QCD Calculations of Decays of Heavy Flavor Hadrons

Matthias Neubert
Institut für Physik (THEP), Johannes Gutenberg-Universität
D-55099 Mainz, Germany

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Precision tests of the Standard Model and searches for New Physics in the quark flavor sector depend on accurate theoretical calculations of decay rates and spectra for rare, flavor-changing processes. The theoretical status and recent developments of techniques allowing such calculations are reviewed. Special attention is paid to the calculation of the $B \to X_s \gamma$ branching ratio, the extraction of the $b$-quark mass from a fit to $B \to X_u l \bar{\nu}_l$ moments, and the determination of $|V_{ub}|$ from spectra in the inclusive decay $B \to X_u l \bar{\nu}_l$. From a reanalysis of different inclusive distributions the updated average value $|V_{ub}| = (3.98 \pm 0.15 \pm 0.30) \cdot 10^{-3}$ is derived. Using only the theoretically cleanest channels, we obtain $|V_{ub}| = (3.70 \pm 0.15 \pm 0.28) \cdot 10^{-3}$.

I. INTRODUCTION

Heavy-quark physics is a corner stone in the ongoing effort to explore the Standard Model of elementary-particle interactions, to determine its parameters with greatest achievable precision, and to search for hints of departures from Standard Model predictions. Thanks to an experimental program at various facilities worldwide that has spanned several decades, accompanied by steady progress in theory, there have been tremendous accomplishments in this field and several important discoveries have been made. While the Standard Model still stands as the most fundamental theoretical structure explaining the fundamental interactions of matter in the Universe, the combined effort of energy-frontier and luminosity-frontier experiments in the coming decade is likely to shed light on the deep questions concerning the physics at the terascale – questions about the origin of mass, the asymmetry between matter and antimatter, and the nature of dark matter.

One of the main challenges in heavy-quark physics is to disentangle the underlying, flavor-changing couplings of the quarks (which in the Standard Model encode all information about CP violation) from the less interesting but all dominating effects of the strong interactions, as described by Quantum Chromodynamics (QCD). Sophisticated theoretical tools going beyond the realm of perturbation theory are required to accomplish percent-level precision in the calculation of decay rates and kinematical distributions for the most interesting processes. Some of the relevant tools and recent developments are discussed in this talk.

II. THEORETICAL FOUNDATIONS

Before discussing specific applications it is important to summarize the theoretical tools we have for calculating decay rates and spectra in heavy-quark physics, and the concepts from which these tools derive. The underlying theme is the separation of short-distance from long-distance physics, which is natural due to the presence of the large mass scale $m_Q \gg \Lambda_{\text{QCD}}$, which is far above the scale of nonperturbative hadronic physics. Factorization theorems state that short- and long-distance contributions to a given observable can be separated into Wilson coefficient functions $C_i$ and nonperturbative matrix elements $M_i$. Generically, up to power corrections of the form $(\Lambda_{\text{QCD}}/m_Q)^n$ one has

$$\text{Observable} \sim \sum_i C_i(m_Q, \mu) M_i(\mu) + \ldots .$$

Such a factorization formula is useful, since by virtue of it the dependence on the high scale $m_Q$ is calculable, and often the number of matrix elements $M_i$ is smaller than the number of observables that can be expressed in the form shown above. Typically, this reduction is accomplished by means of some symmetry. The nonperturbative matrix elements can then be extracted from data or calculated using theoretical approaches such as lattice QCD or QCD sum rules. The factorization scale $\mu$ in (1) serves as an auxiliary separator between the domains of short- and long-distance physics. Observables are formally independent of the choice of $\mu$; however, they inherit some residual dependence once the Wilson coefficients $C_i$ are computed at finite order in perturbation theory. The dependence gets weaker as higher orders in the perturbative expansion are included. The $\mu$-independence of the right-hand side of (1) implies a renormalization-group equation for the functions $C_i$, which can be solved. In the process, large perturbative logarithms of the form $\alpha_s^n \ln^b (m_Q/\mu)$ can be summed to all orders in perturbation theory.

