A natural mechanism to induce an electric charge into a black hole

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We present a natural mechanism which may induce an electric charge into an accreting black hole in the presence of a strong, high energy radiation field. We study this mechanism using Newtonian physics, and we also discuss the process within the context of a Kerr Newman black hole. Finally, we consider possible astrophysical applications in X-ray variability and jet formation in the Active Galactic Nuclei (AGN).

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I. INTRODUCTION

The standard picture for active galactic nuclei (AGN) is based on a black hole surrounded by an accretion disk and, at least for radio loud objects, a jet of plasma ejected probably through magnetohydrodynamical mechanisms (e.g. Contopoulos\textsuperscript{2, 3, 4}; Kudoh & Shibata\textsuperscript{5}; Kudoh et al.\textsuperscript{6, 7}).

Little importance has been given in astrophysics to mechanisms that might produce an electric charge in a black hole, as it has been assumed that a significant charge cannot be achieved. For most physical situations, the electron-proton coupling in a plasma is so strong that radiation pressure cannot break this coupling, although it may introduce some kind of electric charge polarization and plasma oscillations. Levich, Sunyaev & Zeldovich\textsuperscript{8} considered first this problem in AGN, taking into account quantum effects which introduce

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an extra term for the radiation pressure. This new term is proportional to the second power of the radiation flux and is inversely proportional to the sixth power of the distance. In their paper, Levich et al. showed that gravitational force may be in fact smaller than radiation pressure with this additional term, and that matter may escape from the nuclei of Seyfert galaxies and quasars. However, probably due to the poor knowledge of AGN spectral energy distribution (SED) in 1972, they considered sources where infrared luminosity was dominant, and they argued that: ‘It is evident that both the nuclei and the electrons should move simultaneously. Thus, the matter as a whole should move away from the sources.’

Nowadays, we know that the inner funnel and the free falling matter into the black hole must glow with hard radiation, although most AGN are optically deep to $\gamma$-rays. Electron-positron pairs are created, which implies the existence of $\gamma$ photons of at least $10^{-11}$J (i.e., 100 MeV or $10^{22}$Hz) interacting with soft X-ray photons. Some electrons may be Comptonized by hard X-rays and $\gamma$-rays. An electron Comptonized by a $10^{22}$Hz photon would acquire an energy $\sim 10^{-12}$J, i.e. two orders of magnitude larger than the electron energy at rest. Under these circumstances, the velocity of the electron is such that basic ideas of plasma, such as Debye shielding, do not apply. The behaviour of a very fast particle moving through a plasma is an important aspect of plasma physics which has been investigated recently [e.g., Meyer-Vernet, Shivamoggi & Mulser], even in the case of nuclear reactions in stellar plasmas [Shaviv & Shaviv]. In fact, electrons moving with velocities one order of magnitude larger than the thermal electrons of the plasma, are practically not affected by collisions or even magnetic fields. An astronomical example of such fast particles are cosmic rays, both ions and electrons, which can travel through and among the galaxies and reach detectors on earth.

On the other hand, even though the exact solutions for charged black holes, i.e. the Reisnner-Nördstrom and the Kerr Newman metrics, have been known for more than forty years, (see [12]), and several analysis about their properties have been performed (see [13] and references there in), those charged black holes have always been consider of purely theoretical interest, as long as Nature in general is neutral and the charge to mass ratio, even for an electron is so huge:

$$\frac{e}{m_e} = \frac{1.38 \times 10^{-34} \text{cm}}{6.76 \times 10^{-56} \text{cm}} = 2.04 \times 10^{21}, \tag{1}$$

that even if, by some unknown process, there could be a cloud of electrons or even protons
which accreted and that, surmounting the enormous electric repulsion, form a charged black hole, this will not be a black hole but a naked singularity, as long as the horizon radius, even for a non rotating black hole, \( r_H = M \left( 1 + \sqrt{1 - \left( \frac{eQ}{M} \right)^2} \right) \), turns out to be imaginary.

In this paper, we explain how the high energetic radiation field, generated in the inner edge of the accretion disk, can accelerate the electrons in the outer edge of the accretion flow, allowing them to escape from the accreting plasma. Hard X-rays and \( \gamma \)-rays, can accelerate the electrons up to relativistic velocities through Compton scattering. These charges, moving much faster than the thermal electrons, cannot be shielded, and their impact parameter is greatly reduced, moving almost freely through the plasma \([14]\). The released electrons can eventually be incorporated into the outflow material in the inner funnel and reach the disk corona. The efficiency of this process, estimated from the periodicity of the X-ray variability, is very small (approximately one out of \(10^{25}\) electrons can escape). These electrons will be supported in the corona by radiation pressure from the disk, until the electric force from the charged black hole overwhelms this pressure. This black hole charge is produced by the decoupled accreted protons, and it will eventually increase enough to reduce the accretion rate. This, in turn, will reduce the radiation pressure until the electrons can no longer be supported in the corona and fall to the black hole attracted by the electric force, thus neutralizing its charge. This process can be repeated producing low amplitude quasiperiodic or, under some circumstances, strictly periodic variations in the X-ray luminosity originated in the accretion disk, which may be superposed to another component arising, for example, from the disk corona. Moreover, we are able to compute an expression for the ratio of the maximum charge acquired to the mass of the black hole, and show that the black hole conserves a well defined horizon.

