Surface acoustic wave resonators in the quantum regime

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We present systematic measurements of the quality factors of surface acoustic wave (SAW) resonators on ST-X quartz in the gigahertz range at a temperature of 10 mK. We demonstrate an internal quality factor $Q_i$ approaching 0.5 million at 0.5 GHz and show that $Q_i \geq 4.0 \times 10^8$ is achievable up to 4.4 GHz. We show evidence for a polynomial dependence of propagation loss on frequency, as well as a weak drive power dependence of $Q_i$ that saturates at low power, the latter being consistent with coupling to a bath of two-level systems. Our results indicate that SAW resonators are promising devices for integration with superconducting quantum circuits.

We investigate the quality factors of SAW resonators measured inside a dilution refrigerator at high vacuum and a temperature of $T \approx 10$ mK, under which conditions the dissipation of the SAW is very low. We work with the common SAW substrate ST-X quartz due to its known good performance in resonators at room temperature and weak piezoelectric coupling [18], which results in SAWs with little electrical character and hence good prospects for being weakly electrically coupled to environmental sources of dissipation. The reflectors and IDTs are in all cases made from superconducting aluminum such that at 10 mK ohmic losses can be neglected. The resonators were characterized by measuring the complex reflection coefficient $S_{11}(f)$ at the IDT using a vector network analyzer (VNA). Using an RLC equivalent circuit model, one can derive the following expression for the reflection coefficient close to a single resonance:

$$S_{11}(f) = \frac{Q_e - Q_0}{Q_e + 2iQ_e(f - f_0)/f}.$$  

Here $Q_i$ is the internal $Q$ factor of the mode and $Q_e$ is the external $Q$ factor due to the presence of the IDT and measurement port.

The measured single-port SAW resonators can be characterized by a set of geometric parameters illustrated in Fig. 1(a) and listed in Table I. The devices have a frequency response centered at $f_0 = v/\lambda_0$, where $v \approx 3100$ m/s is the SAW velocity and $a = \lambda_0/4$ is the electrode and space width in the lithographically defined mirrors and IDT. The mirrors are separated by an integer number of half wavelengths $d = m\lambda_0/2$ and their reflectivity is given by $R = \tanh(N_g|r_{iI}|) \approx 1$ in the limit $N_g|r_{iI}| \gg 1$, where $N_g$ is the number of electrodes in the mirror and $r_{iI}$ is the reflectivity of each electrode. In this limit, the mirrors have a high reflectivity within their first stopband $\Delta f_{SB} = 2f_0|r_{iI}|/\pi$. The IDT excites and detects SAWs in a broader frequency range than the mirrors (due
to being smaller spatially) and has a bandwidth \( \Delta f_{\text{IDT}} = 1.8 f_0 / N_t \), where \( N_t \) is the number of electrodes in the IDT [18]. Thus within \( |f - f_0| < \Delta f_{\text{FSR}} / 2 \) the resonator supports high-quality resonant modes, measurable via the IDT. Since the electrode reflectivity is small (\( |r_s| \approx 0.2\% \) for our devices), the resonant modes partly penetrate into the mirrors to a penetration depth \( L_p = a / |r_s| \) [18]. The devices therefore behave as acoustic Fabry-Perot cavities, with cavity length \( L_c = d + 2L_p \) and free spectral range \( \text{FSR} = v / L_c = 2 f_0 (d/2a + 1/|r_s|) \). When \( \text{FSR} > \Delta f_{\text{FSR}} \), a single mode resonance is observed within the mirror stopband, whereas for longer resonators, for which \( \text{FSR} < \Delta f_{\text{FSR}} \), multiple resonances are observed (see Fig. 2).

