Room-temperature condensation of photons in a plasma cavity

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Bose–Einstein condensation of photons propagating inside a plasma-filled cavity is investigated.

The finite chemical potential is provided by the electronic density, which induces a finite photon mass that depends on local plasma variables and allows the condensation to occur. We derive an equation that models the evolution of the photon-mode occupancies, with Compton scattering taken into account as the mechanism of thermalization. The kinetic evolution of the photon spectrum is solved numerically, and we find evidences of condensation for realistic plasma parameters, \( n_e \sim 10^{12} - 10^{16} \text{ cm}^{-3} \). The critical temperature scales linearly with the number of photons, and we find high condensate fractions at room temperature, for reasonable cavity lengths \((1 - 10) \text{ mm}\) and photon numbers \((10^4 - 10^6)\).

Introduction — Over the past years, Bose–Einstein condensation has been accomplished with atomic species, including \(^7\text{Li}\) [1], spin-polarized \(^1\text{H}\) [2], metastable \(^4\text{He}\) [3] and \(^{41}\text{K}\) [4]. Despite the remarkable advances on the experimental realization of BECs, the possibility of producing a condensate of photons remained elusive for a long time. The reason relies on the vanishing chemical potential for free photons, which leads to non-conservation of the number of particles during thermalization, thus preventing the second-order phase transition to take place. This problem was first circumvented in Ref. [5], where it was shown that the presence of a cavity induces a finite photon mass, scaling inversely with the cavity length. The establishment of this mass is a direct consequence of the quantization of the photon momentum along the cavity axis. Nevertheless, no thermalization mechanism was proposed therein. This problem has been later addressed in Ref. [6, 7], where experimental evidence for the formation of a photon BEC in dye-filled cavities were first reported. Later on, other authors have observed photon condensation in similar physical setups [8, 9]. Such remarkable findings have motivated a number of theoretical studies, unveiling the mechanisms behind photon condensation with dye molecules [10–14] and atomic media [15, 16].

An alternative physical medium where one could imagine photons to undergo Bose–Einstein condensation is the plasma. As it is well known, photons have an effective mass in a plasma [17], as first suggested by Anderson through a Higgs-like mechanism [18–21]. As such, plasmas naturally solve the aforementioned photon mass problem and provide the finite chemical potential for condensation to occur. Contrary to the case of the experiments with optical cavities, photon condensation in a plasma has already been addressed as a bulk phenomenon, arising in homogeneous and unbounded systems [22]. This is particularly relevant in the astrophysical context, where external trapping potentials are absent. Indeed, the possibility of photon BEC in a plasma was first considered by Zel’dovich and Levich in 1968 [23], in relation to the distortion of the cosmic microwave background radiation through inverse Compton scattering — the so-called Sunyaev-Zel’dovich effect [24, 25]. However, the question of photon condensation inside finite-sized plasma systems have never been proposed.

In this Letter, we investigate the kinetics of photon thermalization in a cavity filled with a plasma, showing evidences of room-temperature Bose-Einstein condensation. The system under study takes advantage of both the photon mass inside a plasma \((m_\gamma = \hbar \omega_p/c^2\text{, with } \omega_p\text{ the plasma frequency})\) and the boundary effects induced by the cavity, which lead to a discretization on the photon momentum modes. The mode discreteness yields a finite critical temperature despite the system being effectively one-dimensional. Here, we derive and solve the kinetic equations accounting for the evolution of the photon spectrum, with Compton scattering taken as the mechanism of interaction, after integrating out the electron degrees of freedom. This results in an effective interaction between the photons, mediated by the plasma. For sufficiently high number of photons, we find high macroscopic fractions occupying the ground state. Moreover, the ground-state energy can be varied by controlling the cavity length and electron density. Remarkably, the critical temperatures above which the condensate fraction vanishes are extremely high \((\sim 10^3 - 10^4 \text{ K})\) when compared to the critical temperatures in customary BEC experiments with identical photon numbers, and also compatible with the typical plasma temperatures. Hence our findings open the possibility of achieving room-temperature condensation and creating a source of coherent photons that, contrary to laser systems, is established through a thermalization process. Such a macroscopic state has a plethora of applications in a future generation of photon-based devices.

