Influence of anisotropic yield criteria on simulation accuracy of the hole-expansion test

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Abstract. Prediction accuracy of the two well-known anisotropic yield criteria, namely, Hill’48 and Yld2004-18p, on plastic deformation of AA5052-O sheet aluminium alloy experiencing compound stress was profoundly examined and contrasted in this work. To obtain the flow behaviour and anisotropy of the test material in varying loading directions, both uniaxial tensile and balanced-biaxial tests were conducted in the corresponding directions. Furthermore, a disc-compression test was carried out to reveal the biaxial $r$-value. Those gained mechanical properties could therefore be used to determine the material constants required by each yield criterion. The standard conical-punch hole expansion test was chosen here to impose complex stress on the sheet specimen. Forming parameters such as the punch load, stroke paths, hole expansion ratio, sheet thickness profile around the hole circumference and flange height were all monitored and investigated. It was shown that the results from the Yld2004-18p model with an exponent of eight apparently better agreed with the experimental data than those from the Hill’48 one. A conclusion could be drawn here that the yield criterion significantly affected the accuracy of the predicted deformation behaviour of the investigated aluminium alloy grade AA5052-O nevertheless in dependence on the degree of material anisotropy.

1. Introduction

In the past, low carbon steel sheets were widely used in the automotive manufacturing industries. Recently, various lightweight materials have nevertheless been increasingly utilised and gradually replacing those conventional steels in many applications such as automotive bodies for the sake of preservation of the Earth’s environment [1]. Aluminium alloy sheets are among the lightweight materials that are critically effective in reducing automotive body weight. Several drawbacks of those alloys such as inferior formability, high degree of springback and high cost have however been reported [2-4]. They are knowingly prone to fracture during common forming processes because of their low ductility. Optimal tool geometries and forming conditions, not causing fracture of the aluminium alloy sheets, are thereby definitely required [1]. In the stretch flanging process of automotive wheel parts like brake panels and hub holes, the hole expansion test has been recognised as a discriminating method for characterising the material forming behaviour. To save time and cost of the experimental procedure, a qualified finite element model, especially that able to accurately anticipate the material yield behaviour, is totally a must [5]. In this work, AA5052-O, sheet aluminium alloy, was of interest. Its anisotropic plasticity was thoroughly investigated. Three famous laboratory tests—uniaxial tension, disc-
compression and hydraulic-bulge tests—were all accomplished to identify the material mechanical characteristics. Such experimentally-determined properties as flow stresses, r-values and yield loci helped formulating all anisotropic coefficients of those two yield models under scrutiny, Hill’48 and Yld2004-18p [6, 7], as well as illustrating the competence of both models in handling the aluminium-sheet anisotropy. A demonstrative hole-expansion test, acknowledged capable of measuring stretch-flangeability of any sheet metal, was then conducted. The test outcomes were eventually compared with those simulatively obtained from both computational models in order to study the obscured effects of both yield criteria over simulated AA5052-O stretch-flangeability.

2. Material characterisation

2.1 Uniaxial tensile test

All uniaxial tensile tests were carried out by a universal testing machine on 1-mm-thick AA5052-O specimens of the ASTM E8 standard shape. The samples were prepared along seven angles—0°, 15°, 30°, 45°, 60°, 75° and 90°—to the rolling direction (RD). Throughout the entire tests, a strain rate of 0.001 s\(^{-1}\) was kept constant. As a result, seven stress-strain curves and r-values were obtained. The flow curve gained from the 0° specimen is exemplarily illustrated in Figure 1.

