Quark Model, Nonperturbative Wave Functions, the QCD Sum Rules and Instantons.

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The main subject of these lectures is the Nonperturbative Wave Functions. We describe some nonperturbative methods (like QCD sum rules, dispersion relations, duality etc) in order to study this object. We also consider some application of the obtained results, such as form factors, inclusive amplitudes and diffractive electroproduction. Finally, we discuss the instanton liquid model which may help us to understand the success of the constituent quark model.

1 Introduction

The problem of bound states in the relativistic quantum field theory with large coupling constant is, in general, an extremely difficult problem. Understanding the structure of the bound state is a very ambitious goal which assumes the solution of a whole spectrum of tightly connected problems, such as confinement, chiral symmetry breaking phenomenon, and many others which are greatly important in the low energy region. Fortunately, a great deal of information can be obtained even in absence of such a detailed knowledge. This happens in high energy processes where the only needed nonperturbative input are the so-called light-cone wave functions (WF’s) with a minimal number of constituents. As is known those WF’s give parametrically leading contributions to hard exclusive processes, hard diffractive electroproduction and many others high energy processes. In all such cases the quark and antiquark are produced at small distances \( z \sim 1/Q \to 0 \), where \( Q \) is the typical large momentum transfer. Thus, we can neglect the \( z^2 \) dependence of the wave function of the meson with momentum \( p \) and can concentrate on the variable \( zp \simeq 1 \) which is order of one. Therefore, the problem is drastically simplified in the asymptotic limit and we end up with the light cone wave function which depends only on one variable \( \phi(zp, z^2 = 0) \).

The corresponding wave functions have been introduced to the theory in the late seventies and early eighties in order to describe the exclusive processes in QCD. We refer to the review papers on this subject for the detail definitions and discussions in the given context. The main idea of the approach
is the separation of the large and small distance physics. At small distances we can use the standard perturbative expansion due to the asymptotic freedom and smallness of the coupling constant. All nontrivial, large distance physics is hidden into the nonperturbative WF in this approach. It can not be found by perturbative technique, but rather should be extracted from elsewhere. The most powerful analytical nonperturbative method for such problems is the QCD sum rules. Therefore, problem becomes tractable within existing nonperturbative methods which are based on the so-called Wilson Operator Product Expansion (OPE). Therefore, the problem is reduced to the analysis of the bound states within OPE. The subject of the Section 2 is the corresponding analysis of the light cone WF’s within QCD.

We have therefore a well-formulated problem of the high energy behavior. The formulation is based on the solid background of QCD. However, calculations of the pQCD approach refer, strictly speaking, only to asymptotically high energies. Therefore, as usual, the main question remains: at what energies do the asymptotic formulae start to work?

This problem has been a controversial issue for the last fifteen years. In order to answer on the question formulated above, we should know the power corrections to an amplitude under consideration. It is commonly believed that the power corrections are related to the transverse momentum distribution in a hadron. Therefore, any dependence on $\vec{k}_\perp^2$ gives some power corrections to the leading terms. Naively one may expect that these corrections should be small enough already in the few GeV$^2$ region. However, this is not the case. In the Section 3 we develop a theory of the transverse momentum distribution in a hadron. This is a key element of the whole approach. The answer on the question about $\vec{k}_\perp^2$ will give us the answer on the formulated above question regarding the onset of the asymptotic regime.

The subject of the Section 4 is an application of the these, apparently pure academic results to the phenomenological needs of the particle physics. Namely, we shall see that the onset of the asymptotic regime is very different for different processes in spite of the fact that WF’s which come into the game are always the same.

It is instructive to recall a history of this development to the problem about an applicability of the QCD-based approach to exclusive amplitudes. As we already mentioned, the dependence on $\vec{k}_\perp^2$ gives some power corrections to the asymptotically leading terms. The expectation that these corrections should be small enough already in the few GeV$^2$ region is mainly based not on a theoretical analysis but rather on the phenomenological observation that the dimensional counting rules, proposed in early seventies, see agree well with the experimental data (such as the pion and nucleon form factors,
large angle elastic scattering cross sections and so on). This agreement can be interpreted as a strong argument that power corrections are small in the few GeV$^2$ region.

However, in mid eighties the applicability of the approach at experimentally accessible momentum transfers was questioned. In these papers it was demonstrated, that the perturbative, asymptotically leading contribution is much smaller than the nonleading "soft" one. Similar conclusion, supporting this result, came from the different side, from the QCD sum rules, where the direct calculation of the form factor has been presented at $Q^2 \leq 3 GeV^2$. This method has been extended later for larger $Q^2 \leq 10 GeV^2$ with the same qualitative result: the soft contribution is more important in this intermediate region than the leading one.

Therefore, nowadays it is commonly accepted that the asymptotically leading contribution to the exclusive amplitudes cannot provide the experimentally observable absolute values at accessible momentum transfers. If we go along with this proposition, then the natural question arises: How can one explain the very good agreement between the experimental data and dimensional counting rules if the asymptotically leading contribution cannot explain the data for experimentally accessible energies? A possible answer was suggested recently and can be formulated in the following way: very unusual properties of the transverse momentum distribution of a hadron lead to the mimicry of the dimensional counting rules by the soft mechanism at the extended range of intermediate momentum transfers. Numerically, the soft term is still more important than the asymptotically leading contribution at rather high $Q^2 \sim 50 \div 100 GeV^2$.

A situation with hard diffractive electroproduction of the $\rho$ meson is quite different. Having the experience with exclusive processes in mind, one could expect a similar behavior (i.e. a very slow approach to the asymptotically leading prediction) for the diffractive electroproduction as well. We shall argue in the Section 4 that this naive expectation is wrong. The asymptotically leading formula starts to work already in the region $Q^2 \sim 10 GeV^2$, where power corrections do not exceed the 20% level in the amplitude. This surprising result is a consequence of very special properties of the WF in the $k_2^2$ variable (as we mentioned earlier, the $k_2^2$ dependence determines corrections to the leading term).

In Section 5 we discuss and formulate some unsolved problems: How one can understand the constituent quark model in terms of QCD (more specifically, in terms of QCD vacuum structure)? We discuss the so-called Instanton Liquid Model which in its present form correctly reproduces multiple mesonic/baryonic/glueball correlation functions, and also has an increasing di-
rect support from lattice studies of instantons (see for review\textsuperscript{[17]}). We find that many properties of the light cone WF’s which have been known for a while, can be understood from the instanton point of view.

2 $\Psi(\xi, k^2_\perp)$ and its longitudinal distribution.

2.1 General constraints

The aim of this section is to provide necessary definitions and establish some essentially model independent constraints on the non-perturbative WF $\Psi(\xi, k^2_\perp)$ which follow from the use of such general methods as dispersion relations, duality and large order perturbation theory.

We define the $\pi$ meson axial wave function in the following gauge-invariant way:

\[ i f_\pi q_\mu \psi_\pi(z_q, z_2^2) = \langle 0 | \bar{d} \gamma_\mu \gamma_5 e^{ig \int_{-z}^{z} A_\mu dz} u(-z) | \pi(q) \rangle = \sum_n \frac{i^n}{n!} \langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 (iz - D_\nu) u(0) | \pi(q) \rangle, \]

where $\hat{D}_\mu \equiv D_\mu - D_\nu$ and $i\bar{D}_\mu = i\partial_\mu + g A_\mu \lambda^a_\mu / 2$ is the covariant derivative. From its definition is clear that the set of different $\pi$ meson matrix elements defines the nonperturbative wave function $\psi_\pi(z_q, z^2)$.

The most important part (at asymptotically high $q^2$) is the one related to the longitudinal distribution. In this case $z^2 \approx 0$ the WF depends only on one $z_q$ variable. The corresponding Fourier transformed wave function will be denoted as $\phi(\xi)$ and its $n$-th moment is given by the following local matrix element:

\[ \langle 0 | \bar{d} \gamma_\mu \gamma_5 (i \hat{D}_\mu z_q) u | \pi(q) \rangle = i f_\pi q_\nu (z_q)^n \langle \xi^n \rangle = i f_\pi q_\nu (z_q)^n \int_{-1}^{1} d\xi \xi^n \phi(\xi), \]

\[ -q^2 \to \infty, \quad z_q \sim 1, \quad \xi = 2u - 1, \quad z^2 = 0. \]

Therefore, if we knew all matrix elements (which are well-defined) we could restore the whole distribution amplitude $\phi(\xi)$. In the infinite momentum frame (IMF) $q_z \to \infty$ the distribution amplitude (DA) $\phi(\xi)$ describes the distribution of the total longitudinal momentum $q_z$ between the quark and antiquark carrying the momenta $uq_z$ and $(1-u)q_z$, respectively. In what follows we will use the both variables $\xi$ and $u$, $\bar{u} \equiv 1-u$ interchangeously.

