NUMERICAL SIMULATION OF HOT ACCRETION FLOWS. I. A LARGE RADIAL DYNAMICAL RANGE AND THE DENSITY PROFILE OF ACCRETION FLOW

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ABSTRACT

Numerical simulations of hot accretion flow, both hydrodynamical and magnetohydrodynamical, have shown that the mass accretion rate decreases with decreasing radius; consequently, the density profile of accretion flow becomes flatter than in the case of a constant accretion rate. This result has important theoretical and observational implications. However, because of technical difficulties, the radial dynamic range in almost all previous simulations usually spans at most two orders of magnitude. This small dynamical range, combined with the effects of boundary conditions, makes the simulation results suspect. In particular, the radial profiles of density and inflow rate may not be precise enough to be used to compare with observations. In this paper, we present a “two-zone” approach to expand the radial dynamical range from two to four orders of magnitude. We confirm previous results and find that from $r_s$ to $10^3 r_s$, the radial profiles of accretion rate and density can be well described by $\dot{M}(r) \propto r^p$ and $\rho \propto r^{-p}$. The values of $(s, p)$ are $(0.48, 0.65)$ and $(0.4, 0.85)$ for the viscous parameters $\alpha = 0.001$ and $\alpha = 0.01$, respectively. More precisely, the accretion rate is constant (i.e., $s = 0$) within $\sim 10 r_s$, but beyond $10 r_s$, we have $s = 0.65$ and $0.54$ for $\alpha = 0.001$ and $0.01$, respectively. We find that the values of both $s$ and $p$ are similar in all numerical simulation works irrespective of whether a magnetic field is included or not and what kind of initial conditions are adopted. Such an apparently surprising “common” result can be explained by the most recent version of the adiabatic inflow–outflow model. The density profile we obtain is in good quantitative agreement with that obtained from the detailed observations and modeling of Sgr A* and NGC 3115. The origin and implications of such a profile will be investigated in a subsequent paper.

Key words: accretion, accretion disks – black hole physics

1. INTRODUCTION

Hot accretion flow, such as advection-dominated accretion flows (ADAFs; Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1994, 1995; Abramowicz et al. 1995; for reviews, see Narayan et al. 1998; Kato et al. 1998; F. Yuan & R. Narayan 2012, in preparation), is of great interest because of its widespread applications in low-luminosity active galactic nuclei (AGNs), including the supermassive black hole in our Galactic center and the quiescent and hard states of black hole X-ray binaries (see reviews by Narayan 2005; Yuan 2007; Narayan & McClintock 2008; Ho 2008; Yuan 2011). In early analytical work, the mass accretion rate was assumed to be independent of radius, $\dot{M}(r) = \text{constant}$. In this case, the radial profile of density satisfies $\rho(r) \propto r^{-3/2}$ (e.g., Narayan & Yi 1994). Numerous hydrodynamical (HD) and magnetohydrodynamical (MHD) numerical simulations have been performed, with most focusing on the dynamics of the accretion flow (e.g., Igumenshchev & Abramowicz 1999, 2000; Stone et al. 1999, hereafter SPB99; Stone & Pringle 2001; Hawley et al. 2001; Machida et al. 2001; McKinney & Gammie 2002; Hawley & Balbus 2002; Igumenshchev et al. 2003; Pen et al. 2003; De Villiers et al. 2003; Proga & Begelman 2003a, 2003b; Pang et al. 2011; McKinney et al. 2012; Narayan et al. 2012). The effect of strong radiation was studied by Yuan & Bu (2010) and, most recently, by Li et al. (2012). One of the most surprising—and perhaps also the most important—findings of these simulations is that the mass accretion rate (or, more precisely, the inflow rate; refer to Equation (7) for its definition) is found to be not a constant but rather it decreases with decreasing radius. Denoting the mass accretion rate as $\dot{M}(r) \propto r^p$, numerical simulations have found that $s \sim 0.5–1$ (see Section 4.1 for a review). Consequently, the density profile flattens compared to the previous $\rho(r) \propto r^{-1.5}$; we now have $\rho(r) \propto r^{-p}$ with $p \lesssim 1$. Such results have obtained strong observational support in the case of the supermassive black hole in our Galactic center, Sgr A* (Yuan et al. 2003; refer to Section 4.2.1 for details), the low-luminosity AGN NGC 3115 (Wong et al. 2011; refer to Section 4.2.2 for details), and black holes in elliptical galaxies (Di Matteo et al. 2000; Mushotsky et al. 2000).

