Cutting forces and vibrations during ball end milling of inclined surfaces

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Abstract

This work concentrates on the analysis of cutting forces and vibrations generated during ball end milling process with variable surface inclination angle (α). The cutting force and tool’s displacements (vibrations) model including surface inclination was formulated. Experiments were carried out on hardened alloy steel X155CrVMo12-1 with sintered carbide (TiAlN coating) monolithic tool. Instantaneous values of cutting forces were measured in the range of variable feed per tooth (fz) and surface inclination angle (α) values. The shear (Ki) and edge (Ke) coefficients in function of analyzed factors were determined using the measured cutting force signals as an input data. The research revealed that cutting forces and vibrations are strongly affected by the surface inclination, both in quantitative and qualitative aspect. This observation is also confirmed by the developed model.

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1. Introduction

Ball end milling process is a very popular technology in the production of drop forging dies and casting molds made from hardened steel [1,2]. This kind of process includes also the machining of curvilinear surfaces in high speed machining (HSM) conditions. During ball end milling of curvilinear surfaces the inclination angle of the machined surface is variable, which affects the active length of cutting edge and working angle values. Consequently, this phenomenon influences cutting forces [3,4] and vibrations generated during machining process. From the literature survey it is also resulting, that surface inclination angle affects surface roughness [5,6] and the tool wear [7] during ball end milling. Therefore, the reliable prediction of milling forces including surface inclination is significant for the simulation of the machinability, cutter wear, vibrations, as well as the surface quality.

There has been much research in the past on many force components models during ball end milling process. The work can be classified into two ground groups: analytical and mechanistic. Analytical methods are usually including thermomechanical phenomena occurring in cutting process and modeling physical mechanisms during work piece decohesion process, e.g. slip stress and strain via intense plastic deformation [8]. Mechanistic models assume that cutting forces are proportional to the cross-sectional area of cut and so called specific cutting force coefficients [9, 10]. These mechanistic models can be classified into two groups. In the first one, the effects of shearing mechanism due to the chip generating process on the tool’s rake face and effects of ploughing mechanisms on the flank face are expressed as the one specific force coefficient for each cutting force component [11]. In the second model, the shearing and ploughing effects are characterized separately by the respective specific shear and edge force coefficients [12, 13].

In this study, the influence of the machined surface inclination angle on the cutting forces and vibrations is investigated, both in the quantitative and qualitative aspect. The cutting force and vibration model including kinematic – geometric parameters is also formulated. The developed model is validated empirically during ball end milling of hardened steel with variable surface inclination.
2. Cutting forces and vibrations model

In order to determine cutter’s instantaneous displacements related to cutter’s deflections, induced by cutting forces $F_i$, one should solve the following differential motion equations:

\[ m_i \ddot{x}(t) + c_i \dot{x}(t) + k_i x(t) = F_i(t) \]  
\[ m_i \ddot{y}(t) + c_i \dot{y}(t) + k_i y(t) = F_i(t) \sin \alpha + F_i(t) \cos \alpha \]  
\[ m_i \ddot{z}(t) + c_i \dot{z}(t) + k_i z(t) = F_i(t) \cos \alpha - F_i(t) \sin \alpha \]

During ball end milling process of inclined surfaces, cutter’s displacements (vibrations) are determined in the directions: perpendicular to the tool’s rotational axis and collinear to the feed motion vector ($\gamma(t)$), perpendicular to the tool’s rotational axis and feed motion vector ($\chi(t)$), parallel to tool’s rotational axis ($\phi(t)$).

In the equation (1) $m_i$, $c_i$, $k_i$ denotes modal parameters ($m_i$ – modal mass, $c_i$ – damping coefficient, $k_i$ – stiffness coefficient), which can be determined using impact test, while $F_i$, $F_y$, $F_z$ instantaneous cutting forces in the machine tool’s coordinates. In order to determine these cutting forces, mechanicistic cutting force model, developed by Lee and Altintas [12] is applied. In this model, a set of curvilinear coordinate system normal to the ball envelope is used to specify the resultant force acting on the $i$-th infinitesimal segment of the cutting edge. Figure 1 depicts cutting forces acting on the cutter and tool’s coordinates for a ball end mill.

