Quantum Geodesics

Daniel C. Galehouse

Physics Department, University of Akron, Akron, Ohio 44325

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Abstract

Classical methods of differential geometry are used to construct equations of motion for particles in quantum, electrodynamic and gravitational fields. For a five dimensional geometrical system, the equivalence principle can be extended. Local transformations generate the effects of electromagnetic and quantum fields. A combination of five dimensional coordinate transformations and internal conformal transformations leads to a quantum Kaluza-Klein metric. The theories of Weyl and Kaluza can be interrelated when charged particle quantum mechanics is included. Measurements of trajectories are made relative to an observers’ space that is defined by the motion of neutral particles. It is shown that a preferred set of null geodesics describe valid classical and quantum trajectories. These are tangent to the probability density four vector. This construction establishes a generally covariant basis for geodesic motion of quantum states.

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I. INTRODUCTION

The fifth dimension has been discussed in the scientific literature for seventy years or more. Historically, the preponderance of attempts to use five dimensions combine a gravitational theory similar to Einstein’s with an electromagnetic theory similar to Maxwell’s. The earliest versions were modeled after the general theory of relativity, while later attempts to combine other interactions are considerably more intricate. The integration of quantum ideas into theories has been frustrating and inconclusive. Quantum mechanics is usually appended by attaching a Hilbert space to each region of space-time.

It is suggested here that there is a wholly geometrical way to describe physical phenomena, including quantum mechanics. The historical exclusion of quantum mechanics from fundamental classical theories is deemed arbitrary. It is proposed that the essential elements of quantum mechanics reside within natural geometrical structures. Quantum mechanics becomes an inherent, inseparable part of the mathematics.

The basis of classical unified field theories rests on the work of H. Weyl and T. Kaluza. The association of these ideas with quantum mechanics has been known for some time. Following earlier studies for the Weyl theories, it has become possible to develop a five dimensional geometrical quantum theory.

II. INITIAL ASSUMPTIONS AND CONCEPTS

Several particular assumptions need to be discussed. By stating unusual starting points, it is hoped to save the reader the difficulty of inferring them from the conclusions.

Much of the Einsteinian viewpoint is adopted. Microscopic trajectories are hypothesized as a universal description of the effects of quantum mechanics, electromagnetism and general relativity. While the success of this approach for quantum theory is not yet established, it allows the direct discussion of gravitational phenomena. In accord with the goal
of a wholly geometrical theory, the mathematics is hard deterministic and time symmetric. Quantum probabilities must be associated with a congruence of statistically populated trajectories. Because of the metaphysical conflicts between quantum theory and relativity, certain concepts must be refashioned to allow the synthesis. The vector potential plays a more prominent role. Metrics in higher dimensions have modified interpretations. The hilbert space becomes a calculational artifact and not a starting point.

Electrodynamics is assumed time symmetric, along the lines of the classical articles by Feynman and Wheeler [24]. For geodetic motion that includes electromagnetic effects, the force of radiative reaction must be due to fields derived from other particles. There can be no force of radiation until the interacting (absorbing) particles are included in the field sources. For a quantum gravitational theory, this time symmetric construction must be supposed to exist at a metaphysical level that is deeper than the macroscopic structure of Maxwell. Electrodynamics is assumed to derive from a primitive classical truth of geometry, comparable with general relativity.

In a microscopically deterministic theory, the question of the separation of causal agents from their effects becomes problematical. This does not appear to contradict the experimental situation. A precise definition of causality probably cannot have a microscopic formulation in a theory that is hard deterministic, time symmetric and quantum mechanical. Without a primitive assumption, it is supposed that time asymmetry and the associated causality follow from macroscopic effects that may depend on other conditions in the universe. These conditions may include statistical mechanics, cosmology, psychology or the real physical effects of other interactions [25]. Microscopic time asymmetry is not eliminated in principle but does not contribute to the discussion.

The geometrical concept of a congruence is associated with the quantum mechanical state function of a particle. Wave particle duality is not considered essential. Since emission and absorption is the only known evidence for photons, they are assumed to be calculational artifacts that derive from charged particle quantum mechanics [26]. Finite mass particles, having time like trajectories, are the only true particles. Interactions are mediated by fields.
Any discreteness in the fields (photons or gravitons) derives from the discreteness of the source particles \[^{[27]}\]. It is assumed that the fields have no dynamical qualities except that which is implied by the sources. Fundamental free fields are rejected.

To describe more than one particle, the mathematical field structure must be augmented. Gravity is conventionally approximated by a single metric tensor that specifies the global distortion of space-time. In such an approximation, a collection of particles \(P_1, P_2, \ldots P_k\) moves in a space-time specified by a metric that is a solution of a set of gravitational field equations. This single metric is insufficient when constructing a combined theory of electromagnetism and quantum mechanics. The structures are not analogous. These latter effects are described by fields \(A_{(1)\mu}, A_{(2)\mu}, \ldots A_{(k)\mu}\) and \(\psi_{(1)}, \psi_{(2)} \ldots \psi_{(k)}\), one each for each particle. Of course the approximation of a universal metric for a collection of simple particles is usually valid, particularly in the laboratory. However when quantum mechanics and electrodynamics are integrated with gravity, an analogous collection of metrics \(g_{(1)\mu\nu}, g_{(2)\mu\nu} \ldots g_{(k)\mu\nu}\) is required. These metrics can be treated as equal under many experimental situations; but in principle, even microscopic gravitational interactions are present and provide interparticle forces. Consequently each particle must have a formal wave function, vector potential and metrical tensor, \((g_{\mu\nu}, A_\mu, \psi)\). An extended principle of equivalence is made possible since the motion of each particle can be the result of different combinations of assigned fields. A special metric \(\hat{g}_{\mu\nu}\), identified with the observer of general relativity, is maintained separately from the individual metrics of the particles. It represents the structure of space-time as measured with neutral particles and the idealized null trajectories of electromagnetic interactions. Multiple metrics have been used at least since Dirac \[^{[28]}\] and are implied by the conformal invariance of Weyl \[^{[22]}\]. Other multiple metric theories of gravitation are listed by Ni \[^{[29]}\].

A classical observer does not perceive quantum mechanics or electromagnetic fields to be part of space-time geometry. If these effects are to be intrinsic, a separate phenomenology of charged and neutral particles must be defined. Hypothetical mirrors are assumed to be neutral objects to the observer. Forces of radiative reaction on a mirror are neglected or
compensated during gravitational space-time measurements and the individual charged particles which make up the mirror are treated collectively. The observer’s metric $\dot{g}_{\mu\nu}$ is used to describe the perceived pattern of the collective motion of many particles. As a mathematical object, it retains multiparticle phenomenology and cannot be measured by using a single particle wave function or quantum state. Observations depend on idealized measurements of composite particles and clocks that traverse each point with different velocities [30]. Each such independent direction requires at least one wave function [31]. Four dimensional space-time, must be derived in this way from a larger geometry as a limit or approximation. Stable, massive, neutral, primitive (non-composite) particles are required. Since there are none known, the phenomenological metric may not have a basis in the motion of elementary particles but may only have a well defined meaning as a description of collective motion.

