The Tolman-Bondi Model in the Ruban-Chernin Coordinates.

1. Equations and Solutions. *

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Abstract

The Tolman-Bondi (TB) model is defined up to some transformation of a co-moving coordinate but the transformation is not fixed. The use of an arbitrary co-moving system of coordinates leads to the solution dependent on three functions $f, F, F'$ which are chosen independently in applications.

The article studies the transformation rule which is given by the definition of an invariant mass. It is shown that the addition of the TB model by the definition of the transformation rule leads to the separation of the couples of functions $(f, F)$ into nonintersecting classes. It is shown that every class is characterized only by the dependence of $F$ on $f$ and connected with unique system of co-moving coordinates. It is shown that the Ruban-Chernin system of coordinates corresponds to identical transformation.

The dependence of Bonnor’s solution on the Ruban-Chernin coordinate $M$ by means of initial density and energy distribution is studied. It is shown that the simplest flat solution is reduced to an explicit dependence on the coordinate $M$. Several examples of initial conditions and transformation rules are studied.

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1 The Introduction

The observations show that in the large scale the Universe is not homogeneous. At the same time it is also supposed that the secreted centre is missing. These two properties separately are presented in the Friedmann-Robertson-Walker (FRW) and Tolman-Bondi (TB) models (Tolman 1934; Bondi 1947): the FRW model is homogeneous and does not include the secreted centre; the TB is nonhomogeneous and includes one.

As the simplest nonhomogeneous model the TB model is used for interpretation of the observation data. The TB model is used to calculate the redshift (Bondi 1947; Moffat 1992; Ribeiro 1992a, 1993) to describe the local void (Moffat 1995; Bonnor & Chamorro 1990; Chamorro 1991) to interpret the fractal structure of the matter distribution in the Universe (Ribeiro 1992a; Ribeiro 1992b; Ribeiro 1993). A place of the TB model among the models consistent with the modern observation data is shown in (Baryshev et al. 1994).

The TB model (Bondi 1947; Tolman 1934) describes the spherical symmetry dust motion with zero pressure in a co-moving system of coordinates. The solution of the Tolman’s
equations obtained by Bonnor (1972; 1974) contains two undetermined functions \( f(r) \) and \( F(r) \) of the co-moving coordinate \( r \) which are defined by an initial conditions of the model. But the co-moving coordinate is defined up to a continuous transformation \( r' = \Phi(r) \), so the functions \( f(r) \) and \( F(r) \) are defined nonuniquely. This makes difficulty in an interpretation of the observation data.

The article is based on the Bonnor’s solution and on the univalent definition of the co-moving coordinate given by Ruban and Chernin (1969).

The article is dedicated to classification of the transformation \( \Phi \), to formulation an initial conditions through density and energy distribution, to representation of the TB model in the Ruban-Chernin system of coordinates and to the study of some examples of function \( f \) and \( F \) and initial conditions.

It is shown that the Bonnor’s flat solution of the TB model is reduced to an explicit dependence on the co-moving coordinate \( M \) in the Ruban-Chernin system of coordinates.

The Newtonian analog of the TB model has been studied by Ruban and Chernin (1969).

2 The TB Model

The TB model is represented by an interval, an equation of motion for the metrical function \( \omega(r, t) \) with initial conditions for it and an equation for density (Tolman 1934).

In the spherical symmetry and co-moving system of coordinates the interval of the TB model has the form

\[
ds^2(r, t) = e^{\omega(r,t)} \left( \frac{\omega'^2(r, t)}{4 f^2(r)} dr^2 + e^{\omega(r,t)} \left( d\theta^2 + \sin^2 \theta d\phi \right) - c^2 dt^2, \right. \]

where \( c \) is the velocity of light, \( t \) is time, \( \omega(r, t) \) is the metrical function, \( f(r) \) is the undetermined function, \( \dot{r} = \frac{\partial}{\partial t} \). The metric (1) is defined up to a continuous transformation \( \Phi \) of the co-moving coordinate \( r \) (Landau & Lifshits 1973).

