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The underinvestment problem under conglomeration

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Abstract: This theoretical paper explores whether the underinvestment problem is aggravated or ameliorated by the formation of a pure conglomerate. It establishes that the answer depends critically on the volatility of corporate assets. If volatility is low, conglomeration ameliorates the underinvestment problem, whereas if volatility is high, conglomeration aggravates the underinvestment problem. These analytical results are then invoked as a potential explanation for the ambiguous conclusions of empirical studies that delve into the existence of a conglomerate discount.

Subjects: Microeconomics; Corporate Finance; Investment & Securities

Keywords: underinvestment; conglomeration; option pricing; diversification discount

1. Introduction

In an iconic paper in the literature of corporate finance, Myers (1977) developed the concept of what has come to be known as the underinvestment problem. The notion has likewise been labeled as the game of refusing to contribute equity capital in the popular textbook co-authored by Myers himself, i.e. by Brealey, Myers, and Allen (2011). This refers to a situation where a leveraged firm, having determined that an investment proposal generates a positive net present value (NPV), nonetheless decides to reject the proposal. As NPV measures the increase in the market value of the firm that
would be occasioned by acceptance of the project, this rejection represents a value loss for all the firm’s investors, i.e. both its bondholders as well as its stockholders considered together as a single group.

Additional details of the firm’s circumstances explain why this seemingly anomalous situation in fact represents a rational outcome. The financing for the investment is to be provided by the firm’s stockholders in the form of earnings that are retained in the firm, rather than paid out as dividends. However, as the firm finds itself in a state of financial distress, meaning that the chances of the firm defaulting on its debt obligations are material, part of the NPV will be appropriated by the firm’s bondholders. Project acceptance significantly reduces the probability of loan default, which redounds significantly to the benefit of the firm’s creditors. Thus, because the stockholders finance the proposed investment but capture only a part of the project’s NPV, the firm’s managers, insofar as they take direction from the stockholders, are motivated to reject the proposal. In short, the stockholders refuse to contribute equity capital.

Another research theme in the same academic literature is the financial rationale for pure conglomerate mergers, which are characterized by the absence of all economies, both of scale and of scope. Thus, value is neither created nor destroyed as a result of conglomeration, the sole consequence being a reduction in the standard deviation of corporate assets. Lewellen (1971) demonstrated that, if the earnings streams of the entities that comprise the conglomerate exhibit less-than-perfect positive correlation, the conglomerate’s probability of loan default will be less than any of the constituent entities, as the diversification effect of conglomeration induces a decline in the volatility of the corporation’s assets. Galai and Masulis (1976) as well as Kim and McConnell (1977) then argued that the foregoing would redound to the benefit of the conglomerate’s bondholders and to the detriment of the conglomerate’s stockholders. Stated another way, a pure conglomerate merger represents a zero-sum game involving a transfer of wealth from the stockholders of the conglomerate’s constituent entities to the creditors of the same entities.

The extant literature’s focus on pure conglomerate mergers as the appropriate setting in which to gage the resulting effect on investment incentives is well taken. If the merger were to generate economies, this would ipso facto create a bias in favor of conglomeration. Contrariwise, if the merger were to generate diseconomies, this would create a bias against conglomeration by the same token. Thus, to obviate falling into the logical fallacy of “begging the question,” whereby the conclusions of the theoretical analysis are preordained by the initial assumptions, this paper likewise stipulates pure conglomerate mergers as the focal point of analysis.

This paper links these two disparate research strands by posing the following query: What is the effect of pure conglomeration on the incidence of underinvestment? By invoking the paradigm of the Black–Scholes option pricing model, it is shown that, if corporate assets exhibit low volatility, conglomeration ameliorates the problem of underinvestment, whereas if corporate assets are sufficiently volatile, conglomeration aggravates the underinvestment problem. By adducing the ambiguous extant empirical evidence on the existence of a diversification discount, which attempts to determine whether a firm’s market value as a separate entity exceeds its value as part of a conglomerate, some substantiation is provided, albeit merely suggestive, for the theoretical results presented here.

2. An algebraic depiction of the underinvestment problem
This section provides a succinct algebraic formulation of the numerical example employed in the Brealey et al. (2011) textbook to illustrate the underinvestment problem, with a view to presenting a more general treatment.

