Generalization of the Truth-relevant Semantics to the
Predicate Calculus

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1. Propositional Logic

1.1 Truth-relevance

There are Boolean formulae such that their value can be determined by a subset of
their variables. Consider for example \( A = P \lor \neg P \lor Q \). When \( \nu(P) = T \) then \( \nu(A) = T \)
regardless of the value of \( Q \). When \( \nu(P) = F \) then \( \nu(A) = T \) also regardless of the value \( Q \).
The set of variables occurring in \( A \) is \( \{P, Q\} \). We say that the subset \( \{P\} \) is truth-
determining for \( A \); for all the valuations of \( \{P\} \), i.e. \( \nu(P) = T \) and \( \nu(P) = F \), we can
determine the value of \( A \) regardless of the other variables.

Definition 1.1.1: A set of propositional variables is truth-determining for a proposition
\( A \) iff the value of \( A \) can be determined as true or false on all assignments of \( T, F \) to the
set.

Another example: \( P \rightarrow (Q \rightarrow P) \) \hfill (1.1)

If \( P \) is true then \( (Q \rightarrow P) \) is true regardless of the value of \( Q \), but then the entire formula
(1.1) is true (regardless of the value of \( Q \).) Suppose \( P \) is false. Then (1.1) is true
regardless of the value of \( Q \). In either case [i.e. \( \nu(P) = T \) or \( \nu(P) = F \)] the value of (1.1)
can be determined without any knowledge of the value of \( Q \). Thus \( \{P\} \) is truth-
determining for (1.1). This is not the case for \( Q \). For assume \( Q \) is true. Then the value
of \((Q \rightarrow P)\) cannot be determined without knowing the value of \(P\). And without knowing the value of \((Q \rightarrow P)\), the value of (1.1) cannot be determined.

**Definition 1.1.2**: Let \(P_i, \ldots, P_n\) be all the variables occurring in \(A\). Then \(P_i\) is *truth-redundant (t-redundant)* in \(A\) iff there is a truth determining set for \(A\) that does not contain \(P_i\).

**Definition 1.1.3**: \(A\) is *truth-relevant* if it contains no truth-redundant variables.

These definitions are due to Diaz (1981, pp. 65-67).

### 1.2 T-relevance with preconditions

Suppose we have a switching circuit with inputs \(P\) and \(Q\) and output equivalent to the Boolean expression \(\neg P + Q\). Suppose further that \(P\) is "stuck" in 0. [Such a fault can actually occur in electronic circuits.] Then the output will always be 1 regardless of the value of \(Q\). Therefore under the condition that \(P = 0\), \(P\) is truth determining and \(Q\) is t-redundant. Note that if \(P\) is "stuck" in 1 then it is not truth determining. Also when \(P\) is permanently equal to 1, \(Q\) is not truth determining because *it is not the case that the output is solely determined by \(Q\) regardless of the value of \(P\).*

**Definition 1.2.1**: Let \(A\) be a proposition such that certain propositional variables occurring in \(A\) can have only one truth value. A set of propositional variables in such a proposition is *truth-determining* for a proposition \(A\) iff it is sufficient to determine the value of \(A\).

The purpose of this concept will become apparent shortly.
2. Predicate Logic with One Variable

2.1 Truth-relevance under any interpretation

Let us now consider a sentence of first order predicate logic:

\((\forall x)((Jx & \neg Jx) \rightarrow Sx)\)  \hspace{1cm} (2.1.1)

The Boolean table of \((Jx & \neg Jx) \rightarrow Sx\) looks as follows:

| Row | Jx | Sx | (Jx & \neg Jx) | (Jx & \neg Jx) \rightarrow Sx |
|-----|----|----|----------------|-------------------------------|
| 1   | 0  | 0  | 0              | 1                             |
| 2   | 0  | 1  | 0              | 1                             |
| 3   | 1  | 0  | 0              | 1                             |
| 4   | 1  | 1  | 0              | 1                             |

Table 2.1.1

(We are using the symbols "0/1" as opposed to "F/T" as in our logic the Boolean values do not necessarily correspond to the truth values.)

We observe that for all assignments of the variable \(x\) to objects, the Boolean value of the expression \((Jx & \neg Jx) \rightarrow Sx\) is 1, it is determined solely by \(Jx\), and this is so because the value of \((Jx & \neg Jx)\) is always 0.

