1. Introduction

It is known that there is an intermediate model resolution in which neither the Reynolds Averaged Navier-Stokes Simulation (RANS) nor the Large-Eddy Simulation (LES) is applicable for a numerical simulation of the atmospheric boundary layer. This model resolution is termed “terra incognita” by Wyngaard (2004) or, more generally, “gray zone” in the atmospheric boundary layer. Honnert et al. (2011) regarded the model resolution corresponding to the terra incognita as the resolution at which the subgrid (SGS) part of the turbulent flux or turbulence kinetic energy (TKE) is comparable to their grid scale (GS) part. Ito et al. (2015) defined the terra incognita as the range of \(0.2 < \frac{\Delta h}{z_i} < 1.5\) (\(\Delta h\): the horizontal grid cell size, \(z_i\): the depth of the boundary layer) in terms of the magnitude of the resolved horizontal heat flux. The corresponding horizontal resolution locates approximately several hundred meters for a typical convective boundary layer. The horizontal resolution for numerical weather prediction (NWP) models is becoming close to the terra incognita range due to increasing computational resources. Therefore, developing a turbulence scheme applicable to the terra incognita range is required to achieve an appropriate NWP with such a high resolution.

In a RANS where the turbulent motion is completely parameterized, a boundary layer approximation (e.g., Mellor and Yamada 1982) is frequently
employed to model the atmospheric boundary layer, in which only the vertical turbulent transport is considered because the effect of the horizontal turbulent flux is negligible when each convective motion cannot be resolved. In other words, the RANS parametrization scheme in the atmospheric boundary layer is regarded as essentially one-dimensional and highly anisotropic. Conversely, a parametrization scheme used for an LES is based on three-dimensional isotropic turbulence; unresolved turbulence is expected to be isotropic in a sufficiently fine model resolution (inertial sub-range) such that the LES approach can be justified. One can expect that a turbulent scheme applicable to the terra incognita range should be three-dimensional and anisotropic if the effect of unresolved turbulence seamlessly varies with model resolution. Wyngaard (2004) indicated the importance of the anisotropic nature of turbulent transport in modelling the turbulent flux for the terra incognita. Honnert and Masson (2014) discussed the applicability of a vertically one-dimensional turbulent scheme from an analysis of the dynamical and thermal production terms in the TKE equation. They found that the contribution of dynamical production was enhanced and that the three-dimensionality of the turbulence scheme became important as the model resolution increased. Kitamura (2015) analyzed the dependence of the mixing lengths used in the formulation of eddy viscosity and thermal eddy diffusivity on the model resolution and reported that a remarkable anisotropic effect of the turbulent flux in subgrid scales appeared in the terra incognita.

Recently, turbulent parameterization schemes for terra incognita resolution have been presented. Boutle et al. (2014) blended the one-dimensional scheme proposed by Lock et al. (2000) and the Smagorinsky model (Smagorinsky 1963) which is usually used as an LES scheme. The weight between these two schemes is determined on the basis of TKE partitioning given in Honnert et al. (2011), who first analyzed the grid size dependence of TKE partitioning and turbulent fluxes using an a priori analysis. In the a priori analysis, a turbulent field with a lower resolution is obtained by removing small scale motions from a high resolution field using a low pass filter. The filtered field provides the resolved flux that should be represented at the target resolution defined by the width of the spatial filter. Varying the filter width allows us to investigate the grid size dependence. Shin and Hong (2015) attempted to incorporate the dependence on the horizontal resolution into the vertical heat transport parameterization for convective boundary layer simulations. Their scheme is based on the result obtained from Shin and Hong (2013), who extended the analysis of Honnert et al. (2011) by including effects of stability and separately treating non-local and local transport terms. Although their model successfully reproduced the vertical heat flux in a convective boundary layer even for the terra incognita range, the three-dimensional Deardorff model (Deardorff 1980) was employed in the evaluation of the horizontal diffusion. Ito et al. (2015) extended the Mellor-Yamada-Nakanishi-Niino (MYNN) model (Nakanishi and Niino 2009) to make it applicable to the terra incognita.

In the present study, I attempt to constitute a new parameterization scheme applicable to the terra incognita range. In the new scheme, the (isotropic) Deardorff model is extended so that the anisotropy of the turbulent flux is naturally considered. The length scales as the functions of the horizontal and vertical grid sizes are empirically determined on the basis of the results obtained from the a priori analysis for a convective boundary layer by Kitamura (2015) and are incorporated into the Deardorff model. The anisotropy is represented by defining the horizontal and vertical components of the turbulence mixing length separately. In contrast to previous studies in which one- and three-dimensional schemes are pragmatically combined, the proposed scheme is formulated in terms of the anisotropic three-dimensional turbulent flux. Numerical simulations for a convective boundary layer are examined for various horizontal grid sizes including the terra incognita range, and the new model are compared with the original Deardorff model to clarify the effect of the proposed length scales. In the present study, I focus on whether the new model is able to appropriately represent the convection and vertical heat flux corresponding to a target resolution including the terra incognita. The GS and SGS partitions of the TKE and vertical heat flux are discussed with special emphasis.

