Charmonium decay widths in matter in a field theoretic model for composite hadrons

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Abstract

We calculate the decay widths of the charmonium states, $J/\psi$, $\psi(3686)$ and $\psi(3770)$, to $D\bar{D}$ pairs in isospin asymmetric strange hadronic matter, using a field theoretical model for composite hadrons with quark constituents. For this purpose we use a quark antiquark pair creation term that arises within the model, and then use explicit charmonium and $D$ and $\bar{D}$ states to evaluate the decay amplitudes. The medium modifications of these partial decay widths, arising from the mass modifications of the $D(\bar{D})$ and the charmonium states calculated in a chiral effective model, are also included. The results of the present investigations are then compared with the decay widths computed earlier, using an ad hoc light quark pair creation in $^{3}P_{0}$ state. The effects of the isospin asymmetry, the strangeness fraction of the hadronic matter on the masses of the charmonium states and $D(\bar{D})$ mesons and hence on the decay widths, have also been studied. The isospin asymmetry effect is observed to be dominant for high densities, leading to appreciable difference in the decay channels of the charmonium to $D^{+}D^{-}$ and $D^{0}\bar{D}^{0}$ pairs. The density modifications of the decay widths of the charmonium states should show up in experimental observables, like the production of the $J/\psi$ and open charm mesons, in asymmetric heavy ion collisions in the compressed baryonic matter (CBM) experiments at the future facility of FAIR, GSI.

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I. INTRODUCTION

The study of the properties of hadrons in the medium is an important and challenging topic of research in strong interaction physics. The topic is of direct relevance in the context of heavy ion collision experiments, which probe matter under extreme conditions, for example at high temperatures and/or densities. The properties of the hadrons as modified in the medium have direct consequences on the experimental observables from the strongly interacting matter produced in heavy ion collision experiments. The properties of the kaons and antikaons in the medium are of relevance in neutron star phenomenology where an attractive interaction of antikaon-nucleon can lead to antikaon condensation in the interior of the neutron stars [1]. The medium modifications of kaons and antikaons can also affect observables like, the production as well as collective flow of the kaons and antikaons in the heavy ion collision experiments. The modifications of the masses of the charm mesons, $D$ and $\bar{D}$ as well as the $J/\psi$ mesons and the excited states of charmonium, can have important consequences on the yield of open charm mesons as well as of $J/\psi$ in heavy ion collision experiments. Also, in high energy heavy ion collision experiments at RHIC as well as LHC, the formation of the quark-gluon-plasma (QGP) [2, 3] can lead to the suppression of $J/\psi$.

The $D$ ($\bar{D}$) mesons are made up of one heavy charm quark (antiquark) and one light (u or d) antiquark (quark). In the QCD sum rule approach, the modifications of the masses of the $D$ ($\bar{D}$) mesons in the hadronic matter are due to the interactions of light antiquark (quark) present in the $D$($\bar{D}$) mesons with the light quark condensate [4, 5]. The properties of the $D$($\bar{D}$) meson have also been studied using the quark meson coupling (QMC) model [6] as well as the coupled channel approach [7–11]. In a recent study [12], the $D$-nucleon interactions have been studied using a quark model for the hadrons, in which the baryons/mesons are constructed as bound states of the constituent quarks (antiquarks). The field operators of the constituent quarks are written in terms of a momentum dependent mass, $M(k)$ which arises from dynamical chiral symmetry breaking [12–15]. The masses of the charmonium states, which are made up of a heavy charm quark and a charm antiquark, are due to their interaction with the gluon condensates. This is because all the heavy quark condensates can be related to the gluon condensates via heavy-quark expansion [16]. Also in the nuclear medium there are no valence charm quarks to leading order in density and any interaction
with the medium is gluonic. The QCD sum rule approach \cite{17} and leading order perturbative calculations \cite{18} to study the medium modifications of charmonium, show that the mass of $J/\psi$ is reduced only slightly in the nuclear medium. In Ref. \cite{19}, the mass modification of charmonium has been studied using leading order QCD formula and the linear density approximation for the gluon condensate in the nuclear medium. There is seen to be a small drop in the mass of the $J/\psi$ in the nuclear medium, whereas the drop in the masses of the excited states of charmonium ($\psi(3686)$ and $\psi(3770)$) are found to be significant. The mass modification of the $J/\psi$ state has also been studied from the self energy of $J/\psi$ through a $D$ meson loop \cite{19}, as well as due to a $D^*$ meson loop \cite{20}. In the present work, we study the medium modification of the decay widths of $J/\psi$ and the excited charmonium states $\psi(3686)$ and $\psi(3770)$ to $D\bar{D}$ pairs in the strange hadronic medium using a field theoretical model for the composite hadrons with quark constituents \cite{21–23}.

The outline of the paper is as follows: In section II, we give a very brief introduction of the field theoretical model for the hadrons with quark constituents using explicit constructions of the charmonium states as well as the $D$ and $\bar{D}$ mesons in terms of the quark and/or antiquark constituents. We then calculate the matrix element of the S-matrix in the lowest order to compute the decay width of the charmonium states to $D^+D^-$ or $D^0\bar{D}^0$ pairs. The decay widths of the charmonium states using a quark pair creation model, namely the $^3P_0$ model, are also briefly described to compare the results of the present investigation with these existing results in the literature. In section III, we briefly describe the chiral effective model \cite{24}, used to investigate the in-medium masses of the open charm mesons ($D(D^0, D^+)$ and $\bar{D}(\bar{D}^0, D^-)$) and of the charmonium states. The in-medium properties of the $D$ and $\bar{D}$ mesons arise due to their interactions with the light hadrons, i.e., the nucleons, hyperons and scalar mesons. Within the chiral effective model, the scale symmetry breaking of QCD has also been incorporated through a scalar dilaton field. The masses of the charmonium states in the hadronic medium have been calculated from the modification of the gluon condensates \cite{24} arising within the effective hadronic model from the modification of the dilaton field in the strange hadronic matter. In section IV, we discuss the results obtained in the present investigation and compare with the existing results of the partial decay widths \cite{24, 25} using a quark pair creation model, the $^3P_0$ model \cite{26, 27}, where a light quark antiquark pair is assumed to be created in the $^3P_0$ state, and the light quark (antiquark) combines with
the heavy charm antiquark (charm quark) of the decaying charmonium state to produce the open charm $D$ and $\bar{D}$ mesons. In section V, we summarize the results for the medium modifications of the charmonium decay widths calculated in the present field theoretic model for the composite hadrons and discuss possible outlook.

II. THE MODEL FOR COMPOSITE HADRONS

We shall very briefly discuss the model to clarify the notations, so as to apply the same in the present problem.

