Related Power-law Growth of Particle Multiplicities near Midrapidity in Central Au+Au Collisions and in $\vec{p}(p) - p$ Collisions

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Abstract

A simple power-law growth of charged-particle multiplicities near midrapidity in central Au+Au collisions at $\sqrt{s_{NN}} = 56$ and 130 GeV, recently measured at RHIC, is derived. We give predictions for the central particle densities up to $\sqrt{s_{NN}} = 1800$ GeV. A strong growth of the Au+Au densities above those for $\vec{p}(p) - p$ collisions is predicted.

Charged-particle multiplicity densities near midrapidity in Au+Au collisions at $\sqrt{s_{NN}} = 56$ and 130 GeV are among the first results[1] from the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory. The most striking feature of the data is the strong rise of the density (normalized to the measured number of participating pairs of nucleons) in going from 56 to 130 GeV, an increase of about 31%. As seen in Fig. 1, the rising curve through the data for Au+Au collisions is much steeper than the curve through the corresponding densities measured for $\vec{p}(p) - p$ collisions over the range of high c. m. collision energies, 22 to 1800 GeV[2, 3, 4, 5]. The latter curve is steeper than growth as $\ln s$. In this paper, we show that a simple power-law growth represents the $\vec{p}(p) - p$ central-density data accurately. The power is slowly increasing; the values are the same as the powers which have been theoretically calculated on the basis of dynamical arguments in an early study[6] of the average charged-particle multiplicities as a function of $\sqrt{s}$. This is the curve through the $\vec{p} - p$ data points shown in Fig. 1. Using this power-law growth, we show that a power-law growth of the density holds for central Au+Au collisions, and we give explicit values for the slowly-increasing powers in terms of those calculated for $\vec{p}(p) - p$ collisions[6]. The power law growth fits the new data[1] for central Au+Au collisions, and allows definite predictions for higher $\sqrt{s_{NN}}$. These are given for $\sqrt{s_{NN}}$ in the energy domain 200 to 1800 GeV, as shown by the curve through the Au+Au data in Fig. 1. We note that Pb+Pb data[7] at $\sqrt{s_{NN}} = 17.8$ GeV is also represented by the curve. It is worth recalling that general theoretical arguments exist for simple power-law growth of multiplicities, with a limiting power approached asymptotically, in particular the arguments of Polyakov[8] from the early days of approximate scale-invariance and self-similar processes. Also, power-law (inverse) behaviour has been derived[9] for the asymptotic S-matrix amplitude whose integration over impact parameter determines the growth of the total $\vec{p}(p) - p$ cross section at very high energies[10].
The growth of the normalized particle density near central pseudorapidity, for \( \bar{p}(p) - p \) collisions, is given by the form,

\[
\frac{dN}{d\eta} = \rho(\sqrt{s}) = A(\sqrt{s})^{2p(s)}
\]  

(1)

where \( A \) is a constant of order of unity\(^{F1}\), and \( p(s) \) is a small power which grows slowly with increasing \( s \), approaching a value of the order of 0.1 at very high \( \sqrt{s} \). This follows from an accurate representation of the average charged-particle multiplicities given by the form\(^{F2}\).

\[
\langle n(s) \rangle = Af(s)(\ln\sqrt{s})^{2p(s)}
\]  

(2)

Here, \( A \) is a normalization constant of order unity\(^{F1}\), \( f(s) \) is a function which approaches approximately unity at large \( s \), and the \( (\ln s)^{2p(s)} \) factor represents the increasing extent of the rapidity “plateau”. Thus the essential dependence is as \( (\ln s)^{2p(s)} \). The slowly increasing \( p(s) \) have been theoretically calculated at 12 values of \( \sqrt{s} \) from 14 to 40,000 GeV (as given in Table I of \(^{F3}\)). The calculation involves a single phenomenological parameter which controls the (slowing) growth of \( p(s) \) toward a limiting value of order of 0.1 (Eq. (5) in \(^{F3}\)). The calculation is based upon the physical idea that collision energy which is progressively removed from the extreme fragmentation region sustains a power-law growth of \( \langle n(s) \rangle \) with a slowly increasing power, i. e. an increasingly high multiplicity of particles over the rapidity plateau. This is consistent with the entire central \( \bar{p}(p) - p \) collision system becoming “blacker” over the impact-parameter plane.\(^{F3}\)

