Development of combined search methods for efficiency indicator extreme in variation statement of forecasting tasks for determine characteristics of composite materials

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Abstract. Modified search methods for global extreme of multiparameter functions are studied, considering the structural features of efficiency criteria used in problems of forecasting residual life and durability of structures made of composite polymeric materials. Comparative and qualitative studies have been carried out and methods have been developed to select the most promising areas of search for the absolute minimum of the studied performance indicators in refined variation formulation of inverse problems of forecasting residual life of composites.

1. Introduction
In recent decades, one of the most important tasks in development of various structures, machines and mechanisms has been creation of reliable methods for quantifying performance of products from polymeric and composite materials [1-13]. Specificity of studied problems of forecasting strength change of polymer composites affected by extreme climatic factors leads to the fact that extrapolation approach in traditional formulation is not effective for these problems. Conceptual improvement of extrapolation approach can be achieved based on choice of a mathematical model with a focus on physical concepts.

Residual life of composite structure can be considered as its defining characteristic.

2. Problem statement
Models that describe dependence of change in one of defining characteristics (for example, strength $R$ of polymer composites) affected by climatic factors generally form can be represented
as functional dependencies of following form:

\[ R(u_1, u_2, \ldots, u_n; t) = R_0 + \Phi(u_1, u_2, \ldots, u_n; t) \]  

We denote \( \Delta R^3 \) as experimental mean values of function \( \Delta R = R - R_0 \) on experimental samples after \( t \)-year (\( t = 1, 2, \ldots, m \)). Parameters of optimal forecasting model \( u^* = (u^*_1, u^*_2, \ldots, u^*_m) \) deliver extreme value to special criterion of efficiency \( J \), which is a complex function of forecasting problem parameters:

\[ J(u) = J \left( \{ \Delta R(u, t) \}_{t=1}^{m}; \{ \Delta R^3 \}_{t=1}^{m} \right) \]

In the case when experimental data adequately reflect change dependence structure in determining characteristics of composite materials (CM) on impact of extreme environmental factors and operational loads, and experimental data were obtained with minor errors that do not distort behavior patterns of real dependencies, task of restoring parameters of CM models from effects of extremely environmental factors can be reduced to finding absolute minimum of quality function (2) in extreme problem:

\[ J(u^*) = \min_u J(u) \]  

Task of restoring parameters of models of changes in residual life of a composite structure affected by extreme environmental factors can be reduced to solving an extreme problem (3). Vector of parameters \( u^* = (u^*_1, u^*_2, \ldots, u^*_n) \) that delivers minimum value to performance indicator \( J(u) \) (3), determines optimal forecast dependence of change in defining characteristics of CM on impact of extreme environmental factors and operational loads.

3. Features of inverse problems of predicting the residual life of composites in the variation formulation

Characteristic feature of inverse problems of forecasting residual resource of composites in a variation formulation with complex efficiency indicators of form (2) is that structure of efficiency indicators has a substantially multi-extremum nature, which does not reliably allow finding absolute minimum of such performance indicators using well-known methods finding a minimum. Simultaneously, conducted studies and corresponding computational experiments (CE) showed that usage of various known methods of finding a minimum can lead to substantially different results. Simultaneously, it is impossible to assess how significantly resulting solutions differ from required global optimal solutions that deliver global minimum to efficiency indicator. Consequence of ineffectiveness of existing approaches to solving inverse problems of forecasting composites in a variation formulation is that predicted solutions of residual resource may differ significantly from actual values of residual resource of composites, even when forecasting for a short period and even when developed predictive mathematical model of durability has a high adequacy of real situation. Nevertheless, conducted studies, CE showed that performance indicators in inverse problems of forecasting residual resources in variation formulation have several specific features. Accounting for these features allow developing solving inverse problems of forecasting residual life of composites in variation formulation more effective methods for finding absolute minimum compared to known approaches, as well as modifying known minimization methods taking into account specific features of considered performance indicators.

