Gamma-ray absorption in the microquasar SS433

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Abstract

We discuss the gamma-ray absorption in the inner region of the microquasar SS433. Our investigation includes several contributions to the opacity of this system. They result from the ambient fields generated by the primary star, possibly an A-type supergiant, and a very extended disk around the black hole. We find an important source for gamma-ray absorption in the UV photon field from the extended disk, which provides a rather significant background opacity. Furthermore, we predict a sharp though dramatic absorption effect every time the companion star crosses the foreground of the emission zone. This establishes a periodic gamma-ray observational signature.

Key words: gamma-rays: theory, X-rays: binaries, radiation mechanisms: non-thermal, stars: winds, outflows

1. INTRODUCTION

SS433 is a particularly interesting X-ray binary system that consists of a donor star feeding mass to a black hole [1] in orbit with a period of 13d. From the vicinity of the compact object two oppositely directed jets are launched developing regular precession with a period of 162d. These jets can be considered as 'dark' [2] because their main power output is given by their kinetic luminosity, \( L_k \sim 10^{39} \text{ erg s}^{-1} \). Indeed, the ejected matter has determined the deformation of the nebula W50 that surrounds the SS433 system [3].

Gamma-ray emission from similar objects has been confirmed very recently [4,5,6], so the study of absorption in the complex case of SS433 is important to characterize the possible high energy emission that this source might present to instruments such as GLAST and the new Cherenkov arrays MAGIC II and VERITAS. For a recent discussion of the absorption in the case of other systems like LS 1 +61 303 and LS 5039, see for instance, Ref. [7,8,9]. In the present case of SS433, we shall base our work on the set of parameters that are currently believed to describe the system after more than 20 years of observation and study. These parameters are summarized in the next section. The resulting absorption signatures are presented in Sect. 3, and a discussion is left for Sect. 4, where we analyze the possible detection of the specific absorption features.

2. THE SOURCE

The mass loss rate in the jets of SS433 is \( \dot{m}_j = 5 \times 10^{-7} M_{\odot} \text{yr}^{-1} \) and their bulk velocity is \( v_b \approx 0.26c \). The normal to the orbital plane makes an angle \( \theta \approx 21^\circ \) with the approaching jet and an angle \( i = 78^\circ \) to the line of sight. The line of sight then makes an angle \( i_j \) with the jet which is time-dependent because of precession (see Fig.1).

A thick expanding disk wind believed to be fed by the supercritical accretion disk encloses also the star [13]. According to Ref. [1], the disk has a half opening angle \( \alpha_w \approx 30^\circ \), a mass loss rate \( M_w \approx 10^{-4} M_{\odot} \text{yr}^{-1} \) and a velocity \( v_w \approx 1500 \text{ km s}^{-1} \).

The spectral identification of the primary star has been
difficult because of the presence of the extended disk, since the star is often partially or totally obscured by it. After convenient observations at specific configurations of precessional an orbital phases it has become quite clear that the star is an A-supergiant [15,16]. We assume the mass of the components as derived from INTEGRAL observations [11], $M_{bh} = 9 M_\odot$ and $M_\star = 30 M_\odot$ for the black hole and the star respectively, which correspond to an orbital separation $a \approx 79 R_\odot$ for a zero-eccentricity orbit as is the case for SS433. Since the star is believed to fill its Roche lobe, the implied radius according to [12] is

$$Q = \frac{3}{8} \frac{M_\star R_\star}{M_{bh}} \approx 38 R_\odot,$$

where $q = M_{bh}/M_\star$.

When a gamma-ray of energy $E_\gamma$ travels a distance $d\rho_\gamma$ in a photon field, there is a differential optical depth associated with it. Absorption correspondingly arises from the gamma-ray interaction with soft photons of energy $E$. Assuming that the former runs following $\hat{e}_\gamma$, and the latter are directed along $\hat{e}_{ph} = (\sin \theta' \cos \phi', \sin \theta' \sin \phi', \cos \theta')$, the corresponding optical depth is given by [14],

$$d\tau_{\gamma\gamma} = (1 - \hat{e}_\gamma \cdot \hat{e}_{ph}) n_{ph} \sigma_{\gamma\gamma} d\rho_\gamma dE d\cos \theta' d\phi'.$$