In addition to the obvious theoretical interest, the radial profiles of accretion rate and density also have important observational applications. This is because they will determine the emitted spectrum and other radiative features of an accretion flow (e.g., Quataert & Narayan 1999; Yuan et al. 2003). For example, in the case of Sgr A*, the mass accretion rate at the Bondi radius can be determined directly by observations. The radiation of the accretion flow may be completely different depending on the radial profile of the accretion rate (or, more exactly, the density). The radial profile of the accretion rate also determines the evolution of black hole mass and spin. In many current numerical simulations, due to resolution difficulties, we can at best resolve the Bondi radius and determine the Bondi accretion rate there. Then, the evolution of the mass and spin of black holes will be determined by the fraction of the Bondi accretion rate that finally falls onto the black hole, which is determined by the radial profile of the accretion rate.

It is thus important to carefully investigate the radial profiles of accretion rate and density. One problem in almost all previous simulations was that the radial dynamical range was rather small, usually at most two orders of magnitude. Technically this is because it would be too time-consuming as to be almost
impossible to simulate an accretion flow if the dynamical range is too large. In addition, as is well known, simulation results are usually not reliable close to the boundary due to boundary condition effects. These cast some doubt on previous simulation results, especially the exact quantitative radial profiles of the physical quantities. The situation is even worse for MHD simulations than for HD ones. Because the Alfvén speed is very large in regions of low density and strong magnetic field, MHD simulations are much more expensive than HD simulations. The radial dynamical range is thus more limited, and it is harder to evolve the simulation for a long time, which further constrains the range over which the steady state is reached.

A large dynamical range is also useful in investigating the following problem. Previous works (SPB99; Igumenshchev & Abramowicz 1999; Yuan & Bu 2010) have found that the Bernoulli parameter of most outflows in their simulations is indeed negative.\(^3\) One may expect that the outflow may not be able to escape to infinity, but may rejoin the accretion flow at a certain distance. The fact that no previous simulations found accumulation of matter in the accretion flow could be for two possible reasons. One is that the simulation time is not long enough for accumulation to occur. However, the radial velocity of the outflow is roughly of the order of the local Keplerian velocity (Yuan et al. 2012, hereafter Paper II). We can therefore expect that if the outflow finally rejoins the accretion flow, the accretion timescale by a factor of \(\alpha\). Thus this possibility is unlikely. Another possible reason is that the radial dynamical range is too small. The accumulation may occur at a radius larger than the outer boundary of all current simulations. To examine this possibility, a larger dynamical range is required.

In this paper, we simulate two-dimensional HD accretion flows. In particular a “two-zone” approach will be adopted which helps us to overcome the technical problem and achieve a large dynamic range spanning four orders of magnitude. It is widely believed that in reality the magnetic stress associated with the MHD turbulence driven by the magnetorotational instability (MRI) transfers angular momentum (Balbus & Hawley 1991, 1998). Therefore, we should, in principle, use MHD simulation. In this paper, we do not include magnetic field but instead include an anomalous shear stress to mimic the magnetic stress. However, as we will describe in Paper II, the mechanisms of producing the accretion rate profile are different for HD and MHD accretion flows. One may ask whether the radial profile of the accretion rate obtained in the present HD simulation is the same as that obtained in more realistic MHD simulations. In this regard, the recent work by Begelman (2012) gives a positive answer. The comparison between our HD simulation with MHD simulations also confirms this point (see Section 4.1 for details).

The structure of this paper is as follows. In Section 2, we introduce the details of our “two-zone” simulation approach. The results of our simulations are presented in Section 3. We find that the profiles of the mass accretion rate and density still follow a perfect power-law form and the power-law index remains almost unchanged compared to previous results. In Section 4, we compare our results with previous HD and MHD simulations (Section 4.1) and observations (Section 4.2). We find broadly good agreement among them. The last section (Section 5) summarizes our results.

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3 In Paper II we find that, depending on the initial condition of the simulation, the Bernoulli parameter can be negative or positive.
Wiita (1980) potential. We adopt the same non-uniform grid in both the radial and angular directions as in SPB99. The resolution is $N_r = 168$ and $N_\theta = 88$.  

**Step 1.** We first simulate the outer zone. Following SPB99, the initial state of our simulation is an equilibrium torus with a constant specific angular momentum. We refer the reader to SPB99 for the description of the torus. We set the radius of the maximum density at $10,000\,r_s$, the maximum density of the torus $\rho_{\text{max}} = 1.0$, the density of the medium $\rho_0 = 10^{-4}$, and pressure $p_0 = \rho_0/r$. The standard outflow boundary conditions (projection of all dynamical variables) at both the inner and outer boundaries are adopted.

