Interaction mediated transport of optical localized states in a trapping potential

B. Garbin,1 J. Javaloyes,2 G. Tissoni,1 and S. Barland∗,1

1Université Côte d’Azur, CNRS, Institut de Physique de Nice, F-06560 Valbonne, France
2Departament de Física, Universitat de les Illes Balears, C/Valldemossa, km 7.5, E-07122 Palma, Spain.

(Dated: Compiled September 11, 2018)

The position and motion of ideally non-interacting localized states of light in propagative geometries can be controlled via an adequate parameter modulation. Here we show theoretically and experimentally that this process can be accurately described as the phase locking of oscillators to an external forcing but that non-reciprocal interactions between light bits can drastically modify this picture. Interactions lead to the convective motion of defects and to unlocking as a collective emerging phenomenon.

PACS numbers:

Several kinds of localized states (LSs) of light have been observed in many propagative experimental configurations including forced nonlinear resonators [1–4] subcritical lasers [5], or lasers with external forcing [6, 7]. In spite of their differences they share many properties, in particular the existence of a neutral mode associated to their translation. Thanks to this feature, any perturbation non-orthogonal to this neutral mode will cause motion of LSs [8] and adequately prepared landscapes can be used to trap localized solutions. This has been realized experimentally in many systems, both in the transverse dimension [9–12] and along propagation [13–15]. In transverse systems, it has already been noted that the dynamics of a cavity soliton nucleating on and escaping from a trapping position in presence of a drift term can be described as a saddle-node or saddle-loop bifurcations, depending on the depth of the trapping site [16–19]. In propagative geometries however, most experiments do not offer access to a complete vision of the phenomenon, first due to the difficulty to resolve completely both the fast intra round trip dynamics and the slow evolution over many round trips and second because interactions are in general not considered. Here we show, on the basis of experimental data and analytical calculations, that a unifying view of the trapping process of a single structure is that of phase locking of an oscillator, but also that interactions between LSs can drastically alter their transport in a periodic landscape.

In propagative geometries, due to the fundamentally periodic nature of optical resonators, each dissipative soliton forming along the direction of propagation (e.g. in Kerr fiber ring cavities or semiconductor lasers with feedback) can be considered as an oscillator whose natural frequency is set by the soliton’s round-trip time in the resonator. Attempting to tweeze or trap these light bits with a parameter modulation simply consists in trying to lock the phase and frequency of each oscillator to that of an external clock. The detuning between the forcing frequency and the nearest multiple of the inverse of the round-trip time breaks the parity symmetry in a very similar way to a drift force in transverse systems.

In the following we study the unpinning transition of a single dissipative soliton and identify experimentally the signature of the saddle-node on a circle (SNIC) bifurcation, which is generic of the oscillator unlocking process. We also analyze in terms of oscillator phases the spatial distribution of LSs in the locked and unlocked regimes, relating their transport to the lack of a common clock (diffusive process), but also to their asymmetrical repulsive interaction. When several LSs are pinned they form a lattice and due to their interactions we observe the propagation of a dislocation in this lattice, i.e. a metastable closely related to supersolitons and soliton Newton cradle predicted theoretically in conservative [20–22] and dissipative [23] settings. These observations can be reproduced numerically and the equation describing the unpinning bifurcation of a single soliton can be established analytically. This equation happens to also be very close to the one that describes the formation of solitons in this system, based on a forced oscillator [6]. When many structures are present, we show that beyond simply altering the diffusive process, soliton interactions can be the fundamental cause of their transport.

