Large Scale Structure:
Setting the Stage for the Galaxy Formation Saga

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Abstract
Over the past three decades the established view of a nearly homogeneous, featureless Universe on scales larger than a few Megaparsec has been completely overhauled. In particular through the advent of ever larger galaxy redshift surveys we were revealed a galaxy distribution displaying an intriguing cellular pattern in which filamentary and wall-like structures, as well as huge regions devoid of galaxies, are amongst the most conspicuous morphological elements.

In this contribution we will provide an overview of the present observational state of affairs concerning the distribution of galaxies and the structure traced out by the matter distribution in our Universe. In conjunction with the insight on the dynamics of the structure formation process obtained through the mapping of the peculiar velocities of galaxies in our local Universe and the information on the embryonic circumstances that prevailed at the epoch of Recombination yielded by the various Cosmic Microwave Background experiments, we seek to arrive at a more or less compelling theoretical framework of structure formation. The main aspects of this framework of the rise of structure through gravitational instability can probably be most readily appreciated through illustrative examples of various scenarios, as for instance provided by some current state-of-the-art N-body simulations.

We will subsequently wrap up the observational and theoretical evidence for the emergence and evolution of structure in the Universe by sketching the stage for the ultimate Holy Grail of late 20th century astrophysics, understanding the saga of the formation of what arguably are the most prominent and at the same time intoxicatingly beautiful and intriguing denizens of our Cosmos, the galaxies.
Figure 1. The Las Campanas redshift survey. The figure displays the three-dimensional position of 26418 galaxies in 6 thin strips on the sky. The total survey comprises nearly 700 square degrees, with each strip measuring a $1.5^\circ \times 80^\circ$ region on the sky, with the survey extending out to an effective depth of approximately 300 Megaparsec. Clearly visible is the spongelike features into which the galaxies have organized themselves, with filaments and walls surrounding void regions with characteristic sizes in the order of 50 Mpc. Courtesy: Shectman, S., Schechter, P., Oemler, G., Kirshner, B., Tucker, D., Landy, S., Hashimoto, Y. & Lin, H.
1 Cosmic Minutiae: the origin of cosmic structure

With the twentieth century drawing to a close we may be melancholic and see that human irrationality and evil ran havoc on an unprecedented scale, exposing the world to almost inconceivable levels of destruction, terror and suffering. On the other hand, we are equally justified in priding ourselves of participating in an era of unparalleled triumph of the human ratio, a century full of tremendous scientific progress and enlightenment. Forever our century will figure as the one in which the human mind finally succeeded in unlocking the seals holding the secret to our cosmic origin. Cosmology finally broke away from its mythological roots, and for the first time since the Dawn of Civilization the centuries old quest for the origin of the world seems to have come across a well-founded and consistent answer, inscribed in the scientific epos of the “Hot Big Bang” theory. The success of these relativistic, homogeneous and isotropic Friedmann-Roberstson-Walker Universe models — including the repercussions for the subsequent unfolding of the physical state of Universe — in describing the structure and evolution properties of the Universe on global scales of many hundreds of Megaparsec and beyond, is truely enchanting.

However, the Friedmann-Robertson-Walker Universe only represents the cosmos in its most global and universal context, it does not contain any explanation of its own origin, nor of the state or even the existence of its constituents. It does not go to any extent in explaining why the Universe is one harbouring a wealthy internal structure. Moreover, the past three decades have seen a radical revision of our view of the structural organization of matter in our Universe. The canonic view of a nearly homogeneuous, featureless Universe on scales larger than a few Megaparsec got completely overhauled into one displaying a baffling richness, populated by an astonishing variety of objects. As a matter of illustration, when turning to Figure 1, a map of the distribution of galaxies yielded by the Las Campanas survey (e.g. Landy et al. 1996), we see that the image of a homogeneous Universe is far besides truth, and cannot be anything but an approximation of reality, valid only for the Universe on global scales. Within cosmology the issue of the formation of structure has therefore gradually manoeuvred itself to the forefront of scientific interest. Not only because it fills in an obvious hiatus in the Friedmann-Robertson-Walker models, but also because we have come to realize that an understanding of the structure formation process is of key significance in unravelling the primordial physical processes determining the evolution and fate of the Universe itself. While we
may have the impression that objects and structures ranging from planets, stars up to superclusters are but cosmic minutiae, we should realize that it is often minutiae that are the punctuation marks enabling us to systematize the flood of information reaching us from cosmic realms into new levels of insight.

The central unsolved riddle of cosmology has therefore become the question of how the near perfectly homogeneous, featureless, extremely hot and dense early Universe gave rise to the wealth and variety of structure which make our cosmos into such a fascinating world to live in. Instrumental in solving this puzzle is the realization that our Universe still contains cosmological fossils, structures and physical properties that still contain traces of the processes that have been responsible for the emergence of all the objects and structures populating our cosmos. The way in which matter has arranged itself on scales of a few up to several hundreds Megaparsec has evolved sufficiently far to yield observable manifestations of the growth process while its matter contents and internal motions have not yet been blended to such an extent that they do no longer contain any directly and objectively retrievable information on the structure formation processes.

