DYNAMICAL STUDY OF THE PENTAQUARK ANTIDECUPLET IN A CONSTITUENT QUARK MODEL *

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Dynamical calculations are performed for all members of the flavor antidecuplet to which the pentaquark \( \Theta^+ \) belongs. The framework is a constituent quark model where the short-range interaction has a flavor-spin structure. From symmetry considerations the lowest state acquires a positive parity. By fitting the mass of \( \Theta^+ \) of minimal content \( uudd\bar{s} \), the mass of \( \Xi^{--} \), of minimal content \( ddss\bar{u} \), is predicted to be approximately 1960 MeV. It is shown that the octet and antidecuplet states with the same quantum numbers mix ideally due to SU(3)\( _F \) breaking.

1. Introduction

At present there is a large variety of approaches to pentaquarks: chiral soliton or Skyrme models, constituent quark models, instanton models, QCD sum rules, lattice calculations, etc. Here I shall discuss the pentaquarks in the framework of constituent quark models. These models describe a large number of observables in ordinary hadron spectroscopy as e. g. spectra, static properties, decays, form factors, etc. Therefore it seems interesting to look for their predictions for exotics. I shall refer to two standard constituent quark models: the color-spin (CS) model where the hyperfine interaction is of one-gluon exchange type and the flavor-spin (FS) where the hyperfine interaction is due to meson exchange. There are also hybrid models where the hyperfine interaction is a superposition of CS and FS interactions.

Presently the main issues of any approach to pentaquarks are:

1. The spin and parity

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(2) The mass of $\Theta^+$ (or $\Theta^+_0$ for heavy pentaquarks)
(3) The splitting between isomultiplets of a given SU(3)$_F$ representation
(4) The mixing of representations due to SU(3)$_F$ breaking
(5) The width
(6) The production mechanism

Here I shall present results for light and heavy pentaquarks obtained in the
FS model. I shall cover all but the last two items.

2. Parity and spin

The antidecuplet to which $\Theta^+$ belongs$^1$ can be obtained from the direct
product of two flavor octets, one representing a baryon ($q^3$) and the other
a meson ($q\overline{q}$)

$$8_F \times 8_F = 27_F + 10_F + \overline{10}_F + 2(8)_F + 1_F \quad (1)$$

The antidecuplet $\overline{10}_F$ can mix with $8_F$, for example, because SU(3)$_F$ is not
exact. This mixing will be considered below.

To find the parity of $\Theta^+$ and of its partners one looks first at the $q^4$
subsystem with $I = 0$ and $S = 0$, i. e. with quantum numbers compatible
with the content $uudd\overline{s}$ of $\Theta^+$. This means that the flavor and spin wave
functions have symmetry $[22]_F$ and $[22]_S$ respectively. Their direct product
can generate the state $[4]_{FS}$. If the orbital wave function contains a unit
of orbital excitation, it is described by $[31]_O$. The color part of $q^4$ is $[211]_C$
in order to give rise to a color singlet state after the coupling to $\overline{q}$. Then
$[31]_O \times [211]_C \rightarrow [1111]_{OC}$ so that the Pauli principle requires $[4]_{FS}$. In
the FS model the contribution of the hyperfine attraction of $[4]_{FS}$ is so
strong that it fully overcomes the excess of kinetic energy due to the orbital
excitation and the lowest state has positive parity irrespective of the flavor
content of the pentaquark$^2$. The spin is either 1/2 or 3/2.

In the CS model a positive parity as lowest state is also possible in
principle$^3$. The orbital and flavor symmetries give $[31]_O \times [22]_F \rightarrow [211]_{OF}$
and, due to Pauli principle, this must combine with $[31]_{CS}$, the lowest
allowed symmetry. Then only if the excess of kinetic energy is compensated
by the attractive hyperfine interaction the parity would be positive. This
does occur in realistic calculations with CS interaction, as implied by Refs.$^4,5$,
so that the lowest state has negative parity. More generally, parity
remains a controversial issue also in QCD lattice calculations$^6,7$. QCD
sum rules lead to negative parity$^8$. 

