Adaptive Fuzzy Robust Control of a Bionic Mechanical Leg With a High Gain Observer

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ABSTRACT In this paper, an observer-based adaptive fuzzy robust controller is proposed for trajectory tracking control of a bionic mechanical leg (BML) with unmeasured system states, dynamic uncertainties and external disturbances. A high gain observer (HGO) is constructed to estimate the unavailable joint velocities using the joint position feedback signals, while an adaptive fuzzy logic system (AFLS) is employed to address the lumped uncertainties. The nonlinear robust controller is then synthesized via backstepping method to improve the position tracking performance. The stability of the closed loop system is mathematically demonstrated via the Lyapunov’s stability theory. It is proven that under the proposed controller all the closed-loop signals are bounded and the trajectory tracking errors converge to a small neighborhood of the origin with appropriate design parameters. The effectiveness of the proposed control scheme is illustrated by simulation studies.

INDEX TERMS Bionic mechanical leg (BML), unmeasured system states, dynamic uncertainty, external disturbance, high gain observer (HGO), fuzzy logic system (FLS), nonlinear robust control.

I. INTRODUCTION

In recent years, with the application of robots spreading not only to industrial fields, but also to many complex and special areas, such as the life service field [1], [2], rehabilitation and medical treatment industry [3], [4], disaster rescue [5], [6], ocean engineering [7], [8], and space operation [9], [10], bionic robots have attracted substantial attention. Multi-leg walking robots as one kind of bionic robot, compared with traditional wheeled robots and caterpillar robots, possess flexible movement performance, a powerful obstacle avoidance ability and an excellent terrain adaptability [11], and have become prominent research object, including bipedal robots (the humanoid robot Atlas of Boston Dynamics [12], the Cassie series of robots of the Agility Robotics [13] and Cassie Blue of Ohio State University and University of Michigan [14]), quadruped robots (the Sport Mini Dog of Boston Dynamics [12], MIT Cheetah of MIT Biomimetics Robotics Lab [15], ANYmal robot of the Autonomous Systems Lab of ETH Zurich [16], and the MIT Mini Cheetah of Massachusetts Institute of Technology of Cambridge and University of Notre Dame [17], [18]) and hexapod robots (CR200 of KRISO of South Korea [7], and LAU- RON of FZI Research Center for Information Technology of German [19], [20]).

As an important component of multi-leg walking robots, the bionic mechanical leg (BML) plays a significant role in the motion of robot, and its control performance directly affects the motion performance of the entire robot. However, designing a high-performance control method for the BML system is not easy in practical applications. Firstly, as a multiple joint serial mechanism, the BML system is usually a high degree-of-freedom (DOF) complicated multi-input and multi-output (MIMO) dynamic system. Secondly, all the system states required in the design of the controller are not easy to obtain simultaneously during practical applications. Thirdly, serious nonlinearities (i.e., parameters of inertia, Coriolis/centrifugal and gravity coupled with system states), uncertainties (i.e., unmodeled dynamics, friction effect, and parameter perturbation) and unknown external disturbances impact the system simultaneously. Therefore, it is significant to design a high-performance trajectory tracking...
controller for the BML system in the presence of unmeasured system states, dynamic uncertainties and external disturbances.

The BML system is a multiple joint serial robotic system, and many studies on control methods have been performed on these complicated robotic systems. Most early researchers applied the proportional-integral-derivative (PID) controller to robotic manipulators [21], [22] since it does not require an exact mathematical model of the dynamic system and is convenient to implement. However, the PID controller is not able to ensure the control performance over a wide range, as it does not consider nonlinearities (i.e., state coupling impact and friction) and uncertainties (i.e., parametric uncertainties, unmodeled uncertainties, and external disturbances) of the system.

To promote the system performance, numerous advanced control techniques have been developed to overcome the aforementioned issues for robotic systems, such as the sliding mode control (SMC) [23], [24], robust control [25], [26], and adaptive control [27], [28], as well as combinations, including adaptive SMC control [29]–[31], robust SMC control [32], [33], adaptive robust control [34], etc. However, most of these controllers require full-state measurement of the dynamic systems to complete the controller design, which are not easy to acquire simultaneously in practical applications, especially in some occasions where weight, volume, research and development costs are under strict restrictions. Therefore, it is meaningful to research the state estimation problem for robotic systems with unavailable system states.

Several state observers, such as extended state observer (ESO) [35]–[38], high gain observer (HGO) [39], [40], neural network-based observer (NNO) [41], [42] and fuzzy logic-based observer (FLO) [43], [44], have been widely applied to control systems with state estimation problems. In [38], an ESO was designed for the control of an underwater robot with unknown disturbances and uncertain nonlinearities, while a HGO was constructed to observe the speed signals for the output feedback control of an autonomous mobile robot in order to prevent the use of high-cost speed sensors in [40]. Due to the universal approximation feature, neural networks (NNs) and fuzzy logic systems (FLSs) are widely used for the design of state observers. In [41], a NNO was applied to acquire the unknown joint velocities by using the joint position measurements during the design of an adaptive NNs based controller for a robot manipulator. An observer-based output feedback control scheme was proposed for a flexible manipulator, in which FLO was employed to provide real-time states to the adaptive fuzzy controller in [44].

