3D visualization of two-phase flow in the micro-tube by a simple but effective method

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Received 22 April 2009
Published 9 July 2009
Online at stacks.iop.org/JMM/19/085005

Abstract
The present study provides a simple but effective method for 3D visualization of the two-phase flow in the micro-tube. An isosceles right-angle prism combined with a mirror located 45° bevel to the prism is employed to synchronously obtain the front and side views of the flow patterns with a single camera, where the locations of the prism and the micro-tube for clear imaging should satisfy a fixed relationship which is specified in the present study. The optical design is proven successfully by the tough visualization work at the cryogenic temperature range. The image deformation due to the refraction and geometrical configuration of the test section is quantitatively investigated. It is calculated that the image is enlarged by about 20% in inner diameter compared to the real object, which is validated by the experimental results. Meanwhile, the image deformation by adding a rectangular optical correction box outside the circular tube is comparatively investigated. It is calculated that the image is reduced by about 20% in inner diameter with a rectangular optical correction box compared to the real object. The 3D re-construction process based on the two views is conducted through three steps, which shows that the 3D visualization method can easily be applied for two-phase flow research in micro-scale channels and improves the measurement accuracy of some important parameters of the two-phase flow such as void fraction, spatial distribution of bubbles, etc. (Some figures in this article are in colour only in the electronic version)

1. Introduction
Flow pattern visualization is essential for understanding the mechanism of the two-phase flow in the micro-scale passages such as micro-tubes, micro-channels, etc. High-speed photography is regarded as one of the most popular visualization techniques in the study of the two-phase flow. Some researchers visualized the two-phase flow in the micro-tube by a backlighting arrangement, in which the glass tube was placed between the light source and the camera [1, 2]. Others used etching micro-channels in the silicon plate to conduct experiments [3, 4], in which coaxial lighting was applied in their optical systems by covering a transparent plate. However, the front view of the two-phase flow is the only information obtained in most flow pattern visualization studies in micro-scale passages, so far. The 2D images can only provide partial information and sometimes important information such as the bubble nucleation sites, bubble shapes and spatial distribution of the bubbles, which can provide an in-depth understanding of the two-phase flow, is missed or not accurately obtained. Therefore, the 3D flow pattern visualization technique, which can help to obtain the accurate spatial details, is indispensible for further quantitative investigation of the two-phase flow.

3D visualization is fulfilled by 3D re-construction which is concerned with the assembly of information obtained in a 2D plane of known orientation into a coherent and comprehensive 3D set of data [5]. The 3D re-construction is widely applied in biomedical imaging and one of the typical examples is CT (computed tomography), by which a large series of 2D x-ray
images are taken to generate a 3D image of the inside of an object. However, in the study of the two-phase flow, there is almost no successful application of 3D visualization in the open literature because of the high expenditure and technical difficulty. Some researchers tried to apply the x-ray tomography in the study of the two-phase flow; however, there was a great limitation in the temporal and spatial resolution, which could only be applicable for the stationary flow [6]. Schleicher et al [7] developed a fast optical tomography sensor to study the two-phase flow, which worked similarly to traditional CT. The information obtained by the sensor was used to re-construct the 2D images (slices). They piled up the slices of a specific cross-section at different time instants and re-constructed the 3D images. Whereas, their 3D images are not virtually the spatial 3D images and could not demonstrate the transient flow characteristics of the whole flow field. Okawa et al [8] used two synchronized video cameras to visualize the bubble dynamics in a tube of 20 mm in inner diameter. The front view and side view of the two-phase flow were recorded simultaneously. However, the conception of re-construction of the 3D images was not reported in their paper. Takamasa et al [9, 10] proposed a kind of stereo image-processing method (SIM) to measure the interfacial configuration of bubbly flow under normal and microgravity conditions. They obtained two views from two normal sides of the test section by using one video camera and four mirrors. The method was an efficient approach in enhancing the accuracy and ranges of the measurement of some important parameters in the two-phase flow such as void fraction, etc. Celata et al [11] modified the optical system of Takamasa et al [9, 10] by using four mirrors to visualize bubbles, while another four mirrors to lighten the two sides of the rectangular test section. But they did not perform further work to extend the method to other cases with different geometrical configurations such as circular tube, etc. Also the 3D re-construction was not conducted, though they assumed the bubble shaped as spheroid around the axis to calculate the volume of the bubbles.

