An Effective Lagrangian with Broken Scale and Chiral Symmetry Applied to Nuclear Matter and Finite Nuclei

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Abstract

We study nuclear matter and finite nuclei with a chiral Lagrangian which generalizes the linear $\sigma$ model and also accounts for the QCD trace anomaly by means of terms which involve the $\sigma$ and $\pi$ fields as well as the glueball field $\phi$. The form of the scale invariant term leading to an omega meson mass, after symmetry breaking, could involve coupling to $\phi^2$ or $\sigma^2$ or some linear combination thereof. In fact an $\omega_\mu \omega^\mu \phi^2$ form is strongly favored by the bulk properties of nuclei, which also rather strongly constrain the other parameters. A reasonable description of the closed shell nuclei oxygen, calcium and lead can be achieved, although the spin-orbit splittings are somewhat smaller than the experimental values.
1 Introduction

We are still far from being able to transform the Lagrangian of quantum chromodynamics (QCD) into an effective Lagrangian involving mesons and baryons which could be used for nuclear matter or finite nuclei. Nevertheless it is possible to incorporate the broken global chiral and scale symmetries of QCD into an effective Lagrangian. In particular the notion of (broken) scale invariance has been modeled by a scalar glueball potential \[ \boxed{} \]. An early attempt to introduce this into an appropriately modified Walecka-type Lagrangian was made in ref. \[ \boxed{} \]. An alternative approach is to include the glueball potential in the Lagrangian of the linear sigma model (supplemented by a contribution from the vector-isoscalar omega field) \[ \boxed{} \], \[ \boxed{} \]. The latter is particularly attractive since it incorporates (spontaneously broken) chiral symmetry, another feature suggested by QCD. A common problem associated with these models is that the compression modulus for equilibrium nuclear matter is at least a factor of two larger than current estimates in the range 200–300 MeV. Of course one can arbitrarily correct this by adding terms to the Lagrangian, for instance \( \sigma^3 \) and \( \sigma^4 \), and choosing the parameters to produce the desired result \[ \boxed{} \]. We have recently suggested \[ \boxed{} \] a more satisfactory approach, however, where the form of the potential which breaks scale invariance is modified in a reasonable way to include a contribution from the \( \sigma \)-field as well as the glueball field, \( \phi \). This was found to yield \( K \approx 390 \) MeV, which, while still on the large side, is decidedly more reasonable.

There still remains uncertainty concerning the form of the omega mass term, \( \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \), where the explicitly dimensionful parameter \( m_\omega^2 \) must be
replaced by the square of a scalar field (multiplied by a dimensionless coupling constant) in order to satisfy the dictates of scale invariance. The question arises as to whether the coupling be to $\sigma^2$ or $\phi^2$ or some linear combination of the two. In refs. [3, 6] the first choice was made. One of the purposes of the present paper is to examine this choice in nuclear matter and finite nuclei; we shall find that the properties of finite nuclei are sensitive to this question. A more basic purpose of this investigation is to determine whether our effective Lagrangian, which saturates nuclear matter in the mean field approximation, is able to give a sensible account of finite nuclei. This is particularly pressing since Furnstahl and Serot [7] have recently suggested that chiral models are inherently unable to give a satisfactory description of nuclei; this work was based on the approach of Boguta [8] and did not include the glueball field, so there was no attempt to incorporate broken scale invariance.

The effective Lagrangian that we employ, together with the necessary formalism for finite nuclei and nuclear matter is given in sec. 2. Our results are displayed in sec. 3, first for nuclear matter and then for the closed shell nuclei $^{16}\text{O}$, $^{40}\text{Ca}$ and $^{208}\text{Pb}$. Our conclusions are given in sec. 4. It is of interest to compare the low density expansion to that of the standard Walecka model [9], this we discuss briefly in the Appendix. This suggests a simple approximation in which the glueball field is frozen at its vacuum value and the formalism is outlined in the Appendix. Examples are given in the text which show that the frozen glueball model reproduces our complete results quite accurately.
2 Theory