The formal basis of the factorization formula (1) is the (euclidean) operator product expansion. Important examples of this type of factorization include the effective weak Hamiltonian [1]

$$H_{\text{eff}} = \sum_i C_i(m_t, m_W, m_Z, m_H, M_{\text{NP}}, \mu) Q_i(\mu),$$

which is the starting point of any calculation in quark flavor physics. In this case the Wilson coefficients $C_i$ con-

*On leave from Laboratory for Elementary-Particle Physics, Cornell University, Ithaca, NY 14853, U.S.A.
tain short-distance physics associated with heavy Standard Model particles such as the top-quark, the electroweak gauge bosons, or the Higgs boson. However, if there exists physics beyond the Standard Model involving some new heavy particles that couple to the Standard Model, then the Wilson coefficients will unavoidably also be sensitive to the masses and couplings of these new particles. An important goal of flavor physics is to search for these kinds of effects. Other important applications of the factorization theorem include the derivation of heavy-quark symmetry relations for the exclusive semileptonic decays $\bar{B} \to D^{(*)} l \bar{\nu}_l$ and $\bar{B} \to \rho l \bar{\nu}_l$, which have been studied in great detail using light-cone QCD sum rules (for recent work, see e.g. [24, 25]). Also, the QCD factorization approach to nonleptonic processes such as $\bar{B} \to D^{(*)} \pi$, $\bar{B} \to \pi K$, and $\bar{B} \to K^* \gamma$ relies on such a factorization theorem [26, 27, 28, 29, 30, 31, 32]. Some important recent accomplishments in this area have been the calculation of the hard-scattering kernels in the QCD factorization formula for rare hadronic decays of the form $\bar{B} \to h_1 h_2$ (where $h_i$ are light pseudoscalar or vector mesons) at $O(\alpha_s^2)$ [33, 34, 35], the proof of factorization for the decay $\bar{B} \to K^* \gamma$ [36], and the calculation of part of the $O(\alpha_s^2)$ corrections for this process [37].

Any systematic theoretical approach has limitations, an understanding of which is necessary in order to gauge the accuracy of calculations and, more generally, understand what can and what cannot be calculated in a model-independent way. An obvious practical limitation of any calculation is the need to truncate the perturbative expansion of the perturbative coefficients $C_i$ in (1), and $H_i$, $J_i$ in (3). State of the art is to compute these objects at next-to-next-to-leading order (NNLO) in perturbation theory. Even if it was possible to calculate the perturbative functions to all orders (and of course we do not have the tools to accomplish this!), then the asymptotic nature of perturbative expansions in quantum field theory would imply an irreducible ambiguity in the precision with which these functions can be determined, an ambiguity that scales like a power of $\Lambda_{QCD}/Q$. In order to do better, it is necessary to include power-suppressed corrections in the calculation. In some cases such as the inclusive decays $\bar{B} \to X_u l \bar{\nu}_l$ and $\bar{B} \to X_{\gamma\gamma}$, a factorization theorem such as (3) can be shown to hold at any order in perturbation theory. In other cases, such as the nonleptonic decays $\bar{B} \to \pi K$ and $\bar{B} \to \pi \pi$, factorization breaks down beyond the leading power in $\Lambda_{QCD}/m_b$ [38, 39, 40]; however, starting already at first order in $\Lambda_{QCD}/m_b$ the number of unknown soft functions exceeds the number of observables. In other cases, such as the one-lepton radiative decays $\bar{B} \to \pi K \gamma$ and $\bar{B} \to \pi \pi \gamma$, factorization breaks down beyond the leading power in $\Lambda_{QCD}/m_b$ [26, 27, 28]. It follows from these remarks that observables very sensitive to higher-order perturbative effects or power corrections cannot be predicted with high accuracy. Well-known examples include some direct CP asymmetries for the decays $\bar{B} \to \pi K$ and $\bar{B} \to \pi \pi$, and the branching fractions for some “color-suppressed” decays such as $\bar{B} \to \pi^0 \pi^0$. The fact that calculations are rather uncertain in these cases does not indicate a failure of the factorization framework (which is rooted in quantum field theory and as such cannot be questioned), but rather that in some cases the expansion in powers of $\alpha_s$ and $\Lambda_{QCD}/m_b$ does not converge well.

Before turning to phenomenological applications we...
should mention an ongoing theoretical debate on the
question of the breakdown of factorization beyond the
leading power in heavy-to-light transitions such as the
$B \rightarrow \pi$ form factor or hadronic processes such as $B \rightarrow \pi \pi$. It is well known that a perturbative analysis of the
form factor is plagued by “endpoint divergent” convolu-
tion integrals. Power corrections to the factorization formula for $B \rightarrow \pi \pi$, which arise from processes such as “weak annihilation”, give rise to sim-
elar endpoint divergences. It has recently been
claimed that these divergences can be removed, and fac-
torization be established, in a systematic way using so-
called “0-bin” subtractions combined with rapidity fac-
torization. Unfortunately this claim has not been
supported by detailed calculations, and in my view it is
unlikely that the problem can be solved following the
approach described in that paper. Nevertheless, based
on these results it has been argued that weak annihila-
tion amplitudes and power-suppressed penguin am-
plitudes should be real to a good approximation, i.e.,
free of large soft rescattering phases. If true, this would
have important implications for phenomenology. Finally,
the prediction of the QCD factorization approach that
charm-penguin contributions to nonleptonic decay am-
plitudes factorize has been challenged in, and a rebuttal has been given in. More theoretical
work on this important issue would be desirable.

