Cosmological phase transitions and gravitational waves in the singlet Majoron model

Youping Wan, a,b Batool Imtiaz, a,b Yi-Fu Cai, a,b

aCAS Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Chinese Academy of Sciences, Hefei, Anhui 230026, China
bSchool of Astronomy and Space Science, University of Science and Technology of China, Hefei, Anhui 230026, China

E-mail: wanyp@ustc.edu.cn, batool24@mail.ustc.edu.cn, yifucai@ustc.edu.cn

Abstract. In this paper, we study the cosmological phase transitions (PTs) and gravitational waves (GWs) in the singlet Majoron model. With one extra Higgs singlet been introduced besides the Standard Model Higgs doublet, the model should be dealt as a two-field problem properly. We firstly calculate the effective potential for the two classical scalars in the framework of finite temperature field theory, then make use of the public available Python package ‘CosmoTransitions’ to study the cosmological phase transitions, finally analyze the properties of gravitational waves generated by first order PTs. We find patterns of cosmological phase transitions in this model turns to be quite fruitful, usually there are multi-step PTs, either being first or higher orders, happening along distinguished directions. By confronting the PTGWs with space-borne gravitational wave interferometers such as LISA, DECIGO, BBO, TAIJI and TianQin, we conclude only these vevs $v_{BL}$ with small values (smaller than $\sim \mathcal{O}(10)$ GeV) may lead to astronomically interested GW signals.

Keywords: cosmological phase transitions, gravitational waves, neutrino mass
1 Introduction

The strong first order phase transitions (PTs) are of great cosmological significance as they are necessary conditions for successful baryogenesis mechanism [1]—which explains why there are more matter than anti-matter in our universe. Since the first discovery of gravitational waves (GWs) by the Advanced Laser Interferometer Gravitational Wave Observatory (aLIGO) [2], a new era of GW astronomy has been opened, we are now at the position of exploring the multi-band GW detections. On the other hand, in literature it is well-studied that stochastic GW backgrounds can be generated during first order cosmological phase transitions [3–6]. Unfortunately, the electroweak phase transition (EWPT) in the standard model (SM) of particle physics turns to be a crossover [7–9], which is far from strong enough to generate the stochastic GW backgrounds. In order to strengthen the EWPT as well as probe new physics, many extensions of the SM have been studied, such as models with extra scalar singlet(s) [10–14], with dimensional six effective operators [15–19], 2HDMs [20–23], NMSSM [24–26], and more other works [27–30].

The singlet Majoron model is another simplest extension of SM [31] (see also the GR model [32]), which is originally proposed to solve the neutrino mass problem. With the introductions of one more complex Higgs singlet conserves the global $U(1)_{B-L}$ symmetry and also the right-handed (RH) neutrinos, people get interested if there exist first order phase transitions [33–36] or testable GW signals in the developing GW astronomy as well as radio astronomy [37]. In Refs. [33, 34] the authors claim there exists a flat direction on the surface of the effective potential, thus the two-field problem can be reduced into a single-field one, they conclude strong first order phase transitions can be realized. Another study in Ref. [35] further confirm that phase transitions typically proceed in two steps, i.e., a very weakly first order transition from the $U(1)_{B-L}$ breaking followed by the EWPT. In these earlier works, the researchers haven’t considered the Yukawa couplings between the singlet Higgs field and the right-handed neutrinos. Later on, a very comprehensive work has been done in Ref. [36], the authors conduct a full numerical simulation to handle the two-field problem directly. Their work shows the existence of two step phase transitions—once again the PT due to the $U(1)_{B-L}$ symmetry breaking is above the EWPT, they also find large values of the two Higgs interactive coupling and the Yukawa coupling between the Higgs singlet and the right-handed neutrinos are expected in order to obtain strong first order phase transitions.
In this paper, we will follow the work of Ref. [36], treat the two-field problem directly by making use of the public available Python package CosmoTransition [38], and go even further by calculating the GWs generated from strong first order phase transitions. We confirm the patterns of phase transitions which have been found in Refs. [35, 36], at the meanwhile we find even more new patterns, for the PTGWs we confront them with space-borne gravitational wave interferometers such as LISA [39], DECIGO [40–42], BBO [43], and also the Chinese projects TAIJI (ALIA descoped) [44] and TianQin [45, 46], we conclude they can be astronomically interested only when the vevs \( v_{BL} \) have small values (smaller than \( \sim O(10) \) GeV). The paper is organized as follows: in section 2 we write down the model of singlet Majoron and unify our notations; in section 3 we calculate the effective potential of the two classic fields within the finite temperature field theory, and show the interesting patterns of cosmological phase transitions; in section 4 we calculate the GW signals generated from first order phase transitions and confront them with space-borne GW detectors; finally in section 5 we make the conclusion.

2 The singlet Majoron model

The singlet Majoron model is among the simplest extensions of the Standard Model (SM) of particle physics. An \( U(1)_{B-L} \) global symmetry, where the \( B \) and \( L \) stand for baryon and lepton numbers, is introduced apart from the original gauge symmetry \( SU_c(3) \times SU(2)_L \times U(1)_Y \). Right-handed neutrinos \( \nu_R \), together with a complex singlet Higgs \( \sigma \) become new members of the high energy physics particles’ zoo. The Lagrangian starts from

\[
L = -f \bar{L} \sigma \nu_R - g \sigma \bar{\nu}_R \nu_R^c + h.c.,
\]

in which \( \Phi \) and \( \bar{L} \) are the doublets of SM Higgs and Light-handed (LH) fermions, \( f, g \) are Yukawa coupling constants.

The potential for the doublet and singlet Higgs at tree level can be written as

\[
V(\sigma, \Phi) = \lambda_s |\sigma|^4 + \mu_s^2 |\sigma|^2 + \lambda_h |\Phi|^4 + \mu_h^2 |\Phi|^2 + \lambda_{sh} |\sigma|^2 |\Phi|^2
\]

in order to ensure today’s vacuum \( (v_{BL}, v_{ew} = 246 \) GeV), we will choose \( \mu_s^2 = -\lambda_s v_{BL}^2 - (1/2)\lambda_{sh} v_{ew}^2 \), \( \mu_h^2 = -\lambda_h v_{ew}^2 - (1/2)\lambda_{sh} v_{BL}^2 \). To be clearer, let’s expand the Higgs Bosons into their real components

\[
\sigma = \frac{1}{\sqrt{2}} (v_{BL} + \rho + i\chi), \quad \Phi^T = \frac{1}{\sqrt{2}} (G_1 + iG_2, \, v_{ew} + H + iG_3)^T.
\]

Just like a Dirac mass with \( m = f v_{ew}/(2\sqrt{2}) \) being obtained after the electroweak symmetry breaking, a Majorana mass with \( M = g v_{BL}/\sqrt{2} \) can also be generated after the breaking of the \( U(1)_{B-L} \) symmetry. If the Majorana mass is much larger than the Dirac one, redefine heavy and light neutrinos as \( N = \nu_R + \nu_R^c + m/M (\nu_L + \nu_L^c) \), \( \nu = \nu_L + \nu_L^c - m/M (\nu_R + \nu_R^c) \), then the masses of light neutrinos \( m_\nu \simeq m^2/M \) are highly suppressed by the heavy neutrinos mass \( m_N = M \), that’s the simplest way to understand why neutrinos’ observational masses are so small.