Here, $n_{ph}$ is the density of the soft photons per solid angle and soft photon energy units. The angles $\theta'$ and $\phi'$ are taken in a convenient coordinate system and the cross section for the process $\gamma \gamma \rightarrow e^+ e^-$ is given by

$$\sigma_{\gamma\gamma}(E_\gamma, E) = \frac{\pi q_e^2}{2} (1 - \xi^2)^2 \times$$

$$\left[2\xi(\xi^2 - 2) + (3 - \xi^2) \ln \left(\frac{1 + \xi}{1 - \xi}\right)\right],$$

with

$$\xi = 1 - \frac{2(m_e c^2)^2}{E_\gamma E(1 - \hat{e}_\gamma \cdot \hat{e}_{ph})}.$$  

3.1. Optical depth due to the companion starlight

Starlight photons coming from the companion mid-A supergiant at a temperature $T \approx 8500$ K [15,16] can cause absorption of gamma-rays. To see this clearly let us consider some simple geometry.

In Fig. 1, the observer is assumed to lay in the $xz$-plane and the gamma-ray path is described by the vector

$$\vec{R}_\gamma = \rho_\gamma (\hat{x} \sin i + \hat{z} \cos i) = \rho_\gamma \hat{e}_\gamma.$$

The position of the star is given by $\vec{R}_\star = a(\hat{x} \cos \phi + \hat{y} \sin \phi)$ and we suppose that the gamma-ray is produced in the jet at

$$\vec{z}_j = z_j (\hat{x} \sin \theta' \cos \psi + \hat{y} \sin \theta' \sin \psi + \hat{z} \cos \theta'),$$

where $\psi$ is the precessional phase. The position where the interaction with the soft photon takes place is indicated by

$$\vec{r}_\gamma = \vec{z}_j + \hat{R}_\gamma.$$  

Integration in $\theta'$ and $\phi'$ of expression (1) gives

$$\frac{d\tau_\star}{dE d\phi} = 2\pi n_\star(E) \sigma_{\gamma\gamma}(E, E_\gamma)$$

$$\times \left[1 - \cos \theta'_{\max} + \cos \theta'_{\min}\right]<br>$$

$$- \frac{\hat{e}_\gamma \cdot \hat{e}_{ph}}{2} \left[\sin^2 \theta'_{\max} - \sin^2 \theta'_{\min}\right].$$

where $\theta'_{\max}$ is the angle of interaction $\sqrt{\gamma^2 - (r_\gamma/R_\star)^2}$, $\theta'_{\min} = 0$ [17]. The density of radiation from the star can be approximated as usual by

$$n_\star(E) = \frac{2E^2}{(h c)^3} (e^{E/kT_\star} - 1) (\text{ph cm}^{-3} \text{erg}^{-1} \text{sr}^{-1}).$$

Further integrating in target photon energy $E$ and along the gamma-ray path $\rho_\gamma$ yields the resulting starlight contribution to the optical depth

$$\tau_\star = \int_{E_{\min}}^{\infty} dE \int_0^{\infty} d\rho_\gamma \frac{d\tau_\star}{dE d\phi}.$$  

where

![Fig. 1. Schematic view of SS433. The approaching jet is most of the time closer to the line of sight and the receding one is oppositely directed.](image-url)
\[ E_{\text{min}} = \frac{2(m_e c^2)^2}{E_{\gamma}(1 - \hat{e}_{\gamma} \cdot \hat{e}_{\text{ph}})}. \]  

(11)

Once one relates the position vectors with time, one can obtain \( \tau_\gamma(E_\gamma, t) \) as shown in Fig 2. These gamma-rays were assumed to be originated mostly near an injection point at the base of the approaching jet, \( z_0 = R_0/\tan \xi \simeq 1.3 \times 10^3 \text{ cm} \), where \( \xi \simeq 0.6 \) is the half opening angle of the jet and the initial jet radius is taken to be \( R_0 = 10G M_{\text{bh}}/c^2 \). It can be seen from Fig 2 that this component of the opacity is relevant just when the star is in the foreground of the emission point, namely when it crosses the \( x \)-axis.

