Statistical monitoring of models based on artificial intelligence

Anna Malinovskaya
Leibniz University Hannover, Institute of Cartography and Geoinformatics, Germany

Pavlo Mozharovskyi
LTCI, Télécom Paris, Institut Polytechnique de Paris, France

Philipp Otto
Leibniz University Hannover, Institute of Cartography and Geoinformatics, Germany

September 16, 2022

Abstract

The rapid advancement of models based on artificial intelligence demands innovative monitoring techniques which can operate in real time with low computational costs. In machine learning, especially if we consider neural network (NN) learning algorithms, and in particular deep-learning architectures, the models are often trained in a supervised manner. Consequently, the learned relationship between the input and the output must remain valid during the model’s deployment. If this stationarity assumption holds, we can conclude that the NN generates accurate predictions. Otherwise, the retraining or rebuilding of the model is required. We propose to consider the latent feature representation of the data (called “embedding”) generated by the NN for determining the time point when the data stream starts being nonstationary. To be precise, we monitor embeddings by applying multivariate control charts based on the calculation of the data depth and normalized ranks. The performance of the introduced method is evaluated using various NNs with different underlying data formats.

Keywords: Data Depth, Latent Feature Representation, Multivariate Statistical Process Monitoring, Neural Networks, Online Process Monitoring.
1 Introduction

The rapid advancement of artificial intelligence (AI) has encouraged practitioners in various scientific fields to examine the benefits and challenges of Machine Learning (ML) algorithms in their research. For instance, in astronomy, the separation of astrophysical objects such as stars and galaxies can be performed by applying decision trees (Ball et al., 2006); a reliable forecast of energy consumption implementing support vector machines is an extensively studied topic in civil engineering (Gao et al., 2019); in biology, neural networks (NNs) are applied for predicting molecular traits (Angermueller et al., 2016). Due to the emergence of big data and the increased capability of computers, the development of NNs receives particularly broad attention (cf. Chiroma et al., 2018). Large NNs, consisting of many hidden layers, are commonly called “deep learning” models. Their application achieves impressive results in many areas, such as finance, linguistics, or photogrammetry (cf. Aldridge and Avellaneda, 2019; Otter et al., 2020; Heipke and Rottensteiner, 2020).

Considering the hierarchical structure of NNs, the provided data sample in the input layer flows through hidden layers, each of them consisting of neurons – mathematical operations that contain non-linear transformations, returning the prediction in the output layer (Hermans and Schrauwen, 2013). Hidden layers recombine the learned data representation from previous layers, meaning that at each layer, a NN abstracts the irrelevant details of the input and extracts only those features which are useful for producing the output.

Supervised learning for tasks such as regression or classification requires sample data with a known relationship between the input and the output during the model’s training. In this phase, the algorithm aims to minimize the error between the model’s output and its target. After estimating the parameters and completing the test phase, we deploy the model on new data expecting a stable performance. However, over time, the distribution of the input data might change, resulting in samples that follow a different distribution than the training and testing data (Huang et al., 2011). Consequently, the violation of the stationarity assumption between the input and the output could negatively affect the model’s predictive performance.

According to the experimental results obtained by Wang et al. (2019), neurons inside a NN activated by different examples of studied data have two important properties: they are stable and distinguishable. In other words, the data samples that belong to identical classes activate the same neurons in a particular numerical range. Consequently, if the data becomes nonstationary, e.g., comes
from an unknown class, the values at the neurons would deviate, exceeding the usual activation range. Despite the data being spurious, the model would return the best possible prediction. However, its interim representation generated by the NN before the output layer would be different compared to the interim representation of the data that correctly belongs to a predicted class. Commonly represented by a low-dimensional vector, the learned latent feature representation also called “embedding” comprises a dense summary of the incoming sample.

In computer science, a problem when the data distribution changes and makes the current prediction model inaccurate or obsolete is known as “concept drift” (Demšar and Bosnić, 2018), while in statistics, time series whose statistical properties change over time are called “nonstationary”. In addition to those distortions of the learned relationship between the input and the output, another phenomenon called “novelty” can occur, particularly in classification tasks (Garcia et al., 2019). In this case, the data stream starts containing instances of a new class that were not present during the training phase. Consequently, if one or several issues exist, the revision of a model is needed to ensure the correctness of its output. A solution can vary from simply running the entire currently existing training procedure with new data up to completely exchanging the algorithm. Although there are strategies that handle nonstationarity in ML applications without its explicit detection (cf. Gama et al., 2014), here, we concentrate on scenarios where we apply a change point detection technique. As a consequence, this strategy minimizes the cost of redundant model adjustments, while preserving the prediction accuracy.

To perform a real-time inspection and promptly detect changes, online monitoring techniques can be applied, such as univariate or multivariate control charts (cf. Celano et al., 2013; Psarakis, 2015; Perdikis and Psarakis, 2019). This monitoring method represents one of the primary techniques of Statistical Process Monitoring (SPM), testing over time whether unusual sources of variability are present (Montgomery, 2012). The combination of control charts and deep learning algorithms to improve the monitoring procedure is already an existing concept (cf. Psarakis, 2011; Weese et al., 2016; Lee et al., 2019; Zan et al., 2020; Sergin and Yan, 2021; Yeganeh et al., 2022). However, the authors are not aware of the reverse usage of control charts, namely to supervise the quality of NN applications by monitoring input data in terms of their generated embeddings – a field that offers new perspectives for SPM.

The majority of multivariate SPM methods are based on the assumption that the observed process follows a specific distribution. In general case of NNs, however, the distribution of embeddings is unknown. Although the network could be trained in a way such that the embeddings follow a cer-
tain distribution, this approach could restrict applications in practice. Thus, we focus on developing a non-parametric SPM approach by using a data depth-based control chart, making no assumption about the type of a NN and the distribution of embeddings.

Following, after a brief summary of related publications in Section 2 and the description of the major components of a NN, we formulate the considered change point detection problem in Section 3. Afterwards, we discuss a notion of data depth and data depth-based control charts in Section 4. The experimental results of our method are provided in Section 5, followed by the discussion about open research directions in Section 6.

2 Related work

The majority of existing monitoring methods of the data stream processed by an AI-based algorithm rely on labeled instances. A considerable group of techniques applies a sliding window approach, comparing the data distribution of the current window to the reference sample obtained from the training dataset (cf. Bifet and Gavalda, 2007). There are several approaches for monitoring the classification error rate or performance metrics (cf. Klinkenberg and Renz, 1998; Klinkenberg and Joachims, 2000 and Nishida and Yamauchi, 2007), for example, by conducting sequential hypothesis tests based on the ideas of control charts (cf. Gama et al., 2004, Baena-Garcia et al., 2006, Kuncheva, 2009 and Mejri et al., 2017). A two-stage time adjusting control chart developed by Mejri et al. (2021) is designed for monitoring misclassification error rates, updating the control limits at first, and validating the detected changes in the second stage.

Regarding deep learning applications, there is a limited number of studies in the area of developing approaches to detect nonstationary samples. Disabato and Roveri (2019) considered the monitoring of the classification error during the application of convolutional NNs (CNNs), proposing a mechanism for partial retraining of the model after nonstationarity was detected. Priya and Uthra (2021) introduced another drift detection framework for deep learning, where the “adaptive sliding window” technique is applied to the input samples from class-imbalanced data.

In contrast to these approaches, first, we assume that the labeled data are available only during the training and test phases of the model, meaning that as soon as a prediction is produced on the new incoming samples, it is not possible to conclude whether the classification was performed correctly. This scenario of only sparsely available labeled data is realistic for many applications, among them
are malware detection, management of power grids, and remote-sensing (Ditzler et al., 2015). Second, the underlying reason related to the notable performance of the NNs is their generalization ability in complex high-dimensional AI tasks such as object or speech recognition (Goodfellow et al., 2016). For such data formats, the detection of nonstationarity from the original data would not be appropriate due to the excessive number of features. Thus, the dimension must be reduced, so that our proposed monitoring technique is based on embeddings.

3 Detection of change points in NN applications

While there are deep learning methods to detect anomalous observations similarly to control charts, we aim to monitor the deep learning algorithm itself. In this work, we focus on classification problems solved by different types of NNs.

3.1 Architecture of a NN

The main goal of training a NN is to estimate the parameters $\vartheta$ of a highly nonlinear function $f(\cdot, \vartheta) : \mathbb{R}^d \rightarrow \mathbb{R}^v$, mapping a $d$-dimensional input (i.e. observed data) to a $v$-dimensional output (i.e. class labels). In general, the proposed monitoring technique can be also applied to NNs which solve regression tasks by grouping the predicted values into a set of classes. In both cases, the dependent variables are class labels $y_t \in \{1, \ldots, v\}$, where the $v$-dimensional output of the algorithm indicates probabilities for the processed data to belong to the given classes. In supervised learning, we assume that true class labels are known for the training period $t = 1, \ldots, T$ which is used for estimating the parameters $\vartheta$. Thus, $\hat{y}_t = \arg \max f(x_t, \hat{\vartheta})$ for a set of input variables $x_t$ is the prediction of the NN, i.e., the most probable class, where $\hat{\vartheta}$ defines the learned parameters during the training, such that $\hat{y}_t$ coincides with $y_t$ in most cases for all $t = 1, \ldots, T$.

