Magnetoplasmon-Polaritons in a Two-Dimensional Electron System with a Back Gate

A. A. Zabolotnykh* and V. A. Volkov*

*Kotelnikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow, 125009 Russia

*e-mail: andrey.zabolotnyh@phystech.edu

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Magnetoplasmon-polariton excitations in a two-dimensional (2D) electron system with a back gate are theoretically studied. The back gate is a metal layer that is parallel to the layer of 2D electrons and is separated from them by a dielectric substrate serving as a waveguide. In the absence of a magnetic field, the interaction of 2D plasmons with the modes of the waveguide limited by the gate from one side results in the formation of a family of waveguide plasmon-polariton modes. The two lowest of these modes are TM modes and have a gapless dispersion relation. As known, a static magnetic field \( B \) perpendicular to the plane of the system hybridizes different modes. The spectra and magnetodispersion of the found 2D modes are determined. The classification of all modes as longitudinal and transverse (TM–TE classification), which is usually valid only in the absence of \( B \), is recovered in the limit of high fields \( B \). The magnetic field dependence of the cutoff frequencies of the considered modes significantly affects the results. Even a low magnetic field opens a frequency gap proportional to the magnetic field magnitude \( B \) in the spectrum of one of the lowest magnetoplasmon-polariton modes. As the magnetic field increases, the gap is saturated and the mode becomes waveguide.

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INTRODUCTION

It is known that plasma oscillations or plasmons in a two-dimensional (2D) electron system (ES) embedded in a dielectric medium with the dielectric constant \( \kappa \) in the quasistatic limit, i.e., disregarding the electromagnetic retardation effects, have a square-root dispersion relation [1] (here and below, we use the cgs system of units)

\[
\omega_q = \sqrt{\frac{2\pi n e^2 q}{\kappa m}}. \tag{1}
\]

Here, \( n \) is the 2D electron density, \( e \) is the elementary charge, \( m \) is the effective mass of the electron, and \( q \) is the magnitude of the wave vector of the plasmon lying in the plane of the 2D ES.

The 2D ES with a metal electrode (gate) situated parallel to it at a distance \( d \) (see Fig. 1) is called the gated 2D ES with, correspondingly, gated plasmons. At zero external magnetic field, in the long-wavelength limit \( (qd \ll 1) \), and neglecting the electromagnetic decay, the dispersion relation of gated plasmons has the linear form [2]

\[
\omega_q = V_p q, \tag{2}
\]

where \( V_p = \sqrt{4\pi n e^2 d/(m \kappa_d)} \) is the (quasistatic) velocity of gated plasmons and \( \kappa_d \) is the dielectric constant of the subgate dielectric substrate.

If the system is placed in an external static magnetic field \( B \) perpendicular to the plane of the 2D ES, plasma oscillations in this case are often called magnetoplasmons and their spectrum \( \omega_{mp}(q) \) in the quasi-static limit is given by the expression

\[
\omega_{mp}^{mp}(q) = \sqrt{\omega_A^2 + \omega_{p,s}^2(q)}, \tag{3}
\]

where \( \omega_A = \sqrt{\frac{4\pi n e^2 d}{m \kappa_0}} \) is the plasma frequency in the absence of the magnetic field, and \( \omega_{p,s}(q) \) is the plasma frequency in the presence of the magnetic field.

Fig. 1. (Color online) Gated 2D electron system in the case of the back-gated structure. It is assumed that \( \kappa_d > \kappa_0 \), where \( \kappa_d \) and \( \kappa_0 \) are the dielectric constants of the subgate dielectric substrate and external medium, respectively.
where $\omega_0(q)$ and $\omega_e(q)$ are the frequencies of the conventional, Eq. (1), and gated, Eq. (2), plasmons in zero magnetic field, respectively; $\omega_c = eB/(mc)$ is the cyclotron frequency of electrons in the magnetic field $B$; and $c$ is the speed of light in vacuum.

Plasmons were experimentally observed for the first time in 2D systems of electrons on the liquid helium surface [3] and in silicon inversion layers [4, 5]. Plasma oscillations are currently studied experimentally in various structures, including GaAs/AlGaAs quantum wells [6, 7] and graphene [8, 9].