2. Model description

I employ an incompressible fluid on an f-plane under the Boussinesq approximation. The governing equations of the GS motion and temperature are written as follows:

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \varepsilon_{3ij} f (\bar{u}_j - U_j^f) + \frac{g}{\theta_0} \bar{\delta}_{ij}$$

$$- \frac{\partial \bar{v}_i}{\partial x_j},$$

(1)
\[
\frac{\partial \overline{\theta}}{\partial t} + \frac{\partial \overline{\theta} \overline{u}_j}{\partial x_j} = -\frac{\partial \tau_{\theta j}}{\partial x_j}, \tag{2}
\]

\[
\frac{\partial \overline{u}_i}{\partial x_i} = 0, \tag{3}
\]

where \( u_i \) is the velocity component in the \( x_i \) direction, \( \theta \) is the potential temperature, \( \theta_0 \) is a reference potential temperature, \( p \) is the pressure divided by the mean density, \( g \) is the acceleration of gravity, \( f \) is the Coriolis parameter, and \( U^p = (U^p_x, 0, 0) \) is the velocity of the horizontally uniform geostrophic flow corresponding to the imposed mean gradient of the pressure. The subscript \( i \) and \( j \) (= 1, 2, 3) represent the \( x, y, \) and \( z \) components, respectively. The overbar \( (\overline{\cdot}) \) denotes spatial filtering to extract the grid scale variable. The subgrid stress terms \( \tau_{ij} \) and the SGS temperature flux \( \tau_{\theta j} \), which represent the effect of SGS components, are defined as follows:

\[
\tau_{ij} = \overline{u_i u_j^\prime} - \overline{u_i u_j}, \tag{4}
\]

\[
\tau_{\theta j} = \overline{\theta u_j^\prime} - \overline{\theta u_j}. \tag{5}
\]

These terms must be parameterized because \( \overline{u_i u_j^\prime} \) and \( \overline{\theta u_j^\prime} \) are unknown variables in the GS Eqs. (1)–(3).

\( \tau_{ij} \) and \( \tau_{\theta j} \) are parameterized on the basis of the SGS scheme proposed by Deardorff (1980), where eddy viscosity and thermal eddy diffusivity are assumed. In the new model introduced here, the horizontal and vertical components of the eddy viscosity and thermal eddy diffusivity are treated separately to consider the anisotropy of the SGS flux in contrast to Deardorff’s original formulation. In the previous formulations for the terra incognita, this anisotropy was represented by combining a vertical one dimensional RANS scheme that is imposed on the boundary layer assumption and a horizontal eddy viscosity (e.g., Boutle et al. 2014; Shin and Hong 2015; Ito et al. 2015). The present formulation provides a more unified framework without combining two different schemes. The SGS flux for momentum is parameterized as follows:

\[
\tau_{ij} = -K_{Mvh} S_{ij} + (2\sigma + \Lambda)/3 \delta_{ij} \quad (i, j = 1, 2), \tag{6}
\]

\[
\tau_{i3} = \tau_{3i} = -K_{Msv} S_{i3} \quad (i = 1, 2), \tag{7}
\]

\[
\tau_{33} = -K_{Msv} S_{33} + (2\sigma + \Lambda)/3. \tag{8}
\]

Here

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right), \tag{9}
\]

\[
\Lambda = (K_{Mvh} - K_{Msv}) S_{33}. \tag{10}
\]

\( \sigma \) denotes the SGS TKE and is defined as

\[
\sigma = \tau_{ij} / 2. \tag{11}
\]

The subscripts \( h \) and \( v \) appended to the eddy viscosity coefficients \( K_M \) denote the horizontal and vertical components, respectively. It should be emphasized that the three types of eddy viscosity coefficients \( K_{Mvh}, K_{Msv} \), and \( K_{Msv} \) are not identical to each other. The term \((2\sigma + \Lambda/3)\) must be included in the diagonal components of the subgrid stress \( \tau_{ij} \) to satisfy Eq. (11). In particular, \( \Lambda/3 \) has to be considered when \( K_{Mvh} \neq K_{Msv} \). The horizontal and vertical components of the SGS temperature flux \( \tau_{\theta j} \) are also formulated separately in the same manner:

\[
\tau_{\theta j} = -K_{Hh} \overline{\theta u_j^\prime} / \overline{\theta x_j} \quad (j = 1, 2), \tag{12}
\]

\[
\tau_{\theta 3} = -K_{Hv} \overline{\theta u_3^\prime} / \overline{\theta x_3}, \tag{13}
\]

where \( K_{Hh} \) and \( K_{Hv} \) are the horizontal and vertical components of the thermal eddy diffusivity coefficients, respectively.