An increase in dimensionality always leads to new quantities and interpretations. It is supposed that each particle, no matter how it might be described ultimately, projects onto a time like trajectory in the observer’s space-time. Thus, the directly observed dimensionality is always three plus one. The effects of new coordinates must be inferred. A similar problem occurs in the transition to four dimensions. Starting with three space dimensions, the metric assigns positions and distances to palpable objects. The pythagorean theorem can be verified (or falsified) by direct measurement. Relativity demonstrates this space to be incomplete. It must be extended to include time. In doing this, the developed procedure of using light beams to find the components of the four metric, is not even qualitatively equivalent to the use of a simple ruler. In space-time, the experimental determination of geodesics is completely different. Presumably, any further extension beyond four dimensions will involve crucial conceptual modifications. By analogy, one would expect five dimensional measurements to be even less pythagorean than four dimensional measurements.

Modifications in the metrical interpretation mitigate the need for cylindricity or dynamical compactification. Instead, the use of null five-vectors is sufficient. Arbitrary non-null five vectors can be constructed but they are not used here to represent physical quantities. As the geodesics of light are null in four-space, the geodesics of particles are null in five-space.
This seems to be important for the relationship between Weyl and Kaluza-Klein theories as well as for the proper inclusion of quantum effects. If the velocity vector is null, then the motion can be represented by four parameters alone. The essential vector length variations of Weyl theory are incompatible with a Riemannian theory unless the vectors map into null Riemannian vectors. A fractional change in a Weyl vector can be mapped onto a null vector because a fraction change in a null Riemannian vector is permitted. In this way, transformations within the Riemannian system can be related to apparent conformal effects. A non-null extension may be possible but such vectors cannot be related to a Weyl theory. A rigid metrical five dimensional structure that is analogous to our perception of three-space is assumed in many other theories. These, even when followed by the application of spontaneous compactification will not allow for the implicit inclusion of quantum mechanics.

Because of the physics of geometrization, the construction of lagrangians has not been found useful. The physical quantities that have reasonable covariance properties are often null or unavailable and the usual suppositions lead to quantities which are identically zero. Lagrangian mechanics was originally motivated by the need to extend calculations to situations involving constraints, such as rigid bodies or contact forces. This motivation is unjustified in relativistic theories since rigid bodies do not exist and contact elasticities are finite. Furthermore, the origin of these contact constraints is directly from the quantum mechanics of collectively interacting particles. This questions the epistemological assignment of lagrangians as a basis for quantum mechanics. There is no apriori reason to believe that theories developed to describe constraints, should provide a basis for microphysics. The convenience of choosing interaction terms in a lagrangian increases descriptive strength but reduces predictive power. It has been found necessary and advantageous to choose differential equations without recourse to any construction of classical mechanics.

A few less controversial questions may be worth mentioning. A full discussion of second quantization is not made here. Such a formalism would represent the description of more than one particle, possibly including particle creation and annihilation. These complexities
will have to be deferred until the one particle system is better understood. At present, only a partial version of a geometrically appropriate quantum electrodynamics is available. A complete systematic discussion is needed. Spin, weak and strong interactions are also deferred. An understanding of elementary quantum methods is deemed a necessary prerequisite.

A number of other constructions found in the literature have not been incorporated. Magnetic monopoles are not used. Five dimensional monopole solutions are supposed non-physical and may be mathematical ghosts [32]. Strings [33] may show similarities but do no follow the formalism. There are no discrete lattices. Solitons are not used. Space-time is not treated stochastically [34]. There is no torsion [35]. There is no topological compactification, dynamical or otherwise [18]. Because of the difficulty of precisely defining the difference between quantum and non-quantum theories, classical mechanics is intended to refer to the limit, as $\hbar \to 0$ and not to any fundamental classical theory.

These particular assumptions should provide a usable basic starting point. Surely reality is more complicated.

III. NEUTRAL FIVE-SPACE

It is useful to associate the fifth dimension of a five dimensional Riemannian space with the proper time. Let the defining equation for $d\tau$ be written as:

$$\dot{\gamma}_{mn}dx^m dx^n = 0 \quad (1)$$

where $dx^5 \equiv d\tau$, $\dot{\gamma}_{55} = -1$, $\dot{\gamma}_{5\mu} = 0$ and $\dot{g}_{\mu\nu} = \dot{\gamma}_{mn}$. The Einstein summation is in effect. Lower case greek indices are summed over four values and lower case latin indices are summed over five. Equation (1) is intended to apply to the standard observer of four dimensional space-time. It is proposed that nullity of five displacements is maintained for primitive particles even when off diagonal terms are appended.

A neutral particle has a fifth coordinate defined by the path integral of $d\tau = dx^5$. The space-time coordinates define $\tau$, up to an additive constant. Further constructions will
indicate that electromagnetic or quantum effects occur when $\gamma_{\mu 5} \neq 0$. It is supposed that the nullity of displacements for charged neutral particles is maintained in the presence of the electromagnetic field. The particle intrinsic fifth coordinate will not necessarily be the same as the neutral space proper time. In this case, an inferred physical comparison of $\gamma_{\mu 5}$, $\gamma_{55}$, and $g_{\mu \nu}$, can determine a physical value for the new terms $\gamma_{m5}$. In the neutral case $\gamma_{55}$ is not well defined and is arbitrarily set to -1. Time like displacements are represented by real proper time values in conventional units.

For classical relativity, the accumulation of differences in the proper time along distinct trajectories is second order. No cross terms, $\gamma_{\mu 5}$, occur. The twins of the twin paradox are neutral objects and experience second order age corrections. The direct observation of first order terms, which are only assigned to primitive particles, is not possible because a real physical clock cannot be attached to the five space trajectory.

IV. EQUIVALENCE FOR CHARGED QUANTUM PARTICLES

The usual principle of equivalence is intended, historically, for uncharged classical particles in a four dimensional theory [15,36]. When attempting to extend this, the existence of particles with different charge to mass ratios causes an essential difficulty. Even more, quantum diffraction depends inversely on the mass and destroys compositional additivity. A principle of equivalence must therefore be constructed for a single isolated particle characterized by particular charge and mass value.

Particles having different interactions, (electromagnetic or quantum) must use tensors of different internal construction. The particle properties are incorporated into the fields that determine the local particle rest frame. The mass must enter with a factor $\hbar$. As will be shown in a paper to follow, this factor is a scale size of the fifth dimension much as the speed of light is the scale size of the fourth dimension. This construction presumes that the mass ratios of fundamental particles are ultimately derivable geometrical quantities. Factors of $\hbar$ are taken to appear in combination with all masses. This factor also appears in the fine
structure constant because the quantum manifestation of the inertial force is a standard quantity against which the electric forces are measured.

