POSSIBLE BLACK UNIVERSES IN A BRANE WORLD

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A black universe is a nonsingular black hole where, beyond the horizon, there is an expanding, asymptotically isotropic universe. Such spherically symmetric configurations have been recently found as solutions to the Einstein equations with phantom scalar fields (with negative kinetic energy) as sources of gravity. They have a Schwarzschild-like causal structure but a de Sitter infinity instead of a singularity. It is attempted to obtain similar configurations without phantoms, in the framework of an RS2 type brane world scenario, considering the modified Einstein equations that describe gravity on the brane. By building an explicit example, it is shown that black-universe solutions can be obtained there in the presence of a scalar field with positive kinetic energy and a nonzero potential.

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1. Introduction

The problem of singularities is one of the long-standing problems in the classical theories of gravity. Singularities are places where general relativity or another classical theory of gravity does not work. Therefore, a full understanding of the physics of phenomena under study (origin and fate of our Universe, gravitational collapse etc.) requires avoidance of singularities or/and modification of the corresponding classical theory or addressing quantum effects. There have been numerous attempts on this trend, some of them suggesting that singularities inside the event horizons of black holes should be replaced with a kind of regular core ([1], see [2] for a recent review), others describing bouncing or “emergent” universes (see, e.g., [3, 4] for reviews).

In our view, of particular interest are models which combine avoidance of singularities in both black holes and cosmology, those which have been termed black universes [5, 6]. These are regular spherically symmetric black holes, with the same causal structure as the Schwarzschild black hole, but where a possible explorer, after crossing the event horizon, gets into an expanding universe instead of a singularity. Thus such hypothetic configurations combine the properties of a wormhole (absence of a center, a regular minimum of the area function) and a black hole (a Killing horizon separating R and T regions). Moreover, the Kantowski-Sachs cosmology in the T region is asymptotically isotropic and approaches a de Sitter regime of expansion, which makes such models potentially viable as models of our accelerating Universe.

Such objects are naturally obtained [5, 6] if one considers local concentrations of dark energy (DE) represented by different forms of phantom matter: phantom scalars in scalar-tensor theories of gravity and the so-called k-essence whose most general Lagrangians have the form $F(\phi, X)$ where $X = \nabla_\alpha \phi \nabla^\alpha \phi$. In all such cases, there is at least one classical field with a negative kinetic term.

The phantom behavior of DE (such that its pressure to density ratio $w = p/\rho < -1$) is favored by modern cosmological observations. More precisely, the “Gold” supernova sample data [7] slightly favor a phantom behavior of DE at small redshifts $z < 0.3$ along with crossing the phantom divide $w = -1$ at larger $z$ [8] (see [9] for further references). Other cosmological data suggest a cosmological constant as the best fit but still do not exclude recent phantom DE behavior, see [10]. The latest estimates of $w$ also peak somewhat near $-1$ and admit $w < -1$ [11].

Meanwhile, there exist models that admit a phantom DE behavior without explicitly introducing phantom fields. Among them the simplest is the generic scalar-tensor gravity with non-zero scalar field potentials (see, e.g., [12]) which has sufficient freedom to describe all observational data.

There are theoretical reasons for considering phantom fields: they naturally appear in some models of string theory [13], supergravities [14] and theories in more than 11 dimensions like F-theory [15].

Nevertheless, one should bear in mind that a classical field with a negative kinetic term can have an arbitrarily large negative energy of high-frequency oscillations, which is quite undesirable from the viewpoint of quantum field theory: it can lead to runaway production of
particles and antiparticles accompanied by production of equal negative energy of the phantom field itself (see, e.g., [16]). Nothing of this kind is observed, which casts serious doubt on possible existence of phantom fields. Moreover, as was recently argued in [12], cosmological models with a phantom scalar field cannot explain the observed large-scale homogeneity and isotropy of the Universe.

Thus there exist arguments both pro et contra phantom fields, and the latter seem somewhat stronger. In any case, it is reasonable to try to avoid such fields in modelling real or hypothetic phenomena.

Accordingly, we here try to show that black-universe models can be obtained without invoking phantom fields. This appears to be possible in the framework of the brane world scenario, using the modified Einstein equations [17] describing gravity on the brane.

