Composite fermions and quartets in the Fermi-Bose mixture with attractive interaction between fermions and bosons.

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We consider a model of Fermi-Bose mixture with strong hard-core repulsion between particles of the same sort and attraction between particles of different sorts. In this case, besides the standard anomalous averages of the type \( \langle b \rangle, \langle bb \rangle \) and \( \langle cc \rangle \), a pairing between fermion and boson of the type \( bc \) is possible. This pairing corresponds to a creation of composite fermions in the system. At low temperatures and equal densities of fermions and bosons composite fermions are further paired in quartets. Our investigations are important for high-\( T_c \) superconductors and in connection with recent observation of weakly bound dimers in magnetic traps at ultralow temperatures.

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I. INTRODUCTION.

A model of Fermi-Bose mixture is very popular nowadays in connection with different problems in condensed matter physics such as high-\( T_c \) superconductivity, superfluidity in \(^3\)He-\(^4\)He mixtures, fermionic superfluidity in magnetic traps and so on.

In high-\( T_c \) superconductivity this model was firstly proposed by J.Ranninger to describe simultaneously high transition temperature and short coherence length of SC pairs on one hand and the presence of well-defined Fermi-surface on the other. Later on P.W.Anderson reformulated this model introducing bosonic degrees of freedom (holons) and fermionic degrees of freedom (spinons), which, according to his ideas, experience in strongly correlated model a phenomena of spin-charge separation.

Since then a lot of prominent scientists try to prove ideas of Anderson in the framework of 2D Hubbard and \( t - J \) models. In this context it is necessary to mention first of all the ideas of Laughlin and Patrick Lee. These ideas are based on an anionic picture or on slave boson method. However, even these nice papers do not contain a rigorous proof of spin-charge separation in the whole parameter region of the phase diagram of high-\( T_c \) superconductors. Moreover, the photoemission experiments and numerical calculations of Maekawa and Eder show that at least at low temperatures the Cooper pairs in high-\( T_c \) materials are very much the same as in ordinary superconductors.

In this paper we show that Fermi-Bose mixture with attractive interaction between fermions and bosons is unstable towards the creation of composite fermions \( f = bc \). Moreover, for low temperatures and equal densities of fermions and bosons the composite fermions are further paired in the quartets \( \langle ff \rangle \). Note that a matrix element \( \langle f \rangle = \langle bc \rangle \neq 0 \) only for the transitions between the states with \( |N_B; N_F\rangle \) and \( |N_B - 1; N_F - 1\rangle \), where \( N_B \) and \( N_F \) are numbers of particles of elementary bosons and fermions, respectively. For superconductive state a matrix element \( \langle ff \rangle \neq 0 \) only for the transitions between the states with \( |N_B; N_F\rangle \) and \( |N_B - 2; N_F - 2\rangle \). Our results are interesting not only for the physics of high-\( T_c \) materials, but also for Fermi-Bose mixtures in magnetic traps where we can easily tune the parameters of the system such as the particle density and the sign and strength of the interparticle interaction.
FIG. 1: The skeleton diagram for the coefficient \( b \) near \( \Psi^4 \) in the effective action. The dashed lines correspond to bosons, the solid lines correspond to fermions.

II. THEORETICAL MODEL.

A model of the Fermi-Bose mixture has the following form on a lattice:

\[
H = H_F + H_B + H_{BF},
\]

\[
H_F = -t_F \sum_{<ij>} c_{i\sigma}^\dagger c_{j\sigma} + U_{FF} \sum_i n_{i\sigma}^F n_{i\sigma}^F - \mu_F \sum_i n_{i\sigma}^F,
\]

\[
H_B = -t_B \sum_{<ij>} b_i^\dagger b_j + \frac{1}{2} U_{BB} \sum_i n_{i\sigma}^B n_{i\sigma}^B - \mu_B \sum_i n_{i\sigma}^B,
\]

\[
H_{BF} = -U_{BF} \sum_{i\sigma} n_{i\sigma}^B n_{i\sigma}^F.
\] (1)

