Combined large-$N_c$ and heavy-quark operator analysis of 2-body meson-baryon counterterms in the chiral Lagrangian with charmed mesons

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Abstract

The chiral $SU(3)$ Lagrangian with pseudoscalar and vector $D$ mesons and with the octet and decuplet baryons is considered. The leading two-body counter terms involving two baryon fields and two $D$ meson fields are constructed in the open-charm sector. There are 26 terms. A combined expansion in the inverse of the charm quark mass and in the inverse of the number of colors provides sum rules that reduce the number of free parameter down to 5 only. Our result shows the feasibility of a systematic computation of the open-charm baryon spectrum based on coupled-channel dynamics.

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I. INTRODUCTION

The chiral $SU(3)$ Lagrangian with the pseudoscalar and vector $D$ mesons has been applied extensively in the literature (see e.g. [1–4]). Most exciting are recent studies on s-wave scattering of Goldstone bosons off $D$ mesons [5–8]. The leading order coupled-channel interaction leads to the formation of scalar and axial-vector $D$ mesons with properties compatible with the known empirical constraints. Such results are similar to findings on the s-wave scattering of the Goldstone bosons off the baryon octet and decuplet states (see e.g. [9–11]), where the coupled-channel dynamics based on the leading order chiral Lagrangian generates s- and d-wave baryon resonances.

The main objective of the present study is to pave the way for systematic coupled-channel computations on open-charm baryon resonances. A first application of the chiral Lagrangian to s-wave baryon resonances considered the coupled-channel interaction of the Goldstone bosons with the ground-state baryons with open charm [12]. A rich spectrum of chiral excitations was obtained. The challenge of the open-charm sector is the possibility of charm-exchange reactions, where the charm of the baryon is transferred to the meson. A first phenomenological case study modeled the coupled-channel force by a t-channel exchange of vector mesons in the static limit [13–15]. Such a t-channel force recovers the leading order predictions of the chiral symmetry whenever Goldstone bosons are involved. It was shown in [13, 14, 16] that a rich spectrum of s- and d-wave baryon resonances is generated dynamically based on such a schematic ansatz. Two types of resonances are generated. The resonances of the first type are formed predominantly by the interaction of the Goldstone bosons with the open-charm baryon ground states, and the second type is a consequence of the coupled-channel interaction of the $D$ mesons with the octet and decuplet baryons. For both types of resonances the exchange of the light vector mesons constitute the driving forces that generate the hadronic molecules. The exchange of charmed vector mesons leads to the typically small widths of the second type resonances.

While the interaction of the Goldstone bosons with any hadron is dictated by chiral symmetry, this is not true for the interaction of the $D$ mesons with e.g. baryons. At leading order the $D$ mesons interact with baryons via local counter terms that are undetermined by chiral symmetry. This resembles the situation encountered in chiral studies of the nucleon-nucleon force (for a review see e.g. [17]). Only the long-range part of the interaction is
controlled by chiral interactions, the short range part needs to be parameterized in terms of a priori unknown contact terms. Needless to state that a reliable coupled-channel computation requires the consideration of both, the short-range and long-range forces.

The purpose of the present work is a systematic construction of the leading order contact terms for the interaction of the $D$ mesons with the baryon octet and decuplet states. Though the chiral symmetry does not constraint such contact terms, there are significant correlations amongst the counter terms implied by the heavy-quark symmetry and the large-$N_c$ limit of QCD. In the limit of a large charm quark mass the pseudoscalar and vector $D$ mesons form a spin multiplet with degenerate properties [1]. Thus, a systematic coupled-channel computation in this limit requires the simultaneous consideration of pseudoscalar and vector $D$ mesons. This leads necessarily to the presence of long-range $t$-channel forces. The transition of a vector $D$ meson to a pseudoscalar $D$ meson involves a Goldstone boson. A first attempt to consider pseudoscalar and vector $D$ mesons on an equal footing, however, assuming zero range forces only, can be found in [18]. Similarly, in the limit of a large number of colors in QCD the baryon octet and decuplet states form a super multiplet [19, 20]. This asks for the simultaneous consideration of the octet and decuplet baryons.

In a recent study two of the authors worked out the implications of large-$N_c$ QCD on the local counter terms of the Goldstone boson interaction with the baryon octet and decuplet states [21]. The technology developed in [22, 23] was applied. Matrix elements of current-current correlation functions were evaluated in the baryon octet and decuplet states. The matrix elements were expanded in powers of $1/N_c$ applying the operator reduction rules of [23]. This technology is well suited for an application to the open-charm sector. The implications of heavy-quark symmetry on the counter terms can be worked out using a suitable multiplet representation of the pseudoscalar and vector $D$ mesons [2–4].

II. CHIRAL LAGRANGIAN WITH MESON AND BARYON FIELDS

The construction rules for the chiral $SU(3)$ Lagrangian density are recalled. For more technical details see for example [24–32]. The basic building blocks of the chiral Lagrangian
where we include the pseudoscalar meson octet fields \( \Phi(J^P = 0^-) \), the baryon octet fields \( B(J^P = \frac{1}{2}^+) \) and the baryon decuplet fields \( B_\mu(J^P = \frac{3}{2}^+) \). Furthermore we introduce the \( SU(3) \) flavor antitriplet fields \( D(J^P = 0^-) \) and \( D_{\mu\nu}(J^P = 1^-) \) to describe the pseudoscalar and vector \( D \) mesons \([6]\). Explicit chiral symmetry-breaking is included in terms of scalar and pseudoscalar source fields \( \chi_{\pm} \) proportional to the quark mass matrix of QCD with \( \chi_0 = 2 B_0 \text{diag}(m_u, m_d, m_s) \). The merit of the particular field combinations in Eq. (1) is their identical transformation property under chiral \( SU(3)_L \otimes SU(3)_R \) rotations.

