Research on Geometric Precision Compensation of NC Machining Tools Based on Modified 9-line method

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Abstract: In the process of precision compensation of NC machine tools, the traditional 9-line method has such principle error as error compensation of the rolling angle, because of difference from the coordinate system of the laser measurement and machine tool manufacturing. Secondly, the method has a poor robustness because the measurement noise and the repeatability of the machine tool are included in the measurement. Thirdly, the compensation model can only be used for the error compensation, and cannot be used for the precision design and the precision control and process optimization in the process of the machine tool assembly. In the paper 9-line method is modified based on error compensation model reconstructed, computer simulation research and empirical analysis. Compared with the experimental results, it is proved that the new method can correct three points of the traditional nine line method, and improve the accuracy of the roll angle error.

1. Preface
In such different stages of NC machine tool design as design, manufacturing, assembly and motion, the precision must be guaranteed to assembly and debugging. The precision compensation is implemented by modifying the parameters of input control system to compensate the controllable precision. The compensation has to go through two stages. One is to establish a six-dimensional position and attitude error model of work-piece, and the other is to estimate error by the model. At present, all the researches are based on the mapping model, which is based on the geometric error source of the machine tool and the tool and work-piece of the end-effector. It is the basis of compensation and the entity of research. Error prediction can be achieved by direct measurement or indirect identification. The accuracy of the results reflects the source of geometric error and the choice of compensation methods. Nine-line method is one of the effective methods for multi-axis NC machine tool detection and error compensation, because it can quickly achieve error measurement and error separation. Based on the principle of laser measurement, the nine-line method can reverse the error of machine tools by measuring the synthetic error between the actual motion of moving parts and the motion of laser beam simulation. However, in the process of error compensation for precision CNC machine tools, the application of the nine-line method and the credibility of the measurement results are limited because of the following three reasons. One is that the coordinate system of laser measurement and machine tool manufacturing is different, for example, there is a principle error in the process of error compensation of roll angle. Secondly, the nine-line method incorporates measurement noise and repetitive positioning accuracy of machine tools, resulting in poor robustness of the identification algorithm. Thirdly, the identification results obtained by the nine-line method can only
be used for error compensation, but cannot be used for precision control and process optimization in the early stage of machine tool precision design and machine tool assembly and debugging, because the model neglects the structural parameters of the machine tool in the design.

2. The principle and its shortcomings of the nine-line method for measuring roll angle

The principle of the nine-line method is derived from the principle of laser interferometry. By measuring the positioning error and part straightness error of nine straight lines in the three-dimensional workspace of the machine tool, 21 geometric errors of the three feed shafts are identified. It is known that the attitude error is positively related to the distance from the axis of the rigid body to the axis according to the Bell principle. The position error and roll angle error of any point in the machine tool coordinate system and the measuring coordinate system are directly proportional to the roll angle. The straightness error based on laser interferometry is caused by the laser beam coordinate system. The non-coincidence of the two coordinates results in the deviation term which varies with the measurement position. And this deviation term is the deviation value neglected by the nine-line method in identifying the roll angle error. The effectiveness of error compensation depends on the accuracy of the deviation measurement.

In order to improve the identification efficiency, the nine-line identification model uses a set of data to observe each error source, and the total data is 6 groups. However, the more data sets, the greater the error identification accuracy is disturbed. For multi-axis high precision CNC machine tools, the above-mentioned deviation terms will be greater, the impact on the identification results will be more obvious, and even lead to the results of the unreliable [3]. Therefore, it is of practical significance to improve the robustness of the nine-line algorithm and improve the stability and accuracy of identification accuracy.

The advantage and disadvantage of the nine-line method is that it does not establish the precision model of the machine tool and ignores the dimension parameters of the machine tool structure. It is easy to identify the geometric source error and separate the error compensation. Due to neglecting the existence of the machine tool itself, the assembly error and its cumulative effect of the machine tool can not be reflected comprehensively in the identification of the nine-line method, so that the error of the error source can not be reflected in the compensation results, and the compensation can not be original. The absence of nine-line identification model in the stage of precision design and process planning of machine tools is a congenital deficiency.

