Hawking-like radiation as tunneling from the apparent horizon in a FRW Universe

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Abstract

We study Hawking-like radiation in a Friedmann-Robertson-Walker (FRW) universe using the quasi-classical WKB/tunneling method which pictures this process as a “tunneling” of particles from behind the apparent horizon. The correct temperature of the Hawking-like radiation from the FRW spacetime is obtained using a canonical invariant tunneling amplitude. In contrast to the usual quantum mechanical WKB/tunneling problem where the tunneling amplitude has only a spatial contribution, we find that the tunneling amplitude for FRW spacetime (i.e. the imaginary part of the action) has both spatial and temporal contributions. In addition we study back reaction and energy conservation of the radiated particles and find that the tunneling probability and change in entropy, $S$ are related by the relationship: $\Gamma \propto \exp[-\Delta S]$ which differs from the standard result $\Gamma \propto \exp[\Delta S]$. By regarding the whole FRW universe as an isolated adiabatic system the change in the total entropy is zero. Then splitting the entropy between interior and exterior parts of the horizon ($\Delta S_{\text{total}} = \Delta S_{\text{int}} + \Delta S_{\text{ext}} = 0$), we can explain the origin of the minus sign difference with the usual result: our $\Delta S$ is for the interior region while the standard result from black hole physics is for the exterior region.

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I. INTRODUCTION

There has been recent work aimed at establishing the relationship between the Friedmann equations and the first law of thermodynamics in the framework of a Friedmann-Robertson-Walker (FRW) universe [1–4]. If one assumes that the apparent horizon of a FRW universe has an entropy $S = A/4$ and a temperature $T = 1/2\pi \tilde{r}_A$, where $A$ and $\tilde{r}_A$ are the area and radius of the apparent horizon, respectively, one is able to derive the Friedmann equations of the FRW universe with any spatial curvature by applying the first law of thermodynamics, $dE = T dS$, to the apparent horizon [2]. Other recent work in this direction can be found in [5–8].

One can derive the Hawking-like temperature, $T = 1/2\pi \tilde{r}_A$, using the first law of thermodynamics and the relationship between entropy and area of the apparent horizon. In this paper we take a different starting point for the problem: we first obtain the temperature using the Hamilton-Jacobi WKB/tunneling method and then we study the entropy change and back reaction using the first law of thermodynamics and energy conservation. The WKB method was already applied to FRW spacetime in a recent paper [9], where the tunneling amplitude for scalar particles across the apparent horizon of the FRW spacetime was investigated. The same approach was applied to fermions tunneling from the apparent horizon [10] and for calculating higher order quantum corrections [11] to the temperature and entropy of the FRW spacetime. A relationship between the tunneling rate through the apparent horizon of the FRW spacetime and the first law of thermodynamics was obtained in [12] using energy conservation.

The tunneling method was initially proposed in [13], [14]. Due to its simplicity this method has attracted a lot of attention and subsequent work [15]. In the tunneling method of [14] one calculates the imaginary part of the action using the null geodesic equation, while [13] employs the Hamilton-Jacobi equations to obtain the particles’ classical action along with detailed balance of the ingoing and outgoing probability amplitudes. The method of [13] has been applied to more general and complicated spacetimes [16] and to dynamical black holes [17]. However, in [18], it was shown that the WKB/tunneling method appeared to give a temperature twice as large as the correct Hawking temperatures. Recently, this problem has been solved [19] when it was shown that in contrast to normal quantum mechanical WKB/tunneling problems that the tunneling probability received a contribution
from the time coordinate upon crossing the horizon. By requiring canonical invariance of the tunneling amplitude and taking into account the temporal contribution one obtains the correct Hawking temperature. Other recent work in this direction can be found in [20]. In this paper, we will revisit the Hawking-like radiation from the apparent horizon of FRW spacetime taking into account these subtle points. Using these results for the radiation we examine the relationship between the temperature and the change in entropy of the interior and exterior of the FRW spacetime. In studying this connection between temperature and entropy we take into account back reaction effects and energy conservation of the radiated particle.

