Brane Content of Branes’ States

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Abstract

The problem of decomposition of unitary irreps of (super) tensorial (i.e. extended with tensorial charges) Poincaré algebra w.r.t. its different subalgebras is considered. This requires calculation of little groups for different configurations of tensor charges. Particularly, for preon states (i.e. states with maximal supersymmetry) in different dimensions the particle content is calculated, i.e. the spectrum of usual Poincaré representations in the preon representation of tensorial Poincaré. At d=4 results coincide with (and may provide another point of view on) the Vasiliev’s results in field theories in generalized space-time. The translational subgroup of little groups of massless particles and branes is shown to be (and coincide with, at d=4) a subgroup of little groups of "pure branes" algebras, i.e. tensorial Poincaré algebras without vector generators. At 11d it is shown that, contrary to lower dimensions, spinors are not homogeneous space of Lorentz group, and one have to distinguish at least 7 different kinds of preons.

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1 Introduction

The study of supersymmetric theories leads to the change of our understanding of space-time symmetry algebras and of space-time itself. Instead of Poincaré algebra, which is a semidirect product of Lorentz algebra on an Abelian algebra of vectorial generators of space-time translations, now we have additional ”translations” by tensorial charges, which are carried by branes [1]. These charges appear in the anticommutator of supercharges, the most general among such an algebras is that of M-theory, where anticommutator of supercharges includes all possible tensors, namely vector, membrane and five-brane:

\[ \{\bar{Q}, Q\} = \Gamma^\mu P_\nu + \Gamma^{\mu\nu} Z_{\mu\nu} + \Gamma^{\mu\nu\lambda\rho\sigma} Z_{\mu\nu\lambda\rho\sigma}, \]  
\[ \mu, \nu, ... = 0, 1, 2, ..10. \]  

Similar algebras exist in lower dimensions, below we shall consider the minimal algebras, i.e. algebras with minimal number of spinors \( Q \). It is shown in [2] that many results in an M-theory can be derived directly from algebra (1). Particularly, properties of brane states of M-theory can be studied, which is natural, since all branes are unitary irreps of (1). Let’s consider the bosonic subalgebras of these super-Poincaré algebras, e.g. for 11d case (1) that is Lorentz \( M_{\mu\nu} \), momenta \( P_\mu \), membrane charge \( Z_{\mu\nu} \) and 5-brane charge \( Z_{\mu\nu\lambda\rho\sigma} \). We shall call such an algebras ”tensorial Poincaré” and denote them \((M_{\mu\nu}; P_\mu, Z_{\mu\nu}, Z_{\mu\nu\lambda\rho\sigma})\). Actually they are a semidirect products of Lorentz algebra \( M_{\mu\nu} \) with Abelian algebra of generators \( P_\mu, Z_{\mu\nu}, ... \). So bosonic subalgebra of M-theory is \((M_{\mu\nu}; P_\mu, Z_{\mu\nu}, Z_{\mu\nu\lambda\rho\sigma})\), in our notations.

The natural approach to (1) from the point of view of modern field theory is to try to construct the field theories, invariant w.r.t. such (super)-algebras, the first step of such approach should be the construction of their unitary irreps. That can be achieved by Wigner’s method of inducing representation from the unitary irreps of little group [3], [4]. Next will be the construction of relativistic free field equations, the space of solutions of which will give, modulo gauge invariance, another description of unitary irreps of tensorial Poincaré. Already at that step the generalization of space-time will be required, because one have to introduce a tensorial coordinates dual to tensorial charges. Such an approach was elaborated in [5], [6] for tensorial Poincaré \((M_{\mu\nu}; Z_{\mu\nu})\) in the space with two times, particularly with the aim of
study the SO(2,10) invariance hypothesis of M-theory \cite{7}. The little groups of branes are calculated for some cases in \cite{5,8}. Then interaction terms have to be constructed, which have to maintain gauge invariances - for conventional Poincaré case they are often determined by that requirement. For the simplest 4d, (actually (2+2)d) case an interaction terms are constructed in \cite{9} just by such requirement. Also, the presence of tensorial charges requires the reconsideration of spin-statistics theorem. That theorem can be considered as a rule, assigning the definite statistics to the irreps of Poincaré algebra. Now, for \( \mathfrak{osp}(2 M) \) type algebras (actually for their bosonic subalgebras, i.e. tensorial Poincaré), since the classification of irreps is substantially different from that for usual Poincaré, one has to rederive spin-statistics theorem, the first steps in that direction were done in \cite{8}, where spin-statistics for preons \cite{12} is considered.

