Unsteady Plasma Flow Near an Oscillating Rigid Plane Plate Under the Influence of an Unsteady Nonlinear External Magnetic Field

TAHA ZAKARAIA ABDEL WAHID, (Member, IEEE), AND ADEL M. MORAD

1Mathematics and Computer Sciences Department, Faculty of Science, Menoufia University, Shebin Elkom 32511, Egypt
2Computational Mathematics and Mathematical Physics Department, Institute of Mathematics, Mechanics, and Computer Science, Southern Federal University, Rostov-on-Don 344090, Russia

Corresponding author: Taha Zakaraia Abdel Wahid (taha.zakaria@science.menofia.edu.eg)

ABSTRACT In an irreversible thermodynamics framework, Stokes’ second problem was examined for unsteady oscillating flow. The objective was to apply that for a plasma near an illimitable oscillating rigid plane plate under the influence of an unsteady nonlinear applied magnetic field. The Bhatnagar-Gross-Krook (BGK) pattern of the Boltzmann kinetic equation supplemented by Maxwell’s equations was investigated. The method of moments was applied with a two-sided distribution function. The exact traveling wave solution was obtained for a system consisting of four non-homogeneous partial differential equations. The velocity, shear stress, viscosity coefficient, generated electric field, applied nonlinear magnetic field, polarization, gyro-radius, and gyro-frequency were calculated. Furthermore, the distinction between the equilibrium velocity distribution function and the perturbed distribution functions was theoretically clarified at distinct time values. The advantage of the Boltzmann equation permitted us to consider irreversible non-equilibrium thermodynamics principles. For that purpose, the calculated distribution functions should be used in the formulae of entropy, entropy flux, thermodynamic forces, and kinetic coefficient. From the analysis of the results, it is found that Boltzmann’s H-theorem, thermodynamics laws, and Le Chatelier’s principle were consistent with our model for the whole system. The distinct contributions of the forces exerted on the system modified its internal energies; they were expressed via the Gibbs formula. The results demonstrated that the proposed model is capable of describing the behavior of plasma helium gas in the upper atmosphere ionic belts. Based on the analytical calculations, 3D-Graphics illustrating the physical quantities were drawn to predict their conduct, and the results are deeply discussed.

INDEX TERMS Plasma materials processing, nuclear and plasma sciences, electrons, atmospheric-pressure plasmas, plasma applications, plasma transport processes, kinetic theory, electrokinetics, fluid dynamics, gases, electromagnetic fields.

I. INTRODUCTION

Plasma (fully or partially ionized gases) based technologies serve some of the world’s huge industries manufacturing as an essential portion of the world’s international commercial products comprehensive cell phone technologies, computers, microelectromechanical systems (MEMS), and nanoelectromechanical systems (NEMS). Although plasma physics is based on the main fields of classical mathematical physics, such as mechanics, electromagnetism, and statistical mechanics, rapid advances in plasma science and technology have been demonstrated. That happened by the introduction of new sources, such as space and astrophysical plasma [1], or new fields, such as the field of plasma medicine, plasma processing of semiconductors, and microelectronics, biological plasma systems, and thermonuclear fusion research [2]–[4]. Plasma physics is thus at the intersection of classical and modern sciences.

The Boltzmann kinetic equation symbolizes a well-defined model for characterizing the motion of a rarefied charged gas or plasma while considering microscopic results. That is the situation of electron gas flow in MEMS and NEMS. The oscillating Couette problem has specific analogies with Stokes’ second problem for planar flows, which include...
rigid plane plate oscillating in an illimitable medium. Since the pioneering work of Stokes [5] and later Rayleigh [6], many studies have been devoted to exploring the second problem of Stokes, which is one of the critical configurations for understanding how a non-equilibrium gas reacts to a rigid flat plate oscillating on its plane [7]. Comprehensive research with the linearized Bhatnagar-Gross-Krook-type equation was carried out over the board ranges of Knudsen number [8].

Furthermore, the effectiveness of the oscillation frequencies on the phases and amplitudes of the velocity field and shear stress was investigated. Developing our comprehension of the time-periodic shear driven gas flows, and oscillatory Couette flow supplies a perfect test case. This case has gained enormous interest in the field of gas flows, and it has been investigated extensively utilizing kinetic theory in Refs. [9]–[11].

Throughout the history of plasma physics, the efforts to understand plasma–surface interactions such as Rayleigh flow problem in unsteady plasma under the effect of an applied nonlinear magnetic field led to the development of the problem. It explains how electromagnetic waves and velocity propagate through the non-uniform magnetized plasma. In the last few decades, the kinetic theory of plasma flow on the oscillating plate played a useful role in the combination of the solution of the Boltzmann equation and the behavior of collision plasma [12]–[21]. The kinetic theory of plasma aims to determine the perturbation velocity distribution functions for each plasma species and their evolution. Macroscopic plasma variables, such as flow velocity, density, and temperature, can be determined from the perturbed velocity distribution functions. The kinetic theory of the Boltzmann equation can tackle the Maxwellian distributions and is often the approach needed when studying plasma waves and their instability. The first attempts to model the solution of Boltzmann kinetic equation used simplifications of the Chapman-Enskog method, which is proposed firstly by Chapman [22], Enskog [23] and developed in [24]. Several methods have been suggested to solve the equation of Boltzmann based on the method of moments, such as the moment method of Grad, Krook model, and the Lee model [25]–[28]. In [27], Donko and Dyatko studied the effects of negative mobility and conductivity for xenon gas using the Boltzmann equation solutions. They neglected Coulomb’s part in collisions and considered Coulomb’s collisions as binary processes. It is well known that the method of moments technique coupled with the two-sided distribution functions, is an effective and flexible method to show existence results for the Boltzmann kinetic equation [12]–[21]. Grad’s pioneering use of this method allowed extensive researches to solve the Boltzmann equation [29]. After that, Chapman and Cawling [30] introduced the so-called Chapman-Enskog method. Kremer [31] proposed the new Chapman–Enskog–Grad model for constitutive equations by improving Chapman–Enskog’s characteristics and the method of moments.

