Quark mass density- and temperature- dependent model for bulk strange quark matter

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Abstract

It is shown that the quark mass density-dependent model can not be used to explain the process of the quark deconfinement phase transition because the quark confinement is permanent in this model. A quark mass density- and temperature-dependent model in which the quark confinement is impermanent has been suggested. We argue that the vacuum energy density $B$ is a function of temperature and satisfies $B = B_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$, where $T_c$ is the critical temperature of quark deconfinement. The dynamical and thermodynamical properties of bulk strange quark matter for quark mass density- and temperature-dependent model are discussed.

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I. INTRODUCTION

It is generally accepted that the fundamental theory of strong interaction is quantum chromodynamics (QCD), however in reality, because of its nonperturbative characters, QCD has very little impact on the study of low and medium energy nuclear phenomena. Many effective models, some of them based on quark and gluon degrees of freedom [1], and the others based on nucleons and mesons [2], or quarks and mesons [3,4], have been employed to investigate the nuclear matter and strange matter. The quark mass density-dependent model (QMDD) suggested by Fowler, Raha and Weiner [5], is one of such candidates of effective models. Though this model involves some arbitrary choices and can not reproduce all conclusions given by lattice calculations, it is introduced as an alternative to the static bag model of confinement and has substantial application in the study of bulk quark matter, especially strange quark matter and strange star.

Recently, since the speculation of Witten [6] that the strange quark matter (SQM) may be more stable than normal nuclei, especially, since the argument given by Greiner and his co-workers [7] that the small lumps of strange quark matter (strangelets) may be produced in relativistic heavy-ion collisions and could serve as an unambiguous signature for the formation of quark-gluon plasma, much theoretical effort has been devoted to studying the properties of SQM. Many investigations have been carried out in the frame of MIT bag model [8,9] or QMDD model. Obviously, a successful effective model should be used to describe not only the dynamical and thermodynamical properties of SQM, but also the phase transitions of QCD.

In this paper, we will focus our attention on the QMDD model. According to the QMDD model, the masses of u, d quarks and strange quarks (and the corresponding anti-quarks) are given by

\[ m_q = \frac{B}{3n_B}, \quad (q = u, d, \bar{u}, \bar{d}), \]
\[ m_{s, \bar{s}} = m_{s0} + \frac{B}{3n_B}, \]

where \( n_B \) is the baryon number density, \( m_{s0} \) is the current mass of the strange quark and \( B \) is the vacuum energy density inside the bag. At zero temperature,

\[ n_B = \frac{1}{3}(n_u + n_d + n_s), \]

\( n_u, n_d, n_s \) represent the density of u-quark, d-quark and s-quark, respectively. The basic hypothesis Eqs. (1) and (2) in QMDD model can easily be understood from the quark confinement mechanism. A confinement potential which is proportional to \( r \) (or \( r^2 \)) must be added to a quark system in the phenomenological effective models because the perturbative QCD can not give us the confinement solution of quarks. The confinement potential \( kr \) prevents the quark goes to infinite or to the very large regions. The large regions or the large volume means that the density is small. This mechanism of confinement can be mimicked through the requirement that the mass of an isolated quark becomes infinitely large so that the vacuum is unable to support it. Thus, for a system of quarks at zero temperature, the energy density tends to a constant value while the mass tends to infinity, as the volume
increases to infinity or the density decreases to zero \[10\]. This is just the picture given by Eqs.\((1)\)-(\(3\)). In fact, the similar confinement mechanism emerges in MIT bag model also. The boundary condition of confinement for MIT bag corresponds to that the quark mass is zero inside the bag but infinity at the boundary or outside the bag \[11\].

