Large-$N$ Solution of the Heterotic $\mathcal{N} = (0, 2)$ Two-Dimensional $CP(N - 1)$ Model

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Abstract

We continue explorations of non-Abelian strings, focusing on the solution of a heterotic deformation of the $CP(N - 1)$ model with an extra right-handed fermion field and $\mathcal{N} = (0, 2)$ supersymmetry. This model emerges as a low-energy theory on the worldsheet of the BPS-saturated flux tubes (strings) in $\mathcal{N} = 2$ supersymmetric QCD deformed by a superpotential of a special type breaking $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$. Using large-$N$ expansion we solve this model to the leading order in $1/N$. Our solution exhibits spontaneous supersymmetry breaking for all values of the deformation parameter. We identify the Goldstino field. The discrete $\mathbb{Z}_2 N$ symmetry is shown to be spontaneously broken down to $\mathbb{Z}_2$; therefore the worldsheet model has $N$ strictly degenerate vacua (with nonvanishing vacuum energy). Thus, the heterotic $CP(N - 1)$ model is in the deconfinement phase. We can compare this dynamical pattern, on the one hand, with the $\mathcal{N} = (2, 2)$ $CP(N - 1)$ model which has $N$ degenerate vacua with unbroken supersymmetry, and, on the other hand, with nonsupersymmetric $CP(N - 1)$ model with split quasivacua and the Coulomb/confining phase. We determine the mass spectrum of the heterotic $CP(N - 1)$ model in the large-$N$ limit.
1 Introduction

This paper continues exploration [1] of the heterotic $\mathcal{N} = (0,2)$ model with the $CP(N-1)$ target space for the bosonic fields. We solve this model at large $N$ in the leading order in $1/N$ using the $1/N$ expansion technique which was designed by Witten for a parent model [2].

Two-dimensional $CP(N-1)$ models emerge as effective low-energy theories on the worldsheet of non-Abelian strings which had been found in a class of four-dimensional gauge theories [3, 4, 5, 6], see also the review papers [7, 8, 9]. The main feature of the above non-Abelian strings is the occurrence of orientational moduli associated with rotations of their color fluxes inside a global SU($N$) group. Internal dynamics of the orientational moduli is described by two-dimensional $CP(N-1)$ model.

The first non-Abelian strings were found, as critical solitonic solutions, in $\mathcal{N} = 2$ supersymmetric gauge theories. These bulk theories have eight conserved supercharges; hence, four of them are conserved on the worldsheet. Thus, if the bulk theory has $\mathcal{N} = 2$ supersymmetry (SUSY), the $CP(N-1)$ model on the string worldsheet is automatically $\mathcal{N} = 2$ supersymmetric (more exactly, it is $\mathcal{N} = (2,2)$).

Then it was shown that non-Abelian BPS-saturated strings [1] survive certain $\mathcal{N} = 1$ preserving deformations of the bulk $\mathcal{N} = 2$ theory. In particular, a mass term for the adjoint fields was considered in [10]. The string solution at the classical level remains BPS-saturated. With four supercharges in the bulk, normally, this would imply conservation of two supercharges on the string worldsheet. Previously it was believed, however, that worldsheet supersymmetry gets an “accidental” enhancement [10]. This is due to the facts that $\mathcal{N} = (1,1)$ SUSY is automatically elevated up to $\mathcal{N} = (2,2)$ on $CP(N-1)$ and, at the same time, there are no “heterotic” $\mathcal{N} = (0,2)$ generalizations of the bosonic $CP(N-1)$ model.

Edalati and Tong noted [11] that the target space is in fact $CP(N-1) \times \mathbb{C}$ rather than $CP(N-1)$. If two fermionic moduli (former supertranslational moduli) become coupled to superorientational moduli, one can built a heterotic $\mathcal{N} = (0,2)$ model with the $CP(N-1)$ target space for the bosonic moduli. Edalati and Tong suggested a general structure of such a model (in the gauged formulation). Later Tong argued [12] that $\mathcal{N} = (0,2)$ supersymmetry of the heterotic model is spontaneously broken at the quantum level.

A geometric representation for the heterotic $\mathcal{N} = (0,2)$ model was obtained in Ref. [1],

\[1\] We mean BPS-saturated at the classical level.
\[ L_{\text{heterotic}} = \xi_R^i \partial_L \xi_R + \left[ \gamma g_0^2 \xi_R G_{ij} (i \partial_L \phi^i \phi_j^j) \psi_{IR}^i + \text{H.c.} \right] \]

\[-g_0^4 |\gamma|^2 \left( \xi_R^i \xi_R \right) \left( G_{ij} \psi_{IR}^j \psi_{IL}^i \right) \]

\[+ G_{ij} \left[ \partial_\mu \phi^i \partial_\mu \phi^j + i \bar{\psi}_i^j \gamma^\mu D_\mu \psi_j^i \right] \]

\[-\frac{g_0^2}{2} \left( G_{ij} \psi_{IR}^j \psi_{IL}^i \right) \left( G_{km} \psi_{IL}^k \psi_{LR}^m \right) \]

\[+ \frac{g_0^2}{2} \left( 1 - 2g_0^2 |\gamma|^2 \right) \left( G_{ij} \psi_{IR}^j \psi_{IL}^i \right) \left( G_{km} \psi_{IL}^k \psi_{LR}^m \right) , \quad (1.1) \]

where \( G_{ij} \) is the \( CP(N-1) \) metric,

\[ G_{ij} = \frac{\partial^2 K(\phi, \phi^j)}{\partial \phi^i \partial \phi^j} , \quad K = \frac{2}{g_0^2} \ln \left( 1 + \sum_{i,j=1}^{N-1} \phi^i \delta_{ji} \phi^j \right) , \quad (1.2) \]

\( K \) is the Kähler potential, \( \gamma \) is the deformation parameter, while the curvature tensor \( R_{ijk\bar{m}} \) can be written as

\[ R_{ijk\bar{m}} = -\frac{g_0^2}{2} \left( G_{ij} G_{km} + G_{im} G_{kj} \right) . \quad (1.3) \]

The first two lines in Eq. (1.1) describe the kinetic term and interactions of an additional field, the right-handed fermion \( \xi_R \), which is the only remnant of the \( C \) factor of the moduli space of the string (i.e. \( CP(N-1) \times C \)). In Ref. [1] \( \gamma \) was obtained in terms of the deformation parameter of the bulk theory. The last three lines describe the \( N = (2,2) \) \( CP(N-1) \) model fields. In fact, putting \( \gamma \) to zero in the last line we get just the standard \( N = (2,2) \) \( CP(N-1) \) model.

Although qualitatively we agree with [1], there are several distinctions in a number of aspects\(^2\). Dynamics of the model (1.1) is intriguing and nontrivial; in particular, we proved [1] the fact that at small \( |\gamma| \) supersymmetry is spontaneously broken, with \( \xi_R \) playing the role of Goldstino. In the limit \( |\gamma| \ll 1 \) the vacuum energy density is proportional to the square of the bifermion condensate\(^3\)

\[ \mathcal{E}_{\text{vac}} = |\gamma|^2 \left| \langle R_{ij} \psi_{IR}^i \psi_{IL}^j \rangle \right|^2 \neq 0 . \quad (1.4) \]

Thus, we confirmed Tong’s conjecture [12] of spontaneous SUSY breaking.

\(^2\) These distinctions are discussed in [1]; we will say more on that below.

