About the effects of diffusion in electric tomography

Alejandro Cabo*, Jorge J. Riera** and Pedro A. Valdés Hernández***

* Department of Theoretical Physics, Instituto de Cibernética, Matemática y Física ICIMAF, La Habana, Cuba.

** Department of Biomedical Engineering, Florida International University, Miami, Florida.

** Departamento de Neuroimágenes, Centro de Neurociencias de Cuba, La Habana, Cuba

The Maxwell equations for an homogeneous medium in which electric currents are composed of Ohmic, diffusive and impressed currents are written in the static approximation in which displacement currents are neglected. Closed exact expressions are given for the solutions of the charge density $\rho$, electric field potential and the magnetic field intensity as functions of the impressed currents and densities ($J_{imp}$, $\rho_{imp}$). Also, approximate formulæ for these magnitudes are derived. The solutions correctly reproduce the conclusions of a previous work in which the relevance of Yukawa like potentials determined by diffusion effects were identified. Within the approximate solutions, the effective potential generated by the impressed sources is given by the superposition of Yukawa potentials created by the impressed charges concentrated in a given volume element. This representation makes evident that at small spatial regions, the experimental detectors are able to measure spherically symmetrical signals, indicating the presence of electric monopoles. The discussion also allows to argue that the magnetic field tomographic signal becomes completely independent of the presence of diffusion in the medium. The analysis also permits to determine the range of space and temporal frequencies in which the diffusion effects might become important in justifying the detection of monopole signals.

I. INTRODUCTION

To clarify the physical role of the diffusion effects in current EEG within small spatial dimensions is a topic of relevance in neurophysiology [1–3]. The work [1] represented a breakthrough in the research activity in this theme. The results indicated the existence of electric monopole components in the current charge densities measured from the tomography of very small samples within mouse brains. After that, the origin of these effects had been intensely debated [1–3]. In reference [2] it was considered the numerical solutions of a set of equations for the charge density in an homogeneous model of the brain including Ohmic as well as diffusive currents in addition to the impressed sources. The results indicated the possibility for detecting spherically symmetric solutions when the electrodes are of the order of distance $\frac{\lambda}{a}$ where the $\epsilon$ is the dielectric constant of the tissue, $d$ is the diffusion constant, $\sigma$ is the tissue conductivity and $a$ is typical size of the spatial region in which the impressed sources are concentrated.

In the present work we present and solve the equations for the model investigated in [2]. Firstly, we derive the set of Maxwell equations by fixing the approximation in which the displacement current contribution to the electric field is disregarded. The linear equations for the charge density, the electric potential and the magnetic field are solved exactly. Afterwards, it is performed an approximation which strongly simplifies the solutions for the frequencies usually measured in electroencephalography when the characteristic frequency $\sqrt{\epsilon}$ and the frequency bandwidth of the measured signals $\Delta \nu$ satisfies $\Delta \nu \sqrt{\epsilon} \ll 1$. The approximation corresponds to disregard the time in which the final static Yukawa solution is established after suddenly adding a free point charge in the medium. After this, the solution for the potential generated by completely general form of impressed currents is expressed as the superposition of Yukawa potentials centered in each element of volume and weighted by the impressed charge density at this point. This expression directly implies that when measures are done at spatial distances of the order of $\frac{\lambda}{a}$, it is possible to detect spherically symmetric potentials indicating the presence of a net charge (electric monopoles) in the interior regions. Further, the solution for the magnetic field is also derived. The result is simpler: the diffusion in no way affects the magnetic encephalography signals. We also investigate the range of spatial regions in which monopole signals could be most easily detected. It follows that for dipole sizes in the range 0-10 $\mu$m , the dipole potential has a ratio with respect to the monopole potential, being smaller than the number 6 which is estimated from the results in [1], for some possible values of the brain tissue parameters.

