Analysis of the strong $D_2^*(2460)^0 \rightarrow D^+\pi^-$ and $D_{s2}^*(2573)^+ \rightarrow D^+K^0$ transitions via QCD sum rules

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Abstract

The strong $D_2^*(2460)^0 \rightarrow D^+\pi^-$ and $D_{s2}^*(2573)^+ \rightarrow D^+K^0$ transitions are analyzed via three point QCD sum rules. First, we calculate the corresponding strong coupling constants $g_{D_2^*D\pi}$ and $g_{D_{s2}^*DK}$. Then, we use them to calculate the corresponding decay widths and branching ratios. Making use of the existing experimental data on the ratio of the decay width in the pseudoscaler $D$ channel to that of the vector $D^*$ channel, finally, we estimate the decay width and branching ratio of the strong $D_2^*(2460)^0 \rightarrow D^*(2010)^+\pi^-$ transition.

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1 Introduction

Following the first observation reported in 1986 [1] the past few decades have been a period for the observations of orbitally excited charmed mesons [2–12]. During this period there have also been several theoretical studies on the masses, strong and electromagnetic transitions of these mesons via various methods (for instance see [13–16] and references therein). Among these orbitally excited mesons are the $D^*_2(2460)$ and $D^*_s2(2573)$ mesons. The $D^*_2(2460)$ state has the quantum numbers $I(J^P) = \frac{1}{2}(2^+)$. Being not known exactly, $I(J^P) = 0(2^+)$ quantum numbers are favored by the width and decay modes of the $D^*_s2(2573)$ state. In this work, it is considered as a charmed strange tensor meson. One can see [17–23] and references therein for some experimental and theoretical studies on the properties of the charmed strange mesons.

In the literature, compared to the other types of mesons, there are little theoretical works on the properties of the tensor mesons. Especially, their strong transitions are not studied much. Studying the parameters of these tensor mesons and the comparison of the attained results with the existing experimental results may provide fruitful information about the internal structures and the natures of these mesons. Considering the appearance of these charmed tensor mesons as intermediate states in studying the $B$ meson decays, the results of this work can also be helpful in this respect. Beside all of these, the possibility for searches on the decay properties of $D^*_2$ and $D^*_s2$ mesons at LHC is another motivation for theoretical studies on these states.

The present work puts forward the analysis of the strong transitions $D^*_2(2460)^0 \rightarrow D^+\pi^-$ and $D^*_s2(2573)^+ \rightarrow D^+K^0$. For this aim, first we calculate the strong coupling form factors $g_{D^*_2D\pi}$ and $g_{D^*_s2DK}$ via QCD sum rules as one of the most powerful and applicable non-perturbative methods to hadron physics [24]. These strong coupling form factors are then used to calculate the corresponding decay widths and branching ratios of the transitions under consideration. Making use of the existing experimental data on the ratio of the decay width in the pseudoscaler $D$ channel to that of the vector $D^*$ channel, finally, we evaluate the decay width of the strong $D^*_2(2460)^0 \rightarrow D^*(2010)^+\pi^−$ transition.

2 QCD sum rules for the strong coupling form factors $g_{D^*_2D\pi}$ and $g_{D^*_s2DK}$

The aim of this section is to present the details of the calculations of the coupling form factors $g_{D^*_2D\pi}$ and $g_{D^*_s2DK}$ for which we use the following three-point correlation function:

$$\Pi_{\mu\nu}(p,p^\prime,q) = i^2 \int d^4x \int d^4y \ e^{-ip\cdot x} e^{i(p^\prime - p)^\prime \cdot y} \langle 0 | T \left( J^D(y) J^{\pi[K]}(0) J^{D^*_2[D^*_s2]}_{\mu\nu}(x) \right) | 0 \rangle,$$  \hspace{1cm}(1)

where $T$ is the time ordering operator and $q = p − p^\prime$ is transferred momentum. The interpolating currents appearing in this three-point correlation function can be written in terms of the quark field operators as

$$J^D(y) = i\bar{d}(y)\gamma_5c(y),$$

$$J^{\pi[K]}(0) = i\bar{u}[\bar{s}](0)\gamma_5d(0),$$

1
\[ J^{D^*_2[D^*_2]}_{\mu\nu}(x) = \frac{i}{2} \left[ \bar{u}(s)(x)\gamma_{\mu}\gamma_{\nu}\bar{u}(s)(x) + \bar{u}(s)(x)\gamma_{\nu}\bar{u}(s)(x) \right], \]  

