Strong Coupling Holography

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Abstract

We show that whenever a 4-dimensional theory with $N$ particle species emerges as a consistent low energy description of a 3-brane embedded in an asymptotically-flat $(4 + d)$-dimensional space, the holographic scale of high-dimensional gravity sets the strong coupling scale of the 4D theory. This connection persists in the limit in which gravity can be consistently decoupled. We demonstrate this effect for orbifold planes, as well as for the solitonic branes and string theoretic $D$-branes. In all cases the emergence of a 4D strong coupling scale from bulk holography is a persistent phenomenon. The effect turns out to be insensitive even to such extreme deformations of the brane action that seemingly shield 4D theory from the bulk gravity effects. A well understood example of such deformation is given by large 4D Einstein term in the 3-brane action, which is known to suppress the strength of 5D gravity at short distances and change the 5D Newton’s law into the four-dimensional one. Nevertheless, we observe that the scale at which the scalar polarization of an effective 4D-graviton becomes strongly coupled is again set by the bulk holographic scale. The effect persist in the gravity decoupling limit, when the full theory reduces to a 4D system in which the only memory about the high-dimensional
holography is encoded in the strong coupling scale. The observed intrinsic connection between the high-dimensional flat space holography and 4D strong coupling suggests a possible guideline for generalization of AdS/CFT duality to other systems.
1 Introduction

An extraordinary example of connection between gravity and quantum fields is AdS/CFT correspondence [1]. According to it the quantum dynamics of fields can be understood in terms of classical gravity in higher dimensions and vice-versa.

In search of the gravity/field theory duals for the other systems, one has to be prepared that the power of extended duality may be more limited than in the AdS/CFT case. The new dualities may be more useful in relating certain quantities and less useful for others. Nevertheless, even a limited generalization of AdS/CFT correspondence would be of fundamental importance.

In trying to discover such generalizations, it is important to identify relations that are not unique to the AdS geometry of the setup. In the present paper we shall identify such a relation, which, although shared by AdS/CFT, is not bounded to the existence of AdS geometry in a near horizon limit.

We shall observe, that an emerging holography [2] will play the crucial role in our findings.

The key quantities of our focus will be, the number of species on 4D field theory side and the holographic scale for $N$ bits of information on the high-dimensional gravity side. Our central finding is, that the strong coupling scale of such 4D theory is determined by the bulk holographic scale and this property extends far beyond the AdS/CFT setup.

Before formulating our results quantitatively, we shall first describe the framework we shall be working in. We start with setting the connection between the number of species and the various scales of the theory.

It has been shown by recent studies [3, 4], that consistency of the well-understood black hole (BH) physics demands that in any $4 + d$-dimensional Einsteinian theory of gravity with $N$ propagating particle species the upper bound on the gravitational cutoff of the theory is given by the following energy scale,

$$M_{N(4+d)} \equiv \frac{M_{4+d}}{N^{\frac{d}{4+d}}} ,$$

where $M_{4+d}$ is the $4 + d$-dimensional Planck mass. The corresponding Planck length we shall denote by $L_{4+d}$. Eq(1) follows from the fact, that the BHs smaller than the length scale

$$L_{N(4+d)} \equiv N^{\frac{1}{4+d}} L_{4+d} ,$$

inevitably violate quasi-classicality conditions (such as, the heating rate not exceeding the temperature squared) and can no longer be treated as such (see [3, 4] for details).
The hierarchy (1) between the Planck mass and the cutoff is expected due to renormalization of the graviton kinetic term by species loops [5,6]. However the power of the BH argument is that it is fully non-perturbative and directly applies to the physical values, regardless of perturbation theory artifacts, such as the possible cancellations or fine tunings of the loop contributions.

Before proceeding we wish to comment on some other bounds on species number and stress physical differences from our case. The black hole quasi-classicality scale was used in [7], but the bound on the cutoff obtained in this work is milder, $M_4/N^{1/4}$, as opposed to $M_4/\sqrt{N}$ in our case. The scale $M_4/\sqrt{N}$ was first obtained as the upper bound on the temperature of radiation-dominated universe in which $N$ species are in thermal equilibrium in [8] (and further generalized in [9]). The crucial difference in the meaning of this scale in our case [3,4] is, that it is an absolute cutoff of the species theory, at which gravity of individual species gets strong and the species resolution is no longer possible. Therefore, for example, one cannot raise the temperature of the system above this scale (while staying in a weak gravity regime) even if only a single flavor of species is in equilibrium and all the remaining $N-1$ species are decoupled.

To summarize, the scale (1) in our case is the scale above which gravitational interactions of elementary particles cannot continue to be in a weakly-coupled classical regime.

A particular usefulness of the relation (1) is that it enables understanding of geometric relations in terms of species number counting. A simple example of such counting is given by Kaluza-Klein (KK) theory of $(D+d)$-dimensional space in which $d$-dimensions are compact, and the metric on non-compact $D$-dimensions is Minkowskian. The relation between the $D$-dimensional and $(D+d)$-dimensional Planck scales is given by,

$$M_D^2 = M_{D+d}^2 V(d),$$

where $V(d)$ is the volume of $d$-dimensional compact space measured in the units of the fundamental Planck length $L_{D+d}$. In each concrete case finding $V(d)$ can be a complicated geometric task, but at the end of the day it is given by the number $N$ of KK species as counted by a $D$-dimensional observer.

We have shown recently [4], that the dependence (1) of the gravitational cutoff of $4+d$-dimensional theory on the number $N$ $(4+d$-dimensional) particle species, can be easily understood from the quantum information point of view. Indeed, having $N$ particle

\footnote{Interestingly, this scale coincides with a three-level unitarity bound on production of species in one-graviton exchange amplitude (see the fifth reference in [3]), although the authors of [7] arrived there by other means.}
species means that we can distinguish and label them by physical measurements. That is, we can encode information in the species label. The critical length scale below which we can no longer read the species label is inevitably a cutoff of the theory. The role of such critical length is precisely played by \(L_{N(4+d)}\). Indeed, the distance \(L_{N(4+d)} \equiv 1/M_{N(4+d)}\) marks the size of a smallest pixel that can decode the species label. Such a pixel has to contain samples of all the \(N\) species in order to identify the label of any incoming particle. Since localization of a particle wave-function in a box of size \(L_{\text{pixel}}\) cost at least energy \(1/L_{\text{pixel}}\), the lowest mass of the pixel is therefore \(m_{\text{pixel}} = N/L_{\text{pixel}}\). The bound on \(L_{\text{pixel}}\) is imposed due to requirement that the pixel does not collapse into a BH. In other words, \(L_{\text{pixel}}\) has to exceed the corresponding Schwarzschild radius of the pixel \((r_{\text{pixel}})\), which in \(4 + d\) dimensional gravity is given by

\[
r_{\text{pixel}} = \left( \frac{N}{L_{\text{pixel}}} \cdot \frac{1}{M_{4+d}^{2+d}} \right)^{\frac{1}{1+d}}.
\]

Requiring \(r_{\text{pixel}} \leq L_{\text{pixel}}\), we get the bound

\[
L_{\text{pixel}} \geq L_{N(4+d)}.
\]

Notice, that the scale \(L_{N(4+d)}\) is at the same time the holographic bound for \(N\) information bits. This is natural, since the species label is a particular form of information, and its storage must obey the general laws of holography.

