Observational Constraints on Varying Alpha in $\Lambda(\alpha)\text{CDM}$ Cosmology

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ABSTRACT

In this work, we consider the so-called $\Lambda(\alpha)\text{CDM}$ cosmology with $\Lambda \propto \alpha^{-6}$ while the fine-structure “constant” $\alpha$ is varying. In this scenario, the accelerated expansion of the universe is driven by the cosmological “constant” $\Lambda$ (equivalently the vacuum energy), and the varying $\alpha$ is driven by a subdominant scalar field $\phi$ coupling with the electromagnetic field. The observational constraints on the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ models with various couplings $B_F(\phi)$ between the subdominant scalar field $\phi$ and the electromagnetic field are considered.

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I. INTRODUCTION

The year of 1998 is amazing in some sense. In this year, the accelerated expansion of the universe was firstly discovered from the observation of distant type Ia supernovae (SNIa) [1]. Later, this amazing discovery was further confirmed by the observations of cosmic microwave background (CMB) [2] and large-scale structure (LSS) [3] (including baryon acoustic oscillation (BAO) [4] especially). This mysterious phenomenon has formed a big challenge to physicists and cosmologists. It hints the existence of dark energy (a new component with negative pressure). The simplest candidate of dark energy is a tiny positive cosmological constant $\Lambda$ (equivalently the vacuum energy). However, it is hard to understand why the observed vacuum energy density is about 120 orders of magnitude smaller than its natural expectation (namely the Planck energy density). This is the so-called cosmological constant problem [5, 6, 45].

In the literature, many attempts have been made to solve (at least alleviate) the cosmological constant problem. Among them, an interesting idea is the so-called axiomatic approach [7]. Based on four natural and simple axioms in close analogy to the Khinchin axioms (which can uniquely derive the Shannon entropy in information theory [8]), Beck [7] derived an explicit form for the cosmological constant, i.e.

$$\Lambda = \frac{G^2}{\hbar} \left( \frac{m_e}{\alpha} \right)^6,$$

in which $\alpha$ is the electromagnetic fine-structure constant, $m_e$ is the electron mass, $G$ is the gravitational constant, $\hbar$ is the reduced Planck constant. Accordingly, the vacuum energy density reads [7]

$$\rho_\Lambda = \frac{c^4 \Lambda}{8\pi G} = \frac{Gc^4}{8\pi\hbar^2} \left( \frac{m_e}{\alpha} \right)^6,$$

where $c$ is the speed of light. Numerically, it gives $\rho_\Lambda \simeq 4.0961 \text{GeV/m}^3$, which can easily pass all the current observational constraints. We refer to [7] for the detailed derivations. Note that Eq. (1) can also be derived in other completely independent approaches (see e.g. [9, 10, 46]). We refer to [23] for a brief review of these approaches.

Coincidentally, in the same year 1998, another amazing discovery was claimed. From the observation of distant quasars, Webb et al. [11] announced the first hint for the varying fine-structure “constant” $\alpha$. While the relevant observational data are accumulating [12–14], a time-varying $\alpha$ has been extensively considered in the literature (see e.g. [18, 20, 21]). That is, the scalar field $\phi$ might be due to a varying speed of light $c$ [20], while Lorentz invariance is broken. Another possibility for a varying $\alpha$ is due to a varying electron charge $e$, which was firstly proposed by Bekenstein [27] in 1982, while local gauge and Lorentz invariance are preserved. This is a dilaton theory with coupling to the electromagnetic $F^2$ part of the Lagrangian, but not to the other gauge fields. It has been generalized to a so-called BSBM model [28, 29] after the first observational hint of varying $\alpha$ from the quasar absorption spectra in 1998 [11]. In fact, the main spirit of Bekenstein-type models is using a scalar field $\phi$ coupling with the electromagnetic field to drive the varying $\alpha$.