We assume that the previously calculated powers, \( p(s) \) used in Eq. (2), are approximately the powers to be used in Eq. (1). Using the calculated \( p(s) \) from Table I in \(^{F3}\), we obtain from Eq. (1) the following \( \rho(\sqrt{s}) \), for \( A = 1.2 \) (with \( \sqrt{s} \) in GeV),

\[
\begin{align*}
p p & \quad \sqrt{s} = 22 \quad p(s) \cong (0.043 - 0.055) \rightarrow \rho(22) \cong (1.6 - 1.7) \\
\bar{p}p & \quad \sqrt{s} = 53, \quad p(s) \cong 0.055 \rightarrow \rho(53) \cong 1.9 \\
& \quad \sqrt{s} = 200, \quad p(s) \cong 0.069 \rightarrow \rho(200) \cong 2.5 \\
& \quad \sqrt{s} = 546, \quad p(s) \cong 0.075 \rightarrow \rho(546) \cong 3.1 \\
& \quad \sqrt{s} = 630, \quad p(s) \cong 0.075 \rightarrow \rho(630) \cong 3.2 \\
& \quad \sqrt{s} = 900, \quad p(s) \cong 0.078 \rightarrow \rho(900) \cong 3.45 \\
& \quad \sqrt{s} = 1800, \quad p(s) \cong 0.081 \rightarrow \rho(1800) \cong 4
\end{align*}
\]  

(3)

The resulting curve shown in Fig. 1 is an accurate representation of the data. Eq. (1) gives agreement with 7 explicit data points with a single parameter\(^{F3}\) controlling the growth of the power \( p(s) \), and the normalization \( A \).

Given the basic power-law behavior, then for central Au+Au collisions, the growth of the normalized particle density near midrapidity can also be given approximately by a simple power-law form, with \( p \)\(^{F3}\)\(^{F1}\)\(^{F2}\)\(\sqrt{s}N_{NN} \) and normalization,

\[
\frac{1}{0.5\langle N_{part} \rangle} \frac{dN}{d\eta} = \rho_{Au}(\sqrt{sN_{NN}})
\]

\[
\cong \left\{ A(\sqrt{sN_{NN}})^{2p(sN_{NN})} \right\} \times \left\{ A \left( \frac{\sqrt{sN_{NN}}}{\langle n(sN_{NN}) \rangle} \right)^{2p} \right\}
\]  

(4)

\(F1\) The physical meaning of this simple normalization is about one particle per unit of rapidity at \( \sqrt{s} \approx 5 \) GeV. \(^{F3}\)

\(F2\) It is noteworthy that analyzes of certain characteristics of cosmic-ray air showers above 10\(^{18}\) eV suggest a more strongly growing hadronic cross section and/or “inelasticity”.
The first quantity in brackets represents the multiplicity from each of 0.5\langle N_{\text{part}} \rangle collisions\footnote{We approximate the multiplicity growth in the same way for hadron-hadron collisions i. e. pion-pion, pion-nucleon, nucleon-nucleon. At high enough energies, one might make explicit reference to partonic collisions. The notion of partonic “saturation” at very high \sqrt{s} can be incorporated in the limiting value of the power \rho(s).} where \langle N_{\text{part}} \rangle is the experimentally estimated\footnote{The last factor in Eq. (4) can be generalized to include neutrals, and a lower, effective \sqrt{s} for centrally-produced particles. As an example, a resulting approximate numerical factor of \sim 1.5 \left( \frac{\sigma_{p\pi}}{\sigma_{NN}} \right)^{0.1} is close to unity. Eq. (4) can be generalized to \rho_{AA}(\sqrt{s}) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.} average number of participants at \sqrt{s_{NN}}. The second quantity in brackets gives rise to an expanded multiplicity, in the approximation in which each particle from the initial multiplicity undergoes an additional collision with a typical collision energy approximated roughly as \sqrt{s_{NN}} = \left( \sqrt{s_{NN}}/\langle n(s_{NN}) \rangle \right). This gives rise to multiplication of the initial multiplicity by the bracketed number which is determined by the power \rho, the value of \rho(s) appropriate to \sqrt{s_{NN}}. Additional collisions can occur, but in the energy range \sqrt{s_{NN}} from 56 to 1800 GeV discussed below, further reduction to a typical subsequent collision energy results in low energy. In calculating \rho_{AA}(\sqrt{s_{NN}}) from Eq. (4), we use the values of the average multiplicity \langle n(\sqrt{s_{NN}}) \rangle as these are tabulated in Table I of\footnote{In our calculations with all relevant \sqrt{s_{NN}} < 50 GeV, we use an approximate \rho(\sqrt{s_{NN}}) = \rho(53) = 0.055. In more detail, use of smaller powers at low energies would tend to be compensated by use of higher values of effective energy for some collisions i. e. nucleon-nucleon, valence quarks.\footnote{Then \left( \sqrt{s_{NN}}/\langle n(s_{NN}) \rangle \right) \approx \sim \sqrt{s}.} we have calculated \rho_{AA}(\sqrt{s_{NN}}). We give the effective power \footnote{The densities as such, make no direct reference to particle momenta.} of s_{NN}, \rho' = (\rho(s_{NN}) + \rho(\sqrt{s_{NN}})) and the results for \rho_{AA}(\sqrt{s_{NN}}),