In the present paper we shall be interested in the situation in which the \(N\) species live in a 4D sub-space and do not propagate in the whole \(4 + d\) space-time. Such a situation is physically motivated, since localized zero modes are generic in theories with branes. We thus consider a situation in which bulk theory is \(4 + d\)-dimensional gravity (with \(\sim 1\) species) propagating on asymptotically-flat background and interacting with a 4D theory with \(N\) particle species. We shall investigate how the 4D species cope with the laws of bulk holography.

We shall discover that the bulk holographic scale \(L_{N(4+d)}\) plays a crucial role in determining the strong coupling scale of the 4D theory \((\Lambda_{\text{str}})\). Namely, the following bound holds,

\[
L_{N(4+d)} \leq L_{\text{str}},
\]

where \(L_{str} \equiv \Lambda_{str}^{-1}\).

This role is robust and persists even for extreme deformations of the 4D action, that seemingly shields the brane theory from the bulk gravitational effects. Even in the limit when one consistently decouples gravity, the system reduces to a pure 4D theory in which the strong coupling scale is set by the bulk holographic bound.
The connection to AdS/CFT is explicit, since as we shall see, the latter is an example of saturation of the above bound. In AdS/CFT the role of our species is played by open string zero modes living on the stuck of $D_3$-branes embedded in asymptotically-flat 10D space. The strong coupling scale of 4D theory is set by the 10D holographic scale. This property turns out not to depend on the AdS nature of the near horizon geometry, which otherwise is crucial for establishing AdS/CFT. The same effect is exhibited by other cases in which species are confined to the branes that do not exhibit AdS properties in near-horizon limit. Our observations suggest a possible high-dimensional holographic origin of the strong coupling scale of 4D theories, which extends beyond the AdS/CFT correspondence.

2 Holography for Localized Species

Consider a 5D space-time with coordinates $x_\mu, y$ ($\mu = 0, 1, 2, 3$), in which $N$ species are localized on a 3-dimensional surface (a 3-brane), located at $y = 0$. We shall assume that the bulk space-time metric, $g_{A,B}(x_\mu, y)$, is asymptotically flat, and the bulk theory is a pure 5D Einsteinian gravity with 5D Planck mass $M_5$. Since below $M_5$ the only species propagating in the bulk is the five dimensional graviton, the natural cutoff of the bulk theory is $M_5$. The corresponding fundamental length scale we shall denote by $L_5 \equiv M_5^{-1}$. In particular, $L_5$ is a lower bound on the size of quasi-classical BHs in the bulk. What is the fundamental scale for the brane theory?

The brane theory is a 4D theory of $N$ species coupled to 5D gravity. The fundamental holographic scale of this theory is set by the shortest length-scale on which the resolution of species is in principle possible. That is, cutoff is set by the size of a smallest pixel in which one is able to localize $N$ species. The requirement that such a pixel does not collapse into a BH under the influence of 5D gravity, fixes the lower bound

$$L_{N5} = N^{1/3} M_5^{-1}, \quad (7)$$

which coincides with the holographic scale of the 5D theory for $N$ bits of information. This coincidence is no accident, since as we have explained, the species flavor is a particular form of the information, and has to obey the same holographic bounds.

Notice, that the same scale represents the lower bound for the BH quasi-classicality, since any 5D BH of size $R < L_{N5}$ intersecting with the brane, would half-evaporate during a time less than its size, which is a clear violation of quasi-classicality conditions.

We are thus lead to the following crucial conclusion:
For a 3-brane theory with \( N \) species, the lower bound on the cutoff scale is set by the holographic bound of 5D bulk theory for \( N \) bits of information storage.

In the other words beyond the scale \( L_{N5} \) some change of the regime must occur in the 4D theory. What is this change of the regime?

There are the following two logical possibilities.

First, let us assume that laws of gravity are essentially unaffected by the presence of the brane, meaning that 4D sources continue interacting via 5D Einsteinian gravity for the distances \( r \ll L_{N5} \). In such a case, we are forced to admit that the species can no longer be emitted as elementary states from the BHs of intermediate size \( L_5 \ll r_g \ll L_{N5} \), because if they were, this would be in conflict with the assumption of 5D BH quasi-classicality all the way till \( L_5 \). Thus, the identities of species must be compromised at shorter distances, and the species theory must be UV completed. For example, species may simply become composite below the scale \( L_{N5} \).

On the other hand, if we assume that species continue to be elementary for distances \( \ll L_{N5} \), then we are inevitably lead to the conclusion that 5D gravity must change the regime beyond this scale.

To summarize, the fact that in the presence of the localized species the 5D theory breaks down at distances \( L_{N5} \), suggests that either effective 4D field theory of \( N \) species or gravity (or both) require an UV completion beyond the bulk holographic scale.

Insisting that species remain elementary below \( L_{N5} \) leaves us with inevitability of the gravity-modification at some crossover distance \( r_c \gg L_{N5} \). We shall now discuss the underlying physics of this modification. For transparency of presentation, where possible, all unessential order-one numeric factors will be absorbed in definition of scales. The reader can easily reproduce them, whenever needed.

3 Gravity of Species on a Tensionless Brane (Localization on Orbifolds)

We shall consider first the case of a tensionless brane. In such a case, the brane is effectively a surface at the \( Z_2 \)-orbifold fixed point.

The curious fact is that, despite the zero tension, such a brane continues to have dramatic gravitational effect. Indeed, as explained above, the classical 5D gravitational description is impossible to be valid at distances shorter than \( L_{N5} \), without compromising the validity of \( N \)-species description above this scale. If we insist on the validity of 5D description all the way till \( L_5 \), this will lead us to the conclusion that the bulk and
the brane observers must experience the different cutoffs. The cutoff of the bulk theory remains $M_5$, whereas the cutoff of the world-volume 4D theory is $L_{N5}^{-1} \ll M_5$, since as explained above, the brane observer cannot resolve species in her own theory above this energy scale.