**Step 2.** We then simulate the inner zone. We inject the gas at the outer boundary. The values of density, specific internal energy, and velocity of the injected flow are taken from the steady simulation results of step 1 at $200\,r_s$. We do not use $100\,r_s$ because we hope to avoid the effect of the inner boundary. We adopt the outflow boundary condition at the inner boundary.

**Step 3.** Obviously, simply connecting the simulation results of the last two steps is generally not self-consistent. This is because, for example, the inner zone will obviously produce some outflow at the outer boundary and these outflows will be injected into the outer zone. This effect has not been taken into account in step 1. We therefore simulate the outer zone again, but this time we use the results of step 2 at $100\,r_s$ as the inner boundary condition. Again, here we do not choose $200\,r_s$ because we hope to avoid the effect of the outer boundary. In this way, we can capture all the outflows produced from the inner region and preserve their properties such as velocity and internal energy, and thus their (negative) Bernoulli parameter. Thus, we should be able to observe whether these outflows will rejoin the accretion flow and accumulate somewhere in the outer zone. After the outer zone reaches the steady state, we simulate the inner zone again, following the method described in step 2.

**Step 4.** We plot the radial profiles of various physical quantities along different $\theta$ of the inner and outer zones. We look at the curves in each zone to judge the convergence by eye, i.e., whether the results in each zone obtained in two adjacent steps are consistent with each other. If not, we perform iterations further to improve the convergence. Usually we get satisfactory convergence after up to three iterations.

However, we note that this iterative approach does not guarantee complete consistency between the two zones. In principle, we should transfer all information on the fluid at each time step from one zone to another. We assume that a steady solution is reached and only transfer part of the information at one snapshot. However, our approach does transfer some information between the two zones, and these pieces of information are perhaps the most important for the dynamics of the accretion flow. Using this “two-zone” approach, we can easily simulate the accretion flow with a dynamic range spanning four orders of magnitude in radius. Compared with the usual “one-zone” approach, the calculation time is about $10^{3}/N \sim$ several hundreds times shorter for our problem (here $N$ is the number of iterations). This approach could potentially be used in other simulation problems that require a large dynamical range. However, note that the underlying assumption of this approach is that steady solutions exist. This is satisfied for our problem.

3. RESULTS

3.1. Model A: The Case of $\alpha = 0.001$

Figure 1 shows the time-averaged radial distributions of physical quantities near the equator of the whole region of the accretion flow (both the inner and outer zones). The results are averaged over time and angle between $\theta = 84^\circ$ and $\theta = 96^\circ$. The solid lines are the final results of step 4. For comparison, we also show the results of steps 1 and 2 by the dot–dashed
lines. We can see that the profiles become slightly flatter after convergence is achieved. The density, gas pressure, rotation velocity, and radial velocity can be described by a power law scaling with radius:

\[ \rho \propto r^{-0.65}, \quad P \propto r^{-1.7}, \quad v_{\phi} \propto r^{-0.5}, \quad v_r \propto r^{-0.55}. \]  

(6)

These scalings are measured away from the innermost region, i.e., \( r \gtrsim 10r_s \). We avoid the region within \( 10r_s \) for two reasons. First, as emphasized by Narayan et al. (2012) and also confirmed by the radial velocity plot in Figure 1, the radial velocity of the flow increases inward much more rapidly close to the black hole because of the strong gravity; thus, it deviates from its scaling extrapolated from the region \( r \gtrsim 10r_s \). Second, as we will see from Figure 2, within \( \sim10r_s \) the inflow rate is a constant and there is little outflow. Therefore, in this sense, this region is also “special” compared with the region outside \( \sim10r_s \). We note that these two “inner boundary effects” do not exist if a Newtonian potential is adopted.

These results are roughly consistent with those of SPB99, who found \( \rho \propto r^{-0.5}, \) \( P \propto r^{-1.5}, \) and \( v_r \propto r^{-0.5} \). We have done some test calculations and found that the small discrepancy is because we adopt the Paczyński & Wiita potential while a Newtonian potential is adopted in SPB99. The scaling of the radial and rotational velocities is also consistent with the self-similar solution of ADAF (Narayan & Yi 1994),\(^2\) where we have \( v_r \propto r^{-0.5} \) and \( v_{\phi} \propto r^{-0.5} \). The deviations of \( \rho \) and \( P \) from the self-similar solution are significant. This is because of the inward decrease of the mass accretion rate, as we will describe below. We note that the scaling we obtain should be more reliable than that of previous work because of our extremely large radial dynamical range.