with \( \bar{D}_\mu(x) \) being the two-side covariant derivative that acts on left and right, simultaneously. The covariant derivative \( \bar{D}_\mu(x) \) is defined as

\[ \bar{D}_\mu(x) = \frac{1}{2} \left[ \bar{D}_\mu(x) - \bar{D}_\mu(x) \right], \]

where

\[ \bar{D}_\mu(x) = \partial_\mu(x) - i \frac{g}{2} \lambda^a A_\mu^a(x), \]
\[ \bar{D}_\mu(x) = \partial_\mu(x) + i \frac{g}{2} \lambda^a A_\mu^a(x). \]

Here \( \lambda^a (a = 1, 2, \ldots, 8) \) are the Gell-Mann matrices and \( A_\mu^a(x) \) stand for the external gluon fields. These fields are expressed in terms of the gluon field strength tensor using the Fock-Schwinger gauge \( x^\mu A_\mu^a(x) = 0 \), i.e.,

\[ A_\mu^a(x) = \int_0^1 d\alpha \alpha x_\beta G^\alpha_{\beta\mu}(\alpha x) = \frac{1}{2} x_\beta G^\alpha_{\beta\mu}(0) + \frac{1}{3} x_\eta x_\beta D_{\eta\beta} G^\alpha_{\beta\mu}(0) + \cdots, \]

where we keep only the leading term in our calculations and ignore from contributions of the derivatives of the gluon field strength tensor.

One follows two different ways to calculate the above mentioned correlation function according to the QCD sum rule approach. It is calculated in terms of hadronic parameters called hadronic side. On the other hand, it is calculated in terms of quark and gluon degrees of freedom by the help of the operator product expansion in deep Euclidean region called the OPE side. The match of the coefficients of same structures from both sides provides the QCD sum rules for the intended physical quantities. By the help of double Borel transformation with respect to the variables \( p^2 \) and \( p'^2 \) one suppresses the contribution of the higher states and continuum.

In hadronic side, the correlation function in Eq. (1) is saturated with complete sets of appropriate \( D^*_2[D^*_2] \), \( \pi[K] \) and \( D \) hadronic states with the same quantum numbers as the used interpolating currents. Performing the four-integrals over \( x \) and \( y \) leads to

\[ \Pi^{had}_{\mu\nu}(p, p', q) = \frac{\langle 0 | J^{\pi[K]} | \pi[K](q) \rangle \langle 0 | J^{D} | D(p') \rangle \langle D^*_2[D^*_2](p, \epsilon) | J_{\mu\nu}^{D^*_2[D^*_2]} | 0 \rangle}{(p^2 - m^{2}_{D^*_2[D^*_2]})(p'^2 - m^{2}_{D})(q^2 - m^{2}_{\pi[K]})} \times \langle \pi[K](q) D(p') | D^*_2[D^*_2](p, \epsilon) \rangle + \cdots, \]

where \( \cdots \) represents the contributions of the higher states and continuum. The matrix elements appearing in this equation are parameterized as follows:

\[ \langle 0 | J^{\pi[K]} | \pi[K](q) \rangle = i \frac{m^2_{\pi[K]} f_{\pi[K]}}{m_d + m_u[s]}, \]
\[ \langle 0 | J^{D} | D(p') \rangle = i \frac{m^2_D f_D}{m_d + m_c}, \]

where \( m_d \) and \( m_c \) are the quark masses. The other matrix elements are obtained by the help of unitarity.
\[ \langle D_2^*[D_{s2}^*](p, \epsilon) | J_{\mu\nu}^D | 0 \rangle = m_{D_2^*[D_{s2}^*]}^3 f_{D_2^*[D_{s2}^*]} \epsilon^{(\lambda)}_{\mu\nu}, \]  

and

\[ \langle \pi[K](q)D(p') | D_{s2}^*[D_{s2}^*](p, \epsilon) \rangle = g_{D_2[D_{s2}^*D_{s2}^*]} f_{\pi[K]} f_{D_2^*[D_{s2}^*]} \]

\[ + C \varepsilon^{\alpha \beta \gamma} \varepsilon_{\mu\nu} \rho_\alpha p_\beta + D \rho' \eta p_\eta + E \rho' \eta p_\eta + F \rho_\eta p_\eta + G \rho_\eta p_\eta + H \rho_\eta p_\eta, \]  

