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Regression Filtration with Resetting to Provide Exponential Convergence of MRAC for Plants with Jump Change of Unknown Parameters

Glushchenko Anton, Member, IEEE, Petrov Vladislav, and Lastochkin Konstantin

Abstract—This paper proposes a new method to provide the exponential convergence of both the parameter and tracking errors of the composite adaptive control system without the regressor persistent excitation (PE) requirement. Instead, the derived composite adaptation law requires the regressor to be finitely exciting (FE) to guarantee the above-mentioned properties. Unlike known solutions, not only does it relax the PE requirement, but also it functions under the condition of a jump change of the plant uncertainty parameters. To derive such an adaptation law, an integral filter of regressor with forgetting and resetting is proposed. It provides the required properties of the control system, and its output signal is bounded even when its input is subjected to noise and disturbances. A rigorous analytical proof of all mentioned properties of the developed adaptation law is presented. Such law is compared with the known composite ones, which also relax the PE requirement. The wing-rock problem is used for the mathematical modeling. The obtained results fully support the theoretical analysis and demonstrate the advantages of the proposed method.

Index Terms—Composite MRAC, exponential convergence, finite excitation, stability analysis, wing rock.

I. INTRODUCTION

Several groups of adaptive observation and control methods have been developed specifically to control the dynamic plants with significant parameter uncertainty [1]. The first one includes adaptive observers of the plant states and schemes to identify the parameters of the plant or the disturbance (Model Reference Adaptive Systems – MRAS) [1-3]. In turn, the second group consists of methods of direct, indirect, and composite Model Reference Adaptive Control (MRAC) [4,5]. Both groups require the estimates of the unknown parameters of the plant uncertainty to be obtained. They are used to estimate the plant states and values of the parameters of the system with uncertainty (MRAS) or compensate for the influence of the uncertainty on the control quality (MRAC). To achieve this, the parameter uncertainty of the plant is, first of all, expressed from the plant equations in the form, usually, of linear regression. Then an algorithm to identify such regression unknown parameters is derived. It allows achieving the objective of MRAC or MRAS [6].

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A. I. Glushchenko is with V.A. Trapeznikov Institute of Control Sciences RAS, Moscow, Russia (phone: +79102266946; e-mail: strondutt@mail.ru).

V. A. Petrov is with Stary Oskol technological institute (branch) NUST “MiSiS”, Stary Oskol, Russia (e-mail: 79040882508@ya.ru).

K. A. Lastochkin is with V.A. Trapeznikov Institute of Control Sciences RAS, Moscow, Russia (e-mail: lastconst@yandex.ru).

A well-known and deeply investigated problem of the gradient-based identification is that the estimates converge exponentially to the ground-truth values only when the requirement of the regressor persistent excitation (PE) is met [6-8]. Generally speaking, the parameter convergence is a very advantageous property considering MRAC and MRAS [8], because it automatically guarantees exponential convergence of the tracking error (between the outputs of the observer and the plant for MRAS and the output of the plant itself and the one of the reference model for MRAC) [6]. It also provides the robustness of the estimation law. If there is no parameter convergence, such a law requires the application of the robust modifications [6,9]. It was proved in [10] that the PE requirement is satisfied if the number of the spectral lines in the setpoint signal coincides with the number of the regression unknown parameters. For many practical applications, the fulfillment of this condition (as a result of the reference signal modification) could be inconsistent with the initial control objective, and also leads to increased power consumption and wear of the actuators. So, the aim to provide exponential parameter convergence under conditions, which are weaker than PE, has become of high interest in recent years. We recommend [11] as a detailed review of most of the known solutions to the problem under consideration. While only the main methods to relax PE in MRAC are considered in the following analysis.

As for the conventional adaptation law, the PE requirement to provide the exponential convergence of the parameter error is caused by a fundamental reason. It is the minimization of the objective function of the instantaneous (proportional) tracking error. In case the PE condition is not met, such function may have a minimum at points, which do not coincide with the one corresponding to the regression ideal parameters [12]. Therefore, the main concept of most of the known methods of PE requirement relaxation is the transformation of the problem from the optimization of the proportional objective function to the proportional-integral one. It has a single minimum at the point corresponding to the ideal parameters even in the finite excitation (FE) case [12].

Considering MRAC, this concept can be implemented using the ideas of the composite adaptation method (CMRAC) [5]. According to it, the algorithm of the unknown parameters identification is the sum of two laws: 1) to minimize the tracking error as the difference between the outputs of the plant and the reference model, 2) to estimate the plant uncertainty. Further modifications of CMRAC are aimed at optimization of the integral error of the uncertainty identification. For this purpose, BackGround [13-15], Concurrent Learning (CL) [11,16,17], and PI adaptation law [18] have been proposed to obtain an integral uncertainty identification error and thereby relax the PE requirement to the finite (FE) or initial (IE) excitation ones.

