Quantum Origin of the Early Universe and the Energy Scale of Inflation

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Abstract

Quantum origin of the early inflationary Universe from the no-boundary and tunnelling quantum states is considered in the one-loop approximation of quantum cosmology. A universal effective action algorithm for the distribution function of chaotic inflationary cosmologies is derived for both of these states. The energy scale of inflation is calculated by finding a sharp probability peak in this distribution function for a tunnelling model driven by the inflaton field with large negative constant $\xi$ of nonminimal interaction. The sub-Planckian parameters of this peak (the mean value of the corresponding Hubble constant $H \simeq 10^{-5} m_P$, its quantum width $\Delta H/H \simeq 10^{-5}$ and the number of inflationary e-foldings $N \simeq 60$) are found to be in good correspondence with the observational status of inflation theory, provided the coupling constants of the theory are constrained by a condition which is likely to be enforced by the (quasi) supersymmetric nature of the sub-Planckian particle physics model.

1. Introduction

It is widely reckognized that one of the most promising pictures of the early universe is a chaotic inflationary scenario \cite{1}. The inflation paradigm is the more so attractive that it allows to avoid the fortune telling of quantum gravity and cosmology because the inflationary epoch is supposed to take place at the energy scale or a characteristic value of the Hubble constant $H = \dot{a}/a \sim 10^{-5} m_P$ much below the Planck one $m_P = G^{1/2}$. The predictions of the inflation theory essentially depend on this energy scale which

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must be chosen to provide a sufficient number of e-foldings $N$ in the expansion law of a scale factor $a(t)$ during the inflationary epoch, $N = \int_{t_{\text{in}}}^{t_{\text{fin}}} dt H = \ln (a_{\text{fin}}/a_{\text{in}})$, and also generate the necessary level of density perturbations capable of the formation of the large scale structure. In the chaotic inflationary model the Hubble constant $H = H(\phi) = \sqrt{8\pi U(\phi)/3m_P^2}$ is effectively generated by the potential $U(\phi)$ of the inflaton scalar field $\phi$ which satisfies the slow-roll approximation \cite{1}, \( \dot{\phi} \simeq -(1/3H)\partial U/\partial \phi \ll H\phi \). The number of e-foldings $N = N(\phi_I)$, the effective Hubble constant $H = H(\phi_I)$ and the generation of the density perturbations, as well as the validity of the slow-roll approximation itself, essentially depend upon one parameter – the initial value of the inflaton field $\phi_I$, and one of the most fundamental observational bounds is the following restriction on this quantity \cite{1}

\[ N(\phi_I) \simeq (4\pi/m_P^2) \int_{\phi_I}^{\phi_0} d\phi H(\phi) \left[ \partial H(\phi)/\partial \phi \right]^{-1} \geq 60. \]  

This quantity, however, is a free parameter in the inflation theory, and, to the best of our knowledge, there are no convincing principles that could fix it without invoking the ideas of quantum gravity and cosmology. These ideas imply that there exists a quantum state of coupled gravitational and matter fields, which in the semiclassical regime generates the family of inflationary universes with different values of $H(\phi)$, approximately evolving at later times according to classical equations of motion. This quantum state allows one to calculate the distribution function $\rho(\phi)$ of these universes and interpret its maximum at certain value of $\phi = \phi_I$ (if any) as generating the quantum scale of inflation. The implementation of this approach, undertaken in the tree-level approximation for the no-boundary \cite{2,3} and tunnelling \cite{4} quantum states of the Universe, was not successful. The corresponding distribution functions turned out to be extremely flat \cite{3,7} for large values of $\phi$ (in the domain of the inflationary slow-roll ansatz) and unnormalizable at $\phi \to \infty$, which totally breaks the validity of the semiclassical expansion underlying the inflation theory, since the contribution of the over-Planckian energy scales is not suppressed to zero. Apart from this difficulty, the only local maximum of $\rho(\phi)$ found for the no-boundary quantum state was shown to be generating insufficient amount of inflation violating the above bound \cite{9}.

In the series of recent papers \cite{10,11,12,13} we proposed the mechanism of suppressing the over-Planckian energy scales by the contribution of the quantum (loop)
part of the gravitational effective action to the distribution function of the above type. This can justify the use of the semiclassical expansion and serve as a selection criterion of physically viable particle models with the normalizable quantum state, suggesting the supersymmetric extension of field models in the theory of the early universe [14]. In [15] this mechanism has been further applied to show that it can also generate the quantum scale of inflation and, in particular, serve as a quantum gravitational ground for the inflationary model of Bardeen, Bond and Salopek [16] with large negative constant $\xi$ of nonminimal inflaton-graviton coupling. In this paper we discuss the result of [15] with a special emphasis on the universality of the effective action algorithm for the above distribution function in both no-boundary and tunnelling quantum states. We also dwell on the advantages of the tunnelling proposal for the cosmological wavefunction, that arise both at the level of applications in the theory of the early Universe and at the conceptual level of the prospects for the third quantization of gravity.

