Computational Fluid Dynamics Modeling Study for the Thermal Performance of the Pin Fins Under Different Parameters

Osamah R Al-khafaji and Audai H Alabbas

1 Power mechanics Techniques Department, Technical instructors training institute, Middle technical university, Baghdad, Iraq. E-mail: osamaraad19@yahoo.com
2 Pumps Engineering Techniques Department, Al-Mossaib Technical Collage, Al-Furat Al-Awst Technical University, Kufa, Iraq. E-mail: aalabbas@atu.edu.iq

Abstract. Computational Fluid Dynamics CFD modeling study was carried out to investigate the effect of the pin fins arrangements over the flat plate heat sink. The pin fins were adopted with in-line array form over the surface of the heat sink with different parameters. The CFD domain was designed by the use of ANSYS FLUENT 2019 R1 commercial software. The Reynolds Average Naiver Stoke RANS turbulent model was used in this study to show accurately capture the effect of alternating acceleration and deceleration and consequent variations in the pressure field. The heat sink is in a rectangular shape with a rib length of 100mm and100mm. This CFD investigation included a simulation of the proposed heat sink under the forced convection with Reynolds number range of 2166.67 ~ 19500 and subjected heat flux of 1000W/m². Pin fins diameters range were 3, 6 and 9mm while the fin lengths range were 5, 10 and 15mm under the same the heat flux and Reynolds number. The predicted results showed an augmentation in the Nusslte number values of the finned heat sink as compared to the flat one. Moreover, this enhancement was associated with the increase of the applied Reynolds number. It was found that the best thermal performance of the heat sink was conducted for the fin length of 15mm and fin diameter of 9mm.

Keywords: Pin fins, CFD, Plate heat sink

1. Introduction
Heat sinks are considered as the most commonly used methods to reject the heat energy which is utilized in the multi applications such as domestic and industrial ones. The enhancement of the heat sinks thermal performance is associated with the increase in the heat transfer coefficient. The last parameter can be increased by fabricating the fins over the heat sink with multi shapes such as plane fins, pin fins and wavy fins. The use of the pin fins array acts on specifying the performance of the heat sink by changing the length, geometry and the shape of the array. Therefore, a lot of studies
tended to investigate the effect of the use of the pin fins to enhance the thermal performance of the heat sink.

An experimental study investigated the effect of the pin fin geometry on the thermal performance by L. Micheli and et al. The conducted results showed that the use of the pin fins gave more thermal effectiveness against the flat plate utilization between 3% – 7% [1]. The increase in the pin fin height and pitch distances tended to enhance the heat transfer coefficient of the film wise condensation [2]. It was found the cleared effect of the perforations addition on the thermal performance of the pin fins [3]. It was also found that the increase in the perforation diameter of the pin fin was associated with the decrease in the pin fin length where all that provides an enhancement in the thermal performance [4].

Regarding with the general geometries of the pin fin, it was conducted that the triangular configuration of the pin fin was found as the best thermal performance related to rectangular and circular configurations. This investigation was done by filling the used heat sink with different types of phase changing materials PCM, where the triangular configuration results provided the best performance with and without the addition of PCM [5]. Furthermore, the effects of the diamond, square, streamline and circular geometries of the micro pin fins for the cases of cooling system and flow boiling showed that the square shape provided the best heat transfer coefficient, while the diamond shape showed the less pressure drop effect of the flow [6]. The previous studies [1] and [4] proved the effectivity of the use of the pin fins for free convection. Furthermore, it was concluded that the pin fins tended to increase the heat transfer coefficient of the condensation in comparison with the smooth tube case [7]. An investigation [8] studied the effect of the pin fin additive on the heat sink performance. The used pin fins were arranged within line and staggered array under the effect of the natural and forced convection. The results showed an augmentation in the heat transfer coefficient of the heat sink by the increase of the pin fin spacing under staggered array. The analytical method found that the pin fin efficiency was increased by the increase of the fin diameter and it was decreased by increasing of the fin length [9]. Moreover, a numerical study found that the increase of the pin fin spacing tended to raise the heat transfer rate of the used heat sink [10].

