Neutrino Oscillations and R-parity Violating Collider Signals

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Abstract

R-parity and L violation in the MSSM would be the origin of the neutrino oscillation observed in Super-Kamiokande. A distinctive feature of this framework is that it can be tested in colliders by observing decay products of the destabilized LSP. We examine all the possible decay processes of the neutralino LSP assuming the bilinear contribution to neutrino masses dominates over the trilinear one which gives rise to the solar neutrino mass. We find that it is possible to probe neutrino oscillations through colliders in most of the R-parity conserving MSSM parameter space.

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I. INTRODUCTION

Lepton (L) or baryon number (B) conservation is not a consequence of gauge invariance in supersymmetric extension of the standard model. The presence of both L and B violation would lead to fast proton decay, and thus one usually imposes a discrete symmetry which ensures L and/or B conservation. A typical example is R-parity discarding renormalizable L and B violating operators, as a consequence of which the lightest supersymmetric particle (LSP) is stable and a good candidate of dark matter. Another interesting possibility is to impose baryon parity (B-parity) which allows the following L violating terms in the superpotential of the minimal supersymmetric standard model (MSSM):

\[ \mu_i \hat{H}_u \hat{L}_i + \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c. \]

For the later use, it is convenient to generalize the bilinear term to include the usual \( \mu \)-term:

\[ \mu_\alpha \hat{H}_u \hat{L}_\alpha \]

where \( \alpha = 0, ..., 3 \) and \( \hat{L}_0 = \hat{H}_d \). It remains an open question what kind of discrete symmetry can be derived from a more fundamental theory beyond the MSSM. In view of gauge symmetry origin of discrete symmetries, it has been shown that the standard R-parity and \( Z_3 \) B-parity can only be consistent with the MSSM and the latter is preferred to arrange for proton stability without fine-tuning taking into account higher dimensional operators [1].

R-parity (and L) violation in the MSSM would be the origin of neutrino masses and mixing, in particular, the recently observed atmospheric neutrino oscillation [2]. In the presence of the \( |\Delta L| = 1 \) operators in the superpotential (1) as well as in the soft terms, neutrinos get nonzero masses through the well-known mechanism of particle-sparticle mixing and one-loop radiative correction [3]. A distinctive feature of this mechanism is that the LSP is destabilized and its decay modes can be easily identified in colliders. The most characteristic events for those L-violating phenomena are same-charge final-state dileptons due to the Majorana properties of two produced LSP’s, which have little Standard Model backgrounds [4]. Therefore, detecting the LSP decay, one can directly measure the quantities related to the L violating parameters [5,4], which might enable us to extract some crucial information on the neutrino sector of the theory and test neutrino oscillations in colliders.

The purpose of this paper is to provide a detailed analysis of the decay processes of the LSP (preferably taken as the lightest neutralino) through both the bilinear and the trilinear L violating couplings in the MSSM whose sizes are determined by current neutrino experiments. The bilinear terms play an important role in the whole discussion. As they are typically the dominant sources for neutrino masses, our analysis will be based on the assumption that the atmospheric neutrino masses and mixing are mainly due to the bilinear terms. The collider signatures of the bilinear terms have been studied in Refs [4, 10] taking only one lepton generation and the corresponding (tau) neutrino with a large mass \( \sim \) MeV. The correlation between collider signals and the atmospheric neutrino oscillation has been first analyzed in Ref. [5] and also in [6]. In this paper, we extend these works to include fully the three lepton generations and analyze both two and three body decays of the LSP. The latter become important when the trilinear couplings are introduced to explain the solar neutrino oscillation.

In the next section, we provide a comprehensive formulation of two and three body decay rates of neutralinos through both the bilinear and trilinear couplings. In section III, we
estimate the sizes of the R-parity and L violating parameters for which the current neutrino data can be accommodated. Then, we examine in section IV the region of neutralino mass parameters for which the LSP decay and its branching modes can be measured. An emphasis is put on how to separate out the information on the bilinear terms from the contribution of the trilinear couplings which are present to give the masses and mixing for the solar neutrino oscillation. Conclusion is given in section V.

II. PARTICLE-SPARTICLE MIXING AND LSP DECAYS

One usually rotates away the term $\hat{H}_u \hat{L}_i$ in Eq. (1) so that L violation only appears in the trilinear couplings whose consequences in collider experiments are studied extensively [11]. But, there also exist soft supersymmetry breaking bilinear terms which cannot be rotated away in general. These terms may play more important roles for the LSP decay [5,7–10]. The relevant soft terms in the scalar potential are

$$V_{\text{soft}} = m^2_{H_d} |H_d|^2 + m^2_{L_i} |L_i|^2 + \left\{ B_H H_d + B_i H_u L_i + m^2_{L_i H_d} L_i H_u^\dagger + \text{h.c.} \right\} + \cdots ,$$

from which it is clear that the L violating soft parameters $B_i, m^2_{L_i H_d}$ cannot be rotated away unless the soft parameters satisfy certain relations called the alignment condition [12]. This kind of alignment condition is usually achieved at the mediation scale of supersymmetry breaking which is presumed to be L conserving. However, at the weak scale where the minimization of the scalar potential takes place, the alignment breaks down by the renormalization group evolution of the soft parameters. Therefore, unless one enforces the alignment at the weak scale by some mechanism, the L violating terms being linear in $L_i$ induce nonzero vacuum expectation values (VEVs) of sneutrino fields which are non-aligned with $\mu_i$. As a result of this misalignment, the mixing between particles and sparticles, in particular, neutrinos and neutralinos, arises to lead important phenomenological consequences.

In the generic presence of the bilinear L violating terms $\mu_i$ and nonzero sneutrino VEVs, the neutralino–neutrino mass matrix in the basis $(\tilde{\nu}_3, \tilde{\nu}^0_u, \nu_\alpha)$ is given by

$$M_N = \begin{pmatrix} M_1 & 0 & M_Z s_W v_\alpha / v \\ M_2 & -M_Z c_W v_\alpha / v & M_Z c_W v_\alpha / v \\ 0 & -M_Z c_W v_\alpha / v & -\mu_\alpha \end{pmatrix} .$$

Here, $M_N$ is a $7 \times 7$ symmetric matrix, $v_u = \langle H_u \rangle$, $v_d = \langle H_d \rangle$, $v_i = \langle \tilde{\nu}_i \rangle$ with $i = 1..3$, and $s_W, c_W$ are the weak mixing elements. The neutralino–neutrino mass matrix can be diagonalized by the rotation $N^* M_N N^\dagger$. That is, the index $i$ of the diagonalization matrix $N_{ij}$ runs for the mass eigenstates, and the index $j$ runs for the weak eigenstates $(\tilde{B}, \tilde{W}_3, \tilde{H}^0_u, \nu_\alpha)$. We would like to remind that for the mass eigenstates $\chi^0_i$, the states with $i = 1, 2, 3$ are the neutrinos and $i = 4$ denotes the lightest neutralino.

