Departure from the Exact Location of Mean Motion Resonances Induced by the Gas Disk in Systems Observed by Kepler

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Received 2020 July 27; revised 2020 November 24; accepted 2020 November 30; published 2021 January 20

Abstract

The statistical results of transiting planets show that there are two peaks, around 1.5 and 2.0, in the distribution of orbital period ratios. A large number of planet pairs are found near the exact location of mean motion resonances (MMRs). In this work, we find that the depletion and structures of the gas disk play crucial roles in driving planet pairs out of the exact location of MMRs. Under such a scenario, planet pairs are trapped into exact MMRs during orbital migration first and keep migrating at the same pace. The eccentricities can be excited. Due to the existence of a gas disk, eccentricities can be damped, leading to a change in orbital period, which will make planet pairs depart from the exact location of MMRs. With depletion timescales larger than 1 Myr, near-MMR configurations are formed easily. Planet pairs have higher possibilities of escaping from MMRs with a higher disk aspect ratio. Additionally, with a weaker corotation torque, planet pairs can depart farther from the exact location of MMRs. The final location of the innermost planets in the systems are directly related to the transition radius from the optically thick region to the outer optically thin disk. While the transition radius is smaller than 0.2 au at the late stage of the star evolution process, the period of the inner planets can reach around 10 days. Our formation scenario is a possible mechanism for explaining the formation of near-MMR configurations with the innermost planet farther than 0.1 au.

Unified Astronomy Thesaurus concepts: Exoplanet dynamics (490); Exoplanet evolution (491); Exoplanet Formation (492)

1. Introduction

The Kepler mission and its follow-up program K2 have released over 6000 planetary candidates, including ∼2700 confirmed planets and more than 3300 candidates yet to be confirmed (Borucki et al. 2010; Batalha et al. 2013; Mazeh et al. 2013; Fabrycky et al. 2014; Dressing et al. 2017; Kunimoto & Matthews 2020). Among them, there are hundreds of confirmed multiple-planetary systems. The population of discovered planets provides us with a good sample to study the dynamics and formation of planetary systems (Moriarty & Ballard 2016; Gong & Ji 2017; Mills & Fabrycky 2017; Wang & Ji 2017; Gong & Ji 2018; He et al. 2019; Yang et al. 2020; Zhang 2020). From the statistical results, we find that there are plenty of planet pairs in near-mean-motion-resonance (MMR) configuration, especially the first-order resonances, such as 2:1 and 3:2 MMRs (Lissauer et al. 2011; Wang et al. 2012; Lee et al. 2013; Wang & Ji 2014, 2017; Antoniadou & Libert 2020). However, the peaks in the distribution of period ratios are not shown to be at the exact location of MMRs, but at a little distance from them (Wang et al. 2012; Lee et al. 2013; Steffen et al. 2013; Wang & Ji 2017; Wu et al. 2019; Pan et al. 2020). Most of the planet pairs pile up at a location farther than the exact MMR. Near the 3:2 MMR, the maximum fraction of planet pairs can reach ∼3% for the period ratio within an interval of 0.025, while the maximum fraction near 2:1 MMRs is about 2% for the period ratio within an interval of 0.025. The Transiting Exoplanet Survey Satellite (TESS) started operations in July 2018 to detect transiting planets in 85% of the sky (Ricker et al. 2015). To date, there have been more than 80 planets confirmed and over 2000 planetary candidates to be confirmed. TOI-125 and TOI-270, which were confirmed by TESS, are multiple-planetary systems bearing planets in near-MMR configurations (Quinn et al. 2019; Nielsen et al. 2020). More systems with similar configurations are expected to be found by the TESS mission to increase the number of planet pairs in near MMRs.

Several mechanisms have been proposed to explain the formation of configurations close to exact MMRs. A major impact of the tidal effect produces near MMRs for those systems with planets very close to the central star (Papaloizou & Terquem 2010; Lithwick & Wu 2012; Batygin & Morbidelli 2013; Delisle et al. 2014). Lee et al. (2013) analyzed systems with planets less than 2R⊕ in radius and showed that if they are rocky, with Q/k2 ~ 100, planet pairs can form in a near-MMR configuration because of tidal damping. But for planets larger than 2R⊕ or with a distance from the central star farther than 0.1 au where the tidal effect is not strong enough, the formation of a near-MMR configuration is not clear.

A magnetospheric cavity with a one-sided torque disk plays an alternative role in the formation of near-MMR configurations (Koenigl 1991; Liu et al. 2015, 2017; Liu & Ji 2020). The effect of various planet masses, disk accretion rates, stellar magnetic field strengths, and depletion timescales of gas disks was explored. The direction of type I migration at the edge of the cavity is outward. The configuration of planet pairs, which can be trapped into MMRs at the very beginning, will be rearranged. However, in this scenario, a special gas disk is required, and the final period ratio is related to the mass ratio of the two planets.

A late orbital instability after disk depletion has been suggested by Izidoro et al. (2017), Ogihara et al. (2018), and Lambrichs et al. (2019) to explain the formation of a near-MMR configuration. They assumed that planets formed a chain resonance
configuration after slow migration with a timescale of about 1 Myr. The orbital instability during the gas depletion phase will make close-in super-Earths form the observed nonresonant configurations. The final period ratio between planets depends on their masses, but they just investigated the influence of planet mass on the final stage. The planet mass-growth process (Petrovich et al. 2013) or interaction with planetesimals (Chatterjee & Ford 2015) can lead to the formation of near-MMR configurations. However, the depletion timescale and structures of gas disks will affect the amplitude that departs from exact MMRs.

