NEWTON’S SECOND LAW : NOT SO ELEMENTARY (AS IT MAY SEEM).

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INTRODUCTION

The adjectivation ‘Newtonian’ is used conventionally to connotate classical mechanics. In reality, during the course of three centuries, the reflection on mechanical phenomena has brought to a deep revision of the principles on which Newton built his theory. In particular, we learnt to recognise the dangers hidden behind the most ingenious anthropomorphic conceptions that constitute the intuitive bases of physics in the entry-level courses. It could be thought that the critical reflections of Mach\(^1\) and of Poincaré\(^2\) could have had some sort of impact on the introductory didactics of classical mechanics that, while still named ‘Newtonian’, shouldn’t certainly be identified with that of the ‘Principia’. However, notwithstanding more than a century has elapsed since Mach and Poincaré, this isn’t so: almost all introductory mechanics textbooks definitely follow the historical path in the exposition of dynamics. The reasons for this conservatism are many and diverse.

The manuals of analytical mechanics generally follow the Kirchhoff\(^3\) and Sommerfeld’s\(^4\) schools of thought, according to which the second law of dynamics is nothing if not a definition of force.

This is countered by another school of thought which says that the principles of newtonian mechanics aren’t definitions at all, but rather actual laws of nature. An illustrative representative of this position was Richard Feynman, who in the twelfth chapter of his ‘Feynman Lectures on Physics’ offered a sharp and witty reflection on the second law.\(^5\)

In this work I propose an interpretation that is also suggested by Galilean Relativity theory. In effect, it isn’t always kept in mind that newtonian force is a legitimate measurement only if placed in a very special class of reference frames - we could also identify it as an absolute reference. Instead, acceleration, as a kinematic quantity, is correct within any reference frame. On the other hand, it isn’t even easy to distinguish the measurement of force from that of acceleration. As a matter of fact it would be proper said that accelerometers measure forces while dynamometers in fact measure accelerations.

1. IS THE ACCELERATION AN OBSERVABLE ?

Let's point out that the formal definition of acceleration as second derivative of position with respect to the time has no practical meaning. The concept is only operational meaningful as extrapolation to the limit of the average acceleration. Indeed, no instrument exists that measures directly the instantaneous acceleration of an object. To measure the acceleration of an object we must connect this with another observable quantity. For example, using a radar technique, we can detect an object coming toward us with an electromagnetic wavepacket of known central frequency and then measure its frequency shift upon reflection. We can thus assume that the variation in frequency of the radar return is proportional to the acceleration of the body. This, however, remains only a definition. Moreover, it doesn’t distinguish if the body or the observer is accelerating. This measurement involves only laboratory instruments, but we could also...
place an accelerometer on the body, as on-bord cars of Formula One Racing. In this case it is the accelerometer that transmits the data to the observer, and whether subjected to acceleration is the observed body or the observer is immediately clear. Nevertheless, even an accelerometer doesn’t give a direct measurement of the acceleration since it also measures a quantity which is defined proportional to the acceleration.

‘Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you some flies, butterflies, and other small flying animals...’ says Galileo in his Dialogue on Two Chief World Systems.

Alternatively, we could provide our friends with a spring balance, comprising a spring with two fixed hooks at the ends; one to anchor to a support, the other to hang the objects on. Its most obvious use is in the laboratory as an instrument which serves to attribute to a body a property named weight. As a consequence, the physicists in the main cabin of a ship out there can use the spring balance to measure the weights of bodies inside their laboratory. Now note: for the inhabitants of the ship laboratory what they measure is weight; for us, in our laboratory, it is instead about the measurement of acceleration.

Problems arise when the physicists in the ship insist to compare their results with those of the other laboratories. If these others are at different heights or latitudes or inside elevators or other spaceships, the results of their work on the weights of the objects will be found different. The same object, weighted under different laboratory conditions, can give different results. In this case we eventually discover a new quantity possessed by the body - name it its mass - such that the body weight measured with the balance depends upon two quantities; \( m \), which only depends on the properties of the body, and \( g \), which depends on the properties of the laboratory. That is to say:

\[
w = g \cdot m \tag{1}
\]

where the constant \( g \) refers to the ‘intensity of the gravitational field’ inside a particular laboratory. This permits us to assign to all bodies a mass and to all laboratories a gravity field.

We admit - and it is not a small hypothesis – that the equivalent sample of mass can be shared between different laboratories (meaning objects that in the same laboratory have the same weight). With these weights we can calibrate the balances and use them as gravimeters: instruments that have the scope to define the quantity that characterises not single objects, but rather the single laboratories. If we weight the same object in two different laboratories and find two different weights \( w_1 \) and \( w_2 \) we can say that this refers to the \( g \) characteristic of the laboratory.

From this assumption the concept of mass gets defined.

If we take a standard body – or define a reference mass – we then have the possibility to define, for every laboratory, the value of the gravitational constant \( g \).