Furthermore, in physical robotic systems, dynamic uncertainties and external disturbances always exist, which not only degrade the tracking accuracy, but also multiply the controller design difficulties. To address these problems, Yao et al. proposed an indirect adaptive robust controller (ARC) for the trajectory tracking of a robotic manipulator driven by electro-hydraulic actuators with parametric uncertainties and uncertain nonlinearities, in which the parametric uncertainties were obtained through an adaptation algorithm, while the uncertain nonlinearities were solved using the robust control terms [45]. However, ARC is difficult to implement in robotic systems with multiple degrees of freedom since many parameters need to be processed through adaptive mechanisms with a significant amount of online process work; This condition considerably increases the complexity of the control law design, and sometimes the design is even impossible because it may cause instability due to the convergence of the parameters not being the real value of the control system. Therefore, to overcome the dynamic uncertainties and external disturbances in the control of robotic systems, other methods are introduced into robotic control systems. Duc et al. employed a nonlinear ESO to the output feedback control of a robot manipulator with time-varying output constraints and external disturbance, utilizing the extra state of the ESO to estimate the generalized disturbances of the manipulator dynamic system [46]. However, from the design procedure of ESO, we can see that it only considers the effect of the system uncertainties and external disturbances, but not consider the effect of the state estimation errors. Due to the advantages of the universal approximation feature and not enquiring much model information, FLSs and NNs are also widely used for dealing with various types of uncertainties in robotic systems [44], [47], [48]. For instance, in [44], FLSs were used to approximate the unknown nonlinear systems for a single-link robotic manipulator coupled to a brushed direct current motor with actuator saturation. In [47], NNs were proposed to handle uncertainties in the manipulator dynamics, actuator dynamics and unknown disturbances for a robotic manipulator with time-varying joint space constraints. In [48], FLSs and NNs were simultaneously employed to combine into a FNN approximator in order to estimate the uncertain dynamics for the control of an uncertain constrained robot with unknown dynamics and constraints. However, the above controllers were almost completed with full system states, which are not always possible in a wide range of robot applications. Therefore, the system performance might be influenced by these methods when there are unavailable system states.

Motivated by the above analysis, a high gain observer-based adaptive fuzzy robust controller (AFRC-HGO) is proposed for the trajectory tracking control of a 3-DOF BML system with the existence of unmeasured system states, dynamic uncertainties and unknown external disturbances. The proposed controller is developed via the backstepping method. The main contributions of this paper are summarized as follows:

1. A novel AFRC-HGO controller is developed for high performance trajectory tracking control of a 3-DOF BML system. This proposed controller integrates the fast convergence property and celebrated separation principle of the HGO and the universal approximation feature of the FLSs with nonlinear robust control action. The HGO is used to estimate the unmeasured system states, while the
AFLS to approximate the lumped uncertainties, including dynamic uncertainties, unknown external disturbances and state estimation errors. Compared with the ESO-based controllers [37], [38], [46], the AFLS of the proposed controller approximates the dynamic uncertainties, unknown external disturbances and state estimation errors simultaneously, in which the influence of the state estimation errors becomes one more piece of information to be compensated in the controller.

(2) The inverse kinematic model and system dynamic model of the 3-DOF BML system are established. To reduce the contact force between the foot and ground during motion, a parabolic-like gait trajectory in Cartesian coordinate system is planned.

(3) In order to demonstrate the effectiveness of the proposed controller, simulation studies are conducted on the BML system, which considers the effects of different working conditions and external disturbances. Additionally, three other control methods (proportional-integral-derivative control (PID), sliding mode control (SMC) and nonlinear extended state observer-based robust control (RC-NESO)) are compared with the proposed AFRC-HGO for trajectory tracking performance evaluations.

The remainder of this paper is organized as follows. Section II gives the problem formulation, including inverse kinematics, gait motion trajectory planning, system modeling, and a fuzzy logic system (FLS). Section III provides the construction of a high gain observer. Section IV presents the adaptive fuzzy robust controller design procedure and its theoretical results. The effectiveness of the proposed controller is demonstrated via simulation studies in Section V, and conclusions and future work are drawn in Section VI.

II. PROBLEM FORMULATION
The BML system studied in this work is inspired by the walking leg of crabs depicted in Fig.1, which is mainly composed of two links (a thigh link and a shank link) and three rotary joints (a hip yaw joint, a hip roll joint and a knee roll joint placed in series), and thus has 3-DOF.

Remark 1: The BML system will work both on land and in deep-sea water, so is watertight and oil compensating design. In order to reduce the weight and the complexity of hollow routing, the BML system is only configured with a position sensor at each joint, which will lead to unmeasured system states. Meanwhile, as the BML system will work both on land and in water, its parameters such as inertia, Coriolis/centrifugal and gravity will vary dramatically under different working conditions, thus resulting in system dynamic uncertainties. Moreover, the hydrodynamic external force and tidal current [7] will act on the body of the BML system when it works in water, which can be seen as unknown external disturbances. Therefore, the BML system will suffer from unmeasured system states, dynamic uncertainties and unknown external disturbances simultaneously when it works.

A. INVERSE KINEMATICS
The BML is a 3-DOF serial mechanism. The D-H coordinate systems and the corresponding D-H parameters [49] used to develop the kinematic model of the BML system are shown in Fig.2 and Table 1, respectively.

| #  | θ   | d   | a   | α   |
|----|------|-----|-----|-----|
| 0-1| θ_1 (90°) | 0   | L_1 (0m) | 90° |
| 1-2| θ_2 (0°)  | 0   | L_2 (0.66m) | 0°  |
| 2-3| θ_3 (-90°) | 0   | L_3 (0.85m) | 0°  |

FIGURE 1. The bionic mechanical leg.

FIGURE 2. D-H coordinate systems of the BML.

The coordinates of the foot tip of the BML with respect to the base frame {0} can be determined as follows:

\[
0_p_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -s_1(L_1 + L_2c_2 + L_3s_23) \\ c_1(L_1 + L_2c_2 + L_3s_23) \\ L_2s_2 - L_3c_23 \end{bmatrix} \] (1)
and the joint angles can thus be derived through the inverse kinematics, as follows:

\[
\begin{align*}
\theta_1 &= \arctan \left( \frac{-z}{\sqrt{M^2 + N^2}} \right) - \varphi \\
\theta_2 &= \arccos \left( \frac{A^2 + \varphi^2 - L_2^2 - L_3^2}{2L_2L_3} \right) \\
\theta_3 &= \arcsin \left( \frac{\varphi}{\rho \cos(\varphi)} \right) \\
\end{align*}
\]  

(2)

where \( x, y \) and \( z \) are the coordinates of the foot tip, \( s_i = \sin(\theta_i) \), \( c_i = \cos(\theta_i) \), \( s_{ij} = \sin(\theta_i + \theta_j) \), \( c_{ij} = \cos(\theta_i + \theta_j) \), \( A = \frac{1}{c_i} - L_1 \), \( M = L_2 + L_3 \), \( \rho = \rho \sin(\varphi) \), \( N = L_3 c_3 = \rho \cos(\varphi) \), and \( \varphi = \arctan(2(M, N)) \).

