Bose-Einstein Condensation of Pions in High Multiplicity Events

V.V. Begun\textsuperscript{1,2} and M.I. Gorenstein\textsuperscript{2,3}

\textsuperscript{1}Museo Storico della Fisica e Centro Studi e Ricerche Enrico Fermi, Rome, Italy
\textsuperscript{2}Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
\textsuperscript{3}Frankfurt Institute for Advanced Studies, Frankfurt, Germany

We present microcanonical ensemble calculations of particle number fluctuations in the ideal pion gas approaching Bose-Einstein condensation. In the samples of events with a fixed number of all pions, \(N_\pi\), one may observe a prominent signal. When \(N_\pi\) increases the scaled variances for particle number fluctuations of both neutral and charged pions increase dramatically in the vicinity of the Bose-Einstein condensation line. As an example, the estimates are presented for \(p+p\) collisions at the beam energy of 70 GeV.

PACS numbers: 24.10.Pa, 24.60.Ky, 25.75.-q

Keywords: Bose-Einstein condensation, high pion multiplicities, anomalous fluctuations

The phenomenon of Bose-Einstein condensation (BEC) was predicted long time ago \cite{1}. Tremendous efforts were required to confirm BEC experimentally. The atomic gases are transformed into a liquid or solid before reaching the BEC point. The only way to avoid this is to consider extremely low densities. At these conditions the thermal equilibrium in the atomic gas is reached much faster than the chemical equilibrium. The life time of the metastable gas phase is stretched to seconds or minutes. This is enough to observe the BEC signatures. Small density leads, however, to metastable state (in thermal, but not chemical equilibrium) to reach the BEC line. This is to consider extremely low densities. At these \cite{4,5,6}, known in high energy nucleus-nucleus collisions, as well as in the elementary particle ones. There were several suggestions to search for BEC of \(\pi\)-mesons \cite{2}. However, complete statistical mechanics calculations of pion number fluctuations have never been presented. There is a qualitative difference in properties of the mean multiplicity and of the scaled variance of multiplicity fluctuations in different statistical ensembles. The results obtained with grand canonical ensemble (GCE), canonical ensemble (CE), and microcanonical ensemble (MCE) for the mean multiplicity approach to each other in the large volume limit. This reflects the thermodynamic equivalence of the statistical ensembles. Recently it has been found \cite{4,5,6} that corresponding results for the scaled variance are different in different ensembles, and this difference is preserved in the thermodynamic limit. To extract the matter properties from analysis of event-by-event fluctuations, one needs to fix the samples of high energy events, and choose the corresponding statistical ensemble for their analysis. This is discussed below.

Let us start with a well known example of non-relativistic ideal Bose gas. The occupation numbers, \(n_p\), of single quantum states, labelled by 3-momenta \(\mathbf{p}\), are equal to \(n_p = 0, 1, \ldots, \infty\). In the GCE their average values, fluctuations, and correlations are the following \cite{7}:

\[
\langle n_p \rangle = \frac{1}{\exp \left( \frac{\mathbf{p}^2}{2m} - \mu \right)/T - 1},
\]

\[
\langle (\Delta n_p)^2 \rangle = \langle n_p \rangle (1 + \langle n_p \rangle) \equiv v_p^2,
\]

\[
\langle n_p n_k \rangle = v_p^2 \delta_{pk},
\]

where \(\Delta n_p \equiv n_p - \langle n_p \rangle\), \(m\) denotes the particle mass, \(T\) and \(\mu\) are the system temperature and chemical potential, respectively (throughout the paper we use the units with \(\hbar = c = k = 1\)). The average number of particles in the GCE reads \cite{7}:

\[
\langle N \rangle \equiv \mathcal{N}(V, T, \mu) = \sum_p \langle n_p \rangle = \frac{V}{2\pi^2} \int_0^\infty \frac{p^2dp}{\exp \left( \frac{\mathbf{p}^2}{2m} - \mu \right)/T - 1},
\]

