A Theoretical Evaluation of the Models for Stock Market Volatility.

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Abstract

Volatility forecasting has been widely debated in empirical finance, nevertheless, studies examining issues in volatility and their resolution through various models has received a scant attention. Therefore, the present study which is purely a review work aims to elucidate volatility stylised facts along with discussion on theoretical foundation and procedure of volatility forecasting approaches. To serve this purpose, about sixty research papers were reviewed to extract meaningful insights on stock market volatility and its measurement methods. As a whole, it is observed that unconditional models that are intuitive and simple in estimation ignore most of well-known ‘stylised facts’ about volatility. GARCH family models though cater to most of volatility stylised facts, yet at the practitioners’ level, EWMA approach appears to be more reliable and worthwhile. Further, studies show that it is difficult to evaluate GARCH models as empirical results of such a model are dependent on the sampling frequency. Hence, choice among such models remains to be an empirical issue sensitive to length and frequency of data. Finally, GARCH family models expected to take care of main stylised facts like, volatility clustering, asymmetric effect, etc., yet models that have a capacity to handle properties like, non-normal behaviour of stock market volatility are beyond the purview of this study, thus represent a future gap for a literature review based research.

Key words: stock market, volatility, stylised facts, GARCH.

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1. Introduction:

In a literal sense, “Volatility is up-and-down movement of the market” (Ibbotson, 2011). However, technically, volatility can be understood as a scale that measures uncertainty. In statistical parlance, volatility is the second moment while expectation or average is the first moment. This essentially implies that, “volatility is a phenomenon which captures the variations around the average of any random parameter or target variable, both upside and downside. Volatility characterises the stability or instability of any random variables” (Bessis, 2002, p.77, 80). A commonly used measure of volatility is the standard deviation, which measures the dispersion of returns around mean. “Financial economists find the standard deviation to be useful because it summarizes the probability of seeing extreme values of returns. When the standard deviation is large, the chance of a large positive or negative return is large” (Schwert, 2011, p. 3-4). Although high volatility of stock markets is generally undesirable, yet a fair level of volatility is preferred by the markets to boost up investment activity. The stock market volatility is caused due to flow of the new information. The market prices change due to the response of agents to the new information, thereby causing volatility. According to efficient market hypothesis, “price changes occur when new information about the true investment value of stocks becomes available to the public; the price changes are big because the information is about something very important” (Shiller, 1987, p. 234). Unlike price changes that trigger volatility, in finance, “volatility in essence is considered as the percentage changes in stock prices or rates of return” (Shwert, 1998, p. 2). Another view on the concept of volatility is that, “volatility can be defined as changeability or randomness of asset prices” (Roy & Karmakar, 1995, p. 38).

The understanding of volatility and its modelling is equally crucial to various stakeholders that include, investors, arbitrageurs, portfolio managers and policy makers. In particular, volatility helps market agents in their investing, trading or hedging activity, stock exchanges in derivative pricing and risk management, portfolio managers to evaluate investment performance and policy makers to sense the direction of economic activity. ‘Volatility’ a concept that represents a comprehensive measure of risk in tradable assets has been of considerable interest equally among the academic researchers and investment practitioners. In general, investment decisions are influenced by two main criteria, i.e., expected return and its volatility or variability through time. The entire spectrum of empirical discussion and evidence on asset or portfolio

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1 See: https://insights.som.yale.edu/insights/why-does-market-volatility-matter
performance hinges upon the idea of volatility. By convention, asset returns should follow a positive relation with volatility. The credibility of any findings about such a relationship would however rely on ability of the model specifications to capture potential risks as well as characteristics of sample data employed to map risk-return dynamics into the asset pricing process. The idea of how volatility should be modelled has considerably evolved over past from volatility being treated as a constant parameter to a measure that varies through time. It is now a widely acknowledged fact that volatility itself undergoes through change over time with characteristics of trends, cycles and shocks that it preserves to create short or long dependence in its evolution through time, thus such elements in it pose serious challenges in deciding about how to approach it for modelling or measurement purpose. The whole academic literature on volatility is divided into two main classes, i.e., unconditional\(^2\) and conditional\(^3\) measures of volatility. In essence, it is very difficult to have a true estimate of volatility, as any model being used will undergo through a measurement error. There has been a huge discussion on validity and appropriateness of different classes of volatility models, however, arguments and findings are mostly relevant and sensitive to the context of empirical investigation and the way data is sampled for it. Despite sophisticated conditional models being claimed as more realistic, however, in practice it seems that unconditional models are still being believed to be very convenient and useful method to deal with measurement of volatility. Although a vast literature is found on the empirical use and evaluation of volatility models, yet there is dearth of studies that would have taken up theoretical discussion on the evaluation and justification of various volatility modelling alternatives. Some of the relevant questions that can raised in this behalf include:

i. What is stock market volatility?

ii. Is volatility forecastable?

iii. Does the time interval (the observation interval) matter in volatility modelling?

iv. What are the various stylised facts that should be incorporated in a model?

v. What are the various volatility capturing methods?

vi. Which method among these provides a best forecast of volatility?

To consider these questions, a number of basic methodological viewpoints need to be discussed, mostly about the evaluation of methods. The present study, therefore, mainly aims

\(^2\) Models that assume volatility is time-invariant.

