Calculation of Astrophysical S-factor and Thermonuclear Reaction Rates for \((p,n)\) Medium Elements Reactions

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Abstract. The cross-sections of \((p,n)\) medium elements reactions as a function of proton energies such as \(^{45}\)Sc\((p,n)^{45}\)Ti, \(^{48}\)Ti\((p,n)^{48}\)V, \(^{51}\)V\((p,n)^{51}\)Cr, \(^{52}\)Cr\((p,n)^{52}\)Mn, \(^{55}\)Mn\((p,n)^{55}\)Fe, \(^{56}\)Fe\((p,n)^{56}\)Co, \(^{59}\)Co\((p,n)^{59}\)Ni, \(^{62}\)Ni\((p,n)^{62}\)Cu, \(^{63}\)Cu\((p,n)^{63}\)Zn, and \(^{66}\)Zn\((p,n)^{66}\)Ga have been interpolated near threshold up to 10 MeV in step of 0.05 MeV using MATLAB program. Weighted averages of cross-sections have been used to calculate the astrophysical S-factor and thermonuclear reaction rates as a function of the center of mass energy, \(E_{\text{c.m.}}\) and \(T_9\) respectively. Polynomial expressions have been used to fit the calculated astrophysical S-factor and thermonuclear reaction rates to determine the astrophysical S-factor at various \(E_{\text{c.m.}}\) and thermonuclear reaction rates at various \(T_9\) from best fitting equations with minimum Chi-Square. Empirical formulae of reactions set \(^{45}\)Sc\((p,n)^{45}\)Ti, \(^{48}\)Ti\((p,n)^{48}\)V, \(^{55}\)Mn\((p,n)^{55}\)Fe, \(^{59}\)Co\((p,n)^{59}\)Ni, \(^{66}\)Zn\((p,n)^{66}\)Ga and reactions set \(^{48}\)Ti\((p,n)^{48}\)V, \(^{51}\)V\((p,n)^{51}\)Cr, \(^{59}\)Co\((p,n)^{59}\)Ni, \(^{63}\)Cu\((p,n)^{63}\)Zn, \(^{66}\)Zn\((p,n)^{66}\)Ga have been used to calculate the astrophysical S-factor as a function of \(E_{\text{c.m.}}\) and \(Z\) and thermonuclear reaction rates as a function of \(T_9\) and \(Z\) of target nucleus. The results have been compared with the adopted data that have been calculated from the fitting equations which have a good agreement.

Keywords: Cross sections; astrophysical S-factor; thermonuclear reaction rates; Gamow factor; Gamow energy; Sommerfeld parameter.

1. Introduction
A star can be defined as a self - gravitating celestial object in which there is, or there once was (in the case of dead stars), sustained thermonuclear fusion of hydrogen in their core [1]. The astrophysical S-factor, \(S(E)\) has extensively been used in the field to remove the energy dependence of the Coulomb barrier penetration from the cross section, \(\sigma(E)\) [2]. As stellar energies are much lower than the Coulomb barrier, the cross sections strongly depend on energy [3]. If all the nuclei in a star fused when the “Coulomb Barrier” was overcome, the star would burn instantaneously, in a ‘flash’ [4], [5]. The astrophysical calculations concerning the synthesis of elements require as input temperature-dependent expressions for thermonuclear reaction rates (TNRR) [6]. The quantity of interest in calculating thermonuclear reaction rates for astrophysical purposes is \(N_A \langle \sigma v \rangle\) which is the product of Avogadro’s number with the average value of the cross section times velocity, averaged over a Maxwell-Boltzmann distribution of temperature [7]. Total cross sections of \((p,n)\) light elements reactions as a function of
center of mass energy have been measured by several authors which are mentioned by different references such as: 45Sc(p,n)45Ti [8–11], 48Ti(p,n)48V[11,12], 51V(p,n)51Cr[8,13–17], 52Cr(p,n)52Mn[11,18], 55Mn(p,n)55Fe[8,19–22], 56Fe(p,n)56Co [11,23], 59Co(p,n)59Ni[19,20,24,25], 62Ni(p,n)62Cu[11,26,27], 63Cu(p,n)63Zn[11,28,29], and 66Zn(p,n)66Ga [11,30,31] respectively. The aim of this work is to determine the empirical formulae to calculate the astrophysical S-factor, S(E) and thermonuclear reaction rates, N Aσν> using the modified cross sections of the above reactions. The results are compared with those published in the literature.

2. Theory

Atomic masses of each medium elements and isotopes related this work has been taken from the nuclear wallet cards published by the National Nuclear Data Center (NNDC) [32].

The Q – value of the reaction

\[ Q = M_n + M_Y - (M_Y + M_n) c^2 \]  

(1)

Where \( M_p, M_X, M_Y, \) and \( M_n \) are the atomic masses of the incident, target particles, product nucleus and neutron (outgoing particle), respectively and \( (c^2 = 931.494013 \text{ MeV/amu}) \) where \( u= \text{atomic mass unit (amu)} = 1.66 \times 10^{-27} \text{ kg} \). From conservation law of energy [33]:

\[ M_p c^2 + T_p + M_n c^2 = M_X c^2 + T_n + M_Y c^2 + T_Y \]  

(2)

Where \( T_p, T_n, \) and \( T_Y \) are the proton, neutron, and heavy product kinetic energies.

In the laboratory system conservations of energy and momentum lead to the following equation [33]:

\[ Q = T_n \left(1 + \frac{M_p}{M_X}\right) - T_p \left(1 + \frac{M_X}{M_p}\right) - \frac{2}{M_Y} (M_p T_p M_n T_n)^{1/2} \cos \theta \]  

(3)

This is called the Q-value equation.

If Q is positive, the reaction is said to be exoergic; if Q is negative, it is endoergic. The amount of energy needed for an endoergic reaction is called the threshold energy and can be calculated easily [34].

\[ E_{th} = Q (1 + \frac{M_p}{M_X}) \]  

(4)

\[ V(r) = \frac{Z_1 Z_2 e^2}{r} \]  

(5)

Where \( Z_1 \) and \( Z_2 \) are the charges of the projectile and target nuclei, and \( r \) and \( r = r_1 + r_2 \) is their separation, \( e \) is the charge of electron \( (e^2 = 1.44 \text{ MeV fm}) \), and the radius of the nucleus is given by \( r = 1.3 \times 10^{-13} A^{1/3} \) cm, where \( A \) is the mass number (atomic weight) [35]. Then Eq. (5) leads to

\[ V(r) = E_c \left( \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}} \right) \]  

(6)

Where \( E_c \) is the coulomb barrier or coulomb energy in MeV, \( A_1^{1/3} \) and \( A_2^{1/3} \) are the mass numbers of the charges of projectile and target nuclei respectively.

