New scalar resonance $X_0(2900)$ as a $\bar{D}^*K^*$ molecule: Mass and width

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We explore features of the scalar structure $X_0(2900)$, which is one of the two resonances discovered recently by LHCb in the $D^-K^+$ invariant mass distribution in the decay $B^+ \rightarrow D^+D^-K^+$. We treat $X_0(2900)$ as a hadronic molecule composed of the conventional mesons $\bar{D}^0$ and $K^{*0}$ and calculate its mass, coupling, and width. The mass and coupling of $X_0(2900)$ are determined using the QCD two-point sum rule method by taking into account quark, gluon, and mixing vacuum condensates up to dimension 15. The decay of this structure to final state $D^-K^+$ is investigated in the context of the light-cone sum rule approach supported by a soft-meson technique. To this end, we evaluate strong coupling $G$ corresponding to vertex $X_0D^-K^+$, which allows us to find width of the decay $X_0(2900) \rightarrow D^-K^+$. Obtained predictions for the mass of the hadronic molecule $\bar{D}^0K^{*0}$ $m = (2868 \pm 198)$ MeV and for its width $\Gamma = (49.6 \pm 9.3)$ MeV can be considered as arguments in favor of molecule interpretation of $X_0(2900)$.

I. INTRODUCTION

The LHCb collaboration recently announced an observation of two resonant structures $X_0(2900)$ and $X_1(2900)$ (hereafter $X_0$ and $X_1$, respectively) in the invariant $D^-K^+$ mass distribution of the decay $B^+ \rightarrow D^+D^-K^+$. In accordance with LHCb, obtained results constitute the clear evidence for exotic mesons with full open flavors. At the same time, the collaboration did not exclude models which explain experimental data using hadronic rescattering effects.

The LHCb extracted the masses and widths of these structures, as well as determined their spin-parities. Thus, it was shown that the $X_0$ is the scalar resonance $J^P = 0^+$ with parameters

$$m_0 = (2866 \pm 7 \pm 2) \text{ MeV},$$
$$\Gamma_0 = (57 \pm 12 \pm 4) \text{ MeV},$$

whereas $X_1$ is the vector state $J^P = 1^-$ and has the mass and width

$$m_1 = (2904 \pm 5 \pm 1) \text{ MeV},$$
$$\Gamma_1 = (110 \pm 11 \pm 4) \text{ MeV}.$$  

From decay channels of these resonances $X_{0(1)} \rightarrow D^-K^+$, it is evident that they are built of four different valence quarks $ud\bar{c}\bar{s}$. These circumstances place $X_0$ and $X_1$ to distinguishable position in the $XYZ$ family of exotic mesons. The LHCb’s discovery is doubly remarkable, because existence of the resonance $X(5568)$, seen by the D0 collaboration [2] and presumably composed of quarks $sdb\bar{c}$, was not later confirmed by other experiments.

The LHCb information generated interesting theoretical investigations to explain structure of new resonances $X_0$ and $X_1$, calculate their masses, and if possible, estimate widths [3]. In these papers the authors made different suggestions about internal organization of these resonances, and used various methods and schemes to compute their parameters. The diquark-antidiquark and hadronic molecule pictures are dominant models to account for collected experimental data. For example, in Refs. [3, 4] $X_0$ was considered as a scalar tetraquark $X_0 = [sc]\overline{[ud]}$ using a phenomenological approach and the sum rule method, respectively. In Ref. [6] $X_0$ was interpreted as $S$-wave hadronic $D^{*-}K^{*+}$ molecule, whereas $X_1$ considered $P$-wave diquark-antidiquark state $X_1 = [ud]\overline{[sc]}$. In Ref. [8], on the contrary, it was asserted that two resonance-like peaks generated in the process $B^+ \rightarrow D^+D^-K^+$ due to rescattering effects may emerge in LHCb experiment as the states $X_0$ and $X_1$.

It is worth noting that the exotic scalar meson with open flavor structure $X_c = [uc]\overline{[ds]}$ was studied in our work [12], in which it was explored as a charmed partner of the resonance $X(5568)$. The mass and width of this tetraquark were calculated using the sum rule method and two interpolating currents. These currents correspond to structures $C_{\gamma_5} \otimes \gamma_5 C$ and $C_{\gamma_\mu} \otimes \gamma^\mu C$, and are scalar-scalar ($S$) and axial-axial currents ($A$), respectively. The width of $X_c$ was evaluated by analyzing decay channels $X_c \rightarrow D^-_s \pi^+$ and $X_c \rightarrow D^0K^0$. Performed studies led to the following results

$$m_S = (2634 \pm 62) \text{ MeV}, \quad \Gamma_S = (57.7 \pm 11.6) \text{ MeV},$$

and

$$m_A = (2590 \pm 60) \text{ MeV}, \quad \Gamma_A = (63.4 \pm 14.2) \text{ MeV}.$$  

The prediction $(2.55 \pm 0.09)$ GeV for the mass of $X_c$ was made in Ref. [10], as well.