The geometrical extension can be motivated by physical argument. Consider a gedanken experiment performed on two idealized particles. Each of these should be isolated, non-composite, stable, and of finite mass. The first one, denoted by $N$ is neutral and does not respond to electromagnetic fields. The second, denoted by $C$ is charged. It responds in the classical limit according to the Lorentz force law. Following figure (I), the two particles begin on the left with coincident motion. A single rest frame making both particles equivalent can be obtained.

The particles separate as they traverse a region where the electromagnetic field tensor is nonzero. The field in this region can be adjusted so that after exiting, both particles converge to intersect at space-time point $P_2$ with distinct velocities $U^\mu(C) \neq U^\mu(N)$. At $P_2$, the particles cannot be made equivalent even though they were equivalent at $P_1$. The usual sense of the equivalence principle dictates that it should be possible to perform continuous coordinate transformations to find an invariant local rest frame for either particle. Within general relativity, there is no way to do this.

To resolve the paradox, additional geometrical quantities can be used to describe velocities that may have electromagnetic (or quantum) origin. Either a four dimensional non-Riemannian Weyl geometry or a five dimensional Riemannian geometry is possible. These supply additional electromagnetic-geometrical transformations which describe the motion of charged particles without affecting the motion of idealized neutral objects. In this way, a principle of equivalence can be used.

The nullity of the displacement vector, as applied to an individual particle is related to the concept of a Killing vector. It will be seen that the equivalence of the trajectories under displacements is related to the equivalence of the different forces that might cause a deflection. Different parts of the trajectory cannot be equivalent as viewed by the neutral observer because the acceleration appears to have different causative explanations. That is, the effective field and source currents vary along the congruence.
Several studies have been made of the intrinsic relationship of the Aharonov-Bohm effect \[37\] with five dimensional theories \[38\]. As a realizable example, applied to this gedanken experiment, it is particularly interesting since neither the neutral nor the charged particle actually enters the region where the field tensor is non-zero. There is however a difference in deflection which is experimentally observable and cannot be described by the conventional principle of equivalence in space-time.

V. FIXED GAUGE CLASSICAL THEORIES

The metric tensor, $g_{\mu\nu}$, and electromagnetic vector potential, $A_{\mu}$, have known classical geometrical interpretations. It is desirable to adjoin a wave function without a first quantization process. As a first step, the classical equations must be rewritten in a fixed gauge form \[39,21,23\]. The usual gauge freedom is assumed non-fundamental and is transformed to a fixed value specified by making the action identically zero. (Any analogous wave function will have its phase removed.) The standard classical five metric, having signature $(+,-,-,-,-)$ is appropriate and can be written in the form

$$
\gamma_{mn} = \begin{pmatrix}
g_{\mu\nu} - A_{\mu}A_{\nu} & A_{\mu} \\
A_{\nu} & -1
\end{pmatrix}.
$$

(2)

where $A_{\mu} = \frac{e}{m} A_{\mu}$. It is sufficient to show that a velocity defined by

$$
\frac{dx^\mu}{dw} = A^\mu
$$

(3)
defines a classical null geodesic. The absolute derivative of the above is

$$
\frac{d^2 x^\mu}{dw^2} + \left\{ \frac{\dot{\mu}}{\nu \beta} \right\} \frac{dx^\nu}{dw} \frac{dx^\beta}{dw} = \dot{g}^\mu_{\beta} \left[ \partial_{A_{\beta}} \frac{dx^\lambda}{dw} - \left\{ \frac{\lambda}{\nu \beta} \right\} A_{\lambda} \frac{dx^\nu}{dw} \right]
$$

(4)

wherein the Christoffel symbols are calculated with respect to the neutral space four metric $\dot{g}_{\mu\nu}$. This can be converted into the conventional form since the Hamilton-Jacobi equation with fixed gauges is $A_{\mu}A_{\nu}\dot{g}^{\mu\nu} = 1$. The covariant derivative of this equation can be used to eliminate the second Christoffel symbol which gives
\[
\frac{d^2 x^\mu}{d w^2} + \left\{ \frac{1}{\nu/\beta} \right\} \frac{d x^\nu}{d w} \frac{d x^\beta}{d w} = g^{\mu\beta} \left( \frac{\partial A_\beta}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\beta} \right) \frac{d x^\nu}{d w}.
\]  
(5)

Here, the path parameter \( w \) is normalized relative to the neutral space four metric. Direct derivations from the five metric are given in references [8], either by the Lagrangian method or by parallel transport. These agree with the above fixed gauge calculation and give the combined force law. This establishes that the fixed gauge equation of motion (3) is a valid construction in the presence of gravitational and electromagnetic fields.

This description defines a congruence of motion which is the set of integral curves of a fixed gauge vector potential field. Such an individual congruence, which is a general solution of the Hamilton-Jacobi system, can be developed into a quantum state. Selection of a whole congruence need not specify a particular trajectory as the unique trajectory of a particle. To completely specify such a quantum state, a quantum field equation is required. For a scalar field, the Klein-Gordon equation is used. Such a quantum solution, when written in a fixed gauge representation cannot in general have a normalized vector potential as in the classical representation. The magnitude \( A^\mu A_\nu g^{\mu\nu} \) is not constant. This is true even though the unnormalized vector potential still satisfies the inhomogeneous Maxwell equations. To make the system amenable to quantization, it is assumed at this point that the unnormalized vector potential is still suitable as a purveyor of the physical quantity within the geometrical system. This identification can be applied to both the Weyl and Kaluza theories.

The resulting congruence contains, under reasonable conditions and with reasonable physical assumptions, the information that was contained in the original wave function. Any phase information has been incorporated directly into \( A_\nu \), only the magnitude of \( \psi \) need be regenerated. This can often be done, in practical cases from the current conservation alone. Global reconstruction may require recourse to the field equation and boundary conditions. The essential idea is that such a congruence is sufficient to represent the essential part of a single particle quantum state function.
VI. LOCAL FIVE TRANSFORMATIONS

The local structure of the five dimensional coordinate system is larger than the four dimensional Lorentz group. The additional transformations are necessary for the principle of equivalence discussed in section IV. An interpretation can be developed by applying the new transformations as local point operations on a charged particle. A related problem is mentioned in [40].

The usual transformations involving the observer’s coordinates \( x_\mu \) still apply and affect the vector potential covariantly. The transformations involving \( \tau \) are new, and because of the fixed gauge assumption, have specific physical effects and interpretations. They are affine and orthogonality is not maintained. The most useful of these is

\[ x'^\mu = x^\mu + A^\mu \tau \] (6)

and changes the four-velocity of the charged particle according to

\[ \frac{dx'^\mu}{ds} = \frac{dx^\mu}{ds} + A^\mu \frac{d\tau}{ds} \] (7)

The electromagnetic vector potential is thereby identified with a shear transformation. If the arbitrary path increment \( ds \) is selected equal to the apparent proper time change, \( dw \), the transformation (6) gives a new velocity for the particle. Such transformations represent a realignment of the five dimensional space. It is used to describe velocity changes that are of quantum or electromagnetic origin as distinguished from those that are gravitational.

