Magnetic dipole absorption at intersubband transitions in quantum wells

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We consider theoretically magnetic dipole mechanism of light absorption at intersubband transitions in wide-gap quantum wells (QW). In contrast to electric dipole resonance, discussed mechanism manifests in the interaction with s-polarized component of electromagnetic radiation. Magnetic dipole resonance leads to relatively weak absorption, but it should be measurable against the background of much stronger electric dipole absorption because of the absence of frequency shift due to collective plasma effects. It also means that the observation of these dipole resonances of both types may become an experimental method of characterization of QW potential profile.

INTRODUCTION

Physical systems with the size quantization (quantum wells, quantum wires and quantum dots) are currently under active investigation both due to the potential applications in nonlinear optics [1-3] and the variety of significant fundamental effects. The latter phenomena are the bosonization of intersubband fermion oscillations [4, 5], the strong Stark splitting [6], the Purcell enhancement of spontaneous or induced quantum-optical processes [7, 8] and others.

In this paper we predict theoretically the effect of the resonance excitation of orbital high-frequency magnetic moment in the plane of a quantum well (QW). In particular, magnetic dipole interaction should lead to relatively weak absorption of s-polarized or normally incident radiation. Note that the absorption of s-polarized waves can be also caused by multiband coupling effects in QW with a strong coupling between the conduction band and the valence band, i.e., in the narrow-gap structures [9-13]. The interaction mechanism considered in the present paper is completely different, and its’ key features are the following:

(i) Intersubband transitions of this type are possible only in the presence of a nonzero electron momentum in the direction of the translational symmetry of the system.

(ii) The intensity of the magnetic dipole energy exchange between the field and the medium is determined by the ratio of the QW thickness to the characteristic spatial scale of the high-frequency field. Absorption rate is proportional to the squared amplitude of the high-frequency magnetic field projected onto the QW plane.

(iii) In the case of degenerate system, the magnetic dipole absorption of high-frequency field is proportional to the difference of the squared populations at the resonant levels (but not to the population difference like in the case of “common” electric dipole absorption).

We also find that magnetic dipole resonances have much smaller frequency shifts due to collective (plasma) effects than the electric dipole resonances. So, magnetic dipole resonant frequencies appeared to be much closer to the corresponding intersubband frequencies. That is why independent measurements of the magnetic dipole and electric dipole resonant frequencies could allow both extracting the QW parameters and experimental verifying of theoretical models of collective fermion interaction.

In Section I the basic mechanism of the magnetic dipole interaction in QW is presented. In Section II we formulate the self-consistent model of the electromagnetic field interaction with free charge carriers in a thin layer. The carrier dynamics is described by the simplest QW model implying rather large energy gap between the conductive band and the valence band. In Section III high-frequency magnetic susceptibility of QW is derived in the rotating wave approximation (RWA) and the key properties of the magnetic dipole and electric dipole resonances are compared.
basing on the self-consisted model. Section IV is devoted to the estimations and comparison of the magnetic dipole absorption with other mechanisms that are possible in the case of an isolated conductive band.

I. The basic model of magnetic dipole resonance in QW

Let us consider a quantum well lying in the plane \((x, y)\), so the size quantization is in the “z” direction (see Fig. 1). Suppose that the conduction band and the valence band are separated by a rather large energy gap. In this case, the following eigenvectors can be attributed to the conduction electrons:

\[ |m, k \rangle = \Phi_m(z)e^{ik_x x + ik_y y}. \]

Here \( \Phi_m(z) \) corresponds to the subbands resulting from the size quantization. Suppose that during the time interval \( t \in [0, T] \) an electron is under the action of electromagnetic field given by the vector potential \( A = \text{Re} x_0 \tilde{A}_x(z)e^{-i\omega t} \) and the frequency \( \omega \) is equal to the frequency of the intersubband transition between the states \( \Phi_1(z)e^{ik_x x} \) and \( \Phi_2(z)e^{ik_x x} \). Solving the Schrödinger equation by the perturbation method, for the initial state \( \Phi_1(z)e^{ik_x x} \) at \( t > T \) we obtain

\[ \Psi(t, T) \approx \left[ \Phi_1(z)e^{-i\frac{E_{1,k}x}{\hbar} - i\frac{E_{2,k}x}{\hbar}} + C_2 \Phi_2(z)e^{-i\frac{E_{2,k}x}{\hbar}} \right] e^{ik_x x} + o\left( \tilde{A}_x^2 \right), \]

where \( E_{2,k_x} \) and \( E_{1,k_x} \) are energies of “2” and “1” states correspondingly, \( \omega = \frac{E_{2,k_x} - E_{1,k_x}}{\hbar}, C_2 = -i \frac{e^{k_x T}}{2m_\parallel} \tilde{A}_{21}, \tilde{A}_{21} = \langle \Phi_2 | \tilde{A}_x | \Phi_1 \rangle \), \( -e \) is the electron charge, \( m_\parallel \) is the effective mass of the charge carrier in the plane of QW. The obtained state function \( \Psi(t, T) \) gives the following current density in the “x” direction:

\[ j_x \approx -\frac{e\hbar k_x}{m_\parallel} |\Phi_1(z)|^2 + \frac{e^2 \hbar^2 k_x^2}{m_\parallel^2 c^2} \text{Re}[i\Phi_1^*(z)\Phi_2(z)\tilde{A}_{21}e^{-i\omega t}] + o\left( \tilde{A}_x^2 \right). \]

