The Influence of the Number of Joints on the Size Effect of Shear Strength and Characteristic Strength

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The number of joints influences the shear and characteristic strength of structural plane; however, the relationship of the influence is yet to be derived. This study formulates 11 simulation programs using numerical simulation and realistic failure process analysis software. The influence of the number of joints and size on the shear strength of structural plane is studied. The stress-strain curves of different numbers of joints and sizes are analyzed. The mathematical models of the shear strength of structural plane and the number of joints and sizes are proposed, and their specific expressions are obtained. The mathematical models of the shear characteristic size and strength of structural plane and the number of joints are established.

1. Introduction

Shear strength is one of the important mechanical properties of rock, and the shear strength of rock structural planes has a size effect [1]. There are rock mass defects with different scales such as pores, voids, and structural planes in the rock mass [2]. Due to the difference in the number of joints existing in the rock, the number of joints will have an impact on the size effect of the shear strength of structural planes. Therefore, it is of great significance to study the influence of the number of joints on the shear strength of structural planes.

The size effect of rock shear strength has been consistently the focus of domestic and foreign research. For example, Ren et al. [3] proposed a systematic method to obtain the actual shear strength of soil-rock mixture (S-RM) by detecting the effect of rock size. Scholars used the Particle Flow Code (PFC) software to study the size effect of shear strength. For example, Chen et al. [4] used the PFC software to conduct direct shear test on different sizes and studied the size effect of the shear mechanical properties of nonpermeable horizontal rocks. Chen et al. [5] used the PFC2D software to carry out numerical experiments on the size effect of shear strength of structural plane. Wang et al. [6] used PFC2D to study the direct shear mechanics and size effect characteristics of jointed rock masses. Scholars used the Realistic Failure Process Analysis (RFPA) software to study the size effect of shear strength. For example, Luo et al. [7] used RFPA2D to numerically simulate the three boundary conditions and obtained the numerical results of the size effect of rock strength under different boundary conditions. The foregoing studies have been mostly performed from the perspective of the size effect of the direct shear test of a single joint. The influence of the number of joints on the size effect was not considered, and the relationship between the number of joints and the size effect of the shear strength was not derived.

The number of joints affects the shear strength of structural plane. For example, Meng et al. [8] and Gao et al. [9] used numerical simulations to study the influence of joint density on rock strength. Lu et al. [10] used the three-dimensional distinct element code (3DEC) software to study
the influence of the number of joints on the size of the shear characteristics of the rock mass. Liu et al. [11] used RFPA to study the influence of the number of joints on rock blasting. Gao et al. [12] used discrete element method to investigate the deformation process and failure mechanism of rock mass with increased joint roughness. Wang [13] used numerical simulations to study the influence of the number of joints on shear strength. Scholars performed numerous studies on the influence of the number of joints on the shear strength. Some of these investigations are related to size effect; the relationship between rock mass size and shear strength of structural planes is rarely derived.

The physical and mechanical parameters of rock vary with rock size. When this size reaches a certain critical value, the mechanical parameters of rock cease to vary with the increase in rock size. This is called the characteristic size of the representative elementary volume (REV) of rock, and the strength corresponding to this size is called the characteristic strength. For example, Xiaoqiang et al. [15] established a discrete element model of fractured rock mass based on the REV size to study the deformation and failure characteristics of fractured rock mass. Scholars studied the impact on rock mass REV. For example, Xia et al. [16] used the GeneralBlock software to study the influence of the dispersion degree of fracture ductility on rock block REV. Na et al. [17] proposed a method to study the influence of confining pressure on REV of different jointed rock masses. Scholars use different methods to evaluate the size of rock REV. For example, Loyola et al. [18] proposed to use the central limit theorem to evaluate the REV size of fractured rocks. Yuan et al. [19] studied the REV of fractured rocks based on a dual-pore model. Ni et al. [20] estimated the REV of fractured rock mass based on the damage coefficient. Ying et al. [21] proposed a method to estimate the REV of fractured rock mass based on volumetric fracture strength P32 and statistical tests. Huang et al. [22] estimated the REV of fractured rock mass based on the geological strength index. Although the above studies have used different methods to evaluate the rock REV, they have not studied it from the perspective of mathematical relationships. The mathematical model representing the characteristic size and the number of joints and that of the characteristic strength and the number of joints have not been established.