The field operator at $t=0$ for a constituent quark for a hadron at rest is taken as

$$\psi(x, t = 0) = Q(x) + \bar{Q}(x),$$  \hspace{1cm} (1)

where $Q(x)$ and $\bar{Q}(x)$ are the quark annihilation and antiquark creation operators and are given as

$$Q(x) = (2\pi)^{-\frac{3}{2}} \int U(k) Q_I(k) \exp(i k \cdot x) d k$$  \hspace{1cm} (2)

and

$$\bar{Q}(x) = (2\pi)^{-\frac{3}{2}} \int V(k) \bar{Q}_I(k) \exp(-i k \cdot x) d k.$$  \hspace{1cm} (3)

In the above, $Q_I(k)$ and $\bar{Q}_I(k)$ are the two component quark annihilation and antiquark creation operators, given as $Q_I(k) = Q_{Ir}(k) u_{Ir}$ and $\bar{Q}_I(k) = \bar{Q}_{Is}(k) v_{Is}$. The summation over the dummy indices is understood. $Q_{Ir}(k)$ annihilates a quark with spin $r$ and momentum $k$ and $\bar{Q}_{Is}(k)$ creates an antiquark with spin $s$ and momentum $k$ and they satisfy the usual anticommutation relations

$$\{Q_{Ir}(k), Q_{Is}(k')\} = \{\bar{Q}_I(k), \bar{Q}_I(k')\} = \delta_{rs} \delta(k - k')$$ \hspace{1cm} (4)

In equations (2) and (3), $U(k)$ and $V(k)$ are given as

$$U(k) = \begin{pmatrix} f(k^2) \\ \sigma \cdot k g(k^2) \end{pmatrix}, \hspace{1cm} V(k) = \begin{pmatrix} \sigma \cdot k g(k^2) \\ f(k^2) \end{pmatrix},$$ \hspace{1cm} (5)

where the equal time anticommutation relation for the four-component Dirac field operators gives the constraint \cite{21}

$$f^2 + g^2 k^2 = 1,$$ \hspace{1cm} (6)
on the functions \( f(k) \) and \( g(k) \). For free Dirac field of mass \( M \), we have

\[
f(k) = \left( \frac{k_0 + M}{2k_0} \right)^{1/2}, \quad g(k) = \left( \frac{1}{2k_0(k_0 + M)} \right)^{1/2},
\]

(7)

where \( k_0 = (k^2 + M^2)^{1/2} \). The field operator expansion for the constituent quark is written as above, where the constituent quark mass, \( M \) obtained from dynamical chiral symmetry breaking, is no longer a constant, but is a momentum dependent function \[12, 13, 15\]. Within an effective model for the quarks, the momentum dependent mass function, \( M(k) \) is obtained by solving a gap equation. However, the constituent quark mass is observed to change appreciably only at large momenta \[12\]. Hence, we shall take a small momentum expansion, which yields the functions \( f(k) \) and \( g(k) \) of the Dirac field operator for the constituent quarks as \[12\]

\[
g(k) \approx \frac{1}{2M}, \quad f(k) \approx 1 - \frac{1}{2}g^2k^2,
\]

(8)

where \( M = M(k = 0) \).

The above field operators for the quarks are for the constituents of hadrons at rest. To describe these as constituents of hadron with finite momentum, we need to suitably Lorentz boost these operators, which requires the knowledge of the time dependence of the field operators in addition to the space dependence as above. This was taken to be given by quarks occupying fixed energy levels \[21, 22\] as in the bag model, so that for the \( i \)-th quark we have

\[
Q_i(x) = Q_i(x) \exp(-i\lambda_i Mt),
\]

(9)

where \( \lambda_i \) is the fraction of the energy (mass) of the hadron carried by the quark, with \( \sum_i \lambda_i = 1 \). Eq. (9) is for hadrons at rest, and, for a hadron in motion with four momentum \( p \), with appropriate Lorentz boosting \[23\] for \( t=0 \) is given as

\[
Q^{(p)}(x, 0) = (2\pi)^{-\frac{3}{2}} \int d\mathbf{k} S(L(p))U(k)Q_I(k + \lambda p) \exp(i(k + \lambda p) \cdot x).
\]

(10)

For the antiquark creation operator we similarly have for \( t=0 \)

\[
\tilde{Q}^{(p)}(x, 0) = (2\pi)^{-\frac{3}{2}} \int d\mathbf{k} S(L(p))V(-k)\tilde{Q}_I(-k + \lambda p) \exp(-i(-k + \lambda p) \cdot x)
\]

(11)

The form of Lorentz boosting that gives the constituent quark field operators as above was chosen in \[23\] as it gave the correct form of diffraction scattering along with the earlier results.
The determination of the $\lambda_i$'s, which correspond to the fractions of energy (mass) carried by the constituent quarks in the hadron, will be explicitly described in the next section when we construct the $D$-meson states with the constituent quarks.

**A. Decay widths of the charmonium states to $D\bar{D}$ pair in the composite model of the hadrons**

In the present work, we shall study the medium dependence of the partial decay widths of the charmonium states, $J/\psi$, $\psi(3686)$ and $\psi(3770)$ to $D\bar{D}(D^+D^-\text{or}D^0\bar{D}^0)$. In the hadronic medium, the masses of the $D$ and $\bar{D}$ are observed to be different due to the difference in their interactions with the hadronic matter. Accounting for this fact, the magnitude of $p$, the 3-momentum of the $D$ ($\bar{D}$) meson, when the charmonium state $\psi$ decays at rest, is given by

$$|p| = \left( \frac{m_{\psi}^2}{4} - \frac{m_{D}^2 + m_{\bar{D}}^2}{2} + \frac{(m_{D}^2 - m_{\bar{D}}^2)^2}{4m_{\psi}^2} \right)^{1/2}. \quad (12)$$

1. $\psi'' \to D^+D^-$

We note that the masses of $\psi'' \equiv \psi(3770)$ and $D^\pm$ in the vacuum are given as

$$m_{\psi''} = 3773 \text{ MeV}; \quad m_{D^\pm} = 1869 \text{ MeV} \quad (13)$$

so that this decay is admissible in vacuum. The in-medium effects of the decay widths will be given by considering the medium modifications of the masses of the charmonium and $D^\pm$ mesons.

We write the state for the charmonium state $\psi''$ with spin projection $m$ at rest as

$$|\psi''_m(\vec{0})\rangle = \int d\mathbf{k}_1 c_I^i(\mathbf{k}_1)^\dagger a_m(\psi'', \mathbf{k}_1) \bar{c}_I(\mathbf{k}_1) |\text{vac}\rangle, \quad (14)$$

where, $i$ is the color index of the quark/antiquark operators, and,

$$a_m(\psi'', \mathbf{k}_1) = \frac{1}{\sqrt{3}} u_{\psi''}(\mathbf{k}_1) \frac{1}{4\sqrt{\pi}} \left( \sigma_m - 3(\sigma \cdot \hat{k}_1) \hat{k}_1^m \right). \quad (15)$$

In the above, we take harmonic oscillator wave functions for the charmonium state $\psi''$ with the wave function for $\psi''$ being given as

$$u_{\psi''}(\mathbf{k}_1) = \left( \frac{16}{15} \right)^{1/2} \pi^{-1/4} (R_{\psi''})^{7/4} \mathbf{k}_1^2 \exp \left( -\frac{1}{2} R_{\psi''}^2 \mathbf{k}_1^2 \right). \quad (16)$$
The $D^+$ and $D^-$ states are explicitly given as

$$|D^+(p)\rangle = \int c_I^{i_1}(k_2 + \lambda_2 p)\dagger u_{D^+}(k_2)\tilde{d}_I^{i_1}(-k_2 + \lambda_1 p)dk_2$$

(17)

and

$$|D^-(p')\rangle = \int d_I^{i_2}(k_3 + \lambda_1 p')\dagger u_{D^-}(k_3)\tilde{c}_I^{i_2}(-k_3 + \lambda_2 p')dk_3.$$ 

(18)

In the above,

$$u_{D^\pm}(k) = \frac{1}{\sqrt{6}} \left(\frac{R_D^2}{\pi}\right)^{3/4} \exp\left(-\frac{R_D^2 k^2}{2}\right).$$

(19)

For the above states we have used alternative Lorentz boosting which is like getting the hadron through translation operator and is as given by equations (10) and (11), and was favoured on experimental grounds [23]. We shall now explicitly calculate $\lambda_1$ and $\lambda_2$. We recall that it was conjectured in Ref. [22] that the binding energy of the hadron as shared by the quarks shall be inversely proportional to the quark masses. Thus we shall explicitly have

$$\omega_1 = M_d + \frac{M_c}{M_c + M_d} \cdot (m_D - M_c - M_d)$$

(20)

and,

$$\omega_2 = M_c + \frac{M_d}{M_c + M_d} \cdot (m_D - M_c - M_d),$$

(21)

with

$$\lambda_i = \frac{\omega_i}{m_D}. $$

(22)

This was conjectured with hydrogen atom in mind [22], and we shall throughout use this.

