Application of high-order differential energy operator in bearing fault diagnosis

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Abstract. An alternative energy operator demodulation method named high-order differential energy operator is proposed in this paper. The high-order differential energy operator, unlike previous demodulation techniques, can detect the weak fault bearing signature from a heavily contaminated signal because it can increase the signal-to-noise ratio. Furthermore, this energy operator is also able to suppress vibration interferences. Thus, the operator is more robust than the classical Teager energy operator (TEO) and its improved versions (DESA-1 and DESA-2 for short). Besides, it is also a non-parameter and non-filtering method, so it is easy and straightforward to apply. Finally, the built-in amplitude demodulation (AD) capability eliminates the enveloping step required by most AD methods like TEO and Hilbert transform (HT). The results of simulation tests and bearing fault experiments demonstrate that this method can effectively extract fault features, certifying its superiority in comparison with previous demodulation methods. Therefore, it is more likely to be useful and practical in the field of bearing fault diagnosis, especially in the presence of intense noise and interferences.

1. Introduction
It is well known that bearings play an essential role in rotating machinery and vibration machinery such as electric motors, wind turbines, and vibrating screen. Meanwhile, bearings are one of the most failure-prone parts owing to severe working condition and heavy load. However, bearing fault signals are so weak that it is impossible to detect the fault signatures directly due to pollution of intense noise and vibration interferences from other vibration resources.

Fortunately, with the development of the non-linear signal processing techniques, this problem has been primarily studied, and many viable solutions have been found. Some of the popular fault detection methods include Wavelet Transform-based methods [1, 2], Mode Decomposition-based methods [3-5], Fast Kurtogram [6, 7], Cyclostationary analysis [8, 9].

As we know, the bearing fault characteristic signals are amplitude-modulated and frequency-modulated (AM–FM) signals in real life [10]. Also, demodulation techniques are theoretically simple and computationally efficient. These advantages, in a way, ensure the premise of application to industry. Accordingly, in this paper, we will be focusing on the demodulation methods and their application to bearing fault detection. However, the classical demodulation techniques, such as Hilbert transform (HT) and Teager energy operator (TEO), are sensitive to noise and vibration interferences like the gear-mesh frequency, gear fault frequency or the turbine and pump flow signals [11]. It is, therefore, necessary to develop an alternative demodulation technique that is similar to the classical TEO but more robust. Based on this, an alternative demodulation technique called high-order differential energy operator (HO_DEO) [12] is presented in this paper. This HO_DEO is not only a
non-parametric and non-filtering method but also a theoretically simple and computationally efficient method.

What is more, as HO_DEO is based on differentiation, it can enhance the weak signal considerably and suppress the vibration interference. It, therefore, can detect the weak signal from a severely contaminated signal, so it has been employed in many research works [13-15]. Furthermore, we found that HO_DEO possesses the built-in amplitude demodulation (AD) capability, so it can directly detect the bearing fault features without computing the envelope. In this paper, HO_DEO is compared with the classical TEO and its improved versions (DESA_1 and DESA_2 for short) [16, 17] and is used in bearing fault diagnosis of rotating machinery. The results indicate that HO_DEO outperforms the TEO and its improved versions.

2. Theoretical background

2.1. Energy measured

The HO_DEO is defined as

$$Y_{k}(x) = x[n]x[n+k-1] - x[n-1]x[n+k]$$

(1)

where $k$ denotes the 15] positive order.

The discrete version of HO_DEO is expressed as

$$Y_{k}[x[n]] = x[n]x[n+k-2] - x[n-1]x[n+k-1]$$

(2)

The mean absolute values of the amplitude estimation error with order $k=2, 3, 4$ are shown in Table 1. We can find that there is no explicit relationship between the order and the performance improvement. Besides, the performance of HO_DEO indicates no noticeable improvement with growth of the order [15]. Thus, $k=3$ is used in this paper, and the discrete version of 3th-order DEO can be expressed as

$$Y_{3}[x[n]] = x[n]x[n+1] - x[n-1]x[n+2]$$

(3)

### Table 1. Mean absolute error.

| Order | Noise free | SNR=30dB | SNR=20dB |
|-------|------------|----------|----------|
| $k=2$ | 2.68       | 25.59    | 37.22    |
| $k=3$ | 0.77       | 8.57     | 18.06    |
| $k=4$ | 0.87       | 8.87     | 19.05    |

2.2. Properties

A. Noise robust

Assume that a discrete input signal is embedded in a zero-mean white Gaussian noise and its mathematical expression can be written as

$$x[n] = s[n] + w[n]$$

(4)

where $s[n]$ represents the noise-free signal, i.e., $s[n] = A \cos(\omega n + \phi)$, and $w[n]$ is the zero-mean white Gaussian noise.

