Characteristics of Air Resistance in Aerostatic Bearings

Jianlong Yin 1,2, Jing Yu 2,* , Pengfei Cao 2, Dongsheng Li 2, Xiaoyan Shen 2☯ and Ming Li 1

1 School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200444, China; 1380201103@cjlu.edu.cn (J.Y.); robotlib@shu.edu.cn (M.L.)
2 College of Metrology and Measurement Engineering, China Jiliang University, Hangzhou 310018, China; 13165995181@163.com (P.C.); lidongsheng@cjlu.edu.cn (D.L.); xyshen@cjlu.edu.cn (X.S.)
* Correspondence: yujing88@cjlu.edu.cn

Abstract: The definition of air resistance is nonuniform when analyzing the bearing capacity, stiffness, and stability of an orifice throttling aerostatic restrictor. In this study, a capillary tube similar to the inlet section of an aerostatic restrictor is used as the research object, and the Bernoulli equation under adiabatic conditions is established. Through an experiment, the pressure and temperature of the capillary tube inlet and outlet and the flow through the capillary tube are measured. Based on the air resistance definition, the empirical formula of the coefficient \( k \) is obtained, and the theoretical air resistance of the capillary path is calculated. The relative error between the theoretical air resistance and experimental air resistance is kept within 10%. The comparison results verify the accuracy of the air resistance theory and provide a basis for the subsequent establishment of a universal definition of air resistance. Subsequently, air resistance can be used to design aerostatic bearings and help improve their characteristics.

Keywords: aerostatic restrictor; air resistance; aerostatic orifice throttling bearings; capillary tube measurement

1. Introduction

With the rapid development of machining technology and precision measurements, the precision and speed of precision products and equipment continue to improve. Because conventional liquid lubrication methods can no longer meet the need for high precision, gas lubrication technology has emerged as an alternative [1]. Gas lubrication technology has been widely applied in the industry owing to its advantages, including high precision, fast movement, less pollution, and low cost [2,3]. A key structure in gas lubrication technology is the throttle [4,5]. The focus of research on throttles has been on the advantages of their design and characteristics, including low cost, large bearing capacity, high stability, and low thermal deformation with a large temperature rise [6]. The bearing capacity and stability of air films are contradictory. An increase in the bearing capacity will inevitably lead to a decrease in the stability of the throttle, whereas an increase in stability will decrease the bearing capacity of the throttle [7,8]. Therefore, balancing the bearing capacity and stability of a restrictor is one of the purposes of studying the characteristics of aerostatic orifice throttling restrictors.

