1. Introduction

Dark matter consists larger part of mass in the universe, which is not included in the standard model. The concept of dark matter (DM) has been developed since 1922, which explains a range of contradict theories in the universe [1]. The hypothesis of dark matters is based on the evidence from the age of universe uniformly scattered cosmic microwave background radiation (CMBR), and the bullet cluster observed [1]. Although DMs are still not being detected right now, the latest researche had introduced a great variety of dark matter models.

Among plenty of DM models, supersymmetry, axions, and neutrinos are widely investigated analytically. Supersymmetry is a hypothesis of dark matter particles, which states that Boson and Fermion can be associated. Neutrino is a zero-charged first elementary particle with zero rest mass, not a constituent of ordinary matter [2]. Axions are naturally occurring subatomic particles that are predicted in particle physics, astrophysics, and condensed matter physics.

As the history of those theories and evidence are seldomly written, this review will briefly arrange the history of those three dark matter particles and going forward to the latest discoveries. The rest of the paper is organized as follow: the definition origination and future development of supersymmetry will be introduced in Sec. 2; the theory and recent progress for neutrino will be presented in Sec. 3, while the discussions for axion are given in Sec. 4; a brief summary is given in Sec. 5 eventually.
2. Supersymmetry
The supersymmetry is a hypothesis of dark matter particles, which states that Boson and Fermion can be associated [2]. Although this symmetry hasn't been observed yet, we introduce the origin, functions, and importance of the supper symmetry.

2.1. Definition of supersymmetry

2.1.1 Definition of supersymmetry
Generally, it is well known that basic particles have two different types: Bosons and Fermions. Particles exhibit boson properties with zero or integer spin values (e.g., photons and He nuclei) while exhibiting fermion properties with integer and half spin values (e.g., leptons, neutrons, and He\(^3\) nuclei) [3]. Supersymmetry is a symmetry between Bosons and Fermions, a very important hypothesis of dark matter proposed by a Japanese physicist Hironari Miyazawa in 1966[4]. Supersymmetry plays an important role in connecting Bosons and Fermions. Recently, there has been an increasing interest in the prove and experiments in supersymmetry.

A major issue of the hypothesis is that it hasn't been proven yet, but many verification approaches are proposed. For example, two typical events (seen from Fig. 1) might occur via Large Hadron Collider (LHC), which is expected to give a strong supersymmetry signal [5].

![Fig. 1. The Feynman diagram of two possible processes [5].](image)

Supersymmetry requires that a boson must exist on the same quantum numbers for every fermion and vice versa. Therefore, it predicts the existence of several new electrically neutrals and non strongly interacting particles, e.g., the super partners of the neutrinos, photon, Z boson, Higgs boson, and graviton. If any of these super partners were stable, they could be cosmological abundant that play an important role in the history and evolution of our Universe [6].

2.1.2 The theory of supersymmetry
Physicists have established an 8-dimension (8-D) supersymmetry theory to unify fermion and boson, which is called a hyperspace. However, the extra four dimensions cannot be understood in either time or space. The 8D universe is inhabited by fermions. Matter can be transferred between the four dimensions and the 8D universe. This means bosons can be changed into fermions, and fermions can be changed into bosons. There is no difference between them. The reason for their different spins is just an illusion that we are limited to the four-dimensional space and cannot see the 8D space.

These pairs of particles supersymmetric are defined as partners, e.g., gravitino, photino, and gluon. A fermion partner is called a super particle, which just adds an ‘s’ in front of the word (e.g., selectron). However, we have not found any of those supersymmetric partners of fermions or bosons. For instance, the electron is not the supersymmetric partner of any known boson. As a result, if every boson or fermions has its supersymmetric partner, the number of particles of the world will be twice.

2.1.3 An indirect support evidence of supersymmetry
A very notable result of supersymmetry is that it has "theoretical evidence" from the grand unified theory, though it has no experimental evidence. Several years ago, researches expected that the strong
interaction and electric weak interaction could be unified together at very high energy (about 10^{15}-10^{16} GeV). Such a theory is known as a grand unified theory. One of the preconditions for the grand unified theory is that the coupling constants of strong, electromagnetic, and weak interactions must all be the same on the grand unified energy scale, which can be proved theoretically. However, the above coupling constants are not the same at any energy level, i.e., the standard model is incompatible with the requirements of the grand unification theory. The supersymmetry gives a new approach to accomplish the grand unified theory because the calculation shows that combine the grand unified theory and the supersymmetry, all these coupling constants are perfectly the same. Although it is only theoretical evidence, it motivates the physicists to prove the supersymmetry.

