Hiding cosmic strings in supergravity D-term inflation

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Abstract

The influence of higher-order terms in the Kähler potential of the supergravity D-term inflation model on the density perturbation is studied. We show that these terms can make the inflaton potential flatter, which lowers the energy scale of inflation under the COBE/WMAP normalization. As a result, the mass per unit length of cosmic strings, which are produced at the end of inflation, can be reduced to a harmless but detectable level without introducing a tiny Yukawa coupling. Our scenario can naturally be implemented in models with a low cut-off as in Type I or Type IIB orientifold models.
I. INTRODUCTION

Inflation in the early Universe not only realizes globally homogeneous and flat space but also provides the seeds of density perturbations \[1\]. To realize successful inflation which matches observational data of large-scale structures and anisotropy of cosmic microwave background radiation (CMB), the potential of the scalar field which drives inflation, the *inflaton*, must be very flat. The required flat potential may be realized with the help of supersymmetry or supergravity. In supersymmetric models, the scalar potential consists of the contribution from F-term and D-term. In F-term inflation models where the vacuum energy to drive inflationary expansion is dominantly provided by the F-term, the inflaton mass is in general of the same order of the Hubble parameter \(H\) during inflation, in other words, a slow roll parameter

\[ \eta \equiv \frac{V''[\sigma]}{V[\sigma]}, \]  

(1)
generally takes a value of the order of unity, although \(\eta \ll 1\) should be satisfied for successful inflation. Here \(V[\sigma]\) is the potential energy density of the inflaton, \(\sigma\), the prime denotes the derivative with respect to the inflaton field and we take the unit with \(8\pi G = 1\) throughout this paper. It is therefore difficult to realize a sufficiently long expansion to solve the horizon and flatness problems. This is the so-called the \(\eta\) problem of inflation models in supergravity.

On the other hand, D-term inflation, where the vacuum energy is provided by D-term, does not suffer from the problem \[2\]. Hence, D-term inflation appears more attractive than F-term inflation from this point of view.

However, it has been revealed that the D-term inflation model also has some problems \[3\]. For example, from the observational point of view, the cosmic strings generated after inflation significantly affect the spectrum of CMB anisotropy \[4\], because this is a kind of hybrid inflation model \[5\]. In addition, one may suspect the potential for D-term inflation not to be valid, because the inflaton needs to have a large initial value of the order of (sub-)Planck scale for a natural model parameter \[6\]. Hence, D-term inflation seems to be under strong pressure. In addition, in the framework of (heterotic) string models, there are two more problems, namely, the runaway behavior of dilaton and too large a magnitude of Fayet-Illiopoulos (FI) term. (See, however, \[7\].)

In this paper, we investigate effects of higher-order terms in the Kähler potential in D-term inflation. So far, little attention has been paid to the effects of the Kähler potential
in the dynamics of D-term inflation, because these terms do not disturb the flatness of the potential unlike in F-term inflation models. However, we should clarify the effects of higher-order terms in the Kähler potential, since the inflaton must take a large initial value close to the Planck scale. We study the effects of higher-order terms in the Kähler potential, unlike Rocher and Sakellariadou who studied D-term inflation recently taking only the leading term in the Kähler potential into account [8].

As we will show, these terms alter the dynamics of the inflation and the resultant constraints on the model parameter, especially the magnitude of FI term. Hence, as the result, the predicted mass per unit length of cosmic strings can be reduced and meet the observational constraints without assuming a very small Yukawa coupling as in previous works [8, 9].

The rest of this paper is organized as follows. After reviewing the D-term inflation model in the minimal supergravity in the next section, in Secs. III and IV, we study the effects of two types of higher-order terms separately. Section V is devoted to conclusions.

