Impurity-induced stabilization of Luttinger liquid in quasi-one-dimensional conductors

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(Dated: March 22, 2022)

Abstract

It is shown theoretically that the Luttinger liquid phase in quasi-one-dimensional conductors can exist in the presence of impurities in a form of a collection of bounded Luttinger liquids. The conclusion is based upon the observation by Kane and Fisher that a local impurity potential in Luttinger liquid acts, at low energies, as an infinite barrier. This leads to a discrete spectrum of collective charge and spin density fluctuations, so that interchain hopping can be considered as a small parameter at temperatures below the minimum excitation energy of the collective modes. The results are compared with recent experimental observation of a Luttinger-liquid-like behavior in thin NbSe$_3$ and TaS$_3$ wires.

PACS numbers: 71.10.Pm, 71.10.Hf, 71.27.+a, 71.45.Lr
Electronic properties of one-dimensional (1D) metals are known to be very different from those of ordinary three-dimensional (3D) metals (for a review see Ref.[1, 2, 3]). 3D electron gas is well described by Landau’s Fermi-liquid picture in which interaction modifies free electrons making them quasiparticles that behave in many respects like non-interacting electrons. In contrast to the 3D case, in 1D electronic systems the Fermi-liquid picture breaks down even in the case of the arbitrarily weak interaction. In 1D metals, the single-electron quasiparticles do not exist, and the only low energy excitations turn out to be charge and spin collective modes with the sound-like spectrum. These modes are dynamically independent giving rise to a spin-charge separation in 1D systems. Furthermore, correlation functions at large distances and times decay as a power law with interaction dependent exponents. Such a behavior has been given a generic name Luttinger liquid [4].

The concept of Luttinger liquid is of great interest in view of its application to real physical systems, such as carbon nanotubes and semiconductor heterostructures with a confining potential (quantum wires and quantum Hall effect edge states). The case of a special interest are quasi-1D conductors, i.e., highly anisotropic 3D conductors with chain-like structure. Numerous experimental studies of both organic and inorganic q1D conductors at low temperatures typically demonstrate broken-symmetry states, like superconductivity, spin- or charge-density wave (CDW) states, and a metallic behavior above the transition temperature with non-zero single-particle density of states at Fermi energy. For instance, the most studied inorganic quasi-1D metals undergo the Peierls transition from metallic state either to a semiconducting CDW state (e.g., blue bronze K$\text{$_0$$\text{$_3$$\text{MoO}_3$}$, TaS$_3$, (TaSe$_4$)$_2$I etc.) or to semimetallic CDW state (NbSe$_3$) [5]. Typically, this transitions occur in the temperature range 50 - 250 K. From the theoretical point of view, the formation of Luttinger liquid in quasi-1D conductors at low enough temperatures is also problematic because of the instability towards 3D coupling in the presence of arbitrarily small interchain hopping [6, 7, 8, 9, 10]. So the interchain hopping induces a crossover to 3D behavior at low energies, while Luttinger liquid behavior can survive only at high enough energy scale where it is not affected by 3D coupling.

In contrast to the interchain hopping, the Coulomb interaction between the electrons at different chains does not destroy the Luttinger liquid state, the main difference from the 1D case being the absence of simple scaling relations between the exponents of the various correlation functions [11, 12, 13, 14].
However, in recent experimental studies of temperature and field dependence of conductivity of TaS$_3$ and NbSe$_3$ in nanoscale-sized crystals a transition from room-temperature metallic behavior to nonmetallic one accompanied by disappearance of the CDW state at temperatures below 50 - 100 K was observed [15, 16, 17]. The low temperature non-metallic state was characterized by power law dependencies of the conductivity on voltage and temperature like that expected in Luttinger liquid, or by more strong temperature dependence corresponding to the variable-range hopping. Resembling dependencies of conductivity were reported although in focused-ion beam processed or doped relatively thick NbSe$_3$ crystals [17]. Hopping conductivity was also detected in dirty quasi-1D conductors KCP and organic TCNQ-based metals [18], while pure materials are known to undergo the Peierls transition to the CDW state.

