NORMALIZATION OF THE MATTER POWER SPECTRUM VIA THE ELLIPTICITY FUNCTION OF GIANT GALAXY VOIDS FROM SDSS DR5

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ABSTRACT

The ellipticity function of cosmic voids exhibits strong dependence on the amplitude of the linear matter power spectrum. Analyzing the most recent void catalogs constructed by Foster and Nelson from the fifth data release of the Sloan Digital Sky Survey, we measure observationally the ellipticity function of giant galaxy voids. Then, we incorporate the redshift distortion and galaxy bias effect into the analytic model of the void ellipticity function and fit it to the observational result by adjusting the value of the power-spectrum normalization with the help of the generalized $\chi^2$-minimization method. The best-fit normalization of the linear power spectrum is found to be $\sigma_S = 0.90 \pm 0.04$. Our result is higher than the WMAP $\sigma_S$-value but consistent with that from the recent work of Liu and Li who have constructed a new improved CMB map independently.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

The normalization amplitude of the linear matter power spectrum is one of the key cosmological parameters that are required to complete the theoretical description of the initial conditions of the universe (Tegmark et al. 2006). It is often quantified in terms of $\sigma_S$, the rms fluctuations of the linear density field within a top-hat spherical radius 8 $h^{-1}$Mpc. Various observables have so far been used to constrain the value of $\sigma_S$: the cluster abundance (e.g., Henry et al. 2009), the weak lensing cosmic shear (e.g., Van Waerbeke et al. 2000), strong lensing arc statistics (e.g., Huterer & Ma 2004), the cluster shapes (Lee 2006), and the cosmic microwave background radiation (CMB) temperature map (e.g., Dunkley et al. 2009; Liu & Li 2009). Yet, these observables depend not solely on $\sigma_S$ but concurrently on the other key parameters such as the matter density parameter $\Omega_m$, primordial non-Gaussianity parameter $f_{NL}$, and dark energy equation of state $w$. Furthermore, it has been realized that complicated systematics involved in the measurement of these observables could bias strongly the estimates of $\sigma_S$. Hence, to break the parameter degeneracy and to diminish any systematic bias, it is very important to consider as many alternative probes as possible.

Recently, (Park & Lee 2007, hereafter, PL07) have proposed the void ellipticity function as another sensitive probe of $\sigma_S$. Noting that the shapes of voids are modulated by the competition between tidal distortion and cosmic expansion, they have analytically derived the void ellipticity function under the assumption that the dynamics of void galaxies can be well described by the Zel’dovich approximation just as that of the dark matter particles are in the linear regime. They have tested their model against the observational data, however, the PL07 model has to be extended to incorporate the redshift distortion effect since in practice the void ellipticities can be measured only in redshift space.

Moreover, there is one condition that the success of the PL07 analytic model is contingent upon. Its validity has been tested only for the case that the voids are found through the specific void-finding algorithm of Hoyle & Vogelezang (2002, hereafter HV02). Since there is no unique definition of voids, the ellipticity distribution may well depend on the way in which voids are identified (Colberg et al. 2008). For the fair comparison with the PL07 model, the HV02 algorithm should be consistently used for the identification of voids from observations. Very recently, Foster & Nelson (2009, hereafter FN09) have constructed a catalog of 232 voids from the Sloan Digital Sky Survey Data Release 5 (SDSS DR5). Now that the voids of the FN09 catalog are identified using the HV02 algorithm, it must provide the most optimal dataset against which the PL07 analytic model can be compared.

In this Letter our goal is to constrain the value of $\sigma_S$ by comparing the extended PL07 analytic model of the redshifted void ellipticity function with the observational result from the FN09 catalog of SDSS voids.

2. REDSHIFTED VOID ELLIPTICITY FUNCTION

Let us first give a brief overview on the PL07 theoretical model. An analytic expression for the probability density distribution of the minor-to-major axial ratio, $\nu$, of a void at redshift $z$ on the Lagrangian scale $R_L$ was found by PL07 as

$$p(\nu; z, R_L) = \int_0^1 d\mu \frac{1}{\sqrt{10\pi \sigma^2_{R_L}}} \exp \left[ -\frac{5\nu^2}{2\sigma^2_{R_L}} + \frac{15\nu^2(\lambda_1 + \lambda_2)}{2\sigma^2_{R_L}} \right] \times \exp \left[ \frac{15(\lambda_1^2 + \lambda_1\lambda_2 + \lambda_2^2)}{2\sigma^2_{R_L}} \right] \times (2\lambda_1 + \lambda_2 - \delta_e)(\lambda_1 - 2\lambda_2 - \delta_e) \frac{4(\delta_e - 3)^2 \mu \nu}{(\mu^2 + \nu^2 + 1)^3},$$

