Finite-element analysis of the fluid temperature distribution in double U-tube Borehole Heat Exchangers

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Abstract. In the evaluation of Thermal Response Tests (TRTs) and in the design of Borehole Heat Exchanger (BHE) fields, the mean temperature of the fluid $T_m$ is usually approximated by the arithmetic mean $T_{ave}$ of inlet and outlet temperatures. This approximation can yield errors in the estimation of the thermal conductivity of the ground, of the BHE thermal resistance, and of the heat pump performance. An expression for the evaluation of $T_m$ has been proposed by Marcotte and Pasquier (Marcotte D, Pasquier P 2008, Renewable Energy, 33 2407) for single U-tube BHEs. In this paper, the difference between $T_m$ and $T_{ave}$ is determined by 3D finite-element simulations for a typical double U-tube BHE in 6 unsteady working conditions. The results are validated qualitatively through an approximate analytical method, and show that the expression proposed by Marcotte and Pasquier underestimates the difference between $T_m$ and $T_{ave}$ when applied to a typical double U-tube BHE. Therefore, new relations to evaluate this difference for double U-tube BHEs would be useful.

1. Introduction
One of the most effective techniques to reduce the use of fossil fuels in building operation is the installation of heat pumps for space heating and cooling. In particular, the installation rate of ground-source heat pumps (GSHPs), which have higher efficiency with respect to air-source heat pumps, is increasing in several countries. According to Lund and Boyd [1], the worldwide installed thermal power of GSHPs increased from 1.854 GW to 49.898 GW from 1995 to 2015. Among GSHPs, Ground-Coupled Heat Pumps (GCHPs) appear as the most promising for future developments, because of their applicability even where regional laws do not allow groundwater extraction. The most diffuse GCHP systems employ vertical ground heat exchangers, also called Borehole Heat Exchangers (BHEs). A BHE is usually composed of either a single U-tube or a double U-tube in high-density polyethylene. A typical external diameter of each tube is 40 mm for single U-tube BHEs and 32 mm for double U-tube BHEs. The length of a BHE is usually between 50 and 150 m, and the most common diameter is about 15 cm. Coaxial BHEs, with an internal circular tube contained within a larger one, are less diffuse. After drilling and inserting the polyethylene tubes, a U-tube BHE is usually filled with a proper sealing grout.