![Figure 1. Experimental and numerically-Swift-fitted flow stress curves of the examined aluminium AA5052-O undergoing uniaxial tension in RD](image)

Meanwhile, Table 1 depicts all yield stresses, extracted from the curves, normalised with respect to the yield stress at the RD and all r-values.

| Degree | 0° | 15° | 30° | 45° | 60° | 75° | 90° | Biaxial |
|--------|----|-----|-----|-----|-----|-----|-----|--------|
| Normalised | 1.0000 | 0.8719 | 0.8883 | 1.0296 | 0.9010 | 0.9182 | 1.0660 | 1.1200 |
| r-value | 0.6230 | 0.7070 | 0.7220 | 0.7290 | 0.7380 | 0.7160 | 0.6670 | 0.9626 |

2.2 Hydraulic bulge test

A hydraulic bulge test was performed on a 200-ton hydraulic press to obtain the biaxial flow behaviour of the AA5052-O sheet. The experimental procedure and tooling can be found in [11] for more details. The drawing-die diameter was 200 mm. The test specimen was a 280-mm-wide square blank.
hydraulic pressure gradually increased until crack was initiated. The pressure, dome height, membrane stress and thickness strain were all online-recorded. The resultant normalised biaxial yield stress is shown in Table 1 while the biaxial flow curve is plot in Figure 1.

2.3 Disc compression test

The through-thickness, single-layer disc compression test was executed to reveal the biaxial r-value ($r_b$). More details on the experimental procedure can be found in [9, 12]. An AA5052-O disc sample had a diameter of 10 mm. A compressive load was applied to the disc, starting from 5 kN up to 12 kN by a stepwise increment of 0.9 kN. The biaxial r-value could be estimated as a ratio of the strain in the transverse direction (TD) versus RD, whose consequence is shown in Table 1.

3. Anisotropic yield criteria and strain hardening model

3.1 Hill’48 yield criterion

A quadratic anisotropic yield criterion, proposed by Hill [8] in 1948, has become one of the most widely used yield functions. The Hill’s 48 model assumes a plane-stress state, an absolutely-proper approximation for typical sheet metal forming. The equation reads

$$2f(\sigma) = (G + H)\sigma_{xx}^2 + (F + H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 1$$

(1)

where $F, G, H$ and $N$ are the computed anisotropic coefficients. Table 2 exhibits all four coefficients of the studied material. They can be derived from a group of four experimental data, demonstrated in Table 1, such as the uniaxial yield stress measured in the RD and r-value in three varying load directions, 0°, 45° and 90° with respect to the RD.

| Table 2. Anisotropic coefficients of the Hill’48 yield criterion for the AA5052-O sheet |
|---------------------------------|----------------|----------------|----------------|
| F                               | G              | H              | N              |
| 0.5755                          | 0.6161         | 0.3839         | 1.4645         |

3.2 Yld2004-18p yield criterion

The anisotropic yield function Yld2004-18p computes the scalar equivalent stress out of a full 3D stress tensor. Eighteen anisotropic coefficients are introduced as entries of the two $6 \times 6$ linear transformation matrices modifying the 3D stress tensor. More details on the definition of those parameters can be found in [9]. The Yld2004-18p yield function can be expressed as

$$\phi = 4\bar{\sigma}^m = |S'_1 - S'_2|^m + |S'_1 - S'_3|^m + |S'_2 - S'_1|^m + |S'_2 - S'_3|^m + |S'_3 - S'_1|^m + |S'_3 - S'_2|^m + |S'_3 - S'_3|^m$$

(2)

where $\bar{\sigma}$ is the equivalent stress; $m$ is the exponent; $S'_i$ and $S''_i$ ($i = 1, 2$ or $3$) are the principal values of the modified deviatoric stress tensors $S' = C'S$ and $S'' = C''S$. The variable S is the deviatoric stress tensor while both $C'$ and $C''$ are the modifying linear transformation matrices mentioned above. All eighteen coefficients in the matrices, describing the anisotropic deformation behaviour of the considered material, can be optimised thru an iterative regression process on Eq. (2) provided a set of measured yield stresses and r-values for several load orientations as shown in Table 1 for the AA5052-O sheet. The exponent $m$ is set to 8 as recommended for the face-centred cubic (FCC) crystal [7, 9], which is the typical microstructure for aluminium alloys. In the meantime, Table 3 shows all those eighteen anisotropic coefficients of the examined aluminium-alloy sheet, resulting from an optimisation process.
### Table 3. Anisotropic coefficients of the Yld2004-18p yield criterion for the AA5052-O sheet

| α₁  | α₂  | α₃  | α₄  | α₅  | α₆  | α₇  | α₈  | α₉  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.1504 | 1.1504 | 1.3241 | 1.1428 | 0.7253 | 0.6311 | 1.0127 | 0.9983 | 0.9983 |

| α₁₀ | α₁₁ | α₁₂ | α₁₃ | α₁₄ | α₁₅ | α₁₆ | α₁₇ | α₁₈ |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.5215 | 0.5434 | 0.5387 | 0.4778 | 1.0140 | 0.9588 | 0.9789 | 1.0042 | 0.5000 |