$x[n]$ is inserted into the expectation operator $E[\Psi(x)]$, the expected value of TEO can be found to be [14]

$$E[\Psi(x)] = E[\Psi(s)] + \sigma^2$$

(5)

in which $\sigma^2$ is variance. This Equation (5) means that TEO is biased by the variance $\sigma^2$ and skews the instantaneous amplitude (IA) and instantaneous frequency (IF) estimators in the presence of noise.

A generalized energy operator was shown to be
\psi_g\{x[n]\} = x[n-l]x[n-p] - x[n-q]x[n-s]
\] \quad l+p=q+s
\] (6)

where the subscript g denotes generalized. The expected value of Equation (6) can be written as [18]
\[ E[\psi_g(x)] = A^2 \sin[\alpha_1(1-p+q-s)/2] \sin[\alpha_2(q-s-1+p)/2]
\] (7)

If we choose l=0, p=-1, q=1, s=-2, Equation (6) becomes 3rd-order DEO
\[ \psi_g\{x[n]\} = \psi_x\{x[n]\} = x[n]x[n+1] - x[n-1]x[n+2]
\] (8)

moreover, due to \(1 \neq p, q \neq s\), the resulting output is
\[ E[\psi_g(x)] = A^2 \sin 2\omega \cdot \sin \omega
\] (9)

Note that in comparison with Equation (5), the outcome of this case does not contain the variance \(\sigma^2\), which shows that HO_DEO is immune to noise.

B. Vibration interferences robust

Similarly, assume that we have the signal
\[ u(t) = Ae^\omega \cos(\omega t + \phi) + \sum_{n=1}^{N} B_n \cos \omega_n t
\] (10)

where \(f_1(t)\) represents an amplitude-modulation signal; \(f_2(t)\) represents multiple vibration interferences.

Insert Equation (10) into Equation (1), when \(k=3\), the \(Y_3\{u(t)\}\) is
\[ Y_3\{u(t)\} = \frac{1}{2} A^2 \omega^2 \sin[2(\omega t + \phi)] + Ae'\cos(\omega t + \phi) + \sum_{n=1}^{N} B_n \cos \omega_n t
\] (11)

in which \(n(t)\) is given as follow
\[ n(t) = \omega \sin(\omega t + \phi) + \sum_{n=1}^{N} B_n \cos \omega_n t \quad \rightarrow \quad \sum_{n=1}^{N} B_n \cos \omega_n t
\] (12)

\[ + \cos(\omega t + \phi) \sum_{n=1}^{N} B_n \cos \omega_n t + \omega \sin(\omega t + \phi) + \sum_{n=1}^{N} B_n \cos \omega_n t
\]

Note that in contrast to Equation (11), the result of HO_DEO contains two terms: the amplitude demodulation term \(1/2 A^2 \omega^2 \sin[2(\omega t + \phi)]\), the high-frequency component \(Ae'\cos(\omega t + \phi)\) and the multiple vibration interferences are offset against each other. This expression indicates that HO_DEO is also able to suppress vibration interferences.

3. Simulation test

The vibration model of a faulty bearing can be displayed trains of impulsive components as follow [19]
\[ y(t) = \gamma_0 e^{-\beta t} \sin \omega_0 \sqrt{1 - \xi^2} t + \delta(t) + \sum_{n=1}^{N} B_n \cos \omega_n t
\] (13)

in which the displacement constant is \(\gamma_0=5\); the damping coefficient is \(\xi=0.1\); the period corresponding to the fault frequency is \(T=0.00995\) s, and the fault frequency is \(f_0=104.86\) Hz; the natural frequency is \(f_n=3000\) Hz, \(\omega_n = 2\pi f_n\). \(\delta(t)\) denotes the additive white Gaussian noise and SNR=-5dB. \(\sum_{n=1}^{N} B_n \cos \omega_n t\) represents four vibration interference frequencies (33Hz, 43Hz, 50Hz and 310Hz).
The time and frequency domains of the bearing fault signal are shown in Figure 1. In subfigure (b), it is seen that the noise and vibration interferences are dominant and it is difficult to distinguish the bearing fault characteristic frequency.

**Figure 1.** Bearing fault signal: (a) time domain; (b) frequency domain.

Now, the four EOs are applied to extract the bearing fault characteristic frequency. The different results are shown in Figure 2.

