Magnetic confinement and the Linde problem

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Abstract

Perturbation theory of thermodynamic potentials in QCD at $T > T_c$ is considered with the nonperturbative background vacuum taken into account. It is shown that the colormagnetic confinement in the QCD vacuum prevents the infrared catastrophe of the perturbation theory present in the case of the free vacuum (the “Linde problem”). A short discussion is given of the applicability of the nonperturbative formalism at large $T$ and of the relation with the HTL theory. The observation of A.D.Linde, that the terms $O(g^n), n > 6$ contribute to the order $O(g^6)$ is confirmed also with the account of the colormagnetic confinement, and it is shown that the latter makes these terms IR convergent, and summable. As a result one obtains the selfconsistent theory of the gluon plasma.
1 Introduction

The Linde problem in the thermal QCD has a long history, see [1, 2] and refs therein. It has occurred in the thermal perturbative QCD, where similarly to QED, the originally massless constituents (gluons) acquire effective perturbative mass operators $m(T)$, which regulate the convergence of $g^n$ terms and of the whole perturbative series. Correspondingly, the colorelectric screening mass $m_D(T)$, obtained from $\Pi_{00}(T)$ (similarly to the QED case) starts from $gT$, however the colormagnetic screening mass does not exist perturbatively [1, 2] (again, as in QED), and if introduced effectively as $O(g^2T)$, the perturbative series is not defined at the order $g^6$ (problem (1)). Linde also remarks, that the higher order diagrams contribute to the same order (problem (2)).

Meanwhile the effective perturbative theory of thermal QCD (the hard-thermal-loop (HTL) theory) was developed in [3, 4], using the colorelectric $m_D(T)$ and the resummation technic through order $g^2, g^3, g^4, g^5$, which appears to be quite successful, see [5] for a review. The nonperturbative nature of the magnetic scale $g^2T$, which appears necessarily at $O(g^6)$ can be connected to the 3d Yang Mills theory, see e.g. [6].

A natural question arises, how this situation can be explained and treated in a 4d approach to QCD, where nonperturbative (np) physics (including confinement) is taken into account?

In what follows we shall consider the np approach to QCD, developed in [7, 8, 9, 10, 11, 12]. For an alternative approach see [13, 14].

Most effects in QCD at low and intermediate energies cannot be explained without np dynamics, which enters in the theory, e.g. the string tension $\sigma$, or a mass of some meson ($\rho, K$), or else the constant $\Lambda_{QCD}$, entering in $\alpha_s(Q)$ in addition to current quark masses $m_q$. The approach of the np vacuum, ensuring confinement, and stabilizing perturbation theory, was developed in [7] for QCD at zero temperature $T$, and in [8, 9] for $T > 0$, see reviews [10, 11] for $T = 0$ and [12] for $T > 0$. The problem of the confinement and deconfinement is treated in our approach, called the Field Correlator method (FCM), taking into account two kinds of colorelectric correlators $D^E(z), D^E_1(z) \sim \langle E_i(x)E_i(y) \rangle$, and two kinds of colormagnetic, $D^H(z), D^H_1(z) \sim \langle H_i(x)H_i(y) \rangle$, where the first ones ($D^E, D^B$) are of purely nonabelian character, while $D^E_1, D^H_1$ exist also in QED.

Assumed in [8] and later confirmed on the lattice [15], that the nonabelian colorelectric correlator $D^E(z)$ vanishes together with confinement at $T_c$, while all others stay nonzero for $T > T_c$, in particular the nonabelian
colorelectric correlator $D^E_1$, responsible for the nonzero Polyakov lines, while the nonabelian colormagnetic correlator $D^H(z)$ ensures the magnetic confinement for the motion in the spatial planes. This property was studied in the FCM formalism in [16], and analytically and numerically in a different approach in [17], see also [18] for later developments.

As a result of magnetic confinement there appears the spatial string tension, which defines the area law of the spatial projection of any Wilson loop in 4d,

$$\langle W(C) \rangle = \exp \left( -\sigma_s A_{3d}(C) \right), \quad \sigma_s = \frac{1}{2} \int d^2 z D^H(z).$$

(1)

One can now consider any QCD diagram and the whole perturbative series as being immersed in the np vacuum, so that all closed loops in the 3d space are covered by the confining film, while for 4d loops covered is its 3d projection. This fact turns over the whole thermal QCD dynamics. Namely, in the diagrams with the temporal gluon ladder evolution, the exchanged gluons automatically acquire the screening mass $m^H_D(T)$, which appears due to the projected spatial confinement, as it was shown in [19]. Indeed, in the place of the colorelectric screening mass $m^D_T \sim gT$, there appears the screening mass $m^H_D(T) \approx 2\sqrt{\sigma_s(T)}$ [19], which agrees well with lattice data for screening masses [20]. It is still an open question how these two effective masses combine in the high $T$ np dynamics.

