Inter-Subset Hamming Distance Maximization for Enhancing the Physical Layer Security of Antenna Subset Modulation

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\textbf{ABSTRACT} In this article, a novel antenna subset selection technique for enhancing the Physical Layer Randomness (PLR) of Antenna Subset Modulation (ASM) has been proposed. Hamming Distance Optimized Antenna Subset Selection (HD-OASS) minimizes the correlation between the antenna subsets at transmitter by maximizing the hamming distance between the successively used antenna subsets. Unlike previously proposed Randomized Antenna Subset Selection (RASS) and Side-Lobe Level Optimized Antenna Subset Selection (SLL-OASS), in which antenna subsets are randomly selected from the codebook, HD-OASS chooses the antenna subsets having optimally maximized hamming distance. It is shown that SLL-OASS has rather unfavorable effects on encryption strength due to considerable reduction of codebook size. Furthermore, HD-OASS has been shown to outperform RASS and SLL-OASS in terms of encryption strength in the unwanted directions of eavesdropper.

\textbf{INDEX TERMS} Antenna subset modulation, directional modulation techniques, eavesdropper, intended receiver, physical layer randomness, physical layer security.

\section{I. INTRODUCTION}

Over the past decade, Directional Modulation (DM) has emerged as strong candidate for providing Physical Layer Security (PLS) against eavesdropping. Several DM techniques which use single or multiple antenna elements include; switched phased-array [1], near-field direct antenna modulation [2], phased-array based DM [3]–[5], 4-D antenna array [6], dual-beam DM [7], [8], and frequency diverse array [9]–[12]. Unlike traditional cryptographic approach for data security [13] in which data is encrypted even for Intended Receiver (IR), all the DM techniques transmit non-encrypted data (plaintext) along the direction of IR and encrypted data (ciphertext) along the unwanted directions. Depending upon the design of transmitter architecture, DM techniques are broadly classified into two classes [14]; radiator-reconfigurable and excitation-reconfigurable techniques.

Antenna Subset Modulation (ASM) is one of the excitation-reconfigurable DM techniques that performs modulation at antenna level [15]. Instead of using the complete antenna array for data transmission, it uses randomly chosen subset of antenna array (through the process of array thinning) that is modulated at symbol rate. Previously, two antenna subset selection techniques have been proposed for ASM; Randomized Antenna Subset Selection (RASS) and Optimized Antenna Subset Selection (OASS). RASS exhibit high average Side-Lobe Level (SLL) due to random selection of antenna subsets. On the other hand, OASS reduces average SLL of antenna subsets using Simulated Annealing (SA) algorithm [16].

Low-Complexity Antenna Subset Modulation (LC-ASM) improves the transmitter architecture of ASM by performing modulation both at baseband and antenna level [17]. Unlike ASM, LC-ASM generates the desired phase and amplitude of digital symbol at baseband. It makes ASM compatible with amplitude modulation schemes like Quadrature Amplitude Modulation (QAM), along with phase modulation schemes.
for which ASM was originally proposed. Interference mitigation techniques for multi-directional ASM are proposed in [18].

In the domain of PLS, Symbol Error Rate (SER) is very commonly used as a measure of wireless communication security. High and relatively stable value of SER in the unwanted directions is considered analogous to good randomization of data constellations and hence good PLS. Therefore, all the proposed techniques for ASM [15], [17]–[20] have focused on increasing SER in the unwanted directions. However, SER is not a direct measure of encryption strength. It is the probabilistic measure of erroneously received symbols. A comparatively recent paradigm in the PLS domain proposes to quantify the wireless communication security in terms of robust and well-adopted statistical randomness tests devised by National Institute of Standards and Technology (NIST) [21]. Physical Layer Randomness (PLR) [22] is one of such parameters which proposes a new model for analyzing the encryption strength of DM techniques in terms of randomness introduced along the undesired directions i.e. towards eavesdropper (Eve). It maps the concepts of PLS techniques to symmetric-key block encryption ciphers, like state-of-the-art Advanced Encryption Standard (AES) [23], to benchmark the encryption strength of DM techniques to strong block ciphers [24]. In this article, this recent paradigm has been adopted.