where \(V\) is the system volume. We consider particles with spin equal to zero, thus the degeneracy factor equals 1. In the thermodynamic limit, \(V \to \infty\), the sum over momentum states is transformed into the momentum integral, \(\sum_p \ldots = (V/2\pi^2) \int_0^\infty \ldots p^2dp\). This substitution, assumed in all formulae below, is valid if the chemical potential in the non-relativistic Bose gas is restricted as \(\mu < 0\) (or \(\mu < m\) in relativistic formulation). When the temperature \(T\) decreases at fixed ratio, \(\langle N \rangle/V\), the chemical potential \(\mu\) increases and becomes equal to zero at \(T = T_C\), known as the BEC temperature. At this point from Eq. (2) one finds, \(\mathcal{N}(V, T=T_C, \mu=0) = V[mT_C/(2\pi)]^{3/2} \zeta(3/2)\), where \(\zeta(3/2) \approx 2.612\) is the Riemann zeta-function.
At $\mu = 0$ and $T < T_C$, a macroscopic part, $N_0$ (called the BE condensate), of the total particle number occupies the lowest energy level $p = 0$

Introducing $\Delta N \equiv N - \langle N \rangle$ one finds the particle number fluctuations in the GCE,

$$\langle (\Delta N)^2 \rangle = \sum_{\mathbf{p}, \mathbf{k}} \langle \Delta n_{\mathbf{p}} \Delta n_{\mathbf{k}} \rangle = \sum_{\mathbf{p}} v_{\mathbf{p}}^2 , \quad (3)$$

$$\omega \equiv \frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle} = \frac{\sum_{\mathbf{p}} v_{\mathbf{p}}^2}{\sum_{\mathbf{p}} \langle n_{\mathbf{p}} \rangle} = 1 + \frac{\sum_{\mathbf{p}} \langle n_{\mathbf{p}} \rangle^2}{\sum_{\mathbf{p}} \langle n_{\mathbf{p}} \rangle} . \quad (4)$$

The limit $-\mu/T \gg 1$ gives $\langle n_{\mathbf{p}} \rangle \ll 1$. This corresponds to the Boltzmann approximation, and then from Eqs. (2) it follows: $N(V, T, \mu) \approx V \exp(\mu/T)(mT/2\pi)^{3/2}$ and $\omega \approx 1$. When $\mu$ increases the scaled variance $\omega$ becomes larger, $\omega > 1$. This is the well known Bose enhancement effect for the particle number fluctuations. From Eq. (4) at $\mu \to 0$ one finds $\omega \to \infty$. Two comments are appropriate. First, for finite systems $\omega$ remains finite, and $\omega = \infty$ emerges from Eq. (4) at $\mu = 0$ in the thermodynamic limit $V \to \infty$, when the sums over $\mathbf{p}$ are transformed into the momentum integrals. Second, the anomalous fluctuations of the particle number at the BEC point correspond to the GCE description. In the CE and MCE, the number of particles $N$ in a non-relativistic system is fixed by definition, thus, $\omega_{\text{MCE}, \text{CE}} = \omega_{\text{MCE}, \text{CE}} = 0$. At $\mu = 0$ and $T < T_C$ the average number of particles in the BE condensate is proportional to $1 - (T/T_C)^{3/2}$ and it remains the same in the CE and MCE at $V \to \infty$. The fluctuations of $N_0$ are, however, very different and strongly suppressed in the CE and MCE.

We consider now the relativistic ideal gas of pions,

$$\langle n_{\mathbf{p},j} \rangle = \frac{1}{\exp((\mathbf{p}^2 + m_{\pi}^2 - \mu_j)/T) - 1} , \quad (5)$$

