An analytical study on effects of viscous dissipation and suction/injection on a steady mhd natural convection couette flow of heat generating/absorbing fluid

Abiodun O Ajibade1, Ayuba M Umar2 and Tafida M Kabir3

Abstract
This work concerns a theoretical investigation on the effects of suction/injection, magnetic field, permeability of porous materials and viscous dissipation on an electrically conducting incompressible fluid passes through a vertical porous channel filled with porous materials. One of the plates moves in the flow direction while the other is stationary. The governing coupled flow equations have been solved analytically using Homotopy Perturbation Method (HPM). The influences of the flow parameters on velocity and temperature were plotted on graphs while numerical values for rate of heat transfer and shear stress on the heated and cold plates were presented in tables. Excellent agreements were found when compared with the previous works. It is noteworthy to mention that the hydrodynamic and thermodynamic distributions of the fluid increase with increase in viscous dissipation ($Ec$). It is also found that the shear stress decreases with increase in the magnetic field ($M$) while a reverse case was observed for growing the permeability of the porous materials ($K$). It is further found that the velocity and temperature distributions decrease with increase in suction ($\lambda < 0$).

Keywords
Porous materials, magnetic field, free convection, viscous dissipation, vertical channel

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Introduction
The study of fluid dynamics nowadays has received a great interest and concern due to its applications in science and engineering. These applications could be found in geothermal energy, gas turbines, plasma physics, petroleum industries, cooling of nuclear reactors, food processing industries, nuclear power plants, gas drainage, lubrication industries etc. Investigation on electrically conducting fluid in the presence of magnetic field: Magnetohydrodynamics (MHD), permeability, suction/injection, heat source/sink and viscous dissipation in different media is of great importance since fluid flow and heat transfer are inevitable transports either naturally or forcibly.

Jha and Ajibade1 studied a transient case of a free convective flow of heat generating/absorbing fluid in a vertical channel. They showed that an increase in heat absorption led to increase in the rate of heat transfer on the moving plate and causes a decrease in the heat transfer on the stationary plate. Effects of chemical reaction and radiation absorption on an unsteady MHD natural convection flow through a semi infinite vertical channel in the presence of heat source and

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suction was studied by Ibrahim et al.\textsuperscript{2} They concluded that the velocity profile decreases with increasing magnetic field. Asadollah et al.\textsuperscript{3} investigated effect of magnetic field on fluid flow in a pipe. He has shown that the magnetic field reduces the axial flow velocity of the central stream of the pipe. Ajibade and Umar\textsuperscript{4} studied effects of chemical reaction and radiation absorption on an unsteady magnetohydrodynamics free convection flow in a vertical channel which is filled with porous materials. They concluded that the velocity profile increases with increase in permeability parameter of the porous materials. Nadeem et al.\textsuperscript{5} studied MHD flow of a three-dimensional casson fluid past a porous sheet. They found that the velocity profile decreases with increase in magnetic field and porosity parameters. Ajibade and Umar\textsuperscript{6} studied combined effects of diffusion-thermo and chemical reaction on an unsteady magnetohydrodynamics fluid past through an inclined channel in the presence of heat and radiation absorption. They found that the velocity profile decreases with increase in magnetic field while it increases with increase in permeability of the porous materials. They also found that the fluid motion decreases with increase in heat absorption.

The internal mechanical energy generated as a result of continuous interaction of fluid particles which is irreversibly converted to kinetic energy is referred to as viscous dissipation. Gebhart\textsuperscript{7} was the first researcher to investigate the effect of viscous dissipation on natural convection flow. He concluded that internally generated energy cannot be neglected in natural convection flow of high gravitational forces or a fluid of high Prandtl number. Recently, Dharmendar et al.\textsuperscript{8} studied MHD flow of a nanofluid and heat transfer over an exponentially stretching porous sheet. They concluded that the temperature profile increases with an increase in viscous dissipation Ec while suction decreases velocity and temperature profiles of the fluid. Sibanda and Makinde\textsuperscript{9} examined an investigation on steady MHD flow and heat transfer through a rotating disk in porous medium in the presence of Ohmic heating and viscous dissipation. It was found that the thermal energy of the fluid increases with increasing Ec and magnetic field M while the velocity profile decreases with increase in M. It’s also concluded in the works of Sarkar et al.,\textsuperscript{10} Endalew and Sarkar,\textsuperscript{11} and Seth et al.\textsuperscript{12,13} that the velocity distributions decrease with increase in magnetic field. MHD effects on heat transfer over a stretching sheet which is embedded in porous medium were studied by Hunegnaw and Naikoti.\textsuperscript{14} They concluded that increase in Eckert number enhances the temperature distribution, whereas an increase in Pr decreases the fluid temperature. They also concluded that temperature profile increases with increasing permeability parameter while the reverse case was observed for velocity profile. Mohamed\textsuperscript{15} investigated flow of a micropolar fluid over a stretching surface in the presence of viscous dissipation. He concluded that the temperature profile increases with increase in Eckert number ($Ec>0$) but decreases when the fluid is being cooled ($Ec<0$).

