Full reference text
indices. Hence they yield a family of diffeomorphism and gauge invariant Lagrangians. Taking into account the antisymmetries and considering the possible combinations of indices, we obtain a linear combination of elementary Lagrangians

\[ L = \frac{1}{2} F_{ab}(\alpha_1 T^{ab} T^{cm} \alpha_2 T_{nm} + \alpha_3 T^{mn} T^{ab} + \alpha_4 T^{ab} T_{nm}). \]  

(iv) Expressions of the types \( A \cdot A \cdot T \) and \( F \cdot F \cdot T \) have an odd number of free indices, so they do not yield diffeomorphism invariant Lagrangians.

(v) Expressions of the type \( A \cdot F \cdot T \) have 6 free indices, so they may serve as building blocks for diffeomorphism invariant Lagrangians. Considering the possible combinations of indices, we obtain the following generic expression

\[ L = F_{ab}(\beta_1 A^a T^{bm} + \beta_2 A_m T^{abm} + \beta_3 A_m T^{abm}). \]  

For imposing gauge invariance on this Lagrangian, it is necessary to require some constraints for the torsion field as, for instance,

\[ \beta_2 T^{abm} + \beta_3 T^{m[ab]} = 0. \]  

However, this would be an ad hoc procedure.

(vi) An expression of the type \( A \cdot A \cdot T \cdot T \) yields a rich family of diffeomorphism invariant Lagrangians. However, because of lack of gauge invariance, it is necessary to apply a constraint of the form \((\{A \cdot T \cdot T\} a)^a\).

If we look back to the Lagrangian \(2\), we recognize that \(2\) is of type (v) and (vi) and thus has to be excluded.

(vii) The expressions of the type \( A \cdot F \cdot T \cdot T \) have an odd number of free indices and, thus, do not yield diffeomorphism invariant Lagrangians.

(viii) The expression of the type \( F \cdot F \cdot T \cdot T \) has an even number of free indices and thus yields a family of diffeomorphism and gauge invariant Lagrangians. We will construct the complete list of such Lagrangians in the following sections.

Our considerations point to the existence of three different types of Maxwell-torsion Lagrangians:

- Diffeomorphism invariant Lagrangians which do not respect gauge invariance, namely the cases (i), (v), and (vi). Observe that even in these cases the situation is not completely hopeless. The gauge invariance could be reestablished by considering a torsion field that fulfills a suitable constraint. For generic elementary torsion such a constraint is unnatural and cannot be justified. However, if the torsion is not an elementary field but rather constructed from some other fields (scalar, vector, spinor, Kalb-Ramond, ...), then a gauge invariant Lagrangian may exist. An interesting possibility of a torsion generated by an antisymmetric Kalb-Ramond field was studied recently [7].
  
- The family (iii) of diffeomorphism and gauge invariant Lagrangians. On the level of the field equation, these models yield an addition term that is independent of the electromagnetic field. Such a term can be treated as an additional current. Consequently, the corresponding model admits the existence of a global (cosmological) electromagnetic field even without charges. Such a modification of classical electrodynamics seems to be unwarranted.
  
- The family of models with a supplementary Lagrangian of type (viii). Eventually, this candidate is left over and will be studied subsequently.

II. QUADRATIC TORSION COUPLED TO MAXWELL’S FIELD

Recently Preuss [8], in the context of a cosmological test of Einstein’s equivalence principle, suggested to investigate a specific nonminimally coupled Lagrangian density \( L = L * 1 \) of the form

\[ (Pr) L = \ell^2 (T_{a \wedge F} T^a \wedge F), \]  

which is supposed to be added to the conventional Maxwell Lagrangian. Here \( \ell \) is a coupling constant with the dimension of length and the star * denotes the Hodge duality operator.