3.2. Optical depth due to the emission from the extended disk

The extended disk wind is believed to be the origin of both mid-IR and UV emission. In the first case, the reported free-free emission was detected in the range of wavelengths 2-12 \( \mu \text{m} \) for which we adopt the following flux density fit

\[ F_{\nu} = 2.3 \times 10^{-23} \left( \frac{\nu}{\mu \text{m}} \right)^{-0.6} \text{ erg s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}. \]  

(12)

With an emitting region of radius \( R_{\text{out}} = 50R_\odot \), we estimate the corresponding density of mid-IR photons with energy \( E \) for \( r_\gamma < R_{\text{out}} \) as

\[ n_{\text{IR}} \approx \frac{hF_{\gamma}}{cE} \left( \frac{d}{R_{\text{out}}} \right)^2 \left( \text{ph cm}^{-3}\text{sr}^{-1}\text{s}^{-1} \right), \]  

(13)

where \( d = 5.5 \text{ kpc} \) is the distance to SS433. Taking \( \Delta E \) as the range of energies of the mid-IR emission, \( \Delta \rho_\gamma \) as the gamma-ray path where absorption is most significant, and considering frontal collisions, the implied contribution to the opacity of this photon field results

\[ \tau_\text{IR} \sim n_{\text{IR}} \sigma_\gamma \Delta \rho_\gamma \Delta E \ll 1, \]  

(14)

and hence, negligible.

As for the UV emission, it was detected in the range of wavelengths \( \sim (1000 \text{ }- \text{ } 10000)\AA \) \[21\] and, in a first conservative approach, we consider an associated black body temperature \( T_{\text{UV}} = 21000 \text{ K} \) as conjectured for the disk in an edge-on state, with a corresponding radius of the emitting zone \( R_{\text{in}} = 33R_\odot \). The inner radius of the disk is taken as \( R_{\text{in}} \approx 2G M_{\text{bh}}/v_w^2 \) as suggested in Ref. \[20\].

A quick estimate as the one just performed above for the mid-IR emission reveals that the UV optical depth is in contrast significant, so we will proceed to adequately integrate it as described in the Appendix. The obtained contribution is shown in Fig 3 as a function of time and gamma-ray energy. It can be noted from this plot that the maximum absorption occurs at the two times \( t \sim 50 \text{ d} \) and \( t \sim 110 \text{ d} \) when the gamma-ray path is parallel to the plane of the disk emitting zone. This implies \( \gamma\gamma \) collisions at larger angles resulting in higher cross sections and smaller threshold energies. Likewise, for intermediate times the absorption is lower because the collision angle is smaller.

![Fig. 2. Starlight optical depth.](image)

3.3. Optical depth due to \( \gamma N \) interactions.

The nucleons of the star and the thick extended disk can also absorb gamma-rays and photo-produce pions. We consider the cross section as in Ref. \[22\],

\[ \sigma_\gamma = \begin{cases} 340 \mu \text{b} & \text{for } 200 \text{ MeV} < E_\gamma < 500 \text{ MeV} \\ 120 \mu \text{b} & \text{for } E_\gamma > 500 \text{ MeV} \end{cases}, \]  

(15)

where the first case corresponds to the single pion channel and the second case to the multi-pion channel.

The density of the extended disk wind at a distance \( r_\gamma \) from the compact object is estimated to be

\[ \rho_\gamma \sim \frac{n_{\text{IR}}}{\xi^2}, \]  

(16)

as suggested in Ref. \[20\].
where the solid angle element is related to the disk wind half opening angle $\alpha_w$ by $\Delta \Omega = 4 \pi \sin \alpha_w$. As for the star, we suppose that it has a density given by

$$\rho_\star(r) = \frac{M_\star}{4 \pi R_\star r^2},$$

where $r$ represents the distance to the center of the star.

The optical depth due to these $\gamma N$ interactions can be estimated as

$$\tau_{\gamma N}(z_j) = \int_0^\infty d\rho_\gamma \sigma_{\gamma N} \frac{(\rho_\star + \rho_w)}{m_p}.$$

The obtained result is shown in Fig. 4 as a function of time for $200 \text{ MeV} < E_\gamma < 500 \text{ MeV}$ and $E_\gamma > 500 \text{ MeV}$. As it can be seen from this plot, periodic peaks of high absorption appear when the star is in the foreground.

