Uncertain mesoscopic parameters in dry woven fabric with Gaussian Emulation Machine for Sensitivity Analysis (GEM-SA)

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Abstract. An approach to investigate the influence of uncertain mesoscopic parameters on the macroscopic stiffness of dry woven fabric is presented. The key input parameters that have been investigated are the yarn spacing, the yarn width, the yarn height, fabric misalignment angle and the friction coefficient between the yarns. A sensitivity analysis based on GEM-SA (Gaussian Emulation Machine for Sensitivity Analysis) is presented to determine the macroscopic stiffness and its probability density function (PDF) with respect to the given set of uncertain input parameters. The uniaxial tensile deformation of a fabric is analysed as a special case of the biaxial stretching theory and expressed in terms of frictional interactions occurring at the crimp interchange points in both warp and weft yarns. The yarn-yarn arrangement is replaced by a model in which the warp and weft yarns are straight lines with a point of inflection at the center of the overlapping yarns. The results show that the frictional component is the dominant component as compared to the adhesion component in yarn-yarn interactions. We also quantify the yarn-yarn interactions among uncertain mesoscopic parameters with respect to the macroscopic stiffness. It is found that the misalignment of yarn angle is the most influential factor followed by the friction coefficient in dry woven fabric stiffness predictions with 0.44 of first sensitivity index contribution for both methods.

1. Introduction

Considerable attention has been devoted to woven fabric composite materials in the past decades and a realistic fabric geometric description is essential for the modelling of the mechanical and physical properties of fabrics and fabric composites. A dry woven fabric investigation is appropriate to define the correlation of yarn surface configuration and fibre alignment in woven fabric composite. The mechanical behaviour of dry woven fabrics is important for the manufacturing process of textile composites wherein it starts with the placement of a dry woven fabric in a preform. Dry fabric refers to a bundle of fibres with no matrix that bond them together. This makes the dry fabric in highly nonlinear and strain dependent transverse stiffness condition and no shear stiffness values in their constitutive models. Attempts to determine the mechanical behaviour of dry woven fabrics start from three major paths. The first is the well-known Peirce geometrical model [1] and has been used for instance by Painter [2] and Love [3]. However, those techniques are lacking in reliability and applicability.

Secondly, analytical approaches developed by assimilating the mechanism of force equilibrium into geometrical models and reducing the number of over-constrained assumptions. Hearle et al. [4] implemented an analytical approach on the mechanical behaviour of woven fabrics by applying both geometrical and mechanical analyse. Conversely, Buet-Gautier and Boisse [5] and Cao et al. [6] conducted experiments to characterize the mechanical behaviour of woven fabrics. Cavallaro et al. [7]
proposed both numerical and experimental methods for determining the elastic and shear moduli of uncoated plain woven fabrics. Takano et al. [8], Skoček et al. [9] and Rabczuk et al. [10] employed computational homogenization in order to predict macroscopic properties based on microscopic models. Takano et al. [8] proposed a four-level hierarchical modelling of textile composite materials and structures. Skoček et al. [9] employed the Mori-Tanaka method in obtaining the effective properties of textile composites. Based on a simple periodic unit cell, Zeman and Šejnoha [11] derived a theoretical formulation for homogenization of balanced plain weave composites with imperfect micro-structure. It is reported that the theoretical formulation is applicable to any material with imperfect micro-structures. Eventually, Tabiei and Yi [12] carried out a comparative study of the four-cell and the simplified methods of cells for woven fabric composite elastic properties. Recently, Beex et al. [13] employed a lattice model in experimental identification of electronic woven textiles.

The third approach is based on continuum mechanics by treating the fabric as a continuum laminate, e.g. the contributional by Kilby [14], Chou [15], Ko and Pastore [16], Pastore and Gowayed [17] and Hearle and Du [18]. Studies on the mechanical behaviour of fabrics under biaxial extension were carried out by Clulow and Taylor [19] and Reichardt et al. [20]. Their studies mainly dealt with fabric constitutive relationships and rarely elaborated on predicting the stiffness as reported by Pan [21]. As a composite of yarns and coating materials, dry woven fabrics are initially modelled as bundles of yarns, and their probabilistic properties are often estimated based on an assumed yarn distribution referring to the underlying stiffness distributions of the warp and weft yarns as delineated by Shahpurwala and Schwartz [22]. By incorporating yarn-yarn interactions, the predicted accuracy of the fabric stiffness is significantly improved. A similar yarn distribution assumption was adopted by Šejnoha and Zeman [23] to determine the yarn-yarn geometrical imperfections performance (imperfections in fabric laminates) using classical micromechanical schemes such as the Mori-Tanaka method.

Studies related to fabric mechanical behaviour on the basis of the structural models were carried out by Badel et al. [24] and Milani et al. [25] for the misalignment fibres, Karaoğlan et al. [26] for frictional impact response, and Badel et al. [27] for large deformation analyses using rate constitutive equations. Stochastic variations of the input parameters like material properties, geometry thicknesses, etc. can significantly affect the macroscopic stiffness. During the fabrication process, uncertainties (e.g. measurement errors) may occur and contribute to the variations of fabric stiffnesses in which follow a normal distribution [28]. Up to now, there are very few studies attempting to quantify the influence of uncertain input parameters with respect to the fabric stiffness.