The brane world concept describes our world as a 4D surface (brane) supporting all or almost all matter fields and embedded in a higher-dimensional space-time (called the bulk). This concept traces back to the early 80s and leads to a variety of consequences in cosmology, gravitational and particle physics, see the reviews [18]. In particular, brane worlds turn out to be a natural framework for wormholes [19, 20] (see also references therein) since there the modified Einstein equations [17] [see Eqs. (13)] contain a source term $E'_{\mu}^{\nu}$ of geometric origin which need not observe the usual energy conditions. And, as we shall see in the present paper, it is this source term that can replace phantom fields in building black-universe models.

The modified Einstein equations (EE4) used here correspond to the so-called RS2 scenario: a single brane in a $Z_2$-symmetric 5-dimensional bulk, with all fields except gravity confined on the brane. It generalizes the second Randall-Sundrum model comprising a single Minkowski brane in an anti-de Sitter (AdS) bulk [21]. However, in other brane-world scenarios, the effective 4D Einstein equations also contain terms similar to $E'_{\mu}^{\nu}$, e.g., on codimension-1 branes without $Z_2$ symmetry [22] and in Dvali-Gabadadze-Porrati brane worlds [23] with different kinds of induced gravity terms [24]. Thus we can anticipate that black-universe solutions similar to ours exist in such brane worlds as well, though probably under some other conditions.

The paper is organized as follows. In Section 2 we briefly discuss the Einstein-scalar field equations for static, spherically symmetric systems in general relativity and present the simplest black-universe solution with a phantom scalar. In Section 3 we analyze the similar set of equations in a brane world and show that for our purpose we can neglect some terms, namely, those quadratic with respect to the stress-energy tensor of matter. Section 4 is devoted to attempts to obtain black-universe solutions by properly employing the freedom that exists in this system. Section 5 contains some concluding remarks.

## 2. Black-universe solutions in general relativity

Consider the general static, spherically symmetric metric

$$ds^2 = A(u) dt^2 - \frac{du^2}{A(u)} - r^2(u)d\Omega^2,$$

(1)

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is the metric on a unit sphere.$^3$ The metric (1) is written in terms of the “quasiglobal” coordinate $u$, which is particularly convenient for dealing with Killing horizons where it behaves in the same way as the manifestly well-behaved Kruskal-like null coordinates. For this reason, in terms of $u$, one may consider regions on both sides of such a horizon remaining in a formally static framework.

The two functions, $A(u)$ (often called the redshift function) and $r(u)$ (the area function, equal to the radius of a coordinate sphere at given $u$) entirely determine the geometry under consideration. Horizons correspond to regular zeros of $A(u)$.

Our interest is to find black-universe solutions which, by definition, must have the following properties:

1. Regularity in the whole range $u \in \mathbb{R}$;
2. Asymptotic flatness as $u \to -\infty$ (without loss of generality), i.e., $r(u) \approx -u, \ A(u) \to 1$;
3. A de Sitter asymptotic as $u \to +\infty$, i.e., a $T^2 \subset S^4$ region ($A < 0$) where $r(u) \sim u, \ -A(u) \sim u^2$;
4. A single simple horizon (i.e., a simple zero of $A(u)$) at finite $u$.

As shown in [6], such solutions can exist in general relativity with various phantom sources. Let us here present the simplest example with a minimally coupled scalar field having the Lagrangian

$$L_\phi = \frac{1}{2} \varepsilon \partial^\alpha \phi \partial_\alpha \phi - V(\phi),$$

(2)

where $\varepsilon = +1$ corresponds to normal scalar fields with positive kinetic energy, $\varepsilon = -1$ to phantom fields, and $V(\phi)$ is a potential. The set of Einstein-scalar equations for the metric (1) and $\phi = \phi(u)$ may be written in the form

$$\varepsilon (A r^2 \phi')' = -r^2 dV/d\phi,$$

(3)

$$A' r^2)' = -2r^2 V;$$

(4)

$$2r'^2/r = \varepsilon \phi'^2;$$

(5)

$$A(r^2)' - r^2 A'' = 2.$$  

$^3$Our sign conventions are as follows: the metric signature $(+---)$; the curvature tensor $R^\mu_{\rho\nu\sigma} = \partial_\rho R^\mu_{\nu\sigma} - \ldots$, so that, e.g., the Ricci scalar $R > 0$ for de Sitter space-time, and the stress-energy tensor (SET) such that $T^\mu_\nu$ is the energy density.
where the prime denotes $d/du$. The scalar field equation (3) is a consequence of Eqs. (23)–(25), which, given a potential $V(\phi)$, form a determined set of equations for the unknowns $r(u)$, $A(u)$, $\phi(u)$.

