Dark Energy and Fate of the Universe

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We explore the ultimate fate of the Universe by using a divergence-free parametrization for dark energy \( w(z) = w_0 + w_a \frac{\ln(2 + z)}{1 + z} - \ln 2 \). Unlike the CPL parametrization, this parametrization has well behaved, bounded behavior for both high redshifts and negative redshifts, and thus can genuinely cover many theoretical dark energy models. After constraining the parameter space of this parametrization by using the current cosmological observations, we find that, at the 95.4% confidence level, our Universe can still exist at least 16.7 Gyr before it ends in a big rip. Moreover, for the phantom energy dominated Universe, we find that a gravitationally bound system will be destroyed at a time \( t \approx P \sqrt{2[1 + 3w(-1)][1 + w(-1)]} \), where \( P \) is the period of a circular orbit around this system, before the big rip.

There are two ultimate questions for human beings: “where do we come from?” and “where are we going?”. For a long time, they have been topics of just religion and philosophy. But in the last three decades, along with the rapid development of modern cosmology, scientists have already obtained some important clues to these two questions. To explain the origin of the Universe, cosmologists have established a standard theoretical framework: Inflation + Hot Big Bang. To foresee the destiny of the Universe, people have realized that the key point is to understand the nature of dark energy [1].

The fact we are facing is the absence of a consensus theory for dark energy, though ad hoc models of dark energy based upon clever ideas are not rare. In the absence of theoretical guidance, the equation-of-state parameter (EOS) of dark energy, \( w = p_a/\rho_a \), provides a useful phenomenological description. If cosmological observations could determine \( w \) in a precise way, then the underlying physics of dark energy would be successfully revealed. However, in fact, extracting the information of \( w \) from the observational data is extremely difficult. Usually, parametrization of dark energy is inevitable. A widely used approach is the binned parametrization [2]. The key idea is to divide the redshift range into several bins and set \( w \) as constant in each bin. Since the current observational data at high redshifts (i.e., \( z > 1 \)) are very rare, for the binned parametrization, only two parameters of EOS \( w \) can be constrained well [3]. So in the literature, a more popular approach is to assume a specific ansatz for \( w \).

Among all the ansatz forms of \( w \), the Chevallier-Polarski-Linder (CPL) parametrization [4, 5] is the most popular one. It has a simple form,

\[
w(z) = w_0 + w_a \frac{z}{1 + z},
\]

where \( z \) is the redshift, \( w_0 \) is the present-day value of the EOS, and \( w_a \) characterizes its dynamics. However, as pointed out in our previous study [6], the CPL description will lead to unrealistic behavior in the future evolution, i.e., \( |w(z)| \) grows rapidly and finally encounters divergence as \( z \) approaches \(-1\) [6]. In order to keep the advantage of the CPL parametrization, and avoid its drawback at the same time, we believe that a divergence-free parameterization is necessary. In Ref. [6], Ma and Zhang (MZ, hereafter, for convenience) proposed the following hybrid form of logarithm and CPL parametrizations:

\[
w(z) = w_0 + w_a \left( \frac{\ln(2 + z)}{1 + z} - \ln 2 \right).
\]

This new parametrization has well behaved, bounded behavior for both high redshifts and negative redshifts. In particular, for the limiting case, \( z \to -1 \), a finite value for EOS can be obtained, \( w(-1) = w_0 + w_a(1 - \ln 2) \).

According to the CPL description, the destiny of the Universe is totally decided by the sign of \( w_a \): if \( w_a < 0 \), then \( w(-1) \to +\infty \), and so in the future the Universe will again become matter dominated and return to decelerated expansion; if \( w_a > 0 \), then \( w(-1) \to -\infty \), and so the Universe will eventually encounter the “big rip” singularity. So, as we have seen, the CPL description is unrealistic for predicting the future of the Universe: the sign of \( w_a \) solely determines the final fate of the Universe that is vastly different for \( w_a < 0 \) and \( w_a > 0 \). The same problem exists in other future-divergent parametrizations, such as \( w(z) = w_0 + b \ln(1 + z) \), \( w(z) = w_0 + w_a z/(1 + z)^2 \), etc. For the MZ description, the future evolution of the Universe is well-behaved [6, 7], thus the current constraints on the parameters might provide the important clue to the ultimate fate of the Universe.

The theme of the fate of the Universe is rather profound and is not testable. Nevertheless, the question of “where are we going?” is so attractive that we would like to make the inquiry. Our real purpose is to highlight the importance of the detection of the dynamics of dark energy. The future of the Universe might become conjecturable if the dynamics of dark energy is identified by the data. Currently, the observational data are not accurate enough to exclude or confirm the cosmological constant, however, we still could infer how far we are from a cosmic doomsday, in the worst case, from the current
data. In this study, we will try to discuss the destiny of the Universe by analyzing the current data. It should be stressed that our discussions depend on specific parametrizations of dark energy. So, we do not claim that we are able to make robust prediction for the future of the Universe. Instead we only use some parametrizations to speculate about the future, basing on the existing observational fact. We also assume that there will not be a sudden change for the property of dark energy in the future.