Hence, this paper aims to conduct a variance-based sensitivity analysis to quantify which variation in the uncertain mesoscopic parameters are driving the macroscopic dry woven fabric stiffness uncertainty using GEM-SA (Gaussian Emulation Machine for Sensitivity Analysis). A uniaxial stretching theory as elaborated by Kawabata et al. [29] is used to calculate the dry woven fabric stiffness functions. The yarn-yarn interactions are characterized in terms of frictional interactions occurring at the crimp interchange points as derived by Pan [21]. A crimp interchange point is defined as the interaction point of the two yarns, which is also known as an undulation point. The mesoscopic uncertainty criteria included in this study are the yarn spacing, yarn height, yarn width, misalignment of the yarns angle and friction coefficient between individual yarns. We also quantitatively determine the yarn-yarn interactions among uncertain mesoscopic parameters with respect to the macroscopic fabric stiffness and provide the lower and upper bounds of our predictions.

This paper is organized as follows: The next section describes the basic equations governing the macroscopic fabric stiffness of the mesoscopic dry-woven unit cell. The sensitivity analysis is explained in Section 3. Section 4 presents the numerical results before the paper is concluded in Section 5.

2. Determination of dry woven fabric strength

Woven fabrics consist of yarns in two principal in-plane directions (the warp and weft direction). The mechanical response of a fabric is therefore governed by the two types of yarns and the interactions between them. It is well-known that the tensile properties (force-deformation relationship) of textile
fabric are non-linear caused primarily by the weave structure and secondly by the geometric non-linearity of the warp and weft yarns. They also depend on the fibre properties for instance the diameter, the coefficient of friction and the initial elastic modulus. We focus on Mata Berkait-dry woven fabric uniaxial tensile loading and determine the macroscopic fabric stiffness properties with respect to specific uncertain mesoscopic parameters. The uniaxial-deformation theory introduced by Kawabata et al. [30] is applied.

2.1. A uniaxial stretching theory and derivation of fabric stiffness relationship

The structure of the Mata Berkait-dry woven fabric is depicted in figure 1. In figure 1(a) the dotted box focuses on a single yarn-yarn interaction of which a zoom is shown in figure 1(b). The fabric stiffness is governed as a limiting state subjected to the imposed loads along the warp and weft yarns [34]. For the direction in which the stress is applied, the yarn is assumed to be perfectly flexible, considering effects due to bending on the transverse yarn. When an external force is applied to the fabric, the tension force rises in the yarns and straightens the initially curved yarn. However, for the sake of simplicity, in our yarn segment models, their lengths in which determined by two consecutive yarn-yarn interaction, are modelled as straight lines.
Figure 1. The structure model of Mata Berkait-dry woven fabric generated with TexGen [31]. The structure is reproduced from Ilyani Akmar et al. [32] and inspired by Malaysian Handicraft Development Corporation [33]. (a) The structure of a Mata Berkait-dry woven fabric (A = weft yarn, B = warp yarn), (b) a yarn-yarn interaction. Reproduced from Kawabata et al. [30].

The lines that represent yarn segments are connected to each other by additional lines in the yarn-yarn interaction points in which mechanically incorporate the effect of undulation (see Figure 1(b)). A Cartesian coordinate system is introduced with the axis $X_1$ in the warp direction and the axis $X_2$ in the weft direction. The origin is located at the point $O$ in Figure 1(b). The warp and weft yarns are assumed to be straight lines which bend at point $p_1$ and $p_2$ in the $X_3$-direction as in Figure 2. Utilizing this simplification, the structural constants for the fabric stiffness prediction for the uni-axial case are derived as follows

\[
\begin{align*}
    y_{01} &= \frac{1}{n_2} ; \quad y_{02} = \frac{1}{n_1} ; \\
    S_1 &= \frac{(l_{01} - y_{01})}{y_{01}} ; \quad S_2 = \frac{(l_{02} - y_{02})}{y_{02}} \\
    \sin \theta_{01} &= \frac{1}{s_{1+1}} ; \quad \sin \theta_{02} = \frac{1}{s_{2+1}} ; \\
    l_{01} &= \frac{y_{01}}{\sin \theta_{01}} ; \quad l_{02} = \frac{y_{02}}{\sin \theta_{02}} \\
    h_{m1} &= \frac{l_{01}}{2} \cos \theta_{01} ; \quad h_{m2} = \frac{l_{02}}{2} \cos \theta_{02}
\end{align*}
\]
Figure 2. The unit structure model in the (a) undeformed state and (b) deformed state as a replacement model of the real model. Reproduced from [30].