We next evaluate the matrix element of the quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium state and the final state for the reaction $\psi'' \rightarrow D^+(p)D^-(p')$.

The relevant part of the quark pair creation term is through the $d\bar{d}$ creation. From equations (10) and (11) we can write down $\mathcal{H}_{d\bar{d}}(x, t = 0)$, and then integrate over $x$ to obtain the expression

$$\int \mathcal{H}_{d\bar{d}}(x, t = 0)dx$$

$$= \int dkdk'd_I^{i_1}(k + \lambda_1 p')\dagger U(k)\dagger S(L(p'))\dagger \delta(-k' + \lambda_1 p + k + \lambda_1 p') \cdot (\alpha + (k + \lambda_1 p') + \beta M_d)$$

$$S(L(p))V(-k')\tilde{d}_I^{i_1}(-k' + \lambda_1 p),$$

(23)
where $M_d$ is the constituent mass of the $d$ quark. In equation (24), the Lorentz boosting factor $S(L(p))$ is given as

$$S(L(p)) = \left( \frac{p^0 + m_h}{2m_h} \right)^{1/2} + \frac{\alpha \cdot p}{(2m_h(p^0 + m_h))^{1/2}}$$

with $m_h$ is the mass of the hadron with momentum $p$, which is the $D$ meson here.

From equation (23) we can then evaluate that

$$\langle D^+(p)|D^-(p')|H_{d1d}(x, t = 0)d\xi|\psi''_m(\vec{0})\rangle = \delta(p + p')\int d\xi_k A''_m(p, k_1),$$

(25)

with appropriate simplifications using equations for the states $|\psi''_m(0)\rangle$, $|D^+(p)\rangle$ and $|D^-(p')\rangle$ given in (14), (17) and (18). In the above equations the spectators $c$ and $\bar{c}$ give that

$$k_2 + \lambda_2 p = k_1; \quad -k_3 + \lambda_2 p' = -k_1.$$

(26)

Also, $d$ and $\bar{d}$ contractions of the above with the same in (23) yield the results

$$k + \lambda_1 p' = k_3 + \lambda_1 p'; \quad -k' + \lambda_1 p = -k_2 + \lambda_1 p.$$

(27)

This enables us to integrate over all momenta except $k_1$. For the evaluation of $A''_m(p, k_1)$, we first note that during the evaluation of the above matrix element, we have a spin matrix given as $a_m(\psi'', k_1)$ as well as a spin matrix in the expression for the pair creation term in equation (23). For the evaluation of $A''_m(p, k_1)$ we get, including summing over color,

$$A''_m(p, k_1) = 3 \cdot \text{Tr}[a_m(\psi'', k_1)u_D^-(k_1 + \lambda_2 p')U(k_1 + \lambda_2 p')\dagger S(L(p'))^\dagger($$$$

(\alpha \cdot (k_1 - p) + \beta M_d)S(L(p))V(-k_1 + \lambda_2 p)u_D^+(k_1 - \lambda_2 p)].$$

(28)

We shall now simplify $A''_m(p, k_1)$. Firstly, since the $D(\bar{D})$ mesons are completely non-relativistic, we shall be taking that $S(L(p))$ and $S(L(p'))$ are identity. $U, V$ are given as in equation (5). We first obtain that in (28) we have, using equation (6),

$$U(k_1 - \lambda_2 p)^\dagger(\alpha \cdot (k_1 - p) + \beta M_d)V(-k_1 + \lambda_2 p)$$

$$= \sigma \cdot (k_1 - p) - 2\sigma \cdot (k_1 - \lambda_2 p) \cdot \left( g^2(k_1^2 - (1 + \lambda_2)(k_1 \cdot p) + \lambda_2 p^2) + gM_d f \right).$$

(29)

These yield that in equation (28) we now have, using (15) for $a_m(\psi'', k_1)$

$$A''_m(p, k_1) = \frac{1}{\sqrt{3}} \cdot \frac{1}{4\sqrt{\pi}} u_{\psi''}(k_1) \cdot u_D^-(k_1 - \lambda_2 p)u_D^+(k_1 - \lambda_2 p) \cdot$$

$$3\text{Tr}[\left( \sigma_m - 3k_1^m(\sigma \cdot \hat{k}_1) \right) \cdot (\sigma \cdot (k_1 - p) - 2\sigma \cdot (k_1 - \lambda_2 p) \cdot$$

$$\left( g^2(k_1^2 - (1 + \lambda_2)(k_1 \cdot p) + \lambda_2 p^2) + gM_d f \right)].$$

(30)
We now note that many terms which are odd in \( q \) rotational invariance we shall have the approximate forms of \( k \) taking the trace, equation (30) simplifies to

\[
A_m''(p, k_1) = \frac{1}{4\sqrt{3\pi}} \left( \frac{16}{15} \right)^{\frac{1}{2}} \cdot \pi^{-\frac{1}{4}} \cdot (R_m''^2)^{\frac{1}{2}} \cdot \frac{1}{6} \cdot \left( \frac{R_D^2}{\pi} \right)^{\frac{3}{2}} \exp \left[ -\frac{1}{2} R_m'' k_1^2 - R_D^2 (k_1 - \lambda_2 p)^2 \right] \cdot 6[(k_{1m} - p_m)k_1^2 - 3k_{1m}(k_1 \cdot (k_1 - p)) - 2 ((k_{1m} - \lambda_2 p_m)k_1^2 - 3k_{1m}(k_1 \cdot (k_1 - \lambda_2 p))] \\
(g^2(k_1^2 - (1 + \lambda_2)(k_1 \cdot p) + \lambda_2 p^2) + \frac{1}{2}(1 - \frac{g^2}{2}(k_1 - \lambda_2 p)^2))].
\] (31)

We now substitute

\[
a = \frac{1}{2} R_m'' + R_D^2; \quad b = R_D^2 \lambda_2 / a
\] (32)

and

\[
c = \frac{1}{4\sqrt{3\pi}} \left( \frac{16}{15} \right)^{\frac{1}{2}} \cdot \pi^{-\frac{1}{4}} \cdot (R_m''^2)^{\frac{1}{2}} \cdot \frac{1}{6} \cdot \left( \frac{R_D^2}{\pi} \right)^{\frac{3}{2}}.
\] (33)

We now change the integration variable to \( q \) in equation (31) with the substitution \( k_1 = q + bp \) and write

\[
\int A_m''(p, k_1) dk_1 = 6c \exp[(ab^2 - \lambda_2^2 R_D^2)p^2] \cdot \int \exp(-aq^2) T_m'' dq,
\] (34)

where, we have, first rearranging in terms of \( k_1 \)

\[
T_m'' = (\lambda_2 - 1)p_m k_1^2 + 3k_{1m}(1 - \lambda_2)k_1 \cdot p + \{k_{1m}(2k_1^2 - 3\lambda_2(k_1 \cdot p)) + \lambda_2 p_m k_1^2\} \\
\cdot g^2\{(3/2)k_1^2 - (2 + \lambda_2)k_1 \cdot p + (2\lambda_2 - (1/2)\lambda_2^2)p^2\}
\] (35)

and then, rearranging in terms of \( q \),

\[
T_m'' = q_m[3(1 - \lambda_2)(q \cdot p + bp^2) + 2q^2 + (4b - 3\lambda_2)q \cdot p + (2b^2 - 3\lambda_2 b)p^2] \\
\cdot g^2\{(3/2)q^2 + (3b - (2 + \lambda_2))q \cdot p + ((3/2)b^2 - (2 + \lambda_2)b + 2\lambda_2 - (1/2)\lambda_2^2)p^2\}] \\
+ p_m[(\lambda_2 - 1)q^2 + b(1 - \lambda_2)q \cdot p + 2b^2(1 - \lambda_2)p^2 + \{(2b + \lambda_2)q^2 \\
+ (4b^2 - b\lambda_2)q \cdot p + (2b^3 - 2\lambda_2 b^2)p^2\}] \cdot g^2\{(3/2)q^2 + (3b - (2 + \lambda_2))q \cdot p \\
+ ((3/2)b^2 - (2 + \lambda_2)b + 2\lambda_2 - (1/2)\lambda_2^2)p^2\}]
\] (36)