It is clear that $X_c$ and $X_0$ are different particles and their decay channels differ from each another. Nevertheless, it is convenient to compare parameters of $X_c$ with...
LHCb data to make some preliminary assumptions on structure of $X_0$. The mass of the ground-state tetraquark $X_0$ is not large enough to account for LHCb data. We must also take into account that, the tetraquark $X_0$ is composed of a relatively heavy diquark $[su]$ and heavy antidiquark $[ar{c}ar{s}]$, whereas $X_0$ would have a light diquark $[ud]$-heavy antidiquark $[car{s}]$ structure. Heavy-light tetraquarks are more compact and lighter than ones with the same quark content but other compositions [17]. Therefore, the mass of the resonance $X_0$ should be within or even below limits (3)-(4) provided we treat it as a ground-state tetraquark: In the diquark-antidiquark picture $X_0$ may be considered as a radially excited $[ud][car{s}]$ state.

Alternatively, one may analyze it as a hadronic molecule, i.e., as a bound state of conventional $D$ and $K$ mesons. Mesons $D^−$ and $K^+$ may form a bound state if the mass of a molecule $D^−K^+$ is less than corresponding two-meson threshold 2365 MeV. But this estimate is considerably below the mass of $X_0$, therefore, it is difficult to expect that the molecule $D^−K^+$ can be considered as the $X_0$ state. For compounds $D^0\bar{K}^*$ (hereafter $D^*K^*$) and $D^{∗−}K^{∗+}$ relevant two-particle thresholds are equal to $\sim 2900$ MeV, and hence they cannot dissociate to vector mesons $D^*$ and $K^*$ provided masses of these molecules are below this limit: An estimation for the mass 2848 MeV of the scalar molecule $D^*\bar{K}^*$ obtained in Ref. [18] supports this assumption. But such hadronic molecules can decay to a pair of pseudoscalar $D$ and $K$ mesons. Then, structures $D^0\bar{K}^*$ and $D^{∗−}K^{∗+}$ may be interpreted as $X_0$ if their masses are compatible with $m_0$. The scenario with $D^{∗−}K^{∗+}$ as $X_0$ was realized in Ref. [19] in which the authors calculated the mass of the molecule $D^{∗−}K^{∗+}$ in the framework of the QCD sum rule method. Result obtained there 2.87$^{+0.19}_{−0.14}$ GeV indicates that an assumption about molecule structure of $X_0$ deserves detailed studies.

In the present work, we treat $X_0$ as a hadronic molecule $D^*K^*$ composed of two vector mesons $D^0 = \pi u$ and $K^{*0} = \pi s$, and compute not only its mass, but also width. The mass of $X_0$ is evaluated in the context of the sum rule method, where we take into account quark, gluon and mixed condensates up to dimension 15. The width of $X_0$ is found by considering the decay channel $X_0 \rightarrow D^−K^+$. To this end, we calculate the coupling $\mathcal{G}$ that describes strong vertex $X_0 D^−K^+$ in the context of the light-cone sum rule (LCSR) approach using technical tools of the soft-meson approximation. Information on this coupling obtained from analysis allows us to estimate the width of $X_0$.

This work is structured in the following manner: In Section II we calculate the mass and coupling of the hadronic molecule $D^*K^*$. In Section III we compute the strong coupling $\mathcal{G}$ by employing the LCSR method and soft-meson technique. In this section we find also the width of the decay $X_0 \rightarrow D^−K^+$. Section IV is reserved for discussion and our conclusions.

II. SPECTROSCOPIC PARAMETERS OF THE $D^*K^*$

The mass $m$, and current coupling $f$ of the hadronic molecule $D^*K^*$ are necessary to check the assumption about a molecule nature of the resonance $X_0$. These spectroscopic parameters are required also to study its strong decay. We compute $m$ and $f$ using the QCD two-point sum rule method [19, 20], which is one of the effective nonperturbative approaches to determine parameters of the ordinary and exotic hadrons.

The required sum rules can be derived from analysis of the two-point correlation function

$$\Pi(p) = i \int d^4xe^{ipx} \langle 0 | T \{J(x)J^\dagger(0)\}|0\rangle,$$  \hspace{1cm} (5)

where $T$ denotes the time-ordered product and $J(x)$ is the interpolating current for the scalar particle $X_0$. For molecule state $D^*K^*$ this current is given by the expression

$$J(x) = [\bar{c}_u(x)\gamma_\mu u_a(x)][\bar{s}_b(x)\gamma_\mu d_b(x)].$$  \hspace{1cm} (6)

In Eq. (6) $c(x), s(x), u(x)$ and $d(x)$ stand for the corresponding quark fields, whereas $a$ and $b$ are color indices.

Within the sum rule method the masses of various tetraquarks were analyzed in numerous articles (see, for example, the review papers [21, 22]), therefore below we present only crucial points of performed analysis. In the sum rule method the correlation function $\Pi(p)$ should be expressed both in terms of physical parameters of $X_0$ and quark-gluon degrees of freedom. In the first case, one finds the phenomenological side of the sum rules $\Pi^{\text{Phys}}(p)$ from Eq. (5) by inserting a complete set of intermediate states. As a result we get

$$\Pi^{\text{Phys}}(p) = \frac{\langle 0 | J X_0 \langle X_0 | J^\dagger |0\rangle}{m^2 - p^2} + \cdots,$$  \hspace{1cm} (7)

where the contribution of only the ground-state particle $X_0$ is shown explicitly: Dots denote effects of higher resonances and continuum states in the $X_0$ channel.

