Energy Efficient Resource Allocation for Time-Varying OFDMA Relay Systems with Hybrid Energy Supplies

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Abstract—This paper investigates the energy efficient resource allocation for orthogonal frequency division multiple access (OFDMA) relay systems, where the system is supplied by the conventional utility grid and a renewable energy generator equipped with a storage device. The optimal usage of radio resource depends on the characteristics of the renewable energy generation and the mobile traffic, which exhibit both temporal and spatial diversities. Lyapunov optimization method is used to decompose the problem into the joint flow control, radio resource allocation and energy management without knowing a priori knowledge of system statistics. It is proven that the proposed algorithm can result in close-to-optimal performance with capacity limited data buffer and storage device. Simulation results show that the flexible tradeoff between the system utility and the conventional energy consumption can be achieved. Compared with other schemes, the proposed algorithm demonstrates better performance.

Index Terms—Cooperative relay, OFDMA, renewable energy, stochastic optimization.

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) is regarded as a promising technology in the future wireless communications, due to its advantages of high spectral efficiency and resistance to multipath fading. On the other hand, cooperative relaying is able to enhance the transmission range, system capacity and reliability. Thus, the relay-assisted OFDMA system has been attracting intensive research interests in both academia and industry since it can provide cooperative diversity, channel diversity and multiuser diversity gains at the same time.

Although the transmission quality of wireless systems can be improved with cooperative communications, the new challenges due to the dramatic increase of power cost and traffic demands in wireless communications have been increasingly prominent. About half of the mobile service provider’s annual operating expenses are power costs and base stations (BSs) consume over 80% of the total energy of the network as claimed by Fehske et al. [1] and Feng et al. [2]. Moreover the largest fraction of greenhouse gas emissions caused by radio networks come from access networks, and mostly from BSs. Then the needs for energy saving of radio networks advocate green radio communications. Green radio communications will not only help mobile operators to enhance the energy efficiency of BSs but also reduce the emissions of CO₂ by using renewable energy sources (e.g., wind and solar power) to supply wireless communications in addition to the constant alternating current (AC) from utility grid, e.g., the works by Ho et al. [3] and Niyato et al. [4]. As to the relay-assisted transmission powered by renewable energy source, like the idea of demand response in [3]-[6] the relay selection and radio resource allocation should be adaptive to the variations in harvested renewable energy in order to ensure the quality of service (QoS) of mobile users with the minimum AC energy. In addition, the joint design of relay selection, sub-carrier allocation and power control in relay-assisted OFDMA networks is a challenge since all the considered variables are coupled together, let alone the mentioned time-varying renewable energy generation.

In this paper, we consider a downlink OFDMA relay-assisted network consisting of multiple relay stations and a BS, which is powered by hybrid energy, i.e., renewable energy and constant AC from the grid. Although the renewable energy provided by harvester is free, it is intermittent over time due to time-varying solar and wind patterns. In order to smooth the uncertain supply of renewable source, an energy storage device is equipped with the BS to buffer the intermittent renewable energy generation. Since the capacity of energy storage device and data buffer at the BS are finite, it is necessary to propose a joint design in order to coordinate the downlink transmission, radio resource allocation and energy management of the storage device in order to maximize the throughput utility with the least grid energy in a long-term manner. In summary, the contributions of this paper are as follows:

- From the perspective of network operator, we formulate a stochastic optimization problem to keep a balance between the throughput utility and on-grid energy consumption, which are usually two conflicting objectives.
- An online algorithm FREE is proposed to regulate the downlink data requests, allocate radio resource and manage energy in the storage device under the constraints of finite buffer size and storage capacity.
- The proposed FREE can arbitrarily approach the optimal tradeoff between system throughput and on-grid energy consumption by theoretic analysis and simulation results.

The rest of this paper is organized as follows. Section II
presents a review of related works. Section III describes the system model and the components of the system. Section IV casts the problem into a stochastic optimization problem. Section V presents an online algorithm to jointly allocate resource and manage energy. Section VI analyzes the performance of the proposed algorithm. Simulations are provided in Section VII. The paper is concluded in Section VIII.

II. RELATED WORKS

Since this paper considers the scenario of relay-based cooperative networks with hybrid energy supply, the related works on cooperative communications are first reviewed and then some latest results on renewable energy powered communications are discussed.

Various cooperative communication schemes such as Amplify-and-Forward (AF) and Decode-and-Forward (DF) have been proposed in the literature [9]–[21]. Compared with a DF relay decoding, re-modulating and retransmitting the received signal, an AF one simply amplifies and retransmits the signal towards destinations without decoding. Wang et al. in [10] and Shen et al. in [11] proposed power control for fairness in AF relay networks. The power control of multiple users with AF relays are considered by Zappone et al. in [12] and by Cheung et al. in [13] for energy-efficiency. Depending on the channel condition and performance criterion, DF may outperform AF, or vice versa. For DF relay strategy, [13]–[18] considered the resource allocation by solving a class of static optimization problem. The joint design of congestion control and relay resource allocation for LTE-A was proposed in [19] by solving a stochastic optimization problem. Tang et al. in [20] proposed three greedy algorithms for multiuser multiple relaying networks to maximize the network utility and comprehensive theoretical analysis was given to measure the worst performance. Different from the assumption of half-duplex at relay, Li et al. in [21] proposed a full-duplex cooperative communication scheme. In this paper, the energy-aware relay-assisted transmission aims to balance the throughput utility and on-grid energy consumption. Since the transmission power of relays is coupled with the power allocation of the BS, which is supplied by hybrid energy, the relay power allocation, transmission mode selection (cooperative transmission or direct transmission) and subcarrier allocation are all affected by queue states, residual energy in storage device. This makes the radio resource allocation problem significantly different from the aforementioned works.

To reduce the consumption of on-grid energy in wireless access networks, there are some works considering that the transmitter is powered by hybrid energy source [22]–[24]. This paper considers that the transmitter is also powered by hybrid energy source in a relay-assisted cooperative communication scenario, where an online joint resource allocation and energy management algorithm is proposed with lower computational complexity compared with the dynamic programming based ones in [22], [23]. Compared against [24] the salient feature of the proposed algorithms in this paper is that its implementation is independent of the energy harvesting profiles. For the delay/throughput optimal transmission with renewable energy sources, the schemes on transmission power control and energy management in battery for point-to-point transmissions were proposed in [25]–[27]. The network planning problem with sustainable energy sources in OFDMA relay networks was considered by Zheng et al. in [28] to ensure network connectivity and users’ QoS requirements. To further reduce the circuit energy consumption of cellular networks, the BSs or relay stations (RSs) can be switched into sleep mode dynamically by utilizing the spatial and temporal fluctuations of traffic loads and inter-cell cooperation [29]–[33]. The most related work also includes [34], where Ahmed et al. consider power control for a cooperative system consisting of one energy harvesting (EH) source and one EH relay communicating to one destination. Almost all the above works assume that either the data buffer or the storage device is infinite to make their problem tractable. Also the prior knowledge on random events is needed in their designs. In this paper, the trade-off between throughput utility and grid energy consumption is considered for multi-user/multi-relay networks under the constraints of finite data and energy buffer. An online algorithm is designed without relying on any statistical information.

III. SYSTEM MODEL

A. Physical Layer Model

We consider the downlink transmission in a two-hop relay-assisted network, which consists of a set of users $\mathcal{N} = \{1, 2, \cdots, N\}$ indexed by $n$, a set of RSs $\mathcal{R} = \{1, 2, \cdots, K\}$ indexed by $i$ and one BS indexed by $B$. OFDMA is employed over both hops and the whole bandwidth $W$ is equally divided into $M$ subcarriers with sub-bandwidth $b = \frac{W}{M}$. The set of subcarriers is $\mathcal{M} = \{1, 2, \cdots, M\}$ and the subcarrier is indexed by $m$. The noise at each receiver on each subcarrier is zero-mean circular symmetric complex Gaussian with variance $bN_0$. The system is assumed to be a time-slotted one with equal time interval normalized to be unit time. Hereafter $b$ is assumed to be one without loss of generality.

