Power Harmonic Emission Level Evaluation Based on Principal Component Analysis Regression Method

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Abstract. Accurate harmonic emission level evaluation is the basis of distinguishing the responsibility of harmonic pollution between power supply system and consumer. A new harmonic emission level evaluation method based on principal component analysis (PCA) regression is proposed in this paper. Firstly, the principle of principal component analysis regression method is stated. Secondly, harmonic impedance of supply system is evaluated by principal component analysis regression method. Finally, harmonic emission level of supply system and consumer is calculated according to the harmonic impedance of supply system. This method has a high accuracy in harmonic emission level evaluation. The effectiveness of the proposed method was verified by computer simulation results.

Introduction

With wide application of nonlinear load and the increase of the distributed powers, harmonic pollution of power system becomes more and more serious [1]. In order to control harmonics, the responsibility of harmonic pollution attribution between supply system and consumer must be clear [2]. Therefore, distinguishing the evaluation of harmonic emission level of supply system and consumer has become an important task [3].

At present, the research of power harmonic emission level evaluation is mainly focused on the harmonic contribution of the power supply system and the consumer to the point of common coupling (PCC) [4-6]. According to the characteristics of harmonic emission level, current evaluation methods mainly construct the Thevenin or Norton equivalent circuit, which is based on system side and consumer side model. And then, harmonic evaluation is performed by applying statistical methods. Binary linear regression [7], robust regression [8], partial least squares regression [9-10] and so on are widely used harmonic emission level evaluation methods currently.

The existing methods improve the accuracy of the evaluation of power harmonic emission level, but there are still some problems that should be solved. In order to represent the problem entirely, the existing methods introduce excessive independent variables, increasing the complexity of the model. In addition, the correlation among independent variables can also make some important data loss. These problems result in the results of harmonic emission level evaluation unsatisfactory. In order to solve the two problems, the method based on PCA regression is proposed in this paper. The principal components are selected according to the amount of the information contained, which effectively overcomes the nonorthogonality among the independent variables in modeling, and the problem that the important information may be deleted by mistake is also avoided.

The Linear Regression Method Based on Principal Component Analysis

PCA is a statistical method that finishes the information expression by using the linear combination of the least variables [11]. It is assumed that there are p indexes which are considered as random
variables, expressed by $X_1, X_2, \cdots, X_p$ respectively. The $p$-dimensional random variables composed of these $p$ indexes can be represented as $X=(X_1, X_2, \cdots, X_p)'$.

The PCA problem of $p$ variables is a linear transformation problem among variables. That is:

$$F_j = a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jp}x_p \quad (j=1,2,\cdots,p).$$

(1)

where $a_{ij}$ is the coefficient of PCA.

If $F_1$ is used to replace the original $p$ variables, $F_1$ need to involve the information of the $p$ variables as much as possible. Thus, the larger the variance $Var(F_1)$, the better. Here, $F_1$ is called the first principal component. If $F_1$ cannot involve all information of variables, the second principal component $F_2$ should be introduced. $F_1$ is not correlated with $F_2$, that is, $Cov(F_1, F_2)=0$. And so on, if there are $p$ principal components, these principal components are not correlated with each other. At the same time, $Var(F_1)>Var(F_2)>\cdots>Var(F_p)$. Usually, the first $q$ principal components are selected in practice according to the cumulative contribution rate of principal component. The formula of contribution rate $\mu$ is

$$\mu = \frac{\lambda_i}{\sum_{i=1}^{p} \lambda_i}$$

(2)

where $\lambda_i = Var(F_i)$.

Generally speaking, only when the cumulative contribution rate is more than 85%\cite{12}, can we ensure that the integrated variables contain the vast majority of information of the original variables. Selecting principal components according to the contribution rate can avoid important information being deleted by mistake. In additional, when the value of $F_i$ is close to zero, the corresponding independent variable is an approximate linear dependence relationship. Therefore, the measure above is able to neglect the smaller principal components and can solve the collinearity problem among independent variables.

After calculating the principal component expression and the sub-variables by using the method of PCA, dependent variables are regressed to sub-variables of principal component. Then, the regression model among dependent variables and sub-variables of principal component is obtained.