The contributions of our paper are summarized below:

1. The inter-subset Hamming Distance Optimized Antenna Subset Selection (HD-OASS) has been proposed as a novel antenna subset selection technique for enhancing PLR (encryption strength at physical layer) of ASM. Maximization of inter-subset hamming distance translates to minimum overlap of antenna positions among the successively used antenna subsets, which results in reduced correlation between successively transmitted symbols.

2. HD-OASS has been shown to perform better than RASS. Random selection of antenna subsets does not ensure that the correlation between the antenna positions of successively used antenna subsets is high.

3. It is shown that SLL optimization through simulated annealing degrades the performance of ASM in terms of randomness due to significant reduction of key space (usable combinations of antenna subsets). HD-OASS has been suggested as an alternate subset selection technique which should be maximized for enhancing PLR rather than SLL reduction.

4. HD-OASS has been shown to generate narrower PLR beamwidth along the direction of Bob compared to RASS and SLL-OASS.

5. The encryption strength of HD-OASS has been benchmarked against state-of-the-art symmetric key block cipher of AES. It is shown that in order to achieve PLR comparable to AES, hamming distance maximization of antenna subsets outperforms previously proposed optimization techniques.

Notations: In this article, \( B \) represents the codebook matrix containing all the \( C^N_M = \frac{2^N}{M^N} \) possible combinations of antenna subsets. The superscript * is the notation for complex conjugate. \( b_i \) denotes the \( i^{th} \) row vector (or \( i^{th} \) antenna subset) from the codebook matrix \( \mathbf{B} \). \( \mod 2 \) is base 2 modulo operation. \( x \bowtie y \) represents Hadamard or element-wise product of two vectors \( x \) and \( y \). \( \sum_i \) indicates row wise summation of elements of a vector. The normalized inter-subset hamming distance between two vectors \( b_i \) and \( b_j \) is represented as \( d_{ij} \).

II. ANTENNA SUBSET MODULATION

In this section, the system model for analyzing ASM is presented. There are two fundamental distinctions between the transmitter architecture of ASM compared to Conventional Phased Array (CPA), as shown in Fig. 1:

1. ASM synthesizes the desired phase of symbol at antenna level using phase shifters, unlike baseband constellation synthesis in CPA.

2. Instead of using the complete antenna array for signal transmission in a pre-specified direction as in CPA, ASM selects a subset of array. This subset is randomly selected and modulated (changed) for every symbol duration.

A. SYSTEM MODEL

Suppose that Alice is equipped with a uniform linear ASM-enabled array comprising of \( N \) isotropically radiating antenna elements separated by \( d \) inter-element distance, as shown in Fig. 1. Intended Receiver (IR) i.e. Bob is located along a pre-specified direction \( (\theta_R) \) known to Alice. At any discrete time \( k \), the signal transmitted by the complete array is represented as a vector \( \mathbf{x}(k) \). The modulation scheme being used is Phase Shift Keying (PSK) and \( \phi(k) \) is the phase of encoded symbols. The array is phase compensated using phase-shifters to direct the main lobe in the direction of Bob. After phase compensation, the signal is amplified using Power Amplifier (PA) at each RF chain. Following it, high speed RF switches are capable of randomly selecting \( M \) (\( M < N \)) antenna elements depending upon the antenna subset (code) from the codebook. Eavesdropper (Eve) is situated outside the main lobe of array the direction of which is...
unknown to Alice. The signal received along any direction \( \theta \) at time \( k \) can be written as:

\[
y(k, \theta) = h^*(\theta)x(k)
\]

where \( x(k) \) is the transmitted signal vector and \( h(\theta) \) is the \( N \times 1 \) channel vector which can be written as:

\[
h(\theta) = \begin{bmatrix} e^{-j\frac{\lambda}{2} \frac{2\pi k d}{\lambda} \cos \theta}, & e^{-j\frac{\lambda}{2} \frac{2\pi (k-1) d}{\lambda} \cos \theta}, & \ldots, & e^{j\frac{\lambda}{2} \frac{2\pi k d}{\lambda} \cos \theta} \end{bmatrix}
\]