Eq. (10) can be written as

\[ \langle \pi[K](q)D(p') | D_{s2}^*[D_{s2}^*](p, \epsilon) \rangle = g_{D_2[D_{s2}^*D_{s2}^*]} f_{\pi[K]} f_{D_2^*[D_{s2}^*]} \]

By the usage of the matrix elements given in Eqs. (7), (8), (9) and (12) in Eq. (6), the correlation function takes its final form in the hadronic side,

\[ \Pi^{\text{had}}_{\mu\nu}(p, p', q) = \frac{g_{D_2[D_{s2}^*D_{s2}^*]} m_D^2 m_{\pi[K]}^2 f_{\pi[K]} f_{D_2^*[D_{s2}^*]}}{(m_c + m_d)(2m_{\pi[K]}^2 - m_D^2)(q^2 - m_{\pi[K]}^2)} \]

\[ \times \left[ m_{D_2^*[D_{s2}^*]} p \cdot p' p_\mu p_\nu - \frac{2 (p \cdot p')^2 + m_{D_2^*[D_{s2}^*]} p^2}{3 m_{D_2^*[D_{s2}^*]}} p_\mu p_\nu - m_{D_2^*[D_{s2}^*]} p' \cdot p'_\mu p'_\nu + m_{D_2^*[D_{s2}^*]} p \cdot p' p_\mu + m_{D_2^*[D_{s2}^*]} p_\nu + \frac{m_{D_2^*[D_{s2}^*]}^2}{3} g_{\mu\nu} + \cdots \right], \]

where the summation over the polarization tensor has been applied, i.e.

\[ \sum_\lambda \varepsilon^{(\lambda)}_{\mu\nu} \varepsilon^{(\lambda)} = \frac{1}{2} T_{\mu\alpha} T_{\nu\beta} + \frac{1}{2} T_{\mu\beta} T_{\nu\alpha} - \frac{1}{3} T_{\mu\nu} T_{\alpha\beta}, \]

and

\[ T_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{D_2^*[D_{s2}^*]}^2}. \]

Following the application of the double Borel transformation with respect to the initial and final momenta squared, we attain the hadronic side of the correlation function as

\[ \hat{\Pi}^{\text{had}}_{\mu\nu}(q) = g_{D_2[D_{s2}^*D_{s2}^*]} m_D^2 m_{\pi[K]}^2 e^{-\frac{m_D^2}{M^2}} e^{-\frac{m_{\pi[K]}^2}{M^2}} \]

\[ \left\{ \frac{1}{12} m_{D_2^*[D_{s2}^*]} \left( m_D^4 + (m_{D_2^*[D_{s2}^*]}^2 - q^2)^2 - 2m_D^2(m_{D_2^*[D_{s2}^*]}^2 + q^2) \right) g_{\mu\nu} \right\}. \]
\[ + \frac{1}{6m_{D_2[D*_{s2}]}} \left[ m_D^4 + m_D^2(4m_{D2[D*_{s2}]} - 2q^2) + (m_{D2[D*_{s2}]} - q^2)^2 \right] p_\mu p_\nu \]

\[ - \frac{1}{2}m_{D_2[D*_{s2}]}(m_D^2 + m_{D2[D*_{s2}]} - q^2)p_\nu p_\nu' + m_{D_2[D*_{s2}]}p_\mu p_\nu' \]

\[ - \frac{1}{2}m_{D_2[D*_{s2}]}(m_D^2 + m_{D2[D*_{s2}]} - q^2)p_\mu p_\nu' \right) \right] + \cdots, \eqno{(16)} \]

where \( M^2 \) and \( M'^2 \) are Borel mass parameters.