An experimental investigation in which implied the hexagonal pin fins led to enhance the heat transfer by convection as compared to square and circular pin fins configurations [11]. Moreover, it was obtained that the use of the pin fins tended to increase the values of the Nusselt numbers against the triangular fin utilization for the same experimental parameters [12]. Another experimental study found that the elliptical shape of the pin fin provided the best thermal performance against the circular, triangular, diamond, square shapes. It is also found that the increase in the Reynolds number tended to increase the performance of the heat sink under all the shapes of the pin fins [13]. The increase in the average values of the Nusselt number was conducted by the increase in the pin fin spacing up to a certain value then it was decreased. This strategy of the study was concluded through an experimental investigation in which discussed the performance of the circular pin fin under the free convection [14]. A numerical investigation found that the perforated pin fins with square cross section on the heat sink acted on eliminating the thermal resistance of that sink under forced convection. Moreover, It was found that the increase in the Reynolds number, fin number and the perforations number had the tendency to reduce the thermal resistance as much as possible [15]. It was found that the increase in the pin fin length and spacing led to increase the of the Nusselt number values of the fined heat sink [16].

The present study tends to simulate the plane heat sink with and without pin fins by the use of a CFD commercial software. The previous studies do not cover the Reynolds number values of less than 3000. Also they don’t reach the higher values such as 17000 or more. Therefore, this investigation is tended to cover a Reynolds number range of 2166.67 ~ 19500. The previous researches present a good investigation for the pin fin array effects and the spacing values among them. Therefore, the pin fin length and diameter values have the tendency to be the investigated parameters in this study. This CFD study revealed that the strategy of this investigation has a great importance on the heat transfer performance of the pin fins in the industrial fields.
2. Geometrical model

ANSYS FLUENT 2019 R1 is used to draw the model of the required heat sink. The heat sink is in a rectangular shape with a rib length of 100mm and 1mm height as shown in Figure 1. The used thickness in the heat sink is depended on the used one for a previous experimental investigation [17]. Circular pin fins are placed on the heat sink with the in-line arrangement and equivalent distances among. The pin fins are modified during the present work to be 3, 6 and 9mm and lengths of 5, 10 and 15mm respectively as shown in Figure 1.

The heat sink is designed inside an enclosure in order to form the air flow channel with different flow velocities as shown in Figure 2. The used dimensions of that channel are selected to form the aspect ratio of 5 and the hydraulic diameter of 30mm. The entrance length of the channel is made to be as a tenth of the hydraulic diameter in order to satisfy the fully developed flow of air [18]. The heat sink and the pin fins are assumed to be made of aluminum during all the simulation.
Figure 2. The air flow path over the heat sink

3. Numerical analysis
The numerical methods present the suitable way to analyze the complicated cases such as the temperature distribution through three dimension plate or extracting the velocity values during the flow of the air inside the rectangular duct. The numerical methods does not present the exact solutions in which are regarded in the common analytical methods but, they give an approximated results with some percentage of error. The computed results by the numerical methods become acceptable when the error percentage is too low so that the numerical methods are acceptable to develop the required results in the scientific research.

It is tended to use the computational fluid dynamics techniques (CFD) to simulate the proposed model where this technique is available by ANSYS FLUENT 2019 R1 software program. The denoted program uses the finite volume numerical method to solve the governing equations in which are discussed in the next term.

3.1 Governing Equations
The used governing equations are specified according to the general statement of the simulation and the assumptions in which are listed below:

1- The flow is steady
2- No flow slip
3- Fully developed flow
4- The bouncy effect is neglected
5- The operated range of the air temperature is limited between (28-38)°C so that the air density has insignificant variation. Thus, the density is assumed to be constant.