This is a lattice analog of the standard Hamiltonian considered for example in Ref. 13 by Efremov and Viverit. Here \( t_F \) and \( t_B \) are fermionic and bosonic hopping amplitudes, \( c_{i\sigma}^\dagger, c_{i\sigma}, b_i^\dagger, b_i \) are fermionic and bosonic creation-annihilation operators. The Hubbard interactions \( U_{FF} \) and \( U_{BB} \) correspond to hard-core repulsions between particles of the same sort. The interaction \( U_{BF} \) corresponds to the attraction between fermions and bosons. \( W_F = 8t_F \) and \( W_B = 8t_B \) are the bandwidths in 2D. Finally, \( \mu_F \) and \( \mu_B \) are fermionic and bosonic chemical potentials. For the square lattice the spectra of fermions and bosons after Fourier transform read: \( \xi_{\sigma} = -2t_F(\cos p_x d + \cos p_y d) - \mu_F \) for fermions, and \( \eta_p = -2t_B(\cos p_x d + \cos p_y d) - \mu_B \) for bosons, where \( d \) is a lattice constant. In the intermediate coupling case \( W_{BF}/\ln(W_{BF}/T_{0BF}) < U_{BF} < W_{BF} \) the energy of the bound state reads:

\[
|E_b| = \frac{1}{2m_{BF}d^2} \exp \left[ \frac{2\pi}{m_{BF}U_{BF}} \right] - 1,
\] (2)

where \( m_{BF} = m_B m_F/(m_B + m_F) \) is an effective mass, \( W_{BF} = 4/m_{BF}d^2 \) and \( T_{0BF} = 2\pi n_B/m_{BF} \). For simplicity we consider a case of equal densities \( n_B = n_F = n \) which is more relevant for physics of holons and spinons.

Note that in intermediate coupling case a binding energy between fermion and boson \( |E_b| \) is larger than bosonic and fermionic degeneracy temperatures \( T_{0F} = 2\pi n_B/m_B \) and \( T_{0B} = 2\pi n_F/m_F \equiv \varepsilon_F \), but smaller than the bandwidths \( W_B \) and \( W_F \). In this case a pairing of fermions and bosons \( \langle bc \rangle \neq 0 \) takes place earlier (at higher temperatures) than both Bose-Einstein condensation of bosons (or bibosons) \( \langle bb \rangle \neq 0 \) or \( \langle cc \rangle \neq 0 \) and Cooper pairing of fermions \( \langle cc \rangle \neq 0 \). Note that in the case of a very strong attraction \( U_{BF} > W_{BF} \) we have a natural result: \( |E_b| = U_{BF} \), and an effective mass \( m_{BF}^* = m_{BF}U_{BF}/W_{BF} \gg m_{BF} \) is additionally enhanced on the lattice. Note also that the Hubbard interactions \( U_{FF} \) and \( U_{BB} \) satisfy the inequalities: \( U_{FF} > W_F/\ln(W_F/|E_0|) \) and \( U_{BB} > W_B/\ln(W_B/|E_0|) \).

Now let us consider a temperature evolution of the system. It is governed by the corresponding Bethe-Salpeter equation. After analytical continuation \( i\omega_n \to \omega + i0 \) (see Ref. 13) the solution of this equation acquires a form:

\[
\Gamma(q, \omega) = \frac{-U_{BF}}{1 - U_{BF} \int \frac{d^2 p}{(2\pi)^2} \frac{1 - n_F(\xi(p)) + n_B(\eta(q-p))}{\xi(p) + \eta(q-p) - \omega - i0}}
\] (3)