Injection simply means administering a fluid into a system as in the case of blood transfusion while suction is the removal of fluid from a system. If the two happen at the same time, then the opposite sides of the plates are porous which allow coming in and out of the moving fluid. Suction/injection of fluid in channels has received a concern due to its application in science and engineering, food processing industries, cooling of electrical appliances, petroleum drilling industries etc. Jha et al.\textsuperscript{16} analysed the effects of suction/injection and wall surface on natural convection flow in a vertical micro-porous channel. They concluded that increase in suction/injection led to decrease in velocity and temperature profiles. Uwanta and Hamza\textsuperscript{17} numerically studied the effects of suction/injection on an unsteady convective flow of viscous reactive fluid past through a vertical porous channel while Falade et al.\textsuperscript{18} found an exact solution of a Magnetohydrodynamics flow through a porous channel saturated with porous medium. The effects of suction/injection on a steady mixed convection flow through a vertical channel were investigated by Jha and Aina.\textsuperscript{19} They found that suction/injection has effects on the micro-porous-channel surfaces. Jha et al.\textsuperscript{20} studied a transient case of hydromagnetic-free convection flow in the presence of suction/injection and found that fluid velocity decreases with increase in suction/injection.

The aim of this research is to analyse the effects of viscous dissipation, suction/injection and magnetic field on a fluid passing through a porous channel. The applications of these fluid properties as they affect the hydrodynamic, thermodynamics, rate of heat transfer and shear stress between the porous plates and the fluids could be found in lubrication industries, food processing and food preserving industries, cooling of electric appliances, drilling of petroleum products, etc. When suction/injection, magnetic field, viscous dissipation are nullified and making the permeability of the porous materials so large, the work of Jha and Ajibade\textsuperscript{1} has been recovered.

**Mathematical analysis**

We consider a fully developed Laminar flow of an incompressible viscous fluid passing through vertical parallel porous plates channel (see the Figure 1). The plates are porous giving rise to uniform suction with velocity $\lambda$ through one porous plate with simultaneous injection through the other plate. One of the plates is heated and moving with a constant velocity $U$ while the other is stationary and kept at ambient
temperature. The flow is subjected to uniform transverse magnetic field in the presence of thermal buoyancy effects. All the fluid properties are assumed to be constants. The effects of variation in density with temperature have been considered only in the body-forced term. The flow variables are functions of space \( y^* \) only. The mathematical model that captures the flow formation and heat transfer in a vertical channel filled with saturated porous materials is formulated as:

\[
\nu \frac{d^2 u^*}{dy^*2} - \lambda^* \frac{du^*}{dy^*} - \left( \frac{\alpha B_0^2}{\rho} + \frac{\nu}{K^*} \right) u^* + gB(T^* - T_0) = 0
\]

(1)

\[
\frac{k}{\rho c_p} \frac{d^2 T^*}{dy^*2} - \lambda^* \frac{dT^*}{dy^*} - \frac{Q_0}{\rho c_p} (T^* - T_0) + \frac{\nu}{c_p} \left( \frac{du^*}{dy^*} \right)^2 = 0
\]

(2)

and the boundary conditions of the model are:

\[
\begin{align*}
    u^* &= U, & T^* &= T_w, & y^* &= 0 \\
    u^* &= 0, & T^* &= T_0, & y^* &= h
\end{align*}
\]

(3)

The first, second, third and fourth terms of equation (1) are viscosity, suction/injection, magnetic/permeability and thermal buoyancy effects of the fluid respectively. The first, second, third and fourth terms of equation (2) are the thermal diffusivity, suction/injection, heat generation/absorption and viscous dissipation effects of the fluid respectively.

