Can Rossby waves explain the cyclic magnetic activity of the Sun and solar-type stars?

E.A. Bruevich \textsuperscript{a}, I. K. Rozgacheva \textsuperscript{b}

\textsuperscript{a}Sternberg Astronomical Institute, MSU, Moscow, Russia
\textsuperscript{b}VINITI RAS, Moscow, Russia

E-mail: \textsuperscript{a}red-field@yandex.ru, \textsuperscript{b}rozgacheva@yandex.ru

ABSTRACT

Magnetic activity is a global property of the Sun; the complex processes of solar activity are connected with the solar magnetic fields. For solar-type stars and the Sun magnetic activity depends on the physical parameters of the star. In this article we study the relationships between the duration of activity cycle and effective temperature for solar-type stars and the Sun. We’ve tried to explain these relationships due to the existence of layer of laminar convection near the bottom of the convective zone of the star. In this layer the Rossby waves are formed. They generate the primary poloidal magnetic field, which is the source of energy of the complex phenomena of magnetic activity.

KEY WORDS: Sun; Magnetic Activity; Solar-Type Stars; Activity Cycles; Convection, Rossby Waves.

1 Introduction

Magnetic activity of the Sun is called the complex of electromagnetic and hydrodynamic processes in the solar atmosphere. The analysis of active regions (plages and spots in the photosphere, flocculae in the chromosphere and prominences in the corona of the Sun) is required to study the magnetic field of the Sun and the physics of magnetic activity. The very important property of solar irradiance is its cyclical nature.

J. Hale and S. Nicholson discovered the solar magnetic field. After systematic studies they stated the magnetic law of polarity of the spots (Hale-Nicholson law) and the changes of their characteristics during the solar cycle. They found that the periodic variation of the number of spots according to a 11 years cycle constitutes half of the 22 years cycle of the evolution of the solar magnetic field. The most outstanding aspect is the inversion of East-West polarity of the magnetic fields of the spots of the active areas which accompanies the 11 years cycle [1].

J. Hale and S. Nicholson found that within one solar cycle in bipolar magnetic fields all the spots \textit{p} from one hemisphere and all the spots \textit{f} from a different hemisphere have the same polarity, and in the next cycle the polarities of all these
spots are changed to the opposite. Change in such a way that each solar cycle is the epoch of the constant polarity of heliomagnetic fields: the change of cycles corresponds to the change of its polarity, and the total magnetic cycle contains two adjacent spot’s cycles. These laws Hale - Nicholson show, on the one hand, that the mechanism of generation of heliomagnetic field affects the oscillatory manner, producing a fairly regular (quasi periodic) reverse generated field. On the other hand, these laws can be seen that the generation as heliomagnetic field and fluctuations of solar activity is performed with one and the same mechanism. Thus, it becomes clear that the theory of the solar cycle is a global problem of magnetic hydrodynamics of the Sun.

The differential rotation of the Sun is accompanied by energy costs for overcoming the forces of viscosity (primarily turbulent viscosity created by small-scale convective motions in granules and supergranuls). The angular velocity of rotation at different heliographic latitudes may be equalized during a few turns of the sun without any supporting mechanism. According to modern concepts [2] such a supporting mechanism is the mechanism of meridional and radial momentum transfers in the convective zone of the Sun with the help of giant cells (which are influenced by the rotation of the Sun). These giant cells form the spiral macro turbulence, in which the velocity vortex is not orthogonal to the velocity. Giant convection cells were detected by magnetic fields observations: the fields with opposite polarities of magnetic field alternate (for different longitudes) with prevailing wave number $m = 6$. They, perhaps, are the manifestation of Rossby waves in the convective layer of the Sun.

The solar cycle (or solar magnetic activity cycle) is the periodic change in the sun’s activity (including changes in the levels of solar radiation and ejection of solar material) and appearance (visible in changes in the number of sunspots, flares, and other visible manifestations). Solar cycles have an average duration of about 11 years. For hundreds of years of solar activity observations the duration of this cycle (called the Schwabe cycle) vary from 9 to 12 years. Differential rotation of the sun’s convection zone consolidates magnetic flux tubes, increases their magnetic field strength and makes them buoyant. For the first time such a model considered by G. Babcok [3]. As they rise through the solar atmosphere they partially block the convective flow of energy, cooling their region of the photosphere, causing ‘sunspots’. The Sun’s apparent surface, the photosphere, radiates more actively when there are more sunspots. Satellite monitoring of solar luminosity since 1980 has shown there is a direct relationship between the solar activity (sunspot) cycle and luminosity with solar cycle peak-to-peak amplitude of about 0.1% [1].

