Effect of dissipative environment on collapses and revivals of a nonlinear quantum oscillator

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We study the dissipative dynamics of a wave packet passing through two different nonlinear media. The effect of dissipation on the known phenomenon of collapses and revivals of a wave packet as it evolves in a Kerr type nonlinear medium represented by the Hamiltonian \((a^d a)^3\) is investigated. We find that partial revivals do take place when dissipation values are moderate. We consider the next order nonlinearity represented by the Hamiltonian \((a^d a)^3\) where we observe the phenomena of super revivals. The effect of dissipation in this case has an additional feature of number dependence for the case of displaced number states. Overall our simulations show the robustness of the phenomena of collapses and revivals in a dissipative environment.

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I. INTRODUCTION

Dynamics of quantum systems in complex environments plays an important role in diverse fields of science [1–6]. A laser beam that is quantum mechanically represented by a coherent state, while passing through nonlinear media, can undergo a variety of complex transformations including collapses and revivals of the quantum state. Over and above any systematic evolution (linear or nonlinear) dissipation is always present and therefore a realistic study must include the effect of a dissipative environment [3, 7, 8]. Classically the dissipative effects lead to diminishing of the amplitude, however, since the interactions occur at atomic scales, quantum effects are important. An important set of quantum states is obtained by a phase space displacement of the number states (states with a fixed number of quanta \(n\)) [9–12]. A reasonable model for dissipation on the other hand is to model the dissipative medium as a set of oscillators that interact with the system [7, 13]. For linear as well nonlinear media the initial motion of the wave packet is periodic with the period of motion termed as the ‘classical period’ \(T_c\) [14], corresponding to the natural frequency of the underlying harmonic oscillator. For a linear medium the periodic character of the motion is preserved; however, for a nonlinear medium, after a few cycles quantum interference takes over leading to a significant spread of the wave packet. The wave packet is no longer recognizable as a packet and is said to have “collapsed”. As time advances, it resurrects itself leading to its “revival”. The time at which this revival takes place is called the ‘revival time’ \(T_{rev}\) [15, 16]. At times that are rational fractions of \(T_{rev}\), the wave packet turns into a series of subsidiary packets and the phenomenon is called fractional revival. For the case of third order nonlinearity, for times beyond revival times, a new sequence of full and fractional revivals start, which are characterized by a longer time scale called the ‘super revival time’ \(T_{sr}\).

The phenomenon of collapses and revivals of the wave packets was first discussed in [19]. Thereafter, a number of authors have discussed this phenomenon [20–27]. In most studies of collapses and revivals the medium is assumed to be non-dissipative. However, for situations of practical interest the medium cannot be assumed to be non-dissipative or ideal. How to incorporate the effects of dissipation on the phenomena of collapses and revivals of wave packets is the central question that we address in the present paper.

Unlike in classical mechanics, the dissipative terms cannot be directly incorporated in quantum equations of motion as this leads to the decay of the Heisenberg uncertainty relation which is absurd [13]. To overcome this difficulty various approaches have been proposed [28–31]. We consider the model for dissipation where the system is considered to interact with the environment via an interaction Hamiltonian. The composite system consisting of the system of interest plus the environment evolves unitarily and the environment degrees of freedom are traced over to arrive at the dynamics of the system alone [3, 4, 32–36]. We study the dissipative dynamics of a wave packet passing through two different nonlinear media.

Our work embarks upon a study of the dynamics of a quantum wave packet through a nonlinear medium in the presence of dissipation. In particular we aim to study the effects of dissipation on the phenomenon of wave packet revival and super-revival. We consider single-mode radiation fields as our system and we consider coherent states and displaced number states as our wave packets. Such packets can arise in many experimental situations [37–39]. For nonlinear media we first consider a Kerr medium...
represented by the Hamiltonian $(a^\dagger a)^2$. The dynamics of the coherent wave packet in this case gives rise to revivals and collapses of the wave packet. To include the effects of dissipation we write down a master equation corresponding to a situation where the coherent wave packet interacts with a set of harmonic oscillators representing thermal bath modes. From the master equation, we derive numerical results for the time evolution of expectation value of amplitude operator of the wave packet. Further, we discuss the model of dissipation and various time scales pertaining to the time evolution of wave packets in nonlinear media.