Moreover, $D^H(z)$ can be calculated via the gluelumps [21], known both analytically [22] and on the lattice [23], which yields the relation

$$\sqrt{\sigma_s(T)} = c_\sigma g^2(T) T,$$

(2)

where $c_\sigma$ is of the np origin, as shown in the appendix. This coincides with the lattice data results [24], where $c_\sigma = 0.566 \pm 0.013$.

The two-loop approximation is generally used for $g^2(T)$ [25].

Note, that Eq. (2) implies that the magnetic mass $m^H_D \sim \sqrt{\sigma_s}$ as a cutoff parameter indeed occurs, which can make the 4 loop integral convergent and is indeed $m^H_D = O(g^2 T)$, which resolves, what can be called the problem 1 of Linde, as will be clarified below. This is purely np result, irrespectively of the appearing $g^6$ factor. The problem (2) of Linde (see (iii) in [1] on p. 290) that the sum of the infinite ladder of gluon loops with $n > 4$ contributes to the same order $g^6 T^4$ will be discussed in section 3.
Above we have discussed the temporal evolving graphs, where confinement is absent in the \((4, i)\) planes, and appears in the projected \((i, j)\) planes.

Coming now to the 3d graphs, or 3d projections of their 4d equivalents, one must understand how confinement changes all dynamics. First of all, the IR divergence in 3d is lifted by spatial confinement, as shown below in the paper. Secondly, strictly speaking the IR regulator has not the form of an effective mass parameter, but the area law factor \(g^2T\), however, as we show in section 3, both cutoffs qualitatively are similar, since \(g^2T\) yields confinement Hamiltonian with lowest masses behaving as \(\sqrt{\sigma_s} \sim g^2T\), which play the role of the cutoff factors. As a result one obtains the overall regulator mass parameter \(g^2T\) both for temporal and spatial evolution (colorelectric and colormagnetic effective masses). One might be worried by the visible contradiction with the HTLpt approach, but one can argue, that incident values of \(gT\) and \(g^2T\) are not far \((g(10T_c) = 1.14, g(2T_c) = 1.39)\) and the consequent optimization can make those even closer.

Still the effective mass \(gT\) can appear quantitatively (e.g. on the lattice), when one can neglect the Wilson loop \(\sqrt{\sigma_s}\), i.e. for very small loops, however it is still an open question, whether this situation is practically realizable.

Comparing HTL results with the lattice calculations, one can conclude, that the \(O(g^6)\) term is basically important for \(T \leq 0.5\) GeV (see e.g. Fig. 1 of \([26]\)). As a result a new HTL version appeared in \([27]\), called “the \(O(g^6)\) fitted” HTL contribution, as well as “the \(O(g^6)\) fitted + nonpert.” version. As we shall show below, the \(O(g^6)\) terms indeed contain the whole series \(O(g^n), n > 6\), as was shown by Linde \([7]\), but in addition the colormagnetic confinement makes these terms finite and summable. All this makes our analysis and discussion of the Linde problems even more timely and relevant.

Coming back to the Linde problem \([\text{I}]\), the standard perturbation theory (without nonperturbative background), which proceeds essentially in 3d, becomes infrared divergent, starting with the 6-th order in \(g\). In essence, the problem occurs due to very weak fall-off of the gluon propagator in 3d without \(\sigma_s\), e.g. in the \(x\)-space

\[
G(x, y) \sim \frac{T}{\pi|x - y|}, \quad |x - y| \gg 1/T
\]  

(3)

As it was suggested in \([28]\), the account of the nonperturbative vacuum with colormagnetic confinement is able to resolve this problem. Indeed, it was found in \([\text{I}]\), that the overall power of coordinates in the integration terms \(d^3x_i\) and vertices \(\frac{\partial}{\partial x_i}\) minus that of gluon propagators is equal to \(x^{\frac{7}{2}} - 3\)
where $n$ is the order of the expansion in $g$. This shows, that all diagrams $\sim g^n$ with $n \geq 6$ are infrared divergent, while the colormagnetic confinement in 3d cuts off all integrals at distances $(x_i - x_j) \gg \sigma_s^{-1/2}$ [28].

It is the purpose of the present paper to study in more detail how magnetic confinement cuts off the IR divergent integrals and how the np effective masses can be defined to match the perturbative approach (e.g. the HTLpt).

The paper is organized as follows. In the next section we write the general background field formalism for the thermodynamic potential, and define its perturbation series.

In section 3 we study the gluonic multiloop diagram with spatial (magnetic) confinement and define its infrared and ultraviolet properties, showing that indeed the presence of $\sigma_s$ prevents the IR divergence of any diagram.

Section 4 is devoted to the summary and prospectives.

In appendix a detailed derivation is given of $\sigma_s$ in terms of gluon propagators and guellumps.