The equation of the SGS TKE can be described as

\[
\frac{d\sigma}{dt} = -\tau_{ij} \frac{\partial \overline{u}_i}{\partial x_j} + \frac{g}{\theta_0} \tau_{\theta 3} - \frac{\partial}{\partial x_i} (\overline{u}_i (e + p') - e), \tag{14}
\]

where \( p' \) is deviation of the pressure divided by the mean density and \( e \) is the TKE dissipation rate. In the Deardorff model, the correlation terms between the velocity and TKE and between the velocity and pressure are assumed to be

\[
\overline{u}_i (e + p') = -2K_{Mvh} \frac{\partial \overline{\theta}}{\partial x_i} \quad (i = 1, 2), \tag{15}
\]

\[
\overline{u}_3 (e + p') = -2K_{Msv} \frac{\partial \overline{\theta}}{\partial x_3}. \tag{16}
\]

The dissipation term \( e \) is parameterized as

\[
e = \frac{C_e \overline{p'}^{3/2}}{l_c}, \tag{17}
\]

which is derived from the Kolmogorov -5/3 power law in the energy spectrum for isotropic turbulence (Moeng and Wyngaard 1988). I set \( C_e = 0.7 \), following Deardorff (1980). \( l_c \) is a length scale characterizing the energy dissipation rate and will be determined later.

Each component of the eddy viscosity and thermal eddy diffusivity coefficients is determined from a
turbulence length scale and the TKE in the Deardorff model:

\begin{align}
K_{Mhh} &= C l_{\text{Mhh}} \sqrt{\overline{\varepsilon}}, \\
K_{Mhv} &= C l_{\text{Mhv}} \sqrt{\overline{\varepsilon}}, \\
K_{Mvv} &= C l_{\text{Mvv}} \sqrt{\overline{\varepsilon}}, \\
K_{hh} &= C l_{hh} \sqrt{\overline{\varepsilon}} / Pr,
\end{align}

where \( Pr \) is a turbulent Prandtl number. \( C = 0.1 \) and \( Pr = 1/3 \) are chosen, which are the same as the values adopted in an unstable condition by Deardorff (1980). Although the dependence of \( Pr \) on static stability or model resolution is not considered, the length scales estimated by Kitamura (2015) are derived from a constant turbulent Prandtl number \( Pr = 1/3 \). Hence, I employ this assumption also in the present study for consistency with the length scales estimated in Kitamura (2015).

All length scales for the LES have been traditionally defined as a geometric average of the grid cell sizes in each direction:

\[ l = (\Delta_x \Delta_y \Delta_z)^{1/3}, \tag{18} \]

where \( \Delta_x, \Delta_y \) and \( \Delta_z \) denote the grid scales in the \( x, y \) and \( z \) directions. In the typical LES parameterizations, the anisotropy of turbulence is not considered in the formulation of length scale \( l \). In contrast, Kitamura (2015) suggested that the anisotropy of the length scale appeared when the aspect ratio between the horizontal and vertical grid cell sizes was not unity. In the present study, I empirically determine the dependence of the turbulence length scales on the model resolution from the results obtained by the a priori analysis (Kitamura 2015) and apply them to the SGS model. Figure 1 indicates the dependence of the length scales on the horizontal grid size. Each component of the length scales is determined so as to fit that estimated from the a priori analysis in Kitamura (2015). The vertical components \( l_{hv} \) and \( l_{Mvv} \) are much larger than the isotropic length Eq. (18), while the horizontal ones \( l_{hh} \) and \( l_{Mhh} \) are slightly different from Eq. (18). The formulae for the length scales are defined as

\[ l = \left( \Delta_x \Delta_y \Delta_z \right)^{1/3}, \tag{18} \]
Here \( \Delta_h = \Delta_c = \Delta_v \) is assumed. I assume that \( l_{Mfv} \) is identical to \( l_{Hv} \) although Kitamura (2015) suggests that the two possess different dependences, because preliminary experiments revealed that \( l_{Mfv} \) estimated from Kitamura (2015) caused an underestimation of the SGS flux in the vertical direction.