**Definition 2.1.1:** We will say that a predicate \(Fx\) is empty if \(\neg(\exists x)Fx\) and that it is universal if \(\forall x Fx\).

In the example above the predicate \((Jx & \neg Jx)\) is empty.

**Definition 2.1.2:** A set of one place atomic predicates for a monadic sentence \(A\) is truth-determining under any interpretation iff it is sufficient to determine the truth value of \(A\) under any interpretation.
Definition 2.1.3: A monadic sentence $A$ is truth-relevant under any interpretation iff it does not have a proper subset of truth-determining atomic predicates.

Our definition of satisfaction for monadic formulas will be identical with the classical definition.

Definition 2.1.4: A monadic sentence $A$ is true under any interpretation if it is satisfied and t-relevant under any interpretation. A negation of a true sentence is false.

In the logic of presuppositions (Strawson, 1952, pp. 173-179) compatible with the semantics presented herein the sentence (2.1.1) is neither true nor false.

2.2 Truth-relevance under Interpretation

By interpretation $U$ we will understand a universe of discourse i.e. a set $|U|$ of objects, plus the extent of all the predicates used in our language.

Let us consider the following interpretation $U$:

$|U| = \{a, b, c\}$

$Jx$: $x$ is John's child

$\neg J_a, \neg J_b, \neg J_c$

$Sx$: $x$ is asleep

$S_a, S_b, \neg S_c$

I.e. Alex and Betty are asleep and John has no children.

The Boolean table of $Jx \rightarrow Sx$ looks as follows:

| Row | $Jx$ | $Sx$ | $Jx \rightarrow Sx$ |
|-----|------|------|---------------------|
| 1   | 0    | 0    | 1                   |
| 2   | 0    | 1    | 1                   |
| 3   | 1    | 0    | 0                   |
| 4   | 1    | 1    | 1                   |
Table 2.2.1

We will now substitute constants corresponding to the elements of $|U|$ one by one thus successively obtaining $Ja \rightarrow Sa$, $Jb \rightarrow Sb$, $Jc \rightarrow Sc$, the corresponding Boolean values being:

|     | $Jx$ | $Sx$ | $Jx \rightarrow Sx$ | Row of Table 1 |
|-----|------|------|----------------------|----------------|
| $Ja \rightarrow Aa$ | 0    | 1    | 1                    | 2              |
| $Jb \rightarrow Ab$ | 0    | 1    | 1                    | 2              |
| $Jc \rightarrow Ac$ | 0    | 0    | 1                    | 1              |

Table 2.2.2

We observe that $Jx$ is always 0, and subsequently the truth value of the implication is always 1 regardless of the value of $Sx$. Thus under interpretation $U$, $Jx$ is truth determining and $Sx$ is t-redundant. Note that $Sx$ is not truth determining; it can be equal to 0 and then the outcome will depend on the value of $Jx$. The fact that John has no children is equivalent to $Jx$ being "stuck" at logical 0.

**Note 2.1:** For the purposes of truth determination, when a predicate $P_x$ is neither empty nor universal (under an interpretation) i.e. the value of $P_x$ is not "stuck" either at 0 or 1, we will nevertheless assume that it could be empty or universal.

Let $A(x)$ be a sentence with one variable. Then the following definitions apply.

**Definition 2.1.2:** A set of one place atomic predicates for a monadic sentence $A$ is truth-determining under interpretation $U$ iff it is sufficient to determine the truth value of $A$ under interpretation $U.$
Definition 2.1.3: A monadic sentence $A$ is truth-relevant under interpretation $U$ iff it does not have a proper subset of truth-determining atomic predicates.

Our definition of satisfaction for monadic formulas will be identical with the classical definition.

Definition 2.1.4: A monadic sentence $A$ is true under interpretation $U$ if it is satisfied and t-relevant under interpretation $U$. A negation of a true sentence is false.