We now note that many terms which are odd in \( q \) in equation (36) will vanish. Also, from rotational invariance we shall have \( q_m(q \cdot p) \equiv \frac{1}{3} q^2 p_m \), and \((q \cdot p)^2 \equiv \frac{1}{3} q^2 \cdot p^2 \). This yields
that in equation (36) we shall have

\[ T_m^\psi (p, q) \equiv [C_0(|p|) + C_1(|p|)q^2 + C_2(|p|)(q^2)^2]p_m, \]

where

\[
C_0 = 2b^2(1 - \lambda_2)p^2 + 2b^2g^2(p^2)^2(3/2)b^2 - (2 + \lambda_2)b + 2\lambda_2 - (1/2)\lambda_2^2
\]

\[
C_1 = g^2p^2[14b^3 - b^2((32/3) + (37/3)\lambda_2) + b((28/3)\lambda_2 - (1/3)\lambda_2^2)]
\]

\[
C_2 = g^2[7b - (2/3)\lambda_2 - (4/3)].
\]

The integrations over \(q\) in equation (34) with (37) is straightforward. On performing this integration, one then obtains that

\[
\int A_m^\psi (p, k_1)dk_1 = A_m^\psi (|p|)p_m,
\]

where, \(A_m^\psi\) above is given as

\[
A_m^\psi (|p|) = 6c \exp[(ab^2 - m_D^2\lambda_2^2)p^2] \cdot \frac{(\pi)}{a} \left[ C_0 + C_1 \frac{3}{2a} + C_2 \frac{15}{4a^2} \right],
\]

with \(C_0, C_1, C_2\) given in equation (38).

With \(\langle f|S|i\rangle = \delta_4(P_f - P_i)M_{fi}\) we then have for \(\psi''\) of spin \(m\),

\[
M_{fi} = 2\pi \cdot (-iA_m^\psi (|p|))p_m.
\]

We shall be investigating the medium effects of the decay width of the charmonium state to the \(D^+D^-\) pair. In the medium, the modifications of the masses of the outgoing states \(D^+\) and \(D^-\) are different because of the difference in their interactions with the hadronic medium. For the charmonium state decaying at rest, taking the average for spin, we obtain the expression for the decay width as

\[
\Gamma(\psi'' \rightarrow D^+D^-) = \frac{1}{2\pi} \int \delta(m_{\psi''} - p_{D^+}^0 - p_{D^-}^0)|M_{fi}|^2_{av} \cdot 4\pi|p_{D^+}^0|^2d|p_{D^+}^0|
\]

\[
= \frac{8\pi^2}{3} \cdot |p|^3 \frac{P_{D^+}^0P_{D^-}^0}{m_{\psi''}} A_m^\psi (|p|)^2
\]

In the above, \(p_{D\pm}^0 = (m_{D\pm}^2 + p^2)^{1/2}\), and, \(|p|\) is the magnitude of the momentum of the outgoing \(D^\pm\) mesons. The decay of \(\psi''\) to \(D^0\bar{D}^0\) proceeds through a \(u\bar{u}\) pair creation and the decay width (42) is modified to

\[
\Gamma(\psi'' \rightarrow D^0\bar{D}^0) = \frac{8\pi^2}{3} \cdot |p|^3 \frac{P_{D^0}P_{D^0}\bar{D}^0}{m_{\psi''}} A_m^\psi (|p|)^2
\]
In the above, \( p^0_{D^0} = (m^2_{D^0} + p^2)^{\frac{1}{2}} \), \( p^0_{\bar{D}^0} = (m^2_{\bar{D}^0} + p^2)^{\frac{1}{2}} \), and, \(|p|\) is the magnitude of the momentum of the outgoing \( D^0(\bar{D}^0) \) mesons.

2. \( J/\psi \to D^+D^- \)

We note that 

\[
m_{J/\psi} = 3097 \text{ MeV}
\]

so that the above decay in vacuum shall not be admissible. However it may become kinematically admissible with in-medium effects. Here we first consider

\[
|\psi_m(\vec{0})\rangle = \int dk_1 c_I(k_1) a_m(\psi, k_1) \tilde{c}_I(-k_1) |\text{vac}\rangle,
\]

where

\[
a_m(\psi, k_1) = \sigma_m \frac{1}{\sqrt{6}} \left( \frac{R^2_\psi}{\pi} \right)^{3/4} \exp \left( - \frac{R^2_\psi k_1^2}{2} \right).
\]

The calculations are completely parallel to the calculation of the matrix element of \( \psi'' \to D^+D^- \). Similar to equation (25), we can evaluate

\[
\langle D^+(p) | D^-(p') \rangle \int \mathcal{H}_{d\bar{d}}(x, t = 0) dx |\psi_m(\vec{0})\rangle = \delta(p + p') \int dk_1 A_m^\psi(p, k_1),
\]

\[
A_m^\psi(p, k_1) = \frac{\sqrt{3}}{\sqrt{6}} \cdot \left( \frac{R^2_\psi}{\pi} \right)^{3/4} \cdot \frac{1}{6} \cdot \left( \frac{R^2_D}{\pi} \right)^{\frac{3}{4}} \exp \left[ - \frac{1}{2} R^2_\psi k_1^2 - R^2_D (k_1 - \lambda_2 p)^2 \right] \cdot \left( k_1 - \lambda_2 p \right)_m \cdot \left( k_1 - \lambda_2 p \right)^2 + 1 + \frac{1}{2} \left. \left( 1 - \frac{g^2}{2} (k_1 - \lambda_2 p) \right) \right].
\]

As earlier, we now substitute

\[
a = \frac{1}{2} R^2_\psi + R^2_D; \quad b = R^2_D \lambda_2 / a \]

and

\[
c = \frac{1}{\sqrt{6}} \cdot \left( \frac{R^2_\psi}{\pi} \right)^{3/4} \cdot \frac{1}{6} \cdot \left( \frac{R^2_D}{\pi} \right)^{3/4}.
\]

We now change the integration variable to \( q \) in equation (31) with the substitution \( k_1 = q + bp \) and write

\[
\int A_m^\psi(p, k_1) dk_1 = 6c \exp[(ab^2 - \lambda_2^2 R^2_D) p^2] \cdot \int \exp(-a q^2) T_m^\psi dq.
\]
Rearranging in terms of $q$, we get

$$T^\psi_m = (\lambda_2 - 1) p_m - 2g^2[q_m + (b - \lambda_2) p_m] \cdot \left[ \frac{3}{4} q^2 - \left(-\frac{3}{2} b + 1 + \frac{1}{2} \lambda_2\right) q \cdot p + \left(\frac{3}{4} b^2 - b \left(1 + \frac{1}{2} \lambda_2\right) + \lambda_2 - \frac{1}{4} \lambda_2^2\right) p^2 \right].$$  \quad (52)