The system of Einstein’s equation is reduced to the equation of motion and the equation of density. The equation of motion is:

\[
e^{\omega(r,t)} \left( \ddot{\omega}(r, t) + \frac{3}{4} \dot{\omega}^2(r, t) - \Lambda \right) + \left[ 1 - f^2(r) \right] = 0 \tag{2}
\]

with initial conditions

\[
\omega(r, t) \bigg|_{t=0} = \omega_0(r), \quad \dot{\omega}(r, t) \bigg|_{t=0} = \dot{\omega}_0(r), \tag{3}
\]

where \( \Lambda \) is a cosmological constant and \( \omega_0(r) \) and \( \dot{\omega}_0(r) \) are given functions.

Using the Bonnor (1972) notation

\[
R(r, t) = e^{\omega(r,t)/2}, \tag{4}
\]

where \( R(r, t) \) is an analog of the Euler coordinate of the particle with co-moving coordinate \( r \), we rewrite the initial conditions in the form

\[
R(r, t) \bigg|_{t=0} = R_0(r) = e^{\omega_0(r)/2}, \tag{5}
\]

\[
\dot{R}(r, t) \bigg|_{t=0} = \dot{R}_0(r) = \frac{1}{2} \dot{\omega}_0(r) e^{\omega_0(r)/2}. \tag{6}
\]
First integral of the equation (2) is:

$$\frac{1}{2} \left( \frac{\partial R(r,t)}{\partial t} \right)^2 = c^2 \frac{f^2(r) - 1}{2} + c^2 \frac{F(r)}{4R(r,t)} + \frac{\Lambda}{6} c^2 R^2(r,t),$$

(7)

where $F(r)$ is the second undetermined function. The function $F(r)$ is defined from (5), (6) and (7). If $\Lambda = 0$ then the equation (7) may be interpreted as an analog of the energy conservation law (Bondi 1947).

The integral of the equation (6) has the form

$$\pm t + F(r) = \int_{R_0(M)}^{R(M,t)} \frac{d\bar{R}}{\sqrt{f^2(r) - 1 + \frac{F(r)}{2R} + \frac{\Lambda}{3} \bar{R}^2}},$$

(8)

where $F(r)$ is the third undetermined function, the sign ‘+’ corresponds to an expanding solution and the sign ‘−’ corresponds to the falling one.

The definition of density is given in the TB model by the formul a

$$8\pi G \frac{8\pi G}{c^2} \rho(r,t) = \frac{dF(r)}{dr} \frac{1}{2 R^2(r,t) \frac{\partial R(r,t)}{\partial r}}.$$  

(9)

3 The Transformation Rules of the Co-moving Coordinates and the Ruban-Chernin System of Coordinates

In accordance with (8) the invariant density has the form

$$\rho(r,t) \sqrt{-g(r,t)} = \frac{c^2}{16 \pi G} \frac{F'(r)}{f(r)},$$

(10)

and does not depend on time. The invariant mass is

$$M(r) = 4 \pi \int_0^r \rho(r,t) \sqrt{-g} \, dr =$$

$$4 \pi \int_0^r \rho(r,t) \frac{R^2(r,t)}{f(r)} \frac{\partial R(r,t)}{\partial r} \, dr = \frac{c^2}{4G} \int_0^r \frac{F'(r)}{f(r)} \, dr.$$

(11)

(11) has the sense of transformation rule from $r$ to $M$ and is specified by the function

$$\Psi(r) = \frac{c^2}{4G} \frac{F'(r)}{f(r)}.$$  

(12)

