Troubles with Global Monopoles in Quantum Gravity

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ABSTRACT

A theory with a global $O(3)$ symmetry broken at a scale $\eta$ admits topological configurations: global monopoles realized by Goldstone fields winding around the core of false vacuum. One may expect them to behave as heavy, big composite objects, difficult to make and therefore mostly harmless. However, after gravity is turned on, as long as Equivalence Principle holds, one finds that global monopoles have a negative mass, $M \sim -\eta/\sqrt{\lambda}$, where $\lambda$ is the field theory coupling. This could catalyze an instability in the space of a global monopole, by the production of additional global monopole-antimonopole pairs along with normal particles, leading to energy production *ab nihilo*, if the pair energy is dominated by their negative rest masses. In a theory with unbroken local Poincaré symmetry, this could lead to a divergent ‘decay rate’ of the global monopole configurations.

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1 Introduction

It has been argued for a long time that quantum gravity does not permit exact global symmetries (see, e.g. [1–8]). An argument often heard is that since black holes do not conserve global charges, and they count among the Hilbert space states in quantum gravity, in processes where they appear as virtual states the global charges cannot be conserved. Instead these processes generate global symmetry violating operators in effective theory. This argument is appealing, but it seems very difficult to implement it directly, for example, as a diagrammatic calculation of a global symmetry breaking operator generated by a loop including a virtual black hole.

More recently, different arguments emerged, invoking weak gravity conjecture (WGC) [9] and AdS/CFT [10, 11]. These arguments rest on ideas on how the theory of gravity may behave in the UV. In particular WGC is interesting since it seems to block trying to realize global symmetries as a zero charge limit of gauge symmetries. Yet while there is evidence supporting these ideas, we still do not know if such specific frameworks for UV completions are unique or generic. Hence it is interesting to pursue possible IR avenues of conflict between global symmetries and gravity, to see if anything can be shaken out from the mix of the two.

An example is provided by an interesting exploration of (in)compatibility of non-compact, continuous shift symmetry with gravity in [12]. There, the positivity of energy (combined with the implicit assumption that the geometry is static) yields a bound on the total field variation exterior to a source. This is despite the fact that in flat space the field variations can be arbitrarily large. Further examples were provided in, for example, [13–15], where it was found that gravity appears to obstruct large field changes, which indirectly implies that the naïve shift symmetry of the flat space limit cannot coexist with gravity.

Here we will pursue a different approach, which will use only very basic IR properties of gravity, employing very few ideas about the UV. We will consider theories which have a spontaneously broken global $O(3)$ symmetry. Theories with such structures seem legitimate in flat space without gravity. They have a nontrivial vacuum manifold, with low energy Goldstone modes described by a nonlinear $\sigma$-model. In addition to local excitations they also admit topological global monopoles, which correspond to nontrivial Goldstone fields winding around the region of false vacuum in the core. They exist when the vacuum manifold is isomorphic to the sphere at infinity. In flat space, global monopoles seem to be nonperturbative structures which are heavy, big composite objects, that tend to be mostly harmless [16]. However even this is not without controversy, since the issue of their stability has already been debated vigorously [17–25]. We will briefly review the status of this debate and outline the features relevant for our argument. We will work with the symmetry breaking scale $\eta \ll M_{Pl}$, to avoid any additional issues concerning large field variations\footnote{It is in fact known that super-Planckian field values induce topological inflation [28, 29], which in light of the results of [30] might even be viewed as a form of instability [31].}. Further we will work in weak coupling.

Turning gravity on lets global monopoles spring several surprises. First, in the case of isolated global monopoles, their Goldstone fields support a solid deficit angle [26]. This appears to regulate the flat space divergent energy density distribution outside of a global
monopole. Second, thanks to this, global monopoles have a negative gravitational mass, 
\( M \sim -\eta/\sqrt{\lambda} \) [27], where \( \lambda \) is the field theory coupling. Finally, we will invoke a simple argument using Equivalence Principle to infer that as a result the inertial mass of global monopoles is also negative. Combining this with unbroken local Poincaré symmetry, and using continuity of dispersion relations, implies that global monopoles should be described in a local theory as ghosts, since they reside on mass shells which are inverted relative to normal matter. To a good approximation they seem to be ‘hard’ at low energies.

This immediately suggests possible catastrophic consequences, just like in the case of theories with hard fundamental ghosts [32–35]: if they are at least metastable in local theory, and their rest energy is dominated by mass terms, global monopole-antimonopole pairs may provide means to catalyze an instability of the background spacetime of a global monopole, due to their pair production along with normal positive energy particles. In this case, at least, the positive energy theorem of [36,37] is not directly applicable since the background has a deficit angle. As long as the process amplitude is not exactly zero, and the net energy of the pair is negative, the ‘decay rate’ of a global monopole would diverge when the theory has unbroken Poincaré symmetry in the UV. This would be reminiscent to how the flat space stable vacuum appears to reject remnants [38].