The experimental setup is based on a single transverse and longitudinal mode Vertical Cavity Surface Emitting Laser (VCSEL) with optical injection and delayed feedback [6]. The VCSEL is biased at very high current (typically seven times the lasing threshold), ensuring high damping of relaxation oscillations. The injection beam frequency is detuned of about 5 GHz from that of the standalone laser and the amount of injected power (a few percent of the emitted power) is set such that the VCSEL field is phase locked to the external forcing, very close to the unlocking transition. In this regime the laser behaves as a neuron-like excitable system, well described by the Adler equation [24]. A very low reflectivity external mirror (typically 1 %) is placed in front of the VCSEL so as to define with the latter a compound cavity (typically 2 m long) in which dissipative solitons based on the SNIC phase space structure are stable and can be independently addressed [6] via a Lithium Niobate phase modulator operating on the driving beam [25, 26]. We

∗Corresponding author: stephane.barland@inln.cnrs.fr
trap the solitons by driving the Lithium Niobate phase modulator, with a sinusoidal signal whose frequency is close enough to a multiple of the inverse of the round-trip time of the soliton in the extended cavity. In the comoving reference frame, this results in a spatially periodic modulation of the background solution. However (unlike the usual situation in transverse systems) this trap continuously shifts towards one or the other side with respect to the stationary soliton, depending on the detuning between the modulation frequency and the nearest multiple of the inverse round-trip time Fig. 1.

In Fig. 1A), the oblique stripes reveal the spatial modulation caused by the trapping modulation. The modulation frequency is slightly higher than 20 times the inverse of the soliton round-trip time and the stripes’ slope shows the nonstationarity of the trap. Here, close to a completely trapped regime, the soliton is most of the time trapped into meta-stable states from which it randomly jumps away into a neighbouring trap. The direction of the jumps is set by the detuning between the modulation frequency and the closest multiple of the soliton inverse round-trip time. On Fig. 1B), we plot the position and speed of the soliton in the reference frame of the trap. The position shows the trajectory of an overdamped particle in a tilted potential in a noisy environment described by the Adler equation [29,30], which also describes the evolution of a forced phase oscillator at the unlocking transition [27,31].

The unpinning transition of many solitons is shown on Fig. 2. Here we place the observer in the reference frame of the trapping landscape. In the first 1000 round-trips, 16 solitons occupy 16 stable traps out of the 20 available. From time to time some solitons escape randomly and fall into the empty neighbouring trap. In the case of the events occurring at (10.5,250) or (9,1200) the neighbouring trap is already occupied and one of the two solitons vanishes because two solitons can not occupy one single narrow ($\approx T/2 = 0.22 \text{ ns}$) trap. This is due to a repulsive interaction between them [6], caused by the refractory time following excitatory spikes in this injected laser (of the order of 0.4 ns [32]).

At around round-trip 1500, all solitons progressively start escaping their traps towards the right, each of them following a periodically oscillating trajectory. Considering the solitons as non interacting oscillators, this change of behavior can be seen as an unlocking transition from the common forcing, but this description is very incomplete as we shall see. We show on Fig. 2B),C) the histograms of the temporal separations between solitons. On panel C) (trapped regime) two peaks are clearly visible and well separated. Their absolutely minimal width (less than 100 ps) results from all the oscillators being locked: their relative phase is defined to the precision of (less than 100 ps) results from all the oscillators being locked: their relative phase is defined to the precision of

Although the statistical sample is not extremely large (about 50 events in total), this histogram can readily be interpreted in terms of an exponential decay at large times (manifestation of Kramer’s law [28]) with a cut-off at short time basically set by the duration of the velocity spikes themselves. This dynamics is perfectly analogue to that of an overdamped particle in a tilted potential in a noisy environment described by the Adler equation [29,30], which also describes the evolution of a forced phase oscillator at the unlocking transition [27,31].

Figure 1: Trajectory of a dissipative soliton in a periodic trap close to the unpinning transition. A) Spatiotemporal diagram in the reference frame of the free running soliton. B) Position (blue) and velocity (green) of the LS in the reference frame of the trap. C) Histogram of residence time.

Figure 2: Unpinning transition. A) Spatiotemporal diagram in the reference frame of the trap. B),C) Histogram of the solitons inter-distances respectively plotted for the last (resp. first) 1000 round-trips.
the forcing deterministically drifts (due to the detuning) and randomly diffuses due to noise. This diffusion is related to the dispersion in the response time of excitable systems near the excitation threshold \cite{1}. Diffusion of overdamped particles in tilted potential is dramatically enhanced close to the drifting threshold \cite{2}, which explains the very large broadening of the second histogram as well as the emergence of a third peak. However, diffusive motion can be expected only when particles do not interact, which is not the case here: the gaps between the first and second (resp. second and third) peak of the histogram tend to close very fast, but the gap before the first peak remains extremely pronounced in the unlocked regime. This gap demonstrates that interactions strongly influence the phase diffusion of the oscillators i.e. the transport of dissipative solitons in the trapping landscape.