Using \( p^\mu = mv^\mu \) for the kinetic momentum,

\[ p_\mu = p'_\mu - mA_\mu \] (8)

identifies the quantum nature of the transformation because of the similarity with the minimal substitution. This assumption identifies the motion as a first order quantity relative to the neutral system. A Lorentz transformation, as a contact transformation in space-time, changes the velocity of the particle and also the value of \( A_\mu \). The new value represents the instantaneous particle motion and also the local effective transformed orientation of the five
dimensional space. The electromagnetic effects are not modeled by Lorentz group elements but by the new aspects of the five dimensional coordinate system.

The gedanken experiment of section (IV) supports this interpretation. Within the interaction region of figure (1), five dimensional effects occur which cause a deflection of the particle relative to the unvarying neutral geodesics. Once free from the interaction, and arriving at $P_2$, the integrated effects of the electromagnetic field leave a residual velocity $U_2(C)$ that is distinct in ontology from the neutral velocity $U_2(N)$. This is implicitly defined by the orientation of the five dimensional system, which has an internal geometrical distortion that is not represented in neutral space-time. This integrated effect persists outside of the interaction region. It can, in fact, describe Aharonov-Bohm type effects. The velocity of the charged particle cannot be reduced to the velocity of the neutral particle by a space-time Lorentz transformation. A similar discussion can be made for a fixed gauge Weyl theory. The charge particle motion is varied by the non-Riemannian part of the connections which are outside the range of space-time Lorentz transformations.

VII. INERTIA AND MEASUREMENT

An important property of this description of a quantum state is that there is no formal identification of the mass. Moreover, various different field values $A_{\mu}$ can correspond to the motion of a particle having possibly continuously varying value. There is no inertial frame because arbitrary transformations are allowed. The motion is inertia free. This mathematical concept of motion is important to allow the resolution of the very different concepts of inertia that prevail for the interactions studied. The relation between the physical inertia and its numerical representation is not the same for classical, quantum, and gravitational theories. For gravity, inertial effects are independent of the value of the mass, as long as it is nonzero. Electromagnetism has a characteristic $e/m$ value and quantum mechanics scales inversely.

For conventional derivations of quantum mechanics, the inertial properties of particles
are assumed to be intrinsic and are eventually introduced through the classical hamiltonian or lagrangian. This, prevents the identification of the mass with the field equations. This epistemology must be changed in a geometrical construction. The inertia is fundamental and originates with the quantum processes of diffraction and interference. These effects must come from the quantum geometrical system without the intervention of classical physics.

Measurement of the inertia of a particle must be considered fundamentally quantum. The classical observation is the good fortune of a simple experiment. It is in reality a measure of the ratio of the Compton wavelength of the test particle to one or more Compton wavelengths in the measuring apparatus. In the classical limit, the diffraction effects become negligible and inertia, as is classically understood, remains.

The observation of inertia depends on having quantum stabilized dimensional standards. The observers coordinate system must be calibrated in a consistent, systematic manner \[41\]. Even the elementary demonstration of Newton using a bucket of water or spheres connected by a cord refers to sized objects. A single length standard is represented by a standard clock that is quantum based and measures in terms of $\hbar/mc^2$. A bouncing light beam clock requires that the mirrors be held at an absolute space-like distance. An apparatus to hold the mirrors must be built with atoms, or other quantum objects of constant size determined equivalently by values of $\hbar/mc$.

The only alternative time scale is one based on the Planck length. The relationship is addressed in the work of Dirac and others on large numbers \[12,17\]. It is supposed here that the Planck length may be cosmological rather than fundamental. The relationship of the Planck clock to the atomic clock remains a goal of theoretical and experimental research \[13\]. It is assumed that the fine structure constant $\alpha$ is an invariant and that any of the time scales which can be formed by $\alpha^k(h/m)$ are the same. For such situations as $\alpha$ might be allowed to vary, $h/m$ is taken as the fundamental quantity. Other interactions, weak and strong, are presumed unessential to chronometry. Figure (2) shows a quantum clock that avoids these problems and provides a theoretical time standard.

Because the mass is taken out of the classical arena, the phenomenology of spacelike
measurements must be carefully reconsidered. The accepted construction is shown in figure (3). Two particles intersect two spacelike separated points \( x_1 \) and \( x_2 \). A light beam is sent from particle 1 at time \( t_A \) to particle 2 so that it arrives at \( t = 0 \). The beam is immediately returned to particle 1 arriving at time \( t_B \). The distance \( x_1 - x_2 \) is to be inferred from the measured delays \( t_A \) and \( t_B \). If, however, the particle 1 is described by quantum mechanics, conventional theory proposes that the trajectory is ambiguously defined if it is defined at all. It may in fact be thought to intersect the \( x \) axis anywhere between \( x_2 \) and \( 2x_1 - x_2 \), which are the relativistically allowed limits. The distance \( x_2 - x_1 \) can no longer be measured. This is a serious metaphysical failure. Even a practical analysis shows that the observational determination of a macroscopic observer’s metric for a particular coordinate system can only be done if the experimental system is sufficiently large to be treated classically [31]. Because of this failure, the classical construction must be replaced if a fundamental sense of space-time is to be maintained. The assumed classical geodesy must be considered a phenomenological result.

Following Mach [46], attempts have been made to associate inertia with gravitational forces by the action of distant objects [44]. From the second order calculation made by Thirring [45], it is known that there is an effect on inertia. Because it is not possible to manipulate the distant objects experimentally, the choice between using a relative concept of inertia and an absolute concept is probably to be made by mathematical convenience. The inertia free description of the five dimensional system can accomplish this. The usual absence of first order structure is remedied by the fixed gauge construction. The boundary conditions of Mach may then be supplied by the standard quantum (or Hamilton-Jacobi) boundary conditions. The resulting fixed gauge system has the mathematical capacity to incorporate a fundamental construction of inertia.

With three fields, there will be three corresponding sets of boundary conditions. Electromagnetic radiation (and the implied acceleration) is to be related to boundary conditions defined by an external absorber. Gravitational radiation is analogous but more complicated. And for quantum mechanics, the sense of inertia implied by the eigenstates is determined
by the space-time boundary conditions. The separation of these effects may depend on the particular choice of a neutral physical space-time. The quantum aspects are the more important lowest order effects. Electromagnetic effects, when separated from quantum effects, are first order and gravitational forces are second order. This supports generally the modern ideas that relate inertia to electromagnetic fields.

