Cosmological constant in broken maximal sugras

Gordon Chalmers

Argonne National Laboratory
High Energy Physics Division
9700 South Cass Avenue
Argonne, IL 60439-4815
e-mail: chalmers@pcl9.hep.anl.gov

Abstract

We examine the form of the cosmological constant in the loop expansion of broken maximally supersymmetric supergravity theories, and after embedding, within superstring and M-theory. Supersymmetry breaking at the TeV scale generates values of the cosmological constant that are in agreement with current astrophysical data. The form of perturbative quantum effects in the loop expansion is consistent with this parameter regime.
1 Introduction

Data from type I redshifted supernova indicate a small but non-vanishing value of the cosmological constant \( \Lambda_c \) [1, 4, 3]. The cosmic microwave background and its anisotropy support this experimental evidence. A non-vanishing value of \( \Lambda_c \) is a theoretical challenge to reconcile with supergravity and superstring (M-) theory, and in this letter we examine this problem in the context of maximal supersymmetry, both spontaneously broken [4, 5] and generally broken. In the former theories a quartic mass relation found by tuning the moduli of the compactification may enforce the leading cancellation to all loop orders. In the latter theories, breaking supersymmetry in a manner in which there are eight independent supersymmetry breaking scales excludes the quartic \( \Lambda_4 \) and sextet \( \Lambda_6 \) term in the expansion of the cosmological constant as we examine in this work; the remaining terms in the expansion are in agreement with data.

There are two forms of the cosmological constant problem: string scale and low scale. In this work we analyze the issue of breaking supersymmetry and solving the cosmological constant problem at the order of a TeV.

The cosmological constant in maximally supersymmetric gravity theories has been examined in detail up to one loop [5, 6]. Spontaneously broken supergravity theories are parameterized by four real numbers, characterizing the mass spectrum [7]; we also consider more general scenarios, as this susy breaking does not contain eight independent scales in four-dimensions in association with the number of supercharges. The former example contains several mass relations such that the graded trace up to cubic order of the particle masses [3, 7],

\[
\sum (-)^F m_i = \sum (-)^F m_i^2 = \sum (-)^F m_i^3 = 0 ,
\]

must be equal to zero. There is a fourth relation [4] beyond that in [3],

\[
\sum (-)^F m_i^4 = 0 .
\]

The cosmological constant is finite to one-loop order and has the form [3],

\[
A = \sum_j (-)^F \left( m_j^4 \ln(m_j^2/\Lambda^2) - \frac{3}{2} m_j^4 \right) = \sum_j (-)^F m_j^4 \ln m_j^2 .
\]

A further relation of the mass spectrum sets this also to zero. This work will derive similar results on the ultraviolet nature of the cosmological constant with very mild analyticity.

\[^1\text{These data are subject to error bars, and we fit to the quoted value, even though the cosmological constant is arguably consistent with zero experimentally.}\]
requirements imposed on the functional form of the cosmological constant in the loop expansion. The series is given and the form is shown to agree remarkably well with astrophysical data.

Maximal supergravity theories \[8, 9, 10\] possess thirty-two conserved components or eight Weyl supercharges in four dimensions. The cosmological constant induced after breaking supersymmetry is generically large in gravitational theories, and of the order \(\Lambda^4\) with \(\Lambda\) the Wilsonian ultra-violet cutoff.\(^2\) However, if we demand that the leading order term in the coupling expansion of the constant vanishes upon restoration of any two amounts of the supersymmetry \((N = 1\) supersymmetry without matter contains a \(\Lambda^6/m^2_{pl}\) term from the D-terms), then the leading unique polynomial describing the cosmological constant is,

\[
\mathcal{L} = \int d^4x \sqrt{g} \frac{1}{m^4_{pl}} \prod_{j=1}^{8} \Lambda_j , \tag{1.4}
\]
in which \(\Lambda_j\) defines the scale of non-conservation of the independent supercharges. The cancellations of the \(\Lambda^4\) and \(\Lambda^6/m^2_{pl}\) terms are associated with approximate supersymmetry restoration at high-energies together with the extraction of eight powers of loop momentum in the tensor integrations of the \(N = 8(32)\) theory. The particle masses are taken to be fractions of the supersymmetry breaking scale, \(m_j = \alpha_j \Lambda\) and the scales as \(\Lambda_j = \beta_j \Lambda\). The two distinct energy scales are associated with: 1) the supersymmetry breaking scales \(\Lambda_j\), 2) the gravitational coupling constant which is taken to be the Planck mass (with the same for the UV cutoff in the field theory).