The present study provides a simple but effective method to obtain the front view and side view of the two-phase flow in the micro-tube synchronously with only a single high-speed video camera. Image deformation due to refraction and geometrical configuration of the test section is quantitatively investigated. With the two views, the 3D re-construction process is demonstrated in detail.

2. Optical system

Figure 1(a) shows the typical visualization system of the two-phase flow in a micro-tube at the cryogenic temperature range. The optical system was placed in a high-vacuum cryogenic dewar, in which case the illumination and magnification for visualization of the two-phase flow in a micro-tube proved to be a demanding work [12]. Figure 1(b) shows the enlarged view of the 3D visualization system. Backlighting is employed, in which case the transparent glass tube is located between the light source and the camera. An isosceles right-angle prism is arranged with one of the right-angle edges close to the micro-tube. On the other side of the micro-tube, a mirror is placed at an angle of 45° bevel to the prism, which reflects the light from the light source to illuminate the side of the tube. On the focal plane of the high-speed camera, the front and side views of the two-phase flow in the micro-tube are imaged simultaneously. Figure 1(c) shows the bird’s eye view of the optical system, in which the image in the prism \( M_1N_1 \) is turned 90° clockwise compared to the object points \( M \) and \( N \), as shown in the visualization field. The image in the prism which is obtained by the camera is actually the side view of the two-phase flow in the micro-tube. With carefully adjusting the position of the prism, the front view \( AB \) and side view \( MN \) can be clearly obtained synchronously by the camera.

2.1. Derivation of the relative position of the micro-tube with respect to the prism

The following paragraphs will derive the exact location of the image with respect to the corresponding object from the perspective of the geometrical optics. Figure 2 shows the optical pathway diagram of the optical system. The incident rays \( OB \) and \( OC \) originate from the object point \( O \). The two
incident rays enter into the prism and reflect off the bevel edge of the prism. The rays exit at the points $F$ and $G$ of the other right-angle edge of the prism. The points $O_1, M_1, N_1, O, M, N$ are the corresponding image points and object points which will be recorded synchronously by the camera.

The object point $O$ and the image point $O_1$ can be clearly recorded synchronously by the camera only when the two points are imaged simultaneously on the focal plane which indicates that the following relation should be met:

$$
\frac{GO}{O_1} = \frac{GD}{D}.
$$

The distance between the exit point $G$ and the image point $O_1$ is

$$
\frac{GO}{O_1} = \frac{a + nL}{n} = a/n + L,
$$

where $n$ is the refractive index of the prism, $a$ is the length of the right-angle edge of the prism and $L$ is the distance between the points $O$ and $B$. The detailed derivation of the expression is shown in appendix A.

Equation (1) derives the following formula governing the location of the axis of the micro-tube, where point $A$ is the origin of the coordinates:

$$
y = a(n - 1)/n - x. \tag{3}
$$

Figure 3 shows the relative position according to equation (3), where the refractive index of the prism $n$ is chosen to be 1.5 as an example.

Because the micro-tube is placed on the right side of the prism and the corresponding image points should not exceed the right-angle edge of the prism, it is easy to acquire the definition domain of equation (3)

$$
x \in [r, a(n - 1)/n - r]. \tag{4}
$$

where $r$ is the outer radius of the micro-tube.

The range of the length of the right angle of the prism $a$ is derived as

$$a(n - 1)/n - r \geq r, \quad a \in [2nr/(n - 1), +\infty). \tag{5}
$$

The location of the axis of the micro-tube corresponds to the specific line segment $O'O''$ as shown in figure 3. The corresponding image location $O_1'O_1''$ is virtually on the bevel side of the prism, as shown by the red line in the figure. Note that the distance between the axis of the object $O$ and the corresponding axis of the image $O'$ is constant $(a(n - 1)/n)$. It is apparent that the image points $M_1$ and $N_1$ are turned $90^\circ$ clockwise compared to the object points $M$ and $N$.