The data of cross-sections of nonresonant reactions, exhibit a dramatic decrease at low energies due to quantum tunneling, as reflected in the energy dependence of the transmission coefficient through the Coulomb barrier [2]: The astrophysical S-factor, \( S(E) \) in unit \( (\text{MeV} - \text{b}) \) is related to the cross-section by [36]:

\[ S(E) = E \sigma(E) \exp(2\pi\eta) \]  

(7)

Where \( E \) is the energy in the center of mass system \( (E_{c.m.}) \) in MeV, \( \sigma(E) \) is the cross section of the reaction in (mb), \( 2\pi\eta \) is the Gamow factor, and \( \eta \) is Sommerfeld parameter [37]:

\[ \eta = \frac{Z_1 Z_2 e^2}{h^2} = 0.1575 Z_1 Z_2 \sqrt{\frac{\mu_{\text{prod}}}{E(\text{MeV})}} \]  

(8)
\( \hbar \) is Planck’s constant over \( 2 \pi \) (1.0546 \times 10^{-27} \text{ ergs}), \( v \) is the relative velocity, \( \mu \) is the reduced mass. And the Gamow factor \( G(E) \) or \( 2\pi\eta \) [2]:

\[
2\pi\eta = 0.98951Z_1Z_2 \sqrt{\frac{\mu(u)}{E(Mev)}} \tag{9}
\]

The reduced mass \( \mu \) in u (amu) is determined by the equation [38]:

\[
\mu = \frac{m_1m_2}{m_1+m_2} \tag{10}
\]

Where \( m_1 \) and \( m_2 \) represents the masses of the projectile and target nucleus in units of (amu), respectively. The energy of a pair of particles in their center of mass \( E_{c.m.} \) is related to the laboratory energy, \( E_{lab.} \) of the incident particle by the relationship [33]:

\[
E_{c.m.} = \frac{m_2}{m_1+m_2}E_{lab} \tag{11}
\]

The Gamow energy \( E_G \), in MeV [39]:

\[
E_G = 2\pi^2 \mu u C^2 \alpha^2(Z_1Z_2)^2 = 0.979\mu(Z_1Z_2)^2 \tag{12}
\]

Where \( \alpha = \frac{1}{137} = \frac{e^2}{\hbar c} \) is the fine-structure constant.

The thermonuclear reaction rates, \( N_A(\sigma v) \) in unit \( (cm^2mol^{-1}s^{-1}) \) [37]:

\[
N_A(\sigma v) = \left( \frac{B}{\mu r} \right)^{1/2} \frac{1}{(k_B T)^{3/2}} N_A \int_0^{\infty} E\sigma(E) \exp\left(-E/k_BT\right) dE \tag{13}
\]

Where \( N_A \) is the Avogadro’s number \( (6.022 \times 10^{23} \text{ mol}^{-1}) \), \( k_B \) is the Boltzmann’s constants \( (1.38 \times 10^{-16} \text{ erg/K}) \), and \( T \) is the temperature respectively. Eq. (13) leads to [37]:

\[
N_A(\sigma v) = 3.7313 \times 10^7 \mu^{-1/2} T_9^{-3/2} \int_0^{\infty} E\sigma(E) \exp\left(-11.605E/T_9\right) dE \tag{14}
\]

Where \( T_9 \) is the temperature in units of \( 10^9 \text{K} \) ( \( T_9 = 10^{-9}T \) )

The weighted averages of the cross sections of light elements whose cross sections \( \sigma_0(mb) \) and the uncertainty (errors) \( \Delta\sigma_0(mb) \) are expressed by the following Eqs. [40]:

\[
\sigma_0(mb) = \sum_i \sigma_i / \sigma_i^2 \tag{15}
\]

Where \( \sigma_i \) is the cross section of \( i^{th} \) reference, and \( \Delta\sigma_i \) is the errors corresponding to each values of \( \sigma_i \),

\[
\Delta\sigma_0(mb) = \pm \frac{1}{\sqrt{\sum_i (1/\Delta\sigma_i^2)}} \tag{16}
\]

The type of formalism has been considered in the present work is the polynomial fit expression of the form:

\[
Y = C_0 + C_1X + C_2X^2 + C_3X^3 + \cdots + C_NX^N = \sum_{i=0}^{M} C_iX^i \tag{17}
\]
This polynomial is obtained by the Excel computer program (Format Trendline). Where \((c_0, c_1, c_2, c_3, \ldots)\) are free parameters, and \((i = 0, 1, 2, 3, \ldots, M)\).

\[
G_i = \sum_{j=0}^{N} C_{ij}K^j
\]  

have been considered in this work. Subsequently, by combining the above equations (17) & (18), the following expression has been obtained:

\[
Y = \sum_{l=0}^{M} \left( \sum_{j=0}^{N} C_{lj}K^j \right)X^l
\]

Where \(Y = \ln[S(E)]\) or \(\ln[N_A<\sigma\nu>]\), \((i=0,1,2,\ldots,M)\), \((j=0,1,2,\ldots,N)\), \((C_{00}, C_{01}, C_{02}, \ldots)\) are free parameters (coefficients of polynomials), \(K\) is the center of mass energy or \(T_s\) according to the \(S(E)\) or \(N_A<\sigma\nu>\), and \(X\) is atomic number \(Z\). The Excel computer program has been used to obtain the best fit formulae corresponding to different energies ranges from the threshold up to \(10^{10} \text{K}\) in the center of mass system or \(T_s\) ranges from 1 to \(10^{10} \text{K}\). The data of these ranges were excluded in each step, till an acceptable value of the coefficient of determination \(R^2 \approx 1\) was reached. The best fit adopted data was obtained with increasing order to provide the minimum value of Chi-Square \((\chi^2)\) by using the Eq. [41]:

\[
\chi^2 = \frac{1}{(N - M)} \sum_{l=0}^{N} \left( \frac{Y_{\text{exp}}^l - Y_{\text{cal}}^l}{\Delta Y_{\text{exp}}^l} \right)^2
\]

Where \(N\) is the data points number, \(M\) is the fitting coefficients number, \(Y_{\text{exp}}^l\) and \(\Delta Y_{\text{exp}}^l\) are the experimental (adopted value) of \(\ln[S(E)]\) or \(\ln[N_A<\sigma\nu>]\) and its error respectively, \(Y_{\text{cal}}^l\) is the calculated \(\ln[S(E)]\) or \(\ln[N_A<\sigma\nu>]\).