Conventional research on the static and dynamic characteristics of aerostatic orifice throttling is mainly based on theoretical calculations, simulation analyses, and experimental observations. However, these methods have disadvantages, including difficulty in solving the Reynolds equation, excessively high simulation time, complicated experimental designs, and low accuracy [9]. Owing to the compressibility of air in turbulence, an accurate solution cannot be obtained through theoretical calculations [10]. Thus, to overcome this shortcoming, aerostatic resistance theory is considered. This theory relates to the air pressure drop with the change in temperature and energy of the aerostatic bearing, and it establishes the equivalent circuit model of the aerostatic restrictor or aerostatic bearing.
Dynamic and static performance indicators, such as flow, bearing capacity, stiffness, and stability of aerostatic orifice throttling, have been analyzed [11], which significantly shorten the lengthy calculation. Theisen et al. experimentally proved that a low-viscosity gas will lead to a poor damping performance, and the damping performance of gas bearings can be enhanced through robust control methods [12,13]. Hsiao et al. comparatively analyzed the flow characteristics of an aerostatic bearing obtained using an experimental method, i.e., the lumped-parameter model, and the dynamic characteristics of gas flowing through a restrictor [14]. Al-Bender used the lumped-parameter model to establish a dynamic model of a thin-film aerostatic bearing and analyzed the factors affecting the stability of a throttle [15]. Kim et al. compared the flow rate, hydraulic resistance, and pressure drop to the current, resistance, and voltage [16]. They established equivalent circuits to measure the flow through microfluidic components and systems (including micropumps and microvalves) and predicted the dynamic response characteristics of the microfluidics of fluid equipment quickly [16]. Li established pressure impedance models of double U-shaped, double-ring, and multi-microchannel aerostatic pressure restrictors and compared their bearing capacity and stiffness, which were calculated using a simulation and the pressure impedance model [17]. Chen proposed an arrayed microhole restrictor (AMR) to suppress the vortex flow and reduce the vibration of aerostatic bearings. Through a computational fluid dynamics analysis, the transient flow features were studied for aerostatic bearings with an AMR and conventional restrictors, and the static performances of bearings were also compared. The vibration strength of the bearing was experimentally measured to validate the effectiveness of an AMR. The results show that vortex shedding in the recess is suppressed and the vibration can be effectively reduced using an AMR, whereas the load capacity and stiffness of the bearing remain unchanged [18]. Belforte presented an experimental study on pneumostatic pads with micro holes realized via laser technology [19]. The minimum orifice diameter was approximately 0.05 mm. To optimize the performance of the pads, the influence of the diameter and the number of holes on the load capacity, stiffness, and consumption versus height air gap was analyzed [19]. However, there is no complete and unified definition of air resistance, air capacity, and air inductance, which play important roles in the equivalent circuit model [20,21]. Essentially, air resistance is the foundation of air capacity and air inductance.

Therefore, the establishment of a complete and unified definition of aerostatic resistance is important in the design of aerostatic restrictors and development of air lubrication technology. With the establishment of the definition and theoretical model of aerostatic resistance, they will support the design and analysis of aerostatic bearings. In this study, air resistance is proposed to illustrate the air flow characteristics in aerostatic systems. In this study, capillary tubes are used to analyze and measure the air drop and determine the mechanism of throttles in defining air resistance. The air resistance can be used to design aerostatic bearings and help improve their characteristics.

2. Materials and Methods

2.1. Orifice Throttling Air Resistance Theory

The Bernoulli equation is one of the basic theories used in fluid mechanics. Its essence is the conservation of fluid mechanical energy, where the sum of the kinetic energy, gravitational potential energy, and pressure potential energy is a constant [22].

Owing to the viscosity of air, the energy loss in the process should consider the actual pipe flow. A compressible fluid flowing at high speeds cannot exchange heat with air, so the flow can be regarded as adiabatic (i.e., isentropic flow) [23]. Assuming that the volume force is only gravity, the Bernoulli equation can be considered a one-dimensional flow with energy loss along the pipeline. Thus, the Bernoulli equation for an inviscid compressible fluid under adiabatic conditions can be expressed as follows:

\[
\frac{V_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_1} + gz_1 = \frac{V_2^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_2}{\rho_2} + gz_2 + gh_f
\]  
(1)
where $V_1$ and $V_2$ are the flow velocity of a unit mass of fluid on cross sections 1 and 2, respectively; $p_1$ and $p_2$ are the pressure of a unit mass of fluid on cross sections 1 and 2, respectively; $z_1$ and $z_2$ are the position potential energy of a unit mass of fluid on cross sections 1 and 2, respectively; and $h_f$ is the energy loss per unit mass of fluid from flow cross sections 1 to 2, also called the head loss, which usually includes the resistance loss and local resistance loss.

Figure 1 presents a schematic diagram of the orifice restrictor. Based on the flow required by the restrictor, the inner diameter is set below 10 mm, forming a horizontal, short, and straight pipe.