\(^3\) The bifermion condensate in Eq. (1.4) must be evaluated at \( \gamma = 0 \), i.e. in the \( N = (2,2) \) model.
Our task in the present paper is to solve the heterotic $\mathcal{N} = (0, 2)$ model for arbitrary values of $\gamma$. Although we cannot do it for arbitrary $N$, at large $N$ powerful methods of $1/N$ expansion do allow us to find a complete solution. Qualitative features of this solution are expected to be valid even for $N = 2$. The representation of the model which is most convenient for the $1/N$ expansion is the so-called gauged formulation \[2, 13\].

The model (1.1) plays a two-fold role. If the deformation parameter is smaller than a critical value, to be discussed in Sect. 6, it describes the worldsheet dynamics of the four-dimensional heterotic string. If it becomes larger than a critical value, the string swells, and two-derivative terms no longer capture its worldsheet dynamics. In this limit the model (1.1) can be considered on its own right, with no reference to non-Abelian strings in four dimensions. We focus on the first aspect. At the same time, dynamics of the heterotic $CP(N-1)$ model in the limit of infinitely large deformation parameter (presumably, conformal) is intriguing and captivating. This is a topic for a separate investigation, though.

Our main results are as follows. We prove spontaneous SUSY breaking for all values of the deformation parameter, identify the Goldstino field and find the mass spectrum of excitations. The bifermion condensate is shown to develop at all finite values of the deformation parameter. It plays the role of the order parameter for the spontaneous breaking of the $Z_{2N}$ symmetry of the heterotic $CP(N-1)$ model. $Z_{2N}$ is broken down to $Z_2$. $N$ vacua of the model have nonvanishing energy but are strictly degenerate. This fact guarantees that the model is in the deconfinement phase.

Organization of the paper is as follows. In Sect. 2 we briefly review the gauged formulation of the standard $\mathcal{N} = (2, 2)$ model, outline the gauged formulation of the $\mathcal{N} = (0, 2)$ model and discuss various regimes for the deformation parameter. In Sect. 3 we calculate the one-loop effective potential in the large-$N$ limit, and then analyze the vacuum structure. Section 4 is devoted to the mass spectrum of excitations. In Sect. 5 we discuss deconfinement regime in the heterotic $CP(N-1)$ model, as opposed to the Coulomb/confinement regime in its nonsupersymmetric “parent.” Section 6 presents arguments regarding the limiting dynamics of the heterotic $CP(N-1)$ model in the limit of infinitely large deformation parameter. Section 7 summarizes our findings.

2 Heterotic $\mathcal{N} = (0, 2)$ model

We will start from reviewing the gauged formulation of the conventional $\mathcal{N} = (2, 2)$ model, and then elaborate a similar formulation for the heterotic $\mathcal{N} = (0, 2)$ model.
2.1 Gauged formulation of the undeformed $\mathcal{N} = (2, 2)$ model

The target space of the orientational moduli of the non-Abelian string is

$$\frac{SU(N)_{C+F}}{SU(N-1) \times U(1)} \sim CP(N-1).$$

(2.1)

With $\mathcal{N} = 2$ SQCD in the bulk, orientational moduli completely decouple from the (super)translational ones, and can be considered in isolation. Here we will briefly describe the conventional $CP(N-1)$ model with $\mathcal{N} = (2, 2)$ supersymmetry (which is a part of string worldsheet theory) in the gauged formulation [13].

This formulation implies introduction of a U(1) gauge field $A$ which gauges the U(1) symmetry of $N$ complex fields $n^l$. The gauge coupling is assumed to be large in the bare Lagrangian, $e^2 \to \infty$, so that the kinetic term of $A$ vanishes. The bosonic part of the action is

$$S_{CP(N-1) \text{ bos}} = \int d^2x \left\{ |\nabla_k n^l|^2 + \frac{1}{4e^2} F_{kl}^2 + \frac{1}{e^2} |\partial_k \sigma|^2 + \frac{1}{2e^2} D^2 + 2 |\sigma|^2 |n^l|^2 + iD(|n^l|^2 - r_0) \right\},$$

(2.2)

where

$$\nabla_k = \partial_k - iA_k$$

(2.3)

while $\sigma$ is a complex scalar field. Moreover, $r_0$ can be interpreted as a coupling constant, and $D$ is a $D$-component of the gauge multiplet. The $i$ factor in the last term of Eq. (2.2) is due to Euclidean notation. (For our conventions and notation see Ref. [1].)

The bare constant $r_0$ of the worldsheet model is related to the coupling constant of the bulk theory at the scale determined by the bulk gauge boson mass $m_W$ (see e.g. [2]),

$$r_0 = \frac{4\pi}{g_2^2(m_W)} = \frac{N}{2\pi} \ln \frac{m_W}{\Lambda},$$

(2.4)

where $\Lambda$ is the dynamical scale of the $CP(N-1)$ model. To keep the bulk theory weakly coupled we must assume that $m_W \gg \Lambda$.

Eliminating $D$ from the action (2.2) leads to the constraint

$$|n^l|^2 = r_0.$$  \hspace{1cm} (2.5)

As was mentioned, in the limit $e^2 \to \infty$ the gauge field $A_k$ and its $\mathcal{N} = 2$ bosonic superpartner $\sigma$ become auxiliary and can be eliminated by virtue of the equations of motion,

$$A_k = -\frac{i}{2r_0} \bar{n}_l \partial_k n^l.$$  \hspace{1cm} (2.6)

\footnote{The coupling constant $r_0$ is related to the coupling $\beta$ used in [1] as $r_0 = 2\beta$.}
With $2N$ complex fields $n^i$, one real constraint \((2.5)\) and one phase “eaten” by gauging the common $U(1)$ symmetry, the model has $2N - 1 - 1 = 2(N - 1)$ real variables. This is precisely the number of the bosonic fields in Eq. \((1.1)\).

Now, let us pass to the fermionic sector of the $\mathcal{N} = (2, 2)$ model. The corresponding part of the action in the gauged formulation takes the form

$$S_{\text{CP}(N)}^{\text{ferm}} = \int d^2x \left\{ \bar{\xi}^l R i(\nabla_0 - i\nabla_3) \xi^l_R + \bar{\xi}^l L i(\nabla_0 + i\nabla_3) \xi^l_L 
+ \frac{1}{e^2} \lambda^l_R i(\nabla_0 - i\nabla_3) \lambda^l_R + \frac{1}{e^2} \lambda^l_L i(\nabla_0 + i\nabla_3) \lambda^l_L + \left[ i\sqrt{2} \sigma \bar{\xi}^l_R \xi^l_L 
+ \frac{i\sqrt{2}}{r_0} (\lambda^l_R \xi^l_L - \lambda^l_L \xi^l_R) + \text{H.c.} \right] \right\}, \tag{2.7}$$

where the fields $\xi^l_{L,R}$ are the fermion superpartners of $n^i$ while $\lambda^l_{L,R}$ belong to the gauge multiplet. In what follows we will introduce a shorthand

$$\nabla_L \equiv \nabla_0 - i\nabla_3, \quad \nabla_R \equiv \nabla_0 + i\nabla_3. \tag{2.8}$$

Integrating the fields $\lambda^l_{L,R}$ in the limit $e^2 \to \infty$ we arrive at the following constraints:

$$\bar{n}^l \xi^l_L = 0, \quad \bar{n}^l \xi^l_R = 0. \tag{2.9}$$

Moreover, integrating over the $\sigma$ field gives

$$\sigma = -\frac{i}{\sqrt{2} r_0} \bar{\xi}^l_L \xi^l_R. \tag{2.10}$$

The $U(1)$ gauge field $A$ now takes the form

$$A_0 + iA_3 = -\frac{i}{2r_0} \bar{n}^l \left( \bar{\zeta}^{\leftrightarrow}_0 + i \bar{\zeta}^{\leftrightarrow}_3 \right) n^l - \frac{1}{r_0} \bar{\xi}^l_R \xi^l_R, \tag{2.11}$$

$$A_0 - iA_3 = -\frac{i}{2r_0} \bar{n}^l \left( \bar{\zeta}^{\leftrightarrow}_0 - i \bar{\zeta}^{\leftrightarrow}_3 \right) n^l - \frac{1}{r_0} \bar{\zeta}^l_L \xi^l_L.$$

When we substitute the above expressions in the Lagrangian we generate the four-fermion interactions $(\bar{\xi}^l_R \xi^l_L)(\bar{\xi}^k_R \xi^k_L)$ and $(\bar{\xi}^l_L \xi^l_L)(\bar{\xi}^k_R \xi^k_R)$, respectively.