The BS can transmit directly to each user or employ one of the RSs for cooperative communications. If the cooperative communication is adopted, each transmission consists of two stages. In the first stage, the BS communicates with RSs and users on some subcarriers. In the second stage, the selected RS on the same subcarrier helps the BS transmit to the user with decode-and-forward (DF) strategy with the same codebook as that of the BS. Let $\{\alpha_{i,n}^m : \alpha_{i,n}^m \in \{0, 1\}, \forall m \in \mathcal{M}, i \in \mathcal{R}, n \in \mathcal{N}\}$ denote the set of RS, user and subcarrier matching indicator, where $\alpha_{i,n}^m = 1$ indicates that the BS can communicate with user $n$ via relay $i$ over the $m$th subcarrier, and $\alpha_{i,n}^m = 0$ otherwise. Note that $\beta_{n}^m = 1$ means that the BS will communicate to user $n$ over subcarrier $m$ directly and $\beta_{n}^m = 0$ otherwise. An exclusive subcarrier assignment is enforced such that at most one user or one user-relay pair can be allocated to a single subcarrier. Also, on one subcarrier,
only one transmission mode (cooperative or direct) is allowed. These constraints are as follows, \( \forall n \in \mathcal{N}, i \in \mathcal{R}, m \in \mathcal{M} \):

\[
\sum_{m} \sum_{i \in \mathcal{R}} \left( \alpha_{i,n}^m + \alpha_{B,n}^m \right) \leq 1, \quad \alpha_{i,n}^m \in \{0, 1\}, \alpha_{B,n}^m \in \{0, 1\}. \tag{1}
\]

We use \( c_{i,n}^m \) to denote the achievable rate by the BS transmitting to user \( n \) directly over the \( m \)th subcarrier. We have

\[
c_{i,n}^m = \log_2 \left( 1 + \frac{p_{B}^m \left| h_{B,i,n}^m \right|^2}{\Gamma N_0} \right), \tag{2}
\]

where \( p_{B}^m \) denotes the BS's transmission power on the \( m \)th subcarrier and \( h_{B,i,n}^m \) is the channel gain from the BS to user \( n \) over subcarrier \( m \). \( \Gamma \) is the gap to Shannon capacity, which is mainly determined by modulation techniques and the target bit-error rate. When the BS is communicating with user \( n \) assisted by RS \( i \) over the \( m \)th subcarrier, the achievable rate is given by

\[
c_{i,n}^m = \frac{1}{2} \min \left\{ \log_2 \left( 1 + \frac{p_{B}^m \left| h_{B,i,n}^m \right|^2}{\Gamma N_0} \right), \log_2 \left( 1 + \frac{p_{i}^m \left| h_{i,n}^m \right|^2 + p_{B}^m \left| h_{B,n}^m \right|^2}{\Gamma N_0} \right) \right\}, \tag{2}
\]

where the "\( \frac{1}{2} \)" in (2) is due to the half duplex forward constraint, \( h_{B,i,n}^m \) is the channel gain from the BS to RS \( i \) over subcarrier \( m \), \( h_{i,n}^m \) is the channel gain from RS \( i \) to user \( n \) on the \( m \)th subcarrier and \( p_{i}^m \) is the allocated relay power by RS \( i \) for user \( n \) over the \( m \)th subcarrier. For notional simplicity we define \( H_{B,n}^m = \left| h_{B,n}^m \right|^2 / \Gamma N_0 \), \( H_{B,i,n}^m = \left| h_{B,i,n}^m \right|^2 / \Gamma N_0 \), \( H_{i,n}^m = \left| h_{i,n}^m \right|^2 / \Gamma N_0 \), \( (\forall n \in \mathcal{N}, i \in \mathcal{R}, m \in \mathcal{M}) \) as the normalized channel gains for the rest of the paper. We use \( \mathcal{I}(t) = \{ H_{B,n}^m(t), H_{B,i,n}^m(t), H_{i,n}^m(t) : \forall n \in \mathcal{N}, i \in \mathcal{R}, m \in \mathcal{M} \} \) to denote the collection of all the channel state information at time slot \( t \). It is assumed that \( \mathcal{I}(t) \) belongs to a finite space that has arbitrarily large size and is independent and identically distributed (i.i.d.) at every slot. Considering the BS may communicate with user \( n \) directly or with the assistance of selected relay over some subcarriers, the transmission rate for user \( n \) is

\[
\mu_n(t) = \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{R}} \left( \alpha_{i,n}^m c_{i,n}^m + \alpha_{B,n}^m c_{B,n}^m \right). \tag{3}
\]

Denote \( p_{B}^{\text{max}} \) and \( p_{i}^{\text{max}} \) as the total transmission power constraint for the BS and RSs, respectively, i.e.,

\[
p_B \leq p_{B}^{\text{max}}, \quad p_i \leq p_{i}^{\text{max}}, \quad \forall i \in \mathcal{R}, \tag{4}
\]

where \( p_{B} = \sum_{m \in \mathcal{M}} p_{B}^m \), \( p_i = \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} p_{i,n}^m \). In addition, there are the power mask constraints \( \hat{P}(m) \) on each subcarrier for the BS and RSs to avoid the adjacent cell interference, i.e.,

\[
0 \leq p_{B}^m \leq \hat{P}(m), \quad 0 \leq p_{i,n}^m \leq \hat{P}(m). \tag{5}
\]

The determination of \( \hat{P}(m) \) is out of the scope of this paper and is given a priori.

\[\text{B. Queueing Model}\]

At the BS there are random arrival packets with rate \( A_n(t) \) packets/slot at the end of time slot \( t \) waiting for transmission to user \( n \). Each user’s packets are stored in one of \( N \) data queues corresponding to each destination before they can be sent out. The packet arrival process \( A_n(t) \in [0, A_n^{\text{max}}] \) is assumed to be i.i.d. over each time slot and \( \mathbb{E}[A_n(t)] = \lambda_n, \forall n \in \mathcal{N} \). Among the arrival packets, only \( R_n(t) \) of \( A_n(t) \) are admitted into each user’s buffer with queue length \( Q_n(t) \) in time slot \( t \) by a flow control mechanism. The data queue is updated as

\[
Q_n(t + 1) = (Q_n(t) - \mu_n(t))^{+} + R_n(t), \quad \forall n \in \mathcal{N}, \tag{6}
\]

where \( [x]^{+} = \max \{0, x\} \), and \( \mu_n(t) \) is the service rate of user \( n \) given in (3). To quantitatively measure the balance between the transmission request \( R_n(t) \) and service rate \( \mu_n(t) \), the definition of stability is given first.

\[\text{Definition I ([3])}: \] A queue \( Q(t) \) is strongly stable if:

\[
\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[Q(\tau)] < \infty.
\]

A network is strongly stable if all individual queues of the network are strongly stable.

In the following, strong stability is also referred to stability.

\[\text{C. Energy Management Model}\]

Actually, the power consumption of BS or RSs consists of two parts. One part is the dynamic transmission power of the power amplifier denoted as \( p \) and the other part is \( \Delta p \), the static power consumption of the radio frequency chains including power dissipation in the mixer, transmit filters and digital-to-analog converters [23] [27]. Thus the power consumption of the BS and relay \( i \) can be expressed as \( P_B = p_B + \Delta p_B \) and \( P_i = p_i + \Delta p_i \), respectively. Since the BS dominates the energy consumption in cellular networks [36], its required energy can be taken not only from the renewable energy generator but also from the power grid. Due to the investment cost each RS is only powered by grid. To “smooth” the intermittent renewable energy supply, we introduce an energy storage device to the BS. The harvested renewable energy is first put into the storage device and then withdrawn to supply the BS. If the residual energy in the storage is not enough, the BS will be supplied by the grid. Since the storage device has a limited size, we have to cope with charging and discharging properly. Let \( w(t) \) with \( w(t) \in [0, w^{\text{max}}] \) denote the harvested energy from the renewable source at time slot \( t \). Only \( \delta(t) \) of \( w(t) \) of the renewable energy is actually charged into the storage device, where \( \delta(t) \in [0, 1] \). The energy level \( S(t) \) at the storage device is updated as an energy queue [37]:

\[
S(t + 1) = S(t) - O(t) + \delta(t)w(t), \tag{7}
\]

where \( O(t) \) is the total power withdrawn from the storage device to supply the BS at time slot \( t \). As it can be found later, the energy management of the BS is designed based on \( S(t) \)
instead of the profile of $w(t)$. There is a maximum discharge rate constraint $O_{\text{max}}^{\text{max}}$ for the storage device. The energy level in the storage device should be always available and can not exceed the storage capacity $S_{\text{max}}$. Therefore, at each time slot, we should ensure that

$$O(t) \leq \min \{S(t), O_{\text{max}}\}.$$  

(8)

$$w(t) \leq \min \{w_{\text{max}}, S_{\text{max}} - S(t)\}.$$  

(9)

The BS’s power consumption $P_B(t)$ is supplied by $O(t)$ from the storage device and $J(t)$ from power grid directly with $0 \leq J(t) \leq J_{\text{max}}$, i.e.,

$$P_B(t) = O(t) + J(t).$$  

(10)

$J_{\text{max}}$ is used to take into account the hardware and line constraint of BS power supply system. Thus the total energy withdrawn from the power grid at time slot $t$ is $P(t) = J(t) + \sum_{i \in \mathcal{R}} P_i(t)$. For the BS being able to be supplied by storage device or power grid independently, it is assumed that $J_{\text{max}} \geq P_{\text{max}}^{\text{max}} + \Delta P_B$ and $O_{\text{max}} \geq P_{\text{max}}^{\text{max}} + \Delta P_B$. The important notations of this paper are listed in Table I.