There are several aspects to consider about the network’s architecture when designing it, for instance, the number of hidden layers and neurons, the connections between the layers, and, fundamentally, which type of NN to use. To introduce these essential elements, an example of a feedforward NN (FNN) which defines the basic family of NN models is presented in Figure 1. The network consists of four layers with two hidden layers, where each circle represents a neuron or node that stores a scalar value. Consequently, a layer is a $k_i$-dimensional vector with $k_i$ being the number of nodes contained in the $i$-th layer. Here, the input layer is a $k_1$-dimensional vector, e.g. $x_t \in \mathbb{R}^6$ (colored in
green), and its first neuron is defined as $x_{t, 1}$. In the output layer (colored in red), assuming we have a classification problem with three classes, we would construct a NN so that we obtain probabilities of the input to belong to class 1, class 2, and class 3. To obtain a broader overview of how to design and train a NN, the reader can refer to Goodfellow et al. (2016). As presented in Figure 1, often the interim representation of the sample produced by hidden layers reduces the original dimensionality of the data. Thus, the interim output generated from the high-dimensional data such as images is valuable not only due to the dimensionality reduction but also due to the compression of knowledge about the sample. This latent representation that we previously defined as “embedding” is the object of interest for change point detection procedure.

### 3.2 Change Point Detection

For monitoring the usage of a NN, we propose to use embeddings that usually have a vector form which we denote by $m_t \in \mathbb{R}^k$ with $k$ being the dimension of the hidden layer. This representation is observed every time the NN is applied, i.e., for both historical (training) and new data. Thus, embeddings implicitly depend on $x_t$, but also on the fitted parameters $\hat{\vartheta}$, so that changes associated with the data quality and performance of a NN can be detected.

In succeeding parts, we refer to the set \( \{m_t : t = 1, \ldots, T\} \) as historical data with correctly known labels \( \{y_t : t = 1, \ldots, T\} \) and to \( \{m_i : i = T + 1, T + 2, \ldots\} \) as incoming data instance with the
predicted class label $\hat{y}_i$ obtained from $f(m_i, \hat{\theta})$. Moreover, we consider that the true label of $y_i$ is not available and historical data are stationary. A stationary process means that the embeddings of each class for both the training and test datasets have constant location and scale.

Furthermore, we assume that $m_t$ follows a certain distribution $F_{m_t|y_t=c}$ depending on the true class $y_t$. These conditional distributions $F_{m_t|y_t=c}$ are denoted by $\Xi_c$ for all classes $c \in \{1, \ldots, v\}$. Based on historical data, it is possible to estimate the distribution $\Xi_c$ empirically that can be used to determine whether a possible change in the data stream occurred, i.e., whether the current observation $m_i$ is generated from a different distribution than $\Xi_{\hat{y}_i}$. Hence, we define a change point $\tau$ to arise if

$$m_i \sim \begin{cases} 
\Xi_{\hat{y}_i} & \text{if } i < \tau \\
\Xi_b & \text{if } i \geq \tau 
\end{cases}.$$ 

In other words, a change point $\tau$ is the timestamp when the analysis of the embedding generated from the incoming data indicates that the data sample does not belong to the class that was predicted by the NN. Consequently, we can regard this situation as a change in the class definition and formulate a sequential hypothesis test of each sample $i$ for detecting changes in the NN application as follows

$$H_{0,i} : \Xi_{\hat{y}_i} = \Xi_{y_i} \quad \text{against} \quad H_{1,i} : \text{there is a location shift and/or a scale increase in } \Xi_{\hat{y}_i}.$$

It is important to note that the alternative hypothesis could be true due to (1) misclassification by the model, i.e., $\hat{y}_i \neq y_i = a$, where $a \in \{1, \ldots, v\}$, leading to $\Xi_{\hat{y}_i} \neq \Xi_a$, or (2) nonstationarity of the data stream, i.e., $\hat{y}_i \neq y_i = b$ with $b \notin \{1, \ldots, v\}$ but $\Xi_{\hat{y}_i} \neq \Xi_b$ because $i \geq \tau$. For a reliable change point detection, it is essential to distinguish between these cases accurately. A practical solution to resolve this issue is presented and discussed in the experimental part, Section 5.3.4.

To test repeatedly whether the process is in-control over time, i.e., whether the data assigned to a particular class does not deviate from the rest in this class, we can apply a multivariate control chart. Since $\{\Xi_c : c = 1, \ldots, v\}$ are unknown and can be estimated only empirically, we need a nonparametric monitoring technique that relaxes any distributional assumptions.

A considerable number of proposed multivariate nonparametric (also called distribution-free) charts are based on the idea of ranks. The rank-based approaches can be classified into control charts using longitudinal ranking and those applying cross-component ranking (Qiu, 2014). Considering the first group, Qiu (2014) defines componentwise longitudinal ranking (cf. Boone and Chakraborti, 2012), spatial longitudinal ranking (cf. Zou et al., 2012), and longitudinal ranking by data depth (cf.
Liu and Singh, 1993). While the charts of the first two subgroups are moment-dependent approaches, the advantage to apply control charts based on the notions of data depth is their flexibility of not posing any moment requirements (cf. Liu and Singh, 1993). Especially considered are those depths that are of the geometric, combinatorial type, for example, Simplicial depth or Halfspace depth introduced in Section 4.1. Moreover, control charts based on the data depth offer simultaneous monitoring of a possible location shift or scale increase in the process.

In general, depth functions discussed in this work are affine invariant and satisfy useful axioms such as monotonicity, convexity (apart from Simplicial depth) and continuity (cf. Mosler and Mozharovskyi, 2022). These characteristics suit well our problem, therefore, to perform online monitoring we use a nonparametric control chart based on a notion of data depth.

4 Monitoring framework based on the data depth and nonparametric multivariate control charts

The application of data depth in quality control was proposed by Liu and Singh (1993), resulting in the design of Shewhart-type multivariate nonparametric control charts based on the Simplicial depth (Liu, 1990, 1995). Usually, the control chart statistic is derived batch-wise. Hence, we compare the performance of the charts when \( n = 1 \) observations at each point of time against the control charts having the batch sizes \( n = 3 \) and \( n = 5 \).

According to recent publications on data depth-based control charts (cf. Cascos and López-Díaz, 2018; Barale and Shirke, 2019; Pandolfo et al., 2021), the careful choice of the data depth is crucial for satisfactory monitoring performance. Thus, we compare several notions of data depth and discuss the compromise between their effectiveness and computational effort in Section 5.

4.1 Notion of data depth

A data depth is a concept for measuring the centrality of a multivariate observation \( m_i \) with respect to a given reference sample \( R_c = \{m_t := 1, \ldots, |R|, y_t = c\} \), where \( |R| \) defines its size which is the same for all considered classes. In other words, it creates a center-outward ordering of points in the Euclidean space of any dimension (Liu et al., 2006). There are various notions of data depth, each of them providing a distinctive center-outward ordering of sample points in a multidimensional space.
In this work, we consider four notions of data depth, namely Halfspace depth, Mahalanobis depth, Projection depth, and Simplicial depth.

First, the Halfspace depth (originally known as “Tukey depth”) introduced by Tukey (1975) and further developed by Donoho and Gasko (1992) is defined as the smallest number of data points in any closed halfspace with boundary hyperplane through \( m_i \) (Struyf and Rousseeuw, 1999). That is,

\[
D_{HC}(m_i, R_c) = \frac{1}{|R|} \min_{\|p\|=1} |\{ b : \langle p, m_b \rangle \geq \langle p, m_i \rangle \}|,
\]

where \(|\cdot|\) denotes the cardinality of the set \( B \) with \( m_b \in R_c \), \( p \) are all possible directions with \( \|p\| = \sqrt{\langle p, p \rangle} \) being the Euclidean norm and \( \langle \cdot, \cdot \rangle \) the inner product. In our work, we consider its robust version HD\(_r\) that is proposed by Ivanovs and Mozharovskyi (2021) and calculated approximately, offering some advantages in being strictly positive and continuous beyond the convex hull of the observed samples.

Second, we consider the Mahalanobis depth that is based on the Mahalanobis distance (cf. Mahalanobis, 1936). It is derived as

\[
D_{MC}(m_i, R_c) = \frac{1}{1 + (m_i - \mu_m)' \Sigma^{-1} (m_i - \mu_m)},
\]

where \( \mu_m \) is the mean vector of the embeddings in the reference sample and \( \Sigma^{-1} \) is the covariance matrix, estimated by the sample mean and the sample covariance matrix, respectively.