These fossils revealed themselves as cosmological research in the second half of the twentieth century came across the existence of a rich and beforehand unexpected organization of matter into structures over a large range of scales. The structure of these matter arrangements has been unveiled through the mapping of the distribution of the galaxy distribution, currently even over a substantial range of cosmic history. Moreover, by virtue of its imprint on passing radiation from objects in the background, snatching its Ly $\alpha$ photons, culminating into a forest of “Ly $\alpha$” lines, we are even obtaining a reasonable idea of the distribution and physical state of diffuse matter populating the regions in between identifiable “objects”. Even the embryonic state of the cosmic matter distribution has been exposed to scientific exploration through the detection of temperature fluctuations in the cosmic background radiation, following the ground-breaking efforts of the COBE satellite. We even have been able to obtain insight into the dynamics underlying the structure formation process, as meticulous and careful measurements of peculiar motions of galaxies yielded information on the forces that have been shaping the organization of matter. Moreover, stakes are high that exploitation of the gravitational impact on the path of photons, usually phrased as gravitational lensing, will enable us to get an unbiased view of the gravitational potential throughout the Universe.

While we are witnessing a continuously expanding inflow of new data on cosmic structure, a sensible interpretation of these data can only be achieved through providing a general physical framework for their formation and evolu-
tion. Although there have been several theories around, over the past decade one has clearly obtained the lead, most data at least partially corroborating its implications, the theory of Gravitational Instability.

2 A Cosmological Footnote: Creation through Gravitational Instability

The finding of COBE of very small fluctuations in the temperature of the microwave background radiation, and its interpretation in terms of slight variations of the gravitational potential at the surface of last scattering, is a remarkable confirmation of the general theoretical framework of "gravitational instability" for cosmic structure formation. According to this theory the early universe was almost perfectly smooth except for tiny density variations with respect to the general background density of the universe and related tiny velocity perturbations with respect to the general Hubble expansion. Because slight density enhancements exert a slightly stronger gravitational attraction on the surrounding matter, they start to accrete material from its surroundings as long as pressure forces are not sufficient to counteract this infall. In this way the overdensity becomes even more overdense, and their gravitational influence even stronger. The denser it becomes the more it will accrete, resulting in an instability which can ultimately cause the collapse of a density fluctuation to a gravitationally bound object. This generic process of structure formation is illustrated in Figure 2, displaying both a density fluctuation field, the corresponding force field and the subsequent displacement of parcels of matter. The size and mass of the object is of course dependent on the scale of the fluctuation. For example, galaxies are thought to have formed out of fluctuations on a scale of $\approx 0.5h^{-1}\text{Mpc}$, while clusters of galaxies have emerged out of fluctuations on a larger scale of $\approx 4h^{-1}\text{Mpc}$. The formation of voids fits in the same general scheme, having grown out of primordial underdensities in the matter distribution.

Providing the general framework, the gravitational instability theory needs lots of details to be filled in before it can be considered a complete theory. There is of course the issue of the amount of matter represented by a density fluctuation, as more massive fluctuations will collapse sooner. Given the amplitude of the fluctuations, their total mass is determined by the average cosmological density, paramerized by $\Omega$. The very low value of the amplitude of the primordial density fluctuations inferred from the COBE MWB measurements is a strong argument in favour of a high overall density of the universe. Otherwise,
density fluctuations would simply not have had sufficient time to collapse on all the scales that nowadays are observed to exhibit so much structure. Also some other observational indications support a high value of $\Omega$, which has the important implication that most likely the major share of matter in the universe does not consist of familiar baryons and leptons but of one or more as yet unidentified species of “dark matter”.

The nature and amount of dark matter is also of substantial influence in determining the character of the initial density and fluctuation field, probably the most crucial issue in the structure formation saga. Rather than consisting of some isolated, well-defined and smooth density peaks and dips, each of its own particular scale, the density field can be thought of as a random superposition of fluctuations of various scales. It will therefore bear the character of a noise field, “a random field”, a random superposition of waves much like the surface of the sea at rough weather. Evidently, the waves with the largest amplitude will collapse first. The character of the density field evolution will then depend on the relative amplitudes of the different waves. One extreme case is that of small scale waves having by far the highest amplitude. They will collapse into virialized objects well before a larger scale perturbation, in which they are possibly embedded, starts to collapse. Consequently, we will see a hierarchical or “bottom-up” build-up of structure, where small objects that formed first merge into larger structures, which themselves merge to form galaxies, cluster of galaxies, and so on. The other extreme is that of the case in which there are only perturbations on large scales, with no contributions from smaller scales. In such a “top-down” scenario the first emerging structures form through the collapse of those large scale perturbations. In the most popular versions of “top-down” theories these objects would correspond to superclusters. Subsequently, smaller objects like galaxies have to form through the fragmentation of these collapsed large objects into smaller pieces, an as yet mostly ununderstood process in which non-gravitational gas processes play a key role.