What if we impose the requirement that the cutoffs of the two observers must be equal? This means, that the brane observer should be able to continue resolving species down to the distances $L_5 \ll L_{N5}$. This is only possible if the brane gravity starts becoming weaker than the 5D Einsteinian gravity at distances smaller than a certain critical size $r_c$, which must be larger than the 5D holographic scale, $r_c > L_{N5}$.

Indeed, if the brane observer were to distinguish species down to the distances $L_5$, the brane gravity must be such that the Schwarzschild radius of a BH of mass $N/L_5$, must not exceed $L_5$. Thus, below some distance scale $r_c \gg L_{N5}$, the brane gravity should cross over to a regime in which the gravitational radius of a given source is shorter than in 5D Einstein gravity. That is, for distances $r < r_c$ the Newtonian potential must become

$$V(r < r_c) = \frac{1}{M_5^3} \frac{1}{r^{2-\alpha}} \ ,$$

where $\alpha > 0$ is a parameter to be determined in a moment. The crossover scale $r_c$ is fixed from the condition that under the gravitational law the gravitational radius of a pixel of size $L_5$ with $N$ localized species must be equal to $L_5$. That is,

$$r_c = N^\frac{1}{\alpha} L_5 ,$$

We shall now see that $\alpha = 1$, because of the following reason. Modification of gravity ($\alpha \neq 0$) implies that the brane has a non-trivial gravitational effect on the probe sources. This effect can only be manifested through the terms in the brane’s gravitational action. Since the brane tension was by construction set to zero, the only remaining relevant term, that respects the world-volume general covariance, is the 4D Einstein-Hilbert term of the induced metric

$$\int d^4x \sqrt{-\tilde{g}} M_4^2 R_4 \ ,$$

where $g_{\mu\nu}(x) \equiv g_{\mu\nu}(x, y = 0)$ is the induced metric on the brane, and we have labeled the coefficient by $M_4^2$. The gravitational effect of this term is well-studied, and it is known to lead to exactly the above-discussed modification of gravity at the crossover scale $r_c = M_4^2/M_5^3$ and with $\alpha = 1$! The brane-brane graviton propagator takes the following form (tensorial structure, which is the one of massive spin-2, is suppressed)

$$G(p) = \frac{1}{M_5^3 r_c p^2 + \sqrt{p^2/r_c}} \ ,$$
where $p$ is the usual four-momentum. This propagator reproduces exactly the above crossover behavior. At distances $r \ll r_c$ gravity crosses over to a 4D regime with the effective 4D Planck scale given by

$$M_4^2 = M_5^2 r_c = N M_5^2.$$

(12)

Notice, that this is exactly the value that is expected from the contribution of the loops of localized $N$ particles [5, 6, 12]. The important input from the holography is, that the crossover behavior has to be there because of fully non-perturbative consistency reasons, irrespective of the loop contribution.

The physics of the above crossover behavior is very transparent in terms of KK-decomposition of the 5D graviton. On any 4D-Lorentz-invariant background such a decomposition has the following general form,

$$h_{\mu\nu}(x, y) = \int_0^\infty dm \psi^{(m)}(y) h_{\mu\nu}^{(m)}(x),$$

(13)

where $h_{\mu\nu}^{(m)}(x)$ are the 4D massive spin-2 fields and $\psi^{(m)}(y)$ are their wave function profiles in $y$-dimension that form a complete orthonormal set. The effective 4D graviton, to which a brane observer couples, is a collective mode given by the superposition of infinite number of KK states,

$$h_{\mu\nu}(x) \equiv h_{\mu\nu}(x, y = 0) = \int_0^\infty dm \psi^{(m)}(0) h_{\mu\nu}^{(m)}(x).$$

(14)

The propagators of $h_{\mu\nu}^{(m)}(x)$-states are the usual 4D propagators of the massive Pauli-Fierz fields, which mediate Yukawa type potentials, $e^{-mr}/r$. The non-trivial information about the changes of the gravitational regime is encoded in the boundary values of $\psi^{(m)}(y)$ functions at $y = 0$. The key point is that because of $R_4$-term on the brane, the wave-functions of $m \gg r_c^{-1}$ modes are suppressed at $y = 0$,

$$|\psi(0)^{(m)}|^2 = \frac{1}{1 + (r_c m)^2},$$

(15)

(for simplicity we have set numeric factors equal to one). As a result of this suppression, the brane observer is effectively decoupled from the KK modes that are heavier than $r_c^{-1}$, and sees the following effective gravitational potential

$$V(r)_{brane} = \frac{1}{M_5^3} \int_0^{r_c} dm |\psi(0)^{(m)}|^2 \frac{e^{-mr}}{r} \simeq \frac{1}{M_5^3} \int_0^{r_c} dm \frac{e^{-mr}}{r},$$

(16)

where in the last term we had to cutoff the integral at $r_c^{-1}$ by taking into the account the suppression of the heavier modes given by (15). Due to this suppression, at the distances $r \ll r_c$ the brane observer sees a weak 4D gravity

$$V_{brane}(r \ll r_c) \simeq \frac{1}{M_5^3 r_c^3}.$$

(17)
This goes in sharp contrast with the experience of the bulk observer locates say at $y \gg r_c$. Such an observer couples to an entire KK tower and experiences 5D gravity for all $r$,

$$V(r)_{\text{bulk}} = \frac{1}{M_5^2} \frac{1}{r^2}.$$  \hspace{1cm} (18)

Thus, because of the modified brane gravitational action, imposed by holography, the brane observer at short distances sees 4D gravity that is much weaker than its would-be 5D counterpart. The crucial fact is, that the crossover scale $r_c = N L_5$ at which the 4D regime sets in is much larger than the 5D holographic scale $L_{N5} = N^{1/3}$, resulting into the following hierarchy of scales

$$(r_c = N L_5) \gg (L_{N5} = N^{\frac{4}{3}} L_5) \gg L_5.$$  \hspace{1cm} (19)

In the other words, the brane observer manages to store the $N$ bits of information in a box of size $L_{\text{pixel}} = L_5$, which is much smaller than the size of the analogous box ($L_{N5}$) required for the storage of exactly the same amount of information by her 5D counterpart. This is possible thanks to stretching the crossover distance $r_c$ much beyond the 5D holographic scale $L_{5N}$.