The design of a BHE field requires the knowledge of the undisturbed ground temperature $T_g$ and of the thermal properties of the ground. Except for very small BHE fields, $T_g$, the thermal conductivity $k_g$ and
the thermal diffusivity $\alpha_g$ of the ground, as well as the BHE thermal resistance per unit length $R_b$, are determined through a Thermal Response Test (TRT) performed as recommended by ASHRAE [2]. In a TRT, after the measurement of $T_g$, hot water produced by electric resistances is circulated through the tested BHE and heat is transmitted to the ground. The basic monitored quantities are the heating power per unit BHE length $q_l$, the inlet water temperature $T_{in}$ and the outlet water temperature $T_{out}$. The evaluation of a TRT is usually performed by the infinite-line-source approximation model, which employs as input data $q_l$ and a plot of the mean fluid temperature $T_m$ versus the natural logarithm of time $t$ [3-5]. In this method, one assumes that the mean fluid temperature is given by the arithmetic mean of $T_{in}$ and $T_{out}$ that we denote by $T_{ave}$. Some authors have pointed out that, on account of the internal thermal short-circuiting, a relevant difference between $T_m$ and $T_{ave}$ can occur [6, 7] and can yield an overestimation of the BHE thermal resistance [6] as well as an underestimation of the thermal conductivity of the ground [7]. Therefore, a correct estimation of $T_m$ from measured values of $T_{in}$ and $T_{out}$ could be useful in the evaluation of TRTs. Another technical problem in which the knowledge of the relation between $T_{in}$, $T_{out}$ and $T_m$ would be useful is the hourly simulation of ground-coupled heat pump systems. In fact, accurate analytical expressions of dimensionless response functions of BHEs, called $g$-functions, are now available [8, 9] and can be used to determine the time evolution of $T_m$ by fast computation codes. Since the difference between $T_{in}$ and $T_{out}$ can be easily determined by an energy balance, the knowledge of the difference between $T_m$ and $T_{ave}$ could allow to determine an accurate value of $T_{out}$, the relevant parameter to determine the heat pump COP. Marcotte and Pasquier [6] performed finite-element evaluations of the mean fluid temperature of single U-tube BHEs in different working conditions and proposed an approximate expression, called p-linear average, to evaluate $T_m - T_g$ as a function of $T_{in} - T_g$ and $T_{out} - T_g$. Li et al. [7] studied the short-circuiting thermal resistance between legs in single U-tube BHEs by means of 2-D numerical simulations in steady state conditions, implemented in COMSOL Multiphysics. Then, they investigated by 3-D simulations the loss of heat transfer between a single U-tube BHE and ground due to thermal short-circuiting. Finally they showed that, when a TRT is evaluated through the infinite-line-source model, the thermal short-circuiting can cause a relevant underestimation of the thermal conductivity of the ground, especially if the flow rate is low. Only single U-tube BHEs were considered in [6, 7]. An analytical study of the temperature distribution both in single and in double U-tube BHEs was presented by Zeng et al. [10]. The authors considered steady-state heat transfer within the BHE and assumed that the temperature $T_s$ of the external surface of the BHE is uniform. They neglected the heat conduction in the vertical direction, but considered the energy balance in the vertical direction for the fluid flow. By this scheme, they determined analytical expressions of the distribution of the difference between the fluid bulk temperature along the channels and $T_s$, both for single U-tube and double U-tube BHEs. Although interesting, these expressions cannot be used directly for practical calculations of $T_m$, because $T_s$ is not known. The aim of this paper is to determine typical values of the difference between $T_m$ and $T_{ave}$ which occur in double U-tube BHEs working in heating mode. The study is performed by employing a 3D finite-element simulation code implemented in COMSOL Multiphysics. The results are compared with those determined by applying the p-linear average proposed in [6] and the analytical method proposed in [10].

2. Numerical method
A double U-tube BHE commonly employed in Northern Italy was considered, with tubes in high density polyethylene PE 100 and the following geometrical parameters: diameter 0.152 m, length 100 m, tubes with external diameter 0.032 m and internal diameter 0.026 m, distance between the axes of opposite tubes 0.085 m. A sketch of the BHE cross section is reported in Figure 1. The inlet fluid temperature was assumed as constant, and an operation time of 100 hours was considered. The analysis was performed for two design choices. In the first choice, the BHE field is designed so that the lowest temperature of the working fluid is 4 °C and the working fluid is water. For this choice, the inlet water temperature was set equal to 4 °C. In the second choice, the BHE field is
designed so that the fluid temperature can be lower than 0 °C and the working fluid is a 20% water-glycol mixture. For this choice, the inlet water temperature was set equal to -2 °C. Three volume flow rates were considered for each choice, namely 12, 16 and 24 liters per minute. The thermophysical properties of the BHE materials (polyethylene and grout), of the ground and of the working fluids are reported in Table 1.

![Figure 1. Illustration of the BHE cross section.](image)

|                         | Water                   | Water/Glycol 20% mixture | PE100 tube | Grout | Ground |
|-------------------------|-------------------------|--------------------------|------------|-------|--------|
| Thermal conductivity [W/(m K)] | 0.569                   | 0.501                    | 0.4        | 1.6   | 1.8    |
| Heat capacity per unit volume [MJ/(m³ K)] | 4.207                   | 4.009                    | 1.824      | 1.600 | 2.500  |
| Dynamic viscosity [mPa s]        | 1.567                   | 3.430                    | -          | -     | -      |

The ground around the BHE was modeled as a cylinder with a diameter of 20 m and length of 110 m, coaxial with BHE and containing it. The lateral and bottom surfaces of the ground were considered as adiabatic, and the top surface, at ground level, was assumed as isothermal at 4°C. As initial condition, the whole domain was set at the undisturbed ground temperature, which was assumed as varying with the depth $z$ according to the equation

$$T = 4 + z, \text{ for } 0 \leq z \leq 10 \text{ m}; \quad T = 14 + 0.03 \ z, \text{ for } z > 10 \text{ m}.$$  

Equation (1) takes into account a geothermal gradient of 0.03 °C/m starting from $z = 10$ m and has the mean value 14.715 °C from the ground surface to a depth of 100 m.