This study formulates 11 simulation programs using numerical simulations and RFPA. The influence of (i) number of joints and (ii) size on the shear strength of structural planes is investigated, and the stress-strain curves of different numbers of joints and sizes are analyzed. Mathematical models of number of joints, size, and the shear strength of structural plane are established. Mathematical models of the shear characteristic size of structural plane, the shear characteristic strength of structural plane, and the number of joints are developed.

2. Numerical Simulation Program and Simulation Parameter Setting

2.1. Numerical Simulation Program and Model Formulation. The numerical simulation contains two research contents. The first pertains to the influence of number of joints on the shear strength of structural plane, and the second relates to the size effect on the shear strength of structural planes. For the first research content, six sets of numerical simulation programs were developed. The model sizes were 200 mm × 200 mm, 400 mm × 400 mm, 600 mm × 600 mm, 800 mm × 800 mm, 1000 mm × 1000 mm, and 1200 mm × 1200 mm. Each set of simulation programs included five types of numbers of joints working conditions; the number of joints were one, three, five, seven, and nine. Numerical simulations of shear were conducted for each working condition. For the second research content, five sets of numerical simulation programs with number of joints values of one, three, five, seven, and nine were developed. Each set of simulation programs included six types of model-size working conditions; the model sizes were 200 mm × 200 mm, 400 mm × 400 mm, 600 mm × 600 mm, 800 mm × 800 mm, 1000 mm × 1000 mm, and 1200 mm × 1200 mm. Numerical simulations of shear were performed for each working condition. The specific numerical simulation programs are summarized in Table 1.

Similarly, although 30 working conditions were involved, the length of this article is limited by only a series of simulation models with a size of 200 mm × 200 mm (Figure 1) and a series of simulation models with a number of joints of 1 bar (Figure 2). Through these two examples, the model formulation process followed when the number of joints and size change are presented.

2.2. Boundary Conditions and Rock Structural Plane Parameters

2.2.1. Rock and Joint Mechanical Parameters. The main research object of this paper is rock with different numbers of joints, and the abovementioned samples of different sizes are input into the RFPA software. The deformation parameters of rock and joints were as follows: the elastic moduli of the rock and joints were 8000 and 0.1 MPa, respectively. Their Poisson ratio and internal friction angle (as a strength parameter) were 0.25 and 30°, respectively. The mechanical parameters of the rock and joints are summarized in Tables 2 and 3, respectively.

2.2.2. Boundary Conditions and Loading Methods. The software used in this simulation is the RFPA software, and 2D numerical models are established for research. The theoretical basis of the simulation was the rock shear deformation theory. The mechanical model of the simulation test was a plane stress model, and the constraint condition was that the two sides of the model were subjected to horizontal shear and the upper and lower sides were subjected to load. The adopted loading method in the simulation was displacement loading on both sides of the model. The initial loading of the model was zero, and the loading increment was 0.01 mm. During the simulation, the upper model moved to the right at a certain speed, whereas the lower model remained fixed. The normal stress on the upper and lower plane of the model remained constant during the simulation. The criterion used in the numerical simulation was Mohr–Coulomb. The loading boundary of the numerical model is shown in Figure 3.
3. Numerical Simulation Results and Analysis

3.1. Influence of the Number of Joints on Shear Strength of Structural Planes

3.1.1. Stress-Strain Curve Analysis with Different Numbers of Joints. According to the simulation program and results of the first research, the numerical simulation results for each working condition were output, and the stress-strain curve for each working condition was drawn. Then the stress-strain curves for all working conditions in each simulation program were drawn in the same coordinate system. Summary diagrams of the stress-strain curves considering different numbers of joints for each simulation program are obtained, as shown in Figure 4.
Figure 3: Load boundary of direct shear numerical model.

Figure 4: Continued.
According to the stress-strain curve for each working condition in the numerical simulation program, the shear strength of structural plane in each working condition was explored. The shear strength of structural plane under all working conditions is summarized in Table 4.