Based on the classical migration model, we present a formation scenario of near-MMR configuration. First of all, planets are formed at the outer region of the system where there is enough material to form terrestrial planets with a couple of Earth masses. Due to the effect between planets and the surrounding gas disk, terrestrial planets will undergo orbital migration (Lin & Papaloizou 1979; Kley & Nelson 2012). During the convergent migration, planet pairs can be captured into exact MMRs (Lee & Peale 2002; Papaloizou & Szuszkiewicz 2005; Pierens & Nelson 2008). If planets migrate to the inner region of the system, the tidal effect raised by the star can destroy the configuration of MMRs and become near-MMR configuration. Based on this formation scenario, we investigated the configuration formation of the KOI-152 system with planets in near 4:2:1 MMRs (Wang et al. 2012), the formation of near 2:1 and 3:2 MMRs brought about by the properties of the star, the speed of type I migration (Wang & Ji 2014), the effect caused by the mass accretion process and the possible outward migration (Wang & Ji 2017), and the existence of giant planets in planetary systems (Sun et al. 2017; Pan et al. 2020).

We analyze the data on the innermost planets in a system farther than 0.1 au. The results are shown in Figure 1 with 1054 planet pairs. We mainly focus on the region near 2:1 and 3:2 MMRs in the period ratio distribution between the planet and its adjacent inner planet in the range of [1.4, 2.2]. Panel (a) shows the fraction distribution. We find that near the exact location of 2:1 MMRs, where a gap is shown between the period ratio of 1.97 and 2.0, few cases are found in this region. A peak appears at the period ratio larger than 2.0, especially in the region between 2.0 and 2.06. The fraction decreases from 0.01 to 0.001 in the region of [2.06, 2.13]. A similar tendency is seen near the exact location of 3:2 MMRs. Few planet pairs are found between the period ratio of [1.4, 1.5]. A peak exists at the period ratio larger than 1.5 and smaller than 1.55. The fraction decreases from 0.015 to 0.004 in the region of [1.55, 1.62]. Another peak appears at about 1.69, which is near the 5:3 MMR. The fraction between 1.63 and 1.68 remains stable and is consistently lower than 0.09. Panel (b) of Figure 1 shows the distribution of the orbital period of the innermost planets versus the period ratio. The periods of most innermost planets are in the range of [10, 100] days, where there are seven planet pairs with the inner planet’s period is larger than 100 days. In this work, our primary goal is to investigate the formation scenario of systems where the semimajor axis of the inner planet is larger than 0.1 au, along with planet pairs in near-exact-MMR configuration.

We suppose that planet pairs can be first captured into exact MMR during the orbital migration process induced by the gas disk. Then, the planet pair will migrate at the same pace, and the eccentricities of the planets can be excited after they are in MMRs. The depletion timescale of the gas disk is estimated to be a million days.
years (Haisch et al. 2001). Here, we assume the gas disk surrounding the star can survive from $10^5$ to $10^7$ yr (Williams & Cieza 2017). If the eccentricity-damping effect is still strong enough, eccentricities that have been excited into MMR will be damped (Choksi & Chiang 2020). Due to the eccentricity damping, the semimajor axis will change a little. Thus, planet pairs will depart from the exact location of MMRs. In this formation scenario, the depletion and structures of gas disks are possible essential factors that influence the final configurations.

The work is structured as follows: in Section 2, we describe the models, including the disk models and numerical methods used in our simulations. The main results of simulations containing some typical cases and the statistical results are summarized in Section 3. Section 4 shows the conclusions and discussions.

2. Models

2.1. Disk Models

The surface density profile of the gas disk $\Sigma_g$ based on the empirical minimum-mass solar nebular model (MMSN; Hayashi 1981) at a stellar distance $r$ is described as

$$\Sigma_g = f_g \Sigma_0 \left( \frac{r}{1 \text{ au}} \right)^{-k} \exp \left( -\frac{r}{\tau_{\text{dep}}} \right) \, \text{g cm}^{-2}$$

$$= 1700 f_g \left( \frac{r}{1 \text{ au}} \right)^{-1} \exp \left( -\frac{r}{\tau_{\text{dep}}} \right) \, \text{g cm}^{-2},$$

where $\tau_{\text{dep}}$ is the gas depletion timescale and $f_g$ is the enhancement factor of the MMSN, $k = 1$ is the power-law index of the gas density we adopted in this work, and $t$ means the time.

2.2. Type I Migration and Gas Damping

We adopt the prescription for type I migration as described in Paardekooper et al. (2010, 2011), where the total torque can be expressed in terms of

$$\Gamma_{\text{tot}} = f_{\text{tot}} \Gamma_0 / \gamma$$

$$f_{\text{tot}} = f_{\text{LB}} + f_{\text{CR}},$$

where $f_{\text{tot}}$, $f_{\text{LB}}$, and $f_{\text{CR}}$ are the coefficients for the total, Linblad, and corotation torques, respectively; $\gamma = 1.4$ is the adiabatic index; and

$$\Gamma_0 = (q/h)^2 \Sigma_p r_p^4 \Omega_p^2,$$

where $\Sigma_p$ and $\Omega_p$ are the disk surface density and angular frequency at the location of the planet $r_p$, $q = M_p/M_\star$, and $h = H/r$ is the disk’s aspect ratio at $r_p$. The magnitude of $f_{\text{tot}}$ is a function of $s \equiv \delta \ln \Sigma / \delta r, \beta \equiv \delta \ln \Omega / \delta r, q, \nu$ (viscosity), and $\xi$ (thermal diffusivity). Herein, we assume an expression to approximate the $f_{\text{tot}}$ as