The remainder of this talk will focus on three im-
portant applications of heavy-quark theory: the calcula-
tion of the partial inclusive $B \rightarrow X_s \gamma$ branching ratio
at NNLO, the extraction of the $b$-quark mass from mo-
ments of inclusive decay spectra, and the determina-
tion of $|V_{ub}|$ from inclusive $B \rightarrow X_u \ell \nu$ decay distributions.

New Physics implications of $B \rightarrow X_s \gamma$ decay have been
discussed by G. Isidori and S. Heinemeyer at this con-
ference, while determinations of $|V_{ub}|$ from exclusive de-
cays have been covered in the talks of E. Barberio and
C. Davis. For a lack of time, this is a very limited sele-
tion of topics from a vast range of possibilities. My
apologies go to all those authors whose important con-
tributions are omitted here.

III. THE $B \rightarrow X_s \gamma$ BRANCHING RATIO

The decay $B \rightarrow X_s \gamma$ is the prototype of a flavor-
changing neutral current process. In the Standard Model
it is mediated by loops involving heavy top quarks and
$W$ bosons, and it is expected to yield unknown heavy
particles, if they exist at the TeV scale, could give con-
tributions to the decay rate not much below the Standard
Model level. A precise study of this process, both exper-
imentally and theoretically, can thus serve as a precision
test of the flavor sector and place constraints on the pa-
rameters of New Physics models. Indeed, it is by now
broadly appreciated that the constraints on model building
from processes such as $B \rightarrow X_s \gamma$ can be as valuable as
constraints from electroweak precision observables, astro-
physical observations, and other low-energy observables
such as electric dipole moments and the anomalous mag-
netic moment of the muon.

About one year ago, the calculation of most NNLO
contributions (all that are believed to be significant) to
the $B \rightarrow X_s \gamma$ decay rate has been completed, and a first
prediction for the branching ratio has been derived in a
paper by Misak at 16 coworkers. This has been a
heroic effort, which would not have been possible with-
out a large collaborative approach. At the same time, a
dedicated, model-independent analysis of the effect of a
cut on the photon energy, $E_\gamma > E_0$ with $E_0$ in the range
between 1.8 and 2 GeV as applied in the experimental
analyses, has been performed. It is conventional (yet somewhat artificial) to write the branching
ratio as $Br(E_\gamma > E_0) = Br(\text{tot}) (E_0)$, where the function
$F(E_0)$ describes the effect of the cut, while $Br(\text{tot})$ is the “total” branching ratio (which in practice refers
to a very low cut $E_\gamma > 1$ GeV, since the total branching
ratio does not exist).

We begin with a brief review of the NNLO calculation
of the total branching ratio. The first task was to com-
pute the Wilson coefficients in the effective weak Hamil-
tonian at NNLO. This required two-loop matching
calculations for the coefficients $C_{1-6}$ at the weak scale
and three-loop matching calculations for the coeffi-
cients $C_{7\gamma}$ and $C_{8\gamma}$. These matching conditions are
the place where New Physics contributions would enter
the theoretical analysis. Next, the Wilson coeffi-
cients have to be evolved down to a scale $\mu \sim m_b$
using the renormalization group. As input, one needs the
three-loop anomalous dimension matrices for the opera-
tors $Q_{1-6}$ and $Q_{7\gamma}$, $Q_{8\gamma}$, as well as the four-loop anomalous dimensions accounting for the mixing of $Q_{1-6}$
into $Q_{7\gamma}, Q_{8\gamma}$. With these results at hand, the second
task was to compute the matrix elements of the various
operators at NNLO. The full two-loop corrections to the
matrix element of the leading dipole operator $Q_{7\gamma}$ were
obtained in. For the required three-loop matrix elements of the operators $Q_{1,2}$ only an extrapolation
in the mass of the charm-quark is available based on
calculations performed in the artificial limit $m_c \gg m_b/2$
One of the dominant uncertainties in the prediction
results from this extrapolation. The remaining matrix
elements are known at $O(\alpha_s^2 \alpha_s^2)$ and $O(\alpha_s^4)$. The resulting prediction for the total branching ratio in the Standard
Model is