The transformation (11) is read now

$$M(r) = \int_0^r \Psi(\bar{r}) \, d\bar{r}.$$  

(13)
We suppose also that \( M^{-1} \) exists and is the only one. The uniqueness is connected to the condition of the particle layers nonintersecting. From (13) it follows that the transformation rule from some coordinate \( r_1 \) to \( r_2 \) one is as follows:

\[
\int_0^{r_1} \Psi_1(r) \, dr = \int_0^{r_2} \Psi_2(r) \, dr.
\] (14)

The transformation (13) introduces a classification of the co-moving coordinates and the functions \( f \) and \( F \). The function \( \Psi \) defines: 1) the transformation \( r \to M \) and position of the system coordinates \( \{r, t\} \) with respect to the system coordinates \( \{M, t\} \); 2) the classes of the couples of the functions \( f(r) \) and \( F(r) \) which are indiscernible form the point of view of the transformation rule (11); 3) the dependence between the functions \( f(r) \) and \( F(r) \) inside every class:

\[
F(r) = \frac{4G}{c^2} \int_0^r \Psi(\tilde{r}) f(\tilde{r}) \, d\tilde{r}.
\] (15)

From (13) it follows that the classes of couples of the functions \( f \) and \( F \) with different functions \( \Psi \) are nonintersecting.

The transformation

\[
\Phi : r \to \tilde{r}
\] (16)

between two arbitrary co-moving coordinates \( r \) and \( \tilde{r} \) has been introduced in the section 1. We can define it now as

\[
\Phi = \psi \circ \tilde{\psi}^{-1},
\] (17)

where

\[
\psi : r \to M, \quad \tilde{\psi}^{-1} : M \to \tilde{r}.
\] (18)

(17) demonstrates that the transformation \( \Phi \) between arbitrary \( r \) and \( \tilde{r} \) is represented as the superposition of two transformations having the form (13). So, the study of the transformation (16) is reduced to the studying the transformation (13) or, which is the same, the function \( \Psi \).

There are three functions \( f(r) \), \( F(r) \) and \( \Psi(r) \), only two of which are independent. The function \( f(r) \) defines the metric (1) and must be used as an independent function. The function \( \Psi(r) \) marks the class of the transformation rules. So, we can use the set of functions \( \Psi, f, F \) instead of the set \( f, F, \Psi \).

The function \( F(r) \) is used as initial condition in a number of articles. So, the use \( F(r) \) or \( \Psi(r) \) as independent function follows from the context of the problem.

We define the Ruban-Chernin system of coordinates as the identical transformation (13):

\[
M = r.
\] (19)

The function \( \Psi \) corresponding to this transformation is

\[
\Psi(r) = 1
\] (20)

so, the dependence between \( F(r) \) and \( f(r) \) in case of identical transformation is

\[
F(M) = \frac{4G}{c^2} \int_0^M f(\tilde{M}) \, d\tilde{M}.
\] (21)
4 The Definition of the Functions \( f, F \) and \( F \) in the Ruban-Chernin Coordinates

First interpretation of the functions \( f(r) \) and \( F(r) \) has been presented in (Bondi 1947). Following Bondi let us compare the equation (7) and the equation of the total energy in the Newtonian theory:

\[
\frac{1}{2} \left( \frac{\partial R(r,t)}{\partial t} \right)^2 = E(m) + \frac{Gm}{R(m,t)}.
\]

(22)

where \( m \) is the mass of the sphere where the particle is located, \( E(m) \) is the full specific energy. In case of \( \Lambda = 0 \), the equations have the unique structure concerning \( R \), so

\[
E_0(r) = c\frac{f^2(r) - 1}{2}
\]

(23)

may be interpreted as an analog of a full specific energy and

\[
\frac{c^2}{4R(r,t)} F(r) = \frac{Gm(r)}{R(r,t)}
\]

(24)

may be interpreted as an analog of a specific potential energy, \( m(r) \) is an effective mass which will be defined.