In OPE side, we calculate the aforesaid correlation function in deep Euclidean region, where \( p^2 \to -\infty \) and \( p'^2 \to -\infty \). Substituting the explicit forms of the interpolating currents into the correlation function Eq. (1) and after contracting out all quark pairs via Wick’s theorem, we get

\[
\Pi_{\mu\nu}^{\text{OPE}}(p, p', q) = \frac{i^5}{2} \int d^4x \int d^4ye^{-ip \cdot x}e^{ip \cdot y} \times \left\{ Tr \left[ \gamma_5 S_d^{ij}(y-x)\gamma_5 D_{ij} (x) S_c^{ij}(x) \right] + [\mu \leftrightarrow \nu] \right\}, \eqno{(17)}
\]

where \( S_c^{ij}(x) \) represents the heavy quark propagator which is given by [26]

\[
S_c^{ij}(x) = \frac{i}{(2\pi)^4} \int d^4ke^{-ik \cdot x} \left\{ \delta^{ij} - \frac{g_s G_{\alpha\beta}}{4} \delta^{ij} \left( \frac{k + m_c}{k^2 - m_c^2} - \frac{k + m_c}{(k^2 - m_c^2)^2} \right) \right\}, \eqno{(18)}
\]

and \( S_{u[i]}(x) \) and \( S_d(x) \) are the light quark propagators and are given by

\[
S_{u[i]}^{ij}(x) = \frac{i}{2\pi^2 x^4} \delta_{ij} - \frac{m_q}{4\pi^2 x^2} \delta_{ij} - \frac{\langle \bar{q} q \rangle}{12} \left( 1 - \frac{m_q}{4} \frac{x^2}{m_q^2} \right) \delta_{ij} - \frac{x^2}{192} m_q \langle \bar{q} q \rangle \left( 1 - \frac{m_q}{6} \frac{x^2}{m_q^2} \right) \delta_{ij}
\]

\[ - \frac{i g_s G_{\alpha\beta}}{32\pi^2 x^2} \left[ \frac{\sigma_{\alpha\beta} + \sigma^{\alpha\beta}}{x} + \cdots. \right. \]

\( \left. \eqno{(19)} \right) \]

After insertion of the explicit forms of the heavy and light quark propagators into Eq. (17), we use the following transformations in \( D = 4 \) dimensions:

\[
\frac{1}{[(y-x)^2]^n} = \left( \frac{\pi^{D/2}}{\Gamma(D/2-n)} \right)^{1/(D/2-n)} \Gamma(D/2-m) \Gamma(m)^{1/(D/2-m)} \int \frac{d^Dy}{(2\pi)^D} e^{-iy \cdot (y-x)} \]

\[
\frac{1}{[y^2]^m} = \left( \frac{\pi^{D/2}}{\Gamma(D/2-m)} \right)^{1/(D/2-m)} \Gamma(D/2-m) \Gamma(m)^{1/(D/2-m)} \int \frac{d^Dy}{(2\pi)^D} e^{-iy \cdot y} \left\{ (-1)^{n+1} 2^{D-2n} \pi^{D/2} \Gamma(D/2-m) \Gamma(m) \right\} \left( -\frac{1}{t^2} \right)^{D/2-n}, \eqno{(20)}
\]

and perform the four-\( x \) and four-\( y \) integrals after the replacements \( x_\mu \to i \frac{\partial}{\partial y_\mu} \) and \( y_\mu \to -i \frac{\partial}{\partial y_\mu} \). The four-integrals over \( k \) and \( t' \) are performed by the help of the Dirac Delta functions which are obtained from the four-integrals over \( x \) and \( y \). The remaining four-integral over \( t \) is performed via the Feynman parametrization and

\[
\int d^4t \frac{(t^2)^\beta}{(t^2 + L)^\alpha} = \frac{i \pi^2 (-1)^{\beta - \alpha} \Gamma(\beta + 2) \Gamma(\alpha - \beta - 2)}{\Gamma(2) \Gamma(\alpha) \Gamma(-L)^{\alpha - \beta - 2}}, \eqno{(21)}
\]
Albeit its smallness we also include the contributions coming from the two-gluon condensate in our calculations.

The correlation function in OPE side is written in terms of different structures as

\[ \Pi_{\mu \nu}^{OPE}(p, p', q) = \Pi_1(q^2)p_{\mu}p_{\nu} + \Pi_2(q^2)p_{\mu}p'_{\nu} + \Pi_3(q^2)p'_{\mu}p'_{\nu} + \Pi_4(q^2)p'_{\mu}p_{\nu} + \Pi_5(q^2)g_{\mu \nu}, \]

(22)

where each \( \Pi_i(q^2) \) function receives contributions from both the perturbative and non-perturbative parts and can be written as

\[ \Pi_i(q^2) = \int ds \int ds' \frac{\rho_i^{pert}(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \Pi_i^{non-pert}(q^2), \]