where \( \lambda \) is the wavelength of the carrier. Equation 1 is the generalized representation of transmitted signal \( y(k, \theta) \). In ASM architecture, subsets of antenna array are randomly selected through high-speed RF switches for every PSK symbol transmission. The cumulative effect of switching in ASM is incorporated in the system model by Hadamard product of binary antenna subset vector \( b(k) \) with channel phase compensation vector \( h(\theta_T) \), where \( \theta_T \) is the direction of intended receiver. The randomly selected code/antenna subset i.e. \( b(k) \) encodes the indices of \( M \) transmit antennas that are selected for \( k^{th} \) discrete symbol duration. Normalized to \( M \) active antenna elements, the resulting input vector \( x(k) \) can be written as:

\[
x(k) = \sqrt{\frac{E_s}{M}} [b(k) \otimes h(\theta_T)] e^{j\phi(k)}
\]

where \( \sqrt{E_s} \) is the signal energy and \( e^{j\phi(k)} \) is the phase of PSK modulated symbols. Incorporating the effect of phase compensation and subset switching in ASM, the system model given by equation 1 becomes:

\[
y(k, \theta) = h^*(\theta)x(k) = \sqrt{\frac{E_s}{M}} e^{j\phi(k)}[b(k) \otimes h(\theta_T)] = \sqrt{\frac{E_s}{M}} e^{j\phi(k)} \sum_{n=0}^{N-1} b_n(k) e^{j(n-\frac{N-1}{2}) \frac{2\pi d}{\lambda} \cos \theta}.
\]

Equation 4 can further be represented as:

\[
y(k, \theta) = \sqrt{\frac{E_s}{M}} \sum_{n=0}^{N-1} b_n(k) e^{j\phi_n(k)} e^{j(n-\frac{N-1}{2}) \frac{2\pi d}{\lambda} \cos \theta}
\]

where,

\[
\beta_n(k) = \frac{\phi(k)}{\text{modulation component}} - \frac{n-N-1}{2} \frac{2\pi d}{\lambda} \cos \theta_T.
\]

In equation 5, \( \beta_n \) is the progressive inter-element phase difference which is applied by adjusting the phase-shifting of each RF chain shown in Fig. 1. It consists of modulation component and beamsteering component. From equation 6, it is evident that modulation and beamsteering are jointly performed in ASM at antenna level.

**Effect of ASM along Bob:**

Consider that Bob is spatially situated along \( \theta_T \) direction with respect to Bob and the direction of Bob is known to Alice. The main beam is pointed in the direction of Bob by array phase compensation i.e. \( \theta = \theta_T \). Therefore, in the direction of Bob, Equation 4 simplifies as:

\[
y(k, \theta_T) = \frac{\sqrt{E_s} e^{j\phi(k)}}{M} \sum_{n=0}^{N-1} b_n(k) e^{j(n-\frac{N-1}{2}) \frac{2\pi d}{\lambda} \cos \theta_T - \cos \theta_T}
\]

From equation 7, it is clear that Bob receives the original non-distorted phase of symbol i.e. \( \phi(k) \). This is because Alice has phase-compensated the array only in the direction of Bob. Hence, Bob receives non-encrypted data (plaintext) at physical layer in ASM.

**Effect of ASM along Eve:**

Consider that Eve is located along \( \theta \neq \theta_T \) and its direction is unknown to Alice. For any \( \theta \), the system model of ASM is given by equation 5 as:

\[
y(k, \theta) = \frac{\sqrt{E_s} e^{j\phi(k)}}{M} \sum_{n=0}^{N-1} b_n(k) e^{j(n-\frac{N-1}{2}) \frac{2\pi d}{\lambda} \cos \theta - \cos \theta_T}
\]

Equation 8 would result in a complex value which would depend on the value of \( \theta \) as well as \( b(k) \). The direction of Bob i.e. \( \theta_T \) is also unknown to Eve. Furthermore, the antenna indices are being randomly modulated after every symbol transmission. For Eve to de-modulate the phase of original transmitted symbol, it would require the exact estimate of not only the direction of Bob but also the information of antenna subsets which are changing for every symbol. Therefore, in the direction of Eve scrambled and encrypted constellation of data (ciphertext) is transmitted.

