Supercurrent-phase relationship of a Nb/InAs(2DES)/Nb Josephson junction in overlapping geometry

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Superconductor/normal conductor/superconductor (SNS) Josephson junctions with highly transparent interfaces are predicted to show significant deviations from sinusoidal supercurrent-phase relationships (CPR) at low temperatures. We investigate experimentally the CPR of a ballistic Nb/InAs(2DES)/Nb junction in the temperature range from 1.3 K to 9 K using a modified Rifkin-Deaver method. The CPR is obtained from the inductance of the phase-biased junction. Transport measurements complement the investigation. At low temperatures, substantial deviations of the CPR from conventional tunnel-junction behavior have been observed. A theoretical model yielding good agreement to the data is presented.

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I. INTRODUCTION

A fundamental item in the theoretical modeling of a Josephson junction is the dependency of the supercurrent $I_S$ flowing through it on the phase difference $\varphi$ across the junction. Golubov, Kupriyanov and Il’ichev published a recent review on the subject. As this relation is necessarily $2\pi$-periodic and odd it can be expressed as the following Fourier series (with the critical current $I_C$):

$$I_S(\varphi) = I_C f(\varphi) = \sum_n n \sin(n \varphi).$$

$I_S(\varphi)$ is known as (super-)current-phase relationship or CPR, the dimensionless term $f(\varphi)$ as normalized CPR. For vanishing transparency (e.g. a tunnel junction), only $I_1$ is relevant, reducing Eq. 1 to the well-known dc Josephson equation

$$I_S(\varphi) = I_1 \sin \varphi = I_C \sin \varphi.$$  \hspace{1cm} (2)

Junctions with direct (i.e. non-tunnel) conductivity, such as the Nb/InAs(2DES)/Nb weak links used in this work, are predicted to show more complex behavior and a non-sinusoidal CPR due to higher order processes of charge transport such as multiple Andreev reflections. Indications of a significant non-harmonic term in our junctions have been found in microwave measurements. We are interested in experimental access to the CPR as a way to test and improve the theoretical models describing our junctions, since the conduction mechanisms in these devices are still not fully understood. The measurements deliver not only the dependency $f(\varphi)$, but also $I_C$, thus offering an independent comparison to transport measurements.

II. THEORY

The first extensive theoretical works concerning high transparency superconducting weak links were published by Kulik and Omel’yanchuk for a short quasiclassical point contact in the dirty limit $l \ll \xi_0$ (with mean free path $l$ and coherence length $\xi_0$) and for the clean limit $l \gg \xi_0$, predicting a CPR $f(\varphi)$ changing from a sinusoidal curve to a saw tooth shape for high transparencies and low temperatures. These idealized model systems serve to understand the basic mechanisms leading to a deviation of the CPR from the well-known Josephson relation (Eq. 2), which we encounter also in complex real-world junctions, though mixed with secondary processes. Mechanically controlled break-junctions come closest to the assumptions of the theory of Kulik and Omel’yanchuk. CPR measurements by a flux-detecting method in this system agreed with the predicted changes in the position of the maximum and curve shape. In contrast to these systems, our SNS junctions exhibit a finite length, some scattering of electrons in the 2DES, and an area of induced superconductivity around the electrodes caused by the proximity effect. To take these properties into account and to describe the non-sinusoidal CPR in our experiments, we used the scattering matrix formalism to derive a formula for the Josephson current through a double-barrier structure.

$$\frac{eI_R}{\pi k_B T_C} = \frac{16 T}{T_C} \sum_{n=0}^{N-1} \sum_{m=0}^{\infty} \left( -A^2|S_{12}|^2 \sin(\varphi) + A^4|S_{11}|^2 + A^4|S_{21}|^2 \right),$$

\hspace{1cm} (3)
where $R_0 = \hbar/e^2$ is the resistance quantum, $A$ is the coefficient of the Andreev reflection at the InAs(2DEG)/InAs interface and $S_{ij}$ are the elements of the scattering matrix for normal reflection in a symmetric double barrier junction. $S_{11} = |r| + |t|^2 p_n/(1 + |r|^2 p_n^2) = S_{22} = |t|^2 p_n/(1 + |r|^2 p_n^2)$. Here $|r|^2, |t|^2$ are the reflection and transmission probabilities of the left and right-hand barrier, $p_n = \exp(i k_F a - \omega_m a/\hbar v_{F,n})$, where $\omega_m = (2n + 1)\pi k_B T$ are Matsubara frequencies, $k_F$ and $v_{F,n}$ are components normal to the barriers of the wave vector and Fermi velocity, respectively, of the $n$-th transverse mode to the barriers and $a$ is the distance between them. Following the BTK approach, the reflection and transmission coefficients can be written in the form

$$|t|^2 = 1 - |r|^2 = \frac{1 - (n/N)^2}{(1 - (n/N)^2)(\eta + 1)^2/4\eta + Z^2} \quad (4)$$

where $Z$ is the dimensionless potential barrier strength and $\eta = v_{F,s}/v_{F,n}$ is the Fermi velocity mismatch.