In order to account for such behavior, we study the possibility of impurity-induced stabilization of a gapless Luttinger liquid state in quasi-1D metals. Impurities in Luttinger liquid are known to act as infinite barriers forming the effective boundaries for low energy excitations [19, 20, 21]. This leads to a dimensional quantization and, consequently, to a minimal excitation energy $\omega_1$. As a result, the interchain hopping does not destroy the Luttinger liquid phase at temperatures $T \ll \omega_1$, producing only small perturbations of the 1D picture. To show this, we consider first the gapless 1D Tomonaga-Luttinger (TL) model with impurity potential included, and make certain that the system with impurities breaks up into a set of independent segments described as bounded Luttinger liquid with discrete spectrum. Then we calculate corrections caused by the interchain hopping to thermodynamic potential and to the one-particle Green’s function, and find that such corrections are small at low temperatures. Finally, we discuss modifications introduced by generalization of the TL model to the more realistic case of Coulomb potential and compare our results with experimental data.

First of all we start with the TL model ignoring interchain hopping integral $t_\perp$ in order to formulate the problem in the zeroth approximation in $t_\perp$. Electronic operators for right ($r = +1$) and left ($r = -1$) moving electrons with spin $s$ are given in terms of phase fields as (see Ref.[1, 3])

$$\psi_{r,s}(x) = \lim_{\alpha \to 0} \frac{e^{i r k_F x}}{\sqrt{2\pi \alpha}} \eta_{r,s} e^{i A_r}, \quad A_r = \frac{1}{\sqrt{2}} [\Theta_\rho - r \Phi_\rho + s(\Theta_\sigma - r \Phi_\sigma)] \quad (1)$$

here phase fields $\Phi_\nu(x)$ are related to charge ($\nu = \rho$) and spin ($\nu = \sigma$) densities, while fields
\( \Theta_\nu(x) \) are related to the momentum operators \( \Pi_\nu = (1/\pi)\partial_x \Theta_\nu \) canonically conjugate to \( \Phi_\nu \). Further, \( \eta_{r,s} \) are Majorana (“real”) Fermionic operators that assure proper anticommutation relations between electronic operators with different spin \( s \) and chirality \( r \), and the cut off length \( \alpha \) is assumed to be of the order of interatomic distance.

We describe the intrachain properties of the system by the standard TL Hamiltonian [1, 3] with added 2\( k_F \) impurity backscattering term [2]. In the bosonized form it reads

\[
H = \sum_{\nu=\rho,\sigma} \int dx \left\{ \frac{\pi v_\nu K_\nu}{2} \Pi_\nu^2 + \frac{v_\nu}{2\pi K_\nu} (\partial_x \Phi_\nu)^2 \right\} + \sum_i V_0 d \delta(x - x_i) \cos (\sqrt{2}\Phi_\rho + 2k_F x) \cos (\sqrt{2}\Phi_\sigma(x)),
\]

where \( v_\nu \) are velocities of the charge (\( \nu = \rho \)) and spin (\( \nu = \sigma \)) modes, \( K_\nu = v_F/v_\nu \) is the standard Luttinger liquid parameter describing the strength of the interaction, \( V_0 \) and \( d \sim \alpha \) are amplitude and radius of the scattering potential, respectively.

Kane and Fischer [19] found that the backscattering impurity potential for repulsive potential (\( K_\rho < 1 \)) flows to infinity under scaling. Their arguments were generalized by Fabrizio and Gogolin [22] to the case of many impurities. It was shown that the impurity potential can be considered as effectively infinite provided that the mean distance, \( l \), between impurities satisfies the condition

\[
l \gg \frac{1}{k_F} \left( \frac{D}{V_0} \right)^{2/(1-K_\rho)},
\]

where \( D \) is the bandwidth. We assume that the impurity potential is of atomic scale, \( V_0 \lesssim D \), and the interaction between electrons is not too weak, (i.e., \( K_\rho \) is not too close to 1). Then condition (3) is satisfied for \( l \gg 1/k_F \sim \alpha \) which is of the order of interatomic distance. So the limit of strong impurity potential should be a good approximation in a wide range of impurity concentrations.

