Evidence for Different Freeze-Out Radii of High- and Low-Energy Pions Emitted in Au+Au Collisions at 1 A·GeV

(A. Wagner, C. Müntz, H. Oeschler, C. Sturm)∗
(R. Barth, M. Cieślak, M. Dębowski, E. Grosse, P. Koczoń, M. Mang, D. Miśkowiec, R. Schicker, E. Schwab, P. Senger)†
(P. Beckerle, D. Brill, Y. Shin, H. Ströbele)‡
(W. Waluś)§
(B. Kohlmeyer, F. Pühlhofer, J. Speer, K. Völkel)¶

March 30, 2022

Abstract

Double differential production cross sections of $\pi^-$ and $\pi^+$ mesons and the number of participating protons have been measured in central Au+Au collisions at 1 A·GeV. At low pion energies the $\pi^-$ yield is strongly enhanced over the $\pi^+$ yield. The energy dependence of the $\pi^-/\pi^+$ ratio is assigned to the Coulomb interaction of the charged pions with the protons in the reaction zone. The deduced Coulomb potential increases with increasing pion c.m. energy. This behavior indicates different freeze-out radii for different pion energies in the c.m. frame.

Relativistic heavy ion collisions are a unique tool to study nuclear matter at high densities and temperatures [1]. At bombarding energies around 1 A·GeV, baryonic densities of 2–3 times normal nuclear matter density ($\rho_0 = 0.17$ fm$^{-3}$) are expected to be reached during a time span of 10-15 fm/c (see e.g.: [2]). Thereafter, the high pressure built up in the reaction zone drives the system apart and particles freeze out according to their mean free path. A major experimental challenge is to determine the baryonic density in the different stages of the reaction.

∗Technische Universität Darmstadt, D-64289 Darmstadt, Germany
†Gesellschaft für Schwerionenforschung, D-64291 Darmstadt, Germany
‡Johann Wolfgang Goethe-Universität, D-60054 Frankfurt, Germany
§Uniwersytet Jagielloński, PL-30-059 Kraków, Poland
¶Phillips-Universität Marburg, D-35037 Marburg/Lahn, Germany
This determination requires to measure both the number of baryons inside the reaction zone and its time-dependent volume. The number of participating nucleons is derived from the projectile spectator charges \[3\], whereas the measurement of their spatial extent is more complex. Two-particle correlations \((\pi\pi, KK, pp)\) can be used to extract information on the size and lifetime of the source at freeze-out \[4\].

As an alternative approach, presented in this Letter, the Coulomb potential of the nuclear fireball is extracted by studying its interaction with oppositely charged pions. The strength of the Coulomb field depends on the number of participating protons and their geometrical configuration (resp. the freeze-out radius) at the time of pion emission. At a beam energy of 1 A·GeV pion production mainly proceeds via the excitation and decay of the \(\Delta^{33}_{1232}\)-resonance. In this case the overall \(\pi^-/\pi^+\)-ratio is determined by the \(N/Z\)-ratio of the colliding nuclei via isospin conservation. The variation of the \(\pi^-/\pi^+\)-ratio as a function of the pion energy is attributed to the Coulomb field of the charge in the reaction volume which de- or accelerates the pions according to their charge. Previously, the influence of Coulomb effects on the emitted pions has been studied for different reaction systems and energies \[4, 5\] without deducing the fireball’s Coulomb field. For example, in Ne+NaF collisions at 0.4 A·GeV a strong peak in the \(\pi^-/\pi^+\)-ratio has been found at laboratory angles close to zero degrees which was explained by Coulomb interaction of the charged pions with the projectile spectator \[8\].