The octet fields may be decomposed into their isospin multiplets,

\[
\Phi = \tau \cdot \pi + \alpha^\dagger \cdot K + K^\dagger \cdot \alpha + \eta \lambda_8, \\
\sqrt{2} B = \alpha^\dagger \cdot N + \lambda_8 \Lambda + \tau \cdot \Sigma + \Xi^t i \sigma_2 \cdot \alpha, \\
\alpha^\dagger = \frac{1}{\sqrt{2}} (\lambda_4 + i \lambda_5, \lambda_6 + i \lambda_7), \quad \tau = (\lambda_1, \lambda_2, \lambda_3),
\]

with \( K = (K^+(+), K^{0(0)})^t \), for instance. The matrices \( \lambda_i \) are the Gell-Mann matrices and \( \sigma_2 \) is the second Pauli matrix. The open-charm fields \( D \) and \( D_{\mu\nu} \) form flavor antitriplets with e.g. \( D = (D^0, -D^+, D_s^+) \). The baryon decuplet fields \( B^{\mu\nu}_{abc} \) are completely symmetric in \( a, b, c = 1, 2, 3 \) and are related to the physical states by

\[
B^{111} = \Delta^{++}, \quad B^{113} = \Sigma^+ / \sqrt{3}, \quad B^{133} = \Xi^0 / \sqrt{3}, \quad B^{333} = \Omega^-, \\
B^{112} = \Delta^+ / \sqrt{3}, \quad B^{123} = \Sigma^0 / \sqrt{6}, \quad B^{233} = \Xi^- / \sqrt{3}, \\
B^{122} = \Delta^0 / \sqrt{3}, \quad B^{223} = \Sigma^- / \sqrt{3}, \\
B^{222} = \Delta^-.
\]

To cope with flavor indices in the products of \( SU(3) \) tensors containing decuplet fields, we shall apply a compact dot-notation suggested in \([33]\):

\[
(\bar{B}^\mu \cdot B^\nu)_k^m \equiv \bar{B}_{ijk}^\mu B^{ijm}_\nu, \quad (\bar{B}^\mu \cdot \Phi)_k^m \equiv \bar{B}_{ijk}^\mu \Phi^i_\ell \epsilon^{jlm}, \quad (\Phi \cdot B^\mu)_k^m \equiv B^{ijm}_\nu \Phi^i_\ell \epsilon_{jlk}.
\]

The chiral Lagrangian consists of all possible interaction terms, formed with the fields \( U_\mu, B, B_\mu, D, D_{\mu\nu} \) and \( \chi_{\pm} \). Derivatives of the fields must be included in compliance with the local chiral \( SU(3) \) symmetry. This leads to the notion of a covariant derivative \( \mathcal{D}_\mu \) which
is identical for all fields in Eq. (1). For baryons, the covariant derivative $\mathcal{D}_\mu$ acts on the octet and decuplet fields as follows:

$$
(\mathcal{D}_\mu B^a_b) = \partial_\mu B^a_b + \Gamma^a_{\mu,l} B^l_b - \Gamma^a_{\mu,b} B^b_l ,
$$

$$
(\mathcal{D}_\mu B^{abc}_\nu) = \partial_\mu B^{abc}_\nu + \Gamma^a_{\mu,l} B^{lbc}_\nu + \Gamma^b_{\mu,l} B^{abc}_\nu + \Gamma^c_{\mu,l} B^{abd}_\nu ,
$$

(5)

with $\Gamma^\mu = -\Gamma^{\dagger}_\mu$ given by

$$
\Gamma^\mu = \frac{1}{2} e^{-i \frac{\Phi}{2}} \left[ \partial_\mu - i (v_\mu + a_\mu) \right] e^{i \frac{\Phi}{2}} + \frac{1}{2} e^{i \frac{\Phi}{2}} \left[ \partial_\mu - i (v_\mu - a_\mu) \right] e^{-i \frac{\Phi}{2}} .
$$

Analogous expressions hold for the covariant derivatives for the open-charm fields.

The chiral Lagrangian is a powerful tool, once it is combined with appropriate counting rules leading to a systematic approximation strategy. We aim at describing hadronic interactions at low-energy by constructing an expansion in small momenta and small quark masses. In the following we construct the leading order two-body counter terms involving two $D$ meson and two baryon fields.

We begin with the terms with two baryon octet fields. There are 12 leading order terms

$$
\mathcal{L} = D \left\{ c_1^{(S)} (\bar{B} B)_+ + c_2^{(S)} (\bar{B} B)_- + \frac{1}{2} c_3^{(S)} \text{tr} (\bar{B} B) \right\} \bar{D} 
- \frac{1}{2} D_{\mu\nu} \left\{ c_1^{(S)} (\bar{B} B)_+ + c_2^{(S)} (\bar{B} B)_- + \frac{1}{2} c_3^{(S)} \text{tr} (\bar{B} B) \right\} \bar{D}^{\mu\nu} 
+ \frac{i}{M_c} D_{\mu\nu} \left\{ c_1^{(A)} (\bar{B} \gamma_5 \gamma^\mu B)_+ + c_2^{(A)} (\bar{B} \gamma_5 \gamma^\mu B)_- + \frac{1}{2} c_3^{(A)} \text{tr} (\bar{B} \gamma_5 \gamma^\mu B) \right\} (\partial^\nu \bar{D}) + \text{h.c.} 
+ \frac{1}{4} M_c \epsilon^{\mu
u\alpha\beta} D_{\mu\nu} \left\{ c_1^{(A)} (\bar{B} \gamma_5 \gamma_\alpha B)_+ + c_2^{(A)} (\bar{B} \gamma_5 \gamma_\alpha B)_- + \frac{1}{2} c_3^{(A)} \text{tr} (\bar{B} \gamma_5 \gamma_\alpha B) \right\} (\partial^\tau \bar{D}_{\tau\beta}) + \text{h.c.} ,
$$

(6)

where $\bar{D} = D^\dagger$, $M_c$ is the mass of the charm quark and $(\bar{B} \Gamma B)_\pm$ stands for (anti-)commutator in flavor space. All structures describe s-wave scattering and therefore we assign the chiral power $Q^0$ to them. The total number of terms in Eq. (6) is readily understood. There are three different flavor $SU(3)$ invariants only implying three types of coupling constants $c_1^{(-)}, c_2^{(-)}$ and $c_3^{(-)}$. It remains to understand the four spin structures. There is one spin structure involving the pseudoscalar $D$ mesons with the coupling constants $c_3^{(S)}$. This follows since a two-body system of a spin zero and a spin one-half particle allows to form one s-wave state only. In contrast, two s-wave states involving the vector $D$ mesons are possible. The relevant coupling constants are introduced with $c_3^{(S)}$ and $c_3^{(A)}$. The terms
parameterized with $c^{(A)}$ describe the s-wave transitions from a pseudoscalar to a vector $D$ meson.

