MASSIVE HALOS IN MILLENNIUM GAS SIMULATIONS: MULTIVARIATE SCALING RELATIONS

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ABSTRACT

The joint likelihood of observable cluster signals reflects the astrophysical evolution of the coupled baryonic and dark matter components in massive halos, and its knowledge will enhance cosmological parameter constraints in the coming era of large, multiwavelength cluster surveys. We present a computational study of intrinsic covariance in cluster properties using halo populations derived from Millennium Gas Simulations (MGS). The MGS are re-simulations of the original 500 Mpc Millennium Simulation performed with gas dynamics under two different physical treatments: shock heating driven by gravity only (GO) and a second treatment with cooling and preheating (PH). We examine relationships among structural properties and observable X-ray and Sunyaev–Zel’dovich (SZ) signals for samples of thousands of halos with \( M_{200} \geq 5 \times 10^{13} h^{-1} M_{\odot} \) and \( z < 2 \). While the X-ray scaling behavior of PH model halos at low redshift offers a good match to local clusters, the model exhibits non-standard features testable with larger surveys, including weakly running slopes in hot gas observable–mass relations and \( \sim 10\% \) departures from self-similar redshift evolution for \( 10^{14} h^{-1} M_{\odot} \) halos at redshift \( z \sim 1 \). We find that the form of the joint likelihood of signal pairs is generally well described by a multivariate, log-normal distribution, especially in the PH case which exhibits less halo substructure than the GO model. At fixed mass and epoch, joint deviations of signal pairs display mainly positive correlations, especially the thermal SZ effect paired with either hot gas fraction \( (r = 0.88/0.69 \text{ for PH}/\text{GO at } z = 0) \) or X-ray temperature \( (r = 0.62/0.83) \). The levels of variance in X-ray luminosity, temperature, and gas mass fraction are sensitive to the physical treatment, but offsetting shifts in the latter two measures maintain a fixed 12\% scatter in the integrated SZ signal under both gas treatments. We discuss halo mass selection by signal pairs, and find a minimum mass scatter of 4\% in the PH model by combining thermal SZ and gas fraction measurements.

Key words: galaxies: clusters: general – galaxies: clusters: intracluster medium

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1. INTRODUCTION

Accurate cosmology using surveys of clusters of galaxies requires a robust description of the relations between observed cluster signals and underlying halo mass. Even without strong prior knowledge of the mass–signal relation, cluster counts, in combination with other probes, add useful constraining power to cosmological parameters (Cunha et al. 2009). However, significant improvements can be realized when the error in mass variance is known (Lima & Hu 2005; Cunha & Evrard 2010; Wu et al. 2010). Improvements can also be gained by extending the model to multiple observed signals (Cunha 2009), especially when an underlying physical model can effectively reduce the dimensionality of the parameter sub-space associated with the model (Younger et al. 2006). The coming era of multiple observable signals from combined surveys in optical, submillimeter and X-ray wavebands invites a more holistic approach to modeling multiwavelength signatures of clusters.

Signal covariance characterizes survey selection, in terms of mass and additional observables. For the case of X-ray-selected samples, Nord et al. (2008) demonstrate that luminosity–temperature covariance can mimic apparent evolution in the luminosity–mass relation under analysis that combines deep, X-ray-flux-limited samples with local, shallow ones.

Employing a selection observable with small mass variance minimizes such errors. Recent work has shown that the total gas thermal energy, \( Y \), observable via an integrated Sunyaev–Zel’dovich (SZ) effect (Carlstrom 2002) or via X-ray imaging and spectroscopy, is a signal that scales as a power law in mass with only \( \sim 15\% \) scatter (White et al. 2002; Kravtsov et al. 2006; Maughan 2007; O’Hara et al. 2007; Zhang et al. 2008; Jeltema et al. 2008). However, unbiased estimates of the mass selection function for \( Y \) or any other signal require accurate knowledge of how the signal–mass scaling relation evolves with redshift. The redshift behavior of signals is generally not well known empirically, although recent work has begun to probe evolution in X-ray signals to \( z \sim 1 \) (Maughan et al. 2006; Vikhlinin et al. 2009). Emerging samples from wide-area SZ surveys should dramatically improve this situation.

One can address signal–mass covariance using hydrostatic, virial or lensing mass estimates from observations, but several sources of systematic and statistical error challenge this approach. For hydrostatic masses, early gas dynamic simulations (Evrard 1990; Navarro et al. 1996; Thomas 1997) suggested that turbulent gas motions drove hydrostatic masses to underestimated true values by \( \sim 20\% \). More sophisticated recent models, with a factor thousand improvement in mass resolution, demonstrate this effect at a similar level in the mean, with \( \sim 15\% \) scatter among individual systems (Rasia et al. 2006; Nagai 2006; Jeltema et al. 2008). Cluster masses can also be measured by the shear induced on background galaxies due to gravitational lensing. With this method, individual cluster masses have mass...
uncertainties $\sim 20\%$ due to cosmic web confusion (Hoekstra 2003; de Putter & White 2005).

The mean scaling behavior of samples binned in some selection signal offers another empirical path to measuring covariance. Non-zero covariance between the selection and an independent, follow-up signal implies that the selection-binned scaling relation of the follow-up signal with mass need not match that signal’s intrinsic mass scaling. Comparison of scaling relations from differently selected samples thereby offers insight into covariance. Rykoff et al. (2008) offer a first attempt at this exercise for X-ray luminosity and optical richness using the optically selected Sloan Digital Sky Survey (SDSS) $\maxbgc$ sample (Koester et al. 2007). The sample contains $\sim 13,000$ clusters for which weak lensing mass estimates have been made by stacking the shear of richness-binned subsamples (Sheldon et al. 2009; Johnston et al. 2007). Rykoff et al. (2008) stack ROSAT All-Sky Survey data (Voges et al. 1999) in the same $\maxbgc$ richness bins, and find that the mean X-ray luminosity–mass relation derived with richness binning is consistent at the $\sim 2\sigma$ level with relations derived solely from X-ray data (Reiprich & Böhringer 2002; Stanek et al. 2006).

A theoretical approach to studying cluster covariance is to realize populations via numerical simulation. While high-resolution treatment of astrophysical processes, including star formation, supernova and active galactic nucleus (AGN) feedback, galactic winds, and thermal conduction have been included in recent simulations (Dolag et al. 2005; Borgani et al. 2006; Kravtsov et al. 2006; Sijacki et al. 2008; Puchwein et al. 2008), the computational expense has limited sample sizes to typically a few dozen objects. A detailed study of population covariance requires larger sample sizes, as can be generated by lower resolution simulations of large cosmological volumes using a more limited physics treatment (Bryan & Norman 1998; Hallman et al. 2006; Gottlöber & Yepes 2007; Yang et al. 2009).

We take the latter approach in this paper, focusing on the bulk properties of massive halos identified the Millennium Gas Simulations (MGS), a pair of resimulations of the original 500$h^{-1}$ Mpc Millennium run (Springel et al. 2005), each with $10^{9}$ total particles, half representing gas and half dark matter. The pair of runs use different treatments for the gas physics—a gravity-only (GD) simulation, sometimes called “adiabatic,” in which entropy is increased via shocks, and a simulation with cooling and preheating (PH). The former ignores galaxies as both a sink for baryons and a source of feedback for the hot intracluster medium (ICM). The latter also ignores the mass fraction contribution of galaxies, but it approximates the hot intracluster medium (ICM). The latter also ignores both a sink for baryons and a source of feedback for the gravitational softening length is 25$h^{-1}$ kpc. The cosmological parameters match the original: $\Omega_m = 1 - \Omega_\Lambda = 0.25$, $\Omega_\Lambda = 0.045$, $h = 0.73$, and $\sigma_8 = 0.9$. While some differences between the simulations are expected due to the difference in mass resolution and gravitational softening length, Hartley et al. (2008) verify the positions of dark matter halos to within 50$h^{-1}$ kpc between the original Millennium and the MGS. The value of $\sigma_8$ is higher than the WMAP3 value (Spergel et al. 2007), but we do not expect it to strongly affect the results of this paper.

In this paper, we consider two models of the MGS: a GD simulation where the only source of gas entropy change is from shocks, and a PH simulation with preheating and cooling along with shock heating. The GD simulation is useful as a base model that can be easily compared to previous hydrodynamic simulations of galaxy clusters. Furthermore, comparing GD simulations to observations highlights the cluster properties that are strongly affected by astrophysical processes beyond gravitational heating.