High-frequency component of the current depends on the quasi-momentum \( k_x \) squared and, therefore, it doesn’t disappear during the averaging over the equilibrium ensemble. If the functions \( \Phi_1(z)e^{ik_x x} \) and \( \Phi_2(z)e^{ik_x x} \) are of the different parity, then the high-frequency current have a stratified two-stream structure with the magnetic moment oriented along the "y" axis (see Fig. 1).

Fig. 1. Schematic picture of magnetic dipole absorption of s-polarized light. The simplest structure of electric currents inside the QW is shown by yellow arrows.

II. Self-consistent model of QW in high-frequency field

II.1 Self-consistent equations of the electromagnetic field in a thin layer

Let us consider a thin layer located in the area \(-L \leq z \leq L\) between two half-spaces with dielectric constants \( \varepsilon_{(\pm)} \) for \( z < -L \) and \( \varepsilon_{(\pm)} \) for \( z > L \). Permittivity of the layer determined by the
valence electrons is $\varepsilon(z)$. Suppose that this QW contains free electrons with the density $n(r)$. The value of $n(r)$ is connected with the free charge density $\rho(r)$ by the relation $\rho(r) = Q_I(z) - e n(r)$, where $Q_I(z)$ is the ion charge density not compensated by the valence electrons.

Keeping in mind further comparison of different interaction mechanisms, we start with considering an $s$-polarized electromagnetic wave (see Fig. 2a). In this case we can use the vector potential $A = \text{Re} \, x_0 \tilde{A}_x(z) e^{i q y - i \omega t}$, which satisfies the wave equation:

$$
\left( \frac{\partial^2}{\partial z^2} - q^2 + \frac{\omega^2 \varepsilon(z)}{c^2} \right) \tilde{A}_x(z) + \frac{4\pi}{c} j_x(z) = 0,
$$

where $j_x(z)$ is the complex amplitude of current $j_x = \text{Re} j_x(z) e^{i q y - i \omega t}$.

Below we discuss quite a common case of a thin QW satisfying inequalities:

$$
q L, L \sqrt{\varepsilon \omega / c} \ll 1
$$

where $\bar{\varepsilon}$ is the average value of $\varepsilon(z)$ into the layer. After double integration on $z$, taking into account inequalities (2), we find:

$$
\tilde{A}_x(z) - \tilde{A}_x(-L) = -\frac{4\pi}{c} \int_{-L}^{z} dz' \int_{-L}^{z'} dz'' j_x(z'') + \tilde{B}_y(-L) \cdot (z + L).
$$

For $p$-polarized wave all the values can be set as $f = \text{Re} \tilde{f}(z) e^{i q x - i \omega t}$ (see Fig. 2b). In this case “$x$” and “$z$” components of the electric field $E$ and current $j$ are nonzero, so the electromagnetic field can be described by the scalar and vector potentials $\varphi = \text{Re} \tilde{\varphi}(z) e^{i q x - i \omega t}$ and $A = \text{Re} x_0 \tilde{A}_x(z) e^{i q x - i \omega t}$. We should note that for the polarizations of both types we may use gauge condition $A_{z,y} = 0$.

Fig. 2. Incidence of (a) s-polarized and (b) p-polarized waves at QW.

The self-consistent system of equations should include:

(i) Gauss’s law $\nabla D = 4\pi \rho$:

$$
\frac{\partial}{\partial z} \left( \varepsilon(z) \frac{\partial \varphi}{\partial z} \right) = \varepsilon(z) q^2 \frac{\partial \varphi}{\partial z} - \varepsilon(z) q \frac{\omega}{c} \tilde{A}_x + 4\pi e \bar{n},
$$

where $D = \varepsilon(z) \left( -\nabla \varphi - \frac{1}{c} \tilde{A} \right)$ is the electric displacement field.

(ii) continuity equation $\dot{\rho} + \nabla \cdot \mathbf{j} = 0$:

$$
i \omega \rho = \frac{\partial}{\partial z} \tilde{j}_z + i q \tilde{j}_x.
$$

(iii) $z$-component of the Maxwell equation $\nabla \times \mathbf{B} = 4\pi \mathbf{j} / c + \dot{\mathbf{D}} / c$:

$$
q \frac{\partial \tilde{A}_x}{\partial z} = -\frac{4\pi i}{c} \tilde{j}_z + \frac{\omega \varepsilon(z)}{c} \frac{\partial \varphi}{\partial z}.
$$

From Eq. (4), taking into account inequalities (2), we obtain the following equation for the complex amplitude of the scalar potential oscillation:
\[
\tilde{\varphi}(z) = -\tilde{D}_z(-L) \int_{L}^{z} dz' + 4\pi e \int_{L}^{z'} dz'' \int_{L}^{z'} \tilde{n}(z'') dz''.
\] (7)

After that we can substitute Eqs. (5), (7) in Eq. (6) and integrate. Using boundary conditions \(j_z(\pm L) = 0\) and \(\tilde{D}_z(\pm L) = -\frac{qc}{\omega} \tilde{B}_y(\pm L)\), we obtain equation for the vector potential in the same form as Eq.(3).