In this Letter we report on the measurement of double differential production cross sections of positively and negatively charged pions emitted in central \(^{197}\text{Au}+^{197}\text{Au}\) collisions at 1 A·GeV incident kinetic energy impinging on a target of 1.93 g/cm\(^{-2}\). The experiment was performed with the Kaon spectrometer \[9\] at the heavy-ion synchrotron SIS at GSI. The spectrometer covers a momentum-dependent solid angle of \(\Delta\Omega = (15-35)\) msr with a momentum resolution of \(\delta p/p \simeq 0.01\) over the full momentum range. The momentum resolution is dominated by multiple scattering. The momentum acceptance is \(p_{\text{max}}/p_{\text{min}} \approx 2\) for a given magnetic field setting. The measured laboratory momenta vary between 0.156 GeV/c and 1.5 GeV/c. The spectra were obtained at laboratory angles of \((44 \pm 4)\) degrees. This angular range corresponds to a range of normalized rapidity of \(0.43 \leq y/y_{\text{beam}} \leq 0.74\) with transverse momenta between 0.12 GeV/c and 1.2 GeV/c. The particle identification is performed by the reconstruction of the trajectory and by the determination of the particle velocity with two time-of-flight arrays. The collision centrality is determined by means of the hit multiplicity of charged particles in a 84-fold segmented scintillation detector close to the target covering polar angles between 12 and 48 degrees. The corresponding number of participating protons is derived from the summed charge of projectile spectator fragments measured at forward angles by a 380-fold segmented hodoscope. The method is discussed in ref. \[10\]. In order to minimize the influence of spectator matter (nucleons not contributing to the reaction zone) we have selected collisions with the highest charged particle multiplicity and
pion emission around mid-rapidity. These central collisions which were selected represent $(14 \pm 4)\%$ of the total reaction cross section. The corresponding number of participating protons is determined to be $< Z_{\text{part}} > = 110 \pm 8$.

Figure 1 shows the $\pi^-$ and $\pi^+$ cross section $d^2\sigma / (dE_{\text{c.m.}}^\text{kin} \, d\Omega_{\text{c.m.}})$ as a function of the kinetic energy $E_{\text{c.m.}}^\text{kin}$ in the center-of-momentum frame for central Au+Au reactions. The $\pi^-$ yield clearly exceeds the $\pi^+$ yield at low kinetic energies whereas at higher energies these two yields are close to each other. Similar results are obtained by the FOPI collaboration, yet over a smaller momentum range [10].

In order to determine the energy-integrated $\pi^-/\pi^+$-ratio $R_{\text{exp}} = (d\sigma(\pi^-)/d\Omega) / (d\sigma(\pi^+)/d\Omega)$ we parameterized the pion spectra by the sum of two Maxwell-Boltzmann distributions and extrapolated the fit into the non-measured energy range (for details see ref. [11]).

The overall $\pi^-/\pi^+$-ratio of $R_{\text{exp}} = 1.94 \pm 0.1$ agrees well with the ratios derived from the isospin decomposition of individual nucleon-nucleon collisions at the given beam energy: Using the parameterization of ref. [12] one obtains a value of $R_{\text{iso}} = 1.90$. The corresponding value from the isobar model which assumes a pion production exclusively via a isospin $3/2$ resonance is $R_\Delta = 1.95$. The agreement between experimental and theoretical values indicates that the $\pi^-/\pi^+$-ratio reflects the N/Z asymmetry of the colliding system.

Figure 2 shows the $\pi^-/\pi^+$-ratio for Au+Au as a function of the pion kinetic energy in the c.m. system. The $\pi^-/\pi^+$-ratio decreases from 2.8 at low pion energies to a nearly constant value of 1.1 for pion energies above 0.4 GeV.

For the following analysis we assume that the $\pi^-/\pi^+$-ratio is independent of the pion energy if Coulomb effects are disregarded. This assumption is supported by transport model calculations (IQMD [13], BUU [14]) which find - within the error bars - a constant $\pi^-/\pi^+$-ratio of $\approx 1.9$ for all pion energies if the Coulomb interaction is switched off. The BUU calculation also includes baryonic resonances heavier than the $\Delta_{33}(1232)$ resonance.