In \cite{10,11} the \( OSp(2M) \) (conformal) invariant approach to (free) higher spin theories is developed, on the basis of generalized space-time. It is interesting and intriguing, that this approach leads to the same kind of space-time, as field theory approach of \cite{5} \cite{6}. As we shall see below, there are more precise connections between these approaches.

The new feature of tensorial Poincaré algebras is that they have subalgebras which itself are tensorial, or sometimes usual, Poincaré algebras. Irreps of this algebra (and corresponding group) can be decomposed into irreps of that subgroups. That will be the subject of study of present paper. This should help for a (future) study of whether superstring/M-theory can be described in this way, as some theory in space-time with coordinates dual to all tensorial central charges. So, we shall study what irreps of say particle Poincaré, or another tensorial Poincaré are making up the given irrep of given tensorial Poincaré. Hence the title of paper: branes content of branes'. In this paper the problem is not solved in whole generality, but, for some interesting cases is reduced to standard problems in group theory (harmonic analysis) and answers are given in simple cases. Actually even in 4d there exist another subalgebras, which we shall call "pure branes" subalgebras (see below) with respect to which irreps can be decomposed, also.

At \( d=4 \) tensorial Poincaré includes vector - an energy-momentum \( P_\mu \), and second-rank antisymmetric tensor - membrane (domain wall) charge \( Z_{\mu\nu} \). The corresponding susy theory includes one Majorana spinor, with susy relation:
\[
\{ \hat{Q}, Q \} = \Gamma^\mu P_\mu + \frac{1}{2} \Gamma^{\mu\nu} Z_{\mu\nu},
\]
\(\mu, \nu, \ldots = 0, 1, 2, 3.\)  

We denote the numeric value of rhs (on the subspace when it has definite value) by \(k_{\alpha\beta}\) (for this and similar relations in other dimensions), and corresponding values for \(P_\mu\) and \(Z_{\mu\nu}\) as \(p_\mu(k), z_{\mu\nu}(k)\):

\[
k = \Gamma^\mu p_\mu(k) + \frac{1}{2} \Gamma^{\mu\nu} z_{\mu\nu}(k)
\]