While adiabatic simulations match the self-similar prediction, $L \sim T^2$, for the X-ray luminosity–temperature relation, observations show a steeper slope (Arnaud & Evrard 1999; Osmond & Ponman 2004). Preheating, the assumption of an elevated initial gas entropy at high redshift, was introduced by Evrard & Henry (1991) and Kaiser (1991) as a means to resolve the discrepancy in shape between the observed X-ray luminosity function and that expected from self-similar scaling of the cosmic mass function. The PH simulation is tuned to match X-ray observations of clusters at redshift zero, particularly the luminosity–temperature ($L$–$T$) relation (Hartley et al. 2008). The preheating is achieved by boosting the entropy of every gas particle to 200 keV cm$^2$ at redshift $z = 4$. Although the preheating dominates in the PH simulation, there is also cooling based on the cooling function of Sutherland & Dopita (1993). Collisionless star particles are formed from gas particles at temperatures of $T < 2 \times 10^4$ K, densities of $n_H > 4.18 \times 10^{-27}$ g cm$^{-3}$, and overdensities of $\Delta > 100\rho_c$ (Hartley et al. 2008). Balogh et al. (2001) present resolution tests for this star formation prescription. In the absence of any feedback mechanism, a simulation at the resolution of the MGS will cool an unphysically high fraction of the baryons. Fewer than 2% of the baryons in the PH model are converted to stars, however, as star formation is essentially halted by the preheating at $z = 4$.

This simple model certainly does not capture all of the complex astrophysical effects associated with star and supermassive black hole formation in clusters. The central entropy structure in local clusters is distinctly bimodal, with roughly half the population centered near the 200 keV cm$^2$ value used in the PH and the remainder centered on an entropy level a factor of 10 smaller. However, the latter reflects potentially cyclical AGN feeding
and feedback (Voit et al. 2002; Sijacki et al. 2007; Puchwein et al. 2008) that strongly influences only a small mass fraction of the total cluster gas. Observational evidence of ubiquitous galactic winds at high and moderate redshifts (Pettini et al. 2001; Weiner et al. 2009) suggest that most of the heating of the ICM occurs at high redshift. Further support for fast feedback comes from red sequence galaxies extending to redshift $z = 1.4$ in the Spitzer/Infrared Array Camera (IRAC) Shallow Survey (Eisenhardt et al. 2008), the colors of which are consistent with passive evolution of a burst of star formation at redshift $z \sim 4$.

### 2.2. Halo Catalog

We identify halos in the simulation as spherical regions, centered on filtered density peaks, that encompass an average density of $\Delta_c \rho_c(z)$, where $\rho_c(z)$ is the critical density of the universe. Both dark matter and baryons are included in the density measurement. We use halos identified at an overdensity of $\Delta_c = 200$ for most of our analysis. Halo centers were identified with an $N$th nearest neighbor approach, which approximates the local density by calculating the distance to the 32nd nearest dark matter particle. The groupfinder begins with the dark matter particle with the highest local density, and works outward in radius particle by particle (including gas and dark matter) until the interior mean density is $200\rho_c(z)$. The algorithm then identifies the densest dark matter particle not already in a halo, and continues iteratively until all overdense regions with more than 100 particles have been identified. Overlapping halos are permitted; however, the center of mass of a halo may not be in another halo.

At redshift zero, in the PH simulation we have approximately 220,000 halos with at least 100 particles, and 4474 over a mass cut of $M > 5 \times 10^{13} h^{-1} M_\odot$. These numbers are higher in the G0 simulation, with approximately 370,000 halos with at least 100 particles, and 5612 over the mass cut of $M > 5 \times 10^{13} h^{-1} M_\odot$. In Figure 1, we plot differential halo counts as a function of total mass from the G0 and PH simulations, as well as the prediction from the Tinker mass function (Tinker et al. 2008) (solid, black line). As discussed in Stanek et al. (2009), preheating causes a decrease in total halo mass of up to $\sim 15\%$ relative to the G0 treatment, with the largest effects at lower masses and higher redshifts. While the G0 halo space density matches the TMF expectations well, the number of halos in the PH case is lower, especially at lower mass.

Among the halos over the mass cut of $M > 5 \times 10^{13} h^{-1} M_\odot$, we identify those that spatially overlap. For a pair of overlapping halos, we denote the less massive halo as a “satellite” and the more massive halo as a “primary” halo. For the rest of the analysis in this paper, we exclude the satellite halos. At redshift zero we are left with a sample of 4404 halos in the PH simulation and 5498 halos in the G0 simulation.

We repeat the halo finding exercise at all redshifts available for each model. In the case of PH, we employ a total of 63 outputs extending to a redshift of two. For the G0, we analyze only a subset of outputs at redshifts, $z = 0, 0.5, 1.0$, and 2.0. Unless otherwise noted, all of our analysis is for primary halos over the total mass limit of $5 \times 10^{13} h^{-1} M_\odot$.

### 2.3. Bulk Halo Properties

With the primary halo samples identified, we calculate bulk properties for them that we roughly classify into “structural” and “observable” categories. The former includes dark matter velocity dispersion, ICM mass fraction, gas mass-weighted temperature, and halo concentration while the latter includes X-ray luminosity and spectroscopic-like temperature, thermal SZ effect, and a dimensionless ICM emission measure.

As the MGS simulations are SPH treatments of the gas, integrals over volume map to summations over all particles, via $\int dV \rho \to \sum_i m_i \rho_i$. We consider two measures of ICM temperature. First, we consider the mass-weighted temperature. As GADGET-2 is a Lagrangian simulation with equal mass gas particles, the mass-weighted temperature is simply the average temperature of the particles in the halo:

$$T_m = \frac{1}{M} \int dV \rho T \rightarrow \frac{1}{N} \sum_{i=1}^N T_i. \quad (1)$$

We also calculate the spectroscopic-like temperature, $T_{sl}$, as defined in Mazzotta et al. (2004),

$$T_{sl} = \frac{\int n^2 T^{\alpha-1/2} dV}{\int n^2 T^{\alpha-3/2} dV} \times \frac{\sum_{i=1}^N \rho_i T_i^{\alpha-1/2}}{\sum_{i=1}^N \rho_i T_i^{\alpha-3/2}}, \quad (2)$$

with $\alpha = 0.75$. The spectroscopic-like temperature offers a good match to the temperature derived from a one-component fit to an X-ray spectrum, but is far simpler to compute.

We calculate X-ray luminosities,

$$L = \int_0^{\infty} dV \rho \rho \Lambda(T) \rightarrow \sum_{i=1}^N \rho_i \tilde{\Lambda}(T_i), \quad (3)$$

using MEKAL tables assuming fixed 0.3 solar metallicity to calculate $\tilde{\Lambda}(T)$. We do this in energy bands of 0.7–2.0 keV, 0.7–5.0 keV, and 0.7–7.0 keV, and also compute a bolometric luminosity, $L_{bol}$ using a wide photon energy range of
The mean behavior of halo properties is characterized by mean scaling relations, in the form of a signal covariance matrix, \( \langle s_i \rangle = \langle s_{i,14}(a) \rangle = \alpha_i(a) \mu \), where \( s_i \) are structural (Section 3.1) and observable (Section 3.2) properties. We find a better fit in terms of a quadratic in \( \ln a \),

\[
\Delta s_{i,14}(a) = \beta_i \ln(E(a)) + \epsilon_{i,1}(\ln(a))^2 + \epsilon_{i,2}(\ln(a))^2,
\]

where \( E(a) = H(a)/H_0 \). For the PH model, this form is a poor fit to \( f_{\text{ICM}} \)-related quantities. We then demonstrate good agreement between X-ray observations and the low-redshift PH model results. Deviations from mean scaling behavior, in the form of a signal covariance matrix, \( \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle \), are presented in Section 4.

### 3.1. Structural Quantities

The left panels of Figure 2 present scaling relations as a function of mass at \( z = 0 \) for subsamples of the G0 and PH halo samples. Four structural measures are presented: dark matter velocity dispersion, \( \sigma_{\text{DM}} \); intracluster gas mass fraction, \( f_{\text{ICM}} \); mass-weighted gas temperature, \( T_m \); and NFW concentration, \( c \). Best-fit parameters to the mass scaling, Equation (9), are presented in Table 1. The right panels show redshift evolution of the slopes and the shifts in normalization, Equation (10), for each signal. Error bars in the fit parameters are derived from bootstrapping resampling of the samples. In general, the uncertainties on the best-fit parameters are very small, \( \sim 0.1 \% \) at redshift \( z = 0 \), and are much smaller than the intrinsic scatter at fixed mass about the median power-law relation. The errors grow larger at higher redshifts; the mass-limited sample size drops below 100 at \( z = 1.8 \) for the PH case. Fits to the redshift evolution of the normalization, Equation (11), are presented in Table 2.