Third, the Projection depth proposed by Zuo and Serfling (2000) is specified as

\[
D_{PC}(m_i, R_c) = \left( 1 + \sup_{\|p\|=1} \frac{|\langle p, m_i \rangle - \text{med}(\langle p, R_c \rangle)|}{\text{MAD}(\langle p, R_c \rangle)} \right)^{-1}
\]

with \( \langle p, m_i \rangle \) denoting the inner product and the projection of \( m_i \) to \( p \) if \( \|p\| = 1 \). The notation \( \text{med}(E) \) defines the median of a univariate random variable \( E \) and \( \text{MAD}(E) = \text{med}(|E - \text{med}(E)|) \) is the median absolute deviation from the median. As the exact computation of the Projection depth is possible only at a very high computational cost (cf. Mosler and Mozharovskyi, 2022), we use the algorithms that enable its calculation approximately. Dyckerhoff et al. (2021) provide the implementation and comparison of various algorithms. In our work, we study the performance of control charts based on three different algorithms to compute Projection depth. In particular, we consider coordinate descent (PD\(_1\)), Nelder-Mead (PD\(_2\)) and refined random search (PD\(_3\)).

Fourth, we calculate the Simplicial depth (Liu, 1990) as

\[
D_{SC}(m_i, R_c) = \left( \frac{|R|}{d + 1} \right)^{-1} \sum_{y \in \mathbb{R}_c} I_S(m_i \mid x \in R_c, y = c)(m_i),
\]
where $S(m_t | t \in R_c, y_t = c)$ defines the open simplex consisting of vertices $\{m_{t,1}, \ldots, m_{t,d+1}\}$ from all observations $t$ in the reference sample $R_c$. The $\Diamond$ notation means that we validate all possible combinations to construct an open simplex with $(d + 1)$ vertices. By $I_A(x)$ we specify the indicator function on a set $A$ returning 1 if $x \in A$ and 0 otherwise. Both Simplicial (SD) and Mahalanobis (MD) Depths are computed with algorithms provided by Pokotylo et al. (2019).

Related to the classification problem of multivariate data, there exist depth-based classifiers (cf. Vencálek, 2017) such as depth-vs-depth plot (DD-plot) (Li et al., 2012) or DD-alpha procedure (Lange et al., 2014). Also, the field of outlier or anomaly detection is a widespread area for data depth usage (cf. Dang and Serfling, 2010; Baranowski et al., 2021). Combining these two perspectives, we apply a data depth-based Shewhart-type $r$ control chart for $n = 1$ and $Q$ control chart for $n > 1$, both developed by Liu (1995), for detecting nonstationarity in the data stream.

### 4.2 The $r$ and $Q$ control charts

A typical control chart is a graphical technique for monitoring processes, recording the performance of quality characteristics of interest over a sample number or time (Kan, 2003). The process is said to be in-control if the test statistic is plotted inside the upper and lower limits ($UCL$ and $LCL$, respectively). As soon as unusual sources of variability occur, points tend to appear outside this area. This procedure is called “signaling”, informing about the out-of-control state of the process and the need for its investigation.

Usually, the application of control charts is divided into Phase I and Phase II. In Phase I, we collect the reference data, examine its quality and verify the process stability, then estimate model parameters if applicable and derive the values for the control limits (Jones-Farmer et al., 2014). The data in Phase I does not have to coincide with the training data of a NN but could also be a subset. That is, the sets $R_c \subseteq \{m_t : t = 1, \ldots, T, y_t = c\}$ of size $|R|$ create the Phase I data. Successively, in Phase II, the control chart statistic is plotted for each embedding $m_i$ with $i > T$. In Figure 2, we display the introduced periods and sets.

Considering the $r$ and $Q$ control charts proposed by Liu (1995), both schemes are based on ranks of multivariate observations which are obtained by computing data depth. To determine whether $m_i$ belongs to $\Xi_c$, we use the following control chart statistic

$$r^c(m_i) = \frac{|\{D^c(m_t) \leq D^c(m_i) : t \in R_c, y_t = c\}|}{|\{t \in R_c : y_t = c\}|}$$
Fig. 2: Summary of the introduced notation and data subdivision.

that defines the rank of the observed depth, related to the observations in the reference sample with the class $c$. Thus, the $r$ control chart monitors the values of $r^c$ over time whereas the test statistic of $Q$ control chart is the average of consecutive subsets of $r^c(m_i)$, being

$$Q^c(m_i) = \frac{1}{n} \sum_{j=1}^{n} r^c(m_{ij})$$

with the batch size $n$ (Liu, 1995). Considering the interpretation of ranks, we can state that $r^c(m_i)$ reflects how outlying $m_i$ is with respect to the reference sample $m$. If $r^c(m_i)$ is high, then there is a considerable proportion of data in the reference sample that is more outlying compared to $m_i$ (Liu, 1995). The same explanation applies to the control statistic $Q^c(m_i)$.

Regarding the control limits, there is no need to introduce the $UCL$ as $r^c$ belongs to the continuous interval $[0;1]$. However, the procedure to determine the lower control limit varies for the presented control charts. In case of the $r$ control chart, the $LCL$ coincides with the significance level of the hypothesis test, here defined as $\alpha$. The choice of $\alpha$ depends on the specification of average run length ($ARL$) – the expected number of monitored data points required for the control chart to produce a signal (Stoumbos et al., 2001). Considering the Shewhart-type control charts, the recipr-
cal of $ARL$ corresponds to the false alarm rate ($FAR$) in the in-control state of a process. Technically speaking, since the $r$ control chart is a Shewhart control chart, $FAR = \alpha$, where $\alpha$ is interpreted as the probability of a false alarm (Stoumbos et al., 2001).

In case of the $Q$ control chart, we can compute the control limit using the equation

$$LCL = \frac{(n!\alpha)^{1/n}}{n},$$

given that $\alpha \leq \frac{1}{n!}$ (Stoumbos et al., 2001). However, if $\alpha > \frac{1}{n!}$, the LCL has to be computed numerically by solving the polynomial equation provided in Liu (1995). In general, the process is considered to be out-of-control if $r^c(m_i) \leq \alpha$ or $Q^c(m_i) \leq LCL$.

According to Liu (1995), both control charts can be applied with affine-invariant notions of data depth, explicitly mentioning Simplicial depth, Mahalanobis depth,, and Halfspace depth. Since Projection depth is also affine-invariant (cf. Mosler, 2013), the $r$ and $Q$ control charts can be directly applied with the introduced data depth functions.

5 Experiments

In the following chapter, we analyze the proposed monitoring framework on three experiments whose complexity is gradually increasing. The discussion begins with the description of the empirical study in Section 5.1 and is followed by a short introduction of the considered performance measures in Section 5.2. Further, the construction of reference samples and the misclassification effect are analyzed in Section 5.3. Afterward, we compare the performance of chosen batch sizes in Section 5.4. In Section 5.5 we study the computation time of respective data depth functions.

5.1 Description of Experiments

In all experiments we consider balanced datasets to train the NNs, meaning that each class is represented by the same number of samples. We start with a binary classification of sonar data performed with FNN, proceeding with a four-class classification of questions, using a NN with a Long Short-Term Memory (LSTM) layer. We conclude the study with the ten-class classification of images, applying CNN. More details of each experiment are provided in Appendix A – C.

Considering the required data, we obtain embeddings for reference samples in Phase I from the training data. The embeddings of the data points used for testing the NN compose the in-control part
Table 1: Summary of the experiments. The data size information corresponds to the total number of samples from all classes.

Regarding the calculation of data depth, we compute the Mahalanobis and Simplicial depths exactly. However, due to the exponential increase in the computational complexity with growing $k$ (cf. Mosler and Mozharovskyi, 2022), we evaluate Simplicial depth for Experiment 1 only. Considering the robust Halfpace and Projection depths, the quality of their approximations highly depends on the parameter values. Thus, in both cases, we set the number of directions to be $10^4$ in Experiments 1 and 2, and $10^5$ in Experiment 3. Regarding the parameter values that are involved in algorithms for computing Projection depth, we follow the recommendations provided in Dyckerhoff et al. (2021).

5.2 Performance measures

A well-operating control chart has a low number of false alarms (i.e., situations when the process is in-control but the control chart falsely signals a change) and a high rate of correctly detected spurious data points (i.e., situations when the process is out-of-control and the control chart correctly signals the change). Usually, for assessing the performance of control charts, the focus lies on measuring and comparing ARL to a signal. That means, when the process remains in-control, the ARL to a false alarm is observed. When the process is out-of-control, the ARL measures the speed of a control chart
to detect the occurred change. Alternatively, we can consider the False Alarm Rate (FAR) to evaluate
the performance in Phase I, when the process is claimed to be in-control. In Phase II, we consider the
Signal Rate (SR) for the in-control part and the Correct Detection Rate (CDR) for the out-of-control
part.

All three metrics are computed identically: To account for a possible discrepancy between the
class proportions of the predicted data in Phase II, we apply the weighted mean of the occurred signals,
weighting the number of data points in each class within the observed period. In case of FAR, we use
the sample mean because the class sizes are identical. If a tested control chart operates as desired, then
FAR of Phase I equals the chosen probability of a false alarm \( \alpha \). In Phase II, ideally we would expect
SR to be similar to FAR for the in-control samples (neglecting potential misclassification), while the
CDR should be as large as possible for the out-of-control samples. If the CDR is low, we conclude
that the control chart does not accomplish its primary purpose – to detect nonstationarity in a data
stream.