\(^3\) Models that consider volatility depends on its past behavior and information content.
to answer these questions by examining the literature on volatility modelling so as to critically evaluate various volatility models that have evolved through past. For this purpose, a total of about forty research papers were thoroughly reviewed to present a concise view of the controversial concept. The rest of the study is summarised as: the second section examines the current literature, the third section takes up discussion on traditional volatility models, followed by segment third and fourth that deals with sophisticated symmetric and asymmetric time-varying volatility models. Finally, the discussion ends up with overall evaluation and concluding summary of the study.

2. Review of Literature:

The review of literature encompasses two important aspects of stock market volatility, i.e. characteristics of stock return volatility and issue concerning volatility forecasting models. As pointed out rightly, “correctly describing the dynamics of the returns is important in order to obtain accurate forecasts of the future volatility which, in turn, is important in risk analysis and management” (McAleer & Medeiros 2006, p.3). The modelling or prediction of stock market volatility has been ever a challenging task. This is so because on one side if simplicity in estimation of unconditional volatility is overshadowed by its inability to handle volatility stylised facts, yet towards the other end, conditional volatility methods that were primarily introduced with main idea of handling time-varying properties of volatility “fail to describe satisfactorily several stylised facts that are observed in financial time series” (McAleer & Medeiros, 2006, p.3). Unconditional volatility though visible but conditional volatility being a latent phenomenon is not directly observable. This further adds to the complicacy involved in the task of volatility forecasting. The understanding of inherent issues associated with stock market volatility is a crucial subject given the role of risk in evaluation of financial securities. What are the various characteristics that an ideal estimate of volatility should qualify? The answer could be found by looking at various unique features or ‘stylised facts’ about stock market volatility. “These ‘stylised facts’ comprises of a set of properties, common across many instruments, markets and time periods, that has been observed by independent studies” (Cont, 2001, p. 223). This study reviews the main stylised facts that include, non-normal returns, mean-reversion, volatility clustering, the leverage effect and the spill over effect. The volatility modelling is seriously troubled when normality assumption of underlying stock returns is violated. Generally volatility is measured on continuous time intra-day or discrete time daily basis. A short interval of time is preferred to avoid presence of noise in the data. According to Cont (2001), as the time scale \( \Delta t \) is increased, the stock return distribution tend to appear
more and more like a normal distribution. In essence, at different time scales, the shape of distributions does not remain the same. This is supported by Carroll and Kearney (2009) who argue that return distributions tend to be leptokurtic with fat tails and excess peakedness at the means relative to the normal distribution. Jegajeevan (2012) also observes that daily and monthly index returns in the Colombo stock exchange follow different distributions. While daily index returns are leptokurtic or fat tailed having larger kurtosis, monthly returns relatively with less kurtosis exhibit normality patterns. G. William Schwert, founder of volatility literature, in his 1990 work argues that volatility is stable over short-term while as it may vary over longer periods of time. Fouque et al. (2000) argue that volatility in case of high frequency S&P 500 index returns in US is slowly mean-reverting but is fast mean-reverting in context of derivative contract. Short-run intraday volatility may be trending, however, it generally reverts to its mean on daily, weekly or long-run basis. Another study points out that “when volatility is disturbed, it tends to return to its normal level, which may itself vary over time” (Carroll and Kearney, 2009, p.73).

Volatility has a property to arise in bunches. This is also referred to as volatility persistence in financial economics. Empirical literature is full of evidence about such phenomenon characterising stock market volatility. Volatility is said be caused due to inflow of information and information generally flows to markets in bunches. This makes volatility to give rise to clusters over time. Put differently, large (small) changes in volatility persist with large (small) changes of either sign leading to formation of clusters in volatility. The “increase in stock market volatility brings an increased chance of large stock price changes of either sign” (Schwert, 1990). From the analysis of the US stock market returns during 1802-1989, the author finds patterns leading to clusters of high and low returns during brief sub periods over the period. Thus, such a behaviour provides the evidence that volatility in stock returns is time varying and clustering with some power of memory, implying that a shock in volatility does not die out immediately as proposed by the theory of efficient market hypothesis. ARCH models were essentially developed as a response to such behaviour of stock return volatility. Another feature that makes volatility interesting is asymmetry. Since volatility is caused due to flow of information, the nature of information, whether bad or good impacts differently making a negative information to produce more volatility than the positive information of similar degree. This is known as ‘asymmetric effect’ or ‘leverage effect’ in volatility. In general, “volatility is characterised as higher in bearish markets than it is in bullish markets (asymmetry), indicating a negative correlation between conditional volatility and current stock
prices” (Black, 1976). Another explanation of asymmetric volatility known as leverage effect hypothesis states that fall in the price of a firm’s stock leads to deterioration in the value of firm, thus increasing the financial leverage. Due to this, the stock becomes riskier and volatility goes up. However, theory of leverage effect doesn’t have a strong empirical backing. Christie (1982) documents a strong cross-sectional associating between asymmetry and leverage but leverage alone is not sufficient to explain the asymmetric effects. An alternative explanation to this phenomenon is ‘volatility feedback hypothesis’ according to which, “causality runs from volatility to prices, i.e., positive shocks to volatility increase future risk premium, and if dividends remain the same, then the stock price should fall” (Campbell and Hentschel, 1992, p. 2). This hypothesis suggests that higher volatility is compensated by higher expected returns. Therefore, when volatility increases, stock prices should fall to produce higher expected returns to the investors. Higher volatility causes fall in price and low volatility makes prices to rise. The authors name this relation as volatility feedback hypothesis. Finally, among the popular stylised facts is spill-over effect of volatility. By this feature, volatility appears to be contagious having tendency to overflow from one market to another market and one region to another region. Mamtha and Srinivasan (2016) argue that this is because strong linkage among the various financial markets enables information of market movements to transmit all around the world. Many empirical studies argue about the presence of a strong spill-over return and volatility effect from US to other markets that includes emerging markets also. At micro market level, many researchers have attempted to investigate the spill-over effects between indices and individual stocks or stocks and their underlying derivative instruments. On one hand, a lot of academic evidence can be found about transmission of volatility from US to Indian stock market. At the same time, Indian market is also believed to effect volatility in some of the Asian markets. Mukherjee and Mishra (2010) argue that almost all Asian markets except China and Malaysia have overnight return and volatility spill-over effects caused by Indian stock market.