**Dark matter velocity dispersion.** The velocity dispersion of the dark matter particles is a fundamental measure of the virial state of a halo. Observationally, the galaxy velocity dispersion tracks ICM temperature in a manner consistent with virial expectations, but the possibility of a \( \sim 10\% \) bias relative to the dark matter is still allowed (Biviano & Katgert 2004; Becker et al. 2007). Figure 2(a) shows that the scaling with mass is slightly steeper than self-similar, \( \sigma_{\text{DM}} \approx M^{1/3} \). The normalization at \( 10^{15} h^{-1} M_\odot \) is in good agreement with the 1082.9 \( \pm 4.0 \) km s\(^{-1} \) value derived from a suite of N-body simulations by Evrard et al. (2008). As discussed in that study, a suppression of the slope is expected when the low-mass halo cutoff corresponds to a few thousand particles, as is the case here. The fact that the PH and G0 slopes are similar (0.341 and
0.345) indicates that the difference in gas physics treatments is not responsible for the shift.

In both PH and GO cases, the slope remains constant to high redshift, as seen in the upper right panel of Figure 2. The evolution of the normalization, $\sigma_{\text{DM,14}}$ shown in Figure 2(b), deviates slightly from the self-similar prediction, $\sigma_{\text{DM}} \propto [E(z)M]^{1/3}$, with the velocity dispersion being 1%–2% higher at $z = 1$ (values of $\beta$ are given in Table 2). Despite strong baryon content differences discussed below, the dark matter virial scaling under the PH and GO treatments remains remarkably consistent.

Mass-weighted temperature. The mass-weighted temperature, $T_m$, a useful probe of the hydrodynamic state of the ICM, is known to have small scatter with respect to mass in simulations (Evrard et al. 1996; Bryan & Norman 1998; Borgani et al. 2004). Figure 2(c) shows that the slope of the GO scaling relation, $T \sim M^{0.56}$, agrees with self-similarity at the few percent level. The PH slope is significantly less steep, $T \sim M^{0.35}$. Since all gas particles receive the same entropy boost at $z \sim 4$, the impact relative to the characteristic virial entropy is larger for low-mass halos than for high-mass halos. This effect tilts the $T_m$ scaling, and leads to a ~35% higher normalization at $10^{14} h^{-1} M_\odot$ in the PH simulation relative to the GO case. The effect diminishes at higher masses, dropping to 10% at $10^{15} h^{-1} M_\odot$.

While the slopes of the $T_m$–$M$ relation do not evolve strongly with redshift in either treatment, the normalizations deviate from self-similar evolution, with the GO simulation lying ~10% low and the PH case 5% high at $z = 1$ relative to $T_m \sim E(a)^{2/3}$ scaling (Kaiser 1986). The result for the GO case is somewhat surprising. We show in the Appendix that the deviation from self-similar evolution in $T_m$ results from increasing importance in bulk kinetic, or turbulent, energy in the ICM at high redshift. This trend is expected from the increasing frequency of mergers in a mass-limited sample at high redshift.

Baryon fraction. Figures 2(e) and (f) show the scaling of the hot gas fraction, $f_{\text{ICM}}$, with mass at $z = 0$ along with the redshift evolution of the slope and normalization. For reasons discussed below, we show evolution in the PH normalization and slope at a mass of $5 \times 10^{14} h^{-1} M_\odot$ as well as the fiducial mass of $10^{14} h^{-1} M_\odot$.

The GO halos at redshift zero have a slightly depleted baryon fraction, with mean $f_{\text{ICM}} = 0.162$, and a dispersion of 0.040. There is no trend with mass, as the slope is 0.001 ± 0.019. This result is consistent with previous SPH simulations of similar resolution and evaluated at $r_{200}$ by Crain et al. (2007) and Ettori et al. (2006). The simulation of the MareNostrum universe (Gottlöber &Yepes 2007) has twice our mass resolution, and a baryon fraction of $f_{\text{ICM}} = 0.167$ at a virial radius, $r_{200}$, that encompasses a mean density of ~100 times the critical density. This small but significant increase is consistent with the radial trend seen in our simulation. Our value of $f_{\text{ICM}}$ is lower than the 0.185 value measured by Kravtsov et al. (2005) at $r_{200}$ in their adiabatic Adaptive Mesh Refinement (AMR) code. That study included a comparison of SPH and AMR simulations evolved under gravity only, and they note a statistically significant offset of ~5%, with the AMR simulation having higher baryon fraction.

As has been shown in previous simulations (Bialek et al. 2001; Borgani et al. 2001; Muanwong et al. 2006; Younger & Bryan 2007), preheating has a dramatic effect on the hot gas fraction of massive halos. Low-mass halos can lose half of their baryons within $r_{200}$, while the most massive halos are depleted by only ~10% relative to the GO case. A power-law form is a poor representation of the PH mean behavior of $f_{\text{ICM}}$ with mass, so we extend the model to a quadratic in $\ln M$ and give best-fit parameters in Table 3. We present a comparison with observed gas fractions in Section 3.3 below.

In the GO simulation, the slope of the gas fraction remains consistent with zero at all redshifts, while the mean gas fraction increases slightly, rising from $f_{\text{ICM}} = 0.162$ at $z = 0$ to 0.1673
the evolution as a quadratic in \( \ln(\sigma) \). For the baryon fraction, and for other halo parameters which depend strongly on the characteristic entropy of the later halo. At the fiducial \( 10^{14} \, h^{-1} M_\odot \) normalization scale, halos at \( z = 1 \) have a \( \sim 20\% \) lower gas fraction compared to \( z = 0 \), and the local slope of the \( f_{\text{ICM}}-M \) relation is also steeper at higher redshift. The effects of preheating dominate over the universal expansion at the \( 10^{14} \, h^{-1} M_\odot \) mass scale; hence the evolution of the normalization cannot be simply described as a power of \( E(z) \). For the baryon fraction, and for other halo parameters which depend strongly on the baryon fraction, we fit the evolution as a quadratic in \( \ln(\sigma) \), with the best-fit parameters presented in Table 4.

At a higher mass scale of \( 5 \times 10^{14} \, h^{-1} M_\odot \), the effects are milder, and the local slope is very close to the slope of the \( \sigma_8 \) model. The most massive halos are less affected by preheating, and their baryon fractions are therefore more appropriate to use as indicators of cosmology (Pen et al. 2003; Allen et al. 2004, 2008). We note that the mean gas fraction shift at \( 5 \times 10^{14} \, h^{-1} M_\odot \) in the \( \Phi \) model is comparable to the 20% uniform prior on gas fraction applied by Allen et al. (2008) in their analysis of a Chandra sample of 42 clusters with \( kT > 5 \, \text{keV} \) extending to \( z = 1.1 \). The cosmological constraints from that work would thus not be strongly affected if a \( \Phi \) model prior on \( f_{\text{ICM}} \) behavior were imposed.

**NFW concentration.** Figures 2(g) and (h) show the behavior of the NFW concentration, \( c \), derived from fits to the total mass density profile of each halo. At \( 10^{15} \, h^{-1} M_\odot \), the mean concentration in the \( \sigma_8 \) simulation, \( c = 3.3 \), is in agreement with values at that mass derived from the original Millennium Simulation with only dark matter (Neto et al. 2007; Gao et al. 2008). However, the \( \sigma_8 \) halos have a flat \( c-M \) relation, in conflict with the expected trend, seen by Gao et al. (2008) and earlier simulations, of higher concentrations at lower mass. This difference is partly due to the lower resolution in the MGS compared to the original Millennium, but back-reaction on the dark matter from the gas may also play a role. We do not investigate the origins of this effect here, but note that the redshift evolution of the concentration at fixed mass, a \( \sim 30\% \) decline at \( z = 1 \), agrees with the level seen by Gao et al. (2008).

For the \( \Phi \) case, the mean concentration of \( c = 2.6 \) at \( 10^{15} \, h^{-1} M_\odot \) is lower than the \( \sigma_8 \) value. As discussed above, the \( \Phi \) halos have a lower baryon fraction than \( \sigma_8 \) halos, and the increasing baryon loss at low masses tilts the \( c-M \) relation to have positive slope. The \( \Phi \) model is not designed to address the physics of cluster cores, so we should not expect the \( \Phi \) halo concentrations to match observed behavior.
3.2. X-ray and \( \Sigma \) Signals

We now turn to common bulk observed properties of clusters: the \( \Sigma \) decrement, \( Y \), and X-ray spectroscopic-like temperature, \( T_{sl} \), luminosity, \( L_{bol} \), and the emission measure \( \hat{Q} \).

Sunyaev-Zeldovich decrement. From Figures 2(c) and (e), we know that preheated halos at fixed mass have, on average, lower ICM mass fractions and higher mass-weighted temperatures than their \( \Phi \) counterparts. Since the integrated thermal \( \Sigma \) decrement, \( Y \), is a product of these two measures, we can anticipate some degree of cancellation coming from this opposing behavior. Figure 3(a) shows that this cancellation is quite close to exact above a mass scale of \( 2 \times 10^{14} \, h^{-1} M_\odot \).