5.3 Results: Choice of reference sample

In this part, we inspect the influence of methods to select data points that form a reference sample,
and whether the increase in its size contributes to a better monitoring performance.

5.3.1 Formation of a reference sample based on model’s confidence

For the control chart to be a useful monitoring technique, its derivation should be based on a well-
chosen reference sample. Referring to Section 3.1, the application of the NN on the incoming data
in the first place results not only in the embeddings but also in the probabilistic predictions. That
means, we construct reference samples by choosing \(|R|\) data points for each class which obtained the
highest probability measure to correctly be a part of a class \( c \). Also, we investigate how the size of
reference samples affects performance. Thus, we arbitrarily selected three different sizes \(|R|\) for each
experiment. The obtained results are provided in Table 2.

Studying the outcomes of Phase I in Experiment 1, we notice that although we chose \( \alpha = 0.05 \),
FAR sometimes equals 0.06. This happens due to rounding, thus, FAR = 0.06 for |R| = 50 and
|R| = 70 coincides with the expected value. Consequently, the expected value of FAR is reached for
each notion of data depth except Simplicial depth. The reason for that is the large number of data
points in the reference sample of class 2 with SD(\( m_c \)) = 0. That can be explained by Simplicial depth

14
| Evaluation | Size | Phase | Observed process | Metric | MD | SD | HD | PD₁ | PD₂ | PD₃ |
|------------|------|-------|-----------------|--------|----|----|----|-----|-----|-----|
| Experiment 1 | 50 | I | In-control | FAR | 0.06 | 0.03 | 0.06 | 0.06 | 0.06 | 0.06 |
| 50 | II | In-control | SR | 0.11 | 0.03 | 0.39 | 0.03 | 0.05 | 0.05 |
| 50 | II | Out-of-control | CDR | 0.70 | 0.00 | 0.90 | 0.13 | 0.13 | 0.13 |
| 60 | I | In-control | FAR | 0.05 | 0.03 | 0.05 | 0.05 | 0.05 | 0.05 |
| 60 | II | In-control | SR | 0.16 | 0.03 | 0.21 | 0.05 | 0.05 | 0.05 |
| 60 | II | Out-of-control | CDR | 0.90 | 0.00 | 0.93 | 0.67 | 0.73 | 0.73 |
| 70 | I | In-control | FAR | 0.06 | 0.03 | 0.06 | 0.06 | 0.06 | 0.06 |
| 70 | II | In-control | SR | 0.00 | 0.03 | 0.11 | 0.00 | 0.00 | 0.00 |
| 70 | II | Out-of-control | CDR | 0.87 | 0.00 | 0.90 | 0.83 | 0.83 | 0.83 |
| Experiment 2 | 400 | I | In-control | FAR | 0.05 | / | 0.05 | 0.05 | 0.05 | 0.05 |
| 400 | II | In-control | SR | 0.69 | / | 0.70 | 0.39 | 0.45 | 0.42 |
| 400 | II | Out-of-control | CDR | 0.88 | / | 0.90 | 0.47 | 0.60 | 0.50 |
| 500 | I | In-control | FAR | 0.05 | / | 0.05 | 0.05 | 0.05 | 0.05 |
| 500 | II | In-control | SR | 0.59 | / | 0.60 | 0.26 | 0.39 | 0.34 |
| 500 | II | Out-of-control | CDR | 0.78 | / | 0.78 | 0.38 | 0.57 | 0.43 |
| 600 | I | In-control | FAR | 0.05 | / | 0.05 | 0.05 | 0.05 | 0.05 |
| 600 | II | In-control | SR | 0.38 | / | 0.39 | 0.21 | 0.25 | 0.26 |
| 600 | II | Out-of-control | CDR | 0.57 | / | 0.63 | 0.25 | 0.43 | 0.33 |
| 2000 | I | In-control | FAR | 0.05 | / | 0.05 | 0.05 | 0.05 | 0.05 |
| 2000 | II | In-control | SR | 0.51 | / | 0.54 | 0.48 | 0.47 | 0.48 |
| 2000 | II | Out-of-control | CDR | 0.92 | / | 0.93 | 0.91 | 0.91 | 0.89 |
| Experiment 3 | 3000 | I | In-control | FAR | 0.05 | / | 0.05 | 0.05 | 0.05 | 0.05 |
| 3000 | II | In-control | SR | 0.43 | / | 0.46 | 0.41 | 0.39 | 0.39 |
| 3000 | II | Out-of-control | CDR | 0.88 | / | 0.90 | 0.87 | 0.86 | 0.84 |
| 4000 | I | In-control | FAR | 0.05 | / | 0.05 | 0.05 | 0.05 | 0.05 |
| 4000 | II | In-control | SR | 0.34 | / | 0.37 | 0.33 | 0.31 | 0.30 |
| 4000 | II | Out-of-control | CDR | 0.79 | / | 0.83 | 0.78 | 0.78 | 0.73 |

**Table 2**: Performance of $r$ control charts ($\alpha = 0.05$) in the presented experiments with $R$ being the predicted class. Green rows highlight Phase I false alarm rates (FAR), and violet define Phase II signal and correct detection rates (SR and CDR), respectively. The underlined numbers indicate the best result for a particular monitoring period excluding the results of Simplicial Depth SD due to the case that $FAR \neq \alpha$. 
assigning zero to every point in the space that lies outside the convex hull of the sample (Francisci et al., 2019). In Figure 3, we observe the dispersion of the data. All out-of-control samples received the predictions of class 1, meaning that their depth is determined with respect to the blue point cloud. Overall, the control charts operate well with $|R| = 70$. Although control charts based on $\text{HD}_r$ obtain higher $SR$ than other depths, it can be deployed in practice due to reliable detection of nonstationarity, achieving $CDR \geq 0.90$. Alternatively, one can choose $\text{MD}$ with $|R| = \{60, 70\}$ because of lower $SR$ and comparably high $CDR$.

Considering the results of Experiment 2, we notice decreasing $SR$s when the size of the reference sample is increasing. Nevertheless, the metric $SR$ remains substantially high for considered control charts. Although the further increase of reference samples might improve monitoring during the in-control state, it would negatively affect the detection of spurious data. As we can observe in case of $\text{HD}_r$, the $CDR$ is reduced by 27%, changing from $|R| = 400$ to $|R| = 600$. Thus, to improve
Fig. 4: An example of the $r$ control chart based on PD$_2$ using the data from Experiment 3, $|R| = 4000$. Colors correspond to the phases illustrated in Figure 2 (green: Reference sample, purple: Phase II, in-control part, red: Phase II, out-of-control part). The dashed line represents the control limit $\alpha = 0.05$, the signals are shown in dark red. Additionally, the misclassified samples in Phase II (in-control) are indicated with a star.

the performance of Phase II in Experiment 2, a paired control chart or another procedure to decide confidently when the data points are in-control is required.

Regarding Experiment 3, we notice promising results for the Projection depth. Figure 4 displays an example of a control chart based on PD$_2$. First of all, two algorithms achieve $CDR = 0.91$ with the reference sample size $|R| = 2000$. However, it should be noticed that SRs stay high. Also, similarly to Experiment 2, there is a decreasing tendency of the SR with a growing reference sample size. In contrast, the decline of $CDR$ is less extreme. Hence, it is best to agree on such a reference sample size that slightly decreases $CDR$ and, at the same time, improves the performance of the control chart during the in-control state.

To better understand nuances of the proposed monitoring framework, we investigate why the SRs in Experiments 2 and 3 are particularly high. Possible reasons could be either a high number of signals due to misclassification or larger dispersion of the test data compared to the reference samples. These points are explored in the subsequent part.
| Evaluation | Size $|R|$ | Metric | MD  | SD  | HD  | PD  | PD  | PD  |
|------------|--------|--------|-----|-----|-----|-----|-----|-----|
| Experiment 2| 400    | SR     | 0.94/0.93 | 0.60 | 0.80 | 0.67 |
|            | 500    | SR     | 0.91/0.82 | 0.41 | 0.74 | 0.72 |
|            | 600    | SR     | 0.85/0.87 | 0.41 | 0.60 | 0.63 |
| Experiment 3| 2000   | SR     | 0.97/1.00 | 0.95 | 0.95 | 0.95 |
|            | 3000   | SR     | 0.94/0.95 | 0.92 | 0.91 | 0.91 |
|            | 4000   | SR     | 0.88/0.90 | 0.86 | 0.83 | 0.83 |

Table 3: Summary of signal rates related to misclassified samples in Phase II, in-control part. The underlined numbers indicate the best result for a particular reference sample size. Here, the SR values correspond to the fraction of misclassified data, being 87 data points in Experiment 2 and 958 data points in Experiment 3.

5.3.2 Effect of misclassification and data dispersion

As Experiment 1 has only 4 misclassified test samples, we examine the effect of misclassification on Experiments 2 and 3, with 87 and 958 misclassified data points, respectively. The results are summarized in Table 3, where the SRs based only on misclassified data samples are shown.

