Resolving the Antibaryon-Production Puzzle in High-Energy Heavy-Ion Collisions

Ralf Rapp and Edward V. Shuryak

Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800, U.S.A.
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We argue that the observed antiproton production in heavy-ion collisions at CERN-SPS energies can be understood if (contrary to most sequential scattering approaches) the backward direction in the process $p\bar{p} \leftrightarrow n\pi$ (with $\bar{n}=5-6$) is consistently accounted for within a thermal framework. Employing the standard picture of subsequent chemical and thermal freezeout, which induces an over-saturation of pion number with associated chemical potentials of $\mu_\pi \simeq 60-80$ MeV, enhances the backward reaction substantially. The resulting rates and corresponding cross sections turn out to be large enough to maintain the abundance of antiprotons at chemical freezeout until the decoupling temperature, in accord with the measured $\bar{p}/p$ ratio in Pb(158 GeV)+Pb collisions.

Over the last decade remarkable progress has been made in the understanding of the dynamics of strong interactions probed through (ultra-) relativistic heavy-ion collisions. Although the main challenge of an unambiguous identification of the QCD phase transition to the Quark-Gluon Plasma (QGP) persists, we have greatly advanced our knowledge on properties of highly excited hadronic matter close to the expected phase boundary. A variety of collective phenomena has been observed indicating that the produced systems have indeed reached macroscopically large sizes, justifying the use of equilibrium techniques such as thermo- and hydrodynamics.

One of the important results that will be used below is that the final-state hadron abundances, including antibaryons, can be rather accurately characterized by the so-called chemical freezeout stage [1] with a common temperature $T_{ch}$ and baryon chemical potential $\mu_B$, the specific values depending on collision energy (note that a precise description of all hadron species requires corrections to an ideal gas ensemble, e.g., excluded (eigen-) volumes to mimic short-range repulsions, or ‘strangeness suppression’ factors [2]). Such corrections are not important for our subsequent analysis and will be neglected. For a contrasting view of hadron production in heavy-ion collisions, see, e.g., ref. [3].

At SpS energies, the chemical freezeout is clearly distinct from the thermal one (with an associated temperature $T_{th} < T_{ch}$), from where on the particles stream freely to the detectors. This follows from the kinetics of hadronic reactions [3], i.e., a significant difference between elastic and inelastic collision rates at low relative energies, and has been confirmed by several experimental evidences (see, e.g., ref. [3]). In central Pb+Pb collisions, e.g., a nucleon at midrapidity is elastically rescattered on average about 10-15 times, but less than once inelastically [3]. Also, as shown in [3], most of the collective flow effects at SpS are generated in between the two freezeouts, and their observation leaves no doubt about the existence of such an intermediate stage.

Another consequence is that abundances of secondary mesons (pions, kaons, etc.) are not subject to significant changes when the system evolves from $T_{ch}$ to $T_{th}$. Again, this is conceivable as the rates for number changing reactions are too small to maintain chemical equilibrium on the time scales of the hadronic fireball lifetime, $t \simeq 10$ fm/c. Since, on the other hand, elastic scattering (e.g., $\pi\pi \rightarrow \rho \rightarrow \pi\pi$) is still effective, thermal equilibrium is approximately maintained. In a statistical mechanics language such a scenario entails additional conservation laws, which can be implemented via effective (pion-, kaon-, etc.) chemical potentials to guarantee fixed particle numbers. For pions (kaons) typical values of $\mu_{\pi} = 60-80$ MeV ($\mu_K = 100-130$ MeV) are reached at $T_{th} = 110-120$ MeV, see [4,5]. The ensuing ‘over-saturation’ of the pion phase space is to play the key role in what follows.

The situation for antibaryons is different from mesonic secondaries as the pertinent inelastic (or annihilation) cross section is not small. At the relevant (thermal) energies in the comoving frame of collective expansion, $\sqrt{s} = 2(m_N + E_{ch}^0) \simeq 2.3$ GeV, one has $\sigma_{\bar{p}p \rightarrow n\pi} \simeq 50$ mb. Taking an average baryon density of $\rho_B = 0.75\rho_0$ ($\rho_0 = 0.16$ fm$^{-3}$) around $T = 150$ MeV in the course of the hadronic evolution and a typical (anti-) proton velocity of $v_{th} = p/E_{tot} = 0.56c$ (from $E_{ch}^0 = (3/2)kT \simeq 225$ MeV), we obtain for the chemical equilibration time scale

$$\tau_{ch} = \frac{1}{\sigma_{ann} \rho_B v_{th}} \simeq 3 \text{ fm/c}.$$ \hfill (1)

This is well below the fireball lifetime in the pure hadronic phase of $\tau_{had} \simeq 7$ fm/c [6], cf. Fig. 1; in other words, only a fraction of $\exp[-\tau_{had}/\tau_{ch}] \simeq 10\%$ does not rescatter towards thermal freezeout. Thus, in spite of the good agreement of the final $\bar{p}$ yields with the standard chemical freezeout predictions, antibaryon production ought to be reconsidered.

