We consider anisotropies in the Cosmic Microwave Background (CMB) generated by spatially limited seeds; these objects could correspond to relics of high energy symmetry breaking in the early universe. It is shown how the CMB perturbation propagate beyond the size of the seed in the form of waves traveling with the CMB sound velocity. Moreover, these waves are the substantial part of the signal, both for polarization and temperature. The explanation of this phenomenon in terms of the CMB equations is given.

Observationally, this effect is threefold promising. First, it enlarges the signal from a seed intersecting the last scattering surface to the scale of the CMB sound horizon at decoupling, that is roughly one degree in the sky. Second, it offers cross correlation possibilities between the polarization and temperature signals. Third, it allows to unambiguously distinguish these structures from point-like astrophysical sources.

I. THE PEBBLES

There is now a lot of interest in the connection between high energy physics and cosmology. It is motivated by the possibility that processes not reproducible here on the Earth actually occurred in the early universe. For this reason, a lot of work is currently in progress to predict in detail the traces that such processes could have left, in order to recognize them and gain insight into physics that is still unknown, or only theoretically approached. The unknown sector of physics extends from the energy scales presently explored by accelerators, described successfully by the standard model of the known elementary particles, up to the scales of more fundamental theories, perhaps supersymmetry and supergravity; such regimes, as thought in the early universe, should have taken place at temperatures $T$ (in energy units) in the interval

$$10^2 \text{GeV} \leq T \leq 10^{19} \text{GeV} \text{ or more .} \tag{1}$$

According to our hypotheses, two main classes of phenomena took place in the early universe: an era of accelerated expansion, the inflation, and the breaking of high energy symmetries, see \cite{1}.

The first process should leave traces in the form of Gaussian and scale-invariant density fluctuations \cite{2}; this visually corresponds to a completely disordered distribution of hills and wells in the density field, covering all the scales.

The second process leaves completely different traces: spatially limited structures, like topological defects \cite{3}, or bubbles made both of true and false vacuum \cite{4}. At the present status of the theoretical knowledge, their production may occur with or without inflation. Models able to produce such structures both during and after inflation have been studied \cite{4,5}. In order to be observable, the first case is most interesting, since the size of the structure is stretched by the stage of inflation after their formation, up to cosmologically interesting scales of tens of comoving Mpc or more.

As well as the Gaussian fluctuations, these structures may be considered as seeds for the CMB perturbations. In the recent past, they have been thought as candidates for the structure formation process with preliminary discouraging results \cite{4}, even if the numerical simulations and the models to explore are far to be exhausted; unfortunately, we do not have a good theory to predict their exact properties and abundance. The only sure thing is that the detection of at least one of them would be the first observational evidence of the existence of high energy symmetries. So the analysis here regards the signal from each single seed, without requiring neither that they dominate the structure formation process, nor that their signature is present on the whole sky CMB power spectrum.

These seeds may also be thought to possess some spatial symmetries, both because appropriate and because the problem becomes simpler. Spherical and cylindrical symmetries are particularly simple and appropriate for bubbles, monopoles and strings, also forming loops \cite{1,2,3}; also they allow to write simple and suitable formulas for the CMB perturbations; we refer to \cite{6} for a more quantitative and detailed exposition of these aspects.

In this work we point out the characteristic signature of these structures on the CMB, in direct connection with the forthcoming whole sky CMB experiments \cite{7}. As we shall see, their spatial shape combined with the undulatory properties of the CMB physics mix and produce their unambiguous signs.

II. THE POND

We begin with some necessary technical detail, but we hope to finish with physically simple and intuitive results.

In linear theory, and assuming a Friedmann Robertson Walker (FRW) background, the equations driving the energy density perturbation and the peculiar motion of photons can be obtained from the linearized Einstein equations \cite{8}. Perturbations may be classified as scalar, vector and tensor with respect to spatial rotations; bub-
bles or topological defects are essentially fluctuations in the energy density composed by matter, radiation as well as scalar fields, therefore the case of interest here is the first one. The linearization implies a gauge freedom with respect to infinitesimal frame transformations; we choose the Newtonian gauge which physically corresponds to observers at rest with respect to the comoving expansion and experiencing the latter isotropic.

