Enhancement of $\text{Br}(B_d \to \mu^+ \mu^-)/\text{Br}(B_s \to \mu^+ \mu^-)$ in Supersymmetric Unified Models

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We explain the 2.3σ deviation in the recent measurements of the neutral $B$ mesons decay into muon pairs from the standard model prediction in the framework of supersymmetric grand unified models using anti-symmetric coupling as a new source of flavor violation. We show a correlation between the $B_d \to \mu^+ \mu^-$ decay and the CP phase in the $B_d \to J/\psi K$ decay and that their deviations from the standard model predictions can be explained after satisfying constraints arising from various hadronic and leptonic rare decay processes, $B$-$B$, $K$-$K$ oscillations data and electric dipole moments of electron and neutron. The allowed parameter space is typically represented by pseudoscalar Higgs mass $m_A \leq 1$ TeV and $\tan \beta H(\equiv v_u/v_d) \lesssim 20$ for squark and gluino masses around 2 TeV.

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INTRODUCTION

The recent measurements of the branching fractions of the rare $B$ meson decays, $B_d^0 \to \mu^+ \mu^-$ and $B_d^0 \to \mu^+ \mu^-$, showcase impressive achievements of the LHC experiments \cite{1}. The ratios of the experimental measurements and the standard model (SM) predictions are

\begin{align*}
\text{Br}(B_s \to \mu^+ \mu^-)_{\exp/\text{SM}} &= 0.76^{+0.20}_{-0.18}, \quad (1) \\
\text{Br}(B_d \to \mu^+ \mu^-)_{\exp/\text{SM}} &= 3.7^{+1.6}_{-1.8}. \quad (2)
\end{align*}

Both measurements are consistent with the SM predictions within the errors, though $\text{Br}(B_d \to \mu^+ \mu^-)$ seems to be a bit larger than the SM prediction. The ratio of the fractions,

$$R \equiv \frac{\text{Br}(B_d \to \mu^+ \mu^-)}{\text{Br}(B_s \to \mu^+ \mu^-)},$$

has less theoretical errors in the SM compared to each fraction, and the prediction for the SM (and for the models with minimal flavor violation [2]) is $R \simeq \frac{\tau_{B_d} M_{B_d} f_{B_d}^2 |V_{ub}|^2}{\tau_{B_s} M_{B_s} f_{B_s}^2 |V_{ub}|^2}$, where $\tau_B$, $M_B$, and $f_B$ are the lifetime, mass, and decay constant of the respective mesons, and

$$R_{\text{SM}} = 0.0295^{+0.0028}_{-0.0025}, \quad R_{\exp} = 0.14^{+0.08}_{-0.06}. \quad (4)$$

The experimental result shows deviation from the SM prediction at 2.3σ.

These rare decays are induced radiatively in the SM, and they are sensitive to the new physics, and their measurements provide us a direction in which the SM can be extended to the models beyond SM. The deviation from the SM prediction is not very significant statistically at present, however, it is meaningful to investigate the models which can enhance the ratio $R$ since the usual source of flavor chaining neutral currents (FCNC) does not produce any enhancement. In this paper, we suggest a possible source to explain the enhancement of the ratio naturally in the supersymmetric (SUSY) standard model, and investigate the implications of the models in the possible unified frameworks, such as SU(5) and SO(10) grand unified models.

In order to modify the ratio $R$ compared to the SM prediction, one needs a new type of FCNC source which is different from Cabibbo-Kobayashi-Maskawa (CKM) flavor mixings. The main concerns regarding the enhancement of $R$ are the followings:

1. How natural is that to have a larger $b \to d$ transition compared to the $b \to s$ transition in the presence of a new FCNC source to enhance $R$? In fact, the $B_s$-$B_d$ mixings and the $B_s \to J/\psi \phi$ decay are consistent with the SM predictions and it seems that there is not a large source of FCNC in the $b \to s$ transition.