Using rotational invariance, we then get, as earlier,

$$T^\psi_m(p, q) \equiv [C_0^\psi(|p|) + C_1^\psi(|p|)q^2 + C_2^\psi(|p|)(q^2)^2] p_m,$$  \quad (53)

where now

$$C_0^\psi = (\lambda_2 - 1) - 2g^2\left(\frac{3}{4} b^2 - (1 + \frac{1}{2} \lambda_2) b + \lambda_2 - \frac{1}{4} \lambda_2^2\right),$$

$$C_1^\psi = g^2 \left[-\frac{5}{2} b + \frac{2}{3} + \frac{11}{6} \lambda_2\right],$$

$$C_2^\psi = 0.$$  \quad (54)

Proceeding as earlier we then get

$$\int A_m^\psi(p, k_1)dk_1 = A^\psi(|p|)p_m$$  \quad (55)

where, $A^\psi$ above is now given as

$$A^\psi(|p|) = 6c \exp[(ab^2 - R_D^2 \lambda_2^2)p^2] \cdot \left(\frac{\pi}{a}\right)^{\frac{3}{2}} \left[C_0^\psi + C_1^\psi \frac{3}{2a} + C_2^\psi \frac{15}{4a^2}\right].$$  \quad (56)

Hence we get, with parallel notations as earlier,

$$\Gamma(J/\psi \to D^+D^-) = \frac{8\pi^2}{3} \cdot |p|^3 p^\psi_{D^+D^-} \cdot A^\psi(|p|)^2$$  \quad (57)

The expression for the decay width of $J/\psi \to D^0\bar{D}^0$, which proceeds through a $u\bar{u}$ pair creation, is obtained by replacing $D^+(D^-)$ by $D^0(\bar{D}^0)$ in expression given by (57).

3. $\psi' \to D^+D^-$

We note that we have here the mass of $\psi' \equiv \psi(3686)$ in vacuum is

$$m_{\psi'} = 3686 \text{ MeV}$$  \quad (58)
so that, this decay is not seen in vacuum, but this may be observed in medium when it is kinematically admissible. The state here is given as

\[ |\psi'_m(0) > = \int dk_1 c^\dagger_1(k_1)^\dagger a_m(\psi', k_1) \bar{d}_1(-k_1)|\text{vac} >, \]  

where

\[ a_m(\psi', k_1) = \sigma_m \frac{1}{\sqrt{6}} \left( \frac{3}{2} \right)^{1/2} \left( \frac{R_{\psi'}^2}{\pi} \right)^{3/4} \left( \frac{R_{\psi'}^2 k_1^2 - 1}{3} \right) \exp \left[ -\frac{1}{2} R_{\psi'}^2 k_1^2 \right]. \]  

We then proceed in the same way as for the calculations for $\psi''$, and substitute $k_1 = q + b p$, with the values of $a, b, c$ now given as

\[ a = \frac{1}{2} R_{\psi'}^2 + R_D^2; \quad b = R_D^2 \lambda_2 / a \]  

and

\[ c = \frac{1}{\sqrt{6}} \left( \frac{3}{2} \right)^{1/2} \left( \frac{R_{\psi'}^2}{\pi} \right)^{3/4} \left( \frac{R_D^2}{\pi} \right)^{3/2}. \]  

Equation (34) now is replaced by

\[ \int A_m^{\psi'}(p, k_1) dk_1 = 6 c \exp[(ab^2 - \lambda_2 R_D^2)p^2] \cdot \int \exp(-a q^2) T_m^{\psi'} dq. \]  

where, from equation (60), $T_m^{\psi'}$ will have an additional factor $\left( \frac{2}{3} R_{\psi'}^2 k_1^2 - 1 \right)$. Including this factor and writing everything in terms of $q$ we now obtain that

\[ T_m^{\psi'} = \left[ \frac{2}{3} R_{\psi'}^2 \left( q^2 + 2 b q \cdot p + b^2 p^2 \right) - 1 \right] \cdot \left[ (\lambda_2 - 1)p_m - 2 g^2[q_m + (b - \lambda_2)p_m] \cdot \left\{ \frac{3}{4} q^2 - \left( -\frac{3}{2} b + \frac{1}{2} \lambda_2 \right) q \cdot p + \left( \frac{3}{4} b^2 - b \left( 1 + \frac{1}{2} \lambda_2 \right) + \lambda_2 - \frac{1}{4} \lambda_2 \right) p^2 \right\} \right]. \]  

With rotational invariance we then have,

\[ T_m^{\psi'}(p, q) \equiv \left[ C_0^{\psi'}(|p|) + C_1^{\psi'}(|p|)q^2 + C_2^{\psi'}(|p|)(q^2)^2 \right]p_m, \]  

where now

\[ C_0^{\psi'} = \left( \frac{2}{3} R_{\psi'}^2 b^2 p^2 - 1 \right) C_0^{\psi}, \]

\[ C_1^{\psi'} = \frac{2}{3} R_{\psi'}^2 C_0^{\psi} + \left( \frac{2}{3} R_{\psi'}^2 b^2 p^2 - 1 \right) C_1^{\psi} - \frac{8}{9} R_{\psi'}^2 b g^2 p^2 \left[ \frac{9}{4} b^2 - b \left( 2 + \frac{5}{2} \lambda_2 \right) + 2 \lambda_2 + \frac{1}{4} \lambda_2 \right], \]

\[ C_2^{\psi'} = \frac{2}{3} R_{\psi'}^2 g^2 \left[ -\frac{7}{2} b + \frac{2}{3} + \frac{11}{6} \lambda_2 \right]. \]
In the above, $C^\psi_0$ and $C^\psi_1$ are given in equation (54). We then have the decay width of $\psi'$ to $D^+D^-$ given as, with parallel notations as earlier,

$$\Gamma(\psi' \rightarrow D^+D^-) = \frac{8\pi^2}{3} \cdot |p| \frac{3P^0_D P^0_{D^+} D^-}{m_{\psi'}} A^{\psi'}(|p|)^2$$

(67)

where, $A^{\psi'}$ is given as

$$A^{\psi'}(|p|) = 6c \exp[(ab^2 - R^2_D \lambda^2_2) p^2] \cdot \left(\frac{\pi}{a}\right)^3 [C^\psi_0 + C^\psi_1 \frac{3}{2a} + C^\psi_2 \frac{15}{4a^2}].$$

(68)

Similarly we have, again with parallel notations,

$$\Gamma(\psi' \rightarrow D^0 \bar{D}^0) = \frac{8\pi^2}{3} \cdot |p| \frac{3P^0_{D^0} P^0_{\bar{D}^0} \bar{D}^0}{m_{\psi'}} A^{\psi'}(|p|)^2$$

(69)

As we see, the decay widths of the charmonium states are given as a polynomial part multiplied by a gaussian part. Hence the medium dependence of the decay width is due to the combined effect of the polynomial and the exponential part of the decay width.

**B. Decay widths of charmonium states to $D\bar{D}$ pairs in $^3P_0$ model**

The medium modifications of the partial decay widths of the charmonium states to $D\bar{D}$ pairs in the hot isospin asymmetric strange hadronic medium have been investigated [24], by accounting for the internal structures of the parent and outgoing mesons using the $^3P_0$ model [25–27]. The charmonium state decays at rest to $D\bar{D}$ pair, with the creation of a light quark ($u,d$) antiquark pair in the $^3P_0$ state and the heavy charm quark (antiquark) combining with the light antiquark (quark) to produce the $D$ and $\bar{D}$ mesons. The decay widths in the medium are calculated by accounting for the medium modifications of the charmonium states, as well as $D$ and $\bar{D}$ mesons calculated within a chiral effective model [24].

