Quark/gluon content of $\eta(1295)$ and $\eta(1440)$

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The quark/gluon content of $\eta(1295)$ and $\eta(1440)$ mesons is discussed, mesons being considered as members of the first radial excitation $2^3 S_0$ $q\bar{q}$ nonet. Recent results on $\eta(1295)$ and $\eta(1440)$ two-photon widths from L3 together with the information on radiative $J/\Psi$ decay allow one to evaluate the $\eta(1295)/\eta(1440)$ mixing angle and the admixture of the glueball component. We found that $\eta(1440)$ is predominantly non-strange $q\bar{q}$ state, with a possible admixture of the glueball component (20 ± 20)%.

I. INTRODUCTION

The investigation of pseudoscalar-isoscalar states ($IJ^{PC} = 00^{-+}$) is an important task for the $q\bar{q}$ nonet classification and search for exotic states. In this way the study of nature of $\eta(1295)$ and $\eta(1440)$ which are considered as members of the first radial excitation nonet is a crucial step.

During last decades $\eta(1440)$ has been considered as a probable candidate for the glueball. This hypothesis is based on the fact that $\eta(1440)$ is produced in a gluon-rich reaction like radiative $J/\Psi$ decay. At present this interpretation does not agree with lattice QCD calculations [1,2] which predict masses of pure gluonic $0^{-+}$ state in the region above 2 GeV.

Although there is more sophisticated interpretation of $\eta(1440)$ as a bound state of gluinos [3], it seems that the interpretation of $\eta(1440)$ as a member of the first radial excitation $2^3 S_0$ $q\bar{q}$ nonet is the most preferable. As was demonstrated in [4] the majority of $q\bar{q}$ trajectories on the $(n, M^2)$ and $(J, M^2)$ plots (n is radial quantum number and J is meson spin) are linear, with nearly the same slope. In this scheme $\eta(1295)$ is considered as a flavon partner of $\eta(1440)$. The recent results from GAMS [5] and E852 [6] Collaborations give a clear confirmation of the existence of $\eta(1295)$ resonance.

The existence of two $0^{-+}$ resonances near 1440 MeV, namely, $\eta_L$ and $\eta_H$, needs special discussion. According to PDG-2000 [7] there is now fairly consistent picture for two pseudoscalars. It was claimed that $\eta_L$ decays mainly into $a_0(980)\pi$ or directly to $K\bar{K}\pi$, while the second one, $\eta_H$, is seen in $KK\pi$ channel only, coming from $K^*(892)K$ intermediate state.

It seems that this opinion is due to the circumstance that most analyses have been done with the Breit-Wigner constant width. In fact, since the $K^*K$ has the threshold at 1394 MeV, its phase space increases rapidly at 1400-1500 MeV, thus rather strong s-dependence of the width in this channel takes place. As was shown in [6], the data on $J/\Psi \to \gamma(K^+K^-\pi^0)$ and $J/\Psi \to \gamma(\eta\pi^+\pi^-)$ can be well fitted as only one Breit-Wigner resonance, with s-dependent width. This gives a mass shift between $\eta\pi\pi$ and $KK\pi$ channels about 40 MeV. Also the analysis of $pp \to \eta\pi^+\pi^-\pi^+\pi^-$ shows an evidence that the $\eta\pi\pi$ channel is fed by triangle diagrams of the form $\eta(1440) \to K^*K$, followed by $K\bar{K}$ rescattering via $a_0(980) \to \eta\pi$.

Let us also note that there is an evidence [8] for the existence of a broad $0^{-+}$ meson which was seen in radiative $J/\Psi$ decay and has the mass around 2200 MeV. It was suggested in [9] that this state displayed exotic characteristics and can be identified as glueball. In this case this glueball state, due to its large width ($\sim$ 800 MeV), can mix with $\eta(1440)$. The question is whether this mixture is small or large.