The Preuss Lagrangian is a specific example of the family (viii) of the last section. In symbolic form, the additional torsion dependent Lagrangian of type (viii) reads

\[ (tor) L = -\frac{1}{8} \ell^2 \sum_{a, \ldots, q} (F_{ab} F_{cd} T_{kln} T_{npq}), \]  

where the summation is performed by contracting the indices with the help of the metric tensor. The indices of the expression \( F_{ab} F_{cd} T_{kln} T_{npq} \) can be contracted in the following three ways:

(i) All indices of the \( F \)-pair and of the \( T \)-pair are contracted separately. The only possibility for the \( F \)-pair is \( F_{ab} F_{ab} \). For the \( T \)-pair we have three possibilities and, accordingly, three independent Lagrangians:

\[ (1) L = F_{ab} F_{ab} T_{mnk} T_{mnp}, \]

\[ (2) L = F_{ab} F_{ab} T_{mnk} T_{nkm}, \]

\[ (3) L = F_{ab} F_{ab} T_{mn} T_{mnp}. \]

Certainly, these three Lagrangians could be rewritten via the irreducible pieces of the torsion [14] or as the independent parts of the teleparallel Lagrangian [15].

(ii) Two free indices of the \( F \)-pair are contracted with two free indices of the \( T \)-pair. The only possible \( F \)-pair is \( F_{am} F_{bn} = F_{bm} F_{an} \). Thus the \( T \)-pairs have to be
symmetric too. Consequently, we have five independent Lagrangians:

\begin{equation}
(4) \ L = F_{ab} F_{cd} T_{mn} T^{ab} T^{cd},
\end{equation}

\begin{equation}
(5) \ L = F_{ab} F_{cd} T_{mn} T^{ab} T^{cd},
\end{equation}

\begin{equation}
(6) \ L = F_{ab} F_{cd} T_{mn} T_{ab} T_{cd},
\end{equation}

\begin{equation}
(7) \ L = F_{ab} F_{cd} T_{mn} T_{kn},
\end{equation}

\begin{equation}
(8) \ L = F_{ab} F_{cd} T_{kn} T_{mn}.
\end{equation}

(iii) The $F$-pair and the $T$-pair have four free indices. Taking the $F$-pair in the general form $F_{ab} F_{cd}$, we find the following possibilities:

\begin{equation}
(9) \ L = F_{ab} F_{cd} T_{mn} T^{ab} T^{cd},
\end{equation}

\begin{equation}
(10) \ L = F_{ab} F_{cd} T_{mn} T^{cd} T^{ab},
\end{equation}

\begin{equation}
(11) \ L = F_{ab} F_{cd} T_{mn} T_{ab} T^{cd},
\end{equation}

\begin{equation}
(12) \ L = F_{ab} F_{cd} T_{mn} T_{ab} T^{cd},
\end{equation}

\begin{equation}
(13) \ L = F_{ab} F_{cd} T_{mn} T^{cd} T^{ab},
\end{equation}

\begin{equation}
(14) \ L = F_{ab} F_{cd} T_{mn} T_{ab} T_{cd},
\end{equation}

\begin{equation}
(15) \ L = F_{ab} F_{cd} T_{mn} T_{ab} T_{cd}.
\end{equation}

Summing up, the explicit form of the general torsion Lagrangian \cite{17} reads

\begin{equation}
\frac{1}{\ell^2} (\text{tor}) L = -\frac{1}{8} \sum_{i=1}^{17} \gamma_i (i) L,
\end{equation}

with the dimensionless constants $\gamma_i$.

Let us rewrite the Preuss Lagrangian \cite{15} in terms of components. With the formula $\ast w \wedge \vartheta = - (e_a \wedge w)$ (see \cite{14}), we find:

\begin{equation}
\frac{1}{\ell^2} (\text{Pr}) L = \frac{1}{16} T_{bca} F_{mn} T_{pq} F_{rs} \ast \vartheta^b c m n \wedge \vartheta^p q r s
\end{equation}

\begin{equation}
= -\frac{1}{16} T_{bca} F_{mn} T_{pq} F_{rs} \ast (e_p \vartheta^b c m n \wedge \vartheta^q r s)
\end{equation}

\begin{equation}
= -\frac{1}{8} T_{bca} F_{mn} T_{pq} F_{rs} \ast \vartheta^b c m n \wedge \vartheta^p q r s.
\end{equation}

\begin{equation}
= \frac{1}{4} (T_{bca} T_{def} F_{mn} F_{ab} + 4 T_{bca} T_{def} F_{mn} F_{ab} + T_{bca} T_{def} F_{mn} F_{ab}) \ast 1.
\end{equation}