The QCD sum rules approach allows one to find the magnitudes only the few first moments\textsuperscript{[3]}. As is known, this information is not enough to
reconstruct the WF; the parametric behavior at $\xi \to \pm 1$ is the crucial issue in this reconstruction.

To extract the corresponding information, we use the following duality argument. Instead of consideration of the pion DA itself, we study the following correlation function with pion quantum numbers:

$$\int dx e^{iqx} \langle 0 | T J^\|_n(x), J_0(0)|0 \rangle = (zq)^{n+2} I_n(q^2), \quad J^\|_n = \bar{d}\gamma_\mu z_\nu \gamma_5 (i \overset{\leftrightarrow}{D}_\mu z_\mu)^n u$$

and calculate its asymptotic behavior at large $q^2$. The result can be presented in the form of the dispersion integral, whose spectral density is determined by the pure perturbative one-loop diagram:

$$\frac{1}{\pi} \int_0^\infty ds \frac{ImI_n^{pert}(s)}{s - q^2}, \quad ImI_n(s)^{pert} = \frac{3}{4\pi(n+1)(n+3)}.$$  \hfill (4)

We assume that the $\pi$ meson gives a nonzero contribution to the dispersion integral for arbitrary $n$ and, in particular, for $n \to \infty$. Formally, it can be written in the following way

$$\frac{1}{\pi} \int_0^{S_n^\pi} ds ImI(s)_n^{pert} = \frac{1}{\pi} \int_0^\infty ds ImI(s)_n^\pi,$$  \hfill (5)

Our assumption means that there are no special cancellations and $\pi$ meson contribution to the dispersion integral is not zero, i.e. $S_n^\pi(\|) \neq 0$, where we specified the notation for the longitudinal distribution. In this case at $q^2 \to \infty$ our assumption (5) leads to the following relation:

$$f_\pi^2 \langle \xi^n \rangle (n \to \infty) \to \frac{3S_n^\infty(\|)}{4\pi^2 n^2}.$$  \hfill (6)

It unambiguously implies the following behavior at the end-point region $\Box$:

$$\langle \xi^n \rangle = \int_{-1}^1 d\xi \xi^n \phi(\xi) \sim 1/n^2, \quad \phi(\xi \to \pm 1) \to (1 - \xi^2).$$  \hfill (7)

Few comments are in order. Because we consider (by definition) the total correlation function $\Box$, the behavior $\Box$ should be fulfilled for any nonperturbative WF no matter what the specific shape of WF is. Thus, our first constraint looks as follows:

$$\bullet \ 1 \quad \phi(\xi \to \pm 1) \to (1 - \xi^2).$$  \hfill (8)
We want to emphasize that the constraint (1) is of very general origin and follows directly from QCD. No numerical approximations were involved in the above derivation. Pre-asymptotic as \( q^2 \to -\infty \) perturbative and non-perturbative corrections are only able to change the duality interval in Eq. (6) (which is an irrelevant issue, anyhow) but not the parametric \( 1/n^2 \) behavior which remains unaffected.

2.2 Numerical constraints in the longitudinal direction

We shall discuss here the \( \pi \) meson case only. Similar consideration can be carried out for other hadrons as well within the same technique. We refer to review article \( \text{(2)} \) for the references.

The QCD sum rules for the moments of the \( \pi \) meson WF defined above (2) have the following form:

\[
\frac{1}{\pi} \int_0^{\infty} ds e^{-s/M^2} ImI_n(s) = \frac{3M^2}{4\pi^2(n+1)(n+3)} + \frac{1}{12M^2} \left( \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right) + \frac{16\pi(11+4n)}{81M^4} (\sqrt{\alpha_s q_2})^2 + \ldots \\
\frac{1}{\pi} ImI_n(s) = f_\pi^2 \langle \xi^n \rangle \delta(s) + \theta(s-s_n) \frac{3}{4\pi^2(n+1)(n+3)} 
\]

where \( M^2 \) is so called Borel parameter which varies in the region where power corrections as well as continuum contribution (modeled by the standard \( \theta(s-s_n) \) function) are small enough, i.e. < 30%. The result of the standard fitting procedure with respect to the Borel parameter \( M^2 \) looks as follows:

\[
\langle \xi^2 \rangle_{\mu^2=1.5GeV^2} = \int_{-1}^{1} d\xi \xi^2 \phi(\xi) \sim 0.38, \quad \langle \xi^4 \rangle_{\mu^2=2.2GeV^2} \sim 0.21 \quad (10)
\]

Now, in order to interpret these results, we should compare them with the corresponding values of the asymptotic DA \( \phi(\xi)^{asym} = 3/4(1 - \xi^2) \):

\[
\langle \xi^2 \rangle = \int_{-1}^{1} d\xi \xi^2 \phi(\xi)^{asym} = 0.20, \quad \langle \xi^4 \rangle = 0.086 \quad (11)
\]

For comparison of those DA’s we should keep in mind that:

a) They both normalized to unity, \( \int_{-1}^{1} d\xi \phi(\xi)^{asym} = \int_{-1}^{1} d\xi \phi(\xi) = 1 \),

b) They both have the same behavior at \( \xi \to \pm 1 \), \( \phi(\xi \to \pm 1) \to (1 - \xi^2) \),

c) They have very different few first moments, see eqs (10), (11).

Such a result was a motivation to suggest the so-called “two-hump” or “CZ”
wave function. Very soon, such a WF became a very popular and controversial issue at the same time, see, e.g. \cite{2,3}.

It is not the purpose of these lectures to comment the corresponding results. Rather, I would like to make a remark, that there are strong arguments in favor as well as against of this two-hump WF. My opinion is: the true result rests somewhere in the middle of those two numbers: \cite{4,5}. In different words, the QCD sum rule predictions \cite{6} give somewhat larger magnitudes for the moments and asymptotic results \cite{7} predict somewhat lower values in comparison with what we believe would be the correct magnitude of the moments.

Unfortunately the information regarding $\langle \xi^2 \rangle$ is very difficult to extract by doing a phenomenological analysis of an experiment. The problem in such of analysis is the following. Before to extract any information from the experiment, we have to make sure that one of the following conditions is satisfied:

a). We are already in the asymptotic regime, i.e. the power corrections are small. Therefore, an asymptotic formula already works and one can extract a relevant information from the corresponding theoretical expression.

b). We are not in the asymptotic regime. However, the power corrections are under control, i.e. we know the origin of those power corrections as well as we know a way of how to estimate them. In this case, again, one could extract the relevant information regarding the longitudinal distribution $\phi(\xi)$.

Unfortunately, none of those conditions is satisfied presently in any experiment. Therefore, one can not make a reliable prediction about the properties of the distribution amplitude $\phi(\xi)$ unless we know precisely the power corrections. That is why an interpretation of an experiment could be very ambiguous and very often it is based on a strong assumption about power corrections.

We can not improve our understanding by going along the condition a). To satisfy this condition we need to go to the much higher energy and momentum transfer in comparison to what is available at the moment. Therefore, we shall go along the line b), i.e. we shall try to understand the power corrections relevant to the problem. As we mentioned in Introduction, those power corrections are tightly connected to the behavior of a $\Psi(\xi, k_2^2)$ in transverse direction $k_2^2$. Actually it was the main motivation for the recent study of the hadron transverse momentum distribution within QCD \cite{14,35,36}. We present a review of those results in the next section.

3 Transverse distribution
3.1 General constraints

The moments in transverse directions are defined analogously to Eq. (2) through gauge invariant matrix elements

\[ \langle 0 | \bar{d} \gamma_{\mu} (i \not{D_\nu} t_\nu)^{2n} u | \pi(q) \rangle = i f_\pi q_\mu (2n - 1)!! \frac{(2n - 1)!!}{(2n)!!} \langle \vec{k}_\perp^{2n} \rangle, \]  

(12)

where transverse vector \( t_\mu = (0, \vec{t}, 0) \) is perpendicular to the hadron momentum \( q_\mu = (q_0, 0_\perp, q_z) \). The factor \( (2n - 1)!! / (2n)!! \) is introduced to (12) to take into account the integration over \( \phi \) angle in the transverse plane: \( \int d\phi (\cos \phi)^{2n} / \int d\phi = (2n - 1)!! / (2n)!! \). By analogy with a non-gauge theory we call \( \langle \vec{k}_\perp^{2n} \rangle \) in this equation the mean value of the quark perpendicular momentum, though it does not have a two-particle interpretation. Indeed, it is very different from the naive, gauge dependent definition like \( \langle 0 | \bar{d} \gamma_{\nu} \gamma_5 \partial_\perp^2 u | \pi(q) \rangle \), because the physical transverse gluon is a participant of this definition. We believe that such definition is the useful generalization of the transverse momentum conception for the interactive quark system. Its relation to the higher Fock components will be discussed at the end of this section. Its relation to constituent quark model and Instanton picture of QCD vacuum will be discussed in Section 5. Here we note that Eq. (12) is the only possible way to define \( \langle \vec{k}_\perp^{2n} \rangle \) in a manner consistent with gauge invariance and operator product expansion.

