On the dominance of non-exotic meson-meson scattering by $s$-channel $q\bar{q}$ confinement states and the classification of the scalar mesons

Eef van Beveren$^a$ and George Rupp$^b$

$^a$ Centro de Física Teórica, Departamento de Física, Universidade, P3004-516 Coimbra, Portugal, (eef@teor.fis.uc.pt)

$^b$ Centro de Física das Interacções Fundamentais, Instituto Superior Técnico, Edifício Ciência, P1049-001 Lisboa Codex, Portugal, (george@ajax.ist.utl.pt)

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Abstract

Non-exotic scalar-meson resonances in $S$-wave meson-meson scattering are studied in the light of a unitarised Schrödinger model. The resulting poles in the scattering matrices, by analytical continuation into the complex-energy plane, are grouped into nonets of isoscalar, isodoublet, and isotriplet resonances. All singularities can be related to quark-antiquark confinement states, the light-quark nonet of which has ground states at 1.3 to 1.4 GeV and level spacings of some 300–400 MeV, except for a nonet of light scalar mesons below 1 GeV. All non-exotic $S$-wave resonances reported by experiment fit into this scheme.

Introduction

Lattice QCD in principle offers the most direct way to link to experiment what we believe to be the fundamental theory of strong interactions [1]. However, in view of the constant evolution of
results from sophisticated lattice QCD solutions \cite{3}, it becomes ever more puzzling why mesonic resonances can be described as simple quark-antiquark systems in effective theories \cite{1}. But apparently, such a scenario works!

Seemingly, the perturbative vacuum states of QCD at low energies are not quarks and gluons, but rather confined constituent quarks and residual interactions, a picture that has been successful for several decades by now. What do lattice calculations teach us about constituent quarks? One might think of colour-triplet configurations of quarks, antiquarks, and glue, maybe with admixtures of higher colour multiplets, which mutually feel colour interactions. Can lattice QCD identify such substructures? Moreover, what happens exactly when those substructures suddenly turn colourless and cease to be confined? Does one observe colourless substructures that drift apart on the lattice?

In unitarised meson models one assumes an effective mass for the constituent quark, a confinement force for the remaining colour interactions, and a mechanism for decay \cite{4, 5}. However, not knowing how the separation into constituent quarks, confinement, and decay can be derived from QCD, each model is the result of educated guesses, rather than of rigorous derivations starting from QCD. This frustrating state of the affairs has in the past three decades led to a proliferation of effective models and theories. However, no model exists so far which completely describes the resonances of meson-meson scattering to a satisfactory degree of accuracy. Nevertheless, some educated guesses are less successful than others. For example, $q\bar{q}$ models for resonances that do not take meson loops into consideration \cite{6} can never be in agreement with experiment, since the resulting spectrum consists of zero-width bound states, whereas large widths are measured.

**Non-exotic meson-meson scattering**

The assumption that non-exotic meson-meson scattering is dominated by the $s$-channel states of the confinement mechanism has been worked out in a series of papers \cite{6, 7, 8, 9}. Here, we will confine our attention to a toy model that we studied in Ref. \cite{9}. There we obtain for the elastic low-energy partial-wave meson-meson scattering matrix,

$$S_\ell(p) = \exp(2i\delta_\ell(p)),$$  \hspace{1cm} (1)

the relation

$$\cotg(\delta_\ell(p)) = \frac{n_\ell(pa)}{j_\ell(pa)} - \left[ 2\lambda^2 \mu pa \sum_{n=0}^{\infty} \frac{|F_n(a)|^2}{E - E_n} \right]^{-1},$$  \hspace{1cm} (2)
where $p$ represents the relative momentum in the CM frame of the two mesons, $\ell$ their relative angular momentum, and $\mu$ their reduced mass; $E_n$ ($n = 0, 1, 2, \ldots$) represent the energy eigenvalues of the constituent $q\bar{q}$ system to which the meson pair couples, and $\mathcal{F}_n$ the corresponding $q\bar{q}$ eigenfunctions; $n_\ell$ and $j_\ell$ stand for the spherical Bessel and Neuman functions, respectively.

The intensity of the coupling between the meson-meson system and the $q\bar{q}$ system is described by the parameter $\lambda$, whereas $a$ stands for the average distance at which the transitions from one system to the other take place [11].

For $\lambda = 0$, we find $S_\ell(p) = 1$, which describes a system of two non-interacting mesons. For small values of $\lambda$, the scattering matrix (1) has poles in the lower half of the complex-energy plane, which can approximately be given by

$$E_{\text{pole}} \approx E_n - \left| \mathcal{F}_n(a) \right|^2 \left[ \sum_{n' \neq n} \frac{\left| \mathcal{F}_{n'}(a) \right|^2}{E_n - E_{n'}} - \frac{i}{2\lambda^2 \mu p a} j_n(pa) k_{\ell n}^{(1)}(pa) \right]^{-1},$$

indicating that to each value of the radial quantum number $n$ corresponds one singularity, i.e., one meson-meson scattering resonance. For higher values of $\lambda$, one can determine the locations of the poles in the scattering matrix by numerical methods.