In the examples below the strings of symbols 'F', 'G', '*', '.' represent objects in a universe of discourse. Each string of symbols represents the entire universe of discourse. 'F' stands for an object in the extent of some predicate $F()$, 'G' stands for an object in the extent of some predicate $G()$, '*' stands for an object in the extent of both $F()$ and $G()$, '.' stands for an object, which is neither in the extent of $F()$ nor in the extent of $G()$. Thus e.g. in example U1 there are some objects in the extent of $F()$, some objects in the extent of $G()$, some objects in neither, and no object is in the extent of both $F()$ and $G()$.

Example U1:

F F F F F F F F F F . . G G G G G G G G

For this example the following sentences are all true:

$$ (x)\neg(Fx \land Gx) = ||= (x)(\neg Fx \lor \neg Gx) = ||= (x)(Fx \rightarrow \neg Gx) = ||= \neg(\exists x)(Fx \land Gx)$$  \hspace{1cm} (2.2.1)

They are basically saying that there is no overlap between $F$ and $G$.

Example E1:

F F F F F F F F F F * * G G G G G G G G

The following sentence is true:

$$ (\exists x)(Fx \land Gx)$$  \hspace{1cm} (2.2.2)

It is a negation of the sentences from the prior example, and it basically says that
there is an overlap between F and G.

**Example U1b:**

\[ \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \]

Here we have a universe of discourse such that there are some objects in the extent of F() but no object in the extent of G(). None of the sentences (2.2.1) are true. While in this case \((x)\neg(Fx \land Gx)\) is satisfied it is not t-relevant. In this case G() is empty, and it suffices to determine that \((x)\neg(Fx \land Gx)\) is satisfied. Consequently \((x)\neg(Fx \land Gx)\) is not true and its negation, \((\exists x)(Fx \land Gx)\) is not false.

Now consider:

\[(\exists x)(Fx \lor Gx)\]  

\((2.2.3)\)

The sentence is satisfied because clearly there is an x such that Fx. Is it t-relevant? The predicate G is empty. But G() is not truth-determining. It would seem that F() alone is sufficient to determine the truth value of \((2.2.3)\). But F() is neither empty nor universal therefore according to Note 2.1 it alone does not determine the truth value of \((2.2.3)\).

For the example U1B the sentence \((2.2.3)\) is true.

### 3 Predicate Logic with Two or More Variables

#### 3.1 Introduction

Consider the case of two universal quantifiers. We intend to say that

\[(x)(y)Bxy\]  

\((3.1.1)\)

is true iff for all \(a_i\) in the range of \(x\)

\[(y)Ba_i y\]

is true. In classical logic that is all there is to it. In the logic of presuppositions now
there are three additional issues.

1. It could be that some of the \((y)Ba_y\) are neither true nor false. We will therefore stipulate that we are quantifying only over such \((y)Ba_y\) that are t-relevant, i.e. true or false.

2. In classical logic 'all' means all of zero, one or more. In the logic of presuppositions 'all' means all of one or more. That is we require that at least one formula \((y)Ba_y\) be t-relevant.

3. It could be the case that while there is at least one t-relevant \((y)Ba_y\), there is no t-relevant \((x)Bxb\). This situation is depicted on Figure 2. There is no t-relevant formula \((x)(Fx \rightarrow \neg Gxb)\). The commutativity of the quantifiers requires that there be at least one t-relevant \((x)Bxb\). (Figure 1)

Below we will attempt to generalize and formalize these notions, as well as provide some examples to illustrate how the entire system operates.
3.2 Definitions

Definition 3.1.1 (t-relevance of universally quantified sentences):
A sentence \((x_1)(x_2)...(x_n)Ax_1x_2...x_n\) in prenex normal form is \(t\)-relevant iff there are \(c_1, ..., c_n\) such that
\((x_1)Ax_1c_2...c_n\) is \(t\)-relevant \([\text{per definition 2.2.2}]\) and
\((x_2)Ac_1x_2...c_n\) is \(t\)-relevant and
...\((x_n)Ac_1c_2...x_n\) is \(t\)-relevant.

Definition 3.1.2 (t-relevance of universally quantified open formulas):
A formula \((y_1)(y_2)...(y_n)Ac_1c_2...c_my_1y_2...y_n\) in prenex normal form is \(t\)-relevant iff there are \(d_1, ..., d_n\) such that
\((y_1)Ac_1c_2...c_my_1d_2...d_n\) is \(t\)-relevant and
\((y_2)Ac_1c_2...cmd_1y_2...d_n\) is \(t\)-relevant and
...
\((y_n)Ac_1c_2...cmd_1d_2...y_n\) is \(t\)-relevant.

