Neuromorphic computing with nanoscale spintronic oscillators

Jacob Torrejón1, Mathieu Riou1, Flavio Abreu Araujo1, Sumito Tsunegi2, Guru Khalsa3†, Damien Querlioz4, Paolo Bortolotti1, Vincent Cros1, Kay Yakushiji2, Akio Fukushima2, Hitoshi Kubota2, Shinji Yuasa2, Mark D. Stiles3 & Julie Grollier1

Neurons in the brain behave as nonlinear oscillators, which develop rhythmic activity and interact to process information1. Taking inspiration from this behaviour to realize high-density, low-power neuromorphic computing will require very large numbers of nanoscale nonlinear oscillators. A simple estimation indicates that to fit $10^8$ oscillators organized in a two-dimensional array inside a chip the size of a thumb, the lateral dimension of each oscillator must be smaller than one micrometre. However, nanoscale devices tend to be noisy and to lack the stability that is required to process data in a reliable way. For this reason, despite multiple theoretical proposals2–5 and several candidates, including memristive6 and superconducting7 oscillators, a proof of concept of neuromorphic computing using nanoscale oscillators has yet to be demonstrated. Here we show experimentally that a nanoscale spintronic oscillator (a magnetic tunnel junction)8,9 can be used to achieve spoken-digit recognition with an accuracy similar to that of state-of-the-art neural networks. We also determine the regime of magnetization dynamics that leads to the greatest performance. These results, combined with the ability of the spintronic oscillators to interact with each other, and their long lifetime and low energy consumption, open up a path to fast, parallel, on-chip computation based on networks of oscillators.

Nanoscale spintronic oscillators (or spin-torque nano-oscillators) are nanoscale pillars composed of two ferromagnetic layers separated by a non-magnetic spacer (Fig. 1a). Charge currents become spin-polarized when they flow through these junctions and generate torques on the magnetizations10,11 that lead to sustained magnetization precession at frequencies of hundreds of megahertz to several tens of gigahertz. Magnetization oscillations are converted into voltage oscillations through magneto-resistance. The resulting radio-frequency oscillations, of up to tens of millivolts (ref. 12), can be detected by measuring voltage amplitude that results when an input signal of $V_{in} = \pm 250$ mV is injected (here for $I_{DC} = 6.5$ mA (vertical dotted line) and $\mu_0H = 430$ mT). d, Schematic of the experimental set-up. A d.c. current $I_{DC}$ and a rapidly varying waveform that encodes the input $V_{in}$ are injected into the spin-torque nano-oscillator. The microwave voltage $V_{osc}$ emitted by the oscillator in response to the excitation is measured with an oscilloscope.

Neurons in the brain exhibit rhythmic activity and interact to process information. Taking inspiration from this behavior, high-density, low-power neuromorphic computing requires large numbers of nanoscale nonlinear oscillators. A simple estimation indicates that to fit $10^8$ oscillators into a chip the size of a thumb, the lateral dimension of each oscillator must be smaller than one micrometre. However, nanoscale devices are noisy and lack the stability required for reliable data processing. For this reason, despite multiple theoretical proposals and several candidates, including memristive and superconducting oscillators, a proof of concept of neuromorphic computing using nanoscale oscillators has not been demonstrated. Here, we experimentally show that a nanoscale spintronic oscillator (a magnetic tunnel junction) can be used to achieve spoken-digit recognition with an accuracy similar to that of state-of-the-art neural networks. We also determine the regime of magnetization dynamics that leads to the greatest performance. These results, combined with the ability of the spintronic oscillators to interact with each other and their long lifetime and low energy consumption, open up a path to fast, parallel, on-chip computation based on networks of oscillators.