![Schematic diagram of the orifice restrictor.](image)

**Figure 1.** Schematic diagram of the orifice restrictor.

Equation (1) can be regarded as the Bernoulli equation when the viscous compressible fluid flows at a high speed under adiabatic conditions. This equation can also be used to derive the pressure loss calculation formula. Equation (1) can then be rewritten as

$$
\frac{\Delta p}{\rho g} = \frac{\gamma - 1}{\gamma} \frac{p_1 - p_2}{\rho_m g} = \left( \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) + (z_2 - z_1) + h_f
$$

(2)

When the flow of the viscous compressible fluid is adiabatic, the change in the kinetic energy and potential energy for a fully developed laminar flow or turbulent flow is small and negligible. Therefore, Equation (2) can be rewritten as

$$
\Delta p = \frac{\gamma - 1}{\gamma} \rho_m gh_f
$$

(3)

The energy loss of the viscous fluid flowing in the pipeline consists of pressure loss and local resistance loss. For the horizontal, short, and straight pipe, the local resistance loss can be ignored. Thus, the energy loss of the viscous fluid while moving in the pipe is equal to the pressure loss along the entire process. The pressure loss along the viscous fluid in the pipe is expressed as

$$
h_f = h_l = \frac{\Delta p}{\rho g} = \lambda \frac{l U^2}{d 2g}
$$

(4)

where $\lambda$ is the resistance coefficient along the path; $l$ is the tube length; $d$ is the equivalent diameter of the pipeline, which is four times the ratio of the cross-sectional area to the circumference; and $U$ is the average flow rate in the tube.

By substituting Equation (4) into Equation (3), the pressure loss in the pipe can be calculated as follows:

$$
\Delta p = \frac{\gamma - 1}{\gamma} \rho_m l U^2 = \frac{\gamma - 1}{\gamma} \rho_m l d^2
$$

(5)

where $A$ is the cross-sectional area of the flow channel in the pipe.

According to Hagen-Poiseuille low, the total flow cross a round tube can be expressed as:

$$
Q = \frac{\pi d^4}{128 \mu l} \Delta p
$$

(6)
So, the pressure loss can be expressed as:

$$\Delta p = \frac{128 \mu l Q}{\pi d^4}$$  \hspace{1cm} (7)$$

The Hagen-Poiseuille low can also provide a calculated method for pressure loss. When the air flows through the pipeline, the gas viscosity and roughness of the pipe wall influence the airflow \[ 24\]. The velocity distribution of the air in the vertical section of the tube is nonuniform. The air close to the boundary layer has a low velocity. The main features of air resistance in a pneumatic system are pressure loss, system heating, and system efficiency reduction. Equation (5) shows that the ratio of pressure loss \( \Delta p \) is related to the heat ratio \( \gamma \), resistance coefficient along the path \( \lambda \), air density \( \rho \), pipeline length \( l \), equivalent diameter of the pipeline \( d \) (as the cross-sectional area \( A \) and equivalent diameter \( d \) can be converted to each other, they are regarded as the same variable), and flow \( q \). For air, the heat ratio \( \gamma \) is 1.4; the air density represents the compressibility. In addition, the air volume is restricted by the shape of the inner flow channel. For horizontal, short, and straight pipes, the cross-sectional area \( A \) of the internal flow and the length of the inner flow channel \( l \) are also important parameters that affect the air resistance and are mostly constant. The only uncertain and variable parameter is the resistance coefficient along the path \( \lambda \), which is easily affected by the roughness of the inner wall of the pipeline. Therefore, when the air flow state does not change, the air resistance is determined by the air density \( \rho \), length of the pipeline \( l \), and equivalent diameter \( d \) of the pipeline. However, when the air flows at a high speed, it is turbulent, and the increase in air pressure not only changes the density \( \rho \) but also causes a change in the resistance coefficient along the path \( \lambda \).

Here, the capillary tube is considered an example. To address the inaccurate calculation of the resistance coefficient along the path, a new parameter is defined, and the symbol \( R_g \) is used to indicate air resistance. The function of air resistance is expressed as

$$R_g = f(k, \rho, l, d)$$  \hspace{1cm} (8)$$

where \( R_g \) is the air resistance and \( k \) is a parameter that changes with the air flow state and includes the resistance coefficient along the path \( \lambda \).