II.2 Dynamics of free charge carriers

Let us define the potential well for electrons as \(U_{qw}(z)\) and use the simplest model for the electron Hamiltonian assuming that the energy gap between the valence and conduction bands is enough large [1,7]:

\[
\hat{H} = -eU_{qw}(z) + \frac{\hat{p}_x^2}{2m_\parallel} + \frac{\hat{p}_y^2}{2m_\parallel} + \hat{V},
\] (8)

where \(\hat{p} = -i\hbar \nabla\) is the momentum operator, \(m_{\perp\parallel}\) are the components of the effective mass tensor, \(\hat{V}\) is the interaction operator. For the chosen gauge \(\hat{V}\) operator has the form:

\[
\hat{V} = -e\varphi + e \frac{\hat{p}_x \Delta_x + \Delta_{xx} \hat{p}_x}{2m_\parallel c} + e^2 \frac{\Delta_z^2}{2m_\parallel c^2},
\] (9)

where \(\hat{p} = -i\hbar \nabla\) is the momentum operator, \(m_{\perp\parallel}\) are the components of the effective mass tensor, \(\hat{V}\) is the interaction operator.

In the absence of perturbative potentials \(\varphi\) and \(A\) the Hamiltonian (8) describes formation of subbands with the energies \(E_{m,k} = E_m + \frac{k^2 \hbar^2}{2m_\parallel}\) and eigenfunctions:

\[
|m, k\rangle = \Psi_{mk}(r) = \Phi_m(z) \exp(ik_x x + ik_y y),
\]

where the functions \(\Phi_m(z)\) and values \(E_m\) are determined by the one-dimensional Schrödinger equation:

\[
E_m \Phi_m = -\frac{\hbar^2}{2m_\parallel} \frac{\partial^2 \Phi_m}{\partial x^2} - eU_{qw}(z) \Phi_m,
\]

while the set of quasi-momenta \(k = (k_x, k_y)\) is determined by the periodic boundary conditions at the border of a unit area.

Under condition \((\varepsilon \omega/c)/k_F \ll 1\) \((k_F\) is the Fermi quasi-momentum) the problem can be reduced to the study of “spinless” carriers, while the spin degeneration can be taken into account just in the macroscopic values calculations (see Appendix for details).

The average carrier density \(n(r)\) and current \(j(r)\) are determined by (e.g., see [17, 34]):

\[
n(r) = \sum_{m} \sum_{\alpha \beta} \Psi_\beta^* (r) \Psi_\alpha (r) \rho_{\alpha \beta}, \quad j(r) = -\frac{e}{\hbar} \sum_{m} \sum_{\alpha \beta} \{ \Psi_\beta^* (r) [\hat{V} \Psi_\alpha (r)] + [\hat{\nabla}^* \Psi_\beta (r) \Psi_\alpha (r)] \}(r) \rho_{\alpha \beta},
\] (10)

where \(\rho_{\alpha \beta}\) is the density matrix and \(|m, k\rangle \equiv |m, k\rangle\); \(\hat{\nabla} = \frac{i}{\hbar} [\hat{H}, r]\) is the velocity operator\(^1\) [14].

The density matrix is described by the von Neumann equation:

\[
\frac{\partial \rho_{\alpha \beta}}{\partial t} + i \frac{E_{\alpha} - E_{\beta}}{\hbar} \rho_{\alpha \beta} + \frac{i}{\hbar} \sum_{\chi} \{ V_{\alpha \chi} \rho_{\chi \beta} - \rho_{\alpha \chi} V_{\chi \beta}\} = 0.
\] (11)

Solving Eq. (10) we will use the simplest model of relaxation processes using the substitution \(\omega \Rightarrow \omega + i\gamma\) in the resonant terms \(\propto \frac{1}{E_{\alpha} - E_{\beta} - \hbar \omega}\) [21, 22]. If the relaxation constant \(\gamma\) is much less than the resonant frequency \(\omega_{\alpha \beta} = \frac{|E_{\alpha} - E_{\beta}|}{\hbar}\), this approach is valid for the description of resonant effects within the framework of RWA approximation applicability – see papers [20, 23, 24] for details.