![Geometry and tool coordinates for a ball milling cutter.](image)

The elemental tangential $dF_{\theta j}$, radial $dF_{r j}$, and axial $dF_{d j}$ cutting forces acting on the $j$-th tooth, are expressed by:

\[ dF_{\theta j} = K_{\theta} d\psi_j + K_{r} dA_j \]  
\[ dF_{r j} = K_{r} d\psi_j + K_{d} dA_j \]  
\[ dF_{d j} = K_{d} d\psi_j + K_{d} dA_j \]

where: $K_{\theta}$, $K_{r}$, $K_{d}$ are the edge specific coefficients [N/mm], $K_{r}$, $K_{d}$, $K_{d}$ are the shear specific coefficients [N/mm²], $d\psi_j$ is the infinitesimal length of cutting edge [mm], $dA_j$ is the cross sectional area of cut [mm²].

In order to calculate cutting forces acting on $i$-th infinitesimal segment of the cutting edge, it is necessary to determine cross sectional area of cut and active length of cutting edge, as well as calibrate specific coefficients.

On the basis of Figure 1 the instantaneous cutting forces in machine tool’s coordinates can be expressed:

\[ F_i = \sum_{j=1}^{z} F_{\theta j} \cdot \sin \phi_j \cdot \cos \phi_j \cdot \cos \phi_{j'} \cdot \cos \phi_{j''} \cdot \cos \phi_{j'\prime} \]  
\[ F_i = \sum_{j=1}^{z} -F_{\theta j} \cdot \sin \phi_j \cdot F_{r j} \cdot \sin \phi_j \cdot F_{d j} \cdot \cos \phi_j \]  
\[ F_i = \sum_{j=1}^{z} F_{r j} \cdot \cos \phi_j \cdot F_{d j} \cdot \sin \phi_j \]

where: $z$ is the active number of teeth.

Positioning angles $\phi_j$ and $\phi_i$ of the $j$-th cutting edge found in equations (3) are expressed by:

\[ \phi_j = \frac{\psi_{ji} + \psi_{fj}}{2} \]  
\[ \phi_j = \frac{\pi \cdot n \cdot t}{30} - \frac{\psi_{ji} + \psi_{fj}}{2} - (j-1) \left( \frac{2\pi}{z} \right) - 2(n(N-1)) \]

where: $\psi_{ji}$, $\psi_{fj}$ are the initial and final positioning angles in the reference plane [rad], $\psi_{ji}$, $\psi_{fj}$ are the initial and final lag angles [rad], $j$ is the ordinal number of tooth, $N$ is the number of tool’s rotation, $n$ is the spindle rotational speed [rev/min], $t$ is the time [s].

Instantaneous cross sectional area of cut can be calculated on the basis of equation:

\[ A_j = R \cdot f_i \cdot \left( 1 - \cos \left( \phi_{dj} - \alpha \right) \right) \cdot \sin \phi_j \]  

where: $R$ is the tool’s radius [mm], $f_i$ is the feed per tooth [mm/tooth], $\alpha$ is the surface inclination angle.

Infinitesimal length of cutting edge can be formulated from the expression proposed by [12] as:

\[ dl = \sqrt{\left( \frac{dr(\psi_j)}{d\psi_j} \right)^2 + r(\psi_j)^2 + \frac{R^2}{\tan^2 \alpha} \cdot d\psi_j} \]

The $r(\psi_j)$ expression in equation (6) can be formulated from:
In this study, two different cases of ball end milling process were investigated, namely upward ramping with \( \alpha > 0 \) (Figure 2a) and slot milling with \( \alpha = 0 \) (Figure 2b).

![Fig. 2. Selected cutting modes of ball end milling: a) upward ramping, b) slot milling.](image)

In this step the border conditions \( \psi_{1a}, \psi_{1b}, \phi_{1r}, \phi_{1s} \) for the investigated cutting modes will be discussed. During slot milling, three phases of tool immersion into the work piece in function of tool rotation angle \( \Omega \) are distinguished (Figure 3).