In the second part of this review, a brief account of efficacy of various volatility models is presented. The literature has a mixed empirical support about usefulness of various volatility forecasting approaches. Nevertheless, choice of model should resolves two competing criteria, i.e. length of time and frequency of measurement to be used. While unconditional models work well even when data observation are just the sufficient, for instance, within month daily data for a monthly estimate, on the contrary, to perform efficiently, conditional models require sufficiently large or a very high frequency data. A daily conditional measure may be
unimaginable unless possibility of high frequency intra-day data. Merton (1980) suggests that variance over a fixed interval can be estimated arbitrarily, although accurately, as the sum of squared realizations, if data are available at a sufficiently high sampling frequency. Hwang and Pereira (2006) suggest estimating GARCH models with a data sample having not at least 500 observations would be prone to convergence errors and persistence may get understated resulting in faster reversal to the unconditional mean. This would mean that an ideal monthly GARCH is possible when there is at least forty years of monthly historical stock return data. The whole literature appears to be inconclusive with respect to ranking of different models for their efficiency. When returns are independently and identically distributed, a simple sum of squares provides the best estimate of short term volatility. Beyond that, conditional GARCH models were actually believed to be very well designed to take care of inconsistencies in time series behaviour of stock return volatility. As suggested by researchers, “GARCH (p,q) model can parsimoniously capture leptokurtosis, volatility clustering, non-trading periods, forecastable events and the relation between macroeconomic uncertainty” (Carroll and Kearney, 2009, p. 74).

The standard GARCH or GARCH (1,1) model followed by Exponential GARCH or EGARCH model has very wide applicability in the empirical volatility modelling irrespective of different return distribution intervals and markets. Nevertheless, “GARCH models undergo through a bias that empirical results obtained are dependent on the sampling frequency” (Engle & Patton 2001, p. 22). Nath and Dalvi (2008) found that realized volatility measures given by sum of squared returns from high frequency data performed better than GARCH model estimates. Kuen and Hoong (1992) and Kumar (2006) in separate studies conducted in Singapore and India found that exponentially weighted moving average or EWMA model performed superiorly well to forecast stock market volatility over the naive volatility and GARCH volatility methods. Starica (2003) argue that GARCH (1,1) provides a poor longer horizon forecasting against simple non-stationary approach based on historical volatility as they found them more than four times bigger than a simple forecast based on historical volatility. Differing from these evidences, Akgiray (1989) suggested that time series of daily stock returns exhibits significant levels of dependence for several time intervals and GARCH (1,1) defined data generating process very well and produced best forecast accuracy. Yu (2002) observed that out of nine different volatility estimates comprising of random walk, historical average, moving average, simple regression, exponential smoothing, EWMA, ARCH, GARCH and stochastic volatility models, stochastic volatility model has been rated best in terms of its
forecasting performance in the New Zealand stock market. Poon & Granger (2003) in their survey of about 66 studies that involved comparing volatility models notice that option implied standard deviation provided the best forecasting, while historical volatility and GARCH roughly performed the same. McMillan, Speight and Apgwilym (2000) argue that after considering asymmetric loss functions, random walk and historical mean is favoured for daily, historical mean, simple regression and exponential smoothing for weekly and random walk for monthly volatility forecasts in the UK stock market. From this discussion, the empirical evidence suggests lack of unanimity on which model should be the right choice. Nevertheless, a model specific to certain data generating process should however, incorporate the given stylised facts concerning stock market volatility. To the best of knowledge of this study, most of works in Indian stock market have emphasised testing of volatility models, a work that reviews various theoretical issues surrounding volatility models is rarely found. Therefore, the present study aims to put a gamut of issues that surround stock market volatility and volatility modelling in a proper perspective so as to present meaningful insights for the academicians and practitioners. Following this, the objectives of the study are underlined as:

i. To understand the concept of stock market volatility.

ii. To examine various stylised facts associated with stock market volatility.

iii. To analyse and evaluate theoretically main models of stock market volatility.