\(^1\) It is easy to see that Eq.(10) corresponds to the standard expression for the matrix elements of the current density operator: \(j_{\alpha \beta} = -\frac{e}{2m_\parallel} \left[ \Psi_\beta^* (r) \hat{p} \Psi_\alpha (r) + (\hat{p}^* \Psi_\beta) \Psi_\alpha (r) \right] - \frac{e^2}{2m_\parallel c} \Psi_\alpha^* \Psi_\beta (r)\) (see [14]).
III. Electric dipole and magnetic dipole oscillations in QW

Let us consider a linear electromagnetic response of QW and neglect quadratic terms $\sim A_x^2$ in the interaction operator $\hat{V}$ (see Eq.(9)). For the harmonic dependence $\propto e^{\pm iq\cdot r}$ (vectors $q$ and $r$ belong to $x,y$ plane) matrix elements of the interaction operator have the form $V_{mn}^{kk'} \propto \delta(k-k')$. Under the condition $q/k_F \ll 1$ interband transitions are “direct” and consequently $V_{mn}^{kk'} \approx \delta_{kk'} V_{mn}$, which can be presented as

$$V_{mn}^{kk'} = \frac{1}{2} \langle \tilde{V}_{mn}^{kk} e^{-i\omega t} + H.C. \rangle,$$

(12)

where $\tilde{V}_{mn}^{kk} = -e\tilde{\Phi}_{mn} + \frac{e\hbar k_F}{m_e} \tilde{A}_{mn}$. $\tilde{f}_{mn} = \langle \Phi_m | f | \Phi_n \rangle$. Since the interaction operator $V_{mn}^{kk'}$ assumed to be diagonal with respect to $k,k'$ indices, the density matrix perturbation can be represented in a similar way:

$$\rho_{mn}^{kk'} = \delta_{mn} \delta_{kk'} n_{mn}^{(0)} + \frac{1}{2} \langle \tilde{\rho}_{mn}^{kk} e^{-i\omega t} + H.C. \rangle,$$

(13)

where $n_{mn}^{(0)}$ are equilibrium (unperturbed) populations. From Eqs. (11) and (12),(13) we find the linear perturbation of the density matrix:

$$\tilde{\rho}_{mn}^{kk} = \frac{n_{mn}^{(0)}}{\omega_{mn} - \omega - i\gamma} \left( \frac{e\tilde{\Phi}_{mn}}{h} - \frac{e\hbar k_F \tilde{A}_{mn}}{m_e c} \right).$$

(14)

Eqs. (10) and (14) allow to obtain complex amplitudes of the longitudinal current and electron density. Still using RWA approximation, we will take into account just one intersubband transition with the frequency $\omega_{mn} \approx \omega$. For distributions $n_{mn}^{(0)}$ symmetric with respect to the longitudinal quasi-momentum $k_x$ we come to the following expressions:

$$\tilde{J}_x(z) = \frac{e^2 \hbar}{m_e c} \sum_k k_k^2 \left( \frac{\tilde{\Phi}_{mn}^{(0)}}{\omega_{mn} - \omega - i\gamma} \right) \Phi_m^{*}(z) \Phi_m(z) \tilde{A}_{mn},$$

(15)

$$\tilde{n}(z) = \frac{e}{\hbar} \sum_k n_{mn}^{(0)} \left( \frac{\tilde{\Phi}_{mn}^{(0)}}{\omega_{mn} - \omega - i\gamma} \right) \Phi_m^{*}(z) \Phi_m(z) \tilde{\Phi}_{mn}.$$

(16)

To find the matrix elements $\tilde{\Phi}_{mn}$ and $\tilde{A}_{mn}$ in a self-consistent way we use Eqs. (3) and (7) and find:

$$\tilde{\Phi}_{mn} = \tilde{D}_z(-L) \frac{d_{mn}^{(eff)}}{e} + 4\pi e \int_{-L}^{L} \left( \frac{dz'}{\epsilon(z')} \right) \int_{-L}^{L} \tilde{n}(z') dz' \left| \Phi_m \right\rangle,$$

(17)

$$\tilde{A}_{mn} = -\left( \frac{4\pi}{c} \right) \left| \Phi_m \right\rangle \int_{-L}^{L} dz' \int_{-L}^{L} dz'' \tilde{J}_x(z'') + \tilde{B}_y(-L) \cdot (z_m + \delta_{mn} L);$$

(18)

here $d_{mn}^{(eff)} = -e \int_{-L}^{L} \left( \frac{dz'}{\epsilon(z')} \right) \Phi_m$. Substitution of Eqs. (15), (16) in Eqs. (17), (18) gives the self-consistent solutions for the matrix elements $\tilde{\Phi}_{mn}$ and $\tilde{A}_{mn}$ which includes the fields at the QW boundary $\tilde{D}_z(-L)$ and $\tilde{B}_y(-L)$.