Perturbations in the CMB photons are coupled to the fluctuations of the other constituents of the cosmic energy density. In particular, Thomson scattering between baryons and photons induces polarization perturbations in the CMB, being an anisotropic process. At early times, the fluid is so dense that the photons free path $\hat{\tau}^{-1}$ vanishes; it is small with respect to the Hubble horizon $H^{-1}$ and the perturbation wavelength $1/k$. Therefore, the CMB equations may be expanded in powers of $k/\hat{\tau}$ and $H/\hat{\tau}$. In practice, the first order terms become important when decoupling, when the photons free path suddenly increases. One can consider CMB photons traveling on a direction $\hat{n}$ in the spacetime point $x \equiv (\eta, \vec{x})$, where $\eta$ is the conformal time defined in terms of the ordinary time $t$ and of the scale factor $a$ by $d\eta = dt/a$. CMB temperature and polarization perturbations are expanded into spherical harmonics describing the dependence on $\hat{n}$. This treatment was firstly used in [4] and recently expanded to include non-flat geometries and non-scalar perturbations [6]. For each Fourier mode, computations are performed in the $\hat{k}$-frame, where the wavevector $\hat{k}$ is the polar axis for the angular expansion; the fixed laboratory frame is instead indicated as the $lab$-frame; this distinction is particularly important for the perturbations considered here.

To fix the ideas, before decoupling the CMB dynamics may be considered at the zeroth order in the Thomson scattering terms. Thus the equations for the energy density fluctuations $\delta$ and peculiar motion $v$ of photons are easily gained by the linearized conservation equations and have the simple following form:

$$\dot{\delta} = -\frac{4k}{3}v - 4\dot{\Phi}, \quad \dot{v} + \frac{\dot{a}}{a}3\rho_b \left(1 + \frac{3\rho_b}{4\rho}\right)^{-1} \cdot v = \frac{k}{4} \left(1 + \frac{3\rho_b}{4\rho}\right)^{-1} \cdot \delta + k\Psi. \quad (2)$$

$\Phi$ and $\Psi$ are the two scalar metric perturbations accounting for fluctuations from all the fluid species, and $a, \dot{a}$ are the cosmic scale factor and its derivative with respect to the conformal time. The terms containing the unperturbed baryon density $\rho_b$ are a residual of the coupling between photons and baryons, at the zeroth order in the Thomson scattering. Also we point out that $\delta, v$ are simply linked to the monopole and the dipole of the CMB temperature fluctuation dependence on the photon propagation direction $\hat{n}$:

$$\left(\frac{\delta T}{T}\right)_0 = \frac{1}{4} \delta, \quad \left(\frac{\delta T}{T}\right)_1 = v. \quad (3)$$

Equations (2) may be put together in the following most simple form:

$$\ddot{\delta} + \frac{\dot{a}}{a}3\rho_b \left(1 + \frac{3\rho_b}{4\rho}\right)^{-1} \cdot \ddot{\delta} + \frac{k^2}{3} \left(1 + \frac{3\rho_b}{4\rho}\right)^{-1} \cdot \delta = -\frac{4k^2}{3}\Psi - \frac{\dot{a}}{a}3\rho_b \left(1 + \frac{3\rho_b}{4\rho}\right)^{-1} \cdot \dot{\Phi} - \dot{\Phi}. \quad (4)$$

This is a wave equation with friction of cosmological origin (proportional to $\dot{a}/a$) and forcing, gravitational terms at the second member. Focus on the friction term. Its effect is simple. Much before the horizon crossing the following conditions are satisfied [5]: the second term of the first member dominates over the third one, the gravitational potentials are constant and $\rho \gg \rho_b$; therefore the solution is trivially $\delta = -4\Psi$ and no friction effect exists at all. At the horizon crossing the last term of the first member becomes important, and $\delta$ starts to oscillate, making $\delta$ not vanishing. Thus the friction becomes active and damps the oscillations. Now we come to the central arguments of the present work.

Differently from analogous problems in cosmology, the friction term is multiplied by $\rho_b/\rho$. In most cosmological models the baryon content is of the order of percent with respect to the dark matter component, because of the severe constraints from nucleosynthesis [11]. Therefore it is evident that

$$10^{-2} \leq \frac{\rho_b}{\rho} \leq 10^{-1} \quad (5)$$

between equivalence and decoupling, reducing substantially the friction term for the oscillations occurring at these epochs.

For the perturbations considered here, the forcing terms in (4) are active on spatially limited regions, occupied by the seed. The arguments above show that outside the seed the equation for $\delta$ is

$$\ddot{\delta} + \frac{k^2}{3}\delta \simeq 0. \quad (6)$$

This says that the oscillations occurring at the horizon crossing are brought outside the seed with the CMB sound velocity. They shall reach the sound horizon at the time considered, given by

$$\eta_s(\eta) \simeq \int_{\eta_0}^{\eta} \frac{1}{\sqrt{3}} d\eta' = \frac{\eta}{\sqrt{3}}. \quad (7)$$

and it can be shown that it corresponds roughly to one degree in the sky; we remark that $\delta$ waves mean $\delta T/T$ waves from (3), and this means polarization waves, since polarization and temperature perturbations are tightly coupled.