Considering that the curves follow the same law, one of the graphs is selected as an example. The curve with seven joints in Figure 4(a) is selected to analyze the stress-strain trend. When the strain is small, the curve of shear stress and shear strain is approximately a straight line, and the deformation of model is in the elastic stage. After the stress reached the peak strength of 199.84 MPa, the curve stress decreased rapidly to 151.56 MPa. As the strain increases, the stress fluctuates around 151.56 MPa.

The analysis of the influence of number of joints on the shear strength of structural planes is also based on Figure 4(a). When the model is 200 mm in fixed size, in the elastic deformation stage, the shear strengths corresponding to the number of joints do not considerably vary. As the number of joints increases from one to nine, the shear strength of structural plane decreases from 224.13 MPa to 197.65 MPa. It can be concluded that as the number of joints increases, the shear strength of structural plane gradually decreases. This is mainly because the more the number of joints, the more the weak surface of the rock and the smaller its peak shear strength.

The influence of the size on the shear strength of structural plane is analyzed next. The statistics summarized in Table 4 indicate that when the number of joints is one, as the model size increases from 200 mm to 1200 mm, the peak strength decreases from 224.13 MPa to 35.37 MPa. When the number of joints is 3, 5, 7, and 9, as the size of the model continues to increase, the peak strengths of the structural plane all exhibit a downward trend. This is mainly because of the influence of the size on the shear strength of structural surface. That is, the shear strength of structural plane has a size effect.

Similarly, when the size of structural plane is the same, but the number of joints increases, the shear strength of the structural plane shows a decreasing trend. When the number of joints is the same, but the size of structural plane increases, the shear strength of the structural plane shows a decreasing trend.

3.1.2. Fitting Method of Relationship between the Shear Strength and the Number of Joints. The statistics listed in Table 4 show that as the number of joints increases, the shear strength of structural planes gradually decreases. The data were imported into Origin software to draw a scatter plot of the shear strength and number of joints of each numerical simulation program. The fitting curve of number of joints and shear strength is drawn, as shown in Figure 5.

The fitting curves in Figure 5 indicate that when the number of joints increases, but the size remains the same, the shear strength of structural planes gradually decreases. Moreover, in spite of the different sizes of structural planes, the law of change is similar. The fitting curve in Figure 5 shows that the relationship between the number of joints and shear strength at different sizes can be derived, as listed in Table 5.

3.1.3. Formulation of Mathematical Model of the Shear Strength of Structural Plane and the Number of Joints

(1) Mathematical Model (1). The function types of these formulas were analyzed according to the relationship between the shear strength of structural planes and the number of joints at different sizes. A mathematical model for shear strength versus number of joints is proposed as follows:

\[ \tau(n) = a + bn, \]  

(1)
Table 4: Shear strength of structural plane with different numbers of joints.

| Simulation program | Size of structural plane (mm) | Shear strength of structural plane with different numbers of joints (MPa) |
|--------------------|-------------------------------|---------------------------------------------------------------------|
|                    |                               | Program 7 | Program 8 | Program 9 | Program 10 | Program 11 |
| Program 1          | 200                           | 224.13    | 206.15    | 203.33    | 199.84     | 197.65     |
| Program 2          | 400                           | 111.34    | 109.28    | 105.14    | 97.73      | 95.64      |
| Program 3          | 600                           | 71.71     | 71.29     | 66.24     | 65.05      | 65.23      |
| Program 4          | 800                           | 56.23     | 52.32     | 51.32     | 50.28      | 48.92      |
| Program 5          | 1000                          | 44.28     | 42.87     | 42.17     | 40.64      | 40.11      |
| Program 6          | 1200                          | 35.37     | 34.43     | 33.94     | 32.86      | 32.45      |

Figure 5: Fitting curves of shear strength of structural plane with different numbers of joints.