Thus, the Preuss Lagrangian is given as a linear combination according to

\begin{equation}
(\text{Pr}) L = \frac{\ell^2}{4} \left( (1) L - 4 (5) L + (16) L \right),
\end{equation}

that is, $\gamma_1 = -2$, $\gamma_5 = 8$, and $\gamma_{16} = -2$, all other $\gamma_i$'s vanish.

\section{Electrodynamics with a Local and Linear "Constitutive Law"}

Now we want to put our findings on the nonminimal coupling of torsion to Maxwell's field into an efficient framework. This coupling can be reformulated as standard Maxwell theory with an effective constitutive tensor $\chi^{abcd} = \chi^{abcd} + \chi^{abcd} + \chi^{abcd}$, which is characterized by the constitutive tensor $\chi^{abcd}$ with 36 independent components \cite{18}. This tensor can be decomposed irreducibly according to

\begin{equation}
\chi^{abcd} = \overline{\chi}^{abcd} + \chi^{abcd} + \chi^{abcd},
\end{equation}

where $\overline{\chi}^{abcd}$ is the principal part (20 independent components), $\gamma^{abcd} = (\chi^{abcd} - \chi^{cda}b) / 2$ is the skew component (15 independent components), and $\chi^{abcd} = \chi^{[abcd]} = \theta^{abcd}$ is the axion part (1 component). The principal part is known to be related to the metric structure of spacetime. Moreover, as it is shown in \cite{18}, the metric can even be derived from the tensor $\overline{\chi}^{abcd}$. So, it is natural to expect that the additional parts of the constitutive tensor may also be related to the geometry of spacetime.

In fact, starting from the standard Lagrangian

\begin{equation}
L = -\frac{1}{2} H \wedge F - \frac{1}{4} H^{ab} F_{ab} \ast 1 = -\frac{1}{8} \overline{\chi}^{abcd} F_{ab} F_{cd} \ast 1,
\end{equation}

the additional constitutive tensor, induced by the torsion, turns out to be

\begin{equation}
(\text{tor}) \chi^{abcd} = \ell^2 \sum_{k=q} \left( T_{klmn} T_{npq} \right) [ab][cd],
\end{equation}

where the summation is understood as a contraction of the indices by means of the metric tensor such that the indices $a, b, c, d$ remain free.
The expression \( \chi_{abcd} \) is derived from a Lagrangian. Hence, in addition to the antisymmetry relations
\[
(\text{tor}) \chi_{abcd} = - (\text{vac}) \chi_{bacd} = - (\text{tor}) \chi_{abcd},
\]
it has to satisfy
\[
(\text{tor}) \chi_{abcd} = (\text{tor}) \chi_{cdab}. \quad \text{Accordingly, a skewon piece does not occur in (32) and the torsion field, in the framework of this Lagrangian approach, will modify the principal part and, additionally, generate an axion part of the constitutive tensor.}
\]
The axion part takes the form
\[
(\text{tor}) \chi_{abcd} = \ell^2 \left[ \sum_{k=1}^{q} \left( T_{klm} T_{npq} \right) \right]^{[abcd]} . \quad (33)
\]
The modification of the principal part
\[
(\text{tor}) \chi_{abcd} = \ell^2 \left[ \sum_{k=1}^{q} \left( T_{klm} T_{npq} \right) \right]^{[ab][cd]} - (\text{tor}) \chi_{abcd} \quad (34)
\]
may yield birefringence of the vacuum. Let
\[
\chi_{abcd} = (\text{vac}) \chi_{abcd} + (\text{tor}) \chi_{abcd} \quad (35)
\]
be prescribed, here \((\text{vac}) \chi_{abcd}\) denotes the vacuum constitutive tensor. Then the light propagation can be determined by means of the generalized Fresnel equation
\[
G^{abcd} q_a b q_c d = 0, \quad (36)
\]
where \(q_a\) is the wave covector of a propagating surface of discontinuity and
\[
G^{abcd} := 14 \epsilon_{mnpq} \epsilon_{rstu} \chi^{mn(r} \chi^{ps]c} \chi^{d)} q_t u \quad (37)
\]
is the Tamm-Rubilar tensor density. Thus additional terms in the constitutive tensor yield, via (37), a modification of the original Fresnel equation. In some cases, the original vacuum light cone is split into two light cones yielding birefrigence, as has been shown recently for the Preuss Lagrangian.

