GENERAL RELATIVISTIC MAGNETOHYDRODYNAMIC SIMULATIONS OF COLAPSARS: ROTATING BLACK HOLE CASES

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Received 2004 April 6; accepted 2004 July 1

ABSTRACT

We have performed 2.5-dimensional general relativistic magnetohydrodynamic (MHD) simulations of collapsars including a rotating black hole. This paper is an extension of our previous paper. The current calculation focuses on the effect of black hole rotation using general relativistic MHD with simplified microphysics; i.e., we ignore neutrino cooling, physical equation of state, and photodisintegration. Initially, we assume that the core collapse has failed in this star. A rotating black hole of a few solar masses is inserted by hand into the calculation. We consider two cases, a corotating case and a counterrotating case with respect to the black hole rotation. Although the counterrotating case may be unrealistic for collapsars, we perform it as the maximally dragging case of a magnetic field. The simulation results show the formation of a disklike structure and the generation of a jetlike outflow near the central black hole. The jetlike outflow propagates outwardly with the twisted magnetic field and becomes collimated. We have found that the jets are generated and accelerated mainly by the magnetic field. The total jet velocity in the rotating black hole case is comparable to that of the nonrotating black hole case, \( v \approx 0.3c \). When the rotation of the black hole is faster, the magnetic field is twisted strongly owing to the frame-dragging effect. The magnetic energy stored by the twisting magnetic field is directly converted to kinetic energy of the jet rather than propagating as an Alfvén wave. Thus, as the rotation of the black hole becomes faster, the poloidal velocity of the jet becomes faster. In the rapidly rotating black hole case the jetlike outflow can be produced by the frame-dragging effect only through twisting of the magnetic field, even if there is no stellar rotation.

Subject headings: accretion, accretion disks — black hole physics — gamma rays: bursts — methods: numerical — MHD — relativity — supernovae: general

Online material: color figures

1. INTRODUCTION

It is now generally believed that “long-soft” gamma-ray bursts (GRBs) are a phenomenon related to the deaths of massive stars. There is direct and indirect observational evidence of the close relationship between GRBs and supernovae, such as the observed association with star-forming regions in galaxies (Vreeswijk et al. 2001; Bloom et al. 2002b; Gorosabel et al. 2003), the “bump” observed in the afterglows of some GRBs (Reichert 1999; Galama et al. 2000; Bloom et al. 2002b, 2004; Garnavich et al. 2003), metal emission lines observed in the X-ray afterglow of GRB 011211 (Reeves et al. 2002), and the association of GRB 980425 with SN 1998bw (Galama et al. 1998). Recently, further clear evidence was found: GRB 030329 was accompanied by a bright energetic Type Ic supernova, SN 2003jd (Price et al. 2003; Hjorth et al. 2003; Stanek et al. 2003). It has been suggested so far that some GRBs are produced when the iron core of a massive star collapses either to a black hole (Woosley 1993; MacFadyen & Woosley 1999) or to a rapidly rotating highly magnetic neutron star (Wheeler et al. 2000), both of which eventually produce a relativistic jet.

The collapsar model is one of the most promising scenarios involving massive stars (Woosley 1993; MacFadyen & Woosley 1999). A collapsar is a rotating massive star that lacks a hydrogen envelope. In this model, the iron core of the rotating massive star collapses to a black hole surrounded by an accretion disk. The accretion through this disk produces outflows via neutrino annihilation and/or magnetohydrodynamic (MHD) processes. They are further collimated by the passage through the stellar mantle. The formation and propagation of relativistic flows from collapsars have been studied numerically by both Newtonian (MacFadyen & Woosley 1999; MacFadyen et al. 2001) and special relativistic hydrodynamic simulations (Aloy et al. 2000; Zhang et al. 2003, 2004). However, these previous numerical simulations of the collapsar model did not fully address the formation mechanism of the relativistic outflow.

It is suspected that magnetic fields may play an important role in the formation and acceleration of relativistic jets. In numerical simulations of the gravitational collapse of massive stars the effect of stellar rotation and intrinsic magnetic fields was studied by several authors (LeBlanc & Wilson 1970; Symbalisty 1984; Ardeljan et al. 2000; Kotake et al. 2004; Yamada & Sawai 2004). Symbalisty (1984) showed the formation of high-density, supersonic jets in the combination of a rapid rotation and a strong dipole magnetic field, which was confirmed by Yamada & Sawai (2004).

