On Some Types of $\alpha rps$-Closed Maps

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Abstract
This paper is continues to study of a new type of closed maps which is called $\alpha rps$-closed map. As well as, we give and study other types of $\alpha rps$-closed maps which are ($\alpha rps$-closed maps, strongly $\alpha rps$-closed maps and almost $\alpha rps$-closed maps) in topological spaces. Also, we will study the relation between these mappings and discussion some properties of these maps.

Keywords: $\alpha rps$-closed; Topology; Mappings; Subset

Introduction
Mappings play as important role, in the study of modern mathematics, especially in topology and functional analysis [1-5]. Different types of closed and open mappings were studied by various researchers [6]. Generalized closed mappings were introduce and studied. After him different mathematicians worked and studied on different versions of generalized maps [7].

Hamed introduced and studied $\alpha rps$-closed sets and also introduce the notion ($\alpha rps$-continuous, $\alpha rps$-irresolute and strongly $\alpha rps$-continuous) functions [8].

In this paper, we introduce and study new types of closed maps namely $\alpha rps$-closed map in topological spaces and we use this maps to give other types of $\alpha rps$-closed map which are ($\alpha rps$-closed maps [9-13], strongly $\alpha rps$-closed maps and almost $\alpha rps$-closed maps) and we discuss the properties of these maps as well as, shows the relationships between some types of these maps [14-18].

Throughout this paper $(X,\tau)$, $(Y,\sigma)$ and $(Z,\mu)$ (or simply $X$, $Y$ and $Z$) represent non-empty topological spaces [19-22]. For a sub set $A$ of a space $X$,$cl(A)$, $int(A)$ and $A^c$ denoted the closure of $A$, the interior of $A$ and the complement of $A$ in $X$ respectively [23].

Preliminaries
Some definition and basic concepts have been given in this section.

Definition
A sub set $A$ of a space $X$ is said to be a:

1. Semi-open [9] If $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
2. $\alpha$-Open set [16] If $A \subseteq int(cl(int(A)))$ and $\alpha$-closed set if $cl(int(cl(A))) \subseteq A$.
3. Preopen set [15] If $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
4. Semi-preopen set [1] If $A \subseteq cl(int(cl(A)))$ and semi-preclosed if $int(Cl(int(A))) \subseteq A$.
5. Regular open [20] If $A=cl(int(A))$ and regular closed if $A=cl(cl(int(A)))$.
6. Regular $\alpha$-open [21] if there is a regular open set $U$ such that $U \subseteq A \subseteq acl(U)$.

The semi-closure (resp. $\alpha$-closure, semi-pre closure), of a sub set $A$ of $X$ is the intersection of all semi-closed (resp. $\alpha$-closed, semi-pre closed) sets containing $A$ and denoted by $spcl(A)$ (resp. $acl(A)$, resp. $spcl(A)$).

Remark: It has been proved that:
1. Every regular closed set and closed set in a space $X$ is an $\alpha rps$-closed set.
2. Every $\alpha rps$-closed set is ($sg$-closed, $gs$-closed, $ag$-closed, $ga$-closed, $rg$-closed, $rga$-closed) set.

Definition
A sub set $A$ of a space $X$ is said to be a:

1. Generalized closed set (briefly, g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an open set in $X$.
2. Generalized semi-closed set (briefly, gs-closed) [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an open set in $X$.
3. Generalized $\alpha$-closed set (briefly, $\alpha g$-closed) [12] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a semi-open set in $X$.
4. Generalized $\alpha$-closed set (briefly, ga-closed) [13] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an $\alpha$-open set in $X$.
5. Generalized $\alpha$-closed set (briefly, g-$\alpha$-closed) [12] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a regular open set.
6. Generalized $\alpha$-closed set (briefly, rg-$\alpha$-closed) [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a regular open set.
7. Regular generalized $\alpha$-closed set (briefly, rga-$\alpha$-closed) [21] if $Cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is a regular open set.
8. Pre-semi closed if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a $g$-open.
9. Regular pre-semi closed (briefly, rps-$\alpha$-closed) [19] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an $rg$-open set in $X$.

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The concepts of g-closed map and almost closed map are independent to αrps-closed map. As show in the following examples.

Example: Let X={a, b, c} with the topology τ={X, θ, {a}, {a, c}}, where αRPSC(X, τ)={X, θ, {b}, {b, c}} and define f: (X, τ)→(Y, τ) by f(a)=a, f(b)=c and f(c)=b, then f is αrps-closed map. Since for the closed set A={b} in (X, τ), but f(A)=f({b})={c} is closed set in (X, τ). Hence, f is not αrps-closed map. As shown in the following example.

