1. Introduction

The phenomenon of phase-coexistence is the typical signature of a first-order transition. The free energy of a system at the coexistence point is the sum of two contributions: the bulk free energy $F_{\text{bulk}}$, scaling like the volume $V$, and the free energy $F_{\Sigma}$ of the interface $\Sigma$ separating the two bulk phases. $F_{\Sigma}$ scales like an area. The interface tension $\sigma$ is defined as $\sigma = \lim_{\Sigma \to \infty} \frac{F(\Sigma)}{|\Sigma|}$ (the reduced tension is defined as $\sigma^R = \frac{\sigma}{T}$). In the context of $SU(N)$ pure gauge theories ($N$ is the number of colors) phase-coexistence occurs in two different situations: one at $T \geq T_c$ between ordered phases pointing in different directions in color space, the other at $T_c$, between confined ('disordered') and deconfined ('ordered') phases. The corresponding interface tensions are indicated $\sigma_{oo}$ and $\sigma_{od}$ respectively.

This paper presents a new algorithm to measure the order-order interface tension. Nevertheless, we can also give an estimate of the order-disorder tension via the so-called perfect wetting hypothesis [1], which states that between two ordered phases a layer of disordered phase can be generated at no free energy cost. Therefore the free energy of one order-order interface is related to that of two order-disorder interfaces, namely $F_{oo} = 2F_{od}$. In general, anyway, $\sigma_{oo} = w\sigma_{od}$, $w \leq 2$, because otherwise order-order interfaces would be unstable. Therefore the choice $w = 2$ gives a lower bound for $\sigma_{od}$.

2. The method

The algorithm we propose is an improvement of the so-called snake algorithm [3]. The ‘snake’ idea is to add ‘by hand’ a 2d interface in the system by progressively flipping the coupling of a set of plaquettes dual to a surface $A$, according to the

Our system is a 4d $SU(N)$ pure gauge theory at finite temperature with spatial periodic boundary conditions (b.c.). $a$ indicates the lattice spacing. The volume is $V = L^3 \cdot L_t$; the Wilson action is used. We will indicate the elements of the center of $SU(N)$ by $\zeta_i$, $i = 1, \ldots, N$. Heat-bath and over-relaxation are applied to $SU(2)$ subgroups of the $SU(N)$ matrices, according to the strategy used in [2].
Figure 2. (Minus log of the) Ratios in Eq. (1) as a function of $r$, after removal of the Lüscher correction. A broad plateau develops from $r \sim \xi$ ($\xi = 1/\sqrt{\sigma} \simeq 3.5$ is the correlation length) to $r \rightarrow L$.

Figure 3. Finite size effects on the interface tension ($L_t = 2$, $\beta = 5.15$). After removal of the Lüscher correction, $\sigma$ is systematically underestimated, unless lattices larger than $L \gtrsim 7/\sqrt{\sigma}$ are used (vertical line). The curve is meant to guide the eye.

identity

$$Z(A) = \frac{Z(A)}{Z(0)} = \frac{Z(A-1)}{Z(A-2)} \cdots \frac{Z(1)}{Z(0)}$$ (1)

where $Z(k)$, $k \in [0, \ldots A]$ indicates the partition function of a system in which only $k$ plaquettes are flipped. The free energy of the interface is $F(A) = \sigma R_{oo} A = -\log \frac{Z(A)}{Z(0)}$. A direct measurement of $\frac{Z(A)}{Z(0)}$ is not possible due to a serious overlap problem, which is alleviated by the factorization Eq. (1). The price to pay is a very large number of independent simulations.

The observable is measured in the following way:

$$\frac{Z(k)}{Z(k-1)} = \frac{\langle e^{\beta \frac{1}{2} \text{Tr}(\zeta \Pi_k)} \rangle_k}{\langle e^{\beta \frac{1}{2} \text{Tr}(\Pi_k)} \rangle_k}$$ (4)

where $\Pi_k$ indicates the $k$–th flipped plaquette, and the average $\langle \cdot \rangle_k$ refers to the ensemble in which the first $(k-1)$ plaquettes are flipped, the $k$–th plaquette has coupling zero, and all the others are unchanged. Further variance reduction methods are described in [3].

The leading finite-size corrections in Eq. (3) come from Gaussian fluctuations of the interface [4]. Our $r \cdot L$ interface (Fig. 1) is periodic in one direction $(L)$, and pinned in the other $(r)$. The corresponding correction, given in terms of the Dedekind $\eta$-function, reduces for $r \ll L$ to the Luscher-like $\sigma_{\text{eff}}(r) \equiv -a^{-2} \log Z(k)/Z(k-1) \approx \sigma + \frac{\pi}{12\sigma}$. In Fig. 2 we show our measurements, after removal of this known correction, as a function of $r$. A broad plateau develops, from small values $r \sim \xi$, where $\xi = 1/\sqrt{\sigma}$ is the correlation length, to large ones $r \rightarrow L$, showing that additional corrections are very small. At very large
3. Results

In Fig. 4 we present preliminary results for the order-disorder interface tension in SU(3), together with a compilation of the published data. While our $L_t = 4$ simulation needs more statistics, our $L_t = 2$ and 3 determinations of $\sigma$ are accurate, and much larger than previous measurements obtained with the histogram method [6][7][8][9]. We assign this discrepancy to the smaller lattice sizes considered previously, which lead to a systematic underestimate of $\sigma$ as in Fig. 3. A discussion of the continuum limit is awaiting completion of the $64^3 \cdot 4$ simulations.

In Tab. 1 we present preliminary results for the case $SU(4)$. In $SU(N)$ with $N > 3$ we have more order-order tensions ($\sigma_k, k = 1, \ldots, \lfloor N/2 \rfloor$). In the weak coupling regime one can show the Casimir relation $\frac{\sigma_k}{\sigma_1} = \frac{k(N-k)}{N-1}$ [10]. This perturbative prediction seems accurate down to temperature $T = 1.2 T_c$. Measurements closer to $T_c$ are in progress.

Table 1

| $T/T_c$ | $L = 16$ | $L = 24$ |
|---------|---------|---------|
| 2.3     | 1.350(20) | 1.342(13) |
| 1.5     | -       | 1.300(18) |
| 1.2     | 1.277(33) | 1.310(30) |

Ratio $\sigma_2/\sigma_1$ of $SU(4)$ interface tensions ($L_t = 5$). The Casimir perturbative value is $4/3$. For the data at $T = 1.2 T_c$ we find again that the F.S. effects are smaller than the statistical error only if $L \gtrsim 7/\sqrt{\sigma}$.

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