The consequences of the exposed arguments are straightforward. Consider a spatially limited seed intersecting the last scattering surface. Its CMB signal is
made of waves extending approximatively over one degree in the sky. This is extremely interesting for the future whole sky, high resolution observations \[7\]. These same waves are a unambiguous proof of the fact that really the seed was generated well before decoupling, simply because any other astrophysical source does not produce them because it formed after decoupling, appearing point-like. These concepts might be observationally decisive if our theoretical thinking is quite right and high energy symmetry breaking traces are really present in the nearby universe.

In the next section we give a numerical example of the realization of the phenomenology exposed here in the context of the standard cosmological scenario. We refer to \[7\] for all the computational and formal details.

III. THE WAVES

We considered spherical and infinitely long cylindrical energy concentrations in a background formed by cold dark matter (\(\Omega_{CDM} = .95\)), baryons (\(\Omega_b = .05\)), photons and massless neutrinos, assumed distributed initially adiabatically. At any time, and for this kind of sources, the expressions of the CMB temperature and polarization perturbations may be written in simple and intuitive forms \[7\]. We give here the expressions for spherical sources.

**Temperature.** The temperature CMB perturbation at any spacetime point \((\eta, \vec{r})\) for a spherical source is given by

\[
\frac{\delta T}{T} = \sum_{l \geq 0} P_l(\hat{n} \cdot \hat{r}) \int_0^\infty \frac{k^2 dk}{2\pi^2} \left( \frac{\delta T}{T} \right)_l(k, \eta) j_l(kr) \, . \tag{8}
\]

\(P_l\) and \(j_l\) are the Legendre polynomials and the fractional order Bessel function respectively. Note how the spatial directions \(\hat{n}\) and \(\hat{r}\), describing geometrically the problem, are outside the Fourier integral. In spite of the virtually infinite series, already the \(l = 0\) term is the substantial component of the signal, as we show below.

**Polarization.** It is described by the Stokes parameters \(Q\) and \(U\) on the plane perpendicular to the photon propagation direction \(\hat{n}\). As for the temperature case, each Fourier mode of the polarization perturbations admit an expansion in spherical harmonics with coefficients \(Q_l\) and \(U_l\). Remembering that these quantities are defined in the \(\vec{k}\)-frame, the problem may be further simplified since \(U_l = 0\) for scalar perturbations. Also we mention that as a distinction with respect to the temperature case, the tensor spherical harmonics are now required, accounting for the tensor properties of the polarization; for this reason, \(Q_l\) and \(U_l\) are defined for \(l \geq 2\). The polarization CMB perturbation at any spacetime point \((\eta, \vec{r})\) in presence of a spherical source is given by

\[
Q = \cos(2\phi_\tau) \cdot \mathcal{I} \quad , \quad U = -\sin(2\phi_\tau) \cdot \mathcal{I} \quad , \tag{9}
\]

\[
\mathcal{I} = \sum_{l \geq 2} \sqrt{\frac{(l-2)!}{(l+2)!}} P_l^2(\hat{n} \cdot \hat{r}) \int_0^\infty \frac{k^2 dk}{2\pi^2} Q_l(k, \eta) j_l(kr) \, .
\]

\(Q\) and \(U\) are essentially identical except for the geometrical dependence on \(\phi_\tau\), that is the angular coordinate of the projection of \(\vec{r}\) on the plane perpendicular to \(\hat{n}\), as seen in the \(lab\)-frame: if the latter is chosen so that \(\phi_\tau = 0\) we have \(U = 0\). Note that now the second order Legendre polynomials compare into the sum. They are meaningful since prevent photons traveling radially \((\hat{n} \cdot \hat{r} = \pm 1)\) to be polarized; this is correct since in that case no preferred axis exists for polarization. As for the temperature case, the substantial component of the perturbation is given by the first term of the sum, \(l = 2\).

Note that these relations are independent on the nature of the seed, which is encoded in the \((\delta T/T)_l, Q_l, U_l\) coefficients and could be made of matter and radiation as well as scalar field, that has the only restriction in its sphericity. Each coefficient \((\delta T/T)_l, Q_l, U_l\) obeys motion equations, and together with the linearized Einstein ones the whole system may be evolved in time, see \[7\] and references therein.

