Constraints on Neutrino Parameters by Neutrinoless Double Beta Decay Experiments

Hiroaki Sugiyama ‡
Department of Physics, Tokyo Metropolitan University
1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan
E-mail: hiroaki@phys.metro-u.ac.jp

Abstract. Allowed regions on the $m_1 - \cos 2\theta_{12}$ plane are extracted from results of neutrinoless double beta decay experiments. It is shown that $0.05 \text{eV} \lesssim m_1 \lesssim 1.85 \text{eV}$ is obtained for the normal (inverted) hierarchy by using the LMA best fit parameters and the $0\nu\beta\beta$ result announced late last year, which is $0.05 \text{eV} \leq \langle m \rangle_{\beta\beta} \leq 0.84 \text{eV}$ with $\pm 50\%$ uncertainty of the nuclear matrix elements.

1. Introduction

Although it is well known that neutrinos are massive by neutrino oscillation experiments [2-4], the values of their masses are still unknown. The answer is never given by oscillation experiments because oscillation probabilities depend on $\Delta m_{ij}^2 \equiv m_j^2 - m_i^2$. Thus, we rely on non oscillation experiments such as single beta decay measurements [5], which are direct measurements of neutrino mass, neutrinoless double beta decay ($0\nu\beta\beta$) searches [6] and cosmological measurements [7] or Z-burst interpretation of the highest energy cosmic ray [8].

Double beta decay experiments seems to have rather higher sensitivities than those of other non oscillation experiments. Recent results of negative observations of $0\nu\beta\beta$ put upper bounds on the observable $\langle m \rangle_{\beta\beta}$ as $\langle m \rangle_{\beta\beta} < 0.35 \text{eV}$ (90\% C.L.) by Heidelberg-Moscow [9] and $\langle m \rangle_{\beta\beta} < 0.33 - 1.35 \text{eV}$ (90\% C.L.) by IGEX [10] §. The energy regions to be probed can reach to the order of $10^{-2} \text{eV}$ by some of the future experiments [11-17]. Such experiments seem to have strong possibility of $0\nu\beta\beta$ observations. Actually, an observation of $0\nu\beta\beta$ was announced late last year as $0.05 \text{eV} \leq \langle m \rangle_{\beta\beta} \leq 0.84 \text{eV}$ (95\% C.L.) with $\pm 50\%$ uncertainty of the nuclear matrix elements (KDHK result [18]; See also comment on the results and the replies [19]). While this result should be checked in the future experiments, it is fruitful to investigate what kind of information can be extracted from $0\nu\beta\beta$ observations.

The constraints imposed on neutrino mixing parameters by $0\nu\beta\beta$ experiments have been discussed by many authors (See the references in [1]). The implications of the KDHK result have been also discussed (See, for example, the references of the second article in [19]). In this talk, the constraints on a neutrino mass and the solar mixing angle are discussed in the generic three flavor mixing framework by observations (as well as non-observations) of $0\nu\beta\beta$.

‡ Invited talk based on [1] at Beyond the Desert 02, Oulu, Finland, 2-7 June 2002.
§ The bounds on $\langle m \rangle_{\beta\beta}$ depend on nuclear matrix elements. The actual observable is the half-life $T_{1/2}^{0\nu}$: $T_{1/2}^{0\nu} > 1.9 \times 10^{25} \text{y}$ was obtained by Heidelberg-Moscow, and $T_{1/2}^{0\nu} > 1.57 \times 10^{25} \text{y}$ by IGEX.
2. Constraints

We use the following standard parametrization of the MNS matrix [20]:

\[
U_{\text{MNS}} \equiv \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{13} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}e^{-i\delta} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}.
\]  

(1)

The mixing matrix for three Majorana neutrinos is

\[
U \equiv U_{\text{MNS}} \times \text{diag}(1, e^{i\beta}, e^{i\gamma}),
\]

(2)

where \(\beta\) and \(\gamma\) are extra CP-violating phases which are characteristic of Majorana particles [21]. In this parametrization, the observable of double beta decay experiments is described as

\[
\langle m \rangle_{\beta\beta} \equiv \sum_{i=1}^{3} m_i |U_{ei}|^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 e^{2i\beta} + m_3^2 s_{13}^2 e^{2i(\gamma-\delta)},
\]

(3)

where \(U_{ei}\) denote the elements in the first low of \(U\) and \(m_i\) (\(i = 1, 2, 3\)) are the neutrino mass eigenvalues. In the convention of this talk, the normal hierarchy means \(m_1 < m_2 < m_3\) and the inverted hierarchy \(m_3 < m_1 < m_2\).