## 2 Background perturbation theory

In this section we exploit the background perturbation theory, developed in [8, 9], to study soft and hard regimes of the internal integrations and to demonstrate the role, which is played in this process by the magnetic confinement. Since we are mostly interested in the high $T$ gluon contributions, we confine ourselves to the case of pure gluodynamics.

We split the gluonic field $A_\mu$ into nonperturbative (NP) background $B_\mu$ and the perturbative part $a_\mu$

$$A_\mu = B_\mu + a_\mu$$

and the partition function $Z$ can be written as a double average, using 'tHooft identity [8, 9]

$$Z \equiv <\!\!\!\!\!\!\!\!\!< \exp(-S(B + a)) >_a >_B$$

where the action $S$ contains the standard gluon, ghost and gauge fixing terms and in particular the triple vertices $a^3, a^2 B$.

The inverse gluon propagator can be written as

$$G^{-1} = -D^2(B)_{ab} \cdot \delta_{\mu\nu} - 2gF^c_{\mu\nu}(B) f^{acb}$$
where
\[(D\lambda)_{ca} = \partial_{\lambda} \delta_{ca} - igT^{b}_{c a} B^{b}_{\lambda}.\] (7)

In what follows we shall for simplicity neglect the gluon spin term – the last term on the r.h.s. of (6) (the latter gives a correction to spatial (magnetic) confinement), and then the gluon propagator can be written as
\[
(-D^{2})^{-1}_{xy} = \langle x | \int_{0}^{\infty} dt e^{i D^{2}(B)} | y \rangle = \int_{0}^{\infty} dt (D z)_{xy}^{w} e^{-K \Phi(x, y)}
\] (8)
where
\[
K = \frac{1}{4} \int_{0}^{s} d\tau \left( \frac{d z_{\mu}}{d\tau} \right)^{2}, \quad \Phi(x, y) = P \exp \int_{x}^{y} B_{\mu} dz_{\mu}
\] (9)
and a winding path measure is
\[
(D z)_{xy}^{w} = \lim_{N \to \infty} \prod_{m=1}^{N} \frac{d^{4} \zeta(m)}{(4\pi \epsilon)^{2}} \sum_{n=0, \pm 1, \ldots} \int \frac{d^{4} p}{(2\pi)^{4}} e^{i p (\sum \zeta(m)-(x-y)-n\delta_{\mu 4})}
\] (10)

In the free case, \(B_{\mu} \equiv 0\), one obtains the gluon propagator
\[
G(x, y) \to (-\partial^{2})^{-1}_{xy} = \sum_{k=0, \pm 1, \ldots} \int \frac{d^{3} p}{(2\pi)^{3}} \frac{e^{-i p(x-y)-i 2\pi k T(x-(x-y))}}{(p^{2} + (2\pi k T)^{2})}.
\] (11)

At large distances the zero mode \((k = 0)\) yields the behavior shown in (3) and this is the origin of the IR divergence of higher order \(g^{n}\) contributions to the free energy, as was shown in [1], while magnetic confinement, contained in \(\Phi(x, y)\), cuts off all divergences, as will be demonstrated below.

One can easily find the lowest order (one loop) \(np\) contribution to the free energy
\[
F_{gl}^{0}(B) = T \left\{ \frac{1}{2} \log \det G^{-1} - \log \det (-D^{2}(B)) \right\},
\] (12)
which can be written as
\[
F_{gl}^{0}(B) = -T \int_{0}^{\infty} \frac{ds}{s} \xi(s) d^{4} x (D z)_{xx}^{w} e^{-K \langle tr_{a} \Phi(x, x) \rangle_{B}}
\] (13)
and finally for the pressure
\[
P_{gl} = (N_{c}^{2} - 1) \int_{0}^{\infty} \frac{ds}{s} \sum_{n \neq 0} G^{(n)}(s),
\] (14)
with
\[ G^{(n)}(s) = \int (Dz)^n e^{-K \langle tr_a \Phi(x,x) \rangle_B} \] (15)

\( \Phi(x,x) \) contains colorelectric fields \( B_4(x) \), which produce Polyakov lines \( L_{adj}(T) \) \[9\], and in addition also colormagnetic fields, which are contained in the spatial Wilson loop, \( \langle tr_a \Phi_s(C_n) \rangle_B \equiv \langle W_s(C_n) \rangle \), which can be written in terms of field correlators \[9, 12, 16\], as an integral over minimal surface inside the loop \( C \)

\[ \langle W_s(C_n) \rangle = tr_a \langle \exp(ig \int_C A_\mu dz_\mu) \rangle = tr_a \langle \exp(ig \int ds_{\mu\nu} F_{\mu\nu}) \rangle \] (16)

and using the cumulant expansion \[7\] and dropping all cumulants except for quadratic, one has

\[ \langle W_s(C_n) \rangle = \exp \left( -\frac{1}{2} \int_S \int_S ds_{\mu\nu}(u) ds_{\lambda\sigma}(v) \langle F_{\mu\nu}(u) F_{\lambda\sigma}(v) \rangle \right). \] (17)