One natural subalgebra is usual (particle, i.e. vectorial) Poincaré, which includes Lorentz plus \(P_\mu\) generators, \((M_{\mu\nu}; P_\mu)\). For that case we show in Sect.2 that 1/2 BPS massive membrane representation contains all representations of particle Poincaré, with given mass and different spins, each spin appearing once. For 3/4 BPS (preon - [12]) representation, characterized by \(k_{\alpha\beta} = \lambda_\alpha \lambda_\beta\), the answer is similar - after decomposition of simplest (scalar) irrep of little group of preons we obtain all massless representations of particle Poincaré, one for each helicity. This last result can be interpreted as a group theory point of view on Vasiliev’s result [10]. Although the context is different - the \(OSp(8)\) invariant equations of motion in generalized space-time are considered, and problem of construction of Cauchy surface is discussed in [10], mathematically the considerations are similar. The little group of preons is \(T_2\) - two-dimensional Euclidean translations. That coincides exactly with the \(T_2\) factor of little group of massless particle with momenta of preon, \(p(k_{\alpha\beta}) = p(\lambda_\alpha \lambda_\beta)\). Remind that little group of massless particles is semidirect product of \(SO(2)\) with \(T_2\), \(SO(2) \ltimes T_2\). So one can say that excited states of preons correspond to the non-trivial representations of that, usually trivially represented, factor of little group of massless particles. Moreover, we can consider the other subalgebra, namely \(M_{\mu\nu}, Z_{\mu\nu}\), i.e. that of Lorentz generators plus membrane charge only. Mathematically it is perfectly possible, but physically considerations of such an algebras has to be justified. Particularly, it seems to be impossible to write down the supersymmetry algebra with such subalgebras, because the requirement of positivity of eigenvalues of corresponding term in [2] can’t be satisfied. This differs from 12d susy algebra [7] by signature, the two time dimension in 12d make it possible to have susy algebra without vector charge. But even in usual one-time signature case, consideration of bosonic algebras is not forbidden.
by any general considerations, and, particularly, the unitary irreps of that
algebras can be constructed. In that case we obtain the result, that the little
group of tensor $z_{\mu\nu}(k_{\alpha\beta}) = z_{\mu\nu}(\lambda_\alpha \lambda_\beta)$ taken for preon representation is same
$T_2$ subgroup of 2d Euclidean translations factor of massless particle’s little
group. So, excited (i.e. with non-trivial representations of little group) repre-
sentations of ”pure membrane” algebra ($M_{\mu\nu}; Z_{\mu\nu}$) corresponds to excited
$T_2$ generators of massless particles’ little group in usual Poincaré-invariant
theories. Usually that generators are represented trivially, by zero operators,
because unitary irreps of little group are required to be finite-dimensional
which leads to non-trivial representation of SO(2) subgroup of $SO(2) \ltimes T2$
little group only, otherwise the unitary representation of this noncompact
group would be infinite dimensional. This sheds some light on that factor
of little group of massless particles and one can conjecture, that correspond-
ing excitations will appear in full theory of 4d super Poincaré with tensorial
charges. One can conjecture, also, that self-consistent ”pure branes” theory
may exist. Similar phenomena happens in higher dimensions. In Sections 2,
3, 4, 5 we consider the decompositions of representations of minimal tenso-
rial Poincaré in dimensions 4,6,10 and 11 for different BPS states, mainly for
preons, i.e. those with maximal number of supersymmetries survived. In 11d
case (Section 5) a new phenomenon appears. In dimensions $d < 11$, preons
are orbits of corresponding Lorentz group, i.e. are quotients $G/H$, where $H$
is the little group, a subgroup of Lorentz group $G$. So, each spinor $\lambda_\alpha$
can be transformed into any other spinor $\sigma_\alpha$, there is no different kinds of pre-
ons, and one can speak about a preon representation of a (super)Poincaré.
At 11d space of spinors is 32 dimensional, but orbits are generically 25 di-
ensional, so there are at least 7 invariants, distinguishing preons in 11d.
Correspondingly, the irreps of 11d (super) algebra will be labelled by that
invariants.

In Conclusion results and prospects are discussed.

2  (1+3)d

Let’s consider the minimal supersymmetry algebra in 4d Minkowski space-
time (2). That includes one Majorana spinor $Q$ (4 real components), Lorentz
generators $M_{\mu\nu}$ (6), energy-momentum vector $P_\mu$ (4) and brane (domain
wall) charge $Z_{\mu\nu}$ (6). Lhs of (2) is 4x4 real (more exactly, with reality condi-
tions, following from those on Majorana spinor $Q$ in a given representation of
gamma matrices, see below) matrix with 10 independent components, which number coincides with the number of independent real generators in rhs.