In our previous paper (Mizuno et al. 2004) we performed 2.5-dimensional general relativistic MHD simulations of the gravitational collapse of a rotating magnetized massive star with a nonrotating black hole at the center as a model for

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collapsars. We showed the formation of a disklike structure and the generation of a mildly relativistic jet (∼0.3c) inside the shock wave launched at the core bounce. We have found that the jet is accelerated by the magnetic pressure and the centrifugal force and is collimated by the pinching force of the toroidal magnetic field amplified by the rotation and the effect of geometry of the poloidal magnetic field. The Poynting flux transports more energy outward than the jet. However, the jet in these simulations is too slow to be applicable to GRBs. In our previous paper, we put a nonrotating black hole at the center, which is not appropriate for rotational collapse. It is natural that the iron core collapses to a rotating black hole. A rotating (Kerr) black hole is useful for forming the relativistic jet. If the black hole is rotating, two kinds of energy are available for generating a jet. One is the rotational energy of the accreted matter, and the other is the rotational energy of the rotating black hole itself. Blandford & Znajek (1977) investigated the magnetospheres of Kerr black holes and derived a force-free solution for the electromagnetic field. Their results showed that electromagnetic energy is radiated from the black hole horizon directly. This is called the Blandford-Znajek mechanism. However, the direct energy emission from the black hole horizon appears to be inconsistent with the causality at the horizon. Punsly & Coroniti (1990) found that if the ergospheric plasma gets frozen onto large-scale magnetic field lines, it can drive a magnetic wind to infinity. Using a general relativistic MHD code, the basic mechanism of energy extraction from the Kerr black hole via magnetic field is investigated numerically (Koide et al. 2002; Koide 2003). The numerical results showed that the rotational energy of the Kerr black hole can be extracted when the magnetic field is strong enough.

Here, as a collapsar model we perform 2.5-dimensional general relativistic MHD simulations of the gravitational collapse of a rotating magnetized massive star with a rotating black hole at the center. We investigate the physics of the formation of jets, the acceleration force on the jets, and the dependence on the rotation parameter of the black hole. This paper is an extension of our previous paper (Mizuno et al. 2004). We describe the numerical method of our simulations briefly in § 2 and present our results in § 3. The summary and discussion are given in § 4.

2. NUMERICAL METHOD

2.1. Basic Equations

In order to study the formation of relativistic jets from collapsars we use a 2.5-dimensional general relativistic magnetohydrodynamics (GRMHD) code with Boyer-Lindquist coordinates (R, θ, φ) (Koide 2003; Mizuno et al. 2004). The method is based on a 3 + 1 formalism of the general relativistic conservation laws of particle number and energy momentum, Maxwell equations, and Ohm’s law with no electric resistance (the ideal MHD condition) on a curved spacetime (Thorne et al. 1986; Koide et al. 2000; Koide 2003; Mizuno et al. 2004).

The spacetime (σ0, x1, x2, x3) = (ct, x1, x2, x3) is described by the metric gμν, where the line element ds is given by (ds)2 = gμν dxμ dxν. Here, c is the speed of light. If we assume that the off-diagonal spatial elements of the metric gμν vanish,

\[ g_{ij} = 0 \quad (i \neq j), \]

and we use the notation,

\[ g_{00} = -h_0^2, \quad g_{ii} = h_i^2, \]
\[ g_{0i} = g_{0i} = -h_i^2 \omega_i / c, \]

then the line element can be written

\[ (ds)^2 = g_{μν} dx^μ dx^ν \]
\[ = -h_0^2 c dt^2 + \sum_{i=1}^3 \left[ h_i^2 (dx^i)^2 - 2h_i^2 \omega_i dt dx^i \right]. \]

When we define the lapse function α and “shift velocity” (shift vector) β as

\[ \alpha = \sqrt{h_0^2 + \sum_{i=1}^3 \left( \frac{h_i \omega_i}{c} \right)^2}, \]
\[ \beta^i = \frac{h_i \omega_i}{c \alpha}, \]

the line element ds is written as

\[ (ds)^2 = -\alpha^2 c dt^2 + \sum_{i=1}^3 \left( h_i dx^i - c \beta^i \alpha dt \right)^2. \]

The contravariant metric is written explicitly as

\[ g^{00} = -\frac{1}{\alpha^2}, \]
\[ g^{0i} = g^{0i} = -\frac{\omega_i}{c}, \]
\[ g^{ii} = \frac{1}{h_i h_j} (\delta^{ij} - \beta^i \beta^j), \]

where δij is the Kronecker’s δ symbol.

In the GRMHD code, a simplified total variation diminishing (TVD) method is employed (Davis 1984). This method is similar to the Lax-Wendroff method with addition of a diffusion term and is useful because it requires only the maximum speed of physical waves but not each eigenvector or eigenvalue of the coefficient matrix of the linearized GRMHD equations.