To find the behavior \( \langle \vec{k}_\perp^{2n} \rangle \) at large \( n \) we can repeat the previous duality arguments with the following result:

\[ f_\pi^2 \langle \vec{k}_\perp^{2n} \rangle \frac{(2n - 1)!!}{(2n)!!} \sim n! \Rightarrow f_\pi^2 \langle \vec{k}_\perp^{2n} \rangle \sim n! \]  

(13)

This behavior has been obtained in Ref. 36 by studying of the large order perturbative series for a proper correlation function. Dispersion relations and duality arguments transform this information into Eq. (13). It is important to stress that any nonperturbative wave function should respect Eq. (13) in spite of the fact that apparently we calculate only the perturbative part (see the comment after Eq. (7)). The duality turn this perturbative information into exact properties of the non-perturbative WF. The most essential feature of Eq. (13) is its finiteness for arbitrary \( n \). This means that higher moments

\[ \langle \vec{k}_\perp^{2n} \rangle = \int d\vec{k}_\perp^2 d\xi \xi^{2n} \Psi(\vec{k}_\perp^2, \xi) \]  

(14)

\[ ^{b} \text{Here and in what follows we ignore any mild (non-factorial) } n\text{-dependence.} \]
do exist for any $n$. In this formula we introduced the non-perturbative
$\Psi(\xi, k_2^\perp)$ normalized to one. Its moments are determined by the local matrix elements $\langle \xi, k_2^\perp \rangle$ which are obviously finite, because they are normalized at low normalization point $\mu$. The relations to Brodsky and Lepage notations $\Psi_{BL}(x_1, k_2^\perp)$ and to the longitudinal distribution amplitude $\phi(\xi)$ introduced earlier, look as follow:

$$\Psi_{BL}(x_1, k_2^\perp) = \frac{f_\pi 16\pi^2}{\sqrt{6}} \Psi(\xi, k_2^\perp), \quad \int d k_2^2 \Psi(\xi, k_2^\perp) = \phi(\xi), \quad \int_{-1}^{1} d \xi \phi(\xi) = 1$$

(15)

where $f_\pi = 133 MeV$. The existence of the arbitrary high moments $\langle \vec{k}_2^{2n} \rangle$ means that the non-perturbative $\Psi(\xi, k_2^\perp)$, defined above, falls off at large transverse momentum $k_2^2$ faster than any power function. The relation (13) fixes the asymptotic behavior of $\Psi(\xi, k_2^\perp)$ at large $k_2^2$. Thus, we arrive to the following constraint:

- $2 \langle \vec{k}_2^{2n} \rangle = \int d k_2^2 d \xi k_2^2 \Psi(\xi, k_2^\perp) \sim n! \quad n \to \infty$. (16)

We can now repeat our duality arguments again for an arbitrary number of transverse derivatives and large ($n \to \infty$) number of longitudinal derivatives. The result reads:

- $3 \quad \int d k_2^2 k_2^{2k} \Psi(k_2^2, \xi \to \pm 1) \sim (1 - \xi^2)^{k+1}$ (17)

The constraint (17) is extremely important and implies that the $k_2^2$ dependence of $\Psi(k_2^2, \xi)$ comes exclusively in the combination $k_2^2/(1 - \xi^2)$ at $\xi \to \pm 1$. This means that the standard assumption on factorizability $\Psi(k_2^2, \xi) = \psi(k_2^2) \phi(\xi)$ is at variance with very general properties of the theory such as duality and dispersion relations. The only form of $\Psi(\xi, k_2^2)$ satisfying all the constraints (7), (16) and (17) is the Gaussian with a very particular argument:

$$\Psi(k_2^2 \to \infty, \xi \to \pm 1) \sim \exp \left( -\frac{k_2^2}{\Lambda^2(1 - \xi^2)} \right)$$

(18)

(here $\Lambda^2$ is a mass scale which can be fixed by calculating the moments $\langle \vec{k}_2^2 \rangle, \langle \vec{k}_4^2 \rangle$ etc.) Strictly speaking, so far we have only established the validity of Eq.(18) in a vicinity of the end-point region $\xi \to \pm 1$. However, one
can argue\footnote{14} that, the behavior \footnote{18} can be approximately valid (with some accuracy) in the whole range of the $\xi$ variable.

We would like to pause here in order to make the following conjecture. The Gaussian $\Psi(\xi, k^2_{\perp})$ (reconstructed above from the QCD analysis) not accidentally coincides with the harmonic oscillator $\Psi(\xi, k^2_{\perp})$ from the constituent quark model. We shall discuss this conjecture within the instanton model in a more detail in Section 5, but now let us recall some results from the constituent quark model.

It has been known for a while\footnote{38} that the equal-time (ET) wave functions

$$\Psi_{ET}(q^2) \sim \exp(-q^2)$$

(19)

of the harmonic oscillator in the rest frame give a very reasonable description of static meson properties. Together with Brodsky-Huang-Lepage prescription\footnote{39} connecting the equal-time and the light-cone wave functions of two constituents (with mass $m \sim 300$ MeV) by identification

$$q^2 \leftrightarrow \frac{k^2_{\perp} + m^2}{4x(1-x)} - m^2, \quad \psi_{ET}(q^2) \leftrightarrow \psi_{LC}\left(\frac{k^2_{\perp} + m^2}{4x(1-x)} - m^2\right),$$

one can reproduce the Gaussian behavior\footnote{18} found from QCD. It means, first of all, that our identification of the moments\footnote{12} defined in QCD with the ones defined in quark model, is the reasonable conjecture. The same method can be applied for the analysis of the asymptotic behavior of the nucleon WF which in obvious notations takes the form:

$$\Psi_{nucleon}(k^2_{\perp i} \to \infty, x_i) \sim \exp(-\sum \frac{k^2_{\perp i}}{x_i}).$$

(20)

However, there is a difference. In quark model we do have a parameter which describes the mass of constituent $m \simeq 300$ MeV. We have nothing like that in QCD. This difference has very important phenomenological consequences which will be discussed in the next section. Here we would like to emphasize that there is no room for such mass term in QCD. Its inclusion violates the duality constraint\footnote{7} since in this case we would have

$$\langle \xi^n \rangle \sim \int_{-1}^{1} d\xi \xi^n \exp\left(-\frac{m^2}{\Lambda^2(1-\xi^2)}\right) \sim \exp(-\sqrt{n}) \quad n \to \infty$$

(21)

instead of the $1/n^2$ behavior\footnote{10}. In other words, a true nonperturbative WF must respect the asymptotic freedom which is incompatible with the quark model -type mass term in the WF. To set this more accurately, one can say
that a possible (scale-dependent) mass term in $\Psi(\xi, k_\perp^2)$ must renormalize to zero at a normalization point $\mu^2 \sim \text{a few GeV}^2$ where the duality arguments apply. This conclusion is not at variance with popular models for the QCD vacuum such as e.g. the instanton vacuum (see [17] for review). Even more than that: The instanton picture does support all properties we have been discussing in these lectures, see the section 5 for details. In no sense we claim that a WF like (18) exhausts the $\pi(\rho)$ -meson properties, or (20) exhausts the nucleon properties.

3.2 Lowest moments $\langle \vec{k}_\perp^2 \rangle$, $\langle \vec{k}_\perp^4 \rangle$ and vacuum condensates.

The general constraints of the previous section are insufficient for building up a realistic non-perturbative WF for the $\rho$, $\pi$ -mesons or nucleon. To fix this, we follow the same logic as in the analysis of the distribution amplitudes and calculate few lowest $\langle \vec{k}_\perp^{2n} \rangle$, $n = 1, n = 2$ moments of $\Psi(\xi, k_\perp^2)$. The physical meaning of the second moment $\langle \vec{k}_\perp^2 \rangle$ is clear: this quantity serves as a common scale for power corrections in physical amplitudes. The fourth moment $\langle \vec{k}_\perp^4 \rangle$ tell us on how strongly $\Psi(\xi, k_\perp^2)$ fluctuates in the $\vec{k}_\perp^2$ plane. In this section we will calculate the lowest moments $\langle \vec{k}_\perp^2 \rangle$ and $\langle \vec{k}_\perp^4 \rangle$ for $\rho$ and $\pi$ meson $\Psi(\xi, k_\perp^2)$.

With the knowledge of $\langle \vec{k}_\perp^2 \rangle$ and $\langle \vec{k}_\perp^4 \rangle$ we then construct a model WF $\Psi(\xi, k_\perp^2)$ which will be used to estimate higher twist effects in different processes in the next section.