The organization of this paper is as follows: In section (II) we derive the Hawking-like temperature for a FRW spacetime using the Hamilton-Jacobi method. If one requires canonical invariance for the tunneling amplitude one apparently finds a temperature that is twice the correct value. Upon taking into account the temporal contribution which occurs from the change in the time coordinate upon crossing the horizon the correct temperature is recovered. In section (III), we consider the back reaction effects and energy conservation of the radiated particles. We also make a connection between the tunneling amplitude and the entropy change. In section (IV) we give our conclusions.

II. HAWKING-LIKE TEMPERATURE WITH CANONICAL INVARIANCE

The standard form of the FRW metric is

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \]

where \( d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2 \) denotes the line element of an unit two-sphere \( S^2 \), \( a(t) \) is the scale factor of our universe and \( k \) is the spatial curvature constant which can take values \( k = +1 \) (positive curvature), \( k = 0 \) (flat), \( k = -1 \) (negative curvature). In a FRW universe, there is a dynamical apparent horizon, which is the marginally trapped surface with vanishing expansion and determined by the relation\[1\]

\[ h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0, \]

(2)
where $\tilde{r} = a(t)r$, $h^{ab} = \text{diag}(-1, \frac{1-k\tilde{r}^2}{a^2})$, and $\partial_a = (\partial_t, \partial_r)$. After a simple calculation one can obtain the radius of the apparent horizon

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}.$$  

(3)

where $H$ is the Hubble parameter, $H \equiv \dot{a}/a$ (the dot represents derivative with respect to the cosmic time $t$).

In the tunneling approach of reference [14] the Painlevé-Gulstrand coordinates are used for the Schwarzschild spacetime. Applying the change of radial coordinate, $\tilde{r} = ar$, along with the above definitions of $H$ and $\tilde{r}_A$ to the metric in (1) one obtains the Painlevé-Gulstrand-like metric for the FRW spacetime

$$ds^2 = -\frac{1-\tilde{r}^2/\tilde{r}_A^2}{1-k\tilde{r}^2/a^2}dt^2 - \frac{2H\tilde{r}}{1-k\tilde{r}^2/a^2}dtd\tilde{r} + \frac{1}{1-k\tilde{r}^2/a^2}d\tilde{r}^2 + \tilde{r}^2d\Omega_2^2.$$  

(4)

These coordinates have been used in both null geodesic method and Hamilton-Jacobi method [9–11] to study the Hawking-like radiation from a FRW metric.

In this paper we will use the Hamilton-Jacobi approach. One starts by considering a massless scalar field $\phi$ in the FRW spacetime. This scalar field obeys the Klein-Gordon equation

$$-\hbar^2 \sqrt{-g} \partial_\mu (g^{\mu\nu} \sqrt{-g} \partial_\nu) \phi = 0.$$  

(5)

Substituting the standard ansatz for scalar wave function $\phi(\tilde{r}, t) = \exp[-\frac{i}{\hbar}S(\tilde{r}, t) + \cdots]$ into (5) and taking the limit as $\hbar \to 0$, gives the Hamilton-Jacobi equation

$$g^{\mu\nu} \partial_\mu S \partial_\nu S = 0.$$  

(6)

Since the FRW universe is spherical symmetric, we only need to examine the $(t-\tilde{r})$ sector of the spacetime. Using the Painlevé-Gulstrand-like coordinates in (4) and solving for $S$, one obtains

$$S(\tilde{r}, t) = -\int \frac{\omega}{\sqrt{1-k\tilde{r}^2/a^2}}dt + \omega \int \frac{-H\tilde{r} \pm \sqrt{1-k\tilde{r}^2/a^2}}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}d\tilde{r},$$  

