Non-local current correlations in ferromagnet/superconductor nanojunctions

C. J. Lambert

Department of Physics, Lancaster University, Lancaster, LA1 4YB, UK

J. Koltai, J. Cserti

Department of Physics of Complex Systems, Eötvös University
H-1117 Budapest, Hungary

When two fully polarized ferromagnetic (F) wires with opposite polarizations make contact with a spin-singlet superconductor, a potential-induced current in wire 1 induces a non-local current of equal magnitude and sign in wire 2. The magnitude of this current has been studied in the tunneling limit and found to decay exponentially with the distance between the contact. In this paper we propose a new structure in which this novel non-local effect is increased by orders of magnitude. We study the spin-dependent electronic transport of a diffusive nanojunction and demonstrate that when a normal diffusive region is placed between the F leads and superconductor, the non-local initially increases with the separation between the F leads, achieving a maximum and decays as a power law with increasing separation.

Experimental studies of electronic transport properties of nanostructures containing both ferromagnets (F) and superconductors (S) reveal novel features, not present in normal-metal/superconductor (N/S) junctions, due to the suppression of electron-hole correlations in the ferromagnet. When spin-flip processes are absent, further effects are predicted, including the suppression of the conventional giant magnetoresistance ratio in diffusive magnetic multilayers and the appearance of non-local currents when two fully polarized ferromagnetic (F) wires with opposite polarizations make contact with a spin-singlet superconductor. The latter effect has been highlighted, because of interest in the possibility of generating entangled pairs of electrons at a superconductor (S) interface. A recent study of such a junction in the tunneling limit predicts that the magnitude of the non-local current will decrease exponentially with the distance between the F contacts. It is therefore of interest to ask how the magnitude of this novel effect can be enhanced.

In this paper we propose a hybrid nanostructure in which the non-local current is enhanced by orders of magnitude compared with the geometry of ref. The proposed structure is sketched in figure 1 and comprises two clean F wires, each of width $M_f$, separated by a distance $M$, in contact with a diffusive, normal metallic region of area $A = L(2M_f + M)$, which in turn makes contact with a spin-singlet superconductor.

In the linear response limit, the current-voltage relation of a such a hybrid structure connected by non-superconducting wires to normal reservoirs was first presented in ref. If $I_1, I_2$ are the currents leaving reservoirs 1 and 2, and $v_1, v_2$
their respective voltages, then at zero temperature \(^{18}\)

\[
I_1 = \left[ 2e^2 / h \right] \left[ (N - R_0 + R_a)(v_1 - v) + (T'_a - T'_0)(v_2 - v) \right] \tag{1a}
\]

and

\[
I_2 = \left[ 2e^2 / h \right] \left[ (T_a - T_0)(v_1 - v) + (N - R'_0 + R'_a)(v_2 - v) \right], \tag{1b}
\]

where \(v\) is the condensate potential. In this expression \(R_0\) is the coefficient for an electron from reservoir 1 to be reflected as an electron back into reservoir 1, while \(T_0\) is the coefficient for an electron from reservoir 1 to be transmitted as an electron into reservoir 2. \(R_a\) is the coefficient for an electron from reservoir 1 to be Andreev reflected as a hole into wire 1 and \(T_a\) is the coefficient for an electron from reservoir 1 to be transmitted as a hole into wire 2. Finally \(R'_0, T'_0, R'_a, T'_a\) are the corresponding coefficients for electrons originating from reservoir 2. All coefficients are evaluated at the Fermi energy and are obtained by summing over the elements of sub-matrices of the quantum mechanical scattering matrix \(S\). As a consequence of unitarity of \(S\), they satisfy \(N = R_0 + R_a + T_a + T_0 = R'_0 + R'_a + T'_a + T'_0\), where \(N\) is the number of open scattering channels in the wires.

As discussed in ref \(^{19}\), equations (1) are a convenient starting point for describing a wide variety of transport phenomena in hybrid superconducting nanostructures.