For construction of unitary irreps of corresponding bosonic algebra \((M_{\mu\nu}; P_\mu, Z_{\mu\nu})\) we have to consider orbits of Lorentz group in the space of vector and tensor, construct a unitary irreps of stabilizer (little group) \(H\) of a given point on that orbit, and induce in a standard way that representation to the representation of the whole Poincaré in the space of functions on the orbit with values in a given irrep of \(H\). For an algebra \((2)\) we shall consider the following orbits: particle, when \(Z_{\mu\nu} = 0\); preon, when \(k_{\alpha\beta} = \lambda_{\alpha}\lambda_{\beta}\); BPS massive membrane; ”pure branes” orbit, with \(P_\mu = 0\) and different \(Z_{\mu\nu}\). Little groups for particle case are well known, some other cases were considered in [8], ”pure branes” will be calculated here for a first time, as well as different decompositions of tensorial Poincaré irreps w.r.t. its different bosonic subalgebras.

Consider first the particle case. It means that we are considering usual Poincaré with \(M_{\mu\nu}\) and \(P_\mu\) generators. We should consider orbits of Lorentz group on the space of momenta \(p\). Different orbits differs by values of Lorentz invariants, \(p^2\) is one of them. The physical cases are \(p^2 \geq 0\). For \(p^2 = m^2 > 0\) little group is \(SO(3)\), representations are characterized by spin, integer or half-integer. For massless case little group is two-dimensional Euclidean Poincaré, i.e. a semidirect product of rotations \(SO(2)\) and translations \(T_2\) of two-dimensional Euclidean plane, \(SO(2) \ltimes T_2\). The known interacting field theories are using the finite-dimensional representations of \(SO(2) \ltimes T_2\), representing translations trivially, and representation of \(SO(2)\) are classified by their (integer or half-integer) helicity. As we shall see below, non-trivial representations of translation generators corresponds to states of ”pure branes” theory.

Now consider the preon’s state, i.e. states with \(k_{\alpha\beta} = \lambda_{\alpha}\lambda_{\beta}\), where \(\lambda_{\alpha}\) is commuting Majorana 4d spinor. One calculation gives us simultaneously the statement that space of \(\lambda_{\alpha}\) is a homogeneous space of Lorentz group \(SO(1,3)\) and an algebra of its little group. Namely, we act by Lorentz generators on \(\lambda_{\alpha}\) and find that its stabilizer is \(T_2\) group, and dimensionality of orbit is 4, i.e. equal to that of whole space of spinors. We use Weyl representation of gamma-matrices:

\[
\Gamma^\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}
\]

(4)

\[
\sigma_\mu = (1, \sigma_i), \bar{\sigma}_\mu = (-1, \sigma_i)
\]

(5)

where \(\sigma^i\) are Pauli matrices. The similar relation defines our gamma matri-
ces in any even dimension through gamma matrices of previous Euclidean dimension. Then (pseudo)Majorana condition on spinor can be deduced (see, e.g. [14]), 4d Majorana spinor $\lambda_\alpha$ satisfies

$$ (\lambda_\alpha)^* = \Gamma^0 \Gamma^1 \Gamma^3 \lambda_\beta $$

(6)

Then stabilizer algebra of e.g. Majorana spinor $(1,0,0,1)$ is

$$ \begin{pmatrix}
0 & a & b & 0 \\
-a & 0 & 0 & a \\
-b & 0 & 0 & b \\
0 & -a & -b & 0
\end{pmatrix} $$

(7)