### 2.2. Maximal magnification

For visualization of the two-phase flow in micro-scale passages, one of the most important requirements is the magnification of the image. Magnification is the value representing the size of the image captured by the camera compared to the real size of the corresponding object. For a certain prism, the maximal magnification of the image including the two views occurs when the fixed focal plane is fulfilled by $O'O_1(a(n - 1)/n)$ as shown in figure 3. Then the magnification is formulated as

$$
M = \frac{e}{OO_1 + 2r} = \frac{ne}{(n-1)a + 2nr}, \tag{6}
$$

where $e$ is the magnification, $e$ is the length of the focal plane of the camera. For prisms with different lengths, the maximal magnification comes when the prism length $a$ has its minimum equal to $2nr/(n - 1)$.
2.3. Experimental results

Figure 4 shows the two-phase flow visualization images recorded by the optical system shown in figure 1(a). The test section is a piece of the quartz glass tube with the inner diameter of 1.46 mm and is coated with a layer of transparent ITO (indium tin oxide) film which acts as the heater to boil the fluid (liquid nitrogen). The parameters of bubble dynamics, such as the nucleation sites, the bubble departure diameter and the spatial distribution of bubbles, etc, are visually measured at a temporal resolution of 1 ms and spatial resolution of 12 μm (according to the specifications of the present camera), as shown in figure 4(a). Figure 4(b) shows the liquid entrainment observed in the experiment, which results from the interface instability caused by the relative motion between the two continuous phases. The liquid phase, which has greater velocity compared to the vapor velocity, tears the interface and flows through the vapor phase when the inertial force exceeds the surface tension force, as shown by the image at 3 ms in the figure. The spatial location and the phase of the lump could not be specified only based on the front view of the two-phase flow. And the incomplete information may lead to the misinterpretation of the observed phenomenon in the experiments. With the two views, the liquid lump can be verified and the location and shape of the liquid lump can be visually measured.

The two-view optical system has been proven successful by the demanding cryogenic visualization work and is of course applicable for room temperature visualization experiments conveniently. The optical system can also be applied for the visualization of the two-phase flow in the micro-channel with other non-circular cross-section configurations like rectangular, etc.

3. Image deformation due to refraction and geometrical configurations of the test section

Kawahara et al [13] investigated two-phase flow characteristics in a 100 μm diameter circular tube and found that the inner diameter was almost enlarged by 50% due to optical distortion.
However, they did not perform further quantitative study of the enlargement in inner diameter. The image deformation, i.e., the deviation of the image from the corresponding object, is quantitatively investigated in the present study. Figure 5 indicates that the obtained images are greatly enlarged in inner diameter compared to the real situation. The inner diameter of the micro-tube obtained from the image is 1.73 mm, which is evaluated by counting the number of pixels that the inner diameter covers in the image, whereas the measured inner diameter is 1.46 mm, indicating that the image is enlarged by about 18.5% in inner diameter. The image enlargement compared to the real situation depends on two factors: (1) the different refractive indexes of the glass wall and the fluid; (2) the geometrical configuration of the test section. Figure 6 shows the bird’s eye view of the optical path concerning the image deformation. The outer diameter of the tube is \( r_o \) and the inner diameter is \( r\), and the refractive indexes of air outside the tube, the glass and the fluid flowing in the tube are \( n_0, n_1 \) and \( n_2 \), respectively. The incident ray \( AB \) from the object point \( A \) undergoes twice refraction on the inner and outer surfaces of the glass tube as it penetrates through the curved glass wall, and then exits the glass tube as ray \( CE \) in the parallel direction. \( s \) denotes the distance between the center \( O \) and the object point \( A \); \( s' \) denotes the distance between the center \( O \) and the image point \( A' \).

