Preliminary study of HVSR forward modeling: parameters properties and non-uniqueness of subsurface models

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Abstract. The application of horizontal to a vertical spectral ratio (HVSR) in Indonesia is often analyzed using theoretical equations that only focus on Vs. or the two rock layers' thickness. On the other hand, there are already HVSR modeling programs that involve other parameters such as Vp, density, Qp, and Qs for multiple layers. Armed with existing modeling, the effect of each parameter and test the possible non-uniqueness of HVSR modeling is found. In the end, the right inversion method to get satisfactory results is found. Modeling is done by calculating the wave amplification of the transfer function, the phenomenon of attenuation, and dispersion. A synthetic model will be made from the modeling scheme, which is approached with various possible parameters using a random test of 200000 models for each test. From the existing parameters, it is found that only the parameters Vs, thickness, and Qs affect the position f0. Meanwhile, the parameters Vp, density, and Qp only affect the amplitude of the curve. The parameters Vp, Vs, density, and thickness have a consistent relationship between parameters, but not for Qp and Qs. From the various tests carried out, it was found that many combinations can produce similar responses, both parameter combinations, and combinations of parameters with a different number of layers. Inversion modeling is needed to produce a precise subsurface model that can reduce the non-uniqueness results, such as optimization with a global approach, a statistical approach, or a hybrid inversion method.

1. Introduction

Horizontal to vertical spectral ratio (HVSR) is one of the geophysical methods used to characterize subsurface conditions. This method is included in the passive method because it takes advantage of physical phenomena without any artificial sources. HVSR was initiated by [1] and has grown rapidly in the world and Indonesia. Broadly speaking, HVSR is a recording of vibrations that affect local effects, such as human activities, wind, and ocean waves. In the process of analyzing HVSR data, the data is transformed into the frequency domain. In the frequency domain, we will know the value of the fundamental frequency which has a high amplitude, f0. These fundamental frequencies are used to characterize rock layer properties, such as the thickness and strength of sedimentary rocks [2]. HVSR commonly used for three scientific purposes that involving f0 evaluation that correlates with the danger due to earthquake, evaluation of various resonance that involving wide area for micro zonation purpose and disaster mitigation (seismic-risk mitigation) and evaluation of sediment layer that comprising the thickness and the depth of bedrock layer [3]. In the application, HVSR method covers various of scientific field, including geology [4].
So far, the process commonly carried out in Indonesia to obtain thickness and $V_S$ values of rocks is to reduce the $f_0$ value by using the formula $f_0 = V_S / (4h) \ [5]$. Another way that can be done is to use the inversion method \[6\]. The HVSR data inversion method is not widely used in Indonesia. The problem that often occurs in the geophysical data inversion method is that there are many possible combinations of parameter models that can form the same response or so-called non-uniqueness. This non-uniqueness phenomenon can lead to misinterpretation of the subsurface model. Therefore, this study was conducted to determine the physical parameters that affect the HVSR curve and the relationship between one parameter and another. In addition, we tested the non-uniqueness of the HVSR modeling which might form a similar response with a combination of different models.

2. Method
We conducted a simulation of the HVSR based on the calculation of the theoretical model of the transfer function \[7\]. This theoretical calculation considers the amplification value in the frequency domain by considering the parameters of the wave velocity ($V_P$ or $V_S$), density, thickness, and quality factor ($Q_P$ or $Q_S$). This calculation begins with calculating each S wave amplification ($AMP_S(V_S, \rho, h, Q_S)$) and the P wave amplification ($AMP_P(V_P, \rho, h, Q_P)$).

To explain a more realistic situation, several things were considered, such as the phenomenon of attenuation and dispersion \[6\]. The attenuation itself must also consider the frequency factor because the higher the frequency, the greater the attenuation. The HVSR value was calculated from the comparison of horizontal amplification (S wave) and vertical amplification (P wave) in the frequency domain (Figure 1a.).

![Figure 1. a) The results of two-layers HVSR modeling in a logarithmic scale. b) A comparison of the HVSR response of the synthetic and one of the random test models with an error weighting.](image)

To determine the nature of the parameters, random tests are carried out, which can map the effect of each parameter. This test is carried out on the sediment layer, namely the layers above the final layer. Each randomized modeling test was carried out with 200000 models. In addition, the model parameters in every layer are greater than the layer above, except for the thickness.

