RETHINKING VALUEDICE: DOES IT REALLY IMPROVE PERFORMANCE?

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May 30, 2022

ABSTRACT

Since the introduction of GAIL, adversarial imitation learning (AIL) methods attract lots of research interests. Among these methods, ValueDice has achieved significant improvements: it beats the classical approach Behavioral Cloning (BC) under the offline setting, and it requires fewer interactions than GAIL under the online setting. Are these improvements benefited from more advanced algorithms designs? We answer this question by the following conclusions.

First, we show that ValueDice could reduce to BC under the offline setting. Second, we verify that overfitting exists and regularization matters in the low-data regime. Specifically, we demonstrate that with weight decay, BC also nearly matches the expert performance as ValueDice does. The first two claims explain the superior offline performance of ValueDice. Third, we establish that ValueDice does not work when the expert trajectory is subsampled. Instead, the mentioned success of ValueDice holds when the expert trajectory is complete, in which ValueDice is closely related to BC that performs well as mentioned. Finally, we discuss the implications of our research for imitation learning studies beyond ValueDice.

1 Introduction

Many practical applications involve sequential decision-making. For these applications, an agent implements a policy to select actions and maximize the long-term return. Imitation learning approaches obtain the optimal policy from expert demonstrations [Argall et al., 2009, Hussein et al., 2017, Osa et al., 2018]. Imitation learning has been successfully applied in game [Ross et al., 2011, Silver et al., 2016], recommendation system [Shi et al., 2019, Chen et al., 2019], and robotics [Levine et al., 2016, Finn et al., 2016], etc.

One of the milestones in imitation learning is the introduction of generative adversarial imitation learning (GAIL) [Ho and Ermon, 2016]. Different from the classical approach Behavioral Cloning (BC) [Pomerleau, 1991] that trains a policy via supervised learning, GAIL performs state-action distribution matching in an adversarial manner [Ghasemipour et al., 2019, Ke et al., 2019, Xu et al., 2020]. Even when expert demonstrations are scarce, GAIL is empirically shown to match the expert performance while BC cannot [Ho and Ermon, 2016]. However, the price is that GAIL requires a number of environment interactions (e.g., 3M for Gym MuJoCo locomotion tasks), which restricts GAIL to be applied

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1This paper is presented at the blog track of the 10th international conference on learning representations (ICLR), 2022. Link: https://iclr-blog-track.github.io/2022/03/25/rethinking-valuedice/.
under the online setting. In contrast, BC does not require any interaction thus BC is widely applied under the offline setting.

**Context.** The remarkable achievements of GAIL attract lots of research interests in adversarial imitation learning (AIL) approaches. Among these methods, ValueDice [Kostrikov et al., 2020] has achieved significant improvements. On the one hand, ValueDice beats BC under the offline setting. On the other hand, ValueDice outperforms GAIL and another off-policy AIL algorithm DAC [Kostrikov et al., 2019] in terms of interaction efficiency. All existing results suggest Value Dice is perfect. This motivates the central question in this manuscript: are these improvements benefited from more advanced algorithm designs?

Before answering the raised question, we have to carefully examine the achievements of Value Dice. For the comparison with BC, the empirical advance of Value Dice seems to contradict the recent lower bound that BC is (nearly) minimax optimal under the offline setting [Rajaraman et al., 2020, Xu et al., 2021]. In other words, this theoretical result suggests that no algorithm can beat BC in the worst case under the offline setting. For the online part, it is well accepted that optimization of Bellman backup objectives (e.g., temporal-difference learning and Q-learning) with function approximation can easily diverge (see the divergence example in [Baird, 1995, Tsitsiklis and Van Roy, 1997]). To address this divergence issue, target network is proposed in [Mnih et al., 2015] and this technique is widely applied in deep RL [Lillicrap et al., 2016, Fujimoto et al., 2018, Haarnoja et al., 2018]; see [Lee and He, 2019, Zhang et al., 2021, Agarwal et al., 2021, Chen et al., 2022, Li et al., 2022] for the explanation about why the target network can address the divergence issue. However, unlike DAC that uses the target network, Value Dice is shown to successfully perform off-policy optimization without the target network. We will provide explanations for these “contradictions” when we answer the raised question.

**Main Contributions.** First, we show that Value Dice could reduce to BC under the offline setting. Second, we verify that overfitting exists and regularization matters in the low-data regime. Specifically, we demonstrate that with weight decay, BC also nearly matches the expert performance as Value Dice does. Actually, Value Dice has implemented the orthogonal regularization [Brock et al., 2018]. The first two claims explain the superior offline performance of Value Dice. Third, we establish that Value Dice cannot successfully optimize Bellman backup objectives when the expert trajectory is subsampled. As a result, Value Dice is inferior to existing methods (e.g., DAC and GAIL) in this case. Instead, the mentioned success of Value Dice holds when the expert trajectory is complete, in which Value Dice is closely related to BC that performs well as mentioned.

For general imitation learning research, our studies have two implications. For one thing, we additionally prove that ℓ1-norm based state-action distribution matching (a general AIL method) is also equivalent to BC under the offline setting under mild conditions. This suggests offline AIL is expected to be minimax optimal under the offline setting as BC does. For another thing, the success of BC (with regularization) discloses limitations of existing MuJoCo locomotion benchmarks: deterministic transitions and limited initial states. For such tasks, BC has no compounding errors and performs well even provided 1 expert trajectory, which is not revealed in the previous research.