where \( \xi(p) = p^2/2m_F - \mu_F; \eta(p) = p^2/2m_B - \mu_B \) are spectrums of fermions and bosons at low densities \( n_F d^2 \ll 1 \) and \( n_B d^2 \ll 1 \). Note that in the pole of BS-equation enters the temperature factor \( 1 - n_F(\xi(p)) + n_B(\eta(q-p)) \) in contrast
with the factor $1 - n_F(\xi(p)) - n_F(\xi(q - p))$ for two-fermion Cooper pairing and $1 + n_B(\eta(p)) + n_B(\eta(q - p))$ for two-boson pairing. The pole of the Bethe-Salpeter equation corresponds to the spectrum of the composite fermions:

$$\omega \equiv \xi^*_p = \frac{p^2}{2(m_B + m_F)} - \mu_{\text{comp}},$$  \hfill (4)

Note that in Eq. (4) $\mu_{\text{comp}} = \mu_B + \mu_F + |E_b|$ is a chemical potential of composite fermions. Note also that composite fermions are well-defined quasiparticles, since the damping of quasiparticles equals to zero in the case of bound state ($E_b < 0$), but it becomes nonzero and is proportional $E_b$ in the case of virtual state ($E_b > 0$). The process of a dynamical equilibrium (boson + fermion $\rightarrow$ composite fermion) is governed by the standard Saha formula. In 2D case it reads

$$\frac{n_B n_F}{n_{\text{comp}}} = \frac{m_B F T}{2 \pi} \exp \left\{ - \frac{|E_b|}{T} \right\}. \hfill (6)$$

The crossover temperature $T_*$ is defined, as usual, from the condition that the number of composite fermions equals to the number of unbound fermions and bosons: $n_{\text{comp}} = n_B = n_F = n$. This conditions yields:

$$T_* \simeq \frac{|E_b|}{\ln(|E_b|/2T_{BBF})} \gg \{T_{0B}:T_{0F}\}. \hfill (7)$$

Note that in Boltzmann regime $|E_b| > \{T_{0B}:T_{0F}\}$, in fact we deal with the pairing of two Boltzmann particles. That is why this pairing does not differ drastically from the pairing of two particles of the same type of statistics. Indeed, if we substitute $\mu_B + \mu_F$ in Eq. (6) on $2\mu_B$ or $2\mu_F$ we will get the familiar expressions for chemical potentials of composite bosons consisting either from two bosons or from two fermions. The crossover temperature $T_*$ plays the role of a pseudogap temperature, so the Green functions of elementary fermions and bosons acquire a two-pole structure below $T_*$ in similarity with Ref. 20.

By performing the Hubbard-Stratonovich transformation, the original partition function $Z = \int Db Db Dc Dc \exp \{-\beta F\}$ can be written in terms of the composite fermions $Z = \int D\Psi D\Psi^* \exp \{-\beta F_{\text{eff}}\}$. This procedure gives the magnitude of the interaction between the composite fermions. The lowest order of the series expansion is given in Fig. 1. Analytically this diagram is given by:

$$-\frac{1}{2} \sum_n \int \frac{d^2p}{(2\pi)^2} \left\{ G_F(p; i\omega_n F) G_B(-p; -i\omega_n B) + G_F(-p; -i\omega_n B) G_B(p; i\omega_n B) \right\}, \hfill (8)$$

where $G_F = 1/(i\omega_n F - \xi(p))$ and $G_B = 1/(i\omega_n B - \eta(p))$ are fermion and boson Matsubara Green functions, $\omega_n F = (2n + 1)\pi T$ and $\omega_n B = 2n\pi T$ are fermion and boson Matsubara frequencies. In fact this integral determines the coefficient $b$ near $\Psi^4$ in the effective action. Evaluation of integral (8) yields:

$$b \simeq -N(0)/|E_b|^2, \hfill (9)$$

where $N(0) = m_{BF}/2\pi$. The corrections to the coefficient $b$ are presented on Fig. 2. They contain explicitly the T-matrices for boson-fermion and fermion-fermion interactions. In the intermediate coupling case these diagrams are small in a small parameters $f_{BBO} \sim 1/\ln(W_b/|E_b|)$ and $f_{FOB} \sim 1/\ln(W_F/|E_b|)$. So the exchange diagram really provides the main contribution to the coefficient $b$.