Recently the $e^+e^- \to e^+e^-K_S^0K^\pm\pi^\mp$ and $e^+e^- \to e^+e^-\eta\pi^+\pi^-$ final states were studied [10] with L3 detector at LEP. The performed analysis shows the clear signal from $\eta(1440)$ in untagged $\gamma\gamma$ collisions in the $K_S^0K^\pm\pi^\mp$ decay channel. The value of its two-photon width is found to be

$$\Gamma_{\gamma\gamma}(\eta(1440)) \times BR(\eta(1440) \to K\bar{K}\pi) = 212 \pm 50(\text{stat}) \pm 23(\text{sys}) \text{ eV}.$$  (1)

In the $\eta\pi^+\pi^-$ decay channel neither $\eta(1440)$ nor $\eta(1295)$ were observed, and the following upper limits for their two-photon widths were determined:

$$\Gamma_{\gamma\gamma}(\eta(1440)) \times BR(\eta(1440) \to \eta\pi\pi) < 95 \text{ eV}.$$  (2)

$$\Gamma_{\gamma\gamma}(\eta(1295)) \times BR(\eta(1295) \to \eta\pi\pi) < 66 \text{ eV}.$$  (3)

The transition amplitudes $\eta(1295) \to \gamma\gamma$ and $\eta(1440) \to \gamma\gamma$ were calculated in [11] under the assumption that the decaying mesons are members of the first radial excitation $2^1 S_0$ $q\bar{q}$ nonet. The calculations show that partial widths strongly depend on the mixing angle between $n\bar{n}$ and $s\bar{s}$ components of the mesons: the main contribution is due to
the $n\bar{n}$ meson component, while the contribution of the $s\bar{s}$ component is small. Neglecting a possible mixture with glueball state, partial widths are found to be of the order of 100 eV.

The purpose of the present paper is to enlighten the quark/gluon content of $\eta(1295)$ and $\eta(1440)$ using the two-photon widths and radiative $J/\Psi$ decays of these mesons.

II. PARTIAL WIDTHS FOR THE DECAYS $\eta(1295) \rightarrow \gamma\gamma$ AND $\eta(1440) \rightarrow \gamma\gamma$

It is well known that isoscalar member of basic pseudoscalar octet mixes with corresponding pseudoscalar singlet to produce $\eta$ and $\eta\prime$. The same situation is expected for members of the first radial excitation nonet, $\eta(1295)$ and $\eta(1440)$. These states can also mix with the glueball component.

Taking into account the mixing of non-strange quark component, $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$, with strange one, $s\bar{s}$, and with glueball component, $G$, the wave functions are determined as follows:

\[
\Psi_{\eta(1440)} = \cos \phi \sin \theta \psi_{n\bar{n}} + \sin \phi \psi_{s\bar{s}}, \quad \text{(4)}
\]

\[
\Psi_{\eta(1295)} = \cos \phi' \sin \theta' \psi_{n\bar{n}} - \sin \phi' \psi_{s\bar{s}} + \sin \phi' \psi_{G}, \quad \text{(5)}
\]

where $\theta$ and $\theta'$ are mixing angles between non-strange and strange components of the wave function, and $\phi$, $\phi'$ determine the admixture of the glueball state. The orthogonality condition reads

\[
\cos \phi \cos \phi' \sin(\theta' - \theta) + \sin \phi \sin \phi' = 0. \quad \text{(6)}
\]

The two-photon decays of $\eta$ and $\eta\prime$ were analysed in [3]. The data on transition form factors $\pi^0 \rightarrow \gamma^\ast(Q^2)\gamma$, $\eta \rightarrow \gamma^\ast(Q^2)\gamma$, $\eta\prime \rightarrow \gamma^\ast(Q^2)\gamma$ over a broad range of photon virtualities, $Q^2 \leq 20$ GeV$^2$ [17], made it possible:

(i) to restore wave functions of $\eta$ and $\eta\prime$ (for both $n\bar{n}$ and $s\bar{s}$ components),

(ii) to estimate gluonium admixture in $\eta$ and $\eta\prime$,

(iii) to restore vertex function for the transition $\gamma \rightarrow q\bar{q}$ (or photon wave function) as a function of the $q\bar{q}$ invariant mass.