2. Tree-level approximation: nonminimal inflaton field

Two known proposals for the cosmological quantum state, which semiclassically generate the families of inflationary universes, are represented by the no-boundary [2, 3] and tunnelling [4] wavefunctions. They describe two different types of nucleation of the Lorentzian quasi-DeSitter spacetime from its Euclidean counterpart which in the context of spatially closed cosmology can be represented by the 4-dimensional Euclidean hemisphere matched across the equatorial section to the Lorentzian expanding Universe. The tree-level no-boundary $\rho_{\text{NB}}(\phi)$ and tunnelling $\rho_{\text{T}}(\phi)$ distribution functions of such universes are just the squares of their wavefunctions

$$
\rho_{\text{NB}}(\phi) \sim e^{-I(\phi)}, \quad \rho_{\text{T}}(\phi) \sim e^{+I(\phi)},
$$

(2.1)

where $I(\phi)$ is a doubled Euclidean action of the theory calculated on such a hemisphere (or the action on the full quasi-spherical manifold). When $\phi$ belongs to the domain of the slow-roll approximation and is practically constant in the solution of both Lorentzian and Euclidean equations of motion, the Euclidean spacetime only
slightly differs from the exact 4-dimensional sphere of the radius \(1/H(\phi)\) – the inverse of the Hubble constant – and \(I(\phi)\) takes the form

\[
I(\phi) = -3m_p^4/8U(\phi).
\]

Thus, \(\rho_{NB}(\phi)\) and \(\rho_T(\phi)\) describe opposite outcomes of the most probable underbarrier penetration: respectively to the minimum and to the maximum of the inflaton potential \(U(\phi) \geq 0\) (although, in the former case the minimum \(U(\phi) = 0\) generally falls out of the slow-roll domain).

The equations above apply to the case of an inflaton field minimally coupled to the metric tensor \(G_{\mu\nu}\) with the Lagrangian

\[
L(G_{\mu\nu}, \phi) = G^{1/2} \left\{ \frac{m_p^2}{16\pi} R(G_{\mu\nu}) - \frac{1}{2} (\nabla \phi)^2 - U(\phi) \right\},
\]

but can also be used in the theory of the nonminimal scalar field \(\varphi\)

\[
L(g_{\mu\nu}, \varphi) = g^{1/2} \left\{ \frac{m_p^2}{16\pi} R(g_{\mu\nu}) - \frac{1}{2} \xi \varphi^2 R(g_{\mu\nu}) - \frac{1}{2} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \varphi^4 \right\},
\]

provided \(L(G_{\mu\nu}, \phi)\) above is viewed as the Einstein frame of \(L(g_{\mu\nu}, \varphi)\) with the fields \((G_{\mu\nu}, \phi) = ((1 + 8\pi|\xi|\varphi^2/m_p^2)g_{\mu\nu}, \phi(\varphi))\) related to \((g_{\mu\nu}, \varphi)\) by the transformation that can be found in [16, 19, 20]. For a negative nonminimal coupling constant \(\xi = -|\xi|\) this model easily generates the chaotic inflationary scenario [21] with the Einstein frame potential

\[
U(\phi) \bigg|_{\phi = \phi(\varphi)} = \frac{m^2 \varphi^2/2 + \lambda \varphi^4/4}{(1 + 8\pi|\xi|\varphi^2/m_p^2)^2},
\]

including the case of a symmetry breaking at scale \(v\) with \(m^2 = -\lambda v^2 < 0\) in the Higgs potential \(\lambda(\varphi^2 - v^2)^2/4\). At large \(\varphi\) it approaches a constant and depending on the parameter \(\delta \equiv -8\pi|\xi| m^2/\lambda m_p^2 = 8\pi|\xi|v^2/m_p^2\) has two types of behaviour at the intermediate values of the inflaton field. For \(\delta > -1\) it does not have local maxima and generates the slow-roll decrease of the scalar field leading to a standard scenario with a finite inflationary stage, while for \(\delta < -1\) it has a local maximum at \(\bar{\varphi} = m/\sqrt{\lambda(1 + \delta)}\) and due to a negative slope of the potential leads to the eternal inflation for all models with the scalar field growing from its initial value \(\varphi_I > \bar{\varphi}\).
The tree-level distribution functions (2.1) for such a potential do not suppress the
over-Planckian scales and are unnormalizable at large $\varphi$, $\int_{-\infty}^{\infty} d\varphi \rho_{NB,T}(\varphi) = \infty$, thus
invalidating a semiclassical expansion. Only in the tunnelling case with $\delta < -1$ the
distribution $\rho_{T}(\phi)$ has a local peak at $\bar{\varphi}$, which could have served as a source of the
inflation energy scale at reasonable sub-Planckian value of the Hubble constant. However,
this peak requires the positive mass of the inflaton field $m^2 > \lambda m_P^2/(8\pi|\xi|)$ which
is too large even for reasonable values $\xi = -2 \times 10^4$, $\lambda = 0.05$ \[16\] and formally gen-
erates an infinite duration of the inflationary stage (because the latter starts from the
maximum of the inflaton potential).

