TIDAL INTERACTION OF M32 AND NGC 205 WITH M31: SURFACE PHOTOMETRY AND NUMERICAL SIMULATIONS

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ABSTRACT

We investigate the interaction history of the M31 subgroup by comparing surface photometry of two of its satellites, M32 and NGC 205, with N-body simulations of satellite destruction. The recent discovery of a giant stream in the outer halo of M31, apparently pointed in the direction of M32 and NGC 205, makes such an investigation particularly relevant. The observational component of this study is based on 1.7 × 5.0 B- and I-band CCD mosaic images centered on M31 and covering both satellites. Standard ellipse-fitting techniques are used to model and remove M31 disk light and to perform surface photometry on the satellites to limiting brightness levels of \((\mu_B, \mu_I) = (27, 25)\) mag arcsec\(^{-2}\), corresponding to isophotal semimajor axis lengths of \(r_{\text{lim}, B} = 420^\prime\) (1.6 kpc) and \(r_{\text{lim}, I} = 720^\prime\) (2.7 kpc). A hint of excess light in the outer parts of M32 noted in earlier studies is confirmed; in particular, clear evidence is seen for a sharp (upward) break in the surface brightness profile at \(r = 150^\prime\) relative to a \(r^{1/4}\) law that fits the inner region of M32. This break is accompanied by a steep increase in isophotal ellipticity \(\epsilon\), as well as position angle \(\phi\) twisting. In addition to this excess, evidence is seen for an inner downward break in the surface brightness profile at \(r = 50^\prime\). The robustness of the M32 isophotal features is demonstrated through their (1) insensitivity to the details of background subtraction, (2) symmetry about M32’s center, and (3) narrow range of \(B-I\) color that is consistent with the interior regions of M32 but not with M31 residual spiral arm/dust lane features. The study of NGC 205 reveals pronounced isophote twisting at \(r \sim 300^\prime\) that is coincident with a subtle downward break in the surface brightness profile, relative to an exponential law fitted to the inner region.

The simulation component of this project is based on the analysis of single-component, spherical satellites that are being tidally disrupted through interactions with their parent galaxy. Generic features of the simulations include an excess in the surface brightness profile at large radii, a depletion zone at intermediate radii, and isophotal elongation and twists that are coincident with breaks in the brightness profile. The two satellites, M32 and NGC 205, display most of these features consistently across the \(B\) and \(I\) bands, which is strongly suggestive of tidal interaction and probable stripping by M31. We discuss what these observed features can tell us about the satellites’ orbital parameters and histories. Specifically, M32 is found to be on a highly eccentric orbit and away from pericenter. Investigating M32’s unusual combination of high surface brightness and low luminosity (the hallmark of compact ellipticals), we make empirical estimates of the galaxy’s intrinsic properties and conclude that it is not likely to be the residual core of a tidally stripped normal elliptical galaxy, as has been suggested, but rather that its precursor was intrinsically compact.

Key words: galaxies: dwarf — galaxies: evolution — galaxies: individual (NGC 205, M32) — galaxies: interactions — galaxies: photometry — Local Group

1. INTRODUCTION

Globular clusters and satellite galaxies serve as convenient tracers of the mass distribution of their parent galaxy. In theory, even a few well-determined satellite orbits can be used to constrain the gravitational potential field of the central galaxy (Evans et al. 2000). Unfortunately, direct measurement of orbital parameters—the proper motion in particular—is difficult. It has long been believed that observable signatures of tidal interaction in the satellites can be used to determine at least some of these critical parameters. For instance, it was proposed that globular cluster profiles are limited by the Galactic tidal field in which they are embedded (von Hoerner 1957) and that the anomalous properties of some peculiar elliptical (E) galaxies could be the result of similar tidal interactions (King 1962; Aguilar & White 1986). Since that time there have been numerous investigations into the dynamics of tidally truncated systems. Objects such as globular clusters and compact elliptical (cE) galaxies, of which M32 is a prototype, have been modeled with the King modification of the von Hoerner tidal radius formula:

\[
R_{\text{peri}} = R_{\text{peri}} \left[ \frac{m_{\text{sat}}}{M_{\text{gal}, \text{peri}}(\epsilon_{\text{orb}} + 3)} \right]^{1/3},
\]

where \(R_{\text{peri}}\) is the tidal radius of the satellite set at pericenter, \(R_{\text{peri}}\) is the satellite’s pericenter distance, \(m_{\text{sat}}\) and
$M_{gal,peri}$ are the mass of the satellite galaxy and the mass of the parent galaxy enclosed within the satellite’s orbit, respectively, and $e_{orb}$ is the satellite’s orbital eccentricity.

Several studies have been based on the assumption that the limiting radius of a truncated object corresponds to its tidal radius at pericenter. Faber (1973) derived perigalacticon distances for a sample of cE galaxies. This was followed by more ambitious attempts to constrain the orbital parameters of Galactic globular clusters (Peterson 1974; Innanen, Harris, & Webbink 1983) and M31 satellite galaxies (Cepa & Beckman 1988). These interpretations provided a qualitative picture of the interactions; however, uncertainties in the determination of the tidal radius prevented the accurate recovery of orbital parameters. In addition, the discovery of extratidal stars around Galactic globular clusters (Grillmair et al. 1995) and dwarf spheroidals (Irwin & Hatzidimitriou 1995; Kuhn, Smith, & Hawley 1996) complicated the notion of a well-defined, observable tidal radius. In response to these findings, a slew of detailed numerical simulations emerged that modeled extratidal features, as well as extended tidal tails (Oh, Lin, & Aarseth 1995; Moore 1996; Combes, Leon, & Meylan 1999; Johnston, Sigurdsson, & Hernquist 1999a). In turn, these motivated further observational studies to more precisely characterize both of these peripheral populations (Majewski et al. 2000; Leon, Meylan, & Combes 2000). Comparisons between observations and models are proving to be powerful tools for probing the Galactic potential (Johnston et al. 1999b) and determining satellite dark matter fractions, mass-loss rates (Johnston et al. 1999a), and orbital parameters.

The proximity of Galactic satellites makes detailed observations possible, but we are more or less limited to viewing them from within the plane of their orbit. External systems, while observationally more challenging, offer the advantage of a global perspective on the parent galaxy and a bird’s-eye view of the satellites’ orbits. Our nearest large galaxy neighbor, the Andromeda spiral (M31), has been the subject of such studies for the last few decades (Byrd 1979; Sato & Sawa 1986). Galaxy interactions in the M31 subgroup have recently been in the limelight as a result of the discovery of a tidal stream in the outer halo of M31 ( Ibata et al. 2001) and hints of tidal debris around its dwarf spheroidal satellites (Ostheimer et al. 2002). In this paper we investigate signatures of tidal interaction in the outskirts of the luminous M31 satellites M32 and NGC 205. This is especially relevant since the Ibata et al. stream lies, at least in projection, along a line intersecting both M32 and NGC 205. Our study uses traditional integrated surface photometry techniques, in contrast to the star-count analyses of Ibata et al. and Ostheimer et al. Studies of satellite interactions well beyond the Local Group will, into the foreseeable future, likely be restricted to the use of surface photometry methods on relatively high surface brightness, luminous satellites; thus our work on M32 and NGC 205 may be viewed as a pilot study for more distant systems.

In addition to probing the parent galaxy potential, it is interesting to investigate the impact of tidal interactions on the morphology and evolution of low-mass satellites. The satellites M32 and NGC 205 represent two distinct classes of low-mass galaxies, cE’s and dwarf ellipticals (dE’s), respectively. Normal E galaxies are found to populate a region in luminosity ($L$), surface brightness ($\mu$), and internal velocity dispersion ($\sigma$) space called the fundamental plane (FP). In the $\mu$-$L$ projection of this space the E galaxy population fainter than $M_B \sim -18$ bifurcates into tracks of (1) high surface brightness, high-metallicity cE’s and (2) low surface brightness, low-metallicity dE’s (Kormendy 1985). There is no clear formation scenario unifying these three classes of galaxies, and there has been a long-standing debate about whether cE’s or dE’s represent the natural low-mass extension of normal E’s (Faber 1973; Wirth & Gallagher 1984; Nieto & Prugniel 1987; Bender & Nieto 1990; Kormendy & Djorgovski 1989).

As a class, cE galaxies have de Vaucouleurs law profiles like normal E’s. Furthermore, they occupy a region of structural parameter space that is the direct low-luminosity extrapolation of the E galaxy fundamental plane (Wirth & Gallagher 1984; Nieto & Prugniel 1987), though well separated from it. On the other hand, the general proximity of cE’s to massive parent galaxies has led to speculation that cE’s are formed through the capture and tidal truncation of satellite galaxies (King & Kiser 1973; Tonry 1984, 1987). The range of proposed cE progenitors includes E’s (Faber 1973), SO’s (Nieto 1990), and spirals (Bekki et al. 2001; Graham 2002). Alternatively, Burkert (1994a) has proposed a model in which cE’s are formed through a starburst and subsequent violent collapse within the potential well of a massive galaxy. If cE’s are the low-mass counterparts of normal E’s, their rarity (Ziegler & Bender 1998; Drinkwater & Gregg 1998) and small range of absolute magnitudes would imply a sharp turnover in the E galaxy mass function.

