Metallic stripes: separation of spin-, charge- and string fluctuation.

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Inspired by the cuprate stripes, we consider the problem of a one dimensional metal living on a delocalized trajectory in two dimensional space: the metallic lattice string. A model is constructed with maximal coupling between longitudinal and transversal charge motions, which nevertheless renormalizes into a minimal generalization of the Luttinger liquid: an independent set of string modes has to be added to the long wavelength theory, with a dynamics governed by the quantum sine-Gordon model.

Evidence is accumulating that the superconducting state in the cuprates is closely related to the stripe phase, where the active charge degrees of freedom confine on anti-phase boundaries in the antiferromagnetic background. The case can be made that the stripes might be internally like one dimensional (1D) metals. Several theoretical works have appeared taking this ‘self-organized’ one dimensionality as a starting point. However, compared to conventional one dimensional metals, stripes are at the least qualitatively different in one regard: the trajectory on which the metal lives is itself delocalized in space, obviously so because of the absence of static striped order in either the superconducting- or normal states. The question arises what can be said about the general nature of a quantum string which is internally a metal. According to the Luttinger liquid theory of 1d metallicity, all what matters at long wavelength are the collective charge- and spin oscillations which are governed by quantum sine-Gordon field theories (QSG). It can be argued that the strings of relevance to cuprates are governed by QSG as well. Here we will demonstrate that at least in principle a fixed point theory exists which is a minimal generalization of the Luttinger liquid: a metallic string can be like a Luttinger liquid, except that a set of string modes has to be added for the theory to be complete. Our construction rests on the assumption that a reference string state exists which is at the same time localized in space and internally charge-incompressible due to a charge density wave (CDW) instability. The CDW solitons emerging under doping fluctuate at the same time the string position (CDW) instability. The CDW solitons emerging under doping fluctuate at the same time the string position (CDW) instability.

In the context of cuprates, it might be that the state at \( x = 1/8 \) corresponds with internally insulating, localized stripes. Since the average stripe separation \( d \) decreases like \( 1/x \) for \( x \leq 1/8 \), the stripes are likely internally charge incompressible. For \( x > 1/8 \), \( d \) becomes approximately \( x \) independent, suggesting that the stripes are ‘doped’ with the additional holes. At the same time, the static stripe phase shows a special stability at \( x = 1/8 \), and this might reflect a tendency towards localization on the single stripe level. For modelling purposes we assume the electronic system on the stripe to be dominated by short range repulsive interactions, favoring a \( 4k_F \) CDW instability as suggested by Nayak and Wilczek (Fig. 1a) – the other possibilities are more complicated, but not necessarily qualitatively different in the present context. Finally, it is assumed that spin separates at the very beginning and can be ignored all along. Since the stripe sweeps through a spin-full background, the neglect of spin is certainly not justifiable, and further work is needed on this fascinating problem.

If the string does not delocalize, the remaining problem of a doped \( 4k_F \) charge density wave is well understood. A representative model, in the sense of adiabatic continuity, is the extended Hubbard model with both \( U \) (on-site repulsion) and \( V \) (nearest-neighbor repulsion) large compared to the band width. At low doping lattice commensuration dominates and the relevant lattice scale physics is that of solitons. Using simple kinematics, Kivelson and Schrieffer pointed out that the injected hole splits into propagating soliton- and anti-soliton excitations, both carrying half the charge of the hole (fig. 1b). The soliton dynamics is described in terms of a spinless fermion problem,

\[
H_{\text{CDW}} = \sum_{ij} t'(ij)c_i^\dagger c_j + \sum_{ij} V'(ij)n_i n_j \tag{1}
\]

where \( c_i^\dagger \) creates a soliton, subject to short range hoppings \( (t') \) and (repulsive) interactions \( (V') \). This problem is dual to a bosonic quantum sine Gordon theory,

\[
H_{\rho,\text{ren}} = \frac{v_\rho}{2} \int dx \left[ K_\rho \Pi_\rho^2 + \frac{1}{K_\rho} (\dot{\phi}_\rho)^2 + g \sin(\alpha \phi_\rho) \right] , \tag{2}
\]

where \( v_\rho \) and \( K_\rho \) correspond with the charge velocity and charge stiffness, respectively. Away from quarter filling, this theory is in the weak coupling regime (the sine interaction is an irrelevant operator) and the long wavelength
dynamics is governed by free field theory (Luttinger liquid), completely specified by the renormalized stiffness and velocity. These parameters have to be calculated numerically, and their behavior is well documented for the extended Hubbard model [8].