Curvature in \( f_{ICM} \) tilts the \( \Phi \) relation from a local slope of \(-1.8 \) to \(-1.6 \) across the mass range \( 10^{14} \rightarrow 10^{15} \, h^{-1} M_\odot \). As with the \( f_{ICM} - M \) relation, we fit \( Y \) to a quadratic in \( \ln M \) for the \( \Phi \) case in order to account for the curvature and to get the best possible measure of the intrinsic scatter about the mean relation. Parameters are given in Table 3. The slope in the \( \Phi \) model is very close to the self-similar evolution of \( 5/3 \).

While the \( Y - M \) normalization at high masses is not sensitive to cluster physics in our models, previously ART simulations with cooling, star formation and supernova feedback (CSF) display a normalization drop by 25% relative to the \( \Phi \) case Nagai (2006). At a basic level, the different behavior between the two studies simply reflects the fact that the thermal pressure support, \(-\nabla P/\rho \), is insensitive to a multiplicative shift in gas density normalization. Our models force all baryons into the hot phase, while a CSF treatment allows some fraction of baryons—40% in the case of Nagai (2006)—to reside in galactic sinks of stars and cold gas. Since observations indicate that the ratio of stellar to hot gas mass declines with increasing mass, from values near 0.5 at \( 10^{14} \, h^{-1} M_\odot \) to 0.1 at \( 10^{15} \, h^{-1} M_\odot \) (Giordani et al. 2009), one would anticipate that the observed \( Y - M \) relation to be steeper than self-similar.

A recent X-ray-based estimate of the \( Y - M \) relation from a \( \text{Chandra} \) archival sample of groups and clusters finds a slope of 1.75 at \( r_{500} \) over the mass range \( 10^{13} \rightarrow 10^{15} \, h^{-1} M_\odot \) (Sun et al. 2009).

Figure 3(b) compares the redshift evolution of the \( Y - M \) relation in the two simulations. The slope in the \( \Phi \) model steepens at higher redshift, and the normalization drifts below self-similar expectations by \( \sim 30\% \) at \( z = 1 \). The evolution at high mass \((\sim 10^{14} \, h^{-1} M_\odot)\) in the \( \Phi \) model is similar to that of the \( \Phi \) model, in both slope and normalization, since redshift \( z \sim 1 \). At our chosen normalization mass of \( \sim 10^{14} \, h^{-1} M_\odot \), the evolution departs from self-similar at \( z > 0.5 \), driven by the evolution of the baryon fraction at that scale. Like the baryon fraction, we fit the evolution of \( Y \) to a quadratic in \( \ln(\alpha) \), with the best fit presented in Table 4. We note that the \( \Phi \) behavior is mildly in conflict with the CSF model evolution of Nagai (2006), which displayed consistency with self-similar evolution at the \( \sim 20\% \) level. With only 11 halos, that study could not address statistical differences at the level we do here.

Spectroscopic-like temperature. We also consider the spectroscopic-like temperature, \( T_{sl} \), an analytic prescription derived by Mazzotta et al. (2004) to approximate observed X-ray spectral temperatures. We find that \( T_{sl} \) agrees well with the X-ray temperatures derived from spectral fits to X-MAS2 mock observations (Rasia et al. 2005). As seen in Figures 3(c) and (d), the slopes in the \( \Phi \) and \( \Phi \) treatments agree well, \( \alpha = 0.57 \), but at fixed mass the \( \Phi \) halos are \( \sim 40\% \) hotter than the \( \Phi \) halos. The distribution of the gas in the \( \Phi \) models contains cool, low entropy cores of accreted sub-halos (Mathiesen & Evrard 2001), and these cool, dense clumps pull down the \( T_{sl} \) measure relative to the \( \Phi \) simulation. The cores in the latter case have been effectively erased by the preheating.

The redshift evolution of \( T_{sl, 14}(\alpha) \) in the \( \Phi \) simulation is very close to self-similar \((\propto [E(\alpha)]^{1/2})\), and tracks well the mass-weighted behavior shown above (see Table 2). The evolution in the \( \Phi \) simulation departs dramatically from self-similar behavior at low redshifts.

BoLometric luminosity and emission measure. The different behaviors of the baryon fraction and temperature in the \( \Phi \) and \( \Phi \) simulations drive X-ray luminosity differences, but the gas clumping, or emission measure, also plays a significant role. Figures 3(e) and (g) show the \( z = 0 \) mass scalings of
the bolometric luminosity, \( L_{\text{bol}} \), and dimensionless emission measure, \( \hat{Q} \). Because of the influence of line emission, the \( \Omega_0 \) scaling is less steep than the self-similar slope of \( L \sim M^{4/3} \), and the emission measure is nearly constant with mass at a value \( \sim 3.5 \). In the \( \Omega_0 \) case, the clumping factor is smaller by a factor of 2 and displays a significant trend with mass. Combined with the \( f_{\text{ICM}} \) behavior, the result is a suppression of \( L_{\text{bol}} \) in the \( \Omega_0 \) case by a factor of 10 at \( 10^{14} \, h^{-1} \, M_\odot \), and a steepening of the slope in mass to 1.87, from 1.08 in the \( \Omega_0 \) case.

Although the \( \Omega_0 \) halos have a higher temperature at fixed mass, this effect is dwarfed by the decreases in \( f_{\text{ICM}} \) and \( \hat{Q} \) between the \( \Omega_0 \) and \( \Omega_0 \) treatments. The lower normalization of the \( \hat{Q} - M \) relation reflects a shallower gas density profile in the \( \Omega_0 \) case. In turn, the lower central gas densities contribute to lowering the overall mass profile, driving the shift in concentrations to lower values for the \( \Omega_0 \) halos discussed above. As in the \( Y - M \) relation, the curvature in the \( f_{\text{ICM}} - M \) relation in the \( \Omega_0 \) model drives curvature in the \( L_{\text{bol}} - M \) relation. Hartley et al. (2008) see this curvature in the \( L - T \) relation in the \( \Omega_0 \) model, as well as in a large, local observed sample of galaxy clusters. We fit the \( L_{\text{bol}} - M \) relation in the \( \Omega_0 \) model to a quadratic in \( \ln M \), presenting the best fit in Table 3.

The evolution of the normalization in \( \Omega_0 \) is not a perfect power of \( E(a) \), due to the complicated evolution of \( f_{\text{ICM}} \) with redshift. We fit the evolution of \( L_{14}(a) \) to a quadratic in \( \ln(a) \) in the \( \Omega_0 \) simulation, and note that it is weaker than the \( \Omega_0 \) evolution. However, as shown in Figure 3(f), this evolution is a function of mass. The larger solid points, for halos at \( 10^{14} \, h^{-1} \, M_\odot \), show a weaker evolution than the evolution of the halos at \( 5 \times 10^{14} \, h^{-1} \, M_\odot \), shown by the smaller solid points. The latter halos evolve similarly to the \( \Omega_0 \) halos, illustrating that the most massive halos in the \( \Omega_0 \) model are similar in structure and history to the \( \Omega_0 \) population. The evolution of the \( L_{\text{bol}} - M \) relation in the \( \Omega_0 \) simulation is driven mainly by the redshift evolution of the hot gas fraction, with \( \hat{Q} \) contributing 10% of the decrease at \( z = 1 \).

### 3.3. Comparison to Observations

As shown by Hartley et al. (2008), the \( \Omega_0 \) simulation offers a good match to the core-excised \( L - T \) relation of local clusters. Here, we briefly explore the level of agreement between the models and observations in this and other scaling relations. While not wishing to oversell the simple physical treatment of the preheated model, which is undoubtedly wrong in detail, we show below that it reproduces several scalings with quite high fidelity. Preheating appears to be a useful effective model. It is important to remember that precise comparison of observations and simulation expectations requires careful modeling of survey selection and projection effects, and we do not treat these effects here.

Figure 4 compares the \( L_{\text{bol}} - \Delta \) relations with the core-excised \( L_{\text{bol}} - T_X \) measurements of the local, representative REXCESS survey (Pratt et al. 2009). The slope of the observed relation is somewhat shallower, and its scatter somewhat larger, than the \( \Omega_0 \) model predictions, but these differences are at the level of a few tens of percent in luminosity, or less than 10% in temperature. The small scatter in the core-excised \( L_{\text{bol}} - T \) relation for REXCESS clusters (and \( \Omega_0 \) halos) indicates that local galaxy clusters are well-behaved outside of the core (Neumann & Arnaud 2001).

At higher redshift, we consider the CCCP clusters, a subset of the 400 square degree survey which has been followed-up with Chandra (Vikhlinin et al. 2009). These clusters range from approximately \( 0.3 < z < 0.8 \), so at fixed mass we scale, in a self-similar manner, the observed luminosities and temperatures to \( z = 0.5 \) for comparison with the model \( L_{\text{bol}} - T \) relations. The latter are measured within \( \Delta = 500 \) to be consistent with the treatment of Vikhlinin et al. (2009). The comparison is presented in Figure 5. The agreement is good, but the temperature errors are larger than for the REXCESS sample. The scatter in the CCCP sample is larger, mainly due to the fact that cores have not been excited in the luminosity measurements of this sample.

