Regulation of dynamic stress-strain state and reliability of deformable systems with vibration dampers under harmonic loads

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Abstract. The problem of analysis linearly deformable systems with a finite number of single-mass dynamic vibration dampers (DVD), used as regulators of the stress-strain state (SSS) with different variants of the harmonic load (by location and type) was considered. Equations of state of the system with dampers have been obtained and a method has been proposed to choice a set of parameters (masses and stiffness) for each of the DVD, taking into account the properties of the system, loads, and parameters of other DVD and the specified requirements of regulation. The developed design apparatus has been used for solving a model problem of a dynamic SSS regulation in a rod system with several dampers and for estimation of its reliability with polycriterial serviceability requirements. The quantitative results of calculations are presented. The comparative analysis of the reliability have been performed for unprotected systems and systems with DVD. The possibility of determining the rational location and parameters of DVD under harmonic non-point loading with the target reliability of the structure is shown.

1. Introduction
One of the most dangerous types of impacts on construction systems (structures, building) is dynamic loading, which can cause unacceptable vibrations. An effective way to oppose vibrations is the use of dynamic vibration dampers [1, 2]. In the case when the dynamic load is applied at one point, then when a DVD is installed at the loading point and in load’s direction, complete damping of the vibration in the system is possible (ideal DVD). If there are several sources of vibration, then the setting of a one single-mass DVD may not be enough for significant reduction of the displacements level in the system. It becomes possible to improve the dynamic SSS of system noticeably with the help of installation of several dampers. The vibration damper, in its essence, is a very powerful mean of regulating the dynamic SSS of a system. As it was shown by previous studies [3], a system protected by a damper is sensitive even to small random deviations of parameters from their design values. This raises the problem of calculating the characteristics of each damper, taking into account the properties of the system, the load and other DVD. It becomes possible to maintain successful work of the system in a frequencies zone, where large gradients of amplitude-frequency characteristic (AFC) of the unprotected structure take place. By reason of the stochastic nature of all design parameters, an estimation of the reliability of systems protected by a DVD complex becomes necessary.
The purposes of the work are: developing a method for analysis regulation of the dynamic state of linearly deformable systems protected by a finite number of vibration dampers (regulators), taking into account the specific SSS parameters requirements on the system; calculation of the modelling systems reliability with a complex of DVDs; verification of the proposed method.

2. The main part

The solution to the problem of the system SSS regulation

A harmonically loaded arbitrary system with the group of \( n_d \) single-mass DVD is considered (figure 1). The number of degrees of freedom for the unprotected system is \( n \), for dampers – \( n_d \).

\[ \begin{bmatrix} \delta^* \end{bmatrix} \cdot [J] + [\Delta_F] = 0, \]

where \( \begin{bmatrix} \delta^* \end{bmatrix} \) – system’s dynamic pliability matrix of size \((n + n_d) \times (n + n_d)\);

\( [J] \) – inertia forces vector; \( [\Delta_F] \) – load caused amplitude displacements vector.

The first \( n \) equations (1) characterize the amplitude state of the system itself (the first group of equations), and the subsequent \( n_d \) equations relate to the dampers (the second group of equations).

Let present the matrix of dynamic pliability of the system in the following form:

\[ \begin{bmatrix} \delta^* \end{bmatrix} = \begin{bmatrix} \delta_{11} & \ldots & \delta_{1n} & \delta_{1,n+1} & \ldots & \delta_{1,n+n_d} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \ldots & \delta_{nn} & \delta_{n,n+1} & \ldots & \delta_{n,n+n_d} \\ \delta_{n+1,1} & \ldots & \delta_{n+1,n} & \delta_{n+1,n+1} & \ldots & \delta_{n+1,n+n_d} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_{n+n_d,1} & \ldots & \delta_{n+n_d,n} & \delta_{n+n_d,n+1} & \ldots & \delta_{n+n_d,n+n_d} \end{bmatrix} \]