The dimensionless quantities used are:

\[
    u = \frac{u^*}{U}, \quad y = \frac{y^*}{h}, \quad T = \frac{T^* - T_0}{T_w - T_0}, \quad \lambda = \frac{\lambda^* h}{\nu}
\]

(4)

Using the dimensionless quantities above, equations (1)–(2) are transformed in dimensionless form as:

\[
\frac{d^2 u}{dy^2} - \lambda \frac{du}{dy} - \left( M + \frac{1}{K} \right) u + GrT = 0
\]

(5)

\[
\frac{d^2 T}{dy^2} - \lambda \Pr \frac{dT}{dy} - ST + EcPr \left( \frac{du}{dy} \right)^2 = 0
\]

(6)

And the boundary conditions are:

\[
\begin{align*}
    u &= 1, & T &= 1, & y &= 0 \\
    u &= 0, & T &= 0, & y &= 1
\end{align*}
\]

(7)

Where \( \lambda \) is the suction/injection parameter, \( Gr \) is the thermal Grashof number, \( M \) is the magnetic field parameter, \( K \) is the permeability parameter, \( Pr \) is the Prandtl number, \( Ec \) is Eckert number and \( S \) is heat source/sink parameter.

\[
Gr = \frac{g \beta (T_w - T_0) h^3}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}, \quad S = \frac{Q_0 h^2}{k},
\]

\[
Ec = \frac{U^2}{c_p (T_w - T_0)}, \quad M = \frac{\sigma B_0^2 h^2}{\nu p}, \quad K = \frac{K^*}{h^2}
\]

**Method of solution**

**Basic idea on homotopy perturbation method**

The momentum and energy equations in the present work are coupled and nonlinear. To obtain a closed form solution of the model is not an easy task. Various solution methods have been derived for such problems ranging from perturbation methods, numerical solutions and several other approximate solution techniques. One of such approximate techniques which was proposed by He\textsuperscript{21} is the homotopy perturbation method. The method is simple, effective and convenient to solve nonlinear and coupled boundary value problems. In addition, the limitation of small parameter that characterised the regular perturbation was overcame by the homotopy perturbation technique. He\textsuperscript{21–24} presented the new method by considering the nonlinear differential equation.

To solve the problem by the homotopy perturbation method, we construct the convex homotopy of the momentum and energy equations. Therefore, in the absence of initial approximation \( v_0 \), the equation becomes

\[
\frac{d^2 u}{dy^2} = p \left[ \lambda \frac{du}{dy} + \left( M + \frac{1}{K} \right) u - GrT \right]
\]

(8)

such that

\[
    u = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + ... \\
    T = u_0 + pT_1 + p^2 T_2 + p^3 T_3 + ...
\]
Substituting equation (9) into equation (8), we have

\[
\frac{d^2 u_0}{dy^2} + p \frac{d^2 u_1}{dy^2} + p^2 \frac{d^2 u_2}{dy^2} + \ldots = p\lambda \frac{du_0}{dy} + p^2 \lambda \frac{du_1}{dy} + \\
+ p^3 \lambda \frac{du_2}{dy} + \ldots + p \left( M + \frac{1}{K} \right) u_0 \\
+ p^2 \left( M + \frac{1}{K} \right) u_1 + p^3 \left( M + \frac{1}{K} \right) u_2 + \ldots \\
- pGrT_0 - p^2 GrT_1 - p^3 GrT_2 + \ldots
\]

(10)

Comparing the coefficients of \( p^0, p^1, p^2, p^3, \ldots \)

\[
p^0 : \frac{d^2 u_0}{dy^2} = 0
\]

(11)

\[
p^1 : \frac{d^2 u_1}{dy^2} = \lambda \frac{du_0}{dy} + \left( M + \frac{1}{K} \right) u_0 - GrT_0
\]