![Fig. 3. Border conditions for the slot milling process.](image)

Phase 1:

\[
\Omega = \pi \frac{n-1}{30} \leq \Omega_1 = \frac{a_p \tan \lambda}{R} + 2\pi(N-1) \\
\begin{align*}
\psi_{1i} &= 0; \\
\psi_{1s} &= \frac{a_p \tan \lambda}{R} + 2\pi(N-1) \\
\phi_{1r} &= \arccos \left( \frac{R-a_p}{R} \right); \\
\phi_{1s} &= \arccos \left( \frac{R-a_p}{R} \right) \\
a_p(\Omega) &= \frac{R}{\tan \lambda} \left( \frac{2(1-j-1)\pi}{z} - 2\pi(N-1) \right)
\end{align*}
\]  

Phase 2:

\[
\Omega_2 \leq \Omega < \Omega_3 = \left[ \frac{(j-1)\pi}{z} - 2\pi(N-1) \right] \\
\begin{align*}
\psi_{1i} &= 0; \\
\psi_{1s} &= \frac{a_p \tan \lambda}{R} + 2\pi(N-1) \\
\phi_{1r} &= \arccos \left( \frac{R-a_p}{R} \right); \\
\phi_{1s} &= \arccos \left( \frac{R-a_p}{R} \right) \\
a_p(\Omega) &= \frac{R}{\tan \lambda} \left( \frac{2(1-j-1)\pi}{z} - 2\pi(N-1) \right)
\end{align*}
\]

In equations (8d) and (10d) \( a_p(\Omega) \) denotes instantaneous depth of cut which depends on tool rotation angle.

Figure 4 depicts phases of tool immersion into the work piece for the upward ramping.

![Fig. 4. Border conditions for the upward ramping process.](image)
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The piezoelectric force dynamometer was used to measure total cutting forces components. Its natural frequency is equal to 1672 Hz. In order to avoid disturbances induced by proximity of forcing frequency to gauge natural frequency, the band – elimination filter was applied. Cutting force components were measured (in machine tool’s coordinates – Figure 5), in following directions: direction X – feed normal force $F_x$ [N], direction Y – feed force $F_y$ [N], direction Z – thrust force $F_z$ [N].

Fig. 5. Cutting force components in machine tool coordinates.

3.3. The calibration of specific cutting force coefficients

In the investigations carried out it was assumed, that maximum instantaneous forces in X and Z direction ($F_x, F_z$) and minimum instantaneous forces in Y direction ($F_y$) per tool revolution are corresponding to maximum instantaneous values of cross sectional area of cut and active length of cut. In order to calibrate specific cutting force coefficients, maximum and minimum instantaneous forces per tool revolution in each direction ($F_x, F_y, F_z$) were acquired. These measured forces were substituted into equations for specific cutting force coefficients:

$$K_x = \frac{2(F_{x\text{cal}} - F_{x})}{A_{\text{max}}} , \quad K_y = \frac{2(F_{y\text{cal}} - F_{y})}{A_{\text{max}}} , \quad K_z = \frac{2(F_{z\text{cal}} - F_{z})}{A_{\text{max}}} \tag{14a}$$

where: $F_{x\text{cal}}, F_{y\text{cal}}, F_{z\text{cal}}$ are the cutting forces in tool’s coordinate system applied in calibration [N], $F_x, F_y, F_z$ are the ploughing forces in tool’s coordinate system applied in calibration [N], $A_{\text{max}}$ is the maximum value of cross sectional area of cut per 1 tooth [mm$^2$], $l_{\text{max}}$ is the maximum length of cut per 1 tool [mm].

Cutting forces in tool’s coordinate system, applied in calibration can be calculated on the basis of equations:

$$F_{x\text{cal}} = \frac{1}{2}(F_x \sin \varphi_{x\text{cal}} - F_z \cos \varphi_{x\text{cal}}) \tag{15a}$$

$$F_{y\text{cal}} = \frac{1}{2}(-F_x \cos \varphi_{x\text{cal}} \cos \varphi_{y\text{cal}} - F_z \sin \varphi_{x\text{cal}} \sin \varphi_{y\text{cal}}) \tag{15b}$$

$$F_{z\text{cal}} = \frac{1}{2}(-F_x \cos \varphi_{x\text{cal}} \sin \varphi_{y\text{cal}} - F_z \sin \varphi_{x\text{cal}} \cos \varphi_{y\text{cal}} + F_y \cos \varphi_{y\text{cal}}) \tag{15c}$$

3. Experimental details

3.1. Work and tool materials

Investigations have been carried out on hardened alloy steel X155CrVMo12-1 plate with hardness approx. 56 HRC and dimensions: 125 x 230 x 160 mm. Monolithic ball end mill made of sintered tungsten carbide (WC) with diameter $d=16$ mm and number of teeth $z=2$ was selected as milling cutter. Milling tool made of fine-grained tungsten carbide had anti-wear TiAlN coating and the following geometry:

$\gamma_x = -15^\circ, \lambda_x = 30^\circ, r_{xs} = 5 \mu m, a_{r} = 6^\circ$.