or
\[
\begin{bmatrix}
\delta^* \\
\delta'^* \\
\vdots \\
\delta'^{n}\n\end{bmatrix} =
\begin{bmatrix}
\delta_{1,i} \\
\delta'_{1,i} \\
\vdots \\
\delta'_{n,i}\n\end{bmatrix} =
\begin{bmatrix}
\delta_{1,j} \\
\delta'_{1,j} \\
\vdots \\
\delta'_{n,j}\n\end{bmatrix},
\]  
\tag{2}

where \(\delta^*_i = \delta_0 \left( m_i \omega_p^2 \right)^{-1} \) \((i = 1, \ldots, n)\); \(\delta^*_{svi,si} = \delta_{svi,si} - \left( m_{d,svi} \omega_p^2 \right)^{-1} \) \((i = 1, \ldots, n_j)\),

\(\delta_{svi,si} = \delta_{svi,si} + \left( c_{d,svi} \right)^{-1} \) \((k = n+n_d+1, \ldots, n+2n_d)\), \(k\) – number of \(i^{\text{th}}\) damper’s attachment point; \(m_i\) – the mass of the unprotected system corresponding to the \(i^{\text{th}}\) degree of freedom; \(m_{d,svi}\) – damper mass; \(\omega_p\) – forcing frequency. The indexation of parameters is adopted unified in all equations for notational convenience: \(i = 1, \ldots, n\) – for main points of system; \(i = n+1, \ldots, n+n_d\) – for damper masses; \(k = n+n_d+1, \ldots, n+2n_d\) – for system points at damper installation places.

In this case the diagonal elements of the block \([\delta'^*_{j,j}]\) contains unknown parameters of damper – mass \((m_j)\) and stiffness \((c_j)\), which must be selected in such a way to fulfill the requirements for improving the indicators of the system’s dynamic SSS. Consequently, the problem of selecting the parameters of the DVD complex is considered as the problem of the system’s SSS regulation.

We’ve used displacement restrictions as the requirements of regulation:

\[
\begin{bmatrix}
\Delta^r_{n+n_d+1} \\
\Delta^r_{n+n_d+2} \\
\vdots \\
\Delta^r_{n+2n_d}\n\end{bmatrix} = \begin{bmatrix}
\Delta^r_{n+n_d+1} \\
\Delta^r_{n+n_d+2} \\
\vdots \\
\Delta^r_{n+2n_d}\n\end{bmatrix},
\]  
\tag{3}

where \(\Delta^r_k\) \((k = n+n_d+1, \ldots, n+2n_d)\) – displacements of system points in places of fastening of DVD; \(\{\Delta^r_i\}\) \((k = n+n_d+1, \ldots, n+2n_d)\) – target displacements of points where DVD are placed.

Thus, the regulation equations are written as

\[
\begin{bmatrix}
\Delta^r \\
\delta^* \\
\vdots \\
\delta'^{n}\n\end{bmatrix} = \begin{bmatrix}
\delta_{1} \\
\delta'^{1} \\
\vdots \\
\delta'^{n}\n\end{bmatrix} \begin{bmatrix}
J^1 \\
J^{n+1} \\
\vdots \\
J^{n+2n_d}\n\end{bmatrix} = \begin{bmatrix}
\Delta^r_{n+n_d+1} \\
\Delta^r_{n+n_d+2} \\
\vdots \\
\Delta^r_{n+2n_d}\n\end{bmatrix},
\]  
\tag{4}

The number of regulation equations corresponds to the number of DVDs. In this case only inertia forces are unknown in equations (4) while the remaining parameters may be calculated or specified previously. Since the inertia forces \([J]\) in (1) and (4) are the same, as well as the numbers of equations of the second group in (1) and of the regulation equations (4) are the same, it is possible to change the mentioned groups of equations in places, as a result we have:
The resulting new system (5) does not include the unknown parameters of dampers. It is important in this case that the inertia forces obtained from (5) correspond to the regulation requirements.