We continue with the leading order terms involving two baryon decuplet fields. At leading order we find the relevance of the following 10 terms

$$\mathcal{L} = -D \left\{ d_1^{(S)} \bar{B}^\sigma \cdot B_\sigma + \frac{1}{2} d_2^{(S)} \text{tr} (\bar{B}^\sigma \cdot B_\sigma) \right\} \bar{D}$$
$$+ \frac{1}{2} D_{\mu \nu} \left\{ d_1^{(S)} \bar{B}^\sigma \cdot B_\sigma + \frac{1}{2} d_2^{(S)} \text{tr} (\bar{B}^\sigma \cdot B_\sigma) \right\} D^{\mu \nu}$$
$$+ \frac{i}{4} \epsilon^{\mu \nu \alpha \beta} D_{\mu \nu} \left\{ d_1^{(E)} \bar{B}_\alpha \cdot B_\beta + \frac{1}{2} d_2^{(E)} \text{tr} (\bar{B}_\alpha \cdot B_\beta) \right\} \bar{D} + \text{h.c.}$$
$$+ \frac{1}{2} D_{\beta \mu} \left\{ d_3^{(E)} \bar{B}_\alpha \cdot B_\beta + \frac{1}{2} d_4^{(E)} \text{tr} (\bar{B}_\alpha \cdot B_\beta) \right\} \bar{D}^{\beta \mu}$$
$$- \frac{1}{2} D_{\alpha \mu} \left\{ d_3^{(E)} \bar{B}_\alpha \cdot B_\beta + \frac{1}{2} d_4^{(E)} \text{tr} (\bar{B}_\alpha \cdot B_\beta) \right\} \bar{D}^{\alpha \mu}.$$  \hspace{1cm} (7)

In this case there are two independent flavor structures. Again, there is one spin structure involving two pseudoscalar $D$ meson fields with coupling constants $d^{(S)}$. Since a two-body system of a spin three-half and a spin one particle allows to form three s-wave states only, there are three different contractions of the Lorentz indices in the terms containing two vector meson fields. There remains the last spin structure parameterized by $d^{(E)}$ describing the s-wave transitions from a pseudoscalar to a vector $D$ meson.

We close the collection of our counter terms with structures involving a baryon octet and a decuplet field. At leading order we find the following 4 terms

$$\mathcal{L} = \frac{i}{4} \epsilon^{\mu \nu \alpha \beta} \left\{ e_1^{(A)} D (\bar{B}_\mu \cdot \gamma_5 \gamma_\nu B) \bar{D} + e_2^{(A)} D_{\alpha \beta} (\bar{B}_\mu \cdot \gamma_5 \gamma_\nu B) \bar{D} + \text{h.c.} \right\}$$
$$+ \frac{e_1^{(A)}}{2} D^{\tau \nu} (\bar{B}_\mu \cdot \gamma_5 \gamma_\nu B) \bar{D}_{\tau} + \text{h.c.}$$
$$- \frac{e_2^{(A)}}{2} D^{\tau \mu} (\bar{B}_\mu \cdot \gamma_5 \gamma_\nu B) \bar{D}_{\tau} + \text{h.c.}.$$  \hspace{1cm} (8)

Altogether we constructed 26 leading order two-body counter terms. In the following we shall use the heavy-quark symmetry in order to correlate the coupling constants introduced above.

### III. HEAVY QUARK MASS EXPANSION

In order to work out the implications of the heavy-quark symmetry of QCD it is useful to introduce auxiliary and slowly varying fields, $P_\pm(x)$ and $P^\mu_\pm(x)$. We decompose the $D$
meson fields into such fields
\[ D(x) = e^{-i(v \cdot x) M_c} P_+(x) + e^{+i(v \cdot x) M_c} P_-(x), \]
\[ D^{\mu\nu}(x) = i e^{-i(v \cdot x) M_c} \left\{ v^\mu P_+^\nu(x) - v^\nu P_+^\mu(x) + \frac{i}{M_c} \left( \partial^\mu P_+^\nu - \partial^\nu P_+^\mu \right) \right\} + i e^{+i(v \cdot x) M_c} \left\{ v^\mu P_-^\nu(x) - v^\nu P_-^\mu(x) - \frac{i}{M_c} \left( \partial^\mu P_-^\nu - \partial^\nu P_-^\mu \right) \right\}, \]
with a 4-velocity \( v \) normalized by \( v^2 = 1 \). The mass parameter \( M_c \) was already introduced in Eq. (6). As a consequence of Eq. (9), time and spatial derivatives of the fields \( \partial_\alpha P_{\pm} \) are small compared to \( M_c v^\alpha P_{\pm} \). In the limit \( M_c \to \infty \) the former terms can be neglected. Note that the fields \( P_{\pm} \) and \( P^\mu_{\pm} \) annihilate quanta with charm content \( \pm 1 \). From the equation of motion for the vector \( D \) mesons it follows
\[ \partial^\mu \partial_\alpha D^{\alpha\nu} - \partial^\nu \partial_\alpha D^{\alpha\mu} + M_c^2 D^{\mu\nu} = 0 \leftrightarrow \]
\[ \left\{ (\partial \cdot \partial) \mp 2 M_c i (v \cdot \partial) \right\} P^\mu_\pm = 0 \quad \& \quad \mp i M_c v_\mu P^\mu_\pm + \partial_\mu P^\mu_\pm = 0, \]
which teaches us that any term \( v_\mu P^\mu_\pm \) vanishes in the heavy-quark mass limit.

In the limit of infinite quark mass, the fields \( P_{\pm} \) and \( P^\mu_{\pm} \) may be combined into an appropriate multiplet field. This reflects the fact that in this limit the 1\(^{-}\) and 0\(^{-}\) mesons are related by a spin flip of the charm quark, which does not cost any energy. Therefore, the properties of pseudoscalar and vector states should be closely related. We follow here the formalism developed in Refs. [2, 3, 34–36] and introduce the multiplet field \( H \), connected to the fields \( P_+ \) and \( P^\mu_+ \) as follows
\[ 1 \quad H = \frac{1}{2} \left( 1 + \gamma_\mu \right) \left( \gamma_\mu P^\mu_+ + i \gamma_5 P_+ \right) \]
\[ \bar{H} = \gamma_0 H^\dagger \gamma_0 = \left( P^\dagger_+ \gamma_\mu + P^\dagger_+ i \gamma_5 \right) \frac{1}{2} \left( 1 + \gamma_5 \right), \]
\[ P^\mu_+ v_\mu = 0, \quad v^2 = 1, \]
in terms of which the interaction should be composed. According to Ref. [1], the field \( H \) transforms under the heavy-quark spin symmetry group \( SU_v(2) \), the elements of which being characterized by the 4-vector \( \theta^\alpha \) with \( \theta \cdot v = 0 \), as follows:
\[ H \to e^{-i S_\alpha \theta^\alpha} H, \quad \bar{H} \to \gamma_0 \left( e^{-i S_\alpha \theta^\alpha} H \right)^\dagger \gamma_0 = \bar{H} e^{+i S_\alpha \theta^\alpha}, \]
\[ S^\alpha = \frac{1}{2} \gamma_5 [\gamma^\mu, \gamma^\alpha], \quad S^\dagger_\alpha \gamma_0 = \gamma_0 S_\alpha, \quad [\gamma^\mu, S_\alpha]_\pm = 0. \]