$$ Br(E_\gamma > 1 \text{ GeV}) = (3.27 \pm 0.23) \cdot 10^{-4}. \quad (4) $$

The combined theoretical uncertainty results from a 3% error due to higher-order perturbative corrections, a 3% error in the extrapolation in the charm-quark mass, another 3% uncertainty from parameter variations (from $m_\tau, m_c, \alpha_s$ etc.), and finally a 5% error assigned to account for nonperturbative power corrections to the total rate.

The result is a milestone in the history of heavy-
flavor theory. Note that the largest contribution to the
theoretical uncertainty is now due to our lack of control
over nonperturbative power corrections. Unfortunately,
it will be very difficult if not impossible to reduce this
error with current technology. A recent analysis has iden-
tified a new class of nonlocal power corrections to the total
decay rate, which are described in terms of matrix ele-
ments of trilocal light-cone operators \[68\]. Naive model
estimates of these effects point to a small reduction of the
total rate with an uncertainty of 5%. However, this num-
ber is only a guess, and it is difficult to substantiate it.
There are other, similar types of nonlocal contributions,
which are currently being analyzed \[69\].

The presence of a cut \(E_\gamma > E_0\) on the photon energy,
which is applied in order to reduce the background in all
measurements of the \(\bar{B} \to X_s \gamma\) branching ratio to date,
leads to significant complications in the theoretical anal-
ysis. The reason is that it introduces a sensitivity to the
low scale \(\Delta = m_b - 2E_0 \approx 1\) GeV, which is barely in the
perturbative domain. One thus needs to deal with a com-
licated many-scale problem characterized by the hierar-
chy \(m_b > \sqrt{m_b \Delta} > \Delta > \Lambda_{\text{QCD}}\). A systematic framework
for this situation is the multi-scale operator product ex-
pansion developed in \[70\]–\[71\]. In this case the theoretical
accuracy that can be achieved is not set by size of the ex-
pansion parameters \(\alpha_s(m_b) \approx 0.2\) and \(\Lambda_{\text{QCD}}/m_b \approx 0.1\),
but rather by \(\alpha_s(\Delta) \approx 0.4\) and \(\Lambda_{\text{QCD}}/\Delta \approx 0.5\) (though
first-order corrections in this ratio are absent). A detailed
analysis at NNLO comes to the conclusion that \[54\]

\[
F(1.6 \text{ GeV}) = 0.93^{+0.04}_{-0.06},
\]

which tends to be a stronger suppression than that pre-
dicted by naive model estimates \[72\]. The above number
includes a perturbative uncertainty of \(1.5\)%, parameter
dependences of 2%, and an error from power corrections
of also 2%.

Combining the results \[41\] and \[54\], one obtains the
NNLO prediction \[53\]

\[
\text{Br}(E_\gamma > 1.6 \text{ GeV}) = (2.98 \pm 0.26) \cdot 10^{-4}.
\]

It is consistent within errors with the result \((3.15 \pm 0.23)\cdot
10^{-4}\) quoted in \[51\], which is based on a naive pertur-
bative expansion in \(\alpha_s(m_b)\). For completeness we also
quote the number \((3.49 \pm 0.49) \cdot 10^{-4}\) obtained in \[73\]
using a renormalon-inspired model for the shape func-
tion in \(B \to X_s \gamma\) decay. Note that the difference with
respect to \(40\) is primarily due to a larger total branching
ratio obtained in this scheme, not so much due to the
treatment of cut-related effects.

The theoretical result \(40\) for the branching ratio is
about 1.4 standard deviations lower than the current
world-average experimental value \(\text{Br}(E_\gamma > 1.6 \text{ GeV}) =
(3.55 \pm 0.26) \cdot 10^{-4}\) \[24\], indicating that there is some room
for possible New Physics effects. We emphasize, however,
that to date no actual measurement has been performed
with a cut as low as 1.6 GeV. Indeed, the quoted experi-
mental number involves a model-dependent extrapola-
tion to low energy. In the future, it would be desirable
if experiment and theory would be confronted at a value
of \(E_0\) that can indeed be achieved experimentally. The
lowest such value at present is \(E_0 = 1.8\) GeV.

