FAST TRACK COMMUNICATION

Time delays across saddles as a test of modified gravity

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Abstract

Modified gravity theories can produce strong signals in the vicinity of the saddles of the total gravitational potential. In a sub-class of these models, this translates into diverging time delays for echoes crossing the saddles. Such models arise from the possibility that gravity might be infrared divergent or confined, and if suitably designed they are very difficult to rule out. We show that Lunar Laser Ranging during an eclipse could probe the time-delay effect within metres of the saddle, thereby proving or excluding these models. Very Large Baseline Interferometry, instead, could target delays across the Jupiter–Sun saddle. Such experiments would shed light on the infrared behaviour of gravity and examine the puzzling possibility that there might be well-hidden regions of strong gravity and even singularities inside the solar system.

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(Some figures may appear in colour only in the online journal)

Even though it is inevitable that Einstein’s theory of general relativity (GR) will not be the final word, it is telling that almost a century after its proposal all theories trying to supersede it have been ruled out or remain beyond detection [1–3]. Nonetheless, it is precisely the experimental misfortunes of ‘modified gravity’ that prove the strength of GR, so it is important to keep pushing the boundaries, constructing and observationally disproving new possibilities. A further motivation derives from attempts to combine quantum theory and GR, a logical (if not an empirical) necessity. Such efforts invariably lead to corrections to GR, often at energy scales beyond the reach of current experiment, but not always. Finally, we should never forget that at face value—taking into account only the matter sources that we do see—the observational status of GR in astrophysics and cosmology is calamitous. This is usually blamed on our imperfect knowledge of the matter content of the Universe, and dismissed by introducing new forms of invisible matter. But it could well be that the discrepancies signal a breakdown in our understanding of gravity.

Among the many theories attempting to extend GR some have tried to address the last issue, doing away with the need for non-visible or ‘dark’ matter to explain anomalies at galactic, cluster and cosmological levels (see e.g. [4, 5]). Such theories have been labelled
'MOdified Newtonian Dynamics' (MOND), even though they have now been embedded into fully relativistic field theories (e.g. [6–12]). In all of them, new effects are triggered below an acceleration scale, $a_0 \sim 10^{-10}$ $\text{ms}^{-2}$, a property suggested by the phenomenology. The fact that MONDian behaviour is physically triggered by an acceleration scale does not preclude writing $a_0$ in terms of length scale:

$$L_0 = \frac{c^2}{a_0}$$

for which $a_0$ could be a proxy. It is interesting that $L_0 \sim 30,000 \text{Mpc}$ is of the order of the current horizon/Hubble radius. Nonetheless, it is the onset of low acceleration that triggers new effects. MONDian theories do not have preferred frames and do not break diffeomorphism invariance; yet new effects emerge in the non-relativistic approximation when the total Newtonian force per unit mass falls below $a_0$. For this reason, one may expect the presence of ‘MONDian habitats’ in the small regions encasing the saddle points of the gravitational potential in the solar system, the points where the Newtonian force vanishes [13]. The prospect of a MONDian saddle test has motivated extensive work [14–19], with LISA Pathfinder (LPF) in mind, but not only [20].

One of the most powerful tests of GR, by now elevated to the category of ‘classical’ test, employs the echo time-delay effect. By flashing ‘light’ (usually a radio wave) at a distant object and catching its reflected ‘echo’, one measures a distinctive delay, if its path intersects a strong gravitational field. This so-called Shapiro effect was first detected with the radar echo off Venus in superior conjunction, leading to stringent constraints on the $\gamma$ PPN parameter [1]. Since then the observational front has improved very fast. These tests use the fact that for a large class of theories (metric theories) the travel time is given by

$$t = \frac{1}{c} \int \left(1 - \frac{2\Phi}{c^2} \right) dz,$$

where $\Phi$ is the total gravitational potential, as obtained in the non-relativistic limit. Whilst the delay along a single path may be gauged away, the variation in delays along neighbouring paths is operationally meaningful, and constitutes a bona fide observational target.

