One-Loop Minimization Conditions
in the Minimal Supersymmetric Standard Model

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Abstract

We study, in the Minimal Supersymmetric Standard Model, the electroweak symmetry breaking conditions obtained from the one-loop effective potential. Novel model-independent lower and upper bounds on $\tan \beta$, involving the other free parameters of the model, are inferred and determined analytically. We discuss briefly some of the related issues and give an outlook for further applications.

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1 Introduction

Breaking the electroweak symmetry through radiative corrections in the MSSM \[1\], is nowadays one fashionable scenario. According to it, the phenomenon of spontaneous electroweak symmetry breaking becomes intimately related to supersymmetry breaking. This scenario has a double merit: –theoretical, as it suggests that very different energy scales could be deeply connected through one single dynamics –phenomenological, as it reduces tremendously the number of free parameters of the MSSM, leading to quantitative estimates of the mass spectrum \[2\].

Yet, the true effective potential in which the vacuum structure is encoded, is a poorly known object beyond the tree-level approximation. One reason for this is the very many different mass scales present in the MSSM, so that a renormalization group (RG) analysis becomes rather tricky. Furthermore, even an RG-unimproved one-loop analysis is made difficult by the presence of many scalar field directions.

In this talk, we report on an analytic study of the one-loop effective potential, and its effects on the minimization conditions which determine the physical electroweak vacuum \[3\]. Naively, one would argue that such effects cannot change much of the qualitative pattern of the tree-level minima. The fact is that, even if the one-loop effective potential is expected to differ, point-to-point, only perturbatively from its tree-level value, it is still possible that its shape be locally modified in such a way that new local minima (or at least stationary points) appear. Furthermore, in the case of vanishing masses, the loop corrections can have an even more drastic effect, generating perturbatively a spontaneous symmetry breaking, the so-called dimensional transmutation \[4\]. The MSSM contains by construction several free non-vanishing mass parameters, either of supersymmetric origin like the \(\mu\) term, or parameterizing the effective supersymmetry breaking through the soft terms. Thus, the radiative electroweak symmetry breaking (EWSB) \[1\] in the MSSM does not generically realize a dimensional transmutation. However, one can still sit at some energy scale where almost-flat directions appear\[4\]. If there is a minimum in such a direction, then a change in the shape of the potential at one-loop in the vicinity of the almost-flat direction, could create perturbatively a supplementary local minimum, which could be lower than that already present at tree-level. The case of almost-flat direction is relevant when \(\tan \beta \sim 1\).

Another interesting feature has to do with the trend of the radiative corrections to the tree-level Higgs boson mass relations. These relations come up as a consequence of

\[4\] The existence of such directions is guaranteed by that of flat directions in the supersymmetric limit.

In our case we will be exclusively concerned with the so-called D-flat directions
(softly broken) supersymmetry and of the minimization conditions assuring the formation of vacuum expectation values associated with $SU(2)_L \times U(1)_Y$ spontaneous symmetry breaking. A particular property of the tree-level Higgs potential in the MSSM, is that the minimization conditions boil down to just the stationarity conditions. That is, at tree-level the vanishing of the $1^{\text{st}}$ order derivatives implies that all the squared Higgs masses are automatically positive. Indeed, starting from the tree-level potential

$$V_{\text{tree}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + \text{h.c.})$$

$$+ \frac{g_1^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g_2^2}{2} (H_1^\dagger H_2)(H_2^\dagger H_1) \quad (1.1)$$

where $m_1, m_2, m_3$ are free mass parameters, $g^2 \equiv g_1^2 + g_2^2$, and $g_1$ (resp. $g_2$) denote the $U(1)_Y$ (resp. $SU(2)_L$ gauge couplings), and defining

$$< H_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \quad < H_2 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad (1.2)$$