Hence we may infer that a consistent theory of quantum gravity with unbroken local Poincaré symmetry in the UV may not support stable global monopoles which arise in field theories with spontaneously broken global symmetries. If this is true, our example, albeit somewhat special, suggests a glimpse into what may go wrong with quantum gravity if we try to push global symmetries onto it. An alternative, which would prevent this conclusion, is that the pair energy is never negative despite the fact that their local rest masses are. For this to be the case, a generalization of the theorems [36,37] to some spaces which are not asymptotically flat could be true, although there is no proof at this time.

2 Global Monopole Solutions and Their Properties

Global monopoles are topological solitons of field theories with spontaneous symmetry breaking, which contain an \( O(3) \) symmetry, with a potential

\[
V(\vec{\Phi}) = \frac{\lambda}{4} \left( \vec{\Phi}^2 - \eta^2 \right)^2.
\]

(1)

Because any vacuum at infinity spontaneously breaks \( O(3) \) to \( O(2) \), the theory has massless Goldstone bosons. They are crucial for the existence of a global monopole because they give long range fields which wind around the global monopole core, providing the topological charge distinguishing a global monopole from the vacuum. In such theories there are configurations where the field in the target space develops a hedgehog configuration around the core of the false vacuum,

\[
\vec{\Phi} = f(r) \frac{\vec{r}}{r},
\]

(2)

wrapping around it (in the example above, exactly once) as it interpolates to the degenerate manifold of true vacua at infinity (for a review see [16]). As the Goldstones range out to
infinity in a global monopole configuration, interpolating between the different true vacua far away, they yield stress energy tensor which vanishes only as \[ T^0_0 = -T^r_r \sim \frac{\eta^2}{r^2}, \] (3) while the angular components vanish much faster. As a result, individual global monopoles formally seem to have a divergent field energy, 

\[ E = 4\pi \int_0^R dr \ r^2 T^0_0 \sim 4\pi \eta^2 R. \] (4)

Hence individual global monopoles cannot be produced in flat space unless infinite energy initial conditions are used. Thus one may imagine that due to such huge energy in the global monopole field, its mass, and therefore its inertia, should be very large. However this single source solution can be regulated by adding a global antimonopole, which is a parity image of a global monopole, at a distance \( \sim R \) from the global monopole [16]. In that case the energy (4) will be cut off at infinity, since the antimonopole serves as a sink of field lines, at a finite distance from the source. This yields a faster dilution of stress energy far away.

Early on it has been asserted that a global monopole may be unstable [17] (see also [18, 19]) since it can unwind by itself, thanks to an angular perturbation of the spherical isopotential shells around it. A small perturbation of the global monopole on, say, the North Pole has a tendency to grow in the axial direction, and pull in the Goldstone field lines towards the vertical axis, where they can unwind.

A more detailed analysis has shown that this instability really induces a force acting on the global monopole, accelerating it in the direction of the perturbation [18, 20–22]. We could imagine this configuration of a “leashed” global monopole as a global monopole-antimonopole pair separated by a large distance. The array breaks spherical symmetry, since the presence of a global antimonopole introduces a direction, and so the field lines tend to align with it and form a global ‘flux tube’ [20, 21]. The initial realignment starts slowly due to a large separation, but then the constant force pulls the two together [22].

Thus we assert that the instability observed in [17] can be viewed as a signal of the onset of the process of global monopole-antimonopole annihilation in the large initial distance limit [18, 20, 22]. Moreover, although the topological charge is accounted for by the boundary conditions at infinity, the energy barrier between configurations with a different charge seems to be finite [23]. This suggests that the number of global monopoles could be ‘rearranged’ with a finite energy investment [23] despite the apparent divergence of field energy (4).

More careful examinations with gravity turned on reveal interesting surprises. To see how the energy (4) behaves, we may use General Relativity as a probe. It turns out that the leading order sources in \( T^\mu_\nu \) do not yield any local gravitational forces. Instead, the leading order \( T^\mu_\nu \) which had yielded the divergence (4) in flat space now sources a global solid deficit angle of the space exterior to a global monopole [26]. The metric of a global monopole outside of the core is

\[ ds^2 = -\left(1 - 8\pi G_N \eta^2 - \frac{2 G_N M}{r}\right) dt^2 + \left(1 - 8\pi G_N \eta^2 - \frac{2 G_N M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2. \] (5)
The solid angle surrounding a global monopole at infinity is \( \Delta \phi = 4\pi (1 - 8\pi G_N \eta^2) \), which follows from the leading order behavior of the metric as \( r \to \infty \). Thus the leading order energy is effectively degravitated. Locally, all it does is enhance the value of Newton’s constant in the space outside of a global monopole. This is obvious from Gauss’ law at infinity, where the angular deficit implies that there is stronger fields for a fixed source mass since there is less spatial volume for the field lines to dilute \([39]\). The long range effects of \( T^{\mu \nu} \) persist only as the deficit angle \([26, 27]\). They are important for the understanding of a global monopole-antimonopole pair creation as we will argue later; they are crucial for understanding their local gravitational and inertial properties.