In section \[\text{II}\] we present the model. In section \[\text{III}\] we write down the Kerr Newman solution and discuss some possible consequences of our model within the context of such an exact solution. In section \[\text{IV}\] we apply the model to an AGN that show X-ray periodic or quasiperiodic variability, and compare our model with other proposed models for explaining the observed variability. And finally our conclusions are summarized in section \[\text{V}\]
II. THE MODEL

The region between the inner edge of an accretion disk and the event horizon of a supermassive black hole contains a very high energy radiation field of X-ray and $\gamma$-ray photons emitted at the inner edge of a thick disk. In the funnel, the radiation is high enough to generate pairs of particles. Thus, in the inner region, a high number of $\gamma$-ray photons is also produced, with energies of at least $10^{-11} J$. Although high energy radiation cannot escape freely from the optically thick accretion flow, an electron near the edge of the infalling plasma has a probability of being scattered by an energetic photon. In this paper we consider that the funnel is virtually empty of matter, as in Begelman, Blandford & Rees [15]. Thus, the released electron can move freely along the funnel. However, the density inside the funnel is an open unsolved question. The formation of an electron-positron jet inside the funnel [15] is compatible with the electron-proton decoupling as far as the $\gamma$ ray collisions which produce electron-positron pairs imply that photons can move freely inside the funnel, thus the funnel has an extremely low particle density. For electron-proton plasma jets formed inside the funnel, however, optical depth effects can become very important.

In this section we will show that, in the radiation field of very high energy photons where the accretion flow is embedded, the pushing force of Compton scattering on electrons and the pulling force of gravity on protons are so intense, that they can occasionally overcome the electron-proton coupling force in the accreting plasma. After being released from the freely falling plasma, the electron may remain as an isolated charge and eventually reach the disk corona, while the decoupled proton is accreted unto the black hole inducing a positive electric charge in the singularity. It must be noted that the main cause underlying this process, apart from the strong gravitational attraction of the black hole, is the high energy of the photons, high enough to induce electron-proton decoupling, which means that this process occurs at a microscopic rather than macroscopic scale, as would be the case for a process induced by radiation pressure. The efficiency of this decoupling process must necessarily be extremely low, since a break of the global electric coupling in a plasma is not possible. From observational data, we will show that approximately one in $10^{25}$ electrons must be scattered in such a way, in order to reach the disk corona and to trigger the process. Once the scattered electron has reached the corona, it will be decelerated by the ambient pressure and subject to a combined pulling electric force from the charged black hole and
a pushing radiation pressure from the disk. At the beginning of the process, the radiation pressure is very intense, while the electric force is weak. As the positive charge in the black hole increase by the infall of decoupled protons, the electric repulsive force on other protons also increase and thus the net attraction on these particles decreases, diminishing the accretion rate and the X-ray luminosity component from the disk. Long before the net force on the protons vanishes preventing the accretion, the luminosity emitted in the thick part of the disk will drop so much that the radiation pressure will no longer support the decoupled electrons in the corona. These electrons will fall onto the black hole at once, neutralizing it and allowing again a higher rate of accretion, thus raising the luminosity to its former high value. The onset of this process is periodic, or quasiperiodic in the presence of small perturbations.

In the next subsections, we shall develop the ideas outlined above adopting a classical or special relativity approach. These approaches are justified because, for electrons escaping through the inner funnel, after moving a few Schwarzschild radii, relativistic corrections are negligible for our purpose. During the scattering process, the electrons receive the photons with the same energy as they had when they were emitted, with negligible gravitational effects on the photon energy. All this allows us to study the Compton scattering of the electrons in the infalling plasma from energy considerations. We consider that, at least in the edge of this plasma, the electrons can escape without any interaction with other particles. Abramowicz & Piran [16] and Sikora & Wilson [17] have calculated the radiative acceleration of a single particle in an empty funnel. However, detailed descriptions of the geodesic trajectories of the matter is beyond the scope of this paper.

A. Compton scattering on the free fall matter

The particles in the inner accretion disk that leave the last stable orbit fall to the black hole embedded in a piece of free fall plasma. An electric charge in this plasma is shielded in a distance called Debye’s wavelength, which is given by:

\[ \lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_e e^2}}. \]

For a black hole of \(10^8 M_\odot\) (\(M_\odot\) being the solar mass), accreting at the Eddington rate, the characteristic particle density near the horizon is \(10^{11} \text{ cm}^{-3}\) [15], and the electron tem-
perature ranges from $10^5$ to $10^7$K. Thus, the Debye’s wavelength is of the order of $10^{-5}$ to $10^{-4}$m. This wavelength implies a binding energy for the electron-proton pairs in the infalling plasma, given by:

$$E = \frac{1}{4\pi \epsilon_0} \frac{e^2}{\lambda_D},$$

which is of the order of $10^{-24}$ to $10^{-25}$J.

On the other hand, the infalling plasma is embedded in a highly energetic radiation field of ultraviolet, X-ray and even $\gamma$ ray photons, emitted from the inner edge of the disk and from the infalling plasma itself. The maximum kinetic energy of the ‘struck’ electron after a Compton scattering process is given by:

$$K_{\text{max}} = \frac{h \nu_0}{1 + 1/2\alpha}, \quad (2)$$

where $\nu_0$ is the frequency of the incident photon and $\alpha$ is the ratio between the energies of the incident photon and the electron in its rest frame:

$$\alpha = \frac{h \nu_0}{m_e c^2}.$$

For X-ray and $\gamma$-ray photons with $\nu_0 \geq 10^{19}$Hz, the electron can acquire an energy $> 10^{-15}J$, which is at least 9 orders of magnitude larger than the binding energy with the proton. Thus, the Debye’s wavelength cannot be applied to such an energetic electron. This moving charge excites electrostatic plasma oscillations along its trajectory, trailing a train of density oscillations of wavelength much larger than the Debye’s wavelength [e.g. Meyer-Vernet [9], Shivamoggi & Mulser [10]].