II. D-TERM INFLATION

Here we give a brief review on the D-term inflation in minimal supergravity in order to make the problem clear and help to compare it with our model in the following section. We consider the $ \mathcal{N} = 1 $ supersymmetric model with $ U(1) $ gauge group and the non-vanishing FI term $ \xi $. The minimal model contains three matter fields $ S $ and $ \phi_{\pm} $. The fields $ \phi_{\pm} $ have $ U(1) $ charges $ q_{\pm} = \pm 1 $ such that $ \xi > 0 $, while $ S $ is neutral for the $ U(1) $. Suppose the following Kähler potential and superpotential,

$$ K = |S|^2 + |\phi_{+}|^2 + |\phi_{-}|^2, \quad \text{(2)} $$

$$ W = \lambda S \phi_{+} \phi_{-}. \quad \text{(3)} $$

Then, the scalar potential is written as

$$ V = \lambda^2 e^{\frac{1}{2}(|S|^2 + |\phi_{+}|^2 + |\phi_{-}|^2)} \left[ |\phi_{+} \phi_{-}|^2 + |S \phi_{-}|^2 + |S \phi_{+}|^2 + (|S|^2 + |\phi_{+}|^2 + |\phi_{-}|^2 + 3)|S \phi_{+} \phi_{-}|^2 \right] $$

$$ + \frac{g^2}{2} \left( \xi + |\phi_{+}|^2 - |\phi_{-}|^2 \right)^2, \quad \text{(4)} $$

where $ g $ is the gauge coupling. The true vacuum of this potential corresponds to

$$ S = \phi_{+} = 0, \quad |\phi_{-}| = \sqrt{\xi}. \quad \text{(5)} $$
For a large value of $S$,

$$|S| > S_c \equiv \frac{g}{\lambda} \sqrt{\xi}, \quad (6)$$

the potential has the local minimum with a non-vanishing vacuum energy density

$$V_0 = \frac{g^2}{2} \xi^2 \quad (7)$$

at

$$|\phi_\pm| = 0, \quad (8)$$

which minimizes the potential in this regime. Then, the radial part of $S$ is a flat direction but it acquires a non-vanishing potential through radiative corrections, for supersymmetry is broken due to the non-vanishing D-term. In this regime the scalar fields $\phi_\pm$ have masses

$$m_\pm^2 = \lambda^2 |S|^2 e^{\pm |S|^2} \pm g^2 \xi, \quad (9)$$

while the mass-squared of their fermionic partner is simply given by $\lambda^2 |S|^2 e^{\pm |S|^2}$. As a result the one-loop effective potential is given as

$$V_{1\text{-}\text{loop}} = \frac{g^2}{2} \xi^2 \left( 1 + \frac{g^2}{8\pi^2} \ln \frac{\lambda^2 |S|^2 e^{\pm |S|^2}}{\Lambda^2} \right), \quad (10)$$

for a large field value $|S|^2 \gg g^2 \xi / \lambda$, where $\Lambda$ is a renormalization scale.

Without loss of generality we can identify the real part of $S$, $\sigma \equiv \sqrt{2} \text{Re} S$, as the inflaton. Thus, the inflaton slowly rolls down the potential from a large initial value during inflation. When the inflaton reaches

$$\sigma_c \equiv \sqrt{2} S_c = \frac{\sqrt{2} g}{\lambda}, \quad (11)$$

where $\phi_-$ becomes tachyonic or

$$\sigma_f \equiv \frac{g}{2\pi}, \quad (12)$$

which corresponds to $\eta = V''[\sigma]/V_0 = -1$, the inflation terminates. Unless the Yukawa coupling $\lambda$ is extremely small, $\lambda \lesssim 10^{-4}$, inflation terminates when the inflaton arrives at $\sigma_f$. In the late stage of inflation when $e^{\sigma^2/2} \approx 1$, the inflaton evolves as

$$\frac{\sigma^2}{2} - \frac{\sigma_e^2}{2} = \frac{g^2}{4\pi^2} N, \quad (13)$$

where $\sigma_e = \max(\sigma_f, \sigma_c)$ is the field value at the end of inflation and $N$ is the number of e-folds acquired between $\sigma$ and $\sigma_e$. For $N = 50 - 60$ and a natural value of gauge coupling
$g$, the right-hand side of Eq. (13) is $O(0.1) - O(1)$. This means that the inflaton must take a large field value of the order of sub-Planckian scale.