Considering only spatial loops \( C \) and surface areas \( S \) for \( k = 0 \), i.e. the term without higher Matsubara frequencies, one has to do with colormagnetic correlators only,

\[ \frac{g^2}{N_c} \langle H_i(u) H_j(v) \rangle = \delta_{ij} D^H(u - v) + O(D_1^H). \] (18)

To find \( \sigma_s \) in \[\Pi\] one can use the connection of \( D^H \) with the gluelump Green's function \[21\], which, as shown in the Appendix, can be written as

\[ D^H(z) = g^4 \left( \frac{N_c^2 - 1}{2} \right) T^2 G_{3d}^{(2g)}(z) \] (19)

where \( G_{3d}^{(2g)} \) is the two-gluon NP Green's function in 3d. As a result using \[\Pi\] one can write the \( T \)-dependent part of \( \sigma_s \) as

\[ \sigma_s(T) = g^4 T^2 c_\sigma^2, \quad c_\sigma^2 = \frac{N_c^2 - 1}{4} \int d^2 z G_{3d}^{(2g)}(z), \] (20)

where \( c_\sigma \) is dimensionless number and a fully np quantity.

Insertion of \( \langle tr_a \Phi \rangle \) in \[15\] as an area law \[\Pi\] yields a loop graph of a gluon, where the string tension \( \sigma_s \) controls the area inside the loop, so that the gluon cannot go far from the initial point \( x \), the maximal distance being
Figure 1: The gluon loop contribution to the pressure to the lowest order with spatial string tension inside the loop

\[ R \lesssim \frac{1}{\sqrt{\sigma_s}} \text{, see Fig. 1.} \]

One can now generalize this picture to the higher terms in the perturbative series \( O(g^n) \), where these terms are formed by applying the term \( L_3 \) in the original QCD Lagrangian

\[ L_3 = g \partial_\mu a^a_\mu f^{abc} a^b_\alpha a^c_\nu \]  \hspace{1cm} (21)

on any gluon line. As a result one obtains e.g. the diagram of Fig. 2 of the order \( g^8 \). It is essential, that each gluon propagator \( G^a_{\mu\nu}(x^{(i)}, x^{(k)}) \equiv \langle a^a_\mu(x^{(i)}) a^a_\nu(x^{(k)}) \rangle \) is proportional to \( \Phi(x^{(i)}, x^{(k)}) \) and the latter after averaging over background fields \( B_\mu \), in the product together with all other gluon propagators, forms the total Wilson loop with the same outer contour \( C_n \), but now with inner lines, dissecting it into a sum of pieces of area \( \Delta A^{(i)} \), \( \mathcal{A} \rightarrow \sum_i \Delta A^{(i)} \), each piece is subject to the area law with the same \( \sigma_s \), so that one obtains the factor \( \exp \left( -\sigma_s \sum_i \Delta A^{(i)} \right) \), which prevents the escape of all gluons from the center of the area, and in this way ensures infrared stability.

One can say, that each gluon is interacting with the closest neighbor via linear confining interaction and therefore the distance between them is of the order of \( (\sqrt{\sigma_s})^{-1} \). On the other hand, in the case, when outer gluons are moving along the fourth axis, \( z_4 \), there is no spatial area law for their trajectories. However, in this case all internal gluons in Fig. 2 can be considered as the gluon exchanges between two outer gluons, and then these gluons acquire Debye masses which prevent IR divergence \([19]\).
3 Perturbative expansion in magnetic confinement

Here we exploit the physical arguments of the previous section, explaining the possible role of the confining interaction at large intergluon distances, and give a quantitative approach to the calculation of 3d perturbative series. It will be done in two different formalisms: first, we calculate the propagator of a gluon, which interacts with a neighboring gluon via confining interaction and demonstrate, that the asymptotics of its Green’s function essentially modifies, as compared to the free case and makes the integrals convergent. In the second step we consider a triangular or rectangular part of the total possibly large multigluon diagram, and demonstrate, how this part changes from the convergent integrals in the case of confinement, to the infrared divergent ones in the free gluon case.