A massless Goldstone boson \( \chi \) appears after the \( U(1)_{B-L} \) global symmetry broken, this is the Majoron field discussed in this paper. Even though the Majoron is massless, small mass term will be generated after one has considered the effective operators with higher
dimensions. The authors of Ref. [47] argue when the effective operators been considered, we’d get a constraint about $v_{BL}$

$$v_{BL} \leq \left( \frac{m_\nu}{25 \text{ eV}} \right)^{4/7} \times 10 \text{ TeV},$$

(2.4)

since the neutrino mass upper bound is smaller than $m_\nu \sim 2 \text{ eV}$ [48], this gives

$$v_{BL} \leq 1,590 \text{ GeV}.$$  

(2.5)

In [49, 50], it is pointed a $v_{BL}$ with a value larger than $v_{ew}$ can be quite dangerous. However, in our numerical simulation we will explore both the scenarios either $v_{BL}$ be larger or smaller than $v_{ew}$, aiming at getting a global overlook on the parameter space which can lead to strong first order PTs and GWs, remarkably we find $v_{BL}$ with higher values are also astronomically uninterested according to the results in Sec.4.

3 Patterns of cosmological phase transitions in the singlet Majoron model

Consider two classic fields

$$\sigma = \frac{s}{\sqrt{2}}, \quad \Phi^T = \left( 0, \frac{h}{\sqrt{2}} \right)^T,$$

(3.1)

the tree level potential can be rewritten as

$$V^{\text{tree}}(s, h) = \frac{1}{4} \lambda_s \left( s^2 - v_{BL}^2 \right)^2 + \frac{1}{4} \lambda_h \left( h^2 - v_{ew}^2 \right)^2 + \frac{1}{4} \lambda_{sh} \left( s^2 - v_{BL}^2 \right) \left( h^2 - v_{ew}^2 \right),$$

(3.2)

whose global minimum clearly shows today’s vacuum. Then consider the one loop correction at zero temperature with MS renormalization

$$\Delta V^{T=0}_1(s, h) = \frac{1}{64\pi^2} \sum_i n_i m_i^4(s, h) \left[ \log \frac{m_i^2(s, h)}{Q^2} - c_i \right],$$

(3.3)

with $(c_1, c_{W_T}, c_{Z_T}) = (1.5, 0.5, 0.5)$, here 'T' stands for the transverse modes, 'L' stands for the longitudinal modes, the degree of freedoms $(n_H, n_G, n_p, n_\chi, n_{\nu_R}, n_t) = (1, 3, 1, 1, -6, -12)$, $(n_{W_T}, n_{W_L}, n_{Z_T}, n_{Z_L}) = (4, 2, 2, 1)$, the mass spectrum can be found in the appendix. We also introduce a counter term $\Delta V^{T=0}_{ct}(s, h) = Ah^2$, use the following renormalization conditions

$$\frac{\partial (V^{\text{tree}} + \Delta V^{T=0}_1 + \Delta V^{T=0}_{ct})}{\partial s} \bigg|_{(s, h) = (v_{BL}, v_{ew})} = 0,$$

(3.4)

$$\frac{\partial (V^{\text{tree}} + \Delta V^{T=0}_1 + \Delta V^{T=0}_{ct})}{\partial h} \bigg|_{(s, h) = (v_{BL}, v_{ew})} = 0,$$

(3.5)

to make sure the vacuums at zero temperature won’t be shifted away, we then solve and substitute the energy scale $Q$ and $A$, see the appendix. The temperature correction reads

$$\Delta V^{T\neq 0}(s, h, T) = \sum_F n_F \frac{T^4}{2\pi^2} J_F \left[ \frac{m_F^2(s, h)}{T^2} \right] + \sum_B n_B \frac{T^4}{2\pi^2} J_B \left[ \frac{m_B^2(s, h)}{T^2} \right]$$

(3.6)
with \( J \left[ m_i^2 / T^2 \right] = \int_0^\infty dx x^2 \log \left[ 1 \pm \exp \left( -\sqrt{x^2 + m_i^2 / T^2} \right) \right] \), here plus sign is for fermions and minus sign for bosons. In order to include the ring (or daisy) contribution, we make the following replacements \([11, 20]\) for bosonic masses

\[
m_B^2(s, h) \rightarrow M_B^2(s, h, T) = m_B^2(s, h) + \Pi_B(T),
\]

(3.7) the expressions for the self-energies and the thermal mass (Debye mass) for the longitudinal component of Z boson can be found in the appendix. We substitute the Debye mass terms into Eq.\((3.6)\), finally, the total effective potential of the classic fields is given by

\[
V_{\text{eff}}(s, h, T) = V_{\text{tree}}(s, h) + \Delta V^T_{\text{eff}}(s, h) + \Delta V^T_{\text{eff}}(s, h) + \Delta V^T_{\text{eff}}(s, h, T).
\]

(3.8)

Obtained the effective potential, bubbles’ profile can be found by solving the bounce equation

\[
\frac{d^2 \hat{\phi}}{dr^2} + \frac{\alpha}{r} \frac{d \hat{\phi}}{dr} = \nabla V_{\text{eff}}(\hat{\phi}), \quad \hat{\phi}(r \rightarrow \infty) = \hat{\phi}_F, \quad \frac{d \hat{\phi}}{dr} \bigg|_{r=0} = 0,
\]

(3.9)

where \( r^2 = |\vec{x}|^2 \), \( \alpha = 2 \) at finite temperature, and \( r^2 = |\vec{x}|^2 - t^2 \), \( \alpha = 3 \) at zero temperature. For single field problems, it can be solved with a ‘overshooting-undershooting’ method\([51, 52]\), however the case for multi fields can be much more complicated, in the reference \([38]\) Carroll L. Wainwright creates a powerful method—path deformation, to overcome the problem. In our project we make use of Wainwright’s public available package CosmoTransition \([38]\) to study the cosmological phase transitions.