Naively one might expect most of the antiprotons to be annihilated, and various transport calculations (e.g., ARC [7] and UrQMD [8]) have indeed been unable to account for the measured number, falling short by significant factors. Consequently, speculations have been raised that the puzzle could be resolved by certain in-medium effects leading to either a reduction of the annihilation rate [8] or an enhanced production [4].
A generic problem with transport/cascade simulations is that, although multi-body resonance decays are included, the inverse reactions are not. As is well-known, this violates detailed balance and, in principle, prevents the simulations from reaching the proper thermodynamic limit [11]. The discussions are typically focused on three-body reactions; in the present context it is even 5- or 6-pion collisions which are most relevant for producing baryon-antibaryon pairs. A widespread belief is that those reactions have insignificant rates under the conditions probed in the hadronic stages of heavy-ion collisions. However, as we will argue below, this is not the case; inverse reactions have to be addressed and are, in fact, capable of explaining the observed antiproton yield (although the \( \bar{p} \) enhancement in nucleus-nucleus over \( pp \) collisions thus looses its proposed direct relation to QGP formation, the latter is by no means excluded by the subsequent arguments).

The expression for the thermal reaction rate for the process \( \bar{p} p \leftrightarrow n \pi \) can be written as (see also Ref. [12])

\[
R_{th} = \int d^3 \vec{k}_p \, d^3 \vec{k}_\bar{p} \, d^3 \vec{k}_\pi_1 \cdots d^3 \vec{k}_\pi_n \, \delta^{(4)}(K_{tot}) \, |M_n|^2 \times \left\{ z_p \, z_{\bar{p}} \, \exp\left[-\frac{E_p + E_{\bar{p}}}{T}\right] - z^n_{\pi} \, \exp\left[-\sum_{i=1}^n \frac{\omega_{\pi,i}^2}{T}\right] \right\},
\]

where \( M_n \) denotes the invariant scattering matrix element (which, of course, is identical for the back- and forward direction due to time-reversal invariance of strong interactions), \( d^3 \vec{k}_x = d^3 \vec{k}_x/(2\pi)^3 \) the phase space integrations and \( K_{tot} = k_p + k_{\bar{p}} - k_{\pi_1} - \cdots - k_{\pi_n} \) the total four-momentum; \( z_x = e^{\mu_x/T} \) are the fugacities of particle species \( x \) (in Boltzmann approximation), and \( n \) is the number of pions produced in a \( \bar{p} p \) annihilation at a given energy (to be discussed in more detail below).

For a nonvanishing net nucleon number and in chemical equilibrium, one has \( \mu_N > 0 \), \( \mu_S = -\mu_N \) and \( \mu_\pi = 0 \). Under SpS conditions, typical values at chemical freeze-out are \( (T_{ch}, \mu_N^ch) \approx (170, 260) \) MeV, which results in an antiproton-to-proton ratio of

\[
\frac{\bar{p}}{p} \propto \exp\left[-\frac{(E_p + \mu_N)/T}{\exp[-(E_p - \mu_N)/T]}\right] = \exp[-2\mu_N/T] = 4.7\%,
\]

consistent with the experimentally measured value of \((5.5 \pm 1)\% \) [13][14]. Note also that at chemical freezeout \( z_p z_{\bar{p}} = 1 \), \( z_{\pi} = 1 \), and the forward and backward rates in eq. (2) are simply equal (the Boltzmann factors in both terms contain the same total energy). This implies, e.g., that cascade simulations ignoring the back-reaction at comparable pion densities \( \langle z_{\pi} \rangle \approx 0.2 - 0.25 \text{ fm}^{-3} \) cannot give a proper account of the antiproton production.

The evolution of the system towards thermal freezeout can be constructed using a thermal fireball model which, based on isentropic expansion, leads to \( (T_{th}, \mu_N^{ch}) \approx (120, 415) \) MeV. At first sight, this gives a \( \bar{p}/p \) ratio of \( \exp[-2\mu_N/T] \approx 0.1 \% \), a factor of \( \sim 50 \) below the experimental result. However, this estimate is lacking an important ingredient. A correct statistical treatment including pion-number conservation forces us to introduce individual chemical potentials/fugacities for anti-protons and pions. Insisting on equilibrium for the reaction in question, the following relation holds:

\[
\langle z_{\pi} \rangle = \frac{z_p}{z_{\bar{p}}} = \langle z_p \rangle^{-1} \langle z_{\pi}^n \rangle.
\]

Since antibaryons constitute only a small fraction of the produced secondaries, they have an insignificant impact on the evolution of nucleon- and pion-chemical potentials. Therefore, eq. (3) provides an estimate of the antiproton fugacity at thermal freezeout. The crucial point here is that pion over-saturation will generate a strong enhancement of the back-reaction \( n\pi \rightarrow \bar{p}p \).

