CONTINUUM COUPLING AND PAIR CORRELATION IN WEAKLY BOUND DEFORMED NUCLEI

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We formulate a new Hartree-Fock-Bogoliubov method applicable to weakly bound deformed nuclei using the coordinate-space Green’s function technique. An emphasis is put on treatment of quasiparticle states in the continuum, on which we impose the correct boundary condition of the asymptotic out-going wave. We illustrate this method with numerical examples.

1. Introduction

The RI-beam facilities in the new generation will increase significantly the number of experimentally accessible nuclei, especially in medium and heavy mass regions. We may reach nuclei close to the neutron drip-line in the $10 \leq Z \leq 20$ and $N \geq 20$ region, where a bunch of deformed neutron-rich nuclei are expected. This will provide us with a new opportunity to study interplay among the presence of weakly bound neutrons, the coupling to nuclear deformation effects, the pairing correlation, and the collective excitations.

A promising theoretical framework to describe this situation may be the self-consistent mean-field approaches. More specifically, we consider here the Hartree-Fock-Bogoliubov (HFB) method to construct the pair-correlated and deformed ground state, and the quasiparticle random phase approximation (QRPA) to describe the excitation modes built on the ground state. As commonly recognized, one has to describe precisely the nucleon wave function of weakly bound and unbound orbits, which should have proper asymptotic behaviours. There exist such formulations for spherical nuclei, but a new challenge here is that we have to do it for deformed nuclei. A method using the Pöschel-Teller-Ginocchio basis is proposed recently. The quasiparticle motion in deformed Woods-Saxon potential is analyzed in detail in the coupled-channel formalism. We here take a slightly different approach based on the coordinate-space Green’s function technique since we plan to apply it also to the continuum QRPA. In the present work, we shall
show that the coordinate-space Green’s function technique enables us to formulate the deformed continuum HFB method in which the nucleon waves satisfy a proper boundary condition of the asymptotic out-going wave.

2. Deformed continuum HFB method using the Green’s function

We first describe the quasi-particle motion in the HFB mean-fields consisting of the particle-hole field and the pair field which are both deformed. The axial symmetry is assumed. The quasiparticles of the Bogoliubov type have two-component wave functions \( \psi_{\sigma}^{(1,2)}(r\sigma) \), for which we use the radial coordinate system and the partial wave expansion

\[
\psi_{\sigma}^{(i)}(r\sigma) = \sum_{L} \phi_{L}^{(i)}(r) y_{L}(\hat{r}\sigma) \tag{1}
\]

with \( L \equiv (jlm) \) and \( y_{L}(\hat{r}\sigma) \) being the spin spherical harmonics. The HFB equation is then written as a coupled-channel Schrödinger equation for the radial wave functions \( \{ \phi_{L}^{(i)}(r) \} \) where the quantum number \( L \) represents the “channel”. Note that the energy spectrum of the quasiparticle consists of discrete and continuum parts, which are separated by the energy condition \( E < |\lambda| \) and \( E > |\lambda| \) (\( \lambda \) is the Fermi energy) as is in the spherical case.

We can construct the exact Green’s function for the quasiparticle motion in our deformed HFB problem. It is an extension of the spherical theory of Ref. 3 to deformed cases, and we accomplished this by employing a general prescription of constructing the exact Green’s function for a deformed potential scatterer. Here the HFB Green’s function (a \( 2 \times 2 \) matrix form combining the normal and abnormal functions) is expanded as

\[
G(r\sigma, r'\sigma', E) = \sum_{L,L'} N_{e} y_{L}(\hat{r}\sigma) y_{L'}(r, r', E)y_{L}^{*}(\hat{r}'\sigma'). \tag{2}
\]

The coupled-channel radial Green’s function \( g_{LL'}(r, r', E) \) is constructed as a linear combination of products of “regular solutions” \( \{ \phi_{L}^{(i)}(r) \} \) (i.e., those satisfying the boundary conditions \( \phi_{L}^{(i)}(r) \rightarrow r^{l} \delta_{L} \delta_{ij} \) at the origin \( r \rightarrow 0 \)) and “out-going wave solutions” \( \{ \phi_{L}^{(O)}(r) \} \) (those connected to the proper asymptotic form \( \phi_{L}^{(O)}(r) \rightarrow r^{-1} H_{L}^{+}(kr) \delta_{L} \delta_{ij} \) for \( r \rightarrow \infty \) where \( H_{L}^{+}(kr) \) is the out-going Hankel function).

We calculate the density \( \rho(r) \) and the pair density \( \tilde{\rho}(r) \) using the HFB Green’s function thus constructed. The generalized density matrix

\[
R(r\sigma, r'\sigma') = \left( \begin{array}{cc}
\rho(r\sigma, r'\sigma') & \tilde{\rho}(r\sigma, r'\sigma') \\
\tilde{\rho}^{*}(r\bar{\sigma}, r'\bar{\sigma}') & \rho(r\bar{\sigma}, r'\bar{\sigma}')
\end{array} \right) = \frac{1}{2\pi i} \int_{C} G(r\sigma, r'\sigma', E) dE,
\]

which is a sum of the wave functions of all the quasiparticle states including the continuum states, is calculated using a contour integral of the HFB Green’s function. Incorporating this way of calculating densities into the standard iterative algorithm, we obtain the HFB ground state after convergence.
3. Numerical analysis