where index $j$ enumerates 3 isospin pion states, $\pi^\pm$, $\pi^0$, and $n_0$, the energy of one-particle states is taken as, $\epsilon_{\mathbf{p}} = (\mathbf{p}^2 + m_{\pi}^2)^{1/2}$ with $m_{\pi} \approx 140$ MeV being the pion mass (we neglect a small difference between the masses of charged and neutral pions). The inequality $\mu_j \leq m_{\pi}$ is a general restriction in the relativistic Bose gas, and $\mu_j \equiv m_{\pi}$ corresponds to the BEC. In Ref. [2] we discussed in details the Bose gas with one conserved charge in the CE ($V, T, Q = \text{const}$), i.e. the $\pi^\pm\pi^-$gas with fixed electric charge. This corresponds to the GCE ($V, T, \mu_Q$), thus, in Eq. (5) $\mu_+ = \mu_Q$ and $\mu_- = -\mu_Q$ for $\pi^+$ and $\pi^-$, respectively. Approaching the BEC of $\pi^+$ at $\mu_+ \to m_{\pi}$, one finds the relation between $T_C$ and $\rho_0 \equiv \rho_{\pi^+} - \rho_{\pi^-}$ (the picture of BEC of $\pi^-$ at $Q < 0$ and $\mu_0 \equiv -m_{\pi}$ is obtained by a mirror reflection). At $T_C/m_{\pi} \ll 1$ it coincides with the non-relativistic formula, $T_C \approx 3.31 \rho_Q^{2/3} m_{\pi}^{-1}$, and at $T_C/m_{\pi} \gg 1$ it gives $T_C \approx 1.73 \rho_Q^{1/2} m_{\pi}^{-1/2}$ (see, e.g., Ref. [8]). The scaled variance $\omega^\pi \equiv \langle (\Delta N_\pi)^2 \rangle/(\langle N_\pi \rangle)$ in the GCE goes to infinity. This is similar to the non-relativistic case. On the other hand, the scaled variance for negative particles, $\omega^- \equiv \langle (\Delta N^-)^2 \rangle/(\langle N^- \rangle)$, remains finite and even decreases with $\mu_Q$. The pion numbers $N_\pi$ and $N^-$ fluctuate in the both GCE and CE. However, the exact conservation imposed in the CE on the system charge, $Q = N_+ - N_-$, suppresses anomalous fluctuations at the BEC point: $\omega^-_{\text{CE}}$ is finite with the upper limit, $\langle \zeta(2) \rangle/\langle \zeta(3) \rangle \approx 3.68$, reached at $T_C/m \to \infty$ (see details in Ref. [3]).

In what follows we discuss a rather different pion system. In the MCE ($V, E, Q = 0, N_\pi = \text{const}$) formulation, the total system energy $E$, electric charge $Q = N_+ - N_-$, and total number of pions $N_\pi = N_0 + N_++N_-$ will be fixed. This corresponds to the GCE ($V, T, \mu_Q = 0, \mu_{\pi}$) description with $\mu_+ = \mu_+ + \mu_Q$, $\mu_- = \mu_- - \mu_Q$, and $\mu_0 = 0$ in Eq. (5). We restrict $\mu_Q = 0$ and consider BEC when $\mu_{\pi} \to m_{\pi}$. The $\mu_0 = 0$ corresponds to zero electric charge, $Q = 0$ or $N_+ = N_{\pi}$, in the pion system.

The pion density is equal to $\rho_{\pi}(T, \mu_{\pi}) = \sum_{\mathbf{p},j} \langle n_{\mathbf{p},j} \rangle/V$. The phase diagram of the ideal pion gas in $\rho_{\pi} - T$ plane is presented in Fig. 4. BEC starts at $T = T_C$ when $\mu_{\pi} = \mu_{\pi_{\pi}} = m_{\pi}$. It gives:

$$\rho_{\pi}(T = T_C, \mu_{\pi} = m_{\pi}) = \frac{3T_C m_{\pi}^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} K_2(n m_{\pi}/T_C) \exp(n m_{\pi}/T_C), \quad (6)$$