Several authors had investigated the Rayleigh flow problem under the influence of the external fields during the last half-century. For example, Drake [32] obtained an approximate solution of the Rayleigh problem for the impulsive motion of a fluid with an applied suction with time on a flat plate. For unsteady and incompressible viscous flow, Yoshizawa [33] analyzed the motion using Navier-Stokes equations for a semi-infinite plate, and they obtained the upstream and downstream solutions with the Reynolds number. While Tokis and Geroyannis [34] investigated the model in 3D for the flow of a rotating fluid near an infinite oscillating plate under the influence of electric and magnetic fields. Rayleigh’s problem with a Maxwellian distribution function for a non-homogeneous system of charged particles in a rarefied gas was studied by Khater and El-Sharif in Ref. [35]. For a historical background of Rayleigh flow, we refer to Lord Rayleigh [6], who was the first innovator of the behavior of the flow of an incompressible fluid close to an infinite flat plate suddenly moving in the plane. The first systematic study of the Rayleigh flow in the interaction between moving plasmas and solid surfaces under the effect of an applied nonlinear magnetic field was undertaken by Chang and Chang [36]. Their generalization of the collision plasma case was considered independently by Abdel Wahid [14]. In addition, Abourabia and Tolba [16] developed Rayleigh’s problem using the BGK type of the Boltzmann collision term using the method of moments in the Liu and Lees model, and they ignored the electrons collision process at large Debye length. While Abdel Wahid and Elagan [15] studied the problem with the precision value of all frequencies electron-electron collision. In 2018, Abdel Wahid [21] extended the traveling wave parameter solution of the unsteady flows of collision plasma on an infinite moving plate using the BGK-type of the Boltzmann equation with four specific collision frequency terms. He obtained an approximate solution for the mean flow velocity and shear stress and treated the immobile ions as a uniform neutralizing background.

In the context of irreversible thermodynamics [37], Abourabia and Abdel Wahid [12] investigated the characteristics of Rayleigh’s flow model of a rarefied electron gas. They provided an exact solution of the Boltzmann kinetic equation but with an approximate solution and a crude approximation in terms of collision frequency. These results were expanded and improved in Refs. [13]–[15], where the exact solution and correct formulae for the electron-electron, electron-ion, and electron-atom collision terms were successfully implemented. The present study provides new theoretical concepts of magnetized plasma gas that concerned with the study of electron-molecule collisions on a flat oscillating plate, which has novel technological applications in many fields.

This article aims to find the exact unsteady solution of the BGK kinetic equation model of an inhomogeneous charged gas bounded by such an oscillating plate suffering from an applied external unsteady non-uniform nonlinear magnetic field. Investigation of the reciprocal collision effects of electrons with electrons, positive ions, and neutral atoms in the form of the electron distribution function is carried out thoroughly. Also, the initial-boundary value problem
of Stokes applied to a helium plasma system is solved to determine the macroscopic parameters such as shear stress, coefficient of viscosity, and mean velocity along with the induced electric and applied nonlinear magnetic fields. In this study, we pay our attention to a helium plasma of noble gases and, more specifically, to helium plasma belt in the upper atmosphere [38], [39]. Besides, this paper focuses on the intriguing effects of the irreversible non-equilibrium properties of the entire system. Using the predicted distribution functions, we find that the irreversible thermodynamic behavior of the paramagnetic plasma is of critical physical importance. The exact calculations are applied to a helium plasma belt in the upper atmosphere. Finally, we discussed the overall results and noted that the agreement of the obtained results with the preliminary studies is confirmed.

II. FORMULATION OF THE KINETIC PROBLEM IN PLASMA

In the kinetic equation of Boltzmann, the average electric and magnetic fields arise, and these average fields should satisfy Maxwell’s equations containing the average charge and flux densities. It is called a class of time-dependent problems in which the set of equations can be reduced to two or three coupled ordinary differential equations. The technique is applied to a plasma located under the effect of an external magnetic field for obtaining nonlinear plasma oscillations. In this study, we consider the plasma under the influence of an applied nonlinear magnetic field and in an equilibrium state in the initial situation. The physical system considered here is a plane to be located at \( y = 0 \), and the half-space \( (y \geq 0) \) be occupied by plasma gas under the effect of an unsteady non-uniform magnetic field, \( B_{z,E} \) acting on the flow in the perpendicular direction. Then the plate begins to oscillate in the plane, and the mixture is set to motion. We use the regular structure, since the direction of the electric field is along the \( x \)-axis, and the direction of the nonlinear magnetic field is along the \( z \)-axis. Since, an illimitable plane plate rested at the level \( y = 0 \), parallels to the \( xz \)-plane. It is fluctuating harmonically in the \( x \)-direction with frequency \( \Omega \). Therefore, the velocity of the infinite oscillating plate can be represented by the relation