### 2 Background

**Markov Decision Process.** Following the setup in [Kostrikov et al., 2020], we consider the infinite-horizon Markov decision process (MDP) [Puterman, 2014], which can be described by the tuple \(\mathcal{M} = (\mathcal{S}, \mathcal{A}, p, r, p_0, \gamma)\). Here \(\mathcal{S}\) and \(\mathcal{A}\) are the state and action space, respectively. \(p_0(\cdot)\) specifies the initial state distribution while \(\gamma \in [0, 1]\) is the discount factor. For a specific state-action pair \((s, a)\), \(p(s'|s, a)\) defines the transition probability of the next state \(s'\) and \(r(s, a) \in [0, 1]\) assigns a reward signal.

To interact with the environment (i.e., an MDP), a stationary policy \(\pi: \mathcal{S} \rightarrow \Delta(\mathcal{A})\) is introduced, where \(\Delta(\mathcal{A})\) is the set of probability distributions over the action space \(\mathcal{A}\). Specifically, \(\pi(a|s)\) determines the probability of selecting action \(a\) on state \(s\). The performance of policy \(\pi\) is measured by policy value \(V(\pi)\), which is defined as the cumulative and discounted rewards. That is,

\[
V(\pi) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_0 \sim p_0(\cdot), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t) \right].
\]

To facilitate later analysis, we need to introduce the (discounted) state-action distribution \(d^\pi(s, a)\):

\[
d^\pi(s, a) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t p(s_t = s, a_t = a \mid s_0 \sim p_0(\cdot), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)).
\]
With a little of notation abuse, we can define the (discounted) state distribution $d^\pi(s)$:

$$d^\pi(s) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t P(s_t = s | s_0 \sim p_0(\cdot), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)).$$

**Imitation Learning.** The goal of imitation learning is to learn a high-quality policy from expert demonstrations. To this end, we often assume there is a nearly optimal expert policy $\pi^{exp}$ that could interact with the environment to generate a dataset $D = \{(s_i, a_i)\}_{i=1}^n$. Then, the learner can use the dataset $D$ to mimic the expert and to obtain a good policy. The quality of imitation is measured by the policy value gap $V(\pi^{exp}) - V(\pi)$ [Abbeel and Ng, 2004, Syed and Schapire, 2007, Ross and Bagnell, 2010]. The target of imitation learning is to minimize the policy value gap between the expert policy $\pi^{exp}$ and the learner’s policy $\pi$:

$$\min_\pi V(\pi^{exp}) - V(\pi).$$

Notice that this policy optimization is performed without the reward signal. In this manuscript, we mainly focus on the case where the expert policy is deterministic, which is true in many applications and is widely used in the literature.

**Behavioral Cloning.** Given the state-action pairs provided by the expert, Behavioral Cloning (BC) [Pomerleau, 1991] directly learns a policy mapping to minimize the policy value gap. Specifically, BC often trains a parameterized classifier or regressor with the maximum likelihood estimation:

$$\min_\theta \sum_{(s,a) \in D} - \log p_\theta(a|s),$$

which can be viewed as the minimization of the KL-divergence between the expert policy and the learner’s policy [Ke et al., 2019, Ghasemipour et al., 2019, Xu et al., 2020]:

$$\min_\pi \mathbb{E}_{a \sim d^{exp}} \left[ D_{KL}(\pi^{exp} || \pi(\cdot|s)) \right],$$

where $D_{KL}(p||q) = \sum_{x \in X} p(x) \log(p(x)/q(x))$ for two distributions $p$ and $q$ on the common support $X$. Under the tabular setting where the state space and action space are finite, the optimal solution to (1) could take in a simple form:

$$\pi^{BC}(a|s) = \left\{ \begin{array}{ll}
\frac{\sum_{(s',a') \in D} 1(s' = s, a' = a)}{\sum_{(s',a') \in D} 1(s' = s)} & \text{if } \sum_{(s',a') \in D} 1(s' = s) > 0 \\
\text{otherwise}
\end{array} \right. (2)$$

That is, BC estimates the empirical action distribution of the expert policy on visited states from the expert demonstrations.

**ValueDice.** With the state-action distribution matching, ValueDice [Kostrikov et al., 2020] implements the objective:

$$\max_\pi -D_{KL}(d^\pi || d^{\pi^{exp}}) := \mathbb{E}_{(s,a) \sim d^\pi} \left[ \log \frac{d^{\pi^{exp}}(s,a)}{d^\pi(s,a)} \right]$$

$$= \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t \log \frac{d^{\pi^{exp}}(s_t, a_t)}{d^\pi(s_t, a_t)} | s_0 \sim p_0(\cdot), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t) \right]. (3)$$

That is, (4) amounts a max-return RL problem with the reward $\tilde{r}(s,a) = \log(d^{\pi^{exp}}(s,a)) / \log(d^\pi(s,a))$. In ValueDice, a dual form of (4) is presented:

$$-D_{KL}(d^\pi || d^{\pi^{exp}}) = \min_{x,s,A \rightarrow R} \left[ \log \mathbb{E}_{(s,a) \sim d^{\pi^{exp}}} [e^{\tilde{r}(s,a)}] - \mathbb{E}_{(s,a) \sim d^\pi} [x(s,a)] \right].$$