Complex amplitudes $\tilde{J}_x(z) \propto \tilde{n}(z) \propto \Phi^{*}_m \Phi_m$ meet the condition $\int_{-L}^{L} \tilde{J}_x(z) dz = \int_{-L}^{L} \tilde{n}(z) dz = 0$, which leads to $\tilde{B}_y(-L) = \tilde{B}_y(+L) = \tilde{H}_y$ and $\tilde{D}_z(-L) = \tilde{D}_z(+L) = \tilde{D}_z$:

$$\tilde{\Phi}_{mn} = \frac{\omega_{mn} - \omega - i\gamma}{\omega_{mn} + \Omega_{mn}^{(0)} - \omega - i\gamma} \frac{d_{mn}^{(eff)}}{e} \tilde{D}_z,$$

(19)

$$\tilde{A}_{mn} = \frac{\omega_{mn} - \omega - i\gamma}{\omega_{mn} + \Omega_{mn}^{(0)} - \omega - i\gamma} \frac{z_m}{\omega_{mn} + \Omega_{mn}^{(0)} - \omega - i\gamma} \tilde{H}_y,$$

(20)

where

$$\Omega_{mn}^{(0)} = \frac{4\pi e^2 (N_n - N_m) G_{mnmn}}{h} \text{ and } \Omega_{mn}^{(A)} = -\frac{4\pi e^2 (N_n - N_m) J_{mnmn} \epsilon_{mn}^{(eff)}}{m_e c^2}$$

(21)

are the frequency shifts due to collective effects;
\[ J_{mmn} = \frac{\hbar^2}{4\omega_{mmn}^2} \int_{-L}^{L} \left| \Phi_m(z) \frac{\partial \Phi_n}{\partial z} - \Phi_n^*(z) \frac{\partial \Phi_m}{\partial z} \right|^2 \, dz, \quad (22) \]

\[ G_{mmn} = \frac{\hbar^2}{4\omega_{mmn}^2} \int_{-L}^{L} \frac{1}{e(x)} \left| \Phi_m(z) \frac{\partial \Phi_n}{\partial z} - \Phi_n^*(z) \frac{\partial \Phi_m}{\partial z} \right|^2 \, dz \]

are so-called overlap integrals\(^2\);

\[ W_{\text{eff}}^{mn} = 2 \frac{N_n(W_{mn} - N_m W_{nm})}{N_n - N_m}, \quad N_n = \sum_k n_{nk}^{(0)}, \quad \langle W_{\|n} \rangle = \frac{1}{N_n} \sum_k \frac{\hbar^2 k_z^2}{2m_1} n_{nk}^{(0)}. \quad (24) \]

The last of Eqs. (24) determines the effective energies of the longitudinal motion (for the band under consideration).

In Eqs. (24) the summation can be replaced by integration: \( \Sigma_k(\ldots) \Rightarrow \eta \int \omega(\ldots) d^2k \), where \( \eta = 1/4\pi^2 \) is the density of 2D states, \( g = 2 \) is the spin degeneracy factor. For a degenerate distribution with the Fermi energy \( E_F \) we obtain:

\[ N_n = \frac{m_1(E_F - E_n)}{\pi \hbar^2}, \quad \langle W_{\|n} \rangle = \frac{\pi \hbar^2 N_n}{4m_1}, \quad W_{\text{eff}}^{mn} = \frac{\pi \hbar^2 N_n + N_m}{2m_1}, \quad (25) \]

while for \( E_m > E_F > E_n \) \( N_m = 0, \quad N_n \neq 0 \).

Note that in the absence of external fields (when \( D_z = H_y = 0 \)) Eqs. (19) and (20) determine the eigenfrequencies of electric dipole and magnetic dipole oscillations respectively:

\[ \bar{\varphi}_{mn} \left( \omega_{mn} + \Omega_{mn}^{(\varphi)} - \omega - i\gamma \right) = 0, \quad (26) \]

\[ \bar{A}_{mn} \left( \omega_{mn} + \Omega_{mn}^{(A)} - \omega - i\gamma \right) = 0. \quad (27) \]

Using Eqsns. (14), (19) and (20) we find the self-consistent solution for the density matrix:

\[ \bar{\rho}_{mnkk} = \frac{d_{mn}^{(eff)}(n_{mk} - n_{nk})}{\hbar(\omega_{mn} + \Omega_{mn}^{(\varphi)} - \omega - i\gamma)} \bar{D}_z + \frac{d_{mn}^{(eff)}(n_{mk} - n_{nk})}{m_1c(\omega_{mn} + \Omega_{mn}^{(A)} - \omega - i\gamma)} \bar{H}_y, \quad (28) \]

where \( d_{mn} = -ez_{mn} \).

The first term in Eq. (28) describes the density matrix perturbation caused by an external electric field. This expression differs from the well-known solution for a two-level system [4]. First, there is a factor \( d_{mn}^{(eff)} \bar{D}_z \) instead of \( d_{mn} \bar{D}_z \) which is due to possible inhomogeneity of the dielectric permittivity into QW [7]. Second, there is a new term \( \Omega_{mn}^{(\varphi)} \) in the denominator – so-called “depolarization shift” of the resonant frequency relative to the intersubband transition frequency [25].

