The thermodynamics of isolated horizon in higher dimensional loop quantum gravity

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The statistical mechanical calculation of the thermodynamical properties of higher-dimensional non-rotating isolated horizons is studied in the loop quantum gravity framework. By employing the Hawking temperature and horizon mass of isolated horizons as physical input, the microcanonical ensemble associated with the system are well established. As a result, the black hole entropy and other thermodynamical quantities can be computed and consistent with well known Hawking’s semiclassical analysis. Moreover, the value of the Immirzi parameter of higher-dimensional loop quantum gravity is also obtained.

PACS numbers: 04.60.Pp, 04.50.Kd

I. INTRODUCTION

Black hole (BH) as one prediction of general relativity supplies a splendid platform for both experimental and theoretical physics. Recently, the Event Horizon Telescope (EHT) observations of the shadows of M87*1 and Sgr A*2 have supported the existence of BH and unveil some mysteries of it. Theoretically, as the simple and strong gravitation object, BH is a practical research object for general relativity (GR), modified gravities, and quantum theories of gravity, especially after its thermodynamics was established by Bekenstein and Hawking 3, 4 in 1970s. Whereafter, the thermodynamics was extended to quasilocally defined boundary of BH, namely isolated horizon (IH) 5. It allows to describe the BH as physical object by local geometry which is independent with the spacetime outside the horizon. The Bekenstein-Hawking formula of BH entropy 6, 7 brings GR, quantum mechanics and statistical mechanics together. How to account for the statistical mechanics origin of BH entropy then becomes a great challenge for quantum theories of gravity.

As a well-known candidate for quantum theory of gravity, there are various attempts made in the framework of loop quantum gravity (LQG) 8–11 to account for the BH entropy applying the notion of IH, for example, from the aspects of boundary topology theories 12–17, state counting methods 18–21, and symmetries of IH 22, 23. Usually, in previous examples, the leading orders of entropy will agree with the Bekenstein-Hawking entropy if and only if Immirzi parameter γ takes some special values γ0 13, 14, 18, 21. It gives one way to fix the free parameter γ in LQG. In Ref. 24, the authors proposed a new method for calculating the entropy by introducing the universal horizon temperature and the energy measured by local observer. In this treatment, the Bekenstein-Hawking entropy could be reached for arbitrary values of γ. And γ occurs in the semiclassical correction term of statistic entropy via chemical potential. If one further requires the classical formula of entropy is exactly the Bekenstein-Hawking one, the value of Immirzi parameter can also be fixed. This requirement is reasonable because if chemical potential is non-vanishing, one could achieve a lower energy by adding or removing particles (named punctures in loop quantized black hole).
Then in 4-dimensional LQG, this method lead to the following equation determines the value of Barbero-Immirzi Parameter as
\[ \sum_j (2j + 1) e^{-2\pi\gamma\sqrt{j(j+1)}} = 1 \]  
(1.1)
with \( j \) being half integer. This equation was also obtained in all previous state counting of black hole entropy in LQG method \[13, 24\].

Higher dimensional gravity as an extension of general relativity has received widely concern. It is derived from the Kaluza-Klein theory \[26\] that the irrelevant physical phenomena, namely gravity and Maxwell theory, in 4-dimensional spacetime could be unified in higher dimensional theory. The loop quantization of higher dimensional spacetime has achieved in Refs. \[27–30\]. It is further applied to higher dimensional cosmological models \[31\] and coherent state for semi-classical analysis \[32–36\]. Even there are many achievement in higher dimensional LQG, however, the first law and entropy of higher dimensional BH in LQG framework is still an open issue. It also leaves the Immirzi parameter uncertain in higher dimensional LQG.

The aim of this article is extending the IH thermodynamics in LQG obtained in Ref. \[24\] into higher dimensional spacetime and fixing the Immirzi parameter in higher dimensions. This article is organized as follows: In Sec. II, we perform the local version of the first law measured by stationary observers in higher dimensional spacetime. By means of the area operator in higher dimensional LQG and the variational method, the quantum corrected entropy and the first law is given in Sec. III. We analyse the value of Immirzi parameter in Sec. IV and conclude in Sec. V.

