Formation of Deeply Bound 1s Pionic States in the $^{206}\text{Pb}(d,^3\text{He})$ Reaction

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ABSTRACT

Recently, deeply bound pionic states were found experimentally in $(d,^3\text{He})$ reactions on $^{208}\text{Pb}$. The observed spectrum showed an excellent agreement with the DWIA calculation and the dominant peak was attributed to the pionic 2$p$ state contribution. We studied theoretically $^{206}\text{Pb}(d,^3\text{He})$ reactions within the same model, and found that it is very likely to observe the pionic 1$s$ state as an isolated peak in the $^{206}\text{Pb}(d,^3\text{He})$ reaction with feasible energy resolution and statistics.

Since the suggestion of Toki and Yamazaki for the formation of deeply bound pionic atoms such as 1$s$ and 2$p$ states in heavy nuclei by direct reactions\cite{1,2}, there have been a number of efforts to find these states both experimentally and theoretically\cite{3-9}. It is worth mentioning that $(n,d)$ reactions\cite{8} and $(p,pp)$ reactions\cite{9} on the $^{208}\text{Pb}$ target were able to identify some strength in the excitation function below the pion production threshold. It was, however, not yet convincing to claim for pionic atom formation due to the lack of good resolution and statistical accuracy. Very recently, Yamazaki et.al. performed an experiment of $(d,^3\text{He})$ reactions on $^{208}\text{Pb}$ with better resolution and statistical accuracy\cite{10}, and succeeded in identifying clearly a peak structure in the bound pion region. We found also that the theoretical predictions\cite{6} made before the experiment agree almost perfectly with the experiment. This agreement provides a strong confidence on the predictability of the theoretical model used.

We have analyzed the latest data in more detail using our model\cite{11} and found that the experimental peak structure consists of several contributions. The largest contributions are due to $[l\pi \otimes j_n^{-1}] = [2p \otimes p_{1/2}^{-1}]$ and $[2p \otimes p_{3/2}^{-1}]$. Contributions from the deepest pionic 1$s$ state, which is the most interesting, can be found only as a skewed shape of the experimental peak because the contributions from the pionic 1$s$ state and 2$p$ state could not be separated with the experimental energy resolution. We expect to identify the pionic 1$s$ contribution as a shoulder with much better energy resolution ($FWHM \approx 200keV$) as shown in Fig. 1 (a). However, it is very difficult to realize such high resolution experimentally. At higher incident energies, because of the matching condition of the momentum transfer, the $[1s \otimes i_{13/2}^{-1}]$ configuration makes an isolated peak. In this case, however, the excitation strength is small and the peak identification will be difficult\cite{11}.

In this paper we study the spectral shape and the excitation function of the
\(^{206}\text{Pb}(d,^3\text{He})\) reaction instead in order to observe the pionic \(1s\) contribution as a peak with feasible experimental energy resolution. Since the contribution of the \([2p \otimes p_{1/2}^-]\) configuration in the case of \(^{208}\text{Pb}\) makes an additional peak between the \([2p \otimes p_{3/2}^-]\) and the \([1s \otimes p_{3/2}^-]\) peaks, we may be able to remove this if we do not have contributions from the \(p_{1/2}^-\) neutron hole. It is expected that the valence two neutron holes in the ground state of \(^{206}\text{Pb}\) are dominantly \(p_{1/2}^-\). Hence, the assumption that the \(p_{1/2}^-\) orbital in the ground state of \(^{206}\text{Pb}\) is empty may hold. This is the reason why we consider \(^{206}\text{Pb}\) as a target nucleus to observe the pionic \(1s\) contributions as an isolated peak.

First we consider the \(^{206}\text{Pb}(d,^3\text{He})\pi^{-205}\text{Pb}\) reaction as the \(^{208}\text{Pb}(d,^3\text{He})\pi^{-207}\text{Pb}\) without contributions from two neutrons in the \(p_{1/2}^-\) orbital for qualitative discussions. We will calculate realistic spectra later. We have used the effective number approach for the theoretical calculation\(^4\),\(^6\),\(^12\). In Fig. 1 (a), the calculated spectrum of \(^{208}\text{Pb}(d,^3\text{He})\pi^{-207}\text{Pb}\) is shown. This calculated spectrum agrees with the data very well\(^10\),\(^11\). We show the spectrum without the contribution from the \(p_{1/2}^-\) neutron orbital in Fig. 1 (b). Clearly we can see the pionic \(1s\) state contributions as an isolated peak at \(Q \approx -134\text{MeV}\) which consists of \([1s \otimes f_{5/2}^-]\) and \([1s \otimes p_{3/2}^-]\).