Table 5: Fitting relationship between shear strength and the numbers of joints.

| Size of structural plane (mm) | Fitting formula |
|-------------------------------|-----------------|
| 200                           | \(\tau(n) = 221.037 - 2.963n\) |
| 400                           | \(\tau(n) = 114.563 - 2.147n\) |
| 600                           | \(\tau(n) = 72.704 - 0.96n\) |
| 800                           | \(\tau(n) = 55.979 - 0.833n\) |
| 1000                          | \(\tau(n) = 44.656 - 0.528n\) |
| 1200                          | \(\tau(n) = 35.662 - 0.37n\) |

where \(\tau(n)\) is the shear strength of the structural plane when the number of joints is \(n\) (unit: MPa); \(n\) is the number of joints (units: bar); and \(a\) and \(b\) are parameters.

The relationship between shear strength and number of joints is given by (1), which contains parameters \(a\) and \(b\). When \(a\) and \(b\) have been determined, the shear strength corresponding to any number of joints can be derived.

(2) Method for Evaluating Parameters. The statistics listed in Table 5 show that the parameter values are related to the size of structural planes, and the parameters under each size are summarized in Table 6.

According to the data therein, the size of the structure plane and the parameter value were considered as abscissa and ordinate, respectively, to plot their scatter diagram of the parameters and the size of structure plane. Then, based on this diagram, parameters \(a\) and \(b\) and the size of structural planes are fitted, as shown in Figures 6 and 7.

According to the fitting curves in Figures 6 and 7, the relationships between parameters \(a\) and \(b\) and the size of structural planes are as follows:

\[
a = 419.219e^{-\frac{l}{242}} + 36.653, \\
b = -4.742e^{-\frac{l}{423.865}} - 0.066, \\
\]

where \(a\) and \(b\) are parameters and \(l\) is the size of the structural plane (unit: mm).

(3) Mathematical Model (2). The mathematical model of parameters and the size of structural planes was brought into the mathematical model of the shear strength of structural plane and the number of joints. Accordingly, a mathematical model of the shear strength and the number of joints is derived as follows:

\[
\tau(n) = 419.219e^{-\frac{l}{242}} + 36.653 + \left(-4.742e^{-\frac{l}{423.865}} - 0.066\right)n, \\
\]

where \(l\) is the size of structural plane (units: mm).

The derived mathematical model of the shear strength of structural plane and the number of joints is suitable for solving the shear strength of the structural plane on the two-dimensional plane. It can be applied to field rock with structural planes. It provides guidance and a method for the quantitative analysis of the number of joints and shear strengths of structural planes for engineering practice.

3.2 Influence of the Size on Shear Strength of Structural Planes

3.2.1 Stress-Strain Curve Analysis of Different Sizes. According to the simulation program and results of the second research, a summary of the stress-strain curves corresponding to different structural plane sizes for each simulation program was derived, as shown in Figure 8.
According to the stress-strain curves, the shear strength under all working conditions is summarized in Table 7.

The model size of 200 mm in Figure 8(e) is selected to analyze the stress-strain trend. When the stress is small, the shear stress increases linearly with the increase of the shear strain. As the strain increases, the shear stress reaches the peak strength of 197.65 MPa, and the stress suddenly drops to 115.95 MPa; the shear stress fluctuates near this value as the strain increases.

The analysis of the impact of size on the shear strength of structural planes is also based on Figure 8(e) as an example. The number of joints is three; as the model size increases from 200 mm to 1200 mm, the shear strength of structural plane decreases from 206.15 MPa to 34.43 MPa. The peak strength of structural plane whose size is 200 mm is 206.15 MPa, whereas that whose size is 400 mm is 109.28 MPa. The peak strengths differ by 96.87 MPa. The peak strength of structural surface whose size is 1000 mm is 42.87 MPa, whereas that whose size is 1200 mm is 34.43 MPa; the difference in peak strength between the two is only 8.44 MPa. The foregoing shows that as the size increases, the shear strength gradually decreases, indicating that the shear strength has a size effect. When the number of joints is one, five, seven, and nine, as the size increases, the shear strengths of structural plane all show a decreasing trend. This is mainly because of the impact of size on the shear strength of structural surface.