With the trick of variable change, Kostrikov et al. [2020] derived the following optimization problem:

$$\max_\pi \min_{\nu:S \times A \rightarrow R} J_{DICE}(\pi, \nu) := \log \left( \mathbb{E}_{(s,a) \sim d^{\pi^{exp}}} [e^{\tilde{r}(s,a) - B^\nu(s,a)}] \right) - (1 - \gamma) \cdot \mathbb{E}_{s_0 \sim p_0(\cdot), a_0 \sim \pi(\cdot|s_0)} [\nu(s_0, a_0)], (5)$$

where $B^\nu(s,a) = \gamma \mathbb{E}_{s' \sim p(\cdot|s,a), a' \sim \pi(\cdot|s')} [\nu(s', a')]$ performs one-step Bellman update (with zero reward). As noted in [Kostrikov et al., 2020], objective (5) only involves the samples from the expert demonstrations to update, which may lack diversity and hamper training. To address this issue, Kostrikov et al. [2020] proposed to use samples from the replay buffer under the online setting. Concretely, an alternative objective is introduced:

$$J_{DICE}^{\text{mix}} := \log \left( \mathbb{E}_{(s,a) \sim d^{mix}} [e^{\tilde{r}(s,a) - B^\nu(s,a)}] \right) - (1 - \gamma) \cdot \mathbb{E}_{s_0 \sim p_0(\cdot), a_0 \sim \pi(\cdot|s_0)} [\nu(s_0, a_0)]$$

$$- \alpha \mathbb{E}_{(s,a) \sim d^{mix}} [\nu(s,a) - B^\nu(s,a)],$$

where $d^{mix}(s,a) = (1-\alpha)d^{\pi^{exp}}(s,a) + \alpha d^{\text{RB}}(s,a)$. In this case, VauDiced is to perform the modified state-action distribution matching:

$$\max_\pi -D_{KL}((1-\alpha) d^\pi + \alpha d^{\text{RB}} || (1-\alpha) d^{\pi^{exp}} + \alpha d^{\text{RB}}).$$

As long as $\alpha \in [0, 1)$, the global optimality of $\pi = \pi^{exp}$ is reserved. Under the offline setting, we should set $\alpha = 0.$
3 Rethinking ValueDice Under the Offline Setting

In this section, we revisit ValueDice under the offline setting, where only the expert dataset is provided and environment interaction is not allowed. In this case, the simple algorithm BC can be directly applied while on-policy adversarial imitation learning (AIL) methods cannot. This is because the latter often requires environment interactions to evaluate the state-action distribution \( d^π(s, a) \) to perform optimization.

Under the offline setting, the classical theory suggests that BC suffers the compounding errors issue [Ross et al., 2011, Rajaraman et al., 2020, Xu et al., 2020]. Specifically, once the agent makes a wrong decision (i.e., an imperfect imitation due to finite samples), it may visit an unseen state and make the wrong decision again. In the worst case, such decision errors accumulate over \( H \) time steps and the agent could obtain zero reward along such a bad trajectory. This explanation often coincides with the empirical observation that BC performs poorly when expert demonstrations are scarce. Thus, it is definitely interesting and valuable if some algorithms can perform better than BC under the offline setting. Unfortunately, the information-theoretical lower bound [Rajaraman et al., 2020, Xu et al., 2021] suggests that BC is (nearly) minimax optimal under the offline setting.

**Theorem 1** (Lower Bound [Xu et al., 2021]). Given expert dataset \( D = \{(s_i, a_i)\}_{i=1}^m \) which is i.i.d. drawn from \( d^{\text{exp}} \), for any offline algorithm \( \text{Alg} : D \to \hat{π} \), there exists a constant \( δ_0 \in (0, 1/10) \), an MDP \( M \) and a deterministic expert policy \( \pi^{\text{exp}} \) such that, for any \( δ \in (0, δ_0) \), with probability at least \( 1 - δ \), the policy value gap has a lower bound:

\[
V(\pi^{\text{exp}}) - V(\hat{π}) \geq \min \left( \frac{1}{1 - \gamma}, \frac{|S|}{(1 - \gamma)^2 m} \right).
\]

**Remark 1.** We note that the sample complexity of BC is \( \tilde{O}(|S|/((1 - \gamma)^2 \varepsilon)) \) [Xu et al., 2021], which matches the lower bound up to logarithmic terms. We emphasize that the minimax optimality of BC does not mean that BC is optimal for each task. Instead, it says that the worst case performance of BC is optimal. The worst case performance of offline imitation learning algorithms can be observed on the Reset Cliff MDPs in [Rajaraman et al., 2020, Xu et al., 2021], which share many similarities with MuJoCo locomotion tasks\(^2\). To this end, we can believe that BC is optimal for MuJoCo locomotion tasks under the offline setting.

![Comparison of ValueDice and BC](image)

Figure 1: Comparison of ValueDice and BC under the offline setting with 1 expert trajectory, which reproduces [Kostrikov et al., 2020, Figure 4]. Dashed lines correspond to the performance of expert policies.

From the empirical side, we realize that Kostrikov et al. [2020] showed that their proposed algorithm ValueDice could be better than BC on many interesting tasks under the offline setting; see [Kostrikov et al., 2020, Figure 4] or the reproduced results\(^3\) in the following Figure 1 (all experiment details can be found in Appendix B). This manifests a gap between theory and practice. In the sequel, we examine the achievement of ValueDice and explain this gap.

### 3.1 Connection Between ValueDice and BC Under Offline Setting

As a first step, let us review the training objective of ValueDice in the offline scenario. For ease of presentation, let us consider the case where the number of expert trajectory is 1, which follows the experimental setting used in Figure 1.

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\(^2\)The similarity is explained as follows. Both tasks involve a bad absorbing state: once the agent takes a wrong action, it goes to this bad absorbing state and obtains a zero reward forever.