Further, \( \Pi_\nu, \Theta_\nu \) and \( \Phi_\nu \) must obey the commutation relations (see Ref.[1, 2, 3]) ensuring anticommutation of electronic operators (1). Using then the analogy of Eq.(2) with the Hamiltonian of an elastic string strongly pinned at impurity sites, we can write down solutions for the phase operators \( \Phi_\nu \) and \( \Theta_\nu \) in the region between impurity positions at \( x = x_i \) and \( x_{i+1} \) as

\[
\Phi_\nu(x) = \sum_{n=1}^\infty \sqrt{\frac{K_\nu}{n}} (b_n + b_n^+) \sin q_n \bar{x} + \frac{\bar{x}}{\sqrt{2l_i}} \Phi_{i+1} - \frac{\bar{x}}{\sqrt{2l_i}} \Phi_i + \sum_{j<i} \pi \Delta N_{\nu j} - \pi \Delta N_{\nu i} \frac{\bar{x}}{l_i},
\]

\[
\Theta_\nu(x) = \sum_{n=1}^\infty \sqrt{-\frac{1}{K_\nu n}} (b_n - b_n^+) \cos q_n \bar{x} + \theta_\nu,
\]

(4)
where $\tilde{x} = x - x_i$, $l_i = x_{i+1} - x_i$, $q_n = \pi n / l_i$, $\Phi_i$ is the modulo $2\pi$ residue of $2k_F x_i$. Further, $\Delta N_{\rho i} = (\Delta N_{\uparrow i} + N_{\downarrow i}) / \sqrt{2}$, $\Delta N_{\sigma i} = (\Delta N_{\uparrow i} - N_{\downarrow i}) / \sqrt{2}$, and $\Delta N_{\uparrow i}$ ($\Delta N_{\downarrow i}$) is the number of extra electrons with spin up (down) in the region between $i$-th and $(i + 1)$-th impurities, and, finally, $\theta_{\nu i}$ is the phase canonically conjugate to $\Delta N_{\nu i}$ ($[\theta_{\nu i}, \Delta N_{\nu i}] = i$).

Excitation spectra of the eigenmodes are $\omega_{\nu} = n \omega_{1,\nu} \equiv v_{\nu} q_n$ where $\omega_{1,\nu} = \pi v_{\nu} / l_i$ is the minimum excitation frequency for mode $\nu$.

Note that if we consider the open boundary conditions at the sample ends, $x = 0$, and, $x = L$, (instead of periodic boundary conditions that are commonly used) then operators $\eta_s$ in Eq.(1) are the same for electrons going right and left. In this case, the electron field operator, $\psi_s(x) = \psi_{s+}(x) + \psi_{s-}(x)$, vanishes at impurity positions, $x = x_i$, and expressions for the phase fields between the impurity sites turn out to be similar to those found for bounded 1D Luttinger liquid in Refs.[22, 23, 24], the main difference being the summation over $j < i$ that insure proper commutation relations between the electron operators related to different segments between impurities. Thus the system breaks up into a set of independent segments described as bounded Luttinger liquid with discrete spectrum.

Now we consider the role of interchain hopping adding to (2) the hopping Hamiltonian

$$H_{\perp} = t_{\perp} \sum_{m,n,r,s} \int dx \psi^{+}_{r,s,m}(x) \psi_{r,s,n}(x) + HC$$

$$= \sum_{m,n,r,s} \int dx \frac{it_{\perp} \eta_{r,s,n} \eta_{r,s,m}}{\pi \alpha} \left[ \sin(A_{r,m} - A_{r,n}) + \sin(A_{r,m} - A_{r,n} + 2irk_F x) \right],$$

where indices $n$ and $m$ denoting the chain numbers related to the nearest neighbors are added.

Arguments by Schulz [8] on instability of the Luttinger liquid in the presence of the interchain hopping were based on calculations of temperature dependence of the thermodynamic potential at low temperatures. So we calculate contribution of the interchain hopping to the thermodynamic potential per unit volume given by the standard expression [25]

$$\Delta \Omega = -T \ln \langle S \rangle / V, \quad S = T_\tau \exp \left( -\int_0^{1/T} H_{\perp}(\tau) d\tau \right),$$

where $V$ is the volume, $T_\tau$ stands for imaginary time ordering, and $\langle \cdots \rangle$ means thermodynamic averaging over the unperturbed state.