Nanoscale spintronic oscillators (or spin-torque nano-oscillators) are nanoscale pillars composed of two ferromagnetic layers separated by a non-magnetic spacer (Fig. 1a). Charge currents become spin-polarized when they flow through these junctions and generate torques on the magnetizations that lead to sustained magnetization precession at frequencies of hundreds of megahertz to several tens of gigahertz. Magnetization oscillations are converted into voltage oscillations through magneto-resistance. The resulting radio-frequency oscillations, of up to tens of millivolts (ref. 12), can be detected by measuring voltage amplitude that results when an input signal of $V_{in} = \pm 250$ mV is injected (here for $I_{DC} = 6.5$ mA (vertical dotted line) and $\mu_0H = 430$ mT). d, Schematic of the experimental set-up. A d.c. current $I_{DC}$ and a rapidly varying waveform that encodes the input $V_{in}$ are injected into the spin-torque nano-oscillator. The microwave voltage $V_{osc}$ emitted by the oscillator in response to the excitation is measured with an oscilloscope.
the voltage across the junction (Fig. 1b). Spin-torque nano-oscillators are therefore simple and ultra-compact: their lateral size can be scaled down to 10 nm and their power consumption reduced to 1 μW (ref. 13). Because they have the same structure as present-day magnetic memory cells, they are compatible with complementary metal–oxide–semiconductor (CMOS) technology, have high endurance, operate at room temperature and can be fabricated in large numbers (currently up to hundreds of millions) on a single chip14. Just as the frequency of a neuron is modulated by the spikes received from other neurons, the frequencies of spin-torque nano-oscillators are highly sensitive to the magnetization dynamics of neighbouring oscillators to which they are coupled15,16. Together, these features of spin-torque nano-oscillators make them promising candidates for use in neuromorphic computing. Their oscillation amplitudes are highly sensitive to the frequency of spin-torque nano-oscillators, which are therefore simple and ultra-compact: their lateral size can be scaled down to 10 nm and their power consumption reduced to 1 μW (ref. 13). Because they have the same structure as present-day magnetic memory cells, they are compatible with complementary metal–oxide–semiconductor (CMOS) technology, have high endurance, operate at room temperature and can be fabricated in large numbers (currently up to hundreds of millions) on a single chip14. Just as the frequency of a neuron is modulated by the spikes received from other neurons, the frequencies of spin-torque nano-oscillators are highly sensitive to the magnetization dynamics of neighbouring oscillators to which they are coupled15,16. Together, these features of spin–torque nano-oscillators make them promising candidates for use in neuromorphic computing with large arrays of coupled oscillators17–21. However, they have yet to be used to perform an actual computing task.

Our idea is to exploit the amplitude dynamics of spin–torque nano-oscillators for neuromorphic computing. Their oscillation amplitude is shown in Fig. 1c, sampled with a time step \( \tau \) that is injected into the oscillator and use the amplitude response \( \tilde{V}(t) \) as the neural output.

Our nano-oscillators consist of circular magnetic tunnel junctions, with a 6-nm-thick free layer of FeB of 375-nm diameter, which have magnetic vortex ground states (see Methods). We measure the dynamics of the signal amplitude \( \tilde{V}(t) \) directly using a microwave diode. In Fig. 1c we show the nonlinear response of the amplitude \( \tilde{V} \) to a d.c. current \( I_{DC} \), \( \tilde{V} \propto (I_{DC} - I_0) \), where \( I_0 \) is the current threshold for steady oscillations to occur15. Using an arbitrary waveform generator, we inject a varying current through the junctions in addition to the d.c. current, using the set-up schematized in Fig. 1d. The resulting voltage oscillations, recorded with an oscilloscope, are shown in Fig. 1e. The amplitude of the oscillator varies in response to the injected d.c. current, with a relaxation time that induces a few hundred nanoseconds memory of past inputs23.

Recent studies have revealed that time-multiplexing can enable a single oscillator to emulate a full neural network24–26. Here we use this approach—a form of “reservoir computing” (see Methods)—to demonstrate the ability of spin–torque nano-oscillators to realize neuromorphic tasks. We perform a benchmark task of spoken-digit recognition. The input data, taken from the TI-46 database27, are obtained by linearly combining the 400 values of \( \tilde{V}_c \) sampled from each interval \( \tau \). e, f, Spoken-digit recognition rates in the testing set as a function of the number of utterances \( N \) used for training for the spectrogram filtering (\( 4 \mu H = 430 \text{ mT}, I_{DC} = 6 \text{ mA} \)) and for the cochlear filtering (\( 4 \mu H = 448 \text{ mT}, I_{DC} = 7 \text{ mA} \)). Because there are many ways to pick the \( N \) utterances, the recognition rate is an average over all \( 100/(10 - N) \) combinations of \( N \) utterances out of the 10 in the dataset. The red curves are the experimental results using the magnetic oscillator. The black curves are control trials, in which the pre-processed inputs are used for reconstructing the output on a computer directly, as described in Methods, without going through the experimental set-up. The error bars correspond to the standard deviation of the recognition rate, based on training with all possible combinations.