1. We start with the free gluon propagator in the 3d Euclidean space in the Feynman gauge, where we choose one of the 3d coordinates as the Euclidean time $t \equiv x_3$, while $x_1, x_2$ are spatial coordinates. Using path integral technic [29], one can write

$$g(x, y) = \left(\frac{1}{-\partial^2}\right)_{xy} = \sqrt{\frac{t}{8\pi}} \int_0^\infty \frac{d\omega}{\omega^{3/2}} (D^2 z) xy e^{-K(\omega)} =$$

$$= \sqrt{\frac{t}{8\pi}} \int_0^\infty \frac{d\omega}{\omega^{3/2}} (x|e^{-H(\omega)t}|y),$$

(22)

where $t = x_3 - y_3$,

$$K(\omega) = \int_0^T dt \left(\frac{\omega}{2} + \frac{\omega}{2} \left(\frac{dz}{dt}\right)^2\right),$$

(23)

$$H(\omega) = \frac{p^2}{2\omega} + \frac{\omega}{2}.$$  

(24)

It is clear, that eigenfunctions of $H(\omega)$, denoted in (22) as $\langle x|k \rangle = \exp(i k x)$, with the assumed summation over all eigenstates of $H(\omega)$, $\langle x|e^{-HT}|y\rangle = \int\frac{dk}{(2\pi)^2} \langle x|k \rangle e^{-H(k)T} \langle k|y \rangle$ lead after integration over $d\omega$ to the final result,

$$g_0(x, y) = \frac{1}{4\pi|x-y|}$$

(25)
Figure 2: The 8-th order graph with the crossed rectangular under study which agrees with (3) in our 3d case, and as was discussed in the previous section, leads to the IR divergence of the perturbative series (the Linde problem). Let us now take into account the magnetic confining interaction, given by (1), which in our notations, can be written as

\[
\langle W(C_n) \rangle = \exp(-tV(z - z_0)), \quad V(z) = \sigma_s |z|
\]  

(26)

where \(z_0\) is a spatial coordinate of the neighboring gluon, see Fig. 2 where the gluon of the line 13 is interacting with that of 24 (a similar situation occurs in the case of two neighboring gluons, as in Fig. 2 for any gluon in vertical line, in which case one should replace \(V(z - z_0)\) by the sum of two potentials, but the final qualitative results will be the same).

In our case the Hamiltonian (24) will be rewritten as

\[
H(\omega) = \frac{P^2}{2\omega} + \frac{\omega}{2} + \sigma_s |z|.
\]  

(27)

Solving the 2d equation \(H(\omega)\varphi_n = (\frac{\omega}{2} + \varepsilon_n(\omega))\varphi_n\), one immediately obtains that \(\varepsilon_n(\omega)\) depends on \(\omega\) as [16]

\[
\varepsilon_n(\omega) = \omega^{-1/3}(2\sigma_s)^{2/3} f_n, \quad f_0 \approx 0.87,
\]  

(28)

while eigenfunctions at large \(|x|\) are

\[
\langle x|n \rangle \equiv \varphi_n(x) \sim \exp\left(-\frac{2}{3} \sqrt{2\omega \sigma_s} |x|^{3/2}\right),
\]  

(29)

and the gluon Green’s function behaves as [29]

\[
g(x, y) = \left(\frac{1}{-D^2}\right)_{xy} = \sqrt{\frac{t}{8\pi}} \int_0^\infty \frac{d\omega}{\omega^{3/2}} \sum_{n=0}^\infty \varphi_n(x) \varphi_n(y) e^{-M_n t}.
\]  

(30)
Here $M_n(\omega) = \omega + \omega^{-1/3}(2\sigma_s)^{2/3}f_n$, $f_n > 0$, and the integral $d\omega$ is convergent at $\omega = 0$, and can be calculated by the stationary phase method, using the extremum of $M_n(\omega)$ at $\omega = \omega_0(n)$. As a result from $\frac{dM_n}{d\omega} |_{\omega=\omega_0} = 0$ one gets

$$\omega_0 = 0.93\sqrt{\sigma_s}, \quad M_0 = 2\omega_0 = 1.86\sqrt{\sigma_s}$$

and the exponent of (29) is $0.81\sigma_s^{3/4}|x|^{3/2}$, while $g(x, y)$ behaves as

$$g(x, y) \sim \sum_n e^{-M_n(\omega_0)t} \varphi_n(\omega_0(n), x) \varphi_n(\omega_0(n), y)$$

and the Green’s function is decreasing at large $x, y$ as in (29), i.e. faster than exponent of $x, y$ assuming convergence of all spatial integrals, so that every gluon will be placed at the distance of $(\sigma_s)^{-1/2}$ from the neighbors. We note here, that $M_0$ in (31) is close to the Debye mass $M_D \approx 2\sqrt{\sigma_s}$, as found in [19] in agreement with lattice data [20].