Based on Equation (5), when the air flow remains unchanged, \( \Delta p \) is proportional to \( q^2 \); thus, Equation (5) can be rewritten as

$$\Delta p = R_g q^2$$  \hspace{1cm} (9)$$

From Equation (9), the air resistance is dimensional, and the unit of air resistance can be determined as kg/m².

In summary, a definition of air resistance applicable to capillary tubes is proposed, whose dimensional formula is ML⁻⁷. Air resistance represents the transport capacity of air in the pipeline. The greater the air resistance, the greater the pressure loss after the air flows through the pipeline. The formula of air flowing at high speeds in the capillary tube is

$$R_g = \frac{k \rho l}{d^5}$$  \hspace{1cm} (10)$$

The characteristics of the parameter \( k \) will be further analyzed based on experiments.

2.2. Simulation Research on the Air Resistance Model

Based on the definition of air resistance, the average flow rate in the pipeline under test can be obtained without using a flow meter. Considering the local pressure loss, the pressure at the outlet of the tested pipeline can be calculated according to the pressure at the inlet of the measuring pipeline. According to the calculated average flow rate and terminal air pressure, when the inlet air pressure of the tested pipeline is known, the air resistance of the tested pipeline can be calculated by substituting \( p_{\text{air}}, p_{\text{ao}}, \) and \( q_m \) into Equation (9). The average flow rate of the air \( U_1 \) in the measured pipeline can be obtained from the average
flow density and average flow rate of the measured pipeline. The mathematical model is as follows:

\[
\left(1 + \frac{\gamma - 1}{2U_1/U_s^2}\right)^{\frac{5.95(p_i + p_o)}{\gamma - 1}} U_1 A_1 = \rho_1 \sqrt{\frac{(p_i - p_o) d_1^5}{k p_s l}}
\]

(11)

where \(d_1\) is the hydraulic diameter of the measured pipeline and \(U_s\) is the local speed of sound.

After obtaining the average flow rate in the pipeline under test \(q_m\) from the average flow rate \(U_1\), the air resistance model of the pipeline under test can be obtained:

\[
R_{gm} = \frac{\Delta p_m}{q_m^2} = \frac{p_{ai} - p_{ao}}{(U_1 A_1)}
\]

(12)

Using COMSOL simulation software, stainless-steel pipes with inner diameters of 2, 3, and 4 mm were selected as the simulation objects. For each diameter, lengths of 0.5 and 1.0 m were selected. The modeling process of the stainless-steel pipe with an inner diameter of 2 mm and a length of 0.5 m was taken as an example.

Air in a turbulent flow state is the medium examined in this study. Accordingly, the fluid-flow single-phase flow-turbulence \(k - \varepsilon\) model is followed stepwise, and the steady state is selected according to the context of this research. The measured pipeline’s internal flow channel model is shown in Figure 2.

![Figure 2](image.png)

**Figure 2.** Measured pipeline’s internal flow channel model.

### 2.3. Orifice Throttle Air Resistance Experimental Setup

A capillary tube with a structure similar to that of the flow channel in the orifice restrictor was selected as the experimental object, and the air resistance of the capillary tube was measured. Turbulent air flow was fully developed in the capillary tube of the measured section. The heat exchange between the air and outside environment was controlled to avoid affecting the original flow state of the air in the tube when the sensor was installed. The experimental device includes a pipeline module, sensor installation module, and support module with a measurement and control system designed for data collection and analysis. Figure 3 shows the setup of the mechanical experiment.