Although the QMDD model can provide a dynamical description of confinement and explain the stability and many other dynamical properties of SQM at zero temperature, when we extend this model to finite temperature and discuss the thermodynamical behaviors of SQM, many difficulties will emerge. Firstly, the thermodynamic potential \(\Omega\) is not only a function of temperature, volume and chemical potential, but also of density, because the quark masses depend on density. How to treat the thermodynamics with density-dependent particle masses self-consistently is a serious problem and has made many wrangles for this model in references \[12\]-\[15\]. Secondly, as will be shown below, it can not reproduce a correct lattice QCD phase diagram qualitatively or give us a successful equation of state when \(n_B \to 0\). It can not describe the phase transitions of quark deconfinement because the quark masses are independent of temperature. To overcome this difficulty, we will suggest a quark mass density- and temperature-dependent model (QMDTD) in this paper. Instead of a constant \(B\) in Eqs.\((1)\) and \((2)\), we argue that \(B\) would be a function of temperature and choose the function \(B(T)\) from Friedberg-Lee model. We will prove that the above difficulty can be overcome in our QMDTD model.

The organization of this paper is as follows. In the next section we review three different treatments concerning the thermodynamics with density-dependent quark mass in refs. \[12\], \[13\] and \[14\] respectively, and prove all treatments can not overcome the second difficulty mentioned above. In section 3, we give detailed arguments on the temperature dependence of vacuum energy density \(B\) and extend QMDD model to a QMDTD model. Our results are summarized in section 4. In this section we prove that the temperature \(T\) vs density \(n_B\) phase diagram for QMDTD model becomes reasonable and it can be employed to mimic the QCD phase transition qualitatively. The comparison of QMDD model and QMDTD model for studying the dynamical and thermodynamical properties of SQM will also present in section 4. The last section is a summary.

**II. THERMODYNAMICAL TREATMENTS**

At finite temperature, the antiquarks must be considered. Eq.\((3)\) becomes

\[
n_B = \frac{1}{3}(\Delta n_u + \Delta n_d + \Delta n_s),
\]

where

\[
\Delta n_i = n_i - n_i = \frac{g_i}{(2\pi)^3} \int_0^\infty d^3k \left( \frac{1}{\exp[\beta(\varepsilon_i - \mu_i)] + 1} - \frac{1}{\exp[\beta(\varepsilon_i + \mu_i)] + 1} \right),
\]

\((n_i)\) \(n_i\) is the number density of the (anti)flavor \(i\) \(i = u, d, s\), \(g_i = 6\) is the degeneracy factor, \(\mu_i\) is the chemical potential (for antiparticle \(\mu_i = -\mu_i\)). Inside SQM, \(s\) (and also \(\bar{s}\) quarks are produced through the weak processes

\[
u_e, u + e^- \to d + \nu_e, d \to u + e^- + \nu_e, s \to u + e^- + \nu_e,
\]

\(u + d \leftrightarrow u + s, s \to u + e^- + \nu_e, d \to u + e^- + \nu_e, u + e^- \to d + \nu_e,\)
and similarly for antiquarks. The system of SQM must satisfy the following constraints. The condition of chemical equilibrium yields

\[ \mu_s = \mu_d, \quad \mu_s = \mu_u + \mu_e. \]  

(7)

The condition of charge neutrality reads

\[ 2\Delta n_u = \Delta n_d + \Delta n_s + 3\Delta n_e. \]  

(8)

The thermodynamic potential of SQM system is

\[ \Omega = \sum_i \Omega_i = -\sum_i \frac{g_i T}{(2\pi)^3} \int_0^\infty d^3k \ln \left(1 + e^{-\beta(\varepsilon_i(k) - \mu_i)}\right), \]  

(9)

where \( i \) stands for \( u, d, s \) (or \( \bar{u}, \bar{d}, \bar{s} \)) and the electron \( e(e^+) \), \( g_i = 2 \) for \( e \) and \( e^+ \). Noting that \( \varepsilon_i(k) = \sqrt{m_i^2 + k^2} \) and \( m_i \) is given by Eqs. (1) and (4), we can calculate the thermodynamic potential \( \Omega \) under the constraints Eqs. (4), (7) and (8), and obtain the thermodynamical quantities such as number density \( n_i \), pressure, internal energy and etc. Due to the density-dependent quark mass, many different treatments had been given in the references.