In the case $\sigma_e = \sigma_f$, by using Eq. (13), the amplitude of the comoving curvature perturbation is given as

$$P_\zeta^{1/2} \equiv \frac{H^2}{2\pi|\sigma|} = \xi \sqrt{\frac{N}{3}} = 4.7 \times 10^{-5} \left( \frac{\xi}{1.1 \times 10^{-5}} \right) \left( \frac{N}{55} \right)^{1/2}.$$  \hspace{1cm} (14)

Thus, the required magnitude of FI term $\xi$ to generate the appropriate amplitude of the density perturbation is estimated as $\xi = 1.1 \times 10^{-5}$, in other words,

$$\sqrt{\xi} \simeq 3.3 \times 10^{-3} \simeq 7.6 \times 10^{15}\text{GeV}. \hspace{1cm} (15)$$

After inflation, the cosmic string with the mass per unit length $2\pi\xi$ is generally formed, although an exceptional model has been proposed [10]. These cosmic strings potentially affect the density perturbation, indeed, this fact is a fatal shortcoming for D-term inflation model [4]. The constraints on cosmic strings have been studied [9, 11]. According to a recent study [9], the constraints on the magnitude of the FI term is derived as

$$\sqrt{\xi} \lesssim 1.9 \times 10^{-3}, \hspace{1cm} (16)$$

and obviously conflicts with Eq. (15).

It has been proposed that this problem might be avoided by considering the other case $\sigma_e = \sigma_c > \sigma_f$ with a very small Yukawa coupling, $\lambda \lesssim 10^{-4}$ [9]. This, however, does not entirely solve the problem, because taking a small $\lambda$ results in a larger value of $S_c$. If its value exceeds unity, the supergravity effect or the exponential factor in the potential plays an important role. Then the above formula of curvature perturbation can no longer be used and we would find a blue-tilted spectrum. As a result, we again need a larger value of $\xi$ to generate the density perturbation with the appropriate magnitude. Indeed, the only way to escape from this difficulty is to adopt an anomalously small value of $g \lesssim 10^{-2}$ [8], as is seen from the relation

$$S_c = \frac{\sqrt{\xi}g}{\lambda} = 1 \left( \frac{\sqrt{\xi}}{10^{-3}} \right) \left( \frac{10^{-5}}{\lambda} \right) \left( \frac{g}{10^{-2}} \right) \lesssim 1. \hspace{1cm} (17)$$
III. D-TERM INFLATION WITH HIGHER-ORDER COUPLING BETWEEN
THE INFLATON AND THE CHARGED FIELDS

In this section, we take higher-order terms in the Kähler potential into account in analyses
of the dynamics of D-term inflation. Since we focus on the inflation regime when \( \phi_+ \) and
\( \phi_- \) are small, the following Kähler potential can be regarded as a sufficiently general one.

\[
K = |S|^2 + |\phi_+|^2 + |\phi_-|^2 + f_+ (|S|^2)|\phi_+|^2 + f_- (|S|^2)|\phi_-|^2, \tag{18}
\]

where \( f_\pm (|S|^2) \) are arbitrary functions of \(|S|^2\). One can also add higher-order terms contain-
ing \( S \) alone, but this can be absorbed by a field redefinition of \( S \). Here we do not consider
such terms and assume that \( S \) has a canonical kinetic term at \( \phi_\pm = 0 \). The effect of such a
redefinition will be studied separately in the next section.

With the superpotential Eq. (3), the Lagrangian density is given by

\[
\mathcal{L} = \mathcal{L}_{\text{kin}} - V \tag{19}
\]

with

\[
\mathcal{L}_{\text{kin}} = - (\partial S, \partial \phi_+, \partial \phi_-)
\times \left( \begin{array}{ccc}
1 + (f_+ + f''_+ |S|^2) |\phi_+|^2 + (f'_+ + f''_+ |S|^2) |\phi_-|^2 & f'_+ S^* \phi_+ & f''_+ S^* \phi_+ \\
f'_+ \phi_+ & 1 + f_+ & 0 \\
f'_- S \phi_- & 0 & 1 + f_-
\end{array} \right) \left( \begin{array}{c}
\partial S^* \\
\partial \phi_+^* \\
\partial \phi_-^*
\end{array} \right), \tag{20}
\]

and

\[
V = V_F + V_D, \tag{21}
\]

