Quantum preferred frame: Does it really exist?

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Abstract – The idea of the preferred frame as a remedy for difficulties of the relativistic quantum mechanics in description of the non-local quantum phenomena was undertaken by physicists such as J. S. Bell and D. Bohm. The possibility of the existence of a preferred frame was also seriously treated by P. A. M. Dirac. In this paper, we propose an Einstein-Podolsky-Rosen–type experiment for testing the possible existence of quantum preferred frame. Our analysis suggests that to verify whether a preferred frame of reference in the quantum world exists, it is enough to perform an EPR-type experiment with a pair of observers staying in the same inertial frame and with use of the massive EPR pair of spin–one-half or spin-one particles.

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Introduction. – Quantum mechanics (QM) is not a local realistic theory. It predicts a strange phenomenon, named by Einstein as “spooky action at a distance”. It was confirmed in many experiments [1–4] which are Bohm’s modifications of the Einstein-Podolsky-Rosen (EPR) famous thought experiment [5]. As a consequence of the ingenious paper by Bell [6], we know that the Einstein’s idea to find a local realistic theory describing quantum phenomena [7] was false: the nature of quantum world is non-local. This is related to the property of multiparticle states known as quantum entanglement. Although quantum non-locality is not in conflict with the Galilean physics, it is rather baffling from the special relativity (SR) point of view. According to Shimony [8] quantum mechanics and special relativity might “peacefully coexist” if they attribute physical meaning to the final probabilities only. However, this “peaceful coexistence” relies on the ignoring some weak points of the relativistic QM, like lack of the covariant notion of localization or problem with spin observable [9–12]. It would be advisable for quantum mechanics if it admit the absolute simultaneity notion and consequently a preferred frame of reference [13,14] (which seems to be a natural framework for the non-locality) while preserving the (well-established) Lorentz covariance. In this paper we suggest how to verify whether a preferred frame of reference in the quantum world exists.

It seems that the present technological progress allows experimental investigations of the relativistic aspects of quantum entanglement. However, in description of the relativistic version of the EPR-Bohm experiment it is necessary to overcome very serious interpretational and theoretical difficulties (e.g. problem of the covariant localization as well as the non-uniqueness and non-covariance of the relativistic spin operator). Another issue is that non-local (instantaneous) reduction of quantum entangled state (if it has a physical meaning) intuitively conflicts with relativity of simultaneity for moving observers [15,16]. Hardy’s Gedankenexperiment suggests that every realistic quantum theory should possesses an absolute notion of simultaneity, or equivalently a preferred frame (PF) of reference [16]. It is worth stressing that also Bell was convinced that a consistent formulation of quantum mechanics requires a preferred frame at the fundamental level as the most natural way to incorporate quantum non-locality:

“...The aspects of quantum mechanics demanding non-locality remain in relativistic quantum mechanics. It may well be that a relativistic version of the theory, while Lorentz invariant and local at the observational level, may be necessarily non-local and with a preferred frame (or aether) at the fundamental level...” [14]

(Bell gives the very clear point of view on this question in [17] too.) Similar statements were formulated by Bohm [17,18]. The hypothesis of the privileged frame was also discussed by Dirac [19].

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In this paper, we show that verification whether a preferred frame of reference in the quantum world exists can be done by experiment of Einstein-Podolsky-Rosen type with relativistic massive entangled pair of spin–one-half or spin-one particles. In the discussion of the possible experiments we used the recently found relativistic EPR correlation functions [11,20]. Our analysis suggests that it is enough to perform the experiment by a pair of observers staying in the same inertial frame. Moreover, even in the case of the negative answer, this experiment can give us hint about the proper choice of relativistic spin operator.