Plasma-wave equations.— We start by revising the theoretical framework of the coupling between photons and electrons in a plasma. Essentially, the effect of the plasma is to modify the refraction index of the medium, which
becomes $n(\omega) = (1 - \omega_p^2/\omega^2)^{1/2}$, with $\omega_p^2 = e^2 n_e^2/\epsilon_0 m_e$ and $e$ being the elementary charge, $n_e$ the electron density, $\epsilon_0$ the vacuum permittivity, and $m_e$ the electron mass. The frequency becomes space and time dependent, through the local electron density $n_e \equiv n_e(\mathbf{r}, t)$. Conversely, the ion motion is negligible due their high inertia, and the photon dynamics is mostly determined by the electrons. The photon dispersion relation follows from $\omega = ck/n(\omega)$ [26].

To derive the details of the photon-plasma coupling, we resort to Maxwell equations. We start by

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \partial_t \mathbf{E} + \mu_0 \mathbf{J},$$

with $\mathbf{B}$ denoting the magnetic field, $\mathbf{E}$ the electric field and $\mathbf{J}$ the charge current density. The last term on the right hand side is responsible for the coupling, through $\mathbf{J} = \sum_j n_j \mathbf{u}_j$, with $j$ running over the different plasma species with charge $Q_j$, density $n_j$ and velocity $\mathbf{u}_j$. In their turn, the plasma fields $n_j$ and $\mathbf{u}_j$ evolve with their own classical equations of motion coupled to the electromagnetic fields,

$$\partial_t n_j + \nabla \cdot (n_j \mathbf{u}_j) = 0 \quad \text{(2)}$$

$$\partial_t \mathbf{u}_j + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = \frac{Q_j}{m_j} (\mathbf{E} + \mathbf{u}_j \times \mathbf{B}) - \frac{1}{m_j n_j} \nabla P_j,$$  \hspace{1cm} \text{(3)}

where $m_j$ is the mass of the specie $j$ and $P_j$ is the pressor tensor. Following the usual prescription, we write the electromagnetic fields in terms of the potentials $\phi$ and $\mathbf{A}$, $\mathbf{E} = -\nabla \phi - \partial_t \mathbf{A}$ and $\mathbf{B} = \nabla \times \mathbf{A}$. Replacing for those in Eq. (1) and choosing the Coulomb gauge, leads to

$$\left(\nabla^2 - \frac{1}{c^2} \partial_t^2\right) A = \mu_0 \mathbf{J}_\perp,$$  \hspace{1cm} \text{(4)}

where $\mathbf{J}_\perp = \mathbf{J} - (\mathbf{e}_k \cdot \mathbf{J}) e_k$ is the transverse component of the density current with respect to $\mathbf{e}_k = \mathbf{k}/k$. Now, we linearize the above equations with respect to small perturbations and neglect second order terms. Since photons are high-frequency waves, we retain only the electron quantities and neglect the effect of the ions, which are slower. This amounts to retain only the electron quantities by taking the limit $m_i \to \infty$, which restricts the sum in $\mathbf{J}$ to $j = e$. Electromagnetic perturbations propagating in the medium verify $\mathbf{k} \cdot \mathbf{E} = 0$ (as opposed to electrostatic perturbations), thus $n_e \approx n_0$, i.e., the first-order perturbation to the electron density vanishes. As a result, Eq. (2) provides $\mathbf{k} \cdot \mathbf{u}_{e,1} = 0$, with $\mathbf{u}_{e,1}$ the first-order perturbation to the electron velocity. Joining this condition to Eq. (4) yields

$$\left(\nabla^2 - \frac{1}{c^2} \partial_t^2\right) A = \frac{\omega_p^2}{c^2} A.$$  \hspace{1cm} \text{(5)}

Equation (5) takes the form of a Klein–Gordon equation with a source term, which can be interpreted as photons acquiring a mass as a result of the interaction with the material medium. To see that, we Fourier transform the latter to obtain the photon dispersion

$$\omega = \omega_k = (\omega_p^2 + c^2 k^2)^{1/2},$$  \hspace{1cm} \text{(6)}

which, when compared to the relativistic formula for the energy leads to the photon mass $m_\gamma = \hbar \omega_p/c^2$. The value of the mass is proportional to the electron density, hence we recover the free dispersion $\omega = ck$ in the absence of the plasma. Equation (6) can now be compared to $\omega = ck/n$, from which we can extract the refraction index $n(\omega) = (1 - \omega_p^2/\omega^2)^{1/2}$.