where:
- \( y_0 \) = the spacing of warp yarn in undeformed state;
- \( y_0 \) = the spacing of weft yarn in undeformed state;
- \( l_0 \) = the yarn length of warp direction in undeformed state;
- \( l_0 \) = the yarn length of weft direction in undeformed state;
- \( h_m \) = the distance between the neutral line and the yarn axis along the \( X_3 \)-axis in the undeformed state for warp yarn (also a maximum value of \( h \) in warp direction);
- \( h_m \) = the distance between the neutral line and the yarn axis along the \( X_3 \)-axis in the undeformed state for weft yarn (also a maximum value of \( h \) in weft direction);
- \( h_1 \) = the deflection of the point at which the warp yarn intersects the \( X_3 \)-axis. The deflection is caused by the stretch ratios \( \lambda_1 \) and \( h_1 \) is the movement of point \( p_1 \);
- \( \theta_0 \) = the angle between the yarn axis and the \( X_3 \)-axis in the unit structure model in the undeformed state;
- \( \theta_1 \) = the angle between the yarn axis and the \( X_3 \)-axis in the unit structure model in the deformed state;
- \( n_1 \) = number of ends/cm (in the undeformed state);
- \( n_2 \) = number of picks/cm (in the undeformed state);
- \( S_1 \) = the yarn crimp due to weaving which defined by \( \frac{(l_01-\gamma_01)}{\gamma_01} \); where \( l_01 \) is the length of warp yarn in the unit structure and \( \gamma_01 = \frac{1}{n_2} \);
- \( S_2 \) = the yarn crimp due to weaving which defined by \( \frac{(l_02-\gamma_02)}{\gamma_02} \); where \( l_01 \) is the length of warp yarn in the unit structure and \( \gamma_02 = \frac{1}{n_1} \).

If we consider that the fabric is stretched in \( X_1 \)-direction, \( \lambda_1 \) is its stretch ratio. We assume the lateral tensile force \( T_{f2} = 0 \) but the value of \( F_1 \) will not be zero due to the contribution from \( T_{f1} \geq 0 \). The tension force \( T_{f1} \) is governed by the stretch ratios of the warp \( \lambda_{w1} \) and weft \( \lambda_{w2} \) where:
\( F_c \) = the compressive force acting along the \( X_3 \)-direction;
\( \lambda_1 \) = the stretch ratio of the fabric along the coordinate axis for warp direction;
\( T_{f1} \) = tension force in warp yarn direction;
\( T_{f2} \) = tension force in weft yarn direction;
\( \lambda_{y1} \) = stretch ratio of the warp yarn,
\( \lambda_{y2} \) = stretch ratio of the weft yarn,
\( \lambda_y \) = stretch ratio of the yarn, where the stretch ratio \( \lambda \) is given by:

\[
\lambda = 1 + \text{strain} \quad (e)
\]

The Mata Berkait-dry woven fabric is a modification of a plain weave. The difference in weaves reflects the interaction points of the yarns and the contact pressure between the yarns. The compressive forces \( F_c \) of the fabric related to the tension forces \( T_{fi} \) in the yarns, are determined through a simple geometrical and mechanical analysis, see Figure 3. \( F_c \) is approximated in terms of \( f_w \) and \( f_w \) with respect to the path lines per yarn-yarn interaction point; \( f_w \) and \( f_w \) are the warp float and weft float wherein the length of a warp/weft yarn on the face of the fabric, measured in number of intersections as indicated in Long [35] and Pan [21]. The compressive force \( F_c \) of 24 yarns that generate the Mata Berkait unit structure can be formulated as shown in table 1.

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**Figure 3.** The cross-sections of each weave arrangement that construct Mata Berkait-dry woven fabric unit structure: (a) for \( A1 = A6 = B7 = B12 \), (b) for \( A2 = A5 = B8 = B11 \), (c) for \( A3 = A4 = B9 = B10 \), (d) for \( A7 = A12 = B1 = B6 \), (e) for \( A8 = A11 = B2 = B5 \), (f) for \( A9 = A10 = B3 = B4 \). The warp yarn is in red whilst the colourful yarns are the weft yarns. Refer to figure 1(a) for the position of A1-A12 and B1-B12.

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**Figure 4.** The compressive force \( F_c \) imposed on Mata Berkait unit structure.
The compressive force, $F_c$ per single yarn ($A =$ weft yarn, $B =$ warp yarn).

| Index no. | Compressive force, $F_c$ relationship ($i = 1$ (warp)) |
|-----------|-----------------------------------------------------|
| A1 = A6 = B7 = B12 | 0.30 $T_f$ cos $\theta$ |
| A2 = A5 = B8 = B11 | 0.27 $T_f$ cos $\theta$ |
| A3 = A4 = B9 = B10 | 2.25 $T_f$ cos $\theta$ |
| A7 = A12 = B1 = B6 | 1.80 $T_f$ cos $\theta$ |
| A8 = A11 = B2 = B5 | 1.50 $T_f$ cos $\theta$ |
| A9 = A10 = B3 = B4 | 1.25 $T_f$ cos $\theta$ |
| Total $F_c | F_c$ | 6.37 $T_f$ cos $\theta$ |

The stretch ratio of the warp yarns, $\lambda_{y1}$ and the angle of warp yarn, cos $\theta$ are calculated as

$$\lambda_{y1} = \frac{\sqrt{4(h_{m1}-h_1)^2+(\lambda_1y_{01})^2}}{\sqrt{4(h_{m1})^2+(\lambda_1y_{01})^2}}; \quad \cos \theta_1 = \frac{2(h_{m1}-h_1)}{\sqrt{4(h_{m1}-h_1)^2+(\lambda_1y_{01})^2}}$$  