(7)

where the $+/-$ sign corresponds to the outgoing/ingoing solutions, respectively. We note that in metric (4), one should use the Kodama vector [21] to define the energy of particle as $\omega = -\sqrt{1-k\tilde{r}^2/a^2}\partial_t S$. Here $\sqrt{1-k\tilde{r}^2/a^2}\partial_t$ is the Kodama vector. It is obvious that
the action $S$ has a pole at the apparent horizon. Through a contour integral, we obtain the imaginary part of the action for both outgoing and ingoing solutions

$$\text{Im} S_{\text{out}} = \text{Im} \int \frac{-H \tilde{r} + \sqrt{1 - k \tilde{r}^2 / a^2}}{(1 - \tilde{r}^2 / \tilde{r}_A^2) \sqrt{1 - k \tilde{r}^2 / a^2}} \omega d \tilde{r} = 0,$$  

(8)

$$\text{Im} S_{\text{in}} = \text{Im} \int \frac{-H \tilde{r} - \sqrt{1 - k \tilde{r}^2 / a^2}}{(1 - \tilde{r}^2 / \tilde{r}_A^2) \sqrt{1 - k \tilde{r}^2 / a^2}} \omega d \tilde{r} = \pi \omega \tilde{r}_A.$$  

(9)

The above results are similar to what one finds in the Schwarzschild case \[14\] except now it is the imaginary part of the *outgoing* action which has zero imaginary contribution. In the Schwarzschild case it was the *ingoing* action which had zero imaginary part. This distinction will be important in the next section when we relate the change in entropy of the FRW spacetime to the tunneling amplitude. Note also that unlike the usual quantum mechanical tunneling problem one finds different contributions to the imaginary part of the action in these coordinates for the outgoing versus the ingoing case.

To obtain the temperature one equates the Boltzmann weight, $\exp[-\omega/T]$, with the tunneling probability $\Gamma$ which is usually written as

$$\Gamma \propto \exp[-2 \text{Im} S].$$  

(10)

In order to obtain the correct temperature (i.e. $T = \frac{1}{2\pi \tilde{r}_A}$) one must choose $\text{Im} S = \text{Im} S_{\text{in}}$ rather than $\text{Im} S = \text{Im} S_{\text{out}}$. Aside from having to make this arbitrary choice in order to obtain the correct result it has been noted \[18\] that (10) is not invariant under canonical transformations. One should instead use the canonically invariant expression

$$\Gamma \propto \exp[-\text{Im} \oint p dr]$$  

(11)

where $\oint p dr = S_{\text{in}} - S_{\text{out}}$. Note that (10) and (11) are numerically the same in the usual case when the action $S$ has the same magnitude for the ingoing and outgoing directions crossing the barrier. This is not the case for the Painlevé-Gulstrand-like coordinates of (4). If one uses (11) and (8), (9) to obtain the temperature one finds twice the temperature $- T = \frac{1}{\pi \tilde{r}_A}$.

To see the resolution of this problem with the temperature we transform the Painlevé-Gulstrand-like coordinates of (4) via the following change of the time coordinate

$$dt \rightarrow dt_* = dt + \frac{H \tilde{r}}{1 - \tilde{r}^2 / \tilde{r}_A^2} d \tilde{r}.$$  

(12)

In this way (4) takes a Schwarzschild-like form

$$ds^2 = -\frac{1 - \tilde{r}^2 / \tilde{r}_A^2}{1 - k \tilde{r}^2 / a^2} dt_*^2 + \frac{1}{1 - \tilde{r}^2 / \tilde{r}_A^2} d \tilde{r}^2 + \tilde{r}^2 d \Omega^2_2.$$  

(13)
For coordinates \((13)\), the Kodama vector is \(K = \sqrt{1 - k\tilde{r}^2/a^2(\partial/\partial t_*)}\). Thus the energy of the particle is defined as

\[
\omega = -\sqrt{1 - k\tilde{r}^2/a^2}\partial t_* S. \tag{14}
\]

Thus one obtains

\[
S(\tilde{r}, t_*) = -\int \frac{\omega}{\sqrt{1 - k\tilde{r}^2/a^2}} dt_* \pm \omega \int \frac{1}{1 - \tilde{r}^2/\tilde{r}_A^2} d\tilde{r}. \tag{15}
\]