The length scales defined in Eqs. (19)–(22) converge to an upper limit \( l_\infty \), which can be interpreted as the length at the RANS limit. It is assumed that all the components converge to a common upper limit, although the upper limit for \( l_{Mfv} \) and \( l_c \) estimated from the a priori analysis is larger than that for the other components, as is seen in Fig. 1. In the present study, \( l_c \) is calculated from the master length employed in the MYNN model (Nakanishi and Niino 2009), which can be calculated from the horizontal mean of the velocity and temperature fields. The master length \( l_{MYNN} \) in the MYNN model is determined as follows (Nakanishi and Niino 2009):

\[
\frac{1}{l_{MYNN}} = \frac{1}{l_s} + \frac{1}{l_T} + \frac{1}{l_B},
\]

\[
l_s = \begin{cases} \kappa z / 3.7, & \zeta \geq 1 \\ \kappa z (1 + 2.7 \zeta)^{-1}, & 0 \leq \zeta < 1 \\ \kappa z (1 - 100 \zeta)^{0.2}, & \zeta < 0, \end{cases}
\]

\[
l_T = 0.23 \int_0^{\infty} qzdz,
\]

\[
l_B = \begin{cases} q / N, & \partial \Theta / \partial z > 0 \text{ and } \zeta \geq 0 \\ [1 + 5(\kappa_r / l_T N)^{1/2}] q / N, & \partial \Theta / \partial z > 0 \text{ and } \zeta < 0, \\ \infty, & \partial \Theta / \partial z \leq 0, \end{cases}
\]

Here \( \kappa = 0.4 \) denotes the von Kármán constant, \( \zeta = z/L \) the nondimensional height normalized by the Monin-Obukhov length \( L \), \( N \) the Brunt-Bäisälä frequency, \( \Theta \) the horizontal mean temperature and \( \bar{q}_e = [(g/\Theta)|\bar{w}| / L_T]^{1/3} (\bar{w}|/|) \) the surface temperature flux. \( l_\infty \) is determined from the condition that the TKE dissipation rate in the Deardorff model (Eq. 17) should coincide with that in the MYNN model at the RANS limit:

\[
l_\infty = B_1 C_e l_{MYNN}/2\sqrt{2},
\]

where \( B_1 = 24 \), which is the constant derived from the MYNN model. Note that \( l_\infty \) is not constant; it depends on time and altitude.

The numerical model employed in the present study is the same as that described by Kitamura (2010, 2015). A pseudospectral method with the 2/3 rule for de-aliasing is adopted in a horizontal plane, and the vertical discretization follows the Lorentz grid, in which \( u, v, \theta \) and \( p \) are located at full levels and \( w \) at half levels. The vertical advection is evaluated with a third-order upwind finite difference scheme. Time integration is performed with the fourth-order Runge-Kutta method.

3. Experimental setup

A set of numerical experiments for several horizontal resolutions shown in Table 1 is examined to investigate dependence on the horizontal model resolution. The model domain is \( 13640 \times 13640 \times 3840 \) m³. It was confirmed that this domain size did not distort the results in comparison with the preliminary experiments with the larger domain size. The vertical grid size \( \Delta_z \) is set to 20 m (192 grids) for all the experiments, because I focus on the horizontal resolution dependence rather than the vertical one. The horizontal resolution \( \Delta_h \) indicated in Table 1 is applied in the calculation of the turbulence length scale. Note that I define \( \Delta_h \) as \( L_h/(2N) \), where \( L_h \) and \( N \) are the horizontal width of the domain and the truncation wavenumber because the smallest horizontal scale that the model is able to resolve is not \( L_h/M \), but is \( L_h/(2N) \) due to the application of the pseudospectral method with the 2/3 rule. The numerical experiments are performed using the length scale originally employed by Deardorff (1980) Eq. (18) (hereafter referred to as “CTRL”) as well as the length scales presented in Eqs. (19)–(22) (referred to as “NEW”).

The length scales in the new formulation are identical
to the original one Eq. (18) in the N341 experiment where Δ is equal to Δ, except for the imposed upper limit l∞.

The results of the experiment with the highest resolution (N341) is also used as a reference in a priori analysis, in which a turbulent field at a coarser resolution is obtained by applying a test filter to a flow field with finer resolution. Applying the test filter to the GS temperature equation Eq. (2) yields

\[
\frac{\partial \tilde{T}_j}{\partial t} + \frac{\partial (\tilde{T}_j \tilde{u}_j)}{\partial x_j} = -\frac{\partial (\tilde{T}_j + \tilde{\tau}_j)}{\partial x_j},
\]

\[
\tilde{T}_j := \tilde{\theta} \tilde{u}_j - \tilde{\theta} \tilde{u}_j.
\]

Here, ( ) denotes a test filter to extract a field with a target resolution. Equation (29) indicates that the GS and SGS temperature fluxes at the resolution defined by the width of the test filter should be represented as \( \tilde{\theta} \tilde{u}_j \) and \( \tilde{T}_j + \tilde{\tau}_j \) in terms of the comparison with Eq. (2). If the field obtained from the N341 experiments is assumed to be true, the GS and SGS fluxes in the test filtered equation Eq. (29) can be used for validating those obtained from the lower resolution experiments.