where $v_1, v_2$ can always be defined to be real valued, one ends up with four free parameters, namely, the $m_i$'s and $\tan \beta \equiv \frac{v_2}{v_1}$. [The combination $v_1^2 + v_2^2$ is assumed to be fixed by the Fermi scale]. Then, these four parameters reduce to two when the stationarity conditions $\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = 0$ are imposed. This is all the input one needs to recover the celebrated Higgs boson mass relations,

$$m^2_{H^0,H^0} = \frac{1}{2} (m_Z^2 + m_{A^0}^2 + \sqrt{(m_Z^2 + m_{A^0}^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}) \quad (1.3)$$

$$m^2_{H^\pm} = m^2_{A^0} + m_W^2 \quad (1.4)$$

where it is usual to choose $m_{A^0}$ and $\tan \beta$ as the two free input parameters. From the above relations one notes that the positivity of the squared Higgs masses comes out automatically, provided that $m_A^2 \geq 0$. Furthermore, the latter is always verified, since $m_A^2 = -(v_1^2 + v_2^2) \frac{m_3^2}{v_1 v_2}$ and the stationarity conditions $\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = 0$ enforce $v_1 v_2$ and $m_3^2$ to have opposite signs \[3\]. It should be clear that such a property is not generally true.

The question we ask here is whether it remains true beyond the tree-level approximation. We will find out that the generic answer to the above question is negative. This means that new extra conditions, on top of the ones usually imposed in radiative EWSB scenarios, should be considered to ensure that a stationary point is indeed a (local) minimum of the effective potential. These conditions will imply some model independent constraints involving $\tan \beta$, thus contrasting with the belief that such constraints are obtainable only in model-dependent situations. Actually our constraints turn out to be even stronger than...
the existing ones. Finally, the analytic study of the vacuum structure of the one-loop effective potential, will also allow us, on one hand, to check whether or not new minima can occur in the regime $\tan \beta \sim 1$, as was previously noted, and on the other, to understand the reason for the instability of the vacuum expectation values against the change of the renormalization scale in the tree-level-RGE-improved approximation, as compared to the one-loop case \[^{3}\].

### 2 The one-loop Effective Potential

The 1-loop effective potential has the well-known form \[^{4}\]

$$
V = V_{\text{tree}} + \frac{\hbar}{64\pi^2} \text{Str}[M^4(\log \frac{M^2}{\mu_R^2} - 3/2)]
$$

in the $\overline{MS}$ scheme. Here $\mu_R$ denotes the renormalization scale, $M^2$ the field dependent squared mass matrix of the scalar fields, and $\text{Str}[...] \equiv \sum_{\text{spin}} (-1)^{2s}(2s+1)(...)_s$, where the sum runs over gauge boson, fermion and scalar contributions. $V_{\text{tree}}$ is the tree-level MSSM potential \[^{3}\]. A full analytic study of the potential Eq.(2.5) in the MSSM is quite involved, especially if one aims at the determination of the structure of the minima in various scalar field directions. For instance, to keep the complete information, the scalar, vector and fermion contributions to $M^2$ lead at best, respectively to $(14 \times 14)$, $(3 \times 3)$ and $(10 \times 10)$ matrices. We have developed some analytic tools which allow to extract information with less approximations than what is usually done, especially in studying the first and second order derivatives of the Logarithmic parts in Eq.(2.5), keeping simultaneously the biggest possible number of scalar directions. In this talk we are mainly interested in the Higgs directions in a given approximation, so we will not need all the above mentioned developments to which we come back briefly in the last section.

