Inheritance Patterns: Probability Rules & Probability Trees

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Abstract

Educators usually teach the Mendelian inheritance model using Punnett squares to determine the probability of an offspring having a particular genotype and phenotype. To find the probability of an outcome of a particular cross, students need to understand the underlying biological concepts of these visual representations. However, this approach becomes more complex for cases with three or more characters and shies away from the authentic integration of mathematical and biological concepts. Therefore it is crucial for students to use mathematical algorithms that Mendel used to understand and solve inheritance problems. In this paper, we propose relating two simple probability rules to the laws of inheritance and using a probability tree diagram to predict the combined frequency of traits in the offspring of crosses. We validate the proposed probability rules for various examples.

Key Words: Mendel; laws of inheritance; law of independent assortment; law of segregation; probability; addition or sum rule; multiplication or product rule.

Introduction

Many science educators strongly support the integration of mathematics and life sciences (Duffus & Olier, 2010; Labov et al., 2010; NRC, 2012; Salsberg, 2009). Additionally, in the reformed undergraduate science courses, educators expect students to reason and to build biological knowledge by using various mathematical representations (Matthews et al., 2010). Similar changes have been proposed in K–12 settings to promote STEM education (Common Core Standards Initiative, 2010; NRC, 2012). However, in life science courses, teachers often opt for easy and noncontextual methods for quick solutions, which can often undermine mathematical concepts (Garfield & Ahlgren, 1988; Liu & Thompson, 2007). A good example is the probability concepts in Mendelian genetics. This topic is being taught using the Punnett square method, a nonmathematical and easily understandable matrix to demonstrate Mendel’s laws of inheritance. Freshman biology majors, in spite of exposure to genetics since primary grades, have difficulty understanding basic concepts (Batzli et al., 2014; Bowling et al., 2008; Colon-Berlingeri & Burrowes, 2011; Shaw et al., 2008; Stewart, 1982). A Punnett square is a checkerboard of squares with the gametes from each parent arranged along the top and sides of the grid. Each box of the grid represents the union of the gametes in the corresponding row and column, which show all possible genotype combinations of offspring that can result from a random crossing. Though the Punnett square method is an excellent visual method for predicting the probability of possible combinations of genotypes for monohybrid or dihybrid crosses, it has some limitations. First, the Punnett square gets complicated as the combination of traits grow exponentially with an increasing number of characteristics. The matrix for a monohybrid cross is 4 (2 × 2), for a dihybrid cross it is 16 (4 × 4), for a trihybrid cross it is 64 (8 × 8), and so on. Second, there are steps in the Punnett square method that can lead to mistakes. For example, one has to set up a matrix, find all possible combinations by visually counting the desired phenotype, and then calculate the frequency. Third, as the process becomes complex, students lose the connection between the process and concepts and tend to solve the problem in a nonmeaningful way.

Providing structured opportunities for students to apply probability rules and algorithms to biological concepts will allow them to experience the interconnectedness of the STEM fields. In this work, we propose a methodology that implements two rules of probability to create probability tree diagrams for students to understand Mendelian genetics and show the effectiveness of this activity when dealing with more complex scenarios.

Prior Knowledge & Background Information

Some working knowledge of the genetic basis for inheritance will be helpful. It is also important that students understand that Mendel’s laws and the rules of probability are applied to single-gene traits that are on different chromosomes. The probability rules involve addition and multiplication of fractions; therefore, a quick review

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of these concepts is recommended. Table 1 includes a summary of Mendel’s laws of inheritance and probability rules relevant to the proposed activities.

- **Overview of the Activity**

  This activity is designed to converge statistics with biology. Students in this investigation will have an opportunity to use probability rules in the context of Mendel’s laws of inheritance and visually represent all potential outcomes and their respective likelihoods by drawing a probability tree. A tree diagram is a special type of graph that is constructed to visually represent possible combinations of two or more events that are random and mutually exclusive while minimizing mathematical errors.

  The proposed lesson consists of two modules with one extension. Module 1 will focus on monohybrid crosses to allow students to work with one character as they get familiar with the algorithm of implementing probability rules to predict the results of a cross. Module 2 will deal with more complex dihybrid crosses. Students will use the algorithms learned in the first activity to predict the inheritance patterns of two characters. The Extension Module 3 includes a challenge for students to study a trihybrid cross. The garden pea plant found in traditional textbooks will be used as a model organism in this lesson. The lesson presented here is intended for two 50-minute class periods. The extension activity can be given as a homework problem.