As is evident from (24), black-universe solutions cannot be obtained with $\varepsilon = +1$ because in this case $r'' \leq 0$ which is incompatible with requirements 1-3 (instead of $u \in \mathbb{R}$, there will be inevitably $r = 0$ at some finite $u$, which is either a singularity or, at best, a regular center).

A particular solution to these equations with $\varepsilon = -1$ is given by [5]

$$ r = (u^2 + b^2)^{1/2}, \quad b = \text{const} > 0; \quad (7) $$

$$ B(u) = \frac{A(u)}{r^2(u)} = \frac{c}{b^2} + \frac{1}{b^2 + u^2} $$

$$ + \frac{u_0}{b^3} \left( \frac{bu}{b^2 + u^2} + \arctan \frac{u}{b} \right), \quad (8) $$

$$ \phi = \pm \sqrt{2} \arctan(\frac{u}{b}) + \phi_0, \quad (9) $$

$$ V = -\frac{c}{b^2} \frac{r^2 + 2u^2}{r^2} $$

$$ - \frac{u_0}{b^3} \left( \frac{3bu}{r^2 + 2u^2} - \arctan \frac{u}{b} \right) \quad (10) $$

with $c$, $u_0$, $\phi_0 = \text{const}$. In particular,

$$ B(\pm \infty) = -\frac{1}{3} V(\pm \infty) = \frac{2bc \pm \pi u_0}{2b^3}. \quad (11) $$

Choosing in (9), without loss of generality, the plus sign and $\phi_0 = 0$, we obtain for $V(\phi)$

$$ V(\phi) = \frac{c}{b^2} (3 - 2 \cos^2 \psi) $$

$$ - \frac{u_0}{b^3} \left[ 3 \sin \psi \cos \psi + \psi (3 - 2 \cos^2 \psi) \right], \quad (12) $$

The solution is asymptotically flat at large negative $u$ ($B \to 0$, $A \to 1$) under the condition $2bc = \pi u_0$, and the Schwarzschild mass, defined in the usual way, is then $m = -u_0/3$. Then, assuming $m > 0$, we find that $B(\pm \infty) = -3\pi m/b^3 = \text{const} < 0$, which corresponds to a de Sitter asymptotic with a cosmological constant $\Lambda > 0$. The horizon position is found by solving the transcendental equation $B(u) = 0$.

This is an example of a black-universe solution. Other cases of the solution (7)–(10) include wormholes and asymptotically AdS configurations, see more details in [5,6].

### 3. Gravity on the brane

#### 3.1. Modified Einstein equations

The gravitational field on the brane is described by the modified Einstein equations [17]

$$ G^\nu_\mu = -\Lambda_5 \delta^\nu_\mu - \kappa_5^2 T^\nu_\mu - \epsilon_5^4 \Pi^\nu_\mu - E^\nu_\mu, \quad (13) $$

where $G^\nu_\mu = R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R$ is the 4D Einstein tensor, $\Lambda_4$ is the 4D cosmological constant expressed in terms of $\Lambda_5$ and the brane tension $\lambda$:

$$ \Lambda_4 = \frac{1}{2} \left( \Lambda_5 + \frac{1}{6} \epsilon_5^4 \lambda^2 \right); \quad (14) $$

$$ \epsilon_5^4 = 8\pi G_N = \epsilon_5^4 \lambda/(6\pi) = m_4^{-2} \quad \text{is the 4D gravitational constant}; \quad G_N \quad \text{is Newton’s constant of gravity, and} \quad m_4 \quad \text{is the 4D Planck mass}; $$

$$ \epsilon_5 = m_5^{-3/2}, \quad m_5 \quad \text{being the 5D Planck energy scale}; $$

