Reconstructing the Source in Heavy Ion Collisions from Particle Interferometry

Urs Achim Wiedemann, Boris Tomášik and Ulrich Heinz

Institut für Theoretische Physik,
D-93040 Regensburg, Germany

The preliminary CERN SPS NA49 Pb+Pb 158 GeV/A one- and two-particle h−-spectra at mid-rapidity are consistent with a source of temperature $T \approx 130$ MeV, lifetime $\tau_0 \approx 9$ fm/c, transverse flow $\eta_f \approx 0.35$, and a transverse geometric size which is twice as large as the cold Pb nucleus.

1. Introduction

Hadronic one- and two-particle spectra provide for each particle species information about the phase-space distribution $S(x, K)$ of the hadronic emission region [1]. Once reconstructed from the measured spectra, $S(x, K)$ allows to distinguish between different dynamical scenarios of heavy ion collisions. It provides an experiment-based starting point for a dynamical back extrapolation into the hot and dense early stage of the collision, where quarks and gluons are expected to be the relevant physical degrees of freedom.

Our reconstruction of $S(x, K)$ is based on the hadronic one- and two-particle spectra

\[ E \frac{dN}{dp} = \int d^4x S(x, p) = \frac{1}{2 \pi p \cdot dp \cdot d\phi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos n(\phi - \psi_R) \right], \tag{1} \]

\[ C(K, q) = 1 + \frac{\int d^4x S(x, K) e^{iq \cdot x}}{\int d^4x S(x, p_1) \int d^4y S(y, p_2)} = 1 + \lambda \exp \left[ - \sum_{ij} R_{ij}^2(K) q_i q_j \right]. \tag{2} \]

The triple-differential hadronic one-particle spectrum (1) tests the momentum-dependence of $S(x, K)$ only. Its azimuthal $\phi$-dependence with respect to the reaction plane $\psi_R$ is parametrized by the harmonic coefficients $v_n$ [2]. Information about the space-time structure of $S(x, K)$ can be obtained from the relative momentum dependence of the two-particle correlator $C(K, q)$, $q = p_1 - p_2$. This $q$-dependence is usually parametrized via the Hanbury-Brown Twiss (HBT) radius parameters $R_{ij}^2(K)$ which depend on the average pair momentum $K = \frac{1}{2} (p_1 + p_2)$. Depending on the Gaussian parametrization adopted, the indices $i, j$ in (2) run either over the Cartesian directions $long$ (parallel to the beam), $out$ (parallel to the transverse component $K_\perp$) and $side$, or over the corresponding Yano-Koonin coordinates $q_\perp = \sqrt{q_i^2 + q_j^2}$, $q_3$ and $q^0 = q \cdot K/K_0$ [3].
Figure 1. LHS: \( \chi^2 \) contour plot of a fit to the NA49 \( h^- \) spectrum. Dashed lines are for constant values of \( \eta_f^2/T \). RHS: different combinations of temperature \( T \) and transverse flow \( \eta_f \) account for the same one-particle slope.

2. Reconstructing an azimuthally symmetric source

A typical data analysis starts from a simple ansatz for the phase space distribution \( S(x,K) \) in terms of very few, physically intuitive fit parameters, \( [1,4] \):

\[
S_\pi(x,p) = S_{\text{dir}}\pi(x,p) + \sum_R S_{R\rightarrow\pi}(x,p),
\]

\[
S_{t}(x,P) = \frac{2J_i+1}{(2\pi)^3} P \cdot n(x) \exp \left( -\frac{P \cdot u(x)}{T} \right) \exp \left( -\frac{r^2}{2R^2} - \frac{\eta^2}{2(\Delta \eta)^2} - \frac{(\tau - \tau_0)^2}{2(\Delta \tau)^2} \right),
\]

This model e.g. assumes local thermalization at freeze-out with temperature \( T \) within a space-time region of transverse Gaussian width \( R \), emission duration \( \Delta \tau \), longitudinal extension \( \eta \Delta \eta \), where \( \eta = \frac{1}{2} \ln [(t + z)/(t - z)] \), and average emission time \( \tau_0 \). The model allows for dynamical source correlations via the hydrodynamic flow 4-velocity \( u_\mu(x) \).