At the subleading level, gravity reveals an even greater surprise \([27]\): a global monopole has a negative gravitational mass! Technically, this rather odd feature can be understood as follows. Inside a global monopole core the field configuration is near the false vacuum \( \Phi = 0 \), where the energy density is \( \rho \sim V \sim \lambda \eta^4 / 4 \), which behaves like a positive cosmological constant, exhibiting repulsion of normal matter due to the negative pressure \( p \sim -\rho \), which induce forces \( \propto (\rho + 3p) < 0 \). The interior geometry is approximately given by de Sitter static patch metric,

\[
ds^2 = -(1 - H^2_{\text{core}} r^2) \, dt^2 + \left(1 - H^2_{\text{core}} r^2\right)^{-1} \, dr^2 + r^2 d\Omega^2, \tag{6}
\]

where

\[
H^2_{\text{core}} \sim \frac{8\pi G_N V}{3} \sim \frac{2\pi G_N \lambda \eta^4}{3}. \tag{7}
\]

Because the exterior metric has a deficit angle, the interior and exterior can be smoothly matched at the “surface” of the global monopole, where the metrics and their derivatives continuously connect. This yields \([27]\), up to corrections \( \propto 8\pi G_N \eta^2 \ll 1 \),

\[
r_0 \simeq \frac{2}{\sqrt{\lambda \eta}}, \quad M \simeq -\frac{16\pi \eta}{3\sqrt{\lambda}}. \tag{8}
\]

The mass in the exterior metric must be negative by Birkhoff’s theorem, since the interior mass \([27, 40]\) is \( M_{\text{core}} \sim (\rho_{\text{core}} + 3\rho_{\text{core}}) \text{Vol}_{\text{core}} \sim -2\rho_{\text{core}} \text{Vol}_{\text{core}} \). Importantly, also note that as long as

\[
\frac{2G_N |M|}{r_0} \sim \frac{2}{3} 8\pi G_N \eta^2 \ll 1, \tag{9}
\]

or so as long as \( 8\pi G_N \eta^2 \ll 1 \), the description of a monopole using flat space physics should be reliable, since gravity is too weak in its core. Clearly, we are in this limit when \( 8\pi G_N \eta^2 \ll 1 \). Further note that by combining Eqs. (8),

\[
r_0 |M| = \frac{r_0}{\lambda_{\text{compton}}} \simeq \frac{32\pi}{3\lambda} \gg 1, \tag{10}
\]

which shows that in perturbation theory, \( 8\pi G_N \eta^2 \ll 1, \lambda \ll 1 \), the global monopole core size is much larger than its Compton wavelength. So, global monopoles are classical, extended
objects\(^2\), which as long as \(8\pi G_N \eta^2 \lesssim 1\), \(\lambda \ll 1\) have gravitational properties that can be consistently described using linearized General Relativity.

But this is not all: as long as gravity is governed by General Relativity, which obeys Equivalence Principle, the global monopole inertial mass \(M_{\text{inertial}}\) must also be negative since it must be equal to its gravitational mass \(M\). To see this, imagine a probe particle near a global monopole, in free fall in the geometry with the metric (5), following the geodesic equation

\[ \ddot{x}^\mu + \Gamma^{\mu}_{\nu\lambda} \dot{x}^\nu \dot{x}^\lambda = 0. \]  

(11)

Let the particle stand to the right of the monopole, moving slowly and being separated a distance \(r\). Because the global monopole gravitational mass \(M\) is negative, the particle experiences a repulsive acceleration controlled by the global monopole’s negative gravitational mass, as the particle mass cancels out of the geodesic equation. So the force pushing the particle away from the global monopole is \(\vec{F} \sim -mM|\vec{r}_0/2 = m|M|\vec{r}_0/2\).

Now: change coordinates from the rest frame of the global monopole to the rest frame of the particle, and look at the motion of the global monopole governed by the gravitational field of the particle, with a Schwarzschild metric of the mass \(m\). To the leading order, the global monopole follows geodesic equation in the new coordinate system, experiencing an attractive acceleration induced by the positive particle mass – since this time the global monopole’s inertial mass cancels from the geodesic equation, the gravitational field and the acceleration imparted by it is the same for all probes, controlled only by the particle’s positive mass \(m\), as follows from Equivalence principle. The force pulling the global monopole towards the particle (on the right of the monopole) is \(\vec{F} \sim mM_{\text{inertial}}\vec{r}_0/2\). Again by Equivalence Principle – since the motion of the particle in the global monopole field must be the same as the motion of the global monopole in the particle’s field – the forces \(F\) and \(\hat{F}\) must be equal and opposite \([41, 42]\), and hence \(M_{\text{inertial}} = -|M| = M \equiv M_{\text{gravitational}}\), as claimed. Therefore

\[ M_{\text{inertial}} \equiv M \simeq -\frac{16\pi \eta}{3\sqrt{\lambda}}. \]  

(12)

The calculation of the mass \(M\) can be made more precise by generalizing the notion of the ADM mass in the presence of the deficit angle, which changes the boundary conditions at infinity. The generalization of the ADM mass for spacetimes with solid deficit angle \([43, 44]\) confirms the intuitive results after the renormalization of \(G_N\) by the deficit angle. This leaves us precisely with the result of (8) to the leading order in the \(8\pi G_N \eta^2\) expansion \([27, 43, 44]\), allowing the mass of a global monopole to be negative.