The coefficient near quadratic term $\Psi^2$ in an effective action in agreement with general rules of diagrammatic technique (see Ref. 14) is given by:

$$a + cq^2/2(m_B + m_F) = 1/\Gamma(q; 0), \hfill (10)$$

where $\Gamma(q; 0)$ is given by (10). The solution of (10) yields $c = N(0)/|E_b|$, $a = N(0)\ln(T/T_*)$. So in spite of the fact that in reality $T_*$ corresponds to a smooth crossover and not to a real second order phase transition, the effective action of composite fermions at temperatures $T \sim T_*$ formally resembles Ginzburg-Landau functional for Grassman field $\Psi_\alpha$. 
If we want to rewrite the effective action with gradient terms

\[ \Delta F = a\bar{\Psi}\alpha\Psi\alpha + \frac{c}{2(m_F + m_B)}(\nabla\bar{\Psi}\alpha)(\nabla\Psi\alpha) + \frac{1}{2} b\bar{\Psi}\beta\Psi\beta\Psi\alpha \]  

in the form of the energy functional of nonlinear Schrödinger equation for the composite particle with the mass \( m_B + m_F \) we have to introduce the effective order parameter \( \Delta_\alpha = \sqrt{\epsilon}\Psi\alpha \). Accordingly in terms of \( \Delta_\alpha \) the new coefficients \( \tilde{a} \) and \( \tilde{b} \) near quadratic and quartic terms read: \( \tilde{a} = a/c \) and \( \tilde{b} = b/c^2 \). Note that Grassman field \( \Delta_\alpha \) corresponds to the composite fermions and is normalized according to the condition \( \Delta_\alpha^+\Delta_\alpha = n_{\text{comp}} \). Hence the coefficient \( \tilde{b} \) plays the role of the effective interaction between composite particles. From Eqs. (11) and (13) \( \tilde{b} = -1/N(0) \).

This result coincides by absolute value, but is different in sign with the results of with Drechsler and Zwerger\(^{21} \), who calculated in 2D case the residual interaction between two composite bosons each one consisting of two elementary fermions. The sign difference between these two results is due to different statistics of elementary particles in both cases. It is also important to calculate \( b(q) \), where the momenta of the incoming composite fermions equal respectively to \( (q, -q) \). It is easy to find that:

\[ b(q) = -\frac{1}{2} \sum_n \int \frac{d^2p}{(2\pi)^2} \left\{ G_B(p; i\omega_n B)G_F(p; -i\omega_n F)G_B(p + q; i\omega_n B)G_F(p - q; -i\omega_n F) + G_B(p; -i\omega_n B)G_F(p; i\omega_n F)G_B(p - q; -i\omega_n B)G_F(p + q; i\omega_n F) \right\}. \]  

Straightforward calculation for small \( q \) yields in the case of equal masses \( m_B = m_F = m \):

\[ b(q) = -\frac{m}{4\pi(|E_b| + q^2/4m)^2}. \]  

Accordingly:

\[ \tilde{b} = \frac{b}{c^2} \approx -\frac{4\pi}{m(1 + q^2/4m|E_b|)^2}, \]  

where \( |E_b| = 1/ma^2 \). Analogous result in a 3D case was obtained by Pieri and Strinati\(^{22} \). Hence, the four particle interaction has a Yukawa-form in momentum space. Therefore: \( U_4(r) \approx -1/ma^2\sqrt{2r/a}\exp(-2r/a) \) corresponds to an attractive potential with the radius of the interaction equal to \( a/2 \). We can calculate now the binding energy of quartets \( |E_4| \). The straightforward calculation absolutely similar to the calculation of \( |E_b| \) yields:

\[ 1 = \frac{\tilde{b}(m_B + m_F)}{2\pi} \int_0^{2\pi} \frac{q dq}{q^2 + (m_B + m_F)|E_4|}. \]  

