THE SUPERCRITICAL PILE GAMMA-RAY BURST MODEL: THE PROMPT TO AFTERGLOW EVOLUTION

A. MASTICHIADIS\textsuperscript{1} AND D. KAZANAS\textsuperscript{2}

\textsuperscript{1} Department of Physics, University of Athens, Panepistimiopolis, GR 15783, Zografos, Greece
\textsuperscript{2} Astrophysics Sciences Division, NASA/GSFC, Code 663, Greenbelt, MD 20771, USA

Received 2008 October 21; accepted 2009 February 6; published 2009 February 27

ABSTRACT

The “Supercritical Pile” model is a very economical gamma-ray burst (GRB) model that provides for the efficient conversion of the energy stored in the protons of a relativistic blast wave (RBW) into radiation and at the same time produces—in the prompt GRB phase, even in the absence of any particle acceleration—a spectral peak at energy $\sim 1\text{MeV}$. We extend this model to include the evolution of the RBW Lorentz factor $\Gamma$ and thus follow its spectral and temporal features into the early GRB afterglow stage. One of the novel features of the present treatment is the inclusion of the feedback of the GRB produced radiation on the evolution of $\Gamma$ with radius. This feedback and the presence of kinematic and dynamic thresholds in the model are sources of potentially very rich time evolution which we have begun to explore. In particular, one can this way obtain afterglow light curves with steep decays followed by the more conventional flatter afterglow slopes, while at the same time preserving the desirable features of the model, i.e., the well-defined relativistic electron source and radiative processes that produce the proper peak in the $\nu F_\nu$ spectra. In this Letter, we present the results of a specific set of parameters of this model with emphasis on the multiwavelength prompt emission and transition to the early afterglow.

\textit{Key words:} gamma rays: bursts

1. INTRODUCTION

The cosmological origin of gamma-ray bursts (GRBs) has by now been firmly established following the discovery of their afterglows and the determination of their redshifts (Costa et al. 1997; van Paradijs et al. 1997) and the launch of \textit{Swift} which increased the number of observed afterglows and redshift determinations. These developments left little doubt that GRB emission is intimately associated with relativistic blast waves (RBW), as proposed by Rees & Mészáros (1992) and at the same time shifted the focus of the study from the prompt GRB to its afterglow (Zhang & Mészáros 2004; Piran 2004).

The early, sparsely sampled GRB afterglow light curves, were well fitted with simple power-law functions, appropriate to emission from either spherical (Sari et al. 1998) or jetlike (Sari et al. 1999) RBW. However, the launch of \textit{Swift} with its prompt, continuous, broad frequency coverage has provided new unexpected (and unexplained) details of the afterglow light curves. Chief amongst them are (1) an early afterglow steep decrease of the flux ($\propto t^{-3}$ to $t^{-6}$) followed often by a period of constant flux (before its eventual power-law decline) in many bursts. (2) Large flares in the X-ray light curves $\sim 10^3$–$10^6$ s after the beginning of the event (see O'Brien et al. 2006, for more details). These were compounded to the already open problems of the prompt emission, namely, (3) the GRB “inner engine.” (4) The nondissipative transport of the GRB energy to the emission region and, most importantly, its efficient dissipation there. (5) The physics behind the characteristic energy of peak GRB emission, $E_p$, and its narrow distribution within the class of the classic GRB (Mallozzi et al. 1995; Preece et al. 2000). (6) The physics that relate GRB to X-ray-rich (XRR) bursts and X-ray flashes (XRF), transients of lower flux and lower $E_p$, recorded by broadband missions such as \textit{BeppoSAX}, \textit{HETE}, and \textit{Swift} (e.g., Yonetoku et al. 2004). Of the above problems, (1) has received no apparent resolution while (2) is loosely attributed to continued activity at the “inner engine”; while not implausible, this demands activity over timescales almost $10^7$ times longer than the characteristic time associated with the “inner engine” dynamics ($\sim 10^{-3}$ s), as the latter is thought to be related to stellar collapse. Issue (4) is considered to be effected either through protons (e.g., Rees & Mészáros 1992) or magnetic fields (Vlahakis & Königl 2001), however, the necessary and efficient dissipation “is one of the least studied aspects of GRB” (Piran 2004); this issue is generally approached by parameterizing the energy density in relativistic electrons to be a given fraction (typically $\sim 50\%$) of that of protons. Issue (5) is generally open, given the absence of an underlying reason for such a characteristic energy. Monte Carlo simulations of a large number of models (Zhang & Mészáros 2003) failed to reproduce the narrow width of the observed distribution because of the large number of parameters involved and/or because of the lack of strong dependence of $E_p$ on any single parameter. Finally, there are a number of proposals concerning (6) (Yamazaki et al. 2002; Dermer et al. 1999), which appear plausible but without a single one of them universally agreed upon.