The price, of course, is the deficit angle, amounting to the alteration of the boundary conditions at infinity. In flat space, the net global monopole number – ie the total winding number – enumerates superselection sectors. To create a single global monopole would presumably be obstructed by the need to ‘bring’ the charge from infinity, and so spend an

\[2\]As a technical aside, to fully describe them we need to keep the radial mode in the theory. Since the mass of the radial mode of \(\Phi\) is \(m_\Phi \sim \sqrt{\lambda} \eta\), we find \(r_0 m_\Phi = O(1)\), implying that global monopoles are about the same size as the Compton wavelength of the radial scalar modes, integrated out in the true vacuum. Hence global monopoles explore the physics above the cutoff of the perturbative \(\sigma\)-model describing only the Goldstones, since the cutoff of the low energy effective theory is set by \(m_\Phi\).
infinite amount of energy, since Goldstone fields dilute too slowly far away. However, these superselection sectors seem to only be separated by finite energy barriers [23]. This, at least, suggests that a global monopole-antimonopole pair can be formed readily. In flat space without gravity, if the initial state is the Minkowski vacuum, or even a configuration with a global monopole on shell, a global monopole-antimonopole pair could be created quantum-mechanically if the conservation laws permit it. In this limit all the energies are dominated by positive rest masses, by the theorems [36, 37] in asymptotically flat spaces. So the pair will quickly fall in and annihilate – with a zero net energy release – existing a mere moment only because of the uncertainty principle. If so, the flat space vacuum would be stable, since these events would merely be one of the many quantum fluctuations that come and go. The diagrams representing their contributions to any observables would be disconnected bubbles, which factor out. In this argument the gravitational effects are completely ignored.

Turning gravity on brings out the negative global monopole masses and deficit angles. Having objects with negative masses, with all local energy densities being positive, is a very unsettling proposition. Already [27] note that if these configurations really existed, they could lead to the formation of Bondi dipoles [45]. After noting this option, [27] step back and dismiss it on the grounds that global monopoles are topological objects which are not ‘isolated point particles, but extended sources.’ A single global monopole is indeed an extended object in the strict geometric sense: it affects the boundary conditions at infinity. Yet at high energies a global monopole can unwind as the symmetry is restored. The charge and the field energy would then flow away to infinity.

On the other hand, global monopole-antimonopole pairs extend far less, with their long distance effects diluting away more quickly. This naturally regulates the energy divergences far away. At distances much larger than the global monopole-antimonopole separation, the net charge of the pair will look to be zero, which means that the field strengths of the Goldstones must diminish faster than in the case of a single global monopole. Therefore in a quantum mechanical process global monopoles could be created as pairs and so a net global monopole number should not even change\(^3\). However the numbers of global monopoles and global antimonopoles may fluctuate.

This should survive the onset of gravity; in pair creation, the curvature deformation mimicking deficit angle may be induced locally near individual sources after the pair emerges, but will not extend to infinity as long as the field energy between the pair is finite. This local distortion of the geometry may subsequently propagate farther – or not – depending on the fate of the pair. However in practice, in the linearized gravity regime applicable to our analysis due to choosing \(8\pi G_N \eta^2 \ll 1\), \(\lambda \ll 1\), inside the region of space where the pair forms, and close to the global monopoles, they will behave as negative mass objects when probed by normal matter for all practical intents and purposes.

We contend that this may allow the global monopole-antimonopole pair creation to drive an instability of a configuration initially containing a global monopole on shell through particle production. As we stressed above, the formation of a global monopole-antimonopole pair out of the flat space vacuum is not prohibited a priori if all conservation laws are satisfied. Since Goldstone fields are cut off, this is a finite energy configuration. As we stated above,

\(^3\)Such events could be extremely rare, but if the rates are not zero the problems will arise.
this configuration should be interpreted as a pair of global monopoles connected with a ‘flux tube’. This is motivated by the studies of Goldhaber’s instability [17, 18], whereby a global monopole unwinds directionally before annihilating with a global antimonopole [18, 20–22]. Since the global antimonopole is needed to regulate the Goldstone field energy (4) anyway, it is natural to interpret Goldhaber’s instability as the tendency of Goldstone fields to collapse to a flux tube [20, 21]. We maximize its energy by assuming no energy loss in the field rearrangement, so that the interaction energy mediated by the flux tube is given by the regulated expression (4). This way we reproduce the formula for the constant force acting on a global monopole (as already noted in [21]).