### IV. PROBLEM FORMULATION

The objective of the network operator is to maximize the total utility of the average throughput for each user with the least on-grid energy consumption. Here the utility function $f(\cdot)$ is assumed to be non-decreasing, non-negative and concave, which is used to maintain fairness in resource allocation [38]. Since there are some random events in the system, such as random arrival packets towards each user and renewable energy generation, it is more reasonable to optimize the objective in a long-term manner. Basically the system stability should be maintained. Since the maximum system utility and the least on-grid energy consumption are usually two conflicting objectives, we will find a policy to solve the following stochastic optimization problem to achieve optimal tradeoff between them,

$$\max \phi \sum_{n \in \mathcal{N}} f(\bar{R}_n) - \varphi \bar{P}$$

s.t. C1: $\bar{R}_n \leq R_n, \forall n \in \mathcal{N}$

C2: $[1, 2, 3, 4, 5] \leq X_n(t), \forall t \in \mathcal{N}$

C3: the system is stable

(P1)

where

$$\bar{R}_n = \lim_{t \to \infty} \frac{1}{t-1} \sum_{\tau=0}^{t-1} E\{R_n(\tau)\}$$

is the time-average admitted rate of user $n$, and

$$\bar{P} = \lim_{t \to \infty} \frac{1}{t-1} \sum_{\tau=0}^{t-1} E\{P(\tau)\}$$

is the time-average on-grid energy consumption and the expectations are taken with respect to the random events. $\phi$ and $\varphi$ are two constants introduced to make a tradeoff between throughput utility and on-grid energy consumption. Denote the optimal objective value of (P1) as $V_1$. Due to C1 and the bounded on-grid energy consumption, we can always find a constant $V_{\text{max}}$ such that $V_1 \leq V_{\text{max}}$. In (P1) if $f(\bar{R}_n)$ is chosen as $\bar{R}_n$, the objective function of (P1) is also known as the parametric form in solving the energy efficiency maximization problem [40].

In problem (P1), the current energy management policy is coupled with future ones due to the iteration [7], i.e., the current energy management policy may have impacts on the future energy charging or discharging action for the energy storage device. This coupling nature together with unknown statistics of renewable energy generation makes (P1) difficult. Denote $\bar{O} \triangleq \lim_{t \to \infty} \frac{1}{t-1} \sum_{\tau=0}^{t-1} E\{O(\tau)\}$, $\bar{\delta} \triangleq \lim_{t \to \infty} \frac{1}{t-1} \sum_{\tau=0}^{t-1} E\{\delta(t) w(\tau)\}$ as the time-average value of the expected energy withdrawn from the storage device and the input renewable energy, respectively. By summing (7) from the initial state to $t - 1$ and taking expectations on both sides, we have

$$E\{S(t)\} - S(0) = \sum_{\tau=0}^{t-1} E\{-O(\tau) + \delta(t) w(\tau)\}.$$  

(11)

Since the energy level in the storage device satisfies $0 \leq S(t) \leq S_{\text{max}}$, we have $\bar{O} = \bar{\delta} \bar{w}$ after dividing both sides of (11) by $t$ and taking limitation of $t \to \infty$. Since the objective function of (P1) is a function of the time average, to make it tractable we transfer it to an optimization problem with a time averaged objective function. We instead consider (P2) to tackle the time-coupling difficulties in (P1) and the objective function in (P1).

$$\max \phi \sum_{n \in \mathcal{N}} f(\bar{X}_n) - \varphi \bar{P}$$

s.t. C1: $\bar{X}_n \leq X_n, \forall n \in \mathcal{N}$

C2: $X_n(t) \leq X_{\text{max}}^n, \forall t, n \in \mathcal{N}$

C3: $[1, 2, 3, 4, 5] \leq X_n(t), \forall t$  

(C2)

C4: the system is stable

(P2)

where $X_n(t)$ is an auxiliary variable, and

$$\bar{X}_n \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{X_n(\tau)\}, \quad \bar{X}_n$$

is the lower bound of $\bar{R}_n$ as shown in C1'. In the following section, we will use virtual queue technique to satisfy the time average constraint C1’. Denote the optimal objective value of (P2) as $V_2$. By Ch5 of [39], we have $V_2 \geq V_1$. Now, there is no time correlation on the energy level of storage device in (P2). Using the theory of Lyapunov optimization,
can be solved by an online algorithm to achieve the close-to-optimal performance subject to system stability.

V. JOINT FLOW CONTROL, RESOURCE ALLOCATION AND ENERGY MANAGEMENT

Since the buffer size at the BS is finite, to ensure that each data queue \( Q_n(t) \) at the BS has a deterministic upper bound \( Q_{\text{max}} \) with \( Q_{\text{max}} \geq A_{\text{max}}^n \), where \( A_{\text{max}}^n = \max_n \{ A_{\text{max}} \} \), we introduce the following virtual queue,

\[
U_n(t + 1) = [U_n(t) - R_n(t)]^+ + X_n(t), \quad \forall n \in \mathcal{N} \tag{12}
\]

whose stability can satisfy the constraint C1’ in (P2).

It is observed that in (P2), the decision variables \{\( R_n(t) \), \{\( \alpha_{B,n}^m \), \( \alpha_{C,n}^m \) \} \}, \{\( P(t) \), \{\( O(t) \) \} and \{\( \delta(t) \) \} are coupled together. We decompose the system problem (P2) into joint Flow control, Resource allocation and Energy management (FREE) by Lyapunov optimization method in each time slot. Its design principle is to minimize the upper bound of Lyapunov drift minus system objective function greedily without knowing the profile of stochastic events. The detailed derivation can be found in the proof of Theorem 9. Fig. 1 illustrates that the coupled dynamics in FREE are coordinated by queue states \{\( U_n(t), X_n(t), S(t) \)\}. Although FREE are designed to solve (P2) online and separately, Theorem 9 demonstrates that it can approach the optimal solution of (P1) asymptotically. Its detailed design is as follows. In the following, the joint flow control, wireless resource allocation and energy management in FREE will be described in detail.

\[ \begin{align*}
\text{Potential admitted rate regulation} & \quad \text{Service rate by relay/direct transmission} \\
\text{Flow control} & \quad \text{Resource allocation} \\
X_n(t) & \quad \text{Energy requirement} \\
R_n(t) & \quad \text{Battery} \\
U_n(t) & \quad \text{Renewable energy} \\
Q_n(t) & \quad \text{Energy management} \\
S(t) & \quad \text{Wireless resource allocation}
\end{align*} \]

Fig. 1. Diagram of coupled problems of flow control, wireless resource allocation and energy management.

A. Flow Control

At each time slot \( t \), the BS decides the admission data rate \{\( R_n(t) \)\} to the data buffer of each user as the solution to the following problem,

\[
\begin{align*}
\min_{R_n(t)} & \quad R_n(t) (Q_n(t) - Q_{\text{max}} + A_{\text{max}}^n) \\
\text{s.t.} & \quad 0 \leq R_n(t) \leq A_n(t)
\end{align*} \tag{13}
\]

The problem (13) is corresponding to one part of the upper bound of Lyapunov drift plus penalty in (38). The solution to the above problem is

\[
R_n(t) = \begin{cases} 
0 & \text{if } Q_n(t) > Q_{\text{max}} - A_{\text{max}}^n \\
A_n(t) & \text{otherwise}
\end{cases} \tag{14}
\]

In (13), the tradeoff between system utility and on-grid energy is not explicitly considered. It can be intuitively understood that more saved on-grid energy will result in a smaller \( \mu_n(t) \) and a larger \( Q_n(t) \) subsequently by (38). Then according to (13), more arrival packets will be dropped in order to mitigate congestion. Thus, on-grid energy is saved with a compromise of throughput.