**A. First treatment**

The first thermodynamical treatment for QMDD model was given by Chakrabarty [12]. After getting the thermodynamic potential \( \Omega \), he used the usual thermodynamical formula to calculate the number density \( n_i \), total pressure \( p \) and the total energy density \( \varepsilon \), and found

\[ n_i = -\frac{1}{V} \left. \frac{\partial \Omega}{\partial \mu_i} \right|_{T,n_B}, \]  

(10)

\[ p = -\frac{\Omega}{V}, \]  

(11)

\[ \varepsilon = \frac{\Omega}{V} + \sum_i \mu_i n_i - \frac{T}{V} \left. \frac{\partial \Omega}{\partial T} \right|_{\mu_i,n_B}. \]  

(12)

After comparison with the results given by MIT bag model, Chakrabarty claimed the properties of SQM given by QMDD model were found to be very different from those predicted by the MIT bag model. Since the density dependence of quark mass has not completely and explicitly taken into account in this thermodynamical calculations, this treatment seems incorrect [13]. But in order to compare with other treatments, we list this treatment here also.
B. Second treatment

The second different treatment is given by Benvenuto and Lugones [13]. They claimed that the features found by Chakrabarty are consequences of an incorrect thermodynamical treatment for QMDD model. In deriving the energy density and the pressure, an extra term appears due to the dependence of the quark mass on the baryon density.

The results become

$$p = -\frac{1}{V} \frac{\partial (\Omega/n_B)}{\partial (1/n_B)} \bigg|_{T,\mu_i} = -\frac{\Omega}{V} + \frac{n_B}{V} \frac{\partial \Omega}{\partial n_B} \bigg|_{T,\mu_i},$$

(13)

$$\varepsilon = -p + \sum_i \mu_i n_i - \frac{T}{V} \frac{\partial \Omega}{\partial T} \bigg|_{\mu_i, n_B},$$

(14)

and $n_i$ still satisfies Eq. (10). The extra term produces significant changes in the energy per baryon, make the pressure take the negative value in the low density region and shift the stability window of strange matter (SM). In almost all cases they found that the properties of SQM in the QMDD model are nearly the same as those obtained in the MIT bag model.

C. Third treatment

The third different treatment is done by Peng and his coworkers [14]. Their improvements include: (1) Based upon a quark condensates argument, instead of Eqs.(1) and (2), they introduce

$$m_q = \frac{D}{n_B^{1/3}}, \quad (q = u, d, \bar{u}, \bar{d}),$$

(15)

$$m_{s,\bar{s}} = m_{s0} + \frac{D}{n_B^{1/3}},$$

(16)

where $D$ is a parameter usually determined by stability arguments. (2) They agree with the second treatment that one must add an extra term to the pressure formula because of the quark mass density-dependence, but do not agree with them for adding an extra term to the expression of the energy density because it can not give a correct QCD vacuum energy. The pressure and the energy density given by this treatment are

$$p = -\frac{1}{V} \frac{\partial (\Omega/n_B)}{\partial (1/n_B)} \bigg|_{T,\mu_i} = -\frac{\Omega}{V} + \frac{n_B}{V} \frac{\partial \Omega}{\partial n_B} \bigg|_{T,\mu_i},$$

(17)

$$\varepsilon = \frac{\Omega}{V} + \sum_i \mu_i n_i - \frac{T}{V} \frac{\partial \Omega}{\partial T} \bigg|_{\mu_i, n_B}. $$

(18)