We assume a linear transverse flow profile \( \eta_t(r) = \eta_f \left( \frac{r}{R} \right) \) with variable strength \( \eta_f \), and Bjorken scaling of the flow component in the longitudinal direction, \( v_L = z/t, \eta_L = \eta/2 \ln [(1 + v_L)/(1 - v_L)] = \eta \). Resonances are produced in thermal abundances with proper spin degeneracy \( 2J_i+1 \) for each particle species \( i \). Their contribution to the pion yield is obtained by propagating them along their classical path \( x^\mu = X^\mu + \frac{P^\mu}{M}\tau \) according to an exponential decay law \( [4] \),

\[
S_{R\rightarrow\pi}(x,p) = \int_R \int d^4 X \int d\tau \Gamma e^{-\Gamma \tau} \delta^{(4)} \left[ x - \left( X + \frac{P}{M}\tau \right) \right] S_{\text{dir}}^R(X,P),
\]

where \( \int_R \) is the integral over the available resonance phase space for isotropic decays. We include all pion decay channels of \( \rho, \Delta, K^*, \Sigma^*, \omega, \eta, \eta', K_0^*, \Sigma \) and \( \Lambda \) with branching ratios larger than 5 percent. The model parameters \( T, \eta_f, \tau, \Delta \eta, \Delta \tau, \tau_0 \) can then be determined via the following strategy:

1. transverse one-particle spectrum \( dN/dM_\perp^2 \) determines blue-shifted temperature \( T_{\text{eff}} \): The slope of \( dN/dM_\perp^2 \) is essentially given by \( T_{\text{eff}} = T \sqrt{(1 + \eta_f)/(1 - \eta_f)} \). Hence, different combinations of \( T \) and \( \eta_f \) can account for the same data, see Fig. 1.

2. \( dN/dM_\perp^2 \) and \( R_\perp(M_\perp) \) disentangle temperature and transverse flow. \( R_\perp \) fixes transverse extension \( R \):
Figure 2. Yano-Koonin-Podgoretskii HBT-radius parameters. The preliminary NA49 Pb+Pb data are for $h^+h^+$ (squares) and $h^-h^-$ (diamonds) correlations.

The $M_\perp$-slope of $R_\perp$ is proportional to $\eta_f^2/T$, $R_\perp^2 \approx R^2/(1 + M_\perp \eta_f^2/T)$ [6,7]. This slope allows in combination with $dN/dM_\perp^2$ to specify $T$ and $\eta_f$, see Fig. 2. The overall size of $R_\perp$ determines the Gaussian width $R$. We find $\eta_f \approx 0.35$, $T \approx 130$ MeV, $R \approx 7$ fm.

3. $R_\parallel$ determines $\tau_0$, width of rapidity distribution $dN/dY$ determines $\Delta \eta$:
In principle, $R_\parallel$ depends on $\tau_0$, $\Delta \eta$ and $\Delta \tau$ [7]. We have fixed $\Delta \eta = 1.3$ by matching the width of the pion rapidity distribution. The data then favour clearly a lifetime of $\tau_0 \approx 9$ fm/c (see Fig. 2) but are not very sensitive to the emission duration $\Delta \tau$ [8]. The present plots are obtained with $\Delta \tau = 1.5$ fm/c.

4. $R_0$ discards opaque sources:
For the model [8-9], the YKP-parameter $R_0$ is mainly sensitive to the temporal aspects of the source. The large statistical uncertainties for $R_0$ do not allow to constrain the model parameter space further, see Fig. 4. Models of opaque sources including an opacity factor in (4) are excluded already by the present data [8].

The radius of a cold Pb nucleus corresponds to $R_{\text{Pb}}(\text{cold}) \approx 3.5$ fm in the Gaussian parametrization [4], i.e., the experimental data indicate a very large source $R \approx 2 R_{\text{Pb}}(\text{cold})$. This is dynamically consistent with a collision scenario in which the initially produced pressure gradients result in a significant transverse flow $\eta_f = 0.35$, driving the expansion of the system over a time of 9 fm/c to twice its initial size. The temperature may have decreased substantially during this expansion; the data indicate 130 MeV at freeze-out. These conclusions are further supported by the analysis presented by G. Roland [9], which is based on approximate analytical formulas. We have extracted the model parameters by comparison to numerical model calculations, following the above strategy. They are not obtained from a simultaneous fit to all observables, and hence we do not quote errors.