The decay widths of the charmonium states have been computed in the $^3P_0$ model using the harmonic oscillator wave functions for the charmonium states [24, 25]. The strength of the harmonic oscillator wave function of the charmonium state, is determined from the mean squared radii $\langle r^2 \rangle$ as 0.47$^2$ fm$^2$, 0.96$^2$ fm$^2$ and 1 fm$^2$ for the charmonium states $J/\psi$, $\psi(3686)$ and $\psi(3770)$, respectively [19, 24]. This gives the value for the parameter $1/R$,
where $R = R_{\psi''}, R_{\psi'}, R_\psi$ of the wave functions of the $\psi'' \equiv \psi(3770), \psi' \equiv \psi(3686)$ and $J/\psi$, given by equations (16), (60) and (46), as 0.37 GeV, 0.39 GeV and 0.52 GeV respectively.

The decay widths for the charmonium states $J/\psi, \psi(3686)$ and $\psi(3770)$ decaying to $DD$ ($D^+D^-$ or $D^0\bar{D}^0$), are given as [25]

$$\Gamma(J/\psi \to D\bar{D}) = \frac{\pi}{2} \frac{E_D E_{\bar{D}} \gamma^2}{2m_{J/\psi}} \frac{2^8 r^3 (1 + r^2)^2}{3(1 + 2r^2)^3} x^3 \exp \left( -\frac{x^2}{2(1 + 2r^2)} \right), \quad (70)$$

$$\Gamma(\psi(3686) \to D\bar{D}) = \frac{\pi}{2} \frac{E_D E_{\bar{D}} \gamma^2}{2m_{\psi(3686)}} \times \frac{2^7 (3 + 2r^2)^2 (1 - 3r^2)^2}{3^2(1 + 2r^2)^7} x^3 \times \left( 1 + \frac{2r^2 (1 + r^2)}{(1 + 2r^2)(3 + 2r^2)(1 - 3r^2)x^2} \right)^2 \exp \left( -\frac{x^2}{2(1 + 2r^2)} \right), \quad (71)$$

$$\Gamma(\psi(3770) \to D\bar{D}) = \frac{\pi}{2} \frac{E_D E_{\bar{D}} \gamma^2}{2m_{\psi(3770)}} \frac{2^{11/2}}{3^2} \left( \frac{r}{1 + 2r^2} \right)^7 x^3 \left( 1 - \frac{1 + r^2}{5(1 + 2r^2)x^2} \right)^2 \exp \left( -\frac{x^2}{2(1 + 2r^2)} \right), \quad (72)$$

In the above, $r = \frac{R_D}{R_\psi}$ is the ratio of the harmonic oscillator strengths of the decaying charmonium state and the produced $D(\bar{D})$-mesons, $\gamma$ is a measure of the strength of the $^3P_0$ vertex, $x = |p| R_D$, where $|p|$ is the magnitude of the 3-momentum of the produced $D(\bar{D})$ meson and $E_{D(\bar{D})} = (|p|^2 + m_{D(\bar{D})}^2)^{1/2}$ is the energy of the $D(\bar{D})$ meson. As we see, the decay widths of the charmonium states are given as a polynomial part multiplied by a gaussian part. In the medium, the masses of the charmonium states and the $D$ and $\bar{D}$ can be such that the polynomial part vanishes, leading to nodes in the decay widths of the charmonium states [24, 25].

III. CHARMONIUM STATES AND $D(\bar{D})$ MESONS IN THE MEDIUM

We study the medium dependence of the partial decay widths of the charmonium decaying to $D\bar{D}$ due to medium modifications of the masses of the $D$ meson, the $\bar{D}$ meson and the charmonium state calculated in a chiral effective model [29]. The model is based on the nonlinear realization of chiral symmetry [30, 32] and broken scale invariance [29, 33, 34]. The properties of the hadrons in the strange hadronic matter within the chiral effective model are investigated using the mean field approximation, where all the meson fields are
treated as classical fields. By solving the equations of motion of these fields, the values of the meson fields are obtained. These are calculated for given values of the isospin asymmetry parameter, \( \eta = -\sum \frac{I_3 \rho_i}{\rho_B} \), and the strangeness fraction, \( f_s = \sum \frac{s_i \rho_i}{\rho_B} \), where \( \rho_i \) is the number density of the baryon of \( i \)-th type (\( i = p, n, \Lambda, \Sigma^+, \Sigma^0, \Xi^-, \Xi^0 \)) and \( s_i \) is the number of strange quarks in the \( i \)-th baryon. Within the chiral effective model, the in-medium masses of the \( D \) and \( \bar{D} \) mesons [35] have been studied in isospin asymmetric nuclear matter at zero [36] and finite temperatures [37] as well as in hot asymmetric hyperonic matter [24]. These arise from the interaction of these mesons with nucleons, hyperons and the scalar mesons. The mass shifts of the charmonium states arise due to interaction with the gluon condensates in QCD, simulated by a scalar dilaton field through a scale symmetry breaking term [29, 37, 38] in the effective hadronic model [24, 37]. Equating the trace of the energy momentum tensor in QCD to the trace of the energy momentum tensor in the chiral effective model corresponding to the scale symmetry breaking gives the relation between the scalar gluon condensate to the dilaton field [24, 37, 39, 40]. In the following section, we shall investigate the medium effects of the charmonium decay widths arising from the in-medium masses of the \( D, \bar{D} \) and charmonium states as calculated in Ref. [24].

IV. RESULTS AND DISCUSSIONS

In this section, we discuss the results of the modifications of the decay widths of the charmonium states, \( J/\psi, \psi(3686) \) and \( \psi(3770) \) to \( D\bar{D} \) pair in isospin asymmetric strange hadronic matter, using the field theoretic model for the composite hadrons described in section II. The medium modifications of the masses of \( D, \bar{D} \) mesons and of the charmonium states calculated in a chiral effective model as in section III have been discussed in detail in Ref. [24]. These masses determine the in-medium decay widths. We therefore plot these in figures 1 and 2 and use them to calculate the decay widths as given in subsequent figures 3 and 4.

In the field theoretic model here the decay widths are through a pair creation Hamiltonian that arises naturally in the Dirac Hamiltonian of the constituent field operators as in equation (23) from [21, 22] with Lorentz boosting parallel to a translation as in Ref. [23]. The constituent masses of the light quarks (u,d) are taken as \( M_u = M_d = 280 \) MeV, which is
FIG. 1: (Color online) The masses of $D$ ($D^+,D^0$) and $\bar{D}$ ($D^-,D^0$) mesons are plotted as functions of baryon density in units of nuclear matter saturation density. These are plotted for the values of the isospin asymmetric parameter $\eta = 0$ and $\eta=0.5$ for the hyperonic matter with strangeness fraction,$f_s=0.5$, and are compared with the masses for the case of nuclear matter ($f_s=0$) shown as dotted lines.

smaller than the canonical value of about 300 MeV. This is logical as here the energy scale is higher. The constituent quark mass of the heavy charm quark is taken as $M_c=1580$ MeV. The parameter $\lambda_2$, which is the energy fraction of the heavy quark (antiquark) in the $D(\bar{D})$ mesons, is calculated as in equations (21) and (22), and is estimated as 0.85. With the above values of the quark masses, one reproduces the vacuum decay widths for the decay channels $\psi'' \to D^+D^-$ and $\psi'' \to D^0\bar{D}^0$ as 12 MeV and 16 MeV respectively, which was a reason for the choice of the above values. There are no other free parameters in the present model, and we take these parameters as fixed for calculating the in-medium decay widths.
FIG. 2: (Color online) The shifts in the masses of the charmonium states ($J/\psi$, $\psi(3686)$ and $\psi(3770)$) are plotted as functions of the baryon density in units of nuclear matter saturation density. These are plotted for the values of the isospin asymmetric parameter $\eta = 0$ and $\eta=0.5$ and are compared with the masses for the case of strangeness fraction, $f_s=0$.