The second term in Eq. (28) describes the excitation of magnetic dipole oscillations in QW. And there is also a frequency shift \( \Omega_{mn}^{(A)} \) caused by collective effects.

Let us calculate complex amplitudes of the resonant oscillations of electric dipole and magnetic dipole moments per unit area of QW:

\[ \tilde{P}_z = -e \int_{-L}^{L} z \tilde{n} \, dz, \quad \tilde{M}_y = \int_{-L}^{L} \tilde{m}_y \, dz, \]

where \( -e \frac{\partial \tilde{m}_y}{\partial z} = \bar{j}_x. \) Using Eq. (29) with Eqsns. (15), (16), (19) and (20) we find:

\[ \tilde{P}_z = \frac{d_{mn}^{(eff)}(N_n - N_m)}{\hbar(\omega_{mn} + \Omega_{mn}^{(\varphi)} - \omega - i\gamma)} \bar{D}_z, \quad (29) \]

\[ \tilde{M}_y = \frac{|d_{mn}|^2(N_n - N_m)}{\hbar(\omega_{mn} + \Omega_{mn}^{(A)} - \omega - i\gamma)} \frac{W_{\text{eff}}^{mn}}{m_1c^2} \bar{H}_y. \quad (30) \]

Representing Eq. (30) as \( \tilde{M}_y = \chi^{(m)} \bar{H}_y \), we obtain the effective magnetic susceptibility at the resonant intersubband transition:

\[ \chi^{(m)} = \frac{|d_{mn}|^2(N_n - N_m)}{\hbar(\omega_{mn} + \Omega_{mn}^{(A)} - \omega - i\gamma)} \frac{W_{\text{eff}}^{mn}}{m_1c^2}. \quad (31) \]

\(^2\)To obtain Eqs. (22), (23) we use the following property of stationary solutions of the Schrödinger equation: \( \Phi_n^* \Phi_m = \frac{\langle h/m_1 \rangle}{2\omega_{mn}} \frac{\partial \Phi_n^*}{\partial z} - \Phi_n^* \Phi_m \frac{\partial \Phi_m}{\partial z} \).
Thus, the resonant magnetic susceptibility $\chi^{(m)}$ differs from the electric susceptibility by the factor $\frac{W_{\text{eff}}}{m_{\text{eff}}^2}$ and by the value of resonant frequency (other differences are not so important). For a degenerate system magnetic susceptibility (31) can be rewritten using Eq. (25):

$$\chi^{(m)} = \frac{\pi h |d_{mn}|^2 (N_n^2 - N_m^2)}{2m_n^2 \varepsilon^2 (\omega_{mn} + \Omega_{mn}^{(A)} - \omega - i\gamma)}. \tag{32}$$

From Eq. (32) it follows that the magnetic susceptibility of QW is proportional to the difference of squared carrier densities ($N_n^2 - N_m^2$), but not to the difference of densities ($N_n - N_m$) as the electric permittivity does. Also note that $\chi^{(m)}$ is grater for charge carriers with small “longitudinal” effective mass $m_\|$.

The power of magnetic dipole resonant losses per unit area of QW can be calculated using the initial Eq. (11) for the density matrix:

$$W = \frac{\partial}{\partial t} \sum_a E_a \rho_{aa} = \frac{2}{\hbar} \text{Im} \sum_{a\beta} E_a V_{a\beta} \rho_{\beta a}. \tag{33}$$

We still take into account just a resonant intersubband transition with $\omega_{mn} \approx \omega$; in this case substitution of Eqs. (12) and (13) in Eq. (33) gives:

$$W = \frac{\omega}{2} \text{Im} \sum_k \tilde{V}_{mnkk} (\tilde{\rho}_{mnkk}). \tag{34}$$

Then, we can use Eqs. (29), (12) and (20) for the density matrix perturbation and the interaction operator and set $\tilde{\varphi} = \tilde{D}_z = 0$. The result confirms the well-known phenomenological formula [26] for high-frequency field losses in magnetic medium:

$$W^{(m)} = \frac{\omega}{2} \text{Im} \chi^{(m)} |\tilde{H}_y|^2. \tag{35}$$

## IV. Estimations

For estimations we consider the wide-gap semiconductor GaAs, having $m_\perp \approx m_\| = m_{\text{eff}} = 0.067m_0$ and $\varepsilon \approx 10$. Let us choose the radiation frequency $\hbar \omega \approx 0.3$ eV (at which $\gamma \approx 10$ meV) and consider the resonant transition between the two lowest levels, so $\omega = \omega_{21}$. We use the relation $|z_{nm}|^2 \omega_{mn} = \frac{\hbar}{m_\perp} U_{mn}$, where $U_{mn}$ is the form-factor depending on the potential well profile and on the chosen transition type, but not on the QW thickness $L$. In particular, for the transition between two lowest levels in a deep “rectangular” well we get $U_{21} = 128/27\pi^2 \approx 0.5$ and $|z_{21}| \approx 1.2 \text{ nm}$.