3. Beyond the tree-level theory: no-boundary vs
tunnelling wavefunctions

Beyond the tree-level approximation the distribution function for the inflaton field
$\varphi$ should be regarded as a diagonal element of the reduced density matrix of this field
$Tr_f|\Psi><\Psi|$. It can be obtained from the full quantum state $|\Psi> = \Psi(\phi, f | t)$ by tracing out the rest of the degrees of freedom $f$

$$\rho(\phi | t) = \int df \Psi^*(\phi, f | t) \Psi(\phi, f | t), \quad (3.1)$$

which does not reduce to a simple squaring of the wavefunction (we begin this section
by considering again the minimally coupled inflaton field). For the inner product in
(3.1) to be unambiguously defined, the wavefunction $\Psi(\phi, f | t)$ should be taken in the
representation of physical (ADM) variables with the time $t$ fixed by a chosen ADM
reduction procedure \[22\]. Strictly speaking this reduction is not generally (globally on
phase space of the theory) consistent, and a complete understanding and the interpre-
tation of the cosmological wavefunction might be reached only in the framework of the
third quantization of gravity theory. Although this framework still does not have a
status of a well-established physical theory, there exists a good correspondence principle
of this framework with the quantization in reduced phase space for systems with a
wide class of special (positive-frequency) semiclassical quantum states. For these states
the conserved current of the Wheeler-DeWitt equations perturbatively coincides with
the inner product of the ADM quantization mentioned above and thus can be used for
the construction of the probability distribution (for a perturbative equivalence of the ADM and Dirac-Wheeler-DeWitt quantization of gravity for such physical states see 23, 24, 11). As we shall see below, the tunnelling wavefunction belongs to such a class of states, while the no-boundary one does not and should be supplied with additional (third quantization) principles to be interpreted in terms of the probability distribution of the above type.

In the approximation of the Robertson-Walker model, the ADM physical variables describing a spatially homogeneous background and inhomogeneous field modes (treated perturbatively) are respectively the inflaton field $\phi$ and linearized transverse (and traceless) modes $f$ of all possible spins 10, 11, 12, while $t$ can be chosen to be a cosmic time with the unit lapse or a conformal time with the lapse $N = a$. In this approximation the semiclassical solution of the Wheeler-DeWitt equations can be given by the linear superposition of the two (decaying and growing) wavefunctions in the underbarrier (Euclidean) regime $a \leq 1/H$

$$
\Psi_{\pm}(a, \phi, f) = \frac{1}{[a^2(1-H^2a^2)]^{1/4}} e^{\pm I(a, \phi)} \prod_n \frac{1}{\sqrt{u_n}} e^{\pm \frac{i}{2} a^k (u_n/u_n)f_n^2}, \quad (3.2)
$$

$$
I(a, \phi) = -\frac{\pi m_p^2}{2H^2} [1 - (1 - H^2a^2)^{3/2}], \quad H^2 = \frac{8\pi U(\phi)}{3m_p^2}, \quad (3.3)
$$

and outgoing and ingoing wavefunctions in the classically allowed (Lorentzian) regime $a > 1/H$

$$
\Psi_{\pm}^L(a, \phi, f) = \frac{1}{[a^2(H^2a^2 - 1)]^{1/4}} e^{\pm S(a, \phi)} \prod_n \frac{1}{\sqrt{v_n}} e^{\pm \frac{i}{2} a^k (v_n/v_n)f_n^2}, \quad (3.4)
$$

$$
S(a, \phi) = -\frac{\pi m_p^2}{2H^2} (H^2a^2 - 1)^{3/2}. \quad (3.5)
$$

Here $I$ and $S$ are the Euclidean and Lorentzian Hamilton-Jacobi functions of a spatially homogeneous superspace background, the products over $n$ denote the quadratic contribution into these Hamilton-Jacobi functions of the spatially inhomogeneous modes $f_n$ enumerated by the collective index $n$. The functions $u_n$ and $v_n$ are their Euclidean and Lorentzian basis functions respectively in the semiclassical Euclidean and Lorentzian times defined according to the classical equations for the minisuperspace background

$$
\frac{\partial I}{\partial a} = -\frac{3m_p^2 a\dot{a}}{4\pi N}. \quad (3.6)
$$
(a similar equation holds for Lorentzian time with $S$ replacing $I$, and for brevity we denote by the dot the derivatives with respect to both Euclidean $\tau$ and Lorentzian $t$ times). In case of gravitons and minimally interacting massless scalar particles, the functions $v_n$ satisfy the wave equation
\begin{equation}
\ddot{v}_n + \frac{k}{a} \dot{v}_n + \frac{n^2 - 1}{a^{2k-4}} v_n = 0,
\end{equation}
where $n = 1, 2, \ldots$ for a scalar, $n = 3, 4, \ldots$ for a graviton and $k$ corresponds to the choice of lapse $N = a^{3-k}$ ($k = 3$ for cosmic time and $k = 2$ for a conformal one). Euclidean basis functions $u_n$ satisfy a similar equation with the negative sign of the potential term.