In most cases, after a Compton scattering in a plasma, another electron will replace the scattered one and radiation will have either negligible effects or, if it is very intense, it may introduce some polarization and plasma oscillations. However, near a black hole, the infalling plasma containing the isolated proton will be accreted and, if the scattered electron escapes from the accretion flow, a net positive charge will be introduced into the black hole. In fact, the electrons will be Compton scattered in all directions, and only a fraction of them will be able to escape from the infall. In this context, we use the efficiency of the process as the probability that an electron escapes from the accretion flow through the inner funnel and reaches the disk corona, where the radiation pressure prevents it from falling again into
the black hole. As we shall see later, this efficiency is very low, and thus the macroscopic state of the infalling plasma remains virtually neutral.

B. Movement of a fast electron across the plasma

The plasma does not have time to respond and shield a fast electron, i.e., an electron that is moving about one order of magnitude faster than the thermal electrons of the plasma [Nicholson [14] and references therein]. This implies that such a fast electron can move almost freely, traveling a large distance before it is stopped. A measure of this distance can be obtained starting from the collision frequency. For an electron moving at a velocity $v$ and interacting with another electron at a minimum distance $r_0$, the impulse (change in the moment of the electron) can be expressed as:

$$\Delta p \approx \frac{e^2}{4\pi\epsilon_0 r_0 v}.$$  \hspace{1cm} (3)

For a relativistic electron, the initial moment $p$ is given by:

$$p = m_e v\gamma,$$  \hspace{1cm} (4)

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the electron Lorentz factor. We are interested in an increase $\Delta p \approx p$. Thus, substituting Equation (4) in Equation (3) we obtain:

$$r_0 \approx \frac{e^2}{4\pi\epsilon_0 m_e v^2\gamma}.$$  \hspace{1cm} (5)

We can consider $r_0$ as the impact parameter for the relativistic electron. Thus, the electron sweeps a volume $\pi r_0^2 v$ in a second. Then, the number of collisions per second for a plasma of number density $n$ is:

$$\nu = \frac{\pi n e^4}{(4\pi\epsilon_0)^2 m_e^2 v^3\gamma^2}.$$  \hspace{1cm} (6)

An estimate of the time that the electron moves freely through the plasma can be obtained integrating Equation (6) over a range of impact parameters, and considering collisions with both electrons and protons. This yields a deflection timescale for the electron given by [cf.
Nicholson [14] for the collision frequency of a non relativistic electron:

\[ t_d = \frac{(4\pi\varepsilon_0)^2 m^2_e v^3 \gamma^2}{8\pi n e^4 \ln \Lambda}, \]  

where \( \ln \Lambda \) is the Coulomb logarithm for a plasma at a given temperature \( T \) and number density \( n \):

\[ \ln \Lambda \approx 10 + 3.45 \log T - 1.15 \log n. \]

Besides the deflection time, there are other two timescales of interest to characterize the free path of a particle through a plasma. In our case, the *slowing-down timescale* is of the same order as the deflection time, and the *energy exchange timescale* is an order of magnitude larger. Thus, the deflection time is an appropriate parameter to calculate the electron free path in our case. Figure (1) shows the deflection timescale for an electron moving through the accretion flow, as a function of the electron velocity. Note that for electrons moving faster than one third of the speed of light this timescale is about one second.

Thus, the free path of the electron is about \( 10^8 m \) (Fig. 2). This free path is then one hundredth of the radius of the last stable orbit around a black hole of \( 10^7 M_\odot \). Thus, for Compton processes near the outer edge of the accretion flow (about the last stable orbit), the Comptonized electrons can reach a going on zone outside the accreting plasma, and not be swallowed by the black hole. As for the accreting plasma, it will sustain a small positive charge which will be accreted unto the black hole.

![Graph](image)

**FIG. 1:** Deflection timescale as a function of the electron velocity moving through a plasma of number density \( 10^{11} cm^{-3} \) and temperature \( 10^7 K \).
FIG. 2: Free path of the electron as a function of its velocity moving through a plasma of number density $10^{11} \text{cm}^{-3}$ and temperature $10^7 \text{K}$.

The photon frequency needed to accelerate the electron through Compton scattering can be calculated from eq. (2) and the expression of the kinetic energy in special relativity, resulting in:

$$\nu = \frac{m_e c^2}{2\hbar} \left[ (\gamma - 1) + \sqrt{(\gamma + 1)(\gamma - 1)} \right].$$  \hspace{1cm} (8)

Fig. (3) shows the velocities acquired by the Comptonized electron as a function of the photon frequency. Note that hard X-ray and $\gamma$-ray photons can accelerate electrons up to one third or more of the speed of light.