---

**Figure 2 | Spoken-digit recognition.** a–d, Principle of the experiment. a, Audio waveform corresponding to the digit 1 pronounced by speaker 1. b, Filtering to frequency channels for acoustic feature extraction. The audio waveform is divided in intervals of duration \( \tau \). The cochlear model filters each interval into 78 frequency channels (65 for the spectrogram model), which are then concatenated as 78 (65) values for each interval, to form the filtered input. c, Pre-processed input (transformed from the purple shaded region in b). The filtered input is multiplied by a randomly filled binary matrix (masking process), resulting in 400 points separated by a time step \( \theta \) of 100 ns in each interval of duration \( \tau \) (\( \tau = 4000 \)). d, Oscillator output. The envelope \( \tilde{V}(t) \) of the emitted voltage amplitude of the experimental oscillator is shown (\( \mu H = 430 \text{ mT}, I_{DC} = 6 \text{ mA} \)). The 400 values of \( \tilde{V}(t) \) per interval \( \tau \) (\( \tilde{V}_c \), sampled with a time step \( \theta \)) emulate 400 neurons. The reconstructed output ‘1’, corresponding to this digit, is
The task consists of recognizing sine waveforms from square ones with the same period. The target for the output that is reconstructed from the oscillator’s response is one for square, zero for sine. We emulate 24 neurons \( \bar{V}_i \), \( \tau = 240 \theta \), a root-mean-square (r.m.s.) deviation of output-to-target deviations: map as a function of d.c. current \( I_{DC} \) and magnetic field \( \mu_0 H \). a, Extraction of parameters from the time traces of the oscillator’s responses. Top, maximum positive \( (V_{up}) \) and negative \( (V_{down}) \) variations in the oscillator’s amplitude in response to the varying pre-processed input. Bottom, noise audio waveforms of isolated spoken digits (0 to 9) pronounced by five different female speakers (Fig. 2a). The goal is to recognize the digits, independent of the speaker.

Neural networks classify information through chain reactions: neuron after neuron, each input undergoes a series of nonlinear transformations. In a trained network, the same digit always triggers a similar chain reaction even if it is pronounced by different speakers, whereas different digits generate different chain reactions, thus allowing pattern recognition. An input can trigger a chain reaction in space by using ensembles of neurons, wherein the state of downstream neurons depends on the state of upstream neurons. But an input can also trigger a chain reaction in time by constantly exciting a single nonlinear oscillator with memory: in this case, the state of the oscillator in the future depends on the state of the oscillator in the past. We use the latter approach, which simplifies the hardware because only one oscillator is needed, but requires preprocessing of the input: each point of the audio waveform is converted into a fast-paced binary sequence that is designed to generate a chain reaction of amplitude variations in the oscillator.

The improvement shown in the experimental results over the control results (see Methods) indicates that the spin-torque nano-oscillator greatly improves the quality of spoken-digit recognition, despite the added noise that is concomitant to its nanometre-scale size. In Fig. 2e (linear spectrogram filtering), we present an example in which the extraction of acoustic features, achieved by Fourier transforming the audio waveform over finite time windows, plays a minimal part in classification. Without the oscillator (black line), the recognition rates are consistent with random choices; with the oscillator (red line), the recognition rate is improved by 70%, reaching values of up to 80%. This example highlights the crucial role of the oscillator in the recognition process. Using nonlinear cochlear filtering (Fig. 2f), which is the standard in reservoir computing and has been optimized on the basis of the behaviour of biological ears, we achieve recognition rates of up to 99.6%, as high as the state-of-the-art. Compared to the control trial, the oscillator reduces the error rate by a factor of up to 15.