By contrast, dE’s tend to (1) be fitted by a King or exponential $\mu$ profile instead of a de Vaucouleurs law and (2) form a track in $\mu$-$L$ space that is perpendicular to the classical E galaxy track. Such structural differences do not, however, rule out the possibility of a connection between dE and E galaxies since different physical processes may be at work in high-versus low-mass galaxies. The conventional wisdom regarding dE formation has been that, given their low binding energies, they are susceptible to supernova-driven galactic winds that regulate star formation, expand the stellar component, and thereby produce diffuse density profiles (Burkert 1994b and references therein). Recent simulations have contested these claims and suggested that mechanisms such as galaxy harassment (Moore et al. 1996; Moore, Lake, & Katz 1998; Moore et al. 1999) and tidal heating (Mayer et al. 2001a, 2001b) may be responsible for the transformation of spiral and dwarf irregular galaxies into dE’s. Far more numerous than cE’s, dE’s populate a wide range of absolute magnitudes fainter than $M_B \sim -18.0$, beyond the sharp faint-end cutoff of the cE luminosity function. If dE’s are the low-mass counterparts of normal E’s, it would imply that the low end of the E galaxy mass function has a smooth extension.

Tidal interactions may also have some bearing on M32’s unusual stellar content. Its stellar mix has been a controversial topic, with suggestions ranging from a pure old population (Cole et al. 1998) to a single coeval intermediate-age population (Vazdekis & Arimoto 1999; del Burgo et al. 2001). More plausibly, M32 seems to contain a small fraction of intermediate-age stars mixed in with an underlying old population (O’Connell 1980; Burstein et al. 1984; Rose 1985; Bica, Alloin, & Schmidt 1990; Davidge et al. 2000). Proposed theories for the origin of this secondary stellar population invoke galaxy interactions as a trigger for star formation within M32 or in the context of the accretion of gaseous material.
The M31 satellites M32 and NGC 205 are thus good test subjects for investigating the formation and evolution of these two classes of low-mass early-type galaxies. In this paper a large-format CCD mosaic image is used to carry out surface photometry of the satellites. Earlier studies of these systems have been plagued by large uncertainties in the measurement of their faint outer isophotes: photographic studies (de Vaucouleurs 1953; Hodge 1973) are hampered by low-level plate-fog variations, while more recent CCD observations (Kent 1987; Peletier 1993) have limited fields of view, making sky subtraction problematic. The situation is complicated by the fact that the satellites’ outer brightness profiles are contaminated by M31 disk light. The large field of view of our CCD mosaic image makes global modeling and subtraction of M31’s disk light possible, thereby allowing for reliable measurement of the satellites’ faint isophotes. These measurements are compared with numerical simulations to place constraints on the orbital parameters and mass-loss rates of the satellites and to estimate the evolution of M32’s luminosity and central surface brightness.

This paper is divided into the following sections. A summary of the observations and an overview of the basic data reduction procedure are given in § 2. The removal of “background” M31 light is discussed in § 3. Details of the surface photometry of M32 and NGC 205 are presented in §§ 4 and 5, respectively. A comparison of the observations to numerical simulations is presented in § 6, and implications for the evolution of M32 are discussed in § 7. The main points of the paper are summarized in § 8.

2. OBSERVATIONS AND BASIC DATA PROCESSING

The observations were carried out over the course of four nights in 1992 October and November using the Kitt Peak National Observatory 0.9/0.6 m (primary mirror/corrector) Burrell Schmidt telescope with a Tektronix ST2KA 2048 × 2048 CCD. Each CCD frame has a field of view of 68’ × 68’ and is slightly vignette at the corners. The pixel scale is 2.03, and the typical FWHM of stellar images is in the range of 2–5 pixels (4′−10′) as a result of seeing and, in the worst cases, poor focus. The data set consists of 22 × 10 minute exposures in the B band and 35 × 10 minute exposures in the I band.

After overscan/bias subtraction, trimming, and flat-fielding, each individual CCD frame is geometrically transformed onto a distortion-free astrometric system defined by the HST Guide Star Catalog. The images in each band are then flux calibrated to a common photometric system, corrected for temporal variations in the night-sky brightness, and mosaicicked into a composite image. The reader is referred to Guhathakurta, Choi, & Raychaudhury (2002) for the details of the mosaicking technique and least-squares method of transparency and sky-brightness corrections.

The final B- and I-band mosaic images each cover a ∼1.7′ × 5′ region centered on M31. Though both mosaics cover all of M32, only the B-band mosaic covers all of NGC 205. The I-band coverage is limited to the southeast half of the galaxy. Due to varying degrees of frame overlap, the effective exposure time is not uniform across the entire field of view; typical effective exposure times are 20 minutes in B and 40 minutes in I.

3. REMOVAL OF M31’S DISK LIGHT

The projected distance between M31 and its two nearest satellites is small enough that there is significant overlap in their light distributions. At the location of M32’s center, for example, M31’s disk light accounts for ∼12% of the background in both B and I bands. In addition, its steeply sloped contribution varies from ∼5%–20% between M32’s southeast and northwest extremities (r = 300′). An investigation of the satellites’ global properties requires a careful treatment of this contamination. Previous attempts to remove the M31 light contribution from the satellite profiles have relied on a simple plane or a low-order polynomial fitted to the local background (Kent 1987; Peletier 1993). While this approach is successful in modeling the smooth contribution of M31’s disk, it is not as effective at removing disk features such as spiral arms. The advantage of using a large-format CCD mosaic image is that it allows for the global modeling and subtraction of M31’s disk light.

For the purpose of modeling the M31 disk only, the original B- and I-band CCD mosaic images are median-filtered using a 30′′ × 30′′ window. The resulting image is largely free of foreground Galactic stars and compact M31 disk features. The implementation of a two-dimensional exponential disk plus de Vaucouleurs bulge model reveals strong departures from global symmetry in the form of large-scale disk warps and spiral arms. By contrast, the more empirical approach of modeling M31 annulus by annulus with elliptical isophotes better reproduces its global light distribution: it provides enough flexibility to fit large- and intermediate-scale structures on scales larger than the angular extent of the two satellites. A series of ellipses is fitted to the M31 disk isophotes by applying the IRAF2/STSDAS task ELLIPSE to the star-removed, median-filtered images in each of the B and I bands. Figure 1 shows only the best-fit ellipses in the semimajor axis range 30′ < r < 70′ overlaid on the original (i.e., unfiltered) B-band CCD mosaic image; the full M31 fit extends to a semimajor axis length r = 138′ and even overlaps with NGC 205. Compared with previous attempts to remove M31’s disk light, the ellipse-based coordinate system is a more natural choice for modeling the spiral arm structure, which is often sharp in the radial dimension but extended in the azimuthal dimension. The best-fit ellipse models are subtracted from the original mosaic images to create residual images containing the satellites that are largely free of M31 disk light. Figure 2 shows B-band images of M32 before and after M31 subtraction, emphasizing the importance of careful subtraction. Though the majority of M31’s disk light is well-subtracted in the latter image, fine-scale residual structure, such as dust lanes and star-forming knots in the spiral arms, are still evident. This residual fine-scale structure is a potential source of systematic error in the surface photometry of M32’s faint outer regions (see § 4.4).

4. M32

Due to its small projected separation of only 5.5 kpc from the large spiral galaxy M31, M32 is a good test case for investigating tidal effects on satellite galaxies. The high sur-

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2 IRAF is distributed by the NOAO, which is operated by AURA, under cooperative agreement with the NSF.
face brightness inner isophotes of M32 are nearly circular and are well characterized by an $r^{1/4}$ law $\mu$ profile. The inner brightness distribution provides a rather simple (extrapolated) baseline with respect to which subtle departures in the outer parts might be identified and measured. These include sharp features or “breaks” in the $\mu$ profile, isophotal elongation and twists, and other signatures of tidal interaction. As discussed in $\S$ 1, such a study is relevant because (1) M32 is the prototype of the rare class of cE galaxies that may result from the tidal truncation of normal E galaxies, (2) its proximity allows for detailed observations that are currently unavailable for any other object in its class, and (3) the recently discovered stream in the outer halo of M31 (Ibata et al. 2001) might be tidal debris from M32 or NGC 205.

A complicating factor in the study of M32 is the fact that it happens to be superposed onto the face of M31. Figure 3 shows M32 at two contrast levels in each of the $I$ and $B$ bands. The high-contrast panels on the right reveal that a significant amount of fine-scale residual structure remains even after our best attempts to model and subtract M31’s disk light. This fine-scale structure is most prominent in the $B$ band on the northwest side of the M32 nucleus, toward the bright inner disk of M31; it is probably associated with dust lanes and star-forming regions. In $\S$ 4.4 tests are carried out to characterize the effect of these residual contaminating M31 disk features on measurements of the faint outer isophotes of M32.