3. Data Reduction and Analysis

The Atomic masses have been taken into consideration to determine the Q-Value, threshold energy, Coulomb barrier, reduced mass, and the ratio between \((E_{c.m.}/E_{\text{lab}})\) of \((p,n)\) medium elements reactions using the Eqs. (1, 4, 6, 10, and 11) respectively; the results are shown in the table (1).

| (p,n) Medium Element Reaction | Q-value (MeV) | \(E_{\text{threshold}}\) (MeV) | Coulomb Barrier \(E_c\) (MeV) | Reduced Mass \((\mu)\) (amu) | \(E_{c.m.}/E_{\text{lab}}\) |
|-------------------------------|---------------|-------------------------------|-----------------------------|-----------------------------|-----------------------------|
| \(^{40}\text{Sc}(p,n)^{38}\text{Ti}\) | -2.844E+00 | 2.908E+00 | 2.844E+00 | 5.105E+00 | 9.857E-01 | 9.781E-01 |
| \(^{40}\text{Ti}(p,n)^{38}\text{V}\) | -4.797E+00 | 4.898E+00 | 4.797E+00 | 5.259E+00 | 9.871E-01 | 9.794E-01 |
| \(^{51}\text{V}(p,n)^{49}\text{Cr}\) | -1.535E+00 | 1.565E+00 | 1.535E+00 | 5.411E+00 | 9.883E-01 | 9.806E-01 |
| \(^{52}\text{Cr}(p,n)^{50}\text{Mn}\) | -5.494E+00 | 5.600E+00 | 5.494E+00 | 1.329E+01 | 9.886E-01 | 9.810E-01 |
| \(^{55}\text{Mn}(p,n)^{53}\text{Fe}\) | -1.013E+00 | 1.032E+00 | 1.013E+00 | 5.766E+00 | 9.897E-01 | 9.820E-01 |
| \(^{56}\text{Fe}(p,n)^{54}\text{Co}\) | -5.349E+00 | 5.445E+00 | 5.349E+00 | 5.968E+00 | 9.900E-01 | 9.823E-01 |
| \(^{59}\text{Co}(p,n)^{57}\text{Ni}\) | -1.855E+00 | 1.887E+00 | 1.855E+00 | 6.112E+00 | 9.909E-01 | 9.832E-01 |
| \(^{60}\text{Ni}(p,n)^{58}\text{Cu}\) | -4.741E+00 | 4.818E+00 | 4.741E+00 | 6.256E+00 | 9.917E-01 | 9.840E-01 |
| \(^{63}\text{Cu}(p,n)^{61}\text{Zn}\) | -4.149E+00 | 4.215E+00 | 4.149E+00 | 6.452E+00 | 9.919E-01 | 9.842E-01 |
| \(^{66}\text{Zn}(p,n)^{64}\text{Ga}\) | -5.957E+00 | 6.048E+00 | 5.957E+00 | 6.592E+00 | 9.927E-01 | 9.849E-01 |
Eqs. (8, 9, 12, and 7) taken into consideration to determine the Sommerfeld parameter ($\eta$), Gamow factor ($G(E)$), Gamow energy ($E_g$), and the astrophysical S-factor, $S(E)$ of the (p,n) medium elements reactions. The results are shown in Table (2).

**Table 2.** The Sommerfeld parameter ($\eta$), Gamow factor ($G(E)$), Gamow energy ($E_g$), and the astrophysical S-factor, $S(E)$ of the (p,n) medium elements reactions

| Medium Element Reaction | Sommerfeld Parameter $\eta$ | Gamow factor $G(E)$ | Gamow Energy $E_g$(MeV) | Astrophysical S-factor $S(E)$ |
|-------------------------|-----------------------------|---------------------|------------------------|-------------------------------|
| $^{45}$Sc(p,n)$^{45}$Ti | 3.282E+00/$\sqrt{E_{c.m.}}$ | 2.063E+01/$\sqrt{E_{c.m.}}$ | 4.256E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.063E+01/$\sqrt{E_{c.m.}}$) |
| $^{46}$Ti(p,n)$^{46}$V | 3.441E+00/$\sqrt{E_{c.m.}}$ | 2.163E+01/$\sqrt{E_{c.m.}}$ | 4.678E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.163E+01/$\sqrt{E_{c.m.}}$) |
| $^{51}$V(p,n)$^{51}$Cr | 3.599E+00/$\sqrt{E_{c.m.}}$ | 2.262E+01/$\sqrt{E_{c.m.}}$ | 5.119E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.262E+01/$\sqrt{E_{c.m.}}$) |
| $^{52}$Cr(p,n)$^{52}$Mn | 3.757E+00/$\sqrt{E_{c.m.}}$ | 2.361E+01/$\sqrt{E_{c.m.}}$ | 5.576E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.361E+01/$\sqrt{E_{c.m.}}$) |
| $^{55}$Mn(p,n)$^{55}$Fe | 3.915E+00/$\sqrt{E_{c.m.}}$ | 2.461E+01/$\sqrt{E_{c.m.}}$ | 6.056E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.461E+01/$\sqrt{E_{c.m.}}$) |
| $^{56}$Fe(p,n)$^{56}$Co | 4.072E+00/$\sqrt{E_{c.m.}}$ | 2.560E+01/$\sqrt{E_{c.m.}}$ | 6.553E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.560E+01/$\sqrt{E_{c.m.}}$) |
| $^{57}$Co(p,n)$^{57}$Ni | 4.121E+00/$\sqrt{E_{c.m.}}$ | 2.659E+01/$\sqrt{E_{c.m.}}$ | 7.073E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.659E+01/$\sqrt{E_{c.m.}}$) |
| $^{60}$Ni(p,n)$^{60}$Cu | 4.389E+00/$\sqrt{E_{c.m.}}$ | 2.759E+01/$\sqrt{E_{c.m.}}$ | 7.613E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.759E+01/$\sqrt{E_{c.m.}}$) |
| $^{61}$Cu(p,n)$^{61}$Zn | 4.547E+00/$\sqrt{E_{c.m.}}$ | 2.858E+01/$\sqrt{E_{c.m.}}$ | 8.168E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.858E+01/$\sqrt{E_{c.m.}}$) |
| $^{66}$Zn(p,n)$^{66}$Ga | 4.705E+00/$\sqrt{E_{c.m.}}$ | 2.958E+01/$\sqrt{E_{c.m.}}$ | 8.747E+02/$\sqrt{E_{c.m.}}$ | $E_{c.m.}\cdot\sigma(E)\cdot\exp(2.958E+01/$\sqrt{E_{c.m.}}$) |

4. Results and Discussion

In general, we can write Eq. (17), instead of $X$ insert center of mass energies $E_{c.m.}$ then the Eq. (17) become:

\[ Y = C_0 + C_1 K + C_2 K^2 + C_3 K^3 + \cdots + C_N K^N = \sum_{i=0}^{M} C_i K^i \tag{21} \]

Where ($C_0, C_1, C_2, \ldots$) are free parameters, $K$ is the center of mass energy or $T_\beta$ ($i = 0, 1, 2, 3 \ldots M$), and $Y = \ln[S\text{-factor (MeV-b)}]$ or $Y = \ln[N_A \cdot \sigma(v) \cdot (\text{cm}^3 \text{mol}^{-1} \text{s}^{-1})]$.