The imaginary parts of the action for outgoing and ingoing particles are

\[
\text{Im}S_{\text{out}} = \text{Im} \int \frac{1}{1 - \tilde{r}^2/\tilde{r}_A^2} \omega d\tilde{r} = -\frac{\pi\omega\tilde{r}_A}{2},
\]

\[
\text{Im}S_{\text{in}} = -\text{Im} \int \frac{1}{1 - \tilde{r}^2/\tilde{r}_A^2} \omega d\tilde{r} = \frac{\pi\omega\tilde{r}_A}{2}. \tag{17}
\]

In this case regardless of if one takes the tunneling probability as \(\Gamma \propto \exp[\mp 2\text{Im}S_{\text{in, out}}]\) or \(\Gamma \propto \exp[-\text{Im} \oint pdr]\) and \(\oint pdr = S_{\text{in}} - S_{\text{out}}\) one obtains \(T = 1/\pi\tilde{r}_A\), which is the twice of the correct value as was the case with the Painlevé-Gulstrand-like coordinates.

The resolution of this factor of two in the temperature lies in an overlooked temporal contribution the tunneling amplitude. For a Schwarzschild black hole [19], the temporal contribution to the action is found by changing Schwarzschild coordinates into Kruskal-Szekeres coordinates and then matching different Schwarzschild time coordinates across the horizon. Things are a bit different in the case of the FRW universe. Since for the FRW metric the metric coefficients depend on both radius and time, there is no time translation Killing vector field as for the static Schwarzschild spacetime. For the FRW metric one should use the Kodama vector in place of the Killing vector. For the FRW spacetime, the Kodama vector is timelike, null and spacelike for the regions outside, on, and inside the apparent horizon, respectively. Because the energy of the particle is defined by the Kodama vector, the discrepancy of Kodama vector inside and outside the horizon will effect the temporal part of the action. In terms of the Kodama vector, the time transformation should be defined by \(\partial_\tau = \sqrt{1 - k\tilde{r}^2/a^2(\partial/\partial t_*)}\) and \(d\tau = dt_*/\sqrt{1 - k\tilde{r}^2/a^2}\). Then the metric \((13)\) becomes

\[
ds^2 = -(1 - \tilde{r}^2/\tilde{r}_A^2)d\tau^2 + \frac{1}{1 - \tilde{r}^2/\tilde{r}_A^2} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2_2. \tag{18}
\]

In order to get the temporal contribution to the imaginary part of action, one must transform above metric into the Kruskal-Szekeres-like coordinates. The transformation can be written
in the following form

\[ T \sim \begin{cases} \left( \tilde{r}_A^2 - \tilde{r}^2 \right)^{1/2} \sinh \frac{\tilde{r}}{\tilde{r}_A}, & \tilde{r} < \tilde{r}_A; \\ \left( \tilde{r}^2 - \tilde{r}_A^2 \right)^{1/2} \cosh \frac{\tilde{r}}{\tilde{r}_A}, & \tilde{r} > \tilde{r}_A. \end{cases} \]  

(19)

\( T \) is the Kruskal-Szekeres-like time coordinate. To match different patches across the horizon, one needs to rotate the time coordinate \( \tau \) as \( \tau \to \tau - i\pi \tilde{r}_A/2 \). Therefore, there is a temporal contribution to the imaginary part of the total action, given by \( \text{Im}(\omega \Delta \tau^{\text{in,out}}) = -\pi \omega \tilde{r}_A/2 \).

Note that unlike the spatial contribution the temporal contribution has the same sign independent of which direction the horizon is crossed. Thus, using (11), (16), (17) and adding the ingoing and outgoing temporal contributions we obtain a tunneling probability of

\[ \Gamma \propto \exp\left[-2\pi \omega \tilde{r}_A\right]. \]  

(20)

On comparing the tunneling probability with the thermal spectrum \( \Gamma \propto \exp\left[-\omega/T\right] \), we recover the correct temperature, \( T = 1/2\pi \tilde{r}_A \), associated with the apparent horizon. One could also use the Painlevé-Gulstrand-like coordinates of (4) all the way through the calculation to obtain the correct temperature, but in this case there will be two temporal contributions. The first temporal contribution is just that already given by (19). The second comes from the pole in the second term of the transformation (12) between the Painlevé-Gulstrand-like coordinates of (4) and the Schwarzschild-like coordinates of (13). The calculation works in either coordinate system but is actually a bit more complicated in the Painlevé-Gulstrand-like coordinates.

