Reconstruction of the acceleration field of ground vibrations taking into account the directivity of vibrations’ propagation

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Abstract. Many times, commonly used attenuation relations do not allow to obtain a sufficiently practicable accuracy of the results’ description of the observations. This may be an evidence of the attenuation directional nature of ground vibrations caused by tremors. In such cases, using the attenuation relationship that takes into account the directivity of the propagation of vibrations may allow to increase the accuracy of the reconstruction of ground vibrations accelerations. In the article there are presented the reconstruction’s results of recorded accelerations of ground vibrations caused by mine tremors, occurring in the area of one of the Polish mine. In all cases, the directivity of vibrations propagation was observed. The results of the calculations show that the direction of lower surface vibrations attenuation is parallel to the direction of the tectonic fault adjacent to the epicentre of the tremor inducing these vibrations.

1. Introduction

In many mining plants, the exploitation of the seams induces high-energy rockmass tremors. These tremors often cause significant ground vibrations. Because it is practically impossible to observe vibrations parameters in all objects subjected to them, it is necessary to have dependencies allowing to determine the magnitude of ground vibrations depending on the seismic energy of the tremor and its epicentral (hypocentral) distance. Most often, the magnitude of vibrations is determined by specifying the maximum amplitude of their acceleration or velocity and the duration of vibrations. In the further part of the article, the maximum amplitudes of ground vibrations acceleration will be briefly referred as ground vibrations acceleration s. As a rule, mentioned earlier relationships are determined by regression analysis. This is due to the significant complexity of the analytical description of the phenomena occurring between the source of vibrations (tremor hypocenter) and the surface area. Regression equations, determined using the results of previous observations are called the attenuation relationship. A very comprehensive list of the various attenuation relationships used in global seismology includes work [1]. In mining problems, the model proposed in [2] is usually used.

Both in global seismology and in mining seismology, attenuation relations that do not take into account the directionality of vibrations, are generally used. Whereas, frequently after the occurrence of vibrations generated by rockmass tremors, large differences in the magnitude of observed accelerations of vibrations at points located at similar epicentral distances are observed, often the magnitude of ground vibrations at points closer to the epicentre of tremor are lower than at observation points located further from the epicentre of considered tremor. This phenomenon cannot always be
explained by changes in the value of the vibrations amplification coefficient. Similar observations are found in global seismology [3]. This may be evidence of the directional nature of the attenuation of ground vibrations caused by tremors. Among the relatively few works in which the possibility of taking into account directionality of ground vibrations attenuation, the following can be mentioned [3, 4]. The model proposed by the authors of the article [3] is characterized by nine parameters. It requires having measurement results from many surface monitoring stations. In the case of observation networks installed by mining plants, we have data on registered ground vibrations determined by a relatively small number of measuring apparatus. For this reason, it is necessary to use simpler models. That is why the model proposed in [5] was used in the article.

Calculations of ground vibrations accelerations, the results of which are given in this paper, were conducted for data from the mining area of one of the Polish coal mine. Due to the difficulty associated with the reliable determination of the value of the vibrations amplification coefficient by loose quaternary formations, the effect of its differentiation on the magnitude of ground vibrations was omitted.

2. Relationships between the accelerations of ground vibrations and tremors’ seismic energies and their epicentral distances

Analyses presented in the article were carried out for selected 4 strongest tremors that occurred in the subject hard coal mine in 2007 - 2017. To describe the fields of ground vibrations accelerations caused by these rockmass tremors, two forms of attenuation relationship were used. As each tremor was considered independently, from the used attenuation relations was removed the component related to the seismic energy of the tremor (using the presented approach, for each of the analysed cases of ground vibrations, energy of tremor is the constant).

The first of attenuation relationship, often used to calculate the parameters of ground vibrations generated by rockmass tremors, has a very simple form [2]. After taking into account the above remark concerning the energy of tremors, it is described by the equation (1):

\[ \log a = a_1 R + a_2 \log R + a_3 + \varepsilon \]  

where: \(a\) – maximum amplitudes of ground vibrations accelerations [m/s²], \(R\) – hypocentral distance [m] \((R = \sqrt{r^2 + h^2})\), \(r\) – epicentral distance [m], \(h\) – depth of tremor [m], \(a_i\) – regression parameters, \(\varepsilon\) – random component.

In the calculations of which the results are presented in the article, the epicentral distance of tremors was taken into account (assuming \(h = 0\) m).