FIG. 3: Comptonized electron velocity as a function of the photon frequency.
C. Force balance and electric discharge

In the previous discussion, we considered Compton scattering on electrons as the mechanism which can induce a positive electric charge in the black hole. However, once the decoupled electrons have reached the disk corona, we must consider the effect of radiation in terms of radiation pressure rather than Compton scattering. This pressure will prevent the infall of the electrons in the corona until the black hole reaches a critical electric charge. For this process to take place, the black hole must not be electrically shielded at short distances, even if the Debye wavelength is very short. The reasons why the black hole cannot be shielded at short distances comes from the Eddington luminosity:

\[ L_E = \frac{4\pi c G M m_p}{\sigma_e}, \]

(9)

which is a measure of the maximum brightness of the disk (although it can be surpassed for thick disks). The Eddington luminosity depends on the mass of the particles accreted, usually considered protons and electrons together, although only the mass of the protons is taken in consideration. But note that if the accreted particles where only of the electron mass, the luminosity would need to be three orders of magnitude lower. This implies that single electrons cannot be accreted alone or even efficiently shield a moderate black hole charge, as the radiation would sweep them out. In other words, the radiation field behaves as an electric resistance that prevents the movement of isolated electrons towards the black hole. Then, the shielding of the black hole would take place both in the accretion disk and in the disk corona. But for an electron at the inner border of the (optically) thick accretion disk, the radiation pressure from the opposite wall overwhelms the radiation pressure from behind. Thus, the electrons would be behind the protons. This implies that the shielding of the black hole is performed behind the last stable orbit. Of course, the matter inside the last stable orbit cannot shield the black hole either, since there are no stable orbits anymore.

Thus, in a classical approximation, and neglecting magnetic effects, there are three forces acting on isolated particles near a black hole: gravitational, Coulombian and radiation pressure. For protons, radiation pressure is negligible, while for electrons, gravity force is very weak compared to the other two. If we consider the absolute values of these three
forces, the conditions for accretion for these particles will be:

\[ F_{r,e} < F_{g,e} + F_{e,e} \approx F_{e,e} , \]

and

\[ F_{g,p} > F_{r,p} + F_{e,p} \approx F_{e,p} , \]

where the first subindex denotes gravity \((g)\), radiation \((r)\) and electricity \((e)\), and the second subindex denotes proton \((p)\) and electron \((e)\).

All these forces are proportional to \(R^{-2}\), where \(R\) is the distance from the central source, and thus a decoupled electron will not fall again until the luminosity decreases and the charge increase up to a certain critical limit. On the other hand, the dependence of all these forces on \(R\) permits to approach the problem independently of \(R\). Thus, eqs. (10) and (11) can be expressed as:

\[ f_{g,e} + Qf_e - f_{r,e} > 0 , \]

\[ f_{g,p} - Qf_e - f_{r,p} > 0 , \]

where \(Q\) denotes the number of free elementary charges. The \(f\) terms are defined as:

\[ f_{g,x} = GMm_x , \]

\[ f_{r,x} = \frac{L \sigma_x}{4\pi c} , \]

\[ f_e = \frac{e^2}{4\pi \epsilon_0} . \]

where \(L\) is the luminosity, and the Thompson cross section \(\sigma_x\) (where \(x\) is the subindex of the particle), instead of the Klein-Nishina cross section, is used for simplicity and because, for the soft X-ray radiation considered, they differ only in the fourth significant digit.

The accretion rate of matter onto the black hole can be expressed in function of the density of the material \((\rho)\), a certain surface around the black hole that the matter will cross \((S)\), and the falling velocity of this matter \((v)\):
\[ \dot{M} = \rho S v. \]

Elementary physics shows that the velocity \( v \) may be expressed as a function of the distance \( R \) to the singularity and the acceleration \( a \) on the particles:

\[ v = \sqrt{2Ra}. \]

As the luminosity is directly proportional to the accretion rate, it will also be proportional to the square root of the acceleration on the falling material and, therefore, to the square root of the forces acting on the falling particles.

If we express the maximum luminosity emitted in the inner (variable) region in terms of the Eddington luminosity \( (L = \kappa L_E) \), and neglect the radiation pressure on the proton, we can express the luminosity in a given moment as a function of the forces acting on the proton:

\[ L = \kappa L_E \sqrt{\frac{f_{g,p} - Q_f e}{f_{g,p}}}. \tag{13} \]

Substituting in this expression eq. (12), and replacing \( L_E \) for its value in function of the mass \( M \) of the black hole, Eq.(9), we write the condition for electron infall:

\[- \left[ GMm_p - \frac{Q e^2}{4\pi \epsilon_0} \right] - \kappa \sqrt{GMm_p} \sqrt{GMm_p - \frac{Q e^2}{4\pi \epsilon_0}} + GM(m_p + m_e) > 0.\]

Solving the quadratic equation for \( \sqrt{GMm_p - \frac{Q e^2}{4\pi \epsilon_0}} \), we obtain:

\[ \sqrt{GMm_p - \frac{Q e^2}{4\pi \epsilon_0}} = \sqrt{GM \frac{(4 + \kappa^2)m_p + 4m_e - \kappa \sqrt{m_p}}{2}}, \]

thus, the ratio of the critical charge to the mass of the black hole, \( \frac{eQ_{\text{crit}}}{M} \), allowed before the free electrons begin to fall to the black hole is:

\[ \frac{eQ_{\text{crit}}}{M} = 4\pi \epsilon_0 G \frac{m_p}{e} \left[ \frac{\kappa}{4} \sqrt{\frac{\kappa^2}{4} + 1} + \frac{m_e}{m_p} - \frac{m_e}{m_p} \right]. \tag{14} \]