The study of the properties of a quantum congruence shows the importance of quantum boundary conditions for the expression of inertia. Consider, as sketched in figure (4), an experiment consisting of particles that traverse a series of unaligned apertures. A certain fraction will diffract through each hole in turn and finally be detected. These cannot be said to move on a classical trajectory. Moreover, the number of apertures can be increased indefinitely as long as they are not too small and the particles are not required to have unattainable velocities. Otherwise, the trajectories are nearly arbitrary. The loss of particle counts is not important and is actually caused by implied absorptive boundary conditions. Rather than have the particles hit the screens, each aperture could be connected to the previous one by a small tube upon which the wave function is set to zero. In either case, the motion is convoluted but does not require external classical fields. If $\Lambda_{\mu}(x, t)$ be chosen everywhere tangent to the probability current density, the five dimensional fixed gauge congruence can describe this motion. By adjusting the five dimensional “cut” coordinate transformation to align the vector potential with the local current, a quantum particle can be followed as it traverses the experiment. The five dimensional geometry is sufficient to describe elementary quantum motion.

Fundamental quantum inertia is the constraint put on the trajectories by the geometry as external forces or boundary conditions are applied through the field equations. These should be constructed from invariants \cite{21,23,47,1}. All of this requires more formalism. Because the trajectories of charged particles are not straight lines with respect to a neutral frame, curvilinear coordinates are required. This elementary picture, that uses straight trajectory segments must be expanded into a full structure capable of describing complex motion.
VIII. THE TRANSITION TO FIVE DIMENSIONS

The particle congruence, as a mathematical representation of a physical object, admits five dimensional coordinate transformations analogous with the four dimensional Lorentz transformations. These can be used to construct and interpret five dimensional metric tensors. To first order, a differential vector \((dx^\mu, d\tau)\), in a coordinate system \((x^\mu, \tau)\), will be mapped linearly onto a differential vector \((dx'^\mu, d\tau')\) in a coordinate system \((x'^\mu, \tau')\). In this approximation,

\[
d x'^\mu = C_5 dx^\mu + C^\mu d\tau \\
d \tau' = D_5 d\tau + D_\mu dx^\mu
\]  

where \(C\) and \(D\) must satisfy integrability conditions.

The coefficient \(C_5\) describes a local conformal transformation for space-time. If \(C_5\) is constrained to unity, conformal effects are removed from the coordinate transformation. The case \(C^\mu \neq 0\) is useful to eliminate \(\tau\) dependence when it is present. The coefficient \(D_5\) may be at most indirectly observed because it involves the normalization of \(\tau\) over extended space-time. The remaining coefficient \(D_\mu\) produces quantum and electromagnetic effects. The invariance properties of this term have been studied by Klein and others \[48\]. It is usually identified as an invariant gauge transformation. In a fixed gauge theory, it operates on the particle state and changes the velocity, mass, and effective wave function. It is an integral part of the extended principle of equivalence.

This last term is best studied separately by examining the shear transformation.

\[
x'^\mu = x^\mu
\]

\[
\tau' = \Phi(x^\mu) + \tau
\]  

which becomes
\begin{align*}
    dx'^\mu &= dx^\mu \\
    d\tau' &= \Phi_\mu dx^\mu + d\tau
\end{align*}

and where the coefficient \( \Phi_\mu \equiv \Phi_{,\mu} \) must always be exact.

The transformed five metric is calculated directly from the invariance of the line element.
\begin{equation}
    dx'^\mu dx'^\nu \gamma'_{mn} = dx^\mu dx^\nu g_{\mu\nu} - d\tau^2 = dx'^\mu dx'^\nu \dot{g}_{\mu\nu} - (d\tau - \Phi_\mu dx^\mu)^2
\end{equation}

\begin{align*}
    \gamma_{mn} &= \left( g^{\mu\nu} - \Phi_\mu \Phi_\nu \Phi_{,\mu} \Phi_{,\nu} \Phi_{\mu\nu} - 1 \right). \\
    \Phi_\mu 
\end{align*}

The local five-Lorentz transformations can also be calculated from
\begin{equation}
    dx'^a = \Lambda^a_b dx^b
\end{equation}

which gives
\begin{equation}
    \Lambda^a_b = \begin{pmatrix}
        \delta^a_b & 0 \\
        \Phi_{,b} & 1
    \end{pmatrix}
\end{equation}

and again demonstrates the shear character.

In five dimensions, two gauge coefficients may be present rather than the single gauge factor \( \lambda \) of the Weyl theory. To make the transition from neutral space, it is necessary to write, in the most general case,
\begin{equation}
    d\tau^2 \chi^2 = \lambda \dot{g}_{\mu\nu} dx'^\mu dx'^\nu
\end{equation}

where \( \chi \) and \( \lambda \) can both be functions of position. The ratio \( \lambda/\chi^2 \) can no longer be treated as one gauge function. When \( \Phi_\mu \neq 0 \), the factors have distinct effects since there are off diagonal terms in \( \gamma_{ab} \). If such a conformal transformation is applied as a point transformation it changes the physical fields. In addition there is possibly an overall conformal multiplier \( \omega \). These factors do not affect the projection of the congruence onto space-time. They must
either be invariance transformations or else must change the identities of the physical fields that accompany the motion.

The application of the conformal transformations along with the extended Lorentz transformations generates a more complicated single particle metric from neutral space.

\[
\gamma'_{mn} = \begin{pmatrix}
(\lambda' g_{\mu\nu} - \Phi_{\mu} \Phi_{\nu})\omega & \omega \Phi_{\mu,\nu}/\chi \\
\omega \Phi_{\nu}/\chi & -\omega/\chi^2
\end{pmatrix}
\] (20)

Setting \( \omega = \chi^2 \) and \( \lambda' = \lambda \omega \) with \( A_\mu = \Phi_{\mu} \chi \) this metric becomes

\[
\gamma'_{mn} = \begin{pmatrix}
\lambda' \dot{g}_{\mu\nu} - A_\mu A_\nu & A_\mu \\
A_\mu & -1
\end{pmatrix}
\] (21)

The new quantity \( A_\mu = \chi \Phi_{\mu} \) is not in general integrable and can be associated with the electromagnetic field.

This metric is a generalization of the standard Kaluza-Klein form. The conformal transformations have been joined with the coordinate transformations to allow the generation of quantum and electromagnetic effects. The resulting five metric, expressed in terms of fixed gauge quantities, is distinct from the conventional Kaluza-Klein theory because it represents the microphysics of a single quantum particle. It is a quantum object, not by any process of quantization, but by the fixed gauge assumption and the inherent quantum nature of geometry.

Because these conformal factors do not change the direction of \( A_\mu \), the five dimensional null geodesics of section V generalize immediately to the same trajectories proposed by the quantum-Weyl theory \[21\]. External interactions including the electromagnetic source terms presumably can be characterized by combinations of the conformal factors \( \chi, \lambda, \) and \( \omega \). It is the transformation of source terms implied by changes in conformal factors that is an essential part of extended equivalence. By setting different conformal factors (as observed from the neutral space) different mechanistic combinations of electromagnetic, quantum and gravitational effects can be ascribed to a given congruence. The subtleties of the existence of a Killing vector are now more apparent. Each point of the congruence is equivalent as far as simple motion is concerned. If however, the particular metric coefficients (that are the
expressed determining influence of that motion) are included, each point can have different local external fields.