This corresponds to non-compact group $T_2$ of translations which coincides with that of massless particle, because little group of particle with $p^2 = p(k)^2$ should contain that for $k$ as subgroup. This is right for any $k$, because stabilizer of $k$ is intersection of stabilizers of $p_\mu(k)$ and $z_\mu(k)$. In our case $p(k)^2 = 0$, so little group for massless particle $SO(2) \ltimes T_2$ contains as subgroup that of preons, i.e. $T_2$. These considerations are confirmed by dimensionality check: the orbit in four-momenta space is 3-dimensional cone $SO(1,3)/(SO(2) \ltimes T_2)$, and preon orbit is four dimensional $SO(1,3)/T_2$. So, the particle representation is induction to the whole Poincaré of the representation of little group in the space of functions on $SO(1,3)/(SO(2) \ltimes T_2)$ with values in unitary irreps of $SO(2) \ltimes T_2$, irreps of preons are induction to the whole tensorial Poincaré from the representation of the little group in the space of functions on $SO(1,3)/T_2$, with values in unitary irreps of $T_2$. This last orbit is a fiber bundle over the first one with $SO(2)$ fiber. For decomposition of representation of tensorial $(M_{\mu\nu}; P_\mu; Z_{\mu\nu})$ Poincaré w.r.t. the usual $(M_{\mu\nu}; P_\mu)$ Poincaré we have to decompose the space of functions on that bundle into the space of functions on the base. So, we have to find the space of spinors, giving the same momenta $p_\mu$, i.e. fiber over given $p_\mu$, and define the action of $SO(2)$ on that fiber. Decomposition of the space of function on that fiber w.r.t. $SO(2)$ gives the helicity content of preon representation. The space of spinors giving same $p_\mu$, say $p_\mu = (1, 0, 0, 1)$ is $\lambda_2 = \lambda_3 = 0, \lambda_4 = \lambda_1^*, |\lambda_1| = 1$. $SO(2)$ transformations are acting on $\lambda_1$ by phase rotations, and space of functions gives the whole integer spectrum of helicities, each one once. For the space of double-valued functions (spinor type functions) the spectrum will consist of all half-integer helicities, each
helicity appearing once. This result actually is identical to that of Vas
iliev [10], as mentioned above.

It is natural to consider another subgroup of 4d tensorial Poincaré, namely
\((M_\mu\nu; Z_\mu\nu)\). In that case we have to calculate \(Z_\mu\nu\) for preons, find its little
group and decompose preon’s little group representation w.r.t. that little
group, i.e. decompose space of functions on a fiber over given \(z_{\mu\nu}(k)\) w.r.t.
that little group. Direct calculation for given \(k_\alpha^\beta\) gives
\[
\begin{pmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}
\]
(8)
The stabilizer of this matrix is the same \(T_2\) \([4]\). So actually there is no fiber
over this point, and irrep of \((M_\mu\nu; P_\mu, Z_\mu\nu)\) gives an irrep of \((M_\mu\nu; Z_\mu\nu)\).

The above analysis can be repeated for other, not preon representations
of tensorial Poincaré \((M_\mu\nu; P_\mu, Z_\mu\nu)\). Take a BPS membrane state, with
\[
P_\mu = (m, 0, 0, 0), Z_{12} = -Z_{21} = m
\]
(9)
This is a 1/2 massive (membrane) BPS of susy algebra \([2]\). One can find a lit-
tle group for that configuration \([9]\), it is \(SO(2)\) (rotations around 3-rd axis),
so orbit is \(SO(1,3)/SO(2)\). The little group for particle subalgebra is \(SO(3)\),
with orbit \(SO(1,3)/SO(3)\). Correspondingly, fiber is \(SO(3)/SO(2) = S^2\).
So, for an e.g. representation, induced from trivial representation of \(SO(2)\)
we have to decompose the space of functions on \(S^2\) w.r.t. its invariance group
\(SO(3)\). That is the sum of all representations of \(SO(3)\), with any spin, with
multiplicity one. Decomposition w.r.t. the ”pure membrane” subalgebra
\((M_\mu\nu, Z_\mu\nu)\) can be done provided we define the little group for tensor \(z_{\mu\nu}\).
That is \(SO(2) \otimes T_1\), so orbit is \(SO(1,3)/SO(2) \otimes T_1\), and fiber for decompo-
sition of membrane representation on a ”pure membrane” representations is
\(T_1\).