(1)

where $\sigma(z, R_L)$ represents the rms fluctuations of the linear density field smoothed on scale $R_L$ at redshift $z$, and...
\{\nu, \mu\}$ (with $\nu \leq \mu$) represent the two axial ratios of cosmic voids that can be obtained from the inertia momentum tensors of the anisotropic spatial positions of void galaxies. The key concept of this analytic expression is that the two axial ratios, $\nu$ and $\mu$, are related to the largest and second to the largest eigenvalues, $\lambda_1$ and $\lambda_2$, of the tidal field smoothed on the scale $R_L$ as

$$\lambda_1(\nu, \mu) = \frac{1 + (\delta_v - 2)\nu^2 + \mu^2}{(\mu^2 + \nu^2 + 1)},$$

$$\lambda_2(\nu, \mu) = \frac{1 + (\delta_v - 2)\mu^2 + \nu^2}{(\mu^2 + \nu^2 + 1)},$$

where $\delta_v$ denotes the critical density contrast of a void linearly extrapolated to $z = 0$.

PL07 calculated the value of $\delta_v$ as the galaxy number density contrast as $\delta_v \equiv (n_{bg} - \bar{n}_g)/\bar{n}_g$ where $n_{bg}$ and $\bar{n}_g$ represent the number density of void galaxies and the mean number density of all galaxies in a given sample. PL09 found that $\delta_v \approx 0.9$ on average but also noted a tendency that $\delta_v$ decreases gradually with the sizes of voids. The Lagrangian scale radius $R_L$ was calculated as $R_L \equiv (1 + \delta_v)^{1/3}R_E/(1 + z)$. Here $R_E$ represents the effective (comoving) spherical radius of a void defined as $4\pi R_E^3/3 = V$ with the void volume $V$. The values of $\delta_v$ and $R_E$ have to be determined from the observed voids that are to be used for comparison. It is worth mentioning here that this relation between $R_L$ and $R_E$ holds good also in redshift space.

Defining the ellipticity of a void as $\varepsilon \equiv 1 - \nu$, the probability density distribution of the void ellipticities on scale $R_L$ at redshift $z$ is calculated as $p(\varepsilon; z, R_L) = p(1 - \nu; z, R_L)$. PL07 originally derived Equations (13) for the present epoch $z = 0$. It was shown that the analytic procedure to higher redshifts can be described by a single linear distortion parameter, $\beta$, which is related to the background cosmology as $\beta = \frac{\Omega_m^2 + \Omega_{\Lambda}(1 + \Omega_m/2)}{b_g}$ (Hamilton 1998, and references therein).

where $b_g$ is a linear galaxy bias factor measured in real space. Since we are interested in the redshift distortion effect on voids, we should use a bias factor of the void galaxies, say $b_{vg}$. Baslakos et al. (2007) have measured the real-space clustering of the HI galaxies and found that the galaxies in low-density regions are anti-biased, exhibiting $b_{vg} \approx 0.68$. Now, the redshifted power spectrum can be approximated at first order as

$$\Delta^2(k) = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right)
$$

where $\beta$ is calculated using the void galaxy bias factor $b_{vg} = 0.68$. Replacing the real-space power spectrum $\Delta^2(k)$ by the redshifted power spectrum $\Delta^2(k)$, we finally obtain an analytic expression for the redshifted void ellipticity function, $f(\varepsilon; z, R_L)$. Figure[3] plots the analytic predictions of the redshifted void ellipticity function for the three different cases of the linear power-spectrum normalization ($\sigma_8 = 0.7, 0.8$ and $0.9$ as dotted, dashed and solid, respectively). The Lagrangian void scale, redshift, and total number of voids are set at the values consistent with the ones used for the void catalog (see §3).

As can be seen, the void ellipticity function depends very sensitively on the $\sigma_8$ value.

### 3. Comparison with Observational Results

FN09 extended the HV02 void-finding algorithm to improve its statistical robustness and applied it to the volume-limited spectroscopic sample of the galaxies from...
listed the redshift number of the void galaxies of each void as $R_v$, assuming a flat $\Lambda$CDM cosmology with $\Omega_m = 0.28$, $\Omega_{\Lambda} = 0.72$ and $H_0 = 100h$. Throughout this paper, we also adopt the same cosmology, setting the other key parameters at $n_s = 0.96$ and $h = 0.71$ from the WMAP priors (Dunkley et al. 2008). The mean values of $z$, $R_v$, and $N_{\text{vg}}$ averaged over all voids are found to be $\bar{z} = 0.114$, $\bar{R}_v = 24.64h^{-1}\text{Mpc}$, and $\bar{N}_{\text{vg}} = 7$, respectively.

Using information on $a$, $b$, and $c$, we measure the ellipticity of each void as $\varepsilon = 1 - c/a$. Binning the ellipticities as $\varepsilon_i$, counting the number of SDSS voids in each ellipticity bin, and dividing the void number counts by $d\varepsilon$, we determine the observational void ellipticity function, $f_\varepsilon(\varepsilon_i)$. To estimate the statistical errors in the measurement of $f_\varepsilon(\varepsilon_i)$, we separate the voids into six subsamples each of which has approximately the same number of voids, and then calculate the void ellipticity function separately from each subsample. The errors at each ellipticity bin $\varepsilon_i$ is now calculated as the standard deviation of $f_\varepsilon(\varepsilon_i)$ between the six subsamples, which include both the cosmic variance and the Poisson noise. Figure 2 plots the resulting void ellipticity function as dots.