Plasmons in 2D ESs based on semiconductor structures are of interest because their frequencies correspond to the characteristic frequencies of the resonant response of the system to electromagnetic radiation, which are in the gigahertz and terahertz ranges interesting for applications [10–21]. However, to determine the frequencies of plasma oscillations in real systems, it is necessary to correctly determine the wave vector in Eqs. (1)—(3). The wave vector is usually determined by the characteristic length of “inhomogeneities”: the period of the exciting metal lattice [4], size of metal gates located near the 2D ES [22–24], size of the structure [25], wavelength of the ultrasonic [26] response to the characteristic frequencies of the resonant response of the system to electromagnetic radiation [27].

The recent development of technologies makes it possible to fabricate high-quality macroscopic semiconductor 2D ESs, e.g., 2D discs based on GaAs/AlGaAs quantum wells with a diameter of $D \approx 5$ mm and a density of $n \approx 3 \times 10^{11}$ cm$^{-2}$ [27]. For plasmons in such structures ($q \approx 2/D = 4$ cm$^{-1}$, $\omega_0(2/D) \approx 4.8 \times 10^{10}$ rad/s for $\kappa = 12.8$), the quasistatic approach and, in particular, Eq. (1) are inapplicable because they are valid under the condition $q \kappa \ll c q / \sqrt{\kappa_0}$, which is no longer satisfied.

For this reason, to describe plasmons and response of such systems, it is important to take into account the electromagnetic retardation effects.

Electromagnetic retardation for plasmons in the ungated 2D ES in the absence and presence of the magnetic field was theoretically described even in the first works on these subjects [1, 28]. More recently, the effect of electromagnetic retardation on plasma oscillations was studied in detail in [29–39]. When electromagnetic retardation is taken into account, plasmons are called plasmon-polaritons.

However, plasmon-polaritons in gated 2D ESs have as yet been poorly studied. As mentioned in [40], the reason is as follows. Retardation effects for plasmons in such systems are characterized by the ratio of the (quasistatic) velocity of gated plasmons $V_p$ appearing in Eq. (2) to the speed of light in the dielectric substrate $c/\sqrt{\kappa_0}$. In standard gated structures with $d \approx 100$ nm, this ratio is small and, correspondingly, the effect of the electromagnetic retardation on gated plasmons is negligibly weak.

Nevertheless, gated plasmons in the regime of the significant electromagnetic retardation with $V_p \sim c/\sqrt{\kappa_0}$, were recently studied experimentally [41]. To this end, back-gated structures (Fig. 1) were used where the thickness of the substrate $d$ reached 640 μm. It is important that the strong screening regime $qd \ll 1$ was implemented in [41]. The authors of [41] showed that experimental results can be described by renormalized plasmon velocity $V_p/\sqrt{1 + A^2}$ [40] and the cyclotron frequency $\omega_c/(1 + A^2)$ [42], where

$$A = \frac{V_p \sqrt{\kappa_0}}{c} = \sqrt{\frac{4\pi e^2 n_d d^2}{mc^2}}$$

is the dimensionless retardation parameter for plasmons in the gated 2D ES. Formula (3) for the frequency of magnetoplasmon-polaritons $\omega_{pp}^{\text{g}}$ in the infinite gated 2D ES takes the form

$$\omega_{pp}^{\text{g}}(q) = \sqrt{\frac{\omega_0^2}{(1 + A^2)} + \frac{V_p^2 q^2}{1 + A^2}}$$

Below, we show that this expression is applicable in the long-wavelength limit $|d q^2 - \omega_0^2 \kappa_0 c^2| \ll 1$ and frequencies much lower than the frequency of light in the “external” part of the system: $\omega \ll c q / \sqrt{\kappa_0}$. We note that Eq. (5) can be obtained from the equation considered in [43], but it was not studied in detail.

This work was motivated by study [41], where an analytical approach to the description of magnetoplasmon-polaritons was developed but for the relatively retardation regime where $\omega \ll c q / \sqrt{\kappa_0}$. Furthermore, only the mode with the lowest frequency was discussed, whereas higher (waveguide) modes were not considered. In this work, we study in detail the entire structure of magnetoplasmon-polariton modes in the infinite gated 2D ES, in particular, in the strong retardation regime $\omega \lesssim c q / \sqrt{\kappa_0}$.

It is also noteworthy that, in contrast to [43], the 2D ES with only one metal gate is considered in this work (see Fig. 1), as in [41], which qualitatively affects the spectrum of desired modes in the strong retardation regime.