We write the total effective Lagrangian in the form

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_0 - V_G , \]  

where in this schematic equation we have separated out the scale invariant part, \( \mathcal{L}_0 \), from the potential \( V_G \) which induces the breaking of scale and chiral invariance. (The effects of explicit chiral symmetry breaking arising from the non-vanishing of current quark masses in the QCD Lagrangian are neglected.) We write \( \mathcal{L}_0 \) in terms of the chiral-invariant combination of sigma and pi fields, \( \sigma \) and \( \pi \), the glueball field \( \phi \), the field of the omega vector meson \( \omega_\mu \) and, since we are interested in finite nuclei, the vector-isovector rho field \( b_\mu \) and the Maxwell field \( A_\mu \). Specifically

\[ \mathcal{L}_0 = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} B_{\mu \nu} \cdot B^{\mu \nu} \]

\[ - \frac{1}{2} f_{\mu \nu} f^{\mu \nu} + \frac{1}{2} \omega_\mu \omega^\mu [G_{\omega \sigma}(\sigma^2 + \pi^2) + G_{\omega \phi} \phi^2] + \frac{1}{2} G_\rho \phi^2 [b_\mu \cdot b^\mu] \]

\[ + \bar{N} \left[ \gamma^\mu (i \partial_\mu - g_\omega \omega_\mu - \frac{1}{2} g_\rho b_\mu \cdot \sigma - \frac{1}{2} e(1 + \tau_3) A_\mu) - g(\sigma + i \pi \cdot \tau \gamma_5) \right] N . \]  

(2)

Here the field strength tensors are defined in the usual way \( F_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \), \( B_{\mu \nu} = \partial_\mu b_\nu - \partial_\nu b_\mu \) and \( f_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). As we have remarked in the introduction, the form of the scale invariant \( \omega \) mass term is not \textit{a priori} obvious, so we have chosen a linear combination \( \sigma^2 \) and \( \phi^2 \) fields (of course more exotic choices are possible) and we shall examine the effect of varying the coefficients. In the corresponding case for the \( \rho \) meson we have simply chosen \( \phi^2 \), we shall comment further upon this later.
In order to obtain the scale breaking term $V_G$ in the Lagrangian, we recall that the divergence of the scale current in QCD is given by the trace of the “improved” energy-momentum tensor and so the scalar potential $V_G(\phi, \sigma, \pi)$ is chosen to reproduce, via Noether’s theorem, the effective trace anomaly [1, 10]

$$\theta^\mu = 4V_G(\Phi_i) - \sum_i \Phi_i \frac{\partial V_G}{\partial \Phi_i} = 4\epsilon_{\text{vac}} \left( \frac{\phi}{\phi_0} \right)^4,$$

(3)

where $\Phi_i$ runs over the scalar fields $\{\phi, \sigma, \pi\}$ and $\epsilon_{\text{vac}}$ is the vacuum energy. The proportionality $\theta^\mu \propto \phi^4$ is suggested by the form of the QCD trace anomaly [11],

$$\theta^\mu(x) = \frac{\beta(g)}{2g} F^a_{\mu\nu}(x) F^{a\mu\nu}(x),$$

(4)

where $F^a_{\mu\nu}(x)$ is the gluon field strength tensor and $\beta(g)$ is the usual QCD beta function. Eq. (3) involves a purely gluonic, color-singlet, scalar, dimension-four operator, and also allows for the recovery of the low energy theorems which follow from broken scale invariance [12]. Then in ref. [6] we suggested the form

$$V_G(\phi, \sigma, \pi) = B\phi^4 \left( \ln \frac{\phi}{\phi_0} - \frac{1}{4} \right) - \frac{1}{2} B\delta\phi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2}$$

$$+ \frac{1}{2} B\delta\zeta \phi^2 \left( \sigma^2 + \pi^2 - \frac{1}{2} \phi^2 \right),$$

(5)

where $\zeta = \frac{\phi_0}{\sigma_0}$ and the subscript 0 indicates the vacuum value. Here the logarithmic terms contribute to the trace anomaly and are such that eq. (3) is satisfied with $\epsilon_{\text{vac}} = -\frac{1}{4} B\phi_0^4 (1 - \delta)$. This requirement uniquely specifies the second term on the right in eq. (5). The third term is needed to ensure
that in the vacuum \( \phi = \phi_0, \sigma = \sigma_0 \) and \( \pi = 0 \). To retain the physically necessary feature that \( \epsilon_{\text{vac}} < 0 \), given that \( B > 0 \), we require that \( \delta < 1 \). Provided \( 0 < \delta < 1 \), \( V_G \to \infty \) for \( \sigma \) or \( \phi \to \infty \) as is physically sensible. For further insight regarding the parameter \( \delta \), we recall that the QCD beta function with \( N_c \) colors and \( n_f \) flavors is given at the one loop level by

\[
\beta(g) = -\frac{11N_cg^3}{48\pi^2} \left( 1 - \frac{2n_f}{11N_c} \right) + \mathcal{O}(g^5),
\]

where the first number in parentheses arises from the (antiscreening) self-interaction of the gluons and the second, proportional to \( n_f \), is the (screening) contribution of quark pairs. Recalling eq. (4), a value of \( \delta = 4/33 \) is suggested for the present case with \( n_f = 2 \) and \( N_c = 3 \). Since one cannot rely on the one-loop estimate for \( \beta(g) \), we shall examine the effect of modifying the value of \( \delta \).