(3)

where, we introduced the misalignment of yarn angle $e$ in the obtained $\theta_1$ as the uncertain mesoscopic parameter in the Mata Berkait-dry woven fabric unit structure. The tensile force of warp yarn, $T_f$ as illustrated in Figure 4 can be written as

$$T_f = E_1 \lambda_{y1} \frac{\bar{\rho}_1}{\rho_1}$$  

(4)

where, $\rho_1$ is the density, $\bar{\rho}_1$ is the linear density and $E_1$ is the Young’s modulus of the warp yarn. Assuming that the cross-section of the yarns remains the same, i.e. yarn diameters do not change due to yarn-yarn interaction force $F_c$, we can conclude that

$$h_1 = h_{m1}.$$  

(5)

By substituting Equation (3) - Equation (5) into the formulation for $F_c$ as given in Table 1, we obtain

$$6.37g(\lambda_{y1}) = \frac{2(h_{m1}-h_1)}{\sqrt{4(h_{m1}-h_1)^2+(\lambda_1y_{01})^2}} = 0.$$  

(6)

By substituting $h_1$ into Equation (3), the value of $\lambda_{y1}$, $\theta_1$ and $T_f$ can be obtained in the equilibrium state. The expression of the warp yarn stiffness in tension $F_1$ along the $X_1$-axis can be expressed for a given set of $\lambda_3$ as

$$F_1 = T_f \sin \theta_1.$$  

(7)

According to Pan [21], the compression forces $F_c$ are exerted on a contact point of two interlacing yarns with perimeter of yarn shape to cause a frictional force, $\mu F_c$ where $\mu$ is the friction coefficient. Pan [21] highlighted that the frictional component is the dominant component in fabric stiffness. The adhesive component is determined by the surface geometry and properties of the yarns. This frictional force is equilibrated by the shear force, $\tau_y$ and given by

$$\tau_y C_y = \mu F_c$$  

(8)

where $C_y$ represents the length of the yarn-yarn contact area and $\mu$, $y_{01}$, $W_0$, $h_{mi}$ and misalignment in yarn angles, $e$ are the uncertain mesoscopic parameters for the sensitivity analysis in Section 3. This
shear force $\tau_y$, is in fact a reflection of the shear bonding strength of the yarn-yarn contact point, and is related to the yarn surface friction behaviour and the tension forces $T_{f1}$ as indicated.

2.2. Dry-woven fabric properties
The geometries of the Mata Berkait-dry woven fabrics are generated in TexGen and imported into ABAQUS in order to predict their mechanical response. Periodic boundary conditions are used to replicate the repetitive nature of the fabric. The implementation of periodic boundary conditions assure that macroscopic deformation modes, or a combination of them, are applied in an average sense to the mesoscopic model. In addition, the limitation of the periodic boundary condition is given by the characteristic size of any meso-structure in the unit cell or the characteristic length-scale of any important effect should be smaller than the size of the computational unit cell. The cross-sections are assumed to have an elliptical shape and have a uniform deformation in the yarns at the meso-scale level of the fabric unit cells. The material of the yarns is considered to be isotropic and linear elastic. A macroscopic uniaxial deformation is imposed to the unit cells and the strain values obtained from the analyses are utilized to define the fabric stiffness as indicated.

The properties of the yarns including the initial geometry and material properties are listed in table 2. These values are utilized as the mean values for the sensitivity analysis. A normal distribution is assumed in order to determine the ranges of the parameters.

Table 2. Mesoscopic parameters under undeformed state ($0 = $undeformed state, $i = 1$ (warp), 2 (weft)) [25][48][49][50].

| Property                                  | Mean Value                 |
|------------------------------------------|----------------------------|
| Yarn spacing, $y_{0i}$                   | 1.0 mm                     |
| Yarn height, $h_{mi}$                    | 0.15 mm                    |
| Yarn width, $w_{0i}$                     | 0.5 mm                     |
| Yarn length, $l_{0i}$                    | 1.04 mm per unit structure |
| Misalignment angle, $\theta$             | 0 rad                      |
| Yarn-yarn friction coefficient, $\mu$    | 0.25                       |
| Yarn density, $\rho_i$                  | $4.15\times10^6 \text{ g/m}^3$ |
| Linear density of yarns, $\tilde{\rho}_i$ | $2.9\times10^2 \text{ g/m}$ |
| Yarn Young’s modulus, $E_i$              | 200 GPa                    |

3. Sensitivity analysis: fabric strength of dry woven fabric unit cells
Sensitivity analysis is an essential approach of model generation and quality assurance for models involving many input parameters [36][41][49]. The sensitivity analysis in our study aims to investigate how the variation of the uncertain mesoscopic parameters affects the macroscopic fabric stiffnesses. Model interactions occur when the change of two or more input parameters simultaneously cause a variation in the output greater than outputs by varying each of the inputs alone. The presence of such interactions occurs in any non-additive models, but these interactions will be neglected by methods such as scatter plots. With this GEM-SA (Gaussian Emulation Machine for Sensitivity Analysis) accounts for such interactions and is used in this paper. Eventually, sensitivity indices (sensitivity index between uncertainty input parameters) are computed based on the results obtained in GEM-SA.