3 6d

The 6d Minkowski space supersymmetry algebra we shall consider includes,
besides Lorentz generators \(M_\mu\nu\), the Weyl spinor \(Q_\alpha\) (8 real components),
vector $P_{\mu\nu}$ (6) and third rank self-dual tensor $Z^+_{\mu\nu\lambda}$ (10). Susy relation is

$$\{\bar{Q}, Q\} = \Gamma^\mu P_\mu + \Gamma^{\mu\nu\lambda} Z^+_{\mu\nu\lambda},$$

$$\mu, \nu, \ldots = 0, 1, 2, 3, 4, 5.$$ (10)

The lhs is 4x4 Hermitian matrix, with 16 real components, which coincides with number of real generators in rhs. Usually in 6d one considers symplectic Majorana-Weyl spinors, which, while having same number of real components, have a bigger invariance group - SU(2) instead of U(1) in our Weyl spinors case. But last one is simpler, and we consider that here.

The preon representation of (11) corresponds to r.h.s. matrix of rank one. That can be parameterized as $k = \lambda \bar{\lambda}$, where Weyl spinor $\lambda$ is defined up to phase transformation. The space of Weyl spinors $\lambda$ is a homogeneous space $SO(1,5)/SO_L(3) \ltimes T_4$, where $SO(3)_L \ltimes T_4$ is the stabilizer of spinors. The little group of preon representation, i.e. stabilizer of r.h.s. of (11), $\lambda \bar{\lambda}$, has an additional $SO(2)$ factor: $(SO_L(3) \times SO_R(2) \ltimes T_4)$. Namely, let’s take a Weyl spinor of e.g. form $(1, 0, 0, 0)$, and transform that under $SO(1,5)$ rotation. Then in our representation of gamma matrices the algebra of stabilizer of this spinor is given by following matrix:

$$\begin{pmatrix}
0 & -w_2 & -w_3 & -w_4 & -w_5 & 0 \\
-w_2 & 0 & w_{23} & w_{24} & w_{25} & w_2 \\
-w_3 & -w_{23} & 0 & w_{25} & -w_{24} & w_3 \\
-w_4 & -w_{24} & -w_{25} & 0 & w_{23} & w_4 \\
-w_5 & -w_{25} & -w_{24} & -w_{23} & 0 & w_5 \\
0 & -w_2 & -w_3 & -w_4 & -w_5 & 0
\end{pmatrix}$$

which is a semidirect product of Abelian algebra of translations $T_4$ (with parameters $w_2, w_3, w_4, w_5$) and algebra $so_R(3)$ (remaining $w$-s). Algebra of phase transformations of a given spinor $(1, 0, 0, 0)$ in our representation of gamma-matrixes are given by $so_L(2)$.

The little group of 6d massless particles is $SO(4) \ltimes T_4$ (given by (11) without restrictions on the elements of middle 4 by for 4 antisymmetric matrix) so fiber for decomposition of preon representation w.r.t. the particles representations is $SO(4)/(SO_R(3) \times SO_L(2) \ltimes SO(3)/SO(2)$. So, we have to decompose the space of functions (considering simplest representation, with little group represented trivially) on $SO(4)/(SO_R(3) \times SO_L(2)) \ltimes SO(3)/SO(2)$ w.r.t. the SO(4) rotations. As we see, one of SO(3) factors of SO(4) (we
consider the covering group) is represented trivially, for other SO(3) factor representations of all spins appear, with multiplicity one.

Similarly to previous Section, we can consider "pure 3-brane" algebra, i.e. \((M_{\mu\nu}, Z_{\mu\nu\lambda})\) algebra. In that case for our spinor \((1, 0, 0, 0)\) the \(Z_{\mu\nu\lambda}\) tensor has non-zero components \(Z_{125} = -Z_{134} = Z_{256} = -Z_{346}\) (and those with permutations of indexes) only, and its little group is \((SO_L(3) \times SO_R(2)) \ltimes T_4\), i.e. coincide with that of preons. So the fiber for decomposition of preon representation w.r.t. 3-brane algebra is trivial, and one has one "pure 3-brane" irrep in decomposition of preon irreps.