In fact, this treatment is a ”mixture” of the first and the second treatment. It chooses the pressure of the second treatment and the energy density of the first treatment as its pressure and energy density respectively.
Now we are in the position to study the thermodynamical behavior of SQM by using the QMDD model. The temperature $T$ vs density $n_B$ curves are shown in Fig.1 by three dashed lines for three treatments respectively where we choose the parameters $B = 170\text{MeVfm}^{-3}, m_{s0} = 150\text{MeV}, D = 140\text{MeVfm}^{-1}$ and $P = 400\text{MeVfm}^{-3}$. We see from Fig.1 that the temperature $T$ tends to infinite when $n_B \to 0$. This result is treatments-independent and can easily be understood if we notice the basic hypothesis of QMDD model, namely, the Eqs.(1) and (2) (or Eqs.(15) and (16)), the quark masses are divergent when $n_B \to 0$. To excite an infinite weight particle, one must prepare to pay the price of infinite energy, i.e. infinite temperature. This result demonstrates that the confinement in QMDD model is permanent. The quark can not be deconfined for any temperature. This model can not describe the quark deconfinement phase transition and give us a correct phase diagram of QCD.

III. QUARK MASS DENSITY- AND TEMPERATURE- DEPENDENT MODEL

Obviously, if we hope to employ the QMDD model to mimic the phase transition of QCD, the first problem is to avoid the permanent confinement mechanism given by Eqs.(1) and (2) (or Eqs.(15) and (16)). It would be useful to recall what happen in MIT bag model and Friedberg-Lee soliton bag model [16]. MIT bag model is a permanent quark confinement model because the confined boundary condition does not change with temperature. The vacuum energy density $B$ is a constant in MIT bag model. Contrary, the Friedberg-Lee soliton bag model is a impermanent quark confinement model. Its confinement mechanism comes from the interaction between quarks and a non-topological scalar soliton field. Since the spontaneously breaking symmetry of scalar field will be restored at finite temperature, the non-topological soliton will disappear and the quark will deconfine at critical temperature. In this model, the vacuum energy density $B$ equals to the different value between the local false vacuum minimum and the absolute real vacuum minimum. This value depends on the temperature. It means that $B$ must be a function of temperature in Eqs.(1) and (2) if we hope to deconfine quark. In order to describe the phase transition of QCD, we must extend the QMDD model to a quark mass density- and temperature- dependent (QMDTD) model and suppose that $B$ is a function of temperature. This is our first argument.

Our second argument comes from the calculations of the effective masses of nucleons and mesons recently. We can sum the tadpole diagrams and the exchange diagrams for mesons by Thermo Field Dynamics and find the masses of nucleons and mesons all decrease with temperature [17–20]. This result for $\rho$-meson is in agreement with recent experiments [21,19]. According to the constitute quark model, the nucleon are constructed by three quarks and the meson by two quarks. It means that in a satisfying quark model we must consider the temperature dependence of the quark mass. But this effect has not been taken into account by Eqs.(1) and (2) if $B$ is a constant.

According to the conclusion of Benvenuto and Lugones [13], the results found by QMDD model are nearly the same as that obtained in the MIT bag model. On the other hand, as was pointed out by [16], the MIT bag model can be obtained from Friedberg-Lee soliton bag model provided fixed some parameters. Then it is natural to choose $B(T)$ given by Friedberg-Lee model as our input.

Introducing an ansatz
\[ B = B_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right], \quad 0 \leq T \leq T_c \]  \hspace{1cm} (19)
\[ B = 0, \quad T > T_c \]  \hspace{1cm} (20)

where \( T_c \) is the critical temperature of deconfinement. When \( 0 \leq T \leq T_c \), Eqs. (1) and (2) become

\[ m_q = \frac{B_0}{3n_B} \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right], \quad (q = u, d, \bar{u}, \bar{d}), \]  \hspace{1cm} (21)
\[ m_{s,\bar{s}} = m_{s0} + \frac{B_0}{3n_B} \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]. \]  \hspace{1cm} (22)

The masses of quarks not only depend on the density \( n_B \), but also on the temperature \( T \). And when \( T \geq T_c \), \( m_q = 0, m_{s,\bar{s}} = m_{s0} \). When \( T = 0 \), Eqs. (21) and (22) reduce to Eqs. (1) and (2), and our QMDTD model reduces to QMDD model.