**Kinetic model.**— In the case of a fully ionized plasma, electron-photon elastic scattering processes are the main source of interaction. We follow the Boltzmann approach and calculate the variation of the number of particles measured by a joint distribution function $\rho(\mathbf{p}, \mathbf{k}, t)$ for electrons in mode $\mathbf{p}$ and photons in mode $\mathbf{k}$ at time $t$. We have

$$\partial_t \rho(\mathbf{p}, \mathbf{k}, t) = J_+(\mathbf{p}, \mathbf{k}, t) - J_-(\mathbf{p}, \mathbf{k}, t),$$  \hspace{1cm} \text{(7)}

with $J_+(\mathbf{k})$ the number of particles per unit volume per unit time that enters (leaves) the joint phase-space cell $\text{d}^3 \mathbf{p} \text{d}^3 \mathbf{k}$ centered in $(\mathbf{p}, \mathbf{k})$ due to the scattering event. The currents can be written as

$$J_+(\mathbf{p}, \mathbf{k}, t) = \int \text{d}^3 \mathbf{p}' \text{d}^3 \mathbf{k}' \rho(\mathbf{p}', \mathbf{k}', t) w(p', k' \to p, k) \times [1 + N(\mathbf{k}, t)][1 - F(\mathbf{p}, t)],$$  \hspace{1cm} \text{(8)}

$$J_-(\mathbf{p}, \mathbf{k}, t) = \int \text{d}^3 \mathbf{p}' \text{d}^3 \mathbf{k}' \rho(\mathbf{p}, \mathbf{k}, t) w(p, k \to p', k') \times [1 + N(\mathbf{k}', t)][1 - F(\mathbf{p}', t)],$$  \hspace{1cm} \text{(9)}

with $N(\mathbf{k}) = n_{e,1}^{-1} \int \text{d}^3 \mathbf{p} \rho(\mathbf{p}, \mathbf{k}, t)$ and $F(\mathbf{p}, t) = n_{e,1}^{-1} \int \text{d}^3 \mathbf{k} \rho(\mathbf{p}, \mathbf{k}, t)$ denoting the photon and electron distributions with densities $n_{e,1}$ and $n_e$, respectively. The factor $w(p, k \to p', k')$ stands for the transition rate for incoming $(p, k)$ and final $(p', k')$ states, with $p$, $k$, $p'$ and $k'$ the four-vector momenta associated with the electron and photon degrees of freedom. The conservation laws for the scattering event are expressed in the transition rates through $w(p, k \to p', k') \propto \delta(p + k - p' - k')$. This ensures momentum and energy conservation in the form $\mathbf{p} + \mathbf{k} = \mathbf{p}' + \mathbf{k}'$ and $\epsilon_p + \hbar \omega_k = \epsilon_{p'} + \hbar \omega_{k'}$, with $\epsilon_p = \gamma_p m_e c^2 \approx \omega_p^2/(2m_e)$ the electron dispersion and $\gamma_p$ the electron Lorentz–factor. The non-relativistic approximation remains valid when $k_B T_e \ll m_e c^2 \approx 0.5$ MeV, with $T_e$ the electron temperature. Additionally, the factors $1 + N$ and $1 - F$ in Eqs. (8) and (9) account for quantum degeneracy of each population, i.e., they ensure that fermions do not occupy the same state and bosons tend to occupy the same state. However, since we are interested in hot electrons, we treat those as point particles and neglect the electron degeneracy by setting $1 - F \approx 1$. On the contrary, the photon degeneracy should not be discarded, for it results in a non-linear term of order $N^2$, responsible for the condensation process.
When only the photon dynamics of interest, it is convenient to integrate out the electron degrees of freedom. This procedure is valid as long as the correlations between electrons and photons can be neglected; in other words, when the following expansion holds,

\[ \rho(p, k, t) \simeq F(p, t)N(k, t) + \text{correlations}, \]