The chargino-charged lepton mass matrix in the basis $(\tilde{W}^+, \tilde{H}^+_u, E^+_k)$ and $(\tilde{W}^-, l^-_\alpha)$ is

$$M_C = \begin{pmatrix} M_2 & \sqrt{2} M_W v_\alpha / v \\ \sqrt{2} M_W v_\alpha / v & -\mu_\alpha \\ 0 & M^l \end{pmatrix} ,$$

where $M^l$ is the lightest charged lepton mass.
where $M'$ is the $3 \times 4$ matrix with the components; $M'_{k0} = -m_k^l v_k / v_0$, $M'_{k1} = \delta_{ik} m_i^l$. Two diagonalization matrices $U_{R,L}$ are defined by $U_R M \chi U_L^T$ where $U_R$ ($U_L$) casts the positively (negatively) charged states into the mass eigenstates. The first index $i$ of the matrix $U_{Lij}$ and $U_{Rij}$ run for the mass eigenstates; the states with $i = 1, 2, 3$ are the charged leptons ($e, \mu, \tau$) and $i = 4$ and $5$ the two charginos.

In terms of the mass eigenstates, the neutral and charged current interactions take the form:

$$
\mathcal{L} = \frac{e}{2s_W c_W} \sum_{i=1}^{7} \epsilon_i N_{i\sigma} N_{j\sigma}^* \bar{\ell}_i \gamma^\mu P_L \chi_j^0 \gamma_\mu \ell_j \, \frac{e}{s_W} \left\{ \bar{\chi}_i \gamma^\mu \left( \theta_{ij}^L P_L + \theta_{ij}^R P_R \right) \chi_j^0 \frac{1}{m_i + m_j} \right\} + \text{h.c.} \right] 
$$

(5)

where

$$
\theta_{ij}^N = \sum_{\sigma=3}^{7} \epsilon_\sigma N_{i\sigma} N_{j\sigma}^* \quad \text{with} \quad \epsilon_\sigma = (1, -1, \ldots, -1),
$$

$$
\theta_{ij}^L = N_{i2} U_{Lj1} + \frac{1}{\sqrt{2}} \sum_{\sigma=4}^{7} N_{i\sigma} U_{Lj \sigma-2}^*,
$$

$$
\theta_{ij}^R = N_{i2} U_{Rj1} - \frac{1}{\sqrt{2}} N_{i3} U_{Rj2}^*.
$$

To denote the mass eigenstates in a conventional way, let us introduce the following notations: $\nu_i = \chi_i^0$ with $i = 1 - 3$ and $\widetilde{\chi}_i^0 = \chi_i^{0,0}$ for $i = 1 - 4$.

From Eq. (5), one gets the following rate of the decay process $\widetilde{\chi}_i^0 \rightarrow \nu_i Z$ or $l_i^\pm W^\pm$ when the neutralino $\chi_i^0$ is heavier than the Z and/or W boson:

$$
\Gamma(\chi_i^0 \rightarrow \nu_i Z) = \frac{G_F M_{\chi_i^0}^3}{16 \sqrt{2\pi}} I \left( \frac{M_Z^2}{M_{\chi_i^0}^2} \right) |\theta_{\chi_i^0}^N|^2,
$$

$$
\Gamma(\chi_i^0 \rightarrow l_i^\pm W^\pm) = \frac{G_F M_{\chi_i^0}^3}{4 \sqrt{2\pi}} I \left( \frac{M_W^2}{M_{\chi_i^0}^2} \right) \left\{ |\theta_{\chi_i^0}^L|^2 + |\theta_{\chi_i^0}^R|^2 \right\}. 
$$

(7)

where $I(x) = (1 - x)^2 (1 + 2x)$. Note that we have neglected the possible neutralino decay into the lightest Higgs $h^0$ whose rate is suppressed compared to the decay into W boson by factor of the order of $M_W^2/M_{h^0}^2$. There could exist other LSP decay modes such as $\chi_i^0 \rightarrow \nu \gamma$ which occurs at the one-loop level and has the branching ratio at most a few percent

For the sake of discussion it is useful to use a seesaw formula valid for $v_i/v_0, \mu_i/\mu \ll 1$. This allows us to decompose the diagonalization matrices into two parts; the usual neutralino and chargino mixing, and the mixing between neutrinos (charged leptons) and neutralino (charginos) which arise from L violation. In this decomposition, the elements $\theta_{\chi_i}$ for the particle–sparticle mixing in Eq. (3) take the following factorized forms:

$$
\theta_{\chi_i}^N = \left[ \sum_{a=1}^{2} N_{\chi_a} c_a^N + 2 \sum_{a=1}^{3} N_{\chi_a} c_3^N \right] \xi_i c_3,
$$

$$
\theta_{\chi_i}^L = \left[ \sum_{a=1}^{2} N_{\chi_a} c_a^L - \frac{1}{\sqrt{2}} \sum_{a=1}^{2} N_{\chi_a} c_a^L + \frac{1}{\sqrt{2}} N_{\chi_4} (c_2^L - c_4^L) \right] \xi_i c_3,
$$

$$
\theta_{\chi_i}^R = N_{\chi_3} \left[ c_1^R - \frac{1}{\sqrt{2}} c_2^R \right] \xi_i c_3. 
$$

