Multiphysic Simulation of molten pool transients for electron beam welding and additive manufacturing processes investigation

A V Shcherbakov, D A Gaponova, A P Sliva, A L Goncharov, A V Gudenko, R V Rodyakina, V A Kostin and V K Dragunov

National Research University “Moscow Power Engineering Institute”, 14, Krasnokazarmennaya str., Moscow, 111250, Russia

E-mail: ShcherbakovAV@mpei.ru

Abstract. Molten pool free surface shape calculation and its evaluation in time for analysis of electron beam melting and welding processes is a very important problem. Unfortunately, experimental methods can provide only a part of the necessary information. Electron beam heating creates the conditions for intense metal transfer, and pool surface shape affects the energy absorption efficiency. This paper reports the method which combines the Monte-Carlo simulation for spatial and energy parameters of heating source calculating with the Volume of Fluid algorithm for liquid free-surface numerical evaluation. The velocity and pressure field technique calculation in a molten pool with the action of surface tension forces and vapor recoil pressure is described. It is shown that the proposed simulation method allows to study heat and mass transfer transient processes, including deep penetration channel formation. Model validation was carried out. Simulation results are shown to be consistent with experimental data obtained using a high-speed camera.

1. Introduction

Investigation of heat and mass transfer processes in a molten pool is extremely important for processes such as electron beam welding and additive manufacturing. This is confirmed by a large number of publications devoted to the simulation of heat and mass transfer for the mentioned technologies [1,2,3], which is also connected with electron beam peculiarities as a heating source. The high intensity of heat source generated by electron beam – metal interaction leads to a high vapor recoil pressure, which deforms the surface of molten pool. And even if the beam intensity is lower, the spatial distribution of heating source will depend on the surface shape of heated object, which is due to the electrons backscattering. Semi-empirical methods have already been proposed to take into account the influence of product shape on the heat source volume distribution [4]. In a keyhole mode welding [5], when a vapor-gas channel arises due to the action of recoil pressure, the absorption of beam energy increases due to re-scattering of electrons inside the channel. Beam oscillations [6,7,8] are often used in the process of wire-based additive manufacturing [6]. When electron beam interacts with wire surface and formed bead, beam energy absorption will also change. Vapor recoil pressure during beam oscillations is also a driving factor for additive production: a directed metal flow is formed, which ensures the formation of a higher and narrower bead [9]. All these facts indicate that when studying heat and mass transfer processes during electron beam welding, melting, and additive
manufacturing, it is necessary to take into account the scattering and re-scattering of electrons from molten pool surface. Since experimental study of these processes is difficult and it is completely impossible to obtain information on heating source distribution for the complex product surface, it is advisable to use methods of mathematical modeling, which will be described below.

2. Heat and mass transfer model description
First of all, heat and mass transfer model must be described. The model was implemented in a three-dimensional space using the control volume method. Figure 1, a, shows geometrical representation of solid/liquid media used in the model. In the centers of cubic cells, scalar values, such as temperature, pressure, cell filling degree, are calculated. On the other hand, in the faces of cubic cells, vector values, such as heat fluxes, flow velocities and fluid flows are calculated (due to energy and liquid mass exchange between the cells occurs just on their faces). Therefore, the model uses “staggered” grid described in the literature [10], and the size of the cells (parameter \( h \) in figure 1, a) assumed to be equal for all of the cells (regular grid is used) and for all directions.

To describe heat transfer in the solid phase, a numerical solution of heat equation was used. Since thermo-physical characteristics of material depend on temperature, heat transfer problem was solved in a nonlinear formulation:

\[
cp \frac{dT}{dt} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + Q_v - \frac{\partial \psi}{\partial t} L_m \rho,
\]

where \( T \) is temperature, \( K; \tau \) is time, \( \sigma \) is specific heat of metal, \( J/(\text{kg} \cdot \text{K}); \rho \) is density of the material, \( \text{kg/m}^3; \lambda \) is thermal conductivity, \( \text{W/(m} \cdot \text{K)}, Q_v \) is volume heat source density, \( \text{W/m}^3 \). For melting, the expenditure of additional energy is required (and energy is released during crystallization). To describe the Stefan problem [11], the following equations were used:

\[
\psi(T) = \begin{cases} 
0, & T < T_S \\
\frac{T-T_S}{T_L-T_S}, & T_S < T < T_L \\
1, & T > T_L
\end{cases}
\]

where \( \psi \) is melt fraction in two-phase region, \( T_S \) is solidus temperature and \( T_L \) is liquidus temperature.

