer et al., Phys. Lett. B 79, 231 (1978); Nucl. Phys. B 147, 105 (1979); E. Witten and J. Bagger, Phys. Lett. B 115, 202 (1982); E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Phys. Lett. B 116, 231 (1982); Nucl. Phys. B 212, 413 (1983); J. Bagger, Nucl. Phys. B 211, 302 (1983).

[24] T.S. Bunch and P.C.W. Davies, Proc. R. Soc. London A 360, 117 (1978); A. Vilenkin and L. Ford, Phys. Rev. D 26, 1231 (1982); A.D. Linde, Phys. Lett. B 116, 335 (1982); A. Vilenkin, Nucl. Phys. B 226, 527 (1983).

[25] J. Garcia-Bellido and A. Linde, preprint CERN-TH/97-329, hep-ph/9711360.

[26] E. Kolb and M. Turner, The Early Universe, Addison-Wesley, Redwood City, California (1990).
Fig. 1: The evolution of $\chi$, $\phi_1^+$, $\phi_2^+$, $\phi^-$ and the Hubble parameter $H$ for a theory described by the tree-level potential of eq. (2.4) with $g = 0.5$, $\xi/m_{Pl}^2 = 10^{-2}$, $\lambda = 0.3$, $\mu/m_{Pl} = 10^{-3}$, $\kappa = 0.04$. The initial values of the fields have been chosen as $\chi/m_{Pl} = 1.1$, $\phi_1^+/m_{Pl} = 0.04$, $\phi_2^+/m_{Pl} = 0.03$, $\phi^-/m_{Pl} = 0.04$, $\sigma/m_{Pl} = 1.2$, $\phi_1/m_{Pl} = 0.4$, $\phi_2/m_{Pl} = 0.2$. Dimensionful quantities are given in units of $m_{Pl}$. 
Fig. 2: Same as in fig. 1 for $\sigma$, $\phi_1$, $\phi_2$. 