(12)

\[
p^2 : \frac{d^2 u_2}{dy^2} = \lambda \frac{du_1}{dy} + \left( M + \frac{1}{K} \right) u_1 - GrT_1
\]

(13)

\[
p^3 : \frac{d^2 u_3}{dy^2} = \lambda \frac{du_2}{dy} + \left( M + \frac{1}{K} \right) u_2 - GrT_2
\]

(14)

The boundary conditions are transformed as

\[
u_0(0) = 1, u_1(0) = u_2(0) = u_3(0) = \ldots = 0
\]

(15)

\[
u_0(1) = u_1(1) = u_2(1) = u_3(1) = \ldots = 0
\]

Similarly, since no initial approximation for equation (6). Therefore, it is transformed as

\[
\frac{d^2 T}{dy^2} = p \left[ ST - EcPr \left( \frac{du}{dy} \right)^2 \right]
\]

(16)

Substituting equation (9) into equation (16), we have

\[
\frac{d^2 T_0}{dy^2} + p \frac{d^2 T_1}{dy^2} + p^2 \frac{d^2 T_2}{dy^2} + p^3 \frac{d^2 T_3}{dy^2} + \ldots
\]

\[
= p\lambda Pr \frac{dT_0}{dy} + p^2 \lambda Pr \frac{dT_1}{dy} + p^3 \lambda Pr \frac{dT_2}{dy} + \ldots
\]

\[
- p\lambda Pr \frac{dT_0}{dy} + p\lambda Pr \frac{dT_1}{dy} + p^2 \lambda Pr \frac{dT_2}{dy} + \ldots
\]

\[
- p\lambda Pr \frac{dT_0}{dy} + p\lambda Pr \frac{dT_1}{dy} + \ldots
\]

\[
- p \left( M + \frac{1}{K} \right) u_0 - p\lambda Pr \frac{dT_0}{dy} + p\lambda Pr \frac{dT_1}{dy} + \ldots
\]

\[
- p \left( M + \frac{1}{K} \right) u_0 - p\lambda Pr \frac{dT_0}{dy} + p\lambda Pr \frac{dT_1}{dy} + \ldots
\]

\[
- p\lambda Pr \frac{dT_0}{dy} + p\lambda Pr \frac{dT_1}{dy} + \ldots
\]

Comparing the coefficients of \( p^0, p^1, p^2, p^3, \ldots \)

\[
p^0 : \frac{d^2 T_0}{dy^2} = 0
\]

(18)

\[
p^1 : \frac{d^2 T_1}{dy^2} = \lambda Pr \frac{dT_0}{dy} + ST_0 - EcPr \left( \frac{du_0}{dy} \right)^2
\]

(19)

\[
p^2 : \frac{d^2 T_2}{dy^2} = \lambda Pr \frac{dT_1}{dy} + ST_1 - 2EcPr \frac{du_0 du_1}{dy}
\]

(20)

\[
p^3 : \frac{d^2 T_3}{dy^2} = \lambda Pr \frac{dT_2}{dy} + ST_2 - 2EcPr \frac{du_0 du_2}{dy} - EcPr \left( \frac{du_1}{dy} \right)^2
\]

(21)

The boundary conditions are transformed as

\[
T_0(0) = 1, T_1(0) = T_2(0) = T_3(0) = \ldots = 0
\]

(22)

\[
T_0(1) = T_1(1) = T_2(1) = T_3(1) = \ldots = 0
\]

solving from above equations (11) and (18),

\[
u_0 = A_1 y + A_2 \]

(23)

\[T_0 = B_1 y + B_2 \]

(24)

Applying the boundary conditions \( u_0(0) = 1, u_0(1) = 0 \) and \( T_0(0) = 1, T_0(1) = 1 \)

\[A_1 = -1, A_2 = 1, B_1 = -1, B_2 = 1 \]

\[
T_0 = 1 - y
\]

(25)

\[T_0 = 1 - y \]

(26)

Solving equations (12) and (19), we have

\[
u_1 = -\lambda \frac{y^2}{2} + \left( M + \frac{1}{K} \right) \left[ \frac{y^2}{2} - \frac{y^3}{6} \right] - Gr \left[ \frac{y^2}{2} - \frac{y^3}{6} \right] + A_3 y + A_4
\]