We have approximated the phenomenological side of the sum rule $\Pi^{\text{Phys}}(p)$ in Eq. (7) using a simple-pole term. But, in the case of the multiquark systems, this approach must be applied with some caution, because the physical side may receive a contribution also from two-hadron reducible terms. In fact, the relevant interpolating current couples not only to the multiquark hadron, but interacts also with two conventional hadron states lying below the mass of the multiquark system [23, 24]. These contributions can be either subtracted from the sum rules or included into parameters of the pole term. In the case of the tetraquarks the second approach is preferable and was applied in articles [27, 28]. It appears that, the two-meson states generate the finite width $\Gamma(p)$ of the tetraquark and lead to modification

$$\frac{1}{m^2 - p^2} \rightarrow \frac{1}{m^2 - p^2 - i\sqrt{p^2\Gamma(p)}},$$ \hspace{1cm} (8)
These effects, properly taken into account in the sum rules, rescale the coupling $f$ leaving stable the mass $m$ of the tetraquark. Detailed analyses proved that two-hadron contributions as a whole, and two-meson ones in particular are small, and can be neglected. Therefore, to derive the phenomenological side of the sum rules, we use in Eq. (7) the zero-width single-pole approximation.

Introducing the spectroscopic parameters of $X_0$ through the matrix element

$$\langle 0 | J | X_0 \rangle = fm,$$

we rewrite $\Pi^{\text{Phys}}(p)$ in the final form

$$\Pi^{\text{Phys}}(p) = \frac{f^2m^2}{m^2 - p^2} + \cdots.$$  \hspace{1cm} (10)

The function $\Pi^{\text{Phys}}(p)$ has a simple Lorentz structure proportional to $\sim I$, and the term in Eq. (10) is the invariant amplitude $\Pi^{\text{Phys}}(p^2)$ corresponding to this structure.

The second component of the sum rules $\Pi^{\text{OPE}}(p)$, is calculated in the operator product expansion (OPE) with some accuracy. To find $\Pi^{\text{OPE}}(p)$, we employ the explicit expression of the interpolating current $J(x)$ in Eq. (5), and contract relevant heavy and light quark fields. After these manipulations, for $\Pi^{\text{OPE}}(p)$ we get

$$\Pi^{\text{OPE}}(p) = i \int d^4 x e^{ipx} \text{Tr} \left[ \gamma^\mu S_u^{aa'}(x) \gamma^\nu S_c^{a'a}(x) \right] \times \text{Tr} \left[ \gamma^\alpha S_d^{bb'}(x) \gamma^\beta S_s^{b'b}(x) \right],$$  \hspace{1cm} (11)

where $S_c(x)$ and $S_u(s,d)(x)$ are the heavy $c$- and light $u(s,d)$-quark propagators, respectively. Their explicit expressions can be found, for instance, in Ref. [24]. The correlation function has also a trivial structure: We do not depend on the Borel and continuum terms from the physical side of quark-hadron duality, one subtracts higher resonances and continuum terms from the physical side of the equality. As a result, the sum rule equality starts to depend on the Borel $M^2$ and continuum threshold $s_0$ parameters.

The second equality required to find sum rules is obtained by applying the operator $d/d(-1/M^2)$ to the first expression. Then the sum rules for $m$ and $f$ are

$$m^2 = \frac{\Pi(M^2, s_0)}{\Pi(M^2, s_0)},$$  \hspace{1cm} (12)

and

$$f^2 = \frac{e^{m^2/M^2}}{m^2} \Pi(M^2, s_0).$$  \hspace{1cm} (13)

Here, $\Pi(M^2, s_0)$ is the Borel transformed and subtracted invariant amplitude $\Pi^{\text{OPE}}(p^2)$, and $\Pi'(M^2, s_0) = d/d(-1/M^2)\Pi(M^2, s_0)$.

The function $\Pi(M^2, s_0)$ has the following form

$$\Pi(M^2, s_0) = \int_{M^2}^{s_0} ds \rho^{\text{OPE}}(s)e^{-s/M^2} + \Pi(M^2).$$  \hspace{1cm} (14)

Throughout this article, we neglect the mass of the quarks $u$ and $d$, and set $M = m_c + m_s$ in Eq. (14). The two-point spectral density $\rho^{\text{OPE}}(s)$ is computed as an imaginary part some of terms in the correlation function $\Pi^{\text{OPE}}(p)$. The component $\Pi(M^2)$ is the Borel transformation of remaining terms in $\Pi^{\text{OPE}}(p)$, and are obtained directly from their expressions. Calculations are carried out by taking into account vacuum condensates up to dimension 15. The dimension-15 contribution to the correlation function is proportional to product of light quark condensates $\langle \sigma Gq \rangle, \langle \overline{q}q \rangle, \langle \overline{q}q \rangle^2$. This and other higher dimensional terms in $\Pi(M^2, s_0)$ are obtained as products of basic vacuum condensates using factorization procedure by assuming that it does not lead to essential ambiguities. Analytical expressions of $\rho^{\text{OPE}}(s)$ and $\Pi(M^2)$ are rather lengthy to be presented here explicitly.