Lunar Laser Ranging (LLR) is a major asset, among other fields, in gravitational physics (see for example [21, 22] as well as [23, 24] where the effect on MONDian theories in also considered). Using lunar retroreflectors one may time very accurately echoes of sharp laser signals. Progress has been made steadily and precisions of tens of picoseconds, corresponding to distances of around 1 mm, can now be achieved [22]. It is immediately obvious (see figure 1) that in principle LLR could probe the Moon saddle during a lunar eclipse, sensing a possible MONDian time-delay effect. Given that the Sun–Earth saddle is within the lunar orbit, it turns out that it can also be targeted by LLR during a solar eclipse (cf figure 1). In practice, an LLR saddle test requires the correct vantage point on Earth or even in its orbit. Although it is obvious that the correct alignment is possible during an eclipse if other perturbations are ignored, the matter is less obvious in the more realistic solar system. However, it turns out that if perturbations to the three-body setup are taken into account, the exact location of the vantage point does changes, but not its presence on Earth or its orbit. Jupiter shifts the saddle location by at most 8 km, and Saturn by 0.5 km. The galaxy displaces the saddle by about a metre and the extra-galactic field by a metre at most (see the discussion in section 7 of [13]). In contrast, the moon can shift the Earth–Sun saddle by as much as 6 000 km (see figure 8 in [15]).

Very Large Baseline Interferometry (VLBI) is another asset in gravitational science (see e.g. [25]) providing another strategy for probing delays across saddles. It correlates images of the same object (say, a quasar) as obtained in different continents. A setup could be arranged
in which one light ray goes through a saddle whilst the other (a few thousand kilometres away) does not. VLBI has the advantages that it could be used to probe other saddles (e.g. the rather large Sun–Jupiter saddle), and that it relies only on the presence of a source behind the saddle, rather than on the vagaries of eclipses. In spite of potential difficulties, a saddle test with LLR or VLBI could be carried out without the logistic overheads associated with the LPF saddle extended mission.

It turns out that the time-delay effect is negligible for the MONDian models usually taken as targets for LPF. But there are also models well beyond the reach of LPF which predict a strong time-delay signal. Therefore, the experimental test examined in this communication is complementary to an LPF test. This is hardly surprising. LPF accelerometers are sensitive to tidal stresses, i.e. the second derivatives of the gravitational potential (they feel their variation on a given frequency range). A time-delay test, instead, is sensitive to the integral of the potential along the line of sight (or rather to its variation across neighbouring paths).
therefore natural to find complementarity between the two measurements. An extension to LPF would constrain modified gravity theories which do away with the need for dark matter. A time-delay measurement would instead probe theories which encode the property that gravity is subject to confinement, as we shall now see.

It has long been speculated that gravity might resemble QCD. This has been found in field theory, string theory, in studies invoking holographic correspondences and in the study of the renormalization group flow (for a sample of references scattered throughout several decades, see [26–30]. In particular, studies of the renormalization group flow have shown a strong parallel between the two theories, suggesting that gravity could be asymptotically free (or ‘safe’) and reciprocally be subject to confinement, with a divergent strength at large distance, or low energy (e.g. [31–33]). The latter is actually what happens in the presence of a negative cosmological constant, inducing an attractive force satisfying Hook’s law (diverging like \( r \)).

But more generally the issue has been raised in the context of the renormalization group flow of quantum GR, where it has been conjectured that the degenerate fixed point, when lifted, contains divergent IR behaviour. An equivalent implementation of this phenomenology is possible in the context of MONDian theories, should these be released from their obligations as dark matter alternatives, but with a key property kept and exacerbated. Existing MONDian theories already have the peculiarity that they enhance the strength of gravity in situations where the standard Newtonian force becomes weak. Specifically, for astrophysical applications, as \( F_N \ll a_0 \), we have that the MONDian force goes like \( F_\phi \sim \sqrt{F_N} \), at least in spherically symmetric situations. Thus, the MONDian force still drops to zero with \( F_N \), albeit slower. But what if it diverged instead? For example, we could imagine the ‘dual’ behaviour \( F_\phi \propto \frac{1}{F_N} \), and once this possibility is considered we could consider sharper divergences, such as exponentials, or

\[
F_\phi \sim \frac{1}{F_N^p},
\]  

(3)
The with \( p > 0 \) very large. If one is to avoid appealing to dark matter, clearly \( a_0 \) would have to be smaller than the usual one, but not otherwise. It turns out that it would be very difficult to rule out theories of this sort, except for their echo and VLBI saddle delays, as we will now show.

