A Quantitative Study on the Cycle Length of Refracted Rays Affected By Ocean Eddies

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Abstract. A local linear layered (LLL) model is proposed in this paper and the underwater sound channel is described by a segmental linear function. Taking advantage of the LLL model, cycle length of refracted rays can be derived as an explicit formula and the factors that could affect cycle length can be reduced to several parameters with intuitive physical meanings. Numerical simulations have shown that the results of the formula agree well with Bellhop.

1. Introduction

Ocean eddies are common phenomena in the ocean, which have attracted the attention of both oceanographers and underwater acousticians for decades. A primary environmental characteristic of ocean eddies is a large distortion of the normally horizontal isotherms. Studies have shown that cold eddies are responsible for isothermal uplifts of 500 m or more, with similar depressions for the warm case [1, 2, 3]. Due to the large temperature variations, ocean eddies cause substantial perturbations in sound speed, resulting in conspicuous fluctuations of underwater acoustic field.

The impacts of ocean eddies on acoustic propagation have been intensely studied and modeled because of their importance to underwater engineering and sonar detection. Vastano [4] and Gemmill [5] used ray-theoretical models to investigate the effects of eddies on transmission loss and ray geometry. Baer [6] utilized a primary 3D version of the parabolic equation to study propagation through an eddy. Lu [7] took advantage of experimental results to illustrate the influence of a cold eddy on acoustic transmission. Jian [8] investigated the effects of sound speed perturbations induced by three cold eddies on long-range sound propagation. Li [9] studied the influence of eddies on the shifting of convergence zones (CZ). All investigators indicated that major physical properties of the sound speed profile (SSP), such as sound speed, sound speed gradient, depth of SOFAR axes, would be influenced by ocean eddies, resulting in the shifting of convergence zones, altered multipath arrival sequences and horizontal refraction. Then questions arise: Among those physical properties, which one plays the dominate role in the fluctuations of acoustic field? Is there any way to quantitatively...
describe the impacts of each property on the fluctuations? Unfortunately, previous researches about acoustic propagating through ocean eddies were mostly studied qualitatively.

This article is devoted to solving the above questions. Taking into account of the fact that ray trace is an intuitive way to reflect the acoustic field and horizontal cycle length is a representative characteristic of the ray trace, the horizontal cycle length seems to be an effective parameter to intuitively reflect the fluctuations of the acoustic field. Therefore, this paper will take the horizontal cycle length of ray trace as the research object and quantitatively study the impacts of the physical properties of SSP on the cycle length in an eddy environment.

First and most important, one needs to explicitly formulate the relationship between the cycle length and the SSP. To achieve this goal, a local linear layered (LLL) model is proposed in this paper. The LLL model can be considered as an approximation of the real sound speed profile in the ocean. Numerical studies demonstrate that the cycle length in the LLL model is in good agreement with that in the real SSP. The LLL model consists of a sequence of parallel layers with constant sound speed gradients and the sound speed profile of the LLL model can be described by a segmental linear function. The LLL model has two distinct advantages: (1) The cycle lengths of rays can be explicitly formulated in the LLL model; (2) Parameters in the formula has intuitive physical meanings and correspond specific physical properties of SSP. Taking advantage of the LLL model, the formula of cycle lengths is derived and impacts of the physical properties of SSP on the cycle lengths of rays is quantitatively studied.

In this paper, numerical simulations are carried out to verify the cycle length formula. The results in an eddy environment are compared with those obtained in the absence of eddies to demonstrate the effects of eddies on the cycle lengths of ray traces. Besides, the impacts of the parameters in the formula on the cycle lengths have also been numerically studied and the physical mechanism is explained briefly. As it will be shown later, results of the cycle length formula in the LLL model are in good agreement with those calculated by Bellhop program in the real SSP.

This paper is organized as follows. In Sec. II, the local linear layered model is introduced. In Sec. III, the cycle length of rays in the LLL model is formulated using Snell's law. In Sec. IV, numerical simulations are carried out in an eddy environment. The performance of the cycle length formula is compared intuitively with the results obtained by Bellhop program. The conclusions follow in Sec. V.