Besides orientational and superorientational moduli, the BPS-saturated non-Abelian strings have (super)translational moduli too. They are related to the possibility of shifting the string center $x_{0i}$ in the plane orthogonal to its axis, $i = 1, 2$. The corresponding supertranslational moduli are $\zeta_R$, $\zeta_L$. In the $\mathcal{N} = 2$ bulk theory the worldsheet fields $x_{0i}(t, z)$, $\zeta_R(t, z)$ and $\zeta_L(t, z)$ are just free fields decoupled from the orientational sector.
2.2 Heterotic $\mathcal{N} = (0, 2)$ model

After we break $\mathcal{N} = 2$ supersymmetry of the bulk model by switching on the deformation superpotential for the adjoint fields,

$$\mathcal{W}_{3+1} = (\mu/2) \left( A^2 + (A^a)^2 \right)$$

(2.12)

(see [1]), the above decoupling is no longer valid. The classical string solution still remains $1/2$ BPS-saturated [10] (see also [11, 11]). Two supercharges that survive on the string worldsheet still protect $x_0i$ and $\zeta_L$. The worldsheet fields $x_0i(t, z)$ and $\zeta_L(t, z)$ remain free fields decoupled from all others. This is no longer the case with regards to $\zeta_R$ which gets an interaction with $\xi$’s.

As a result, the heterotic $\mathcal{N} = (0, 2)$ model in the gauged formulation takes the form

$$S = \int d^2 x \left\{ \frac{1}{2} \bar{\xi}_R i \partial_L \xi_R + [\sqrt{2} i \omega \bar{\lambda}_L \xi_R + \text{H.c.}] \right. \right. + |\nabla_L n_l|^2 + \frac{1}{4e^2} F_{kl}^2 + \frac{1}{e^2} |\partial_k \sigma|^2 + \frac{1}{2e^2} D^2 + 2|\sigma|^2 |n_l|^2 \right. + i D(|n_l|^2 - r_0) \right. \left. + \bar{\xi}_R i \nabla_L \xi_R^l + \bar{\xi}_R i \nabla_R \xi_R^l + \frac{1}{e^2} \bar{\lambda}_R i \partial_L \lambda_R + \frac{1}{e^2} \bar{\lambda}_L i \partial_R \lambda_L \right. \left. \right. + \left( [i \sqrt{2} \sigma \bar{\xi}_R \xi_L^l + i \sqrt{2} \bar{n}_l (\lambda_R \xi_L^l - \lambda_L \xi_R^l) + \text{H.c.}] + 4 |\omega|^2 |\sigma|^2 \right\},$$

(2.13)

where we omitted the fields $x_0i(t, z)$ and $\zeta_L(t, z)$ as irrelevant for the present consideration. This is the action obtained in [11]. Here $\partial_{L,R} = \partial_0 \mp i \partial_3$. The terms containing $\zeta_R$ and/or $\omega$ break $\mathcal{N} = (2, 2)$ supersymmetry down to $\mathcal{N} = (0, 2)$. The parameter $\omega$ is complex and dimensionless.\footnote{The relation of $\omega$ to the $\mathcal{N} = (0, 2)$ deformation parameter $\delta$ used in [1] is $\omega = \sqrt{\tau_0} \delta$.}

Integrating over the axillary fields $\lambda$ we arrive at the constraints

$$\bar{n}_l \xi_L^l = 0, \quad \bar{\xi}_R n_l = \omega \xi_R,$$

(2.14)

replacing those in Eq. (2.9). We see that the constraint (2.9) is modified for the right-handed fermions $\xi_R$ implying that the supertranslational sector of the worldsheet theory is no longer decoupled from the orientational one. The general structure of the deformation in (2.13) is dictated by $\mathcal{N} = (0, 2)$ supersymmetry.

\footnote{The relation of $\omega$ to the $\mathcal{N} = (0, 2)$ deformation parameter $\delta$ used in [1] is $\omega = \sqrt{\tau_0} \delta$.}
2.3 On the value of the deformation parameter

Edalati and Tong conjectured [11] that the worldsheet deformation parameter $\omega$ is proportional to the bulk deformation parameter $\mu$ (see Eq. (2.12)),

$$\omega \sim \mu.$$ (2.15)

In the previous paper [1] we derived the worldsheet theory (2.13) directly from the bulk theory. This derivation provides us with a relation between the bulk and worldsheet deformation parameters, namely,

$$\omega = \begin{cases} 
\text{const } \sqrt{r_0 \frac{g_5^2 \mu}{m_W}}, & \text{small } \mu, \\
\text{const } \sqrt{r_0 \ln \frac{g_5^2 \mu}{m_W}}, & \text{large } \mu. 
\end{cases}$$ (2.16)

Here $g_5^2$ is the SU(2) gauge coupling of the bulk theory. The worldsheet deformation parameter $\omega$ is determined by the profile functions of the string solution [1]. For simplicity we assume that $\mu$ and $\omega$ are real. In the general case $\arg \omega = \arg \mu$.

While the small-$\mu$ result in (2.16) is in accordance with the Edalati–Tong conjecture, the large-$\mu$ behavior ($g_5^2 \mu \gg m_W$ where $m_W$ is the W-boson mass) indicated in Eq. (2.16) is in contradiction, since we get logarithmic rather than power behavior.

The physical reason for the logarithmic behavior of the worldsheet deformation parameter with $\mu$ is as follows. In the large-$\mu$ limit certain states in the bulk theory become light [10, 11]. This reflects the presence of the Higgs branch in $\mathcal{N} = 1$ SQCD [14] to which our bulk theory flows in the $\mu \to \infty$ limit. The argument of the logarithm in (2.16) is the ratio of $m_W$ and a small mass of the light states associated with this would-be Higgs branch [1].

Now, let us discuss $N$ counting. How all expressions relevant to the problem at hand depend on $N$ at large $N$? It is obvious that

$$r_0 \sim N,$$ (2.17)

while the masses of physical states and the scale of the theory do not depend on $N$,

$$m_W \sim N^0, \quad g_5^2 \mu \sim N^0, \quad \Lambda \sim N^0.$$ (2.18)

This, in turn, implies that the deformation parameter $\omega$ behaves as $^6$

$$\omega \sim \sqrt{N}.$$ (2.19)

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$^6$In Ref. [1] the bulk theory with the U(2) gauge group was studied. Thus, strictly speaking, the result (2.16) for the dependence of $\omega$ on $\mu$ was derived only in the $N = 2$ case. However, the $N$ dependence of $\omega$ is captured correctly by the factor $\sqrt{r_0}$ in these equations, therefore we generalize here (2.16) to arbitrary $N$. 