Example: Let X={a, b, c} with the topology τ={X, θ, {a}, {a, c}}, where αRPSC(X, τ)={X, θ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}, where αRPSC(X, τ)={X, θ, {a}, {b, c}}, where αRPSC(X, τ)={X, θ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}, and define f: (X, τ)→(Y, τ) by f(a)=a, f(b)=c and f(c)=b, then f is αrps-closed map, since for the closed set A={b} in (X, τ), but f(A)=f({b})={c} is closed set in (X, τ). Hence, f is αrps-closed map.

In this section, we introduce a new type of closed sets namely arps-closed maps in topological spaces and study some of their properties.

Definition: A map f: (X, τ)→(Y, σ) is called arps-closed map if f(A) is arps-closed set in (Y, σ), for every closed set A in (X, τ).

Proposition: Every arps-closed map is arps-closed map.

Proof: It follows from definition of closed map and fact that every closed set is arps-closed map.

Remark: The converse of above proposition need not be true as seen from the following example.

Example: Let X={a, b, c} with the topology τ={X, θ, [a], [a, c]}, where αRPSC(X, τ)={X, θ, [b], [c], [b, c]} and define f: (X, τ)→(Y, τ) by f(a)=a, f(b)=c and f(c)=b, then f is arps-closed map, but f is not closed map, since for the closed set A={b} in (X, τ), but f(A)=f({b})={c} is closed set in (X, τ). Hence, f is not closed map.

The proof of steps 3, 4, 5, and 6 are similar to step 1 and 2. The following example show the converse of proposition need not be true in general.

Example: Let X={a, b, c} with the topology τ={X, θ, [a], [b, c]}, where αRPSC(X, τ)={X, θ, [a], [b, c]}, where αRPSC(X, τ)={X, θ, [b], [c], [b, c]} and define f: (X, τ)→(Y, τ) by f(a)=a, f(b)=c and f(c)=b, then it is clear that f is gα-closed map. Since for the closed set A={b} in (X, τ), but f(A)=f({b})={c} is closed set in (X, τ). Hence, f is not arps-closed map.

Remark: The concepts of g-closed map and almost closed map are independent to arps-closed map. As shown in the following examples.

Example: Let X={a, b, c} with the topology τ={X, θ, [a], [a, c]}, where
Proposition: A map \( f : (X, \tau) \to (Y, \sigma) \) is \( \alpha rps \)-closed map, if for every closed set \( A \subset X \), \( f(A) \) is \( \alpha rps \)-closed set in \( Y \).

Proof: Let \( A \) be a closed set in \( (X, \tau) \). Then, \( f(A) \) is a closed set in \( (Y, \sigma) \), since \( f \) is a closed map. Therefore, \( f(A) \) is \( \alpha rps \)-closed set in \( Y \). Hence, \( f \) is \( \alpha rps \)-closed map.

Example

Let \( X = \{a, b, c\} \) with the topology \( \tau = \{\emptyset, \{a\}, \{b, c\}\} \), where \( \alpha rps(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}\} \) and \( \alpha a rps(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}\} \). Thus, \( \alpha rps(X, \tau) \) is \( \alpha rps \)-closed map.

Proposition

If \( f : (X, \tau) \to (Y, \sigma) \) is \( \alpha rps \)-closed map and \( Y \) is a \( T^{1/2} \)-space, then \( f \) is closed map.

Proof: Let \( A \) be a closed set in \( (X, \tau) \). Then, \( f(A) \) is a closed set in \( (Y, \sigma) \), since \( f \) is \( \alpha rps \)-closed map. Hence, \( f \) is closed map.

Example

Let \( X = \{a, b, c\} \) with the topology \( \tau = \{\emptyset, \{a\}, \{b\}, \{c\}\} \), where \( \alpha rps(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}\} \) and \( \alpha a rps(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}\} \). Thus \( \alpha rps(X, \tau) \) is \( \alpha rps \)-closed map.
Remark

If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is \( \alpha rps \)-closed map and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) is a closed map, then \( g \circ f: (X, \tau) \rightarrow (Z, \mu) \) need not be \( rps \)-closed map, and this is shown by the following example:

Example

Let \( X=Y=Z=\{a, b, c\} \) with the topologies \( \tau=\{X, \emptyset, \{a\}, \{a, c\}\}, \sigma=\{Y, \emptyset, \{a\}, \{b, c\}\} \) and \( \alpha rps C(Y, \sigma)=\{Y, \emptyset, \{b\}, \{c\}\} \) and \( \alpha rps C(Z, \mu)=\{Z, \emptyset, \{a\}, \{b, c\}\} \). Let \( f: (X, \tau) \rightarrow (Y, \sigma) \), and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) be two the identity maps then it is easy to see that \( f \) is a \( rps \)-closed map and \( g \) is a closed map, but \( g \circ f: (X, \tau) \rightarrow (Z, \mu) \) is not \( rps \)-closed map, since for the closed set \( A=\{b\} \) in \( X, \tau \), then \( g \circ f \) \((\{b\})=g \circ f \) \((\{b\})=\{b\} \), which is not \( rps \)-closed set in \( (Z, \mu) \). Therefore, \( g \circ f: (X, \tau) \rightarrow (Z, \mu) \) is not \( rps \)-closed map.