In Section III, we present the results for the time evolution of expectation value of amplitude operator in a dissipative environment and our main results in this context. In Section IV we present some concluding remarks.

II. DISSIPATION MODEL AND REVIVAL TIMES FOR THE NONLINEAR OSCILLATOR

We study system-environment interaction in the Born-Markov approximation which yields the quantum master equation for the complete system. Consider a quantum system $(S)$ interacting with its environment $(E)$, which can be treated as a heat bath or reservoir. The total system-environment Hamiltonian for this system can be written in the form

\[ H = H_S + H_E + H_{\text{int}} \tag{1} \]

where $H_S$, $H_E$, and $H_{\text{int}}$ describe the Hamiltonian for the system, environment and interaction between them, respectively. For one mode radiation field represented by bosonic annihilation operator $a$ with $[a,a^\dagger] = 1$, traversing through a nonlinear medium the above Hamiltonians are given by the following expressions

\[ H_S = \hbar \omega_0 a^\dagger a + H_{\text{NL}} \tag{2} \]

\[ H_E = \sum_j \hbar \omega_j e_j^\dagger e_j \]

\[ H_{\text{int}} = \sum_j \hbar g_j [ae_j^\dagger + a^\dagger e_j]. \tag{3} \]

Here $\omega_0$ is the frequency corresponding to the harmonic oscillator representing the mode and $H_{\text{NL}}$ is the nonlinear part of Hamiltonian being considered. The dissipation is modeled as a set of harmonic oscillators with frequencies $\omega_j$. $e_j^\dagger$ and $e_j$ are the creation and annihilation operators of the environment. The interaction Hamiltonian is in the rotating wave approximation and $g_j$ represents the coupling strength of the $j$th bath mode with the system.

In the Schrödinger picture, reduced density matrix $\rho_S$ for the system is described by the master equation

\[ \frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H_S,\rho_S] + \gamma(N+1)[a\rho a^\dagger - \frac{1}{2}(a^\dagger a\rho + \rho a^\dagger a)] + \gamma N[a^\dagger \rho a - \frac{1}{2}(aa^\dagger \rho + \rho aa^\dagger)]. \tag{4} \]

Here $\gamma$ is the damping rate related to the coupling strengths $g_j$ of the bath oscillators with the system. The parameter $N = (\exp(\frac{\hbar \omega}{k_B T}) - 1)^{-1}$ represents the bath quanta at frequency $\omega_0$ for a bath in thermal equilibrium at temperature $T$.

In the present work, we model the coupling strengths as coming from a Gaussian distribution function centered at the natural frequency $\omega_0$ of the system i.e.

\[ g_j = \sqrt{\frac{\gamma}{2\pi}} \exp\left[-K(\omega_0 - \omega_j)^2\right], \tag{5} \]

and $K$ is a parameter defining the shape of the Gaussian. We have chosen $K = 500$. This model is justified as for an environment with discrete distribution of oscillator frequencies the most relevant mode is the one which is at resonance with the frequency $\omega_0$ of the system. The more the environment’s mode is detuned from $\omega_0$, lower is its probability to interact with it. For cases where $\hbar \omega_0 >> K_B T$ (for instance, a photon of red light traversing an environment at room temperature), the energy will only flow from the system to the environment. This leads to further simplification of the master equation where we drop the last term in Equation (4) to obtain

\[ \frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H_S,\rho_S] + \gamma(N+1)[a\rho a^\dagger - \frac{1}{2}(a^\dagger a\rho + \rho a^\dagger a)]. \tag{6} \]

The system Hamiltonian $H_S$ has two parts, the first being the linear part represented by $H_{S_0} = \hbar \omega_0 a^\dagger a$ and the second being the nonlinear part $H_{\text{NL}}$ leading to

\[ \frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H_{S_0},\rho_S] - \frac{\gamma}{2}[H_{\text{NL}},\rho_S] + \gamma(N+1)[a\rho a^\dagger - \frac{1}{2}(a^\dagger a\rho + \rho a^\dagger a)]. \tag{7} \]

We consider two cases of nonlinear terms $H_{\text{NL}}$ as given below

\[ H_{\text{NL1}} = b_1(a^\dagger a)^2, \]

\[ H_{\text{NL2}} = b_2(a^\dagger a)^3. \tag{8} \]