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2.4 Axial U(1)

The model (2.13) has a U(1) axial symmetry which is broken by the chiral anomaly down to the discrete subgroup \(Z_{2N}[2]\). Now, the \(\sigma\) field is related to the fermion bilinear operator by the following formula:

\[
\sigma = -\frac{i}{\sqrt{2}(r_0 + 2|\omega|^2)} \bar{\xi}_L \xi_R^l
\]

(cf. (2.10)). Moreover, under the above \(Z_{2N}\) symmetry transformation it transforms as

\[
\sigma \rightarrow e^{\frac{2\pi i}{2N}} \sigma,
\]

(2.21)

We will show below that the \(Z_{2N}\) symmetry is spontaneously broken by the condensation of \(\sigma\), down to \(Z_2\), much in the same way as in the conventional \(\mathcal{N} = (2,2)\) model [2]. This is equivalent to saying that the fermion bilinear condensate \(\langle \bar{\xi}_L \xi_R^l \rangle\) develops, breaking the discrete \(Z_{2N}\) symmetry down to \(Z_2\).

3 One-loop effective potential

The \(\mathcal{N} = (2,2)\) model as well as nonsupersymmetric \(CP(N-1)\) model were solved by Witten in the large-N limit [2]. The same method was used in [15] to study nonsupersymmetric \(CP(N-1)\) model with twisted mass. In this section we will generalize Witten’s analysis to solve the \(\mathcal{N} = (0,2)\) theory.

Since the action (2.13) is quadratic in the fields \(n^l\) and \(\xi^l\) we can integrate over these fields and then minimize the resulting effective action with respect to the fields from the gauge multiplet. The large-N limit ensures the corrections to the saddle point approximation to be small. In fact, this procedure boils down to calculating a small set of one-loop graphs with the \(n^l\) and \(\xi^l\) fields propagating in loops. After integrating \(n^l\) and \(\xi^l\) out, we must check self-consistency.

Integration over \(n^l\) and \(\xi^l\) in (2.13) yields the following determinants:

\[
\left[ \det \left( -\partial_k^2 + iD + 2|\sigma|^2 \right) \right]^{-N} \left[ \det \left( -\partial_k^2 + 2|\sigma|^2 \right) \right]^N,
\]

(3.1)

where we dropped the gauge field \(A_k\). The first determinant here comes from the boson loops while the second from fermion loops. Note, that the \(n^l\) mass is given by \(iD + 2|\sigma|^2\) while that of fermions \(\xi^l\) is \(2|\sigma|^2\). If supersymmetry is unbroken (i.e. \(D = 0\)) these masses are equal, and the product of the determinants reduces to unity, as it should be.
Calculation of the determinants in Eq. (3.1) is straightforward. We easily get the following contribution to the effective action:

\[
\frac{N}{4\pi} \left\{ \left( i D + 2 |\sigma|^2 \right) \ln \frac{M_{uv}^2}{i D + 2 |\sigma|^2 + 1} - 2 |\sigma|^2 \ln \frac{M_{uv}^2}{2 |\sigma|^2 + 1} \right\},
\]

where quadratically divergent contributions from bosons and fermions do not depend on \( D \) and \( \sigma \) and cancel each other. Here \( M_{uv} \) is an ultraviolet (UV) cutoff. Remembering that the action in (2.13) presents an effective low-energy theory on the string worldsheet one can readily identify the UV cutoff in terms of bulk parameters,

\[
M_{uv} = m_W. \tag{3.3}
\]

Invoking Eq. (2.4) we conclude that the bare coupling constant \( r_0 \) in (2.13) can be parameterized as

\[
r_0 = \frac{N}{4\pi} \ln \frac{M_{uv}^2}{\Lambda^2}. \tag{3.4}
\]

Substituting this expression in (2.13) and adding the one-loop correction (3.2) we see that the term proportional to \( i D \ln M_{uv}^2 \) is canceled out, and the effective action is expressed in terms of the renormalized coupling constant,

\[
r_{\text{ren}} = \frac{N}{4\pi} \ln \frac{i D + 2 |\sigma|^2}{\Lambda^2}. \tag{3.5}
\]

Assembling all contributions together we get the effective potential as a function of the \( D \) and \( \sigma \) fields in the form

\[
V_{\text{eff}} = \int d^2 x \, \frac{N}{4\pi} \left\{ - \left( i D + 2 |\sigma|^2 \right) \ln \frac{i D + 2 |\sigma|^2}{\Lambda^2} + i D \\
+ 2 |\sigma|^2 \ln \frac{2 |\sigma|^2}{\Lambda^2} + 2 |\sigma|^2 u \right\}, \tag{3.6}
\]

where instead of the deformation parameter \( \omega \) we introduced a more convenient (dimensionless) parameter \( u \) which does not scale with \( N \),

\[
u = \frac{8\pi}{N} |\omega|^2; \tag{3.7}
\]

see Eq. (2.19).

Minimizing this potential with respect to \( D \) and \( \sigma \) we arrive at the following relations:

\[
r_{\text{ren}} = \frac{N}{4\pi} \ln \frac{i D + 2 |\sigma|^2}{\Lambda^2} = 0,
\]

\[
\ln \frac{i D + 2 |\sigma|^2}{2 |\sigma|^2} = u. \tag{3.8}
\]
Equations (3.8) represent our master set which determines the vacua of the theory. Solutions can be readily found,

\[ 2|\sigma|^2 = \Lambda^2 e^{-u}, \quad \sigma = \frac{1}{\sqrt{2}} \Lambda \exp \left( -\frac{u}{2} + \frac{2\pi i k}{N} \right), \quad k = 0, \ldots, N-1, \]

\[ iD = \Lambda^2 \left( 1 - e^{-u} \right). \quad (3.9) \]

The phase factor of \( \sigma \) does not follow from (3.8), but we know of its existence from the fact of the spontaneous breaking of the discrete chiral \( Z_{2N} \) down to \( Z_2 \), see Sect. 2.4. Substituting this solution in Eq. (3.6) we get the expression for the vacuum energy density,

\[ \mathcal{E}_{\text{vac}} = \frac{N}{4\pi} iD = \frac{N}{4\pi} \Lambda^2 \left( 1 - e^{-u} \right). \quad (3.10) \]

Note that at small \( u \) the vacuum energy density reduces to

\[ \mathcal{E}_{\text{vac}} \propto u N |\sigma|^2, \quad (3.11) \]

in full accord with Eq. (1.4). On the other hand, at large \( u \)

\[ \mathcal{E}_{\text{vac}} \to \frac{N}{4\pi} \Lambda^2. \quad (3.12) \]

This value is of the order of \( N\Lambda^2 \). Needless to say, the linear \( N \) dependence was expected.

It is instructive to discuss the first condition in (3.8). That \( r_{\text{ren}} = 0 \) was a result of Witten’s analysis [2] too. This fact, \( r_{\text{ren}} = 0 \), implies that in quantum theory (unlike the classical one)

\[ \langle |n|^2 \rangle = 0, \quad (3.13) \]

i.e. the global \( \text{SU}(N) \) symmetry is not spontaneously broken in the vacuum and, hence, there are no massless Goldstone bosons. All bosons get a mass.

If the deformation parameter \( u \) vanishes, the vacuum energy vanishes too and supersymmetry is not broken, in full accord with Witten analysis [2] and with the fact that the Witten index is \( N \) in this case [16]. The \( \sigma \) field develops a vacuum expectation value (VEV) breaking \( Z_{2N} \) symmetry (2.21). As we switch on the deformation parameter \( u \), the \( D \) component develops a VEV; hence, \( \mathcal{N} = (0,2) \) supersymmetry is spontaneously broken. The vacuum energy density no longer vanishes.

\[ ^7 \text{The vacuum structure (3.9) of the } \mathcal{N} = (2,2) \text{ model at } u = 0 \text{ was also obtained by Witten for arbitrary } N \text{ in [13] using a superpotential of the Veneziano–Yankielowicz type [17].} \]
$u=0.5$

$u=2.5$

Figure 1: The potential \((3.15)\) as a function of \(|\sigma|^2\).