The horizontal mean of the initial potential temperature profile is given as \( \Theta(z) = 288.15 - \Gamma z \), where \( \Gamma \) is the lapse rate of the potential temperature. I set \( \Gamma = -4 \times 10^{-3} \) K m\(^{-1}\) to impose weak stratification for the initial basic state in the present experiments. The lateral boundary condition is periodic, and the free slip boundary condition is assumed at the top. The Rayleigh damping and Newtonian cooling with a relaxation time of 500 seconds were imposed in the upper one-fourth region to suppress the reflection of gravity waves at the top of the model. The surface momentum flux is diagnosed using the stability function of Beljaars and Holtslag (1991) based on the Monin-Obukhov similarity theory. The Coriolis parameter is set to \( f = 1.031 \times 10^{-4} \) s\(^{-1}\), which corresponds to the latitude 45°N. I examine four types of benchmark simulation, following Shin and Hong (2013). The imposed surface temperature

| Number of horizontal grids M | N341 | N170 | N85 | N42 | N21 | N10 | N5 |
|------------------------------|------|------|-----|-----|-----|-----|----|
| Truncation wavenumber N      | 341  | 170  | 85  | 42  | 21  | 10  | 5  |
| Horizontal resolution Δh [m] | 20   | 40.12| 80.24| 162.38| 324.76| 682 | 1364|

Table 2. The surface temperature flux and geostrophic flow velocity imposed in the four benchmark simulations. The acronyms BT, BF, SW and SS stand for buoyancy-driven (B) thermals (T), buoyancy-driven (B) wind-forced (F), weaker-shear (SW) and stronger-shear (SS), respectively (Shin and Hong 2013).

| Case | BT | BF | SW | SS |
|------|----|----|----|----|
| \( \tilde{\theta}_w \) [K m s\(^{-1}\)] | 0.2 | 0.2 | 0.05 | 0.05 |
| \( U_g \) [m s\(^{-1}\)] | 10 | 10 | 15 | 15 |

4. Results

Figure 2 shows the spectra of the kinetic energy for the BF and SW cases. The spectra in the BF and SW cases qualitatively resemble each other except that the spectral power in SW is smaller than that in BF. In the highest resolution experiment (N341), the energy spectrum follows a –5/3 slope for the wavenumbers from approximately \( 2 \times 10^{-3} \) to \( 5 \times 10^{-2} \). Moreover, the energy spectrum becomes steeper in the higher wavenumber region due to numerical diffusion arisen from the third-order advection scheme used for the vertical advection and due to the energy dissipation derived from the SGS scheme. Namely, the wave-number of the effective resolution is smaller than that of the grid cell size. For CTRL, however, the region in which the energy dissipation occurs tends to disap-
appear as the model resolution is coarser and artificial energy accumulation is observed in N5, N10 and N21 experiments. Takemi and Rotunno (2003) reported that a spurious build up of energy at high wavenumbers is seen in the LES with a 1 km resolution, unless the eddy viscosity coefficient is larger than that used in a standard LES simulation. The energy spectrum in the CTRL simulation is consistent with their result. Insufficient SGS diffusion results in energy accumulation, which should be avoided because it causes a reflection to the motion in the lower wavenumber modes (Skamarock 2004). The original Deardorff model cannot appropriately represent a resolved field when the horizontal grid size is larger than that in the N42 experiment (Δ ~ 160 m) for both the BF and SW cases. In contrast, artificial energy accumulation is not observed for all the experiments in NEW, although the spectral magnitude in the energy containing range is insufficient for N5 and N10 experiments in the SW case.

The ratio of the GS or SGS TKE to the total TKE is frequently used as a measure to validate resolution dependence of the resolved turbulence in a model (e.g., Honnert et al. 2011; Shin and Hong 2015; Ito et al. 2015). Honnert et al. (2011) reported that the TKE partition between the GS and SGS part was well represented as a function of the horizontal grid size normalized by the height of the boundary layer. Horizontal grid size dependence of the SGS TKE normalized by the total TKE is shown in Fig. 3. The height of the boundary layer $z_i$ in the present analysis is defined as the height at which the vertical gradient of the horizontal mean potential temperature has a maximum. In this figure, the similarity function applied in the range of $0.2 \leq z/z_i \leq 0.8$ presented by

![Energy spectra](image)

Fig. 2. Energy spectra for the kinetic energy averaged over $z = 500 – 1000$ m (BF) and $z = 300 – 600$ m (SW) at $t = 5$ h. The abscissa is the horizontal wavenumber $k_h = (k_x^2 + k_y^2)^{1/2}$ ($k_x, k_y$: wavenumbers in the x and y direction). The reference line represents a $-5/3$ slope.