In the literature, there exist two main types of varying $\alpha$ models driven by a scalar field $\phi$, depending on the role played by $\phi$. The first one is using the scalar field $\phi$ to simultaneously drive the accelerated expansion of the universe and the varying $\alpha$ (see e.g. [13, 20, 21]). That is, the scalar field $\phi$ also plays the role of dark energy, and it is the dominant component of the universe. On the contrary, the second one is using the scalar field $\phi$ to drive only the varying $\alpha$. The accelerated expansion of the universe is instead driven by the cosmological constant $\Lambda$ (equivalently the vacuum energy), while the scalar field $\phi$ is subdominant and its only role is to drive the varying $\alpha$ (see e.g. [13, 28, 29]). In the present work, we...
adopt the second perspective naturally. For simplicity, we consider a subdominant quintessence with a canonical kinetic energy as the simplest scalar field $\phi$ to drive the varying $\alpha$.

The rest of this paper is organized as followings. In Sec. II we setup the varying $\alpha$ model driven by a subdominant quintessence $\phi$ in $\Lambda(\alpha)$CDM cosmology with $\Lambda \propto \alpha^{-6}$. Then, in Sec. III we consider the observational constraints on the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ models with various couplings $B_F(\phi)$. The brief concluding remarks are given in Sec. IV.

**FIG. 1:** The relations between the scalar field $\phi$, the varying fine-structure “constant” $\alpha$, and the Hubble parameter $H$. See the text for details.

## II. VARYING ALPHA DRIVEN BY QUINTESSENCE IN $\Lambda(\alpha)$CDM COSMOLOGY

Following e.g. [20, 21, 30, 31], the relevant action reads

$$S = -\frac{m_p^2}{2} \int d^4x \sqrt{-g} (R - 2\Lambda) + \int d^4x \sqrt{-g} \mathcal{L}_\phi - \frac{1}{4} \int d^4x \sqrt{-g} B_F(\phi) F_{\mu\nu} F^{\mu\nu} + S_m,$$

(3)

where $R$ is the Ricci scalar; $g$ is the determinant of the metric $g_{\mu\nu}$; $m_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass; $F_{\mu\nu}$ are the components of the electromagnetic field tensor; $S_m$ is the action of pressureless matter; we have set the units $\hbar = c = 1$; we can safely ignore the contribution of radiation; $\mathcal{L}_\phi$ is the Lagrangian of the subdominant scalar field $\phi$. For a subdominant quintessence, it is given by

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),$$

(4)

where $V(\phi)$ is the potential. Noting that the coupling $B_F$ takes the place of $e^{-2}$ in Eq. (3) actually [32], one can easily see that the effective fine-structure “constant” is given by [20, 21, 27, 30]

$$\alpha = \frac{\alpha_0}{B_F(\phi)},$$

(5)

and then

$$\frac{\Delta \alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = B_F^{-1}(\phi) - 1,$$

(6)

where the subscript “0” indicates the present value of corresponding quantity. By definition, the present value of $B_F$ should be equal to 1. In general, $\phi$ and hence $\alpha$ are functions of spacetime. However, as is well known, we can safely ignore their spatial variation, and only consider the homogeneous $\phi$ and $\alpha$ in...
the present work. Throughout, we assume that only the electron charge $e$ is varying, and all the other fundamental constants $\hbar, G, c, m_e$ are true constants. Thus, $\rho_\Lambda \propto \Lambda \propto \alpha^{-6}$. Using Eq. (5), we get

$$\rho_\Lambda = \rho_\Lambda (\frac{\alpha}{\alpha_0})^{-6} = \rho_\Lambda B_F^2(\phi).$$  

(7)

Considering a flat Friedmann-Robertson-Walker (FRW) universe, the corresponding Friedmann equation and Raychaudhuri equation are given by

$$H^2 = \frac{1}{3m_p^2}(\rho_\Lambda + \rho_m),$$  

(8)

$$\dot{H} = -\frac{1}{2m_p^2}(\rho_\Lambda + \rho_m + p_\Lambda + p_m) = -\frac{\rho_m}{2m_p^2},$$  

(9)

respectively, where $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1+z)^{-1}$ is the scale factor (we have set $\alpha_0 = 1$); $z$ is the redshift; a dot denotes a derivative with respect to the cosmic time $t$; $\rho_m$ is the energy density of dust matter; $\rho_\Lambda = -\rho_\Lambda$ and $p_m = 0$ are the pressures of the vacuum energy and dust matter, respectively. We have safely ignored the subdominant scalar field $\phi$ and the electromagnetic field in Eqs. (8) and (9).