In section 2, we derive the set of Maxwell equations in the static approximation. Section 3 is devoted to expose the solution for the electron charge density. First, the exact solution for the density is written in terms of a Green function for the obtained linear equation, acting as a kernel over the charge density. In addition, the approximate
expression for the solution is also derived here. The section 4 consider the determination of the total electrostatic potential also as a linear expression of the impressed sources. In the next section 5, the solution for the magnetic field is presented. Section 6 discuss the dimensions of the spatial regions in which monopole signals could be observed in dependence of the parameters of the brain tissue. Finally, the results are reviewed in the summary in last section.

II. MAXWELL EQUATIONS FOR AN HOMOGENEOUS BRAIN MODEL

Let us present in this section the basic equations to be employed in describing the homogeneous model of the brain tissue including Ohmic, diffusion and impressed currents. Since the static approximation will be used, the Gauss Law equations becomes

\[- \nabla^2 \phi = \frac{1}{\epsilon} \rho + \frac{1}{\epsilon} \rho_{imp}, \quad (1)\]

where \(\rho_{imp}\) is the impressed charge densities assumed to be created by the neurons through their active covering membranes, \(\rho\) is the total charge density in the brain medium being external to the cells \(\phi\) is the electric potential which negative gradient gives the electric field intensity. Further, the total electric current will be written in the form

\[\vec{J}_T = -(\sigma \nabla \phi + d \nabla \rho) + \vec{J}_{imp}, \quad (2)\]

in which the first term is the Ohmic current, the second is the current associated to the diffusion (given as the negative of the gradient of the density \(\rho_{imp}\)) and the last term is the impressed current created at the neuron’s surfaces. Let us assume now that the impressed current and sources satisfy the local charge conservation condition

\[0 = \frac{\partial}{\partial t} \rho_{imp} + \nabla \cdot \vec{J}_{imp}. \quad (3)\]

Then, the conservation condition for the total current \(\vec{J}_T\) and the total density \(\rho + \rho_{imp}\) leads to

\[\nabla \cdot \vec{J}_T + \frac{\partial}{\partial t} (\rho + \rho_{imp}) = 0, \quad (4)\]

\[\frac{\partial}{\partial t} \rho + \sigma \epsilon \rho - d \nabla^2 \rho + \sigma \epsilon \rho_{imp} = 0, \quad (5)\]

which is a closed equation for the determination of the volume charge density \(\rho\) as a functional of the impressed charge density \(\rho_{imp}\). The electric and magnetic fields are defined as usual by

\[\vec{E} = -\nabla \phi, \quad (6)\]

\[\vec{B} = \nabla \times \vec{A}. \quad (7)\]

The Ampere’s Law can be written as

\[\nabla \times \vec{B} = \vec{J}_T + \epsilon \frac{\partial}{\partial t} \vec{E} = \vec{J}_T - \epsilon \frac{\partial}{\partial t} \nabla \phi, \quad (8)\]

in which the displacement current \(\epsilon \frac{\partial}{\partial t} \vec{E}\) should not be neglected in order to be able to satisfy the local current conservation condition

\[\nabla \cdot \vec{J}_T + \epsilon \frac{\partial}{\partial t} \nabla \cdot \vec{E} = 0, \quad (9)\]

\[\nabla \cdot \vec{J}_T + \frac{\partial}{\partial t} \rho + \frac{\partial}{\partial t} \rho_{imp} = 0. \quad (9)\]

The set of equations for the determination of the density \(\rho\), the electric potential \(\phi\) and the magnetic field in terms
of the impressed sources generating them, can be summarized as follows

\[-\nabla^2 \phi = \frac{1}{\epsilon} \rho + \frac{1}{\epsilon} \rho_{\text{imp}}, \quad (10)\]

\[0 = \frac{\partial}{\partial t} \rho + \frac{\sigma}{\epsilon} \rho - d \nabla^2 \rho + \frac{\sigma}{\epsilon} \rho_{\text{imp}}, \quad (11)\]

\[\nabla \times \vec{B} = -(\sigma \nabla \phi + d \nabla \rho) + \frac{\epsilon}{\epsilon} \frac{\partial}{\partial t} \vec{E} + \vec{j}_{\text{imp}}, \quad (12)\]

\[\vec{E} = -\nabla \phi, \quad \vec{B} = \nabla \times \vec{A}, \quad (13)\]

\[0 = \frac{\partial}{\partial t} \rho_{\text{imp}} + \nabla \cdot \vec{j}_{\text{imp}}. \quad (14)\]