The careful reader will have noticed that \( \mathcal{L}_{\text{eff}} \), given by eqs. (2) and (5), does not contain the standard potential term of the linear sigma model which we have previously written \[3, 6\] in scale invariant form as \( \frac{1}{4} \lambda \left( \sigma^2 + \pi^2 - \frac{\phi^2}{\tau} \right)^2 \). We found that very small values of \( \lambda \) were favored by the predicted compression modulus in nuclear matter. We have also found that departures from small values yield binding energies of nuclei which are much too low. Therefore in this presentation we make the welcome simplification of setting \( \lambda = 0 \), thus eliminating this term. Note that while the term is present in the standard linear sigma model, it is not necessary for the breaking of chiral invariance. This can be carried out just as well by \( V_G \) in eq. (5).
It is economical to write the sigma and glueball fields in terms of their ratios to the vacuum values, viz.

\[ \chi(r) = \frac{\phi(r)}{\phi_0}, \quad \nu(r) = \frac{\sigma(r)}{\sigma_0}. \]  

For \( r \to \infty \) we obtain the vacuum values \( \chi = \nu = 1 \). Now, in the vacuum, we require that the rho and the omega masses take their physical values so that we can write the mass terms in the form

\[ \frac{1}{2} m^2_\rho \chi^2 b_\mu \cdot b^\mu \quad \text{and} \quad \frac{1}{2} m^2_\omega \left[ R_\omega \nu^2 + (1 - R_\omega) \chi^2 \right] \omega_\mu \omega^\mu. \]

Thus, for the omega, taking \( R_\omega \) to be 1 gives a pure \( \nu \) coupling, whereas the value 0 yields a pure \( \chi \) coupling. The effective mass of the nucleon, in the usual way, is \( M^*(r) = M \nu(r) \). The field equations are obtained from Lagrange’s equations and of course \( \pi = 0 \) in the mean field approximation.

We specialise to the time independent, spherically symmetric case. The Dirac equation for the nucleon and the equation for the photon field are of the form given by Horowitz and Serot \[13] and do not need to be repeated here. As usual for the vector fields only the time-like components survive and also just the isovector \( z \)-component for the \( \rho \) field. The equations for the \( \phi, \sigma, \omega \) and \( \rho \) can be written

\[
\begin{align*}
\phi_0^2 D\chi - 2B_0(2 - \delta)\chi + 2B_0 \delta \nu &= 4B_0[\chi^3(\ln \chi - \delta \ln \nu) - \chi + \delta \nu] \\
+ B_0 \delta[\chi(\nu^2 - \chi^2) + 2(\chi - \nu)] - m^2_\omega(1 - R_\omega)\omega_0^2 \chi - m^2_\rho b_0^2 \chi, \\
\sigma_0^2 D\nu - 2B_0 \delta \nu + 2B_0 \delta \chi &= M \rho_s - B_0 \delta \left[ \frac{\chi^4}{\nu} - \chi^2 \nu + 2(\nu - \chi) \right] \\
- m^2_\omega R_\omega \omega_0^2 \nu, \\
D\omega_0 - m^2_\omega \omega_0 &= -g_\omega \rho_B + m^2_\omega [R_\omega(\nu^2 - 1) + (1 - R_\omega)(\chi^2 - 1)]\omega_0, \\
Db_0 - m^2_\rho b_0 &= -g_\rho \rho_3 - m^2_\rho (1 - \chi^2)b_0,
\end{align*}
\]

(9)
where the densities $\rho_s = \langle NN \rangle$, $\rho_B = \langle N^0 N \rangle$ and $\rho_3 = \frac{1}{2}\langle N^0 \tau_0 N \rangle$ can be expressed in terms of the components of the nucleon Dirac spinors in the usual way [13]. In eq. (9) we have made the definitions $D \equiv \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}$ and $B_0 \equiv B \phi_0^4$. The terms linear in the fields, i.e., the kinetic energy and mass terms, have been separated out on the left of these equations. The equations of ref. [6] for nuclear matter are regained by setting the derivatives to zero so that the results are independent of $\phi_0$ and $\sigma_0$, however these quantities are needed for finite nuclei. They are also needed to obtain the vacuum scalar masses and we note that the mass matrix is not diagonal. In order to solve the equations (9) by an iterative Green’s function technique [7, 13] it is necessary to go to a representation which is diagonal in the limit that $r \to \infty$. We solve in terms of the quantities $(1 - \chi)$ and $(1 - \nu)$ which go to zero in the same limit.