Now let us study the case of two system of coordinates: a system of an arbitrary coordinates \( \{r,t\} \) and the Ruban-Chernin system of coordinates \( \{M,t\} \). Suppose the functions \( f(r) \) and \( F(r) \) are specified. The transformation rule from \( \{r,t\} \) to \( \{M,t\} \) is given by the formula (11), where \( M(r) \) is the continuous transformation up to which the co-moving coordinate is defined. So, we have three functions \( f(r), \Psi(r) \) and \( M(r) \), any two of which define the third function.

From (10) and (21) it follows that the Ruban-Chernin system of coordinates is singled out by the fact that the invariant density is constant:

\[
\rho(M,t) \sqrt{-g(M,t)} = \frac{1}{4\pi}.
\]

(25)

From (24) it follows that the effective mass \( m(M) \) is:

\[
m(M) = \int_0^M f(\tilde{M}) \, d\tilde{M}.
\]

(26)

The initial condition \( E_0(M) \) defines the function \( f(M) \) by the formula (23), so the formula (21) becomes the definition of the function \( F(M) \). The effective mass is equal to \( M \) in case of \( f = 1 \). We can now rewrite the equation (5) in the form

\[
\frac{1}{2} \left( \frac{\partial R(M,t)}{\partial t} \right)^2 = E_0(M) + \frac{Gm}{R(M,t)} \int_0^M \sqrt{1 + \frac{2}{c^2} E_0(M) \, dM} + \frac{\Lambda}{6} c^2 R^2(M,t),
\]

(27)

where

\[
E_0(M) \geq -\frac{c^2}{2}.
\]
Tolman (1934) also uses the third undetermined function \( F \) to solve the equation (7). The function \( F(M) \) in the Ruban-Chernin coordinates has the form
\[
F(M) = \int_0^{R_0(M)} \frac{d\tilde{R}}{\sqrt{2 E_0(M) + 2 \frac{G m(M)}{R} + \frac{c^2 \Lambda}{3} \tilde{R}^2}}.
\] (28)

5 The Initial Conditions for the TB Model in the Ruban-Chernin Coordinates

The equation of motion (2) for metrical function \( \omega(M,t) \) requires two initial conditions: \( \omega(M,0) \) and \( \dot{\omega}(M,0) \). These functions are obtained in this section.

Let us suppose that an initial profile of the density is given as the function of the co-moving coordinate \( M \). From (11) and (19) it follows that
\[
R(M,0) = R_0(M) = \left[ \frac{3}{4 \pi} \int_0^M \frac{f(\tilde{M})}{\rho(\tilde{M},0)} d\tilde{M} \right]^{1/3}.
\] (29)

Substituting (4) into (29) we obtain the first initial condition
\[
\omega(M,0) = \frac{2}{3} \ln \left[ \frac{3}{4 \pi} \int_0^M \frac{f(\tilde{M})}{\rho(\tilde{M},0)} d\tilde{M} \right].
\] (30)

The second initial condition is obtained by the substitution (4), (5) and (6) into (27):
\[
\left( \frac{\partial \omega(M,t)}{\partial t} \right)^2 \bigg|_{t=0} = c^2 \dot{\omega}^2(M,0) =
\]
\[
= 8 \frac{E_0(M)}{R^2(M,0)} + 8 \frac{G m(M)}{R(M,0)} + \frac{4}{3} c^2 \Lambda.
\] (31)

For the initial profile of the velocity we obtain:
\[
\frac{\partial R(M,0)}{\partial t} = \pm \sqrt{2 E_0(M) + 2 \frac{G m(M)}{R(M,0)} + \frac{c^2 \Lambda}{3} R^2(M,0)}.
\] (32)

The equations (29) - (32) represent the initial conditions of the TB model through the initial profiles of density and energy. The last equation defines the initial profile of velocity. The full specific energy \( E_0(R) \) is limited by the meaning \( E_{\text{min}}(R) \) when \( \frac{\partial R(M,0)}{\partial t} = 0 \):
\[
E_0(R) \geq E_{\text{min}}(R) = - \frac{G m(M)}{R(M,0)} - \frac{c^2 \Lambda}{6} R^2(M,0).
\] (33)