We first briefly lay down the formalism for defining MONDian theories, without wedding ourselves to a specific formulation. As explained in [14], in spite of the large number of MONDian theories, the expression for the non-relativistic potential invariably satisfies three types of equations only (which may be formally reduced to two). The dynamics may be written as resulting from the usual Newtonian potential \( \Phi_N \) and a ‘fifth force’ field, \( \phi \), responsible for MONDian effects, with total potential \( \Phi = \Phi_N + \phi \). For ‘type I’ theories, \( \phi \) is ruled by a nonlinear Poisson equation

\[
\nabla \cdot (\mu(z) \nabla \phi) = \kappa G \rho ,
\]  

(4)

where, for convenience, we pick the argument of the free function \( \mu \) as

\[
z = \frac{\kappa}{4\pi} \frac{|\nabla \phi|}{a_0} ,
\]  

(5)

where \( \kappa \) is a dimensionless constant. For ‘type II’ theories, we have instead

\[
\nabla^2 \phi = \frac{\kappa}{4\pi} \nabla \cdot (\nu(v) \nabla \Phi_N),
\]  

(6)

where the argument of the free function \( \nu \) is given by

\[
\nu = \left( \frac{\kappa}{4\pi} \right)^2 \frac{|\nabla \Phi_N|}{a_0} .
\]  

(7)
Should these theories serve their duties as dark matter alternatives we would require \( \mu \sim z \) for \( z \ll 1 \) and \( v \sim 1/\sqrt{v} \) for \( v \ll 1 \), and \( a_0 \) would be the usual MONDian acceleration. However, these theories have an interest in their own right: they may generally be regarded as theories with a preferred acceleration scale. More general functions \( \mu \) or \( \nu \) and values of \( a_0 \) should then be considered. If we want to keep the alternative to dark matter rationale, then the \( a_0 \) used here must be at least one order of magnitude smaller than that employed in traditional MONDian theories. However, if we detach these theories completely from that role, and if dark matter does exist and play a role in the dynamics, this is not true.

We illustrate our calculations using type II theories, because they are simpler. For the purpose of investigating confined gravity, we shall consider free functions which for \( v \ll 1 \) are power laws:

\[
\nu \propto 1/v^n.
\]

From (6) and (3), we have \( p = n - 1 \) and so for \( n > 1 \) (to be contrasted with the MONDian \( n = 1/2 \)) we obtain confinement behaviour. For \( n = 3/2 \), the theory mimics a negative cosmological constant in spherically symmetric situations. The dual behaviour suggested in (3) with \( p = 1 \) follows from \( n = 2 \). Note that a negative Lambda is the perfect dual to standard MOND behaviour. The potential \( \phi \) diverges around a saddle if \( n \geq 2 \). However, this only translates into a divergent time delay at the saddle if \( n \geq 3 \), with \( n = 3 \) representing a logarithmic divergence.

Even though the parameter \( \kappa \) will not appear in the final answer for the time delay, it is important for justifying an approximation and setting a scale. The rationale for its appearance in (7) and the proportionality constant in (6) is as follows [14]. If \( \nu \to 1 \) at large argument, then Newton’s constant \( G \) is renormalized by \( \kappa/(4\pi) \), and this should be small. But in order that \( F_{\phi} \sim a_0 \) when \( F_N \sim a_0 \), we should use (7) for the argument of \( \nu \), if \( \nu \sim 1/\sqrt{v} \) is to be triggered at \( v \sim 1 \) in the usual MONDian theory. With (8) the same requirement becomes