1-The continuity equation is [19]
\[
\frac{\partial u}{\partial x} i + \frac{\partial v}{\partial y} j + \frac{\partial w}{\partial z} k = 0
\] (1)

2-The momentum equations for the three dimensions are [19]
\[
-\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)
\] (2)
\[-\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \quad (3)\]

\[-\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \rho \left( \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) \quad (4)\]

3. The fluid energy equation is [19]

\[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (5)\]

4. The solid energy equation [19]

\[0 = \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (6)\]

RANS (Reynolds Average Naiver Stoke) model is used to analyze the turbulent flow where this model provides a good accuracy to the present turbulent flow case. It is also found that the studied domain includes a circular pin fins the order that makes the air flow moves in a wavy form. Therefore, it is recommended to use this model [20].

\[\rho \left( \frac{\partial u_i}{\partial t} + U_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu + \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial R_{ij}}{\partial x_j} \quad (7)\]

\[R_{ij} = -\rho u_i'u_j' \quad (8)\]

where \( R_{ij} \) is called as the Reynolds stress tensor and it is unknown parameter in the equation (8) where it depends on the Boussinesq assumption to be solved as follow:

\[R_{ij} = -\rho u_i'u_j' = \mu \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] - \frac{2}{3} \mu \frac{\partial U_i}{\partial x_j} \delta_{ij} - \frac{2}{3} \rho \delta_{ij} \quad (9)\]

Where \( u_t \) is the turbulent viscosity while \( S \) describe the term of mean flow

3.2 Mesh processing

In order to get on high accuracy in the computing of the domain in thermal performance by the use of the numerical methods. Therefore, the subdividing of the geometry into fine scale elements assists the analysis to reach the required accuracy. ANSYS FLUENT 2019 R1 helps the user to subdivide the studied geometrical domain into any specified number of the elements by depending on the volume of each element. The selection of the elements number is done based on the grid independence procedure as it shown in the table 1. The average values of the used elements size are 0.75*0.75*0.75mm, as shown in the Figure 3 [21]. The meshed domain is processed by the use of 3 cores of CPU with 2.3GHz.

In order to reduce the required time to finalize the numerical analysis under an adequate result. Therefore, the selected number of the elements in the present domain should be tested for many trials. It is tended to use the mean surface temperature of the finned heat sink as an effected factor to study the changes of its values when the elements number are varied. The selection of the elements number should be as minimum as possible and it corresponds to the minimum error percentage for the mean surface temperature values. This is enabled the soft-ware program to present the suitable results at least time therefore. The selected elements number is 200,000 when the pin fin diameter and length are 9mm and 15mm respectively. It is noticed that the variation of the mean surface temperature starts
with 38.5° C and it approached to be fixed at 34.1°C. Therefore, it is obvious that the error percentage gets reduction during the analysis and thus the used elements are 200,000.

![Image](image_url)

**Figure 3.** The Meshing of the heat sink

| Elements number | Mean surface temperature (°C) |
|-----------------|------------------------------|
| 180,000         | 38.5                         |
| 200,000         | 34.6                         |
| 220,000         | 34.4                         |
| 240,000         | 34.1                         |

### 3.3 Boundary conditions

The present study acts on simulating the pin fin heat sink which is subjecting to the air flow path. In the real case, the flowing of the air through a certain path should be done by a channel where their walls are fully thermal insulated. Therefore, the boundary conditions of the air flow path side walls enclosure are selected as insulated. This one is accepted with a small area at the bottom wall which represents the wall of the heat sink as shown in Figure 4.

The actual case of the studied domain assumes that the air flow enters from one side and exits from the other side by means of a blower or centrifugal fan. Therefore, the showed face in the Figure 3 represents the inlet velocity as a boundary condition. While another face from the other side has an outlet flow option as a boundary condition. The inlet velocity option requires the specifying of the velocity value at each analysis case while the outlet flow option enables the ANSYS FLUENT 2019 R1 to forward the flow path outside of the domain. The left area at the bottom wall from the enclosure which is a part of the heat sink is fabricated to be a wall option with a specified value of the heat flux as a boundary condition.
4. Data processing

The predicted results of the numerical analysis such as temperature distribution over the heat sink and the pressure difference between inlet and outlet of the air should be processed to investigate the overall performance by computing the required parameters as follows.

