Parity-violating asymmetries in elastic $\vec{e}p$ scattering in the chiral quark-soliton model: Comparison with A4, G0, HAPPEX and SAMPLE.

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Abstract

We investigate parity-violating electroweak asymmetries in the elastic scattering of polarized electrons off protons within the framework of the chiral quark-soliton model ($\chi$QSM). We use as input the former results of the electromagnetic and strange form factors and newly calculated SU(3) axial-vector form factors, all evaluated with the same set of four parameters adjusted several years ago to general mesonic and baryonic properties. Based on this scheme, which yields positive electric and magnetic strange form factors with a $\mu_s = (0.08 - 0.13)\mu_N$, we determine the parity-violating asymmetries of elastic polarized electron-proton scattering. The results are in a good agreement with the data of the A4, HAPPEX, and SAMPLE experiments and reproduce the full $Q^2$-range of the G0-data. We also predict the parity-violating asymmetries for the backward G0 experiment.

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1. The complex structure of the nucleon goes well beyond its simplest description as a collection of three valence quarks moving in some potential. The sea of gluons and $q\bar{q}$-pairs that arises in quantum chromodynamics is expected to play an important role even at long distance scales. As the lightest explicitly non-valence quark the strange quark provides an attractive tool to probe the $q\bar{q}$-sea, since any strange quark contribution to an observable must be the effect of the sea. Thus the strange quark contribution to the distributions of charge and magnetization in the nucleon has been a very important issue well over decades, since it provides a vital clue in understanding the structure of the nucleon. For recent reviews, see, for example, Refs. [1, 2, 3, 4, 5]. Recently, the strangeness content of the nucleon has been studied particularly intensively since parity-violating electron scattering (PVES) has demonstrated to provide an essential tool for probing the sea of $s\bar{s}$ pairs in the vector channel [6, 7]. In fact, various PVES experiments have been already conducted in order to measure the parity-violating asymmetries (PVAs) from which the strange vector form factors can be extracted [8, 9, 10, 11, 12, 13, 14, 15, 16]. While PVES experiments have direct access to the PVA with relatively good precision, a certain amount of uncertainties arise in the flavor decomposition for the nucleon vector form factors. As a result, the strange vector form factors extracted so far from the data have rather large errors [8, 9, 10, 11, 12, 13, 14, 15].

The chiral quark-soliton model ($\chi$QSM) is an effective quark theory of the instanton-degrees of freedom of the QCD vacuum. It results in an effective chiral action for valence and sea quarks both moving in a static self-consistent Goldstone background field [17, 18] originating from the spontaneous chiral symmetry breaking of the QCD. It has successfully been applied to mass splittings of hyperons, to electromagnetic and axial-vector form factors [17] of the baryon octet and decuplet and to forward and generalized parton distributions [19, 20, 21] and has led even to the prediction of the heavily discussed pentaquark baryon $\Theta^+$ [22]. The present authors have recently investigated in the $\chi$QSM model the strange vector form factors [23, 24] and they presented some aspects of the SAMPLE, HAPPEX, and A4 experiments. The results have shown a good agreement with the available data, though the experimental uncertainties are rather large, as mentioned above. Thus, it is theoretically more challenging to calculate directly the PVAs and to confront them with the more accurate experimental data. Moreover, since the G0 experiment has measured the PVA over a range of momentum transfers $0.12 \leq Q^2 \leq 1.0$ GeV$^2$ in the forward direction [16], the check of the theory is on much firmer ground.

Actually, the PVA contains a set of six electromagnetic form factors ($G_{E,M}^{u,d,s}$) and three axial-vector ones ($G_{A}^{u,d,s}$). In fact, all these form factors have already been calculated within the SU(3)-$\chi$QSM [23, 24, 25, 26] by using the well established parameter set consisting of $m_s = 180$ MeV and the other three parameters having been adjusted some years ago to the physical values of $f_\pi$, $m_\pi$ and baryonic properties as e.g. the charge radius of the proton and the delta-nucleon ($\Delta - N$) mass splitting. Apart from reproducing the existing experimental data on the PVAs, we will predict the PVAs of the future G0 experiment at backward angles.