### B. GAIT MOTION TRAJECTORY PLANNING

The desired gait motion trajectory of the foot tip of the BML system is planned as a composite cycloid trajectory [50], [51], of which the expressions of the swinging phase and supporting phase are illustrated in (3) and (4), respectively.

\[
\begin{align*}
x(t) &= \frac{S_0}{2\pi} \left( \frac{2\pi t}{T_y} \sin \left( \frac{2\pi t}{T_y} \right) \right) - 0.2, \\0 \leq t < T_y \\
y(t) &= 0.45, \\
0 \leq t < T_y \\
z(t) &= \begin{cases} 
2H_0 \left( \frac{t}{T_y} - \frac{1}{4\pi} \sin \left( \frac{4\pi t}{T_y} \right) \right) - 0.61, \\
0 \leq t < \frac{T_y}{2} \\
-2H_0 \left( \frac{t}{T_y} - \frac{1}{4\pi} \sin \left( \frac{4\pi t}{T_y} \right) \right) + 2H_0 - 0.61, \\
\frac{T_y}{2} \leq t < T_y 
\end{cases}
\end{align*}
\]

(3)

\[
\begin{align*}
x(t) &= S_0 - \frac{S_0}{2\pi} \left( \frac{2\pi t - T_y}{T - T_y} \sin \left( \frac{2\pi \left( t - \frac{T_y}{2} \right)}{T - T_y} \right) \right) - 0.2, \\T_y \leq t < T \\
y(t) &= 0.45, \\
T_y \leq t < T \\
z(t) &= -0.61, \\
T_y \leq t < T 
\end{align*}
\]

(4)

where the walking step \( S_0 = 0.4m \), the leg moving height \( H_0 = 0.2m \), the gait cycle \( T = 4s \) and the swing phase cycle \( T_y = 2s \).

### C. SYSTEM MODELING

The dynamic model of the BML system can be described as follows [49], [52]:

\[
\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + d
\]

(5)

where \( \tau, q, \dot{q}, \ddot{q}, d \in \mathbb{R}^{3 \times 1} \) are vectors of the joint driving torque, joint position, joint velocity, joint acceleration and external disturbances, respectively; \( M(q) \in \mathbb{R}^{3 \times 3} \) is the inertia matrix; \( C(q, \dot{q}) \in \mathbb{R}^{3 \times 3} \) is the centrifugal and Coriolis matrix; and \( G(q) \in \mathbb{R}^{3 \times 1} \) is the gravitational vector. For convenience, \( M, C \) and \( G \) are substituted for \( M(q), C(q, \dot{q}) \) and \( G(q) \) later in this work, respectively.

As the precise values of \( M, C \) and \( G \) are usually unknown in practical applications, they can be expressed as the combination of nominal values and uncertain values as follows:

\[
\begin{align*}
M &= M_n + \Delta M \\
C &= C_n + \Delta C \\
G &= G_n + \Delta G 
\end{align*}
\]

(6)

where \( M_n, C_n \) and \( G_n \) are the nominal values of the model, while \( \Delta M, \Delta C \) and \( \Delta G \) are the values of the modeling uncertainties. The expressions of the elements of \( M_n, C_n \) and \( G_n \) have been determined, which are included in Appendix A.

Substituting (6) into (5), the system model (5) can be expressed as follows:

\[
\tau = M_n\ddot{q} + C_n\dot{q} + G_n + (\Delta M\ddot{q} + \Delta C\dot{q} + \Delta G + d)
\]

(7)

Defining \( x_1 = [q_1, q_2, q_3]^T \), \( x_2 = [\dot{q}_1, \dot{q}_2, \dot{q}_3]^T \), and \( x = [x_1, x_2]^T \), the dynamic Eq. (7) of the BML system can be rewritten in state-space form:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u + f_1(x_1, x_2) + f_2(x_1) + f_3(x, t) \\
y &= x_1 
\end{align*}
\]

(8)

where \( u = M_n^{-1}\tau, f_1(x_1, x_2) = -M_n^{-1}C_n\dot{q}, f_2(x_1) = -M_n^{-1}G_n\dot{q}, f_3(x, t) = -M_n^{-1}(\Delta M\ddot{q} + \Delta C\dot{q} + \Delta G + d) \), with \( y \) as the system output.

**Assumption 1:** The function \( f_1(x_1, x_2) \) is set to meet the Lipschitz condition with respect to the state variables \( x_1 \) and \( x_2 \). The function \( f_3(x, t) \) is bounded by the upper boundary \( D \), i.e., \( \| f_3(x, t) \| < D \).

### D. FUZZY LOGIC SYSTEM (FLS)

To approximate the lumped uncertainties of the BML system, a fuzzy logic system (FLS) is employed.

**Lemma 1** ([51]): Let \( y(x) \) be a continuous function defined on a compact set \( \Omega \). Then, for any constant \( \eta > 0, \xi > 0 \), there exists a FLS \( \hat{y}(x|\theta) = \xi^T(x)\theta \) and an optimal parameter vector \( \theta^* \) defined on a compact set \( U \), such that

\[
\sup_{x \in \Omega} |y(x) - \hat{y}(x|\theta)| \leq \eta
\]

(9)

\[
\|y(x) - \hat{y}(x|\theta^*)\| \leq \xi
\]

(10)

where \( \hat{y}(x|\theta) = \frac{1}{N} \sum_{i=1}^{N} \mu_{F_i}(x) \), \( \xi^T(x)\theta = \left[ \sum_{i=1}^{N} \mu_{F_i}(x_i) \right] \left[ \sum_{i=1}^{N} \xi_i(x_i) \right] \) is the approximated function, \( \theta = [\theta_1, \theta_2, \ldots, \theta_N]^T \) and \( \xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_N(x)]^T \) are vectors of
adjustable parameters and fuzzy basis functions, respectively, \( \xi_i(x) = \prod_{j=1}^{n} \mu_{P_i}(x_j) / \sum_{i=1}^{N} \prod_{j=1}^{n} \mu_{P_i}(x_j) \), and \( \theta^* = \arg \min_{\theta \in U} \left[ \sup_{x \in \Omega} |y - \hat{y}(x|\theta)| \right] \).

According to Lemma 1, FLSs are universal approximators, i.e., they can approximate any smooth function on a compact space [54].