**EPR correlations in preferred-frame quantum mechanics.** Motivated by the above ideas, in the papers [21,22] Lorentz-covariant quantum mechanics with a preferred frame built in was formulated. The crucial point in this construction is some freedom in formulation of SR, known as the conventionality of distant clocks synchronization [23–27]. It is related to the fact that only round-trip velocity of light is testable without prior synchronization of distant clocks. In that scheme relativity principle may be broken without destroying Lorentz covariance. Quantum theory formulated in different synchronization schemes is unirary non-equivalent to the standard relativistic quantum mechanics and leads to different physical predictions for non-local phenomena, while it gives the same results for the local ones. As was shown in [22], in the Lorentz-covariant preferred-frame quantum mechanics (shortly: preferred-frame quantum mechanics, PFQM) it is possible to calculate the EPR correlation function Lorentz-covariantly and uniquely. For the spin-1/2 particles in the singlet state the correlation function reads

\[
C^{1/2}_{\text{c.m.}}(\mathbf{a}, \mathbf{b}) = \frac{1}{\sqrt{1-v_A^2 + (\mathbf{a} \cdot \mathbf{v}_A)^2}} \frac{1}{\sqrt{1-v_B^2 + (\mathbf{b} \cdot \mathbf{v}_B)^2}} \left\{ - \mathbf{a} \cdot \mathbf{b} \sqrt{1-v_A^2} \sqrt{1-v_B^2} + (\mathbf{a} \cdot \mathbf{v}_A)(\mathbf{b} \cdot \mathbf{v}_B) \right. \\
- \left. \left[ \mathbf{a} \cdot (\mathbf{v}_A \sqrt{1-v_B^2} + \mathbf{v}_B \sqrt{1-v_A^2}) \right] \left[ \mathbf{b} \cdot (\mathbf{v}_A \sqrt{1-v_B^2} + \mathbf{v}_B \sqrt{1-v_A^2}) \right] \right\}. 
\]  

(5)

The PF is identified with the cosmic background radiation frame then for the observers on the Earth |\(u_{A,B}\)| \(\approx 0.001\) and the correction to the non-relativistic formula is of the order 10⁻⁶. It is important to stress that in the case when Alice and Bob are at rest in the same inertial frame (\(u_A = u_B\), \(\Lambda = I\)), this correlation function takes exactly non-relativistic form irrespectively of the PF speed [22], i.e. regardless of the choice of this inertial frame:

\[
C(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}. 
\]

(3)

The same holds for correlations of spin-one particles in a singlet state; the correlation function in this case is given again by the eq. (3) (with the additional factor 2/3 in the front). Thus for observers in the same inertial frame PFQM predictions in these cases are identical as in the non-relativistic QM.

**Relativistic EPR correlations for spin–one-half particles.** Completely different situation takes place in the standard relativistic quantum mechanics. Firstly, there is a problem with the choice of the spin operator. Indeed, spin is defined as a difference of the total angular momentum (generator of rotations) and the orbital angular momentum \(Q \times P\) where the position operator \(Q\) is non-covariant and non-unique. If it is taken as the famous Newton-Wigner position operator [9,28] then, for two EPR spin–one-half particles with velocities \(v_A\) and \(v_B\) (so being in sharp momentum states) and mass \(m\), the correlation function in the Lorentz singlet state, for observers staying in the same inertial frame, can be derived from the formula obtained in [29,30] and reads

\[
C^{1/2}_{\text{FW}}(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b} + \frac{(\mathbf{v}_A \times \mathbf{v}_B)}{1 - \mathbf{v}_A \cdot \mathbf{v}_B + \sqrt{(1-\mathbf{v}_A^2)(1-\mathbf{v}_B^2)}} \left[ \mathbf{a} \times \mathbf{b} + \frac{\mathbf{a} \cdot \mathbf{v}_A (\mathbf{b} \times \mathbf{v}_B) - \mathbf{b} \cdot \mathbf{v}_B (\mathbf{a} \times \mathbf{v}_A)}{(1 + \sqrt{1-\mathbf{v}_A^2})(1 + \sqrt{1-\mathbf{v}_B^2})} \right]. 
\]  

(4)

Notice that in the center-of-mass frame of EPR particles (i.e. for \(v_A = -v_B\)) the correlation function takes its non-relativistic form (3).