\[
V_w = \text{Re} (U_0 \exp(-i\Omega t)) = U_0 \cos(\Omega t),
\]

with the velocity amplitude \( U_0 \) that will be small compared with the thermal molecular velocity \( V_{th} \), and the real part of the complex expression is represented by \( \text{Re} \). Since the electron to ion mass ratio is sufficiently small, i.e., \( \frac{m_i}{m_e} \ll 1 \) in plasma gases, the ions will then be considered as an immobile neutralizing background. Therefore, the charged gas is initially in the case of absolute equilibrium; the plate then suddenly begins to oscillate in its plane at a velocity \( U_0 \cos(\Omega t) \) along \( x \)-axis \( (U_0 \) and \( \Omega \) are constants). The plate is also treated uncharged, impermeable, and as an insulator. For the better design of the whole system (electrons + ions + plate), it is necessary to maintain the temperature constant in the model. Unlike electron currents, we treat a frequency system so that we can ignore ion currents. Also, the plate is treated as impermeable, uncharged, and an insulator material. Thus, one should disregard the ions’ motion and focus our attention on the motion of the electrons.

The plasma gas for the electrons and ions can be represented by the phase-space distributions function \( \varphi_e(r, \tilde{r}, \tilde{t}) \) of the electrons, which satisfies the Boltzmann transport equation with the terms of collisions. Thus, the time-dependent Boltzmann equation based on the BGK type describing this behavior of the plasma gas can be written as

\[
\frac{\partial \varphi_e}{\partial t} + \vec{v}_e \cdot \frac{\partial \varphi_e}{\partial \vec{r}} + \frac{\vec{F}_e}{m_e} \cdot \frac{\partial \varphi_e}{\partial \tilde{r}} = \nu_{ee}(\varphi_{0e} - \varphi_e) + \nu_{ei}(\varphi_{0i} - \varphi_e) + \nu_{en}(\varphi_{0n} - \varphi_e),
\]

where

\[
\varphi_{0a} = n_a(2\pi RT_a)^{-\frac{3}{2}} \exp \left( -\left( \frac{\tilde{r}}{2RT_a} \right)^2 \right).
\]

Here \( \varphi_{0a} \) is the local Maxwellian distribution function, which has the same three first moments as \( \varphi_a \). The plasma physical properties \( n_a, T_a \) and \( \vec{V}_a \), which represent the number density, the temperature, and the mean flow velocity, respectively, can get by taking the moments of the distribution functions, as will be discussed below in this context. For full velocity accommodation, the whole particles are reflected.
from the plane plate, i.e., the plasma particles are reflected with a velocity equal to the velocity of the plate. Thus, we can treat the boundary conditions of the velocity of the moving plate as follows

\[ V(x, 0, t) = U_0 \cos(\Omega t), \]  

(2)

where \( V_0 \) is finite at \( t > 0 \), \( Y \to \infty \), and \( t > 0 \).

The force exerted on the charged particle \( e \) passing through the electric field \( \vec{E} \) and magnetic field \( \vec{B} \) with velocity \( \vec{v} \) is the Lorentz force \( \vec{F}_L \) which can be written in the following formula

\[ \vec{F}_L = -e\vec{E} - \frac{e}{c}(\vec{v} \times \vec{B}) \]  

(3)

The magnetic field in Lorentz formula can be expanded and written in the form below

\[ \vec{B} = B_0 + B_{2y} \hat{y} + B_{3x} \hat{x}, \]  

(4)

where \( B_{2y} \) represents the unsteady non-uniform external non-linear magnetic field and \( B_{3x} \) represents the induced non-linear magnetic field that are the functions of space \( y \) and time \( t \), whereas \( \Phi, \omega, \kappa \) are constants.

The properties of the problem are extended and written in the following form

\[ \vec{V} \equiv (V_x, 0, 0), \quad \vec{J} \equiv (q_nV_x, 0, 0), \]  

\[ \vec{E} \equiv (E_x, 0, 0), \quad \vec{B} \equiv (0, 0, B_z) \]  

(5)

From the viewpoint of electromagnetic theory, these properties must satisfy Maxwell’s equations, so the components \( V_x \), \( E_x \), and \( J_y \) will be expressed as functions of position \( y \) and time \( t \).

To describe the kinetic motion of the charged particles in plasma, using the relations \( (2)-(5) \) into \( (1) \), we get

\[ \frac{\partial \vec{v}_e}{\partial t} + \vec{v}_e \frac{\partial \vec{v}_e}{\partial y} - \frac{eB_{2y}}{m_e} (\vec{\xi}_y \frac{\partial \vec{v}_e}{\partial \xi_y} - \vec{\xi}_x \frac{\partial \vec{v}_e}{\partial \xi_x}) = \frac{eE_{2x}}{m_e} \frac{\partial \vec{v}_e}{\partial \xi_x} = v_{ee}(\vec{v}_e - \vec{v}_e) + v_{en}(\vec{v}_e - \vec{v}_e) \]

Finding a solution to this complicated integro-differential equation has been a challenge for scientists. Among the exact methods of solution, the most important one is the method of moments, which analytically solved by the relaxation time approximation in the BGK model, which replaces the collision term by the most proper form for the solution.