### III. HIGH GAIN OBSERVER (HGO) DESIGN

Designing an adaptive fuzzy robust controller for the BML system requires a full state feedback. However, only the joint position sensor is installed in the joint of the BML system. Therefore, the task of the observer is to estimate the unmeasured system state \( x_2 \) for the later controller design of the BML system.

HGO has many advantages than other observers, for instance, it possesses not only disturbance rejection property and fast dynamics feature, but also the celebrated separation principle that it can make the output feedback controller recover the stability and performance properties of the state feedback controller by choosing appropriate parameter [39]. Therefore, HGO has been an effective tool in the estimation and control of nonlinear systems.

According to the structure of the system model (8), the HGO is designed as [39], [40]

\[
\hat{x}_1 = \hat{x}_2 + l_1(x_1 - \hat{x}_1) \\
\hat{x}_2 = u + \hat{f}_1(x_1, \hat{x}_2) + f_2(x_1) + l_2(x_1 - \hat{x}_1)
\]

where \( \hat{x} = [\hat{x}_1, \hat{x}_2]^T \) is the estimated state vector, and \( \hat{f}_1(x_1, \hat{x}_2) \) is the estimation value of \( f_1(x_1, x_2) \).

In order to guarantee stability, the gains of the observer are designed as

\[
L = [l_1, l_2]^T = [2\omega_0, \omega_0^2]^T
\]

where \( \omega_0 = 1/O_0 \) is the only design parameter of the observer, and \( O_0 \) is a small positive constant.

Define estimation error \( \hat{x} = x - \hat{x} = [\hat{x}_1, \hat{x}_2]^T \), and the scaled estimation error is described as \( e_i = \hat{x}_i/\omega_0^{-1}, i = 1, 2 \). From (8) and (11), the dynamic equation of the scaled estimation errors can be shown as

\[
\dot{e} = \omega_0 A e + \frac{B\hat{f}_1}{\omega_0} + \frac{Bf_3}{\omega_0}
\]

where \( e = [e_1, e_2]^T, A = \begin{bmatrix} -2I_{3 \times 3} & I_{3 \times 3} \\ -I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \), and \( B = \begin{bmatrix} O_{3 \times 3} \\ I_{3 \times 3} \end{bmatrix} \).

According to Assumption 1, there exist some known constants \( c_1 \) and \( c_2 \) that satisfy the following conditions:

\[
\| \hat{f}_1 \| \leq c_1 \| e_1 \| + c_2 \| e_2 \| \leq (c_1 + c_2) \| e \|
\]

Theorem 1: Considering HGO (11), the dynamic equation of the scaled estimation errors (13), and inequation (14), the estimation errors of the HGO are bounded, and their upper bound can be compressed to an arbitrarily small range by adjusting the parameter \( \omega_0 \), namely

\[
\lim_{\omega_0 \to \infty} \hat{x}_1 = x_1, \quad \lim_{\omega_0 \to \infty} \hat{x}_2 = x_2.
\]

Proof of Theorem 1: Since matrix A is Hurwitz, a positive definite matrix \( P \) exists, such that

\[
A^T P + PA = -I_{6 \times 6}
\]

Consider the following Lyapunov function:

\[
V_0 = \frac{1}{2} e^T P e
\]

Using (13) and (15), the derivative of \( V_0 \) is

\[
\dot{V}_0 = \frac{1}{2} e^T P e + \frac{1}{2} e^T P \hat{f}_1 = -\frac{1}{2} \omega_0 e^T e + \frac{e^T P \hat{f}_1}{\omega_0} + \frac{e^T P Bf_3}{\omega_0}
\]

From (14) and the boundary of \( f_3 \), we have

\[
\dot{V}_0 \leq -\frac{1}{2} \omega_0 e^T e + \frac{\|PB\| \|\hat{f}_1\|}{\omega_0} \| e \| + \frac{\|PB\| \|f_3\|}{\omega_0} \| e \|
\]

\[
\leq -\frac{1}{2} \omega_0 \| e \|^2 + \frac{\lambda_0 (c_1 + c_2)}{\omega_0} \| e \|^2 + \frac{\lambda_0^2 d^2}{2 \omega_0^2} + \frac{1}{2} \| e \|^2
\]

\[
= -\lambda \| e \|^2 + \frac{\lambda_0^2 d^2}{2 \omega_0^2}
\]

where \( \lambda_0 = \|PB\| \) and \( \lambda = \frac{1}{2} \omega_0 - \frac{1}{2} - \frac{\lambda_0 (c_1 + c_2)}{\omega_0} \) are both positive constants. In order to ensure \( \lambda > 0 \), we must choose \( \omega_0 > \frac{1}{\lambda} \left( 1 + \sqrt{1 + 8\lambda_0 (c_1 + c_2)} \right) \).

The results of (18) indicate that the estimation errors of the HGO are bounded with appropriate parameter \( \omega_0 \), and the upper bound of the estimation errors can be compressed to an arbitrarily small range by adjusting the parameter \( \omega_0 \). This indicates that \( \hat{x}_1 \) and \( \hat{x}_2 \) will converge to \( x_1 \) and \( x_2 \), respectively, if \( \omega_0 \) is large enough.

### IV. CONTROLLER DESIGN

To improve the tracking performance of the BML system, an adaptive fuzzy robust controller is described in this section. The control object is to make the three joints track the desired trajectory as close as possible under the effect of system dynamic uncertainties and unknown external disturbances with only the joint position feedback. The block diagram of the proposed controller is shown in Fig.3, which mainly consists of a trajectory generator, an inverse kinematic calculator, a HGO observer, an AFLS, a nonlinear RC controller and the BML plant. The HGO is constructed to estimate the unavailable joint velocities using the joint position feedback signals, while the AFLS is employed to handle the lumped uncertainties. The nonlinear robust controller is then synthesized via backstepping method based on full state feedback and approximated lumped uncertainties.

The following changes of coordinates are defined

\[
z_1 = y - x_1d = x_1 - x_1d
\]

\[
z_2 = x_2 - \alpha
\]
where $x_{1d}$ is the desired joint position, and $\alpha$ is the virtual control function.