6 The Initial Conditions for the FRW Model in the Ruban-Chernin Coordinates

The FRW model is the special case of the TB model which is specified by the condition
\[
\frac{\partial \rho(M,t)}{\partial M} = 0.
\] (34)
So, only the first initial condition is changed and reads:

\[ R_{FRW}(M,0) = \left[ \frac{3}{4 \pi \rho_{FRW}(0)} \int_0^M f(\tilde{M}) \, d\tilde{M} \right]^{1/3}, \]  

(35)

\[ \omega_{FRW}(M,0) = \frac{2}{3} \ln \left[ \frac{3}{4 \pi \rho_{FRW}(0)} \int_0^M f(\tilde{M}) \, d\tilde{M} \right]. \]  

(36)

7 The Bonnor’s Solution in the Ruban-Chernin Coordinates

The solution of the equation of the TB model with \( \Lambda = 0 \) has been obtained by Bonnor (1972; 1974). The solution of the Newtonian analog of the TB model has been obtained by Ruban and Chernin and has the same form. We represent here the solution in the Ruban-Chernin coordinates.

From the equation (27) it follows that

\[ R(M,t) \left( \frac{\partial R(M,t)}{\partial t} \right)^2 - 2 E_0(M) R(M,t) = 2 G m(M). \]  

(37)

This equation is studied together with the initial conditions (29).

The flat Bonnor’s solution is obtained by the substitution of \( E_0(M) = 0 \) and brings to the equation

\[ R(M,t) \left( \frac{\partial R(M,t)}{\partial t} \right)^2 = 2 G M \]  

(38)

which has the solution

\[ R^{3/2}(M,t) = R_0^{3/2}(M) \pm 3 \sqrt{\frac{G M}{2}} t, \]  

(39)

satisfying the initial condition (29).

In case of \( E_0(M) \neq 0 \) the solution has the following form depending on the sign of the \( E_0(M) \). For \( E_0(M) < 0 \):

\[ R(M,t) = R_m(M) \{ 1 - \cos[\eta(M,t)] \}, \quad \pm t + F(M) = \frac{R_m(M)}{\sqrt{2|E_0(M)|}} \{ \eta(M,t) - \sin[\eta(M,t)] \}, \quad E_0(M) < 0, \]  

(40)

where

\[ R_m(M) = \frac{G m(M)}{|E_0(M)|}, \]  

(41)

\[ \eta(M,t) = 2 \arcsin \sqrt{\frac{R(M,t)}{R_m(M)}}. \]  

(42)

In the Ruban-Chernin system of coordinates the function \( F(M) \) has the form:

\[ F(M) = \frac{R_m(M)}{\sqrt{2|E_0(M)|}} \{ \eta_0(M) - \sin[\eta_0(M)] \}, \]  

(43)
where

$$\eta_0(M) = \eta(M, 0)$$  \hspace{1cm} (44)

In case of $E_0(M) > 0$:

$$R(M, t) = R_m(M) \left\{ \cosh[\eta(M, t)] - 1 \right\},$$

$$\pm t + F(M) = \frac{R_m(M)}{\sqrt{2E_0(M)}} \left\{ \sinh[\eta(M, t)] - \eta(M, t) \right\},$$

where

$$R_m(M) = \frac{Gm(M)}{E_0(M)},$$  \hspace{1cm} (46)

$$\eta(M, t) = 2 \text{arcosh}\sqrt{\frac{R(M, t)}{R_m(M)}}.$$  \hspace{1cm} (47)

In the Ruban-Chernin system of coordinates the function $F(M)$ has the form:

$$F(M) = \frac{R_m(M)}{\sqrt{2E_0(M)}} \left\{ \sinh[\eta_0(M)] - \eta_0(M) \right\},$$  \hspace{1cm} (48)

where

$$\eta_0(M) = \eta(M, 0).$$  \hspace{1cm} (49)

8 The Examples of Functions $f(r)$, $F(r)$, $\Phi(r)$ and Initial Conditions

Up to this moment we have used one transformation $\Phi$, which has been produced by the definition of the invariant mass. In Table 1 we show several examples of co-moving coordinates given by Bonnor.