The energy-momentum tensor can be used to obtain the total energy of the system in the standard way [9]. Subtracting constants so that the energy is measured relative to the vacuum, we obtain

$$E = \sum_{\alpha}^{occ} \epsilon_\alpha (2j_\alpha + 1) - 2\pi \int_0^\infty dr r^2 \left\{ M \nu \rho_s + g_\omega \omega_0 \rho_B + g_\rho \rho_3 \right. + 2B_0 \left[ \chi^4 (\ln \chi - \delta \ln \nu + \frac{1}{4}) - \frac{1}{4} \right] + \frac{1}{2} B_0 \delta [\chi^2 (2\nu^2 - 3\chi^2) + 1] \left. - m_\omega^2 R_\omega \nu^2 + (1 - R_\omega) \chi^2 \omega_0^2 - m_3^2 \chi^2 \chi_0^2 \right\}. \quad (10)$$

In the first term on the right the $\epsilon_\alpha$ are the Dirac single particle energies and $j_\alpha$ is the total angular momentum of the single particle state. In nuclear matter this term becomes $4 \sum_k \left( g_\omega \omega_0 + \sqrt{k^2 + M^2} \right)$. Making this replacement and using eq. (9) the expression for the energy density of nuclear matter given in ref. [13] can be obtained.
3 Results

3.1 Nuclear Matter

In nuclear matter we have four parameters to consider. We will choose $B_0$ and $C_\omega^2 \equiv \frac{g_\omega^2 M^2}{m_\omega^2}$ to fit the saturation properties. The binding energy/nucleon we take to be 16 MeV. In refs. [3, 6] we took a saturation density of 0.16 fm$^{-3}$, but this yields central densities in lead which are too high. We have therefore used 0.148 fm$^{-3}$ ($k_F = 1.3$ fm$^{-1}$), which is the value favored by Serot and coworkers in mean field calculations [13, 7]. Having fixed these two parameters, we then have the parameter of the $\omega$ mass term, $R_\omega$, and the coefficient of the quark contribution to the trace anomaly, $\delta$, which can be varied.

We show in table 1 several parameter sets that we have considered (the designation F here indicates the frozen glueball model where $\chi$ is kept equal to 1; see discussion in the Appendix). One might hope that the parameters would show a reasonable correspondence with the independently determined quantities, although precise agreement would not be expected for an effective theory such as this. In this sense the values for $C_\omega^2$ in table 1 are in acceptable agreement with the value $103 \pm 36$ deduced from data on the $\omega$ coupling constant [14] and the known mass. Turning to $|\epsilon_{\text{vac}}|$, QCD sum rule studies suggest [15] a value of $(240 \text{ MeV})^4$ which would not favor the extreme variations of $\delta$ (sets IV and VII), although bag model estimates [16] range as low as $(146 \text{ MeV})^4$. The values of $C_\rho^2$, given in table 1 for later use, have been obtained by fitting to a symmetry energy of 35 MeV. Since exchange
effects are known to make a significant contribution to the symmetry energy we shall not make a direct comparison to experiment.

Turning to the derived quantities, nuclei generally favor effective masses below 0.7 \cite{7}, which is the case for sets VI and VII. However, here the compression moduli, $K$, are in excess of 400 MeV. Pearson \cite{17} has made the point that the leptodermous expansion, which is one method used to fit the data, permits values of $K$ up to about 400 MeV with little difference in the $\chi^2$ fits. Defining the third derivative of the binding energy per particle as

$$S = k_F^3 \frac{d^3}{dk_F^3} \left( \frac{E}{A} \right)$$

(evaluated at equilibrium), the data indicate a linear relation between $S/K$ and $K$ (see ref. \cite{18}). We have listed values of $S/K$ in table 1. Cases VI and VII are well off the allowed error band, but the remaining cases are quite close to, although slightly off, the band.