where $K_2$ is the modified Hankel function. The Eq. (6) gives the BEC line shown by the solid line in Fig. 1. If $T_C/m_{\pi} \ll 1$, from Eq. (6) one finds, $T_C \approx 2\pi [(\zeta(3)/2)]^{-2/3} \rho_{\pi}^{2/3} m_{\pi}^{-1}$. This corresponds to the non-relativistic limit discussed above, but with a degeneracy factor $g_{\pi} = 3$. In the ultrarelativistic limit, $T_C/m_{\pi} \gg 1$, from Eq. (6) it follows: $T_C \approx [\pi^2/3 \zeta(3)]^{1/3} \rho_{\pi}^{1/3}$. We consider the region in $\rho_{\pi} - T$ plane between the $\mu_{\pi}=0$ and $\mu_{\pi} = m_{\pi}$ lines. The lines of fixed energy density, $\varepsilon(T, \mu_{\pi}) = \sum_{\mathbf{p},j} \epsilon_{\mathbf{p}} \langle n_{\mathbf{p},j} \rangle/V$, are shown as dotted lines in Fig. 4 inside this region for three fixed values of $\varepsilon$. An increase of $\rho_{\pi}$ at constant $\varepsilon$ leads to the increase of $\mu_{\pi}$ and decrease of $T$. In this letter we discuss how the system approaches the BEC line ($\mu_{\pi} = m_{\pi}, T = T_C$), and do not touch the region ($\mu_{\pi} = m_{\pi}, T < T_C$) below this line where the non-zero BE condensate is formed. The GCE ($V, T, \mu_Q, \mu_{\pi})$, MCE ($V, E, Q, N_\pi$), and CE ($V, T, Q, N_\pi$) are equivalent for average quantities, including average particle multiplicities, in the thermodynamic limit. Thus, Eq. (4) and phase diagram in Fig. 1 remain the same in all statistical ensembles. However, the pion number fluctuations are very different in different ensembles. As an example we consider the high multiplicity events in $p + p$ collisions at RIA accelerator with the beam energy of 70 GeV (see Ref. [14] on the experimental project “Thermalization”, team leader V.A. Nikitin). In the reaction $p + p \rightarrow p + p + N_\pi$ with small final proton momenta in the c.m.s., the total c.m. energy of created pions is $E \approx \sqrt{s} - 2m_p \approx 9.7$ GeV. The trigger system designed at JINR (Dubna) selects the events with $N_\pi > 20$ in this reaction. This makes it pos-
Bose-Einstein Condensate

FIG. 1: The phase diagram of the pion gas with $\mu_Q = 0$. The dashed line corresponds to $\rho_\pi(T, \mu_\pi = 0)$, and the solid line to BEC [6]. The dotted lines show the states with fixed energy densities: $\varepsilon = 6, 20, 60$ MeV/fm$^3$. The $N\pi$ numbers in the figure correspond to $\mu_\pi = 0$ and $\mu_\pi = m_\pi$ at these energy densities for the total pion energy, $E = 9.7$ GeV.

Possible to accumulate the samples of events with fixed $N\pi = 30 \div 50$ and the full pion identification during the next 2 years [1]. Note that for this reaction the kinematic limit is $N_{\pi,\text{max}} = E/m_\pi \approx 70$. The pion system in the thermal equilibrium is expected to be formed for high multiplicities. The volume of the pion gas system is estimated as, $V = E/\varepsilon(T, \mu_\pi)$, and the number of pions equals to $N_\pi = V\rho_\pi(T, \mu_\pi)$. The values of $N_\pi$ at $\mu_\pi = 0$ and $\mu_\pi = m_\pi$ for 3 different values of energy density $\varepsilon$ are shown in Fig. 1.

For $\Omega = 0$, the average pion multiplicities, $\langle N_\pi \rangle = \langle N_{\pi,\pm} \rangle = N_\pi/3$, are the same in all statistical ensembles for large systems. This thermodynamic equivalence is not, however, valid for the scaled variances of pion fluctuations. The system with the fixed electric charge, $Q = 0$, the total pion number, $N_\pi$, and total energy of the pion system, $E$, should be treated in the MCE. The volume $V$ is one more (and unknown) MCE parameter. The calculations below are carried out in a large volume limit, thus, parameter $V$ does not enter explicitly in the formulae for the scaled variances. The microscopic correlators for the MCE $(V, E, Q = 0, N_\pi)$ equal to

\[
\langle \Delta n_{p,j} \Delta n_{k,i} \rangle_{\text{m.c.e.}} = \nu_{p,j}^2 \delta_{pk} \delta_{ji} - \nu_{p,j}^2 \nu_{k,i}^2
\]

\[
\times \left[ \frac{q_i q_j}{\Delta(q^2)} + \frac{\Delta(q^2) + c_p c_k}{\Delta(q^2)} \Delta(\pi^2) - \frac{(c_p + c_k)(\Delta(\pi^2))}{\Delta(\pi^2)} \right]
\]

(7)

where $q_i = 1, q_j = -1, q_0 = 0; \nu_{p,j}^2 = (1 + \langle n_{p,j} \rangle) \langle n_{p,j} \rangle; \Delta(q^2) = \sum_{p,j} q_i^2 \nu_{p,j}^2, \Delta(\pi^2) = \sum_{p,j} \nu_{p,j}^2, \Delta(\pi^2) = \sum_{p,j} \nu_{p,j}^2 \delta_{j,k}; \Delta(\pi) = \sum_{p,j} c_p c_k \nu_{p,j}^2$. Note that the first term in the r.h.s. of Eq. (7) corresponds to the GCE. Correlations between differently charged pions, $j \neq i$, and between different single modes, $p \neq k$, are absent in the GCE. It then follows: $\omega^+ = \omega^- = \omega^0 = \omega^1 = 1 + \sum_p \langle n_p \rangle^2 / \sum_p \langle n_p \rangle$, similar to Eq. (4), but with $\langle n_p \rangle = \{ \exp[(\sqrt{p^2 + m_\pi^2} - \mu_\pi)/T] - 1 \}^{-1}$.