\begin{align*}
\sqrt{s_{NN}} &= 56, & \rho' \approx 0.11 &\to \rho_{AA}(56) \approx 2.6 \\
\sqrt{s_{NN}} &= 130, & \rho' \approx 0.12 &\to \rho_{AA}(130) \approx 3.4 \\
\sqrt{s_{NN}} &= 200, & \rho' \approx 0.125 &\to \rho_{AA}(200) \approx 4 \\
\sqrt{s_{NN}} &= 1800, & \rho' \approx 0.135 &\to \rho_{AA}(1800) \approx 7.1
\end{align*}

As a check, for Pb+Pb collisions at a relatively low \footnote{Then \left( \sqrt{s_{NN}}/\langle n(s_{NN}) \rangle \right) \approx 3 \text{ GeV.}} \sqrt{s_{NN}} = 17.8, \rho' \approx 0.11 \to \rho_{Pb}(17.8) \approx 2.\footnote{The \rho_{Pb} data is parameterized} The resulting curve in Fig. 1 represents the present data and predicts the densities at higher \sqrt{s_{NN}}, from 200 to 1800 GeV. With the measured \footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.} \langle N_{\text{part}} \rangle = 330 at 56 GeV and 343 at 130 GeV, our calculated values of \langle dN/d\eta \rangle are 430 and 580, respectively. The experimental numbers for \langle dN/d\eta \rangle are 408 and 580, respectively. The experimental numbers for \langle dN/d\eta \rangle are 408 \pm 40 (stat) \pm 35 (syst). The predicted \langle dN/d\eta \rangle are 680 at 200 GeV (for \langle N_{\text{part}} \rangle = 343), and 1220 (1350) at 1800 GeV (for \langle N_{\text{part}} \rangle = 343(380)). In Eq. (4), the quantity \left( \sqrt{s_{NN}}/\langle n(s_{NN}) \rangle \right)^{2p} gives an increase in the multiplicity by about 20% at 56 GeV. This increases to \sim 30% at 130 GeV, and to \sim 50% at 1800 GeV. It is useful to compare these multiplicities with another recent calculation\footnote{\rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.} which gives a low value of \sim 945 at 1800 GeV (i. e. \rho_{Pb}(1800) \approx 5.5). This calculation involves applying a large “correction”, motivated by speculative dynamics, to reduce excessive multiplicities from a dual model (already necessary at 56 GeV, i. e. note Fig. 1 in\footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.}. This results in a very strong suppression of multiplicities in the TeV range\footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.}.

In conclusion, although scale invariance\footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.} is not exact, a global\footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.} collision property like particle densities near midrapidity for Au+Au collisions and \bar{p}(p) – p collisions can be directly related and quantitatively represented by a physically motivated, simple power-law growth with energy\footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.} This is relevant to simply estimating the maximum energy density\footnote{\rho_{Pb}(200) \approx 3. Our measured \rho_{Pb}(130) \approx 2. The data is parameterized as \rho_{Pb}(s) = \rho_{Pb}(53) \times (s/50)^{0.15}, which is close to unity. Eq. (4) can be generalized to \rho_{Pb}(s) \left\{ 1 - x + x A \left[ (\rho(s))/\langle n(s) \rangle \right]^{2p} \right\}, where the function x \equiv 0 for \rho(p) – p collisions goes continuously to x \equiv 1 for central collisions of two heavy ions, at present \sqrt{s}.}. which is reached only for a very brief collision time.
Added note