To calculate the velocity and pressure fields, the Navier-Stokes equations for the Newtonian fluid were numerically solved:

\[
\frac{\partial v_x}{\partial \tau} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \frac{f_{sur,x}}{\rho}
\]

\[
\frac{\partial v_y}{\partial \tau} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) + \frac{f_{sur,y}}{\rho},
\]

\[
\frac{\partial v_z}{\partial \tau} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \frac{f_{sur,z}}{\rho} + g
\]

where \( p \) is pressure, \( \text{Pa; } v \) is velocity, \( \text{m/s, } \mu \) is viscosity, \( \text{m}^2/\text{s}; f_{sur} \) is volume equivalent of the resultant forces acting on the free surface of molten pool, \( \text{N/m}^3 \) and \( g \) is the gravity acceleration, \( \text{m/s}^2 \). To satisfy the continuity condition for an incompressible fluid
\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0, \]  

(4)

the iterative predictor-corrector method was used. The pressure field satisfying the incompressibility condition is described by the Poisson equation

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\rho}{\Delta t} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right). \]  

(5)

The last equation is a second-order elliptic differential equation and it is solved numerically for selected time step \( \Delta t \) until velocity field satisfies the incompressibility condition. The temperature field in the liquid was calculated using the heat transfer equation

\[ c_\rho \left( \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + Q_y. \]  

(6)

An extremely difficult task is to simulate the motion of molten pool free surface. To solve it, the volume of fluid method [12] was used. If we denote the fraction of control volume as \( L (0 \leq L \leq 1) \), then we can write

\[ \frac{\partial L}{\partial t} + v_x \frac{\partial L}{\partial x} + v_y \frac{\partial L}{\partial y} + v_z \frac{\partial L}{\partial z} = 0. \]  

(7)

The last equation is solved by numerical methods, but one must take into account that \( L \) cannot be negative or greater than 1. To ensure these conditions, the donor-acceptor method [12] was implemented. The surface tension force and vapor recoil pressure force act on the free surface of liquid.

The main factor in the surface tension force is free surface curvature, which can be defined as divergence of normal vectors field. The product of curvature and surface tension coefficient \( \sigma \), N/m, gives the additional pressure \( p_{st} \) that occurs on liquid surface [13]

\[ p_{st} = \sigma \cdot \left( -\nabla \cdot \frac{\nabla L}{|\nabla L|} \right), \]  

(8)

where the tilde sign means that the smoothed \( L \)-field is used. For vapor recoil pressure on the molten pool surface \( \Gamma \) (where \( \nabla L \neq 0 \)) we can use the expression

\[ p_{rec} = 133 \cdot 10^A - B, \]  

(9)

where A and B are evaporation constants (these constants, or vapor pressures for different materials are given in literature [14]). Since the pressures created by surface forces are known (see equations (8)–(9)), it is possible to calculate the volume equivalent of these forces by the expression

\[ f_{sur} = -\nabla p|_{(x,y,z)\in \Gamma}. \]  

(10)
Figure 1. Control volumes with a shifted grid (a) and graphical representation of electron trajectories (b) in the developed model.

