Slow deformation waves in an elastoplastic medium with faults

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Abstract. Slow deformation waves are excited mainly by natural processes in the Earth's crust and lithosphere and are manifested in changes in the seismic regime and geophysical fields. There are two main types of slow deformation waves: inter-fault and intra-fault. The principal difference in the nature of the propagation of these zones is that in the case of transmission of deformation excitation from fault to fault (inter-fault wave) the speed value is in the range from 20 km/year to 30 km/year or more. If the excitation of processes occurs within a single fault zone (intra-fault wave) the speed varies from 10 km/year to 4 km/year or less. Structural maps of the model medium with faults were used to study the generation and propagation of inter-fault slow deformation waves. A mathematical model proposed by P. V. Makarov was used to perform test calculations for the generation and propagation of slow deformation waves in nonlinear elastic-plastic media. The features of the propagation of deformation waves are investigated for different fault positions in the calculation region. It is shown that the propagating fronts of slow perturbations in the medium have different shapes depending on the location of the faults in the calculation regions. The zigzag nature of the wave front is revealed in a medium with faults in the center. When fault ends are located near the boundary of the calculation region, they have a quasi planar shape.

1. Introduction

Modern geodynamics is a part of general geodynamics that studies the movements of the Earth's interior and the reasons that cause them when the time of the latter's actions is commensurate with the duration of the observation process itself. In this case, the duration of observations is understood as either the interval between repeated (geodetic, geophysical) measurements or the period of continuous recording of parameters by geophysical (hardware) methods [1].

The main phenomena studied in modern geodynamics are deformation and seismic processes, as well as related variations of geophysical and fluid-geochemical fields. Seismicity is often attributed to modern geodynamics. This is true because seismicity is a fast component of the geodynamic process. Actual movements (deformations) belong to the slow part of the spectrum of geodynamic phenomena. Therefore, in recent years, there has been a tendency among experts to refer to slow movements as modern geodynamics (or deformation processes) and to define everything related to earthquakes as seismic processes. Consequently, the current geodynamics of active faults is studied both in seismically active and aseismic regions. However, if in seismically active regions the identified faults represent a potential activity or threat by the very fact of their existence, this is not obvious in platform, weakly seismic regions [2]. As a rule, the term “fault” or “fault zone” is used as a kind of interface...
between blocks that differ in various mobility or other characteristics. The other point of view is that faults should be considered as specific geological bodies, a certain volume of the Earth's crust that has an anomalous structure and increased jointing, resulting from linear destruction of the medium. These ideas correspond to a significant extent with the concepts of S. I. Sherman and the tectonophysical school headed by him. A fault zone is an area containing rocks with anomalous physical-mechanical, geogeophysical, fluid-geochemical, and other characteristics [1].

Deformation waves of the Earth were one of the most striking guesses in theoretical geophysics of the last third of the XX century. The successes of theoretical research have generated great interest in the search for possibilities of experimental detection of the effects of wave propagation of this type, and, first of all, in the intensive study of the migration of earthquakes. It was from attempts to explain the cause of the directional migration of earthquakes discovered by C. Richter in 1958 in the North Anatolian fault in Turkey [3], and thereby to resolve one of the emerging problems of seismology, that the active formation of the concept of deformation waves of the Earth began. Earthquake migration is associated with the propagation of tectonic stresses which cause additional stress and, as a consequence, the successive occurrence of strong earthquakes in fault segments with a high concentration of elastic stresses [4].

The formation of the concept of deformation (tectonic) waves of the Earth was largely developed on the basis of two discoveries made by that time: migration of sources of strong earthquakes along deep faults and global plate tectonics. Representations of the lithospheric plates separated by powerful faults and underlain viscous asthenosphere, led to the construction of three types of deformation waves theoretical models: 1) layered models (lithosphere—asthenosphere) [5, 6]; 2) layered models with the addition of the effect of bending of the rigid lithospheric plate [7]; 3) models of faults with a viscous interlayer between the sides (viscoelastic). These models were designed to describe slow stress waves corresponding to the migration of strong earthquakes along transform faults and troughs (valleys).