4 10d

For 10d Minkowski space minimal supersymmetry algebra includes, besides Lorentz generators \(M_{\mu\nu}\), Majorana-Weyl spinor \(Q\) (16 real components), vector \(P_\mu\) (10) and fifth rank self-dual tensor \(Z_{\mu\nu\lambda\rho\sigma}^+\) (126). Susy relation is given by

\[
\{Q, Q\} = \Gamma_\mu P_\mu + \Gamma_{\mu\nu\lambda\rho\sigma} Z_{\mu\nu\lambda\rho\sigma}^+,
\]

\[
\mu, \nu, \ldots = 0, 1, 2, ..., 9
\]

The lhs is 16x16 symmetric (after right multiplication on charge conjugation C matrix) matrix, with 136 real components, which coincide with count of real generators in rhs. Consider corresponding bosonic algebra \(M_{\mu\nu}; P_\mu, Z_{\mu\nu\lambda\rho\sigma}^+\) and its preon representation, when rhs (after C multiplication) is \(\lambda_\alpha \lambda_\beta\). The 16d space of spinors \(\lambda_\alpha\) is homogeneous space \(SO(1, 9)/SO(7) \ltimes T_8\), where action of \(SO(7)\) on \(T_8\) is defined by its insertion, in its spinorial representation, into adjoint representation of \(SO(8)\). So, the little group is \(SO(7) \ltimes T_8\). In our gamma matrices representation the above statements appear as follows: the Weyl condition is selecting first 16 components of general 32d spinor, the Majorana condition requires \(Q_1 = Q_6^*, Q_2 = -Q_5^*, Q_3 = -Q_8^*, Q_4 = Q_7^*\). Then calculation of stabilizer of specific spinor with, e.g. non-zero and equal to 1 first and sixth components only gives a Lorentz
The strong difference with previous cases is in that space of 11d spinors is not a rank tensor \(M\) of scalar functions on orbit, with respect to particle subalgebra (the decomposition of simplest representation of preons, i.e. that in the space position of preon representation of is (8). The little group for massless particle is \(SO(8) \ltimes T_8\), the fiber for decomposition of preon representation of is \(SO(8)/SO(7) = S^7\). Correspondingly, the decomposition of simplest representation of preons, i.e. that in the space of scalar functions on orbit, with respect to particle subalgebra \(\{M_{\mu\nu}; P_\mu\}\) is given by the sum of symmetric tensor representations with multiplicity 1 (4, Section 10.3).

\[ (13) \]

where \(c_1 = -w_{46} - w_{57} + w_{89}, c_2 = w_{36} + w_{58} + w_{79}, c_3 = w_{37} - w_{48} - w_{69}, c_4 = -w_{34} + w_{59} - w_{78}, c_5 = -w_{35} - w_{49} + w_{68}, c_6 = -w_{39} + w_{45} - w_{67}, c_7 = w_{38} + w_{47} - w_{56}.\)

The structure of algebra (13) is the following: it is the semidirect product of \(T_3\) - Abelian algebra of matrices (13) with non-zero first row, last row, first and last columns only, and remaining matrices. This last algebra, represented by middle 8x8 matrices of (13) is spinor representation of \(SO(7)\), based on a real representation of 7d gamma matrices. Such representation can be constructed with the help of multiplication table of octonions, as shown in (15).