3.1. Core methodology in sensitivity analysis
The following steps are required:
(a) Estimate the mesoscopic uncertainty in each input parameter (e.g. variation, probability distributions).
(b) Identify the model output to be analysed.
(c) Run the model as dictated by the method of choice and the uncertain mesoscopic parameters.
(d) Calculate the sensitivity measures.
3.2. GEM-SA (Gaussian Emulation Machine for Sensitivity Analysis)

The Gaussian Emulation Machine for Sensitivity Analysis (GEM-SA) is developed by Kennedy [37]. This tool builds an emulator of a computer code from a set of input and output points. It also performs uncertainty and sensitivity analyses of the code using far fewer code runs than Monte-Carlo based methods. The GEM-SA (Gaussian Emulation Machine for Sensitivity Analysis) uses a Gaussian process prior probability distribution to describe an unknown code output, as a function of the code inputs. It is compatible with the Bayesian Analysis of Computer Code Outputs (BACCO) to get a wide definition of the probability. The reason for this is that the Bayesian methods are more efficient than other existing approaches in which the efficiency is achieved through emulation. According to Hagan [38], the two-step approach is used in BACCO where firstly an emulator is built as a statistical representation of the simulator and then this emulator is used to derive relevant analyses.

3.3. Principles of emulation

An emulator is a statistical approximation of the simulator. An approximation to \( f(x) \) for any input configuration \( x \) is a single value \( \hat{f}(x) \). An emulator provides an entire probability distribution for \( f(x) \). We can regard the mean of that distribution as the approximation \( \hat{f}(x) \), but the emulator also provides a distribution around that mean which describes how close it is likely to be to the true \( f(x) \). In fact, Hagan [38] stated that an emulator is a probability distribution for the entire function \( f(.) \).

3.4. Emulators usage in sensitivity analysis

A sensitivity analysis is used to determine what effect each individual input, or pair of inputs, has on the output. The uncertainty of the input parameters is specified using a probability distribution for each input. This is known as the prior distribution, because it corresponds to an uncertainty before obtaining any output. In GEM-SA, a global sensitivity analysis is carried out, meaning that all of the nuisance parameters are integrated over their range. By default, this integration is with respect to a uniform prior distribution for each of the inputs. If a different prior distribution is specified within the project, the integration is performed with respect to this distribution. The prior distribution is selected from the Project command. The mathematical details used by GEM-SA are elaborated below as in Kennedy [39]. GEM-SA provides a standardisation of the set of inputs and outputs for a convenient usage. Let denote the transformed input \( x \), and initial input \( x_i \), as the input component in each set as follows;

\[
x_i = \frac{x_i' - x_{i\text{min}}}{x_{i\text{max}} - x_{i\text{min}}} \tag{9}
\]

where \( x_{i\text{min}} \) and \( x_{i\text{max}} \) denotes the minimum and maximum values of all the \( i \)-th input components. Then the transformed output \( y \) can be retrieved by calculating the sample mean \( m_y \) and variance \( v_y \) of the output points with the initial output \( y_i \) and number of samples \( n \) as

\[
m_y = \frac{1}{n} \sum_{i=1}^{n} y_i' \tag{10}
\]

we let the output \( y \)

\[
y = \frac{y' - m_y}{\sqrt{v_y}} \tag{11}
\]

3.5. Prior Gaussian process model

Once the input space \( \chi \subseteq \mathbb{R}^p \) has been identified, a smooth function \( f : \chi \rightarrow \mathbb{R} \) representation is utilized which derived one dimensional output vector. Lets assume \( \mathbf{h}(x) \in \mathbb{R}^m \) as the vector of arbitrary regression functions for any \( x \in \chi \). Then, the following Gaussian process model is considered; 

\[
[f(\cdot)|\beta, \sigma, \mathbf{r}] \sim N(\mathbf{m}(\cdot), \sigma^2 \mathbf{c}(\cdot, \cdot)) \tag{12}
\]
where the mean \( m(\cdot) \) and correlation functions \( c(x_1, x_2) \) can be written as:

\[
m(\cdot) = \beta^T h(\cdot)
\]

\[
c(x_1, x_2) = \exp\{-(x_1 - x_2)^T R (x_1 - x_2)\} \forall x_1, x_2 \in \chi.
\]  \hspace{1cm} (13)

Based on Equation (13), the variables are delineated as follows:

(a) The vector of regression coefficients \([\hat{\beta}_1, ..., \hat{\beta}_q]\) \(\in \mathbb{R}_q\).

(b) The variance \(\sigma^2\).

(c) The positive definite roughness matrix \(R = \text{diag}\{r_i\} \in \mathbb{R}_{p,p}\).

The diagonal form of \( R \) implies a correlation structure between any pair \( f(x_i) \) and \( f(x_2) \) being insensitive to inputs interactions. Hence, the prior specification is completed by

\[
p(\beta, \sigma^2) \propto \sigma^{-2},
\]  \hspace{1cm} (14)

and for each \( i = 1, \ldots, p \) independently

\[
r_i \sim \mathcal{N}(0, 0.01).
\]  \hspace{1cm} (15)