The parameter sets designated with an F in table 1 correspond to the frozen glueball model ($\chi = 1$) and we see that they give a close correspondence with the complete results. The point is further made in figs. 1 and 2 where the agreement is seen to be particularly good at low density, as we argue in the Appendix. The lower panel of fig. 2 shows that in the complete model $\chi$ remains very close to unity over the whole region of densities considered. We remark that the curves for $\nu$ will increase linearly with $k_F$ for sufficiently large density (a discussion of the high density behavior was given in ref. \cite{19}), although it is debatable whether this theory is applicable in that region.
3.2 Finite Nuclei

When we turn to finite nuclei two extra parameters are involved, namely the vacuum values of the scalar fields, $\phi_0$ and $\sigma_0$. Thus we have a total of four free parameters. Let us first eliminate two of these. We shall discuss changes relative to parameter set I of table 1 with $\sigma_0 = 110$ MeV, which will turn out to be our preferred parameter set. First consider reducing $\delta$ by using sets IV and V of table 1 (and reasonable values for the parameters not specified). We find little sensitivity in the predicted properties of nuclei. However, if we increase $\delta$ using sets VI and VII, for which $K$ is large, we find a significant reduction in the binding energy, particularly for set VII ($^{40}$Ca is less bound by $2\frac{1}{4}$ MeV, for example). We conclude that a value of $\delta$ in the neighborhood of $4/33$ is reasonable and fix on this value, although nuclei are not sensitive to the precise figure. Next consider the ratio of the vacuum scalar fields, $\zeta = \phi_0/\sigma_0$. Using set I we have varied $\zeta$ in the range 0.7–2.1, which corresponds to varying the higher scalar mass, $m_\sigma$, between 3 and 1 GeV. Very little change is observed in the nuclear properties. The salient point is that the mixing between the glueball and the sigma is small, $\leq 3\%$, and it is the sigma which directly couples to nucleons. Henceforth we choose $\zeta$ such that $m_\sigma = 1.5$ GeV because this seems reasonable in view of QCD sum rule estimates for a scalar glueball mass in the range 1–2 GeV [13]. The values of $\zeta$ are listed in table 2.

We are then left to consider the variation of $R_\omega$ and $\sigma_0$: a few comments on the expected magnitude of $\sigma_0$ are in order here. In the original Gell-Mann-Lévy linear sigma model [20], a calculation of the axial vector current
matrix element responsible for pion decay led to the identification $\sigma_0 = f_\pi = 93$ MeV. In a more general chiral effective Lagrangian incorporating vector mesons (ref. [21] for an early review) this identification no longer follows due to the necessity of including the $a_1$ axial vector meson, together with the $\rho$.

Briefly, cross-terms of the form $a_1^\mu \cdot \partial_\mu \pi$ appear in the full Lagrangian which are eliminated by the replacement $a_1^\mu \rightarrow a_1^\mu + \xi \partial^\mu \pi$, with an appropriate parameter $\xi$. This, in turn, leads to a change in the coefficient of the pion kinetic term, $\partial_\mu \pi \cdot \partial^\mu \pi$, and a rescaling of the pion field is necessary to bring this to canonical form. One defines a renormalized field $\pi_R = Z_\pi^{-\frac{1}{2}} \pi$.

Now a calculation of the full axial current yields $\sigma_0 = Z_\pi^{-\frac{1}{2}} f_\pi$. The precise form of $Z_\pi$ is model dependent. In the simplest model, which does not include the physics of broken scale invariance one finds [21], using the KSRF relation, that $Z_\pi^{\frac{1}{2}} = m_{a_1}/m_\rho = \sqrt{2}$. We do not expect either this precise functional form or value to be maintained in more realistic models, but we do anticipate $Z_\pi^{\frac{1}{2}} > 1$. Accordingly, $\sigma_0$ is kept as a free parameter, which we can presume to be larger than 93 MeV. Two further remarks are in order. First, the additional terms involving the $a_1$ will not contribute in the mean-field approximation. Second, from eq. (2) after symmetry breaking, the nucleon mass is given by $M = g\sigma_0$. The physical pion-nucleon coupling is defined in terms of the renormalized pion field to be $g_{\pi NN} = Z_\pi^{\frac{1}{2}} g$. So, in fact, one still has $M = g\sigma_0 = g_{\pi NN} f_\pi$ as the approximate form of the Goldberger-Treiman relation.