In cgs units, \( 4\pi \epsilon_0 = 1 \), \( G = 6.6 \times 10^{-8} \text{cm}^3/\text{grs}^2 \), \( m_p = 1.6 \times 10^{-24} \text{grs} \), \( m_e = 9.1 \times 10^{-28} \text{grs} \), and the charge of the electron is \( e = 4.8 \times 10^{-10} \text{gr cm}^3/\text{s} \), recalling that 1Coul = \( 3 \times 10^9 \text{gr cm}^3/\text{s} \).
Thus, Eq.(14) takes the form:

\[
\frac{eQ_{\text{crit}}}{M} = 2.2 \times 10^{-22} \left[ \kappa \sqrt{\frac{\kappa^2}{4} + 1 + 5.68 \times 10^{-4} - \frac{\kappa^2}{2} - 5.68 \times 10^{-4}} \right] \frac{\text{gr cm}^3}{\text{s}} \frac{1}{\text{gr},} \quad (15)
\]

Now it is easy to express this ratio in geometric units, where \( c = 1 \), and \( G = 1 \), thus \( 1s = 3 \times 10^{10} \text{cm} \), and \( 1gr = 1.3 \times 10^{-28} \text{cm} \), thus obtaining:

\[
\frac{eQ_{\text{crit}}}{M} = 6.43 \times 10^{-17} \left[ \kappa \sqrt{\frac{\kappa^2}{4} + 1 + 5.68 \times 10^{-4} - \frac{\kappa^2}{2} - 5.68 \times 10^{-4}} \right], \quad (16)
\]

where now the charge and the mass are measured in centimeters. Note that for very small values of \( \kappa \), the equations for the ratio mass to charge are not very precise, due to the simplifications done in the calculus. Thus, we expect \( \frac{eQ_{\text{crit}}}{M} \) to become zero as \( \kappa \to 0 \), but the small term dependent of the electron mass prevents \( Q_{\text{crit}} \) from reaching this limit.

![Graph](image)

FIG. 4: The contours show the dependence of \( Q_{\text{crit}} \) on the mass of the black hole and the variable component of the luminosity in Eddington’s limit units \( \kappa \).

To calculate the time required by the black hole to reach the critical number of charged particles \( Q_{\text{crit}} \), we express the derivative of the charge with respect to time as a function of the accretion rate and an efficiency \( \varepsilon \) for the process. This efficiency gives the ratio between the number of scattered electrons that will escape from the infalling plasma and the total number of electrons that would fall to the black hole if there were no scattering. The efficiency \( \varepsilon \) may depend on quantum effects and variables such as the binding force between charges, and the radiation field. The evaluation of the latter is particularly difficult since it depends on the energy distribution and can be very inhomogeneous and anisotropic.
For this reason, we leave $\varepsilon$ as a parameter to be fixed from the periodicity estimated from observations. Thus, the time derivative of the charge is expressed by:

$$\frac{dQ}{dt} = \frac{\dot{M}\varepsilon}{m_p}.$$  

Substituting $\dot{M} = L/\eta c^2$, where $\eta \approx 0.4$ is the accretion radiating efficiency (Box 33.3), from eq. (13), and re-arranging we obtain:

$$\frac{dQ}{(1 - Q_f f_{\text{g,p}})^{1/2}} = \frac{\kappa \varepsilon L_E}{\eta m_p c^2} dt.$$  

Integrating from zero to $Q_{\text{crit}}$, we obtain the necessary time, $\tau$, to get the critical black hole charge,

$$\tau = \frac{\sigma \varepsilon_0 \eta m_p c}{\kappa \varepsilon^2} \left[ 2 - \left( 2\kappa^2 - 2\kappa\sqrt{4 + \kappa^2} + 4 \right)^{1/2} \right].$$  

(17)

Thus, after the lapse of time $\tau$, enough electrons will fall onto the black hole, in order to neutralize it. In this way, the physical initial conditions are restored and the process is repeated. The process will be periodic or quasiperiodic, depending on the presence of other mechanisms that may produce instabilities or variations of the accretion rate.

Equation (17) can be approximated by:

$$\tau = (1.153 - 0.308\kappa + 0.034\kappa^2) \frac{10^{-20}\eta}{\varepsilon},$$  

(18)

which is a decreasing function for relevant values of $\kappa$ (between 0 and 3).

D. Amplitude of the variability

Finally, we must consider the variation of the luminosity that this process will produce. We define the index of variability as the ratio between the maximum and the minimum luminosities, $L_{\text{max}}$ and $L_{\text{min}}$ respectively:

$$R_L = \frac{L_{\text{max}}}{L_{\text{min}}}.$$  

(19)

The luminosity measured by an observer on Earth may have an underlying, constant term ($L_c$). This constant term can also be expressed as a function of the Eddington luminosity:
\[ L_c = \chi L_E , \]

Thus, eq. (19) becomes,

\[ R_L = \frac{(\chi + \kappa)L_E}{L_E\left\{ \chi + \kappa\left[ (f_{g,p} - Q_{\text{crit},f_e})/f_{g,p}\right]^{1/2} \right\} . \]

Substituting terms and simplifying, we find that:

\[ R_L = \frac{2(\chi + \kappa)}{2\chi + \kappa(\sqrt{4 + \kappa^2} - \kappa)} . \]  \hspace{1cm} (20)

Fig. (5) shows a contour map for eq. (20) as a function of the luminosities of the underlying component \( \chi \) and the variable component \( \kappa \) in Eddington’s units. In the disk model, the inner and the outer part of the disk have similar contributions to the total luminosity \[18\], but the inner region is responsible for most part of the energetic radiation. Although the Eddington limit can be violated in some circumstances \[\text{Beloborodov} \[19\] \text{and references therein}\], it must be noted that, for a X-ray luminosity of \( \kappa = 1 \) and \( \chi = 0 \), the ratio between the maximum and minimum luminosities \( R_L \) has a maximum value of approximately 1.6. This value gives an estimate of the amplitude of the X-ray variability that our model can account for.