To obtain the integral error, it is proposed [13-17] to save the data on the uncertainty in the DataStack. For the same purpose, it is proposed to use an integrator in the memory regression extension (MRE) procedure [11,19] instead of the Kreisselmeier filter [19] to
obtain the PI law of adaptation [18]. The main common problems of these two methods are, firstly, unbounded growth of the information matrix (integral regressor) in the case of noise and disturbances or when the PE requirement is met. And, secondly, inaccurate identification of the piecewise-constant unknown uncertainty parameters. To solve the first problem, it is proposed in [20] to use, considering MRE, not a purely integral filter, but the one with exponential forgetting. It always allows, contrary to [13-18], to obtain a bounded regressor. But this method, as well as [13-18], is not able to identify the piecewise-constant uncertainty parameters correctly. Further this problem of methods [13-18, 20] is considered in detail.

Inaccurate identification of the piecewise-constant parameters is caused by the fact that, in this case the integral objective function includes information not about one uncertainty (regression), but about some superposition of several of them. Consequently, the minimum of such objective function is at the point, which is the averaged value of the ideal parameters of all accumulated regressions [12]. Such superposition occurs because the methods [13-18, 20] do not have an algorithm to forget completely the already used and outdated data about uncertainty. As a solution to this problem, let the methods be considered, which relax the PE requirement and simultaneously provide some algorithm to forget the outdated data.

First of all, it is the study [21], in which it has been proposed, following some production rule, to switch from integral filtering [18] to aperiodic Kreisselmeier’s one [19]. The disadvantage of this method is that the outdated data will start to be forgotten only when a special condition on the product of the parameter error and the value of the unknown parameters change holds. This significantly reduces the applicability domain of such method. Secondly, it is [22], in which it has been proposed to use a variable forgetting factor in the Kreisselmeier filter [19] and a regression to identify the plant parameters, which is obtained when some filtered regressor metric has maximum value. Such a metric is calculated on the finite excitation intervals. Since this approach does not provide strict guarantees on the improvement of the maximum value of the chosen metric, then the update of regressions, which are used for the parameter estimation, may not happen. So this method provides exponential parameter error convergence if the plant unknown parameters are constants, but not piecewise-constant ones. Thus, as far as MRAC schemes are concerned, none of the above-mentioned ones guarantees the exponential convergence of the estimation process of the plant piecewise-constant parameters without PE.

Therefore, in this research, a new method is proposed, which guarantees exponential convergence of the estimation process of the plant unknown piecewise-constant parameters in case of the finite excitation (FE) of the regressor. It is proposed to reset the filter output [20] to zero instantaneously at time points corresponding to a change of the value of a piecewise-constant reference signal \( r \). The need for instantaneous, rather than aperiodic, resetting of the filter output is motivated by studies [21, 22], which results make it clear that, when aperiodic forgetting is applied, often the filter output does not have enough time to reach zero (or a low value). This leads to inaccurate identification of the plant piecewise-constant parameters.

A similar idea has been applied to implement Concurrent Learning Model Predictive Control system [23]. In this approach when some kind of plant identification error metric is low, the model is known with high accuracy and MPC is applied. If such metric becomes high, i.e. the ideal parameters of the plant have changed and need to be identified again, the data stack of CL is cleared, and CMRAC is used again. But this approach is not able to track piecewise-constant unknown parameters without switching to MPC stage. The algorithm may stick at CMRAC stage in case the ideal values of the unknown parameters change in the course of it. Therefore, unlike [23], we propose to apply the resetting procedure, using the reference signal \( r \) as a signal to reset the filter, to relax the PE requirement for MRAC.

The main contributions of this paper are: 1) the application of a reset procedure to update dynamically the output of an integral filter with exponential forgetting [20]; 2) the PE requirement is relaxed to the FE one to provide the exponential convergence of the estimates of the plant unknown piecewise-constant parameters for MRAC.

The rest of the paper is organized as follows. Section II presents the basic definitions and notations used in the paper; Section III gives a generalized formal statement of the problem; Section IV presents the procedure of filtering with resetting and the final adaptation law; in Section V the system stability is analyzed and the exponential convergence of the MRAC control scheme with the developed adaptation law is proved; Section VI compares it with the known methods; Section VII presents the results of numerical simulation.

II. PRELIMINARIES

The following notations will be used throughout the paper: \( L \), \( \Lambda \) is the space of the essentially bounded functions, \( \lambda_{\text{min}}(\cdot) \) and \( \lambda_{\text{max}}(\cdot) \) are the minimal and maximal eigenvalues of a matrix, \( \text{vec}(\cdot) \) is the operation of a matrix vectorization, \( \|\| \) is the Euclidean norm of a vector, \( \|\|_{\infty} \) is the Frobenius norm of a matrix \( (\|\|_{\infty}=(\text{vec}(\cdot))^{\top}(\cdot)\|\|_{\text{F}}) \), \( f(t) \) is a function, which depends on time (the time argument is sometimes omitted for the sake of simplicity). Also, the following definitions will often be used throughout the paper:

Definition 1: A regressor \( q(t) \in \Lambda \) is finitely exciting (FE) over a time set \([t_s, t_T]\) if there exist \( t_s \geq t_0 \geq 0 \) such that

\[
\int_{t_s}^{t_T} q(t)\phi_i(t)\,dt \geq aI,
\]

where \( I \) is a unity matrix and \( a \) is a level of excitation.