The sum rules (12) and (13) depend on universal quark $(\overline{q}q) = \langle 0 | \overline{q}q | 0 \rangle$, gluon $(\alpha_s G^2/\pi) = \langle 0 | \alpha_s G^2/\pi | 0 \rangle$ and mixed quark-gluon $(\overline{q}Gq) = \langle 0 | \overline{q}Gq | 0 \rangle$ vacuum condensates, $(q = u, d)$, and similar expressions for the strange quark $s$ and masses of $c$ and $s$ quarks

$$\langle \overline{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3, \langle \overline{s}s \rangle = (0.8 \pm 0.1)(\overline{q}q),$$

$$\langle \overline{q}Gq \rangle = m_0^2(\overline{q}q), \langle \overline{q}G \overline{q} \rangle = m_c^2(\overline{q}q), \langle \overline{q}Gq \rangle = m_s^2(\overline{q}q),$$

$$m_0 = (0.8 \pm 0.2) \text{ GeV}^2,$$

$$\overline{q}q = (0.012 \pm 0.004) \text{ GeV}^4,$$

$$m_s = 93^{+11}_{-9} \text{ MeV}, m_c = 1.27 \pm 0.2 \text{ GeV}.$$  \hspace{1cm} (15)

As is seen, the vacuum condensate of strange quarks is different from $0 \overline{q}q | 0 \rangle$. The mixed condensates $\langle \overline{q}Gq \rangle$ and $\langle \overline{q}Gq \rangle$ can be expressed in terms of the corresponding quark condensates and parameter $m_s^2$, numerical value of which was extracted from analysis of baryonic resonances.
The $m$ and $f$ are functions of the parameters $M^2$ and $s_0$, as well. The correct choice for $M^2$ and $s_0$ is an important problem of sum rule computations. The working regions for $M^2$ and $s_0$ should satisfy usual constraints imposed on the pole contribution (PC) and convergence of the operator product expansion. To analyze these questions, we introduce the quantities

$$PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}.$$  

(16)

and

$$R(M^2) = \frac{\Pi^{\text{dimN}}(M^2, s_0)}{\Pi(M^2, s_0)}.$$  

(17)

In Eq. (17) $\Pi^{\text{dimN}}(M^2, s_0)$ is a last term (or a sum of last few terms) in the correlation function. In the present analysis, we use the sum of last three terms in OPE, and hence DimN $\equiv$ Dim($13 + 14 + 15$).

The PC is used to fix upper bound for $M^2$, whereas $R(M^2)$ is necessary to find low limit for the Borel parameter. These two values of $M^2$ fix the boundaries of a region where the Borel parameter can be varied. Our analysis shows that the working regions for the parameters $M^2$ and $s_0$ are

$$M^2 \in [2, 3] \text{ GeV}^2, \ s_0 \in [11.3, 13.3] \text{ GeV}^2,$$  

(18)

and they obey restrictions on PC and convergence of OPE. Thus, at $M^2 = 3 \text{ GeV}^2$ the pole contribution is 0.5, whereas at $M^2 = 2 \text{ GeV}^2$ it becomes equal to 0.8. At the minimum of $M^2 = 2 \text{ GeV}^2$, we find $R \approx 0.01$, which guarantees the convergence of the sum rules. We extract the parameters $m$ and $f$ approximately at a middle point of the window (18), $M^2 = 2.5 \text{ GeV}^2$ and $s_0 = 12 \text{ GeV}^2$, where the pole contribution is PC $\approx 0.65$. This fact ensures the ground state nature of $X_0$.

Our predictions for $m$ and $f$ are

$$m = (2868 \pm 198) \text{ MeV},$$

$$f = (3.0 \pm 0.7) \times 10^{-3} \text{ GeV}^4.$$  

(19)

The sum rule results, in general, should not depend on the parameter $M^2$. But in real calculations $m$ and $f$ are sensitive to the choice of $M^2$. From inspection of Eq. (19) it is seen, that theoretical uncertainties in the case of $m$ equal to $\pm 6.9\%$, whereas for the coupling $f$ they amount to $\pm 23.3\%$. Theoretical ambiguities of $f$ are larger, because $f$ is determined directly in terms of $\Pi(M^2, s_0)$, whereas the sum rule for $m$ depends on the ratio of such functions and is exposed to smaller variations. Nevertheless, uncertainties even for the coupling $f$ remain within limits accepted in sum rule computations. In Fig. 1 we display the sum rule’s predictions for $m$ as a function of $M^2$, where one can see its residual dependence on the Borel parameter.

The continuum threshold parameter $s_0$ separates a ground-state contribution from effects due to higher resonances and continuum states. In other words, $\sqrt{s_0}$ has to be smaller than the mass $m^*$ of the first excitation of the $X_0$. The self-consistent sum rule analysis implies that the difference $\sqrt{s_0} - m$ is around or less than $m^* - m$. Excited states of conventional hadrons and their parameters are known either from experimental measurements or from alternative theoretical studies. In the case of the multiquark hadrons there is a deficiency of relevant information. The mass gap $\sqrt{s_0} - m \approx (500 - 600) \text{ MeV}$ found in the present work can be considered as a reasonable estimate $m^* \approx (m + 500) \text{ MeV}$ for the hadronic molecule $\overline{D}^* K^*$ containing one heavy quark. Dependence of extracted value of $m$ on the scale $s_0$ is also shown in Fig. 1.

