Quark Confinement in Multi-Flavor Quantum Chromodynamics*

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It is investigated how quark confinement depends on the number of flavors, \( N_f \), in QCD on the lattice with Wilson quarks. We strengthen and extend the conclusion reported at Lattice 91: (1) For \( N_f \leq 6 \) the finite temperature deconfining transition/crossover line crosses the chiral limit at finite \( \beta \). We identify the crossing point for \( N_f = 2 \) and \( 6 \) on a \( T = 4 \) lattice, where \( T \) is the lattice size in the temporal direction. We find the phase transition at the crossing point is continuous for \( N_f = 2 \), while it is of first order for \( N_f = 6 \). (2) For \( N_f \geq 7 \), the \( T \)-independent deconfining transition observed at \( \beta = 0 \) extends up to \( \beta = 4.5 \) with the critical quark mass \( m_{\text{quark}} = O(a^{-1}) \), where \( a \) is the lattice spacing.

1. Introduction

It is well known that if the number of flavors \( N_f \) exceeds 17, asymptotic freedom of QCD is lost. Then a natural question is whether or not there is a constraint on the number of flavors for other fundamental properties of QCD: quark confinement and spontaneous breakdown of chiral symmetry. To investigate this problem, we study the phase structure of lattice QCD with degenerate \( N_f \) Wilson quarks in the space of 4 parameters: \( N_f \), \( \beta \) (gauge coupling), \( K \) (hopping parameter), and \( T \) (lattice size in the temporal direction). For \( T = 4 \), 6, 8, the spatial lattice is chosen to be \( 8^2 \times 10 \), while for \( T = 18 \), \( 18^2 \times 24 \). We generate gauge configurations by the hybrid-molecular-dynamics R algorithm with molecular dynamics time step \( \Delta \tau = 0.01 \), unless otherwise stated.

In this article we report the results obtained after Lattice 91, with brief description of those \[1\] reported at Lattice 91 to make this article self-contained.

2. \( \beta = 0 \) case

For \( N_f \geq 7 \), a strong first order phase transition occurs when we vary the hopping parameter. We denote the transition point as \( K_d \). The behavior of physical quantities implies that quarks are deconfined and chiral symmetry is recovered for \( K > K_d \). This transition is remarkably stable under the increase of \( T \) up to 18. Therefore the transition at \( K_d \) is a \( T \)-independent bulk transition. Thus when \( N_f \) exceeds 7, quarks are not confined and chiral symmetry is not spontaneously broken for light quarks even in the strong coupling limit \[1,3\].

For \( N_f = 6 \), on the other hand, we conclude that quarks are confined in the chiral limit \( K_c \) (\( K_c = 0.25 \) at \( \beta = 0 \)) from the following facts: 1) The number of iterations for the quark matrix inversion (by CG), \( N_{\text{inv}} \), increases and exceeds 10,000 at the first stage of the simulation for \( N_f = 6 \) at \( T = 4 \) with \( K = K_c \). This is in clear contrast with the case of \( N_f \geq 7 \), where \( N_{\text{inv}} \) is \( O(100) \). This difference can be attributed to the existence/absence of zero eigenvalues of the quark matrix \[3\]. We will use this difference later as an indicator to discriminate the two phases. 2) The finite temperature deconfining/crossover transition line \( K_c(\beta) \) on the \( T = 4 \) lattice crosses the chiral limit line \( K_c(\beta) \) at finite \( \beta \) as discussed in detail below.

3. \( \beta > 0 \) case (i) \( N_f \leq 6 \)

The u and d quarks are very light in the energy scale of QCD and they are confined. Therefore, we expect that quarks are confined in the chiral limit

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the cases of dynamics time. We find a clear difference between on the $K_N$ obtained in this way for terms of $1$ ing phase vanishes with a linear extrapolation in $K$. we have done a series of simulations exactly on the Fukugita et al. several years ago [3]. We have studied this problem both by following the $K$ line to a smaller $\beta$ region and by monitoring the number of CG iterations, $N_{inv}$, on the $K_c$-line.

3.1. $N_f=2$

In Fig.2 are shown the results for hadron masses and $2m_q$ at $\beta = 4.3$ for $N_f = 2$ on the $T = 4$ lattice by the hybrid MC algorithm. Here the quark mass $m_q$ is defined through an axial-vector current Ward identity [5]. We identify the location of $K_t$ by a sudden change of the behavior of physical observables such as $m_{\pi}^2$. $K_c(\beta)$ is defined as the point where $m_{\pi}^2$ in the confining phase vanishes with a linear extrapolation in terms of $1/K$. Fig 2 is the phase diagram obtained in this way for $N_f = 2$ at $T = 4$.

To identify the location of the crossing point, we have done a series of simulations exactly on the $K_c(\beta)$-line. Fig 3 is the result of $N_{inv}$ for $N_f = 2$ on the $K_c$-line as a function of the molecular-dynamics time. We find a clear difference between the cases of $\beta \leq 3.9$ and $\beta \geq 4.0$. Therefore we identify the crossing point $\beta_{ct}$ as $\beta_{ct} \sim 3.9 - 4.0$. This $\beta_{ct}$ is consistent with the linear extrapolation of the $K_t$-line as is shown in Fig.2.