**FIG. 5:** This figure shows the index of variability (ratio between maximum and minimum luminosities expected from the black hole charging process), as a function of the variable component of the luminosity in Eddington’s units \( \kappa \), and the underlying steady luminosity \( \chi \) in the same units. The numbers identify isometrics of the ratio between luminosities.
It is very important to stress that, in those cases where enough data are available to compute the bolometric luminosity, amplitude and period of the variability, this model needs only three free parameters, namely the efficiency of the mass-energy conversion for the accretion process (in this paper we use for this efficiency \( \eta = 0.4 \)), the total X-ray and greater frequency luminosities in Eddington units, and the efficiency \( \varepsilon \) of the electron-proton decoupling process. It is also worth stressing that the model proposed in this paper does not require of any mechanism to enhance the variable component of the luminosity.

III. KERR NEWMAN SPACE TIME

It is not known an exact solution to the Einstein Maxwell equations which models the dynamical process of accretion and charge of the black hole. Actually there is not an exact solution for just a dynamical process of accretion in Schwarzschild! The black hole solutions are final states of such processes. However, we think it can give us a clue of what might be happening to consider a charged rotating black hole.

The Kerr Newman space time, which describes such a charged rotating black hole is given by

\[
d s^2 = \frac{\Delta}{\Sigma} (d t - a \sin^2 \theta d \varphi)^2 - \frac{\sin^2 \theta}{\Sigma} \left[ (r^2 + a^2) d \varphi - a d t \right]^2 - \frac{\Sigma}{\Delta} d r^2 - \Sigma d \theta^2,
\]

(21)

where \( \Delta = r^2 - 2M r + a^2 + (eQ)^2 \), \( \Sigma = r^2 + a^2 \cos^2 \theta \), \( M \) being the mass of the black hole, \( a \) its angular momentum per mass, and \( eQ \) its charge.

This expression is a solution to the Einstein equations, \( G_{\mu \nu} = 8 \pi T_{\mu \nu} \), with \( T_{\mu \nu} \) the stress energy tensor generated by the electromagnetic field, (in turn generated by the electromagnetic tensor of the black hole, \( F_{\mu \nu} \)), given by:

\[
T_{\mu \nu} = \frac{1}{8\pi} (F^\sigma_{\mu \sigma \nu} - \frac{1}{4} g_{\mu \nu} F_{\alpha \beta} F^{\alpha \beta}).
\]

(22)

The electromagnetic tensor \( F_{\mu \nu} \) is given by:

\[
F_{\mu \nu} = \begin{pmatrix}
  0 & -E_r & -E_\theta & 0 \\
  E_r & 0 & 0 & -B_\theta \\
  E_\theta & 0 & 0 & B_r \\
  0 & B_\theta & -B_r & 0
\end{pmatrix},
\]

(23)
where
\[
E_r = \frac{\sqrt{2} eQ \left( r^2 - a^2 \cos^2 \theta \right)}{\Sigma^2},
\]
\[
E_\theta = \frac{2 \sqrt{2} eQ a^2 r \cos \theta \sin \theta}{\Sigma^2},
\]
\[
B_r = \frac{2 \sqrt{2} eQ a \left( r^2 + a^2 \right) \cos \theta \sin \theta}{\Sigma^2},
\]
\[
B_\theta = \frac{\sqrt{2} eQ \left( r^2 - a^2 \cos^2 \theta \right) a \sin^2 \theta}{\Sigma^2},
\]
and is also a solution of the Maxwell equations without currents: \( F^\mu_{\nu,\nu} = 0 \), and \( F^\mu_{\nu,\lambda} + F^\lambda_{\nu,\mu} = 0 \).

As mentioned in the introduction, a major concern with a model that deals with charged black holes is that the charge must remain small enough to avoid the situation of a naked singularity, i.e., a black hole with no horizon [12]. For the Kerr-Newman metric the external horizon is given by:
\[
r_+ \equiv M \left( 1 + \sqrt{1 - \left( \frac{a}{M} \right)^2 - \left( \frac{eQ}{M} \right)^2} \right),
\]
where \( \frac{a}{M} \) takes the canonical value \( \frac{a}{M} = 0.998 \). We recall the reader that in this section we are working in the usual geometric units. The naked singularity would be produced when \( \frac{Q}{M} > 3.996 \times 10^{-3} \), and from Eq. (16), we see that this is far from taking place as the maximum value of the charge allowed is several order of magnitude below.