We can then view the global monopoles at the end points of the tube as locally negative mass objects, when we get near them (see also [46] for a similar point of view). The question then is, what is the energy of the pair, and could it be negative. This question is subtle. Although it may appear that we could make the energy negative for a pair even in vacuum, when they are initially sufficiently close, so the flux tube between them is short, and they are moving, so their kinetic energy may add to their negative rest masses, the overall global monopole-antimonopole resonance energy in asymptotically flat space is prohibited from being negative by the theorems of [36,37]. This would block the instability in the vacuum.

However if the background already contains global monopoles, and therefore has a deficit angle, the case is not so clear cut. The theorems [36, 37] do not automatically guarantee that the pair energy must be positive. The total energy of the system might not be positive definite due to different boundary conditions at infinity. In this case, if a global monopole-antimonopole pair pops out in a background already containing a global monopole, with an overall negative energy, it must be accompanied by at least one positive mass particle-antiparticle pair, such that the total system satisfies energy-momentum conservation enforced by local Poincaré invariance – that the net energy change is zero. Hence the production rate may be kinematically allowed. Once the positive mass particles get on shell, they will fly away, since they can be light – or even massless, being photons or even gravitons themselves.

In many cases the global monopole-antimonopole pairs which are created will fall in and unwind away, absorbing some of the positive mass particles floating around; in some, they may even end up wiggling around for a while and increasing their (negative) kinetic energy by emitting positive mass particles. But their ultimate fate past the fact that they could be created seems largely irrelevant for the instability. If they are on shell, the positive mass particles will escape regardless of the fate of the global monopole-antimonopole pair, and stress-energy may flow out of the space which is asymptotically locally the same as the vacuum, except for the deficit angle. When local Poincaré symmetry is unbroken in the UV, the phase space for the outgoing decay products has divergent volume just like similar processes for the production of hard elementary ghosts, where the energy-momentum conservation holds [32–34]. As a result, as long as the decay rate is not exactly zero, it should diverge. The suppression due to the compositeness of the global monopoles may make their pair production rate per unit phase space volume exponentially small, but as long as it is not zero the disaster is lurking. All it takes is a pair with a net negative energy and local Poincaré invariance in the UV.

\footnote{We thank A. Vilenkin for useful communications about this issue.}
An alternative outcome is that even though the space has a deficit angle, and so is not asymptotically flat, the results of [36, 37] still hold and exclude negative energy pairs. If this is true then our discussion below may be viewed as an explanation why [36, 37] need to extend to this case, despite the global redefinition of the ADM mass [43, 44]. In this case reconciling the negative mass of an isolated global monopole with the positive energy of a pair would be quite interesting to see.

3 The Global Monopoles’ Fizz

Now we turn to the details of how global monopoles could trigger the instability in the space surrounding them. As we noted in the beginning, we will be working with safely subcritical global monopoles, where $8\pi G_N\eta^2 \ll 1$, $\lambda \ll 1$ so that we can for the most part ignore the deficit angle. Locally, it only renormalizes $G_N$ in this limit; however it is crucial for the existence of negative masses.

Consider now a global monopole-antimonopole pair, separated by a distance $R$, in a locally asymptotically flat space of a distant global monopole. The global monopole far away sets up the deficit angle at infinity. In the least favorable case to our argument, when the force is attractive, since it is constant [16, 22], the time it takes for the pair to annihilate is controlled by the initial separation,

$$\Delta T \sim \sqrt{\frac{R|M|}{4\pi\eta^2}}.$$  \hspace{1cm} (13)

At distances $r > r_0$ we will treat individual global monopoles comprising the pair as hard particles, with negative masses $M$ of Eq. (12). To the leading order, when they are close enough their total energy may be dominated by the kinetic terms due to negative masses. The negativity of the mass combined with unbroken local Poincaré symmetry implies that global monopoles would behave as ghosts, with negative energy

$$E = -\sqrt{\vec{p}^2 + M^2},$$  \hspace{1cm} (14)

for a given momentum $\vec{p}$. To see this, imagine an observer boosted relative to a global monopole at rest. In the rest frame of the moving observer, it appears that the global monopole is moving in the opposite direction, with the relativistic energy $E \neq M$. By continuity of boosts, we can dial down the relative velocity $\vec{v}$ to zero smoothly, which means that in the limit $\vec{v} = 0$ we must reproduce $E = -M$. Therefore $E = -\sqrt{\vec{p}^2 + M^2}$. This means that the mass shells of the global monopoles are inverted relative to normal particles.