Figure 6 compares the mass scaling of ICM mass fractions at redshift zero in the models to \( \text{XMM} \) measurements for local clusters (Arnaud et al. 2007; Sun et al. 2009). Values are measured within \( \Delta = 500 \). To estimate observed cluster masses, hydrostatic estimates that include a radial temperature gradient are employed. We note that Arnaud data agree with other observational determinations at high mass (Vikhlinin 2006; Giodini et al. 2009), while the Sun et al. (2009) data extend to lower mass, \( \sim 10^{13} \, h^{-1} \, M_\odot \). The \( \Omega_0 \) model matches the observed \( f_{\text{ICM}} - M \) relation well, with hot gas fractions a factor 2 less than the cosmic ratio, \( \Omega_h/\Omega_m \) at \( 10^{14} \) and a strongly increasing trend toward higher masses. Larger observed samples are needed to test the curvature and degree of scatter.

In Figure 7, we compare the \( \Delta - M \) relation of the models to that from Arnaud et al. (2007). The \( \Omega_0 \) model, with its cool sub-halo cores, is strongly offset from the data. The \( \Omega_0 \) model relation is much closer, with a similar slope and scatter. There is, however, a consistent offset of \( \sim 15\% \) in mass toward lower values in the observed sample. The magnitude of this offset is consistent with the level of expected bias from hydrostatic mass estimates, which simulations show tend to underestimate true masses by approximately 20% (Rasia et al. 2006; Nagai et al. 2007).

Overall, the bulk X-ray properties of the \( \Omega_0 \) simulation match the local scaling relations quite well. This is particularly
true after excising the core from the observations, and after considering the mass bias introduced by hydrostatic mass estimates. Although the physics of the PH model is simple, it appears to provide useful representation for the behavior of the bulk of the hot ICM lying outside the cool core regions.

4. COVARIANCE OF BULK PROPERTIES

In this section, we explore the second moment of the halo scaling relations. Understanding the variance at fixed mass is necessary for calculating the mass selection properties of signal-limited samples, and survey counts alone typically constrain only a linear combination of signal normalization and variance (Stanek et al. 2006). In addition, the signal covariance determines the precise structure of scaling relations in signal-selected samples. A worked example of the $L$–$T$ relation expected for X-ray flux-limited samples is given by Nord et al. (2008).

After defining terms, we begin by presenting the covariance of signals at fixed mass at the present epoch, then demonstrate that redshift evolution in most elements is weak. We close with analysis of the mass selection properties of signal pairs.

4.1. Signal Covariance Matrix

For a set of halos of mass $e^\mu$ at expansion epoch $a$, we define a symmetric covariance matrix, with elements

$$\Psi_{ij} \equiv \langle (s_i - \bar{s}_i(\mu, a))(s_j - \bar{s}_j(\mu, a)) \rangle,$$  \hspace{1cm} (13)

where the mean values are determined by Equation (9) and the brackets represent an ensemble average. The $j$th diagonal element of the covariance matrix is the $j$th signal variance, $\sigma_j^2 \equiv \Psi_{jj}$. We present these measures along with the correlation matrix, $C_{xy} = \Psi_{xy}/(\sigma_x\sigma_y)$, which expresses the covariance in normalized terms.

Figure 8 presents a graphical representation of the data comprising the covariance matrix for a subset of signals at redshift zero. The diagonal panels show the distribution of signal deviations (in the natural log of the measured signal) about the mean for the PH (shaded) and GO (lines) treatments. Panels off the diagonal plot the normalized deviations $((s_i - \bar{s}_i)/\sigma_i$ versus $(s_j - \bar{s}_j)/\sigma_j$ for signal pairs, with the lower and upper triangles showing PH and GO cases, respectively. The
orientation and spread of halos in each panel determines the correlation coefficient. For instance, the tight ellipse formed by the population in the $Y-f_{\text{ICM}}$ panel indicates a high correlation coefficient between this pair of signals.

The $z = 0$ values of the correlation coefficients, along with uncertainties from bootstrap resampling, are presented in Table 6. As in the mean fit parameters, typical statistical uncertainties are on the order of 1%. Before exploring the off-diagonal terms, we first examine the variance and distribution function shape of individual signal deviations.

### 4.2. Signal Variance at Fixed Mass

The assumption of log-normal variance is a common element of the likelihood analysis used in cluster cosmology studies (see references in Section 1). Both the absolute variance and the full distribution shape affect the expected signal counts derived from mass function convolution.

In Table 5, we list root mean square deviations for the full set of signals at $z = 0$. To test the log-normal expectation, we list the normalized deviates at which each signal’s ranked distribution reaches fixed percentile values, taken to be $\pm 2\sigma$, $\pm 1\sigma$ and median/mean of a Gaussian distribution. The difference between the listed values and their integer counterparts is a measure of the degree of local deformation from Gaussian in the frequency distribution.

While deviations from Gaussianity are apparent in essentially all measures, the typical percentile shifts in the PH model are only a few percent. Exceptions are a significant positive skew in the dark matter velocity dispersion, with median location of

\[-0.12\sigma\] and shifts in the $\pm 2\sigma$ Gaussian tails to $-1.7\sigma$ and $2.5\sigma$, respectively. Most distributions are slightly leptokurtic, especially the mass weighted temperature. Worth noting is

![Graphical representation of the data comprising the covariance matrix for the two simulations at $z = 0$. Diagonal panels plot the distribution of deviations from mean mass scaling behavior, $(s_i - \bar{s}_i)$, where $s_i$ is the natural log of the $i$th signal. Shaded histograms show PH results and solid lines show G0 data. Each off-diagonal panel plots the normalized deviations, $(s_i - \bar{s}_j)/\sigma_i$, for an $(i, j)$ pair of properties. The lower triangle shows PH and the upper triangle shows G0 behavior. (A color version of this figure is available in the online journal.)](image)

| Signal | Scatter | 2.275% | 15.865% | 50% | 84.135% | 97.725% |
|--------|---------|--------|---------|-----|---------|---------|
| Gaussian | ... | -2.00 | -1.00 | 0.00 | 1.00 | 2.00 |
| PH$\sigma_{\text{DM}}$ | 0.042 ± 0.001 | -1.66 | -0.90 | -0.11 | 0.85 | 2.43 |
| PH $f_{\text{ICM}}$ | 0.086 ± 0.001 | -2.14 | -0.93 | 0.03 | 0.93 | 1.93 |
| PH$T_{\text{dm}}$ | 0.058 ± 0.002 | -1.90 | -0.84 | -0.04 | 0.75 | 2.11 |
| PH$T_{\text{ICM}}$ | 0.069 ± 0.001 | -2.17 | -0.93 | 0.04 | 0.90 | 1.88 |
| PH $Y$ | 0.125 ± 0.002 | -2.04 | -0.97 | 0.04 | 0.90 | 1.95 |
| PH$Q$ | 0.193 ± 0.002 | -1.98 | -0.98 | 0.004 | 1.00 | 1.94 |
| PH$c$ | 0.116 ± 0.001 | -1.89 | -0.99 | -0.04 | 0.97 | 2.19 |
| PH$\sigma_{\text{DM}}$ | 0.300 ± 0.008 | -1.27 | 0.43 | 0.13 | 0.89 | 1.31 |
| G0 $\sigma_{\text{DM}}$ | 0.042 ± 0.001 | -1.65 | -0.88 | -0.12 | 0.77 | 2.54 |
| G0 $f_{\text{ICM}}$ | 0.036 ± 0.001 | -2.12 | -0.98 | 0.03 | 0.99 | 1.90 |
| G0 $T_{\text{dm}}$ | 0.102 ± 0.001 | -2.23 | -1.00 | 0.12 | 0.93 | 1.69 |
| G0 $T_{\text{ICM}}$ | 0.219 ± 0.002 | -2.32 | -1.06 | 0.19 | 0.96 | 1.49 |
| G0 $Y$ | 0.123 ± 0.001 | -2.17 | -1.00 | 0.10 | 0.95 | 1.75 |
| G0 $Q$ | 0.282 ± 0.003 | -2.19 | -1.03 | 0.10 | 0.97 | 1.70 |
| G0 $c$ | 0.109 ± 0.001 | -2.59 | -0.91 | 0.19 | 0.85 | 1.55 |
| G0 $0.280 ± 0.008 | -2.42 | -0.97 | 0.13 | 0.86 | 1.28 |

**Notes.**

* The distribution of deviates shows locations, in terms of normalized deviates, $b/\sigma$, at which the ranked distributions reach the listed percentiles. Values for a Gaussian distribution are listed in the first row for each signal. Error estimates on the scatter come from bootstrap resampling.
the fact that the shapes of two important cluster selection observables, $Y$ and $L_{\text{bol}}$, do not deviate by more than 0.1 from the Gaussian expectations.

