Study on a 2D flow and sediment mathematical model for the reach where a bridge is located

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Abstract. In order to evaluate the feasibility of constructing a bridge in the end of Ganjiang River, a two-dimensional mathematical model was established in this paper. In order to verify the reliability of the mathematical model, based on the water and sediment processes from 2013 to 2018, the riverbed erosion and deposition were calculated. According to the measured terrain in 2013 and 2018, the calculated amount and distribution of erosion and deposition were verified. The verification results show that the mathematical model established in this paper can better simulate the erosion and deposition amount and distribution of the river reach, which can provide a reliable technical support for the construction of bridges.

1. Introduction

Nanchang, the capital city of Jiangxi Province is located in the end of Ganjiang River. With the rapid development of Nanchang City, the urban traffic pressure is increasing day by day. In order to solve the traffic difficulties, it is necessary to build new bridges in the south of Nanchang city. At present, the two-dimensional mathematical model of water and sediment is widely used in the calculation of bridges [1-2]. In order to explore the development trend of river bed erosion and deposition at the bridge site and to report the safety of the bridge, this paper established a two-dimensional mathematical model of water and sediment in the river reach where the bridge is located, and the mathematical model was verified by using the measured water and sediment data and the river bed topography data. This study can provide technical support for the bridge demonstration.

2. Study reach and method

2.1. Study reach

The study reach in this paper is the tail Ganjiang river reach where the bridge will be located. The reach is slightly curved and straight with U-shaped section. The water surface width of the river reaches 1500m at a high-water level, 700m at a dry water level. The annual average runoff was 69.06 billion m$^3$, with an annual average runoff of 75.99 billion m$^3$ in recent 10 years (2010-2019). The annual average sediment discharge was about 7.7 million tons with an annual average value of 2.3 million tons in recent 10 years (2010-2019) [3]. The location and topography of the study reach is shown in Figure 1.
2.2. Method
In this paper, a two-dimensional mathematical model of water and sediment in plane was used as the research means. The governing equations of the mathematical model is [4]:

\[
\frac{\partial Z}{\partial t} + \frac{\partial uH}{\partial x} + \frac{\partial vH}{\partial y} = 0 
\]  

(1)

\[
\frac{\partial uH}{\partial t} + \frac{\partial uH}{\partial x} = \frac{\partial}{\partial y} \left[ \frac{n^2 \sqrt{u^2 + v^2}}{H^2} u - gh \frac{\partial Z}{\partial x} + v\left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right] 
\]  

(2)

\[
\frac{\partial vH}{\partial t} + \frac{\partial vH}{\partial x} = \frac{\partial}{\partial y} \left[ \frac{n^2 \sqrt{u^2 + v^2}}{H^2} v - gh \frac{\partial Z}{\partial y} + u\left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \right] 
\]  

(3)

\[
\frac{\partial S}{\partial t} + \frac{\partial uS}{\partial x} + \frac{\partial vS}{\partial y} = \frac{1}{\rho} \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) - \alpha S - S^*_L 
\]  

(4)

\[
\gamma \frac{\partial S_L}{\partial t} = \alpha \omega_L (S_L - S^*_L) 
\]  

(5)

\[
Z = \sum_{i=1}^{N} z_i 
\]  

(6)

Among them, \( H \) is water depth (m); \( u \) and \( v \) are velocity (m/s) in \( X \) and \( Y \) direction; \( Z \) is water level (m); \( n \) is roughness coefficient of Manning; \( \nu_T \) is turbulent viscosity coefficient of water flow; \( g \) is gravity acceleration; \( S \) is sediment concentration; \( S^* \) is sediment carrying capacity; \( \omega \) is settling velocity of sediment particles (m/s); \( \gamma ' \) is dry density of sediment (kg/m\(^3\)); \( \omega_L \) is restoring saturation coefficient; and \( \omega_L, S_L, S^*_L \) are sediment particle settling speed, sediment concentration and sediment carrying capacity of suspended load groups, respectively.

In this paper, the structural mesh is used to divide the study area [5], the finite volume method is used to discretize the governing equations [6], and the ADI method is used to solve the discrete equations [7].

2.3. Data
The validation of this model is based on the water and sediment process from 2013-1-1 to 2018-12-31. In this process, the maximum, minimum and average flow discharge were 14600 m\(^3\)/s, 359 m\(^3\)/s and 2290 m\(^3\)/s. And the maximum, minimum and average sediment concentration were 0.148 kg/m\(^3\), 0.001
kg/m$^3$ and 0.0176 kg/m$^3$. By calculating the riverbed scouring and silting of the above processes, the model was then validated by using the calculation results and the actual scouring and silting thickness of the two measured topographies in 2013 and 2018. The water and sediment processes and the grading of the input sediment are shown in Figure 2.

![Figure 2](image)

**Figure 2** Flow discharge and sediment concentration process from 2013 to 2018.

### 3. Results

#### 3.1. Validation of scouring and silting amount

The scouring and silting volumes at different water levels were given in Figures 3 and Table 1. It can be seen that the measured riverbed of the main channel (below 8m contour) in the calculation reach from 2013 to 2018 was generally in the state of silting. The amount of silting is about 7.618 million m$^3$. According to the results of the numerical simulation, the silting is mainly accrued in the calculation period. The amount of calculated silting of the riverbed below 8 m elevation is about 7.220 million m$^3$. Compared with the measured value, the calculation error is about 5.49%, and the calculation error is small.

![Figure 3](image)

**Figure 3** Measured and calculated scouring and silting amount under different river bed elevation.

**Table 1** Comparison of calculated and measured scouring and silting amount.

| Elevation (m) | Measured ($10^4$m$^3$) | Calculated ($10^4$m$^3$) | Error |
|---------------|-------------------------|---------------------------|-------|
| 4             | 65.3                    | 60.0                      | -8.10%|
| 8             | 761.8                   | 720.2                     | -5.49%|

#### 3.2. Verification of scouring and silting distribution

Figure 4 shows the measured and calculated distribution of riverbed scouring and silting from 2013 to 2018. According to the measured distribution of riverbed scouring and silting, the main channel of the calculated reach is mainly slight silting from 2013 to 2018. The silting is generally in the range of 0.3m~1.2m, and the average silting range is about 0.5m. There was an obvious decrease in river bed elevation above 8 m contour in some local reach. For example, the left bank at the entrance of the study river section and the local offshore area, the decrease was generally more than 10m and the down cut is about 15.59 million m$^3$. Considering the existence of human activities such as channel regulation and
sand excavation within the calculation reach in recent years, the obvious downward area of the river bed above may be formed by human activities. By judging the calculation results, the distribution of calculated scouring and silting was mainly slight silting, and the silting range is generally in the range of 0.4m to 1.1m. In general, regardless of human activities such as sand mining, the calculated scouring and silting distribution, as well as the scouring and silting thickness were basically in agreement with the measured values, and the calculation accuracy well meets the requirements of sediment numerical simulation verification calculation.

Figure 4 Measured and calculated scouring and silting distribution.

4. Conclusions
In this paper, a two-dimensional water and sediment mathematical model of a river section was established and validated based on a 6-year water and sediment process and two topographic measurements. The validation results show that the two-dimensional water-sediment mathematical model established in this paper has high calculation accuracy and can be used for the study of riverbed scouring and silting trend in the reach where the bridge project will be located.

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