\[
V_F = \lambda^2 \frac{e^K}{\det K_i} \left[ \{(1 + f_+)(1 + f_-) - [(1 + f_+)(1 + f_-)]' |S|^2 \} |\phi_+ \phi_-|^2 \\
+ \{(1 + f_-)(1 + f''_- |S|^2) |\phi_+|^2 \} + [(1 + f_-)(f'_+ + f''_- |S|^2) - f'_+ f''_- |S|^2] |\phi_-|^2 + f'_+ f''_- |S \phi_-|^2 \right] |S \phi_-|^2 \\
+ \{(1 + f_+)(1 + f''_+ |S|^2) |\phi_-|^2 \} + [(1 + f_+)(f'_+ + f''_+ |S|^2) - f'_+ f''_+ |S \phi_+|^2] |\phi_+|^2 + f'_+ f''_+ |S \phi_+|^2 \right] |S \phi_+|^2 \\
- 3 \lambda^2 e^K |S \phi_+ \phi_-|^2, \tag{22}
\]

\[
V_D = \frac{g^2}{2} (q_+ (1 + f_+)|\phi_+|^2 + q_- (1 + f_-)|\phi_-|^2 + \xi) \tag{23}
\]

where

\[
f'_\pm \equiv \left. \frac{d f_\pm (x)}{dx} \right|_{x = |S|^2}, \quad f''_\pm \equiv \left. \frac{d^2 f_\pm (x)}{dx^2} \right|_{x = |S|^2}. \tag{24}
\]
Here the determinant of the Kähler metric is given by

\[
\det K = [1 + (f'_+ + f''_+ |S|^2)|\phi_+|^2 + (f'_- + f''_- |S|^2)|\phi_-|^2] (1 + f_+)(1 + f_-) \\
-(1 + f_+)f''_- |S\phi_-|^2 - (1 + f_-)f''_+ |S\phi_+|^2.
\] (25)

Here and hereafter, we assume that both \(f_+\) and \(f_-\) are well behaved in the sense that the kinetic terms have the correct signature.

Equations of motion of \(\phi_\pm\) are given by

\[
-(1 + f_\pm)\partial^2 \phi_\pm - 2f'_\pm S^* \partial S \partial \phi_\pm - f''_\pm \phi_\pm \partial^2 S - f''_\pm (S^* \partial S)^2 + \frac{\partial V}{\partial \phi_\pm} = 0. \tag{26}
\]

The second, third, and fourth terms are additional terms through the higher-order terms and regarded as the additional friction term and the additional mass terms, respectively. Since both are negligible during inflation by the slow roll conditions \(|\dot{S}/S| \ll H\) and \(|\ddot{S}/\dot{S}| \ll H\), they do not affect the dynamics of inflation. Then, the equation of motion of \(\phi_+\) is given as

\[
(1 + f_+)(\ddot{\phi}_+ + 3H \dot{\phi}_+ + q_+ g^2 \xi \phi_+) + e|S|^2 \frac{\lambda^2 |S|^2}{(1 + f_-)} \phi_+ \simeq 0, \tag{27}
\]

near \(\phi_+ = 0\), and \(\phi_+\) settles at \(\phi_+ = 0\). Similarly, the equation of motion of \(\phi_-\) is given as

\[
(1 + f_-)(\ddot{\phi}_- + 3H \dot{\phi}_- + q_- g^2 \xi \phi_-) + e|S|^2 \frac{\lambda^2 |S|^2}{(1 + f_+)} \phi_- \simeq 0. \tag{28}
\]

Now the critical point \(S_c\) at which \(\phi_- = 0\) becomes unstable is given by a solution of

\[
e|S|^2 \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)} = -q_- g^2 \xi. \tag{29}
\]

If we assume \(f_\pm\) are small for small \(S\), \(S_c\) is practically the same as Eq. (11) for \(\lambda = \mathcal{O}(1)\) because \(\xi \ll 1\). Thus, we find that inflation would successfully proceed in the same manner as the simple model which we have reviewed in the previous section.