(27)

\[
T_1 = -\lambda Pr \frac{y^2}{2} + S \left[ \frac{y^2}{2} - \frac{y^3}{6} \right] - EcPr \frac{y^2}{2} + B_3 y + B_4
\]

(28)

Applying the boundary conditions \( u_1(0) = 0, u_1(1) = 0 \) and \( T_1(0) = 0, T_1(1) = 0 \)

\[A_3 = \frac{\lambda}{2} + Gr \frac{1}{3} \left( M + \frac{1}{K} \right) \]

(29)

\[A_4 = 0 \]

\[B_3 = \frac{\lambda Pr}{2} + EcPr \frac{S}{2} - \frac{S}{3} \]

(30)

\[B_4 = 0 \]
Applying the boundary conditions $u_2(0) = 0$, $u_2(1) = 0$ and $T_2(0) = 0$, $T_2(1) = 0$

\[
A_5 = \frac{GrEcPr}{24} + \frac{GrS}{45} + \frac{GrApPr}{24} - \frac{Gr}{45} \left( \frac{M + 1}{K} \right)^2 - \frac{Gr\lambda}{24} - \frac{\lambda^2}{12} \\
A_6 = 0
\]

\[
B_5 = \frac{S^2}{12} - \frac{\lambda^2 Pr^2}{12} - \frac{EcPr^2 \lambda}{12} - \frac{EcPr S}{24} - \frac{Gr Ec Pr}{12}
\]

\[
B_6 = 0
\]

Therefore, the approximate solution of equations (5) and (6) are:

\[
u = u_0 + u_1 + u_2 + u_3 + \ldots
\]

\[
T = u_0 + T_1 + T_2 + T_3 + \ldots
\]

The physical quantities of interest are the skin friction and rate of heat transfer. Therefore the shear stress between the fluid at the heated and cold plate are given as:

\[
\tau_0 = -1 + \frac{\lambda}{2} + \frac{Gr}{3} - \frac{1}{3} \left( \frac{M + 1}{K} \right) + \frac{Gr Ec Pr}{24} - \frac{Gr S}{45} + \frac{Gr Ap Pr}{24} - \frac{Gr}{45} \left( \frac{M + 1}{K} \right)^2 - \frac{Gr\lambda}{24} - \frac{\lambda^2}{12}
\]

\[
\tau_1 = -1 - \frac{\lambda}{2} + \frac{1}{6} \left( \frac{M + 1}{K} \right) - \frac{Gr}{6} - \frac{\lambda^2}{12}
\]

while the rate of heat transfers between the fluid and the heated and cold plate are given by:

\[
Nu_0 = -1 + \frac{\lambda Pr}{2} + \frac{Ec Pr}{3} - \frac{S}{3} + \frac{S^2}{45} - \frac{\lambda^2 Pr^2}{12} - \frac{Ec Pr^2 \lambda}{12}
\]

\[
Nu_1 = -\frac{Ec Pr}{24} - \frac{Ec Pr}{6} + \frac{Ec Pr}{12} \left( \frac{M + 1}{K} \right) - \frac{Gr Ec Pr}{12}
\]
Validation

The present work has been validated by comparing with the work of Jha and Ajibade\textsuperscript{1} when the effects of viscous dissipation, channel plates porosity and transverse magnetic field are completely suppressed and the comparison which is presented in Table 1 is made possible when the permeability of the porous media is increased to the extent that its effect on the fluid flow is negligible. The table clearly shows an excellent agreement between the present problem and that of Jha and Ajibade.\textsuperscript{1}

Results and discussion

Effects of magnetic field, permeability parameter and suction/injection in the presence of viscous dissipation on an incompressible fluid passing through a vertical channel filled with porous materials were investigated. The working fluid is considered as air with ($\text{Pr} = 0.71$). The values of the governing flow parameters are carefully selected to agree with the previous researches. $\lambda > 0$ and $\lambda < 0$ signify injection and suction respectively while $S > 0$ and $S < 0$ represent heat sink and source respectively.