4.1. Astrophysical S-factor Empirical Formulae

The adopted astrophysical S-factor has been used to obtain the fitting parameters by using the polynomial expressions (18), (20) and (21) by the following step:

1. The polynomial expressions which are used in eq. (21) to fit the calculated natural logarithm of astrophysical S-factor, $S(E)$ of the studied medium elements to determine the adopted natural logarithm of astrophysical S-factor from the best fitting with minimum Chi-Square using Eq. (20). The obtained best fitting Equations of the mentioned reactions were presented in equations (22, 23, 24, 25, 26, 27, 28, 29, 30, and 31) for the reactions $^{45}$Sc(p,n)$^{45}$Ti, $^{46}$Ti(p,n)$^{46}$V, $^{51}$V(p,n)$^{51}$Cr, $^{52}$Cr(p,n)$^{52}$Mn, $^{55}$Mn(p,n)$^{55}$Fe, $^{56}$Fe(p,n)$^{56}$Co, $^{59}$Co(p,n)$^{59}$Ni, $^{62}$Ni(p,n)$^{62}$Cu, $^{63}$Cu(p,n)$^{63}$Zn, and $^{66}$Zn(p,n)$^{66}$Ga respectively.

\[ 45\text{Sc}(p,n)\quad \chi^2 = 0.206 \]

\[ \ln[S\text{-factor(MeV-b)}] = -0.0097E^3 + 0.1908E^2 - 1.3719E + 11.975 \tag{22} \]
\[ ^{48}\text{Ti}(p,n)^{48}\text{V} \quad x^2 = 0.855 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = -0.1004E^4 + 3.1142E^3 - 35.854E^2 + 181.26E - 330.11 \quad (23) \]

\[ ^{51}\text{V}(p,n)^{51}\text{Cr} \quad x^2 = 1.006 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = 0.0024E^4 - 0.0687E^3 + 0.7405E^2 - 3.8774E + 16.191 \quad (24) \]

\[ ^{52}\text{Cr}(p,n)^{52}\text{Mn} \quad x^2 = 0.793 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = -0.1933E^4 + 6.1958E^3 - 73.88E^2 + 388.15E - 748.49 \quad (25) \]

\[ ^{55}\text{Mn}(p,n)^{55}\text{Fe} \quad x^2 = 0.0649 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = -0.0034E^3 + 0.083E^2 - 0.9498E + 14.193 \quad (26) \]

\[ ^{56}\text{Fe}(p,n)^{56}\text{Co} \quad x^2 = 0.0014 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = -0.0014E^2 - 0.1037E + 10.509 \quad (27) \]

\[ ^{59}\text{Co}(p,n)^{59}\text{Ni} \quad x^2 = 0.075 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = -0.0047E^4 + 0.123E^3 - 1.144E^2 + 4.0873E + 7.6684 \quad (28) \]

\[ ^{62}\text{Ni}(p,n)^{62}\text{Cu} \quad x^2 = 0.0628 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = 0.0356E^3 - 0.8537E^2 + 6.4351E - 4.0352 \quad (29) \]

\[ ^{63}\text{Cu}(p,n)^{63}\text{Zn} \quad x^2 = 0.0026 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = 0.0208E^3 - 0.4423E^2 + 2.6549E + 7.4516 \quad (30) \]

\[ ^{66}\text{Zn}(p,n)^{66}\text{Ga} \quad x^2 = 0.079 \]

\[ \ln\left[ S^{-} \text{-factor (MeV - b)} \right] = -0.114E^2 + 1.8612E + 3.8573 \quad (31) \]

2. At fixed values of center of mass energy, the variation of the natural logarithm of the astrophysical S-factor with the physical parameter atomic number $Z$ has been fitted to the polynomial expression using Eq. (21). The obtained results were used to determine the free parameters ($C_i$).

3. The free parameters $C_i$ were plotted against each value of the center of mass energies and fitted to adequate the polynomial expression were presented in Eq. (18).

4. The final formula of a set of reactions has been determined by using the combination of the two polynomials to show the systematic behavior of the reactions which are shown in Eq. (19). The $Y$ Variable is the astrophysical S-factor.

4.1.1 The Empirical Formulae Relating the Astrophysical S-factor to Center of Mass Energy and the Atomic Number $Z$ of the Target Nucleus

The empirical formulae relating to the astrophysical S-factor (MeV-b) with both of center of mass energies $E_{cm}$, and the target atomic number $Z$, were performed as follows: 1- At fixed values of the center of mass energies from 6 to 10 MeV in steps of 0.25 MeV for the $^{45}\text{Sc}(p,n)^{45}\text{Ti}$, $^{48}\text{Ti}(p,n)^{48}\text{V}$, $^{55}\text{Mn}(p,n)^{55}\text{Fe}$, $^{59}\text{Co}(p,n)^{59}\text{Ni}$, and $^{66}\text{Zn}(p,n)^{66}\text{Ga}$ reactions, the natural logarithm of the astrophysical S-factor will vary with the atomic number($Z$) this shown in Fig. (1). The data have been fitted to the following polynomial expression:

\[ Y = \sum_{i=0}^{2} C_i X^i \quad (32) \]

Where $Y = \ln[S(E)]$, and $X=Z$, with free parameters $C_i$ ($C_0$, $C_1$, and $C_2$).
The variation of the natural logarithm of the astrophysical S-factor \( S(E) \) with the atomic number \( Z \) for the \( ^{45}\text{Sc}(p,n)^{45}\text{Ti} \), \( ^{48}\text{Ti}(p,n)^{48}\text{V} \), \( ^{55}\text{Mn}(p,n)^{55}\text{Fe} \), \( ^{59}\text{Co}(p,n)^{59}\text{Ni} \), and \( ^{66}\text{Zn}(p,n)^{66}\text{Ga} \) reactions at fixed values of center of mass energy.

2- The adopted astrophysical S-factor, \( S(E) \) has been used as a function of target atomic number \( Z \) at the fixed center of mass energies using the Excel computer program to get the fitting expressions and then used to calculate the fitting parameters. The obtained results are presented in Table 3.