We have initially performed a comprehensive study of SAW resonators at a wavelength of \( \lambda_0 = 6 \mu m \) \( (f_0 = 524 \text{ MHz}) \) to determine how \( Q_e \) and \( Q_i \) depend on transducer and grating geometry. This frequency was chosen for compatibility with standard photolithography for which the feature sizes of \( a = 1.5 \mu m \) are achievable and a large number of devices could be fabricated on a single wafer. In this initial investigation we determined that, in accordance with SAW theory [20], the external quality factor follows \( Q_e \propto L_c / N_t^2 \) and the internal quality factors were limited by grating reflectivity, following

\[
Q_g = \frac{\pi (d + 2L_p)}{\lambda_0 [1 - \tanh(|r_s| N_t)]}.
\]

Based on these observations, we designed a device to realize a high \( Q_i \) (within the confines of our chip geometry) with widely spaced long gratings (device \( p_1 \) see Table I). This device exhibits \( Q_e = 1.16 \times 10^5 \), \( Q_i = 4.53 \times 10^5 \) and the frequency response is shown in Fig. 2(a). Note that this measurement is not in the quantum regime, since \( k_B T \approx h f_0 \).

We next proceeded to fabricate SAW resonators at higher frequencies, using electron beam lithography. In Fig. 3(a) we plot \( Q_i \) for a series of resonators with \( \lambda_0 = 1.0 \mu m \) \( (f_0 \approx 3.1 \text{ GHz}) \), for which the distance between the two gratings \( d \) was varied over the range 0.05–1.8 mm with all

![Fig. 1.](image1)

**FIG. 1.** (a) Schematic of a one-port SAW resonator connected to the measurement setup (A: cryogenic amplifier, B: cold attenuators, C: circulator, and VNA: Vector Network Analyzer). (b) Optical microscope image of a SAW resonator (device \( r_1 \); see Table I). Inset: magnification of the IDT electrodes of a similar device (\( q_1 \)).

| Device | \( a (\text{nm}) \) | \( f_0 (\text{GHz}) \) | \( d / 2a \) | \( Q_e / 10^3 \) | \( Q_i / 10^3 \) | \( Q_i f_0 / 10^{14} \) |
|--------|----------------|----------------|--------------|----------------|----------------|------------------|
| \( p_1 \) | 1500 | 0.52 | 1051 | 116 | 453 | 2.36 |
| \( r_1 \) | 250 | 3.11 | 109 | 24 | 88 | 0.27 |
| \( r_2 \) | 250 | 3.12 | 229 | 18 | 10.4 | 0.32 |
| \( r_3 \) | 250 | 3.11 | 429 | 98 | 18.8 | 0.62 |
| \( r_4 \) | 250 | 3.11 | 829 | 167 | 38.4 | 1.19 |
| \( r_5 \) | 250 | 3.10 | 1229 | 363 | 54.5 | 1.74 |
| \( r_6 \) | 250 | 3.09 | 1929 | 657 | 74.7 | 2.32 |
| \( r_7 \) | 250 | 3.09 | 2429 | 473 | 81.0 | 2.52 |
| \( r_8 \) | 250 | 3.10 | 2829 | 843 | 79.6 | 2.45 |
| \( r_9 \) | 250 | 3.11 | 3229 | 1230 | 103 | 3.18 |
| \( r_{10} \) | 250 | 3.08 | 3629 | 927 | 109 | 3.23 |
| \( q_1 \) | 390 | 2.01 | 1929 | 242 | 171 | 3.43 |
| \( q_2 \) | 340 | 2.29 | 1929 | 499 | 126 | 2.88 |
| \( q_3 \) | 300 | 2.60 | 1929 | 174 | 108 | 2.81 |
| \( q_4 \) | 275 | 2.81 | 1929 | 232 | 78.8 | 2.21 |
| \( q_5 \) | 225 | 3.44 | 1929 | 445 | 47.3 | 1.63 |
| \( q_6 \) | 200 | 3.83 | 1929 | 358 | 41.0 | 1.57 |
| \( q_7 \) | 175 | 4.42 | 1929 | 528 | 40.2 | 1.78 |

![Fig. 2.](image2)

