Mixing foams and grains in Hele-Shaw cells

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Abstract. We have observed some features of the coexistence of foams and granular materials in Hele-Shaw cells. The most part of the liquid and granular material stays at the bottom of the cell, with only a small quantity of the mixture resting on the froth. The fractal dimensions of the final states of the foams are close to the values obtained from the Random Apollonian Packing model. The disperse structure of the granular material affects the probability distribution of number of sides of the foam bubbles. The nearest neighbor distances between the peaks of the sand piles at the bottom of the cell are close to a lognormal distribution.

1. Introduction
When granular materials are mixed with foams, the grains tend either to float and stick to the bubbles or to sink in the liquid. Although some physical mechanisms of granular materials and foams mixtures are partially explored in the studies of granular media [1], wet granular matter [2], capillarity [3], foams [4], a better understanding of the behavior of granular materials in foam systems still remains an important prospect [5]. The understanding of the coexistence of foams and granular matter is difficult because the detailed knowledge of the components is insufficient to describe the whole system. For example, we have to take into account the transport of the three immiscible phases (air, liquid, and grains), or the interplay of the morphology of polyhedral bubbles and sand piles. For that reason, we have explored experimentally some aspects of this coexistence of foams and granular matter in vertical Hele-Shaw cells. We have compared two setups. The first one is based on the foam formation, while the second one is composed by the mixture of foam and granular matter. After that, we also compared some fractal properties of space filling models with the experimental data of both setups. Some adhesive properties of wet granular systems were explored in order to have an idea of the quantity of grains in these froths. Additionally, we have obtained the distribution of nearest-neighbor distances between the sand piles that form at the bottom of the cell.

2. Experimental Apparatus
The experiments were performed in two transparent Hele-Shaw cells, each one consisting of two plain parallel Plexiglas plates separated by a gap (20 x 20 x 0.2 cm³). The first cell contains only air and an amount of commercial dishwashing liquid (V = 6 cm³), and this cell is referred in this letter as setup I [6]. This liquid is manufactured by Bombril, and is used without dilution. The essential surfactant is Linear Alkylbenzene Sulfonate (LAS). The surface tension is \( \gamma = 25 \text{ dyne/cm} \), and the density of this detergent is \( \rho = 0.95 \text{ g/cm}^3 \). In addition to the dishwashing liquid and air, setup II is a similar system containing 10 g of granular material. The granular material consists of polydispersed spherical glass...
beads with diameters $d$ varying from 500 μm to 700 μm. The density of the glass beads is 2.6(1) g/cm$^3$. In both experiments, the cell is flipped upside-down generating the foam. Let us consider each flip of the Hele-Shaw cell as one iteration, and $n_f$ is the number of successive iterations. After each iteration in setup II, part of the granular material tends to remain in the foam layer because of the competition of the involved forces. Considering a single spherical particle moving through the foam, the capillary force $\gamma (d/2)$ is twenty-five times stronger than the inertial force $\rho g (d/2)^3$. The pictures of the foam were recorded with a digital camera. These pictures were converted into binary images and analyzed with a computer.

![Figure 1](image.png)

**Figure 1.** (a) Time evolution of the liquid film length after the first flip ($n_f = 1$) is represented by the solid circles for setup I (just foam), and the open circles for setup II (foam and granular matter). The film network reorganizes with time following a relaxation process in (ii). (b) Two photos of the cell for setup I in the liquid stretching regime (i). (c) Time evolution of the liquid film length after each flip, with $n_f = 20$ represents the full developed foam. (d) Two photos of the cell for setup II during the stretching regime.

3. Liquid film evolution
First, we compare the first flip in both setups, in order to have a glimpse of the involved phenomena. In figure 1(a) the solid circles represent the evolution of the liquid film length as a function of time for setup I. As soon as the cell is flipped, there is an unstable arrangement between the two phases causing the main part of the liquid to fall down through the cell wetting the walls. The expansion of the liquid filaments forming a grid of parallel lines is shown in figure 1(b). This interface expansion lasts 108 seconds in the plot of figure 1(i). After that, there is a rearrangement of the network of filaments shown in figure 1(ii). The film network reorganizes with time following a relaxation process, with smooth decay to the minimal surface. The relaxation time of setup I for the first flip is around 33 minutes. In short, the diagram for a sequence of flips is shown in figure 1(c), on which the mechanism of interface length growing is a cumulative process that depends on the number of iterations, as reported in a previous paper [6]. On the other hand, a different evolution of the foam interface in the
presence of granular material for the first flip is represented by the open circles of figure 1(a). In the case of setup II, part of the liquid goes down under gravity to the bottom of the cell, as in the previous case, but a mixture of liquid and granular material remains at the top (figure 1(d)). This mixture engenders an effective cohesive interaction between the glass beads, forming slurry confined between the plates. This slurry moves down slowly through the cell. Consequently, the foam is affected by the motion of this slurry during the relaxation time, and the film length of the foam for setup II does not follow a smooth decay. The relaxation time of setup II is around 40 minutes, due to the motion of the granular material.

\[D_f = 1.6(2) \quad \text{setup II}\]

\[D_f = 1.58(7) \quad \text{setup I}\]

Figure 2. (a) Fractal dimension (solid circles) versus the number of edges for the Random Apollonian Packing. The continuous line is the fitted function as in equation (1). The dotted line represents the value of fractal dimension for the experimental results of setup I, and the dashed line of setup II. The insets are the fractal dimension versus number of flips for setup I (b), and for setup II (c), with the values of the fractal dimension for the foams in the stationary states in both cases.