### 4.1. Surface Brightness Profile

Surface photometry is carried out using standard ellipse-fitting techniques with the IRAF task ELLIPSE, independently in $B$ and $I$ bands. Measurements are made out to a semimajor axis length of $r \approx 425''$ (1.6 kpc), which corresponds to a limiting surface brightness level of $(\mu_B, \mu_I) = (27, 25)$ mag arcsec$^{-2}$. Ellipse fits are performed in three ways: on the entire galaxy, on the northwest half only, and on the southeast half only. Unless otherwise noted, the measurements of M32’s surface brightness ($\mu$),
isophotal ellipticity ($\epsilon$), and isophotal position angle ($\phi$ or $\phi'$)\(^3\) presented in the rest of this paper are based on ellipse fits to M32’s southeast half, as it is least susceptible to contamination from M31’s inner disk; the global and northwest-half ellipse fits are only used to test the symmetry of M32’s isophotes (§ 4.4.1). The central positions of the fitted ellipses are held fixed at the nominal value determined from the innermost isophotes (the obvious nucleus of M32), while their $\epsilon$ and $\phi'$ are allowed to vary with semimajor axis from ellipse to ellipse. The best-fit ellipses with semimajor axis length $100'' < r < 300''$ are overlaid on the $B$- and $I$-band M32 images in Figure 3, illustrating the radial extent of the low surface brightness region.

Radial profiles of $\mu$, $\epsilon$, and $\phi'$, derived from the best-fit elliptical isophotes, are presented in Figure 4 in $r^{1/4}$ (left) and log-linear coordinates (right). The $B$- and $I$-band profiles are all seen to be in good agreement with each other. A comparison with the $R$-band study of Kent (1987), shows that the $\mu$ profiles are consistent out to his limiting measured isophote of $r \sim 300''$. By contrast the $\epsilon$ and $\phi'$ profiles are consistent only out to $r \sim 200''$; beyond this radius, these isophotal shape parameters are frozen in Kent’s study at their last fitted values because of insufficient signal-to-noise. Figure 5a shows an $I$-band image of M32, in contrast to an M32 residual image (Fig. 5b), in which the best-fit ellipse model for M32 has been subtracted. The smoothness of the latter residual

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\(^3\)The position angle $\phi$ is defined following the usual observational convention: counter-clockwise from N (i.e., N through E). For the numerical simulations, however, the position angle ($\phi$ in Paper I) is defined with respect to the satellite → parent line increasing toward the satellite’s projected direction of motion, this being the natural coordinate system for the simulations. By analogy, we define the quantity $\phi_{M32} = \pm (\phi_{M32} - \Gamma)$, where the positive sign is adopted corresponding to a clockwise projected orbit for M32 around M31 (\(\Gamma \approx 6.3–6.4\)).
image provides a measure, albeit qualitative, of the goodness of the M32 ellipse fits.

A de Vaucouleurs $r^{1/4}$ law profile is a convenient way to parameterize M32’s radial $\mu$ distribution. Independent fits to the $B$- and $I$-band data over a 5.5 mag range in $\mu$ from $10'' < r < 140''$ yield best-fit $r^{1/4}$ law profiles with $r^{1/4}_{B,I} = 29''$, $r^{1/4}_I = 17.53$ mag arcsec$^{-2}$, and $r^{1/4}_B = 19.43$ mag arcsec$^{-2}$. This “standard” fit is in general agreement with Kent’s $R$-band fit over the semimajor axis range $15'' < r < 100''$: $r^{1/4}_R = 32''$ and $r^{1/4}_R = 18.79$ mag arcsec$^{-2}$. While there is excellent overall consistency across BRI bands, close inspection reveals systematic differences between this standard $r^{1/4}$ law fit and M32’s actual $\mu$ profile. These differences, as well as alternative $r^{1/4}$ law fits, will be explored further in § 4.3; for the sake of comparison to previous analyses, the standard $r^{1/4}$ law fit is adopted in the following section.

4.2. A de Vaucouleurs Profile Excess: The “Faint Diffuse Plume” Revisited

From Figure 3 it is clear that, although M32 appears truncated and predominantly spherical in low-contrast images, it is surrounded by a skirt of low surface brightness material that becomes increasingly elongated at large radii. This was originally detected in photographs as a “faint diffuse plume curved away from M31’s disk” by Arp (1966) and later described by Kent (1987) as “an excess of light at large radii.” Detailed characterization of this region, however, has proved to be elusive until
now. The onset of this faint diffuse plume in the $\mu$ profile of Figure 4 is marked by a clear upward break at $r \sim 150''$ with respect to the standard $r^{1/4}$ law profile. The excess is coincident with sharp shifts in $\epsilon$ and $\phi'$ in both $B$ and $I$ bands, and is measurable to $r > 300''$, with a peak departure of $\Delta \mu = 0.5$ mag above the extrapolation of the standard fit. The semimajor axis range of the isophotes plotted in Figure 3 ($100'' < r < 300''$) is marked with a double line in the top panel of Figure 4 in order to illustrate the region over which the excess is found. Inspecting the relevant portion of the image (Fig. 3), it is clear why this excess feature was previously classified by Arp as a “diffuse plume.” In early photographic studies, which were sensitive to blue light, the excess region was swamped by M31’s disk structure on the northwest side of M32, leading to a one-sided detection. Coupled with sharp changes in $\epsilon$ and $\phi'$ (Fig. 4), the excess appeared to be an asymmetric and disjoint feature alongside an otherwise well-behaved E galaxy.

Uncertainties in the surface photometry of the M32 outskirts are dominated by systematic errors, which are difficult to quantify. Our finding of sudden elongations and twists in the $\epsilon$ and $\phi'$ profiles at radii coincident with the excess in the $\mu$ profile indirectly indicates that our measurements are reliable. This is in contrast to the Kent (1987) study: although Kent’s $\mu_R$ measurements are in general agreement with ours, the undetermined values for $\phi'$ beyond $r \gtrsim 200''$ in his study made it difficult at the time to draw any firm conclusions about the low surface brightness features in M32.

Figure 4 also reveals a previously undetected feature in the form of a subtle downturn in the $\mu$ profile at $r \sim 250''$. Although this is near the reliability limit of our data, it is seen in both colors and is accompanied by another isophote twist in $\phi'$, as well as a flattening in the $\epsilon$ profile.

A note of caution may be in order here. Ellipse parameters $\phi'$, $\epsilon$, and to a lesser extent $\mu$ are all coupled, so the mere coincidence of the profile features does not eliminate the possibility that they are the result of an M31 disk residual or
an asymmetric feature in M32. This possibility can, however, be ruled out via additional tests of the background subtraction and isophotal symmetry and color; these tests will be discussed in §4.4.

4.3. Evidence for an Inner Break and Depletion Zone

Decoupling M32’s tidal interaction features from its intrinsic profile requires some a priori assumptions about its unperturbed properties. The standard $r^{1/4}$ law fit is a good global fit to M32’s current $\mu$ profile; however, if tidal interactions have affected its outer isophotes, alternative fits that are limited to M32’s inner regions should be more representative of its intrinsic profile.

In the left panels of Figure 6, the $\mu$ profile is plotted with the standard fit overlaid (top) along with the $\Delta\mu$ residuals of this fit (bottom). Systematic differences are seen between the measured profile and this best-fit $r^{1/4}$ law. Within the radius range $10'' < r < 140''$ (indicated by double lines) over which the $r^{1/4}$ law is fitted, these departures correspond to at least one and possibly two additional $\mu$ profile breaks.

In the middle and right panels the same $\mu$ profiles are shown with “inner” and “extreme inner” $r^{1/4}$ laws that are fitted to more restricted radius ranges of $10'' < r < 65''$ and $10'' < r < 30''$, respectively. These alternative profiles are shallower than the standard fit, have larger values of $r_{\text{eff}}$ ($r_{\text{eff, inner}} \sim 37''$ and $r_{\text{eff, extreme inner}} \sim 44''$), and fainter effective surface brightnesses. Table 1 summarizes the parameters of the different $r^{1/4}$ law fits in the different bands: the radial range over which the data are fitted, $r_{\text{eff}}$, and $\mu_{\text{eff}}$.

These alternative $r^{1/4}$ law fits bring a new feature to light in M32. In addition to the upward break at $r \sim 150''$ and its

![Figure 6](image-url)
associated excess region at large radii, there is evidence for an inner radius *downward* break at \( r \sim 50'' \) and a “depletion zone” in which the surface brightness is diminished with respect to the extrapolated \( r^{1/4} \) law profile. Though the measured \( \mu \) profile is the same in each panel, its interpretation depends on which \( r^{1/4} \) law is adopted as the intrinsic profile. For instance, going from the standard to the extreme inner fit, the residuals exhibit the general trend of a de Vaucouleurs law profile with an excess region at large radii to one with an increasingly significant depletion zone at intermediate radii. In particular, a downward break in the \( \mu \) profile can be clearly identified in the last two residual plots at \( r \sim 50'' \). This previously unrecognized break is coincident with an inner twist in the \( \varphi' \) profile, supporting the theory that they have a common, presumably tidal, origin.

