Design and Analysis of a Total-Internal-Reflection (TIR) Structure Based on Ray-Mapping Method for Tailored Illumination

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ABSTRACT Tailored illumination have widespread applications such as road lighting. Most road lighting only serves the road but neglects the surroundings which are also important in illumination quality. Therefore, the background illumination is created to solve this problem, finding a balance between road and surroundings. However, optical structures in existing methods still need further processing, and details on energy efficiency are not disclosed either, resulting in a limited practicality. An improved total-internal-reflection (TIR) structure is proposed to generate a complex illumination including a pattern and a circular background. By solving a nonlinear equation which is obtained by converting the pattern problem into the boundary surface problem, the profile curves are optimized, and is then combined into a Lens by a commercial CAD software. The irradiance of the pattern is designed to be twice as much as the background, which can be adjusted by the boundary conditions. The results show that the method can generate the tailored illumination, with the uniformity of pattern and background being 95.3% and 89.3%, respectively, while the light efficiency reaches 86.8%. It can be extended to generate different patterns by modifying the boundary surface applied to road lighting, advertising, entertainment, etc.

INDEX TERMS Light emitting diodes, LED lamps, lighting, optical design techniques, optical devices.

I. INTRODUCTION

The problem for the lighting quality is always concerned by optical designers. The design or optimal of the LED arrays is the one of the common ways to improve the lighting quality, Moreno et al. [1] and Moreno and Munoz [2] proposed a special LED arrays with spherical structure, which can generate a wider ranging distribution compared with plane array, while improving the uniformity of target plane. However, the disadvantage of Moreno’ method is its low efficiency, while optical lens or reflectors can be used to improve the efficiency. Zhu et al. [3] proposed an improved reflector with freeform surface, which can be used to generate a rectangular illumination with high uniformity and efficiency, Ding et al. [4] obtained a freeform surface by solving a partial differential equation, which has important sense to non-imaging optical design. However, Ding’s method can be only used to point light sources, and the lighting quality may decline when it comes to the extended sources. Then an innovative method called simultaneous multiple surface (SMS) was proposed [5]–[7], which can solve the problem mentioned above, Benitez et al. [7] designed double freeform surfaces that deflected the rays of the input bundles into the rays of the corresponding output bundles and vice versa, which proved the good light control performance of SMS method. Ray mapping method is also an important method which has been proved to be an efficient method to control the irradiance distribution accurately [8], [9], Parkyn [8] proposed a ray-mapping method, which can be used to calculate each normal of the freeform surfaces, Feng et al. [9] extended this method to road lighting. Some models and structures have also been proposed [10]–[16], such as TIR structures [10]–[12], which can be introduced to illumination systems due to its high efficiency, but most of the existing TIR lenses only serve the beam collimating or uniform illumination. Therefor the modification of total-internal-reflection
(TIR) structures are needed to obtain the tailored illumination patterns. Ma et al. proposed a modified TIR structure which can be used to obtain the rectangular illumination [10], but the obtained patterns can be further expanded. Other rectangular illumination designs were proposed or optimized [13]–[15], most of which begun to focus on the shapes of the irradiance distribution.