(8)
where $\xi_i \equiv v_i/v_0 - \mu_i/\mu$. Note that no mixing effect can arise if the sneutrino VEVs are aligned with $\mu_i$, that is, $\xi_i = 0$ as discussed in the beginning of this section. The coefficients $c^N, c^L$ and $c^R$ are given by 

$$
c_i^N = \frac{M_Z}{F_N} \left(-s_W \frac{M_2}{M_\gamma}, c_W \frac{M_1}{M_\gamma}, -s_\beta \frac{M_Z}{\mu}, c_\beta \frac{M_Z}{\mu}\right),
$$

$$
c_i^L = \frac{M_W}{F_C} \left(\sqrt{2}, 2s_\beta \frac{M_W}{\mu}\right),
$$

$$
c_i^R = \frac{M_W m_i^2}{F_C^2} \left(\sqrt{2}(\frac{M_2}{\mu} + \frac{1}{t_\beta}), -2c_\beta (\frac{M_2^2}{\mu M_W} + c_\beta^2 \frac{M_W}{\mu})\right),
$$

where $F_N = M_1 M_2 / M_\gamma - M_2^2 s_2 \beta / \mu$, $F_C = M_2 - M_2^2 s_2 \beta / \mu$, and $M_\gamma = c_\gamma^2 M_1 + s_\gamma^2 M_2$. In Eq. (9), $c_4^N$ and $c_2^L$ have extra $\mu_i/\mu$ contributions [14] which are irrelevant since they are canceled out in the physical process of neutralino decays as can be seen from the expressions in Eq. (8). Eqs. (8) and (9) provide clear understanding of a few properties. The ratios $\xi_i / \xi_j$ can be determined by measuring the branching fractions of the decay modes $\tilde{\chi}^0 \rightarrow l_j W$ independently of the other (L conserving) supersymmetric parameters like $M_2$ and $\mu$ also independently of the sizes of the trilinear couplings. The rate $\Gamma(\tilde{\chi}^0 \rightarrow l_i W)$ depends on the charged lepton masses through $c^R$ (9). This mass dependence is suppressed by $m_i^2/F_C$, but can be significant for Higgsino–like LSP and for large $\tan \beta$ as $c^R_4$ is proportional to $1/c_\beta$.

When the LSP is lighter than $W$ boson, it has only three body decay modes which have two major contributions: one is the bilinear contribution coming from the mixing $N, U_{R,L}$ through the exchange of $W$ or $Z$ boson and the other comes from the trilinear $L$ violating couplings $\lambda, \lambda'$ through exchanges of squarks or sleptons. For the latter contributions, there exists a complete formulation in the literature [13]. There are also contributions due to the charged and neutral Higgs exchanges which is neglected in our computation since they are generically smaller than the corresponding contributions from the $W, Z$ exchanges. More important one comes from the slepton-Higgs mixing [14,15] whose size is determined by the bilinear parameters $\mu_i, B_i$ and $m_{L,H_d}^2$. This mixing effect can be translated to the trilinear couplings in the mass basis, which is included in our analysis.

In order to calculate the rates of the neutralino decay $\tilde{\chi}^0 \rightarrow \tilde{f}_i \tilde{f}_j k_k$, we recast the decay amplitudes in terms of the 12 matrix elements into the general form:

$$
\mathcal{M}(\tilde{\chi}^0 \rightarrow \tilde{f}_i \tilde{f}_j k_k) = S_{IJ}(s, t, u)[\bar{v}_\chi P_I v_j][\bar{u}_k P_J v_j] + V_{IJ}(s, t, u)[\bar{v}_\chi \gamma^\mu P_I v_j][\bar{u}_k \sigma_\mu v_J] + T_{IJ}(s, t, u)[\bar{v}_\chi \sigma^{\mu\nu} P_I v_j][\bar{u}_k \sigma_{\mu\nu} v_J],
$$

where $I, J = L, R$, and $s = (p_{\tilde{\chi}} - p_i)^2$, $t = (p_{\tilde{\chi}} - p_j)^2$ and $u = (p_{\tilde{\chi}} - p_k)^2$. In our case, the neutralino decay has the modes; $\tilde{\chi}^0 \rightarrow \nu \bar{\nu}, \nu l \bar{\nu}, \nu q \bar{q}$ and $l q \bar{q}$. For each mode, nonvanishing matrix elements are presented in Appendix. The modes $\tilde{\chi}^0 \rightarrow \nu \bar{\nu}$ and $\nu l \bar{\nu}$ come solely from the bilinear contribution, and thus has one and two matrix elements, respectively. Note that the bilinear contributions appear in $S_{LR}$ or $V_{IJ}$, and only $S_{LL}$ and $T_{LL}$ terms can interfere with each other neglecting the final state masses in our cases. It turns out this interference is negligible in the cases we will consider.
The mode $\tilde{\chi}^0 \rightarrow \nu l \bar{l}$ contains 9 flavor modes of the charged leptons. Thus one could get information on the flavor structure of the trilinear couplings $\lambda_{ijk}$ by identifying dilepton signals with missing energy. In a detector, the modes $\tilde{\chi}^0 \rightarrow \nu q q$ and $l q q'$ can be seen as two jets with missing energy ($\nu jj$) and charged lepton with two jets ($l jj$). In the case of three body decays of the LSP, the values $\xi_i$ could be measured by separating out the bilinear contribution in the $l jj$ decay modes. But the three body LSP decay length from the bilinear contribution alone is typically larger than a few meters. The LSP could decay fast enough when there are relatively large trilinear couplings, in which case a large trilinear contribution to a decay mode can dominate over a bilinear contribution making it impossible to measure $\xi_i$.

One of the most promising ways of measuring $\xi_i$ through various L-violating channels is to look for same-charge dilepton events [4], $l_i l_j + 2W$ or $l_i l_j + 4j$, from two LSP’s produced in, e.g., $e^+ e^-$ collisions. As will be discussed in the following sections, the recent Super-Kamiokande data imply that one should observe comparable numbers of the same-sign ($\mu \mu$), ($\mu \tau$), and ($\tau \tau$) events [4] except for large $\tan \beta$ and small $|\mu|$.

III. NEUTRINO OSCILLATIONS AND L VIOLATING PARAMETERS

Let us estimate the sizes of the R-parity and L violating parameters for which the current neutrino data can be accommodated. In particular, our discussion is based on the assumption that R-parity violation gives rise to the neutrino masses and mixing explaining the atmospheric and solar neutrino oscillations.

