Abstract

We show that a left-handed neutrino $\nu_L$ can oscillate into its $CP$-conjugated state $\bar{\nu}_R$ with maximal amplitude, in direct analogy with $K^0 - \bar{K}^0$ oscillations. Peculiarities of such oscillations under different conditions are studied.
1 Introduction

35 years ago Pontecorvo suggested the possibility of neutrino oscillations [1, 2] (see also [3]) by analogy with the oscillations of neutral $K$ mesons. The question he raised was “...whether there exist other mixed neutral particles besides the $K^0$ mesons which differ from their antiparticles and for which the particle $\rightarrow$ antiparticle transitions are not strictly forbidden” [1].

Direct analogy with the $K^0$–$\bar{K}^0$ case would imply oscillations of a neutrino into its CP-conjugated state with large amplitude. We shall refer here to such a process as ”Pontecorvo’s original oscillations”.

The essential difference between the $K^0$ mesons and neutrinos is related to the spin of neutrinos. It was realized after the $V–A$ structure of weak interactions had been established that neutrinos are produced and interact in chiral states. In particular, only left-handed neutrinos $\nu_L$ have been observed. The $CP$ conjugation transforms $\nu_L$ into a right-handed antineutrino $\bar{\nu}_R$, and so the realization of the Pontecorvo’s original idea would mean the existence of the oscillations

$$\nu_L \leftrightarrow \bar{\nu}_R.$$  \hspace{1cm} (1)

Strictly speaking, transitions (1) are not just a process of lepton number oscillations, but also simultaneously neutrino spin precession.

Since the helicity of a free particle is conserved, in vacuum the oscillations (1) cannot occur. Flavour oscillations between the neutrinos of the same chirality are possible in vacuum [4, 3], but in this case the transitions take place between the neutrino states which are not related by $CP$ conjugation. In addition, the mixing of these states need not be large.

Particle-antiparticle transitions in vacuum can in principle take place for 4-component neutrinos. In terms of chiral states these oscillations would imply transitions of $\nu_L$ into $\bar{\nu}_L$ (or $\bar{\nu}_R$ into $\nu_R$) [3], so that the neutrino helicity is conserved. However, such transitions
are not analogous to the $K^0 - \bar{K}^0$ oscillations: $\bar{\nu}_L$ is not the true antiparticle of the left-handed neutrino, the existence of which is required by the $CPT$ invariance, but rather a different neutrino state. In the ultrarelativistic limit $\nu_L$ and $\bar{\nu}_L$ can be considered as independent particles with quite different interactions ($\nu_L$ is active whereas $\bar{\nu}_L$ is sterile in the standard model). This is in contrast with the hypothetical neutron-antineutron oscillations, since at low energies the different helicity components of the neutron are strongly coupled through its large mass. Moreover, left-handed neutron and antineutron have the same strong interaction.

For the above reasons it was generally supposed that Pontecorvo’s original oscillations are just the oscillations of active neutrinos into sterile states, whereas the true neutrino-antineutrino oscillations (1) were considered impossible. In this paper we show that under certain conditions maximal-amplitude $\nu_L \leftrightarrow \bar{\nu}_R$ oscillations can nevertheless occur.

## 2 General conditions for $\nu_L \leftrightarrow \bar{\nu}_R$ transitions

Consider for definiteness the transitions involving electron neutrinos, $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$. As we already stressed, oscillations (1) imply helicity flip of the neutrino states. Such a flip can be induced, for example, by interactions of neutrinos with external magnetic fields provided the neutrinos have magnetic (or electric) dipole moments. However, the magnetic-moment interaction cannot transform a neutrino into its own antineutrino because of $CPT$ invariance. Nevertheless, it can convert a neutrino into an antineutrino of another species [5]. From this fact two conclusions follow: (i) in addition to the neutrino of a given flavour, one needs at least one more neutrino state $\nu_x$ to be involved in the process, and (ii) an additional interaction which mixes $\nu_x$ with $\nu_e$ is required. If these conditions are satisfied, the $\nu_{eL} \rightarrow \bar{\nu}_{eR}$ transition can proceed via $\nu_x$ in the intermediate state, and $\nu_{eL} - \bar{\nu}_{eR}$ mixing appears as a second-order effect.