### III. SOLUTION FOR THE CHARGE DENSITY

The equation for the density (11),

\[\frac{\partial}{\partial t} \rho + \frac{\sigma}{\epsilon} \rho - d \nabla^2 \rho = \frac{\sigma}{\epsilon} \rho_{\text{imp}} \]

can be solved by acting on it on the left with the inverse kernel \(G\) of the differential operator \(\frac{\partial}{\partial t} + \frac{\sigma}{\epsilon} \rho - d \nabla^2\), leading to

\[\rho(\vec{x}, t) = -\frac{\sigma}{\epsilon} \int d\vec{x}' dt' \ G(\vec{x}, t; \vec{x}', t') \rho_{\text{imp}}(\vec{x}', t'), \quad (15)\]

in which the mentioned kernel can be written in the Fourier representation as

\[G(\vec{x}, t; \vec{x}', t') = \int \frac{d\vec{k} \ dw}{(2\pi)^4} \ \exp(i (\vec{x} - \vec{x}') \cdot \vec{k} - i (t - t') w) \frac{-i w + \frac{\sigma}{\epsilon} + d \ \vec{k}^2}{-i w + \frac{\sigma}{\epsilon} + d \ \vec{k}^2}. \quad (16)\]

But, the following integral identity can be employed

\[\int i \ dw \ \frac{\exp(-i \ \tau \ w)}{w + i(\frac{\sigma}{\epsilon} + d \ \vec{k}^2)} = 2\pi \ \theta(\tau) \ \exp(-\frac{\sigma}{\epsilon} + d \ \vec{k}^2 \ \tau), \]

where \(\theta(\tau)\) is the Heaviside unit step function. This allows to rewrite for \(G\) the expression

\[G(\vec{x}, t; \vec{x}', t') = \theta(t - t') \exp(-\frac{\sigma}{\epsilon} (t - t')) \int \frac{d\vec{k}}{(2\pi)^3} \ \exp(i (\vec{x} - \vec{x}') \cdot \vec{k} - d \ \vec{k}^2 (t - t')). \quad (17)\]

In the case of a very rapid decaying with the time difference \(t - t'\) of \(G\) allows to determine a solution representing a very good approximation for the exact charge density \(\rho\). For this purpose let us express the integral over \(t'\) in (15) (after substituting the Fourier expansion for \(G\)) by an equivalent integral over the difference \(\lambda = (\frac{\sigma}{\epsilon} + d \ \vec{k}^2)(t - t')\) in the form

\[\rho(\vec{x}, t) = -\frac{\sigma}{\epsilon} \int d\vec{x}' dt' \ \theta(t - t') \exp(-\frac{\sigma}{\epsilon} (t - t')) \times \]

\[\int \frac{d\vec{k}}{(2\pi)^3} \ \exp(i (\vec{x} - \vec{x}') \cdot \vec{k} - d \ \vec{k}^2 (t - t')) \rho_{\text{imp}}(\vec{x}', t') \quad (18)\]

\[= -\frac{\sigma}{\epsilon} \int d\vec{x}' \int \frac{d\vec{k}}{(2\pi)^3} \ \exp(i (\vec{x} - \vec{x}') \cdot \vec{k}) \times \]