As is well known [3, 16–20], the mixing between neutrinos and neutralinos gives rise to the so-called tree mass. The neutrino–neutralino mass matrix $M_N$ has two massless modes corresponding to the lightest neutrinos and the only massive neutralino which is clear from the following seesaw-reduced rank–one neutrino mass matrix;

$$m^{\text{tree}}_{ij} = \frac{M_Z^2}{F_N} c_\beta \xi_i \xi_j.$$  \hspace{1cm} (11)

Let us first suppose that the tree mass gives rise to the largest neutrino mass $m_{\nu_3}$ that is responsible for the atmospheric neutrino oscillation, namely, $m_{\nu_3} = \sqrt{\Delta m^2_{\text{atm}}} \approx 0.05$ eV [2]. Then the overall size of $\xi \equiv \sqrt{\sum_i \xi_i^2}$ is determined from Eq. (11):

$$\xi = 0.7 \times 10^{-6} \frac{1}{c_\beta} \left( \frac{F_N}{M_Z} \right)^{\frac{1}{2}} \left( \frac{m_{\nu_3}}{0.05 \text{eV}} \right)^{\frac{1}{2}}.$$ \hspace{1cm} (12)

Given the mass matrix (11), one can find the following expressions for the oscillation amplitudes [8]:

$$\sin^2 2\theta^\text{atm}_{\mu \tau} = 4 \frac{\xi^2}{\xi^3 \xi^3},$$ \hspace{1cm} (13)

$$\sin^2 2\theta^\text{atm}_{\text{ef}} = 4 \frac{\xi^2}{\xi^2 (1 - \frac{\xi^2}{\xi^2})}.$$
The first quantity is measured to be \( \sin^2 2\theta_{\odot} \approx 1 \) in the Super-Kamiokande which implies \( \xi_2 \approx \xi_3 \) [2]. The second one is measured in the CHOOZ experiment for \( \nu_e \) disappearance experiment [21] from which one finds \( \xi^2_1/\xi^2 \lesssim 0.05 \) in the region \( \Delta m^2_{\text{sol}} > 2 \times 10^{-3} \text{eV}^2 \). Therefore, as realized in Ref. [3], colliders may provide independent checks of the Super-Kamiokande and CHOOZ results by measuring the ratios \( \xi_i/\xi \) through the LSP decays discussed in the previous section. In other words, combining the results from the neutrino experiment and collider signatures, one could prove R-parity violation as a source of neutrino masses. For this to happen, of course, the LSP has to decay inside a detector.

When the LSP is heavier than the W boson and thus the two body decays \( \chi^0 \rightarrow l^\pm W^\mp \) are allowed kinematically, the decay length is typically less than 10 cm [see the next section]. For the LSP mass below \( M_W \), the LSP can have only three body decay modes. The LSP decay through \( W, Z \) exchanges can make the decay length smaller than 1 m for the LSP mass close to \( M_W \) even ignoring the effect of the trilinear couplings. With larger trilinear couplings, the LSP decay through sfermion exchanges can be detectable in colliders. As will be seen in the next section, the contribution of the trilinear couplings with \( \lambda, \lambda' \sim 10^{-6} \) to the three body decay rates becomes comparable to the bilinear contribution in the case of common sfermion masses, \( \tilde{m}_f = 300 \text{ GeV} \). For the explicit calculation, we will take the universal trilinear couplings.

The trilinear couplings of the order of \( 10^{-6} \) are too small to be influential in neutrino experiments. More interesting case is when the trilinear couplings are large enough to produce the neutrino masses and mixing explaining the solar neutrino deficit [22]. To estimate their sizes, let us recall that the trilinear couplings generate the one-loop neutrino mass through the neutrino masses and mixing explaining the solar neutrino deficit [22]. To estimate their sizes, let us recall that the trilinear couplings generate the one-loop neutrino mass through squark and slepton exchanges [3]. Two neutrinos which remain massless at tree level become massive through the loop contribution. The loop mass takes the form,

\[
m^\text{loop}_{ij} = \frac{3m^2_{\nu}(\tilde{A}_b + \mu \tilde{t}_\beta)}{8\pi^2 \tilde{m}_f^2} \lambda'_{333} \lambda'_{j33} + \frac{m^2_{\nu}(\tilde{A}_\tau + \mu \tilde{t}_\beta)}{8\pi^2 \tilde{m}_f^2} \lambda_{333} \lambda_{j33},
\]

(14)

where \( A_{b,\tau} \) are trilinear soft parameters and \( \tilde{m}_{b,\tau} \) are typical soft masses of sbottom and stau fields. Here we keep only \( \lambda_{333}, \lambda'_{333} \) assuming that they give the largest contribution which is the case when the trilinear couplings follow the usual hierarchy of the Yukawa couplings, that is, the couplings involving the third generations are larger than the others. This would be a consequence of a flavor symmetry dictating the hierarchies among the Yukawa and trilinear couplings [23,24]. Still, having the arbitrariness of the L violating parameters \( \xi_i, \lambda_{333} \) and \( \lambda'_{333} \), the loop mass can be made larger than the tree mass. But in the context of minimal supergravity [20,25] or gauge mediated supersymmetry breaking [26–28] where the misalignment \( \xi_i \neq 0 \) is generated radiatively, the tree mass is typically larger than the loop mass. This is our motivation to assume that the largest mass from Eq. (11) explains the atmospheric neutrino oscillation, and the second largest mass due to the loop correction accounts for the solar neutrino oscillation. Suppose now that \( \lambda_{333} \) gives the second largest eigenvalue \( m_{\nu_2} \), which will be the case with the model discussed below. Then, one needs to have \( m_{\nu_2} = \sqrt{\Delta m^2_{\text{sol}}} \approx 2 \times 10^{-3} \text{eV} \) for the MSW solution of the solar neutrino deficit [22]. Therefore, the size of \( \lambda_{233} \) has to be

\[
\lambda_{233} \approx 1.2 \times 10^{-4} \left( \frac{3}{t_\beta} \right)^{1/2} \left( \frac{100 \text{GeV}}{|\mu|} \right)^{1/2} \left( \frac{m_{\nu_2}}{2 \times 10^{-3} \text{eV}} \right)^{1/2},
\]

(15)
taking $\tilde{m}_f = 300$ GeV and ignoring the trilinear soft terms $A$. Let us note for the later use that the required value of $\lambda_{233}$ becomes smaller for larger $\tan \beta$. It is expected that $\lambda_{133}$ is smaller than $\lambda_{233}$ by one or two orders of magnitude in the case of the favorable small mixing MSW solution [22]. As discussed in the previous section, one can use the decay modes $\tilde{\chi}_1^0 \rightarrow \nu l_i l_j$ to get information on the flavor structure of $\lambda_{ijk}$.