The above situation creates an impression that thanks to crossover modification of gravity the 4D-observer managed to resolve species all the way to distances $L_{\text{pixel}} = L_5$, and in this way bypass the constraints of 5D holography. We are now ready to ask, whether because of modification of gravity the memory about the 5D holographic cutoff is completely erased in 4D theory. Remarkably, the answer to this question is negative. Although crossover modification of gravitational regime extends the species-resolution scale to the distances much shorter than $L_{N5}$, nevertheless, the latter scale still manifests itself through the strong coupling scale in the 4D theory! The key for establishing this connection is the appearance of a new intermediate strong coupling scale in 4D theory, which we shall now consider.

### 3.1 Strong coupling scale of scalar gravitons and $r_s$-phenomenon

By now it is very well understood \[13, 14, 15\] that in the above model the scalar polarization of an effective 4D graviton (14) has self-interactions that are suppressed by the new scale,

$$\Lambda_{\text{str}} \equiv (r_c^{-2} M_P)^\frac{1}{2}.$$  \hspace{1cm} (20)

The origin of the latter strong coupling scale is the following. As we have seen, the 4D graviton (14) that is sourced by 4-dimensional energy-momentum tensors living in the brane world-volume theory is not a mass-eigenstate, but a resonance representing a linear
superposition of the continuum of the massive KK modes. Since each KK graviton is
a massive spin-2 boson, it contains five physical degrees of freedom, and so does their
composite 4D graviton (14). In fact, from the 4D perspective the decomposition (16)
is nothing but a spectral representation of the resonance, with the spectral function given
by (15).

One of the five physical degrees of freedom is an extra scalar polarization, which
we shall denote by \( \pi \). Ignoring the spin-1 helicity, in terms of canonically normalized
spin-2 Einsteinian helicity \( \tilde{h}_{\mu\nu} \) and the scalar helicity \( \pi \), the 4D graviton (14) can be
decomposed as,

\[
h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{1}{6} \eta_{\mu\nu} \pi + \frac{r_c}{3} \frac{\partial_{\mu} \partial_{\nu}}{\sqrt{\Box}} \pi.
\]

(21)

Straightforward analysis [13] shows, that while the Einsteinian spin-2 helicity \( \tilde{h}_{\mu\nu} \)
experiences the usual \( 1/M_4 \)-suppressed interactions, the scalar mode \( \pi \) becomes self-strongly
coupled at much lower scale \( \Lambda_{\text{str}} \), given by (20).

The effect of this strong coupling on the gravitational field created by localized sources
is rather profound and leads to creation of a new intermediate gravitational scale [13, 15, 
17, 16, 18, 19, 21, 22, 20, 26],

\[
r_s \equiv \left( \frac{r_c^2 r_g}{3} \right)^{\frac{1}{3}}.
\]

(22)

Obviously, for any source with the Schwarzschild radius \( r_g \ll r_c \), we have the following
hierarchy of scales,

\[
r_g \leq r_s \leq r_c.
\]

(23)

The notion of \( r_s \) for the above 5D theory [10] is analogous to the so-called Vainshtein
effect [23] that avoids van Dam -Veltman-Zakharov discontinuity [11] in non-linear theory
of Pauli-Fierz massive graviton.

The physical meaning of the scale \( r_s \) can be defined as the distance from the source
at which the non-linear interactions of the scalar helicity \( \pi \) become important. In this
sense, the radius \( r_s \) plays the same role for \( \pi \) as the ordinary Schwarzschild radius \( r_g \)
plays for Einsteinian spin-2 helicity \( \tilde{h}_{\mu\nu} \). The existence of \( r_s \) was demonstrated by explicit
solutions [13, 17, 16, 18, 19, 21, 22, 20, 26]. However, emergence of this scale can be
understood already from the simple power counting arguments [13, 15].

Consider a 4-dimensional static source of the Schwarzschild radius \( r_g \) and imagine
that we are trying to probe its metric. At sufficiently large distances \( r \gg r_s \) (low momenta
\( p \ll r_s^{-1} \)) the interaction between the source and the probe are dominated by one-graviton

\(^2\text{However, as shown in [26, 25, 27], for non-linear Pauli-Fierz case the Vainshtein effect is intertwined}
\text{with the appearance of Boulware-Deser type [24] ghost, which is absent in the present case.}\)
exchange. At some shorter distances (high momenta) non-linear self-interactions of gravitons become important. This critical distances will define the value of $r_\ast$. The leading non-linear coupling comes from the trilinear interaction of the longitudinal gravitons ($\pi$) with the coefficient $r_\ast^2/M_4$, and the corresponding vertex has a momentum dependence of the form

$$\frac{r_\ast^2}{M_4} p^4,$$

where $p$ is a typical energy-momentum flowing through the vertex. The scale $r_\ast$ corresponds to the distance from the source at which the contribution from the above trilinear vertex becomes as important as the linear contribution given by one graviton exchange. This fixes \(22\).

In order to single out the leading non-linearity, we can also take an useful limit $r_c, M_4 \to \infty$ and keeping $\Lambda_{\text{str}}$ fixed [14, 27]. In the language of species, this corresponds to taking $N, M_5 \to \infty$. In this limit, 5D gravity decouples. However, the resulting theory is non-trivial. It is a 4D theory of a single scalar field $\pi$ with the Lagrangian

$$\pi \Box \pi + \frac{1}{\Lambda_{\text{str}}^3} (\partial_\nu \pi)^2 \Box \pi + \frac{\pi}{M_4} T,$$

where the strong coupling scale $\Lambda_{\text{str}}$ is given by \(20\), and $T \equiv T^\mu_\mu$ is an external source. As argued in [28], this form may not capture some essential properties of 5D theory, but this difference is inessential for our purposes, since as explained above, the strong coupling effect is well established already in the full theory.

Notice that in order to maintain a non-trivial source in the decoupling limit, we should keep $T/M_4$ fixed. This means that in this limit $r_g = T/M_4^2 \to 0$, but $r_\ast$ remains fixed. The above-explained physical meaning of $r_\ast$ can be made explicit by exactly solving the equation for $\pi$, which is effectively an algebraic equation on the derivative of $\pi$ [20]. We shall not do this, since it is enough to notice that at sufficiently large distances (where the non-linear term is unimportant) the solution must be the usual $\pi = r_g/r$. Then, comparing the strengths of the variations of the two terms in \(25\) on this function, it is easy to see that the linearized solution breaks down and gets modified exactly at the scale $r_\ast$, where the non-linear term in the equation of motion becomes dominant. We shall now see that the scale $r_\ast$ is the holographic bridge between the 4D and 5D theories.