At the present time in solar and stellar physics also study multiple and changing cycles with relatively small-amplitude: quasi-biennial, semi centennial and century activity cycles [4 - 7]. In a simple model of a stochastically excited solar
It was shown that the modern observations of the Sun have established the profiles of the flows in the solar interior. It was found that the differential rotation of the sun has a strong shear in a thin layer at the base of the convective region, called the tachiocline. This layer comprises less than 3% of the solar radius. The strong magnetic fields observed in the solar surface must be stored in this deep layer of the sun for a timescale of the order of the solar cycle. In this region, the magnetic flux tubes must be intense enough as to travel through the convection zone without being destroyed by the turbulent movements. In the modern models of heliomagnetic dynamo it is believed that the toroidal field is created from the poloidal by differential rotation of the convective zone of the Sun as was first considered at [3]. In his model after many rotations, the field lines become highly twisted and bundled, increasing their intensity, and the resulting buoyancy lifts the bundle to the solar surface, forming a bipolar field that appears as two spots, being kinks in the field lines. The leading spot of the bipolar field has the same polarity as the solar hemisphere, and the trailing spot is of opposite polarity. The leading spot of the bipolar field tends to migrate towards the equator, while the trailing spot of opposite polarity migrates towards the solar pole of the respective hemisphere with a resultant reduction of the solar dipole moment. This process of sunspot formation and migration continues until the solar dipole field reverses (after about 11 years).

Thus the solar magnetic fields are created by dynamo processes. In them the poloidal and toroidal fields are changed to each other; in $\alpha$-effect the poloidal field lines are stretched out and wound around the Sun by differential rotation forming toroidal field lines; twisting of the toroidal field lines into poloidal filed lines is caused by effects of solar rotation, so called $\Omega$-effect.

The $\alpha \Omega$ - dynamo theory which is based on the hypothesis about the generation of the magnetic field due to the differential rotation of the Sun in the turbulent convective shell can describe the main features of solar magnetic activity [5, 6]. Solar cycle models based on what is now called the Babcock-Leighton mechanism: 1) generation of toroidal field from the poloidal field due to the differential rotation of the convective shell ($\Omega$- effect); 2) generation of poloidal field from bipolar magnetic regions of toroidal field due to the differential rotation and the turbulent viscosity ($\alpha$- effect). The $\alpha \Omega$ - dynamo theory which is based on the hypothesis about the generation of the magnetic field due to the differential rotation of the Sun in the turbulent convective shell can describe the main features of solar magnetic activity [7, 9]. This theory is simulates well the following phenomena of magnetic activity such as the formation of strong local bipolar magnetic fields (0,1 Tesla), the cyclicity of magnetic activity and Sporer’s law. According to the theory of $\alpha \Omega$ - dynamo the faster the star rotates, the higher the value of parameter of differential rotation (the difference between the periods of rotation of the polar regions and of the equator) and also the magnetic activity is higher. However,
were found that the observations do not confirm this conclusion [10].

We use the HK Project data [10, 11]. These observations are the longest-running program to monitor stellar activity cycles similar to the 11-year sunspot cycle. Almost 100 stars have been observed continuously since 1966; at present the project is monitoring long-term changes in chromospheric activity for approximately 400 dwarf and giant stars. The HK-Project uses a specially-designed instrument to measure the amount of light from active magnetic regions in stars. This light comes from calcium atoms that have lost one electron each. The different wavelengths of light emitted by these atoms were labeled long ago. The $H$ and $KCaII$ light gave this project its name. This light comes from the upper levels of the Sun near active magnetic regions that we can see, like sunspots. Other stars are too far away to see these features on their surfaces. Studying the relative strength of these two wavelengths of calcium light from distant stars similar to our Sun gives an indirect measure of the amount of surface activity on the stars - "starspots". Using this method, astronomers have been able to follow cycles similar to the sunspot cycle that has been observed on the Sun for centuries. For solar-type stars from HK-project [10, 11] were found that the differential rotation increases with the growth of the average period of rotation. This feature of the rotation of stars creates problems for the theory of $\alpha\Omega$ - dynamo [12].