In fact, interactions can have a dominant role in the transport of ensembles of dissipative solitons. On Fig. 3 we show the trajectories of seven solitons coexisting in a (stationary) six-traps landscape. Here the trap is shallow and repulsive interactions prevent the coexistence or two LS inside one single trap. Upon the arrival of one soliton the other is expelled and falls into the neighboring trap, where the process repeats itself. Thus, the flow of solitons is fundamentally due to their interactions. The disturbance propagating from left to right in Fig. 3 is a dislocation in a soliton lattice due to the period mismatch between the lattice and the underlying potential: a metastable soliton. This is very similar to the supersolitons described theoretically in \cite{3} but here the collisions result from the non-commensurability of the interacting soliton solution with the trap’s period.

By noting that LS are periodic solutions whose period is close to the delay time, these experimental results can be explained by a partial differential equation derived in \cite{3} for the evolution of the laser phase relative to that of the optical injection. Assuming that $\alpha$ is the Henry linewidth enhancement factor of the laser and the delayed feedback has a phase $\Omega$, we define $\psi = \Omega + \arctan \alpha$ to find that the equation governing the evolution of $\theta = \phi_{\text{Laser}} - \phi_{\text{inj}} + \arctan \alpha$ reads

$$ \frac{\partial \theta}{\partial \xi} = \Delta - \sin[\theta + \varphi(x)] - \frac{\partial^2 \theta}{\partial x^2} + \tan\psi \left( \frac{\partial \theta}{\partial x} \right)^2, \quad (1) $$

where we introduced the effective detuning $\Delta$ and the pseudo-space variable $(x)$ representing the local position of the light bits within a period. The slow temporal variable $\xi$ contains the residual evolution induced e.g. by interactions between LSs, noise and the action of the modulated injected field, see \cite{3} for details. We assume the latter to be of the following form $\varphi(x) = m \sin(\omega x)$.

Considering $(\Delta, \psi, m) \ll 1$, one can reconstruct analytically the effective equations of motion of several distant LSs using a variational approach. We evaluate the interactions between distant kinks by projecting their residual interactions on the dual of their neutral translation mode. When $(\Delta, \psi, m) = 0$, analytical $2\pi$ homoclinic orbits corresponding to kink and anti-kink solutions of Eq. 1 are known and read $\Theta(x) = 4 \arctan \exp(x)$. When $(\Delta, \psi, m) \neq (0, 0, 0)$ the perturbed solution takes the form

$$ \theta(x, \xi) = \sum_{j=1}^{N} \Theta(x - a_j(\xi)) + \cdots \quad (2) $$

where the dots represent higher order corrections. Multiplying by the adjoint eigenvector of the goldstone (translation) mode and integrating over $\mathbb{R}$ allows us to find the following dynamical equations for the positions $a_j(\xi)$

$$ \frac{da_j}{d\xi} = -\frac{\pi}{4} \left( \Delta + 2\psi \right) - M(\omega) \sin(\omega a_j) - F(a_{j+1} - a_j) - F(a_{j-1} - a_j) + \sigma \xi(t) \quad (3) $$

with $M(\omega) = m(\pi \omega^2/4 \sech(\pi \omega/2))$ and $\xi(t)$ is a white Gaussian noise accounting for experimental fluctuations. The first term in Eq. 3 represents the additional drift imposed by the combined presence of the detuning and of the gradient squared terms. As such the exact period of the perturbed orbits is slightly different from the one found in the unperturbed case. The second term stems from the action of the modulated potential. As the modulation is averaged over the extent of the LSs, its effective strength $M(\omega)$ is a function of the frequency $\omega$, as expected if one remembers that it actually corresponds to the parametric forcing of an oscillator. The third and fourth terms represent the force between nearest neighbors. In deriving Eq. 3 we assumed that the kinks are not too close, and that the interaction terms are small, so that we can safely neglect second order additional coupling arising via the inertia terms. The effective force $F$ contains interactions mediated by all the terms of Eq. 1 and, although cumbersome, the expression of $F$ can be found analytically. However, as soon as the distance between the kinks is larger than a few times their typical width, one can use an asymptotic approximation that simply reads