We start our discussion from the the second moment of the $\Psi(\xi, k_\perp^2)$ in the transverse direction defined by equation (12). For the $\pi$ meson it was calculated for the first time in [4] and independently (using a quite different technique and very different motivation) in [40]. Both results are in a full agreement to each other:

$$\langle \vec{k}_\perp^2 \rangle_\pi = \frac{5}{36} \frac{\langle \bar{q}i\gamma_\mu G_{\mu
u}^a \Lambda^a_{\nu,q} \rangle}{\langle \bar{q}q \rangle} \simeq \frac{5m_0^2}{36} \simeq (330 \text{MeV})^2, \quad m_0^2 \simeq 0.8 \text{GeV}^2. \quad (22)$$

A very similar calculation can be done for the $\rho$ meson as well [15]. The result looks like this:

$$\langle \vec{k}_\perp^2 \rangle_\rho = (420 \text{MeV})^2 \quad (23)$$

Essentially, the result given by equations (22) and (23) defines the general scale of all nonperturbative phenomena for the pion and $\rho$ meson correspondingly. Those numbers are very much the same. It is not accidentally coincides with $300 \div 400 \text{MeV}$ scale which is the typical magnitude in the hadronic physics.

To study the fine properties of the transverse distribution it is desired to know the next moment. The problem can be reduced to the analysis of the
mixed vacuum condensates of dimension seven:

\[
\langle \vec{k}^4 \rangle_\pi = \frac{1}{8} \left\{ -\frac{3}{4} \langle \bar{q}q \rangle^2 G_{\mu\nu} G_{\mu\nu} q \rangle + \frac{13}{9} \langle \bar{q}q \rangle \right\}.
\]

A similar formula for the \(\rho\) meson has been derived in and takes the following form:

\[
\langle \vec{k}^4 \rangle_\rho \approx \frac{3}{10} m^4 \rho \langle u \rangle^4 + \frac{1}{4} \frac{\langle 0| \langle q^2 G_{\mu\nu} G_{\mu\nu} q \rangle \rangle}{\langle 0| \bar{q}q \rangle \rangle} - \frac{1}{8} \frac{\langle 0| \langle q^2 G_{\mu\nu} G_{\mu\nu} q \rangle \rangle}{\langle 0| \bar{q}q \rangle \rangle} = -\frac{1}{3} \frac{\langle 0| \langle q^2 G_{\mu\nu} G_{\mu\nu} q \rangle \rangle}{\langle 0| \bar{q}q \rangle \rangle}.
\]

Therefore, we have explicitly expressed \(\langle \vec{k}^4 \rangle\) in terms of the vacuum expectation values (VEV’s) of the dimension 7 operators. This is very important result: we essentially say that the properties of a hadron are determined by the QCD vacuum structure! One can say even stronger: the gluon fluctuations inside of a hadron are essentially nothing, but the vacuum fluctuations. They are very strong (they have in general 1GeV scale) but they are approximately the same for all hadrons. This observation gives a chance to understand a constituent quark model: all hadrons are build up from the same constituents which originated from QCD vacuum, see section 5 for details.

Coming back to (24) and (25) one could estimate these condensates naively, by factorizing them into the products of the quark \(\langle \bar{\psi} \psi \rangle\) and gluon \(\langle g^2 G^2 \rangle\) condensates. This procedure, based on the factorization hypothesis, does not work in the given case: there are essential deviations from the factorization prediction in VEV’s (23,24). The non-factorizability of mixed quark-gluon matrix elements of such type has been studied in by two independent methods with full agreement in estimates between them. The first one was based on the analysis of heavy-light quark systems, while the second method has related the vacuum condensates of the form (23,24) to some pion matrix elements known from PCAC. Here we only formulate the result of this analysis. A measure of non-factorizability is introduced by the correction factors \(K_1, K_2\) in the matrix elements

\[
\langle 0| \bar{q}q^2 G_{\mu\nu} G_{\mu\nu} q \rangle = \frac{1}{6} K_1 \langle 0| g^2 G_{\mu\nu} G_{\mu\nu} |0\rangle \langle 0| \bar{q}q |0\rangle \langle 0| \bar{q}q |0\rangle
\]

\[
\langle 0| \bar{q}q^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q |0\rangle = -\frac{1}{3} K_2 \langle 0| g^2 G^2 |0\rangle \langle 0| \bar{q}q |0\rangle.
\]

(\(K_1 = K_2 = 1\) in the factorization limit). Those factors are approximately the same \(K_1 = K_2 = K \simeq 3\) for the both mixed operators appearing in Eq.(26). A possible uncertainty of this estimate does not exceed 30 \%. In this case, formulae (23,24) give the following estimate:

\[
\langle \vec{k}^4 \rangle_\rho \approx 0.14 GeV^4.
\]
We would like to present the result of these calculations for $\langle \vec{k}^4 \rangle_\rho$ by introducing a dimensionless parameter which is a quantitative measure of the fluctuations in the transverse direction:

$$R \equiv \frac{\langle \vec{k}^4 \rangle_\rho}{\langle k^2 \parallel \rangle^2_\rho} \simeq 4 \sim 5.$$  \hfill (28)

For the $\pi$ and $\rho$ mesons parameter $R$ is almost the same and it is very large. It corresponds to an unexpectedly large hadronic matrix elements of the high dimensional operators. We have explicitly calculated those m.e. for the operators of dimension seven. However, we expect that a similar conclusion also takes place for arbitrary higher dimensional operators. Such a large value of the parameter $R$ corresponds to the strong fluctuations in the transverse direction. In terms of wave function this property means a very inhomogeneous distribution in the transverse direction.

In terms of the QCD vacuum structure these fluctuations are due to the numerical enhancement of the high dimensional quark gluon mixed condensates or, what is the same, the large magnitude of the parameter nonfactorizability $K$ in Eq.(26). On the microscopical level such an inhomogeneous distribution corresponds to some fluctuations of the strong gluon fields with a small size. In this case, factorization prescription (which essentially suggests a homogeneous distribution of the gluon vacuum fields) clearly does not work. As a consequence of this, high dimensional operators do violate a factorization. We identify these vacuum fluctuations with instantons, see section 5.

An additional evidence in favor of this viewpoint is the recent calculation of the $\eta'$ matrix element (which can be measured experimentally!) of a high-dimensional gluon operator within an instanton model. In general, it is very difficult to have a direct experimental measurement of a matrix element of a high dimensional operator. In most cases, we use some indirect experiments (see section 4: Applications) to establish the properties of those matrix elements. Fortunately, due to the uniqueness of the $\eta'$ meson a high dimensional operator can be measured in the CLEO directly! Such a matrix element can be explicitly extracted from the $B \to \eta'$ decays. This matrix element indeed is very large in agreement with our analysis.

As we shall discuss in section 5, in the constituent quark model those properties correspond to the small size ($1 GeV^{-1}$) of the constituent quark in comparison with the hadron size ($1 fm$).

### 3.3 Model wave function $\Psi(\xi, k^2_\perp)$

The results obtained so far are essentially model independent. We have fixed the form of the high-$k^2_\perp$ tail of the true nonperturbative WF $\Psi(\xi, k^2_\perp)$ Eq.(13).
by the use of the quark-hadron duality and dispersion relations. Furthermore, we calculated the lowest moments $\langle \vec{k}_2^2 \rangle$ and $\langle \vec{k}_4^2 \rangle$ using the equations of motion and QCD sum rules. Now our purpose is to build some model for the true non-perturbative WF $\Psi(\xi, k_2^2)$ which would respect all general constraints of the previous Sections and incorporate the effect of strong fluctuations in the transverse $k_2^2$ plane found in the previous section (see Eq. (28)).

We start our discussion from the analysis of the $\Psi(\xi, k_2^2)$ motivated by constituent quark model [28, 39] (CQM). Such a function is known to give a reasonable description of static hadron properties. The Brodsky-Huang-Lepage prescription [39] leads to the following form for the pion $\Psi(\xi, k_2^2)$:

$$\Psi(k_2^2, u)_{CQM} = A \exp(-k_2^2 + m^2/8\beta^2 u\bar{u}),$$  \hspace{1cm} (29)

We call this function as the constituent quark model WF. As we already discussed before, it satisfies two general constraints (\bullet 2, \bullet 3), but not to (\bullet 1) because of the nonzero magnitude for the constituent mass $m$. We make the standard choice for the parameter $m \simeq 330 MeV$ in accordance with its physical meaning. The parameter $\beta$ can be determined from the numerical constraint for the mean value $\langle \xi^2 \rangle$ for $\pi$ meson (22) and $\rho$ meson (23) correspondingly. They do not differ much and we shall not distinguish them for qualitative discussions. Parameter $A$ is determined by the normalization eq.(15).