The influence of the number of joints on the shear strength of structural plane is also based on Figure 8(e) as an example. As the model size increases from 200 mm to 1200 mm, the shear strength of structural plane decreases from 206.15 MPa to 34.43 MPa. The peak strength of structural plane whose size is 200 mm is 206.15 MPa, whereas that whose size is 400 mm is 109.28 MPa. The peak strengths differ by 96.87 MPa. The peak strength of structural surface whose size is 1000 mm is 42.87 MPa, whereas that whose size is 1200 mm is 34.43 MPa; the difference in peak strength between the two is only 8.44 MPa. The foregoing shows that as the size increases, the shear strength gradually decreases, indicating that the shear strength has a size effect. When the number of joints is one, five, seven, and nine, as the size increases, the shear strengths of structural plane all show a decreasing trend. This is mainly because of the impact of size on the shear strength of structural surface.

3.2.2. Fitting Method of the Relationship between the Shear Strength of Structural Plane and the Size

The statistics summarized in Table 7 indicate that as the size increases, the shear strength of structural plane gradually decreases. The fitting curve of shear strength and the size was drawn based on the scatter diagram, as shown in Figure 9.

The fitting curves in Figure 9 indicate that the shear strength of structural plane gradually decreases as the size increases under the same number of joints. For different numbers of joints, the law of change is similar. The fitting relationship between the shear strength and the size at different numbers of joints listed in Table 8 can be obtained based on the fitting curve in Figure 9.

3.2.3. Formulation of a Mathematical Model of the Shear Strength of Structural Planes and the Size

(1) Mathematical Model (1). According to the relationship between the shear strength of structural planes and the size considering different numbers of joints, the function types of formulas were analyzed. A mathematical model for the shear strength of structural planes versus the size is proposed, as follows:

\[ \tau(l) = d + f e^{(-l/g)} \]

where \( \tau(l) \) is the shear strength of structural plane when the size is \( l \) (unit: MPa); \( l \) is the size of structural plane (unit: mm); and \( d, f, \) and \( g \) are parameters.
Figure 8: Stress-strain curves of different sizes of rock in different number of joints. (a) $n = 1$, (b) $n = 3$, (c) $n = 5$, (d) $n = 7$, and (e) $n = 9$. 
The relationship between the shear strength of structural plane and the size is given by (4), which contains the parameters \(d\), \(f\), and \(g\). When \(d\), \(f\), and \(g\) are determined, the relationship of the shear strength of structural plane under any size can be obtained.

(2) Method for Evaluating Parameters. The statistics summarized in Table 8 indicate that the parameter value is related to the number of joints, and the parameters under each number of joints are shown in Table 9.

Based on the data listed in this table, the number of joints and parameters' values were considered as abscissa and ordinate, respectively, to plot a scatter diagram. Based on this diagram, the relationship between the parameters \(d\), \(f\), and \(g\) and the number of joints was fitted as shown in Figures 10–12.

The relationship between the shear strength of structural plane and the size is given by (4), which contains the parameters \(d\), \(f\), and \(g\). When \(d\), \(f\), and \(g\) are determined, the relationship of the shear strength of structural plane under any size can be obtained.

(2) Method for Evaluating Parameters. The statistics summarized in Table 8 indicate that the parameter value is related to the number of joints, and the parameters under each number of joints are shown in Table 9.

Based on the data listed in this table, the number of joints and parameters' values were considered as abscissa and ordinate, respectively, to plot a scatter diagram. Based on this diagram, the relationship between the parameters \(d\), \(f\), and \(g\) and the number of joints was fitted as shown in Figures 10–12.

According to the fitting curves shown in Figures 10–12, the relationships between parameters \(d\), \(f\), and \(g\) and the size of structural plane are as follows:

\[
\begin{align*}
\tau(l) &= 37.955 + 416.213e^{(-l/241.602)} \\
\tau(l) &= 36.254 + 392.184e^{(-l/240.234)} \\
\tau(l) &= 36.019 + 391.276e^{(-l/226.262)} \\
\tau(l) &= 35.623 + 385.011e^{(-l/220.296)} \\
\tau(l) &= 35.257 + 382.604e^{(-l/218.895)}
\end{align*}
\]

where \(n\) is the number of joints (units: bar).