**FIG. 2.** (a) Magnitude (blue) and phase (green) of the measured reflection coefficient \( S_{11} (f) \) of the SAW resonator \( p_1 \). Solid lines are a fit to Eq. (1). (b) Frequency response of device \( q_7 \) showing 18 supported high-\( Q \) modes. The solid red box indicates the resonant mode measured in Fig. 4. A background due to the measurement setup has been subtracted in both (a) and (b).
other geometric parameters fixed (see Table I). The number of modes seen increases from 1 for device \( r_1 \) to 65 for device \( r_{10} \). For devices with more than five modes \( (r_5-r_{10}) \), we took the average \( Q_i \) for the five modes at the center of the grating stopband where the reflectivity is highest. We expect \( Q_i \) to be dominated by the grating reflectivity at low \( d \) and saturate at high \( d \) due to propagation losses. The data follow this trend and they can be fitted with the following relation:

\[
Q_i = \left( \frac{1}{Q_0^2} + \frac{\nu v_p}{\pi f_0} \right)^{-1}.
\]

From a fit to Eq. (3), we determine the electrode reflectivity to be \( \sigma_r = 0.002 \), and we can place an upper limit on the propagation loss at 3.1 GHz of \( \sigma_{\rho}|_{10 mK} < 0.015 \text{ mm}^{-1} \) corresponding to a phonon mean free path of \( l = 1/\sigma_r \approx 6 \text{ cm} \).

We then moved on to investigate the frequency dependence of \( Q_i \), using a device geometry with large \( d = 1929 \lambda_0/2 \), over the range 2.0–4.4 GHz (devices \( q_1-q_7 \); see Table I). These long devices all display around 20 resonant modes within the stopband of the reflectors [see Fig. 2(b) for the frequency response of device \( q_7 \)]. In Fig. 3(b) we plot the dependence of the average internal quality factor \( \overline{Q}_i \), of all modes of each device against average mode frequency \( f_0 \). Error bars represent the standard deviation of \( Q_i \) from the set of resonant modes. The quality factor is seen to decrease with frequency, and the linear trend observed on a log-log scale indicates a polynomial dependence. We fit the data to \( \overline{Q}_i = \pi f f / \nu v_p = c_1 f^{-c_2} \) and find \( c_1 = 719 \pm 85 \), \( c_2 = 2.07 \pm 0.13 \). This strongly suggests \( \alpha_p \propto f^3 \), agreeing with a model developed for SAW propagation loss in the high frequency and low temperature limit [21].

We conclude our investigation with a measurement of the internal quality factor of one of the modes of device \( q_7 \) (\( f_0 = 4.449 \text{ GHz} \)) at low drive powers such that the average resonator phonon population reaches the regime \( \bar{n} \ll 1 \). Figure 4 shows the dependence of \( Q_i \) on drive power \( P \). In this case, the VNA was connected to device \( q_7 \) through highly attenuated microwave lines with an overall attenuation from the instrument to the sample of \( -67 \pm 1 \text{ dB} \). This allows us to accurately estimate the average phonon population \( \bar{n} \) resulting from the coherent drive, shown in the upper scale of Fig. 4. A clear reduction in \( Q_i \) is observed as \( P \) is reduced, saturating at low power at a value of \( Q_0 = 34500 \). A similar dependence has been observed in bulk mechanical resonators [22,23] and electromagnetic coplanar waveguide resonators (CPWRs) [24] at low temperature, and has been attributed to coupling to a bath of two-level systems (TLS). The analytical expression of the loss rate caused by a TLS bath is given by [25]

\[
\alpha_{\text{TLS}} = F \frac{2\pi^2 f_0 n_0 \gamma^2}{\rho \nu^2} \tanh \left( \frac{h f_0}{2k_B T} \right) \left( 1 + \frac{P}{P_c} \right)^{-0.5},
\]