Apparently, the image point \( A' \) is deviated farther away from the center \( O \) compared to the object point \( A \). The detailed derivation of the relationship between \( s \) and \( s' \) is shown in appendix B. The result is shown in figure 7(a), which demonstrates the relationship between the image points and the corresponding object points along the radius. The refractive indexes of the glass wall \( (n_1) \) and liquid nitrogen \( (n_2) \) are 1.517 and 1.2053, respectively. In the figure, the image is enlarged by about 20% in inner diameter compared to the real situation, which agrees well with the experimental results shown in figure 5. It is also shown that the deviation of the image point from the corresponding object point can be reduced as the refractive index of the fluid decreases. The deviation could be neglected when the fluid is gaseous nitrogen \( (n_2 = 1.000297) \) as shown in the figure. The effect of the ratio of the wall thickness to the inner radius \( (d/ri = 0.1, 0.01) \) is plotted in the figure. And it is found that the ratio has no apparent effect on the deviation and can be neglected when the object point is in the range of \( s/r_i < 0.7 \). However, when the object point approaches to the inner wall \( (0.7 < s/r_i < 1.0) \), the effect of the ratio on the deviation is notable.

Kawahara et al. [13] suggested an optical rectangular box to correct the image deformation, which was filled with the same fluid or with some material of the same refractive index as that flowing in the tube. This method is mostly adopted in the conventional cases, in which the deformation due to the curved wall can be almost avoided by this method. However, in the micro-scale cases, installing an optical box filled with some kind of material is complicated and even unacceptable in some cases as the glass tube coated with a transparent heating.
layer. So the optical correction box without filling material, which is more easily realized, is chosen for analysis. The optical path and detailed derivation process for the case of a rectangular optical correction box are shown in appendix B.

Figure 8(a) shows the results for the cases with the rectangular optical correction box. Inversely, the image is reduced in inner diameter compared to the real object. For the case of $n_2 = 1.2053$, the image points have about 20% deviation from the corresponding object points. While for the case of $n_2 = 1.000297$, the image points could even be deviated by 35% in inner diameter. Figure 8(b) indicates that the size of a radius has no effect on the deviation. For radius as small as 0.73 mm and as large as 100 mm, the deviation of image points from the real object points is the same.

Monochromatic light is the ideal option for the illumination in the optical system with the prism. If white light is employed, it should be addressed that the chromatic aberration occurs due to the decomposition of white light by the prism. Actually, the white light is employed in the present study which may bring about chromatic aberration. For the prism used in the present study, the refractive indexes of red, yellow and blue light are 1.514, 1.517 and 1.522, respectively. The effect of chromatic aberration on the clear imaging is obtained by applying the three refractive indexes into equation (3), detailed derivation of which is shown in appendix C. It is seen in figure 9 that the chromatic aberration causes 0.59% deviation from the case of yellow light, where the length of the right angle edge of the prism is 12 mm. The calculated deviation distance of 0.048 mm is within the range of depth of field (DOF) (0.396 mm) in the present optical system. Therefore, the unsharpness of the image caused by chromatic aberration is imperceptible and the white light can be selected as the light source in the present study.

The region, where the front and side views of the flow patterns can be clearly recorded synchronously, can be extended based on equation (3) according to the depth of field:

$$ y = a(n - 1)/n - x \pm k, $$

where $k$ is the depth of field of the lens in the present study. The corresponding two curves are plotted in figure 9 in the case of $k$ equal to 0.396 mm. It shows that clear image can be obtained in the region bordered by the two curves of $\pm 9.6\%$.

