Ridge extraction algorithms for one-dimensional continuous wavelet transform: a comparison

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Abstract. This paper compares between three different algorithms that are used in detecting the phase of a fringe pattern from the ridge of its wavelet transform. A Morlet wavelet is adapted for the continuous wavelet transform of the fringe pattern. A numerical simulation is used to perform this comparison.

1. Introduction
Recently, there has been much interest in demodulating fringe patterns using continuous wavelet transform (CWT) where the demodulation process includes extracting the phase information encoded into the deformed fringe patterns. Ridge extraction from the CWT map is considered to be the heart of the phase demodulation stage. In the literature, many ridge extraction algorithms have been proposed. In this paper, three algorithms will be explained, analyzed and compared to each other. The first two algorithms use the amplitude of the CWT to extract the ridge [1] and they will be called in this paper: direct maximum and cost algorithms. The third algorithm extracts the ridge from the phase of the CWT [2] and will be named phase-map algorithm. This paper is organized as follows: section 2 introduces wavelet transform, section 3 explains the principle of the three ridge extraction algorithms using a noisy simulated object and section 4 compares the results of the algorithms.

2. Wavelet Transform (WT)
Wavelet transform is a suitable tool to analyze non-stationary signals and thus it has been developed as an alternative approach to the current available transforms, such as Fourier transform, to analyze fringe patterns. Moreover, it is worth mentioning that WT has a multiresolution property in the time and frequency domains which overcomes the resolution problem in other transforms.

The term wavelet means a small wave of limited duration and it can be real or complex. However, two conditions must be satisfied in any wavelet which are: the wavelet must have an average value of zero and must have a finite energy [3]. Many different types of mother wavelets are available and for phase evaluation application, the most suitable mother wavelet is probably the complex Morlet, because it has better localization in time and frequency domain [4]. The Morlet wavelet is a plane wave modulated by a Gaussian function, and is defined as

$$
\psi(x) = \pi^{1/4} \exp(i \omega_0 x)\exp(-x^2 / 2)
$$

(1)
where $\omega_0$ is the fre
imaginary part (solid
$\omega_0$)

The one-dimensional continuous wavelet transform (1D-CWT) of a row $f(x)$ of a fringe pattern is obtained by translation on the $x$ axis by $b$ (with $y$ fixed) and dilation by $a$ of the mother wavelet $\psi(x)$ as given by

$$ W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \psi^* \left( \frac{x-b}{a} \right) dx \quad (2) $$

where * denotes complex conjugation and $W(a,b)$ is the calculated CWT coefficients which refers to the closeness of the signal to the wavelet at a particular scale.

Extracting the phase distribution from a fringe patterns is as follows. After applying the complex Morlet wavelet to a row of the fringe pattern, the resultant wavelet transform is a two dimensional complex array. Hence, the modulus and the phase arrays can be calculated by the following equations

$$ \text{abs}(a,b) = |W(a,b)| \quad (3) $$

$$ \phi(a,b) = \tan^{-1} \left( \frac{\Im[W(a,b)]}{\Re[W(a,b)]} \right) \quad (4) $$

where $\Im[W(a,b)]$ and $\Re[W(a,b)]$ represent the complex and real part of the wavelet transform respectively.

These two arrays will be used to extract the ridge of the CWT and hence the phase of the row of the fringe pattern can be determined. The ridge can be defined as the location where the modulus of the CWT reaches its local maximum along the scaling direction[5] and the modulus of the transform is maximum when the analysis frequency equals the signal frequency [6].

3. Ridge extraction algorithms

Ridge detection has an important role in the fringe analysis process using wavelet transform and more specifically, it is the fundamental tool in the phase extraction stage. In the literature, many ridge extraction algorithms have been proposed and in this work three methods will be explained and compared. However, ridge extraction does not present any particular problem with noiseless signals. Therefore, in this section we rather present a noisy simulated object and its fringe pattern as shown in Fig. 2. The object is represented by a 512×512 pixels and it is given by
\[
\phi(x, y) = 0.1[(x - 256)^2 + (y - 256)^2]^{1/2}
\]
and its fringe pattern is given by
\[
I(x, y) = \cos(2\pi f_o x + \phi(x, y)) + 1.5*\text{NOISE}
\]
where NOISE represents a normally distributed random noise with standard deviation equals to one.

The complex Morlet one-dimensional continuous wavelet transform (1D_CWT) is applied to the fringe pattern row by row. The resultant wavelet transform for each row is a two dimensional complex array, hence the modulus and the phase of the CWT can be determined. Fig. 3 shows the absolute and the phase of row 120 of the fringe pattern. Here, the white color indicates large values, whereas the black color indicates small values. The horizontal axis is the translation \(b\), and the vertical axis is the scale \(s\). Now as the image has been processed with CWT, each ridge extraction algorithm can apply its procedure to extract the ridge either from the modulus or from the phase of the CWT.

Figure 2: (a) A simulated object and (b) its fringe pattern.

Figure 3: (a) The modulus and b) the phase of row 120 of the fringe pattern.
3.1. Direct maximum

This algorithm was proposed by Carmona et al. [1]. It extracts the ridge from the amplitude of the CWT as follows. The maximum value of each column in the modulus is determined and then the corresponding phase is chosen from the phase array. This process is repeated for all the rows of the fringe pattern and the final result is a wrapped phase, Fig. 4(a), which needs to be unwrapped, Fig. 4(b).