$$f_{\text{tot}} = f_{\text{nec}} Q(q) + f_{\text{LB}}$$

$$f_{\text{nec}} = \text{coef} \times \left( 1 - \frac{2 \left( \frac{q}{q_1} \right)^2}{1 + \left( \frac{q}{q_1} \right)^2} \right)$$

$$Q(q) = \frac{1}{2} \left( \frac{q}{q_1} \right)^2 \left( \frac{q}{q_2} \right)^{-2} + \left( \frac{q}{q_1} \right)^2 + \left( \frac{q}{q_2} \right)^{-2}$$

where $r_{\text{t}}$ is the transition radius between the inner optically thick region and the outer optically thin disk—it decreases with the mass accretion rate, gas density, and time. $r_m$ is the magnetosphere radius. $f_{\text{nec}}$ is the coefficient of the fully nonsaturated component of the corotation torque, the saturation parameter $Q(q)$ represents a range of $q_1 < q < q_2$ over which the corotation torque is not saturated. coef is the coefficient of $f_{\text{nec}}$. With a different coef, the strength of the corotation torque will change.

If $f_{\text{tot}}$ is negative, the planet will undergo inward migration with the negative torque. On the contrary, if $f_{\text{tot}}$ is positive, the planet will suffer from outward migration with the positive torque. Figure 2 shows the value of $f_{\text{tot}}$ changing with the mass of the planets and the distance away from the central star. If the planet is massive enough ($m > M_{\text{crit}}$, Ida & Lin 2008),

$$M_{\text{crit}} \approx 30 \left( \frac{\alpha}{10^{-3}} \right) \left( \frac{a}{1 \text{ au}} \right)^{1/2} \left( \frac{M_\star}{M_\odot} \right) M_\odot,$$

a gap will form around it. The torque on the planet is nonlinear due to gap opening, and the planet will undergo type II migration instead of type I migration. $M_{\text{crit}}$ is about 30 $M_\odot$ for the system with a solar-like star. In Figure 2, the gray dashed line shows the location of 30 $M_\odot$. The torque we estimated in this work is suitable for the region below the gray line. In Figure 2, the red color means that $f_{\text{tot}}$ is positive, while the blue color indicates that $f_{\text{tot}}$ is negative. As observed from Figure 2, we can conclude that planets will experience outward migration within 0.25 au when the planet mass is about 10 $M_\oplus$. Here we define this boundary between inward and outward migration as $r_{\text{boundary}}$, where $r_{\text{boundary}}$ changes with the transition radius $r_{\text{t}}$. Meanwhile, $r_{\text{boundary}}$ is related to the planet mass. In Figure 2, we assume $r_{\text{t}} = 0.5$ au, coef $= 5$, $q_1 = 3 \times 10^{-5} M_\odot$, and $q_2 = 10^{-3} M_\odot$. $r_{\text{boundary}}$ appears to remain between 0.2 and 0.32 au when the planetary mass ranges from 10 to 300 $M_\oplus$ as shown by the blue line. The timescale of type I migration is

$$\tau_0 = \frac{m_p \sqrt{GM_\star r}}{2 \Omega_{\text{total}}} = \frac{m_p \gamma \sqrt{GM_\star r}}{2 \Omega \Sigma_g r^4 \Omega^2} = \frac{\tau_0}{f_a h^2}.$$

Herein, $f_a = f_{\text{tot}}$. The timescale of gas damping is

$$\tau_\nu = \frac{h^2 f_a \tau_0}{\nu} = \frac{m_p h^2 \gamma \sqrt{GM_\star r}}{2 \Omega \Sigma_g r^4 \Omega^2} = \frac{\tau_\nu}{f_a h^2},$$

where $M_\star$ represents the mass of the central star, and we choose $M_\star = 1 M_\odot$ in this work. $\Omega$ is the Kepler angular velocity, and $G$ is the gravitational constant.

The force of eccentricity damping is expressed as

$$F_{\text{damp}} = \frac{(v \cdot r) r}{r^2 r_e}.$$

The force of type I migration is

$$F_{\text{mig1}} = \frac{\Gamma_{\text{tot}}}{m_p r}.$$
The acceleration of the planetary embryos with mass $m_i$ is described as
\[
\frac{d}{dt} V_i = - \frac{\ldots}{r_i^2} \left( \frac{r_i}{\ldots} \right) + \sum_{j=1}^{n} \ldots + \ldots + E_{\text{damp}} + E_{\text{migl}}. \tag{13}
\]

In this work, we integrate Equation (13) to explore the dynamical evolution of planets in the system using the time symmetric integrator Hermite scheme (Aarseth 2003). In our numerical simulations, all planets are initially assumed to occupy coplanar and circular orbits. The mean anomaly and the argument of pericenter are generated between 0° and 360° randomly. Each case is integrated to 10 Myr.

3. Numerical Simulation Results

In our model, we consider the planetary systems to be composed of two planets of a couple of Earth masses and a solar-mass central star. Herein we perform extensive numerical simulations to investigate the dynamical evolution of planet pairs in the system, where we take into account the combined parameters of a wide variety of gas disk densities $\Sigma_g$, which reflect the depletion timescale of the gas disk $\tau_{\text{depl}}$, the coefficient of the torque coef, the transition radius $r_t$, and the disk aspect ratio $h$. $p_{10}$ and $p_{20}$ are the initial orbital periods of the two planets, subscripts 1 and 2 represent the values of the inner planet $P_1$ and outer planet $P_2$, and $m_1$ and $m_2$ respectively denote the planetary masses. This allows us to extensively explore the dynamical nature of planet pairs at specific MMRs using the above-mentioned parameters.