Let \( \dot{x}(t) = f(x(t)) \) be a linear dynamic system, which is globally Lipschitz continuous. Its origin is stable. Then:

Definition 2: A system equilibrium point is globally exponentially stable \( (\in GES) \) if for \( \kappa > 0 \) and \( \rho > 0 \) there exists a time point \( T_s \), such that

\[
\|x(t)\| \leq \rho \|\phi(0)\| e^{-\kappa t} \text{ for } t \geq T_s \text{ and any initial state } x(0).
\]

Definition 3: The solution \( x(t) \) is exponentially ultimately bounded \( (\in EUB) \) with uniform ultimate bound \( R \) if for \( \kappa > 0, \rho > 0, \) and \( R > 0 \) there exists a time point \( T \), such that \( \|x(t)\| \leq \rho \|\phi(0)\| e^{-\kappa t} + R \text{ for } t \geq T \) and any initial state \( x(0) \).

III. PROBLEM STATEMENT

Let a problem of adaptive control of a class of linear time-invariant (LTI) plants be considered:

\[
\dot{x}(t) = Ax(t) + B(u(t) + \Delta(t)),
\]

where \( x \in \mathbb{R}^n \) is a state vector, \( u(t) \in \mathbb{R}^m \) is a control action, \( \Delta \in \mathbb{R}^{m \times n} \) is a parameter uncertainty, \( A \in \mathbb{R}^{n \times n} \) is a state matrix, and \( B \in \mathbb{R}^{m \times n} \) is an input matrix of full column rank. \( (A, B) \) is a controllable pair such that \( m \leq n \). Elements of \( A \) and \( B \) are known. State vector \( x \) is considered to be measurable.

The parameter uncertainty \( \Delta \) is linearly parametrized as follows:

\[
\Delta(t) = \Theta(t) \bar{\Phi}(x(t)),
\]

where \( \Theta(t) \in \mathbb{R}^{m \times n} \) is a vector of unknown parameters; \( \Phi(x(t)) \in \mathbb{R}^m \) is a bounded regressor vector.

Assumption 1. The ideal parameters of parameter uncertainty \( \forall t \geq 0 \) can be written such as

\[
\hat{\Theta} = \Theta_0 + \sum_{j=1}^{\infty} h(t - t_j),
\]

where \( h(t) \) is bounded piecewise-constant.
where $\Phi(\mathbf{x}) \in \mathbb{R}^{m \times n}$ is the value of the unknown parameter change, $b_l(t)$ is the step function, $t$ is the unknown time point when the unknown parameters change, a.e. is the abbreviation of almost everywhere.

Thus, in this paper, the problem of the adaptive control of the LTI plant (2) with the piecewise-constant parameters of uncertainty $\Delta$ is considered. The reference model for (2) is written as:

$$\dot{x}_r(t) = A_{r\Delta}x_r(t) + B_{r\Delta}y_r(t),$$

where $x_\Delta \in \mathbb{R}^n$ is the reference model state vector, $r \in \mathbb{R}^m$ is the reference values of (9) is piecewise-constant $r(t) = \sum r_k h(t - t_k)$.

$$\dot{x}_r(t) = (A + BK_x)x_r(t) + BK_f r(t) + B(\hat{\Theta}^T + \sum_{i=1}^{m} k_i \Phi(x(t)))$$

where $\hat{\Theta} \in \mathbb{R}^{m \times n}$ is the parameter error between the ideal values of the parameter uncertainty and their estimates.

**Assumption 2.** There exist such ideal values of the parameters of the baseline controller $K_x \in \mathbb{R}^{m \times n}$ and $K_f \in \mathbb{R}^{m \times n}$ so as the following equalities hold true:

$$A + BK_x = A_{r\Delta}; BK_f = B_{r\Delta}.$$

Then the closed-loop equation (7) is written as

$$\dot{x}_r(t) = A_{r\Delta}x_r(t) + B_{r\Delta}y_r(t) + B(\hat{\Theta}^T + \sum_{i=1}^{m} k_i \Phi(x(t))).$$

**Assumption 3.** The reference $r$ of (9) is piecewise-constant $r(t) = \sum r_k h(t - t_k)$.

where $\Delta \in \mathbb{R}^n$ is the value of the reference change, $t_0 \geq 0$ is a known time point of the reference change.

**Assumption 4.** If $t_0 \notin \mathbb{R}$ then $\Phi(\mathbf{x}) \in \mathbb{E}^n$ for $[t_0; t_0 + T]$.

**Remark 1.** Assumption 3 is met for most real plants, which function not in the inner loop of a cascade control system. The meaning of Assumption 4 is that the change of the setpoint in accordance with (10) results in the finite excitation of the regressor $\Phi(x)$. From the practical point of view, the value of $t_0$ can be obtained online with the help of the implication $r(t) \neq r(t - T_0) \Leftrightarrow t = t_k$, where $T_0 > 0$ and is sufficiently close to zero.

The error equation between (9) and (5) is written as

$$\dot{e}_{r\Delta}(t) = A_{r\Delta}e_{r\Delta}(t) + B\hat{\Theta}^T(t)\Phi(x(t)).$$

Let the notion of the augmented error $\xi = [\dot{e}_{r\Delta} \text{ vec}(\hat{\Theta}^T)]^T$ be introduced into (11). Then the adaptive control goal is defined.

