Quantum-enhanced interferometry for axion searches

Denis Martynov and Haixing Miao
University of Birmingham, Birmingham B15 2TT, United Kingdom
(Dated: November 4, 2019)

We propose a table-top experiment to search for axions and axion-like-particles in the galactic halo using quantum-enhanced interferometry. This proposal is related to the previously reported ideas (Phys. Rev. D 98, 035021, Phys. Rev. Lett. 121, 161301, Phys. Rev. D 100, 023548) but enhance the gain-bandwidth product of optical cavities but these schemes were not experimentally demonstrated yet. Authors in [32] found a different approach to increase the range of axion masses in their proposal. They utilize a folded optical cavity with non-degenerate eigen P- and S-polarisation modes. The frequency difference between these polarisations is tuned to a particular axion mass. By changing the frequency between P- and S-pol in the optical cavity, we can increase the range of probed axion masses up to $10^{-8}$ eV for $\sim 10$ m long interferometers.

The key question addressed in this paper is how to tune the setup to a particular axion mass. Authors in [32] propose to change angles of incidence of the laser beam on the cavity mirrors. Indeed, this approach will change the frequency separation between P- and S-polarisation but can also dramatically reduce a quality factor of the optical cavity making it insensitive to the axion field. In this paper, we propose a different solution and introduce a coupled cavity configuration. In our scheme, we scan for axions of different masses in a deterministic way by changing a small longitudinal offset in the auxiliary optical cavity as discussed in Sec I.

In Sec II we analyze quantum noises of our detector. We propose to further enhance its sensitivity by injecting squeezed states of light to the readout port, similar to the gravitational-wave detectors [33, 34]. We consider different lengths of the interferometer and derive constraints on the axion-photon coupling coefficients. The current LIGO, Virgo, and GEO facilities are of particular interest in this paper. Once new third generation facilities, such as Einstein Telescope [35] and Cosmic Explorer [36], are built, the current facilities can be utilized to search for dark matter using a proposed layout. We summarise our conclusions in Sec IV.

I. INTRODUCTION

Laser interferometry found a number of applications in the fundamental physics research. Modern gravitational-wave detectors, such as LIGO [1] and Virgo [2], are extremely precise km-scale instruments which routinely observe signals from black holes [3–5] and neutrons stars [6, 7] in the audio frequency band. Precise laser interferometers are also utilized in opto-mechanics to study quantum behaviour of macroscopic objects, such as opto-mechanical cooling [8, 9] and quantum correlations [10–12]. Precise laser gyroscopes operate on the Sagnac principle [13] and study geophysics [14, 15], test general theory of relativity [16] and improve low-frequency performance of the gravitational-wave detectors [17–19]. Large gyroscopes have recently become sensitive enough to detect the Chandler and the annual wobble of the earth [15].

In 2018, a new configuration of interferometers was proposed to search for axions [20] and axion-like-particles (ALPs) [21] in the local galactic halo [22]. The axion field behaves as a classical field due to its high occupation number and is approximately

$$a(t, z) = a_0 \cos(\Omega_a t + k_a z), \quad (1)$$

where $\Omega_a = m_a c^2 / h$, $m_a$ is the axion mass and $k_a$ is the wavenumber of the axion field. Searches for ALPs [23–24] as possible dark matter candidates become more appealing in the recent years since ultra-sensitive detectors of weakly interacting massive particles, such as XENON [25], LUX [26], and PandaX [27] will reach neutrino background in the near future [28]. The key principle of ALPs searches with laser interferometry proposed in [22] is a difference in phase velocities between left- and right-handed circularly polarized light which propagates in the presence of the ALPs field. This phase difference can be precisely measured using laser interferometers.

The 10 m detector proposed in [29] has a potential to search for axions with masses below $10^{-13}$ eV. This limitation comes from the constant gain-bandwidth product of optical cavities: increase of the interferometer sensitivity on resonance leads to reduction of its bandwidth. Some schemes were proposed in the literature [30–31] to enhance the gain-bandwidth product of optical cavities but these schemes were not experimentally demonstrated yet. Authors in [32] found a different approach to increase the range of axion masses in their proposal. They utilize a folded optical cavity with non-degenerate eigen P- and S-polarisation modes. The frequency difference between these polarisations is tuned to a particular axion mass. By changing the frequency between P- and S-pol in the optical cavity, we can increase the range of probed axion masses up to $10^{-8}$ eV for $\sim 10$ m long interferometers.