3. The orbital wave function

There are four internal Jacobi coordinates \( \vec{x} = \vec{r}_1 - \vec{r}_2, \vec{y} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{3}, \vec{z} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4)/\sqrt{6} \), and \( \vec{t} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5)/\sqrt{10} \) where 5 denotes the antiquark. The total wave function is a linear combination of three independent orbital basis vectors contributing with equal weight:

\[
\psi_1 = \psi_0 z Y_{10}(\hat{z}), \quad \psi_2 = \psi_0 y Y_{10}(\hat{y}) \quad \text{and} \quad \psi_3 = \psi_0 x Y_{10}(\hat{x})
\]

where \( \psi_0 = \left[ \frac{1}{48\pi^5\alpha\beta^2} \right]^{1/2} \exp \left[ -\frac{1}{4\alpha^2} (x^2 + y^2 + z^2) - \frac{1}{4\beta^2} t^2 \right]. \) (2)

contains two variational parameters: \( \alpha \), the same for all \( q^4 \) coordinates \( \vec{x}, \vec{y}, \) and \( \vec{z} \), and \( \beta \), related to \( \vec{t} \), the relative coordinate of \( q^4 \) to \( q^1 \).

4. The antidecuplet

The pentaquark masses are calculated by using the realistic Hamiltonian of Ref. 9, which leads to a good description of low-energy non-strange and strange baryon spectra. It contains an internal kinetic energy term \( T \), a linear confinement potential \( V_c \) and a short-range flavor-spin hyperfine interaction \( V_\chi \) with an explicit radial form for the pseudoscalar meson exchange. Details of these calculations are given in Ref. 10. The expectation

| \( q^4 \) | \( I, I_3 \) | \( V_\chi \) |
|-------|---------|-------|
| uudd  | 0, 0    | \( 30 V_\pi - 2 V_{uu} - 4 V_{uu}^{\eta} \) |
| uuuds | 1/2, 1/2| \( 15V_\pi - V_{uu}^{\eta} - 2 V_{uu}^{\eta'} + 12 V_{K} + 2 V_{us}^{uu} - 2 V_{us}^{uu}^{\eta} \) |
| ddss  | 1, -1   | \( V_\pi + \frac{1}{3} V_{uu}^{\eta} + \frac{2}{3} V_{uu}^{\eta'} + \frac{4}{3} V_{us}^{uu} + \frac{2}{3} V_{us}^{uu}^{\eta} + 20 V_{K} \) |
|       |         | \( -\frac{16}{3} V_{us}^{uu} - \frac{16}{3} V_{us}^{uu}^{\eta} \) |

values of the hyperfine interaction \( V_\chi \) integrated in the flavor-spin space, for some \( q^4 \) subsystems (for notation see text).

The SU(3)_F is explicitly broken by the quark masses and by the meson masses. By taking \( V_{uu}^{uu} = V_{uu}^{us} = V_{uu}^{ss} \) and \( V_{uu}^{uu} = V_{uu}^{uu} = 0 \), one recovers the simpler model of Ref. 12 where one does not distinguish between the uu, us or ss pairs in the \( \eta \)-meson exchange.
Moreover, in Ref. 12, for every exchanged meson, the radial two-body matrix elements are equal, irrespective of the angular momentum of the state, $\ell = 0$ or $\ell = 1$. This is because one takes as parameters the already integrated two-body matrix elements of some radial part of the hyperfine interaction, fitted to ground state baryons. Here one explicitly introduces radial excitations at the quark level. Table 2 contains the partial contributions