In the simplest case, the additional interaction should not change the helicity of neu-
trinos, but it must change their lepton numbers. Indeed, the $\nu_L \leftrightarrow \bar{\nu}_R$ oscillations imply $\Delta L_L = 2$ transitions, whereas in the magnetic-moment induced transitions the individual lepton charges $L_L$ change by only one unit. The additional interaction one needs can then be just the one which generates the flavour (mass) mixing of neutrinos.

In all cases at least two chains of transitions contribute to the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ transition, and one should make sure that no cancellation between the corresponding amplitudes occurs. Indeed, let $\nu_e$ be a left-handed neutrino; then the $\nu_{eL} \to \bar{\nu}_{eR}$ transitions can proceed through the $\nu_{eL} \to \nu_{xL} \to \bar{\nu}_{eR}$ chain, where the first transition is due to the mass mixing and the second one is induced by the magnetic-moment interaction. From CPT invariance it follows that the antiparticle of $\nu_{xL}$ exists, and is a right-handed neutrino $\bar{\nu}_{xR}$. It also mediates the $\nu_{eL} \to \bar{\nu}_{eR}$ transitions through the chain $\nu_{eL} \to \bar{\nu}_{xR} \to \bar{\nu}_{eR}$. Now the first transition is due to the magnetic-moment interaction, and the second one results from the mass mixing. The amplitudes of the helicity-flipping transitions $\nu_{eL} \to \bar{\nu}_{xR}$ and $\nu_{xL} \to \bar{\nu}_{eR}$ in these two chains have opposite signs due to the CPT symmetry\(^1\) while the amplitudes of the transitions induced by the mass mixing coincide. Therefore, if $\nu_{xL}$ and $\bar{\nu}_{xR}$ are degenerate in energy, the contributions of the two chains to the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ amplitude exactly cancel each other and the $\nu_{eL} - \bar{\nu}_{eR}$ mixing does not appear. Obviously, the cancellation takes place for any number of additional neutrinos. Moreover, this result holds true for any number of the transitions in the chains: the crucial points are that (i) there should be an odd number of transitions induced by the magnetic-moment interaction, and (ii) for a given chain, another one with CP-conjugated particles and inverted order of transitions in the intermediate states always exists.

To induce the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ transitions, one should lift the degeneracy of the intermediate states. This can be realized if the transitions take place in matter (provided the intermediate neutrino is not sterile) and/or in a magnetic field whose direction changes along the neutrino trajectory. Indeed, matter affects neutrinos and antineutrinos differently (the corresponding

\(^1\)The matrix of transition magnetic moments is antisymmetric.
forward scattering amplitudes have opposite signs [4], and rotating magnetic fields affect differently left-handed and right-handed states [7, 8].

The possibility of $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ transitions in matter and transverse magnetic field was first pointed out in [9]. However, for fixed-direction magnetic fields the transition probability was shown to be small even for large neutrino mixing and magnetic moments [10]. As we shall see, the magnetic field rotation can change the situation drastically.

The $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations could in principle be generated by the magnetic-moment interaction alone (i.e. could occur even in the absence of mass mixing) if at least two additional neutrino states, say, $\nu_\mu$ and $\nu_\tau$, and three transition magnetic moments are involved. However, as it was shown in [11], for massless neutrinos the $\nu_{eL} - \bar{\nu}_{eR}$ mixing vanishes identically in this case even in the presence of rotating magnetic field. The reason is that equal numbers of left-handed and right-handed neutrinos are present in the intermediate states, and so the cancellation of amplitudes is not destroyed even by the field rotation. The same conclusion can be shown to hold true also in matter since the properties of $\nu_\mu$ and $\nu_\tau$ in matter are identical. This result changes if neutrinos possess nonzero masses and vacuum mixing.

In what follows we shall discuss the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations induced by flavour mixing and transition magnetic moment with one additional active neutrino, say, $\nu_\mu$. The case in which $\nu_x$ is a sterile neutrino will also be commented on.