IV. AXION FIELD INDUCED BY TORSION

The axion, that is, a pseudo-scalar field, was extensively studied in different contexts of field theory. Its quantized version is believed to provide a solution to the strong CP problem of QCD. The emergence of axions is a general phenomenon in superstring theory. For the role of the axion in inflationary models, see e.g., [22]. The axion as a classical field also appears in various discussions of the equivalence principle in gravitational physics [23, 24, 25]. In the context discussed in this paper, the axion field is induced by the nonminimal coupling of Maxwell’s field to the torsion of spacetime according to [26]. Explicitly, we find the following:

(i) The Lagrangians [8, 9] yield
\[
(\text{tor}) \chi_{abcd} = \ell^2 \epsilon^{abcd} (T \cdot T) = 0. \quad (38)
\]
(ii) The Lagrangians [11]--[15] yield
\[
(\text{tor}) \chi_{abcd} = \ell^2 \epsilon^{abcd} (T \cdot T) = 0. \quad (39)
\]
(iii) As for the Lagrangians from the third group [16]--[21], we find the general form of the axion field \(\theta = \epsilon_{abcd} \chi_{abcd} / 4!\) as
\[
\theta = -\ell^2 \epsilon_{abcd} \left( \alpha_1 T_{mab} T_{cdm} + \alpha_2 T_{mab} T_{cdm} + \alpha_3 T_{mab} T_{cdm} + \alpha_4 T_{mab} T_{cdm} \right), \quad (40)
\]
where \(\alpha_1, \ldots, \alpha_4\) are free dimensionless parameters, which are linear combinations of \(\gamma\)’s.

The axion field of the Preuss Lagrangian reads explicitly
\[
(\text{Pr}) \chi_{abcd} = 2\ell^2 T_{[ab} T_{cd]} m \quad (41)
\]
or
\[
(\text{Pr}) \theta = \ell^2 \epsilon_{abcd} T^{[ab} T_{cd]} m. \quad (42)
\]
If in vacuum an axion field \(\theta\) emerges, then the Lagrangian picks up an additional piece \(\sim \theta F \wedge F\), see [18]. Accordingly, the inhomogeneous Maxwell equation reads
\[
d \epsilon F + d \theta \wedge F = J. \quad (43)
\]
Obviously, only a nonconstant axion field contributes. The homogeneous Maxwell equation remains untouched: \(d F = 0\). Hence charge conservation is guaranteed, that is, \(d J = 0\).

The axion field doesn’t influence the light cone structure, see [17, 18]. However, as was shown by Haugan and Lämmertz, see also the literature given there, the coupling of Maxwell’s field to a nonconstant axion field, in the case of a plane electromagnetic wave, amounts to a rotation of the polarization vector of the wave, i.e., the axion field induces an optical activity (similar to a solution of sugar in water [27]).

V. PRINCIPAL PART OF THE CONSTITUTIVE TENSOR \(\chi_{abcd}\) AND TORSION

A modification of the principal part of the constitutive tensor may yield crucial changes in the structure of the light cone, as we discussed at the end of Sec.III. In particular Preuss studied recently possible observational consequences. In the framework of our model, the constitutive tensor is determined by the torsion of spacetime, inter alia. Consequently, if the appropriate changes in the propagation of light were observed, this would represent evidence for the existence of the torsion field.