2. The PVA in polarized $\vec{e}p$ scattering is defined as the difference of the total cross sections for circularly polarized electrons with positive and negative helicities divided by their sum:

$$A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}. \quad (1)$$

Denoting, at the tree level, the amplitudes for $\gamma$ and $Z$ exchange by $M_\gamma$ and $M_Z$, respec-
FIG. 1: The electroweak neutral axial-vector form factors $G_A^e$ and $G_A^{pZ}$ as functions of $Q^2$ calculated in the $\chi$QSM.

respectively, the total cross section for a given polarization is proportional to the square of the sum of the amplitudes, which indicates the interference between the electromagnetic and neutral weak amplitudes:

$$\sigma_\pm \sim |M^\gamma + M^Z|^2.$$ (2)

The PVA comprises three different terms:

$$A_{PV} = A_V + A_s + A_A,$$ (3)

where

$$A_V = -a\rho' \left[ (1 - 4\kappa' \sin^2 \theta_W) - \frac{\varepsilon G_E^p G_E^p + \tau G_M^p G_M^p}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2} \right],$$

$$A_s = a\rho' \left[ \frac{\varepsilon G_E^s G_E^s + \tau G_M^s G_M^s}{\varepsilon (G_E^s)^2 + \tau (G_M^s)^2} \right],$$

$$A_A = a \frac{\varepsilon (1 - 4\sin^2 \theta_W)\varepsilon' G_M^p G_A^p}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$a = G_F Q^2 / \left( 4\sqrt{2}\pi\alpha_{EM} \right),$$

$$\tau = Q^2 / (4M_N^2),$$

$$\varepsilon = \left[ 1 + 2(1 + \tau) \tan^2 \theta/2 \right]^{-1},$$

$$\varepsilon' = \sqrt{\tau(1 + \tau)(1 - \varepsilon^2)}.\quad (4)$$

The $G_{E,M}^p$, $G_{E,M}^s$, and $G_A^p$ denote, respectively, the electromagnetic form factors of the proton, strange vector form factors, and the axial-vector form factors. The $G_F$ is the Fermi constant as measured from muon decay, $\alpha_{EM}$ the fine structure constant, and $\theta_W$ the electroweak mixing angle given as $\sin^2 \theta_W = 0.2312$ [27]. The $Q^2$ stands for the negative square of the four momentum transfer. The parameters $\rho'$ and $\kappa'$ are related to electroweak radiative corrections [1, 28].
FIG. 2: The parity-violating asymmetries as a function of $Q^2$, compared with the SAMPLE measurement \[9\]. The dotted curve is calculated without the s-quark contribution. The dashed curve is obtained by using the form factors from the $\chi$QSM without the electroweak radiative corrections, while the solid one ($\chi$QSM) includes them and is our final result.

FIG. 3: The parity-violating asymmetries as a function of $Q^2$, compared with the A4 measurement \[12\]. The dotted curve is calculated without the s-quark contribution. The dashed curve is obtained by using the form factors from the $\chi$QSM without the electroweak radiative corrections, while the solid one ($\chi$QSM) includes them and is our final result.

Factoring out the quark charges, we can express the electromagnetic and electroweak neutral axial-vector form factors of the proton in terms of the flavor-decomposed electromagnetic form factors:

$$G_{E,M}^p = \frac{2}{3} G_{E,M}^u - \frac{1}{3} \left( G_{E,M}^d + G_{E,M}^s \right),$$
$$G_{A}^{pZ} = G_A^d - (G_A^u + G_A^s).$$

(5)
FIG. 4: The parity-violating asymmetries as a function of $Q^2$, compared with the HAPPEX measurement [11]. The dotted curve is calculated without the s-quark contribution. The dashed curve is obtained by using the form factors from the $\chi$QSM without the electroweak radiative corrections, while the solid one ($\chi$QSM) includes them and is our final result.

Including the electroweak radiative corrections [1, 28], we find that the electroweak axial-vector form factors of the proton can be written as [30]:

$$G_A^p(Q^2) = -(1 + R_A^1)G_A^{(3)}(Q^2) + R_A^0 + G_A^s,$$

with the values for the electroweak radiative corrections [28]:

$$R_A^1 = -0.41 \pm 0.24, \quad R_A^0 = 0.06 \pm 0.14.$$

Figure 1 depicts the electroweak neutral axial-vector form factors expressed in Eqs. (5, 6), which is obtained in the $\chi$QSM [26]. We will use $G_A^{pZ}$ in Fig. 1 to yield the PVA.