  Instructors can choose to modify the pace and depth of the content depending on their time frame, comprehension level, and class size. Mendel’s inheritance and probability are middle school and high school concepts in the Next Generation Science Standards and the Common Core State Standards. The modular nature of the lesson allows easy adaption to the grade level. At the middle school level, teachers can use Module 1, however both Modules 1 and 2 can be implemented for high school and introductory college courses. The lesson can be extended to non-Mendelian inheritance such as codominance, complete dominance, and multiple alleles for more advanced students.

  The process of predicting the outcome of a cross is divided into three steps. In each step the relevant biological principles, probability rules, and the steps in creating graphical representations, such as tree diagrams, are discussed.

  **Step 1: Gamete formation.** This step involves predicting the types and probabilities of gametes that can form from each parent. The law of segregation, or the separation of the allelic pair with only one of the allelic pairs passing to the gametes, will be applied. In crosses involving two or more characteristics, the law of independent assortment, which states that allelic pairs of different characters assort independently of each other, will be applied. Since each character is inherited independently, the inheritance pattern of the allelic pairs of each character will be calculated separately. For easy visualization of the outcomes, results will be represented as a pattern of branches with probabilities labeled on the branches and types of gametes at the ends of the branches.

  **Step 2: Fertilization.** In this step the combined probabilities of the offspring genotypes and phenotypes are predicted. Sexual reproduction involves a random fertilization of a gametes from each parent. Gamete formation and fertilization are consecutive, independent, and mutually exclusive processes, and therefore the multiplication and addition rules of probability can be applied. A probability tree diagram will be constructed by connecting the branches from step 1.

  **Step 3: Outcome.** The final step deals with calculating the outcome of the genetic cross by using the multiplication and/or addition rules of probability. The genotypes, the combination of alleles of a specific gene, and phenotypes, or any observable characteristics, of the offspring are calculated by multiplying the allele combinations and the frequencies along the branches. If there are branches with the same results, the frequencies of those branches are added.

- **Implementation of the activity**

  The instructor should guide the entire class through the first monohybrid cross. After this demonstration, students should organize and work in small groups. Student groups will be guided by a worksheet that details the steps for each mechanism (provided as Supplemental Material with the online version of this article). The parent generation is labeled P, and the offspring generation is labeled F.

  **Module 1: One-Character Inheritance or Monohybrid Crosses**

  Students will study the inheritance pattern of a single-trait seed shape. Two possible crosses will be presented to the students.

  **Monohybrid cross 1:** This is an instructor-led whole-class activity. Students will be presented with a cross between a heterozygous P-Female strain, with a different form of alleles (Rr), and a homozygous recessive P-Male strain, with the same form of alleles (rr). Students will be asked to draw the shape of the seeds and

| Laws and Rules               | Description                                                                 |
|------------------------------|-----------------------------------------------------------------------------|
| Law of segregation           | During gamete formation, each pair of alleles segregate unchanged and pass into different gametes, so that each gamete (egg or sperm) receives only one allele of a pair. |
| Law of independent assortment| Each allelic pair separates independently during gamete formation, and therefore traits are passed on to offspring independent of each other. |
| Probability rule of addition | The probability of either of two mutually exclusive events occurring is equal to the sum of their individual probabilities. P(A) or P(B) = P(A) + P(B) |
| Probability rule of multiplication | The probability of two independent events both occurring is the product of their individual probabilities. P(A) and P(B) = P(A) × P(B) |
label the appropriate genotype (Figure 1). Students should notice that the parent genotype is diploid and that the shape of the seed depends on the two alleles.

In step 1, students are to draw branches to visually represent the frequency and types of gametes that can form from each parent based on the law of segregation and the rules of probability. A heterozygous (Rr) P-Female will produce two types of gametes, each with 1/2 probability. A homozygous recessive (rr) P-Male will produce only one type of gamete (r) with a probability of 1 (Figure 1).

In step 2, the possible combination of gametes to form the zygote and their frequencies is predicted by drawing a tree diagram representing the gamete of P-Male as the primary branches. The secondary branches, representing P-Female, are added to the ends of each primary branch. Reversing the primary and secondary branches yields the same results. With two types of eggs and one type of sperm, offspring with two genotypes, each with a probability of 1/2, will result from the cross (Figure 2).