\section*{IV. EXTRACTION OF \(m_b\) FROM A FIT TO \(\bar{B} \to X_s \ell \bar{\nu}_\ell\) MOMENTS}

The operator product expansion allows for a model-
dependent calculation of moments of decay distribu-
tions in the inclusive semileptonic process \(\bar{B} \to X_s \ell \bar{\nu}_\ell\)
\[3\]. Hadronic physics is encoded in a few parameters
defined in terms of forward \(B\)-meson matrix elements of
local operators. They are called \(\mu_2^s\) (or \(\lambda_1\)), \(\mu_2^G\) (or \(\lambda_2\)),
etc. The most important parameters in the prediction
are, however, the heavy-quark masses \(m_b\) and \(m_c\). The
only assumption underlying the theoretical calculations
is that of quark-hadron duality. It is believed to be reli-
able for the relevant energy release \(\Delta E \approx m_b - m_D\).

Traditionally, the parameters \(\{\{V_{cb}, m_b, m_c, \mu_2^s, \mu_2^G\}\}
are extracted from a global fit to experimental data on
moments of the lepton energy and invariant hadronic
mass spectra in \(\bar{B} \to X_s \ell \bar{\nu}_\ell\) decay and of the photon
energy spectrum in \(\bar{B} \to X_s \gamma\) decay. The experimental
data include measurements reported by the BaBar, Belle,
CLEO, CDF, and DELPHI collaborations \[74\], while the
teoretical calculations are based on formulae derived in
\[75\]–\[78\]. The global fit takes account of the strong
correlations between the various quantities. The status of
the theoretical calculations is such that the leading terms
in the operator product expansion for the moments are
known at \(O(\beta_0 \alpha_s^2)\) but not yet at \(O(\alpha_s^3)\). Power-suppressed
contributions are only known at tree level. It will be
important in the future to increase the accuracy of the
calculations. The technology for obtaining the complete
\(O(\alpha_s^2)\) corrections to the leading term exists \[79\], and it
is expected that results will soon be published. Also,
the \(O(\alpha_s)\) corrections to the leading power-suppressed
effects, those proportional to the parameters \(\mu_2^s\) and \(\mu_2^G\),
are currently being calculated. Results have already been
published for the terms proportional to \(\mu_2^s\) \[80\], but they
have not yet been included in the fits.

The fit strategy is as follows: A set of experimental re-
results, including errors and correlations, is fitted to a set of
theoretical equations derived using the operator product
expansion. The fit is performed in the space of parame-
ters \(\{\{V_{cb}, m_b, m_c, \mu_2^s, \mu_2^G\}\}\) mentioned above. Besides
\(\{V_{cb}\},\) these parameters must be defined in a particular
renormalization and subtraction scheme. The most pop-
ular schemes are the kinetic scheme \[81\], the 1S scheme
\[82\], and the shape-function scheme \[22\]–\[24\]. While in
principle it is merely a matter of choice which of these
schemes one prefers, fitting in scheme A and convert-
ing the results to scheme B might give slightly differ-
ent results from fitting directly in scheme B, if truncated
perturbative expressions are employed. This fact
should be taken into account when deriving estimates
for the theoretical uncertainties; specifically, the quoted
uncertainties should not be less than the uncertainties inherent in the scheme translations. To illustrate this point, we quote results derived by the Heavy Flavor Averaging Group (HFAG) for this conference. They obtain $m_b^{\text{kin}} = (4.613 \pm 0.035)$ GeV from a fit performed in the kinetic scheme and $m_b^{\text{IS}} = (4.701 \pm 0.030)$ GeV from a fit performed in the 1S scheme \cite{74}. On the other hand, translating the first number from the kinetic to the 1S scheme at $O(\beta_0 \alpha_s^2)$ would give $m_b^{\text{IS}}$ from kin $= (4.745 \pm 0.036)$ GeV, which is about 1.5 standard deviations higher than the value obtained from the fit in the 1S scheme. This indicates that the uncertainty is larger than the 30 MeV quoted from the fit.

Table I collects results obtained by translating the central values $m_b^{\text{kin}} = 4.613$ GeV (upper portion) and $m_b^{\text{kin}} = 4.677$ GeV (lower portion) from the kinetic scheme into other schemes (MS, 1S, and shape-function scheme). The formulae translating from one scheme to another involve perturbative expansions, and following common practice we choose to evaluate the coupling $\alpha_s$ in these expansions at a scale $\mu$ varied between $m_b/2$ and $2m_b$. This results in the quoted uncertainties. A few comments are worth mentioning: First, note that the scale uncertainty from the conversion into the MS scheme remains significant even at two-loop order. This unfortunate fact prevents us from using the very precise value for $m_b^{\text{kin}}$ recently obtained from an analysis of the total $e^+e^- \to \text{hadrons}$ cross section in the threshold region \cite{84} to derive similarly precise values for the $b$-quark mass in other schemes.

