A novel evaluation on fracture pressure in depleted shale gas reservoir

Kai Zhao¹,², Junliang Yuan³, Yongcun Feng⁴, Chuanliang Yan⁵

¹College of Petroleum Engineering, Xi’an Shiyou University, Xi’an 710065, China
²Shaanxi Key Laboratory of Advanced Stimulation Technology for Oil and Gas Reservoirs, Xi’an Shiyou University, Xi’an 710065, China
³CNOOC Research Institute, Beijing 100000, China
⁴Department of Petroleum and Geosystems Engineering, The University of Texas at Austin, Austin, Texas 78712, USA
⁵School of Petroleum Engineering, China University of Petroleum (East China), Qingdao 266580, China

Abstract

Fracture pressure is the key parameter both for horizontal drilling and hydraulic fracturing in the shale gas reservoir. Reservoir depletion will push for the change of the fracture pressure. Previous studies showed that fracture pressure will be decreased with reservoir depletion, which is not suitable for the anisotropic shale gas reservoir. Aiming at resolving this problem, a novel evaluation was established, which can be used to evaluate the influence on fracture pressure of the reservoir depletion, the anisotropy, and the well inclination. The results showed that the fracture pressure will be increased for the strongly anisotropic reservoir with reservoir depletion, and the fracture pressure has a tiny difference between the various anisotropic reservoirs before reservoir depleted, but a larger difference will appear after reservoir depleted, at the later depletion period, the fracture pressure is higher for the stronger anisotropic one. When the anisotropy and the depletion are both the same, the fracture pressure for the higher deviated well is lower. The method and the results can provide a theoretical basis both for the drilling and hydraulic fracturing in depleted shale gas reservoir.

Introduction

In recent years, unconventional gas dispersed in shale as the potential resource has gradually become the focus of energy development due to itself large reserves and the depletion of conventional energy. The greatest growth is predicted for shale gas production, increasing from 16% of total production in 2009 to an expected 45% in 2035 in the United States. [1–5]. However, shale gas reservoir has very low permeability, so the horizontal drilling and the hydraulic fracturing technique have been widely used for increasing the production of shale gas, by increasing the drainage area and creating the artificial fractures, respectively [6]. In particular, stimulated reservoir volume (SRV) was more widely applied in the shale gas reservoir by means of multistage fracturing of horizontal wells due to its large fracture network, Mayerhofer indicated that the complex network structures in multiple planes are created in shale reservoirs, the concepts of single-fracture half-length and conductivity are insufficient to describe stimulation performance, and the evaluation methodology on SRV using microseismic-mapping data was shown [7].
Later, fractal geometry was proposed to apply in evaluation of effective stimulated reservoir volume in shale gas reservoirs, and the mechanism of fracture propagation in shale and the control on morphology of fracture network were researched through laboratory experiments [8, 9].

Fracture pressure is the key parameter both for horizontal drilling and hydraulic fracturing [10–12]. In the process of horizontal drilling, a higher pressure generated by drilling fluid may cause the exposed formations to be fractured, resulting in dangerous mud loss, which is a very troublesome and expensive problem. Furthermore, the difficulty of the evaluation on fracture pressure in horizontal well was increased due to well orientation rotation, compared with vertical well. Hence, an accurate knowledge of fracture pressure gradients is the key for the safe and economical drilling operation. Unlike horizontal drilling to avoid fracturing the formations, the goal of hydraulic fracturing is to fracture the reservoir. However, the question, how much construction pressure can be used to fracture the reservoir, is also the most wanted to be solved. Therefore, the accurate evaluation of fracture pressure is also critical to run a successful and economical well stimulation. As a whole, the bottom-hole pressure should be controlled below the fracture pressure in the process of horizontal drilling, but above the fracture pressure in the process of hydraulic fracturing, in order to avoid accidents and economic loss. And the valuation of fracture pressure is the basis of the horizontal drilling and the hydraulic fracturing operations.

Considerable work has been done in the last few years to determine fracture pressure, and the relation between fracture pressure and other parameters, such as in situ stress, pore pressure, effective stress coefficient, Poisson’s ratio, tectonic stress, and tensile strength etc. have been already analyzed in depth [13–16]. They found that the fracture pressure had a close correlation with the in situ stress and pore pressure. Matthews and Kelly introduced an effective stress coefficient to determine fracture pressure, but the effective stress coefficient was determined by the ratio of the minimum effective horizontal stress to effective overburden pressure, which cannot show its physical significance, and the influence of tensile strength, tectonic stress and stress concentration effect cannot be included [13]. Eaton developed Matthews and Kelly’s method, but only the physical significance of the effective stress coefficient was proposed [14]. Anderson proposed a method to determine fracture pressure which can include the influence stress concentration effect, but the tectonic stress was neglected [15]. Then, a new method was proposed by Stephen which can include the influence of tectonic stress and stress concentration effect, but the difference between the maximum and minimum horizontal stress was neglected, and the tensile strength was not included [16]. Later, some scholars revealed the mechanism of formation breakdown based on the theory of rock mechanics, and gradually developed the above methods, most of the factors including pore pressure, tensile strength, tectonic stress, anisotropy of in situ stress, stress concentration effect [17] were included. But the later studies found that after a long-term production of hydrocarbon resources, reservoir pressure will be depleted, in turn, a reduction of in situ stress acting within the reservoir will generate. Therefore, fracture pressure will be changed finally after reservoir depletion [18–22]. The results showed that fracture pressure will be decreased due to reservoir depletion. However, most of the previous research assumed the depleted reservoir as isotropic medium. In fact, shale gas reservoir has a significant anisotropic characteristic due to bedding plane act [23–27]. In this case, fracture pressure in depleted reservoir is a function of anisotropy, well inclination and other factors. Therefore, conventional methods cannot evaluate fracture pressure effectively in depleted shale gas reservoir. Based on this, the anisotropic mechanical parameters of shale gas reservoir were tested by laboratory experiments, then the evaluation method of in situ stress and fracture pressure in depleted shale gas reservoir was derived, and the influence on fracture pressure of reservoir depletion was analyzed in depth, which can provide theoretical support for the well drilling and stimulation in depleted shale gas reservoir.