In Figure 2, we plot the time-averaged and angle-integrated mass accretion rate of the whole region. Following SPB99, the mass inflow and outflow rates, \( \dot{M}_{\text{in}} \) and \( \dot{M}_{\text{out}} \), are defined as follows:

\[ \dot{M}_{\text{in}}(r) = 2\pi r^2 \int_0^\pi \rho \min(v_r, 0) \sin\theta d\theta, \]  

(7)

\[ \dot{M}_{\text{out}}(r) = 2\pi r^2 \int_0^\pi \rho \max(v_r, 0) \sin\theta d\theta. \]  

(8)

The net mass accretion rate is defined as

\[ \dot{M}_{\text{acc}}(r) = \dot{M}_{\text{in}}(r) + \dot{M}_{\text{out}}(r). \]  

(9)

The rates of inflow and outflow and the net rate are denoted by the solid, dashed, and dotted lines, respectively. Note that the results are obtained by time averaging the integral rather than integrating the time averages. Also shown in the figure by the dot–dashed line is the inflow rate obtained in steps 1 and 2. We see that the final curves for the inflow rate after step 4 become steeper than the results of steps 1 and 2. The net rate close to the outer boundary of the outer zone is not constant, indicating that it requires a longer time to reach the fully steady solution there. This is because the viscosity coefficient is very small, so the accretion timescale is very long. The radial profile of the inflow rate from \( \sim2r_s \) to \( 10^4r_s \) can be described by

\[ \dot{M}_{\text{in}}(r) = \dot{M}_{\text{in}}(r_{\text{out}}) \left( \frac{r}{r_{\text{out}}} \right)^{0.48}, \]  

(10)

or more precisely by

\[ \dot{M}_{\text{in}}(r) = \dot{M}_{\text{in}}(r_{\text{out}}) \left( \frac{r}{r_{\text{out}}} \right)^{0.65}. \]  

(11)

\( \sim10r_s \) to \( 10^4r_s \), while it is almost constant within \( 10r_s \). This is different from SPB99. In that work, the authors found that the inflow rate keeps decreasing inward until the inner boundary, and the slope is steeper than ours, which is \( \dot{M}_{\text{in}} \propto r^{0.75} \) (“Run K” in SPB99). Again, the reason is that we adopt a different gravitational potential for the black hole. Actually, we can see from the figure that beyond \( 100r_s \), where the two types of potential are basically identical, the power-law index

\(^2\) ADIOS has the same scaling as ADAF for the radial and rotational velocities. ADIOS differs from ADAF only in the scaling of density and accretion rate. See Equations (14) and (16) in Narayan et al. (2012).
Thus, few outflows are produced. The timescale required for the formation of outflow due to the strong gravitational force is because of the gradient of the gas pressure, i.e., the buoyant force. When the gravitational force is stronger, the gas pressure plays a relatively minor role. In other words, in the innermost region the accretion timescale is shorter than the range of the whole simulation domain usually spans only two orders of magnitude and the steady solution is only reached within one order of magnitude because of the effect of the outer boundary condition. This result indicates that although the Bernoulli parameter of most of the mass outflow is negative (SPB99; Yuan & Bu 2010), the outflow does not rejoin the accretion flow and accumulates somewhere within 10^4r_s. Physically, this is likely because the flow is viscous; thus, there is an associated energy flux.

3.2. Model B: The Case of α = 0.01

We simulate model B to examine the effect of varying the amplitude of the shear stress. The viscosity coefficient in model B is 10 times larger than in model A. Igumenshchev & Abramowicz (1999) and SPB99 have already investigated this issue. They found that when the stress becomes stronger, the convective instability becomes weaker, in the sense that the radial profile of the inflow rate becomes flatter. However, they did not present qualitative results for the profiles of density and inflow rate when the stress is increased. In the extreme case, Igumenschev & Abramowicz (1999) found that when α ≥ 0.1 the flow becomes almost laminar, and a bipolar outflow structure very close to the rotation axis, ~8r_s, is produced.

Here we try to give more qualitative results in order to compare with observations (see Section 4 below). Figure 3 is similar to Figure 1, presenting the radial profiles of density, pressure, rotational velocity, and radial velocity. Figure 4 shows the radial profiles of inflow and outflow rates and the net rate. From Figure 3, we see that the equatorial density, gas pressure, rotation velocity, and radial velocity can again be well described by a power-law form. The slopes of the profiles of rotation and radial velocity remain the same as in model A. The normalization of the radial velocity is 10 times larger, as expected. Compared with model A, the profiles of density and pressure are moderately steeper, while that of the inflow rate is ~0.75, in good agreement with SPB99. The deeper potential adopted in our work suppresses convection to some degree and makes the outflow weaker. In Paper II, we analyze the origin of the inward decrease of the accretion rate. We find that it is because of the mass loss in the outflow, and the production of outflow is because of the gradient of the gas pressure, i.e., the buoyant force. When the gravitational force is stronger, the gas pressure plays a relatively minor role. In other words, in the innermost region the accretion timescale is shorter than the timescale required for the formation of outflow due to the strong gravity; thus, few outflows are produced.

