Computation of Nirmala Indices of Some Chemical Networks

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Abstract

Chemical graph theory is a branch of graph theory whose focus of interest is to finding topological indices of chemical graphs which correlate well with chemical properties of the chemical molecules. In this paper, we compute the Nirmala index, first and second inverse Nirmala indices for some chemical networks like silicate networks, chain silicate networks, hexagonal networks, oxide networks and honeycomb networks along with their comparative analysis.

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1. Introduction:

In this paper, we consider finite, simple, connected graphs. Let \( G = (V, E) \) be a graph. The degree \( d_G(v) \) of a vertex \( v \) is the number of vertices adjacent to \( v \). The edge connecting the vertices \( u \) and \( v \) will be denoted by \( uv \). We refer to¹ for undefined term and notation.

A molecular graph or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. Several topological indices² are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties.³,⁴,⁵

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In\(^6\), the Nirmala index and Nirmala exponential of a molecular graph \(G\) were introduced and they are
defined as
\[
N(G) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \quad \text{and} \quad N(G, x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u) + d_G(v)}}.
\]

In\(^7\), the first and second inverse Nirmala indices of \(G\) were introduced and they are defined as
\[
IN_1(G) = \sum_{uv \in E(G)} \left( \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right)^{1/2} \quad \text{and} \quad IN_2(G) = \sum_{uv \in E(G)} \left[ \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^{1/2}.
\]

Recently, some Nirmala indices were studied in\(^8, 9, 10\). Many other topological indices were studied, for example, in\(^11, 12, 13, 14, 15, 16, 17, 18\).

In this paper, we compute the Nirmala index, inverse Nirmala indices for some chemical networks. For more details on some chemical network, we refer to\(^9\).

2. Results for Silicate Networks:

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network is symbolized by \(SL_n\), where \(n\) is the number of hexagons between the center and boundary of \(SL_n\). A silicate network of dimension two is depicted in Figure 1.

![Figure 1. Silicate network of dimension two](image)

In the following theorem, we compute the Nirmala index and its exponential of \(SL_n\).

**Theorem 1.** Let \(SL_n\) be the family of silicate networks. Then
(i) \(N(SL_n) = (1 + \sqrt{2})54n^2 + (6\sqrt{6} + 18 - 36\sqrt{2})n\).
(ii) \(N(SL_n, x) = 6nx^\sqrt{6} + (18n^2 + 6n)x^3 + (18n^2 - 12n)x^{3\sqrt{2}}\).

**Proof:** Let \(G\) be the graph of a silicate network \(SL_n\) with \(|V(SL_n)| = 15n^2 + 3n\) and \(|E(SL_n)| = 36n^2\). By algebraic method, in \(SL_n\) there are three types of edges based on the degrees of end vertices of each edge as follows:
\(E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}\), \(|E_3| = 6n\).
\[ E_3 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}, |E_3| = 18n^2 + 6n. \]
\[ E_9 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}, |E_9| = 18n^2 - 12n. \]

By using the definitions and cardinalities of the edge partition of \( SL_n \), we deduce

(i) \[ N(SL_n) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} = (3 + 3\sqrt{3})^2 6n + (3 + 6\sqrt{3})^2 (18n^2 + 6n) + (6 + 6\sqrt{3})^2 (18n^2 - 12n). \]

After simplification, we get the desired result.

(ii) \[ N(SL_n, x) = \sum_{uv \in E(G)} x^{d_G(u) + d_G(v)} = 6nx^{(3+3\sqrt{3})^2} + (18n^2 + 6n)x^{(3+6\sqrt{3})^2} + (18n^2 - 12n)x^{(6+6\sqrt{3})^2}. \]

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of \( SL_n \).

**Theorem 2.** Let \( SL_n \) be the family of silicate networks. Then

(i) \[ \text{IN}_1(SL_n) = \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \right) 18n^2 + \left( \frac{\sqrt{2}}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \right) 6n. \]

(ii) \[ \text{IN}_2(SL_n) = \left( \sqrt{2} + \sqrt{3} \right) 18n^2 + \left( \frac{\sqrt{5}}{\sqrt{2}} + \sqrt{2} - 2\sqrt{3} \right) 6n. \]

**Proof:** From definitions and by cardinalities of the edge partition of \( SL_n \), we deduce

(i) \[ \text{IN}_1(SL_n) = \sum_{uv \in E(G)} \left[ \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^{\frac{1}{2}} = \left( \frac{1}{3} + \frac{1}{3} \right)^{\frac{1}{2}} 6n + \left( \frac{1}{3} + \frac{1}{6} \right)^{\frac{1}{2}} (18n^2 + 6n) + \left( \frac{1}{6} + \frac{1}{6} \right)^{\frac{1}{2}} (18n^2 - 12n). \]

After simplification, we get the desired result.