Let’s consider the second group of equations, replaced in (1) by the regulation equations (4):

$$
\left[ \frac{\delta^*_j}{\delta^*} \right] \cdot \left[ J^n \right] + \left[ \frac{\Delta^n}{\Delta'_{p}} \right] = 0.
$$

(5)

Each equation (6) contains unknown damper parameters (stiffness and mass), so the solution of the equation leads to the relationship expression between them taking into account the target displacements in the places of dampers fastenings:

$$
G_{d,n+i} = \frac{1}{c_{d,n+i}} - \frac{1}{m_{d,n+i} \omega_{p}^2} \frac{\sum_{j=n+i}^{n+i+1} \delta_{n+i,j} J - \sum_{j=n+i}^{n+i+1} \delta_{n+i,j} J - \Delta_{n+i,F}}{J_{n+i+1}} - \delta_{n+i},
$$

(7)

where $i = 1, ..., n_d$; $k = n + n_d + 1, ..., n + 2n_d$, $k$ – number of $i$th damper’s attachment point.

If one of the damper parameters is preset, the value of the other one may be determined from (7). Thus, the proposed algorithm makes it easy to choose the characteristics for the entire complex of dampers in accordance with the certain initial requirements.

Example

The two degree-of-freedom beam under the uniformly distributed harmonic load (figure 2, (a)) was considered with specified data: $a = 1$ m, $EI = 22500$ kN·m², $m_1 = 1000$ kg, $m_2 = 2000$ kg, $q = 20$ kN/m, $\omega_p = 90$ s⁻¹.

The results of realized calculations are presented in Table 1 where the displacements of masses and points $A$, $B$ also their dynamic magnification factors (DMF) are given. The bending moment diagrams are shown in figure 2, (b), (c).

| Table 1. Displacements and DMFs. |
|----------------------------------|
| Point | $y_{1i}$ (m) | $y_{2i}$ (m) | DMF |
|-------|--------------|--------------|-----|
| 1     | 0.001833     | 0.006327     | 3.45|
| 2     | 0.001722     | 0.006633     | 3.85|
| A     | 0.002519     | 0.008984     | 3.57|
| B     | -0.00167     | -0.007269    | 4.36|
Note that displacements and forces during dynamic loading increase by more than 3 times in comparison with the corresponding static ones. To improve the system’s SSS two DVD are used. They install at the places of maximum dynamic displacements (points A, B) – see figure 3, (a).

![Figure 2](image1.png)

**Figure 2.** Design model (a); bending moments caused by the conventionally static amplitude harmonic load (b); dynamic bending moments (c).

![Figure 3](image2.png)

**Figure 3.** Design model with DVDs (a); dynamic bending moment diagram (b).
The selection of the damper parameters was made according to the above presented algorithm with the requirement that the displacements are equal zero in the points where the dampers are placed (points A and B): \( \Delta \xi = 0; \Delta \zeta = 0 \). From (7) \( G_{d3} = G_{d4} = 0 \) were obtained in this case. The following parameters of dampers were used in the calculation: 
\[
\begin{align*}
3 & \quad m_{d3} = 250 \text{ kg} \quad c_{d3} = 2025 \text{ kN} \cdot \text{m} \\
4 & \quad m_{d4} = 250 \text{ kg} \quad c_{d4} = 2025 \text{ kN} / \text{m}
\end{align*}
\]

The distribution of the dynamic bending moments is presented in figure 3, (b). Note that their maximum absolute values have decreased significantly – almost 13 times.

Varying the parameters of regulation it is possible to choose a combination in which the distribution of forces will be even better. It will positively affect the reliability and durability of the system under multi-cycle loading by reducing the amplitude of the dynamic component of the total stresses [5].

Calculation of reliability
The reliability calculation was performed according to the method described in [6], using the concepts of resistance \( R \), load effect \( Q \) and generalized serviceability reserve \( S = R - Q \).

The following requirements for non-failure performance of system were used:
- for the bending dynamic moments in the pre-specified design sections of beam \( M_d < [M] \) \( (\bar{\varrho}_m = [M] = 140 \text{ kN} \cdot \text{m} \); \( \bar{\varrho}_s = \bar{M}_s(x_j) \));
- for the dynamic displacements of characteristic points \( y_d < [y] \) \( (\bar{\varrho}_y = [y] = 0.01 \text{ m} \); \( \bar{\varrho}_y = \bar{y}_y(x_j) \)).