To verify the significance of the departures discussed above, the two-dimensional surface brightness distribution of M32 is studied. Residual images shown in Figures 7b–7d are generated by subtracting a de Vaucouleurs law model of M32’s light distribution from the original M32 image. The models are based on the standard, inner, and extreme inner \( r^{1/4} \) law fits and the measured \( \epsilon \) and \( \varphi' \) ellipse fit profiles. In each panel a pair of concentric circles mark the inner and outer radius limits of the associated \( r^{1/4} \) law fit. As a contrast, the residual image of M32’s *actual* \( \mu \) profile is shown in Figure 7a (same as Fig. 5b), with all of the aforementioned radius limits overlaid.

The images in Figure 7 reveal that the departures from the various \( r^{1/4} \) law profile fits are not only systematic in radius, as illustrated in the Figure 6 plots, but also azimuthally symmetric. The systematic nature of the residuals indicate that they cannot be simply reconciled with localized features such as star formation regions. This global symmetry also reaffirms the investigation of the alternative \( r^{1/4} \) law profile fits. Going from the standard to extreme inner fit, the trend to a more prominent depletion zone and a less prominent excess region seen in the azimuthally averaged residual profiles is clearly evident in the images as well.

A priori there is little reason to assume that one de Vaucouleurs law fit is more representative of the intrinsic profile than any other; however, as will be seen in § 6, the simulations provide some useful hints. They indicate that generic profiles of interacting satellites all show evidence of depletion and excess regions. This implies that the interpretation of M32 based on the inner and extreme inner fits may be the most physically significant. The implications for the evolution and tidal interaction history of M32 will be addressed in § 7.

### Table 1

| Band    | \( r_{\text{inner}} \) (arcsec) | \( r_{\text{outer}} \) (arcsec) | \( r_{\text{eff}} \) (arcsec) | \( \mu_{\text{eff}} \) (mag) | \( \Delta \mu_{\text{eff}} \) (mag) | Comments          |
|---------|-------------------------------|-------------------------------|-------------------------------|------------------|------------------|-----------------|
| Standard fit | 15                             | 100                           | 32.0                          | 18.79            |                  | Kent 1987 data  |
| Standard fit | 10                             | 140                           | 32.5                          | 18.64            |                  | Kent 1987 data  |
| Extreme inner fit | 10                             | 140                           | 28.5                          | 17.53            |                  | Standard fit    |
| Extreme inner fit | 10                             | 140                           | 28.5                          | 19.43            |                  | Standard fit    |
| Extreme inner fit | 10                             | 65                            | 28.5                          | 19.43            |                  | Standard fit    |
| Extreme inner fit | 10                             | 65                            | 36.8                          | 18.00            | 0.47             | Inner fit       |
| Extreme inner fit | 10                             | 65                            | 36.4                          | 19.90            | 0.47             | Inner fit       |
| Extreme inner fit | 10                             | 30                            | 46.8                          | 18.41            | 0.88             | Extreme inner fit|
| Extreme inner fit | 10                             | 30                            | 42.0                          | 20.15            | 0.72             | Extreme inner fit|

### 4.4. Testing the Robustness of the Measured Brightness Distribution

Comparison of the observations with \( N \)-body simulations hinges on the reliability of the quantities derived from the isophote fits. It is critical to test for potential systematic errors that may bias the photometry. Two approaches are taken to convince ourselves and the reader that the measured faint features in the M32 outskirts are not artifacts of the reduction procedure or M31 contamination. The first is an investigation of background subtraction errors and isophote symmetry. The second is a color test of M32’s extended isophotes. Discussions of these are presented below.

#### 4.4.1. Background Subtraction and Symmetry

Accurate measurement of low surface brightness, extended isophotes requires a careful characterization of the sky and spatially variable M31 contribution. In order to verify that features such as the upward break in the depletion zone \( (r \sim 150'') \) are not relics of the reduction, the robustness of M32’s \( \mu, \epsilon, \text{ and } \varphi' \) profiles against various background errors is tested. The B- and I-band \( \Delta \mu \) plots in Figure 8 and the \( \epsilon \) and \( \varphi' \) plots in Figure 9 illustrate the results of these tests.

The primary concern regarding the measurement of M32’s outer isophotes is the accurate removal of M31’s disk contribution. The ideal M31 disk fit minimizes residuals in the regions around M32 and not necessarily over M31’s entire disk. A simple global model allows for the fitting of large-scale background features; however, it also introduces the problem that asymmetries in the disk can produce a biased subtraction near M32. A range of different rejection thresholds and weighting functions for the M31 fit are tested to minimize the impact of such asymmetries. Ultimately, the best fit is based on the visual inspection of the background after M31 subtraction. In Figure 8a, the M32 \( r^{1/4} \) law residuals based on the inner de Vaucouleurs law are compared for three images with different M31 subtractions. The best-fit M31 subtraction is adopted in one (case A, triangles), while an undersubtraction (case B, squares) and an oversubtraction (case C, circles) of M31 features in the vicinity of M32 are adopted in the others. It is interesting to note that it is only beyond \( r \sim 250'' \) that the residuals start to diverge. This illustrates the robustness of the surface photometry over the depletion zone and excess region. The \( \epsilon \) and \( \varphi' \) profiles for the different M31 subtractions shown in Figures 9a and 9c are also seen to be fairly insensitive to the M31 subtraction in both the B and I band. For the remaining...
de Vaucouleurs law residual plots, the best-fit M31 subtraction is adopted.

The second concern is the careful treatment of M31’s residuals around M32 after the best-fit M31 subtraction has been performed. Although the M32 isophote fit allows for the spatial filtering of background sources, it is difficult to filter nonuniform variations that, because of the steep slope in M31’s disk, systematically increase in magnitude from one side of the galaxy to the other. The northwest side of M32 tends to suffer from more severe contamination than the southeast side even in the M31 subtracted image. The impact of this variable contamination on the M32 isophote fits is tested by dividing the galaxy along its minor axis into halves—toward and away from the nucleus of M31—and performing independent ellipse fits to both halves, as well as to the whole. Figure 8b shows the de Vaucouleurs law $\mu$ profiles that result from the ellipse fitted to the southeast half (triangles), the northwest half (squares), and the entire body of M32 (circles). Out to a distance of $r \sim 250''$ the scatter in this plot is low, indicating that the $\mu$ profiles of the different fit regions are in good agreement. Beyond this radius the M31 residuals on the northwest side start to visibly affect the M32 surface brightness measurement. In Figure 9b and Figure 9d, similar results are found for the $\epsilon$ and $\phi'$ profiles, with one exception. The northwest half $B$-band ellipse fit produces $\epsilon$ and $\phi'$ profiles that are noisier than the others and fixed beyond $\sim 250''$. The $I$-band fits of this same region, however, are in agreement with those of the southeast half, indicating that the true profile is symmetric and that the departures seen in the $B$-band are most likely due to M31 disk residuals. Further evidence reinforcing the symmetry of $\epsilon$ and $\phi'$ is seen in the final two panels of Figure 5. In these residual images the best-fit ellipse model for M32 is modified to have constant $\phi'$ (Fig. 5c) and constant $\epsilon$ (Fig. 5d), with both held at their inner radius values. The systematic features visible in these images in comparison to the best-fit...
residual image (Fig. 5b) illustrates the significance and symmetry of the $e$ and $\phi$ variations with radius. Based on these tests, the cleaner, southeast half of the galaxy is ultimately adopted for the best-fit $r^{1/4}$ profiles.

The final concern relates to the accurate determination of the sky, since the outer isophotes of M32 have surface brightnesses that are a small fraction of the sky level ($\sim 1\%-2\%$ at $r \approx 300'$). Regions well away from M32 that appear to be clean of contaminating spiral arms or dust lanes are used to estimate the sky. To investigate potential systematic errors that this may introduce, the robustness of the $\mu$ profiles is tested in the final two panels of Figure 8. In Figure 8c residuals computed with our best estimate of the sky background (triangles) are compared with those that would result from a misestimate of the sky (squares, circles). This extreme example shows that even a large $\pm 1\%$ sky error cannot account for the low surface brightness depletion and excess features. This point is reinforced in Figure 8d, in which M32 residuals for the best-case sky subtraction (triangles) are plotted against curves that represent the expected residuals for a theoretical de Vaucouleurs law profile with various degrees of sky subtraction error. The predictable effect of sky errors does not match the M32 residuals, indicating that sky misestimates are not responsible for the low surface brightness features.