where \( n_0 \) is the density of states of the TLS, \( \rho \) is the density of the crystal, \( P_c \) is a critical power, and \( \gamma \) describes the strength of the coupling between the TLS and the phonons. The filling factor \( F \) takes into account spatial variation of \( \rho \) and \( \gamma \), due to the geometry of the SAW mode [26]. The internal quality factor is related to \( \alpha_{\text{TLS}} \) by \( Q_i = (\nu n_{\text{TLS}} f_0 / \pi f_0 + 1/Q_0)^{-1} \), where \( Q_0 \) takes into account remaining losses. We find that our data fits well to this expression with \( P_c = -65.7 \text{ dBm} \) and \( F n_0 \gamma^2 = 4.5 \times 10^3 \text{ J/m}^3 \). In order to completely validate this model and to estimate the value of \( F \), temperature dependent measurement will be required [24,26]. The ratio \( Q_0/Q_{\text{TLS}} \approx 0.6 \) is much higher than typically seen in CPWRs [27], indicating that any TLS contribution to the loss is considerably less in the SAW case. Such a difference is qualitatively in agreement with an electric field coupling to the TLS bath, since in a weak piezoelectric such as quartz, only a small fraction of the SAW energy is electrical.

We finally comment on the prospects for realizing strong coupling cavity QED between a SAW resonator and a superconducting qubit, which requires a coupling strength \( g \) between qubit and resonator exceeding the linewidths of both [28]. The linewidth for resonator \( q_7 \) in the quantum regime is \( \kappa = 4 f / Q_i \approx 130 \text{ kHz} \), while a superconducting qubit on quartz may be expected to have linewidth \( \gamma < 1 \text{ MHz} \) from experiments on GaAs, a similar piezoelectric

![FIG. 3. (a) \( \overline{Q}_i \) versus \( d \) for devices \( r_1 - r_{10} \). Solid line is a fit to Eq. (3). Shaded area indicates one standard deviation of \( \alpha_p \), and the dashed line is the linear part of this fit (\( \alpha_p = 0 \)). (b) \( \overline{Q}_i \) versus \( \bar{f}_0 \) for devices \( q_1 - q_7 \) and \( r_6 \). Dashed line is a fit to \( \overline{Q}_i = c_1 f^{-c_2} \).](image-url)

![FIG. 4. \( Q_i \) versus drive power \( P \) for the resonant mode \( f_0 = 4.449 \text{ GHz} \) of device \( q_7 \). Solid line: fit to \( Q_i = [\nu n_{\text{TLS}}/(\pi f_0) + 1/Q_0]^{-1} \); \( \bar{n} \) is the mean number of phonons occupying the resonator due to the coherent drive \( P \).](image-url)
substrate [16]. The coupling strength can be estimated as $h g = e \beta V_{\text{rms}}^m$ [29] where the electric potential due to the vacuum fluctuations of the SAW mode $V_{\text{rms}}^m \approx 20$ nV for a geometry similar to device $q_7$, and $\beta$ is a dimensionless parameter that takes into account the geometric match of the qubit to the SAW. For a superconducting qubit with well chosen geometry, $g \approx (0.2) e V_{\text{rms}}^m / \hbar \approx 2 \pi \times 1$ MHz should be easily achievable. Coupling strengths in the 10–100 MHz range could be achieved in stronger piezoelectrics such as LiNbO$_3$ [18] or ZnO [30]. It should also be possible to couple in a similar way a wide variety of other solid-state quantum systems, such as quantum dots and crystal defect center spins, to SAW devices [31].

In summary, we have fabricated SAW resonators with a range of geometries and frequencies in the gigahertz range and measured them at cryogenic temperatures, demonstrating quality factors up to $4.5 \times 10^5$. By measuring a range of different resonator lengths, we are able to place an upper limit on propagation loss at 3.1 GHz and 10 mK of $\alpha_p \lesssim 0.015$ mm$^{-1}$. We observed a frequency dependence of $Q_0$ for long resonators that suggests $\alpha_p \propto f^3$. In a highly isolated measurement setup, we observed a clear power dependence of the quality factor of a 4.4 GHz resonator consistent with coupling to a two-level system bath. We have demonstrated internal quality factors in the $10^8$ range up to 4.4 GHz, which should provide motivation for experiments that integrate SAW resonators with quantum coherent devices such as superconducting qubits.

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