It is considered as the most affective factor in the thermal investigations as a result of being it specifies the enhancement or decreasing of the applications the thermal performance. This term can utilize depending on the final equation of the following derivation

\[ Q = h \cdot A \cdot (T_s - T_\infty) \]  
\[ Nu = \frac{h \cdot D_h}{K} \]  

By systemizer equation (10) and equation (11) for the same heat transfer. Thus, Nusslte number should be as the following [18]

\[ Nu = \frac{Q \cdot D_h}{A \cdot K \cdot (T_s - T_\infty)} \]  

Moreover, it is found that the surface area of the pin fins and the occupied area by the pin fins, as shown in Figure 5 should be taken in the consideration of computing the Nusslte number. Therefore a ratio between the total surface area of all the pin fins to the surface area of the heat sink without the occupied area by the pin fins must be multiplied to the Nusslte number relation. This relation is used in the calculations of the Nusslte number of the present study [18].

\[ Nu = \frac{\text{Fin number} \cdot (A_t + A_c)}{A_{ss}} \cdot \frac{Q \cdot D_h}{A \cdot K \cdot (T_s - T_\infty)} \]
Environmental temperature \( T_{\infty} \) can be computed by the use of the equation below

\[
T_{\infty} = \frac{T_{\text{air,in}} + T_{\text{air,out}}}{2}
\]  (14)

Reynolds number \( (Re) \) is used to examine the flow region type and it can be computed by

\[
Re = \frac{\rho u D_h}{\mu}
\]  (15)

Hydraulic diameter \( (D_h) \) expresses the active diameter of flowing the air inside the flow path enclosure and it can be computed by the relation below [22]

\[
D_h = \frac{4 A_{\text{cross section}}}{F_{\text{cross section}}}
\]  (16)

Friction factor can be computed by using the Darcy Weisbach relation as it shown below [22]

\[
f = \frac{2A p D_h}{\rho L u^2}
\]  (17)

It is important to notice that all the physical properties such as air density and conductivity are achieved by the use of Engineering Equation Solver (EES) software under the dependence of the film temperature.

Reynolds Performance Parameter is used to give an optimization for any thermo-hydraulic system in which operate under the forced convection [23]

\[
\text{Reynolds Performance Parameter} = \frac{Nu}{f_{\text{flat}}} \frac{f_{\text{flat}}}{Nu}
\]  (18)
Dittus–Bolter correlation can be written as: [16]

\[ Nu_{\text{Dittus–Bolter}} = 0.2333 \cdot Re^{0.8} \cdot Pr^{0.4} \]  

(19)

5. Validation
In order to get on the confidence that improves the accuracy of the computed results. Therefore, a validation process is done by depending on multi results in which are achieved in the previous work. Dittus–Boilte correlation [23] in which acts on computing the Nusslte number values under the same as values of the Reynolds and Prantdl numbers. It is found that the computed results by Dittus–Bolte correlation give a good verification as it shown below in Figure 6.

![Figure 6. Validation of the Nussltt number](image)

The experimental study of Ref. [23] provides a good verification to the present work by considering that the operated Reynolds number range is within. It is also noticed that a part of this experimental modification includes the use of plane heat sink with in-line pin fins array. It is found that the CFD results differ from that results of the experiment due to reach the CFD to the ideality. It is also known that any experimental investigation includes some of the heat losing, flow slipping and uncertainty in the measurement instruments. The use of the three increments in this figure is due to the used range of Reynolds number in which corresponds the utilized range in the present study.

![Figure 7. Comparison between CFD result at 5mm pin fin length and 3mm as diameter and Experimental results [23] under same pin dimensions](image)
6. Results and Discussion

Figures 8, 9 and 10 are showing the increase in the Nusselt number against the applied Reynolds number for all cases of the present study. This is because of being the increase in the velocity of flow acts on eliminating the thermal sublayers and then reducing the thermal resistance over the heat sink. The figures mentioned explain the difference of the Nusselt number as compared to the used fin diameter where they are obviously shown that the pin fin diameter of 9mm presents the best thermal performance. The larger pin fin diameter of 9mm gives the highest Nusselt number under different Reynolds number and pin fin length. This is because of being the area of flow between the parallel lines of the pin fins adoption is reduced as compared to other used pin fins diameters.