Hence:

\[ |E_4| = \frac{4}{a^2(m_B + m_F)\left[\exp\left(\frac{4\pi}{|\tilde{b}(m_B + m_F)|}\right) - 1\right]} \]  

FIG. 2: The corrections to the coefficient \( b \) containing boson-boson and fermion-fermion interactions.
For equal masses \( m_B = m_F \) a coupling constant \( \tilde{b} (m_B + m_F) / 4\pi = 1/2 \) and thus:

\[
|E_4| = \frac{2|E_b|}{(e^{1/2} - 1)} \approx 3|E_b|.
\] (18)

The process of dynamical equilibrium (composite fermion + composite fermion \( \rightleftharpoons \) quartet) is again governed by the Saha formula of the type:

\[
\frac{n_{\text{comp}}^2}{n_4} = \frac{m_4 T}{2\pi} \exp \left\{ \frac{|E_4|}{T} \right\}.
\] (19)

where \( m_4 = (m_B + m_F)/2 \). The number of composite fermions equals to half a number of quartets \( n_4 = n_2/2 \) for the crossover temperature:

\[
T_{4^*}^{(4)} = \frac{|E_4|}{\ln(|E_4|/2T_0)}.
\] (20)

Below this temperature the quartets of the type \( \langle f_i^b i; f_j^b j \rangle \) play the dominant role in the system. Note that \( T_{4^*}^{(4)} > T_4 \), so quartets are dominant over pairs (composite fermions) in all the temperature interval. Note also that the quartets are in spin-singlet state. The creation of spin-triplet quartets is prohibited or at least strongly reduced by the Pauli principle. The triplet \( p \)-wave pairs of composite fermions are possibly created in a strong coupling case \( |E_b| > W \), when the corrections to the coefficient \( b \) given by the diagrams on Fig. 2 are large and repulsive. However in this case the small parameters are absent and it is very difficult to control the diagrammatic expansion.

### III. THREE PARTICLE PROBLEM.

If we consider a scattering process of an elementary fermion on a composite fermion, we get a repulsive sign of the interaction regardless of the relative spin orientation of composite and elementary fermions. The same result in 3D for scattering of elementary fermion on dimer consisting of two fermions was obtained by Shlyapnikov et al.\(^{23}\). However, for a scattering process of elementary boson on a composite fermion, we get an attractive sign of the interaction. Moreover, a fourier-component of the three-particle interaction for \( m_B = m_F = m \) reads in 2D case:

\[
U_3(q) = -\frac{8\pi}{m(1 + q^2 a^2)}
\] (21)

Hence

\[
U_3(r) \sim \frac{1}{ma^2} K_0(r/a) \sim -\frac{1}{ma^2} \sqrt{\frac{a}{r}} e^{-r/a}.
\] (22)

again corresponds to an attractive potential of the Yukawa type, but now with a range of the interaction equals to \( a \). Calculation of the three-particle bound-state energy yields:

\[
1 = \frac{|U_3(0)|}{2\pi} \int_0^{1/a} dq \frac{qdq}{q^2/2m_B + q^2/2(m_B + m_F) + |E_3|}.
\] (23)

Hence for \( m_B = m_F = m \):

\[
|E_3| = \frac{3}{4ma^2} \left[ \exp \left( \frac{3\pi}{m|U_3|} \right) - 1 \right] = \frac{3|E_b|}{4(e^{3/8} - 1)} \approx 1.65|E_b|.
\] (24)

The dynamical equilibrium of the type: composite fermion + boson \( \rightleftharpoons \) trio is governed by the following Saha formula:

\[
\frac{n_{B\text{comp}}}{n_3} = \frac{m_3 T}{2\pi} \exp \left\{ \frac{|E_3|}{T} \right\}.
\] (25)
where $m_3 = m_B(m_F + m_B)/(2m_B + m_F)$. Accordingly, trios dominate over unbound bosons for temperatures $T < T_{c*}^{(3)}$, where:

$$T_{c*}^{(3)} = \frac{|E_3|}{\ln(|E_3|/2T_0)}.$$  \hspace{1cm} (26)

Note that $T_{c*}^{(3)} < T_{c*}^{(4)}$, so trios are not so important as quartets.