Next, note that the value of $m_b$ at NLO in the 1S scheme has a tiny scale uncertainty, yet the value obtained at NNLO differs from the NLO value by more than 40 MeV. This illustrates that scale variation can sometimes underestimate the true theoretical uncertainty. Finally, note that for the translation into the shape-function scheme two-loop corrections not associated with running coupling effects appear to be important. It would therefore be desirable to include the full $O(\alpha_s^2)$ corrections in the moment fit.

A serious problem with the way the fit is currently performed by HFAG is that besides the moments of semileptonic spectra in $\bar{B} \to X_c l \bar{\nu}_l$ decay also moments of the $\bar{B} \to X_c \gamma$ photon energy spectrum are used. Based on what we know about $\bar{B} \to X_c \gamma$ decay, this treatment can no longer be justified theoretically! Whereas a clean theoretical approach to the $\bar{B} \to X_c \gamma$ decay is provided by the operator product expansion, the theoretical basis for the calculation of the $\bar{B} \to X_c \gamma$ moments is far more complicated and subject to uncontrollable theoretical uncertainties. First, it is unquestionable that the existing measurements of the $\bar{B} \to X_c \gamma$ moments are performed in a kinematic region where shape-function effects are still important \cite{85}. To correct for these effects, a (shape-function) model-dependent “bias correction” is applied to the data \cite{86}. More generally, it has now been established that as soon as operators other than the dipole operator $Q_{7\gamma}$ are not ignored, then there is no local operator product expansion for the $\bar{B} \to X_c \gamma$ decay rate and moments, even outside the shape-function region \cite{63,69}. Indeed, a novel factorization formula can be derived for the photon energy spectrum, which is graphically represented in Figure 1. Besides a “direct photon” contribution (first term), there appear “single resolved” and “double resolved photon” contributions, which are impossible to calculate with present tools. Their impact on the moments of the photon spectrum is currently under investigation.

When the goal is to extract heavy-quark parameters such as $m_b$ in a model-independent way, one should thus not include the $\bar{B} \to X_c \gamma$ moments in the fit. An analysis based only on the $\bar{B} \to X_c l \bar{\nu}_l$ moments leads to $m_b^{\text{kin}} = (4.677 \pm 0.053)$ GeV in the kinetic scheme and $m_b^{\text{IS}} = (4.751 \pm 0.058)$ GeV in the 1S scheme \cite{74}. The increase of the uncertainties can be understood by observing that in the combined fit there was a slight tension between the results from the $\bar{B} \to X_c l \bar{\nu}_l$ and $\bar{B} \to X_c \gamma$ moments, as illustrated in Figure 2. The increased errors shown above more properly reflect the true uncertainties. These errors will be reduced (perhaps to 30–40 MeV) once the $O(\alpha_s^2)$ and $O(\alpha_s/m_b^2)$ corrections will be included in the fit.

| $m_b^{\text{kin}} = 4613$ MeV | $O(\alpha_s)$ | $O(\beta_0 \alpha_s^2)$ | $O(\alpha_s^2)$ | Reference values |
|-----------------------------|---------------|-----------------------|---------------|-----------------|
| $\overline{m}_b(m_b)$ [MeV] | $4317^{+55}_{-88}$ | $4159^{+67}_{-96}$ | $4195^{+55}_{-72}$ | $4164 \pm 25$ ($e^+e^- \to \text{hadrons}$ \cite{84}) |
| $m_b^{\text{IS}}$ [MeV]   | $4693^{+6}_{-8}$ | $4745^{+5}_{-9}$ | $4742^{+13}_{-9}$ | $4701 \pm 30$ (fit in 1S scheme \cite{74}) |
| $m_b^{\text{SF}}$ [MeV]   | $4690^{+1}_{-2}$ | $4584^{+6}_{-16}$ | $4643^{+24}_{-11}$ | $4630 \pm 60$ (used by HFAG update for LP07) |

| $m_b^{\text{kin}} = 4677$ MeV |
|-----------------------------|---------------|-----------------------|
| $\overline{m}_b(m_b)$ [MeV] | $4377^{+55}_{-89}$ | $4217^{+67}_{-97}$ |
| $m_b^{\text{IS}}$ [MeV]   | $4761^{+6}_{-8}$ | $4807^{+8}_{-9}$ |
| $m_b^{\text{SF}}$ [MeV]   | $4671^{+1}_{-2}$ | $4648^{+6}_{-16}$ | $4706^{+24}_{-11}$ | $4707 \pm 56$ (HFAG update for LP07) |
V. INCLUSIVE DETERMINATION OF $|V_{ub}|$

