Influence of the geometric and physical parameters of the dielectric optical micro-waveguides with rectangular cross section on their losses

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Abstract. Formation of losses in the dielectric optical micro-waveguides of rectangular cross section is analytically studied. It is shown that the two processes are involved in the formation of the frequency characteristic of the attenuation constant: the redistribution of energy in the waveguide-environment system and the group velocity dispersion. The group velocity dispersion determined by the geometry of the waveguiding structure leads to the formation of a local maximum of the attenuation constant in the frequency response.

1. Introduction
Optical waveguides play a crucial role in the modern integrated microwave photonics. Using those, different passive and active components are constructed. One of the most important characteristics of the optical waveguides, in addition to dispersion, is their losses that affect on the propagating signal-wave decay. Note that the optical losses in the dielectric waveguides are determined by several components. Firstly, since in a dielectric waveguide a part of the optical wave propagates inside the waveguide core, and a part in the cladding, the insertion losses are determined by absorption of the materials from which the waveguiding structure is made. Secondly, due to imperfect of the technology, there are losses on the surface inhomogeneities at the boundary of media.

As the literature analysis shows, the losses in the optical dielectric micro-waveguides of rectangular cross sections have been mainly investigated in two ways: experimentally and by computer modeling. In the computer modeling, the waveguides properties have been studied using various software packages, for example, COMSOL Multiphysics. We note that an analytical theoretical study of the influence of the geometric and physical parameters of the optical waveguides on their loss was not carried out. The purpose of this work was an investigation of the media parameters influence on the electromagnetic wave attenuation constant.

2. Analytical theory of loss of electromagnetic waves in a rectangular dielectric micro-waveguide
In order to study the effect of absorption of the micro-waveguide material on the losses of the waves propagating in it, in the first step we construct an analytical dispersion theory for optical waveguiding modes. In the second step, we use this dispersion theory to calculate numerically an attenuation constant of optical waves propagating in a micro-waveguide. When constructing the analytical
Dispersion theory, we modify the approximate mode analysis proposed in [1]. The modification of [1] is that our analytical theory takes into account the losses of the waveguide materials and the surrounding space by introducing into the Maxwell equations and the material equations the complex dielectric constant of the media. Considering the boundary value problem, we assume that inside the waveguide the fields are described by trigonometric functions, and outside by exponential functions. Note also, we ignore the fields existing in the outside corner regions (see Fig. 1).

After applying the boundary conditions of electrodynamics to the corresponding components of the electric and magnetic fields, four cases can be distinguished in the solution, each of which describes a specific set of modes propagating in a rectangular waveguide along the z axis.

The character of the modes in a dielectric waveguide of rectangular cross section has a hybrid structure, i.e., there are all three components of the electric and magnetic fields. The modes in such a waveguide are usually denoted as \( E_{x}^{m,n} \) and \( E_{y}^{m,n} \), where \( m \) and \( n \) are the number of variations of the electric field in the \( x \) and \( y \) coordinates, and \( E_{x} \) and \( E_{y} \) are the transverse components of the electric field having the maximum value in the waveguide section. In the following, we will be interested only in the two lowest modes with orthogonal polarization - \( E_{x}^{1,1} \) and \( E_{y}^{1,1} \).

The dispersion equations for the lowest \( E_{x}^{1,1} \) - and \( E_{y}^{1,1} \) - modes have the following forms:

\[
\begin{align*}
\frac{k_x}{\kappa_1} \alpha_{13} \operatorname{ctg} \left( \frac{a}{2} k_{x} \right) - k_x \alpha_{12} \operatorname{tg} \left( \frac{b}{2} k_{y} \right) + \left( \varepsilon_1 - \varepsilon_2 \right) k_0^2 - k_{x}^2 - k_{y}^2) &= 0, \\
k_y \alpha_{13} \operatorname{ctg} \left( \frac{a}{2} k_{x} \right) - k_y \alpha_{12} \operatorname{tg} \left( \frac{b}{2} k_{y} \right) - \frac{\varepsilon_1}{\varepsilon_2} \left( \varepsilon_1 - \varepsilon_2 \right) k_0^2 - k_{x}^2 - k_{y}^2) &= 0, \\
k_x \alpha_{13} \operatorname{tg} \left( \frac{a}{2} k_{x} \right) - k_x \alpha_{12} \operatorname{ctg} \left( \frac{b}{2} k_{y} \right) - \left( \varepsilon_1 - \varepsilon_2 \right) k_0^2 - k_{x}^2 - k_{y}^2) &= 0, \\
k_y \alpha_{13} \operatorname{tg} \left( \frac{a}{2} k_{x} \right) - k_y \alpha_{12} \operatorname{ctg} \left( \frac{b}{2} k_{y} \right) + \frac{\varepsilon_1}{\varepsilon_2} \left( \varepsilon_1 - \varepsilon_2 \right) k_0^2 - k_{x}^2 - k_{y}^2) &= 0,
\end{align*}
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the complex dielectric permeability of the material of the waveguide and cladding; \( a \) and \( b \) are the micro-waveguide width and height; \( \alpha_{12} = ik_{x,y} = \sqrt{k_0^2 \left( \varepsilon_1 - \varepsilon_2 \right) - k_{x,y}^2} \); \( \alpha_{13} = ik_{x,y} = \sqrt{k_0^2 \left( \varepsilon_1 - \varepsilon_2 \right) - k_{x,y}^2} \); \( k_{x} \) and \( k_{y} \) are the transverse wave numbers inside the waveguide along the \( x \) and \( y \) axes, which are solutions of the systems of equations (1) and (2). Unlike the theory of approximate mode analysis [1], the transverse wave numbers are complex values. The imaginary parts of \( k_{x} \) and \( k_{y} \) characterize the loss of the mode in question in the structure.