As presented in Ref. [40], [41], electron-ion, electron-electron, and neutral-electron collision frequencies \( v_{ei}, v_{ee}, \) and \( v_{en} \) are, respectively, written in the form

\[ v_{ei} = \frac{4\sqrt{2\pi}n_e\gamma_1^2\gamma_2^3\Lambda_{ei}}{3m_e k_B^2 T^3}, \quad v_{ee} = \frac{4\sqrt{2\pi}n_e\gamma_3^2\Lambda_{ee}}{3m_e k_B^2 T^3}, \quad v_{en} = \frac{4\sqrt{2\pi}n_e\gamma_1^2\gamma_2^3\Lambda_{en}}{3m_e k_B^2 T^3} \]  

(7)

Here \( \text{Log}(\Lambda) = \text{Log}[4\pi n\lambda_c^3] \) is the Coulomb logarithm and \( Z \) is the degree of ionization, with the Debye length \( \lambda_D \).

Now, let us show that the Maxwell–Boltzmann velocity distribution functions represent the solution of \( (6) \) along with the properties \( (7) \).

\[ \begin{align*}
\varphi_1 &= n(2\pi RT)^{-\frac{3}{2}} \left( 1 + \frac{\xi}{\eta} e_c \right) \exp \left( -\frac{\xi^2}{2RT} \right) \\
&\text{for } \xi_y < 0 \downarrow \\
\varphi_2 &= n(2\pi RT)^{-\frac{3}{2}} \left( 1 + \frac{\xi}{\eta} e_c \right) \exp \left( -\frac{\xi^2}{2RT} \right) \\
&\text{for } \xi_y > 0 \uparrow
\end{align*} \]  

(8)

for the downward and upward going particles.

Multiplying Eq. \( (6) \) by \( \psi_j(\xi) \) and integrating over \( \xi \) as in the Grad method [29], the resulting form will represent the transport equations of electrons as follows

\[ \begin{align*}
\frac{\partial}{\partial t} \int \psi_1 \varphi_{1} d\xi + \frac{\partial}{\partial y} \int \psi_2 \varphi_{2} d\xi &+ \frac{eE_{2x}}{m_e} \int \psi_1 \varphi_{1} \frac{d\xi}{\xi_y} + \frac{eB_{2y}}{m_e} \int (\xi_y \frac{d\psi_1}{d\xi_y} - \xi_x \frac{d\psi_1}{d\xi_x}) \\
&\quad + \frac{-\xi_y \frac{d\psi_1}{d\xi_y}}{\xi_x} \varphi_{1} d\xi = v_{ee} \int (\varphi_0_e - \varphi_e) \psi_1 d\xi \\
&\quad + v_{en} \int (\varphi_0_e - \varphi_e) \psi_2 d\xi.
\end{align*} \]  

(9)

Using the integration over the velocity dimensions [37], [42], we can get the following relation:

\[ \int \psi_j(\xi) \varphi_{1} d\xi = \int_{\xi_y = -\infty}^{\infty} \int_{\xi_x = -\infty}^{\infty} \int_{\xi_z = -\infty}^{\infty} \int_{\xi_y = -\infty}^{\infty} \int_{\xi_x = -\infty}^{\infty} \int_{\xi_z = -\infty}^{\infty} \psi_j(\xi) \varphi_{1} d\xi \\
\]  

(10)

where \( \psi_j = \psi_j(\xi), \quad j = 1, 2 \).

Here \( d\xi = d\xi_y d\xi_x d\xi_z, \xi_y, \xi_x, \) and \( \xi_z \) are the components of the particle velocity along in all directions. Moreover, electromagnetic fields \( E \) and \( B \) can get from the equations of Maxwell for electrons as follows:

\[ \begin{align*}
\frac{\partial E_{2x}}{\partial y} - \frac{1}{c^2} \alpha \frac{\partial B_{2y}}{\partial t} &= 0, \\
\frac{\partial B_{2y}}{\partial y} - \frac{1}{c} \alpha \frac{dE_{2x}}{dt} - \frac{4\pi n_e c \alpha}{c_0} V_{ke} &= 0,
\end{align*} \]  

(11)

(12)

where the number density and the flow velocity of the plasma gas are \( n = \int \varphi \varphi d\xi, \quad nV_x = \int \vec{v}_e \psi d\xi \).

The initial and boundary conditions can be written in the following form

\[ V_x(y, 0) = \tau_y(y, 0) = E_x(y, 0) = 0, \quad B_y(y, 0) = B_0, \quad 2V_x(0, t) + \tau_y(0, t) = 2M_0 \cos(\Omega t), \quad \text{for } t > 0 \]  

(13)

where the quantities \( V_x, \tau_y, E_x \) and \( B_c \) are finite at \( y \to \infty \).