With this in mind, we first consider variation of $\sigma_0$ keeping $R_\omega = 0$. The results are given in table 2, where the notation $I/93$, for example, implies use of parameter set I from table 1 with $\sigma_0 = 93$ MeV. The theoretical values of
the binding energy/particle include a correction for the c.m. kinetic energy \[22\]. The charge densities, and therefore the radii quoted, are corrected for the finite size of the proton \[13\] and for c.m. effects \[23\]. We see that the choice \(\sigma_0 = 93\) MeV (fourth row in table 2) yields poor binding energies, radii which are too small and, as shown in fig. 3 for \(^{40}\)Ca, an oscillatory charge density. This figure indicates that increasing \(\sigma_0\) to 110 MeV improves matters greatly. One can view this as reducing the magnitude of the derivative of \(\nu\) in eq. (9) or as reducing \(m_<\) to \(\sim 500\) MeV (Furnstahl and Serot \[7\] stress that values in this neighborhood are to be preferred). In fact as regards the charge radii and densities the value 120 MeV gives even better results, however we see from table 2 that in this case the binding energies/particle are \(1\frac{3}{4}\) MeV too low. Therefore we prefer the parameter set I/110.

Now we consider variation of \(R_\omega\) with \(\sigma_0 = 110\) MeV. As \(R_\omega\) is increased from zero to 0.5 or 1.0 we see that the binding energies become too large and shows the wrong \(A\)-dependence and also the radii become too small. Table 2 also shows that the mass of the lighter scalar meson, \(m_<\), increases sharply. Further, as we see for \(^{208}\)Pb in fig. 4 the charge density begins to develop oscillations which are not present in the data. Similar problems were noted by Furnstahl and Serot \[7\] who used \(R_\omega\) values of 0.76 or 1, although their model differs significantly from ours. As pointed out by Price, Shepard and McNeil \[24\] this is likely to signal the onset of a situation where the ground state of nuclear matter is no longer uniform, but exhibits periodic density fluctuations. We conclude that nuclei strongly favor \(R_\omega = 0\).

We have also compared the case I/110 with the frozen glueball approximation, case IF/110, in table 2. There is a close correspondence between the
results, as there is for the predicted charge densities which we illustrate for $^{16}$O in fig. 5.

Finally we turn to the single particle energies. The levels near the Fermi surface in lead are shown in figs. 6 and 7 for neutrons and protons respectively. The relative positions of the neutron and proton levels are sensitive to the treatment of the $\rho$ meson [13]. We have used a scale-invariant $\phi^2$ coupling in the mass term, but the results differ negligibly if a standard mass term is used. However there is a difference if a $\sigma^2$ coupling is used because this reduces $C_\rho^2$ and leads to $\sim 2$ MeV less (more) binding for protons (neutrons). Since figs. 6 and 7 clearly show that this is undesirable we have not considered this possibility further. The results with the sets II/110 ($R_\omega = 0.5$) and I/93 show immediate problems since the major shell closure is incorrectly given due to the position of the proton $1h_{9/2}$ and the neutron $1i_{11/2}$ levels. This problem disappears with the preferred parameter set, I/110, and the single particle spectrum is much more reasonable, although a larger proton $1h_{9/2} - 3s_{1/2}$ splitting would be desirable. This may reflect the spin-orbit splittings which are, on average, 65% of the experimental values here and also in the other nuclei we have considered. They could be improved by reducing the effective masses in table 1 below 0.7. This can be achieved by taking a small negative value for $R_\omega$, although we find that this leads to a larger compression modulus and lower binding energies.
4 Conclusions

Our primary purpose has been to see whether an effective Lagrangian which includes the breaking of scale and chiral invariance can make sensible predictions for finite nuclei. Finite nuclei are expected, and found, to be a much more severe test than nuclear matter. In this endeavour the form of the scale invariant mass term for the $\omega$ meson was found to be critical and a pure coupling to the glueball field, $\phi^2 \omega_\mu \omega^\mu$, was required, although a small effect from the sigma field cannot be ruled out. A similar coupling for the $\rho$ mass term was also favored. It is interesting that the properties of nuclei can be used to provide such strong constraints on the form of the couplings of the scalar glueball to other mesonic fields appearing in the chiral effective Lagrangian.

It was also found to be necessary to increase the value of the vacuum value of the sigma field, $\sigma_0$, from the naive expectation of 93 MeV to somewhere in the neighborhood of 110 MeV. With these caveats the bulk properties of O, Ca and Pb were quite well accounted for and, while the spin-orbit splittings are about 65% of the desired values, the basic structure of the single particle spectra was reasonable. As a by-product we were able to define a simplified, frozen glueball model which was able to reproduce the results of the complete model quite accurately, but, by itself, would seem highly arbitrary.