Complex relationship of poloidal and toroidal magnetic fields indicates that in the convective shell of the Sun there are not less than two working mechanisms of the solar dynamo. They have a variety of spatial scales and characteristic times of their formation. In the framework of the hypothesis about the only one turbulent convective shell, it is impossible to ensure sustainable separation of small-scale and large-scale hydromagnetic dynamo.

In this paper we study the relationships between the duration of activity cycle $T_{\text{cyc}}$ and effective temperature $T_{\text{eff}}$ for solar-type stars and the Sun. We explain these relationships due to the existence of Rossby waves which are formed at the bottom of the convective zone of the stars and the Sun. The Rossby waves conserve vorticity and owe its existence to the variation of the Coriolis force with latitude. The Rossby waves are connected with the primary poloidal magnetic field of a star, which is the source of energy of the complex phenomena of magnetic activity.

The main our assumption is: the time of generation of Rossby waves $t_g$ corresponds to duration of the activity cycle $T_{\text{cyc}}$. We show that theoretical dependence of the time of generation of Rossby waves $t_g$ versus $T_{\text{eff}}$ (the basic parameter of a star) describes well the connection between the star’s duration of the activity cycle $T_{\text{cyc}}$(obtained from observations of solar-type stars and the Sun) and their $T_{\text{eff}}$. 
2 "The rotation period - the effective temperature" and "the duration of the cycle of activity - the effective temperature" relationships

To refine the theory of the solar dynamo search of relationships between the physical parameters of solar-type stars is necessary. These are the effective temperature, the rotation period, the duration of cycles, the magnetic field and the age.

In the Table 1 we present the information about the periods of rotation $P_{\text{rot}}$ and effective temperatures of stars $T_{\text{eff}}$, about the duration of their 11-year $T_{11}$ and quasi-biennial $T_{2}$ cycles.

The periods of cycles of activity are given according to the calculations of cyclic periodicities of HK-project stars by the authors of this work with help of Fourier analysis of light curves of stars [5]. We also used the earlier definition of "11-year" periods by the authors of HK-project ($T_{11}^{HK}$) [11] for a sample of 52 stars and the Sun.

We used the data of Table 1 and found the statistically significant relationships "the rotation period - the effective temperature" and "the duration of activity cycle - the effective temperature", see Fig. 1-3.

Linear regression equation has the following form:

$$\log P_{\text{rot}} = 15.7 - 3.87 \cdot \log T_{\text{eff}}$$ (1)

The linear correlation coefficient (Pearson’s correlation coefficient) in the regression equation (1) is equal to 0.73. According to Pearson’s cumulative statistic test (which asymptotically approaches a $\chi^2$ distribution) the linear correlation between the $P_{\text{rot}}$ and $T_{\text{eff}}$ is statistically significant at a 0.05 level of significance.

Thus our data set of the rotation periods and the effective temperatures of stars shows the following power-law dependence

$$P_{\text{rot}} \sim T_{\text{eff}}^{-3.9}$$ (2)

This ratio follows from the law of radiation Stefan - Boltzmann, from the well known "mass-luminosity" relationship and from the conservation of the angular momentum of the stars [7].

The linear regression equation for points of diagram at the Fig. 2 is of the form:

$$\log T_{11} = 5.15 - 1.11 \cdot \log T_{\text{eff}}$$ (3)

The linear correlation coefficient (Pearson’s correlation coefficient) in the regression equation (3) is equal to (- 0.67). According to Pearson’s cumulative
Table 1: Results of our calculations of $T_{11}$ and $T_2$ values and HK-project $T_{11}^{HK}$ calculations for 52 stars and the Sun