We entirely carry out 288 runs in two groups with the combination parameters as mentioned above. The details of the parameters we adopted in each group are shown in Table 1. In the following, we show five typical runs (Cases 1–5), revealing the evolution scenario that two planets deviate from exact MMRs. Table 2 summarizes the adopted initial parameters for three runs, where $p_{1e}$ and $p_{2e}$ represent the resultant orbital periods of the pairs, respectively.

3.1. Escape Process from the Exact Location of MMR

After two planets are trapped in the exact location of the MMR during orbital migration, they will keep migrating at the same pace. Their eccentricities can be excited in this process. Due to the existence of a gas disk, eccentricities can be damped, leading to the change of orbital period at the same time. The change in angular momentum caused by the eccentricity-damping force is zero; therefore, we can get
\[
\Delta a \propto ae\Delta e/(1 - e^2) \approx ae^2, \tag{14}
\]
where $\Delta a$ is the change in semimajor axis caused by the eccentricity-damping force. If the eccentricities of the two planets are excited to comparable values, the change in their semimajor axes $\Delta a$ are mainly proportional to $a$. The outer planet will change at a larger distance than the inner one due to the eccentricity-damping process. If the eccentricity of the inner
planet is excited to a value that is much larger than the outer one, the change of the period ratio is related to $ae^2$, which depends on the relative space between the two planets and the amplitude of eccentricity after they are excited.

### 3.2. Case 1: Departure from the Exact Location of the 2:1 MMR

Panels (a1), (b1) and (c1) of Figure 3 show a typical run for two planets departing from 2:1 MMR. Here, the depletion timescale of the gas disk is assumed to be 1 Myr, and the gas density is 1/50 of the standard MMSN model. According to description of surface density of the gas disk in Equation (1), $\Sigma_g$ will decrease to $\Sigma_{g0}/e$ at 1 Myr at the same location, but the gas still exists and the eccentricity-damping effect is working in the next few million years. As noted from Figure 3, two planets undergo an inward type I migration before the inner planet reaches ~50 days at about 2 Myr. The planet pair is captured into exact 2:1 MMR quickly. Planets will remain at the exact location of 2:1 MMR if no eccentricity damping exists. According to the estimation of equilibrium eccentricity in MMR (Equation (A27) of Papaloizou & Szuszkiewicz 2005), the amplitude of eccentricity is affected by the mass ratio, period ratio, and the ratio between the timescale of eccentricity damping and orbital migration (Murray & Dermott 1999; Nelson & Papaloizou 2002; Papaloizou & Szuszkiewicz 2005; Liu et al. 2015). Eccentricity tends to be excited to a higher value for a less massive inner planet. In case 1, in the first two million years, the eccentricity of the inner planet can be excited to be approximately 0.04, whereas that of the outer planet is not radically stirred up but performs a slight fluctuation about 0.005. In panel (b1) of Figure 3, we observe that the eccentricity of $P_1$ falls down to 0.01 gradually due to the eccentricity damping from 2 to 4 Myr. In this case, the excited eccentricity of $P_1$ when two planets are trapped into exact 2:1 MMR is much higher than that of $P_2$, $e_1/e_2 \sim 10$, while $a_1/a_2$ almost remains at 0.63 for the 2:1 MMR. Thus, according to Equation (14), $\Delta\theta_1$ is larger than $\Delta\theta_2$ with the damping of eccentricities—the space between two planets becomes larger and larger. Panel (c1) of Figure 3 displays the evolution of the ratio of the orbital period between two planets $P_2/P_1$, which was started at about 2.13 (not in 2:1 MMR), then temporarily becoming 2.0 (captured into exact 2:1 MMR with resonant angles librating in a very small amplitude), and a final value of 2.0317 (leaving the exact location of the 2:1 MMR with resonant angles librating in a large amplitude). Clearly, this gives a dynamical portrait for elucidating the resonant evolution history for two planets in the system. As given in Table 2, we have their resultant orbital periods of 51.69 and 104.99 days, respectively, suggesting that the two planets do slightly deviate from the exact 2:1 MMR.

From the Kepler data, we can present several systems that hold planets near the exact location of 2:1 MMRs, such as the systems Kepler-328 and Kepler-56. Kepler-328 (Xie 2014) harbors two planets confirmed at 34.92 and 71.31 days; their period ratio is 2.0421. Kepler-56 (Steffen & Hwang 2015) owns two planets at 10.50 and 21.41 days, with an orbital period ratio of 2.0390. These two systems may be formed through a similar scenario Case 1 describes.