\(^1\) Note that \( \text{tr} \gamma_5 \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta = -4 i \epsilon_{\mu\nu\alpha\beta} \) in the convention used in this work.
Under a Lorentz transformation, characterized by the antisymmetric tensor \( \omega_{\mu\nu} \), the spinor part of the field transforms as

\[
H \rightarrow e^{i S_{\mu\nu} \omega_{\mu\nu}} H e^{-i S_{\mu\nu} \omega_{\mu\nu}}, \quad \bar{H} \rightarrow e^{i S_{\mu\nu} \omega_{\mu\nu}} \bar{H} e^{-i S_{\mu\nu} \omega_{\mu\nu}},
\]

\( S_{\mu\nu} = \frac{i}{4} [\gamma_\mu, \gamma_\nu] \).

It follows that only combinations of the form where the Dirac matrices are right of the field \( H \) or left to the field \( \bar{H} \) are invariant under the spin group \( SU_v(2) \). Now it is straightforward to construct the \( SU_v(2) \)-invariant effective Lagrange density bearing the structures detailed in Eqs. (6-8). We introduce

\[
\mathcal{L} = -\frac{1}{2} \text{tr} H \left\{ f_1^{(S)} (\bar{B}B)_+ + f_2^{(S)} (\bar{B}B)_- + \frac{1}{2} f_3^{(S)} \text{tr}(\bar{B}B) - f_4^{(S)} (\bar{B}_\alpha B^\alpha) - \frac{1}{2} f_5^{(S)} \text{tr}(\bar{B}_\alpha B^\alpha) \right\} \bar{H}
- \frac{1}{2} \text{tr} H \left\{ f_1^{(A)} (\bar{B} \gamma_5 \gamma^\mu B)_+ + f_2^{(A)} (\bar{B} \gamma_5 \gamma^\mu B)_- + \frac{1}{2} f_3^{(A)} \text{tr}(\bar{B} \gamma_5 \gamma^\mu B) \right\} \gamma_5 \gamma_\mu \bar{H}
+ \frac{i}{4} \text{tr} H \left\{ f_1^{(T)} (\bar{B}_\mu B_\nu) + \frac{1}{2} f_2^{(T)} \text{tr}(\bar{B}_\mu B_\nu) + f_3^{(T)} (\bar{B}_\mu \gamma_5 \gamma_\nu B - \bar{B} \gamma_5 \gamma_\nu B_\mu) \right\} \sigma^{\mu\nu} \bar{H}.
\]

We recall that the field \( H \) is a three-dimensional row in flavor space, each of its components consisting of a 4 dimensional Dirac matrix. With the help of Eq. (9) and Eq. (11) the chiral and the \( SU_v(2) \)-invariant effective interactions can be expressed in terms of the fields \( P_+ \) and \( P^\mu \). By matching these expressions to each other we obtain 15 relations

\[
\begin{align*}
c_1^{(S)} &= \bar{c}_1^{(S)} = f_1^{(S)}, & c_2^{(S)} &= \bar{c}_2^{(S)} = f_2^{(S)}, & c_3^{(S)} &= \bar{c}_3^{(S)} = f_3^{(S)},
d_1^{(S)} &= \bar{d}_1^{(S)} = f_4^{(S)}, & d_2^{(S)} &= \bar{d}_2^{(S)} = f_5^{(S)},
c_1^{(A)} &= \bar{c}_1^{(A)} = f_1^{(A)}, & c_2^{(A)} &= \bar{c}_2^{(A)} = f_2^{(A)}, & c_3^{(A)} &= \bar{c}_3^{(A)} = f_3^{(A)},
d_1^{(E)} &= \bar{d}_1^{(E)} = d_3^{(E)} = f_1^{(T)}, & d_2^{(E)} &= \bar{d}_2^{(E)} = d_4^{(E)} = f_2^{(T)},
e_1^{(A)} &= \bar{e}_2^{(A)} = \bar{e}_1^{(A)} = \bar{e}_2^{(A)} = f_3^{(T)}.
\end{align*}
\]

Having implemented the heavy-quark symmetry, out of 26 parameters from Section III there remain 11 independent parameters only.

IV. LARGE-\(N_c\) OPERATOR ANALYSIS

In this section we further correlate the parameters of the effective interaction introduced in Section II. We follow the works of Luty and March-Russell [22] and of Dashen, Jenkins and Manohar [23]. These works introduced a formalism for a systematic expansion of baryon
matrix elements of QCD quark operators in powers of $1/N_c$. In a previous work two of the authors detailed the application of this formalism to correlation functions involving a product of two axial-vector quark currents \[21\]. Here we further adapt the formalism for the case where charm changing quark currents occur. We consider baryon matrix elements of the following products of two quark currents:

$$
C_{AA,\mu\nu,a}(q) = i \int d^4 x e^{-i q \cdot x} \langle 0 | T \bar{A}_\mu(0) \lambda_a \bar{A}_\nu(x), 
$$

$$
C_{VV,\mu\nu,a}(q) = i \int d^4 x e^{-i q \cdot x} \langle 0 | T \bar{V}_\mu(0) \lambda_a \bar{V}_\nu(x), 
$$

$$
C_{VA,\mu\nu,a}(q) = i \int d^4 x e^{-i q \cdot x} \langle 0 | T \bar{V}_\mu(0) \lambda_a \bar{A}_\nu(x),
$$

with the quark field operators $u(x), d(x), s(x), c(x)$ of the up, down, strange and charm quarks. With $\lambda_a$ we denote the Gell-Mann matrices supplemented with a singlet matrix $\lambda_0 = \sqrt{2/3}$.