With such large trilinear couplings (15), the three body decay rate can become even larger than the two body decay rate. To figure out when this can happen, let us note that the two body decay length for $M_{\tilde{\chi}_1^0} \sim 150$ GeV is about 0.1 cm [see FIG. 1]. Then using the Dawson formula for the three body decay rate of the photino–like LSP [29], we find that the same decay length can be obtained when a single large coupling takes the value,

$$\lambda, \lambda' \sim 1 \times 10^{-4} \left( \frac{\tilde{m}_f}{300 \text{ GeV}} \right)^2 \left( \frac{150 \text{ GeV}}{M_{\tilde{\chi}_1^0}} \right)^{\frac{3}{2}} \left( \frac{0.1 \text{ cm}}{l} \right)^{\frac{1}{2}},$$

where $\tilde{m}_f$ is the mass of squark and $l \equiv 1/\Gamma$ denotes the LSP decay length. We find the above value is in the range of the required value (14) for small $\tan \beta$. Therefore, the effect of trilinear couplings could surpass that of bilinear couplings so that it would be hard to separate out the information on $\xi_i$ alone. Furthermore, the loop masses from the trilinear couplings $\lambda'_{333}$ in Eq. (14) may affect the largest neutrino mass $m_{\nu_3}$. For instance, taking the value of $\lambda'_{333}$ as in Eq. (15), one finds that the loop mass is comparable to the tree mass (11). Therefore, the expressions (11), (13) should be altered to include the loop mass as well. In the context of supergravity where nonaligned $\xi_i$ are generated from the trilinear couplings, the loop mass can be comparable to the tree mass to account for both the atmospheric and solar neutrino oscillations [30]. In this case, it would be nontrivial to probe the mixing angle through L violating decays of the LSP unless all the trilinear couplings are known. However, there is a class of models which are more predictive and do not possess the above difficulties. This is the so-called bilinear model [7,16,17,26,28].

The bilinear model does not have trilinear R-parity violating couplings. A classic example would be the minimal SU(5) grand unified theory where generalized $\mu$ terms, $\mu_{\alpha} \hat{H}_u \hat{L}_\alpha$, are allowed but the trilinear couplings cannot be present to ensure proton longevity. As discussed in the previous section, in order to calculate the LSP decays, one has to go to the mass basis diagonalizing the neutralino/neutrino, chargino/charged lepton and slepton/Higgs mixing [16]. This amounts to generating effectively L violating trilinear couplings. Not bothering with details about bilinear and scalar mass parameters, it is expected that the trilinear couplings take the values, e.g., $\lambda_{ijk} \approx \xi_i \delta_{jk} m_j^d / \langle H_d \rangle$. The dominant couplings are then

$$\lambda'_{i33} \approx 2 \times 10^{-8} \frac{1}{c_\beta} \left( \frac{F_N}{M_Z} \right)^{\frac{1}{2}},$$

$$\lambda_{i33} \approx 7 \times 10^{-9} \frac{1}{c_\beta} \left( \frac{F_N}{M_Z} \right)^{\frac{1}{2}}. \quad (17)$$

As was discussed in the previous section, these couplings are negligible in our discussion for low $\tan \beta$ but become important for large $\tan \beta$. The other couplings are negligible for all $\tan \beta$ in view of our previous discussions. An interesting feature is that the loop mass can
account for the solar neutrino deficit through the MSW solution for large $\tan \beta$. For instance, taking $\tan \beta = 50$, we get $\lambda_{33} \approx 2 \times 10^{-5}$ which agrees with the value given in Eq. (15). We note that, in the large $\tan \beta$ region, the value of $\xi$ in Eq. (12) becomes comparable to $\lambda_{233}$. In this case, the loop mass matrix has a large contribution from $\lambda'_{33}$ but it also takes the form $m_{ij}^{\text{loop}} \propto \xi_i \xi_j$ with the change of its overall size. Thus, the relations in Eq. (13) remain untouched. As will be shown explicitly in the next section, the induced trilinear couplings are large enough to make the LSP decay visible in colliders, but not large enough to change the two body decay length. Furthermore, the numbers $\xi_i$ can be measured without any contamination from the trilinear couplings.

**IV. NUMERICAL ANALYSIS OF LSP DECAYS**

Before presenting our numerical results, let us recapitulate the crucial points for testing neutrino oscillations through the detection of the R-parity violating LSP decays in colliders. First of all, the LSP has to decay inside a detector. So, we require the LSP decay length $l$ to be less than 1 m and show the corresponding region of neutralino mass parameters for given L violating parameters and sfermion masses. We also assume the unification relation

$$M_1 = \frac{5}{3} t_w^2 M_2,$$

and restrict ourselves to the region satisfying the recent LEP constraint on the mass of the lightest chargino: $M_{\tilde{\chi}^\pm} > 90$ GeV [31]. These restrictions rule out the region of $M_2$ and $\mu$ for $M_{\tilde{\chi}_1^0} \lesssim 50$ GeV and the possibility of the chargino LSP.

Since the mode $l_i W$ or $l_i jj$ for the two or three body decay case is important to determine the parameter $\xi_i$ [see Eqs. (8,11)], we will present several figures showing the branching ratios of all possible decay modes on the $M_2 - \mu$ plane, as well as with fixed $M_2$ and varying $\mu$. In the definition of e.g. the $l_i jj$ branching ratio, we include both $l_i^\pm$. The quantities $\xi_i$ are measured by the bilinear contribution to the decay processes we need to separate out this from the trilinear contribution. For this purpose, we will present the branching ratios coming from bilinear terms and trilinear terms separately. To get an idea of when the trilinear contribution becomes visible, we will show the decay length and branching ratios taking the universal trilinear couplings of $10^{-6}$. As discussed in the previous section, when the trilinear couplings are of the order $10^{-4}$ which may be required to produce the solar neutrino mass, the trilinear contribution to the two body decay rate becomes comparable to the bilinear contribution. In this case, it would be meaningless to separate out the two contributions making it complicated to probe the neutrino oscillations through collider signals. However, it is possible to have a good separation between the bilinear and trilinear contributions while generating both the atmospheric and solar neutrino masses. This is the case in the bilinear model with large $\tan \beta$, where the order of the magnitudes of the induced trilinear couplings are given in Eq. (17). Therefore, taking the bilinear model as a specific example for the atmospheric and solar neutrino oscillations, we will show the decay length and branching ratios.

In our computations, we take the following input values as a reference:
\[ \xi_1 << \xi_2 = \xi_3 = \frac{\xi}{\sqrt{2}}, \]  
\[ A_f = m_f = 300 \text{ GeV} \quad \text{for all sfermions}, \]

while \( \xi \) fulfills the relation (14). We vary \( M_2 \) and \( \mu \) in the range of \( 50 \sim 500 \text{ GeV} \) and \(-300 \sim 300 \text{ GeV} \). Here, we take \( \xi_2 = \xi_3 \) reflecting the large mixing angle of the atmospheric neutrino oscillation as in Eq. (13).