In order to utilize the theoretical lower bound on \(\langle m \rangle_{\beta\beta}\), we derive a constraint on \(\langle m \rangle_{\beta\beta}\). An appropriate choice of the phase-factor \(e^{2i(\gamma-\delta)}\) in eq. (3) leads to an inequality

\[
\langle m \rangle_{\beta\beta} \geq \langle m \rangle_{\beta\beta} \geq c_{13}^2 |m_1 c_{12}^2 + m_2 s_{12}^2 e^{2i\beta}| - m_3 s_{13}^2.
\]

(4)

Strictly, the right-hand side (RHS) should be the absolute value of it. It is, however, not necessary to consider the absolute value because \(s_{13}^2\) has a very small value. The RHS of (4) is minimized by replacing \(e^{2i\beta}\) with \(-1\) and \(s_{13}^2\) with the largest value \(s_{\text{CH}}^2\) which is determined by reactor experiments [22]. Thus, we obtain

\[
\langle m \rangle_{\beta\beta} \geq c_{\text{CH}}^2 |m_1 c_{12}^2 - m_2 s_{12}^2| - m_3 s_{\text{CH}}^2.
\]

(5)

Next, we derive a theoretical upper bound on \(\langle m \rangle_{\beta\beta}\) to utilize an experimental lower bound \(\langle m \rangle_{\beta\beta}\). Since the RHS of (3) is maximized by setting the phase-factors unity, we obtain

\[
\langle m \rangle_{\beta\beta} \leq \left( m_1 c_{12}^2 + m_2 s_{12}^2 \right) c_{13}^2 + m_3 s_{13}^2.
\]

(6)

Furthermore, \(s_{13}^2\) is replaced by \(s_{\text{CH}}^2\) (zero) for the normal (inverted) hierarchy in order to set the RHS to be the largest value with respect to \(s_{13}^2\). Then, the inequality results in

\[
\langle m \rangle_{\beta\beta} \leq \left( m_1 c_{12}^2 + m_2 s_{12}^2 \right) c_{\text{CH}}^2 + m_3 s_{\text{CH}}^2
\]

(7)

for the normal hierarchy, and

\[
\langle m \rangle_{\beta\beta} \leq m_1 c_{12}^2 + m_2 s_{12}^2
\]

(8)

for the inverted hierarchy.

Constraints (5), (7) and (8) determine an allowed region on the plane of a neutrino mass versus the mixing angle. In this talk, we use the \(m_1 - \cos 2\theta_{12}\) plane, where \(m_1\) denotes the lightest neutrino mass for each hierarchy.
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3.

Discussion

In this section, we analyze the constraints obtained in the previous section. Two example cases of experimental results are considered below; the case 1 is $0.1 \, \text{eV} \leq \langle m \rangle_{\beta\beta} \leq 0.3 \, \text{eV}$ which is within the region of the KDHK result, and the case 2 is $0.01 \, \text{eV} \leq \langle m \rangle_{\beta\beta} \leq 0.03 \, \text{eV}$ which is outside of the region of the KDHK result. The LMA solution of the solar neutrino problem, which is only one allowed at 99% C.L. [23], is considered mainly. Therefore, the mass square difference are fixed here after as $|\Delta m_{12}^2| = 5.0 \times 10^{-5} \, \text{eV}^2$ and $|\Delta m_{23}^2| = 3.0 \times 10^{-3} \, \text{eV}^2$.