2. The mass difference of $B_d$ from the measurements of $B_d$-$B_s$ mixings and the $B_d \to J/\psi K$ decay are consistent with SM. How can large modifications of them be avoided if the $\text{Br}(B_d \to \mu^+ \mu^-)$ is enhanced by a new FCNC source? The SM prediction of $\sin 2\beta$ has a slight difference from the experimental measurements from the $B_d \to J/\psi K$ decay [3]:

$$\sin 2\beta = 0.692^{+0.029}_{-0.018} \text{ (exp)}, \quad (5)$$
$$\sin 2\beta = 0.774^{+0.017}_{-0.036} \text{ (SM prediction)}. \quad (6)$$

Can the slight difference be consistent with a modification of $R$?

3. The $b \to s\gamma$ and $b \to d\gamma$ decays are consistent with the SM prediction. The ratio $\text{Br}(b \to d\gamma)/\text{Br}(b \to s\gamma)$ is also related to $|V_{td}/V_{ts}|^2$ up to hadronic uncertainty [4]. The ratio, by using the experimental measurements [3, 6], is $\text{Br}(b \to d\gamma)/\text{Br}(b \to s\gamma) = 0.040 \pm 0.009 \pm 0.010$. How natural can the enhancement of $B_d \to \mu^+ \mu^-$ be without enhancing $b \to d\gamma$?
FCNC INDUCED BY ANTI-SYMMETRIC COUPLINGS

In SUSY models, too much FCNCs are generated in general, and thus, the flavor universality of the SUSY breaking mass parameters are often assumed. In such a framework, the renormalization group evolution can generate off-diagonal elements in the fermion mass matrices and FCNCs are induced. If the CKM quark mixing is the only source to generate the off-diagonal elements, $R$ is not modified even though the individual branching fractions are modified. In unified models, new particles can propagate in the loops and it can generate a new type of flavor violation source \[ \frac{a}{b} \]. In simple models, this new FCNC source can induce $b\to s$ transitions, and thus, the ratio $R$ is rather reduced. In this paper, we suggest a model to provide a possible explanation for enhancing $R$.

We consider the following anti-symmetric coupling matrix $h'$ under the flavor indices for the left-handed quark doublet $q$:

$$ h'_{ij} g_{ij}(3, 3, -\frac{1}{3}), \text{ or } h'_{ij} g_{ij}(6, 1, -\frac{1}{3}). $$

(7)

The Yukawa coupling $h'$ can induce off-diagonal elements in the left-handed squarks mass matrices by RGE in the form of $\alpha - h'h'^t(3m_0^2 + A_0^2)/(8\pi^2)$, where $m_0$ is a universal scalar mass and $A_0$ is a universal scalar trilinear coupling. Denoting

$$ h' = \begin{pmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{pmatrix}, $$

(8)

one obtains

$$ h'h'^t = \begin{pmatrix} |a|^2 + |b|^2 & -bc^* & -ac^* \\ -bc^* & |a|^2 + |c|^2 & -ab^* \\ -ac^* & -ab^* & |b|^2 + |c|^2 \end{pmatrix}. $$

(9)

We then find interesting features in the off-diagonal elements arising from the anti-symmetric coupling:

1. In the case of a naive hierarchy $|b| < |c|$ in the $h'$ coupling, one obtains an inverted hierarchy in the off-diagonal elements using the RGEs since $|\langle h'h'^t \rangle_{13}| > |\langle h'h'^t \rangle_{23}|$, and thus it can be expected that the $b\to d$ FCNC is larger than the $b\to s$ FCNC.

2. The magnitudes of two out of three off-diagonal elements (12, 13 and 23) in $h'h'^t$ are correlated. For example, if 12 element is zero, one of 13 and 23 elements of $h'h'^t$ is zero. One can easily enhance $b \to d$ transition (but not $b \to s$) after satisfying $K\bar{K}$ data.

These two features nicely explain how $R$ is enhanced naturally using the RGE-induced FCNC. In order to illustrate these features, we plot $B_d \to \mu^+\mu^-$ vs $B_s \to \mu^+\mu^-$ by imposing the anti-symmetric coupling $h'$ (Fig.1). We use universal scalar masses for squark and slepton fields, $m_0 = 2$ TeV and the unified gaugino mass, $m_{1/2} = 1$ TeV. The universal scalar trilinear coupling $A_0$ is chosen to make the Higgs mass to be 125 GeV ($A_0 \simeq 5$ TeV). However, we use non-universal SUSY breaking Higgs masses, and we choose the Higgsino mass $\mu = 3$ TeV and the CP odd Higgs mass $m_A = 1$ TeV. The ratio of the Higgs vacuum expectation values, $\tan \beta_H$ is chosen not to be very large (here we use $\tan \beta_H = 20$) to make it consistent with the experimental measurement. We will explain the reason for these choices later. In this plot, the naive hierarchies among $a$, $b$ and $c$ (such as $|a|, |b| < |c|$) is not assumed to illustrate the second feature. As it is expected, the green points (which correspond to the choice of small 12 off-diagonal element) appear like a cross or $\dagger$ symbol. We note that even in the case where there is no new 23 FCNC source, $B_s \to \mu^+\mu^-$ is enhanced due to the chargino contribution.