For a more quantitative assessment it is important to have a rather accurate determination of the pion multiplicity distributions in \( pp \) annihilation. A nice discussion of its systematics has been given in ref. [15], which will follow here to incorporate the empirical knowledge. For \( pp \) annihilation at rest \((\sqrt{s} = 2m_N)\), the data can be represented by a least-square fit to a Gaussian probability distribution (normalized to one),

\[
P(n) = \frac{1}{\sqrt{2\pi} \sigma} \exp[-(n - \langle n \rangle)^2/2\sigma^2],
\]

with a mean \( \langle n \rangle = 5.02 \) and width \( \sigma = 0.90 \) (see also ref. [16]). At larger c.m.-energies both the average number and width increase. For the former, linear [17] and logarithmic [18] dependencies have been proposed,

\[
\langle n \rangle = c_1 + c_2 \, s^{0.5},
\]

\[
\langle n \rangle = \tilde{c}_1 + \tilde{c}_2 \, \ln(s)
\]
elastic potential close to thermal freezeout can indeed be easily
found. The latest data point is from Ref. [13].

For our application in a thermal environment at $T = 150$ MeV (implying $\sqrt{s} = 2.33$ GeV) we fix $\langle n \rangle = 5.65$ (in accordance with eq. (6)) together with a 10% increase in $\sigma$ (as suggested by eq. (7)) to extract discrete weights $w_n = P(n; \langle n \rangle, \sigma)$. The averaged pion-fugacity enhancement factor then follows as

$$\langle z^n \rangle = \sum_{n=2}^{n_{\text{max}}} w_n \exp[n\mu_\pi/T],$$

where $n_{\text{max}} = 9$ for any practical purpose. Inserting now thermal freezeout values $T_{th} = 120$ MeV and $\mu_\pi^{th} \simeq 65$ MeV (as arising in a thermal fireball model [8]), yields $\langle z^n \rangle = 25$. This entails a large enhancement of the antiproton-to-proton ratio, from 0.1% to 2.5%. In fact, owing to the high power of the pion fugacity, slightly larger chemical potentials of $\mu_\pi = 75-80$ MeV result in an enhancement factor of 42-54, rendering the pertinent $\bar{p}/p$-ratio in line with the observed (chemical freezeout) value, cf. Fig. 2.

Let us finally comment on implications of our observations for RHIC. Close to the expected chemical freezeout the pion density is very similar to SpS conditions. Thus the rate of producing antiprotons through multipion annihilation per unit time and volume is essentially the same in both cases. The crucial difference is, however, that the total density of antiprotons is much larger around midrapidity at RHIC due to substantially smaller baryon chemical potentials. More quantitatively, using typical thermal model estimates [20] with $\sigma_{\pi N}^{FB} \simeq 0.2 g_0$ shortly after chemical freezeout (further reduced thereafter), one obtains $\tau_{\chi}^{RHIC} \simeq 11$ fm/c. With the lifetime of the hadronic phase at RHIC being comparable to that at SpS energies, chemical equilibrium in the $\bar{p}p \leftrightarrow n\pi$ reaction cannot be maintained until thermal freezeout (also, the emerging pion oversaturation is less pronounced in a baryon-poor regime). The observed antiprotons at RHIC should therefore mostly originate from earlier stages, corresponding to the standard hadrochemical freezeout in the vicinity of the phase boundary. Nevertheless, our time scale estimate indicates that even under RHIC conditions, antibaryon annihilation will be partially compensated by the inverse reactions.

To summarize, we have analyzed the $\bar{p}/p$-ratio at SpS energies employing a thermal approach. So far this observable has been difficult to understand within, e.g., transport models which only included the annihilation channel, causing doubts whether the latter is actually active, or unconventional mechanisms for enhanced production need to be invoked. We have shown, however, that the ‘puzzle’ can be resolved in a rather standard statistical-mechanics framework upon inclusion of the inverse process of multipion scattering into $\bar{p}p$ pairs, which can be supported until thermal freezeout. Our main ingredient was that effective pion-number conservation generates pion over-saturation at the later stages of a heavy-ion collision, as described by the build-up of appreciable pion chemical potentials. Raised to a large power ($n \sim 6$) the corresponding pion fugacities sustain a high antibaryon fraction, thus counter-balancing the loss from $BB$ annihilation. This mechanism also complies with the measured centrality dependence being essentially constant, as to be expected from a hadro-chemistry varying little with impact parameter (for sufficiently peripheral collisions the applicability of thermal model analyses, of course, ceases and $\bar{p}$ production, normalized to the number of participant nucleons, approaches its value in $p-p$ collisions, which lies about 30% below the one in central nucleus-nucleus reactions [13]).

Finally we should note again that our findings are not

![FIG. 2. Antiproton-to-proton ratio as a function of (decreasing) temperature in an isentropically expanding fireball. The dashed curve represents the naive ratio, $\exp[-2\mu_\pi/T]$, whereas the full curves are for finite pion chemical potentials indicating uncertainties as discussed in the text. The experimental data point is from Ref. [13].](image)
contradictory to an earlier QGP formation. They rather
give further support to the equilibrium concept of subse-
quent chemical and thermal freezeout stages, which has
already proven successful in the explanation of a vari-
ety of observables, such as hadron abundances, collective
flow, HBT radii and electromagnetic radiation.

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