### 3.3. Case 2: Departure from the Exact Location of the 3:2 MMR

The parameters used in this case are shown in the second line of Table 2. Comparing with Case 1, the density of the gas disk is larger in this case, using the standard MMSN model, and the coefficient of $f_{\text{esc}}$ is 5, which is larger than that of Case 1. Therefore, the boundary $r_{\text{boundary}}$ is much larger in Case 2 than in Case 1. The red, blue, and yellow lines in Figure 2 show the distribution of $r_{\text{boundary}}$ with coef = 2, coef = 5, and coef = 10, respectively. At the boundary location, the coefficient of the total torque is 0, and when the planet approaches the boundary, the speed of orbital migration will be reduced. We note that the boundary line declines as coef decreases. For the inner planet which has 12 $M_{\oplus}$, $r_{\text{boundary}}$ moves from $\sim$0.08 au (corresponding to the orbital period of 8.5 days) to 0.27 au (corresponding to the orbital period of 51 days) when we change coef from 2 to 5 as shown in the black diamonds in Figure 2. $r_{\text{boundary}}$ is largest when

### Table 1

| Group | $\tau_{\text{dep}}$ (Myr) | $r_0$ (au) | coef | $h$ | $f_g$ | $P_{\text{10}}$ (days) | $P_{\text{20}}$ (days) | $m_1$ ($M_\oplus$) | $m_2$ ($M_\oplus$) |
|-------|-----------------|----------|-----|-----|------|------------------|------------------|-----------------|-----------------|
| 1     | 0.1, 0.5, 1, 3  | 0.5      | 2   | 5, 10| 0.02 | 0.05/0.1, 0.055/4 | 1/50, 1/10, 1     | 150             | 320             |
| 2     | 0.1, 0.5, 1, 3  | 0.2      | 2   | 5, 10| 0.02 | 0.05/0.1, 0.055/4 | 1/50, 1/10, 1     | 150             | 320             |

Note. $P_{\text{10}}$ and $P_{\text{20}}$ are the initial orbital period of two planets in the systems. $m_1$ and $m_2$ are the masses of planets. The main difference between groups 1 and 2 is the transition radius $r_r$. 

### Table 2

| Case | $\tau_{\text{dep}}$ (Myr) | $r_0$ (au) | coef | $h$ | $f_g$ | $P_{\text{10}}$ (days) | $P_{\text{20}}$ (days) | $m_1$ ($M_\oplus$) | $m_2$ ($M_\oplus$) |
|------|-----------------|----------|-----|-----|------|------------------|------------------|-----------------|-----------------|
| 1    | 0.1             | 0.5      | 2   | 0.05| 1/50 | 150              | 320              | 12              | 24              |
| 2    | 0.1             | 0.5      | 5   | 0.02| 1/1  | 150              | 320              | 12              | 24              |
| 3    | 1               | 0.5      | 2   | 0.1 | 1/10 | 150              | 320              | 12              | 24              |
| 4    | 1               | 0.5      | 5   | 0.05| 1/500| 300              | 540              | 24              | 12              |
| 5    | 1               | 0.5      | 5   | 0.05| 1/500| 300              | 640              | 12              | 24              |

Note. $P_{\text{10}}$ and $P_{\text{20}}$ are the initial orbital period of two planets in the systems. $m_1$ and $m_2$ are the masses of planets. $P_{\text{10}}$ and $P_{\text{20}}$ represent each of the final orbital periods, respectively. $f_g$ is the enhanced factor of the standard gas disk.
orbital periods for the inner planet evolution of the orbital periods, semimajor axes, eccentricities, and period ratios, respectively. The black line and red lines, respectively, represent the evolutions of the

Figure 3. The evolution of Case 1–3. Three typical runs show that the two planets finally deviate from exact MMR. Panels (a1)-3, (b1)-3, and (c1-3) display the evolution of the orbital periods, semimajor axes, eccentricities, and period ratios, respectively. The black line and red lines, respectively, represent the evolutions of the orbital periods for the inner planet \( (P_1) \) and the outer planet \( (P_2) \). In panel (a), the gray lines are associated with the evolution of the pericenter and apocenter, respectively.

coeff = 10, the boundary is located at about 0.38 au (corresponding to the orbital period of 85 days). For the outer planet with mass 24 \( M_⊕ \), \( r_{\text{boundary}} \) moves near \( \sim 0.1 \) au (10 days) with coeff = 2 to 0.32 au (66 days) with coeff = 5 as shown by the white dots in Figure 2. Therefore, it seems possible for the inner planet whose semimajor axis is smaller than 0.27 au to migrate outward in Case 2.

The evolution process of Case 2 is shown in panels (a2), (b2), and (c2) of Figure 3. In such a case, \( f_{\text{E}} \) = 1, the gas density is 50 times the value given in Case 1, suggesting a much faster inward orbital migration. As a result, this contributes to the major reason that induced two planets to pass through the 2:1 MMR and trapped into the 3:2 MMR. When two planets are in the location near the 2:1 MMR, the torque on the planet is still strong enough, and thus they will continue migrating inward until they stop at the region near 43.8 and 66.6 days, respectively. Different from Case 1, the inner planet runs through the location of \( r_{\text{boundary}} \) to the inner region, which may cause outward orbital migration. Within 0.9 Myr, they are kept at the exact location of the 3:2 MMR with a small amplitude of resonant angles as shown in panel (c2) of Figure 3. The eccentricities of the inner planet can be excited to be about 0.01. Subsequently, the decrease in semimajor axis is caused by eccentricity damping. According to Equation (14), the variation of the semimajor axis of \( P_1 \) caused by eccentricity damping is larger than that of \( P_2 \). Considering outward orbital migration and the effect of eccentricity damping simultaneously, two planets migrate outward in 0.1 Myr, and the outer planet migrates outward for a longer distance than the inner planet. Finally, the two planets eventually arrive at 57.85 and 92.23 days, respectively, with a ratio of orbital periods of 1.5943, which deviates from the exact location of the 3:2 MMR with large amplitude resonant angles. This indicates a scenario of planet pairs departing from the exact location of the 3:2 MMR owing to eccentricity damping and outward orbital migration.