Our focus will be on the MZ description since it is a divergence-free parametrization. Of course, we will still discuss the CPL description as a comparison. First, we will constrain the cosmological models by using the current data. Next, we will discuss the implications—from analyzing the data describing the past history of the Universe—for the future of the Universe. The data we used include the type Ia supernova (SN) data from the 3-yr SNLS (SNLS3) observations [8], the “WMAP distance prior” data from the 7-yr WMAP (WMAP7) observations [9], the baryon acoustic oscillation (BAO) data from the SDSS Data Release 7 (SDSS DR7) [10], and the latest Hubble constant measurement from the Hubble Space Telescope (HST) [11]. We perform a χ^2 analysis on the MZ and CPL models by using a Markov Chain Monte Carlo technique [12].

According to the joint data analysis, in Fig. 1 we plot the probability contours at 68.3% and 95.4% confidence levels (CL) in \( w_0 - w_a \) plane, for the MZ parametrization. For 95.4% CL, the values of the model parameters are \( \Omega_{m0} = 0.2641^{+0.0242}_{-0.0342} \), \( w_0 = -1.0862^{+0.4869}_{-0.2993} \), \( w_a = -0.0567^{+0.8399}_{-0.2345} \), and \( h = 0.7235^{+0.0436}_{-0.0420} \) giving \( \chi^2_{\text{min}} = 423.444 \). Moreover, according to the properties of dark energy, we divide the \( w_0 - w_a \) plane into four parts: quintessence, phantom, quintom A (with big rip), and quintom B (without big rip). It is seen that the quintessence is almost disfavored (at 1σ CL) by the current observations, while both the cosmological constant, the phantom, and the quintom are consistent with the current observational data.

To make a comparison, we also constrain the CPL parametrization by using the same observational data, and find that the corresponding model parameters are \( \Omega_{m0} = 0.2646^{+0.0417}_{-0.0340} \), \( w_0 = -1.0665^{+0.5169}_{-0.4131} \), \( w_a = -0.0911^{+1.5033}_{-3.4714} \), and \( h = 0.7242^{+0.0440}_{-0.0420} \) giving \( \chi^2_{\text{min}} = 423.432 \). Although the CPL parametrization can also fit the current data well, its EOS will finally encounter divergence as \( z \) approaches −1. So we do not think that the CPL parametrization is a realistic description for the future evolution of the Universe. Notwithstanding, we will still discuss the big rip feature of the Universe using this description.

If in the future the EOS of dark energy \(-1 \leq w < -1/3\), then the fate of the Universe is obvious: the expansion continues forever; though galaxies disappear beyond the horizon and the Universe becomes increasingly dark, structures that are currently gravitationally bound still remain unaffected. This possibility exists in our fitting result, but this case is too fascinating, and so we have nothing to discuss for this case. What we are really interested in is the existence of the possibility of the “cosmic doomsday”. We want to infer, from the current data, how the worst situation would happen in the future of the Universe. If the doomsday exists, how far are we from it? Before the big rip, what time would the gravitationally bound systems be torn apart?

Adopting the MZ description, we can find out the parameter space where the big rip would happen. In Fig. 1, we have denoted the region with a big rip that is below the dashed line. A numerical calculation will easily tell us the time of the doomsday: \( t_{\text{BR}} - t_0 = 103.5 \) Gyr for the best-fit result, and \( t_{\text{BR}} - t_0 = 16.7 \) Gyr for the 95.4% CL lower limit, where \( t_{\text{BR}} \) denotes the time of big rip (when the scale factor blows up). In other words, for the worst case (2σ CL lower limit), the time remaining before the Universe ends in a big rip is 16.7 Gyr. As a comparison, we also consider the case of CPL description, though it is unrealistic for the future evolution of the Universe, as discussed above. For the CPL description, we find that \( t_{\text{BR}} - t_0 = 9.6 \) Gyr for the 95.4% CL lower limit.

Bound objects in the Universe, such as stars, globular clusters, galaxies, and galaxy clusters, are stabilized since they have detached from the Hubble flow, and so their internal dynamics are independent of the cosmic expansion. However, if in the future \( w < -1 \), the density of dark energy will grow so that eventually the internal dynamics of bound objects will be influenced by dark energy. Once the density of dark energy exceeds that of any object, the repulsive gravity of phantom
energy overcomes the forces holding the object together, and the object is torn apart. For a gravitationally bound system with mass \( M \) and radius \( R \), the period of a circular orbit around this system at radius \( R \) is \( P = 2\pi(R^3/GM)^{1/2} \), where \( G \) is the Newton’s constant. As pointed out by Caldwell, Kamionkowski and Weinberg [13], this system will become unbound roughly when \(-4\pi/3\rho_M + 3p_{de})R^2 \approx M\). So one can determine the corresponding redshift \( z_{tear} \) when this system is destroyed, by solving the equation \([1 + 3w(z_{tear})]/f(z_{tear}) = -8\pi^2/(\Omega_{m0}H_0^2P^2)\), where \( f(z) = \exp[3 \int_0^z (1 + w(z'))/(1 + z') \, dz' \] characterizing the dynamics of dark energy. According to the fitting results of the MZ parametrization, we plot the relation between the characteristic time scale of a gravitationally bound system \( P \) and \( t−t_0 \) in Fig. 2. In fact, we are interested in the time interval between the big rip and the event that a specific structure is destroyed by phantom energy, which can be calculated by the integral \( t_{BR}−t_{tear} = \int_0^t (1 + z)H(z)\, dz \). In Ref. [13], Caldwell, Kamionkowski and Weinberg have tackled this issue under the framework of the constant \( w \) model, and found a simple analytical formula \( t_{BR}−t_{tear} \approx P \sqrt{2[1 + 3w(−1)]}/[6\pi(1 + w(−1)) \] , where \( w(−1) = w_0 + w_a(1 − \ln 2) \). Then, we can easily derive a simple analytical formula, 

