The Hyperfine Splittings in Bottomonium and the $B_q (q = n, s, c)$ Mesons

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A universal description of the hyperfine splittings (HFS) in bottomonium and the $B_q (q = n, s, c)$ mesons is obtained with a universal strong coupling constant $\alpha_s(\mu) = 0.305(2)$ in a spin-spin potential. Other characteristics are calculated within the Field Correlator Method, taking the freezing value of the strong coupling independent of $n_f$. The HFS $M(B^*) - M(B^0) = 45.3(3)$ MeV, $M(B^* - M(B^0) = 46.5(3)$ MeV are obtained in full agreement with experiment both for $n_f = 3$ and $n_f = 4$. In bottomonium, $M(\Upsilon(9460)) - M(\eta_b) = 70.0(4)$ MeV for $n_f = 5$ agrees with the BaBar data, while a smaller HFS, equal to 64(1) MeV, is obtained for $n_f = 4$. We predict HFS $M(\Upsilon(2S)) - M(\eta_b(2S)) = 36(1)$ MeV, $M(\Upsilon(3S)) - M(\eta_b(3S)) = 27(1)$ MeV, and $M(B^+ - M(B^0) = 57.5(10)$ MeV, which gives $M(B^*_c) = 6334(1)$ MeV, $M(B_c(2S^3S_1)) = 6865(5)$ MeV, and $M(B^*_c(2S^3S_1)) = 6901(5)$ MeV.

I. INTRODUCTION

Recently, $\eta_b(1S)$ has been discovered by the BaBar collaboration in the radiative decays $\Upsilon(3S) \rightarrow \gamma \eta_b(1S)$ and $\Upsilon(2S) \rightarrow \gamma \eta_b(1S)$, with a mass (averaged over two results) $M(\eta_b) = 9391.1 \pm 3.1$ MeV. It gives a rather large hyperfine splitting (HFS), $\Delta(\eta_b) = M(\Upsilon(1S)) - M(\eta_b(1S)) = 69.9 \pm 3.1$ MeV. Later this mass was confirmed by the CLEO collaboration also in the radiative $\Upsilon(3S) \rightarrow \gamma \eta_b(1S)$ decay. This important new information allows to test again our understanding of the hyperfine (HF) interaction in QCD.

Although a spin-spin potential between heavy quarks was used in numerous studies, the parameters defining this potential significantly differ in different models. As a result, theoretical predictions for the mass difference $\Delta(\eta_b) = M(\Upsilon(9460)) - M(\eta_b(1S))$ vary in a wide range: $35 - 90$ MeV and in most cases they are smaller than the experimental number. On a fundamental level the spin-spin potential $V_{ss}(\text{lat})$ has been studied in detail in quenched QCD (see references therein). This lattice potential appears to be compatible with zero at distances $r \geq 0.3$ fm and (for unknown reasons) has negative sign at smaller $r$ (with a large magnitude); in any case the lattice potential does not contradict the Fermi-Breit potential with $\delta^3(r)$, although the behavior of the spin-spin potential at $r \leq 0.3$ fm remains uncertain.

On the other hand, a detailed phenomenological analysis given in Ref. has demonstrated the importance of the smearing of the $\delta^3(r)$-function, from which one may expect that for heavy mesons, containing a $b$-quark, the use of $\delta^3(r)$ may be a good approximation. For lighter mesons, like $D, D_s$, and charmonium, a nonperturbative spin-spin potential may be essential, giving a contribution $\sim 10\%$. Here we concentrate on bottomonium and the $B_q (q = n, s, c)$ mesons, for which nonperturbative contributions are small, and neglect the smearing effect, in this way avoiding to introduce several unknown parameters.

Our main goal here is the extraction of the strong coupling $\alpha_s(\mu)$ from known HFS. In theoretical models two typical choices of $\alpha_s(\mu)$ are used:

1. In the first one, “a universal” $\alpha_s(\mu)$ is used. For example, in Ref. $\alpha_s(\mu) = 0.36$ taken, was obtained from a fit to the mass difference $M(J/\psi) - M(\eta_b(1S)) = 117$ MeV, but their HFS, $M(\Upsilon(9460)) - M(\eta_b) = 87$ MeV, is $\sim 25\%$ larger than the experimental number. In Ref. using a smaller $\alpha_s(\mu) = 0.339$ a good description of the HFS of the $B$ and $B_c$ mesons was obtained. However, a comparison of their and our results is difficult, because a large string tension, $\sigma = 0.257$ GeV$^2$, was taken in [9], while here and in Ref. the conventional value $\sigma = 0.18$ GeV$^2$ is used.