For now, and in order to illustrate the possibility of extracting fully analytic results, we will make a simplifying working assumption, which is not devoid of some physical relevance. We will assume that the logarithms in Eq.(2.5) are re-absorbed in the running of all the parameters in $V_{\text{tree}}$. This assumption would be exact, in the context of renormalization group analysis, if there were only one physical mass scale in the model. In our case, it amounts to the rough approximation of neglecting altogether all field and mass scale differences in the logs. We will also sit in the Higgs directions, that is all scalar fields are put to zero except for the two Higgs doublets

$$
H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \quad \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}
$$

(2.6)
The effective potential takes then the form

\[ V = \nabla_{\text{tree}}(\mu_R^2) + \frac{h}{64\pi^2}(-3/2)\text{Str}M^4 \]  

(2.7)

where \( \nabla_{\text{tree}}(\mu_R^2) \), obtained from \( V_{\text{tree}} \) given in Eq.(1.1) by the replacements \( m_i^2 \rightarrow m_i^2(\mu_R^2), g_i \rightarrow g_i(\mu_R^2) \), is the so-called RGE-improved tree-level effective potential\(^5\). The above rough assumptions about the Logs can be improved by switching on more and more different mass scales, and treating separately different regions of the scalar field space \([4]\). At each energy scale, the one-loop effective potential would still have the form of Eq.(2.7) except that its validity would be limited to a given region of the scalar field space, and accordingly the running of the various parameters absorbs the decoupling effects of higher mass scales. It follows, since we are interested here mainly in the interplay between the finite (non-logarithmic) terms and the functional form of \( \nabla_{\text{tree}}(\mu_R^2) \), that our approximation in Eq.(2.7) should be reasonably illustrative.

3 The one-loop Minimization Conditions

Putting together the contributions of the full-fledged MSSM, we find the following form for the one-loop effective potential in the Higgs directions (and in the Landau gauge),

\[ V_{\text{tree}} + \kappa \text{Str}M^4 = X_{m_1}^2 |H_1|^2 + X_{m_2}^2 |H_2|^2 + X_{m_3}^2 (H_1.H_2 + h.c.) \\
+ X(|H_1|^2 - |H_2|^2)^2 + \tilde{\beta}|H_1^3H_2|^2 + \tilde{\alpha}(|H_1|^4 - |H_2|^4) + \Omega_0 \]  

(3.8)

where \( H_1,H_2 \equiv \epsilon_{ij}H_1^iH_2^j \) and

\[ \tilde{\alpha} = \frac{3}{2}\kappa g^2(Y_t^2 - Y_b^2) \]  

(3.9)

\[ \tilde{\beta} = \frac{g_2^2}{2} + \kappa g_2^2(g_1^2 + 5g_2^2 - 6(Y_t^2 + Y_b^2)) \]  

(3.10)

\[ X = \frac{g_2^2}{8} + \kappa(g_1^2g_2^2 + \frac{23g_1^4 + 5g_2^4}{4} - \frac{3}{2}g_2^2(Y_t^2 + Y_b^2)) \]  

(3.11)

\[ X_{m_1}^2 = m_{H_1}^2 + \mu^2 + \kappa[-4g_2^2M_1^2 + 3g_2^2(m_{H_1}^2 - 3\mu^2 - 4M_2^2)] \]

\(^5\)In principle one should also take into account the running of the Higgs fields, however the corresponding effect is negligibly small at the energy scale we consider. Note also that the other running scalar fields remain consistently vanishing if their initial value is zero.
\[ +12(Y_b^2(A_b^2 + m_{bR}^2 + m_T^2) + \mu^2 Y_t^2) \]
\[ + g_1^2(3m_{H_2}^2 - 2m_{H_1}^2 - 3\mu^2) \]
\[ - \sum_{i=\text{generation}} 2(m_{d_{R,i}}^2 + m_{t_{R,i}}^2 + m_{Q_{i}}^2 - m_{L_{i}}^2 - 2m_{u_{R,i}}^2)) \]  
\[ \tag{3.12} \]