The formation of anisotropic structural patterns in these random density fields is the consequence of an additional characteristic property of gravitational collapse. Overdensities, on any scale and in any scenario, always collapse such that they become increasingly anisotropic. At first they turn into a flattened “pancake”, later possibly followed by contraction into an elongated filament or by full collapse into a virialized clump like a galaxy or a cluster. This tendency to collapse anisotropically is caused by the intrinsic primordial flattening of the overdensity as well as by the anisotropy of the gravitational force field induced by the external matter distribution, i.e. by tidal forces. In the case of a pure hierarchical scenario the amplitude of large scale overdensities will be so low
Figure 2. Illustration of structure formation through gravitational instability. A cut through a random density field realization is displayed in the upper lefthand frame. The corresponding force field is shown in the upper righthand frame. This force field induces matter displacement, leading up to a distribution of matter shown in the lower lefthand corner. Compare this with the corresponding streaming velocities in the lower righthand frame.
that they will not really have started their anisotropic collapse before the small scale overdensities have turned into high-density virialized clumps. Instead of appearing like a large coherent anisotropic structure the resulting large scale matter distribution will therefore more resemble a mere incoherent and shapeless density enhancement in the number of small clumps. On the other hand, in less extreme hierarchical scenarios large scale density fluctuations will have an amplitude high enough such that by the time small scale clumps have completely collapsed the large scale structure in which they are embedded will already have contracted substantially. In those cases we expect to see more or less coherent walls and filaments in which the small scale clumps stand out like beads on a string. Finally, in the most extreme “top-down” case we will only see the anisotropic contraction of a large scale object like a supercluster. The resulting pattern will be one of a network of filaments and walls without any internal structure.

3 Cosmic Symbiotics: Large Scale Structure and Galaxies

From the preceding it has already been clear that galaxies play a central role in the efforts to map the structure of the Universe and to come to an encompassing theory of structure formation in the Universe. This immediately exposes a precarious issue within the whole framework of structure formation. What is the role of galaxies? What is their nature?

Although obviously a subjective view, there is some right in considering galaxies are amongst the most beautiful and mesmerizing objects in the Universe. To some extent autarctic entities, cosmic cities harbouring and organizing all the ingredients necessary for bringing forth highly complex states of matter organization, like stars, planets, and even something we describe as “life”, they are at the same time the beacons of the Universe. Mainly through their existence have we been able to study the structure of the Universe.

Obviously, the hope is that by mapping the galaxy distribution we at the same time obtain a representative map of the matter distribution. However, this is only an assumption, a crucial one that theoretically has still not been justified. There are some a posteriori indications that it is indeed true on scales of a few Megaparsec and larger. However, no compelling theory exists of how and where galaxies would form within the large scale organization of matter. In order to extract firm conclusions in our search for cosmic structure formation,
we therefore need to get a better understanding of the biased view that the
distribution of galaxies represents, and hence of the process of galaxy formation.
While this obviously is of prime importance in relating the galaxy and matter
distribution, it is even true for the more objective, but less detailed, probe
of the matter distribution offered by measured peculiar velocities. While we
are probably not far from reality assuming that galaxies float along with all
other matter currents in the Universe and therefore that their peculiar velocities
represent excellent probes of the underlying velocity field, there still remains the
possibility there is some level of "velocity bias".

Here nature plays a trick on us. The formation of galaxies is not a purely
gravitational matter, and therefore not one producing configurations readily
retraceable to its cosmic origin and shaping agents. On the contrary, it is a
highly complex and dissipative business consisting of a subtle interplay between
gravitational, radiative and hydrodynamic processes on a range of scales. This
complex interaction incorporates cooling processes of gas, ultimately leading
to the formation of stars, feedback processes of exploding stars, enriching gas
by heavier elements, while radiation emitted by stars and galactic and cosmic
background will counter the cooling of gas. Hence rendering it impracticable to
try to understand galaxy formation on the basis of the structure and kinematics
of galaxies themselves alone, the hope is that inferences about larger structures
over a range of scales may be extrapolated to galaxy scale, providing the initial
setting of protogalaxies.

While this is arguably one of the most important yields of large scale struc-
ture studies, we are at the same time caught in a web as we have already seen
that a complete and objective assessment makes it necessary to understand
the process of galaxy formation. Big strides towards the ultimate resolution
of the structure formation riddle therefore implies a hand-in-hand progress in
theoretical understanding and observational indications.