If the flat density profile of \( \rho \propto r^{-0.65} \) is because of the inward decrease of the mass accretion rate, we should expect that the power-law index \( \rho \propto r^{-p} \) should be related to \( s(M \propto r^s) \) by \( p = 1.5 - s \). From Equations (6) and (11), we have \( p = 0.65 < 1.5 - s = 0.85 \). The small deviation is mainly because the density profile depends not only on the inflow profile, but on the sum of the inflow and outflow rates, i.e., \( M_{out}(r) - M_{in}(r) \). The radial profile of the outflow rate is steeper than that of the inflow rate,

\[
M_{out}(r) \propto r^{0.87}
\]

throughout the radius, or

\[
\dot{M}_{out}(r) \propto r^{0.8}
\]

for the region beyond \( r = 100r_s \). At large radii, \( r \gtrsim 100r_s \), the profiles of the inflow and outflow rates are almost identical; thus, the relation \( p = 1.5 - s \) is better satisfied, where \( p \sim 0.65 \) and \( s \sim 0.75 - 0.8 \).

The most notable result from Figures 1 and 2 is that the profiles of physical quantities, such as density and mass inflow rate, are well described by a single power-law form, extending from \( 10^9r_s \) to ~\( 2r_s \). This confirms previous simulation results such as SPB99, although in those simulations the dynamical range of the whole simulation domain usually spans only two orders of magnitude and the steady solution is only reached within one order of magnitude because of the effect of the outer boundary condition. This result indicates that although the Bernoulli parameter of most of the mass outflow is negative (SPB99; Yuan & Bu 2010), the outflow does not rejoin the accretion flow and accumulates somewhere within \( 10^4r_s \).

\[\text{Figure 3.} \text{Radial structure of the accretion flow (two zones) of model B. All quantities have been averaged over time and the polar angle between } \theta = 84^{\circ} \text{ and } \theta = 96^{\circ}. \text{ The dashed line in the plot of rotational velocity } v_{\phi} \text{ denotes Keplerian rotation at the equator. The density and pressure profiles are slightly steeper than those of model A.}\]
Figure 4. Time-averaged and angle-integrated mass accretion rate of model B. The solid, dashed, and dotted lines are for the mass inflow rate $\dot{M}_{\text{in}}$, outflow rate $\dot{M}_{\text{out}}$, and the net rate $\dot{M}_{\text{acc}}$, respectively.

profiles become flatter:

$$\rho(r) \propto r^{-0.85}, \quad p(r) \propto r^{-1.85},$$

(14)

$$\dot{M}_{\text{in}}(r) \propto r^{0.4}.$$  

(15)

Again, more precisely, the inflow rate is described by

$$\dot{M}_{\text{in}}(r) \propto r^{0.54}$$

(16)

from $\sim 10r_s$ to $10^4r_s$, while it is almost constant within $10r_s$. The outflow rate is much steeper. Beyond $r = 100r_s$ it can be described by

$$\dot{M}_{\text{out}}(r) \propto r^{0.73}.$$  

(17)

Again, we see that the relationship of $p = 1.5 - s$ is reasonably satisfied beyond $10r_s$, as in the case of $\alpha = 0.001$.

We have also conducted simulations with $\alpha = 0.05$. We found that convective outflow becomes significantly weaker. The density profile becomes steeper while the accretion rate profiles become flatter.

3.3. Effect of Changing the Initial Conditions

In models A and B, a rotating torus is adopted as the initial condition for our simulation. We also study the cases of other initial conditions. One is to inject gas from an outer boundary, with the properties of the injected gas, including temperature, rotation, and radial velocity, being determined by the self-similar solution of an ADAF (Narayan & Yi 1994). Another model is one in which the initial condition is expanded from the one-dimensional global solution of ADAFs. We find that the radial profiles of inflow rate and density are very similar (Paper II). However, this will not be the case if the angular momentum of the injected gas in the initial condition is very low. In that case, the profile of the accretion rate will become flatter and the density profile steeper. A full discussion of the effect of the initial and boundary conditions will be presented in a subsequent paper (D. Bu et al. 2012, in preparation).