Step 1: The time derivative of the tracking error $z_1$ yields

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} = z_2 + \alpha - \dot{x}_{1d} \quad (21)$$

Define $V_1 = \frac{1}{2} z_1^T z_1$ as a Lyapunov function of the first subsystem of (8), then the derivative of $V_1$ is

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T (z_2 + \alpha - \dot{x}_{1d}) \quad (22)$$

Choosing the following virtual control function

$$\alpha = -\lambda_1 z_1 + \dot{x}_{1d} \quad (23)$$

where $\lambda_1 > 0$ is a constant.

Substituting (23) into (22) yields

$$\dot{V}_1 = -\lambda_1 z_1^T z_1 + z_1^T z_2 \quad (24)$$

Step 2: From $z_2 = x_2 - \alpha$, we obtain the time derivative of the tracking error $z_2$ as

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}$$

$$\dot{z}_2 = u + f_1(x_1, x_2) + f_2(x_1) + \lambda_1 (x_2 - \dot{x}_{1d}) - \dot{x}_{1d}$$

Choosing the adaptation law as

$$\dot{\lambda}_1 = \gamma_1 (\eta_1(x_1, \dot{x}_2) z_2 - 2k_1 \lambda_1) \quad (32)$$

where $\gamma_1 > 0$ and $k_1 > 0$ are constants, $i = 1, 2$.

Step 3: Choose the following Lyapunov function when considering the impact of the approximation errors

$$V_3 = V_2 + \frac{1}{2\gamma_1} \beta_1^T \dot{\beta}_1 + \frac{1}{2\gamma_2} \beta_2^T \dot{\beta}_2 \quad (34)$$

where $\beta_i = \theta_i^* - \theta_i$, $i = 1, 2$.

From (27) and (34), the time derivative of $V_3$ is

$$V_3 = \dot{V}_2 + \frac{1}{2\gamma_1} \beta_1^T \dot{\beta}_1 + \frac{1}{2\gamma_2} \beta_2^T \dot{\beta}_2$$

Choosing the adaptation law as

$$\dot{\theta}_1 = \gamma_1 (\eta_1(x_1, \dot{x}_2) z_2 - 2k_1 \lambda_1) \quad (32)$$

$$\dot{\theta}_2 = \gamma_2 (\eta_2(x_1, \dot{x}_2) z_2 - 2k_2 \theta_2) \quad (33)$$

where $\gamma_i > 0$ and $k_i > 0$ are constants, $i = 1, 2$.

Step 4: To prove the stability of the entire closed loop system, consider the following Lyapunov function based on (15) and (34) as

$$V = V_0 + V_3 \quad (36)$$
From (18), (28) and (35), the time derivative of $V$ is

$$
\dot{V} = \dot{V}_0 + \dot{V}_3 \\
\leq -\left(\frac{1}{2}\omega_0 - \frac{1}{2} - \frac{\lambda_0(c_1 + c_2)}{\omega_0}\right) \| \varepsilon \|^2 + \frac{\lambda_0^2 D^2}{2\omega_0^3} - \lambda_1 z_1^T z_1 \\
+ \frac{1}{2}\eta_1^2 + \frac{1}{2}\eta_2^2 + \frac{1}{2}\theta_1^2 + \theta_1^T \eta_1 z_1 - \frac{1}{\gamma_1} \theta_1 \\
+ \theta_1^T \eta_2 z_2 - \frac{1}{\gamma_2} \theta_2
$$

(37)

Substituting adaptation functions (32) and (33) into (37) yields

$$
\dot{V} \leq \left(\frac{1}{2}\omega_0 - \frac{1}{2} - \frac{\lambda_0(c_1 + c_2)}{\omega_0}\right) \| \varepsilon \|^2 + \frac{\lambda_0^2 D^2}{2\omega_0^3} - \lambda_1 z_1^T z_1 \\
- (\lambda_2 - 1) z_2^T z_2 \\
+ \frac{1}{2}\eta_1^2 + \frac{1}{2}\eta_2^2 + \frac{2k_1}{\gamma_1} \theta_1^T \eta_1 - \frac{1}{\gamma_2} \theta_1 \\
+ \frac{2k_2}{\gamma_2} \theta_2^T \theta_2 - \frac{1}{\gamma_2} \theta_2 \\
= \frac{-\omega_0^2 - \lambda_0 - \lambda_0(c_1 + c_2)}{2\omega_0} \| \varepsilon \|^2 - \lambda_1 z_1^T z_1 \\
- (\lambda_2 - 1) z_2^T z_2 - \frac{k_1}{\gamma_1} \theta_1^T \theta_1 - \frac{k_2}{\gamma_2} \theta_2^T \theta_2 + \Omega
$$

(38)

Theorem 2: For the BML dynamic system (8), under assumption 1, with the high gain observer (10), and the parameter adaptive law (32) and (33), the adaptive fuzzy robust control method (41) guarantees that the closed-loop system is stable and the output position tracking error converges to a small neighborhood of the origin by appropriately choosing the fuzzy membership functions, the observer parameter $\omega_0$, the adaptive law parameters $\gamma_1, \gamma_2, k_1$ and $k_2$, and the controller design parameters $\lambda_1$ and $\lambda_2$.

V. SIMULATION

A. SIMULATION SETUP

To verify the effectiveness of the control strategy proposed in this paper, a simulation model of the BML trajectory tracking control system is set up, which is depicted in Fig. 4. The nominal parameters of the BML system are listed in Table 2. The simulations are performed using MATLAB 2020a, in which the solver is selected as the ode4 (Runge-Kutta) type with a fixed step and a 1ms sampling time. The initial states of the BML system are set as $x_1 = [0.418, 0.036, -0.507]^T$ rad, $x_2 = [0, 0, 0]^T$ rad/s, while the initial states of the observers are set as $\tilde{x}_1 = [0, 0, 0]^T$ rad, $\tilde{x}_2 = [0, 0, 0]^T$ rad/s, and $\tilde{x}_3 = [0, 0, 0]^T$ rad/s.

| # | mass (kg) | mass center (m) | inertia matrix (kg m²) |
|---|---|---|---|
| 0-1 | 10.758 | [0, 0.001, -0.017]^T | diag(0.044, 0.039, 0.032) |
| 1-2 | 19.261 | [-0.154, 0, -0.014]^T | diag(0.135, 1.455, 1.375) |
| 2-3 | 10.375 | [-0.391, 0, -0.045]^T | diag(0.138, 2.437, 2.327) |

TABLE 2. The nominal parameters of the BML system.