3. Improvement of the nine-line identification model

The improved nine-line identification model should solve the problem that the difference between machine tool coordinates and laser measuring coordinates leads to measurement deviation. And it can optimize the identification results, improve the robustness of identification by enhancing the denoising performance, and reflect the element error in the process of machine tool manufacturing and assembly. Taking X to feed axis as an example, the principle and application of error identification are described as bellows.

As a straight-line feed motion, the X-direction feed shaft produces six geometric errors, which are known as three translational errors and three angular errors [3]. Its vector expression is
Fig. 1 setting of coordinate system and coordinate system of machine tool

\[ e(x) = \left( \begin{array}{ccc} \rho_x(x) & \rho_y(x) & \rho_z(x) \\ \theta_x(x) & \theta_y(x) & \theta_z(x) \end{array} \right) \]  

(1)

\( \rho_x(x), \rho_y(x), \rho_z(x) \) are translational errors of X axis respectively, \( \theta_x(x), \theta_y(x), \theta_z(x) \) are rotation error of X axis respectively. It can be seen that these 6 errors are related to the measuring axis X.

The mechanical coordinate origin of the translational axis is set by the machine tool manufacturer and is generally located at 0 at the end of the feed shaft (see Fig. 1, the machine coordinate system is selected at 0 starting point of the effective stroke of the feed shaft). Three points \( p_1, p_2, p_3 \) are selected in three straight lines parallel to the X axis respectively. Their position coordinates in the machine tool coordinate system can be expressed as:

\[ r_{pi} = (x_i, y_i, z_i) \]

\( \Delta r_{pi} \) represents position error, any position is caused by measurement error.

\[ \Delta x_{xj} = \begin{bmatrix} \Delta r_{p1}^{(xj)} \\ \Delta r_{p2}^{(xj)} \\ \Delta r_{p3}^{(xj)} \end{bmatrix} \]  

(2)

In the X direction, there is a deviation \( K_{xj} \) in the Y direction straightness error of a certain point, which is based on the value \( \Delta \theta_x^{(xj)} \) obtained by laser measurement and the value \( \Delta \theta_x^{(xj)}_0 \) obtained by measuring in machine coordinate system. The value determines the degree of roll angle error [4].

\[ K_{xj} = \Delta \theta_x^{(xj)} - \Delta \theta_x^{(xj)}_0 \]  

(3)

According to formula (3), the K value can be calculated by increasing the number of measurement groups and using the least square method. The K value calculated on part of the stroke can be used in the whole stroke roll angle measurement, which greatly improves the accuracy of the measurement.

Fig. 2 the deviation coefficient K value is established when the worktable moves to different positions.

When measuring the straightness error of the feed shaft with a dual-frequency laser measuring instrument, a laser receiver (or reflector) is moving with the worktable, which can be regarded as a
working coordinate system. It is a moving coordinate system. N measuring points are selected equidistantly on the measuring axis, and the position errors of any point $p_i$ (the position straightness errors distributed in three directions are expressed as positioning accuracy errors $\Delta x^{(i)}_{p_i}$, $\Delta y^{(i)}_{p_i}$, $\Delta z^{(i)}_{p_i}$) include translation errors and rotation errors.

$$\Delta x^{(i)}_{p_i} = l_3 \cdot \rho (x) + (\Delta x_{p_i} \times)^* \theta (x)$$  \hspace{1cm} (4)

$$\Delta y^{(i)}_{p_i} = l_3 \cdot \rho (y) + (\Delta y_{p_i} \times)^* \theta (y)$$  \hspace{1cm} (5)

$$\Delta z^{(i)}_{p_i} = l_3 \cdot \rho (z) + (\Delta z_{p_i} \times)^* \theta (z)$$  \hspace{1cm} (6)

$$(\Delta x_{p_i} \times) \cdot (\Delta y_{p_i} \times)^* \cdot (\Delta z_{p_i} \times)$$ are the component of the position vector in three directions of the anti-symmetric matrix, which can be expressed as:

$$\begin{bmatrix}
0 & -z_l & y_l \\
1 & 0 & -1 \\
-1 & 1 & 0 \\
0 & -1 & 1
\end{bmatrix} \hspace{1cm} (7)$$

$$\begin{bmatrix}
z_l & 0 & -x_l \\
-1 & 1 & 0 \\
0 & -1 & 1
\end{bmatrix} \hspace{1cm} (8)$$

$$\begin{bmatrix}
1 & 0 & -1 \\
-y_l & x_l & 0
\end{bmatrix} \hspace{1cm} (9)$$

From the above (3) ~ (8) formula, 9 identification equations can be obtained. For the measurement position error vector $\Delta_x(x_j)$ at any position $x_j$ of the feed shaft, the value equals the identification matrix $H$ and the position error matrix $e(x_j)$ of the point multiplies, the expression as bellow in the form of a matrix.