Next, we include the radiative correction by \(\phi_\pm\) and derive the one-loop effective potential. Here, we should notice that \(\phi_\pm\) are not canonical any longer owing to the mixing terms in the Kähler potential. While the effective masses of charged scalar fields \(\phi_\pm\) is given as

\[
m^2_{\phi_\pm} = q_\pm g^2 \xi + e|S|^2 \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)} \tag{30}
\]

for the canonically normalized variables \(\varphi_\pm\), the mass of their fermionic superpartners is given as

\[
m^2_{\text{fermion}} = e|S|^2 \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)} \tag{31}
\]
for the canonical variable. The correction from one-loop contributions to the potential is expressed as

$$\delta V = \frac{1}{32\pi^2} \left( \frac{q + g^2 \xi + e|S|^2}{(1 + f_+)(1 + f_-)} \right)^2 \ln \left( \frac{q + g^2 \xi + e|S|^2}{\Lambda^2} \right)$$

We can identify the real part of $S$, $\sigma = \sqrt{2\text{Re}S}$, with the inflaton as before and we obtain the one-loop effective potential

$$V_{1-loop}(\sigma) = \frac{g^2 \sigma^2}{2} \left( 1 + \frac{g^2}{8\pi^2} \left[ \ln \left( \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)\Lambda^2} + \frac{\sigma^2}{2} \right) \right] \right),$$

with $f_\pm = f_\pm (\sigma^2/2)$ for a large field value of $\sigma$. The last term comes from the exponential factor of the mass of charged fields.

Field equations during inflation with $\phi_\pm = 0$ read

$$H^2 = \frac{1}{3} \frac{g^2 \xi^2}{2},$$

and

$$\ddot{\sigma} + 3H \dot{\sigma} + \frac{g^2 \xi^2}{2} \frac{g^2}{8\pi^2} \left( \frac{2}{\sigma} - \frac{f_+\sigma}{1 + f_+} - \frac{f_-\sigma}{1 + f_-} + \sigma \right) = 0.$$}

The additional new terms from the higher coupling between the inflaton and charged fields in the Kähler potential can be important when $f_\pm \sigma(1 + f_\pm)$ are comparable to $2/\sigma$ or $\sigma$. Here the suffix $\sigma$ denotes differentiation with respect to $\sigma$. In order for $\sigma$ to evolve towards $\sigma_e$,

$$V'[\sigma] = \frac{g^2 \xi^2}{2} \frac{g^2}{8\pi^2} \left( \frac{2}{\sigma} - \frac{f_+\sigma}{1 + f_+} - \frac{f_-\sigma}{1 + f_-} + \sigma \right),$$

should be positive definite. The amplitude of the comoving curvature perturbation in this

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1 However, if the local maximum is located on a larger field value $\sigma > \sigma_{\sim 55}$, it might not be necessary for this requirement to be strict. In fact, the eternal inflation scenario could be realized around the very flat hill [12]. One may find a similar discussion in Ref. [13].
case reads
\[ P_{\zeta}^{1/2} = \frac{H^2}{2\pi|\dot{\sigma}|} = \frac{g^3 \xi^3}{4\sqrt{6\pi}|V'[\sigma]|} \]
\[ = \frac{4\pi \xi}{\sqrt{6}g} \left( \frac{2}{\sigma} + \frac{f_{+}\sigma}{1 + f_{+}} - \frac{f_{-}\sigma}{1 + f_{-}} \right)^{-1}, \tag{37} \]
under the slow-roll approximation.

First let us incorporate lowest order correction to the Kähler potential,
\[ f_{\pm} = \frac{c_{\pm}}{2} \sigma^2, \tag{38} \]
with \( c_{\pm} \) being positive constants. If we take \( c_{+} = c_{-} \equiv c \), the positivity condition \( V'[\sigma] > 0 \) requires \( c < 3 + 2\sqrt{2} \). Conversely, if we take \( c = 3 + 2\sqrt{2} \approx 5.83 \), \( V'[\sigma] \) vanishes at \( \sigma = \sqrt{2(\sqrt{2} - 1)} \approx 0.91 \). This means that if we take \( c \) slightly smaller than the critical value \( 3 + 2\sqrt{2} \) the potential is very flat near \( \sigma \approx 0.91 \). Then the amplitude of density fluctuation is enhanced due to this flatness of the potential and we can achieve the desired amplitude, \( \approx 10^{-5} \) with smaller values of \( \xi \). From \( (37) \) we find that if we take \( c = 5.5 \) and \( g = 0.7 \), the amplitude of curvature fluctuation meets the CMB normalization with a small enough value of \( \xi \), \( \xi = 2.7 \times 10^{-6} \), as is seen in Figs. 1 and 2, where the spectral index takes \( n_s \approx 0.96 \).