The emphasis of the existing non-imaging systems mainly focused on how to obtain an irradiance distribution with high uniformity and high efficiency on the target plane, but neglected to tackle the complex illumination, it does not have enough evidence that the complex illumination can be generated by these methods or structures. The customizability of irradiance distribution is also important, so the tailored illumination gradually becomes one of the most advanced topics in non-imaging optics, which has progressed beyond traditional non-imaging systems due to its superior controllability of irradiance distribution. The tailored illumination should be adapted to the complex road condition of the city park or residential area. There might be some roadside facilities such as park chairs, trash cans, some rest place etc., which need low illumination to ensure that the pedestrians can still use or enjoy. Therefore, the background illumination needs to be generated to satisfy some situations that need different illumination. Plenty of work has been undertaken in this field. Ries and Winston [17] and Ries and Muschaweck [18] are the forerunners of this field to propose some innovative methods to obtain the required patterns, such as tailoring a reflector based on the edge-ray principle, which was proved to be an efficient approach to obtain the uniform illumination with required patterns on the targets [17], or generating a special patterns ‘oec’ on the target plane [18], which enormously advanced the development of tailored illumination. Wang et al. [19] proposed an irregular surface, which can be used to generate a pattern ‘E’ on the target plane, but the freeform surface was non-continuous and overcomplicated, which was difficult to measure and process. Based on the problem mentioned above, Wu et al. [20], [21] converted the tailored illumination into a nonlinear boundary problem for the elliptic Monge–Ampere (MA) equation, obtained a continuous double freeform lens generating the desired patterns (such as “ZJU” and “UPM”) with background illumination on the target plane, the irradiance of the patterns was twice as much as background. Then the MA method presented by Wu was applied to road lighting systems [22], generating different uniform patterns with background for different road, such as bending road and straight road. However, some details such as energy efficiency in these publications need to be further disclosed. Some previous similar research works such as nonplanar illumination [23] and ringed distribution [24] have been proposed. Supporting quadratic method (SQM) [25]–[27] improved by Image Processing Systems Institute of the RAS is also a versatile method for tailored illumination, in which the freeform surface is represented as a set of quadric segments with certain parameters. It is essentially the combination of numerous microlens which is used as collimation, then the microlens array can be calculated into a continuous freeform surface by SQM. SQM can be taken as one of the most effective method for tailored illumination, and the most advanced technology in non-imaging optical design.

In order to further exploit the potential performance of the TIR structures, and realize the tailored illumination by using a simple ray-mapping method, in this paper, a modified total-internal-reflection (TIR) structure generating a tailored illumination is proposed, with the advantages of simple ray mapping relationship, accurate light irradiation control, and easier to reprocess the existing large quantity products to satisfy the diverse needs. Two modified TIR lenses were designed as examples for road lighting by this method, the road lighting based on LEDs are used here due to their significant advantages for the energy conservation. The installation errors on the lighting quality, including illumination uniformity and light efficiency were also discussed. The results show that the proposed lenses generated a complex irradiance distribution, including a rectangular pattern or a bend pattern with circular background illumination, of which uniformity and light efficiency agreed quite well with the desired lighting quality.

II. MATHEMATICAL MODELS

The modification of the TIR structures is mainly on the bottom parts of the lenses, as Figure 1 shows, the bottom parts of modified TIR lenses are freeform surfaces while the traditional TIR lenses are flats. The bottom surfaces based on the determined edge of the illumination patterns, which is the key to obtain the tailored patterns on the target plane. The edge of the patterns can be used as limiting conditions of the mathematical models, so the bottom surfaces can be calculated using the Matlab, and the modified TIR lenses can be obtained using the commercial CAD tools.

The TIR structure is shown in the Figure 2, the desired irradiation distribution is generated by two independent non-imaging systems, the plano-convex structure and the total-internal-reflection (TIR) structure, the plano-convex structure provides a uniform background illumination, and the patterns is generated by the TIR structure.

We choose the LED with lambertian distribution, which can be expressed as,

\[ I_\theta = I_{\text{axis}} \cdot \cos \theta \] (1)
where the $I_{\text{axis}}$ represents the axial intensity of the LED. As Figure 2 shows, the $\alpha$ represents the edge ray of plano-convex structure, the intersection point $A$ between the $\alpha$ and the target plane determines the size of the background distribution. The key to determine uniformity of background irradiance distribution is the plano-convex surface, it can be calculated by the law of energy conservation and the edge ray principle [17], $Q$ is the intersection point between the optical axis and the target plane, the energy of the $Q$ and $A$ can be calculated, then set these two equal to each other, which can be expressed as,

$$I_{\text{axis}} \sin^2 \theta \int_0^y 2\pi y dy/R_{\text{max}}^2 = \int_0^\theta 2\pi I_{\text{axis}} \sin \theta \cos \theta d\theta \tag{2}$$

where $\theta_1$ is the critical angle between the plano-convex structure and the TIR structure, $R_{\text{max}}$ represent the maximum radius of the background illumination, $\theta$ is the angle between the emitted light of LED and optical axis. Due to the law of the Snell, The slope of the refractive surface can be expressed as,

$$dy/dx = (n \cdot \cos \mu - \cos \theta)/(\sin \theta - n \cdot \sin \mu) \tag{3}$$