$$ F_\psi(\delta) = (4 \, \text{sgn}(\delta) + 6\pi \psi) \exp(-|\delta|), \quad (4) $$
with \( \delta \) the distance between consecutive LSs.

Distant LSs interact via their exponentially decaying tails. While many salient examples of Solitons are even functions, as e.g solutions of the Nonlinear Schrödinger, Ginzburg-Landau or Lugiato-Lefever equations, the left and right exponential tails are not necessarily identical. As such, the interactions between LSs may not be reciprocal and, as such, they do not obey the action-reaction principle, as demonstrated recently in passively mode-locked laser [15]. This is also the case here. This fact is not inconsistent with the parity symmetry of Eq. 1. As we have two solutions branches, the symmetry only maps the kink over the anti-kink solution while both solutions are neither even nor odd in general.

From Eq. 3 in the case of a single LS, we can clearly see the possibility of an Adler Locking-UnLocking transition for the LS drifting speed depending on the precise value of \( M \) as compared to the two critical values \( M_c = \pm \left( \frac{3}{2} (\delta + 2\psi) \right) \). The system possesses a single stable (and an unstable) equilibrium point, provided that \( |M| \geq |M_c| \). In this case, the position evolves into a washboard potential that exhibits for each period a minimum and a maximum corresponding to each of these fixed points. Sufficiently close to the unlocking transition, excitability is found and noise can generate random sequence of excursions where the LS “falls” from one weakly stable potential minimum toward the next, as depicted experimentally in Fig. 1. In the presence of an external force field, we show numerical simulations in Fig. 3b) demonstrating the interplay between the action of the modulation potential and the repulsive forces between nearest neighboring solitons, which result, as in the experiment, in the propagation of a meta-soliton.

This fundamentally collective phenomenon can be explained by interactions included in Eq. 3. Interactions between LSs renormalize their drifting speed which is therefore a function of \( N \), the number of LSs. As the drifting speed (or equivalently, the repetition period of the oscillator) of \( N_1 \) LSs is slightly different from that of \( N_2 \) LSs, one foresees that a periodic solution with \( N_1 \) LSs could be locked to the frequency of the external modulation while a solution with \( N_2 \) LSs would not, as is exemplified on Fig. 3. While global coupling between LSs due to the presence of a slow variable, as e.g. thermal effects, is common in the framework of spatial and temporal dissipative solitons, this effect is absent from the simple framework of Eq. 3. Yet, a variation of the drift speed as a function of \( N \) is still fundamentally present, as a consequence of the non-reciprocity of the interactions. As such, starting from a state with \( N_1 \) LSs, the ensemble may pass from a globally locked state to a drifting one as shown on Fig. 4a,b) if the number of LS changes sufficiently. As in the case of the meta-soliton, interactions are therefore fundamentally at the origin of the dynamics. This vision is confirmed by the experimental analysis of the drifting speed as a function of the number of LSs shown in Fig. 4c).

In conclusion we have presented exhaustive experimental data regarding the optical trapping of dissipative solitons which are based on the phase space topology of an optical excitable system. This realization may in itself be potentially useful in terms of storage or routing of optical phase bits in coherent communication schemes [14, 15]. We have provided a unifying view of the trapping and escape processes in the very simple terms of locking of oscillators, shedding new light on related experiments and bridging the gap with the extensive literature of transversally extended systems. We have demonstrated the impact of dissipative solitons interactions on their transport in propagative geometries, relating it to a dependence of their round-trip frequency on their number via their non-reciprocal interactions. As a particular example of collective dynamics in a mesoscopic ensemble of light bits, we have also demonstrated the existence of a defect propagating in a lattice of dissipative optical solitons, \textit{i.e.} of a metasoliton.