with the second term on the right hand side being much smaller than the first, so it can be discarded. In fact, correlations are small whenever there is a separation of time scales, which in the present case amounts to have the electron gas equilibrated much faster than the photons. Under those assumptions, and invoking dynamical reversibility in the form \( w(p, k \to p', k') = w(p', k' \to p, k) \), the Boltzmann equation reduces to an equation for the photon distribution function,

\[ \partial_t N(k, t) = \frac{1}{n_e} \int d^3p \int d^3p' d^3k' \ w(p, k \to p', k') \times \{ F(p', t)N(k', t)[1 + N(k, t)] - F(p, t)N(k, t)[1 + N(k', t)] \}. \] (11)

The explicit form of \( w(p, k \to p', k') \) depends on the microscopic scattering process. In this case we consider Compton scattering, which reads as a special case of the complete Klein-Nishina scattering in the limit of non-relativistic electrons and soft photons (\( \hbar \omega \ll m_e c^2 \)). The Compton differential cross-section reads

\[ \frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{16\pi} \left(1 + \cos^2 \theta\right), \] (12)

where \( \sigma_T \approx 6.65 \times 10^{-29} \text{ m}^2 \) is the Thompson cross-section and \( \theta \) is the photon scattering angle. At this point it is convenient to introduce the discretized photon momenta \( k \equiv k_\ell = \pi \ell \epsilon_e / d \), where \( \ell \) is an integer, \( d \) is the cavity length and \( \epsilon_e \) is directed along the longitudinal axis of the cavity. The discretized photon frequencies will be denoted by \( \omega_\ell \equiv \omega_{k_\ell} \). We will further suppose that the photons are confined to move in one dimension along the cavity axis. This amounts to the replacement

\[ \int d^3k' \ w(p, k \to p', k') \to \sum_\ell \tilde{w}(p, k_\ell \to p', k_\ell), \] (13)

where \( \tilde{w} \) is the appropriate transition rate in terms of the discretized photon momenta. Deviations from the \( z \)-direction are suppressed due to the \( \cos^2 \theta \) dependence in Eq. (12). For sufficiently high temperature, the electron distribution function can be approximated by the time-independent Maxwell-Boltzmann function at temperature \( T_e \), \( F(p) = F_0 \exp(-\epsilon_p / k_B T_e) \), where \( F_0 \) ensures the normalization \( \int d^3p \ F(p) = n_e \). The electron equilibrium is assumed to be maintained throughout the experiment, such that \( \partial_t F \simeq 0 \) is valid during the photon equilibration process.

The Boltzmann equation can be simplified to the following balance equation

\[ \partial_t N_\ell = \sum_{\ell'} \left[ N_{\ell'} W_{\ell \ell'} - N_\ell - (1 - W_{\ell \ell'}) N_\ell N_{\ell'} \right], \] (14)

with \( N_\ell \equiv N(k_\ell, \tau) \) the photon-mode occupancies. We further defined the normalized energy shift as \( \Delta_{\ell \ell'} = \hbar(\omega_{\ell'} - \omega_\ell) / (k_B T_e) \), such that \( W_{\ell \ell'} = \exp(\Delta_{\ell \ell'}) \). Moreover, \( \tau = t / t_0 \) is the normalized time and \( t_0 = 8\pi / (\sigma_T n_e c) \) is the typical time scale for the thermalization in the cavity. Equation (14) defines a system of coupled first-order differential equations, that can be solved after an initial distribution \( N_\ell(0) \) is provided. The last term vanishes as \( T_e \to \infty \), and the equation becomes linear in \( N_\ell \), which prevents the formation of a condensate [27, 28]. Hence there must be a critical temperature \( T_c \) above which the condensate no longer develops. In cold atom experiments, the value of \( T_c \) typically ranges between a few \( nK \) and \( \mu K \), depending on the density and mass of the atomic cloud. In fact, theoretical calculations reveal that \( T_c \) scales as \( 1/m \), with \( m \) typically in the range \( 10^{-27} - 10^{-26} \text{ kg} \) for atomic BECs. In the present case, an estimate for the photon mass is given by \( \hbar \omega_p / c^2 \), which is about 12 to 13 orders of magnitude below the typical atomic masses. We can thus anticipate much higher critical temperatures.