3 $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations

Consider a system of four neutrino states $\nu_{eL}$, $\bar{\nu}_{eR}$, $\nu_{\mu L}$, and $\bar{\nu}_{\mu R}$ with vacuum mixing and transition magnetic moment $\mu$ relating $\nu_{eL}$ with $\bar{\nu}_{\mu R}$. The evolution of this system in matter and magnetic field can be described by the Schroedinger-like equation $i(d/dt)\nu = H\nu$, where
\[ \nu = (\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu)^T \] and \( H \) is the effective Hamiltonian of the system:

\[
H = \begin{pmatrix}
0 & 0 & s_2 \delta & \mu B_\perp \\
0 & -2N - \dot{\phi} & -\mu B_\perp & s_2 \delta \\
s_2 \delta & -\mu B_\perp & H_\mu & 0 \\
\mu B_\perp & s_2 \delta & 0 & H_\bar{\mu}
\end{pmatrix}
\] (2)

Here the angle \( \phi(t) \) defines the direction of the magnetic field \( B_\perp(t) \) in the plane orthogonal to the neutrino momentum, \( B_\perp(t) = |B_\perp(t)|, \phi \equiv d\phi/dt, N \equiv \sqrt{2}G_F(n_e - n_n/2), \delta \equiv \Delta m^2/4E, s_2 \equiv \sin 2\theta_0, c_2 \equiv \cos 2\theta_0 \), where \( G_F \) is the Fermi constant, \( n_e \) and \( n_n \) are the electron and neutron number densities, \( E \) is the neutrino energy, \( \Delta m^2 = m_2^2 - m_1^2, m_1, m_2 \) and \( \theta_0 \) being the neutrino masses and mixing angle in vacuum. The matrix elements \( H_\mu \) and \( H_\bar{\mu} \) in (2) read

\[
H_\mu \equiv -(1 + r)N + 2c_2 \delta, \quad H_\bar{\mu} \equiv -(1 - r)N + 2c_2 \delta - \dot{\phi}
\] (3)

where \( r \equiv n_n/(2n_e - n_n) \). The diagonal elements of \( H \) define the energies of the "flavour levels", i.e. of \( \nu_eL, \bar{\nu}_eR, \nu_\muL \) and \( \bar{\nu}_\muR \). According to our previous discussion, direct \( \nu_eL - \bar{\nu}_eR \) mixing is absent in the Hamiltonian (2), but can be induced in higher orders.

To have practically pure \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) transitions one should find the conditions under which the \( (\nu_{eL}, \bar{\nu}_{eR}) \) subsystem approximately decouples from the rest of the neutrino system. As we have indicated earlier, this decoupling should not be complete, otherwise the effective \( \nu_{eL} - \bar{\nu}_{eR} \) mixing would disappear. We shall assume that the following decoupling conditions are satisfied:

\[
|H_\mu|, |H_\bar{\mu}| \gg |2s_2 \delta|, 2\mu B_\perp.
\] (4)

This allows one to block-diagonalize the Hamiltonian (2); the resulting effective Hamiltonian

\[ In deriving eq. (2) we have moved to the reference frame rotating with the same angular velocity as the transverse magnetic field \[ \mathbf{B}_\perp(t) \] and also subtracted a matrix proportional to the unit one so as to make the first diagonal element equal to zero. These transformations amount to multiplying neutrino states by certain phase factors and thus do not affect the transition probabilities.
of the \( (\nu_{eL}, \bar{\nu}_{eR}) \) subsystem is

\[
H' = \begin{pmatrix}
0 & H_{e\bar{e}} \\
H_{e\bar{e}} & -2N - \dot{\phi} + \eta
\end{pmatrix}
\]  

(5)

Here the nondiagonal (mixing) matrix element is given by

\[
H_{e\bar{e}} = s_2 \delta \mu B_\perp \left( \frac{1}{H_\mu} - \frac{1}{H_{\bar{\mu}}} \right) \simeq s_2 \delta \mu B_\perp \frac{\dot{\phi} - 2rN}{[(-rN + \dot{\phi}/2)^2 - (2c_2\delta)^2]}.  
\]  

(6)

and \( \eta \equiv (\tan \omega - \cot \omega)H_{e\bar{e}} \), where

\[
\tan \omega \equiv s_2 \delta/\mu B_\perp.
\]  

(7)