The “Supercritical Pile” model (SPM; Kazanas et al. 2002; Mastichiadis & Kazanas 2006, henceforth KGM02 and MK06), adapted from active galactic nuclei (AGNs; Kazanas & Mastichiadis 1999), has been introduced to provide a resolution to (4). The compelling arguments in favor of the SPM are (1) its economic (nonthermal particles not necessary), efficient conversion of the RBW relativistic proton energy into photons through a radiative instability akin to that of a supercritical nuclear pile. (2) Its spectra which exhibit a characteristic value for $E_p \approx 1 \text{MeV}$ (in the lab frame) irrespective of the RBW Lorentz factor $\Gamma$, in agreement with observation (Mallozzi et al. 1995), produced as an “unintended consequence” of the dissipation process. Crucial in addressing these issues has been the presence of an upstream medium which scatters the RBW photons (a “mirror”) and allows them to be re-intercepted by the RBW, while boosted in energy by $\Gamma^2$.

In MK06, we have explored numerically the SPM assuming a constant Lorentz factor $\Gamma$ for the RBW, confirmed the efficiency of proton energy conversion into radiation and the presence of a well-defined value for $E_p$, reflecting the kinematic threshold
of the reaction $p\gamma \rightarrow p e^+ e^-$. The present treatment is far more realistic: (1) it computes the evolution of the RBW Lorentz factor $\Gamma$ through a medium of density $n(r) \propto R^{-2}$, thought to represent the wind of a Wolf–Rayet (W–R) star, also including the effects of the radiative drag of the bulk-Comptonized photons. (2) Replaces the upstream “mirror” required by the model by scattering the RBW photons in this medium. The combination of these effects can result in a rich GRB time evolution, but we presently restrict ourselves to a specific example of a GRB light curve, deferring the broader exploration of other models to a future publication. Despite this limited scope, we can reproduce some of the salient features of the GRB in the afterglow evolution, such as their steep decrease in flux following the termination of their prompt phase, an effect traceable in this specific case to the kinematic threshold of the model.

In Section 2, we provide the general framework of our model with emphasis on its novel aspects compared to previous treatments. In Section 3, we present the results of our calculations, and finally in Section 4, the results are summarized and conclusions are drawn.

2. THE COUPLED RADIATIVE–DYNAMICAL EVOLUTION

We consider a RBW of speed $v_0 = \beta c$ and Lorentz factor $\Gamma$. Its radius $R(t)$ is measured from the center of the original explosion and it is sweeping up the circumstellar medium (CSM) of density $\rho_{\text{CSM}}$. The evolution of $\Gamma$ as a function of radius is given by the combination of the conservation laws of mass

$$\frac{dM}{dR} = 4\pi R^2 \Gamma \rho_{\text{CSM}} - \frac{1}{c^2 \Gamma} \dot{E},$$

and energy–momentum

$$\frac{d\Gamma}{dR} = -\frac{4\pi R^2 \rho_{\text{CSM}} \Gamma^2}{M} - \frac{F_{\text{rad}}}{M c^2},$$

(Chiang & Dermer 1999; Mastichiadis & Kazanas 2008; Boettcher & Principe 2009). Here, $\dot{E}$ is the radiation emission rate as measured in the comoving frame and $F_{\text{rad}}$ is the radiation drag force exerted on the RBW by any radiation field exterior to the flow. Given that the RBW velocity $v_0$ is very close to the speed of light $c$, the entire radiative history of the RBW lies just ahead of it at a distance $D \sim R/\Gamma^2$; therefore, isotropization of this radiation by scattering in the ambient medium (the action of the “mirror”) will lead to its re-interception by the RBW to thus contribute to $F_{\text{rad}}$. This is given by the expression

$$F_{\text{rad}} = \frac{64 \pi}{9 c} \tau_{\text{CM}} n_{e}^{\text{CSM}} \sigma_{T} R \Gamma^4 \dot{E},$$