The optimal operating conditions for pattern recognition are therefore comparable to the cochlear filtering used in the experimental results.

The optimal operating conditions for pattern recognition with our spin-torque nano-oscillator are determined by the oscillation amplitude and noise. We use a simpler task, classification of sine and square waveforms with the same period, to investigate the ability of the oscillator to classify waveforms in a wide range of injected d.c. currents \( I_{DC} \) and applied magnetic fields \( \mu_0 H \) (see Methods). As can be seen in Fig. 3a, the quality of pattern recognition, characterized by the root-mean-square of deviations between the reconstructed output and the target, varies from 10% to more than 30% depending on the bias conditions.

The oscillator performs well when it responds strongly to the time-varying preprocessed input, with large amplitude variations in both the current and magnetic field.
positive and negative directions, $V_{up}$ and $V_{dn}$ respectively (Fig. 3b, top). On the other hand, it performs poorly when the noise in the oscillator $\Delta V$ (the standard deviation of the noise in the voltage amplitude) is high (Fig. 3b, bottom). As shown in Fig. 3b, we extract these parameters from the time traces of the voltage emitted from the oscillator at each bias point, and plot $V_{up}V_{dn}/(\Delta V)$ (Fig. 3d) as a function of the d.c. current $I_{DC}$ and field $\mu_0H$. The red regions of large oscillation amplitudes in Fig. 3c correspond to low magnetic fields, in which the magnetization is weakly confined, and to high currents, for which the spin torque on magnetization is maximal. The blue regions of high noise in Fig. 3d correspond to areas just above the threshold current $I_{th}$ for oscillation, in which the oscillation amplitude $V$ is growing rapidly as a function of current and is becoming sensitive to external fluctuations. As can be seen by comparing Fig. 3c and d, the range of bias conditions highlighted by the dotted white boxes (currents of 6–7 mA and magnetic fields of 350–450 mT) features wide variations in oscillation amplitudes and low noise. In this region, root-mean-square deviations below 15% are achieved, and there are no classification errors between sine and square waveforms. The similarity between the map of $V_{up}V_{dn}/\Delta V$ (Fig. 3e) and that of the classification performance (Fig. 3a) confirms that the best conditions for classification correspond to regions of optimal compromise between low noise and large amplitude variations. The necessity of a high signal-to-noise ratio for efficient neuromorphic computing, highlighted here for magnetic oscillators, is a general guideline that applies to any type of nanoscale oscillator.

As a conclusion, our pattern-recognition results show that simple, ultra-compact spintronic oscillators have all of the properties that are needed to emulate collections of neurons: nonlinearity, memory and stability. The ability of groups of these oscillators to mimic neural connections by influencing the behaviour of one another through current and magnetic-field coupling opens up a route to realizing large-scale neural networks in hardware, which exploit magnetization dynamics for computing.