Obtained prediction for the mass of the state $\overline{D}^* K^*$ is in a nice agreement with new LHCb measurements.
This is necessary, but not enough to interpret $X_0$ as the hadronic molecule. For reliable conclusions, we need to calculate the width of the molecule $D^- K^+$ and confront it with data: only together these parameters can support or not assumptions about the structure of $X_0$.

III. THE DECAY $X_0 \to D^- K^+$

In this section we explore the strong decay $X_0 \to D^- K^+$ in order to find width of the resonance $X_0$. Strictly speaking, there are other decay channel of the scalar state $X_0$, namely the $S$-wave decay to a pair of mesons $D^0 K^0$. Because $X_0$ was observed as enhancement in the $D^- K^+$ mass distribution, we concentrate on the first process and saturate full width of the resonance $X_0$ by this channel.

The width of the decay $X_0 \to D^- K^+$ is determined by the strong coupling $G$ corresponding to the vertex $X_0 D^- K^+$. We are going to calculate $G$ using method of the QCD sum rule on the light-cone [34] and methods of the soft-meson approximation [34]. To this end, we start from analysis of the correlation function [33, 35].

\[ \Pi(p, q) = i \int d^4x \epsilon^{ip\cdot x} \langle K(q)|\mathcal{T}\{J^D(x)J^I(0)\}|0\rangle, \]  

where $K$ and $D$ we denote, in short forms, the mesons $K^+$ and $D^-$, respectively. In the correlation function $\Pi(p, q)$, the interpolating current $J(x)$ is given by Eq. (6), whereas for $J^D(x)$ we use

\[ J^D(x) = \overline{c}_n(x) i\gamma_5 d_n(x), \]  

with $n$ being the color index. It is not difficult to determine $\Pi(p, q)$ in terms of the physical parameters of the particles involved into the decay [33]. By taking into account the ground states in the $D$ and $X_0$ channels, we get

\[ \Pi^{\text{Phys}}(p, q) = \frac{\langle 0|J^D|D(p)\rangle}{p^2 - m_D^2} \frac{\langle D(p)K(q)X_0(p')\rangle}{m_c^2 - m_K^2} \times \frac{\langle X_0(p')|J^I|0\rangle}{p'^2 - m_c^2} + \ldots, \]  

where $p, q$ and $p' = p + q$ are the momenta of the particles $D, K$, and $X_0$, and $m_D$ is the mass of $D$ meson. The ellipses in Eq. (22) refer to contributions of higher resonances and continuum states in the $D$ and $X_0$ channels.

To continue, we have to calculate $\Pi^{\text{QCD}}(p, q)$ in terms of the quark-gluon degrees of freedom and find the QCD side of the sum rule. Contractions in Eq. (20) of $c$ and $d$ quark and antiquark fields yield

\[ \Pi^{\text{OPE}}(p, q) = \int d^4x \epsilon^{ip\cdot x} \left[ \gamma^\mu S_c^\mu (-x) \gamma_5 \right. \times S_d^\mu(x) \gamma_{\mu\alpha\beta}(K(q)\pi^p_\alpha(0)s^\alpha_\beta(0)|0\rangle, \]  

where $\alpha$ and $\beta$ are the spinor indexes.

Apart from quark propagators the function $\Pi^{\text{OPE}}(p, q)$ depends also on local matrix elements of the quark operator $\pi$s sandwiched between the vacuum and $K$ meson. To express $\langle K(q)\pi^p_\alpha(0)s^\alpha_\beta(0)|0\rangle$ using the $K$ meson’s local matrix elements, we expand $\langle 0|\pi|0\rangle$ over the full set of Dirac matrices $\Gamma^J$ and project them onto the color-singlet states

\[ \pi^p_\alpha(0)s^\alpha_\beta(0) \rightarrow \frac{1}{12} \delta^{\alpha\beta} \Gamma^J \left[ \pi(0)\Gamma^J s(0) \right], \]  

where

\[ \Gamma^J = 1, \quad \gamma_5, \quad \gamma_\mu, \quad i\gamma_5\gamma_\mu, \quad \sigma_{\mu\nu}/\sqrt{2}. \]  