4. Combined methods of search of extreme, of performance indicators in the refined variation formulation of the task of remaining life prediction of composites

In [7], method for searching extreme of multiparameter functions with an optimal choice of parameters for solving reverse problems of forecasting strength change of polymer composites
affected by extreme climatic factors and operational loads was proposed. Technique has been developed for optimal choice of parameters, which makes it possible to increase efficiency of using gradient methods for solving inverse problem of forecasting determining characteristics of polymer composites and to give them a non-local character. For tasks in which absolute minimums of efficiency indicators are located on directions along which their most significant decrease is observed, proposed approach will be sufficiently effective. However, in the most common situations related to solving forecasting problems, dependence of performance indicators $J(u)$ on determining parameters has rather complicated structure, in which absolute minimum may not be along directions calculated in this way. In [9-10] to solve a more complex range of inverse problems of forecasting changes in strength of polymer composite structures affected by extreme climatic factors of Far North, Arctic zone, described by models in which dependence of efficiency indicators $J(u)$ on determining parameters - parameters $u_1, u_2, ..., u_n$ - have a more complicated structure, modifications of methods of possible directions with definition of optimal search directions were developed. Structure of modified methods of possible directions includes such additional features as additional procedures for generating a significant number of possible search directions, given by unit vectors $\vec{l} \in \Omega = \{\vec{l}\}$, whose angles $\varphi_1, \varphi_2, ..., \varphi_n$ with coordinate axes represent set of random variables with a uniform distribution law; additional procedures for comparative evaluation of prospects of selected possible directions according to specially constructed criteria $W(\vec{l})$ and finding optimal search direction. Structure of modification of methods of possible directions includes such additional elements that increase their efficiency, such as global optimization of efficiency parameter along the chosen conditionally optimal search direction $l^*_p$; constructing next $(p + 1)$-th approximation to global optimal solution associated with decomposition of efficiency criteria $J(u)$ in Taylor series in vicinity of $p$th approximation along conditionally optimal search direction $l^*_p$.

$$J(u) = J(u^p) + \frac{\partial J(\vec{\alpha})}{\partial \alpha_p} (u - u^p) + \frac{1}{2} (u - u^p)^T \frac{\partial^2 J(\vec{\alpha})}{\partial \alpha^2_p} (u - u^p) + \ldots$$

$$\frac{\partial J(\vec{\alpha})}{\partial \alpha} = (\nabla J(u^p), \vec{\tau}) = \sum_{i=1}^n \frac{\partial J(u^p)}{\partial u_i} \alpha_i = \sum_{i=1}^n \frac{\partial J(u^p)}{\partial u_i} \cos \varphi_i$$

$$\frac{\partial^2 J(\vec{\alpha})}{\partial \alpha^2} = \sum_{i=1}^n \sum_{k=1}^n \frac{\partial^2 J(u^p)}{\partial u_i \partial u_k} \alpha_i \alpha_k = \vec{\tau}^T \nabla^2 J(u^p) \vec{\tau}, \quad p = 0, 1, 2, \ldots$$

5. Promising ways to increase the potential in improving the efficiency of methods for predicting the determining characteristics of composites

Question about ways of further effective expansion of potential possibilities in increasing forecasting methods efficiency for solving problems of estimating residual life, reliability, durability of CM and structures for extreme climatic conditions of Far North and Arctic zone is investigated. One of the promising ways to increase potential opportunities is associated with improvement of methods for constructing optimal search directions for absolute minimum of performance indicators (2). And methods outlined in [9-10], which form the basis of combined approach, comparative qualitative analysis of methods of choosing the most promising search directions for absolute minimum of studies performance indicators and mutually conjugate search directions was conducted. Advantages of search methods for absolute extreme of performance indicators, when mutually conjugate directions are selected as promising areas of search, are analyzed. Analysis showed that effectiveness of conjugate directions’ method application for solving problems of forecasting residual life of composites is caused by following design features, which make it possible to adequately consider specific features of considered problems of forecasting composites affected by extreme environmental factors.

In methods of combined directions, sufficiently successful selection of next promising search directions is carried out, which quite effectively takes into account specificity of performance indicators in tasks of forecasting residual life of composites. In methods of conjugate directions,
next search directions selected as promising, are characterized by property that if vector $u^*$ is a minimum point of performance indicator $J(u)$ (2) along direction $\epsilon$, and vector $u^{**}$ is minimum point of performance indicator $J(u)$, obtained from a different starting point along direction $\xi$ parallel to vector $(u^{**} - u^*)$ will be $G$-conjugate to direction $\xi$ [14]. Established important properties of $G$-conjugate directions are applied in studies inverse problems of forecasting residual resource of composites to construct the most promising search directions for absolute minimum of complexly constructed efficiency indicators of form (2).

6. Summary

Conducted research and development of known methods for finding a minimum of multiparameter functions, a comparative assessment of their effectiveness, development of recommendations for their optimal layout and effective design modification in accordance with specific features of considered performance indicators in inverse problems of forecasting residual resource of variation formulation can significantly expand potential for improving effectiveness of forecasting residual life methods, durability and CM structures affected by extreme environmental factors specific to Far North and Arctic areas.

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