\(^3\)Our implementation of BC is different from the one in [Kostrikov et al., 2020]. In particular, our BC policy is deterministic and does not learn the covariance, while the BC policy in [Kostrikov et al., 2020] is stochastic. We note that although we may learn a stochastic policy, it is important for the evaluation performance to output a deterministic action (i.e., the mean) [Ho and Ermon, 2016, Haarnoja et al., 2018]. In fact, we observe that learning a stochastic policy by sharing the hidden layers really hurts the performance of BC. Nevertheless, we would like to clarify that the implementation of BC in [Kostrikov et al., 2020] aims for a “fair” comparison because ValueDice uses a stochastic policy.
As a result, the training objective of ValueDice with $T$ samples is formulated as

$$J_{\text{DICE}}^{\text{offline}}(\pi, \nu) := \log \left( \sum_{t=1}^{T} \left[ e^{\nu(s_t, a_t)} - \gamma E_{a_{t+1} \sim \pi(s_{t+1})} [\nu(s_{t+1}, \tilde{a}_{t+1})] \right] \right) - (1 - \gamma) \sum_{t=1}^{T} E_{\tilde{a}_{t} \sim \pi(s_t)} [\nu(s_t, \tilde{a}_t)]. \tag{6}$$

**Remark 2.** Even derived from the principle of state-action distribution matching (i.e., objective (3)), under the offline setting, we argue that ValueDice cannot enjoy the benefit of state-action distribution matching. This is because objective (6) is performed only on states collected by the expert policy. That is, there is no guidance for policy optimization on non-visited states. This is the main difference between the online setting and offline setting. As a result, we cannot explain the experiment results in Figure 1 by the tailored theory for online state-action distribution matching in [Xu et al., 2022].

To obtain some intuition about the optimal solutions to the empirical objective (6), we can further consider the case where $\gamma = 0$. Note that even this variant could also outperform BC under the offline setting (see Figure 2).

![Comparison of BC and ValueDice's variants](image)

Figure 2: Comparison of BC and ValueDice’s variants (with different discount factors $\gamma$ and orthogonal regularization coefficients $\lambda$) under the offline setting with 1 expert trajectory. The default configuration for ValueDice is $\gamma = 0.99$ and $\lambda = 0.0001$. Dashed lines correspond to the performance of expert policies.

**Theorem 2.** Consider the tabular MDPs. If $\gamma = 0$, objective (6) reduces to

$$J_{\text{DICE}}^{\text{offline, }\gamma=0}(\pi, \nu) := \log \left( \sum_{t=1}^{T} e^{\nu(s_t, a_t)} \right) - \sum_{t=1}^{T} E_{\tilde{a}_{t} \sim \pi(s_t)} [\nu(s_t, \tilde{a}_t)]. \tag{7}$$

Importantly, the BC’s solution as in (2) is also one of the globally optimal solutions for ValueDice’s objective in (7).

**Proof.** For tabular MDPs, each element is independent. As a consequence, for a specific state-action pair $(s_t, a_t)$ appeared in the expert demonstrations, its optimization objective is

$$\max_{\pi(\cdot|s_t) \in \Delta(A)} \min_{\nu(\cdot, a_t) \in \mathbb{R}} \nu(s_t, a_t) + b - E_{\tilde{a}_{t} \sim \pi(\cdot|s_t)} [\nu(s_t, \tilde{a}_t)],$$

which has equilibrium points when $\pi(a_t|s_t) = 1$ and $\pi(a|s_t) = 0$ for any $a \neq a_t$. To this end, the “counting” solution in (2) for BC is also one of the globally optimal solutions for the empirical objective (7). \qed

We remark that Theorem 2 provides a restricted case where ValueDice could degenerate to BC under the offline setting; see the experimental verification in Figure 3. As we have analyzed in Remark 2, the same intuition is expected to hold when $\gamma > 0$ as ValueDice does not have guidance on non-visited states. However, it is technically challenging to draw this conclusion. Instead, we consider another state-action distribution matching method with the $\ell_1$-norm metric. In particular, we claim that this conclusion still holds for tabular MDPs with finite horizons.

**Theorem 3 (Informal version of Theorem 5 in Appendix).** Consider the tabular MDPs with finite horizons. Assume that there is 1 expert trajectory provided. Under the offline setting, we have that BC’s solution is the unique globally optimal solution to $\ell_1$-norm based state-action distribution matching (i.e., $\min_{\pi} \| d^\pi - \hat{d}^{\text{exp}} \|_1$), where $\hat{d}^{\text{exp}}$ is the estimated expert state-action distribution from expert demonstrations.

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4The benefit is explained as follows. Under the online (or the known transition) setting, state-action distribution matching methods can perform policy optimization on non-visited states. With additional transition information, state-action distribution matching methods could have better sample complexity than BC under mild conditions [Xu et al., 2022].
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Figure 3: Comparison of BC and ValueDice in terms of MSE (i.e., BC’s loss) under the offline setting with 1 expert trajectory. This result suggests that ValueDice is closely related to BC under the offline setting.

See Appendix A.2 for the formal argument and proof. Note that the assumption of 1 expert trajectory is mainly to make the analysis simple and ℓ₁-norm based state-action distribution matching is to ensure that the objective is well-defined. Actually, the experiment results in Figure 1 satisfy the assumptions required in Theorem 3. We remark that Theorem 3 establishes a one-to-one connection between BC and ValueDice under the offline setting, which adds a constraint on the optimization procedure (i.e., optimization is defined over visited states). We conjecture that this conclusion holds for other AIL methods. Finally, we note that the one-to-one relationship is not disclosed by Theorem 2.

In summary, we could believe that under the offline setting, AIL methods could not perform optimization on non-visited states. As a result, the BC policy is sufficient to be the globally optimal solution of AIL methods. We highlight that this insight is not illustrated by the lower bound argument [Rajaraman et al., 2020, Xu et al., 2021], which only tells us that one specific algorithm, BC, is nearly minimax optimal under the offline setting. From another viewpoint, our result also suggests that state-action distribution matching is expected to be a “universal” principle: it is closely related to BC under the offline setting while it enjoys many benefits under the online setting.