The 11d Minkowski space supersymmetry algebra includes Lorentz generators \(M_{\mu\nu}\), the Majorana spinor \(Q_\alpha\) (32 real components), vector \(P_\mu\) (11), second rank tensor \(Z_{\mu\nu}\) (55) and fifth rank tensor \(Z_{\mu\nu\lambda\rho\sigma}\) (462). Anticommutator of supercharges is given by (1). The lhs is 32x32 real symmetric matrix, with 528 real components, which coincide with count of real generators in rhs. The strong difference with previous cases is in that space of 11d spinors is not a...
homogeneous manifold of Lorentz group $SO(1,10)$, so it is not correct to define the representation by simply stating that rank of rhs of (1) is one, hence it is equal to $\lambda_\alpha \lambda_\beta$ with some spinor $\lambda_\alpha$. One has to define the values of additional invariants which separate the homogeneous sub-manifolds in the space of spinors $\lambda_\alpha$. The number of such invariants should be at least 7, because stabilizer of e.g. spinor with non-zero (and equal to 1) entries at first and sixth places only (this is a particular Majorana spinor in our gamma matrices representation, see previous Section) is 30-dimensional group $SO(7) \ltimes T_9$, so dimensionality of quotient $SO(1,10)/SO(7) \ltimes T_9$ is $55 - 30 = 25$, which is 7 units less than dimensionality of spinor’s space. So, one has at least 7 kinds of preons, the values of invariants, which distinguish them, simultaneously are labelling irreps of tensorial Poincaré $(M_{\mu\nu}; P_\mu, Z_{\mu\nu}, Z_{\mu\nu\lambda\rho\sigma})$. For the orbit considered we can look for a stabilizer of vector $P_\mu$ and second-rank tensor $Z_{\mu\nu}$, i.e. consider the decomposition w.r.t. the $(M_{\mu\nu}; P_\mu, Z_{\mu\nu})$ subalgebra. The corresponding stabilizer is $SO(8) \ltimes T_9$, so fiber is $SO(8)/SO(7) = S^7$, and according to general results (4, Section 10.3), for the simplest case of trivial representation of little group, the space of functions on $S^7$ contains all symmetric tensors representations (one row Young diagram), each one once. Next, it is not difficult to define stabilizer of second rank membrane tensor $Z_{\mu\nu}$, which is necessary for decomposition w.r.t. the ”pure membrane” subalgebra $(M_{\mu\nu}, Z_{\mu\nu})$. That is again $SO(8) \ltimes T_9$. Finally, we comment on the decomposition of preon’s state w.r.t. the massless particle (which corresponds to preons). Since little group for massless particle is $SO(9) \ltimes T_9$ for decomposition of preons w.r.t. particles we obtain a fiber $SO(9)/SO(7)$. The final answer can be obtained both directly, by decomposing the space of functions on $SO(9)/SO(7)$ w.r.t. $SO(9)$ group, or, using previous result, in two stages - first decomposing w.r.t. particle + membrane subalgebra, i.e. taking fiber $SO(8)/SO(7)$ and then decomposing the space of functions on fiber $SO(9)/SO(8)$, with values in each of representations, obtained on a previous stage. According to the general theorem (4), results should be the same.

6 Conclusion

We have calculated little groups (algebras) for different orbits of tensorial Poincaré algebras at different dimensions. That groups are useful in construction of irreps of corresponding (super)Poincaré algebras with tensorial
the proper subalgebras with highest rank tensor removed. I.e. for $d=4,6,10$ it is the decomposition w.r.t. the subalgebra $M_{\mu \nu}; P_{\mu}$, at 11d complete result is obtained for subalgebra $M_{\mu \nu}; P_{\mu}, Z_{\mu \nu}$. Results obtained permit the consideration of other cases, also. It seems that present approach provide group's theory point of view on Vasiliev's results \cite{10}, and can be helpful there in higher dimensions. We show, that at $d=11$ preon \cite{12} states are not defined by simply stating that r.h.s of \cite{11} has rank one, and is square of some spinor, but one should define the specific orbit to which that spinor belongs, which generally requires definition of 7 parameters. This fact requires further study of what is real difference in representations of tensorial Poincaré for different preon orbits. Also results on a little groups can be extended to other subalgebras of tensorial Poincaré algebras, as well as to other algebras, corresponding to theories with extended supersymmetries, or (6d case) formulations of the same algebra with different symmetry group. The most relevant next steps are extension of construction \cite{5}, \cite{6}, \cite{9} of field theories with tensorial Poincaré algebra as space-time symmetry on some of representations, discussed in the present paper.

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