![FIGURE 4. Simulation model of the BML trajectory tracking control system.](image-url)

To reduce the impact force between the foot tip and ground during motion, a commonly used composite cycloid gait trajectory is implemented on the BML, of which the expressions are shown in (3) and (4). This desired gait trajectory is a parabolic-like trajectory in Cartesian coordinate system, where the velocity and acceleration of the motion beginning and end are all zero at each period, as seen in Fig.5. The planned gait trajectory of the foot tip is transformed into the desired joint trajectory through the inverse kinematic algorithm (2), while the actual joint positions are considered as the system outputs.
proportional-integral-derivative control (PID), sliding mode control (SMC), and nonlinear extended state observer-based robust control (RC-NESO).

1) The PID controller is derived as

$$\tau = k_p(x_{1d} - x_1) + k_d(\dot{x}_{1d} - \dot{x}_2)$$  \hspace{1cm} (42)

where $k_p$ and $k_d$ are positive design parameters.

2) The SMC controller is derived as

$$\tau = M_n(\ddot{x}_{1d} + k \dot{e}) + C_n(\dot{x}_{1d} + k e) + G_n + \Gamma \text{sgn}(e)$$  \hspace{1cm} (43)

where $e = x_{1d} - x_1$, $s = \dot{e} + K e$, $K = \text{diag}(k_1, k_2, k_3)$ and $\Gamma = \text{diag}(\gamma_1, \gamma_2, \gamma_3)$ are positive design parameters. In order to suppress the chattering effect caused by the signum function in Eq. (43), the saturation function is used to replace the sign function in the implementation (the boundary layer $\delta = 0.1$). The stability analysis is included in Appendix B.

3) The RC-NESO controller is derived as [37]

$$\tau = M_n a + C_n \dot{r} + G_n - kr - M_n x_3$$  \hspace{1cm} (44)

where $e = x_1 - x_{1d}$, $\dot{v} = \dot{x}_{1d} - \Delta e$, $a = \dot{v} + \Delta e$, $\Lambda$ and $k$ the positive design parameters.

Remark 2: In this paper, only the joint position $x_1$ is measured, thus, the joint velocity $x_2$ used in the PID controller, SMC controller and proposed controller is provided by the HGO observer, while the joint velocity $x_2$ and general disturbance $x_3$ used in the RC-NESO controller are derived from the NSEO observer.

Remark 3: The parameters of the PID controller are selected by trial and error, but the parameter selection of the other three controllers is implemented based on the following two steps. The first step is to choose a set of specific values for the parameters to ensure that the time derivative of the Lyapunov function of each controller is a negative function in order to meet the theoretical stringency with various prerequisites. However, it is often difficult to obtain a satisfactory control performance with such selected parameters. Alternatively, in implementation the second step is to simply choose sufficiently large parameters without worrying about the specific prerequisites. In this way, prerequisites will be satisfied for a certain set of parameters at least locally around the desired trajectory to be tracked. This is a practical method and has been used by other researchers to synthesize control methods for complex systems [35], [36]. The parameters of the HGO observer and the NSEO observer are chosen to ensure the stability of the observer and desired state convergence performance. All the design parameters of the controllers and observers are shown in Table 3.

The membership functions of the FLSs are chosen as

$$\mu_1^x(x_1(i)) = e^{-\frac{1}{2}[\frac{\delta}{(\gamma_1/2)^2}]^2},$$

$$\mu_2^x(x_2(i)) = e^{-\frac{1}{2}[\frac{\delta}{(\gamma_2/2)^2}]^2},$$

$$\mu_3^x(x_3(i)) = v e^{-\frac{1}{2}[\frac{\delta}{(\gamma_3/6)^2}]^2}, \quad i = 1, 2, 3, \quad j = 1, 2, 3.$$
TABLE 3. The design parameters of the controllers and observers.

| Controller | Parameters |
|------------|------------|
| PID        | $k_p = \text{diag}(2000,9600,3200)$, $k_q = \text{diag}(50,250,25)$, $\omega_0 = 500$ |
| SMC        | $K = \text{diag}(45,180,180)$, $\Gamma = \text{diag}(160,160,160)$, $\omega_0 = 500$ |
| RC-NESO    | $\hat{k} = \text{diag}(224,448,224)$, $\Lambda = \text{diag}(640,1920,640)$, $\varepsilon = 0.002$ |
| AFRC-HGO   | $\lambda_1 = \text{diag}(500,500,1000)$, $\lambda_2 = \text{diag}(500,500,2000)$, $\gamma_1 = \gamma_2 = 100$, $k_1 = k_2 = 1.5$, $\omega_0 = 500$ |

Remark 4: As the BML system will work both on land and in deep-sea water, the influence of different work conditions on system parameters belongs to the system dynamic uncertainties, which are set with the values of $M_n$, $C_n$, and $G_n$ changing in accordance with the law of $1 + 0.5\sin(0.5\pi t)$ times the nominal values. Additionally, the hydrodynamic force and tidal current acting on the body of the BML system when it works in water belong to the unknown external disturbances, which are set to a simplified periodically varying signal $d = [50\sin(0.5\pi t), 50\sin(0.5\pi t), 50\sin(0.5\pi t)]^T$ relative to the results in [55].

Remark 5: As the joints of the BML system are filled with compensating oil and will work in deep-sea water, high water pressure will exacerbate the impact of the sealing force and the viscous oil drag force on the joint, thus deteriorate the control performance of the BML system. In this paper, these influences are not considered, and we only focus on the influence of different work conditions on the system parameters and the hydrodynamic force and tidal current force impact on the body.

C. SIMULATION STUDIES

Simulation studies are performed on the BML system for trajectory tracking control in the presence of unmeasured system states, dynamic uncertainties and unknown external disturbances.