2. The second choice, with a scale $\mu$ dependent on the quark mass, is mostly used in pQCD, where $\alpha_s(\mu_b) \sim 0.18$ and $\alpha_s(\mu_c) \sim 0.26$. Just due to such a small value of $\alpha_s(\mu_b)$, taken in bottomonium, small HFS were obtained in Ref. [12], although their wave functions (w.f.) at the origin gave excellent descriptions of the dielectron widths for $\Upsilon(nS)(n = 1, 2, 3)$.

Here we use instead of the Fermi-Breit potential a spin-spin potential derived using the Field Correlator Method.
(FCM) \cite{14}, where relativistic corrections are taken into account and with the mass of a light quark \(m_n = 5\) MeV \((n = u, d)\) and \(m_s = 200\) MeV for an s-quark, the \(B, B_s\) mesons can be considered on the same footing as the \(B_c\) mesons.

It can be shown that the HFS are sensitive to the value of \(\Lambda_{\overline{MS}}(n_f)\) taken. Since \(\Lambda_{\overline{MS}}\) is known only for \(n_f = 5\) and \(\Lambda_{\overline{MS}}\) used for \(n_f = 3, 4\), varies in a wide range, we make here the assumption, already used in \cite{4}, that the freezing value of the vector coupling constant (denoted as \(\alpha_{\text{crit}}(n_f)\)) is the same for \(n_f = 3, 4, 5\).

Then it appears possible to obtain a good description of the HFS for the \(B_s\) mesons \((q = n, s, c)\) and bottomonium, taking a universal \(\alpha_s(\mu) = 0.305(2)\) (with small one-loop corrections): its value is smaller than in Refs. \cite{2} and \cite{3}.

We also predict the HFS of the as yet undiscovered \(\eta_b(2S)\) and \(\eta_b(3S)\), and the mass of \(B_c^+\).

II. THE HF POTENTIAL IN THE FIELD CORRELATOR METHOD

The Fermi-Breit potential,

\[
\hat{V}_{ss}(r) = s_1 \cdot s_2 - \frac{32\pi}{9} \frac{\alpha_{\text{hf}}(\mu)}{\bar{m}_1 \bar{m}_2} \delta^3(r),
\]

(1)

widely used in heavy quarkonia, contains the constituent quark masses \(\bar{m}_1\) and \(\bar{m}_2\), which are very much model-dependent.

In Eq. (1) the strong coupling constant \(\alpha_{\text{hf}}(\mu)\) may differ from \(\alpha_s(\mu)\) (in the \(\overline{\text{MS}}\) renormalization scheme) due to higher order perturbative corrections. Here we take the one-loop corrections from \cite{13},

\[
\alpha_{\text{hf}}(\mu) = \alpha_s(\mu) \left[ 1 + \frac{\alpha_s(\mu)}{\pi} \rho(n_f) \right],
\]

(2)

where

\[
\rho = \frac{5}{12} \beta_0 - \frac{8}{3} - 3 \frac{3}{4} \ln 2.
\]

(3)

Although higher corrections are small: \(\sim 6\%\) for \(n_f = 3, \sim 3\%\) for \(n_f = 4\), and \(\leq 0.1\%\) for \(n_f = 5\), still they are needed to improve the accuracy and have a more self-consistent picture.

The important role of relativistic corrections, even for the \(B_c\) meson, has already been underlined in Refs. \cite{4} and \cite{14}, and in the lattice calculations of the \(B_c\) mass \cite{16, 17}. We take them into account using the spin-spin potential (without smearing), derived in the FCM \cite{14, 18}:

\[
\hat{V}_{ss}(r) = s_1 \cdot s_2 - \frac{32\pi}{9} \frac{\alpha_{\text{hf}}(\mu)}{\omega_1 \omega_2} \delta^3(r),
\]

(4)

for which the HFS is

\[
\Delta_{\text{hf}}(nS) = \frac{8}{9} \frac{\alpha_{\text{hf}}(\mu)}{\omega_1 \omega_2} |R_n(0)|^2.
\]

(5)

In Eqs. (4) and (5) the variables \(\omega_1(nS), \omega_2(nS)\) are the averaged kinetic energies of a quark 1 and an antiquark 2, which play a role of the dynamical masses:

\[
\omega_1(nS) = \sqrt{p^2 + m_1^2}, \quad \omega_2(nS) = \sqrt{p^2 + m_2^2}.
\]

(6)

The important point is that in Eq. (6) the masses \(m_1\) and \(m_2\) are well defined; they are the pole masses of \(c\) and \(b\) quarks (now known with an accuracy of \(\sim 70\) MeV for a \(b\) quark and \(\sim 100\) MeV for a \(c\) quark \cite{19}). In leading order, the pole mass does not depend on the number of flavors, while to order \(\alpha_s(\bar{m}_Q)\) it slightly depends on \(n_f\).