BASIC EQUATIONS AND APPROACH

We consider the 2D ES occupying the $z = 0$ plane and the surface of the ideally conducting gate located on the $z = -d$ plane. The dielectric constant of the medium between the 2D ES and back gate ($-d < z < 0$) is $\kappa_0$, and the dielectric constant beyond the system ($z > 0$) is $\kappa_0$. We assume that $\kappa_0 > \kappa_0$. The system is placed in the external static magnetic field $B$ perpendicular to the 2D ES plane (see Fig. 1).
We search for solutions in the form of waves propagating along 2D ES, \( \exp(\mathbf{q} r - i \omega t) \), where \( \mathbf{r} \) is the vector in the 2D ES plane and \( \mathbf{q} \) is the 2D wave vector of the plasmon-polariton. We are interested in the spectrum in the long-wavelength limit \( q \ll k_f \), where \( \hbar k_f \) is the Fermi momentum, because the effect of electromagnetic retardation is the strongest in this limit.

To determine the spectrum, we use the classical approach based on the solution of Maxwell’s equations for self-consistent electromagnetic fields of the plasmon-polariton and local Ohm’s law \( \mathbf{j} = \mathbf{\sigma} \mathbf{E} \) relating the current \( \mathbf{j} \) in the 2D ES and the electric field \( \mathbf{E} \), where \( \mathbf{\sigma} \) is the dynamic conductivity tensor of the 2D ES in the magnetic field, for which we use the Drude model. We also assume that the wave vector of the plasmon-polariton is directed along the \( x \) axis: \( \mathbf{q} = (q, 0) \).

To obtain the dispersion relation, we use the standard procedure [28, 44, 45]. The solutions of Maxwell’s equations for the electric field components of the plasmon-polariton \( E_x \) and \( E_y \) in the regions \( z > 0 \) and \( 0 > z > -d \) have the form \( E^{(0)}_{x,y}(z) \exp(-\beta_0 z) \) and \( E^{(1)}_{x,y}(z) \exp(i q_{zf} z) + E^{(2)}_{x,y}(z) \exp(-i q_{zf} z) \), respectively, where \( q_{zf} = \sqrt{\omega^2 \kappa_d / c^2 - q^2} \) and \( \beta_0 = \sqrt{q^2 - \omega^2 \kappa_0 / c^2} \). The condition \( \Re \beta_0 \geq 0 \) should be satisfied because we search for only the solutions decreasing at \( z \to +\infty \). Below, we use the conventional electrodynamic boundary conditions for \( E_{x,y}(z) \): (i) \( E_{x,y}(z = -d) = 0 \); (ii) continuity in the \( z = 0 \) plane of the 2D ES; and (iii) discontinuity of the derivative with respect to \( z \) at \( z = 0 \), which is related to the current \( j \) in the 2D ES:

\[
\frac{\kappa(z)}{q^2 - \omega^2 \kappa(z) / c^2} \frac{\partial_z E_x(z)}{\partial_z E_y(z)} = \frac{4\pi}{i \omega} j_x, \tag{6}
\]

\[
\frac{\partial_z E_y(z)}{\partial_z E_x(z)} = -\frac{4\pi i \omega}{c^2} j_y. \tag{7}
\]

Using the boundary conditions, we arrive at the dispersion relation for the desired magnetoplasmon-polariton modes

\[
\left( \frac{\kappa_0 - \kappa_d \cot q_{zf} d}{\beta_0 - q_{zf} \cot \frac{q_{zf} d}{c}} - \frac{4\pi \sigma_{xx}}{i \omega} \right) \left( \beta_0 + q_{zf} \cot q_{zf} d - \frac{4\pi i \omega \sigma_{xx}}{c^2} \right) + \left( \frac{4\pi i \omega \sigma_{xy}}{c} \right)^2 = 0, \tag{8}
\]

where \( \sigma_{xx} \) and \( \sigma_{xy} \) are the longitudinal and transverse (Hall) conductivities of the 2D ES, respectively.

We note that Eq. (8) was derived for the 2D ES with an arbitrary conductivity \( \mathbf{\sigma} \); the only condition is the applicability of Ohm’s law \( \mathbf{j} = \mathbf{\sigma} \mathbf{E} \). Further, to obtain the dispersion relations in the explicit form, we use the simple dissipationless isotropic Drude model for the conductivity. However, Eq. (8) with the appropriate conductivity tensor describes magnetoplasmon-polaritons in a wider class of 2D ESs, including 2D ESs in a high magnetic field [28] and graphene [45, 46].