When constructing an arbitrary vector potential from a gradient, a single integrating factor may not always exist. In this case, a more complex approach to interaction is necessary. A complete physical determination awaits a set of quantum-Einstein-Maxwell source equations. For a single external source particle, the integrating factor can be found in the rest frame of the source. For multiple source particles, additivity fails and a more involved analysis is needed. It is conjectured that the requirement of multiple integrating factors is naturally satisfied by the use of multiple source particles.

IX. QUANTUM GEODESICS

The geodesic system of section (V), describing classical motion, can be applied directly to the quantum case. From standard theory, it is known that the Klein-Gordon conserved current in fixed gauge form is

\[ P^\mu = \frac{e}{m} \psi^* \psi A^\mu \]

This defines a congruence for a solution of the quantum field equation. It is to be associated with one or more 5-metrics. The trajectories are to be defined by a first order fixed gauge equation, in form identical to the classical case. The changing length of \( A_\mu \) is now essential to the quantum and electrodynamic observations.

A modified normalization can be defined by \( A^*_\mu = \xi A_\mu \). Let the factor \( \xi \) be chosen so that \( e^2/m^2 A^*_\nu A^*\mu \dot{g}^{\nu\mu} = 1 \). This starred vector potential can be used to define a trajectory with a parameter \( w \) that is entirely analogous to the classical case except that \( A^*_\mu \) is not a solution of Maxwell’s equations.

\[ \frac{dx^\mu}{dw} = \frac{e}{m} A^\nu A^*_{\nu} \dot{g}^{\nu\mu} \]

The \( dx^5 \) dependence can be chosen to keep the five displacement null.
From the arguments of section (VIII) it is expected that an appropriate five dimensional metric is of the form
\[
\gamma_{mn} = \omega \begin{pmatrix}
g_{\mu\nu} - A_\mu A_\nu & A_\nu \\
A_\nu & -1
\end{pmatrix}
\] (24)
where \(g_{\mu\nu}\) and \(A_\mu\) are fixed gauge quantum fields. To show that these are geodesics, a coordinate transformation can be defined by \(dx^5' = \xi dx^5\). This choice depends on the particular congruence. It is not integrable generally but can be chosen uniquely by executing the integration along the congruence.

Choosing \(\omega = \xi^2 \omega'\), the five dimensional metric now becomes
\[
\gamma_{mn} = \begin{pmatrix}
\xi^2 g_{\mu\nu} - A_\mu^* A_\nu^* & A_\mu^* \\
A_\mu^* & -1
\end{pmatrix}
\] (25)
and has the same form as the classical theories. It must have geodesics as given by (23). These are quantum geodesics because the motion they predict gives a correct description of statistical measurements of quantum states in combined gravitational and electromagnetic fields. They are tangent to the accepted probability density current in space-time. By the argument of section (V), the observed second order equation can be found by taking the absolute derivative of the quantum trajectory relative to the observers’ metric \(\dot{g}_{\mu\nu}\). This gives
\[
\frac{d^2 x^\mu}{dw^2} + \left\{ \frac{\dot{\mu}}{\epsilon \lambda} \right\} \frac{dx^\epsilon}{dw} \frac{dx^\lambda}{dw} = \frac{e}{m} \tilde{g}^{\mu\beta} \left[ \frac{\partial (\xi A_\beta)}{\partial x^\lambda} - \frac{\partial (\xi A_\lambda)}{\partial x^\beta} \right] \frac{dx^\lambda}{dw}. \tag{26}
\]
Quantum forces are included by way of the rescaled vector potential. The particle motion is always tangent to \(A_\mu\) but the second order derivatives are attributed to different physical fields by the measuring process. Resubstituting \(A_\mu^* = \xi A_\mu\),
\[
\frac{d^2 x^\mu}{dw^2} + \left\{ \frac{\dot{\mu}}{\epsilon \lambda} \right\} \frac{dx^\epsilon}{dw} \frac{dx^\lambda}{dw} = \frac{e}{m} \tilde{g}^{\mu\lambda} \left[ \frac{\partial (\xi A_\beta)}{\partial x^\lambda} - \frac{\partial (\xi A_\lambda)}{\partial x^\beta} \right] = \frac{e}{m} \tilde{g}^{\mu\lambda} \left( \frac{\partial A_\beta}{\partial x^\lambda} - \frac{\partial A_\lambda}{\partial x^\beta} \right) \frac{dx^\lambda}{dw} + \dot{g}^{\mu\beta} (\xi A_\nu - \xi_\beta A_\lambda) \frac{dx^\lambda}{dw} + \frac{e}{m} \tilde{g}^{\mu\beta} (\xi - 1) \left( \frac{\partial A_\beta}{\partial x^\lambda} - \frac{\partial A_\lambda}{\partial x^\beta} \right) \frac{dx^\lambda}{dw}. \tag{27}
\]
The first term on the left is the acceleration of the particle and the second includes all classical gravitational and fictious (inertial) forces. The right hand side splits into three parts. The
first is the conventional electromagnetic force. The second and third are quantum forces which can be named respectively convected and parametric because of the dependence on the factor $\xi$. The representation is of the motion of a single quantum particle interacting through fields alone. With this formalism, it is easy to see that an electron diffraction pattern should be distorted smoothly by an external field, either electromagnetic or gravitational.

Much like the null four geodesics of light beams, the null five geodesics do not change under an overall conformal transformation. The first order defining equation for a quantum geodesic can be written as

$$\frac{dx^\mu}{ds} = g^{\mu\nu} \Lambda_\nu$$

(28)

where $ds$ is not in general affine. The above result still holds because the derivative of the projected trajectory is made by the observer in terms of the neutral space metric. The path parameter $w$, normalized by $g_{\mu\nu}$, is used to display the apparent acceleration of the inferred four-velocity of the particle.