**Mechanical Properties Test of Gas Shale**

**Structure characteristics analysis from the macroscopic and microcosmic perspective**

The mechanical properties of gas shale have a close correlation with its structure characteristics. During the diagenetic process, with increasing depth, physical and chemical compaction on sediment rock is gradually replaced by gravitational compaction. As a consequence, shale particle swarms are arranged along the fixed orientation, forming the bedding structure [28]. From a macroscopic perspective, the obvious bedding structure is observed in the macroscopic photos of shale gas reservoir (Fig. 1), and the spaces between the adjacent beddings are different. Just like the bedding shale from Northeastern British Columbia, the range of the spacing is also different, from 0.01 to 0.40 m, with a mean of 0.11 m [29]. Furthermore, the bedding plane observed in Figure 1 is nearly parallel.

In addition, the thin section observation and the scanning electronic microscope were applied to observe its microstructure (Fig. 2). The results indicate that the clay mineral is obviously arranged along the fixed orientation.
Some studies have shown that the directionally arranged clay mineral is an important factor that causes rock anisotropy, for shale stone, the degree of anisotropy is dependent on the clay mineral content, and for gas shale with high carbon, the degree of anisotropy is dependent on the clay mineral and organic matter content, and the higher the content of these two components, the greater the anisotropy [31, 32]. Furthermore, the clay minerals often have flaky structures, so the microcracks were developed along the fixed orientation in the layers and interlayer [30], and the width of the microcracks observed in the core samples ranges from 1 to 25 um, which also causes rock anisotropy.

Anisotropic elastic deformation test

To analyze the mechanical anisotropy of gas shale, the rock samples were cored in different directions (Fig. 3), and the Rapid Triaxial Rock Testing System produced by GCTS company in the United States was applied to test the stress–strain curves, then, the elastic parameters are calculated. In the coring process, the smallest diameter of standard core size is 25 mm and the slenderness ratio 1.8~2.0 [33]. In order to make sure the two ends of the samples are smooth and the nonparallelism is <0.02 mm, the end face should be perpendicular to the central axis, and the angular deviation <0.25°. In addition, because the shale is extremely fragile during the coring process, low speed and smooth operation should be maintained to avoid disturbance damage. Before the experiment, the core was wrapped with thermoplastic pipe. The displacement loading mode was selected to apply axial stress, and the constant strain rate $10^{-3}$ mm/sec was adopted before shale break down, but the rate is changed into 0.1 mm/min after shale break down. In the triaxial compression test, confining pressure is 10 MPa and the loading rate is 0.035 MPa/sec.

The elastic parameters of gas shale in different direction are shown in Figure 4. The experiment results indicated the mechanical properties have a significant difference in...
different directions. Hence, the anisotropy of the gas shale is obvious. When the $\gamma$ is increased, the modulus of elasticity is increased, so the difference of the modulus of elasticity between the parallel and vertical bedding directions is maximum, and the anisotropy ratio is about 1.55. When the confining pressure (10 MPa) is exerted, the modulus of elasticity in different directions is all increased, and the maximum anisotropy ratio is about 1.53. The Poisson’s ratios have also showed obvious anisotropy. When the $\gamma$ is increased, the Poisson’s ratios have a decreasing trend on the whole, but the trend is not fitted in some directions.

**Tension strength test**

In order to simulate borehole fracture failure and test its tension strength, the original rock samples were cut into small disks with a diameter of approximately 100 mm and a thickness of approximately 25 mm. A hole at the center of the disks is created, and the diameter of the center hole is approximately 16 mm, then a prefabricated crack is created on the center hole symmetrically, and the length is approximately 2 mm. Then, the external loads were increased gradually along the radial directions until the crack extends, and the tension strength test can be calculated as:

$$S_t = \frac{2P}{\pi D t}$$

where $S_t$ is the tension strength; $P$ is the external loads when the crack extends; $D$ is the diameter of the disk; $t$ is the thickness of the disk.

Using the above test method, the shale core was processed and tested, the shale core photo after the test of tension strength was shown in Figure 5. The diameter and thickness of the disk is processed as 99.30 and 23.67 mm respectively, and the external loads when the crack extends is 5.6260 KN; hence, the ultimate tension strength is calculated as 1.52 MPa.

**In Situ Stress in Depleted Shale Gas Reservoir**

In situ stress at a certain depth is usually described by three orthogonal principal stress components, that is, the vertical stress, the maximum and the minimum horizontal stress, respectively. In situ stress can be measured directly or indirectly by various methods, such as hydraulic fracturing, leak-off test, acoustic emission testing, differential strain analysis, analysis of wellbore failures (wellbore breakouts and drilling-induced tensile fractures), and so on [34–37]. The in situ stress in some fixed depths can be obtained by these methods. However, the continuous stress profile cannot be derived. The effective method is to establish the prediction model based on tested data.
Isotropic reservoir

The vertical stress is always assumed to be the principle stress, and it is equal to the weight of overlying rocks, it can be obtained easily through the integral of formation density. The key is how to derive the maximum and the minimum horizontal stress through logging data.