**III. INTER-SUBSET HAMMING DISTANCE MAXIMIZATION**

In this section, the algorithm of HD-OASS is discussed. Suppose that \( a_{in} \) represents \( n^{th} \) antenna position of \( i^{th} \) antenna subset. Then, any antenna subset \( b_i \) can be written as:

\[
b_i = [a_{i1} \ a_{i2} \ a_{i3} \ldots \ a_{iN}].
\]

Similarly, the antenna subset for the \( j^{th} \) symbol duration would be:

\[
b_j = [a_{j1} \ a_{j2} \ a_{j3} \ldots \ a_{jN}].
\]
For any two binary vectors \( \mathbf{b}_i \) and \( \mathbf{b}_j \), the normalized inter-subset hamming distance is defined as:

\[
d_{ij} = \frac{1}{N} \sum_r \text{mod}_2(\mathbf{b}_i + \mathbf{b}_j)
\]  

where \( \sum_r \) denotes row wise summation of elements of a vector and \( \text{mod}_2 \) is base 2 modulo operation. The equation can further be modified as:

\[
d_{ij} = \frac{1}{N} \sum_r \text{mod}_2(\mathbf{a}_i + \mathbf{a}_j \mod 2)
\]  

\[
d_{ij} = \frac{1}{N} \sum_r \text{mod}_2(\mathbf{a}_i + \mathbf{a}_j + \mathbf{a}_j \mod 2)
\]

\[
d_{ij} = \frac{1}{N} \sum_r \text{mod}_2(\mathbf{a}_i + \mathbf{a}_j + \mathbf{a}_j + \mathbf{a}_j)
\]

**Algorithm 1** Hamming Distance Maximization

1: procedure Antenna_Subset_Selection (N,M)
2: Initialize (\( \mathbf{b}_i, \mathbf{b}_j \))
3: for \( c = 1 \) to iter_count do
4: for \( i = 1 \) to codebook_size do
5: if \( d_{ij} = \frac{1}{N} \sum_r \text{mod}_2(\mathbf{b}_i + \mathbf{b}_j) \geq 0.5 \)
6: \( \mathbf{b}_{i+1} = \mathbf{b}_j \)
7: else
8: \( \mathbf{b}_{i+1} = \phi \)
9: end if
10: \( j = j + 1 \)
11: end for
12: end for
13: end procedure

The normalized hamming distance constraint value of greater than or equal to 0.5 means that for any successive symbol transmissions, the antenna positions remain uncorrelated by at least 50%. Antenna subsets having low inter-subset hamming distance are discarded by the algorithm.

**B. OPTIMIZATION ALGORITHM**

The algorithm for antenna subset selection using inter-subset hamming distance maximization is as follows:

1: For an array of N antenna elements in which \( M (M < N) \) are randomly turned on, there are \( \binom{N}{M} \) possible combinations of antenna subsets. All the \( \binom{N}{M} \) combinations of antenna subsets of ASM are stored in a cookbook.
2: A reference antenna subset \( \mathbf{b}_i \) and a test antenna subset \( \mathbf{b}_j \) are randomly selected from the codebook.
3: Run the optimization equal to the number of pre-specified iter_count starting from \( c = 1 \). Each round of optimization iteratively discards antenna subsets which have \( d_{ij} \leq 0.5 \).
4: For every iteration, the inter-subset hamming distance of complete codebook is calculated with respect to reference antenna subset.
5: Calculate the hamming distance between \( \mathbf{b}_i \) and \( \mathbf{b}_j \). For binary codebook (as for ASM), the normalized inter-subset hamming distance is calculated by \( d_{ij} = \frac{1}{N} \sum_r \text{mod}_2(\mathbf{k}_i, \mathbf{k}_j) \). After calculation, the condition is checked whether the hamming distance between the two codes is greater than or equal to 0.5. The value of \( d_{ij} \geq 0.5 \) means that we are discarding all the antenna subsets which have hamming distance less than 0.5.
6: If \( d_{ij} \geq 0.5 \), then the test antenna subset \( \mathbf{b}_j \) is selected as the next antenna subset i.e. \( \mathbf{b}_{i+1} = \mathbf{b}_j \) after \( \mathbf{b}_i \).
7: Furthermore, the subset \( \mathbf{b}_j \) is updated as the reference subset for next calculation.
8: If \( d_{ij} \leq 0.5 \), then the algorithm rejects the antenna subset \( \mathbf{b}_j \).
9: The test antenna subset is incremented by one in the codebook.
IV. PHYSICAL LAYER RANDOMNESS (PLR)