A capital budget proposal requires an upfront investment, denoted $I$, the financing for which is to be provided by the stockholders, and generates a positive NPV. The change in the market value of the firm, inclusive of the additional equity financing obtained by foregoing a dividend payout, is denoted...
\[ \Delta MV_{\text{Firm}} = I + NPV. \] The change in the market value of the firm's bonds is given as \( \Delta MV_{\text{B}} = \alpha (I + NPV) \), where \( \alpha \) represents the fraction of the firm's market value increase that is captured by the bondholders. Ignoring momentarily the required investment of the stockholders, the fraction of the market value gain experienced by the stockholders is represented by \( \Delta MV_{\text{S}} = (1 - \alpha) (I + NPV) \). The stockholders' market value gain calculated net of the requisite investment is thus calculated as \( \text{Net} \Delta MV_{\text{S}} = -I + \Delta MV_{\text{S}} \).

A situation of underinvestment occurs when \( \text{Net} \Delta MV_{\text{S}} < 0 \) notwithstanding the positive \( NPV \), i.e. although all the firm's investors viewed as a single group experience a wealth gain, the stockholders themselves suffer a wealth loss. By substituting the formulas found in the preceding paragraph, it is easy to show that this inequality simplifies to \( \frac{NPV}{I + NPV} < \alpha \). Thus, the ratio \( \frac{NPV}{I + NPV} \) defines a critical value of \( \alpha \), below which underinvestment is avoided and above which underinvestment is encountered. For example, when \( \alpha = 0 \), i.e. the bondholders do not capture any of the wealth gain from a positive \( NPV \) proposal, the inequality is violated and the problem of underinvestment vanishes.

Thus, any corporate action that induces an increase in \( \alpha \), e.g. more reliance on debt financing and/or a higher probability of debt default aggravates the underinvestment problem. Contrariwise, any corporate action that causes \( \alpha \) to fall, e.g. more reliance on equity financing or a reduced probability of default, ameliorates the underinvestment problem.

3. The Black–Scholes option pricing paradigm
In their seminal article on option valuation, Black and Scholes (1973) posit the isomorphic relationship that exists between a call option and the commons shares of a firm that has debt outstanding, a paradigm that is further articulated in Galai and Masulis (1976). This approach is exploited here to determine the effect of conglomeration on the parameter \( \alpha \). Does conglomeration, interpreted in accordance with Lewellen (1971) as solely a reduction in the volatility of corporate assets with no concomitant change in the market value of those assets, result in a decrease or an increase in the indicated parameter? On the one hand, if the former is the answer, then conglomeration ameliorates the underinvestment problem, whereas, on the other hand, if the latter is the answer, then conglomeration aggravates the underinvestment problem.

Galai and Masulis (1976) show that the effect on the market value of debt of a firm that experiences a marginal increase in its market value, which is merely the \( \alpha \) parameter defined earlier, equals \( N(-d) \). \( N(*) \) is the cumulative probability distribution of the unit normal random variable and \( d \) is defined as follows.

\[
\frac{\ln(V/B) + (r + \frac{1}{2}\sigma^2)T}{\sigma \sqrt{T}} \tag{1}
\]

where \( V \) is the market value of the firm, \( B \) is the promised payment to the bondholders, \( r \) is the risk-free rate, \( T \) is the remaining life of the firm, and \( \sigma \) is the volatility or standard deviation of the firm’s assets. It is convenient to assume that \( B \) is strictly less that \( V \), for otherwise the firm would be insolvent and the problem of underinvestment would be rendered moot. The foregoing implies that \( \ln(V/B) \) is either positive or zero.

Consider what happens to \( \alpha = N(-d) \) when \( \sigma \) approaches the lower bound value of zero. \(-d \) approaches the value of \(-\infty \), which means that \( N(-d) \) approaches the value of zero. Thus, at extremely low levels of corporate asset volatility, \( \alpha \) approaches zero. The claim of bondholders on the \( NPV \) of an investment proposal is virtually nil, thus making the problem of underinvestment disappear. From the foregoing, it can be concluded that, at low levels of corporate asset volatility, conglomeration, insofar as it results in a reduced \( \alpha \), tends to ameliorate the underinvestment problem.