A. Wave functions and transition form factors

Both non-strange and strange components of the $\eta(M)$-meson wave functions are parametrized in the exponential form. For the basic and first radial excitation nonets, the wave functions are determined as follows:

\[
\Psi^{(0)}_\eta(s) = Ce^{-bs}, \quad \Psi^{(1)}_\eta(s) = C_1 (D_1 s - 1)e^{-b_1 s}, \quad \text{(7)}
\]

where $s$ is the $q\bar{q}$ invariant mass squared. Parameters $b$ and $b_1$ are related to the radii squared of corresponding $\eta(M)$-mesons. Then the other constants ($C$, $C_1$, $D_1$) are fixed by the normalization and orthogonality conditions:

\[
\Psi^{(0)}_\eta \otimes \Psi^{(0)}_\eta = 1, \quad \Psi^{(1)}_\eta \otimes \Psi^{(1)}_\eta = 1, \quad \Psi^{(0)}_\eta \otimes \Psi^{(1)}_\eta = 0. \quad \text{(8)}
\]

The convolution of the $\eta(M)$-meson wave function determines the form factor of $\eta$-meson, $f^{(n)}_\eta(q^2) = \left[ \Psi^{(n)}_\eta \otimes \Psi^{(n)}_\eta \right]$, thus allowing us to relate the parameter $b$ (or $b_1$) at small $Q^2$ to the $\eta$-meson radius squared: $f_q(q^2) \simeq 1 - \frac{1}{2} R^2_\eta Q^2$.

It is useful to compare the results of calculation of the transition form factor approximated by simple exponential wave function with those obtained using more sophisticated wave function. Such a comparison can be done for basic $1S_0$ $q\bar{q}$ nonet. The $\eta$ and $\eta\prime$ wave functions (or those for its $n\bar{n}$ and $s\bar{s}$ components) were found in [18] basing on the data for the transitions $\eta \rightarrow \gamma\gamma^\ast(Q^2)$, $\eta \rightarrow \gamma\gamma^\ast(Q^2)$ at $Q^2 \leq 20$ GeV$^2$. The calculated decay form factors $F^{(n)}_{n\bar{n} \rightarrow \gamma\gamma}$ and $F^{(0)}_{s\bar{s} \rightarrow \gamma\gamma}$ at $Q^2 = 0$ for these wave functions are marked in Fig. 1b by rhombuses. The wave functions found in Ref. [18] give the following mean radii squared for $n\bar{n}$ and $s\bar{s}$ components: $R^2_{n\bar{n}} = 13.1$ GeV$^{-2}$ and $R^2_{s\bar{s}} = 11.7$ GeV$^{-2}$.

Solid curves in Fig. 1a represent $F^{(0)}_{n\bar{n} \rightarrow \gamma\gamma}$ and $F^{(0)}_{s\bar{s} \rightarrow \gamma\gamma}$ calculated by using exponential parametrisation (8) we see that both calculations coincide with each other within reasonable accuracy. The coincidence of the results justifies exponential approximation for the wave function used in the calculation of transition form factors at $Q^2 \sim 0$.

The Fig. 1b shows the calculation results obtained in [13] for transition form factors of the first radial excitation nonet at $Q^2 = 0$. The form factor of the $n\bar{n}$ component, $F^{(1)}_{n\bar{n} \rightarrow \gamma\gamma}$, depends strongly on the mean radius squared, increasing rapidly for $R^2_{n\bar{n}} \sim 14 - 24$ GeV$^{-2}$ (0.7-1.2 fm$^2$). As to $s\bar{s}$ component, the form factor $F^{(1)}_{s\bar{s} \rightarrow \gamma\gamma}$ is small and it changes sign at $R^2_{s\bar{s}} \simeq 15$ GeV$^{-2}$. 

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The results shown in Fig. need to be commented. The dominant contribution from the non-strange component of the $\eta$-meson wave function comes mainly from the transition form factors. For case of $F_{n\bar{n}\to\gamma\gamma}^{(n)}$ this factor is equal to $(q_u^2 + q_d^2)/\sqrt{2}$, while for $F_{n\bar{s}\to\gamma\gamma}^{(n)}$ it is $q_s^2$, where $q_u$, $q_d$, $q_s$ are the quark charges. It is clear that for basic nonet the ratio between $n\bar{n}$ and $s\bar{s}$ transition form factors is just the ratio between charge factors. In case of radial excitation nonet the wave function can change sign and the difference between non-strange quark mass ($m_n = 0.350$ GeV) and strange quark mass ($m_s = 0.500$ GeV) affect different behaviour of the transition form factors.