\[ X_{m_2}^2 = m_{H_2}^2 + \mu^2 + \kappa[-4g_1^2 M_1^2 + 3g_2^2(m_{H_2}^2 - 3\mu^2 - 4M_2^2) \]
\[ + 12(Y_t^2(A_t^2 + m_{t_{R}}^2 + m_T^2) + \mu^2 Y_t^2) \]
\[ + g_1^2(3m_{H_2}^2 - 2m_{H_1}^2 - 3\mu^2) \]
\[ + \sum_{i=\text{generation}} 2(m_{d_{R,i}}^2 + m_{t_{R,i}}^2 + m_{Q_{i}}^2 - m_{L_{i}}^2 - 2m_{u_{R,i}}^2)) \]  
\[ \tag{3.13} \]

\[ X_{m_3}^2 = -B\mu + \kappa [g_1^2(B + 4M_1) + 3g_2^2(B + 4M_2) \]
\[ - 12(A_t Y_t^2 + A_b Y_b^2)] \]  
\[ \tag{3.14} \]

\[ \kappa = (-\frac{3}{2}) \frac{\hbar}{64\pi^2} \]  
\[ \tag{3.15} \]

\[ g^2 \equiv g_1^2 + g_2^2 \]  
\[ \tag{3.16} \]

The above expressions are exact, apart from the fact that we kept only the top/bottom Yukawa couplings \( Y_t \) and \( Y_b \) for simplicity, the generalization to the other Yukawa couplings being straightforward. The \( X_{m_i}^2 \)'s are functions of the various soft susy breaking masses and couplings, associated with all squarks doublets masses, \( m_{Q,i}, m_{T}, \) and singlets masses, \( m_{a_{R,i}}, m_{d_{R,i}}, m_{b_{R}}, m_{t_{R}} \), all sleptons doublets masses, \( m_{l_{R,i}} \) and singlets masses \( m_{l_{R,i}} \) (no right-handed \( \nu \)), gaugino soft masses \( M_1, M_2 \) (no gluino contributions at this level), Higgs soft masses \( m_{H_1}, m_{H_2} \) and the supersymmetric \( \mu \)-term, with \( m_1^2 = m_{H_1}^2 + \mu^2, m_2^2 = m_{H_2}^2 + \mu^2 \), as well as \( B\mu(\equiv m_3^2) \) and the soft trilinear couplings \( A_t, A_b \). Note also that \( \Omega_0 \) in Eq.(3.8) is a field independent additive constant depending exclusively on soft susy breaking terms. It contributes to the cosmological constant and will be discarded throughout. We stress that no model-dependent assumptions are needed to establish the above expressions. The results which will follow from them will thus be applicable either in the context of SUGRA-GUT scenarios, where eventually universality may be assumed, or in a gauge-mediated supersymmetry breaking context, or for that matter in any fully model-independent analysis.

Starting from Eq.(3.8) we can determine both the stationarity conditions

\[ \frac{\partial V}{\partial H_a}|_{H_a=\langle H_a \rangle} = 0 \]  
\[ \tag{3.17} \]

\[ ^6 \text{see however } [8, 7] \text{ for relevant issues} \]
and stability conditions

\[
\text{eigenvalues of } \left. \frac{\partial^2 V}{\partial H_a \partial H_b} \right|_{H_a = <H_a>} \geq 0 \tag{3.18}
\]

where \(a, b = 1, 2\) label the two Higgs doublets and \(i, j = 1, \ldots 4\) label the four real valued fields within each of the doublets. From Eq. (3.17) one finds the following two conditions

\[
X_{m_1}^2 v_1 + X_{m_3}^2 v_2 + X v_1 (v_1^2 - v_2^2) + \bar{\alpha} v_1^3 = 0 \quad , \quad X_{m_2}^2 v_2 + X_{m_3}^2 v_1 + X v_2 (v_2^2 - v_1^2) - \bar{\alpha} v_2^3 = 0
\]

(3.19)

which we recast for later convenience in the following form:

\[
X_{m_3}^2 (\bar{\alpha} - X)t^4 + (\bar{\alpha} X_{m_1}^2 - X (X_{m_1}^2 + X_{m_2}^2))t^3
\]

\[
+ (\bar{\alpha} X_{m_2}^2 + X (X_{m_1}^2 + X_{m_2}^2))t + X_{m_3}^2 (\bar{\alpha} + X) = 0
\]

(3.20)

\[
u = \frac{1}{\bar{\alpha}(t^2 - 1)} (X_{m_3}^2 (t^2 + 1) + (X_{m_1}^2 + X_{m_2}^2))
\]

(3.21)

where \(t \equiv \frac{v_2}{v_1} = \tan \beta\) and \(u \equiv v_1 v_2\).