It is natural to compare this approach to the standard Walecka model extended to include non-linear $\sigma^3$ and $\sigma^4$ terms (as for example in refs. [27, 4]). The agreement is better than found here, particularly as regards the single particle energies. Nevertheless, it is remarkable that the present effective chiral Lagrangian, whose form is motivated by QCD, can give an acceptable
description of both nuclear matter and finite nuclei in the mean field approximation. It remains to be seen whether fine tuning can further improve the agreement here. For example, a term of the form \((\omega_\mu\omega^\mu)^2\) is the simplest among many possibilities, consistent with scale invariance, which could be added to the Lagrangian. We are currently investigating the effects of including such a term.

After this work was completed we received a preprint from Furnstahl and Serot \cite{28} who have studied the same problem, restricting themselves to an \(\omega\) mass term of the form \(\sigma^2\omega_\mu\omega^\mu\). In agreement with our conclusions, they point out that the results for nuclei are poor. As we have stressed, nuclei require that the coupling be to the glueball field rather than the sigma field.

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Appendix
Low Density Expansion and Frozen Glueball Model

It is of interest to study the low density expansion of eqs. (9) and (10) in the case of nuclear matter. Examination of eq. (9) indicates that we can expand

\[ \nu = 1 + a \rho_B (1 + b k_F^2) + \mathcal{O}(k_F^7) , \]
\[ \chi = 1 + c \rho_B (1 + d k_F^2) + \mathcal{O}(k_F^7) , \]  \( (11) \)

where \( \rho_B \) is the baryon density and \( k_F \) the Fermi momentum. Further to this order the coupling between \( \omega_0 \) and \( \nu \) and \( \chi \) can be neglected. Then the equation of motion for \( \chi \) indicates that

\[ d = b \quad , \quad \frac{c}{a} = \frac{\delta}{2 - \delta} , \]  \( (12) \)

The equation of motion for \( \nu \) yields

\[ a = - \frac{M(2 - \delta)}{4 B_0 \delta (1 - \delta)} , \quad b = - \frac{3}{10 M^2} . \]  \( (13) \)

Substituting into eq. (10), we find the energy/particle

\[ \frac{E}{A} = M + \frac{3k_F^2}{10M} - \frac{3k_F^4}{56M^3} + \frac{g_\omega^2 \rho_B}{2m_\omega^2} \]
\[ - \frac{M^2(2 - \delta)}{8B_0 \delta (1 - \delta)} \rho_B \left( 1 - \frac{3k_F^2}{5M^2} \right) + \mathcal{O} \left[ \left( \frac{k_F}{M} \right)^6 \right] . \]  \( (14) \)

The first three terms are the energy of a Fermi gas and the succeeding terms are the \( \omega \) and scalar meson contributions. This is of exactly the same form
as the standard Walecka model \[9\], except that the scalar meson mass is replaced by

\[
m = \left( \frac{4B_0\delta(1-\delta)}{\sigma_0^2(2-\delta)} \right)^{1/2} = m_\sigma m_\pi \frac{\phi_0}{[2B_0(2-\delta)]^{1/2}} \simeq m_<. \tag{15}
\]

where \(m_<\) (\(m_>\)) are the lower (upper) eigenvalues of the vacuum mass matrix (see \[8\] for an explicit discussion). The mass, \(m\), is approximately \(m_<\), provided that \(\phi_0/\sigma_0\) is not too large so that the mixing of the sigma and glueball states is small.

Now the fact that \(d = b\) in eq. (12), means that, to the order we are considering, we can obtain the correct result for \(\nu\) with \(\chi = 1\), provided that \(B_0\) is replaced by

\[
B'_0 = B_0 \frac{2(1-\delta)}{2-\delta}. \tag{16}
\]

In this approximation, we see that the scalar potential, \(V_G\), takes the simple form

\[
V_G(\phi, \sigma, \pi = 0) - \epsilon_{\text{vac}} = \frac{1}{2}B_0\delta(\nu^2 - 1 - \ln \nu^2). \tag{17}
\]

In eq. (9), with \(\chi = 1\), the glueball equation of motion can be dropped and we are left with the single scalar equation

\[
\sigma_0^2D\nu - 2B_0\delta\nu = M\rho_s - B_0\delta \frac{(1+\nu^2)}{\nu} - m_\omega^2\omega^2\omega_0^2\nu. \tag{18}
\]

