Vacuum fluctuations for spherical gravitational impulsive waves

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Abstract We propose a method for calculating vacuum fluctuations on the background of a spherical impulsive gravitational wave which results in a finite expression for the vacuum expectation value of the stress-energy tensor. The method is based on first including a cosmological constant as an auxiliary constant. We show that the result for the vacuum expectation value of the stress-energy tensor in second-order perturbation theory is finite if both the cosmological constant and the infrared parameter tend to zero at the same rate.
1. Introduction

It has been known for a long time that plane waves do not give rise to vacuum fluctuations [1,2]. One may check whether the same result is true for spherical waves as well. One possible check is to use the Nutku-Penrose metric [3] for spherical impulsive gravitational waves and to study whether vacuum fluctuations arise when a scalar particle is coupled to the gravitational field through this metric.

We applied this procedure to different holomorphic warp functions [4-8] and found no finite fluctuations in the Minkowski case. We must stress the fact that in these calculations first-order perturbation theory was used. This method gave a null result for the Minkowski case. Taking only first order perturbation theory was, perhaps, the weak point in these calculations. One conjecture [9] was that the null result would change in second-order calculations. Here our results on this problem, performed in second-order perturbation theory, will be reported.

It is common knowledge that massless particle production is prohibited by the presence of conformal symmetry [10]. One may argue that this fact is not relevant here since the Nutku-Penrose metric is not conformally related to Minkowski space. This would be true if an exact calculation were made. One must note at this point that we are using only approximate methods to solve this problem. We are perturbing around the Minkowski space to calculate the fluctuations in the new space. This perturbation does not change the essential properties of the solutions of the Minkowski space, but just modulates them by multiplying by a factor. The Minkowski Green function is just a monomial over distance. We could not find a way to regularize it and still retain non-zero terms, since there are none left when the worst divergence is subtracted. Our perturbative calculation just modifies the Minkowski expression by multiplying it by a finite term. As a result we could not get any finite result in this case in first-order perturbation theory for the cases studied.

If we had a mass parameter in the problem, we could get an expansion whose second or third term could give finite fluctuations. This was not possible here. We still hoped that the second-order perturbation calculation might give a signal for a possible way out of this impasse.

One way to introduce a mass parameter without changing the Nutku-Penrose solution too much is to study a similar case in de Sitter space where one obtains an impulsive wave solution by a simple manoeuvre [11]. The scalar curvature of the de Sitter space has dimensions of mass squared. Here the calculation in first order gives finite results for all of the different choices of the warp functions studied [8,12]. If we carry the calculations to second order, at least for the special choice studied here, we get infrared divergences. We conjecture that this behaviour is generic for a wider class of warp functions, if not for all possible forms, since in first-order calculations we get the same general behaviour for all the different choices we have studied. We verify this conjecture for another warp function which has shown to be representative for a wider class of warp functions.
We take these divergences for the signal we are looking for. The Green function we calculate includes terms which are inversely proportional to the square of an infrared mass which should be taken to zero at the end of the calculation. One natural way to get rid of these two parameters (the curvature of the de Sitter space and the infrared parameters) is to set them proportional to each other. In one sense both terms are introduced for technical reasons. We went to de Sitter space to perturb around a space which is different from the Minkowski in the first place, but one that still retains the Nutku-Penrose solution. The infrared parameter was introduced just to be able to calculate the Green function unambiguously. There were no massive parameters in the model initially, and they should not be present in the end result. Taking these two terms proportional to each other and then sending them to zero cancels these two auxiliary parameters completely, still retaining a finite contribution to $< T_{\mu\nu} >$.

At the end of the calculation all reference to the de Sitter universe is gone and our result is what one would expect to get in the Minkowski case. We have the vacuum expectation value of one component of the stress-energy tensor,

$$< T_{\mu\nu} > \propto \frac{\delta(v)}{u^2}$$

where $\delta(v)$ is the Dirac delta function. In the Nutku-Penrose metric the only non-zero component of the curvature tensor is proportional to the same function. One may anticipate, following Deser [1], that any non-zero result may be proportional to the same form.