The strength of phase transitions can be illustrated by the parameter \( \alpha \), which is the ratio between the latent heat and the radiation energy

\[
\alpha \equiv \frac{\rho_{\text{vac}}}{\rho_r} \bigg|_{T=T_*},
\]

(3.10)

where the latent heat is evaluated from \( \rho_{\text{vac}} = [T(d\Delta V_{\text{eff}}/dT) - \Delta V_{\text{eff}}] \bigg|_{T=T_*} \), \( \Delta V_{\text{eff}} \) is the potential difference between the true and false vacuums; the radiation energy density is \( \rho_r = g_* \pi^2 T^4 / 30 \) with \( g_* \) being the relativistic degree of freedoms. The time scale of phase transitions is the inverse of the parameter \( \beta \)

\[
\beta = -\frac{d(S_3/T)}{dt} \bigg|_{t=t_*} \approx \frac{1}{\Gamma} \frac{dT}{dt} \bigg|_{t=t_*},
\]

(3.11)

where the Euclid action \( S_3 = \int d^3 x \left[ (1/2)(\nabla s)^2 + (1/2)(\nabla h)^2 + V_{\text{eff}}(s, h, T) \right] \), \( \Gamma \) is the bubble nucleation rate defined by \( \Gamma = \Gamma_0 \exp (-S_3/T) \). In the actual calculation, a renormalization of \( \beta \) could be quite useful

\[
\tilde{\beta} = \frac{\beta}{H_*} = T_* \frac{d(S_3/T)}{dT} \bigg|_{T=T_*}.
\]

(3.12)

By definition, a larger value of \( \alpha \) means a stronger phase transition, and a larger \( \beta \) means a faster phase transition. The phase transition temperature \( T_* \) is estimated when the bubble nucleation probability in unit time and unit volume \( \simeq 1 \),

\[
1 \simeq \int_0^{t_*} \frac{\Gamma dt}{H^2} = \int_{T_*}^\infty dT \left( \frac{g_{\text{eff}}}{8\pi^2} \right)^2 \left( \frac{M_{\text{pl}}}{T} \right)^4 \exp \left( -S_3/T \right), \quad \text{(13.13)}
\]

- 4 -
Fig. 1. An example of a first order phase transition along the h-direction followed by a higher order phase transition along the s-direction in the contour plot of the effective potential Eq.(3.8). This is given by the parameters $v_{BL} = 5 \, \text{GeV}, \lambda_s = 1, \lambda_{sh} = 0.1, g = 0.6$. Here the h-direction phase transition happen at temperature $T \approx 137.88 \, \text{GeV}$, the s-direction phase transition happen at the temperature $T \approx 10 \, \text{GeV}$.

this gives us a simple criterion to estimate $T_s$:

$$
\frac{S_3}{T_s} \simeq \ln \left[ \frac{1}{4} \left( \frac{90}{8\pi^2 g_{\text{eff}}} \right)^2 \right] + 4 \ln \left[ \frac{M_{\text{pl}}}{T_s} \right].
$$

(3.14)

Notice when we derive this criterion, we have assumed the energy density of the universe is dominated by radiation, so it is not gonna be applicable when $\alpha$ is very large—which means the radiation dominated condition is spoiled—such as the case when bubbles have runaway in vacuum [53].

In the singlet Majoron model, parameters can be divided into two distinguished groups: the SM ones and the beyond-SM ones. For SM parameters, we take the gauge couplings $g_1 = 0.65$ and $g_2 = 0.35$, the heaviest fermion top quark’s Yukawa coupling constant $y_t = 0.989$, the Doublet Higgs boson’s self coupling constant $\lambda_H = 0.13$, today’s electroweak vacuum expected value(vev) $v_{\text{ew}} = 246 \, \text{GeV}$. The beyond-SM parameters are the newly introduced singlet Higgs and the right-handed neutrinos’ self coupling $\lambda_s$, mixing coupling $\lambda_{sh}$, Dirac mass term Yukawa coupling $f$, Majorana mass term Yukawa coupling $g$, and vev of the global symmetry $v_{BL}$. It easily to find the parameter $f$ is degenerated with $g$ and $v_{BL}$, we
Fig. 2. An example of a higher order s-direction phase transition followed by a hybrid first order phase transition. This is given by the parameters $v_{BL} = 1$ GeV, $\lambda_s = 0.1, \lambda_{sh} = 0.175, g = 1.1$. Here the higher order phase transition happen around $T \simeq 175$ GeV, the hybrid phase transition happen at the temperature $T \simeq 86.6$ GeV.

thus won’t set it as a free parameter. Finally, we have four parameters to be constrained: $\lambda_s, \lambda_{sh}, g, v_{BL}$. For the plotting convenience, we will use a numeric string such as ’100-0.8-0.1-0.4’ to represent $v_{BL} = 100$ GeV, $\lambda_s = 0.8, \lambda_{sh} = 0.1, g = 0.4$. In this paper we will scan the following parameter space

$$0 \text{ GeV} \leq v_{BL} \leq 1000 \text{ GeV}, \quad 0 \leq \lambda_s \leq 1, \quad -1 \leq \lambda_{sh} \leq 1, \quad 0 \leq g \leq 2.$$  \hspace{1cm} (3.15)

Since there are two scalar fields, the effective potential Eq.(3.8) is a 2D surface evolves with the changing of temperature. At early universe the temperature is high, symmetries are unbroken, the zero point of the fields space turns to be the only vacuum. As the universe expanding its temperature goes down, structure of the effective potential Eq.(3.8) becomes nontrivial and different local minimums will be formed, among them the lowest one is the true vacuum. Phase transitions can happen by tunneling from the higher vacuum(s) to lower ones, sometimes there are barriers between the distinguished vacuums which corresponds to the first order phase transitions, while at some other times phase transitions can happen very smoothly without any potential barriers being formed, they are referred as higher order phase transitions. In our numerical simulation, we find patterns of phase transitions in the singlet Majoron model can be quite fruitful, usually there are multi-step PTs, some of them are along
the $s$ field direction, some are along the $h$ field direction, and some others are along a mixing direction, and the meanwhile first and/or higher PTs may occurs under the same parameters. Since we are mostly interested in the strong first order PTs, we will show some examples in the figure plots.