In order to calculate the Josephson current from Eq. 3, one must determine the Andreev reflection coefficient $A$ for the InAs(2DEG)/InAs interface. Since superconductivity is induced in InAs by the proximity effect, the coefficient $A$ may be written as $A = iF/(1 + G)$, where $F$ and $G$ are Green functions in the inversion layer of InAs. Due to the low electron density of the inversion layer the suppression of the pair potential in Nb can be neglected and $F$ and $G$ can be expressed by the McMillian equations

$$G = \omega / \sqrt{\omega^2 + \Phi^2}, \quad F = \Phi / \sqrt{\omega^2 + \Phi^2},$$

and $\Phi = \Delta/(1 + \gamma B \sqrt{\omega^2 + \Delta^2})$, where $\gamma B$ is a dimensionless parameter characterizing the transparency between Nb and InAs, $\Delta = \Delta/\pi k_B T_C$, $\omega = \omega/\pi k_B T_C$, $\Delta$ is the superconducting energy gap of bulk Nb, and $T_C$ its critical temperature. Both the critical temperature and the energy gap of the Nb can be suppressed near the interface because of disorder, but $\Delta \approx 0.6$ remains constant. Since $T_C$ can be determined from temperature measurement, there are only two free fitting parameters in the model, the carrier density $n_s$ and $\gamma B$ (we assume $Z = 0$, i.e. no real barrier). The Fermi velocity $v_F$ and Fermi wave vector $k_F$ are calculated for a given value of $n_s$. Since the normal resistance depends on $n_s$, as well, the $n_s$ obtained from the fit can be verified comparing the theoretical and experimental value of the resistance.

III. SAMPLE PREPARATION

The superconductor used in our SNS junctions is a Nb thin film, the normal conductor is the two-dimensional electron system (2DES) that forms as naturally occurring inversion layer at the surface of bulk p-type InAs. The InAs surface is cleaned in situ using low-energy Ar etching prior to the deposition of a 100 nm thick Nb film by magnetron sputtering, yielding the highly transparent interfaces required to observe deviations from sinusoidal CPR. The structures are patterned using standard optical and electron-beam lithography. We are using an overlapping sample geometry (see inset of Fig. 1), where we can set the electrode separation $a$ with nm accuracy using anodic oxidation to grow the insulating Nb$_2$O$_5$ interlayer. At a typical electron density of $n_s = 1.2 \times 10^{12}$ cm$^{-2}$ and mobility of $\mu \approx 10^4$ cm$^2$/Vs, the mean free path of $l = 240$ nm is much longer than the electrode separation $a = 40$ nm. From the dependence of the critical current on the channel length, we estimate the coherence length $\xi_N \approx 145$ nm at 1.8 K. Thus we have a ballistic ($a \ll l$) superconducting weak link in the short limit ($a \ll \xi_N$). More details have been published by Chrestin et al.

The use of the native 2DES on p-type InAs is not without difficulties, as it forms on every surface of the crystal. Thus it offers parallel conduction paths, resulting in significant bypass currents around the junction, which reduce the normal resistance $R_N$.