In the limit \(\mu \to \infty\), the deformation parameter \(u\) behaves logarithmically with \(\mu\),

\[
u = \text{const} \left( \ln \frac{m_W}{\Lambda} \right) \left( \ln \frac{g_2^2 \mu}{m_W} \right),
\]

where the constant above does not depend on \(N\). At any finite \(u\) the \(\sigma\)-field condensate does not vanish, labeling \(N\) distinct vacua as indicated in Eq. \((3.9)\). In each vacuum \(\mathbb{Z}_{2N}\) symmetry is spontaneously broken down to \(\mathbb{Z}_2\); the order parameter is \(\langle \sigma \rangle\). We will discuss physics of the model in the large-\(\mu\) limit in more detail in Sects. 5 and 6.

To conclude this section let us integrate over the axillary field \(iD\) in \((3.6)\) to get the effective potential of the \(\mathcal{N} = (0, 2)\) model as a function of the physical field \(\sigma\),

\[
V(\sigma) = \frac{N}{4\pi} \left\{ \Lambda^2 + 2|\sigma|^2 \left[ u + \ln \frac{2|\sigma|^2}{\Lambda^2} - 1 \right] \right\}.
\]

Clearly, the minimum of this potential is at \(|\sigma|^2\) given in the first line of Eq. \((3.9)\). The plot in Fig. 1 illustrates the tendency of the growth of \(E_{\text{vac}}\) and decrease of \(\langle \sigma \rangle\) as the deformation parameter \(u\) increases.

### 4 The physical spectrum

Our next task is calculation of the mass spectrum of the theory \((2.13)\). To this end we start from the one-loop effective action and analyze it as a function of both the “extra” field \(\zeta_R\) and the boson and fermion fields from the gauge supermultiplet \((A_k, \sigma, \lambda)\). After integration over \(n'\) and \(\xi'\) we obtain
$$S_{\text{eff}} = \int d^2 x \left\{ \frac{1}{4e_\gamma^2} F_{kl}^2 + \frac{1}{e_{\sigma_1}^2} |\partial_k (\text{Re} \, \sigma)|^2 + \frac{1}{e_{\sigma_2}^2} |\partial_k (\text{Im} \, \sigma)|^2 ight. \\
+ \frac{1}{e_\lambda^2} \bar{\lambda}_R i \partial_L \lambda_R + \frac{1}{e_\lambda^2} \bar{\lambda}_L i \partial_R \lambda_L + \frac{1}{2} \bar{\zeta}_R i \partial_L \zeta_R \\
+ \left. V(\sigma) + i \frac{N}{\pi} \frac{\text{Im} \sigma}{|\sigma|} F^* + \left[ i \sqrt{2} \bar{\sigma} \bar{\lambda}_L \lambda_R + \sqrt{2} i \bar{\omega} \bar{\lambda}_L \zeta_R + \text{H.c.} \right] \right\}, \quad (4.1)$$

where it is anticipated that in the vacuum under consideration we will have $\text{Im} \, \sigma = 0$, i.e. we consider the vacuum given by Eq. (3.9) with $k = 0$. This condition determines the form of the $F^* \, \sigma$ coupling, namely $\text{Im} \, \sigma F^*$. If necessary, it is not difficult to modify the expression (4.1) for other vacua. Moreover,

$$\partial_L \equiv \partial_0 - i \partial_3, \quad \partial_R \equiv \partial_0 + i \partial_3, \quad (4.2)$$

$V(\sigma)$ is given in Eq. (3.15) while $F^*$ is the dual gauge field strength,

$$F^* = \frac{1}{2} \varepsilon_{kl} F_{kl}. \quad (4.3)$$

Here $e_\gamma^2$, $e_\sigma^2$ and $e_\lambda^2$ are the coupling constants which determine the wave function renormalization for the photon, $\sigma$, and $\lambda$ fields, respectively. Moreover, $\Gamma$ is the induced Yukawa coupling. These couplings are given by one-loop graphs which we will consider below.

The $(\text{Im} \, \sigma) \, F^*$ mixing was calculated by Witten in [2] for $\mathcal{N} = (2, 2)$ theory. This mixing is due to the chiral anomaly which makes the photon massive in two dimensions. In the effective action this term is represented by the mixing of the gauge field with the imaginary part of $\sigma$. Since the anomaly is not modified by $\mathcal{N} = (2, 2)$ breaking deformation, we can use Witten’s result in the deformed theory.

The wave function renormalizations of the fields from the gauge supermultiplet are, in principle, momentum-dependent. We calculate them below in the low-energy limit assuming the external momenta to be small.

The wave function renormalization for $\sigma$ is given by the $n$ loop graph and a similar graph with the $\xi$ fermions (Fig. 2). A straightforward calculation yields

$$\frac{1}{e_{\sigma_1}^2} = \frac{N}{4\pi} \frac{1}{2|\sigma|^2} \left( \frac{1}{3} + \frac{2}{3} \frac{|\sqrt{2} \sigma|^4}{(2|\sigma|^2 + iD)^2} \right),$$

$$\frac{1}{e_{\sigma_2}^2} = \frac{N}{4\pi} \frac{1}{2|\sigma|^2}. \quad (4.4)$$

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The integral is saturated at momenta of the order of $\xi$ mass $\sqrt{2}|\sigma|$.

The wave function renormalization for the gauge field was calculated by Witten in [2]. The result is

$$\frac{1}{e^{2}_\gamma} = \frac{N}{4\pi} \left[ \frac{1}{3} iD + 2|\sigma|^2 + \frac{2}{3} \frac{1}{2|\sigma|^2} \right].$$

(4.5)

The right-hand side in Eq. (4.5) is given by two graphs in Fig. 3 with bosons $n^l$ and fermions $\xi^l$ in the loops. The first term in (4.5) comes from bosons while the second one is due to fermions.

The renormalization for the $\lambda$ fermions is shown in Fig. 4. This graph gives

$$\frac{1}{e^{2}_\lambda} = 2 \frac{N}{4\pi} \int \frac{dk^2}{(k^2 + iD + 2|\sigma|^2)(k^2 + 2|\sigma|^2)},$$

(4.6)

Note that the gauge supermultiplet was introduced in (2.13) as axillary fields, with no kinetic terms ($e^2\rightarrow\infty$ in (2.13)). We see that the kinetic terms for these fields are generated at the one-loop level. Therefore, these fields become physical [2].

The Yukawa coupling is determined by the one-loop graph in Fig. 5 which gives

$$\Gamma = 2 \frac{N}{4\pi} \int \frac{dk^2}{(k^2 + iD + 2|\sigma|^2)(k^2 + 2|\sigma|^2)},$$

(4.7)

where one propagator comes from the bosons $n^l$ while the other from the fermions $\xi^l$. 

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The subsequent analysis is straightforward if we limit ourselves to two limits: small and large values of the bulk deformation parameter $\mu$. These are the limits which were considered in [1].