In Fig.1 we show an example of a first order phase transition along the $h$-direction followed by a higher order phase transition along the $s$-direction. As the universe cools near to the phase transition temperature $T \approx 137.88$ GeV, there exist multi-vacuums and the tunneling happen along the Higgs direction firstly; another smoother phase transition happen when the temperature goes down to $T \approx 10$ GeV, however it can not be seen in Fig.1 due the smallness of $v_{BL}$. In this example the $h$-direction first order phase transition turns to be too weak to generate detectable GW signals.

In Fig.2 a higher order PT happen around $T \approx 175$ GeV along the $s$-direction, followed by a strong first order PT at the temperature $T \approx 86.6$ GeV. We see as temperature goes down, the original vacuum located at the origin point splitting into two symmetric new vacuums on the axes of Higgs by a higher order phase transition. When the temperature goes even lower, two new vacuums on the $h$ axes are formed, they coexist with the old vacuums until the tunneling happen at $T \approx 86.6$ GeV. The first order PT is very strong, as a matter of
Fig. 4. An example of two first order phase transitions, the first is along the s-direction followed by the second which is along a mixing direction. This is given by parameters $v_{BL} = 100 \text{ GeV}, \lambda_s = 0.3, \lambda_{sh} = 0.2, g = 0.3$. Here the s-direction phase transition happen at temperature $T \approx 364.07 \text{ GeV}$, the hybrid phase transition along the mixing direction happen at $T \approx 122.7 \text{ GeV}$.

fact the GWs generated can be detected by the U-DECIGO detector [54], see the first plot in Fig.5. This strong first order phase transition may be regarded as the generalization analogy to the EWPT in the standard model. However, they are somehow different due to the mixing effect between Higgs and the s field, here we’d like to regard this case as an example of ‘hybrid phase transition’.

Fig.3 shows a first order phase transition along the s-direction followed by a higher order PT along the Higgs direction. This s-direction PT proceeds very fast, leads to very high frequency GWs which go beyond the detectable range of spatial GW interferometers. The final vacuums locate at $(\pm v_{BL}, \pm v_{ew})$ when the temperature gets closed to 0.

– 8 –
It is even more interesting to find multi first order PTs from some parameters, in Fig. 4 we show an example with a h-direction first order PT happen after a s-direction first order PT. Near $T \simeq 364.07$ GeV, there form three vacuums separated by the barriers, one locating at the zero point, the other two are the nonzero ones with opposite signs according to the symmetry. As temperature goes lower, the true vacuum tunnels from the zero to the nonzero ones. Notice here we have employed zoomed-in plots to show the s-direction first PT better, as we have claimed, the PTGWs have too high frequency and too weak amplitude to be detectable. The other first order PT happen to be a hybrid one at $T \simeq 122.7$ GeV, whose GWs are also too weak to be detectable.

Therefore we see the cosmological phase transitions patterns in singlet Majoron model are quite fruitful, which should be the distinguishable feature for 2D problems compared with single field ones. We thus believe the single Majoron model is a useful laboratory to study the properties of cosmological PTs and PTGWs.

4 Phase transition gravitational waves in the singlet Majoron model

There are various GW sources generated during strong first order phase transitions, we will follow the paper Ref. [53] by considering the contributions from the scalar fields, the sound waves [55–57], and the magnetohydrodynamic (MHD) turbulence [58]. The first GW source comes from the vacuum bubbles collision [6, 58, 59], the GW energy spectrum and peak frequency are

$$\Omega_{\text{env}}(f_{\text{env}}) h^2 \simeq 1.67 \times 10^{-5} \left( \frac{1}{\beta} \right)^2 \left( \frac{\kappa_s \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{\frac{1}{2}} \left( \frac{0.11 v_0^3}{0.42 + v_0'} \right)$$

(4.1)

$$f_{\text{env}} \simeq 1.65 \times 10^{-5} \text{Hz} \left( \frac{0.62}{1.8 - 0.1 v_0' + v_0''} \right) \tilde{\beta} \left( \frac{T_s}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{2}},$$

(4.2)

this experience formulae are summarized from numerical simulations with envelope approximation (also see the work with analytic derivation [60, 61] or beyond the envelope approximation [62, 63]). The second source is generated by the sound waves of the bulk motion [55–57]

$$\Omega_{\text{sw}}(f_{\text{sw}}) h^2 \simeq 2.65 \times 10^{-6} \left( \frac{1}{\beta} \right) \left( \frac{\kappa_s \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{\frac{1}{2}} v_0' \left[ \frac{7 (f/f_{\text{sw}})^{\frac{3}{2}}}{4 + 3 (f/f_{\text{sw}})^{\frac{3}{2}}} \right]^{\frac{1}{2}},$$

(4.3)

$$f_{\text{sw}} \simeq 1.9 \times 10^{-5} \text{Hz} \left( \frac{1}{v_0} \right) \tilde{\beta} \left( \frac{T_s}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{2}}.$$  

(4.4)

The third are GWs generated by MHD turbulence [58] with energy spectrum and peak frequency

$$\Omega_{\text{tu}}(f_{\text{tu}}) h^2 \simeq 3.35 \times 10^{-4} \left( \frac{1}{\beta} \right) \left( \frac{\kappa_{tu} \alpha}{1 + \alpha} \right)^{\frac{3}{2}} \left( \frac{100}{g_*} \right)^{\frac{1}{2}} v_0' \frac{(f/f_{\text{tu}})^{\frac{3}{2}}}{\left[ 1 + (f/f_{\text{tu}}) \right]^{\frac{1}{2}}} \left( 1 + 8 \pi f/h_* \right)$$

(4.5)

$$f_{\text{tu}} \simeq 2.7 \times 10^{-5} \text{Hz} \left( \frac{1}{v_0} \right) \tilde{\beta} \left( \frac{T_s}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{2}}.$$  

(4.6)
Fig. 5. Gravitational wave signals from the model parameters with \( v_{BL} \leq v_{ew} = 246 \) GeV, compared with the detectability of future spatial interferometers LISA, DECIGO, BBO, TAIJI and TianQin. The benchmark energy scales are taken \( v_{BL} = 1 \) GeV, 5 GeV, 10 GeV, 50 GeV, 100 GeV, 246 GeV, with the notation \( 'v_{BL} - \lambda_e - \lambda_{th} - g' \) in the plots (here 'm0.3' refers to \(-0.3\)). The solid lines refer to GWs from h-direction or hybrid first order PTs, the dotted lines refer to those from s-direction PTs. For parameters which lead to twice first order PTs, we have use suffixes (h) or (s) to make it clearer.

where \( h_\nu = 1.65 \times 10^{-5} \text{Hz} (T_\nu/100 \text{ GeV}) (g_\nu/100)^{\frac{1}{4}} \).