Let us now present our results in FIGs. 1–8. In FIG. 1, we show the decay lengths (a) including only the LSP decays to \( W, Z \) bosons or through \( W, Z \) exchanges, (b) taking the universal trilinear couplings \( \lambda, \lambda' = 10^{-6}, \) and (c) considering the bilinear model, for \( \tan \beta = 3 \). The thick lines in the \( M_2 - \mu \) planes correspond to the points for \( M_{\tilde{\chi}^0_1} = M_W \). The shaded region is excluded by the constraint on the lightest chargino mass. One finds that there is a very small region with the two body decay length larger than 1 m (denoted by filled stars just above the thick line) for negative \( \mu \). It is clear from FIG. 1a and 1c that the trilinear couplings in the bilinear model (17) are negligible for the small \( \tan \beta \). There is also a little change in FIG. 1b.

The situation changes for \( \tan \beta = 50 \) as in FIG. 2. The decay lengths become shorter than 10 cm in the two body decay region. The \( M_2 - \mu \) plane is completely covered for the bilinear model as shown in FIG. 2c. This is mainly due to large \( \lambda'_{333} \) as in Eq. (17). Also more region is covered in FIG. 2b with \( \lambda, \lambda' = 10^{-6} \).

In FIG. 3, we present (a) the \( lW \) branching ratio adding all charged leptons, and (b) each branching ratio fixing \( M_2 = 300 \text{ GeV} \), in the case of \( \tan \beta = 3 \). We do not show the region with \( M_{\tilde{\chi}^0_1} < M_Z \) for which the \( \nu Z \) channel is forbidden. As can be seen in FIG. 3a, there is a limited region (denoted by dots) where the \( lW \) mode is strongly suppressed, which corresponds to the point with \( \mu = -140 \text{ GeV} \) in FIG. 3b. This implies that a cancellation among various terms in \( \theta_{\tilde{\chi}^0_1}^{L,R} \) in Eq. (8) takes place. As we put \( \xi_2 = \xi_3 \), the \( \mu W \) (solid line) and \( \tau W \) (dashed line) branching ratios exactly coincide for large \( |\mu| \) and thus for large \( M_{\tilde{\chi}^0_1} \) \((M_2 = 300 \text{ GeV})\). But for smaller \( |\mu| \) the two ratios deviate from each other which contradicts with the result of Ref. [4]. This is a consequence of the charged lepton mass dependence of \( \theta_{\tilde{\chi}^0_1}^{R} \) in Eqs. (8) and (9) which becomes very important for the Higgsino–like LSP. Let us mention as a reference that the valley at \( \mu = -140 \text{ GeV} \) corresponds to the LSP mass \( M_{\tilde{\chi}^0_1} = 123 \text{ GeV} \). Our result is larger than that of Ref. [4] in the \( \nu Z \) decay rate by factor of 2, coming from the fact that the \( \Delta L = -1 \) and \( \Delta L = 1 \) modes are to be summed. The \( eW \) branching ratio is negligibly small as we put \( \xi_1 << \xi_{2,3} \). If \( \xi_1 \) is comparable to \( \xi_{2,3} \), the curve for the \( eW \) branching ratio is in parallel with that of the \( \mu W \) branching ratio with the constant deviation by factor of \( \xi_1^2/\xi_2^2 \) as the dependence on \( m_e \) or \( m_\mu \) is negligible. However, the universal property does not remain for the \( eW \) and \( \tau W \) modes especially for large \( \tan \beta \).

The same set of the branching ratios as FIG. 3 are shown in FIG. 4 with \( \tan \beta = 50 \). The \( lW \) branching fraction becomes larger than that with smaller \( \tan \beta \) and it can be bigger than 40%. From FIG. 4b, one finds that as \( \tan \beta \) increases the deviation between \( \mu W \) and \( \tau W \) branching ratios greatly magnifies as discussed in section II.

FIG. 5 shows three body branching ratios for \( \tilde{\chi}^0_1 \rightarrow \nu \nu \nu, \nu l l, \nu jj \) and \( l jj \) without trilinear couplings. \( M_2 \) is fixed to be 150 GeV. For \( \tan \beta = 3 \) in FIG. 5a, the \( \mu jj \) (solid line) and \( \tau jj \) (dashed line) branching ratios appear to be the same. On the contrary, there is a
big deviation between two branching ratios for $\tan \beta = 50$ and small $|\mu|$ as shown in FIG 5b. This is also due to the large dependence of $\theta^R_{\tilde{\chi}_i}$ on the lepton masses for the Higgsino–like LSP.

In FIG. 6, three body branching ratios with universal trilinear couplings of $10^{-6}$ with $M_2 = 150$ GeV and $\tan \beta = 3$ are shown by separating (a) the bilinear and (b) the trilinear contributions. As there is almost no interference between the two contributions, the full branching ratios can be obtained just by adding these two contributions as in FIG 6c. The trilinear contribution to the $ljj$ modes is shown to be negligible for positive $\mu$ in FIG. 6b. As can be seen from FIG. 6d, the bilinear contribution (solid line) dominates over the trilinear contribution (dashed line) except for $\mu \sim -100$ GeV.

For $\tan \beta = 50$ in FIG. 7, trilinear contribution becomes larger but still the bilinear contribution to $ljj$ mode dominates. One again sees the big deviation between $\mu jj$ and $\tau jj$ modes for smaller $|\mu|$.

Finally, the three body branching ratios of the bilinear model with $\tan \beta = 50$ are shown in FIG. 8. Here the trilinear contribution dominates but it comes mostly from the $\nu jj$ mode. This is because the coupling $\lambda'_{i33}$ do not contribute to the mode $ljj$ since it involves heavy top production. Therefore, one can directly measure the $ljj$ branching ratios from the bilinear contribution which have branching ratios of $1 - 6\%$ as shown in FIG. 8c. Another important property is that the $\nu ll$ branching ratio from the trilinear contribution is about $1\%$ and thus one can also probe the flavor structure of the trilinear couplings $\lambda$. As a consequence, collider experiments can provide an independent check of whether the bilinear model explains the solar and atmospheric neutrino oscillations.