Equation (7) can be written in the super-operator form as

\[ \frac{d\rho_S}{dt} = \mathcal{L}\rho_S, \tag{9} \]
where \( \mathcal{L} \) is the Liouvillian given by
\[
\mathcal{L} = -\frac{i}{\hbar}[H_S + H_{NL}, \rho_S] \\
+ (N + 1)\gamma[a\rho a^\dagger - \frac{1}{2}(a^\dagger a\rho - \rho a^\dagger a)]
\] (10)

For the case of a time independent Hamiltonian the solution of Equation (9) can be formally written as
\[
\rho_S(t) = \exp[\mathcal{L}(t)]\rho_S(t_0).
\] (11)

From the above equation one can compute the time evolution of the expectation value of any physical quantity of interest. In particular, when we have a coherent wave packet, we would like to calculate the time varying expectation value of the wave packet amplitude \( \langle a(t) \rangle \).

Coherent wave packet that we consider is obtained by a phase space displacement of the vacuum state of the harmonic oscillator [9][12],
\[
|\alpha\rangle = D(\alpha)|0\rangle \\
D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)
\] (12)

The quantity \( \alpha \) is a complex number. These are well known coherent states and typically represent laser light in quantum optics.

The action of the displacement operator \( D(\alpha) \) on number states \( |n\rangle \) (states with fixed number of photons) gives rise to displaced number states given by
\[
|\alpha, n\rangle = D(\alpha)|n\rangle
\] (13)

We will also consider these states in our analysis while dealing with higher order nonlinearity.

Starting with the initial density operator corresponding to the relevant system state we will numerically compute the solution of Equation (11) and then compute the relevant expectation values.

### A. Revival Times

For a nonlinear oscillators when the Hamiltonian commutes with the number operator, the eigen vectors are same as those of the underlying linear oscillators. The energy of the \( n^{th} \) level on the other hand can be expanded as a Taylor series [23] around the central energy \( n_0 \) when the energy spread is not too much as follows

\[
E_n \approx E_{n_0} + (n - n_0)E'_{n_0} + \frac{1}{2}(n - n_0)^2E''_{n_0} \\
+ \frac{1}{6}(n - n_0)^3E'''_{n_0} + ....
\] (14)

where the primes over the energy terms represent derivatives which define the various time scales. The first time scale is given by
\[
T_{cl} = \frac{2\pi}{E_{n_0}}
\] (15)

and is the ‘classical’ time period for the shortest closed orbit [14]. It controls the initial behavior of the wave packet. The second time scale
\[
T_{rev} = \frac{2\pi}{\frac{1}{2}E''_{n_0}}
\] (16)

is the revival time [45][46]. It governs the appearance of the fractional revivals and the full revivals. The third time scale
\[
T_{sr} = \frac{2\pi}{\frac{1}{6}E'''_{n_0}}
\] (17)

is the super revival time. It is a larger time scale as compared to the classical time period and the revival time period. Relevance of these times depends upon the kind of nonlinearity under consideration.

For the Kerr type nonlinearity the Hamiltonian is given by
\[
H_S = \hbar \omega_0 a^\dagger a + b_1(a^\dagger a)^2
\] (18)

This Hamiltonian has same eigen states as the original (linear) oscillator namely, \( |n\rangle \) and the corresponding energy eigen values are given by
\[
E_{n_1} = \hbar \omega_0 n_1 + b_1 n_1^2
\] (19)

It is clear that the revival time is an important parameter here and is given
\[
T_{rev} = \frac{2\pi}{\frac{1}{2}(2b_1)} = \frac{2\pi}{b_1}.
\] (20)

Similarly, for a nonlinear medium with nonlinearity proportional to \( (a^\dagger a)^3 \) the Hamiltonian and energy are given as
\[
H_{S_2} = \hbar \omega_0 a^\dagger a + b_2(a^\dagger a)^3
\] (21)

and corresponding energy eigen states are given by
\[
E_{n_2} = \hbar \omega_0 n_2 + b_2 n_2^3.
\]

We can compute the revival time as
\[
T_{rev} = \frac{2\pi}{\frac{1}{4}(6b_2 n_2)} = \frac{2\pi}{3b_2 n_2}
\] (22)

and the super-revival time as
\[
T_{sr} = \frac{2\pi}{\frac{1}{6}(6b_2)} = \frac{2\pi}{b_2}.
\] (23)

It is evident the super-revival time becomes relevant in this case because of the presence of higher order nonlinear terms in the Hamiltonian.
FIG. 1. $\langle a \rangle$ plotted as a function of time for coherent wave packet of red light in the presence of dissipation and without any nonlinearity.