3.1. Case 1: $0.1 \, \text{eV} \leq \langle m \rangle_{\beta\beta} \leq 0.3 \, \text{eV}$

The bounds for this case are presented in Fig. 1. It is remarkable that the bounds (7) and (8), which are obtained with $\langle m \rangle_{\beta\beta}^{\text{min}}$, are almost vertical because of small $\Delta m_{12}^2$. The lines cross the horizontal axis at $m_l \simeq \langle m \rangle_{\beta\beta}^{\text{min}}$ for not very small $\langle m \rangle_{\beta\beta}^{\text{min}}$ (See the next subsection for very small $\langle m \rangle_{\beta\beta}^{\text{min}}$). Therefore, $\langle m \rangle_{\beta\beta}^{\text{min}}$ is approximately regarded as the lower bound on $m_l$; $\langle m \rangle_{\beta\beta}^{\text{min}} \leq m_l$.

On the other hand, since there are asymptotes $\cos 2\theta_{12} = \pm t_{\text{CH}}^2$ for the bounds (5), no upper bound on $m_l$ exists for $|\cos 2\theta_{12}| \leq t_{\text{CH}}^2$. The LMA solution is fortunately outside of the region. Note that the bounds (5) for the normal and inverted hierarchy are very similar to each other. It means that the degenerate mass approximation $m_i \simeq m_\nu$ is very good in this case. In this approximation, the constraint (5) becomes

$$\langle m \rangle_{\beta\beta}^{\text{max}} \geq m_\nu \left( t_{\text{CH}}^2 |\cos 2\theta_{12}| - s_{\text{CH}}^2 \right). \quad (9)$$

We see that $m_\nu / \langle m \rangle_{\beta\beta}^{\text{max}}$ is a good parameter. In Fig. 2, the bounds (5) are presented on the $m_l / \langle m \rangle_{\beta\beta}^{\text{max}} - \cos 2\theta_{12}$ plane for $\langle m \rangle_{\beta\beta}^{\text{max}} = 0.1 \, \text{eV}$. The similarity between the bounds for two hierarchies means the goodness of the degenerate mass approximation for $\langle m \rangle_{\beta\beta}^{\text{max}} \simeq 0.1 \, \text{eV}$ and also for larger values of $\langle m \rangle_{\beta\beta}^{\text{max}}$. Thus, the same bounds as those in Fig. 2 can be used.
for $\langle m \rangle_{\beta\beta}^{\text{max}} \gtrsim 0.1$ eV. For example, the simple constraint $m_l \leq 2.2 \times \langle m \rangle_{\beta\beta}^{\text{max}}$ is read off in the figure for the LMA best fit value $\cos 2\theta_{12} = 0.49$, which corresponds to $\tan^2 \theta_{12} = 0.34$ given by the second article in [23].

3.2. Case 2: $0.01$ eV $\leq \langle m \rangle_{\beta\beta} \leq 0.03$ eV

The bounds for this case are presented in Fig. 3. It is clear that the degenerate mass approximation is no longer good because the relevant energy scale is smaller than the atmospheric one $\sqrt{\Delta m_{23}^2} \simeq 0.05$ eV. The bounds for the normal and inverted hierarchy differ significantly. One of the important point is the disappearance of the bound for the inverted hierarchy by (8). It means that the constraint (8) is satisfied even for $m_l = 0$. It is also possible for the normal hierarchy with smaller $\langle m \rangle_{\beta\beta}^{\text{min}}$. There are the smallest $\langle m \rangle_{\beta\beta}^{\text{min}}$ needed for the existence of the bound by (7) and that by (8). The values are extracted approximately from the RHS of (7) and (8) with $m_l = \cos 2\theta_{12} = 0$ as

$$\frac{1}{2} c_{\text{CH}}^2 \sqrt{\Delta m_{12}^2 + s_{\text{CH}}^2 \Delta m_{23}^2} \simeq 0.005 \text{ eV} \sim \sqrt{\Delta m_{12}^2}$$

(10)

for the normal hierarchy, and

$$\frac{1}{2} c_{\text{CH}}^2 \left(\sqrt{\Delta m_{23}^2 - \Delta m_{12}^2} + \sqrt{\Delta m_{23}^2}\right) \simeq 0.053 \text{ eV} \simeq \sqrt{\Delta m_{23}^2}$$

(11)

for the inverted one. In the case 2, $\langle m \rangle_{\beta\beta}^{\text{min}} = 0.01$ eV is smaller than 0.053 eV, and that is why there is no lower bound on $m_l$ for the inverted hierarchy in Fig. 3.