The real parts of the singularities roughly correspond to the central resonance positions, $E_r$, whereas the moduli of the imaginary parts approximately equal half the resonance widths, $\Gamma_r$. In short,

$$E_{\text{pole}} \approx E_r - i \frac{\Gamma_r}{2}.$$  

(4)

Singularities may be located on the real energy axis below threshold, representing stable (with respect to strong decay) mesons (e.g. $K$, $J/\Psi$, $\Upsilon$ [7]).

For practical purposes, one might truncate the sum in formula (2) and substitute the truncated part by a constant [9].

For $\lambda \to \infty$ one finds

$$\cotg (\delta_\ell(p)) = \frac{n_\ell(pa)}{j_\ell(pa)},$$

(5)

which represents scattering from a hard sphere of radius $a$. In this case, the interior of the $q\bar{q}$ state becomes unobservable and no resonance spectrum can be deduced from meson-meson scattering.

**Comparison with experiment**

In Ref. [9] we compare the predictions of formula (1) for $K\pi$ $S$- and $P$-wave scattering in $I = 1/2$ with the experimental cross sections. For $P$ waves, in the region of the $K^*(892)$ resonance, we
find that the lowest-lying state of the confinement spectrum is at some 945 MeV, whereas the corresponding pole comes out at \((887 - 27i)\) MeV.

In a more refined model [7], which also takes inelasticity into account by considering the coupling to all channels with allowed initial and final states of pseudoscalar and vector mesons, and furthermore employs a more sophisticated mechanism for the coupling of \(q\bar{q}\) confinement states to meson-meson scattering channels, it is found that the ground state of the confinement spectrum in this case comes out at 1.19 GeV, some 300 MeV above the position of the \(K^*(892)\) pole. It shows that bare states can be several hundreds of MeVs away from the actual central resonance positions, and, moreover, that such conclusions are model dependent. The toy model of Ref. [9] yields a shift of only some 60 MeV. In this perspective, the question as to where a bound-state model should find its bare states is hard to be answered.

The latter question becomes even more difficult in the case of \(S\)-wave scattering. There we find, in the toy model of Ref. [9], that the ground state of the confinement spectrum is at 1.31 GeV and the corresponding pole at \((1.46 - 0.12i)\) GeV. However, further inspection of the singularity structure of the scattering matrix reveals another pole far below this energy region, namely at \((714 - 228i)\) MeV. The latter singularity has no direct relation to any of the bare states. Hence, the \(I = 1/2, J^P = 0^+\) ground state of bound-state models should be at some 1.3 GeV and not below 1 GeV. This result is confirmed by the full model [6] (pole at \((727 - 263i)\) MeV), in which also the poles belonging to the two isoscalars \(f_0(980)\) and the rather controversial \(f_0(470 - 208i)\) (\(\sigma\) meson), as well as to the isovector \(a_0(980)\), have no direct relation with the ground states of the corresponding confinement spectrum at about 1.2 GeV.

A model study of poles

The relation between the \(q\bar{q}\) bare states and the poles in the corresponding scattering matrix can be found in the coupled-channel model [1] by considering the process of stepwise reducing the coupling constant \(\lambda\). Such a study has been performed in detail in the toy model of Ref. [3], with the following result. All poles move towards a corresponding \(q\bar{q}\) bare state on the real axis, as predicted by formula (3), except for the \(S\)-wave singularity below 1 GeV. The negative imaginary part of this pole grows inversely proportionally to \(\lambda^2\), implying that the corresponding “resonance” disappears into the background of \(K\pi\) scattering. Unfortunately, such processes cannot be tested in experiment, since Nature corresponds to a fixed value for \(\lambda\).
Constituent-quark-pair creation

For $P$- and higher-wave meson-meson scattering, we do not find singularities other than those which can be related to a $q\bar{q}$ state of the confinement spectrum. The extra poles below 1 GeV, described through *pole doubling* in Ref. [11], exclusively appear in $S$-wave meson-meson scattering. Hence, we must conclude that the latter “resonances” are a consequence of the mechanism of constituent-quark-pair annihilation and creation, which couples meson-meson initial and final states to the $q\bar{q}$ confinement states.