V. CONCLUSIONS

Recent observation of neutrino oscillation in Super-Kamiokande would require physics beyond the standard model. Supersymmetric extension of the standard model allowing L (and thus R-parity) violation is one of the well-motivated theoretical frameworks for nonzero neutrino masses and mixing. That is, the current atmospheric and solar neutrino data may be explained by R-parity violation in the MSSM. A distinctive feature of this picture is that it could be tested in colliders through L violating signatures of the LSP decay. In generating neutrino masses from R-parity violation, one has to take into account both trilinear and bilinear terms in the superpotential and in the soft breaking scalar potential. Typically the bilinear contribution to neutrino masses through nonaligned sneutrino VEVs dominates over the trilinear contribution. Based on this aspect, we have analyzed collider signals for the L violating decays of the LSP (taken as the lightest neutralino) which has two body and/or three body decay channels depending on the LSP mass.

The results of our analysis are shown in FIGs. 1–8 presenting the decay lengths and branching ratios as functions of the supersymmetric parameters $M_2$ and $\mu$ under the assumption of gaugino mass unification. When the trilinear L violating couplings are too small, there is some region of the LSP mass where the LSP decay cannot be seen in a detector. However, if the trilinear couplings are large enough to give rise to the solar neutrino mass while the atmospheric neutrino mass comes from the bilinear parameters, the LSP is shown to decay inside a detector for the whole R-parity conserving MSSM parameter space.
restricted by the recent lower bound on the chargino mass $m_{\tilde{\chi}_1^+} > 90$ GeV and the unification relation of gaugino masses. As a specific example of realizing both the atmospheric and solar neutrino oscillations, we have taken the bilinear model with large $\tan \beta$ which has no arbitrariness in the trilinear couplings. In such a model, the neutrino oscillation parameters measured in the atmospheric neutrino experiments can be directly probed in colliders by measuring the branching ratios of the LSP decays into $l_iW$ or $l_ijj$. This opens a possibility to test the assumption of the R-parity violating MSSM as the origin of neutrino masses and mixing.

In a more general case with arbitrary trilinear couplings, it would be impossible to directly test the neutrino oscillations in colliders. But if future colliders measure the $l_iW$ or $l_ijj$ branching ratios it would provide a hint for or against a specific mechanism predicting the structures of the trilinear couplings and/or soft supersymmetry breaking parameters.

VI. APPENDIX

In order to express the amplitude of the decay $\chi^0 \to \bar{f}_i f_j f_k$ in a compact form, we introduce the notations:

$$g_f^x = \frac{\sqrt{2}e}{s_W c_W} [s_W N^*_{\chi_1^0} Y_f + c_W N^*_{\chi_2^0} T^3_f],$$

$$h_f^x = \begin{cases} N^*_{\chi_3^0} \frac{e}{\sqrt{2} s_W} \frac{m_f}{M_W c_\beta} & \text{for } T^3 = -\frac{1}{2} \text{ fermions}, \\ N^*_{\chi_4^0} \frac{e}{\sqrt{2} s_W} \frac{m_f}{M_W s_\beta} & \text{for } T^3 = \frac{1}{2} \text{ fermions}, \end{cases}$$

$$l_i^x = \frac{e^2}{\sqrt{2} s_W^2} \theta_{\chi_i}^L, \quad r_i^x = \frac{e^2}{\sqrt{2} s_W^2} \theta_{\chi_i}^R, \quad n_{ij}^x = -\frac{e^2}{2 s_W^2 c_W} \theta_{\chi_i}^N [T^3_f - s_W^2 Q_f],$$

$$P_V(x) = \frac{1}{x - M_V^2} \quad \text{for } V = Z, W, \; x = s, t, u,$$

$$P_f(x) = \begin{cases} g_f^x (x - m_{\tilde{f}_1}^2) + h_f^x \Delta \tilde{f} & \text{for } x = s, t, \\ (x - m_{\tilde{f}_1}^2) (x - m_{\tilde{f}_2}^2) & \text{for } x = t, \\ m_{\tilde{f}_R} m_{\tilde{f}_L} & \text{for } x = u, \end{cases}$$

$$Q_f(x) = \begin{cases} h_f^{x*} (x - m_{\tilde{f}_1}^2) + g_f^{x*} \Delta \tilde{f} & \text{for } x = s, t, \\ (x - m_{\tilde{f}_1}^2) (x - m_{\tilde{f}_2}^2) & \text{for } x = t, \\ m_{\tilde{f}_R} m_{\tilde{f}_L} & \text{for } x = u, \end{cases}$$

Here $m_{\tilde{f}_1, \tilde{f}_2}^2$ are the eigenvalues of the sfermion mass-squared matrix,

$$\begin{pmatrix} m_{\tilde{f}_L}^2 & \Delta \tilde{f} \\ \Delta \tilde{f} & m_{\tilde{f}_R}^2 \end{pmatrix},$$

where $\Delta \tilde{f} = m_f (A_f + \mu t_\beta)$ or $m_f (A_f + \mu / t_\beta)$ for $T^3 = \mp \frac{1}{2}$ sfermions, respectively.

For the amplitude of the decay $\chi^0 \to \bar{\nu}_i e_j^+ e_k^-$ coming from the bilinear contribution and the trilinear coupling $\lambda_{ijk}$, we find the following 5 nonvanishing matrix elements:
We find that the decay $\chi \to \bar{\nu}_e \nu_x$ is possible for the decay $\chi \to \bar{\nu}_e ar{\nu}_e$ for the decay $\chi \to \bar{\nu}_e ar{\nu}_e$. Similarly, there are 5 nonvanishing matrix elements for the decay $\chi \to e^+ \bar{\nu}_d d_k$:

$$S_{LL}^{\nu e} = \delta_{ij}k [P_{\bar{\nu}_e}(s) - \frac{1}{2} P_{\nu_e}(t) - \frac{1}{2} P_{\bar{\nu}_e}(u)],$$

$$S_{LR}^{\nu e} = \delta_{ij}k r_{\bar{\nu}_e} P_{\nu_e}(t),$$

$$V_{LL}^{\nu e} = \delta_{ij}n_{\nu_e} P_{\nu_e}(s) + \delta_{ij}r_{\bar{\nu}_e} P_{\nu_e}(t) + \frac{1}{2} \lambda_{ij}k Q_{\nu_e}(u),$$

$$V_{LR}^{\nu e} = -\delta_{ij}n_{\nu_e} P_{\nu_e}(s) - \frac{1}{2} \lambda_{ij}k Q_{\nu_e}(t),$$

$$T_{LL}^{\nu e} = -\frac{1}{8} \lambda_{ij} [P_{\nu_e}(t) - P_{\bar{\nu}_e}(u)].$$