Another important point of Fig. 3 is that the bound (5) for the inverted hierarchy crosses the vertical axis at $\cos 2\theta_{12} = 0.57$. It means that a small $\langle m \rangle_{\beta\beta}$ excludes large values of $\cos 2\theta_{12}$ for the hierarchy. Conversely, a $\cos 2\theta_{12}$ larger than $t_{\text{CH}}^2$ gives a theoretical minimum of $\langle m \rangle_{\beta\beta}$[24].
Figure 3. The bounds (5), (7) and (8) are shown for $0.01 \, \text{eV} \lesssim \langle m \rangle_{\beta \beta} \lesssim 0.03 \, \text{eV}$. The solid (dashed) lines are for the normal (inverted) hierarchy. The inside of those bounds are allowed. The LMA region is superimposed with shadow.

Finally, let us extract the bound on $m_l$ from the KDHK result. Since $\langle m \rangle_{\beta \beta}^{\max} = 0.84 \, \text{eV}$ is large enough, we can use Fig. 2. Therefore, the upper bound on $m_l$ is extracted as

$$m_l \lesssim 2.2 \times \langle m \rangle_{\beta \beta}^{\max} = 1.85 \, \text{eV}$$

for the LMA best fit parameters. On the other hand, $\langle m \rangle_{\beta \beta}^{\min} = 0.05 \, \text{eV}$ is larger than (10) but smaller than (11). Thus, although there is the lower bound on $m_l$ for the normal hierarchy, not for the inverted one. Roughly, $\langle m \rangle_{\beta \beta}^{\min}$ is the lower bound on $m_l$ for the normal hierarchy. By combining those results, we obtain

$$0.05 \, \text{eV} (0 \, \text{eV}) \lesssim m_l \lesssim 1.85 \, \text{eV}$$

for the normal (inverted) hierarchy with the LMA best fit parameters. If $\langle m \rangle_{\beta \beta}^{\min}$ becomes a little larger, $0\nu\beta\beta$ observations can give the first exclusion of $m_l = 0$ for both hierarchies.

4. Conclusions

Allowed regions on the plane of a neutrino mass versus the solar mixing angle $\theta_{12}$ were obtained by using $\langle m \rangle_{\beta \beta}^{\max}$ ($\langle m \rangle_{\beta \beta}^{\min}$), which is an experimental upper (lower) bound on the observable $\langle m \rangle_{\beta \beta}$ of double beta decay experiments. For given $\theta_{12}$, these become constraints on a neutrino mass such as the lightest mass $m_l$; Roughly, $\langle m \rangle_{\beta \beta}^{\min} \lesssim m_l \lesssim 2.2 \times \langle m \rangle_{\beta \beta}^{\max}$ for the LMA best fit parameters.

It became clear that the condition $|\cos 2\theta_{12}| > t_{\text{CH}}^2 \simeq 0.03$ was necessary for the upper bound on $m_l$ to exist. On the other hand, the condition $\langle m \rangle_{\beta \beta}^{\min} \simeq 0.005 \, \text{eV} (0.053 \, \text{eV})$ needs to be satisfied for the normal (inverted) hierarchy, for the lower bound on $m_l$ to exist.

For example, $0.05 \, \text{eV} \leq \langle m \rangle_{\beta \beta} \leq 0.84 \, \text{eV}$ gives $0.05 \, \text{eV} (0 \, \text{eV}) \lesssim m_l \lesssim 1.85 \, \text{eV}$ for the normal (inverted) hierarchy.
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References