As a result for $T < T_{c*}^{(3)}$ there are mostly quartets in the system. The quartets are bose-condensed at the critical temperature: $T_c = T_0/(8 \ln(4/na^2))$ in the case of equal masses. It is important to note that in Feshbach resonance scheme, we are usually in the regime $T \sim T_0$, where quartets prevail over trios and pairs. Note also that octets are not formed in the system because two quartets repel each other due to Pauli principle in similarity with the results of Ref. 21,22.

In conclusion we considered an appearance and pairing of composite fermions in Fermi-Bose mixture with an attractive interaction between fermions and bosons.

In Fermi-Bose mixture our investigations enrich superfluid phase diagram in magnetic traps and are important in connection with recent experiments, where weakly bound dimers $^6$Li$_2$ and $^{40}$K$_2$, consisting of two elementary fermions, were observed. Note that in a magnetic trap it is possible to get an attractive scattering length of fermion-boson interaction with the help of Feshbach resonance. Note also that even in the absence of the Feshbach resonance it is experimentally possible now to create Fermi-Bose mixture with attractive interaction between fermions and bosons. For example in Ref. 28 and 29 such mixture of $^{87}$Rb (bosons) and $^{40}$K (fermions) was experimentally studied. Moreover, the authors of Ref. 28 and 29 experimentally observed the collapse of Fermi-gas with a sudden appearance of fermionic $^{40}$K atoms when the system enters into the degenerate regime. We cannot exclude in principle that it is just manifestation of the creation of the quartets $\langle bc; bc \rangle$ in the system. Note that in the regime of strong attraction between fermions and bosons the phase-separation with the creation of larger clusters or droplets is also possible. Note also that much slower collapse in boson subsystem of $^{87}$Rb atoms can be possibly explained by the fact that the number of Rb atoms in the trap is much larger than the number of K atoms, so after the formation of composite fermions a lot of residual bosons are still present in the system. The more thorough comparison of our results with an experimental situation will be subject of a separate publication. Here we would like to mention only that for experiments performed in Ref. 28 and 29 a 3D case is more actual. In the 3D case an attractive interaction between composite fermions acquires a form

$$\tilde{b}(q) = -\frac{\pi a_{ef}}{m_B F[1 + q^2/2(m_B + m_F)|E_0|]},$$ \hspace{1cm} (27)

where $|E_0| = 1/2m_B a^2$ is a shallow level of a fermion-boson bound state. Note that in the case of the repulsive interaction between two bosons, each one consisting of two fermions, $a_{ef} = 2a$ in the mean-field theory of Haussmann, $a_{ef} = 0.75a$ in the calculations of Strinati et al. and $a_{ef} = 0.6a$ in the calculations of Shlyapnikov et al. The shallow bound state of quartets exists in the 3D case only if

$$a_{ef} > 2\pi a \left(\frac{m_B F}{m_B + m_F}\right)^{3/2}.$$ \hspace{1cm} (28)

For $m_B = m_F = m$: $a_{ef} > \pi a/4$. 

In conclusion we considered an appearance and pairing of composite fermions in Fermi-Bose mixture with an attractive interaction between fermions and bosons.