3- The obtained free parameters \( C_i \) (\( C_0 \), \( C_1 \), and \( C_2 \)), as shown in Table (3) are plotted against with the prefixed values of center of mass energies from 6 to 10 MeV in step of 0.25 MeV as shown in Fig(2), and then the obtained free parameters \( C_i \) have been fitted to the following polynomial expression:

\[
C_i = \sum_{j=0}^{2} C_{ij}E^j
\]  

(33)

Table 3. Free parameters \( C_i \) (\( C_0 \), \( C_1 \), and \( C_2 \)) as a function of the center of mass energy.

| \( E_{c.m.} \) (MeV) | \( C_0 \)    | \( C_1 \)    | \( C_2 \)    |
|---------------------|-----------|-----------|-----------|
| 6                   | -38.595   | 3.6166    | -0.0655   |
| 6.25                | -34.076   | 3.2484    | -0.0581   |
| 6.5                 | -30.24    | 2.9271    | -0.0516   |
| 6.75                | -26.973   | 2.647     | -0.0458   |
| 7                   | -24.172   | 2.403     | -0.0407   |
| 7.25                | -21.755   | 2.1908    | -0.0362   |
| 7.5                 | -19.654   | 2.0066    | -0.0324   |
| 7.75                | -17.822   | 1.8475    | -0.0291   |
| 8                   | -16.225   | 1.7112    | -0.0263   |
| 8.25                | -14.848   | 1.5961    | -0.024    |
| 8.5                 | -13.692   | 1.5012    | -0.0221   |
| 8.75                | -12.776   | 1.4263    | -0.0207   |
| 9                   | -12.136   | 1.372     | -0.0196   |
| 9.25                | -11.823   | 1.3394    | -0.0189   |
| 9.5                 | -11.907   | 1.3303    | -0.0186   |
| 9.75                | -12.475   | 1.3474    | -0.0186   |
| 10                  | -13.63    | 1.3939    | -0.019    |

The combination of the two polynomials Eq. (32) and Eq. (33) takes the form of the following formula of energy range from 6 to 10 MeV in the step of 0.25 MeV:
\[ Y = \sum_{i=0}^{2} \left( \sum_{j=0}^{2} C_{ij}E^j \right)X^i \]  

(34)

Where \( Y = \ln[S(E)] \), \( X = \text{atomic number } Z \)

\[ Y = \sum_{i=0}^{2} (C_{00}E^0 + C_{11}E^1 + C_{22}E^2)X^i \]

\[ Y = C_{00}E^0X^0 + C_{01}E^1X^0 + C_{02}E^2X^0 + C_{10}E^0X^1 + C_{11}E^1X^1 + C_{12}E^2X^1 + C_{20}E^0X^2 + C_{21}E^1X^2 + C_{22}E^2X^2 \]

(35)

Where (\( C_{00}, C_{01}, C_{02}, C_{10}, C_{11}, \ldots, C_{22} \)) are free parameters and their values are shown in the below:

\[
\begin{bmatrix}
C_{00} & C_{01} & C_{02} \\
C_{10} & C_{11} & C_{12} \\
C_{20} & C_{21} & C_{22}
\end{bmatrix}
= \begin{bmatrix}
-221.1 & 45.151 & -2.4379 \\
18.619 & -3.6921 & 0.1971 \\
-0.3699 & 0.0748 & -0.004
\end{bmatrix}, \quad R^2 = \begin{bmatrix}
0.9988 \\
0.9991 \\
0.9988
\end{bmatrix}
\]

\( R^2 = 0.9988 \)

\( R^2 = 0.9991 \)

\( R^2 = 0.9988 \)

\( R^2 = 0.9991 \)

\( R^2 = 0.9988 \)

\[ C_z = -2.4379E^2 + 45.151E - 221.1 \]

\( R^2 = 0.9988 \)

\[ C_z = 0.1971E^2 - 3.6921E + 18.619 \]

\( R^2 = 0.9991 \)
Fig 2. $C_i$ coefficients against the center of mass energy, for $C_0$, $C_1$, and $C_2$ respectively. The solid line represents the fitted curve through the data.

The values of free parameters of the above and the atomic number $Z$ of each targets nuclei for each reactions $^{45}$Sc(p,n)$^{45}$Ti, $^{48}$Ti(p,n)$^{48}$V, $^{55}$Mn(p,n)$^{55}$Fe, $^{59}$Co(p,n)$^{59}$Ni, and $^{66}$Zn(p,n)$^{66}$Ga are substitute into Eq. (35) the results are shown in the following formulae:

$^{45}$Sc(p,n)$^{45}$Ti
\[
\chi^2 = 3.266
\]
\[
\ln[S - factor(\text{MeV} - b)] = -0.0628E^2 + 0.6037E + 6.7731
\]

$^{48}$Ti(p,n)$^{48}$V
\[
\chi^2 = 4.936
\]
\[
\ln[S - factor(\text{MeV} - b)] = -0.0377E^2 + 0.128E + 9.4864
\]

$^{55}$Mn(p,n)$^{55}$Fe
\[
\chi^2 = 1.088
\]
\[
\ln[S - factor(\text{MeV} - b)] = -0.0104E^2 - 0.4015E + 13.1875
\]

$^{59}$Co(p,n)$^{59}$Ni
\[
\chi^2 = 2.347
\]
\[
\ln[S - factor(\text{MeV} - b)] = -0.0322E^2 - 0.0065E + 11.9559
\]

$^{66}$Zn(p,n)$^{66}$Ga
\[
\chi^2 = 2.814
\]
\[
\ln[S - factor(\text{MeV} - b)] = -0.1249E^2 + 1.708E + 4.56
\]

These above formulae have been used to calculate the astrophysical S-factor $S(E)$ in natural logarithm for each of the $^{45}$Sc(p,n)$^{45}$Ti, $^{48}$Ti(p,n)$^{48}$V, $^{55}$Mn(p,n)$^{55}$Fe, $^{59}$Co(p,n)$^{59}$Ni, and $^{66}$Zn(p,n)$^{66}$Ga reactions and compared with the adopted astrophysical S-factor calculated from the fitting expressions which indicated in Eqs. (22, 23, 26, 28, and 31) respectively which are in a good agreement. The obtained data are presented in Table (4).

4.2. Thermonuclear Reaction Rates Empirical Formulae

The adopted thermonuclear reaction rates $N_A<\sigma v>$ have been used to obtain the fitting parameter by using the polynomial expressions (18), (20) and (21) by the following steps:

1. The polynomial expressions were used in eq. (21) to fit the calculated natural logarithm of thermonuclear reaction rates $N_A<\sigma v>$ of the studied medium elements to determine the adopted natural logarithm of thermonuclear reaction rates $N_A<\sigma v>$ from the best fitting with minimum Chi-Square using Eq. (20). The obtained best fitting equations of the mentioned reactions are present in equations (41, 42, 43, 44, 45, 46, 47, 48, 49, and 50) for the reactions $^{45}$Sc(p,n)$^{45}$Ti,
$^{48}$Ti(p,n)$^{48}$V, $^{51}$V(p,n)$^{51}$Cr, $^{52}$Cr(p,n)$^{52}$Mn, $^{55}$Mn(p,n)$^{55}$Fe, $^{56}$Fe(p,n)$^{56}$Co, $^{59}$Co(p,n)$^{59}$Ni, $^{62}$Ni(p,n)$^{62}$Cu, $^{63}$Cu(p,n)$^{63}$Zn, and $^{66}$Zn(p,n)$^{66}$Ga respectively.