Note that the second equality in eq. (6) holds only for \(|2N + \dot{\phi}| \ll \max \{ (1+r)N, |2c_2\delta| \} \) (see the discussion below). It follows from (6) that the effective \( \nu_{eL} - \bar{\nu}_{eR} \) mixing is caused by an interplay of flavour mixing and the one induced by the interaction of the magnetic moment with magnetic field. In accordance with our general discussion, it arises due to the transitions through the \( \nu_{\mu L} (\bar{\nu}_{\mu R}) \) states: \( \nu_{eL} \rightarrow \nu_{\mu L} (\bar{\nu}_{\mu R}) \rightarrow \bar{\nu}_{eR} \). In fact, \( H_{e\bar{e}} \) in eq. (6) exactly coincides with the result of the calculations in the second-order perturbation theory, and the values \( H^{-1}_\mu \) and \( H^{-1}_{\bar{\mu}} \) are just the propagators of the Schroedinger equation corresponding to \( \nu_{\mu L} \) and \( \bar{\nu}_{\mu R} \) in the intermediate state. They enter eq. (6) with opposite signs because of the antisymmetry of the matrix of transition magnetic moments. In vacuum \( H_\mu = H_{\bar{\mu}} \) and the contributions of \( \nu_\mu \) and \( \bar{\nu}_\mu \) cancel each other. Matter \( (n_n \neq 0) \) and magnetic field rotation \( (\dot{\phi} \neq 0) \) lift the degeneracy of the \( \nu_\mu \) and \( \bar{\nu}_\mu \) levels and so give rise to the \( \nu_{eL} - \bar{\nu}_{eR} \) mixing.

It follows from eqs. (6) and (4) that the \( \nu_{eL} - \bar{\nu}_{eR} \) mixing term is always smaller than each of the generic first order mixings \( s_2 \delta \) and \( \mu B_\perp \). The better the decoupling, the smaller \( H_{e\bar{e}} \). The mixing term \( H_{e\bar{e}} \) increases with decreasing \( H_\mu \) or \( H_{\bar{\mu}} \), but this enhancement is limited by conditions (4). The \( \nu_{eL} - \bar{\nu}_{eR} \) mixing angle \( \theta_m \) is defined as

\[
\tan 2\theta_m = \frac{2H_{e\bar{e}}}{-2N - \dot{\phi} + \eta}.
\]  

(8)

In medium with constant \( N, r, B_\perp \) and \( \dot{\phi} \), the evolution of the \( \nu_{eL} - \bar{\nu}_{eR} \) system will
have a character of oscillations with constant amplitude and length:

\[ P(\nu_{eL} \rightarrow \bar{\nu}_{eR}; t) = \frac{(2H_{ee})^2}{(2H_{ee})^2 + (2N + \phi - \eta)^2} \sin^2 \left( \frac{1}{2} \sqrt{(2H_{ee})^2 + (2N + \phi - \eta)^2} t \right) \] (9)

Let us stress that in nonrotating magnetic fields the \( \nu_{eL} - \bar{\nu}_{eR} \) mixing is always strongly suppressed. Indeed, for \( \dot{\phi} = 0 \) one gets \( \tan 2\theta_m \simeq 2r(\mu B_\perp s_2 \delta)/(H_\mu H_\bar{\mu}) \ll 1 \). On the contrary, in a twisting field the \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) oscillations can be resonantly enhanced. For

\[ \dot{\phi} = -2N + \eta \simeq -2N \] (10)

the \( \nu_{eL} - \bar{\nu}_{eR} \) mixing becomes maximal (\( \sin^2 2\theta_m = 1 \)) and the oscillations proceed with maximal depth. Eq. (10) is nothing else but the resonance condition for the \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) oscillations. It implies that the field rotation compensates for the energy splitting of the \( \nu_{eL} \) and \( \bar{\nu}_{eR} \) levels caused by their interaction with matter. Up to the small term \( \eta \sim H_{ee} \), it does not depend on the neutrino energy.

The necessity of matter and field rotation for strong \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) oscillations can therefore be understood as follows. Matter is needed to avoid the cancellation of the amplitudes with \( \nu_\mu \) and \( \bar{\nu}_\mu \) intermediate states in (6). However, matter lifts the degeneracy of the \( \nu_e \) and \( \bar{\nu}_e \) as well, and so the \( \nu_{eL} \leftrightarrow \bar{\nu}_{eL} \) oscillations will not proceed with maximal amplitude. Field rotation can restore the degeneracy of \( \nu_e \) and \( \bar{\nu}_e \) while keeping the \( \nu_\mu \) and \( \bar{\nu}_\mu \) energies split, or vice versa. This comes about because the energies of the electron and muon neutrinos in matter have different density dependence. For the same reason the field rotation alone cannot lead to large-amplitude \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) oscillations.