Similarly, there are 5 nonvanishing matrix elements for the decay $\chi \to e^+ \bar{\nu}_d d_k$:

$$S_{LL}^{\nu d} = -\lambda'_{ij} [P_{\bar{\nu}_d}(s) - \frac{1}{2} P_{\nu_d}(t) - \frac{1}{2} P_{\bar{\nu}_d}(u)],$$

$$S_{LR}^{\nu d} = -\lambda'_{ij} k Q_{\nu_d}(s),$$

$$V_{LL}^{\nu d} = V_{\nu d}^{\nu e} P_{\nu_e}(s) - \frac{1}{2} \lambda'_{ij} k Q_{\nu_d}(u),$$

$$V_{LR}^{\nu d} = V_{\nu d}^{\nu e} P_{\nu_e}(s),$$

$$T_{LL}^{\nu d} = +\frac{1}{8} \lambda'_{ij} [P_{\nu_d}(t) - P_{\bar{\nu}_d}(u)].$$

We find that the decay $\chi \to \bar{\nu}_i \bar{\nu}_j d_k$ has 4 nonzero matrix elements:

$$S_{LL}^{\nu d} = \lambda'_{ij} [P_{\bar{\nu}_d}(s) - \frac{1}{2} P_{\nu_d}(t) - \frac{1}{2} P_{\bar{\nu}_d}(u)],$$

$$V_{LL}^{\nu d} = \delta_{ij}k n_{\nu_d} P_{\nu_d}(s) + \frac{1}{2} \lambda'_{ij} k Q_{\nu_d}(u),$$

$$V_{LR}^{\nu d} = -\delta_{ij}k n_{\nu_d} P_{\nu_d}(s) - \frac{1}{2} \lambda'_{ij} k Q_{\nu_d}(t),$$

$$T_{LL}^{\nu d} = -\frac{1}{8} \lambda'_{ij} [P_{\nu_d}(t) - P_{\bar{\nu}_d}(u)].$$

For the decay $\chi \to \bar{\nu}_i \bar{\nu}_j u_k$, we find that there are only two nonvanishing matrix elements:

$$V_{LL}^{\nu u} = \delta_{ij}k n_{\nu_u} P_{\nu_u}(s),$$

$$V_{LR}^{\nu u} = -\delta_{ij}k n_{\nu_u} P_{\nu_u}(s).$$

Finally, the decay $\chi \to \bar{\nu}_i \bar{\nu}_j \nu_k$ has only one nonvanishing term:

$$V_{LL}^{\nu \nu} = \delta_{ij}k n_{\nu_u} P_{\nu_u}(s) + \delta_{ik}n_{\nu_u} P_{\nu_u}(t).$$
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FIGURE CAPTIONS

FIG. 1. Two body and three body decay lengths of the LSP on the \( M_2 - \mu \) plane with (a) the vanishing trilinear couplings, (b) the universal trilinear couplings of \( \lambda, \lambda' = 10^{-6} \), and (c) the trilinear couplings given by the bilinear model when \( \tan \beta = 3 \). The shaded area is excluded by the restriction \( m_{\tilde{\chi}_1^\pm} > 90 \) GeV and the two body decay is allowed only in the above the thick solid line corresponding to \( m_{\tilde{\chi}_2^0} = M_W \). Different markers are used to denote the region with the decay length of \( l < 10^{-3} m \) (dot), \( 10^{-3} < l < 10^{-2} m \) (open circle), \( 10^{-2} < l < 10^{-1} m \) (asterisk), \( 10^{-1} < l < 1 m \) (filled circle), \( l > 1 m \) (filled star) and \( l < 1 m \) (filled box).

FIG. 2. The same as FIG. 1, but with \( \tan \beta = 50 \).

FIG. 3. (a) Two body branching ratio of the LSP decay into \( lW \) on the \( M_2 - \mu \) plane with \( \tan \beta = 3 \). Different markers are used to denote the region with the branching ratio of \( 0 \sim 0.2 \) (dot), \( 0.2 \sim 0.4 \) (open circle), \( 0.4 \sim 0.6 \) (asterisk), \( 0.6 \sim 0.8 \) (filled star), \( 0.8 \sim 0.9 \) (filled triangle) and \( 0.9 \sim 1 \) (filled box). (b) The same branching ratios when \( M_2 = 300 \) GeV. In the shaded area, \( M_{\tilde{\chi}_1^0} < M_Z \). The four different lines correspond to the decay mode of \( \tilde{\chi}_1^0 \rightarrow \mu W \) (solid line), \( \tilde{\chi}_1^0 \rightarrow \tau W \) (dashed line), \( \tilde{\chi}_1^0 \rightarrow lW \) (dash-dotted line) and \( \tilde{\chi}_1^0 \rightarrow \nu Z \) (dotted line), respectively.

FIG. 4. The same as FIG. 3, but with \( \tan \beta = 50 \).

FIG. 5. (a) Three body branching ratios with the vanishing trilinear couplings when \( M_2 = 150 \) GeV and \( \tan \beta = 3 \). The shaded area is excluded due to the constraint \( M_{\tilde{\chi}_1^\pm} > 90 \) GeV. The five lines correspond to the decay mode of \( \tilde{\chi}_1^0 \rightarrow \mu jj \) (solid line), \( \tilde{\chi}_1^0 \rightarrow \tau jj \) (dashed line), \( \tilde{\chi}_1^0 \rightarrow \nu jj \) (dash-dotted line), \( \tilde{\chi}_1^0 \rightarrow \nu \nu \nu \) (thick dash-dotted line), and \( \tilde{\chi}_1^0 \rightarrow \nu \nu \) (dotted line). (b) The same as (a), but with \( \tan \beta = 50 \).

FIG. 6. Three body branching ratio with universal trilinear couplings of \( 10^{-6} \) when \( \tan \beta = 3 \) and \( M_2 = 150 \) GeV. Bilinear and trilinear contributions are shown in (a) and (b), respectively, and the sum of these two contributions is shown in (c). The line convention for each decay mode in (a), (b) and (c) is the same as in FIG. 5. In (d), we show the branching ratio summed over the five decay modes. The solid line denotes the bilinear contribution and the dashed line the trilinear contribution.

FIG. 7. The same as FIG. 6, but with \( \tan \beta = 50 \).

FIG. 8. The same as FIG. 7, but with the trilinear couplings given by the bilinear model.
FIGURES

FIG. 1.
FIG. 2.
FIG. 3.
FIG. 4.

(a) M2

(b) BR

1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 0 100 200 300 $\mu$

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19
FIG. 6.
FIG. 7.