The stainless-steel round tubes used in the experiment had inner diameters of 2 and 4 mm and lengths of 0.5 and 1.0 m. The pressure at the input end of the tested pipeline increased from 0.125 MPa to 0.425 MPa, with an increase interval of 0.025 MPa. Temperature and pressure sensors measured the temperature and pressure at the input and output ends of the pipeline under test, respectively, and a gas mass flow meter measured the air flow in the pipe. The data measured by the sensor and flow meter were recorded using computer software through a data acquisition card. The air resistance under different air pressure inputs was then calculated according to Equation (7) and saved.
The air pressure and temperature at the input and output ends of the capillary tube under test were measured, and the air resistance of the capillary tubes with different parameters at different pressure inputs was calculated. Based on the definition of air resistance, a new parameter \( k \) was defined, and the characteristics of the parameter \( k \) were further analyzed.

3. Results and Discussion

3.1. Simulation Results and Analysis

Pipes with the following dimensions (inner diameter [mm] \( \times \) length [m]) were modeled in this study: \( 2 \times 0.5, 2 \times 1.0, 3 \times 0.5, 3 \times 1.0, 4 \times 0.5, \) and \( 4 \times 1 \). The air pressure at the end of the pipeline under test and the average flow velocity in the pipeline under test were obtained via a simulation, and the air resistance of the pipeline under test was calculated using Equation (9). The tetrahedral mesh was chosen, where the max cell is 0.02 mm and the min cell is \( 1.53 \times 10^{-4} \) mm. The air pressure and average flow at the inlet and outlet of the measurement pipeline were simulated in the air resistance model of the pipeline under test, and the air resistance of the pipeline under test was calculated. The results were compared, as shown in Figure 4.

Overall, the simulation and theoretical air resistance have the same trend, which proves that the air resistance model can be used to calculate the air resistance of the pipeline under test. Furthermore, the principle of the air resistance model is also proven correct.

3.2. Experimental Results and Analysis

By substituting the experimental air resistance calculated by Equation (9) into Equation (10), the experimental values of the parameter \( k \) can be obtained, as shown in Figure 5.

From Figure 5, the following observations were derived: (1) For pipelines with different specifications, as the input air pressure increases, the change trend of the parameter \( k \) remains the same; it decreases with a negative exponential trend, and \( k \) finally tends to stabilize. Moreover, under the same air pressure input, regardless of the pipe inner diameter or pipe length, the values of the parameter \( k \) are similar. (2) When the air pressure at the inlet end of the tested pipeline is less than 0.25 MPa, the curve shows a rapid downward trend. When the inlet pressure is greater than 0.25 MPa, the curve tends to be flat. This result is due to the changes in the air flow rate, which leads to a continuous increase in the Reynolds number and continuous thinning of the viscous bottom layer. Therefore, the parameter \( k \) is a function related to the Reynolds number.
Figure 4. Comparison curve between the simulated and theoretical air resistance values. (a) Pipe with 2 mm inner diameter and 0.5 m length. (b) Pipe with 2 mm inner diameter and 1.0 m pipe length. (c) Pipe with 3 mm inner diameter and 0.5 m pipe length. (d) Pipe with 3 mm inner diameter and 1.0 m pipe length. (e) Pipe with 4 mm inner diameter and 0.5 m pipe length. (f) Pipe with 4 mm inner diameter and 1.0 m pipe length.

Figure 5. $k$ curve.
Theoretically, the parameter \( k \) changes with the Reynolds number \( Re \), showing a negative exponential relationship, setting the parameter expression as \( k = a \cdot Re^{-b} \). Using the least-squares method, the parameter \( k \) and Reynolds number \( Re \) were curve-fitted to obtain the best estimated value of the coefficients \( a \) and \( b \) in the expression of the parameter \( k \):

\[
k = 14.77Re^{-0.75}
\]  

(13)

where \( Re \) is the Reynolds number.

To verify the accuracy of the air resistance definition applicable to the capillary tube proposed in this paper, the air resistance calculated using the air resistance definition formula was experimentally compared with the measured air resistance, as shown in Figures 6–9.

![Figure 6](image6.png)

**Figure 6.** Comparison curve between the theoretical and experimental values of the gas resistance and input pressure of the tube with an inner diameter of 2 mm and length of 0.5 m.