3.2 Overfitting of ValueDice and BC

Based on the above conclusion that AIL may reduce to BC under the offline setting, it is still unsettled that why ValueDice outperforms BC in Figure 1. If we look at training curves carefully, we would observe that BC could perform well in the initial learning phase. As the number of iterations increases, the performance of BC tends to degenerate. This is indeed the overfitting phenomenon.

To overcome the overfitting in this low-data regime, we can adopt many regularization techniques such as weight decay [Krogh and Hertz, 1991], dropout [Srivastava et al., 2014], and early stopping [Prechelt, 1996]. For simplicity, let us consider the weight decay technique, i.e., adding a squared ℓ₂-norm penalty for the training parameters to the original training objective. The empirical result is shown in Figure 4. Surprisingly, we can find that even with small weight decay, BC improves its generalization performance and is on-par with ValueDice.

The perspective of overfitting motivates us to carefully examine the performance of ValueDice. In particular, we realize that in practice ValueDice uses the orthogonal regularization [Brock et al., 2018], a kind of regularizer from the GAN literature. Without this regularizer, the performance of ValueDice is poor as shown in Figure 2.

Observation 1. Regularization is important for offline imitation learning algorithms such as BC and ValueDice in the low-data regime.

Up to now, we know that ValueDice degenerates to BC under the offline setting and the experiment results suggest that regularization matters. With proper regularization, BC and ValueDice are able to match the expert performance.

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5In contrast, the KL-divergence used in ValueDice is not well-defined for some state-action pair \((s, a)\) with \(d^\pi(s, a) = 0\) but \(d^{\text{exp}}(s, a) > 0\).

6Recently, Swamy et al. [2021] showed an interesting conclusion that BC and ValueDice can be viewed as “off-Q” algorithms. Based on this conclusion, Swamy et al. [2021] argued that BC and ValueDICE should have similar policy value gaps. We note that this conclusion mainly holds in the population level (informally speaking, “population” means that there are infinite samples). In contrast, we focus on the finite-samples setting and and validate that offline AIL methods can directly reduce to BC. One important message that we want to convey is that in the offline setting with finite samples, the policy optimization only involves visited states so the fundamental limit (i.e., the compounding errors issue) cannot be overcome.
3.3 Discussion: Why BC Performs Well?

One related question about previous experiment results is that why BC (with regularization) performs so well (i.e., BC nearly matches the expert performance) even provided with 1 expert trajectory as in Figure 4. It seems that there are no compounding errors for BC. Furthermore, the presented results seem to contradict the common impression that BC performs poorly but AIL performs well [Ho and Ermon, 2016, Kostrikov et al., 2019]. We discuss this mismatch in the following part.

First, the compounding errors argument for BC relies on the assumption that the agent makes a mistake in each time step as in [Ross et al., 2011]. As such, decision errors accumulate over time steps. However, the assumption may not hold for some cases. Considering MDPs with deterministic transitions, the BC’s policy does not make any mistake by starting from a visited initial state. As a consequence, there are no compounding errors along such a “good” trajectory. Formally, the policy value gap of BC is expected to be $\tilde{O}(|S|H/m)$ under MDPs with deterministic transitions, where $H$ is the finite horizon and $m$ is the number of expert trajectories; see [Xu et al., 2022].

**Theorem 4 (BC for Deterministic MDPs [Xu et al., 2022]).** Consider the tabular MDPs with finite horizons and deterministic transitions. Given $m$ expert trajectories with length $H$, the policy value gap of BC is upper bounded by:

$$V(\pi^{exp}) - \mathbb{E} [V(\pi^{BC})] \leq \left(H, \frac{|S|H}{m}\right),$$

where the expectation is taken over the randomness when collecting the expert dataset and $\pi^{BC}$ is the “counting” solution similar to that in (2).

Here the dependence on $|S|$ is due to the random initial states. The policy value gap becomes smaller if the initial states are limited. We highlight that this upper bound is tighter than the general one $\tilde{O}(|S|H^2/m)$, which are derived from MDPs with stochastic transitions [Rajaraman et al., 2020]. The fundamental difference is that stochastic MDPs ensure BC to have a positive probability to make a mistake even by starting from a visited initial state, which leads to the quadratic dependence on $H$. For our purpose, we know that MuJoCo locomotion tasks have (almost) deterministic transition functions and the initial states lie within a small range. Therefore, it is expected that BC (with regularization) performs well as in Figure 4.

**Observation 2.** For deterministic tasks (e.g., MuJoCo locomotion tasks), BC has no compounding errors if the provided expert trajectory are complete.

Second, the worse dependence on the planning horizon for BC is observed based on subsampled trajectories in [Ho and Ermon, 2016, Kostrikov et al., 2019]. More concretely, they sample non-consecutive transition tuples from a complete trajectory as the expert demonstrations. This is different from our experiment setting, in which we use complete trajectories. It is obvious that subsampled trajectories artificially “mask” some states. As a result, BC is not ensured to match the expert trajectory even when the transition function is deterministic and the initial state is unique. This explains why BC performs poorly as in [Ho and Ermon, 2016, Kostrikov et al., 2019]. We note that this subsampling operation turns out minor for AIL methods (see the explanation in [Xu et al., 2022]).

4 Rethinking ValueDice Under the Online Setting

In this section, we consider the online setting, where the agent is allowed to interact with the environment. In particular, it is empirically shown that ValueDice outperforms another off-policy AIL algorithm DAC [Kostrikov et al., 2019]
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in terms of environment interactions; see [Kostrikov et al., 2020, Figure 2] or the reproduced results in the following Figure 5. More surprisingly, ValueDice successfully performs policy optimization without the useful technique target network. This contradicts the common sense that optimization of Bellman backup objectives (e.g., temporal difference learning and Q-learning) with function approximation can easily diverge (see the divergence example in [Baird, 1995, Tsitsiklis and Van Roy, 1997]. To address this divergence issue, target network is proposed in [Mnih et al., 2015] and this technique is widely applied in deep RL [Lillicrap et al., 2016, Fujimoto et al., 2018, Haarnoja et al., 2018]; see [Lee and He, 2019, Zhang et al., 2021, Agarwal et al., 2021, Chen et al., 2022, Li et al., 2022] for the explanation about why the target network can address the divergence issue.