In an effort to paint the cosmic environment in which galaxies are assem-
bled, in the hope of clearing up the environmental impact on the emergence of
galaxies, we will first provide a description of the observed large scale patterns
in the galaxy distribution. This will be followed by a short discussion of the
efforts towards explaining the formation of these patterns through gravitational
instability, utilizing ever more intricate simulations of the evolution of representa-
tive distributions of particles. In this way we hope to offer a framework for
the incorporation and interpretation of the role of the extreme representatives
of the galaxy population within the structure formation saga, so that we may
have a better understanding of how to utilize the subjects of this meeting, radio
galaxies at high redshift, as probes of their large scale environment.
4 Foaming Delights: patterns in the cosmic matter distribution

During the past three decades it were in particular major advances in telescope and detector technology that instigated a continuously stronger effort towards surveying and mapping the matter and galaxy distribution in the Universe. Penetrating previously unexplored swathes of the local Cosmos, systematic galaxy redshift surveys have uncovered the existence of an hitherto unexpected richly patterned and fascinating organization of matter on scales ranging from a few to even several hundred Megaparsec. Early hints (e.g. de Lapparent et al. 1986) for the existence of a foamlike textured galaxy distribution got strongly corroborated as larger and more ambitious surveys expanded their reach, establishing the image of a vast cosmic foamlike network ostensibly pervading nearly all of the visible Universe (see Fig. 1, Landy et al. 1996).

The frothy geometry is evidently one of the most prominent aspects of the cosmic fabric, highlighted by galaxies populating huge filamentary and wall-like structures, the sizes of the most conspicuous one regularly exceeding $100h^{-1}$ Mpc. The closest and best studied of these massive anisotropic matter concentrations can be identified with known supercluster complexes, enormous structures comprising one or more rich clusters of galaxies and a plethora of more modestly sized clumps of galaxies. Both our Local Group and the Virgo cluster are members of such a structure, the Local Supercluster, a huge flattened concentration of about fifty groups of galaxies in which the Virgo cluster is the dominant and central agglomeration. The Local supercluster is but a modest specimen of its class, dominated by only one rich cluster. A far more prominent example of a supercluster, and arguably more canonic in terms of morphological character, is the Perseus-Pisces supercluster (see Fig. 3). Due to its relative closeness (approximately $55h^{-1}$ Mpc), its characteristic and salient filamentary geometry, and its favourable orientation perpendicular to the line of sight, it has become one of the best mapped and meticulously studied superclusters. It is a huge conglomeration of galaxies that clearly stands out on the sky. The boundary of the supercluster on the northern side is formed by the filament running southwestward from the Perseus cluster, a majestic chain of galaxies of truly impressive proportions. It has a length of at least $50h^{-1}$ Mpc and a width of about $5h^{-1}$Mpc. The ridge possibly extends even further out to a total length of $140h^{-1}$ Mpc, although obscuration by the Galactic Disk prevents firm conclusions on this point. Along the major ridge we see a more or less continuous arrangement of high density clusters and groups, of which the
Figure 3. The Perseus-Pisces supercluster chain of galaxies. Separate two-dimensional views of the galaxy distribution in the northern region of the Pisces-Perseus region. The upper panel shows the sky distribution of all galaxies in the overall northern survey sample of Wegner, Haynes & Giovanelli (1993). The region believed to contain the Pisces-Perseus main ridge is outlined. The lower panel shows the two dimensional redshift distribution (right ascension-recession velocity $V_0$) for galaxies in the ridge region highlighted in the upper panel. From Giovanelli & Haynes 1996, kindly provided by M. Haynes.
most notable ones are the Perseus cluster itself (Abell 462), Abell 347 and Abell 262. An exquisit impression of its structure can be obtained from the 21 cm line redshift survey of some 5000 late-type galaxies in the Perseus region, by Giovanelli, Haynes and collaborators (see e.g. Wegner, Haynes and Giovanelli, 1993, and Fig. 3), the most detailed study of the region currently available. In addition to the presence of such huge filaments we can also discern vast planar assemblies in the galaxy distribution. A striking example of its kind is the Great Wall which was identified through the CfA2 survey (Geller & Huchra 1989). It constitutes a huge planar assembly of galaxies with dimensions that are estimated to be of the order of $60h^{-1} \times 170h^{-1} \times 5h^{-1}$ Mpc, which has the Coma cluster of galaxies as its most prominent density enhancement. Another huge wall of galaxies in our cosmic neighbourhood has been found on the southern hemisphere (e.g. Da Costa 1993), adding to the impression of them being ubiquitous elements of cosmic structure. This impression got even more convincing support after the publication of the results of the deeper Las Campanas redshift survey (Fig. 1). Its chart of 26,000 galaxy locations in six thin strips on the sky, extending out to a redshift of $z \sim 0.1$, currently represents the best and most representative impression of cosmic structure available. In the near future we can look forward to considerable extensions of the cosmic atlas. The 2dF and Sloan redshift surveys have embarked on a majestic enterprise to probe the galaxy distribution of the Universe in hitherto unexplored regions of cosmic territory, out to scales of $\sim 1000h^{-1}$Mpc (see e.g. Lahav 1995, and website [http://msowww.anu.edu.au/~colless/2dF] for further details and even some recent results of the 2dF survey, and Gunn & Weinberg 1995, Margon 1998 and website [http://www-sdss.fnal.gov:8000/] for details and updates of the Sloan SDSS redshift survey). The compilation of more than a million galaxies they strive after will for the first time produce truly uniform and representative samples of our cosmic environment, a true voyage of discovery...