We now turn to the more formal procedure to define the properties of the one-loop part of the complicated diagrams, shown in Fig.3. At each vertex of this diagram enters the operator (21), which generates 3g vertex $\Gamma_i$ with momentum operator $p_i$, so that the quadratic loop diagram in the 3d space can be written as
\[ G(p^{(1)}, p^{(2)}, p^{(3)}, p^{(4)}) = \prod_{i=1}^{4} \Gamma_i \int_{0}^{\infty} ds_i (Dz^{(i)})_{x^{(i)}, x^{(i-1)}} e^{-K_i \Phi^{(i)} e^{ip^{(i)} x^{(i)}}} dx^{(i)}. \] (33)

Here we have introduced the phase factors

\[ \Phi^{(i)}(x^{(i)}, x^{(i-1)}) = P_A \exp(i g \int_{x^{(i-1)}}^{x^{(i)}} A_{\mu} dz_{\mu}), \] (34)

omitting for simplicity the gluon spin phase factor, originating from the last term in (13), since it is inessential in the asymptotic limit for large \( |x^{(i)} - x^{(i-1)}| \). Here \((Dz^{(i)})\) is

\[ (Dz^{(i)})_{xy} = \lim_{N \to \infty} \prod_{k=1}^{N} \frac{d^{3} \xi^{(i)(k)}(k) d^{3} q^{(i)(k)}}{(4\pi \varepsilon)^2 (2\pi)^4} e^{i q^{(i)(k)}(\sum_{k} \xi^{(i)(k)}(k) - x - y)}, \quad N \varepsilon = s. \] (35)

It is essential that the product of all phase factors \( \Phi^{(i)} \) in the whole diagram of Fig. 3 should be averaged over vacuum configurations, yielding 3d confinement, and each gluon line is in adjoint representation, and can be represented as the double fundamental line in the simple case of the large \( N_c \) limit, so that one finally has a product of independent closed fundamental lines, circumvented by a common line in the outer contour. In the same large \( N_c \) limit the average of this product can be represented as the product of averages of individual loops time the average of the outer contour, which yields the overall confining factor. In what follows we shall be interested in the properties of one rectangular loop and demonstrate its spatial convergence, while the overall confining loop will exhibit additional convergence.

The rather complicated calculations, given in the Appendix B of the paper [30] for the case \( d = 4 \), can be done in an analogous manner for the case \( d = 3 \), and one obtains the following form of the rectangular diagram of Fig. 3 with account of the spatial confinement

\[ G_4(p_i) = (2\pi)^3 \delta^{(3)} \left( \sum p^{(i)} \right) \prod_{i=1}^{4} \int \frac{d^3 q_i \Gamma_i}{q_i^2} I_4(b), \] (36)

where

\[ I_4(b) = \int \frac{d^3 P}{(2\pi)^3} \left( \frac{4\pi}{\sigma} \right)^6 \exp \left( -\frac{2}{\sigma} \sqrt{b_1^2 b_2^2 - (b_1 b_2)^2} \right) \exp \left( -\frac{2}{\sigma} \sqrt{b_3^2 b_4^2 - (b_3 b_4)^2} \right), \] (37)
and $b_i$ are

$$b_1 = q_1 - p_2 - p_3 + P, \quad b_2 = q_2 - p_3 + P,$$

$$b_3 = q_3 + P, \quad b_4 = q_4 + p_4 + P.$$  \hfill (38)

One can check, that at large momenta, (the hard regime) when $b_i^2 \gg \sigma$, $i = 1, 2, 3, 4.$

$$I_4(b) \rightarrow \prod_{i=1,2,4} \delta^{(3)}(b_i).$$  \hfill (39)

and the product of four factors $d^3q_i$ is reduced to a single integration $d^3q_3$, as it should be in the free case without confinement.

As a result one has in (36) for one loop in Fig. 3 the combination

$$d^3q_i \prod_{i=1}^4 \frac{\Gamma_i}{q_i^2},$$

and for the whole chain of $n$ loops, as in Fig. 3, one obtains an estimate (see [1])

$$M_n(T) \sim g^{2(n-1)} \left( T \int_{a\sqrt{\sigma}}^T d^3q \right)^n \frac{q^{2(n-1)}}{(q^2)^{3(n-1)}}.$$  \hfill (40)

Here we have used the hard limit condition, $q \geq a\sqrt{\sigma}, a \gg 1.$

Integration in (40) yields the result $(n > 4)$

$$M_n^{\text{hard}}(T) \sim g^{2(n-1)} T^n \sigma^{\frac{a^4-n}{a^3-1}}\sim g^6 T^4 \frac{\sigma}{(c_\sigma a)^{n-4}}.$$  \hfill (41)

where we have exploited (2). This, apart from the $c_\sigma a$ factor, is the problem (2) of Linde [1]: all terms with $n > 4$ contribute to the order $g^6 T^4$, however with decreasing magnitude for $c_\sigma a \gg 1.$