In QCD the charm changing vector and axial-vector currents are accessed by means of classical source functions $V_\mu(x)$ and $A_\mu(x)$ that couple to the currents directly. Our effective Lagrangian may be considered to be a function of such classical sources. All what we need in the following is the existence of a direct coupling of these sources to the $D$ meson fields

$$
\mathcal{L} = f_A \left[ A_\mu \left( \partial_\mu \bar{D} + (\partial_\mu D) \bar{A}_\mu \right) \right] + \frac{f_V}{M_V} \left[ V_\mu \left( \partial_\mu \bar{D}_{\mu\nu} + (\partial_\mu D_{\mu\nu}) \bar{V}_\nu \right) \right],
$$

with some coupling constants $f_A$ and $f_V$. With Eq. (17) the counter terms introduced in Section II contribute to the matrix elements of $C_{\mu\nu,a}^{XY}(q)$ ($X, Y = V, A$) in the baryon octet and decuplet states. According to the LSZ reduction formalism, such contributions entail the corresponding contributions to the on-shell $D$ meson baryon scattering amplitudes. They are identified unambiguously by taking the residuum of poles generated by the incoming and outgoing $D$ meson. For the sake of more compact notation in what follows, it is convenient to introduce

$$
\tilde{C}_{XY,\mu\nu,a}(\bar{q}, q) = \frac{\bar{q}^2 - M_X^2}{f_X} C_{\mu\nu,a}^{XY}(q) \frac{q^2 - M_Y^2}{f_Y},
$$
with $X, Y = V, A$ and $M_A$ and $M_V$ the masses of the pseudoscalar and vector $D$ mesons in the flavor $SU(3)$ limit. In Eq. (18) we identify $q_\mu$ and $\bar{q}_\mu$ with the 4-momenta of the incoming and outgoing $D$ mesons.

Our strategy is to first evaluate baryon matrix elements of the correlation function (18) with the help of the chiral Lagrangian. Since each parameter in the chiral Lagrangian can be dialed freely without violating any chiral Ward identity, we may focus on the contributions of the two-body counter terms. Detailed results are collected in the Appendix. In a second step we work out the $1/N_c$ expansion of the matrix elements of Eq. (18). Finally, we perform a matching of the two results. A correlation of the parameters of the chiral Lagrangian arises.

Following Ref. [21], the baryon octet and decuplet states

$$ |p, \chi, a\rangle, \quad |p, \chi, ijk\rangle, \quad (19) $$

are specified by the momentum $p$ and the flavor indices $a = 1, \cdots, 8$ and $i, j, k = 1, 2, 3$. The spin-polarization label is $\chi = 1, 2$ for the octet and $\chi = 1, \cdots, 4$ for the decuplet states. At leading order in the $1/N_c$ the baryon-octet and decuplet states are degenerate. Thus, it is sometimes convenient to suppress the reference to the particular state and use in the following $|p, \chi\rangle$ as a synonym for either a baryon-octet or baryon-decuplet state.

The large-$N_c$ operator analysis rewrites matrix elements in the physical baryon states $|p, \chi\rangle$ in terms of auxiliary states $|\chi\rangle$ that reflect the spin and flavor structure of the baryons only. All dynamical information is moved into effective operators. The $1/N_c$ expansion takes the generic form

$$ \langle \bar{p}, \bar{\chi} | \mathcal{O}_{\mu\nu,a}(\bar{q}, q) | p, \chi \rangle = \sum_n c_n(\bar{p}, p)(\bar{\chi})|O^{(n)}_{\mu\nu,a}|\chi\rangle, \quad (20) $$

where we assume all 4-momenta to be on-shell. In the the center-of-mass frame the two three vectors $\bar{p}$ and $p$ characterize the kinematics. The effective operators $O^{(n)}_{\mu\nu,a}$ are composed out of spin, flavor and spin-flavor operators, $J_i, T^a$ and $G^a_i$, which act on the auxiliary states $|\chi\rangle$. 

10
For $N_c = 3$ we recall the results of Refs. [21, 33]. It holds

\begin{align*}
J_i |a, \chi\rangle &= \frac{1}{2} \sigma_{\chi\overline{\chi}}^{(i)} |a, \overline{\chi}\rangle, \\
T^a |b, \chi\rangle &= i f^a_{bca} |c, \chi\rangle, \\
G^a_i |b, \chi\rangle &= \sigma_{\chi\overline{\chi}}^{(i)} \left( \frac{1}{2} d_{bca} + \frac{i}{3} f^a_{bca} \right) |c, \overline{\chi}\rangle + \frac{1}{2\sqrt{2}} S^{(i)}_{\chi\overline{\chi}} \Lambda_{ab}^{kln} |kln, \overline{\chi}\rangle, \\
J_i |klm, \chi\rangle &= \frac{3}{2} \left( \vec{S} \cdot \vec{S}^\dagger \right)_{\chi\overline{\chi}} |klm, \overline{\chi}\rangle, \\
T^a |klm, \chi\rangle &= \frac{3}{2} \Lambda_{kln}^{a, nop} |nop, \chi\rangle, \\
G^a_i |klm, \chi\rangle &= \frac{3}{4} \left( \vec{S} \cdot \vec{S}^\dagger \right)_{\chi\overline{\chi}} \Lambda_{kln}^{a, nop} |nop, \overline{\chi}\rangle + \frac{1}{2\sqrt{2}} \left( S_i^\dagger \right)_{\chi\overline{\chi}} \Lambda_{kln}^{ab} |b, \overline{\chi}\rangle, 
\end{align*}

with the Pauli matrices $\sigma_i$ and the transition matrices $S_i$ characterized by

\begin{align*}
S_j^\dagger S_j &= \delta_j - \frac{1}{3} \sigma_i \sigma_j, \\
S_i S_j - S_j S_i &= -i \varepsilon_{ijk} S_k, \\
\vec{S} \cdot \vec{S}^\dagger &= 1_{(4 \times 4)}, \\
\vec{S}^\dagger \cdot \vec{S} &= 2 \mathbf{1}_{(2 \times 2)}, \\
\vec{S} \cdot \vec{\sigma} &= 0, \\
\varepsilon_{ijk} S_i S_j &= i \vec{S} \sigma_k \vec{S}^\dagger.
\end{align*}