2.2. Origin of supersymmetry
Physicists’ imagination would not remain confined to the standard model for long. Instead, it would turn to the contemplation of many speculative and yet undiscovered candidates for the dark matter of our Universe. In particular, plenty of studies began to consider the possibility that nature may contain a space-time symmetry relating fermions to bosons, dubbed "supersymmetry in the early 1970s [1]. Y. A. Gol'fand and E. P. Likhtman proposed simple models in mathematical physics [7] in the early 1970s. Subsequently, J. Wess and B. Zumino presented Super gauge transformations in four dimensions [8], which first applied supersymmetry to four-dimensional space-time. In their studies, the super gauge transformations are defined in four space-time dimensions. Their commutators are shown to generate γ-5 transformations and conformal transformations. Various kinds of multiplets are described, and examples of their combinations to new representations are given. The relevance to super gauge transformations for Lagrangian field theory is explained. Finally, the abstract group-theoretic structure is discussed. This paper is usually regarded as the year when supersymmetry was born (1974).

2.3. The practical function of supersymmetry
One of the attractive sources of supersymmetry is the complementarity of Boson and Fermion in physical properties. This theory can be used to solve some extremely difficult problems in the high energy physics field. Another attractive source of supersymmetry is that a large number of divergent results in general quantum field theory can be eliminated by the contribution to supersymmetric partners in supersymmetric theory. Therefore, a supersymmetric theory has a very superior regularization property. Besides, there are many research aspects and motivations for supersymmetry: the construction of supersymmetric Lagrangians, superspace and superfields, soft supersymmetry-breaking interactions, the Minimal supersymmetric Standard Model (MSSM), R-parity and its consequences, the origins of supersymmetry breaking, the mass spectrum of the MSSM, decays of supersymmetric particles, experimental signals for supersymmetry, and some extensions of the minimal framework [9].

2.4. The importance of the supersymmetry
It has been 47 years since supersymmetry was proposed, but no supersymmetric partner of any known particle has been observed in experiments, and even no conclusive indirect evidence has been found. Nevertheless, the extraordinary charm of supersymmetry in theory still makes it have a high status of theoretical physics. Moreover, the concept of supersymmetry can be seen in almost all the frontier fields of physics. Once experiments prove it, it will have a significant influence on the field of theoretical physics. On the other hand, if it is not proved, the entire field of theoretical physics is disappointed [10].

2.5. Future expectation of the supersymmetry
The most important expectation in this field absolutely is the hypothesis of supersymmetry was proved. Nevertheless, it is not simple to verify the theory, i.e., more experimental data and results are expected. However, the most important weakness for this hypothesis is that none of the supersymmetric partners have been observed, i.e., the most important work to find even one of the supersymmetric partners in the next few years. That is a big step forward to the truth to the world.
3. Neutrino

3.1 The theory of neutrino

Wolfgang E. Pauli propounds the first prediction of the existence of neutrino in 1930. This theory is based on the non-conserved mass-energy during the decay. The theory predicts that the neutrino is a neutral charged first elementary particle with zero rest mass, which is formed by radioactive decays and is not a constituent of ordinary matter [2].

Neutrino hot dark matter can be investigated with respect to the structure formation and anisotropies in the cosmic microwave background radiation (CMBR) [11]. It is widely believed that the hot dark matters (HDM) are consists of massive neutrinos. The term HDM is used as they have much more energy than cold dark matter (CDM).

While the Beta decay emits neutrinos, the proton decay emits antineutrinos with opposite electron spin with the neutrino. Recent research presents a new method of measuring neutrino masses according to the dark matter-neutrino relative theory [12].

3.2 Oscillations and fast conversion of neutrino

Massive stars can end their lives with the core-collapse supernova explosions (CCSN) by losing energy dramatically in terms of the emission of neutrino-antineutrino pairs [13]. A recent study has shown the characteristics that the fast flavor oscillations in the dense neutrino gases, which influences the evolution of its surroundings [14]. The results of most literature are prone to a condition that the angular distribution of and cross each other and the same emission properties of other species of neutrinos [15], necessary for the occurrence of fast instabilities of the neutrino. The reason for the name is that their growth rates are proportional to the self-coupling potential \( \mu = \sqrt{2} G_F n_\nu \) (where \( G_F \) is the Fermi coupling constant and \( n_\nu \) is the neutrino density) and has a much greater oscillation frequency than the vacuum oscillation frequency [16]. The occurrence of fast flavor conversion (FFC) will alter the current supernovae theory [17].