FIG. 2. Collapse and revival of a coherent wave packet of red photon passing through medium with nonlinearity proportional to $(a^\dagger a)^2$ for $\gamma = 0.0, 0.0001, 0.001$ and 0.008.

III. RESULTS AND DISCUSSION

We carry out the numerical solution of Equation (11) to find out the time evolved state of the system. The simulation is done for a variety of parameters to explore the effects of dissipation on the process of collapses and revivals for both types of nonlinearities. We finally calculate the expectation value of oscillator amplitude $\langle a \rangle$ as a function of time.

FIG. 3. Effect of weak and strong nonlinearity represented by parameter $b_1$, on the revival behavior for a coherent wave packet of red photon passing through medium without dissipation the values of nonlinearity are (a) 0.00002, (b) 0.0002, (c) 1 and (d) 10.

FIG. 4. Super-revival pattern of coherent state wave packet. Amplitude $\langle a \rangle$ is plotted as function of time in a medium with nonlinearity proportional to $(a^\dagger a)^3$ for $\gamma = 0, 0.000004, 0.0004, 0.004$.

The dynamics is observed at a time scale much smaller than the classical time period, revival time period and super revival time period. The parameters in the numerical calculations are chosen to match a physically viable situ-
nction corresponding to red light with angular frequency $\omega_0 = (0.15\pi)/2$ atomic unit (a.u.), passing through a nonlinear medium. The initial amplitude of the coherent state pertaining to photon of red light is taken to be $\alpha = -1.9$. As mentioned earlier we take discrete set of modes for the bath oscillators and the coupling strengths given by the Gaussian model given in Equation (5). The relevant frequencies for which coupling is appreciable are between $-2\omega_0$ to $2\omega_0$, and the coupling of the system with modes outside this range is negligible for $K = 500$. The damping parameter $\gamma$ represents the overall coupling strength between the system and environment and can be computed from Equation (3) by substituting the value of $\omega_j = \omega_0$. The strength of the nonlinearity in the medium is defined by parameters $b_1$ and $b_2$ appearing in Equation (1). We study the effect of system-environment coupling on collapses and revival of the wave packet by varying the dissipation parameter $\gamma$. In addition to this, we have also studied the behavior of revival and collapses as a function of nonlinearity of the medium.

In the rest of this section, we present the results obtained for dynamics of a coherent wave packet traversing a dissipative medium in the absence of any nonlinearity. Thereafter, we discuss the findings for the media with nonlinearity proportional to $(a^\dagger a)^2$ and $(a^\dagger a)^3$. In case of nonlinear term proportional to $(a^\dagger a)^2$, the medium behaves as an optical Kerr medium. The initial coherent state wave packet is constructed by displacing the vacuum state $|0\rangle$ as described in Equation (12). In case of nonlinearity proportional to $(a^\dagger a)^3$, the dynamics of coherent wave packet reveals the existence of revivals as well as super-revivals of the expectation values of oscillator amplitude operator $\langle a \rangle$. In this case, the revival time depends upon the principle quantum number $n$. Therefore we also study the effect of dissipative medium on the revivals of coherent wave packet corresponding to displaced number states for different values of $n$ as defined in Equation (13). The analysis of displaced number states is important only in the case of higher-order nonlinearity.

As a first computation we consider the effect of dissipation on the linear oscillator without any nonlinear terms. The results are displayed in Fig. 1 where we have plotted the amplitude $\langle a \rangle$ as a function of time. The energy spectrum is equidistant and there is only one time associated with motion namely the classical time $T_{cl}$. Dissipation caused by environment is evident through the damping and ultimate decay of the expectation value of coherent wave packet amplitude $\langle a \rangle$ as time evolves. Time evolution is observed for 15 classical time periods. The decay of amplitude is exponential as expected.