We will call all such \(n\)-tuples \(<c_1, ..., c_m,d_1, ..., d_n>\) \(t\)-relevant assignments.

Definition 3.1.3 (t-relevance of existentially quantified sentences):
A sentence \((x_1)(x_2)...(x_n)Ax_1x_2...x_n\) in prenex normal form is \(t\)-relevant iff
\((x_1)(x_2)...(x_n)~Ax_1x_2...x_n\) is \(t\)-relevant.

NOTE:
\((x)~Ax\) is a negation of \((x)Ax\), but a sentence is \(t\)-relevant iff its negation is \(t\)-relevant.

If \((\exists x)Ax\) is satisfied than it is always \(t\)-relevant, however if it is not satisfied it is not necessarily false.

Definition 3.1.4: (t-relevance of existentially quantified open formulas)
A formula \((\exists y_1)(\exists y_2)...(\exists y_n)A_c_1c_2...c_my_1y_2...y_n\) in prenex normal form is \(t\)-relevant iff
\((y_1)(y_2)...(y_n)~A_c_1c_2...c_my_1y_2...y_n\) is \(t\)-relevant.

Definition 3.1.5: (t-relevance - other cases)
In all other cases i.e. the quantifiers are not part of the first homogeneous block from the right:

The sentence \((\exists y_1)(\exists y_2)Axy\) is \(t\)-relevant if at least one \((y)Axy\) is \(t\)-relevant.
The sentence \((x)(y)Axy\) is \(t\)-relevant if at least one \((y)Axy\) is \(t\)-relevant.
The sentence \((x)(\exists y)Axy\) is \(t\)-relevant if at least one \((\exists y)Axy\) is \(t\)-relevant.
The sentence \((\exists x)(\exists y)Axy\) is \(t\)-relevant if at least one \((\exists y)Axy\) is \(t\)-relevant.

To define satisfaction we will use an approach similar to the one used by Gerald.
Definition 3.2.1 (satisfaction of unquantified formulas)
Our definition of satisfaction for monadic formulas will be identical with the classical definition.

Definition 3.2.2: (satisfaction of universally quantified formulas)
If $A$ is a polyadic $(b)B$, we say that $A$ is satisfied if all the t-relevant assignments of $B$ are satisfied.

Here the term 'all' means all of one or more, i.e. if there is no t-relevant assignments of $B$ then $(b)B$ is not considered satisfied.

Definition 3.2.3: (satisfaction of existentially quantified formulas)
If $A$ is a polyadic $(\exists b)B$, we say that $A$ is satisfied if at least one t-relevant assignment of $B$ is satisfied.

Definition 3.3.1: (truth - universal quantifiers)
A sentence $(x_1)(x_2)...(x_n)Ax_1x_2...x_n$ in prenex normal form is true iff it is satisfied and t-relevant.

Definition 3.3.2: (truth - other cases)
In all other cases a sentence is true iff it is satisfied.

Definition 3.3.3: (falsehood)
A sentence $(x_1)(x_2)...(x_n)Ax_1x_2...x_n$ in prenex normal form is false iff its negation is true.
3.3 Examples

Example 1: \((x)(y)(Fxy \rightarrow \neg Gxy)\)

* According to definition 3.2.2 \((x)(y)(Fxy \rightarrow \neg Gxy)\) is satisfied if all the t-relevant assignments of \((y)(Fxy \rightarrow \neg Gxy)\) are satisfied. On figure 1 these are highlighted in yellow and they are all satisfied.

* According to definition 3.1.1 \((x)(y)(Fxy \rightarrow \neg Gxy)\) is t-relevant iff there are \(a, b\) such that \((x)(Fx_b \rightarrow \neg Gx_b)\) is t-relevant and \((y)(Fy_a \rightarrow \neg Gy_a)\) is t-relevant. On figure 1 these are highlighted in blue and yellow respectively.

* Thus for the scenario on figure 1 the sentence is true since it is satisfied and t-relevant as required by definition 3.3.1.