At temperatures $T \gg \omega_{1,\nu}$, the discreteness of the excitation spectrum can be neglected, hence, according to Refs. [6, 7, 8, 9, 10], interchain hopping is expected to give significant
contributions destroying the Luttinger liquid. We examine the opposite limit, $T \ll \omega_{1,\nu}$, which does not exist in pure infinite Luttinger liquid.

Consider first the second order correction in $t_\perp$. The leading contribution to $\langle S \rangle$ in Eq.(6) is given by items in which the term related to a given chain contains contributions from the electrons with the same chirality, $r$, only,

$$\sum_{m,r,r'} \frac{t_\perp^2}{8\pi^2\alpha^2} \int d1d2 \langle T_x e^{i[A_r(m(1)-A_r,m(1)]} e^{-i[A_r(m(2)-A_r,m(2)]} \rangle e^{i(r-r')k_F(x_1-x_2)},$$

where $\{x_1, \tau_1\}$ and $\{x_2, \tau_2\}$. Other items in which the terms related to the same chain contain contributions from electrons moving both left and right give small contribution, and we do not discuss them in details.

Then we use Eq.(4) in (1) and calculate average in (7) using the relation

$$\langle T_x e^{iA_r,m(1)} e^{-iA_r,m(2)} \rangle = e^{-\frac{1}{2} \{ A_r^2(m(1)+A_r^2(m(2)-2T_x[A_r,m(1)A_r,m(2)]\}}.$$ 

Neglecting small corrections $\propto \exp[-\frac{\omega_{1,\nu}}{T}]$ due to Planck’s distribution functions, we find for the average in the exponent

$$\langle T_x A_r(1)A_r(2) \rangle = \frac{1}{8} \ln \left[ \frac{\cosh z - \cos y_+}{\cosh z - \cos y_-} \right] + i \tan^{-1} \left[ \frac{\sin y_-}{e^z - \cos y_-} \right],$$

where $y_\pm = \frac{\pi(x_1 \pm x_2)}{l_i}$, $z = \frac{\pi(\alpha + \nu_1 |\tau_1 - \tau_2|)}{l_i}$ (chain indices are dropped for brevity here).

In the integrations over 1 and 2, the leading contributions comes from region $1 \approx 2$, i.e., $|y_-| \ll 1$, $z \ll 1$ where expression (7) reduces to

$$\frac{t_\perp^2 mL}{\pi^2\alpha^2 T} \int \frac{\cos[(r-r')k_F(x_-)] dx_- d\tau_-}{\prod_{\nu=\rho,\sigma} [(1 + \epsilon_{\nu}\tau_-)^2 + (x_-/\alpha)^2]^{1+2\delta_{\nu}}},$$

where $\epsilon_{\nu} = \frac{\omega_{\nu}}{\pi\alpha}$, $\delta_{\nu} = \frac{1}{4}(K_{\nu} + 1/K_{\nu} - 2)$, $\tau_- = |\tau_1 - \tau_2|$, $x_- = x_1 - x_2$, and $m$ is the number of the nearest-neighbor chains. Contribution to expression (7) from integration over region $|y_-| \gg 1$, $z \gg 1$, is small, $\sim (\alpha/l_i)^{2d}$, $\delta = \delta_{\rho} + \delta_{\sigma}$, because $\delta$ is not too small in the assumed case of the not too small interaction (cf. Eq. (3)).

Additional items in $\langle S \rangle$ in Eq.(6) in which the terms related to the same chain contain contributions from electrons moving both left and right is smaller than that given by Eq. (10) by factor $\sim (\alpha/l_i)^{K_{\rho}+K_{\sigma}}$. For reasonable values of $K_{\nu}$, this contribution is small and can be neglected.

