Abstract

We study the Electric Dipole moment of $b$ quark in the general two Higgs Doublet model (model III) and three Higgs Doublet model with $O(2)$ symmetry in the Higgs sector. We analyse the dependency of this quantity to the new phase coming from the complex Yukawa couplings and masses of charged and neutral Higgs bosons. We see that the Electric Dipole moment of $b$ quark is at the order of $10^{-20} \text{e cm}$, which is an extremely large value compared to one calculated in the SM and also two Higgs Doublet model (model II) with real Yukawa couplings.
1 Introduction

The study of CP violating effects provides comprehensive informations in the determination of free parameters of the various theoretical models. Non-zero Electric Dipole Moment (EDM) for elementary particles is the sign of such violation. Neutron EDM has a special interest and the experimental upper bound has been found as $d_N < 1.1 \times 10^{-25} \text{ e cm}$ \[1\]. EDM of electron and muon have been measured experimentally as $d_e = (-2.7 \pm 8.3) \times 10^{-27} \text{ e cm}$ \[2\] and $d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm}$ \[3\]. These measurements can also give powerful clues about the internal structure of the particles, if it exists.

The source of CP violation in the SM is complex Cabbibo-Kobayashi- Maskawa (CKM) matrix elements. The quark EDM vanishes at one loop order in the SM, since moduli of matrix element is involved in the relevant expression. Further, it also vanishes at two loop order after sum over internal flavours \[4, 5\]. When QCD corrections are taken into account, nonzero EDM exists \[6\]. However, EDM for quarks is very small in the SM and need to be enhanced with the insertion of new models beyond. In the literature, quark EDM is calculated in the multi Higgs doublet models \[6, 8, 9\]. In \[8\], EDM due to the neutral Higgs boson effects is obtained in the two Higgs doublet model (2HDM) and \[8\] discusses the necessity of more scalar fields than just two Higgs doublets for non-zero EDM when only the charged Higgs boson effects are taken into account. In \[10\], EDM of $b$-quark in the case of $Z$-boson and photon are calculated in the 2HDM and models with three and more Higgs doublets. EDM of $b$-quark for $Z$-boson is predicted in the range $10^{-21} - 10^{-20} \text{ e cm}$, following the scenario where CP violation may only come from the neutral Higgs sector. Further, $b$-quark EDM is obtained in the range $10^{-23} - 10^{-22} \text{ e cm}$ when the CP violating effect comes from the charged sector. \[11\] is devoted to the study of $t$-quark EDM, which is predicted at the order of the magnitude of $10^{-20} \text{ e cm}$, arises from the neutral Higgs sector in the framework of the multi Higgs doublet model. In \[12\], quark EDM is calculated in the 2HDM if the CP violating effects come from CKM matrix elements and it is thought that $H^\pm$ particles also mediate CP violation besides $W^\pm$ bosons, however, at the two loop order these new contributions vanish. The electric and weak electric dipole form factors for heavy fermions in a general two Higgs doublet model is studied in \[13\] and it is concluded that the enhancement of three orders of magnitude in the electric dipole form factor of the $b$ quark with respect to the prediction of 2HDM I and II is possible.

In our work, we study EDM of $b$-quark in the general 2HDM (model III) and the general 3HDM with $O(2)$ symmetry in the Higgs sector (3HDM($O_2$)) \[14\]. In this case, CP violation comes from complex Yukawa couplings and even at one loop, it is possible to get extremely large
EDM, at the order of $10^{-20} \, e \, cm$. Since the theoretical values of quark EDM’s are negligible in the SM and also in the model II 2HDM with real Yukawa couplings, model III and 3HDM($O_2$) results can give important information about the new parameters, namely masses of charged and neutral Higgs particles, Yukawa couplings, beyond the SM. Note that, non-zero EDM can be obtained due to the charged Higgs boson effects even in the 2HDM in this case.

The paper is organized as follows: In Section 2, we present EDM for $b$ quark in the framework of model III and 3HDM($O_2$). Section 3 is devoted to discussion and our conclusions.