3.4. Total optical depth

Analyzing the different contributions, it can be noted that below $E_\gamma = 1 \text{ GeV}$, the only relevant source of absorption is due to the $\gamma N$ interactions with the nucleons of the extended disk and the star.

If we add up all the contributions calculated above for the case of gamma-rays originated at $z_j = z_0$ in the approaching jet, we can obtain the total optical depth which is shown in Fig. 5 as a function of time and gamma-ray energy. It can be seen clearly from this plot that the absorption caused by $\gamma N$ interactions with the nucleons of the star is important along all the range of gamma-ray energies studied (200 MeV - 20 TeV).

4. SIGNA TURES ON A GAMMA-RAY SIGNAL

In the present work we have not specified any particular gamma-ray emission process. It is reasonable to expect, however, that the absorption effects here presented may be observable as long as the putative gamma-rays are produced mostly at the inner regions of SS433, i.e. in the vicinity of the black hole, near the base of the jets. This possibility seems likely, especially in view of the recent detection of gamma-rays from similar objects, such as LS I +61 303, LS 5039, and Cyg X-1 [40].

In an attempt to give an idea of the effect that the obtained optical depth may cause on an out-coming gamma-ray flux, we can, indeed, assume that most of the radiation is produced near the base of the jets (at $z_j \sim z_0$). In these conditions, if the spectrum of produced gamma-rays follows a power law as

$$J_\gamma = K_\gamma E_\gamma^{-2} \text{ (ph erg}^{-1}\text{sr}^{-1}\text{cm}^{-2}\text{-s}^{-1})$$

where $K_\gamma$ is a constant, the total luminosity within an energy range $(E_{\gamma \min}, E_{\gamma \max})$ is

$$L_\gamma = \Delta A \int_{E_{\gamma \min}}^{E_{\gamma \max}} dE_\gamma E_\gamma J_\gamma(E_\gamma),$$

where $\Delta A$ is the element of area of the emitting region near the injection point. It follows then, that

$$K_\gamma \Delta A = \frac{L_\gamma}{\ln \frac{E_{\gamma \max}}{E_{\gamma \min}}},$$

which allows us to estimate the corresponding photon flux to be detected at the Earth as

$$\Phi_\gamma(t) = \frac{\Delta A}{4 \pi d^2} \int_{E_{\gamma \min}}^{E_{\gamma \max}} dE_\gamma J_\gamma(E_\gamma) e^{-\tau_{\text{tot}}(t, E_\gamma)}.$$
For illustration, we assume that the equivalent isotropic gamma-ray luminosity between $E_{\gamma}^{\text{min}} = 200$ MeV and $E_{\gamma}^{\text{max}} = 20$ TeV is $L_\gamma \approx 10^{36} \text{ erg s}^{-1}$. The resulting flux is shown in the upper panel of Fig.6 whereas the flux corresponding to energies $E_{\gamma} > 800$ GeV is shown in the lower panel as compared to the upper limit given by HEGRA. In the first case the obtained mean photon flux is

$$\langle \Phi_\gamma \rangle|_{E_\gamma > 200 \text{MeV}} = 4.4 \times 10^{-6} \text{ ph cm}^{-2} \text{s}^{-1}, \quad (23)$$

and in the second case,

$$\langle \Phi_\gamma \rangle|_{E_\gamma > 800 \text{GeV}} = 5 \times 10^{-13} \text{ ph cm}^{-2} \text{s}^{-1}, \quad (24)$$

which is below the HEGRA cut, $\Phi_\gamma^{\text{lim}} = 8.9 \times 10^{-13} \text{ ph cm}^{-2} \text{s}^{-1}$.

We emphasize that these values are assumed only to make a qualitative description of a possible gamma-ray signal and that their true values may be obtained in a more detailed study regarding the emission process that could operate.

Fig. 6. Gamma-ray flux as a function of time for $E_\gamma > 200$ MeV (upper panel) and for $E_\gamma > 800$ GeV (lower panel) as compared to the upper limit given by HEGRA. The assumed gamma-ray luminosity is $L_\gamma = 10^{36} \text{ erg s}^{-1}$ and the maximum energy is $E_{\gamma}^{\text{max}} = 20$ TeV.