In Sec II we analyze quantum noises of our detector. We propose to further enhance its sensitivity by injecting squeezed states of light to the readout port, similar to the gravitational-wave detectors [33, 34]. We consider different lengths of the interferometer and derive constraints on the axion-photon coupling coefficients. The current LIGO, Virgo, and GEO facilities are of particular interest in this paper. Once new third generation facilities, such as Einstein Telescope [35] and Cosmic Explorer [36], are built, the current facilities can be utilized to search for dark matter using a proposed layout. We summarise our conclusions in Sec IV.

II. OPTICAL LAYOUT

In the proposed experiment we measure a difference in phase velocities between left- and right-handed circularly polarized light which propagates in the ALPs field. This
The phase difference is given by the equation \[ \frac{\Delta \nu_{\text{ph}}}{c} = \frac{g_{\alpha \gamma} \hat{a}}{k}, \] (2)

where \( c \) is the speed of light, \( g_{\alpha \gamma} \) is the axion-photon coupling coefficient, \( k \) is the wavenumber of the laser field. Eq. (2) implies that the phase accumulated by the left- and right-handed circular polarized light by travelling distance \( L \) in the ALPs field will be different by \( \Delta \phi = \phi_0 \Delta \nu_{\text{ph}}/c \), where \( \phi_0 = 2\pi L/\lambda \) and \( \lambda \) is the laser wavelength.

We propose to measure the phase difference given by Eq. (2) with two folded optical resonators: main and auxiliary cavities (see Fig. 1). The main optical cavity is pumped with a strong laser field with frequency \( \omega_l \) in the P-polarisation which is partially converted to the field in the S-polarisation by the ALPs field inside the main cavity. The frequency separation by the pump and signal fields \( \omega_p - \omega_s \) is determined by the ALPs frequency \( \Omega_s \). The key challenge is to build a cavity which amplifies both pump and signal fields for a set of ALP masses.

In this section we discuss how the signal in the S-polarisation is produced using Jones calculus, discuss how the auxiliary cavity tunes eigen mode \( \omega_s \) of the main cavity, derive fields in the main cavity and discuss the measurement technique.

A. Propagation of the fields in the main cavity

We now consider how linearly polarized light propagates in the ALPs field between two points separated by distance \( L \). We adopt Jones calculus with the electric field vector given by \((E_p, E_s)^T\), where \( E_p \) and \( E_s \) are the horizontal and vertical components of the field. The Jones matrix \( P \) for propagation of light in the ALPs field is given by the equation

\[
P = A^{-1} \begin{pmatrix} e^{i(\phi_0 + \Delta \phi/2)} & 0 \\ 0 & e^{i(\phi_0 - \Delta \phi/2)} \end{pmatrix} A \approx \begin{pmatrix} 1 & \Delta \phi/2 \\ -\Delta \phi/2 & 1 \end{pmatrix},
\]

(3)

where matrices \( A \) and \( A^{-1} \) convert electric fields from the linear to circular bases and back. In Eq. (3) we assume that the distance \( L \) satisfies \( e^{i\phi_0} = 1 \) and \( \Delta \phi \ll 1 \). Eq. (3) implies a slow rotation of the polarisation angle of a linearly polarized light in the ALPs field.

Phase shift \( \Delta \phi \) can be amplified in a high-finesse optical cavity. However, since P- and S- polarisation acquire phase difference of \( \pi \) upon reflection from a mirror under normal angle of incidence, rotation of the polarisation angle will be cancelled after the round trip propagation of the field inside the linear optical cavity. Mathematically, the round trip Jones matrix for a linear cavity is \( R_1 P R_2 P = I \), where \( R_1 = R_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \) are Jones matrices for the mirrors at normal incidence.

In order to accumulate \( \Delta \phi \) over many bounces inside an optical cavity, we introduce folding in the cavity as shown in Fig. 1. Distance between the mirrors 1 and 4 and mirrors 2 and 3 is significantly smaller compared to the distance between mirrors 1 and 2 and mirrors 3 and 4. Therefore, we can neglect any rotation of the polarisation angle between these mirrors and the round trip Jones matrix is given by equation

\[ Q = M_1 M_4 P M_3 M_2 P, \]

(4)

where matrices \( M_1, M_3, M_4 \) correspond to reflection of the laser field from each of the four mirrors. Matrix \( M_2 \) describes reflection of the laser light from auxiliary cavity. We can express Jones matrices of the mirrors as

\[ M_i = \begin{pmatrix} e^{i \theta_i} & 0 \\ 0 & e^{i \alpha_i} \end{pmatrix}, \]

(5)

where \( \theta_i \) and \( \alpha_i \) are phases accumulated by the P- and S-polarised beams during propagation inside optical coatings of the mirrors.