3. Methodology:

This is a study based on review of literature on stock market volatility. The work is expected to present a bird’s-eye view of stock market volatility, its characteristics and various volatility models. The main purpose of this review work is to fill the gap in the literature for the researchers and practitioners, to discuss various volatility modelling approaches, and to present the strengths and limitations of each that are available in the literature. Therefore, literature survey based research approach is specifically followed. Towards this end, a thorough theoretical scrutiny of a good number of research papers has been pursued to understand in the first step stock market volatility and associated issues thereof. In the second step, mechanism of main volatility models along with theoretical evaluation of each is also conducted.

4. Volatility Models:

In finance, volatility refers to the degree of variability of returns (of financial assets) around its mean value. The mean of the returns may be itself constant or varying over time. Volatility as a measure of risk is also very much instrumental in pricing, valuation, and investment decisions
of financial assets. With the early developments in finance during the last century, be it Markowitz's Modern Portfolio Theory of 60's or Sharpe-Lintner's Capital Asset Pricing Model introduced in 70’s, volatility or variance remained an equally important component along with asset returns for evaluating efficiency of financial assets. Most of these models assume volatility as time invariant, i.e. change in the value of returns measured through volatility remains constant over time. For example, asset pricing models including Capital Asset Pricing Model or CAPM assume residuals errors are homoscedastic or follow a time invariant variance. In reality, we find that error terms of a return series whose volatility measurement we may be interested in vary over time and therefore, characterise what is popularly called in econometrics as Heteroscedasticity.

4.1 Historical or Realised Volatility measures:

By definition, historical or realised volatility is the simplest of volatility measurement approaches. Pandey (2002) argues that volatility of asset returns is by convention being calculated through standard deviation of daily closing stock returns that belong to certain time length from past. Such a daily estimate is scalable to any period, such as, annual, monthly, etc. following square root of time rule, provided asset returns were from an i.i.d distribution. Let’s consider stock returns \( r_t \) at time \( t = 1, 2, 3, \ldots, T \), then realised volatility or standard deviation of returns can be shown as:

\[
\sigma^2(r) = \frac{1}{T-1} \sum_{t=1}^{T} (r_t - \bar{r})^2
\]

Where

\[
\bar{r} = \frac{1}{T} \sum_{t=1}^{T} r_t
\]

To calculate time invariant measure of variance or volatility, the whole sample returns are used. However, if the sample is split in two halves to have two variance measures for comparison, the estimated variances will likely be different, indicating early signs of the fact that the variance or volatility is possibly time-varying. One of the modifications to overcome this problem is taking realised volatility from a rolling sample window. The rolling window technique can either adopt overlapping or non-overlapping data sample to forecast volatility.

\[\text{A condition in a series of a random variables of not every variable having the same finite variance.}\]
For instance, some previous studies like Officer (1973) uses rolling 12-month observation to work out monthly standard deviation, while as French, Schwert and Stambaugh (1987) use daily observations in each month to estimate monthly standard deviation of stock market returns. The later approach of French, Schwert and Stambaugh (1987) has been adopted by numerous studies arguing two of its advantages. First, sampling frequently increase the accuracy of the standard deviation estimate for any particular interval and the second it takes care of time invariant nature of volatility. Historical volatility that is captured from closing auction prices\(^5\) is not susceptible to manipulation or a “fat fingered\(^6\)” trade, unless other proxies of prices are used as input for volatility measurement.

French, Schwert and Stambaugh (1987) provide an alternative method to simple standard deviation that overcome problems of autocorrelation particularly in daily return observations due non-synchronous trading. According to this method, variance of the monthly return is the sum of the squared daily returns plus twice the sum of the products of adjacent returns during a certain sample window expressed as:

\[
V_{it} = \frac{D_t}{d} \sum_{d=1}^{D_t} r_{id}^2 + 2 \sum_{d=1}^{D_t} r_{id} r_{id-1}
\]  

(2)

Where \(D_t\) is the number of trading days in month \(t\) and \(r_{id}\) is the return of stock \(i\) in day \(d\). Since variance is measured on daily basis, detaching of mean returns is not important. The second term of equation adjusts the variance to the autocorrelation of stock returns. The second term of autocorrelation is dropped if volatility measure \(V_{it}\) turns out to be \(<1\) to force non-negativity condition of volatility.

In addition to close-to-close measure, various models that use different price proxies for measurement of volatility have been developed. Among these, Parkinson’s measure (1980), Garman and Klass measure (1980), Rogers Satchell measure (1994), Yang Zhang measure (2000) have been widely used in the academic literature. Parkinson’s method instead of close-to-close prices uses high and low price during a volatility measurement horizon, generally a day.

\(^5\) The closing price of securities at Indian’s main stock exchange, BSE is computed on the basis of weighted average price of all trades executed during the last 30 minutes of a continuous trading session. In case there is no trade recorded during the last 30 minutes, then the last traded price of security in the continuous trading session is taken as the official closing price.