The \((8 \times 8)\) matrix of second derivatives

\[
\left. \frac{\partial^2 V}{\partial H_a \partial H_b} \right|_{H_a = <H_a>}
\]

has, (after having eliminated \(X_{m_1}^2, X_{m_2}^2\) through Eqs. (3.19)), five non-zero eigenvalues and three vanishing ones corresponding to the goldstone modes. It is technically easier and theoretically equivalent, to express the stability conditions, Eq. (3.18) in terms of some invariants of the matrix, rather than in terms of the eigenvalues themselves. One then finds the following conditions,

\[
-(v_1^2 + v_2^2) \frac{X_{m_3}^2}{v_1 v_2} \geq 0
\]

(3.23)

\[
2\bar{\alpha}(v_1^2 - v_2^2) + (v_1^2 + v_2^2)(2X - \frac{X_{m_3}^2}{v_1 v_2}) \geq 0
\]

(3.24)

\[
-4\bar{\alpha}^2 v_1^2 v_2^2 + 2(v_2^2 - v_1^2)(v_1^2 + v_2^2)\bar{\alpha} - (v_2^2 - v_1^2)X \frac{X_{m_3}^2}{v_1 v_2} \geq 0
\]

(3.25)
\[ (-\frac{X_{m_3}^2}{v_1 v_2} + \tilde{\beta})(v_1^2 + v_2^2) \geq 0 \quad \text{(twice)} \quad (3.26) \]

We are now ready to discuss some features of the difference between tree-level and one-loop minimization conditions. We note first that the only parameter appearing in Eq.(3.8) and which is purely one-loop is \( \tilde{\alpha} \). In the tree-level limit \( \tilde{\alpha} \to 0 \), \( X_{m_i}^2 \to m_i^2 \), \( X \to g^2/8 \) and \( \tilde{\beta} \to g_2^2/2 \) and Eqs.(3.19) yield the usual tree-level minimization conditions. Furthermore, in this limit Eqs.(3.23 - 3.26) become automatically satisfied once Eq.(3.23) is, that is when \( \frac{m_i^2}{v_1 v_2} \leq 0 \). However, the previous inequality is itself a consequence of the tree-level version of Eqs.(3.19), as can be seen from solving Eq.(3.20) in terms of \( t \) in the limit \( \tilde{\alpha} \to 0 \), [3]. This explains the automatic positivity of the squared Higgs masses stated in the introduction.

At one-loop level \( \tilde{\alpha} \neq 0 \). Even if \( \tilde{\alpha} \) remains small, it leads to a drastic change in the structure of Eqs.(3.19) and (3.23 - 3.26). The stationarity equations can lead now to four different values for \( \tan \beta \) (instead of two at tree-level), while the stability equations are not all satisfied when

\[ \frac{X_{m_3}^2}{v_1 v_2} \leq 0 \quad (3.27) \]

Actually Eq.(3.27) guarantees Eqs.(3.23, 3.24, 3.26) while Eq.(3.25) will bring a new constraint. Moreover, an important difference from the tree-level case is that Eq.(3.27) is generically no more a consequence of the stationarity conditions Eqs.(3.19).