(3) Mathematical Model (2). The mathematical model of the number of joints and the parameters was brought into the
mathematical model of the shear strength of structural plane and the size. Accordingly, a mathematical model of the shear strength of structural plane and the size is derived, as follows:

$$\tau (l) = 37.853n^{-0.032} + (414.142n^{-0.037})e^{-l/247.795-3.327n}. \tag{6}$$

The derived mathematical model of the shear strength of structural plane and the size is suitable for solving the shear strength on the two-dimensional plane. It provides guidance and a method for the quantitative analysis of the size and the shear strength of structural plane for engineering practice.

### Formulation of Mathematical Model of the Characteristic Size

#### Establishment of Mathematical Model of the Shear Characteristic Strength of Structural Plane and the Number of Joints

As the size of structural plane and the number of basic units increase, the shear strength of structural plane tends to a stable value, which is the characteristic strength value of structural plane. At this strength, the related size is the characteristic size of structural plane.

The exact characteristic size is difficult to assess quantitatively. The absolute value of the slope of the curve is derived using the two sides of (2) in the following equation:

$$\tau’ (l) = \frac{f}{g} e^{-l/g}. \tag{7}$$

Formulate

$$|\tau’ (l)| \leq \alpha \tag{8}$$

to obtain

$$l \geq g \ln \frac{f}{g} - g \ln \alpha, \tag{9}$$

where $\alpha$ is the acceptable absolute value of the slope, which can be an extremely small value.

Herein we can think that what (7) gets is the characteristic size and bring the fitting parameters into it. By assuming that $\alpha = 0.05$, the characteristic size of each structural plane can be obtained. The relationship between the characteristic size and the number of joints can be obtained by calculation, as summarized in Table 10.

The statistics listed in Table 10 show that as the number of joints gradually increases, the characteristic size of structural plane gradually decreases. The relationship between characteristic size and the number of joints can be analyzed qualitatively.

The number of joints and the characteristic size were considered as abscissa and ordinate, respectively, to draw a
The fitting curve was drawn based on the scatter plot, as shown in Figure 13. The fitting curve in Figure 13 shows that the shear characteristic size of structural planes exhibit a decreasing trend when the number of joints increases. The curve conforms to a linear relationship. A mathematical model for the characteristic size and the number of joints is as follows:

\[
D = 868.317 - 10.444n, \quad (10)
\]

where \(D\) is the characteristic size of structure plane (unit: mm).

Equation (10) provides a quantitative description of the relationship between the number of joints and the characteristic size. It is suitable for solving the characteristic size of structural surface on the two-dimensional plane. In engineering, if the number of joints can be measured, then the characteristic size, which has important engineering application value, can be obtained more accurately using equation (10).

### 3.3.2. Formulation of Mathematical Model of the Shear Characteristic Strength of Structural Plane and the Number of Joints

According to equation (6), the characteristic strength of structural plane can be obtained based on characteristic size. The calculated relationship between the shear characteristic strength and the number of joints is summarized in Table 11.

The statistics listed in Table 11 show that as the number of joints increases, the shear characteristic strength of structural plane gradually decreases. The fitting curve was drawn based on the scatter plot of the shear characteristic strength and the number of joints, as shown in Figure 14.

Figure 14 shows that the shear characteristic strength of structural plane tends to decrease as the number of joints increases. The curve conforms to a linear relationship. A mathematical model of the shear characteristic strength of structural plane and the number of joints is

\[
\tau_w = 50.118 - 0.464n, \quad (11)
\]

where \(\tau_w\) is the shear characteristic strength of structural plane (units: MPa) and \(n\) is the number of joints (units: bar).

Equation (11) provides a quantitative description of the relationship between the shear characteristic strength of structural plane and the number of joints. It is suitable for solving the shear characteristic strength of structural surfaces on a two-dimensional plane. In engineering, if the number of joints can be measured, the shear characteristic strength, which has important engineering application value, can be obtained more accurately using (11).

### 4. Discussion

The number of joints and size have an impact on the shear strength, shear characteristic size, and shear characteristic strength of structural planes, but the relationship is yet to be obtained. In this study, mathematical models of the shear strength of structural plane and the number of joints and the size were established. A mathematical model of the shear characteristic size, the shear characteristic strength of structural plane and the number of joints were developed.