The potential admitted rate \( X_n(t) \) is the solution to the following problem,

\[
\begin{align*}
\min_{X_n(t)} & \quad \frac{Q_{\text{max}} - A_{\text{max}}^n}{Q_{\text{max}}} U_n(t) \cdot X_n(t) - V \phi (X_n(t)) \\
\text{s.t.} & \quad 0 \leq X_n(t) \leq A_{\text{max}}^n
\end{align*} \tag{15}
\]

If \( f(\cdot) \) is differentiable, \( X_n(t) \) is updated as

\[
X_n(t) = \min \left\{ \max \left\{ 0, f^{-1} \left( \frac{Q_{\text{max}} - A_{\text{max}}^n}{Q_{\text{max}}} U_n(t) \right) \right\}, A_{\text{max}}^n \right\}, \tag{16}
\]

where \( V \geq 0 \) is a parameter set according to the system performance requirement and is to be specified in Section VI.

B. Wireless Resource Allocation

We determine the power control, RS selection and subcarrier allocation by solving the following optimization problem based on the measurement of channel states and queue states at the beginning of each time slot.

\[
\begin{align*}
\max & \quad \sum_{n \in \mathcal{N}} \left( \frac{U_n(t) Q_n(t)}{Q_{\text{max}}} \mu_n(t) + (S(t) - \theta) p_B(t) \right) \\
\text{s.t.} & \quad -V \phi \sum_{i \in \mathcal{R}} p_i(t) \\
& \quad \mathbf{11}, \mathbf{12}, \mathbf{13}
\end{align*} \tag{17}
\]

The optimization problem (17) has zero-duality-gap if the number of subcarrier goes to infinity \( \mathbf{41}, \mathbf{42} \). We introduce the Lagrange multipliers \( \lambda_i \) and \( \lambda_B \) to relax the power constraints \( \mathbf{14} \) and solve (17) by Lagrangian dual decomposition. The Lagrange function is

\[
L (\alpha, p, \lambda) = \sum_{n=1}^{N} \frac{U_n(t) Q_n(t)}{Q_{\text{max}}} \mu_n(t) + (S(t) - \theta) p_B(t) - V \phi \sum_{i=1}^{K} \lambda_i (p_i^{\text{max}} - p_i(t)) + \lambda_B (p_B^{\text{max}} - p_B(t)) + \lambda_j \sum_{i=1}^{K} \lambda_i (p_i^{\text{max}} - p_i(t)). \tag{18}
\]

The dual function is \( D (\lambda) = \max_{\alpha, p} L (\alpha, p, \lambda) \mathbf{3} \), which can be decomposed into \( M \) independent subproblems,

\[
\max \quad \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{R}} \lambda_m(t), \forall m \in \mathcal{M}, \tag{19}
\]

where

\[
\begin{align*}
\Omega_m & = \frac{U_n(t) Q_n(t)}{Q_{\text{max}}} \alpha_{B,n}^m \mu_n(t) + \alpha_{C,n}^m (S(t) - \theta - \lambda_B) p_B(t) \\
& = \Omega_m (B, n)
\end{align*} \tag{20}
\]

\(^3\text{Here, we use bold letter to denote a vector, e.g., } \lambda = [\lambda_1, \lambda_2, \cdots, \lambda_K, \lambda_B].\)
if direct transmission towards user $n$ is adopted over subcarrier $m$, or
\[
\Omega_m = \frac{U_m Q_m}{\lambda m} \alpha_{m}^m(t) + \alpha_{m}^m(S(t) - \theta - \lambda_B) p_B^m(t) \\
- \alpha_{m}^m(V \varphi + \lambda_i) p_i^m(t) \\
\leq \Omega_m(i,n),
\]
if relay-assisted transmission is chosen.

We first consider the direct transmission (20) to find the optimal $\Omega_m^*(B,n)$,
\[
\max_{0 \leq p_B^m \leq P(m)} \frac{U_m Q_m}{\lambda m} \alpha_{m}^m(t) + (S(t) - \theta - \lambda_B) p_B^m(t) \\
\text{s.t.} \ H^m_{B,n}(t) \geq \hat{H}^m_{B,n}(t), \ P(m), \ 0 \leq p_i^m(t) \leq P(m)
\]
The optimal power of the BS over subcarrier $m$ is obtained as
\[
p_B^m(t) = \begin{cases} \hat{P}_B(t) & \text{otherwise} \\
\frac{U_m Q_m}{\lambda m} + \frac{U_m Q_m}{\lambda m} \alpha_{m}^m(t) - \frac{1}{H^m_{B,n}(t)} \hat{P}(m) & \text{if } \theta + \lambda_B - S(t) \leq 0 \end{cases}
\]
where $\hat{P}_B(t) = (\max_{0 \leq t \leq T} \frac{U_m Q_m}{\lambda m} \alpha_{m}^m(t) - \frac{1}{H^m_{B,n}(t)} \hat{P}(m))$.

Then substituting (22) into (21) we can obtain the optimal $\Omega_m^*(B,n)$ given that the BS transmits to user $n$ over subcarrier $m$ directly.

In the case of cooperative transmission, we consider the following two cases (23) and (24) for the objective function (22).
\[
\max_{0 \leq p_B^m \leq P(m)} \frac{U_m Q_m}{\lambda m} \alpha_{m}^m(t) + (S(t) - \theta - \lambda_B) p_B^m(t) \\
+ (V \varphi + \lambda_i) p_i^m(t) \\
\text{s.t.} \ H^m_{B,n}(t) \geq \hat{H}^m_{B,n}(t), \ P(m), \ 0 \leq p_i^m(t) \leq P(m)
\]
The optimal solution to problem (24) is
\[
\begin{align*}
\{ & \alpha_{m}^m(t) \} \\
\text{s.t.} & \ H^m_{B,n}(t) \geq \hat{H}^m_{B,n}(t), \ P(m), \ 0 \leq p_i^m(t) \leq P(m)
\end{align*}
\]
where $q_n(t) = \frac{U_m Q_m}{\lambda m} \alpha_{m}^m(t) \log 2$.

Theorem 2: The optimal transmission power allocation for problem (23) is as follows:
\[
\begin{align*}
\text{If } S(t) - \theta - \lambda_B + (V \varphi + \lambda_i) H^m_{B,n} & \geq 0, \ p_B^m(t) = \\
\hat{P}(m), \ p_i^m(t) = \frac{d_{i,n}^m - \hat{P}(m) H^m_{B,n}}{H^m_{B,n}}, & \text{where } \alpha_{m}^m = \\
\max \{0, \min \{ \frac{q_n(t) H^m_{B,n}}{\lambda_B - S(t) + \theta} - 1, \hat{P}(m) H^m_{B,n}, \hat{P}(m) H^m_{B,n} \} \}.
\end{align*}
\]

Remark 4: By the above description, it may be concluded some subcarriers will be abandoned temporarily due to their low energy efficiency. At the same time the abandoned subcarriers can be used by other cells belonging to the same operator. The abandoned subcarriers can also be re-used by the same cell after a while when the channel conditions turn to be better. Therefore, the network operator will not waste any resource or revenue from a long term and global point of view.

By substituting the optimal transmission power in Theorem 2 the optimal $\Omega_m^*(i,n)$ is obtained. Then the subcarrier allocation and relay selection can be achieved by
\[
\begin{align*}
\{ & \alpha_{j,n}^m(t) \} \\
\text{if } (j,n) = \arg \max_{(j,n) \in \mathbb{N}} & \Omega_m^*(B,n) \\
0 & \text{otherwise}
\end{align*}
\]
Note that in $\{25\}$ $j$ can be the index of a RS or referred to the BS if direct transmission is adopted.

The associated dual problem of (17) is $\min_{\lambda \leq 0} D(\lambda)$, which can be solved by standard subgradient method (26) and (27) for $\forall i \in \mathbb{R}$.

Proof: The proof can be found in Appendix A. ■

Some intuitive explanations for Theorem 2 are given below.

Remark 3: For the solution to problem (23), it can be found that in the case of $S(t) - \theta - \lambda_B + \frac{(\varphi + \lambda_i) H^m_{B,n}}{H^m_{B,n}} \geq 0$ if the second hop channel gain $H^m_{B,n}$ is relatively low compared with $H^m_{B,n}$, which may result in $d^m_{i,n} = 0$, the BS will ask no relay for help transmission to avoid more electricity withdrawn from power grid to supply the relay. Moreover, for the same case, if $q_n(t)$ is large enough, which may result in a large $d^m_{i,n}$, the BS has to ask relay $i$ for helping transmission to alleviate congestion of user $n$’s buffer. In the aforementioned situation, the BS transmits with $\hat{P}(m)$ since there is enough residual energy in the storage device and the objective function of (23) is increasing with respect to $p_B^m$. If $S(t) - \theta - \lambda_B + \frac{(\varphi + \lambda_i) H^m_{B,n}}{H^m_{B,n}} < 0$ due to less residual energy in the storage device, the combination of relay $i$, the BS and user $n$ is not suitable for transmission over subcarrier $m$ to avoid withdrawing too much electricity from the grid.