\[\int_{0}^{\infty} \ \frac{d\lambda}{\frac{\sigma}{\epsilon} + d \ \vec{k}^2} \ \exp(-\lambda) \rho_{\text{imp}}(\vec{x}', t - \frac{\lambda}{\frac{\sigma}{\epsilon} + d \ \vec{k}^2}), \quad (19)\]
But, it can be noticed when the screening time \( \frac{1}{\sigma} \) is very much larger than the unit, the integrand over the dimensionless quantity \( \lambda \) should be concentrated near the values of \( \lambda \) being of the order of the unit. This is because in this case \( \rho_{\text{imp}}(\vec{x}', t - \frac{\lambda}{\sigma + d \sqrt{k^2}}) \to \rho_{\text{imp}}(\vec{x}', t) \) becomes independent of \( \lambda \). In this approximation, it follows

\[
\rho(\vec{x}, t) = -\int d\vec{k} \frac{\exp(i \vec{x} \cdot \vec{k})}{1 + \frac{d \sigma}{\sigma} \vec{k}^2} \rho_{\text{imp}}(\vec{k}, t).
\]  

(22)

which defines an approximate solution for the volumetric density \( \rho \) as a function of the impressed density.

IV. SOLUTION FOR THE ELECTRIC POTENTIAL

The solution determining the electric tomographic signals is the one corresponding to the electric potential. It can be derived by using the already obtained solution for the volume charge density

\[
\rho(\vec{x}, t) = -\frac{\sigma}{\varepsilon} \int d\vec{x}' dt' G(\vec{x}, t ; \vec{x}', t') \rho_{\text{imp}}(\vec{x}', t')
\]  

(23)

Then, the Gauss Law in differential form

\[-\nabla^2 \phi = \frac{1}{\varepsilon} \rho + \frac{1}{\varepsilon} \rho_{\text{imp}}, \]

(24)

and the inverse kernel of the Laplacian allows to write the potential as follows

\[
\phi(\vec{x}, t) = \int d\vec{x}' \frac{1}{4\pi|\vec{x} - \vec{x}'|} \left( \frac{\sigma}{\varepsilon} \rho(\vec{x}', t) + \frac{1}{\varepsilon} \rho_{\text{imp}}(\vec{x}', t) \right).
\]  

(25)

By substituting the solution for the density, the exact expression for the electric potential as a functional of the impressed sources becomes

\[
\phi(\vec{x}, t) = \int d\vec{x}' \frac{1}{4\pi\varepsilon|\vec{x} - \vec{x}'|} \left( \frac{\sigma}{\varepsilon} \int d\vec{x}'' dt' G(\vec{x}', t ; \vec{x}'', t') \rho_{\text{imp}}(\vec{x}'', t') + \rho_{\text{imp}}(\vec{x}', t) \right).
\]  

(26)

A. The approximate expression for the total potential

Let us perform again the previous approximation in which the screening times are assumed to be very much rapid, but in an alternative way. Remembering that

\[-\nabla^2 \phi = \frac{1}{4\pi\varepsilon|\vec{x} - \vec{x}'|} = \delta^{(3)}(\vec{x} - \vec{x}'), \]

and acting with the operator \(-\nabla^2 \) over \( \phi(\vec{x}, t) \) permits to rewrite the known expression

\[-\varepsilon \nabla^2 \phi(\vec{x}, t) = \rho(\vec{x}, t) + \rho_{\text{imp}}(\vec{x}, t), \]

which after Fourier transformed gives

\[
\varepsilon k^2 \phi(\vec{k}, w) = \rho(\vec{k}, w) + \rho_{\text{imp}}(\vec{k}, w)
\]

\[= (1 - \frac{1}{-i \frac{\sigma}{\varepsilon} w + 1 + \frac{d \sigma}{\sigma} k^2}) \rho_{\text{imp}}(\vec{k}, w). \]  

(27)