The problem which we considered is important for theoretical understanding of HTSC materials and for the investigation of Fermi-Bose mixtures of neutral particles at low and ultralow temperatures. In high-$T_c$ superconductors the role of bosons is played by holons, the role of fermions is played by spinons. At high temperatures spinons and holons are unbound. At lower temperatures they are bound in composite fermions and, moreover, the composite fermions are further paired in quartets (singlet Cooper pairs). The radius of the quartets (the coherence length of the Cooper pair) is governed by the binding energy of the quartets $|E_4|$. If $|E_4|$ is larger that $T_0$, then the quartets are local: $p_F a < 1$. Finally for $T_c = T_0/(8 \ln(4/na^2))$ the local quartets are bose-condensed and the system becomes superconductive.

Note that we consider a low density limit $|E_0| \gg T_0$. In the opposite case of higher densities $T_0 \gg |E_0|$, Bose-Einstein condensation of holons or biholons (see Ref. 11,12,24) takes place earlier than a creation of composite fermions and quartets. Such a state can be distinguished from the ordinary BCS-superconductor by measuring a temperature dependence of the specific heat and the normal density.

IV. CONCLUSIONS.
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1. J. Bardeen, G. Baym, and D. Pines, Phys. Rev. 156, 207 (1967).
2. J. Ranninger and S. Robasztkiewicz, Physica B 135, 468 (1985).
3. B. K. Chakraverty, J. Ranninger, and D. Feinberg, Phys. Rev. Lett. 81, 433 (1998).
4. P. W. Anderson, Science 235, 1196 (1987).
5. P. A. Lee and N. Nagaosa, Phys. Rev. B 46, 5621 (1992).
6. P. A. Lee, N. Nagaosa, T. K. Ng, and X. G. Wen, Phys. Rev. B 57, 6003 (1998).
7. R. B. Laughlin, Phys. Rev. Lett. 60, 2677 (1988).
8. A. L. Fetter, C. B. Hanna, and R. B. Laughlin, Phys. Rev. B 39, 9679 (1989).
9. H. Ding et al., Phys. Rev. B 54, R9678 (1996), and references therein.
10. Y. Ohta, T. Shimozato, R. Eder, and S. Maekawa, Phys. Rev. Lett. 73, 324 (1994).
11. W. Hofstetter, J. I. Cirac, P. Zoller, E. Demler, and M. D. Lukin, cond-mat/0204237.
12. J. N. Milstein, S. J. J. M. F. Kokkelmans, and M. J. Holland, cond-mat/0204334.
13. D. V. Efremov and L. Viverit, Phys. Rev. B 65, 134519 (2002).
14. P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985).
15. A. A. Abrikosov, L. F. Gorkov, and I. E. Dzyaloshinskii, Methods of Quantum Field Theory in Statistical Physics (Dover, New York, 1999).
16. L. D. Landau and E. M. Lifshitz, Statistical Physics (Course of Theoretical Physics, Volume 5) (Butterworth-Heinemann, 1999).
17. P. Nozieres and D. Saint James, J. Phys. (Paris) 4, 1133 (1982).
18. M. Yu. Kagan and D. V. Efremov, Phys. Rev. B 65, 195103 (2002).
19. M. Yu. Kagan, R. Fresard, M. Capezzali, and H. Beck, Phys. Rev. B 57, 5995 (1998).
20. M. Yu. Kagan, R. Fresard, M. Capezzali, and H. Beck, Physica B 284-288, 447 (2000).
21. M. Drechsler and W. Zwerger, Ann. Phys. 1, 15 (1992).
22. P. Pieri and G. C. Strinati, Phys. Rev. B 61, 15370 (2000).
23. D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, cond-mat/0300010.
24. E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Physics Reports 315, 199 (1999).
25. R. Haussmann, Z. Phys. B: Condens. Matter 91, 291 (1993).
26. C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Nature 424, 47 (2003).
27. B. G. Levi, Physics Today October, 18 (2003).
28. G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Phys. Rev. Lett. 89, 150403 (2002).
29. G. Mondugno, G. Roati, F. Ferlaino, R. J. Brecha, and M. Inguscio, Science 297, 2240 (2002).