The presence of strange quark pairs in the sea may have a significant impact of the pattern of chiral symmetry breaking: in particular large differences can occur between the chiral limits of two and three massless flavours (i.e., whether $m_s$ is kept at its physical value or sent to zero). This may induce problems of convergence in three-flavour chiral expansions. To cope with such difficulties, we introduce a new framework, called Resummed Chiral Perturbation Theory. We exploit it to analyse $\pi\pi$ and $\pi K$ scatterings and match them with dispersive results in a frequentist framework. Constraints on three-flavour chiral order parameters are derived.

1. Two chiral limits of interest
Because of its intermediate mass, the strange quark has a special status in low-energy QCD. It is light enough to allow for a combined expansion of observables in powers of $m_u, m_d, m_s$ around the $N_f = 3$ chiral limit (meaning 3 massless flavours): $m_u = m_d = m_s = 0$. But it is sufficiently heavy to induce significant changes from the $N_f = 3$ limit to the $N_f = 2$ limit: $m_u = m_d = 0$ and $m_s$ physical. Each limit can engender its own version of Chiral Perturbation Theory ($\chi$PT). In the $N_f = 2$ limit, the pions are the only degrees of freedom, whereas $N_f = 3$ $\chi$PT deals with pions, kaons and $\eta$. This second version of $\chi$PT is richer, discusses more processes in a larger range of energy, but contains more unknown low-energy constants (LECs) and may have a slower convergence. The details of the connection between the two theories remain under debate.

Indeed, due to $s\bar{s}$ sea-pairs, order parameters such as the quark condensate and the pseudoscalar decay constant, $\Sigma(N_f) = -\lim_{N_f} \langle \bar{u}u \rangle$ and $F^2(N_f) = \lim_{N_f} F^2_\pi$, can reach significantly different values in the two chiral limits ($\lim_{N_f}$ denoting the chiral limit with $N_f$ massless flavours) [1]. An illustration is provided by the quark condensate:

$$\Sigma(2) = \Sigma(2; m_s) = \Sigma(2; 0) + m_s \frac{\partial \Sigma(2; m_s)}{\partial m_s} + O(m_s^2)$$

$$\Sigma(3) + m_s \lim_{m_u, m_d \to 0} \int d^4x \langle 0|\bar{u}u(x)\bar{s}s(0)|0\rangle + O(m_s^2)$$

Here, $s\bar{s}$-pairs are involved through the two-point correlator $\langle (\bar{u}u)(\bar{s}s) \rangle$, which violates the Zweig rule in the vacuum (scalar) channel. One expects [1] that this effect should suppress order parameters when $m_s \to 0$: $\Sigma(2) \geq \Sigma(3)$ and $F^2(2) \geq F^2(3)$. Since the quark condensate(s) and the pseudoscalar decay constant(s) are the leading-order LECs for the two versions of $\chi$PT, a strong decrease from $N_f = 2$ to 3 would have an impact on the structure of the two theories.
2. Two- and three-flavour chiral expansions

A few years ago, the E865 collaboration provided new data on \( K_{\ell 4} \) decays \[2\]. Building upon the dispersive analysis of \( \pi\pi \) scattering \[3\], we extracted the two-flavour order parameters \[4\]:

\[
X(2) = (m_u + m_d)\Sigma(2)/(F_\pi^2 M_\pi^2) = 0.81 \pm 0.07 \quad Z(2) = F^2(2)/F_\pi^2 = 0.89 \pm 0.03 \quad (3)
\]

A different analysis, with an additional theoretical input from the scalar radius of the pion, led to an even larger value for \( X(2) \) \[5\]. The situation is somewhat modified by new data from the NA48 collaboration \[6\], which show some discrepancy with the E865 phase shifts in the higher end of the allowed phase space. The role of isospin breaking corrections is under discussion currently. The preliminary values of the phase shifts \[6\] tend to increase the value of the \( I = J = 0 \) \( \pi\pi \) scattering length, and to decrease the value of the two-flavour quark condensate, pushing \( X(2) \) down to 0.7. In any case, one would expect values closer to 1, since \( X(2) \) and \( Z(2) \) monitor the convergence of \( N_f = 2 \) chiral expansions of \( F_\pi^2 M_\pi^2 \) and \( F_\pi^2 \) respectively. Such expansions in powers of \( m_u \) and \( m_d \) only should exhibit smaller NLO corrections (below 10\%) \[4\].