To make function wider in the longitudinal direction, one can insert into formula (29) an additional factor

$$(1 + g(\mu_1)(\xi^2 - 1/5)), \quad g(\mu_1) = g(\mu_2) \cdot (\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)})^{50/96} \quad (30)$$

with additional parameter $g(\mu)$. This new parameter $g(\mu)$ allows to adjust $\langle \xi^2 \rangle$ as appropriate. For $\pi$ and $\rho$ meson WF’s this parameter is quite different. For the asymptotic distribution amplitude parameter $g = 0$.

Now we go from Constituent Quark Model to QCD-based WF. Before to design $\Psi_{QCD}(k_2^2, u)$, let us explain what do we mean by that. We define the non-perturbative wave function $\Psi(k_2^2, u, \mu)_{QCD}$ through its moments which can be expressed in terms of the non-perturbative matrix elements (12). As is known, all non-perturbative matrix elements are defined in such a way that all gluon’s and quark’s virtualities smaller than some parameter $\mu$ (point of normalization) are hidden in the definition of the “non-perturbative matrix

\footnote{Here we neglect all terms in QCM related to spin part of constituents. In particular, we do not consider Melosh transformation and other ingredients of the light cone⇒equal time connection. It does not effect any qualitative results presented here.}
elements” (12). All virtualities larger than that should be taken into account explicitly (perturbatively, due to the asymptotic freedom). In particular, all perturbative tails like $1/k_\perp^2$ should be subtracted from the non-perturbative WF by definition. The same procedure should be applied for the calculation of non-perturbative vacuum condensate $\langle G_{\mu\nu}^2 \rangle$, where the perturbative part related to free gluon propagator $1/k^2$ is also should be subtracted.

With these general remarks in mind we propose the following form for the non-perturbative wave function $\Psi(k_\perp^2,u,\mu_0)$ QCD at the lowest normalization point:

$$\Psi(k_\perp^2,u,\mu_0) = A \exp\left(-\frac{k_\perp^2}{8\beta^2 u \bar{u}}\right) \cdot \{1 + g(\mu_0)[\xi^2 - \frac{1}{5}]\}. \quad (31)$$

Parameter $\beta$ for this parametrization is found to be $\beta \approx 0.3$ GeV (it corresponds to $R = 2.2$ and $\langle k_\perp^2 \rangle = 0.14$ GeV$^2$). In comparison with the constituent quark model the “only” difference is the absence of the mass term $\sim m$ in the exponent. This difference is a key element. We discussed this point earlier and we would like to emphasize this point again: the nonzero mass in $\Psi(k_\perp^2,u)_{CQM}$ was unavoidable part of the wave function within a quark model. As we discussed earlier we do not see any room for such a term in QCD, because its presence corresponds to the behavior

$$\langle \xi^n \rangle = \int_{-1}^{1} d\xi \xi^n \phi(\xi) \sim \int_{-1}^{1} d\xi \xi^n \exp\left(-\frac{1}{1 - \xi^2}\right) \sim \exp\left(-\sqrt{n}\right), \quad n \to \infty, \quad (32)$$

which is in contradiction to $1/n^2$ result found earlier (7).

One can implement the effect of strong $k_\perp^2$ fluctuations discussed previously and which is quantitatively expressed in terms of parameter $R$ (28). It can be done in a number of ways. We shall not discuss this point here referring to the original literature (38). However, we note, that this effect leads to the existence of two characteristic scales in $\Psi(\xi,k_\perp^2)$ which has its explanation within instanton vacuum liquid model (37). As we already mentioned, such a property of the $\Psi(\xi,k_\perp^2)$ could be viewed as an explicit manifestation of the complexity of the hadronic structure: one scale $\sim 1$ fm determines the hadron size itself; another, new scale $\sim 1$ GeV$^{-1}$ determines the hadron substructure, the size of the constituent quark.

Our conclusion is that the transverse momentum distributions for the $\pi$ and $\rho$-mesons are to a large extent alike. However, the distribution in the longitudinal direction is very different for those WF’s.

Let us summarize. We constructed two different types of wave functions. The first one, $\Psi_{CQM}$ is motivated by quark model with its specific mass parameters. The second type is motivated by QCD consideration. All these
WFs have Gaussian behavior at large $\vec{k}_\perp^2$. However, in the case of $\Psi_{\text{CQM}}$ this behavior is related to the nonrelativistic oscillator model, while for QCD motivated models this behavior is provided by constraints discussed in the previous section.

Contrary to the CQM, the QCD motivated wave functions do not contain the mass parameter $m \simeq 300\text{MeV}$ which is an essential ingredient of any quark model. Such a term is absolutely forbidden from the QCD point of view.

In the next section we discuss some applications. In particular, we calculate the contribution to the pion form factor caused by these wave functions. We shall find the qualitative difference in behavior on $Q^2$, which is our main point. We shall also discuss the behavior of the hard diffractive electroproduction of $\rho$ meson as a function of $Q^2$. Again, as in $\pi$ meson form factor case we shall see a strong influence of the transverse momentum distribution on this behavior. We shall also discuss nucleon form factor.

4 Applications

4.1 $\pi$ meson form factor

The first application we consider is the pion form factor. The starting point is the famous Drell-Yan formula (for modern, QCD-motivated employing of this formula, see [2]), where the $F_\pi(Q^2)$ is expressed in terms of full wave functions:

$$F_\pi(Q^2) = \int \frac{dxdq^2_{\perp}}{16\pi^3} \Psi_{\text{BL}}(x, \vec{k}_\perp + (1 - x)\vec{q}_\perp)\Psi_{\text{BL}}(x, \vec{k}_\perp), \quad (33)$$

where $q^2 = -q^2_{\perp} = -Q^2$ is the momentum transfer. In this formula, the $\Psi_{\text{BL}}(x, \vec{k}_\perp)$ is the full wave function; the perturbative tail of $\Psi_{\text{BL}}(x, \vec{k}_\perp)$ behaves as $\alpha_s/\vec{k}_\perp^2$ for large $\vec{k}_\perp^2$ and should be taken into account explicitly in the calculations. This gives the one-gluon-exchange (asymptotically leading) contribution to the $\pi$ meson form factor in terms of distribution amplitude $\phi(x)$.

In terms of QCD, the formula (33) is an assumption. However, we expect, that by taking into account only "soft" gluon contribution (hidden in the definition of $\vec{k}_\perp^2$ (3)), we catch the main effect of the soft physics. There is no proof for that within QCD. The only argumentation which can be delivered now to support this assumption is based on the intuitive picture of quark model, where the current quark and soft gluons form a constituent quark with original quantum numbers. No evidence for the gluon playing the role of a valence participant with a finite amount of momentum is seen.
In terms of vacuum structure those soft gluons are nothing but vacuum fluctuations. Therefore, those gluons are very strong in amplitude (they are soft in a sense of the momentum they carry on which is small). Those vacuum fluctuations (which are classical configurations like instantons) are much stronger than the quantum fluctuations carrying a nonzero momentum.

From the viewpoint of the operator product expansion, the assumption formulated above, corresponds to the summing up a subset of higher-dimension power corrections. This subset actually is formed from the infinite number of soft gluons and unambiguously singled out by the definition of non-perturbative $\Psi(\xi, k^2_\perp)$ (1), (12).

We refer to the original paper for details. Here we formulate the result of calculations. The main qualitative difference between quark model and QCD- motivated wave functions is as follows: a much slower fall off at large $Q^2$ is observed for the $\Psi(\xi, k^2_\perp)$ motivated by QCD. The qualitative reason for that is the absence of the mass term, see discussion after the formula (32). Precisely this term was responsible for the very steep behavior in all previous calculations based on a quark model wave function. The declining of the form factor getting even slower if one takes into account the property of the broadening of $\Psi(\xi, k^2_\perp)$ in transverse direction. This property corresponds to the strong fluctuations in the transverse direction and quantitatively is related to the large parameter $R$ discussed in the previous section.

Therefore, our main observation is that the QCD based WF’s could mimic the dimensional counting rules by the soft mechanism at the extended range of intermediate momentum transfers. Numerically, the soft term is still more important than the asymptotically leading contribution at rather high $Q^2 \sim 50 \div 100\text{GeV}^2$.

4.2 Nucleon form factor

Now we would like to extend our previous analysis to the nucleon form factor. The starting point, as before, is the fundamental constraints (1 − 3), which being applied to the nucleon wave function imply the Gaussian behavior with the specific argument (10). With these constraints in mind one can model the nucleon WF in the same way as we did for the pion. Having modeled the nucleon wave function, one can calculate a soft contribution to different nucleon amplitudes. The corresponding analysis was carried out in the ref. Here we quote some results from this paper.