### A. Results for nonlinearity proportional to $(a^\dagger a)^2$

For this case the results are presented in Fig. 2 where we display expectation value of amplitude $\langle a \rangle$ as a function of time for different values of dissipation. The value of the nonlinearity parameter $b_1$ is taken to be 0.005. It is evident from Fig. 2(a) that in the absence of coupling between system and environment (marked by zero value of $\gamma$) full revival of amplitudes takes place with a periodicity of $kT_{rev}/2$ with $k = 1, 2, 3 \cdots$. At these revival times all eigenstates accumulate a phase of $2\pi k$. This behavior has already been pointed out both experimentally [14] and theoretically [15]. For finite but small damping parameter $\gamma = 0.0001$ as shown in Fig. 2(b), due to damping a reduction in revived amplitude is noticeable although the revivals do take place. As the value of $\gamma$ increases the revivals become weaker and weaker as seen in (Fig. 2(c)) & (d)). For a certain value of $\gamma$ approximately equal to 0.008, the revivals disappear altogether. Our results on the one hand show robustness of the revival phenomena in a dissipative environment and on the other hand points towards a threshold level of dissipation after which no revivals occur whatsoever.

In Fig. 3 we study the effect of weak and strong nonlinearity on the revival behavior. For zero nonlinearity there are perfect revivals with a time period of $T_{cl}$. Inclusion of nonlinear term in the Hamiltonian results in an erratic energy spectrum leading to the collapse and subsequent revival of the wave packet. For very small values of the nonlinear parameter there is no perceptible effect on the pattern as seen in fig. 3(a) where we have chosen a value $b_1 = 0.00002$. However, as seen from fig. 3(b) the packet undergoes a revival for small values of nonlinearity at time $T_{rev}$. As the value of the nonlinearity parameter is increased the revivals survive up to some value (Fig. 3(c)) and then disappear (Fig. 3(d)). For Fig. 3(d), the value of the nonlinearity parameter is
chosen to be large \((b_1 = 10)\) and the pattern is irregular.

As we scan different values of the nonlinearity parameter \(b_1\) we find that there is a threshold value of \(b_1\) for which the state actually collapses and then revives at \(T_{\text{rev}}\). We call this value of nonlinearity parameter as \(b_{\text{onset}}\) and in our case \(b_{\text{onset}} = 0.0002\). As the value of the nonlinearity parameter is further increased the revivals survive up to some value \((\text{Fig. 3(c)})\) and then disappear \((\text{Fig. 3(d)})\). The value of nonlinearity parameter above which the revival pattern disappears is called \(b_{\text{offset}}\) and approximately \(b_{\text{offset}} = 1.0\) in our case.

B. Results for nonlinearity proportional to \((a^\dagger a)^3\)

To begin with we start with a coherent state wavepacket and study its behavior in the presence of nonlinearity and for different values of the dissipation parameter as was done for the case of Kerr nonlinearity. Figures 4(a), 4(b), 4(c) and 4(d) show the super-revival pattern of coherent wave packet amplitude \(\langle a \rangle\) for \(\gamma = 0, 0.0004, 0.0004,\) and 0.004, respectively. The super-revival pattern is evident in these figures. Here the super-revival time \(T_{\text{sr}}\) is independent of the principle quantum number \(n\) \((\text{Equation (23)})\) similar to the case studied in the previous section where the revival time was independent of \(n\). Within one super revival time some full revivals and some partial revivals come into the picture. The revival of the state occurs at the super revival time which is longer time scale compared to the revival time. For the sake of comparison with the previous section we further study the effect of strength of nonlinearity in the system given by the parameter value \(b_2\). From fig. 5 one can conclude that for a medium with nonlinear term proportional to \((a^\dagger a)^3\) revivals are present up to a certain value of nonlinearity beyond which the pattern becomes irregular. For values of \(b_2\) \((\text{figs. 5(b) & (c)})\) the revivals are present. For \(b_2\) values greater then 0.6 the revivals are irregular. For strong nonlinearity \(b_2 = 10\), the revivals are not only irregular but also almost stop appearing, this is depicted in fig. 5(d). As we discussed for the earlier case we have the notions of \(b_{\text{onset}}\) and \(b_{\text{offset}}\) here too.

Figure 5(a) \((b_2 = 0.00004)\) pertains to a value of nonlinearity below \(b_{\text{onset}}\). In this case too as scan through different value of \(b_2\) we find that \(b_{\text{onset}} = 0.0004\) and \(b_{\text{offset}} = 0.06\) approximately.

C. The effect of \(n\)

The fact that revival time in this case shows dependence on principle quantum number \(n\) \((\text{Equation (22)})\), provides an opportunity to explore the revival pattern for various values \(n\) for displaced number states. As defined in Equation 13 we considered displaced number states \(|\alpha, n\rangle\).