To study the nature of the transition at $\beta_{ct}$, we measure $m_{\pi}^2$ and $m_q$ on the $K_c$-line (Fig.3). When we decrease $\beta$ toward the crossing point $\beta_{ct}$, $m_{\pi}^2$ decreases linearly and is consistent with zero at $\beta_{ct}$. This implies that the chiral phase transition is continuous (second-order or crossover).

For $T = 18$, we find $\beta_{ct} \sim 4.5 - 5.0$. Note that the shift of $\beta_{ct}$ with $T$ is small. This means that if one wants to get a confining chiral limit for $\beta > 5.0$, one needs $T > 18$.

3.2. $N_f=6$

Monitoring $N_{inv}$ in the simulations exactly on the $K_c(\beta)$-line for various $\beta$’s, we find a two-state signal in the time development of $N_{inv}$ at $\beta = 0.3$, depending on the choice of the initial configuration. When the initial configuration is chosen to be a deconfining configuration at $\beta = 0.4$, $N_{inv}$ at $\beta = 0.3$ is quite stable and small ($\sim 600$) and $m_{\pi}^2$ is large. On the other hand, $N_{inv}$ for a trajectory starting from a mixed configuration shows a rapid increase with molecular-dynamics time and in accord with this, $m_{\pi}^2$ decreases with molecular-dynamics time. Upper bound for $m_{\pi}^2$ in this case is given in Fig.4, where the result of $m_{\pi}^2$ and $2m_q$ on the $K_c$-line at $T = 4$ is shown. We identify the crossing point $\beta_{ct} \sim 0.2 - 0.3$, taking account...
of other results also. We see that the results are in clear contrast with the case of $N_f = 6$: $m^2_\pi$ is approximately a constant and large near the crossing point $\beta \sim 0.3 - 0.5$. At $\beta = 0.3$ we have two values for $m^2_\pi$ depending on the initial configuration. The smaller one is consistent with the vanishing of $m^2_\pi$. Therefore the chiral transition is of first order for $N_f = 6$. The phase diagram for $N_f = 6$ at $T = 4$ is shown in Fig. 5.

4. $\beta > 0$ case (ii) $N_f \geq 7$

Repeating similar studies for various $\beta$’s and for various $N_f$’s to that in the case of $\beta = 0$, we find that the gross feature of the phase transition is quite similar to that at $\beta = 0$. As a typical example, are shown the results of plaquette and Polyakov loop at $\beta = 4.5$ for $N_f = 12$ in Fig. 5. As in the case of $\beta = 0$, the deconfining transition point is stable for changing $T$ for large $T$. For $\beta \geq 2.0$, however, we find a small upward $K$ shift of the deconfining transition point with $T$ for small $T$ ($\sim 4$).

Summarizing these results for $N_f \geq 7$, we have the following phase diagram: First we have a $T$-independent deconfining transition line $K_d$, which starts from a finite $K$ at $\beta = 0$ and extends to larger $\beta$. In addition to this line, we have a $T$-dependent deconfining transition/crossover line $K_t$, which reaches the $K_d$-line without crossing the $K_c$-line. With increasing $T$, this $K_t$-line moves toward larger $\beta$. Up to $\beta = 4.5$ for $N_f = 12$ and 7, we find that $K_d$ does not approach $K_c$ with increasing $\beta$ in the sense that the critical $m_q$ at $K_d$ is $O(1)$ in units of the inverse lattice spacing and is rather increasing slightly with $\beta$.

5. Conclusion

We have studied the phase diagram of QCD with $N_f$ degenerate Wilson quarks. For $N_f \leq 6$, the finite temperature deconfining transition/crossover line $K_t$ crosses the chiral limit $K_c$ at finite $\beta$ and shifts toward larger $\beta$ with increasing $T$. At the crossing point, the data imply that the transition is continuous for $N_f = 2$ at $T = 4$, while it is of first order for $N_f = 6$ at $T = 4$. When $N_f$ exceeds 7, we have a $T$-independent deconfining transition line $K_d$ separating the chiral limit $K_c$ from the confining region. The finite temperature transition $K_t$-line reaches the $K_d$-line without crossing the $K_c$-line. Up to $\beta = 4.5$ for $N_f = 12$ and 7, we find that the critical $m_q$ at $K_d$ is $O(1)$ in units of the inverse lattice spacing. If this fact holds in the continuum limit, it implies that quarks are not confined in the continuum limit for $N_f \geq 7$. Therefore it is important.
to study the behavior of the $K_d$-line at larger $\beta$ to conclude the implications in the continuum limit. A similar bulk deconfining transition is reported recently in lattice QCD with staggered quarks for $N_f = 8$ [7]. It is not clear whether the phenomenon reported is related to that observed by us.

The simulations on the $8^2 \times 10 \times T$ ($T = 4\sim 8$) lattices and the $18^2 \times 24 \times T$ ($T=18$) lattice have been performed with HITAC S820/80 at KEK and with QCDPAX at the University of Tsukuba, respectively. We would like to thank members of KEK for their hospitality and strong support and the other members of QCDPAX collaboration for their help. We also thank Sinya Aoki and Akira Ukawa for valuable discussions. This project is in part supported by the Grant-in-Aid of Ministry of Education, Science and Culture (No.62060001 and No.02402003).

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