\(^6\) A human error that is caused by inadvertent use of computer or other devices while placing an order in the market. The order may hence occur into a wrong type (buy or sell) than what is desired or may be of far greater size than intended. Such errors can potentially impact market prices significantly.
\[
Volatility_{parkinson} = \sigma_p = \sqrt{\frac{F}{N} \left( \frac{1}{4\ln(2)} \sum_{i=1}^{N} \left( \ln \left( \frac{h_i}{l_i} \right) \right)^2 \right)} \tag{3}
\]

Since this approach could underestimate volatility because it assumes trading occurs continuously while in reality market is halted on certain days in a week. Garman and Klass is just an improvement over Parkinson measure that is based on open and close prices in a day. In case of unavailability of open prices, previous day close price is used in that case.

\[
Volatility_{Garman-Klass} = \sigma_{GK} = \sqrt{\frac{F}{N} \left( \sum_{i=1}^{N} \left( \frac{1}{2} \left[ \ln \left( \frac{h_i}{l_i} \right) \right]^2 + (2\ln(2)-1) \left[ \ln \left( \frac{c_i}{o_i} \right) \right]^2 \right) \right)} \tag{4}
\]

Under this method, as overnight jumps (assumption that opening price is not different from the previous days close) of prices are not considered, as such, it also leads to underestimation of volatility. The methods discussed so far do not include mean or average return (assuming it zero) for volatility measurements, while as it may be significantly different from zero as return frequency moves from high to low. This aspect of volatility is taken care by Rogers-Satchell measure that accounts for volatility of securities with a mean different from zero. However, the drawback of this method is that it cannot capture jumps, hence, could produce under-estimated volatility.

\[
Volatility_{Rogers-Satchell} = \sigma_{RS} = \sqrt{\frac{F}{N} \left( \sum_{i=1}^{N} \left( \frac{h_i}{c_i} \ln \left( \frac{h_i}{c_i} \right) + \frac{h_i}{c_i} \ln \left( \frac{c_i}{o_i} \right) + \frac{l_i}{c_i} \ln \left( \frac{l_i}{c_i} \right) + \frac{l_i}{c_i} \ln \left( \frac{l_i}{o_i} \right) \right) \right)} \tag{5}
\]

Finally, among the proxy based time-varying volatility methods, Yang Zhang method measures volatility as weighted average of close-open volatility and the open-close volatility. This method overcomes the main drawbacks of previous models by allowing non-zero drift and opening jumps. However, tends to slightly underestimate the volatility following assumption of continuous prices. It also assumes that past and current volatility are independent of each. According to Bennett & Gil (2012), in presence of correlation between daily and overnight return, Yang-Zhang approach is likely to underestimate volatility.

\[
Volatility_{Yang-Zhang} = \sigma_{YZ} = \sqrt{F \left( \sigma_{overnight volatility}^2 + k \sigma_{open-close volatility}^2 + (1-k)\sigma_{RS}^2 \right)} \tag{6}
\]

Where,

\[
k = \frac{0.34}{1.34 + \frac{N+1}{N-1}} \quad \text{and;}
\]
One of the major drawbacks of historical volatility is that it assumes equal weights to individual observations while calculation mean-square deviations. While as the memory of market says that a recent observation may be more relevant than a distant one. Along with these models, linear regressions models that consider volatility as time invariant have been in use ever since introduction of CAPM during 1960’s. CAPM and other factor asset pricing models that are based on ordinary least square regression assume to have residuals as white noise with $N(0, \sigma^2)$. In other words, it follows that asset returns variance is time invariant, as such, exhibits what is popularly known in financial economics as, “homoscedasticity”. In reality, volatility of stock returns would be heteroscedastic and exhibiting positive auto correlations (persistence). Three main properties that are most prevalent in case of financial time series of stock returns include: volatility clustering\(^7\) or volatility changing widely across the time that leads to creation of volatility bunches, leptokurtosis\(^8\) and leverage effect\(^9\). Jegajeevan, 2012 suggests that it would be inappropriate to employ models that are based on constant variance when a financial time series is fraught with such characteristics.

### 4.2 EWMA, a simple time-varying Volatility Model:

A simple and convenient method to deal with irregular behavior of volatility is given by Exponentially Weighted Moving Average or (EWMA) approach. This method models variance or volatility by assigning more importance to recent observations over past observations unlike close-to-close realised or historical volatility measure which follows that all observations should have an equal weight. “By using a large number of past observations, assigning more recent ones greater weight, a volatility estimate which has high information content and is more sensitive to recent shocks will be obtained. Hence, the volatility estimate will respond to market factor shocks more rapidly and a marked market factor shock will

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\(^7\) A phenomenon in volatility that is characterised by “large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes” (Mandelbrot 1963). Alternatively, volatility in price of financial assets tends to exhibit in bunches of small or large form over time.

\(^8\) A characteristic in stock returns where tails in its distribution are much longer and fatter than the normal Gaussian type distribution.

\(^9\) A characteristics under which volatility responds asymmetrically to the information, i.e., the potential of a negative price change to influence volatility more than a positive price change of similar magnitude.
“leave” the volatility estimate gradually, avoiding an echo effect” (Sironi & Resti, 2007, p. 168). EWMA measure can be expressed as:

\[ \sigma_i^2 = \lambda \sigma_{i-1}^2 + (1 - \lambda) x_i^2 \]  

(7)