For $P$ and higher waves, the centrifugal barrier prevents the formation of such resonances in meson-meson scattering. But in the absence of a centrifugal barrier for $S$ waves, resonances are formed that in the cases of the $f_0(980)$ and the $a_0(980)$ are narrow enough to be clearly observable, but which for the $f_0(470 - 208i)$ and $K^*_0(727 - 263i)$ are too broad to be firmly established.

One should note here that, when a pole with a large imaginary part lies close to threshold — close meaning that the distance from threshold to the real part of the singularity is smaller than or of the same order as the imaginary part — then the corresponding cross section has a shape which is very different from a standard Breit-Wigner. In the Argand plot, one finds a resonance motion that rapidly slows down for higher energies. If then, moreover, new thresholds get open and other rapid Breit-Wigner resonances show up, its appearance can hardly be recognised as that of a resonance, within the experimental accuracy.

Nevertheless, whether or not one associates a resonance with the controversial $J^P = 0^+$ singularities is of *no importance*. What is crucial in the above observations is the fact that it settles the classification of the $f_0(980)$ and $a_0(980)$ resonances in a nonet scheme for mesons rather more naturally than other proposals.

Resonance shapes

The model result that some of the light scalar resonances are broad, while others are narrow, has its origin in the effects of inelasticity. In Ref. [8], Table 1 of the Appendix, a list of inelasticity channels for the three scalar-meson isomultiplets is presented, as well as the intensities of the relative couplings.

We learn from this table that the isotriplet couples twice as strongly to $\eta_n\pi$ as to $K\bar{K}$, other thresholds lying at higher or much higher energies, which makes their effect hardly relevant here. However, only a small part of the $\eta_n\pi$ channel decays into $\eta\pi$, the rest into $\eta'\pi$. This implies that, of the lowest channels, the $K\bar{K}$ and also the $\eta'\pi$ channel are far stronger than the $\eta\pi$. 
channel (see also Ref. [12]). Elastic S-wave $\eta \pi$ scattering in the absence of inelasticity can be described by the toy model of formula (2). In Ref. [9], formula (2) has been applied to elastic S-wave $K \pi$ scattering. Now, when we substitute there the $K$ mass by the $\eta$ mass, then we obtain a toy model for elastic S-wave $\eta \pi$ scattering. With this substitution, we find for the model of formula (2) indeed a pole close to the $\eta \pi$ threshold, and with a relatively large imaginary part ($763 - 199i$ MeV). Moreover, the related toy-model prediction for the $\eta \pi$ elastic S-wave scattering cross section does not show a clear resonance, exactly as in the case of the $K^*_0(727 - 263i)$ pole in $K \pi$ elastic scattering.

Inelasticity, which has been taken into account in Ref. [6], has two consequences here: first, the pole moves close to the $K \bar{K}$ threshold, with a smaller imaginary part ($968 - 28i$ MeV), and, second, since the $\eta \pi$ threshold is far enough below that pole, its resonance shape turns more Breit-Wigner-like (see e.g. Fig. 2 of Ref. [6]). Nevertheless, upon reducing the coupling constant, the modulus of the imaginary part of this pole increases in a similar way as does the lower pole, the $K^*_0(727 - 263i)$, in $K \pi$ elastic S-wave scattering. Consequently, the two poles have a similar origin, not directly related to the bare spectrum. It is only through the strong interference of the $K \bar{K}$ and the $\eta' \pi$ channels that a reasonable Breit-Wigner-like shape appears for the light isotriplet resonance $a_0(980)$.

We also observe from the above-referred table of Ref. [6] that, of the isoscalar complex ($n \bar{n}$ coupled to $s \bar{s}$), the $n \bar{n}$ couples strongly to $\pi \pi$, whereas the $s \bar{s}$ couples strongly to $K \bar{K}$, with, again, other thresholds lying higher or much higher, thus making their influence of little importance here. Furthermore, $n \bar{n}$ and $s \bar{s}$ are coupled to one another through the $K \bar{K}$ channel, which implies that the $s \bar{s}$ component of the isoscalar complex also couples to $\pi \pi$, but quite weakly. Hence, for the $s \bar{s}$ resonance $f_0(980)$ we can now repeat the arguments we gave for the isotriplet resonance shape, with $\eta \pi$ replaced by $\pi \pi$. Moreover, since the $\pi \pi$ threshold in the isoscalar case lies much lower than the $\eta \pi$ threshold in the isotriplet case, we find a more convincing Breit-Wigner-like shape for the $f_0(980)$ in $\pi \pi$ scattering [13] than for the $a_0(980)$ in $\eta \pi$ scattering.

However, the $n \bar{n}$ component of the isoscalar complex, which yields a pole close to the $\pi \pi$ threshold, has no further strong-inelasticity channel to allow for a Breit-Wigner-like shape for the corresponding resonance $f_0(470 - 208i)$. The same happens to the isodoublet, which, according to the table of coupling constants mentioned before, couples strongly to the $K \pi$ channel. No further lower-lying inelasticity channel exists in this case. Consequently, also the $K^*_0(727 - 263i)$ has no Breit-Wigner-like shape.