To visualize free surface of molten pool, the Simple Line Interface Calculation (SLIC) method was used [15]. Discretization of differential equations (1), (3), (5), (6)–(8) and (10) was carried out on the basis of the finite difference method (FDM) and fully explicit Euler scheme. For example, the numerical form of heat transfer equation (6) can be written as follows:

\[
T_{(x,y,z)}^{t+1} = T_{(x,y,z)}^t + \Delta t \left( \frac{\lambda}{c_p} \frac{T_{(x+1, y, z)}^t - 2T_{(x, y, z)}^t + T_{(x-1, y, z)}^t}{h^2} + \frac{T_{(y+1, x, z)}^t - 2T_{(x, y, z)}^t + T_{(y-1, x, z)}^t}{h^2} + \frac{T_{(z+1, x, y)}^t - 2T_{(x, z, y)}^t + T_{(z-1, x, y)}^t}{h^2} - \frac{v_{x(x,y,z)}T_{(x+1/2, y, z)}^t - T_{(x-1/2, y, z)}^t}{h} - \frac{v_{y(x,y,z)}T_{(x, y+1/2, z)}^t - T_{(x, y-1/2, z)}^t}{h} - \frac{v_{z(x,y,z)}T_{(x, y, z+1/2)}^t - T_{(x, y, z-1/2)}^t}{h} + \frac{Q_v}{c_p} \right).
\]
where \( x, y, \) and \( z \) are the indices of computational grid nodes. Since the staggered grid is used, linear interpolation is required to calculate velocity projections in these grid nodes:

\[
\begin{align*}
\nu^t_x(x,y,z) &= \frac{\nu^t_{x(x+1/2,y,z)} + \nu^t_{x(x-1/2,y,z)}}{2} \\
\nu^t_y(x,y,z) &= \frac{\nu^t_{y(x,y+1/2,z)} + \nu^t_{y(x,y-1/2,z)}}{2} \\
\nu^t_z(x,y,z) &= \frac{\nu^t_{z(x,y,z+1/2)} + \nu^t_{z(x,y,z-1/2)}}{2}.
\end{align*}
\] (12)

And the same interpolation is required to calculate temperature in “fractional” nodes of equation (11). For example,

\[
T^t_{(x+1/2,y,z)} = \frac{T^t_{(x,y,z)} + T^t_{(x+1,y,z)}}{2},
\] (13)

or

\[
T^t_{(x-1/2,y,z)} = \frac{T^t_{(x,y,z)} + T^t_{(x-1,y,z)}}{2}.\] (14)

The numerical implementation of predictor-corrector scheme implies calculating velocity projections at the predictor step, and then estimation of pressure field, satisfying the continuity condition. The corrector step consists in recalculating velocity field under the action of calculated pressure field.

The very first step is following. The pressure field is unknown, so at the first step it is not taken into account and we assume that the fluid flows only under the action of gravity. When using the finite-difference approximation of differential operators, equation (3) is written in projection form, and each of three velocity projections is calculated separately. So, for the predictor step with index \( k \), we can use the expressions (15)–(17).