(ii) \[ \text{IN}_2(SL_n) = \sum_{uv \in E(G)} \left[ \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^{\frac{1}{2}} = \left( \frac{1}{3} + \frac{1}{3} \right)^{\frac{1}{2}} 6n + \left( \frac{1}{3} + \frac{1}{6} \right)^{\frac{1}{2}} (18n^2 + 6n) + \left( \frac{1}{6} + \frac{1}{6} \right)^{\frac{1}{2}} (18n^2 - 12n). \]

After simplification, we get the desired result.

3. Results for Chain Silicate Networks:

We now consider a family of chain silicate networks. This network is symbolized by \( CS_n \) and is obtained by arranging \( n \) tetrahedral linearly, see Figure 2.

![Figure 2. Chain silicate network](image)
In the following theorem, we compute the Nirmala index and its exponential of $CS_n$.

**Theorem 3.** Let $CS_n$ be the family of chain silicate networks. Then

(i) $N(CS_n) = (\sqrt{6} + 12 + 3\sqrt{2})n + 4\sqrt{6} + 6 - 6\sqrt{2}$.

(ii) $N(CS_n, x) = (n + 4)x^\frac{3}{2} + (4n - 2)x^3 + (n - 2)x^\frac{6}{5}$.

**Proof:** Let $G$ be the graph of chain silicate networks $CS_n$ with $|V(CS_n)| = 3n + 1$ and $|E(CS_n)| = 6n$. By algebraic method, in $CS_n$, $n \geq 2$, there are three types of edges based on the degree of the vertices of each edge as follows:

- $E_6 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, |E_6| = n + 4$.
- $E_9 = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}, |E_9| = 4n - 2$.
- $E_{12} = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}, |E_{12}| = n - 2$.

By using the definitions and cardinalities of the edge partition of $SL_n$, we deduce

(i) \[ N(CS_n) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} = (3 + 3\frac{1}{2})(n + 4) + (3 + 6\frac{1}{2})(4n - 2) + (6 + 6\frac{1}{2})(n - 2). \]

After simplification, we get the desired result.

(ii) \[ N(CS_n, x) = \sum_{uv \in E(G)} x^{d_G(u) + d_G(v)} = (n + 4)x^{(3+3)\frac{1}{2}} + (4n - 2)x^{(3+6)\frac{1}{2}} + (n - 2)x^{(6+6)\frac{1}{2}}. \]

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of $CS_n$.

**Theorem 4.** Let $CS_n$ be the family of chain silicate networks. Then

(i) $IN_1(CS_n) = \left(\frac{\sqrt{2}}{\sqrt{3}} + \frac{4}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)n + 4\sqrt{6} - 2\sqrt{2} - 2\sqrt{3}$.

(ii) $IN_2(CS_n) = \left(\frac{\sqrt{5}}{\sqrt{2}} + 4\sqrt{2} + \sqrt{3}\right)n + 4\sqrt{6} - 2\sqrt{2} - 2\sqrt{3}$.

**Proof:** From definitions and by cardinalities of the edge partition of $CS_n$, we derive

(i) \[ IN_1(CS_n) = \sum_{uv \in E(G)} \left[ \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^\frac{1}{2} = (\frac{1}{3} + \frac{1}{3})^2(n + 4) + (\frac{1}{3} + \frac{1}{6})^2(4n - 2) + (\frac{1}{6} + \frac{1}{6})^2(n - 2). \]

After simplification, we get the desired result.

(ii) \[ IN_2(CS_n) = \sum_{uv \in E(G)} \left[ \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^\frac{1}{2} = (\frac{1}{3} + \frac{1}{3})^2(n + 4) + (\frac{1}{3} + \frac{1}{6})^2(4n - 2) + (\frac{1}{6} + \frac{1}{6})^2(n - 2). \]

After simplification, we get the desired result.
4. Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of same kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is symbolized by $HX_n$, where $n$ is the number of vertices in each side of hexagon. A hexagonal network of dimension six is shown in Figure 3.

In the following theorem, we compute the Nirmala index and its exponential of $HX_n$.

**Theorem 5.** Let $HX_n$ be the family of hexagonal networks. Then

(i) $N(HX_n) = 54n^2 + (12\sqrt{10} - 186\sqrt{2})n + 12\sqrt{7} + 18 + 144\sqrt{2} - 24\sqrt{10}$.