It is important that the signs of $I$ and $S$ in (3.2) and (3.4) are correlated with the signs of $f^2$ terms in the exponentials. Therefore, to provide the normalizability of the wavefunctions in the space of $f$,
\begin{equation}
\text{Re} \left( \pm \frac{\dot{u}_n}{u_n} \right) < 0, \quad \text{Re} \left( \pm i \frac{\dot{v}_n}{v_n} \right) < 0,
\end{equation}
for different ($\pm$) branches of the quantum state one should choose different basis functions among the two independent solutions of eq (3.7) and its Euclidean analogue: $v_n^\pm$ and $u_n^\pm$. In the branch of the growing underbarrier wavefunction $\Psi_-$ (which is just the case of a pure no-boundary state of Hartle and Hawking) this uniquely leads to the regularity of $u_n^-$ on the Euclidean section of the spacetime background, while for the decaying wavefunction $\Psi_+$ (the dominant contribution to the tunnelling state) $u_n^+$ is singular at the pole of the Euclidean hemisphere [8].

The wavefunctions (3.2) and (3.4) are the building blocks of the semiclassical tunnelling and no-boundary wavefunctions. The no-boundary wavefunction prescribed by the Euclidean path integral turns out to be the following in the Euclidean and Lorentzian regimes [8]
\begin{equation}
\Psi_{NB} = \Psi_-, \quad a < 1/H,
\end{equation}
\begin{equation}
\Psi_{NB} = e^{\pi m^2/2H^2} \left[ e^{i\pi/4} \Psi_+^L + e^{-i\pi/4} \Psi_-^L \right], \quad a > 1/H,
\end{equation}
while the tunnelling wavefunction $\Psi_T$ in the prescription of the outgoing wave of ref.[7] looks as follows (the overall normalization of $\Psi_T$ is fixed by the requirement that $\Psi_T$ is $\phi$-independent at $a = 0$):
\begin{equation}
\Psi_T = e^{-\pi m^2/2H^2} \Psi_+^L, \quad a > 1/H,
\end{equation}
\[ \Psi_T = \Psi_+ - \frac{i}{2} e^{-\pi m_P^2/H^2} \Psi_- \quad a < 1/H. \] (3.12)

As shown in [7, 8], the requirement of the normalizability of the wavefunction in \( f \) (3.8) (in both branches of the underbarrier wavefunction (3.12)) and the matching conditions at the nucleation point \( a = 1/H \) uniquely singles out \( v_n^+(t) \) in (3.11) to be the negative frequency basis function of the DeSitter invariant vacuum in the Lorentzian expanding Universe. This differs from the proposal in ref. [18] where the vacuum state was specified at \( a = 0 \) instead of \( a > 1/H \) (see also [9]). In the conformal time \( t (k = 2) \), in which the DeSitter Lorentzian background has the form

\[ a = (H \cos t)^{-1}, \quad 0 \leq t < \pi/2, \] (3.13)

this basis function is given by

\[ v_n^+(t) = \frac{(z - 1)^{(n-1)/2}}{(z + 1)^{(n+1)/2}} \left( 1 + \frac{z}{n} \right), \quad z = -i \tan t. \] (3.14)

The Euclidean basis functions \( u_n^\pm(\tau) \) in (3.12) are the analytic continuation

\[ u_n^\pm(\tau) = v_n^\mp(\mp i\tau), \quad 0 \leq \tau < \infty, \] (3.15)

to the imaginary values of the Lorentzian time. The analytic continuation \( t = i\tau \) matches at \( \tau = t = 0 \) the Lorentzian DeSitter Universe with the Euclidean DeSitter hemisphere having the scale factor

\[ a = (H \cosh \tau)^{-1}, \quad 0 \leq \tau < \infty. \] (3.16)

Note that \( u_n^+(\tau) \) is singular at the pole of this hemisphere \( \tau = +\infty \), while \( u_n^-(\tau) \) is regular there. On the contrary, these two functions are correspondingly regular and singular at the point \( \tau = -\infty \) which can be regarded as a pole of the hemisphere \( -\infty < \tau \leq 0 \) complimentary to the above one on the full 4-dimensional sphere – the DeSitter gravitational instanton obtained by glueing the Euclidean hemisphere with its double [10, 12].