The Nusselt number values for the 6mm and 3mm pin fins are high as compared to the flat heat sink due to the reduction in the area of flow. The reduction in that area is due to the increase of the volume of the single pin fin the order that occupied some of the flow area. Therefore, and according to the continuity principle where the reduced area of flow acts on increasing the velocity of flow as it is shown in the Figure 11. The last one tends to eliminate the thermal sublayers and consequently increase the rate of heat transfer. It is also that the arrangement of the pin fins array over the heat sink tends to increase the overall surface area of the studied heat sink. Therefore, a lot of the heat energy can be transferred to the air through the flow.

The Nusselt number values of the 6mm pin fin diameter are high as compared to the 3mm pin fin diameter as it is shown in the figures above. This investigation is as a result of being the volume of the 6mm pin fin is more than the 3mm pin fin the order that tends to reduce the area of flow. Consequently, the velocity of flow has the tendency to be increased. The flat heat sink case study is observed to have the lower values of the Nusselt number in the Figures 8, 9 and 10, where the missing of the pin fins conduct a reduction in the overall surface area. Thus, a reduction in the heat transfer is conducted for the flat heat sink. The average values of the Nusselt number are observed to be increased by the increase of the pin fins length values as it is shown in the Figures 8, 9 and 10. This one is because of the increase in the overall surface area of the heat sink which are done by the increase of the pin fin lengths as it shown in the Figure 12 where it explains the increase in the surface area of the used heat sink with and without pin fins adoption. The previous observations ensure the strong association between the pin fin length values with the pin fin diameter values in order to specify the heat sink thermal performance.

![Figure 8. Nusselt number Vs. Reynolds number for the fin length of 5mm](image-url)
Figure 9. Nusslte number Vs. Reynolds number for the fin length of 10mm

Figure 10. Nusslte number Vs. Reynolds number for the fin length of 15mm

The Figures 13, 14 and 15 show the values of the pressure drop which are calculated between the inlet and the outlet of the supposed enclosure. As a result of being the conditions of the inlet and outlet of the enclosure are the same for all the case studies of the pin fins, Therefore, the achieved results of the pressure drop which are explained below can indicate the effects of the pin fins on the hydraulic losses. The predicted results explain the maximum values of the pressure drop for the 9mm pin fin case, while the intermediate values are observed for the 6mm pin fin. Furthermore, the lowest values of the pressure drop are achieved for the 3mm pin fin case. These outlines are due to the flow blocking in which is made by the pin fins arrangements on the heat sink, where there is a higher volume for the 9mm the order that from a higher blocking to the air flow.
Figure 11. The velocity vector over the heat sink for the fin length 15mm and fin Dia. of 3mm under different Reynolds number

Figure 12. The surface area Ratio
The fabricated volume by the 6mm pin fin is less than the previous pin fin and more than 3mm pin fin volume. Therefore, this one tends to make the intermediate blocking case. Consequently, the showed variations among the pin fin pressure drop in the Figures 13, 14 and 15 are as a result to the previous investigation. It is noticed that the flat heat sink case study has the minimum values for the pressure drop because of missing the flow blocking where there are no pin fins over. The augmentation of the pressure drop against the increase in the Reynolds number are done due to the hydraulic losses. The Figure 16 presents the pressure drop contours between the inlet and the outlet of the enclosure and at the middle section.

According to the Darcy Wiesbach relation where the observed behavior of pressure drops in the Figures 13, 14 and 15 should provide good confidence with the behavior of the friction factor under the same cases of the present study. Therefore, the Figures 17, 18 and 19 satisfy the denoted assumption before with the saving of the fin diameter and length effects.

![Figure 13. The pressure drop for Pin fin length of 5mm](image)

![Figure 14. The pressure drop for Pin fin length of 10mm](image)
Figure 15. The pressure drop for Pin fin length of 15mm

Figure 16. The contours pressure drop for Pin fin length of 15mm
Figure 15. The Friction Factor vs Reynolds number for Pin fin length of 5mm