In Ref. [53] the authors discuss three different kinds of bubbles, i.e. the non-runaway bubbles, the runaway in plasma bubbles, and the runaway in vacuum bubbles. In this paper
Fig. 6. Gravitational wave signals from the model parameters with $v_{BL} > v_w = 246$ GeV, compared with the detectability of future spatial interferometers LISA, DECIGO, BBO, TAIJI and TianQin. The benchmark scales are taken $v_{BL} = 500$ GeV, 750 GeV, 1000 GeV, with the notation ‘$v_{BL} – \lambda_s – \lambda_{th} – g’$ in the plots (here ‘m0.3’ refers to $–0.3$). The solid lines refer to GWs from h-direction or hybrid first order PTs, the dotted lines refer to GWs from s-direction PTs. For parameters which lead to twice first order PTs, we have used the tracer suffixes (h) or (s).

we will follow them directly. For non-runaway bubbles, we evaluate the total GW energy spectrum from $\Omega_{sw}(f_{sw})h^2 + \Omega_{tu}(f_{tu})h^2$, take the efficiency factor $\kappa_{sw} = \alpha (0.73 + 0.083\sqrt{\alpha} + \alpha)^{–1}$ when the bubble wall velocity is quite close to 1, or $\kappa_{sw} = v_w 6.9\alpha (1.36 – 0.037\sqrt{\alpha} + \alpha)^{–1}$ when $v_w$ is smaller than 0.1; we take the turbulence efficiency factor $\kappa_{tu} = 0.05\kappa_{sw}$. For the case of runaway bubbles in plasma, the contributions from s field itself is also included, the smallest $\alpha$ in this case can be found by calculating

$$\alpha_\infty \simeq \frac{30}{24\pi^2} \sum_b c_b \Delta m_b^2(s, h_s) / g_{eff} T_s^4$$

where the squared mass difference between the two phases can be read from Eq.(A.1), the coefficients $c_b$ satisfy $(c_H, c_G, c_P, c_X, c_W, c_Z, c_t, c_{v_R}) = (1, 3, 1, 1, 6, 3, 6, 3)$. The efficiency factor is evaluated from $\kappa_{sw} = (\alpha_\infty / \alpha)\kappa_{\infty}$, with $\kappa_{\infty} = \alpha_\infty / (0.73 + 0.083\sqrt{\alpha_\infty} + \alpha_\infty)$. When $\phi_s / T_s$ is very large, the phase transition may haven’t happened until the universe is super-cooled, which could be quite different from the non-runaway or the runaway in plasma cases. In our paper, we will not deal with runaway in vacuum bubbles, since the criterion
of $T_*$ Eq.(3.14) is spoiled when the total energy density of the universe is dominated by the vacuum energy.

We scan the parameter space in Eq.(3.15) by splitting $v_{BL}$ into two groups: $v_{BL} \leq v_{ew}$ and $v_{BL} > v_{ew}$. As there are four free parameters $v_{BL}$, $\lambda_s$, $\lambda_{sh}$, $g$, we will firstly fix the value of $v_{BL}$ as the benchmark energy scale, then increase the other parameters from their minimum to their maximum by a step length each time. For parameters which have the the same values of $(v_{BL}, \lambda_s)$, we pick out the ones which can produce the largest GW signals and display their GW signals in Fig.5 and Fig.6 by confronting them with the space-borne interferometers such as LISA, DECIGO, BBO, TAIJI and TianQin. Since there are so many curve lines in the plots we also take a unique notation ‘$v_{BL} - \lambda_s - \lambda_{sh} - g$’ to save the plots space.

Fig.5 shows the GWs from the singlet Majoron model with $v_{BL} \leq v_{ew}$, where the benchmarks are chose to be $v_{BL} = 1$ GeV, 5 GeV, 10 GeV, 50 GeV, 100 GeV, 246 GeV. It is clear that the detectable GW signals appears only when the $v_{BL}$ is small (smaller than $\sim O(10)$ GeV), on the other hand most of the GW curves are untouchable by interferometers since their amplitudes are not large enough while their peak frequencies are too high. By considering the different patterns of PTs discussed in Sec.3, we remarkably find the fact that all the detectable GWs belongs to the PTs along mixing directions, i.e. the hybrid phase transitions (HPTs). To get a better understanding about these HPTs, let’s have a closer look at Fig.2, at the phase transition temperature $T_* \simeq 86.6$ GeV the PT happens due to the tunneling from the false vacuum locates at $(232$ GeV, $-5.43 \times 10^{-6}$ GeV) to the true vacuum locates at $(6.62 \times 10^{-6}$ GeV, 223 GeV), giving a $\phi_* / T_* \simeq \sqrt{232^2 + 223^2} / 86.6 \simeq 3.72$ which undoubtedly shows a very strong first order phase transition. Other than HPTs, usually the first order PTs along h or s directions turns to produce GWs which are hardly detect by future interferometers. In our simulation, the strongest GW signals are given by the parameters $v_{BL} = 1$, $\lambda_s = 0.1$, $\lambda_{sh} = 0.2$, $g = 1.1$, which has reached the detectivities of LISA, DECIGO, BBO and TAIJI. However, this sample may be quite closed to the parameters which can generate runaway in vacuum bubbles, as when we adjust parameters around it we continually meet very large value of $\phi_*/T_*$ (larger than $\sim O(10)$) and spoilings of the criterion Eq.(3.14). Since we haven’t dealt with the bubbles runaway in vacuum properly in this work, we may have lost some parameters which can generated detectable GW signals.

Fig.6 shows GWs from vev $v_{BL} > v_{ew}$, where benchmarks energy scales are taken $v_{BL} = 500$ GeV, 750 GeV, 1000 GeV. This time there exist none detectable signals, all the GWs are too faint while their frequencies are too high. And for larger a $v_{BL}$, first order PTs along the s-direction tends to generate stronger GWs compared with the those from PTs along h-direction.