### 3.6. The posterior distribution for condition on data

The simulation output \( d = [f_i(s_i)] \in \mathbb{R}_n \) is obtained by running the computer code on a pre-selected design set \(\{s_1, ..., s_n\} \subset \chi\). By taking into account the assumptions as previously listed, the joint distribution of the outcomes in \( d \) conditional on nuisance parameters \( \beta, \sigma^2 \) and \( R \) is the normal distribution. Hence, the expression can be expressed as:

\[
[d|\beta, \sigma^2, R] \sim N_n(H\beta, \sigma^2 A)
\]  \hspace{1cm} (16)

where \( H = [h(s_1), ..., h(s_n)] \in \mathbb{R}_{m,n} \) and \( A = [c(s_r, s_i)] \in \mathbb{R}_{n,n} \). Due to the presence of nugget term (due to numerical errors in the solver), the nugget variance \( \lambda^{-1} \) is added to each diagonal element of \( A \). By letting \( t^T(\cdot) = [c(\cdot, s_1), ..., c(\cdot, s_n)] \in \mathbb{R}^n \), the following expression can be formed for the posterior conditional distribution as

\[
[f(\cdot)|\beta, \sigma^2, R, d] \sim N_q(m^*(\cdot), \sigma^2 c^*(\cdot; \cdot))
\]  \hspace{1cm} (17)

where

\[
m^*(x) = \beta^T [h(x) - H^T A^{-1} t(x)] + d^T A^{-1} t(x)
\]

\[
c^*(x_1, x_2) = c(x_1, x_2) - t^T(x_1) A^{-1} t(x_2).
\]  \hspace{1cm} (18)

The posterior distribution of \( f(\cdot) \) conditional on the roughness matrix \( R \) alone is found by integrating Equation 17 with respect to the posterior distribution of \( \beta \) and \( \sigma^2 \). This yields \( f(\cdot) \) with conditional upon the roughness matrix \( R \) as follows;

\[
[f(\cdot)|R, d] \sim t_{n-q}(m^{**}(\cdot), \hat{\sigma}^2 c^{**}(\cdot; \cdot))
\]  \hspace{1cm} (19)

in which

\[
m^{**}(x) = \hat{\beta}^T h(x) + (d - H\hat{\beta})^T A^{-1} t(x)
\]

\[
c^{**}(x, x') = c^*(x, x') - [h(x) - H^T A^{-1} t(x)]^T (H^T A^{-1} H)^{-1} \cdot [h(x') - H^T A^{-1} t(x')].
\]  \hspace{1cm} (20)

The GLS estimator of \( \beta, \hat{\beta} \) is expressed as \( \hat{\beta} = \left(H^T A^{-1} H\right)^{-1} H^T A^{-1} d \) and \( \hat{\sigma}^2 \) can be taken as \( \hat{\sigma}^2 = d^T (A^{-1} - A^{-1} H(H^T A^{-1} H)^{-1} H^T A^{-1}) d \). It can also expressed as
\[ S_i^{\text{GEM}} = \hat{\beta}_i, \quad i = 1, 2, \ldots, n. \]  \hfill (21)

The inferences needed for the uncertainty and sensitivity analyses are found directly from these posterior quantities for \( f (\cdot) \). Detailed elaboration can be found in Kennedy [39] and Hagan [38]. Through GEM-SA, the main effect and total-effect sensitivity indices are directly calculated from the posterior quantities.

### 3.7. Main effect and total-effect sensitivity indices

In addition, the main effect and total-effect sensitivity indices are also calculated. The main effect sensitivity index can be written as

\[ S_i = \frac{V(E(Y|X_i))}{V(Y)}, \]  \hfill (22)

where \( V(E(Y|X_i)) \) is the variance of the expected value of \( Y \) when conditioning with respect to each uncertainty criteria \( X_i \) and \( V(Y) \) is the unconditional variance of \( Y \). The index \( S_i \) is a measure for the exclusive influence of uncertainty criteria \( X_i \). If the sum of all \( S_i \) is close to one, the model is additive, and no interaction of the uncertainty criteria exists. Keitel et al. [40] and Vu-Bac et al. [41] mentioned that in complex engineering problems, interactions between input parameters may exist, thus, higher order sensitivity indices provide additional information. The total-effect index \( S_{Ti} \) is used to present the total contribution of the uncertainty criteria, \( X_i \), to the output, i.e. main effects in addition to all higher order effects and the expression of \( S_{Ti} \) is defined as

\[ S_{Ti} = 1 - \frac{V(E(Y|X_i))}{V(Y)}, \]  \hfill (23)

Due to the parameter interaction in the uncertainty criteria, the value of \( S_{Ti} \) of a parameter in uncertainty criteria increases, therefore \( \sum S_{Ti} \geq 1 \). The difference of \( S_{Ti} - S_i \) measures how \( X_i \) interacts with other input parameters in the uncertainty criteria as found in Keitel et al. [40]. In addition, \( S_i \) and \( S_{Ti} \) demand a large number of samples which leads to an expensive computational analysis.

### 3.8. Mesoscopic uncertainty criteria

Five mesoscopic uncertainty criteria are investigated:

(a) **Yarn spacing**, \( y_i \)

(b) **Yarn width**, \( w_i \)

(c) **Yarn height**, \( h_i \)

(d) **Misalignment of the yarn angle**, \( e \)

(e) **Friction coefficient**, \( \mu \).