PLR is cryptography-inspired metric that has been proposed for analyzing randomness introduced by a PLS technique [22]. PLR also enables direct comparison of encryption strength of physical layer techniques to that of upper layer block ciphering techniques like AES, a strong encryption algorithm.
PLR consists of 15 standard randomness tests devised by NIST, namely; frequency monobits test, block frequency test, runs test, longest runs of ones in a block test, binary matrix rank test, discrete fourier transform test, non-overlapping template matching test, overlapping template matching test, universal statistical test, linear complexity test, serial test, approximate entropy test, cumulative sums test, random excursion test, and random excursion variant test. The results of each test is recorded as a p-value and accordingly denoted by: $P_F$, $P_B$, $P_L$, $P_R$, $P_D$, $P_N$, $P_O$, $P_U$, $P_C$, $P_T$, $P_A$, $P_S$, $P_E$ and $P_V$. A rank $\zeta$ is designated to each p-value depending upon its magnitude, as shown in Table 1. The cumulative sum of ranks of all the tests is defined as PLR:

$$PLR = \sum_{\zeta=1}^{N_T} \zeta$$  \hspace{1cm} (16)

where $N_T$ represents the total number of tests that has been performed for the analysis of randomness of ciphertext. It is equal to 15 in our case, as we are performing all the NIST tests.

V. SIMULATION RESULTS

In this section, simulation results for hamming distance codebook optimization and its effect on PLR are discussed. A comparison of PLR of HD-OASS with RASS and SLL-OASS is also presented to show the effectiveness of hamming distance maximization.

Consider a uniform linear array of $N = 24$ elements, of which $M = 16$ antennas are randomly turned on for each symbol transmission. The size of codebook (total possible combinations in which antenna subsets could be selected) is thus equal to $C_{16}^{24} = \frac{24!}{16! \cdot 8!} = 7.35 \times 10^5$. The direction of IR (i.e. Bob) is known to Alice, which has been assumed to be equal to $\theta_{IR} = 60^\circ$. The progressive inter-element phase difference ($\beta$) is adjusted accordingly at the transmit side to point the main beam in the direction of Bob. The modulation scheme is Quadrature Phase Shift Keying (QPSK).

A. INTER-SUBSET HAMMING DISTANCE OPTIMIZED CODEBOOK

In Fig. 6, a simplified example of ASM with $N = 6$ and $M = 4$ focusing on the calculation of inter-subset hamming distance is presented. At $T = 0$, $M = 4$ antennas are randomly turned ON for transmission of one symbol. The antenna subset for $T = 0$ can be written as:

$$b_0 = [1 \ 1 \ 0 \ 1 \ 0 \ 1].$$  \hspace{1cm} (17)

For $T = 1$, the antenna subset is:

$$b_1 = [0 \ 1 \ 1 \ 0 \ 1 \ 1].$$  \hspace{1cm} (18)

The normalized inter-subset hamming distance between the two antenna subsets is readily calculated, using equation 13, as:

$$d_{01} = \frac{1}{6} \sum_r (mod_2 [1 + 0 + 1 + 0 + 1 + 0 + 1 + 1 + 1])$$

$$= \frac{1}{6} \sum_r [1 + 0 + 1 + 1 + 1 + 0]$$

$$= \frac{1}{6} \times 4$$

$$= 0.66$$  \hspace{1cm} (19)

Similarly, for the next symbol duration $T = 2$, the hamming distance with respect to previous antenna subset is calculated as:

$$d_{12} = \frac{1}{6} \sum_r (mod_2 [1 + 2 + 1 + 1 + 2])$$

$$= \frac{1}{6} \sum_r [1 + 0 + 1 + 1 + 0]$$

$$= \frac{1}{6} \times 2$$

$$= 0.33$$  \hspace{1cm} (20)