4. Re-construction of 3D image

3D image is re-constructed by taking a series of 2D images (slices). Generally, there are two methods to take slices: one is to get the images of a specific cross-section at different time instants, then re-construct the 3D image by piling up these images; the other is to obtain the images of different cross-sections at different time instants, which is mainly applied to micro-CT in biomedical science. However, the former one is less used in the two-phase flow of the micro-tube because the re-constructed image is not actually spatial 3D image and beyond the interest of the study of the two-phase flow. The latter method is actually static 3D re-construction and can only be applicable for the stationary flow because of the limitation of spatial and temporal resolution. There is almost no report about the application of the latter method to the transient two-phase flow study in the micro-scale passages. The present study provides a simple but effective method to produce slices.
Figure 10. 3D re-construction step 1: the raw image is segmented first and then binarized by a series of image processing algorithms. The 3D re-construction process is divided into three steps: (1) image segmentation and binarization; (2) taking elliptical slices; and (3) piling up slices and re-constructing 3D image.

(1) For raw images shown in figure 10(a), the background luminance for the front view and side view can be apparently different due to different illumination intensity. Therefore, the image is segmented and different thresholds are applied to the two views in the binary image processing, as shown in figure 10(b). Then a series of image processing algorithms are adopted. Median filter and other filtering algorithms are combined to eliminate the background noises including the stains and scratch on the tube wall. The border of the binary image is extracted. Then the expansion matching method based on the morphology is applied to close the border. Afterward, the existing holes in the contour are filled. There are still other isolated lines or points connected to the border. The border clearing algorithm is used to clear the border and the final result of step 1 is shown in figure 10(c). It is seen that vapor regions such as small bubbles and elongated bubbles are specified by white color, which are ready for taking slices in the next step. It should be pointed out that some uncertainties may be introduced in step 1. The uncertainty is mainly caused by different thresholds for the image binarization, which may cause a maximal uncertainty of 2.3% in major and minor axes.

(2) The shape of the bubble of the two-phase flow is mainly determined by the surface tension force, which acts to minimize the interfacial area and tends to keep bubbles retaining the spheroid shapes. Therefore, it is reasonable to suppose that the circumference of the horizontal cross-section of the bubble shapes elliptically. For obtaining the bubble slices, each row of the binary image is scanned from left to right at the spatial resolution of one pixel, during which the lengths of the continuous white pixels are calculated. For the case in figure 10(c), there are

Figure 11. 3D re-construction step 2: elliptical slice is drawn based on the major axis and minor axis obtained from figure 10(c); the two slices shown in (b) are the corresponding bird’s eye views in (a); all slices are produced in the pixel coordinate system. (The units of the co-ordinates are in pixels.)
two segments of continuous white pixels for each row of scan, by which the major axis and minor axis can be determined along with the origin of the ellipse. One slice of the bubble is then accomplished by drawing elliptical contour, as shown in figure 11(a). And the process is repeated for each row from top to bottom until the whole image is scanned. There are total 716 scans, in which 457 elliptical slice planes obtained. The spatial distribution of some slices is shown in figure 11(b). The data typically containing all the slice planes are saved for further process.

(3) The 3D image can be obtained by piling up these slices around a specified axis using the patch algorithm. The 3D image can be displayed in any orientation. Smooth lighting is added and the tube wall is incorporated to locate the two-phase flow, as shown in figure 12(a).

The image deformation correction demonstrated in section 3 can be incorporated in the 3D re-construction process. The elliptical slices taken in step 2 are corrected according to the deviation calculation in section 3. So the modified slices of the real object are obtained for 3D re-construction in step 3. Figure 12(b) shows the corrected 3D real object. There are other methods to correct the image deformation such as adding a rectangular optical box to the test section, designing special lens for the image recording system, etc. Compared to these methods, the 3D re-construction method shown here is simpler and does not need complicated modification to the test section. Moreover, the re-constructed 3D image can demonstrate more detailed information in three-dimensional space, which could be valuable in improving the measurement accuracy of some parameters such as void fraction, spatial distribution of bubbles, etc.

The advantage and limitation of the above 3D re-construction method should be addressed. Generally, the surface tension force is dominant compared to other forces in the two-phase flow in micro-scale passages. So ellipse assumption of the bubble shape and stacking enough slices can be used to approximate the bubble, and 3D re-construction method can have good accuracy and benefits to quantitative investigation of bubble dynamics in micro-tubes. The method can also be extended to the two-phase flow system in which the shape of the dispersed phase can be predicted. However, the above method is limited in some unusual cases of a two-phase flow system, in which the dispersed phase such as bubbles or droplets has irregular and unpredictable shapes.