The expression (26) reveals a main difference between vertices composed of conventional mesons and vertices containing a tetraquark and two ordinary mesons. Indeed, in the vertices of ordinary mesons the correlation function depends on distribution amplitudes (DAs) of one of the final-state mesons, for example, on DAs of $K$ meson. The DAs of the mesons are determined as non-local matrix elements of relevant quark fields placed between the meson and vacuum states. In the case under discussion, it is evident that instead of non-local matrix elements, we have $\Pi^{\text{QCD}}(p, q)$ that contains local matrix elements of $K$ meson. The reason is that $X_0$ and interpolating current Eq. (10) are built of four quark fields at the same space-time location. Substitution of this current into the correlation function and contractions of $c$ and $d$ quark fields yield expressions, where the remaining light quarks are sandwiched between the $K$ meson and vacuum states, forming local matrix elements. In other words, we encounter the situation when dependence of the correlation function on the meson DAs disappears and integrals over the meson DAs reduce to overall normalization factors. In the framework of the LCSR method such situation is possible in the kinematical limit $q \to 0$, when the light-cone expansion is replaced by the short-distant one. As a result, instead of the expansion in terms of DAs, one gets expansion over the local matrix elements [33]. The limit $q \to 0$ is known as the soft-meson approximation. In this approximation $p = p'$ and invariant amplitudes $\Pi^{\text{Phys}}(p^2)$ and $\Pi^{\text{OPE}}(p^2)$ depend only on the variable $p^2$. For our purposes a decisive fact is the observation made in Ref. [33]: the soft-meson approximation and full LCSR treatment of the conventional mesons’ vertices lead to predictions which are numerically very close to each other.
The soft-meson approximation simplifies the QCD component of the sum rule, but leads to additional complications in its phenomenological side. In this limit for invariant amplitude $\Pi^{\text{Phys}}(p^2)$ we get

$$\Pi^{\text{Phys}}(p^2) = G \frac{m_f m_D m_D^2 m^2}{m_c (p^2 - m^2)^2} + \cdots,$$

where $\bar{m}^2 = (m^2 + m_T^2)/2$. This amplitude contains the double pole at $p^2 = \bar{m}^2$, therefore its Borel transformation is given by the formula

$$\Pi^{\text{Phys}}(M^2) = G \frac{m_f m_D m_D^2 \bar{m}^2 e^{-\bar{m}^2/M^2}}{M^2} + \cdots. \quad (29)$$

In the standard approach the invariant amplitude $\Pi^{\text{Phys}}(p^2, p'^2)$ depends on two variables $p^2$ and $p'^2$, and the Borel transformations over $p^2$ and $p'^2$ suppress contributions of higher resonances and continuum states. The suppressed terms afterwards can be subtracted using assumption on quark-hadron duality. But in the soft limit even after Borel transformation besides ground-state term $\Pi^{\text{Phys}}(M^2)$ contains additional unsuppressed contributions. This is a price to be paid in the soft approximation for simple $\Pi^{\text{OPE}}(p^2)$ expression. To remove contaminating contributions from the phenomenological side of the sum rule, one has to act on $\Pi^{\text{Phys}}(M^2)$ by the operator $\mathcal{P}(M^2, m^2)$ that singles out the ground-state term. Contributions remaining in $\Pi^{\text{Phys}}(M^2)$ after this prescription can be subtracted by the usual way. Then the sum rule for the strong coupling $G$ reads

$$G = \frac{m_c \bar{m}^2}{f m_f m_D} \mathcal{P}(M^2, \bar{m}^2) \Pi^{\text{OPE}}(M^2, s_0). \quad (30)$$

Returning to the calculation of $\Pi^{\text{OPE}}(p, q)$, it is worth noting that by substituting the expansion (28) into Eq. (20), one has to perform summations over color indices and fix local matrix elements of $K$ meson that contribute to $\Pi^{\text{OPE}}(p^2)$ in the soft limit. There are only a few such elements: Two-particle matrix elements of twist-2 and twist-3

$$\langle 0 | \bar{\psi} \gamma_\mu \gamma_5 q | K(q) \rangle = i f_K q_\mu, \quad \langle 0 | \bar{\psi} \gamma_5 q | K \rangle = f_K m_K^2 m_s. \quad (32)$$

There are also three-particle local matrix elements of $K$ meson, for an example,

$$\langle 0 | \bar{\psi} \gamma^\nu \gamma_5 i g G_{\mu
u} s | K(q) \rangle = i q_\mu f_K m_K^2 \kappa_{4K}. \quad (33)$$

The suppression of terms afterwards can be subtracted by the usual way. Then the sum rule for the remaining in $\Pi^{\text{OPE}}(M^2)$ is given by the formula

$$\Pi^{\text{OPE}}(M^2, s_0) = \frac{\mu_K}{8\pi^2} \int_{M^2}^{s_0} dM^2 \frac{dM^2}{s} e^{-s/M^2} + \Pi_{\text{NP}}(M^2). \quad (34)$$

The nonperturbative component of the correlation function $\Pi_{\text{NP}}(M^2)$ has the following form

$$\Pi_{\text{NP}}(M^2) = -\frac{\langle \bar{d}d \rangle}{3} \frac{\mu_K m_c}{\pi} e^{-m_z^2/M^2} \left[ \frac{\alpha_s G^2}{\pi} \frac{\mu_K m_z^4}{72 M^4} \int_0^1 dz \frac{dM^2}{sz} e^{-m_z^2/[M^2z(1-z)]} + \frac{\langle \bar{d}g\sigma G d \rangle}{6 M^4} e^{-m_z^2/M^2} \right]$$

$$\times \frac{\mu_K m_c (m_c^2 + 3M^2)^2}{54 M^6} e^{-m_z^2/M^2} + \frac{\langle \bar{d}g\sigma G d \rangle}{216 M^{10}} e^{-m_z^2/M^2}. \quad (35)$$

In expressions above, we introduce $\mu_K = f_K m_K^2/m_s$. It turns out that only the twist-3 matrix element from Eq. (32) contributes to the function $\Pi_{\text{NP}}(M^2)$. The last term in $\Pi_{\text{NP}}(M^2)$ is proportional to $\langle \bar{d}g\sigma G d \rangle$ with dimension 9, and is suppressed additionally by the factor $1/M^6$. Therefore, dimension-9 accuracy for computation of $\Pi^{\text{OPE}}(M^2, s_0)$ adopted in the present article is high and enough to get reliable result.