4.1 Small-$\mu$ limit

Let us first put $\omega = 0$ and reproduce the mass spectrum of the theory in the $\mathcal{N} = (2, 2)$ limit that had been obtained by Witten in [2]. If $\omega = 0$ then $D = 0$ and supersymmetry is unbroken. Masses of the $n^l$ bosons and $\xi^l$ fermions coincide. They are given by the following formula:

$$m_n = m_\xi = \sqrt{2}|\sigma| = \Lambda,$$

where we used Eq. (3.9) at $u = 0$. The wave function renormalizations are also equal in this limit,

$$\frac{1}{e_\sigma^2} = \frac{1}{e_\gamma^2} = \frac{1}{e_\lambda^2} = \frac{N}{4\pi} \frac{1}{2|\sigma|^2} = \frac{N}{4\pi} \frac{1}{\Lambda^2},$$

while the Yukawa coupling (4.7) is

$$\Gamma = \frac{N}{4\pi} \frac{2}{\Lambda^2}.$$

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Since $\sigma$ develops a VEV, the Yukawa term in (4.1) gives a mass to the $\lambda$ fermion. Using (4.9) and (4.10) we get

$$m_{\lambda R} = m_{\lambda L} = 2\sqrt{2}|\sigma| = 2\Lambda . \quad (4.11)$$

Minimizing the potential $V(\sigma)$ in Eq. (3.15) we calculate the mass of the real part of the $\sigma$ field,

$$m_{\text{Re } \sigma} = 2\Lambda . \quad (4.12)$$

where we also used Eq. (4.9). The anomalous (Im $\sigma$) $F^*$ mixing in (4.1) gives masses to both photon and the imaginary part of $\sigma$. Using (4.9) we get

$$m_{\text{ph}} = m_{\text{Im } \sigma} = 2\Lambda . \quad (4.13)$$

We see that all fields from the gauge multiplets have the same mass $2\Lambda$ in accordance with $\mathcal{N} = (2, 2)$ supersymmetry. The factor of 2 in Eq. (4.13) is easy to understand if we take into account that, say, $\lambda$ is a bound state of $n$ and $\xi$, each of them has mass $\Lambda$ and all interactions are $O(1/N)$. The binding energy $O(1/N)$ is not seen in the leading order in the large-$N$ expansion. The field $\zeta_R$ is massless and sterile at $\omega = 0$.

Now let us switch on a small deformation parameter $u$. The field $\zeta_R$ is no longer sterile. This explicitly breaks $\mathcal{N} = (2, 2)$ supersymmetry down to $\mathcal{N} = (0, 2)$. Moreover, $\mathcal{N} = (0, 2)$ supersymmetry gets spontaneously broken due to the VEV of the $D$ component in (3.9). The vacuum energy no longer vanishes, it becomes proportional to the deformation parameter $u$, see Eq. (3.11). The spectrum of fields from the former $\mathcal{N} = (2, 2)$ gauge multiplet does not change very much: superpartners split acquiring mass differences linear in $u$ around the average value $2\Lambda$.

Due to the spontaneous supersymmetry breaking we have a massless Goldstino fermion in the theory. To check this explicitly we diagonalize the mass matrix for the $\zeta_R$, $\lambda_R$ and $\lambda_L$ fermions in Eq. (4.1). Equating the determinant of this matrix to zero produces the following equation for the mass eigenvalues $m$:

$$m^3 - m \left[ 2|\sigma|^2 \Gamma^2 \epsilon_\lambda^4 + 4\omega^2 \epsilon_\lambda^2 \right] = 0 . \quad (4.14)$$

At any $\omega$ we have a vanishing eigenvalue. It corresponds to a massless Goldstino. Clearly, at small $\omega$ this fermion coincides with $\zeta_R$ (with an $O(\omega)$ admixture of the $\lambda$ fermions). At small $\omega$ we can neglect the second term in the square brackets in Eq. (4.14). Then substituting (4.9) and (4.10) into the first term we reproduce the result (4.11) for the masses of the $\lambda$ fermions.
4.2 Large-$\mu$ limit

As we increase the bulk deformation parameter $\mu$ so does the worldsheet deformation parameter $u$. Spontaneous supersymmetry breaking in the worldsheet model gets stronger (the strings are no longer BPS). The $\mathcal{N} = (0, 2)$ supermultiplet splittings grow.

In this regime the masses of the $n^l$ bosons and $\xi^l$ fermions become essentially different. They are

$$m_n = \sqrt{iD + 2|\sigma|^2} = \Lambda, \quad m_\xi = \sqrt{2|\sigma|} = \Lambda \exp\left(-\frac{u}{2}\right), \quad (4.15)$$

where we used Eqs. (3.9). The fermions are much lighter than their bosonic counterparts. The mass split is unsuppressed by $1/N$ since it is due to $\xi \zeta n$ coupling which is of order unity in the regime under consideration.

The mass of the real part of $\sigma$ can be readily calculated using the potential $V(\sigma)$ (see (3.15)),

$$m_{\text{Re}\sigma} = 2\Lambda \exp\left(-\frac{u}{2}\right) \left\{\frac{1}{3} + \frac{2}{3} e^{-2u}\right\}^{-1/2}, \quad (4.16)$$

where we also invoked Eq. (4.4). Moreover, diagonalizing the photon-Im $\sigma$ mixing in Eq. (4.1) we get

$$m_{\text{ph}} = m_{\text{Im}\sigma} = \sqrt{6} \Lambda \exp\left(-\frac{u}{2}\right). \quad (4.17)$$

The binding between the constituents is again due to $\xi \zeta n$ coupling and is unsuppressed by $1/N$.

The above masses no longer coincide with the mass of the real part of $\sigma$. Technically the difference arises due to the difference in the coupling constants $e_\sigma$ in (4.4) and $e_\gamma$ in (4.5). When the $\sigma$ VEV is small the second term in (4.5) dominates, and $e_\gamma$ becomes

$$\frac{1}{e_\gamma^2} = \frac{N}{6\pi} \frac{1}{2|\sigma|^2} = \frac{N}{6\pi} \frac{1}{\Lambda^2} e^u, \quad (4.18)$$

while $e_\sigma$ is given by Eq. (4.4).

The fermion masses can be obtained from Eq. (4.14). Clearly, at large $\mu$ the $\lambda_L \zeta_R$ mixing dominates in Eq. (4.1). In this limit $\lambda_R$ (the bound state of $\xi$ and $n$) becomes the massless Goldstino state,

$$m_{\lambda_R} = 0, \quad (4.19)$$

while the masses of $\lambda_L$ and $\zeta_R$ are given by non-zero roots of Eq. (4.14),

$$m_{\lambda_L} = m_{\zeta_R} = \Lambda \sqrt{u}, \quad (4.20)$$
where we used the fact that the coupling $e^r_\lambda$ in (4.6) reduces in this limit to
\[
\frac{1}{e^r_\lambda} = \frac{N \, 2}{4\pi \Lambda^2}.
\]

We see that these two fermions become heavy in the limit $u \gg 1$. Thus, the low-energy effective theory contains the light (but massive!) photon, two light $\sigma$ states and only fermion: the massless Goldstino $\lambda_R$.

5 Kink deconfinement vs. confinement

As was already mentioned, both the $\mathcal{N} = (2, 2)$ and nonsupersymmetric $CP(N - 1)$ models were solved by Witten at large $N$ [2]. Witten showed that $n$’s are in fact kinks, $\xi$’s their superpartners, and they are confined in nonsupersymmetric version while adding supersymmetry converts confinement into deconfinement. This is in one-to-one correspondence with the existence of $N$ degenerate vacua in the latter case. These vacua become nondegenerate quasivacua in nonsupersymmetric $CP(N - 1)$ models [15].