5. DISCUSSION

The most noticeable absorption signature that can be imprinted on a gamma-ray flux from the inner regions of SS433 is given by the regular and dramatic absorption caused by the star when it crosses the line of sight. This effect takes place approximately once every 13 days during the time that the star blocks the emitting region (≈ 3 d). In practice, this could serve to determine the size of the companion star as well as the mass distribution of the stellar atmosphere as long as sufficient time resolution can be achieved. It seems possible that, given the little absorption that corresponds to sub-GeV energies and the fact that more events are expected for lower energies, the GLAST instrument could detect a signal possibly with the time behavior predicted here. Another detectable absorption feature to observe at these energies corresponds to the $\gamma N$ interactions with the nucleons of the extended disk. This is expected to cause, because of the precession, a long term modulation of the signal reducing the maximum flux by $\sim 50\%$ as shown in the upper panel of Fig.6.

As for higher energies ($E_\gamma > 10$ GeV), absorption through $\gamma\gamma$ interactions also becomes important, mainly that caused by the UV emission from the extended disk wind, and secondly, the absorption due to the starlight photons. If the assumed luminosity in gamma-rays is similar to the real intrinsic luminosity, a detection at these energies seems more difficult but not impossible with the forthcoming Cherenkov telescopes MAGIC II and VERITAS.

Finally we remark again that the conclusions drawn in the present study are independent of the gamma-ray producing mechanism. Taking into account reprocessing of gamma-rays due to cascading effects [24], would not either alter the absorption patterns here predicted.

A comprehensive study, including the description of the physical mechanisms allowing the high-energy emission processes in SS433 will be presented elsewhere.

Acknowledgements We thank V. Bosch-Ramon for useful comments. G.E.R. is supported by the Argentine agencies CONICET (PIP 5375) and ANPCyT (PICT 03-13291 BID 1728/OC-AR). H.R.C. is supported by CNPq and FINEP, Brazil, and M.M.R. is supported by CONICET, Argentina. M.M.R. is also grateful to O. A. Sampayo for useful discussions relevant to the present work.

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Appendix A. Calculation of the gamma-ray absorption near a disk-like emitting region

We consider the absorption of gamma-rays as they travel near a disk that emits soft photons. This disk, with an inner radius $R_{in}$ and an outer radius $R_{out}$, lies in a plane perpendicular to the jets, where the gamma-rays emerge. For gamma-rays produced with energy $E_{\gamma}$, traveling in a direction $\hat{e}_{\gamma}$, the differential optical depth due to photons with a radiation density $n_{ph}$ and a direction given by $\hat{e}_{ph}$, can be written as (e.g. [19])

$$d\tau_{\gamma\gamma} = (1 - \hat{e}_{ph} \cdot \hat{e}_{\gamma}) n_{ph} \sigma_{\gamma\gamma} \cos \theta \frac{\cos \eta R_{d} dR_{d} d\phi_{d}}{l_{ph}^{2}} d\rho_{\gamma} dE. \quad (A.1)$$

Here the $R_{d}$ is the length of the vector $\vec{R}_{d}$ signaling a point in the disk, and $\phi_{d}$ is its corresponding azimuthal angle, and $l_{ph}$ is the magnitude of the vector $\vec{l}_{ph}$ connecting the point on the disk to the gamma-ray position. The angles and vectors involved in (A.1) are then to be expressed in a new coordinate system with its $Z$-axis oriented along the axis of the approaching jet.

In the system fixed to the compact object the unit vector $\hat{e}_{\gamma}$ is given by Eq. (1) in terms of the unit vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$, and the basis of the new system can be obtained by transforming according to

$$R = \begin{pmatrix} \cos \psi \cos \theta - \sin \psi \cos \sin \theta \\ \sin \psi \cos \theta \cos \psi - \sin \psi \sin \sin \theta \\ - \sin \theta & 0 & \cos \theta \end{pmatrix}. \quad (A.2)$$