Table 2 shows the transformation rules from some system of coordinates $\{r, t\}$ defined by two functions $f(r)$ and $F(r)$, to the Ruban-Chernin system of coordinates $\{M, t\}$. The transformation rule is produced by the given functions $f$ and $F$.

We will analyze now two examples of initial conditions given by Bonnor (1974) and Ribeiro (1993). They consider the flat Bonnor’s solution in the form

$$R(r, t) = \frac{1}{2} \left[ 9F(r) \right]^{1/3} \left[ t + \beta(r) \right]^{2/3}.$$  \hspace{1cm} (50)

Independently from the fact how the function $F$ depends on $r$, it follows from (11) that in case of $f(r) = 1$

$$F(M) = \frac{4G}{c^2} M.$$  \hspace{1cm} (51)

Comparing (50), (51) and (39) we find out that

$$R(M, t) = \frac{9^{1/3}}{2} \left( \frac{4GM}{c^2} \right)^{1/3} \left[ t + \beta(M) \right]^{2/3},$$  \hspace{1cm} (52)
Table 1: Examples of co-moving coordinates.

| author | $\Phi : r \rightarrow \text{new coordinate}$ |
|--------|---------------------------------------------|
| Ruban and Chernin, 1969 | $r \rightarrow M(r)$ |
| Bonnor, 1972 | $r \rightarrow \alpha R_0(r)$ |
| Bonnor, 1972 | $r \rightarrow \alpha (1 + kr^3)^{-1/6}$ |
| Bonnor, 1974 | $r \rightarrow \alpha F(r)$ |

Table 2: Examples of the functions $f, F, F$ and transformation rules.

| author | $f(r)$ | $F(r)$ | $F$ | $M = \psi(r)$ |
|--------|--------|--------|-----|----------------|
| Bonnor, 1972 | 1 | $\frac{\alpha R_0^2(r)}{\sqrt{1 + k R_0^3(r)}}$ | $\sim r^{3/2}$ | $4\pi \frac{\alpha R_0^2(r)}{\sqrt{1 + k R_0^3(r)}}$ |
| Bonnor, 1974 | 1 | $\alpha r^3$ | $F = 0$ | $4\pi \alpha r^3$ |
| Bonnor, 1974 | $\sqrt{1 + r^2}$ | $\alpha r^3$ | $r F' \rightarrow 0$ | $6\pi \alpha \left( r \sqrt{1 + r^2} - \text{arcsinh}(r) \right)$ |
| and Chamorro, 1991 | $\sin^2(r)$ | $k \sin^3(r)$ | $\beta_0 + \eta_0 r^q$ | $12 \pi k \sin(r)$ |
| Ribeiro, 1993 | 1 | $\alpha r^p$ | $\ln(e^{\beta_0} + \eta_1 r)$ | $4\pi \alpha p \int_0^r \frac{\tilde{r}^{p-1} d\tilde{r}}{\cos(\tilde{r})}$ |
| Ribeiro, 1993 | $\cos(r)$ | $\alpha r^p$ | $\beta_0 + \eta_0 r^q$ | $4\pi \alpha p \int_0^r \frac{\tilde{r}^{p-1} d\tilde{r}}{\cosh(\tilde{r})}$ |
| Ribeiro, 1993 | $\cosh(r)$ | $\alpha r^p$ | | |
where
\[ \beta(M) = \frac{1}{3} \sqrt{\frac{2 R_0^3(M)}{GM}}. \]  