We improve, then, the calculation by taking into account the realistic ground state configuration of \(^{206}\text{Pb}\), and the realistic excitation energies and strengths of \(^{205}\text{Pb}\). The experimental data show that the \(^{206}\text{Pb}\) ground state is not a pure \(p_{1/2}^-\) state but contains admixtures of other two neutron hole configurations and can be written as

\[
\Psi^{(206\text{Pb})_{g.s.}} = a(2p_{1/2})^{-2} + b(1f_{5/2})^{-2} + c(2p_{3/2})^{-2} + d(0i_{13/2})^{-2}
\]  

(1)

with

\[
a^2 = 0.54, \quad b^2 = 0.20, \quad c^2 = 0.12, \quad d^2 = 0.12.
\]

In our theoretical calculation, we assume the pion wavefunctions are the same as those of \(\pi^{-208}\text{Pb}\) system and the neutron single particle wave functions are those of \(^{208}\text{Pb}\). We calculate the effective numbers and normalize them using the occupation probability of each neutron state in the ground state of \(^{206}\text{Pb}\). For example, there are \(2 (= 2j + 1)\) neutrons in the \(2p_{1/2}\) orbital in the ground state of \(^{208}\text{Pb}\), while in the ground state of \(^{206}\text{Pb}\) there are only 0.92 \((= 2j + 1 - 2a^2)\) neutrons in average, where \(a\) is the \(2p_{1/2}\) \(-^2\) expansion coefficient in eq. (1). Hence, for the \(2p_{1/2}\) neutron orbital, the normalization factor is 0.46 \((= 0.92/2)\). This normalization is necessary to get realistic total strength for all transitions corresponding to the pickup of a neutron in each orbital. We show in Table 1 the normalization factor for each neutron state which appeared in eq. (1).

As for the excited levels and excitation strengths of \(^{205}\text{Pb}\), we use experimental excitation energies and spectroscopic factors obtained by \(^{206}\text{Pb}(d,t)^{205}\text{Pb}\)\(^4\). Since one neutron pickup reaction from a certain orbital in \(^{206}\text{Pb}\) can be coupled to some
excited states of $^{205}\text{Pb}$, we need to distribute the effective numbers to each excited levels of $^{205}\text{Pb}$ in proportion to the experimental spectroscopic factors. In Table 2 we show the relative strength of one neutron pickup processes from the same initial neutron state in the $^{206}\text{Pb}$, which should be multiplied to the effective numbers in order to include the effect of level splitting in the final $^{205}\text{Pb}$. We should mention here that we treat the $2p_{1/2}$ and $2p_{3/2}$ neutron states together as one neutron state since the spin of the two $p$ wave excited states of $^{205}\text{Pb}$, whose excitation energies are 0.80 MeV and 0.98 MeV, were not determined. Thus, we normalized the total effective numbers of these two states together using the factor in Table 1 and, then, distribute them to five $p$ wave excited states of $^{205}\text{Pb}$ in proportion to the experimental spectroscopic factors. In Table 3 we show the calculated binding energies and widths of $\pi^{-205}\text{Pb}$ atom using the same theoretical model as in ref. 2.

We will show an example using $[l_\pi \otimes l_{n^{-1}}] = [2p \otimes p_{1/2}^{-1}]$ and $[2p \otimes p_{3/2}^{-1}]$ configurations. Their effective numbers at $T_d = 600\text{MeV}$ were calculated to be $7.37 \times 10^{-3}$ and $1.47 \times 10^{-2}$. First they are normalized together for the ground state of $^{206}\text{Pb}$ as $[7.37 \times 10^{-3} + 1.47 \times 10^{-2}] \times 0.78 = 1.72 \times 10^{-2}$ using the normalization factor in Table 1. This effective number is distributed to each excited state of $^{205}\text{Pb}$ using the relative strength shown in Table 2. For example, the effective number for a state with $E_x = 0.002\text{MeV}$ is $1.72 \times 10^{-2} \times 0.234 = 4.02 \times 10^{-3}$. We performed similar calculations for all other configurations.

For pion quasi-free production and contributions from deeper neutron holes, we use the same spectrum as in Fig. 1 since it is not essential in this paper. Our model also implicitly postulates that the configuration mixing of the neutron-hole and pion-particle state in pionic atoms in $^{205}\text{Pb}$ is small as in the case of pionic atoms in $^{207}\text{Pb}$. This assumption can be checked by knowing the width of the peak since the mixing must broaden the width. In the latest data, the peak width is consistent with this assumption within the experimental energy resolution.

We show the calculated results in Fig. 2 where negative pion production threshold with ground state of $^{205}\text{Pb}$ corresponds to $Q = -140.87\text{MeV}$. We see that the pionic $1s$ contribution is also seen as an isolated peak in this realistic calculation. The peak includes contributions from $(1s_\pi (^{205}\text{Pb})_{g.s.}$ and $(1s_\pi (^{205}\text{Pb})_{0.26}\text{MeV}$. These two contributions can’t be distinguished in this case since the difference of the excitation energies of these two neutron levels (0.26MeV) are smaller than the width of the $1s$ pionic state due to the strong absorption effect. In Fig. 3 we show the same result as in Fig. 2 except for the different instrumental energy resolution. Even with the feasible energy resolution, we can see the pionic $1s$ contributions as a peak as we expected in the beginning of this paper. Hence, this should be a good reaction to observe the deepest pionic state.