**Constraints from various FCNC processes**

The $B_d\bar{B}_d$ mixing amplitude is given as

$$ M_{12} = M^\text{SM}_{12} + M^\text{SUSY}_{12}, $$

and the mass difference of the $B_d$ meson and a CP phase of $B_d \to J/\psi K$ decay are obtained as $\Delta M = 2|M_{12}|$, and $2\beta = \arg M_{12}$. To satisfy the experimental results, $|M^\text{SUSY}_{12}|$ has to be sufficiently small, but a contribution of small size can be expected due to a slight discrepancy of the sin2$\beta$ measurement in Eqs. (8), (9).

The SUSY contributions to the $B \to \mu^+\mu^-$ amplitudes are dominated by the Higgs-penguin diagram \[ 10 \]. Those amplitudes can be enhanced (compared to the SM amplitude) even if there is only left-handed quark FCNCs arising due to the off-diagonal elements in the squark mass matrices. On the other hand, the SUSY contributions to the $B\bar{B}$ mixing amplitudes $M^\text{SUSY}_{12}$ are enhanced (especially by the double Higgs-Penguin mediated
The fields $\left(3,3,-\frac{1}{3}\right)$, $\left(\bar{3},1,-\frac{1}{3}\right)$ which provide the anti-symmetric couplings to the left-handed quark doublets can be unified with the Higgs representations in unified theories, such as SU(5) and SO(10). In SU(5),

$$10 \times 10 = 5_s + 45_a + 50_s,$$

and in SO(10),

$$16 \times 16 = 10_s + 120_a + 126_s,$$

where $s$ and $a$ stand for symmetric and anti-symmetric, respectively. Therefore, the Yukawa couplings with 45 in SU(5) and 120 in SO(10) can provide the anti-symmetric sources for the FCNCs.
In Table 1, we list five possible bi-fermion couplings to the Higgs representations for the anti-symmetric couplings to generate the off-diagonal elements in the left-handed squarks in SU(5), flipped-SU(5) (whose gauge symmetry is SU(5) × U(1)X) and SO(10). If the Higgs representation is not 45 (for example, 10 for (3, 2, −1)), the Yukawa coupling is not anti-symmetric, and one should choose another gauge symmetry in the same row of the list.

As described, the right-handed FCNC should be small due to the constraints arising from the $B_s - B_d$ mixing amplitudes. Since each Higgs representation has a conjugate representation which can also have bi-fermion coupling, we list the corresponding conjugate bi-fermion couplings. The conjugate bi-fermion couplings include the right-handed down-type quark $d^c$, baryon $(3, 2, -\frac{1}{2})$ and $(\bar{3}, 2, -\frac{3}{2})$. In SU(5) or flipped-SU(5) model, the conjugate bi-fermion couplings are $10 \cdot 45 \cdot 45$ and they are not necessarily unified to the anti-symmetric coupling, and the right-handed FCNC can be free in principle. In SO(10), the conjugate coupling matrices are unified to the 120 Higgs coupling, and thus, the antisymmetric couplings are naively given by $(3, 3, -\frac{1}{2})$ and $(\bar{3}, 2, -\frac{3}{2})$ reps in SO(10) model. However, since the same reps. are included in $\mathbf{T}_{26}$ and 126 $(45 \subset 126, \bar{T}_{3} \subset T_{26})$, and $45 + \bar{45} \subset 120$ and they can mix, the linear combination of the light fields to generate the FCNC can be different between the conjugate and unconjugate reps., by adjusting the $\lambda_{120} \cdot 126 \cdot 210$ and $\lambda_{120} \cdot \bar{T}_{26} \cdot 210$ couplings using $\lambda \gg \lambda$. Therefore, in general, all the five cases are possible in SO(10).