Now we can calculate the probability distribution \( \rho (\phi | t) \) for the tunnelling and no-boundary quantum states. This distribution makes sense only in the Lorentzian domain \( a > 1/H \) and in the tunnelling case consists in squaring the single outgoing component (3.12), dropping the first preexponential factor in eq.(3.4) (this corresponds
to calculating the needed conserved current of the Wheeler-DeWitt equation or the reduction to the physical state of the ADM quantization [24] and taking the gaussian integral over \( f \). The result looks as follows

\[
\rho(\phi|t) = \prod_n [\Delta_n]^{-1/2} e^{-\pi m_p^2 / H^2}, \tag{3.17}
\]

\[
\Delta_n = ia^k (v_n^+ v_n^{+*} - v_n^{++*} v_n^+). \tag{3.18}
\]

The preexponential factor here is given by the product of the Wronskians of the Lorentzian basis functions \( v_n^+ \) and \( v_n^{+*} \), which are \( t \)-independent and can be calculated at the nucleation point \( t = 0 \). As it was shown in [10, 11, 12, 13] this product coincides with the product of Wronskians of the pairs of Euclidean basis functions \( u_{n}^{\pm} \) regular on the opposite (complementary) hemispheres of the DeSitter gravitational instanton and, actually, comprises the exponentiated one-loop Euclidean effective action of the full set of fields \( f \), \( \Gamma_{1\text{-loop}}(\phi) = (1/2) \sum_n \ln \Delta_n \) so that the probability distribution takes the form

\[
\rho_T(\phi) \approx \frac{1}{H^2(\phi)} e^{I(\phi) - \Gamma_{1\text{-loop}}(\phi)}. \tag{3.19}
\]

Here \( I(\phi) = -\pi m_p^2 / H^2(\phi) \) is the classical action on the DeSitter instanton, the effective action is chosen to include also the contribution of the quantum inflaton mode (which is compensated by additional multiplier \( 1 / H^2(\phi) \)) and can be represented in the form of the functional trace of the logarithm of the inverse propagator of the full system of fields \( g(x) \) inhabiting the Universe

\[
\Gamma_{1\text{-loop}}(\phi) = \frac{1}{2} \text{Tr} \ln \frac{\delta^2 I[g]}{\delta g(x) \delta g(y)} \bigg|_{\text{DS}}. \tag{3.20}
\]

It is calculated on the quasi-DeSitter gravitational instanton \( \text{DS} \) – the 4-dimensional quasi-sphere of radius \( 1/H(\phi) \) – and, therefore, parametrized by \( \phi \).

In the case of the no-boundary state, the Lorentzian wavefunction consists of the outgoing and ingoing components which describe the expanding and contracting Universes. Therefore, the reduced phase space quantization does not work, as well as the current of the Wheeler-DeWitt equation turns out to be zero in view of the reality of the wavefunction. Therefore, the only hope to interpret this situation is to invoke certain ideas of the third quantization and regard the wavefunction as describing the two
coexisting Universes propagating in opposite directions in the minisuperspace of the scale factor. Then the calculation of the probability distribution of every of such Universes heuristically implies projecting the full wavefunction on one of the components and then repeating the same calculations that obviously lead to the same algorithm in terms of the effective action of the theory on the DeSitter instanton. The fact that the regular and singular Euclidean modes get interchanged as compared to the tunnelling case does not change the result because they both symmetrically enter the final algorithm.

Thus the probability distribution for no-boundary and tunnelling quantum states can be both represented in one equation and applied to the case of the nonminimal inflaton field [10, 11, 12, 15]

\[ \rho_{NB,T}(\phi) \approx \frac{1}{H^2(\phi)} e^{\pm I(\phi) - \Gamma_{1\text{-loop}}(\phi)}, \]  

(3.21)

where \( I(\phi) = I(\phi(\phi)) \) is the Euclidean action rewritten in the frame of the original Lagrangian (2.4), \( H(\phi) \) is a Hubble constant in the same frame related by the equation

\[ H(\phi) = H(\phi) \sqrt{1 + 8\pi|\xi| \varphi^2/m_p^2}, \]  

(3.22)

to the Hubble constant \( H(\phi) \) in the Einstein frame and \( \Gamma_{1\text{-loop}}(\varphi) \) is the effective action in the original field frame (remember that we denote the quantities in this frame by boldface letters to distinguish them from those of the Einstein one). Note that in contrast to the classical action the effective actions calculated in different field frames are numerically different and do not differ only by the redefinition of their field argument (even on shell). This would be the case for the local reparametrizations of fields only up to the renormalization procedure which induces the dimensional cutoff defined relative to a given parametrization of the spacetime metric. In what follows we consider as physical (that is defining the renormalization scale) the metric \( g_{\mu\nu} \) in the original parametrization, which means that \( \Gamma_{\text{loop}}(\varphi) \) should be defined and renormalized in the original field frame. This is of crucial importance for obtaining the correct scaling behaviour of the effective action and the probability distribution.