In general, \( \theta_i \neq \alpha_i \) since reflectively of each coating layer is different for P- and S-polarisation according to the Fresnel equations. This inequality leads to non-generate frequencies of the P- and S-polarised modes \( \omega_p \neq \omega_s \). We propose to design stacks of the optical coating such that \( e^{i(\theta_i + \theta_3)} \approx e^{i(\alpha_1 + \alpha_3)} \) and \( e^{i(\theta_2 + \theta_3)} \approx e^{i(\alpha_2 + \alpha_3)} \). In this case, \( M_1 M_4 = I \) and \( M_2 M_3 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i \beta} \end{pmatrix} \)

where \( \beta \) is an extra phase accumulated by the S-polarised beam inside the auxiliary cavity.

B. Auxiliary cavity

We now discuss how the frequency difference \( \omega_p - \omega_s \) is dynamically tuned by the auxiliary cavity. If phases accumulated by the fields in P- and S-polarisation are \( \xi_p \) and \( \xi_s \) then the reflection coefficient from the auxiliary cavity is given by the equation

\[ r_{p,s} = \frac{-r_2 + e^{i \xi_{p,s}}}{1 - r_2 e^{i \xi_{p,s}}}, \]

(6)

FIG. 1. Layout of the proposed experiment which consists of the main and auxiliary optical cavities. Digits show number of the mirrors accoring to the discussion in the text.
where \( r_2 \) is the amplitude reflectivity of the mirror 2. Fig. 2 shows the argument of \( r_2 \) for different phases accumulated in the auxiliary cavity. We control this cavity such that \( \xi_p \approx \pi \). In this case, \( r_p = -1 \) even for small changes of \( \xi_p \). S-polarised beam is close to the resonance in the auxiliary cavity \( \xi_s \ll 1 \) and we can write the argument of \( r_s \) as

\[
\beta = \frac{4}{T_2} \xi_s. \tag{7}
\]

Small tuning of \( \xi_s \) leads to significant shift of the S-polarised beam phase in the main cavity \( \beta \). We use this procedure to control the frequency of the eigen mode \( \omega_s \) as discussed below.

### C. Resonating fields

We use Eq. (4) and (7) to calculate the round trip Jones matrix \( Q \) for P- and S-polarised fields in the main cavity:

\[
Q = \left( \begin{array}{c} 1 \end{array} \right) \frac{\Delta \phi}{e^{i\beta}(1 + e^{i\beta})}, \tag{8}
\]

We propose to pump the cavity with the P-polarised light which builds up in the main cavity and does not resonate in the auxiliary one. In the further analysis we neglect the time dependence of the pump field since it is not affected by the ALP field. S-polarised light builds up in the main cavity due to the ALPs field according to the equations

\[
E_{p,\text{cav}} = \sqrt{B_p} E_{p,\text{in}},
\]

\[
E_{s,\text{cav}}(t) = E_{s,\text{cav}}(t - \tau) e^{i\beta(1 - \frac{Y}{2}) - \Delta \phi(t)} E_{p,\text{cav}}, \tag{9}
\]

where \( B_p \) is the power build-up factor of the pump field in the main cavity. \( Y \) is the total round trip loss of the S-polarised light and \( \tau \) is the round trip time of the main cavity. Fourier transform of Eq. (9) for the S-polarised light is given by equation

\[
E_{s,\text{cav}}(\Omega) = a_0g_{a\gamma}L \rho_{DM} \sqrt{(1 - \Omega_s)^2} E_{p,\text{cav}}, \tag{10}
\]

which implies that \( E_{s,\text{cav}} \) is resonantly enhanced if the following condition is satisfied

\[
\Omega = \Omega_s = \frac{\beta}{\tau}, \tag{11}
\]

which implies that the frequency of the axion is equal to the frequency separation of the P- and S-polarisation eigen modes in the interferometer. The bandwidth of the resonance is \( \Delta f = \frac{\gamma L}{2\pi} \). Resonant amplification of the S-polarised light is schematically shown in Fig. 3.

### D. Optical readout

The signal field in the S-polarisation can be measured in transmission of the mirror 4 using heterodyne readout. P-polarised field serves as a local oscillator in our readout scheme. Half waveplate is introduced at the readout port to convert a small fraction of the P-polarised light to S-polarised light \( P_{4,\text{LO}} = 2P_{p,\text{cav}}T_{p,4} \), where \( \zeta \) is the rotation angle of the half waveplate, \( P_{p,\text{cav}} \) is resonating power in the P-polarisation in the main cavity. Power at the readout port at the beat frequency is given by equation

\[
P_{\text{out}} = \frac{2}{Y} a_0g_{a\gamma}L \rho_{DM} \sqrt{P_{p,\text{cav}}T_{4,\text{LO}}}, \tag{12}
\]

where we used the expression for the local dark matter density \( \rho_{DM} = m_\chi^2 g_{a\gamma}^2 / 2 \). Eq. (12) shows that the signal at the readout point of the interferometer is proportional to the length of the optical cavity and axion-photon coupling coefficient. We note that this equation is only valid for \( \omega_s / (2\pi) < 1/\tau \) which implies that light should be able to make more than one round trip in the main cavity during the period of the ALP field.
III. SENSITIVITY