To complete our study we consider now the soft regime, all momenta $q_i, p_i$ in (38) are small, $q_i, P \lesssim \sqrt{\sigma}.$ In this case every loop integration $d^3q_i$ in (40) is replaced by

$$d^3q_i \rightarrow \prod_{i=1}^4 d^3q_i I_3(b) = \sigma_s^{3/2} f \left( \frac{q_i}{\sqrt{\sigma}} \right)$$  \hfill (42)

and as a result one obtains in the soft regime

$$M_n^{\text{soft}}(T) \sim g^{2(n-1)} T^n \sigma_s^{\frac{4-n}{4-1}} \varphi_n \sim g^6 T^4 \varphi_n.$$  \hfill (43)

where $\varphi_n$ is a converging integral of dimensionless ratios $q_i/\sqrt{\sigma}$. One can see, that (43) yields qualitatively the same result as in (41), supporting the
effective mass notion \( (m_{H}^{D} \sim g^{2}T \text{ both in the hard and soft regimes}) \) for the order of magnitude estimates. However quantitatively one should calculate nonperturbatively the whole series \( n \geq 4 \) to recover the \( O(g^{6}) \) contribution. This situation is similar to the solution of the relativistic problem of two potentials: one confining and another gluon exchange but without small parameters.

4 Summary and prospectives

We have considered above the gluon thermodynamics with the nonperturbative background fields, which ensure spatial confinement due to colormagnetic correlators \([18]\). As a result one obtains the area law of the spatial Wilson loop with the nonzero spatial string tension. Qualitatively it is clear, that all multigluon diagrams in 3d would be convergent at large spatial distances, and this property was used in \([28]\) to argue that the Linde problem is absent in the confining vacuum. In the present paper this qualitative argument was given a more quantitative foundation, and in particular it was shown, that the spatial string tension \( \sigma_s(T) \) generates the corresponding effective gluon mass \( M_0 \approx 2\sqrt{\sigma_s(T)} \) \([31]\), and the latter is proportional to \( T \) at large \( T \), \( M_0 \approx 2T \) at \( T > 300 \text{ MeV} \). It is interesting, that \( M_0 \) appears to be very close to the nonperturbative Debye mass, calculated in \([19]\), in good agreement with lattice data \([24]\).

As a result one can exploit the universal effective gluon mass \( m_{H}^{D}(T) \approx 2\sqrt{\sigma_s(T)} \) both for colorelectric and colormagnetic gluons as a first approximation in the effective perturbation theory up to the \( g^{6} \) order.

From this point of view we have stressed the existence of the effective screening mass parameter also for magnetic components in 3d, which is of np origin and occurs due to magnetic confinement string tension \( \sigma_s \) – this is the answer to what we call the problem 1 of Linde. The second problem of Linde, the infinite loop series with \( n > 4 \) contributing to the order \( g^{6}T^{4} \), is confirmed above in the np approach, which actually tells that all terms starting with \( n = 1 \) should be treated nonperturbatively and this yields again the Linde answer \( \mathcal{M}_n \sim g^{2(n-1)}T^{4} \left( \frac{T}{m_{D}^{H}} \right)^{n-4} \), which is appropriate for \( T \sim T_c, m_{D}^{H} = \text{const.} \), and catastrophic for \( T \gg T_c \), when \( m_{D}^{H} \sim g^{2}T \), and all terms in the series are \( O(g^{6}T^{4}) \).

As was mentioned above, the whole dynamics of loop diagrams with \( n \geq 4 \)
lies in the soft np region, where the magnetic confinement and not gluon exchange mechanism plays the most important role. It is an open question what is the sum of \( n > 4 \) np loops with magnetic confinement, which is equivalent to the \( gg \) amplitude in the case of two interactions in 3d: confining \( V_{\text{conf}} \) and gluon exchange \( V_{\text{OGE}} \), but the answer is possible to obtain.

Coming to the case \( n < 4 \), one can see from the structure of diagrams (see e.g. Eq. (40)), that effective internal momenta are in the hard regime, which implies the subleading role of np effects for \( T \gg T_c \). In this case the extension of loops is small and effects of magnetic confinement can be neglected, so that the colorelectric \( m_D^E \sim gT \) enters the game, and the HTLpt is effective.

At the same time the background perturbation theory \([8, 9]\) with magnetic confinement included, can be a useful instrument for the dynamics of the quark-gluon medium at all \( T \), including the region near \( T = T_c \). Indeed, the same background correlators, but now of colorelectric character, \( D^E(z) \), give rise to the Polyakov loops, \( L_q, L_g \) and ensure the correct behavior of the pressure and trace anomaly near \( T_c \) \([9]\), while the asymptotics at large \( T \) is some 20\% off without the account of magnetic confinement and Debye mass. As it was recently shown in \([32]\) the inclusion of colormagnetic confinement helps to achieve a very good agreement for all thermodynamic quantities and in particular to explain the remarkable “shoulder” in the behavior of trace anomaly \( \frac{I(T)}{T^2} \).

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Appendix 1

Calculation of the spatial string tension via two-gluon Green’s function

To calculate \( D^H(z) \) one can use the technic, developed in \([21]\) for \( D^E(z) \), which allows to express it via two-gluon Green’s function \( G_{4d}^{(2g)}(z) = G_{4d}^{(g)} \otimes G_{4d}^{(g)} \), where two gluons interact nonperturbatively.