**Figure 2.** Energy spectra: (a) DESA; (b) DESA_1; (c) DESA_2; (d) HO_DEO.

The four subfigures illustrate the energy spectra derived by the four EOs, respectively. It is evident that TEO and its improved versions produce unacceptable results. As indicated in Figure 2, the noise and vibration interferences, as well as their combinations, are still predominant and it is impossible to identify the bearing fault characteristic frequency, which means that the three methods fail to detect the bearing fault characteristic frequency from a heavily contaminated signal. On the contrary, as shown in Figure 2(d), it is easy to distinguish the bearing fault characteristic frequency and its harmonics, which shows that HO_DEO is capable of detecting the bearing fault characteristic frequency in the presence of substantial noise and vibration interferences. Besides, it is seen from Figure 2(d) that the vibration interferences almost disappear. The result shows that HO_DEO can suppress the vibration interferences, which is identical to theoretical analysis in section 2.

From the results of the tests above, it confirms that HO_DEO is superior to TEO and its improved versions and also prove that the theoretical analysis is correct.
4. Application of the real bearing fault signals

4.1. Bearing fault detection for the rotating machine

In this section, HO_DEO is applied to two groups of real signals from the bearing inner race and bearing outer race respectively to verify its usefulness.

The bearing fault data is provided by Machinery Failure Prevention Technology Society (MFPTS). Note that the fault data is extracted under the 300 lbs of load. The related parameters of the damaged bearing are listed in Table 2.

Table 2. Parameters of the damaged bearing.

| Roller diameter (mm) | Pitch diameter (mm) | Number of rolling elements | Contact angle |
|----------------------|---------------------|-----------------------------|---------------|
| 5.97                 | 31.62               | 8                           | 0°            |

Table 3 displays these frequencies related to the experiments.

Table 3. Characteristic frequencies related to the experiments.

| Defect location | Rotational speed (r/min) | Sampling frequency (kHz) | Rotational frequency f (Hz) | Fault characteristic frequency (Hz) |
|-----------------|--------------------------|--------------------------|-----------------------------|-----------------------------------|
| Inner ring      | 1500                     | 48.828                   | 25                          | 118                               |
| Outer ring      | 1500                     | 48.828                   | 25                          | 81                                |

In this real test, we also added the additional background noise (SNR=-5dB) and four vibration interferences (33, 43, 50 and 310Hz) to the measured signal in order to increase the difficulty of fault detection, to better present the superiority of the proposed method.

4.1.1. Fault detection for the outer race. The outer race of the damaged bearing is shown in Figure 3.

Figure 4 shows the time waveform and frequency spectrum of the heavily corrupted signal. We can see that the fault characteristic frequency is buried by noise and vibration interferences in subfigure (b).

Now, HO_DEO is used to process the fault signal. Meanwhile, TEO and its improved versions are still employed to compare with HO_DEO. The different results derived by the four methods are illustrated in Figure 5.

Similar to the simulation test in section 3, TEO and its improved versions still cannot extract the outer race fault characteristic frequency in this experiment thanks to the existence of additive noise and vibration interferences. On the contrary, HO_DEO produces a satisfactory result. It is found in Figure 5(d) that the fault characteristic frequency at 81Hz is not only at a higher magnitude but also its harmonics are easily discernible.

Figure 3. Outer race fault.
4.1.2. Fault detection for the inner race. The inner race of the damaged bearing is shown in Figure 6.

Likewise, the time waveform and the frequency spectrum of the inner race fault signal with the additional background noise (SNR=-5dB) and four vibration interferences (33, 43, 50, and 310Hz) are displayed in Figure 7. Like the outer race fault signal, the fault characteristic frequency is overwhelmed by noise and interfering components, and it is difficult to be distinguished.

Now, the four approaches are still employed to process the inner race fault signal. The results derived by them are shown in Figure 8. The results of Figure 8 are virtually similar to those of Figure 5. TEO and its improved versions similarly fail to detect the inner fault characteristic frequencies, and the fault characteristic frequencies are all masked by noise and interfering components. However, HO DEO is still capable of distinguishing the fault characteristic frequency and its harmonics from the heavily contaminated signal. According to the real tests, indeed, it is convincing that HO_DEO outperforms TEO and its improved versions.
Nevertheless, it should be noted that the sidebands in subfigure (d) show a higher peak, and even magnitudes of some sidebands are the same as those of some harmonics, such as the sidebands of harmonics at 354Hz and 471Hz. The sideband phenomenon in the envelope analysis is unavoidable for inner race fault diagnosis of rotating machinery [20]. Moreover, HO_DEO can boost the weak signal, but at the same time it can also level up the magnitude of the sideband. So, the two factors lead to the sideband phenomenon in Figure 8(d). For comparison, we also chose the other two inner race fault signals extracted under the 0lbs and 50lbs of the load. The comparison results are shown in Figure 9.