5 Conclusion

In this paper, we have studied cosmological phase transitions and gravitational waves in the singlet Majoron model, by treating it as a two-field problem fully without freezing any one of them. We confirm the phase transition patterns properties have been found in Refs. [35, 36], i.e. usually the EWPT happens after the global $U(1)$ symmetry breaking PT, further more we also find other patterns with different orders or along distinguished directions. Thanks to the 2D structure of the effective potential, PTs in the singlet Majoron model are much more fruitful compared with the ones in single field problems.
We further study the GWs generated during first order phase transitions, by separating the parameters with $v_{BL}$ which is lower and greater than electroweak $v_{ew}$, we have scanned the parameter space, calculate and confront the GW signals with future space-borne interferometers such as LISA, DECIGO, BBO, TAIJI and TianQin. We find there could be astronomically interested GW signals, most of them are coming from first order phase transitions along the Higgs direction or a mixing direction, the space-borne GW interferometers may have chance to touch the parameters with small $v_{BL}$ such as smaller than $\sim O(10)$ GeV. For parameters with larger $v_{BL} > v_{ew}$ the GWs from s-direction PTs become the domination source, however, they are too faint and with too high frequency to be detected.

We’d like to point out the fact that in this paper we just consider the bubbles which are non-runaway or runaway in plasma in Ref. [53], while the runaway in vacuum bubbles (see [64]) are abandoned, for whom the criterion Eq.(3.14) is spoiled. By doing so we may lost some parameters which may lead to detectable GWs, we will leave this for future’s work.

Acknowledgments

We are grateful to Andrea Addazi, James M. Cline, Ryusuke Jinno and Antonino Marciano for valuable discussions and comments. YPW would like to thank Carroll L. Wainwright for useful discussion about the package of CosmoTransitions. YPW is supported in part by the Fundamental Research Funds for the Central Universities (WK2030220018) and the China Postdoctoral Science Foundation (2017M621999). BI and YC are supported in part by the Chinese National Youth Thousand Talents Program, by the NSFC (Nos. 11722327, 11653002, 11421303, J1310021), by the CAST Young Elite Scientists Sponsorship Program (2016QNRC001), and by the Fundamental Research Funds for the Central Universities. Part of numerical simulations are operated on the computer cluster LINDA in the particle cosmology group at USTC.

A Mass spectrum, cut-off, and Debye masses

The mass spectrum can be found as

\[ m_{pp}^2(s, h) = \lambda_s (3s^2 - v_{BL}^2) + \frac{1}{2} \lambda_{sh} (h^2 - v_{ew}^2); \tag{A.1} \]
\[ m_{\chi\chi}^2(s, h) = \lambda_s (s^2 - v_{BL}^2) + \frac{1}{2} \lambda_{sh} (h^2 - v_{ew}^2); \tag{A.2} \]
\[ m_{HH}^2(s, h) = \lambda_h (3h^2 - v_{ew}^2) + \frac{1}{2} \lambda_{sh} (s^2 - v_{BL}^2); \tag{A.3} \]
\[ m_{GG}^2(s, h) = \lambda_h (h^2 - v_{ew}^2) + \frac{1}{2} \lambda_{sh} (s^2 - v_{BL}^2); \tag{A.4} \]
\[ m_{\nu_R}^2(s, h) = \frac{1}{2} g_1^2 s^2; \quad m_{t}^2(s, h) = \frac{1}{2} g_t^2 h^2; \tag{A.5} \]
\[ m_{W}^2(s, h) = \frac{1}{4} g_1^2 h^2; \quad m_{Z}^2(s, h) = \frac{1}{4} (g_1^2 + g_2^2) h^2; \tag{A.6} \]
\[ m_{\rho H}^2(s, h) = \lambda_{sh} s h; \tag{A.7} \]

in which the ‘h’ and ‘G’ are the Higgs Boson and Goldstones of the Higgs doublet; and we can see when $s = v_{BL}$, $m_{\chi}^2 = 0$, so the mass of the Goldstone of $\sigma$ is vanishing when there are
no higher dimensional effective terms. We can diagonalize the mass matrix of neutral bosons, and thus redefine the $\rho$ particle and the Higgs particle

$$m_{1,2}^2 = \frac{1}{2} \left[ m_{\rho}^2 + m_{H}^2 \right] \pm \frac{1}{2} \sqrt{\left( m_{\rho}^2 - m_{H}^2 \right)^2 + 4(m_{\rho})^2}. \quad (A.8)$$

Consider a counter term

$$\Delta V^{T=0}_{ct}(s,h) = A h^2, \quad (A.9)$$

from the renormalization conditions Eq.(3.4), we can get the energy scale cut-off and coefficient $A$

$$\log Q^2 = \left( \sum_i n_i m_i^2 \frac{\partial m_i^2}{\partial s} \right)^{-1} \left[ \sum_i n_i m_i^2 \frac{\partial m_i^2}{\partial s} \left( \log m_i^2 - c_i + \frac{1}{2} \right) \right] \bigg|_{(s,h) = (v_{BL}, v_{ew})} \quad (A.10)$$

$$A = - \frac{1}{64\pi^2 h} \left[ \sum_i n_i m_i^2 \frac{\partial m_i^2}{\partial h} \left( \log m_i^2 - \log Q^2 - c_i + \frac{1}{2} \right) \right] \bigg|_{(s,h) = (v_{BL}, v_{ew})} \quad (A.11)$$

where \((c_i, c_{W_T}, c_{Z_T}) = (1.5, 0.5, 0.5)\).

The self-energies are given by

$$\Pi_{\rho}(T) = \Pi_{\chi}(T) = \left( \frac{1}{3} \lambda_s + \frac{1}{6} \lambda_{sh} + \frac{1}{8} g^2 \right) T^2, \quad (A.12)$$

$$\Pi_{H}(T) = \Pi_{G}(T) = \left[ \frac{1}{16} \left( 3g_1^2 + g_2^2 \right) + \frac{1}{2} \lambda_h + \frac{1}{4} g_1^2 + \frac{1}{12} \lambda_{sh} \right] T^2, \quad (A.13)$$

$$\Pi_{W_L}(T) = \frac{11}{6} g_1^2 T^2, \quad \Pi_{W_T}(T) = \Pi_{Z_T}(T) = \Pi_{\gamma_T}(T) = 0, \quad (A.14)$$

the Debye mass for the longitudinal component of Z boson is

$$M_{Z_L}^2(s,h,T) = \frac{1}{2} \left[ m_Z^2(s,h) + \frac{11}{6} \frac{g_1^2}{\cos^2 \theta_w} T^2 + \Delta(s,h,T) \right], \quad (A.15)$$

$$\Delta(s,h,T) = \left[ m_Z^2(s,h) + \frac{11}{3} \frac{g_1^2 \cos^2 2\theta_w}{\cos^2 \theta_w} \left( m_Z^2(s,h) + \frac{11}{12} \frac{g_1^2}{\cos^2 \theta_w} T^2 \right) T^2 \right]^{\frac{1}{2}}. \quad (A.16)$$

References

[1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, “On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe,” Phys. Lett. 155B, 36 (1985).