4. Foam structure

For several iterations the developed foam is a planar network of filaments extending throughout the Hele-Shaw cell, with the main part of the liquid resting at the bottom of the cell for both setups. The completely developed foam is called a stationary state, represented by bubbles having a characteristic length related to the capillary length and by spatial homogeneity, following a kinematic model for bubble splitting process [7]. In order to capture the general features of the evolution of the foam structure, we have used some space filling models to compare fractal dimensions and differentiate the structures of foams. We can find similar approaches in Refs. [6] [8]. The Random Apollonian Packing is the case in which circles touching one another, with different sizes, are placed randomly in the plane [9]. This packing was further extended by Delaney [10] to consider a model in which cells can have a range of polygonal shapes. Considering the smoothness term \[\phi/(n-k)\] for polygonal bubbles, we obtained a general expression relating the fractal dimension and number of bubble edges \(n\) filling the plane given by
with fitting parameters \( \alpha = 17, \beta = 25.1, \phi = 360, \) and \( k = 2.9. \) The parameter \( k \) is related to the number of side of a triangle, and is almost 3, and \( \phi \) is the angle for the circumference in degrees. A strong dependence of scaling properties on the bubble shape was obtained, and it is shown in figure 2, along with the fractal dimension. For example, the fractal dimension \( D_f = 1.634 \) for squares, \( D_f = 1.612 \) for pentagons, and \( D_f = 1.597 \) for hexagons. In the case of the fractal dimension of the experimental data, we applied the box counting method to measure the fractal dimension formed by the foam for sequences of iterations. The estimates for the fractal dimension for setup I are shown in figure 2(b), and the analogous measurement of setup II is shown in figure 2(c). The dispersion of fractal dimensions of the foam in presence of the granular material is higher than in the case of pure foam. The foam structures in the stationary state in both cases have almost the same fractal dimension values, with \( D_f = 1.58(7) \) for setup I, and \( D_f = 1.6(2) \) for setup II. These values of fractal dimension for the foam in the stationary state are close to those obtained for the case of Random Apollonian Packing of hexagons, taking into account the error bars.

The presence of the granular material increases the standard deviation for the value of the fractal dimension of the foam, from \( \sigma = 0.07 \) for setup I to \( \sigma = 0.2 \) for setup II. A possible reason for that effect is because the particles could represent some kind of defect in pattern of the foam structure. These imperfections also have influence in the box-counting algorithm used for the computation of the fractal dimension \( D_f. \)

**Figure 3.** The probability distribution, \( \rho(n) \), of the number of bubble edges is shown in (a) for setup I, and for setup II in (b). The insets are photographs of the foam shown in (a) and for the case of the foam with sand in (b), where horizontal bars are 1 cm long. Although the appearance of the foam seams the same, the quality of a foam is represented by the width of \( \rho(n) \) distribution. The thick lines represent lognormal functions fitted to the experimental data. The addition of the sand changes the quality of the foam by spreading the \( \rho(n) \) distribution. Both setups have \( \pi \) around 5.7, hence the fitted functions agree with the fractal dimensions obtained for the Random Apollonian Packing for both setups, taking into account the error bars.
We also have performed a quantitative comparison between the bubble geometry in both setups. The main difference was observed in the probability distribution $\rho(n)$ of the bubble sides. For the case of setup I, the probability distribution of the number of sides of the bubbles is the discrete one shown in figure 3(a), while the data of setup II is shown in figure 3(b). By inspection, we have observed the existence of skewed size probability distributions in both cases. The thick line in each case represents the best fit of a lognormal distribution to the discrete distributions. Although the data are related to discrete distributions, the fit of a lognormal distribution in terms of a log-transformed variable enables us to compare typical parameters, such as the mean value $n$ and standard deviation $\sigma$. For the case of setup I we have obtained $\bar{n} = 5.72(2)$ and $\sigma = 0.192(3)$. For setup II, the values are $\bar{n} = 5.74(3)$ and $\sigma = 0.227(4)$. In both cases the reduced chi-square is equal to 0.996.

The disperse structure of the granular material affects the probability distribution $\rho(n)$, indicated by the fact that the standard deviation of the bubble sides distribution increases in the presence of granular materials.