The overall conformal invariance can be used to provide an additional demonstration that the probability current is directed along null five geodesics. One can consider a particular factor $\zeta$ such that it is the integrating factor of the electromagnetic vector potential. As before let $A_\mu \zeta = \Phi,_{\mu}$ for some scalar $\Phi$ and factor $\zeta$. In addition, the fifth coordinate can be transformed by $dx^5 = \zeta dx^5$ using again the integration along the congruence. The metric after the transformation is

$$\gamma_{mn} = \left( \begin{array}{ccc} \zeta^2 g_{\mu\nu} - \Phi,_{\mu} \Phi,_{\nu} & \Phi,_{\mu} \\ \Phi,_{\nu} & -1 \end{array} \right)$$

(29)

and gives after the cut transformation $x'^5 = x^5 - \Phi$ the diagonal form

$$\gamma_{mn} = \left( \begin{array}{ccc} \zeta^2 g_{\mu\nu} & 0 \\ 0 & -1 \end{array} \right)$$

(30)

Again, more than one conformal-coordinate transformation may be needed to integrate the vector potential. Geodesics of this diagonal metric include the point solutions $x^m = 0$ that satisfy $dx^\mu/ds = 0$. These transform to the initial geodesics by the inverse of the above
sequence. The residual factor $\zeta^2 g_{\mu\nu}$ contains information on interactions and probability densities. In this coordinate system, each geodesic is represented by one point which is apparently the hidden variable. This point can be fixed at a certain value of the proper time relative to an arbitrary initial surface. The entire past and future history of the particle congruence can be calculated from the five metric by integrating along geodesics in five-space. For given determining external fields, the intersection point of this trajectory with a particular space like surface can be found. Each possible particle position measurement can be thought to result from one geodesic in the collected congruence. The congruence as a whole represents the motion generated by the specific given quantum state.

X. ON VON NEUMANN’S THEOREM

A few comments may clarify the issue of hidden variables. It is necessary because in a fundamentally geometric theory, it may be essential to describe particles with geodesics. Such specific identification of point particle motion is adverse to conventional interpretations of quantum mechanics. A complete critical discussion of the standard objections to hidden variable theories is outside the scope of this paper. When precisely applied, the usual objections are valid for many alternate theories. Some of these alternate theories may also disagree with experiment or may have some mathematical error. Overall, there is no generally accepted way of avoiding the objections to hidden variables. For the purposes of this article it is sufficient to point out a class of geometrical theories that are not subject to the accepted objections.

One of the most serious practical problems is that an assumption of the existence of hidden variables provides no guidance as to how to find them. Under these circumstances, the question of definition is most important. It is usually supposed that any reasonable choice would be derived from the classical theory through the system of observables constructed by Von-Neumann [49]. This is not the case here. The mathematics of differential geometry substitutes an inequivalent fundamental structure. Since the quantum system is not to be
derived from classical physics, the use of differential operators as physical concepts is not required. For the geometrical theories of this paper, classical mechanics is fully rejected as a fundamental theory. The conditions of Von Neumann’s theorem fail because the hidden variables no longer have to be defined according to his prescription.

While de Broglie’s hidden variable concept, [50], is still popular in the minds of many physicists [51], his use of the operator substitution method allows Von Neumann’s argument to prevail. This suppressive theorem can be applied when the derivative as a representative of the physical momentum is presumed to be a legal substitution into the equations of classical mechanics.

Without bare physical operators, all physical quantities commute, and Von Neumann’s theorem is avoided. The concept of the classical canonical momentum matched with the analogous operator oriented quantum concept fail together. There is no quantum measurement theory in the usual sense. It is worth noting that if a quantum theory is defined by a system of differential equations alone, then there is no unambiguous way to define the canonical momentum. All classical theory is avoided.

The process of operator substitution has no mathematical precedent and no accepted mathematical justification. The metaphysics of quantization fails and the arguments of Von Neumann may not hold. First quantization, in actuality, serves the purpose of recreating differential factors that are neglected in the phenomenological perception of classical mechanics. It is an essential method in an historical context. All of the accepted derivations, [52], of the Klein-Gordon equation, except possibly the one by Klein, proceed along these lines.

Thus the geometrical view presented here is fundamentally different. It is not a reformulation and new results are possible. In particular, the capability to address gravitational problems is added over the conventional wisdom. The rejection of first quantization may have effects that extend beyond immediate considerations. The repeated circular derivation of quantum from classical and then classical from quantum is implicit in much of modern physics. This cyclic logic should be broken at the point of first quantization. Geometrical
theories or any other new theory may need to be evaluated against experiment and not current phenomenology.

**XI. ON BELL’S THEOREM**

Modern criticism of alternative quantum theories and hidden variables usually is presented in the context of Bell’s theorem [53]. Comparisons of theory and experiment have been found to support the conventional formalism. A number of measured results [54] lie outside the predictions of a large class of alternate theories. Bell’s theorem is motivated partly from the work of Von Neumann. The accepted quantum discussion depends on the use of operators as physical quantities and neglects relativistic effects. A careful consideration of the assumptions and arguments of Bell’s theorem shows under what conditions alternate constructions might be mathematically possible and experimentally acceptable.

A serious weakness of Bell’s analysis centers around the approximations that are necessary to make a manifestly relativistic or covariant theory non-relativistic. When applied to alternate theories, Bell’s objections depend on the presence of an implied classical basis. This has subtle implications that become part of the quantum interpretation. Moreover, the wave function is hypothesized without any predecessor. Better that the wave function should be part of a geometrical space and not an esoteric representation of some sort of fundamental statistical essence. In addition, some of the classical properties appended during quantization, especially those relating to causality, are misleading and inappropriate.

Radiation is manifestly relativistic and the mathematics and sense of causality that deal with it should be covariant. The justification of the use of a nonrelativistic limit for radiation comes only from classical physics. It is this metaphysics of motion in a relativistic field that causes trouble. It is known that Bell’s theorem cannot be applied to a theory like quantum electrodynamics wherein advanced potentials are used. Such fields, while undesirable in a classical theory, are not forbidden by any concept of fundamental geometry. A fully covariant geometrical theory in which advanced fields are implicit, may be capable of predicting the
non-local results typical of photon correlation experiments. Conversely it is not possible to derive a non-relativistic quantum mechanics from either quantum electrodynamics or geometrical theory without serious conceptual compromises.

The implications of such classically imposed causality are subtle. For a fully microscopically deterministic theory, an experiment with complete arbitrary initial conditions (in the sense of the Einstein, Rosen, and Podolsky [53]) is not apriori possible. It is believed that almost any experiment can be done, but only a the small number of initial conditions allowed by the current state of the universe are possible. The temperature of the experiment must be greater than the ambient noise. Any experiment, because of electromagnetic radiation and distant gravitational interaction may connect with particles at the farthest reaches, past and future. Our sense of causality is a large scale observation of these events. A microscopic electromagnetic field that does or does not go backwards in time can only be included or excluded in so far as it does or does not explain scientific experience. Moreover, since the microscopic structure of quantum mechanics does not have an intrinsic direction, formal symmetry between advanced and retarded potentials must be possible.

For a photon correlation experiment, the calculation using the propagators of quantum electrodynamics has essential terms with advanced dependency. Because of implicit instantaneous interactions, non-relativistic quantum mechanics implies the use of such fields but also represses any explicit reference to them. Since Bell’s theorem requires that all fields be retarded, the conclusions of this theorem are avoided for quantum systems. Even an implicit advanced interaction voids Bell’s theorem. It is still unknown, whether in a more complete theory, the presence of advanced potentials must be explicitly displayed or whether they are an artifact of the mathematical methods. Because of the demonstration by experiment, such interactions must be integrated into the theory despite the counter-intuitive indications of classical physics.