(23)

where the spectral densities \( \rho_i(s, s', q^2) \) are given by the imaginary parts of the \( \Pi_i \) functions, i.e., \( \rho_i(s, s', q^2) = \frac{1}{\pi} Im[\Pi_i] \). In present study, we consider the Dirac structure \( p_{\mu}p_{\nu} \) to obtain the QCD sum rules for the considered strong coupling form factors. The \( \rho_1(s, s', q^2) \) and \( \Pi_1^{non-pert}(q^2) \) corresponding to this Dirac structure are obtained as

\[ \rho_1^{pert}(s, s', q^2) = \int_0^1 dx \int_0^{1-x} dy \left[ \frac{\alpha_s G^2}{\pi} \right] \frac{1}{8L^4} m_c m_d m_q (1 - x - y) \left[ m_c m_d m_q (1 - x - y) \right] + m_c \left( p^2 x + q^2 y - 1 \right) (x + y - 1)(x + y) + m_c p^2 x (x + y - xy - y^2 - 1) + \left( m_q (x + y - 1) - m_d (x + y) \right) \left( p^2 (x - 1) (x + y - 1) + y(p'^2 (1 - x)) + q^2 (x + y - 1) \right) + \frac{1}{24L^2} (x - 1)^2 x^2 (2x - 1) (p'^2 - q^2 + p^2 (3x - 2)) + xy (x - 1) \left( q^2 (x - 1) (4 - 13x + 6x^2) + p^2 (x - 1) (2 - 17x + 24x^2) + p'^2 (3 - 11x + 15x^2 - 6x^3) \right) + q^2 y^2 (3 - 32x + 81x^2 - 75x^3 + 24x^4) + xy^2 \left( p^2 (57x - 90x^2 + 42x^3 - 10) + p'^2 (11 - 40x + 50x^2 - 18x^3) \right) + q^2 y^3 \times (x - 1) (15 - 62x + 42x^2) + xy^3 \left( p^2 (x - 1) (42x - 19) + 48xp^2 - 24x^2 p^2 - 19p'^2 \right) + xy^4 \left( p'^2 (17 - 18x) - p^2 (17 - 24x) \right) + q^2 y^4 (27 - 73x + 42x^2) + 6xy^5 (p'^2 - p^2) + 3y^5 q^2 (8x - 7) + 6y^5 q^2 - m_c x^3 (1 + 8x^2 - 7y + 8y^2 - 7x) + 16xy - m_c m_q x (x + y - 1) (8x^3 - 3x^2 - 2x - 5y + 10xy + 8x^2 y + 8y^2) + m_c m_q x (8x^4 - 11x^3 + 8x^2 - 3x - 3y + 14xy - 19x^2 y + 16x^3 y + 7y^2 - 12xy^2 + 8x^2 y^2 - 4y^3) + \frac{1}{48L^2} \left[ 24x^4 + x^3 (72y - 55) + 3x^2 (13 - 48y + 32y^2) + (y^2 - y) (8 - 31y + 24y^2) - 8x + 75xy - 144xy^2 + 72y^3 \right]\]
\[ + \frac{m_0^2 \langle dd \rangle m_q}{24 q^2 (m_c^2 - p^2)^4} \left( 9 m_c^4 - 8 m_c^3 m_d - 12 m_c^2 p^2 + 2 m_c m_d p^2 + 3 p^4 \right) \\
+ \frac{m_0^2 \langle qq \rangle m_d}{24 q^2 (m_c^2 - p^2)^4} \left( 9 m_c^4 + 8 m_c^3 m_q - 12 m_c^2 p^2 - 2 m_c m_q p^2 + 3 p^4 \right), \tag{25} \]

where \( \langle \bar{q} q \rangle = \langle \bar{u} u \rangle \), \( m_q = m_u \) and \( \langle \bar{q} q \rangle = \langle \bar{s} s \rangle \), \( m_q = m_s \) for the initial \( D_s^{*} \) and \( D_{s2}^{*} \) states, respectively and
\[
L(x, y, q^2) = -m_c^2 x + sx - sx^2 + q^2 y - q^2 xy - sxy + s'xy - q^2 y^2. \tag{26} \]