3.2 Holographic Meaning of $r_\ast$

We are now ready to discuss how the bulk holographic scale manifests itself in an effective 4D theory. The fundamental length scale of 5D theory is $L_5$. This scale represents an absolute lower bound on the localization length of a single information bit, since it is a
critical distance at which the size of the bit ($L_{\text{bit}}$) and its 5D gravitational radius ($r^{(5)}_{\text{bit}}$) cross. In other words, the Schwarzschild radius of any localized bit of length $L_{\text{bit}} = L_5$ is the same as the length itself.

The energy of such an information bit is $m_{\text{bit}} = M_5$, whereas the 5D gravitational radius is $r^{(5)}_{\text{bit}} = L_5$. Since in 5D the Schwarzschild radius scales as the cubic root of the mass, the holographic bound on localization of $N$ information bits is $L_{N5} = N^{\frac{2}{3}} L_5$.

Now, as discussed above, by populating the 4D world-volume theory by $N$ particle species and requiring that species identities stay resolved all the way to distance $L_5$, we are forced to the conclusion that at short distances gravity on the brane must be weaker than 5D gravity. This implies the existence of 4D Einstein-Hilbert term (10) with the coefficient $M^2_4 = N M^2_5$.

Thus, from the first glance it seems that the 4D observer has avoided the 5D-holographic constraint on the information storage. Indeed, the mass of an elementary information bit in 4D and 5D theories is the same, simply given by the inverse localization length, $m_{\text{bit}} = 1/L_{\text{bit}} = M_5$. As explained above, in 5D gravity the Schwarzschild radius of such information quantum would be $r^{(5)}_{\text{bit}} = L_5$. However, in 4D brane theory the Schwarzschild radius of the same bit ($r^{(4)}_{\text{bit}}$) is now set by the weaker 4D gravity, $r^{(4)}_{\text{bit}} = \frac{M_5}{M^2_4} = \frac{r^{(5)}_{\text{bit}}}{N}$, (26) and correspondingly is much shorter than its 5D counterpart. This creates an impression that 4D theory is completely liberated from the 5D holographic bound, and has no memory about $L_{N5}$.

Remarkably, this is just an illusion, and the 4D system still keeps a clear memory about the 5D holography. This memory manifests itself through the fact that the $r_*$ radius of the above elementary information bit, with the Schwarzschild radius $r^{(4)}_{\text{bit}}$ given by (26), is $r_* = (r^{(4)}_{\text{bit}} r_c^2)^{\frac{1}{3}} = N^{\frac{2}{3}} L_5$. (27) This is exactly equal to the 5D holographic scale $L_{N5}$!

More generally, irrespective of an origin or a value of $r_c$, the $r_*$ radius of $N$ information bits in 4D theory scales with $N$ exactly as the 5D holographic scale, $r_*(N) \propto L_{N5} \propto N^{\frac{1}{2}}$. (28)

Thus, what we are finding is, that 5D holographic scale secretly penetrated in 4D theory as a scale of strong interactions.

The generalization of the above holographic counting to higher co-dimension tensionless rigid branes is straightforward. For a 3-brane with localized $N$ species embedded in
4 + d-dimensional space, the holographic scale that sets the 4D cutoff is given by (2). The BH quasi-classicality arguments tell us that resolution of species identities at distances shorter than $L_{N(4+d)}$ is incompatible with 5D gravitational regime beyond this scale. And again, should we demand that 4D observer be able to resolve species all the way down to the $(4 + d)$ Planck length, we shall be confronted with the necessity of the crossover modification of gravity at some scale $r_c \gg L_{N(4+d)}$, with the subsequent strong coupling effect at the short distances.

4 Solitonic Branes

We shall now consider another example, in which the cutoff of species theory is again bounded by the bulk holographic scale. We shall see, that by consistency of the underlying theory and brane’s inner structure, the distance below which species can be considered as 4D elementary particles ($l_{\text{species}}$) is secretly constrained by the bulk holographic scale $L_{N(4+d)}$. We then try to deform the theory by taking $l_{\text{species}} \to 0$ and keeping $L_{N(d+4)}$ fixed. Again, we shall discover that theory responds by creating a strong coupling scale ($L_{\text{strong}}$) in 4D theory which is automatically bounded by the bulk holographic scale, according to (6). Even in the gravity-decoupling limit, 4D theory modes appear with the strong coupling scale bounded by $L_{N(4+d)}^{-1}$.

We shall consider solitonic branes that are solutions of classical field equations of a high-dimensional theory. Since we are working in asymptotically flat space-times, we shall choose the codimension-two branes, which are known to produce asymptotically-flat spaces. We shall consider a solitonic brane produced by a 6D scalar field $\Phi$, charged under an $U(1)$ gauge symmetry. The Lagrangian of interest is a trivial generalization of the Abelian Higgs model to 6D:

$$|D_A \Phi|^2 - \frac{\lambda}{2} (|\Phi|^2 - v^2)^2 - \frac{1}{4} F_{AB}^2. \quad (29)$$

Here $A,B$ are the six dimensional indexes, and the minimal coupling to 6D gravity is assumed. Notice, that both, the Higgs coupling $\lambda$ as well as the gauge coupling ($g$), have inverse-mass dimensionality. The Higgs field develops a vacuum expectation value at the scale $v$. The brane solution in question is a well-known Nielsen-Olesen flux tube [29] lifted to 6D. The solution, in the cylindrical coordinates $\rho, \phi$, has the following asymptotic form. For $\rho \to \infty$, $\Phi = ve^{i\phi}$, which due to non-trivial topology forces $\Phi = 0$ at $\rho = 0$. The brane core supports the magnetic flux in the direction $F_{56}$. The brane has two cores, the magnetic core (the region with localized magnetic flux) and the Higgs core (the region with non-zero Higgs energy). The widths of these cores are set by the inverse masses of
the gauge \((m_{\text{gauge}}^{-1} = (gv)^{-1})\) and the Higgs \((m_{\text{Higgs}}^{-1} = (\lambda v)^{-1})\) particles respectively. In the limit when \(g = \lambda\), known as Bogomolnyi-Prasad-Sommerfield (BPS) limit, the two cores are of the same size. As it is well known, in this limit, the gauge repulsion between the two vortexes is exactly compensated by the Higgs attraction, and the vortexes are in a neutral equilibrium. For simplicity, we shall adopt this limit. The brane tension, energy per unit 4D volume, is given by \(T \sim v^2\) (or to be more precise, \(T = 2\pi v^2\) in BPS limit), and is independent of the the gauge and Higgs couplings.