Partial width for $\eta(M)$ decaying into $\gamma\gamma$ is given by

$$
\Gamma_{\eta(M)\to\gamma\gamma} = \frac{\pi}{4} \alpha^2 M^3 (F_{\eta(M)\to\gamma\gamma}^{(1)})^2 ,
$$

where $\alpha = 1/137$ and

$$
F_{\eta(1295)}^{(1)} = \cos \phi' \left[ \cos \theta' F_{n\bar{n}}^{(1)} - \sin \theta' F_{s\bar{s}}^{(1)} \right] ,
$$
$$
F_{\eta(1440)}^{(1)} = \cos \phi \left[ \sin \theta F_{n\bar{n}}^{(1)} + \cos \theta F_{s\bar{s}}^{(1)} \right] .
$$

III. $J/\Psi \to \gamma\eta(M)$ DECAY

The radiative $J/\Psi$ decay is a source of additional information about quark/gluon content of $\eta$ mesons. The radiative $J/\Psi$ decay is dominated by the annihilation of $c\bar{c}$ quarks, with the production of photon and two gluons which form $q\bar{q}$ mesons. The amplitude for such a process is proportional to the convolution of two wave functions $A \sim \Psi_{J/\Psi\to\gamma\Psi_{meson}}$, and corresponding branching ratio is $BR = | A |^2 \times phase\ volume$. So the square of the production amplitude is proportional to the probability of the glueball component $R$ in the meson. For $\eta(M)$ meson we have

$$
R = \frac{BR(J/\Psi \to \gamma\eta(M))}{(M_{J/\Psi}^2 - M^2)^3} .
$$

Let us compare probabilities $R$ for $\eta'$ and $\eta(1440)$. Following we use

$$
BR(J/\Psi \to \gamma\eta') = 4.3 \times 10^{-3}
$$
and

$$
BR(J/\Psi \to \gamma\eta(1440)) = 3.0 \times 10^{-3}.
$$

The calculations show that for $\eta'$ and $\eta(1440)$ the values of $R$ are close to each other. The result of Ref. shows that $\eta'$ contains $(10 \pm 10)\%$ of the glueball component. It means that we can expect the same amount of glueball for $\eta(1440)$.

The $\eta(1295)$ meson is not seen in radiative $J/\Psi$ decay. The assumption that

$$
BR(J/\Psi \to \gamma\eta(1295)) = 0
$$
allows us to estimate the mixing angle between non-strange and strange $q\bar{q}$ component in $\eta(1295)$ and $\eta(1440)$. Since there is no glueball admixture in $\eta(1295)$ and $\sin \phi' = 0$, the orthogonality condition gives us the same mixing angles for $\eta(1295)$ and $\eta(1440)$: $\theta = \theta'$. As a result, we come to the following mixing scheme:

$$
\Psi_{\eta(1295)} = \cos \theta \psi_{n\bar{n}} - \sin \theta \psi_{s\bar{s}} ,
$$
$$
\Psi_{\eta(1440)} = \cos \phi [\sin \theta \psi_{n\bar{n}} + \cos \theta \psi_{s\bar{s}}] + \sin \phi \psi_G .
$$

Then transition form factors are equal to

$$
F_{\eta(1295)}^{(1)} = \cos \theta F_{n\bar{n}}^{(1)} - \sin \theta F_{s\bar{s}}^{(1)} ,
$$
$$
F_{\eta(1440)}^{(1)} = \cos \phi \left[ \sin \theta F_{n\bar{n}}^{(1)} + \cos \theta F_{s\bar{s}}^{(1)} \right] .
$$
The assumption (13) also implies that \( \eta(1295) \) is close to the SU(3) octet because the glueball is close to the SU(3) singlet. In fact there is no SU(3) flavour symmetry, and the suppression parameter \( \lambda \) for the production of \( s\bar{s} \) pair is as follows [19,20]:

\[
\lambda = 0.6 \pm 0.2. 
\]  