None of the poles of this nonet of scalar resonances has a direct relation to the bare spectrum. By stepwise reducing the model coupling constant, all nine poles stepwise disappear into the
complex plane with increasing negative imaginary part, whereas the corresponding structures in the meson-meson scattering cross sections stepwise disappear into the background.

**The $K$-matrix**

As one can easily observe from formula (2), we have no poles in the $K$-matrix at the energy eigenvalues $E_n$ of the confinement spectrum, since the hard-sphere-scattering part in the expression for the cotangent of the phase shift does not vanish at energies $E_n$. This contradicts the observation of Sarantsev and collaborators (Ref. [14] and references therein) that bare states are the singularities of the $K$-matrix, so this issue deserves further study.

In the limit of an infinitely strong coupling between the confinement and scattering sectors, formula (2) predicts that no bare spectrum can be observed in meson-meson scattering other than the hard-sphere spectrum. This is reasonable, since in that limit the mesons become impenetrable and thus do not allow the observation of the interior dynamics. However, when the hard-sphere-scattering part of formula (2) is removed, then one just obtains stronger resonances close to the poles of the $K$-matrix when the coupling constant $\lambda$ is increased.

For small coupling, both models give similar results, except that the real shifts for formula (2) can be much larger when the hard-sphere-scattering part is present. This is probably the reason why the bare states of Ref. [14] are always close to the central resonance energies.

We may therefore conclude that the behaviour of both models for moderate coupling is very similar, except for the interpretation of the bare states. In Ref. [15], we studied other consequences of the fact that mesons are not point particles, but finite distributions of constituent quarks.

Nevertheless, the fits of Ref. [16] to the data are too good for the corresponding model to be totally wrong. It might be that, with a small modification, the latter model would also yield the extra $J^P = 0^+$ nonet of singularities and no related bare states, without destroying the excellent fits to the data.

$J^P = 0^+$ resonances nonets

In Table 1, we classify the experimentally observed non-exotic scalar mesons into nonets.

The $f_0(470 − 208i)$ comes in the Tables of Particle Properties, Ref. [17], under $f_0(400–1200)$, but is not well established as a resonance, while the $K^*_0(727 − 263i)$ is not even mentioned in Ref. [17], although evidence for the existence of structure in that energy region has been reported in Ref. [18]. Moreover, a pole in the $S$-matrix is not necessarily observable as a clear resonance in meson-meson scattering, as we have argued before. Furthermore, the $s\bar{s}$ assignment of the

7
radial excitation | isotriplets | isodoublets | isoscalars
--- | --- | --- | ---
pole doubling | $a_0(980)$ | $K_0^*(727 - 263i)$ | $f_0(470 - 208i)$ and $f_0(980)$
ground state | $a_0(1470)$ | $K_0^*(1430)$ | $f_0(1370)$ and $f_0(1500)$
first | | $K_0^*(1950)$ | $f_0(1710)$ and $f_0(?)$
second | | | $f_0(2020)$ and $f_0(2200)$

Table 1: The nonet classification [6] of the $S$-matrix poles for $J^P = 0^+$ meson-meson scattering.

$f_0(980)$ [20, 21, 14] hints at the existence of a corresponding, most probably lower-lying, $n\bar{n}$ structure in $\pi\pi$ scattering [22].

The $f_0(1370)$ and $f_0(1500)$ resonances have been studied in many works [21, 14, 23], with a diversity of explanations as to their nature, out of which the above nonet classification is the most comprehensive.

Of all resonances in Table 1, the $K_0^*(1950)$ does not seem to be well in place: the general level splittings of some 300 – 400 MeV do not agree with the jump of 520 MeV from the ground state to the first radial excitation of the scalar isodoublet. However, in the analysis of Ref. [19] one finds in Table 2 a set of possible singularities (in the third Riemann sheet) related to the $K_0^*(1950)$ resonance, which all have real parts in the energy region 1.7 – 1.77 GeV. Moreover, in Ref. [16] a central resonance position of $1.82 \pm 0.04$ GeV is reported for this resonance [17].

Furthermore, the $f_0(1710)$ is placed at 1.77 GeV in Ref. [24], which, nevertheless, does not alter the above classification, whereas both the $f_0(2020)$ and the $f_0(2060)$ need confirmation and might very well represent the same resonance. However, if there really exist two resonances in this energy region, then our classification indicates that one of them must be of a nature other than $q\bar{q}$.

In conclusion, one should note that several analyses find too many $f_0$ resonances, whereas in our analysis we lack an $f_0(1840)$.

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