**II. THE CLASSICAL CONFIGURATION OF BLACK HOLE IN HIGHER DIMENSION**

Let us consider the \( D + 1 \) dimensional Schwarzschild spacetime. The metric expressed under spherically symmetric coordinates is \[37, 38\]
\[ ds^2 = -(1 - \left(\frac{r_H}{r}\right)^D)dt^2 + (1 - \left(\frac{r_H}{r}\right)^D)^{-1}dr^2 + r^2d\Omega_{D-1}^2, \]  \(2.1\)
where \( r_H \) is the horizon radius and \( d\Omega_{D-1}^2 \) is the line element of unit \( (D - 1) \)-sphere. Without ambiguity, we use \( \Omega_{D-1} = \frac{2\pi^{D/2}}{\Gamma(D/2)} \) to denote the area of unit \( (D - 1) \)-sphere, where \( \Gamma(n) \) is the Euler’s Gamma function with \( \Gamma(n) = (n-1)! \) and \( \Gamma(1/2) = \sqrt{\pi} \). The black hole horizon radius \( r_H \) matches the mass parameter \( M \) through
\[ r_H = \left[ \frac{16\pi G}{(D-1)\Omega_{D-1}} M \right]^{\frac{1}{D-2}}, \]  \(2.2\)
where the \( D + 1 \) dimensional gravitational constant is defined by \( G = c^2\ell_{P}^{D-1}/\hbar \). The horizon area is \( A_{D-1} = \Omega_{D-1} r_H^D \). And the surface gravity is defined by \( \kappa_H^2 = -\frac{1}{4} \nabla_a \xi_b \nabla^a \xi^b \), where \( \xi = \partial_t \) is the timelike Killing vector. Substituting the metric \(2.1\) into the above definition, one could get that
\[ \kappa_H = \frac{D - 2}{2} \frac{1}{r_H}. \]  \(2.3\)
The Komar mass is
\[ E_H \equiv -\frac{D - 1}{16\pi G(D-2)} \int_S \nabla^a \xi^b dS_{ab} = M, \]  \(2.4\)
where \( S \) is an arbitrary \((D-1)\)-sphere surrounding BH and the area element \( dS_{ab} \) is defined by the unit normals \( t_a \) and \( r_b \). The classical version of the first law of black hole thermodynamics have been extended to higher dimensions \[37, 39–42\]. Apart from the work term, we can obtain
\[ dE_H = \frac{\kappa_H}{8\pi G} dA. \]  \(2.5\)
And the integral formula of the first law, namely, the Smarr’s formula is
\[ E_H = \frac{D - 1}{D - 2} \kappa_H A. \]  
(2.6)
Considering the Hawking radiation, the Schwarzschild BH will evaporate. The equilibrium could be achieved by imbedding the black hole into the Hartle-Hawking vacuum with Hawking temperature
\[ T_H = \frac{\hbar \kappa_H}{2\pi}. \]  
(2.7)
at null infinity, which leads to a time symmetric thermal bath of radiation. Since the statistical mechanical descriptions of BH in loop quantum gravity is executed in quasilocal horizon, namely isolated horizon. Comparing the first law of BH with the one in thermodynamics i.e., \( dE = T dS \) and replacing \( T \) by \( T_H \), one can get the entropy of BH as
\[ S = \frac{A}{4l_p^{D-1}}. \]  
(2.8)
It is nothing but the extension of the BH entropy in 4-dimensional case.

We introduce the classical configuration of the system by the definitions of surface gravity and Komar mass based on Killing horizon for simplicity. Those quantities can also be defined on IH, since the IH is the extension of Killing horizon, which does not require a global Killing vector. The absent of global structure results in that one couldn’t choose a unique null norm of IH by requiring the normalization of the Killing vector at null infinity as Killing horizon did. Hence, there is a free positive constant parameter for the horizon null norm \( l^a \), such that \( l^a = \alpha \xi^a \) when the IH reduces to Killing horizon. Therefore, the surface gravity defined by \( l^a \) on IH will be multiplied by \( \alpha \). The horizon mass can also be defined using Hamiltonian methods as the generator of a preferred time translation on IH. In this method, the horizon mass is primary quantity expressed as a specific function of the fundamental quantities defined intrinsically at the horizon, namely area, angular momentum and charges. For non-rotating IH, the horizon mass equals to Komar mass multiplied by \( \alpha \). The first law of black hole thermodynamics have been extended to higher dimensions IH. Since \( \alpha \) is an overall factor in the expression of the first law, we will drop it when using the notion of isolated horizon. The advantages of IH that the quantities defined on it are independent with the spacetime outside horizon, allows that the horizon and spacetime outside horizon can be quantized independently, in the framework of LQG which we will employ in the next section.