The amplitude of the \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) oscillations in eq. (9) has a resonant dependence on the parameters of the problem. The width of the resonant peak at half-height is

\[ \frac{\Delta \dot{\phi}}{|(\dot{\phi})_{res}|} = \frac{\Delta N}{N_{res}} = \frac{2|H_{ee}|}{N_{res}} \simeq 4(1 + r) \frac{\mu B_\perp s_2 \delta}{[H_\mu H_\bar{\mu}]} \ll 1, \] (11)

i.e. the density and \( \dot{\phi} \) widths of the resonance are very small. On the contrary, due to the fact that the resonance condition is almost energy-independent, the energy width of the
resonant peak can be fairly large:

\[
\frac{\Delta E}{E_{\text{res}}} = \begin{cases} 
2(s_2\delta_0/\mu_B) \approx 2 \left[ 1 - \frac{2(2N+\phi)c^2}{(1+r)N\delta^2} \right]^{-1/2}, & (1+r)N \ll 2c_2\delta, \\
2(\mu B_{\perp}/s_2\delta_0) \approx 2 \left[ 1 - \frac{(1+r)N(2N+\phi)}{2(\mu B_{\perp})^2} \right]^{-1/2}, & (1+r)N \gg 2c_2\delta \end{cases}
\]  

(\delta_0 is the value of \(\delta\) at resonance). This means that for a neutrino beam with continuous energy spectrum a large fraction of neutrinos can undergo resonantly enhanced \(\nu_{eL} \leftrightarrow \bar{\nu}_{eR}\) oscillations.

From eq. (9) it follows that the \(\nu_{eL} \leftrightarrow \bar{\nu}_{eR}\) oscillation length at resonance \((l_{ee})_{\text{res}} = \pi/H_{ee}\) is much bigger than the lengths of both the flavour oscillations \(\nu_{eL} \leftrightarrow \nu_{\mu L}\) and the spin-flavour precession \(\nu_{eL} \leftrightarrow \bar{\nu}_{\mu R}\) caused by the generic first-order mixings. The latter two processes will proceed with small amplitudes \(\sim (s_2\delta/\mu)^2\) and \((\mu B_{\perp}/H_{\mu})^2\) respectively. Therefore the \(\nu_e\) transition probability will be described by a superposition of maximal-amplitude long-wavelength and small-amplitude short-wavelength oscillations (Fig. 1a).

4 \(\nu_{eL} \leftrightarrow \bar{\nu}_{eR}\) oscillations influenced by a third neutrino state

If the flavour energy levels \(H_{\mu}\) or \(H_{\bar{\mu}} \rightarrow 0\), the \(\nu_e - \bar{\nu}_e\) mixing term \(H_{ee}\) and the width of the \(\nu_e - \bar{\nu}_e\) resonance increase, but the influence of the \(\nu_{\mu}\) or \(\bar{\nu}_{\mu}\) levels on the \(\nu_{eL} \leftrightarrow \bar{\nu}_{eR}\) transition probability becomes stronger. In particular, the amplitudes of the \(\nu_{eL} \leftrightarrow \nu_{\mu L}(\bar{\nu}_{\mu R})\) transitions increase. In this case the \(\nu_{eL} \leftrightarrow \bar{\nu}_{eR}\) resonance density approaches the MSW one [6] or that of the resonant spin-flavour precession [9, 12].

Let us assume that

\[
\frac{\dot{\phi}}{2} = -N = -\frac{2c_2\delta}{1+r},
\]

which corresponds to \(H_{\mu} = 0\). Now the energy levels of three neutrino states, namely, those of \(\nu_{eL}, \bar{\nu}_{eR}\) and \(\nu_{\mu L}\), cross in one point (this means that the resonances of the \(\nu_{eL} \leftrightarrow \bar{\nu}_{eR}\),
\( \nu_{eL} \leftrightarrow \nu_{\mu L} \) and \( \nu_{\mu L} \leftrightarrow \bar{\nu}_{eR} \) transitions merge in one point). The influence of the \( \nu_{\mu} \) state on the \( (\nu_{eL}, \bar{\nu}_{eR}) \) system becomes maximal. However, if \( H_{\bar{\mu}} \) satisfies eq. (4) (which is realized for \( s_2 \ll 1 \) and \( \mu B_\perp \ll 4 c_2 \delta \), the \( \bar{\nu}_{\mu} \) state will still be decoupled, and so one can perform a 3–1 block-diagonalization of \( H \). The resulting effective Hamiltonian of the strongly coupled \( (\nu_{eL}, \bar{\nu}_{eR}, \nu_{\mu L}) \) system is just given by the 3 \( \times \) 3 matrix in the upper left corner of \( H \) in eq. (2) with the following modifications: (i) the diagonal terms acquire small corrections which are not important for our consideration, and (ii) the \( H_{ee} = H_{\bar{e}e} \) terms are no longer zero but rather are equal to \( (-\epsilon) \), where