3.4. The New Scaling Law for Hot Accretion Flow

Narayan & Yi (1995; see also Narayan et al. 1998) presented the radial scaling of many quantities of the hot accretion flow based on the self-similar solution. These solutions are very useful in estimating approximately the properties of the hot accretion flow. However, at that time, the inward decrease of the accretion rate had not been found and taken into account in the solution. For the convenience of future use, here we present the new scaling law. In terms of interpreting observations, the most prominent effect of the outflow is on the profile of density. Therefore, compared with Narayan & Yi (1995), only the scaling of density-related quantities such as the number density, magnetic field strength, pressure, and viscous dissipation rate is changed. The scaling of other quantities is also presented for completeness. In the original scaling of Narayan & Yi (1995), the parameter $\beta = 0.5$ ($\equiv p_{\text{gas}}/(p_{\text{gas}} + p_{\text{mag}})$, with $p_{\text{mag}} = B^2/8\pi$). Here we do not specify a value for $\beta$. The new results are:

$$v \approx -1.1 \times 10^{11} \alpha r^{-1/2} \text{ cm s}^{-1},$$

$$\Omega \approx 2.9 \times 10^4 m^{-1} r^{-3/2} \text{ s}^{-1},$$

$$c_s^2 \approx 1.4 \times 10^{20} r^{-1} \text{ cm}^2 \text{ s}^{-2},$$

$$n_e \approx 6.3 \times 10^{10} \alpha^{-1} m^{-1} \dot{m}_\text{out} r^{-3/2} \left( \frac{r}{r_{\text{out}}} \right)^{-2} \text{ cm}^{-3},$$

$$B \approx 6.5 \times 10^8 (1 - \beta)^{1/2} \alpha^{-1/2} m^{-1/2} \dot{m}^{1/2} \dot{m}_\text{out}^{-3/4} r^{-1/2} \text{ G},$$

$$p \approx 1.7 \times 10^6 \alpha^{-1} m^{-1/2} \dot{m}_\text{out}^{-3/2} r^{-1/2} \text{ g cm}^{-1} \text{ s}^{-2},$$

$$q^+ \approx 5.0 \times 10^{33} m^{-2} \dot{m}_\text{out} r^{-5/2} p^{-3/2} \text{ erg cm}^{-1} \text{ s}^{-2},$$

$$\tau_{\text{es}} \approx 24\alpha^{-1} \dot{m}_\text{out}^{-1} p^{-3/2} r^{-3/2}.$$

As in Narayan & Yi (1995), all quantities are written in scaled units, $M = m \dot{M}_{\odot}$, $r$ is the radius in units of $r_s$, $\dot{M} = \dot{m} \dot{M}_{\text{Edd}}$ ($\dot{M}_{\text{Edd}} \equiv 10L_{\text{Edd}}/c^2$), and $\dot{m}_\text{out}$ is the mass accretion rate at the outer boundary $r_{\text{out}}$.

4. COMPARISON WITH PREVIOUS WORK AND OBSERVATIONS

4.1. Comparison with Previous Simulations: A Common Radial Density Profile?

SPB99 found that, for the $\alpha$ description (Run K in SPB99) we adopt here, $\rho(r) \propto r^{-0.5}$. Igumenshchev & Abramowicz (2000)
performed two-dimensional HD simulations, considering a larger range in the parameter space spanned by $\alpha$ and the adiabatic index $\gamma$. They found that when $\gamma = 5/3$, for both $\alpha = 0.01$ and 0.03 the density profile is roughly the same, i.e., $\rho(r) \propto r^{-0.5}$. This result is approximately consistent with SPB99. The density profile in these two works is flatter than our result since a pseudo-Newtonian potential was adopted there. McKinney & Gammie (2002) obtained $\rho \propto r^{-0.6}$ and $v_r \propto r^{-2}$. However, this result suffers greatly from the “inner boundary effect,” since a pseudo-Newtonian potential was adopted and the above scaling was measured over $1.3 r_s \lesssim r \lesssim 10 r_s$.

Many MHD numerical simulations of hot accretion flow have been performed since the pioneering HD work of Igumenshchev & Abramowicz (1999, 2000) and SPB99. For two-dimensional simulations, since turbulence and accretion die away because there is no dynamo action to maintain the poloidal magnetic field, the steady state cannot be reached at large radii. This effect, combined with the inner boundary effect, severely restricts the measurement of the radial profiles of dynamical quantities such as density. For a three-dimensional simulation, it is very time-consuming to simulate a large dynamical range. Perhaps for these reasons only a few works have presented radial profiles of density and inflow rate. For example, Stone & Pringle (2001), Hawley et al. (2001), and Hawley & Balbus (2002) performed two- and three-dimensional MHD simulations of accretion flow. They all found that the mass accretion rate decreases with decreasing radius, but the quantitative radial profile was not given.