**Thermalization process and condensation.**—We now proceed to solve Eq. (14) numerically. The occupancies are initiated as an excitation (e.g., a laser) with Lorentzian distribution centered at \( \ell = \ell_0 \), with bandwidth \( \Gamma \) and total number of photons \( N \),

\[ N_\ell(0) = \frac{N}{\pi (\ell - \ell_0)^2 + (\Gamma / 2)^2}. \] (15)

The solution of Eq. (14) depends essentially on the electron temperature, total number of photons and cavity length. Fig. 1 shows the initial, intermediate and steady-state occupancies as a function of the mode number, for the case of thermal and condensate regimes. We choose to show only the case of \( \ell > 0 \) since the steady-state solution is symmetric with respect to the sign of \( \ell \), due to the degenerate energy dispersion. We verified that, regardless of the external parameters, the initial distribution approaches a Bose-Einstein distribution,

\[ f(\epsilon, T, \mu) = \frac{1}{\exp(\epsilon / k_B T) - 1}, \] (16)

with \( T \) being a function of time that approaches \( T_e \) as \( t \to \infty \). Hence we say that the effect of the electron bath is to thermalize the photon gas: after each collision with the electrons, photons tend to distribute according to Eq. (16) with \( \epsilon = \hbar \omega_\ell \). The time of formation of the condensate is of the order of tens of milliseconds, which is comparable to the time of formation of atomic BECs.

The crossover between the BEC and thermal phases is governed by the chemical potential, which is defined by the temperature and total numbers of particles through \( N = \sum_\ell f(\hbar \omega_\ell, T, \mu) \). The latter bears a solution of the form \( \mu \equiv \mu(N, T) \). When the number of photons surpasses a critical number \( N_c \), the excess particles occupy the ground state, and we have \( \mu(N > N_c, T) \) of the order of \( \epsilon_0 \), with \( \epsilon_0 \) the ground-state energy, so that Eq. (16)
\[ N = 1000 \text{ and } T = 0.2 \text{ eV}, \] which yields \( T > T_c \), and the initial distribution approaches a thermal Bose-Einstein distribution with \( \mu \ll E_0 \). (b) \( N = 3000 \) and \( T = 0.01 \text{ eV}, \) which leads to \( T < T_c \) and the formation of a condensate, \( \mu \simeq E_0 \). The solid black lines shows the Bose-Einstein distribution with \( T = T_c \) for each case, after the system had reached thermal equilibrium. Other parameters are: \( d = 1 \text{ mm}, n_e = 10^{15} \text{ cm}^{-3}, \ell_0 = 20, \) and \( \Gamma = 5.\)

FIG. 2. Condensate fraction as a function of the electron temperature, (a) for different values of the reduced photon number \( n = N/1000 \) and cavity length, and (b) for different values of the cavity length with \( N = 1000.\)

attains large values at the origin (the numeric result for \( \mu \) is shown in the left panel of Fig. 3). We verified that, for fixed number of photons, there is a temperature below which the occupancies of the ground states \( (\ell = \pm 1) \) become of the order of the total number of photons, and the systems enters in the BEC phase. As the temperature is increased, the ground-state occupancies decrease, as the states become equiprobable. In Fig. 2 we depict the condensate fraction for different configurations. Those profiles are similar to the ones given by the standard non-interacting case: for small \( T_c \), the photons are mainly in the ground state, and as the temperature increases, the function rounds off and falls to zero, at about \( T \sim T_c.\)

FIG. 3. (a) Chemical potential normalized to the ground-state energy \( (\epsilon_0 = 7.41 \times 10^{-3} \text{ eV}) \) as a function of the electron temperature, using \( N = 10^5 \) and \( d = 10^{-4} \text{ m}.\) The BEC phase is characterized by a region of almost constant \( \mu \), of the order of the ground-state energy; when \( \mu \) decreases abruptly, the system enters in the thermal phase, with distribution spectrum of a thermal Bose gas. The result was calculated by fitting a Bose-Einstein distribution to the steady-state solutions of Eq. (14). (b) Critical temperature as a function of the total number of photons for different values of the cavity length. The critical temperature approaches a straight line for \( d \gg c/\omega_p.\)