The term in (1.4) vanishes upon restoration of any two (or one) of the conserved supercharges. Having only seven independent supersymmetry breaking scales would allow the function

\[
(c_1 \Lambda_1 \ldots \Lambda_6 + \text{perms})/m^2_{pl} \tag{1.5}
\]
which would not vanish upon taking one of the supersymmetry breaking scales to be zero. This is compatible with the \(\Lambda^6/m^2_{pl}\) term in \(N = 1\) supergravity. If there are eight independent scales (all of which are present in the expansion), then this sextet term would vanish upon restoration of any two supersymmetries.

The numerical value, modulo a suppressed coefficient of order unity, agrees very well with the experimental value of the cosmological constant when the supersymmetry breaking is at the TeV scale,

\[
\frac{\Lambda^8}{m^4_{pl}} \sim 2 \times 10^{-10} \text{erg/cm}^3 , \tag{1.6}
\]

\(^2\)A review of the cosmological constant in gravitational theories may be found in \[11\].
with $\Lambda \sim 3 \times 10^3 \text{ GeV}$ (and $m_{\text{pl}} = 10^{19} \text{ GeV}$). The constant is sensitive to the supersymmetry breaking scale, which receives a $2^8$ factor from doubling all the scales. The naive leading order terms,

$$\alpha \Lambda^4 \beta \frac{\Lambda^6}{m_{\text{pl}}^2}, \quad (1.7)$$

generically present, are absent in this supersymmetry breaking. In the spontaneously broken $\mathcal{N} = 8$ theory, mass relations enforce these terms to vanish.

We consider next the series expansion of the cosmological constant in the general maximal supersymmetric theories with multiple supersymmetry breaking scales $\Lambda_j$; we do not give the explicit mechanism in this work, however explicit supersymmetry breaking terms. The effective action we consider is obtained by integrating modes up to the highest energy scale $\Lambda_1 < \Lambda_2 \ldots < \Lambda_8$, with no particular range of the $\Lambda_j$ (such as holding all $\Lambda_j \ll \Lambda_8$). Then the physics of the theory is governed by $\Lambda_j$ together with, in the field theory, the ultra-violet Wilsonian cutoff $\Lambda$, and the masses $m_j$ of the particles (if considering the string). The gravitational coupling constant is $m_{\text{pl}}$. In the string models, we may simply induce the supersymmetry breakings below the string scale.

We analyze the form according to the two sectors: 1) loop diagrams containing a propagating gravitational mode with coupling $m_{\text{pl}}^{-2}$ and 2) loop diagrams containing only the matter degrees of freedom. (Theories with spontaneously broken supersymmetries of the Scherk-Schwarz type, possessing four scales may, up to the $L^{\text{th}}$-loop order have canceled quartic and sextet terms if we consider mass relations similar to the one in (1.3). This mass relation is the only condition we impose on the spontaneously broken expansions, and we assume that a solution exists for mass parameters in these breaking scenarios.)

Inverse powers of the susy breaking scale do not occur in the expansions of diagrams containing massive particles, as analyzed below. The primitive divergences of multi-loop diagrams are polynomials in $\Lambda$, modified by logarithms, and upon taking the particle masses to be proportional to $\Lambda$, there are no dimensionless expansion parameters; the ratios $\Lambda_i/\Lambda_j$ within logarithms are of order unity upon taking the scales to be the same and do not produce an expansion variable. At values of $\Lambda_j = \beta_j \Lambda$ this enforces the general expansion to be of the form, (with the parameters $\alpha_j$ labeling masses in terms of scales)

$$\mathcal{L}_{N=8} = \sum_{n=0}^{\infty} \int d^4x \sqrt{g} \ c_n(\beta_j, \alpha_j) \frac{\Lambda^8}{m_{\text{pl}}^4} \left( \frac{\Lambda}{m_{\text{pl}}} \right)^{2n}, \quad (1.8)$$

in which supersymmetry breaking is parameterized by eight independent scales. Incorporating a particle mass $M$ into the expansion alters the form of the $n^{\text{th}}$ coefficient into the
expansion and generates contributions,
\[ c_n \Lambda^4 \left( \frac{L^2}{m^2_{pl}} \right)^n [\alpha_0^{(n)} \frac{\Lambda^4}{m^4_{pl}} + \alpha_1^{(n)} \frac{\Lambda^4}{m^4_{pl}} \ln^L \frac{\Lambda}{M} + \ldots + \beta_0^{(n)} \frac{\Lambda^3 M}{m^4_{pl}} + \ldots] + \ldots \]

\[ + c_n M^4 \left[ \frac{\beta_0^{(n)} M}{m^4_{pl}} \ln^L \frac{\Lambda}{M} + \ldots \right] + \ldots \]