In the present model the in-medium decay widths are given by equations (42), (67) and (57), their medium dependence being through the magnitude of $p$, the 3-momentum of $D(\bar{D})$ mesons, given in terms of the masses of the charmonium, the $D$ and the $\bar{D}$ mesons, as in equation (12). The expressions for these decay widths are given as a polynomial part multiplied by a gaussian contribution. The density dependence of the decay widths of the charmonium states ($J/\psi$, $\psi' \equiv \psi(3686)$ and $\psi'' \equiv \psi(3770)$) to $D\bar{D}$ for isospin symmetric ($\eta=0$) hyperon matter with $f_s=0.5$ are shown in figure 3 and compared to the case of symmetric nuclear matter ($\eta=0, f_s=0$). In isospin symmetric matter, the masses of the $D^0$ and $D^+$ of the $D$ meson doublet, as well as the masses of the $\bar{D}^0$ and $D^-$ of the $\bar{D}$ doublet, remain almost degenerate with the small mass difference between them through solution of their dispersion relations arising due to the small mass difference in their vacuum masses.
FIG. 3: (Color online) The partial decay widths of the charmonium states, calculated using the present model for composite hadrons, to (i) $D^+D^-$, (ii) $D^0\bar{D}^0$ and (iii) the sum of the two channels ((i)+(ii)) in the isospin symmetric strange hadronic matter ($\eta=0, f_s=0.5$), accounting for the medium modifications of the $D(\bar{D})$ mesons. These are shown in subplots (a), (c) and (e), when the mass modifications of the charmonium states are neglected and (b), (d) and (f), the partial decay widths are shown when the mass modifications of the charmonium states are also taken into account. These results are compared to the case of nuclear matter ($f_s=0$), shown as dotted lines.

Hence, the partial decay widths of the charmonium state decaying to $D^+D^-$ and $D^0\bar{D}^0$ are almost identical in isospin symmetric matter, as in figure 3. For hyperonic matter, the decay channel of $J/\psi$ to $D\bar{D}$ ($D^+D^-$ or $D^0\bar{D}^0$) becomes possible at densities higher than 4.3 $\rho_0$ when the in-medium masses of $D$ and $\bar{D}$ mesons are taken into account, but the mass of $J/\psi$...
FIG. 4: (Color online) The partial decay widths of the charmonium states, calculated using the present model for composite hadrons, to (i) $D^+D^-$, (ii) $D^0ar{D}^0$ and (iii) the sum of the two channels ((i)+(ii)) in the isospin asymmetric strange hadronic matter ($\eta=0.5$, $f_s=0.5$), as functions of the baryon density, in units of the nuclear matter saturation density, accounting for the medium modifications of the $D(\bar{D})$ mesons. These are shown in subplots (a), (c) and (e), when the mass modifications of the charmonium states are neglected and (b), (d) and (f), the partial decay widths are shown when the mass modifications of the charmonium states are also taken into account. These results are compared to the case of nuclear matter ($f_s=0$), shown as dotted lines.
FIG. 5: (Color online) The partial decay widths of the charmonium states, calculated using the \( ^3P_0 \) model, to (i) \( D^+D^- \), (ii) \( D^0\bar{D}^0 \) and (iii) the sum of the two channels ((i)+(ii)) in the isospin symmetric strange hadronic matter \( (\eta=0, f_s=0.5) \), accounting for the medium modifications of the \( D(\bar{D}) \) mesons. These are shown in subplots (a), (c) and (e), when the mass modifications of the charmonium states are neglected and (b), (d) and (f), the partial decay widths are shown when the mass modifications of the charmonium states are also taken into account. These results are compared to the case of \( f_s=0 \), shown as dotted lines.

is taken to be its vacuum value. This value of the threshold density is modified only slightly (to \( 4.5\rho_0 \)) when the mass shift of the \( J/\psi \) is also included. This is due to the fact the \( J/\psi \) has only a small modification in the medium, as can be seen from the figure. For \( J/\psi \), there is seen to be an increase of the polynomial part of the decay width with density, which
FIG. 6: (Color online) The partial decay widths of the charmonium states, calculated using the $^3P_0$ model, to (i) $D^+D^-$, (ii) $D^0\bar{D}^0$ and (iii) the sum of the two channels ((i)+(ii)) in the isospin asymmetric strange hadronic matter ($\eta=0.5$, $f_s=0.5$), as functions of the baryon density, in units of the nuclear matter saturation density, accounting for the medium modifications of the $D(\bar{D})$ mesons. These are shown in subplots (a), (c) and (e), when the mass modifications of the charmonium states are neglected and (b), (d) and (f), the partial decay widths are shown when the mass modifications of the charmonium states are also taken into account. These results are compared to the case of $f_s=0$, shown as dotted lines.
dominates over the gaussian part, thus leading to a monotonic rise in the decay of $J/\psi$, as seen in the subplots (a) and (b) of figure 3. However, these decay widths are observed to be very small in magnitude, being of the order of around 0.1 MeV at a density as large as $6\rho_0$.

The decay widths of $\psi(3686)$ and $\psi(3770)$ to $D\bar{D}$ are illustrated in subplots (c) and (e) of figure 3 for isospin symmetric hadronic matter ($\eta=0$), with the medium modifications of the masses of the $D$ and $\bar{D}$ mesons, but not of the charmonium state. Due to drop in the mass of $DD$ pair in the medium, the decay of $\psi(3686)$ to $D\bar{D}$ becomes kinematically admissible above a density of around 0.6$\rho_0$, as seen in subplot (c). The decay of $\psi(3770)$ to $D\bar{D}$ is already possible in vacuum, which however is modified in the medium due to mass drop of the $D\bar{D}$ pair. As has already been mentioned, the medium modifications of the decay widths are through the magnitude of $p$, the 3-momentum of the $D(\bar{D})$ meson. For both of the excited charmonium states, $|p|$ is seen to increase with density, when only the mass modifications of the $D$ and $\bar{D}$ mesons are considered. This leads to an increase in the polynomial part of the decay width upto a density of about $2\rho_0$ ($1.5\rho_0$) for $\psi(3686)$ ($\psi(3770)$), when the decay widths of $\psi(3686)$ and $\psi(3770)$ attain values of around 30 MeV and 150 MeV respectively. As the density is further increased, the gaussian parts dominate due to increase in $|p|$ with density, thus leading to a drop in their decay widths. The fall off with density is observed to be slower for the nuclear matter ($f_s=0$) as compared to for hyperonic matter. This is due to the fact that $|p|$ has a higher value in hyperonic matter as compared to in nuclear matter, since the masses of the $D$ and $\bar{D}$ mesons have smaller in-medium masses with strangeness in the medium. When the in-medium masses of the charmonium states are also included, the partial decay widths of the excited charmonium states, $\psi(3686)$ and $\psi(3770)$ (plotted in subplots (d) and (f)), are seen to be modified significantly. For hyperonic matter, there is seen to be initially a rise in the decay width with density, which however, is observed to remain almost constant, as the density is further increased. This is due to the fact that $|p|$ becomes smaller with the in-medium charmonium mass, which leads to much lesser suppression arising from the gaussian part of the decay width at high densities, as compared to the case when the charmonium mass modification is not taken into account.