Overall, we observe that the quality of a model based on AI affects the performance of the monitoring procedure, frequently leading to an alarm when a sample was misclassified. Particularly when MD or HD$_r$ is applied, the majority of misclassified data points obtain a low rank, leading to signals and contributing to a higher SR. At the same time, the results in Experiment 2 show that there can exist a substantial fraction of misclassified samples that are treated as in-control observations. For instance, the lowest SR for PD$_1$ was obtained with $|R| = \{500, 600\}$, being 41% that corresponds to 36 data points. This fact supports the expectation of a lower contribution to SR through wrong predictions as hypothesized in Section 3.2. That means, depending on the chosen framework, the misclassification might have only a moderate negative effect on the monitoring procedure. Hence, misclassification does not ultimately cause a signal.

To investigate another reason of high SR values, we visualize the in-control data in Figures 5 and 6 of Experiments 2 and 3, respectively. We use Radial Coordinate visualization (Radviz), where the variables are referred to as anchors, being evenly distributed around a unit circle (cf. Hoffman et al., 1999; Caro et al., 2010; Abraham et al., 2017). Their order is optimized, so that highly correlated variables are placed next to each other. Correspondingly, data points are projected to positions that are close to the variables which have higher influence on them.
Comparing both plots, we notice a considerable difference in the location of reference samples representing the corresponding classes. The reason is that the embeddings were generated differently in Experiment 3, where, in contrast to Experiment 2, each class is determined by all available variables, i.e., neurons. In general, a high dispersion of test data can be noticed in Experiment 2, leading to more signals during the monitoring. In Experiment 3, the subset of data from Phase II (in-control) that is not placed at any class area is considerably smaller, explaining lower $SR$ values than in Experiment 2. In both cases, the reference sample data and the test data indicate different statistical properties, rising the question about the reliable choice of reference samples and, generally, about the division of data into training and testing sets.
Fig. 6: Visualization of the Reference Samples (RS) $\{R_1, \ldots, R_{10}\}$ with $|R| = 2000$ and the data from Phase II, in-control part (test data) from Experiment 3. $V_1, \ldots, V_{16}$ define anchors which correspond to neurons that produced embeddings $m_i \in \mathbb{R}^{16}$. Density contour plots outline respective classes. To improve the visualization, we rescaled the circle, choosing a radius of 0.4.
To conclude, the choice of data depth as well as of the size $|R|$ significantly influences the performance. In all three experiments, $HD_r$ achieves the highest $CDR$. However, to obtain good performance during the entire monitoring period in tasks similar to Experiments 1 and 3, it is advisable to consider Mahalanobis Depth and Projection depth, especially $PD_2$.

5.3.3 Random formation of a reference sample

To examine the influence of how the reference samples are formed, we conducted a Monte Carlo study with the data from Experiment 3. Here, we created reference samples for each class by randomly picking data points from correctly classified training data without considering probabilistic outcomes of the NN.

The obtained results based on 10 runs are summarized in Table 4. For each choice of the data depth notion, the standard deviation does not exceed 0.01, meaning that the small number of iterations is sufficient for our investigation. The control charts based on $HD$ achieve the highest $CDR$ among all proposed control charts. Furthermore, the control charts based on $PD_2$ and $PD_3$ are more reliable during Phase II (In-control), resulting in a $SR$ of 0.18.

In contrast to the results in Section 5.3.1, we observe low fluctuation in performance with the changing size $|R|$. However, the $CDR$ results are considerably lower than in the previous application. Hence, although satisfactory $SR$s are reached, the control charts with randomly created reference samples are insufficient for detecting out-of-control data.

5.3.4 Reference sample in form of merged classes

In this analytical part, we monitor the data in each of three experiments by creating the reference samples as a combination of single reference samples that were constructed in Section 5.3.1. The benefit of this approach is that no predictions are needed, meaning that if the NN model provides an incorrect class label, the possible negative effect of misclassification is excluded. Additionally, the application of merged reference samples implies that if a data point is spurious, it would remain spurious compared to the entire reference data.

Reporting only the strongest case for each experiment in Table 5, we can observe satisfactory performance in Experiment 1, where also Simplicial depth reaches the expected $FAR$. In Experiments 2 and 3, the results are less convincing during the out-of-control part. The reason is lower data depth values of reference sample points in a merged case than in a case of individual classes, leading to poor
Table 4: Monte Carlo study: Performance of $r$ control charts ($\alpha = 0.05$) in Experiment 3 with reference samples $R$ being predicted classes that were randomly constructed. Green rows highlight Phase I false alarm rates ($FAR$), and violet define Phase II signal and correct detection rates ($FAR$ and $CDR$), respectively. The underlined numbers indicate the best result for a particular monitoring period.

detection of nonstationary samples. On the contrary, the $SR$ values are reduced compared to the case when the reference sample relates to one class only.

Although the calculation of the data depth with respect to a merged version of a reference sample eliminates the misclassification problem, in our empirical study the detection results of spurious data by using predicted classes on their own are substantially better for higher-dimensional problems. For low-dimensional cases such as Experiment 1, we recommend first examining the performance of the monitoring based on the merged reference sample and then, if it is not operating acceptably, applying the method with the reference samples of predicted classes. Also, it is advisable to experiment with the size $|R|$, choosing other values than presented in the examples.

5.4 Results: Batch size

To evaluate the performance of $Q$ control charts, we chose those $|R|$ which achieved the most satisfactory performance in Section 5.3.1. Due to the small size of the dataset in Experiment 1, we computed the $Q$ control charts with the batch size $n = 5$ only for Experiments 2 and 3.

Considering the results in Table 6, we notice that (apart from $SD$) the $SR$ values are excessively high. The increase can be explained by a substantial change in control limits. Referring to Section 4.2, for $n = 3$, we obtain $LCL = 0.22$ and for $n = 5$, $LCL = 0.29$. At the same time, in all possible consolidations, the $Q$ control chart achieves notable results during the out-of-control period, consid-
Table 5: Performance of $r$ control charts ($\alpha = 0.05$) in the presented experiments with $R$ being merged classes. Green rows highlight Phase I false alarm rates ($FAR$), and violet define Phase II signal and correct detection rates ($SR$ and $CDR$), respectively. The underlined numbers indicate the best result for a particular monitoring interval.

Table 5: Performance of $r$ control charts ($\alpha = 0.05$) in the presented experiments with $R$ being merged classes. Green rows highlight Phase I false alarm rates ($FAR$), and violet define Phase II signal and correct detection rates ($SR$ and $CDR$), respectively. The underlined numbers indicate the best result for a particular monitoring interval.

erably improving the $CDR$s in Experiment 2 for Projection depth. Thus, if a supplementary procedure can be developed for detecting and filtering signals successfully during the process to be in-control, the $Q$ control chart would outperform the $r$ control chart.

5.5 Results: Computation time

For performing online surveillance, the computation time of the monitoring statistic is of particular importance. As the most time-consuming part of our approach is the derivation of data depth values, we compare the execution time of different algorithms to obtain the depth of one data point. To provide a concise summary, we compare running times for the middle sizes of reference samples, namely $|R| = \{60, 500, 3000\}$, and for cases displayed in Table 5.

In Figure 7, we can see that Simplicial depth requires considerably longer to be computed than other notions of data depth. Regarding the algorithms to approximate Projection depth, the running times remain similar also in Experiments 2 and 3, Figures 8 and 9, respectively. Despite the increased complexity of experiments, Mahalanobis depth is characterized by a stable and low running time. On the contrary, the running time of $HD_r$ increases noticeably with the growing size of reference samples as well as additional dimensions.