On the other hand, from the total energy conservation equation $\dot{\rho}_\text{tot} + 3H(\rho_\text{tot} + \rho_\text{tot}) = 0$, we find that $p_m + 3H\rho_m = -\dot{\rho}_\Lambda \neq 0$. So, $\rho_\Lambda$ is no longer proportional to $a^{-3}$ (see [23] for a detailed discussion on this issue). The equation of motion (EoM) for the subdominant scalar field $\phi$ reads

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0,$$  

(10)

where the subscript “$\phi$” denotes the derivative with respect to $\phi$. In principle, there should be an additional term proportional to $F_{\mu\nu}F^{\mu\nu}$ and the derivative of $B_F$ in the right hand side of Eq. (10), due to the coupling between the scalar field and the electromagnetic field. However, it could be safely ignored thanks to the following facts: (i) the derivative of $B_F$ is actually equivalent to $\dot{\alpha}$, which is very tiny (given equivalence principle constraints [30, 31]); (ii) the statistical average of the term $F_{\mu\nu}F^{\mu\nu}$ over a current state of the universe is zero [21].

In Fig. 1 we show the relations between the scalar field $\phi$, the varying fine-structure “constant” $\alpha$, and the Hubble parameter $H$. The subdominant scalar field $\phi$ drives the varying $\alpha$ through the coupling $B_F$ according to Eq. (5). The varying $\alpha$ affects the Hubble parameter $H$ (which characterizes the cosmic expansion) through $\rho_\Lambda \propto \alpha^{-6}$ in Eq. (5). The Hubble parameter $H$ affects the evolution of the scalar field $\phi$ through the friction term proportional to $H$ in the EoM given by Eq. (10).

For convenience, we recast the evolution equations with dimensionless quantities. Substituting Eqs. (4) and (6) into Eq. (5), we obtain

$$H^2 = \frac{\rho_\Lambda B_F^6}{3m_p^2} \frac{2}{3} \ddot{t}.$$  

(11)

Using the relation $\dot{t} = -(1+z)Hf'$ (where a prime denotes a derivative with respect to the redshift $z$), we recast Eq. (11) as

$$E^2 = (1 - \Omega_m0)B_F^2 + \frac{2}{3}(1+z)EE',$$  

(12)

where $E \equiv H/H_0$ and $\Omega_m0 \equiv \rho_m/3m_p^2H_0^2 = 1 - \rho_\Lambda/(3m_p^2H_0^2) \equiv 1 - \Omega_m0$.

Introducing $\ddot{\phi} \equiv \phi/m_p$ and $U(\phi) \equiv V(\phi)/(m_p^2H_0^2)$, we recast Eq. (10) as

$$(1+z)^2E^2\dot{\phi}' + (1+z)E[(1+z)E' - 2E]\dot{\phi}' + U,\phi = 0.$$  

(13)

For simplicity, following e.g. [18, 28, 29], we only consider the scalar field $\phi$ without potential in this work, and hence Eq. (13) becomes

$$(1+z)^2E^2\dot{\phi}' + (1+z)E[(1+z)E' - 2E]\dot{\phi}' = 0.$$  

(14)

Once the coupling $B_F(\ddot{\phi})$ and the initial conditions are given, one can numerically solve the coupled 2nd order differential equations (12) and (14) to obtain $\dot{\phi}$ and $E$ as functions of the redshift $z$. Then, $\Delta\alpha/\alpha$ as a function of the redshift $z$ is on hand by using Eq. (6).
III. OBSERVATIONAL CONSTRAINTS ON THE VARYING ALPHA MODELS

One can constrain the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ models by using the observational data, if the theoretical $\Delta \alpha/\alpha$ as a function of the redshift $z$ is known. Here, we consider the observational $\Delta \alpha/\alpha$ dataset given in [14, 33, 34], which consists of 293 usable $\Delta \alpha/\alpha$ data from the absorption systems in the spectra of distant quasars (note that two outliers should be removed from the full numerical data of 295 quasar absorption systems [14, 33, 34]), over the absorption redshift range $0.2223 \leq z_{abs} \leq 4.1798$. Note that all the 293 $\Delta \alpha/\alpha$ data are of $O(10^{-5})$. The $\chi^2$ from these 293 $\Delta \alpha/\alpha$ data is given by