Outside of the core, the metric of the brane is a generalization of Vilenkin’s cosmic string metric \([30]\),

\[
d^2s = -dx^2_\mu + d\rho^2 + (1 - \frac{\delta}{2\pi})\rho^2d\phi^2, \tag{30}
\]

where \(\delta = 2\pi\frac{T}{M_6}\) (where a factor of 4 has been absorbed in renaming \(M_6\)). We thus see that metric produced by brane is an asymptotically-flat metric with the angular deficit set by \(\delta\). Critical value of the tension for which the brane over-closes the space (in fact, turns it into a a cylinder) is \(T = M_6^4\) corresponding to the angular deficit of \(\delta = 2\pi\). Each brane supports the two localized zero modes, \(\pi^a (a = 1, 2)\), which are the Nambu-Goldstone bosons of the spontaneously broken translation invariance in extra directions. Their localization width is set by the brane core thickness \(l_{\text{core}} \equiv (\lambda v)^{-1}\).

Consider now \(N\) coincident branes. In the BPS limit, such a system continues to support \(2N\) massless Goldstone bosons. Since the tensions add up, the deficit angle produced by such a system is \(\delta_N = 2\pi N\frac{T}{M_6}\). Defining the scale \(\Lambda_{\text{str}} \equiv L_{\text{str}}^{-1} \equiv T^{\frac{1}{4}}\) and requiring that the deficit is less than \(2\pi\), we get that the bound on the tension scale,

\[
\Lambda_{\text{str}}^{-1} \geq \frac{N^\frac{1}{4}}{M_6}. \tag{31}
\]

Thus, for \(N\) coincident branes, the bound on the brane tension scale (the Higgs VEV) is set by the bulk holographic scale for \(N\) bits of the information,

\[
\Lambda_{\text{str}}^{-1} \geq L_{N6}. \tag{32}
\]

The localization width of the species \(l_{\text{species}} = l_{\text{core}} = m_{\text{Higgs}}^{-1}\) automatically sets the cutoff on the species theory for the 4D brane observer, since for shorter distances species are no longer point-like. Notice, that for the weakly coupled bulk theory \(l_{\text{core}} > L_{\text{strong}}\), since the Higgs and gauge particles are lighter then the soliton mass scale. Thus, for the weakly coupled bulk theory, \(l_{\text{species}} > \Lambda_{\text{str}}^{-1}\), and the holographic bound is automatically satisfied.

Remarkably, the bulk holographic scale continues to bound the strong coupling scale of the 4D theory even in the limit in which the particle localization width goes to zero.
Indeed, let us take the limit $g, \lambda \to \infty, \ N \to \infty, \ M_6 \to \infty$ while keeping $\delta, v (\Lambda_{str})$ and $L_{N6}$ fixed. In this limit Higgs and gauge bosons become infinitely heavy and decouple from the low energy theory. Gravity decouples as well. So the bulk low energy theory in this limit is trivial, but the solitonic sector delivers massless modes. Notice that in this limit the resolution length scale $l_{\text{species}}$ for 4D species goes to zero. So naively (just as in 5D example) one may think that the surviving 4D theory has no memory about the 6D holographic bound. However, this is not the case. The 4D theory gets strongly-coupled at the brane tension scale. The low energy fields are the $\pi$-fields with the Nambu-type action,

$$\int d^4x \Lambda_{str}^4 \sqrt{\text{det} \partial_\mu \pi^a \partial_\nu \pi^a}. \quad (33)$$

Notice, that because of (31), the strong coupling scale of the surviving 4D theory is again set by the bulk holographic scale.

We thus see, that whenever we try to decouple the species resolution scale, $l_{\text{species}}$, from $L_{N6}$, the 4D theory responds by creating a sector with the strong coupling scale bounded by $\Lambda_{str} = L_{N6}^{-1}$.

5 Species on D-Branes

We shall now consider a case when $N$ 4D species are localized on $D_3$-branes embedded in an asymptotically flat 10D space. As we shall see, the same property holds true in this case also. Namely, the strong coupling scale of the 4D theory is bounded by the 10D holographic scale, $L_{N10}$. This follows from AdS/CFT correspondence.

Indeed, in order to get $N$ 4D species embedded in 10D asymptotically-flat space, we start by considering a stuck of $N_D \equiv \sqrt{N} \ D_3$-branes. This is exactly the setup of AdS/CFT correspondence. Because of their BPS properties, $D_3$-branes produce a static asymptotically-flat metric with the 10D gravitational radius given by,

$$R = (gN_D)^{\frac{1}{4}} l_s \equiv \lambda^{\frac{1}{4}} l_s, \quad (34)$$

with $l_s$ denoting the string length. In the last expression we have defined the ’t Hooft coupling $\lambda$. The first hint towards the holographic connection is already given by the fact that $R$ coincides with the 10-dimensional holographic scale $L_{N10}$ for the storage of $N$ bits of information, and thus is solving the bound

$$N = (RM_{10})^8, \quad (35)$$

with the ten dimensional Planck mass $M_{10} = \frac{1}{g^2 l_s}$. The effective near-horizon gravity is
AdS$_5 \otimes S^5$ with metric

$$ds^2 = \frac{r^2}{R^2} dx^2 + \frac{R^2}{r^2} (dr^2 + r^2 d\Omega_5).$$

(36)

We shall now repeat some usual steps for establishing AdS/CFT in order to show how the connection between the 4D strong coupling scale and $L_{N10}$ emerges. Since in the case of the AdS/CFT correspondence we are interested in a duality between a gravity theory in ten dimensions and a non-gravitational theory in four dimensions we should proceed to analyze the gravity decoupling limit of the holographic correspondence discussed above. Generically the gravity decoupling limit can be defined in two different ways. Namely as the limit of $l_s \to 0$, or instead as the limit $g \to 0$ but keeping the string length $l_s$ finite. Let us first consider the decoupling of gravity in the limit $l_s = 0$. By introducing the holographic energy variable $U = \frac{r}{l_s}$ we transform the metric (36) into

$$ds^2 = l^2 (\frac{U^2}{\lambda^2} dx^2 + \lambda^2 dU^2 + \lambda^2 d\Omega_5).$$

(37)

Nicely enough string theory is perfectly well defined in this background even in the limit $l_s = 0$, therefore we get a full fledged gravity theory in ten dimensions. However, in this limit gravity in the four dimensional theory is completely decoupled since $M_{10} = \infty$. Moreover the scale $R = L_{N10}$ goes to zero in this limit leading to the well known CFT in four dimensions with infinite cutoff.

