In large-mass diffraction in deep inelastic scattering, we study the gluon jet close to the rapidity gap and show that its transverse momentum spectrum is peaked around a value which characterizes the onset of S-matrix unitarity. We argue that such a measurement of diffractive jets could help understanding if at the present energies, unitarity comes as a consequence of saturation.

1 Introduction

One of the most successful approaches to the understanding of hard diffraction in DIS is the QCD dipole picture which naturally describes both inclusive and diffractive events within the same theoretical framework. It expresses the scattering of the photon through its fluctuation into a color singlet $q\bar{q}$ pair (dipole) and the same dipole scattering amplitude enters in the formulation of the inclusive and diffractive proton structure functions. This approach revealed that unitarity and the way it is realized should be important ingredients of the description of diffractive cross-sections, making those ideal places to look for saturation effects at small-$x$.

In this study, we analyse hard diffraction when the proton stays intact after the collision and the mass of the diffractive final state is much bigger than the virtuality of the photon. This process is called diffractive photon dissociation. We propose the measurement of the final state configuration $X + \text{jet} + \text{gap} + p$ in virtual photon-proton collisions. We show that in the context of saturation theory, the transverse momentum distribution of the measured jet is resonant with the scale at which the contributions of large-size dipoles start to be suppressed, called the saturation scale. That exhibits the potential of the diffractive-jet measurement for testing saturation and extracting the saturation scale.

2 Diffractive photon dissociation

In deep inelastic scattering, a photon of virtuality $Q^2$ collides with a proton. In an appropriate frame called the dipole frame, the virtual photon undergoes the hadronic interaction via a fluctuation into a dipole; the dipole then interacts with the target proton and one has the following factorization

$$\sigma^{\gamma^*p} = \int d^2r \int_0^1 d\alpha \left( |\psi_T^-(r, \alpha; Q)|^2 + |\psi_L^-(r, \alpha; Q)|^2 \right) \sigma(r) \quad (1)$$
which relates a cross-section for an incident photon $\sigma^{\gamma p}$ to the corresponding cross-section $\sigma(r)$ for an incident dipole of transverse size $r$. In the leading logarithmic approximation we are interested in, the dipole cross-sections do not depend on $\alpha$, the fraction of photon longitudinal momentum carried by the antiquark. The wavefunctions $|\psi^\gamma_T|^2$ and $|\psi^\gamma_L|^2$ describing the splitting of the transversely (T) or longitudinally (L) polarized photon on the dipole are well known.

In diffractive deep inelastic scattering, the proton gets out of the collision intact and there is a rapidity gap between that proton and the final state $X$, see the left plot of Fig. 1. If the final state diffractive mass $M_X$ is much bigger than $Q$, then the dominant configurations of the final state come from the $q\bar{q}g$ component of the photon wavefunction or from higher Fock states, i.e. from the photon dissociation. In this study, we consider this kinematical regime where $\beta \equiv Q^2/(Q^2 + M_X^2) \ll 1$ and we investigate the $q\bar{q}g$ component. In this case, due to the infrared singularity of QCD, the final-state configuration that gives the dominant contribution to the cross-section corresponds to the gluon jet being the closest to the gap. We shall study the transverse momentum spectrum of this gluon jet within high-energy QCD.

The right plot of Fig. 1 represents the diffractive production of a gluon with transverse momentum $k$ and rapidity $y = \log 1/\beta$ off a $q\bar{q}$ dipole of size $r_0$.

![Diagram](image.png)

Figure 1: Left side: diffractive dissociation of a photon of virtuality $Q$ into a final state of mass $M_X$. Right side: diffractive production of a gluon with transverse momentum $k$ and rapidity $y = \log 1/\beta$ off a $q\bar{q}$ dipole of size $r_0$.
usual kinematics of diffractive DIS: $Y = \log(1/x)$ and $\Delta \eta = \log(1/x_P)$ with $x = Q^2/(Q^2 + W^2)$ and $x_P = x/\beta$. $W^2$ is the center-of-mass energy of the photon-proton collision. Independently of the form of the $S-$matrices, one can show that the behavior of the observable as a function of the gluon transverse momentum $k$ is the following: it is going to rise as $k^2$ for small values of $k$ and fall as $1/k^2$ for large values of $k$. A maximum will occur for a value $k_0$ which is related to the inverse of the typical size for which the $S-$matrices approach zero; in other words, the maximum $k_0$ will reflect the scale at which unitarity sets in. The question is whether unitarity is due to non-perturbative physics or if it rather comes as a consequence of parton saturation. We shall explore the latter possibility, in which case $k_0$ is identified to the saturation scale $Q_s$.