A 3-D unsteady model was implemented in the finite-element code COMSOL Multiphysics. The conjugate conduction-convection heat transfer problem in the tubes was studied by means of the Pipe Flow Module. The time-dependent problem was solved with time steps varying from 0.1s to 3600s.

In order to check the mesh independence of results, computations were performed with three different unstructured meshes, denoted as Mesh 1, Mesh 2 and Mesh 3, with increasing numbers of tetrahedral elements up to 346309. Values of the mean fluid temperature at three different times evaluated with these meshes, for water with volume flow rate 16 liters per minute, were compared. The results of the mesh independence check are reported in Table 3. The highest deviation from Mesh 3, which was
adopted for final computations, is 0.042 °C for Mesh 1 and 0.026 °C for Mesh 2. Figure 2 illustrates a part of the adopted mesh for the BHE and the surrounding ground.

Table 2. Results of the mesh independence check: water with volume flow rate 16 liters per minute

| Mesh | Elements | $T_m$ [°C] | Discrepancy from Mesh 3 [°C] |
|------|----------|------------|-----------------------------|
|      |          | 5 h  | 20 h  | 100 h | 5 h  | 20 h  | 100 h |
| 1    | 178561   | 6.539 | 6.007 | 5.626 | 0.042 | 0.030 | 0.024 |
| 2    | 218735   | 6.523 | 5.993 | 5.614 | 0.026 | 0.016 | 0.012 |
| 3    | 346309   | 6.497 | 5.977 | 5.602 | ---   | ---   | ---   |

Figure 2. 3-D mesh of the BHE and the surrounding ground.

3. Results and discussion

Plots of the power extracted from the ground per unit BHE length versus time for the case of water with inlet temperature 4 °C are reported in Figure 3, where $q_{12}$, $q_{16}$ and $q_{24}$ denote the power obtained with a volume flow rate of 12, 16 and 24 liters per minute, respectively. Power values higher than 60 W/m were not reported, to improve the readability of the plots. The figure shows that the power is a decreasing function of time and that a higher flow rate yields a higher power, mainly because the mean fluid temperature is lower. A flow rate increase from 12 to 16 liters per minute yields a power enhancement of about 6% (from 28.20 to 29.94 W/m at $t = 100$ hours); a similar power enhancement (from 29.94 to 31.74 W/m at $t = 100$ hours) is obtained with a flow rate increase from 16 to 24 liters per minute; the latter, however, is a very high flow rate.

Plots of the power extracted from the ground per unit BHE length versus time for the case of water-glycol with inlet temperature -2 °C are reported in Figure 4, with the same symbols. Power values higher than 90 W/m were omitted to improve the readability. A comparison between Figure 4 and Figure 3 shows that the decrease in inlet temperature from 4 °C to -2 °C yields a relevant increase in power extracted from the ground (from 28.20, 29.94 and 31.74 W/m to 42.34, 45.58 and 49.03 W/m, for $t = 100$ hours).
The main results of this study concern the analysis of the difference between the mean fluid temperature $T_m$ and the arithmetic mean of inlet and outlet temperature, $T_{ave}$, which is commonly used as an approximation of $T_m$. The values of $T_m - T_{ave}$ determined through the finite element simulations performed in this investigation have been first compared with those obtainable by applying the p-linear average recommended in [6] to determine the mean fluid temperature for single U-tube BHEs, denoted by $T_{mMP}$, namely

$$T_{mMP} = T_g + \lim_{p \to 0} \frac{p}{(1 + p)} \left[ (T_m - T_g)^p + (T_g - T_e)^p \right].$$

(2)