Our conclusion is that to obtain a non-zero result for the vacuum fluctuations in the case studied here, first we have to work in de Sitter space, and thus explicitly break the conformal symmetry in the metric we perturb about. When this calculation is carried to second order, infrared divergences are traded for the for the curvature of the space to get a finite result. At the end we are back in the metric which describes an impulsive wave in Minkowski space.

In section 2 we describe the second-order calculation in Minkowski space. We pass to the de Sitter case just by multiplying the Minkowski result by a conformal factor. Before we go to the coincidence limit we set the two auxiliary parameters proportional to each other. We then take the coincidence limit and perform differentiations on the Green function to obtain a finite value for the vacuum expectation value of the stress-energy tensor. We conclude with a few remarks.

2. Minkowski calculation

We start with the Nutku-Penrose metric [3]

$$ds^2 = 2dudv - u^2d\zeta + v\Theta(v0f(\zeta)d\zeta)^2.$$  \hspace{1cm} (1)

Here $v$ is the retarded time, $u$ is similar to the radial distance and $\zeta$ is the angle of the stereographic projection, $f(\zeta)$ is the Schwarzian derivative of $h(\zeta)$ which is the holomorphic
warp function describing the impulsive wave, and $\Theta(v)$ is the Heavyside unit step function. For different choices of the warp function, the non-zero component of the curvature tensor is multiplied by a different function, but the essential characteristics of the metric do not change. The Nutku-Pentrose solution corresponds to the snapping of a cosmic string, giving rise to a spherical impulsive wave.

In the past we calculated vacuum fluctuations by computing the stress-energy tensor of a scalar field in the background of this metric. Our warp functions $h(\zeta)$ included a parameter which is related to the string parameter $G \mu u \approx 10^{-6}$ that we wrote as $\delta$ or $\epsilon$. We used $h(\zeta) = \left( \frac{A\zeta + B}{C\zeta + D} \right)^{1+\delta+i\epsilon}$ for different values of $(A,B,C,D)$ [4-8]. We got the null result in all of these cases if we perturbed around the Minkowski space. If we multiply this metric by a conformal factor, $(1 + \frac{\Lambda uv}{6})^2$ we get a spherical wave in de Sitter space [11]. Then there are finite fluctuations even in first-order perturbation theory that are proportional to the square of the scalar curvature [12].

To investigate the same phenomena in second-order perturbation theory, here we use a different choice of the warp function. We take $h = e^{\alpha \zeta}$. There are two reasons why this function is chosen. First, this is an important special case, first discussed by Gleiser and Pullin [13]. It has the special property that $f(\zeta)$ is independent of zeta although $h(\zeta)$, the decisive term in the solution, depends on the same parameter. This fact allows us to write our solutions as sums of functions of $\zeta$ and $\bar{\zeta}$. Our essential motive for studying this case is not this technical point, though. A similar problem was solved exactly [14], in another context. There were somewhat vague indications of particle production. Here we want to see whether the vacuum expectation value of one component of the stress-energy tensor is proportional to the non-zero component of the curvature tensor. Although the result concerning particle production would not allow us to conclude anything definite about our new calculation, we may still argue that an unambiguous result should exist for our latter calculation as well. We will just use the exact result as a guard against possible infrared problems. If such problems arise in our expansion we will know that they are due to technical factors, since the exact solution does not have them, and we will try to regularize them.

We start with $h = e^{\alpha \zeta}$. This gives $f = -\frac{\alpha^2}{2}$ resulting in a metric

$$ds^2 = 2dudv - \frac{1}{4} \left( dx^2 (2u - \alpha^2 \Theta(v))^2 + dy^2 (2 + \alpha^2 \Theta(v))^2 \right).$$