Similarly, the leading contribution to $\Delta \Omega$ from higher-order terms in series expansion of the exponential in Eq.(6) was found to come from even powers $2n$ in $t_\perp$ that can be
represented as a sum of \((2n-1)!!\) items like (7) with almost coinciding times and coordinates. Therefore, summing up the leading contributions and inserting them into Eq.(6) we can calculate the variation of the thermodynamic potential per single chain and per unit length

\[
\Delta \Omega = -a \frac{t_{\perp}^2 m}{\pi^2 v_F}, \quad a = \int \frac{(1 + \cos 2k_F \alpha x)}{\prod_{\nu=\rho,\sigma}[(1 + \tau/K_{\nu})^2 + x^2]^{1+2\delta}} \, dx \, d\tau.
\]

For moderate repulsion, \(\delta \sim 1\), \(a \sim 1\). In the limit of strong repulsion, \(K_\rho \ll K_\sigma \sim 1\), \(a\) is small, \(a \sim K_\rho^2\).

Thus \(\Delta \Omega\) is much smaller than the thermodynamic potential of purely 1D Luttinger liquid, \(\Omega_0 \sim \left(\frac{1}{K_\rho} + \frac{1}{K_\sigma}\right) \varepsilon_F k_F\),

\[
\Delta \Omega / \Omega_0 \sim \left(\frac{t_{\perp}}{\varepsilon_F}\right)^2,
\]

and temperature-dependent corrections to Eq.(11) are determined by small thermally activated contributions \(\propto \exp\left[-\frac{\omega_{1,\nu}}{T}\right]\).

Now we calculate modification of the one-particle Green’s function due to the interchain hopping.

\[
G(1, 1') = -\langle T_\tau \psi(1) \bar{\psi}(1') S \rangle / \langle S \rangle.
\]

Again, we consider the low-temperature limit, \(T \ll \omega_{1,\nu}\), non-existing in pure infinite system. Consider first the second order correction in \(t_{\perp}\) to the Green’s function of pure 1D system, \(G_0(1, 1')\).

\[
G_2(1, 1') = -\langle T_\tau \psi(1) \bar{\psi}(1') S_2 \rangle + \langle T_\tau \psi(1) \bar{\psi}(1') \rangle \langle S_2 \rangle.
\]

Calculation is similar to that considered above, \((cf. \ (7-9))\). However, in contrast to the case of the thermodynamic potential where the leading contribution was given by regions of almost coinciding values of times and coordinates, such contributions from two terms in (9) cancel each other. So the second-order correction is estimated as

\[
G_2(1, 1') \lesssim \left(\frac{t_{\perp}l}{v_F}\right)^2 \left(\frac{\alpha}{l}\right)^{2\delta} G_0(1, 1').
\]

Estimation of the fourth order correction in \(t_{\perp}\) gives \(G_4 \sim (t_{\perp}l/v_F)^2(\alpha/l)^{2\delta} G_2\). Therefore, we conclude that at \(T \ll \omega_{1,\nu}\) the interchain hopping gives small corrections to the one-particle Green’s function, provided

\[
\left(\frac{t_{\perp}}{v_F}\right)^2 \left(\frac{\alpha}{l}\right)^{2\delta} \sim \left(\frac{t_{\perp}}{\varepsilon_F}\right)^2 \left(\frac{\alpha}{l}\right)^{2\delta-2} \ll 1,
\]

where we estimated the cut-off parameter as \(\alpha \sim 1/k_F\).
So far we considered the TL model in which interaction is described by coupling constants related to forward- and backscattering. In order to make comparison with experimental data we must consider a more realistic Coulomb potential. It is reasonable to assume that the long-range part of the interaction is dominated by the Coulomb potential, while the backscattering is described by relatively small coupling constant $g_1$. This enables us to concentrate on the spin isotropic case and to ignore the possibility of the spin gap. The problem of the long-range Coulomb potential on an array of chains was solved in Refs. [11, 12, 13]. It was found that interaction of electrons on a given chain is screened by the electrons on other chains, and the Coulomb interaction can be described by the TL Hamiltonian with coupling constants dependent on wave vector,

$$g_2 = g_4 = \frac{4\pi e^2}{s^2(q^2_{\|} + \epsilon q_{\perp}^2)},$$  \hspace{1cm} (15)

where $s$ is the lattice period in the direction perpendicular to the chains, and $\epsilon$ is a background dielectric constant for the transverse direction. Coupling constants in spin channel remain unaffected. In principle, the coupling constants must be determined by matrix elements of Coulomb potential that depend on details of wavefunctions and on chain arrangement, and must contain an infinite sum over transversal reciprocal lattice vectors. So expression (15) is not universal and depends on material.