We do not consider the evolution of the metric because the accreted mass is sufficiently small on the timescale of the simulations. Previous works of collapsar simulation included microphysics. MacFadyen & Woosley (1999), for example, took into account a physical equation of state (EOS), photodisintegration, and neutrino cooling. Proga et al. (2003) implemented them in MHD. In this paper we neglect these microphysics entirely and concentrate on the general relativistic MHD, particularly the effect of the rotation of black hole. We assume, mainly for numerical simplicity, that matter can be described as an ideal gas with a gamma-law EOS (p ∝ ργ), although we know that the gamma-law EOS is not a very good approximation for the gravitational collapse of massive stars. We further assume that Γ = 5/3 in the simulations for numerical reasons, although it is more appropriate to adopt Γ = 4/3 for the gas of current interest.
2.2. Initial Condition

As for the initial model, we have the collapsar model in mind. In principle, we should start calculations from a realistic progenitor model with rotation and magnetic field. We take, however, the following pragmatic approach as a rough guide: we use the postbounce profile of Bruenn’s realistic one-dimensional supernova model (Bruenn 1992). As a supernova model, this model is failed. Nominally believed, this model will produce a black hole later. Since our computation is scale free, we employ only the profiles of the density, pressure, and radial velocity as our initial condition. Furthermore we put a few \( M_\odot \) black hole at the center. The initial condition in this paper is the same as our previous paper except for the metric providing the background spacetime (see Mizuno et al. 2004). In this way, we can discuss generic features of the dynamics.

A Kerr black hole has two characteristic parameters: Its mass \( M \) and angular momentum \( J \). We often use the rotation parameter \( a = J / |J| \), where \( |J| = GM^2 / c \) is the angular momentum of a maximally rotating black hole with mass \( M \). In the Boyer-Lindquist coordinates, the metric of Kerr spacetime is written as

\[
h_0 = \sqrt{1 - \frac{2mr}{\Sigma}}, \quad h_1 = \sqrt{\frac{\Sigma}{\Delta}}, \quad h_2 = \sqrt{\Sigma},
\]

\[
h_3 = \frac{A}{\Sigma} \sin \theta,
\]

\[
\omega_1 = \omega_2 = 0, \quad \omega_3 = \frac{2cr^2aR}{A},
\]

where \( r_g \equiv GM / c^2 \) is the gravitational radius, \( \Delta = R^2 - 2mr + (ar)^2 \cos^2 \theta, \Sigma = R^2 + (ar)^2 \cos^2 \theta \) and \( A = R^2 + (ar)^2 \sin \theta \). In this metric, the lapse function is \( \alpha = (\Delta \Sigma / A)^{1/2} \). The radius of the event horizon is \( r_H = r_g[1 + (1 - a^2)^{1/2}] \), which is found by setting \( \alpha = 0 \). We also use the Schwarzschild radius of the black hole, \( r_S = 2GM / c^2 = 2r_g \) as a unit of length in this paper.

We add by hand the stellar rotation and intrinsic magnetic field to the originally nonrotating, nonmagnetic model as an initial condition. The initial distribution of rotational velocity is assumed to be a function of the distance from the rotation axis, \( r = R \sin \theta \), only:

\[
v_0 = v_0 \frac{x_0}{r^2 + x_0^2} r.
\]

Here \( v_0 \) is a model parameter for rotational velocity. We fix \( x_0 = 100r_\odot \) in this paper. We include the stellar rotation only up to \( 18r_\odot \). Because the accreted mass for stellar matter in preshock region is sufficiently small on the timescale of our interest, the neglect of rotation in the preshock region does not affect the dynamics. The rotation profile is similar to the previous simulations for the gravitational collapse of a rotating core (Mönchmeyer & Müller 1989; Yamada & Sato 1994). The initial magnetic field is assumed to be uniform and parallel to the rotational axis. This is known as the Wald solution (Wald 1971; Koide 2003). We use \( B_0 \) as a model parameter for magnetic field strength.

We emphasize that our simulations are scale free. The normalization units and typical values for normalizations are found in Mizuno et al. (2004). The models computed in this paper are summarized in Table 1.

### Table 1: Models and Parameters

| Case   | \( a \)  | \( B_0 \) | \( v_0 \) |
|--------|---------|---------|---------|
| KA1    | 0.999   | 0.05    | 0.01    |
| KA2    | -0.999  | 0.05    | 0.01    |
| KB3    | 0.0     | 0.05    | 0.01    |
| KB4    | 0.3     | 0.05    | 0.01    |
| KB5    | 0.5     | 0.05    | 0.01    |
| KB6    | 0.8     | 0.05    | 0.01    |
| KB7    | 0.9     | 0.05    | 0.01    |
| KC8    | 0.999   | 0.05    | 0.0     |

Notes.—Corotating case KA1 \((a = 0.999, B_0 = 0.05, \text{ and } v_0 = 0.01)\) and counterrotating case KA2 \((a = -0.999, B_0 = 0.05, \text{ and } v_0 = 0.01)\) are considered to be the standard cases in our simulations. The “B” cases differ from case KA1 only in the value of the rotation parameter of the black hole. In case KC8 there is no stellar rotation.