For $T = 2$, the antenna subset is:

$$b_2 = [1 \ 1 \ 0 \ 0 \ 1 \ 1].$$  \hspace{1cm} (21)

The normalized inter-subset hamming distance between the two antenna subsets is:

$$d_{12} = \frac{1}{6} \sum_r (mod_2 [1 + 2 + 1 + 0 + 0 + 1 + 1 + 1])$$

$$= \frac{1}{6} \sum_r [1 + 0 + 1 + 0 + 0 + 1]$$

$$= \frac{1}{6} \times 2$$

$$= 0.33$$  \hspace{1cm} (22)
In this example, $d_{01}$ is twice the magnitude of $d_{12}$. This means that the first two subsets have twice the antenna positions that are non-overlapping compared to antenna positions of later two subsets. In HD-OASS, it is desirable to choose only those antenna subsets which have minimum overlap of antenna positions for successively transmitted symbols. Thus, only those antenna subsets which have high $d_{ij}$ are selected and antenna subsets having low value of $d_{ij}$ are discarded.

Calculations of inter-subset hamming distance of 5 randomly selected antenna subsets for different codebooks of ASM are shown in Fig. 3, 4, and 5 for HD-OASS, RASS, and SLL-OASS respectively. Shaded boxes represent the antennas that are ON for that particular symbol duration, while unshaded boxes represent the positions of OFF antennas. For every symbol duration, $M = 16$ antennas are turned on out of $N = 24$ antenna elements. From subset 1 to subset 2 in Fig. 4, the antenna positions that differ with each other are 10. Therefore, the normalized inter-subset hamming distance is

$$d_{ij} = \frac{1}{24} \times 10 = 0.416.$$  

In a similar fashion, the calculations are performed for all three codebooks, as shown in Fig. 3, 4, and 5. Notice that HD-OASS, in Fig. 3, has only those antenna subsets which have $d_{ij} \geq 0.5$. Antenna subsets having $d_{ij} \leq 0.5$ are discarded. For SLL-OASS in Fig. 5, the antenna subsets are optimized based on SLL, not hamming distance. Therefore, it contains antenna subsets having $d_{ij}$ as low as 0.25 as well. This imply that the first two antenna subsets of SLL-OASS differ only by 25% with respect to antenna positions.

Refer to Fig. 7, 8, and 9, in which the histograms of normalized inter-subset hamming distance are plotted. Fig. 7 shows the histogram of RASS codebook. It can be seen that it contains all the $\binom{24}{16} = \frac{24!}{16!8!} = 7.35 \times 10^5$ possible antenna subsets ranging from $d_{ij} = 0.15$ to $d_{ij} = 0.65$. The peak of histogram lies at $d_{ij} = 0.4$. In Fig. 8, the histogram of SLL optimized codebook is shown. SLL optimized codebook, using simulated annealing algorithm, contains
TABLE 2. Comparison of PLR of plain, AES, and ASM encrypted image along Eve direction of 0°.  

| Image Data | $P_F$ | $P_L$ | $P_R$ | $P_O$ | $P_D$ | $P_N$ | $P_O$ | $P_T$ | $P_A$ | $P_S$ | $P_P$ | $P_V$ | PLR |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| Plain text | 0.044 | 0.046 | 1.0   | 0.0   | 0.258 | 2.0   | 0.0   | 0.0   | 0.0   | 0.0   | 1.0   | 0.599 | 4   |
| AES        | 0.126 | 0.406 | 0.253 | 1.0   | 0.311 | 3.03  | 1.0   | 0.364 | 0.41  | 1.0   | 0.124 | 0.564 | 4   |
| HD-OASS - 0° | 0.423 | 0.405 | 0.253 | 3.06  | 0.027 | 1.0   | 0.565 | 5.0   | 0.337 | 3.028 | 1.0   | 0.422 | 4   |
| RASS - 0°  | 0.406 | 0.399 | 0.225 | 2.094 | 2.028 | 1.0   | 0.561 | 5.0   | 0.165 | 3.0   | 1.0   | 0.156 | 3   |
| SLL-OASS - 0° | 0.332 | 0.357 | 0.150 | 1.252 | 2.032 | 1.0   | 0.552 | 5.0   | 0.0   | 0.0   | 0.0   | 0.376 | 3   |