In order to solve differential motion equation (1), modal parameters ($m, c, k$) were determined using impact test, and thus the following parameters were received:

$m = 0.079 N s^2/m$, $c = 40.8 Ns/m$, $k = 19492469 N/m$.

3.2. Research range and method

The measured quantities in the carried out research were cutting forces ($F_x, F_y, F_z$), fixed in machine tool’s coordinate system. Cutting parameters applied in the research are presented in the Table 1. Experiments were conducted on 5-axes CNC milling workstation (DMU 60monoBLOCK), in upward ramping and slot milling conditions. In all investigated cases tool’s effective diameter was lower than the value of pick feed $-D_s < b$.

| $\alpha$ [°] | $f_z$ [mm] | $a_n$ [mm] | $v_c$ [m/min] | $n$ [rev/min] |
|--------------|------------|-------------|---------------|--------------|
| 0-60         | 0.02-0.1   | 0.2         | 100           | 2297-8953    |
| interval     | 0.15       | interval    | 0.02          |              |

Table 1. Cutting parameters applied in the research

Phase 2:

$$\Omega_2 < \Omega < \Omega_3 = \frac{\pi}{2} + \frac{2(j-1)\pi}{z} (1 - \cos \alpha) + \arccos \left(1 - \frac{a_{\gamma x}}{R \sin^2 \alpha}\right)$$

$$+ (1 - \cos \alpha) \tan \lambda_x + 2\pi(N - 1)$$

$$\psi_{\Omega 2} = (1 - \cos \alpha) \tan \lambda_x ; \quad \psi_{\Omega 3} = (1 - \cos \varphi_{\Omega}) \tan \lambda_x$$

$$\phi_{\Omega 2} = \alpha ; \quad \phi_{\Omega 3} = \alpha + \arccos \left(\frac{R - a_{\gamma x}(\Omega)}{R}\right)$$

$$- R \sin^2 \alpha + a_{\gamma x}$$

Phase 3:

$$\Omega_3 < \Omega < \Omega_4 = \frac{3\pi}{2} + \frac{2(j-1)\pi}{z} (1 - \cos \alpha) + \arccos \left(1 - \frac{a_{\gamma x}}{R \sin^2 \alpha}\right)$$

$$+ (1 - \cos \alpha) \tan \lambda_x + 2\pi(N - 1)$$

$$\psi_{\Omega 4} = 0 ; \quad \psi_{\Omega 3} = 0 ; \quad \varphi_{\Omega 3} = 0 ; \quad \varphi_{\Omega 2} = 0$$

From the above deliberations and Figure 4 it is resulting that in case of upward ramping, tool cuts only when the phase 2 of tool immersion into the work piece occurs. It means that in milling with surface inclination angle, the active number of teeth can be less than one, and thus pulsating forces can occur.
Positioning angles: \( \phi_{\text{cal}}, \phi_{\text{rcal}} \) applied for calibration, found in equations (15) can be calculated from the equations:

\[
\phi_{\text{cal}} = \phi_{\text{min}} - \frac{\phi_{\text{max}} - \phi_{\text{min}}}{4}, \quad \phi_{\text{rcal}} = \phi_{\text{min}} - \frac{\phi_{\text{max}} - \phi_{\text{min}}}{2} \tag{16}
\]

where: \( \phi_{\text{min}}, \phi_{\text{max}} \) are namely, minimal and maximal positioning angles per 1 tooth [rad], \( \phi_{\text{rmin}}, \phi_{\text{rmax}} \) are namely, minimal and maximal positioning angles in the reference plane per tooth [rad],

Estimated values of specific cutting force coefficients are expressed in function of cutting parameters (Table 2).