We use the Zero Angular Momentum Observer (ZAMO) system for the 3-vector quantities, such as velocity \( v \), magnetic field \( B \), electric current density \( J \), and so on. For scalars, we use the frame comoving with the fluid flow. The simulations are done in the region \( 1.4r_\odot \) (KA1, KA2, KB7, and KC8), \( 1.6r_\odot \) (KB6), \( 1.8r_\odot \) (KB4 and KB5), and \( 2.0r_\odot \) in \( < 60r_\odot \). Initial rotation is included, and the matter near the rotation axis is ejected closer to the rotation axis. Comparing the jetlike outflow for the rotating black hole with the one in the previous simulations, we find that our jets are similar to those in Newtonian MHD simulations (LeBlanc & Wilson 1970; Symbalisty 1984), which show the formation of a jetlike outflow close to the rotation axis. The jet in the pseudo-Newtonian MHD simulations (Proga et al. 2003) is more similar to the jet...
We emphasize that the generation mechanism of jets in those MHD simulations are common. Figure 2 shows the time evolution of plasma beta \( \beta = P_{\text{gas}}/P_{\text{mag}} \), where \( P_{\text{gas}} \) is gas pressure and \( P_{\text{mag}} \) is magnetic pressure) and toroidal magnetic field for the corotating case KA1, again with the counterrotating case KA2. In both cases, the magnetic field is twisted as a result of the differential rotation of accreted matter and the frame-dragging effect of the rotating black hole and is amplified significantly near the central black hole. The amplified magnetic field expands outward with the jetlike outflow and collimates it. In the case of rotating black holes, it is not clear that the amplified magnetic field propagates as an Alfvén wave, in contrast to the case of nonrotating black holes, but the amplified magnetic field launches an outgoing shock wave. The plasma beta distribution is a little different between the corotating and the counterrotating cases. In the corotating case, the plasma beta inside the jetlike outflow is generally low except in the region close to the rotational axis. It implies that the jetlike outflow is generated and accelerated mainly by the magnetic field. In the counterrotating case, on the other hand, the plasma beta inside the jetlike outflow is complicated. The plasma beta is low near the central black hole. At the edge of the jetlike outflow, the plasma beta is high. When the black hole rotation is fast, the magnetic field is twisted counter to the rotation of stellar matter in the counterrotating case. Hence, the direction of the toroidal magnetic field generated near the black hole is opposite to that in the accreting matter far from the black hole. Such counter twisted magnetic field propagates outward to collide and release magnetic twist in the accreting matter in the distant region. As a result of this release, the region of weak toroidal magnetic field is produced locally. This region becomes a high plasma beta region. This high plasma beta region seems inherent in the counterrotating case.

For the rotating black hole, we cannot see a gas-pressure-driven jet that was found in the general relativistic MHD simulations of a black hole (nonrotating and rotating) with an accretion disk (Koide et al. 1998, 1999, 2000; Aoki 2004). The gas-pressure–driven jet is generated by the shock produced in the equatorial plane of an accretion disk by the centrifugal barrier. Since we assume in our simulations almost rigid rotation, the centrifugal barrier is not so effective in the initial accretion phase. The matter falls to the central black hole smoothly without generating a shock. If our simulations run long enough to form an accretion disk around the central black hole, the gas-pressure–driven jet may be generated.

**3.2. Properties of the Jet**

We discuss here the properties of the jet found in the rotating black hole case. Figures 3 and 4 show the distributions of...
various physical quantities along the jetlike outflow at $t/r_S = 136$ in the corotating case KA1 and in the counterclockwise case KA2, respectively.

It is easily seen in the velocity distribution that the jet has a mildly relativistic velocity, $\sim 0.3c$ in the corotating case and $\sim 0.25c$ in the counterclockwise case. It is supersonic in both cases. The total velocity of jet in the corotating case is clearly larger than the velocity of circular orbit of a particle around Kerr black hole (Kepler velocity). This means that the jetlike outflow in the corotating case is likely to get out of the stellar remnant. On the other hand, the total velocity of jet for the counterclockwise case is comparable to the Keplerian velocity, which is as high as that for the nonrotating black hole case. When the magnetic twist is large, magnetic pressure becomes large. The large magnetic pressure pushes up the accreting stellar matter as the jetlike outflow rather than rotates along the magnetic field. Therefore the poloidal component of velocity becomes dominating.