Fig. 8.—Surface brightness residuals $\Delta \mu$—original minus inner fit $r^{1/4}$ law—for M32 in $B$ (open symbols) and $I$ bands (filled symbols). Various tests of the robustness of M32’s surface brightness profile are illustrated. (a) Different degrees of removal of M31 disk light: best subtraction (case A; triangles), extreme undersubtraction (case B; squares), and extreme oversubtraction (case C; squares) in the general vicinity of M32. (b) Ellipse fits to different parts of M32: southeast half of major axis toward M31’s outer disk (triangles), northwest half of major axis toward M31’s inner disk (squares), and the entire galaxy (circles). (c) Different degrees of removal of a constant sky background: best subtraction (triangles), 1\% undersubtraction (squares), and 1\% oversubtraction (squares). (d) Expected effect of sky subtraction error on a galaxy with an intrinsic $r^{1/4}$ law brightness profile: perfect sky estimate (dashed), $0.5\%$ error (dotted), and $2\%$ error (dot-dashed). The best estimate of M32’s residual profile (triangles) shows a distinct shape that cannot simply be the result of sky subtraction error on a galaxy with an $r^{1/4}$ law profile. The consistency of the M32 residuals across $B$ and $I$ bands for these various tests (a–c) for $r \lesssim 250'$ demonstrates the robustness of the surface photometry and the significance of the depletion zone and the onset of the excess region (upward break).
Together, Figures 8 and 9 show the reliability of the $C_{15}$ and $C_{30}$ profiles out to at least $r_{2500}$ and demonstrate that the profile features associated with the depletion zone and excess region cannot be reconciled with background subtraction errors. To further illustrate this point, a color comparison of the residuals is investigated.

4.4.2. Color Comparison of Extended Isophotes

It is impossible to remove every small-scale M31 disk feature; therefore, the possibility that the extended isophotes of M32 are dominated by a chance superposition of these residual features is investigated. Although background contamination would generally produce asymmetric features, a color comparison of the extended isophotes with those of the sky and the M31 disk residuals provides a useful complementary test. Using the $B$- and $I$-band images, a color-index map is made of M32 and its surroundings. In Figure 10 different sections of this map are sampled in $10'' \times 10''$ boxes and plotted on a $B-I$ versus $m_H$ diagram. Finely sampling each isophote of M32, from the core ($r < 10''$) to the most extended isophotes ($r > 300''$), the mean value of the M32 color index is determined to be $B-I = 1.9$, with relatively small scatter for regions within $r < 150''$. An envelope enclosing the locus of points representative of M32 is plotted in Figure 10a. The points in this panel sample three distinct M31 residual regions (stars, crosses, squares), as well as smooth patches of uncontaminated sky (triangles).

In the three remaining panels (Figs. 10b–10d) this background sample (small crosses) is plotted against points (circles) that represent three subregions within M32’s measured isophotes: the main body ($r < 150''$), the depletion and excess regions ($150'' < r < 300''$), and the most extended measured isophotes ($r > 300''$). The consistency of

![Graphs showing the reliability of the isophotal shape/orientation parameters.](image-url)
the $B-I$ color index in regions out to $r \sim 300''$, compared with the colors of the field samples, supports the claim that the excess light is truly associated with M32, and not with M31 or sky variations. Even beyond $r > 300''$ the color spread around the M32 mean color is relatively small down to $I_{\lim} = 23.5$ mag arcsec$^{-2}$.

How does the broadband color of the tidal debris in M32’s outskirts compare with that of the stellar stream found by Ibata et al. (2001) in the M31 halo? The $B-I$ color is about 2 for M32’s outer tidal excess (Figs. 4 and 10). Since $V-I \approx 0.5(B-I)$ across a wide range of stellar types (Bergbusch & VandenBerg 2001), the $V-I$ color index for M32 is estimated to be about 1. At face value the luminous giants near the tip of the red giant branch detected by Ibata et al. are redder than this with $V-I$ spanning the range 1–3 (see Fig. 2c of their paper). However, it should be noted that the integrated $V-I$ color of an old stellar population is comparable to the color of stars near the base of the red giant branch (see Guhathakurta et al. 1998), which corresponds to $V-I \sim 1$ for the stream. Thus, the expected $V-I$ color of the integrated stream agrees quite well with that of M32’s outer tidal material.

5. NGC 205

The observational data for NGC 205 are similar to those presented for M32 above; the same surface photometry and analysis methods are used. The situation is somewhat easier for NGC 205 with regard to contamination by M31 disk light, so extensive tests of the robustness of the results are not carried out for this galaxy. The interpretation of the NGC 205 data, however, is far more complicated for a few
reasons: (1) the satellite is significantly flattened, making it difficult to draw conclusions from our simulations of spherical, nonrotating satellites; (2) the inner \( \mu \) profile is complicated, intermediate in Sersic index \( n \) between an exponential law \( (n = 1) \) and a de Vaucouleurs law \( (n = 4) \) but not a good fit to any \( n \) value, making it difficult to estimate the intrinsic profile of NGC 205; and (3) the intrinsic brightness distribution is patchy in places, with hints of dust lanes and star-forming regions, complicating the search for subtle departures from isophotal symmetry due to tidal effects.

Surface photometry is performed using the task ELLIPSE on M31-subtracted images; however, unlike the M32 isophotes, the two bands are not fitted independently. Due to incomplete coverage in the \( I \) band, only the \( B \) band is used for the determination of the best-fit elliptical isophotes. The \( \epsilon \) and \( \phi' \) (\( |\phi'_{\text{NGC 205}}| = |\phi'_{\text{NGC 205}} - 132.9'| \)) profiles from this fit are then used to compute the \( I \)-band photometry. Though this is not expected to have a major impact on the resulting profiles, it does mean that the ellipticity \( \epsilon \) and position angle \( \phi' \) profiles are only fitted in one band. Surface photometry is measured out to a limiting semimajor axis length of \( r \sim 720'' \) (2.7 kpc) with a limiting surface brightness of \( \mu''_B = 27.0 \text{ mag arcsec}^{-2} \). In Figure 11 a \( B \)-band image of NGC 205 is shown at two different contrast levels, with elliptical isophotes ranging from \( 140'' < r < 660'' \) overlaid to illustrate the low surface brightness region in which pronounced isophote twisting is observed. In Figure 12 the results of the best-fit elliptical isophotes are presented as radial profiles of \( \mu, \epsilon \) and \( \phi' \) in log-linear and de Vaucouleurs coordinates.

Comparisons of the measured NGC 205 brightness profile with those of Hodge (1973) and Kent (1987) show general agreement. The data confirm that the profile is not well fitted by any one analytic profile, but instead is intermediate between an exponential and \( r^{1/4} \) law, as noted by Kent. As in the case of M32, the profile fits are highly dependent on the radius range chosen. As is evident in the log-linear plot of Figure 12, the profile is well fitted by an exponential law with a scale length \( r_{\exp}^{B,R,I} = 150'' \) over the range \( 75'' < r < 250'' \) and \( r_{\exp}^{B,R,I} = 170'' \) over the range \( 150'' < r < 250'' \). With respect to the \( r_{\exp}^{B,R,I} = 150'' \) profile, a subtle downward break is detected at \( r = 300'' \) in both the \( B \) and \( I \) bands of the current data set, as well as in Kent’s \( R \)-band data. The degree of the departure is inconsistent between the two sets. This may be due to increasing magnitude errors at isophotes that are approaching the Kent brightness limit. At radii beyond the limits of either Kent or Hodge \( (r > 500'') \), the surface brightness returns to the projected exponential profile. Although the magnitude of this break is subtle enough that neither Kent nor Hodge make note of it, its coincidence with shifts in \( \epsilon \) and \( \phi' \) provide compelling evidence for its significance.

The general shape of the measured \( \epsilon \) curve, which rises with increasing radius to a maximum value of \( \epsilon = 0.52 \) at \( r = 260'' \) and then dips at larger radii, is in agreement with past results (Richter & Hogner 1963; Hodge 1973; Kent 1987). Beyond the peak at \( r = 260'' \) there is some discrepancy between the curves of Hodge and Kent. Kent shows a sharply dropping ellipticity out to the limit of his data at \( r = 460'' \), while Hodge sees only a slight dip at \( r = 400'' \), followed by a continued rise to the end of his data at \( r = 480'' \). The M31-removed CCD measurements confirm a slight dip beyond the maximum and then a gradual decline of the ellipticity out to and beyond \( 480'' \). In the inner regions the large amplitude ellipticity fluctuation at \( r \sim 20'' \) seen by both Richter & Hogner and Kent is also confirmed. Contrary to Hodge’s speculation that this feature is an artifact of “a combination of poor statistics and systematic effects,” our data indicate that it is significant.

The major-axis \( \phi' \) profile shows a gradual increase out to a radius of \( r = 260'' \) and then a steady drop corresponding to a \( 30' \) twist out to the last measured isophote. This is in good agreement with Hodge, who measures continuous twisting out to the limit of his data.
6. INTERPRETATION OF OBSERVATIONS IN LIGHT OF N-BODY SIMULATIONS

The previous sections focused on the detailed characterization of the M32 and NGC 205 observations. In the following section these characteristics will be compared with numerical simulations in the hope of determining whether they are tidally induced and, if so, using them to constrain the satellites’ orbital parameters. A brief description of the simulations and their analysis is presented, along with results of their application to M32 and NGC 205. The details of the simulations and the general trends of satellite interaction can be found in Johnston, Choi, & Guhathakurta (2002, hereafter Paper I).