Within the Drude model in the “pure” limit, where the frequency \( \omega \) is much higher than the inverse relaxation time of 2D electrons, the conductivity tensor components \( \sigma_{xx} \) and \( \sigma_{xy} \) have the form

\[
\sigma_{xx} = \frac{e^2}{m} \frac{n - i \omega}{\omega^2 + \omega_p^2}, \quad \sigma_{xy} = \frac{e^2}{m} \frac{-\omega}{\omega^2 + \omega_p^2}. \tag{9}
\]

Substituting Eqs. (9) into the dispersion relation given by Eq. (8), we obtain the spectra of electromagnetic modes in the gated 2D ES in the magnetic field. We note that the frequencies of these modes in the considered dissipationless 2D ES are real and, thereby, \( \beta_0 \) is positive real, whereas \( q_{zf} \) is real if \( \omega \sigma_{xx} \) is positive real and imaginary if \( \omega < \omega \sigma_{xx} \).

**SPECTRA AND MAGNETODISPERSION OF GATED MAGNETOPLASMON-POLARITONS**

Before analyzing spectra in the magnetic field, we consider the limiting cases of zero and high magnetic fields. In zero magnetic field, \( \sigma_{xy} = 0 \) and the spectrum consists of TM and TE modes. The spectrum of TM modes, which have the components \( (E_x, H_y, E_z) \), is determined from the condition that the expression in the first parentheses of in the first term of Eq. (8) is zero. The spectrum of TE modes with the components \( (H_x, E_y, H_z) \) is determined from the condition that the expression in the second parentheses of in the first term of Eq. (8) is zero. It is important that two gapless TM modes exist in the system. The first mode is a plasmon mode with the asymptotic behavior (5) at \( qd \ll 1 \) and \( \omega_0 = 0 \) [40]. The second mode is a waveguide mode with the frequency \( \omega \approx \sqrt{\omega_d} \ll \omega < \omega \approx \sqrt{\omega_d} \) between light cones and the asymptotic behavior at \( qd \ll 1 \)

\[
\omega = \frac{c q}{\sqrt{\omega_0}} - \frac{c q}{2 \sqrt{\omega_0} \left( A^2 + \kappa_d / (\kappa_d - \kappa_0) \right)^2}, \tag{10}
\]

where \( A \) is the parameter given by Eq. (4).

Higher TM modes, as well as all TE modes, are gapped. They have cutoff frequencies and wave vectors and begin on the “outer” light cone similar to, e.g., electromagnetic modes of a dielectric waveguide [47]. Transverse magnetic modes with the numbers \( N = 1, 2, \ldots \) exist at \( \omega > \omega_{TM,N} \) and \( q > q_{TM,N} \), where

\[
\omega_{TM,N} = \frac{\pi N c}{d \sqrt{\kappa_d - \kappa_0}}, \quad q_{TM,N} = \frac{\pi N}{d \sqrt{\kappa_d / \kappa_0 - 1}}. \tag{11}
\]
The cutoff frequencies for TE modes with the numbers $N = 1, 2, \ldots$ are determined from the implicit equation

$$
\left( \frac{\omega_k d}{c} \right)^2 - \frac{1}{\kappa_0} \cot \left( \frac{\omega_k d}{c} \sqrt{\kappa_0} \right) + \Lambda^2 = 0.
$$

With the increase in the wave vector, all waveguide modes tend to the dispersion relation for light in the substrate $\omega = cq/\sqrt{\kappa_d}$. The characteristic spectrum of modes in zero magnetic field is shown in Fig. 2a.

We now discuss the formal limiting case of infinitely high magnetic field $\omega_0 \rightarrow \infty$ (assuming that the Drude model given by Eqs. (9) is still applicable). In this case, the conductivity of the 2D ES vanish ($\vec{\sigma} \rightarrow 0$), and the 2D ES itself no longer affects modes. Correspondingly, the spectrum in this limit again formally consists of TM and TE modes, which are waveguide modes and their parameters are determined by the parameters $d$, $\kappa_d$, and $\kappa_0$. The conditions that the first and second parentheses in the first term of Eq. (8) are zero at $\sigma_{xx} = \sigma_{yy} = 0$ give the dispersion relations for TM and TE modes, respectively. The characteristic form of the spectrum in a high magnetic field is shown in Fig. 2b. It is important that only one gapless mode exists in this limit.