1) CASE 1

In this case, the simulation is conducted only with the proposed AFRC-HGO controller. The physical parameters of the BML system are set as the nominal values shown in Table 2, and the parameters of the proposed controller are set with the values listed in Table 3. Suppose dynamic uncertainties and external disturbances are acted on the system only in the last 15 seconds of the simulation time. The simulation results are illustrated in Fig.6–Fig.11 and Table 4, where the desired joint position and the actual joint position are abbreviated as $q_{id}$ and $q_i$, the joint position tracking error and joint velocity estimation error are abbreviated as $e_i$ and $\epsilon_i$, and the joint driving torque and the approximated lumped uncertainty are abbreviated as $\tau_i$ and $f_i$, respectively.

As seen, in Fig.6 and Fig.7, the joint position tracking performance is excellent, although with physical parameters changing up to 50% of the nominal values and external disturbances up to 50 Nm, the proposed controller can still ensure that the actual joint positions almost overlap with the desired joint positions, with tracking errors within $[1.05 \times 10^{-4}, 1.86 \times 10^{-4}, 2.32 \times 10^{-4}]$ rad. The reason for such a
good control performance is that the proposed AFRC-HGO controller employs both the system model information to achieve accurate model-based compensation and the additional approximation signal to reduce the uncompensated term.

Fig. 8 shows the joint velocity observation errors, which indicate that with HGO the estimated joint velocities can converge to the actual values with a fast dynamic, where the estimation errors are within [0.012, 0.009, 0.131] rad/s. However, comparing the estimation errors before and after dynamic uncertainties and external disturbances inserted into the system, we can see that the estimation errors change dramatically. This may be explained by the fact that before the dynamic uncertainties and external disturbances applied to the system, the observer dynamic model is accurate with the system dynamic model, so the velocity estimation values are identical to the actual velocity values of the system with respect to the same input signals, thus the estimation errors are zero. However, after the dynamic uncertainties and external disturbances acted on the system, the observer dynamic model is different from the system dynamic model, therefore leading the estimation errors to seriously change.

Moreover, from the results in Fig. 9 and Fig. 10, it can be seen that the joint driving torque and the lumped uncertainty (the robust term, denoted by $f = [f_1, f_2, f_3]^T = M_n \dot{\hat{\phi}}$, for the convenience of a later comparative analysis) approximated by the AFLS increase with the increasing of the dynamic uncertainties and external disturbances. However, before the dynamic uncertainties and external disturbances acted on the system, the lumped uncertainty $f$ is nonzero due to the joint position tracking errors and the state estimation errors. Additionally, comparing the results of Fig. 9 and Fig. 10, it can be found that the quantities of the joint torque variance are much larger than those of the approximated lumped uncertainty after dynamic uncertainties and external disturbances acted on the system, which implies that the model compensation term and the stable feedback term of the actual control signal have realized the main control function of the system, while the robust term is the supplement to reduce the influence of various uncertainties on the system in order to improve the performance of the controller.

Fig. 11 shows the foot tip trajectory of the BML in the Cartesian coordinate system, which illustrates that, no matter whether the dynamic uncertainties and external disturbances...
acted on the system, the proposed controller can make the foot tip of the BML move in a parabolic-like trajectory precisely, thus demonstrating the effectiveness of the proposed controller for accurate trajectory tracking capability.

2) CASE 2

To further verify the performance of the proposed AFRC-HGO controller, three other controllers introduced in V.B are compared in this case. The studies are performed under the same conditions as in case 1. The parameters of different controllers and observers are set with the values given in Table 3. The simulation results are shown in Fig.12-Fig.15, where the desired joint position, the actual joint position, the joint position tracking error, and the joint driving torque are abbreviated the same as those in case 1 but with the corresponding controller name in order to distinguish one from the other. The approximated lumped uncertainty and estimated general disturbance are abbreviated as $f_i$ and $g_i$, respectively.

**TABLE 5. Joint position tracking error of the four controllers.**

| Error (rad) | PID       | SMC       | RC-NESO   | AFRC-HGO  |
|-------------|-----------|-----------|-----------|-----------|
| $\theta_1$  | 0.036     | 1.5×10^-3 | 1.5×10^-4 | 1.0×10^-4 |
| $\|e_1\|_{\text{max}}$ | 0.042     | 2.1×10^-3 | 2.8×10^-4 | 1.5×10^-4 |
| $\theta_3$  | 0.041     | 1.2×10^-3 | 3.0×10^-4 | 2.4×10^-4 |

The joint position tracking performance of the four controllers is shown in Fig.12, and the corresponding tracking errors are given in Fig.13 and Table 5. As shown, the PID controller and SMC controller cannot handle such an aggressive movement very well, with maximum tracking errors approximately equal to [0.036, 0.042, 0.041] rad and [1.5×10^-3, 2.1×10^-3, 1.2×10^-3] rad, respectively. In contrast, the joint position tracking performance of the proposed AFRC-HGO controller and the RC-NESO controller is improved significantly, where the tracking errors are maintained within [1.0×10^-4, 1.5×10^-4, 2.4×10^-4] rad and [1.5×10^-4, 2.8×10^-4, 3.0×10^-4] rad respectively, which decrease nearly two orders of magnitude compared with the PID controller, and one order of magnitude compared with the SMC controller.

The main reason for such a large difference in the tracking performance of the four controllers is that, as explained in the previous case, the AFRC-HGO controller and RC-NESO controller both employ the system model to achieve an accurate model-based compensation and the additional estimation or approximation information to deal with the lumped uncertainties and compensate them in the feedforward way to reduce the uncompensated term, whereas the SMC controller only employs the system model information to achieve an accurate model-based compensation, and the PID controller does not consider these at all.
Fig. 14 shows the driving torque (system control input) from the AFRC-HGO controller and RC-NESO controller. Fig. 15 presents the approximation lumped uncertainty \( f \) from the AFLS in the AFRC-HGO controller and the estimated generalized disturbance \( g \) derived from the extended state of the NESO in the RC-NESO controller. As can be seen, the system control input and the approximated lumped uncertainty /estimated general disturbance obviously change when the dynamic uncertainties and external disturbance are inserted into the BML system. This is mainly because that the approximation or estimation information are taken into effect with the compensation mechanisms of the two controllers. Moreover, by analyzing the difference of the approximation or estimation information before and after the dynamic uncertainties and external disturbance added into the BML system, we can find that their values are different, and more interestingly, the estimated values of the AFRC-HGO controller are all nonzero before the dynamic uncertainties and external disturbance added, while those of the RC-NESO controller are all zero. This is because the generalized disturbance \( g \) derived from the extended state of the NESO only includes the dynamic uncertainties and the unknown external disturbances, but the lumped uncertainty \( f \) derived from the AFLS consists of the dynamic uncertainties, unknown external disturbances and the effects of the estimation errors, in which the effects of the estimation errors become one more piece of information to be compensated in the controller, thereby leading to a light performance improvement compared with the RC-NESO controller.