If the weights are measured in Hooke’s \( (H) \) and the mass in kilogrammes \( (kg) \), to the constant of gravity we give the dimensions of \( H/kg \).

If we want to construct a mechanics shared among all laboratories, it is necessary that we all measure our own gravitational constant \( g \).

Among all the laboratories that perform this measurement, there are some that possess a singular property – that their gravimeters always give a null result. This result selects a very peculiar class of laboratories since for these the construction of a shared mechanics is particularly simple. We call these laboratories characterised by a zero gravitational constant inertial reference frames, examples being artificial satellites and all vehicles in free fall.

2. SPRING BALANCES AS ACCELEROMETERS.
To single out laboratories that deserve the qualification of inertial we have seen that it is enough to have a spring balance. Only laboratories in which the spring balance presents a negative response can be classified as inertial. If physicists belonging to two different inertial laboratories also want to use a radar detector, aside from the nullity of their own gravitational constant, they disclose another physical property: the speed. Every observer judges another laboratory to be in uniform rectilinear motion with respect to their own. The status of inertial laboratories, derived by their zero value of the gravitational constant, confers to them, among all other possible laboratories, the privileged property that it is possible to construct a simple mechanics with exactly the same laws obeyed in each of them. These laws are, however, not applicable to other laboratories – characterised by their different values of the gravitational constant. If from an inertial laboratory we observe a non-inertial laboratory, and study its motion with the technique of radar detection, we find that the non-inertial laboratory moves with an accelerated motion in respect to ours. The physicists that reside in that laboratory are already aware that they are in a non–inertial laboratory (in fact, their balances indicate values instead of a zero for $g$) and they observe that we (who reside in an inertial laboratory) also move in an accelerated motion. As we said, it is because of our privileged status and because our shared mechanics laws that give us the right to declare that all non-inertial laboratories are, in reality, laboratories in accelerated motion, while inertial laboratories don’t accelerate at all. The important fact is that, from a broad point of view, if laboratory A is accelerating with respect to laboratory B it should follow that B is accelerating in respect to A. This isn’t true within the sphere of dynamics: if A is an inertial laboratory only laboratory B can be subject to an accelerated motion. Therefore, the laboratories in which the balance indicates non-zero weights are, in reality, the laboratories subjected to acceleration with respect to the class of inertial laboratories. In this proposition is contained the so-called ‘Principle of equivalence’, which is one of the founding assumptions of the Theory of General Relativity.

The acceptance of this principle projects a different meaning on the observations taken from the balance inside the non-inertial laboratory. To the physicists working within a non-inertial laboratory their dynamometers signal the gravity of a body. In this framework, the fact that the weight is proportional to the mass is of no surprise. The concept of mass arises from the requirement of a quantity that characterises a single body, that states invariant among laboratories of either type. It is this need, at the base of definition (2), which defines the mass.

For the physicists who work in an inertial laboratory, however, things aren’t so. That which is measured by the balance in a non-inertial laboratory is not gravity but rather the acceleration of the laboratory itself. Therefore the balance acts not as a gravitometer but as an accelerometer. Every measurement of acceleration made by observers in inertial laboratories corresponds to a measure of force made by observers in accelerated laboratories. The deeper meaning of Newton’s second law is based on the affirmation of proportionality between gravity measured in a non-inertial laboratory and the acceleration of a laboratory measured by an inertial laboratory:

$$g = Ka \quad (2)$$

where $g$ is measured with a gravitometer and $a$ by kinematic methods. The proportional constant $K$ has the dimensions:

$$[K] = \left[ \frac{H}{kg \cdot m^3} \right] \quad (3)$$

Like all the universal constants, it too plays the role of ‘conceptual synthesizer’, which expresses the unification of two distinct physical concepts into a single one of more general validity.

This is not new in physics. The mechanical equivalent of heat – the constant $J$ – gives us just another example of the role of ‘conceptual synthesizer’ of universal constants, as is the Boltzmann’s constant $k$, which takes care of synthesizing temperature and kinetic energy.

This is exactly the right time to re-read the Newton formulation:
Mutationem motus proportionalem esse vi motrici impressae, & fieri secundum lineam rectam qua vis illa imprimitur.’

In the current formulation the adjective ‘proportionalem’ is cheerfully substituted by ‘aequalem’, that is the proportionality constant has been hidden under the rag. This should not dispense us to state that the unity of force was chosen especially to this scope. That is, the Newton is defined in such a way that the proportionality constant between the gravity $g$ and the acceleration $a$ of the laboratory disappears. In reality, this is the fate of universal constants, to become invisible.

As their nature of conceptual synthesizers becomes diffuse, and they become a constituent of the normal physics background, the universal constants reduce themselves to simple conversion of unity and, finally, with an appropriate choice of units, disappear. It is this that has happened to the equivalent mechanic of heat $J$ and, in many respects, to the velocity of light, $c$.

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