Under the $\text{G}_0$ treatment, the shape of the dark matter velocity dispersion is the same as in the PH case, showing positive skew at the 10% level. However, the shapes of the hot gas properties of $\text{G}_0$ halos generally differ, to a slight degree, from the PH shapes; all measures tend to be slightly skew negative. The differences can be subtle, as close inspection of the $Y$ histograms in Figure 8 confirms.

Comparing the PH and $\text{G}_0$ distributions, we conclude that a log-normal approximation is a fairly accurate description, but calculations demanding better than $\sim 10\%$ precision in shape will require an expanded treatment, either via direct Monte Carlo from simulations, or from analytic extensions, such as an Edgeworth series in Hermite polynomials (Kofman et al. 1993; Shaw et al. 2009). We leave detailed analysis of the physical mechanisms driving these distribution shapes to future investigations. In the case of $\sigma_{\text{DM}}$, we have preliminary evidence that mergers drive the positive skew tail of the deviations; nearly all halos with a deviation of $>3\sigma$ have undergone a merger since redshift $z = 0.2$.

The amplitudes of the scatter in the set of signals range from a low of 0.036 for $f_{\text{ICM}}$ to a high of 0.28 for $L_{\text{bol}}$, both under the $\text{G}_0$ treatment. For the PH case, $L_{\text{bol}}$ is highest, at 0.19 while $\sigma_{\text{DM}}$ is lowest at 0.042. The latter value matches the $\text{G}_0$ case, and both agree with the scatter derived from the ensemble value presented by Evrard et al. (2008). While the dark matter virial scaling is robust to simple physical treatments for the baryons, Lau et al. (2009) find that strongly dissipative baryon physics depresses the slope of the $\sigma_{\text{DM}}-M$ relation by introducing mass- and redshift-dependent increases in halo velocity dispersion.

By raising the halo sound speed, which drives the shock radius to larger values and lowers the Mach number of infalling material (Voit et al. 2002), preheating leads to more thermally regular halo gas, with smaller scatter in mass-weighted temperature, $T_m$, compared to the $\text{G}_0$ simulation. Naively, in a virialized cluster, one expects $\sigma_{\text{DM}}^2 \sim T_m$, and so the scatter temperature should be double that of the velocity dispersion. While the $\text{G}_0$ simulation nearly follows this expectation, halos in the PH treatment have a $T_m$ scatter of 5.8%, only 1.4 times the scatter in dark matter velocity dispersion. The $T_m$ scatter in this case does increase with mass, however, reaching 9% $\pm$ 2% at $10^{15} h^{-1} M_\odot$.

The scatter in $f_{\text{ICM}}$ shows the reverse behavior. In the $\text{G}_0$ simulation, the scatter is very small, 3.6%, consistent with other $\text{G}_0$ SPH simulations (Ettori et al. 2006; Crain et al. 2007). When only gravity drives gas thermodynamics, the gas distribution traces the dark matter distribution very well. The PH model scatter of 8.6% is more than double that of the $\text{G}_0$ case, but this difference is mass-dependent. As seen in Figure 9, the $f_{\text{ICM}}$ scatter for PH halos decreases with mass, dropping to 4% above $5 \times 10^{14} h^{-1} M_\odot$. There is no appreciable mass trend in the $\text{G}_0$ treatment.

The opposing statistical shifts in $f_{\text{ICM}}$ and $T_m$ effectively cancel when combined to form the thermal SZ signal. Not only does the mean $Y-M$ relation in Figure 3 agree well between the two simulations, but both simulations have a similar scatter, $\sim 12\%$, a value consistent with previous simulations (Evrard et al. 1990; da Silva et al. 2000; Kay et al. 2004; Modl et al. 2005; Nagai 2006; Kravtsov et al. 2006; Hallman et al. 2007). As discussed below, the combination of low scatter in $Y$ and the steep slope of the $Y-M$ relation make the thermal SZ effect, or its X-ray equivalent, an excellent mass proxy for cluster surveys. Furthermore, as shown above, the distribution of $\ln(Y)$ is very close to a Gaussian. In the PH simulation, the scatter in $Y$ decreases with mass increases, driven by the behavior of $f_{\text{ICM}}$. The scatter drops below 10% above $2 \times 10^{14} h^{-1} M_\odot$.

In both models, the scatter in $T_d$ is higher than that of $T_m$ at fixed mass. The difference is slight in PH, but over a factor of 2 in $\text{G}_0$, a reflection of the larger amount of cool substructure in the latter treatment. This sensitivity of $T_d$ to gas physics treatment highlights opportunities to constrain gas physics with future high-quality X-ray spectroscopy of large samples. Such studies would be enabled by the proposed WFXT mission.7

After NFW concentration, the X-ray luminosity has the highest level of scatter at fixed mass in both the PH ($\sigma_{nL} = 0.19$) and $\text{G}_0$ ($\sigma_{nL} = 0.28$) simulations. These values are consistent with the core extracted value, $\sigma_{nL} = 0.27 \pm 0.06$, observed for REXCESS (Pratt et al. 2009).

Notes.

The redshift zero correlation coefficients, with the results from the PH simulation in the lower triangle and the results from the $\text{G}_0$ simulation in the upper, as in Figure 8. Uncertainties from bootstrapping resampling are on the order of 0.01 and are not shown.

### Table 6

| Signal | $\sigma_{\text{DM}}$ | $T_m$ | $T_d$ | $f_{\text{ICM}}$ | $Y$ | $L_{\text{bol}}$ | $Q$ | $c$ |
|--------|---------------------|-------|-------|-----------------|-----|---------------|-----|-----|
| $\sigma_{\text{DM}}$ | $\ldots$ | 0.55 | 0.81 | 0.28 | 0.54 | 0.51 | 0.17 | 0.19 |
| $T_m$ | 0.35 | $\ldots$ | 0.85 | 0.48 | 0.97 | 0.67 | 0.38 | 0.49 |
| $T_d$ | 0.86 | 0.5 | $\ldots$ | 0.42 | 0.83 | 0.67 | 0.47 | 0.64 |
| $f_{\text{ICM}}$ | $-0.10$ | 0.42 | 0.37 | $0.69$ | 0.60 | 0.32 | 0.37 |
| $Y$ | 0.079 | 0.74 | 0.62 | 0.88 | $\ldots$ | 0.73 | 0.40 | 0.51 |
| $L_{\text{bol}}$ | 0.26 | 0.50 | 0.73 | 0.76 | 0.78 | $\ldots$ | 0.65 | 0.70 |
| $Q$ | 0.32 | 0.029 | 0.56 | 0.15 | 0.12 | 0.59 | $\ldots$ | 0.71 |
| $c$ | 0.15 | 0.053 | 0.39 | 0.29 | 0.26 | 0.51 | 0.64 | $\ldots$ |

Figure 9. Scatter as a function of mass in the PH (filled, black points) and $\text{G}_0$ (open, red points) simulations for $f_{\text{ICM}}$, $T_m$, and $Y$ (left to right panels). (A color version of this figure is available in the online journal.)

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7 http://wfxt.pha.jhu.edu/
4.3. Off-diagonal Elements of the Correlation Matrix

There is much information about the physical processes driving the ICM encoded in the correlation matrix of Table 6. For cosmological studies, it would be useful to identify pairs of cluster properties whose correlation coefficient is insensitive to gas physics modeling. High values of the correlation coefficient identify cluster properties with strong mass selection potential, as shown in Section 4.5.

The highest measures of correlation are between \( T_m \) and \( Y \) in the \( \text{GO} \) case (0.97), and between \( f_{\text{ICM}} \) and \( Y \) in the \( \text{PH} \) run (0.88). The SZ–X-ray correlation of \( Y \) and \( L_{\text{bol}} \) is also large, \( \sim -0.7 \), in both treatments. This robust behavior is promising for cross-calibrations of future, combined X-ray and SZ surveys (Younger et al. 2006; Cunha 2009).

The covariance between \( \sigma_{\text{DM}} \) and \( T_m \) at fixed mass is an indicator of halo virialization. At redshift zero, this correlation is much higher, \( C = 0.56 \) in \( \text{GO} \) than in \( \text{PH} \), where it is 0.35. The halos in the \( \text{GO} \) simulation are governed by gravitational effects only, so \( T_m \) and \( \sigma_{\text{DM}} \) excursions track each other closely. In \( \text{PH} \), however, the preheating raises the sound speed in the halos, making the thermalization due to mergers less pronounced, thereby diminishing (though not eliminating) the coupling of \( T_m \) and \( \sigma_{\text{DM}} \) deviations.