Not only do we come across filamentary and planar mass concentrations. In fact, perhaps one of the most intriguing discoveries emanating from extensive redshift surveys has been the existence of large voids in the galaxy distribution, enormous regions, sometimes up to tens of Megaparsec in extent, wherein few or no galaxies are found. The Boötes voids in the KOSS redshift surveys (Kirshner et al. 1981, 1987) was the first of its kind to attract the attention. It is an almost completely empty spherical region (however, see Szomoru 1995) with a diameter of around $60h^{-1}$Mpc and is still regarded as the canonical example. Various redshift surveys covering large parts of the local Universe have shown that voids with sizes typically in the range of $20 - 50h^{-1}$ Mpc are a common feature in the galaxy distribution, at least up to a redshift of $z \sim 0.5$ (e.g. see
Vogeley, Geller & Huchra 1991 and Bellanger & De Lapparent 1995).

We may therefore conclude that filaments, walls and voids are eminent structural elements of the galaxy distribution. Moreover, a careful assessment of their distribution throughout space also shows them not to be merely independently and randomly scattered objects. On the contrary, the galaxy maps clearly reveal the voids to be generically associated with surrounding density enhancements. In other words, the voids, filaments and walls are not only outstanding components of the galaxy distribution. They also conspire by weaving themselves into the beautiful *foamlike* tapestry that permeates our universe wherever we turn our gaze (e.g. Fig. 1). Within the framework traced out by the galaxy distribution they are both contrasting as well as complementary ingredients, with the vast under-populated regions, (the voids), being surrounded by walls and filaments. At the intersections of the latter we often find the most prominent density enhancements in our universe, the *clusters* of galaxies.

Within the scheme of galaxy clustering and structure formation these dense and rich clusters stand out as the apogee of objects that can still be considered individually distinguishable entities. Being the dwelling sites of sometimes up to thousands of galaxies, they constitute the most massive collapsed and virialized matter condensations in the Universe. They appear to populate the high-mass tail of a wide spectrum of galaxy assemblies, from small groups of a few galaxies, via somewhat more substantial groups like our own Local Group up to the true giants like the Virgo Cluster or the even more majestic Coma Cluster. The majority of these groups and clusters are strewn over the foamlike network of filaments and walls, constituting the occasional density enhancements rendering these structures their often irregular appearance. Generically, these groups are therefore seen to concentrate along the high density ridges of the cosmic network, leading up to the sites where several filaments and walls intersect, often highlighted by the presence of one or more massive clusters. Moreover, the fact that the groups and clusters seem to display a more pronounced concentration towards the walls and filaments of the cosmic foam than the galaxies themselves do is reflected quantitatively in the higher amplitude of their two-point correlation function. In other words, they seem to represent a more biased tracer of the underlying distribution of mass.

The ubiquity of the characteristic frothy cosmic structure over vast expanses of the visible Universe has already been confirmed by the results of redshift surveys in very small regions of the sky out to huge depths, the “pencil beam redshift surveys” in some cases probing to redshifts in the order of \( z \sim 0.5 \). Most famed amongst its peers is the pencil beam redshift survey by Broadhurst et al. (1990), whose conspicuous spiky redshift distribution along a direction towards
the North Galactic and South Galactic pole at the time got an ambivalent reception of surprise mixed with scepticism. Nonetheless, later work only strengthened the impression of huge peaks in the one-dimensional redshift probes. Comparison with shallower wide-angle surveys made clear that the identification of these redshift spikes with Great Walls such as the one revealed by the CfA2 survey was fully warranted, the spikes coinciding with the locations where the narrow redshift probes were piercing through the cosmic walls. Moreover, such deep pencil beam probes make clear that pronounced structures on scales in the order of 100 Mpc already existed at surprisingly early cosmic epochs and therefore argue for a surprisingly early development of structure organization in the Universe. The most astonishing recent corroboration for such early action is the recent statistical evidence (Steidel et al. 1998) for a substantial level of clustering in the population of the so-called Lyman break galaxies, at redshifts of even $z \sim 3$.

5 The Cosmic Abacus: quantifying structure

Hence, a substantial amount of observational evidence seems to indicate that already at a remarkably young age the Universe shed its primordial featureless and pristine complexion, and set out to forging the scaffolds for the construction of the patterns that pervade our Universe on Megaparsec scales. In order to turn this qualitative conclusion to further use, and decide which theoretically proposed scenario lay at the basis of the observed patterns, we evidently need to find and/or find measures to quantify those aspects of the matter and galaxy distribution that are as strongly discriminative as possible. Shedding aside the highly complex anisotropic patterns, most work has concentrated on the first orders of the clustering process, effectively describing the likelihood and frequency of over- and underdensities over a range of scales, as well as their mutual spatial correlation.