We would like to comment here how we can choose the best incident energy for the pionic $1s$ state formation. In our theoretical model the elementary cross
section and the effective numbers provide the incident energy dependence of the $^3$He spectrum. The elementary cross section begins to be finite from the pion production threshold and takes the maximum value around $T_d = 600 MeV$. Then, it decreases with increasing energies. The energy dependence of the effective numbers was studied systematically for (n,d) reactions and it was found that the matching condition plays an important role. In the $^{206}$Pb(d,$^3$He)$\pi^{-}$-$^{205}$Pb reaction, the effective numbers for the pionic 1$s$ state formation with 2$p$ and 1$f$ neutron hole states takes maximum value around $T_d = 600 MeV$ where the elementary cross section is maximum. Thus, we think this incident energy for deuteron beam is best for the present case. For the (d,$^3$He) reaction with target nuclei which have valence neutrons in the s state, the best energy will be smaller than 600 MeV since the effective numbers for $[1s \otimes s^{-1}]$ configuration takes maximum value at zero momentum transfer.

In summary we have calculated the excitation spectrum of the $^{206}$Pb(d,$^3$He) reactions at $T_d = 600 MeV$ in order to observe the deepest pionic 1$s$ state. We have shown that the contributions from the pionic 1$s$ state can be seen as an isolated peak with the feasible experimental energy resolution. The peak includes both contributions from $[1s \otimes 2p^{-1}]$ and $[1s \otimes 1f_{5/2}^{-1}]$. Since the excitation energies of neutron hole states are known experimentally, we can get information on the pionic 1$s$ state from this peak. We believe that this theoretical work motivates further experimental efforts to develop the physics of pionic atom spectroscopy.

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Table 1
The occupation probabilities of neutron single particle states in the ground state of $^{206}$Pb, which are used as the normalization factors of the effective numbers for $(d,^3\text{He})$ reactions. The combined occupation probability of the $p$ state ($p_{1/2} + p_{3/2}$) is 0.78.

| State | Probability |
|-------|-------------|
| $2p_{1/2}$ | 0.46 |
| $2p_{3/2}$ | 0.94 |
| $1f_{5/2}$ | 0.93 |
| $0i_{13/2}$ | 0.98 |

Table 2
Excitation energies and spectroscopic factors in $^{205}$Pb from $^{206}$Pb(d,t)$^{205}$Pb data [13]. Relative excitation strengths of excited levels from the same initial neutron state in $^{206}$Pb are also shown. Each level is assigned according to the TABLE V in ref. 13.

| Level | $E_x$ [MeV] | S | Relative Strength |
|-------|-------------|---|------------------|
| $p_{1/2}$ | 0.002 | 1.67 | 0.234 |
| $p_{3/2}$ | 0.26 | 4.69 | 0.658 |
| $p_{3/2}$ | 0.58 | 0.13 | 0.018 |
| $p_{5/2}$ | 0.80 | 0.03 | 0.004 |
| $p_{9/2}$ | 0.98 | 0.61 | 0.086 |
| $f_{5/2}$ | 0.0 | 5.60 | 0.974 |
| $f_{5/2}$ | 0.75 | 0.15 | 0.026 |
| $f_{7/2}$ | 1.61 | 0.71 | 0.119 |
| $f_{7/2}$ | 1.77 | 5.27 | 0.881 |
| $i_{13/2}$ | 1.01 | 11.5 | 1.000 |
Table 3
Calculated binding energies and widths of $\pi^{-205}Pb$ atom in unit of keV.

| $nl$ | $E$  | $\Gamma$ |
|------|------|----------|
| 1s   | 7012 | 648      |
| 2s   | 2977 | 188      |
| 2p   | 5187 | 433      |
| 3s   | 1648 | 81       |
| 3p   | 2427 | 159      |
| 3d   | 2857 | 92       |
| 4s   | 1048 | 41       |
| 4p   | 1413 | 75       |
| 4d   | 1608 | 53       |
| 4f   | 1575 | 1.0      |
| 5s   | 727  | 24       |
| 5p   | 926  | 41       |
| 5d   | 1029 | 30       |
| 5f   | 1010 | 0.9      |
| 6s   | 536  | 15       |
| 6p   | 655  | 24       |
| 6d   | 716  | 18       |
| 6f   | 704  | 0.6      |
Fig. 1
(a) Calculated forward cross sections of the $^{208}$Pb(d,$^3$He) reactions at $T_d = 600 MeV$ with $200 keV$ experimental resolution. (b) Same figure as (a) without the contributions from $2p_{1/2}$ neutron state. In both figures, the vertical dashed line denotes the $\pi^-$ emission threshold energy and the flat background is assumed to be $20[\mu b/sr/MeV]$.

Fig. 2
Calculated forward cross sections of $^{206}$Pb(d,$^3$He) reactions at $T_d = 600 MeV$ with $200 keV$ experimental resolution, and with the use of the experimental neutron hole configurations and the excitation strengths. Thin solid line shows the contributions from the neutron $2p$ states ($p_{1/2}$ and $p_{3/2}$), and dashed curve those from the neutron $1f$ states ($f_{5/2}$ and $f_{7/2}$).

Fig. 3
Calculated forward cross sections of the $^{206}$Pb(d,$^3$He) reactions at $T_d = 600 MeV$ with $400 keV$ experimental resolution.
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