For different elementary Lagrangians we find the following effects on the light cone structure (we put \(\ell = 1\) for simplicity):

...
(i) The Lagrangians (39–41) yield
\[ (\text{tor}) \chi^{abcd} = S(g^{ac}g^{bd} - g^{ad}g^{bc}) = 2Sg^{[ac}g^{db]}, \]
where $S$ is a scalar function quadratic in torsion. Therefore, the Tamm-Rubilar tensor density changes only by a conformal factor. Accordingly, the structure of the light cone is preserved for such models.

(ii) For the Lagrangians (41–43), we introduce the abbreviation $S^{ab} = S^{ba} := [T \cdot T]^{(ab)}$. In this group of models, the axion field is absent. Thus
\[ (\text{tor}) \chi^{abcd} = \frac{1}{4} (g^{ac,S}g^{bd} + g^{ad,S}g^{bc} - g^{bc,S}g^{ad}) = g^{[ac}[S^{db]}], \]
In 6 × 6 matrix form, the 3 × 3 constitutive matrices, in an orthogonal frame, read (49):
\[ (\text{tor}) A^{\alpha\beta} := (\text{tor}) \chi^{\alpha\beta 00} = \frac{1}{4} (g^{00}S_{\alpha\beta} + g^{\alpha\beta}S^{00}), \]
\[ (\text{tor}) B_{\alpha\beta} := \frac{1}{4} \epsilon_{\beta\gamma\delta} \epsilon_{\alpha\mu\nu} (\text{tor}) \chi^{\gamma\delta\mu\nu} = \frac{1}{4} (g_{\alpha\beta}S_{\gamma} - S_{\alpha\beta}), \]
\[ (\text{tor}) C^{\alpha\beta} := \frac{1}{2} \epsilon_{\beta\gamma\delta} (\text{tor}) \chi^{\delta\alpha 0\gamma} = \frac{1}{4} \sqrt{\epsilon_{\alpha\gamma}S^{0\gamma}}, \]
\[ (\text{tor}) D_{\alpha\beta} := \frac{1}{2} \epsilon_{\alpha\gamma\delta} (\text{tor}) \chi^{0\beta\gamma\delta} = \frac{1}{4} \sqrt{\epsilon_{\alpha\beta}S^{\gamma\delta}}. \]
These matrices obey
\[ (\text{tor}) A = (\text{tor}) A^T, \quad (\text{tor}) B = (\text{tor}) B^T, \quad (\text{tor}) C = (\text{tor}) D^T, \]
where $T$ denotes the transposed of a matrix. As shown in [15], these relations, together with the closure condition for the constitutive tensor, guarantee uniqueness of the light cone. Thus no birefringence emerges also in this group of models.

(iii) For the Lagrangians from the third group [16–21], birefringence is a generic property. An example of this effect, for Preuss Lagrangian in the case of spherically symmetric torsion, was given in [8].

VI. EXAMPLE: SPATIALLY HOMOGENEOUS TORSION IN A FRIEDMANN COSMOS

As an example, we consider a spatially homogeneous torsion field appearing in a Friedmann type solution of gauge theories of gravity [22, 30, 31]. The independent nonvanishing components of the torsion are
\[ T_{0\alpha\beta} = \frac{u}{\ell} \delta_{\alpha\beta}, \quad T_{\alpha\beta\gamma} = \frac{v}{\ell} \epsilon_{\alpha\beta\gamma}, \]
where $u = u(t)$ and $v = v(t)$ are functions of time the explicit forms of which depend on the specific cosmological model under consideration [29, 30, 31]. We chose an orthonormal frame with metric $g_{ab} = \text{diag}(+1, -1, -1, -1)$.

The general Lagrangian [29], generated by torsion, yields, together with the constitutive tensor of the vacuum, the following nonvanishing components of the total constitutive tensor:
\[ \chi^{0\alpha\beta\gamma} = (c_1 uv) \epsilon^{\alpha\beta\gamma}, \]
\[ \chi^{0\alpha 0\beta} = - (1 + c_2 u^2 + c_3 v^2) \delta^{\alpha\beta}, \]
\[ \chi^{\alpha\beta\gamma\delta} = (1 + c_4 u^2 + c_5 v^2)(\delta^{\alpha\gamma}\delta^{\beta\delta} - \delta^{\alpha\delta}\delta^{\beta\gamma}). \]
Here $c_1, \ldots, c_5$ are linear combinations of $\gamma$'s.