**Figure 4.** Close up photographs showing nodes, where the edges of the bubbles touch each other. (a) A perspective view of two nodes of degree three, following the rules of Plateau; there is a grain in one of them. All nodes observed for pure foam have degree three, as well as the majority of nodes with grains or cluster of grains. Nevertheless, as it is shown in (b), we have observed a node degree four. In (c) is shown the close up of the picture in (b). In (d) there is a perspective view of this node, showing that it is really a node connecting four bubble edges. From this view we can see the grain in the opposite plate behind the three grains seen in (c).

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The change of the detergent film configuration is an interesting example of this result. The place where detergent films meet is a Plateau border. Considering the Plateau border as a node, the degree of a node is the number of liquid films touching it. The two nodes of figure 4(a) have degree three. For the case of system I, the degree of every node is always three, following the rules of Plateau. On the other hand, the granular material in dry foam occasionally changes the degree of some nodes in the soap film, because part of the granular material spreads out through the network of liquid film, creating some grain clusters caused by the interstitial liquid bridges between grains. Figure 4(b) shows the case of a node with degree four apparently not consistent with Plateau’s laws. The zooming of this node is shown in figure 4(c), and there is the perspective view of the node in which there are four films meeting at the center of figure 4(d), anchored by the glass beads. The explanation for this node with degree four is related to the spherical geometry of the grains, enabling several films touching the same grain at various angles, compatible with Plateau rules. So these nodes can be considered an emergent property due to the combination of foams and grains.

We have considered the nodes with liquid films with three grains between the plates, because nodes containing many particles do not allow defining the geometry of the Plateau borders precisely.

**Figure 5.** The probability distribution, $\rho(dp)$, of nearest neighbor distances between the peaks of the sand piles at the bottom of the Hele-Shaw cell. The lognormal function is fitted to the experimental data. The inset picture shows the sand piles totally immersed in the liquid.

In addition to the previous comments, an intriguing sand pile distribution was observed. The nearest neighbor distances $dp$ between the two peaks of the sand piles at the bottom of the cell (see the inset of figure 5) exhibit a statistical distribution $\rho(dp)$, represented by the fit of the thick line of figure 5. We have obtained the plot of a lognormal distribution with $\overline{dp} = 1.43(6)$ cm, $\sigma = 0.70(4)$, and reduced chi-square equal to 0.976. Due to the fact that this probability distribution was obtained from the experiment, we can conjecture that $\rho(dp)$ somehow represents the net result of the dynamics of wet granular materials passing through the foam structure.

### 5. Conclusion

In conclusion, we have observed some features of foams and granular materials in Hele-Shaw cells. We have found that the general evolution of the foam in the presence of the granular material is different from the case without grains, while the foam structure in the stationary state in both cases is
almost the same, with their fractal dimension close to the values obtained from the Random Apollonian Packing of hexagons. The foam forms a network of ducts which transport the grain, and it is affected by the motion of this slurry during the relaxation time. Most part of the liquid and granular material stays at the bottom of the cell, with only a small quantity of the mixture resting on the froth. We have observed that at least 22% of the foam profile is composed by granular matter. Our results indicate that granular matter can alter some aspects of pattern formation in foams, such as the emergence of nodes with degree four. Plots of the number of bubble-sides followed a lognormal distribution. This distribution spans for the case of the coexistence of granular material and foams. We have measured the distance of the first neighbors of the sand pile peaks, and once again we have obtained a lognormal distribution.

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References
[1] Jaeger H M, Nagel S R and Behringer R P 1996 Phys. Today 49(4) 32
[2] Hornbaker D J, Albert R, Albert I, Barabási A L and Schifer P 1997 Nature (London) 387 795
Herminhaus S 2005 Adv. Phys. 54 221
Mitarai N and Nori F 2006 Adv. Phys. 55 1
Andersen K H, Abel M, Krug J, Ellegaard C, Sondergaard L R and Udesen J 2002 Phys. Rev. Lett. 88 234302
[3] Pomeau Y and Villernaux E 2006 Phys. Today 59(3) 39
[4] Pugh R J 1996 Adv. Colloid Interface Sci. 64 67
[5] Rubio J, Souza M L and Smith R W 2002 Min. Eng. 15 139
[6] Tufaile A and Tufaile A P B 2008 Phys. Lett. A 372 6381
[7] Caps H, Vandewalle N and Broze G 2006 Phys. Rev. E 73 065301(R)
[8] Sauerbri S, Hass E C and Plath P J 2006 Discrete Dynam. Nat. Soc. 2006 79117
Herdtle T and Aref H 1991 Phil. Mag. Lett. 64 335
[9] Manna S S and Herrmann H J 1991 J. Phys. A 24 L481
[10] Delaney G W, Hutzler S and Aste T 2008 Phys. Rev. Lett. 101 120602
[11] de Gennes P-G, Wyart F B, Quere D and Reisinger A 2004 Capillarity and Wetting Phenomena: Drops, Bubbles, Pearls, Waves (New York: Springer)