| No | Star on the HD catalog | Spectral class | $P_{rot,\, days}$ (Soon et al 1996) | $T_{eff}, \, K$ (Allen 1977) | $T_{11}^{HK}$, years | $T_{11}$, years | $T_2$ years |
|----|------------------------|----------------|---------------------------------|-----------------------------|----------------------|----------------|-------------|
| 1  | Sun                    | G2-G4          | 25                              | 5780                        | 10,0                 | 10,7           | 2,7         |
| 2  | HD1835                 | G2,5           | 8                               | 5750                        | 9,1                  | 9,5            | 3,2         |
| 3  | HD3229                 | F2             | 4                               | 7000                        | 4,1                  | -              | -           |
| 4  | HD3651                 | K0             | 45                              | 4900                        | 13,8                 | -              | -           |
| 5  | HD4628                 | K4             | 38,5                            | 4500                        | 8,37                 | -              | -           |
| 6  | HD20630                | G5             | 9,24                            | 5520                        | 10,2                 | -              | -           |
| 7  | HD26913                | G0             | 7,15                            | 6030                        | 7,8                  | -              | -           |
| 8  | HD26965                | K1             | 43                              | 4850                        | 10,1                 | -              | -           |
| 9  | HD32147                | K5             | 48                              | 4130                        | 12,1                 | -              | -           |
| 10 | HD10476                | K1             | 35                              | 5000                        | 9,6                  | 10             | 2,8         |
| 11 | HD13421                | G0             | 17                              | 5920                        | -                    | 10             | -           |
| 12 | HD18256                | F6             | 3                               | 6450                        | 6,8                  | 6,7            | 3,2         |
| 13 | HD25998                | F7             | 5                               | 6320                        | -                    | 7,1            | -           |
| 14 | HD35296                | F8             | 10                              | 6200                        | -                    | 10,8           | -           |
| 15 | HD39587                | G0             | 14                              | 5920                        | -                    | 10,5           | -           |
| 16 | HD75332                | F7             | 11                              | 6320                        | -                    | 9              | 2,4         |
| 17 | HD76151                | G3             | 15                              | 5700                        | -                    | -              | 2,52*       |
| 18 | HD76572                | F6             | 4                               | 6450                        | 7,1                  | 8,5            | -           |
| 19 | HD78366                | G0             | 9,67                            | 6030                        | -                    | 10,2           | -           |
| 20 | HD81809                | G2             | 41                              | 5780                        | 8,2                  | 8,5            | 2,0         |
| 21 | HD82885                | G8             | 18                              | 5490                        | 7,9                  | 8,6            | -           |
| 22 | HD100180               | F7             | 14                              | 6320                        | 12                   | 8              | -           |
| 23 | HD103095               | G8             | 31                              | 5490                        | 7,3                  | 8              | -           |
| 24 | HD114710               | F9,5           | 12                              | 6000                        | 14,5                 | 11,5           | 2           |
| 25 | HD115383               | G0             | 12                              | 5920                        | -                    | 10,3           | 3,5         |
| 26 | HD115404               | K1             | 18                              | 5000                        | 12,4                 | 11,8           | 2,7         |
| 27 | HD120136               | F7             | 4                               | 6320                        | 11,6                 | 11,3           | 3,3         |
| 28 | HD124570               | F6             | 26                              | 6450                        | -                    | -              | 2,7         |
| 29 | HD129333               | G0             | 13                              | 5920                        | -                    | 9              | 3,2         |
| 30 | HD131156               | G2             | 6                               | 5780                        | -                    | 8,5            | 3,8         |
| No | Star on the HD catalog | Spectral class | $P_{\text{rot}}, \text{days}$ | $T_{\text{eff}}, \text{K}$ | $T_{11}, \text{years}$ | $T_{11}, \text{years}$ | $T_{11}, \text{years}$ |
|----|-------------------------|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 31 | HD143761                | G0             | 17                          | 5920                        | -                           | 8,8                         | -                           |
| 32 | HD149661                | K2             | 21                          | 4780                        | 14,4                        | 11,5                        | 3,5                         |
| 33 | HD152391                | G7             | 11                          | 5500                        | 10,7                        | -                           | -                           |
| 34 | HD154417                | F8             | 7,8                         | 6100                        | 7,4                         | -                           | -                           |
| 35 | HD155875                | K1             | 30                          | 4850                        | 5,7                         | -                           | -                           |
| 36 | HD156026                | K5             | 21                          | 4130                        | -                           | 11                          | -                           |
| 37 | HD157856                | F6             | 4                           | 6450                        | -                           | 10,9                        | 2,6                         |
| 38 | HD158614                | G9             | 34                          | 5300                        | -                           | 12                          | 2,6                         |
| 39 | HD160346                | K3             | 37                          | 4590                        | 7                           | 8,1                         | 2,3                         |
| 40 | HD166620                | K2             | 42                          | 4780                        | 15,8                        | 13,7                        | -                           |
| 41 | HD182572                | G8             | 41                          | 5490                        | -                           | 10,5                        | 3,1                         |
| 42 | HD185144                | K0             | 27                          | 5240                        | -                           | 8,5                         | 2,6                         |
| 43 | HD187681                | F8             | 10                          | 6100                        | 7,4                         | -                           | -                           |
| 44 | HD188512                | G8             | 17                          | 5490                        | -                           | 4,1                         | -                           |
| 45 | HD190007                | K4             | 29                          | 4500                        | 10                          | 11                          | 2,5                         |
| 46 | HD190406                | G1             | 14                          | 5900                        | 8                           | -                           | -                           |
| 47 | HD201091                | K5             | 35                          | 4410                        | 13,1                        | 3,6                         | -                           |
| 48 | HD201092                | K7             | 38                          | 4160                        | 11,7                        | 2,5                         | -                           |
| 49 | HD203387                | G8             | 30                          | 5490                        | -                           | -                           | 2,6                         |
| 50 | HD206860                | G0             | 9                           | 6300                        | 6,2                         | -                           | -                           |
| 51 | HD216385                | F7             | 7                           | 6320                        | 7                           | 2,4                         | -                           |
| 52 | HD219834                | K2             | 43                          | 4780                        | 10                          | 11                          | 2,5                         |
| 53 | HD224930                | G3             | 33                          | 5750                        | 10,2                        | -                           | -                           |
Figure 1: Diagram "the rotation period $P_{rot}$ - the effective temperature $T_{eff}$". The line shows the linear regression on a data set. The error bars correspond to $1\sigma$ scatter of linear regression.