Figures 2 and 3 show the effects of ($Pr$) on velocity and temperature profiles respectively. The velocity profile increases with increase in ($Pr$). This is physically true since Prandtl number is the ratio of viscous diffusivity to thermal diffusivity rate. Prandtl number increases with increase in viscous diffusivity or decrease in thermal diffusivity rate. On the temperature

\begin{table}[h]
\centering
\caption{Comparison between the present work and Jha and Ajibade.\textsuperscript{1}}
\begin{tabular}{c|c|c|c|c}
\hline
\textbf{S} & \textbf{Jha and Ajibade}$^1$ & \textbf{Present Work} \\
& \textbf{Gr = 4, y = 0.5} & \textbf{Gr = 4, Pr = 0.71, y = 0.5,} \\
& & \textbf{M = 0, K = 10000, $\lambda = 0$, Ec = 0,} \\
\hline
\textbf{Velocity} & 0.7263622323059852 & 0.723957447917318 & 0.723957447917318 & 0.444010416666667 \\
\textbf{Temperature} & 0.443409441985037 & 0.444010416666667 & 0.444010416666667 & 0.444010416666667 \\
\hline
1 & 0.7263622323059852 & 0.443409441985037 & 0.723957447917318 & 0.444010416666667 \\
0.5 & 0.737609131457281 & 0.470298858567840 & 0.73698281250651 & 0.470377604166667 \\
-0.5 & 0.76371806093636 & 0.532964757624204 & 0.763019947917318 & 0.532877604166667 \\
-1 & 0.778987854649098 & 0.569746963662275 & 0.77604781250651 & 0.569010416666667 \\
\hline
\end{tabular}
\end{table}
distribution, the temperature of the fluid decreases near the heated plate with increase in Prandtl number while a reverse case was observed near the cold plate. The observed trend towards the heated plate is due to decrease in thermal diffusivity which reduces the diffusion of the applied heat into the system thereby causing a decrease in temperature as \( Pr \) increases. The influence of growing \( Pr \) towards the cold plate is to reduce the diffusion of heat generated due to viscous dissipation thereby causing heat accumulation towards the cold plate which resulted in thermal boundary layer thickness of the fluid to rise up thereby causes increase in the thermodynamics and hydrodynamics as well.

For different values of \( (S) \), Figures 4 and 5 were plotted for velocity and temperature profiles respectively. It has been seen that the velocity and temperature profiles decrease with increase in heat sink \( S > 0 \). This is physically true since, when heat is absorbed, the fluid becomes dense and the convection current is weakened, therefore the fluid temperature decreases which in return decreases the fluid flow as well. It is further observed that both the velocity and temperature profiles increase with increase in heat generation \( (S < 0) \). Increase in heat generation strengthens the convection current thereby decreases the density of the fluid which results to raise in temperature and thus increases the fluid velocity.

Figures 6 and 7 depict the velocity and temperature profiles respectively. Both Velocity and temperature
profiles increase with increase in viscous dissipation ($Ec$). This is due to the fact that, the increase in hydrodynamic and thermodynamics of the fluid is resulted by an increase in internal energy generated as a result of fluid particles’ interactions which leads to decrease in fluid density and increase in convection within the channel thereby causing increase in fluid flow and energy transfer easily.

Figures 8 and 9 display the influences of ($Gr$) on velocity and temperature profiles respectively. The velocity profile increases with increase in Grashof number ($Gr$). Increase in the thermal buoyancy results to increase in the convection current and decrease in fluid density within the channel. Figure 9 shows that the temperature profile decreases with an increase in ($Gr$) near the heated plate while a reverse case was observed near the cold plate.

Figures 10 and 11 show the effects of suction/injection parameter ($\lambda$) on velocity and temperature profiles. It is observed that the velocity and temperature profiles increase with increase in injection ($\lambda>0$) through the heated plate. This is physically true, since the existence of porosity of the plates, allows fluid flow through the system which eventually leads to increase in the fluid motion and the energy dissipated diffuses to other parts of the system which increases the fluid temperature as
well. It is also found that the temperature and velocity distributions within the channel decrease with increase in the suction ($\lambda < 0$) through the cold plate. This is due to the fact that heated fluid escapes through the heated plate and replaced with cold fluid through the cold plate causing a decrease in the temperature within the system which in turn weakens the convection current and decreases the hydrodynamics of the fluid.