In the practice of utilizing HVSR data, the primary concern is the value of the fundamental frequency ($f_0$). Therefore, a weighting formula is created that focuses the random test evaluation in the area around the fundamental frequency. This is also applied to the Open HVSR program \[8\]. The weighting formula used is a simple low pass filter equation with a cutoff limit (Figure 1b.), as follows

$$w = \frac{1}{1 + \left( \frac{f}{f_{\text{cutoff}}} \right)^w}$$ \tag{1}

Considering the weighting value, the following formula is used to calculate the error value for the synthetic data and the test data.

$$\text{error} = \sum_{i=1}^{N} \sqrt{(d_{\text{synthetic}}^i - d_{\text{test}}^i) \times w^i}$$ \tag{2}
3. Result and Discussion

We simulated the changes in the response of the HVSR curve to the addition in each parameter. The following is the result of two-layers synthetic modeling with the first layer being the sedimentation layer and the second layer being the bedrock (Table 1). To determine the effect of each parameter, modeling is carried out by changing one of the parameters of the synthetic model. Each parameter in layer 1 and layer 2 is changed to be 20% larger than the parameter of the synthetic model.

Table 1. Parameters of the two-layers synthetic model.

| Layer  | \(V_p\) (m/s) | \(V_s\) (m/s) | \(\rho\) (g/cm\(^3\)) | \(h\) (m) | \(Q_p\) | \(Q_s\) |
|--------|----------------|--------------|------------------------|--------|--------|--------|
| Layer 1| 1200           | 500          | 1.7                    | 70     | 15     | 5      |
| Layer 2| 3400           | 2000         | 2.0                    | \(\infty\) | \(\infty\) | \(\infty\) |

Figure 2. a) HVSR response for each 20% change of the synthetic data at layer 1 and b) at layer 2.

Figure 2a. are curves of parameter change in the sediment layer and Figure 2b. are curves of parameter change in the bedrock layer. It appears that \(V_s\) and the thickness of the sediment layer resulted in a significant shift in the value of \(f_0\). The parameter of the S wave quality factor (\(Q_s\)) also affects the position \(f_0\), but the resulting change is smaller. From these models, the greater the \(V_s\) and \(Q_s\) values, the greater the \(f_0\) value. This is consistent with the principle that the stronger the rock, or the greater \(V_s\) and \(Q_s\) values, the greater the \(f_0\) [9]. On the other hand, the thicker a layer of sediment is, the smaller the value of \(f_0\) will be. The curve maximum amplitude is affected by the other parameters, namely \(V_p\), \(\rho\), and \(Q_p\). This amplitude value can be described from the impedance ratio between layer \(j\) and layer \(j+1\), namely \(\alpha_j = \frac{V_j\rho_j}{V_{j+1}\rho_{j+1}}\).

Figure 3. Random test results for model A1 (\(V_s^* = 600\) m/s) and model A2 (\(h^* = 56\) m) compared to the synthetic model.

Furthermore, we want to know how the ability of \(V_s\) and layer thickness match the response of the synthetic model. For this reason, two models were carried out, the A1 and A2 models. Model A1’s all parameters are fixed and the same as the synthetic model, except for \(V_s\) which is larger by 20% (600 m/s). The A1 model test aims to find the layer thickness that can match the curve of the synthetic
model with different $V_S$. Meanwhile, model A2’s all parameters are fixed and the same as the synthetic, except that the layer thickness which is smaller by 20% (56 m). In this A2 model test, we look for $V_S$ which can match the curve of the synthetic model with different thicknesses. The results obtained are that the A1 model produces a thickness that is 17% (82.1 m) greater than the synthetic thickness, as the A2 model produces a $V_S$ that is 20% (398.6 m) smaller than the synthetic data (Figure 3). Both models cannot produce a model that is similar to the synthetic data or produce small errors.

What is interesting about the A1 and A2 models is that the properties of $V_S$ and thickness results are almost proportional to the change in thickness and $V_S$. When $V_S$ increases, the thickness increases, and vice versa. From this if we only make a model with a combination of $V_S$ and thickness, it will be difficult to produce a model that is similar to the synthetic model. Although they cannot produce a similar curve, both models have an $f_0$ position that is close to the $f_0$ position of the synthetic model (1.45 Hz). Model A1 has a $f_0$ value of 1.45 Hz, while model A2 has a $f_0$ of 1.46 Hz. This indicates that the combination of $V_S$ and thickness can only produce $f_0$ which is close to the synthetic model but cannot match the amplitude of the curve.

Many studies use $f_0$ as the basis for the analysis with the theoretical equation $f_0 = V_S/(4h)$. From formula, we can see that $V_S$ and thickness will compensate for one another. To get a unique value, one of the parameters must be known, such as using $V_S$ data from the attenuation function method [10]. If not, then the result is not unique, there are many possibilities if you want to match the value of $f_0$ as in the case of models A1 and A2.