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**Figure 5.** Uncertainty criteria designation. (a) Basic yarn geometrical designation. Reproduced from [42], (b) a woven composite with misalignment of yarn angles. Reproduced from [25].
Yarn spacing, height and width are the most important dimensions for dry woven fabric unit cells. The configuration of yarn geometry is given in Figure 5(a). The detail of misalignment fibre is depicted in Figure 5(b) and Figure 6. Referring to Islam and Bandara [43], spacing in yarns essentially delineates to the fabric sett and has a major influence on the appearance. Uniform yarn spacing in fabrics is prominent and excessive variation of yarn spacing can be considered as a fabric imperfection. Commonly, a yarn spacing variation exists due to the yarn count variation, improper settings and adjustments of the machine parameters, eccentricity and wear in the motion transmission system.

![Figure 6](image.png)

**Figure 6.** Misalignments in yarns: (a) positive misalignment, (b) negative misalignment.

The fabric alignment depends on the way the fabric is placed in the fixture or how it was prepared. The experimental work by Lussier and Chen [44] confirmed that misalignment in a yarn under tension condition causes large increases in load and stiffness rather than under shear condition. Milani et al. [25] added that the effect of misalignment of yarn angles becomes more significant as the weave structure becomes more complicated.

Gupta and Mogahzy [45] reported that the friction coefficient between the yarns is a significant importance, since it affects both the efficiency of the manufacturing process and the quality and strength of the resulting end product. Testa and Yu [46] reported that if hardly any friction between yarns is present because the yarns are not coated, a coated fabric offers no resistance to yarn rotation and therefore has no shear stiffness. Hence the resistance to shear deformation results almost entirely from the coating. It has been demonstrated that some new frictional parameters are strongly correlated with some pertinent fabric properties. Ajayi and Elder [47] delineated that an increase of the static frictional force may result from increasing mechanical interlocking of surface protuberances and an increasing fabric compression.

Correct estimates of the tolerance value intervals are crucial for reliable output predictions. The intervals are presented in table 3.

| Mesoscopic uncertainty | Base criteria Value | Lower bound | Upper bound | Data resources |
|------------------------|---------------------|-------------|-------------|----------------|
| Yarn spacing, yi       | 1.0 mm              | -2.5 %      | 2.5 %       | Peng and Cao [48] |
| Yarn width, wi         | 0.5 mm              | -3.55 %     | 3.55 %      | Peng and Cao [48] |
| Yarn height, hi        | 0.15 mm             | -4.0 %      | 4.0 %       | upper estimation of Komeili and Milani [49] |
| Misalignment - yarn angle, e | 0 rad | 0.087 rad | 0.087 rad | Milani et al. [25] |
| Friction coefficient, μ | 0.25                | 0           | 0.5         | modified estimation of Lin et al. [50] |
4. Numerical results

The influence of each input parameter on the fabric stiffness is illustrated in Figure 7 and Table 4. For GEM-SA outputs, there are two ways of result illustrations for the effects of the uncertainty parameters in the performed fabric stiffness response i.e. graphically method, as shown in Figure 7 (line plots) and emulation diagnostics. Based on this line plots, we could estimate the expected value of the output (accumulated by averaging over all other inputs). It can be observed that, the effect of misalignment and friction coefficient are highly considerable. A decrease of \( t_C \) occurs under a negative misalignment or vice versa. This shows that a symmetrical curve is obtained under the variation of yarn angle misalignment. The misalignment parameter has been a significant criterion due to the low shear stiffness values used in the analysis. With the use of low shear stiffness materials, the yarns tend to be aligned first along the loading direction with no high resistance before other effect of uncertainty criteria.

Contradictorily, the use of high shear stiffness materials allows the yarns to behave like a beam which is able to resist shearing during the loading condition. In addition, it is shown that \( t_C \) will gradually increase with an increasing friction coefficient during uniaxial loading. This proves that the apparent contact area is important in fabric frictional characteristics. The important factors of the frictional resistance in woven fabrics are the moving direction and the fabric structure wherein concurrently give a large impact to the fabric stiffness. Effect of yarn height, yarn width and yarn spacing are equivalently unimportant. This can be seen from each pair-wise of uncertainty criteria slopes. A slight increment of \( t_C \) can be observed as the yarn width and yarn spacing variations become higher.

A comparison of the GEM-SA results with the results of the full quadratic regression mode analysis is shown in Table 4. It can be observed that the misalignment of yarn angle behaves dominantly in prediction of fabric stiffness of Mata Berkait-dry woven fabric with 0.44 of total sensitivity index contribution for both methods. We conclude that the misalignment has a large influence on the mechanical response compared to the other parameters that we have varied. When misalignment is present, the yarns elongate, as well as reorient at the same time. It is followed by friction coefficient with 0.41 and 0.35 for both evaluated methods. It has been demonstrated that some frictional parameters are strongly correlated with some pertinent fabric properties. In addition, the friction plays an important role through the applied forces at the contact surfaces.
Figure 7. The t,C realisation of Mata Berkait-dry woven fabric in uni-axial loading case.
Table 4. Comparison of the GEM-SA results with the full quadratic regression mode analysis results (1 = Yarn spacing, 2 = Yarn width, 3 = Yarn height, 4 = Misalignment of yarn angle, and 5 = Friction coefficient)