In our later calculations, we prefer to the thermodynamical treatment of Eqs. (17) and (18), because they consider the density-dependent masses and the QCD vacuum energy explicitly. Substituting Eqs. (21) and (22) into Eq. (9), under the constraints Eqs. (4), (7) and (8), and using the same argument as that of the third treatment, we find the thermodynamical quantities \( n_i, p, \varepsilon \), which is still expressed by Eqs. (10), (17) and (18) because Eqs. (21) and (22) have the same density-dependence as that of Eqs. (1) and (2). Even though the expressions of \( n_i, p \) and \( \varepsilon \) are the same, but we would like to emphasize that the results given by our model and QMDD model are different because the thermodynamical potentials calculated from Eqs. (1) and (2) and Eqs. (19)-(22) are quiet different at finite temperature. Our results are summarized in next section.

IV. RESULTS AND DISCUSSION

The numerical calculations have been done by adopting the parameters \( B_0 = 170 \text{MeVfm}^{-3}, m_{s0} = 150 \text{MeV}, T_c = 170 \text{MeV} \). The temperature \( T \) vs baryon number density \( n_B \) figure is shown in Fig. 1 where the pressure \( P \) is fixed to be \( 400 \text{MeVfm}^{-3} \). The three dashed lines refer to three different treatments of QMDD models, respectively, and the solid line refers to our QMDTD model. We see from Fig. 1 that the basic difference between QMDTD model and QMDD model is: when \( n_B \to 0 \), the temperature approaches to a critical temperature \( T_c = 170 \text{MeV} \) in our model, and diverges in QMDD model. It means that QMDTD model is a impermanent confinement model. It can be employed to describe the phase transition of QCD qualitatively.

The same curves of QMDTD model but for different pressures \( p = 400, 300, 200 \) and \( 150 \text{MeVfm}^{-3} \) are shown in Fig. 2 respectively. We see from Fig. 2 that the conclusion \( T \) approaches to \( T_c \) when \( n_B \to 0 \) will not change with pressure. The basic physical reason is that in QMDTD model, \( B \) is a monotonously decreasing function in the region \( 0 \leq T < T_c \) and becomes zero when \( T \) approaches to \( T_c \). The singularity of quark mass at zero density of QMDD model has been wiped out in QMDTD model. When \( T = T_c \), \( m_{q,\bar{q}} = 0 \) and \( m_{s,\bar{s}} = m_{s0} \). It guarantees that the divergence of temperature at zero density will not happen in QMDTD model no matter the values of pressure be.
To compare our model with QMDD model further, we investigate the thermodynamical stability of SQM at finite temperature. The energy per baryon vs baryon density curves at \( T = 50\text{MeV} \) for QMDTD model and QMDD model are shown in Fig.3 where the solid line refers to QMDTD model and the three dashed lines for three different treatments of QMDD model respectively. We see that the solid line is lower than the others. It means that the SQM described by QMDTD model is more stable than that by QMDD model. The values of \( n_{B0} \) and \( (\varepsilon/n_{B})_0 \) at the saturation point are summarized at Table 1. We see from Table 1 that the value of \( n_{B0} \) for QMDTD model is situated between the maximum value 0.55fm\(^{-3}\) and the minimum value 0.36fm\(^{-3}\) for the second and the third treatments of QMDD model, but the energy per baryon \( (\varepsilon/n_{B})_0 = 1006\text{MeVfm}^{-3} \) is lower than all treatments. The point marked with a heavy dot in the solid line is the zero pressure point for QMDTD model, as can be seen clearly, which matches the lowest-energy state and satisfies the basic requirement of thermodynamics pointed by ref. [14]. The results obtained from Fig.3 represent that our model can reproduce all thermodynamical properties of SQM which has been explained by QMDD model.