4 The \( \tan \beta \) model-independent bounds

In this section we present the new constraints on \( \tan \beta \) ensuing from Eqs.(3.19) and (3.23 - 3.26). We will give just a heuristic argument and make some general remarks concerning these constraints. (The interested reader is referred to [3] for full details of the study). To start with, we should stress that the solution \( \tan \beta = 1 \) is from general considerations always unphysical and should not be considered. Indeed it is easy to see from Eqs.(3.19) that this solution would require

\[ X_{m_1}^2 + X_{m_2}^2 + 2X_{m_3}^2 = 0 \quad (4.28) \]

The above equation and \( v_1 = v_2 \) imply that the effective potential Eq(3.8) is vanishing along this direction. This corresponds actually to a D-flat direction together with a cancellation of the soft-susy terms against some F-terms in Eq.(4.28). In any case, the vanishing of the effective potential means that all values of \( v_1 = v_2 \) are degenerate.

\footnote{and the \( X_{m_i}^2 \)'s, \( X \) and \( \tilde{\beta} \) close to their tree-level values so that the change in the effective potential at each point remains perturbatively small.}
Thus \( \tan \beta = 1 \) cannot lead to a preferred electroweak scale! The above is true at least up to one-loop order within our approximation\(^8\). Having barred this solution, Eq.(3.22) becomes well-defined. Actually it is also well-defined in the limit \( \tilde{\alpha} \to 0 \) since in this limit the numerator also goes to zero as can be seen from Eq.(3.20), so that \( u \) goes actually to a \textit{finite} computable quantity.

Let us now give a heuristic argument showing that the interplay between Eqs.(3.22), (3.27) and (3.9) forbids \( \tan \beta \) to be too large or too small. [We restrict ourselves throughout the discussion to positive \( \tan \beta \). The fact that this choice does not reduce the generality of the argument is somewhat tricky at the one-loop level \[3\]. If the region \( \tan \beta >> 1 \) were allowed, then Eq.(3.22) would behave like

\[
v_1 v_2 \sim \frac{X^2}{m_t} \tilde{\alpha} \tag{4.29}
\]

in that region. Taking into account Eq.(3.27) one then must have

\[
\tilde{\alpha} \leq 0
\]

However, due to the form of \( \tilde{\alpha} \) Eqs.(3.9,3.15) and to the fact that \( \tan \beta \simeq \frac{Y_t m_t}{Y_b m_b} \) one sees easily that \( \tilde{\alpha} \leq 0 \) and \( \tan \beta \) arbitrarily large (i.e. \( \frac{Y_t}{Y_b} >> 1 \)) cannot be simultaneously satisfied. We are thus lead to the conclusion that an arbitrarily large \( \tan \beta \) would give a contradiction, i.e. \textit{there should be a theoretical upper bound on} \( \tan \beta \).

Similarly, in the region \( |\tan \beta| << 1 \) one has

\[
v_1 v_2 \sim \frac{X^2}{m_t} \tilde{\alpha} \tag{4.30}
\]

which, together with Eq.(3.27) implies that \( \tilde{\alpha} \geq 0 \), the latter inequality being in contradiction with \( |\tan \beta| << 1 \). Thus \( \tan \beta \) cannot be arbitrarily small and \textit{a theoretical lower bound should exist}. To determine the actual upper and lower bounds on \( \tan \beta \) requires much more work. Here we only state the results.

As we said in the previous section, there are only two independent constraints from Eqs.(3.23 - 3.26), namely Eq.(3.27) and Eq.(3.25). The first one of these constraints combined with Eq.(3.22) yields the following bounds,

\[\text{a) if } \tan \beta > 1 \text{ then } \tan \beta_- \leq \tan \beta \leq \tan \beta_+ \tag{4.31}\]

where \( \tan \beta_- = \text{Min}(T_+, \frac{m_t}{m_b}) \)