In Eq. 21 we introduced further the notation

\begin{align*}
\Lambda_{kln}^{ab} &= \left[ \varepsilon_{ijk} \lambda_{li}^{(a)} \lambda_{mj}^{(b)} \right]_{\text{sym}(klm)}, \\
\delta_{nop}^{kln} &= \left[ \delta_{kn} \delta_{lo} \delta_{mp} \right]_{\text{sym}(nop)}, \\
\Lambda_{kln}^{a, nop} &= \left[ \lambda_{kn}^{(a)} \delta_{lo} \delta_{mp} \right]_{\text{sym}(nop)}, \\
\Lambda_{kln}^{a, klm} &= \left[ \lambda_{k}^{(a)} \lambda_{lm}^{(b)} \right]_{\text{sym}(klm)},
\end{align*}

which proved convenient in various derivations [21]. The symbol ‘sym(nop)’ in Eq. 23 asks for a symmetrization of the three indices nop, i.e. take the six permutations and divide out a factor 6.

We return to the expansion (20). There are infinitely many terms one may write down. At a given order in the $1/N_c$ expansion a finite number of terms is relevant only. The counting is intricate since there is a subtle balance of suppression and enhancement effects. An $r$-body operator consisting of the $r$ products of any of the spin and flavor operators receives the suppression factor $N_c^{-r}$. On the other hand baryon matrix elements taken at $N_c \neq 3$ are enhanced by factors of $N_c$ for the flavor and spin-flavor operators. The counting advocated in [23] may be summarized by the effective scaling laws

\begin{align*}
J_i &\sim \frac{1}{N_c}, \\
T^a &\sim N_c^0, \\
G^a_i &\sim N_c^0.
\end{align*}

The counting rules (24) by themselves are insufficient to arrive at significant results. At a given order in the $1$ over $N_c$ expansion there still is an infinite number of terms contributing.
Taking higher products of flavor and spin-flavor operators does not reduce the \( N_c \) scaling power. The systematic \( 1/N_c \) expansion is implied by a set of operator identities \([21, 23]\) which leads to the two reduction rules:

- All operator products in which two flavor indices are contracted using \( \delta_{ab} \), \( f_{abc} \) or \( d_{abc} \) or two spin indices on \( G' \)’s are contracted using \( \delta_{ij} \) or \( \varepsilon_{ijk} \) can be eliminated.

- All operator products in which two flavor indices are contracted using symmetric or antisymmetric combinations of two different \( d \) and/or \( f \) symbols can be eliminated. The only exception to this rule is the antisymmetric combination \( f_{acg} d_{bch} - f_{bcg} d_{ach} \).

As a consequence the infinite tower of spin-flavor operators truncates at any given order in the \( 1/N_c \) expansion.

We turn to the \( 1/N_c \) expansion of the baryon matrix elements of the operators in Eq. \((18)\). We focus on the space components of the correlation functions. In application of the operator reduction rules, the baryon matrix elements of the product of two quark currents are expanded in powers of the effective one-body operators according to Eq. \((20)\). The ansatz for the momentum dependence of the expansion coefficients in \((20)\) is furnished by the leading order terms in the corresponding low-energy expansion stated in Eqs. \((31-33)\). In the course of the construction of the various structures, parity and time-reversal transformation properties are taken into account. To the subleading order in the \( 1/N_c \)-expansion we find the relevance of the following 11 effective operators

\[
\langle \bar{p}, \bar{\chi} | \bar{C}_{ij,a}^{\text{AA}} | p, \chi \rangle = \bar{p}_i p_j (\bar{\chi} | g_2^{\text{AA}} \left[ J_k, G_k^a \right]_+ | \chi), \\
\langle \bar{p}, \bar{\chi} | \bar{C}_{ij,a}^{\text{VV}} | p, \chi \rangle = \delta_{ij} (\bar{\chi} | g_2^{\text{VV}} \left[ J_k, G_k^a \right]_+ | \chi) \\
+ i \epsilon_{ijk} (\bar{\chi} | g_3^{\text{VV}} G_k^a + \frac{1}{2} g_4^{\text{VV}} \left[ J_k, T^a \right]_+ | \chi) \\
+ (\bar{\chi} | \frac{1}{2} g_5^{\text{VV}} \left[ J_i, G_j^a \right]_+ + \frac{1}{2} g_6^{\text{VV}} \left[ J_j, G_i^a \right]_+ | \chi), \\
\langle \bar{p}, \bar{\chi} | \bar{C}_{ij,a}^{\text{VA}} | p, \chi \rangle = p_j (\bar{\chi} | g_1^{\text{VA}} G_i^a + \frac{1}{2} g_2^{\text{VA}} \left[ J_i, T^a \right]_+ + \frac{1}{2} g_3^{\text{VA}} i \epsilon_{ikl} \left[ J_k, G_l^a \right]_+ | \chi). \tag{25}
\]

In contrast to Eq. \((21)\), where the flavor index \( a \) takes the values 1, ..., 8, the flavor index \( a \) in Eq. \((25)\) runs from 0 to 8 with the identification\(^2\)

\[
T^0 = \sqrt{\frac{1}{6}}, \quad G_i^0 = \sqrt{\frac{1}{6}} J_i. \tag{26}
\]

\(^2\) The factor \( 1/\sqrt{6} \) in Eq. \((20)\) follows from the normalization of \( \lambda_0 \) and from the definition of the effective operators \( T^a \) and \( G_i^a \).
The matrix elements of the operators in Eq. (25) are obtained by consecutive applications of the results in Eq. (21). The matrix elements of all symmetric combinations of two one-body effective operators can be found in the Appendix of Ref. [21].