For different values of magnetic field parameter ($M$), Figures 12 and 13 are displayed for velocity and temperature profiles respectively. It can be seen that the velocity profile decreases with increasing ($M$). This is due to the fact that the fluid velocity is retarded by a force called Lorenz force. On the other hand, temperature profile increases near the heated plate with increase in ($M$). This is due to the fact that, more energy is generated near the heated plate which strengthens the convection current and weakens the density of the fluid. It’s further observed that the temperature distribution reduces near the cold plate with increase in ($M$).

Figures 14 and 15 display the influences of the permeability of the porous material ($K$) on velocity and temperature distributions. The velocity distribution increases with increase in the permeability of the porous materials. This is physically true, since the resistance posed by porous materials weakens as the permeability is increased. The Figure 15 shows that the temperature
distribute near the heated plate with increase in the permeability of the porous materials while a reverse case was observed near the cold plate. The increase in temperature near the heated plate is caused as a result of foreign-heated-fluid injected through the heated plate which rises the temperature of the adjacent fluid.

Table 2 shows the shear stress between the fluid and the plates. It is found that the shear stresses on both the heated and cold plates decrease with increase in the magnetic field (M). This is physically true since increase in magnetic field increases the drag force which retards the fluid flow thereby causing reduction in the shear stress. It is also observed that the shear stresses on both the plates increase with increase in the permeability (K) of the porous materials. This is due to the fact that the increase in permeability increases more free flow of the fluid within the channel. It is further observed that the shear stresses on both the plates reduce with growing injection (λ > 0) while increase were observed on both the plates with growing suction (λ < 0).

Table 3 displays the rate of heat transfer between the plates and the fluid. The rate of heat transfer on both the plates decreases with increase in magnetic field (M) while a reverse case was observed for an increase in the permeability (K) of the porous materials. The reduction in the rate of heat transfer was associated with drag force caused by magnetic field which slows down the free flow of the fluid within the channel. However, when the permeability (K) is widened, there observed smooth flow of the fluid which results in the rate of heat transfer to occur rapidly between the fluid and the plates. It is also observed that the rate of heat transfer on the heated plate increases with increase in suction (λ < 0) while a reverse case was observed on the cold plate. It is further observed that the rate of heat transfer on the heated plate decreases with increase in the injection (λ > 0) while a reverse case was observed on the cold plate. This is due to the fact that, more energetic fluid particles from the heated domain move down to the cold one which raises the temperature of the fluid near the cold plate, thus increases the rate of heat transfer on the cold plate.

**Conclusion**

The present work theoretically investigated the effects of magnetic field, suction/injection and permeability of porous materials on a steady natural convection flow of a viscous incompressible heat generating/absorbing fluid passes through a porous vertical channel. The work concluded that the velocity and temperature profiles increase with increase in Prandtl number Pr, Eckert number Ec, Grashof number Gr, while the temperature profile decreases with increase in heat absorption (S > 0). It is also concluded that velocity profile decreases with increase in Pr and Ec near the cold plate. The shear stresses on both the plates increase with increase in the permeability (K) of the porous materials. If the viscous
dissipation $Ec = 0$, suction/injection $\lambda = 0$, magnetic field $M = 0$ and making the permeability parameter $K$ so large, the work of Jha and Ajibade$^1$ has been recovered.

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### Appendix

#### Notation

| Symbol | Description |
|--------|-------------|
| $\lambda^*$ | Dimensional suction/injection parameter |
| $u^*$ | Dimensional velocity |
| $y^*$ | Dimensional distance between the plates |
| $\nu$ | Kinematic viscosity |
| $B_0$ | Magnetic induction |
| $\rho$ | Fluid density |
| $K^*$ | Dimensional permeability of the porous material |
| $g$ | Acceleration due to gravity |
| $\beta$ | Thermal expansion coefficient |
| $T^*$ | Dimensional temperature |
| $k$ | Thermal conductivity of the fluid |
| $\sigma$ | Electrical conductivity of the fluid |
| $c_p$ | Specific heat at constant pressure |
| $Q_0$ | Heat absorption coefficient |
| $T_0$ | Temperature at the cold plate |
| $T_w$ | Temperature at the heated plate |
| $h$ | Channel width |
| $U$ | Velocity of the heated plate |