To summarize, if we exclude the performance of $MD$, the computation of data depth in $\mathbb{R}^{16}$ for one point with $|R| = 3000$ would usually take more than 10 seconds. In statistics, such results seem to be acceptable. However, taking into account the current applications of NNs, for example, the image
| Evaluation | Batch size | Phase | Observed process | Metric | MD | SD | HD | PD1 | PD2 | PD3 |
|------------|------------|-------|------------------|--------|----|----|----|-----|-----|-----|
| Experiment 1 | 3          | I     | In-control       | FAR    | 0.13 | 0.02 | 0.17 | 0.13 | 0.13 | 0.13 |
|            | 3          | II    | In-control       | SR     | 0.57 | 0.16 | 0.50 | 0.50 | 0.50 | 0.50 |
|            | 3          | II    | Out-of-control   | CDR    | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 |
|            | m_i ∈ R^3 |       |                  |        |     |     |     |     |     |     |
| | | | | | | | | | | | |
| Experiment 2 | 3          | I     | In-control       | FAR    | 0.05 | 0.14 | 0.15 | 0.17 | 0.17 | 0.17 |
|            | 3          | II    | In-control       | SR     | 0.47 | 0.58 | 0.62 | 0.66 | 0.64 | 0.64 |
|            | 3          | II    | Out-of-control   | CDR    | 0.68 | 0.89 | 0.89 | 0.84 | 0.84 | 0.84 |
|            | m_i ∈ R^8 |       |                  |        |     |     |     |     |     |     |
| | | | | | | | | | | | |
| Experiment 3 | 3          | I     | In-control       | FAR    | 0.19 | 0.17 | 0.21 | 0.22 | 0.22 | 0.22 |
|            | 3          | II    | In-control       | SR     | 0.62 | 0.75 | 0.76 | 0.84 | 0.83 | 0.83 |
|            | 3          | II    | Out-of-control   | CDR    | 0.90 | 0.90 | 0.90 | 1.00 | 1.00 | 1.00 |
|            | m_i ∈ R^{16} |       |                  |        |     |     |     |     |     |     |
| | | | | | | | | | | | |
| Experiment 3 | 3          | I     | In-control       | FAR    | 0.08 | 0.09 | 0.08 | 0.08 | 0.08 | 0.08 |
|            | 3          | II    | In-control       | SR     | 0.58 | 0.59 | 0.55 | 0.54 | 0.55 | 0.55 |
|            | 3          | II    | Out-of-control   | CDR    | 0.98 | 0.98 | 0.98 | 0.96 | 0.97 | 0.97 |
|            | m_i ∈ R^{16} |       |                  |        |     |     |     |     |     |     |
| | | | | | | | | | | | |
| Experiment 3 | 3          | I     | In-control       | FAR    | 0.10 | 0.13 | 0.10 | 0.10 | 0.11 | 0.11 |
|            | 3          | II    | In-control       | SR     | 0.75 | 0.76 | 0.71 | 0.71 | 0.71 | 0.71 |
|            | 3          | II    | Out-of-control   | CDR    | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
|            | m_i ∈ R^{16} |       |                  |        |     |     |     |     |     |     |
| | | | | | | | | | | | |

Table 6: Performance of Q control charts (LCL = 0.22 for n = 3 and LCL = 0.29 for n = 5) in the presented experiments with R being predicted class. Green rows highlight Phase I false alarm rates (FAR), and violet define Phase II signal and correct detection rates (SR and CDR), respectively. The underlined numbers indicate the best result for a particular monitoring period.

classification applying a CNN, the time required to process one image is under 0.1 second (cf. Shi et al., 2022). Thus, we should critically consider the running times for the computation of data depth, striving for their improvements and guarantee the applicability of the proposed framework to monitor state-of-the-art models based on AI.

6 Conclusion and Discussion

The models based on AI, especially NNs, have contributed to recent applications in various disciplines. However, the impressive results that were achieved with their deployment mask the necessity of the model’s control and monitoring, meaning that critical flaws could occur without any notice by the expert.

In this work, we propose a monitoring procedure designed for deep learning applications that applies a nonparametric multivariate control chart based on ranks and data depth. To be precise, the method monitors the low-dimensional representation of input data called embeddings that are generated by NNs. It could be illustrated that for two out of three experiments, namely for the least
(a) Single class reference samples of size $|R| = 60$  \hspace{1cm} (b) Merged reference samples of size $|R| = 140$

**Fig. 7:** Distribution of computation time for different data depths in Experiment 1 on a logarithmic scale. The order is Projection Depth with Coordinate Descent, Nelder-Mead and Refined Random algorithms, Robust Halfspace, Simplicial and Mahalanobis Depths.

(a) Single class reference samples of size $|R| = 500$  \hspace{1cm} (b) Merged reference samples of size $|R| = 1600$

**Fig. 8:** Distribution of computation time for different data depths in Experiment 2 on a logarithmic scale. The order is Projection Depth with Coordinate Descent, Nelder-Mead and Refined Random algorithms, Robust Halfspace and Mahalanobis Depths.
(a) Single class reference samples of size $|R| = 3000$  
(b) Merged reference samples of size $|R| = 30000$

**Fig. 9:** Distribution of computation time for different data depths in Experiment 3. The order is Projection Depth with Coordinate Descent, Nelder-Mead and Refined Random algorithms, Robust Halfspace and Mahalanobis Depths.
and most complicated cases, the proposed monitoring methodology has great potential. We explained
the unsatisfactory performance in Experiment 2 with the insufficient similarity between the reference
sample data and test data that creates Phase II (In-control). Also, in our experiments we randomly
choose the data points that create the training and the test datasets, meaning that the statistical identity
is not guaranteed. Thus, in the future, we recommend examining whether an optimal splitting of
data into training and testing sets that preserve distributional similarity, for instance, by applying an
efficient algorithm called “Twinning” (cf. Vakayil and Joseph, 2022), improves the performance of
models based on AI as well as leads to more reliable monitoring.

Additionally, with large data streams another problem arises – an increased number of outliers
(Porzio and Ragozini, 2008). Although they might not be an issue for models based on AI, in SPM
they could lead to unreliable change detection. Thus, the practice of identifying and eliminating
outliers before the monitoring should be explored in further studies.

Furthermore, we investigated the influence of the reference sample by size and by the method
it was constructed. Overall, we could conclude that the choice of data points which form a reference
sample and its size significantly influence monitoring performance. Also, it could be shown that a
bigger reference sample is not automatically related to a better detection capability. Still, there are
open questions about how to attain a reliable reference sample from Phase I. It is advisable to re-
search in more detail how the SPM techniques such as the multivariate mean-rank chart (Bell et al.,
2014) could support the analysis of Phase I data. Also, it is subject to future research when the refer-
ence sample should be updated or continually augmented with recent observations while preventing
contamination with the out-of-control data.

Regarding the misclassification problem, we can state that the wrongly predicted samples do not
directly lead to false signals. However, we can also conclude that a better-trained model would lead to
fewer situations related to misclassification, automatically improving the entire monitoring procedure.
Nevertheless, understanding when a data point has obtained a wrong prediction is indispensable and
requires an additional approach to be developed.

Concerning the type of applied control charts, we used the $r$ and $Q$ control charts proposed by
Liu (1995). As indicated by Chakraborti and Graham (2019) and Qiu (2019), who comprehensively
review the current state of nonparametric SPM, this area remains open for further theoretical and ex-
perimental development. With a wide variety of the existing techniques and their practical relevance,
particularly favorable would be their systematic comparison.
In the discussed experiments we considered well-balanced classification problems. However, the class imbalance is a frequent challenge in training the NNs and needs to be considered in the future research (cf. Ghazikhani et al., 2013). Moreover, it remains open whether the proposed methodology could be applied for monitoring semi-supervised or unsupervised learning models. Also, it is relevant to examine other types of concept drift or nonstationarity.

To summarize, the choice of a particular control chart depends on the specific problem, requiring a compromise between the number of signals in Phase II (In-control) and of correctly detected out-of-control samples. For example, control charts based on Robust Halfspace depth are conservative, meaning there is a substantial fraction of alarms when the process is in-control, but they detect reliably nonstationary samples. However, the calculation costs of Robust Halfspace depth could be considerably greater than of its alternatives, meaning that another aspect of deciding on the best method is the computation time. As soon as one concludes what is important, namely computation time, robustness, or low variance, the proposed monitoring approach can be customize so that it satisfactorily supports applications that involve AI.

References

Abraham, Y., Gerrits, B., Ludwig, M.-G., Rebhan, M., and Gubser Keller, C. (2017). Exploring Glucocorticoid Receptor Agonists Mechanism of Action Through Mass Cytometry and Radial Visualizations. Cytometry Part B: Clinical Cytometry, 92(1):42–56.

Aldridge, I. and Avellaneda, M. (2019). Neural networks in finance: Design and performance. The Journal of Financial Data Science, 1(4):39–62.

Angermueller, C., Pärnamaa, T., Parts, I., and Stegle, O. (2016). Deep learning for computational biology. Molecular systems biology, 12(7):878.

Baena-Garcia, M., del Campo-Ávila, J., Fidalgo, R., Bifet, A., Gavalda, R., and Morales-Bueno, R. (2006). Early drift detection method. In Fourth international workshop on knowledge discovery from data streams, volume 6, pages 77–86.

Ball, N. M., Brunner, R. J., Myers, A. D., and Tcheng, D. (2006). Robust machine learning applied to astronomical data sets. I. star-galaxy classification of the Sloan Digital Sky Survey DR3 using decision trees. The Astrophysical Journal, 650(1):497.
Barale, M. and Shirke, D. (2019). Nonparametric control charts based on data depth for location parameter. Journal of Statistical Theory and Practice, 13(3):1–19.

Baranowski, J., Dudek, A., and Mularczyk, R. (2021). Transient Anomaly Detection Using Gaussian Process Depth Analysis. In 2021 25th International Conference on Methods and Models in Automation and Robotics (MMAR), pages 221–226. IEEE.

Bell, R. C., Jones-Farmer, L. A., and Billor, N. (2014). A distribution-free multivariate phase I location control chart for subgrouped data from elliptical distributions. Technometrics, 56(4):528–538.

Bifet, A. and Gavalda, R. (2007). Learning from time-changing data with adaptive windowing. In Proceedings of the 2007 SIAM international conference on data mining, pages 443–448.

Boone, J. and Chakraborti, S. (2012). Two simple Shewhart-type multivariate nonparametric control charts. Applied Stochastic Models in Business and Industry, 28(2):130–140.