### III. BACK REACTION EFFECTS AND ENTROPY

In this section we study back reaction effects of the radiated particle on the FRW space-time and show the relationship to the tunneling picture of the previous section. Many earlier works have stressed the back reaction effects of the tunneling paradigm in black hole physics [22]. In the present section we re-examine these issues but now taking into account the correct canonically invariant tunneling probability \( \Gamma \propto \exp[-\text{Im} \oint p \, dr] \) and the temporal contribution to the imaginary part of the action.

The canonically invariant tunneling probability \( \Gamma \propto \exp[-\text{Im} \oint p \, dr] \), contains two contributions: from particles crossing the horizon from exterior region to interior region, and also from interior region to exterior region. The self-gravitation effects of these two processes
should both be taken into account. In the FRW spacetime, the energy inside the apparent horizon is defined by a quasi-local mass: the Misner-Sharp mass, \( M = \tilde{r}_A/2 \). We note that the tunneling of a particle from the apparent horizon is an instantaneous process. In this process one can naturally fix the energy inside the apparent horizon as the Misner-Sharp mass \( M \). Therefore, when the the radiated particle moves across the apparent horizon, the energy inside the apparent horizon changes to \( M + \omega \) where \( \omega \) is the energy flux which has past through the apparent horizon. Also, as a result of tunneling, the radius of the apparent horizon, \( \tilde{r}_A \), should change to \( \tilde{r}_A + \delta \tilde{r}_A \). In constructing the first law of thermodynamics on the apparent horizon, a key point is to calculate the amount of energy crossing the apparent horizon in an infinitesimal time interval. By using the Misner-Sharp mass \( M \), the energy flux passed through the apparent horizon is defined as

\[
dE = (k^t \partial_t M + k^r \partial_r M) dt = d\tilde{r}_A, \tag{21}
\]

where \( k^t, r = (1, -H r) \) is the (approximate) generator of the apparent horizon and satisfies \( k^r \partial_r \tilde{r} + k^t \partial_t \tilde{r} = 0 \). Applying the above relation to the radiation process and considering that we have fixed the Misner-Sharp mass \( M \), one finds

\[
\omega = \int_{\tilde{r}_A}^{M + \omega} dE = \int_{\tilde{r}_A}^{\tilde{r}_A + \delta \tilde{r}_A} d\tilde{r}_A = \delta \tilde{r}_A. \tag{22}
\]

In this case, the imaginary part of the action can be rewritten as

\[
\text{Im} S_{\text{in}} = \int_0^\omega \pi (\tilde{r}_A + \omega') d\omega' = \int_M^{M + \omega} \pi (\tilde{r}_A + \omega') dE, \\
\text{Im} S_{\text{out}} = -\int_0^\omega \pi (\tilde{r}_A + \omega') d\omega' = -\int_M^{M + \omega} \pi (\tilde{r}_A + \omega') dE. \tag{23}
\]

(In this section one needs to be careful to distinguish action, \( S \), from entropy \( S \)). In the above expressions, the temporal contribution to the imaginary part of the action has been incorporated.