![Figure 4](image)

Figure 4: (a) The wrapped phase and (b) the unwrapped phase of the fringe pattern using direct maximum method.

3.2. Cost algorithm

The cost method was implemented by Liu et al. [5] and they proposed the dynamic optimization algorithm for cost function ridge detection. In this method, the cost function is introduced to select the ridge from the magnitude of the CWT

\[
Cost[\phi(b), b] = -C_o \int_b |S[\phi(b), b]|^2 \, db + C_i \left| \frac{\partial \phi(b)}{\partial b} \right|^2 \, db \tag{7}
\]

Where \( \phi(b) \) represents any value of the scaling parameter \( a \), \( b \) is the shifting parameter and \( S[\phi(b), b] \) is the modulus value at both \( \phi(b) \) and \( b \). \( C_o \) and \( C_i \) are two weighting coefficients of the modulus and the phase of the wavelet transform respectively [5]. The algorithm selects the local maximum points of the modulus for each column instead of the global ones. These local maxima will be considered as the candidates of the ridge points of this column. Fig. 5(a) shows a curve of the magnitude versus the scaling parameter array, which represents column 50 of the modulus of row 120 of the fringe pattern. The curve shows six local maximum values at the scale values 3, 6, 11, 20, 30 and 51. By repeating this process for all the columns of the modulus, we get complete candidate ridge points for the whole modulus; see Fig. 5(b). The cost function along any ridge can be expressed as

\[
Cost = \sum_{b=2}^{W} \left\{ -|S[\phi(b), b]|^2 + |\phi(b) - \phi(b-1)|^2 \right\} \tag{8}
\]
where $W$ is the width of the modulus map and $C_o$ and $C_f$ will be set to 1 in this work. The cost function is applied to find many different paths and at the end the path with minimum cost will be chosen as the optimal path and therefore the corresponding phase is chosen from the phase array. A wrapped phase will be resulted from repeating the previous steps for all the rows of the fringe pattern and again it needs to be unwrapped, as illustrated in Fig. 6.

**Figure 5:** (a) Column 50 of the modulus of row 120 of the fringe pattern and (b) candidate ridge points for the modulus.

**Figure 6:** (a) The wrapped phase and (b) the unwrapped phase of the fringe pattern using cost method.
3.3. Phase-map algorithm

In 1990, the Marseille group proposed the Phase-map algorithm which is based on a study of the phase of the wavelet transform rather than the modulus and it seems to yield more precise estimates [1]. The algorithm is explained in much detail by Xavier [7]. The method is based on the following principle. When the analysis frequency \( \omega = \omega_o / a \) is close or equal to the signal frequency \( \omega_s \), the rate of variation of the phase of the wavelet transform, \( d\phi_{S(b)}/db \), is actually equal to \( \omega \). Finding this frequency yields \( \omega_s \). The algorithm starts with a rough estimation of the initial scale value \( a_0 \). A new frequency \( \omega_1 \) is then determined as \( d\phi_{S(a_0,b)}/db \). The corresponding scale value \( a_1 = \omega_o / \omega_1 \) is then used to find \( \omega_2 = d\phi_{S(a_1,b)}/db \). The iteration stops when \( (a_{i+1} - a_i)/a_i \) gets lower than a predetermined threshold [7]. Once \( \omega_s(b) \) has been found, the algorithm continues at time \( b+1 \) with \( a_o = \omega_o / \omega_s(b) \). By repeating the previous scenario for all the time shift values, we get the ridge of the first row of the fringe pattern and hence the phase of the whole fringe pattern can be determined. Fig. 7 shows the wrapped and the unwrapped phase resulted from this algorithm.

![Figure 7](image.png)

Figure 7: (a) The wrapped phase and (b) the unwrapped phase of the fringe pattern using Phase-map method.

4. Discussion

As described in Section 3, once the image has been processed with CWT, the three algorithms can be employed to extract the ridge of the wavelet transform. Direct maximum method seems to be quite time consuming as the whole time-frequency domain would have to be explored. Moreover, due to the strong noise components, many local maxima existed. The presence of this noise creates a situation where a spurious peak is mistakenly identified in place of the actual one. However, this algorithm is more robust than phase-map method and that is due to the localization properties of the modulus. Fig. 4(b) shows a better unwrapped phase using direct maximum than the phase-map one shown in Fig. 7(b). On the other hand, the phase-map technique provides more precise estimates and voids the computation of the whole wavelet transform and thus it is faster than the others. However, the cost method was able to extract a much better ridge and hence getting a better unwrapped phase than the two algorithms as shown in Fig. 6. Moreover, in term of execution time, the cost method is slower than the other two algorithms as it needs to search for the optimal ridge. All the previous algorithms were programmed in Matlab with the aid of the YAWTB toolbox [8] and the unwrapped phases were produced using Itoh’s algorithm [9].
5. Conclusion

Ridge extraction is a fundamental tool in fringe pattern demodulation. In this paper, three ridge detection algorithms have been explained and compared with each other in terms of reliability and execution time. With the help of the simulated object, we show that the cost method outperforms the direct maximum and phase-map algorithms in detecting the ridge of the CWT.

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