Lots of logging interpretation models for the in situ stress in isotropic formation have been proposed in the last few years. Some models thought horizontal in situ stresses are produced only by the overburden stress, and the horizontal stresses in the two orientations are equal, in this case, the horizontal stresses are a function of the vertical stress, Poisson’s ratio and pore pressure [13, 15]. But later research found that the two horizontal stresses were often different from each other. The tectonic stress, produced by plate subduction and compression movements, is another source of the horizontal in situ stress. Based on this, an integrated model was proposed. This model assumes that the formation rock is an isotropic, homogeneous and linear elastic medium. Under the action of tectonic movements, no relative displacement between the different stratum was produced. Therefore, the maximum and the minimum horizontal stress can be evaluated as follows [17, 38]:

\[
\begin{align*}
\sigma_H &= \frac{1}{1-\nu} (\sigma_v - \delta P_p) + \frac{\nu E}{1-\nu} + \frac{\nu V_p}{1-\nu} + \delta P_p \\
\sigma_h &= \frac{1}{1-\nu} (\sigma_v - \delta P_p) + \frac{\nu E}{1-\nu} + \frac{\nu V_p}{1-\nu} + \delta P_p
\end{align*}
\]  

(2)

where \(\sigma_H\) and \(\sigma_h\) are the maximum and minimum horizontal stress, respectively; \(\sigma_v\) is the vertical stress; \(P_p\) is the pore pressure; \(\nu\) is the static Poisson’s ratio; \(E\) is the static modulus of elasticity; \(\delta\) is Boit’s coefficient; \(\varepsilon_H\) and \(\varepsilon_h\) are the maximum and minimum horizontal tectonic strain, respectively.

For isotropic reservoir, it can be assumed to be an unbounded isotropic elastic body, then the dynamic elastic modulus can be expressed in terms of the compressive and transversal wave velocities and formation density, which can be derived from logging data [38]:

\[
\begin{align*}
E_d &= \rho V_p^2 (3V_s^2 - 4V_p^2)/(V_p^2 - 2V_s^2) \\
\nu_d &= (V_p^2 - 2V_s^2)/(2(V_p^2 - 2V_s^2))
\end{align*}
\]  

(3)

where \(E_d\) is the dynamic elastic modulus; \(\nu_d\) is the dynamic Poisson’s ratio; \(\rho\) is the formation density; \(V_p\) is the compressive wave velocity; \(V_s\) is the shear wave velocity.

The dynamic elastic modulus and Poisson’s ratio calculated by the acoustic velocity reflect the formation mechanical properties under transient load. However, the load applied to the formation is static [39]. According to the laboratory tests, the static Young’s modulus and Poisson’s ratio were obtained based on the dynamic Young’s modulus and Poisson’s ratio [40]:

\[
\begin{align*}
E &= 0.2526 + 0.7095E_d \\
\nu &= 0.1268 + 0.25\nu_d
\end{align*}
\]  

(4)

After a long-term production of hydrocarbon resources, the pressure in isotropic reservoir will be depleted, in turn, a reduction of in situ stress acting within the reservoir will generate. The change of in situ stresses, when the reservoir pressure has been depleted by \(dP_p\), can be derived by taking the derivatives of Equation (2) for \(P_p\):

\[
\begin{align*}
d\sigma_H &= \delta \frac{1-\nu}{1-\nu} \frac{dP_p}{1} \\
d\sigma_v &= \delta \frac{1-\nu}{1-\nu} \frac{dP_p}{1}
\end{align*}
\]  

(5)

Anisotropic shale reservoir

Shale reservoir with bedding planes can be regarded as a transverse isotropy vertical formation, compared to the conventional isotropic model of the in situ stress, the transverse isotropy vertical model is more suitable for shale reservoir with horizontal bedding [41, 42]:

\[
\begin{align*}
\sigma_H &= \frac{E}{E_v} \frac{1}{1-\nu_v} (\sigma_v - \delta P_p) + \frac{\nu_v E}{1-\nu_v} + \frac{\nu_v V_p}{1-\nu_v} + \delta P_p \\
\sigma_v &= \frac{E}{E_v} \frac{1}{1-\nu_v} (\sigma_v - \delta P_p) + \frac{\nu_v E}{1-\nu_v} + \frac{\nu_v V_p}{1-\nu_v} + \delta P_p \\
\sigma_h &= \frac{E}{E_v} \frac{1}{1-\nu_h} (\sigma_v - \delta P_p) + \frac{\nu_h E}{1-\nu_h} + \frac{\nu_h V_p}{1-\nu_h} + \delta P_p
\end{align*}
\]  

(6)

where \(E_v\) is the vertical Yang’s modulus; \(E_h\) is the transverse Yang’s modulus; \(\nu_v\) is the vertical Poisson’s ratio; and \(\nu_h\) is the transverse Poisson’s ratio.

The stress–strain constitutive equations can be described with Cartesian tensors as follow:
The elastic parameters can be derived in terms of the wave velocities and formation density, which can be derived from logging data [44–46]:

\[
C_{ij} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & C_{44} & C_{66} \\
0 & 0 & 0 & C_{44} & C_{44} & C_{66} \\
0 & 0 & 0 & C_{44} & C_{44} & C_{66}
\end{bmatrix}
\]

(9)

The elastic parameters can be derived in terms of the wave velocities and formation density, which can be derived from logging data [44–46]:

\[
\begin{align*}
C_{11} &= \rho V_p^2 + C_{33} = \rho V_p^2 + C_{33} \\
C_{12} &= \rho V_p^2 \\
C_{13} &= \rho V_p^2 \\
C_{11} &= \rho V_p^2 \\
C_{12} &= \rho V_p^2 \\
C_{13} &= \rho V_p^2
\end{align*}
\]

(10)

where \( V_{p90} \) and \( V_{p0} \) are the P wave velocity in the parallel and vertical bedding directions, respectively; \( V_{s90} \) and \( V_{s0} \) are the S wave velocity in the parallel and vertical bedding directions, respectively; \( V_{p45} \) is the P wave velocity in the 45° direction.