![Figure 1. Cross section of the dielectric waveguide.](image-url)
The propagation constant in the waveguide is defined as follows:

$$\gamma = \beta - i\alpha = \sqrt{\omega^2 \varepsilon_i \mu_0 - k_{t_x}^2 - k_{t_y}^2}.$$  

(3)

The attenuation constant for power (measured in dB/m) is

$$\alpha_{\text{p}, \text{db}} = 20 \cdot \log(e^{-\alpha}),$$  

(4)

and the effective refractive index is

$$n_{\text{eff}} = \frac{\gamma}{k_0}.$$  

(5)

3. Results of numerical simulation

At the next step, an influence of the parameters of micro-waveguides of rectangular cross section on the formation of losses in them was investigated. To do this, the constructed analytical theory was used. In the Matlab mathematical package, a program was written that produced a numerical solution of the dispersion equations (1) and (2) with the given parameters of the medium, which made it possible to determine the losses of propagating modes in the optical micro-waveguides.

In contrast to the COMSOL Multiphysics package, the obtained program requires considerably less computational and time-consuming. The cost reduction is because the basis of the work of COMSOL Multiphysics is the finite element method, which requires the creation of a computational grid, which must be rebuilt with each change of input conditions in the waveguiding structure under consideration. Such input conditions include the geometric dimensions of the micro-waveguide, the refractive indices of the media, the frequency of optical radiation, and others.

Note that in the numerical simulation, the silicon oxide (SiO2) was chosen as the material of cladding and the three materials were used as the waveguide material: silicon (Si), silicon nitride (Si3N4), and ultra-silicon-rich nitride (SiNx). These materials are the most promising for development of the integrated microwave photonics components and devices. Speaking in more details, silicon oxide is the most commonly used dielectric to isolation optical integrated circuits, silicon is already implemented to construct a number of different devices with electronic control [2], silicon nitride has the lowest optical loss [3, 4], and ultra-silicon-rich nitride has the highest nonlinearity coefficient [4].

Figure 2 shows the simulation results obtained for the effective refractive index, attenuation constant, and group velocity of the $E_{x,11}$- and $E_{y,11}$-modes versus the electromagnetic wave frequency. The value of the group velocity in the simulation was determined by the following expression:

$$V_g = \frac{\partial \omega}{\partial \beta}.$$  

(6)

The simulations were performed for the Si3N4 waveguide (a), Si2N3 waveguide (b), and Si waveguide (c). In the modeling, the cross-sectional size of the waveguide was 1.5x0.3 µm, the value of the attenuation constant of the waveguide material was 0.5 dB/cm, and the attenuation constant SiO2 was 0.25 dB/cm. From the obtained characteristics, it is clear that with increasing frequency, the value of the effective refractive index tends from the refractive index of the cladding to the refractive index of the core. Respectively, the value of the attenuation constant tends from the value of the attenuation coefficient of the cladding to the attenuation coefficient of the core with the formation of a local maximum. An appearance of the local maximum is explained by the fact that the two processes are involved in the formation of the frequency characteristic of the attenuation constant: the redistribution of energy in the core-cladding structure and the group velocity dispersion.

It should be noted that the higher is the difference in the refractive indices of the core-cladding system, the greater becomes the steepness of the bending of the dispersion branches. Owing to it, the maximum value of the loss in the micro-waveguide becomes higher. The numerical calculations show that when the influence of the group velocity is negligible, the value of the attenuation constant of the $E_{y,11}$-mode is lower than that of the $E_{x,11}$-mode, and with frequency increasing it tends to the damping
Figure 2. Frequency dependences of the effective refractive index, attenuation constant, and group velocity in the micro-waveguide of rectangular cross-section; the solid line corresponds to the $E_{y_{11}}$-mode, the dotted line corresponds to the $E_{x_{11}}$-mode.

decrement (see, Fig. 2(a)). Such a course of characteristics is because the $E_{y_{11}}$-mode has a larger effective area than $E_{x_{11}}$-mode and, accordingly, most of its energy is distributed in the space surrounding the waveguide core, which has smaller losses. With an increase in the difference of the refractive indices, the group velocity effect on the frequency characteristic of the damping decrement becomes significant and, accordingly, the loss of the $E_{y_{11}}$-mode begins to increase and becomes higher than the loss of the $E_{x_{11}}$ mode with a local maximum (see Fig. 2).

4. Conclusion
In conclusion, the paper theoretically studies on influence of the geometric and physical parameters of the rectangular optical micro-waveguides on the insertion losses of the two lowest modes with orthogonal polarization. A simple yet accurate method to determine the attenuation constant of the waves propagating in the micro-waveguides of the rectangular cross-section is formulated. It is shown that the two processes are involved in the formation of the frequency dependence of the attenuation constant: a redistribution of energy in the core-cladding structure and the group velocity dispersion. The redistribution of energy in the waveguide-environment structure leads to the fact that with an
increase in the frequency of the optical signal, the attenuation constant tends from the value determined by the cladding material absorption loss to the value determined by the microwaveguide core material loss. The group velocity dispersion leads to the formation of a local maximum of the attenuation constant on its frequency dependence. Moreover, the greater is the contrast value of the refractive indices of the structure, the higher is the value of this maximum.

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