There are two invariants of the electromagnetic field, those quantities independent of the observer [21]:
\[
E^2 - B^2 = -\frac{1}{2} F^\mu_{\nu} F^{\mu\nu} = \frac{4 (eQ)^2 \left( r^4 - 6 r^2 a^2 \cos^2 \theta + a^4 \cos^4 \theta \right)}{(r^2 + a^2 \cos^2 \theta)^4},
\]
\[
E \cdot B = \frac{1}{4} \epsilon^{\mu\nu\alpha\beta} F^\mu_{\nu} F^{\alpha\beta} = \frac{4 (eQ)^2 r a \cos \theta \left( r^2 - a^2 \cos^2 \theta \right)}{(r^2 + a^2 \cos^2 \theta)^4},
\]
from which can be constructed a third one which relates the energy density of the electromagnetic field, \( \mathcal{E} \), and the magnitud of the Pointer vector of the flux, \(|S|\):
\[
64 \pi \left( \mathcal{E}^2 - |S|^2 \right) = (B^2 - E^2)^2 + 4 (E \cdot B)^2
= \frac{16 (eQ)^4 \left( r^8 - 8 r^6 a^2 \cos^2 \theta + 30 r^4 a^4 \cos^4 \theta - 8 r^2 a^6 \cos^6 \theta + a^8 \cos^8 \theta \right)}{(r^2 + a^2 \cos^2 \theta)^8}.
\]

These invariant quantities might prove usefull in further studies of the mechanism present in this work within the context of the Kerr Newman solution. For now, we only call the
reader’s attention to the fact that for small angles, which is the region where we expect the mechanism to happen, the first invariant takes the form (we are approximating the value $\frac{n}{\pi} \sim 1$, and expressing the $r$-coordinate as multiples of the external horizon radius, $r_+$, which we are also approximating to be equal to $M$, thus $r = nM$): 

$$E^2 - B^2 = \left(\frac{eQ}{r^2}\right)^2 \frac{4\left(1 - \frac{6}{n^2} + \frac{1}{n^4}\right)}{(1 + \frac{1}{n^2})^4}.$$  \hspace{1cm} (27)

In Eq. (27) we recognize the leading term $\left(\frac{eQ}{r^2}\right)^2$, which is the square of the magnitude of the classical electric field, Eq. (12). Notice however that near the last stable orbit, $n \sim 3$, the relativistic corrections for the magnitudes of the electromagnetic field are important, tending to decrease the magnitude of the first invariant near the horizon. Thus, as expected, our analysis is a first approximation of the phenomena.

These facts suggest that the mechanism present would be accompanied with variable electric and magnetic fields, which in turn might trigger some other phenomena, such as the proposed model of jet production by means of a magnetic field switched on and off.

**IV. Comparison with Observational Data**

Analysis of the X-ray light curves of AGN’s have shown many cases of short time variability. For some objects, these variations are periodic or pseudo periodic (e.g. NGC 4151, 3C 273, NGC 3516, NGC 5548). We will discuss the case of the galaxy IRAS 18325-5926 in detail because it has a very well sampled curve.

IRAS 18325-5926, was observed by ASCA for 5 days between 1997 March 27 and 31. Analysis of these data has shown a periodicity of $5.8 \times 10^4$ s in the 0.5-10 keV band with an amplitude, obtained from the folded curve, of 15%. However, a visual exam of the light curve in Fig. (6) shows that the amplitude of the variability is rather a factor of two larger.

In order to compare the observed light curve with the prediction of our model, we first fit a second order polynomial to account for long term variations. The long term variations are included in the variable luminosity term $\kappa$ during the calculations. Allowing a super Eddington luminosity by a factor of 2, $\kappa$ and $\chi$ are adjusted simultaneously to produce the observed variability and luminosity in Eddington’s units. We have included in Fig. (6) both the observed light curve from Iwasawa et al. and the expected variability of the X-ray luminosity from our model. We must point out that this fit is meant only to illustrate the
general expected behavior. It is not a rigorous fit to the data in the sense of a least square or $\chi^2$ fit. In all this process we have only set one free parameter for this source, the luminosity in Eddington’s units. Once this parameter is selected, the others are adjusted from the observational data.

Fig. (6) shows the results from the behavior predicted by the model compared to the observations. It is worth noticing that we are fitting the observed light curve, rather than the folded curve. Folded curves tend to underestimate the amplitude and structure of the variations [cf. Kotov et al. [23], [24]; Iwasawa et al. [25]; Abramowicz et al. [26]; Fiore et al. [27]]. Note the lag in the fitting results in Fig. (6) from approximately $3 \times 10^5$ to $3.8 \times 10^5$s. In this time interval, the periodical pattern is lost. The first, small peak in this interval at $3 \times 10^5$s, may be still considered in phase with the previous peaks, but with a much smaller amplitude. Then, two more small amplitude peaks follow and the pattern is recovered at $3.8 \times 10^5$s, but with a different phase. This result can be expected if the electrons fall into the central object as a consequence of some kind of disturbance. The accumulation of electric charge is basically an unstable process and a small perturbation (for example, the long term variation of the source) may produce the infall of the electronic cloud. Alternatively, the electron distribution may be inhomogeneous. In such a case, a Self-Organized Criticality (SOC) model [Bak, Tang, & Wiesenfeld, [28]; Mineshige, Ouchi,& Nishimori, [29]] may be applicable. The SOC model comprises numerous reservoirs. When a critical density is reached in a reservoir, an instability appears causing an avalanche and emptying the reservoir. Adjacent reservoirs are coupled, and the instability in one of them may extend to a few or many reservoirs, resulting in a small or large flare. Small flares are basically random, since they originate from a few reservoirs. The reservoir model in AGN has been discussed by Begelman & de Kool [30]. It is worth noticing that for other models (hot spots and lighthouse), the disappearance of the periodic variability and its recovery with another phase (but the same period and amplitude) may only be explained by invoking too many coincidences.