Kepler-276 and Kepler-279 (Rowe et al. 2014) are systems with planets near the exact location of 3:2 MMRs. There are three planets confirmed in both systems. Planets are located at 14.13, 31.88, and 48.65 days in Kepler-276. The outer two planets are in near 3:2 MMRs. Their period ratio is about 1.5260. The configuration of Kepler-279 is similar to that of Kepler-276. Three planets are at 12.31, 35.74, and 54.42 days, respectively. The period ratio of the outer two planets is about 1.5227. The formation process shown in Case 2 may be a possible explanation for the formation of these two systems.

3.4. Case 3: Departure from the Exact Location of the 5:3 MMR

Compared with Case 1, we adopt a disk aspect ratio \( h = 0.1 \) and the gas density herein \( f_{\text{E}} = 0.1 \), which is 1/10 of the standard MMSN model in Case 3. According to Equations (9) and (10), the migration timescale \( \tau_{\text{m}} \) in Case 3 is 0.8 times of that in Case 1, and the eccentricity-damping timescale \( \tau_{\epsilon} \) in Case 3 is 3.2 times of that in Case 1. Thus, the eccentricity-damping process can exist for a longer time in Case 3.

Similarly to Case 1, we retain coeff = 2 in the calculation. \( r_{\text{boundary}} \) is located at about 0.08 au for the inner planet and 0.095 au for the outer planet. Panels (a3), (b3), and (c3) of Figure 3 show the evolution of two planets. From the evolution of the period ratio in panel (c3), we note that two planets are
trapped into 2:1 MMR at the very beginning. With a faster speed of orbital migration, two planets break through the 2:1 MMR and continue migrating into the inner region, and they will be further captured into 5:3 MMR with resonant angles librating at a very small amplitude at \( \sim 1 \) Myr. Because the inner planet is less massive, its eccentricity should be excited to a larger value than the outer one. However, the ratio of eccentricities between the outer and inner planet \( e_2/e_1 \) increases with the decrease of \( a_2/a_1 \) (Nelson & Papaloizou 2002; Papaloizou & Szuszkiewicz 2005). Therefore, in this case, the eccentricities of both planets can be stirred up to comparable values, about 0.03. Under such circumstance, both planets smoothly migrate when the eccentricities damp down. And at this time, the two planets are located at the outer region of \( r_{\text{boundary}} \). Therefore, they will migrate inward. Due to their similar \( e \) and \( \Delta e \), \( \Delta e \) is proportional to \( a \) according to Equation (14). Thus, the outer planet will move inward to a longer distance than the inner planet. The simulation results are consistent with the theoretical estimation. Consequently, the final ratio of their orbital periods is 1.6099 as shown in Case 3. Planets d and e are located at about 15.68 and 25.21 days, making their period ratio equal to 1.6078. The system Kepler-154 has a planet pair in near 5:3 MMRs. There are five planets in the system (Rowe et al. 2014; Morton et al. 2016). The third planet, Kepler-154 d, and the fourth planet, Kepler-154 b, have periods of 20.55 and 33.04 days, respectively. Their period ratio is about 1.6078, which is the same as the planet pair in Kepler-276. These systems may share a similar formation scenario to Case 3.

3.5. Case 4: Departure from the Exact Location of the 2:1 MMR with Divergent Migration

The systems in Groups 1 and 2 are initially set to have two planets, and the mass of the outer planet is larger than that of the inner one. Based on the estimation of the isolated mass of planet embryos (Ida & Lin 2004), \( m \) is proportional to \( a^{3/2} \). The outer planet tends to contain more material than the inner one. According to the statistics on the Kepler candidates (Wang & Ji 2017), there is opportunity for the stars to bear a larger inner planet in a system. Herein, we run a case in the system with a higher-mass inner planet to test the effect of the eccentricity damping.

In this case, the mass of the inner planet is 24 \( M_\oplus \), and the mass of the outer one is 12 \( M_\oplus \). The results are shown in Figure 4. The transition radius is 0.5 au, the scale height of the
The disk is 0.05, coef = 5, $f_g = 0.002$, and the depletion timescale of the gas disk is 1 Myr. Details of the initial settings in this case are shown in Table 2 as Case 4. Due to the low gas density of the disk, the migration process is very gentle. The initial periods of the two planets are 300 and 540 days, respectively. Their period ratio is smaller than 2.0 initially. After about 3 Myr of divergent migration, the period ratio of the planet pair increase from 1.8 to 2.0, the two planets are trapped into 2:1 MMRs, and the resonant angle librates with a small amplitude as shown in panel (d1) of Figure 4. As the inner planet is more massive, its eccentricity should be excited to a smaller value than that of the outer one. However, $e_2/e_1$ is also related to $g = (m_2a_2)/(m_1a_1)$ (Nelson & Papaloizou 2002; Papaloizou & Szuszkiewicz 2005). With the increase of $g$, $e_2/e_1$ decreases. $g$ in Case 4 is larger than that in Case 1. Thus, the eccentricities of the two planets become more comparable in Case 4 than in Case 1. Their eccentricities can be excited to be 0.03 as shown in panel (c), and the eccentricity of the inner planet is a little higher than that of the outer one. Then, the eccentricities can be damped due to the gas disk, and finally, the period ratio of the planet pair can reach 2.0175, which is slightly larger than 2.0. The resonant angles become circular, and the planet pair is out of MMR as shown in panel (d1) of Figure 4. Comparably, we run Case 5 with similar initial conditions to those in Case 4, but we switch the order of the two planets. The initial conditions are shown in Table 2, and the evolution process is shown in Figure 4. The two planets are captured into 2:1 MMR at about 1 Myr. The eccentricity of the inner planet can be excited to about 0.05. The 2:1 MMR is more robust through convergent migration than through divergent migration, thus the eccentricity-damping effect with lower gas density ($f_g = 1/500$) cannot make the planet pair in Case 5 leave the exact location of the MMR, but they can destroy the MMR configuration in Case 4.