\[
\begin{align*}
\nu^t_{x(x,y,z)} &= \nu^t_{x(x,y,z)} + \Delta t \left( -\nu^t_{x(x,y,z)} \frac{\nu^t_{x(x+1/2,y,z)} - \nu^t_{x(x-1/2,y,z)}}{h} - \\
-\nu^t_{y(x,y,z)} \frac{\nu^t_{x(x,y+1/2,z)} - \nu^t_{x(x,y-1/2,z)}}{h} - \nu^t_{z(x,y,z)} \frac{\nu^t_{x(x,y,z+1/2)} - \nu^t_{x(x,y,z-1/2)}}{h} + \\
+ \mu \left( \frac{\nu^t_{x(x+1,y,z)} - 2\nu^t_{x(x,y,z)} + \nu^t_{x(x-1,y,z)}}{h^2} + \frac{\nu^t_{x(x,y+1,z)} - 2\nu^t_{x(x,y,z)} + \nu^t_{x(x,y-1,z)}}{h^2} + \\
+ \frac{\nu^t_{x(x,y,z+1)} - 2\nu^t_{x(x,y,z)} + \nu^t_{x(x,y,z-1)}}{h^2} + \frac{\nu^t_{x(x,y+1,z)} - 2\nu^t_{x(x,y,z)} + \nu^t_{x(x,y-1,z)}}{h^2} \right) + \rho f^t_{ext(x,y,z)} \right)
\end{align*}
\]
\[\begin{align*}
v^{(t+1)k}_{y(x,y,z)} &= v^t_{y(x,y,z)} + \Delta t \left( -v^t_{x(x,y,z)} \frac{v^t_{y(x+\frac{1}{2},y,z)} - v^t_{y(x-\frac{1}{2},y,z)}}{h} - v^t_{y(x,y,z)} \frac{v^t_{y(x,\frac{1}{2},z)} - v^t_{y(x,-\frac{1}{2},z)}}{h} \right) + \\
-\nu^t_{y(x,y,z)} \frac{v^t_{y(x+\frac{1}{2},z)} - v^t_{y(x-\frac{1}{2},z)}}{h} - v^t_{z(x,y,z)} \frac{v^t_{x(x,y,\frac{1}{2})} - v^t_{x(x,y,-\frac{1}{2})}}{h} + \\
+\mu \left( v^t_{y(x+1,y,z)} - 2v^t_{y(x,y,z)} + v^t_{y(x-1,y,z)} \right) + v^t_{y(x,y+1,z)} - 2v^t_{y(x,y,z)} + v^t_{y(x,y-1,z)} + \\
+ v^t_{y(x,y,z+1)} - 2v^t_{y(x,y,z)} + v^t_{y(x,y,z-1)} + \frac{1}{\rho} f^t_{\text{sur,y}(x,y,z)} \right) \\
\frac{v^{(t+1)k}_{z(x,y,z)}}{h} &= v^t_{z(x,y,z)} + \Delta t \left( -v^t_{x(x,y,z)} \frac{v^t_{z(x+\frac{1}{2},y,z)} - v^t_{z(x-\frac{1}{2},y,z)}}{h} - v^t_{z(x,y,z)} \frac{v^t_{z(x,\frac{1}{2},z)} - v^t_{z(x,-\frac{1}{2},z)}}{h} \right) + \\
-\nu^t_{z(x,y,z)} \frac{v^t_{z(x+\frac{1}{2},z)} - v^t_{z(x-\frac{1}{2},z)}}{h} + \mu \left( v^t_{z(x+1,y,z)} - 2v^t_{z(x,y,z)} + v^t_{z(x-1,y,z)} \right) + v^t_{z(x,y+1,z)} - 2v^t_{z(x,y,z)} + v^t_{z(x,y-1,z)} + \\
+ v^t_{z(x,y,z+1)} - 2v^t_{z(x,y,z)} + v^t_{z(x,y,z-1)} + \frac{1}{\rho} f^t_{\text{sur,z}(x,y,z)} + g \right) \\
\end{align*}\]

Then the scalar pressure field is calculated using equation (5). In the case of using an explicit difference scheme to calculate pressure at the \((k+1)\)-th iteration, we can use the expression

\[\begin{align*}
\frac{p^{(t+1)k+1}_{(x,y,z)}}{h} &= \frac{1}{6} \left( \frac{p^{(t+1)k}_{(x+1,y,z)} + p^{(t+1)k}_{(x-1,y,z)} + p^{(t+1)k}_{(x,y+1,z)} + p^{(t+1)k}_{(x,y-1,z)} + p^{(t+1)k}_{(x,y,z+1)} + p^{(t+1)k}_{(x,y,z-1)}}{h^2} + h^2 \rho \right) \times \\
\frac{v^{(t+1)k}_{x(x+\frac{1}{2},y,z)} - v^{(t+1)k}_{x(x-\frac{1}{2},y,z)}}{h} + \frac{v^{(t+1)k}_{y(x,y+\frac{1}{2},z)} - v^{(t+1)k}_{y(x,y-\frac{1}{2},z)}}{h} + \\
+ \frac{v^{(t+1)k}_{z(x,y,z+\frac{1}{2})} - v^{(t+1)k}_{z(x,y,z-\frac{1}{2})}}{h} + \left( \frac{v^{(t+1)k}_{x(x+1,y,z)} - v^{(t+1)k}_{x(x,y,z+1)}}{h} + \frac{v^{(t+1)k}_{x(x-1,y,z)} - v^{(t+1)k}_{x(x,y,z-1)}}{h} \right) \right) \right) \\
\end{align*}\]