Eq. (15) leads to $q$-dependent velocities

$$\omega_\rho = \frac{v_F}{K_\rho} q_{\|}, \quad \frac{1}{K_\rho} = \sqrt{1 + \frac{\mathcal{K}}{s^2(q^2_{\|} + \epsilon q_{\perp}^2)}}, \quad \mathcal{K} = \kappa^2 s^2 = \frac{8e^2}{\hbar v_F},$$  \hspace{1cm} (16)

where $\kappa$ is the inverse Thomas-Fermi screening length.

We do not perform explicit calculations restricting ourselves to estimations. For the case of $q$-dependent coupling expressions for the thermodynamic potential and Green’s functions contain various integrals of correlation functions over $q_{\perp}$. One can show that the results obtained above can be generalized qualitatively to the case of long-range Coulomb interaction if we substitute $q_{\perp}$ in Eq. (16) for its characteristic value, $q_{\perp} \sim \pi/s$. For example, integrals for corrections to the thermodynamic potential are dominated by close values of coordinates and times, similar to Eq. (10), and coupling parameters should be substituted by their averages over $q_{\perp}$,

$$\delta = \frac{1}{4} \left( \frac{1}{K_\rho} + \frac{1}{K_\rho} + \frac{1}{K_\sigma} + \frac{1}{K_\sigma} - 4 \right) \sim \frac{1}{4} \left[ \int \frac{s^2 d^2 q_{\perp}}{(2\pi)^2} \left( K_\rho + \frac{1}{K_\rho} \right) - 2 \right].$$
Note that $\aleph = 8\alpha(c/v_F)$, where $\alpha$ is the fine structure constant. Since $v_F$ is much smaller than the velocity of light, the factor $\aleph$ is large. For $v_F \approx 2 \times 10^7$ cm/s, which is typical value for transition metal trihalogenides, $\aleph \sim 90$. This corresponds to the case of strong interaction and leads to quite large values of coupling parameters, $1/K_{\rho} \sim \sqrt{\aleph/\varepsilon_\perp} \sim 3 \div 8$, $\delta \sim 1/2 \div 2$.

Now we discuss conditions for observation of the Luttinger liquid in quasi-1D conductors stabilized by impurities. First we discuss the condition for the temperature limiting from above the region where the Luttinger liquid can exist. This condition reads, $T \ll \omega_{1,\nu}$. The minimal excitation energies, $\omega_{1,\nu}$, can be estimated as

$$\omega_{1,\nu} = \frac{\hbar v_F}{l} \frac{1}{K_{\nu}} \sim \varepsilon_F \left( \frac{1}{k_F l} \right) \frac{1}{K_{\nu}} \sim \varepsilon_F c_i \frac{1}{K_{\nu}},$$

where $c_i$ stands for dimensionless impurity concentration corresponding to number of impurities per one electron. As Fermi energy in NbSe$_3$ and TaS$_3$ is about 1 eV, we obtain that $\omega_{1,\nu}$ is about 100 K for impurity concentration $c_i \sim 10^{-2} \div 10^{-3}$.

Another condition to be fulfilled is smallness of corrections to the Green’s function due to interchain hopping. According to Eq.(14) the corrections are small provided

$$\left( \frac{t_{\perp}}{\varepsilon_F} \right)^2 c_i^{2\delta-2} \ll 1.$$

If the interaction is strong enough, $\delta \gtrsim 1$, this condition is not more strict than the condition for the limiting temperature discussed above. For lower strength of interaction, $\delta < 1$, this condition reads

$$c_i \gg \left( \frac{t_{\perp}}{\varepsilon_F} \right)^{1/(2-2\delta)}.$$

Estimating $t_{\perp}$ as being of the order of the Peierls transition temperature, $T_P \sim 100 \div 200$ K $\sim 0.01\varepsilon_F$, we find that this condition can be fulfilled easily even at $\delta = 1/2$ for relatively small impurity concentration, $c_i \gg 10^{-2}$.