The final form of the OPE side of the correlation function is obtained after double Borel transformation as
\[
\hat{\Pi}_{\mu\nu}^{\text{OPE}}(q^2) = \left\{ \int ds \int ds' e^{-\frac{m}{M^2} - \frac{m'}{M'^2}} \rho_{\mu}^\text{pert}(s, s', q^2) + \hat{\Pi}_{\mu\nu}^{\text{non-pert}}(q^2) \right\} p_\mu p_\nu + \cdots, \tag{27} \]
where
\[
\hat{\Pi}_{\mu\nu}^{\text{non-pert}}(q^2) = \int_1^0 dx \exp \left[ \frac{m_c^2 M^4 + m_c^2 M^4 + M^2 M^2 \left( -q^2 (x-1) r + 2 m_c^2 x \right)}{M^2 M^2 (M^2 + M'^2) x (x-1)} \right] \frac{\alpha_s G^2}{\pi} \\
\times \frac{1}{48} \sqrt{\frac{1}{(x-1)^2}} \left\{ \frac{M^4 (x-1)^6 (M^2 + M'^2) x}{x^3 u^6 (M^2 + M'^2)^{10}} \left[ x m_c^2 (M^4 + M'^4) - M^2 M^2 \right] \right. \\
\left. \times \left( q^2 (x-1)^2 - 2 m_c^2 x \right) + \frac{M^4 (x-1)^6 (M^2 + M'^2) x}{x^3 u^6 (M^2 + M'^2)^9} \left( M^2 q^2 (x-1) \right) \right. \\
\left. + 4 M^4 x + M^4 \left( q^2 + 2 M^2 x - q^2 x \right) + \frac{M^8 (x-1)^4}{x^2 u^4 M^2 (M^2 + M'^2)^7} \left[ m_c m_d M^6 \right] \right. \\
\left. + M^6 x \left( M^2 (x-1) + m_c m_d x \right) + M^4 M^2 \left( 4 M^2 (1-x) + m_c m_d (1+2x) \right) \right. \\
\left. + M^2 M^4 \left( m_c m_d x (2+x) + M^2 (7x-5x^2-2) \right) \right) - \frac{M^8 (M^2 + M'^2 x)}{x^2 u^3 M^2 (M^2 + M'^2)^5} \\
\times \left( x-1 \right)^4 \left[ m_c m_d + M^2 \right] \right\} \theta \left[ \frac{M^2 - M^2 x}{M^2 + M'^2} \right]. \tag{28} \]

with
\[
u = -1 + x + \frac{M^2 - M^2 x}{M^2 + M'^2}. \tag{29} \]

Equating the coefficients of the same Dirac structure from both sides of the correlation function, we get the following sum rules for the coupling form factors \( g_{D_2^* D_\pi} \) and \( g_{D_{s2}^* D_K} \):
\[
g_{D_2^* D_\pi[D_{s2}^* D_K]} = \frac{\langle \bar{d} d \rangle}{m_c^2 - p^2} \frac{\langle \bar{u} u \rangle}{m_u^2 - p^2} \frac{m_c^2}{4 \pi^2} \frac{6 (m_c + m_d) (m_d + m_u) (m_u^2 - q^2) m_{D_2^* [D_{s2}^*]}}{f_{D_2^* [D_{s2}^*]} f_{D_K} m_{D_2^* [D_{s2}^*]}^2 m_{D_2^* [D_{s2}^*]}^2} \\
\times \left\{ \int_{(m_c + m_d)^2}^{s_0} ds \int_{(m_c + m_d)^2}^{s'_0} ds' e^{-\frac{m_c^2}{M^2} - \frac{m_u^2}{M'^2}} \rho_{\mu}^\text{pert}(s, s', q^2) + \hat{\Pi}_{\mu\nu}^{\text{non-pert}}(q^2) \right\}, \tag{30} \]
where $s_0$ and $s'_0$ are continuum thresholds in $D_2[D_{s2}]$ and $D$ channels, respectively and we have used the quark-hadron duality assumption.