Remark 5: In practice, the subproblems (23) and (24) are solved by each RS locally for $N \times M$ times during one iteration $k$ and the computational complexity at each relay is $O(NM)$, which is the same as (41). And then each relay transmits the optimal $\Omega_m^*(i,n)$ to the BS for the final subcarrier allocation.
The deployment of a certain amount RSs to proper place is also important in utilizing the cooperative diversity and has been extensively investigated in [28] and [31]. A candidate place is where a RS can not only serve users better than direct transmission but also be easy to access on-grid power. Candidate place can be the roof of a building, where RSs serve users in the building. Thus the number of candidate places in a considered area is finite. Since the joint deployment of RS and resource allocation problem is NP-hard, heuristic algorithm will be considered. Firstly, all candidate places are deployed with RSs. Users requiring relay are associated to the RS with the minimum RTE which is defined as the quotient of user’s throughput requirement and achievable throughput with unit transmission power and subcarrier. Then subcarriers are allocated among RSs and BS to satisfy each user’s throughput requirement proportionally. The transmission power of BS and RSs can be obtained by waterfilling. Secondly, calculate each RS’s energy efficiency value, which is defined as the total achievable throughput divided by the RS’s energy consumption. Sort all RSs in an ascending order based on its energy efficiency value. Thirdly, try to delete the first RS from and candidate place and re-connect its users to other RSs based on the RTE criterion in the first step. Subcarriers and transmission power are re-allocated. If each user’s throughput requirement is satisfied, try to delete the second RS and repeat. If the energy level in the energy storage device satisfies

\[ S(t) \in [0, S_{\text{max}}], \forall t. \]

**Remark 6:** The detailed implementation of the proposed joint policy FREE can be found in Fig. 2 Although the FREE policy seems to be implemented separately, the proof of Theorem 6 demonstrates that the solutions to subproblems (13), (14), (15), and (17) minimize the upper bound of system Lyapunov drift plus cost function greedily. Thus, the FREE can solve the system optimization problem (P1) while maintaining stability at the same time, which is formally stated in Proposition 8 and Theorem 9.

**VI. ALGORITHM PERFORMANCE**

This section states the properties of proposed algorithm FREE.

**Theorem 7:** For \( \theta = \phi V + O_{\text{max}} \) and

\[ 0 < V \leq \frac{S_{\text{max}} - w_{\text{max}} - O_{\text{max}}}{\phi}, \]

the energy level in the energy storage device satisfies

\[ S(t) \in [0, S_{\text{max}}], \forall t. \]

**Proof:** The proof can be found in Appendix B. ■

The above results demonstrate that although FREE solves (P2) without considering the constraints (7), the energy level in storage device is feasible in each time slot. The following proposition characterizes the boundedness of data queue. By recalling (21), it is noted that \( \theta \) can be regarded as a charging threshold for the battery and be implemented by the algorithm. Based on (28) and (31), the maximum V is mostly determined by \( S_{\text{max}} \), which should be at least larger than the summation of the maximum charging rate \( w_{\text{max}} \) and the maximum withdrawing rate \( O_{\text{max}} \). We can understand the lower bound of \( S_{\text{max}} \) by the following observation. To maintain the feasible value of residual energy in the battery, we can observe that \( S_{\text{max}} \) is obtained by adding (8) and (9) together, i.e., \( w(t) + O(t) \leq S_{\text{max}} - S(t) + S(t) = S_{\text{max}}, \forall t. \)

**Proposition 8:** Initialize \( Q_n(0) = 0, \forall n \). The data buffer backlogs yield the deterministic bounds \( Q_n(t) \leq Q_{\text{max}} \) for \( \forall t \geq 0, \forall n \).

**Proof:** The proof can be found in Appendix C. ■

In the above results, given \( Q_{\text{max}} \) the designed FREE can guarantee that the actual queue length is limited by \( Q_{\text{max}} \) in each time slot and the virtual queue also has a deterministic upper bound \( U_{\text{max}} \). Theorem 9 further proves FREE designed for the relaxed problem (P2) can approach the optimal solution of original problem (P1) asymptotically.
**Theorem 9:** Given $Q_{\max}^\alpha > A_{\max}^2 + \frac{A_{\max}^2 + \mu_{\max}^2}{2\epsilon}$, the proposed algorithm FREE can achieve the near optimal performance:

$$\liminf_{t \to \infty} \sum_{n \in N} \left( \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[R_n(\tau)] \right) - \frac{1}{2} \sum_{\tau=0}^{t-1} \mathbb{E}[P(\tau)]$$

$$\geq V_1 - \frac{\epsilon}{V}. \quad (32)$$

and a bounded virtual queue

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[U_n(\tau)]$$

$$\leq \frac{1}{2} N A_{\max}^2 \frac{Q_{\max}^\alpha}{Q_{\max}} + N \frac{Q_{\max}^\alpha - A_{\max}^2}{Q_{\max}} A_{\max}^2 \quad (33)$$

where $\epsilon$ is a positive variable, $V_1$ is the optimal objective value of $(P1)$ and $\Xi$ is defined as

$$\Xi \equiv \frac{1}{2} N A_{\max}^2 \frac{Q_{\max}^\alpha}{Q_{\max}} + N \frac{Q_{\max}^\alpha - A_{\max}^2}{Q_{\max}} A_{\max}^2 (34)$$

Before we give the formal proof, the following lemma is needed.

**Lemma 10:** If the packet arrival process $\{A_n(t), \forall n \in N\}$, channel state $I(t)$, and renewable energy generation $w(t)$ are i.i.d., then there exists a randomized stationary policy $\Phi^{RS}$ that takes feasible flow, radio resource allocation and energy management only based on the current system state $\{A_n(t), I(t), w(t)\}$ at each time slot $t$ while satisfying the constraints in problem $(P2)$ and results in the following conditions:

$$\mathbb{E}\{O^{RS}(t)\} = \mathbb{E}\{\delta^{RS} (t) w(t)\},$$

$$\mathbb{E}\{X_n^{RS}(t)\} = \mathbb{E}\{\mu_n^{RS} (t)\} = \mathbb{E}\{R_n^{RS}(t)\} = \kappa_n, \forall n \in N,$$

$$\mathbb{E}\{\phi \sum_{n=1}^{N} f(R_n^{RS}(t))\} - \mathbb{E}\{\varphi P^{RS}(t)\} = V_2,$$

where $\kappa_n$ belongs to the network region $\Lambda$, which is the set of network capacities under all possible control policies. There also exists another randomized stationary policy $RS'$ that only depends on the system state and the following conditions hold, $\forall n \in N$.

$$\mathbb{E}\{O^{RS'}(t)\} = \mathbb{E}\{\delta^{RS'} (t) w(t)\},$$

$$\mathbb{E}\{X_n^{RS'}(t)\} = \mathbb{E}\{\mu_n^{RS'} (t)\} = \mathbb{E}\{R_n^{RS'}(t)\} = \kappa_n - \epsilon,$$

$$\mathbb{E}\{\phi \sum_{n=1}^{N} f(R_n^{RS'}(t))\} - \mathbb{E}\{\varphi P^{RS'}(t)\} = V_2,$$

where $\epsilon \in \Lambda$ is a positive value that can be chosen arbitrarily close to zero. According to [38], we have $V_2 \to V_2$ as $\epsilon \to 0$.

The proof process of Theorem 9 is described in the following.