**Application of Linear Regression Method Based on Principal Component Analysis in Evaluation of Power Harmonic Emission Level**

**Evaluation of System and Consumer Harmonic Emission Level**

The power distribution network system can be equivalent to the circuit shown in Fig. 1 at the point of common coupling of the system and the harmonic consumer.

![Figure 1. The equivalent circuit of system and consumer at the PCC.](image)

In Fig. 1, $U_{sh}$ is the harmonic voltage source of system side; $I_{ch}$ is the equivalent harmonic current source of consumer side; $Z_{sh}$ is the equivalent harmonic impedance of system side; $Z_{ch}$ is the equivalent harmonic impedance of consumer side; $I_{poch}$ is the harmonic current of the point of common coupling; $U_{poch}$ is the harmonic voltage of the point of common coupling.

According to the circuit principle:

$$U_{poch} = U_{sh} - I_{poch}Z_{sh}.$$

(3)
Expanding Eq. 3 according to the real part and imaginary parts to get:

\[ U_{pccx} = U_{shx} - I_{pccx} Z_{shx} + I_{pchy} Z_{shy}. \]  

(4)

\[ U_{pchy} = U_{shy} - I_{pccx} Z_{shy} - I_{pchy} Z_{shx}. \]  

(5)

Eq. 4 and Eq. 5 are the multiple linear regression equations with the real part \( I_{pccx} \) and imaginary part \( I_{pchy} \) of \( I_{pcc} \) as the independent variables and the real part \( U_{pccx} \) and imaginary part \( U_{pchy} \) of \( U_{pcc} \) as the dependent variables. The two equations can be solved by using the principal component regression analysis method to obtain regression coefficients \( Z_{shx}, Z_{shy}, U_{shx} \) and \( U_{shy} \). And then, their respective statistical mean values are obtained.

\[ \bar{Z}_{sh} = \frac{1}{M} \left( \sum_{m=1}^{M} Z_{shm} \right), \]  

(6)

\[ \bar{U}_{sh} = \left( \frac{1}{M} \sum_{m=1}^{M} U_{shm} \right)^2 + \left( \frac{1}{M} \sum_{m=1}^{M} U_{shm} \right)^2. \]  

(7)

Usually, \( |Z_{ch}| \gg |Z_{sh}| \). So the consumer side of the harmonic emission level \( U_{ch} \) can be expressed as:

\[ U_{ch} = I_{ch} \frac{Z_{ch} Z_{sh}}{Z_{ch} + Z_{sh}} = U_{pcc} - \frac{Z_{ch} U_{sh}}{Z_{ch} + Z_{sh}} \approx U_{pcc} - U_{sh}. \]  

(8)

**Evaluation Process of Harmonic Emission Level**

Based on the analysis of above, the following steps should be taken to determine the effect of consumer harmonic source for harmonic emission levels of the point of common coupling.

1) Measure each harmonic voltage and amplitude and phase of current at PCC point.
2) Evaluate the harmonic impedance \( Z_{sh} \) of system side at PCC point.
3) Obtain each harmonic voltage components \( U_{ch} \) and \( U_{sh} \) according to the corresponding equivalent circuit.

**Simulation Examples**

**Simulink Model of Single-Phase Example**

Power System Blockset of Simulink is used to conduct simulation. Simulation circuit is a single-phase system model, as shown in Fig. 2. The power supply system is equivalent to the Thevenin model while the harmonic consumer is equivalent to the Norton model. The supply system consists of fundamental voltage source and harmonics voltage source which contains 3, 5, 7, 9, 11, 13 and 15 times harmonic voltage. The consumer side current source contains 3, 5, 7, 9 and 11 times harmonic current.

![Figure 2. Simulink simulation circuit.](image-url)
The harmonic voltage source is shown in Table 1. Fundamental wave Impedance of system side is set to 0.5 + j1.371. Each harmonic current of the harmonic current source on the consumer side is set as follows: $I_3=60A$, $I_5=20A$, $I_7=10A$, $I_{11}=5A$. Fundamental wave Equivalent impedance of consumer side is set to 2.7229 + j11.3453.

The principal component analysis regression method is used to analyze the original data, and the results are shown in Table 1.