The density contrast of each void is computed as $\delta_v = (n_{\text{vg}} - \bar{n}_{\text{vg}})/\bar{n}_{\text{vg}}$ where $n_{\text{vg}} = N_{\text{vg}}/V$ with $V = (4\pi abc/3)$. And the mean density contrast averaged over all voids is determined to be $\delta_v \approx -0.97$. Using the values of $\bar{z}$, $\bar{R}_v$ and $\delta_v$, the Lagrangian void scale is obtained to be $R_v = 8.534 h^{-1}\text{Mpc}$. We fit the analytic model of the redshifted void ellipticity function to the observational results by adjusting the value of $\sigma_8$ with the help of the generalized $\chi^2$ minimization method where $\chi^2$ is given as

$$\chi^2 \equiv \left[ f^O_s(\varepsilon_i) - f^T_\varepsilon(\varepsilon_i; \sigma_8) \right] C^{-1}_{ij} \left[ f^O_s(\varepsilon_j) - f^T_\varepsilon(\varepsilon_j; \sigma_8) \right],$$

where $f^O_s(\varepsilon_i)$ and $f^T_\varepsilon(\varepsilon_i; \sigma_8)$ denote the observed and the theoretical ellipticity function at the $i$-th ellipticity bin, respectively. The covariance matrix $(C_{ij})$ is defined as $C_{ij} \equiv \langle \Delta f^O_s(\varepsilon_i) \Delta f^O_s(\varepsilon_j) \rangle$ where the ensemble average is taken over the six subsamples. Since the number of the subsamples (six) is larger than the number of the ellipticity bins (five), it is guaranteed that $(C_{ij})$ is invertible (Hartlap et al. 2007). The best-fit $\sigma_8$ that minimizes $\chi^2$ is found to be $\sigma_8 = -0.897 \pm 0.037$ where the errors are calculated as $\sqrt{\chi^2 / \text{dof}}$. Figure 3 plots the analytic model with best-fit $\sigma_8$, demonstrating that the analytic model agrees well with the observational results.

Although a fixed value, $\Omega_m = 0.28$, is assumed for the construction of the FN09 void catalog, it is worth examining the parameter degeneracy between $\sigma_8$ and $\Omega_m$ when the void ellipticity function is used as a probe of $\sigma_8$. Varying the values of $\sigma_8$ and $\Omega_m$ simultaneously, we repeat the whole fitting process. Figure 3 plots the contours of $\chi^2 = 1, 2, 3$ (thick solid, thin solid and dotted lines, respectively) in the $\Omega_m - \sigma_8$ plane, where $\chi^2$ is the reduced $\chi^2$. As it can be seen, the best-fit $\sigma_8$ decreases very mildly as $\Omega_m$ increases.

4. DISCUSSION AND CONCLUSION

We have constrained the matter power spectrum normalization as $\sigma_8 = 0.90 \pm 0.04$ by comparing the void ellipticity function from SDSS DR5 with the extended PL07 model. It is intriguing to note that our result is higher than the WMAP5 value ($\sigma_8 = 0.796 \pm 0.036$) but consistent with that from the recent work of Lin & Lu (2009) who casted a doubt on the accuracy of the cosmological parameters estimated by the WMAP team. Using a new improved CMB map constructed by employing
an independent software scheme, Liu & Li (2009) have found $\sigma_8 = 0.921 \pm 0.036$.

The advantage of using the void ellipticity function as a probe of the power spectrum normalization is that it is purely analytical, free from any nuisance parameters and ad-hoc assumptions. Besides, as shown recently by Lam et al. (2009), the void ellipticity function does not depend on the primordial non-Gaussianity parameter, unlike the other prominent probe of $\sigma_8$, the cluster mass function. It depends most sensitively on $\sigma_8$ among the key cosmological parameters and thus it is in principle one of the most optimal probes of the power spectrum normalization. Yet, it is worth noting that there is one weak point about using the void ellipticity function as a cosmological probe. The number of observable voids at different redshifts is relatively small compared with that of observable clusters, which means that it tends to suffer from small-number statistics. The future galaxy surveys may allow us to overcome this limitation. Our future work is in the direction of forecasting constraints on $\sigma_8$ and $w$ from the forthcoming galaxy surveys by exploiting the void ellipticity function (C. Cunha and J. Lee in preparation).

I thank C. Foster, C. Cunha and D. Huterer for many inspiring comments. I also thank A. E. Evrard and the Physics Department of University of Michigan and Michigan Center for Theoretical Physics at Ann Arbor for the warm hospitality during my Sabbatical when this research is conducted. I acknowledge financial support from the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korean Government (MOST, NO. R01-2007-000-10246-0).

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