**Proof:** The average performance bound (32) is obtained by comparing the Lyapunov drift of FREE with the aforementioned randomized stationary policy. We use Lyapunov optimization technique to derive the performance bound. Let $\Theta(t) = (S(t), U(t), Q(t))$, where $U(t) = (U_n(t), n \in N)$, $Q(t) = (Q_n(t), n \in N)$. Define the Lyapunov function as

$$L(\Theta(t)) \equiv \frac{1}{2} \sum_{n \in N} \left[ \frac{Q_{\max} \max_{x \in \mathcal{X}} U_n^2(t)}{Q_{\max}^\alpha} + \frac{U_n(t)Q_n^2(t)}{Q_{\max}^\alpha} \right] + \frac{1}{2} (S(t) - \theta)^2 \quad (35)$$

Define the conditional Lyapunov drift as follows,

$$\Delta L(\Theta(t)) \equiv \mathbb{E} \left[ L(\Theta(t+1)) - L(\Theta(t)) \mid \Theta(t) \right], \quad (36)$$

where the conditional expectation is taken over the system queue. The drift of the first two items in (35) can be found in [45]. According to the battery dynamics in [7], the drift of the third item in (35) is

$$\begin{align*}
\frac{1}{2} & \left( (S(t+1) - \theta)^2 - (S(t) - \theta)^2 \right) \\
= & \frac{1}{2} \left( 2S(t+1) - S(t+1) - S(t) \right) \\
= & \frac{1}{2} \left( (S(t) - O(t) + \delta(t) w(t))^2 - S(t)^2 - 2\theta(S(t+1) - S(t)) \right) \\
= & \frac{1}{2} (w_{\max}^2 + O_{\max}^2) (S(t) - \theta)(O(t) - \delta(t) w(t)) \\
= & \frac{1}{2} (w_{\max}^2 + O_{\max}^2) (S(t) - \theta)(O(t) - \delta(t) w(t))
\end{align*}$$

By subtracting $\mathbb{E}\{ \sum_{n \in N} \phi f(X_n(t)) - \varphi P(t) \mid \Theta(t) \}$ from the conditional drift of (35), we have

$$\Delta L(\Theta(t)) - \mathbb{E}\{ \sum_{n \in N} \phi f(X_n(t)) - \varphi P(t) \} \mid \Theta(t) \} \quad (38)$$

$$\leq \Xi + \sum_{n \in N} \mathbb{E}\{ \Phi_{1n}(t) + \Phi_{2n}(t) - \Phi_{3n}(t) + \Phi_4(t) \mid \Theta(t) \}$$

$$- (S(t) - \theta) \Delta p_B(t) + \mathbb{E}\{ \varphi V(t) \Delta p_i(t) \mid \Theta(t) \}$$

$$+ \frac{1}{2} \sum_{n \in N} U_n(t) \left[ \mu_{\max}^2 + A_{\max}^2 \right]$$

where $\Xi$ is defined in (34) and $\mathcal{R}(n)$ is the set of RSs that assist the transmissions towards user $n$,

$$\Phi_{1n}(t) = \frac{Q_{\max} \max_{x \in \mathcal{X}} U_n(t) X_n(t) - \phi f(X_n(t))}{Q_{\max}^\alpha}$$

$$\Phi_{2n}(t) = R_n(t) \frac{U_n(t)}{Q_{\max}^\alpha} (Q_n(t) - (Q_{\max}^\alpha - A_{\max}^2))$$

$$\Phi_{3n}(t) = \frac{U_n(t)Q_n(t) \mu_{\max}^2}{Q_{\max}^\alpha} + (S(t) - \theta) p_B(t) - \mathbb{E}\{ \varphi V(t) \sum_{i \in \mathcal{R}(n)} p_i(t) \}$$

$$\Phi_4(t) = (\varphi V + S(t) - \theta) J(t) + (S(t) - \theta) \delta(t) w(t)$$

It can be found that the subproblems [15], [19], and [28] in FREE minimizes

$$\sum_{n \in N} \mathbb{E}\{ \Phi_{1n}(t) + \Phi_{2n}(t) - \Phi_{3n}(t) + \Phi_4(t) \mid \Theta(t) \}$$

on the right hand side of (38) over all possible solutions including the randomized stationary policy. Substituting the
randomized stationary policy \( RS' \) into \( \Phi_{1n}(t) \) and substituting policy \( RS \) into \( \Phi_{2n}(t), \Phi_{3n}(t) \) and \( \Phi_{4n}(t) \) on the right hand side of (38) it is obtained that,

\[
\Delta \mathcal{C}(\Theta(t)) - V \mathbb{E}\left\{ \left( \sum_{n \in N} \phi_f(X_n(t)) - \varphi P(t) \right) | \Theta(t) \right\} \\
\leq \mathbb{E} \left\{ - \epsilon \sum_{n \in N} U_n(t) \right\} \left( 2Q_{\text{max}}^n - A_{\text{max}}^n \right) \\
- \epsilon^{-1} \left( \mu_{\text{max}}^n + A_{\text{max}}^n \right) - V \nu_2 e
\]

(39)

If \( Q_{\text{max}}^n \geq \frac{\mu_{\text{max}}^n + A_{\text{max}}^n}{2} \), by Theorem 5.4 in [38], it is obtained that (38) and

\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{ \sum_{n=1}^{N} \phi_f(X_n(\tau)) - \varphi P(\tau) \right\} \geq V \nu_2 - \frac{\mathbb{E} \sum_{n \in N} U_n(t)}{V
(39)

hold. Since the virtual queue \( U_n(t) \) is stable by (38), we have

\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{ R_n(\tau) \right\} \geq \lim_{t \to \infty} \frac{1}{t} \mathbb{E}\left\{ X_n(\tau) \right\}. \\
\text{Hence we have}
\]

\[
f(\bar{R}_n) \geq f(\bar{X}_n) \geq f(X_n), \\
(40)
\]

where the first inequality is due to the non-decreasing property of \( f(\cdot) \) and the second inequality is Jensen’s inequality. Note that (39) holds for all \( \epsilon \). Due to \( V_2 \to V_2 \) as \( \epsilon \to 0 \), and \( V_2 > V_1 \) we have (40) in Theorem 9 by substituting (40) into (39).

Remark 11: Although the designed per-slot sub-problems [13], [15], [17], [28] without considering the future network performance, jointly solve (P2) instead of (P1), the resulted solution can approach the optimal objective value of (P1) arbitrarily by regulating \( V \). This result is reasonable since a larger storage (a larger \( V \)) can store more renewable energy to save on-grid energy to achieve a larger objective value.

Remark 12: The result in Theorem 9 is based on the assumption of i.i.d stochastic process. It can be generalized to non i.i.d. Markov modulated scenarios using the techniques developed in [38], [39].

### VII. SIMULATION RESULTS

This section presents the simulation results for the proposed algorithm FREE. A cell with 2 km radius is considered, where eight mobile users are uniformly distributed in the cell and four RSs are assumed to be uniformly placed in the midway of BS and cell boundary. The simulation parameters are set as follows except for other specification. In every time slot, the BS receives new packets with destinations of the corresponding users according to an i.i.d Poisson arrival process with average rate of 8 packets/s. The packet size has an exponential distribution with mean packet size of 5000 bits/packet. The buffer size for each user is 10 packets. The total bandwidth is 10 MHz with 128 subcarriers. The channel gain of any transmission pair in networks consists of a small-scale Rayleigh fading component and a large-scale path loss component with path loss factor of 4. The noise level is assumed to be \( 10^{-10} \) W and the gap to capacity is set to be 1. The static power consumption of the BS is 194.24 W and its maximum transmission power is 20 W without particular specification. For any RS, its static power consumption is 40 W and the maximum transmission power is 10 W. The power mask on each subcarrier is \( P(m) = 0.2 \) W. The capacity of the BS storage device is 3000 Wh unless otherwise specified. The maximum discharge rate of the storage device is \( O_{\text{max}} = 1.5 \times (P_{\text{fg}}^{\text{max}} + \Delta P_B) \) W. The maximum power from the grid to supply the BS directly is \( J_{\text{max}} = O_{\text{max}} \). The average generated solar power has two states corresponding to sunny day and cloudy day: 195 Wh with probability 0.6 and 100 Wh with probability 0.4, respectively. The parameter \( \theta \) is set as \( \varphi V + O_{\text{max}} \). The utility function is chosen as \( f(x) = \log(1 + x) \).

Firstly, we verify the stability of proposed algorithms. From Fig. 3 to Fig. 5 it is observed that all the actual queues \( \{Q_n(t)\} \), virtual queues \( \{U_n(t)\} \) and energy level \( \{S(t)\} \) in the storage device have the trend of stability. Especially Fig. 5 shows that all actual queues are smaller than the upper bound of 10 packets and Fig. 4 demonstrates that all virtual queues are bounded, which verifies the results in Proposition 8 and (38). Fig. 5 demonstrates the results in Theorem 7 that the energy level in the storage device of the BS is always nonnegative and below the capacity of storage device.

The simulations are run for \( 1.2 \times 10^4 \) iterations and the results in Fig. 6 to Fig. 9 are obtained by averaging the last 5000 iterations. It is shown that the time-average tradeoff value in (P1) grows with the increase of \( V \), which verifies (32) . It is further noticed that with \( N = 130 \) the time average objective value increases slower than that with \( N = 8 \), since a larger \( N \) in \( \Xi \) (41) brings a larger gap between the optimal
objective value and actual value. To decrease the gap, a larger $V$ is needed. Fig. 6 also demonstrates that even with the same $N$ the system objective value degrade a little when all the channel state is measured with $30\%$ uncertainty, i.e. $\hat{H}(t) = H(t) + \Delta H(t)$, where $H(t)$ is the real channel gain, $|\Delta H(t)/H(t)| = 30\%$ is channel uncertainty and $H(t)$ the measured channel gain.