We take here \(m_1 = m_n = 5\) MeV for a light quark \((n = u, d); m_s = 200\) MeV for an \(s\) quark; the pole mass \(m_c = 1.41\) GeV; \(m_b = 4.79\) GeV for \(n_f = 3\) and \(m_b = 4.82\) GeV for \(n_f = 4.5\).

The quantities \(\omega_1\) and the w.f. are calculated with the use of the relativistic string Hamiltonian (RSH), also derived in the FCM \cite{19}.

\[
H_0 = \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{p^2}{2\omega_{\text{red}}} + V_B(r),
\]

(7)

where the variables \(\omega_1\) enter as the kinetic energy operators. However, if one uses an einbein approximation \cite{18, 21} and considers the spin-dependent potential as a perturbation, then \(\omega_1\) should be replaced by its matrix elements (m.e.) \cite{4}.

A simple expression for the spin-averaged mass \(M(nS)\) follows from the RSH \cite{21}:

\[
M(nS) = \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{p^2}{2\omega_{\text{red}}} + V_B(r),
\]

(8)

Here, the excitation energy \(E_{nS}(\omega_{\text{red}})\) depends on the reduced mass: \(\omega_{\text{red}} = \frac{\omega_1 + \omega_2}{2\omega_1 + 2\omega_2}\). The mass formula \(\omega_1\) does not contain any additive constant in the case of bottomonium, while for the \(B\) and \(B_s\) mesons a negative (not small) self-energy term, proportional to \((\omega_1)^{-1}\) \((q = n, s)\), has to be added to their masses \cite{22}.

Then the variables \(\omega_i(nS)\), the excitation energy \(E_{nS}(\omega_{\text{red}})\), and the w.f. are calculated from the Hamiltonian \(\hat{H}_0\) and two extremum conditions, \(\partial M(nS)/\partial \omega_i = 0\) \((i = 1, 2)\), which are put on the mass \(M(nS)\) \cite{18}:

\[
\left[ \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{p^2}{2\omega_{\text{red}}} + V_B(r) \right] \varphi_{nS}(r) = E(nS) \varphi_{nS}(r),
\]

(9)

\[
\omega_i^2(nS) = m_i^2 - \frac{\partial E(nS, \mu_{\text{red}})}{\partial \omega_i(nS)}, \quad (i = 1, 2).
\]

(10)

In a Hamiltonian approach the choice of the static potential \(V_B(r)\) is of great importance; we take it as a sum of a linear confining term and the OGE-type term: this property of additivity is well established now in analytical studies \cite{14} and on the lattice \cite{23, 24}:

\[
V_B(r) = \sigma r + \frac{4\alpha_B(r)}{3} r.
\]

(11)
For the string tension a conventional value, $\sigma = 0.18 \text{ GeV}^2$, is used here for all mesons.

The main uncertainty comes from the vector coupling $\alpha_V(r)$, which is taken here from Refs. [23, 24] and denoted as $\alpha_V(r)$. Two important conditions have to be put on the vector coupling:

(i) As in pQCD, it must possess the property of asymptotic freedom (AF); precisely owing to this property the static interaction depends on the number of flavors.

(ii) The vector coupling freezes at large distances. The property of freezing was widely used in phenomenology [1-7, 27] and observed in lattice calculations of the static potential [23, 24].

Unfortunately, one cannot use the static potential and the freezing (critical) constant from lattice studies, where the latter is found to be significantly smaller than in phenomenology and background perturbation theory (BPT). There $\alpha_B(\text{crit}) = 0.58 - 0.60$ is used (these numbers are close to the value from [4]). On the lattice, $\alpha_B(\text{lat}) \sim 0.30$ in full QCD ($n_f = 3$) [24] and $\alpha_B(\text{lat}) \sim 0.22$ in quenched calculations [10, 23] were obtained.