The ICM mass fraction behavior in the two simulations is quite different, as noted above, and this difference is apparent in the covariance of \( \sigma_{\text{DM}} \) and \( f_{\text{ICM}} \). In the \( \text{PH} \) simulation, the two properties are anti-correlated, \( C = -0.10 \) at redshift zero. In fact, this is the only negatively correlated pair of signal deviations exhibited by the models. It is likely that this negative correlation is driven by the behavior of mergers. Halos in the early stages of a merger will have a higher \( \sigma_{\text{DM}} \) at fixed mass than the mean relation. If, during these merger events, the collisionless dark matter accretes faster than the baryons (since the extended \( \text{PH} \) gas envelopes of accreting satellites will be more easily ablated during the merger encounter), then the ICM mass fraction will be locally depressed, driving an anti-correlation between \( \sigma_{\text{DM}} \) and \( f_{\text{ICM}} \). These effects are absent in the \( \text{GO} \) case, where the correlation of that model is positive, \( C = 0.28 \).

The concentration and ICM emission measure are measures that are sensitive to formation history (Wechsler et al. 2002; Busha et al. 2007) and substructure driven by merging. The correlations between these parameters and \( \sigma_{\text{DM}} \) are surprisingly modest, \( \sim 0.2 \), perhaps indicating that formation history is a more important driver, compared to recent mergers, for these measures. There is not a strong sensitivity to gas physics in the \( c-\sigma_{\text{DM}} \) correlation, but the \( \text{PH} \) \( \hat{Q} - \sigma_{\text{DM}} \) correlation of 0.32 is significantly larger than the \( \text{GO} \) value. In fact, the only two signals with lower \( \sigma_{\text{DM}} \) correlations in \( \text{GO} \) than in \( \text{PH} \) are \( T_\text{sl} \) and \( \hat{Q} \), both measures that are sensitive to small regions of cool, dense gas.

Both \( Y \) and \( L_{\text{bol}} \) have a weaker correlation with \( \sigma_{\text{DM}} \) in \( \text{PH} \) than in \( \text{GO} \). As \( Y \propto f_{\text{ICM}}T_m \), the scatter in \( Y \) at fixed mass is

\[
\sigma_Y^2 = \sigma_T^2 + \sigma_f^2 + 2C_{TF}\sigma_T\sigma_f \tag{14}
\]

Examining Table 5, we see that, because the scatter in \( Y \) is largely dominated by \( T_m \) in the \( \text{GO} \) model and \( f_{\text{ICM}} \) in the \( \text{PH} \) case, the value of the correlation coefficient, \( C_{TF} \), is not an important factor. Our value for the scatter in \( Y \) is higher than that measured in the simulations of Kravtsov et al. (2006), as they see an anti-correlation in the X-ray inferred deviations of gas mass and temperature at fixed mass. In the intrinsic measures of Table 6, we find positive correlation between \( f_{\text{ICM}} \) and \( T_m \) with values of 0.42 (\( \text{PH} \)) and 0.48 (\( \text{GO} \)).

It is worth noting the interesting case of \( C = 0.08 \) between \( Y \) and \( \sigma_{\text{DM}} \) in the \( \text{PH} \) simulation. Algebraically, this is unsurprising due to the positive correlation between \( \sigma_{\text{DM}} \) and \( T_m \) and the negative correlation with \( f_{\text{ICM}} \). This lack of correlation suggests that SZ surveys should produce cluster samples that are unbiased with respect to dynamical state. Similarly, the low correlation between \( Y \) and \( \hat{Q} \), \( C = 0.12 \), shows that SZ surveys should not be strongly biased by gas clumpiness.

When metals are ignored, the bolometric luminosity scaling, \( L \propto f_{\text{ICM}}^{-2}\hat{Q}_T^{1/2} \), implies that the scatter in luminosity at fixed mass follows:

\[
\sigma_L^2 = 4\sigma_f^2 + \sigma_Q^2 + \frac{1}{3}\sigma_T^2 + 4\Psi_f + 2\Psi_T + \Psi_Q \tag{15}
\]

For the \( \text{PH} \) case, the scatter in ICM mass fraction is the primary contributor, responsible for 90% of the variation in \( L_{\text{bol}} \). In contrast, \( f_{\text{ICM}} \) variations account for only 7% of the scatter in the \( \text{GO} \) case, where the majority contributors are variations in \( T_\text{sl} \) and \( \hat{Q} \). Our results are consistent with the work of Balogh et al. (2006), who use an analytic model to show that variation in halo structure cannot completely account for the observed variance in \( L_{\text{bol}} \) at fixed mass.

In both simulations, the correlation between \( T_m \) and \( T_\text{sl} \) at fixed mass is very high. Although a given halo will not have the same \( T_m \) and \( T_\text{sl} \), the high correlation coefficients indicate that the two temperature measures similarly trace the thermal state of a halo. The correlation is higher in the \( \text{PH} \) simulation than in the \( \text{GO} \) case, and cool cores in the latter model also drive higher correlations between \( T_\text{sl} \) and \( \hat{Q} \) than between \( T_m \) and \( \hat{Q} \).

Both simulations also have a significantly non-zero correlation coefficient between \( L_{\text{bol}} \) and \( T_\text{sl} \), 0.73 in \( \text{PH} \) and 0.67 in \( \text{GO} \). As shown by Nord et al. (2008), the \( L-T \) scaling relation expected from X-ray flux-limited surveys is sensitive to the value of \( C_{\text{TL}} \), and studies of current and future samples will be able to place limits on this correlation, using techniques similar to those employed by Rozo et al. (2009) to constrain the correlation of mass and X-ray luminosity at fixed optical richness for the SDSS maxBCG sample.

4.4. Redshift Evolution of the Signal Covariance

We plot the time evolution of the signal scatter in Figure 10, with error bars from bootstrap resampling. Few properties show any evolution in scatter back to \( z = 2 \). In the \( \text{PH} \) case, the scatter in baryon fraction slightly increases with redshift, causing an increase in the scatter of \( Y \) and \( L \). On the other hand, the scatter in emission measure decreases at higher redshift. Note that, at redshift two, there are only 62 halos in the \( \text{PH} \) simulation above our mass cut of \( 5 \times 10^{13} \) \( h^{-1} \) \( \odot \). At redshift zero, there is a higher scatter at the low-mass end of our mass range, as seen in Figure 2. Since the halos at redshift \( z = 2 \) are only slightly over the mass cut, their higher scatter may reflect this mass dependence rather than pure redshift evolution.

Moving on to the off-diagonal elements, most pairs of signals show little evolution in the correlation coefficient with redshift. Figure 11 shows four typical pairs with little evolution. Even as the physical density of halos changes with redshift, we see that the interplay of \( Y \) and \( L \) or \( T_\text{sl} \) and \( f_{\text{ICM}} \) does not change. We see particularly little evolution in pairs of signals in the \( \text{PH} \) simulation.

We do see evolution in a few signal pairs, notably between \( \sigma_{\text{DM}} \) and other signals in the \( \text{GO} \) simulation. Several pairs
Figure 10. Evolution of the scatter in eight bulk cluster properties. (A color version of this figure is available in the online journal.)

Figure 11. Four pairs of signals which show little to no evolution in the correlation coefficient. The black, solid line denotes the PH simulation, and the red, open points denote the GO simulation. (A color version of this figure is available in the online journal.)

Figure 12. Four pairs of signals which show some degree of evolution in the correlation coefficient, particularly in the GO simulation. The black, solid line denotes the PH simulation, and the red, open points denote the GO simulation. (A color version of this figure is available in the online journal.)

Table 7

| Signal   | $\sigma_{\text{DM}}$ | $T_\text{sl}$ | $f_{\text{ICM}}$ | $Y$          | $L_{\text{bol}}$ |
|----------|----------------------|---------------|-------------------|---------------|------------------|
| $\sigma_{\text{DM}}$ | 0.12 (0.12)       | 0.12          | 0.12              | 0.075         | 0.12             |
| $T_\text{sl}$     | 0.10                | 0.12 (0.38)   | 0.35              | 0.050         | 0.26             |
| $f_{\text{ICM}}$  | 0.11                | 0.12          | 0.28 (0.12)       | 0.054         | 0.21             |
| $Y$              | 0.062               | 0.069         | 0.041             | 0.069 (0.075) | 0.066            |
| $L_{\text{bol}}$  | 0.09                | 0.10          | 0.09              | 0.069         | 0.10 (0.26)      |

Notes.
- The diagonal elements give the mass scatter for individual signals for the PH, GO cases. Off-diagonal elements give the halo mass scatter for signal pairs, with the PH case in the lower triangle and the GO case in the upper, as in Figure 8.