**Goal.** Let the regressor be finite exciting $\Phi(\mathbf{x}) \in \mathbb{E}^n$. The task is to derive a law to adjust the parameters of the adaptive controller part $u_{\Delta}$. Such an adaptation law is to provide the global exponential stability of the augmented error $(\xi \in \mathbb{E}^n \in \mathbb{GES})$ under the condition that the uncertainty parameters are piecewise-constant (4).

**IV. MAIN RESULT**

In this section, adaptation law is to be derived to meet stated Goal.

**A. Plant Uncertainty Calculation**

First of all, the filtered value of uncertainty $\Delta$ is to be obtained from the error equation (11). For this purpose, let all dynamic quantities of (11) be passed through the aperiodic links:

$$\hat{\mu}_f(t) = -k\mu_f(t) + \hat{e}_{r\Delta}(t), \quad \hat{\mu}_f(0) = 0,$$

$$\hat{\varepsilon}_f(t) = -k\varepsilon_f(t) + \hat{e}_{r\Delta}(t), \quad \hat{\varepsilon}_f(0) = 0,$$

$$\hat{u}_{adj}(t) = -k_u\hat{u}_{adj}(t) + \hat{u}_{adj}(0), \quad \hat{u}_{adj}(0) = 0,$$

$$\Phi_f(x(t)) = -k\Phi_f(x(t)) + \Phi_f(x(t)), \quad \Phi_f(x(0)) = 0,$$

where $k > 0$ is the filter time constant.

The quantities $e_{r\Delta}, \Phi(\mathbf{x})$ and $u_{\Delta}$ are found as solutions of the differential equations (12) and (13). As $e_{r\Delta}$ is unknown, let the following lemma be introduced to found $\mu_f$:

**Lemma 1.** The filtered value $\hat{\mu}_f(t)$ of $\mu_f$ is found as:

$$\mu_f(t) = e^{-\gamma t}(\mu_f(0) - \hat{e}_{r\Delta}(0) + ke_f(0)) + e_{r\Delta}(t) - ke_f(t)$$

For proof please refer to the Supplementary Material [24].

Considering (12)-(13), the error equation (11) is rewritten as:

$$\dot{\Delta}_f(t) = B^T(\hat{\Theta}^T + \sum_{i=1}^{m} k_i \Phi(x(t)))u_{\Delta}(t), \quad \forall t \geq 0.$$
C. Resetting Filtration and Adaptation Law Derivation

The scalar regressor \( \omega \) obtained by DREM is filtered to achieve the stated goal. For this purpose, the integral filter with exponential forgetting from [20] will be used:

\[
\dot{v}_j(t) = \exp\left(-\frac{t}{\sigma^2}\right) v_j(t), \quad v_j(t_f) = 0, \quad \text{(23)}
\]

where \( \sigma > 0 \) is the adjustable parameter, \( v \) and \( \dot{v}_j \) are the filter input and output respectively.

**Remark 2.** According to [20], the filtration (23) allows one to obtain a nonsingular regressor and thereby, as proved in [20], relax the PE requirement for exponential convergence of the parameter error. In contrast to other methods, e.g. the integral filter [18], the filter (23) guarantees a bounded \( v \) in case of noise and disturbances.

The values of \( v \) in (23) are to be reset to zero when \( r \) value is changed. This is implemented as

\[
\dot{v}_j(t) = \exp\left(-\frac{t}{\sigma^2}\right) v_j(t), \quad v_j(t_f) = 0, \quad \text{(24)}
\]

The MRE [11] is applied to the scalar regression (20). For this purpose, the equation (20) is multiplied by the regressor \( \omega(t) \):

\[
\omega(t) Y(t) = \omega^2(t) \Theta, \quad \text{(25)}
\]

The extended regressor \( \Omega \) and function \( \omega Y \) are filtered with (24):

\[
\Omega(t) = 0, \quad \text{if } t = t',
\]

\[
\frac{\partial}{\partial t} \int_{t_k}^{t} \sigma dt, \quad \text{otherwise};
\]

\[
Y(t) = 0, \quad \text{if } t = t',
\]

\[
\frac{\partial}{\partial t} \int_{t_k}^{t} \sigma dt, \quad \text{otherwise},
\]

where \( \forall \in R^{m \times m}, \Omega \in R \).

According to Assumption 3, if \( t = t_k \), then \( \Phi(x) \in \Omega \) for \( t_k \), \( t + T \). According to Propositions 1 and 2, if \( \Phi(x) \in \Omega \), then \( \omega Y \). So, taking into account the definition of \( t_k \) and \( t' \) in (24), \( \omega Y \) over the interval \( [t_k; t'] \), \( t \geq t' \). So, the following proposition can be introduced and proved.

**Proposition 3.** If \( \omega Y \) over the interval \( [t_k; t'] \), then

1) \( \forall t \geq t' \), \( \Omega(t) \subseteq L_\infty, \Omega(t) \geq 0 \);
2) \( \forall t \geq t' \), \( \Omega(t) \geq 0 \), \( \Omega_{UB} \leq \Omega(t) \leq \Omega_{LB} \).

For proof please refer to the Supplementary Material [24].