It can be found that the fault characteristic frequency and its harmonics are more recognizable under the light load than under the heavy load. Thus, the sideband phenomenon has to be taken into account when using HO_DEO in the case of the heavy load.

Figure 7. Inner race fault signal: (a) Time domain; (b) Frequency domain.

Figure 8. Energy spectra: (a) DESA; (b) DESA_1; (c) DESA_2; (d) HO_DEO.

Figure 9. Comparison results: (a) Under 0lbs of the load; (b) Under 50lbs of the load.
4.2. Bearing fault detection for the vibrating machine
The bearing fault signals extracted from the vibrating machine are usually more complicated, which consist of a mixture of different oscillations and more intensive noise, than that extracted from the rotating machine [19]. Consequently, diagnosing the faulty bearing for the vibrating machine requires that the bearing fault detection techniques have more excellent performance.

In this section, we took a vibrating screen as a typical example of a vibrating machine to verify the practicality of HO_DEO under the real working condition. The vibrating screen and data acquisition system used in this work are displayed in Figure 10.

Type 1308 bearings are applied to these real experiments, and their specifications are displayed in Table 4.

| Defect location | Rotational speed (r/min) | Sampling frequency (kHz) | Rotation frequency (Hz) | Fault characteristic frequency (Hz) |
|-----------------|--------------------------|--------------------------|------------------------|----------------------------------|
| Inner ring      | 1000                     | 12                       | 16.67                  | 145.85                           |
| Outer ring      | 1000                     | 100                      | 16.67                  | 104.24                           |

4.2.1. Fault detection for the outer race. Figure 11 shows the time-domain waveform of the outer race fault signal and its corresponding Fourier spectrum. It can be noticed that the fault characteristic frequency is buried by noise and a vibration interference generated by imbalance.

Figure 11. Outer race fault signal: (a) The time-domain waveform and (b) Fourier spectrum.
Now, TEO and its improved versions, as well as HO_DEO, are used to detect the fault characteristic frequency from the fault signal. The results of the three techniques on the fault signal are illustrated in Figure 12.

![Figure 12. Energy spectra: (a) DESA; (b) DESA_1; (c) DESA_2; (d) HO_DEO.](image)

It can be seen that Energy spectra obtained by TEO and its improved versions are all dominated by the noise, so the three techniques are utterly incapable of detecting the fault characteristic frequency in this case. In comparison with the three techniques, HO_DEO still succeeds in extracting the fault characteristic frequency and its associated harmonics with visible amplitudes.

4.2.2. Fault detection for the outer race. The inner race fault signal and its corresponding Fourier spectrum are shown in Figure 13. Like the case of the outer race fault signal, we cannot distinguish the fault characteristic frequency of 145.85Hz at all in the Fourier spectrum.

As before, the inner race fault signal is analyzed by the four methods. The results are shown in Figure 14.

Similar to the case of the detection of the outer race fault, TEO and its improved versions still fail to identify the fault characteristic frequency and its associated harmonics. However, HO_DEO produces an acceptable result. The fault characteristic frequency and its associated harmonics are all at a high amplitude that can be easily recognized.

![Figure 13. Inner race fault signal: (a) time-domain waveform and (b) Fourier spectrum.](image)
5. Conclusions
In this study, an alternative energy operator called HO_DEO is proposed. Meanwhile, the effects of the HO_DEO method are investigated. The results of simulation and real bearing fault diagnosis tests provide a clear distinction among HO_DEO, TEO, and its versions. Our data indicate compelling evidence that HO_DEO outperforms these conventional demodulation techniques. Moreover, HO_DEO can detect the bearing fault characteristic frequency from a heavily contaminated signal independently without the help of pre-processing or other methods. These advantages are more likely to encourage this method to apply to industry.

However, as mentioned before, the sideband phenomenon has to be taken into consideration, especially under heavy load. It may give rise to misdiagnosis when the sideband is at a higher magnitude. Thus, the issue should be explored in the future.

Acknowledgments
This work was supported by the National Natural Science Foundation of China (No. 51705030) and the Fundamental Research Funds for the Central Universities, CHD (No. 300102258714).

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