The most important qualitative result of these calculations is similar to what we already observed previously in the $\pi$- meson case: namely, the combination $Q^4 F^{\text{nuc.}}(Q^2)$ is almost constant in the extent region of $Q^2$ in spite
of the fact that the corresponding “soft” contribution naively should be decreasing function of $Q^2$. The qualitative explanation of this phenomenon is the same as before and is related to the absence of the mass term in the QCD-motivated wave function.

The next observation is related to the longitudinal distribution and can be formulated as follows: A fit to the different experimental data leads to a wave function which has the same type of asymmetry which was found previously from the QCD sum rules. The asymmetry is however much more moderate numerically than QCD sum rules indicate.

In particular, one can calculate the valence quark distribution functions $u_p(x)$ and $d_p(x)$ at large $x$ in terms of the non-perturbative nucleon WF. Two properties of the WF are important to provide such a fit: The absence of the mass term in the formula (20) (this leads to the correct power behavior at $x \to 1$) and a moderate asymmetry in the longitudinal direction (this provides an observed ratio for the $\frac{u_p(x)}{d_p(x)} \simeq 5$ at $x \to 1$ in the contrast with the asymptotic formula prediction which gives value of 2 for the same ratio).

We already mentioned earlier that the similar conclusion is likely to have place for the pion wave function also. Therefore, the general moral, based on already completed calculations, can be formulated in the following way. There is a standard viewpoint for the phenomenological success of the dimensional counting rules: it is based on the prejudice that the leading twist contribution plays the main role in most cases. This outlook, as we mentioned earlier, is based on the experimental data, where the dimensional counting rules work very well. We suggest here some different explanation for this phenomenological success. Our explanation of the slow falling off of the soft contribution with energy is due to the specific properties of non-perturbative $\Psi(\xi, k_\perp^2)$. In particular, we argued that the absence of the of the mass parameter in the corresponding $\Psi(\xi, k_\perp^2)$ is the strict QCD constraint. At the same time this property is responsible for the behavior mentioned above. Besides that, we found a new scale ($\sim 1\text{GeV}^2$) in the problem, in addition to the standard low energy parameter $\langle \vec{k}^2\rangle \simeq 0.1\text{GeV}^2$. Both these phenomena lead to the temporarily mimicry of the leading twist behavior in the extent region of $Q^2$.

We believe that this is a new explanation of the phenomenological success of the dimensional counting rules at available, very modest energies. Besides that, as we shall see in the last section, the new scale we found has very natural explanation in terms of the specific vacuum configurations, instantons\cite{3,4,11,12}. They have small size $\sim 1/3\,\text{fm}$ which is very different from the hadron size $\sim 1\,\text{fm}$. The last scale is determined by the density of instantons. The new $1/3\,\text{fm}$ scale, we believe, should appear in the quark model. However, in the constituent quark model this new scale can be nothing, but the size of a
4.3 Hard diffractive electroproduction

In this section we apply our model WF for the study of the pre-asymptotic effect due to the Fermi motion in diffractive electroproduction of the $\rho$-meson. Our prime goal is to get an estimate for the onset of the asymptotic regime in this problem.

The applicability of perturbative QCD (pQCD) to the asymptotic limit of the hard diffractive electroproduction of vector meson was established in Ref. 50 using the light-cone perturbation theory. The authors of 50 have proved that for the production of longitudinally polarized vector mesons by longitudinally polarized virtual photons the cross section can be consistently calculated in pQCD. There was found that at high $Q^2$ the amplitude factorizes in a product of DA’s of the vector meson and virtual photon, the light-cone gluon distribution function of a target, and a perturbatively calculable on-shell scattering amplitude of a $q\bar{q}$ pair off the gluon field of the target. After the factorization of gluons from the target the problem seems to be tractable within OPE-like methods.

Let us start with the general form of the amplitude as a matrix element of the electromagnetic current $j_\mu = e(2/3\bar{u}\gamma_\mu u - 1/3\bar{d}\gamma_\mu d)$:

$$M = \epsilon_\mu \langle N(|p-r|)\rho(q+r)|j_\mu(q)|N(p)\rangle,$$

(34)

where $\epsilon_\mu$ is the polarization vector of the photon (only the longitudinal polarization is considered, see 50) and $r$ stands for the momentum transfer. We will neglect the masses of the nucleon and $\rho$-meson in comparison with the photon virtuality $Q^2$: $m^2_N, m^2_\rho \ll Q^2$. For the momentum transfer $r$ we consider the limit $r^2 = 0$, but $r_\mu \neq 0$. We next note the following. The factorization of the gluon field of the target 50, 51 means that that these gluons act as an external field on highly virtual quarks produced by the photon with $Q^2 \to \infty$. Retaining only the leading contribution, we arrive to the asymptotic formula of Ref. 50 in the form suggested in 52:

$$M = \frac{4\pi \sqrt{2} \alpha_s f_\rho}{N_c Q} \int_0^1 dX \frac{F^\rho_\xi(X) \sqrt{1 - \xi}}{X(X - \xi + i\delta)} \int_0^1 \frac{\phi(u)}{u\bar{u}},$$

(35)

where $\phi(u)$ is the standard $\rho$ meson light-cone DA and $F^\rho_\xi(X)$ is so-called asymmetric gluon distribution which becomes the usual gluon distribution function $X F_g(X)$ in the symmetric limit $\xi \to 0$.

The most important corrections to the asymptotic formula 52 are due to the quark transverse degrees of freedom (the Fermi motion). Calculating
only this contribution we make an educated guess on the scale of higher twist corrections in the diffractive electroproduction. Technically, this correction can be written in the following form:

\[
\sqrt{T(Q^2)} = Q^4 \frac{\int_0^1 du \int_0^{Q^2} d^2\vec{k}_1 \Psi(u, \vec{k}_1^2) \frac{1}{(Q^2 + k_1^2)/(u \bar{u})}}{\int_0^1 du \int_0^{Q^2} d^2\vec{k}_1 \Psi(u, \vec{k}_1^2)} \left(1 - 2 \frac{E_1^2/(u \bar{u})}{(Q^2 + k_1^2)/(u \bar{u})}\right)
\]

where \(\Psi(u, \vec{k}_1^2)\) was defined earlier in terms of the local matrix elements (14).

By definition, for the asymptotically large \(Q^2 \to \infty\) we have \(\sqrt{T(Q^2)} = 1\). Deviations from \(\sqrt{T(Q^2)} = 1\) determine a region of applicability of the asymptotic formula (35).

Now we are in position to discuss numerical estimates for the correction factor (36) in order to answer the main question formulated above. The authors of the original paper have observed that the choice of \(\Psi(u, \vec{k}_1^2)\) in a factorized form \(\Psi(u, \vec{k}_1^2) = \phi(u)\psi(\vec{k}_1^2)\) leads to a very slowly raising function \(\sqrt{T(Q^2)}\) which approaches 0.8 at rather high \(Q^2 \geq 40\) GeV\(^2\) depending on the model chosen for \(\Psi(u, \vec{k}_1^2)\).

Using the QCD motivated WF we see a very fast approach to the asymptotic regime for the hard diffractive electroproduction. The technical reason for this behavior is quite clear: large values of \(\vec{k}_1^2/(u \bar{u})\) are exponentially suppressed by the QCD motivated WF and thus these extra terms in the integral (36) give a small contribution for \(Q^2 > 10\) GeV\(^2\), see original paper\(^37\) for details.

Thus our final conclusion is that the onset of the asymptotic regime for diffractive electroproduction of the longitudinally polarized \(\rho\)-meson is approximately \(Q^2 \simeq 10\) GeV\(^2\) where corrections due to the quark transverse degree of freedom constitute less than 20 % in the amplitude. This can be traced back to the fact that in the case at hand the power corrections are given by the matrix elements of local operators and in fact are fixed completely by the independent calculation of the moments. This is the consequence of the structure of Eq.(36) and the fact that the WF is a function of the single variable \(\vec{k}_1^2/(u \bar{u})\). Only global, but not local characteristics of \(\Psi(u, \vec{k}_1^2)\) in the \(\vec{k}_1^2\) plane are important. This situation can be confronted with the case of exclusive processes. In the most well studied problem of the pion form factor the asymptotic regime has been found to be pushed further to \(Q^2 \gg 10\) GeV\(^2\)\(^{13,14}\), where the sub-leading ”soft” contribution is still larger than the leading asymptotic one. There are

\(^d\) We follow the notations of Ref\(^{53}\) by reserving the symbol \(T(Q^2)\) for the correction in the cross section.
no reasons to expect the onsets of the asymptotic regime to be similar in the
diffractive electroproduction and exclusive processes. On contrary, they differ
parametrically in $1/\alpha_s$. Moreover, the explicit calculations suggest that the
asymptotic regime in the $\rho$-meson diffractive electroproduction starts already
at $Q^2 \simeq 10 \text{ GeV}^2$. We stress that this conclusion refers only to the diffractive
electroproduction of the longitudinally polarized $\rho$-meson, the situation with
the transverse polarization or diffractive charmonium production can be quite
different.