$^{45}$Sc(p,n)$^{45}$Ti $\chi^2 = 0.024$

\[
\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.001870_9^5 - 0.063479_4^4 + 0.886670_9^3 - 6.268370_9^2 + 23.57870_9 - 25.628
\]  

$^{48}$Ti(p,n)$^{48}$V $\chi^2 = 0.372$

\[
\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.003270_9^5 - 0.112970_9^4 + 1.584270_9^3 - 11.25870_9^2 + 42.5470_9 - 59.805
\]  

$^{51}$V(p,n)$^{51}$Cr $\chi^2 = 0.4$
Table 4. Comparison between polynomial fitting expressions (Best Fitting) of the adopted astrophysical S-factor with those calculated from Eqs. (36) to (40).

| Ec.m. (MeV) | \(^{45}\)Sc(p,n)\(^{45}\)Ti | \(^{48}\)Ti(p,n)\(^{48}\)V | \(^{55}\)Mn(p,n)\(^{55}\)Fe | \(^{59}\)Co(p,n)\(^{59}\)Ni | \(^{66}\)Zn(p,n)\(^{66}\)Ga |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|            | ln[S-factor (MeV-b)] (Best Fitting) | ln[S-factor (MeV-b)] (Formula) | ln[S-factor (MeV-b)] (Best Fitting) | ln[S-factor (MeV-b)] (Formula) | ln[S-factor (MeV-b)] (Best Fitting) |
| 6          | 8.51±0.475  | 8.135 | 9.25±0.349 | 8.897 | 10.74±1.077 | 10.404 | 11.48±0.853 | 10.758 | 10.92±0.826 | 10.312 |
| 6.25       | 8.486±0.474 | 8.093 | 9.32±0.351 | 8.814 | 10.66±1.069 | 10.272 | 11.38±0.846 | 10.657 | 11.03±0.835 | 10.356 |
| 6.5        | 8.45±0.472  | 8.044 | 9.26±0.349 | 8.726 | 10.59±1.061 | 10.138 | 11.29±0.839 | 10.553 | 11.13±0.843 | 10.385 |
| 6.75       | 8.42±0.470  | 7.987 | 9.13±0.344 | 8.633 | 10.51±1.054 | 10.004 | 11.20±0.832 | 10.445 | 11.22±0.849 | 10.398 |
| 7          | 8.39±0.469  | 7.922 | 8.97±0.338 | 8.535 | 10.44±1.047 | 9.867 | 11.12±0.827 | 10.333 | 11.30±0.855 | 10.396 |
| 7.25       | 8.36±0.467  | 7.849 | 8.81±0.332 | 8.433 | 10.37±1.039 | 9.730 | 11.05±0.821 | 10.216 | 11.35±0.860 | 10.378 |
| 7.5        | 8.32±0.465  | 7.768 | 8.68±0.327 | 8.326 | 10.30±1.032 | 9.591 | 10.99±0.817 | 10.096 | 11.40±0.863 | 10.344 |
| 7.75       | 8.28±0.463  | 7.680 | 8.59±0.324 | 8.214 | 10.23±1.026 | 9.451 | 10.93±0.812 | 9.972 | 11.43±0.865 | 10.295 |
| 8          | 8.24±0.460  | 7.584 | 8.54±0.322 | 8.098 | 10.16±1.019 | 9.310 | 10.87±0.808 | 9.843 | 11.45±0.866 | 10.230 |
| 8.25       | 8.19±0.458  | 7.479 | 8.54±0.322 | 7.976 | 10.09±1.012 | 9.167 | 10.81±0.804 | 9.711 | 11.45±0.867 | 10.150 |
| 8.5        | 8.14±0.454  | 7.367 | 8.56±0.323 | 7.851 | 10.02±1.005 | 9.023 | 10.76±0.799 | 9.574 | 11.44±0.866 | 10.054 |
| 8.75       | 8.08±0.451  | 7.247 | 8.58±0.324 | 7.720 | 9.99±0.998 | 8.878 | 10.69±0.795 | 9.434 | 11.41±0.864 | 9.942 |
| 9          | 8.01±0.447  | 7.120 | 8.58±0.324 | 7.585 | 9.89±0.991 | 8.732 | 10.62±0.789 | 9.289 | 11.37±0.861 | 9.815 |
| 9.25       | 7.93±0.443  | 6.984 | 8.50±0.321 | 7.445 | 9.81±0.984 | 8.584 | 10.53±0.782 | 9.141 | 11.31±0.857 | 9.672 |
| 9.5        | 7.84±0.438  | 6.841 | 8.39±0.313 | 7.300 | 9.74±0.977 | 8.435 | 10.42±0.775 | 8.988 | 11.25±0.851 | 9.514 |
| 9.75       | 7.74±0.432  | 6.689 | 7.92±0.299 | 7.151 | 9.67±0.969 | 8.284 | 10.29±0.765 | 8.832 | 11.17±0.845 | 9.340 |
| 10         | 7.63±0.426  | 6.530 | 7.29±0.275 | 6.996 | 9.59±0.961 | 8.133 | 10.14±0.753 | 8.671 | 10.69±0.838 | 9.150 |
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.00127\sigma_9^5 - 0.0422T_9^4 + 0.5916T_9^3 - 4.2008T_9^2 + 15.932T_9 - 13.576 \]  
\[x^2 = 0.203\]  
(43)

52Cr(p,n)52Mn  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.0077T_9^4 + 0.2342T_9^3 - 3.0337T_9^2 + 19.048T_9 - 36.889 \]  
(44)

55Mn(p,n)55Fe  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.0087T_9^4 + 0.2366T_9^3 - 2.4329T_9^2 + 11.77T_9 - 8.4719 \]  
(45)

56Fe(p,n)56Co  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.0216T_9^4 + 0.6298T_9^3 - 6.9401T_9^2 + 35.748T_9 - 64.334 \]  
(46)

59Co(p,n)59Ni  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.0021T_9^5 - 0.0711T_9^4 + 0.9356T_9^3 - 6.1675T_9^2 + 21.596T_9 - 20.117 \]  
(47)

62Ni(p,n)62Cu  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.004T_9^5 - 0.1371T_9^4 + 1.8612T_9^3 - 12.693T_9^2 + 45.546T_9 - 60.46 \]  
(48)

63Cu(p,n)63Zn  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.0083T_9^4 + 0.2608T_9^3 - 3.1324T_9^2 + 18.009T_9 - 28.88 \]  
(49)

66Zn(p,n)66Ga  
\[\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.0207T_9^4 + 0.6043T_9^3 - 6.6657T_9^2 + 34.434T_9 - 61.501 \]  
(50)

2- At fixed values of the T_9, the variation of the natural logarithm of the thermonuclear reaction rates with the physical parameter atomic number Z has been fitted to the polynomial expression using Eq. (21).