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**Fig. 2.** (Color online) Blue solid lines are the spectra of electromagnetic modes in the gated 2D electron system in (a) zero, (b) very high, and (c) finite ($A = 1, \kappa_d/\kappa_0 = 12.8$) magnetic fields obtained with the parameters $A = 1$ and $\kappa_d/\kappa_0 = 12.8$. The red dashed-dotted lines are $\omega = cq/\sqrt{\kappa_0}$ and $\omega = cq/\sqrt{\kappa_d}$ light cones. The green dashed line plots the asymptotic equation (5). The inset of panel (c) shows an enlarged fragment of this plot in the interval $0 \leq qd \leq 0.25$, where $\omega_0$ is the frequency given by Eq. (13).
We consider the system in zero magnetic field and the wave vector corresponding to, e.g., only the two lowest gapless modes. How does a transition to the high-field regime, where only one mode corresponds to the same wave vector, occur with increasing magnetic field? To answer this question, we will analyze spectra in finite magnetic fields.

The spectrum of plasmon-polaritons in a finite magnetic field is shown in Fig. 2c. In this case, there is only one gapless mode with the asymptotic behavior given by Eq. (10) with \( A = 0 \) in low-frequency, \( \omega \ll \omega_c \), and long-wavelength, \( qd \ll 1 \), limits. The asymptotic expression (5) describes this mode under the conditions \( \omega \ll \frac{c q d}{\sqrt{\kappa_0}} \). The next mode is gapped with the gap \( \omega_0 \) depending on \( \omega_c \):

\[
\omega_0 = \frac{\frac{\omega_c}{\sqrt{1 + A^2}}}{c \sqrt{\kappa_0}} \leq \frac{\omega_c}{c \sqrt{\kappa_0}}. \tag{13}
\]

In zero magnetic field, \( \omega_c = 0 \) and the cutoff frequency vanishes.

According to Eq. (13), the applied magnetic field indeed opens the frequency gap in the spectrum and, therefore, can change the number of modes at the given wave vector.

Figure 3 shows the magnetodispersion of the cutoff frequencies of different modes. The black long dashed line is the plot of Eq. (13). The blue short dashed line is the line \( \omega_\text{cut} = \omega_c \) near which calculations are inapplicable in the range of about \( 1/\tau \), where \( \tau \) is the electron relaxation time, because the Drude model specified by Eqs. (9) is used at \( \tau \to \infty \). Marks TM and TE indicate the type of the mode corresponding to the cutoff frequency in zero and high magnetic fields. The plots are obtained with the parameters \( A = 1 \) and \( \kappa_0/\kappa_0 = 12.8 \).

We briefly discuss the magnetodispersion dependences, which are often investigated experimentally [6, 7, 41]. The characteristic plot of the magnetodispersion in the long-wavelength limit \( qd \ll 1 \) is shown in Fig. 4.

Two magneto-plasmon modes exist in low magnetic fields at the chosen parameters \( A = 1 \) and \( qd = 0.3 \). The upper mode is near the outer light cone; for this reason, its frequency is almost independent of the magnetic field. As expected from the analysis of spectra, this mode disappears at a sufficiently high magnetic field because of an increase in the cutoff frequency. The chosen parameters, the mode disappears in the magnetic field at which \( \omega_\text{cut} = 1 \). The lower (magneto-plasmon-polariton) mode is well described by the asymptotic expression (5) when \( \omega \ll \frac{c q d}{\sqrt{\kappa_0}} \), i.e., in sufficiently low magnetic fields. As mentioned above, the 2D ES in high magnetic fields has no effect and the dependence on the magnetic field disappears. In this

Fig. 3. (Color online) Blue solid lines are the magnetic field dependences of the cutoff frequencies \( \omega_\text{cut} \) of different modes. The black short dashed line is the plot of Eq. (13). The blue short dashed line is the line \( \omega_\text{cut} = \omega_c \) near which calculations are inapplicable in the range of about \( 1/\tau \), where \( \tau \) is the electron relaxation time, because the Drude model specified by Eqs. (9) is used at \( \tau \to \infty \). Marks TM and TE indicate the type of the mode corresponding to the cutoff frequency in zero and high magnetic fields. The plots are obtained with the parameters \( A = 1 \) and \( \kappa_0/\kappa_0 = 12.8 \).