$$\Delta_x(x_j) = H \cdot e(x_j) = l_3 - [r_{p_1} \times] \begin{bmatrix} (x_j) \rho y \rho z \\
\rho x \theta y \theta z \\
\rho x \rho y \theta z \end{bmatrix}$$  \hspace{1cm} (10)

To satisfy the error identification condition, the rank of matrix $\Delta_x(x_j)$, the value can be obtained by formula (1) and (10).

$$e(x_j) = (H^TH)^{-1}H^T\Delta_x(x_j)$$  \hspace{1cm} (11)

From the matrix of formula (11), the stability of the solution depends on the measurement information established in the machine tool coordinate system. The influence of accidental factors such as measurement noise on the error value is reduced. Formula (10) shows that with the increase of the number of measuring lines and points, the stability of the angle can be further increased, and the influence of the geometric accuracy of the translational axis on the measurement will be further deepened, but the efficiency of the solution will be further reduced [5]. Based on the motion coordinate system established by the translational axis of the machine tool on the working table, the identification results should directly reflect the precision design model of the machine tool and effectively solve the shortcomings of the traditional nine-line method.

The theoretical derivation is based on the measurement of the worktable by the micrometer. It is considered that the influence of the measuring error of the micrometer itself is neglected when the flatness of the worktable is absolutely ideal, and the friction between the micrometer measuring rod and flatness and the measuring noise are neglected.

4. Simulation Analysis

The simulation is mainly to analyze the advantages and disadvantages of the traditional nine-line method and the improved nine-line method to reflect the test results, and lay a foundation for the next step of experimental verification. Firstly, given the translation error and roll angle error of the machine tool moving to a certain point, this simulation gives the translation error $\rho_x^{(i)}$, $\rho_y^{(i)}$, $\rho_z^{(i)}$ whose values are all 8μm, and roll angle error $\theta_x^{(i)}$, $\theta_y^{(i)}$, $\theta_z^{(i)}$ whose values are all 8μm/m. According to the formula (11), the error mapping model of point $p_1$, $p_2$, $p_3$ in the ideal state is obtained. And then add the disturbance in the test process, such as measurement noise, machine tool positioning accuracy and
repetitive positioning precision test on the impact of the test, you can preset a value in advance, and then according to the test results after the event, adjust the results. The perturbation amount given in this simulation is linear Gauss noise disturbance with a mean value of 0 and a mean square error of 1µm. The simulation parameters are shown in the table below.

| Measuring point | X coordinates | Y coordinates | Z coordinates |
|-----------------|---------------|---------------|--------------|
| p_1             | 100           | 200           | 50           |
| p_2             | 200           | 300           | 100          |
| p_3             | 300           | 400           | 150          |

The traditional and improved nine-line methods are used to simulate 100 times each according to the above measurement steps. The probability density functions of measurement errors are fitted by Lagrange algorithm. The translation errors and roll angle errors obtained from the simulation results are listed in Fig. 2 and Table 2 below.