The theoretical basis for the analysis of differential decay distributions in the inclusive semileptonic process $\bar{B} \to X_u l \bar{\nu}_l$ is a factorization formula of the form \cite{18}, in which the soft functions $S_i$ are generalized parton distribution functions for the $B$ meson called shape functions \cite{19,20,21,22}. The leading-order shape function is a universal, process-independent quantity, which also (to a large extent) determines the shape of the $\bar{B} \to X_u \gamma$ photon energy spectrum. The strategy is therefore to extract the shape function from the $\bar{B} \to X_u \gamma$ photon spectrum and then use this information to predict $\bar{B} \to X_u l \bar{\nu}_l$ decay distributions \cite{22,57,58}. The functional form of the shape function is constrained by moment relations, which relate weighted integrals over the shape function to the heavy-quark parameters $m_b$, $\mu^2$ etc. extracted from the $\bar{B} \to X_u l \bar{\nu}_l$ moment fit. An alternative is to employ shape-function independent relations between weighted $\bar{B} \to X_u \gamma$ and $\bar{B} \to X_c l \bar{\nu}_l$ spectra \cite{19,89,90,91,92}. Both approaches are equivalent; yet, not all predictions that have been obtained using the shape-function independent relations are up to the standard of present-day calculations.

The elimination of background events from $\bar{B} \to X_u l \bar{\nu}_l$, in which the charm quark is misidentified, is accomplished by means of different experimental cuts. The most common ones are a cut $M_X < m_D$ on the hadronic invariant mass of the final state, a cut $E_l > (m_B^2 - m_X^2)/2m_B$ on the energy of the charged lepton, a cut $q^2 > (m_B - m_D)^2$ on the invariant mass squared of the lepton-neutrino pair, or a cut $P_+ < m_D^2/m_B$ on the plus component of the total momentum of the final-state hadrons. All are designed such that they eliminate charm background while keeping some portion of the signal events. From a theoretical perspective the ideal cut is that on hadronic invariant mass, followed by the $P_+$ cut. They keep more than 50% of the signal events. Cutting on lepton energy or leptonic mass is far less efficient.

Until this summer, the most complete theoretical analysis of inclusive $\bar{B} \to X_u l \bar{\nu}_l$ spectra was based on calculations described in \cite{22,58} and referred to as the BLNP approach. It includes complete perturbative calculations at NLO with Sudakov resummation, subleading shape functions at tree level, and kinematical power corrections at $O(\alpha_s)$. The calculation of $O(\alpha_s^2)$ corrections is in progress and could have a significant impact \cite{93}. An alternative scheme called “Dressed Gluon Exponentiation” (DGE) \cite{94,95,96} employs a renormalon-inspired model for the leading shape function, which is less flexible in its functional form that the forms used by BLNP. Also, no
TABLE II: Compilation of $|V_{ub}|$ values obtained using different partial $B \to X_u \ell \bar{\nu}_\ell$ decay rates, analyzed using the BLNP approach [22, 38]. The first column of $|V_{ub}|$ values is obtained by extracting the $b$-quark mass from the combined fit to $B \to X_s \ell \bar{\nu}_\ell$ and $B \to X_d \gamma$ moments [74]. The second column is obtained by only using the theoretically clean information from $B \to X_u \ell \bar{\nu}_\ell$ moments (courtesy of HFAG).