In order to extract the strength of the Coulomb force we use a static approximation for the Coulomb field. This assumption is justified as we consider only pion kinetic energies above 0.14 GeV. Their velocities ($\beta > 0.87$) are significantly larger than the expansion velocity of the nuclear fireball ($\beta \approx 0.4$). (Recently, the influence of an expanding source has been studied for charged kaons showing a negligible effect at higher kinetic energies [15].) According to ref. [16], the Coulomb force distorts the pion spectra by modifying both the kinetic energies of the particles and the available phase space. One derives the following relations for the measured cross sections at a given momentum $p$

$$\sigma(\vec{p}) \equiv \frac{d^3\sigma}{dp^3} = \sigma_0(\vec{p}_0(\vec{p})) \cdot \left| \frac{\partial^3 p_0}{\partial^3 p} \right|$$

with the undisturbed momentum $\vec{p}_0$ and the undisturbed differential cross section $\sigma_0(\vec{p}_0(\vec{p}))$ taken at the momentum $\vec{p}_0(\vec{p})$. 

3
The Jacobian in eq. (1) (Coulomb phase-space factor) is evaluated assuming spherical symmetry with the relativistic energy-momentum relation and by using the identity \[ \frac{\partial^3 p_0}{\partial^3 p} = \frac{p_0 E_0 \partial E_0}{p E}. \] (2)

As the pion spectra are not described by a single Maxwell-Boltzmann distribution, the undisturbed differential cross section is parameterized by \[ d^3 \sigma_0 / dp^3_0 \propto \exp(-\beta(E_0) \cdot E_0) \] with \[ \beta^{-1} = 32 \text{ MeV} + 7 \cdot 10^{-2} \cdot E_0. \] This function has been obtained as the average of the functions \( \beta(E_0) \) obtained for the measured \( \pi^- \) and \( \pi^+ \) spectra. For inclusive reactions the function \( \beta \) determined in this way describes well the curvature of neutral pion spectra measured in the same reaction \[ 17. \]

With \( c=1 \)

\[ E_{\pm} = E_0 \pm V_{\text{Coul}}; \quad p_{\pm} = \sqrt{E_{\pm}^2 - m^2} \] (3)

one writes for the spectra of charged pions

\[ \sigma_{\pm}(p_{\pm}) \equiv f_{\pm} \cdot \sigma_0(p_{\pm}) \cdot \frac{p_0 E_0}{p_{\pm} E_{\pm}}. \] (4)

The plus (minus) sign refers to \( \pi^+(\pi^-) \), and \( f_{\pm} \) reflects the weights due to the isospin coefficients. With \( R_{\text{iso}}^{\text{tot}} \equiv f_- / f_+ \) the \( \pi^- / \pi^+ \) ratio can be written as

\[ R = \frac{\sigma_-(p_-)}{\sigma_+(p_+)} = R_{\text{iso}}^{\text{tot}} \cdot \frac{\sigma_0(p_0(p_-^\pi))}{\sigma_0(p_0(p_+^\pi))} \cdot \frac{p_+ E_+}{p_- E_-}. \] (5)

It is worth noticing that for \( p \to \infty \) the value of \( R \) does not necessarily approach unity.

In a first attempt we determine the Coulomb energy \( V_{\text{Coul}} \) using eq. (5). However, with the assumption of a constant Coulomb potential \( V_{\text{Coul}} \) one cannot describe the \( \pi^- / \pi^+ \) ratio in the entire energy range. According to eq. (5), a \( \pi^- / \pi^+ \) ratio of around 1.1 as measured for pion kinetic energies around 0.6 GeV requires a Coulomb potential of 22 MeV. This value, however, is at variance with the \( \pi^- / \pi^+ \) ratio at lower pion energies (see the solid line in fig. 2). Here, the observed ratio clearly exceeds the solid line suggesting a smaller value for the Coulomb potential at lower pion energies. Other attempts to describe the measured \( \pi^- / \pi^+ \) ratio with a constant \( V_{\text{Coul}} \) failed as well \[ 18, 19 \].

One can estimate the freeze-out radius of high-energy pions \( (E_{\text{kin}}^{\text{c.m.}} = 0.6 \text{ GeV}) \) from the determined Coulomb potential of 22 MeV. We assume that the pions are emitted from the surface of a charged sphere, containing all participating protons. The assumption of surface emission is justified by the large pion re-absorption.
probability inside the reaction volume. The Coulomb potential is then given by \( V_{\text{Coul}} = Z_{\text{part}} \cdot e^2 \cdot < r^{-1} > \). With \( < Z_{\text{part}} > = 110 \pm 8 \) and with \( V_{\text{Coul}} = 22 \text{ MeV} \) one obtains an effective radius of

\[
r_{\text{eff}} \equiv < r^{-1} >^{-1} = (7.2 \pm 1.1) \text{ fm}.
\]

According to \( \rho_{\text{eff}} = 3A_{\text{part}}/(4\pi r_{\text{eff}}^3) \) with the number of participating nucleons \( A_{\text{part}} \) this value corresponds to an effective density of \( \rho_{\text{eff}} = (1.1 \pm 0.2) \cdot \rho_0 \).