2. The Local Linear Layered Model
In this section, the local linear layered (LLL) model is introduced.

Typical North Pacific winter profile of sound speed versus depth is shown in Fig. 1, which is representative of those encountered in many tropical and sub-tropical deep-ocean areas. There have already existed several analytical models about the profile that have been widely used, such as Munk profile [10], GDEM model [10] and LSSPM [12]. However, the horizontal cycle lengths of the ray traces cannot be formulated explicitly in the above models. To overcome the problem, the LLL model is proposed in this paper.

Since the horizontal gradient of the sound velocity is high-order smaller than the vertical one in most circumstance, the LLL model regards the local SSP as an linear layered medium. According to the characteristics of the sound speed gradients, the local linear layered model further divides the local SSP into five layers by five characteristic depths.

Just below the sea surface is the mixed layer where the sound speed increases with depth. The base of this layer is termed the mixed layer depth (MLD) denoted by $z_1$, which is associated with the near-surface maximum in sound speed. In the LLL model, sound speed in the mix layer is approximated as a linear function.

$$C(z) = c_i + a_i(z - z_1), 0 \leq z \leq z_1$$ (1)
Where \( a_0 \) is used to denote the average gradient of sound speed in the mix layer and \( c_1 \) represents the sound speed at the MLD \( z_1 \).

Below the mixed layer lies the transition layer, a region where the sound speed decreases slowly with increasing depth. Sound speed in the transition layer can be expressed as

\[
C(z) = c_1 + a_1(z - z_1), z_1 \leq z \leq z_2
\]  

(2)

Where \( a_1 \) is used to denote the average gradient of sound speed in the transition layer.

Below the transition layer is the thermocline layer, a region of the water column where the temperature and sound speed decrease rapidly with depth. This region is characterized by a negative sound-speed gradient. Sound speed in the thermocline layer is approximated by a linear function.

\[
C(z) = c_2 + a_2(z - z_2), z_2 \leq z \leq z_3
\]  

(3)

Where \( a_2 \) represents average gradient of sound speed in this layer. \( z_2 \) and \( z_3 \) are used to denote the depths of the top and bottom boundary of the thermocline layer respectively, and corresponding sound speeds are termed \( c_2 \) and \( c_3 \).

Between the conjugate depths of \( c_3 \) is the axis layer. The conjugate depths refer to a pair of depths that have the same value of sound speed but which lie on opposite sides of the sound channel axis. Here the pair of conjugate depths of \( c_3 \) is denoted by \( z_3 \) and \( z_4 \). In the axis layer, sound speed decreases with depth first from \( z_3 \) to the sound channel axis \( z_a \), then increases from the axis to \( z_4 \). Sound speed in this layer can be approximated by a segmental linear function,

\[
C(z) = \begin{cases} 
  c_a + a_3^+(z - z_a), & z_3 < z \leq z_a \\
  c_a + a_3^-(z - z_a), & z_a < z \leq z_4 
\end{cases}
\]  

(4)

Where \( a_3^+ \) represents the average gradient of sound speed between \( z_3 \) and \( z_a \), and \( a_3^- \) represents the average vertical gradient of sound speed between \( z_a \) and \( z_4 \). \( c_a \) is the sound speed of the sound channel axis.

Below the axis layer, and extending to the sea floor, is the deep isothermal layer. This layer has a nearly constant temperature in which the speed of sound increases with depth due to the effects of pressure. Sound speed can be expressed as,

\[
C(z) = c_3 + a_4(z - z_4), z_4 < z \leq z_5
\]  

(5)

Where \( a_4 \) is the average gradient of sound speed in this layer.
3. Horizontal cycle lengths of ray traces in the LLL model

In this section, the basic equations of the horizontal cycle lengths of ray traces in the LLL model is derived by using of Snell’s law.

3.1. Horizontal distance of a ray in a horizontally stratified medium

In a horizontally stratified medium, at every \( z \), a ray must satisfy the Snell’s law,

\[
\frac{c(z)}{\cos \theta(z)} = \frac{c(z_0)}{\cos \theta_0} = \text{const}
\]

(6)

Where \( \theta_0 \) is the grazing angle which a ray makes with planes \( z=\text{const.} \) The constant in Eq. (6) is determined by the grazing angle \( \theta_0 \) of a ray at the source \( z_0 \).