The input values of $T_{out}$ for Eq. (2) have been taken from the present simulations. The values of $T_m - T_{ave}$ obtained by our finite element simulations have been compared also with the values of $T_m - T_{ave}$ obtainable, for double U-tube BHEs, by applying the analytical method developed in [10].
method yields the temperature distributions in the descending and in the ascending tubes for given: BHE geometry and materials, mass flow rate and specific heat capacity at constant pressure of the working fluid, mean convection coefficient between fluid and pipes, ground thermal conductivity, and mean temperature of the BHE surface. The latter has been deduced, as a function of time, by integrating on the BHE surface the temperature distribution yielded by the present simulations. Reference has been made to the BHE configuration in parallel denoted as 1-3, 2-4 in [10], corresponding to that applied in our simulations. The convection coefficient between fluid and pipes is calculated automatically by the Pipe Flow Module and is not given as an output. For the comparison with the analytical method of Zeng et al. [10] the values of the mean convection coefficient have been deduced through those of the mean fluid temperature, of the mean temperature of the inner surfaces of the tubes, and of the power per unit BHE length.

Plots of $T_m - T_{ave}$ versus time for the case of water-glycol with inlet temperature -2 °C and volume flow rate 12 liters per minute reported in Figure 5, where $T_m - T_{ave}$ without further specifications denotes the values of that difference obtained by our simulations, $(T_m - T_{ave})_{MP}$ denotes the values obtained by applying the p-linear average proposed by Marcotte and Pasquier [6], and $(T_m - T_{ave})_{Z}$ denotes the values obtained by applying the analytical method of Zeng et al. [10].

![Figure 5.](image)

Figure 5. $T_m - T_{ave}$ versus time according to present paper, [6] and [10]: water-glycol with inlet temperature -2 °C and volume flow rate 12 liters per minute

The figure shows an excellent agreement between the values obtained through our simulations and those obtained through the analytical method presented in [10]. On the other hand, the values obtained by applying the p-linear average recommended in [6] are considerably lower, except for a few hours from the operation start. Indeed, the case considered in Figure 5 is that with the highest values of $T_m - T_{ave}$, due to the highest difference between fluid inlet temperature and undisturbed ground temperature and the lowest volume flow rate. In cases with lower values of $T_m - T_{ave}$, some discrepancies were found between the outcomes of our simulations and those of the analytical method developed in [10]. The most relevant discrepancies occurred for water with volume flow rate 24 liters per minute, i.e. in the case with the lowest values of $T_m - T_{ave}$, which is illustrated in Figure 6.
Figure 6. $T_m - T_{ave}$ versus time according to present paper, [6] and [10]: water with inlet temperature 4 °C and volume flow rate 24 liters per minute.

The figure shows that the values obtained through our simulations are the highest, while those obtained through the p-linear average proposed in [6] are the lowest. Although the percent discrepancy between our results and the outcomes of the method of Zeng et al. [10] is considerable, the absolute discrepancy is small: 0.03 °C for $t = 100$ h and nearly constant. It is probably due to a combination of approximations in the analytical method and numerical errors in the evaluation of the mean temperature of the BHE surface, an input needed to apply the method presented in [10].

The values of $T_{out}$, $T_m$, $T_{ave}$, $T_m - T_{ave}$, $(T_m - T_{ave})_{MP}$ and $(T_m - T_{ave})_Z$ for all flow rates, at $t = 5$ hours, $t = 20$ hours and $t = 100$ hours, are reported in Table 3 for water and in Table 4 for the water-glycol 20% mixture. In all cases, especially for $t = 20$ h and $t = 100$ h, the values obtained through the p-linear average are appreciably lower than those obtained by our simulations. Most probably, the relevant differences are due to the different BHE kind and to the higher distance between the centers of opposite tubes considered in [6] (single U-tube BHE instead of double U-tube, 11 cm instead of 8.5 cm).

Since the results obtained here can be considered as validated by the comparison with the outcomes of the analytical method of Zeng et al. [10], it is possible to conclude that the p-linear average expressed by Eq. (2) yields too low values of $T_m - T_{ave}$ with respect to the real ones for a typical double U-tube BHE. Therefore, other expressions for this kind of BHEs seem valuable.