The resonant frequency shifts $\Omega^{(\varphi, A)}_{mn}$ are related as $\Omega^{(A)}_{mn} \approx -\Omega^{(\varphi)}_{mn} \frac{\tilde{W}_{\text{eff}}}{m_{\text{eff}}^2}$ and $\Omega^{(\varphi)}_{mn}$ can be expressed as

$$\frac{\Omega^{(\varphi)}_{mn}}{\omega_{mn}} \approx \alpha \frac{\lambda_{mn}^2}{2} \frac{m_i N_n - N_m}{m_\perp N^*},$$

where $\alpha = \frac{e^2}{4\pi \varepsilon_0 c}$ is the fine structure constant, $\lambda_{mn} = 2\pi c / \omega_{mn}$ is the resonant radiation wavelength, $N^* = \frac{m_i \omega_{mn}}{\pi \hbar}$ is the maximal electron density in the $n$-th (lower) subband under condition that the $m$-th subband is empty, $F_{mn} = (G_{mnmm} / \varepsilon L) \times (\omega_{mn} L^2 m_\perp / \hbar)$ is the form-factor depending on the potential well profile and the specific transition. The transition between two lowest levels in a deep “rectangular” well corresponds to $F_{21} = 5/12\pi \approx 0.13$. For the ratio $2L/\lambda_{mn} \sim 10^{-3}$, assuming that $N_1 - N_2 / N^* \approx 1$, we find $\frac{\Omega^{(\varphi)}_{21}}{\omega_{21}} \approx 0.12 \frac{m_i}{m_\perp}$. One can see that the frequency shift of the electric dipole resonance can be significantly larger than the absorption line width even when the lower subband has low population.

In the papers [4, 5, 27] the expression for “intersubband” plasmon frequency was obtained:

$$\omega_{mn} = \sqrt{\omega_{mn}^2 + \omega_{p_mn}^2}, \text{ where } \omega_{p_mn}^2 = 2\omega_{mn} \Omega^{(\varphi)}_{mn},$$

assumed to be comparable to $\omega_{mn}$. In the case of nonzero population at several subbands, the effect
of plasmon hybridization takes place. This leads to so-called “multisubband” plasmon excitation which cause even stronger shift of electric dipole resonances [27-29].

At the same time, the resonant frequency shift of magnetic dipole resonance \( \Omega_{mn}^{(A)} \sim \Omega_{mn}^{(q)} \frac{W_{mn}}{m_ic^2} \) should be much less than the intersubband transition frequency even at high carrier densities, when \( \Omega_{mn}^{(q)} \sim \omega_{mn} \). Moreover, for the values \( \omega_{mn}/2 \gamma \sim 10 \div 20 \) [30] the shift is several orders of magnitude smaller than the resonance width.

Ratio of the magneto- and electric dipole losses can be calculated from Eqs. (35) and (29):

\[
\frac{W^{(m)}}{W^{(e)}} \approx \frac{\omega_{mn}}{m_ic^2} \left| \frac{\tilde{E}_x}{|\tilde{E}_x|} \right|^2.
\]

To make the simplest estimation we may consider a single propagation of the electromagnetic wave through QW, assuming \( \varepsilon_{(-)} \approx \varepsilon_{(+)} \approx \varepsilon \). In the case of close to normal incidence of the \( p \)-polarized wave (see Fig. 1, b) we have \( |\tilde{H}_y| \approx |\tilde{E}_x| \). For the listed parameters we obtain \( \varepsilon_{(-)} \omega_{mn} \approx 10^{-4} \), so the presence of a relatively small normal component of the high-frequency electric field will make electric dipole absorption predominant.

To estimate the absolute value of magnetic dipole absorption we consider a normally incident electromagnetic wave with the resonant frequency \( \omega = \omega_{mn} + \Omega_{mn}^{(A)} \). The energy flux can be written as \( \Pi = c \sqrt{\varepsilon} \frac{|\tilde{E}_x|^2}{8\pi} \) and \( |\tilde{H}_y|^2 \approx |\tilde{E}_x|^2 \). From Eqs.(35) and (32) it follows:

\[
\frac{W^{(m)}}{\Pi} \approx 2 \alpha \sqrt{\varepsilon} \frac{|z_{21}|^2 \omega^3}{c^2 \gamma} \frac{N_2^2 - N_1^2}{(N^*)^2}.
\]

Rewrite Eq. (36) for further estimations:

\[
\frac{W^{(m)}}{\Pi} \approx 4 \cdot 10^{-5} \sqrt{\varepsilon} \frac{|z_{21}|^2}{1 \text{ mm}} \left( \frac{\hbar \omega_{21}}{1 \text{ eV}} \right)^3 \left( \frac{10 \text{ meV}}{\hbar \gamma} \right) \frac{N_2^2 - N_1^2}{(N^*)^2}.
\]

For the parameters listed above and \( \frac{N_2^2 - N_1^2}{(N^*)^2} \approx 1 \) we find \( \frac{W^{(m)}}{\Pi} \sim 10^{-5} \) which is equivalent to 0.1% absorption in 100-layer QW structure. Also note that from \( |z_{21}|^2 \approx 1/\omega_{21} \) it follows that \( \frac{W^{(m)}}{\Pi} \propto \omega_{21}^2 \), so we obtain \( \sim 1\% \) absorption in 100-layer QW at the transition with 1 eV photon energy.