(ii) $N(HX_n,x) = 12x^{\sqrt{7}} + 6x^3 + (6n - 8)x^{2\sqrt{2}} + (12n - 24)x^{\sqrt{15}} + (6n^2 - 33n + 30)x^{6\sqrt{2}}$.

**Proof:** Let $G$ be the graph of hexagonal network $HX_n$ with $|V(HX_n)| = 3n^2 - 3n + 1$ and $|E(HX_n)| = 9n^2 - 15n + 6$. In $HX_n$, by algebraic method, there are five types of edges based on the degree of the vertices of each edge as follows:

- $E_7 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 4\}, |E_7| = 12$.
- $E_8 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}, |E_8| = 6$.
- $E_9 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}, |E_9| = 6$.
- $E_{10} = \{uv \in E(G) \mid d_G(u) = 4, d_G(v) = 6\}, |E_{10}| = 12n - 24$.
- $E_{12} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}, |E_{12}| = 9n^2 - 33n + 30$.

By using the definitions and cardinalities of the edge partitions of $HX_n$, we deduce

(i) $N(HX_n) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}$

$$= (3 + 4)^{\frac{1}{2}} 12 + (3 + 6)^{\frac{1}{2}} 6 + (4 + 4)^{\frac{1}{2}} (6n - 18) + (4 + 6)^{\frac{1}{2}} (12n - 24) + (6 + 6)^{\frac{1}{2}} (9n^2 - 33n + 30).$$

After simplification, we get the desired result.
\(N(HX_n, x) = \sum_{uv \in E(G)} x^{|d_G(u)+d_G(v)|}\)
\[
= 12x^{(3+4)n} + 6x^{(3+6)n} + (6n-18)x^{(4+4)n} + (12n-24)x^{(4+6)n} + (9n^2 - 33n + 30)x^{(6+6)n}.
\]
After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of \(HX_n\).

**Theorem 6.** Let \(HX_n\) be the family of hexagonal networks. Then

(i) \(IN_1(HX_n) = \frac{9}{\sqrt{3}}n^2 + \left(\frac{6}{\sqrt{2}} + \frac{6\sqrt{5}}{\sqrt{5}} - \frac{33}{\sqrt{3}}\right)n + \frac{6\sqrt{7}}{\sqrt{3}} - \frac{12\sqrt{5}}{\sqrt{3}} + \frac{30}{\sqrt{3}}\)

(ii) \(IN_2(HX_n) = 9\sqrt{3}n^2 + \left(\frac{6\sqrt{2}}{\sqrt{2}} + \frac{24\sqrt{3}}{\sqrt{5}} - 33\sqrt{5}\right)n + \frac{24\sqrt{3}}{\sqrt{7}} - 12\sqrt{2} - \frac{48\sqrt{5}}{\sqrt{5}} + 30\sqrt{3}\)

**Proof:** From definitions and by cardinalities of the edge partitions of \(HX_n\), we derive

(i) \(IN_1(HX_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]\)
\[
= \left(\frac{1}{3} + \frac{1}{4}\right)^2 12 + \left(\frac{1}{3} + \frac{1}{6}\right)^2 6 + \left(\frac{1}{4} + \frac{1}{4}\right)^2 (6n-18) + \left(\frac{1}{4} + \frac{1}{6}\right)^2 (12n-24)
\]
\[
+ \left(\frac{1}{6} + \frac{1}{6}\right)^2 (9n^2 - 33n + 30).
\]
After simplification, we get the desired result.

(ii) \(IN_2(HX_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]\)
\[
= \left(\frac{1}{3} + \frac{1}{4}\right)^\frac{1}{2} 12 + \left(\frac{1}{3} + \frac{1}{6}\right)^\frac{1}{2} 6 + \left(\frac{1}{4} + \frac{1}{4}\right)^\frac{1}{2} (6n-18) + \left(\frac{1}{4} + \frac{1}{6}\right)^\frac{1}{2} (12n-24)
\]
\[
+ \left(\frac{1}{6} + \frac{1}{6}\right)^\frac{1}{2} (9n^2 - 33n + 30).
\]
After simplification, we get the desired result.

5. Results for Oxide Networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension \(n\) is denoted by \(OX_n\). A 5-dimensional oxide network is shown in Figure 4.
In the following theorem, we compute the Nirmala index and its exponential of $OX_n$.