According to our formation scenario, systems that hold an inner planet larger than the outer one can form near-MMR configurations. Noticeably, two planets are located at about 0.8 au and 1.2 au at the end of the simulation. This case can explain the system’s near-MMR configuration, with the innermost planet quite far away from the central star. Additionally, if planets are located in the inner region of the system initially where planets will undergo outward migration, they can also get out of the exact MMR to be near MMR. The process is similar to that shown in Cases 1–3. The difference is that with outward orbital migration, the innermost planet can reach a location farther away from the central star. Considering all of the cases in this work, with the eccentricity damping of the gas disk, planet pairs can get out of exact MMRs and to the configuration of near MMRs.

3.6. Statistical Results

We entirely perform a set of 288 simulations by considering a combination of gas densities, coefficients, timescales of the gas disk, disk aspect ratios, and transition radii. Detailed initial settings are given in Table 1. Figure 5 shows the distribution of the period ratio between two planets at the end of the simulations. Panel (a) displays the fraction of period ratio in the range of [1.4 1.8], which means that the two planets are in near 3:2 and 5:3 MMRs. Panel (b) exhibits the fraction of planet pairs in near 2:1 MMR, where the period ratio is in the range of [1.95, 2.2]. From Figure 5, we find that two peaks appear to be near 3:2 and 2:1 MMRs obviously. In particular, there is a deficit of planet pairs with period ratio smaller than exact 2:1 and 3:2 MMRs and an excess of pairs with period ratio larger than 2:1 and 3:2 MMRs. The results are consistent with the observational distribution shown in Figure 1 (Lissauer et al. 2011). Another small peak appears at the position where the period ratio is slightly larger than 1.66. The fraction of period ratios between 1.5 and 1.66 experiences a decrease from 8% to 1%, while the fraction of period ratios larger than 2.0 sees a decrease from 15% to 1%. These trends can be seen from the observation data in Figure 1. From Table 1, we know that the initial periods of the two planets are 150 and 320 days. Their period ratio is about 2.13, which is larger than 2.0. Therefore, planet pairs are trapped into 2:1 MMR first and are easier to keep in near 2:1 MMR, leading to the absence of data between 1.77 and 2.0, which is inconsistent with the observation result. With the same reason, few systems in our simulations hold planets with period ratio larger than 2.1. In summary, the distribution of period ratios obtained through the formation scenario we proposed is consistent with the observation results.

Figures 6 and 7 show that the distribution of period ratios varies with different parameters of the gas disk. Figure 6 shows the period ratio distribution between 1.4 and 1.8, while Figure 7 displays the period ratio distribution from 1.95 to 2.2. The period ratios of a few cases are less than 1.4; we choose 1.4 to be the minimum period ratio as shown in these figures. Panels (a)–(d) of the two figures exhibit the results obtained from 144 runs in Group 1 with $r_f = 0.5$. The overall trend with data from Group 2 with different disk parameters is similar to that in Group 1. There are 288 dots in panel (e) with all results from Groups 1 and 2, which reveal major differences with different $r_f$.

From Figures 6 and 7, we find that a great number of planet pairs are involved in the position very close to the exact locations of the 2:1 and 3:2 MMRs, which is consistent with...
the results shown in Figure 5. From panel (a) of Figures 6 and 7, we see that planet pairs with a longer disk depletion timescale, especially for systems with $\tau = 5$ Myr, more easily go into near 3:2 MMR rather than in the 2:1 MMR labeled in green dots, while more planet pairs are in the near 2:1 MMR configuration when $\tau \leq 0.5$ Myr. The possibilities that planet pairs are in near 3:2 and 2:1 MMRs are almost even. The results demonstrate that planet pairs have a long time to get rid of the 2:1 MMR, which is the first low-order MMR they meet. With a longer disk depletion timescale, $\tau > 1$ Myr, the period ratio of the planet pair has a wide spread in whole region from 1.5 to 2.15, with most of them in the range of [1.5, 1.7] and a small number of them distributed at the period ratio of [2.0, 2.15]. With $\tau \leq 0.5$ Myr, most planet pairs contribute to a pile up of period ratio near 2.0, especially from 2.0 to 2.02.

Based on the formation scenario, because a planet pair is trapped into MMR quickly through orbital migration process, and their eccentricities will be excited, the eccentricity-damping effect is still strong enough to work on planets, making them depart from the exact location of the MMR over a longer disk depletion timescale. Therefore, a longer disk depletion timescale is more helpful in the departure of planet pairs from the exact location of MMRs.

In all cases, planets located less than 1 au thus have $h = 0.05 r_i^{1/4}$, which is the smallest one among the $h$ values we have chosen. In panel (b), we show that, with a higher $h$ value, longer timescale of eccentricity damping, planet pairs have more opportunities to escape to a configuration that departs farther from the exact location of MMRs.

Moreover, we notice that most of these cases with coef = 10, which are marked by blue dots in panel (c), are in the region with a period ratio less than 1.55 (or 2.04 for the planet pair in near 2:1 MMR). With the decrease of coef, planet pairs move far from the exact region of the MMRs.