Let us compare magnetic dipole losses with the nonresonant Drude losses \( W^{(Dr)} \). For this we consider normal incidence of the electromagnetic wave on QW which is deposited on the perfectly reflective (metallic) substrate. If \( |\tilde{E}_x^{(max)}| \) is the electric field amplitude in a standing wave, then near QW we have \( |\tilde{H}_y|^2 \approx |\tilde{E}_x^{(max)}|^2 \) while the characteristic electric field into the QW can estimated as \( |\tilde{E}_x|^2 \approx \varepsilon_{(-)} \frac{\omega^2}{c^2} L^2 |\tilde{E}_x^{(max)}|^2 \). Using well-known relation \( W^{(Dr)} \approx \frac{\gamma e^2 \Sigma_j N_j}{2 m_i \omega^2} (|\tilde{E}_x|)^2 \) (valid when \( \gamma \ll \omega \)), we came to

\[
\frac{W^{(m)}}{W^{(Dr)}} \approx \frac{|z_{21}|^2 L^2 \omega^2 N_2^2 - N_1^2}{2 l^2 \gamma^2 N^* \Sigma_j N_j}.
\]

When \( \frac{N_2^2 - N_1^2}{N^* \Sigma_j N_j} \approx 1 \) and for the values \( \frac{|z_{21}|}{L} \sim 1, \frac{\omega}{\gamma} \sim 30 \) we find \( \frac{W^{(m)}}{W^{(Dr)}} \approx 5 \cdot 10^2 \). It means that the magnetic dipole absorption is detectable against the background of Drude losses, even in the case of not high contrast of permittivities between the substrate and QW.

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SUPPLEMENTARY MATERIALS

Influence of the spin effects on the absorption of normally incident radiation.

Taking into account spin effects leads to modification of the perturbation operator. First of all, the energy of the spin magnetic moment in a magnetic field should be included: \( \hat{V}_B = -\mu_B (\hat{s} \cdot \mathbf{B}) \), where \( \mu_B \) – Bohr magneton, \( \hat{s} = \frac{1}{2} (\mathbf{x}_0 \hat{\sigma}_x + \mathbf{y}_0 \hat{\sigma}_y + \mathbf{z}_0 \hat{\sigma}_z) \) – spin operator. In the case under consideration the direction of the magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) is time independent, so the operator of the spin projection on the quantization axis commutes with the Hamiltonian \( \hat{H} \) (see [14]).

Further, consider spin-orbital interaction. We will use a well-known expression for the corresponding interaction operator [15, 32]: \( \hat{V}^{(s-o)} = \frac{e\hbar}{2m_0^2c^2} [\mathbf{E} \times \hat{\mathbf{p}}] \cdot \hat{s}. \) For the electric field \( \mathbf{E} = x_0 \text{Re} \hat{E}_x e^{-i\omega t} \) we obtain:

\[
\hat{V}^{(s-o)} = \frac{e\hbar^2\mathbf{E}_x}{4m_0^2c^2} \left( -i\hat{s}_y \frac{\partial}{\partial z} + \hat{s}_z k_y \right) e^{-i\omega t} + H. C.
\]

The simplest estimation of the absorption due to this effect is based on the Einstein coefficient method\(^3\) [33]. Probability of the photon absorption and induced emission can be expressed through the spontaneous emission probability. To find the latter we use the Fermi golden rule [1]; this probability should be proportional to \( |V^{(s-o)}_{mns'}_{kk}|^2 \) (where \( s, s' \) are the spin state indices).

Defining the semi-width of the resonant frequency band as \( \gamma \), we get the following power loss per unit area:

\[
W^{(s-o)} \approx \frac{\omega}{2} \sum_{kss'} |V^{(s-o)}_{mns'kk}|^2 \left( n^{(0)}_{nsk} - n^{(0)}_{ms'k} \right).
\]

(A2)

Using Eq. (A2) and Eq. (35) for the power loss at magnetic dipole resonance we find:

\[
\frac{W^{(s-o)}}{W^{(m)}} \approx \frac{m_s}{16 \varepsilon(\omega)} m_0 \frac{1}{m_0 c^2} \left( m_0^2 \hbar^2 e^2 k_0^2 \right)^2 + \frac{\hbar^2}{m_0^2 \omega_{mn}^2 |\omega_{mn}|^2}.
\]

(A3)

From Eq. (A3) it follows that the absorption due to spin-orbital interaction is comparable with the magnetic dipole absorption just in the case of near-zero carrier density.

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\(^3\) Of course, in this way we could also obtain Eq. (35)