Fig. 4. (Color online) Blue solid lines are the magnetodispersions of gated plasmon-polaritons plotted from the dispersion relation (8) taking into account Eq. (9). The green dashed line plots the asymptotic equation (5). The red dash-dotted line corresponds to the limiting frequency in a high magnetic field; see the vertical dashed line in Fig. 2b. The plots are obtained with the parameters \( A = 1, \kappa_0/\kappa_0 = 12.8, \) and \( qd = 0.3 \).

Near the frequency \( \omega \approx \omega_c \), it is necessary to take into account the finiteness of the relaxation time, which prevents vanishing of the denominator in the Drude model (9).

The next mode becomes gapped when \( \omega \approx \omega_0 \). The gap frequency is given by the implicit equation

\[
\frac{\omega_\text{cut} d}{c \sqrt{\kappa_0 - \kappa_0}} \cot \left( \frac{\omega_\text{cut} d}{c \sqrt{\kappa_0 - \kappa_0}} \right) + \frac{A^2 \omega_\text{cut}^2}{\omega_\text{cut}^2 - \omega_c^2} = 0. \tag{14}
\]

The solutions of the other cutoff points vary with the magnetic field, satisfying the implicit equation

\[
\frac{\omega_\text{cut} d}{c \sqrt{\kappa_0 - \kappa_0}} \cot \left( \frac{\omega_\text{cut} d}{c \sqrt{\kappa_0 - \kappa_0}} \right) + \frac{A^2 \omega_\text{cut}^2}{\omega_\text{cut}^2 - \omega_c^2} = 0.
\]

Near the frequency \( \omega \approx \omega_c \), it is necessary to take into account the finiteness of the relaxation time, which prevents vanishing of the denominator in the Drude model (9).
limit, the frequency is determined by the frequency of the waveguide mode (see Fig. 2b).

**DISCUSSION OF THE RESULTS**

We now discuss the case where the dielectric constants beyond the system and between the 2D ES and the gate are the same: \( \kappa_d = \kappa_0 = \kappa \). In this case, light cones coincide with each other (see Fig. 2), and all waveguide modes existed between them become light with the spectrum \( \omega = cq/\sqrt{\kappa} \) and are not strictly speaking localized near the 2D ES. The lower mode (corresponding to the plasmon-polariton in zero magnetic field) is still described by the asymptotic expression (5) and has the cutoff frequency (13) on the light cone. Thus, the case \( \kappa_d > \kappa_0 \) is much more abundant than the case \( \kappa_d = \kappa_0 \), because the structure of waveguide modes disappears in the latter and it is impossible to determine in detail the interaction of the fundamental (magnetoplasmon) mode with waveguide modes.

We also note that the absorption of the electromagnetic wave normally incident on the gated 2D ES in the magnetic field (which formally corresponds to \( q = 0 \) in Fig. 2) exhibits a peak at the frequency \( \omega_0/(1 + A^2) \) [42] corresponding to the cyclotron resonance in the gated 2D ES. Thus, Eq. (5) correctly describes the frequency of the resonant response of the system in the region above the light cone \( \omega = cq/\sqrt{\kappa_0} \) at \( q = 0 \).

**CONCLUSIONS**

The spectra and magnetodispersion of electromagnetic modes traveling along the 2D ES with the back gate placed in the perpendicular magnetic field have been analyzed. It is important that the retardation parameter given by Eq. (4) in such structures is not small. Particular attention has been paid to the interaction of magnetoplasmons with modes of the waveguide formed by a dielectric substrate with the metal gate on the one side. Two gapless modes (in addition to the waveguide family of TE and TM gapped modes) exist in such a system in zero magnetic field. The application of a perpendicular magnetic field results in the opening of the frequency gap for one of the modes. In a low magnetic field and in the long-wavelength limit, this frequency gap is given by Eq. (13) and increases linearly with the magnetic field. At a high magnetic field, the gap is saturated and the corresponding mode becomes purely waveguide. Furthermore, the magnetic field leads to a change in the cutoff frequencies of the higher modes (see Fig. 3).

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**CONFLICT OF INTEREST**

The authors declare that they have no conflicts of interest.

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