![TKE partition](image)

Fig. 3. Horizontal grid size dependence of the SGS TKE normalized by the total (sum of the GS and SGS parts) TKE. The horizontal grid size in the abscissa is scaled by the height of the boundary layer. The results averaged over the last one hour of the simulations in the mixed layer ($0.2 \leq z/z_i \leq 0.8$) are shown. The open and filled black markers indicate the CTRL and NEW experiments, respectively. The gray markers show the results obtained from the N341 experiments by reducing the resolution. The solid line and the two dashed lines represent the similarity function and the first and last vigintiles for the TKE partition presented by Honnert et al. (2011), respectively.
Honnert et al. (2011) is drawn as a solid line. Their similarity function has been compared with results of benchmark simulations using a terra incognita scheme (Ito et al. 2015). The TKE partition obtained from the highest resolution data (N341) by the procedure described in Section 3 is shown as the gray marker. This comparison between the similarity function and the result of the a priori analysis allows us to confirm the robustness of the resolution dependence of the TKE partition. The result of the a priori analysis is well consistent with the similarity function by Honnert et al. (2011), while it tends to be slightly smaller than the similarity function as the horizontal grid size increases. Hence, the solid line or the gray markers would be regarded as a reference to validate the resolution dependence of the TKE partition. The original Deardorff model apparently underestimates the SGS TKE for $\Delta h/z_i > 0.2$, while it is consistent with the similarity function for the LES regime ($\Delta h/z_i < 0.1$); the original model cannot be applied to a simulation for the terra incognita range. Conversely, the TKE partition in the new model well coincides with the similarity function across a wide range of $\Delta h/z_i$ for all the benchmark cases, while the spread among the test cases is magnified around $\Delta h/z_i \sim 1$. In particular, the new model improves the representation of the TKE partition in the terra incognita ($0.2 < \Delta h/z_i < 1.5$), while it slightly overestimates the SGS TKE in the LES regime.

Note that the ratio of the SGS TKE in the SW and SS cases is higher than that in the BF and BT cases, while the results in the a priori analysis shows the opposite tendency as reported by Shin and Hong (2013). The resolution dependency of the length scales by Kitamura (2015) was estimated only from the experiments in which coherent structure is characterized as convective cells. In the present benchmark simulations, a roll-type convection is dominant in SS and SW and a convective cell pattern is dominant in BF and BT. The SGS diffusion calculated from the present length scales might be excessive for roll convection cases.

The horizontal mean profiles of the temperature for the BF and SW cases are presented in Fig. 4. The boundary layer height in SW is lower than that in BF due to the weaker heat flux imposed at the surface. Slight resolution dependence of the boundary layer height is observed; i.e, it becomes lower as the horizontal resolution is coarser. Nevertheless, its spread is within 80 m. The temperature profile in the mixed layer is almost independent of the horizontal resolution for both CTRL and NEW. In the experiments with the N5 setting for the SW case, the temperature profile in the upper part of the mixed layer is unstable and deviated from that in the higher resolution experiments, while the new model alleviates this deviation than the original one. Positive SGS heat flux is directly related to unstable stratification when a SGS parameterization scheme is formulated by the thermal eddy diffusivity form, which is assumed
in the Deardorff model. Therefore, the temperature profile is prone to be unstable with coarse resolution in which the SGS component in the heat flux is dominant. This is a deficiency commonly seen in schemes based on the eddy viscosity and thermal eddy diffusivity (Wyngaard 2010), and it is difficult to avoid this problem without introducing a counter-gradient term or a non-local scheme. The normalized grid size \( \Delta_h/\bar{z}_i \) in the SW case is greater than that in the BF case for the same resolution, because the boundary layer height is lower in the SW. This fact suggests that the temperature profile in the SW case tends to become more unstable.

Note that the insensitivity in the vertical structure of the temperature does not justify the correctness of the parameterization schemes because this insensitivity would be attributed to the constant surface heat flux regardless of the air temperature. The thickness of the mixed layer is controlled primarily by the sensitive heat supplied from the surface. In contrast, the temperature gradient in the surface layer becomes weaker as the grid size becomes larger. This resolution dependence in the surface layer is commonly observed in CTRL and NEW. The mixing length in the new model is determined by the imposed upper limit \( l_\infty \) rather than the model resolution in the vicinity of the surface because \( l_\infty \) determined by the MYNN model is restricted by the height from the surface due to the effect of \( l_S \) defined as Eq. (24). Nevertheless, the new model does not sufficiently improve the temperature profile in the surface layer.