statistic test (which asymptotically approaches a $\chi^2$ - distribution) the linear correlation between the $T_{11}$ and $T_{eff}$ is statistically significant at a 0,05 level of significance.

Thus, for the investigated sample of stars the periods of "11-year" cycles $T_{11}$ and their effective temperatures $T_{eff}$ are connected in power-law dependence:

$$T_{11} \sim T_{eff}^{-1.1}$$  \hspace{1cm} (4)

We used the data of observations of variations of chromospheric radiation of solar-type stars and the Sun from the Table for statistical analysis and the search if there is a possible linear relationship between the duration of the quasi-biennial cycles of the stars $T_2$ and their effective temperatures $T_{eff}$.

The diagram of "the duration of the cycle - the effective temperature" ($T_2$ is the period of quasi-biennial cycles) for the Sun and stars from the Table is shown in Fig. 3.

Linear regression equation for points of diagram at the Fig. 3 is of the form:

$$\log T_2 = 3,46 - 0,79 \cdot \log T_{eff}$$  \hspace{1cm} (5)
The linear correlation coefficient (Pearson’s correlation coefficient) in the regression equation (4) is equal to (-0.51). According to Pearson’s cumulative statistic test (which asymptotically approaches a $\chi^2$ - distribution ) the linear correlation between the $T_2$ and $T_{eff}$ is statistically significant at a 0.1 level of significance.

Thus, for the investigated sample of stars their periods of quasi-biennial cycles $T_2$ and their effective temperatures $T_{eff}$ are connected as power-law dependence:

$$T_2 \sim T_{eff}^{-0.79}$$

A numerical MHD simulation of hydro magnetic dynamo in case of the fully turbulent, the fully laminar convective shells and also for the convective shell which consists of two layers (the turbulent and the laminar) was made in [13]. We have taken into account these results of numerical MHD simulation for our model estimations of solar magnetic cycle’s duration.
3 The model of solar magnetic fields generation

These properties of the global magnetic activity of the Sun and the exponential dependencies of Fig. 2, 3 can be understood in the framework of the scheme of generation of magnetic fields: at the bottom of convective zone that are heated plasma with help of photons from radiation zones the giant convection cells are formed. There are conditions for convective transfer from the account of only radiant plasma viscosity [6]. The rising element of volume creates a pressure gradient in plasma. The direction depends on the direction of the velocity of an element, the Coriolis acceleration and differential rotation. The pressure gradient will be spread in the spherical shell along the lines of latitudes as Rossby wave.

P. Gilman [14] considered the model of heliomagnetic dynamo, in which Rossby waves that formed in the convective zone of the Sun serve the elements of helical turbulence due to the presence of significant temperature gradient. The vertical movements of the Rossby waves create the toroidal magnetic field from large-scale vertical magnetic fields. Rossby waves transfer them to the poles thus
creating the poloidal field. A new toroidal field of opposite sign is formed with help of this poloidal field. In this simplified Gilman’s model the quasiperiodic reverse of heliomagnetic fields were obtained (with the highest distinction from the observed solar cycle period - instead of 22 years is equal to about 2 years). This disagreement is probably caused by the fact that in the Sun there are not usual Rossby waves that are formed due to the presence of the latitudinal temperature gradient, but there are the giant convection cells which are rather twisted by the Coriolis force, formed due to vertical temperature gradient in the convective zone of the Sun. Later P. Gilman [15] has modified this model with the use of Rossby waves in the theory of hydromagnetic dynamo. It was also analyzed in detail the dynamo action from a typical Rossby wave motion and compared it with the solar cycle. But the quasiperiodic reverse of heliomagnetic fields by new Gilman’s model was also obtained as equal to 2 years.