The standard contention is that structure grew from a random distribution of density fluctuations $\delta(x)$ whose statistical properties are described by a Gaussian distribution function. In other words, all its Fourier components $\delta(k)$ are mutually independent and have a Gaussian distribution, with its average amplitude determined by the $\sigma(k)$, usually denoted by the name of “power spectrum” $P(k)$. Physically, the power spectrum expresses the relative average magnitude of the Fourier waves at every relevant scale in the constituent spatial density field realization. The relative clout of the various waves is of crucial importance determining the outcome and character of the final matter organization. A pri-
Figure 4. A density field with a characteristic cellular geometry (left), and its counterpart with the same power spectrum $P(k)$, yet scrambled phases. The contour levels of the righthand frame are chosen such that only positive $\delta$ levels are indicated. At the lefthand side the density contours range from $\delta = 0.75$ to $\delta = 10$. in 20 steps. Linear contours in both left and right frame. After a suggestion of Alex Szalay.
The primordial field with a blue spectrum with high amplitudes of small scale waves will lead to a scenario in which small scale clumps will spring up as first discernible objects, while on the other hand a red spectrum will yield an evolutionary scenario more resembling the “top-down” unfolding described earlier. In fact, theoretical work has come up with a hoard of analytical power spectra whose shape and amplitude are determined by various cosmological factors, global cosmological parameters like $\Omega$ and the Hubble parameter $H_o$, but also by the nature of the matter, curvature of space, and several other factors. Hence, its determination from the observations has such a high priority in cosmological research. Hence, the strong incentive towards recovering the power spectrum from observations relating to the matter distribution over scales ranging from those of the large Gigaparsec scales discerned in the microwave background temperature fluctuations, through the hundred Megaparsec scales whose fluctuations imprints can be measured from the large-scale velocity fields and cluster clustering, down to nonlinear scales of a couple of Megaparsec, where it is hoped the galaxy distribution still contains sufficient information.

Determinations of the power spectrum have been preceded by and still go hand-in-hand with a huge amount of effort in describing the fluctuation field in terms of the spatial Fourier transform of the power spectrum, the correlation function $\xi(r)$. In principal, $\xi$ contains exactly the same amount of information as $P(k)$, although observational errors make it more practical to determine $\xi$ on small scales, while its drowning in noise at larger scales make the power spectrum the quantity of preference on scales exceeding some 10 Megaparsec.

However, once gravitational instability gets hold of the primordial field and starts moulding it into density field realizations in which higher and higher density peaks collapse to smaller and smaller parts of space and low density regions empty themselves while seizing larger and larger chunks of the Universe, nonlinear gravitational processes start to evoke larger and larger deviations from the initial Gaussian distribution function. It therefore becomes more and more elaborate to characterize the clustering of matter. Fluctuations over a range of scales start to influence each other, with for instance small-scale density enhancements in large scale overdense regions collapsing earlier than those in more barren regions of space. Hence, the various waves start to interact, and transfer power between the different scales. Another process contributing to power transfer between different scales is the tendency of density enhancements and depressions in changing shape, high density regions will collapse to more and more anisotropic configurations in the generic situation of them not being spherical while low density regions expand to a more spherical shape. The ultimate outcome of the gravitational evolution is therefore a field that becomes increas-
ingly non-Gaussian, in the sense of developing a larger and larger amplitude of higher order correlation functions besides the second order correlation functions. A lot of work was therefore devoted to determining some higher orders of the correlation function hierarchy, but at some point this becomes an almost impossible task, the signal being drowned in the noise of the observations. Only in the early quasi-linear stages of gravitational evolution, when density fluctuations are still in the order of $\delta \sim 1$ it is still reasonable to expect the lower order correlation functions more or less fully quantifying structure.