It’s no hard to get the relations between the y-coordinate of the surface $y_{\text{surface}}$ and x-coordinate $x$,

$$y_{\text{surface}} = (d + h + x) \cdot \tan \theta \tag{4}$$

Differentiate the Eq. (4), we have,

$$\frac{dy_{\text{surface}}}{d\theta} = d + h + x \cdot \frac{\cos \theta}{\cos^2 \theta} + \tan \theta \cdot \frac{dx}{d\theta} \tag{5}$$

So the $F_1 = dx/d\theta$ can be obtained by Eq. (3) and Eq. (5), which is expressed as,

$$F_1 = (-d - h - x)/[\cos^2 \theta \cdot (\tan \theta + \cos \mu - 1/n \cdot \cos \theta/\sin \mu - 1/n \cdot \sin \theta)] \tag{6}$$

where the $\mu$ is the angle between the incident ray and the horizontal line, it should be expressed by $x, \theta$ so that the Eq. (6) can be solved by numerical methods. The relations among the $\mu, x, \theta$ can be obtained by deriving the expression of $y$, which can be obtained by Eq. (2), so we have,

$$y = R_{\text{max}} \cdot \sin \theta / \sin \theta_1 \tag{7}$$

Due to the geometrical relations among the $\mu, x, \theta$, $y$ can be also expressed as,

$$y = x \cdot \tan \mu + (d + h + x) \cdot \tan \theta + \frac{l \cdot n \cdot \sin \mu}{\sqrt{1 - n^2 \sin^2 \mu}} \tag{8}$$

So the $F_2 = 0$ can be obtained by Eq. (7) and Eq. (8), which is expressed as,

$$F_2 = x \cdot \tan \mu + (d + h + x) \cdot \tan \theta + \frac{l \cdot n \cdot \sin \mu}{\sqrt{1 - n^2 \sin^2 \mu}} \cdot \frac{R_{\text{max}} \cdot \sin \theta}{\sin \theta_1} \tag{9}$$

where the $d$ represents the thickness at optical axis, $h$ is the distance between the LED and the Lens, $\mu$ is incident angle on point $C$, $l$ is the distance between the target and the Lens, it’s no hard to see that the $F_1$ is a nonlinear differential equation, while the $F_2$ is an implicit equation, these two equations constitute a differential-algebraic equations (DAEs), which is composed of a nonlinear differential equation and an implicit function. The DAEs can be efficiently solved by the Runge-Kutta method, which can be implemented by the “ODE Solver” in Matlab or other variable-order methods.

Similar procedures can be also applied to the mathematical model of TIR structure. As mentioned, the pattern is determined by the TIR structure, which is generally designed in a non-rotationally symmetric structure. As Figure 2 shows, the $\beta$ represents the edge ray of TIR structure, incidence in the edge of the target pattern $B$. At this time, the energy of the $Q$ and $B$ can be calculated. Then just setting these two to be equal to each other, a DAEs can be obtained based on the law of energy conservation, the theory of Snell and the edge ray principle, as follows,

$$\begin{cases}
F_3 = -H \cdot \frac{L_1}{\cos \theta \sin^2 \theta} + \frac{n^2 \sin \theta}{L_2} \\
F_4 = R_{\text{max}} \cdot \frac{\sin^2 \theta - \sin^2 \theta_1}{\sin^2 \theta_1 - \sin^2 \theta_{\text{max}}} - \frac{L_1 \cdot L_2}{\cos \theta} + L_3 \\
L_1 = \sqrt{n^2 - \cos^2 \theta} \\
L_2 = d + h + x - H/\tan \theta \\
L_3 = -\frac{l \cdot n \cdot \sin \mu}{\sqrt{1 - n^2 \sin^2 \mu}} + x \tan \mu - H
\end{cases} \tag{10}$$

where the $H$ represents the radius of the plano-convex surface, $\theta_{\text{max}}$ is the maximum emitted angles of LED, then a matrix $M$ can be obtained due to the characteristics of Eq. (6), Eq. (9) and Eq. (10),

$$M \cdot F = \begin{bmatrix} De & 0 & 0 & 0 \\
0 & If & 0 & 0 \\
0 & 0 & De & 0 \\
0 & 0 & 0 & If \end{bmatrix} \cdot \begin{bmatrix} F_1 \\
F_2 \\
F_3 \\
F_4 \end{bmatrix} \tag{11}$$
where the \( F = [F_1 F_2 F_3 F_4]' \) includes two nonlinear differential equations ([\( F_1 F_3 \)']) and two implicit equations ([\( F_2 F_4 \)']). Therefore, \( M \cdot F \) can be used to identify the nonlinear differential equations (\( De \)) and the implicit functions (\( If \)).