Hereinafter, the symbol « \( \bar{\cdot} \) » denotes the mean, « \( \hat{\cdot} \) » – the standard deviation; \( A_x \) – coefficient of variation of random variable \( X \).

The results of calculations of reliability and failure probability for the unprotected system (figure 2 (a)) for the first criterion are presented in Table 2, and for the second – in Table 3. The calculations were performed with the following probabilistic characteristics: 
\[
\begin{align*}
\bar{\varrho}_m &= 1000 \text{ kg} \quad A_{\varrho_m} = 0.03 \quad \bar{m}_2 = 2000 \text{ kg} \quad A_{\varrho_m} = 0.02 \quad \bar{E}I &= 22500 \text{ kN} \cdot \text{m}^2 \quad A_{\bar{E}I} = 0.04 \quad \bar{q} &= 20 \text{ kN/m} \quad A_{\bar{q}} = 0.06 \quad \bar{\omega}_y &= 90 \text{ s}^{-1} \quad A_{\bar{\omega}_y} = 0.01
\end{align*}
\]

All input random parameters are normally distributed.

The probabilistic characteristics of the output parameters were determined by the statistical linearization method (SLM) [7].

Table 2. Results of calculations for the unprotected system (figure 2) (by bending dynamic moments in beam).

| \( x_j \) (m) | \( \bar{M}_d \) (kN \cdot m) | \( \bar{S} \) (kN \cdot m) | \( \hat{S} \) (kN \cdot m) | \( \beta_s \) | \( P_s \) | \( P_f \) |
|----------------|-----------------|-----------------|-----------------|--------------|---------|---------|
| 0.5            | 48.900          | 91.0996         | 4.3316          | 21.0314      | ~1      | ~0      |
| 1              | 92.801          | 47.1991         | 8.4629          | 5.5772       | 0.999999 | 1.22E-08 |
| 1.5            | 106.075         | 33.9247         | 9.5051          | 3.5691       | 0.999821 | 0.0001791 |
| 2              | 114.350         | 25.6502         | 10.3629         | 2.4752       | 0.993342 | 0.0066582 |
| 2.5            | 117.624         | 22.3758         | 11.0387         | 2.0270       | 0.978670 | 0.0213298 |
| 3              | 115.899         | 24.1014         | 11.5492         | 2.0868       | 0.981548 | 0.0184515 |
| 3.5            | 55.449          | 84.5507         | 5.6856          | 14.8711      | ~1      | ~0      |
| 4              | -10.000         | 130.0000        | 0.6000          | 216.6666     | ~1      | ~0      |
| 4.5            | -2.500          | 137.5000        | 0.1500          | 916.6666     | ~1      | ~0      |

The total probability of failure by the criterion for limiting bending moments – \( P_f = 0.0459546 \).
Table 3. Results of calculations for the unprotected system (figure 2) (by dynamic displacements of beam’s points).

| Point | S (m) | Ŝ (m) | S (m) | β_s | P_s | P_f |
|-------|-------|-------|-------|------|-----|-----|
| 1     | 0.00633 | 0.003673 | 0.000584 | 6.289054 | ~1 | ~0 |
| 2     | 0.00663 | 0.003367 | 0.000628 | 5.359272 | ~1 | ~0 |
| A     | 0.00898 | 0.001016 | 0.000748 | 1.358642 | 0.912870 | 0.087130 |
| B     | -0.00727 | 0.002731 | 0.000649 | 4.209778 | 0.999987 | 0.000013 |

The total probability of failure by the criterion for target displacements – \( P_f \approx 0.1290917 \).

According to the data of tables 2 and 3, the total failure probability under the multi-criteria serviceability requirements is \( P_f \approx 0.16911 \), that is unacceptable according to [8].