Figure 5 shows the horizontal grid size dependence of the ratio of the SGS vertical heat flux to the total one. Each plot is drawn as an average in \( 0.2 \leq z/\bar{z}_i \leq 0.55 \), because the similarity function by Honnert et al. (2011) is defined in the range of \( 0.2 \leq z/\bar{z}_i \leq 0.55 \). The ratio of the SGS heat flux obtained by the coarse graining from the highest resolution is consistent with the similarity function by Honnert et al. (2011). In the LES range (\( \Delta_h/\bar{z}_i < 0.1 \)), the CTRL experiments are consistent with the similarity function rather than the NEW experiments, in which the SGS ratio is overestimated. This overestimation seen in NEW would be partly ascribed to a decline of the effective resolution due to the energy dissipation range at higher wavenumbers (Fig. 2). The SGS component in CTRL is underestimated with increasing \( \Delta_h/\bar{z}_i \) and is apart from the similarity function for \( \Delta_h/\bar{z}_i > 0.1 \). The new model improves the GS and SGS partition of the vertical heat flux in the terra incognita range. However, a wide spread is observed around \( \Delta_h/\bar{z}_i \approx 1 \), i.e., the SGS flux is underestimated for the BF and BT cases and overestimated for the SS case. The wide spread in the heat flux partition might result from these difference in the coherent structure. Further improvement in the resolution dependence of the length scales would be necessary to apply the turbulence scheme to a wider range of \( \Delta_h \).

Figure 6 displays the vertical profile of the temperature flux divided into the GS and SGS parts for the BF and SW cases. The left panels show the results obtained from the coarse graining of the N341 experiment used as a reference field (hereafter, “REF”). As the horizontal grid size becomes larger, the GS part of the temperature flux decreases and the SGS part increases as expected. Decrease (increase) in the GS (SGS) part of the flux in the lower layer would be attributed to restrictions on the resolved vertical motions imposed by the surface. The altitude where the GS and SGS parts are comparable is higher with the increase in the horizontal grid size. In CTRL, the GS (SGS) part of the temperature flux is apparently overestimated (underestimated) in comparison with REF. Note that the total flux is insensitive to the horizontal resolution even for CTRL due to the constant
surface sensitive heat flux. Although the insensitivity of the total flux is insufficient to judge the convergence of LES solutions (Sullivan and Patton 2011), dividing the flux into the GS and SGS parts reveals inadequateness of a SGS model. An overestimation of the GS flux means that the artificial resolved flux is enhanced and the representation of the SGS flux is insufficient. Therefore, the length scale defined in the original Deardorff model is inappropriate for coarser resolution although the total flux is consistent under a constant surface heat flux condition. The NEW experiments well reproduce the GS and SGS fluxes. Each part of the temperature flux is consistent with that in REF for both the BF and SW cases, whereas

Fig. 6. Vertical profile of the temperature flux. The results for the BF and SW cases are shown in the upper and lower panels. The solid and dashed lines indicate the GS and SGS components of the flux, respectively. The left panel displays the flux obtained from the N341 experiment using the procedure of a priori analysis.
the GS part tends to be underestimated in the surface layer. This underestimation in the surface layer can be directly confirmed by the cospectra between the potential temperature $\theta$ and the vertical velocity $w$ in the lower layer shown in Fig. 7. The dissipation range near the truncation wavenumber contributes to the underestimation. Moreover, the decrease of the cospectra in the lower wavenumbers ($k_h \leq 10^{-3} \text{ m}^{-1}$) is remarkable for N42, N85 and N170 in NEW.

The total flux in the entrainment zone, in which the temperature flux is negative, is slightly underestimated with coarser resolution for both CTRL and NEW. The negative SGS flux in CTRL vanishes except for N5 and N10 in the SW case, although the GS flux apparently overestimated; this distribution is quite unrealistic. Conversely, resolution dependence of the SGS flux in NEW is well reproduced for the BF case in comparison with that of REF. However, the GS flux tends to be underestimated as the resolution is coarser. The SGS flux for the SW case is less improved than that for the BF case; it is underestimated with increase of the grid size. This insufficient heat transfer in the entrainment zone seems to cause decrease in the boundary layer height in the coarse resolution cases as seen in Fig. 4.

The variance of the resolved vertical velocity is a good indicator for the magnitude of the resolved convection. Figure 8 displays the resolved vertical velocity variance for the CTRL and NEW experiments in the BF and SW cases. The resolution dependence is similar between the BF and SW cases, while the variance in the SW has a secondary peak at the top of the boundary layer in contrast to that in the BF, which is a unimodal profile. In CTRL, the variance is almost independent of the model resolution in contrast to REF, in which the resolved convection decreases with increasing $\Delta h$. This result directly shows predominance of the artificial convection in the coarse resolutions for the CTRL experiments. The magnitude and peak altitude of the vertical velocity variance in NEW are consistent with REF, while the magnitude of the vertical velocity variance is underestimated in the lower layer.