| $q^4q$ | $\sum_{n=1}^{5} m_i$ | $\langle T \rangle$ | $\langle V_c \rangle$ | $\langle V_\chi \rangle$ | $E$ | $M$ | $\alpha(fm)$ | $\beta(fm)$ |
|--------|-----------------|-----------------|-----------------|-----------------|-----|-----|-------------|-------------|
| uudd$\bar{\tau}$ | 1700 | 1864 | 442 | -2044 | 1962 | 1452 | 0.42 | 0.92 |
| uudd$\bar{\pi}$ | 1800 | 1848 | 461 | -2059 | 2050 | 1540 | 0.42 | 1.01 |
| uuds$\bar{\tau}$ | 1800 | 1535 | 461 | -1563 | 2233 | 1732 | 0.45 | 0.92 |
| uuds$\bar{\pi}$ | 1900 | 1634 | 440 | -1663 | 2310 | 1800 | 0.44 | 0.87 |
| ddss$\bar{\pi}$ | 1900 | 1418 | 464 | -1310 | 2472 | 1962 | 0.46 | 0.92 |
| uuuss$\bar{\pi}$ | 2000 | 1410 | 452 | -1310 | 2552 | 2042 | 0.46 | 0.87 |

Table 3. The antidecuplet mass spectrum (MeV) for $P = +1$.

| Pentaquark | $Y$, $I$, $I_3$ | Present results | Carlson et al. |
|------------|----------------|----------------|---------------|
| $\Theta^+$ | 2, 0, 0 | 1540 | 1540 |
| $N_{10}^{-}$ | 1, 1/2, 1/2 | 1684 | 1665 |
| $\Sigma_{10}^{-}$ | 0, 1, 1 | 1829 | 1786 |
| $\Xi^{-}$ | -1, 3/2, -3/2 | 1962 | 1906 |

and the variational solution $E$ of the Hamiltonian $^9$ resulting from the trial wave function introduced in Sec. 3. All specified $q^4q$ systems are needed to construct the antidecuplet and the octet. One can see that, except for the confinement contribution $\langle V_c \rangle$, all the other terms break SU(3)$_F$: the mass term $\sum_{n=1}^{5} m_i$ increases, the kinetic energy $\langle T \rangle$ decreases and the short range attraction $\langle V_\chi \rangle$ decreases with the quark masses. For reasons explained in Refs. $^{10,11}$ 510 MeV are subtracted from the total energy $E$ in order to reproduce the experimental $\Theta^+$ mass.
For completeness, in the last two columns of Table 2 the values of the variational parameters $\alpha$ and $\beta$ of the radial wave function (Sec. 3) are indicated. The parameter $\alpha$ takes values around $\alpha_0 = 0.44$ fm. This is precisely the value which minimizes the ground state nucleon mass when the trial wave function is $\phi \propto \exp[-(x^2 + y^2)/4\alpha_0^2]$ where $\vec{x}$ and $\vec{y}$ are the Jacobi coordinates of Sec. 3. The quantity $\alpha_0$ gives a measure of the quark core size of the nucleon because it is its root-mean-square radius. The parameter $\beta$ is related to the coordinate $\vec{t}$ of the center of mass of $q^4$ relative to $\bar{q}$. It takes values about twice larger than $\alpha$, which implies that the four quarks cluster together, whereas $\bar{q}$ remains separate in contrast to the diquark Ansatz. Table 3 reproduces the antidecuplet mass spectrum obtained from the masses $M$ of Table 2. The masses of $\Theta^+$ and $\Xi^{--}$ can be read off Table 2 directly. The other masses are obtained from the linear combinations

\begin{align}
M(N_{10}) &= \frac{1}{3}M(uds\bar{d}) + \frac{2}{3}M(udd\bar{s}), \\
M(\Sigma_{10}) &= \frac{2}{3}M(uds\bar{d}) + \frac{1}{3}M(uss\bar{s}).
\end{align}

In comparison with Carlson et al. 12, where the mass of $\Theta^+$ is also adjusted to 1540 MeV, here the masses of $N_{10}$, $\Sigma_{10}$, and $\Xi^{--}$ are higher. In the lowest order of SU(3)$_F$ breaking, one can parametrize the present result by the Gell-Mann-Okubo (GMO) mass formula, $M = M_{10} + cY$. This gives $M \simeq 1829 - 145 Y$. The nearly equal spacing between isomultiplets is illustrated in Fig. 1 a).