where $\tau_{\text{CM}}$ is the RBW Thomson depth, $n_{e}^{\text{CSM}} = \rho_{\text{CSM}}/m_{H}$ is the CSM electron density, and $\sigma_{T}$ is the Thomson cross section. In the above expression, two powers of $\Gamma$ are due to the increase of the photon energy density upon its scattering on the “mirror” while the other two to the usual radiative loss rate (an analogous term due to the $p\gamma \rightarrow p e^+ e^-$ reaction was found to increase $F_{\text{rad}}$ by 20% for the specific parameter values discussed herein but it may be more important for different values). The calculation of the $\dot{E}$ and $F_{\text{rad}}$ terms is done by implementing the numerical code used in MK06 to compute the radiation of the SPM. This is done by solving the simultaneous equations

$$\frac{dn_i}{dt} + L_i + Q_i = 0.$$  

The unknown functions $n_i$ are the differential number densities of protons, electrons, and photons while the index $i$ can be any one of the subscripts “p,” “e,” or “\gamma” referring to each species. The operators $L_i$ denote losses or escape of each species from the system, while $Q_i$ denote injection and source terms of each species by each of a number of processes which are described in detail in MK06. The above equations are solved in the fluid frame in a spherical volume of radius $R_0 = R/\Gamma$. This can be justified by the fact that due to relativistic beaming an observer receives the radiation coming mainly from a small section of the RBW of lateral width $R/\Gamma$ and longitudinal width $R/\Gamma^2$ in the lab but $R/\Gamma$ on the comoving frame.

The present treatment differs from that of MK06 in two important aspects.

1) (Hot protons accumulate continuously on the RBW as it sweeps the CSM. This then sets the source terms of the protons ($Q_{\text{p}}^{\text{inj}}$) and electrons ($Q_{\text{e}}^{\text{inj}}$) (with units particles/energy/volume/time) to

$$Q_{\text{p}}^{\text{inj}} = \frac{\rho_{\text{CSM}} c^2}{m_p c^2} (\Gamma^2 - \Gamma) \delta(\gamma_p - \Gamma) \quad \text{(5)}$$

and

$$Q_{\text{e}}^{\text{inj}} = \frac{\rho_{\text{CSM}} c^2}{m_p m_e c^2} (\Gamma^2 - \Gamma) \delta(\gamma_e - \Gamma),$$

i.e., we assume that at each radius $R$ the RBW picks up an equal amount of electrons and protons from the circumstellar medium which have, upon injection, energies $E_p = \Gamma m_p c^2$ and $E_e = \Gamma m_e c^2$, respectively. Consequently, the proton energy injection rate is given by (Blandford & McKee 1976)

$$\left(\frac{dE}{dt}\right)^{\text{inj}} = 4\pi R^2 \rho_{\text{CSM}} (\Gamma^2 - \Gamma) c^3,$$

while a fraction $m_e/m_p$ of the above goes to electrons.

2) (The scattering of the RBW photons takes place on the CSM ahead of the advancing RBW (rather than an ad hoc mirror) and, as such, its photon scattering column is uniquely determined by the initial conditions and, like all other parameters, is a function of time (or equivalently position).

Equations (1), (2), and (4), along with Equations (3), (5), and (6) form a set which can be solved to yield simultaneously the evolution of the RBW dynamics and luminosity. This approach is self-consistent in that the “hot” mass injected through Equations (5) and (6) shows up at right-hand side of Equation (1), while the radiated luminosity $\dot{E}$ feedbacks onto the energy–momentum equation through the definition of the radiative force, $F_{\text{rad}}$, of Equation (3). The free parameters of this system are (1) the total energy of the explosion $E_{\text{tot}}$, (2) the CSM density profile $n(r)$, and (3) the magnetic field as a function of radius $B(r)$. To avoid computation of the evolution during the RBW acceleration phase when it is likely to produce little radiation, we have chosen to begin our calculations (and the accumulation of matter by the RBW) at a radius $R_0$ at which it has already achieved its asymptotic Lorentz factor $\Gamma_0 = \Gamma(R_0)$.

As proposed in KGM02 and shown explicitly in MK06, the relativistic protons accumulated in the RBW can become supercritical to the network of $p\gamma \rightarrow p e^+ e^-$, $eB \rightarrow \gamma$ once kinematic and dynamic thresholds are simultaneously fulfilled. The kinematic threshold simply reflects the kinematic threshold of the $p\gamma \rightarrow p e^+ e^-$ reaction and reads

$$b \frac{\Gamma^2}{2} \geq 1 \text{ or } \Gamma \geq 214 (n_0)^{-1/12} \text{ for } B = B_0 \simeq \left(8\pi m_p c^2 n_0\right)^{1/2} \Gamma.$$
me/mp energy flux through the shock by a factor only that of the swept-up electrons, which is smaller than the relativistic proton column is small and little emission is possible, of the shock.