\[
t_{BR}−t_{tear} \approx P \sqrt{2[1 + 3w(−1)]}/[6\pi(1 + w(−1)) \] (3)

Interestingly, this formula is very similar to the formula of Ref. [13], except that the constant \( w \) is replaced with the value of \( w(−1) \). This time interval is independent of \( H_0 \) and \( \Omega_{m0} \) as expected. Numerical calculation shows that this result is very precise. So we have shown that the formula of Ref. [13] can be extended to the case of a dynamical dark energy (behaving as a phantom in the future).

In the following, let us speculate on a series of possible consequences before the end of time. We shall describe when the gravitationally bound systems (such as the Milky Way, the solar system, the Earth-moon system, the Sun, and so on) would be destroyed. Utilizing the best-fit results of the MZ parametrization, we plot the relation between the characteristic time scale of gravitationally bound system \( P \) and the time interval \( t_{BR}−t_{tear} \) in Fig. 3. As is seen in this figure, the Milky Way will be destroyed 210 Myr before the big rip; 13 months before the doomsday, the Earth will be ripped from the Sun; 30 days before the doomsday, the moon will be ripped from the Earth; the Sun will be destroyed three hours before the end of time; and 100 min before the end, the Earth will explode.

Actually, we are more interested in the worst situation. In
Table I, we list the corresponding events in the worst case concerning the future of the Universe, where the results of the 95.4% CL lower limit are used. In this case, the Milky Way will be destroyed 32.9 Myr before the big rip; 2 months before the doomsday, the Earth will be torn apart from the Sun; 5 days before the doomsday, the moon will be ripped from the Earth; the Sun will be destroyed 28 min before the end of time; and 16 min before the end, the Earth will explode. Even microscopic objects cannot escape from the rip. For example, the hydrogen atom will be torn apart 3 × 10^{-17} s before the ultimate singularity.

In summary, we explored the ultimate fate of the Universe by using the MZ parametrization. Unlike the CPL parametrization, the MZ parametrization has well behaved, bounded behavior for both high redshifts and negative redshifts, and thus can genuinely cover many theoretical dark energy models. After constraining the parameter space of this parametrization by using the current cosmological observations, SN (SNLS3) + CMB (WMAP7) + BAO (SDSS DR7) + H₀ (HST), we found that, at the 95.4% CL, our Universe can still exist at least 16.7 Gyr before it ends in a big rip. Moreover, we also discussed when a gravitationally bound system will be destroyed, if our Universe is dominated by a phantom in the future. It is found that a gravitationally bound system will be ripped apart at a time \( t \approx \frac{P \sqrt{2}[1 + 3w(-1)]}{[6w(1 + w(-1))]^3} \) before the big rip. This means that the conclusion of Ref. [13] can be extended to the case of dynamical dark energy.

For the CPL parametrization that is a future-divergent description, we showed that the fate of the Universe is solely decided by the sign of \( w_0 \). Obviously, this is unnatural. Nonetheless, we also tested this possibility. For the CPL description, we found that, at the 95.4% lower limit, the big rip is from today by 9.6 Gyr; all the bound systems in the Universe will be torn apart almost at the same time—the big rip.

Of course, one might criticize that any prediction of the future of the Universe is not testable and cannot be truly model-independent. Nonetheless, we feel that, since the ultimate fate of the Universe is closely interrelated to the nature of dark energy, it is fairly natural to infer the future of the Universe from the current detection of the dynamical property of dark energy. The question of “where are we going” is an eternal theme for human beings, so we should have courage to explore this theme. Our attempt implies that a rational parametrization for dynamical dark energy is important. Whether a parametrization is divergence-free may or may not be an important principle, but we have shown that a future divergent description will lead to a strange future for the Universe, while a divergence-free description can be used to foresee the cosmic destiny in a rational way.

In addition to the MZ parametrization, one may also construct some other parametrization forms to avoid the future divergence problem. We only took the MZ form as a typical example. We also tested the 3-parameter form \( w(z) = w_0 + w_a z/(n + z) \). However, we found that \( n \) cannot be constrained well by the data. To our current knowledge, the MZ parametrization is a fairly good ansatz form to explore the dynamics of dark energy as well as the fate of the Universe. Of course, we expect to find out a better parametrization for probing the dynamics of dark energy.

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