**A. THE MODEL EQUATIONS IN NON-DIMENSIONAL FORM**

The exact solution of the model equations and their application to laboratory helium plasma will be considerably improved if they can be written in a subsequently scaled
non-dimensional shape. Moreover, in this non-dimensional technique, the solution of equations is presented in terms of the non-dimensional variables that are physically more to represent.

The dependent variables are scaled using the dimensionless fields $F$ that defined in terms of the physical quantities $F^*$ with constant dimensional scaling factors, as follows:

\[
y = y^* (\tau_{ee} c), \quad t = t^* \tau_{ee}, \quad V_x = V_x^* c, \quad \tau_{xy} = \tau_{xy}^*,
\]

\[
E_x = E_x^* \left( \frac{m_e c}{e\tau_{ee}} \right), \quad B_z = B_z^* \left( \frac{m_e c}{e\tau_{ee}} \right),
\]

\[
\varphi_j = \varphi_j^* n_e (2\pi RT_e)^{-\frac{3}{2}}, \quad \rho = nm, \quad M_a = \frac{V_0}{c},
\]

\[
\gamma = \frac{m_e}{m_i}, \quad \text{and} \quad dU = dU^* (K_B T_e).
\]

It is convenient at this point to suppose that $M_a^2 \ll 1$, i.e., at small Mach number, so that we can consider the change of density and temperature are negligible, which gives $n_a = 1 + O(M_a^2)$ and $T_a = 1 + O(M_a^2)$. Let

\[
\tau_{xy} = \frac{P_{xy}}{\rho V_0 \sqrt{RT_e}} = (V_{x2} - V_{x1}), \quad V_x = \frac{1}{2} (V_{x1} + V_{x2})
\]

Using the dimensionless quantities (14) with (15), the electrons' transport equations for $\psi_1 = \xi_x$ and $\psi_2 = \xi_x \xi_y$, in the integral formula (10) take the following form

\[
\frac{\partial V_{ex}^*}{\partial t^*} + \frac{\partial \tau_{exy}^*}{\partial y^*} - E_{ex}^* = 0,
\]

\[
\frac{\partial \tau_{exy}^*}{\partial t^*} + 2\pi \frac{\partial V_{exy}^*}{\partial y^*} + \tau_{exy}^* = 0,
\]

for the following boundary and initial conditions

\[
V_{ex}^*(y^*, 0) = \tau_{exy}^*(y^*, 0) = 0,
\]

\[
2V_{ex}^*(0, t^*) + \tau_{ex}^*(0, t^*) = 2M_e \cos (\Omega t^*), \quad \text{for} \ t^* > 0
\]

where $V_{ex}^*$ and $\tau_{exy}^*$ are finite when $y \rightarrow \infty$.

The governing equations in the initial-boundary value issue for electrons can be written, after dropping the stars, in the following non-dimensionalized form:

\[
\frac{\partial V_{ex}}{\partial t} + \frac{\partial \tau_{xy}}{\partial y} - E_{ex} = 0,
\]

\[
\frac{\partial \tau_{xy}}{\partial t} + 2\pi \frac{\partial V_{ex}}{\partial y} + \left(1 + \frac{v_{ei}}{v_{ee}} + \frac{v_{en}}{v_{ee}}\right) \tau_{xy} = 0,
\]

\[
\frac{\partial E_{ex}}{\partial y} - \frac{\partial B_{ex}}{\partial t} = 0,
\]

\[
\frac{\partial B_{ex}}{\partial y} - \frac{\partial E_{ex}}{\partial t} - w_{eo} V_{ex} = 0
\]

with the initial and boundary conditions

\[
V_{ex}(y, 0) = \tau_{exy}(y, 0) = E_{ex}(y, 0) = 0,
\]

\[
B_{ex}(y, 0) = B_0
\]

\[
2V_{ex}(0, t) + \tau_{exy}(0, t) = 2M_e \cos (\Omega t), \quad \text{for} \ t > 0
\]

are finite as $y \rightarrow \infty$.

**III. TRAVELING WAVE PARAMETER SOLUTION METHOD**

In the present section, we introduce the algorithm for the well-known solution technique of traveling wave parameters. A transformed coordinate along the direction of wave propagation concerning time can be written in the form [43]–[45]

\[
\zeta = ky - \omega t.
\]

This parameter will transform all the physical dependent variables in the problem as functions of $\zeta$, such that $k$ and $\omega$ are underdetermined transformation constants. Such constants do not depend on the fluid’s properties, but they are parameters that will be determined from the initial and boundary conditions. Following the traveling wave parameter process, as described in (24) leads to the change of derivatives below:

\[
\frac{\partial}{\partial t} = -\omega \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial y} = k \frac{\partial}{\partial \zeta},
\]

\[
\frac{\partial}{\partial t^n} = (-1)^n \omega^n \frac{\partial}{\partial \zeta^n}, \quad \frac{\partial}{\partial y^n} = k^n \frac{\partial}{\partial \zeta^n}
\]

here $n$ is a positive integer.