![Diagram](image)

**FIG. 1.** Soliton dynamics in a strongly coupled doped $4k_F$ stripe. (a) The reference state: localized stripe with $4k_F$ charge density wave. (b) If the stripe is rigid, the doped hole separates in a left- and right moving soliton, both carrying half the electron charge. (c) When the curvature energy becomes less than the charge compressibility energy, the hole can escape ‘sideways’. (d) As a result, the solitons now carry a transversal (step up/down) flavor, which is like a spin degree of freedom. Holes tunneling through the stripe fluctuate the transversal flavor, see (c).

The most elementary physical interpretation of the quantum sine Gordon model, Eq. (1), actually corresponds with a free string moving on a lattice: the field $\phi$ is the transversal displacement ($z(l)$) at point $l$ of the string, while the cosine term describes the lattice washboard on which the string moves ($\alpha \phi \to 2\pi z(l)/a$). The weak- and strong coupling limits are easily understood as a freely meandering string and one which is fully localized due to the lattice potential. As we discussed elsewhere in detail [9], this notion is of relevance in the context of transversally fluctuating insulating stripes. In analogy with the charge density wave problem, the relevant lattice scale dynamics is that of transversal solitons or ‘kinks’. Consider the vicinity of the string delocalization transition. Because of the domination of the lattice potential the string tends to be localized on a particular lattice row $n$, and the exceptions are where the string jumps to neighboring rows $n \pm 1$ (Fig. 1c,1d). The origin of the collective motions of the string lies in the microscopic dynamics of these kink-excitations, while the tightly localized kinks of Fig. 1 represent a legitimate starting point, connected by adiabatic continuation to a class of string microscopies.

The existence of a localized stripe with internal $4k_F$ density wave allows for a simple unification of the microscopic string- and internal charge dynamics. Obviously, the fixed 1D electron trajectory assumed in the Luttinger liquid is no longer a given for electronic stripes. For a fixed trajectory, it costs an energy equal to the jump in the thermodynamic potential $\delta\mu$ to dope the charge density wave with an additional carrier. On the stripe, this (‘longitudinal’) energy cost can be reduced by letting the charge escape ‘sideways’, causing a transversal displacement, at the expense of paying a curvature energy. Hence, when this curvature energy becomes less than the energy cost associated with compressing the charge, the doped holes will ‘carry a string fluctuation’.

In terms of the strong coupling kinks/solitons, the microscopic mechanism of transversal relaxation is obvious: the doped hole corresponds with a double kink in the string which is at the same time a soliton-antisoliton pair in the on-string charge density wave – see Fig. 1. Starting from the CDW/localized string reference state, the kinks and the solitons are the same objects, and this corresponds with the strongest possible microscopic coupling between the on-string metallicity and the string fluctuation. Due to the string fluctuation, the CDW solitons acquire a transversal flavour: the (anti) soliton can move the string either in an ‘upward’ (↑) or ‘downward’ (↓) direction (Fig. 1c,1d). This transversal freedom is like a $s = 1/2$ iso-spin degree of freedom. Since the CDW solitons can be described in terms of spinless fermions, the string soliton dynamics relates to a spinful fermion system. Since the string dynamics is like the spin dynamics in a standard 1D metal, it follows that the separation of charge- and string dynamics is generic.

The qualitative nature of the long wavelength physics can be deduced from the strong coupling cartoon of Fig. 1, leaving the non-universal parameters of the theory to be determined from a more realistic microscopic theory. We seek a generalization of the spinless fermion model, incorporating the string flavor in terms of isospin labels $\uparrow$ and $\downarrow$ attached to the fermions (see Fig. 1c,1d). As a first guess, one could take the spin-full version of Eq. (3) with a hard-core ($U \to \infty$) condition: the string flavor is conserved under the hopping of the solitons. However, this neglects the specifics of the transversal sector: (i) curvature energy is associated with the order of the iso-spins. Obviously a $\uparrow\downarrow$ isospin configuration of neighboring solitons involves a different curvature energy than parallel configurations. These curvature energies can be absorbed in isospin-only Ising terms $\sim S_i^z S_j^z$. 

\[ F \sim \frac{1}{2} \sum_{\langle i,j \rangle} S_i^z S_j^z \]
\( \bar{S}_k = \sum_{\alpha, \beta} c_\alpha^\dagger (\sigma^\alpha_0 c_\beta \sigma^\beta_0). \) (ii) The overall transversal string displacement \( u \) after arclength \( r \) becomes (\( a_0 \) is the lattice constant),

\[
u(r) - u(0) = a_0 \int_0^r dx \sigma^z(x) \tag{3}\]

where \( \sigma(x) = \sum_m \bar{S}_m^z \delta(x - x_m) \) (\( x_m \) is the position of the \( m \)-th kink). As long as this quantity is conserved the string remains localized. Since \( U \to \infty \) there is no kinetic exchange, and Ising isospin terms do not fluctuate \( u(t) \) either. In order to fluctuate the string displacement the isospins should be exchanged and this is possible if and only if two kinks recombine into a hole, because the hole can tunnel through the string, see Fig. 1c. The simplicity of the argument is deceptive: \textit{this is an explicit realization of the idea of topological confinement} \[1]. \ Because of their topological nature, the kinks are strictly limited to the 1D string trajectory. In order to sweep the string through 2D space, the kinks have to pair up in holes, because the latter can propagate in 2D.