The elastic modulus and Poisson’s ratio can be calculated by the above stiffness matrix [47]:

\[
\begin{align*}
E &= C_{11} - \frac{2}{3} C_{33} \\
\nu &= \frac{C_{11} - C_{12}}{2 (C_{11} + C_{12})}
\end{align*}
\]

(11)

Based on laboratory testing on gas shale samples in QY1 well in the Sichuan Basin of China by TerraTek, the dynamic properties are converted to static properties with a conversion factor of 0.78 [42].

Using the same methods with Equation (5), the change of in situ stresses in depleted shale reservoir with bedding planes can be derived by taking the derivatives of Equation (6) for \( P'_p \):

\[
\begin{align*}
\delta P'_p &= \frac{d}{dP'} \left( \frac{1}{E} \right) \\
\delta P'_p &= \frac{d}{dP'} \left( \frac{1}{E} \right)
\end{align*}
\]

(12)

**Fracture Pressure in Depleted Shale Gas Reservoir**

The concentrated stress surrounding a borehole in an anisotropic shale reservoir was generated from two parts, one is the far field in situ stress before drilling in outer boundary, and the other is the bottom-hole stress in inner boundary, as shown in Figure 6(A). According to the
concept of generalized plane strain and linear elastic solid, the ultimate stress distribution surrounding a drilled borehole can be given by [48, 49]:

\[
\begin{align*}
\sigma_{ij} &= \sigma_{ij}^b + \sigma_{ij}^h = \sigma_{i,j} + 2\Re \{ \mu_j^b \Phi_j^b(z_j) + \mu_j^h \Phi_j^h(z_j) \} \\
\sigma_{ij} &= \sigma_{ij}^b + \sigma_{ij}^h = \sigma_{i,j} + 2\Re \{ \Phi_j^b(z_j) + \Phi_j^h(z_j) \} \\
\tau_{ij} &= \tau_{ij}^b + \tau_{ij}^h = 2\Re \{ \mu_j^b \Phi_j^b(z_j) + \mu_j^h \Phi_j^h(z_j) \} \\
\end{align*}
\]

(13)

where \( \sigma_{ij} \) (\( i, j = x, y, z \)) represents the ultimate stress components on the wall of borehole in the borehole coordinate system; \( \sigma_{ij}^h \) (\( i, j = x, y, z \)) represents the far field in situ stress in the borehole coordinate system; \( \sigma_{ij} \) (\( i = 1, 2, 3 \)) represents the stress components generated by the bottom-hole stress \( \sigma_{ij} \) in inner boundary in the borehole coordinate system; \( \mu_j \) (\( i = 1, 2, 3 \)) represents the characteristic roots in the characteristic equation of the stress function; \( \lambda_j \) (\( i = 1, 2, 3 \)) represents the complex numbers; \( s_{ij} \) (\( i, j = 1, 2, 3, 4, 5, 6 \)) represents the component of the flexibility matrix; \( \Phi_j(z_j) \) represents the analytic functions for independent variable \( z_j \).

The analytic function \( \Phi_j(z_j) \) surrounding a drilled borehole was given as [12]:

\[
\begin{align*}
\Phi_j^b(z_j) &= \frac{-1}{2x} IY \left( \mu_j - \lambda_j, \lambda_j, \lambda_j, \lambda_j - 1 + F(\lambda_j, \mu_j - \mu) \right) \\
\Phi_j^h(z_j) &= \frac{-1}{2x} IY \left( \lambda_j, \lambda_j, \mu_j, \mu_j - \mu \right) \\
\Phi_j^f(z_j) &= \frac{-1}{2x} IY \left( \lambda_j, \mu_j, \lambda_j, \mu_j - \mu \right)
\end{align*}
\]

(14)

where

\[
\begin{align*}
IY &= 4t_{xy} - \sigma_{xy} + P_w \\
E &= \tau_{xy} - \sigma_{xy} + iP_w \\
F &= \tau_{xy,0} - \sigma_{xy,0} + iP_w \\
\Delta_{ij} &= \left( \frac{\mu_j}{\gamma_j} \right)^{\frac{1}{2}} - 1 - \mu_j, (k = 1, 2, 3) \\
\Delta &= \mu_j - \mu_j + \lambda_j, \lambda_j (\mu_j - \mu_j), (\mu_j - \mu_j) \\
\end{align*}
\]

For a vertical well, the far field in situ stress \( \sigma_{ij,0} \) can be written as:

\[
\begin{bmatrix}
\sigma_{ij,0} \\
\tau_{ij,0} \\
\sigma_{ij,0}
\end{bmatrix} = \begin{bmatrix}
\sigma_{ij,0} & 0 & 0 \\
0 & \sigma_{ij,0} & 0 \\
0 & 0 & \sigma_{ij,0}
\end{bmatrix}
\]

(15)

At present, many wells are being drilled for oil and gas production are either horizontal, highly deviated from vertical, or have complex trajectories, the far field in situ stress in an arbitrarily deviated borehole coordinate system can be given through transforming in situ stress coordinate system to borehole coordinate system (Fig. 6B):

\[
\sigma_{ij} = [L] \begin{bmatrix}
\sigma_{ij,0} \\
\sigma_{ij,0} \\
\sigma_{ij,0}
\end{bmatrix} [L]^T
\]

(16)

\[
L = \begin{bmatrix}
\cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\
-\sin \beta \cos \alpha & \sin \alpha \sin \beta & \cos \alpha \\
\sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha
\end{bmatrix}
\]

(17)

where \( \alpha \) is the well deviation angle; \( \beta \) is the well azimuth angle relative to the maximum horizontal principal stress orientation.