Based on the above transformation, we would like to seek for the dominant behavior of the traveling wave solution of the model equations. Substituting (24) and (25) into (19)-(22) and using (23) leads to reduce the obtained model, after various calculus manipulations, to one ordinary differential equation in the form:

\[
\left(\frac{\omega^2 - k^2}{\omega} \left(\frac{2\pi k^2}{\omega} - \omega\right)\right) \frac{d^3 V_{ex}(\zeta)}{d\zeta^3} + w_c \left(\omega^2 - k^2\right)
\]

\[
\frac{d^2 V_{ex}(\zeta)}{d\zeta^2} + w_{eo} \omega \frac{dV_{ex}(\zeta)}{d\zeta} + w_{eo} V_{ex}(\zeta) = 0
\]

where $w_c = (1 + \frac{v_{ei}}{v_{ee}} + \frac{v_{en}}{v_{ee}})$ contains all collision frequencies.

The boundary and initial conditions become:

\[
2V_{ex}(\zeta = -\omega) + \tau_{exy}(\zeta = -\omega) = 2M_e \cos (\Omega)
\]

\[
at \ y = 0, \ \text{e.g.,} \ t = 1
\]

\[
E_{ex}(\zeta = 0) = \tau_{exy}(\zeta = 0) = 0, \quad B_{ex}(\zeta = 0) = B_0
\]

where $V_{ex}$, $\tau_{exy}$, $E_{ex}$, and $B_{ex}$ are finite when $\zeta \rightarrow -\infty$.

Via the above procedure and using standard mathematical (symbolic) package software for (26) with the boundary and initial conditions (27), one can obtain various solutions that will be applied to a typical pattern of laboratory helium plasma. All calculations are carried out and presented in the graphics below.
IV. NON-EQUILIBRIUM THERMODYNAMICS AND KINETIC PROPERTIES

We follow the kinetic theory to investigate the irreversible thermodynamic properties of the system [24], [37]. As a consequence, the entropy of the system can either be maintained near the equilibrium state, as presented below.

A. ENTROPY AND ENTROPY PRODUCTION PROFILES

In thermodynamics, two important physical properties are characterizing the study of non-equilibrium thermodynamics. One of them is a thermodynamic state variable called the entropy which is denoted by \( S \) as follows:

\[
S = - \int \varphi_e \ln \varphi_e \varphi_d \xi - \int \varphi_e \ln \varphi_e \xi + \int \varphi_e \ln \varphi_e \varphi_d \xi = - \pi \left( \frac{V_1^2}{x_1^2} + \frac{V_2^2}{x_2^2} - \frac{3}{2} \right). \tag{28}
\]

The entropy generation (production) \( \sigma \) is the other intensive function to understand the fluctuations of a system at equilibrium, which can be written in the following form:

\[
\sigma = \frac{dS}{dt} \quad \text{or} \quad \sigma = \frac{dS}{dt} + \nabla \cdot \mathbf{J}(S). \tag{29}
\]

B. ENTROPY FLUX IN THE VERTICAL DIRECTION

For the system under study, the \( y \)-component of the entropy flux resulting from the velocity exchange constrains the internal entropy production rate of the plasma system is given by

\[
J_y^{(S)} = - \int \xi \varphi_e \ln \varphi_e \varphi_d \xi = \left[ \pi (V_1^2 + V_2^2/x_2^2) \right]. \tag{30}
\]

In order to investigate the internal energy variation for the system, we offered the Gibbs formula, which contains all contributions in the internal energy balance, such as the electromagnetic energy, which distinguishes the atoms as diamagnetic in which the extracted electrons in the paired form [24], [37].

In the plasma diamagnetic state, the internal energy variation is expressed in terms of the extensive quantities \( S, P, \) and \( V \). Those are the extended thermodynamics coordinates related to the intensive quantities \( T \), \( E \) and \( M_B \), respectively. It is noted that the internal energy variation has three participations in the Gibbs energy. The equation groups those: \( dU = dU_S + dU_{pol} + dU_{dia} \). The variation in the internal energy arising from the entropy change is given by \( dU_S = TdS \). In contrast, the variation in the internal energy arising from the electric polarization change is given by \( dU_{pol} = Edp \). Finally, from the thermodynamic relation of the induced electric dipole moment \( M_B = T \delta S \), the variation of the internal energy arising from the change in the nonlinear magnetic field is given by \( dU_{dia} = -mdB \).

Now, we can use the following dimensionless thermodynamic variables

\[
S = S^* K_B, \quad dU = dU^* T K_B, \quad m = m^* \frac{q}{\tau_{ee} c_0} (\lambda_{De})^2, \quad p = p^* \phi_{De} \tag{31}
\]

Thus, after dropping the stars, we will obtain the relations below:

\[
dU_S = dS = \left( \frac{\partial S}{\partial r} \right) \delta y + \left( \frac{\partial S}{\partial t} \right) \delta t, \quad dU_{pol} = f_1 Edp, \quad dU_{dia} = -f_1 M_B dB \tag{32}
\]

where \( \delta y = 1 \) and \( \delta t = 1 \).