We show the distributions of various physical quantities on the surface of $z/r_S = 10$ at $t/r_S = 136$ for the corotating case KA1 (Fig. 5). In order to confirm the jet acceleration mechanism, we evaluate the power, $W_{\text{EM}}$, by the electromagnetic force and that, $W_{\text{gp}}$, by the gas pressure as

$$W_{\text{EM}} \equiv \alpha \mathbf{v} \cdot \mathbf{E} + J \times B,$$

$$W_{\text{gp}} \equiv -\mathbf{v} \cdot \nabla (\alpha p),$$

respectively. The jetlike outflow in the corotating case is located in the region $1.0r_S < r < 10r_S$. In this region, the density and the pressure are higher than those of the surrounding region and the toroidal magnetic field and the vertical velocity are the dominant components of magnetic field and
velocity, respectively. The jetlike outflow in the corotating case is mainly accelerated by the electromagnetic force because the electromagnetic force is high in this region. Hence, it is a magnetically driven jet. The edge of the expanding amplified magnetic field is located in the region $10r_s < r < 15r_s$ outside the jet. In this region, the toroidal velocity is the dominant component of the velocity and the power of gas pressure is high.

We show the time variations of the mass flux of the jet, accretion rate, kinetic energy, and Poynting flux at $z/r_s \approx 15$ for the corotating case KA1 and the counterrotating case KA2 in Figure 6. In the corotating case, the kinetic energy flux is comparable to the Poynting flux, while, in the counterrotating case, the kinetic flux is about twice as large as the Poynting flux. These results differ from those for the nonrotating black hole. This is because the jetlike outflow for the rotating black hole is faster in the vertical direction and denser than for the nonrotating black hole. The Poynting flux for the rotating black hole is also larger, because the magnetic field is twisted strongly owing to the frame-dragging effect of the rotating black hole.

3.3. Dependence on the Rotation Parameter

The dependence of the jet properties on the rotation parameter of the black hole has been investigated. Figure 7 shows the snapshots of density and plasma beta in the corotating cases with different rotation parameters, $a = 0.0$ (KB3), $a = 0.5$ (KB4), $a = 0.8$ (KB5), and $a = 0.9$ (KB6) at $t/r_s = 136$. The difference between KB4 ($a = 0.5$) and KB6 ($a = 0.8$) is not seen clearly. On the other hand, the difference between KB6 ($a = 0.8$) and KB7 ($a = 0.9$) is clear. For smaller values of the rotation parameter, the jet is ejected from more outer regions and the propagation of the amplified magnetic field as Alfvén waves is faster and is seen more clearly. This implies that the inner magnetic field is amplified strongly by the frame-dragging effect.

Figure 8 shows the distribution of velocity ($v_r$, $v_\phi$, and $v_z$) and the ratio of toroidal to poloidal magnetic field components ($B_\phi/B_p$) along the jet in the corotating cases with different rotation parameters $a = 0.0$ (KB3), $a = 0.5$ (KB4), $a = 0.8$ (KB5) and $a = 0.9$ (KB6) at $t/r_s = 136$. The differences between cases are apparent. As the rotation of the black hole becomes slower, the toroidal component of the velocity becomes faster and the poloidal component of the velocity becomes slower. The ratio of toroidal to poloidal magnetic field components gets larger as the rotation of black hole becomes slower. As the rotation of the black hole is faster, the magnetic field is twisted more strongly in shorter times owing to the frame-dragging effect. Hence, a stronger and faster jet is produced near the central black hole.
The dependence on the rotation parameter of the black hole in the corotating case is shown in Figure 9. As the rotation parameter of the black hole increases, the poloidal velocity of the jet and the magnetic twist increase gradually and the toroidal velocity of the jet decreases. These results are understood from how effective the frame-dragging effect is. As the rotation of the black hole is faster, the magnetic energy stored by the twisted magnetic field is converted to kinetic energy of the jet more directly rather than to the propagation of Alfvén waves. As a result, the poloidal velocity of the jet becomes higher. Let us examine the above statement more quantitatively. In the Newtonian approximation, the time evolution of the toroidal magnetic field is given as
\[
\frac{\partial B_\phi}{\partial t} \sim \omega B_p, \tag{17}
\]
where \(\omega\) is the angular velocity, which includes the rotation of both matter and the frame (spacetime),
\[
\omega \propto a. \tag{18}
\]
From this, we obtain
\[
\frac{B_\phi}{B_p} \sim \omega t \propto a. \tag{19}
\]

The upward motion of the fluid is induced by the \(J \times B\) force. If we neglect other forces, the equation of motion for the fluid element in the \(z\)-direction becomes
\[
\rho \frac{\partial v_z}{\partial t} \sim -\nabla \frac{B_\phi^2}{4\pi} \sim \frac{1}{z} \frac{B_\phi^2}{4\pi}, \tag{20}
\]
which can be rewritten as
\[
v_z \sim \frac{1}{\rho z} \frac{B_\phi^2}{4\pi}. \tag{21}
\]
In the current situation, the timescale is given by the propagation time of the Alfvén wave and the Alfvén velocity is written as
\[
v_A \sim v_{A_0} = \frac{B_\phi}{(4\pi \rho)^{1/2}} \text{ because } B_\phi/B_p > 1.\]
Thus, we have \(z/t \sim v_{A_0}\). Then, equation (21) can be rewritten as
\[
v_z \sim \frac{1}{\rho v_{A_0}} \frac{B_\phi^2}{4\pi} \propto B_\phi \propto a. \tag{22}
\]
This explains the dependence of \(v_z\) on \(a\) in Figure 9a, that is, the vertical component of the jet velocity increases as the initial magnetic field strength increases.
On the other hand, the rotation of the fluid is also induced by the $\mathbf{J} \times \mathbf{B}$ force. The equation of motion for the fluid element in the toroidal direction becomes