[2] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], “Observation of Gravitational Waves from a Binary Black Hole Merger,” Phys. Rev. Lett. 116, no. 6, 061102 (2016) [arXiv:1602.03837 [gr-qc]].

[3] E. Witten, “Cosmic Separation of Phases,” Phys. Rev. D 30, 272 (1984).

[4] C. J. Hogan, “Gravitational radiation from cosmological phase transitions,” Mon. Not. Roy. Astron. Soc. 218, 629 (1986).

[5] A. Kosowsky, M. S. Turner and R. Watkins, “Gravitational radiation from colliding vacuum bubbles,” Phys. Rev. D 45, 4514 (1992).
[6] M. Kamionkowski, A. Kosowsky and M. S. Turner, “Gravitational radiation from first order phase transitions,” Phys. Rev. D 49, 2837 (1994) [astro-ph/9310044].

[7] K. Kajantie, K. Rummukainen and M. E. Shaposhnikov, “A Lattice Monte Carlo study of the hot electroweak phase transition,” Nucl. Phys. B 407, 356 (1993) [hep-ph/9305345].

[8] Z. Fodor, J. Hein, K. Jansen, A. Jaster and I. Montvay, “Simulating the electroweak phase transition in the SU(2) Higgs model,” Nucl. Phys. B 439, 147 (1995) [hep-lat/9409017].

[9] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, “The Electroweak phase transition: A Nonperturbative analysis,” Nucl. Phys. B 466, 189 (1996) [hep-lat/9510020].

[10] G. W. Anderson and L. J. Hall, “The Electroweak phase transition and baryogenesis,” Phys. Rev. D 45, 2685 (1992).

[11] J. R. Espinosa and M. Quiros, “The Electroweak phase transition with a singlet,” Phys. Lett. B 305, 98 (1993) [hep-ph/9301285].

[12] J. R. Espinosa and M. Quiros, “Novel Effects in Electroweak Breaking from a Hidden Sector,” Phys. Rev. D 76, 076004 (2007) [hep-ph/0701145].

[13] S. Profumo, M. J. Ramsey-Musolf and G. Shaughnessy, “Singlet Higgs phenomenology and the electroweak phase transition,” JHEP 0708, 010 (2007) [arXiv:0705.2425 [hep-ph]].

[14] F. P. Huang and C. S. Li, “Electroweak baryogenesis in the framework of the effective field theory,” Phys. Rev. D 92, no. 7, 075014 (2015) [arXiv:1507.08168 [hep-ph]].

[15] X. m. Zhang, “Operators analysis for Higgs potential and cosmological bound on Higgs mass,” Phys. Rev. D 47, 3065 (1993) [hep-ph/9301277].

[16] C. Grojean, G. Servant and J. D. Wells, “First-order electroweak phase transition in the standard model with a low cutoff,” Phys. Rev. D 71, 036001 (2005) [hep-ph/0407019].

[17] C. Delaunay, C. Grojean and J. D. Wells, “Dynamics of Non-renormalizable Electroweak Symmetry Breaking,” JHEP 0804, 029 (2008) [arXiv:0711.2511 [hep-ph]].

[18] F. P. Huang, Y. Wan, D. G. Wang, Y. F. Cai and X. Zhang, “Hearing the echoes of electroweak baryogenesis with gravitational wave detectors,” Phys. Rev. D 94, no. 4, 041702 (2016) [arXiv:1601.01640 [hep-ph]].

[19] R. G. Cai, M. Sasaki and S. J. Wang, “The gravitational waves from the first-order phase transition with a dimension-six operator,” JCAP 1708, no. 08, 004 (2017) [arXiv:1707.03001 [astro-ph.CO]].

[20] J. M. Cline and P. A. Lemieux, “Electroweak phase transition in two Higgs doublet models,” Phys. Rev. D 55, 3873 (1997) [hep-ph/9609240].

[21] G. C. Dorsch, S. J. Huber and J. M. No, “A strong electroweak phase transition in the 2HDM after LHC8,” JHEP 1310, 029 (2013) [arXiv:1305.6610 [hep-ph]].

[22] P. Basler, M. Krause, M. Mühlleitner, J. Wittbrodt and A. Wlotzka, “Strong First Order Electroweak Phase Transition in the CP-Conserving 2HDM Revisited,” JHEP 1702, 121 (2017) [arXiv:1612.04086 [hep-ph]].

[23] P. Basler, M. Mühlleitner and J. Wittbrodt, “The CP-Violating 2HDM in Light of a Strong First Order Electroweak Phase Transition and Implications for Higgs Pair Production,” JHEP 1803, 061 (2018) [arXiv:1711.04097 [hep-ph]].

[24] R. Aprea, M. Maggiori, A. Nicolis and A. Riotto, “Gravitational waves from electroweak phase transitions,” Nucl. Phys. B 631, 342 (2002) [gr-qc/0107033].

[25] S. J. Huber and T. Konstandin, “Production of gravitational waves in the nMSSM,” JCAP 0805, 017 (2008) [arXiv:0709.2091 [hep-ph]].
[26] S. J. Huber, T. Konstandin, G. Nardini and I. Rues, “Detectable Gravitational Waves from Very Strong Phase Transitions in the General NMSSM,” JCAP 1603 (2016) no.03, 036 [arXiv:1512.06557 [hep-ph]].

[27] P. Huang, A. J. Long and L. T. Wang, “Probing the Electroweak Phase Transition with Higgs Factories and Gravitational Waves,” Phys. Rev. D 94, no. 7, 075008 (2016) [arXiv:1608.06619 [hep-ph]].

[28] P. Schwaller, “Gravitational Waves from a Dark Phase Transition,” Phys. Rev. Lett. 115 (2015) no.18, 181101 [arXiv:1504.07263 [hep-ph]].

[29] W. Chao, H. K. Guo and J. Shu, “Gravitational Wave Signals of Electroweak Phase Transition Triggered by Dark Matter,” JCAP 1709, no. 09, 009 (2017) [arXiv:1702.02698 [hep-ph]].

[30] Y. Chen, M. Huang and Q. S. Yan, “Gravitation waves from QCD and electroweak phase transitions,” arXiv:1712.03470 [hep-ph].

[31] Y. Chikashige, R. N. Mohapatra and R. D. Peccei, “Are There Real Goldstone Bosons Associated with Broken Lepton Number?,” Phys. Lett. 98B (1981) 265.