It is worth noting that the discussion in this section is based on the similarity in appearance between the observations and simulations, rather than a definitive proof that tides are responsible for the observed features. The models are not specifically tailored to match the intrinsic properties of M32 and NGC 205, or the precise potential of M31. Despite this, they provide a qualitative understanding of the physical mechanism that drives the tidal signatures. It has been shown that the two observed satellites have very different structural parameters. The current set of simulated spherically symmetric satellites are well suited for the analysis of M32; however, they are less applicable to NGC 205 because of its flattened structure. As a result, the bulk of the quantitative analysis is performed for M32 and a more conservative approach is taken for NGC 205. Fine-tuning of the models and spectroscopic observations to determine satellite internal kinematics, both of which are in progress, will

![Diagram of NGC 205 observations and simulations](image-url)
provide leverage to further refine orbital parameters and allow for a more comparable analysis of the two satellites in the future.

6.1. The Simulations

In the numerical simulations 64,000-particle, one-component, spherical satellites are followed for five radial oscillations as they orbit in a fixed three-component potential, representative of the disk, bulge, and halo of a parent galaxy. Particle interactions are computed using code developed by Hernquist & Ostriker (1992) and based on the basis-function-expansion technique. Of the five simulated satellites four have Plummer initial density profiles and orbital eccentricity ranging from 0.10 < e < 0.88 (models 1–4). The fifth has a shallower Hernquist initial profile and an eccentricity of e = 0.88 (model 5).

The analysis of the simulations is performed with a parallel and complementary approach to that of the observations. This facilitates direct comparison between the two. Snapshot “images” of each simulated satellite are generated by projecting the satellite particles onto a two-dimensional plane and smoothing the resulting distribution. The images for a range of orbital phases and viewing angles are then analyzed with the same ellipse-fitting technique that is used for the M32 and NGC 205 images. The resulting trends in the surface brightness µ, ellipticity ε, and position angle φ’ profiles with orbital eccentricity, phase, and viewing angle are used to guide the interpretation of the M31 satellite observations.

6.2. Viewing Angle

The detection of isophote twists in the simulated satellites is a signature of an inclined orbital plane. By contrast, when viewed from within the orbital plane, the fitted ellipses line up along the direction of motion, and twists are not observable. Comparing simulations viewed at angles of 0°, 30°, 60°, and 90° from the orbital plane (edge-on to face-on), isophote twists are measurable for i ≥ 30°, indicating that, even at low inclinations, the effects of tidal twisting are observable. Our study shows that, for i ≥ 30°, the observed quantities have a negligible dependence on the viewing angle. This simplifies the analysis, but it also limits the viewing angle determination to “edge-on” versus “face-on.”

The observed satellites, M32 and NGC 205, exhibit varying degrees of isophote twisting, indicating face-on viewing angles; however, in the case of NGC 205, where the assumption of intrinsic spherical twisting may break down, a note of caution must be added. An intrinsically nonspherical satellite can exhibit isophote twists for i = 0° if the satellite itself is inclined with respect to its orbital plane. NGC 205’s flattened structure may have been tidally induced; but, if not, little can be concluded about its orbital inclination.

6.3. Orbital Eccentricity and Phase

As defined in Paper I, the position angle φ’ is the angle of the satellite semimajor axis with respect to the satellite → parent galaxy vector. It is measured on the side of the satellite closer to the parent galaxy, so that −90° < φ’ < 90°. As will be shown in § 6.4, the probable sense of M32’s projected orbit is clockwise around M31, so this is hereby adopted for the sign convention of φ’ for both galaxies.

For circular orbits of spherical satellites φ’(r_break) the position angle of the r_break isophote and dφ’/dr(r_break) describing the isophote twist, both have the same orientation (Johnston et al. 1999a). The fact that neither M32 nor NGC 205 exhibits this trend between φ’(r_break) and dφ’/dr(r_break) reveals that these satellites are not likely to be on circular orbits. In addition to the relationship between φ’(r_break) and dφ’/dr(r_break), in the case of M32 three other profile features are suggestive of a highly eccentric orbit: (1) the triple break in the φ’ profile, (2) the ratio r_break/r_tide, and (3) the ratio r_break/r_distort. The diagnostics r_break and r_distort are empirically measured radii that characterize the μ and ε profiles. Specifically, r_break is the radius at which a sharp change is measured in the slope of the μ profile, and r_distort is the corresponding radius for the ε profile (Paper I). By contrast, r_tide is an estimate for the theoretical King tidal radius.

In Figure 13 the best-fit elliptical isophote profiles for M32 are compared with those of a simulated satellite on a highly eccentric orbit (e = 0.88) that is approaching apocenter (model 4). Striking similarities seen in the μ, Δµ, ε, and φ’ profiles of the observed and the simulated satellite imply that the M32 features have tidal interaction origins. The Δµ profile is based on the inner de Vaucouleurs law fit for M32 and the intrinsic μ profile for the simulated satellite. The M32 μ, Δµ, and ε profiles have generic shapes that are common for many of the simulated snapshots, independent of the satellite’s orbit or phase. The φ’ profile, however, with its multiple twists—each of which is coincident with either a μ or ε feature—is more atypical. The triple twist in φ’, which is seen only in simulated satellites approaching apocenter of highly eccentric orbits, provides a clue not only about M32’s orbital eccentricity, but also its orbital phase.

The second signature of an eccentric orbit is r_break/r_tidal, the ratio of the observed break in the μ profile and the classically defined theoretical King tidal radius. In the simplifying case of a circular orbit, r_tide,peri (defined in eq. [1]) depends only on the mass of the satellite galaxy, the enclosed mass of the parent, and the distance between them. To investigate the likelihood that M32 is on such an orbit, its tidal radius is calculated based on the following: M32 is assumed to have a circular orbit; the projected distance between M32 and M31 is adopted as their separation; M_M32 = 2.1 × 10^10 M⊙ is adopted; and M31’s enclosed mass is calculated by modeling it as an isothermal sphere, M_M31 = v_circ^2 R_{proj}/G, with v_circ = 240 km s^{-1}. The resulting R_{proj} = 310’’ (1.2 kpc) is only weakly dependent on M_M32 and M_M31, so the main uncertainty is in the assumption that R_{proj} is the true separation. A measured r_break = 140’’ (0.54 kpc) results in r_break/R_{proj} = 0.5, which is a conservative upper limit since R_{proj} is a lower limit to the true separation. The top panels of Figure 14 show the orbital eccentricity and phase dependence of this ratio. The ratio r_break/r_tide typically has values of unity or greater for near-circular orbits. Only in highly eccentric orbits with e ≥ 0.5 does it drop as low as r_break/r_tide = 0.5, suggesting that M32 is on this latter type of orbit.

The final clue to M32’s orbital eccentricity is the coincidence of r_break to r_distort, the radius associated with the onset of isophotal elongation. For the intrinsically spherical, simulated satellites r_distort is defined as the radius at which e > 0.02; however, for the observations, because of the nonspherical nature of real galaxies, r_distort is modified to a more general definition of the radius at which the ellipticity departs sharply from the inner radius value. For M32 this is
seen to occur at \( r_{\text{M32}} = 150'' \) (0.57 kpc), resulting in \( \frac{r_{\text{break}}}{r_{\text{distort}}} \approx 1.0 \). In Figure 13 the locations of \( r_{\text{break}} \) and \( r_{\text{distort}} \) are shown as dotted vertical lines in the \( \Delta \mu \) and \( \epsilon \) plots, respectively. In the lower panels of Figure 14 the orbital eccentricity and phase dependencies of the ratio \( \frac{r_{\text{break}}}{r_{\text{distort}}} \) indicate that \( \frac{r_{\text{break}}}{r_{\text{distort}}} \geq 2.0 \) for near-circular orbits and approaches unity only for the most eccentric orbits, again supporting the theory that M32 is on such an orbit. Unlike \( r_{\text{tidal}} \), both \( r_{\text{distort}} \) and \( r_{\text{break}} \) are directly observable, making this deduction less model dependent and more robust than the previous one about \( \frac{r_{\text{break}}}{r_{\text{tidal}}} \).