Next we consider the case \( c_{+} \neq c_{-} \). Without loss of generality we can assume that \( c_{+} > c_{-} \). In this case the potential of \( \sigma \) can remain monotonic even if \( c_{+} \) is larger than \( 3 + 2\sqrt{2} \). In particular, if we take \( c_{-} = 0 \), \( c_{+} \) can be arbitrary large.

For small \( \sigma \), \( \sigma \ll \sqrt{2/c_{+}} \), the solution of the equation of motion is the same as Eq. \( (33) \) with \( \sigma_{e} = \sigma_{f} \), since we consider the case of \( \lambda = O(1) \). On the other hand, for a region of a large field value, \( \sigma \gg \sqrt{2/c_{+}} \), the effect from the coupling in the Kähler potential becomes significant. For \( \sigma \gg \sqrt{2/c_{+}} \), the slow-roll equation \( (33) \) is rewritten as
\[ 3H\dot{\sigma} + \frac{g^2 \xi^2}{2} \frac{g^2}{8\pi^2} \left( \frac{2}{c_{+}^{2} \sigma^2} - \frac{c_{-}\sigma}{1 + c_{+}^{2} \sigma^2} + \sigma \right) \approx 0. \tag{39} \]
Here, as we expect, the additional term with \( c_{+} \) makes the potential flatter by cancelling out the leading term \( 2/\sigma \) in \( V'[\sigma] \). The approximate solution is given as
\[ \frac{\sigma^4}{4} - \frac{\sigma_{e}^4}{4} = \frac{g^2}{4\pi^2 c_{+}} (N - N_{*}), \tag{40} \]
for the region where the first term dominates other terms in \( V'[\sigma] \), where \( \sigma_{*} \approx \frac{\sqrt{2}}{c_{+}} \) and the corresponding number of e-folds \( N_{*} \approx 4\pi^2/(g^2 c_{+}) \).
FIG. 1: The amplitude of density perturbation for parameters \( g = 0.7, c_+ = c_- = 5.5 \) and \( \xi = 2.7 \times 10^{-6} \). A horizontal axis represents the number of e-folds \( N \) and a vertical axis represents the amplitude of density perturbation \( \mathcal{P}_{\zeta}^{1/2} \). \( N \sim 55 \) would correspond to the present horizon scale.

FIG. 2: The spectral index for the same parameters in Fig.1. A horizontal axis represents the number of e-folds \( N \) and a vertical axis represents the spectral index \( n_s \).

We turn to the generated density perturbation. Here we consider the case that the present horizon scale corresponds to a large field value of \( \sigma, \sigma^2 \gg 2/c_+ \). Then the amplitude of the
FIG. 3: The amplitude of density perturbation for parameters $g = 0.9$, $c_+ = 8$, $c_- = 3$ and $\xi = 2.7 \times 10^{-6}$. A horizontal axis represents the number of e-folds $N$ and a vertical axis represents the amplitude of density perturbation $\mathcal{P}_\zeta^{1/2}$. $N \sim 55$ would correspond to the present horizon scale.

density perturbation is estimated as

$$\mathcal{P}_\zeta^{1/2} = \frac{\sqrt{g^2 \xi^2}}{2\pi} \left( \frac{2}{c_+ \sigma^2} \right)^{1/2} \left( 1 - \frac{c_-}{1 + \frac{c_-}{2\sigma^2}} \frac{c_+ \sigma^4}{4} + \frac{c_- \sigma^4}{4} \right)^{-1} \quad (41)$$

$$\simeq \xi \sqrt{N} \left( \frac{4g^2 c_+}{\pi^2} N \right)^{1/4} \quad (42)$$

$$\simeq 4.5 \times 10^{-5} \left( \frac{\xi}{3.6 \times 10^{-6}} \right) \left( \frac{N}{55} \right)^{1/2+1/4} \left( \frac{g^2}{0.8} \right)^{1/4} \left( \frac{c_+}{8} \right)^{1/4} \quad (43)$$

where we omitted the $(...)^{-1}$ factor after Eq. (42) for simplicity. Comparing Eq. (42) with Eq. (14), we find that, if $(2g^2 c_+ N / \pi^2)$ is greater than unity, the factor enhances the amplitude of the density perturbation. Indeed, as is shown in Eq. (43), it is possible for natural values of parameters. Then, the magnitude of FI term is significantly reduced and satisfies the condition Eq. (16) from the cosmic string constraint. The results of more exact numerical analysis are shown in Figs. 3 and 4.