5. Summary

The present study provides a simple but effective method for 3D visualization of the two-phase flow in the micro-tube. Installing an isosceles right-angle prism and a mirror on the two sides of the micro-tube, the front view and side view of the two-phase flow in the micro-tube are obtained synchronously by a single high-speed video camera. The relative position of the micro-tube with respect to the prism is specified, where the axis of the image is on the bevel edge of the prism and the distance between the axis of the object and the corresponding

![Figure 12. 3D re-construction step 3: the 3D image (left) is re-constructed by piling up slices from figure 11; according to the deviation calculation in section 3, the corresponding major axis and minor axis obtained from figure 10(c) can be rectified to produce new elliptical slices, which are piled up to produce the corrected 3D real object (right). (The units of the co-ordinates are in pixels.)](image)
axis of the image is constant \((a(n-1)/n)\). The optical design is proven successfully by the visualization study of flow boiling of liquid nitrogen in a micro-tube.

The image deformation due to refraction and geometrical configuration of the test section is quantitatively investigated. The image is enlarged by about 20% in inner diameter compared to the real situation for the case of liquid nitrogen in the present study. As the refractive index of the fluid increases, the image is enlarged more greatly for the case of a circular tube. The image deformation by adding a rectangular optical correction box outside the circular tube is comparatively investigated. It is found that the image is reduced by about 20% in inner diameter with a rectangular optical correction box compared to the real situation. Moreover, the image is reduced more greatly in inner diameter as the refractive index of the fluid decreases. The size of the inner radius is found to have no apparent effect on the image deformation.

The 3D re-construction process is divided into three steps. First, the image is segmented into two regions and then binarized. Afterward, slices are taken by scanning the whole image based on the ellipse assumption of the bubble shapes. Finally, the 3D image is obtained by piling up slices. The deformation can be corrected in the 3D re-construction, which can improve the measurement accuracy of some important parameters of the two-phase flow such as void fraction, spatial distribution of bubble, etc.

Acknowledgments

This research is supported by National Natural Science Foundation of China under contract no 50776057 and NSFC-JSPS co-operative project under contract no 50911140104.

Appendix A

Figure A1 shows the optical pathway diagram of the optical system. The incident rays \(OB\) and \(OC\) originate from the object point \(O\). The two incident rays enter into the prism and reflect off the bevel edge of the prism. Finally, the rays exit at the points \(F\) and \(G\) of the right-angle edge. The points \(O'\) and \(O_2\) are two image points that can be observed in the prism. The points \(O_1\) and \(O\) are the corresponding image point and the object point which will be recorded by the camera.

The ray \(OB\) is normal to the receiving surface and the corresponding incident angle and the refractive angle are equal. For the ray \(OC\), the incident angle and the refractive angle are \(i\) and \(i'\) respectively, satisfying the law of refraction as below:

\[
\frac{\sin i}{\sin i'} = n, \quad (A.1)
\]

where \(n\) is the refractive index of the prism.

The distance between points \(B\) and \(C\) is \(h\), which is involved in the following expressions:

\[
\tan i = \frac{h}{L} \quad \text{and} \quad \tan i' = \frac{h}{L'}, \quad (A.2)
\]

where \(L\) is the distance between the points \(O\) and \(B\), and the distance from the image point \(O'\) to the right-angle edge of the prism is \(L'\), which is expressed as

\[
L' = \frac{\tan i}{\tan i'}L = L\frac{n\sqrt{1-n\sin^2 i}}{\cos i}. \quad (A.3)
\]

For paraxial rays \((i \approx 0), \cos i \approx 1 \text{ and } \sin^2 i \approx 0\).

Equation (A.3) can be reduced to

\[
L' = nL. \quad (A.4)
\]

Both rays reflect off the bevel edge of the prism and follow the law of reflection:

\[
O_2D = O'D = L' + y, \quad (A.5)
\]

where \(y\) is the distance between the points \(A\) and \(B\). \(A\) is the origin of the coordinates, where \(y\) increases downward.