Here $\zeta = x + iy$. If we write the d’Alembertian operator in this metric, we get

$$\frac{1}{\sqrt{-g}} \partial_\mu (g^{\mu\nu} \sqrt{-g}) \partial_\nu = 2\partial_\mu \partial_\nu + \left( \frac{1}{u - \frac{\alpha^2}{2} v^2} + \frac{1}{u + \frac{\alpha^2}{2} v^2} \right) \partial_v$$

$$\frac{\alpha^2}{2} \left( \frac{1}{u + \frac{\alpha^2}{2} v^2} - \frac{1}{u - \frac{\alpha^2}{2} v^2} \right) \partial_u - \frac{1}{(u - \frac{\alpha^2}{2} v^2)^2} \partial_x^2 - \frac{1}{(u + \frac{\alpha^2}{2} v^2)^2} \partial_y^2.$$

(3)
for the exact operator. We multiply the d’Alembertian operator given in equation (3) by $u^2$ and expand the operator up to second order in $\alpha^2$:

$$L^{II} = 2u^2 \partial_u \partial_v + 2u \partial_v - \partial_x^2 - \partial_y^2 - \frac{\alpha^2 v}{u} (\partial_x^2 - \partial_y^2) - \alpha^4 \left( \frac{v}{2} \partial_u - \frac{v^2}{u} \partial_v + \frac{3}{2} \frac{v^2}{u^2} (\partial_x^2 + \partial_y^2) \right).$$ (4)

We can construct the vacuum expectation value of the stress-energy tensor from the two-point function through differentiation, after the coincidence limit is taken and the appropriate regularization is done. The two-point function $G_F$ is found by summing the eigenfunctions of the related Sturm-Liouville problem.

In zeroth order we get the empty space solution from the solution $\phi^{(0)}$ [5]:

$$G^{(0)}_F = \sum_{\lambda} \frac{\phi^{\ast(0)}_{\lambda}(x) \phi^{(0)}_{\lambda}(x')}{\lambda} = \frac{A}{(u - u')(v - v') - \frac{uv'}{2} ((x - x')^2 + (y - y')^2)}.$$ (5)

Here $A$ is a constant. The first order solution is written as $\phi^{(1)} = \phi^{(0)} f$. Here $\phi^{(0)}$ is the zeroth-order solution. The first order solution is the product of the zeroth-order solution and another function. This ansatz for $\phi^{(1)}$ is dictated by the differential equations, and is not just an ad hoc guess. We find that $f$ just modulates the zeroth-order solution, and does not essentially change it. The signature of $\phi^{(0)}$ is seen in the ultra-violet behaviour of $G_F$ to a large extent.

We find that $f$ obeys the differential equation

$$L_2 f = \frac{v}{u} (k_2^2 - k_1^2)$$ (6)

where $L_2$ is defined as

$$L_2 = \left( -2i R \frac{\partial}{\partial s} - 2i \left( k_1 \frac{\partial}{\partial x} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - 2 \frac{\partial^2}{\partial s \partial v} + \frac{i K}{R} \frac{\partial}{\partial v} \right).$$ (7)

Here $k_1, k_2, R$, and $K$ are parameters of the zeroth-order solution, $s = \frac{1}{u}$. They are integrated over to get the two-point function $G_G$.

To calculate $f$ we make the ansatz $f = vf_1(s, x, y) + f_2(s, x, y)$ as suggested by the form of equation (6). This ansatz yields two equations, partly coupled:

$$L_3 f_1 = s(k_2^2 - k_1^2),$$ (8)

and

$$L_3 f_2 = \left( 2 \frac{\partial}{\partial s} + \frac{i K}{R} \right) f_1$$ (9)

where

$$L_3 = \left( -2i R \frac{\partial}{\partial s} - 2i \left( k_1 \frac{\partial}{\partial x} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right).$$ (10)
These equations are simply integrated over by multiplying the right-hand side by the inverse of the operator $L_3$.

