The spatial-temporal instability behavior of an electrified viscoelastic liquid sheet

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Abstract. The spatial-temporal instability of an electrified viscoelastic liquid sheet injected into air in a vertical electric field is analysed, in which the complete electrical force is considered. A new mode induced by the electric field (referred to as electrical mode) is found in the long wave region. The results show that the absolute-convective instability (AI-CI) of the electrical mode is only affected by the electric field, while that of the hydrodynamic mode is significantly influenced only when the electric field exceeds a threshold value. In most cases, the electrical mode is the dominant mode unless the Euler number is very small or large. The dominant mode first transforms from the hydrodynamic mode to the electrical mode in a weak electric field, when the flow is convectively unstable. However, it would change back in absolute instability region with a strong electric field. In addition, the competition between the two modes is also influenced by the Weber number and Reynolds number.

1. Introduction

In many problems of instability, the amplitude of a disturbance develops in both time and space, which requires to use spatial-temporal instability analysis. If the amplitude of an initially localized disturbance envelope eventually decays at any fixed location while it grows when observed in a coordinate system moving with its group speed, the system is convectively unstable. Otherwise, the system is absolutely unstable [1]. A methodology for distinguishing the convective and absolute nature of the instability was applied to two-dimensional hydrodynamic instabilities by Huerre and Monkewitz [2, 3]. The AI-CI characteristic of the flow is usually determined by detecting saddle points which satisfies the Briggs-Bers collision criterion [4, 5]. If the growth rate at the saddle point, i.e. the absolute growth rate \( \omega_0 < 0 \), the flow is convectively unstable, contrarily, the flow is absolutely unstable. Recently, more and more physically consistent analysis in terms of spatial-temporal instability has been extensive used in liquid sheet. [6,7]

Recently, the study of electrified viscoelastic liquid flows has received more and more attention. El-Saied and Syam [8] investigated the electrohydrodynamic instability. The results demonstrated that increasing the Mach number from subsonic to transonic leads to the increase in the maximum growth rate and dominant wave number and the dominant wave number to increase and the electric field further accelerate their growth. Yang et al. [9] utilized a simplified linear stability model to analyse the instability characteristics of the electrified viscoelastic liquid cylindrical jet. In their work, the linear viscoelastic constitutive relation is used to describe the viscoelasticity of dilute polymer solutions under small or moderate deformation. Recently, Ruo et al. [10] performed a three-dimensional
instability analysis of an electrified non-Newtonian liquid jet subjected to unrelaxed axial tension. Liu et al. [11] Studied the atomization mechanism of a charged viscoelastic liquid sheet in a stationary gas. The effects of dimensionless parameters on the instability of the flow is studied. Lauricella [12] investigated the effects of dissipative air drag on the dynamics of electrified jet in the electrospinning process, utilizing a Brownian noise model. They examined the role of air drag force in the electrospinning process, and provided prospective beneficial implications for improving forthcoming electrospinning experiments. Tong et al. [13] performed an investigation about the viscoelastic liquid sheet subjected to noticeable unrelaxed axial tension. In this work, we study the spatial-temporal instability of an electrified viscoelastic liquid sheet under the complete electrical force, which is more close to practical conditions.

2. Equations

The schematic of the problem analysed is shown in figure 1.

Figure 1. Sketch of the electrified viscoelastic liquid sheet. The liquid sheet has a density of $\rho$, a thickness of $2a$ and its surface tension is $\tau$. The liquid flow moves with a horizontal velocity $U$. Two flat electrodes are positioned on the top and bottom surface of the liquid sheet, and the distance between them is $2b$. A voltage $U_0$ is imposed on the sheet surface.

An electrical potential function $\phi$ is introduced, which satisfies the Laplace equation $\Delta \phi = 0$.

The equations governing the motion of liquid in the sheet are the conservation laws of mass and momentum, i.e.

$$\nabla \cdot \mathbf{v} = 0$$

$$\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \mathbf{v} = -\frac{1}{\rho} \nabla \cdot \mathbf{\pi}$$

Where $\mathbf{\pi} = p\mathbf{\delta} + \mathbf{\tau}$, $\mathbf{v}$ is the liquid velocity vector, $\mathbf{\pi}$ is the total stress tensor, $p$ is the pressure, and $\mathbf{\tau}$ is the extra stress tensor.

The viscoelastic property of the liquid is described by the Oldroyd-B constitutive equation, i.e.

$$\mathbf{\tau} + \lambda_1 \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \mathbf{\tau} = -\eta_0 \left( \dot{\gamma} + \lambda_2 \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \dot{\gamma} \right)$$

where $\dot{\gamma} = \nabla \mathbf{v} + (\nabla \mathbf{v})^T$ is the rate of strain tensor, and $\eta_0$ is the zero shear viscosity, $\lambda_1$ is the stress relaxation time, and $\lambda_2$ is the deformation retardation time.