The results of this study are shown in Figs. 6 and 7. It is seen that the revival patterns for displaced number states are dependent upon \(n\). We have taken the values \(n = 1, 2, 3, 4\), with nonlinearity \(b_2 = 0.005\), in the absence and presence of the system-environment coupling, respectively. Fig. 6 represents the results without dissipation and Fig. 7 represents the results in the presence of dissipation. The revival pattern is clearly affected by the dissipative effects.

Theoretically calculated revival times for \(n = 1, 2, 3, 4\) are around 400, 200, 133 and 100 a.u. respectively. However, from Fig. 6 one can notice that the revival of the wave packet occurs around 200 a.u. independent of the value of the state \(|\alpha, n\rangle\). This behavior for \(n = 1\) case was discussed in 48 and 47 where it was shown that revivals appear only when all eigenstates have accumulated a phase of \(2\pi k\) and in the process some extra revivals may appear as presented in 6(a). However, from our Figs. 6(c) & 6(d) we also observe that there are cases where some revivals are missing for higher \(n\) values. A comparison of Fig. 6 with Fig. 7 also indicates that the wave packet constructed around the higher displaced states are dissipated more prominently. In order to elucidate this finding, we extend the value of \(n\) for the state \(|\alpha, n\rangle\) upto \(n = 10\) and study the effect of dissipative medium at the first revival time.

The results are shown in Fig. 8 the displaced number states constructed for higher \(n\) witness more dissipation. This is a very interesting finding which is related to the fact that the displaced number states with higher \(n\) are more nonclassical and hence the effect of dissipation is rapid. Similar results were found for a related phenomenon of decoherence in 49, 50.
FIG. 7. Collapse and revival pattern of displaced number states with amplitude $\langle a \rangle$ being plotted as a function of time with nonlinearity proportional to $(a a^\dagger)^3$ and in the presence of dissipation. The pattern is shown for states constructed around $n = 1, 2, 3, 4$ at $\gamma = 0.0001$. The theoretical $T_{\text{rev}}$ for $n = 1, 2, 3, 4$ are around 400, 200, 133, 100 au respectively.

FIG. 8. Effect of dissipative medium on the displaced number states constructed around various $n$ in a medium with nonlinearity proportional to $(a a^\dagger)^3$. The amplitude of revived wave packets at their first revival time instant are recorded for various values of $n$. The dissipation of amplitude is more vigorous for higher values of $n$.

IV. CONCLUDING REMARKS

In this work the quantum master equation approach has been used to analyze the dissipative dynamics of a coherent wave packet passing through nonlinear media of two different types. The effect of a dissipative bath on the phenomena of collapses and revivals is evident. However, it is observed that the basic features of the revival process survives some amount of dissipation. Although the revival is never complete in the presence of dissipation, its signatures are present. This is relevant for any attempts to observe these revivals experimentally.

The other prominent role played by the dissipative medium is that of an environment which soaks the energy of the wave packet. Dissipation is observed in the damping of successive revived coherent wave packet amplitudes $\langle a \rangle$. The controlling element for dissipation is the damping factor $\gamma$ which further depends upon the coupling between the coherent wave packet and the medium. With increase in the strength of damping factor $\gamma$, the amplitude of the successively revived wave packet decreases and ultimately no revivals occur for a particular value of $\gamma$.

For a medium with nonlinearity proportional to $(a a^\dagger)^3$, signatures of super revivals are evident. Theoretically, revivals are predicted at different times for wave packets constructed around different displaced quantum numbers $n$, however, we observe that the revival of the wave packet occurs at a fixed time independent of the $n$. This behavior sheds light on a very fundamental aspect that revivals appear only when all eigenstates have accumulated a phase of $2\pi k$ and in the process some extra revivals may appear and some predicted ones may be missing.

We also observe that the wave packet build around the higher values of principle quantum number are more prone to the dissipative effects of the medium as compared to the low lying states. The extent of nonlinearity of the medium also has an effect on the dynamics of the wave packet. We come across a value of the nonlinearity parameter $b_{\text{onset}}$ below which no collapses and hence no revivals occur. Similarly, there is an upper limit of $b$ that we call $b_{\text{offset}}$ above which the pattern of revivals is irregular.

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