Where \( \sigma_i^2 \) denotes current conditional volatility, \( \sigma_{i-1}^2 \) is past volatility and \( x_i^2 \) is the contemporaneous return term. The parameter lambda or \( \lambda \) is known as decaying factor whose value range between 0-1, 0 if current volatility matters and 1, if current volatility has no role. The value of lambda\(^{10}\) being used should take care of speed of decaying as well as size of sample. Whatever the case, the value of lambda is set in a manner so that sum of total weights in a given sample of data sums to unity. A high lambda would lead to slower decay indicating that there are more data points in the series which are going to die out more slowly. Thus, the effect of past volatility would fade out gradually. Contrary to this, a low lambda leads to faster decay indicating that there are fewer number observations being used for volatility estimation. Bennett & Gil (2012) argue that EWMA volatility is preferable over realised volatility because it takes into account past influence of volatility on current volatility, hence instead of effect of spike getting suddenly disappeared in historic volatility, it gradually fades off under EWMA volatility. The EWMA seems to have much practical appeal as National Stock Exchange (NSE)\(^{11}\) and the Bombay Stock Exchange (BSE)\(^{12}\), the two premier stock exchanges of India, use EWMA volatility to measure Value at Risk (VaR)\(^{13}\) margin for listed equity securities and commodity derivative contracts.

5. Symmetric time-varying volatility models:

5.1 ARCH Model

To counter the issues of times series of stock returns as listed above, Robert F. Engle in the year 1982 for the first time ever introduced Auto Regressive Conditional heteroscedasticity (ARCH), a volatility method that provides for estimation of time-varying variance (conditional heteroscedasticity) and volatility persistence in stock returns. The ARCH model uses heteroscedasticity in stock returns as a property to capture variance more appropriately instead of treating it as a problem to be corrected. “Essentially, the conditional variance \( h_t \) depends

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\(^{10}\) Risk Metrics uses 0.94 weight for \( \lambda \) in case of daily volatility estimates (trading portfolio), and 0.97 for monthly volatility estimates (investment portfolio).

\(^{11}\) See: https://www.nseindia.com/products/content/equities/equities/margins.htm

\(^{12}\) See: https://www.bseindia.com/static/markets/equity/EQReports/risk_management.html#her3

\(^{13}\) Value at Risk or VaR quantifies the risk exposure into a numeric measure in terms of certainty and the time interval.
on past news, hence, can be modeled as a function of the lagged $e_i$” (Engle 1982). Assuming
that investors make their investment decision on the basis of information set $\Psi_{t-1}$ which they
have in time $t-1$, the expected value, $m_t$ and expected volatility, $h_t$ of stock return, $r_t$ can be
shown as a conditional function given $\Psi_{t-1}$, i.e., $m_t = E(r_t / \Psi_{t-1})$ and $h_t = Var(E(r_t / \Psi_{t-1}))$” (Engle
1982). More specifically, it can be defined as:

$$r_t = E(r_t / \Psi_{t-1}) + e_t = m_t + e_t$$  \hspace{1cm} (8)

From this, $e_t = r_t - m_t$ can be shown as the unexpected return at time, t. $e_t$ represents change in
return due to shock or innovation and $h_t$, the conditional variance is a lagged function of these
shocks over time. Hence, conditional variance under ARCH framework is expressed as an auto
regressive process of the lagged squared innovations or error terms. To begin with:

$$e_t^2 = \omega + \alpha e_{t-1}^2 + \nu_t$$  \hspace{1cm} (9)

The above equation is originally an Auto Regressive or AR(1) model that assumes $\nu_t$ is an $i.i.d$
process$^{14}$. Following this equation, ARCH (1) can estimated as:

$$r_t = c + \rho r_{t-1} + e_t$$  \hspace{1cm} (10)

$$e_t = \sqrt{h_t} \text{ where } z_t = N(0,1)$$  \hspace{1cm} (11)

$$h_t = \omega + \alpha e_{t-1}^2$$  \hspace{1cm} (12)

Equation (10) is called the mean equation, which shows that returns follow an AR (1) process.
Equation (11) and (12) jointly constitute the variance equation. The comprehensive model,
Engle developed is the $qth$ order ARCH model, the ARCH ($q$) can be expressed as:

$$h_t = \omega + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \ldots + \alpha_p e_{t-q}^2$$  \hspace{1cm} (13)

Where $\omega > 0, \alpha_1, \alpha_2, \ldots, \alpha_p \geq 0$

The effect of a shock or innovation $i$ periods ago ($i \leq q$) on current volatility is governed by
the parameter $\alpha_i$ which is expected to be $\alpha_i < \alpha_j$ for $i > j$. As time lapses, the effect of news
diminishes, i.e., “the older the news gets, the less effect it would have on current volatility. In

$^{14}$ Independently and identically drawn, i.e., a series of random variables, each variable is uncorrelated with past
and having uniform variance.
an ARCH \((q)\) model, a news that arrived markets more than \(q\) periods ago will have no effect at all on current volatility. Put differently, if a major market movement occurred yesterday or up to \(q\) days ago, the effect will be to increase today’s conditional variance” (Karmakar 2005, p. 23).