![Figure 7](image7.png)

**Figure 7.** Comparison curve between the theoretical and experimental values of the gas resistance and input pressure of the tube with an inner diameter of 2 mm and length of 1.0 m.
Accordingly, the following observations are obtained from the figures:

1. An inflection point exists in the theoretical and experimental air resistance values in Figures 6–9, and the change trend of the air resistance value $R_g$ varies from gradually decreasing to gradually increasing. The Mach number at the turning point is approximately 0.3, which is the boundary point between the low-speed flow and high-speed flow.

2. From Equation (10), the air resistance $R_g$ is proportional to $l$ and inversely proportional to $d^5$. In the capillary tubes of equivalent diameters, the tube lengths experimentally measure 0.5 and 1.0 m. Although the tubes have equivalent diameters, the theoretical air resistance is twice the measured results, and the actual calculation is 1.6–1.8 times the measured results because the density $\rho$ and parameter $k$ are actually different. For capillary tubes with the same tube length, the experimentally measured equivalent diameters are 2 and 4 mm. The theoretical air resistance is 0.03 times the measured results, and the actual calculation is 0.8–1 times the measured results, showing a deviation from theory. This is because although the parameter $k$ is similar for pipelines with different equivalent diameters, the density and flow are completely different, and the variables cannot be controlled.

3. The relative error between the air resistance calculated using the air resistance definition formula and the air resistance measured experimentally is below 10%, which
proves that the definition of air resistance proposed in this paper is correct and universal for the subsequent establishment of the air resistance theory.

4. Based on the curves in the figures, the air resistance is a quantity that changes with the flow state. With different pressure inputs, the air resistance value changes. Nonetheless, some errors still occur in the air resistance values obtained from the measurement and theoretical calculation. The reasons are as follows:

5. The results are affected by the accuracy of the flowmeter. In an actual measurement, with the increase in the input air pressure, the airflow in the pipeline is in a turbulent state. A vortex is formed at a location that intensifies the fluctuation of the pressure at the inlet of the flowmeter and reduces the measurement accuracy of the flowmeter.

6. The pipeline used in the experiment is a stainless-steel capillary tube fabricated by cold drawing. The actual roughness is nonuniform, and the flow state may change along the axis.

7. The experimental environment is not completely adiabatic. The outlet gas flows directly into the atmosphere, and the heat distribution in the tube is uneven. The temperature drop between the inlet and outlet of the tested pipeline is approximately 2°C, which slightly affects the calculation results.

4. Conclusions

In this research, the Bernoulli equation suitable for high-speed compressible flows was derived, and it was used to solve the pressure loss calculation formula in a horizontal circular tube. The definition of resistance and calculation formula for pipe flow pressure loss was referred to, and a definition of air resistance applicable to capillary tubes was established. A type of experimental measurement device was designed to verify the air resistance definition. Through the experiments, the air resistance of stainless-steel capillary tubes with inner diameters of 2 and 4 mm and lengths of 0.5 and 1.0 m were measured, and the characteristics of the parameter k were analyzed. The experimental results show that the relative error between the theoretical and experimental air resistance values was less than 10%. The air resistance of the tested pipeline was derived from the air resistance of the measured pipeline, and the air resistance model of the tested pipeline was established. The simulated and theoretical air resistance were compared and analyzed, and the results show that the two were very close, which proved that the air resistance model of the pipeline under test can be used to calculate air resistance.

The air resistance of six round pipes with different specifications was measured. The air resistance of the tested pipeline decreases with the increase in the input air pressure, which gradually trends toward a stable value. However, differences exist in the decreasing trend of the air resistance of pipelines with different specifications, indicating that the inner diameter of the tube has a greater influence on the air resistance than the length of the tube.