In the following step we determine the Coulomb potential for lower pion energies. The disagreement between the solid line (calculated with a constant Coulomb potential) and the data in fig. 2 is a signature for a different Coulomb field acting on low-energy pions. In order to describe the measured ratio \( R \) over the entire energy range we vary in eq. (5) the Coulomb potential \( V_{\text{Coul}} \) as a function of the pion kinetic energy. The phase space factor is modified accordingly. The resulting dependence of \( V_{\text{Coul}} \) on \( E_{\text{kin}}^{\text{c.m.}} \) is shown in fig. 3. \( V_{\text{Coul}} \) changes from about 22 MeV for high-energy pions to less than 10 MeV for pion energies below 0.2 GeV. At even lower pion kinetic energies pions have de Broglie wavelengths comparable to the size of the emitting source; this problem requires treatment in terms of quantum dynamics which is beyond the scope of this letter.

A value of 10 MeV for pion energies below 0.2 GeV corresponds to an effective freeze-out radius of \( r_{\text{eff}} = (16 \pm 2) \text{ fm} \) and an effective density of \( \rho_{\text{eff}} = (0.1 \pm 0.03) \cdot \rho_0 \). The reduction of \( V_{\text{Coul}} \) therefore indicates a more dilute charge distribution at freeze-out for low-energy pions and consequently larger effective freeze-out radii.

A dilute nucleon distribution is expected for the late stage of the reaction whereas higher densities are reached in an earlier stage. This interpretation is in agreement with microscopic transport models which predict that high-energy pions (transverse momenta \( p_T \geq 0.5 \text{ GeV/c} \)) are emitted during an early stage of the collision \( t \leq 15 \text{ fm/c} \) where the central baryon density is above \( 2 \cdot \rho_0 \) while low-energy pions are emitted later during expansion [20, 21]. This scenario is also supported by the experimental observation, that the multiplicity of high-energy pions \( (E_{\text{kin}}^{\text{c.m.}} \geq 0.45 \text{ GeV}) \) increases more than linearly with the number of participating nucleons [22]. This behavior is a signature of pion production via multiple baryon-baryon collisions which occur more frequently at higher densities. For the system Ar+KCl at an incident energy of 1.5 A·GeV a variation of the source size with the pion energy has been found previously from studying two-pion correlations [23].

Recently \( \pi^-/\pi^+ \)-ratios have also been measured at much higher bombarding energies. In Au+Au collisions at 10.7 A·GeV the \( \pi^-/\pi^+ \)-ratio is found to be 1.5 for low transverse momenta \( p_T \) and it approaches unity at higher \( p_T \) [24]. In Pb+Pb collisions at 158 A·GeV the \( \pi^-/\pi^+ \)-ratio exceeds one only for very small values of \( p_T \). This weak energy-dependence is explained by Coulomb interaction with a rather small co-moving charge of \( Z_{\text{eff}} = 40 \) [25, 26]. Two features of the measurements at the higher incident energies differ from our observations: at
high bombarding energies the number of pions per baryon is significantly higher and due to charge conservation the overall $\pi^-/\pi^+$-ratio tends to be close to unity [2]. The small energy dependence reveals a rather weak Coulomb effect (low number of charges $Z_{eff}$) likely due to a fast disintegration of the source. In contrast, the large variation of $\pi^-/\pi^+$ from 2.8 to about 1.1 at high pion energies as found in the Au+Au system at 1 A-GeV (see fig. 2) indicates a strong Coulomb effect due to a slowly expanding source (compared to the pion velocity) with a large effective charge $Z_{eff}$.