Acknowledgments

J.J. acknowledges financial support project COMBINA (TEC2015-65212-C3-3-P AEI/FEDER UE) and the Ramón y Cajal fellowship. S.B., G.T. and B.G. acknowledge the support from Région Provence Alpes Côte d’Azur through grant number DEB 12-1538.

[1] F. Leo, S. Coen, P. Kockaert, S. P. Gorza, P. Emplit, and M. Haelterman, Nature Photonics 4, 471 (2010), ISSN 1749-4885.

[2] T. Herr, V. Brasch, J. D. Jost, C. Y. Wang, N. M. Kondratiev, M. L. Gorodetsky, and T. J. Kippenberg, Nature Photonics 8, 145 (2014).
[3] V. Brasch, M. Geiselmann, T. Herr, G. Lihachev, M. H. P. Pfeiffer, M. L. Gorodetsky, and T. J. Kippenberg, Science 351, 357 (2016).

[4] K. E. Webb, M. Erkintalo, S. Coen, and S. G. Murdoch, Optics Letters 41, 4613 (2016).

[5] M. Marconi, J. Javaloyes, S. Balle, and M. Giudici, Physical Review Letters 112, 223901 (2014).

[6] B. Garbin, J. Javaloyes, G. Tissoni, and S. Barland, Nature communications 6 (2015).

[7] F. Gustave, L. Columbo, G. Tissoni, M. Brambilla, F. Prati, B. Kelleher, B. Tykalewicz, and S. Barland, Phys. Rev. Lett. 115, 043902 (2015), URL http://link.aps.org/doi/10.1103/PhysRevLett.115.043902.

[8] T. Maggipinto, M. Brambilla, G. K. Harkness, and W. J. Firth, Phys. Rev. E 62, 8726 (2000).

[9] B. Garbin, J. Javaloyes, G. Tissoni, M. Giudici, and S. Barland, Laser Dynamics and Nonlinear Photonics, 2013 Sixth” Rio De La Plata” Workshop on (2013), pp. 1–3.

[10] U. Bortolozzo and S. Residori, Physical Review Letters 96, 037801 (2006), URL http://link.aps.org/abstract/PRL/v96/e037801.

[11] F. Pedaci, P. Genevet, S. Barland, M. Giudici, and J. R. Tredicce, Applied Physics Letters 89, 221111 (2006).

[12] B. Gütlich, H. Zimmermann, C. Cleff, and C. Denz, Chaos: An Interdisciplinary Journal of Nonlinear Science 17, 037113 (2007).

[13] J. K. Jang, M. Erkintalo, S. Coen, and S. G. Murdoch, Nature communications 6 (2015).

[14] J. K. Jang, M. Erkintalo, J. Schröder, B. J. Eggleton, S. G. Murdoch, and S. Coen, Optics Letters 41, 4526 (2016).

[15] J. Javaloyes, P. Camelin, M. Marconi, and M. Giudici, Physical review letters 116, 133901 (2016).

[16] P. Parra-Rivas, D. Gomila, M. A. Matías, and P. Colet, Phys. Rev. Lett. 110, 064103 (2013), URL http://link.aps.org/doi/10.1103/PhysRevLett.110.064103.

[17] E. Caboche, F. Pedaci, P. Genevet, S. Barland, M. Giudici, J. Tredicce, G. Tissoni, and L. A. Lugato, Physical Review Letters 102, 163901 (pages 4) (2009), URL http://link.aps.org/abstract/PRL/v102/e163901.

[18] E. Caboche, S. Barland, M. Giudici, J. Tredicce, G. Tissoni, and L. A. Lugato, Phys. Rev. A 80, 053814 (2009).

[19] M. Turconi, M. Giudici, and S. Barland, Phys. Rev. Lett. 111, 233901 (2013), URL http://link.aps.org/doi/10.1103/PhysRevLett.111.233901.

[20] D. Novoa, B. A. Malomed, H. Michinel, and V. M. Pérez-García, Physical review letters 101, 144101 (2008).