The tradeoff between the system throughput and on-grid energy consumption is shown in Fig. 7 which is depicted by decreasing $\varphi$ gradually while $\phi$ and $V$ are fixed. In order to measure the effects of renewable energy and cooperative relay, $FREE$ is compared with the other two schemes: the scheme with hybrid energy powered BS without relay, and the scheme with both RSs and BS powered by on-grid energy alone. The compared schemes are also designed by Lyapunov optimization. In Fig. 2 it is found that for on-grid energy consumption smaller than 182 the system throughput of $FREE$ is even smaller than the scheme with hybrid energy powered BS only. This range corresponds to a larger $\varphi$, which forces $FREE$ to give up using most RSs to save on-grid energy. However the RSs are still there generating constant power consumption $\Delta P_i$ without any throughput contribution. As $\varphi$ continues to decrease, more RSs are employed by $FREE$ to obtain more throughput than the scheme with hybrid energy powered BS only. It is also found that mostly the scheme with cooperative RSs powered by on-grid energy alone is not energy-efficient compared with the other two schemes. When more on-grid energy is consumed, its throughput is larger than the scheme with BS only but still smaller than $FREE$. Thus, we can see that the benefits due to renewable energy and cooperative relay depend on proper parameter setting.

In Fig. 8 we compare $FREE$ with the other two related works in [41] and [44]. Lagrange dual decomposition method and discrete particle swarm optimization method are adopted respectively by [41] and [44] for resource allocation in cooperative relay networks, i.e., to pursue

$$\max \phi \sum_n f(\mu_n(t)) - \varphi P(t)$$

For fair comparison, we assume that the BSs in [41] and [44] are also powered by hybrid energy. We assume there is a heuristic renewable energy management scheme for [41] and [44]. With the heuristic scheme, the harvested energy is always charged into the battery as long as there is enough space, and the BS always uses the saved renewable energy with priority. Since the two related works do not consider flow control, we measure the time average service rate and the time average on-grid energy consumption with the decreasing $\varphi$. It is found in Fig. 8 that the performance of $FREE$ outperforms the other two schemes, since it adapts the resource allocation to the arrival packets and battery states. Moreover, the performance of [44] is better than [41], since the former takes into account dynamic channel state explicitly.

In Fig. 9 we use the fairness index $FI = \left(\sum_{n=1}^{N} R_n^2\right) / \left(N \sum_{n=1}^{N} R_n^2\right)$ to measure the fairness. If all users have the same throughput, $FI$ is 1. It is found that with the number of user from 10 to 60, the proposed scheme maintains better fairness compared with the throughput maximization scheme.

**VIII. Conclusion**

In this paper, we consider to optimize the tradeoff between downlink throughput utility and on-grid energy consumption of an OFDMA cellular network with the assistance of multiple
RSSs. The BS is powered by the conventional utility grid and the renewable energy. A joint design of flow control, radio resource allocation and energy management is proposed to handle the coupling between the energy consumption of the cooperative network and the storage energy allocation. The joint design scheme, namely FREE can be implemented by exploiting the local channel states, buffer states and energy states. Technical proof is established to ensure that FREE can produce a close-to-optimal performance while allowing finite data buffer and energy storage device. Simulation results show that FREE demonstrates better performance compared with other schemes.

Appendix A

1. Solution to (23)

We first pursue the solution to (23). To address the variable coupling in the objective function, we let
\[ d_{i,n}^m = p_{B,n}^m H_{B,n}^m + p_{B,n}^m H_{i,n}^m. \]

The problem (23) is rewritten as
\[
\begin{align*}
\max & \quad Y_1 = q_n(t) \log \left(1 + d_{i,n}^m\right) \\
\text{s.t.} & \quad 0 \leq d_{i,n}^m \leq \min\left\{p_{B,n}^m H_{B,n}^m, \hat{P}(m) H_{i,n}^m + p_{B,n}^m H_{B,n}^m\right\}, \\
& \quad 0 \leq p_{B,n}^m \leq \hat{P}(m), \quad (41)
\end{align*}
\]

If \( S(t) - \theta - \lambda_B + \frac{(V+\lambda)t}{H_{B,n}^m} \geq 0 \), then the optimal transmission power of the BS over subcarrier \( m \) is \( p_{B,n}^m = \hat{P}(m) \). Substituting \( p_{B,n}^m = \hat{P}(m) \) into (42), the optimal \( d_{i,n}^m \) is
\[ d_{i,n}^m = \max\left\{0, \min\left\{p_{B,n}^m H_{B,n}^m, \hat{P}(m) H_{i,n}^m + p_{B,n}^m H_{B,n}^m\right\}\right\}, \]

where \( \hat{d}_{i,n}^m = \frac{q_n(t) H_{i,n}^m}{H_{B,n}^m} - 1 \) is the solution to \( \nabla d_{i,n}^m Y_1 = 0 \). Then based on the definition of \( d_{i,n}^m \) in (41) the optimal \( p_{B,n}^m \) is
\[ \left(d_{i,n}^m - \hat{P}(m) H_{B,n}^m\right)^+. \]

If \( S(t) - \theta - \lambda_B + \frac{(V+\lambda)t}{H_{B,n}^m} < 0 \), then the optimal solutions to (42) are \( p_{B,n}^m = 0 \) and \( d_{i,n}^m = 0 \) and then \( p_{B,n}^m = 0 \).

2. Solution to (24)

Let the objective of (24) be \( Y_2 \). Since the objective function of (24) is decreasing with respect to \( p_{i,n}^m \), the optimal value is \( p_{i,n}^m = 0 \). There are two cases to be considered for the optimal \( p_B^m \). If \( H_{B,n}^m > H_{B,n}^m \), then the optimal transmission power of the BS is \( p_B^m = 0 \). If \( H_{B,n}^m \leq H_{B,n}^m \), the optimal value is
\[ p_B^m = \begin{cases} 
\hat{P}(m) & \text{if } S(t) - \theta - \lambda_B \geq 0 \\
\left(\frac{q_n(t)}{\lambda_B - \lambda_B} - \frac{1}{H_{B,n}^m}\right) \hat{P}(m) & \text{otherwise ,}
\end{cases} \]

where the optimal \( p_B^m \) in the case of \( S(t) - \theta - \lambda_B < 0 \) is obtained by solving \( \nabla p_B^m Y_2 = 0 \) considering the power mask and non-negative constraints.

Appendix B

Proof of Theorem 7

We now use the induction method to derive (31). Initially, without any action on the storage device, \( 0 \leq S(0) \leq S_{\text{max}} \) holds. Supposing (31) holds for time slot \( t \), we will prove it holds for slot \( t + 1 \). There are the following three cases to be considered.

i) If \( 0 \leq S(t) < O_{\text{max}} \), then
\[
S(t+1) = S(t) + \delta^* w(t) - O^*(t)
\]

ii) If \( O_{\text{max}} \leq S(t) < S_{\text{max}} - u_{\text{max}} \), we have
\[
S(t+1) = S(t) + \delta^* w(t) - O^*(t) \geq O_{\text{max}} - O^*(t) \geq 0.
\]

Moreover,
\[
S(t+1) \leq S(t) + w_{\text{max}} < O_{\text{max}} + w_{\text{max}} \quad (44)
\]

By substituting the upper bound of \( V \) (30) into (41), we have
\[
S(t+1) < S_{\text{max}} - V \varphi < S_{\text{max}}
\]

iii) If \( S_{\text{max}} - u_{\text{max}} \leq S(t) \leq S_{\text{max}} \), it means \( S(t) \geq \theta \), which is obtained by substituting the upper bound of \( V \) (30) into \( \theta \equiv \varphi V + O_{\text{max}} \). Thus, we have \( \delta^* = 0 \) and
\[
S(t+1) = S(t) + \delta^* w(t) - O^*(t) > S_{\text{max}} - u_{\text{max}} - O_{\text{max}} > 0. \quad (45)
\]

To summarize, \( S(t+1) \in [0, S_{\text{max}}] \) holds if \( S(t) \in [0, S_{\text{max}}] \).