Therefore, the unit vectors of the new coordinate system can be expressed in terms of fixed basis as

$$\hat{X} = \hat{x} \cos \theta \cos \psi + \hat{y} \cos \theta \sin \psi - \hat{z} \sin \theta \quad (A.3)$$
$$\hat{Y} = - \hat{x} \sin \psi + \hat{y} \cos \psi \quad (A.4)$$
$$\hat{Z} = \hat{x} \sin \theta \cos \psi + \hat{y} \sin \theta \sin \psi + \hat{z} \cos \theta, \quad (A.5)$$

and the unit vector $\hat{e}_{\gamma}$ can be written as

$$\hat{e}_{\gamma} = \hat{X} (\hat{e}_{\gamma} \cdot \hat{X}) + \hat{Y} (\hat{e}_{\gamma} \cdot \hat{Y}) + \hat{Z} (\hat{e}_{\gamma} \cdot \hat{Z})$$

$$\hat{e}_{\gamma} = \hat{X} (\rho_{\gamma} \cos \theta \sin \sin i - \rho_{\gamma} \sin \theta \cos i)$$
$$- \hat{Y} (\rho_{\gamma} \sin \theta \sin i)$$
$$+ \hat{Z} (\rho_{\gamma} \sin \theta \cos \sin i + \rho_{\gamma} \cos \theta \cos i). \quad (A.6)$$

Since the position of the gamma-ray is $\vec{r}_{\gamma} = \hat{e}_{\gamma} \rho_{\gamma} + \hat{z} z_{0}$ and the position on the disk is $\vec{R}_{d} = R_{d} (\hat{X} \cos \phi_{d} + \hat{Y} \sin \phi_{d})$, the vector $\vec{l}_{ph}$ can be obtained as $\vec{l}_{ph} = \vec{r}_{\gamma} - \vec{R}_{d}$.

The resulting optical depth is given by the quadruple integral

$$\tau_{\gamma\gamma} = \int_{0}^{\infty} d\rho_{\gamma} \int_{0}^{2\pi} d\phi_{\gamma} \int_{R_{in}}^{R_{out}} dR_{d} \int_{E_{min}}^{E_{max}} dE \frac{d\tau_{\gamma\gamma}}{d\rho_{\gamma} d\phi_{d} dR_{d} dE}. \quad (A.8)$$

where

$$E_{min} = \frac{2(m_{e} c^{2})^{2}}{E_{\gamma}(1 - \hat{e}_{\gamma} \cdot \hat{e}_{ph})}. \quad (A.9)$$

The integral can be performed using a Monte Carlo method, that is, introducing the variables $x_{\rho}$, $x_{\phi}$, $x_{R}$, and $x_{E}$ as

$$\rho_{\gamma} = \rho_{1} x_{\rho} \quad (A.10)$$
$$\phi_{d} = 2\pi x_{\phi} \quad (A.11)$$
$$R_{d} = R_{in} + (R_{out} - R_{in}) x_{R} \quad (A.12)$$
$$E = E_{min} + (E_{max} - E_{min}) x_{E}, \quad (A.13)$$

then the integral (A.8) can be written as

$$\tau_{\gamma\gamma} = \int_{0}^{1} dx_{\rho} \int_{0}^{1} dx_{\phi} \int_{0}^{1} dx_{R} \int_{0}^{1} dx_{E} f(x_{\rho}, x_{\phi}, x_{R}, x_{E}), \quad (A.14)$$

where

$$f(x_{\rho}, x_{\phi}, x_{R}, x_{E}) = 2\pi \rho_{1} (R_{out} - R_{in}) \times$$
$$\left( E_{max} - E_{min} \right) \frac{d\tau_{\gamma\gamma}}{d\rho_{\gamma} d\phi_{d} dR_{d} dE}. \quad (A.15)$$

Here the upper limit of the integration in $\rho_{\gamma}$ is taken as $\rho_{1} \approx 10 R_{out}$ where the integrand is significative.

The method relies on the fact that, by the Mean-Value Theorem, the integral can be approximated by

$$\tau_{\gamma\gamma} \approx \frac{1}{N} \sum_{i=1}^{N} f(x_{\rho}(i), x_{\phi}(i), x_{R}(i), x_{E}(i)), \quad (A.16)$$

where each variable takes a random number between 0 and 1. The right hand side is then the statistical average of $N$ evaluations of the function $f$. The error goes like $1/\sqrt{N}$, which in this case of four integration variables makes it a quite accurate method as compared to the iterative ones (see e.g. [25]).