| Method         | Sensitivity indices \( \tau_i C_i \) (N) | Interaction values: |
|----------------|----------------------------------------|---------------------|
| GEM-SA         | First-order sensitivity indices, \( S_i \) | Interaction values: |
|                | \( S_1 = 0.00 \)                        | Yarn height, \( h_i \) versus misalignment in yarn angle, \( e = 0.01 \) |
|                | \( S_2 = 0.00 \)                        | Misalignment in yarn angle, \( e \) versus friction coefficient, \( \mu = 0.12 \) |
|                | \( S_3 = 0.00 \)                        | \( \sum_{i=1}^{5} S_i = 0.85 \) |
|                | \( S_4 = 0.44 \)                        | \( \sum_{i=1}^{5} S_{Ti} = 1.14 \) |
|                | \( S_5 = 0.41 \)                        | |
|                | Total effect indices, \( S_{Ti} \)      | |
|                | \( S_{T1} = 0.00 \)                     | |
|                | \( S_{T2} = 0.00 \)                     | |
|                | \( S_{T3} = 0.02 \)                     | |
|                | \( S_{T4} = 0.58 \)                     | |
|                | \( S_{T5} = 0.54 \)                     | |
| Full quadratic regression mode analysis | First-order sensitivity indices, \( S_i \) | Interaction values: |
|                | \( S_1 = 0.01 \)                        | \( S_{T1} - S_i = 0.11 \) |
|                | \( S_2 = 0.02 \)                        | \( R^2 = R_{adj} = 0.83 \) |
|                | \( S_3 = 0.02 \)                        | |
|                | \( S_4 = 0.44 \)                        | |
|                | \( S_5 = 0.35 \)                        | |
|                | \( \sum_{i=1}^{5} S_i = 0.84 \)        | |
|                | Total effect indices, \( S_{Ti} \)      | |
|                | \( S_{T1} = 0.03 \)                     | |
|                | \( S_{T2} = 0.06 \)                     | |
|                | \( S_{T3} = 0.05 \)                     | |
|                | \( S_{T4} = 0.45 \)                     | |
|                | \( S_{T5} = 0.36 \)                     | |
|                | \( \sum_{i=1}^{5} S_{Ti} = 0.95 \)     | |

Clearly, comparing the GEM-SA with full quadratic regression mode analysis results allows us to draw the following conclusions: (i) the GEM-SA method appears as a reliable approach to the response of fabric stiffness based on the uncertain mesoscopic parameters, (ii) the choice of the full quadratic regression mode analysis is sufficient to replicate the sensitivity analysis of fabric stiffness response. The difference, \( S_{T_i} - S_i \), is a measure for how the uncertain mesoscopic parameter \( i \) is related to the interactions of the other uncertain mesoscopic parameters. It should be noted that the interaction values correspond to the uncertain mesoscopic parameters are more detailed in the GEM-SA than the full quadratic regression mode analysis wherein the uncertain mesoscopic pair values are given.

For instance, the pair-wise interaction of the yarn height with the misalignment angle contributes a small interaction value with 0.01. Consequently, the largest contribution comes from the parameter pair-wise of friction coefficient and misalignment in yarn angle. Therefore, we conclude that the major pair-wise contributions are due to the high first-order sensitivity indices as shown in both methods.

5. Conclusion
The prediction of the fabric stiffness in warp direction using uniaxial deformation to a predefined mechanical model is presented by configuring the sensitivity of uncertain mesoscopic parameters. The main aim is to see if the full quadratic regression mode analysis is as good for the investigation of uncertain mesoscopic parameters on the macroscopic stiffness of fabrics as GEM-SA. It is shown that the full quadratic regression mode analysis is preferred as it is computationally more efficient. A secondary goal is that by performing this investigation, we have figured out which of the five (5)
uncertain mesoscopic parameters is the most important. The application of GEM-SA is utilized to quantify the direct and mutual sensitivity of a number of uncertain mesoscopic parameters on macroscopic stiffness of the Mata Berkait fabric. The yarn spacing, yarn width, yarn height, misalignment of the yarn angle and the friction coefficient of the yarn surfaces are the uncertain mesoscopic parameters investigated in this study.

It is found that the misalignment of the yarn angle is the most influential uncertain mesoscopic parameter for the macroscopic stiffness with 0.44 of first sensitivity index in both methods. The friction coefficient also has a substantial influence to the fabric stiffness with 0.41 of first sensitivity index in GEM-SA and 0.35 in full quadratic regression mode, respectively. The interaction between these two uncertain mesoscopic parameters shows a large influence to macroscopic stiffness. The other uncertain mesoscopic parameters are demonstrated to be unimportant. This is also proven that there is no net effects on fabric stiffness with the variation of weave patterns used due to the equality relation in $t_i$ evaluation. The results of the sensitivity analysis are strongly dependent on the assumptions of the variability of the uncertain mesoscopic parameters. Their properties could be updated in the future works by means of Bayesian theory.

Overall, GEM-SA and full quadratic regression mode analysis are reliable predictions for uncertainty and sensitivity analyses of fabric stiffness. The sensitivity indices of both methods are very similar. It is discovered that the use of emulator allows the detailed, albeit approximate, exploration of the output over the entire analyses. These emulators also allow the assessment of sensitivity and uncertainty of the output related to individual inputs by conducting the sensitivity analysis precisely. Note that the sensitivity indices, especially the total-effect sensitivity indices, are one of the tools for detecting the influential inputs that have major impacts on an input-output process model.

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