In the case when we choose the so called center-of-mass (c.m.) position operator [9,31–33] we obtain an alternative spin operator used by Czachor [34]. The corresponding correlation function \(C_{\text{c.m.}}^{1/2}(\mathbf{a}, \mathbf{b})\) can be derived from [11] see eq. (5) above.
Indeed, from the paper [20] it can be deduced that in this case the relativistic-correlation functions are of the form

\[ C^{1/2}_{NW}(\mathbf{a}, \mathbf{b}) = \frac{2(1 - v^2)}{3 - 2v^2 + 3v^4} \times \left[ -\mathbf{a} \cdot \mathbf{b}(1 + v^2) + 2\mathbf{a} \cdot \mathbf{v}(\mathbf{b} \cdot \mathbf{v}) \right], \]

where \( v \) is the average particle velocity given in the units \( c \) and \( \mathbf{a} \), \( \mathbf{b} \) are the spin–one-half EPR particles. Moreover, there are also significant differences between relativistic and PF correlations as a function of \( v \) (in the units \( c \)) for selected values of the angle \( \theta \) is shown in fig. 2.

Now, let us consider the Clauser-Horne-Shimony-Holt (CHSH) inequality

\[ \text{CHSH} = |C(\mathbf{a}, \mathbf{b}) - C(\mathbf{a}, \mathbf{d}) + C(\mathbf{c}, \mathbf{b}) + C(\mathbf{c}, \mathbf{d})| \leq 2, \]

where Alice measures spin projections along the directions \( \mathbf{a} \) and \( \mathbf{c} \) while Bob along the directions \( \mathbf{b} \) and \( \mathbf{d} \). This inequality is maximally violated by preferred-frame quantum correlations for, e.g., \( \mathbf{a} = (-1/\sqrt{2}, 1/\sqrt{2}, 0) \), \( \mathbf{b} = (0, 1, 0) \) and \( \mathbf{d} = (1, 0, 0) \), i.e. when \( \text{CHSH} = 2\sqrt{2} \). In fig. 3 we have shown both PF and relativistic CHSH as functions of the particle velocity.

Summarizing, in all considered situations with spin–one-half EPR particles we observe for ultra-relativistic velocities significant differences between predictions of relativistic and preferred-frame quantum mechanics. Moreover, there are also significant differences between correlation functions \( C^{1/2}_{NW} \) and \( C^{1/2}_{c.m.} \) (see fig. 2(b) and fig. 3).

Relativistic EPR correlations for spin–one particles. – In the case of the spin-one particles, the relativistic EPR correlations differ from the PFQM ones also in the center-of-mass frame of reference, i.e. for \( \mathbf{v}_A = -\mathbf{v}_B \equiv \mathbf{v} \). Indeed, from the paper [20] it can be deduced that in this case the relativistic-correlation functions are of the form

\[ C^{1}_{c.m.}(\mathbf{a}, \mathbf{b}) = \frac{2(1 - v^2)^2}{3 - 2v^2 + 3v^4} \times \frac{-\mathbf{a} \cdot \mathbf{b}(1 + v^2) + (\mathbf{a} \cdot \mathbf{v})(\mathbf{b} \cdot \mathbf{v})}{\sqrt{(1 - v^2 + (\mathbf{a} \cdot \mathbf{v})^2)(1 - v^2 + (\mathbf{b} \cdot \mathbf{v})^2)}. \]
We observe a maximal deviation from the PF correlations in the coplanar configuration with \( \mathbf{a} \cdot \mathbf{v} = 1 \) (or \(-1\)), \( a \cdot v / |v| = b \cdot v / |v| = \cos \omega \) (see fig. 4).

