Instanton vacuum at finite density of quark matter

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We study light quark interactions in the instanton liquid at finite quark/baryon number density analyzing chiral and diquark condensates and investigate the behaviors of quark dynamical mass and both condensates together with instanton liquid density as a function of quark chemical potential. We conclude the quark impact (estimated in the tadpole approximation) on the instanton liquid could shift color superconducting phase transition to higher values of the chemical potential bringing critical quark matter density to the values essentially higher than conventional nuclear one.

Aiming to provide a good insight into the real world we are interested in deriving the QCD phase diagram from the first principles as a function of temperature, chemical potential and number of flavors. While much was has been learned about the phase structure of QCD at finite temperature (combining the perturbative theory analysis and lattice simulations), this structure at nonzero quark/baryonic densities has been less explored. The former discovered that with the temperature dropped below the critical value, chiral symmetry is broken and quarks become confined. Moreover, both fundamental phenomena, seems, take place at the same temperature indicating they are tightly knitted. This result is considered as one of the most indicative to investigate the QCD dynamics. Indeed, the lattice studies allow us to compare the changing behavior of physical observables on both $T_c$ sides providing very important information on the underlying excitations responsible for confinement or chiral symmetry breaking. And if the letters are more understandable being, according the common wisdom, the instanton-like excitations, different ideas for those responsible for confinement are still under debates. We believe the recent developments could be illuminating on the way of resolving this fundamental problem.

The factorized form of the generating functional of instanton liquid (IL) model $Z = Z_g \cdot Z_\psi$, where the factor $Z_g$ provides nontrivial gluon condensate while the fermion part $Z_\psi$ is responsible to describe the chiral and diquark condensates in instanton medium, allows us to investigate the condensate excitations. $Z_g$ is supposed to be saturated by the superposition of the pseudo-particle (PP) fields ((anti-)instantons).

The quark fields are considered to be influenced by the certain stochastic ensemble of PPs while calculating the quark determinant. The fermion field action is approached by the zero modes which are the solutions of the Dirac equation $[i\hat{D}(A_{II}) - i\mu\gamma_4] \Phi_{II} = 0$ with chemical potential $\mu$. The fermion field action is approximated by the zero modes which are the solutions of the Dirac equation $[i\hat{D}(A_{II}) - i\mu\gamma_4] \Phi_{II} = 0$ with chemical potential $\mu$. In the field of PP. With the auxiliary integration over the parameter $\lambda$ the quark determinant $Z_\psi$ may be exponentiated to the following form with dimensionless variables for the massless quarks of two flavors and three colors

$$Z_\psi \sim \int d\lambda \int D\psi^\dagger D\psi \exp \left\{ N \left( \ln \frac{n\rho^4}{\lambda R} - 1 \right) \right\} \times \times \exp \left\{ \int \frac{dk}{\pi^2} \sum_{f=1}^{2} \psi_{\text{L}}^f(k)(-\hat{k} - i\mu)\psi_{f}(k) + V \right\},$$

$V = V_L + V_R$, $V_L = \lambda (\psi_1^L L_1 \psi_1^L)(\psi_2^L L_2 \psi_2^L)$, and to get the right hand chiral components one
Figure 1. Saddle-point $\lambda$ (free energy, practically) as the function of chemical potential $\mu$. The upper solid and lower dashed lines correspond to the solutions with zero diquark condensate and zero chiral condensate, respectively. The lower solid and upper dashed lines correspond to the same solutions but including the quark interaction with IL when the modified functional integral is evaluated in tadpole approximation.

should change $L \to R$, $\psi_f^T = (\psi_f^R, \psi_f^L)$, and interaction terms are defined by the corresponding Fourier-components of zero modes $\psi^0$, $\phi^0$, $\mu_\nu = (0, \mu)$. Then we are well armed to calculate the chiral condensate

$$\langle \psi^\dagger(k) \psi(l) \rangle = -\pi^4 \delta(k-l) \text{Tr} S(k),$$

if the quark Green function $S(k)$ is known and the diquark condensate also

$$\langle \psi_{1 \alpha i}^L(k) \psi_{2 \beta j}^R(l) \rangle = \epsilon_{\alpha\beta} \pi^4 \delta(k+l) F_{ij}^{L,R}(k),$$

as the solutions of the Gorkov-Dyson-Schwinger equations. The corresponding solutions for the saddle point $\lambda$ of $Z_\psi$ are shown in Fig. 1 (practically free energy in one loop approximation). The quark feedback upon the instanton background is pretty limp and could be perturbatively incorporated as a small variation of instanton liquid parameters $\delta n$ and $\delta \rho$ around their equilibrium values of $n$ and $\bar{\rho}$. We describe further the quark feedback dealing with PPs changing their sizes adiabatically i.e. $\rho \to \rho(x,z)$. It results to the interaction of quarks and scalar field of the $\delta \rho$ deformation what in turn modifies the $L$ and $R$ kernels in the generating functional. The corresponding solutions of the modified (in tadpole approximation) saddle point equations are also exposed in Fig. 1. The dynamical quark mass $M$ (solid lines) and the quark condensate $-i \langle \psi^\dagger \psi \rangle = i \text{Tr} S(x)|_{x=0}$ (dashed lines) are presented in Fig. 2 where the lower curves correspond to the calculation with modified generating functional. Through this paper the value of renormalization constant is fixed by $\Lambda = 280$ MeV. Fig. 3 shows the instanton liquid density for the phase with non-zero chiral condensate (diquark condensate absent). Let us notice that all the curves give the reliable estimates in the interval extending till the vicinity of crossing points.

Figure 2. The dynamical quark mass (solid lines) and chiral condensates (dashed lines) as function of chemical potential $\mu$. The lower solid and dashed curves correspond to the calculation with the modified generating functional.
Figure 3. The instanton liquid density in the phase of broken chiral symmetry.

\[ \mu_c \approx 340 \text{ MeV} \]. Meanwhile, in \[\text{[6]}\] it was mentioned that such an estimate leads to non-realistic values of critical quark density for the transition to color superconducting phase. That estimate occurs to be quite comparable with conventional nuclear density \( n_0 = 0.45 \text{ fm}^{-3} \). Our approach teaches including the quark interaction with IL certainly increases \( \mu_c \). It is clear already from the second \( \mu_c \approx 700 \text{ MeV} \) (the corresponding quark density is exhibited in Fig. 4). Moreover, the position of upper dashed curve in Fig. 4 (the perturbated IL) signals that the \( \mu_c \) for transition to color superconducting phase might be shifted to very high values laying, rather, beyond the validity interval of IL model. It happens because the interquark Coulomb field strengths are comparable or even exceed the instanton ones and therefore the assumption about the saturating configurations becomes invalid.

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