Machida et al. (2001) performed three-dimensional MHD simulations of hot accretion flows, with a toroidal initial magnetic field configuration. A Newtonian potential was adopted; thus their results are not affected by the “inner boundary effect.” They found that the radial structure of the accretion flow is very similar to that obtained by HD simulations. Two fits were obtained. In the “early stage” of the simulation, they found that the radial profiles of the density and radial velocity can be described by $\rho \propto r^{-0.5}$ and $v_r \propto r^{-1.3}$. At the “late stage” of their simulation, they found somewhat different results, $\rho \propto r^{-0.8}$ and $v_r \propto r^{-1.3}$. While the profile of density is very similar to our result, their profile of radial velocity is much steeper. This may be because the averaging approach adopted in their work is different from ours. They average the quantity over the whole range because the averaging approach adopted in their work is different. This result suffers greatly from the “inner boundary effect,” since a pseudo-Newtonian potential was adopted and the above scaling was measured over $1.3 r_s \lesssim r \lesssim 10 r_s$.

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Overall, almost all current numerical simulations give a similar radial density profile, with $\rho \propto r^{-0.5-1}$. The results depend weakly on the presence or absence of a magnetic field (HD or MHD), viscous parameter $\alpha$, strength of the magnetic field (weak or strong), the initial configuration of the magnetic field (toroidal or poloidal or tangled), and the dimension of the calculation (two or three dimensions). If this is true, an interesting question is then: what causes such a result? To answer this question, one first needs to understand what causes the inward decrease of the mass accretion rate. This question is studied in Paper II. Here we briefly summarize the results. Two scenarios have been proposed to explain this result. One is the convection-dominated accretion flow (CDAF) model (Narayan et al. 2000; Quataert & Gruzinov 2000a). This model proposes that both HD and MHD accretion flows are convectively unstable. In this case, the inward decrease of the accretion rate occurs because the fluid circulates in the convective eddies. In contrast to the CDAF model, the adiabatic inflow–outflow solution (ADIOS) model (Blandford & Begelman 1999, 2004; Begelman 2012) suggests that the inward decrease of the accretion rate occurs because of mass loss in the outflow. Our analysis in Paper II favors the latter. Moreover, we find that outflow is driven by the buoyant force and the magnetic centrifugal force in HD and MHD accretion flows, respectively.

Then, an interesting question is: why do different physical mechanisms cause roughly the same profiles of accretion rate and density? Begelman (2012) gives an answer to this question. In the early version of the ADIOS model (Blandford & Begelman 1999, 2004; Begelman 2012) suggests that the inward decrease of the accretion rate occurs because of mass loss in the outflow. Our analysis in Paper II favors the latter. Moreover, we find that outflow is driven by the buoyant force and the magnetic centrifugal force in HD and MHD accretion flows, respectively.

4.2. Comparisons with Observations

4.2.1. Sgr A*

Because of its proximity, the supermassive black hole located at the center of our Galaxy is regarded as the unique laboratory for the study of black hole accretion (see Yuan 2011 for a review). Abundant data have been obtained, placing the most strict constraints on the theory of black hole accretion. One of these is the constraint on the mass accretion rate at both the Bondi radius and the region close to the black hole horizon. The high spatial resolution of Chandra can well resolve the Bondi radius and infer the gas density and temperature there (Baganoff et al. 2003). The values of the Bondi radius and the Bondi accretion rate are then well determined, $r_{\text{Bondi}} \approx 10^5 r_s$ and $M_{\text{Bondi}} \approx 10^{-3} M_\odot$ yr$^{-1}$, respectively. Although the value of mass accretion rate is obtained by the simple analytical Bondi theory, it was found to be in good agreement with the detailed three-dimensional numerical simulations focusing on the fueling of the supermassive black hole in Sgr A* by Cuadra et al. (2006). In fact, the numerical simulations obtained in this work give $M \approx 3 \times 10^{-6} M_\odot$ yr$^{-1}$, only a factor of three lower than $M_{\text{Bondi}}$. This is likely because of the inclusion of angular momentum of the accretion flow in the simulation.