From Equation (9), we learn that $f_g$ is related to the density of the gas disk, which determines the speed of orbital migration directly. With higher gas density, thus larger $f_g$, planet pairs more easily break through the 2:1 MMR to be captured in the 3:2 MMR (Wang & Ji 2014). Panel (d) shows that the results are consistent with the estimation. Most of the cases with $f_g = 1/50$, labeled in blue dots, are distributed near the 2:1 MMR.

Based on our analysis of the torque acting on the planets, with the decrease of the transition radius $r_t$, the orbital periods of the inner planets will decline. Combined with 144 cases with $r_t = 0.2$, we can conclude that most of the inner planets lie in the period range [20, 250] days with $r_t = 0.5$, while the periods of inner planets can be much closer to 10 days with $r_t = 0.2$, as shown in panel (e) of Figures 6 and 7. With the evolution of the star, the transition radius will migrate inward. Therefore, systems with inner planets lying near 10 days may form at the late stage of the star.

Figure 6. Distribution of planet pairs in the space of period ratio between 1.4 and 1.8 vs. the period of the inner planet. Panels (a)–(d) show the distribution of planet pairs changing with the depletion timescale of the gas disk $\tau$, disk aspect ratio $h$, coefficient of $f_{\text{esc}}$ coef, enhancement factor of gas density $f_g$, and transition radius $r_t$, respectively.
Additionally, from Figures 6 and 7, we can obtain that the resonant offset from 2.0 can extend to almost 2.2, and most planet pairs pile up in the region $2.0 \leq P_2/P_1 \leq 2.1$. The period ratios of planet pairs, which are near 1.5 and 1.667, can extend to the range of [1.3, 1.7]. Only a few cases with a larger scale height can be maintained in the region between 1.8 and 2.0.

4. Conclusions and Discussions

In this work, we mainly investigate the formation of planetary systems in the configuration of near MMRs, especially for systems with innermost planets whose semimajor axis is larger than 0.1 au. Considering the eccentricity-damping effect induced by the gas disks, we perform in entirety 288 runs of simulations under a wide variety of depletion timescales of the gas disk, the disk aspect ratio, the enhancement factor of gas density, the coefficient of the corotation torque, and the transition radius of the disk. From the simulations, we conclude that with a proper eccentricity-damping effect, planet pairs can deviate from perfect MMRs and tune in near-MMR configurations. From our simulations, we show that the depletion of the gas disk can make the distribution of the period ratio comparable to the statistics of the observation.

1. According to our formation scenario, planet pairs can be trapped into 2:1, 3:2, or 5:3 MMRs through orbital migration with different migration speeds. Specifically, a large number of simulations linked to 5:3 MMRs are produced with $h = 0.05$ or $h = 0.1$. For those systems harboring innermost planets at a distance larger than 0.1 au, the formation of near MMRs can be elucidated by the existence of proper depletion timescales of the gas disks. Due to the eccentricity damping induced by the gas disk, planet pairs can move out of the exact location of MMRs and be involved in near-MMR configurations. From our simulations, we show that the depletion of the gas disk can make the distribution of the period ratio comparable to the statistics of the observation.

2. With depletion timescales larger than 1 Myr, we find that near-MMR configurations easily form. After a planet pair is trapped into MMR, eccentricity damping is still strong enough to make planets move an adequate distance to depart from exact MMR. Eccentricity damping plays a crucial role in leading to the deviation of planet pairs from exact MMRs. Additionally, with the decrease in coef, which represents the strength of the corotation torque, planet pairs can depart farther from exact MMRs. Moreover, planet pairs with a higher disk aspect ratio, which means longer eccentricity-damping timescales, have higher possibilities of escaping from MMR configurations.

3. The final orbital periods of the innermost planets are directly related to the transition radius. If the innermost planets are in the range [50, 200] days, the transition
radius of the disk is probably at about 0.5 au. In the case of a transition radius near 0.2 au, the innermost planets can arrive at the orbital period of 10 days. This suggests that a system with the inner planet closer to the central star holds stable planets at a later stage of the star evolution process.

4. If the period ratios of planet pairs are located in the range of \([1.8, 2.0]\), planets may be formed in a system whose gas disk has a larger disk aspect ratio.

Tidal effect plays a significant role in giving rise to the departure from exact MMRs of systems with inner planets extremely close to the central star (Lee et al. 2013). From our work, we can conclude that if the planet pairs are captured into MMRs due to the migration process, planet pairs can depart from exact-MMR locations because of the eccentricity damping caused by the gas disk for systems with their inner planets farther than 0.1 au, in which the tidal effect of the star is not strong enough to affect the final configuration of the systems. Through our simulations, we can explain the distribution of the period ratio between adjacent planets observed by the Kepler mission. The timescale for the depletion of the gas disk and the disk aspect ratio are related to how far the planet pair departs from exact MMRs; the location of the transition radius is connected with the final location of the inner planets. The scenario can also be applied to explain the formation of planetary systems observed by TESS, which may find a number of planet pairs in near MMRs (Quinn et al. 2019; Nielsen et al. 2020).

This work is supported by the B-type Strategic Priority Program of the Chinese Academy of Sciences, grant No. XDB41000000, National Natural Science Foundation of China (grants No. 12033010, 11573009, 11773081, 11761131008), CAS Interdisciplinary Innovation Team, Young Innovation Promotion Association and the Foundation of Minor Planets of Purple Mountain Observatory.

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