A nonlinear equation can be obtained by analyzing the mapping relations between TIR surface and patterns, such as,

\[
x_p = -h - d + \frac{H}{\tan \left[ \arcsin \left( \sqrt{\frac{R_{2}^{2} \sin^{2} \theta_{1} - \sin^{2} \theta_{\text{max}}}{R_{\text{max}}^{2}} + \sin^{2} \theta_{1}} \right) \right]}
\]

(12)

The Eq. (12) represents the relations between the bottom surface of the lens and the edge of the desired pattern. As Figure 3 shows, the \( x_p \) is the x-coordinates of bottom surface, while the \( R_F \) is the edge of the pattern. It should be noted that the Figure 3 only shows a profile in a direction, but we can get the corresponding \( x_p \) in each direction can be calculated by Eq. (12). Finally, a series of lines in different directions (\( x_p, H \)) can be obtained, which can be combined into a freeform surface using commercial CAD tools, the obtained freeform surface is the bottom surface of the TIR lens, while the TIR structures is calculated by Eq. (6), Eq. (9) and Eq. (10), which can be solved by Runge-Kutta method effectively. It should be noted that the patterns can be customized by controlling boundaries of TIR structure in each direction. Theoretically, this method can be tailored to any patterns. Generally, the proposed TIR lenses are all non-rotationally symmetric structure. Combining obtained curves into a TIR Lens through commercial CAD tools, an obtained bottom surface can then be processed into diffuser or other structure to improve the energy efficiency. The obtained Lens can be validated by using raytracing software Lightools 8.4.0 or TracePro 7.4.3. Figure 4 shows the flowchart of this method.

## III. SOLVED EXAMPLES

In this paper, LightTools software is used to simulate and verify the performance of tailored illumination. The purpose of this research is to generate a uniform irradiance distribution with background illumination for straight or curve road. Two optical elements were designed to verify the lighting quality for straight road and curve road, respectively. The transmittance of lens is 1, while the refractive index is 1.643. The selected source wavelength is white light spectrum, and the number of rays used in ray tracing is 1000000, and the FFT grid of the target plane is 128*128. Two indicators are selected to evaluate the lighting quality, which are light efficiency and illumination uniformity. As the common evaluation index of illumination systems, the light efficiency reflects the output efficiency of designed optical system directly, which can be understood as the ratio of the received flux over target region to emitted flux from LED, in another word, the light efficiency can also be understood as meaning that the ratio of the number of the rays arriving at the right place to the total number of the rays, so it can be used to reflect the efficiency of the tailored illumination, of which the patterns are clearer while more rays reach the right place. The uniformity is used to reflect the distribution effect of patterns or backgrounds, which is an important index of illumination system. The key to measure the lighting quality is the irradiance data arrays over the target plane, which can be obtained by LightTools, so the uniformity is defined as

\[
U = 1 - \frac{U_{\text{std}}}{U_{\text{ave}}}
\]

(13)

where the \( U_{\text{std}} \) is the standard deviation of the obtained irradiance distribution, the \( U_{\text{ave}} \) represents the average irradiance of target plane, the light efficiency is defined as

\[
E = \frac{\text{Flux}_{\text{Received}}}{\text{Flux}_{\text{Emitted}}}
\]

(14)

A non-uniform background may have a huge impact on the pattern, even change the shape of the pattern, which is not allowed in some applications such as advertising, urban beautification and entertainment, so the uniformity of the patterns (\( U_P \)) and backgrounds (\( U_B \)) are represented with a single uniformity (\( U_S \)) to reflect the uniformity of the whole distribution, defined as \( U_S = (U_P + U_B)/2 \). Figure 5 depicts the tailored illumination generated by the proposed TIR lens for straight-road lighting (SRL), the target plane with the radius of 1500 mm is 1000 mm away from the LED.