\[
v = \left(\frac{\kappa}{4\pi}\right)^{\frac{1}{2}} \frac{|\nabla\Phi_N|}{a_0},
\]

with \( v \propto 1/v^n \) triggered at \( v \sim 1 \). The region where field \( \phi \) goes strongly MONDian is now of size

\[
r_0 = \frac{a_0}{A} \left(\frac{\kappa}{4\pi}\right)^{\frac{1}{2}},
\]

where \( A \) is the diagonal tidal stress at the saddle along the line connecting the two bodies. However, the field \( \phi \) is subdominant with respect to the Newtonian potential until we get to a distance

\[
\tilde{r} = \frac{a_0}{A}
\]

from the saddle. It is only inside this inner bubble that \( \phi \) is both MONDian and dominant.

For orientation purposes, we first evaluate the delay effect in Newtonian theory. Introducing cylindrical coordinates, \([z, \rho, \theta]\), the delay is obtained by integrating equation (2) along the paths of the constant \( \rho = b \). (This is valid for eclipse LLR only; the geometry is more complex for VLBI.) In the linear approximation, the Newtonian potential is \( \Phi^N = A \left( -\frac{z^2}{2} + \frac{\rho^2}{2} \right) \) (see for example the discussion in [13] for the validity of this approximation). For the Earth–Sun saddle, the tidal stress is \( A \approx 4.6 \times 10^{-11} \text{s}^{-2} \). Evaluating (2) over \( z \in (-L/2, L/2) \) in these coordinates and with this potential is straightforward algebra, and the only term that varies with the impact parameter \( b \) (and so is observable) is \( \Delta t = -\frac{L}{2} \frac{db}{A} \). Even if the linear approximation were valid throughout the whole flight (an assumption which provides an upper bound on the real effect), we would get variations of the
order of $10^{-16}$ s for impact parameters $b \sim 1000$ km (with a delay at the centre with respect to outer trajectories). For the Jupiter saddle, the effect is even smaller since $A \sim 1.8 \times 10^{-14}$ s$^{-2}$. The Newtonian delay is therefore negligible.

We now repeat this calculation for type II theories, mimicking the calculation of $\phi$ in [17], for a free function of form (8). The potential satisfies ansatz

$$\phi = -\frac{a_0^6}{A^{n-1}} \frac{1}{r^{n-2}} (f_0 + f_2 \cos(2\psi) + f_4 \cos(4\psi) + \cdots),$$

where the parameters $f$ have to be determined numerically. A particularly simple case follows from $n = 4$. The integration should be performed along $z$ within the region of strong MONDian behaviour, delimited by $r_0$ as defined above. For $n = 4$, the integration can be carried out explicitly, with the three terms in (12) integrating into

$$\Delta t \approx \frac{4 a_0^6}{c^3 A^4 b} \left( f_0 \arctan x - \frac{f_2 x}{1 + x^2} + \frac{f_4 x(1 - x^2)}{(1 + x^2)^2} \right),$$

(13)

(where $x = r_0/b$), where the first term always dominates. If we can assume that $r_0 \gg b$ (always true if $\kappa \ll 1$), this becomes asymptotically

$$\Delta t \approx \frac{2 \pi f_0 a_0^6}{b c^3 A^4},$$

(14)

(i.e. only the first term survives). The parameter $\kappa$ does not appear in the final answer, as long as it is small enough to justify taking $r_0/b \to \infty$. For more general $n$, the calculation is more elaborate, leading to the asymptotic result ($b/r_0 \to 0$)

$$\Delta t \approx \frac{C b}{c^{n-3} A^{n-1}},$$

(15)

where $C$ is given by

$$C = \frac{2 \sqrt{\pi} \Gamma \left( \frac{n-2}{2} \right)}{\Gamma \left( \frac{n+2}{2} \right)} \left( f_0 + \frac{n-4}{n-2} f_2 + \frac{(n-4)(n-6)}{n(n-2)} f_4 \right).$$

As we see, $n = 4$ is a particularly simple limit of this expression. We note that for $n > 3$ the relative time variation at the saddle diverges, decreasing as a power law in $b$ as we move away from the ‘bull’s eye’. In principle, the constant $C$ can be positive or negative, leading to a delay or an advance at the bull’s eye, but we shall call it delay for definiteness.