The second stage in the development of the theory of deformation waves of the Earth began after the publication of V. N. Nikolaevsky in the article [8], in which, in fact, the sin-Gordon equation was postulated for modeling slow solitary waves in a block geological medium. Evidence of these waves was observed in the form of migration of anomalies of geophysical fields near faults. The observed behavior of spatiotemporal migration of modern deformations in fault zones [9, 10] and dynamics of seismic activity has a qualitative similarity with general ideas about excitable active medium, admits of proposal about the autowave character of the deformation process in the fault-block geosphere and can serve as a physical motivation for the use of “autowave” analogies in mathematical modeling of directed migration of deformations and earthquakes [4]. P. V. Makarov in his works [11–13] proposed a version of the general physical and mathematical theory of dynamic processes that can describe the joint generation and propagation in an elastic-plastic medium of both ordinary stress waves and slow deformation autowave and soliton-like perturbations. To carry out this operation, it is necessary to perform calculations and compare the results with experimental data.

The aim of the present work is to study numerically the features of the formation and propagation of slow deformation disturbances in a medium containing faults or foci of inelastic deformation.

The present paper will mainly describe and discuss the generation and propagation of slow deformation waves in nonlinear elastic-plastic media with faults. The paper is structured as follows: the mathematical model used for numerical simulation of slow deformation fronts is stated in section 2; section 3 discusses the results of simulations on the propagation of deformation fronts from faults of different orientation and location; section 4 presents the conclusions derived from the results of the study.

2. **Mathematical model of slow deformation autowave processes**

In the model by P. V. Makarov [11] the methodology of synergetics is laid down: autowave deformation processes in geo-media are considered as the phenomena of self-organization of deformations at different scale levels. The proposed model is based on the most general fundamental concept of solids under loading:
1. All solids, including geomedia, are hierarchically organized multiscale nonlinear dynamic systems. In such a multi-scale media, it is possible to generate deformation perturbations of various physical nature including slow waves of plastic deformation—Lüders fronts in metallic samples, deformation perturbations, as well as waves of damage and/or destruction in quasi-brittle and brittle materials and geomedia caused by perturbations of stress-strain behavior. The multiscale hierarchical organization of a solid allows the generation of slow wave trains of different scales from microscales of elementary inelastic (plastic) events to scales of global tectonic flows and large tectonic plates, and due to deformation self-similarity on different scales, they can be described by a unified mathematical model. These wave disturbances are generated by various physical reasons, in particular, by movements at the boundaries of the blocks.

2. The process of generating slow deformation waves is a cooperative self-consistent deformation response to the external action to the loaded medium as a nonlinear dynamic system with the property of self-organized criticality. Thus, the formation of fronts of slow deformation movements is a process of self-organization in the loaded medium.

3. The scales, long-range action, and velocities of the forming deformation fronts are completely determined by the amount of energy delivered to the medium, the level of generated stress, and the rates of kinetic processes of generation of inelastic (plastic) deformations and/or damage in the loaded medium.

The mathematical model includes the fundamental equations of solid mechanics and constitutive relations that specify the elastoplastic response of the medium to loading (equations (1)–(10)). Combined with the cellular automat method, these equations allow implementing in calculations a cooperative coordinated response of a medium to loading [12–15]. This procedure allows for simulating numerically the self-organization of individual inelastic events in the form of localized plasticity fronts or damage waves.