It is also convenient to obtain an analytical estimate for \( T_c.\) The exact definition requires separating the contribution of the number of particles in the ground-state \((N_0)\) from the remaining states, \( N - N_0 = \sum_{\ell \neq \pm 1} f(\hbar \omega_\ell, T, \mu).\) At the critical temperature, we replace \( \mu \) by \( \epsilon_0 \) and neglect \( N_0,\) to get

\[ N = g \sum_{\ell = 2} f(\hbar \omega_\ell, T_c, \epsilon_0), \tag{17} \]

where \( g = 2 \) is the degeneracy factor. We extract the relation \( T_c = T_c(N) \) by solving Eq. (17) numerically, which we plot in Fig. 3. Typically, an analytical estimate for \( T_c \) is available in the thermodynamic limit (in this case, that is \( d \to \infty \) and \( N \to \infty \) while \( N/d \) is maintained finite). However, as it has been recognized, the thermodynamic limit yields \( T_c = 0 \) when the spacial dimension of the condensate is less than three [29]. Although this prevents condensation from developing in very large systems, the result is modified when the system is considered finite. Therefore, instead of taking the thermodynamic limit, we simply assume that \( d \gg c/\omega_p, \) while being finite. The summation can be replaced by an integral (which remains valid as long as the energy spacing is negligible compared to the temperature), and Eq. (17) becomes

\[ N \approx \frac{gd}{\pi} \int_{\omega_0}^{\infty} dk f(\hbar \omega_k, T_c, \epsilon_0). \tag{18} \]

Performing the integration yields

\[ T_c = \frac{\hbar^2 k_0^2}{\xi m_\gamma k_B} N, \tag{19} \]
with \( k_0 = \pi / d \) the ground-state wavevector and \( \xi = 2\pi - 4 \arctan 2 \simeq 1.9 \) is a constant. Inserting numerical values, and using \( d = 1 \text{mm} \), gives \( T_c = 1.76N \times 10^{-4} \text{eV} \), which is in good agreement with the exact result of Fig. 3 [panel (b)].

As anticipated above, the value of \( T_c \) is much higher than the typical values of atomic BECs. The photon mass is present in the relation for the critical temperature, since for large \( d \), the photon dispersion approaches \( \epsilon_k = m_\gamma c^2 + \frac{h^2 k^2}{2 m_\gamma} \), which quadratic with a cut-off that can be absorbed into the chemical potential. Additionally, Eq. (19) provides \( T_c = 0 \) when \( d \to \infty \) and \( N/d \) is finite because it depends on \( N/d^2 \). This is consistent with previous investigations on finite-sized BECs [30].

**Conclusions.**— We derived a kinetic model for the evolution of the photon-mode occupations in contact with a plasma, starting from the Boltzmann equation. The electron population is considered to be in constant equilibrium at temperature \( T_e \), which modifies the photon dispersion (by opening a gap of \( h\omega_p \) at \( k = 0 \)) and thermalizes the photon gas due to multiple Compton scattering. After integrating out the electron degrees of freedom, we obtained an effective equation for the photons which resembles a balance equation of statistical physics, that we solved numerically. The solution showed that the photon gas approaches a Bose-Einstein distribution at the thermalization of the photon-mode occupancies in contact with a plasma at temperature \( T_e \), which modifies the photon dispersion — the photon mass is determined by the cavity cut-off frequency, typically in the range of \( 10^{14} \text{Hz} \). Here, the plasma frequency establishes an even smaller photon mass, which yields higher critical temperatures. On the other hand, the ground-state wavelength is determined by the cavity distance, which gives an extra degree of control over the BEC parameters. This opens the possibility of setting the BEC wavelength over a wide range of values, which may have technological applications on the search for new light sources. Moreover, extensions to the case of solid-state degenerate plasmas — eventually leading to condensation of photons in a partially filled cavity —, as well accounting for the dynamics of the reservoir, deserve further investigation.

**Acknowledgments.**— J. L. F. and H. T. acknowledge Fundação da Ciência e a Tecnologia (FCT-Portugal) through the Grants No. PD/BD/135211/2017, UI/BD/151557/2021, and through Contract No. CEECIND/00401/2018 and Project No. PTDC/FIS-OUT/3882/2020, respectively.

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