Let us introduce flavour non-symmetrical singlet and octet states:

\[
\Psi_1 = \frac{1}{\sqrt{3}} (\Psi_{u\bar{u}} + \Psi_{d\bar{d}} + \sqrt{\lambda} \Psi_{s\bar{s}}), 
\]

\[
\Psi_8 = \frac{1}{\sqrt{2 + 4/\lambda}} (\Psi_{u\bar{u}} + \Psi_{d\bar{d}} - 2 \sqrt{\lambda} \Psi_{s\bar{s}}). 
\]

Glueball may turn into \( \Psi_1 \) while the transition "glueball \( \rightarrow \Psi_8 \)" is forbidden. This means that \( \Psi_{\eta(1295)} = \Psi_8 \) and therefore

\[
\cos \theta = \sqrt{\frac{\lambda}{2 + \lambda}}. 
\]

**IV. RESULTS**

Since the two-photon width of \( \eta(1440) \) was found in \( KK\pi \) decay channel we can estimate this branching ratio. The BES collaboration claimed [6] that \( \eta\pi\pi \) decay is weak: 10 – 20% of \( K\bar{K} \). Similar ratios have been found in the by Mark III [13] and DM2 [14].

On the opposite, the data on \( p\bar{p} \rightarrow \eta(1440)\sigma \) from [16] give the following ratio:

\[
BR(\eta(1440) \rightarrow KK\pi)/BR(\eta(1440) \rightarrow \eta\pi\pi) = 0.61 \pm 0.19. 
\]  

A large discrepancy between these two samples of data can be explained by the fact that in \( p\bar{p} \) annihilation the phase space available in the \( \sigma \) amplitude is limited; furthermore, its amplitude goes to zero for low \( \sigma \) mass. The result is a strong suppression of the upper side of \( \eta(1440) \). As was shown in [6], the suppression increases in a factor 3 between 1440 MeV and 1465 MeV which makes weaker the process \( \eta(1440) \rightarrow K^*\bar{K} \) in \( p\bar{p} \) annihilation.

Using eq.(22), we assume the following limits for the \( \eta(1295) \) and \( \eta(1440) \) two-photon widths:

\[
160 \text{ eV} < \Gamma_{\gamma\gamma}(\eta(1440)) < 300 \text{ eV}, 
\]

\[
\Gamma_{\gamma\gamma}(\eta(1295)) < 66 \text{ eV}. 
\]  

Fig. 2 shows us the allowed region for mixing angle \( \theta \) and glueball admixture \( \phi \) which comes from the constraints [23] and [24]. Since the transition form factors depend strongly on \( R^2 \), the allowed \((\theta, \phi)\) region depends on \( R^2 \). For larger \( R^2 \) it is possible that \( \eta(1440) \) has more glueball admixture. The condition [24] restricts the amount of \( n\bar{n} \) component in \( \eta(1295) \) as well, thus \( \eta(1440) \) is expected to be mainly \( n\bar{n} \) state for different \( R^2 \). One can expect that the first radial excitation state is larger than the basic one, and it seems natural to consider mean radius squared in the region \( R^2 \sim 15 – 20 \text{ GeV}^{-2} \). Together with additional limits (21) for mixing angle, we have:

\[
0.84 < \sin \theta < 0.91, 
\]

and we come to the following limits for the glueball admixture in \( \eta(1440) \):

\[
0 < \sin^2 \phi < 0.40. 
\]  

Let us note that this result does not contradict the naive estimation which follows from the comparison of radiative \( \eta' \) and \( \eta(1440) \) decays.