In order to trace the holographic meaning of the 4D strong coupling scale, we need a limit in which $R$ is kept finite. As already pointed out a different gravity decoupling limit could be defined by sending $g_s$ to zero, but keeping the string length $l_s$ finite. This gravity decoupling limit becomes very useful for our purposes if at the same time we send $N$ to $\infty$ with $gN$ (and thus $R$ and $L_{N10}$) finite. The holographic meaning of this t’Hooft limit for decoupling of gravity is that - even with vanishing gravity- we still get a non vanishing gravitational radius for the storage of the $N$ bits of information. In the other words, the memory about the bulk holography persists in 4D theory in form of the strong coupling scale. The connection now is clear, the same scale $R$ sets both the strong-coupling scale of 4D theory with $N$ species as well as the 10D holographic scale for $N$ information bits.

Let us now briefly discuss the holographic correspondence in this particular limit. In terms of the holographic energy variable $U$ and for finite $l_s$ the condition of wave length smaller than $R$ induces the existence of a UV wall in the holographic direction at $U = \frac{\lambda^2 l_s}{l}$. Notice that this UV wall goes to infinity in the limit $l_s = 0$. But, if we keep $l_s$ finite, the scale $R$ in the dual four dimensional theory still survives although gravity is completely decoupled. Indeed, this scale is just $\lambda^2 l$ that, as already pointed out, is the gravitational radius for the storage of $N^2$ bits localized in four dimensions. In
summary, when we introduce an UV wall in the holographic direction at $U = \lambda^{\frac{1}{4}} l$ the ten dimensional gravity theory becomes dual to a four dimensional theory with gravity completely decoupled, but with a dynamical scale $R = \lambda^{\frac{1}{4}} l$ set by the bulk holographic scale. This is of course a sort of a "QCD" like theory with the QCD scale given by $\Lambda_{QCD} = R^{-1}$. This "holographic" confinement is simply identifying the size of the singlet glueball with its gravitational radius, that, in the t’Hooft limit, is non-vanishing even if gravity is completely decoupled.

6 Duality and Localization: Attempting a Synthesis

Generically holographic duality maps a higher dimensional bulk gravity theory into a lower dimensional theory on the boundary [2]. For the AdS/CFT correspondence the map is between type IIB string bulk theory on $AdS_5 \otimes S^5$ with length scale $R$ and $N = 4$ four dimensional SSYM with gauge group $SU(N_D)$ and t’Hooft coupling $\lambda$, where $N^{\frac{1}{2}}_D = \frac{R}{l_{pl}}$ and $\lambda^{\frac{1}{4}} = \frac{R}{l_{st}}$. The bulk semiclassical gravity requires both $N$ and $\lambda$ to be large. From the ten dimensional point of view the holographic meaning of $N$ is to define a bound on the amount of information we can store in a ten dimensional space region of size $R$ [31].

An alternative but equivalent interpretation of $N$ is the maximal number of different particle species compatible with Einsteinian ten dimensional quasi-classical black holes of size bigger or equal to $R$. Or equivalently, $N$ is a maximal number of 10D particle species compatible with the cutoff length $R$.

Therefore, a general feature of any holographic duality is to relate a $4 + d$ gravity theory characterized by a scale $R$ to a four dimensional field theory with $N$ species, where $N$ is determined by the holographic relation $N^{\frac{1}{2}}_{4+d} = \frac{R}{L_{4+d}}$, with $L_{4+d}$ being the Planck length in $4 + d$ dimensions. This general feature of holographic dualities leads us to focus our attention on the dynamical mechanism of localization of particle species on a lower dimensional topological defect. Very likely we could be able to unravel from the dynamics of this localization mechanism the basic elements of the holographic correspondence between physical theories in different dimensions.

As we have described in this note the first consequence of localization of species on a lower four dimensional manifold is to set the strong coupling scale of the low energy 4D theory by bulk holographic scale. The strong coupling comes either from the fact that species can no longer be considered as elementary above that scale, or from the modification of gravitational dynamics at the crossover scale $r_c \ll R$ where gravity actually changes from 4 into $4 + d$. In the simplest case of $d = 1$ the dynamical origin of this crossover scale is physically quite transparent and can be uniquely traced to the
existence of 4D Einstein-Hilbert term. The required magnitude of this term is exactly compatible with the one-loop contribution of localized species to the graviton propagator. However, our conclusions hold regardless of its precise origin. Independently of the actual mechanism of generation of the four dimensional IR scale $r_c$ this scale is holographically bounded by the higher dimensional scale $R = L_{N(4+d)}$:

$$r_c \geq R$$

(38)

The origin of this bound lies in the fact that the number of species is precisely defined by the value of $R$ in $4 + d$ Planck units. The modified gravity on the lower dimensional theory necessarily involves the propagation of extra modes that become strongly coupled at a certain scale $r_s$ \cite{13, 15}. This strong coupling scale is determined by the trilinear coupling of the extra scalar helicity of effective 4D graviton.

For making the parallel with string theory case, let us generalize the notion of $r_s$ and $L_{str}$ distance, by defining the length scale at which this trilinear coupling will be order $g^2$,

$$L_{str} = (r_c^2 L_4)^{\frac{4}{3}} g^{-\frac{2}{3}},$$

(39)

with the threshold of strong coupling at $g = 1$. Using now the value of the scale $r_c = \frac{N}{M_5}$ we easily get $L_{str}(N) = \frac{N^2}{M_5} g^{-\frac{2}{3}}$. Now it is easy to observe that the strong coupling scale is playing a role similar to the string scale in the higher dimensional theory. Indeed $\frac{r_c}{L_{str}(N)} = N^{\frac{1}{3}} g^{-\frac{2}{3}} = \lambda^{\frac{2}{3}}$ for $\lambda = g^2 N$. Moreover the semiclassical bulk gravity regime defined by $\lambda > 1$ corresponds to the regime $r_c > L_{str}(N)$.

In summary, it appears that there could be defined a general holographic analogy between a $(4+d)$-dimensional string theory with scale $R$, on one side, and four dimensional modified gravity with $N$ localized species, on the other side, where the number of species is determined by $R$ in Planck units and where the scales $r_c$ and the strong coupling scale $L_{str}(N)$ are in one to one correspondence with the string theoretic scales $R$ and $l_s$.

Notice that the above discussion is completely general and only uses the fact that the higher dimensional gravity theory is characterized by the scale $R = L_{N(4+d)}$ with the holographic meaning of fixing the number of species in the lower dimensional theory.

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3 The $L_4$ in equation (39) is the Planck length in four dimensions.