Taking into consideration the filtration (26), the regression equation (25) is rewritten as follows:

\[
Y(t) = \Omega(t) \Theta. \quad \text{(27)}
\]

According to Proposition 3 and in contrast to \( \omega \), the regressor \( \Omega \), which is obtained by filtering with resetting (26), does not vanish. It allows us to derive the law of adjustment of the adaptive controller parameters \( u_{ad} \) according to the composite adaptation method [5]:

\[
\dot{\Theta}(t) = \Gamma_\xi \Phi(x(t)) e_{ref}(t) PB + \Gamma_\omega(t) \Omega(t) (Y(t) - \Omega(t) \Theta(t)), \quad \text{(28)}
\]

where \( \Gamma_\xi \Phi(x(t)) e_{ref}(t) PB + \Gamma_\omega(t) \Omega(t) (Y(t) - \Omega(t) \Theta(t)), \quad \text{and } \Gamma_\omega(t) \Omega(t) \Theta(t) = 0 \),

\[\Gamma_\xi \in R^{n \times p} \text{ and } \Gamma_\omega \in R, \lambda_\omega > 0, \lambda_\xi > 0 \text{ is the forgetting factor.}
\]

The properties of the regressor \( \Omega \), which are proved in Proposition 3, give the opportunity to use adjustable adaptation gain \( \Gamma_\xi \) in (28).

**Proposition 4.** If \( \omega Y \) over the interval \( [t_k; t'] \), then \( \forall t \geq t' \) the adaptation gain \( \Gamma_\xi \) is bounded so that the following inequality holds true:

\[
\Gamma_{min} \leq \Gamma_\xi \leq \Gamma_{max}.
\]

For proof please refer to the Supplementary Material [24].

**Remark 3.** According to [20, 28, 29], for the boundedness of \( \Gamma_\xi \), it is sufficient that \( \Omega \) is bounded by its lower \( \Omega_{LB} \) and upper \( \Omega_{UB} \) bounds, both of which are above zero, and does not convergence to zero \( \Omega \rightarrow 0 \). Therefore, the proof of Proposition 4 [24] is given in brief version. We recommend to refer to the Supplementary Material [24] for more details.

In (28) the first summation is to provide the convergence of the tracking error \( e_{ref} \), and the second one – of the integral error of the uncertainty identification, and hence – the parameter error \( \Theta \).

**Remark 4.** In the equations (28) of the adaptation loop, there is a point of discontinuity due to filtering (26). However, the time moment \( t^* \), at which the discontinuity occurs, is determined by the external signal \( r \) and does not depend on the internal signals \( x \) and \( e_{ref} \) of the closed-loop system (11). This completely excludes the possibility of chattering in (26), (28) and allows analyzing the stability of the closed-loop system (11), assuming the time point \( t^* \) is initial one.

V. STABILITY ANALYSIS

The scope of this section is to analyze the stability of the system (11) when the law (28) is applied to adjust \( u_{ad} \). In this case, the behavior of the error \( \xi \) is completely defined by the mutual relation of \( t \) and the interval \( [t_k^*; t'_k] \). Three situations are possible:

1) \( t_k^* \leq t'_k \) – the unknown parameters change has happened before the change of the setpoint value \( r \); 2) \( t_k^* \leq t'_k \) – the unknown parameters change has happened when the regressor is finitely exciting because of the setpoint value change; 3) \( t_k^* \geq t'_k \) – the unknown parameters change has happened when the finite excitement of the regressor, caused by the setpoint value change, is over. The boundedness of \( \Gamma_\xi \) for all three cases allows us to introduce the following Lyapunov function to analyze the stability of the system (11):

\[
V(\xi) = e_{ref}^2 / \Omega_{LB} + \Theta(\Gamma_\xi \Gamma_\xi^T \Theta), \quad \text{(29)}
\]

The derivative of (29) with respect to the equations of the system (11) and the adaptive loop (28) is written as:

\[
\dot{V}(\xi) = -e_{ref}^2 \Omega_{UB} - 2 \Theta(\Gamma_\xi \Gamma_\xi^T \Omega(\Gamma_\xi \Theta)), \quad \text{(30)}
\]

For further analysis, let each case be considered separately.

**Theorem 1:** (Case 1) If \( t_k^* \leq t'_k \) and \( \Phi(x) \in \Omega \), then the augmented error \( \xi \) is exponentially stable (\( \xi \in \Omega \)GES).

**Theorem 2:** (Case 2) If \( t_k^* \leq t'_k \) and \( \Phi(x) \in \Omega \), then the augmented error \( \xi \) is exponentially ultimately bounded (\( \xi \in \Omega \)EUB) with ultimate bound \( R \).

**Theorem 3:** (Case 3) If \( t_k^* \geq t'_k \) and \( \Phi(x) \in \Omega \), then the augmented error \( \xi \) is exponentially ultimately bounded (\( \xi \in \Omega \)EUB) with ultimate bound \( R \).

For proof Theorems 1-3 please refer to the Supplementary Material [24].