Accordingly, the torsion induces an axial field $\theta = c_1 w$. In general, the time derivative $\dot{\theta}$ of this axial field doesn’t vanish. Thus in equation (40) only an additional time derivative shows up and the optical activity induced by the torsion of the Friedmann cosmos is proportional to $\theta = c_1 (\dot{w} + uv)$. The detailed form depends on the specific Friedmann model under consideration.

A further physical effect originates from the principal part $\chi^{abcd}$ of the constitutive tensor. For [19]–[31], the 3 × 3 constitutive matrices read
\[ A^{\alpha\beta} = - f \delta^{\alpha\beta}, \quad B_{\alpha\beta} = c^{\beta\alpha} = \sqrt{\epsilon^{\beta\alpha}S^\alpha}, \quad B_{\alpha\beta} = h \delta_{\alpha\beta}, \]
where $f = 1 + c_2 u^2 + c_3 v^2, g = c_1 uv, h = 1 + c_4 u^2 + c_5 v^2$. In a tedious calculation, the Tamm-Rubilar tensor density $G^{abcd}$ can be determined. With
\[ M := G^{0000}, \quad M^{\alpha\beta} := 6g^{00\alpha\beta}, \quad M^{\alpha\beta\gamma\delta} := G^{\alpha\beta\gamma\delta}, \]
we find the nonvanishing coefficients
\[ M = - f^3, \quad M^{\alpha\beta} = 2f^2 h\delta^{\alpha\beta}, \quad M^{\alpha\beta\gamma\delta} = - f^2 h^2 \delta^{(\alpha\beta\gamma\delta)}. \]
Then the generalized Fresnel equation (30) can be written as
\[ M q_0^4 + M^{\alpha\beta} q_0 q_\beta q_\gamma q_\delta + M^{\alpha\beta\gamma\delta} q_0 q_\beta q_\gamma q_\delta = 0. \]
For $f \neq 0$, this is a quadratic equation for $q_0^2$. Its solution reads
\[ q_0^2 = - M^{\alpha\beta\gamma\delta} q_0 q_\beta q_\gamma q_\delta \pm \sqrt{\Delta}, \]
with the discriminant
\[ \Delta := (M^{\alpha\beta\gamma\delta} q_0 q_\beta q_\gamma q_\delta)^2 - 4M M^{\alpha\beta\gamma\delta} q_0 q_\beta q_\gamma q_\delta. \]
This discriminant, upon substitution of (44), vanishes. Therefore we obtain a unique light cone
\[ f q_0^2 - h (q_1^2 + q_2^2 + q_3^2) = 0. \]
The light velocity is positive, provided $f/h > 0$. Thus, in the general model as well as in the Preuss model, the light cone is single and the effect of birefringence absent, see [15]. Certainly, this is a result of the isotropy of the Friedmann model. Accordingly, photons would propagate isotropically but with a torsion dependent velocity $v^2 = f/h$. In astrophysical observations this effect could shows up in a certain deviation from the cosmological redshift predictions of general relativity.

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Here $a, b, \ldots = 0, 1, 2, 3$ are frame indices and $\vartheta^a$ represents the coframe.

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[20] See Eq. (128). The Lagrangian is $\sim (1/2L^2) \left( T^a \wedge \delta b \right) \wedge \left( T_b \wedge \vartheta_a \right) + (1/2c) R^{ab} \wedge \vartheta_{\alpha \beta} + \vartheta F \wedge F$, with $R^{ab}$ as curvature, i.e., no explicit interaction term between gravity and electrodynamics was introduced. Still, the torsion turns out to be $T = (Mr - 2q) r^3 \delta \wedge \delta \vartheta$, etc., with $t$ and $r$ as temporal and radial Schwarzschild coordinates, respectively. Obviously, the torsion does depend on the electric charge $q$.

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