4.4 Concluding remarks

To conclude this section we would like to make a comment regarding the shape
of the $\pi$ meson distribution amplitude. This issue has been discussing for quite
a while. We already mentioned at the end of section 2 that a direct extraction
of the corresponding information from the experimental data is an extremely
difficult (if possible at all) problem. Only indirect analysis is available at the
moment. The point is that the power corrections (which are very difficult to
estimate) could be very important at present energies. To be more specific,
and in order to clarify our point, let us consider amplitude $\gamma\gamma^* \rightarrow \pi$ which
has received some attention recently. Some theoretical calculations claim that the "two-hamp" wave function disagrees with CLEO results; other
calculations conclude that either the asymptotic or the "two-hamp" wave
function is sufficient to describe the data. Much more sophisticated method
which avoids any assumptions regarding a shape of the wave function, gives a
very good description of the experiment. However, in spite of the power and
generality of the method advocated in ref. it does not allow us to extract
an information regarding the shape of the wave function. In order to obtain
such an information we have to make some additional strong assumptions
regarding the power corrections (as well as perturbative corrections) within
the standard scheme of PQCD calculations originated in ref. Therefore, our
point is that a direct analysis of the available experimental data can not provide
an unambiguous information about the properties of the wave functions.

5 Instantons and the Constituent Quark Model.

Let us remind that the purpose of these lectures is twofold. First, these lectures
have strong applied (or phenomenological) direction. The main result obtained
in this direction could be formulated in the following way: there are many theo-
retical, phenomenological and experimental evidences that the “soft” contribu-
tion (being un-leading parametrically) nevertheless can temporarily mimic the
leading twist behavior in the extent region of $Q^2 : \ 3\text{GeV}^2 \leq Q^2 \leq 40\text{GeV}^2$. 

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This is due to the specific properties of $\Psi(\vec{k}_2^\perp, x)$ we have been discussing at length in these lectures. Such a mechanism, if it is correct, would be an explanation of the phenomenological success of the dimensional counting rules at available, very modest energies for many different processes. At the same time, the same properties of $\Psi(\vec{k}_2^\perp, x)$ applied to the hard electroproduction predict the onset of the asymptotic regime at the relatively small $Q^2 \approx 10 GeV^2$. We believe that these are very new results and very new understanding which deserve for the further studying.

The second goal is much more theoretical at the present level of understanding, but it may become a profound phenomenological tool in the hadronic physics in future (at least we hope so). We have in mind the QCD-based description of the WF properties within the instanton model. A reader probably has noticed that almost in each section while we discuss a result obtained in a phenomenological way, we also try to explain the same result in terms of the QCD vacuum structure which could provide such a behavior. We also have been trying to explain those results in terms of the specific (but important) vacuum configurations which could be responsible for the aforementioned properties. We believe that those vacuum configurations are instantons.

Now we would like to collect all relevant information from the previous sections here, in one place, in order to formulate our understanding of the quark model in terms of the instanton vacuum configurations. This presentation unavoidably can not be self-contained, but rather it is a very fragmentary one.

5.1 Instantons. Historical Remarks.

The instanton solution as the classical solution with nontrivial topology was invented in 1975 motivated by the discovery of the ’t Hooft-Polyakov monopole. Shortly after its discovery, the physical meaning of the instanton as a tunneling event between degenerate classical vacua was understood. We refer to a pedagogical review for details. Here we shortly introduce some relevant notations by emphasizing on the physical meaning of the instanton solution.

The starting point is the transition from Minkowski to Euclidean space. In this case the best tunneling path in gauge theory which connects topologically different classical vacua is the solution of classical equation of motion in Euclidean space. To find these solutions, it is convenient to exploit the following identity

$$S = \frac{1}{4g^2} \int d^4x G_{\mu\nu}^a G_{\mu\nu}^a = \frac{1}{4g^2} \int d^4x [\pm G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \frac{1}{2}(G_{\mu\nu}^a \mp \tilde{G}_{\mu\nu}^a)^2], \quad (37)$$

\footnote{See, in particular, a discussion after eq. (21); the end of section 3.2; a discussion after eq. (32); the end of section 4.2.}
where $\tilde{G}^a_{\mu\nu} = 1/2\epsilon_{\mu\nu\lambda\sigma}G^a_{\lambda\sigma}$. Since the first term is a topological charge $Q$ of the configuration and must be an integer, see \[ (38) \]

one can argue that the action is minimal if the field is self-dual or antiself-dual $G^a_{\mu\nu} = \pm \tilde{G}^a_{\mu\nu}$. We call these solutions as an instanton or antiinstanton correspondingly. From (37) we have

$$S = \frac{8\pi^2|Q|}{g^2}$$

for (anti)self-dual fields, implying that the tunneling probability is

$$P_{\text{tunneling}} \sim \exp \left( -\frac{8\pi^2|Q|}{g^2} \rho \right),$$

where the coefficient in front of the exponent is determined by a one-loop calculation. The corresponding calculations have been completed by ’t Hooft in 1976. We do not need in our discussions an explicit formula of the ’t Hooft calculations, however we do need to know few general results which follow from his formula, see \[ 55 \] for details:

1. Each instanton is accompanied by $4N_c$ zero modes ($N_c = 3$ is a number of colors), which are related to the so-called collective coordinates describing the field of the instanton. Those degrees of freedom are: center of the instanton $x_0$ (4 modes) size of the instanton $\rho$ (1 mode) orientations of the instanton in the color space $\Omega_i$ ($4N_c - 5$ modes).

2. Density of instantons $dn_I$, or what is the same, tunneling probability is divergent at large $\rho$:

$$P \sim dn_I d^4x_0 \sim \exp \left( -\frac{8\pi^2|Q|}{g^2(\rho)} \right) d^4x_0 d\rho \sim \frac{d\rho}{\rho^b} (\Lambda\rho)^b, \quad b = 11N_c/3 = 11. \quad (39)$$

Such a result is a consequence of the unjustified perturbative calculations of the $\beta$ function at large size $\rho$. We come back to this point later.

3. Each instanton is accompanied by the fermionic zero mode as a consequence of the very general so-called Index theorem.

5.2 Chiral symmetry breaking and instantons.

It is believed that the one of the most profound features of QCD, the chiral symmetry breaking phenomenon, can be explained by instantons. This idea has been under discussion for a quite a while. The modern approach to the problem has been suggested in \[ 16 \] see also review \[ 17 \]. We refer to the original papers on the subject for details, here we formulate the main ideas and results of this study.

As we mentioned above, in the presence of an instanton, there is an exactly one zero mode. This zero mode is the chiral mode: it is right-handed for
instantons and left-handed for antiinstantons. As we shall see, this fact has
the profound phenomenological consequences for many aspects of the theory:
a description of the chiral symmetry breaking phenomenon, the formation of
the chiral condensate, there appearance of a nonzero constituent quark mass
and many others.

Suppose we have infinitely separated \( I \) (instanton) and \( \bar{I} \) (antiinstanton)
which can be thought as two degenerate states with two zero modes. Like in
the standard quantum mechanical calculations, when we decrease a distance
between \( I \) and \( \bar{I} \), this degeneracy is lifted through the diagonalization of the
hamiltonian. As the result, the obtained two new states have non-zero eigen-
values \( \lambda_{\pm} \) which are equal to the overlap integral \( T_{II} \) between the original
states \( \lambda_{\pm} = \pm |T_{II}| \). When one adds more \( I \)'s and \( \bar{I} \)'s, each of them brings in
a would be zero mode. After the diagonalization they get split symmetrically
with respect to zero point \( \lambda = 0 \). Eventually, for an instanton-antiinstanton
ensemble one gets a continuous band spectrum with a spectral den-
sity \( \nu(\lambda) \) which is finite at \( \lambda = 0 \). Such a spectrum implies a nonzero magnitude for the
chiral condensate \( \langle \bar{q}q \rangle \sim \pi \nu(\lambda = 0) \).