In spite of this ‘divergence’, the observational implications are less dramatic than might be expected. Using the cosmological length scale $L_0$ defined in equation (1) and the strong MOND bubble scale $\tilde{r}$ (in equations (11)), we can rearrange equation (14) in the suggestive form

$$\Delta t \approx C b \tilde{r} \left( \frac{\tilde{r}}{L_0} \right)^{n-2}. $$

(16)

The first factor (besides $C$) is just the time it takes to cross the region closest to the saddle. Unlike time delays caused by the Sun this is small, because the distances involved are small: $b$ should be smaller than $\tilde{r}$ and $\tilde{r} \sim 2.2$ m for the Earth–Sun saddle and $\tilde{r} \sim 5.5$ km for the Jupiter saddle. In addition, the second factor relates the MOND bubble size to the horizon scale, introducing a tiny factor. Therefore, even though the third factor predicts a divergence, this will happen very close to the saddle and be observable only for very steeply diverging functions.

We spell out this expectation in figures 2 and 3, which describes the situation for the Earth–Sun saddle (as a target for LLR during a solar eclipse) and the Jupiter–Sun saddle (as a
Figure 2. The Log10 of the delay in picoseconds as a function of the impact parameter $b$ and exponent $n$ for the Earth–Sun saddle, as probed, say, by LLR during a solar eclipse. As we can see the delay goes very quickly from very small to very large. Realistically, with current technology, only an integrated effect might be observable, and even then only for large $n$.

target for VLBI), respectively. In both of these figures, we have plotted the (base 10) logarithm of the time delay in picoseconds, as a function of the impact parameter $b$ (in metres) and the exponent $n$ used in the free function $\nu$. In both cases, we observe a very abrupt transition from the very small to the very large, with the contour labelled zero denoting the rough borderline for observability with current technology. Typically, the Earth saddle would have to be probed closer to a metre and even then assuming large values of $n$ (in the range 20–30). The Jupiter saddle might be more forgiving, and small values of $n \sim 5$ could come within reach of $b$ of the order of a metre, with $n \sim 20$ still constrained even for impacts of the order of a kilometre. In all fairness, we cannot be overenthusiastic about the detectability of this effect in the first setting, where with current technology it would be seen at best as an integrated effect (the wavepackets often have a width of about 200 m). The next generation of lunar retroreflectors could be necessary. The second situation might be more hopeful. We illustrated our conclusions with the Earth–Sun saddle during a solar eclipse but similar results apply for the Moon saddle, as targeted by LLR during a lunar eclipse. Likewise what we have shown for the Jupiter saddle has a closely related counterpart with the Saturn saddle. Incidentally, equation (16) can be used to prove that the effect for standard MONDian functions ($n = 1/2$) is negligible ($\Delta t \sim 10^{-34}$ s with $b$ about a metre from the saddle). Likewise it can be shown that the functions considered here would have negligible effect for an LPF test.

What about other MONDian theories? As an example, we briefly discuss what was labelled type I theories in [14]. For these, the situation is more complex due to the well-known presence of a curl field, softening the divergence [13, 14]. Under strict spherical symmetry
Figure 3. The Log10 of the delay in picoseconds, as a function of impact parameter \( b \) and exponent \( n \) for the Jupiter–Sun saddle, as potentially probed by VLBI. Again the delay goes very quickly from very small to very large. Here, the boundary is closer to realistic experimental parameters, and lower exponents \( n \) come within reach.