In Eq. (11) the vector coupling $\alpha_B(q^2)$ is defined via the vector coupling $\alpha_B(q^2)$ in momentum space [26]:

$$\alpha_B(q) = \frac{4\pi}{\beta_0 t_B} \left( 1 - \frac{\beta_1 \ln t_B}{\beta_0 t_B} \right)$$  \hspace{1cm} (13)

with the logarithm

$$t_B = \frac{q^2 + M_B^2}{\Lambda_B^2}$$ \hspace{1cm} (14)

containing the vector constant $\Lambda_B(n_f)$, which differs from the QCD constant $\Lambda_{\text{MS}}(n_f)$. The relation between them has been established in Ref. [28]:

$$\Lambda_B(n_f) = \Lambda_{\text{MS}} \exp \left( -\frac{a_1}{2\beta_0} \right),$$  \hspace{1cm} (15)

with $\beta_0 = 11 - \frac{2}{3} n_f$ and $a_1 = \frac{31}{3} - \frac{10}{3} n_f$. Therefore the constant $\Lambda_B$ is always larger than $\Lambda_{\text{MS}}$.

$$\Lambda_B^{(5)} = 1.3656 \Lambda_{\text{MS}}(n_f = 5),$$

$$\Lambda_B^{(4)} = 1.4238 \Lambda_{\text{MS}}(n_f = 4),$$

$$\Lambda_B^{(3)} = 1.4753 \Lambda_{\text{MS}}(n_f = 3).$$ \hspace{1cm} (16)

At present, only the QCD constant $\Lambda_{\text{MS}}(n_f = 5)$ is known with a good accuracy, while for $n_f = 3, 4$ it is defined with an accuracy $\sim 10\%$ [19]. For a given

| Meson | $B$ | $B_a$ | $B_c$ |
|-------|----|------|------|
| $m_q$ | 5  | 200  | 1410 |
| $\omega_q - m_q$ | 611 | 489  | 248  |
| $\omega_b$ | 4805 | 4825 | 4888 |
| $\omega_b - m_b$ | 25  | 25   | 83   |

TABLE II: The ratios $g_{B_q}(10)$ (in GeV) and $|R(0)|^2$ (in GeV$^3$) for the $B_q(1S)$ mesons ($\alpha_{\text{crit}} = 0.58, n_f = 4$)

$$g_{B_q} = 0.161 \frac{|R(0)|^2}{\omega_1(n_S)\omega_2(n_S)},$$  \hspace{1cm} (17)

which directly enters the HFS [5] and appears to be weakly dependent on small variations of the masses $m_1$ and $m_2$, which are compatible with a good description of the meson spectrum.

The w.f. at the origin are sensitive to the values of $A_{\text{HFS}}(n_f)$ (in [30] only the case with $n_f = 3$ was considered). However, if the same freezing value of the coupling constant is taken for $n_f = 3, 4, 5$, then the differences between the w.f. at the origin for $n_f = 3$ and $n_f = 4$ are $\leq 3\%$.

We use here two values for $\alpha_{\text{crit}}$: $\alpha_{\text{crit}} = 0.580$ and $0.604$, for which corresponding values of $A_{\text{HFS}}(n_f)$, $\Lambda_{\text{MS}}(n_f)$ are given below in Eq. (19).

In bottomonium the difference between $g_b$ for $n_f = 4$
and \( n_f = 5 \) appears to be larger, \( \sim 10\% \) (see Table III), where in both cases \( \alpha_{\text{crit}} = 0.604 \) is used.

For the values of \( \alpha_{\text{crit}}(n_f) = 0.604(0.58) \) and \( \Lambda_{\text{MS}}(n_f = 5) = 0.245(0.236) \) GeV, the following vector constants: \( \Lambda_B(n_f = 3) = 0.40(0.389) \) GeV; \( \Lambda_B(n_f = 4) = 0.372(0.360) \) GeV, \( \Lambda_B(n_f = 5) = 0.335(0.323) \) GeV are obtained. Then from the relation (15) we have

\[
\begin{align*}
\Lambda_{\text{MS}}(n_f = 3) &= 271(264) \text{ MeV}, \\
\Lambda_{\text{MS}}(n_f = 4) &= 261(253) \text{ MeV}, \\
\lambda_{\text{MS}}(n_f = 5) &= 245(236) \text{ MeV}.
\end{align*}
\]

For \( n_f = 5 \) it gives \( \alpha_s(M_Z) (\text{two-loop}) = 0.1194 \) for \( \alpha_{\text{crit}} = 0.604 \) and \( \alpha_s = 0.1188 \) for \( \alpha_{\text{crit}} = 0.58 \); both numbers agree with the world averaged value, \( \alpha_s = 0.1176 \pm 0.0020 \) within its error bar.

### III. RESULTS

The experimental error in the HFS

\[
\Delta_{\text{HFS}}(b\bar{b}) = M(\Upsilon(9460)) - M(g_b) = 69.9 \pm 3.1 \text{ MeV} \tag{19}
\]
is small, \( \pm 3 \) MeV, and it is even smaller, \( \leq 1 \) MeV, for the mass differences \( M(B^*) - M(B), M(B^*_s) - M(B_s) \) [19]. Therefore, we expect that the coupling constant \( \alpha_{\text{HFS}} \) can be extracted with a good accuracy from these data.