### 3.3 Swift hardening model

The Swift hardening law [10], characterising the strain hardening behaviour, reads

\[
\bar{\sigma} = K \left( \bar{\varepsilon}_0 + \bar{\varepsilon}_p \right)^n
\]

where \( \bar{\sigma} \) and \( \bar{\varepsilon}_p \) are the effective flow stress and plastic strain, respectively. \( K, n \) and \( \varepsilon_0 \), whose values for the AA5052-O sheet are exposed in Table 4, are all material constants. They can be figured out through curve fitting on the RD uniaxial tensile true stress-strain measurement data as illustrated in Figure 1.

### Table 4. The Swift hardening model’s material constants for the AA5052-O sheet

| K     | \( \varepsilon_0 \) | n     |
|-------|---------------------|-------|
| 741.6476 | 0.0088              | 0.1866 |

### 3.4 Validation of anisotropic yield criteria

To prove the capability of both Hill’48 and Yld2004-18p criteria in coping with anisotropy of the AA5052-O sheet, the normalised flow stress and \( r \)-value at different loading angles numerically derived from both models will be explicitly compared with those gained from the experiments as shown in Figure 2 (a).

![Figure 2. Comparison between the experimental and model-based deformation data of the AA5052-O sheet in terms of (a) normalised flow stress and \( r \)-value and (b) yield locus](image-url)
It can be obviously seen that the Yld2004-18p model provides a perfect agreement for both flow stress and r-value for all loading directions. However, the Hill’48 model overestimates the flow stress in a wide angle range from 30° up to 90° despite a faultless prediction of the r-value. This could be caused by the lack of consideration of the yield stress in all other angles except at the rolling direction during anisotropic coefficient determination. For better understanding, the comparison is also illustrated in the yield locus form on the normalised RD and TD stress space as can be seen in Figure 2 (b). Therein, the Hill’48 model slightly overvalues the uniaxial flow stress at the TD but greatly undervalues the biaxial flow stress. In contrast, the Yld2004-18p model is doing a superb flow-stress evaluation job for all loading directions.

4. Experimental hole expansion test and FE model formulation

The hole expansion test is an approved procedure for evaluating stretch-flangeability of material, in which radial compressive and tangential tensile stresses coexist simultaneously in the vicinity of the hole area [12]. In this work, a hole expansion test was conducted on an Erichsen formability tester with a conical punch. A schematic view of the hole expansion test setup is illustrated in Figure 3 (a). During the test, a square specimen, pierced with a 10-mm-diameter circular hole at the centre, was firmly clamped between a lower ring-shaped blank holder and upper cup-shaped die. Possible draw-in of the outer portion of the sample was prevented through a static blank holder force of 220 kN. The conical punch moved upwards with a constant velocity of 15 mm/min. The forming punch was suddenly stopped approximately after 17 mm once an initial crack had been detected through an instantaneous punch-force drop. On one hand, the punch force-displacement development was online monitored throughout the forming process. On the other hand, the deformed specimen’s final local dimensions measured circumferentially around the flange tip such as hole diameter, flange height and sheet thickness was identified with the help of an offline 3D laser scanner.

![Figure 3. Hole-expansion test setup of (a) the experiment and (b) the finite element simulation model](image)

Apart from the experiment, the hole-expansion test was correspondingly carried out on the ABAQUS2017 finite element platform to study the numerical influence of different material models. Two implicit FE models were formulated. One was based on the Hill’48 model; the other on the Yld2004-18p model. All FE simulation setups like boundary conditions, workpiece meshes and process equipment were defined precisely in accordance with the committed experiment as can be seen in Figure 3 (b). The elastoplastic material model was taken into account and the 3D hexahedral solid...
elements (C3D8R) were entirely applied to the AA5052-O sheet mesh. Nevertheless, the punch and dies were all modelled as an analytical rigid surface. The Coulomb’s friction coefficient of 0.175 was applied to all contact pairs. During the hole expansion test, once the punch force had started to decline as a result of radial crack initiation, the punch was automatically halted. This stopping point was considered the state of fracture. The punch displacement at the point was correspondingly viewed as the simulation termination criterion. Experimental verification of the proposed FE models can therefore be realised thru comparison among the obtained force-displacement curves.