**Online Content** Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

*Received 25 January; accepted 2 June 2017.*

1. Buzsáki, G. *Rhythms of the Brain* (Oxford Univ. Press, 2011).
2. Hoppensteadt, F. C. & Izhikevich, E. M. Oscillatory neurocomputers with dynamic connectivity. *Phys. Rev. Lett.* **82**, 2983–2986 (1999).
3. Aonishi, T., Kurata, K. & Okada, M. Statistical mechanics of an oscillator associative memory with scattered natural frequencies. *Phys. Rev. Lett.* **82**, 2800–2803 (1999).
4. Jaeger, H. & Haas, H. Harnessing nonlinearity: predicting chaotic systems and saving energy in wireless communication. *Science* **304**, 78–80 (2004).
5. Maass, W., Natschlager, T. & Markram, H. Real-time computing without stable states: a new framework for neural computation based on perturbations. *Neural Comput.* **14**, 2531–2560 (2002).
6. Pickett, M. D., Medeiros-Ribeiro, G. & Williams, R. A. A scalable neuron built with Mott memristors. *Nat. Mater.* **12**, 114–117 (2013).
7. Segall, K. et al. Synchronization dynamics on the picosecond time scale in coupled Josephson junction neurons. *Phys. Rev. E* **95**, 032220 (2017).
8. Kiselev, S. I. et al. Microwave oscillations of a nanomagnet driven by a spin-polarized current. *Nature* **425**, 380–383 (2003).
9. Rippard, W. H., Fufall, M. R., Kaka, S., Russek, S. E. & Silva, T. J. Direct-current induced dynamics in Co$_8$Fe$_{19}$/Ni$_8$Fe$_{20}$ point contacts. *Phys. Rev. Lett.* **92**, 027201 (2004).
10. Slonczewski, J. C. Current-driven excitation of magnetic multilayers. *J. Magn. Magn. Mater.* **159**, L1–L7 (1996).
11. Berger, L. Emission of spin waves by a magnetic multilayer traversed by a current. *Phys. Rev. B* **54**, 9333–9338 (1996).
12. Tsunegi, S., Yakushiji, K., Fukushima, A., Yuasa, S. & Kubota, H. Microwave emission power exceeding 10 $\mu$W in spin torque vortex oscillator. *Appl. Phys. Lett.* **109**, 252402 (2016).
13. Sato, H. et al. Properties of magnetic tunnel junctions with a MgO/CoFeB/MgO recording structure down to junction diameter of 11 nm. *Appl. Phys. Lett.* **105**, 062403 (2014).
14. Apalikov, D., Dieny, B. & Slaught, J. M. Magnetoresistive random access memory. *Proc. IEEE* **104**, 1796–1874 (2016).
15. Slavin, A. & Tiberkevich, V. Nonlinear auto-oscillator theory of microwave generation by spin-polarized current. *IEEE Trans. Magn.* **45**, 1875–1918 (2009).
16. Kiselev, S. I. et al. Spin-wave-beam driven synchronization of nanocontact spin-torque oscillators. *Nat. Nanotechnol.* **11**, 280–286 (2016).
17. Macià, F., Kent, A. D. & Hoppensteadt, F. C. Spin-wave interference patterns created by spin-torque nano-oscillators for memory and computation. *Nanotechnology* **22**, 095301 (2011).
18. Pufall, M. R. et al. Physical implementation of coherently coupled oscillator networks. *IEEE J. Explor. Solid-State Comput. Devices Circuits* **1**, 76–84 (2015).
19. Soriano, D. et al. Coupled-oscillator associative memory array operation for pattern recognition. *IEEE J. Explor. Solid-State Comput. Devices Circuits* **1**, 85–93 (2015).
20. Yogendra, K., Fan, D. & Roy, K. Coupled spin torque nano oscillators for low power neural computation. *IEEE Trans. Magn.* **51**, 405909 (2015).
21. Grollier, J., Querlioz, D. & Stiles, M. D. Spintronic nanodevices for bioinspired computing. *Proc. IEEE* **104**, 2024–2039 (2016).
22. Grimaldi, E. et al. Response to noise of a vortex based spin transfer nano-oscillator. *Phys. Rev. B* **89**, 104404 (2014).
23. Soriano, D. et al. Delay-based reservoir computing: noise effects in a combined analog and digital implementation. *IEEE Trans. Neural Netw. Learn. Syst.* **26**, 388–393 (2015).
24. Appeltant, L. et al. Information processing using a single dynamical node as complex systems. *Nat. Commun.* **3**, 1139 (2012).
25. Paquot, Y. et al. Optoelectronic reservoir computing. *Sci. Rep.* **2**, 287 (2012).
26. Martinenghi, R., Rybalko, S., Jacquot, M., Chembo, Y. K. & Larger, L. Photonic nonlinear transient computing with multiple-delay wavelength dynamics. *Phys. Rev. Lett.* **108**, 244101 (2012).
27. Texas Instruments. 46-Word Speaker-Dependent Isolated Word Corpus (TI-46), https://catalog.ldc.upenn.edu/LDC93S9 (NIST, 1991).
28. LeCun, Y., Bengio, Y. & Hinton, G. Deep learning. *Nature* **521**, 436–444 (2015).
29. Yildiz, I. B., von Kriegstein, K. & Kiebel, S. J. From birdsong to human speech recognition: Bayesian inference on a hierarchy of nonlinear dynamical systems. *PLoS Comput. Biol.* **9**, e1003219 (2013).
30. Lyon, R. A computational model of filtering, detection, and compression in the cochlea. in *IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP 82)* Vol. 7, 1282–1285 (IEEE, 1982).