Having explained the physical mechanism of the chiral symmetry bre-
aking phenomenon we proceed with the discussion of some quantities which are much
closer to the phenomenological needs of the hadronic physics. Specifically,
let us consider the quark propagator in the instanton vacuum we dis-
cussed above. The corresponding formula has been derived in ref. [16]. The result of
this calculation has the form of a massive propagator with a momentum-
dependent dynamical mass:

\[
S(k) = \frac{k + iM(k^2)}{k^2 + M^2(k^2)} , M(k) = M_0 F^2(k) , \quad M_0 \simeq 350 MeV
\]

\[
F(z) = 2z [I_0(z)K_1(z) - I_1(z)K_0(z) - \frac{1}{\pi} I_1(z)K_1(z)] , \quad F(0) = 1
\]

In this formula \( z = 1/2k\rho \) (\( \rho \) is the size of instanton, \( \bar{\rho} = 1/3 fm \)); \( M(k \to \infty) \sim \frac{6}{k\rho} \to 0 \) as it should due to the asymptotic freedom. However, at small
\( k \), the mass is not zero but finite number \( M(k \to 0) = M_0 \simeq 350 MeV \) which is
close to the phenomenological value for the constituent quark mass. The value
of the chiral condensate is also can be calculated in this model in a similar way
with the following result

\[
\langle \bar{q}q \rangle = i \int \frac{d^4k}{(2\pi)^4} Tr S(k) = -4N_c \int \frac{d^4k}{(2\pi)^4} \frac{M(k)}{M(k)^2 + k^2} \simeq -(255 MeV)^3 , \quad (42)
\]

which is again very close to the phenomenological magnitude. We conclude this
short historical introduction into instanton physics with the following general
remark: A systematic numerical study of various correlation functions, amplitudes, hadronic characteristics (like charge radius of a hadron, constant $f_\pi$, etc) in the instanton vacuum demonstrates a good agreement with experiments and phenomenology, see review 17. Similar conclusion has been recently obtained from direct lattice measurements 57. Therefore, it is fair to say that instantons explain the basic properties of light hadrons and large distance dynamics. With this optimistic note about instantons we proceed with the discussion of our main subject: the light cone wave functions.

5.3 Low energy: how to formulate the problem?

We start from the following simple observation: At low energies it is very difficult to define a WF in terms of the original quark and gluon fields. The problem is that infinite number of different matrix elements are equally important in such a would be definition. All of them are not zero and all of them have a standard hadronic scale $1\text{GeV}$. Therefore, there is no any special reason to select a quark-aniquark WF in comparison, let us say, a quark-aniquark and ten-gluon-field WF. To be more specific and in order to explain the main point, let us consider few possible choices. One could start from the simplest two-particle WF (we consider pseudoscalar WF for simplification) which is normalized to the following matrix element:

$$\langle 0 | \bar{d} i \gamma_5 u | \pi \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \simeq -\frac{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{f_\pi}.$$  \hspace{1cm} (43)

One could make one step further and add an arbitrary number of gluons into this definition:

$$\langle 0 | \bar{d} i \gamma_5 (ig\sigma_{\mu\nu}G^a_{\mu\nu})^n u | \pi \rangle = -\frac{2\langle \bar{q}(ig\sigma_{\mu\nu}G^a_{\mu\nu})^n q \rangle}{f_\pi}.$$  \hspace{1cm} (44)

where in the last equation we have used PCAC to reduce the matrix element to the vacuum condensates. All those vacuum condensates are not known exactly, but it is believed that they are not zero and have a normal hadronic scale which is about $1\text{GeV}$. One could define a WF, based on these matrix elements 44 in the same way as we did in eq.(43). none of them is better or worse than others at low energy. Moral of this exercise is simple: There is no any reason to start from the simplest WF at low energy, because an arbitrary number of gluons are not suppressed at all. Therefore, all of them go on the same footing.
5.4 High energy: twist classification

Situation at high energies, as we explained in Introduction is quite different: there is a twist classification which makes possible to select the leading twist WF uniquely. Specifically, for the $\pi$ meson WF it is given by eq.(31) with the axial Lorentz structure. All other WFs give parametrically smaller contribution $\sim (1/Q^2)^n$ to any amplitude. Therefore, we can neglect all of them except the leading ones, which by dimensional reasons, are exactly the WFs with a minimal number of constituents. In different words, precisely those WFs represent a constituent nature of the hadrons.

What happens when we go to the lower and lower energies? We do not have the complete answer on this question, but we hope we convinced a reader (see section 5.1) that the most important vacuum gluon fluctuations in hadronic low-energy physics are the instantons. Once we accept this, we know the answer on the question formulated above: The most important quantitative change which instantons make is the complete reconstruction of the relevant degrees of freedom: gluons have disappeared; instead a constituent quark with a momentum-dependent mass appears $M(k) = M_0 F^2(k)$.\(^{[10]}\)

5.5 Next steps.

Having explained the non-perturbative physics which is related to the instantons, we now in position to answer on the question (at least qualitatively) formulated above: what happens to the leading twist WFs when energy is getting smaller and smaller? Our proposal for the answer is: All those WFs acquire a momentum-dependent mass term. In particular, $\Psi(\vec{k}_\perp^2, u, \mu_0)_{QCD}^{[31]}$ takes the form:

$$\Psi(\vec{k}_\perp^2, u, \mu_0)_{QCD} = A \exp\left(-\frac{\vec{k}_\perp^2 + M^2(\vec{k}_\perp^2)}{8\beta^2 u\bar{u}}\right) \cdot \left\{1 + g(\mu_0)\left[\xi^2 - \frac{1}{5}\right]\right\}, \quad (45)$$

with function $M(z)$ given by eq.\((40)\). A similar replacement should be made for all WFs we have discussed previously. The main feature of this replacement is clear: At high energies when a typical $k$ is the same order of magnitude as the external $Q$, we have $M(\vec{k}_\perp^2 \to \infty) \to 0$. In this region, WF $^{[13]}$ transforms into the leading twist WF given by formulae $^{[31]}$. At small energies when $M(\vec{k}_\perp^2 \to M_0)$, our WF $^{[13]}$ transforms into the constituent quark model WF given by eq.\((28)\). Most important question: What happens to a variety of WFs we mentioned above $^{[44]}$ in such a transition? Where do they go?

Before to answer on this question, we should remark that a large magnitude for all hadronic matrix elements with gluons (similar to $^{[44]}$), is related to the
strong vacuum fields (which we identify specifically with instantons in this section) and not to a specific properties of a hadron. In the case of the $\pi$ meson this relation follows from the PCAC. For different hadrons a similar relation is not so obvious as for pion. Nevertheless, experience with $\rho$ meson shows
\footnote{We checked this explicitly only for the $\pi$ and $\rho$ mesons, but we believe that a situation will be the same for all hadrons.} that all gluonic matrix elements are large and they are related to the vacuum condensates, i.e. to the vacuum fields. If it is so, all would be the new WFs (similar to eq. (44)) describe nothing, but some vacuum fluctuations. The main effect related to those vacuum fluctuations is well known: a quark becomes a constituent quark with the momentum dependent mass $M(\vec{k})$. In different words, vacuum gluons are transformed into different powers of $\vec{k}$.

Therefore, the answer on the formulated above question regarding the numerous of WFs is: they transform into the only relevant degrees of freedom, the massive strongly interacting constituent quarks with dependent on $\vec{k}$ mass. In terms of these constituent quarks, an apparent variety of those WFs simply disappears. This effect can be explained using an analogy with the harmonic oscillator problem in the external electric field. As is known, in this case, the whole problem is reduced to the change of variable (one should make a shift of the coordinate on the amount which is proportional to the external electric field). It is exactly what happens in our case: variable $\vec{k}_\perp^2$ get shifted on the amount of $M(\vec{k}_\perp^2)$. Instanton fields are not constant fields, therefore, the value of a shift $\sim M$ depends on momentum $\vec{k}_\perp^2$. This analogy is even closer to the real situation, because a Gaussian dependence which we derived from the QCD earlier (see section 3.1) is exactly the solution of the Schrödinger equation for the harmonic oscillator potential.

Now we can reverse our arguments in order to explain the reason for the matrix elements (44) to be large. Those gluon fields $G_{\mu\nu}$ which are present in the definition of the hadronic matrix elements have nothing to do with the specific hadronic state, but rather, they are related to some strong vacuum (instanton) fluctuations. In this picture we also understand some relations, like (28) which have been obtained in a pure phenomenological way. Namely, the fluctuations in the transverse direction for the different hadrons alike and numerically those fluctuations are very large. In the picture we suggest this phenomenon has its natural explanation: All matrix elements which describe transverse momentum distribution are related to the vacuum (instanton) fields; therefore they are the same for any hadron. Secondly, large magnitude for the parameter $R$ (28) is related to the nonfactorizability of the vacuum condensates which is also has a natural explanation from the instanton point of view. New scale we discussed in the previous sections is nothing but the size of the in-
stanton $\bar{\rho} \sim 1/3 fm$ in the formula for $M(k)$. This scale is much smaller than the quark mass $M_0 \simeq 350 MeV$.

To end this section let me remark that WF (45) with the momentum-dependent mass may help to resolve many long-standing problems in the intermediate region of the nuclear and low-energy particle physics. With this hope we conclude our lectures.

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