We shall demonstrate with numerical examples how the deformed continuum HFB works. We adopt for simplicity a deformed Woods-Saxon potential as the particle-hole field, but we perform the HFB iteration to obtain the self-consistent pair field. We use the density-dependent delta interaction (DDDI) acting in the singlet pair, \( v_{\text{pair}} = v_0 \left( 1 - \eta \rho_n(r)/0.08 \right)^{0.59} \delta(r - r') \), (for neutrons) where \( v_0 \) is fixed to reproduce the \( nn \)-scattering length \( a = -18 \, \text{fm} \). We consider \(^{38}\text{Mg}\) and assume a deformation \( \beta = 0.3 \). Using the Runge-Kutta-Nystrom method we solve numerically the coupled-channel equation within an interval \( r = [0, r_{\text{max}}] \) \( (r_{\text{max}} = 15 \, \text{fm}) \) with a step size \( \Delta r = 0.2 \, \text{fm} \). At the outer boundary the wave functions are connected to the asymptotic forms. The parameter \( \eta = 0.76 \) is chosen to produce the neutron pairing gap around \( \Delta \sim 1.5 \, \text{MeV} \). The cut-off in the quasiparticle energy is 60 MeV, and the maximum \( \Omega \) (\( j_z \)) quantum number is \( \Omega_{\text{max}} = 21/2 \). For comparison, we performed also the HFB calculation using the same model but with a box boundary condition assuming an infinite wall at \( r = r_{\text{max}} \). In the following we show results for neutrons.

Figure 1 shows the radial profile of the monopole and quadrupole parts \( \rho_0(r) \) and \( \rho_2(r) \) of the neutron density \( \rho(r) = \sum_\lambda \rho_\lambda(r) Y_\lambda(\hat{r}) \), and the corresponding \( \tilde{\rho}_0(r) \) and \( \tilde{\rho}_2(r) \) of the neutron pair density \( \tilde{\rho}(r) \). We obtain exponential asymptotics here thanks to the proper boundary condition, and it is in contrast to the results obtained with the box boundary condition (the dashed curves in Fig. 1). \( \rho_0(r) \) and \( \rho_2(r) \) have the same exponential slope, indicating that we can define the deformation of the equi-density surfaces in the asymptotic region. This kind of deformed exponential tail is also seen in the neutron pair density. But the ratio of \( \tilde{\rho}_2(r) \) against \( \tilde{\rho}_0(r) \) is significantly smaller than that of the normal density. This points to that the pair density in the tail has smaller deformation than that of the normal density.

The quasiparticle spectrum above the threshold energy \( E_{\text{th}} = |\lambda| \) should be con-
The occupation number density $n(E)$ (left panel) and the pair number density $\tilde{n}(E)$ (right panel) for neutrons, plotted with the solid curves. The results obtained with the box boundary condition are also plotted with the dashed curves. The inset is a magnification of $\tilde{n}(E)$, and we compare it with the result (the dotted curve) obtained with a Woods-Saxon potential whose bottom is shifted up by +2 MeV.

The quasiparticle states corresponding to deep hole Woods-Saxon orbit appear both in $n(E)$ and $\tilde{n}(E)$ as narrow resonances. It is observed also that the non-resonant continuum states have a significant contribution to the pair number density $\tilde{n}(E)$. This figure shows also that the box-discretized calculation has difficulty to describe the non-resonant continuum states. The lowest energy resonance is not described well by a single state in the box-discretized calculation, and it is because this resonance has a rather large width. This resonance corresponds to the $[310]_{1/2}$ orbit in the deformed Woods-Saxon potential, which is, in the absence of the pairing, a bound state with the single-particle energy $e = -798$ keV.

Naturally we expect most dramatic effect of the weak binding on this state. The inset shows how the spectrum $\tilde{n}(E)$ changes when the depth of the Woods-Saxon potential is artificially shifted (made shallower) by 2 MeV. The Woods-Saxon single-particle energy $e$ and the Fermi energy $\lambda$ changes from $e, \lambda = -798, -889$ keV to $e, \lambda = -40, -88$ keV. We see in the inset of Fig.2 a dramatic increase in the width of the lowest-energy resonance, which apparently originates from the...
weak binding. Note however that the peak energy of the resonance stays almost constant. This implies that the effective pairing gap of this resonant quasiparticle state is unchanged, if we estimate the effective pairing gap through the relation $E_{qp} = \sqrt{(e - \lambda)^2 + \Delta^2}$. This observation is different from that in Ref. 9 claiming a reduction of the effective pairing gap due to the weak binding effect. The difference originates from the fact that we here use the selfconsistent pairing field generated from the DDDI, whose force strength becomes large at low densities. The pair field extending to far outside the nucleus plays important roles.

4. Conclusions

We have formulated the deformed continuum HFB method which is designed for weakly bound deformed nuclei. We utilized here the exact construction of the quasiparticle Green’s function for deformed HFB mean-fields on the basis of the coupled-channel representation. The proper boundary condition of the out-going wave is imposed on the continuum quasiparticles. We have analyzed numerically effects of the continuum coupling and the weak binding on the pair correlation. It is found that the quasiparticle states in the non-resonant continuum play significant role to generate the pair correlation. It is also suggested that the effective pairing gap of weakly bound orbits is not reduced very much, provided that the pairing interaction has the surface enhancement.

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