**Acknowledgements** This work was supported by the European Research Council (ERC) under grant bioSPI/Nispire 682955. We thank L. Larger, B. Penkovsky and F. Duport for discussions.

**Author Contributions** The study was designed by J.G. and M.D.S., samples were optimized and fabricated by S.T. and K.Y., experiments were performed by J.T. and M.R., numerical studies were optimized and fabricated by S.T. and K.Y., experiments were performed by J.T. and M.R., numerical studies were realized by F.A.A., M.R. and G.K., and all authors contributed to analysing the results and writing the paper.

**Author Information** Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Publisher’s note: Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations. Correspondence and requests for materials should be addressed to J.G. (julie.grollier@cnrs-thales.fr).

**Reviewer Information** Nature thanks F. Hoppensteadt and the other anonymous reviewer(s) for their contribution to the peer review of this work.
METHODS

Samples. Magnetic tunnel junction (MTJ) films with a stacking structure of buffer/Pt/Mn(15)/Co$_2$Fe$_2$O$_4$(2.5)/Ru(0.9)/Co$_2$Fe$_2$O$_4$(1.6)/Co$_2$Fe$_2$O$_4$(0.8)/MgO(0.1)/FeO$_2$(6)/MgO(0.1)/Ta(8)/Ru(7) (with thicknesses given in parentheses in nanometres) were prepared by ultrahigh vacuum (UHV) magnetron sputtering. After annealing at 360 °C for 1 h, the resistance–area products (RA) were approximately 3.61 Ω m$^2$. Circular-shape MTJs with a diameter of approximately 375 nm were patterned using Ar ion etching and e-beam lithography. The resistance of the samples is close to 0.8 kΩ. The magnetoresistance ratio is about 135% at room temperature. The FeO$_2$ layer forms a vortex structure as the ground state for the dimensions used here. In a small region called the core of the vortex, the magnetization spirals out of plane. Under d.c. current injection, the core of the vortex steadily gyrates around the centre of the dot with a frequency in the range 250–400 MHz for the oscillators we consider here. Vortex dynamics driven by spin torque are well understood, well controlled and have been shown to be particularly stable.

Measurement set-up. The experimental implementation for spoken-digit recognition and sine/square classification tasks is illustrated in Fig. 1d. The pre-processed input signal $V_{in}$ is generated by a high-frequency arbitrary-waveform generator and injected as a current through the magnetic nano-oscillator. The sampling rate of the source is set to 200 MHz (20 points per interval of time $\theta$) for the spoken-digit recognition task and 500 MHz (50 points per interval of time $\theta$) for the classification of sines and squares. The peak-to-peak variation in the input signal is 500 mV, which corresponds to peak-to-peak current variations of 6 mA, as illustrated in Fig. 1c (part of the incoming signal is reflected owing to impedance mismatch). The bias conditions of the oscillator are set by a d.c. current source and an electromagnet which applies a field perpendicular to the plane of the magnetic layers. The oscillating voltage emitted by the nano-oscillator is rectified by a planar tunnel microwave diode, with a bandwidth of 0.1–12.4 GHz and a response time of 5 ns. The input dynamic range of the diode is between 1 mW and 3.15 mW, corresponding to a d.c. output level of 0–400 mV. We use an amplifier to adjust the emitted power of the nano-oscillator to the working range of the diode. The output signal is then recorded by a real-time oscilloscope. In Figs 1b, c, e, 2d and 3b–e, the amplitude of the signal emitted by the oscillator is shown without amplification (the signal measured after the diode has been divided by the total amplification of the circuit, about $+21$ dB). If, owing to sampling errors, the measured envelope of the oscillators is shifted with respect to the input, classification accuracy can be degraded. We use alignment marks to align our measurements with the input when we reconstruct the output. The alignment precision is $\pm 1$ ns.