$$\chi^2 = \sum_i \frac{[(\Delta \alpha/\alpha)_{th,i} - (\Delta \alpha/\alpha)_{obs,i}]^2}{\sigma^2_i},$$

where $\sigma_i^2 = \sigma_{stat,i}^2 + \sigma_{rand,i}^2$ (see Sec. 3.5.3 of [14] and the instructions of [33, 34] for the technical details of $\sigma_{rand}$ and the error budget). Note that these $\Delta \alpha/\alpha$ data can tightly constrain the model parameters in the coupling $B_F$, but the constraints on the model parameter $\Omega_{m0}$ are too loose. Therefore, the data from the other cosmological observations, such as SNIa, CMB and BAO, are required to properly constrain the model parameter $\Omega_{m0}$. Here, we consider the same SNIa [33], CMB [37, 40] and BAO [3] data as in [23], and the corresponding $\chi^2$ are given with detail in Sec. 3.1 of [23]. The total $\chi^2$ from the combined $\Delta \alpha/\alpha$, SNIa, CMB and BAO data is given by

$$\chi^2 = \chi^2_\alpha + \chi^2_\mu + \chi^2_R + \chi^2_A,$$

where $\chi^2_\alpha, \chi^2_R$ and $\chi^2_A$ are all given in Sec. 3.1 of [23].

A. Linear coupling

At first, we consider the linear coupling [20, 21]

$$B_F(\phi) = 1 - \zeta (\phi - \phi_0),$$

where $\zeta$ is a constant. In fact, this is the mostly considered coupling in the literature, since it is the simplest one. To simplify the initial conditions, we redefine $\varphi \equiv \phi - \phi_0$, and then $B_F(\varphi) = 1 - \zeta \varphi$. Now, the evolution equations [12] and [14] become

$$E^2 = (1 - \Omega_{m0}) B_F^2(\varphi) + \frac{2}{3} (1 + z) E E',$$

$$(1 + z) E^2 \varphi'' + E [ (1 + z) E' - 2 E ] \varphi' = 0,$$

which are the coupled 2nd order differential equations. By definition, the corresponding initial conditions are given by $E(z = 0) = 1$, $\varphi(z = 0) = 0$ and $\varphi'(z = 0) = \varphi_0' = v_0$, where $v_0$ is a constant and will be determined by the observational data. In this case, there are three free model parameters, namely $\Omega_{m0}$, $\zeta$ and $v_0$. Note that if $\zeta = 0$, we have $B_F \equiv 1$ and then $\alpha = const., \Lambda \propto \alpha^{-6} = const.$, namely the model reduces to a constant $\alpha$ in ordinary $\Lambda$CDM cosmology. We can numerically solve the coupled 2nd order differential equations [18] and [19] with the initial conditions mentioned above to obtain $\varphi$ and $E$ as functions of the redshift $z$. Then, $\Delta \alpha/\alpha$ as a function of the redshift $z$ is on hand by using Eq. (6). By minimizing the corresponding total $\chi^2$ in Eq. (15), we find the best-fit model parameters $\Omega_{m0} = 0.2787$, $\zeta = 0.4995 \times 10^{-5}$, and $v_0 = -0.0435$, while $\chi^2_{min} = 869.6$ and $\chi^2_{min}/dof = 0.9972$. In Fig. 2 we also present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \zeta$, $\Omega_{m0} - v_0$ and $\zeta - v_0$ planes. From Fig. 2 it is easy to see that $\zeta$ is tightly constrained to a narrow range of $O(10^{-5})$, thanks to the 293 $\Delta \alpha/\alpha$ data of $O(10^{-5})$. On the other hand, $\zeta = 0$ is within the 1$\sigma$ region, and hence a constant $\alpha$ in ordinary $\Lambda$CDM cosmology is fully consistent with the observational data.
FIG. 2: The 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \zeta$, $\Omega_{m0} - v_0$ and $\zeta - v_0$ planes for the case of linear coupling. Note that $\zeta$ is given in units of $10^{-5}$. The best-fit parameters are also indicated by the black solid points.