In borehole cylindrical coordinate system, the stress distribution can be derived from the coordinate transformation (Fig. 6C):

\[
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_z \\
\tau_{r\theta} \\
\tau_{r\phi} \\
\tau_{\theta\phi}
\end{bmatrix} = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 0 & 0 & 0 & \sin 2\theta \\
\sin^2 \theta & \cos^2 \theta & 0 & 0 & 0 & -\sin 2\theta \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & \cos \theta & -\sin \theta & 0 & 0 \\
0 & 0 & \sin \theta & \cos \theta & 0 & 0 \\
-\sin 2\theta / 2 & \sin 2\theta / 2 & 0 & 0 & 0 & \cos 2\theta
\end{bmatrix}
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_z \\
\tau_{r\theta} \\
\tau_{r\phi} \\
\tau_{\theta\phi}
\end{bmatrix}
\]

(18)

Tensile failure will be generated when the minimum effective principal stress on the rocks surrounding the wellbore exceeds the tension strength; at this time, borehole fracture occurs, and the corresponding bottom-hole pressure is defined as the fracture pressure. Generally, the tensile failure was described as follows [17]:

\[
\sigma_z - \delta P_r = -S_i
\]

(19)

where \( \sigma_z \) is the minimum principal stress on the rocks surrounding the wellbore; \( S_i \) is the tension strength.

The minimum principal stresses \( \sigma_z \) can be calculated by solving the three roots of the characteristic equation:

\[
\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0
\]

(20)

where

\[
\begin{align*}
I_1 &= \sigma_r + \sigma_\theta + \sigma_z \\
I_2 &= \sigma_r \sigma_\theta + \sigma_\theta \sigma_z + \sigma_z \sigma_r - \tau_{r\theta}^2 - \tau_{r\phi}^2 - \tau_{\theta\phi}^2 \\
I_3 &= \tau_{r\theta} \sigma_r + \tau_{r\phi} \sigma_\theta + \tau_{\theta\phi} \sigma_z
\end{align*}
\]

(21)

The cubic characteristic equation has three solutions, which are the three principal stresses, respectively, and the
smallest one is the minimum principal stresses $\sigma_3$. The fracture can be derived by inserting $\sigma_3$ into Equation (19) by an iteration method. The flow chart of calculation was shown in Figure 7.

**Analysis and Discussion**

The above studies suggested that the fracture pressure in depleted shale gas reservoir depends not only on the depletion degree, but also other factors, such as the anisotropic elastic property of the shale, in situ stress and borehole inclination. In order to reveal the mechanism and the law of the fracture pressure change with reservoir depletion, the in situ stress and stress distribution surrounding borehole were analyzed in depth later by case calculation, based on this, the fracture pressure in depleted reservoir and its influence factors were analyzed and discussed. The fundamental parameters were derived from the above test results and on site data, in which the elastic parameters such as $E_h$, $E_v$, $\nu_h$, $\nu_v$ were derived from the experimental results of the anisotropic elastic deformation test, and the tension strength $S_t$ was derived from the tension strength test. In addition, the stress parameters such as $\sigma_{\text{res}}$, $\sigma_{\text{tension}}$, $\sigma_{\text{in situ}}$, $P_w$ were field measured data before reservoir depletion, the drilling parameters such as $P_w$, $r_w$ were from field drilling operations. The values of the parameters were shown in Table 1.

**In situ stress in the depleted shale gas reservoir**

Based on the previous analysis, the vertical in situ stress keeps constant when reservoir depletion, and the variation...
between the maximum and minimum horizontal in situ stress is the same; therefore, the change of the minimum horizontal in situ stress with the depletion of shale gas reservoir was selective analyzed. At different anisotropies of the Yang’s modulus and Poisson’s ratio, the change laws were calculated as shown in Figures 8 and 9, respectively. The results indicate that, for the isotropic reservoir, the minimum horizontal in situ stress was decreased with reservoir depletion, but when the anisotropy was increased, the decreasing amount was smaller, and when the anisotropy of the Yang’s modulus is beyond 4, the minimum horizontal in situ stress was increased with reservoir depletion, which is not fully realized in the previous studies. For the same anisotropy between the Yang’s modulus and Poisson’s ratio, the variation amount is

Table 1. Parameters used in the analysis.

| Parameter                                           | Value   |
|-----------------------------------------------------|---------|
| Initial maximum horizontal in situ stress, $\sigma_H$ | 57 MPa  |
| Initial minimum horizontal in situ stress, $\sigma_h$ | 45 MPa  |
| Initial vertical in situ stress, $\sigma_V$         | 63 MPa  |
| Initial reservoir pore pressure, $P_p$               | 30 MPa  |
| The bottom-hole stress, $P_w$                        | 39 MPa  |
| Borehole radius, $r_W$                               | 0.1 m   |
| Borehole deviation, $\alpha$                         | 30°     |
| Borehole azimuth, $\beta$                            | 0°      |
| The vertical Yang’s modulus, $E_v$                   | 34.48 GPa|
| The transverse Yang’s modulus, $E_h$                 | 53.60 GPa|
| The transverse Poisson’s ratio, $\nu_h$              | 0.15    |
| The vertical Poisson’s ratio, $\nu_v$                | 0.23    |
| The tension strength, $S_t$                          | 1.52 MPa|
| The Boit’s coefficient, $\delta$                    | 0.85    |

Figure 8. The change of minimum horizontal in situ stress with the depletion of shale gas reservoir at different anisotropies of the Yang’s modulus.

Figure 9. The change of minimum horizontal in situ stress with the depletion of shale gas reservoir at different anisotropies of the Poisson’s ratio.
essentially the same, but due to the anisotropy of the Poisson’s ratio is difficult to more than 4, the minimum horizontal in situ stress is almost always deceased with reservoir depletion at different anisotropies of the Poisson’s ratio.