TABLE 3. Comparison of PLR of plain, AES, and ASM encrypted image along Eve direction of 40°.  

| Image Data | $P_F$ | $P_L$ | $P_R$ | $P_O$ | $P_D$ | $P_N$ | $P_O$ | $P_T$ | $P_A$ | $P_S$ | $P_P$ | $P_V$ | PLR |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| Plain text | 0.044 | 0.046 | 1.0   | 0.0   | 0.258 | 2.0   | 0.0   | 0.0   | 0.0   | 0.0   | 1.0   | 0.599 | 4   |
| AES        | 0.126 | 0.406 | 0.253 | 1.0   | 0.311 | 3.03  | 1.0   | 0.364 | 0.41  | 1.0   | 0.124 | 0.564 | 4   |
| HD-OASS - 40° | 0.420 | 0.404 | 0.248 | 2.06  | 0.027 | 1.0   | 0.561 | 5.0   | 0.324 | 3.094 | 1.0   | 0.469 | 4   |
| RASS - 40°  | 0.263 | 0.285 | 0.114 | 1.0   | 0.191 | 0.024 | 1.0   | 0.514 | 5.0   | 0.0   | 0.0   | 0.0   | 3   |
| SLL-OASS - 40° | 0.337 | 0.355 | 0.172 | 1.0   | 0.238 | 0.030 | 1.0   | 0.560 | 5.0   | 0.0   | 0.0   | 0.0   | 3   |

FIGURE 10. Image reconstructed in the direction of IR along $\theta_{IR} = 60°$.  

only those antenna subsets which have low SLL properties. Antenna subsets with high SLL are discarded. Significant reduction of codebook size can be observed in Fig. 8 for SLL-OASS compared to RASS in Fig. 7. The total number of antenna subsets (key space) has declined from $7.35 \times 10^5$ for RASS to 9,242 for SLL-OASS. This means that limited configurations of antenna subsets for SLL-OASS are to be repeatedly used, the effect of which will be seen in diminished randomness, as discussed later in the paper.

According to the algorithm discussed in section III for inter-subset hamming distance maximization, the codebook is iteratively optimized until all the subsets in the codebook have $d_{ij} \geq 0.5$. In Fig. 9 (a)-(d), histograms of HD-OASS after iteration 1, iteration 3, iteration 5, and iteration 9 are shown respectively. During each iteration, the antenna subsets having $d_{ij} \leq 0.5$ are recursively discarded. After 9th iteration, all the antenna subsets have $d_{ij} \geq 0.5$ as shown in Fig. 9(d). This is the final HD optimized codebook.

B. DIRECTIONAL ENCRYPTION OF IMAGE USING ASM

In this section, the encryption strength of various antenna subset selection techniques of ASM for image data is evaluated. According to NIST recommendations [21], an image of minimum size of 16 MB is transmitted in the intended direction of Bob along $\theta_{IR} = 60°$. An eavesdropper, situated outside the mainlobe of the antenna array, receives QPSK symbols that are distorted both in phase and amplitude. The adverse effects of SLL-OASS due to codebook size reduction is discussed. Furthermore, HD-OASS has been shown to outperform RASS and SLL-OASS codebooks in terms of physical layer randomness.

The plaintext image transmitted in the direction of Bob, shown in Fig. 10, has the least magnitude of p-values for all the randomness tests, as indicated by red line in Fig. 11. The calculation of PLR of plaintext is given in Table 2. Plaintext has the PLR of 18 and failure of 8 randomness tests, obviously since it is non-encrypted and hence least randomized. AES encrypted image has high magnitude of p-values for all the tests, as indicated by blue line in Fig. 11. It has the PLR of 44 and no $F$ rank, as suggested by calculations in Table 2. It is the highest value of PLR and serves as a benchmark for comparing randomness of physical layer security techniques like ASM.