The $(3, 3, -\frac{1}{2}) + c.c.$ can give rise to the proton decay operator $qqq\ell$, and it may not be a good choice to make this rep. light to generate FCNC. In flipped-SU(5), $(\bar{3}, 2, -\frac{3}{2}) + c.c.$ have the same quantum numbers as the would-be-Goldstone modes to be eaten by the SU(5) gauge bosons. Therefore, in the flipped-SU(5)-like vacua in SO(10), this rep. can be a good candidate to generate the FCNC. We point out that $(\bar{T}, 1, -\frac{1}{3})$ and $(8, 2, \frac{1}{2})$ reps. are good candidates to increase the unification scale and relax the bound due to the proton lifetime [4].

We note that the 45 rep. in SU(5) and 120 rep. in SO(10) contain the MSSM Higgs doublets and the anti-symmetric coupling is a part of the linear combination of the Yukawa couplings to generate the quark and lepton masses. Since the mixings of the doublets are multiplied in the linear combination of the Yukawa couplings, the original anti-symmetric couplings can be $O(1)$ and can provide large off-diagonal elements via RGE.

The enhancement of the ratio $R$ can impact the $t \rightarrow u\gamma(g)$ and $\tau \rightarrow e\gamma$ decays rather than the $t \rightarrow c\gamma(g)$ and $\tau \rightarrow \mu\gamma$ decay processes. However, both $t \rightarrow c\gamma(g)$ and $t \rightarrow u\gamma(g)$ branching fractions are tiny due to the current bound on gluino and squark masses, and it will be hard to observe them. The $\tau \rightarrow e\gamma$ process can be generated for $(3, 3, -\frac{1}{2})$ and $(\bar{3}, 2, -\frac{3}{2})$ couplings. If the 13 off-diagonal element in the left-handed slepton mass matrix is turned on, the chargino loop contribution can generate the branching fraction of $\tau \rightarrow e\gamma$ to be several times $10^{-9}$, by using the parameters from Figs. 1 and 2.

The other impact of 13 generation mixings is that they can generate neutron and electron electric dipole moments (EDM). The up and down quark EDM can be induced from the chargino diagrams due to the 13 off-diagonal elements in left-handed squark mass matrix. Since $V_{ub}$ or $V_{ud}$ is multiplied to the amplitude, the SUSY contribution is not very large ($\sim 10^{-28}\text{ e\cdot cm}$), but it is much larger than the SM predictions. If there are 13 off-diagonal elements in both left- and right-handed squark mass matrices due to the $(8, 2, \frac{1}{2})$ FCNC source, the amplitude can be much enhanced by a gluino diagram, and it can make the neutron EDM comparable to the current experimental bound, $|d_n| < 2.9 \times 10^{-26}\text{ e\cdot cm}$ [13]. The electron EDM can also be enhanced by a neutralino diagram (for Bino components), if FCNC contributions arise from both left- and right-handed charged-slepton mass matrices induced by $(3, 2, -\frac{1}{6}) + c.c.$ couplings, and it can be comparable to the current experimental bound $|d_e| < 8.7 \times 10^{-29}\text{ e\cdot cm}$ [14].

CONCLUSION

The anti-symmetric Yukawa interaction as a new source of FCNC can explain the enhancement for the ratio of the branching fractions $Br(B_d \rightarrow \mu^+\mu^-)/Br(B_s \rightarrow \mu^+\mu^-)$ and the deviation of the experimental result from the SM prediction of the CP phase in the $B_d \rightarrow J/\psi K$ decay. The new interactions can be described by grand unified models, e.g., SO(10), SU(5), flipped SU(5) etc. The enhancement of the ratio and natural realization of the $b \rightarrow d\gamma$ and $b \rightarrow s\gamma$ data force the choice of the CP odd Higgs mass to be less than 1 TeV and $\tan\beta_H \lesssim 20$ for squark and gluino masses to be around 2 TeV. The allowed parameter space satisfies constraints arising from various hadronic and leptonic rare decay processes, $\bar{B} - B$, $K - K$ oscillations data and electric dipole moments of electron and neutron.

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