The other six electromagnetic form factors, $G_{E,M}^{p,n,s}$ can be read out from Refs. [23, 24, 25].

3. We discuss now the results of the PVA obtained from the $\chi$QSM. In detail, the model has the following parameters: The constituent quark mass $M$, the current quark mass $m_u$, the cut-off $\Lambda$ of the proper-time regularization, and the strange quark mass $m_s$. However, these parameters are not free but has been fixed to independent observables in a very clear way [17]: For a given $M$ the $\Lambda$ and the $m_u$ are adjusted in the mesonic sector to the physical pion mass $m_\pi = 139$ MeV and the pion decay constant $f_\pi = 93$ MeV. The strange quark mass is selected to be $m_s = 180$ MeV throughout the present work, with which the mass splittings of hyperons are produced very well. The remaining parameter $M$ is varied from 400 MeV to 450 MeV. However, the value of 420 MeV, which for many years is known to produce the best fit to many baryonic observables [17], is chosen for our final result in the baryonic sector. We always assume isospin symmetry. With these parameters at hand, we can proceed to derive the form factors of the proton required for the PVA. On obtaining these form factors, we use the symmetry conserving quantization scheme [29] and take into account the rotational $1/N_c$ corrections, the explicit SU(3) symmetry breaking in linear
order, and the wave function corrections, as discussed in Ref. [17, 23] in detail. With this scheme, we have obtained the results [23, 24] for the strange vector form factors in good agreement with the data of the A4, SAMPLE and HAPPEX experiments as far as they were available.

![Graph showing parity-violating asymmetries as a function of Q^2](image)

**FIG. 5:** The parity-violating asymmetries as a function of Q^2, compared with the forward G0 measurement [16]. The dotted curve is calculated without the s-quark contribution. The dashed curve is obtained by using the form factors from the χQSM without the electroweak radiative corrections, while the solid one (χQSM) includes them and is our final result.

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1 The value of the strange electric form factor at Q^2 = 0.091 GeV^2 is newly extracted by the HAPPEX experiment [14]: G_E^s = (-0.038 ± 0.042 ± 0.010) n.m. which is consistent with zero. The G0 experiment indicates that G_E^s may be negative in the intermediate region up to Q^2 ∼ 0.3 GeV^2. The present model predicts G_E^s ≃ 0.025 at Q^2 = 0.091 GeV^2 which is positive and slightly outside the error margins of HAPPEX.
FIG. 6: The parity-violating asymmetries as a function of $Q^2$. They are the predictions for the backward G0 experiment ($\theta = 108$). The dotted curve is calculated without the s-quark contribution. The dashed curve is obtained by using the form factors from the $\chi$QSM without the electroweak radiative corrections, while the solid one ($\chi$QSM) includes them and is our final result.

FIG. 7: The values of $G_E^s(Q^2) + \beta(Q^2, \theta)G_M^s(Q^2)$ as a function of $Q^2$. The dotted fields are the $\chi$QSM-predictions for the A4 experiment at $\theta = 35$ and $\theta = 145$. The theoretical error fields are given by assuming the Yukawa mass of the solitonic profile in the $\chi$QSM to coincide with the pion mass or the kaon mass, respectively.

FIG. 8: The values of $G_E^s(Q^2) + \eta G_M^s(Q^2)$ with $\eta = 0.94Q^2$ as a function of $Q^2$. They are the predictions for the G0 experiment at $\theta = 10$. The theoretical error field is given by assuming the Yukawa mass of the solitonic profile in the $\chi$QSM to coincide with the pion mass or the kaon mass, respectively.
We present our numerical results in Figs. 2-6 at relevant kinematics to the A4, G0, HAPPEX, and SAMPLE experiments in comparison with the data. The dotted curves depict the PVA without the strange quark contribution. This means we put $A_s = 0$ in Eq. 3. The dashed ones are obtained by using the form factors from the SU(3)-$\chi$QSM without the electroweak radiative corrections, i.e. with $\rho'$ and $\kappa'$ set equal to zero, while the solid ones ($\chi$QSM) are our final theoretical asymmetries including those corrections. One notices that the effect of the electroweak radiative corrections is rather tiny. One also notices that with increasing $Q^2$ the PVA without strange contribution deviates more and more from the experiments, which means that with increasing $Q^2$ the contribution of the strange quarks gets larger and larger reaching in the end an amount up to 40% in the present model.