In the direction of Eve, the images reconstructed along $\theta = 0°$ and 40° are shown in Fig. 12 and 13 respectively. Following observations can be made in Fig. 12:

1. The image for HD-OASS, in Fig. 12 (a), can be seen to be strongly randomized. Its p-values are plotted as yellow line in Fig. 11 (a) and can be seen to be comparable to AES. It has the PLR of 44 and no $F$ ranks, as calculated in Table 2. The encryption strength in this direction is, therefore, equal to that of AES.

2. The image in Fig. 12 (b) for RASS is mildly randomized. The features of the image are visible and reduced p-values can be observed in Fig. 11 (a). It has the PLR of 35 and failure of 2 randomness tests of overlapping template matching and serial test.
3. The image is least randomized for SLL-OASS, as shown in Fig. 12 (c). The image features are the most prominent for SLL optimized codebook. Significantly declined p-values are shown as green line in Fig. 11 (a). It has the PLR of 31 and 4 F ranks, signifying the failure of non-overlapping template matching, overlapping template matching, universal statistical test, and serial test. Calculations of PLR are summarized in Table 2.

Similar observations can be made for image along $\theta = 40^\circ$ in Fig. 13, its p-values in Fig. 11 (b), and PLR calculations in Table 3. In both cases, the performance of HD optimized codebook is better than RASS and SLL optimized codebook performs the worst.

C. PLR RESULTS AND PLOTS

As discussed in Section IV, PLR comprises of two components; the magnitude of PLR and the number of failed tests. While high value of PLR is indicative of high encryption strength and good randomness, failed tests signify the existence of patterns of binary data making the data susceptible to eavesdropping. Therefore, for high communication security, it is desirable to have high PLR and least number of failed tests (ideally zero). How much high PLR is sufficient for data security against eavesdropper? AES is the strongest block cipher which is commercially used today. Therefore, the PLR of AES has been used in this work to benchmark the encryption strength of different codebooks of ASM.

Refer to Fig. 14 in which the PLR of HD-OASS is compared to that of RASS and AES for a range of transmit angle ($\theta_{IR} = 60^\circ$ being the direction of IR). On left y-axis the magnitude of PLR, while on right y-axis the number of failed tests are plotted. The PLR of AES, shown as dotted green line, does not vary with direction because AES is not a directional modulation technique. It provides equally strong encryption for all directions. It can be seen that HD-OASS codebook of
ASM has better PLR along several directions compared to RASS. In some directions, its PLR is comparable to that of AES. Furthermore, in the vicinity of the direction of IR along $\theta_{IR} = 60^\circ$, the beamwidth of PLR is about $10^\circ$ narrower compared to RASS. Therefore, HD-OASS provides higher directional communication security by producing narrow randomness beamwidth about the desired direction.

A similar comparison of HD-OASS with SLL-OASS can be seen in Fig. 15. For all directions, the PLR of SLL optimized ASM is much lower than HD optimized codebook and the number of failed tests are higher for SLL-OASS. PLR beamwidth of HD-OASS around the intended direction of $\theta_{IR} = 60^\circ$ is narrower compared to SLL-OASS.

VI. CONCLUSION

In this work, the physical layer encryption strength of ASM has been analyzed. Inter-subset hamming distance maximization has been proposed as a new antenna subset selection technique for ASM which has been shown to significantly outperform previously proposed techniques in terms of PLR. The performance of ASM has been benchmarked against the strong cryptographic standard of AES. HD-OASS has been shown to provide encryption strength comparable to AES along several eavesdropper directions. Furthermore, it is shown that the conventional approach to increase physical layer security by decreasing sidelobe levels (and increasing SER in the undesired directions) using SLL optimized antenna subset selection exhibit rather poor randomness performance compared to randomized antenna subset selection. It renders SLL as inappropriate parameter of optimization for enhancing physical layer security. Diminished PLR of SLL-OASS is attributed to the reduced number of antenna subsets. By discarding antenna subsets having high SLL in the unwanted directions, usable combinations of antenna subsets are significantly reduced, same antenna subsets are used repeatedly, resulting in degraded encryption strength.

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