The starting point for the gluon propagator \( G_{4d}^{(g)} \) is the integration in the
4 th direction in \( \mathbb{R}^4 \) with the exponent \( K_4 = \frac{1}{4} \int_0^s d\tau \left( \frac{dz}{d\tau} \right)^2 \), which gives for the spatial loop with \( x_4 = y_4 \),

\[
J_4 = \int (Dz_4)_{x_4} e^{-K_4} = \sum_{n=0, \pm 1, \ldots} \frac{1}{2\sqrt{\pi s}} e^{-\frac{(n\beta)^2}{4s}} + \frac{1}{2\sqrt{\pi s}} \left( 1 + \sum_{n=\pm 1, \pm 2, \ldots} e^{-\frac{(n\beta)^2}{4s}} \right).
\]

(A1.1)

The second term in (A1.1) at large \( T \gg 1 \) yields \( 2 \sqrt{\pi sT} \), which gives

\[
J_4 = \frac{1}{2\sqrt{\pi s}} + T.
\]

The same linear in \( T \) term is obtained using the Poisson relation [8, 9].

As a result the 4d gluon propagator is reduced to the 3d one,

\[
G_{4d}^{(g)}(z) = TG_{3d}^{(g)}(z) + K_{3d}(z)
\]

(A1.2)

where \( K_{3d}(z) \) does not depend on \( T \). In what follows we consider only the first term in (A1.2), keeping in mind, that \( G_{4d}^{(g)}(z) \) at small \( T \) has a nonzero limit. Substituting this term in the general expression for \( D^E(z) (D^H(z)) \) obtained in [21], one has

\[
D^H(z) = \frac{g^4(N_c^2 - 1)}{2} \langle G_{4d}^{(2g)}(z) \rangle \to \frac{g^4(N_c^2 - 1)T^2}{2} \langle G_{3d}^{(2g)}(z) \rangle,
\]

(A1.3)

where \( G_{3d}^{(2g)} \) is the two-gluon Green’s function in 3d with all interaction between gluons taken into account

\[
\langle G_{3d}^{(2g)} \rangle = \langle G_{3d}^{(g)}(x, y)G_{3d}^{(g)}(x, y) \rangle_B.
\]

(A1.4)

In terms of the gluellump phenomenology, studied in [22, 23] (A1.4) is called the two-gluon gluellump, computed on the lattice in [23] and analytically in [22]. In our case we are interested in the 3d version of the corresponding Green’s function. Choosing in 3d the \( x_3 \equiv t \) axis as the Euclidean time, we proceed as in [16], exploiting the path integral technic [7, 29], which yields

\[
G_{3d}^{(2g)}(x - y) = \frac{t}{8\pi} \int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}} \int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}} (D^2z_1)_{xy} (D^2z_2)_{xy} e^{-K_1(\omega_1) - K_2(\omega_2) - V_t},
\]

(A1.5)

where \( V \) includes spatial confining interaction between the three objects: gluon 1, gluon 2 and the fixed straight line of the parallel transporter, which
makes all construction gauge invariant (see [22, 21] for details). In (A1.5) 
\( t = |x - y| \equiv |w| \); and finally
\[
\sigma_s(T) = \frac{g^4(N_c^2 - 1)T^2}{4} \int \langle G_{3d}^{(2g)}(w) \rangle d^2w. \quad (A1.6)
\]

Constructing in the exponent of (A1.5) the 3 body Hamiltonian in the 2d spatial coordinates
\[
H(\omega_1, \omega_2) = \frac{\omega_1^2 + P_1^2}{2\omega_1} + \frac{\omega_2^2 + P_2^2}{2\omega_2} + V(z_1, z_2), \quad (A1.7)
\]
one can rewrite (A1.5) as follows, see [29]
\[
G_{3d}^{(2g)}(t) = \frac{t}{8\pi} \int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}} \int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}} \sum_{n=0}^{\infty} |\psi_n(0,0)|^2 e^{-M_n(\omega_1, \omega_2)t}. \quad (A1.8)
\]
Here \( \Psi_n(0,0) \equiv \Psi_n(z_1, z_2)|_{z_1 = z_2 = 0} \); and \( M_n \) is the eigenvalue of \( H(\omega_1, \omega_2) \). The latter was studied in [22] in 3 spatial coordinates. For our purpose here we only mention, that \( G_{3d}^{(2g)}(z) \) has the dimension of the mass squared and the integral in (A1.6) is therefore dimensionless. Hence one obtains
\[
\sqrt{\sigma_s(T)} = g^2 T c_\sigma + \text{const}, \quad \text{as was stated above in (3), where}
\]
\[
c_\sigma^2 = \frac{(N_c^2 - 1)}{4} \int d^2w \langle G_{3d}^{(2g)}(w) \rangle, \quad (A1.9)
\]
and const is obtained from the second term in (A1.2).

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