New results on heavy hadron spectroscopy with NRQCD

NRQCD collaboration presented by C. T. H. Davies

Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK

We present results for the spectrum of \( \bar{b}b \) bound states in the quenched approximation for three different values of the lattice spacing. Results for spin-independent splittings are shown to have good scaling behaviour; spin-dependent splittings are more sensitive to discretisation effects. We discuss what needs to be done to match the experimental spectrum.

1. INTRODUCTION

Accurate calculations of the hadron spectrum in lattice QCD require control of systematic errors, and a major source of these errors arises from the use of a space-time lattice with finite lattice spacing. Recent understanding of these discretisation errors has meant that the ‘brute force’ approach of extrapolating to zero lattice spacing has been replaced by the requirement of improving the lattice action to the point where results at different values of \( a \) agree to within specified precision. This also allows us to use effective actions for QCD for which a continuum extrapolation cannot in any case be done, but which are nevertheless good for particular physics.

The approach of Non-relativistic QCD fits into this picture. The NRQCD action is well-suited to a description of non-relativistic quarks such as \( b \) quarks within bottomonium or \( b \)-light mesons. In fact we believe that it is the best approach for these systems. The operators of the action are classified by the powers of \( v^2/c^2 \) that they contain with coefficients that can be matched to full QCD in perturbation theory. The renormalisability of QCD is lost, however, so an explicit momentum cut-off (non-zero lattice spacing) is necessary for NRQCD and the coefficients will contain radiative corrections which diverge as powers of this cut-off, \( \Lambda/m_Q \), so \( \Lambda/m_Q \to \infty \) is not possible. Within a range of \( \Lambda \) around \( m_Q \) physical results independent of \( \Lambda \) are possible, however. Here we will discuss to what extent this situation is realised for current results.

1.1. Simulation Details

We use an evolution equation for quark propagators

\[
G_{t+1} = \left(1 - \frac{aH_0}{2n}\right)^n U_4 \left(1 - \frac{aH_0}{2n}\right)^n \times (1 - a\delta H) G_t. \tag{1}
\]

\( H_0 \) is the lowest order (in \( v^2/c^2 \)) term in the Hamiltonian, the kinetic energy operator:

\[
H_0 = \frac{\Delta^{(2)}}{2M_b^0}. \tag{2}
\]

The correction terms that we include in \( \delta H \) are \( \mathcal{O}(v^4/c^4) \). They comprise relativistic corrections to the spin-independent \( H_0 \) as well as the first spin-dependent terms that give rise to spin-splittings in the spectrum.

\[
\delta H = - c_1 \frac{(\Delta^{(2)})^2}{8(M_b^0)^2} + c_2 \frac{ig}{8(M_b^0)^2} (\Delta \cdot E - E \cdot \Delta) - c_3 \frac{g}{8(M_b^0)^2} \sigma \cdot (\Delta \times E - E \times \Delta) - c_4 \frac{g}{2M_b^0} \sigma \cdot B + c_5 \frac{a^2 \Delta^{(4)}}{24M_b^0} - c_6 \frac{a(\Delta^{(2)})^2}{16n(M_b^0)^2}. \tag{3}
\]

The last two terms in \( \delta H \) come from finite lattice spacing corrections to the lattice laplacian and
the lattice time derivative respectively. We tadpole-improve our lattice action by dividing all the U's that appear by $u_0$, which we take from the fourth root of the plaquette ($u_{0P}$). We then work with tree-level values for the $c_i$, i.e. 1.

Table 1 shows the parameters used in the calculation at 3 different values of $\beta$, for the standard Wilson gauge action.

| $\beta$ | $aM_b^2$ | $n$ | $u_{0P}$ | $V$ |
|---------|---------|----|---------|-----|
| 5.7     | 3.15    | 1  | 0.861   | $12^3 \times 24$ |
| 6.0     | 1.71    | 2  | 0.878   | $16^3 \times 32$ |
| 6.2     | 1.22    | 3  | 0.885   | $24^3 \times 48$ |

Table 1
The parameters used. We are grateful to the UKQCD collaboration and to Kogut et al for the use of their gauge field configurations.

2. RESULTS

2.1. Radial and orbital splittings
Radial and orbital splittings (when spin-averaged to remove spin effects) are calculated at next-to-leading order both in terms of a non-relativistic expansion and in terms of discretisation corrections by the action of eq. We find very little remaining a dependence when we take dimensionless ratios of spin-independent splittings, as in Figure 1. The disagreement with the experimental results, shown as lines is presumably an error from the quenched approximation since higher order (physical) relativistic corrections should have a $\sim 1\%$ effect. This independence from the lattice spacing allows us then to extrapolate quenched and dynamical ($n_f = 2$) results to the real world.

2.2. Spin splittings
The spin splitting that can be calculated most accurately is the hyperfine splitting between $\Upsilon$ and $\eta_b$. No experimental value is known and so the lattice results can make a useful prediction if we can reduce systematic errors below 10%. Our aim here is to check the scaling of the quenched results before attempting an extrapolation in $n_f$.

Because the spin splittings are sensitive to the heavy quark mass it is important that this is tuned correctly. For our simulation parameters the quark masses we use are closest to the correct ones if we fix the scale from the $2S-1S$ splitting. Figure 2 then shows the hyperfine splitting in MeV using this scale at the 3 different lattice spacings. Our results are given by the plain squares, and indicate strong scaling violations, as might be expected of such a short distance quantity with $O(a^2)$ errors. The fancy squares are rescaled by $(u_{0P}/u_{0L})^6$ to give an indication of the results that would be obtained by using $u_0$ from the Landau gauge link rather than the plaquette. They are rather flatter.

The plain diamonds show results from Manke.
et al. [8] using an action improved to the next order for spin-dependent terms [2]. This means adding additional relativistic and discretisation corrections which however will not change the spin-independent spectrum (or lattice spacing). Since these calculations use the same parameters as ours, we have shown them in Figure 2 using our 2S-1S splitting to set the scale. The plain diamonds show much better scaling behaviour than the plain squares and illustrate the fact that the discretisation and relativistic corrections act in different directions [9]. The discretisation corrections tend to increase the hyperfine splitting (obvious from the behaviour of the plain squares) but the relativistic corrections tend to reduce the splitting (so that the plain diamonds are flatter and below the plain squares).

The fancy diamonds illustrate approximately what would happen to the plain diamonds if $u_{0L}$ had been used instead of $u_{0P}$, using the leading order rescaling above. Now very good scaling is observed and this is encouraging, but must be checked in a complete calculation using $u_{0L}$. Note that the SESAM collaboration have used exactly this action with extra spin-dependent corrections and $u_{0L}$ in their comparison of results with $n_f = 0$ and 2 [10].

Statistical errors in the $p$ fine structure in current calculations make it hard to reach clear conclusions about the size of scaling violations [5].

3. CONCLUSIONS

NRQCD actions can give physical results for bottomonium splittings which scale accurately for a reasonable range of lattice spacings. This is true for spin-independent splittings for the action of equation 3; for spin-dependent splittings it is likely that this will happen with the next order of relativistic and discretisation corrections [8,10]. At this level radiative corrections to the leading order terms also have an effect and it will be necessary to decide which $u_0$ to use and what the radiative corrections beyond tadpole-improvement are [11].

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