To describe the vibrations of the ground caused by the considered tremors, a model was also used to take into account the directionality of vibrations’ propagation in the rockmass. This model, proposed in [5], makes it possible to take into account the anisotropy of attenuation the vibrations by assuming that the isoseisms have the shape of an ellipse, rotated around a vertical axis Z by angle q. It is described by the equation (2):

\[ \log a = a_1 R^* + a_2 \log R^* + a_3 + \varepsilon \]  

where: \(R^* = \sqrt{l^2 + m^2}\), \(l = p (x_t - x_{st}) \cos q + (y_t - y_{st}) \sin q\), \(m = (x_{st} - x_w) \sin q + (y_{st} - y_{st}) \cos q\), \(x_t, y_t, z_t\) – tremor coordinates, \(x_{st}, y_{st}, z_{st}\) – coordinates of the seismic station, \(p, q\) – parameters, other designations - as above.

The model (2) was developed using the relations describing the transformation of the coordinate system [6], i.e.: scale change along the X axis (parameter \(p\)) and rotation around the Z axis by angle \(q\).

In case of model 2, the following restrictions on their variability were imposed on the parameters \(a_1, a_2, a_3, p\) and \(q\):
\[-1 \leq a_1 \leq 0, \quad -10 \leq a_2 \leq 0, \quad -100 \leq a_3 \leq 100, \quad 0 \leq p \leq 100, \quad 0 \leq q \leq 2\pi \quad (3)\]

The parameters of model 1 were estimated using the regression analysis method \([7]\). Due to restrictions (3), determining the parameters of model 2 was treated as a task of nonlinear programming with linear restrictions. The objective function \(f_c\) has been defined as follows:

\[
f_c = \left(\sum_{i=1}^{N_{\text{obs}}} (A_{\text{obs}i} - A_{\text{cal}i})^2\right)^{1/2} \quad (4)
\]

where: \(N_{\text{obs}}\) – number of observations, \(A_{\text{obs}i}\), \(A_{\text{cal}i}\) – respectively observed and calculated maximum amplitudes of ground vibrations accelerations at the i-th seismic station.

Parameter values were determined by an evolutionary algorithm implemented in the Solver Add-in for Microsoft Office Excel.

3. Observations of accelerations of ground vibrations caused by rockmass tremors in the area of research

Registrations of ground vibrations in the area are carried out using 14 sets of the seismic apparatus. The location of the measurement stations is shown in figure 1. In the analyses, the results of which are presented in the article, data on maximum amplitudes of ground vibrations accelerations were used, which were caused by the strongest tremors with seismic energy equal and more than \(3 \times 10^7\) J. Additionally, it was assumed that vibrations had to be registered at least 10 observation network sites. Finally, four tremors were considered, whose energy and location of the epicentres are given in figure 1. Observation results are presented in table 1. Ground vibrations caused by discussed tremors were characterized by vibrations accelerations exceeding \(0.3\) m/s\(^2\) (\(0.292\) m/s\(^2\) in the case of tremor 1).

The range of variability of epicentral distances was respectively – for tremor 1: from 2074 m to 8354 m; for tremor 2: from 1902 m to 8908 m; for tremor 3: from 1049 m to 7727 m and for tremor 4: from 927 m to 7109 m. Performing the reconstruction of the fields of accelerations of vibrations caused by considered tremors, the values of the epicentral distances, not less than that used during the estimation of the model parameters, were assumed.

| Seismic station | Tremor’s number |
|-----------------|-----------------|
|                 | 1               |
|                 | 2               |
|                 | 3               |
|                 | 4               |
| 1               | 28.33           |
| 2               | 134.54          |
| 3               | -               |
| 4               | -               |
| 5               | 292.43          |
| 6               | 64.86           |
| 7               | 8.04            |
| 8               | 12.11           |
| 9               | 7.54            |
| 10              | -               |
| 11              | 13.59           |
| 12              | 13.79           |
| 13              | 25.46           |
| 14              | -               |

**Table 1.** Accelerations of ground vibrations caused by considered mine tremors [x10\(^{-3}\) m/s\(^2\)].
4. Reconstruction of the acceleration field of ground vibrations caused by the considered mine tremors

Using the observational material characterized in the previous point, the fields of acceleration of ground vibrations caused by the analysed tremors were reconstructed. For this purpose, the parameters of model 1 (table 2) and model 2 (table 3) were determined.