By the same token, consider what happens to \( \alpha = N(-d) \) when \( \sigma \) approaches infinity. \(-d \) likewise approaches the value of \(-\infty \), which means that \( N(-d) \) approaches the value of zero. Thus, at extremely high levels of corporate asset volatility, \( \alpha \) similarly approaches zero which again means that the claim...
of bondholders on the NPV of an investment proposal is virtually nil, thus also making the problem of underinvestment disappear. Parenthetically, this tactic of harming bondholders by increasing the volatility of corporate assets is well-known in the academic literature and is labeled the risk-shifting game in the popular Brealey et al.’s (2011) textbook. Since conglomeration reduces \( \sigma \), it has the effect of increasing the claim of bondholders on the NPV of an investment proposal. From the foregoing, it can be concluded that, at high levels of corporate asset volatility, conglomeration, insofar as it results in a reduced \( \sigma \), tends to aggravate the underinvestment problem.

To determine the critical value for \( \sigma \), above which the volatility of corporate assets is deemed high and below which the volatility of corporate assets is deemed low, requires that the partial derivative \( \frac{\partial \alpha}{\partial \sigma} \) be evaluated as follows.

\[
\frac{\partial \alpha}{\partial \sigma} = -N'(-d) \frac{\partial d}{\partial \sigma} \tag{2}
\]

In Equation 2, \( N'(\cdot) \) is the probability density function of the unit normal random variable, which is uniformly positive. The following partial derivative is assessed.

\[
\frac{\partial d}{\partial \sigma} = \frac{5\sigma^2 T \sqrt{T} - |\ln(V/B) + r T| \sqrt{T}}{\sigma^2 T} \tag{3}
\]

Substituting Equation 3 into Equation 2 and simplifying results in the conclusion that \( \frac{\partial \alpha}{\partial \sigma} > 0 \) if and only if the following inequality is satisfied.

\[
\sqrt{2 \left[ \frac{\ln(V/B)}{T} + r \right]} > \sigma \tag{4}
\]

The left-hand side of inequality (Equation 4) defines the critical value of corporate asset volatility, below which conglomeration ameliorates the problem of underinvestment because reducing \( \sigma \) likewise reduces the claim of bondholders on NPV. Above that critical value of corporate asset volatility, \( \frac{\partial \alpha}{\partial \sigma} < 0 \), which means that reducing \( \sigma \) increases the claim of bondholders on NPV, conglomeration aggravates the underinvestment problem.

To provide a numerical illustration of inequality (Equation 4), consider a levered corporation whose \( V = 1.58 \), i.e. the market value of the firm is 50% higher than the promised payment to bondholders. It is projected that the firm will be dissolved 10 years hence. Furthermore, assume that the risk-free rate equals 3%. Inequality (Equation 4) states that the critical value for \( \sigma \) equals 37.56%. In this situation, if the value of corporate asset volatility is less than 37.56%, conglomeration ameliorates the underinvestment problem, whereas if \( \sigma \) exceeds 37.56%, conglomeration exacerbates the underinvestment problem.

4. Suggestive empirical support
The theoretical results presented here provide a potential explanation for the equivocal statistical evidence regarding the existence of a diversification or conglomerate discount, i.e. the putative tendency of financial markets to undervalue the shares of conglomerates. If conglomeration aggravates the underinvestment problem, then the diversification discount should increase, whereas if conglomeration ameliorates the underinvestment problem, then the diversification discount should tend to disappear, turning into a diversification premium.

While earlier empirical studies, exemplified by Lang and Stulz (1994) and Berger and Ofek (1995), argue for the existence of such a discount, the later investigations of Maksimovic and Phillips (2001) and Graham, Lemmon, and Wolf (2002) repudiate these findings. This paper posits improper accounting for the volatility of corporate assets as a potential cause for these ambiguous empirical findings. On the one hand, if corporate assets exhibit high volatility, conglomeration will aggravate
the underinvestment problem and thus cause the conglomerate to trade at a discount. On the other hand, if corporate assets exhibit low volatility, conglomeration will ameliorate the underinvestment problem and thus cause the conglomerate to trade at a premium.

5. Conclusion
This paper has explored the effect of conglomeration on the severity of the underinvestment problem. It was shown that, if corporate assets are highly volatile, conglomeration exacerbates the underinvestment problem, whereas if the volatility of corporate assets is below a critical value, conglomeration remediates the problem of underinvestment. If a detailed determination of corporate asset volatility is absent, the valuation effects of conglomerate activity are indeterminate. Thus, the results of this paper provide a potential explanation for the contradictory empirical results of extant studies that attempt to establish the existence of a conglomerate discount. Future statistical research into the latter should account for the disparate valuation effects of conglomerate activity as a result of the underinvestment problem.