We also took into consideration the main assumptions which were postulated in [15]. We know that around the base of convective shell the plasma temperature reaches $T \approx 2 \cdot 10^6\, K$. Plasma is fully ionized at such temperatures. Interaction of radiation with plasma is carried out by the scattering of photons by electrons; if the characteristic energy of photons does not exceed $kT$ (where $k$ is the Boltzmann constant). The motion of the plasma in the Rossby wave is slowed by the radiant viscosity of plasma. This viscosity inhibits the directed motion of electrons (the characteristic time is a fraction of a second), faster than motion of protons. Therefore, in the Rossby wave the electric current appears. This current creates a poloidal field. Lines of force of this poloidal field have the wave-like structure. This field is concentrated around the base of convective envelope in the equatorial and medium-sized heliolatitudes. In the polar heliolatitudes lines of force of the poloidal field come to the atmosphere of the Sun, see Fig. 4.

On the length of the latitude the whole number of Rossby waves is packed. Therefore, the length of these waves should be of the order of $\lambda \approx (1/m)2\pi R_I \cos \phi$ where $\phi$ is the latitude of parallels, along which the wave applies. $R_I \approx 0.7R_{\text{Sun}}$ is the radius of the base of the convective envelope, $m$ is the integer number of Rossby waves. For giant cells which are observed in photosphere the wave number is of order $m = 6$, see [2]. At different latitudes the various Rossby waves and different poloidal magnetic fields are generated. The ordered geometric structure of the spicules indicates the regularity of poloidal field. Therefore, the lines of force of poloidal fields, created by Rossby waves, must be regular and not be entangled due to the large-scale convection. So the convection around the base of convective envelope must be laminar. The length of the Rossby wave is approximately equal to the thickness of the shell of laminar convection.

The characteristic time $t$ of the convective ascent of the plasma element which
Figure 4: The scheme of the Rossby waves and the poloidal magnetic field

is heated by photons from the radiation zone as was shown in [6] is equal to:

$$ t \approx \frac{\nu}{g h \left( \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right) \Delta T} \sim \text{const} $$

where \( \nu \) is the coefficient of viscosity of the plasma, \( g \) is the free fall acceleration, \( h \) is the thickness of the shell of laminar convection, \( \left( \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right) \) is coefficient of thermal expansion of the plasma, \( \Delta T \) is the gradient in the layer.

The average Archimedean acceleration of a plasma element is approximately equal to

$$ a \approx g \left( \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right) \Delta T $$

Then the average Archimedean speed of a plasma element is \( V \approx at \approx \frac{\nu}{h} \).

The Coriolis forces turn and stretch the convective cell along a parallel, so that the length of the path of element of the plasma is comparable with the length of
parallels \( l = 2\pi \cos \varphi \).

Average acceleration of the Coriolis force on this Parallels is in the order of magnitude is equal to \( a_c = 2V\Omega_{Sun}\sin \varphi \approx 2\frac{\varphi}{\Pi}\Omega_{Sun}\sin \varphi \), where \( \Omega_{Sun} \) is the angular velocity of rotation of the Sun around the base of convective shell.

The time interval which is necessary for the generation of Rossby waves and poloidal magnetic field on the order of magnitude is equal to:

\[
t_g \approx \sqrt{\frac{2I}{a_c}} \approx \sqrt{\frac{\pi R_I \cos \varphi}{\Omega_{Sun}^2 \sin \varphi}}
\]

(7)

We assumed that the time of generation of Rossby waves \( t_g \) is the characteristic time parameter of solar magnetic dynamo \( t_g \) and corresponds to duration of the activity cycle \( T_{cyc} \).

The generation time increases with decreasing of helio-latitude. Therefore, the first waves appear at high latitudes. We’ll consider that the poloidal field, connected with Rossby waves have to be the primary for the magnetic activity. \( \Omega \)-effect generates a toroidal field with help of the primary poloidal in average turbulent layer of the convective shell. Magnetic loops of toroidal fields are generated because of the turbulence. Rossby wave compresses these loops and stimulates the formation of spots. So Rossby waves lead to the formation of ”active latitudes”. The meridional fluxes around the base of convective zone are directed from the poles to the equator [16]. They move Rossby waves closer to the equator. The consequence of this will be the slope of the lines of force of poloidal field in the direction to the equator (the axis of symmetry of the field turns). A toroidal field moves too, so in the photosphere there is Sp?rer’s law. The \( \alpha \)-effect generates the poloidal field from the toroidal field with the opposite polarity in relation to the primary poloidal field.