Indeed, in the high-energy limit of the large inflaton field, including the slow-roll domain and corresponding in the model (2.4) to the Hubble constant \( H(\phi) \approx \sqrt{\lambda/12|\xi|\varphi \to \infty} \), the effective action is calculated and renormalized on the DeSitter
instanton of vanishing physical size $H^{-1}$ and, therefore, is determined asymptotically by the total anomalous scaling $Z$ of the theory on such a manifold

$$\Gamma_{\text{1-loop}} \bigg|_{H \to \infty} \simeq Z \ln \frac{H}{\mu}. \quad (3.23)$$

Here $\mu$ is a renormalization mass parameter or a dimensional cutoff generated by the fundamental and finite string theory, if the model (2.4) is regarded as its sub-Planckian effective limit. This is in sharp contrast to the renormalization in the unphysical metric $G_{\mu\nu}$ that would have led to the above equation with $H(\varphi)$ replaced by $H(\phi) \rightarrow \sqrt{\frac{\lambda}{96\pi^2} m_P}$, featuring absolutely different asymptotic behaviour in $\varphi$. In the one-loop approximation the parameter $Z$ is determined by the total second DeWitt coefficient $Z$ of all quantum fields $g = (\varphi, f)$, integrated over the DeSitter instanton,

$$Z = \frac{1}{16\pi^2} \int_{\Delta S} d^4x \frac{g^{1/2}a_2(x, x)}{(x, x)} \quad (3.24)$$

and, thus, crucially depends on the particle content of a model including as a graviton-inflaton sector the Lagrangian (2.4). This quantity, in particular, determines the one-loop divergences of the field model and the set of corresponding $\beta$-functions.

The use of eqs. (3.21) and (3.23) shows that the quantum probability distribution acquires in contrast to its tree-level approximation (2.1) extra $Z$-dependent factor

$$\rho_{\text{NB,T}}(\varphi) \cong e^{\pm 3m_P^4/8U(\varphi)} \varphi^{-Z-2}, \quad (3.25)$$

which can make the both no-boundary and tunnelling wavefunctions normalizable at over-Planckian scales provided the parameter $Z$ satisfies the inequality

$$Z > -1 \quad (3.26)$$

serving as a selection criterion for consistent particle physics models with a justifiable semiclassical loop expansion [10, 14]. Although this equation is strictly valid only in the limit $\varphi \to \infty$, it can be used for a qualitatively good description at intermediate energy scales. In this domain the distribution (3.25) can generate the inflation probability peak at $\varphi = \varphi_I$ with the dispersion $\sigma$, $\sigma^{-2} = -d^2 \ln \rho(\varphi_I)/d\varphi^2_I$,

$$\varphi^2_I = \frac{2|I_1|}{Z + 2}, \quad \sigma^2 = \frac{|I_1|}{(Z + 2)^2}, \quad I_1 = -24\pi \frac{|\xi|}{\lambda} (1 + \delta) m_P^2, \quad (3.27)$$
where $I_1$ is a second coefficient of expansion of the Euclidean action in inverse powers of $\varphi$, $I(\varphi) = -3m_k^2/8U(\phi(\varphi)) = I_0 + I_1/\varphi^2 + O(1/\varphi^4)$. For the no-boundary and tunnelling states this peak exists in complimentary ranges of the parameter $\delta$. For the no-boundary state it can be realized only for $\delta < -1 (I_1 > 0)$ and, thus, corresponds to the endless inflation with the field $\varphi$ on the negative slope of the inflaton potential (2.5) growing from its starting value $\varphi_I > \bar{\varphi}$. For a tunnelling proposal this peak takes place for $\delta > -1$ and generates the finite duration of the inflationary stage with the number of e-foldings in the original frame $N(\varphi_I) = (\varphi_I/m_P)^2\pi(1/6)/(1 + \delta) = 8\pi^2|\xi|(1+6|\xi|)/\lambda(Z+2)$. In what follows we consider the latter case because it describes the conventional scenario with the matter-dominated stage following the inflation.

4. Nonminimal inflation and particle physics of the early Universe

The status of the inflation theory has recently been strongly confirmed by the observations of the cosmic microwave background radiation anisotropy in the COBE [27] and Relikt [28] satellite experiments. In the chaotic inflationary model with a nonminimal inflaton field (2.4) the spectrum of perturbations compatible with these measurements can be acquired in the range of coupling constants $\lambda/\xi^2 \sim 10^{-10}$ [16, 29] (the experimental bound on the gauge-invariant [31] density perturbation $P_\zeta(k) = N_k^2(\lambda/\xi^2)/8\pi^2$ in the $k$-th mode ”crossing” the horizon at the moment of the e-foldings number $N_k$). The main advantage of this model is that it allows one to avoid an unnaturally small value of $\lambda$ in the minimal inflaton model [1] and replace it with the GUT compatible value $\lambda \simeq 0.05$, provided $\xi \simeq -2 \times 10^4$ is chosen to be related to the ratio of the Planck scale to a typical GUT scale, $|\xi| \sim m_P/v$. For these coupling constants the bound $N(\varphi_I) \geq 60$ on the duration of the inflation, generated by the probability peak (3.27), results in an enormous value of the anomalous scaling $Z \sim 10^{11}$. A remarkable feature of the proposed scheme is that this huge value can be naturally induced by large $\xi$ already in the one-loop approximation. Indeed, the expression for $Z_{1\text{--}\text{loop}}$, well known for a generic theory [24], has a contribution quartic in effective
masses of physical particles easily calculable on a spherical DeSitter background

\[ Z_{1\text{-loop}} = (12H^4)^{-1}(\sum_{\chi} m_{\chi}^4 + 4 \sum_{A} m_{A}^4 - 4 \sum_{\psi} m_{\psi}^4) + ..., \]  