Author Contributions: Conceptualization, D.L. and M.L.; methodology, J.Y. (Jianlong Yin); software, P.C.; validation, J.Y. (Jianlong Yin), P.C. and J.Y. (Jing Yu); formal analysis, P.C.; investigation, J.Y. (Jianlong Yin); resources, J.Y. (Jianlong Yin); data curation, J.Y. (Jianlong Yin); writing—original draft preparation, J.Y. (Jing Yu); writing—review and editing, D.L.; visualization, X.S.; supervision, D.L.; project administration, D.L.; funding acquisition, D.L. and J.Y. (Jing Yu) All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Natural Science Foundation of China (grant number 51775529) and the Zhejiang Natural Science Foundation (grant number LY19E050002).

Conflicts of Interest: The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.
Nomenclature

| Symbol | Name (Units)                           |
|--------|---------------------------------------|
| $V_1$  | flow velocity of a unit mass of fluid on cross Section 1 |
| $V_2$  | flow velocity of a unit mass of fluid on cross Section 2 |
| $p_1$  | pressure of a unit mass of fluid on cross Section 1 |
| $p_2$  | pressure of a unit mass of fluid on cross Section 2 |
| $l$    | tube length (mm)                       |
| $U$    | average flow rate in the tube          |
| $\Delta p$ | pressure loss                           |
| $R_g$  | air resistance ($\text{kg/m}^2\text{s}$) |
| $d_1$  | hydraulic diameter of the measured pipeline |
| $q_m$  | average flow rate in the pipeline under test ($\text{m}^3/\text{s}$) |
| $\lambda$ | resistance coefficient along the path |
| $h_f$  | head loss                              |
| $z_1$  | position potential energy of a unit mass of fluid on cross Section 1 |
| $z_2$  | position potential energy of a unit mass of fluid on cross Section 2 |
| $d$    | equivalent diameter of the pipeline (mm) |
| $A$    | cross-sectional area of the flow channel in the pipe ($\text{mm}^2$) |
| $\rho$ | air density ($\text{kg/m}^3$)          |
| $k$    | parameter that changes with the air flow state and includes the resistance coefficient along the path $\lambda$ |
| $U_s$  | local speed of sound ($\text{m/s}$)    |
| $Re$   | Reynolds number                        |