2 Electric Dipole moment of $b$ quark in the general two Higgs Doublet and three Higgs Doublet models

In this section, we calculate $b$-quark EDM in the model III and extend our results to the general 3HDM. Since nonvanishing EDM is the sign for the CP violation, we assume that there exist complex Yukawa couplings which are possible sources of such violations. We start with the Yukawa interaction in the model III

$$\mathcal{L}_Y = \eta^U_{ij} \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta^D_{ij} \bar{Q}_{iL} \phi_1 D_{jR} + \xi^U_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + h.c. ,$$

where $L$ and $R$ denote chiral projections $L(R) = 1/2 (1 \mp \gamma_5)$, $\phi_i$ for $i = 1, 2$, are the two scalar doublets, $\bar{Q}_{iL}$ are left handed quark doublets, $U_{jR}(D_{jR})$ are right handed up (down) quark singlets, with family indices $i, j$. The Yukawa matrices $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ have in general complex entries. Here $\phi_1$ and $\phi_2$ are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} 0 \\ v + H^0 \end{array} \right] + \left[ \begin{array}{c} \sqrt{2} \chi^+ \\ i \chi^0 \end{array} \right] ; \phi_2 = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \sqrt{2} H^+ \\ H_1 + iH_2 \end{array} \right] .$$

with the vacuum expectation values,

$$< \phi_1 > = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} 0 \\ v \end{array} \right] ; < \phi_2 > = 0 ,$$

for the FC charged interactions can be written as

$$\mathcal{L}_{Y,FC} = \xi^{U,D}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^{D,U}_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + h.c. ,$$

where the couplings $\xi^{U,D}_{ij}$ for the FC charged interactions are

$$\xi^{U,D}_{ch} = \xi^{U,D}_N V_{CKM} ,$$

$$\xi^{D,U}_{ch} = V_{CKM} \xi^{D,U}_N .$$
and $\xi^{U,D}_N$ is defined by the expression (for more details see [15])

$$
\xi^{U(D)}_N = (V_{R(L)}^{U(D)})^{-1} \xi^{U(D)} V_{L(R)}^{U(D)} .
$$

Note that the index ”N” in $\xi^{U,D}_N$ denotes the word ”neutral”.

The effective EDM interaction for $b$-quark is defined as

$$
L_{EDM} = i d \bar{b} \gamma_5 \sigma_{\mu\nu} b F^{\mu\nu} ,
$$

where $F^{\mu\nu}$ is the electromagnetic field tensor and ”$d$” is EDM of $b$-quark. Here, ”$d$” is a real number by hermiticity. In model III, the charged $H^\pm$ and neutral Higgs bosons $h^0, A^0$ can induce CP violating interactions which can create EDM at loop level. Note that, we take $H_1$ and $H_2$ as the mass eigenstates $h^0$ and $A^0$ respectively and do not take the mixing of two CP-even neutral bosons $H_0$ and $h_0$ into account, since no mixing occurs at tree level for our choice of Higgs doublets (eq. (2)). We present the necessary 1-loop diagrams due to charged and neutral Higgs particles in Figs. [1] and [2]. Since, in the on-shell renormalization scheme, the self energy $\Sigma(p)$ can be written as

$$
\Sigma(p) = (\not{p} - m_b) \Sigma(p)(\not{p} - m_b) ,
$$

diagrams $a$, $b$ in Figs. [1] and [2] vanish when $b$-quark is on-shell. However the vertex diagrams $c$, $d$ in Fig. [1] and $c$ in Fig. [2] give non-zero contributions. The most general vertex operator for on-shell $b$-quark can be written as

$$
\Gamma_\mu = F_1(q^2) \gamma_\mu + F_2(q^2) \sigma_{\mu\nu} q^\nu
$$

$$
+ F_3(q^2) \sigma_{\mu\nu} \gamma_5 q^\nu
$$

where $q_\nu$ is photon 4-vector and $q^2$ dependent form factors $F_1(q^2)$ and $F_2(q^2)$ are proportional to the charge and anomalous magnetic moment of $b$-quark respectively. CP violated interaction exists with the non-zero value of $F_3(q^2)$ and it is proportional to EDM of $b$-quark. By extracting the CP violating part of the vertex, $b$-quark EDM ”$d$” (see eq.(7)) is calculated as a sum of contributions coming from charged and neutral Higgs bosons,

$$
d = d^{H^\pm} + d^{h^0} + d^{A^0} ,
$$

where $d^{H^\pm}$, $d^{h^0}$ and $d^{A^0}$ are

$$
d^{H^\pm} = \frac{4 G_F}{\sqrt{2}} \frac{e}{32 \pi^2} \frac{1}{m_t} \xi^{U}_{N,tt} \text{Im}(\xi^{D}_{N,bb}) |V_{tb}|^2 \frac{y((-1 + Q_t (-3 + y) - y)(y - 1) + 2 (Q_t + y) \ln y)}{(y - 1)^3} ,
$$