It follows from the proofs that if the unknown parameters \( \Theta \) change their values: 1) before the finite excitation, caused by the change of the setpoint value \( r \), then \( \xi \in \Omega \)GES; 2) during or after the finite excitation time range, then \( \xi \in \Omega \)EUB with the ultimate bound \( R \) or \( R_1 \) [24].

For cases 2 and 3 let the ways to minimize the ultimate bound of the tracking and parameter errors be analyzed separately for each of them. For this purpose, taking into account the definitions of \( \lambda_{min}, \lambda_{max}, \ldots \) [24] and assuming \( \lambda_{max}(\Gamma_\xi) = \lambda_{min}(\Gamma_\xi) \), the equations of bounds of both errors \( e_{ref} \) and \( \Theta \) are obtained from proofs of Theorems 2 and 3:
the application of the DREM procedure, it is proposed (28) to parameters to their ideal values when such values have changed convergence of the estimates of the piecewise-constant uncertainty concerned, the obtained system allows one to provide the exponential which satisfies the requirement stated in Assumption 3, are

and λ2 (by the change of λ and λ2) results in the decrease of the convergence speed to the ideal parameters of the uncertainty in case 1. So, in practice, λmax(Γ1), λ1 and λ2 must be chosen as a compromise between the ultimate bounds R, R1 and the quality of the transient process of θ and uad.

Thus, according to the conducted stability analysis, the obtained adaptation law (28) provides the properties required by Goal when the uncertainty parameters change before the setpoint r change.

VI. COMPARISON WITH KNOWN ADAPTATION LAWS

The main difference of the developed system from the majority of the known ones is the requirement (10) of a certain type of the reference signal r, which is necessary to guarantee the stable implementation of the resetting procedure for the filtering (26). This fact does not allow one to apply the obtained system to control systems, in which r is the output of the command pre-filter (power-up sensor) or the outer loop controller. But, as far as the plants, for which r satisfies the requirement stated in Assumption 3, are concerned, the obtained system allows one to provide the exponential convergence of the estimates of the piecewise-constant uncertainty parameters to their ideal values when such values have changed before the change of the setpoint r and the regressor is FE.

In contrast to existing CMRAC schemes, in this paper, owing to the application of the DREM procedure, it is proposed (28) to augment the basic adaptation law \( \Phi(x)e_{ref}^2PB \) with not a matrix law, which ensures convergence of the integral error of the uncertainty identification, but with a scalar one to provide monotonic estimation of each \( \dot{\theta} \) element. So, being used together with \( \Phi(x)e_{ref}^2PB \), it does not cause additional fluctuations of the transient curves of \( \dot{\theta} \) values.

The properties of the regressor \( \Omega \), which is obtained using the filtration with resetting (26), also make it possible, unlike in other CMRAC schemes, to use a dynamically adjustable \( \Gamma \) gain in (28) for the summand, which is responsible for the uncertainty identification.

Next, we briefly compare the developed law (28) with some previously proposed in the literature.

A. Comparison with Basic Robust Adaptation Law

In practice, the conventional adaptation law \( \Phi(x)e_{ref}^2PB \) is always augmented with the robust modifications. For example, if the \( \sigma \)-modification is used, then it is written as [6,9]:

\[
\hat{\theta}(t) = \Gamma_1 \Phi(x(t))e_{ref}^2(t)PB - \sigma \hat{\theta}(t)
\]  

The robust adaptation law (32) guarantees \( \xi \)EUEB for the plant (2) in the presence of the bounded disturbances. As for the proposed adaptation law (28), it is proved in the theorems 1-3 that \( \xi \)EUBS and \( \xi \)EUEB when there are no disturbances. This is both necessary and sufficient to provide robustness (\( \xi \)EUEB) of (28) against bounded disturbances [6,8,9]. So, in contrast to the conventional adaptation law \( \Phi(x)e_{ref}^2PB \), not only does the proposed (28) guarantee \( \xi \)EUBS in case 1, but also it does not require additional robust modifications.

B. Comparison with Switched MRAC

The adaptation loop with switching described in [30] also uses filtering (12), (13), (18) to obtain the extended linear regression equation (19). Using (12), (13), and (18), it is proposed in [30] to apply the following adaptation law:

\[
\dot{\hat{\theta}}(t) = \Gamma_1(T_e + T_i + T_d + T_w); \quad T_e = k_e(y(i) - \phi(\hat{\theta})(i)) \frac{\dot{\theta}}{(33) T_w = k_w(y(t) - \phi_w(\hat{\theta}))(41)} \]

Here \( k_e > 0, k_d > 0, k_w > 0, y_i \in R^{p \times m} \) and \( \phi_i, \phi_w \in R^{p \times p} \) are defined as:

\[
y_i(t) = \begin{cases} 0, \text{ if } \text{det} \left( \Phi(\theta_x)(\tau)^{n} \right) = 0, & \text{for } y(T) \text{ otherwise,} \\ \phi \left(41\right) \end{cases}
\]

where \( T \) is a time point, when the determinant in (34) becomes positive, \( \phi(\hat{\theta}) \) is a filtered regressor of full rank.

According to the proof in [30], the law (34) guarantees \( \xi \)EUBS when \( \Phi(x) \in \mathbb{E} \). As \( \Phi(\theta_x) \) is the positive semi-definite matrix, then the switching in (34) is possible only once. Consequently, in contrast to the proposed law (28), the adaptation law (34) guarantees \( \xi \)EUBS only when the uncertainty parameters are constant.