References

1. Mondal, N.; Saha, B.K.; Saha, R.; Sanyal, D. High-thrust aerostatic bearing design through transient perturbation modelling with numerical validation. *J. Dyn. Syst. Meas. Control* 2017, 140, 041012. [CrossRef]
2. Miyatake, M.; Yoshimoto, S. Numerical investigation of static and dynamic characteristics of aerostatic thrust bearings with small feed holes. *Tribol. Int.* 2010, 43, 1353–1359. [CrossRef]
3. Minseok, S.; Siraj, A.; Jaehyeok, J.; Park, S.S. Effect of shape errors on the stability of externally pressurized air journal bearings using semi-implicit scheme. *Tribol. Int.* 2017, 115, 580–590. [CrossRef]
4. Belforte, G.; Raparelli, T.; Viktorov, V.; Trivella, A. Discharge coefficients of orifice-type restrictor for aerostatic bearings. *Tribol. Int.* 2007, 40, 512–521. [CrossRef]
5. Sharma, S.C.; Yadav, S.K. A comparative study of full and partial textured hybrid orifice compensated circular thrust pad bearing system. *Tribol. Int.* 2016, 95, 170–180. [CrossRef]
6. Mondal, N.; Saha, R.; Sanyal, D. Designing aerostatic bearing with counterbalancing gaps for lifting a heavy payload. *J. Tribol.* 2014, 136, 031701. [CrossRef]
7. Li, Y.J.; Zhou, M.; Shamoto, E. Simulation-based analysis of the gas bearing utilizing traveling waves. *Key Eng. Mater.* 2013, 579–580, 192–196. [CrossRef]
8. Nguyen, A.V.H.; Dang, T.N. An approach to improve on stiffness for aerostatics bearing. *Appl. Mech. Mater.* 2019, 889, 448–454. [CrossRef]
9. Feng, K.; Zhao, X.Y.; Huo, C.J.; Zhang, Z. Analysis of novel hybrid bump-metal mesh foil bearings. *Tribol. Int.* 2016, 103, 529–539. [CrossRef]
10. Zhang, H.B.; Zhou, L.; Yao, Y.X.; Qin, D.L. Modeling and simulation of an externally pressurized spherical gas bearing. *Appl. Mech. Mater.* 2008, 98, 151–165. [CrossRef]
11. Eleshaky, M.E. CFD investigation of pressure depressions in aerostatic circular thrust bearings. *Tribol. Int.* 2009, 42, 1108–1117. [CrossRef]
12. Theisen, L.R.S.; Niemann, H.H.; Santos, I.F.; Galeazzi, R.; Blanke, M. Modelling and identification for control of gas bearings. *Mech. Syst. Signal Process.* 2016, 70–71, 1150–1170. [CrossRef]
13. Theisen, L.R.S.; Niemann, H.H.; Galeazzi, R.; Santos, I.F. Enhancing damping of gas bearings using linear parameter-varying control. *J. Sound. Vib.* 2017, 395, 48–64. [CrossRef]
14. Renn, J.C.; Hsiao, C.H. Experimental and CFD study on the mass flow-rate characteristic of gas through orifice-type restrictor in aerostatic bearings. *Tribol. Int.* 2004, 37, 309–315. [CrossRef]
15. Al-Bender, F. On the modelling of the dynamic characteristics of aerostatic bearing films: From stability analysis to active compensation. *Precis. Eng.* 2009, 33, 117–126. [CrossRef]
16. Kim, D.; Chesler, N.C.; Beebe, D.J. A method for dynamic system characterization using hydraulic series resistance. *Lab Chip* 2006, 6, 639–644. [CrossRef]
17. Li, J.F. *Theory Research and Experimental Verification on Gas-Impedance Model of Aerostatic Restricter System* [D]; China Jiliang University: Hangzhou, China, 2014. (In Chinese)

18. Chen, X.; Chen, H.; Zhu, J.; Jiang, W. Vortex suppression and nano-vibration reduction of aerostatic bearings by arrayed microhole restrictors. *J. Vib. Control* 2017, 23, 842–852. [CrossRef]

19. Belforte, G.; Colombo, F.; Raparelli, T.; Trivella, A.; Viktorov, V. Experimental analysis of air pads with micro holes. *Tribol. Trans.* 2013, 56, 169–177. [CrossRef]

20. Jia, C.H.; Pang, H.J.; Ma, W.S.; Qiu, M. Analysis of dynamic characteristics and stability prediction of gas bearings. *Ind. Lubr. Tribol.* 2017, 69, 123–130. [CrossRef]

21. Jin, Z.C.; Lin, B.; Zhang, X.F.; Jing, H.L.; Shuang, Q. Research on the influence of different surface machining states on the inclination of the porous gas bearing. *Mater. Sci. Forum.* 2014, 770, 244–247. [CrossRef]

22. Tomohiko, I.; Naoyuki, A.; Toshihiko, A.; Takenori, N.; Iwao, K.; Tadashi, M. Development of externally pressurized small-size conical-shaped gas bearings for micro rotary machines. *Precis. Eng.* 2014, 38, 15–22. [CrossRef]

23. Renn, J.C.; Chiou, Y.M. *Active Closed-Loop Gap Control for Aerostatic Bearing*; Springer International Publishing: Berlin/Heidelberg, Germany, 2016.

24. Kim, M.; Lee, S.M.; Lee, D.W.; Simon, P.; Seonghwan, K. Tribological effects of a rough surface bearing using an average flow analysis with a contact model of asperities. *Int. J. Precis. Eng. Manuf.* 2017, 18, 99–107. [CrossRef]