In step 3, the outcome of the cross is predicted by multiplying the alleles and the probabilities along each branch to predict the frequency and possible offspring genotype and phenotype. The addition rule does not apply in this case because there are no branches with identical results. Since there are two types of gamete combinations, 1/2 of the offspring will have heterozygous round seeds (Rr) and 1/2 of the offspring will have homozygous wrinkled seeds (rr).

Monohybrid cross 2: In this example a cross between two heterozygous parent strains is conducted. Students break into small groups for this activity. Each group will work through the worksheet on their own by following the three steps. Students will identify the parents’ phenotype and genotype to determine the types and probabilities of gametes for each parent. Both the P-Female and the P-Male plants are heterozygous (Rr), so the frequencies and types of gametes for both parents will be the same. Two types of eggs and two types of sperms will form, and the probability will be 1/2 for each gamete (Figure 4).

Students will draw a tree diagram with two branches for P-Female and two branches for P-Male and calculate the outcomes. At this point the instructor should direct the students to look at the probability tree in Figure 5 and apply the addition rule, P(A or B) = P(A) + P(B). Students will add the probabilities of the branches with the same outcomes (Figure 5).

Module 2: Two-Character Inheritance or a Dihybrid Cross
In this module students will work with two traits, flower color and seed shape, to predict the possible combinations of traits in the offspring (F1) generation.

This is a cross between a P-Female strain with heterozygous genotype for both purple flowers and round seeds (FfRr) and a
Appendix). Once the inheritance pattern of each trait is determined (Figure 6), students will calculate the combined inheritance pattern for two traits by using the multiplication rule.

\[ P(A \text{ and } B) = P(A) \times P(B) \]

**Example 1.** What is the probability of having an offspring with purple flowers and round seeds?

\[ P(\text{purple flower and round seeds}) = P(\text{purple flowers}) \times P(\text{round seeds}) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8} \]

**Example 2.** What is the probability of having an offspring with \( ffrr \) genotype?

\[ P(ffrr) = P(ff) \times P(rr) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \]

**Extension Module 3—Three-Character or Trihybrid Crosses**

In this example we will consider a cross between two parents with three traits: plant height, flower color, and seed shape. Both parents are tall with purple flowers and round seeds. They are heterozygous for all three traits; P-Female: \( Tt \) and P-Male: \( Tt \).

Students will separate each trait for both parents based on the law of independent assortment. They will follow the three steps and calculate the probability and the inheritance pattern of the allelic pairs for each trait. Since both parents are heterozygous for all three traits, the frequencies and the inheritance patterns of all three traits will be the same (Figure 7). Using the inheritance pattern of each trait students will calculate the frequencies and types of combinations of the genotype or phenotype of the offspring using multiplication rules.

\[ P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C) \]

\[ P(\text{height and flower color and seed shape}) = P(\text{height}) \times P(\text{flower color}) \times P(\text{seed shape}) \]
Example 1: What is the probability of having a short offspring that produces purple flowers and round seeds?

\[ P(\text{short, purple flower and round seeds}) = P(\text{short}) \times P(\text{purple}) \times P(\text{round}) \]

\[ = \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{64} \]

Example 2: Find the probability of forming an offspring with genotype TtFFrr

\[ P(\text{TtFFrr}) = P(\text{Tt}) \times P(\text{FF}) \times P(\text{rr}) \]

\[ = \frac{2}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32} \text{ or } \frac{2}{64} \]

The three-step process presented in this paper can be applied to crosses or crossovers of multiple traits, incomplete dominance, codominance, and traits with multiple alleles.

○ Discussions

The activity was implemented in a science concept class for middle grade preservice teachers. After completing the module, students were comfortable relating the mathematical steps with biological concepts. Students had a better understanding of the three sequential steps, as each step included the relevant biological principles, probability rules, and the steps in creating graphical representations or the tree diagram. For example, in step 1 students used the principle of segregation to visually represent (branching) the probabilities of different types of gamete formations, and when studying two or more traits together, students used the principle of independent assortment to separate and combine probabilities of different types of gamete formations. Unlike the traditional Punnett square approach that can become more complex as students derive probabilities for cases with more traits, the probability rule-based approach remains simple with three easy-to-follow steps. Once students are comfortable with using the rules of probability, instructors can then extend the probability rule-based approach for di- and trihybrid crosses. The lesson can also be extended to non-Mendelian inheritance such as codominance, complete dominance, and multiple alleles for more advanced students.