So far we considered the case $f_\pm$ has only the lowest-order term $f_\pm = c_\pm \sigma^2 / 2$. One can also incorporate even higher-order terms in $f_\pm$ such as $f_\pm = c_{n\pm} (\sigma^2 / 2)^n$ with $n \geq 2$. These terms can also help to realize flatter potential. At the same time their coefficients $c_{n\pm}$ are also constrained from the requirement that the potential should be monotonic. For example,
FIG. 4: The spectral index for the same parameters in Fig.3. A horizontal axis represents the number of e-folds $N$ and a vertical axis represents the spectral index $n_s$.

if we take $n = 2$, $c_{2+} = C$, and $c_{2-} = 0$, $C$ should satisfy

$$C \leq \frac{11 + \sqrt{125}}{2} \simeq 11. \quad (44)$$

IV. HIGHER-ORDER TERMS OF THE INFLATON SUPERFIELD

Here we study the case the Kähler potential for the inflaton superfield $S$ has a non-minimal structure. Specifically we study the case $K$ is given by a well behaved function $h(|S|^2)$ as

$$K = h(|S|^2) + |\phi_+|^2 + |\phi_-|^2, \quad (45)$$

with the same superpotential $W = \lambda S \phi_+ \phi_-$. The scalar Lagrangian reads

$$\mathcal{L} = -(h' + h''|S|^2)|\partial S|^2 - |\partial \phi_+|^2 - |\partial \phi_-|^2 - V_F - V_D, \quad (46)$$

$$V_F = \lambda^2 e^K \frac{(1 + h'|S|^2)^2}{h' + h''|S|^2} |\phi_+\phi_-|^2 + |S\phi_+|^2 + |S\phi_-|^2 + |S\phi_+\phi_-|^2 + |S|^2(|\phi_+|^4 + |\phi_-|^4),$$

$$V_D = \frac{g_2}{2}(q_+|\phi_+|^2 + q_-|\phi_-|^2 + \xi)^2,$$

where

$$h' = \frac{dh(x)}{dx}\bigg|_{x=|S|^2}, \quad h'' = \frac{d^2h(x)}{dx^2}\bigg|_{x=|S|^2}. \quad (47)$$
We consider the case $S$ serves as the inflaton and $\phi_{\pm}$ settle down to zero during hybrid inflation. As before let us identify the real part of $S$, $\sigma = \sqrt{2}\text{Re}\, S$ with the inflaton. The equation of motion for its homogeneous part reads

$$\left(h' + \frac{\sigma^2}{2}h''\right)\ddot{\sigma} + 2\left(\sigma h'' + \frac{\sigma^3}{4}h'''ight)\dot{\sigma}^2 + 3H\left(h' + \frac{\sigma^2}{2}h''\right)\dot{\sigma} + V'_{\text{1-loop}}[\sigma] = 0. \tag{48}$$

Here $V'_{\text{1-loop}}[\sigma]$ is the derivative of the scalar potential after one-loop correction given in Eq. (33). During slow-roll inflation the first two terms in Eq. (48) are negligible. The amplitude of quantum fluctuation acquired during the Hubble time, $\delta\sigma$ is given by

$$\delta\sigma = \left(h' + \frac{\sigma^2}{2}h''\right)^{-1/2}H \frac{2\pi}{\xi}, \tag{49}$$

due to the non-canonical kinetic term of $\sigma$.

From Eqs. (48) and (49) we find that the amplitude of comoving curvature perturbation is given by

$$P^{1/2}_\zeta = \frac{3H^3}{2\pi|V'_{\text{1-loop}}[\sigma]|} \left(h' + \frac{\sigma^2}{2}h''\right)^{1/2}. \tag{50}$$

Thus one could obtain a larger amplitude of perturbation with the same $\xi$ is the factor $h' + \frac{\sigma^2}{2}h''$ takes a value larger than unity and this may also help to reduce $\xi$.