Due to conditions, $N_+ \equiv N_-$ and $N_+ + N_- + N_0 \equiv N_\pi$, it follows, $\omega_{\text{m.c.e.}} = \omega_{\text{m.c.e.}}^0 / 4$ and $\omega_{\text{m.c.e.}} = \omega_{\text{m.c.e.}}^0 / 2$, where $N_{\text{ch}} = N_+ + N_-$. The behavior of $\omega_{\text{m.c.e.}}^0$ is shown in Fig. 2. To make a correspondence with $N_\pi$ values, we consider again the $p + p \rightarrow p + p + N_\pi$ collisions at the beam energy of 70 GeV and take the pion system energy to be equal to $E = 9.7$ GeV. Despite of the MCE suppression the scaled variances for the
number fluctuations of $\pi^0$ and $\pi^\pm$ increase dramatically and abruptly when the system approaches the BEC line.

As an instructive example let us consider the MCE ($V, E, Q = 0, N_{ch} = \text{const}$), i.e. fixed $N_{ch}$ instead of $N_\pi$. The corresponding GCE formulation gives the following pion chemical potential: $\mu_+ = \mu_-, \mu_0 = 0$, $\mu_- = \mu_+$ in Eq. (5) ($\mu_Q = 0$, as before, because of $Q = 0$ condition). When $\mu_\pi \to m_\pi$ the system approaches the BEC line for $\pi^+$ and $\pi^-$. The thermodynamic behavior and position of this BEC line can be easily approached. Approaching the BEC line one can also find $\omega^{\pi,\text{c.e.}} \to \infty$. The pion number fluctuations are, however, very different in the MCE ($V, E, Q = 0, N_{ch}$) in the statistical ensembles with fixed $N_{ch}$ and $Q$ no anomalous BEC fluctuations are possible. The numbers of $N_\pi$ and $N_\pi$ are completely fixed by the conditions $Q = N_\pi - N_\pi = 0$ and $N_{ch} = N_\pi + N_\pi = \text{const}$, thus, $\omega^{\pi,\text{c.e.}} = \omega^{\pi,\text{g.c.e.}} = 0$. The number $N_0$ fluctuates, but $\mu_0 = 0$, thus, neutral pions are far away from the BEC line and their fluctuations are small, $\omega^0 \approx 1$ in all statistical ensemble formulations.

The broad distributions over $N_0$ and $N_{ch}$ close to the BEC line also implies large fluctuations of the $f \equiv N_0/N_{ch}$ ratio. These large fluctuations were suggested (see, e.g., Ref. [12]) as a possible signal for the disoriented chiral condensate (DCC). The DCC leads to the distribution of $f$ in the form, $dW(f)/df = 1/(2\sqrt{7})$. The thermal Bose gas corresponds to the $f$-distribution centered at $f = 1/3$. Therefore, $f$-distributions from BEC and DCC are very different, and this gives a possibility to distinguish between these two phenomena.