5. Representation mixing

The present model contains SU(3)$_F$ breaking so that representation mixing appears naturally and it can be derived dynamically. Recall that Table 3, column 3 gives the pure antidecuplet masses. The pure octet masses are easily calculable using Table 2. These are

\begin{align}
M(N_8) &= \frac{2}{3}M(udd\bar{d}) + \frac{1}{3}M(udd\bar{s}) = 1568 \text{ MeV}, \\
M(\Sigma_8) &= \frac{1}{3}M(uds\bar{d}) + \frac{2}{3}M(uss\bar{s}) = 1936 \text{ MeV}.
\end{align}

The octet-antidecuplet mixing matrix element $V$ has two non-vanishing contributions, one coming from the mass term and the other from the ki-
netic energy + hyperfine interaction. Its form is

\[ V = \begin{cases} \frac{2\sqrt{7}}{3}(m_s - m_u) + \frac{\sqrt{2}}{3} [S(uud\bar{s}) - S(uudd\bar{d})] = 166 \text{ MeV} & \text{for } N \\ \frac{2\sqrt{7}}{3}(m_s - m_u) + \frac{\sqrt{2}}{3} [S(uuss\bar{s}) - S(uuds\bar{d})] = 155 \text{ MeV} & \text{for } \Sigma \end{cases} \]  

(5)

where \( S = \langle T \rangle + \langle V_\chi \rangle \). The numerical values on the right hand side of Eq. (5) result from the quark masses \( m_{u,d} = 340 \text{ MeV} \), \( m_s = 440 \text{ MeV} \) and from the values of \( \langle T \rangle \) and \( \langle V_\chi \rangle \) exhibited in Table 2. One can see that the mass-induced breaking term is identical for \( N \) and \( \Sigma \), as expected from simple SU(3) considerations, and it represents more than 1/2 of \( V \).

The masses of the physical states, the “mainly octet” \( N^* \) and the “mainly antidecuplet” \( N_5 \), result from diagonalizing a \( 2 \times 2 \) matrix in each case. Accordingly, the nucleon solutions are

\[ N^* = N_8 \cos \theta_N - N_{10} \sin \theta_N, \]
\[ N_5 = N_8 \sin \theta_N + N_{10} \cos \theta_N, \]  

(6)

Figure 1. Comparison between a) the pure antidecuplet spectrum of Table 3 and b) the “mainly antidecuplet” solutions after the mixing with the octet.
with the mixing angle defined by
\[
\tan 2\theta_N = \frac{2V}{M(N^{10}) - M(N^8)}.
\]
(7)

The masses obtained from this mixing are 1451 MeV and 1801 MeV respectively and the mixing angle is \(\theta_N = 35.34^0\), which means that the “mainly antidecuplet” state \(N_5\) is 67 \% \(N^{10}\) and 33 \% \(N^8\), and the “mainly octet” \(N^*\) state is the other way round. The latter is located in the Roper resonance mass region 1430 - 1470 MeV. However this is a \(q^4\bar{q}\) state, i. e. it is different from the \(q^3\) radially excited state obtained in Ref. 9 at 1493 MeV. A mixing of the \(q^3\) and the \(q^4\bar{q}\) states could possibly be a better description of reality. The “mainly antidecuplet” solution at 1801 MeV is 70 MeV above the higher option of Ref. 13, at 1730 MeV, interpreted as the \(Y = 1\) narrow resonance partner of \(\Theta^+\).

In a similar way one obtains two \(\Sigma\) resonances, the “mainly octet” being at 1719 MeV and the “mainly antidecuplet” at 2046 MeV. The octet-antidecuplet mixing angle is \(\theta_\Sigma = -35.48^0\). The lower state is somewhat above the experimental mass range 1630 - 1690 MeV of the the \(\Sigma(1660)\) resonance. As the higher mass region of \(\Sigma\) is less known experimentally, it would be difficult to make an assignment for the higher state. The pentaquark spectrum resulting from the octet-antidecuplet mixing is illustrated in Fig. 1 b). One can see that the order of the last two levels is reversed with respect to case a).