$$\frac{\rho}{\rho} \frac{\partial v_{\phi}}{\partial t} \sim \nabla \left( \frac{B_\phi B_\phi}{4\pi} \right) \sim \frac{1}{z} \left( \frac{B_\phi B_\phi}{4\pi} \right). \quad (23)$$

Using $z/t \sim v_{A\phi}$ in equation (23) leads to

$$v_\phi \sim \frac{1}{\rho} \frac{1}{v_{A\phi}} \left( \frac{B_\phi B_\phi}{4\pi} \right) \propto B_z = \text{const.} \quad (24)$$

This approximately explains the dependence of $v_\phi$ on $a$ for $a < 0.8$ in Figure 9b. However, the exact relation depends on the region where the jet is ejected. In fact, the jet ejected from a deeper region has a lower toroidal velocity owing to the conservation of angular momentum.

We computed the case KC8 in which the accreting matter is not rotating to investigate the effect of frame dragging. Figure 10 shows the snapshots of density and plasma beta at $t/\tau_s = 136$. The simulation results show that the jetlike outflow can be formed solely by the frame-dragging effect. Thus the rapid rotation of black hole will help form the jetlike outflow due to the twisting of the magnetic field by the frame-dragging effect. We also see a disklike structure on the equatorial plane, which is not rotating. Hence this disk is not an accretion disk. The disklike structure is produced by the effect of magnetic field and supported by the gas pressure. All disklike structures seen in other cases are basically the same as this nonrotating disk, and thus are not an accretion disk.

### 4. SUMMARY AND DISCUSSION

We have performed general relativistic MHD simulations of collapsars, paying particular attention to the rotation of the central black hole. We considered not only the corotating but also the counterrotating cases with respect to the black hole rotation to elucidate its effect on the dynamics. Our results are summarized as follows:

1. The formation mechanism of the jet for the rotating black hole is the same as that for the nonrotating black hole. When stellar matter falls onto the black hole, a disklike structure is formed in the vicinity of the black hole horizon and a jetlike outflow is formed near the black hole mainly by the magnetic field. The jetlike outflow propagates outward with...
twisted magnetic fields and becomes collimated. The shock waves (MHD fast shock and slow shock) are also formed near the black hole and propagate outward. Figure 11 shows the schematic picture of our simulation results.

2. The total velocity of jet for the rotating black hole is comparable to that for the nonrotating black hole, \( \sim 0.3c \). However, the poloidal velocity of the jet in the case of rotating black hole is about 3 times higher than that in the case of nonrotating black hole. The poloidal velocity is the dominant component of the jet. Because the magnetic field is more strongly twisted because of the frame-dragging effect of the rotating black hole, more magnetic energy is converted to kinetic energy of the jet than to the propagation of Alfven waves.

3. In the corotating case, the kinetic energy flux is comparable to the Poynting flux. In the counterrotating case, on the other hand, the kinetic energy flux is about twice as large as the Poynting flux. This is because the jet for the rotating black hole has a higher density.

4. As the rotation parameter of the black hole increases, the poloidal velocity of the jet and the magnetic twist increase gradually and the toroidal velocity of the jet decreases. These results are related to how effective the frame-dragging is. As the rotation of black hole is faster, the magnetic field is twisted more strongly owing to the frame-dragging effect. The magnetic energy stored by the twisted magnetic field is converted to the kinetic energy of the jet directly rather than leading to the propagation of Alfven waves. Thus, the poloidal velocity of the jet becomes faster as the rotation of the black hole becomes faster.

5. In the rapidly rotating black hole case the jetlike outflow can be formed by the frame-dragging effect through the twisting of magnetic field even if there is no stellar rotation.

It is noted that in the present simulations, the boundary of the calculation region is located outside of the ergosphere of the Kerr black hole. To evaluate the validity of the boundary location, we checked the Alfven surface. We found that it is
always located at $R > 2r_s$ in the calculation regions in all cases. This means that the plasma in the ergosphere does not influence the bulk plasma of the calculation region in the present cases and supports the validity of the boundary location near the horizon.