For an arbitrary small element of a ray, the total horizontal distance covered by the ray is,

\[
D = \int_{z_0}^{z} \frac{dz}{\tan \theta}
\]

(7)

Here \( D \) is assumed to be a single valued function of \( z \). If this is not the case (for instance, in the underwater sound channel where a ray repeatedly returns to some fixed depth), Eq. (7) is applied to those parts of the ray for which it is single valued.

Considering the simplicity and wide application of the constant gradient approximation, the medium is divided into a set of layers where the dependence of velocity on depth is linear. For an arbitrary layer, we denote layer boundaries by \( z_{i-1}, z_i \). Let \( c_i \) and \( \theta_i \) be the sound velocity and the grazing angle at the lower boundary of the layer, respectively. The horizontal distance \( D_i \) covered by the ray in the layer can be expressed as,

\[
D_i = \frac{c(z_{i-1})}{\cos \theta_{i-1}} \left| \frac{\sin \theta_i - \sin \theta_{i-1}}{a_i} \right|
\]

(8)
Where $a_i = \frac{c(z_i) - c(z_{i-1})}{z_i - z_{i-1}}$ is the sound velocity gradient in the layer. Substitute Eq. (6) into Eq. (8), one gets.

$$D_i = \frac{c(z_0)}{\cos \theta_0} \left| \frac{\sin \theta_i - \sin \theta_{i-1}}{a_i} \right|$$

(9)

For the total horizontal distance covered by the ray, we have.

$$D = \sum_i D_i = \frac{c(z_0)}{\cos \theta_0} \left| \frac{\sin \theta_{b_2} - \sin \theta_{b_1}}{a} \right|$$

(10)

Where

$$a = \frac{1}{\sum a_i (\sin \theta_{b_2} - \sin \theta_{b_1})} \sum \frac{1}{(\sin \theta_i - \sin \theta_{i-1})}$$

(11)

Here $\theta_{b_1}$ and $\theta_{b_2}$ are the grazing angles at the boundaries of the medium, respectively. The effective average sound speed gradient of the medium $a$ can be regarded as the weighted mean of the sound speed gradients in these layers. It is worth noting that Eq. (11) is suitable in the situation where $D$ is a single valued function of $z$. If this is not the case, Eq. (11) is applied to those parts of the ray for which it is single valued.

3.2. Horizontal distance of channel rays in the LLL model

In the deep ocean, studies have shown that channel rays trapped below the transition layer play a dominate role in the convergence zone. In this section, we derive the horizontal distances of a channel ray in the thermocline layer, the axis layer and the deep isothermal layer, which are denoted by $D_2$, $D_3$ and $D_4$ respectively in Fig.2.

**Figure 2.** The diagram of a channel ray.
Combining Eq. (3) and Eq. (10), the horizontal distance of a channel ray from the up turning depth (corresponding grazing angle is 0) in the thermocline layer to \( z_3 \) can be express as

\[
D_2(r) = \frac{c_0}{\cos \theta_0} \left| \frac{\sin \theta_3(r)}{a_2(r)} \right| .
\]

Here \( \theta_3 \) is the grazing angle at \( z_3 \). Since the sound speed varies with range in real ocean environment, the parameters in eq. (12) varies with horizontal range \( r \).

Similarly, horizontal distance of a ray from \( z_4 \) to the down turning depth in the deep isothermal layer can be described as

\[
D_4(r) = \frac{c_0}{\cos \theta_0} \left| \frac{\sin \theta_4(r)}{a_4(r)} \right| .
\]

In the axis layer, the sound speed profile is approximated by a segmental linear function Eq. (4), the horizontal length of a ray in this layer consists of two parts.

\[
D_3(r) = D_3^+(r) + D_3^-(r)
\]

Where the horizontal distance from \( z_3 \) to \( z_a \) is.

\[
D_3^+(r) = \frac{c_0}{\cos \theta_0} \left| \frac{\sin \theta_3 - \sin \theta_a(r)}{a_3^-(r)} \right|
\]

And the horizontal distance from \( z_a \) to \( z_4 \) is.

\[
D_3^-(r) = \frac{c_0}{\cos \theta_0} \left| \frac{\sin \theta_3 - \sin \theta_a(r)}{a_3^+(r)} \right|
\]

Here \( \theta_a \) is the grazing angle at the sound channel axis \( z_a \).