We end up with

$$f = \frac{-ivs^2}{4R}(k_1^2 - k_2^2) + i(k_1^2 - k_2^2) \left( \frac{is^2}{4R^2} + \frac{Ks^3}{24R^3} \right).$$  \hspace{1cm} (11)

To get $G_F$, we have to calculate $O[f]$ where the operator $O$ is given by

$$O = \frac{i}{(2\pi)^2u'u'} \int_{-\infty}^{\infty} \frac{dR}{2|R|} \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} dK \int_{0}^{\infty} d\alpha \times e^{ik_1(x-x')} e^{ik_2(y-y')} e^{ir(v-v')} e^{\frac{iK}{2R}(s-s')} e^{i\alpha(K-k_1^2-k_2^2)}.$$  \hspace{1cm} (12)

When we perform this operation, we get $G_F$ equal to

$$[(x-x')^2 - (y-y')^2] \left( A_1 \frac{s^2v\Theta(v) - s^2v'\Theta(v')}{(s-s')} \right) + A_2 \left( \Theta(v)s^2 + \Theta(v')s'^2 \right) \left( \frac{1}{(s-s')^2} \right) + A_3 \left( \frac{s^3\Theta(v) - s^3\Theta(v')}{(s-s')^3} \right)$$  \hspace{1cm} (13)

where $[(s-s')^2 = (u-u')(v-v') - \frac{uu'}{2} ((x-x')^2 + (y-y')^2)$, and $A_1, A_2, A_3$ are constants. We see that this result is of the Hadamard form. No problems seem to arise in the infrared region. We find that all these terms have the same type of ultraviolet singularity as the flat part. We could not find a finite part of this expression.

If we go one order higher, we end up with the differential equation

$$L_2g = v^2 \left( \frac{iRs}{2} + 3s^2(k_1^2 + k_2^2) + (k_1^2 - k_2^2)^2 \left( \frac{-is^3}{4R} \right) \right)$$

$$+ v \left( -\frac{s}{2} + \frac{iKs^2}{4R} - \frac{(k_1^2 - k_2^2)^2s^3}{4R^2} + \frac{iK(k_1^2 - k_2^2)^2s^4}{24R^3} \right)$$  \hspace{1cm} (14)

when we make the ansatz $\phi_2 = \phi_0g$. We take $g = v^2g_1(x, y, s) + vg_2(x, y, s) + g_3(x, y, s)$. Going through similar steps as described above, we find

$$g_1 = \frac{-s^2}{8} + \frac{is^3(k_1^2 + k_2^2)}{2R} + \frac{(k_1^2 - k_2^2)^2s^4}{32R^2}.$$  \hspace{1cm} (15)

All these terms give two-point functions $G_F$, in the Hadamard form. All are finite in the infrared sense.
We use $g_1$ to get $g_2$:

$$g_2 = \frac{-i3s^2}{8R} - \frac{13Ks^3}{12R^2} + \frac{is^4}{32R^3} \left( (k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2) \right)$$

\[+ \frac{s^5K}{160R^7}(k_1^2 - k_2^2)^2. \tag{16}\]

At this point we start getting problems. All the terms in this expression are divergent in the infrared region. When we apply the operator $\frac{\partial}{\partial x}$ to $g_2$ and take $x = x', y = y'$, we get a term proportional to

$$\int_0^\infty \frac{d\alpha}{\alpha} \exp \left[ -i(u - u')(v - v') \frac{uu'}{uu'\alpha} \right].$$

We can start with a massive scalar particle and set the mass equal to zero at the end of the calculation. Then the above integral reduces to $H_0(m\sqrt{(u - u')(v - v')})$ where $H_0$ is the Hankel function of order zero. It degenerates to a logarithm when $m$ tends to zero. We have to take derivatives of $G_F$ to find the vacuum expectation value of the stress-energy tensor. The term with $m$ decouples if we differentiate the logarithm with respect to $u$ or $v$. The finite part of $<T_{\mu\nu}>$ will not have $\log m$ if there is a finite part. We call this type of divergence 'harmless', in this sense.

For $g_3$, we get

$$g_3 = \frac{3s^2}{8R^2} - \frac{i55Ks^3}{16R^3} - \frac{s^4}{32R^4} \left( (k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2) + \frac{13K}{3} \right)$$

\[+ \frac{is^5K}{320R^5} \left( 3(k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2) \right) - \frac{K^2s^6(k_1^2 - k_2^2)^2}{1920R^6}. \tag{17}\]

We see that when $x = x', y = y'$ all these terms give rise to expressions that are proportional to

$$\int_0^\infty d\alpha \exp \left[ -i(u - u')(v - v') \frac{uu'}{uu'\alpha} \right]$$

which are linearly divergent at the upper limit. If we use a massive field as an infrared cut-off, we get terms that go as $\frac{1}{m^2}$ as $m$ tends to zero. This term multiplies the whole expression and does not drop out on differentiation.