Since completion of this paper, new results have appeared from RHIC in this rapidly developing field. First, data on the “centrality” dependence for Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV, i.e. $\rho_{Au}$ versus $\langle N_{part} \rangle$ appears in Fig. 4 of [13]. This can be approximately, but simply described by the generalized formula in footnote F3. At $\sqrt{s_{NN}} = 130$ GeV, write $\rho_{Au} \approx (2.25) \{(1 - x) + x(1.2)(1.3)\}$, with the parameterization $x = a \ln(N_{P}/b) \approx \ln(N_{P}/10)$, where $N_{P} = \langle N_{part} \rangle$. Then,

$$
\rho_{Au} \approx 2.25 \quad \text{at} \quad N_{P} \approx 10 \quad (x \rightarrow 0)
$$

$$
\rho_{Au} \approx 2.75 \quad \text{at} \quad N_{P} \approx 40
$$

$$
\rho_{Au} \approx 3.05 \quad \text{at} \quad N_{P} \approx 100
$$

$$
\rho_{Au} \approx 3.3 \quad \text{at} \quad N_{P} \approx 200
$$

$$
\rho_{Au} \approx 3.45 \quad \text{at} \quad N_{P} \approx 300
$$

$$
\rho_{Au} \approx 3.5 \quad \text{at} \quad N_{P} \approx 350 \quad (x \rightarrow 1)
$$

We assume that $x$ remains $\sim 1$ for $N_{P} \geq 350$ at $\sqrt{s_{NN}} > 130$ GeV. It is noteworthy that $d\rho_{Au}/dN_{P} \propto 1/N_{P}$, is similar to the $d\rho_{Au}/dN_{P} \propto 1/N_{P}^{0.84}$ obtained from the phenomenological fit $\rho_{Au}N_{P} \propto N_{P}^{(1.16 \pm 0.04)}$ quoted in [13]. The above representation of $\rho_{Au}(N_{P})$ does not necessarily require a markedly increased fraction of “hard” processes at $\sqrt{s_{NN}} = 130$ GeV. In this connection, there is new data on the transverse energy density $dE_{T}/d\eta$, which indicates [14] that $(dE_{T}/d\eta)(dN_{ch}/d\eta)^{-1}$ does not increase markedly in going down from $\sqrt{s_{NN}} \approx 17$ GeV to 130 GeV. The present ideas are consistent with this behavior, because the energy is going mainly into a marked increase in particle production (relative to $p-p$)–not into a substantially larger fraction of “hard” processes.

Finally, with respect to Fig. 1, we note that a recent measurement [13] for Pb+Pb at $\sqrt{s_{NN}} \approx 17$ GeV, when scaled to Au+Au, gives a higher value $\rho_{Au} \sim 2.46$. Also, we note that our curve for $p-p$ reaches the value of $\rho \sim 1.6$ at $\sqrt{s} = 22$ GeV, near to the value measured for $pp$-collisions [2].

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References

[1] PHOBOS Collab., B. B. Back et. al., Phys. Rev. Lett. 85, (2000) 3100.
[2] NA22 Collab., M. Adamus et al., Z. Phys. C37, (1988) 215.
[3] UA5 Collab., G. J. Alner et. al., Phys. Rep. 154, (1987) 247.
[4] UA5 Collab., R. E. Ansorge et al., Z. Phys. C43, (1989) 357.
[5] CDF Collab., F. Abe et al., Phys. Rev. D41, (1990) 2330.
[6] S. Barshay and P. Heiliger, Phys. Rev. D47, (1993) 4150.
[7] J. Bächler et al., Nucl. Phys. A661, (1999) 45.
[8] A. M. Polyakov, Zh. Eksp. Teor. Fiz. 59, (1970) 542 [ Sov. Phys. JETP 32, (1971) 296].
[9] S. Barshay, P. Heiliger, and D. Rein, Mod. Phys. Lett. A7, (1992) 2259. This paper also contains a previously unpublished derivation due to R. P. Feynman.
[10] S. Barshay, P. Heiliger, and D. Rein, Z. Phys. C\textbf{52}, (1991) 415; Z. Phys. C\textbf{56}, (1992) 77.

[11] J. Dias de Deus and R. Ugoccioni, Phys. Lett. \textbf{B491}, (2000) 253.

[12] C. Pajares, D. Sousa, and R. A. Vázquez, Phys. Rev. Lett. \textbf{86}, (2001) 1674.

[13] PHENIX Collab., K. Adcox et al., Phys. Rev. Lett. \textbf{86} (2001) 3500.

[14] PHENIX Collab., K. Adcox et al., nucl-ex/0104015.

[15] WA98 Collab., M. M. Aggarwal et al., Eur. Phys. J. C\textbf{18} (2001) 651.
Figure 1: Measured pseudorapidity density near midrapidity normalized per measured participant pair, for central Au+Au collisions at $\sqrt{s_{NN}} = 56$ and 130 GeV. The point at 17.8 GeV is for Pb-Pb collisions. The predicted curve is calculated from Eq. (4). For comparison, $\bar{p} - p$ data points from [3, 4, 5] are shown. This curve is from Eq. (1). Both curves are simple power laws with related, theoretically-calculated powers [6], and related normalization.