The accretion rate at the innermost region of the accretion flow, on the other hand, is strongly constrained by radio polarization observations (e.g., Quataert & Gruzinov 2000b; Macquart et al. 2006; Marrone et al. 2007). The high level of linear polarization detected at frequencies higher than ~150 GHz sets an upper limit to the rotation measure, which then requires that the accretion rate at the innermost region of the accretion flow must be in the range between $2 \times 10^{-7}$ and $2 \times 10^{-6} M_\odot$ yr$^{-1}$, depending on the assumed configuration of the magnetic field. This implies that more than 99% of the gas captured at the Bondi radius must be lost and does not fall onto the black hole. Correspondingly, the density profile of the accretion flow is significantly flatter than the prediction of $\rho \propto r^{-3/2}$. The detailed modeling of Sgr A* presented in Yuan et al. (2003) has shown that the radial profiles of accretion rate and density are described by $M \propto r^{-0.3}$ and $\rho \propto r^{-1}$.

This density profile is somewhat steeper than that obtained in our simulation. This is likely because the specific angular momentum of the accretion flow at the Bondi radius is significantly sub-Keplerian, ~0.3 times Keplerian, as the numerical simulation has indicated (Cuadra et al. 2006). In this case, our study (D. Bu et al. 2012, in preparation) indicates that the outflow becomes significantly weaker and the density profile correspondingly steeper.

4.2.2. NGC 3115

NGC 3115 is a low-luminosity AGN. The mass of the black hole in its center is $M = (1-2) \times 10^9 M_\odot$. The source is very dim. Chandra observations provide only an upper limit of $10^{38}$ erg s$^{-1} \sim 10^{-9} L_{\text{Edd}}$ (Wong et al. 2011 and references therein). Therefore, the accretion mode in this source must be an ADAF. At a distance of 9.7 Mpc, NGC 3115 is the nearest $>10^9 M_\odot$ black hole. Therefore, the angular size of the accretion flow in this source is very large, which is very helpful in detecting the accretion flow directly. Wong et al. (2011) recently conducted Chandra observations of this source and determined that the Bondi radius is about $4''-5''$. The most prominent result is that, for the first time, they have resolved the accretion flow within the Bondi radius. They found that the temperature is rising toward the black hole, as expected in all accretion flow...
models. However, more importantly, the radial density profile of the accretion flow within 4'' is found to be well described by a power-law form:

$$\rho(r) \propto r^{-1.03^{+0.23}_{-0.11}}.$$  \hspace{1cm} (20)

This result is similar to Sgr A* and is in reasonable agreement with our numerical simulations.

5. SUMMARY

One of the most important findings of both HD and MHD numerical simulations of hot accretion flow in recent years is that the mass accretion rate decreases with decreasing radius. Correspondingly, the density profile becomes flatter than the previous prediction when the accretion rate is constant with radius. One main problem with previous simulations is that, for reasons of technical difficulty, the radial dynamical range has usually been very limited, typically spanning less than two orders of magnitude. The boundary condition effect makes the radial range over which we can reliably measure the profile of physical quantities even smaller. The previous simulation results are therefore somewhat suspect. In this paper, we adopt a “two-zone” approach to simulate an axisymmetric accretion flow, extending the dynamical range to over four orders of magnitude, i.e., from $r_{\text{in}}$ to 40,000$r_{\text{in}}$. We confirm previous results that the profiles of inflow rate and density can be well described by power-law forms. Within $10r_{\text{in}}$, $M_{\text{in}}(r) \sim$ constant. Beyond 10$r_{\text{in}}$, the power-law slopes are a function of the viscous parameter $\alpha$. For $\alpha = 0.001$, they are described by $\rho(r) \propto r^{-0.65}$ and $M_{\text{in}}(r) \propto r^{0.65}$. For $\alpha = 0.01$, the results are $\rho(r) \propto r^{-0.85}$ and $M_{\text{in}}(r) \propto r^{0.54}$.

We also combine all available numerical simulations from the literature which have presented the radial profile of density, both two-dimensional and three-dimensional, HD and MHD. We find that all these simulations give somewhat similar results, $\rho \propto r^{-0.5-1}$, and this is also consistent with our results. The diversity of the power-law index seems to come from the differences of the value of $\alpha$, the gravitational potential of the black hole, the initial condition of the simulation, and the strength of the magnetic field. The rough consistency among various simulations can be explained by the most recent ADIOS model (Begelman 2012). In this work, it was found that after considering both the inflow and outflow zones on an equal footing and with a conserved outward energy flux, the value of $\alpha$ is well constrained to be in a narrow range.

The radial profiles of accretion rate and density obtained by numerical simulations are in good agreement with observations. In the case of Sgr A*, detailed modeling of the multiband spectrum gives $\dot{M}(r) \propto r^{\alpha}$ and $\rho \propto r^{-1}$. In the case of NGC 3115, for the first time, Chandra observations resolve the accretion flow within the Bondi radius and find $\rho \propto r^{-1.03^{+0.23}_{-0.11}}$.

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