Figures 3 and 4 show the time evolution of the CMB temperature and polarization perturbations. In each panel the perturbation profile is shown as a function of the distance from the seed center and symmetry axis respectively for the spherical and cylindrical case. The perturbations are normalized with the density contrast at decoupling \(\delta\), taken at the center for the spherical seed and on the symmetry axis for the cylindrical one. At the horizon crossing, CMB waves form and travel outward, just like the waves from a pebble thrown in a pond.

The upper panels regards the temperature signals, while the lower ones shows the polarization amplitude. The thin lines represents the signal only from the monopole term in \[8\], while the thick ones contains all the contributions; this shows how the monopole is the dominant term. As indicated, photons travel perpendicularly to the radial distance \(\vec{r}\) for the spherical case and on the plane perpendicular to the symmetry axis for the cylindrical case. The symmetries of the seed allow to choose the \(lab\)-frame axes so that the polarization is given by a pure \(Q\) term \[8\]. In the spherical case they are simply parallel and perpendicular to the plane formed by \(\hat{n}\) and the radial direction \(\hat{r}\) in the scattering point; in the cylindrical case they are the symmetry axis \(\hat{z}\) and the direction perpendicular to \(\hat{n}\) and \(\hat{z}\).

Several interesting comments may be made on these graphs.

The waves at any time occupy the position of the CMB sound horizon \[7\], and have roughly the same amplitude of the perturbation inside the seed. Really, in the polarization case, they are the very dominant component of the signal. Also a marked correlation is evident between the polarization and temperature waves.

The symmetries of the seeds constrain the signal. Photons traveling radially in a spherical seed must be not
polarized since this problem is symmetric with respect to rotations around the propagation direction $\hat{n}$. In the cylindrical case the symmetry axis itself is a preferred direction and polarization perturbations may affect photons traveling away from the symmetry axis.

Again we remark that this undulatory behavior of the CMB perturbations occurs for seeds existing since the beginning, $\eta = 0$; no waves at all arise from sources formed after decoupling.

These figures represent the CMB perturbations around spatially limited seeds at different times. Now we show in the spherical case the real simulation of the CMB anisotropies, in order to show how the exposed behavior is maintained [7]. Figure 8 shows the CMB anisotropy from the spherical seed if it intersects the last scattering surface, as a function of the angle $\theta$ from the center. The Sachs-Wolfe effect regarding the zone physically occupied by the seed, $\theta \leq 10^\prime$ has been included. The CMB perturbation waves have been photographed by the decoupling photons. Of course the signal is a function of the relative disposition of the seed with respect to the last scattering surface. Different cases have been displayed: the solid line represents the case in which the seed center lies exactly on the last scattering surface; the dashed and dotted dashed lines shows the signals if the seed lies $30h^{-1}$ Mpc within or outside our Hubble sphere. In particular, if the distance between seed and last scattering surface is much larger then a CMB sound horizon at decoupling, roughly $100h^{-1}$ Mpc, the seed is visible only through the integrated Sachs-Wolfe effect if it lies within our Hubble sphere; in this case however the CMB waves could not be detected, even if distinctive signals would arise [8].

The amplitude of the signal is roughly as expected for linear perturbations with density contrast $\delta$ and size $L$ smaller than $H^{-1}$ at decoupling: $\delta T/T \simeq \delta \cdot (LH)^2$ and a few percent of this for the polarization amplitude.

Of course a lot of work has to be done to predict in detail the signs from any relic of the early universe; at the moment such prediction exists only for the temperature signals from inflationary bubbles [8].

The final general statement that we make here is the following. If the forthcoming high resolution CMB maps should contain perturbed spots surrounded by anisotropy waves reaching the distance of the CMB sound horizon at decoupling, then we could conclude that the processes that generated such signals belong to the unknown very high energy sector of physics. This would open new possibilities for testing fundamental physical theories.

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FIG. 1. CMB perturbation waves from a spherical seed at different times and as a function of the radial distance. The seed has the indicated comoving radius. Waves form at the horizon crossing and travel outward, both for temperature (up, indicated as $\Theta$) and polarization (down, indicated as $Q$). They are well visible just before decoupling and are the unique sign of the previous history of the seed itself.

FIG. 2. CMB perturbation waves from a cylindrical seed at different times and as a function of the distance from the symmetry axis. Waves form at the horizon crossing and travel outward like in figure 1.
FIG. 3. CMB anisotropy from a spherical seed as a function of the angular distance $\theta$ from the central direction. The waves as well as the central temperature spot evident in figure [have been photographed by the decoupling photons and are visible to us.}