The negation of this sentence, i.e. \((\exists x)(\exists y)(Fxy \& Gxy)\) is false.
Example 2: (x)(y)(Fxy → ~Gxy)

* According to definition 3.2.2 (x)(y)(Fxy → ~Gxy) is satisfied if all the t-relevant assignments of (y)(Fxy → ~Gxy) are satisfied. On figure 1 these are highlighted in yellow and they are all satisfied.

* According to definition 3.1.1 (x)(y)(Fxy → ~Gxy) is t-relevant iff there are a, b such that (x)(Fx b → ~Gxb) is t-relevant and (y)(Fay → ~Gay) is t-relevant.

On figure 1 this condition is not satisfied, hence for the scenario on figure 2 the sentence (x)(y)(Fxy → ~Gxy) is not t-relevant.

* Since the sentence is not t-relevant as required by definition 3.3.1 it is not true.

The sentence is neither true nor false and the same applies to its negation (∃x)(∃y)(Fxy & Gxy).
Example 3: $(\exists x)(\exists y)(F_{xy} & G_{xy})$

Here an asterisk '*' means that $F$ and $G$ overlap.

* According to definition 3.2.3 we say that $(\exists x)(\exists y)(F_{xy} & G_{xy})$ is **satisfied** if at least one t-relevant assignment of $(\exists y)(F_{ay} & G_{ay})$ is satisfied. On figure 1 it is highlighted in orange.

* According to definition 3.3.2 the sentence is **true** if satisfied.

The negation of this sentence, namely $(x)(y)(F_{xy} \rightarrow \neg G_{xy})$ is **false**.
**Example 4**: $(\exists x)(y)(Fxy \rightarrow \neg Gxy)$

The sentence $(\exists x)(y)(Fxy \rightarrow \neg Gxy)$ is true.

* There are t-relevant formulae $(y)(Fay \rightarrow \neg Gay)$ [highlighted in yellow and orange.]

* In addition one such sentence is satisfied [highlighted in orange] as required by definition 3.2.3.

* The sentence $(\exists x)(y)(Fxy \rightarrow \neg Gxy)$ is true according to definition 3.3.2.

Its negation, $(x)(\exists y)(Fxy \& Gxy)$ is false.

Compare to example 5.
Example 4b: $(\exists x)(y)(Fx \rightarrow \neg Gxy)$

The sentence $(\exists x)(y)(Fx \rightarrow \neg Gxy)$ is false.

Its negation is $(x)(Ey)(Fx & Gxy)$

* According to definition 3.1.5 the sentence $(x)(Ey)(Fx & Gxy)$ is t-relevant if at least one $(Ey)(Fx & Gxy)$ is t-relevant. That is the case.

* According to definition 3.2.2 the sentence $(x)(Ey)(Fx & Gxy)$ is satisfied if all the t-relevant assignments of $(Ey)(Fx & Gxy)$ are satisfied. All the t-relevant assignments are highlighted in yellow and they are indeed all satisfied.

* According to definition 3.3.1 the sentence $(x)(Ey)(Fx & Gxy)$ is true if satisfied and t-relevant. That is the case.

* According to definition 3.3.3 its negation $(\exists x)(y)(Fx \rightarrow \neg Gxy)$ is false.
Example 5: $(\exists x)(y)(Fxy \rightarrow \neg Gxy)$

* There is no t-relevant formula $(y)(Fay \rightarrow \neg Gay)$.

* Consequently no such sentence is satisfied as required by definition 3.2.3

* According to definition 3.1.5 the sentence $(\exists x)(y)(Fxy \rightarrow \neg Gxy)$ is t-relevant if at least one $(y)(Fxy \rightarrow \neg Gxy)$ is t-relevant. This is not the case. Hence for the scenario on figure 5 the sentence $(y)(Fay \rightarrow \neg Gay)$ is neither true nor false.
Example 6: \((y)(\exists x)(F_{xy} \& G_{xy})\)

The sentence \((y)(\exists x)(F_{xy} \& G_{xy})\) is true.

* According to definition 3.1.5 the sentence \((y)(\exists x)(F_{xy} \& G_{xy})\) is t-relevant if at least one \((\exists x)(F_{xb} \& G_{xb})\) is t-relevant. That is the case.