But, in biological tissues the term \( \frac{\sigma}{\varepsilon} \) is a frequency parameter much might be higher than the measured frequencies \( w \) in usual tomographic experiments. Thus, when the ratio \( \frac{1}{\sigma} w \) might be very much smaller than the unit, ignoring
This term in the denominator of the above expression, the Fourier transform of total value of the electrostatic potential generated by the impressed densities can be written as

\[
\phi(\mathbf{k}, w) = \frac{1}{\epsilon k^2} (\rho(\mathbf{k}, w) + \rho_{\text{imp}}(\mathbf{k}, w))
\]

\[
= \frac{1}{\epsilon k^2} \left( 1 - \frac{1}{1 + \frac{d}{\epsilon k^2}} \right) \rho_{\text{imp}}(\mathbf{k}, w)
\]

\[
= \frac{\frac{d}{\epsilon}}{1 + \frac{d}{\epsilon k^2}} \rho_{\text{imp}}(\mathbf{k}, w).
\]

This relation can be inversely Fourier transformed to the spatial representation to write

\[
\phi(\mathbf{x}, t) = \int d^3 k \int d w \phi(\mathbf{k}, w) \exp(i \mathbf{k} \cdot \mathbf{x} - i w t)
\]

\[
= \int d^3 k \int d w \frac{d}{1 + \frac{d}{\epsilon k^2}} \rho_{\text{imp}}(\mathbf{k}, w) \exp(i \mathbf{k} \cdot \mathbf{x} - i w t)
\]

\[
= \int d^3 \mathbf{x} \frac{1}{\epsilon |\mathbf{x} - \mathbf{x}^\prime|} \exp(-\sqrt{\frac{\sigma}{\epsilon d}} |\mathbf{x} - \mathbf{x}^\prime|) \rho_{\text{imp}}(\mathbf{x}^\prime, t).
\]

This approximate solution gives the measured electrostatic potential as a superposition of Yukawa potentials generated for the impressed charges contained in each volume element. This conclusion reproduces in a generalized way the results previously derived in reference [3]. The expression also tells that, under the assumed rapid screening approximation, the neural impulses detected in the EEG experiments have dependences which are coincident with ones associated to the impressed sources. That is, there are no time delay effects if the rapid screening occurs.

However, as noted above in biological tissues, at difference with usual metals, the characteristic screening frequencies may lie in the interval \(10^5\) to \(10^7\) seconds, which are by now mean higher than the usual measured signals. This difference with usual metals occurs mainly because the high values of the relative dielectric constant \(\epsilon\) ranging from \(10^5\) to \(10^7\). Such large values are determined by macroscopic properties associated to capacitive effect.

\[
\sqrt{\frac{\sigma}{\epsilon}} = 0.000119024 \text{ s which for } w \text{ of order } 100 \text{ Hz satisfies } w \sqrt{\frac{\sigma}{\epsilon}} < 1. \text{ Therefore, the approximated solution can be employed only when brain tissue satisfies } w \sqrt{\frac{\sigma}{\epsilon}} < 1. \text{ Thus, to employ the above approximation it should be verified that the screening frequencies } \frac{\sigma}{\epsilon} \text{ are very much larger than the width of the frequency spectrum of the measured signals, that is } w \sqrt{\frac{\sigma}{\epsilon}} < 1.
\]

V. SOLUTION FOR THE MAGNETIC FIELD

The solution for magnetic field intensity can be determined from the Ampere’s Law by transforming it as follows

\[
\nabla \times \mathbf{B} = -\sigma \nabla \phi + d \nabla \rho + \epsilon \frac{\partial}{\partial t} \mathbf{E} + \mathbf{J}_{\text{imp}}
\]

\[
= \nabla \times \nabla \times A = -\nabla^2 A + \nabla (\nabla \cdot A), \tag{28}
\]

which by after employing the Coulomb gauge \(\nabla \cdot A = 0\), can be written as follows

\[
-\nabla^2 A = -\nabla (\sigma \phi + d \rho + \epsilon \frac{\partial}{\partial t} \phi) + \mathbf{J}_{\text{imp}}. \tag{29}
\]