| Method                        | $m_b^{SF}$ [GeV]: | $a$ | $|V_{ub}| \times 10^{-3}$ (old) | $|V_{ub}| \times 10^{-3}$ (new) |
|-------------------------------|-------------------|-----|-------------------------------|-------------------------------|
| CLEO, $E_l > 2.1$ GeV         | 4.63 ± 0.06       | 14.4 | 3.91 ± 0.46 ± 0.44            | 3.52 ± 0.41 ± 0.35           |
| Belle, $E_l > 1.9$ GeV        | 4.71 ± 0.06       | 9.5  | 4.67 ± 0.43 ± 0.36            | 4.35 ± 0.40 ± 0.33           |
| BaBar, $E_l > 2.0$ GeV        | 11.4              | 11.4 | 4.23 ± 0.24 ± 0.39            | 3.89 ± 0.22 ± 0.33           |
| BaBar, $E_l > 2.0$ GeV, $s_h^\text{max} < 3.5$ GeV$^2$ | 13.8              | 13.8 | 4.37 ± 0.29 ± 0.49            | 3.94 ± 0.27 ± 0.39           |
| Belle, $M_X < 1.7$ GeV        | 10.5              | 10.5 | 3.92 ± 0.26 ± 0.32            | 3.66 ± 0.24 ± 0.27           |
| BaBar, $M_X < 1.55$ GeV       | 13.6              | 13.6 | 4.09 ± 0.20 ± 0.39            | 3.74 ± 0.18 ± 0.31           |
| Belle, $M_X < 1.7$ GeV, $q^2 > 8$ GeV$^2$ | 9.3               | 9.3  | 4.23 ± 0.45 ± 0.36            | 3.97 ± 0.42 ± 0.31           |
| Average                      |                   | 4.31 ± 0.17 ± 0.35 | 3.98 ± 0.15 ± 0.30 |

![Figure 3: Compilation of $|V_{ub}|$ determinations from inclusive spectra in $B \to X_u \ell \bar{\nu}_\ell$ decay, using $m_b^{SF} = (4.71 ± 0.06)$ GeV for the $b$-quark mass, as determined using only model-independent information for $B \to X_s \ell \bar{\nu}_\ell$ moments (courtesy of HFAG).](image)

It is significantly lower than the previous value. A graphical representation of the individual determinations is shown in Figure 3. Note that there is rather good agreement among the different measurements. From a theoretical perspective, one would give preference to the extractions based on the most efficient cuts, such as the

![Figure 3: Compilation of $|V_{ub}|$ determinations from inclusive spectra in $B \to X_u \ell \bar{\nu}_\ell$ decay, using $m_b^{SF} = (4.71 ± 0.06)$ GeV for the $b$-quark mass, as determined using only model-independent information for $B \to X_s \ell \bar{\nu}_\ell$ moments (courtesy of HFAG).](image)

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\[ |V_{ub}| = (3.98 ± 0.15 ± 0.30) \times 10^{-3}. \]
cuts on $M_X$ or $P_{y}$ (not reported in the table). For these quark-hadron duality is minimal. Averaging the BaBar and Belle results derived form the $M_X$ cut, my personal “best guess” for $|V_{ub}|$ is

$$|V_{ub}| = (3.70 \pm 0.15 \pm 0.28) \cdot 10^{-3}. \quad (8)$$

The most important conclusion from this discussion is this: When only model-independent information is used to determine the $b$-quark mass and thereby derive constraints on the leading shape function, then the value extracted for $|V_{ub}|$ from inclusive decays is in rather good agreement with that derived from exclusive decays. For instance, recent analyses of the $q^2$ distribution in $B \rightarrow \pi l \nu$ decays using unquenched lattice QCD or light-cone QCD sum rules yield values in the range between $(3.3-3.6) \cdot 10^{-3}$, with uncertainties of about $\pm 0.4 \cdot 10^{-3}$. In my view there is no obstacle to averaging the “inclusive” and “exclusive” results to obtain a single value for $|V_{ub}|$, but this is best done by HFAG.

VI. CONCLUSIONS

An important theoretical challenge in heavy-flavor physics is to disentangle strong-interaction effects from the underlying weak-interaction transitions, which probe the flavor structure of the Standard Model and measure fundamental parameters such as the elements of the CKM matrix. Tremendous progress in theory has been made based on heavy-quark expansions and factorization techniques combined with effective field-theory methods. Thanks to these developments, and to the hard work of many theorists, we now have accurate predictions, including high-order perturbative effects and nonperturbative power corrections, for a number of important decay processes, including in particular the inclusive processes $B \rightarrow X_s \gamma$ and $B \rightarrow X_c l \nu_l$. The $|V_{tb}|$ vs. $\sin 2\beta$ puzzle, i.e., the fact that there used to be a tension between the determination of $|V_{ub}|$ from inclusive semileptonic decays as compared with the indirect value inferred from the measurement of the angle $\beta$ of the unitarity triangle, is resolved when only model-independent information is used in the extraction of the $b$-quark mass and other heavy-quark parameters. The new value of $|V_{ub}|$ obtained in this way is in good agreement with the determination from exclusive decays.

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