Table 2. Equations of specific cutting force coefficients

| Form of equation | Edge coefficients: | Shear coefficients: |
|------------------|--------------------|--------------------|
| \( k_e \)        | \( k_e = 0.00005a^2 -0.0073 a^3 + 0.33 a^4 -5.6 a^{49} \); | \( k_{ac} = -2128.1 + 74.7 a + 49541 f z -0.872 a^2 -408 a f z -2.99E5 f z^2; \) |
| \( k_c \)        | \( k_c = 26e-0.026 a; \)          | \( k_{rc} = 1782.1 - 53.9 a + 11713.5 f z +0.7 a^2 -16.1 a f z -85602.6 f z^2; \) |
| \( k_r \)        | \( k_r = 0.00003 a^2 +0.0039 a^3 + 0.17 a^4 +2.58 a^{10.6}. \) | \( k_{tc} = 8780.6 - 368.4 a + 37397.9 f z +4.6 a^2 +39.2 a f z -2.8E5 f z^2. \) |

3.4. Results and discussion

Figures: 6–8 depict the comparison of measured and estimated cutting forces \( (F_x, F_y, F_z) \) in function of time for the different surface inclination angle \( \alpha \) values. From the figures 6–8 it is resulting that surface inclination angle \( \alpha \) significantly affects cutting forces, both in quantitative and qualitative aspect. The growth of \( \alpha \) angle decreases the limitary area of tool immersion into the work piece (defined by the working angle \( \psi \) and active number of teeth \( z_c \)) - see Figure 3 and Figure 4. Consequently, pulsating forces can be generated during finishing ball end milling, because for the surface inclination angle \( \alpha=0 \) number of active teeth is very often less than unity \( (z_c<1) \). It can be also seen, that surface inclination angle growth induces the decline in cutting force amplitudes (both estimated and measured ones). Furthermore, the \( F_x \) force is the most sensitive for the surface inclination angle variations. The highest cutting force amplitudes in three investigated directions can be observed for the slot milling \( (\alpha=0) \) process. This dependency confirms the fact that in case of plain milling, cutting speed near cutter’s free end is close to zero, and thus the appearance of ploughing mechanism induces large elastic and plastic deformations of work material. From the Figures 6–8 it can be also seen that cutting forces estimated on the basis of the developed model stay in a good agreement with measured ones. Nevertheless some discrepancies between these courses are found. The highest differences between the calculated and the measured forces are observed for the slot milling \( (\alpha=0) \). The maximal error in this case is equal to about 35% (for the X direction). In the upward ramping process \( (\alpha>0) \) maximal errors of cutting force estimation are lower than 8%. These discrepancies are resulting probably from the accuracy of specific cutting force coefficients calibration. Furthermore, maximal instantaneous values of measured forces per consecutive teeth are not uniform. Alterations of these instantaneous maximal forces produce the envelope, which has a period equal to tool revolution time. This is probably caused by the cutter’s radial run-out phenomenon, related directly to the tool revolution period. However, run-out component was not included in the model. Figure 9 reveals that \( \alpha \) angle has significant influence also on cutter’s displacement form and amplitude.
The highest displacement amplitudes appear for the slot milling, which is induced by the highest cutting force values in this cutting mode, in comparison to the remaining modes with $\alpha > 0$.

![Image](image1)

Fig. 8. (a) measured time courses of cutting forces for $\alpha = 60^\circ$; (b) calculated time courses of cutting forces for $\alpha = 60^\circ$.

It was also observed, that for the slot milling, the highest displacement amplitude appears in the Y direction, whereas for the upward ramping with $\alpha = 60^\circ$ in the X direction.

This can be attributed to the distribution of cutting force components along the cutting edge, which depends on surface inclination. It is worth indicating that, cutter’s displacements affect surface texture. Therefore, the appropriate selection of surface inclination angle can improve the surface quality.

4. Conclusions

In this work, the influence of the machined surface inclination angle on the cutting forces and vibrations was investigated. The cutting force and vibration model including kinematic – geometric parameters was formulated.

The research revealed that cutting forces and vibrations are strongly affected by the surface inclination angle, both in quantitative and qualitative aspect. The lowest cutting force and displacement amplitudes were generated during upward ramping with the highest investigated surface inclination angle. The cutting forces estimated on the basis of the developed model stay in a good agreement with measured ones. However some discrepancies between these courses results from the appearance of cutter’s radial run-out phenomenon. Therefore, in order to improve the accuracy of the developed force model, one should include the radial run-out phenomenon. Deliberations presented in this study can be also the starting point to the formulation of surface texture model for a ball end milling of inclined surfaces.

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