Table 2. Results of the model 1 estimation parameters.

| Tremor’s number | The value of the parameter | The standard deviation of the parameter |
|-----------------|----------------------------|-----------------------------------------|
|                 | $a_1$                      | $a_2$                                   | $a_3$                      | $a_4$                      | $a_5$                      |
| 1               | -1.5E-05                   | -2.35087                                | 10.33754                   | 0.000104                   | 1.226449                   | 3.979695                   |
| 2               | 0.000302                   | -5.8637                                 | 21.25901                   | 0.000203                   | 2.183626                   | 7.015072                   |
| 3               | 9.41E-05                   | -2.34003                                | 9.435975                   | 0.000111                   | 0.863739                   | 2.616008                   |
| 4               | 3.9E-05                    | -1.40472                                | 6.548711                   | 0.000177                   | 1.276707                   | 3.870995                   |

At the assumed significance level of 0.05, according to Fisher-Snedecor F test results, there is no base to reject null hypothesis states that regression is statistically significant. The carried out analysis of residuals distribution also did not show the existence of models errors requiring their modification [7].
Table 3 presents the results of parameters’ estimation of the model 2. In the seventh column of table 3 the size of the angle between the axis X and the direction of the lowest attenuation of ground vibrations is given.

| Tremor’s number | The value of the parameter | Angle [deg] |
|-----------------|---------------------------|-------------|
| 1               | 0.00000                   | -2.41500    | 11.28223 | 2.77940 | 0.96242 | -34.9 |
| 2               | 0.00000                   | -3.19212    | 13.54961 | 2.17157 | 1.29679 | -15.7 |
| 3               | 0.00000                   | -2.15416    | 9.59345  | 2.63703 | 1.41551 | -8.9  |
| 4               | 0.00000                   | -0.78764    | 4.82985  | 3.38634 | 2.41930 | 48.6  |

The parameters identification of the models 1 and 2 for each of the considered tremors made it possible to reconstruct the field of accelerations of ground vibrations caused by these tremors. The results of the models calculations are given in table 4. These results were given only for those positions where ground vibrations caused by a given tremor were recorded.

Table 4. Calculated accelerations of ground vibrations caused by the considered tremors [x10^{-3} m/s^2].

| Seismic station | Model 1 | Model 2 | Tremor’s number |
|-----------------|---------|---------|-----------------|
|                 | 1 2 3 4 | 1 2 3 4 |                 |
| 1               | 49.87   | 26.86   | 55.87           | 261.11 | 25.27 | 15.09 | 45.45 | 302.48 |
| 2               | 71.79   | 28.05   | 11.62           | 29.25  | 133.75 | 64.14 | 9.55  | 48.87 |
| 3               | -       | -       | 292.22          | 34.34  | -     | 181.07 | -     |
| 4               | -       | -       | 43.34           | -      | 18.84 | -     |
| 5               | 323.35  | 13.92   | 34.30           | 292.33 | -     | 9.81  | 78.34 |
| 6               | 49.34   | 34.22   | 15.25           | 54.87  | 65.98 | 24.28 | 20.90 | 46.38 |
| 7               | 8.07    | -       | -               | 30.10  | 6.15  | -     | -     | 32.90 |
| 8               | 9.89    | 6.26    | 14.82           | 36.52  | 5.70  | 1.32  | 3.90  | 60.59 |
| 9               | 8.33    | 6.10    | 18.16           | 26.02  | 8.67  | 4.49  | 36.02 | 26.83 |
| 10              | -       | 11.67   | 16.10           | 56.40  | -     | 4.70  | 8.82  | 47.68 |
| 11              | 14.05   | 7.42    | 15.00           | 43.85  | 8.97  | 2.11  | 5.09  | 48.33 |
| 12              | 13.28   | 7.28    | 12.15           | 35.41  | 12.75 | 2.42  | 4.69  | 32.83 |
| 13              | 35.75   | 11.49   | 190.06          | 47.25  | 32.60 | 31.64 | 471.81 | 40.09 |
| 14              | -       | 402.49  | 30.49           | 49.72  | -     | 404.34 | 19.13 | 61.91 |

In table 5, the analysis’ results of the reconstitution’s accuracy of the observed values of the maximum amplitudes of ground vibrations accelerations are summarized. The results obtained using model 1 and model 2 were taken into account. The values of the mean square error of reconstruction of the observed vibrations accelerations, as well as the value of the correlation coefficient between the observed values and the predicted values of vibrations accelerations were characterized. The values of the maximum underestimation and overestimation of the observed values are also given. Due to the credibility of the reconstruction of vibrations accelerations fields, as well as the safety of building structures, the value of the maximum underestimation of the observation results is particularly important.