B. Power-law coupling

Here, we consider the power-law coupling \cite{20,21}

$$B_F(\dot{\phi}) = \left(\frac{\dot{\phi}}{\dot{\phi}_0}\right)^\zeta,$$

where $\dot{\phi}_0 \neq 0$, and $\zeta$ is a constant. To simplify the initial conditions, we redefine $\varphi \equiv \dot{\phi}/\dot{\phi}_0$, and then $B_F(\varphi) = \varphi^\zeta$. However, this form is pathological. Noting $B_F(\varphi) = \varphi^\zeta = \exp(\zeta \ln \varphi)$, it will become complex number for $\varphi < 0$. To avoid this problem, we should instead consider another form,

$$B_F(\varphi) = |\varphi|^\zeta,$$

which is equivalent to $B_F(\dot{\phi}) = |\dot{\phi}/\dot{\phi}_0|^\zeta$, where $|x|$ denotes the absolute value of $x$. With $\varphi \equiv \dot{\phi}/\dot{\phi}_0$, the evolution equations \cite{12} and \cite{14} become the ones given in Eqs. \cite{18} and \cite{19}. By definition, the corresponding initial conditions are given by $E(z = 0) = 1$, $\varphi(z = 0) = 1$ and $\varphi'(z = 0) = \varphi'_0 = v_0$, where $v_0$ is a constant and will be determined by the observational data. Note that the initial condition $\varphi(z = 0) = 1$ is different from the case of linear coupling by definition. In this case, there are three free
FIG. 3: The same as in Fig. 2 except for the case of power-law coupling. Note that the “gap” in the $\Omega_{m0} - v_0$ plane corresponds to the “hollow” in the $\zeta - v_0$ plane.

model parameters, namely $\Omega_{m0}$, $\zeta$ and $v_0$. Note that if $\zeta = 0$, we have $B_F \equiv 1$ and then $\alpha = \text{const.}$, $\Lambda \propto \alpha^{-6} = \text{const.}$, namely the model reduces to a constant $\alpha$ in ordinary $\Lambda$CDM cosmology. We can numerically solve the coupled 2nd order differential equations (18) and (19) with the initial conditions mentioned above to obtain $\varphi(z)$, $E(z)$, and then $\Delta\alpha/\alpha(z)$. By minimizing the corresponding total $\chi^2$ in Eq. (16), we find the best-fit model parameters $\Omega_{m0} = 0.2786$, $\zeta = 0.0672 \times 10^{-5}$, and $v_0 = -2.2871$, while $\chi^2_{\text{min}} = 868.527$ and $\chi^2_{\text{min}}/\text{dof} = 0.9960$. In Fig. 3, we also present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \zeta$, $\Omega_{m0} - v_0$ and $\zeta - v_0$ planes. From Fig. 3, it is easy to see that $\zeta$ is tightly constrained to a narrow range of $O(10^{-6})$, thanks to the 293 $\Delta\alpha/\alpha$ data of $O(10^{-5})$. On the other hand, $\zeta = 0$ is within the 1$\sigma$ region, and hence a constant $\alpha$ in ordinary $\Lambda$CDM cosmology is fully consistent with the observational data. Note that the observational data cannot well constrain the parameter $v_0$.

C. Exponential coupling

Let us turn to the exponential coupling \cite{20, 21}.

$$B_F(\dot{\varphi}) = \exp (-\zeta(\dot{\varphi} - \dot{\varphi}_0)) ,$$  (22)
where $\zeta$ is a constant. We redefine $\varphi \equiv \varphi' - \varphi_0$, and then $B_F(\varphi) = \exp(-\zeta \varphi)$, while the evolution equations (12) and (13) become the ones given in Eqs. (15) and (19). By definition, the corresponding initial conditions are given by $E(z=0) = 1, \varphi(z=0) = 0$ and $\varphi'(z=0) = \varphi'_0 = v_0$, where $v_0$ is a constant and will be determined by the observational data. In this case, there are three free model parameters, namely $\Omega_{m0}$, $\zeta$ and $v_0$. Note that if $\zeta = 0$, we have $B_F \equiv 1$ and then $\alpha = \text{const.}, \Lambda \propto \alpha^{-6} = \text{const.}$, namely the model reduces to a constant $\alpha$ in ordinary $\Lambda$CDM cosmology. We can numerically solve the coupled 2nd order differential equations (18) and (19) with the initial conditions mentioned above to obtain $\varphi(z), E(z)$, and then $\Delta \alpha/\alpha(z)$. By minimizing the corresponding total $\chi^2$ in Eq. (16), we find the best-fit model parameters $\Omega_{m0} = 0.2787, \zeta = 0.4994 \times 10^{-5}$, and $v_0 = -0.0435$, while $\chi^2_{\text{min}} = 869.6$ and $\chi^2_{\text{min}}/\text{dof} = 0.9972$. In Fig. 4 we also present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0}-\zeta, \Omega_{m0}-v_0$ and $\zeta-v_0$ planes. It is worth noting that the best-fit model parameters and the contours are almost the same as in the case of linear coupling, while the differences are very tiny. This is not surprising, since $B_F(\varphi) = \exp(-\zeta \varphi) \approx 1 - \zeta \varphi + O(\zeta^2)$ for $\zeta \ll 1$.