General concepts of reservoir computing. In machine learning, a reservoir is a network of recurrently and randomly connected nonlinear nodes. When an input signal is injected in the reservoir, it is mapped to a higher-dimensional space in which it can become linearly separable. The key insight behind reservoir computing is that the network does not need any tuning: all connections inside the reservoir are kept fixed. Only external connections (between the reservoir and an output layer) are trained to achieve the desired task.

In other words, reservoir computing requires the generation of complex nonlinear dynamics but, as a trade-off, learning is greatly simplified. For efficient reservoir computing, several requirements related to the dynamical properties of the network should be satisfied. First, different inputs should trigger different dynamics (separation property) and similar inputs should generate similar dynamics (approximation property), enabling efficient classification. Second, the reservoir state should not depend only on present inputs but also on recent past inputs. This short-term memory, called fading memory, is essential for processing temporal sequences for which the history of the signal is important.

A single nonlinear oscillator can emulate a reservoir when it is set in transient dynamics by a rapidly varying input\textsuperscript{23}. The loss of parallelism is compensated by an additional pre-processing input step: the input is multiplied by a rapidly varying d.c. current source. The first half of the signals with periods of 1 and $–1$ values is fed into the oscillator during the resultant oscillator dynamics. This approach provides a marked simplification of the reservoir scheme for hardware implementations, and has been realized in hardware with optical or electronic oscillators assembled from several components.

Spoken-digit recognition. For this task, the inputs are taken from the NIST TI-46 data corpus\textsuperscript{24}. The input consists of isolated spoken digits said by five different female speakers. Each speaker pronounces each digit ten times. The 500 audio waveforms are sampled at a rate of 12.5 kHz and have variable time lengths. The task is the recognition of isolated spoken digits in a real-time processing on a nanosecond timescale could be realized using fully parallel networks of interacting nano-oscillators.

During the classification phase, the ten reconstructed outputs corresponding to one digit are averaged over all of the time steps $\tau$ of the signal, and the digit is identified by taking the maximum value of the ten averaged reconstructed outputs. The post-processing of the output consists of two distinct steps. The first is the called the training or learning process and the second is called the classification (or recognition) process. The goal of training is to determine a set of weights $w_{ij}$, where $i$ indexes the desired digit. These weights are used to multiply the output voltages to give 10N, output values, which are then averaged over the $N$ time intervals to give 10 output values $y_j$, which should ideally be equal to the target values $y_j = 1.0$ for the appropriate digit and 0.0 for the rest. In the training process, a fraction of the utterances are used to train these weights; the rest of the utterances are used in the classification process to test the results.

The optimum weights are found by minimizing the difference between $y_j$ and $y_j$ for all of the words used in the training. In practice, optimal values are determined by using techniques for extracting meaningful eigenvalues from singular matrices such as the linear Moore–Penrose pseudo-inverse operator (denoted by a dagger symbol $\dagger$). If we consider the target matrix $Y$, which contains the targets $y_j$ for all of the time steps $\tau$ used for the training, and the response matrix $X$, which contains all neuron responses for all of the time steps $\tau$ used for the training, then the matrix $W$, which contains the optimal weights, is given by $W = \dagger X^t Y$. This step is performed on a computer and takes several seconds. In the future, real-time processing on a nanosecond timescale could be realized using fully parallel networks of interacting nano-oscillators.

Sine- and square-wave classification. For this classification task, the input is a random sequence of 160 sines and squares with the same period—the first half of the sequence for training and the second half for classification. Each period is discretized into eight points separated by a time step $\tau$. The pre-processing consists of multiplying the value of each point by the same binary sequence that is used for the training and also the second and third, or the sixth and tenth, or any other of the 10!/(8!2!) combinations of 2 picked out of 10. To avoid this bias, the recognition rates that we present are an average of the results over all possible combinations. The error bars correspond to the standard deviation of the word recognition rate. The raw spectrogram is not complex enough to allow a correct reconstruction of the target during the training. Adding the oscillator brings complexity and suppresses this phenomenon.

© 2017 Macmillan Publishers Limited, part of Springer Nature. All rights reserved.