The calculations presented in this letter should be improved by taking into account the finite size effects, pion-pion interactions, and some other effects. However, the described BEC scenario may survive the complications. A crucial point is the analysis of the samples of high $N_\pi$ events. The following inequalities are always hold for particle number fluctuations in different ensembles: $\omega^{\pi,\text{m.c.e.}}_f < \omega^{\pi,\text{c.e.}}_f < \omega^{\pi,\text{g.c.e.}}_f$. Therefore, if the anomalous BEC fluctuations are present in the MCE, they are also exist (and even larger) in the CE and GCE. The reverse statement is not true. The anomalous BEC fluctuations of the GCE may disappear in the CE or MCE. We found that for the system with $N_\pi = \text{const}$ and $Q = 0$ the anomalous BEC fluctuations do not wash out by exact conservation laws of the CE and MCE. The required $N_\pi$ values for the BEC are much larger than the average pion multiplicity per collision, thus, these high $N_\pi$ events are rather rare and give negligible contributions to inclusive observables in high energy collisions. With increasing of $N_\pi$ in the sample with fixed total energy, the temperature of the pion system has to decrease and it approaches the BEC line. This can happen in different ways: at constant energy density $\varepsilon$, at constant pion density $\rho_\pi$, or with decreasing of both $\varepsilon$ and $\rho_\pi$. The pion system should move to the BEC line one way or another. In the vicinity of the BEC line (no BE condensate is yet formed) one observes an abrupt and anomalous increase of the scaled variances of neutral and charged pion number fluctuations. This could (may be even should) be checked experimentally.

Acknowledgments. We would like to thank F. Becattini, K.A. Bugaev, A.I. Bugrij, I.M. Dremin, M. Gaźdзicki, W. Greiner, K.A. Gridnev, M. Hauer, I.N. Mishustin, St. Mrówczyński and Yu.M. Sinyukov for discussions and comments. We are also grateful to E.S. Kokouline and V.A. Nikitin. They informed us on the experimental project [10], and this stimulated the present study. We thank S.V. Chubakov for help in the preparation of the manuscript. The work was supported in part by US CRDF, project agreement UKP1-2613-KV-04, and by Ukraine-Hungary cooperative project M/101-2005.

[1] S. N. Bose, Z. Phys. 26, 178 (1924); A. Einstein, Sitz. Ber. Preuss. Akad. Wiss. (Berlin) 1, 3 (1925).
[2] M.H. Anderson, et al., Science 269, 198 (1995); K.B. Davis, et al., Phys. Rev. Lett. 75, 3969 (1995).
[3] J. Zimanyi, G. Fai, and B. Jakobsson, Phys. Rev. Lett. 43, 1705 (1979); I. N. Mishustin, et al., Phys. Lett. B 276, 403 (1992); C. Greiner, C. Gong, and B. Müller, Phys. Lett. B 316, 226 (1993); S. Pratt, Phys. Lett. B 301, 159 (1993); T. Csörgő and J. Zimanyi, Phys. Rev. Lett. 80, 916 (1998); A. Białas and K. Zalewski, Phys. Rev. D 59, 097502 (1999); Yu.M. Sinyukov, S.V. Akkelin, and R. Lednicky, nucl-th/0604015, R. Lednicky, et al., Phys. Rev. C 61, 034901 (2000).
[4] V.V. Begun, et al., Phys. Rev. C 70, 034901 (2004); ibid 71, 054904 (2005); ibid 72, 014902 (2005); J. Phys. G 32, 935 (2006); A. Keränen, et al., J. Phys. G 31, S1095 (2005); F. Becattini, et al., Phys. Rev. C 72, 064904 (2005); J. Cleymans, K. Redlich, and L. Turko, Phys. Rev. C 71, 047902 (2005); J. Phys. G 31, 1421 (2005).
[5] V.V. Begun and M.I. Gorenstein, Phys. Rev. C 73, 054904 (2006).
[6] V.V. Begun, et al., Phys. Rev. C 74, 044903 (2006); V.V. Begun, et al., nucl-th/0611075.
[7] L.D. Landau and E.M. Lifschitz, Statistical Physics (Fizmatlit, Moscow, 2001).
[8] M. Gajda and K. Rzazewski, Phys. Rev. Lett. 78, 2686 (1997).
[9] H.E. Haber and H.A. Weldon, Phys. Rev. Lett. 46, 1497 (1981); J.I. Kapusta, Finite-Temperature Field Theory, (Cambridge, 1989).
[10] P.F. Ermolov, et al., Phys. At. Nucl., 67, 108 (2004); V.V. Avdeichikov, et al., JINR-P1-2004-190, 45 pp (2005).
[11] V.A. Nikitin, private communication.
[12] J.P. Blaizot and A. Krzywicki, Phys. Rev. D 46, 246 (1992); Acta Phys. Pol. B 27, 1687 (1996); K. Rajagopal and F. Wilczek, Nucl. Phys. B 399, 395 (1993); J. Bjorken, Acta Phys. Pol. B 28, 2773 (1997).