\[
\frac{dp}{dt} + \rho \text{div} \mathbf{v} = 0 \tag{1}
\]

\[
\rho \frac{du_i}{dt} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho F_i \tag{2}
\]

\[
\sigma_{ij} = -P \delta_{ij} + S_{ij}; \quad P = -\frac{1}{3} \sigma_{ii} \tag{3}
\]

\[
P = -K(\dot{\varepsilon}_{ii}) \tag{4}
\]

\[
\dot{S}_{ij} = 2\mu(\dot{\varepsilon}_{ij}^e - \frac{1}{3} \delta_{kk} \delta_{ij}) - S_{ik} \dot{\omega}_{kj} + S_{kj} \dot{\omega}_{ik} \tag{5}
\]

\[
\dot{\varepsilon}_{ij}^e = \dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^p \tag{6}
\]

\[
\dot{\varepsilon}_{ij}^p = \lambda \frac{\partial g(\sigma_{ij})}{\partial \varepsilon_{ij}}, \text{ if } f(\sigma_{ij}) \geq 0; \text{ otherwise } \dot{\varepsilon}_{ij}^p = 0 \tag{7}
\]

\[
g(\sigma_{ij}) = \Lambda P + \sqrt{\frac{1}{2} S_{ij} S_{ij}} + \text{const} \tag{9}
\]

\[
f(\sigma_{ij}) = \alpha P + \sqrt{\frac{1}{2} S_{ij} S_{ij}} - Y \tag{10}
\]

Here equations (1)–(2) express conservation of mass and conservation of momentum laws, \( \rho \) is the density, \( u_i \) are the components of the velocity vector, \( \sigma_{ij} \) are the components of the stress tensor, \( F_i \) are the components of the vector of the bulk forces. Equation (3) means the decomposition of the stress tensor to the hydrostatic pressure and deviatoric stresses. \( P \) is the pressure, \( S_{ij} \) are the components of the deviatoric stress tensor, \( \delta_{ij} \) is the Kronecker delta. Equations (4) and (5) are the constitutive...
evolutionary equations of the first group. Equations (6) mean that the total strain rates are the sum of the elastic and plastic parts of the strain rates. Equations (7) are the geometrical relations for the strain rate and spin tensors. The constitutive evolutionary equations of the second group define the plastic strain rate using the flow rule (8) and functions of plastic potential (9) and the yield surface (10). The following notation is also used: $K$ is the bulk modulus of elasticity, $\mu$ is the shear modulus, $\dot{\varepsilon}_{ij}^e$ and $\dot{\varepsilon}_{ij}^p$ are the elastic and plastic components of the total strain rate $\dot{\varepsilon}_{ij}$, respectively, the dot above the symbol means the material time derivative, $S_{ik}\dot{\omega}_{kj} + S_{kj}\dot{\omega}_{ik}$ is the correction for the rotation that occurs when using corotational Jaumann derivative of the stress tensor, $f(\sigma_{ij})$ is the yield function, $g(\sigma_{ij})$ is the plastic potential function, $\Lambda$ is the dilatancy coefficient, $\alpha$ is the internal friction coefficient, $\lambda$ is the plastic multiplier.

The model with the Drucker–Prager yield criterion combined with the method of cellular automata was used for describing slow deformation fronts of inelastic nature in nonlinear elastic-plastic media. According to the algorithm of cellular automata [12–14], fault zones—narrow elongated areas inclined at some angle to the axis of the load application—were defined as the areas where plastic deformation can be generated.

The behavior of a region of the medium with faults in the conditions of uniaxial compression along the vertical axis was studied using an implementation of the algorithm of cellular automata for transmitting slow perturbation. Specifically, two variants of the neighborhood exist in two-dimensional models of cellular automata: von Neumann and Moore. In the von Neumann neighborhood, the neighbors of a given cell are cells that have a common edge with the given cell. Thus, in our case of the quadrangular cell, each cell has exactly four neighbors. In the Moore neighborhood, two cells are adjacent if they have either a common edge or a common vertex. That is, there are eight cells in the local neighborhood of each cell (apart from itself). Namely, in our research here, we used a variant of the von Neumann neighborhood in two-dimensional models of cellular automata.