As a result of using the vibrations accelerations model 1 (table 5) to reconstitution the recorded values, the coefficient of determination was from 0.55 (tremor 3) to 0.98 (tremor 2). This means that the determined regression equations can be explained from 29% to 99.6% of the observed variation of ground vibrations accelerations caused by the tremors. In the case of tremor number 3 relatively high mean square error values (above 0.08 m/s^2) were found and significant underestimation of the observed value of vibrations accelerations, equal 0.283 m/s^2.
Reconstitution of the results of observations with using the model 2 allowed to obtain much smaller values of the mean square error. The values of the determination coefficient range from 0.90 to 0.998 – the determined equations can be explained from 90% to 99.8% of observed variation of ground vibrations accelerations caused by considered tremors. The underestimations of observed values are much lower (except for one case - tremor 4), compared to the assessment obtained in the result of using model 1.

**Table 5. Characteristics of the accuracy of the observed values’ reconstruction of ground vibrations accelerations.**

| Parameter                          | Model 1 | Tremor’s number | Model 2 |
|-----------------------------------|---------|-----------------|---------|
| Mean square error [x 10^{-3} m/s²] | 23.90   | 14.29 84.90 31.31 3.61 | 4.21 10.13 24.46 |
| Correlation coefficient           | 0.97    | 0.99 0.74 0.93 0.999 0.999 0.998 0.95 |
| The maximum underestimation [x 10^{-3} m/s²] | 62.75   | 35.35 282.90 57.19 6.41 | 9.22 19.85 64.35 |
| Maximum overestimation [x 10^{-3} m/s²] | 30.92   | 15.96 112.92 36.88 7.14 | 6.02 12.92 28.39 |

The results of the reconstitution of ground vibrations acceleration fields caused by the rockmass tremors considered in the article, using the model 1, are shown in figures 2 (a) ÷ 5 (a). In figures 2 (b) ÷ 5 (b) the results of application of the model 2 are shown. In these figures, the location of the tremor epicenters, the observation grid position, the isolines of the calculated ground vibrations accelerations [x10^{-3} m/s²], as well as the course of the main tectonic faults were plotted.

Analysing the directions of reduced attenuation of ground vibrations, determined for the analysed tremors, one can indicate tectonic faults adjacent to epicenters of these phenomena, which run approximately parallel to the directions of anisotropy of vibrations attenuation. These faults have been marked in figures 2 ÷ 5 (b) in bold red line.

**Figure 2.** Reconstruction of the accelerations of ground vibrations [x10^{-3} m/s²] caused by the tremor 1 – (a) using the model 1 and (b) using the model 2.
Figure 3. Reconstruction of the accelerations of ground vibrations \([x10^{-3} \text{ m/s}^2]\) caused by the tremor 2 – (a) using the model 1 and (b) using the model 2.

Figure 4. Reconstruction of the accelerations of ground vibrations \([x10^{-3} \text{ m/s}^2]\) caused by the tremor 3 – (a) using the model 1 and (b) using the model 2.

Figure 5. Reconstruction of the accelerations of ground vibrations \([x10^{-3} \text{ m/s}^2]\) caused by the tremor 3 – (a) using the model 1 and (b) using the model 2.
5. Summary
Examples of ground vibrations caused by mining tremors presented in the article confirm the desirability of including in the attenuation relationship the directional effect of this phenomenon. With a modified regression model that takes into account the directional effect of seismic wave attenuation, a more accurate reconstruction of the distribution of ground vibrations accelerations has been obtained compared to the commonly applied attenuation relation. As a result of the model’s application taking into account the anisotropy of the vibrations attenuation, the mean square error of reproducing the results of the four recorded ground vibrations accelerations has decreased by 85%, 71%, 88% and 22%, respectively, compared to commonly used form of the attenuation relationship. In addition, an approximate, qualitative agreement was found between the direction of the lowest attenuation of ground vibrations and the direction of the tectonic faults adjacent to the epicenter of the tremor.

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