[1] H. Minakata and H. Sugiyama, Phys. Lett. B532 (2002) 275-283.
[2] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B335 (1994) 237;
Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81 (1998) 1562; ibid. 85 (2000) 3999.
[3] Homestake Collaboration, K. Lande et al., Astrophys. J. 496 (1998) 505;
SAGE Collaboration, J.N. Abdurashitov et al., Phys. Rev. C60 (1999) 055801;
GALLEX Collaboration, W. Hampel et al., Phys. Lett. B447 (1999) 127;
Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 86 (2001) 5651; ibid. 86 (2001) 5656;
SNO Collaboration, Q.R. Ahmed et al., Phys. Rev. Lett. 87 (2001) 071301.
[4] K2K Collaboration, S.H. Ahn et al., Phys. Lett. B511 (2001) 178;
See also http://neutrino.kek.jp/news/2001.07.10.News/index-e.html.
[5] R.G.H. Robertson and D.A. Knapp, Ann. Rev. Nucl. Part. Sci. 38 (1988) 185.
[6] P. Vogel, in Current Aspects of Neutrino Physics pp. 177, edited by D.O. Caldwell (Springer-Verlag, Berlin, 2001) [nucl-th/0005020].
[7] D.J. Eisenstein, W. Hu, and M. Tegmark, Astrophys. J. 518 (1999) 2;
M. Fukugita, Talk at Frontiers in Particle Astrophysics and Cosmology; EuroConference on Neutrinos in the Universe, Lenggries, Germany, September 29 - October 4, 2001.
[8] T.J. Weiler, Phys. Rev. Lett. 49 (1982) 234; Astroparticl Phys. 11 (1999) 303;
H. Päs and T.J. Weiler, Phys. Rev. D63 (2001) 113015;
D. Fargion, B. Mele, and A. Salis, Astrophys. J. 517 (1999) 725;
Z. Fador, S.D. Katz, and A. Ringwald, Phys. Rev. Lett. 88 (2002) 171101.
[9] Heidelberg-Moscow Collaboration, H.V. Klapdor-Kleingrothaus et al., Eur. Phys. J. A12 (2001) 147.
[10] IGEX Collaboration, C.E. Aalseth et al., Phys. Rev. D65 (2002) 092007.
[11] GENIUS Collaboration, H.V. Klapdor-Kleingrothaus et al., hep-ph/9910205.
[12] Majorana Collaboration, C.E. Aalseth et al., hep-ex/0201021.
[13] E. Fiorini et al. Phys. Rep. 307 (1999) 309;
A. Bettini, Nucl. Phys. Proc. Suppl. 100 (2001) 332.
[14] H. Ejiri et al., Phys. Rev. Lett. 85 (2000) 2919.
[15] F. Piquemal (for the NEMO Collaboration), hep-ex/0205006.
[16] S. Moriyama, Talk at International Workshop on Technology and Application of Xenon Detectors (Xenon01), ICRR, Kashiwa, Japan, December 3-4, 2001.
[17] S. Waldman, Talk at International Workshop on Technology and Application of Xenon Detectors (Xenon01), ICRR, Kashiwa, Japan, December 3-4, 2001.
[18] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harnay, and I. Krivosheina, Mod. Phys. Lett. A16 (2001) 2409, [hep-ph/0201231].
[19] C.E. Aalthes et al., hep-ex/0202018;
H.V. Klapdor-Kleingrothaus, hep-ph/0205228;
H.L. Harnay, hep-ph/0205293.
[20] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870;
Particle Data Group, K. Hagiwara et al., Phys. Rev. D66 (2002) 010001.
[21] J. Schechter and J.W.F. Valle, Phys. Rev. D22 (1980) 2227;
S.M. Bilenky, J. Hosek, and S.T. Petcov, Phys. Lett. B94 (1980) 495;
M. Doi et al., Phys. Lett. B102 (1981) 323.
[22] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B420 (1998) 397; ibid. B466 (1999) 415;
Palo Verde Collaboration, F. Boehm et al., Phys. Rev. D64 (2001) 112001.
[23] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89 (2002) 011301;
SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89 (2002) 011302;
V. Barger et al., Phys. Lett. B537 (2002) 179-186;
A. Bandyopadhyay et al., Phys. Lett. B540 (2002) 14-19;
J.N. Bahcall, M.C. Gonzalez-Garcia and C. Peña-Garay, JHEP 0207 (2002) 054;
P.C.de Holanda and A.Yu. Smirnov, hep-ph/0205241.
[24] H.V. Klapdor-Kleingrothaus, H. Päs and A.Yu. Smirnov, Phys. Rev. D63 (2001) 073005.