At first glance the obtained results, which comes from the two-photon decay and radiative \( J/\psi \) decay, contradict the experimental branching ratios for \( \eta(1295) \) and \( \eta(1440) \). The \( \eta(1440) \) has a strong \( KK\pi \) decay mode. For example, BES Collaboration found [6] the branching ratios to \( KK^*, \bar{K}K_\eta \) and \( \eta\pi\pi \) in the ratios 1.92: 0.83: 1. Meanwhile, all recent observations of \( \eta(1295) \) have been done in \( \eta\pi\pi \) decay mode. If \( \eta(1295) \) has a large \( s\bar{s} \) component, one can expect the dominant decay mode with strange mesons.
Let us discuss in detail the meson decay mechanism. Experimental data indicate that the decays of $\eta(1295)$ and $\eta(1440)$ are the cascade reactions so first the two-meson state is produced. In our case it can be $KK^*$, $KK_0$ or $a_0(980)\pi$. The quark combinatorial rules [21] may be applied to the calculation of corresponding decay couplings. There exist two types of transitions "$qq \rightarrow \text{two mesons}$" shown in Fig. 3. The type of process represented by the diagram of Fig. 3a is leading one, according to the rules of $1/N$ expansion. The coupling constants for $\eta$ meson decaying into two strange mesons ($KK^*$ or $KK_0$) in the leading order of $1/N$ expansion are:

$$g_{\eta(1440)} \rightarrow KK = g^L(\sqrt{\lambda} \sin \theta + \sqrt{2} \cos \theta),$$

$$g_{\eta(1295)} \rightarrow KK = g^L(\sqrt{\lambda} \cos \theta - \sqrt{2} \sin \theta),$$

(27)

where the parameter $g^L$ hides the unknown dynamics of the decay. With $\sin \theta = 0.85$ and $\lambda = 0.6$ the flavour factor gives the suppression of the $\eta(1295)$ coupling constant by a factor 2 (factor 4 in branching ratio) as compared with $\eta(1440)$ coupling constant. Another suppression comes from the kinematical factor, since phase volume for $KK^*$ and $KK_0$ in $\eta(1295)$ decay is much smaller than in $\eta(1440)$ decay. These arguments can explain the fact why no $KK\pi$ is observed in $\eta(1295)$ decay.

Finally, we need to comment the fact that $\eta(1440)$, with large non-strange $qq$ component, has a higher mass than $\eta(1295)$ state which is $ss$-dominant one. This can be the result of the $qq \rightarrow \text{gluons}$ transitions which can be different for singlet and octet states, thus shifting the mass of meson. Let us remind that similar situation takes place in the ground nonet where $\eta'$ is heavier than other nonet members (so called $\text{U}_A(1)$ problem [22]). There are different models which explain this phenomenon, for example, instanton approach [23], and it is possible that the instanton-induced forces can also give such effect for $\eta(1295)$ and $\eta(1440)$.

V. CONCLUSION

Recent results on $\eta(1295)$ and $\eta(1440)$ two-photon widths from L3 [12] together with the information from radiative $J/\Psi$ decay support the hypothesis that these resonances are members of the first radial excitation $2^1S_0$ $qq$ nonet. We have determined the $\eta(1295)/\eta(1440)$ mixing angle and admixture of the glueball component using the calculation scheme developed in [15]. We argue that $\eta(1440)$ is dominantly non-strange $qq$ state, with possible admixture of glueball component (20 ± 20)% while $\eta(1295)$ is mainly $ss$ state with small gluonium component. These estimations are essentially based on the fact that there is just one resonance state near 1440 MeV, so further analysis of the experimental data is needed to solve this problem.

VI. ACKNOWLEDGEMENTS

I thank V.V Anisovich, D. Bugg and I. Vodopianov for useful discussions and comments. Special thank M. Kienzle-Focacci for invitation to visit Geneva University where most of the work was done.

The work was partly supported by the RFBR grant 01-02-17861.

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FIG. 1. (a) Transition form factors as function of mean radius squared of $\eta(M)$ for the basic nonet. Solid curves present calculations with exponential parametrization $[\mathbf{13}]$; the rhombuses give values calculated with the wave function found in $[\mathbf{18}]$. (b) The transition form factors as function of mean radius squared of $\eta(M)$ for the first radial-excitation nonet.
FIG. 2. Allowed $(\theta, \phi)$-region following from the conditions (23) and (24) at different mean radius squared of $\eta(1440)$. 
FIG. 3. Diagrams for the decay of the $q\bar{q}$-state into two mesons.