We discuss the application of our results to the central engine of GRBs. Using the simulation data, we can estimate the kinetic energy of the jet, $E_{\text{jet}}$. If the mass of the disk formed in the initial accretion phase is comparable to the central black hole mass, e.g., several solar masses (almost all stellar matter except the central core are actually blown away by the presupernova), the density of the disk can be estimated from

$$\pi r_3^2 H \rho_d = 4 M_\odot \approx 10^{34} g.$$  \hfill (25)

![Fig. 7.—Snapshots of density and plasma beta in the corotating cases having different rotation parameter KB3 ($a = 0.0$; $a$ and $e$), KB5 ($a = 0.5$; $b$ and $f$), KB6 ($a = 0.8$; $c$ and $g$), and KB7 ($a = 0.9$; $d$ and $h$) at $t/\tau_3 = 136$. The color scales show the values of the logarithm of density and plasma beta. The white curves represent ($a$–$d$) the magnetic field lines and ($e$–$h$) the contour of the toroidal magnetic field. The contour level step width is 0.025 for ($e$), ($f$), and ($g$) and 0.05 for ($h$) in units of the toroidal magnetic field ($B_T$). Arrows depict the poloidal velocities normalized by the light velocity. For smaller values of the rotation parameter, the jet is ejected more from the outer region and the expansion of the amplified magnetic field as an Alfvén wave is seen more clearly.](image)

![Fig. 8.—Distribution of velocity ($v_r$, $v_\phi$, and $v_z$) and the ratio of toroidal to poloidal magnetic field components ($B_T/B_p$) along the jet in the corotating cases having different rotation parameter KB3 ($a = 0.0$; $a$ and $e$), KB5 ($a = 0.5$; $b$ and $f$), KB6 ($a = 0.8$; $c$ and $g$), and KB7 ($a = 0.9$; $d$ and $h$) at $t/\tau_3 = 136$. For smaller values of the rotation parameter, the toroidal component of velocity is larger, the poloidal component of velocity is smaller and the ratio of toroidal to poloidal magnetic field components becomes smaller.](image)
where \( r_d \) is the outer radius, \( H \) the thickness and \( \rho_d \) the density of the disk, respectively. We can estimate \( r_d \) and \( H \) from the simulation data as \( r_d = 10r_S \simeq 10^7 \text{cm}, \ H = 0.1r_d = 1r_S \simeq 10^6 \text{cm}, \) respectively. Therefore, the density of the disk is \( \rho_d \simeq 3 \times 10^{13} \text{g cm}^{-3}. \) Because the jetlike outflow for the rotating black hole is generated very close to the central black hole, the density of the jet is comparable to the density of the disk. Thus, using the estimated density of the disk, the kinetic energy of the jet is given as \( E_{\text{jet}} \simeq 10^{54} \text{ergs}. \) Although it may somewhat overestimate, this is large enough energy to explain the standard energy \((\sim 10^{51} \text{ergs})\) of GRBs. However, the maximum jet velocity in our simulations including a rotating black hole was about \( 0.3c. \) This velocity is too slow compared to the velocity inferred for the jet of GRBs. We have to consider other acceleration mechanisms. This is the most difficult and fundamental problem in all types of ultrarelativistic outflow, e.g., active galactic nucleus jets, pulsar winds, and GRBs. In order to obtain not only a large energy extraction but also the observed large bulk Lorentz factors, the Poynting flux must be converted to kinetic energy. From the steady solution of relativistic outflow in ideal MHD, it is found that the energy conversion from the Poynting flux to the kinetic energy flux occurs and the outflow is highly accelerated, if the magnetic field lines diverge with the radius more quickly than in the monopole field (Begelman & Li 1994; Takahashi & Shibata 1998; Daigne & Drenkhahn 2002). However, this solution is not self-consistent because the geometry of the magnetic field is not solved. Moreover, the relativistic outflow may not maintain the collimated structure in this situation. Some authors propose a dissipation-induced acceleration mechanism (Spruit et al. 2001; Drenkhahn 2002; Drenkhahn & Spruit 2002; Sikora et al. 2003). Although the centrifugal acceleration may also contribute to the early stages of acceleration of the flow, this is not a magnetocentrifugal acceleration process. If the magnetic fields in the outflow change the direction on sufficiently small scales, a part of the magnetic energy can be released locally by magnetic reconnections. This process has two useful effects. First, it converts Poynting flux directly into radiation, without the intermediate step with internal shocks. Second, it leads a to steeper decline of magnetic pressure, which causes great acceleration of the flow and enhanced conversion of the Poynting flux to kinetic energy. Since in the corotating case of our simulations, the Poynting flux is comparable to the kinetic energy flux, the dissipation-induced acceleration mechanism that might occur later on may be the key to solve the acceleration problem of our simulations.