3.Going to de Sitter space Up to this point we have studied a metric for an impulsive wave propagating through minkowski space. We ran into problems which indicate that we may be perturbing around the wrong vacuum. Since the exact treatment of a related problem has no infrared divergences, we know that these divergences are spurious.

As a possible way out we take the impulsive waves propagating in a de Sitter universe. Since the de Sitter space has a parameter with the dimensions of mass, one may think of trading these two parameters for one another.
we try to perturb around the de Sitter space. Since the impulsive wave solution in the de Sitter space [11] is just the Minkowski solution multiplied by the factor \(1 + \frac{\Lambda uv}{6}\), we get the de Sitter two-point function just by multiplying the Minkowski case by \(1 + \frac{\Lambda uv'}{6}(1 + \frac{\Lambda u'v'}{6})\). This expression can be expanded as

\[
\left(1 + \frac{\Lambda uv}{6}\right)\left(1 + \frac{\Lambda u'v'}{6}\right) = \left(1 + \frac{\Lambda UV}{6}\right)^2 + \frac{\Lambda}{12}(u - u')(v - v') + \Lambda^2(...). \tag{19}
\]

Here \(U = \frac{u+u'}{2}, V = \frac{v+v'}{2}\). We multiply the Green function obtained in Minkowski space by this expression to get the de Sitter expression. In the Minkowski space \(g_3\) is given above gives rise to \(G_F\) whose first term goes as

\[
\frac{\Theta(v)s^2}{(s-s')^2m^2} + \frac{\Theta(v')s'^2}{(s-s')^2m^2} \tag{20}
\]

for \(x=x', y=y'\). Here \(m^2\) is an infrared parameter. We introduced the infrared parameter by adding \(2m^2u^2\) to our operator in equation (4). When the calculation is done in the presence of this additional term, we get the expressions given in equation (20) when we perform the summation operation to obtain the propagator function. In the presence of the new term the operators given in expression (18) are modified and result in terms given in (20). When we go to de Sitter space we have to multiply them by the expression given in (19).

Note that going to de Sitter space was only a technical trick. We will take \(\Lambda\) equal to zero at the end of the calculation and land in Minkowski space. We see that many of the terms that part of Minkowski space is subtracted for regularization are set to zero when \(\Lambda\) goes to zero. The terms given above are undetermined since they are multiplied by \(\frac{\Lambda}{m^2}\), where both \(m^2\) and \(\Lambda\) tend to zero. At this point we choose \(\Lambda\) proportional to \(m^2\). This choice is dimensionally correct. Any other choice, say, \(\Lambda\) proportional to the first power of \(m\) times \(s\), would be unnatural. To obtain the vacuum expectation value of the stress-energy tensor, we have to differentiate the Green function with respect to the coordinates. We are particularly interested in \(\langle T_{\mu\nu} \rangle\). We first take \(m\) and \(\Lambda\) going to zero limit, and then differentiate with respect to \(v\) and \(v'\) symmetrically, and then take the coincidence limit. We see that at the end of this calculation \(\langle T_{\mu\nu} \rangle\) turns out to be proportional to \(\frac{\delta(v)}{u^4}\). Here \(\delta\) is the Dirac delta function. The proportionality constant between the scalar curvature and the infrared mass squared can be absorbed in the perturbation constant \(\alpha^2\).

**Conclusion** We tried to calculate here the quantum fluctuations resulting from snapped cosmic strings, by perturbing around the vacuum, for a warp function that corresponds to the Gleiser-Pullin solution [14]. We could not separate a finite part in the first-order calculation. In the second order, we ran into infrared divergences. If we use the same warp function in the calculation where we perturb around the de Sitter space and take the scalar curvature \(\Lambda\) proportional to \(m^2\), we can get finite results in second-order perturbation theory. In the above expression \(m\) is the infrared cut-off. At the end we get \(\langle T_{\mu\nu} \rangle\).
proportional to $\frac{\delta(v)}{u}$. This is just the result we anticipated in the Minkowski case. At the end we are back in Minkowski space.