In addition to constraining M32’s orbital eccentricity, the three arguments above indicate that M32 is currently in an orbital phase away from pericenter. In particular, the fact that the lower limit for \( r_{\text{tidal}} \) is a factor of 2 greater than both \( r_{\text{break}} \) and \( r_{\text{distort}} \) provides robust evidence that M32 cannot be at pericenter. Severe tidally induced distortions are not expected to be seen interior to the tidal radius of a satellite at pericenter, \( r_{\text{tidal,peri}} \). Following this line of reasoning, \( r_{\text{tidal,peri}} \) can be estimated using \( r_{\text{break}} \) and \( r_{\text{distort}} \) alone. As is shown in Figure 15 of Paper I, \( r_{\text{tidal,peri}} \approx 0.5r_{\text{break}} \) for \( r_{\text{break}} \approx r_{\text{distort}} \). This corresponds to \( r_{\text{tidal,peri}} \approx 0.3 \) kpc, and
translates to a M32-M31 pericenter separation of \( R_{\text{peri}} \sim 0.7 \) kpc via the King formula. Even the most conservative estimate of \( r_{\text{tide,peri}} \approx r_{\text{break}} \) implies an upper limit \( R_{\text{peri}} \lesssim 1.7 \) kpc, which is much less than \( R_{\text{proj}} = 5.5 \) kpc. Adopting \( R_{\text{peri}} = 0.7 \) kpc and an orbital eccentricity \( e = 0.88 \) (based on model 4 of the simulations), M32’s apocenter is estimated to be \( R_{\text{apo}} \approx 10.5 \) kpc. If M32 is currently near apocenter, as the \( \phi' \) triple twist suggests, M32 must be at least \( 8-9 \) kpc in the foreground or background of M31’s core. This is well within the current \( \pm 100 \) kpc uncertainty in the relative distances to M32 and M31.

### 6.4. Direction of Motion

For circular orbits \( \phi'(r_{\text{break}}) \) and \( d\phi'/dr(r_{\text{break}}) \) are related to the direction of the orbit and can therefore be used to constrain the satellite’s projected motion. Unfortunately, these relationships have a phase dependence for eccentric orbits. As a result, the projected motions of M32 and NGC 205 are indeterminate from their isophote orientations alone. In the case of M32, the orbital direction can be recovered since its phase has been independently determined.

If M32 is indeed on a highly eccentric orbit approaching apocenter, as suggested in §6.3, then the simulations indicate that \( \phi'_{\text{M32}}(r_{\text{break}}) \) should be negative (Fig. 11 of Paper I). The orientation of \( \phi' \) is defined with respect to the direction of motion, implying that M32’s \( r_{\text{break}} \) isophote, on the inner side of its orbit, should be pointed away from its direction of motion.

One caveat of the above argument is that the nonspherical nature of real galaxies implies that there is a nonzero intrinsic value for the position angle \( \phi'_{\text{inner}} \). Instead of simply looking at the sign of \( \phi'(r_{\text{break}}) \), one must instead consider the sign of the change in position angle relative to the interior intrinsic value, \( \Delta \phi'(r_{\text{break}}) \equiv \phi'(r_{\text{break}}) - \phi'_{\text{inner}} \), where \( \phi'_{\text{inner}} = -20^\circ \) for M32’s inner isophotes. For M32 \( \Delta \phi'(r_{\text{break}}) \) has an absolute value of \( 5^\circ \). As discussed above, prior knowledge of this satellite’s orbital phase and eccentricity indicates that \( \Delta \phi'(r_{\text{break}}) \) must be negative, and this implies that M32’s projected orbit is clockwise about M31, as indicated in Figure 15.

### 6.5. Comparison of M32 and NGC 205 Intrinsic Profiles

The generic characteristics shared by all of the simulated satellites are a depletion zone at small radii and an excess region at large radii (Fig. 6 of Paper I). This is an expected consequence of tidal stripping and flux conservation that is independent of orbital parameters or the satellite’s initial profile. Both regions have \( \mu \) profile breaks associated with their onset: a gradual downward (negative) break in the case of the depletion zone and an abrupt upward (positive) break in the case of the outer excess region.

Though not easily discernible from the \( \mu \) profiles, both breaks are generally evident in the \( \Delta \mu \) residual plots. To mimic the analysis of real galaxies, for which intrinsic profiles are unknown, \( r_{\text{break}} \) is measured using only the \( \mu \) profile. The detection criteria for \( r_{\text{break}} \) is not biased toward either positive or negative departures; however, it does depend on the sharpness and magnitude of the profile slope change. As a result, \( r_{\text{break}} \) tends to be preferentially associated with the sharp, outer break. Only in the simulated satellite of model 5, which has a shallow initial density profile, is the inner "depletion zone" break detected as \( r_{\text{break}} \). The satellites in models 4 and 5 have identical orbital parameters and differ only in their initial density profile, revealing a connection between the intrinsic profile and its measured parameters after interaction.

In the observed \( \mu \) profiles \( r_{\text{break}} \) is positive for M32 and negative for NGC 205. The difference in the intrinsic profiles of the two satellites—NGC 205’s is much shallower than M32’s—hints at a profile-dependent detection bias, as suggested by the simulations. For M32 it is evident from the \( \Delta \mu \) profiles (Fig. 13) that \( r_{\text{break}} \) corresponds to an outer "excess region" break. For both M32 and the model 4 satellite, though not initially identified because of its gradual nature, the inner depletion zone break is clearly visible. By contrast, because of NGC 205’s shallow intrinsic profile, the stripping of material results in an inner depletion zone break that is sharp enough to be measured (Fig. 12). Unfortunately, there is no evidence for the accompanying outer excess region break that would reinforce its identification. This may simply be beyond the current sensitivity limit of the observations.

### 6.6. Constraints on Mass Loss

It is shown in Paper I that constraints can be placed on satellite mass-loss rates from surface photometry alone. Measurements of the extratidal population of M32 and NGC 205 are used to make an order-of-magnitude estimate for their instantaneous, fractional mass-loss rate per orbital period,

\[
\frac{df}{dt} = \frac{\pi^2 r_{\text{break}}^2 \Sigma_{\text{break}}}{m_{\text{break}}},
\]

as discussed in Paper I. The ratio of \( \Sigma_{\text{break}}/m_{\text{break}} \), the surface density at \( r_{\text{break}} \) to the mass enclosed within this point, can be calculated from the \( \mu \) profile, assuming a constant mass-to-light ratio. The derived rates for M32 and NGC
205, $df/dt_{\text{M32}} = 0.38$ and $df/dt_{\text{NGC 205}} = 2.95$, are shown in the last column of Table 2.

The high apparent destruction rates for both satellites should be qualified by two factors. The first is that, although the simulations show that surface brightness derived rates are accurate to within order unity for near-circular orbits, this relationship degenerates with eccentricity. For eccentric orbits $df/dt$ is phase dependent, as illustrated in Figure 16 of Paper I. Mass-loss estimates are reliable near pericenter, where the bulk of mass loss occurs; however, away from this phase, they are systematically high by up to half an order of magnitude. The reason for this overestimate is that, away from pericenter, only a fraction of the extra-break material that is generally heated, yet bound, will be lost on the current orbit. The second factor is that $df/dt$ is an instantaneous, phase-dependent rate that provides a direct measure of the total mass loss per orbit only when $df/dt$ is constant, as in the circular case. For eccentric orbits it must be integrated over the entire orbit to calculate the total orbital mass loss. Given these caveats, the presented $df/dt$ should be considered only as upper limits for the instantaneous fractional mass-loss rate. As such, they should not be used to extrapolate a destruction rate.

6.7. Future Directions

The simulations presented in this paper and Paper I provide useful pointers about the nature of tidal interaction in M32 and NGC 205, despite the fact that they are not tuned to mimic these satellites. Future simulations will explore combinations of satellite orbital eccentricities and phases that are constrained by the actual distances and radial velocities measured for M32, NGC 205, and M31. Furthermore, the simulated satellites almost certainly depart from real galaxies in the assumption that mass follows light. In the future two-component (stars and dark matter) model satellites will be incorporated into the simulations.

7. IMPLICATIONS FOR M32’S SURFACE BRIGHTNESS AND LUMINOSITY EVOLUTION

In a plot of $L$ versus $\mu E$’s lie on the extension of the giant E galaxy track, typically $\sim 2$–3 mag fainter in luminosity and $\sim 1$–2 mag brighter in surface brightness (Ziegler & Bender 1998). At the faint, low surface brightness extreme is M32. Because of its proximity to M31, most formation theories suggest that M32 is the remnant of a galaxy that has been stripped through tidal interaction. Numerous galaxy types have been proposed as possible precursors; however, given its location in $\mu L$ space, the most intuitive of these is a normal E galaxy. This theory can be investigated directly using our surface brightness observations.

The simulations presented in §6 indicate that, despite the loss of a substantial amount of mass in their outer regions, the interior portions of the dwarf satellites’ $\mu$ profiles remain largely unaffected (Fig. 13, top two panels on the right). While the simulations are admittedly simplistic, in the case of M32 this assumption is probably a reasonable one. One particular concern is that the present simulations involve single-component satellites in which mass follows light. By contrast, real satellites are expected to have extended dark halos. Fortunately, such a halo would tend to further buffer the interior of the satellites from tidal stripping, thereby reinforcing this finding.