D. Polynomial coupling

Finally, we consider the polynomial coupling $[20, 21]$

$$B_F(\varphi) = 1 - \zeta (\varphi' - \varphi_0)^\beta,$$  \hspace{1cm} (23)
where $\zeta$ and $\beta$ are both constants. Again, we redefine $\varphi \equiv \dot{\varphi} - \dot{\varphi}_0$, and then $B_F(\varphi) = 1 - \zeta \varphi^\beta$. Similar to the case of power-law coupling, this form is pathological. Noting $B_F(\varphi) = 1 - \zeta \varphi^\beta = 1 - \zeta \exp(\beta \ln \varphi)$, it will become complex number for $\varphi < 0$. To avoid this problem, we should instead consider 

$$B_F(\varphi) = 1 - \zeta |\varphi|^\beta,$$  

which is equivalent to $B_F(\dot{\varphi}) = 1 - \zeta |\dot{\varphi} - \dot{\varphi}_0|^\beta$. With $\varphi \equiv \dot{\varphi} - \dot{\varphi}_0$, the evolution equations (12) and (14) become the ones given in Eqs. (13) and (19). By definition, the corresponding initial conditions are given by $E(z = 0) = 1$, $\varphi(z = 0) = 0$ and $\varphi'(z = 0) = \varphi'_0 = v_0$, where $v_0$ is a constant and will be determined by the observational data. In this case, there are four free model parameters, namely $\Omega_{m0}$, $\zeta$, $\beta$ and $v_0$. Noting the initial condition $\varphi(z = 0) = 0$, if $\beta < 0$, we find that $B_F$ will diverge at $z = 0$ (however, this is not a problem for the case of power-law coupling, since its corresponding initial condition is $\varphi(z = 0) = 1$, rather than 0). On the other hand, if $\beta = 0$ exactly, $B_F = 1 - \zeta \neq 1$ at $z = 0$ for a non-zero $\zeta$ (note that the present value of $B_F$ should be equal to 1 by definition). Thus, we should require

$$\beta > 0.$$  

Note that if $\zeta = 0$, we have $B_F \equiv 1$ and then $\alpha = \text{const.}$, $\Lambda \propto \alpha^{-6} = \text{const.}$, namely the model reduces to a constant $\alpha$ in ordinary $\Lambda$CDM cosmology. We can numerically solve the coupled 2nd order differential equations (18) and (19) with the initial conditions mentioned above to obtain $\phi(z)$, $E(z)$, and then $\Delta \alpha/\alpha(z)$. By minimizing the corresponding total $\chi^2$ in Eq. (13), we find the best-fit model parameters $\Omega_{m0} = 0.2786$, $\zeta = -0.2161 \times 10^{-5}$, $\beta = 0.4$, and $v_0 = 3.5588$, while $\chi^2_{\text{min}} = 563.909$ and $\chi^2_{\text{min}}/\text{dof} = 0.9919$. Note that the best-fit $\beta$ is not exactly equal to 0, but it is extremely close to 0. In Fig. 5 we also present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \zeta$, $\Omega_{m0} - \beta$, $\Omega_{m0} - v_0$, $\zeta - \beta$, $\zeta - v_0$ and $\beta - v_0$ planes. From Fig. 5 it is easy to see that $\zeta$ is tightly constrained to a narrow range of $O(10^{-6})$, thanks to the 293 $\Delta \alpha/\alpha$ data of $O(10^{-5})$. On the other hand, $\zeta = 0$ deviates from the best fit beyond 1$\sigma$, but it is still within the 2$\sigma$ region. So, the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ are slightly favored by the observational data, while a constant $\alpha$ in ordinary $\Lambda$CDM cosmology is still consistent with the observational data within the 2$\sigma$ region. Note that the observational data cannot well constrain the parameter $v_0$, while $\beta$ is constrained to $\lesssim O(1)$.