The interaction of the new poloidal field with a primary poloidal field stimulates the polar magnetic activity, that weakens these fields. Therefore the new a toroidal field, which appears from the weakened poloidal field, will be weaker than the previous one toroidal field.

The local magnetic activity, which the new a toroidal field will cause in the second half of the magnetic cycle also will be weaker. A new a toroidal field is directed opposite to the direction of Rossby wave propagation. It weakens the electric currents that exist in these waves.

This process leads to a weakening of the primary poloidal field. At the end of the previous stage of the magnetic cycle the \( \alpha \)-effect generates the poloidal field. Its direction is close to the direction of primary poloidal field; if the latter does not experienced significant changes.

In this case, the primary poloidal will increase. From the primary poloidal the \( \Omega \)-effect generates a toroidal field, which is stronger than the previous toroidal
field. Stronger field stimulates stronger activity. So the Gnevyshev-Ol’s rule is explained by the contribution of the primary poloidal field to the evolution of the toroidal field.

If the primary poloidal field has experienced a significant turn, the rule Gnevyshev-Ol’s rule is violated as it was observed for the cycles 22 and 23. The energy of the Rossby waves is the source of magnetic activity. Therefore, these waves gradually decrease. The beginning of a new cycle can be delayed until the formation of the new Rossby waves. This explains the changes the durations of the cycles. If the generation of Rossby waves will slow down in the mid-latitudes due to the penetration of turbulent motions in the lower laminar layer, it will accelerate the meridional transfer of momentum, which will speed up the rotation of the mid-latitudes.

The time of generation of Rossby waves $t_g$ according (7) depends on the thickness of the layer of laminar convection and plasma viscosity. We estimate the viscosity coefficient taking into account the fact that the time of the generation of waves equals the length of the cycle [14-16].

Taking into account that

$$h \approx \lambda \approx \frac{l}{m}$$

we can find the coefficient of viscosity:

$$\nu \approx \frac{\pi^2 R_{\text{Sun}}^2 \cos^2 \varphi}{m \Omega_{\text{Sun}} t_g^2 \sin \varphi}$$

where $R_{\text{Sun}}$ is the radius of the Sun, $m$ is the order of Rossby waves. For "active latitudes" the value of the coefficient of viscosity can be compared with the value of the coefficient of radiant viscosity $\nu_\gamma$ around the base of convective envelope (the region of Rossby wave’s generation):

$$\nu_\gamma \approx \frac{1}{3 n \sigma_T} \approx (1.5 - 2) \cdot 10^{12} \frac{sm^2}{sec}$$

where $c$ is speed of light, $\sigma_T$ - is the Thomson’s cross-section of scattering of photons by electrons and the concentration of plasma $n$ around the base of convective envelope is equal to: $n \approx 5 \cdot 10^{21} \text{sm}^{-3}$. It is a coincidence speaks in favor of the above-described the physical nature of the global magnetic activity. Approximately the same value of turbulent viscosity is used in the $\alpha \Omega$-dynamo model. The model of multi-layered laminar convection using radiant viscosity (9) is described in [15].

Rossby waves at different helio-latitudes and with different wavelengths generate a very complex structure and evolution of the poloidal field. Check of the physical picture of this poloidal field formation according to the observations of
the Sun only is difficult. So we need the study of solar-type stars which are analogs of the Sun of different stages of evolution [10, 11].

We estimate the relationship between the duration of the cycle of activity and the effective temperature of the stars. The angular velocity of rotation of the star, and its effective temperature are related by the ratio \( \Omega \sim T_{\text{eff}}^{-\frac{1}{2}} \), see Fig. 1.

In fully ionized hydrogen plasma its concentration depends on the temperature, as \( n \approx T^{-\frac{3}{2}} \).