The sum rule (31) depends on the various vacuum condensates written down above (15). It contains the masses and decay constants of the final-state mesons $D^-$ and $K^+$: relevant spectroscopic parameters are collected in Table I: Parameters of the mesons $D^-$ and $K^+$ used in numerical computations.

| Quantities | Values (in MeV units) |
|------------|----------------------|
| $m_D$      | 1869.61 ± 0.10       |
| $m_K$      | 493.68 ± 0.02        |
| $f_D$      | 211.9 ± 1.1          |
| $f_K$      | 155.6 ± 0.4          |
To carry out numerical analysis one also needs to fix $M^2$ and $s_0$. The restrictions imposed on these auxiliary parameters are standard for sum rule computations and have been discussed above. Our analysis demonstrates that working regions (15) meet all constraints necessary for computations of $\Pi^{\text{ope}}(M^2, s_0)$. Numerical calculations lead to the following result

$$|G| = (0.66 \pm 0.06) \text{ GeV}^{-1}. \quad (36)$$

This prediction for the strong coupling $G$ is typical for tetraquark-meson vertices. Its numerical value and dimension are determined by definition of matrix element $\langle D(p) K(q)|X_0(p')\rangle$: Modification of the vertex $DKX_0$ in Eq. (23) changes the value and dimension of $G$. In general, it is possible to rewrite $\langle D(p) K(q)|X_0(p')\rangle$ in such a way that to make $G$ dimensionless. In our analysis $G$ is an intermediate parameter, whereas the physical quantity to be found is the width $\Gamma$ of the decay $X_0 \to D^- K^+$. The $\Gamma$ is calculated by taking into account Eq. (23), and its expression depends on these matrix elements. But regardless used convention for the vertex $DKX_0$ and analytical form of the width, numerical computations lead to the same final result with correct dimension, as it should be for a physical quantity.

Having used the matrix elements given by Eq. (23), we derive for the width of the decay $X_0 \to D^- K^+$

$$\Gamma [X_0 \to D^- K^+] = \frac{G^2 m_D^2 \lambda}{8 \pi} \left(1 + \frac{\lambda^2}{m_D^2}\right), \quad (37)$$

where $\lambda = \lambda(m, m_D, m_K)$ and

$$\lambda(a, b, c) = \frac{1}{2a} \left(a^4 + b^4 + c^4 - 2(a^2 b^2 + a^2 c^2 + b^2 c^2)^{1/2}\right). \quad (38)$$

Then it is not difficult to find that

$$\Gamma [X_0 \to D^- K^+] = (49.6 \pm 9.3) \text{ MeV}. \quad (39)$$

This prediction for the width of the resonance $X_0$ is in a reasonable agreement with new LHCb data (1).

The molecule $D^0 K^{*0}$ is composed of two neutral vector mesons, which are strong-interaction unstable particles. The meson $D^0 (2007)^0$ is relatively narrow state $\Gamma_{D^0} \approx 2.1$ MeV, whereas the width $\Gamma_{K^{*+}} = (47.4 \pm 0.5)$ MeV of $K^{*(892)^0}$ within experimental errors is comparable with LHCb data $\Gamma_0$. In Ref. (6) $X_0$ was modeled as hadronic $D^{*-} K^{*+}$ molecule, and width of the meson $K^{*+}$ was used to estimate roughly the $X_0$ resonance’s width. The hadronic molecules $D^{*-} K^{*+}$ and $\bar{D}^0 K^{*0}$ are bound states, and partial widths of their decay channels are determined by quark-gluon interactions inside of these particles. Due to multiquark nature of molecules their internal dynamics obviously differs from those of free mesons $D^*$ and $K^*$. Therefore, estimation of the $D^{*-} K^{*+}$ and $\bar{D}^0 K^{*0}$ molecules’ widths using directly widths of constituent mesons seems us to be problematic. One can suggest, that after dissociation of $\bar{D}^0 K^{*0}$ to two vector mesons, decays of $K^{*0}$ may be used for such analysis. But the mass of the $X_0$ is below relevant two-meson thresholds in both pictures, i.e., $X_0$ does not decay to mesons $\bar{D}^0 K^{*+}$ or $D^{*-} K^{*+}$. The $P$-wave decay $\bar{D}^0 K^{*0} \to D^0(2400)^0 + K^{*0}$ with the vector meson $K^{*0}$ in the final state is forbidden kinematically. Another two-body $P$-wave decays of $\bar{D}^0 K^{*0}$, for example, to mesons $\bar{D}^0(2007)^0 + K^{*0}_0(1430)$ and $\bar{D}^0(2420)^0 + K^{*0}$ are not allowed because of the same reasons. Nevertheless, there are multibody decays of $X_0$ which contribute to its full width. For example, processes $X_0 \to D^- K^+ \pi^0 \pi^0$ and $X_0 \to D^- K^+ \pi^+ \pi^-$ can improve our prediction for $\Gamma$. But these processes imply production of 4 new valence quarks through different mechanisms, which suppress relevant amplitudes by the factor $\alpha_s$ or additional strong couplings. As a result, widths of such subdominant decays would be small.