In the above expression, an important issue to notice is that the non-impressed components of the current have gradient structures. After using the inverse kernel for the Laplacian, the expression for the vector potential can be written as

\[
\mathbf{A}(\mathbf{x}, t) = \int d^3 \mathbf{x}^\prime \frac{1}{4\pi |\mathbf{x} - \mathbf{x}^\prime|} (-\nabla^\prime (\sigma \phi(\mathbf{x}^\prime, t) + d \rho(\mathbf{x}^\prime, t) + \epsilon \frac{\partial}{\partial t} \phi(\mathbf{x}^\prime, t)) + \mathbf{J}_{\text{imp}}(\mathbf{x}^\prime, t)). \tag{30}
\]
Therefore, after taking into account the gradient nature of the non-impressed components of the currents, it can be written the following solution for the magnetic field intensity

\[ \vec{B}(\vec{x}, t) = \nabla \times \int \frac{1}{4\pi|\vec{x} - \vec{x}'|} \vec{J}_{\text{imp}}(\vec{x}', t) \, d^3 \vec{x}' - \int \frac{1}{4\pi|\vec{x} - \vec{x}'|} \nabla \times \vec{J}_{\text{imp}}(\vec{x}', t). \]  

(31)

A central property to notice in this expression is that it is completely independent of the presence of diffusion in the system. Thus diffusion does not affect the magnetic tomographic images.

VI. DISCUSSION ABOUT MONOPOLE DETECTION WITHIN THE MODEL

Let us further discuss the possibilities for the model of predicting the observation of electric monopoles. This point was started to be analyzed in reference [3]. The discussion presented there is supported by the results of this work. The signal for the 1 Coulomb value charge screened by the Yukawa potential, but now written in International System of Units (ISU), has the form

\[ \phi_m(\vec{x} - \vec{x}') = \frac{1}{4\pi \epsilon \epsilon_0 |\vec{x} - \vec{x}'|} \exp(- \sqrt{\frac{\sigma_{\text{ISU}}}{\epsilon \epsilon_0 |\vec{x} - \vec{x}'|}}), \]  

(32)

where \( \epsilon_0 = 8.8542 \times 10^{-12} \, \text{F/m} \) is the vacuum permittivity and \( \epsilon \) is the relative dielectric constant and \( \sigma_{\text{ISU}}, D \) are the electric conductivity and the diffusion constant respectively, but taken in the ISU units. This solution can be interpreted as the stationary response of the medium to the instantaneous implantation of a 1 C point charge at the coordinates \( \vec{x}' \). Let us call this solution as the monopole potential.

Let us now compare the magnitude of the monopole potential with the one associated to a dipole potential, defined (for \( |\vec{d}| \ll |\vec{x} - \vec{x}'| \)) as

\[ \phi_d(\vec{x} - \vec{x}') = \phi_m(\vec{x} - (\vec{x}' + \vec{d})) - \phi_m(\vec{x} - \vec{x}') \]

\[ \simeq -\phi_m(\vec{x} - \vec{x}') \frac{\vec{d}}{|\vec{x} - \vec{x}'|} \frac{1}{|\vec{x} - \vec{x}'|} + \sqrt{\frac{\sigma_{\text{ISU}}}{\epsilon \epsilon_0 |\vec{x} - \vec{x}'|}}. \]  

(33)

That is a less symmetrical solution corresponding to the stationary response of the medium to implant two 1 C point charges, one positive and another negative, separated a distance \( \vec{d} \) between them. The spatial variation of the potential with the distance \( \vec{y} = \vec{x} - \vec{x}' \) between the generation point \( \vec{x}' \) and the observation point \( \vec{x} \) is depicted in figure [I] for two extremal values of the screening distance \( \lambda = \sqrt{\epsilon_0 \frac{D}{\sigma_{\text{ISU}}}} \). The minimum value was chosen by calculating \( \lambda \) for the maximal value of the ISU conductivity \( \sigma_{\text{ISU}} \) in the range of its typical values for tissues

\[ \sigma_{\text{ISU}}^{\text{min}} = 0.05 \, \frac{\text{S}}{\text{m}} \leq \sigma_{\text{ISU}} \leq \sigma_{\text{ISU}}^{\text{max}} = 0.3 \, \frac{\text{S}}{\text{m}}, \]  