**5.2 GARCH Model**

GARCH \((p,q)\) or Generalised ARCH model proposed by Bollerslev in 1986 is most popular among the models of conditional volatility. According to GARCH model, volatility is a function of its own past and also past random shocks. In other words, conditional volatility given by GARCH \((p,q)\) is determined from lagged conditional volatility \(h_{-r}\) and lagged squared innovations in mean equation \(\varepsilon_{r-m}^2\). The simplest yet widely recognised GARCH \((1,1)\) specification is shown as:

\[
\begin{align*}
 r_t &= \mu + \varepsilon_t \\
 \varepsilon_t &= \sqrt{h_t} \\ 
 z_t &\sim N(0,\sigma^2_t) \\
 h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}
\end{align*}
\]

Equation (14) is the known as mean equation of GARCH model and equation (16) is the variance equation. The variance equation comprises of two terms, the ARCH term and the GARCH term. “The size of the parameters \(\alpha\) and \(\beta\) determine the short-run dynamics of the resulting volatility time series. Large GARCH coefficient \(\beta\) indicates that shocks to conditional variance do not vanish quickly, hence volatility is ‘persistent’ and large ARCH coefficient \(\alpha\) means that volatility reacts quite strongly to market movements and if \(\alpha\) is relatively high and \(\beta\) is relatively low then volatilities tend to be more ‘spiky’. \(\alpha+\beta\) being close to unity indicates a ‘shock’ at time \(t\) will persist for many future periods implying that volatility has a ‘long memory’. If \(\alpha+\beta = 1\) then any shock will lead to a permanent change in all future values of conditional volatility” (Karmakar 2007, p. 104). Kalotychou and Staikouras (2009) argue that volatility persistence has certain advantages like, they can be useful in predictability of future economic growth and risk-return trade-off dynamics over business cycles. GARCH model has certain restrictions. First, \((\alpha+\beta)\) has to be < 1 to make volatility estimates meaningful. As already mentioned, \((\alpha+\beta)\) being close to 1 indicates long memory (high persistence) in volatility of asset returns. If \((\alpha+\beta)\) is \(\geq 1\), the effect of shock will be permanent. Hence, condition of \((\alpha+\beta) < 1\) is imposed in estimating GARH model. Second,
since variance can’t be negative, as such, another parameter restriction in GARCH estimation is non-negativity of $\omega, \alpha, \beta$ coefficients. “The GARCH $(p,q)$ model described below can prudently capture leptokurtosis, volatility clustering, non-trading periods, forecastable events and the relation between macroeconomic uncertainty.

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}$$

(17)

Where conditional variance $h_t$ is a weighted function of its long-run value (dependent on $\alpha_0$), information that caused volatility during previous periods, $\alpha_i \varepsilon_{t-i}^2$, and the fitted variance from previous periods, $\beta_j h_{t-j}$. The model has similar parameter restrictions as GARCH (1,1) to ensure strictly non-negativity of the variance” (Carroll and Kearney 2009, p.74-75).

5.3 GARCH in Mean (GARCH-M) Model

Asset Pricing theories suggest a principal positive risk and return relation in case of financial assets. This indicates that increase in the risk of the assets should be followed by increase in their returns. The theories in finance literature suggest that if volatility of the asset returns is time varying, then during high period of volatility, the expected returns should be higher and vice-versa. The ARCH-in-mean (ARCH-M) model proposed by Engle, Lilien and Robins (1987) establishes relation between risk and conditional variances in an intertemporal setup. This much extensively used version from GARCH family can be expressed as:

$$r_t = c + \delta \sqrt{h_{t-1}} + \varepsilon_t$$

(18)

$$h_t = \alpha_0 + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

(19)

The $\delta$ delta term signifies the risk premium which accounts for the stylised fact\textsuperscript{15} that stocks having higher variance are likely to have higher expected return. When $\delta$ is significantly positive, it indicates that a higher conditional variance generates a higher return. In other words, returns move with change in the volatility. “In some empirical applications, the conditional variance term $h_{t-1}$, appears directly in the conditional mean equation, rather than in square root form $\sqrt{h_{t-1}}$. Also, in some applications the term is contemporaneous, $h_t$ rather than lagged” (Brooks 2014, p. 445).

\textsuperscript{15} Stylised facts are in general statistical properties in a variable that tend to be consistent across time and space (markets).
6. Asymmetric time-varying Models:

Asymmetry, persistence (long memory) and volatility transmissions are now globally known stylised facts about the behavior of volatility. The GARCH models suffer through a serious limitation, i.e., the assumption that volatility reacts to positive and negative shocks symmetrically, which essentially may not be a reality. A number of variants of the standard GARCH model that are believed to deal with serious asymmetric limitations of the GARCH models have been proposed. These include Nelson’s (1991) EGARCH or Exponential GARCH model, GJR model by Glosten, Jagannathan and Runkle (1993) and TGARCH or Threshold GARCH model of Glosten et al., (1993) Zakoian (1994). A brief description of these volatility model is given under.

6.1 GJR Model:

The GJR model, as a simple improvement of the GARCH model has an additional term meant to account for possible asymmetries in volatility. The conditional variance as per GJR is expressed as:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma \varepsilon_{t-1}^2 I_{t-1}$$

(20)

Where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ or $= 0$ otherwise

The $\gamma$ coefficient captures the asymmetric or leverage effect. This coefficient is expected to have a value of $> 0$ to satisfy the presence of leverage effect. Here the condition of non-negativity is $\alpha_0 > 0$, $\alpha_1 > 0$, and $\beta \geq 0$, and $\alpha_1 + \gamma \geq 0$. The model is still valid, even if $\gamma < 0$, provided that $\alpha_1 + \gamma \geq 0$.