(4.1)

where the summation goes over all Higgs scalars \( \chi \), vector gauge bosons \( A \) and Dirac spinors \( \psi \). Their effective masses for large \( \varphi \) are dominated by the contributions

\[ m_{\chi}^2 = \lambda_{\chi} \varphi^2 / 2, \quad m_{A}^2 = g_{A}^2 \varphi^2, \quad m_{\psi}^2 = f_{\psi}^2 \varphi^2 \]  

(4.2)

induced via the Higgs mechanism from their interaction Lagrangian with the inflaton field

\[ L_{\text{int}} = \sum_{\chi} \frac{\lambda_{\chi}}{4} \chi^2 \varphi^2 + \sum_{A} \frac{1}{2} g_{A}^2 A_{\mu}^2 \varphi^2 + \sum_{\psi} f_{\psi} \varphi \bar{\psi} \psi + \text{derivative coupling}. \]  

(4.3)

Thus, in view of the relation \( \varphi^2 / H^2 = 12|\xi| / \lambda \), we get the leading contribution of large \( |\xi| \) to the total anomalous scaling of the theory

\[ Z_{1\text{-loop}} = 6 \frac{\xi^2}{\lambda} A + O(\xi), \]  

(4.4)

\[ A = \frac{1}{2\lambda} \left( \sum_{\chi} \lambda_{\chi}^2 + 16 \sum_{A} g_{A}^4 - 16 \sum_{\psi} f_{\psi}^4 \right), \]  

(4.5)

which contains the same large dimensionless ratio \( \xi^2 / \lambda \simeq 10^{10} \) and the universal quantity \( A \) determined by a particle physics model (gravitons and inflaton field do not contribute to \( A \), as well as gravitino in case when the latter is decoupled from the inflaton).

For such \( Z_{1\text{-loop}} \) the parameters of the inflationary peak express as

\[ \varphi_{I} = m_P \sqrt{\frac{8\pi(1 + \delta)}{\xi |A|}}, \quad \sigma = \frac{\varphi_{I}}{\sqrt{12A |\xi|}}, \]  

(4.6)

\[ H(\varphi_{I}) = m_P \frac{\sqrt{\lambda}}{\xi} \sqrt{\frac{2\pi(1 + \delta)}{3 A^2}}, \quad N(\varphi_{I}) = \frac{8\pi^2}{A} \]  

(4.7)

and satisfy the bound \( N(\varphi_{I}) \geq 60 \) with a single restriction on \( A, A \leq 1.3 \). This restriction justifies a slow-roll approximation, because the corresponding smallness parameter (in the original frame of the Lagrangian (2.4)) is \( \dot{\varphi} / H \varphi \simeq -A / 96 \pi^2 \sim -10^{-3} \).
For a value of $\delta \ll 1$ ($\delta \sim 8\pi/|\xi|$ for $|\xi| \sim m_P/v$) and $A \simeq 1$, the obtained numerical parameters describe extremely sharp inflationary peak at $\varphi_I$ with small width and sub-Planckian Hubble constant

$$\varphi_I \simeq 0.03m_P, \quad \sigma \simeq 10^{-7}m_P, \quad H(\varphi_I) \simeq 10^{-5}m_P,$$

which is the most realistic range of the inflationary scenario. The smallness of the width does not, however, lead to its quick quantum spreading: the commutator relations for operators $\hat{\varphi}$ and $\hat{\dot{\varphi}}$, $[\hat{\varphi}, \hat{\dot{\varphi}}] \simeq i/(12\pi^2|\xi|a^3)$, give rise at the beginning of the inflation, $a \simeq H^{-1}$, to a negligible dispersion of $\dot{\varphi}$, $\Delta \dot{\varphi} \simeq H^3/12\pi^2|\xi|\sigma \simeq (8/A)(\sqrt{\lambda}/|\xi|)|\dot{\varphi}| \ll |\dot{\varphi}|$. It is remarkable that the relative width

$$\sigma/\varphi_I \sim \Delta H/H \sim 10^{-5}$$

corresponds to the observable level of density perturbations, although it is not clear whether this quantum dispersion $\sigma$ is directly measurable now, because of the stochastic noise of the same order of magnitude generated during the inflation and superimposed upon $\sigma$.