Thus we find that Luttinger liquid can be stabilized by impurities in relatively pure linear-chain compounds at rather high temperatures corresponding to experimental observation Refs.[15, 16, 17] of the transition from metallic to non-metallic conduction characterized by power law dependencies of conductivity and by conductivity resembling the variable-range hopping. However, in order to make detailed comparison with the experimental data, calculation of the conductivity in a random network made of weakly coupled bounded Luttinger liquids is needed.
The author is grateful to S.V. Zaitsev-Zotov for useful discussions. The work was supported by Russian Foundation for Basic Research, by INTAS, CRDF, and by collaborative program with the Netherlands Organization for Scientific Research.

[1] J. Voit, Rep. Prog. Phys. 58, 977 (1995).
[2] M.P.A. Fisher and L.I. Glazman, in Mesoscopic Electron Transport, edited by L.L. Sohn, L. Kouwenhoven, and G. Schön (Kluwer Academic Publishers, Dordrecht, 1997).
[3] H.J. Schulz, G. Cuniberti and P. Pieri, Field Theories for Low-Dimensional Condensed Matter Systems, (Springer, Berlin, 2000); cond-mat/9807366.
[4] F.D.M. Haldane, J. Phys. C 14, 2585 (1981).
[5] G. Grüner, Density Waves in Solids, (Addison-Wesley, Reading, 1994).
[6] V.N. Prigodin and Yu.A. Firsov, Zh. Eksp. Teor. Fiz. 76, 736, 1602 (1979) [Sov. Phys. JETP 49, 369, 813 (1979)]; Yu.A. Firsov, V.N. Prigodin, and Chr. Seidel, Phys. Reports, 126, 245 (1985).
[7] S.A. Brazovskii and V.M. Yakovenko, Zh. Eksp. Teor. Fiz. 89, 2318, (1985) [Sov. Phys. JETP 62, 1340(1985)].
[8] H.J. Schulz, Int. J. Mod. Phys. B 5, 57 (1991)
[9] D. Boies, C. Bourbonnais, and A.-M.S. Tremblay, Phys. Rev. Lett 74, 968 (1995); cond-mat/9604122.
[10] E. Arrigoni, Phys. Rev. Lett 83, 128 (1999).
[11] S. Barišić, J. Physique 44, 185 (1983).
[12] S. Botrić and S. Barišić, J. Physique 44, 185 (1984).
[13] H.J. Schulz, Int. J. Phys. C: Solid State Phys. 16, 6769 (1983).
[14] R. Mukhopadhyay, C.L. Kane and T.C. Lubensky, Phys. Rev. B 64, 045120 (2001).
[15] S. V. Zaitsev-Zotov, V.Ya. Pokrovskii, and P. Monceau, Pis’ma v ZhETF 73, 29 (2001) [JETP Lett. 73, 25 (2001)].
[16] S.V.Zaitsev-Zotov, Microelectronics Engineering, 69, 549 (2003).
[17] S.V.Zaitsev-Zotov, M.S.H.Go, E.Slot, H.S.J. van der Zant, Physics of Low-Dimensional Structures 1/2, 79 (2002); cond-mat/0110629.
[18] A.N. Bloch et. al., Phys. Rev. B 28, N 12, 753 (1972).
[19] C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett 68, 1220 (1992).
[20] K.A. Matveev and L.I. Glazman, Phys. Rev. Lett 70, 990 (1993).
[21] A. Furusaki and N. Nagaosa, Phys. Rev. B 47, 4631 (1993).
[22] M. Fabrizio and A.O. Gogolin, Phys. Rev. B 51, 17827 (1995).
[23] S. Eggert, H. Johannesson, and A. Mattsson, Phys. Rev. Lett 76, 1505 (1996).
[24] A.E. Mattsson, S. Eggert, and H. Johannesson, Phys. Rev. B 56, 15615 (1996).
[25] A.A. Abrikosov, L.P. Gorkov and I.E. Dzyaloshinski, *Methods of quantum field theory in statistical physics*, (Dover, New York, 1963).