In summary, we have measured $\pi^-$ and $\pi^+$ spectra in central Au+Au collisions at 1 A-GeV together with the number of participating protons. For this heavy system we found a strong enhancement of the $\pi^-$ yield over the $\pi^+$ yield at low pion kinetic energies. Based on the assumption that the $\pi^-/\pi^+$ ratio is given by Coulomb interaction and isospin conservation, the Coulomb potential of the reaction volume is determined. With the additional experimental information on the number of participating protons, effective source radii are derived. They are found to be smaller for high-energy pions than for low energy pions. This result can be interpreted as a consequence of a pion emitting source which expands: high-energy pions are emitted more likely in the dense and hot phase of the reaction while low-energy pions leave the system after expansion at a more dilute phase.

We wish to thank M. Gyulassy (Columbia Univ., New York) for valuable discussions. This work is supported by the German Bundesministerium für Bildung und Wissenschaft, Forschung und Technologie under contract 06 DA 473, by the Polish Committee of Scientific Research under contract 2P03B11109, and by the Gesellschaft für Schwerionenforschung under contract DA OEK.

References

[1] R. Stock, Phys. Rep. 135 (1986)259.
[2] J. Aichelin, Phys. Rep. 202 (1991)233.
[3] D. Brill et al., Z. Phys. A 355 (1996) 61.
[4] D.H. Boal et al., Rev. Mod. Phys. 62 (1990)553.
[5] D. l’Hôte, Nucl. Phys. A 545 (1992)381c; and references therein.
[6] J. Miller et al., Phys. Rev. Lett. 58 (1987)2408.
[7] J.W. Harris et al., Phys. Rev. C 41 (1990)147.
[8] W. Benenson et al., Phys. Rev. Lett. 43 (1979)683.
[9] P. Senger et al., Nucl. Instr. Meth. A 327 (1993)393.
[10] D. Pelte et al., Z. Phys. A 357 (1997)215.
[11] C. Müntz et al., Z. Phys. A 357 (1997)399.
[12] B.J. VerWest and R.A. Arndt, Phys. Rev. C 25 (1982)1979.
[13] S.A. Bass et al., Phys. Rev. C 51 (1995)3343.
[14] S. Teis et al., Z. Phys. A 356 (1997)421.
[15] A. Ayala and J. Kapusta, Phys. Rev. C 56 (1997)407.
[16] M. Gyulassy and S.B. Kauffmann, Nucl. Phys. A 362 (1981)503.
[17] O. Schwalb et al., Phys. Lett. B 321 (1994)20.
[18] B.A. Li, Phys. Lett. B 346 (1995)5.
[19] M.I. Gorenstein and H.G. Miller, Phys. Rev. C 55 (1997)2002.
[20] B.A. Li and W. Bauer, Phys. Rev. C 44 (1991)450.
[21] S.A. Bass et al., Phys. Rev. C 50 (1994)2167.
[22] C. Müntz et al. Z. Phys. A 352 (1995)175.
[23] D. Beavis et al., Phys. Rev. C 27 (1983)910.
[24] L. Ahle et al., Nucl. Phys. A 610 (1996)139c.
[25] H. Bøggild et al., Phys. Lett. B 372 (1996)339.
[26] T. Osada et al., Phys. Rev. C 54 (1996) R2167.
[27] M. Gorenstein, Phys. Rev. C 51 (1995)1465.
Figure 1: Production cross section of negatively and positively charged pions from central collisions of the reaction system $^{197}\text{Au}+^{197}\text{Au}$ at an incident beam energy of 1 A·GeV and at an emission angle of $\theta_{\text{LAB}} = (44 \pm 4)$ degrees.
Figure 2: $\pi^-/\pi^+$ ratio for the reaction system $^{197}$Au+$^{197}$Au as a function of the pion kinetic energy for central collisions. The solid (dotted) curve shows the calculated ratio for a fixed Coulomb energy of 22 MeV (10 MeV). The error bars reflect statistical errors only; a systematic error of ±5% has to be added.
Figure 3: Variation of $|V_{\text{Coul}}|$ with the pion kinetic energy as deduced from the measured $\pi^-/\pi^+$ ratio for central $^{197}\text{Au}+^{197}\text{Au}$ collisions.