Appendix C

Proof of Proposition 8

\( Q_n(0) = 0 < Q_{\text{max}} \). Supposing that \( Q_n(t) \leq Q_{\text{max}} \), we have two cases. (a) \( Q_n(t) \leq Q_{\text{max}} - A_{\text{max}} \). Obviously, \( Q_n(t+1) \leq Q_n(t) + A_{\text{max}} \leq Q_{\text{max}} \) according to the
dynamics of $Q^* (t)$. (b) If $Q^* (t) > Q^* (t) > A^\text{max}$, then $R_n (t) = 0$ according to (13). Thus, we have $Q_n (t+1) \leq Q_n (t) \leq Q^* (t)$.

**REFERENCES**

[1] A. Fehske, J. Malmodin, G. Biçokz, and G. Fettweis, “The global carbon footprint of mobile communications—the ecological and economic perspective,” *IEEE Commun. Mag.*, vol. 49, no. 8, pp. 55-62, Aug. 2011.

[2] D. Feng, C. Jiang, G. Lim, Leonard J. Cimini, Jr., G. Feng, and G. Y. Li, “A survey of energy-efficient wireless communications,” *IEEE Commun. Surveys & Tuts.*, vol. 15, no. 1, pp. 167-178, First Quarter 2013.

[3] C. K. Ho, R. Zhang, “Optimal energy allocation for wireless communications with energy harvesting constraints,” *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808-4818, Sept. 2012.

[4] D. Niyato, L. Xiao, and P. Wang, “Adaptive power management for wireless base station in smart grid environment,” *IEEE Wireless Commun. Mag.*, vol. 19, no. 6, pp. 44-51, Dec. 2012.

[5] Y. Huang, S. Mao, “On quality of usage provisioning for electricity scheduling in microgrids,” *IEEE Syst. J.*, vol. 8, no. 2, pp. 619-628, Jun. 2014.

[6] Y. Yu, C. Zhang, X. Zhang, L. Zhou, K. Yang, “Hybrid spectrum access in cognitive-radio-based smart-grid communications system,” *IEEE Syst. J.*, vol. 8, no. 4, pp. 577-587, Jun. 2014.

[7] Y. Wu, V.K.N. Lau, D.H.K. Tsang, L.P. Qian, L. Meng, “Optimal energy scheduling for residential smart grid with centralized renewable energy source,” *IEEE Syst. J.*, vol. 8, no. 2, pp. 562-576, Jun. 2014.

[8] B. Chai, J. Chen, Z. Yang and Y. Zhang, “Demand response management with multiple utility companies: a two-Level game approach,” *IEEE Trans. Smart Grid*, vol.2, pp.722-731, Mar. 2014.

[9] G. A. S. Sidhu, F. Gao, W. Wang, W. Chen, “Resource allocation in relay-aided OFDM cognitive radio networks,” *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, pp. 3700-3710, Oct. 2013.

[10] D. Wang, Z. Li, and X. Wang, “Joint optimal subcarrier and power allocation for wireless cooperative networks over OFDM fading channels,” *IEEE Trans. Veh. Technol.*, vol.61, no.1, pp.249-257, Jan. 2012.

[11] Y. Yang, H. Zhang, B. Wang, F. B. Yang, X. G. Han, “Fair resource allocation and admission control in wireless multi-user amplify-and-forward relay networks,” *IEEE Trans. Veh. Technol.*, vol. 63, no.3, pp.1383-1397, Mar. 2012.

[12] A. Zappone, Z. Chong, E. Jorswieck, S. Buzzi, “Energy-aware communications with a full-duplex relay,” *IEEE Trans. Wireless Commun.*, Jun. 27-28 2012.

[13] D. Wang, Z. Li, and X. Wang, “Optimal transmission scheduling of cooperative communications with a full-duplex relay,” *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 9, pp. 2353-2363, Sept. 2014.

[14] Y. Zhang, A. Ahmed, and S. Ulukus, “Power allocation for an energy harvesting transmitter with hybrid energy sources,” *IEEE Trans. Wireless Commun.*, vol.12, no.12, pp.6255-6267, Dec. 2013.

[15] D. W. K. Ng, E. S. Lo, R. Schober, “Energy-efficient resource allocation in OFDMA systems with hybrid energy harvesting base station,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3412-3427, Jul. 2013.

[16] P. He, L. Zhao, S. Zhou, Z. S. Niu, “Recursive water-filling for wireless links with energy harvesting transmitters”, *IEEE Trans. Veh. Technol.*, vol. 63, no. 3, pp.1232-1241, Mar. 2014.

[17] O. Ozel, K. Tunucuguhi, J. Yang, S. Ulukus, A. Yener, “Transmission with energy harvesting nodes in fading wireless channels; optimal policies,” *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732-1743, Sept. 2011.

[18] T. Zhang, W. Chen, Z. Han, Z. Cao, “A Cross-layer Perspective on Energy Harvesting Aided Green Communications over Fading Channels,” *Proc. IEEE INFOCOM*, Turin, Italy, Apr. 14-19, 2013.

[19] H. Huang, V. K. N. Lau, “Decentralized delay optimal control for interference networks with limited renewable energy storage,” *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2522-2561, May 2012.

[20] Z. Zheng, L.X. Cai, R. Zhang, X. Shen, “RNP-SA: joint relay placement and sub-carrier allocation in wireless communication networks with sustainable energy,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 10, pp. 3818-3828, Oct. 2012.

[21] M. Marsan, M. Meo, “Energy efficient management of two cellular access networks;” *ACM SIGMETRICS Performance Evaluation Review*, vol. 37, no. 4, pp. 69-73, Mar. 2010.

[22] K. Son, H. Kim, Y. Yi, B. Krishnamachari, “Base station operation and user association mechanisms for energy-delay tradeoffs in green cellular networks,” *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1525-1536, Aug. 2011.

[23] S. Zhou, A. J. Goldsmith, Z. Niu, “On optimal relay placement and sleep control to improve energy efficiency in cellular networks,” *Proc. IEEE ICC*, pp. 1-6, 2011.

[24] E. Oh, K. Son, B. Krishnamachari, “Dynamic base station switching-on/off strategies for green cellular networks,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2126-2136, May 2013.

[25] T. Han, N. Ansari, “On optimizing green energy utilization for cellular networks with hybrid energy supplies,” *IEEE Trans. Wireless Commun.*, vol. 12, No. 8, pp. 3872-3882, Aug. 2013.

[26] I. Ahmed, A. Ikhlef, R. Schober, R. K. Mallik, “Power allocation for direction-dependent and bi- directional Link adaptive relay systems with energy harvesting nodes,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp.1182-1195, Mar. 2014.

[27] J. Laneman, D. Tse, G. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[28] C. Gao, W. Zhang, J. Tang, C. Wang, S. Zou, S. Su, “Relax, but do not sleep: a new perspective on green wireless networking,” in Proc. *IEEE INFOCOM*, Toronto, Canada, Apr. 2014.

[29] W. Liao, M. Li, S. Salinas, P. Li, M.S. Pan, “Energy-source-aware cost optimization for green cellular networks with strong stability,” *IEEE Trans. Emerging Topics Comput.*, no. 1, pp. 1-6, PrePrints, doi:10.1109/ETC.2014.6786912.

[30] L. Georgiadis, M. J. Neely, L. Tassiulas, Resource allocation and cross-layer control in wireless networks, Foundations Trends Netw., 2006.

[31] M. Neely, Stochastic network optimization with application to communication and queueing systems. Morgan & Claypool Publishers, 2010.

[32] Y. Li, M. Sheng, Y. Shi, X. Ma, W. Jiao, “Energy efficiency and delay tradeoff for time-varying and interference-free wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 5921-5931, Nov. 2014.

[33] T. C.-Y. Ng and W. Yu, “Joint Optimization of Relay Strategies and Resource Allocations in a Cooperative Cellular Network,” *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 328-339, Feb. 2007.

[34] J. Huang, V. G. Subramanian, R. Agrawal, R. Berry, “Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 2, pp. 226-234, Feb. 2009.

[35] B. Johansson, P. Soldati, M. Johansson, “Mathematical decomposition techniques for distributed cross-layer optimization of data networks,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1535-1547, Aug. 2006.

[36] W. Li, J. Lei, T. Wang, et al., “Dynamic Optimization for Resource Allocation in Relay Aided OFDMA Systems Under Multi-Service,” *IEEE Veh. Technol. Mag.*, to be published. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=7056500&tag=1

[37] X. Zhu, B. Yang, C. Chen, L. Xue, X. Guan, F. Wu, “Cross-layer scheduling for OFDMA-based cognitive radio systems with time delay and security constraints,” *IEEE Trans. Veh. Technol.*, to be published. [Online]. Available: http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=7012108