3 Numerical Results

In this part, we numerically analyze the obtained sum rules for the strong coupling form factors in the previous section and search for the behavior of those couplings with respect to $Q^2 = -q^2$. The values of the strong coupling form factors at $Q^2 = -m^2_{\pi[K]}$ give the strong coupling constants whose values are then used to find the decay rate and branching ratio of the strong transitions under consideration. To go further, we use some input parameters presented in Table 1.

| Parameters | Values |
|------------|--------|
| $m_c$ | $(1.275 \pm 0.025)$ GeV [27] |
| $m_d$ | $4.8^{+0.5}_{-0.3}$ MeV [27] |
| $m_u$ | $2.3^{+0.7}_{-0.5}$ MeV [27] |
| $m_s$ | $95 \pm 5$ MeV [27] |
| $m_{D(2460)}$ | $(2462.6 \pm 0.6)$ MeV [27] |
| $m_{D_{s2}(2573)}$ | $(2571.9 \pm 0.8)$ MeV [27] |
| $m_D$ | $(1869.62 \pm 0.15)$ MeV [27] |
| $m_{\pi}$ | $(139.57018 \pm 0.00035)$ MeV [27] |
| $m_K$ | $(493.677 \pm 0.016)$ MeV [27] |
| $f_{D(2460)}$ | $0.0228 \pm 0.0068$ [14] |
| $f_{D_{s2}(2573)}$ | $0.023 \pm 0.0011$ [15] |
| $f_D$ | $206.7 \pm 8.9$ MeV [27] |
| $f_{\pi}$ | $130.41 \pm 0.03 \pm 0.20$ MeV [27] |
| $f_K$ | $156.1 \pm 0.2 \pm 0.8 \pm 0.2$ MeV [27] |
| $\langle \alpha_s G^2 \rangle$ | $(0.012 \pm 0.004)$ GeV$^4$ [28] |

Table 1: Input parameters used in calculations.

The next task is to find the working regions for the auxiliary parameters $M^2$, $M'^2$, $s_0$ and $s'_0$. Being not physical parameters, the strong coupling form factors should roughly be independent of these parameters. In the case of the continuum thresholds, they are not completely arbitrary but are related to the energy of the first excited states with the same quantum numbers as the considered interpolating fields. From numerical analysis, the working intervals are obtained as $7.6[8.5]$ GeV$^2 \leq s_0 \leq 8.8[9.4]$ GeV$^2$ and $4.7$ GeV$^2 \leq s'_0 \leq 5.6$ GeV$^2$ for the strong vertex $D_2^*D\pi[D_{s2}^*DK]$. In the case of Borel mass parameters $M^2$ and $M'^2$, we choose their working windows such that they guarantee not only the pole dominance but also the convergence of the OPE. If these parameters are chosen too large, the convergence of the OPE is good but the continuum and higher state contributions exceed the pole contribution. On the other hand if one chooses too small values, although the pole dominates the higher state and continuum contributions, the OPE have a poor convergence. By considering these conditions we choose the windows $8 \text{ GeV}^2 \leq M^2 \leq 16 \text{ GeV}^2$ and
4 GeV² ≤ M² ≤ 10 GeV² for the Borel mass parameters. Our analysis shows that, in these intervals, the dependence of the results on the Borel parameters are weak.

Now we proceed to find the variations of the strong coupling form factors with respect to Q². Using the working regions for the auxiliary parameters we observe that the following fit function well describes the strong coupling form factors in terms of Q²:

\[ g_{D_2^*D\pi[D_2^*DK]}(Q^2) = c_1 \exp \left[ -\frac{Q^2}{c_2} \right] + c_3, \]

(31)

where the values of the parameters c₁, c₂ and c₃ are presented in Table 2. From this fit parametrization we obtain the values of the strong coupling constants at Q² = −m²[π[K]] as presented in Table 3. The errors appearing in our results belong to the uncertainties in the input parameters as well as errors in determination of the working regions for the auxiliary parameters.

|       | c₁ (GeV⁻¹) | c₂ (GeV²) | c₃ (GeV⁻¹) |
|-------|------------|-----------|------------|
| g_{D_2^*D\pi}(Q^2) | 5.17 ± 1.50 | 13.21 ± 3.84 | −(0.54 ± 0.16) |
| g_{D_2^*DK}(Q^2) | 6.43 ± 1.92 | 13.31 ± 3.98 | −(0.79 ± 0.24) |

Table 2: Parameters appearing in the fit function of the coupling form factors.

|       | Q² = −m²[π] | Q² = −m²[K] |
|-------|-------------|-------------|
| g_{D_2^*D\pi} | 4.63 ± 1.39 | g_{D_2^*DK} | 5.76 ± 1.84 |

Table 3: Value of the \( g_{D_2^*D_2^*[D_2^*DK]} \) coupling constant in GeV⁻¹ unit.