**Stress distribution surrounding the borehole in the depleted shale gas reservoir**

The stress distribution surrounding the borehole at different anisotropies in the borehole cylindrical coordinate system was shown in Figure 10. The results present that the anisotropy has the least impact on the radial normal stress (when the anisotropy is increased from 1 to 3, the radial normal stress is all almost during 56 and 40 MPa), and has the greatest impact on the tangential normal stress. When the anisotropy is increased, the tangential normal stress was decreased (When the anisotropy is increased from 1 to 3, the largest value of the tangential normal stress is decreased from 90 to 85 MPa, and the least value of the tangential normal stress is decreased from 40 to 35 MPa). For the radial normal stress surrounding the borehole, the least value is on the borehole wall, but for the tangential normal stress, the largest and the least value is both on the borehole wall, and the position is perpendicular to each other. Furthermore, the shear stress always exists on the borehole wall, which implies the normal stresses on the borehole wall were not the principle stresses.

The change of the minimum effective principal stress surrounding the borehole with reservoir depletion at different anisotropies was shown in Figure 11. The results showed that, the least value of the minimum effective principal stress surrounding the borehole is always on the borehole wall, and for the isotropic reservoir, the least value of the minimum effective principal stress was decreased from 12 to −5 MPa when reservoir depletion was increased from 0 to 30 MPa, and the tension stress was induced, when it is beyond the tension strength, fracture failure will generate on the borehole. However, for the shale gas reservoir with the high anisotropy (for example, the anisotropy is 3), the least value of the minimum effective principal stress was decreased from 10 to 20 MPa when reservoir depletion was increased from 0 to 30 MPa, and the compressional stress was induced, the reservoir was more difficult to be fractured. Furthermore, by the lateral comparison, when the reservoir depletion is the same, the least value of the minimum effective principal stress was bigger if the anisotropy is higher, which implies the shale gas reservoir with the higher anisotropy was more difficult to be fractured.

**Fracture pressure in the depleted shale gas reservoir**

The anisotropy was quantitative evaluated by the coefficient “K_a”, which is defined by “K_a = E_h/E_v = v_h/v_v”.

At the different well inclinations and anisotropies, the changes of fracture pressure with reservoir depletion were shown in Figures 12–15. When the well was drilled as a vertical hole, that is to say the well inclination is 0° (Fig. 12), the fracture pressure is almost the same between the various anisotropic reservoirs before reservoir depleted, since the borehole axis is almost perpendicular to the plane of isotropy and the anisotropic stress solutions are similar to the isotropic ones. With reservoir depletion, the fracture pressure was linearly decreased for the reservoir with the lower anisotropy (K_a < 2.0), but when the anisotropy is beyond 2.0, the fracture pressure will be linearly increased with reservoir depletion. At the later period of the reservoir depletion, the reservoir with the higher anisotropy was more difficult to be fractured. When the well was drilled as a small deviated hole (the well inclination is 30°, as shown in Fig. 13), the fracture pressure has a little difference between the various anisotropic reservoir before reservoir depleted, and the fracture pressure is the largest for the isotropic one. With reservoir depletion, the same change law of the fracture pressure was presented with the vertical well.

When the well was drilled as a high deviated hole (the well inclination is 60°, as shown in Fig. 14), the fracture pressure has a larger difference between the various anisotropic reservoirs before reservoir depleted, and the fracture pressure is the least for the highest anisotropic one. With reservoir depletion, the change law of the fracture pressure has some differences with the vertical and small deviated well: (1) when the anisotropy is 2.0, the fracture pressure almost keeps constant with reservoir depletion; (2) when the anisotropy is increased to 2.5, the fracture pressure is not linearly increased, but significantly faster in earlier period. When the well was drilled as a horizontal hole (the well inclination is 90°, as shown in Fig. 15), the fracture pressure is quickly decreased to zero for the isotropic one, which implies that it is not possible to drill the well without experiencing fracturing when the reservoir depletion is beyond a certain degree, but this will make the hydraulic fracturing to be easier.

Furthermore, by comparing the results in Figures 12–15, some laws can be found, that the fracture pressure in the well with a higher inclination is lower, when the anisotropy and the reservoir depletion are both the same. And for the isotropic reservoir (K_a = 1.0), the fracture pressure decreased due to reservoir depletion for arbitrary borehole, but when the anisotropy K_a is beyond 2.0, the
Figure 10. The stress distribution surrounding the borehole at different anisotropies in the borehole cylindrical coordinate system. The first to the third lines represent the radial normal stress $\sigma_r$, the tangential normal stress $\sigma_\theta$, and the axial normal stress $\sigma_z$, respectively. The fourth to the sixth lines represent the shear stresses $\tau_{rz}$, $\tau_{r\theta}$, and $\tau_{\theta z}$ in different planes, respectively.
Figure 11. The change of the minimum effective principal stress surrounding the borehole with reservoir depletion at different anisotropies.
fracture pressure increased due to reservoir depletion for arbitrary borehole.