References

Batzli, J. M., Smith, A. R., Williams, P. H., McGee, S. A., Dósa, K. & Pfammatter, J. (2014). Beyond Punnett squares: student word association and explanations of phenotypic variation through an integrative quantitative genetics unit investigating anthocyanin inheritance and expression in Brassica rapa fast plants. *CBE—Life Sciences Education*, 13(3), 410–424. https://doi.org/10.1187/cbe.13-12-0232.

Bowling, B. V., Huether, C. A., Wang, L., Myers, M. F., Markle, G. C., Dean, G. E., Acr, E. E., Wray, F. P. & Jacob, G. A. (2008). Genetic literacy of undergraduate non-science majors and the impact of introductory biology and genetics courses. *BioScience*, 58(7), 659–660. https://doi.org/10.1641/BS080712.

Colon-Berlinger, M. & Burrowes, P. A. (2011). Teaching biology through statistics: application of statistical methods in genetics and zoology courses. *CBE—Life Sciences Education*, 10(3), 259–267. https://doi.org/10.1187/cbe.11-0137.

Common Core State Standards Initiative. (2010). National Governors Association Center for Best Practices & Council of Chief State School Officers. http://www.corestandards.org.

Duffus, D. & Oliker, A. (2010). Introductory life science mathematics and quantitative neuroscience courses. *CBE—Life Sciences Education*, 9(3), 370–377.

Garfield, J. & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: implications for research. *Journal for Research in Mathematics Education*, 19(1), 44–63.

Labov, J. B., Reid, A. H. & Yamamoto, K. R. (2010). Integrated biology and undergraduate science education: a new biology education for the twenty-first century? *CBE Life Sciences Education*, 9(1), 10–16. https://doi.org/10.1187/cbe.09-12-0092.

Liu, Y. & Thompson, P. (2007). Teachers’ understanding of probability. *Cognition and Instruction*, 25(2), 113–60.

Matthews, K. E., Adams, P. & Goos, M. (2010). Using the Principles of B12010 to develop an introductory, interdisciplinary course for biology students. *CBE—Life Sciences Education*, 9(3), 290–97. https://doi.org/10.1187/cbe.10-03-0033.

NRC, N. R. (2012). *A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas*. National Academies Press. https://doi.org/10.17226/13165.

Salsberg, E. (2009). 2009 State Physician Workforce Data Book, 60. Association of American Medical Colleges. https://www.aamc.org/media/8396/download.

Shaw, K. R. M., Horne, K. V., Zhang, H. & Boughman, J. (2008). Essay contest reveals misconceptions of high school students in genetics content. *Genetics*, 178(3), 1157–68. https://doi.org/10.1534/genetics.108.091494.

Stewart, J. (1982). Difficulties experienced by high school students when learning basic mendelian genetics. *American Biology Teacher*, 44(2), 80–89.
### Appendix: Probability Tree

#### Figure A1. Construction of inheritance patterns of flower color using a probability tree.

| Gamete formation - Mendel's Law | Fertilization - Probability Rules | Outcome of the Cross |
|----------------------------------|----------------------------------|----------------------|
| P-Female                         | Multiplication Rule              | Genotype             |
| P-Male                           | Addition Rule                    | Phenotype            |
|                                  | Multiply the probabilities       | ff 1/4               |
|                                  | along the branch                 | Ff 1/4               |
|                                  | 1/2 x 1/2 = 1/4                  | Ff 2/4               |
|                                  |                                  | FF 1/4               |
| 1/2                              |                                  | Ff 1/4               |
|                                  |                                  | Ff 3/4               |
|                                  |                                  |                     |

#### Figure A2. Construction of inheritance patterns of seed shape using a probability tree.

| Gamete formation - Mendel's Law | Fertilization - Probability Rules | Outcome of the cross |
|----------------------------------|----------------------------------|----------------------|
| P-Male                           | Multiplication Rule              | Genotype             |
| P-Female                         | Addition Rule                    | Phenotype            |
|                                  | Multiply the probabilities       | Rr 1/2               |
|                                  | along the branch                 | r 1/2                |
|                                  | 1 x 1/2 = 1/2                    | Rr 1/2               |
|                                  |                                  | r 1/2                |
| 1/2                              |                                  |                      |

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