III. THE STATISTIC MECHANICS OF QUANTUM IH IN HIGHER DIMENSION

We review the classical description of thermodynamics of higher dimensional IH in the last section. Now we use the classical configuration as physical input to investigate the statistic mechanics of higher dimensional quantum IH via microcanonical ensemble. For \( D + 1 \) dimensional spacetime, we can construct \( D + 1 \) dimensional LQG with structure group \( SO(D + 1) \). After solving the so-called “simplicity constraint”, the rest degrees of freedom for every edges are label by a non-negative integer \( J \). The dimensions of the representation space reads
\[ \dim(\pi_J) = \left(\frac{J + D - 2}{J!(D-1)!}\right). \]  
(3.1)
Here the role \( J \) is somewhat like the half-integer \( j \) representation of the 4-dimensional LQG with structure group \( SU(2) \). In \( D + 1 \) dimensional LQG, the discrete spectrum of this \( D - 1 \) dimensional area operator reads
\[ \Delta_{D-1} = 8\pi G\hbar \gamma \sum J \sqrt{J(J+D-1)}, \]  
(3.2)
According to the similar strategy [24], we let $A$ denotes the classical area of the Isolated Horizon (IH) with a total number of punctures $N$. A quantum configuration $\{s_J\}$ is given by the number of punctures $s_J$ for all possible values of $J$. The black hole configuration in higher dimensions must obey the following two constraints

$$C_1 : \sum_J \sqrt{J(J+D-1)}s_J = \frac{A}{8\pi\gamma\ell_p^{D-1}}, \quad (3.3)$$

$$C_2 : \sum_J s_J = N. \quad (3.4)$$

where $\ell_p^{D-1} = G\hbar$. Then the number of states $d\{\{s_J\}\}$ associated with a configuration $\{s_J\}$ is

$$d\{\{s_J\}\} = (N)! \prod_J \frac{1}{s_J!} \left( \frac{(J+D-2)!(2J+D-1)}{J!(D-1)!} \right)^{s_J}. \quad (3.5)$$

The configuration which maximizes the entropy $\log(d\{\{s_J\}\})$ subject to the above two constraints is exactly what we are looking for. To this aim, we have the following variational equation

$$\delta \log(d\{\{s_J\}\}) - \lambda \delta C_1 - \sigma \delta C_2 = 0 \quad (3.6)$$

where $\lambda$ and $\sigma$ are the two Lagrange multipliers. By using Stirling’s approximation formula

$$\log N! \approx N(\log N - 1) \quad (3.7)$$

we can obtain the dominant configuration

$$\frac{s_J}{N} = \frac{(J+D-2)!(2J+D-1)}{J!(D-1)!} e^{-\lambda\sqrt{J(J+D-1)}}. \quad (3.8)$$

Summing over all possible $J$ gives

$$e^{-\sigma} \sum_J \frac{(J+D-2)!(2J+D-1)}{J!(D-1)!} e^{-\lambda\sqrt{J(J+D-1)}} = 1. \quad (3.9)$$

Denoting by $\tilde{d}$ the number of the states for the dominant configuration, and omitting the reminder term, the entropy is approximated by

$$S = \log \tilde{d} = \frac{A}{8\pi\gamma\ell_p^{D-1}} + \sigma N, \quad (3.10)$$

where $\sigma$ could be solved from Eq. (3.5) expressed as

$$\sigma = \log \left[ \sum_J \frac{(J+D-2)!(2J+D-1)}{J!(D-1)!} e^{-\lambda\sqrt{J(J+D-1)}} \right]. \quad (3.11)$$

Substituting Eqs. (3.10) and (2.4) into the inverse temperature $\beta = \left( \frac{2\sigma}{\pi\gamma} \right)_N$, the Lagrange multiplier $\lambda$ can be expressed as a function of $\beta$, that is, $\lambda = \frac{(D-2)\gamma\hbar}{2\pi\ell_p^{D-1}} \beta$. Setting temperature as $T_H$ in Eq. (2.7), one could determine the Lagrange multipliers $\lambda$ as $2\pi\gamma$ and $\sigma$ through Eq. (3.11). Except the free parameter $\gamma$ in LQG, we fix the expression of entropy as

$$S = \frac{A}{4\ell_p^{D-1}} + N\sigma(\gamma), \quad \text{with} \quad \sigma(\gamma) = \log \left[ \sum_J \frac{(J+D-2)!(2J+D-1)}{J!(D-1)!} e^{-2\pi\gamma\sqrt{J(J+D-1)}} \right]. \quad (3.12)$$
One could see the role of $\sigma(\gamma)$ played through the definition of chemical potential

$$\mu = -T_H \frac{\partial S}{\partial N} \Big|_E = -\frac{\hbar \kappa_H}{2\pi} \sigma(\gamma)$$

(3.14)

The chemical potential depends on $\gamma$ through Lagrange multiplier $\sigma$. Now that $\gamma$ influences the summation in Eq. (3.11) as the exponential decay factor, larger values of $\gamma$ means the terms in summation decay faster as the increase of spin $J$, and hence means smaller $\sigma$. Since $\mu = -T \sigma(\gamma)$, the chemical potential could be negative, vanish or positive as the value of $\gamma$ increases. In this sense, the correct first law of a quantum IH mechanics should be

$$dE = TdS + \mu dN.$$ 

(3.15)