**Theorem 7.** Let $OX_n$ be the family of oxide networks. Then

(i) \[ N(OX_n) = 36\sqrt{2}n^2 + (12\sqrt{6} - 24\sqrt{2})n. \]

(ii) \[ N(OX_n, x) = 12nx\sqrt{5} + (18n^2 - 12n)x^{2}\sqrt{2}. \]

**Proof:** Let $G$ be the graph of oxide network $OX_n$ with $|V(OX_n)| = 9n^2 + 3n$ and $|E(OX_n)| = 18n^2$. In $OX_n$, by algebraic method, there are two types of edges based on the degree of the vertices of each edge as follows;

\[ E_6 = \{ uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4 \}, |E_6| = 12n. \]

\[ E_8 = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 4 \}, |E_8| = 18n^2 - 12n. \]

By using the definitions and cardinalities of the edge partition of $OX_n$, we deduce

(i) \[ N(OX_n) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} = (2 + 4)\frac{1}{2} 12n + (4 + 4)\frac{1}{2} (18n^2 - 12n). \]

After simplification, we get the desired result.

(ii) \[ N(OX_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u) + d_G(v)}} = 12x^{(2+4)\frac{1}{2}} + (18n^2 - 12n)x^{(4+4)\frac{1}{2}}. \]

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of $OX_n$.

**Theorem 8.** Let $OX_n$ be the family of oxide networks. Then

(i) \[ IN_1(OX_n) = \frac{18}{\sqrt{2}}n^2 + \left(6\sqrt{3} - \frac{12}{\sqrt{2}}\right)n. \]

(ii) \[ IN_2(OX_n) = 18\sqrt{2}n^2 + \left(\frac{24}{\sqrt{3}} - 12\sqrt{2}\right)n. \]

**Proof:** From definitions and by cardinalities of the edge partitions of $OX_n$, we derive
\( IN_1(OX_n) = \sum_{uv \in E(G)} \left( \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right)^{\frac{1}{2}} = \left( \frac{1}{2} + \frac{1}{4} \right)^{\frac{1}{2}} 12n + \left( \frac{1}{4} + \frac{1}{4} \right)^{\frac{1}{2}} (18n^2 - 12n). \)

After simplification, we get the desired result.

\( IN_2(OX_n) = \sum_{uv \in E(G)} \left( \frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right)^{\frac{1}{2}} = \left( \frac{1}{2} + \frac{1}{4} \right)^{\frac{1}{2}} 12n + \left( \frac{1}{4} + \frac{1}{4} \right)^{\frac{1}{2}} (18n^2 - 12n). \)

After simplification, we get the desired result.

6. Results for Honeycomb Networks

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are very useful in chemistry and also in computer graphics. A honeycomb network of dimension \( n \) is denoted by \( HC_n \), where \( n \) is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 5.

![Honeycomb network of dimension four](image)

Figure 5. Honeycomb network of dimension four

In the following theorem, we compute the Nirmala index and its exponential of \( HC_n \).

**Theorem 9.** Let \( HC_n \) be the family of honeycomb networks. Then

(i) \( N(HC_n) = 9\sqrt{6n^2} + (12\sqrt{5} - 15\sqrt{6})n + 12\sqrt{5} + 6\sqrt{6}. \)

(ii) \( N(HC_n,x) = 6x^2 + (12n - 12)x^{\sqrt{5}} + 6 \times 2^n x^{\sqrt{11}} + (9n^2 - 15n + 6)x^{\sqrt{8}}. \)

**Proof:** Let \( G \) be the graph of honeycomb network \( HC_n \) with \( |V(HC_n)| = 6n^2 \) and \( |E(HC_n)| = 9n^2 - 3n \). In \( HC_n \), by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

\[
E_4 = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 2 \}, \quad |E_4| = 6.
\]

\[
E_5 = \{ uv \in E(G) \mid d_G(u) = 2, \ d_G(v) = 3 \}, \quad |E_5| = 12n - 12.
\]

\[
E_6 = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 3 \}, \quad |E_6| = 9n^2 - 15n + 6.
\]
By using the definitions and cardinalities of the edge partition of $HC_n$, we deduce

(i) $N(HC_n) = \sum_{u \in E(G)} \sqrt{d_G(u) + d_G(v)} = (2 + 2)\frac{1}{2} 6 + (2 + 3)\frac{1}{2} (12n - 12) + (3 + 3)\frac{1}{2} (9n^2 - 15n + 6)$.

After simplification, we get the desired result.

(ii) $N(HC_n, x) = \sum_{u \in E(G)} x^{d_G(u) + d_G(v)} = 6x^2 + (12n - 12)x\sqrt{x} + (9n^2 - 15n + 6)x\sqrt{x}$.