Figure 5 (a) shows the bottom of obtained TIR lens, the size of the lens is 40.0mm*40.0 mm*15.0 mm along the X, Y, and Z axis, respectively. It can be found that the TIR lens
FIGURE 5. The tailored illumination for straight-road lighting (SRL): (a) the bottom parts of modified TIR lens; (b) 3-D irradiance distribution on the target plane; (c) 2-D irradiance distribution with profiles on the target plane; (d) The target pattern and standard deviation on each position.

FIGURE 6. The tailored illumination for curve-road lighting (CRL): (a) the bottom parts of modified TIR lens; (b) 3-D irradiance distribution on the target plane; (c) 2-D irradiance distribution with profiles on the target plane; (d) The target pattern and standard deviation on each position.

The irradiance ratio (Pattern : Background) is designed to 2:1, and the simulation results show that the irradiance of curve-shaped pattern is also nearly twice as much as the irradiance of background, which validates the mathematical model.

Figure 5 (d) and Figure 6 (d) show the irradiance distribution of each position on target planes, the patterns are framed in red, while the backgrounds are framed in blue. The red-tinted or blue-tinted regions represent the positions of which standard deviation are over 1 standard deviation of the mean in patterns or backgrounds, while other regions represent the positions of which standard deviation are within 1 standard deviation of the mean, the non-uniform irradiances of patterns are scattered across the whole pattern region, while that of backgrounds are focused on the border of the pattern and background.

A. TOLERANCE ANALYSIS

The tolerance is an important issue [28] for non-imaging optical design. The irradiance distribution on the target plane highly depends on the installation errors including horizontal errors and vertical errors of the lenses, which are unavoidable. To analyze the relationship between the horizontal errors and lighting quality, eleven different horizontal errors (−0.10 mm ~ +0.10 mm) are simulated, and both “straight
road’’ and ‘‘curve road’’ are selected to the sensitivity studies on effects of horizontal and vertical errors. The irradiance distribution on the target plane with different horizontal errors are shown in Figure 7, while the Figure 7 (a) depicts the schematic of the horizontal errors, the Figure 7 (b) depicts the trends of the light efficiency with different horizontal errors, and the Figure 7 (c) shows the trends of the uniformity ($U_S$, $U_P$, $U_B$) with different horizontal errors ($-0.10mm$ to $+0.10mm$).

The vertical errors which are also the major errors in non-imaging systems and its influence on the lighting quality are shown in Figure 8. The lighting quality ($U_S$, $U_P$, $U_B$, $E$) under different vertical errors ($-0.10mm$ to $+0.10mm$) and its trends are also shown in Figure 8 (b) and (c), respectively. From the results it is easy to find that the relationship between the lighting quality ($U_S$, $U_P$, $U_B$, $E$) and the vertical errors is similar with that of horizontal errors. The uniformity $U_S$ of the lenses for SRL/CRL range from 89.2%/84.2% to 92.4%/88.5% with the change of the vertical errors ($-0.10mm$ to $+0.10mm$), while the light efficiency (E) of SRL and CRL range from approximately 85.7%/82.0% to 87.8%/84.2%, respectively.

From the comparisons shown in Figure 7 and Figure 8, it can be found that different installation errors cause different effects on the lighting quality ($U_S$, $U_P$, $U_B$, $E$). The $U_B$ by the horizontal errors is more susceptible, while the $U_P$ is more sensitive to the vertical errors, which is the main cause of the failing in the total internal reflection. It seems that the vertical errors have more effect on the whole tailored quality, not only the more deterioration of the $U_P$, but also the faster drop of efficiency than that from the horizontal errors, for example, in the CRL, the $U_S/U_P/U_B/E$ range from 87.9%/95.6%/80.2%/83.2% to 84.2%/89.8%/78.6%/84.2%, respectively, with the vertical errors increase from $0mm$ to $-0.10mm$. However, based on the results obtained, it is easy to find that the lighting quality is not seriously affected by vertical errors and horizontal errors within limits, for example, although the vertical errors seem to be more influential in light efficiency, it still maintains more than 80%, which means the proposed TIR lenses have certain fault tolerance. Other installation errors and correlations would be discussed in the future study, which are out of the scope of this paper.