5. FE simulation results

First of all, the measured and simulated punch force-displacement developments are compared as shown in Figure 4. Both Hill’48 and Yld2004-18p models seem to perform well regarding the force-displacement aspect although the latter slightly outperforms the former, a little underestimating the punch force.

![Figure 4. Experimental and numerical punch force-displacement developments](image)

Figure 5 compares the hole expansion ratio of the initial to final hole radii along the hole fringe. The ratio usually represents formability of a blank in a hole flanging process. The greater the ratio, the greater the formability and thus expansion. Overall, the Yld2004-18p model performance surpasses that of the Hill’48 model. The Hill’48 thoroughly undercalculates the ratio. Nonetheless, the trend of its curve is still in good agreement with that of the experiment. The reason behind would be that the model considers the r-value at various loading orientations to account for material anisotropy. When compared at different angles, the ratio is maximum (most expanded) at 90º whilst minimum (least expanded) at 60º. This result well corresponds with the r-values of the AA5052-O sheet shown in Table 1. The r-value is maximum (0.738) at 60º while minimum (0.623 and 0.667) at 0º (RD) and 90º (TD). Therefore, the hole at 60º creeps in-plane the most and displaces inwards, while the hole at 0º and 90º is stiffer in-plane and displaces outwards. Such deformation actions lead to a bigger diameter at 0º and 90º but smaller at 60º. The next investigation point is the final altered sheet thickness along the hole circumference. According to Figure 6 (a), the predicted thicknesses from both models deviate marginally from the measured ones. However, the Yld2004-18p model once again produces more realistic results than the Hill’48 does. Considering the thickness at each angle, the trend is very much like that of the hole expansion ratio. So, the sheet tends to be thinner at the location where it in-plane extends the most on account of volume conservation.
The last point to be analysed is the flange height along the hole circumference. As expected, the Yld2004-18p model can generate closer consequences at every measurement point to the experimental ones than the Hill’48 can as demonstrated in Figure 6 (b). The flange height at 15º, 30º, 60º and 75º resulting from the Hill’48 model appears to be more deviated from the experimental one than that at 0º, 45º and 90º. This might probably be due to the inclusion of the r-value of only those three latter orientations when determining the anisotropic coefficients $F$, $G$, $H$ and $N$.

6. Summary

In this work, a conical-punch hole expansion test was accomplished to investigate performance of the two favourite anisotropic yield criteria, namely, Hill’48 and Yld2000-18p, in modelling plasticity of AA5052-O aluminium alloy sheet. The uniaxial tension, disc compression and hydraulic bulge tests were all carried out to obtain required material properties to develop the two yield models. Normalised flow stresses, r-values and yield loci predicted by the Yld2004-18p model were in very good agreement.
with those from the experiments. Furthermore, FE simulations coupled with the yield criteria were carried out for the hole expansion test. The experimental and predicted hole radii in terms of the hole expansion ratio, sheet thickness and flange height all along the hole circumference were compared. It was found that the Yld2004-18p model undoubtedly provided overall simulation results with closer agreement to the measured ones than the Hill’48 model did. Nevertheless, in terms of anisotropy (curve shape), Yld2004-18p edged out Hill’48 by a very narrow margin although involving more r-values from eight different sheet directions. This would most likely be due to small anisotropy of the material as shown in TABLE 1, in which all r-values are a close range. A legitimate conclusion can thus be drawn that, under combined stress of radial compression and tangential tension brought about by a standard conical-punch hole expansion process, the Yld2004-18p model is preferred over the Hill’48 model due to its superior competence in characterising anisotropic plastic deformation.

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