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of $HC_n$.

**Theorem 10.** Let $HC_n$ be the family of honeycomb networks. Then

(i) $IN_1(HC_n) = \frac{9\sqrt{2}}{5} n^2 + 12\sqrt{5} - \frac{15\sqrt{2}}{\sqrt{5}} n + 6 - \frac{12\sqrt{5}}{\sqrt{5}} n + 6\sqrt{\frac{2}{\sqrt{5}}}$.

(ii) $IN_2(HC_n) = \frac{9\sqrt{3}}{\sqrt{2}} n^2 + 12\sqrt{6} - 15\sqrt{3} n + 6 - \frac{12\sqrt{6}}{\sqrt{2}} n + 6\sqrt{\frac{3}{\sqrt{2}}}$.

**Proof:** From definitions and by cardinalities of the edge partition of $HC_n$, we derive

(i) $IN_1(HC_n) = \sum_{u \in E(G)} \left[ \frac{1}{2} d_G(u) + \frac{1}{2} d_G(v) \right]^{\frac{1}{2}} = \left( \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} 6 + \left( \frac{1}{2} + \frac{1}{3} \right)^{\frac{1}{2}} (12n - 12) + \left( \frac{1}{3} + \frac{1}{3} \right)^{\frac{1}{2}} (9n^2 - 15n + 6)$.

After simplification, we get the desired result.

(ii) $IN_2(HC_n) = \sum_{u \in E(G)} \left[ \frac{1}{2} d_G(u) + \frac{1}{2} d_G(v) \right]^{\frac{1}{2}} = \left( \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} 6 + \left( \frac{1}{2} + \frac{1}{3} \right)^{\frac{1}{2}} (12n - 12) + \left( \frac{1}{3} + \frac{1}{3} \right)^{\frac{1}{2}} (9n^2 - 15n + 6)$.

After simplification, we get the desired result.

7. Data Set of Computed Values:

In order to find the usefulness of topological index, we have to predict the coefficient of correlation between the physico-chemical properties and the calculated topological indices. For different values of $n$, the indices are calculated and tabulated below. By using these values, we plot the below graphs and find the coefficient of correlation.

| Nirmala type of Indices | $n = 1$        | $n = 2$        | $n = 3$        | $n = 4$        | $n = 5$        | $n = 6$        | $n = 7$        |
|------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $N(SL_n)$              | 112.1528      | 485.0406      | 1118.6635     | 2013.0215     | 2013.0215     | 4583.9427     | 6260.5058     |
| $N(CS_n)$              | 26.0048       | 44.6999       | 63.3891       | 82.0812       | 100.7733      | 119.4655      | 138.1576      |
| $N(HX_n)$              | 6.4047        | -56.6917      | -11.7881      | 141.1155      | 402.0192      | 770.9228      | 1247.8264     |
| $N(OX_n)$              | 46.3644       | 194.5523      | 444.5634      | 796.3980      | 1250.0560     | 1805.5373     | 2462.8420     |
| $N(HC_n)$              | 12.0000       | 68.2267       | 168.5442      | 312.9525      | 501.4517      | 734.0416      | 1010.7224     |
8. Comparative Analysis:

In this section, we will compare the result of Nirmala type of indices of certain networks in graphical form. Different colors have been used to represent the behavior of the indices for certain networks in the form of graphical lines. And these graphs have been generated by putting different values of $n$ and $x$, mentioned in above Table 1. A comparison is made in figure 6 and figure 7, the above mentioned networks are very close at the beginning and then grew. Among the five structure of networks, silicate network ($SL_n$) is most powerful compare to other structure of networks. The chain silicate ($CS_n$) grew more slowly than other networks. From the vertical axis of the graph $N(G)$, $IN_1(G)$ and $IN_2(G)$, it is clear that the indices for different networks grew in the following order.

$$CS_n < HX_n \leq HC_n < OX_n < SL_n$$

![Figure 6: Graphical representation of N(G) and N(G,x) of certain networks.](image-url)
9. Conclusion

In this paper, we have computed Nirmala type indices of certain networks. These results are helpful to understand the deep behavior of the networks. The coefficient of correlation of $N(CS_n)$, $IN_1(CS_n)$ and $IN_2(CS_n)$ is 1 which shows that the line is linearly fitted. This indicates that the Nirmala and inverse Nirmala indices are theoretically fit for the chain silicate network $(CS_n)$.

10. Open Problem

Find the values of different types of Nirmala indices of certain classes of chemical graphs and explore some results towards QSPR/QSAR/QSTR Model.

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