This field vanishes, so although this is not applicable to a saddle, we can gain some intuition. Parametrizing the free function in (4) as

\[
\mu \propto \frac{1}{z^m},
\]

for \( z \ll 1 \), we see that ignoring the curl field, the exponent \( p \) in (3) is \( p = 1/(m - 1) \), so that \( m > 1 \) becomes the condition for confining behaviour. Now, \( m = 3 \) is equivalent to a negative cosmological constant \( \Lambda \), and \( m = 2 \) leads to a perfect dual (\( p = 1 \)). However, conclusions about the conditions for a divergence at the saddle are more subtle, because the magnetic field cannot be neglected. If this were the case, then \( 1 < m \leq 2 \) would lead to a diverging \( \phi \) and \( 1 < m < 3/2 \) to a diverging delay. However, now we can only try out the more flexible ansatz

\[
\phi = -C_1 \frac{1}{r^\alpha} (f_0 + f_2 \cos(2\psi) + f_4 \cos(4\psi) + \cdots)
\]

and search for a solution numerically (using techniques presented, in a different context, in [34]). We find for \( m = 1.1 \), for example, that \( \alpha = 1.25 \) (instead of \( \alpha = 9 \), expected if the curl field could be neglected). Thus, these theories are even more difficult to constrain than type II. The situation is similar to type III (which also have a curl field).

It is interesting that something as dramatic as this divergence can be so elusive. Furthermore, we have only solved the problem to linear order, and many questions can be raised beyond the scope of the calculation presented in this paper. For example, if gravity is confined and infrared divergent, as envisaged here, could there be a singularity at the saddle? If so, would this singularity be naked, or rather, would there be a horizon? Whilst a positive
answer to the first question is plausible, the answer to the second question is far from obvious. In both cases, the detectability of the time-delay effect for the free functions used above, as calculated here, is unlikely to improve. The field is not attractive, so the usual arguments about accretion discs and x-ray emission do not apply (even considering, e.g. the solar winds). One may think it odd that naked singularities or horizons could be floating around in the solar system, but in practice the regions where such extreme behaviour is felt are very small, and they could pass unnoticed.

Of course, one could fluff up the divergence region by introducing the functions of the form

\[ \nu = \frac{1}{(v - 1)^n} \]  

in type II theories, for example. Then, the nonlinear regime would be entered close to the ellipsoid \( z^2 + \rho^2/2 = \tilde{r}^2 \), and depending on the details of the full relativistic theory, this could signal the formation of a horizon or a naked singularity. Either way, assuming LLR geometry, any photons with \( b < 2\tilde{r} \) would be lost, i.e. they would have an infinite time delay. Close to the disc defined by \( b = 2\tilde{r} \), the time delay would diverge as

\[ \Delta t = C \frac{\tilde{r}}{c L_0} \left( \frac{\tilde{r}}{b - 2\tilde{r}} \right)^{n-2} \]  

(written in a format to allow easy comparison with (16)). We would now need to be glued to the ‘horizon’ for the effect to be measurable, unless \( n \) is very large. However, we would also have a ‘black spot’ comprising the disc \( b < 2\tilde{r} \). This rather extreme free function is the only possibility we found for rendering these theories more tangible, and clearly a large \( a_0 \) is then promptly ruled out.

In summary, we hope that in this communication we have stressed the radical difference between the gravitational physics probed by LLR or VLBI on the one hand, and LPF on the other, regarding saddle points. With LPF one probes second derivatives of the potential, locally. With LLR and VLBI one probes the integral of the potential, at the end points, as a cumulative effect. Therefore, with LPF, for standard MONDian functions, we find a distinctly changing tidal stress at the saddle (to be contrasted with an essentially DC Newtonian background). We cannot realistically get close to the saddle, but even far out we can expect signals with large SNRs. With a delay test, we can potentially probe the region very close to the saddle; however, the predicted effects for standard MONDian functions are tiny. Nevertheless, we become sensitive to functions which diverge at low accelerations, associated with confinement and strong infrared behaviour for gravity. Such theories predict extreme behaviour very close to the saddle, raising the possibility of singularities. They are beyond the reach of LPF and do not purport to present an alternative to dark matter. However, they have an interest in the own right, and are targets for a time-delay test as performed by the current or next generation of lunar retroreflectors, and by VLBI.

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