We realize that the result in [Kostrikov et al., 2020, Figure 2] is based on the setting where complete expert trajectories are provided. In this case, we have known that ValueDice could degenerate to BC, and BC (with regularization) performs well. Furthermore, by comparing learning curves in Figure 1 (offline) and Figure 5 (online), we find that the online interaction does not matter for ValueDice. This helps confirm that the reduction is crucial for ValueDice. Thus, we know the reason why ValueDice does not diverge in this case is that there is no divergence issue for BC.

Now, how about the case where expert trajectories are incomplete/subsampled? That is, the expert state-action pairs are no longer temporally consecutive, which is common in practice. The corresponding results are missing in [Kostrikov et al., 2020] and we provide such results in the following Figure 6. In particular, we see that ValueDice fails but DAC still works.

Observation 3. For MuJoCo locomotion tasks, ValueDice performs well in the complete trajectory case while it does not perform well in the subsampled case even though it can interact with the environment.

Since ValueDice and DAC use the same principle (i.e., state-action distribution matching), we believe that the poor performance of ValueDice in Figure 6 is mainly caused by optimization issues. In terms of optimization, one major difference between ValueDice and DAC is that DAC uses the target network technique while ValueDice does not. As mentioned, without a target network, optimization of Bellman backup objectives with function approximation can easily diverge. Note that the divergence issue discussed here does not contradict the good results in Figure 4 and Figure 5 because in that case, ValueDice is closely related to BC that performs well.

In summary, our experiment results support the claim that ValueDice cannot succeed to perform policy optimization with function approximation under the subsampled cases. Moreover, our results suggest that the mentioned success of ValueDice may rely on its connection to BC.
5 Conclusion

In this manuscript, we rethink the algorithm designs in ValueDice and promote new conclusions under both offline and online settings. We clarify that our results do not indicate that ValueDice is “weaker” than BC or ValueDice does not show any algorithmic insights. Instead, our studies highlight the connection between adversarial imitation learning algorithms (including ValueDice) and BC under the offline setting, point out the instability of Dice-based technique under the certain scenarios, and clarify some confusing results in the literature. We notice that the ideas from ValueDICE are still valuable, which can be extended to other imitation learning and offline reinforcement learning algorithms. Perhaps, some conclusions in this manuscript could provide insights to examine the algorithmic advances and understand the reported results in the related works such as SoftDice [Sun et al., 2021], OptDice [Lee et al., 2021], SmoDice [Ma et al., 2022], and DemoDice [Kim et al., 2022].

For the general imitation learning studies, our work has the following implications.

- **Algorithm Evaluations.** Our experiment results show a clear boundary between the complete trajectory and subsampled trajectory cases. Two cases have dramatically different characteristics, results and explanations. As a result, we must be cautious to evaluate algorithms by identifying the context. Without this, some arguments may be misleading for future studies.

- **Benchmarks.** Our study points out several drawbacks of existing MuJoCo locomotion benchmarks: deterministic transitions and limited initial states. For one thing, provided 1 complete expert trajectory, simple algorithm BC is competent for many tasks, which is not discovered in the previous research. That is, current tasks are not even “hard”. For another thing, in addition to large-scale studies in [Hussenot et al., 2021, Orsini et al., 2021] that mainly focus on existing MuJoCo locomotion benchmarks, future imitation learning studies could benefit from more challenging benchmarks with stochastic transitions and diverse initial states.

Acknowledgements and Disclosure of Funding

We thank anonymous reviewers and Gokul Swamy for the helpful comments. The work of Yang Yu is supported by National Key R&D Program of China National Key Research and Development Program of China (2020AAA0107200), NSFC(61876077), and Collaborative Innovation Center of Novel Software Technology and Industrialization. The work of Zhi-Quan Luo is supported by the National Natural Science Foundation of China (No. 61731018) and the Guangdong Provincial Key Laboratory of Big Data Computation Theories and Methods.

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7We notice that Spencer et al. [2021] also considered the limitation of MuJoCo locomotion benchmarks. Specifically, they argued that these benchmarks are realizable (i.e., no approximation error) and thus are not sufficient to cover some hard regimes in real-world imitation learning applications.
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A Missing Proofs

A.1 Episodic Markov Decision Process

In this part, we briefly introduce the episodic Markov decision process. We remark that episodic MDP is introduced to facilitate analysis since the horizon is finite.

An episodic Markov decision process (MDP) can be described by the tuple \( \mathcal{M} = (S, A, P, r, p_0, H) \). Here \( S \) and \( A \) are the state and action space, respectively. \( H \) is the planning horizon\(^8\), \( p_0 \) is the initial state distribution, and \( r : S \times A \to [0, 1] \) is the reward function. Different from the infinite-horizon discounted MDP in Section 2, transition function \( P = \{ p_1, \cdots, p_H \} \) is non-stationary. That is, \( p_h(s_{h+1}|s_h, a_h) \) determines the probability of transiting to state \( s_{h+1} \) conditioned on state \( s_h \) and action \( a_h \) in time step \( h \), for \( h \in [H] \). Since the optimal policy is not stationary, we need to consider the non-stationary policy \( \pi = \{ \pi_1, \cdots, \pi_h \} \), where \( \pi_h : S \to \Delta(A) \) and \( \Delta(A) \) is the probability simplex. \( \pi_h(a|s) \) gives the probability of selecting action \( a \) on state \( s \) in time step \( h \), for \( h \in [H] \).