After calculating the pressure field, it is necessary to correct the velocity field, i.e. to implement “corrector” step. For example, the velocity projection on the x-axis at the \((k+1)\)-th iteration is calculated using the expression
\[ v^{(t+1)}(x,y,z) = v^{(t)}(x,y,z) \frac{1}{\rho} \left( \frac{p^{(t+1)}_{x+1,y+1,z} - p^{(t+1)}_{x-1,y-1,z}}{h} \right) \Delta t. \] (19)

The donor-acceptor algorithm for solving equation (7) and numerical algorithms for calculating gradients and divergence in equations (8) and (10) were also based on the FDM method. Using the terms in figure 1 for the velocity projections on the centers of control volume cells’ faces, we can write

\[ L^{t+1}_{x,y,z} = L^t_{x,y,z} + \Delta t \times \left( \frac{L^t_{x-1,y,z} v^t_{left} - L^t_{x,y,z} v^t_{right}}{h} + \frac{L^t_{x,y-1,z} v^t_{down} - L^t_{x,y,z} v^t_{up}}{h} \right) + \frac{L^t_{x,y,z-1} v^t_{front} - L^t_{x,y,z} v^t_{front}}{h} = L^t_{x,y,z} + \Delta t \cdot \Omega. \] (20)

These calculations must be carried out iteratively for time step selection to avoid the control volume fraction function out of range 0 ... 1.

3. Monte-Carlo electron scattering simulation

The next and most important feature of this model is a method for calculating electron beam interaction with metal. In most cases, the heating source is taken as a heat flux distributed over the molten pool surface described by a Gaussian function. Such an approach does not allow one to consider important effects, such as electron re-scattering inside a deep penetration channel, or cavity. In the process of additive manufacturing, electron scattering by the surface of a product or wire also plays a large role for heating efficiency. To consider electron re-scattering processes, it is proposed to use the Monte-Carlo method for Lagrangian particles moving in stationary Eulerian cells.

At this stage, electrons are believed to pass through a space which is not occupied by solid or liquid metal without losses and along straight paths. The Monte-Carlo method is excellent for studying the scattering of electrons passing through a gaseous medium, such as metal vapor steam. However, at this stage, electron scattering in a gaseous medium was not considered.

To implement the Monte-Carlo method for electron transport in condensed matter (solid and liquid metal), the well-known single scattering model was used [16]. The decision to use such a model was made taking into account the fact that stopping power changes insignificantly when metal transforms from a solid state to a liquid one. Indeed, published studies [17, 18] show that stopping power is primarily determined by medium density and characteristics of scattering centres; therefore, the violation of long-range order, occurring during melting, has little effect on the transport properties. Another assumption is the use of one-element model of medium at the current stage of research. The studies were carried out for steels with the Fe as a main element; therefore, such a simplification also seems to be acceptable. The stopping power was described by the Bethe equation [19] as

\[ \frac{dE}{ds} = -2\pi e^4 N \frac{Z\rho}{AE} \ln \left( \frac{1.166E}{J} \right), \] (21)

where \( E \) is initial electron energy, keV; \( s \) is electron path, cm; \( e \) is the electron charge, C; \( N \) is the Avogadro’s number, \( N = 6.022\times10^{23} \), mole\(^{-1}\); other parameters relate to material are following: \( Z \) is atomic number, \( \rho \) is density, g/cm\(^3\); \( A \) is atomic weight, g/mole, \( J \) is mean ionization potential, keV.
\[
J = (9.76Z + 58.5Z^{0.19}) \cdot 10^{-3}.
\]  

(22)

According to the accepted approximation [16,19], it was assumed that only acts of elastic scattering led to the deviation of electrons at significant angles. The elastic scattering cross section was calculated by the expression

\[
\sigma_E = 5.21 \cdot 10^{-21} \frac{Z^2}{E^2} \cdot \frac{4\pi}{\alpha(1 + \alpha)} \left( \frac{E + m_0c^2}{E + 2m_0c^2} \right)^2,
\]