$T^\nu_\mu$ is the SET of matter trapped on the brane;

$$ \Pi^\nu_\mu \quad \text{is a tensor quadratic in} \quad T^\nu_\mu, \quad \text{obtained from matching the 5D metric across the brane:} $$

$$ \Pi^\nu_\mu = \frac{1}{4} T^\nu_\mu T^\rho_\sigma - \frac{1}{2} TT^\nu_\mu - \frac{1}{4} \delta^\nu_\sigma (T^{\alpha\beta} T_{\alpha\beta} - \frac{1}{3} T^2), \quad (15) $$

where $T = T^\alpha_\alpha$; lastly, $E^\nu_\mu$ is the “electric” part of the 5D Weyl tensor projected onto the brane: in proper 5D coordinates,

$$ E^\nu_\mu = \delta^A_\mu \delta^C_D C_{ABCD} B^A n^D \quad (16) $$

where $A, B, \ldots$ are 5D indices and $n^A$ is the unit normal to the brane. By construction, $E^\nu_\mu$ is traceless, $E^\nu_\nu = 0$.

Other characteristics of $E^\nu_\mu$ are unknown without specifying the 5D metric, hence the set of equations (13) is not closed. In isotropic cosmology this leads to an additional arbitrary constant in the field equations, connected with the density of “dark radiation” [18]. For static, spherically symmetric systems to be discussed here, this freedom is expressed in the existence of one arbitrary function of the radial coordinate.

#### 3.2. Reasons for neglecting $\Pi^\nu_\mu$

Let us show that under quite reasonable conditions we can neglect the tensor $\Pi^\nu_\mu$ in (13).

We put $\Lambda_4 = 0$, so that

$$ |\Lambda_5| = \frac{1}{6} \epsilon_5^4 \lambda^2 = 6\pi^2 (\epsilon_4/\epsilon_5)^4, \quad (17) $$

and use the observational restriction on the bulk length scale $\ell$ which follows from the recent short-range Newtonian gravity tests [25], showing that Newton’s inverse-square law hold at length scales greater than about 0.1 mm. This means that if we live on an RS2-like brane, the bulk length scale can be estimated as

$$ \ell = (6/|\Lambda_5|)^{1/2} \lesssim 10^{-2} \text{ cm}. \quad (18) $$
Note that the 4D Planck scale in our notation is
\[ m_4 = \sqrt{4\pi G_N} \approx 2.4 \times 10^{18} \text{ GeV}, \]
\[ l_4 = 1/m_4 = \sqrt{\pi} \approx 8 \times 10^{-33} \text{ cm}. \]

Combining (17) and (18), we obtain
\[ m_5/m_4 = (\pi \ell/l_4)^{-1/3} \gg 10^{-10}, \]
so that the 5D Planck energy scale in this scenario is at least about \( 10^8 \) GeV.

Now, we can assert that the term with \( \Pi''_\nu \) is negligible in (13) as compared with the \( T''_\nu \) term as long as
\[ \kappa_4^4 W^2 \ll \kappa_5^4 W, \]
where \( W \) characterizes the magnitude of \( T''_\nu \), say, the absolute value of the largest component of \( T''_\nu \).

\[ W \ll m_5^2/m_4^2 = m_4^4 (m_5/m_4)^6, \]
where \( m_4^4 \approx 3.5 \times 10^{32} \text{ GeV}^4 \approx 8.4 \times 10^{90} \text{ g} \cdot \text{cm}^{-3} \) is the Planck density while the second factor is, according to the experimental bound (19), about \( 10^{-60} \) or larger. As a result, for the “density” \( W \) we have
\[ W \ll 10^{30} \text{ g} \cdot \text{cm}^{-3}. \]

Recalling that the density of nuclear matter is about \( 10^{15} \text{ g} \cdot \text{cm}^{-3} \), it is clear that this bound certainly holds for any thinkable matter.

### 3.3. Brane gravity with a scalar field

Consider Eqs. (13) for static, spherically symmetric configurations of a normal scalar field with an arbitrary potential, neglecting the term \( \Pi''_\nu \). So the scalar field Lagrangian has the form (2) with \( \varepsilon = 1 \). The general static, spherically symmetric metric is again taken in the form (1), with the quasiglobal coordinate \( u \).