We have explored the dominant decay channel $X_0 \to D^- K^+$ of the resonance $X_0$. The result for its width in Eq. (39) has been obtained in the context of the LCSR method by applying first principles of the QCD, and is reliable prediction for this parameter. It can be improved further by including into analysis other decay channels of the $X_0$, which are beyond the scope of the present article.

IV. DISCUSSION AND CONCLUSIONS

In the present work we have explored one of two new resonances $X_0$ and $X_1$ reported by the LHCb collaboration. Namely, we have considered $X_0$ as a scalar hadronic molecule $D^0 K^*$, and calculated its mass and width. For these purposes, we have used the QCD sum rule method. The spectroscopic parameters of the state $D^0 K^*$ have been extracted from two-point sum rules, whereas for analysis of its strong decay channel, we used LCSR method and soft-meson approximation. Obtained predictions for $m$ and $\Gamma$ are in nice agreement with reported LHCb data, which can be interpreted in favor of molecule nature of the resonance $X_0$.

The suggestion about molecular structure of $X_0$ was made in Ref. (4), in which the authors computed the mass of the molecule $D^{*-} K^{*+}$ using the sum rule method. Calculations were done by taking into account nonperturbative terms up to dimension 8. Prediction obtained there for $\tilde{m}$

$$\tilde{m} = 2870^{+190}_{-140} \text{ MeV} \quad (40)$$

is very close to our result. The molecule composition for the $X_0$ in different frameworks was proposed in Refs. (3, 8, 12), as well.

Alternatively, the resonance $X_0$ was analyzed in Refs. (3, 4, 7, 11) by assuming that it is a diquark-antidiquark
state. The sum rule prediction for the mass of the scalar tetraquark $T_1 = [sc][sc]$ with an axial-axial $C\gamma_\mu \otimes \gamma^\mu C$ type structure reads \[4\]

$$m_{T_1} = (2910 \pm 120) \text{ MeV.}$$

(41)

The similar sum rule investigations were performed in Ref. [11]. Results for masses of the scalar tetraquark $T_2 = [ud][\bar{c}\bar{s}]$ with scalar-scalar and axial-axial structures are equal to

$$m_{T_{2s}} = 2750^{+180}_{-120} \text{ MeV},$$

$$m_{T_{2a}} = 2770^{+190}_{-180} \text{ MeV},$$

(42)

respectively. By taking into account uncertainties of calculations, the authors concluded that $X_0$ could be interpreted as a tetraquark $J^P = 0^+$ with either scalar-scalar or axial-axial configurations. It is seen that states $T_1$ and $T_2$ are connected by the relations $T_1 = T_2$ or $T_1 = T_2$ as conventional mesons, for example, $D^0 = c\bar{u}$ and $D^{*0} = c\bar{s}$. Masses of such particles should be equal to each other, which is not the case for $m_{T_1}$ and $m_{T_{2a}}$. In our view, additional studies are necessary to solve problems existing in the QCD sum rule analyses of the resonance $X_0$ in the diquark-antidiquark picture.

Interesting analysis of the ground-state and radially excited tetraquark $T_2$ was performed in Ref. [7]. In this paper the mass of the particles $T_2(1S)$ and $T_2(2S)$ were found equal to 2360 MeV and 2860 MeV, respectively. As a result, the resonance $X_0$ was interpreted there as the excited tetraquark $T_2(2S)$.

The enhancements in the $D^- K^+$ mass distribution labeled by $X_0$ and $X_1$ may have alternative origin [8]. Thus, the authors of Ref. [8] investigated the process $B^+ \rightarrow D^+ D^- K^+$ via $\chi_{c1} D^+ K^{*+}$ and $D_s J/D_s K^{0}$ rescattering diagrams. It was argued that, two resonance-like peaks obtained around thresholds $D^* K^{*+}$ and $D_s J/D_s K^{0}$ may simulate the states $X_0$ and $X_1$ without a necessity to introduce genuine four-quark mesons. The observed experimental peaks were explained there by the triangle singularities in the scattering amplitudes located in the vicinity of the physical boundary.

Experimental results obtained by the LHCb collaboration do not raise doubts about existence of the resonance-like enhancements $X_0$ and $X_1$ in the $D^- K^+$ mass distribution. These structures were already considered as meson molecules, diquark-antidiquark systems, rescattering effects. In other words, there are different and controversial interpretations of the structures $X_0$ and $X_1$ in the literature. Additional theoretical efforts seem are required to clarify nature of these states.

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