From the above derivation, it should be noted that the sound speed gradients \( a_2, a_3^-, a_3^+, a_4 \) in the LLL model actually refer to the effective average sound speed gradients of corresponding layers.

3.3. Cycle length of channel rays in the LLL model

In the context, horizontal distance between two successive up turning points of a channel ray, is named cycle length of the ray. As shown in Fig.2, cycle length of channel rays can be expressed as a summation of horizontal distances in the thermocline layer, the axis layer and the deep isothermal layer.

\[
T(r) = 2[D_2(r) + D_3(r) + D_4(r)]
\]
Substituting Eqs. (12)- (16) Into Eq. (17) and considering the grazing angle at the turning depth is zero, one obtains.

\[
T = 2 \frac{c_0}{\cos \theta_0} \left[ \sin \theta_1(r) + \frac{\sin \theta_2 - \sin \theta_1(r)}{a_2(r)} + \frac{\sin \theta_3 - \sin \theta_1(r)}{a_3^+(r)} + \frac{\sin \theta_4 - \sin \theta_1(r)}{a_4(r)} \right]
\]  
(18)

Here the range parameter is omitted for simplicity. Eq. (18) will be simplified in the rest of this section.

According to Eq. (6), one obtains.

\[
\sin \theta = \sqrt{1 - \frac{c^2}{\rho^2}}
\]  
(19)

Where \( \rho = \frac{c_0}{\cos \theta_0} \) is the sound speed at the turning depth.

Then divide the sound speed into two parts,

\[
c(z) = \rho - \Delta c(z), 0 \leq \Delta c(z) \ll \rho
\]  
(20)

Where \( \Delta c(z) \) is the deviation of sound speed at \( z \) with respect to \( \rho \).

Substituting Eq. (20) into Eq. (19) and adopt the first order approximation, one gets.

\[
\sin \theta \approx \sqrt{\frac{2\Delta c}{\rho}}
\]  
(21)

Finally, combining Eq. (18) and Eq. (21), one gets.

\[
T = 2\sqrt{\rho} \left[ \frac{\sqrt{2\Delta c(z_1)}}{a_2} + \frac{\sqrt{2\Delta c(z_2)}}{a_4} + \frac{2c_3 - 2c_4}{\sqrt{2\Delta c(z_0^2) + 2\Delta c(z_0)}} \left( \frac{1}{a_3^+} + \frac{1}{a_3^-} \right) \right]
\]  
(22)

As mentioned in Section III.B, the physical meanings of \( a_3^+ \) and \( a_3^- \) are the effective average sound speed gradients of the axis layers. To simplify the above formula, we adopt the following approximation.

\[
a_3^+ \approx \frac{c_3 - c_a}{z_3 - z_a}, a_3^- \approx \frac{c_3 - c_a}{z_3 - z_a}
\]  
(23)

Combining Eq. (22) and Eq. (23), the cycle length of channel rays can be expressed as.

\[
T = 2\sqrt{\rho} \left[ \frac{\sqrt{2\Delta c(z_1)}}{a_2} + \frac{\sqrt{2\Delta c(z_2)}}{a_4} + \frac{H}{B} \right]
\]  
(24)

Where
\[ B = \frac{\sqrt{2\Delta c(z_a)}}{2} + \frac{\sqrt{2\Delta c(z_f)}}{2} \]  
\[ H = z_4 - z_3 \]  

Eq. (24) is the explicit formula for the cycle length of channel rays. Obviously, the formula is derived briefly and easy to be understood by taking advantage of the LLL model. What's more, each parameter in Eq. (24) has an intuitive physical meaning and reflect a specific physical property of the sound speed profile. In the following, the physical meanings of these parameters are discussed in detail.