The scalar field EMT is conservative, so the same is required for \( E''_\nu \). If we take it, for convenience, in the form
\[ E''_\nu = \text{diag}(-P-Af, -P, P+Af/2, P+Af/2), \]
where \( P \) and \( f \) are some functions of the radial coordinate \( u \) (so that, as required, its trace is zero), then the conservation law \( \nabla_\alpha E''_\alpha \) is written as
\[ (Pr^4)' = \frac{1}{2r} f(A/r^2)'. \]

Eqs. (13) may be written in the form
\[ \frac{1}{2r^2} (A' r^2)' = -V - P - Af; \]  \hspace{1cm} (23)
\[ 2r''/r = -\phi^2 + f; \]  \hspace{1cm} (24)
\[ A(r^2)'' - r^2 A'' - 2 = 2P + \frac{3}{2} Af. \]  \hspace{1cm} (25)

The scalar field equation \( (Ar^4 f')' = r^2 dV/d\phi \) follows from (23)–(25) combined with (22).

Thus, if \( V(\phi) \) is specified, we have four independent equations (22)–(25) for five unknown functions of \( u: \phi, A, r, f \) and \( P \). The system becomes still more underdetermined if \( V(\phi) \) is not specified: in this case we can choose as many as two functions by hand to obtain a solution.

### 4. Attempts to obtain black-universe solutions

#### 4.1. Models with \( r'' > 0 \)

To have a regular positive minimum of \( r(u) \), we must have \( r'' > 0 \) at least in some range of \( u \). By (24), this can be achieved only with \( f > 0 \). We first try to obtain such a solution with \( r'' > 0 \) everywhere, i.e., \( f > \phi^2 \).

To integrate (22), let us suppose
\[ f = 2C/r^6, \quad C = \text{const}. \]  \hspace{1cm} (26)
Then we have
\[ Pr^4 = CB + C_1, \quad B(u) \equiv A/r^2, \quad C_1 = \text{const}. \]  \hspace{1cm} (27)

Putting for simplicity \( C_1 = 0 \) and substituting (27) into (25), we obtain an equation connecting \( B(u) \) and \( r(u) \):
\[ (r^4 B')' + 2 + 10CB/r^2 = 0. \]  \hspace{1cm} (28)

If both \( r \) and \( B \) are known, the quantities \( \phi \) and \( V \) are found from (23) and (24). Thus it remains to choose \( r \) and \( B \) satisfying (28) and providing the above properties 1–4.

This task turns out to be hard if at all solvable. Thus, if we choose \( r^2 = u^2 + b^2 \ (b = \text{const} > 0) \) with good asymptotics at large \( u \), we obtain \( r''/r = b^2/r^4 \), whence (24) combined with (26) leads to \( \phi^2 < 0 \) at large \( u \), i.e., a contradiction.

Another choice, \( r^4 = u^4 + b^4 \), is compatible with (24) if \( C > 3b^4 \). However, a numerical solution of Eq. (28) leads to functions \( B(u) \) with at least 3 zeros, in other words, at least 3 horizons in space-time (see an example in Fig. 1).

The nature of Eq. (28) apparently implies an oscillatory behavior of \( B(u) \) if the coefficient \( 10C/r^2 \) (originating from \( E''_\nu \)) is large enough in a sufficiently wide range of \( u \). A possible way of avoiding such a behavior is to try a “more concentrated” distribution of \( f \) and \( P \).

#### 4.2. Models with \( f \sim \delta(u) \)

Let us try to find a black-universe solution assuming that \( f(u) \) is concentrated on a single sphere, e.g., \( u = 0 \).
We now make the equations dimensionless by putting

\[ x = u/b, \quad Pr^4 = Q(x)b^2, \quad f r^4 = 2F(x)b^2, \]

\[ r = b r(x), \quad B = B(x)/b^2, \quad (29) \]

where \( b = \text{const} > 0 \) specifies a length scale of the configuration, and to justify our rejection of \( \Pi^\mu_{\nu} \) we assume \( b \gg \ell \) (see above).

In what follows we omit the bars over \( r \) and \( B \). Eqs. (22), (24) and (25) take the form

\[ Q' = r^2 FR, \]

\[ r'' = -\psi^2 + \frac{F}{r'}, \quad (30) \]

\[ (r^4 B')' + 2 + \frac{4Q}{r^2} + 6BF = 0, \quad (31) \]

where the prime denotes \( d/dx \) and \( \psi = \phi/\sqrt{2} \).