IV. CPR MEASUREMENT TECHNIQUE

The CPR of the Josephson junction is determined by an inductive rf frequency method requiring no galvanic contacts to the sample, thus reducing noise. This technique is based on the work of Rifkin and Deaver. The junction to be investigated is incorporated into a superconducting loop. This loop forms an rf SQUID of inductance $L$, which is coupled inductively to a high quality tank circuit in resonance. The phase difference $\phi$ across the junction can be biased by an external magnetic flux $\Phi$ via a dc current $I_{dc}$. Changes in the impedance of the coupled system are measured and can be used to reconstruct the CPR. Further details are given in Ref. [1].
This method requires low critical currents $I_C$ of the junction and low inductances $L$ of the SQUID washer, as the SQUID enters the hysteretic regime when $\beta f'(\varphi) > 1$, where $\beta = 2\pi LI_C/\Phi_0$ is the normalized critical current and $f'(\varphi) = df(\varphi)/d\varphi$. In the hysteretic mode the internal magnetic flux as a function of the external one becomes multivalued, preventing us to reconstruct the CPR in the complete phase range $(0, 2\pi)$. The challenge is to reduce $I_C$ and $L$ while maintaining excellent interface transparency, large $I_C R_N$ products, and sufficient coupling to the tank circuit. We achieve this by reducing the geometric dimensions of the junction and the loop and by complex washer designs. The presented junction is connected to six SQUID washers in parallel with an integrated flux transformer, for an inductance of $L = 17$ pH and optimized coupling to the tank circuit. A similar transformer is described and depicted in Ref. 1.

V. RESULTS

The experiments are performed on an overlapping Josephson junction of width $w = 1$ $\mu$m and electrode separation $a = 40$ nm. Transport measurements at 1.8 K result in $I_C = 3.8$ $\mu$A, $R_N = 58$ $\Omega$, $I_C R_N = 220$ $\mu$V, and show clearly developed subharmonic gap structures (SGS) in the differential resistance of the junction. This indicates high interface quality and transparency of the junctions.

Figure 2 shows the recorded phase shift $\alpha$ in the tank circuit as a function of the applied quasi-dc current $I_{dc}$ ramping the external flux $\Phi_x$ through the SQUID loop. This signal was averaged for 20 periods to reduce noise and then used to reconstruct the CPR depicted in Fig. 3. For decreasing temperatures, gradual deviation of the CPR from conventional sinusoidal behavior towards a sawtooth-shaped curve is observed. The position of maximum current is shifted to 0.63$\pi$ at 1.3 K. At the same time, the critical current increases substantially, so that at some temperature the SQUID enters the hysteretic regime as $\beta f'(\varphi)$ approaches unity. In that case we are no longer able to reconstruct the CPR at lower temperatures.

The $I_C - T$ dependence was fitted by the model described above using the least squares method with two fitting parameters $n_s$ and $\gamma_B$. The other parameters used in the calculation were determined independently as critical temperature $T_C = 6.5$ K, width $w = 1.0$ $\mu$m, electrode separation $a = 40$ nm, interface parameter $Z = 0$, and Fermi velocity mismatch $\eta = 0.93$. The fitting procedure yields the parameters $n_s = 3.75 \times 10^{12}$ $\text{cm}^{-2}$ and $\gamma_B = 2.4$ and good agreement between theoretical and experimental data (see Figs. 1 and 3). Note that the model fits the current-phase characteristic quite well for the parameters determined from the temperature dependence of the critical current. Only the curve taken at $T \approx 2.0$ K exhibits some deviation from the theoretical one. However, the temperature dependence of $I_C(T)$ is rather steep at low temperatures and a small difference between the temperature of the thermometer and the sample could be responsible. From the above parameters we calculate the junction resistance of $R_N = 83$ $\Omega$. Transport measurements yield a slightly lower value of 58 $\Omega$, but as we measure a parallel connection between substrate and junction, a higher junction resistance is to be expected. A realistic substrate resistance of 180 $\Omega$ produces the measured value of 58 $\Omega$. The interface parameters indicate a highly transparent junction. The assumed carrier density is about three times the result $n_s = 1.2 \times 10^{12}$ $\text{cm}^{-2}$ from Shubnikov-de Haas measure-

**FIG. 2:** Measured phase shift at various temperatures.

**FIG. 3:** CPR (red) at temperatures between 1.3 K and 6.0 K, reconstructed from the data in Fig. 2. The solid lines are a fit according to the model described in the text. For comparison, a pure sine curve is included.
ments on comparable samples. Nevertheless, $n_s$ as determined from our model is consistent with the smaller experimental value of the junction resistance. A possible explanation for both effects is an effective widening of the contact due to the proximity effect in the surrounding 2DES.

VI. CONCLUSION

We have successfully measured the current-phase relationship (CPR) of a Nb/InAs(2DES)/Nb Josephson junction in dependence of temperature. At low temperatures, substantial deviations of the CPR from a sinusoidal behavior towards a saw tooth shape are observed. This is in qualitative agreement with predictions of the Kulik-Omel'yanchuk theory for highly transparent SNS junctions and the measurements on Josephson field effect transistors presented in Ref. 11. A model yielding good quantitative agreement to the results is presented. Transport measurements support the results gained by phase sensitive measurements.

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