We then consider the effects of the isospin asymmetry on the charmonium decay widths. The masses of the $D^+$ and $D^0$, as well as for $D^-$ and $\bar{D}^0$ are no longer degenerate in
the asymmetric hadronic medium, thus leading to the charmonium decays to $D^+D^-$ and $D^0\bar{D}^0$ to be different. In figure 4, the decay widths of the charmonium states are shown for asymmetric hadronic matter with $\eta=0.5$. These are plotted for the hyperonic matter with strangeness fraction $f_s=0.5$ and compared to the values for nuclear matter ($f_s=0$). When the medium modifications of the $D$ and $\bar{D}$ mesons are considered, but not of $J/\psi$, then the decay channels to (i) $D^+D^-$ and (ii) $D^0\bar{D}^0$ are observed to be admissible at densities higher than around $3.6\rho_0$ and $4\rho_0$ respectively, as can be seen from subplot (a) of figure 4. The density dependence of the decay widths still remains similar to the case for the isospin symmetric hadronic matter plotted in figure 3. There is seen to be very small change in the decay widths when the in-medium mass for $J/\psi$ is included as can be seen in subplot (b) of figure 4.

The decay widths of the excited charmonium states $\psi(3686)$ and $\psi(3770)$ are plotted as functions of density in (c) and (e) of figure 4 including the medium modifications of the masses of the $D$ and $\bar{D}$ mesons, but not of the charmonium states. The density dependence of these decay widths are seen to be similar to the isospin symmetric matter shown in figure 3. In nuclear matter, the isospin asymmetry dependence is observed to be quite appreciable leading to very different values for the charmonium decay widths to $D^+D^-$ and $D^0\bar{D}^0$. This is due to the reason that in the presence of hyperons, the mass modifications of the $D$ and $\bar{D}$ masses are lessened, as can be seen from figure 1. This leads to a much lesser effect from the isospin asymmetry in the hyperonic medium as compared to in nuclear matter, as can be seen from the figure. With in-medium charmonium masses, the value of $|p|$ becomes smaller, leading to slower drop of the decay width of the charmonium states at higher densities, as can be seen from the subplots (d) and (f) of figure 4. In asymmetric nuclear matter ($\eta=0.5$), the charmonium state $\psi(3686)$ is observed to decay to $D^+D^-$ at densities higher than about $0.8\rho_0$, whereas the decay to $D^0\bar{D}^0$ becomes possible at densities higher than about $3.4\rho_0$. The decay of $\psi(3770)$ to $D^+D^-$ is seen to have an initial rise up to a density of about $0.6\rho_0$ followed by a drop and then again a slow increase as the density is further increased. On the other hand, the decay to $D^0\bar{D}^0$ does not become admissible above a density of about $2.5\rho_0$.

In hyperonic matter, the density behaviour of the decays of $\psi(3686)$ and $\psi(3770)$ remain similar to the case of symmetric hadronic matter plotted in figure 3 and with inclusion of in-medium charmonium masses, the isospin dependence still remains small for the hyperonic
The partial decay widths of the charmonium states as calculated in the present composite hadronic model are now compared with the earlier results using the $^3P_0$ model [24, 25, 27]. For this purpose we show the decay widths obtained from the $^3P_0$ model [24] in figures 5 and 6 for the isospin symmetric ($\eta=0$) and isospin asymmetric ($\eta=0.5$) strange hadronic matter for $f_s=0.5$. As earlier, the medium modifications of the decay widths are through $|p|$, which is given in terms of the masses of the charmonium state, $D$ and $\bar{D}$ mesons by equation (12). The expressions for the decay widths of the charmonium states in the $^3P_0$ model are of the form of a polynomial part multiplied by a gaussian part, as can be seen from the equations (70), (71) and (72). The density dependence of the charmonium decay width is thus due to the combined effect of these contributions. For the excited charmonium states, $\psi(3686)$ and $\psi(3770)$, when the medium modifications of the $D$ and $\bar{D}$ mesons are considered, but not of the charmonium state, then the charmonium decay widths are initially seen to increase with density and then drop with further increase in the density. This behaviour on density leads to even vanishing of the decay widths at certain densities [24]. There is seen to be an increase again at still higher densities. For both symmetric as well as asymmetric matter, the decay widths of $\psi(3686)$ as well as $\psi(3770)$ in the $^3P_0$ model, are seen to vanish (see (c) and (e) of figures 5 and 6). Such nodes in the decay widths arising due to mass drop of the $D$ and $\bar{D}$ mesons in the medium, have been discussed earlier in the literature [25] for excited charmonium states decaying to $D\bar{D}$ pairs. A similar trend of the decay width with density is still observed when the mass modifications of the charmonium states are considered. However, no nodes are any longer observed even upto a density of about $6\rho_0$. The decay width of $J/\psi$ becomes kinematically accessible at densities higher than about $4\rho_0$ ($4.5\rho_0$) with the in-medium masses of the $D$ and $\bar{D}$ mesons, and without (with) the mass modification of $J/\psi$. In isospin asymmetric hadronic matter, the decay widths for the charmonium state to $D^+D^-$ is observed to be different from the decay to $D^0\bar{D}^0$ pair, due to difference in the masses of $D^+$ and $D^0$ of the $D$ meson doublet and of $D^-$ and $\bar{D}^0$ of the $D$ doublet in the medium. Similar to the charmonium decay widths of the present model for asymmetric matter shown in figure 4, the $^3P_0$ also has a much stronger dependence on the isospin asymmetry in nuclear matter as compared to in hyperonic matter. This is due to the fact that the isospin asymmetry effect decreases with strangeness in the hadronic medium.
The qualitative behaviour of the partial decay widths of the charmonium states to $D\bar{D}$ with density, calculated within the present composite hadron model, remains similar to their density dependence as calculated using the $^3P_0$ model. The values obtained for the decay width of $\psi(3770)$ calculated in the present investigation are very similar to those obtained by using the $^3P_0$ model. However, the values of the decay widths of the charmonium states, $\psi(3686)$ and $J/\psi$, are observed to be much smaller in magnitude as compared to those obtained in the $^3P_0$ model. Within the $^3P_0$ model, there are seen to be nodes in the decay widths of the charmonium states, $\psi(3686)$ as well as $\psi(3770)$ when the in-medium masses of the $D(\bar{D})$ mesons are considered, but the medium modification of the charmonium mass is neglected [25, 37]. However, in the present work, the decay widths do show a behaviour of an initial increase with density and then a drop, but no nodes are observed even up to a density of around $6\rho_0$.

V. SUMMARY AND OUTLOOK

We now very briefly summarise the results. We have here investigated the in-medium partial decay widths of the charmonium states to $D\bar{D}$ pair, by using a composite model for the hadrons with quark and/or antiquark constituents. The decay width is calculated from a quark pair creation term which arises within the composite hadron model. The medium modifications of these partial decay widths are studied from the mass modifications of the charmonium and $D(\bar{D})$ mesons within a chiral effective model [24]. When the mass modifications of the $D$ and $\bar{D}$ mesons are considered, but the charmonium mass taken as its vacuum value, one finds for the excited charmonium states $\psi(3686)$ and $\psi(3770)$ that the decay width increases initially with density, followed by a drop when the density is further increased. There is seen to be a modification of these decay widths when the modified charmonium masses are also taken. The isospin asymmetry effects are observed to be more dominant for the $D^+$ and $D^0$, as compared to $D^-$ and $\bar{D}^0$. This leads to the partial decay widths of charmonium states to $D^+D^-$ and $D^0\bar{D}^0$ to be different in asymmetric matter. The isospin asymmetry effects are however observed to be quite dominant in nuclear matter and these effects are observed to be much less in the presence of hyperons in the medium. The density effects on the charmonium decay widths seem to be the dominant medium effect as
compared to the effects of the isospin asymmetry and strangeness in the hadronic matter. This should show up in experimental observables like production of the $J/\psi$ as well as $D(\bar{D})$ mesons from the strongly interacting matter arising from the compressed baryonic matter (CBM) experiment in the future facility at GSI. We note that parallel calculations could be useful for bottomonium in the context of future experiments.

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