**VI. CONCLUSION AND FUTURE WORK**

In this paper, a high gain observer-based adaptive fuzzy robust controller is proposed for the trajectory tracking control of a BML system, which considers not only the unmeasured system states, but also dynamic uncertainties and external disturbances. The unmeasured system states are estimated via the HGO, while the lumped uncertainties (including the dynamic uncertainties, unknown external disturbances and the effects of the estimation and approximation errors) are handled by the AFLS. The nonlinear robust controller is then synthesized via the backstepping method with the combination of the model-based compensation, the stable feedback term and the robust term. Finally, comparative simulation studies are conducted to illustrate the effectiveness of the proposed control method, which show that a superiority trajectory tracking performance can be achieved under the proposed controller.

Our future work will focus on the following two aspects: (1) As discussed in remark 5, high water pressure will exacerbate the impact of the sealing force and the viscous oil drag force on the joint, thus deteriorating the control performance of the BML system. Therefore, next we will explore the complicated variational rule on these forces and search for applicable solutions to manage them. (2) In this paper, we only propose the control algorithm from the perspective of the entire BML dynamics, but do not provide an insight into the single joint control, which is also a complex dynamic system that is worth studying.

**APPENDIX A**

The dynamic equation of the BML system is rewritten as follows:

\[
\tau = M_n \ddot{q} + C_n \dot{q} + G_n + (\Delta M \ddot{q} + \Delta C \dot{q} + \Delta G + d) \tag{45}
\]

where \( \tau = [\tau_1, \tau_2, \tau_3]^T \), \( q = [q_1, q_2, q_3]^T \), and \( \dot{q} = [\dot{q}_1, \dot{q}_2, \dot{q}_3]^T \).
With the nominal values of the BML system, the elements of the matrices $M_n$, $C_n$ and $G_n$ are presented below

$$M_n = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix},$$

$$C_n = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix},$$

$$G_n = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix},$$

where

$$m_{11} = 6.928 + 5.157 \cos(2q_2) + 3.157 \cos(q_3) + 3.157 \cos(2q_2 + q_3) + 1.459 \cos(2q_2 + 2q_3)$$

$$m_{12} = 0.486\sin(q_2) + 0.398\sin(q_2 + q_3)$$

$$m_{13} = 0.398\sin(q_2 + q_3)$$

$$m_{21} = 0.486\sin(q_2) + 0.398\sin(q_2 + q_3)$$

$$m_{22} = 13.310 + 6.313\cos(q_3)$$

$$m_{23} = 2.946 + 3.157 \cos(q_3)$$

$$m_{31} = 0.398\sin(q_2 + q_3)$$

$$m_{32} = 2.946 + 3.157 \cos(q_3)$$

$$m_{33} = 2.946$$

$$c_{11} = -2.897 \sin(2q_2)q_2 - 1.578 \sin(q_3)q_3 - 1.578 \sin(2q_2 + q_3)q_3 - 1.459 \sin(2q_2 + 2q_3)q_3$$

$$c_{12} = -2.897 \sin(2q_2)q_2 + 0.178 \cos(q_2)q_2 + 0.398 \cos(2q_2 + q_3)q_3$$

$$c_{13} = -1.578 \sin(q_3)q_1 - 1.578 \sin(2q_2 + q_3)q_1 - 1.459 \sin(2q_2 + 2q_3)q_1 + 0.398 \cos(q_2 + q_3)q_2$$

$$c_{21} = 2.897 \sin(2q_2)q_1$$

$$c_{22} = -3.157 \sin(q_3)q_3$$

$$c_{23} = -3.157 \sin(q_3)q_2 - 3.157 \sin(q_3)q_3$$

$$c_{31} = [1.578 \sin(q_3) + 1.578 \sin(2q_2 + q_3) + 1.459 \sin(2q_2 + 2q_3)]q_1$$

$$c_{32} = 3.157 \sin(q_3)q_2$$

$$c_{33} = 0$$

$$g_1 = 0$$

$$g_2 = 162.6 \cos(q_2) + 46.870 \cos(q_2 + q_3)$$

$$g_3 = 46.870 \cos(q_2 + q_3)$$

**APPENDIX B**

Proof of the stability of the SMC controller: Define $e = x_{1d} - x_1$, and $s = \dot{e} + Ke$. From (45), the time derivative of the nonnegative function $V = \frac{1}{2}s^T M_n s$ is derived as

$$\dot{V} = \frac{1}{2}s^T \dot{M}_n s + s^T \dot{M}_n s$$

$$\dot{V} = \frac{1}{2}s^T (\dot{M}_n - 2C_n s + s^T C_n s + s^T M_n s)$$

$$\dot{V} = s^T (C_n \dot{e} + C_n Ke + M_n \dot{x}_{1d} - (M_n \dot{x}_1 + C_n \dot{k}_1)$$

$$+ C_n \dot{x}_1 + M_n \dot{K}_e)$$

$$\dot{V} = s^T (C_n (\dot{x}_{1d} + Ke) + M_n (\dot{x}_{1d} + K\dot{e}) + G_n + A - \tau)$$

The following SMC control function is designed

$$\tau = M_n (\dot{x}_{1d} + Ke) + C_n (\dot{x}_{1d} + Ke) + G_n + \Gamma \sgn(s)$$

where $K = \text{diag}(k_1, k_2, k_3)$ and $\Gamma = \text{diag}(\gamma_1, \gamma_2, \gamma_3)$ are the positive design parameters. Substituting (47) into (46) yields

$$\dot{V} = s^T (A - \Gamma \sgn(s))$$

Choose $\gamma_i \geq |\Delta_i|_{\text{max}}, i = 1, 2, 3$. Then we have

$$\dot{V} \leq 0$$

Therefore, the closed loop system is stable.

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