What the proposed adaptive control system and Switched MRAC have in common is the very idea of using switching. However, in Switched MRAC switching is used to detect a full-rank regressor, while in this study it is used to reset the filter output and completely forget the outdated data on the uncertainty and its parameters.

C. Comparison with FE CMRAC

The FE CMRAC adaptation loop [22] also uses filtering, which is similar to (12), (13), to obtain the numerical value of the filtered uncertainty (17). It also uses the variable coefficient \( l(t) \) of the Kreisselmeier filter (18):

\[
l(t) = l_w + (l_d - l_w) tanh \left( \frac{\Phi(x)(x)}{l} \right)
\]

where \( l_w \) and \( l_d \) are minimum and maximum values of the parameter \( l, \theta \), \( \theta = \text{tan}h(x) \) is the hyperbolic tangent function. The law (35) to adjust \( l(t) \) allows one to set the higher weight to the uncertainty data, which are obtained when the regressor \( \Phi(x) \) changes rapidly.

The main point of the novelty of the study [22] is the developed algorithm to obtain the full-rank regressor:

\[
t_e = \max \left\{ \arg \max \left\{ \lambda_{\min}(\phi(x)) \right\}, \frac{\phi_u(t) - \phi_u(t - \lambda_{\min}(\phi(x)))}{\phi_u(t) - \phi_u(t - \lambda_{\min}(\phi(x)))} \right\}
\]

It allows one to choose the adaptation law as:

\[
\dot{\hat{\theta}}(t) = \Gamma_1 \Phi(x(t))e_{ref}^2(t)PB + \Gamma_2(y(t) - \phi_w(\hat{\theta})(i)) \frac{\dot{\theta}}{(37)}
\]

In contrast to switching (34) in Switched MRAC, algorithm (36)
allows obtaining a full-rank regressor every time when FE exists. However, the values of $\phi_t$ and $y_t$ will be updated according to (36) only if the minimum eigenvalue of the matrix $\phi$ over the new finite-excitation time range is higher than it has been over the previous one.

In addition, in contrast to the filtering with the resetting procedure (24) used in this study, the Kreisselmeier filtration (18) with a variable parameter (35) does not allow one to forget completely the outdated information about $\phi$ and $y$. Therefore, the adaptation law (37) is applicable to piecewise-constant uncertainty parameters only if: 1) a new finite excitation leads to an improvement of the minimum excitation time range is higher than it has been over the previous one.

for $1 \leq j \leq 2$.

$\mathbf{A}_j = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$, $\mathbf{B}_j = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The setpoint signal was implemented as a square wave (10):

\[ r(t) = \begin{cases} 1, & 0 < t \leq 8, \\ 0, & 8 < t \leq 16, \\ 1, & 16 < t \leq 24. \end{cases} \]

To demonstrate the performance of the system in all three cases considered in the stability analysis, at $t_1 = 4$ seconds the parameter $\theta_0$ was changed by the value of $\theta_1$, and at $t_2 = 17$ the sum of $\theta_0 + \theta_1$ was changed by the value of $\theta_0$. The parameters of the baseline controller $u_{base}$ were calculated according to the method of the LQ synthesis by optimization of the integral-quadratic quality criterion:

\[ J = \int_0^\infty \left( x^T Q L Q x + u_{base}^T R L Q u_{base} \right) dt, \]

where $Q_{LQ} = \text{diag}[2800.1]$, $R_{LQ} = 100$. The values of other constants of the adaptive control system are shown in Table 1.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $\Theta (0)$ | $\theta_{base}$ | $G_1(0)$ | 1 |
| $Q$ | $\text{diag}[100,10]$ | $\lambda_2$ | 450 |
| $PB$ | [9.45,4.43] | $L$ | 10 |
| $\Gamma_1$ | $500_{LQ}$ | $K$ | 10 |
| $\lambda_1$ | $1100_{LQ}$ | $\Sigma$ | 5 |

The aim of the experiment was to compare the developed system (CMRAC) with the solution based on the conventional adaptation law $\Gamma_1\phi(x)\Theta PB$ (MRAC) when the same value of the adaptation gain $\Gamma_1 = 500_{LQ}$ was used in both loops. Fig.1 shows the change of the reset time $\tau_j^*$ value in the course of the experiment.

\[ \Theta_0 = \begin{bmatrix} 3.63 & -8.58 & 20.2 & -21.9 & -51.88 \end{bmatrix}^T; \]

\[ \theta_0 = \begin{bmatrix} -22.22 & 23.74 & -82.66 & 31.45 & 73.33 \end{bmatrix}^T; \]

\[ \theta_2 = -\theta_1; \quad j = \{1; 2\}. \]

It follows from Fig. 1 that the reset of the filtering procedure (26) was made strictly at the moments of the setpoint $r$ change.

Fig. 2 shows the transients of the regressor $\Omega$ at time intervals that correspond to a change either of $r$ or the uncertainty parameters.