Finally, we consider the case of the most general Kähler potential

$$K = h(|S|^2) + |\phi_+|^2 + |\phi_-|^2 + f_+ (|S|^2)|\phi_+|^2 + f_- (|S|^2)|\phi_-|^2. \tag{51}$$

Eqs. (30) and (31) are replaced with

$$m^2_{\phi_{\pm}} = q_{\pm} g^2 \xi + e^{h(|S|^2)} \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)} \tag{52}$$

and

$$m^2_{\text{fermion}} = e^{h(|S|^2)} \frac{\lambda^2 |S|^2}{(1 + f_+)(1 + f_-)}. \tag{53}$$

Repeating similar calculations, we obtain

$$P^{1/2}_\zeta = \frac{4\pi \xi}{\sqrt{6g}} \left(\frac{2}{\sigma} + \sigma h' - \frac{f_+}{1 + f_+} - \frac{f_-}{1 + f_-}\right)^{-1/2} \left(h' + \frac{\sigma^2}{2}h''\right)^{1/2}, \tag{54}$$

which shows the above mentioned two features, namely, the enhancement by the mixing terms $f_{\pm}$ and the canonical normalization of the inflaton. Thus these effects can cooperate with each other to further reduce the constraint on $\xi$.

For a specific example, if $h(|S|^2)$ contain higher-order terms such as $|S|^{2+2n}$, ($n = 1, 2, \ldots$), in addition to the canonical part $|S|^2$, they will help to allow a lower value of $\xi$. 

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as long as their coefficients are positive. If some of the higher-order term had a negative contribution, the resultant amplitude of perturbation might be somewhat lowered. Such cases with negative coefficients may well result in positive non definite kinetic term causing instability. Hence it is unlikely that these cases are phenomenologically viable.

V. CONCLUSION

In this paper, we have shown that the higher-order terms in the Kähler potential in general affect the dynamics of hybrid inflation and the generated density perturbation significantly. Although these terms do not lead the \( \eta \) problem, they alter the slope of the potential and the dynamics of the inflaton. This means that it is indispensable for a quantitative analysis of D-term inflation to take account of these terms.

Note that the modification is caused by the higher term with a coefficient \( c_+ \) of the order of unity. It means that these higher terms become more significant as the cut-off scale of higher dimensional operators decreases, because the coefficients of these term \( c_\pm \) in the Kähler potential is replaced by \( c_\pm /M^2 \) with a cut-off scale \( M \ll 1 \). For example, this situation can be realized in Type I or Type IIB orientifold models \[14, 15\], since the string scale is not necessarily the Planck scale in these theories. As another noteworthy fact, in the case that the leading term for the inflaton in the Kähler potential \( |S|^2 \) is replaced with \( |S|^2 /M^2 \), the slow roll parameter \( \eta \) is given as

\[
\eta \simeq \frac{g^2}{8\pi^2} \frac{1}{M^2} = 1 \left( \frac{g^2}{0.8} \right) \left( \frac{10^{-1}}{M} \right)^2 ,
\]

hence we find that the D-term inflation is also faced with \( \eta \) problem by a supergravity effect and is impossible for such a low cut-off model.

The higher-order terms in the Kähler potential can make the potential for the inflaton flatter than logarithmic. The flat potential enables the reduced FI term to accomplish the generation of the appropriate density perturbation and yields the different constraint on the magnitude of the FI term. Then, the influence of the cosmic string formed after inflation on CMB spectrum can be suppressed to an acceptable level.

Thus, we can conclude that the D-term inflation model can be consistent with the absence of CMB signature from cosmic strings, even if the Yukawa coupling \( \lambda \) is not extremely small. Remarkably, our proposal predicts the existence of cosmic strings unlike other solutions to
the cosmic string problem in the literature where the model was modified so as not to form cosmic strings \[10, 15\]. Since cosmic strings with \( G_{\mu} = \mathcal{O}(10^{-7}) \) in our model is detectable, our model is testable by observations in near future. Although the model with a very small Yukawa (and a gauge, if acceptable,) coupling also predicts the existence of cosmic strings, these models are distinguishable because they predict different spectral indexes. While the spectral index of the model with a very small Yukawa coupling \[8, 9\] is estimated as

\[
n_s - 1 \simeq -\frac{\lambda^2}{4\pi^2\xi} = -0.0003 \left( \frac{\lambda}{10^{-4}} \right)^2 \left( \frac{10^{-6}}{\xi} \right), \tag{56}
\]

that of our model is \( n_s \sim 0.97 \) as in Figs. 2 and 4.

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