Next, we estimate how far the monopoles can be so that the net energy is negative, using the energy balance formula based on (4). Note that if the initial state was empty Minkowski space the net energy would have to be non-negative by [36, 37]. However if there is a deficit angle due to a global monopole far away, this need not be true. Note also that the formula (4) is at best only an approximation in a conical space, but we will nevertheless use it to illustrate the issues. So for a global monopole-antimonopole at rest, with negative masses (8), to the leading order we take the excess energy over the energy of the background
monopole far away to be roughly given by the flat space formula for the flux tube energy, but with negative monopole masses,

\[ E_{\text{total}} \simeq 4\pi \eta^2 R + 2M = 4\pi \eta^2 R - \frac{32\pi \eta}{3\sqrt{\lambda}} , \tag{15}\]

and allowing for it to be negative if the monopole-antimonopole separation \( R \) is small enough,

\[ R_{\text{cr}} \simeq \frac{8}{3\sqrt{\lambda \eta}} \simeq \frac{4}{3} r_0 . \tag{16}\]

The distance \( R_{\text{cr}} \) is just-so larger than the global monopole core size, meaning the global monopole-antimonopole pair are in contact. They will not unwind right away, but may do so, at a rate depending on \( r_0 \). Still, even this may suffice to trigger a cataclysm. Note that as we take \( \lambda \to 0 \) the danger from long lived negative energy pairs seems to grow.

Moreover, the monopoles are initially moving, having additional kinetic energy so that \(-M \to -M/\sqrt{1 - v^2}\), so the critical separation might be larger, and the net energy of the pair might still be negative. This opens a window for a global monopole-antimonopole separation to be at least a few times larger than \( r_0 \), improving the approximation where the monopoles are treated as hard ghosts. Thus when the pair are moving initially, by causality they will survive for at least a time \( \Delta T \sim (R_{\text{cr}}/\sqrt{\lambda \eta})^{1/2} > \text{few} \times r_0 \). Such global monopole-antimonopole pairs would be approximately on shell: quantum production of photons will be faster than global monopole-antimonopole annihilation. Hence these global monopole-antimonopole pairs ought to be included among those states which the space that contains a global monopole can decay into, at least as metastable resonances.

Since in locally Poincaré invariant theories we must enforce energy conservation, additional positive energy particles would be produced, such that the total energy change is zero [32, 33]. A simple cartoon for a process describing this is a spontaneous creation of a bound system of scalar quanta forming the global monopole-antimonopole pair along with a swarm of free streaming ‘photons’ mediated by gravitational interactions (or any other particle-antiparticle pair, due to the universality of gravity, including gravitons or even the \( O(3) \) Goldstones). The global monopole setting the background can be taken to be far enough that its only effect on local physics is via the deficit angle, which invalidates positive energy condition. Otherwise the dynamics is as in any locally flat space. Treating the background as a ‘vacuum’, the total process would involve many different internal scalar excitations, whose number would be at least as large as \( M/m_\Phi \sim Mr_0 \gg 1 \), with an arbitrary number of external photons popping out, as long as the net energy is zero. A channel with a given number of scalars and two photons is sketched in Fig. 1.

Clearly, there is an exponentially large number of such channels. The total amplitude of the process is obtained by summing amplitudes for each channel. A large number of channels means that they would sum up mostly out of phase, yielding destructive interference and suppressing the amplitude. The rate would be effectively suppressed by compositeness of the global monopole-antimonopole pair. However even an exponential suppression \( \sim \exp(-S) \) is not enough to stop the disaster, as long as the pair energy could be negative.

Indeed, this could trigger a catastrophe because the Lorentz-invariant phase space for such processes is infinite. This is regardless of whether global monopole-antimonopole pair
end up going to infinity, or not. The important point is, there is no stopping the photons, which would carry energy away with them. If realized, this leads to asymptotic space instability, resulting in energy production *ab nihilo*.

Although the initial states which contain a global monopole and deficit angle break the local Poincaré spontaneously, this breaking is in the IR and does not help with UV divergences. The reason that the phase space diverges for a process like

\[
|0_{asymptotic}\rangle \rightarrow |M \bar{M} \gamma \gamma \ldots \rangle
\]

is that the global monopole masses are negative, due to deficit angle. The divergence can be readily inferred from the transition amplitude following the analyses in [32–35]. Let’s consider the simplest divergent contribution coming from the process \(0_{asymptotic} \rightarrow M \bar{M} \gamma \gamma\) whose one channel is depicted in Fig. 1. The contribution to the decay rate of the asymptotically flat space far from the global monopole source via the production of a global monopole-antimonopole pair plus two photons is

\[
\Gamma \equiv \int d^4p_1 d^4p_2 d^4k_1 d^4k_2 |A|^2 \delta(p_1^2 - M^2)\delta(p_2^2 - M^2)\delta(k_1^2)\delta(k_2^2) \times \\
\times \theta(-p_{10})\theta(-p_{20})\theta(k_{10})\theta(k_{20})\delta^4(p_1 + p_2 + k_1 + k_2), \tag{17}
\]

Here \(A\) is the transition amplitude for the process \(0 \rightarrow M \bar{M} \gamma \gamma\), defined in the standard way by \(\langle p_1, p_2, k_1, k_2|0_{asymptotic}\rangle = A(p_1, p_2, k_1, k_2) \delta^{(4)}(p_1 + p_2 + k_1 + k_2)\) (thanks to the local Poincaré symmetry), \(p_j\) are the global monopole 4-momenta and \(k_j\) are the photon 4-momenta, and the signs in the step functions \(\theta(-p_{j0}), \theta(k_{j0})\) take care of the mutual reversal of the mass shells, accounting for the negative inertial mass (12).