V. PHYSICS RESULTS

Typical figures for plasma helium gas are considered in the case of an oscillating solid body in the upper atmosphere ionic belts [38]; all calculations are carried out using the following values:

\[
\begin{align*}
\nu_e &= 10^{17} \text{cm}^{-3}; & n_i &= n_e; & e &= 4.8032 \times 10^{-10} \text{esu}; \\
\varepsilon_m &= 9.1094 \times 10^{-28} \text{gm}; & c &= 2.9979 \times 10^{10} \text{cm sec}^{-1}; \\
\lambda_B &= 1.3807 \times 10^{-16}; & d_{He} &= 10^{-8} \text{cm}; & T &= 200; \\
\mu_i &= 6.633 \times 10^{-23} \text{gm}; & Z &= 1; \\
M_{de} &= \frac{U_0}{\sqrt{2RT}} = 10^{-2}; & \Omega &= 1.5. \tag{33}
\end{align*}
\]

Also, the mean free path of the electron gas \( \lambda_{He} = \frac{1}{\sqrt{2 \pi n_e d_e^2}} = 2.2510^{-2} \text{cm} \) compared to the electron Debye length \( \lambda_{De} = \sqrt{\frac{\mu_B \lambda_{De}}{\pi n_e e^2}} = 3.08610^{-7} \text{cm} \). The collision frequencies for electron-electron, electron-ion, and electron-atom are: \( \nu_{ee} = 1.033 \times 10^{15} \text{sec}^{-1}, \nu_{ei} = 1.461 \times 10^{15} \text{sec}^{-1}, \) and \( \nu_{im} = 1.189 \times 10^4 \text{sec}^{-1} \), respectively.

Based on the concept behind the shooting method along with the numerical values in (33), we can determine the transformation constants to get \( \kappa = 0.85, \omega = 0.08 \), and the Mach number of the plate is \( M_{de} = 10^{-2} \). It is noted that all the variable terms of the problem satisfy the boundary and initial conditions, as seen in Figs. (2)–(6).

FIGURE 2. Velocity profile \( V_x \) vs. position \( y \) and time \( t \).

It can be seen from Fig. (1) that the deviation from equilibrium is small and, over time, the perturbed velocity distribution functions \( \varphi_1 \) and \( \varphi_2 \) approach to the velocity distribution function \( \varphi_{0e} \) in equilibrium case, which gives a sound consistent with the equilibrium law of Le Chatelier. The mean velocity of electrons, near the oscillating plate, is displayed in Fig. (2). We can see that, at \( (y, t) = (0, 0) \), the velocity takes a value that equal to the plate Mach number.
(\(M_a = 10^{-2}\)), which fits the problem conditions. Besides, we noted that the shear stress decreases with time due to the mean velocity behavior, and the observed negative sign of the shear stress reflects its path, from down to up, as shown in Figs. (2) and (3).

The behavior of the coefficient of viscosity for a plasma gas in a slow flow for a Newtonian fluid as

\[
\mu = \tau_{xy} \left( \frac{\partial V_x}{\partial y} \right)^{-1}
\]

is displayed in Fig. (4), which gives a reasonable interpretation of the resistance to the motion that increases gradually over time near the solid oscillating plate. That is because, if any change is forced on an equilibrium system, the system tends to adjust a new equilibrium state of that counteracts the transition, thus agreeing with the theory of Le Chatelier. Figures (5)–(6) show the manner of the electric and nonlinear magnetic field patterns close to the oscillating plate, respectively. We show that the induced electric field increases with time, while the induced nonlinear magnetic field decreases with time. This manner indicates a good agreement with the conditions and physical interpretations of the problem. According to the second law of thermodynamics, the entropy \(S\) should be increased with time; Fig. (7) confirms this fact, which gives an extensive interpretation of the system. Also, the entropy production behavior is accomplished with the celebrated H-theorem for all values of \(y\) and \(t\), where \(\sigma \geq 0\), see Fig. (8).

It is well known that we can determine the energy loss (or gain) of an electron by the action of the forces acting on the electrons in the plasma gas. That is happened by the electromagnetic field produced by the moving particle, because the suddenly oscillating plate causes the work done on the gas, and causing changes in the gas’s internal energy \(U\). When a charged particle (electron) moves through the plasma, part of its energy is loosened (or gained) by impact with the surroundings corresponding to collision processes and plasma polarization. Figures (9)–(11) shown the internal energy change owing to the entropy variation increases in a nonlinear manner with time. That is due to energy loss or gains from ions and plate, respectively. Even though the change in internal energy decreases gradually with time because of the
Entropy production $\sigma$ vs. position $y$ and time $t$.

Internal energy change $dU_s$ vs. position $y$ and time $t$.

Internal energy change $dU_{pol}$ vs. position $y$ and time $t$.

Internal energy change $dU_{dia}$ vs. position $y$ and time $t$.

Gyro-frequency $\Omega_c$ versus position $y$ and time $t$.

Gyro-radius $\rho$ versus position $y$ and time $t$.

in the gyro-frequency, $\Omega_c = \frac{eB_z}{m_e \omega_p}$ and gyro-radius, $\rho = \frac{V_T}{\Omega_c}$ for electrons in plasma physics.

Using the dimensionless variables:

$$\Omega_c = \Omega_c^* \omega_p \quad \text{and} \quad \rho = \rho^* \frac{\lambda_D e}{4\pi}, \quad (35)$$

we can formulate the angular frequency of the electron’s circular motion in the perpendicular plane of the nonlinear magnetic field defined as the gyro-frequency: $\Omega_c = \sqrt{\frac{5}{6\pi}} B_z$.