In fig. 5 we have shown the dependence of \( \Delta C^{1}_{SW} \) on velocity for selected values of the angle \( \omega \), where the difference \( \Delta C^{1}_{SW,c.m.} = C^{1}_{NW,c.m.}(\mathbf{a}, \mathbf{b}) - (-2a \cdot \mathbf{b} / 3). \)

We can also observe a discrepancy between predictions of the relativistic and preferred-frame quantum mechanics in the case of Bell-type inequalities given in Mermin’s paper [35] for particles with spin one in a singlet state. According to this paper, in every local realistic theory the following inequality must be satisfied:

\[
\text{Bell-Mermin} = C^1(\mathbf{a}, \mathbf{b}) + C^1(\mathbf{b}, \mathbf{c}) + C^1(\mathbf{c}, \mathbf{a}) \leq 1. \quad (9)
\]

One can show that in the PFQM case as well as for non-relativistic QM this inequality holds for each configuration of \( \mathbf{a}, \mathbf{b} \) and \( \mathbf{c} \). However, relativistic-correlation functions (7), (8) can violate Bell-Mermin inequality. Indeed, in fig. 6 we observe that the standard relativistic QM breaks this inequality for a particular configuration, while PFQM does not.

**Summary.** From the above discussion it is clear that to test the hypothesis of existence of the quantum preferred frame it is enough to perform an EPR-type experiment with entangled relativistic massive particles by observers staying in the same inertial frame. If they observe no deviation from the non-relativistic EPR correlations and/or Bell-type inequalities, then there is a serious indication that the quantum preferred frame does exist. However, even in the opposite case, the experiment can be helpful in deciding which choice of relativistic spin operator is correct. To the best of our knowledge no such experiment has been yet performed — it must involve ultra-relativistic massive spin–one-half or spin-one particles. This excludes any experiment with photons as well as with kaons (or \( \bar{B} \) mesons) or non-relativistic heavy ions and/or atoms. A serious advantage of using massive fermions to test Bell-type inequalities lies in the fact that these particles, contrary to the photons, are well localized (its coherence length is of the order of \( 10^{-15} \) m, while for photons \( \sim 1 \) m). Moreover, the singlet state of the EPR pair is well defined by measuring the internal energy of the system.

As far as we know, up to date there have been only three experiments testing Bell-type inequalities by means of massive relativistic spin–one-half particles: the Lamelh-Rachti–Mittig (LRM) experiment [36] performed about thirty years ago in CEN-Saclay and two recent experiments: the first one given at the Kernfysisch Versneller Instituut (KVI, Holland) by Hamieh et al. [37] and the second one performed by the Sakai et al. [38] in RIKEN Accelerator Research Facility (Japan). In all three experiments the proton-proton spin correlations was measured. The LRM team tested Bell’s inequalities with the use of the low-energy (13.5 MeV) proton beam which corresponds to the proton velocity \( v \sim 0.17c \). On the other hand, in the KVI experiment, the spin correlations of proton pairs in a \( ^1S_0 \) intermediate state, obtained from \( ^{12}C(d,^2\text{He})^{12}\text{B} \) nuclear charge-exchange reaction, were measured for protons with the kinetic energy \( \sim 86 \text{ MeV} \ (v \sim 0.4c) \). Finally, in the RIKEN experiment the proton pair was created in the \( ^1\text{H}(d,^{2}\text{He})n \) charge-exchange reaction with the proton energy \( \sim 135 \text{ MeV} \ (v \sim 0.5c) \). All these experiments were in agreement with the
non-relativistic quantum mechanics predictions. However, the particles were too slow to give a significant difference between predictions of relativistic QM and PFQM. From our estimation, it is clear that in a conclusive experiment the proton energy should be larger than 800 MeV ($v \sim 0.85c$). In general, correlation experiments with EPR particles with kinetic energies of the order of their masses are waiting to settle the posed question.

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