The following propositions give the condition to make remark true:

Proposition

If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an \( \alpha rps \)-closed map and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) is a closed map and let \( Y \) be \( T^{1/2} \)-space, then \( g \circ f: (X, \tau) \rightarrow (Z, \mu) \) is \( \alpha rps \)-closed map.

Proof

Let \( A \) be a closed set in \( (X, \tau) \), Thus \( f(A) \) is \( \alpha rps \)-closed set in \( (Y, \sigma) \), since \( Y \) is a \( T^{1/2} \)-space, then by proposition. we get \( f \) is a closed map. Now, \( g \circ f(A) = g \circ f \circ f^{-1}(A) \) is a closed set in \( (Z, \mu) \). Therefore, \( g \circ f: (X, \tau) \rightarrow (Z, \mu) \) is \( \alpha rps \)-closed map.

Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) be two maps, such that their composition \( g \circ f: (X, \tau) \rightarrow (Z, \mu) \) is \( \alpha rps \)-closed map.

1. If \( f \) is a continuous and subjective, then \( g \) is \( rps \)-closed map.
2. If \( g \) is \( \alpha rps \)-continuous and injective, then \( f \) is \( \alpha rps \)-closed map.

Proof (i):

Let \( A \) be a closed set in \( (Y, \sigma) \), thus \( f^{-1}(A) \) is a \( \alpha rps \)-closed set in \( (X, \tau) \). Also, since \( f \) is a \( \alpha rps \)-closed map, then \( g \circ f \) \((f^{-1}(A)) = g \circ f \) \((A) \) is a \( \alpha rps \)-closed set in \( (Z, \mu) \). Therefore, \( f \) is \( \alpha rps \)-closed map.

Proof (ii):

Let \( E \) be a closed set in \( (X, \tau) \). Since \( g \circ f \) \((E) \) is a \( \alpha rps \)-closed set in \( (Z, \mu) \), thus \( g \circ f \) \((E) \) is an \( \alpha rps \)-closed map in \( (Z, \mu) \). Therefore, \( f \) is \( \alpha rps \)-closed map.

Some Types of \textit{rps-Closed Maps}

Some other types of \textit{rps-closed} maps are given in this section such as \( \alpha rps \)-closed maps, \( rps \)-closed maps and almost \( rps \)-closed maps) with study the relations between these types of maps.

Definition

A map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called \( \alpha rps \)-closed map if \( f(A) \) is \( \alpha rps \)-closed set in \( (Y, \sigma) \), for every \( \alpha rps \)-closed set \( A \) in \( (X, \tau) \).

Proposition

Every \( \alpha rps \)-closed map is \( rps \)-closed map.

Proof

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be \( \alpha rps \)-closed map and let \( A \) be a closed set in \( (X, \tau) \), by remark \( \forall \) closed set is a \( \alpha rps \)-closed set. Thus, \( A \) is \( \alpha rps \)-closed set in \( (X, \tau) \). Since \( f(A) \) is \( \alpha rps \)-closed map, then \( f(A) \) is a \( \alpha rps \)-closed set in \( (Y, \sigma) \). Therefore, \( f \) is \( \alpha rps \)-closed map.

Corollary

\[ \text{Every} \alpha rps \text{-closed map is} \]

1. \( \text{ag-Closed map.} \)
2. \( \text{ga-Closed map.} \)
3. \( \text{sg-Closed map.} \)
4. \( \text{gs-Closed map.} \)
5. \( \text{rg-Closed map.} \)
6. \( \text{rga-Closed map.} \)

Proof

It is follows from proposition.

Remark: The converse of proposition are not true in general. It is easy to see that in example, \( f \) is \( \alpha rps \)-closed map, but is not \( \alpha rps \)-closed, and in example it is clear that \( f \) is \( \alpha rps \)-closed map, \( gc \)-closed map, \( sg \)-closed map, \( rg \)-closed map and \( rga \)-closed map), but is not \( \alpha rps \)-closed map.

The following propositions give the condition to make the proposition, corollary and Remark are true:

Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an \( \alpha rps \)-closed map and \( Y \) is \( T^{1/2} \)-space then \( f \) is a

1. \( \text{Closed map.} \)
2. \( \text{Almost-closed map.} \)

Proof (i):- It is follows from proposition, we get \( f \) is a closed map.

Proof (ii): It is follows from the fact (\( \forall \) closed map is almost-closed map [17]).

Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be any map, then \( f \) is \( \alpha rps \)-closed map, if \( X \) is \( T^{1/2} \)-space and \( f \) is a

1. \( \text{ag-closed map and Y is a } \alpha T_s \text{-space.} \)
2. \( \text{ga-closed map and Y is a } \alpha T_s \text{-space.} \)

Proof (i)

Let \( A \) be an \( \alpha rps \)-closed set in \( (X, \tau) \), since \( X \) is a \( T^{1/2} \)-space, then by proposition. we get \( f \) is \( \alpha rps \)-closed map, \( \alpha T_s \)-closed map and \( Y \) is a \( \alpha T_s \)-space.

Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be any map, then \( f \) is \( \alpha rps \)-closed map, if \( X \) is \( T^{1/2} \)-space and \( f \) is a

1. \( \text{ag-closed map and Y is a } \alpha T_s \text{-space.} \)
2. \( \text{ga-closed map and Y is a } \alpha T_s \text{-space.} \)

Proof (ii)

It is follows from the fact (\( \forall \) \( \alpha T_s \)-closed map is \( \alpha rps \)-closed map [6]), and Similarly, we proof the following proposition.
2. sg-Closed map and Y is a $T_i$-space.
3. rg-Closed map and Y is a $T^{1/2}$-space.
4. rga-closed map and Y is $T^{1/2}$-space
5. Closed map.
6. arps-Closed map.
7. g-Closed map.

Proposition

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a closed map and $X$ is a $T^{1/2}$-space and locally indiscrete, then $f$ is $\alpha*rps$-closed map.

Proof

Let $A$ be a closed set in $(X, \tau)$. Since $X$ is a $T^{1/2}$-space, then by using proposition. We get $A$ is a closed set in $X$. Also, since $X$ is a locally indiscrete, then by definition of locally indiscrete we have, $A$ is a regular closed set in $X$, since $f$ is a almost-closed map. Thus, $f(A)$ is an almost-closed set in $(Y, \sigma)$. Also, since $g$ is an $\alpha*rps$-closed map.

Thus, $g(f(A))$ is $\alpha*rps$-closed set in $(Z, \mu)$. That is $g(f(A))=g$ of $(A)$ is a $\alpha*rps$-closed set in $(Z, \mu)$.

Therefore, $g$ of $(X, \tau) \rightarrow (Z, \mu)$ is $\alpha*rps$-closed map.

Proposition

The composition of two $\alpha*rps$-closed maps is also $\alpha*rps$-closed map.

Proof

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \mu)$ be two $\alpha*rps$-closed map, and $A$ be $\alpha*rps$-closed set in $X$, since $f$ is $\alpha*rps$-closed, then $f(A)$ is an $\alpha*rps$-closed set in $(Y, \sigma)$. Also, since $g$ is an $\alpha*rps$-closed map.

Thus, $g(f(A))$ is $\alpha*rps$-closed set in $(Z, \mu)$. That is, $g(f(A))=g$ of $(A)$ is a $\alpha*rps$-closed set in $(Z, \mu)$.

Therefore, $g$ of $(X, \tau) \rightarrow (Z, \mu)$ is a $\alpha*rps$-closed map.

Proposition

If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \mu)$ be two $\alpha*rps$-closed map, and then $g$ of $(X, \tau) \rightarrow (Z, \mu)$ is $\alpha*rps$-closed map.

Proof

Let $A$ be a closed set in $(X, \tau)$, then $f(A)$ is $\alpha*rps$-closed set in $(Y, \sigma)$. Also, since $g$ is $\alpha*rps$-closed map.

Thus, $g(f(A))$ is $\alpha*rps$-closed set in $(Z, \mu)$. That is, $g(f(A))=g$ of $(A)$ is a $\alpha*rps$-closed set in $(Z, \mu)$.

Therefore, $g$ of $(X, \tau) \rightarrow (Z, \mu)$ is $\alpha*rps$-closed map.

Similarly, we proof the following corollary.

Corollary

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a closed map and $g: (Y, \sigma) \rightarrow (Z, \mu)$ is a $\alpha*rps$-closed map, then $g$ of $(X, \tau) \rightarrow (Z, \mu)$ is $\alpha*rps$-closed map.

Now, we give another type of $\alpha*rps$-closed map is called strongly $\alpha*rps$-closed map.

Definition

A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called strongly $\alpha*rps$-closed map if $f(A)$ is closed set in $(Y, \sigma)$, for every $\alpha*rps$-closed set $A$ in $(X, \tau)$.

Proposition

Every strongly $\alpha*rps$-closed map $f: (X, \tau) \rightarrow (Y, \sigma)$ is

i. Closed map.

ii. Almost-closed map.

iii. g-Closed map.

iv. arps-Closed map.

v. $\alpha*rps$-Closed map.

Proof

i. Let $A$ be a closed set in $(X, \tau)$, by using remark, step $\forall$ closed set is an $\alpha*rps$-closed we get, $A$ is an $\alpha*rps$-closed set in $(X, \tau)$.