**IV. CONCLUDING REMARKS**

In the present work, we extend the work of [23] by considering the mechanism to drive the varying fine-structure "constant" $\alpha$. In [23], the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ were studied only in a phenomenological manner, from an interacting vacuum energy perspective. Instead, here we further consider the scalar field $\phi$ coupling with the electromagnetic field, and hence it could drive the variation of $\alpha$. The scalar field $\phi$ is subdominant, and it is used to drive only the varying $\alpha$. The accelerated expansion of the universe is instead driven by the cosmological constant $\Lambda$ (equivalently the vacuum energy). On the other hand, $\Lambda \propto \alpha^{-6}$ was derived from three completely independent approaches in the literature, especially the so-called axiomatic approach [6]. So, the two amazing discoveries in 1998 are connected in this way. The coupling $B_F(\phi)$ between the scalar field $\phi$ and the electromagnetic field plays an important role. In this work, we consider various forms of the coupling $B_F(\phi)$, and confront the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ models with the observational data. We find that the key model parameter $\zeta$ in the coupling $B_F$ can be tightly constrained to the very narrow ranges of $O(10^{-5})$ or $O(10^{-6})$, thanks to the 293 $\Delta \alpha/\alpha$ data of $O(10^{-5})$. In the cases of linear, power-law and exponential couplings, a constant $\alpha$ in ordinary $\Lambda$CDM cosmology is fully consistent with the observational data. There is no evidence for the varying $\alpha$ and $\Lambda$. In the case of polynomial coupling, the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ are slightly favored beyond 1$\sigma$.

Some remarks are in order. Firstly, it is worth noting that 3 of 4 models considered in [23] favor the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$, while a constant $\alpha$ in ordinary $\Lambda$CDM model deviates from the best fit beyond 2$\sigma$ or at least 1$\sigma$. So, the results obtained in the present work are quite contrary to the ones of [23], while the same observational datasets are used. The main difference between this work and [23] is that different perspectives and then different parameterizations are taken. In [23], the varying $\alpha$ and $\Lambda \propto \alpha^{-6}$ are studied from an interacting vacuum energy perspective. Thus, the corresponding parameterizations are performed in the interaction between the vacuum energy and pressureless matter. On the other hand, in
FIG. 5: The 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \zeta$, $\Omega_{m0} - \beta$, $\Omega_{m0} - \nu_0$, $\zeta - \beta$, $\zeta - \nu_0$ and $\beta - \nu_0$ planes for the case of polynomial coupling. Note that $\zeta$ is given in units of $10^{-5}$. The best-fit parameters are also indicated by the black solid points.
TABLE I: Comparing the eight models considered in [23] and the present work. See the text for details.

| Model       | IIcon | ICPL | IIcon | ICPL | BFlin | BFpl | BFexp | BFpoly |
|-------------|-------|------|-------|------|-------|------|-------|--------|
| $\chi^2_{\text{min}}$ | 868.149 | 856.005 | 870.391 | 857.605 | 869.6 | 868.527 | 869.6 | 863.90 |
| $k$         | 2     | 3    | 2     | 3    | 3     | 3    | 3     | 4      |
| $\frac{\chi^2_{\text{min}}}{\text{dof}}$ | 0.9944 | 0.9817 | 0.9970 | 0.9835 | 0.9972 | 0.9960 | 0.9972 | 0.9919 |
| $\Delta\text{BIC}$ | 5.370 | 0    | 7.612 | 1.6  | 13.595 | 12.522 | 13.595 | 14.678 |
| $\Delta\text{AIC}$ | 10.144 | 0    | 12.386 | 1.6  | 13.595 | 12.522 | 13.595 | 9.904  |
| Rank        | 3     | 1    | 4     | 2    | 7     | 6    | 7     | 5      |