The liquid sheet is perturbed by a sinuous mode infinitesimal disturbance. To linearize the equations, the normal mode decomposition is substituted. Finally, we get the dimensionless dispersion relation:

$$\frac{4\lambda K^2 We \left( 1 + i M Re E_i (K - \Omega) \right)^2}{Re^2 \left( 1 + i M Re E_i (K - \Omega) \right)^2} \tanh(\lambda) = -\frac{We \left( \lambda^2 + K^2 \right) \left( 1 + i M Re E_i (K - \Omega) \right)^2}{K Re^2 \left( 1 + i M Re E_i (K - \Omega) \right)^2} \tanh(K) + \frac{K We E_u}{D^2} \coth(DK) - K^2 = 0$$
set The dimensionless wave number $K = ka$, the dimensionless frequency $\Omega = (a/U)\omega$, the dimensionless parameters involved include the Reynolds number $Re = \rho Ua/\eta_0$, the Weber number $We = \rho U^2 a/\sigma$, the Euler number $Eu = \varepsilon_0 U_0^2 / \rho U^2 a^2$, where $\varepsilon_0$ is vacuum dielectric constant; the elasticity number $El = \lambda_2 \eta_0 / \rho a^2$, the ratio of the distance between horizontal electrodes and the liquid sheet-to-sheet thickness $D = (b - a)/a$, the ratio of the deformation retardation time to the stress relaxation time $M = \lambda_2 / \lambda_4$. In the calculation, $El = 1$, $D = 40$, and $M = 0.1$. Yang et al. [17] obtained a similar dispersion relation, in which the electric field was approximated by neglecting the horizontal electrical force. However, in our dispersion relation, the complete electrical force is particularly considered.

### 3. Results and Discussions

A linear stability analysis is applied firstly. In figure 2, the increase of the electric field strength enhances the disturbance growth rate, and meanwhile enlarges the corresponding instability domain of wave number. In figure 3, the increase of the Reynolds number enhances the instability of the non-Newtonian fluid.

**Figure 2.** The effect of electric field on the disturbance growth rate of the viscoelastic fluid, where $Re=1000$, $We=400$.

**Figure 3.** The effect of the Reynolds number on the disturbance growth rate of the viscoelastic fluid, where $We=400$, $Eu=1$.

In figure 4, the increase of the Weber number enhances the disturbance growth rate, and enlarges the corresponding instability domain of wave number as well.

**Figure 4.** The effect of the Weber number on the disturbance growth rate of the viscoelastic fluid, where $Re=1000$, $Eu=1$.

**Figure 5.** Contours of $\omega_1 = constant$ in the complex $k$ plane, where $We = 6$, $Re = 80$, $Eu = 0$. 
The maps of the saddle point on the complex wave number plane for We = 6, Re = 80, Eu = 100 are shown in figure 6. A saddle point marked 1 is located in the long wave region, and the other saddle point marked 2 is located in a shorter wave region. In figure 5, when the electric field is absent, the long wave mode disappears, indicating that it is induced by the electric field, so we call it as the electrical mode.

![Figure 6](image1.png)

**Figure 6.** Maps of saddle points (a) Contours of \(\omega_r = \text{constant in the complex } k \text{ plane, where } We = 6, Re = 80, Eu = 100. (b) The amplification of small wave number region of (a).**

The absolute growth rates of the two modes’ the saddle points with increased electric field is shown in figure 8. The instability of the electrical mode is only affected by the electric field, which is enhanced as the increases of Euler number. For the hydrodynamic mode, the instability is significantly influenced only when the electric field exceeds a threshold value.

In addition, the competition for dominant instability mode is also shown in figure 7. In most cases, the electrical mode is the dominant mode unless the Euler number is very small or large. In a weak electric field, the electrical mode will become the dominant mode as the flow is convectively unstable. However, the dominant mode would transform from the electrical mode to the hydrodynamic mode, with the increase of electric field in the absolute instability region.

![Figure 7](image2.png)

**Figure 7.** The absolute growth rates of the two modes versus the Euler number. (a) When Re = 80, (b) When We = 6.

For the hydrodynamic mode, larger Weber numbers weaken the instability in a weak electric field, but intensify the instability in a strong electric field, implying that surface tensor has both destabilizing and stabilizing effects. The increase of Reynolds number enhances the instability of the hydrodynamic mode indicating that viscosity has a destabilizing effect. In terms of dominant mode competition, the increase of Weber number promotes the transition to occur under a smaller Euler number, favoring the
hydrodynamic mode to be dominant, but the increase of Reynolds number has a negative effect on the transition.

The AI-CI transition boundary of the two modes on Re – We plane is separately shown in figure 8. The boundary of electrical mode exists only when it is under the AI-CI critical Euler number Eu = 137. The upper right region is the CI region and the lower left region is the AI region. For the hydrodynamic mode, the left region is the CI region, and the right region is the AI region. As the electric field increasing, the boundary moves left and tends to be a vertical line, which indicates that the AI region is enlarged and its instability becomes less affected by Reynolds number.

![Figure 8](image_url)

**Figure 8.** The AI-CI transition boundary of the two modes. (a) The electrical mode under Eu=137. (The CI region is in upper right, the AI region is in lower left region) (b) The hydrodynamic mode. (the CI region is in left, and the AI region is in right).

In summary, the spatial-temporal instability behavior of an electrified viscoelastic liquid sheet is investigated. A new long-wave unstable mode is found, besides a hydrodynamic mode. For the electric mode, its instability only depends on the electric field. And only when Eu = 137, which is the critical Euler number, Weber number and Reynolds number play roles in the AI-CI transition. For the hydrodynamic mode, the Euler number is able to affect the instability only when it exceeds a threshold value. Surface tensor shows destabilizing and stabilizing effects, while viscosity shows a destabilizing effect. The dominant instability mode mainly depends on the electric field, but the Weber number and Reynolds number can play roles in the mode competition.

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