Using radiant viscosity \( \nu \approx \nu_i \sim T^{-\frac{3}{2}} \sim T_{\text{eff}}^{-\frac{3}{2}} \) and under assumption that the time of generation of Rossby waves \( t_g \) corresponds to duration of the activity cycle \( T_{\text{cyc}} \), from the formula (7) we obtain the following connection between the star’s duration of the activity cycle and its effective temperature:

\[
T_{\text{cyc}} \approx t_R \sim T_{\text{eff}}^{-\frac{3}{2}}
\]

So we can see that relation (10) is consistent (within our assumptions) with the dependencies which are showed at Fig. 2 and Fig. 3.

Thus we show that theoretical dependence of the time of generation of Rossby waves \( t_g \) versus \( T_{\text{eff}} \) (the basic parameter of a star) describes well the connection between the star’s duration of the activity cycle \( T_{\text{cyc}} \) (obtained from observations of solar-type stars and the Sun, see Table 1) and their \( T_{\text{eff}} \).

We have also to take into consider that solar cyclic activity is the very important factor in learning of space weather behavior. Thus, it becomes important to explain the nature of so-called "11-year" cycles on the Sun and solar-type stars.

And so, for the quantitative analysis of the proposed physical picture it is necessary to conduct the numerical experiments of the magnetic hydrodynamics of a multilayer convective shell of the Sun.

The time interval \( t_g \) which is necessary for the generation of Rossby waves (see formula (1)) is equal to 2 years approximately when we use the suitable values of the coefficient of viscosity (3). But if of the coefficient of viscosity is ten times less then (3): this value is not impossible, see [17] we get the period of the Rossby waves generation is equal to 8 -10 years.

4 Conclusions

We offer the physical picture of the interrelationship of observed properties of the local and global magnetic activity of the Sun. The main new element in this picture is a hypothesis about the possibility of the existence of at least two layers of the convective shell of the solar-type stars. Around the ground level the convective shell is the laminar convection layer, which consists of giant convective cells. Thanks to the rotation on the surface of this layer Rossby waves are formed.
These waves have spiral structure due to differential rotation. The formation of the laminar convection is caused by a strong heating of plasma photons from the zone of radiant heat transfer and high viscosity of radiant fully ionized dense plasma. Viscosity of radiant plasma effectively inhibits the directed motion of electrons more than directed motion of protons. Therefore, in the Rossby waves the current appears, and this current generates the poloidal field. This field is a primary reason of the whole magnetic activity of stars. Above the layer of the laminar convection the layer of turbulent convection extends. In the layer of turbulent convection the primary poloidal field generates a toroidal field of starts and process of generation of a strong local magnetic activity solar-type stars in the medium and equatorial latitudes starts. Local and global magnetic activities are interconnected thanks to the existence of internal Rossby waves and the primary poloidal field.

It is shown that in the framework of the hypothesis proposed in our work about the existence of internal Rossby waves you can explain the dependence of "the duration of the activity cycles - the effective temperature” for 11-year and quasi-biennial cycles, see Fig. 2, 3. The duration of the activity cycles according to the order of magnitude is equal to the characteristic time of generation of Rossby waves. Under the assumption that the time of generation of Rossby waves \( t_g \) corresponds to duration of the activity cycle \( T_{cyc} \) it was shown that empirical dependencies \( T_{11} \sim T_{eff}^{-1} \) and \( T_2 \sim T_{eff}^{-0.79} \) describes well the dependencies between the star’s duration of the activity cycle \( T_{cyc} \) and their \( T_{eff} \) estimated by our model \( T_{cyc} \approx t_R \sim T_{eff}^{-\frac{2}{3}} \).

Our study is in good agreement with the fact of existence of the set of periods of cyclical activity on the Sun. There exist the cyclicities of activity with the next durations: \( T_{11/2} \)- "5,5-year cycle”, \( T_{11/4} \)- quasi-biennial cycle, \( T_{11/8} \)- "1,3-year cycle” and \( 2T_{11} \)- "22-year”, \( 4T_{11} \)- semi-century, \( 8T_{11} \)- century cyclicities. Observations of this set of periods of cyclical activity (equal to some parts of the main "11-year" period) show us that a wave nature in the phenomenon of cyclicity of solar activity takes place. It confirms our assumption that the duration of the activity cycles is determined by the length of the Rossby waves.

Also it was shown that when we use the different estimations of the value of the coefficient of viscosity at the bottom of convective zone of the Sun, the time of generation of Rossby waves \( t_g \) (which corresponds to duration of the activity cycle\( T_{cyc} \)) may have the duration is approximately of 2 - 10 years.

Thus we emphasized the significant contribution of Rossby waves in formation of magnetic cycles of stars and the Sun.

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