* According to definition 3.2.2 the sentence \((y)(\exists x)(F_{xy} \& G_{xy})\) is satisfied if all the t-relevant assignments of \((\exists x)(F_{xb} \& G_{xb})\) are satisfied. All the t-relevant assignments are highlighted in yellow and they are indeed all satisfied.

* According to definition 3.3.1 the sentence \((x)(\exists y)(F_{xy} \& G_{xy})\) is true if satisfied and t-relevant. That is the case.

The negation of this formula, \((\exists y)(\exists x)(F_{xy} \rightarrow \sim G_{xy})\) is false.

In case of \((x)(y)(F_{xy} \rightarrow \sim G_{xy})\) there was the additional requirement that there be b such that \((x)(F_{xb} \rightarrow \sim G_{xb})\) is t-relevant. In this case, for \((y)(F_{ay} \& G_{ay})\) the requirement is satisfied automatically.
Example 6b: \((y)(\exists x)(Fxy \land Gxy)\)

The sentence \((y)(\exists x)(Fxy \land Gxy)\) is false because its negation, \((\exists y)(x)(Fxy \rightarrow \lnot Gxy)\) is true.

* There are \(t\)-relevant formulae \((x)(Fxb \rightarrow \lnot Gxb)\) [highlighted in yellow and orange.]

* In addition one such sentence is satisfied [highlighted in orange] as required by definition 3.2.3.

* The sentence \((\exists x)(y)(Fxy \rightarrow \lnot Gxy)\) is true according to definition 3.3.2.
**Example 7: (y)(∃x)(Fxy & Gxy)**

The sentence (y)(∃x)(Fxy & Gxy) is neither true nor false.

* Firstly there is no b such that (∃x)(Fxb & Gxb) is t-relevant as required by definition 3.1.5.

* Consequently there is no t-relevant assignment such that the sentence (∃x)(Fxb & Gxb) is satisfied. So definition 3.2.2 is not satisfied.

The negation, (∃y)(x)(Fxy -> ¬Gxy) is not false as there is no t-relevant sentence (∃x)(Fxb -> Gxb) as required by definition 3.2.3.
Example 8: (x)(y)(z)(Fx\text{xyz} \rightarrow \neg Gx\text{xyz})

* Suppose figure 8 depicts the state of affairs for a particular \( z = c \). We observe that there are some \( a, b \) such that both

\[(x)(Fxbc \rightarrow \neg Gxbc)\]

\[(y)(Fayz \rightarrow \neg Gayz)\]

It means that \((x)(y)(Fxyc \rightarrow \neg Gxyc)\) is t-relevant per definition 3.1.2.

* We can read from the picture that \((x)(y)(Fxyc \rightarrow \neg Gxyc)\) is satisfied. So suppose that in all the cases when \((x)(y)(Fxyci \rightarrow \neg Gxyci)\) is t-relevant for some \( z = c_i \), it is also satisfied. Then \((z)(x)(y)(Fxyz \rightarrow \neg Gxyz)\) is satisfied according to definition 3.2.2.

* Suppose that in addition \((z)(Fabz \rightarrow \neg Gabz)\) is also t-relevant. Then \((z)(x)(y)(Fxyz \rightarrow \neg Gxyz)\) is true according to definition 3.1.1.

The sentence \((\exists x)(\exists y)(\exists z)(Fxy \& Gxy)\), which is a negation of \((z)(x)(y)(Fxyz \rightarrow \neg Gxyz)\), is false.
Example 9: \[(z)(y)(\exists x)(F_{xy} \& G_{xy})\]

* We say that \[(z)(y)(\exists x)(F_{xyz} \& G_{xyz})\] is satisfied if all the t-relevant assignments of \[(y)(\exists x)(F_{xyc} \& G_{xyc})\] are satisfied (definition 3.2.2).

* But \[(y)(\exists x)(F_{xyc} \& G_{xyc})\] is t-relevant if at least one \[(\exists x)(F_{xbc} \& G_{xbc})\] is t-relevant (definition 3.1.5). So assuming that for all z we have picture similar to 9, \[(z)(y)(\exists x)(F_{xyz} \& G_{xyz})\] is satisfied.