**Conclusions**

The clay mineral is obviously arranged along the fixed orientation within the shale gas reservoir, the macroscopic bedding planes and microcosmic cracks were developed, and the mechanical property shows a strong anisotropy, which can be treated as a transversely isotropic body. Hence, the fracture pressure in the depleted shale gas reservoir has a strong correlation with the anisotropy and the well inclination. The paper established a novel evaluation of the fracture pressure in the depleted shale gas reservoir, which can be used to analyze the influence on fracture pressure of the reservoir depletion, the anisotropy and the well inclination. The results showed that for the strongly anisotropic reservoir, the in situ stress and the fracture pressure will be linearly increased with reservoir depletion, which is opposite to the isotropic one. But when the well was drilled as a high deviated hole in the strongly anisotropic reservoir, the fracture pressure may be not linearly increased, significantly faster in earlier period. For the vertical well, the fracture pressure is almost the same between the various anisotropic reservoirs before reservoir depleted, but after reservoir depletion generated, the fracture pressure will show a stronger difference, and the fracture pressure is higher for the anisotropic one.
For the deviated well, the fracture pressure have a larger difference between the various anisotropic reservoirs before reservoir depleted, and when the anisotropy and the depletion are both the same, the fracture pressure in the well with a higher inclination is lower. The method and the results can provide a theoretical basis both for the drilling and hydraulic fracturing in depleted shale gas reservoir.

Acknowledgments

The authors gratefully acknowledge the support of National Natural Science Foundation of China (Grant No. 51604225), PetroChina Innovation Foundation (Grant No. 2017D-5007-0104), and the National Science and Technology Major Special Project (Grant No. 2017ZX05069004).

Conflict of Interest

None declared.

References

1. Yan, C. L., Y. F. Cheng, F. C. Deng, and J. Tian. 2017. Permeability change caused by stress damage of gas shale. Energies 10:1350.
2. Shen, W., X. Li, Y. Xu, Y. Sun, and W. Huang. 2017. Gas flow behavior of nanoscale pores in shale gas reservoirs. Energies 10:751.
3. Zou, C. N., D. Z. Dong, S. J. Wang, J. Z. Li, X. J. Li, Y. M. Wang et al. 2010. Geological characteristics and resource potential of shale gas in China. Pet. Explor. Dev. 37:641–653.
4. Robert, W. H., S. Renee, and I. Anthony. 2011. Methane and the greenhouse-gas footprint of natural gas from shale formations. Clim. Change. 106:679–690.
5. Wang, J., H. Q. Liu, L. Wang, H. L. Zhang, H. S. Luo, and Y. Gao. 2015. Apparent permeability for gas transport in nanopores of organic shale reservoirs including multiple effects. Int. J. Coal Geol. 152:50–62.
6. Denney, D. 2009. Evaluating implications of hydraulic fracturing in shale-gas reservoirs. J. Petrol. Technol. 61:53–54.
7. Mayerhofer, M. J., E. Lolon, N. R. Warpinski, C. L. Cipolla, D. W. Walser, and C. M. Rightmire. 2010. What is stimulated reservoir volume? SPE Prod. Oper. 25:89–98.
8. Sheng, G. L., Y. L. Su, W. D. Wang, F. Javadpour, and M. R. Tang. 2017. Application of fractal geometry in evaluation of effective stimulated reservoir volume in shale gas reservoirs. Fractals 25:1–13.
9. Guo, T. K., S. C. Zhang, Z. Q. Qu, T. Zhou, Y. S. Xiao, and J. Gao. 2014. Experimental study of hydraulic fracturing for shale by stimulated reservoir volume. Fuel 128:373–380.
10. Zhao, J. Z., L. Ren, Y. Q. Hu, and L. Wang. 2012. A calculation model of breakdown pressure for perforated wells in fractured formations. Acta Petrol. Sin. 33:841–845.
11. Ren, L., J. Z. Zhao, Y. Q. Hu, and Y. J. Ran. 2009. Numerical calculation of rock breakdown pressure during hydraulic fracturing process. Chin. J. Rock Mech. Eng. 28:3417–3422.
12. Lu, Y. H., M. Chen, J. B. Yuan, Y. Jin, and X. Q. Teng. 2013. Borehole instability mechanism of a deviated well in anisotropic formations. Acta Petrol. Sin. 34:563–568.
13. Matthews, W. R., and J. Kelly. 1967. How to predict formation pressure and fracture gradient from electric and sonic logs. Oil Gas J. 65:92–106.
14. Eaton, B. A. 1969. Fracture gradient prediction and its application in oilfield operations. J. Petrol. Technol. 21:1353–1360.
15. Anderson, R. A., D. S. Ingram, and A. M. Zanier. 1973. Determining fracture pressure gradients from well logs. J. Petrol. Technol. 25:1259–1268.
16. Stephen, R. D. 1982. Prediction of fracture pressures for wildcat wells. J. Petrol. Technol. 34:863–872.
17. Chen, M., Y. Jin, and G. Q. Zhang. 2008. Petroleum engineering related rock mechanics. Science Press, Beijing.
18. Addis, M. A. 1997. Reservoir depletion and its effect on wellbore stability evaluation. Int. J. Rock Mech. Min. Sci. 34:1–17.
19. Davison, J. M., R. Leaper, M. B. Cauley, B. Bennett, A. Mackenzie, C. J. Higgin et al. 2004. Extending the drilling operating window in brent: solutions for infill drilling in depleted reservoirs. Proceedings of the IADC/SPE Drilling Conference, Dallas, TX, USA, 2–4 March 2004.
20. Zhang, R., X. Y. Shi, R. F. Zhu, C. Zhang, M. Z. Fang, K. H. Bo et al. 2016. Critical drawdown pressure of sanding onset for offshore depleted and water cut gas reservoirs: modeling and application. J. Nat. Gas Sci. Eng. 34:159–169.
21. Addis, M. A., N. C. Last, and N. A. Yassir. 1996. Estimation of horizontal stresses at depth in faulted regions and their relationship to pore pressure variations. SPE Formation Eval. 11:11–18.
22. Kang, X. Q., and Y. J. Xue. 2009. Applied rock mechanics in drilling of depleted reservoirs in deepwater Gulf of Mexico. Petrophysics 50:231–236.
23. Liu, X. W., Z. Q. Guo, C. Liu, and Y. W. Liu. 2017. Anisotropy rock physics model for the longmaxi shale gas reservoir, Sichuan Basin, China. Appl. Geophys. 14:21–30.
24. Guo, Z. Q., X. Y. Li, and C. Liu. 2014. Anisotropy parameters estimate and rock physics analysis for the Barnett Shale. J. Geophys. Eng. 11:1–10.
25. Cho, J. W., H. Kim, S. Jeon, and K. B. Min. 2012. Deformation and strength anisotropy of Asan gneiss, Boryeong shale, and Yeoncheon schist. Int. J. Rock Mech. Min. Sci. 50:158–169.
26. Heng, S. A., C. H. Yang, B. P. Zhang, Y. T. Guo, L. Wang, and Y. L. Wei. 2015. Experimental research on anisotropic properties of shale. Rock Soil Mech. 36:614–616.
27. Niandou, H., J. F. Shao, J. P. Henry, and D. Fourmaintraux. 1997. Laboratory investigation of the mechanical behaviour of Tournemire shale. Int. J. Rock Mech. Min. Sci. 34:3–16.
28. Rickie, H. H., and G. V. Chilingarian. 1974. Compaction of argillaceous sediments. Elsevier Scientific Publishing Company, Amsterdam.
29. McElhan, P. J., and K. Cormier. 1996. Borehole instability in fissile, dipping shales, Northeastern British Columbia. Proceedings of the SPE Gas Technology Symposium, Calgary, Alberta, Canada, 28 April–1 May 1996.
30. Cao, W. K., J. G. Deng, B. H. Yu, W. Liu, and Y. Li. 2016. Effect of anisotropy of elastic parameters of shale formation. J. Xi’an Shiyou Univ. 31:27–35.
31. Li, Y. 2006. An empirical method for estimation of anisotropic parameters in clastic rocks. Lead. Edge 25:706–711.