All these conclusions are rather universal and (apart from the choice of $|\xi|$ and $\lambda$) universally depend on one parameter $A$ of the particle physics model. This quantity should satisfy the bound

$$0 < A \leq 1.3$$

in order to render $Z$ positive, thus suppressing over-Planckian energy scales, and provide sufficient amount of inflation ($A$ should not, certainly, be exceedingly close to zero, not to suppress the dominant contribution of large $|\xi|$ in (4.5)). This bound again suggests the quasi-supersymmetric nature of the particle model, although for reasons different from the conclusions of [14]. It is only supersymmetry that can constrain the values of the Higgs $\lambda_\chi$, vector gauge $g_A$ and Yukawa $f_\psi$ couplings so as to provide a subtle balance between the contributions of bosons and fermions in (4.5) and fit the quantity $A$ into a narrow range. In contrast to ref. [14], this conclusion is robust against the subtleties of the definition of $Z$ (related to the treatment of zero modes on DeSitter background) because it probes only the large limit of $Z \gg 1$. 

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5. Conclusions

Thus, the same mechanism that suppresses the over-Planckian energy scales also generates a narrow probability peak in the distribution of tunnelling inflationary universes and strongly suggests the (quasi)supersymmetric nature of their particle content. It seems to be consistent with microwave background observations within the model with a strongly coupled nonminimal inflaton field. A remarkable feature of this result is that it is mainly based on one small parameter – the dimensionless ratio of two major energy scales, the GUT and Planck ones, given by the combination of the coupling constants $\sqrt{\lambda}/|\xi| \approx 10^{-5}$. This result is independent of the renormalization ambiguity, which gives a hope that it is also robust against inclusion of multi-loop corrections. It is usual to be prejudiced against a large value of the nonminimal coupling $|\xi|$ which generates large quantum effects leaving them uncontrollable in multi-loop orders. This is not, however, quite correct, because the effective gravitational constant in such a model is inverse proportional to $m_\gamma^2 + 8\pi|\xi|\varphi^2$ and, thus, large $|\xi|$ might improve the loop expansion $[20]$. Obviously, the large value of $|\xi|$ at sub-Planckian (GUT) scale requires explanation which might be based on the renormalization group approach (and its extension to non-renormalizable theories $[20]$). As shown in $[20]$, quantum gravity with nonminimal scalar field has an asymptotically free conformally invariant ($\xi = 1/6$) phase at over-Planckian regime, which is unstable at lower energies. It is plausible to conjecture that this instability can lead (via composite states of the scalar field) to the inversion of the sign of running $\xi$ and its growth at the GUT scale, thus making possible the proposed inflation applications $[32]$.

As far as it concerns the GUT and lower energy scales, the ground for supersymmetry of the above type looks very promising in the context of a special property of supersymmetric models to have a single unification point for weak, electromagnetic and strong interactions (the fact that has been discovered in 1987 and now becoming widely recognized after the recent experiments at LEP $[33]$).

From the viewpoint of the theory of the early universe, the obtained results give a strong preference to the tunnelling quantum state. The debate on the advantages of the tunnelling versus no-boundary wavefunction has a long history $[1, 4, 17, 18, 5]$. At present, in the cosmological context the tunnelling proposal seems to be more useful
and conceptually clearer than the no-boundary one, because for its interpretation one should not incorporate vague ideas of the third quantization of gravity which inevitably arise in the no-boundary case: splitting the Lorentzian wavefunction in positive and negative frequency parts and separately calculating their probability distributions. On the other hand, the formulation of the tunnelling proposal is not so aesthetically closed, for it involves imposing outgoing condition after the potential barrier, the unit normalization condition – before the barrier at \( a = 0 \), the requirement of the normalizability in variables \( f \), etc. And all this in contradistinction to the closed path-integral formulation of the no-boundary proposal, automatically providing many of the above properties. On the other hand, outside of the cosmological framework, in particular, within the scope of the wormhole and black hole physics, the tunnelling proposal seems to be helpless. Moreover, at the overlap of the cosmological framework with the theory of the virtual black holes it leads to contradictions signifying that the quantum birth of bigger black holes is more probable than the small (Planckian) ones [34]. All these arguments can hardly be conclusive, because it might as well happen that the difference between the no-boundary and tunnelling wavefunctions should be ascribed to the open problem of the correct quantization of the conformal mode. Note that the normalizability criterion for the distribution function and its algorithm (3.21) do not extend to the low-energy limit \( \varphi \to 0 \), where the naively computed no-boundary distribution function blows up to infinity, the slow-roll approximation becomes invalid, etc. This is a domain related to a highly speculative (but, probably, inevitable) third quantization of gravity [35], which goes beyond the scope of this paper. Fortunately, this domain is separated from the obtained inflationary peak by a vast desert with practically zero density of the quantum distribution, which apparently justifies our conclusions disregarding the ultra-infrared physics of the Coleman theory of baby universes and cosmological constant [35].

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