The installation errors directly influence the h (distance between LED and Plano-convex structure) and the H (distance between optical axis and TIR structure), respectively, while these two parameters directly determine the $\theta$ (angle of the emitted lights from LED). The entrance lights redistributed by freeform surfaces are not in the right place due to the non-ideal angle $\theta$, which brings some non-uniform region of target plane. In another word, the H and the h can influence the uniformity through the angle $\theta$, so the relations between these two parameters and angle $\theta$ are the key to address the problem. According to the mathematical model of the TIR structure, it’s no hard to get the relations between the $h$ and $\theta$,

$$\gamma_{surface} = H + (d + h + x - H/tan\theta) \cdot \sqrt{n^2 - \cos^2 \theta \over \cos \theta}$$

(15)

We assume that the parameters of the TIR structure ($x$, $\gamma_{surface}$ or $\gamma'_{surface}$, $d$) are constant, so the relations among the $H$, $h$ and $\theta$ are depicted in Figure 9.
It’s no hard to see that of the rate (Absolute value) of the $\theta$ declines with the increase of the $h$ and $H$, both in the plano-convex structure and TIR structure. Therefor enhancing the $H$ and $h$ can reduce the sensitivity of $\theta$, which determines the uniformity, while these two parameters determine the size of the lenses. In another word, the final results can be optimized by enlarging the size of the TIR lenses.

**B. ADDITIONAL SIMULATIONS**

It is adjustable that the irradiance ratio of the pattern and background by modifying the boundary conditions between the plano-convex structure and the TIR structure. Figure 10 shows an example (SRL) with 5:1 ratio, where the irradiance of the background ($\sim 3.8E - 06W/m^2$) is almost one fifth of that of the pattern ($\sim 1.9E - 05W/m^2$) the $U_S/U_P/U_B/E$ of which are $85.1%/85.4%/84.8%/82.4%$ respectively.

Compared to the example with 2:1 ratio ($U_S/U_P/U_B/E = 92.3%/95.3%/89.3%/89.3%$), the quality indicators of example with 5:1 ratio declined by $7.8%/10.4%/5.0%/7.7%$ respectively.

In order to show the overall effect of road lighting, a simulation is carried out.

Figure 11 depicts a scale model of road lighting and its detailed lighting performance, where the road with the length of 5560mm contains one curve region (1000mm*820mm) and two straight region (2*1830mm*820mm), so 3 lenses are calculated from 2 kinds of models, which are lined one by one over the road, the lens-road distance and lens-lens distance are 1000mm and 1894mm, respectively. Simulation results shows that the irradiance of the background ($\sim 8E - 06W/m^2$) is almost the half as that of the pattern ($\sim 1.6E - 05W/m^2$), where the main indicators $U_P/U_B/E$ are $90.5%/89.8%/81.8$, which means that the combination of this lenses can still retain most of the performance when it comes to complex target, in this case, the method proposed in this paper makes possible to be applied in the tailored road lighting.

**IV. CONCLUSION**

This paper presents a ray mapping method generating tailored illumination for road lighting. Using the desired patterns on the target plane as constrains, a mathematical model of an improved TIR lens with nonlinear boundary surfaces is established. Two lenses are designed by solving the numerical results of the proposed mathematical models. The time to solve this mathematical model by the software MATLAB using a computer with a 2.60 GHz Intel Core CPU (i7-6700HQ) is less than 10s. For straight road lighting (SRL), the results show that the light efficiency reaches 86.8% and the uniformity of pattern and background are 95.3%, 89.3%, respectively, while those for curve-road lighting (CRL) reach 83.2%, 95.6%, and 80.2%, respectively. Compared with the existing methods, the proposed method is easy to achieve, which can be useful for the direct processing of the existing TIR lens into the proposed TIR lens for tailored illumination. The influences of the installation errors including the horizontal errors and vertical errors are also discussed, which verify the fault tolerance of this method. The derivation of the mathematical models of tailored illumination design is the main focus of this paper, while the experiments would be included into the future work, the processed lenses can be applied to many occasions, such as bend of road, road junction, advertising, urban beautification, entertainment etc., and will broaden the outlook of tailored illumination.

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