Using the initial conditions (29) we obtain
\[ \beta(M) = \left( \frac{1}{6 \pi GM} \int_0^M \frac{d\tilde{M}}{\rho_0(M)} \right)^{-1/3}. \]  

Bonnor (1974) studies one particular case of initial conditions
\[ \beta_0 = \beta(0) = 0. \]  

It follows from (54) that
\[ \beta(0) = \frac{1}{\sqrt{6 \pi G \rho_0(0)}} \]  
so, (53) means
\[ \rho_0(0) = \infty. \]  

At the same time it follows from (52) and (55) that
\[ R(M,0) = 0, \]  
so, there are no particles in the area \( M > 0 \). This means that the initial condition (55) produces the TB model with delta-like distribution of dust:
\[ \rho_0(M) = \delta(M). \]  

The second example which we study is given by Ribeiro (1993). Ribeiro puts
\[ F(r) = \alpha r^p, \quad \beta(r) = \beta_0 + \eta_0 r^q, \]  
where \( \alpha, \beta_0, \eta_0, p \), and \( q \) are constants. From (11) it follows that
\[ M = 4 \pi \alpha r^p \]  
so,
\[ \beta(M) = \beta_0 + \eta_0 \left( \frac{M}{4 \pi \alpha} \right)^{q/p}. \]  

From (62) and (54) the equation for initial density distribution it follows:
\[ \rho_0(M) = \]  
\[ \left\{ 6 \pi G \left[ \beta_0 + \eta_0 \left( \frac{M}{4 \pi \alpha} \right)^{q/p} \right]^{2} + \frac{12 \pi G \eta_0 q}{(4 \pi \alpha)^{q/p} p} \left[ \beta_0 + \eta_0 \left( \frac{M}{4 \pi \alpha} \right)^{q/p} \right] M^{q/p} \right\}^{-1}. \]  

We obtain for \( \rho_0(0) \):
\[ \rho_0(0) = \frac{1}{6 \pi G \beta_0^2}. \]  

We also note that it follows from (63) that
\[ \left. \frac{\partial \rho_0(M)}{\partial M} \right|_{M=0} = 0 \]  
in case of \( q/p > 0 \).
9 Results

The article studies the properties of the TB model with extra definition of the transformation rule of the co-moving coordinates.

It is shown that the application of the definition (11) of invariant mass as the transformation rule of co-moving coordinates allows to separate the couples of functions $f$ and $F$ into the nonintersecting classes and to fix the dependence $F$ on $f$ in every class. It is shown that every class connected with the only system of coordinates and characterized by the only function $\Psi$. The Ruban-Chernin system of coordinates corresponds to the identical transformation.

The Ruban-Chernin system of co-moving coordinates $\{M, t\}$ is used to describe the TB model.

The functions $f$, $F$ and $F$ are used in the TB model (Tolman 1934; Bondi 1947) as independent and undetermined. We have shown that the transformation rule (11) leads to the classification of the set of couples of the functions $(f, F)$ into nonintersecting classes, so the three undetermined functions now are $f$, $\Psi$ and $F$. The Ruban-Chernin system of coordinates is marked by the equality $\Psi(M) = 1$, so only two functions are undetermined: $f(M)$ and $F(M)$.

Two forms of the initial conditions for the TB model are presented in the section (5): we can fix the initial coordinate $R(M)$ and initial velocity $\frac{\partial R(M,t)}{\partial t} \bigg|_{t=0}$ or, which is the same, the initial profile of the density $\rho_0(M)$ and energy $E_0(M)$ distribution. In the first case of initial conditions the function $F(M)$ is changed by the function $R_0(M)$ and the function $f(M)$ by the function $\frac{\partial R(M,t)}{\partial t} \bigg|_{t=0}$. In the second one the function $F$ is changed by the function $\rho_0(M)$ and the function $f(M)$ by the function $E_0(M)$.

The univalent definition of the Ruban-Chernin co-moving system of coordinates allows to simplify the interpretation of the observations using the model.

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