3- The free parameters C_i are plotted against each value of T_9 and fitted to adequate the polynomial expression were presented in Eq. (18).

4- The final formula of a set of reactions has been determined by using the combination of the two polynomials to show the systematic behavior of the reactions which is shown in Eq. (19).

4.2.1. The Empirical Formulae Relating the Thermonuclear Reaction Rates to T_9 and the Atomic Number Z of the Target Nucleus

The empirical formulae relating to the thermonuclear reaction rates N_A<\sigma v> (cm^3 s^{-1} mol^{-1}) with both T_9 and Z were performed as follows:

1- At fixed values of the T_9 from 6 to 10 10^9 K in steps of 0.25 10^9 K for the 40Ti(p,n)40V, 51V(p,n)51Cr, 59Co(p,n)59Ni, 63Cu(p,n)63Zn, and 66Zn(p,n)66Ga reactions, the natural logarithm of the thermonuclear reaction rates will vary with the atomic number Z this shown in Fig. (3). The data have been fitted to the polynomial expression as the same as Eq. (32), Where Y = ln[N_A(\sigma v)], X=Z, with free parameters C_i (C_0, C_1, and C_2).
Fig 3. The variation of the natural logarithm of the thermonuclear reaction rates with the atomic number $Z$ for the $^{48}$Ti($p,n)^{48}$V, $^{51}$V($p,n)^{51}$Cr, $^{59}$Co($p,n)^{59}$Ni, $^{63}$Cu($p,n)^{63}$Zn, and $^{66}$Zn($p,n)^{66}$Ga reactions at fixed values of $T_9$.

2- The adopted thermonuclear reaction rates have been used as a function of $Z$ at fixed $T_9$ using the Excel computer program to obtain the fitting expressions and then used to calculate the fitting parameters. The obtained results are presented in Table (5).

3- The obtained free parameters $C_i$ ($C_0$, $C_1$, and $C_2$), as shown in Table (5) are plotted against with the prefixed values of $T_9$ from 6 to 10 10^9K in steps of 0.25 10^9K as shown in Fig.(4), and then the obtained free parameters $C_i$ have been fitted to the polynomial expression:

$$C_i = \sum_{j=0}^{2} C_{ij} T_9^j$$

(51)

Table 5. Free parameters $C_i$ ($C_0$, $C_1$, and $C_2$) as a function of $T_9$.

| $T_9$ (10^9 K) | $C_0$     | $C_1$     | $C_2$     |
|----------------|-----------|-----------|-----------|
| 6              | -171.44   | 14.564    | -0.2848   |
| 6.25           | -159.79   | 13.667    | -0.2673   |
| 6.5            | -148.42   | 12.79     | -0.2503   |
| 6.75           | -137.07   | 11.913    | -0.2332   |
| 7              | -125.52   | 11.019    | -0.2158   |
| 7.25           | -113.62   | 10.094    | -0.1977   |
| 7.5            | -101.26   | 9.1317    | -0.1789   |
| 7.75           | -88.414   | 8.1281    | -0.1593   |
| 8              | -75.103   | 7.0856    | -0.139    |
| 8.25           | -61.413   | 6.0114    | -0.118    |
| 8.5            | -47.496   | 4.9181    | -0.0966   |
| 8.75           | -33.567   | 3.8239    | -0.0752   |
| 9              | -19.911   | 2.7526    | -0.0544   |
| 9.25           | -6.8799   | 1.7341    | -0.0346   |
| 9.5            | 5.1043    | 0.804     | -0.0167   |
| 9.75           | 15.548    | 0.0044    | -0.0015   |
| 10             | 23.885    | -0.6164   | 0.0099    |
Fig 4. $C_i$ coefficients against $T_9$, for $C_0$, $C_1$, and $C_2$ respectively. The solid line represents the fitted curve through the data.

The combination of the two polynomials Eq. (32) and Eq. (51) takes the form of the following formula range $T_9$ from 6 to 10 $10^9$K in steps of 0.25 $10^9$K:

- $C_0 = 0.46T_9^2 + 43.22T_9 - 449.15$
  $R^2 = 0.9988$

- $C_1 = -0.0387T_9^2 - 3.3255T_9 + 36.068$
  $R^2 = 0.9986$

- $C_2 = 0.0006T_9^2 + 0.0672T_9 - 0.7128$
  $R^2 = 0.9984$
\[
Y = \frac{2}{\sum_{i=0}^{2} \left( \sum_{j=0}^{2} C_{ij} T_{ij} \right) X^{i}}
\]

Where \( Y = \ln[N_{A} < \sigma v >] \), \( T_{ij} \) is the temperature in \( 10^{9} \)K, and \( X = \) atomic number \( Z \)

\[Y = \sum_{i=0}^{2} (C_{00} T_{00}^{0} + C_{01} T_{01}^{1} + C_{12} T_{12}^{2}) X^{i}\]

\( Y = C_{00} T_{00}^{0} X^{0} + C_{01} T_{01}^{1} X^{0} + C_{02} T_{02}^{2} X^{0} + C_{10} T_{10}^{0} X^{1} + C_{11} T_{11}^{1} X^{1} + C_{12} T_{12}^{2} X^{1} + C_{20} T_{20}^{0} X^{2} + C_{21} T_{21}^{1} X^{2} + C_{22} T_{22}^{2} X^{2}\)

Where \( (C_{00}, C_{01}, C_{02}, C_{10}, C_{11}, ..., C_{22}) \) are free parameters and their values are shown in the below:

\[
\begin{bmatrix}
C_{00} & C_{01} & C_{02} \\
C_{10} & C_{11} & C_{12} \\
C_{20} & C_{21} & C_{22} \\
\end{bmatrix}
= 
\begin{bmatrix}
-449.15 & 43.22 & 0.46 \\
36.068 & -3.3255 & -0.0387 \\
-0.7128 & 0.0672 & 0.0006 \\
\end{bmatrix}
\]

\[R^2 = 0.9988\]

\[R^2 = 0.9986\]

\[R^2 = 0.9984\]