#### Table 10: The relationship between shear characteristic size and number of joints.

| Number of joints (bar) | 1     | 3     | 5     | 7     | 9     |
|------------------------|-------|-------|-------|-------|-------|
| Shear characteristic size (mm) | 855.1835 | 837.4197 | 826.9616 | 782.9391 | 777.9836 |

**Figure 13:** Fitting curve between the characteristic size of structural plane and the number of joints.

3.4. Experimental Comparison and Verification Analysis. In order to verify the general applicability of (1), Figures 3–13 in Section 3 of Wang [23] are cited; as shown in Figure 15, the corresponding sample size is 200 mm. According to Figure 15, the shear strength of different numbers of joints is obtained in Table 12.

According to the data in Table 12, a scatter plot of shear strength and number of joints is drawn, and their fitting curves are drawn, as shown in Figure 16.

The relationship between shear strength of structural plane and number of joints is obtained in Figure 16 as follows:

\[
\tau(n) = 4.751 - 0.824n, \quad (12)
\]

where \(\tau(n)\) is the shear strength of the structural plane when the number of joints is \(n\) (unit: MPa) and \(n\) is the number of joints (units: bar).

The function type of (12) conforms to the mathematical model proposed in (1). Therefore, the numerical simulation and experimental conclusions are consistent. This verification shows that the mathematical model proposed in (1) is applicable to the solution of shear strength with respect to the number of joints.

**Figure 16:** Scatter plot of shear strength and number of joints.
With the formulation of these four mathematical models, the relationships between the shear strength, shear characteristic size of structural plane, shear characteristic strength of structural plane, and the number of joints were quantified. Concurrently, the relationship between the shear strength and the size was quantified; this relationship has important scientific significance. For the engineering site, once the number of joints and the size are obtained, the shear strength, shear characteristic size, and shear characteristic strength of the structural plane can be facilely obtained; all of these can provide considerable engineering guidance.

The method for solving the shear strength of structural plane and the number of joints, the size is derived. Moreover, the method for solving the relationship between the shear characteristic size, the shear characteristic strength of structural plane, and the number of joints is obtained. In addition, we verified the applicability of (1) in this paper by citing the experimental results of the published literature and making an in-depth analysis of the experimental data.

However, this study still has remaining deficiencies. Because this study is performed using numerical simulation, the simulation conditions have been simplified. When the numerical model was formulated, the roughness of structural surface in the numerical model was the same, and the influence of different roughness was not considered. At the same time, the effects of other small cracks and water content that may exist in the rock are ignored. Evidently, the factors considered are incomplete, and the study can be improved. Moreover, when the research is extended to the engineering site, it can only be applied to specific rocks with a structural plane.

### 5. Conclusions

This work investigates the influence of the number of joints and the size on the shear strength of structural plane. Through the stress-strain curves of different sizes and the
number of joints, the size effect of the shear strength of structural plane is analyzed. Based on the numerical simulations, the following conclusions are obtained:

1. The relationship between the shear strength of structural plane and the number of joints is

\[ \tau (n) = a + bn. \]  \hspace{1cm} (13)

Parameters \( a \) and \( b \) are solved, and a particular formula is

\[ \tau (n) = 419.219e^{-0.372424} + 36.653 + (4.742e^{-0.865238} - 0.066)n. \]  \hspace{1cm} (14)

2. The relationship between the shear strength of structural plane and the size is

\[ \tau (l) = d + fe^{-l/g}. \]  \hspace{1cm} (15)

Parameters \( d \), \( f \), and \( g \) are solved, and a particular formula is

\[ \tau (l) = 37.853n^{-0.032} + 414.142n^{-0.037}e^{-l/247.795-3.267n}. \]  \hspace{1cm} (16)

3. The shear characteristic size of structural plane is related to the number of joints, and its particular form (obtained by simulation) is

\[ D = 868.317 - 10.444n. \]  \hspace{1cm} (17)

4. The shear characteristic strength of structural plane is related to the number of joints, and its particular form (obtained by simulation) is

\[ \tau_w = 50.118 - 0.464n. \]  \hspace{1cm} (18)

Data Availability

The data that support the findings of this study are available from the corresponding author, Gaojian Hu, upon reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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