Since $f$ is strongly $\alpha*rps$-closed map. Thus, $f(A)$ is a closed set in $(Y, \sigma)$. Therefore, $f$ is a closed map.

ii. It is clear that from step $\forall$ strongly $\alpha*rps$-closed map is a closed and the fact (closed map is almost closed, [17]).

iii. It is clear that from step $\forall$ strongly $\alpha*rps$-closed map is a closed and the fact (closed map is $g$-closed, [4])

iv. It is clear that from step and the proposition.

v. Let $A$ be an $\alpha*rps$-closed set in $(X, \tau)$. Since $f$ is strongly $\alpha*rps$-closed map.

Thus, $f(A)$ is a closed set in $(Y, \sigma)$, by using remark, $\forall$ closed set is an $\alpha*rps$-closed set, then $A$ is an $\alpha*rps$-closed set in $(Y, \sigma)$. Therefore $f$ is a $\alpha*rps$-closed map

Corollary

Every strongly $\alpha*rps$-closed map $f: (X, \tau) \rightarrow (Y, \sigma)$ is

1. $\alpha*g$-Closed map.
2. $g$-Closed map.
3. sg-Closed map.
4. gs-Closed map.
5. rg-Closed map.
6. $rg\alpha$-Closed map.

Proof

It is clear that from proposition. The following examples show the converse of above proposition and corollary need not be true in general.

Example

Let $X=[a, b, c]$ with the topology $\tau=[X, \emptyset, \{a\}]$ and let $f: (X, \tau) \rightarrow (X, \tau)$ be an identity map. Then, it is clear that $f$ is a closed map, almost-closed map and $g$-closed map] but is not strongly $\alpha*rps$-closed, since for closed set $A=[b], f(A)=f([b])=[b]$ is not closed set in $(X, \tau)$.

Example

Let $X=Y=[a, b, c]$ with the topologies $\tau=[X, \emptyset, \{a\}]$, and $\sigma=[X, \emptyset, \{a\}, \{a, c\}]$, where $\alpha\text{RPS}C(X, \tau)=[X, \emptyset, \{b\}, \{c\}, \{b, c\}]$ and let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Then, it is clear that $f$ is $\alpha*rps$-closed map $\alpha*rps$-closed map but is not strongly $\alpha*rps$-closed map. Since for closed set $A=[c], f(A)=f([c])=[c]$ is not closed set in $(Y, \sigma)$.

Example

Let $X=Y=[a, b, c]$ with the topologies $\tau=[X, \emptyset, \{a\}]$, and $\sigma=[X, \emptyset, \{a, b\}, \{b, c\}]$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=a$ and $f(c)=c$. Then, it is clear that $f$ is $\alpha*g$-closed map (ga-closed map, sg-closed map, gc-closed map, rg-closed map and rga-closed map), but is not strongly $\alpha*rps$-closed map, since for closed set $A=[c], f(A)=f([c])=[c]$ is not closed set in $(Y, \sigma)$.

The following condition to make proposition and corollary are true
Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be any map, then \( f \) is a strongly \( \alpha rps \)-closed map if \( X \) is a \( T^{1/2} \)-space and

1. Closed map
2. Almost- closed map.
3. \( g \)-Closed map.
4. \( ga \)-Closed map.
5. \( rg \)-Closed map.
6. \( rga \)-Closed map.
7. \( \alpha rps \)-Closed map.

Proof

It follows from proposition and step and proposition.

Proposition

If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an \( \alpha rps \)-closed map and \( Y \) is a \( T^{1/2} \)-space, then \( f \) is a strongly \( \alpha rps \)-closed map.

Proof

Let \( A \) be an \( \alpha rps \)-closed set in \( (X, \tau) \). Since \( f \) is \( \alpha rps \)-closed map. Thus \( f(A) \) is \( \alpha rps \)-closed set in \( (Y, \sigma) \). Also, since \( Y \) is a \( T^{1/2} \)-space, then \( f(A) \) is a closed set in \( (Y, \sigma) \). Therefore, \( f \) is strongly \( \alpha rps \)-closed map.

Proposition

If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is almost-closed map and \( X \) is a locally - indiscrete and \( T^{1/2} \)-space, then \( f \) is a strongly \( \alpha rps \)-closed map.

Proof

Let \( A \) be an \( \alpha rps \) - closed set in \( (X, \tau) \). Since \( X \) is a \( T^{1/2} \)-space. Then, \( A \) is a closed set in \( (X, \tau) \), hence \( f(A) \) is a \( \alpha rps \)-closed set in \( (Y, \sigma) \). Therefore, \( f \) is a strongly \( \alpha rps \)-closed map.

Next, we give some proposition and results about the composition of strongly \( \alpha rps \)-closed map.

Proposition

The composition of two strongly \( \alpha rps \)-closed maps is also strongly \( \alpha rps \)-closed map.