To find out what parameters can match the curve amplitude and $f_0$ values, we include the $V_P$ and density parameters. Based on the fact in the first experiment, both parameters show the ability to change the amplitude of the curve without changing the value of $f_0$. For this reason, two test models were made, models B1 and B2. Model B1 has a constant $V_S$ of 600 m/s and the parameters of $V_P$, density and thickness of the sediment are made free. Meanwhile, in the B2 model the thickness parameter value is fixed to 56 m, but the $V_P$, density, and $V_S$ parameters are made free. With that, whether the $V_S$ value and different thickness parameters can be formed a curve with an amplitude and $f_0$ similar to the synthetic data response by a combination of other parameters.

![Figure 4](a) Comparison of model B results to the synthetic model. (b) Comparison of model C results with to the synthetic model.

The results obtained in Figure 4a. provide a small calculation error value, much smaller compared to the $V_S$ or thickness approach alone. This small error value still follows the existing weighting, so the error calculation focuses on the curve around $f_0$. From the two models, we get a model that produces an amplitude, $f_0$, and an overall curve similar to the synthetic data. This shows that an HVSR
curve is also influenced by the $V_P$ and density parameters. Both models give a small error, but each one has the opposite parameter values. Model B1 has greater $V_P$, $V_S$, and depth than the synthetic models. On the other hand, the B2 model has smaller $V_P$, $V_S$, and depth than the synthetic model. The interesting thing is that the density parameter is the opposite of the other parameters.

From this result, the general relationship between the parameters follows: $V_S \approx V_P \approx h \approx 1/\rho$. The values of density and wave velocity are inversely proportional, following the formula $V_P = \sqrt{(\mu + \kappa \kappa)/\rho}$ and $V_S = \sqrt{\mu/\rho}$. It is also found that there is a high degree of non-uniqueness because the resulting error is small even though the models used are far different.

At this stage, a random test was carried out for all parameters. With this test, we try to find models with small error values, namely five models with the smallest error (Figure 4b.). It appears that the C1 model is the most similar to the synthetic model, and the error value is small ($5 \times 10^{-4}$). The C2 and C3 models also produce similar models even though the models used are different from the synthetic models. Meanwhile, the C4 model has a model with the deepest first layer thickness, 88 m, but has the smallest error ($3 \times 10^{-4}$) and is smaller than the C1 model. Again, this indicates that there is a unique model that can produce a similar response to the synthetic data response. Meanwhile, the relationship between the parameters $V_P$, $V_S$, density, and thickness of the sediment layer is still consistent, that is, it follows the relationship $V_S = V_P = h = 1/\rho$, on the other hand, the parameter values of $Q_P$ and $Q_S$ have no consistent relationship with other parameters.

**Figure 5.** Comparison of model D results to the synthetic model.

Furthermore, a randomized test was carried out with a three-layers model to get the response of the two-layers synthetic model. It can be seen from Figure 5., that the three-layers approach can produce a small error. The three-layers combination that can produce a small error is a combination that is still similar to the C model. This is evident with the second layer being relatively the same as the first layer and having a thinner thickness so that it gives a relatively low response effect than the first layer. From these results, the response can be approached with a model that has more layers, but still could produce the model non-uniqueness.

Practitioners or researchers should be careful in interpreting the HVSR results. We should be aware of these limitations and reduce this ambiguity wherever possible by measuring or calculating one or more parameters from other methods. An appropriate inversion method is an inversion approach that can reduce the effect of non-uniqueness, namely optimization with a global approach [11]. In this random test 200000 models have been used, but it is still difficult to get the optimal model, therefore
the global optimization used must be able to regulate the optimization process so that the optimization process runs faster. Another method that can be used to reduce non-uniqueness and finish faster is the hybrid inversion method, by combining the optimization with a global approach with least-square inversion [12]. Another method that can be used is to perform statistical calculations so that the existing model parameters can be estimated along with the error range of the resulting model.

4. Conclusion

Based on the synthetic data modeling and random testing, the parameters $V_S$, thickness, and $Q_s$ affect the position of $f_0$, but the effect of $Q_s$ is small. Meanwhile, the $V_S$ and density, greatly influences the amplitude of the HVSR curve. Furthermore, when $V_S$ and thickness combination are used, only the $f_0$ value is approximated, but the amplitude of the curve is difficult to be approached. To produce the similar response to the synthetic model, $V_S$ and thickness will compensate one another. The relationship can be described as $V_S$ is directly proportional to thickness, meaning that if $V_S$ is large, thickness is large. In other cases, if the all-parameters approach is used, to obtain a similar response result, the modeling parameters will have the relationship $V_S \approx V_p \approx h \approx 1/\rho$, while the $Q_p$ and $Q_s$ do not have a consistent relationship with other parameters. With the combination of many parameters in each model, it is highly likely to form the same HVSR curve response from different parameter combinations. In addition to the parameter combinations of a single layer, these combinations can be formed with more or less layers from the synthetic model. Therefore, the HVSR response model has a high non-uniqueness.

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