In isospin language, the hole tunneling corresponds with spin-flip (XY) terms \( \sim \bar{S}_m^+ S_{m+1}^- + h.c. \). Notice that the energy barrier involves the difference in curvature energy and the charge-compression energy. This might well be a small number, and the hole-tunneling rate can in principle be large. Assuming everything to be short ranged, we arrive at the following model, in standard notation (\( n = n_\uparrow + n_\downarrow \)),

\[
H_0^{str} = -t \sum_{n, \sigma} \left( c_{n+1, \sigma}^\dagger c_{n, \sigma} + c_{n, \sigma}^\dagger c_{n+1, \sigma} \right) + V \sum_n n_n n_{n+1} + U \sum_n n_{n+1} n_n + J_{//} \sum_{<nm>} S_n^x S_m^x + J_\perp \sum_{<nm>} \left( S_n^+ S_m^- + S_n^- S_m^+ \right) \tag{4}
\]

\( U \) should be taken to infinity, while \( t, V, J_{//} \) and \( J_\perp \) parametrize the kink kinetic energy, the ‘string neutral’ kink repulsion, the curvature energy and the hole tunneling rate, respectively.

Although we are not aware of explicit calculations on this particular model, the structure of the long wavelength dynamics can be deduced directly \[3\]. When \( J_{//} = J_\perp \), Eq. (4) is like the extended Hubbard model model with finite \( U \), at a low carrier density. The general case \( J_{//} \neq J_\perp \) corresponds with an interacting electron system with a spin-orbit coupling causing uniaxial spin anisotropy. The structure of the long wavelength dynamics can be obtained from the weak-coupling limit, and we refer to the extensive g-ology analysis including spin-orbit coupling by Giamarchi and Schulz \[12\]. Quite generally:

(i) we already mentioned that charge and string flavor will separate always. (ii) The charge dynamics is described by QSG, Eq. (3), and away from the quarter-filled point Umklapp becomes irrelevant. The charge dynamics at long wavelength is therefore described by free fields characterized by the fully renormalized charge-velocity and -stiffness, \( \tilde{v}_c \) and \( \tilde{K}_c \), which will behave similarly as to the ones of the extended Hubbard model in the strongly coupled regime. (iii) A crucial observation is that the gross behavior in the string sector is determined by the ‘isospin-only’ problem \[3\].

The isospin dependencies of the interactions are explicit in Eq. (4), and the isospin-only problem is nothing else than a XXZ problem with \( S = 1/2 \), which has been solved a long time ago \[3\]. If \( -1 < J_{//}/J_\perp < 1 \), the Ising interaction is irrelevant and the system falls in XY universality, as described by free-field theory - the free string is recovered. When \( |J_{//}| > |J_\perp| \) the Ising anisotropy takes over and the string modes acquire a mass – metalliclicity is a necessary but insufficient condition for the string delocalization. Physically, strings in this regime are of the ‘disordered flat’ variety \[1\]. Although kinks proliferate and delocalize, their internal string flavor (isospin) is ordered, as a compromise between kinetic energy and lattice commensuration energy. The ‘ferromagnetic’ case \( (J_{//} < |J_\perp|) \) corresponds with a ‘slanted’ phase \[3\]: the string is still localized, but it takes some direction in space determined by the density of kinks. For \( J_{//} > |J_\perp| \) the string is at average bond centered, and this phase is related to the hidden order present in Haldane spin chains \[3\].