The study of the equation of state for QMDTD model will show that this model is suitable for describing the thermodynamical behavior of SQM. The curves of pressure vs energy density are shown in Fig.4 where the solid line refers to QMDTD model and the dashed lines for different treatments of QMDD model respectively. We see from Fig.4 that the behavior of solid curve is very similar to that of the second treatment. It is monotonous. The values of pressure become negative at low density region and asymptotically tends to the ultrarelativistic case at high density as expected, because of the asymptotic freedom of quark [13].

Finally, we hope to investigate the so called ”stability window” of SQM at zero temperature [13]. According to the argument of Farhi and Jaffe [22], the conditions under which the strange matter be a true hadronic ground state read: at \( P = 0, E/n_B \leq 930\text{MeV} \) for strange matter and \( E/n_B > 930\text{MeV} \) for two flavor quark matter. Noting that even at zero temperature, the formulae of QMDTD model are still different from that of the second and the third treatments of QMDD model, because instead of Eqs.(15), (16) for the third treatment, and Eq.(14) for the second treatment, we have Eqs.(1), (2) and Eq.(18), respectively. Our result is shown in Fig.5, where for comparison, the stability window of the second treatment of QMDD model is also plotted. We see from Fig.5, the regions of \( B_0 \) for stable quark matter are different for these two cases. \( B_0 \) is limited narrowly in \( 69.05\text{MeVfm}^{-3} \leq B_0 \leq 111.6\text{MeVfm}^{-3} \) for the second treatment of QMDD model, but widely in \( 168.7\text{MeVfm}^{-3} \leq B_0 \leq 273.3\text{MeVfm}^{-3} \) for QMDTD model. The stability window is still trianglelike but the adjusted parameters \( B_0 \) and \( m_{s0} \) can take more values for which the system is stable in our model.

V. SUMMARY

In summary, it is found that the QMDD model can not be used to describe the quark deconfinement phase transition because the temperature diverges when baryon number density approaches to zero. The quark confinement in this model is permanent. In order to overcome this difficulty we suggest a QMDTD model in which the quark confinement is impermanent. We argue that the vacuum energy density inside the bag \( B \) be a function
of temperature and prove that the divergence difficulty of temperature dose not emerge in QMDTD model. This model can mimic phase transition of SQM qualitatively. Finally, we compare the dynamical and thermodynamical properties of QMDTD model with three treatments of QMDD model, and find that our QMDTD model is useful to describe the properties of SQM.