Although they may not be the main solution of the acceleration problem, we further discuss possible acceleration mechanisms from other simulations of relativistic jets. The first possibility is a disk jet, especially a gas-pressure–driven jet. The gas-pressure–driven jet was seen in general relativistic MHD simulations of the black hole–accretion disk system (Koide et al. 1998, 1999, 2000; Aoki et al. 2004) and has higher velocity than the magnetically driven jet. It is generated by the shock produced by the centrifugal barrier in the equatorial plane. Since, in our simulations, we initially assumed that the rotation is almost rigid, the centrifugal barrier is not so effective in the initial accretion phase. The accreted matter falls to the central black hole smoothly without the generation of a shock. We expect that if our simulations run long enough to form the accretion disk around the central black hole, the gas-pressure–driven jet will be generated. The second possible mechanism is the break out of the jet from the stellar surface. When the jet emerges from the stellar surface, the steep density gradient accelerates the jet. Some authors (Aloy et al. 2000; Zhang et al. 2003, 2004) have showed numerically that the jet is accelerated significantly by this process, and the terminal Lorentz factor becomes more than 100. Since the jet in our simulations contains large energy, this mechanism may work.
effectively. We will address the propagation of the jet outside the stellar surface in a forthcoming paper.

Our results can be also applied to baryon-rich outflows associated with failed GRBs. The baryon-rich outflow is a fireball with high baryonic load and mildly relativistic velocities. The jet velocity is so slow that it cannot produce GRBs. Such failed GRBs are supposed to occur at higher event rates than successful GRBs (Woosley et al. 2003; Huang et al. 2002). Some failed GRBs may be observed as "hypernovae." In particular, SN 2002ap is a candidate for such an event. Although the association with a GRB has not been found, it is inferred to have a jet with a velocity of \( \sim 0.23c \) and estimated kinetic energy of \( \sim 5 \times 10^{50} \) ergs (Kawabata et al. 2002; Totani 2003). The jet found in our simulations for rotating black holes has comparable velocity and large enough energy to explain the jet of SN 2002ap.

In our simulations, we have neglected microphysics (photodisintegration, treatment of neutrinos, etc.) and concentrated on the general relativistic MHD, particularly the effect of rotating black hole. Such treatment may be reasonable as a first step. However, microphysics are critically important for realistic GRMHD simulations of collapsars. We will address these issues in a forthcoming paper.

We have assumed a uniform global magnetic field in the simulations. It is inferred, however, that rotating compact stars in general have a dipole-like magnetic field. Hence, it may be more likely that the rotating stars collapse with a dipole-like magnetic field. Although we think that the assumption of a uniform global magnetic field will not be so bad for the discussion of the generic aspect of collapsars dynamics, the dependence on the configuration of magnetic field is also currently being investigated and will be presented elsewhere.

We have used the Boyer-Lindquist coordinates for the Kerr metric in the simulations. Although they are one of the most familiar coordinates for the Kerr metric, they are singular on the event horizon. Therefore, we have not been able to treat the event horizon in the simulations. McKinney & Gammie (2004) have performed axisymmetric numerical simulations of a black hole surrounded by a magnetized plasma with Kerr-Schild coordinates to investigate energy extraction by the Blandford-Znajek process. The Kerr-Schild coordinates are regular on the event horizon and thus may be more appropriate to investigate events very close to the black hole. The coordinates will be used in our GRMHD simulations in the future.

Fig. 10.—Snapshots of density (a) and plasma beta (b) for the no stellar rotation case KC8 at \( t/\tau_s = 136 \). The color scales show the values of the logarithm of density and plasma beta. The white curves represent (a) the magnetic field lines and (b) the contour of the toroidal magnetic field. The contour level step width is 0.067 for (b) in units of the toroidal magnetic field (\( B_t \)). Arrows depict the poloidal velocities normalized by the light velocity. Although there is no stellar rotation, the jetlike flow is produced near the black hole.

Fig. 11.—Schematic picture of our simulation results. A disklike structure and a jetlike outflow are formed in the vicinity of the black hole.
Y. M. appreciates many helpful conversations on GRBs and relativistic effects with S. Aoki, T. Totani, R. Yamazaki, M. Takahashi, and K. Nishikawa. He also thanks K. Uehara, H. Kigure, T. Haugbøelle, and Y. Kato. This work was partially supported by Japan Science and Technology Cooperation (ACT-JST), Grant-in-Aid for the 21st Century COE “Center for Diversity and Universality in Physics” and Grants-in-Aid for scientific research from the Ministry of Education, Science, Sports, Technology, and Culture of Japan through grants 14079202, 14540226 (PI: K. Shibata), and 14740166. The numerical computations were partly carried out on the VPP5000 at the Astronomical Data Analysis Center of the National Astronomical Observatory, Japan (yym17b), and partly on the Alpha Server ES40 at the Yukawa Institute for Theoretical Physics of Kyoto University, Japan.

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