(34)

by also using the minimal values of the dielectric and the diffusion constants within the ranges of their typical values

\[ \epsilon_{\text{min}} = 5 \times 10^5 \leq \epsilon \leq \epsilon_{\text{max}} = 8 \times 10^7, \]  

(35)

\[ D_{\text{min}} = 0.45 \times 10^{-9} \leq D \leq D_{\text{max}} = 1.33 \times 10^{-9} \]  

(36)

This defines the minimal value of \( \lambda \) as

\[ \lambda_{\text{min}} = \sqrt{\frac{\epsilon_{\text{min}} \epsilon_0 D_{\text{min}}}{\sigma_{\text{ISU}}^{\text{max}}}} \]

\[ = 8.14902 \times 10^{-8} \, \text{m}. \]  

(37)

In a similar way the maximal value of \( \lambda \) becomes

\[ \lambda_{\text{max}} = \sqrt{\frac{\epsilon_{\text{max}} \epsilon_0 D_{\text{max}}}{\sigma_{\text{ISU}}^{\text{min}}}} \]

\[ = 4.34071 \times 10^{-6} \, \text{m}. \]  

(38)
FIG. 1: The spatial dependence of the stationary Yukawa potential generated by 1 Coulomb point charge
instantaneously inserted in the medium. Two plots are shown: a) the top curve at large distances, is potential
produced in a tissue showing a largest values of the dielectric and diffusion constants and the lower conductivity
value, in the ranges defined in (34), (35), (36). This determines the largest value for the Yukawa length. b) The lower
curve at large distances, related with the opposite situation in which the conductivity and diffusion constant are the
lowest values and the conductivity the largest one.

The two extremal values of $\lambda$ illustrate that for the whole of set of tissues showing the above range of data, the
electrostatic fields have decay lengths of the order of $4 \mu m$ at most. The figure 1 shows $\phi_m(\vec{x} - \vec{x}')$ as function of the
observation distance measured in meters, for the larger value of the decay distance $\lambda_{max} = 4.34071 \times 10^{-6}$ m.

For the comparison between $\phi_m$ and $\phi_d$ let us consider that the vector $\vec{d}$ is parallel with the observation one
($\vec{x} - \vec{x}'$). Thus the ratio of the two potential signals become

$$|\frac{\phi_d(y)}{\phi_m(y)}| = d \left( \frac{1}{y} + \sqrt{\frac{\epsilon_{ISU} \sigma_{max}}{\epsilon_0 D}} \right),$$

$$y = |\vec{x} - \vec{x}'|.$$  

This ratio is plotted in figure 2 as a function of the size of the dipole $d$ and the distance $y$ between its position to
the measurement point. The parameters selected for this plot were

$$D_{max} = 1.33 \times 10^{-9},$$

$$\epsilon_{max} = 8 \times 10^7,$$

$$\sigma_{ISU}^{min} = 0.05.$$  

They corresponds to the larger value of the length $\lambda_{max} = \sqrt{\frac{\epsilon_{max} \epsilon_0 d_{max}}{\sigma_{ISU}^{min}}} = 4.34071 \times 10^{-6}$ generated a material
showing values of the conductivity the dielectric and the diffusion constants restricted to be in the usual experimental
ranges (34), (35), (36) for biological tissues. The range of values for dipole size $d$ (which we interpret as qualitatively
representing the size of the region in which the impressed currents are defined) in the plot of 2 runs from zero to 20
$\mu m$. The other axis is related with the distance at which the potential is measured $y$. The range of values for $y$ of
this axis is similar to the spatial extension of the array of electrodes employed in [1].