The predominance of the artificial convection can be directly confirmed by a snapshot of the resolved vertical velocity. The horizontal cross section of the vertical velocity for the BF case is displayed in Fig. 9. A convective cell pattern is observed in the highest resolution experiment (N341) and it also remains in the field obtained by reducing resolution from the highest resolution experiment (Fig. 9b). However, the convection with the grid size, which is quite different from the structure seen in Fig. 9b, is dominant in CTRL (Fig. 9c), and the vertical velocity is much stronger than the filtered one. The cell structure can be reproduced in the new model. The vertical velocity is comparable to that in the filtered field, while the width of the convective cells is slightly wider than the reference.

Fig. 7. Cospectra between the potential temperature $\theta$ and the vertical velocity $w$ averaged over $z = 50–250 \text{ m}$ for the BF case. Negative values are not plotted in the figure.
The variance of the resolved velocity in the $x$ direction is shown in Fig. 10. The vertical profile has two peaks at the bottom and top of the boundary layer in which horizontal convergence and divergence induced by convection are enhanced. The horizontal velocity variance as well as the vertical one tends to be overestimated in CTRL except for SW with the N5 setting. The excessive amplification with lower resolutions is seen in the lower layer for the BF case. Although the new model improves the magnitude of the variance, it is still overestimated at the bottom and top of the boundary layer. In particular, the peak seen in the top of the boundary layer is remarkably enhanced for the SW case. The enhancement of the peak might be due to the effect of $l_B$ defined in Eq. (26), which make the length at the RANS limit $l_\infty$ smaller in the stable stratification. The vertical component of the mixing lengths $l_{Mhv}$ is sensitive to $l_\infty$ in the present formula-

![Graph showing the variance of the resolved vertical velocity fluctuation for BF, REF, CTRL, and NEW cases.](image)

Fig. 8. Variance of the resolved vertical velocity fluctuation. The line types are the same as in Fig. 2.
The upper limit of the lengths would be excessively suppressed for an entrainment zone with stable stratification.

5. Concluding remarks

To investigate a turbulence closure model applicable to the terra incognita range, I introduced the anisotropic length scales estimated by Kitamura (2015) and incorporated them into the Deardorff model. The concept of the eddy viscosity and thermal eddy diffusivity in the Deardorff model was extended to consider their anisotropy. This approach is different from that in the previous studies (Boutle et al. 2014; Shin and Hong 2015; Ito et al. 2015), in which the effect of the unresolved fluctuations is represented by combining a vertical one-dimensional RANS and a horizontal eddy viscosity. Introducing anisotropic length scales in the present formulation provides a unified framework for a turbulence parameterization without pragmatically combining two different schemes. The new scheme was examined with various horizontal resolutions in the range of $0.025 \leq \Delta_h/z_i \leq 1.8$ for four types of the ideal unstable boundary layer cases presented in Shin and Hong (2013) and
compared with the original Deardorff model. The new model improved the representation of the GS and SGS parts of the temperature flux and the magnitude of the resolved convection even for the coarse resolution including the terra incognita range, whereas the original model was not capable of appropriately representing the SGS flux in this range. The original model tended to underestimate the SGS component and cause artificial energy accumulation at higher wave-numbers. The energy dissipation range near the truncation wavenumber remained regardless of the model resolution in the new model.

However, some problems remain unresolved. The SGS component in the new model tends to be overestimated in the LES range ($\Delta h/z_i < 0.1$), in which the original Deardorff model is applicable. This might be attributed to the overestimation of $l_{Mhv}$ and $l_{Mv}$, because these lengths are sensitive to the horizontal
grid size and are much larger than the geometric average of the grid sizes used as the isotropic length scale in the LES range. Modifying these length in the LES range would improve the partition between the GS and SGS components.

The temperature gradient in the surface layer was weakened as the grid cell size increased. Table 3 summarizes the friction velocity \( u_* \), the Obukhov length \( L \), and the surface temperature \( \theta_s \) calculated from \( u_* \) and the imposed surface heat flux, which characterize the surface flux property. The friction velocity \( u_* \) is larger and the (negative) Obukhov length \( L \) is smaller as the grid cell size increases. However, these variables should be insensitive to the resolution. This dependence still remains in the new model, although it is alleviated than the original Deardorff model. This finding suggests that the eddy viscosity and thermal eddy diffusivity in the vertical direction are still overestimated in the vicinity of the surface, although the mixing length is restricted by the height from the surface. The presented formulations for the length scales would be insufficient in the surface layer; the length scales for the surface layer should be determined independently of those in the mixed layer.

It is also noted that the present experiments are confined to several cases of an ideal convective boundary layer. Various simulations for a boundary layer would be required to confirm the validity of the proposed scheme. Moreover, investigating a stable boundary layer is necessary because the dependence of the length scales in a stable boundary layer could be considerably different from that in an unstable one. Nevertheless, the new model proposed in the present study is the first step towards developing a turbulence scheme applicable to the terra incognita range and is also feasible. Further consideration is required for developing a closure model for the terra incognita.

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