Notice that from Eq. (12) and the value \( \alpha_s(\mu) = 0.305(2) \) one obtains the following coupling constants with one-loop corrections: \( \alpha_{\text{HFS}}(n_f = 5) = \alpha_s(\mu) = 0.305(2), \alpha_{\text{HFS}}(n_f = 4) = 0.314(2), \) and \( \alpha_{\text{HFS}}(n_f = 3) = 0.323(2) \). Precisely these values are used in our analysis.

For \( g_b = 0.230 \text{ GeV} \) \( (n_f = 4) \) and \( g_b = 0.258 \text{ GeV} \) \( (n_f = 5) \) (see Table III) and taking \( \alpha_{\text{HFS}}(n_f = 5) = \alpha_s(\mu) = 0.305(2) \) and \( \alpha_{\text{HFS}}(n_f = 4) = 0.314(2) \), we obtain \( \Delta_{\text{HFS}}(b\bar{b}) = 64.2(4) \text{ MeV} \) \( (n_f = 4) \) and 70.0(4) MeV for \( n_f = 5 \). The difference between them, \( \sim 10\% \), is not small and one may conclude that the HFS in bottomonium is in full agreement with the BaBar data [1, 2] only for \( n_f = 5 \).

For the 2S and 3S bottomonium states the difference between the cases with \( n_f = 4 \) and \( n_f = 5 \) is small; their HFS coincide within 2 MeV. Therefore we predict that \( \Delta(\bar{b}b) \) is equal to 36(2) MeV and 27(2) MeV, respectively, for the 3S and 2S states.

For the \( B_s \) mesons, the cases with \( n_f = 3, 4 \) have been considered and in both cases agreement with experiment is obtained (see Table IV); it happens because the by \( \sim 3\% \) smaller value of \( g_{b\bar{b}} \) is compensated by the \( \sim 3\% \) larger value of \( \alpha_{\text{HFS}}(n_f = 3) \). Thus for \( B_s \) the number of flavors cannot be fixed, if only data on the HFS (and the spectrum) are fitted; therefore some additional information, like decay constants, is needed to fix \( n_f \).

### IV. CONCLUSION

Our study of the HFS is performed assuming that the freezing value of the coupling constant is the same for \( n_f = 3, 4, 5 \) and considering \( \alpha_{\text{crit}} = 0.58 \) and 0.60. The calculated HFS of the \( B \) and \( B_s \) mesons are in good agreement with experiment for both freezing constants. It happens that the HFS for \( n_f = 3 \) and \( n_f = 4 \) coincide with each other (within 0.5 MeV), if the one-loop correction is taken into account in \( \alpha_{\text{HFS}}(n_f) \). The HFS, averaged over two results, are \( M(B^*_s(2^3S_1)) = 6901(5) \text{ MeV} \) and \( M(B_s(2^1S_0)) = 6865(5) \text{ MeV} \), which are calculated in single-channel approximation.

Notice that the extracted strong coupling constant, \( \alpha_s(\mu) = 0.305(2) \), is smaller than the one used in Refs. [2] and [3]. The renormalization scale, \( \mu \sim 1.6 - 1.65 \text{ GeV} \), corresponding to this coupling, is rather large but agrees with the existing interpretation of the spin-spin potential as dominantly perturbative, thus partly justifying the use of the \( \delta^3(r) \)-function.

| Table IV: The HFS (in MeV) of the \( B_s \) mesons with \( \alpha_s(\mu) = 0.307 \) \( \alpha_{\text{HFS}}(n_f = 4) = 0.316, \alpha_{\text{HFS}}(n_f = 3) = 0.324 \) |
|-----------------|-----------------|-----------------|
| \( B \)         | \( B_s \)       | \( B_s(2S) \)   |
| \( \Delta_{\text{HFS}}(n_f = 4) \) | 45.2            | 46.5            |
| \( \Delta_{\text{HFS}}(n_f = 3) \) | 45.4            | 46.1            |
| \( \Delta_{\text{HFS}}(\text{exp}) \) | 45.78 \pm 0.35   | 46.5 \pm 1.25   |
The extracted coupling, $\alpha_\text{s}(\mu) = 0.305(2)$, is smaller than in many other analyses with a universal coupling; it determines the characteristic scale of the spin-spin interaction, being $\mu \sim 1.6 - 1.65$ GeV. Knowledge of this scale can help to better understand the behavior of the spin-spin potential at small distances.

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