It is easy to show that the common geometrical theories already contain advanced potentials. Since a congruence of motion is to be well defined, it must represent all physical effects. In particular, the force of radiative reaction must be included. This must be intro-
duced into the vector potential by the advanced potentials of other interacting particles. The final overall prediction need not be interpreted as an advanced propagation of information or energy. A fixed gauge system that includes all electromagnetic forces must be exempt from Bell’s theorem.

The relationship of geometrical theories to these accepted experiments is important. Most measurements use photon correlations. For the present development of the geometrical description there is no reason to consider calculations which are not equivalent to quantum electrodynamics. The geometrically preferred scheme is to use time symmetric potentials without free fields. Theoretical predictions of this type have already been shown to agree with the standard versions [27].

The experiment by Aspect is representative of this class and is straightforward to analyze. A limitation to retarded potentials is not required. For a two photon correlated emission, the emitting atom cannot execute the state transition without the presence of the simultaneous advanced electrodynamic fields from two or more absorbing particles [56]. The correlation can be observed only if the experiment is arranged so that both emitted photons are collected in the experiment rather than in other parts of the universe.

There are also correlation experiments that involve particles. In the case where the interaction between the particles is mediated by electromagnetic fields, the elementary geometrical theory should be sufficient. The nuclear experiments that use spin polarization [57] give important results for weak and strong interactions. Formal predictions for these fields are not possible because the geometrical structure used here is insufficient. Physically, though it is possible to argue that the results are not incongruent. The use of advanced potentials for other fields should be adequate to predict the observed result. In particular, the effects of spin during scattering do not seem to be time asymmetric. These results should be explainable using the nuclear equivalent of time symmetric potentials. The principle of equivalence is expected to have further extensions so that even spin couplings are replaced by the effects of covariant geometrical fields.

The entanglement of multiparticle states is assumed to be the integrated effect of the
physical interaction that causes the state to form. The entangled states described early by Schrödinger, [58], that are so characteristic of quantum mechanics are interpreted as a persistent geometrical distortion that is inseparable from the interaction that connects the particles.

Time symmetric electrodynamics, developed as a classical theory [59, 24] must be modified to allow for quantum effects. The macroscopic classical approximation must be the limit of the quantum mechanical transitions that really occur. The use of the classical theory as the metaphysical precursor of the quantum field leads to confusion. This sort of quantum electrodynamics has been applied to cosmological situations [60]. Therein, the quantization of time symmetric electrodynamics, [51] is based on classical physics. The unusual predicted results with respect to absorption and emission seem to be due to the assumptions of those classical properties before the quantization is accomplished. The confusion is related to the similarity of classical electrodynamics to possible primitive geometrical forms of electrodynamics. A precise notion of classical absorption or emission cannot be defined until the fundamental processes of discrete quantum absorption or emission are understood. More recent experiments on quantum electrodynamics support the concept that the pre-quantization assumption of absorption and emission is not be justified [62].

**XII. DISCUSSION**

A number of important issues are raised by the application of covariant geodesics to quantum particles. This possibility is not part of the accepted formulation of quantum theory. It is of some interest because the fundamental construction is simpler than other quantum gravitational theories. The real issue is whether the use of such a construction can produce the essential results and complex phenomenology of modern quantum experiments. This article specifies how to begin such a theory and how the usual hard objections to trajectories can be avoided.

The conformal parameters and their relation to source terms are an important develop-
ment. The conformal transformations take a place with the coordinate transformations as a means of generating a principle of equivalence. The usual curvilinear transformations allow the interchange of gravitational and inertial forces. The quantum and electrodynamic forces are now to be included. The conformal factors are essential for the description in the frame of the neutral observer. These are also important in the study of the field equations that begins in a following article.

XIII. SUMMARY

A number of concepts have been developed which allow the description of quantum phenomena to be done with differential geometry. The application is to the combined effects of gravity, electromagnetism and quantum mechanics without weak or strong interactions.

A number of currently accepted beliefs are discounted, particularly those concerning the epistemology of quantum mechanics and the metaphysical basis of general relativity. A new way of thinking about these fields of physics is devised. The general theory of relativity must be treated as a phenomenological result of a deeper theory. The quantum mechanical theory must not be derived from classical physics. A fundamental geometrical electrodynamics is introduced.

The actual fifth dimension is associated with the proper time of charged, isolated, point particles of finite mass. Dependence on the fifth parameter is not apparent to a real observer because the physical laws, when expressed in five dimensional form, predict precisely what would be expected according to common observation. The proper values of the fifth coordinate of a particle are not absolutely determinate but are only defined differentially and with a gauge factor that is not observable. This approach might be called kinematic dimensional reduction.

The classical principle of equivalence is extended. This concept, required by the fundamental notion of trajectory motion, provides a guide for mathematical development. An extension of the geometry either as to the number of dimensions or the use of non-Riemannian
effects is required. The distinction between the effect of the three fields on a primitive particle is only defined after the relation between the particle and a neutral space-time observer is specified.

Fixed gauge methods are used. The individual geometrical fields, are not subject to the variability of most common gauge transformations. The vector potential represents both the velocity of the particle and the relationship of the five space to space-time. Null geodesics are everywhere tangent to the associated probability current and can represent a quantum state. The usual problems with Von Neumann’s theorem and Bell’s theorem are avoided. The geometrical theories are in agreement with experimental results that have been found to limit other alternate quantum theories. The implications for a five dimensional geometrical theories are profound and extensive.
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FIGURES

FIG. 1. A charged particle and a neutral particle start in coincidence at $P_1$. They move through an interaction region where they separate onto distinct trajectories. If these trajectories be allowed to reintersect at $P_2$, the principle of equivalence for both particles cannot be valid if attempted in space-time. A geometrical extension is needed.

FIG. 2. A quantum clock provides a universal measure of time in a quantum-gravitational-electromagnetic theory. A fixed but identified real particle is transmitted with a nominal fixed speed through a diffraction apparatus. By adjusting the slit size, the character of the diffraction pattern can be observed on the screen. For an arbitrary but fixed angular size of the pattern, the slit width provides a dimension usable for construction of a light pulse clock.

FIG. 3. The usual determination of the spacelike distance between two points fails in a fundamentally quantum theory. The separation $x_A - x_B$ is to be found by measuring the transit delays of light beams between the two particles that move through the points $A$ and $B$. The photon timing provides a well defined quantity but the straightness of the intermediate trajectory of particle $A$ is not established from conventional quantum phenomenology.

FIG. 4. A particle is projected through a series of unaligned apertures. If the apertures are sufficiently small but finite, some of the particles will traverse the experiment and strike the screen. The boundary conditions are critical. If one or more particles get through, then they must be supposed to move on trajectories that cannot be described by classical mechanics.
Fig. 1  galehouse-quantum geodesics
Fig. 2  galehouse quantum geodesics
Fig. 3    galehouse quantum geodesics
Fig. 4  galehouse quantum geodesics