The parameters used for the calculations are: The index $R_X$ for the X-ray variability, $R_X = 2$, obtained from observations. The luminosity ($L_{X+}$) at X-ray and higher frequencies in Eddington units, $L_{X+} = 2$, which is a free parameter in the model. The observed period, taken from observation to be $\tau = 43,309$s. The variable and constant X-ray components in Eddington units $\kappa$, and $\chi$ respectively, are taken as $\kappa = 1.8$ and $\chi = 0.2$. These components
have been computed to fit \( R_X \) and \( L_{X+} \) simultaneously, and thus they are not free parameters. The mass of the central body, \( M = 2 \times 10^7 M_\odot \), which is set as a free parameter, as long as the observations do not allow to give a definite value, so we are taking the expected value for AGN’s. Finally, the efficiency of the charging process (\( \varepsilon \)), which is computed as a free parameter to fit the observed period, and we obtain \( \varepsilon = 6.5 \times 10^{-26} \).

And from Eq. (16), we obtain for the maximum value of the charge \( \frac{eQ_{\text{crit}}}{M} = 5.15 \times 10^{-17} \), that is, \( eQ_{\text{crit}} = 2.5 \times 10^9 \) Coul. Notice that although it is a huge value for the charge, the geometry of the space time remains unaffected.

We end this section with some comments about some other models which have been proposed to explain periodic variability in the X-ray light curves. Perhaps the most popular is the ‘hot spots’ model, which proposes that, in the innermost part of a thick accretion disk, hot spots appear due to shocks or other instabilities producing observable periodic variations. The inclination of the thick disk (with respect to the observer) has to be such that the disk itself periodically occults the spots in the inner border: e.g. Bao & Abramowicz [32]. However, this model requires a high number of free parameters to be adjusted to fit the folded light curves, which make these models difficult to be testable; it requires \( 3n + 4 \) free parameters, where \( n \) is the number of spots [27]. In contrast, the model presented in this paper may be expressed in function of only three free parameters, namely the mass-to-energy conversion factor \( \eta \), which is deduced from the accretion theory, the high energy luminosity in Eddington’s units \( L_{X+} \), and the efficiency (\( \varepsilon \)) of the local electron-proton decoupling events.

FIG. 6: IRAS 18325-5926 X-ray light curve from Iwasawa et al. (26). Overlapped, the model fit (see main text for explanation).
that push the released electrons away from the black hole. We have applied the model to the IRAS 18325-5926 case where X-ray periodic or quasiperiodic low amplitude variability has been reported, and found that the model fits reasonably well the observational data, even for unfolded light curves. Moreover, this is the only model that naturally explains the disappearance of the periodicity and later reappearance with a change of phase.

V. CONCLUSIONS

In this paper, we have studied a process that may induce electrical charge in a black hole and we have explained how this process can produce low amplitude, periodic or quasiperiodic X-ray variability. We have shown that high energy radiation along with a large gravitational field generated by a super massive black hole may be able to break a few proton-electron couplings in the infalling plasma due to Compton scattering. Some free electrons may reach the disk corona while the unbound protons are accreted into the black hole. With this mechanism, the black hole will acquire a net positive electric charge. Released electrons will remain as isolated charges which, once in the corona, are continuously pushed by radiation pressure. We have shown that it is sufficient that about one out of $10^{25-26}$ electrons is Compton scattered away from the infalling plasma to trigger this mechanism. The electric repulsion force between protons and the positively charged black hole will hamper further infall, decreasing the accretion rate and the energy output. As the radiation diminishes, its pressure decays and the electronic cloud in the disk corona will infall and neutralize the black hole. The X-ray variable component of the observed luminosity, which had been decaying gradually during the black hole charging process, will increase dramatically in a short time interval when the charge is neutralized, yielding a sawtooth light curve in X-rays. The process is periodic if there are no disturbances, but even small perturbations may provoke a significant part of the electrons to infall. However, the periodicity will be recovered after the disturbance ceases.

We mentioned examples of AGN’s where a periodic or semi periodic variability in the X-ray spectra has been observed, and applied our model to the case of the galaxy IRAS 18325-5926, where we are able to reproduce the overall behavior of the light curve. This result, along with the fact the model presented in this paper has only two free parameters, (considering the efficiency of the mass-energy conversion a data), are evidence that a simple
mechanism, like the one that we propose, may produce the observed X-ray variability in AGN accreting black holes. Even though there is no exact solution to the Einstein-Maxwell equations to model any accreting black hole, comparing with the Kerr Newman solution, we notice that this phenomenon in turn generates variability in the Electromagnetic fields which might trigger other events as the jet production by means of a magnetic field switched on and off.

Perhaps the main result of the present work is the fact that the model presented in this work opens the possibility that charged black holes may indeed exist in Nature, not isolated though, but with an accreting disk. The mechanism to generate such charged black holes seems plausible and the model describes general features of physical objects.

With respect to this last point, we recall that, as mentioned in the work, there are other models proposed to explain the observed variability in the X-ray light curve of several AGN’s. Compared with those, our model needs a much smaller number of free parameters and, as explained, it is stable in the sense that after a perturbation, it naturally recovers the variability in the X-ray light curve with the same characteristics. However, we do not discard other models, but rather propose another one which can be also occurring.

The possibility that there is a natural mechanism to induce an electric charge into an accreting black hole surrounded by a luminous accretion disk, could be a fruitful subject for further studies of charged black holes and its application to astrophysics.

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