It is smaller than the plasma frequency by two orders of magnitude, as seen in Fig. (12). Whereas in the plane, the radius of an electron’s circular motion perpendicular to the nonlinear magnetic field defined as the gyro-radius: $\rho = \sqrt{\frac{5}{12} (B_z)^{-1}}$.

Furthermore, Figs. (12) (13) verify that the gyro-frequency $\Omega_c$ is decreasing nonlinearly with time. That is due to the behavior of the magnetic field itself where they are linearly dependent, as shown in figures (6) and (12).

The gyro-radius $\rho$ is increasing with it. It is vanishing outside the vicinity of the oscillating solid plate. Also, this is due to the value of the nonlinear magnetic field as they related by the previous relation $\rho = \sqrt{\frac{5}{12} (B_z)^{-1}}$, as shown in figures (6) and (13).

VI. CONCLUSION AND OUTLOOK

The solution of the unsteady BGK-type model, together with Maxwell’s equations in the case of plasma gas, was introduced. We did this applying the method of the moments of the two-sided distribution function within the exact traveling intensive variables, which corresponds to either diamagnetic plasma or polarization, figures (12) and (13) characterize a two productive dynamical behavior of the system represented.
wave parameter method and the accurate values of collision frequencies of electron-electron, electron-ion, and electron-atom. Tackling this allowed us to calculate the components of the flow velocity, which were inserted into the suggested two-sided distribution functions. Such results have the advantage of addressing our problem directly in the laboratory helium plasma case.

Furthermore, we evaluated the entropy, entropy production, and the ratios between the various internal energy variation contributions based on the derivatives of the comprehensive variables. Also, the thermodynamic predictions of the problem were calculated using Gibbs equations, which indicate the following order of the maximum number magnitude ratios between the various contributions to the change in the internal energy, based on the extensive parameter in total derivatives:

\[ dU_p : dU_d : dU_s = 1 : 6.71 \times 10^{-1} : 2.0. \]

Finally, we concluded that the internal energies change effect \( dU_p \) and \( dU_d \) due to electric and nonlinear magnetic fields, was extremely significant compared to \( dU_s \), which in recognition of the fact that these fields were applied in a nonlinear magnetic field and its induced electric field. Those participants’ values in the internal energy could be compared with their corresponding values in the absence of external fields, which consistent with the results presented in Ref. [37]. As \( dU_p : dU_d : dU_s = 1 : 2.5 \times 10^{-1} : 3.11 \times 10^{-4} \), it can be concluded that the influences of the variations of the internal energies \( dU_p \) and \( dU_d \) owing to the electric and nonlinear magnetic fields are tiny compared to \( dU_s \), which in recognition of the fact that the fields are self-induced by the plate’s sudden movement.

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TAHA ZAKARAIA ABDEL WAHID (Member, IEEE) was born in Ashmoon, Menoufia, Egypt, in 1976. He received the B.S. degree in mathematics, the M.S. degree in applied mathematics, and the Ph.D. degree in plasma mechanics from the University of Menoufia, Menoufia Government, Egypt, in 1999, 2004, and 2011, respectively.

From 1999 to 2004, he was a Research Assistant with the Mathematics and Computer Sciences Department, Faculty of Science, Menoufia University, Shebin Elkom, Egypt. Since 2011, he has been an Assistant Professor with the Mathematics Department, in many universities, in Egypt, Libya, and Saudi Arabia. He is the author of four international books. He published more than 80 articles in reputed journals and conferences that cited more than 1410 times. His research interests are mathematics, plasma mechanics, mems, quantum mechanics, mathematical physics, partial and ordinary differential equations, thermodynamics, thermal radiation, macro, micro, and nano fluid mechanics, astrophysics, numerical methods, and uranium enrichment. He is an Editorial Board Member and a Referee of more than 35 international journals. His name included in the famous international universal encyclopedia (Who is Who in The World?) five times 2015, 2016, 2018, 2019, and 2020. He awarded the 2018 Albert Nelson Marquis Lifetime Achievement Award. He was the Quality Manager of many colleges.

ADEL M. MORAD was born in Tanta, Egypt, in 1980. He received the B.S. and M.S. degrees in mathematics from the Menoufia University, Egypt, in 2009, and the Ph.D. degree in mathematical modeling, numerical methods, and software from Southern Federal University, Russia, in 2015.

From 2004 to 2009, he was an Assistant Lecturer with the Department of Mathematics and Computer Science, Menoufia University, where he has been a Lecturer with the Mathematics and Computer Science Department, since 2016. Since 2019, he has also been working as a Visiting Research Professor with the Computational Mathematics and Mathematical Physics Department, Southern Federal University. He is the author of two books, more than 40 articles, and has participated and invited speaker in many international conferences. His research interests include plasma physics, thermodynamics and applications, thin films, and computational fluid dynamics. He is manuscripts’ refereed for many profoundly impacted journals. He was a recipient of the Golden Menoufia University Award for Young Scientist, in 2016, and the International Conference of Nuclear Sciences and Applications Best Paper Award, in 2008. He has a strong record of supervision and advice to postgraduate and honors students from various institutions. He is also good command in English, Deutsch, and Russian languages.

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