However, once gravitational clustering starts to transform the density field into one exhibiting a variety of interesting patterns over a vast range of scales, once collapsed and virialized density clumps start to pop up, any hope of a full statistical quantification gets lost. That the power spectrum and correlation functions are not fully equipped in quantifying the most conspicuous aspects of the emerging large scale structure can be discerned from Figure 4. Even only a superficial look at Figure 1 shows how much of an essential aspect of the matter distribution is then swept under the carpet. In fact, the power spectrum does not contain any information on the foamlike morphology of the matter distribution. Figure 4 displays two density fields with exactly the same power spectrum. However, while the lefthand one exhibits a beautiful foamlike morphology, the righthand one is but a featureless Gaussian field. A lot of effort has therefore been devoted to developing and defining statistical quantities that characterize various aspects of the matter distribution, in the hope of them being strongly discriminatory. However, a lot of these attempts produce merely heuristic measures, which have a poorly understood relationship to the underlying scenario of clustering. For instance, minimum spanning trees and percolation measures have gone some way into quantifying filamentary structures, but as yet their relation to the initial power spectrum of fluctuations is unclear. Topology measures do have some quantified relation to the power spectrum, but it’s discriminatory virtues and visual clarity are still contentious, while the same can probably be stated about Minkowski functionals. Attempts in quantifying structure through a hierarchy of fractal dimensions are interesting, but useful only over a limited range of scales where the various moments of the density field exhibit scaling behaviour. Perhaps one of the most interesting and promising approaches, but as yet not fully understood, is the description of structure in terms of wavelets. Although at first received in the cosmological community with a huge grain of scepticism, there are strong indications from other fields of physics that they indeed are very suitable characterizations of non-trivial patterns in nature (see e.g. Bowman & Newell 1998). In addition though, we should continue to study in more detail the structure of the density fields within the well-known realms
of Fourier space, and assess in a more systematic way the evolution of phases, phase distributions and phase correlations through the action of gravity, and fill in the meanings behind the buzz-words so often employed. In other words, we should start thinking about a holographic analysis of the observed as well as simulated matter distributions.

While cosmologists are pursuing the search for transparent “measures of reality” (Crosby 1997), a host of information about the structure formation scenario casting our Universe can also be obtained by a complementary approach, producing theoretical realizations of nonlinear matter distributions for a host of scenarios and comparing them both by their visual impression as well as through the various statistical measures that we can find in our toolbox.

6 Cosmic Pretensions: simulating structure formation

As it appears to be a forbidding task to infer direct inferences from the observed nonlinear galaxy distribution about the valid scenario of structure formation, one can also pursue another approach. This approach comprises the simulation of nonlinear mass distributions in one or more structure formation scenarios. An additional advantage is that it not allows a quantitative comparison with reality, through a comparison of similar statistical quantifications, but also a still very useful qualitative assessment by comparing the visual impression of the observed Universe with that in the simulated portion of the Universe. In fact, while we have seen that we are still failing to characterize striking patterns and properties of for instance cellular matter configurations, this is an essential tool in the study of large scale structure.

Progress in these structure evolution simulations have been tremendous by sheer of a continous and an acceleraring increase in available computer power and available memory space. Several decades ago the early modest particle simulations counted at best a few hundred particles, and the first influential structure formation simulations of the Cold Dark Matter scenario comprised 32,000 particles (Efstathiou et al. 1985), the standard simulation at present already contains at least several million particles, with state-of-the-art simulations exceeding even a billion particles.

The basics of structure formation simulations can be shortly summarized. A realization of a primordial density and velocity fluctuation field for a specific scenario is generated. Its structure is subsequently discretized by a finite,
Figure 5. An illustration of gravitational clustering and collapse. The development of a small region in a \(100h^{-1}\text{Mpc}\) box, within the standard CDM scenario, is followed in a sequence of 12 time steps, going from left to right, top to bottom. The final timestep, bottom right, should correspond to the present epoch, \(a = 1\) (From Van de Weygaert & van Albada 1996).
yet very large, number of particles, usually of equal mass. Evolving this distribution slightly through an analytical approximation of matter displacements in the early quasi-linear stage of clustering, the Zel’dovich approximation, the stage is set for an elaborate sequel through an N-body code that is capable of following the full nonlinear evolution by solving for each individual particle the equations of motion at a sequence of timesteps. Oversimplifying various different ways of performing these N-body simulations, the usual strategy is to interpolate the particle distribution to yield a density field which in turn yields the underlying gravitational potential field through solving the Poisson equation. After having done so, we can interpolate back to the particles to yield their gravitational acceleration, and subsequently their velocity. By doing this at a myriad of timesteps, for a huge number of particles, we hope to obtain an idea of the intricacies of nonlinear gravitational clustering. An illustration of such a sequence of 12 timesteps is given in Figure 5, showing the evolution of a clump of matter with a mass in the range of that of groups of galaxies like our own Local Group. The continuous accretion and concentration of ever larger amounts of matter is clearly born out, as is the rather anisotropic nature of the whole process.

A lot of effort is devoted to simulate the Universe to great detail over a range as large as possible, in an attempt to adhere to the purest definition of “simulation”, trying to reproduce reality in all its aspects (see e.g. the work of the Virgo consortium, e.g. Pearce & Couchman 1997). While this obviously is necessary if ever we wish to have confidence in our model Universes describing the real Universe, it automatically biases scientific efforts towards trying to concentrate huge amounts of manpower and financial resources towards that one goal. However, there is still ample space for a complementary approach, arguably as necessary towards obtaining an understanding of the action of gravity in shaping our cosmic environment.

While the massive state-of-the-art simulations try to reproduce the real Universe, and therefore comprise all the different detailed processes whose interactions conspire to produce the final mass distribution, a full understanding cannot be reached without trying to isolate and understand the various relevant processes and physical factors. In other words, is it possible to pursue a more laboratory oriented approach, not trying to reproduce the Universe in all its charms, but rather concentrating of one or a few supposedly relevant factors. The insight provided through such an approach will reveal more clearly the relative importance of physical processes, quantities and matter configurations in the full-blown large simulations.