(23)

where \(m_0\) is electron rest mass, \(kg\); \(c\) is the speed of light; \(m/s\) \((m_0c^2 = 511 \text{ keV})\). If we substitute the value of \(E\) in keV into this expression, the resulting cross section \(\sigma_E\) will be in \(cm^2\). Value \(\alpha\) is the shielding factor determined by the Bishop formula \((E\) must be also substituted in keV)

\[
\alpha = 3.4 \cdot 10^{-3} \frac{Z^{0.67}}{E}.
\]

(24)

Using the parameters described above, it is possible to calculate electron mean free path in the alloy between acts of elastic scattering [19].

\[
\lambda_E = \frac{A}{N \cdot \rho \cdot \sigma_E}.
\]

(25)

Electron path between the elastic scattering events in accordance with the Monte Carlo method is determined by the expression

\[
s_E = -\lambda_E \cdot \ln R_1,
\]

(26)

Finally, we must obtain expressions for calculating scattering angle \(\theta\) and azimuthal angle \(\psi\) (figure 1, b). The equation for evaluating scattering angle was derived by Newbury et al. [21] and Joy [19] from the angular differential form of Rutherford cross-section

\[
\cos \phi = 1 - \frac{2\alpha R_2}{1 + \alpha - R_2}.
\]

(27)

Azimuthal scattering angle is determined completely randomly:

\[
\psi = 2\pi R_3.
\]

(28)

In the last three expressions, \(R_1, R_2\) and \(R_3\) are independent random numbers generated by the program in the ranges from 0 to 1.

4. Simulation Results and Validation

Based on the expressions and descriptions given in sections 2 and 3, a computer program was developed in the Microsoft Visual Studio. The processes of electron beam welding and wire-based additive manufacturing were simulated. In this work, all processes were simulated for AISI 316L stainless steel as a work-piece and feed material. Properties of 316L stainless steel are shown in figure 2. \(T_S\) and \(T_L\) are solidus and liquidus temperatures, respectively.

Electron beam was simulated by sequentially calculating 800 electron trajectories with an initial energy of 60 keV. For each particle with a Lagrangian coordinate system, at each rectilinear segment of the trajectory, the stopping power (equation 21) was used to calculate the fraction of energy absorbed in the control volume in which this segment of trajectory is located. The energy losses of all simulated electrons were summed up in the data array, and in this way the volumetric distribution of heat flux was calculated. To obtain a beam current density distribution close to normal (Gaussian), coordinates in the \(xy\)-plane were specified in accordance with the expressions
\[ x_n = h \cdot 0.018 \cdot n^{0.8} \cdot \cos \left( \frac{n}{\sqrt{n+1}} \right) + x_{pos}, \]
\[ y_n = h \cdot 0.018 \cdot n^{0.8} \cdot \sin \left( \frac{n}{\sqrt{n+1}} \right) + y_{pos}, \]

where \( x_{pos} \) and \( y_{pos} \) are the current coordinates of the beam center. Figure 3 shows the initial \( xy \)-coordinates of 800 electron trajectories calculated during simulation.

\[ \text{Figure 3.} \]

Figure 2. Properties of 316L stainless steel vs temperature: a – density (\( \rho \)), specific heat (\( c \)) and heat conductivity (\( \lambda \)); b – vapour recoil pressure (\( P_{\text{rec}} \)), viscosity (\( \mu \)) and surface tension (\( \sigma \)).

Figure 4 represents a model of thin-walled product electron beam penetration process. Product dimensions are 0.6×1.5×2.5 mm. Electron beam moved along \( y \)-axis with a speed of 70 mm/s (250 m/h). The high selected value of welding speed (typical welding speeds for steels are 30-100 m/h) was due to the need to reduce the calculation time for observing the initial process of keyhole penetration channel formation under the influence of vapour pressure. Total beam power was 340 W.
Figure 3. Initial xy-coordinates of 800 electron trajectories of the beam simulated

It can be seen that the size of molten pool is continuously increasing. At the moment of time $t = 0.003$ s (figure 4, b), a small cavity is formed on the pool surface, but then it increases in depth (figure 4, c) at a time moment $t = 0.006$ s. And finally, the recoil pressure forces lead to through penetration of the plate (figure 4, d) at a time moment $t = 0.008$ s. As the beam moves, the displacement of metal from vapor pressure zone becomes more intense in the rear part of the pool and a vortex flow arises from the bottom of the channel to its rear wall, which makes the formed channel narrower. Streamlines are shown in figure 4, c.