The policy value is defined as the cumulative rewards over \( H \) time steps:

\[
V(\pi) := E \left[ \sum_{t=1}^{H} r(s_t, a_t) \mid s_0 \sim p_0(\cdot), a_t \sim \pi_t(\cdot|s_t), s_{t+1} \sim p_t(\cdot|s_t, a_t) \right].
\]

Again, we can define the non-stationary state-action distribution \( d_t^n \):

\[
d_t^n(s, a) := \mathbb{P}(s_t = s, a_t = a),
\]

which characterizes the probability of visiting the state-action pair \((s, a)\) in time step \( t \).

A.2 Reduction from AIL to BC

Theorem 5. For episodic MDP, consider the following optimization problem:

\[
\min_{\pi} \sum_{t=1}^{H} \sum_{(s,a) \in S \times A} \left| d_t^n(s, a) - \hat{d}_t^\text{exp}(s, a) \right|,
\]

such that \( \pi_t(a|s) = 1/|A|, \forall s, t \) with \( \hat{d}_t^\text{exp}(s) = 0 \),

Assume that there is only one expert trajectory. Then, we have that the BC’s policy is the globally optimal solution.

Proof. To facilitate later analysis, we will use \( a^\text{exp} \) to denote the expert action on a specific state \( s \) (recall that we assume the expert policy is deterministic). Without loss of generality, let \( \tilde{s} \) be visited state occurred in the single expert trajectory.

We want to mimic the classical dynamic programming proof as in [Bertsekas, 2012]. Specifically, we aim to prove that the BC’s policy is optimal in a backward way. That is, we will use the induction proof. To this end, we define the cost function in time step \( h \) as

\[
\text{Loss}_h = \sum_{(s,a) \in S \times A} \left| d_t^n(s, a) - \hat{d}_t^\text{exp}(s, a) \right|.
\]

Then, we can define the cost-to-go function:

\[
\ell_h = \sum_{t=h}^{H} \sum_{(s,a) \in S \times A} \left| d_t^n(s, a) - \hat{d}_t^\text{exp}(s, a) \right|.
\]

For the base case where time step \( h = H \), we have that

\[
\ell_H = \sum_{(s,a)} \left| d_H^n(s, a) - \hat{d}_H^\text{exp}(s, a) \right|
\]

\[
= \sum_a \left| d_H^n(\tilde{s}, a) - \hat{d}_H^\text{exp}(\tilde{s}, a) \right| + \sum_{s \neq \tilde{s}} \sum_a \left| d_H^n(s, a) - \hat{d}_H^\text{exp}(s, a) \right|
\]

\(^8H\) corresponds to the effective horizon \( 1/(1 - \gamma) \) in the infinite-horizon discounted MDP.

\(^9[x]\) denotes the set of integers from 1 to \( x \).
where the left hand side is the unique globally optimal solution. We want to prove that

\[ \pi_{h+1}(a^{\text{exp}}|\bar{s}) = 1, \pi_{h+2}(a^{\text{exp}}|\bar{s}) = 1, \cdots, \pi_{H}(a^{\text{exp}}|\bar{s}) = 1 \]

where the left hand side is the unique globally optimal solution. We want to prove that

\[ (\pi_h(a^{\text{exp}}|\bar{s}) = 1, \pi_{h+1}(a^{\text{exp}}|\bar{s}) = 1, \cdots, \pi_{H}(a^{\text{exp}}|\bar{s}) = 1) = \arg\min_{\pi_{h+1}, \pi_{h+2}, \cdots, \pi_{H}} \ell_h(\pi_{h+1}, \pi_{h+2}, \cdots, \pi_{H}) \]

where the left hand side is the unique globally optimal solution.

For time step \( h \), we have that

\[
\text{Loss}_h = \sum_{(s,a)} |d^\pi_h(s, a) - d^\text{exp}_h(s, a)|
\]

\[ = 2(1 - d^\pi_h(\bar{s})\pi_h(a^{\text{exp}}|\bar{s})) \]

For time step \( h + 1 \leq h' \leq H \), we have that

\[
\text{Loss}_{h'} = \sum_{(s,a)} |d^{\pi}_{h'}(s, a) - d^\text{exp}_{h'}(s, a)|
\]

\[ = 2(1 - d^{\pi}_{h'}(\bar{s})\pi_{h'}(a^{\text{exp}}|\bar{s})) \]

where the last step follows our assumption. We have the following decomposition for \( d^{\pi}_{h'}(\bar{s}) \),

\[
d^{\pi}_{h'}(\bar{s}) = \sum_{(s,a)} p_{h' - 1}(\bar{s}|s, a)d^{\pi}_{h' - 1}(s, a)
\]

\[ = p_{h' - 1}(\bar{s}|\bar{s}, a^{\text{exp}})d^{\pi}_{h' - 1}(\bar{s}, a^{\text{exp}}) + \sum_{a \neq a^{\text{exp}}} p_{h' - 1}(\bar{s}|\bar{s}, a)d^{\pi}_{h' - 1}(s, a) + \sum_{s \neq \bar{s}} \sum_{a} p_{h' - 1}(\bar{s}|s, a)d^{\pi}_{h' - 1}(s, a)
\]

\[ = p_{h' - 1}(\bar{s}|\bar{s}, a^{\text{exp}})d^{\pi}_{h' - 1}(\bar{s}, a^{\text{exp}}) + \sum_{a \neq a^{\text{exp}}} p_{h' - 1}(\bar{s}|\bar{s}, a)d^{\pi}_{h' - 1}(s, a)\pi_{h' - 1}(a|\bar{s})
\]

\[ + \sum_{s \neq \bar{s}} \sum_{a} p_{h' - 1}(\bar{s}|s, a)d^{\pi}_{h' - 1}(s, a)\pi_{h' - 1}(a|\bar{s})
\]

\[ = p_{h' - 1}(\bar{s}|\bar{s}, a^{\text{exp}})d^{\pi}_{h' - 1}(\bar{s}, a^{\text{exp}}) + \sum_{s \neq \bar{s}} \sum_{a} p_{h' - 1}(\bar{s}|s, a)d^{\pi}_{h' - 1}(s, a)\pi_{h'}(a|\bar{s})d^\pi_h(\bar{s}) \]

where the last equation again follows our assumption.