### Table 1. The system side of each harmonic of impedance and voltage and error.

| Order of harmonics | Impedance True value(Ω) | Impedance Evaluated value(Ω) | Relative error (%) | Voltage True value(V) | Voltage Evaluated value(V) | Relative error (%) |
|--------------------|-------------------------|-----------------------------|--------------------|-----------------------|---------------------------|--------------------|
| 3 times            | 0.6+j1.382              | 0.6+j1.382                  | 0.1                | 400                   | 405.5                     | 1.36               |
| 5 times            | 0.6+j6.9100             | 0.6101+j6.6096              | 1.09               | 20                    | 19.6959                   | 1.52               |
| 7 times            | 0.6+j9.6740             | 0.6076+j9.7626              | 0.92               | 10                    | 9.8245                    | 1.75               |
| 9 times            | 0.6+j12.4380            | 0.5898+j12.2084             | 1.85               | 20                    | 19.7259                   | 1.37               |
| 11 times           | 0.6+j15.2020            | 0.6029+j15.3963             | 1.28               | 5                     | 5.0653                    | 1.31               |
| 13 times           | 0.6+j17.9660            | 0.6221+j18.1525             | 1.04               | 20                    | 20.0146                   | 0.073              |
| 15 times           | 0.6+j20.7300            | 0.5838+j20.6927             | 0.18               | 10                    | 10.1609                   | 1.6                |

From the simulation results in Table 1, it is easy to see that the relative error of the harmonic impedance of the system is evaluated by linear method based on principal component analysis is about 1% except for the fifth harmonic slightly higher, compared with the true values of harmonic impedance of system side. And the maximum relative error of the background harmonics evaluated by the proposed method is only 1.75%. Therefore, the accuracy of estimating harmonic impedance and background harmonics of the system side based on the linear method of principal component analysis is high, which can meet the needs of practical engineering.

### Comparison with Existing Algorithms

Simulation circuit is shown in Fig. 3. On system side, fundamental wave(50Hz) voltage is 100∠0°V and the third harmonic voltage is 30∠60°V. On consumer side, fundamental wave current is 70∠0°A and the third harmonic current source is 10∠30°A. System side third harmonic impedance is 20 + j12.5Ω while consumer side is 100 + j200Ω. The current and voltage are sampled with 2560 sampling points at the PPC which divided into 10 sections of 256 points each. According to the regression equation, binary linear regression, robust regression, partial least squares method and principal component analysis regression are used to get the real and imaginary parts of the third harmonic impedance of system side, as shown in Table 2.

### Table 2. Various methods harmonic impedance evaluated results comparison.

| method                  | The calculated value of real part of third harmonic equivalent impedance /Ω | The calculated value of imaginary part of third harmonic equivalent impedance /Ω |
|-------------------------|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| binary linear regression| 21.7                                                                         | 13.1                                                                         |
| robust regression       | 22.1                                                                         | 13.3                                                                         |
| partial least squares method| 21.5                                                                       | 12.9                                                                         |
| algorithm proposed by the paper | 21.3                                                                      | 12.6                                                                         |

Figure 3. Simulation circuit.
As can be seen from Table 2, it is more accurate to evaluate the results of regression coefficient using the principal component analysis method than other methods of the current. The data shows that the principal component analysis regression method can effectively extract the components of dependent variables that have powerful explanatory ability. Therefore, it is better than the existing regression method and the harmonic emission level evaluation value is closer to the true value.

Conclusion

In this paper, principal component analysis regression method is proposed to evaluate harmonics emission level of supply system and consumer. Under the assumption that the system harmonic emission level is stochastically stable, the regression equation of PCA is obtained by using the plural relationship among voltage, current and harmonic impedance of system side at the PCC. The harmonic emission levels of the system and consumer sides are then deduced from regression coefficients. The proposed method can extract integrated variables which have powerful explanatory ability to dependent variables, and are able to identify useful information and noise in the system. In addition, the correlation analysis of statistical properties was strengthened for variables. The method effectively solved the multiple correlations among the independent variables while avoiding the possibility that the important information was deleted mistakenly. Simulation results show that the proposed method can accurately evaluate the harmonic emission level of power system.

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