Taking care that the energy is consistently obtained we find

\[
E = \sum_\alpha \epsilon_\alpha (2j_\alpha + 1) - 2\pi \int_0^\infty dr r^2 \left\{ M\nu\rho_s + g_\omega\omega_0\rho_B + g_\rho b_0\rho_3 \\
+ 2B_0\delta \ln \nu - m_\omega^2\omega^2\omega_0^2 \right\}. \tag{19}
\]
Using the low density expansion for nuclear matter and replacing \( B_0 \) by \( B'_0 \) in eqs. (18) and (19) this expression gives the same result as the complete result of eq. (10). In practice to saturate nuclear matter correctly a small adjustment of \( C_2 \) is needed and \( B'_0 \) is only approximately given by (16).
Table 1
Values of the parameters and the derived quantities for nuclear matter

| Set | $R_\omega$ | $33\delta$ | $|\epsilon_{\text{vac}}|^\frac{4}{3}$ | $C_\omega^2$ | $C_\rho^2$ | $\frac{M^*_\text{sat}}{M}$ | $K$ | $\frac{S}{K}$ |
|-----|----------|------------|-------------------------------|-------------|-----------|----------------|-----|----------|
| I   | 0.0      | 4          | 235                           | 156         | 116       | 0.71           | 383 | 6.5      |
| IF  | 0.0      | 4          | 231                           | 152         | 123       | 0.72           | 341 | 5.6      |
| II  | 0.5      | 4          | 255                           | 82.6        | 127       | 0.80           | 356 | 5.9      |
| III | 1.0      | 4          | 269                           | 51.3        | 132       | 0.84           | 377 | 6.1      |
| IIIF| 1.0      | 4          | 265                           | 49.2        | 135       | 0.85           | 356 | 5.7      |
| IV  | 0.0      | 1          | 336                           | 153         | 122       | 0.72           | 350 | 5.8      |
| V   | 0.0      | 2          | 281                           | 154         | 119       | 0.72           | 360 | 6.0      |
| VI  | 0.0      | 8          | 194                           | 158         | 107       | 0.69           | 442 | 7.5      |
| VII | 0.0      | 16         | 160                           | 156         | 88        | 0.65           | 666 | 10.6     |
Table 2

Bulk properties of nuclei for various parameter sets

| Set   | $m_<$ (MeV) | $\zeta$ | O \(\frac{BE}{A}\) (MeV) | \(r_{ch}\) (fm) | Ca \(\frac{BE}{A}\) (MeV) | \(r_{ch}\) (fm) | Pb \(\frac{BE}{A}\) (MeV) | \(r_{ch}\) (fm) |
|-------|------------|---------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Experiment | 7.98       | 2.73    | 8.45            | 3.48            | 7.86            | 5.50            |
| I/110  | 506        | 1.4     | 7.35            | 2.64            | 7.96            | 3.41            | 7.33            | 5.49            |
| IF/110 | 508        | –       | 7.86            | 2.62            | 8.35            | 3.40            | 7.54            | 5.49            |
| I/93   | 598        | 1.4     | 9.41            | 2.52            | 9.38            | 3.32            | 8.04            | 5.45            |
| I/120  | 464        | 1.4     | 6.25            | 2.72            | 7.16            | 3.47            | 6.92            | 5.52            |
| II/110 | 598        | 1.6     | 10.08           | 2.49            | 9.85            | 3.29            | 8.22            | 5.45            |
| III/110| 664        | 1.8     | 10.98           | 2.44            | 10.45           | 3.26            | 8.48            | 5.44            |
Figure Captions

Fig. 1. The binding energy/particle as a function of density for nuclear matter; density $\rho$ is given as the ratio to the equilibrium value $\rho_0 = 0.148 \text{ fm}^{-3}$. The parameter sets used are indicated and the designation F indicates the frozen glueball model.

Fig. 2. The fields $\nu$ and $\chi$ corresponding to fig. 1.

Fig. 3. Comparison of the experimental charge density [25] for $^{40}\text{Ca}$ with theoretical predictions for the parameter sets indicated.

Fig. 4 As for fig. 3, but for $^{208}\text{Pb}$.

Fig. 5 As for fig. 3, but for $^{16}\text{O}$. The data are from ref. [26].

Fig. 6. Occupied and unoccupied neutron levels near the Fermi energy in $^{208}\text{Pb}$. The experimental data are compared with predictions for the parameter sets indicated.

Fig. 7. As for fig. 6, but for protons.