[32] G. B. Gelmini and M. Roncadelli, “Left-Handed Neutrino Mass Scale and Spontaneously Broken Lepton Number,” Phys. Lett. 99B (1981) 411.

[33] Y. Kondo, I. Umemura and K. Yamamoto, “First order phase transition in the singlet Majoron model,” Phys. Lett. B 263, 93 (1991).

[34] N. Sei, I. Umemura and K. Yamamoto, “Constraints on the electroweak phase transition in the singlet majoron model,” Phys. Lett. B 299, 286 (1993).

[35] K. Enqvist, K. Kainulainen and I. Vilja, “Phase transitions in the singlet majoron model,” Nucl. Phys. B 403, 749 (1993).

[36] J. M. Cline, G. Laporte, H. Yamashita and S. Kraml, “Electroweak Phase Transition and LHC Signatures in the Singlet Majoron Model,” JHEP 0907, 040 (2009) [arXiv:0905.2559 [hep-ph]].

[37] A. Addazi, Y. F. Cai and A. Marciano, “Testing Dark Matter Models with Radio Telescopes in light of Gravitational Wave Astronomy,” arXiv:1712.03798 [hep-ph].

[38] C. L. Wainwright, “CosmoTransitions: Computing Cosmological Phase Transition Temperatures and Bubble Profiles with Multiple Fields,” Comput. Phys. Comm. 183, 2006 (2012) [arXiv:1109.4189 [hep-ph]].

[39] H. Audley et al., “Laser Interferometer Space Antenna,” arXiv:1702.00786 [astro-ph.IM].

[40] N. Seto, S. Kawamura and T. Nakamura, “Possibility of direct measurement of the acceleration of the universe using 0.1-Hz band laser interferometer gravitational wave antenna in space,” Phys. Rev. Lett. 87, 221103 (2001) [astro-ph/0108011].

[41] S. Kawamura et al., “The Japanese space gravitational wave antenna DECIGO,” Class. Quant. Grav. 23, S125 (2006).

[42] S. Kawamura et al., “The Japanese space gravitational wave antenna: DECIGO,” Class. Quant. Grav. 28, 094001 (2011).

[43] V. Corbin and N. J. Cornish, “Detecting the cosmic gravitational wave background with the big bang observer,” Class. Quant. Grav. 23, 2435 (2006) [gr-qc/0512039].

[44] X. Gong et al., “Descop of the ALIA mission,” J. Phys. Conf. Ser. 610, no. 1, 012011 (2015) [arXiv:1410.7296 [gr-qc]].

[45] J. Luo et al. [TianQin Collaboration], “TianQin: a space-borne gravitational wave detector,” Class. Quant. Grav. 33, no. 3, 035010 (2016) [arXiv:1512.02076 [astro-ph.IM]].

[46] X. C. Hu et al., “Fundamentals of the orbit and response for TianQin,” Class. Quant. Grav. 35, no. 9, 095008 (2018) [arXiv:1803.03368 [gr-qc]].
[47] E. K. Akhmedov, Z. G. Berezhiani, R. N. Mohapatra and G. Senjanovic, “Planck scale effects on the majoron,” Phys. Lett. B 299 (1993) 90 [hep-ph/9209285].

[48] C. Patrignani et al. [Particle Data Group], “Review of Particle Physics,” Chin. Phys. C 40, no. 10, 100001 (2016).

[49] M. Fukugita and T. Yanagida, “Sphaleron Induced Baryon Number Nonconservation and a Constraint on Majorana Neutrino Masses,” Phys. Rev. D 42, 1285 (1990).

[50] J. M. Cline, K. Kainulainen and K. A. Olive, “Constraints on majoron models, neutrino masses and baryogenesis,” Astropart. Phys. 1 (1993) 387 [hep-ph/9304229].

[51] S. R. Coleman, “The Fate of the False Vacuum. 1. Semiclassical Theory,” Phys. Rev. D 15 (1977) 2929 Erratum: [Phys. Rev. D 16 (1977) 1248].

[52] C. G. Callan, Jr. and S. R. Coleman, “The Fate of the False Vacuum. 2. First Quantum Corrections,” Phys. Rev. D 16 (1977) 1762.

[53] C. Caprini et al., “Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions,” JCAP 1604, no. 04, 001 (2016) [arXiv:1512.06239 [astro-ph.CO]].

[54] H. Kudoh, A. Taruya, T. Hiramatsu and Y. Himemoto, “Detecting a gravitational-wave background with next-generation space interferometers,” Phys. Rev. D 73, 064006 (2006) [gr-qc/0511145].

[55] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, “Gravitational waves from the sound of a first order phase transition,” Phys. Rev. Lett. 112, 041301 (2014) [arXiv:1304.2433 [hep-ph]].

[56] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, “Numerical simulations of acoustically generated gravitational waves at a first order phase transition,” Phys. Rev. D 92, no. 12, 123009 (2015) [arXiv:1504.03291 [astro-ph.CO]].

[57] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, “Shape of the acoustic gravitational wave power spectrum from a first order phase transition,” Phys. Rev. D 96, no. 10, 103520 (2017) [arXiv:1704.05871 [astro-ph.CO]].

[58] A. Nicolis, “Relic gravitational waves from colliding bubbles and cosmic turbulence,” Class. Quant. Grav. 21, L27 (2004) [gr-qc/0303084].

[59] S. J. Huber and T. Konstandin, “Gravitational Wave Production by Collisions: More Bubbles,” JCAP 0809, 022 (2008) [arXiv:0806.1828 [hep-ph]].

[60] C. Caprini, R. Durrer and G. Servant, “Gravitational wave generation from bubble collisions in first-order phase transitions: An analytic approach,” Phys. Rev. D 77, 124015 (2008) [arXiv:0711.2593 [astro-ph]].

[61] R. Jinno and M. Takimoto, “Gravitational waves from bubble collisions: An analytic derivation,” Phys. Rev. D 95, no. 2, 024009 (2017) [arXiv:1605.01403 [astro-ph.CO]].

[62] D. Cutting, M. Hindmarsh and D. J. Weir, “Gravitational waves from vacuum first-order phase transitions: from the envelope to the lattice,” arXiv:1802.05712 [astro-ph.CO].

[63] R. Jinno and M. Takimoto, “Gravitational waves from bubble dynamics: Beyond the Envelope,” arXiv:1707.03111 [hep-ph].

[64] D. Bodeker and G. D. Moore, “Can electroweak bubble walls run away?,” JCAP 0905, 009 (2009) [arXiv:0903.4099 [hep-ph]].