To begin with, we choose \( r(x) \) so that \( r'' < 0 \) at all \( x \neq 0 \):

\[ r^2(x) = (|x| + c)^2 - 1, \quad c = \text{const} > 1. \quad (32) \]

This conforms to both flat and de Sitter asymptotics. Then, in Eq. (31), the quantity \( r''/r \) has a delta-like singularity at \( x = 0 \). To compensate it and make \( \psi' \) continuous at \( x = 0 \), we put

\[ F(x) = 2cr_0^2 \delta(x), \quad r_0 = \sqrt{c^2 - 1}. \quad (33) \]

We have then

\[ \psi' = \pm \frac{1}{r^2} = \frac{1}{(|x| + c)^2 - 1}. \quad (34) \]

whence, without loss of generality,

\[ \psi = \begin{cases} h(c - x), & x < 0, \\ 2h(c) - h(c + x), & x > 0, \end{cases} \]

\[ h(x) := \frac{1}{2} \ln \left| \frac{x + 1}{x - 1} \right|. \quad (35) \]

Eq. (30) gives for \( Q(x) \):

\[ Q(x) = Q_0 + 2cr_0^2 B_{x0} \theta(x), \quad (36) \]

where \( Q_0 = \text{const} \), \( B_{x0} = B'(0) \) and

\[ \theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \]

is the Heaviside function.

It remains to solve Eq. (32) and to find \( V \). Eq. (32) now has the form

\[ (r^4 B')' = -2 + \frac{4Q}{r^2} - \frac{8cr_0^2 B_{x0}}{r^2} \theta(x) - 12B_0cr_0^2 \delta(x), \quad (37) \]

where \( B_0 = B(0) \). We solve (38), choosing the initial conditions at large negative \( x \) in the Schwarzschild form

\[ A(x) = r^2(x)B(x) = 1 - \frac{2m}{|x|}, \quad m = \text{const}. \quad (38) \]

Integrating (38) once, we obtain

\[ r^4 B' = 2(x_0 - x) - 4Q_0 h(c - x) - 8\theta(x)cr_0^4 B_{x0}[h(c) - h(x + c)] - 12B_0cr_0^2 \theta(x), \quad (39) \]

where \( x_0 = -m/3 \), see (39). The constant \( B_{x0} \) is found as

\[ r_0^4 B_{x0} = 2x_0 - 4Q_0 h(c) - 6B_0cr_0^2, \quad (40) \]

where we have used the principal value of \( \theta(0) \) equal to 1/2. Note that the term with \( B_{x0} \) in (40) does not
contribute to the expression (41) because it vanishes at \( x = 0 \).

To find \( B(x) \), we first integrate (40) from \(-\infty\) to \( x \leq 0 \), obtaining

\[
B(x) \big|_{x \leq 0} = \frac{1}{r^2} + (x_0 - c) \left[ \frac{c - x}{r^2(x)} - h(c - x) \right] + Q_0 \left[ 1 - 2(c - x)h(c - x) \right] + h^2(c - x). \tag{42}
\]

This also gives the constant \( B_0 = B(0) \),

\[
B_0 = \frac{1}{r_0^2} + (x_0 - c) \left[ \frac{c}{r_0^2} - h(c) \right] + Q_0 \left[ 1 - 2ch(c) + h^2(c) \right] \tag{43}
\]

which may be used to obtain \( B(x) \) at positive \( x \) from (40) by integration from 0 to \( x > 0 \):

\[
B(x) \big|_{x \geq 0} = B_0 + \frac{1}{r_0^2} - \frac{1}{r^2} + [(x_0 + c) - 4h(c)(Q_0 + cr_0^4 B_0) - 60cr_0^2] \times \left[ \frac{c + x}{r^2} + h(c + x) \right]_0^x + (Q_0 + 2cr_0^4 B_0) \left[ 1 - 2(c + x)h(c + x) \right] \times \left[ \frac{1}{r^2(x)} \right]_0^x + h^2(c + x), \tag{44}
\]

where \( [f(x)]_a^b := f(b) - f(a) \). This solution (see Fig. 2) really describes a black universe, but the delta-like distribution of the effective exotic matter, characterized by \( f(u) \), causes an undesirable discontinuity of \( B' \) at \( x = 0 \).