To include \( K^- \) and \( \eta \)-mesons dynamically, one must use three-flavour \( \chi PT \) around the \( N_f = 3 \) chiral limit. Strange sea-quark loops may affect chiral series by damping the leading-order (LO) term, which depends on \( F_\pi^2(3) \) and \( \Sigma(3) \), and by enhancing next-to-leading-order (NLO) corrections, in particular when violating the Zweig rule in the scalar sector. Take for instance:

\[
F_\pi^2 = F^2(3)^2 + 16(m_s + 2m)B_0 \Delta L_4 + 16mB_0 \Delta L_5 + O(m_q^2)
\quad (4)
\]

where \( B_0 = -\lim_{m_u,m_d,m_s \to 0}(\bar{u}u)/F_\pi^2 \), and we have put together NLO low-energy constants and chiral logarithms \( \Delta L_5 = L_5(M_\rho) + 0.67 \cdot 10^{-3}, \Delta L_4 = L_4(M_\rho) + 0.51 \cdot 10^{-3} \) (enhanced by \( m_s \)). If we assume that the LO contribution is numerically dominant (i.e., \( F_\pi^2 = F(3)^2 \) to a very good approximation), we can perform the following manipulations:

\[
\frac{F(3)^2}{F_\pi^2} = \frac{F^2(3)}{F^2(3) + O(m_q^2)} = 1 - 8 \frac{2M_K^2 + M_\pi^2}{F_\pi^2} \Delta L_4 - 8 \frac{M_\pi^2}{F_\pi^2} \Delta L_5 + O(m_q^2) = 1 - 0.51 - 0.04 + O(m_q^2)
\quad (5)
\]

where we have used \( 1/(1 + x) = 1 - x \) and eq. \[4\] at the second step, and the second and third terms of the last equality are obtained using \( L_4(M_\rho) = 0.5 \cdot 10^{-3} \) and \( L_5(M_\rho) = 1.4 \cdot 10^{-3} \) respectively \[7\]. This is clearly in contradiction with the origanal assumption \( F_\pi^2 \simeq F(3)^2 \). The available dispersive estimates of \( L_6 \) \[9\] yield a similar situation for \( F_\pi^2 M_\pi^2 \) citterusum. Therefore, a small positive value of \( L_4(M_\rho) \) or \( L_6(M_\rho) \) is enough to spoil the rapid convergence of \( N_f = 3 \) chiral series and to induce a numerical competition between formal LO and NLO contributions.

3. Constraints from \( \pi\pi \) and \( \pi K \) scatterings

The potentially “large” values of \( L_4 \) and \( L_6 \) lead to a numerical competition between formal LO and NLO contributions in chiral series. To deal with such a situation, we have introduced a framework, called Resummed Chiral Perturbation Theory (Re\( \chi PT \)) \[10\], where we define the appropriate observables to consider and the treatment of their chiral expansion \[10\] \[8\]. It allows for a a resummatiuon of the potentially large effect of the Zweig-rule violating couplings \( L_4 \) and \( L_6 \) \[10\] \[8\]. Since this framework copes with the possibility of a numerical competition between (formal) LO and NLO terms in chiral series, some usual \( O(p^4) \) results are not valid any longer: for instance \( r = m_s/m \) is not fixed by \( M_K^2/M_\pi^2 \) and may vary from 8 to 40.

One can apply this framework to \( \pi\pi \) and \( \pi K \) scatterings, which provide information on \( N_f = 2 \) and \( N_f = 3 \) patterns of chiral symmetry breaking respectively, and in particular on \( r = m_s/m \), the quark condensate \( X(3) = 2m\Sigma(3)/(F_\pi^2 M_\pi^2) \) and the decay constant \( Z(3) = F^2(3)/F_\pi^2 \). One can exploit dispersive relations, such as Roy equations \[3\] and Roy-Steiner equations \[7\], to reconstruct the amplitudes from the phase shifts from threshold up to energies around 1 GeV.
Figure 1. Profiles for the confidence levels of $r = m_s/m$ (left) and $X(3) = 2m\Sigma(3)/(F_\pi^2M_\pi^2)$ (right). In each case, the results are obtained from $\pi\pi$ scattering only (dashed line), $\pi K$ scattering only (dotted line), or both sources of information (solid line).

Matching the dispersive and chiral representations of the amplitude in a frequentist framework provides constraints (in terms of confidence levels) on the main parameters of interest for three-flavour $\chi$PT \cite{10}. Fig. 1 shows the situation for $r = m_s/m$ and $X(3) = 2m\Sigma(3)/(F_\pi^2M_\pi^2)$.

The main impact of $\pi\pi$ scattering consists in constraining $r = m_s/m$: indeed, $\pi\pi$ scattering pins down the two-flavour condensate $X(2)$ rather accurately, which can be related to $r$ through the spectrum of pseudoscalar mesons \cite{1, 8}. The combination of $\pi\pi$ and $\pi K$ scatterings yields:

$$r \geq 14.8, \quad X(3) \leq 0.83, \quad Y(3) \leq 1.1, \quad 0.18 \leq Z(3) \leq 1. \quad [68\% CL]$$ (6)

4. Conclusion

The presence of massive $s\bar{s}$-pairs in the QCD vacuum may induce significant differences in the pattern of chiral symmetry breaking between the $N_f = 2$ and $N_f = 3$ chiral limits. This effect, related to the violation of the Zweig rule in the scalar sector, may spoil the convergence of three-flavour chiral expansions. We introduce Resummed Chiral Perturbation Theory to deal with such a problem, and apply it to our experimental knowledge on $\pi\pi$ and $\pi K$ scatterings. The outcome does not favour the usual picture of a large quark condensation independent of the number of massless flavours. Further experimental information is needed to constrain the pattern of chiral symmetry breaking efficiently and learn more about its variation with $N_f$.

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