Proof

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) be two strongly \( \alpha rps \)-closed maps, and \( A \) be \( \alpha rps \)-closed set in \( (X, \tau) \), since \( f \) is strongly \( \alpha rps \)-closed map, then \( f(A) \) is a closed set in \( (Y, \sigma) \), by remark \( \forall \) closed set is \( \alpha rps \)-closed set, so we get \( f(A) \) is \( \alpha rps \)-closed set in \( (Y, \sigma) \). Also, since \( g \) is an \( \alpha rps \)-closed map. Thus, \( g(f(A)) \) is a closed set in \( (Z, \mu) \). That is \( g(f(A)) = g(A) \) is \( \alpha rps \)-closed set in \( (Z, \mu) \). Therefore, \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is strongly \( \alpha rps \)-closed map.

Similarly, we proof the following proposition.

Proposition

1. If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a strongly \( \alpha rps \)-closed map and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) is closed map, then \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is a strongly \( \alpha rps \)-closed map.
2. If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a \( \alpha rps \) - closed map and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) is strongly \( \alpha rps \)-closed map, then \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is a strongly \( \alpha rps \)-closed map.

Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) be two maps, then \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is a \( \alpha rps \)-closed map, if \( f \) is strongly \( \alpha rps \)-closed map and

i. \( g \) is \( \alpha rps \)-closed map.
ii. \( g \) is \( \alpha rps \)-closed map.

Proof

(i) Let \( A \) be \( \alpha rps \)-closed set in \( X \), since \( f \) is strongly \( \alpha rps \)-closed map, then \( f(A) \) is a closed set in \( Y \). Also, since \( g \) is \( \alpha rps \)-closed map. Thus, \( g(f(A)) \) is a \( \alpha rps \)-closed set in \( Z \). That is \( g(f(A)) = g(A) \) is a \( \alpha rps \)-closed set in \( Z \), Therefore, \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is a \( \alpha rps \)-closed map.

The proof of steps.

Remark

In the proposition the composition \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) need not be in general strongly \( \alpha rps \)-closed map. As shows in the following example:

Example

Let \( X=Y=Z=\{a, b, c\} \) with the topologies \( \tau=\{X, \emptyset, \{a\}, \{b, c\}\} \), \( \sigma=\{Y, \emptyset, \{a\}, \{a, c\}\} \), \( \mu=\{Z, \emptyset, \{a\}, \{a, c\}\} \) and let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an identity map and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) be a mapping defined by \( g(a)=g(b)=c \) and \( g(c)=b \), then it is easy to see that \( f \) is strongly \( \alpha rps \)-closed map and \( g \) is \( \alpha rps \)-closed map.

Proposition

Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \mu) \) be two any maps then \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is \( \alpha rps \)-closed map, if \( g \) is strongly \( \alpha rps \)-closed and

i. \( f \) is a closed map.
ii. \( f \) is a \( \alpha rps \)-closed map.

Proof

(i) Let \( A \) be a closed set in \( X \), since \( f \) is closed map, then \( f(A) \) is a \( \alpha rps \)-closed map. Thus, \( g(f(A)) \) is a \( \alpha rps \)-closed set in \( Z \). That is \( g(f(A)) = g(A) \) is a \( \alpha rps \)-closed set in \( Z \), Therefore, \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) is \( \alpha rps \)-closed map.

The proof of steps.

Remark

In the proposition the composition \( g \) of \( (X, \tau) \rightarrow (Z, \mu) \) need not be in general strongly \( \alpha rps \)-closed map. As shows in the following example:

Example

Let \( X=Y=Z=\{a, b, c\} \) with the topologies \( \tau=\{X, \emptyset, \{a\}, \emptyset, \{b, c\}\} \), \( \sigma=\{Y, \emptyset, \{a\}, \{a, c\}\} \), \( \mu=\{Z, \emptyset, \{a\}, \{a, c\}\} \) and let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a mapping defined by \( g(a)=g(b)=c \) and \( g(c)=b \), then it is easy to see that \( f \) is a closed map and \( g \) is \( \alpha rps \)-closed map.
strongly αrps-closed map, but g of: \((X, \tau) \rightarrow (Z, \mu)\) is not strongly αrps-closed map, since for the closed set \(A=\{b\}\) in \((X, \tau)\), then g of \((A)=g of \((A)=g of \((\{b\} \rightarrow g of \((\{b\}))=g of \((\{c\})\), which is not closed set in \((Z, \mu)\).

The following proposition give the condition to make Remark true:

**Proposition**

Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be any two any maps, then g of \((X, \tau) \rightarrow (Z, \mu)\) is a strongly αrps-closed map, if f is a strongly αrps-closed map and \((Z, \mu)\) is a \(T^{\alpha rps}\)-space

1. g is αrps-closed map.
2. g is α*rps-closed map.