\(^8\)D-flatness can easily suggest that it would also be true to any order of perturbation theory, however as we said, one also has to deal with the need to cancel the F-terms against the soft-terms.
\[ \text{and} \quad \tan \beta_+ = \text{Max}(T_+, \frac{m_t}{m_b}) \]

if \( \tan \beta < 1 \) then \( T_- \leq \tan \beta < 1 \) \quad (4.32)

\[ \text{where} \quad T_\pm = \frac{-X_{m_1}^2 - X_{m_2}^2 \mp \sqrt{(X_{m_1}^2 + X_{m_2}^2)^2 - 4X_{m_3}^4}}{2X_{m_3}^2} \quad (4.33) \]

while Eq.(3.25) can be re-expressed in the form

b) \[ \tan^2 \beta \leq t_- \text{ or } \tan^2 \beta \geq t_+ \quad (4.34) \]

where
\[ t_\pm = \frac{\tilde{\alpha}^2 v_1 v_2 - X \pm \sqrt{(X - \tilde{\alpha}^2 v_1 v_2)^2 + \tilde{\alpha}^2 - X^2}}{\tilde{\alpha} - X} \quad (4.35) \]

leading to the second set of bounds. \( t_\pm \) have an implicit dependence on \( \tan \beta \) through \( v_1 v_2 \), the latter being related to \( \tan \beta \) through Eq.(3.22). [A preliminary version of these constraints was already given in \[ ]\], however, in a less general form for a) with no upper bound. It is the realization that the sign of \( \tilde{\alpha} \) places readily \( \tan \beta \) on one side or the other of \( \frac{m_t}{m_b} \) which lead to the new form with the upper bound\[ ]\).

Constraints a), b) are necessary model-independent conditions for the existence of an electroweak (local) minimum. They can be readily implemented in any phenomenological analysis of the MSSM and allow to reduce from the start, the allowed \( \tan \beta \) domain. The actual analysis will of course depend on the values of \( t_\pm \) and \( T_\pm \), these being calculable in terms of the other parameters of the MSSM. They have, however, some general features:

\[ T_- \leq 1 \leq T_+ \quad (4.36) \]
\[ t_- \leq 1 \leq t_+ \quad (4.37) \]

\( T_\pm \) exist as far as the one-loop effective potential is bounded from below, i.e.

\[ X_{m_1}^2 + X_{m_2}^2 \pm 2X_{m_3}^2 \geq 0 \quad (4.38) \]

\[ ^9 \text{One should note that we rely here on the tree-level relations } m_t = \frac{Y_t v_2}{\sqrt{2}}, m_b = \frac{Y_b v_1}{\sqrt{2}}, \text{ which, apart from leading log corrections in the running of } Y_t, Y_b, v_1, \text{ might suffer from some small corrections. Nonetheless, the qualitative form of the constraint will not be altered} \]
while $t_{\pm}$ are effective all the time. In any case, Eq.(4.37) in conjunction with b) shows that a region around $\tan \beta = 1$ should be always excluded. Furthermore, when $\tan \beta < 1$ then a) and b) imply

$$T_- \leq \tan \beta \leq t_- \quad (4.39)$$

Thus this window will be theoretically closed if $t_- \sim T_-$. This possibility of forbidding model-independently $\tan \beta < 1$ should be contrasted for instance with the usual argument in the context of SUGRA-GUT and requiring $b - \tau$ unification [12].

On the other hand, when $\tan \beta > 1$, a) and b) imply

$$\text{Max}(\sqrt{t_+}, \tan \beta_-) \leq \tan \beta \leq \tan \beta_+ \quad (4.40)$$

It is interesting to note here that when $T_+ < \frac{m_t}{m_b}$ one has the model-independent bounds

$$1 \leq \text{Max}(\sqrt{t_+}, T_+) \leq \tan \beta \leq \frac{m_t}{m_b} \quad (4.41)$$

These bounds are similar to those obtained in minimal sugra, namely, $1 < \tan \beta < \frac{m_t}{m_b}$, [13], but happen to have a more restrictive lower bound! There is of course no contradiction here; it only means that using the tree-level (actually tree-level-RGE-improved) minimization conditions together with some model-dependent assumptions like the universality of the soft-susy masses at the unification scale, is weaker than just using the model-independent one-loop minimization conditions. Of course, one can still make the same model-dependent assumptions on top of the one-loop minimization conditions and get even stronger bounds than in Eq.(4.41).