Table 3. Values of $T_{out}$, $T_m$, $T_{ave}$, $T_m - T_{ave}$, $(T_m - T_{ave})_{MP}$, $(T_m - T_{ave})_Z$ for water, with $T_m = 4$ °C

| °C  | 12 liters/minute | 16 liters/minute | 24 liters/minute |
|-----|------------------|------------------|------------------|
|     | 5 h   | 20 h  | 100 h | 5 h   | 20 h  | 100 h |
| $T_{out}$ | 9.089 | 8.088 | 7.354 | 8.174 | 7.304 | 6.679 |
| $T_m$     | 7.199 | 6.570 | 6.108 | 6.497 | 5.977 | 5.602 |
| $T_{ave}$ | 6.544 | 6.044 | 5.677 | 6.087 | 5.652 | 5.339 |
| $T_m - T_{ave}$ | 0.655 | 0.526 | 0.431 | 0.410 | 0.325 | 0.263 |
| $(T_m - T_{ave})_{MP}$ | 0.543 | 0.329 | 0.213 | 0.345 | 0.206 | 0.132 |
| $(T_m - T_{ave})_Z$ | 0.536 | 0.435 | 0.358 | 0.321 | 0.257 | 0.208 |
### Table 4. Values of $T_{out}$, $T_m$, $T_{ave}$, $T_m - T_{ave}$, $(T_m - T_{ave})_{MP}$, $(T_m - T_{ave})_Z$ for water-glycol, with $T_m = -2 \, ^\circ C$

| °C | 12 liters/minute | 16 liters/minute | 24 liters/minute |
|----|------------------|------------------|------------------|
|    | 5 h | 20 h | 100 h | 5 h | 20 h | 100 h | 5 h | 20 h | 100 h |
| $T_{out}$ | 5.856 | 4.385 | 3.284 | 4.572 | 3.245 | 2.278 | 2.869 | 1.812 | 1.064 |
| $T_m$ | 2.804 | 1.905 | 1.231 | 1.887 | 1.102 | 0.530 | 0.753 | 0.156 | -0.267 |
| $T_{ave}$ | 1.928 | 1.192 | 0.642 | 1.286 | 0.622 | 0.139 | 0.434 | -0.094 | -0.468 |
| $T_m - T_{ave}$ | 0.876 | 0.712 | 0.589 | 0.600 | 0.480 | 0.391 | 0.318 | 0.250 | 0.201 |
| $(T_m - T_{ave})_{MP}$ | 0.820 | 0.508 | 0.333 | 0.542 | 0.327 | 0.210 | 0.278 | 0.164 | 0.103 |
| $(T_m - T_{ave})_Z$ | 0.874 | 0.717 | 0.595 | 0.538 | 0.434 | 0.354 | 0.256 | 0.202 | 0.162 |

### 4. Conclusions

Accurate 3D simulations of a typical double U-tube BHE working with a constant inlet fluid temperature have been performed through a finite element model implemented in COMSOL Multiphysics. The model has allowed to determine the temperature distribution along the tubes by taking into account the thermal short-circuiting between the descending and the ascending fluid. The time evolution of the difference between the mean fluid temperature $T_m$ and the arithmetic mean of inlet and outlet temperatures $T_{ave}$ has been determined, for three values of the flow rate and two design conditions: water as working fluid, with inlet temperature $4 \, ^\circ C$; water-glycol $20\%$ mixture as working fluid, with inlet temperature $-2 \, ^\circ C$. The results obtained have been validated by comparison with those determined by applying the analytical method developed by Zeng et al. (Zeng H, Diao N and Fang Z 2003, *International Journal of Heat and Mass Transfer* 46 4467–81) and have been compared with those yielded by the p-linear average proposed by Marcotte and Pasquier for single U-tube BHEs (Marcotte D and Pasquier P 2008 *Renewable Energy* 33 2407–15). The comparison reveals that the p-linear average underestimates $T_m - T_{ave}$ when applied to a typical double U-tube BHE, so that specific expressions for double U-tube BHEs would be useful.

### 5. References

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