with \( M \sim \Lambda_j \). The form of the expansion is controlled by the fact that every diagram is multiply differentiable in the masses of the internal propagators,

\[ \prod \left\{ \frac{\partial}{\partial m_{\sigma (j)}} \right\} A_L \text{ loops} \big|_{M=0} , \]

which exists and has the structure in field theory of

\[ \prod \frac{m_{\sigma (j)}}{m^4_{pl} \Lambda^2} , \]

with \( \Lambda \) the ultra-violet cutoff. In field theory we take this scale to be of order of the \( m_{pl} \). In string theory the form is the same with \( \Lambda = m_{pl} \). This form in (1.11) shows that the loop expansion of the cosmological constant is a Taylor series expansion in the supersymmetry breaking scales.

Next we comment on the loop expansion and the tower of terms in (1.8) for the gravitational sector followed by the matter sector. The iteration of a \( L \)-loop contribution to \( L + 1 \)-loops follows by inserting an additional propagator and two graviton three-point vertices, or by higher-point vertices. The gravitational vertices are found by expanding the Einstein-Hilbert term, \( m^2_{pl} \int d^4x \sqrt{g} \ R \), and we do not list the diagrammatic rules here. The insertion of two three-point vertices introduces four-derivatives into the loop integration (the graviton \( n \)-point vertices contain two derivatives), two propagators, and an additional loop integration. This iteration is

\[ A_{L+1} = \frac{1}{m^4_{pl}} \int \frac{d^4l}{(2\pi)^4} A_{L-1}^{\mu_1 \nu_1, \mu_2 \nu_2} (l, l + p) \times 

V_{\mu_1 \alpha_1 \beta_1, \nu_1 \gamma_1 \delta_1} (l, l + p) V_{\mu_2 \alpha_2 \beta_2, \nu_2 \gamma_2 \delta_2} (-l, -l - p) \Delta^{\alpha_1 \gamma_1; \alpha_2 \gamma_2} (l + p) \Delta^{\beta_1 \delta_1; \beta_2 \delta_2} (l) . \]

and with a four-point vertex is,

\[ A_{L+1} = \frac{1}{m^4_{pl}} \int \frac{d^4l}{(2\pi)^4} A_L^{\mu \nu, \alpha \beta} (l, -l) \Delta^{\mu \nu; \alpha \beta} (l) . \]

The integral in (1.12) generates \( \Lambda^{4 - 6 + 4} = \Lambda^2 \) additional occurences of the ultra-violet cutoff. The two graviton three-point vertices come with \( 1/m^2_{pl} \) as the gravitational coupling
constant is in terms of the Planck mass. Thus the relative weight between $L$-loops and $L + 1$ loops differs by a factor of $\Lambda^2/m_{pl}^2$. The power series in (1.8) reflects the loop expansion in this manner. Higher point vertices do not change the form in (1.8), but rather mix different loop orders in the coefficients $c_n$, as may be deduced by similar counting. For example, the four-point vertex iteration in (1.13) gives the power count $\Lambda^{2+4-4}$, with two powers of $\Lambda$ from the derivatives from the vertex, four from the loop integration, and minus two from the propagator. Higher-point vertices and other particle modes do not change the analysis.

The pure matter sector, which does not contain explicit occurrences of the gravitational coupling, may be similarly analyzed. The expansion of the diagrams in the pure matter sector, however, do possess inverse powers of the gravitational coupling upon expanding the diagrams at low-energy about an ultra-violet cutoff on the order of the Planck mass, as in (1.11). We consider iterating an $L$-loop scalar field theory graph in $\lambda_1(\Lambda)^{ijk}\phi_i\phi_j\phi_k + \lambda_2(\Lambda)^{ijkl}\phi_i\phi_j\phi_k\phi_l$ theory. Gluing two cubic vertices and three propagators iterates a $L$-loop graph to $L + 1$ loops, but does not raise the degree of primitive divergence. The logarithmic factors increase by one unit, however, in the primitive divergence. The form of the result is

$$c_L(m_i^4 \ln^L m_i + \ldots + m_i^2 \Lambda^2 + \ldots + \Lambda^4 + \ldots), \quad (1.14)$$

and subleading powers in the cutoff, generating terms containing factors of $m_{pl}^{-4+k}$. The indices on the masses represent arbitrary combinations. The four-point vertices in the scalar field theory give the same result. An iteration to higher loop order containing a gravitational mode with two couplings and a factor $m_{pl}^{-2}$; these terms are classified in (1.9) and are explicitly suppressed by an acceptable power of the Planck mass. The functions (1.14) to $L$ loop orders are in principle computable.