The most interesting phase is the delocalized string, and we will now show that the asymptotic structure of Luttinger liquid theory implies a rather weak influence of the string-metallicity on the string fluctuation. A quantity of physical interest is the mean square transversal displacement of the string \[3\], using Eq. (3),

\[
\langle (u(r) - u(0))^2 \rangle = a_0^2 \int_0^r dx dx' \langle \sigma^z(x) \sigma^z(x') \rangle \tag{5}
\]

The spin-spin correlation function of a one dimensional metal has the asymptote \( (K_\sigma \text{ is the spin-stiffness}),\)

\[
\langle \sigma^z(x) \sigma^z(0) \rangle = \frac{C_1}{x^2} + \frac{C_2 \cos(2k_F x)}{|x|^{\eta}} \tag{6}
\]

where \( \eta = K_\sigma + K_p \). Although \( \eta \geq 1 \), it can be less than two and the staggered component of the spin-spin correlator could dominate the behavior of the string correlator, Eq. (5). However, it is easy to see that in the additional integrations in Eq. (5) the staggered component behaves as if it falls off by one power more than \( \eta \) \( (\int dx x^n \cos(2k_F x) x^n \to \int dx x^{n+1}) \). Since \( \eta \geq 1 \) it follows that the large \( r \) asymptote of Eq. (6) is governed by the uniform component \( C_1 \) in Eq. (5). Using that \( \int_0^r dx dx' f(x - x') = \int_x^r (2r - x) f(x) \) and the fact that \( \int_0^\infty dx (\sigma^z(x) \sigma^z(0)) = 0 \) it follows that the metallic string behaves asymptotically as a free string \[3\].

\[
\langle (u(r) - u(0))^2 \rangle = -2a_0^2 C_1 \ln(r/r_c) + \text{const.} \tag{7}
\]
with a constant coming from short wavelength physics and introducing a microscopic cut-off $r_c$.

Although not often discussed, the amplitude $C_l$ of the uniform component of the spin-spin correlation is also in the metal entirely determined by the spin sector, which implies in the present context that the strength of the string fluctuation is determined primarily by the transversal sector. This can be easily understood from the insight by Schulz [15] that the charge sector of the Luttinger liquids is nothing else than a 1D harmonic (‘floating’) Wigner crystal, realizing that the cut-off for the string delocalization. It follows that the exponent in the staggered component of Eq. (6) is the sum of the charge- and spin stiffnesses because the spin system ‘rides’ on the harmonically fluctuating charge solid. Following the same alley, it is straightforward to show [16] that this charge fluctuation is quite insensitive to the internal metallicity. As applied to cuprates, this observation offers a rational for the surprising insensitivity of the static stripe phases in e.g. LTT materials against stripe doping.

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[1] J.M. Tranquada, cond-mat/9709352 and ref.’s therein.
[2] J.M. Tranquada et al., Phys. Rev. Lett. 78, 338 (1997).
[3] J. Zaanen, J. Phys. Chem. Sol. (in press, cond-mat, 9711009).
[4] J. Zaanen et al, J. Low Temp. Phys. 105, 569 (1996); V. J. Emery, S. A. Kivelson, and O. Zachar, Phys. Rev. B 56, 6120 (1997); Yu. A. Krotov, D.-H. Lee and A. V. Balatsky, Phys. Rev. B 56, 8367 (1997); S. A. Kivelson, E. Fradkin and V. J. Emery, cond-mat/9707322.
[5] J. Voit, Rep. Prog. Phys. 57, 977 (1994) and ref.’s therein.
[6] H. Eskes et al, Phys. Rev. B 54, R724 (1996); ibid. cond-mat/9712316.
[7] Y. Nayak and F. Wilczek, Phys. Rev. Lett. 78, 2465 (1997).
[8] F. D. M. Haldane, Phys. Rev. Lett. 45, 1358 (1980); M. P. M. den Nijs, Phys. Rev. B 23, 6111 (1981); V. J. Emery, Phys. Rev. Lett. 65, 1076 (1990).
[9] K. Penc and F. Mila, Phys. Rev. B 49, 9670 (1994); K. Sano and Y. Ono, J. Phys. Soc. Jpn 63, 1250 (1994).
[10] S. A. Kivelson and J. R. Schrieffer, Phys. Rev. B 25, 6447 (1982); J. R. Schrieffer, Proc. Int. School. Phys. Enrico Fermi LXXXIX, F. Bassani ed. (Elsevier, New York, 1985).
[11] S. Chakravarty and P. W. Anderson, Phys. Rev. Lett. 72, 3859 (1994).
[12] T. Giamarchi and H. J. Schulz, Phys. Rev. B 33, 2066 (1986); ibid., J. Phys. France 49, 819 (1988).
[13] A. Luther and I. Peschel, Phys. Rev. B12, 3908 (1975).
[14] J. Zaanen, M. Horbach and W. van Saarloos, Phys. Rev. B 53, 8671 (1996).
[15] H. J. Schulz, in Correlated Electron Systems, ed. V. J. Emery (World Scientific, Singapore, 1993)
[16] J. Zaanen, O. Y. Osman and W. van Saarloos, unpublished.
[17] F. D. M. Haldane, Phys. Rev. Lett. 47, 1840 (1981).