To proceed, it is convenient [34] to rewrite the integral in (17) by introducing an auxiliary 4-momentum \(P\), and integrate over it to enforce \(P = p_1 + p_2 = -k_1 - k_2\), which executes
4-momentum conservation in two steps. This yields [34]

\[ \Gamma \equiv \int d^4P \gamma(P), \]  

where

\[ \gamma(P) = \int d^4p_1d^4p_2d^4k_1d^4k_2 |A|^2 \delta(p_1^2 - M^2)\delta(p_2^2 - M^2)\delta(k_1^2)\delta(k_2^2) \times \]
\[ \times \theta(-p_{10})\theta(-p_{20})\theta(k_{10})\theta(k_{20})\delta(4)(P + k_1 + k_2) \delta(4)(P - p_1 - p_2). \]  

(19)

In this integral, the auxiliary variable \( P \) appears as an external momentum. The local Poincaré symmetry then asserts that – since there are no preferred directions in the UV – \( \gamma \) can depend only on the magnitude \( P^\mu P_\mu = -s \). Note that this is a consequence of the Lagrangian of the theory. Then changing the integration measure in (18) to \( \sqrt{s}dH \) yields

\[ \Gamma \equiv \int \frac{1}{2} \frac{d^3\vec{v}}{\sqrt{1 + \vec{v}^2}} \int ds s \gamma(s). \]  

(20)

This change of variables is analogous to spherical polar coordinates in 4D. However the difference is that \( P \) is a vector in a 4D Minkowski space, and thus in the decomposition \( P^\mu P_\mu = P^2(1 - \vec{v}^2) = -s \) the manifold of \( \vec{v} \) is not compact. Indeed, using \( d^4P = dP P^3 dH_3 = dss dH_3/2 \), and realizing that \( dH_3 \) is the measure on the mass shells defined by \( v_0^2 = 1 + \vec{v}^2 \), we see that \( \int dH_3/2 = \frac{1}{2} \int dv_0 d^3\vec{v} \delta(v_0^2 - 1 - \vec{v}^2) = \int \frac{1}{2} \frac{d^3\vec{v}}{\sqrt{1 + \vec{v}^2}} \sim 2\pi \int dv v \). If this were an integration over euclidean vectors, it would correspond to integrating over a hemisphere from the pole to the equator, which would give a finite answer since the sphere is compact\(^5\).

However, the Lorentzian signature implies that the integration is over an infinite range of the lengths of \( \vec{v} \). Thus as long as \( |A|^2 \neq 0 \) the integral diverges.

We can understand this by considering 4-momentum conservation \( p_1 + p_2 + k_1 + k_2 = 0 \). Since the photons are null, \( k_1^2 = k_2^2 = 0 \). For the global monopoles, we have \( p_1^2 = p_2^2 = M^2 \). Rewriting the conservation equation as the expression for \( p_1 \) in terms of the rest then yields, after a simple manipulation, \( p_2 \cdot k_1 + p_2 \cdot k_2 + k_1 \cdot k_2 = 0 \). Next, we can perform a boost to the rest frame of \( p_2 \), where \( p_2 = (M, 0) \). In this frame \( k_2 \cdot p_2 = ME_2 \). Using the projection constraint and defining the angle \( \theta \) between the two photons via \( k_1 \cdot k_2 = E_1 E_2 \cos(\theta) \), we finally obtain the equation

\[ \cos(\theta) = 1 + M \left( \frac{1}{E_1} + \frac{1}{E_2} \right). \]  

(21)

Since the photons are on the positive mass shell, the term in the parenthesis is positive definite. Thus if \( M > 0 \), there would be no solutions for \( \theta \), since the RHS would exceed unity. However since the global monopole mass is negative, \( M < 0 \), the RHS will be smaller than unity at energies \( \gtrsim \) few \( \times |M| \), and the kinematical constraint (21) will be satisfied for some scattering angle \( \theta \). This will happen not only in the rest frame of \( p_2 \), but in any frame boosted relative to it, with the same value of \( s = -(p_1 + p_2)^2 \), which must also be

\(^5\) In that case, the sign under the square root in the denominator of (20) would be negative.
included among the final decay states. Since there is infinitely many such frames the rate will diverge. This last step is precisely the integration over $\vec{v}$ above.

As long as we maintain that the local Poincaré symmetry is unbroken, the only way to reverse this conclusion seems to be to have $|A|^2 \equiv 0$, or to block the global monopole-antimonopole pair from negative values. But this is precisely the point we want to make: as long as a global monopole-antimonopole pair with negative energy can be formed in the background with an initial global monopole, however unlikely this outcome might be, the decay rate will blow up due to unbroken local Poincaré invariance. It appears that one way to accomplish this switch-off of $A$ is to decouple gravity completely, sending $M_{Pl} \to \infty$. The world without gravity is not what we are after, however.