32. Vernik, L., and A. Nur. 1992. Ultrasonic velocity and anisotropy of hydrocarbon source rocks. Geophysics 57:727–735.

33. Tan, Q., J. G. Deng, and B. H. Yu. 2010. Wellbore instability and countermeasures in offshore bedding shale formations. Pet. Sci. Technol. 28:1712–1718.

34. Aadnoy, B. S. 1990. Inversion technique to determine the in situ stress field from fracturing data. J. Pet. Sci. Eng. 4:127–141.

35. Brudy, M., and M. D. Zoback. 1999. Drilling-induced tensile wall-fractures: implications for the determination of in situ stress orientation and magnitude. Int. J. Rock Mech. Min. Sci. 36:191–215.

36. Lund, B., and M. D. Zoback. 1999. Orientation and magnitude of in situ stress to 6.5 km depth in the Baltic Shield. Int. J. Rock Mech. Min. Sci. 36:169–190.

37. Zoback, M. D., D. Moos, L. Mastin, and R. N. Anderson. 1985. Well bore breakouts and in situ stress. J. Geophys. Res. 90:5523–5530.

38. Jin, Y., J. B. Yuan, M. Chen, K. P. Chen, Y. H. Lu, and H. Y. Wang. 2011. Determination of rock fracture toughness KIIC and its relationship with tensile strength. Rock Mech. Rock Eng. 44:621–627.

39. Yan, C. L., J. G. Deng, Y. F. Cheng, X. J. Yan, J. L. Yuan, and F. C. Deng. 2017. Rock mechanics and wellbore stability in Dongfang 1-1 Gas Field in South China Sea. Geomech. Eng. 12:465–481.

40. Lin, Y. S., H. K. Ge, and S. C. Wang. 1998. Testing study on dynamic and static elastic parameters of rocks. Chin. J. Rock Mech. Eng. 17:216–222.

41. Thiercelin, M. J., and R. A. Plumb. 1994. A core-based prediction of lithologic stress contrasts in East Texas formations. SPE J. 9:251–258.

42. Yang, L., H. K. Ge, Y. H. Shen, J. J. Zhang, W. Yan, S. Wu et al. 2015. Imbibition inducing tensile fractures and its influence on in-situ stress analyses: a case study of shale gas drilling. J. Nat. Gas Sci. Eng. 26:927–939.

43. Song, L. T., Z. H. Liu, C. L. Li, and S. Hu. 2015. Geostress logging evaluation method of tight sandstone based on transversely isotropic model. Acta Petrol. Sin. 36:707–714.

44. Deng, J. G., Z. R. Chen, Y. N. Geng, S. J. Liu, and H. Y. Zhu. 2013. Prediction model for in-situ formation stress in shale reservoirs. J. China Univ. Petrol. 37:59–64.

45. Norris, A., and B. Sinha. 1993. Weak elastic anisotropy and the tube wave. Geophysics 58:1091–1098.

46. Walsh, J., B. Sinha, and A. Donald. 2006. Formation anisotropy parameters using borehole sonic data. Proceedings of the SPWLA 47th Annual Logging Symposium, Veracruz, Mexico, 4–7 June 2006.

47. Kozlowski, K., A. Donald, P. Shotton, J. Jocker, M. Fidan, H. Nielsen et al. 2011. Overburden characterization for geomechanics and geophysical applications in the Eldfisk Field: a North Sea case study. Proceedings of the SPWLA 52nd Annual Logging Symposium, Colorado Springs, Co, USA, 14–18 May 2011.

48. Ong, S. H. 1994. Borehole stability. PhD diss. University of Oklahoma, Norman, OK.

49. Jin, Y., J. B. Yuan, B. Hou, M. Chen, Y. H. Lu, S. Li et al. 2012. Analysis of the vertical borehole stability in anisotropic rock formations. J. Petrol. Explor. Prod. Technol. 2:197–207.