Figure 1(a)), a fact that according to the SPM marks the end on the expanding RBW; if R > R_{\text{dec}}, then Γ will continue its decline at the much slower conventional level of afterglow theory.

The detailed, long-term evolution of the GRB flux depends on E_{\text{tot}}, n(r), and B(r) that determine the values of r and Γ at which the RBW becomes supercritical—it is conceivable that for certain parameter combinations supercriticality can be reached at more than one radius, with the released energy being proportional to the time between the corresponding bursts, see, e.g., Ramirez-Ruiz & Merloni (2001). In Figures 1 and 2, we present the evolution of an RBW with n = n_0(R_0/R)^2 and B = B_0(R_0/R). The parameters are R_0 = 10^{15} cm, n_0 = 8 \times 10^8 cm^{-3}, Γ_0 = 100, B_0 = 4.410^4 G, and total isotropic energy E_{\text{tot}} = 10^{54} erg.

Figure 1(a) depicts the evolution of Γ as a function of radius in this medium with (thin line) and without (thick line) the radiative feedback. The drop in Γ corresponds to the explosive energy release in the protons and the slow down of the RBW due to the radiation drag. As deduced from this figure, R_0 \approx R_{\text{dec}}, since for R > R_0, Γ \propto R^{-1/2}, as expected for adiabatic propagation in a wind density profile (thick line). After the decrease in Γ due to the radiative feedback and after the nonadiabatic effects have died out, the evolution of Γ follows a similar track of lower normalization.

Figure 1(b) shows the multiwavelength spectra at various instances as perceived by the observer. As it was shown in MK06 the spectrum consists of two components, one that is due to the primary particle emission by particles on the RBW and one due to the bulk Comptonization of the upstream-reflected primary radiation by the cold pairs of the RBW. This latter component
peaks early on at 1 MeV, but as the burst evolves moves to lower energies since both \( \Gamma \) and \( B \) drop outward.

Figure 2(a) shows the corresponding apparent isotropic bolometric luminosity as a function of time. This consists of the internally produced luminosity (dashed) and that due to bulk Comptonization of the mirror-scattered radiation by the RBW (dotted) with the thick line representing their sum. As it can also be seen from Figure 1(b), most of the luminosity, is by far contained in the bulk-Comptonized component (at \( E \sim 1 \text{ MeV} \)) and exhibits the steepest decrease due to the decrease in \( \Gamma \) and the arrest of additional pair injection from the protons. At longer timescales, the only injection available is that of the ambient electrons and the emission exhibits the \( \propto t^{-1} \) behavior of “standard” afterglows.

Finally, Figure 2(b) depicts the luminosity at various energy bands as a function of time—here we make no distinction between the direct and the bulk-Comptonized components, but instead we exhibit their sum. As a rule higher frequencies dominate more at the early stages of the burst but drop faster due to a combination of faster cooling and the decrease in \( \Gamma \). This is consistent with observations: the BAT flux (that receives its major contribution from the bulk Comptonized component) decreases much faster than the flux in the other bands and its level defines, in effect, the prompt GRB phase (see also the next section).

3. SUMMARY AND DISCUSSION

We have presented above a first attempt at an integrated version of the SPM, complete with the coupled RBW dynamics, radiation production, and accumulation of hot protons on the RBW from the swept-up matter. The latter process is fundamental as the increase of the hot proton column to supercritical values is necessary for the explosive energy release seen in GRB. Another important feature is the coupling of the radiation to the dynamics of the RBW, the cause of the abrupt decrease in \( \Gamma \) seen in Figure 1(a). Because this can reduce \( \Gamma \) below the SPM kinematic threshold, it can severely reduce the observed flux, especially its bulk-Comptonized spectral component that peaks at \( E_p \sim 1 \text{ MeV} \) and constitutes the main GRB channel. The existence of the kinematic threshold value for \( \Gamma \) and its intimate association with the radiation emission near \( E_p \sim 1 \text{ MeV} \), the defining GRB property, affords for the SPM an operational definition of the GRB prompt phase, a feature unique among the GRB models: as such, the prompt GRB phase is the stage in its evolution during which the kinematic threshold condition of Equation (8) is fulfilled, accompanied by severe reduction in the GRB flux following this stage, as observed.