The pump and signal fields follow the same path in the proposed setup. Therefore, displacement noises, such as ground vibrations, coating thermal noises, and gravity gradient noises, will be common to P- and S-polarized beams. The main source of the classical noises comes from intensity fluctuations of the pump beam. These fluctuations will be measured in transmission of the polarising beam splitter (see Fig. 1) and fed back to the laser in a high bandwidth loop.

The main source of quantum noises comes from vacuum fluctuations which enter the interferometer from the readout port and through optical losses inside the cavity. Power spectrum density of the shot noise is given by the equation

\[ S_{\text{shot}} = 2h\nu P_{\text{LO}} e^{-2r}, \]

where \( P_{\text{LO}} \) is power of the local oscillator on the photodetector, \( \nu = c/\lambda \) is the frequency of laser light, and \( r \) is the squeezing factor.

The signal-to-noise (SNR) ratio of the setup is given by 24

\[ \text{SNR} = \frac{P_{\text{out}}}{\sqrt{S_{\text{shot}}}(T\tau_a)^{1/4}}, \]

where \( T \) is the measurement time, \( \tau_a = \frac{2\pi}{m_v\nu^2} \) is the coherence time of the axion field. We use Eqs. (12) and 12 to calculate SNR of our experiment to ALPs in the galactic halo

\[ \text{SNR} = 2\sqrt{\frac{g_{\text{DM}}P_{\text{p,cav}}}{\hbar \nu T_{s,4}}} g_{\gamma\gamma} L(T\tau_a)^{1/4}, \]

where \( T_{s,4} \equiv 10^{14} \) is power transmission of the mirror 4 for the S-polarised light.

Sensitivity of the experiment to the axion-photon coupling \( g_{\gamma\gamma} \) is shown in Fig. 4 for SNR=1 and different lengths of the interferometer. We choose integration time for each ALP mass \( T \gg \tau_a \) and according to 32, 37, 38, 39, we scan over a range of ALPs masses by changing longitudinal offset of the auxiliary cavity. Every step we shift the frequency difference between eigen modes of the main cavity by its bandwidth to keep resonance enhancement of the signal field in the S-polarisation. Given the total integration time of 1 year, we allocate the integration time logarithmically among the frequency bins.

The key property of this proposal is that the setup does not require strong magnets. Instead, ALPs dark matter converts the strong optical field in one polarisation into a field with an orthogonal polarisation. This property implies that the current gravitational-wave facilities can host dark matter detectors in the future. Once new third generation facilities, such as Einstein Telescope and Cosmic Explorer, are built we can use the current facilities to search for dark matter.

IV. CONCLUSIONS

We proposed a new interferometer which consists of two coupled cavities and can be tuned to search for ALPs in the mass range \( 10^{-17} \) up to \( 10^{-8} \) eV. The sensitivity of the interferometer is enhanced using squeezed states of light similar to the gravitational-wave detectors. Further steps include building a table-top prototype which can already improve over CAST limits in the ALPs mass range from \( 10^{-17} \) up to \( 10^{-8} \) eV. Once the technology is tested, the detector length can be scaled up. In particular, current gravitational-wave facilities are of significant interest of this experiment since it does not require strong magnets.

ALPs searches in the km-scale facilities have a potential to improve over CAST limits by 5-8 orders of magnitude in the mass range \( 10^{-17} \) up to \( 10^{-10} \) eV or detect physics beyond the standard model. Current gravitational-wave facilities are ideal locations for axion interferometry since they already have vacuum envelopes, seismic isolation system, and high-power lasers. New third generation gravitational-wave facilities will not only reach cosmological distances but also create oppor-

![Fig. 4. Sensitivity of the proposed experiment to the axion-photon coupling coefficient after one year of integration and scanning through the ALP masses. The prototype is a table-top experiment with length of 2 m. Existing limits from CAST 39 are shown for comparison.](image-url)
tunities to use existing facilities for new experiments with dark matter.

ACKNOWLEDGMENTS

We thank Matthew Evans and Hartmut Grote for useful discussions. D.M. and H.M. acknowledge the support of the Institute for Gravitational Wave Astronomy at University of Birmingham.