The laboratory approach hinted at above is what we strive for through sim-
Figure 6. The variance of constrained random field realizations illustrated by means of contour density maps in a $5h^{-1}\text{Mpc}$ thick central slice in a $100h^{-1}\text{Mpc}$ box. See text (from Van de Weygaert & Bertschinger 1996).
Figure 7. Four constrained realizations of the nonlinear evolution of our local Universe (see text, Van de Weygaert & Hoffman 1998). Based on a Wiener reconstruction of the local density field on the basis of the Mark III catalogue of galaxy peculiar velocities (Willick et al. 1997). The extended concentrations of mass discernable towards the upper lefthand quarter of all four realizations may be regarded as the kins of the Great Attractor, while to the lower righthand side something akin to the Perseus-Pisces supercluster can be seen.
ulations based on constrained field realizations (see Bertschinger 1987, Hoffman & Ribak 1991, Van de Weygaert & Bertschinger 1996). Figure 6 illustrates the basic ideas behind such constrained field realizations. Its top left panel illustrates the imposed set of constraints by means of the mean field $\bar{f}$, which is set by this particular set of constraints and constraint values (see Van de Weygaert & Bertschinger 1996). The constraint set involves the peculiar velocity and tidal field at the central location of the simulation box. To yield genuine realizations of a random field obeying the constraint set, a residual field is added to the mean field. This residual field contains the fluctuations intrinsic to a field with the relevant power spectrum. The four central and right-hand panels display four different random realizations adhering to these constraints. While the overall pattern of the mean field can clearly be recognized in all four random realizations, they also show where and on what scale the realizations can differ from one to the other. Clearly, around the location at which the constraints have been specified, the variation amongst the realizations is negligible. Further out it approaches the generic variation expected for unconstrained random fields. Also, smaller scales are far less affected than the scale at which the constraints is set, in this case a Gaussian scale of $R_f = 5h^{-1}\text{Mpc}$. To quantify and summarize the variations between the different realizations, the bottom left panel is the contourmap of the value of the variance of the field realizations inside the slice, running from 0.0 at the centre to $\sigma_9 \approx 0.95$ at the edge of the box. Note that in this particular case, with the constraint concerning the value and configuration of the tidal field at the centre of the box, the presence of a strong straining tidal component along the $x$-axis, in combination with compensating compressing components along the $y$- and $z$-axis automatically implies a pronounced quadrupolar mass distribution, ultimately evolving into a configuration of two massive clumps connected by an elongated thinner bridge in between them. Hence, this explains the frequently mentioned and observed connection between clusters of galaxies and filaments (see e.g. Bond, Kofman & Pogosyan 1996)!

A particular insightful application of this idea is by taking the constraints from observed reality. For example, the density field in the local Universe as implicated by the local cosmic flow field. This local flow field can for instance be obtained through interpolation of the measured galaxy peculiar velocities listed in the Mark III catalogue (e.g. Willick et al. 1997). Arguably the best estimate of the corresponding linear density field in the local Universe, the one with the highest signal-to-noise level, is obtained through the application of the Wiener filter technique developed by Hoffman, Zaroubi and collaborators (see Zaroubi et al. 1995). The latter is the de facto mean field of all resulting realizations im-
plied by the measured cosmic flow field and corresponding measurement errors. Subsequently, on the premise of a CDM Universe with $\Omega = 1$, four different realizations were generated by adding appropriately constrained noise fields to the Wiener filter reconstructed field. To this end we invoked the Hoffman-Ribak constrained random field recipe (Hoffman & Ribak 1991, van de Weygaert & Bertschinger 1996). The resulting linearly extrapolated density fields are used as initial density field realizations, and their further nonlinear evolution is followed by means of a P$^3$M N-body code. The outcome at an expansion factor $a = 0.8$ for the four different realizations are displayed in the four panels, each panel representing the particle distribution in a $10h^{-1}$ Mpc thick slice through the centre of the box, which corresponds to our position in the cosmos. The extended mass concentration to the lefthand side should correspond to possible realizations of the Great Attractor region, while towards the lower righthand side one can recognize a matter concentration that in the case of our real Universe is called Perseus-Pisces supercluster (see Van de Weygaert & Hoffman 1998).

Clearly, in this way we will be able to systematically explore various physical effects at work in our local neighbourhood. Knowledge obtained from these assessments can then be incorporated in an analysis of a full-blown superduper simulation. In this way we hope to crawl further and further towards a configuration resembling as closely as possible the large-scale environment in which we live, setting the scene for solving maybe the most mesmerizing riddle of them all, the Holy Grail of 20th century cosmogony, the creation, the rise, evolution and growth of those jewels in the “crown of creation” (Jefferson Airplane 1968), the galaxies.

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