![Graphs](a) Positioning error $\rho_x^{(x)}/\mu m$  
(b) Straightness error $\rho_y^{(x)}/\mu m$  
(c) Straightness error $\rho_z^{(x)}/\mu m$  
(d) Roll angle error $\theta_x^{(x)}(\mu m/m)$  
(e) Roll angle error $\theta_y^{(x)}(\mu m/m)$  
(f) Roll angle error $\theta_z^{(x)}(\mu m/m)$

**Fig. 3 (a) ~ (f) Error measurement and identification results based on the traditional nine line method (dashed line) and the improved nine line method (solid line).**
### Table 2 Analysis of traditional nine line method and improved nine line method based on computer simulation

| Error code | mean value | standard deviation | Contrast |
|------------|------------|-------------------|----------|
|             | traditional nine line method | improved nine line method | traditional nine line method | improved nine line method |         |
| $\rho_x^{(x)} (\mu m)$ | 8.0121 | 8.0056 | 0.6549 | 0.4846 | 26% ↓ |
| $\rho_y^{(x)} (\mu m)$ | 8.0034 | 8.0007 | 0.8127 | 0.4226 | 48% ↓ |
| $\rho_z^{(x)} (\mu m)$ | 8.0308 | 8.0071 | 0.4358 | 0.2963 | 32% ↓ |
| $\theta_x^{(x)} (\mu m/m)$ | 8.0033 | 8.0009 | 1.9643 | 0.8446 | 57% ↓ |
| $\theta_y^{(x)} (\mu m/m)$ | 8.0057 | 8.0019 | 1.5248 | 0.7344 | 52% ↓ |
| $\theta_z^{(x)} (\mu m/m)$ | 7.9983 | 8.0128 | 1.192 | 0.2265 | 81% ↓ |

As can be seen from the above chart, compared with the error identification results of the traditional nine-line method, the error distribution of improved positioning error, straightness error, roll angle error, pitch angle error and yaw angle is more concentrated, the probability distribution of the identification is closer to the true value, that is, the stability of the corresponding matrix model solution is higher.

### 5. Experimental Verification

This research is carried out on a multi-axis high-precision machining center to measure and evaluate the Y-direction error of the feed shaft. The test environment temperature $T=23.01^\circ C$., relative humidity is 75%, material temperature $T=23.01^\circ C$. The test is carried out in two steps. The first step is to confirm the roll angle deviation coefficient and to correct the roll angle calculated and analyzed according to the above error model. The second step is to compare the results of the first step with the results of the electronic precision measuring instrument to find out the advantages and disadvantages. Based on the above formula (11), we can deduce:

$$e(y_j) = (H^T H)^{-1} H^T \Delta \Delta_y$$ (12)

The selection of points $p_1, p_2, p_3$ directly determines the accuracy of measurement and the average standard deviation of identification results, and optimizes the measurement objectives so as to minimize the standard deviation of the six errors obtained from the above three points. After calculation and optimization, three points $p_1(100, 100, 100), p_2(150, 150, 150), p_3(200, 200, 200)$ were selected in the experiment. Figure 4 below gives the average of three points for error measurement, and calculates the number of times for each point to travel.

![Fig. 4 Measurement of dual frequency laser interferometer](image)
The deviation coefficient $K$ can be calculated by comparing the results of physical measurement (e.g., precision measuring instrument micrometer) and laser measurement, and substituting equation (3). The measuring results of the dial gauge are selected when the workbench is in the position of $z=50,100,150,200$. The specific operation is to suck the micrometer on the spindle head, the contact touches the worktable, the worktable moves along the X axis, and the micrometer is read. And calculate the $K = 1.908 \mu m/m^2$. The deviation value of the roll angle $\Delta \theta_z^{(2j)}$ can be obtained by substituting formula 3.

For the comparison result, the roll angle error is measured by the electronic level measuring instrument (measurement accuracy $1 \mu m/m$). The result after comparison is shown in Figure 8 (a) ~ (c).
6. Conclusion

As can be seen from Fig. 10, the corrected roll angle error curve is closer to the electronic level measurement error curve, but there is still some deviation between them. This is because the geometric errors in the manufacturing process, the operational errors in the running process (such as repetitive positioning accuracy, reverse clearance, etc.), and the environmental factors in the testing process inevitably cause irremovable interference to the machine tool operating accuracy. Without increasing the number of measuring lines, the measurement error is increased from 6 items to 9 items after improvement. The simulation results show that the stability of error identification results is improved and the robustness of identification results is improved. Because the identification model is built on the solid plane of the machine tool, the accuracy of machine tool manufacturing and installation is included in the range of measurement and identification error, which can be used for precision control and process optimization in the early stage of machine tool precision design and machine tool assembly and debugging.
Fig. 10 Comparison of three precision compensation methods

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