The ray \(OB\) exits at point \(G\) in the direction of normal to another right-angle edge of the prism, and the other ray \(OC\) exits at an angle of \(i\):

\[
\tan i = \frac{GF}{GO_1}, \quad \tan i' = \frac{GF}{GO_2} \quad (A.6)
\]

and \(GO_2\) is given by:

\[
GO_2 = \frac{\tan i}{\tan i'}GO_1 = GO_1\frac{n\sqrt{1-n\sin^2 i}}{\cos i} \quad (A.7)
\]

The distance between the exit point \(G\) and the image point \(O_2\) is

\[
GO_2 = GD + O_2D = GD + L' + y = a + nL, \quad (A.9)
\]

where \(a\) is the length of the right-angle edge of the prism.

The distance between the exit point \(G\) and the image point \(O_1\) is

\[
GO_1 = (a + nL)/n = a/n + L. \quad (A.10)
\]
The object point $O$ and the image point $O_i$ can be clearly recorded synchronously by the camera when the two points are at the same horizontal line, i.e.

$$GO_i = GD$$  \hspace{1cm} (A.11)

$$a/n + L = a - y.$$  \hspace{1cm} (A.12)

Then the following formula, which governs the location of the axis of the micro-tube, can be written as

$$y = a(n - 1)/n - x.$$  \hspace{1cm} (A.13)

**Appendix B**

Figure 6 shows the bird’s eye view of the optical path showing the image deformation. The angle of incidence $i$ is given by

$$i = \arcsin(s'/r_o).$$  \hspace{1cm} (B.1)

For the ray $EC$, the incident angle and the refractive angle are $i$ and $i_1$ respectively, satisfying the law of refraction as below:

$$\sin i \times n_0 = \sin i_1 \times n_1$$ \hspace{1cm} (B.2)

$$i_1 = \arcsin((s'/r_o) \times n_0/n_1),$$  \hspace{1cm} (B.3)

where $n_0$ and $n_1$ are the refractive indexes of air and the glass, respectively.

Prolong the lines $EC$ and $CB$ toward east to intersect the line $OF$ at the points $A'$ and $A''$. Then apply the law of sines to the triangle $OCA''$,

$$\frac{x''}{\sin i_1} = \frac{r_o}{\sin \theta},$$  \hspace{1cm} (B.4)

where

$$\theta = \pi/2 - i_1 + i$$  \hspace{1cm} (B.5)

and $x''$ is given by

$$x'' = \frac{r_o \sin i_1}{\cos(i_1 - i)}.$$  \hspace{1cm} (B.6)

The ray $CB$ is refracted at the inner surface of the tube,

$$\sin i_2 \times n_1 = \sin i_3 \times n_2.$$  \hspace{1cm} (B.7)

Applying the law of sines to the triangle $OBA''$,

$$\frac{s''}{\sin i_2} = \frac{r_i}{\sin \theta}.$$  \hspace{1cm} (B.8)

Obtaining

$$i_2 = \arcsin\left(\frac{s'' \sin \theta}{r_i}\right)$$  \hspace{1cm} (B.9)

$$i_3 = \arcsin\left(\frac{n_1 \sin i_2}{n_2}\right).$$  \hspace{1cm} (B.10)

$s$ is obtained by applying the law of sines to the triangle $OBA$:

$$s = \frac{r_i \sin i_3}{\sin(\theta + i_2 - i_3)}.$$  \hspace{1cm} (B.11)

Figure B1 shows the optical path of the method with a rectangular optical correction box. The parallel ray $EB$ enters into the glass box and experiences refraction at the inner curve wall, and finally intersects the line $OF$ at the point $A$. The point $A$ is the object point.