Substituting energy (2.4) into (3.12), one could get the entropy in quantum level as $S = \frac{D-2}{D-1} \beta E + \sigma N$. The coefficient containing dimension is caused by the dependence of temperature on horizon mass, which is coincident with the one in classical formula (2.6). It is also the reason why we get the different integral formula of the first law with the one in Ref. 24 when $D + 1 = 4$, since the universal temperature is employed in the article. Comparing the classical and quantum entropies (2.8) and (3.12), there is an extra term corresponding to $N$, which is regarded as quantum hair in Ref. 24. This quantum hair is convincing, since $N$ is the total number of punctures in quantum description of IH, which is no analog at classical level. Except the term of quantum hair, the entropy (3.12) is coincident with the semiclassical Bekenstein-Hawking formula.

For the large black hole, the $\sigma$ is proportional to chemical potential $\mu$ and therefore in the classical limit should approach to 0 to achieve equilibrium 24. Hence we obtain

$$\sum_J \frac{(J + D - 2)! (2J + D - 1)}{J! (D - 1)!} e^{-2\pi \gamma \sqrt{J(J+D-1)}} = 1$$

(3.16)

which determines the value of $\gamma$ in higher dimensional LQG.

IV. THE ANALYSIS OF IMMIRZI PARAMETER IN HIGHER DIMENSIONAL LOOP QUANTUM GRAVITY

As the most crucial free parameter in LQG, the Immirzi parameter occurs in the spectrums of geometric operators and in the dynamics driven by Spin Foam. Immirzi parameter can’t be fixed by LQG individually. In last section, we obtain the equality (3.16) that the Immirzi parameter should satisfy in higher dimensions through comparing statistic entropy and Bekenstein-Hawking entropy. Now we analyse the value of Immirzi parameter in higher dimensions. When $D = 3$, $\gamma = 0.276103$. This result is different with the one obtained in Refs. 14, 19, 24, 47, because we use a different gauge group in order to extend LQG to higher dimension. The Immirzi parameter increases first and then decreases as the dimension increases as shown in Fig. 1. When $D = 5$, $\gamma$ takes the maximal value $\gamma_{\text{max}} = 0.281395$.

V. CONCLUSION

The statistical mechanical calculation of the thermodynamical properties of non-rotating isolated horizons is studied in loop quantum gravity framework. By employing the Hawking temperature and horizon mass of isolated horizons, we establish the microcanonical ensemble associated with the system and extend the first law of quantum IH in 4-dimensional spacetime to higher dimensional case, that is, Eq. (3.15). It turns out that the higher dimensional black hole entropy and other thermodynamical quantities can be computed and consistent with well known Hawking’s semiclassical analysis. As a by-product, the quantum hair of puncture $N$ has originated from the underlying quantum geometry and hence, the first law of classical isolated horizons do not possess this term. Therefore, the only natural
value of the chemical potential is zero in classical level. This in turn fixes the value of the Immirzi parameter $\gamma$ in higher dimensional LQG.

Non-perturbative quantum theories of gravity with different values of $\gamma$ are not equivalent with each other, which could emerge the same classical theory. It means that physical input is needed to fix this free parameter. The role of Immirzi parameter played in quantum gravity itself has already been an research topic [48–50], which is still under studying. In this paper, the Immirzi parameter affect the black hole entropy through the quantum hair term, instead of the term proportional to the horizon area as the earlier methods for calculating black hole entropy in LQG did in 4-dimensional cases, for example [15, 18] and higher dimensional cases [51, 52]. The primary reason for this difference is the various state counting method used in the literatures. Let us try to give a reasonable understanding of this new role. Since different quantum theories with different values of $\gamma$ should emerge the same classical theory, a naive perspective is that $\gamma$ should affect entropy as quantum correction, which should disappear in classical limit. This is exactly the new role of $\gamma$ played in entropy (3.12), which is first proposed in Ref. [24] and extended to higher dimensional non-rotating IH in this article.

This article only concerns about the equilibrium state of BH, we embed the BH into the Hartle-Hawking vacuum and keep its area and hence mass fixed. Therefore, black hole evaporation is not involved. LQG and its symmetric reduced models give the discrete BH mass [53, 54]. It offers an possibility of realizing Hawking radiation in LQG framework.

Acknowledgments

This work is supported by National Natural Science Foundation of China (NSFC) with No.11775082. SS is also supported by NSFC with No. 12147167 and the Project funded by China Postdoctoral Science Foundation with No. 2021M700438. GL is supported by the project funded by China Postdoctoral Science Foundation with Grant No. 2021M691072, and the NSFC with Grant No. 12047519. CZ also acknowledges the support by the Polish Narodowe
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