According to the first law of thermodynamics applied to the FRW universe after the radiation of the particle, we have

\[
\frac{1}{2\pi (\tilde{r}_A + \omega)} dS^f = dE. \tag{24}
\]

Substituting (24) into (23), one can obtain

\[
\text{Im} S_{\text{in}} = \int \frac{1}{2} dS^f = \frac{1}{2} (S^f - S^i) = \frac{1}{2} \Delta S, \\
\text{Im} S_{\text{out}} = -\int \frac{1}{2} dS^f = -\frac{1}{2} (S^f - S^i) = -\frac{1}{2} \Delta S. \tag{25}
\]
Here \( S^f = \pi (\tilde{r}_A + \omega)^2 \) and \( S^i = \pi \tilde{r}_A^2 \) are the entropy of the apparent horizon after and before the particle radiation, respectively. Using these considerations of the self-gravitational effects of the radiated particle in the FRW universe one can immediately find the tunneling probability

\[
\Gamma \propto \exp[-\Delta S] = \exp[-2\pi \omega (\tilde{r}_A + \frac{\omega}{2})].
\]

(26)

Neglecting the quadratic term in \( \omega \), the above result reduces to (20), the result obtained without considering the self-gravitation of radiated particle. It should be noted that the expression (20) for the tunneling probability implies a pure thermal spectrum for the apparent horizon radiation. When back reaction effects and energy conservation are taken into account, (26) indicates that the radiation spectrum deviates from the pure thermal spectrum.

Comparing this result to the well known tunneling probability \( \Gamma \propto \exp[\Delta S] \), which is obtained in [14] for a general, stationary, spherically symmetric black hole background, we find that there is an additional minus in (26) compared with the result in [14]. We show here how these two different results are in fact consistent. In the Schwarzschild spacetime the observer of the radiation is outside the horizon. In the FRW universe the observer (i.e. the Kodama observer who sees the radiation), is inside the apparent horizon. In Schwarzschild spacetime, the tunneling probability is related to the change in the entropy of the horizon through particles which tunnel from inside to outside the horizon. In contrast for the FRW spacetime the tunneling probability is related to the change in the entropy of the apparent horizon via particles which tunnel from outside to inside the apparent horizon.

We assume the whole FRW spacetime is an isolated adiabatic system which can be split into two parts: the interior and the exterior of the apparent horizon. The entropy of the interior is just the entropy of the apparent horizon \( S_{int} \). We use \( S_{ext} \) as the entropy of the exterior of the apparent horizon. As an adiabatic system, the total entropy of the FRW universe is conserved, thus

\[
\Delta S_{int} + \Delta S_{ext} = 0,
\]

(27)

where \( \Delta S_{ext} \) is the change of the entropy of the exterior of the apparent horizon due to the tunneling of a particle from outside to inside the apparent horizon. Taking this viewpoint, the tunneling probability can now be written as \( \Gamma \propto \exp[\Delta S_{ext}] \), which is then consistent with the result in [14].
IV. CONCLUSION

In this paper, we have revisited the derivation of the Hawking-like radiation from the apparent horizon of a FRW universe. By taking into account the canonical invariant tunneling probability and the imaginary parts of both the spatial and temporal contributions of the tunneling amplitude we recover the correct temperature $T = 1/2\pi \tilde{r}_A$.

We also showed how to obtain the relationship between the tunneling probability and the change in entropy, $\Gamma \propto \exp[-\Delta S_{\text{int}}]$, by taking into account back reaction effects and energy conservation of the radiated particles. At first glance this result seems to be wrong by a minus sign as compared to a similar result for spherically symmetric, static black hole spacetimes (i.e. $\Gamma \propto \exp[\Delta S_{\text{BH}}]$ obtained in [14]). This apparent inconsistency arises because in the different spacetimes (FRW versus Schwarzschild) the observer of the radiation is located on different sides of the (apparent) horizons. In the FRW spacetime the observer sees radiation which tunnels from outside to inside the apparent horizon; in the Schwarzschild spacetime the tunneling is from inside to outside the horizon. If we regard the whole FRW universe as an isolated adiabatic system and take into account the total entropy conservation of the FRW universe, this inconsistency is resolved. In this way, the tunneling probability for the FRW universe can be written as $\Gamma \propto \exp[\Delta S_{\text{ext}}]$ which fits well with the black hole result. $S_{\text{ext}}$ is the entropy change of the exterior of the apparent horizon and satisfies $\Delta S_{\text{int}} + \Delta S_{\text{ext}} = 0$.

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