Caro, L. D., Frias-Martinez, V., and Frias-Martinez, E. (2010). Analyzing the role of dimension arrangement for data visualization in radviz. In Pacific-Asia Conference on Knowledge Discovery and Data Mining, pages 125–132. Springer.

Cascos, I. and López-Díaz, M. (2018). Control charts based on parameter depths. Applied Mathematical Modelling, 53:487–509.

Celano, G., Castagliola, P., Fichera, S., and Nenes, G. (2013). Performance of t control charts in short runs with unknown shift sizes. Computers & Industrial Engineering, 64(1):56–68.

Chakraborti, S. and Graham, M. A. (2019). Nonparametric (distribution-free) control charts: An updated overview and some results. Quality Engineering, 31(4):523–544.

Chiroma, H., Abdullahi, U. A., Alarood, A. A., Gabralla, L. A., Rana, N., Shuib, L., Hashem, I. A. T., Gbenga, D. E., Abubakar, A. I., and Zeki, A. M. (2018). Progress on Artificial Neural Networks for Big Data Analytics: A Survey. IEEE Access, 7:70535–70551.

Dang, X. and Serfling, R. (2010). Nonparametric depth-based multivariate outlier identifiers, and masking robustness properties. Journal of Statistical Planning and Inference, 140(1):198–213.
Demšar, J. and Bosnić, Z. (2018). Detecting concept drift in data streams using model explanation. 
Expert Systems with Applications, 92:546–559.

Disabato, S. and Roveri, M. (2019). Learning convolutional neural networks in presence of concept 
drift. In 2019 International Joint Conference on Neural Networks (IJCNN), pages 1–8. IEEE.

Ditzler, G., Roveri, M., Alippi, C., and Polikar, R. (2015). Learning in nonstationary environments: 
A survey. IEEE Computational Intelligence Magazine, 10(4):12–25.

Donoho, D. L. and Gasko, M. (1992). Breakdown properties of location estimates based on halfspace 
depth and projected outlyingness. The Annals of Statistics, pages 1803–1827.

Dua, D. and Graff, C. (2019). UCI machine learning repository.

Dyckerhoff, R., Mozharovskyi, P., and Nagy, S. (2021). Approximate computation of projection 
depths. Computational Statistics & Data Analysis, 157:107166.

Francisci, G., Nieto-Reyes, A., and Agostinelli, C. (2019). Generalization of the simplicial depth: no 
vanishment outside the convex hull of the distribution support. arXiv preprint arXiv:1909.02739.

Gama, J., Medas, P., Castillo, G., and Rodrigues, P. (2004). Learning with drift detection. In Brazilian 
symposium on artificial intelligence, pages 286–295. Springer.

Gama, J., Žliobaitė, I., Bifet, A., Pechenizkiy, M., and Bouchachia, A. (2014). A survey on concept 
drift adaptation. ACM computing surveys (CSUR), 46(4):1–37.

Gao, X., Pishdad-Bozorgi, P., Shelden, D. R., and Hu, Y. (2019). Machine learning applications in 
facility life-cycle cost analysis: A review. Computing in Civil Engineering 2019: Smart Cities, 
Sustainability, and Resilience, pages 267–274.

Garcia, K. D., Poel, M., Kok, J. N., and de Carvalho, A. C. (2019). Online clustering for novelty 
detection and concept drift in data streams. In EPIA Conference on Artificial Intelligence, pages 
448–459. Springer.

Ghazikhani, A., Monsefi, R., and Yazdi, H. S. (2013). Ensemble of online neural networks for non-
stationary and imbalanced data streams. Neurocomputing, 122:535–544.

Goldberg, Y. (2016). A primer on neural network models for natural language processing. Journal of 
Artificial Intelligence Research, 57:345–420.
Goodfellow, I., Bengio, Y., and Courville, A. (2016). Deep learning book. MIT Press, 521(7553):800.

Gorman, R. P. and Sejnowski, T. J. (1988). Analysis of hidden units in a layered network trained to classify sonar targets. Neural networks, 1(1):75–89.

Heipke, C. and Rottensteiner, F. (2020). Deep learning for geometric and semantic tasks in photogrammetry and remote sensing. Geo-spatial Information Science, 23(1):10–19.

Hermans, M. and Schrauwen, B. (2013). Training and analysing deep recurrent neural networks. Advances in neural information processing systems, 26:190–198.

Hoffman, P., Grinstein, G., and Pinkney, D. (1999). Dimensional anchors: a graphic primitive for multidimensional multivariate information visualizations. In Proceedings of the 1999 workshop on new paradigms in information visualization and manipulation in conjunction with the eighth ACM international conference on Information and knowledge management, pages 9–16.

Huang, L., Joseph, A. D., Nelson, B., Rubinstein, B. I., and Tygar, J. D. (2011). Adversarial machine learning. In Proceedings of the 4th ACM workshop on Security and artificial intelligence, pages 43–58.

Ivanovs, J. and Mozharovskyi, P. (2021). Distributionally robust halfspace depth. arXiv preprint arXiv:2101.00726.

Jones-Farmer, L. A., Woodall, W. H., Steiner, S. H., and Champ, C. W. (2014). An Overview of Phase I Analysis for Process Improvement and Monitoring. Journal of Quality Technology, 46(3):265–280.

Kan, S. H. (2003). Metrics and models in software quality engineering. Addison-Wesley Professional.

Klinkenberg, R. and Joachims, T. (2000). Detecting concept drift with support vector machines. In ICML, pages 487–494.

Klinkenberg, R. and Renz, I. (1998). Adaptive information filtering: Learning in the presence of concept drifts. Learning for Text Categorization, pages 33–40.

Krizhevsky, A. and Hinton, G. (2009). Learning multiple layers of features from tiny images. Technical report.
Kuncheva, L. I. (2009). Using control charts for detecting concept change in streaming data. Technical report.

Lange, T., Mosler, K., and Mozharovskyi, P. (2014). Fast nonparametric classification based on data depth. Statistical Papers, 55(1):49–69.

Lee, S., Kwak, M., Tsui, K.-L., and Kim, S. B. (2019). Process monitoring using variational autoencoder for high-dimensional nonlinear processes. Engineering Applications of Artificial Intelligence, 83:13–27.

Li, J., Cuesta-Albertos, J. A., and Liu, R. Y. (2012). DD-classifier: Nonparametric classification procedure based on DD-plot. Journal of the American statistical association, 107(498):737–753.

Liu, R. Y. (1990). On a notion of data depth based on random simplices. The Annals of Statistics, pages 405–414.

Liu, R. Y. (1995). Control charts for multivariate processes. Journal of the American Statistical Association, 90(432):1380–1387.

Liu, R. Y., Serfling, R. J., and Souvaine, D. L. (2006). Data depth: robust multivariate analysis, computational geometry, and applications, volume 72. American Mathematical Soc.

Liu, R. Y. and Singh, K. (1993). A quality index based on data depth and multivariate rank tests. Journal of the American Statistical Association, 88(421):252–260.

Mahalanobis, P. C. (1936). On the Generalised Distance in Statistics. In Proceedings of the National Institute of Sciences of India, volume 2, pages 49–55.

Mejri, D., Limam, M., and Weihs, C. (2017). Combination of Several Control Charts Based on Dynamic Ensemble Methods. Mathematics and Statistics, 5(3):117–129.

Mejri, D., Limam, M., and Weihs, C. (2021). A new time adjusting control limits chart for concept drift detection. IFAC Journal of Systems and Control, 17:100170.

Montgomery, D. C. (2012). Statistical Quality Control. Wiley Global Education.

Mosler, K. (2013). Depth statistics. In Robustness and complex data structures, pages 17–34. Springer.
Mosler, K. and Mozharovskyi, P. (2022). Choosing among notions of multivariate depth statistics. Statistical Science, 37(3):348–368.

Nammous, M. K. and Saeed, K. (2019). Natural language processing: speaker, language, and gender identification with LSTM. In Advanced Computing and Systems for Security, pages 143–156. Springer.

Nishida, K. and Yamauchi, K. (2007). Detecting concept drift using statistical testing. In International conference on discovery science, pages 264–269. Springer.

O’Shea, K. and Nash, R. (2015). An introduction to convolutional neural networks. arXiv preprint arXiv:1511.08458.

Otter, D. W., Medina, J. R., and Kalita, J. K. (2020). A survey of the usages of deep learning for natural language processing. IEEE Transactions on Neural Networks and Learning Systems, 32(2):604–624.

Pandolfo, G., Iorio, C., Staiano, M., Aria, M., and Siciliano, R. (2021). Multivariate process control charts based on the \( l^p \) depth. Applied Stochastic Models in Business and Industry, 37(2):229–250.

Perdikis, T. and Psarakis, S. (2019). A survey on multivariate adaptive control charts: Recent developments and extensions. Quality and Reliability Engineering International, 35(5):1342–1362.

Pokotylo, O., Mozharovskyi, P., and Dyckerhoff, R. (2019). Depth and Depth-Based classification with R Package ddalpha. Journal of Statistical Software, 91(5):1–46.