6.2 Exponential GARCH Model:

The EGARCH model given by Nelson in 1991 is the most popular among asymmetric models. This model uses log transformation of conditional variance to ensure that conditional variance always remains non-negative without imposing any constraints, thus overcoming the major limitation of the GARCH model. This method allows the conditional variance to deal with both the sign and size attributes of the past residuals. EGARCH model can be expressed as:

$$\ln(h_t) = \alpha_0 + \beta \ln(h_{t-1}) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + a \left[ \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} - \frac{\sqrt{2}}{\sqrt{\pi}} \right]$$

(21)
Where, beta term, \( \beta \) is meant to include the influence of past volatility on the current conditional volatility and the delta term, \( \gamma \) captures the effect of asymmetry. A negative value of \( \gamma \) is consistent to what is known in the literature as, “leverage effect”. “If there is a negative relation between returns and volatility, \( \gamma \) will be negative. The absolute value the standardised error terms, \( \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \) have an expected value \( \sqrt{\frac{1}{\pi}} \) assuming the standardised errors are distributed as \( N(0,1) \). If the absolute standardised errors are greater (less) than expected, the conditional variance will rise (fall). Hence, the fourth term in the model captures the magnitude of the lagged error terms” (Carroll and Kearney 2009, p.76).

6.3 Threshold GARCH Model:

The threshold GARCH or TGARCH model proposed by Zakoin in 1994. This model like EGARCH accounts for asymmetric effect in volatility. It can be expressed as:

\[
h_t = \alpha_0 + \alpha \varepsilon_{t-1}^2 + \delta \varepsilon_{t-1}^2 D_{t-1} + \beta h_{t-1}
\]

(22)

Where, \( D_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{if } \varepsilon_{t-1} > 0 \end{cases} \)

The dummy variable \( D_{t-1} \) constitutes bad news (\( \varepsilon_{t-1} < 0 \)), a positive value of \( \delta \) would signify an asymmetric response of the volatility. When the change in \( \varepsilon_{t-1} \) is positive, the total effect in variance would be \( \alpha \varepsilon_{t-1}^2 \), and when the return shock is negative, the total effect in the variance would be \( (\alpha + \delta) \varepsilon_{t-1}^2 \).

7. Conclusion:

This study presents a comprehensive account on the properties of stock market volatility and also theories on various conditional and unconditional estimation approaches to volatility. It must be noted that all the models discussed here depend on historical movements of a variable to forecast its future volatility. The parametric volatility modelling undergoes through a serious bias. In absence of a true volatility model, different models would end up giving different measures of volatility about a similar random variable of interest. While volatility forecasting is sensitive to model factor loadings, the choice of a model itself depends on the length and interval of data being considered for estimation of volatility. This study begins with the description of standard deviation measure which is considered as a simple and basic model of volatility, as such enjoys an intuitive appeal. This method depends on past information
assuming that volatility does not change over time, a condition which is highly unrealistic in
case of stock returns and in particular when a long series of data is available to model volatility.
The introducing of a rolling window sample that accounts for a moving average is a naive
initiative to have an intertemporal measure of volatility. Moreover, adjusting realized volatility
models to accommodate first order autocorrelation in stock returns overcomes effects of non-
synchronous trading in returns data. Next, variants of historical volatility that use different
proxies for measurement of volatility moving from close-to-close price quote to open-close
and high-low prices are examined. The close-to-close price quotes are more widely used in low
frequency data intensive volatility models as they are not biased due to fat finger errors.
Among these models, most are fraught with under or over-estimation issues of intended
volatility of a given variable. Parkinson’s measure ignores the effects due to unavailability of
close price quotes on off-trading days in a week. Similarly, Garman and Klass method does
not take care of over-night jumps happening in price quotes. Likewise, Rogers-Satchell
volatility though takes care of drift in returns on the time-series but it also lacks ability to handle
jumps in prices. Yang Zhang measure though overcomes issues of price jumps and non-zero
mean component, yet it also is likely to underestimate volatility for its reliance on the
assumption of continuous prices. EWMA measure is simplest version of conditional models
that accounts for time-varying properties of time series of stock returns. This method does so
by assigning higher weights to the recent shocks and returns over past ones assuming that
volatility is effected both by drift and the past information about volatility. ARCH model given
by Engle is by far considered a revolutionary breakthrough in volatility modeling. Under the
conditional models, symmetric GARCH model of Bollerslev has a wide applicability in the
academic literature followed by asymmetric EGARCH model given by Nelson. These models
are flexible to accommodate any form of persistence or spikiness in the variable series, yet for
obvious reasons they are hard to be evaluated for efficiency. Considering the peculiarities of
the data series, unfolding of rare economic events having a capacity to disturb the normal
behavior of markets, the journey for search of ideal conditional models to capture dynamic
nature of volatility seems to be a never ending pursuit. However, it must be put on record that
conditional models discovered so far have been able to answer to most of the stylised facts
concerning time series of stock returns volatility. As a final note, realised volatility being
simple in calculation enjoys a wider faith among researchers in finance, nevertheless, EWMA
that is being followed by practioners seems to be an ideal response to intertemporal nature of
volatility. This present study represents a humble attempt to fill the gap in stock market
volatility literature by focusing on volatility stylised facts and evaluation of models that claim to accommodate such underlying properties in stock returns volatility.

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