In this section we will compare the large-$N$ solutions for all three theories: with $\mathcal{N} = (2, 2)$ and $\mathcal{N} = (0, 2)$ supersymmetry as well as the nonsupersymmetric version.

One common feature of all three cases is that spontaneous breaking of the global SU($N$) (flavor) symmetry present at the classical level disappears when quantum effects are taken into account. There are no massless Goldstone bosons in the physical spectra of all three theories. The $n^I$ fields acquire mass of the order of $\Lambda$. Another common feature is that the U(1) gauge field introduced as an axillary field at the classical level develops a kinetic energy term and becomes propagating.

In the $\mathcal{N} = (2, 2)$ $CP(N - 1)$ model supersymmetry is not spontaneously broken and the model has $N$ strictly degenerate vacua. The order parameter which characterizes these vacua is the vacuum expectation value of the $\sigma$ field given in the first line of Eq. (3.9) for $u = 0$, or, which is the same, the bifermion condensate (2.10).

Since we have $N$ different vacua and $Z_{2N}$ symmetry is spontaneously broken down to $Z_2$ we have kinks interpolating between these vacua. These kinks are described by the fields $n^I$ belonging to the fundamental representation of the SU($N$) group [2, 18]. From the standpoint of the underlying bulk theory these kinks are interpreted as confined monopoles [19, 5, 6]. In the bulk theory we have the Higgs phase; thus, the monopoles are confined in the four-dimensional sense, i.e. they are attached to strings. It is easy to show that the values of the magnetic charges of the monopoles from the SU($N$) subgroup of the gauge group ensure that these monopoles

\[8\text{Confinement in two dimensions is confinement along the string.}\]
are the string junctions of two elementary non-Abelian strings, see the review paper [2] for details.

As was shown above, in the $\mathcal{N} = (0, 2)$ theory supersymmetry is spontaneously broken. The vacuum energy density does not vanish, see (3.10). This means that strings under consideration are no longer BPS and their tensions get a shift (3.10) with respect to the classical value $T_{cl} = 2\pi \xi$. However, this shift is the same for all $N$ elementary strings. Their tensions are strictly degenerate; $Z_{2N}$ symmetry is spontaneously broken down to $Z_2$. The order parameter (the $\sigma$ field VEV) remains nonvanishing at any finite value of the bulk parameter $\mu$.

The kinks that interpolate between different vacua of the worldsheet theory are described by the $n^l$ fields. Their masses are given in Eq. (4.15). In $\mathcal{N} = (0, 2)$ theory the masses of the boson and fermion superpartners are split. The bosonic kinks have masses $\sim \Lambda$ in the large-$\mu$ limit, while the fermionic kinks become light. Still their masses remain finite and nonvanishing at any finite $\mu$.

We already know that, from the standpoint of the bulk theory, these kinks are confined monopoles [10, 20]. The fact that tensions of all elementary strings are the same ensures that these monopoles are free to move along the string, since with their separation increasing, the energy of the configuration does not change. This means they are in the deconfinement phase. The kinks are deconfined both in $\mathcal{N} = (2, 2)$ and $\mathcal{N} = (0, 2) \, CP(N - 1)$ theories. In other words, individual kinks are present in the physical spectrum. The monopoles although attached to strings are free to move on the strings.

The main distinction of the nonsupersymmetric $CP(N - 1)$ model from its supersymmetric cousins is that the U(1) gauge field remains massless in the absence of SUSY [2]. The reason is that the nonsupersymmetric version does not have fermions $\xi^l$, while in the supersymmetric versions these fermions provide the photon with a mass via the chiral anomaly. The presence of massless photon ensures long range forces in the nonsupersymmetric $CP(N - 1)$ model. The Coulomb potential is linear in two dimensions leading to the Coulomb/confinement phase [2]. Electric charges are confined. The lightest electric charges are the $n^l$ kinks. Confinement of kinks means that they are not present in the physical spectrum of the theory in isolation. They form bound states, kink-antikink “mesons.” The picture of confinement of $n$’s is shown in Fig. 6.

The validity of the above consideration rests on large $N$. If $N$ is not large the solution [2] ceases to be applicable. It remains valid in the qualitative sense, however. Indeed, at $N = 2$ the model was solved exactly [21, 22] (see also [23]). Zamolodchikovs found that the spectrum of the O(3) model consists of a triplet of degenerate states

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9We stress again that these monopoles are confined in the bulk theory being attached to strings.
(with mass $\sim \Lambda$). At $N = 2$ the action (2.2) is built of doublets. In this sense one can say that Zamolodchikovs’ solution exhibits confinement of doublets. This is in qualitative accord with the large-$N$ solution [2].

Inside the $n\bar{n}$ mesons, we have a constant electric field, see Fig. 6. Therefore the spatial interval between $\bar{n}$ and $n$ has a higher energy density than the domains outside the meson.

Let us reiterate the above picture in somewhat different terms [24, 25]. In the nonsupersymmetric model $N$ degenerate vacua present in supersymmetric versions of the theory are split. At large $N$, along with the unique ground state, the model has $\sim N$ quasistable local minima, quasivacua, which become absolutely stable at $N = \infty$. The relative splittings between the values of the energy density in the adjacent minima is of the order of $1/N$, while the probability of the false vacuum decay is proportional to $N^{-1} \exp(-N)$ [26, 27].

The $n$ quanta (kinks) interpolate between the adjacent vacua. They are confined monopoles of the bulk theory. Since the excited string tensions are larger than the tension of the lightest one, these monopoles, besides four-dimensional confinement, are confined also in the two-dimensional sense: a monopole is necessarily attached to an antimonopole on the string to form a meson-like configuration [24, 25]. Otherwise, the energy of the configuration will be infinitely higher (in a linear manner).

### 6 Conformal fixed point

In this section we discuss what happens if we send $\mu$ to infinity in the bulk theory. This issue was addressed in [12] where it was argued that in the $\mu \to \infty$ limit the $\sigma$-field VEV vanishes, the U(1) gauge field is massless and the theory is in the Coulomb/conf confinement phase (much in the same way as in nonsupersymmetric $CP(N - 1)$ models [2, 15]).

In the $\mu \to \infty$ limit the adjoint fields decouple and the bulk theory flows to $\mathcal{N} = 1$ SQCD. It is well known that this theory has a Higgs branch see, for example, [14]. As was explained in [10], the presence of the Higgs branch in the $\mu \to \infty$ limit is quite an unpleasant feature of the bulk theory. The presence of massless states associated with the Higgs branch obscures physics of the non-Abelian strings.
In particular, the strings swell and become infinitely thick. This means that higher derivative corrections in the effective theory on the string become important. In [10] the maximal critical value of the parameter $\mu$ was estimated beyond which one can no longer trust the effective two-derivative theory on the string worldsheet, 

$$g_2^2 \mu \ll \frac{m_W^3}{(\Lambda_{\text{bulk}}^N)^2},$$

(6.1)

where $\Lambda_{\text{bulk}}^N$ is the scale of $\mathcal{N} = 1$ SQCD. We assume weak coupling in the bulk theory, i.e. $m_W \gg \Lambda_{\text{bulk}}^N$.

Thus, we cannot go to the limit $\mu \rightarrow \infty$, to begin with. Higher derivative corrections to the worldsheet theory (2.13) blow up. We still have a large window in the values of the $\mu$ parameter, with $\mu$ staying below the upper bound (6.1), but, on the other hand, large enough to ensure the decoupling of the adjoint fields, namely,

$$m_W \ll g_2^2 \mu \ll m_W \frac{m_W^2}{\Lambda_{\mathcal{N}=1}^2}.$$

(6.2)

Inside this window the deformation parameter $u$ is finite (see (3.14)). Our results show that the $\sigma$-field VEV does not vanish and we have $N$ strictly degenerate vacua. Moreover, the U(1) gauge field always has a small mass implying that the kinks are in the deconfinement phase. As we explained above the mass generation for the photon field is in one-to-one correspondence with the existence of $N$ distinct degenerate vacua.