The angle of incidence $i$ is as follows:

$$i = \arcsin(s'/r).$$  \hspace{1cm} (B.12)

For the ray $EB$, the incident angle and the refractive angle are $i$ and $i_1$ respectively, satisfying the law of refraction as below:

$$\sin i \times n_1 = \sin i_1 \times n_2$$  \hspace{1cm} (B.13)

$$i_1 = \arcsin((s'/r) \times n_1/n_2).$$  \hspace{1cm} (B.14)

Prolong the line $EB$ toward east to intersect the line $OF$ at the point $A'$, which is the image point. Then apply the law of sines to the triangle $OAB$,

$$\frac{s}{\sin i_1} = \frac{r}{\sin \theta}.$$  \hspace{1cm} (B.15)

$$s = \frac{r \sin i_1}{\sin \theta},$$  \hspace{1cm} (B.16)

where

$$\theta = \pi/2 - i_1 + i.$$  \hspace{1cm} (B.17)

**Appendix C**

The Abbe number ($V$) is adapted to assess the chromatic aberration,

$$V = \frac{n_D - 1}{n_F - n_C},$$  \hspace{1cm} (C.1)

where $n_D$ is the standard refractive index which corresponds to yellow light to which the naked eye is most sensitive; $n_F$ is the refraction index of blue light; and $n_C$ is the refraction index of red light.

The Cauchy equation is as follows:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}.$$  \hspace{1cm} (C.2)

where

$$A = n_D - \frac{B}{\lambda_D^2}, \hspace{1cm} B = \frac{(n_D - 1)\lambda_C^2}{V(\lambda_C - \lambda_F)}.$$  \hspace{1cm} (C.3)

$\lambda_C$ is the wavelength of red light, $\lambda_F$ is the wavelength of blue light and $\lambda_D$ is the wavelength of yellow light.
According to the Schott® Glass database, which offers the Abbe number and standard refractive index for various types of glass, the parameters for the prism of the present study are Type: BK7, \( n_D = 1.51680 \) and \( V = 64.17 \). Other parameters are calculated by neglecting the third term on the right-hand side of equation (C.2) as follows:

\[
A = \frac{4.2173773 \times 10^3}{587.5618^2} = 1.5045838 \quad \text{(C.4)}
\]

\[
B = \frac{(1.51680 - 1) \times 656.2725^2 \times 486.1327^2}{64.17 \times (656.2725^2 - 486.1327^2)} = 4.2173773 \times 10^3 \text{ mm}^2 \quad \text{(C.5)}
\]

\[
n_C = 1.5045838 + \frac{4.2173773 \times 10^3}{656.2725^2} = 1.514 \quad \text{(C.6)}
\]

\[
n_F = 1.5045838 + \frac{4.2173773 \times 10^3}{486.1327^2} = 1.522. \quad \text{(C.7)}
\]

Substituting the calculated refractive indexes \((n_C, n_D, n_F)\) to equation (A.13) in appendix A results in three equations. The corresponding three curves for the equations are shown in figure 9, where the length of the right-angle edge of the prism is 12 mm. Maximal deviation due to chromatic aberration is estimated as

\[
\overline{GO_1(n_C)} - \overline{GO_1(n_F)} = a(1/n_C - 1/n_F) = 0.048 \text{ mm}, \quad \text{(C.8)}
\]

where \(\overline{GO_1}\) is according to equation (A.10) in appendix A.

For the chosen macro lens in the present optical system, the DOF is 0.396 mm for the case of \( f/2.8 \) aperture and the magnification of \( 1 \times \). The calculated deviation distance of 0.048 mm is within the range of DOF (0.396 mm) in the present optical system. Therefore, the unsharpness of the image caused by chromatic aberration is imperceptible and the white light can be selected as the light source in the present study.

The uncertainty of the region where the micro-tube can be installed for clear imaging is estimated based on DOF.

\[
y = a(n - 1)/n - x \pm k, \quad \text{(C.9)}
\]

where \( k \) is the DOF of the lens in the present study. The corresponding two curves are plotted in figure 9 in the case of \( k \) equal to 0.396 mm. It shows that clear image can be obtained in the region bordered by the two curves of \( \pm 9.6\%\).

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