As shown in Figs. 2-5, the present results are in a good agreement with the experimental data from A4, HAPPEX, and SAMPLE at small and intermediate $Q^2$. However, since the G0 experiments have measured the PVA over the range of momentum transfers $0.12 \leq Q^2 \leq 1.0 \text{GeV}^2$, it is more interesting to compare our results with them. In fact, the predicted PVA in the present work describes remarkably well the G0 data over the full range of $Q^2$-values. It indicates that the present model produces the correct $Q^2$-dependence of all the form factors relevant for the PVA.

Figure 6 depicts the prediction for the backward G0 experiment at $\theta = 108^\circ$ whose data are announced to be available in near future.

4. The Figures 7-9 yield further data which allow a detailed comparison between experiment and theory. Fig. 7 shows the typical combination $G_E^a(Q^2) + \beta(Q^2, \theta)G_M^a(Q^2)$ playing a key role in the experiments. In forward direction A4 has measured two points of this observable at small $Q^2$-values, which are both well reproduced by the $\chi$QSM calculations. The dotted error band indicates a systematic error of the $\chi$QSM, since the soliton is bound
FIG. 10: The world data for $G_E^s$ and $G_M^s$ from A4, HAPPEX, SAMPLE and G0 experiments at $Q^2 = 0.1\text{ GeV}^2$. The plot is taken from HAPPEX \cite{15} and the ellipse reflects the 95\% confidence level. The theoretical number obtained by the $\chi$QSM is indicated by a cross which reflects the theoretical errors. The dots indicate the center of the ellipse and the point with vanishing strange form factors.

FIG. 11: The hydrogen and deuterium data for $G_M^s$ and $G_A^s(T=1)$ from HAPPEX at $Q^2 = 0.1\text{ GeV}^2$. The ellipse represents the 1\% overlap of the two measurements. The theoretical number obtained by the $\chi$QSM is indicated with the bar which reflects the theoretical error. The data-plot is taken from Ref. \cite{31} to have the same profile function in the up-, down- and strange direction, see ref. \cite{23} for details. Fig. \ref{fig:8} shows a similar combination for G0, where the $\beta$ is assumed to be equal to $\eta = 0.94\, Q^2$. In this plot the experimental data are again resonably well reproduced by the $\chi$QSM.

Actually one can see at Fig. \ref{fig:10} how the $\chi$QSM values for $G_E^s$ and $G_M^s$ fit into the present
world data at $Q^2 = 0.1 \text{GeV}^2$. The plot is taken from HAPPEX. and the ellipse reflects the 95 % confidence level. Apparently there is good agreement between the $\chi$QSM and the data. A similar conclusion can be drawn from Fig. 11 in which for $G_s^M$ and $G_s^E(T = 1)$ the $\chi$QSM is confronted with the data. Here the ellipse represents the 1-σ overlap of the deuterium and hydrogen measurements. This figure is taken from Beise et al. of the HAPPEX collaboration.

In Fig. 9 the PVAs of the various experiments are presented focusing on the strange contribution. Following Eq. (1) plotted are $A_{\text{phys}} - A_0 = A_s$. The curves are from the $\chi$QSM. Actually the calculations yield for the HAPPEX-experiments and the G0-experiment nearly identical curves which cannot be distinguished in Fig. 9. One notes for this sensitive quantity, originating solely from the strange quarks of the Dirac sea, a good agreement between theory and experiment.

5. In the present work, we have investigated the parity-violating asymmetries in the elastic scattering of polarized electrons off protons within the framework of the chiral quark-soliton model ($\chi$QSM). We used as an input the electromagnetic and strange vector form factors calculated in the former works [23, 24, 25], yielding both positive magnetic and electric strange form factors, and the axial-vector form factors [26] from a recent publication. All these form factors, incorporated in the present work, were obtained with one fixed set of four model parameters, which has been adjusted several years ago to basic mesonic and baryonic observables. In fact, the parity-violating asymmetries obtained in the present work are in a remarkable agreement with the experimental data, which implies that the present model ($\chi$QSM) produces reasonable form factors of many different quantum numbers. We also predicted in the present work the parity-violating asymmetries for the future G0 experiment at backward angles. Altogether, comparing the results of the $\chi$QSM with the overall observables of SAMPLE, HAPPEX, A4 and G0 one observes a remarkable agreement.

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