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REFERENCES

[1] For example, see S. Narison, QCD spectral sum rules, World Scientific, Singapore (1989).
[2] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E6, 515 (1997).
[3] X. Jin and B. K. Jennings, Phys. Rev. C54, 1427 (1996).
[4] P. Wang, R. K. Su, H. Q. Song and L. L. Zhang, Nucl. Phys. A653, 166 (1999) and refs. herein.
[5] G. N. Fowler, S. Raha and R. M. Weiner, Z. Phys. C9, 271 (1981).
[6] E. Witten, Phys. Rev. D30, 272 (1984).
[7] C. Greiner, P. Koch and H. Stöcker, Phys. Rev. Lett 58, 1825 (1987), C. Greiner and H. Stöcker, Phys. Rev. D44, 3517 (1991).
[8] E. P. Gilson and R. L. Jaffe, Phys. Rev. Lett. 71, 332 (1993).
[9] J. Madsen, Phys. Rev. D47, 5156 (1993); Phys. Rev. D50, 3328 (1994).
[10] S. Chakrabarty, S. Raha and B. Sinha, Phys. Lett. B229, 112 (1989).
[11] See A. W. Thomas, Advanced in Nuclear Physics vol. 13, page 1, ed. J. W. Negele and E. Vogt, Plenum press, (1984).
[12] S. Chakrabarty, Nuovo Cimento B106, 1023 (1991). Phys. Rev. D43, 627 (1991); Phys. Rev. D48, 1409 (1993).
[13] O. G. Benremuto and G. Lugones, Phy. Rev. D51, 1989 (1995), G. Lugones and O. G. Benremuto, Phy. Rev. D52, 1276 (1995).
[14] G. X. Peng, H. C. Chiang, B. S. Zou, P. Z. Ning and S. J. Luo, Phy. Rev. C62, 025801 (2000), G. X. Peng, H. C. Chiang, P. Z. Ning, B. S. Zou, Phy. Rev. C59, 3452 (1999).
[15] P. Wang, Phy. Rev. C62, 015204 (2000).
[16] T. D. Lee, Particle Physics and Introduction to Field Theory, Hawood Academic Pub. (1981).
[17] Y. J. Zhang, S. Gao and R. K. Su, Phys. Rev. C56, 3336 (1997).
[18] S. Gao, Y. J. Zhang and R. K. Su, Phys. Rev. C53, 1098 (1996).
[19] P. Wang, Z. Chong, R. K. Su and P. K. N. Yu, Phys. Rev. C59, 928 (1999).
[20] S. Gao, Y. J. Zhang and R. K. Su, Nucl. Phys. A593, 362 (1995).
[21] G. J. Lolos et. al., Phys. Rev. Lett. 80, 241 (1998).
[22] E. Farhi and R. L. Jaffe, Phys. Rev. D 30, 2379 (1984).

VI. TABLE

Table 1, The value of saturation point for different models:

| Model                  | \(n_{B_0} \text{ fm}^{-3}\) | \((\varepsilon/n_B)_0 \text{ MeV fm}^{-3}\) |
|------------------------|-----------------------------|---------------------------------------------|
| 1st treatment of QMDD model | 0.46                        | 1023                                         |
| 2nd treatment of QMDD model | 0.55                        | 1083                                         |
| 3rd treatment of QMDD model | 0.36                        | 1120                                         |
| QMDTD model            | 0.45                        | 1006                                         |

VII. FIGURE CAPTIONS

Fig.1 The temperature \(T\) as a function of baryon density \(n_B\) with a fixed pressure \(P = 400 \text{MeVfm}^{-3}\), three dashed lines are for the first, second and three treatments of QMDD
models respectively, and the solid line is for QMDTD model.

Fig.2 The temperature $T$ as a function of baryon density $n_B$ for QMDTD model with four different fixed pressure $P = 400, 300, 200$ and $150 \text{MeVfm}^{-3}$ respectively.

Fig.3 The energy per baryon $\varepsilon/n_B$ as a function of baryon density $n_B$ for QMDD model (dashed line), and for QMDTD model (solid line).

Fig.4 The pressure $P$ as a function of energy density $E/V$ for QMDD model (dashed line), and for QMDTD model (solid line).

Fig.5 The stability windows of SM for the second treatment of QMDD model and for QMDTD model.
Fig. 1

QMDTD model

1st treatment of the QMDD model

3rd treatment of the QMDD model

2nd treatment of the QMDD model

$T(\text{MeV})$

$n_B(\text{fm}^{-3})$

FIG. 1
FIG. 2

$P = 400 \text{MeVfm}^{-3}$

$P = 300 \text{MeVfm}^{-3}$

$P = 200 \text{MeVfm}^{-3}$

$P = 150 \text{MeVfm}^{-3}$
Fig. 3

\[ \frac{\varepsilon}{n_B} (\text{MeV}) \]

\[ n_B (\text{fm}^{-3}) \]
Fig. 4

- **QMDTD model**
- **1st treatment of QMDD model**
- **2nd treatment of QMDD model**
- **3rd treatment of QMDD model**

- **P (MeV fm\(^{-3}\))**
- **E/V (MeV fm\(^{-3}\))**
Fig. 5: Stability window for QMDTD model and stability window for 2nd treatment of QMDD model.