4.5. Implications for Multi-Signal Mass Selection

At fixed signal, the scatter in halo mass is $\sigma_{\mu i} = \sigma_i / \alpha_i$, where $i$ labels the particular signal and $\alpha_i$ is the slope of that signal–mass relation. From the analysis above, we compute the mass scatter for a subset of $z = 0$ signals and present the data in the first two columns of Table 7. We see that $Y$ provides the best mass selection under both physical treatments, with scatter $\sim 7\%$. For the PH case, the X-ray luminosity is quite good, with a 10% scatter, while $f_{\text{ICM}}$ is the worst selector, with the weak $f_{\text{ICM}}$–$M$ slope producing a 28% scatter in mass. We will see below that this large scatter can actually be used to improve selection when paired with more precise, correlated signals.

When selecting halos using multiple signals, the mass variance is $\Sigma^2 = (\alpha^T \Psi^{-1} \alpha)^{-1}$, where $\alpha$ is a vector of slopes with elements $\alpha_i$. Consider the two-signal case, and let $r (\equiv C_{12})$ be the correlation coefficient between signals 1 and 2.
measurements is log-normal with variance

\[ \sigma_{\mu} \]

the second signal is a better mass proxy, \( \sigma_{\mu2} \) becomes independent of sign when the second signal is much noisier than the first, \( \sigma_{\mu2}/\sigma_{\mu1} \approx 1 \).

(A color version of this figure is available in the online journal.)

Within the context of the log-normal model with signal covariance, Equation (13), the mass selection of a pair of signal measurements is log-normal with variance

\[ \Sigma^2 = (1 - r^2)(\sigma_{\mu1}^{-2} + \sigma_{\mu2}^{-2} - 2\sigma_{\mu1}^{-1} \sigma_{\mu2}^{-1})^{-1}. \] (16)

The improvement in mass selection to be gained by a pair of signals, relative to a single measurement, is displayed in Figure 13. Here, we plot \( \Sigma/\sigma_{\mu1} \) as a function of the scatter ratio \( \sigma_{\mu2}/\sigma_{\mu1} \) for several values of the correlation coefficient. If the second signal is a better mass proxy, \( \sigma_{\mu2} \ll \sigma_{\mu1} \), then it dominates the selection. Combining signals with comparable mass selection, \( \sigma_{\mu2} \sim \sigma_{\mu1} \), can result in anything from no improvement in the degenerate case \( r \to 1 \) to the \( \sqrt{2} \) improvement for uncorrelated signals \( r = 0 \) to dramatic improvement in the anti-correlated case \( r \to -1 \). Non-intuitively, when an intrinsically “noisy” mass proxy is added, \( \sigma_{\mu2} \gg \sigma_{\mu1} \), one still achieves significant improvement in mass selection as long as the signal pair correlation is large in an absolute sense.

We evaluate Equation (16) for a set of observable signal pairs and present the resultant mass scatter values in the right-hand columns of Table 7. In the PH simulation, the correlation coefficient between \( L \) and \( Y \) is \( r = 0.78 \), and the scatter in mass for the pair of signals is 6.9%, a mild improvement over the scatter using only \( L_{\text{bol}} \) (11%) or only \( Y \) (7.5%). Given the high covariance between \( L_{\text{bol}} \) and \( Y \), future multiwavelength surveys that join SZ and X-ray detections will have good mass selection properties.

Finally, we note that the best mass selection comes in the PH case from combining \( Y \) with \( f_{\text{ICM}} \). The combination of a strong correlation, \( r = 0.88 \), and, especially, the relatively large degree of scatter in mass at fixed \( f_{\text{ICM}} \) (0.28) produces a scatter in mass of only 4.1% when measurements of these two signals are joined. We caution that this analysis does not take measurement errors into account, and it is clear that high data quality will be important to realize this level of mass selection.

5. CONCLUSION

We analyze the scalings of multiple properties of massive halos taken from a pair of gas dynamic simulations with different gas physics treatments. Our samples contain tens of thousands of halos with masses \( M_{200} > 5 \times 10^{13} h^{-1} M_\odot \) at redshifts \( z \lesssim 2 \). The physical treatments of GO or PH, while both idealized, represent extremes of no galactic feedback or coordinated, early feedback at \( z = 4 \). We summarize our results as follows.

1. The entropy injection of the PH model drives deviations from self-similar scaling, mainly by reducing the gas fraction and increasing the gas temperature in lower mass halos. The effects are at the few percent level at \( 10^{15} h^{-1} M_\odot \), but become stronger at lower masses and higher redshifts. We provide fits of \( f_{\text{ICM}}, Y, \) and \( L_{\text{bol}} \) to quadratic forms in \( \ln M \) and \( \ln a \) that describe the model evolution at the few percent level.

2. Preheating at a level of 200 keV cm\(^2\) reproduces the scaling behavior of core-extracted X-ray measures of local clusters.

3. The effects on \( f_{\text{ICM}} \) and \( T_m \) nearly cancel when combined to form the thermal SZ signal within \( r_{\text{200}} \), leading to similar \( Y-M \) scaling relations above \( 3 \times 10^{14} \) for both PH and GO cases at low redshift.

4. The second moments about the mean scaling relations generally support a multivariate log-normal form for the joint distribution of signals at fixed mass and redshift. Normalized percentiles of the one-point distributions show deviations from Gaussianity at roughly the level of 0.1 or smaller.

5. We present the first systematic investigation of property covariance for massive halos. Most signal pairs exhibit positive correlations, with the lone exception of \( -0.1 \) between \( \sigma_{\text{GO}} \) and \( f_{\text{ICM}} \) in the PH case. The thermal SZ signal displays a robust 13% scatter that is strongly correlated with variations in both ICM gas mass and temperature, with \( f_{\text{ICM}} \) dominating in the PH case and \( T_m \) being more important in the GO treatment.

6. Combining multiwavelength observations offers an opportunity to improve selection of clusters by their intrinsic mass. We derive the mass variance of signal pairs and show that combining strongly correlated signals always improves mass selection, even when one of the signals by itself is a comparatively poor mass proxy. The combination of thermal SZ and ICM mass fraction in the PH case selects halo mass with just 4% intrinsic rms scatter.

7. The dark matter velocity dispersion scales with mass and redshift according to self-similar expectations, indicating that the virial theorem is respected regardless of gas treatment. However, the gas temperature in both treatments differs from self-similarity. The deviations in the GO case are driven by larger turbulent gas motions in the mass-limited sample at high-z, consistent with that expected from enhanced merger activity.

Identifying the root causes behind the terms in the covariance matrix is a considerable task that we leave for future work. Mergers (Roettiger et al. 1997; Ricker & Sarazin 2001) and assembly bias (Boylan-Kolchin et al. 2009, and references therein) will surely play important roles for many cluster signals. Studies of merger history behavior will shed light on survey...
selection properties, particularly potential biases related to the dynamical state of a halo.

The simple treatment of baryon physics in our simulations limits our investigation to the hot, thermal ICM of clusters. More extensive physical treatments that incorporate galaxy and supermassive black hole formation and other physics such as MHD and non-thermal plasmas will ultimately extend the set of observable halo signals into the optical/NIR and radio wavebands.

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APPENDIX A

SELF-SIMILAR SCALING OF TOTAL ENERGY

Self-similar evolution in gas temperature requires halos to respect hydrostatic and virial equilibrium to the same degree at all epochs. In our mass-limited samples, the amount of gas turbulent pressure evolves with redshift, driving departures from self-similarity scaling in $T_m$.

Define a dimensionless measure of the energy content in gas motion, $\beta_{\text{gas}} = \sigma_{\text{gas}}^2 / (kT_m/\mu m_p)$, where $\mu$ is the mean molecular weight and $\sigma_{\text{gas}}$ is the velocity dispersion of the gas relative to the mean halo velocity. Figure 14 shows values of $\beta_{\text{gas}}$ for a subsample of halos at $z = 0$. The values differ strongly in the two simulations, with mean values of 0.16 in the GO simulation and 0.06 in the PH model at $10^{14} h^{-1} M_\odot$. The increasing trend with mass reflects the later formation epoch of higher mass halos. Next generation X-ray spectroscopy is needed to measure $\sigma_{\text{gas}}$, enabling discrimination between the model predictions of Figure 14.

Figure 15 shows the redshift evolution of the combined thermal and kinetic energies

$$E_{\text{tot}} = \frac{kT_m}{\mu m_p} (1 + \beta_{\text{gas}}),$$

as well as fits to the standard form, Equation (9). The GO simulation respects the self-similar scaling, $E_{\text{tot}} \sim [M/E(a)]^{2/3}$ at the 2% level out to $z = 1$. At a given mass, the turbulent contribution increases with redshift, and so the mass-weighted temperature grows more slowly than self-similar, as seen in Figure 2. For the PH case, the level of turbulence is much lower. Because of the early energy injection, the thermal energy dominates at all redshifts.

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