The results of calculations reliability and probability of failure for the system with DVDs (figure 3, (a)) for the first criterion are shown in table 4, and for the second – in table 5. The calculations were made taking into account the following probabilistic characteristics of the original system (see above) and dampers: \( \bar{m}_{d,3} = 250 \text{ kg}; \ A_{m_{u,3}} = 0.001; \ \bar{c}_{d,3} = 2025 \text{ kN/m}; \ A_{c_{u,3}} = 0.001; \ \bar{m}_{d,4} = 250 \text{ kg}; \ A_{m_{u,4}} = 0.001; \ \bar{c}_{d,4} = 2025 \text{ kN/m}; \ A_{c_{u,4}} = 0.001 \).

Table 4. Results of calculations for the system with DVDs (figure 3) (by bending dynamic moments in beam).

| \( x_i \) (m) | \( \bar{M}_y \) (kN·m) | \( S \) (kN·m) | \( \dot{S} \) (kN·m) | \( \beta_s \) | \( P_s \) | \( P_f \) |
|--------------|-----------------|--------------|---------------|------|-----|-----|
| 0.5          | 5.408           | 134.592      | 3.064         | 43.922 | ~1  | ~0  |
| 1            | 5.815           | 134.185      | 6.104         | 21.982 | ~1  | ~0  |
| 1.5          | 0.861           | 139.139      | 7.678         | 18.121 | ~1  | ~0  |
| 2            | -9.094          | 130.906      | 9.278         | 14.109 | ~1  | ~0  |
| 2.5          | -9.325          | 139.675      | 5.318         | 26.265 | ~1  | ~0  |
| 3            | 3.445           | 136.555      | 1.390         | 98.211 | ~1  | ~0  |
| 3.5          | 1.913           | 138.087      | 4.607         | 29.976 | ~1  | ~0  |
| 4            | -4.620          | 135.380      | 10.587        | 12.788 | ~1  | ~0  |
| 4.5          | 0.190           | 139.810      | 5.292         | 26.421 | ~1  | ~0  |

The total probability of failure by the criterion for limiting bending moments – \( P_f \approx 0 \).

Table 5. Results of calculations for the system with DVDs (figure 3) (by dynamic displacements of beam’s points).

| \( y_i \) (m) | \( S \) (m) | \( \dot{S} \) (m) | \( \beta_s \) | \( P_s \) | \( P_f \) |
|--------------|------------|-----------------|------|-----|-----|
| 1            | 0.000090   | 0.00991         | 0.0001813 | 54.665 | ~1  | ~0  |
| 2            | 0.000037   | 0.00996         | 0.0001344 | 74.144 | ~1  | ~0  |
| A            | 0.000000   | 0.01000         | 0.0001429 | 69.983 | ~1  | ~0  |
| B            | 0.000000   | 0.01000         | 0.0000968 | 103.288 | ~1  | ~0  |

The total probability of failure by the criterion for target displacements – \( P_f \approx 0 \).

According to the data of tables 4 and 5, the total failure probability under the multi-criteria serviceability requirements is \( P_f \approx 0 \), which corresponds to a very high level of reliability.
The comparison of the reliability indices of the unprotected system and the system with DVDs makes it clear that the setting of vibration dampers causes a significant increasing of the reliability in accordance with the design requirements [8, 9]. In addition to the performance criteria considered in the problem solved above, the determination of reliability and durability by the limiting of the material fatigue strength is vital; for this case the technique described in [10] can be used.

3. Conclusion
1. The solution of a problem on determining the characteristics of the DVD complex as a regulators of the dynamic SSS of a deformable multi-degree-of-freedom system under vibration loads is given. The design expressions for calculations the needed masses and stiffnesses of dampers in accordance with the target limitations of structure points’ displacements are obtained in the convenient form for practical use.
2. The proposed methodology and algorithm allow calculating and optimizing the DVD parameters using the specified (required) values of the SSS parameters.
3. The use of a DVD complex significantly improves the dynamic SSS of a multi-degree-of-freedom system and, as a result, leads to the increasing reliability according to the design criteria and requirements of serviceability.

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