**Proof**

Let \(A\) be a αrps-closed set in \(X\), then \(f(A)\) is a closed set in \((Y, \sigma)\), by Remark \((\forall \) closed set is an αrps-closed set), since \(g\) is αrps-closed map. Thus \(g(f(A))\) is αrps-closed set in \((Z, \mu)\). That is \(g(f(A))=g of \((A)\) is an αrps-closed set in \((Z, \mu)\). Also, since \(Z\) is \(T^{\alpha rps}\)-space, so we get g of \((A)\) is a closed set in \((Z, \mu)\). Therefore, \(g of: (X, \tau) \rightarrow (Z, \mu)\) is strongly αrps-closed map.

**Proposition**

Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be any two any maps, then g of \((X, \tau) \rightarrow (Z, \mu)\) is strongly αrps-closed map, if f is strongly αrps-closed map and \((X, \tau)\) is a \(T^{\alpha rps}\)-space and

1. \(f\) is a closed map
2. \(f\) is a αrps-closed map.

**Proof**

(i) Let \(A\) be αrps-closed set in \(X\), since \(X\) is a \(T^{\alpha rps}\)-space, then by using proposition we get \(A\) is a closed set in \(X\). Thus, \(f(A)\) is a closed set in \(Y\), by remark, if \((A)\) is an αrps-closed set in \(Y\). Also, since \(Z\) is \(T^{\alpha rps}\)-space, so we get g of \((A)\) is a closed set in \((Z, \mu)\). Therefore, \(g of: (X, \tau) \rightarrow (Z, \mu)\) is strongly αrps-closed map.

(ii) \(f\) is a αrps-closed map.

\(g of: (X, \tau) \rightarrow (Z, \mu)\) is strongly αrps-closed map, since, \(f\) is subjective. Therefore, \(g\) is αrps-closed map. Thus, \(g(f(A))\) is a closed set in \((Z, \mu)\). That is \(g(f(A))=g of \((f(A))=g of \((A)\) is a closed set in \((Z, \mu)\). Hence, \(g of: (X, \tau) \rightarrow (Z, \mu)\) is strongly αrps-closed map.

**Definition**

A map \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called almost αrps-closed map if \(f(A)\) is αrps-closed set in \((Y, \sigma)\), for every regular closed set \(A in (X, \tau)\).

**Proposition**

Every almost closed map is almost αrps-closed map.

**Proof**

Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a almost closed map and \(A\) be a regular closed set in \((X, \tau)\). Then, \(f(A)\) is a closed set in \((Y, \sigma)\) and by using remark we get \(f(A)\) is a αrps-closed set in \((Y, \sigma)\). Hence, \(f\) is almost αrps-closed map.

**Corollary**

1. Every closed map is almost αrps-closed map.
2. Every α*rps-closed map is almost αrps-closed map.
3. Every strongly αrps-closed map is almost αrps-closed map.

**Proof**

The converse of proposition and corollary need not be true in general.

**Example**

Let \(X=\{a, b, c\}\) with the topologies \(\tau=\{X, \emptyset, \{a\}\}\) and \(\sigma=\{Y, \emptyset, \{a, b\}\}\), where αRPSC(\(X, \tau\)=\(X, \emptyset, \{a\}, \{b\}, \{a, b\}\)) and αRPSC(\(Y, \sigma\)=\(Y, \emptyset, \{a, b\}\)). Define \(f(X, \tau) \rightarrow (Y, \sigma)\) by \(f(a)=c\), \(f(b)=a\) and \(f(c)=b\). Then, it is clear that \(f\) is almost αrps-closed map but is not closed map (αrps-closed, α*rps-closed and strongly αrps-closed map), since for closed set \(A=\{b, c\}\) in \((X, \tau)\), \(f(A)=\{f(b), c\}=\{a, c\}\) is not closed and \((\alpha rps-powered closed \) set in \(Y)\).

**Example**

Let \(X=\{a, b, c\}\) with the topologies \(\tau=\{X, \emptyset, \{a\}\}\) and \(\sigma=\{Y, \emptyset, \{a, b\}\}\), where RC(\(X, \tau\)=\(X, \emptyset, \{b\}, \{c\}\)) and αRPSC(\(Y, \sigma\)=\(Y, \emptyset, \{b\}\)). Define \(f(X, \tau) \rightarrow (Y, \sigma)\) by \(f(a)=b\), \(f(b)=c\) and \(f(c)=c\). Then, it is clear that \(f\) is almost closed map but is not almost closed map, since for regular closed set \(A=\{b, c\}\) in \((X, \tau)\), \(f(A)=\{f(b), c\}=\{c\}\) is not closed in \((Y, \sigma)\).

The following proposition give the condition to make, proposition and corollary are true:

**Proposition**

If \(f: (X, \tau) \rightarrow (Y, \sigma)\) is almost αrps-closed map and \((Y, \sigma)\) is a \(T^{\alpha rps}\)-space, then \(f\) is an almost-closed set.
Proposition

If f: (X,τ)→(Y, σ) is almost arps-closed map and X is a locally indiscrete space, then f is arps-closed set.