The complete system of equations of continuum mechanics was solved using the finite difference method [16]. Simulations were fulfilled using a computer code written by the authors. The computer code has been used previously for simulating various ductile and brittle materials, rocks, geomedia [17–20]. We have performed the corresponding test calculations. Concerning the modeling of slow deformation fronts, note that analytical solutions for testing software implementation do not exist. Therefore, for testing, we repeated the calculations of other authors [12, 13, 21] and obtained good agreement between the results.

To carry out the research calculations of structurally inhomogeneous materials within the framework of the problem under consideration, we have to choose the computational grid that meets some necessary conditions. The conditions are as follows: (i) the number of cells occupying the minimum size of a structure element (a fault width) should not be less than 3; (ii) the results of calculating the stress and strain must converge with the grid refinement. In our case, the Drucker–Prager stress and effective plastic strains in the zones of stress concentrators (the edges of the faults) are of interest. It turned out that a uniform computational grid with 300,000 cells satisfies well with these conditions. We performed our calculations on a personal computer with AMD Ryzen Threadripper 1950X processor, and one calculation took about 8–12 hours, depending on the features of the task.

3. Results and discussion

To describe slow deformation fronts of an inelastic nature in nonlinear elastoplastic media, a model combining the idea of cellular automata and the corresponding constitutive equations that take into account the effects of internal friction and dilatancy is used [12, 13]. Two model structural maps of medium with two faults were constructed. Specifically, the first is a model map, where two faults are located in the center of the calculation region (figure 1 a), the second is a model map where the ends of two faults are placed near the boundary of the calculation region (figure 1 b).
Figure 1. Examples of structural maps of a model medium with faults and loading conditions.

Figure 2 shows a chronogram of the propagation of slow deformation waves between faults located in the center of the calculation region. It can be seen that two fronts of a slow deformation wave are generated at the tops of the faults, which propagate in the inter-fault zone towards the opposite fault.

Figure 2. Propagation of deformation perturbations in the medium when faults are located in the center.
The main advances of the front are in two orthogonally related directions, and the vertical advance is always greater than the horizontal one. In doing that, several angular fronts are formed at some points along a vertical line that stems from the fault’s top. Taking into account the shearing nature of plastic deformations, the wave front is not straight, but zigzag. Depending on the values of the rheological parameters of the medium, the number of zigzags changes.

The chronogram of propagation of slow deformation waves from faults (depicted as thin lines in figure 3 at time $t_1$) located near the boundary of the medium is shown in figure 3.

![Figure 3. Chronogram of propagation of inter-fault deformation perturbations.](image-url)
One can see that the fronts of deformation perturbations head towards each other at approximately the same velocities, their shapes are close to planar but gently arched, they change slightly in time. When the fronts meet, they annihilate, reducing the area not covered by inelastic deformation yet.

A detailed study showed that the speed of slow deformation perturbations exceeds the value loading velocities applied at the boundaries of the calculated region by about 300 times. It should be noted that this is the average speed and the propagation of the deformation fronts is not steady but intermittent.

4. Conclusion
Thus, combined with the cellular automaton method, the equations of solid mechanics allow describing of slow deformation fronts of inelastic nature in nonlinear elastic-plastic media. The features of the propagation of deformation waves are investigated for different fault positions. As a result of the research, it is shown that the shapes of propagating fronts of slow perturbations in the medium are different depending on the position of the faults in the calculation regions. The wave front in a medium with faults in the center has a zigzag character due to the action of uniaxial external loading and maximum shear stresses. The main advances of the front are in two orthogonally related directions, and the vertical advance is always greater than the horizontal one. In the second case, when the faults stem from the boundaries of the calculation regions, the deformation perturbation fronts have a quasi planar shape, approach each other, and annihilate when they meet. In both cases, the fronts do not move at a constant speed but alternating periods of stopping and advancing.

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