For example if lead 2 is employed as a voltage probe, with \(I_2\) set to zero, then equation (1b) yields

\[
(v_2 - v)/(v_1 - v) = (N - R'_0 + R'_a)/(T_0 - T_a). \tag{2}
\]

In the absence of Andreev processes, this ratio is positive. However if Andreev transmission dominates normal transmission, the ratio can be negative, leading to novel negative 4-probe conductances in hybrid superconducting structures \(^{20}\). The appearance of non-local currents is similarly obtained from equations (1) by setting \(v_2 - v = 0\), which yields

\[
I_1 = \left[ 2e^2 / h \right] [N - R_0 + R_a](v_1 - v) = \left[ 2e^2 / h \right] [2R_a + T_0 + T_a](v_1 - v) \tag{3}
\]

and

\[
I_2 = \left[ 2e^2 / h \right] [T_a - T_0](v_1 - v). \tag{4}
\]

When all Andreev processes are absent, this yields the expected result \(I_2 = -I_1 = -(2e^2 / h)T_0(v_1 - v)\), whereas when Andreev reflection and normal transmission are completely suppressed, \(I_2 = +I_1 = (2e^2 / h)T_a(v_1 - v)\). The latter occurs when the F wires are completely spin polarized and the polarization in wire 1 is the opposite of that in wire 2. More generally \(I_1\) and \(I_2\) will have opposite signs whenever Andreev transmission dominates normal transmission. As noted in ref \(^{20}\), this can occur even if the current carrying wires are not ferromagnetic.

In the absence of a normal region in front of the superconductor (ie \(L = 0\)) and in the presence of tunnel junctions at the F-S interfaces, the analysis of ref \(^{13}\) predicts that both \(I_1\) and \(I_2\) decay as \(\exp - (2L / \pi \xi)\), where \(\xi\) is the superconducting coherence length. To analyze this structure for finite \(L\) we solve the Bogoliubov-de Gennes equation on a square tight binding lattice and compute the scattering matrix using an exact recursive Green’s function technique.
We start by defining a tight-binding lattice of sites with the geometry of figure 1. Each site is labeled by a lattice vector \( \vec{l} \) and possesses particle (hole) degrees of freedom \( \psi^\sigma(\vec{l}) \) \( (\phi^\sigma(\vec{l})) \), with spin \( \sigma = \pm 1 \). Ferromagnetism is incorporated via a Stoner model, with an exchange splitting \( h_0(\vec{l}) \) on site \( \vec{l} \). In the presence of local s-wave pairing described by a superconducting order parameter \( \Delta(\vec{l}) \), the Bogoliubov equation takes the form

\[
E\psi^\sigma(\vec{l}) = [\epsilon(\vec{l}) - \sigma h_0(\vec{l})]\psi^\sigma(\vec{l}) - \sum_\delta \gamma \psi^\sigma(\vec{l} + \vec{\delta}) + \sigma \Delta(\vec{l}) \phi^{-\sigma}(\vec{l}) \tag{5}
\]

\[
E\phi^\sigma(\vec{l}) = -[\epsilon(\vec{l}) - \sigma h_0(\vec{l})]\phi^\sigma(\vec{l}) + \sum_\delta \gamma \phi^\sigma(\vec{l} + \vec{\delta}) - \sigma \Delta^*(\vec{l}) \psi^{-\sigma}(\vec{l}), \tag{6}
\]

where \( \vec{l} + \vec{\delta} \) sums over the nearest neighbors of \( \vec{l} \).