4 Notice that if we identify the parameter $g$ with the string coupling constant and the number of species with $N_g^2$ for $N_g$ the number of colors $\lambda = g^2 N$ is just the square of the standard t’Hooft coupling and the relation $\frac{r_c}{L_{str}(N)} = \lambda^{\frac{2}{3}}$ is just the expected AdS/CFT correspondence in five dimensions, namely $R = \lambda_g^{\frac{2}{3}} l_s$ for $\lambda_g = N_g g$ the standard t’Hooft coupling for a gauge theory with $N_g$ colors.
7 Summary and Outlook

In this work we have identified the holographic origin of the strong coupling scale in 4D theories with \( N \) particle species that originate as world-volume theories on 4D surfaces (3-branes) embedded in asymptotically-flat \((4 + d)\)-dimensional spaces with Einsteinian gravity in the bulk.

We have seen, that by consistency of the theory, either the species-theory or the gravity of the 4D sources (or both) necessarily require an UV completion above the bulk holographic scale \( L_{N(4+d)} \). In this way, bulk holography dictates the laws of 4D physics.

In some examples reconciliation with the bulk holography happens via species becoming effectively composite (non-point-like) at distances \( l_{\text{species}} \) shorter than \( L_{N(4+d)} \).

However, even when the species compositeness scale is pushed all the way down to the fundamental Planck length \( L_{4+d} \), the strong coupling is not avoided. In such a case, the gravitational sector responds by crossover modification of Newtonian force law from \( 4 + d \)- to 4-dimensional regime at some distance \( r_c \gg L_{N(4+d)} \). This automatically implies that the scalar polarization of graviton becomes strongly coupled at distance \( r_* \), determined precisely by the bulk holographic scale. In fact, the gravitational field of a single elementary information bit becomes strong exactly at the scale \( r_* = L_{N(4+d)} \).

In all the considered examples there is a consistent gravity-decoupling limit in which the system is reduced to a pure 4D theory with the strong coupling scale determined by the holographic scale of the high-dimensional gravity theory.

We shall now briefly summarize possible implications of our findings.

1. Towards Generalizations of AdS/CFT Correspondence.

Our observation, that \( 4 + d \)-dimensional holographic scale \( L_{N(4+d)} \) sets the bound on the strong coupling scale of the 4D theory, partially relies on the asymptotic flatness of the embedding space. To be more precise, the curvature radius of the high-dimensional space must exceed the holographic scale \( L_{5(4+d)} \).

For \( D \)-branes the feature of asymptotic flatness rests on their BPS properties. The same property guarantees that the near-horizon limit is AdS geometry. As we have shown, the latter fact automatically makes the holographic origin of the 4D strong coupling understandable in terms of AdS/CFT correspondence phenomenon.

On the other hand, we have seen that 4D strong coupling exhibits exactly the same holographic origin in the examples in which the asymptotic flatness of the embedding space does not necessarily imply any AdS type near-horizon geometry. For example, neither for the species on the tensionless orbifold planes, nor for the 6D vortexes this property was there. Yet, the fact that the 4D strong coupling scale \( L_{\text{strong}} \) was bounded
by the bulk holographic scale $L_{N(4+d)}$ persisted also in these examples in a rather profound way.

It thus emerges, that that the high-dimensional holographic origin of 4D strong coupling is a result of species number and asymptotic flatness of the brane metric, rather than necessarily of the AdS-type properties of the near-horizon geometry, or even of the BPS properties. Although for the $D$-branes the latter properties happen to follow from the former (and vice-versa), in general this is not the case.

These findings indicate that the relation $L_{N(d+4)} \leq L_{(4)}^{(strong)}$ may be a way to extend certain properties of AdS/CFT duality to other spaces. Of course, in each particular case, the extended duality may not be as powerful as it is in the AdS limit, however it can still be extremely useful in understanding certain phenomena (such as the appearance of the strong coupling considered in this work) in terms of gravity duals.

2. In Search for Gravity-Duals.

In general, search for gravity-duals of strongly coupled gauge systems may be an extremely complicated task. Our findings provide evidence that the strong coupling scale is linked to the holographic scale of the gravity-duals. A toy explicit model for exploring such a connection may be a generalization of our vortex example to the non-Abelian string case with $CP_n$ world volume theory[32]. An indication for the gravitational origin of the gauge theories on the branes, comes from the general fact that localization of the perturbatively-massless gauge fields on the branes requires condensation of magnetic charges in the bulk, which automatically implies the existence of both open and closed strings in such a theory[33]. The strings in question are the electric flux tubes (or glueballs) whose existence follows from the gauge invariance and the charge conservation on the brane. In this respect, any brane with localized gauge fields shares similarity with $D$-branes, and thus secretly knows about the bulk gravity.

3. Implications for Large Distance Modification of Gravity.

One of the byproducts of our studies is that any crossover modification of classical gravity at distance $r_c$ can be understood in terms of holography, even without any reference to particle species. Indeed, such a modification automatically follows from the requirement that a short-distance observer be able to store $N$ information bits in the box of the size of far-infrared Planck length. In our 5D example of [10] the far-infrared Planck length is the 5D Planck length $L_5$. This requirement fixes the crossover scale to be $r_c = NL_5$. Holographic origin of the 4D strong coupling is then encoded in $r_s$-phenomenon. Indeed, as we have seen, the $r_s$-radius of a single elementary information bit is always given by $L_{N5}$.
It has been shown [15], that any crossover modification of 4D Newtonian gravity beyond some IR scale \( r_c \), results in the strong coupling and \( r_c \)-phenomenon at intermediate distances \( \ll r_c \). This conclusion follows from the general covariance and the fact that gravitons in such theories always contain extra scalar helicities by condition of absence of ghosts. Our suggestion about the holographic nature of the strong coupling in such theories provides evidence of their high-dimensional origin. This evidence is also supported by the known fact that all sensible theories of IR-modified gravity contain continuum (or finely spaced) tower of spin-2 (and possibly spin-0) states. This fact on its own serves as indication of the high-dimensional origin of such theories in which the spin-2 tower is identified with KK states. In combination with the strong coupling holography, the evidence for high-dimensional origin becomes even stronger.

Another interesting question is a generalization of our results for theories with more than one crossover scale \( r_c \). In this respect we note, that some progress has been made recently [34] in identifying class of theories with multiple stages of crossover modifications of gravity within the framework of so-called “cascading” generalizations of [10]. The idea is to have a sequence of branes of decreasing dimensionality embedded within each other in such a way, that each subsequent brane changes the dimensionality of Newton’s law by one. This framework can be a convenient laboratory for studying the holographic origin of the 4D strong coupling in the presence of several crossover distances, and providing holographic dictionary in terms of number of species propagating in various dimensionalities.

Finally, the holographic-equivalence of the large distance modified gravity theories to the theories with \( N \) species suggests that certain gravitational properties can be understood in terms of species dynamics. Some equivalence between the cosmological dynamics on the two sides was already demonstrated in [35].

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