\[
\epsilon = \frac{(s_2 \delta) (\mu B_\perp)}{H_{\bar{\mu}}}. \tag{14}
\]

The eigenvalues of this Hamiltonian are

\[
H_1 \cong f - \frac{\epsilon}{2}, \quad H_2 \cong -f - \frac{\epsilon}{2}, \quad H_3 \cong \epsilon', \tag{15}
\]

where

\[
f = \sqrt{(\mu B_\perp)^2 + (s_2 \delta)^2}, \quad \epsilon' = -\epsilon \sin 2\omega, \tag{16}
\]

and \( \omega \) is defined by eq. (7). The orthogonal matrix diagonalizing the effective Hamiltonian is given to leading order in \( \epsilon/f \) by

\[
S_m = \begin{pmatrix}
\frac{\sin \omega}{\sqrt{2}} & \frac{\sin \omega}{\sqrt{2}} & \cos \omega \\
-\frac{\cos \omega}{\sqrt{2}} & -\frac{\cos \omega}{\sqrt{2}} & \sin \omega \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{\epsilon}{f} \cos 2\omega
\end{pmatrix}. \tag{17}
\]

For constant \( N, r, B_\perp \) and \( \dot{\phi} \) one obtains from (15)–(17) the following probability of the \( \nu_{eL} \leftrightarrow \bar{\nu}_{eR} \) oscillations:

\[
P(\nu_{eL} \rightarrow \bar{\nu}_{eR}; t) = \sin^2 2\omega \sin^4 \frac{1}{2} f t + \sin^2 2\omega \sin^2 \frac{3}{4} \epsilon' t \cos ft + d \sin \epsilon' t \sin ft, \tag{18}
\]

where \( d = O(\epsilon) \). The first term in (18) corresponds to the limit \( \epsilon \rightarrow 0 \). We see that in the three-level crossing point the depth of the \( \nu_{eL} - \bar{\nu}_{eR} \) oscillations, \( \sin^2 2\omega \), does not exhibit any suppression related to the higher order direct \( \nu_{eL} - \bar{\nu}_{eR} \) mixing. Moreover, in the symmetric
case $s_2\delta = \mu B_\perp$, when the vacuum mixing is equal to that induced by the magnetic moment interaction, one has $\sin 2\omega = 1$ and the oscillation depth becomes maximal. For $s_2\delta \neq \mu B_\perp$ the oscillation depth is less than unity and it decreases when the difference between $s_2\delta$ and $\mu B_\perp$ increases.

The $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillation length is given by

$$l_{ee} = \frac{2\pi}{f} = \frac{2\pi}{\sqrt{(s_2\delta)^2 + (\mu B_\perp)^2}}. \quad (19)$$

For $s_2\delta = \mu B_\perp$ it is only $\sqrt{2}$ times bigger than flavour oscillation or spin-flavour precession lengths at the resonance point: $l_{ee} = \sqrt{2}l_{osc} = \sqrt{2}l_p$ ($l_{osc} = 4\pi E/(\Delta m^2 \sin 2\theta_0)$, $l_p = \pi/\mu B_\perp$). In contrast with the case of pure $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations discussed above, $l_{ee}$ does not contain any big factor like $f/\epsilon$; $P(\nu_{eL} \to \bar{\nu}_{eR})$ depends on the fourth power of $\sin(\pi t/l_{ee})$ rather than on the second power (see Fig. 1b). These features are related to the fact that there are three levels involved and that the splitting between the levels is determined by $f$. Now three neutrino species oscillate into each other with comparable amplitudes. For example, the probability of the $\nu_{eL} \to \nu_{\mu L}$ oscillations is $P(\nu_{eL} \to \nu_{\mu L}; t) \approx \sin^2 \omega \cdot \sin^2 ft$. Note that the oscillation length for this mode is two times as small as $l_{ee}$. For maximal depth of the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations the amplitude of the $\nu_{eL} \leftrightarrow \nu_{\mu L}$ oscillations is $\sin^2 \omega = 1/2$.