The horizontal plane depicted in figure 2 is plotted at a value of $|\frac{\phi_d}{\phi_m}| = 6$. This number approximately describes
the ratio between the amount of electric signals produced by pyramidal cells and the corresponding amount defined by
spin stellate cells, at the array of detectors employed in reference [1] (See figure 2 in that reference). The rising with
$d$ surface defines the values of $|\frac{\phi_d}{\phi_m}|$ as a function of $d$, which we interpret as representing the size of non symmetrical
impressed sources; and as a function of the distance $y$ between the point in which these sources are localized and the
observation points at the electrodes. These electrodes are separated by a 50 $\mu m$ in the experiments done in [1].

Then, the intersection curve between the plane and plotted surface corresponds to pairs $(y, d)$, of measuring distances
$y$ and dimensions of the impressed sources $d$, at which the ratio between the dipole and monopole signals shows the
value observed in the experiments done in [1]. Thus, it follows that for sources sizes in the range 0-10 $\mu m$ the dipole
FIG. 2: The plot shows the ratio the dipole and monopole potentials as a function of the measuring distance and the sizes $d$ of the dipole. The horizontal plane is plotted at the value equal to $6$ of the estimated ratio between the dipole and monopole measured potentials in reference [1]. It follows that mainly in all the region $d < 10 \mu m$ the monopole to dipole ratio $\frac{\phi_m}{\phi_d}$ tends to increase with respect to the value $1/6$ estimated in [1]. It can be observed that a normal dendrite width is $1 \mu m$. Thus, the impressed charge densities can be seen as a superposition of densities of this typical width of $1 \mu m$. The evaluation again was done for a tissue showing a largest values of the dielectric and diffusion constants and the lower conductivity value, in the ranges defined in (34),(35),(36).

VII. SUMMARY

We investigated the Maxwell equations for an homogeneous medium in which electric currents are composed of Ohmic, diffusive and impressed currents. The static approximation in which displacements currents are neglected was employed. Solutions of the metallic charge density $\rho$, electric field potential and the magnetic field intensity as functions of the impressed currents and densities ($\vec{J}_{\text{imp}}$, $\rho_{\text{imp}}$) are derived. In addition, approximate formulae for these quantities are also obtained. The analysis reproduce the conclusions of a previous work, by also indicating the relevance of Yukawa like potentials. This representation indicates that for spatial regions of reduced dimensions, the experiments can be able to detect monopole like signals. The spatial sizes of the regions in which this turn to be possible depend on a single parameter: the Yukawa decay length $\lambda = \sqrt{\frac{\rho_{\text{imp}} D}{\sigma_{\text{ISU}}}}$, which grows for larger values of the relative dielectric constant and the diffusion parameter and decreases for larger conductivities of the tissue medium. It also argued that the magnetic tomographic signals result to be independent of the existence of diffusion in the medium. The analysis also allowed to study the ranges of spatial distances in which monopole signals can be detected.

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[1] J. J. Riera, T. Ogawa, T. Goto, A. Sumiyoshi, H. Nonaka, A. Evans, H. Miyakawa and R. Kawashima, J. Neurophysiol. 108, 956 (2012).
[2] J. J. Riera and A. Cabo, J. Neurophysiol. 109, 1694 (2013).
[3] A. Cabo and J. R. Riera, How the active and diffusional nature of brain tissues can generate monopole signals at micrometer sized measures, http://arxiv.org/submit/1079813/pdf 1 Oct. 2014.
[4] C. Bédard and A. Destexhe, Phys. Rev. E84, 041909 (2011).
[5] S. L. Gratiy, K. H. Pettersen, G. T. Einevoll and A. M. Dale, J. Neurophysiol. http://dx.doi.org/10.1152/jn.01047.2012 (2012).
[6] P. L. Nunez and R. Srinivasan, Electric fields of the brain. The neurophyscis of the EEG, 2nd Edition, Oxford University Press, 2006.
[7] S. Gabriel, R. W. Lau, C. Gabriel, Phys. Med. Biol. 41, 2251 (1996).
[8] H. Bateman, Tables of integral transforms: Fourier, Laplace and Mellin transforms, V. 1, McGraw-Hill, New York 1954.