3. Particle interferometry for collisions with finite impact parameter
The reaction plane analysis of triple-differential one-particle spectra [1] has been discussed extensively in this conference e.g. by A. Poskanzer, J.-Y. Ollitrault, and S.
Voloshin. The general strategy for linking this analysis to an azimuthally sensitive particle interferometry is based on the $\Phi$-dependence of the HBT-radii \cite{10}, where $\Phi$ measures the azimuthal angle of $\vec{K}_\perp$ relative to the reaction plane,

$$R^2_{ij}(K_\perp,\Phi,Y) = R^2_{ij,0}(K_\perp,Y) + 2 \sum_{n=1}^\infty R^{c}_{ij,n} 2(K_\perp,Y) \cos n\Phi + 2 \sum_{n=1}^\infty R^{s}_{ij,n} 2(K_\perp,Y) \sin n\Phi . \tag{6}$$

Here, the many harmonic coefficients $R^{*,*}_{ij}$ make a direct comparison to experimental data impossible. However, various relations hold amongst these coefficients, since the leading anisotropy in realistic source models $S(x,K)$ can be quantified by very few parameters only. Using the symmetries of the system and assuming that elliptic deformations dominate we find \cite{10}

$$0 \approx R^{c}_{i,m}^2 \approx R^{s}_{i,m}^2 \approx R^{c}_{d,m}^2 \approx R^{s}_{d,m}^2 \approx R^{c}_{s,l,m}^2 \approx R^{s}_{s,l,m}^2 , \quad m \geq 1 , \tag{7}$$

$$\alpha_1 \approx \frac{1}{3} R^{c}_{o,1}^2 \approx R^{c}_{s,1}^2 \approx -R^{s}_{o,1}^2 , \tag{8}$$

$$\alpha_2 \approx -R^{c}_{o,2}^2 \approx R^{s}_{s,2}^2 \approx R^{s}_{o,2}^2 . \tag{9}$$

A violation of Eqs. (8)-(9) by experiment would indicate strong higher order deformations and rule out many model scenarios. On the basis of Eqs. (7)-(9), an azimuthally sensitive parametrization of the two-particle correlator involves only two additional fit parameters,

$$C_{\psi_R} (K, q) \approx 1 + \lambda \exp[-R_{o,0}^2 q_o^2 - R_{s,0}^2 q_s^2 - R_{l,0}^2 q_l^2 - 2 R_{d,l,0}^2 q_o q_l] \times \exp[-\alpha_1 (3 q_o^2 + q_s^2) \cos(\Phi - \psi_R) + 2 \alpha_1 q_o q_s \sin(\Phi - \psi_R)] \times \exp[-\alpha_2 (q_o^2 - q_s^2) \cos 2(\Phi - \psi_R) + 2 \alpha_2 q_o q_s \sin 2(\Phi - \psi_R)] \tag{10}$$

The anisotropy parameter $\alpha_1$ vanishes at mid-rapidity or if the source contains no dynamical correlations. It characterizes anisotropic dynamics. The parameter $\alpha_2$ characterizes the elliptic geometry. The parameters $\alpha_1$ and $\alpha_2$ can be determined from event samples in spite of the uncertainty in the eventwise reconstruction of the reaction plane. For details, see Ref. \cite{10}.

This work is supported by BMBF, DAAD, DFG and GSI.

REFERENCES

1. U. Heinz, nucl-th/9609029.
2. S.A. Voloshin and Y. Zhang, Z. Phys. C 70 (1996) 665.
3. P. Jones for the NA49 Coll., Quark Matter ’96, Nucl. Phys. A610 (1996) 188c.
4. U.A. Wiedemann and U. Heinz, Phys. Rev. C 56 (1997) 3265; ibidem R610.
5. H. Appelshäuser, NA49 PhD-thesis.
6. S. Chapman, R. Nix and U. Heinz, Phys. Rev. C 52 (1995) 2694.
7. U.A. Wiedemann, P. Scotto and U. Heinz, Phys. Rev. C 53 (1996) 918.
8. B. Tomášik and U. Heinz, nucl-th/9707001, and poster at this conference.
9. G. Roland for the NA49 Coll., these proceedings.
10. U.A. Wiedemann, Phys. Rev. C 57 (1998) 266.