The second term in (18) is generated by the direct $\nu_{eL} - \bar{\nu}_{eR}$ mixing $\sim \epsilon$ in the $3 \times 3$ effective Hamiltonian. It gives a long-period modulation of the oscillation probability. The corresponding modulation length is $l_{mod} \approx 4\pi/(3\epsilon') \gg l_{ee}$. The first two terms in (18) can also be written as

$$\frac{1}{4} \sin^2 2\omega (1 + \cos^2 ft - 2 \cos ft \cos \frac{3}{2} \epsilon' t),$$

which implies that the modulation leads to the oscillation depth varying between $\sin^2 2\omega$ and $\frac{1}{2} \sin^2 2\omega$ (Fig. 1b).

Let us stress that the maximal-amplitude short-wavelength oscillations described by eq. (18) are only possible in a rotating field, when the merging condition (13) can be fulfilled. The merging condition and $l_{ee}$ depend on the neutrino energy, and so the enhancement of the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations in this case has a resonant character, too.
5 Discussion and conclusions

We have discussed so far $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ transitions with an active neutrino in the intermediate state. Evidently, instead of $\nu_\mu$ or $\nu_\tau$, a sterile neutrino could play the role of the additional neutrino required for the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations. All the above results hold in this case as well; the corresponding analytical expressions can be obtained from those already derived by setting $r = 0$. Now, according to (6), $H_{ee} \sim \dot{\phi}$ and the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ transitions can only occur in twisting magnetic fields: matter alone is not sufficient to induce the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations. Sterile neutrinos do not interact with matter and so the degeneracy of $\nu_{xL}$ and $\bar{\nu}_{xR}$ can only be lifted by the magnetic field rotation.

Our previous discussion was mainly constrained to the case of constant $N, r, B_\perp$ and $\dot{\phi}$. In a medium with matter density, and magnetic field varying along the neutrino path, the maximum-amplitude $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations will take place if the functions $N(t)$ and $\phi(t)$ satisfy the resonance condition (10) over a sufficiently large space interval $\Delta t$.

Eq. (10) is in fact just the condition of crossing of the $\nu_e$ and $\bar{\nu}_e$ levels. Thus, if the parameters of the medium vary in such a way that at a certain point $t_r$, eq. (10) holds, resonant $\nu_{eL} \rightarrow \bar{\nu}_{eR}$ conversion may take place in the resonant region $[13]$. The conversion can be nearly complete if the matter density and $\dot{\phi}$ vary slowly enough (adiabatically) along the neutrino path.

The maximum-amplitude $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations we have discussed, especially those in the merging point, can provide an efficient mechanism of generation of $\bar{\nu}_e$ flux in the sun. They can also have important consequences for neutrinos created in collapsing stars.

In conclusion, we have shown that the original Pontecorvo’s idea of large-amplitude neutrino-antineutrino oscillations can be realized provided the following conditions are satisfied:

(i) at least one additional neutrino $\nu_x$ is involved in the transitions, which is mixed with the initial neutrino $\nu_\ell$ through flavour (mass) mixing;
(ii) there exists a transition magnetic moment which connects the $\nu_l$ with $\bar{\nu}_x$;

(iii) neutrino propagates in matter and transverse magnetic field, and

(iv) the direction of magnetic field changes along the neutrino path.

There are two distinct cases of large-amplitude $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations: (a) practically pure $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations when $\nu_{eL}$ and $\bar{\nu}_{eR}$ decouple from the rest of the system (the resonant condition (10) should be satisfied); (b) $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations in the presence of strong transitions into a third neutrino. In this case the merging condition (13) should be fulfilled and, in addition, the flavour mixing should be approximately equal to the magnetic-moment induced one ($\mu B_\perp \simeq s_2 \delta$). The two cases are characterized by different oscillation lengths and different forms of dependences of the oscillation probability on the distance.

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Figure caption

Fig. 1. Dependence of the $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillation probabilities on the distance travelled by neutrinos: (a) the case of isolated $\nu_{eL} - \bar{\nu}_{eR}$ system, (b) $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ oscillations in the presence of strong transitions into a third neutrino state.