The time evolution of the flux in Figure 2 bears great resemblance to that of many *Swift*-XRT GRBs, that exhibit a very steep declining profile followed by a less steep or flat section in their light curves (O’Brien et al. 2006), related, as discussed above, to the relation between \( R_0 \) and \( R_{\infty} \). We believe that the straightforward way that the SPM addresses these vexing, for the standard model, questions attests to its relevance to the GRB underlying physics and phenomenology. It should be noted at this point that the efficiency of conversion of kinetic energy to radiation depends on the value of the ambient density \( n_0 \). This dependence comes through the dynamic threshold of the SPM, as \( n_0 \) also determines the value of the upstream albedo (i.e., of the “mirror,” whose assumption is now obviated), to which the dynamic threshold is proportional. We plan to explore the effects of this parameter on the GRB properties in a future publication.

The duration of the burst shown in Figure 2 is of the order of a few seconds. As such it would be classified as a short burst, despite the fact that the RBW is assumed to propagate in a medium with properties akin to the wind of a W–R star. We therefore have presented an explicit model that produces a short burst from an object of a young stellar population. While it was originally proposed and supported by the earlier observations that short bursts are associated with old stellar populations (implying neutron star collisions as their source of energy), it was shown (Berger 2009) that \( \lesssim 1/3 \) of them are in

---

**Figure 2.** (a) Bolometric burst light curve. The dashed line corresponds to the internal RBW luminosity, while the dotted line is the bulk-Comptonized component. The thick line is the sum of the two. (b) The corresponding burst luminosity for various energy bands as a function of time. The long dashed line is at energy of 1 MeV, the short dashed line is at 10 keV, dotted line at 100 eV, and dot-dashed line at 1 eV. The thick full line is the bolometric light curve. Parameters are as given in the text.
fact associated with stellar populations similar to those of the long GRBs.

The outlook from this first time-dependent treatment of the SPM replete with the CSM distribution and radiation emission and feedback is that this model can potentially produce a great variety of GRB light curves (in agreement with the GRB phenomenology) which it can relate to global parameters of the system. We plan to explore thoroughly these parameters in future publications.

We would like to thank the referee for comments that helped improve this Letter. This research was funded in part by a grant from the Special Funds for Research (ELKE) of the University of Athens. D.K. acknowledges support by an INTEGRAL GO grant.

REFERENCES

Berger, E. 2009, ApJ, 690, 231
Blandford, R. D., & Mc Kee, C. 1976, Phys. Fluids, 19, 1130
Boettcher, M., & Principe, D. 2009, ApJ, 692, 1374
Chiang, J., & Dermer, C. D. 1999, ApJ, 512, 699
Costa, E., et al. 1997, Nature, 387, 783
Dermer, C. D., Chiang, J., & Böttcher, M. 1999, ApJ, 513, 656
Kazanas, D., Georganopoulos, M., & Mastichiadis, A. 2002, ApJ, 587, L18
Kazanas, D., & Mastichiadis, A. 1999, ApJ, 518, L17
Kazanas, D., Mastichiadis, A., & Georganopoulos, M. 2007, in AIP Conf. Proc. 921, The First Glast Symposium, 458
Mallozzi, R. S., et al. 1995, ApJ, 454, 597
Mastichiadis, A., & Kazanas, D. 2006, ApJ, 645, 416
Mastichiadis, A., & Kazanas, D. 2008, Proc. 30th Int. Cosmic Ray Conf., ed. R. Caballero, J. C. D’Olivo, G. Medina-Tanco, L. Nellen, F. A. Sanchez, & J. F. Valdes-Galicia, Vol. 3, 1175
O’Brien, P. T., et al. 2006, New J. Phys., 8, 121, arXiv:astro-ph/0605230
Piran, T. 2004, Rev. Mod. Phys., 76, 1143
Preece, R. D., et al. 2000, ApJS, 126, 19
Ramirez-Ruiz, E., & Merloni, A. 2001, MNRAS, 320, L25
Rees, M. J., & Mészáros, P. 1992, MNRAS, 320, L25
Sari, R., Piran, T., & Halpern, J. P 1999, ApJ, 519, L17
Sari, R., Piran, T., & Narayan, R. 1998, ApJ, 497, L17
van Paradijs, J., et al. 1997, Nature, 386, 686
Vlahakis, N., & Königl, A. 2001, ApJ, 563, L129
Yamazaki, R., Ioka, K., & Nakamura, T. 2002, ApJ, 571, L31
Yonetoku, D., et al. 2004, ApJ, 609, 935
Zhang, B., & Mészáros, P. 2003, ApJ, 581, 1236
Zhang, B., & Mészáros, P. 2004, Int. J. Mod. Phys. A, 19, 2385 (arXiv:astro-ph/0311321)