The values of free parameters of the above and the atomic number \( Z \) of each targets nuclei of each reactions \( ^{48}\)Ti(p,n)\( ^{48}\)V, \( ^{51}\)V(p,n)\( ^{51}\)Cr, \( ^{59}\)Co(p,n)\( ^{59}\)Ni, \( ^{63}\)Cu(p,n)\( ^{63}\)Zn, and \( ^{66}\)Zn(p,n)\( ^{66}\)Ga are substitute into Eq. (53) the following formulae results:

\( ^{48}\)Ti(p,n)\( ^{48}\)V \quad \chi^2 = 0.123

\[\ln[N_{A} (\sigma v) \text{ (cm}^{3} \text{s}^{-1} \text{mol}^{-1})] = -0.1017 T_{00}^{0} + 2.58387 T_{00}^{0} - 0.6492\]

\( ^{51}\)V(p,n)\( ^{51}\)Cr \quad \chi^2 = 2.098

\[\ln[N_{A} (\sigma v) \text{ (cm}^{3} \text{s}^{-1} \text{mol}^{-1})] = -0.1127 T_{00}^{0} + 2.28237 T_{00}^{0} + 3.3428\]

\( ^{59}\)Co(p,n)\( ^{59}\)Ni \quad \chi^2 = 0.362

\[\ln[N_{A} (\sigma v) \text{ (cm}^{3} \text{s}^{-1} \text{mol}^{-1})] = -0.1475 T_{00}^{0} + 2.42037 T_{00}^{0} + 5.0548\]

\( ^{63}\)Cu(p,n)\( ^{63}\)Zn \quad \chi^2 = 0.719

\[\ln[N_{A} (\sigma v) \text{ (cm}^{3} \text{s}^{-1} \text{mol}^{-1})] = -0.1577 T_{00}^{0} + 3.2957 T_{00}^{0} - 2.6428\]

\( ^{66}\)Zn(p,n)\( ^{66}\)Ga \quad \chi^2 = 0.517

\[\ln[N_{A} (\sigma v) \text{ (cm}^{3} \text{s}^{-1} \text{mol}^{-1})] = -0.1617 T_{00}^{0} + 3.9357 T_{00}^{0} - 8.63\]

These above formulae has been used to calculate the thermonuclear reaction rates \( N_{A} < \sigma v > \) in natural logarithm for each of the \( ^{48}\)Ti(p,n)\( ^{48}\)V, \( ^{51}\)V(p,n)\( ^{51}\)Cr, \( ^{59}\)Co(p,n)\( ^{59}\)Ni, \( ^{63}\)Cu(p,n)\( ^{63}\)Zn, and \( ^{66}\)Zn(p,n)\( ^{66}\)Ga reactions and compared with the adopted thermonuclear reaction rates calculated from the fitting expressions of Eqs. (42, 43, 47, 49, and 50) respectively which are in a good agreement, the obtained data are presented in table (6).
Table 6. Comparison between polynomial fitting expressions (Best Fitting) of the adopted thermonuclear reaction rates with those calculated from Eqs. (54) to (58).

| T9 (109 K) | \( ^{40}\text{Ti}(p,n)^{40}\text{V} \) | \( ^{51}\text{V}(p,n)^{51}\text{Cr} \) | \( ^{59}\text{Co}(p,n)^{59}\text{Ni} \) | \( ^{63}\text{Cu}(p,n)^{63}\text{Zn} \) | \( ^{66}\text{Zn}(p,n)^{66}\text{Ga} \) |
|------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 6          | 10.899±0.411                    | 11.218                          | 12.213±0.102                    | 13.703±1.018                    | 11.984±0.819                    |
| 6.25       | 11.318±0.427                    | 11.554                          | 13.391±0.103                    | 13.895±1.032                    | 12.324±0.843                    |
| 6.5        | 11.711±0.441                    | 11.878                          | 13.560±0.105                    | 14.067±1.045                    | 12.641±0.864                    |
| 6.75       | 12.080±0.455                    | 12.190                          | 13.721±0.106                    | 14.217±1.056                    | 12.939±0.885                    |
| 7          | 12.423±0.468                    | 12.488                          | 13.874±0.107                    | 14.342±1.065                    | 13.222±0.904                    |
| 7.25       | 12.741±0.480                    | 12.775                          | 14.018±0.108                    | 14.439±1.073                    | 13.492±0.923                    |
| 7.5        | 13.032±0.491                    | 13.048                          | 14.153±0.109                    | 14.507±1.078                    | 13.753±0.940                    |
| 7.75       | 13.296±0.501                    | 13.309                          | 14.280±0.110                    | 14.542±1.080                    | 14.006±0.958                    |
| 8          | 13.533±0.510                    | 13.557                          | 14.398±0.111                    | 14.545±1.081                    | 14.251±0.974                    |
| 8.25       | 13.744±0.518                    | 13.793                          | 14.509±0.112                    | 14.516±1.078                    | 14.489±0.991                    |
| 8.5        | 13.932±0.525                    | 14.016                          | 14.613±0.113                    | 14.454±1.074                    | 14.718±1.006                    |
| 8.75       | 14.101±0.531                    | 14.226                          | 14.711±0.113                    | 14.363±1.067                    | 14.937±1.021                    |
| 9          | 14.259±0.537                    | 14.424                          | 14.808±0.114                    | 14.755                          | 15.248±1.058                    |
| 9.25       | 14.413±0.543                    | 14.609                          | 14.906±0.115                    | 14.811                          | 15.344±1.049                    |
| 9.5        | 14.576±0.549                    | 14.782                          | 15.011±0.116                    | 14.853                          | 15.506±1.060                    |
| 9.75       | 14.764±0.556                    | 14.942                          | 15.128±0.117                    | 14.882                          | 15.653±1.070                    |
| 10         | 14.995±0.565                    | 15.089                          | 15.264±0.118                    | 14.896                          | 15.770±1.078                    |
5. Conclusions
1- The astrophysical S-factor, S(E), was starting with increase and then decrease irregularly (fluctuate) by increasing the center of mass energy, this because of Coulomb barrier penetration \( \exp(2\pi\eta) \).
2- The astrophysical S-factor increased with increasing atomic number \( Z \) of target nuclei at fixed center of mass energy.
3- The thermonuclear reaction rates, \( N_A\langle\sigma v\rangle \), were increased with increasing \( T_9 \) because by increasing the \( T_9 \) the Charged interacting particles need to overcome the existing Coulomb barrier.
4- The thermonuclear reaction rates decreased with increasing atomic number \( Z \) of target nuclei at fixed \( T_9 \) because as \( Z \) increase Coulomb barrier increased.
5- The astrophysical S-factor and Thermonuclear reaction rates calculated in the present work are in good agreement with those measured previously by other works.

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