We would be remiss to ignore the possibility that the positive energy theorems from asymptotically flat space [36,37] somehow extend to the conical space of a global monopole. This could preclude the situation that the global monopole rest masses ever dominate the flux tube energy. We cannot exclude this, although it is difficult to see how this could occur\(^6\) for all values of $\lambda$. Further if this is the case, then reconciling such a statement with the negative mass of an isolated global monopole which follows from the redefinition of the ADM mass to conical spaces [43,44] would be quite interesting to see.

Alternatively, if the theory has a UV completion which is not locally Poincaré invariant, the instability rate can be cut off and global monopoles may coexist with gravity. An example might be global monopole-like configurations in ‘emergent gravity’ models involving superfluid Helium [47]. Since the UV completion is just the standard nonrelativistic quantum mechanics, the absence of local Poincaré invariance in the UV introduces a cutoff to the integrals contributing to the decay rate.

Another alternative is to declare that stable global monopole states cannot exist in a world with gravity. E.g. those states must include an arbitrary number of excitations and hence may not be normalizable in the standard way. This would be in line with the lore that global symmetries cannot coexist with quantum gravity. Thus our example, while special to an extent since it requires a global spontaneously broken symmetry, may be a piece of evidence supporting this lore.

Simply put, in the presence of a spontaneously broken global symmetry in a world with General Relativity the solitonic structures supported by spontaneous breaking of global symmetry provide long range Goldstone fields sourced by the topological charge. They backreact on the geometry and induce a local distortion around the global monopole cores, which behaves like a ‘cavity’ that traps a section of de Sitter space in the core of the monopole. This region is nonsingular, smooth and completely ‘naked’ to the exterior: it is not hiding behind a horizon, and is not covered by a shell of material. Thus the relativistic pressures of the vacuum energy, which also gravitate, allow the mass of the configuration to be negative. A deficit angle at infinity may permit configurations of multiple such objects where the negative masses dominate. If so, without a strict cutoff, the disaster looms. A space of

\(^6\)Note that if we could take the limit $\lambda \to 0$, we might think the negative mass monopoles would decouple (they would in flat space without gravity). However this would enhance the global symmetry from $O(3)$ to the full symmetry group of the $3D$ Euclidean space by adding three commuting continuous translation shift symmetries. From the point of view of quantum gravity this would seem to be an even worse outcome.
single global monopole becomes crowded quickly.

4 Summary

In summary, we have argued that global monopole configurations in General Relativity with unbroken local Poincaré invariance might be unstable to quantum effects. They have a deficit angle and as a consequence, a negative mass, both gravitational and inertial [27,41]. Although their production rate may be very small, if they can be on shell when paired up with global antimonopoles, and have the pair energy which is negative, which might happen when an initial global monopole is already present, obstructing the theorems of [36,37], normal particles which conserve total 4-momentum could be emitted. The rate for these processes is divergent because the unbroken local Poincaré symmetry in the UV implies that the phase space accessible to the out states is infinite.

One possible ‘obstruction’ to our argument is that the result of [36,37] might still be valid despite the fact that a global monopole sets up a conical space. If so, our discussion could be taken as an indirect argument to this extent, since otherwise a rapid instability will occur. A more general positive energy theorem would exclude the pair production instability. If on the other hand even a single pair might pop up with a negative energy, this would raise an obstacle to having global symmetries in a QFT coupled to universal gravity, with the only invocation of UV being to require that the single-excitation states in asymptotically locally flat space exist forever\(^7\) and that the theory is locally Poincaré invariant. Everything else rests on the IR properties of gravity.

If the latter option is the outcome, then how does gravity exorcise global monopoles, and a global symmetry needed to produce them, out of the spectrum of the theory? At this point, this is a wide open question. One option is that those states simply do not belong to the theory since they would not be normalizable in the usual sense, as the number of particles diverges. Yet another possibility is that gravity further destabilizes global monopole-antimonopole pairs (unlike individual global monopoles which seem to be more stable in classical gravity than without it [24]), and hence they never get on shell with negative energy. Either way, the global monopoles merely flag disconnected superselection sectors determined by boundary conditions at infinity. This is unlike what happens with topological couplings in gauge theories, where gauge symmetry allows sectors with fixed winding numbers to mix, since after all there is no gauge symmetry here. Restricting the theory to a single sector would effectively break global symmetry.

A question then is how this could manifest locally. Perhaps quantum gravity, via, e.g. wormhole effects, such as those proposed long ago in [1,3–5], induces explicit global symmetry breaking terms such as \(\sum_j (\vec{A}_j \cdot \vec{\Phi})^2\) that would lift the Goldstones, and preclude global monopoles altogether even in the classical limit. It seems interesting to explore this further.\(^7\)

\(^7\)A pragmatic demand might only be that the locally flat space, and isolated individual monopoles, exist for \(\gtrsim 10^{10}\) years, to fit the observations. If so, the finiteness of the observable universe might help regulate divergence of the decay rate.
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