3 Phenomenology

The exact form of the $S-$matrices is unknown, and we have to consider models in order to produce values of the observable at any value of $k$. For this purpose we consider the following models, inspired by the GBW parametrization of parton saturation effects:

$$S(x_0, x_1; \Delta \eta) = \Theta(R_p - |b|) e^{-Q^2(x_P)(x_0-x_1)^2/4} + \Theta(|b|-R_p),$$

$$S^{(2)}(x_0, z, x_1; \Delta \eta) = \Theta(R_p - |b|) e^{-Q^2(x_P)(x_0-z)^2/4} e^{-Q^2(x_P)(z-x_1)^2/4} + \Theta(|b|-R_p),$$

where $R_p$ is the radius of the proton. The saturation scale $Q_s$, the basic quantity characterizing saturation effects, is a rising function of energy through its $x_P$—dependence; this can be tested using the observable and the phenomenon explained above.

In Fig. 2 we plot the observable as a function of $k$. The left plot is with fixed $Q^2 = 1$ GeV$^2$ and four values of the saturation scale, $Q_s = 0.5, 1, 2, 3$ GeV. As discussed in section 2, we check that $k^2 d\sigma_{diff}/d^2kdM_X$ grows as $k^2$ at small momentum and decreases as $1/k^2$ at large momentum. We also see that the transition region between the two distinct behaviors at small and large $k$, which features a marked bump, is linked to the value of $Q_s$ as expected. This is confirmed on the right plot where the same observable is plotted as a function of the rescaled transverse momentum $k/Q_s$ for two extreme values of the photon virtuality, $Q^2 = 0.1$ and 100 GeV$^2$. Clearly the maximum for each curve is independent of $Q_s$ and $Q^2$ in a broad range of considered values: one has $k_{max}/Q_s \sim 1.4$. Therefore diffractive gluon production in the domain

![Figure 2: The observable as a function of the gluon transverse momentum $k$. Left plot: for a fixed value of $Q^2 = 1$ GeV$^2$ and four indicated values of the saturation scale $Q_s$. Right plot: as a function of $k/Q_s$ for two extreme values of $Q^2$ equal to 0.1 and 100 GeV$^2$ and the four values of the saturation scale used for the left plot.](image)
of large diffractive mass offers a unique opportunity to determine the saturation scale $Q_s$ and its dependence on $x_F$. Note that since $k_{max}$ is independent of $Q^2$, a wide range of photon virtuality could be used to carry out this measurement, as long as one keeps $\beta \ll 1$.

However, from the experimental point of view there exists an important limitation related to the minimal value of the transverse momentum which can be measured for a jet. In the most pessimistic scenario, considering even rather high values of the saturation scale, $Q_s(x_F) \sim 1$ GeV, it is unlikely that the maximum $k_{max}$ of the cross section (4) can be seen at HERA. Thus, to see the transition between the two different behaviors of the cross section (4) seems like a major experimental challenge. In Fig. 3 we illustrated such a situation, where the cross-section $k^2 M_X d\sigma/dk^2 dM_X$ is plotted in the HERA energy range with the saturation scale taken from Ref. 3 and the overall normalization $\alpha_s = 0.15$ taken from Ref. 6. One sees that, unfortunately, the data should always lie on the perturbative side of the bump. However, it is not necessary to see the whole bump to confirm the influence of the saturation scale on the results. In particular, there is a big difference in the rise towards the bump between the highest $x_F$–bin ($M_X = 40$ GeV and $W = 100$ GeV) and the lowest $x_F$–bin ($M_X = 5$ GeV and $W = 245$ GeV). An experimental confirmation of such a behavior would be in favor of saturation and could lead to the determination of the saturation scale. If however this trend is not observed, it could mean that in this process, unitarity does not come from saturation, but rather from soft physics.

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