Backscattering and re-scattering of electrons influence on the parameters of heat source is extremely important question. It is obvious that deepening of penetration channel will lead to more efficient absorption of beam electrons. Electrons will be reflected inside the channel in the same way as in the Faraday cup. The time dependence of the ratio of power absorbed in the product material $P_a$ to total beam power $P_b$ calculated using the Monte Carlo method is shown in figure 5. It can be seen that during channel formation and transition to through penetration, the amount of absorbed power increases by about 10%, and can be up to 97.5% of the total beam power. It is clear that during the welding process this amount may periodically decrease due to the channel overlap by liquid metal, or due to the passage of some electrons through the product. These phenomena can also be investigated using the proposed model.

At this stage of research, the proposed model was verified only for one mode of electron-beam surfacing (the model was not verified for welding processes). Figure 6 shows a comparison of electron-beam surfacing modeling results with the images, obtained using a high-speed camera VideoSprint. Figure 6, a, shows a longitudinal section of the bead being formed. This mode of bead formation is obtained for surfacing rate (speed of surface moving) $V_{surf} = 0.5$ m/min and feed rate (speed of wire moving) of $V_{wire} = 6$ m/min. The beam power is 2.1 kW. The wire feed speed is usually 3–10 times higher than the deposition rate. It can be seen that a vortex flow is formed in the tail part of the pool, which is deteriorated by the difference in the flow and motion velocities of the substrate, as well as by the action of surface tension forces and thermo-capillary forces (arising from the temperature dependence of surface tension). During surfacing, the electron beam oscillates in a circle, which in practice is necessary to create a sufficiently large heating spot. Figure 6, b shows molten pool shape in cross section. In this plane, vortex flows are also noticeable near the contact of free surface with substrate. Figure 6, c, represents series of images obtained with high-speed camera. The brightest areas show the current position of electron beam.
Figure 4. Penetration channel formation by moving electron beam at different time moments:

\[ a - 0 \text{ s}; b - 0.003 \text{ s}; c - 0.006 \text{ s}; d - 0.008 \text{ s} \]
Figure 5. Time dependence of the ratio of power absorbed in the product material $P_a$ to total beam power $P_b$

It can be seen that molten pool shape corresponds to calculated in the model: under the influence of gravity and vapour recoil pressure forces, liquid metal receives additional acceleration towards the substrate and therefore, cavity forms on the pool surface near the wire. On the contrary, in the rear part of the molten pool hemispherical reinforcement is formed. This is due to the action of surface tension forces and presence of an excess liquid metal volume, since $V_{wire}$ is greater than $W_{surf}$. In the process of additive manufacturing, the pool lengthens and a quasi-stationary mode of liquid metal transfer obtains. In this mode, oscillations of liquid surface can be observed near the melting wire region; in the tail part of the pool, the transfer process is stationary.

Conclusions
1. A mathematical model of heat and mass transfer processes under electron beam heating has been proposed. The model includes the numerical solution of the Navier-Stokes equations for the Euler coordinate system and the Statistical method for calculating electron trajectories in the Lagrangian coordinate system. It is shown that such a solution makes it possible to take into account effects of scattering and re-scattering of electrons during deep penetration channel formation in electron-beam welding.
2. Using the model, the process of penetration channel formation in a thin-walled part (0.6 mm) was studied. It was shown that as a result of electron re-scattering in the channel, beam power fraction absorbed by the channel can increase up to 97.5%.
3. Preliminary verification of developed mathematical model for the process of wire-based electron-beam additive manufacturing has been carried out. By comparing the simulation results and the images obtained using a high-speed camera, a satisfactory coincidence between the nature of metal transfer and molten pool surface shape is substantiated.

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Figure 6. Results of electron-beam surfacing process modeling (a and b), and frames of the process high-speed filming (c)
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