The situation with the decoupling of the adjoint fields we encounter here seems counterintuitive, at least at first sight. Indeed, on physical grounds we can say that once $g_2^2 \mu$ becomes larger than $m_W$ by a factor of, say, 5 or so the adjoint fields are already decoupled, and the subsequent evolution of their mass from $5m_W$ to infinity should have no impact in the bulk as well as on the string worldsheet. However, Eq. (2.16) shows that this is not the case. The logarithmic growth at large $\mu$ seems to be a typical massless particle effect. If the theory had no Higgs branch in the limit of the large bulk deformation parameter, one can expect the worldsheet deformation parameter to be frozen at a finite value. We conjecture that that’s what happens in the $M$ model [20]. In this model the Higgs branch does not develop.

Now, let us abstract ourselves from the fact that the theory (2.13) is a low-energy effective model on the worldsheet of the non-Abelian string. Let us consider this model per se, with no reference to the underlying four-dimensional theory. Then, of course, the parameter $u$ can be viewed as arbitrary. One can address a subtle question: what happens in the limit $u \rightarrow \infty$? In this limit the $\sigma$ field VEV tends to zero (see Eq. (3.9)) and $N$ degenerate vacua coalesce. Moreover, the U(1) gauge field, $\sigma$ and the fermionic kinks $\xi$ become massless (in addition to the $\lambda_R$ field which, being Goldstino in this limit, is necessarily massless). The low-energy theory seemingly
becomes conformal. It is plausible to interpret this conformal fixed point as a phase transition point from the kink deconfinement phase to the Coulomb/confining phase.

A similar phenomenon occurs in two-dimensional conformal $\mathcal{N} = (4, 4)$ supersymmetric gauge theory [28]. In this theory the same tube metric $|d\sigma|^2/\sigma^2$ appears (as in (4.1), (4.3)) and the point $\sigma = 0$ is interpreted as a transition point between two distinct phases.

7 Conclusions

In this paper we discussed dynamics of the heterotic $\mathcal{N} = (0, 2)$ $\text{CP}(N-1)$ model. Besides all fields of the conventional $\mathcal{N} = (2, 2)$ $\text{CP}(N-1)$ model the heterotic one contains a single extra right-handed fermion $\zeta_R$. Interaction of the latter with other fields is characterized by a single dimensionless parameter (3.7) which grows logarithmically with $\mu$, see Eq. (3.14). Using the large-$N$ expansion, we solved the heterotic model in the leading order of this expansion. The proof of the spontaneous supersymmetry breaking which for small deformation parameters was given in [1] is extended to arbitrary values of the deformation parameter. We find the vacuum energy density for $N$ degenerate vacua present in the model. Lifting the vacuum energy from zero makes the $\mathcal{N} = (0, 2)$ model akin to nonsupersymmetric $\text{CP}(N-1)$ model.

The $Z_{2N}$ symmetry is broken down to $Z_2$ much in the same way as in the $\mathcal{N} = (2, 2)$ model. The vacua are labeled by the nonvanishing expectation values $\langle \sigma \rangle$, see Eq. (3.9), in the allowed window (6.2) of the values of the deformation parameter $u$. The presence of $N$ distinct degenerate vacua guarantees the theory to be in the deconfining phase. Correspondingly, the mass of the two-dimensional photon is nonvanishing. This makes the $\mathcal{N} = (0, 2)$ model akin to the $\mathcal{N} = (2, 2)$ model.

We found the mass spectra at small and large values of the deformation parameter. The small-$\mu$ case is rather selfevident. At large $\mu$ we encounter a rather intriguing situation: the only field whose mass is $\sim \Lambda$ is the $n$ field. Others are either much lighter or much heavier.

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References

[1] M. Shifman and A. Yung, *Heterotic Flux Tubes in $\mathcal{N} = 2$ SQCD with $\mathcal{N} = 1$ Preserving Deformations*, arXiv:0803.0158 [hep-th].

[2] E. Witten, Nucl. Phys. B 149, 285 (1979).

[3] A. Hanany and D. Tong, JHEP 0307, 037 (2003) [hep-th/0306150].

[4] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, Nucl. Phys. B 673, 187 (2003) [hep-th/0307287].

[5] M. Shifman and A. Yung, Phys. Rev. D 70, 045004 (2004) [hep-th/0403149].

[6] A. Hanany and D. Tong, JHEP 0404, 066 (2004) [hep-th/0403158].

[7] D. Tong, *TASI Lectures on Solitons*, arXiv:hep-th/0509216.

[8] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, J. Phys. A 39, R315 (2006) [arXiv:hep-th/0602170].

[9] M. Shifman and A. Yung, *Supersymmetric Solitons*, Rev. Mod. Phys. 79 1139 (2007) [arXiv:hep-th/0703267].

[10] M. Shifman and A. Yung, Phys. Rev. D 72, 085017 (2005) [hep-th/0501211].

[11] M. Edalati and D. Tong, JHEP 0705, 005 (2007) [arXiv:hep-th/0703045].

[12] D. Tong, JHEP 0709, 022 (2007) [arXiv:hep-th/0703235].

[13] E. Witten, Nucl. Phys. B 403, 159 (1993) [hep-th/9301042].

[14] K. Intrilligator and N. Seiberg, Nucl. Phys. (Proc. Suppl.) 45BC, 1 (1996) [hep-th/9509066].

[15] A. Gorsky, M. Shifman and A. Yung, Phys. Rev. D 73, 065011 (2006) [hep-th/0512153].

[16] E. Witten, Nucl. Phys. B 202, 253 (1982). [Reprinted in Supersymmetry, Ed. S. Ferrara (North/Holland/World Scientific, 1987), Vol. 1, p. 490].

[17] G. Veneziano and S. Yankielowicz, Phys. Lett. B 113, 231 (1982).

[18] K. Hori and C. Vafa, *Mirror Symmetry*, hep-th/0002222.

[19] D. Tong, Phys. Rev. D 69, 065003 (2004) [hep-th/0307302].

[20] A. Gorsky, M. Shifman and A. Yung, Phys. Rev. D 75, 065032 (2007) [hep-th/0701040].
[21] A. Zamolodchikov and Al. Zamolodchikov, Ann. Phys. 120 (1979) 253.
[22] A. B. Zamolodchikov and Al. B. Zamolodchikov, Nucl. Phys. B 379 (1992) 602.
[23] S. R. Coleman, Annals Phys. 101, 239 (1976).
[24] V. Markov, A. Marshakov and A. Yung, Nucl. Phys. B 709, 267 (2005) [hep-th/0408235].
[25] A. Gorsky, M. Shifman and A. Yung, Phys. Rev. D 71, 045010 (2005) [hep-th/0412082].
[26] E. Witten, Phys. Rev. Lett. 81, 2862 (1998) [hep-th/9807109].
[27] M. Shifman, Phys. Rev. D 59, 021501 (1999) [hep-th/9809184].
[28] E. Witten, JHEP 9707 003 (1997) [arXiv:hep-th/9707093].
[29] P. Koroteev and A. Monin, Large-N Solution of the Heterotic $\mathcal{N} = (0,1)$ Two-Dimensional $O(N)$ Sigma Model, arXiv: hep-th/1003.2645.