This result suggests that to get finite results in perturbation theory for the case studied we have first to break the conformal invariance which does not allow vacuum fluctuations for the massless particles [10], by hand, by going to de Sitter space, and then come back to Minkowski space after the de Sitter space parameter cancels the infrared divergence. As a result of this operation we get a non-zero contribution for vacuum fluctuations.

We anticipate somehow detecting the presence of cosmic strings [15]. We were expecting to get a finite vacuum expectation value for the stress-energy. We could not get finite contributions in our previous calculations when we stayed in Minkowski space [4-7]. When similar calculations were performed for the de Sitter case, we got finite results which were proportional to positive powers of the curvature of the de Sitter space. [12,8]. The contributions were zero when the curvature was set to zero.

In second-order calculations we encountered infrared divergences, which made the expansion ambiguous at this order. We can give a meaning to these calculations by first starting with the de Sitter impulsive wave where the curvature of the space is proportional to the square of the mass of the scalar field whose stress-energy is calculated in the background metric of the impulsive wave. This mass is used as an infrared parameter further in the calculation.

Our method may be criticized because it was applied only to a single choice for the warp function. although our choice was a very important case, we tried to perform the same calculation for the case where

$$h(\zeta) = \left(\frac{1}{\zeta}\right)^{1+i\epsilon} \quad (21)$$

and expanded the operator in powers of $\epsilon$. This choice is known to behave exactly the same way as the more general case,

$$h = \left( \frac{A\zeta + B}{C\zeta + D} \right)^{1+\delta+i\epsilon} \quad (22)$$

the second-order equation reads

$$(L_0 - \lambda_0)\phi^{(0)} = -L_2'\phi^{(0)} - L_1'\phi^{(1)} \quad (23)$$

where

$$L_2' = \frac{8v}{u} \left( v \frac{\partial}{\partial v} - u \frac{\partial}{\partial u} \right) + F[v, u, x, y, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}]. \quad (24)$$

Here $\phi_0, \phi_1$ and $\phi_2$ are the zeroth-, first- and second-order solutions. The first term on the right-hand side of this expansion gives terms that read as

$$\frac{-8v}{u} \left[ iRv + 1 - \frac{iK}{2uR} \right] \quad (25)$$
which are of the same form as the terms in equation (16). When these terms are integrated, we obtain
\[
\phi^{(2)}(2) = \phi^{(0)}(2) \left( \frac{2v^2}{u^2} + \frac{2iv}{Ru^2} - \frac{1}{R^2u^2} - \frac{iK}{3R^3u^3} \right) + \ldots \quad (26)
\]
where dots represent the additional terms we get from those included in F in equation (24).

Doing the same calculation to obtain Green function, we get an expression that behaves exactly as those given in equation (20). There are no terms which cancel these terms. At the end \( < T_{\mu\nu} > \) comes out to be proportional to \( \frac{\delta(v)}{u^4} \). The details of this calculation will be published elsewhere.

We think that this second calculation shows that our result is not restricted to one particular case, but illustrates the general behaviour in this problem. By starting with a massive field in the background metric of the impulsive wave in de Sitter space, we are breaking conformal invariance by hand. In the absence of the impulsive wave, i.e. in the zeroth-order perturbation theory, we do not have conformal invariance in the model, in contrast to the case when we have a massless scalar field and an impulsive wave in the minkowski space. Then we set the two parameters proportional to each other and send them to zero. As a result of this operation we get a finite contribution for \( < T_{\mu\nu} > \).

Our problem did not have conformal invariance in the first place. Using perturbative methods we could not convey this information to our solutions. We propose, then, first introducing additional parameters where this invariance is broken by hand in the perturbative calculation. At the end we send these additional parameters to zero in the same manner. We found that the peerturbative calculation gave the same qualitative answer, at least, that an exact solution would have given.

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