The mixing angles \(\theta_N\) and \(\theta_\Sigma\) are nearly equal in absolute value, but they have opposite signs. The reason is that \(M(N^{10}) > M(N^8)\) while \(M(\Sigma^{10}) < M(\Sigma^8)\). Interestingly, each is close to the value of the ideal mixing angle \(\theta_N^{id} = 35.26^0\) and \(\theta_\Sigma^{id} = -35.26^0\). This implies that in practice the “mainly antidecuplet” \(N_5\) state carries the whole hidden strangeness and that \(N^*\) has a simple content, for example \(uudd\bar{c}\) when the charge is positive.

6. Heavy pentaquarks

Based on the same constituent quark model 9, positive parity heavy charmed pentaquarks of minimal content \(uudd\bar{c}\) have been proposed 2 long before the first observation 16 of \(\Theta^+(uudd\bar{s})\). Table 4 reproduces the results of Ref. 2 where the masses represent the binding energies \(\Delta E\) (Table II) to which threshold energies \(E_T\), (Table I) have been added. These results are compared with the only lattice calculations which predict positive parity 14. Interestingly the masses are quite similar in the two approaches. In
Table 4. Masses (MeV) of the positive parity antisextet charmed pentaquarks.

| Pentaquark | I  | Content       | FS model Ref. [2] | Lattice Ref. [14] |
|------------|----|---------------|--------------------|-------------------|
| $\Theta^0_c$ | 0  | u u d d c    | 2902               | $2977 \pm 104$    |
| $N^0_c$    | $1/2$ | u u d s c | 3161               | $3180 \pm 69$    |
| $\Xi^0_c$  | 1   | u u s s c   | 3403               | $3650 \pm 95$    |

the FS model 9 the lightest negative parity pentaquark is a few hundreds MeV heavier than $\Theta^0_c$ of Table 4. The experimental search for charmed pentaquarks is contradictory so far 18.

7. Conclusions

In the new light shed by the pentaquark studies, the usual practice of hadron spectroscopy is expected to change. There are hints that the wave functions of some excited states might contain $q^4 q$ components. These components, if obtained quantitatively, would perhaps better explain the widths and mass shifts in the baryon resonances. In particular the mass of the Roper resonance may be further shifted up or down. Also it is important to understand the role of the chiral symmetry breaking on the properties of pentaquarks 15, inasmuch as the predictions of Ref. 1, which motivated this new wave of interest, are essentially based on this concept.

References

1. D. Diakonov, V. Petrov and M. Polyakov, Z. Phys. A359, 305 (1997).
2. Fl. Stancu, Phys. Rev. D58, 111501 (1998).
3. B. Jennings and K. Maltman, Phys. Rev. D69, 094020 (2004).
4. Y. Kanada-Enyo et al., hep-ph/0404144 and these proceedings
5. S. Takeuchi and K. Shimizu, these proceedings.
6. S. Sasaki, hep-lat/0310014 and these proceedings.
7. T. -W. Chiu and T. -H. Hsieh, hep-ph/0403020 and these proceedings.
8. J. Sugiyama, T. Doi and M. Oka, Phys. Lett. B381, 167(2004); M. Oka, hep-ph/0409295 (these proceedings), S. H. Lee et al. hep-ph/0411104 and these proceedings.
9. L. Ya. Glozman, Z. Papp and W. Plessas, Phys. Lett. B381, 311 (1996).
10. Fl. Stancu, Phys. Lett. B595, 269 (2004); ibid. Erratum, B598, 295 (2004).
11. Fl. Stancu and D. O. Riska, Phys. Lett. B575, 242 (2003).
12. C. E. Carlson et al., Phys. Lett. B579 (2004) 52.
13. R. A. Arndt et al., nucl-th/0312126.
14. T. -W. Chiu and T. -H. Hsieh, hep-ph/0404007.
15. A. Hosaka, *Phys. Lett.* **B571**, 55 (2003); hep-ph/0409101.
16. T. Nakano et al., (LEPS Coll.) *Phys. Rev. Lett.* **91**, 012002 (2003).
17. M. Genovese et al., *Phys. Lett.* **425**, 171 (1998).
18. A. Aktas et al. (H1 Coll.), hep-ex/0403017; M. -J. Wang, these proceedings.