Guided by the notion that the interior brightness profile of M32 is pristine and the assumption that the original profile (prior to tidal stripping) obeyed a de Vaucouleurs law, one can quantitatively address the question of whether the unusual location of this galaxy in a $\mu L$ plot could be the result of tidal stripping of a normal E galaxy. Of the three $r^{1/4}$ law fits presented in Figure 6 of §4.3, the one labeled “extreme inner” (fit to inner $r = 10''$–$30''$) is most likely to represent the intrinsic profile of M32. The resulting estimates of M32’s intrinsic effective surface brightness are $\mu_{r_{\text{eff}}}^I = 18.41$ mag arcsec$^{-2}$ and $\mu_{B_{\text{eff}}}^I = 20.15$ mag arcsec$^{-2}$. Adopting the standard ($r_{\text{eff}} = 29''$) fit as representative of M32’s current de Vaucouleurs law profile leads to current M32 values of $\mu_{r_{\text{eff}}}^I = 17.53$ and $\mu_{B_{\text{eff}}}^I = 19.43$, in good agreement with historical results. These values imply an evolution of $\Delta \mu_{r_{\text{eff}}}^I = 0.88$ and $\Delta \mu_{B_{\text{eff}}}^I = 0.72$.

The luminosity evolution is estimated by comparing its current luminosity with that of the intrinsic $r^{1/4}$ law profile fit assumed for M32. The three de Vaucouleurs law fits are integrated to estimate the intrinsic luminosity of M32 as a function of enclosed radius. These “curves of growth” are shown in Figure 16 (long-dashed, short-dashed, and dotted lines, for standard, inner, and extreme inner fits, respectively), along with the curve of growth based on the actual brightness profile of M32 (solid line).

These curves overlap with one another interior to the inner break $r = 50''$ and diverge beyond this radius, consistent with the expectation that the majority of the luminosity evolution occurs in the depletion zone. The standard fit, which most closely follows M32’s observed integrated luminosity curve through the depletion zone, underestimates the total magnitude at large radii because of the tidal excess feature discussed in §4.2. The curves of the two inner-radius fits, on the other hand, are less biased by the depletion zone and therefore provide a more conservative estimate for M32’s intrinsic luminosity. Adopting the extreme inner fit as M32’s intrinsic profile results in a modest luminosity evolution of $\Delta B_{\text{M32}} \sim 0.1$ and $\Delta I_{\text{M32}} \sim 0.15$, based on aperture photometry out to $r = 300''$. The adoption of total versus isophotal magnitudes would only impact the luminosity evolution by an additional $\sim 10\%$.

A comparison of M32’s presently observed properties with estimates of its intrinsic properties indicates a relatively

| Name          | $R_{\text{break}}$ (kpc) | $r_{\text{break}}$ (kpc) | $r_{\text{dist}}$ (kpc) | $\epsilon_{\text{break}}$ | $\phi_{\text{break}}$ | $r_{\text{d}}/r_0$ | $df/dt$ |
|---------------|--------------------------|---------------------------|-------------------------|---------------------------|------------------------|----------------------|--------|
| M32............ | 5.5                      | 1.2                       | 0.54                    | 0.57                      | $-25.24$               | 12.60                | 0.38               |
| NGC 205 ...... | 8.3                      | 1.0                       | 1.07                    | ...                       | 0.52                   | $-36.32$             | 66.26              | 2.95               |

* Based on an assumed distance to M31, M32, and NGC 205 of 780 kpc.
small amount of evolution due to tidal effects. Although it is
in the right direction—away from the family of E galaxies in
the $\mu$-$L$ projection—the magnitude of this shift falls far
short of explaining M32’s position in terms of a tidally
stripped/truncated normal E galaxy. Put another way,
intermediate E galaxies have typical effective radii of
$1.2 < r_{\text{eff}} < 8.0$ kpc (Bender, Burstein, & Faber 1993),
whereas estimates of M32’s intrinsic effective radius are in
the range 37”–47” (0.14–0.18 kpc). This implies that M32
was intrinsically “compact” even before any tidal stripping
by M31, supporting Burkert’s (1994a) theory that cE’s are
formed in a compact state—as opposed to being evolved
into one. The bulges of spiral or S0 galaxies, typically inter-
mediate in compactness between E’s and cE’s, cannot be
ruled out based on the current analysis.

It is interesting to note that the integrated absolute mag-
nitude of the Ibata et al. (2001) stream, estimated at
$M_\mu(\text{stream}) \approx -14$, is approximately 10% that of M32:
$M_\mu(\text{M32}) \lesssim -16$. The M32 value is derived from its curve
of growth in the $B$ band, which yields $M_\mu(\text{M32}) \lesssim -15$
(Fig. 16) and an interpolated $B-V$ color of about unity (see § 4.4.2). Moreover, the estimated amount of luminosity evolu-
tion in M32 due to tidal stripping is about 0.1 mag (see
above). Thus accumulated tidal debris from M32 can
adequately account for the overall brightness of the stream.

8. SUMMARY

This paper presents surface photometry of M31’s two
nearest satellites, M32 and NGC 205, and a comparison to
N-body simulations. Details of the simulations are in the
companion paper Johnston, Choi, & Guhathakurta (Paper
I). The primary objectives of this work are to investigate the
impact of tidal interactions on the morphology and evolu-
tion of dwarf satellite galaxies and to place constraints on
the satellite orbital parameters. The main points are out-
lined below.

1. Large-format $B$- and $I$-band CCD mosaic images of
the M31 subgroup form the basis of this study. Global
eclipse fits are used to model and subtract M31’s contami-
nating disk light, enabling measurement of the faint outer
isophotes of M32 and NGC 205, where tidal signatures are
most prominent.

2. The surface brightness profile of M32 has traditionally
been fitted by a de Vaucouleurs $r^{1/4}$ law, but there is a clear
excess of light in the outer parts ($r \gtrsim 140''$) relative to the
standard fit. The excess is coincident with elongation and
twisting of the isophotes. There is also a downward break in
the $\mu$ profile at $r \sim 50''$ in the inner region of M32; this too is
accompanied by isophote twists. The intrinsic $\mu$ profile of
NGC 205 is more complex than M32’s, intermediate
between a simple exponential and $r^{1/4}$ laws, and is in good agreement with previous measurements.

3. The robustness of the M32 results is demonstrated through a series of tests. The measured isophotal parameters—surface brightness, ellipticity, and orientation—are robust out to at least $r \sim 250'$ and share the following characteristics: (a) they are insensitive to details of M31 disk modeling and sky subtraction errors; (b) they are symmetric about M32 despite the stark difference in the quality of the inner versus outer M31 disk list subtraction; and (c) they have a $B-I$ color index that is consistent with the inner parts of M32.

4. The M32 and NGC 205 measurements are compared with numerical simulations of single-component, spherical, nonrotating, satellites orbiting in a fixed, three-component parent galaxy potential. The simulations provide insight into the nature of tidal interaction even though they are not tailored precisely to M32 and NGC 205.

5. The surface brightness profiles of tidally disrupted simulated satellites contain certain generic features reminiscent of those seen in the M31 satellites. These features include an excess region at large radii, a depletion zone at intermediate radii, and a central region that is largely unaffected by tidal interaction. Isophote elongation and twists are also common, though the details of the $\epsilon$ and $\phi$ radial profiles are strongly dependent on orbital phase.

6. A comparison between the observations and numerical simulations indicates that M32 and NGC 205 are likely both on highly eccentric orbits, away from pericenter, and that they are being viewed from outside their orbital plane. The sense of M32's projected orbit around M31 appears to be clockwise. M32 has a simpler (intrinsic) brightness distribution in its inner parts than NGC 205 and is a better match to our current suite of simulations; its orbital parameters are therefore better constrained.

7. Empirical estimates are made of the effect of tidal stripping on M32's luminosity and effective surface brightness, based on an extrapolation of its inner surface brightness profile. The estimated amount of change in $L$ and $\mu_g$ is far too small to be consistent with the theory that M32 evolved from a normal elliptical and suggests instead that M32's precursor was intrinsically more compact than a typical E galaxy. This supports Burkert's (1994a) formation scenario for compact ellipsoids such as M32 through a starburst and subsequent violent collapse within the potential well of a massive galaxy, though spiral or S0 bulges cannot be ruled out as possible precursors.

8. While the current numerical simulations provide qualitative insight into the nature of tidal interaction in the M31 subgroup, future simulations will be tailored specifically to match the observed radial velocities, line-of-sight distances, and dynamical masses of M31, M32, and NGC 205. Other planned improvements include two-component satellites (stars and dark matter). Keck spectroscopy of individual red giant stars in the tidal region of M32 is being used to measure velocity and velocity dispersion profiles, which should better constrain the details of its interaction with M31.

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Peletier, R. F. 1993, A&A, 271, 51
Peterson, C. J. 1974, ApJ, 190, L17
Richter, N., & Hogner, W. 1963, Astron. Nachr., 287, 267
Rose, J. A. 1985, AJ, 90, 1927
Sato, N. R., & Sawa, T. 1986, PASJ, 38, 63
Tonry, J. L. 1984, ApJ, 283, L27

Tonry, J. L. 1987, ApJ, 322, 632
Vazdekis, A., & Arimoto, N. 1999, ApJ, 525, 144
von Hoerner, S. 1957, ApJ, 125, 451
Wirth, A., & Gallagher, J. S. 1984, ApJ, 282, 85
Ziegler, B. L., & Bender, R. 1998, A&A, 330, 819