On the other hand, it is worth noting that one can enforce $\tan \beta$ to be greater or smaller than $\frac{m_t}{m_b}$ by choosing the free parameters of the MSSM in such a way that $T_+ < \frac{m_t}{m_b}$ or $> \frac{m_t}{m_b}$. In the second case one has,

$$\frac{m_t}{m_b} \leq \tan \beta < T_+ \quad (4.42)$$

Again, all the above bounds should be contrasted with the more qualitative ones, derived in the literature from the requirement of perturbativity of the Higgs-top or Higgs-bottom Yukawa couplings, [14]. It is clear that the requirement of perturbativity is more of practical than theoretical relevance, while our constraints are directly related to the physical requirement of symmetry breaking.

To summarize, we have established fully model-independent theoretical bounds on $\tan \beta$ which are necessary conditions for electroweak symmetry breaking. They are actually a consequence of the conditions for (local) minima, before imposing other physical requirements such as relating the vev’s to the correct electroweak scale. This shows that the structure of the MSSM, even when freed from further model assumptions about the
origin of electroweak symmetry breaking, still leads to non-trivial constraints which should be taken into account, both in fully model-independent or model-dependent phenomenological analyses.

5 Outlook

In this talk, we studied the structure of the (local) minima of the effective potential only in the neutral Higgs fields directions. Other minima can occur outside these directions and can lead to spontaneous breaking of the electric charge and/or color symmetries. The conditions for the occurrence of such minima, or at least requiring a relatively short life-time of the associated states, constitute important supplementary constraints. Further conditions from the requirement of boundedness from below can also occur. However, all these issues are usually addressed in the literature, overlooking the potential modification the detailed structure of the one-loop corrections can bring. We illustrated in this talk an example of such effects. An analytic study, along the same lines, is now being pursued in directions involving squark and slepton fields.

Another issue concerns the gauge-fixing dependence of the results. This is safe inasmuch as the results involve only stationary points of the effective potential. In this case, a local minimum remains so, even if the magnitude of the curvature is gauge-fixing dependent. However, in some circumstances, boundedness from below conditions are written at non-stationary points, in which case much more care is required. For instance one can show that such conditions can at best be necessary, but certainly not sufficient, unless they are verified for a ‘large’ class of gauge-fixing terms or –which is more convincing– if boundedness from below is assured at the level of a more fundamental theory than the MSSM, at the relevant very high energy scale.

Finally, it is important to take into account the detailed structure of the logarithmic terms, keeping as much as possible the information simultaneously from different scalar fields and mass scales. The main technical difficulty in obtaining analytic information from the logs, is the need to diagonalize the “mass” matrix $M^2$ in Eq.(2.5). To make this tractable, one often resorts to some simplifying approximations, like keeping just the top-stop and bottom-sbottom contributions to the effective potential. As these approximations are justified as far as one is concerned with the leading loop corrections to tree-level observables (apart perhaps in some non-generic regions of the parameter space), it is less clear that by doing so one does not overlook some theoretical constraints about the (local) minima, precisely in the field directions which are neglected. This question is of particular relevance to the stability conditions, since a straightforward analytic determination of the second order derivatives in a sufficiently large scalar field space can quickly become intractable, unless the field space is sufficiently reduced in order to diagonalize...
$M^2$. But in this case a corresponding information is lost in the reduced directions! The trick is to use a resummed formula which we derived for the second order derivatives, valid before reducing the field space. Then the field space is reduced accordingly in order to obtain (analytically) diagonalizable matrices, and in the same time keep the full information of the second order derivatives in the initial field space. These issues are now under consideration.

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