Considering the regressor, its transients in Fig. 2 proved the conclusions about its properties drawn in Proposition 3. The low value ($\approx 10^{-6}$) of $\Omega$ is explained by the multiplication of (20) by the adjoint matrix. The difference of about ($\approx 10^{4}$) between $\Omega$ values at different stages of the experiment verified the need to adjust $\Gamma_2$.

The transients of the control action $u$ (a), states $x$ (b) and the estimates of the unknown parameters (c) are shown in Fig. 3.

So, for the developed system, the oscillations of curves of the control action $u$ and the state vector $x$ had existed exactly till the moment when the ideal uncertainty parameters were identified. Using the developed loop (28), the parameter error was always bounded, and it was exponentially stable over the intervals $[0;4]$ and $[8;16]$. 

VII. NUMERICAL SIMULATION

The wing-rock phenomenon in the roll motion of slender delta wings has been chosen as the plant for the experiments.

\[ \dot{x}_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left( u(t) + \Theta^T \Phi \left( x(t) \right) \right). \]

\[ \dot{x}_2 = \begin{bmatrix} 1 \end{bmatrix} \left( u(t) + \Theta^T \Phi \left( x(t) \right) \right). \]

Fig. 1. Reset time $\tau_j^*$ curve in the course of the experiment.

$\dot{x} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left( u(t) + \Theta^T \Phi \left( x(t) \right) \right). \]
This verified the proof of Theorems 1-3 and the conclusions drawn in Remark 3. Figure 4 shows the transients for the state $x_1$ and the augmented tracking error $\xi$ obtained by application of the developed adaptation law (28), the classical law $\Gamma_1\Phi(x)e_{ref}PB$, and the composite laws (33), (37), and (39).

Table 2 contains the values of the laws (33), (37) and (39) parameters, which were used for the simulation. The values of all other parameters were set according to Table 1.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $k_1$     | 5     | $k_1$     | 1000  |
| $k_2$     | 9000  | $\Gamma_1$ for (62) | 10    |
| $k_w$     | 50000 | $\Gamma_1$ for (61) | 2500  |
| $l_w$     | 0.1   | $\Gamma_1$ for (63) | 2500  |
| $l_0$     | 10    | $\Gamma_1$ for (63) | 2500  |

As follows from the transients presented in Fig. 4, over the interval $[0; 4]$ all composite laws (28), (33), (37) and (39) ensured exponential convergence of the augmented error to zero. At the same time, when the uncertainty parameters had changed, only the developed adaptation law (28), on account of the previously accumulated data resetting, allowed keeping the property of exponential convergence of the augmented tracking error to zero after the setpoint value change. Here, it should be specially noted that, considered time $t \geq 17$, the exponential stability of the augmented error $\xi$ was provided by (33) owing to the fact that $y_{sw}$ stored the data about the true regression parameters $\Theta_0$, which were recorded to $y_{sw}$ over the interval $[0; 4]$ according to (34) at time point $T$. If the unknown parameters $\Theta$ took a value other than $\Theta_0$ over $[17;25]$, then the law (33) would not provide exponential stability to the error.

Also, the transients shown in Fig. 4 demonstrated the advantages of the proposed system of composite adaptive control over the known ones. Only the developed system allowed one to avoid superpositional mixing of the data of the regressions with different parameters and ensure exponential convergence of the augmented tracking error to zero each time after the reference $r$ value change (if the uncertainty parameters had changed their values before that).
mixing happened, was sufficiently worse than the one by the conventional MRAC. Under such conditions, the loop (28) deteriorated the quality of the transients insignificantly.

In general, the results of the experiments validated the analytically proved property of the developed system to guarantee exponential convergence of the parameter error when the FE requirement of the regressor was met and the uncertainty parameters changed before the change of the setpoint. Also, in comparison with the conventional law, it was possible to improve the quality of the transients of \( e_{\text{ref}} \) and \( \mu \) after the completion of the uncertainty parameters identification.

VIII. CONCLUSION AND FUTURE WORK

In this paper, in order to relax PE requirement for the MRAC scheme with the piecewise-constant uncertainty parameters of the plant, the method was proposed, which was based on the application of MRE procedure with integral filter with input forgetting and resetting. This filter made it possible to develop a CMRAC adaptation loop, which guaranteed exponential convergence of the parameter error to zero if FE requirement was met and the following conditions were satisfied: 1) the setpoint was a piecewise-constant signal; 2) a change of \( r \) caused the regressor FE; and 3) the unknown piecewise-constant parameters had already changed before the change of \( r \). The analytical stability analysis, as well as the conducted numerical experiments, demonstrated the main properties of the obtained system and verified the conclusions made. The proposed method differs from the known ones, which are also used to relax PE requirement for MRAC, by application of the integral filter with input forgetting (23), which is robust to disturbances, and the algorithm to reset its output (24). It relaxes PE requirement not only for the constant unknown uncertainty parameters, but also for the piecewise-constant ones under some weak additional assumptions.

In further research, we plan: 1) to improve the transient response of the obtained system over the intervals when the ideal uncertainty parameters have not been found yet; 2) to develop a robust resetting scheme for the filter (24) based on an algorithm to detect excitation, which is caused by changes of uncertainty parameters rather than the setpoint \( r \) (to extend the obtained results to Case 2 and 3).

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