Combining the large $N_c$ and the low-energy expansions for the baryonic matrix elements of quark operators (or products of them) under consideration, leads to the correlations between the unknown parameters in both expansions. In our special case, matching of the three lines in Eq. (25) to the corresponding expressions in the Appendix provides the following three sets of matching results:

\[ c_1^{(S)} = \frac{3}{8} g_{2AA}, \quad c_2^{(S)} = \frac{1}{2} g_{1AA} + \frac{1}{4} g_{2AA}, \quad c_3^{(S)} = g_{1AA} - \frac{1}{4} g_{2AA}, \]
\[ d_1^{(S)} = \frac{3}{2} g_{1AA} + \frac{15}{8} g_{2AA}, \quad d_2^{(S)} = 0, \]  

(27)

and

\[ \tilde{c}_1^{(S)} = \frac{3}{8} g_{2VV}, \quad \tilde{c}_2^{(S)} = \frac{1}{2} g_{1VV} + \frac{1}{4} g_{2VV} + \frac{1}{6} g_{VV}, \]
\[ \tilde{c}_3^{(S)} = g_{1VV} - \frac{1}{4} g_{2VV} - \frac{1}{6} g_{VV}, \]
\[ \tilde{c}_1^{(A)} = -\frac{1}{6} g_{3VV}, \quad \tilde{c}_2^{(A)} = -\frac{1}{4} g_{3VV} - \frac{1}{4} g_{4VV}, \quad \tilde{c}_3^{(A)} = -\frac{1}{6} g_{3VV} - \frac{1}{2} g_{4VV}, \]
\[ d_1^{(S)} = \frac{3}{2} g_{1VV} + \frac{15}{8} g_{2VV} + \frac{9}{4} g_{VV}, \quad d_2^{(S)} = 0, \]
\[ d_1^{(E)} = \frac{3}{2} g_{3VV} + \frac{3}{2} g_{4VV} - \frac{3}{2} g_{VV}, \quad d_2^{(E)} = \frac{3}{2} g_{4VV}, \]
\[ d_3^{(E)} = \frac{3}{2} g_{3VV} + \frac{3}{2} g_{4VV} + \frac{3}{2} g_{VV}, \quad d_4^{(E)} = \frac{3}{2} g_{4VV}, \]
\[ \tilde{e}_1^{(A)} = g_{3VV} + \frac{3}{2} g_{VV} - \frac{1}{2} g_{VV}, \quad \tilde{e}_2^{(A)} = \frac{1}{2} g_{VV} + \frac{3}{2} g_{VV} + \frac{1}{2} g_{VV}, \]

(28)

with $g_{VV} = \frac{1}{2} (g_{5VV} \pm g_{6VV})$, and

\[ e_1^{(A)} = \frac{1}{4} g_{1VA}, \quad e_2^{(A)} = \frac{1}{6} g_{1VA} + \frac{1}{4} g_{2VA}, \quad e_3^{(A)} = -\frac{1}{6} g_{1VA} + \frac{1}{2} g_{2VA}, \]
\[ e_1^{(A)} = -g_{1VA} + \frac{3}{2} g_{3VA}, \]
\[ e_2^{(E)} = -\frac{3}{2} g_{1VA} - \frac{9}{4} g_{2VA}, \quad e_2^{(E)} = -\frac{3}{2} g_{2VA}. \]

(29)

Note that the parameter $e_2^{(A)}$ does not occur in the presented results here. The large-$N_c$ operator expansion for this parameter is based on the analysis of the matrix elements of $\tilde{C}^{AV}$, which may be decomposed in terms of additional parameters $g_{1,2,3}^{AV}$ in analogy to the decomposition of the matrix elements of $\tilde{C}^{VA}$ in Eq. (25). The corresponding results follow.
from Eq. (29) by the replacements \( e_1^{(A)} \rightarrow e_2^{(A)} \) and \( g_{1,2,3}^{VA} \rightarrow g_{1,2,3}^{AV} \). As a consequence it follows that \( g_{1,2}^{VA} = g_{1,2}^{AV} \), but not necessarily \( g_3^{VA} = g_3^{AV} \).

Altogether we find 12 large-\( N_c \) parameters relevant at leading order. Compared with the 26 chiral parameters we expect a set of 14 sum rules. We group the sum rules into three parts

\[
\begin{align*}
\tilde{c}_3^{(S)} &= 2 \left( c_2^{(S)} - c_1^{(S)} \right), & d_1^{(S)} &= 2 \left( c_1^{(S)} + c_2^{(S)} \right), & d_2^{(S)} &= 0, \\
\tilde{c}_3^{(A)} &= 2 \left( c_2^{(A)} - c_1^{(A)} \right), & d_1^{(E)} &= -9 c_2^{(A)}, & d_2^{(E)} &= 2 \left( 2 c_1^{(A)} - 3 c_2^{(A)} \right), \\
\tilde{c}_3^{(S)} &= 2 \left( c_2^{(S)} - c_1^{(S)} \right), & \tilde{d}_1^{(S)} &= 3 \left( c_1^{(S)} + c_2^{(S)} \right) + \left( \tilde{c}_2^{(A)} - \tilde{c}_1^{(A)} \right), & \tilde{d}_2^{(S)} &= 0, \\
\tilde{c}_3^{(A)} &= 2 \left( c_2^{(A)} - c_1^{(A)} \right), & \tilde{d}_1^{(E)} &= -9 \tilde{c}_2^{(A)} - 3 \left( \tilde{c}_2^{(A)} - \tilde{c}_1^{(A)} \right), & \tilde{d}_2^{(E)} &= \tilde{d}_4^{(E)} = 2 \left( 2 \tilde{c}_1^{(A)} - 3 \tilde{c}_2^{(A)} \right), \\
\tilde{c}_3^{(E)} &= -9 \tilde{c}_2^{(A)} + 3 \left( \tilde{c}_2^{(A)} - \tilde{c}_1^{(A)} \right), & \tilde{d}_3^{(E)} &= \tilde{d}_5^{(E)} = 2 \left( 2 \tilde{c}_1^{(A)} - 3 \tilde{c}_2^{(A)} \right),
\end{align*}
\]

where the first and the third parts correlate the coupling constants describing the interactions of pseudoscalar and vector \( D \) mesons, respectively. The second part provides the analogous relations for the transition interactions, i.e. terms with one pseudoscalar and one vector \( D \) meson field.

We observe that given the third set of the sum rules in Eq. (30), the first two parts are recovered by applying the results of the heavy-quark mass expansion as summarized in Eq. (15). This is a remarkable result demonstrating the consistency of a combined heavy-quark and large-\( N_c \) expansion.

V. SUMMARY

We derived sum rules for the leading order two-body counter terms of the chiral Lagrangian as implied by a combined heavy-quark and large-\( N_c \) analysis. There are altogether 26 independent terms in the chiral Lagrangian with baryon octet and decuplet fields that contribute to the \( D \) and \( D^* \) meson baryon scattering process at chiral order \( Q_0^0 \).