Proof

Let A be a regular closed set in (X, τ), since X is a locally indiscrete, then by definition. We get, A is a regular closed set in X. Also, since f is a almost arps-closed map, then f(A) is an arps-closed set in (Y, σ). Therefore, f is an arps-closed map.

Proposition: Let f: (X, τ)→(Y, σ) be almost arps-closed map and X be a locally indiscrete space and Y be a T*1/2-space, then f is a closed set.

Proof: Let A be a closed set in (X, τ), since X is a locally indiscrete, then by definition. We get, A is a regular closed set in X. Also, since f is a almost arps-closed map. Then, f(A) is an arps-closed set in (Y, σ). Therefore, f is an arps-closed map.

Proposition: Let f: (X, τ)→(Y, σ) be almost arps-closed map and X be a locally indiscrete space and Y be a T*1/2-space, then f is a closed set.

Proof: Let A be a closed set in (X, τ), since X is a locally indiscrete, then by definition. We get, A is a regular closed set in X. Also, since f is a almost arps-closed map. Then, f(A) is an arps-closed set in (Y, σ) and since Y is a T*1/2-space, then by proposition we get f(A) is a closed set in Y.

Proposition

Let f: (X, τ)→(Y, σ) be almost arps-closed map and X be a locally indiscrete space and T*1/2-space, then

1. f is an α*arps-closed set.
2. f is a strongly arps-closed set if Y is a T*1/2-space.

Proof: (i) Let A be a arps-closed set in (X, τ), since X is a T*1/2-space, then by proposition. we have, A is a closed set in X and since X is a locally indiscrete, then by definition. We get, A is a regular closed set in X. Also, since f is a almost arps-closed map. Then, f(A) is an arps-closed set in (Y, σ). Therefore, f is an α*arps-closed set.

Proof: (ii) Let A be a arps-closed set in (X, τ), since X is a T*1/2-space, then by proposition. we have, A is a closed set in X and since X is a locally indiscrete, then by definition. We get, A is a regular closed set in X. Also, Thus, f(A) is an arps-closed set in (Y, σ). Also, since Y is a T*1/2-space Hence, f(A) is a closed set in Y. Therefore, f is a strongly arps-closed set.

Remark: The composition of two strongly arps-closed maps need not be strongly arps-closed map in general, the following example to show that.

Example: Let X=[a, b, c, d], Y=Z=[a, b, c] with the topologies τ=X, θ=[X, θ], {a, b, c}, {a, b, d}, θ, RC(X, τ)=RC(τ,X), RC(Y, σ)=RC(σ,Y), RC(Z, ρ)=RC(ρ,Z), where RC(X, τ)=RC(τ,X), RC(Y, σ)=RC(σ,Y), RC(Z, ρ)=RC(ρ,Z), and ARPCSC(Y, σ)={Y, θ}, ARPCSC(X, τ)={X, θ}, ARPCSC(Z, ρ)={Z, θ, [a, b, c]} Define f(X, τ)→(Y, σ) by f(a)=f(d)=b, f(b)=f(c)=c and g: (Y, σ)→(Z, μ) be an identity map, then it is easy to see that f and g are almost arps-closed maps, but g of: (X, τ)→(Z, μ) is not almost arps-closed map, since for the regular closed set A=[a, d] in [X, τ] g of (A)=g of (a,d)=g of ([a, d])=g of (b)=g of ([b], which is not arps-closed set in (Z, μ). Hence, g of is not almost arps-closed map the following proposition give the condition to make remark is true:

Proposition

If f(X, τ)→(Y, σ) and g: (Y, σ)→(Z, μ) are two almost arps-closed maps and Y is locally indiscrete and T*1/2-space, then g of: (X, τ)→(Z, μ) is α*arps-closed map.

Proof: Let A be a regular closed set in X, then f(A) is an arps-closed set in Y. Also, since Y is a T*1/2-space, then by proposition. we get f(A) is a closed set in Y. Also, since Y is a locally indiscrete, hence f(A) is a regular closed set in Y, since g is almost arps-closed map. then g(f(A)) is a arps-closed set in Z. But g of (f(A))=g of (A). Therefore, g of: (X, τ)→(Z, μ) is α*arps-closed map. The proof of the following proposition it is easy.

Proposition

Let f(X, τ)→(Y, σ) and g: (Y, σ)→(Z, μ) be two maps, then g of: (X, τ)→(Z, μ) is α*arps-closed map, if f is almost arps-closed and g is Figure 1.

1. α*rps-closed map.
2. Strongly arps-closed map.

Remark: Here in the following diagram illustrates the relation between the arps-closed mapping types (without using condition), where the converse is not necessarily true.

Figure 1: Illustrates the relation between the arps-closed mapping types.

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