this work, the parameterizations are performed in the coupling $B_F(\phi)$ between the scalar field $\phi$ and the electromagnetic field. So, it is of interest to compare the eight models considered in [23] and the present work. We label the four models characterized by Eqs. (37), (39), (40), (43) of [23] as IIcon, ICPL, IIcon, ICPL, respectively. We also label the four models characterized by the linear, power-law, exponential, polynomial couplings in the present work as BFlin, BFpl, BFexp, BFpoly, respectively. Since these models have different free parameters and the correlations between model parameters are fairly different, it is not suitable to directly compare their confidence level contours. Instead, it is more appropriate to compare them from the viewpoint of goodness-of-fit. A conventional criterion for model comparison in the literature is $\frac{\chi^2_{\text{min}}}{\text{dof}}$, in which the degree of freedom $\text{dof} = N − k$, while $N$ and $k$ are the number of data points and the number of free model parameters, respectively. On the other hand, there are other criteria for model comparison in the literature. The most sophisticated criterion is the Bayesian evidence (see e.g. [42] and references therein). However, the computation of Bayesian evidence usually consumes a large amount of time and power. As an alternative, one can consider some approximations of Bayesian evidence, such as the so-called Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). The BIC is defined by [43]

$$BIC = -2 \ln L_{max} + k \ln N,$$

where $L_{max}$ is the maximum likelihood. In the Gaussian cases, $\chi^2_{\text{min}} = -2 \ln L_{max}$. So, the difference in BIC between two models is given by $\Delta\text{BIC} = \Delta\chi^2_{\text{min}} + \Delta k \ln N$. The AIC is defined by [44]

$$\text{AIC} = -2 \ln L_{max} + 2k.$$

The difference in AIC between two models is given by $\Delta\text{AIC} = \Delta\chi^2_{\text{min}} + 2\Delta k$. In Table I we present $\chi^2_{\text{min}}/\text{dof}$, $\Delta\text{BIC}$ and $\Delta\text{AIC}$ for the eight models considered in [23] and the present work. Note that the IntICPL model has been chosen to be the fiducial model when we calculate $\Delta\text{BIC}$ and $\Delta\text{AIC}$. Clearly, all the four models considered in [23] are better than all the four models considered in the present work. The IntICPL model is the best from the viewpoint of all the three criteria $\chi^2_{\text{min}}/\text{dof}$, BIC and AIC.

Secondly, in this work we consider the scalar field $\phi$ without potential for simplicity. In general, the potential could be included. However, including the potential will significantly increase the model’s degree of freedom, since the forms of potential can be diverse, and the number of model parameters used to define the potential might be large. All these will make the constraints fairly loose. Similarly, one can consider the other complicated scalar fields, such as $k$-essence, Dirac-Born-Infeld scalar field, tachyon, in place of quintessence considered in this work. But this will significantly increase the model’s degree of freedom, and greatly decrease the constraining ability.

Thirdly, from Figs. 2–5 it is interesting to find that the contours in the $\Omega_{m0} - \zeta$ plane are nearly symmetric and their shapes are close to circles (we thank the referee for pointing out this issue). In fact, this shows that the coupling constant $\zeta$ is largely uncorrelated with $\Omega_{m0}$. Thus, it further justifies our parameterizations (17), (21), (22) and (23) for the coupling $B_F$.

Finally, we again advocate the idea of $\Lambda(\alpha)$CDM cosmology with $\Lambda \propto \alpha^{-6}$ while the fine-structure “constant” $\alpha$ is varying. In fact, although $\Lambda \propto \alpha^{-6}$ could be derived from various completely independent approaches, it has not attracted considerable attention in the community so far. But it is impressive that
the numerical value of $\Lambda = G^2 m_e^6 / (\hbar^4 \alpha^6)$ from Eq. (1) is very close to the observational value. There might be a profound reasoning, other than just a coincidence. On the other hand, if $\alpha$ is varying, the well motivated $\Lambda(\alpha) \propto \alpha^{-6}$ gives a novel realization of $\Lambda(t)$, different from the ones purely written by hand in the literature. We consider that it deserves further investigation.

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