Figure 16. The Friction Factor vs Reynolds number for Pin fin length of 10mm

The used optimization method in this study is not a mathematical procedure such as genetic algorithms but, it is a thermo-hydraulic method. It is known as Reynolds Performance Parameter [18] where the optimum cases of this method are specified when the values become more than unity. This method depends on the conducted results from the analysis such as Nusselt number and the friction factor for all the cases in the present study. The conducted results which are shown in Figures 20, 21 and 22 present the best operational region is at Reynolds number of 6500 for all the cases of the pin fins. This is provided in spite of being the Nusselt number values have a continues augmentation with respect to the used Reynolds number values as it is shown in Figures 8, 9 and 10. The previous observations mean that the friction play the active role in the changing of the optimum zone more than the Nusselt number effects. This is may be due to the increase in the velocity of flow where this tends to enhance the friction losses for the higher values as it is shown in Figures 13, 14 and 15. The Figure 23 shows that the Reynolds performance parameter values are highly at the Reynolds number region of 19500 for all cases of the pin fin diameter. Moreover, the Reynolds number region of 6500 is selected as the optimum operation zone. This is because of considering that the friction are the most prime mover factor to change the Reynolds performance parameter as it is discussed before.
Figure 19. The Friction Factor vs Reynolds number for Pin fin length of 15mm

Figure 20. The Reynolds Performance Parameter for Pin fin length of 5mm

Figure 21. The Reynolds Performance Parameter for Pin fin length of 10mm
Figure 22. The Reynolds Performance Parameter for Pin fin length of 15mm

7. Conclusions
This study includes a numerical investigation regarding the use of the in-line pin fins array over a heat sink and for different dimensions. It was aimed to get the best operation parameters such as Reynolds number and pin fin dimensions which presents the optimum conditions. This research provides the following conclusions:

1- The augmentation of the Nusslte number is done with the increase of the Reynolds number for all cases of the present study.

2- The maximum enhancement in the heat sink thermal performance is found with 66.6% when the used pin fin diameter is 9mm and fin length is 15mm. This one is specified according to the Nusslte number values.

3- The minimum pressure drop is found for the heat sink when the pin fin length is 5mm and the diameter is 3mm. This reduction is done with 65.7% from the maximum pressure drop.

4- The friction factor are observed to be reduced gradually with the increase of the Nusslte number for all the cases of the current study where the minimum friction factor is reduced with 63.63% from the maximum one.

5- It is found that the best operational conditions for the studied heat sink are conducted when the Reynolds number is 6500 for all cases. This finding is achieved by depending on the Reynolds performance parameter values with an enhancement of 58.33% as compared to the unity case.
8. Appendix

Re=2166.67, Flat Heat Sink
Re=6500, Flat Heat Sink
Re=10833.3, Flat Heat Sink
Re=15166.67, Flat Heat Sink
Re=19500, Flat Heat Sink
Re=2166.67, Fin length=3mm, Fin Dia=3mm
Re=6500, Fin length=3mm, Fin Dia=3mm
Re=10833.3, Fin length=3mm, Fin Dia=3mm
Re=15166.67, Fin length=3mm, Fin Dia=3mm
Re=19500, Fin length=3mm, Fin Dia=3mm
Re=2166.67, Fin length=3mm, Fin Dia=6mm
Re=6500, Fin length=3mm, Fin Dia=6mm
Figure A1: Some of the temperature distribution over the heat sink for different cases

9. Nomenclature
A: Surface area (m²)
CFD: Computational fluid dynamic
D: Diameter (m)
F: Friction factor
H: Heat transfer coefficient (W/m². °C)
K: Thermal conductivity (W/m. °C)
Q: Heat transfer rate (W)
RANS: Reynolds Average Naiver Stoke
Re: Reynolds number
S: Turbulent viscosity (m^2/s)
Nu: Nusselt number
R: Reynolds stress tensor
T: Temperature (°C)
C: Specific heat of fluid (J/kg.K)
P: Pressure of the fluid (Pa.)
U: Velocity of fluid in x-axis (m/s)
V: Velocity of fluid in y-axis (m/s)
W: Velocity of fluid in z-axis (m/s)

Greek Letters
∞: Ambient
Μ: Dynamic viscosity of fluid (Pa.s)
Ρ: Density of fluid (kg/m^3)
σ: Deformed stress by fluid flow (Pa.)

Subscript
i: x-axis
j: y-axis
k: z-axis
s: Surface of heat sink
t: Tip of the fin
h: Hydraulic

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