![Fig. 2. (a) Different growth rates of temporal fast instabilities and the corresponding values of the real wave number K of the fast modes propagating in the radial direction. (b) An example of the angular distributions of the eigenvectors of unstable modes, under the condition where \( Q_\nu \) is normalized to one [18].](image)

As illustrated in Fig. 2(a), the growth rate of the temporary fast instabilities against the real wave number K spreads fast in the radial direction [18]. The red line represents the crossings in the backward direction, while the blue line denotes the crossings in the forward direction. Although it shows a relatively small value for backward directions, the growth rate is still significant for larger densities. Fig. 2 (b) gives an example of the angular distributions of the eigenvectors of unstable modes, under the condition that \( K = 1.01 \xi \), where the growth rates are close to their maxima. It is noticeable that the eigenvectors peak in the backward direction, where the population of travelling neutrinos is extremely
small. Whereas the value of $Q_v$ is very small for the forward direction. These results under the condition that the axial symmetry is preserved in the neutrino gas.

### 3.3 Producing the LHC neutrinos

Lately, research has discovered the potential for the neutrino to be created by LHC, which is the particle accelerator with the highest energy in the world as the hadrons produced by the proton-proton collisions decays to emit beams of high-energy neutrinos along the beam collision axis [19]. The measurements of neutrino interactions involve coherent scattering, neutrino capture, inverse beta decay, low-energy nuclear interactions, quasi-elastic scattering, resonant pion production, kaon production, deep inelastic scattering, and ultrahigh energy interactions [20-25].

After 37 years since the possibility of LHC neutrino is discussed, it has not been detected, while the idea of FASER experiment may increase the possibility of successful detection. This experiment covers the blind spot, where the beam lines that produce neutrinos have holes along with them. The FASER experiments are able to measure charged current (CC) scattering and neutral current (NC) scattering [26]. Compared with the different combination of up and down quark neutrino nonstandard interactions (NSI) probed in the CHARM experiment [27], there is a combined constraint from neutrinos and antineutrinos.

### 4. Axions

#### 4.1 The origination of axion

In 1977, the idea of axion was invented by Steven Weinberg as they go through the new global $U(1)$ symmetry [28]. To be more specific, an issue named strong-CP problem consists with the fact that QCD Lagrangian is the following term in quantum chromodynamics (QCD) theory:

$$\mathcal{L}_{QCD} \ni \frac{g^2}{32\pi^2} G_{\mu
u}\mathcal{G}^{\mu\nu}$$

With spontaneously broken, the quantity $\Theta$ in equation (1) can be driven to nearly zero. Meanwhile, it explained the observed value introduced by Roberto Peccei and Helen Quinn early in the same year [29], which implies the existence of a Nambu-Goldstone boson called axion in this broken global symmetry. Besides, the term in Eq. (1) shows the gluon field strength tensor while $\Theta$ illustrates the quantity related to the phase of the QCD vacuum. Based on the previous expectation of $\Theta$, it would lead to a large charge-parity (CP) violating effect [29, 30], which causes the moment of electric dipole neutron much larger than the experimental permit. In other words, the quantity of $\Theta$ must be smaller than a certain threshold.

In retrospect, the concept of axion is developed to solve the strong-PC problem. In 1978, researchers pointed out this broken global symmetry could lead to axion, which is also a kind of Nambu-Goldstone boson [28, 31]. Primarily, this kind of part is excluded. Nevertheless, it is predicted to produce sizable rats of exotic meson decays after relating with existing astrophysical constraints (e.g., contradiction with observation axions heavier than ~10KeV).

Besides, the axions heavier than ~1eV would lead to the cooling of red giant stars, which contradicts current observations. Moreover, the observation data from Supernova 1987A is also strong evidence for the constraints were placed on axion mass (less than or equal to $10^{-3}$ eV). We're still unable to observe axion directly due to the unknown inherited couplings within axion, but plenty of analytical studies predict it present in our universe.

#### 4.2 Detection approaches

As a matter of fact, there are plenty of experiment schemes for detection. The most famous one is the axion dark matter experiment (ADMX) [32-36] based on photon-photon axion coupling, which requires a strong and static magnetic field to transfers axions to transfers axions to monochromatic microwave photons signal. This kind of scenario is proposed by Sikivie (1983) [34] and developed by [35] and others [36]. The ADMX reported a relatively narrow range of masses 1.9–3.3 μeV in 2003 based on
constrained realistic axion dark matter models [37]. With ambitious upgrades, a much larger range of axion detection ought to be expected [38-40].

5. Conclusion
In summary, we present a systematic review for the progress of supersymmetry, axion. Specifically, as for supersymmetry, the definition, origination, and detecting schemes have been discussed to prove. With regard to neutrinos, the theories and proposed proof-of-principal experiments based on LHC are demonstrated. Moreover, the proposal and detection approaches of axion are introduced. Although these three kinds of DM haven’t been observed yet, it is widely believe to be detected some day in future with the state-of-art facilities and updating detection scenarios. These results summarize the recent developments of these three kinds of DM and pave a path for further investigation.

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