The parameter \( p \) corresponds to the sound speed at the turning depth of a channel ray. Given the grazing angle of a ray omitted from a source is unchanged, the parameter \( p \) could be influenced by the sound speed disturbance at the source. Parameters \( a_3 \) and \( a_4 \) are the effective average sound speed gradients of the thermocline layer and the deep isothermal layer respectively, which have already been illustrated in Section III.B. According to the definition of Eq. (20), the parameter \( \Delta c(z_f) \) represents the deviation of sound speed at the boundary of the axis layer with respect to the parameter \( p \). Similarly, the parameter \( B \) reflects the average deviation of sound speed in the axis layer with respect to the parameter \( p \). At last, the parameter \( H \) is actually the thickness of the axis layer, as shown in Eq. (26). From a certain point of view, the above six parameters could be regarded as six physical properties of the SSP. And the sound speed profile just affects the cycle length of channel rays through the six physical properties. Taking advantage of Eq. (24), the impacts of the physical properties on the cycle length can be quantitatively studied in an eddy environment.

It should be emphasized that the parameters in Eq. (24) is actually range dependent and that the values of these parameters should corresponding to the local SSP where the ray propagates. Besides, Eq. (24) is suitable for the situation where the horizontal gradient of sound speed is high-order smaller than the vertical one. If this is not the case, the incline of the sound velocity isoclines due to the horizontal gradient would not be neglect, and the LLL model would be inapplicable. In the environment of ocean eddies, Eq. (24) is shown to be valid by numerical simulations, which will be illustrated in detail in the next section.

4. Numerical simulations

In this section, numerical simulations in an cold eddy environment are conducted to verify the validation of Eq. (24). Taking advantage of the equation, the impacts of the physical properties of SSP on the cycle length of channel rays are quantitatively studied.

4.1. Sound speed environment in a cold eddy

![Figure 3](image-url)  
**Figure 3.** Sound speed disturbance (unit: m/s) in the cross section of sound propagation (left). Sound speed profile at several distances with respect to the eddy core (right).
The sound speed structure of ocean eddies is obtained by ocean reanalysis. Ocean reanalysis is a method of combing historical ocean observations with a general ocean model (typically a computational model) driven by historical estimates of surface winds, heat, and freshwater, by way of a data assimilation algorithm to reconstruct historical changes in the state of the ocean. In this paper, hycom global ocean reanalysis data (downloaded from the website https://hycom.org) is adopted, which can provide 3-dimensional fields of temperature, salinity and density. The hycom data has a uniform 0.08 degree lat/lon grid between 80.48S and 80.48N and is interpolated to 40 standard z-levels in depth, which is sufficient to extract the sound speed structure for ocean eddies.

Then a typical cold eddy in Northwest Pacific is extracted from the sound speed field. Sound speed disturbance in the cross section of sound propagation and several sound speed profiles are shown in Fig.3.

4.2. The cycle length of a channel ray
In this section, the cycle length formula is tested in the background sound speed profile and the cold eddy environment, respectively. The validity of the cycle length formula is tested by Bellhop model in the following way:

1) Ray traces are calculated by the Bellhop model in a real sound speed field and the cycle lengths of channel rays are extracted.

2) The real sound speed field is approximated by a LLL model and the cycle lengths of channel rays are estimated by Eq. (24).

3) The two kinds of cycle length are compared to test the validity of Eq. (24).

![Figure 4. The values D1, D2, D3 in the background environment (left) and in the eddy core (right). Red stars denote Bellhop results, blue circles denote the values of eq. (24).](image)

![Figure 5. The variations of D1, D2, D3 in the cold eddy.](image)
Sound source is posed at 600m. Fig.4 demonstrates the values D1, D2, D3 for a channel ray with grazing angle -9 degree. Relative deviation of cycle length T is less than 3%, which demonstrates that the estimated values of eq. (24) agree well with the true values both in eddy environment and background circumstance.

Fig.5 shows the variations of D1, D2, D3 with respect to the background environment when the ray propagates through the cold eddy. It can be found obviously that D2 and D3 play dominate role. But they counteract with each other since their signs are not the same.

5. Conclusion
A local linear layered (LLL) model is proposed in this paper and the underwater sound channel is described by a segmental linear function. Taking advantage of the LLL model, cycle length of refracted rays can be derived as an explicit formula and the factors that could affect cycle length can be reduced to several parameters with intuitive physical meanings. Numerical simulations have shown that the results of the formula agree well with Bellhop. In the simulation circumstance, the ray traces in the axis layer and the deep isothermal layer dominate the variations of cycle length when a ray propagates through a cold eddy.

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