The expression for the potential \( V(x) \) is rather cumbersome and will not be presented here.

### 4.3 A model with smoothed \( f(u) \)

Evidently the qualitative behavior of the model will not change if we replace the delta-like distribution of \( f(x) \) with a smooth one but sufficiently peaked near \( x = 0 \). We will present an example of such a solution. Namely, let us preserve the notations (29), so that the field equations have the form (30)–(32); however, instead of (33), we choose the following function \( r(x) \):

\[
r^2(x) = (|x| + 1)^2 - 1, \quad |x| > d, \tag{45}
\]

\[
r^2(x) = ax^2 + s = \frac{d+1}{d} x^2 + d, \quad |x| < d, \tag{46}
\]

where \( d = \text{const} > 0 \) is sufficiently small and the constants in (46) are chosen to make \( r \) and \( r' \) continuous at \( x = \pm d \), as shown in Fig. 3.

Let us take \( f(x) \equiv 0 \) at \( |x| > d \) and, for \( |x| < d \), as in (26), \( f = 2C/r^6 \). Then from (30) we find \( Q(x) \) as follows:

\[
Q(x) = \begin{cases}
Q_0, & x < -d; \\
Q_0 + [B(x) - B(-d)]C, & |x| \leq d; \\
Q_0 + [B(d) - B(-d)]C, & x > d,
\end{cases} \tag{47}
\]

where \( Q_0 \) is an integration constant. The constant \( C \) (more precisely, its dimensionless counterpart \( \bar{C} = C/b^4 \)) is determined from Eq. (31), where we require continuity of \( \psi' \) at \( x = \pm d \). Eq. (02') for the function \( B(x) = A/r^2 \) is then solved analytically for \( |x| > d \) but only numerically for \( |x| < d \).

As a result, at \( x \leq -d \) we obtain \( B(x) \) in the form (42) with \( c = 1 \). At \( x \geq d \), an expression for \( B(x) \) is similar to (44) but obtained with initial conditions for \( B \) and \( B' \) at \( x = d \) that follow from numerical integration of Eq. (32) in the range \( |x| < d \).
Figure 5: The potential $V(x)$ obtained from a smoothed but peaked $f(u)$

are shown in Fig. 4. Fig. 5 shows the corresponding potential $V(x)$. Clearly these are black-universe models, where the asymptotic behavior of $V(x)$ approaching a positive constant as $x \to \infty$ corresponds de Sitter expansion with a positive cosmological constant.

5. Concluding remarks

We have built a family of black-universe solutions to the modified Einstein equations valid in an RS2 type brane world. They have been obtained without explicitly introducing any phantom matter. Just as was the case with wormhole solutions [19, 26], the role of exotic matter in the field equations is played by the “tidal” term of geometric origin, which has no reason to respect the energy conditions known for physically plausible matter fields.

This new kind of solutions, having a black hole nature as seen from large negative $x$, supplements the sets of known examples of both spherically symmetric brane-world black holes (see, e.g., [27, 28]) and black holes with scalar “hair” (see, e.g., [5, 6, 29]).

Let us recall that the existence of black-universe models suggests the idea that our Universe could appear from another, “mother” universe and undergo isotropization (e.g., due to particle creation) soon after crossing the horizon. The Kantowski-Sachs nature of our Universe, as opposed to the more popular spatially flat models, is not excluded observationally [31] if its isotropization had happened early enough, before the last scattering epoch (at redshifts $z \gtrsim 1000$). A tentative estimate obtained for cosmological evolution beginning from a horizon (the so-called Null Big Bang) in another kind of model, that with a vacuumlike static core, has shown [30] that such models isotropize very quickly and can now be quite observationally indistinguishable from isotropic ones. One can also notice that we are thus facing one more mechanism of universes multiplication, in addition to the well-known mechanism existing in the chaotic inflation scenario.

The presently obtained models do not pretend to be quite realistic, they simply show the possibility of such a scenario in principle. As any solutions to the effective 4D equations describing gravity on the brane, they certainly need extension to the bulk, whose finding is quite a challenging problem, although an extension always exists due to the known embedding theorems. Another problem to be solved is that of stability of such solutions.

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