At leading order in the heavy-quark expansion we find the relevance of 11 operators only. Additional sum rules were derived from the \( 1/N_c \) expansion. Combining both expansions, the number of unknown parameters is further reduced to 5. At present such sum rules can not be confronted directly with empirical information. They are useful constraints in establishing a systematic coupled-channel effective field theory for \( D \) meson baryon scattering beyond the threshold region.
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VI. APPENDIX

With the help of the effective interaction discussed in Section II we evaluate the leading order contributions to the matrix elements of the correlator function in Eq. (18) for the baryon-octet and baryon-decuplet states. As in the main part of the text, we focus on the space components only and specify the kinematics in the center of mass system.

The leading terms in the low-momentum expansion of the matrix elements of the product of two axial-vector currents are

\[
\langle \bar{p}, \chi, c | \bar{C}_{ij,a}^{AA} | p, \chi, b \rangle = \bar{p}_i p_j \delta_{\chi\chi} \times \left\{ \begin{array}{l}
2 \sqrt{\frac{3}{2}} c_1^{(s)} + \sqrt{\frac{3}{2}} c_3^{(s)} \\
2 c_1^{(s)} d^{abc} + 2 c_2^{(s)} i f^{abc}
\end{array} \right. : a = 0
\]

\[
\langle \bar{p}, \chi, nop | \bar{C}_{ij,a}^{AA} | p, \chi, b \rangle = 0,
\]

\[
\langle \bar{p}, \chi, nop | \bar{C}_{ij,a}^{AA} | p, \chi, klm \rangle = \bar{p}_i p_j \delta_{\chi\chi} \times \left\{ \begin{array}{l}
2 \sqrt{\frac{3}{2}} d_1^{(s)} + \sqrt{\frac{3}{2}} d_2^{(s)} \\
d_1^{(s)} \Lambda_{klm} \Lambda_{rst}^{nop}
\end{array} \right.
\]

Here and in the following the upper row corresponds to the singlet component of the correlation function, $\bar{C}_{ij,0}$, whereas the second row specifies the matrix elements of its octet components with $a = 1, \ldots 8$. Furthermore, the flavour summation indices are $r, s, t = 1, 2, 3$. The expansion for the product of two vector currents reads

\[
\langle \bar{p}, \chi, c | \bar{C}_{ij,a}^{VV} | p, \chi, b \rangle = \delta_{ij} \delta_{\chi\chi} \times \left\{ \begin{array}{l}
2 \sqrt{\frac{3}{2}} c_1^{(s)} + \sqrt{\frac{3}{2}} c_3^{(s)} \\
2 c_1^{(s)} d^{abc} + 2 c_2^{(s)} i f^{abc}
\end{array} \right.
\]

\[
\langle \bar{p}, \chi, nop | \bar{C}_{ij,a}^{VV} | p, \chi, b \rangle = -\frac{1}{2} \sqrt{2} \left( S_i \sigma_j \right)_{\chi\chi} \times \left\{ \begin{array}{l}
0 \\
\tilde{c}_1^{(A)} \Lambda_{nop}^{ab}
\end{array} \right.
\]

\[
+ \frac{1}{2} \sqrt{2} \left( S_j \sigma_i \right)_{\chi\chi} \times \left\{ \begin{array}{l}
0 \\
\tilde{c}_2^{(A)} \Lambda_{nop}^{ab}
\end{array} \right.
\]

\[
\langle \bar{p}, \chi, nop | \bar{C}_{ij,a}^{VV} | p, \chi, klm \rangle = \delta_{ij} \delta_{\chi\chi} \times \left\{ \begin{array}{l}
2 \sqrt{\frac{3}{2}} \tilde{d}_1^{(s)} + \sqrt{\frac{3}{2}} \tilde{d}_2^{(s)} \\
\tilde{d}_1^{(s)} \Lambda_{klm} \Lambda_{rst}^{nop}
\end{array} \right.
\]

\[
+ \frac{1}{2} \left( S_i S_j \right)_{\chi\chi} \times \left\{ \begin{array}{l}
0 \\
\tilde{d}_1^{(E)} \Lambda_{klm} \Lambda_{rst}^{nop}
\end{array} \right.
\]
\[ -\frac{1}{2} \left( S_j S_i^\dagger \right)_{\bar{\chi}\chi} \times \left\{ \left( \sqrt{\frac{2}{3}} \tilde{d}_3(E) + \sqrt{\frac{3}{2}} \tilde{d}_4(E) \right) \delta_{klm}^{nop} + \tilde{d}_3(E) \Lambda_{a, rst}^{nop} \delta_{rst}^{nop} \right\}. \] (32)

Finally, for the product of a vector and an axial-vector currents we obtain:

\[
\langle \bar{p}, \bar{\chi}, c| \bar{C}^{VA}_{ij,a} | p, \chi, b \rangle = p_j S^{(i)}_{\bar{\chi}\chi} \times \left\{ \left( 2 \sqrt{\frac{2}{3}} c_1^{(A)} + \sqrt{\frac{3}{2}} c_3^{(A)} \right) \delta^{abc} \frac{2}{c_1^{(A)}} d_{abc} + 2 c_2^{(A)} i f_{abc} \right\},
\]

\[
\langle \bar{p}, \bar{\chi}, nop| \bar{C}^{VA}_{ij,a} | p, \chi, b \rangle = -\frac{1}{2 \sqrt{2}} p_j S^{(i)}_{\bar{\chi}\chi} \times \left\{ 0 \frac{\Lambda_{nop}^{(A)}}{e_1^{(A)}} \right\},
\]

\[
\langle \bar{p}, \bar{\chi}, nop| \bar{C}^{VA}_{ij,a} | p, \chi, klm \rangle = -\frac{1}{2} p_j \left( \tilde{S} \sigma^{(i)} \tilde{S}^\dagger \right)_{\bar{\chi}\chi} \times \left\{ \left( \sqrt{\frac{2}{3}} d_1(E) + \sqrt{\frac{3}{2}} d_2(E) \right) \delta_{klm}^{nop} \frac{d_1(E)}{\Lambda_{a, rst}^{nop}} \delta_{rst}^{nop} \right\}. \] (33)

The spin-transition matrices \( \sigma \) and \( \tilde{S} \) and the flavor transition tensors \( \delta_{klm}^{nop}, \Lambda_{ab}^{nop} \) and \( \Lambda_{klm}^{c, nop} \) used above, were already introduced in Eq. (22) and Eq. (23), respectively.
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