If \( \vec{l} \) belongs to the normal diffusive region of figure 1, \( \epsilon(\vec{l}) \) is chosen to be a random number, uniformly distributed over the interval \( \epsilon_0 - W/2 \) to \( \epsilon_0 + W/2 \), whereas in the clean F and S regions \( \epsilon_j = \epsilon_0 \). In the S region, the order parameter is set to a constant, \( \Delta(\vec{l}) = \Delta_0 \), while in all other regions, \( \Delta(\vec{l}) = 0 \). If \( \vec{l} \) is located in one of the clean ferromagnetic leads, then Stoner splitting is \( h_0(\vec{l}) = h_j \), where \( j = 1, 2 \) labels the lead containing site \( \vec{l} \). For aligned moments \( h_1 = h_2 = \epsilon_M \) and for anti-aligned moments, \( h_1 = -h_2 = \epsilon_M \). If \( \vec{l} \) is not located in one of the ferromagnetic leads then \( h_0(\vec{l}) = 0 \). The nearest neighbor hopping element \( \gamma \) fixes the energy scale (ie the band-width), whereas \( \epsilon_0 \) determines the band-filling. In what follows, to model the experimentally-relevant regime of \( \Delta_0 \ll \text{Fermi energy}(= 4\gamma - \epsilon_0) \) and \( M_f \gg \text{Fermi wavelength} \), we set \( \gamma = 1, \epsilon_0 = 0.2, \epsilon_M = 3.8, M_f = 10 \) and \( \Delta_0 = 0.1 \).

By numerically solving the Bogoliubov - de Gennes equation for a given energy \( E \), the scattering matrix \( S \) can be computed for a given realization of the disorder and average values of measurable quantities obtained ensemble averaging the quantity of interest. For a given value of disorder \( W \), the elastic mean free path \( \lambda_{el} \) of the normal region is obtained by from a separate calculation of the ensemble-averaged dc conductivity of a rectangular normal bar, connected to normal reservoirs. \( \lambda_{el} \) is then obtained by comparing the result with the Drude formula. In what follows, the lattice constant is set to unity and \( W \) chosen such that \( \lambda_{el} = 10 \).

For \( E = 0 \), figures 2 and 3 show the dependence on \( L \) and \( M \) of the ensemble-averaged Andreev transmission coefficient in the presence of non-magnetic wires, while figures 4 and 5 show the corresponding quantities in the presence of fully spin-polarized wires with opposite magnetizations. For small \( L \) (eg \( L = 1 \)) and finite \( M \), figures 3 and 5 show that \( T_a \) is exponentially small, as predicted by \(^{13}\). However as \( L \) is increased to a finite value, \( T_a \) shows a dramatic enhancement.

Although this structure is difficult to describe analytically, the qualitative behaviour of \( T_a \) is clear and can be understood in terms of a simple resistor model, in which \( T_a \) is proportional to the conductance of electrons from lead 1 travelling first to the N-S interface and then to lead 2. Consider for example figure 2, which for \( M \neq 0 \) shows the expected exponential suppression of \( T_a \) at \( L = 0 \). For small \( L \), \( T_a \) increases linearly with \( L \). The slope of this linear regime is independent of \( M \) for small \( M < \lambda_{el} \), reflecting the ballistic nature of the N region in this limit, whereas
for the opposite limit of $M > \lambda_{cl}$, the slope decreases with increasing $M$, reflecting the diffusive behaviour of the N region. In the diffusive regime, for $M << L$, this leads us to expect the $T_a$ is proportional to $L/(M + 2M_f)$, whereas for $L << M$ it is proportional to $(M + 2M_f)/L$, which suggests that the maximum value of $T_a$ occurs when $L \approx M + 2M_f$. Figure 4 shows that same qualitative behaviour also persists in the presence of ferromagnetic leads.

In conclusion, we have shown that in contrast with the tunneling regime, where $T_a$ is exponentially small, the non-local current can be dramatically enhanced by including a normal region between the F leads and superconductor. In this case the non-local current no longer decays monotonically with $M$ or $L$, but instead exhibits a maximum corresponding to a square normal region with $L \approx M + 2M_f$.

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Figure 2: For normal leads (i.e. when $\epsilon_M = 0$) this figure shows Andreev transmission against length $L$ for different contact separations $M$.

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Figure 3: For normal leads, this figure shows Andreev transmission against contact separation $M$ for different lengths $L$.

Figure 4: For ferromagnetic leads (i.e. $\epsilon_M = 3.8$) with anit-aligned magnetizsations, this figure shows Andreev transmission length $L$ for different contact separations $M$. 
Figure 5: For ferromagnetic leads (i.e. $\epsilon_M = 3.8$) with unit-aligned magnetizations, this figure shows Andreev transmission against contact separation $M$ for different lengths $L$. 