Porzio, G. C. and Ragozini, G. (2008). Multivariate control charts from a data mining perspective. Recent Advances in Data Mining of Enterprise Data: Algorithms and Applications, 6:413.

Priya, S. and Uthra, R. A. (2021). Deep learning framework for handling concept drift and class imbalanced complex decision-making on streaming data. Complex & Intelligent Systems, pages 1–17.

Psarakis, S. (2011). The use of neural networks in statistical process control charts. Quality and Reliability Engineering International, 27(5):641–650.

Psarakis, S. (2015). Adaptive control charts: recent developments and extensions. Quality and Reliability Engineering International, 31(7):1265–1280.
Qiu, P. (2014). Introduction to statistical process control. CRC press.

Qiu, P. (2019). Some recent studies in statistical process control. In Statistical quality technologies, pages 3–19. Springer.

Sergin, N. D. and Yan, H. (2021). Toward a better monitoring statistic for profile monitoring via variational autoencoders. Journal of Quality Technology, pages 1–46.

Shi, C., Zhang, X., Sun, J., and Wang, L. (2022). Remote sensing scene image classification based on self-compensating convolution neural network. Remote Sensing, 14(3):545.

Stoumbos, Z. G., Jones, L. A., Woodall, W. H., and Reynolds, M. R. (2001). On nonparametric multivariate control charts based on data depth. In Frontiers in Statistical Quality Control 6, pages 207–227. Springer.

Struyf, A. J. and Rousseeuw, P. J. (1999). Halfspace depth and regression depth characterize the empirical distribution. Journal of Multivariate Analysis, 69(1):135–153.

Tukey, J. W. (1975). Mathematics and the picturing of data. In Proceedings of the International Congress of Mathematicians, Vancouver, 1975, volume 2, pages 523–531.

Vakayil, A. and Joseph, V. R. (2022). Data twinning. Statistical Analysis and Data Mining: The ASA Data Science Journal.

Vencálek, O. (2017). Depth-based classification for multivariate data. Austrian Journal of Statistics, 46(3-4):117–128.

Voorhees, E. M. and Harman, D. (2000). Overview of the sixth text retrieval conference (TREC-6). Information Processing & Management, 36(1):3–35.

Wang, X., Wang, Z., Shao, W., Jia, C., and Li, X. (2019). Explaining concept drift of deep learning models. In International Symposium on Cyberspace Safety and Security, pages 524–534. Springer.

Weese, M., Martinez, W., Megahed, F. M., and Jones-Farmer, L. A. (2016). Statistical learning methods applied to process monitoring: An overview and perspective. Journal of Quality Technology, 48(1):4–24.
Yeganeh, A., Abbasi, S. A., Pourpanah, F., Shadman, A., Johannsen, A., and Chukhrova, N. (2022). An ensemble neural network framework for improving the detection ability of a base control chart in non-parametric profile monitoring. Expert Systems with Applications.

Yin, Z. and Shen, Y. (2018). On the dimensionality of word embedding. Advances in Neural Information Processing Systems, 31.

Yu, F., Wang, D., Shelhamer, E., and Darrell, T. (2018). Deep layer aggregation. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 2403–2412.

Zan, T., Liu, Z., Su, Z., Wang, M., Gao, X., and Chen, D. (2020). Statistical process control with intelligence based on the deep learning model. Applied Sciences, 10(1):308.

Zou, C., Wang, Z., and Tsung, F. (2012). A spatial rank-based multivariate EWMA control chart. Naval Research Logistics (NRL), 59(2):91–110.

Zuo, Y. and Serfling, R. (2000). General notions of statistical depth function. Annals of Statistics, pages 461–482.

Appendix

A Experiment 1: Binary Classification of Sonar Signals

In the first experiment, we consider a binary classification problem of sonar data (Dua and Graff, 2019). This dataset summarizes sonar signals collected from metal cylinders and cylindrically shaped rocks (Gorman and Sejnowski, 1988). There are 208 samples in total, comprising 111 metal cylinder and 97 rock returns. Each sample consists of a series of 60 numbers ranging from 0.0 to 1.0, representing a normalized spectral envelope. The task of the classifier is to distinguish which samples are from scanning a rock and which are from a metal cylinder.

Our model is a FNN with the architecture 60 → 30 → 10 → 3 → 1 that comprises four fully connected layers reducing the complexity 1 × 60 of the input data by first processing it through the hidden layers that have 30 and 10 neurons. Afterwards, the compressed data representation enters the layer with 3 neurons whose output is also used as embeddings of size 1 × 3 for the monitoring procedure. Then, to obtain the class label, we transform the interim output from size 1 × 3 to 1 × 1.
As we have a binary classification problem, we use only one neuron in the output layer together with the sigmoid activation function that is centered around 0.5, returning the probability of the processed sample belonging to class 2. Thus, if the result of the output layer is 0.5 or higher, we conclude that the processed sample is a part of class 2 (rock) and of class 1 (metal cylinder) otherwise. Due to the small size of the dataset, only 38 samples (24 of class 1 and 14 of class 2) are allocated to the testing stage which is later used in Phase II as the in-control data. Consequently, other samples, 85 metal cylinders and 85 rock examples, are taken for training the NN. The performance metrics such as validation loss and accuracy are used to determine the number of epochs, i.e., training cycles in which the model learns from the data and updates the parameters. Following that, the NN model was trained for 35 epochs and achieved 89.50% accuracy on the test data.

To create out-of-control samples, we estimate parameters of the beta distribution for each class (i.e., \( \tilde{\alpha}_1, \tilde{\beta}_1 \) of class 1, and \( \tilde{\alpha}_2, \tilde{\beta}_2 \) of class 2) and randomly sampled from a beta distribution with parameters \( \tilde{\alpha}_v = \tilde{\alpha}_1 + \tilde{\alpha}_2 \) and \( \tilde{\beta}_v = \tilde{\beta}_1 + \tilde{\beta}_2 \) to generate 30 out-of-control observations.

### B Experiment 2: Multiclass Classification of Questions

For the second experiment, we use the Text REtrieval Conference (TREC) dataset which consists of fact-based questions divided into six broad semantic categories\(^1\) (cf. Voorhees and Harman, 2000). The model was trained with the classes: “Numeric values”, “Description and abstract concepts”, “Entities” and “Human beings”. Examples of such questions can be found in Table 7. The classification task is to assign an incoming question to one of four categories.

The trained neural network contains three hidden layers: a word embedding layer, a Long Short-Term Memory (LSTM) layer and a fully connected layer which we use as the embedding generator of the size \( 1 \times 8 \). After that, the output layer returns probability vector \( 1 \times 4 \), where the maximum value corresponds to the label of the predicted category. It is worth noting that here the word embedding layer is not a part of our monitoring approach but a Natural Language Processing (NLP) technique that enables the model to associate a numerical vector to every word so that the distance between any two vectors is related to the semantic meaning of the encrypted words (cf. Yin and Shen, 2018). Due to the higher complexity of the neural network model than in the first experiment, we are not going further into its architecture’s details. Instead, we recommend referring to publications that offer

\(^1\)https://cogcomp.seas.upenn.edu/Data/QA/QC/
| Class                          | Example                                      |
|-------------------------------|----------------------------------------------|
| Numeric values                | What is the size of Argentina?               |
| Description and abstract concepts | What is artificial intelligence?            |
| Entities                      | What is the tallest piece on a chessboard?  |
| Human beings                  | Who invented basketball?                     |

**Table 7:** Question examples and respective four categories of the TREC dataset used for training the NN in Experiment 2.

In total, 700 data points of each category were used as the training data. The achieved accuracy on the test dataset that contains 150 unseen samples for each class is 85.50% after 25 training epochs. The 60 out-of-control samples were taken from two other semantic categories that were not used for training, namely “Abbreviations” and “Locations”.

## C Experiment 3: Multiclass Classification of Images

In our third experiment, we work with the CIFAR-10 dataset containing color images of the size 32×32 pixels (Krizhevsky and Hinton, 2009). In total, there are 60000 images which correspond to 6000 pictures per class. Example images of each category can be found in Figure 10.

To construct a classifier for predicting to which of the ten groups an input image belongs, we trained a CNN that has a deep layer aggregation structure as proposed by Yu et al. (2018). A specification of such architecture is the tree-structured hierarchy of operations to aggregate the extracted features from different model stages. For a detailed introduction to CNNs, we refer to O’Shea and Nash (2015). The layer used for monitoring has 16 neurons so that we obtain a monitoring task of $m_i \in \mathbb{R}^{16}$.

Regarding the results of the training that reached 88 epochs, the achieved accuracy on the test images is 90.43%. For out-of-control samples, we considered the CIFAR-100 dataset that has 100 image groups, selecting four distinctive classes, namely “Kangaroo”, “Butterfly”, “Train” and “Rocket”. From each category we randomly chose 100 images, having in total 400 samples for the out-of-control part in Phase II.
Fig. 10: Image examples and class labels of the CIFAR-10 dataset.