The final task in present work is to calculate the decay rates and branching ratios for the strong \( D_2^*(2460)⁰ \to D^+\pi⁻ \) and \( D_{s2}^*(2573)⁺ \to D^⁺K⁰ \) transitions. Using the amplitudes of these transitions we find

\[ \Gamma = \frac{|M(p')|^2}{40\pi m_{D_2^*[D_2^*]}^2} |p'|, \]

(32)

where

\[ |M(p')|^2 = \frac{g_{D_2^*D\pi[D_2^*DK]}^2}{3m_{D_2^*[D_2^*]}^4} \left[ \frac{2}{m_{D_2^*[D_2^*]}^2} \sqrt{p'^2 + m_D^2} \right]^4 \]

\[ - \frac{4m_D^2}{3m_{D_2^*[D_2^*]}^2} \left( m_{D_2^*[D_2^*]} \sqrt{p'^2 + m_D^2} - 2m_D^2 \right)^2 + \frac{2m_D^4}{3} \]

(33)

and

\[ |p'| = \frac{1}{2m_{D_2^*[D_2^*]}^2 \sqrt{m_{D_2^*[D_2^*]}^4 + m_D^4 + m_π^4 - 2m_D^2 m_{D_2^*[D_2^*]}^2 m_π^2[K] - 2m_D^2 m_π^2[K] - 2m_{D_2^*[D_2^*]}^2 m_D^2}}. \]

(34)
The numerical values of the decay rates for the transitions under consideration are depicted in Table 4. Using the total widths of the initial particles as \( \Gamma_{D^*_2(2460)^0} = (49.0 \pm 1.3) \text{ MeV} \), \( \Gamma_{D^*_2(2573)^0} = (17 \pm 4) \text{ MeV} \) [27] we also find the corresponding branching ratios that are also presented in Table 4.

| \( D^*_2(2460)^0 \rightarrow D^+\pi^- \) | \( \Gamma(\text{GeV}) \) | \( BR \) |
|-----------------------------------|-----------------|-------|
| \( D^*_2(2460)^0 \rightarrow D^+\pi^- \) | \( 6.27 \pm 1.94 \times 10^{-4} \) | \( 1.27 \pm 0.40 \times 10^{-2} \) |
| \( D^*_{s2}(2573)^+ \rightarrow D^+K^0 \) | \( 3.70 \pm 1.14 \times 10^{-4} \) | \( 2.17 \pm 0.68 \times 10^{-2} \) |

Table 4: Numerical results of decay widths and branching ratios.

Using the following experimental ratio in \( \pi \) channel [27, 29]:

\[
\frac{\Gamma[D^*_2(2460)^0 \rightarrow D^+\pi^-]}{\Gamma[D^*_2(2460)^0 \rightarrow D^+\pi^-] + \Gamma[D^*_2(2460)^0 \rightarrow D^*(2010)^+\pi^-]} = 0.62 \pm 0.03 \pm 0.02, \tag{35}
\]

we also get

\[
\Gamma[D^*_2(2460)^0 \rightarrow D^*(2010)^+\pi^-] = (0.38 \pm 0.12) \times 10^{-3} \text{ GeV} \tag{36}
\]

which leads to

\[
Br[D^*_2(2460)^0 \rightarrow D^*(2010)^+\pi^-] = (7.80 \pm 2.43) \times 10^{-3}. \tag{37}
\]

To sum up, in the present study, we calculated the strong coupling form factors \( g_{D^*_2D\pi} \) and \( g_{D^*_2DK} \) in the frame work of QCD sum rules. Using the obtained working regions for the auxiliary parameters entered the sum rules for the strong form factors, we found the behavior of those form factors in terms of \( Q^2 \). Using \( Q^2 = -m_{\pi[K]}^2 \), we also found the values of the strong coupling constants \( g_{D^*_2D\pi} \) and \( g_{D^*_2DK} \) which have then been used to calculate the decay widths and branching ratios of the strong \( D^*_2(2460)^0 \rightarrow D^+\pi^- \), \( D^*_2(2460)^0 \rightarrow D^*(2010)^+\pi^- \) and \( D^*_{s2}(2573)^+ \rightarrow D^+K^0 \) transitions.
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