Finally, we should have that\(^{10}\) \( \ell_h(\pi_h) = (a) + (b) + \text{constant} \), where constant is unrelated to \( \pi_h(\bar{s}) \) and

\[
(a) = - \prod_{\ell = h + 1}^{h' - 1} p_e(\bar{s}|\bar{s}, a^{\text{exp}}) \left[ p_h(\bar{s}|\bar{s}, a^{\text{exp}})\pi_h(a^{\text{exp}}|\bar{s})d^\pi_h(\bar{s}) + \sum_{a \neq a^{\text{exp}}} p_h(\bar{s}|\bar{s}, a)\pi_h(a|\bar{s})d^\pi_h(\bar{s}) \right] - 2d^\pi_h(\bar{s})\pi_h(a^{\text{exp}}|\bar{s}),
\]

\(^{10}\)We have four types of transition flows: \([1]\) \((\bar{s}, a^{\text{exp}}) \rightarrow (\bar{s}, a^{\text{exp}}) \rightarrow \cdots \rightarrow (\bar{s}, a^{\text{exp}}) \rightarrow \bar{s} ; [2] (\bar{s}, a) \rightarrow (\bar{s}, a^{\text{exp}}) \rightarrow \cdots \rightarrow (\bar{s}, a^{\text{exp}}) \rightarrow \bar{s} ; [3] (\bar{s}, a^{\text{exp}}) \rightarrow (s, a) \rightarrow \cdots \rightarrow (s, a) \rightarrow \bar{s} ; [4] (\bar{s}, a) \rightarrow (s, a) \rightarrow \cdots \rightarrow (s, a) \rightarrow \bar{s} \). The first two terms arise in \((a)\) and the last two terms arise in \((b)\).
\[(b) = -\sum_{s \neq \tilde{s}} \sum_{a} \frac{1}{|A|} p_{h'-1}(\tilde{s}|s,a) \left[ \mathbb{P}(s_{h'\mid 1} = s|s_h = \tilde{s}, a_h = a^{\exp}) \pi_h(a^{\exp} | \tilde{s}) d_h^h(\tilde{s}) \right. 
\quad \left. + \sum_{a \neq a^{\exp}} \mathbb{P}(s_{h'\mid 1} = s|s_h = \tilde{s}, a_h = a) \pi_h(a | \tilde{s}) d_h^h(\tilde{s}) \right].\]

Let us compare the coefficient for \(\pi_h(a^{\exp} | \tilde{s})\) and \(\pi_h(a | \tilde{s})\) with \(a \neq a^{\exp}:\)

\[
c^{\exp} = -d_h^h(\tilde{s}) \left\{ \prod_{\ell=h+1}^{h'-1} p_{\ell}(\tilde{s}|s, a^{\exp}) p_h(\tilde{s}|s, a^{\exp}) + \sum_{s \neq \tilde{s}} \sum_{a} \frac{1}{|A|} p_{h'-1}(\tilde{s}|s,a) \mathbb{P}(s_{h'\mid 1} = s|s_h = \tilde{s}, a_h = a^{\exp}) + 2 \right\},
\]

\[
c^a = -d_h^h(\tilde{s}) \left\{ \prod_{\ell=h+1}^{h'-1} p_{\ell}(\tilde{s}|s, a^{\exp}) p_h(\tilde{s}|s, a) + \sum_{s \neq \tilde{s}} \sum_{a} \frac{1}{|A|} p_{h'-1}(\tilde{s}|s,a) \mathbb{P}(s_{h'\mid 1} = s|s_h = \tilde{s}, a_h = a) \right\}. \]

Hence, \(c^{\exp} < c^a\) and we know that \(a^{\exp}\) is the optimal action in time step \(h\). This proves the induction case.
B Experiment Details

Algorithm Implementation. Our implementation of ValueDice and DAC follows the public repository https://github.com/google-research/google-research/tree/master/value_dice by Kostrikov et al. [2020]. Our implementation of BC is different from the one in this repository. In particular, Kostrikov et al. [2020] implemented BC with a Gaussian policy with trainable mean and covariance as in [Kostrikov et al., 2020, Figure 4]. However, we observe that the performance of this implementation is very poor because the mean and covariance share the same hidden layers and the covariance affects the log-likelihood estimation. Instead, we use a simple MLP architecture without the output of the covariance. This deterministic policy is trained with mean-square-error (MSE):

$$\min_\theta \sum_{(s, a) \sim D} (f_\theta(s) - a)^2.$$ 

The hidden layer size and optimizer of our BC policy follow the configuration for ValueDice.

Benchmarks. All preprocessing procedures follow [Kostrikov et al., 2020]. The subsampling procedure follows [Kostrikov et al., 2019]; please refer to https://github.com/google-research/google-research/blob/master/dac/replay_buffer.py#L154-L177. The expert demonstrations are from https://github.com/google-research/google-research/tree/master/value_dice#install-dependencies.

Experiments. All algorithms are run with 5 random seeds (2021-2025). For all plots, solid lines correspond to the mean, and shaded regions correspond to the standard deviation.