* Hence \[(z)(y)(\exists x)(F_{xyz} \& G_{xyz})\] is true per definition 3.3.2.

The negation of this formula, \[(\exists z)(\exists y)(\forall x)(F_{xyz} \rightarrow \neg G_{xyz})\] is false.

In case of \[(z)(x)(y)(F_{xyz} \rightarrow \neg G_{xyz})\], example 8 there was the additional requirement that \[(z)(\exists b)(\exists z)(F_{abz} \& G_{abz})\] be t-relevant. In this case, for \[(z)(\exists b)(\exists z)(F_{abz} \& G_{abz})\], the requirement is satisfied automatically.

Furthermore the universal quantifiers do commute. For it is apparent that if for all z and all y there is an x with asterisk then also for each y and each z there is an x with asterisk.
Example 10: \((z)(\exists x)(\exists y)(F_{xy} \& G_{xy})\)

Figure 10

We say that \((z)(\exists x)(\exists y)(F_{zxy} \& G_{zxy})\) is satisfied if all the t-relevant assignments of \((\exists x)(\exists y)(F_{zxy} \& G_{zxy})\) are satisfied (definition 3.2.2).

Suppose that figure 10 depicts the situation for a particular \(z = c_1\). And suppose further that for other values of \(z\) some scenarios will look similar to figure 10 and some similar to figure 2. The formula \((\exists x)(\exists y)(F_{cxy} \& G_{cxy})\) for figure 2 is not t-relevant per definition 3.1.4; therefore it does not "count". But based on our stipulation above all the t-relevant ones look like figure 10. As a result the sentence \((z)(\exists x)(\exists y)(F_{zxy} \& G_{zxy})\) would be satisfied and hence true.

The negation of this sentence, namely \((\exists x)(\exists y)(F_{zxy} \rightarrow \neg G_{zxy})\) is false.

In case of \((z)(x)(y)(F_{xyz} \rightarrow \neg G_{xyz})\), example 8 there was the additional requirement that \((z)(F_{abz} \rightarrow \neg G_{abz})\) be t-relevant. In this case, for \((z)(F_{zab} \& G_{zab})\) it is implied automatically.
Example 11: $(\exists z)(x)(y)(F_{xy} \rightarrow \neg G_{xy})$

Suppose that figure 11 depicts the scenario for some $z = c$.

* According to definition 3.2.3 $(\exists z)(x)(y)(F_{zxy} \rightarrow \neg G_{zxy})$ is satisfied if at least one $t$-relevant assignment of $(x)(y)(F_{cxy} \rightarrow \neg G_{cxy})$ is satisfied.

* But $(x)(y)(F_{cxy} \rightarrow \neg G_{cxy})$ is satisfied (example 1).

* Hence according to definition 3.3.2 $(\exists z)(x)(y)(F_{zxy} \rightarrow \neg G_{zxy})$ is true.

Its negation $(z)(\exists x)(\exists y)(F_{zxy} \land G_{zxy})$ is false.
Example 12: $(z)(\exists x)(y)(Fzxy \rightarrow \neg Gzxy)$

* We say that $(z)(\exists x)(y)(Fzxy \rightarrow \neg Gzxy)$ is satisfied if all the t-relevant assignments of $(\exists x)(y)(Fzxy \rightarrow \neg Gzxy)$ are satisfied (definition 3.2.2).

* On figure 12 the formula $(\exists x)(y)(Fzxy \rightarrow \neg Gzxy)$ is satisfied (example 4).

So suppose that for each $z_i$ we have picture similar to figure 12 with an orange vertical line, but for at a different $x_i$ for each $z_i$. Then $(z)(\exists x)(y)(Fzxy \rightarrow \neg Gzxy)$ is satisfied.

* But then $(z)(\exists x)(y)(Fzxy \rightarrow \neg Gzxy)$ is true according to definition 3.3.2.

The negation, $(\exists z)(x)(\exists y)(Fzxy \& Gzxy)$ is false.

In case of $(z)(x)(y)(Fxyz \rightarrow \neg Gxyz)$, example 8 there was the additional requirement that $(z)(Fabz \rightarrow \neg Gabz)$ be t-relevant. In this case there is not any implication that there is an $a$ such that $(z)(Fzab \rightarrow \neg Gzab)$. 
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