The expansion parameter in the quantum series is $\Lambda/m_{pl}$ and is a small dimensionless number, $10^{-15}$. The series in (1.8) makes sense as an expansion in loops or coupling for this reason, and the leading value in (1.4) is stable under quantum corrections.

For comparison we discuss the form of the cosmological constant in general non-supersymmetric examples (for example, breaking $\mathcal{N} = 1$ supersymmetry). The loop expansion contains the series with the two additional terms $\Lambda^4$ and $\Lambda^6$,

$$\mathcal{L} = \int d^4x \sqrt{g} \left[ \alpha \Lambda^4 + \beta \frac{\Lambda^6}{m_{pl}^2} \right] + \sum_{n=0}^{\infty} \int d^4x \sqrt{g} \ c_n \frac{\Lambda^8}{m_{pl}^4} \left( \frac{\Lambda}{m_{pl}} \right)^{2n}. \quad (1.15)$$

These two additional terms do not agree with experimental values of the cosmological constant without renormalization.
The loop expansion of the supersymmetrically broken $\mathcal{N} = 8$ supergravity or IIB superstring theory naturally produces values of the cosmological constant, and a loop expansion, that is in agreement with current astrophysical data if the susy scales are of the order of a TeV. This analysis carries through in the two scenarios: 1) susy breaking scales are all independent, in which case (1.8) holds by imposing analytic requirements, and 2) in spontaneous susy breaking in which mass relations are required to have (1.8). In the former approach no tuning is required. In both examples, the non-vanishing cosmological constant arises from both the gravitational and matter sector.

M-theory has $\mathcal{N} = 1$, $d = 11$ supergravity as its low-energy limit, which describes M-theory graviton scattering to high orders in the derivative expansion, and contains $\mathcal{N} = 8$ supergravity upon toroidal compactification or dimensional reduction. The symmetries of the maximally supersymmetric theory are responsible for the leading order cancellations of terms that spoil cosmological predictions of supergravity theories. Furthermore, the quantum corrections do not alter the semi-classical prediction.

The $\mathcal{N} = 8$ maximally supersymmetric theory, and the IIB superstring (M-) theory that contains it as its massless sector, have two properties that have been explored recently:

1) The theory appears finite in perturbation theory according to the modular properties of the scattering inherited from S- and U-duality and the AdS/CFT duality \cite{10, 11, 12} (after decoupling of the massive modes). The cancelations resulting in finiteness may occur in a string inspired regulator preserving these properties, and are beyond the cancelations at two-loops in the expansion of the graviton scattering \cite{13, 14}.

2) Upon breaking supersymmetry at the TeV scale, the theory produces a cosmological constant that agrees with current experimental data and which is stable under quantum corrections.

The two properties warrant further phenomenological investigations of both IIB superstring (and M-) and $\mathcal{N} = 8$ theories, and in particular to answer if the standard model may be accomodated in its low-energy physics.

Acknowledgements

The work of GC is supported in part by the US Department of Energy, Division of High Energy Physics, contract W-31-109-ENG-38. GC thanks Gia Dvali and Emil Martinec for relevant discussions and correspondence.
References

[1] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Bull. Am. Astron. Soc. 29, 1351 (1997) [astro-ph/9812473].

[2] A.G. Riess et. al., astro-ph/0001384.

[3] S. M. Carroll, astro-ph/0004075.

[4] J. Scherk and J. H. Schwarz, Phys. Lett. B 82, 60 (1979).

[5] E. Cremmer, J. Scherk and J. H. Schwarz, Phys. Lett. B 84, 83 (1979).

[6] R. Rohm, Nucl. Phys. B 237, 553 (1984).

[7] S. Ferrara and B. Zumino, Phys. Lett. B 86, 279 (1979).

[8] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B 76, 409 (1978); E. Cremmer and B. Julia, Phys. Lett. B 80, 48 (1978).

[9] B. de Wit and D. Z. Freedman, Nucl. Phys. B 130, 105 (1977).

[10] B. de Wit and H. Nicolai, Nucl. Phys. B 208, 323 (1982).

[11] S. Weinberg, Rev. Mod. Phys. 61:1 (1989).

[12] G. Chalmers, Nucl. Phys. B 580, 193 (2000) [hep-th/0001190].

[13] G. Chalmers and J. Erdmenger, Nucl. Phys. B 585, 517 (2000) [hep-th/0005192].

[14] G. Chalmers, hep-th/0008162.

[15] Z. Bern, L. Dixon, D. C. Dunbar, M. Perelstein and J. S. Rozowsky, Nucl. Phys. B 530, 401 (1998) [hep-th/9802162].

[16] Z. Bern, hep-th/0102186.