$$
d^{h^0} = -\frac{4 G_F}{\sqrt{2}} \frac{e}{16 \pi^2} \frac{Q_b}{m_b} \text{Im}(\xi^{D}_{N,bb}) \text{Re}(\xi^{D}_{N,bb}) (1 - \frac{r_1 (r_1 - 2)}{\sqrt{r_1 (r_1 - 4)}} \ln \frac{\sqrt{r_1} - \sqrt{r_1 - 4}}{2} - \frac{1}{2} r_1 \ln r_1) ,
$$

$$
d^{A^0} = -d^{h^0} (r_1 \to r_2) ,
$$

(11)
with \( r_1 = m^2_{b\ell}/m^2_b, r_2 = m^2_{t\ell}/m^2_t \) and \( y = \frac{m^2_\tau}{m^2_{W^\pm}} \). Here \( Q_b \) and \( Q_t \) are charges of \( b \) and \( t \) quarks respectively and \( \xi_{N,ij}^{U(D)} = \sqrt{\frac{4G_F}{\sqrt{2}}} \xi_{N,ij}^{U(D)} \). In eq. (11) we take into account only internal \( t \)-quark contribution for charged Higgs and internal \( b \)-quark contribution for neutral Higgs interactions since we assume that the Yukawa couplings \( \bar{\chi}_{N,ib}, i = u, c, \bar{\chi}_{N,ij}^D, j = d, s \) and \( \bar{\chi}_{s_{N,tc}}^U \) are negligible compared to \( \xi_{N,tt}^U \) and \( \xi_{N,bb}^D \) (see [10]). Further, we choose \( \xi_{N,tt}^U \) real and \( \xi_{s_{N,bb}}^D \) complex,

\[
\xi_{N,bb}^D = |\xi_{N,bb}^D| e^{i\theta}.
\]

Finally, using the parametrization eq.(12), we get the EDM of \( b \)-quark in model III as

\[
d = \frac{4G_F}{\sqrt{2}} \frac{e}{32\pi^2} |\xi_{N,bb}^D| \sin \theta \left( \frac{1}{m_t} \xi_{N,tt}^U |V_{tb}|^2 \frac{y((-1 + Q_t (-3 + y) - y)(y - 1) + 2 (Q_t + y) \ln y)}{(y - 1)^3} + \frac{1}{2} \ln r_1 \right) + 2 \frac{Q_b}{m_b} |\xi_{N,bb}^D| \cos \theta \left( \frac{m_t}{r_1 (r_1 - 2)} \ln \frac{r_1 - 4}{2} + \frac{1}{2} \ln r_1 \right) - \frac{r_2 (r_2 - 2)}{\sqrt{r_2 (r_2 - 4)}} \left\{ \ln \frac{r_2 - 4}{2} - \frac{1}{2} r_2 \ln r_2 \right\}.
\]

Now, we would like to extend our result to the general 3HDM\((O_2)\) [14]. The new general Yukawa interaction is

\[
\mathcal{L}_Y = \eta_{ij}^U Q_{iL} \tilde{\phi}_1 U_{jR} + \eta_{ij}^D Q_{iL} \phi_1 D_{jR} + \xi_{ij}^{U+} Q_{iL} \tilde{\phi}_2 U_{jR} + \xi_{ij}^{D+} Q_{iL} \phi_2 D_{jR} + \tilde{\rho}_{ij}^U Q_{iL} \tilde{\phi}_3 U_{jR} + \rho_{ij}^D Q_{iL} \phi_3 D_{jR} + h.c.,
\]

where \( \phi_i \) for \( i = 1, 2, 3 \), are three scalar doublets and \( \eta_{ij}^{U,D}, \xi_{ij}^{U,D}, \tilde{\rho}_{ij}^{U,D} \) are the Yukawa matrices having complex entries, in general. With the choice

\[
\phi_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + H^0 \end{array} \right) + \left( \begin{array}{c} \sqrt{2} \chi^+ \\ i \chi^0 \end{array} \right),
\]

\[
\phi_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} H^+ \\ H^1 + iH^2 \end{array} \right), \quad \phi_3 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} F^+ \\ H^3 + iH^4 \end{array} \right),
\]

and the vacuum expectation values,

\[
< \phi_1 >= \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right) ; < \phi_2 >= 0 ; < \phi_3 >= 0
\]

the information about new physics beyond the SM is carried by the last two doublets \( \phi_2 \) and \( \phi_3 \), while the first doublet \( \phi_1 \) describes only the SM part. The Yukawa interaction for the Flavor Changing (FC) part is

\[
\mathcal{L}_{Y,FC} = \xi_{ij}^{U+} Q_{iL} \tilde{\phi}_2 U_{jR} + \xi_{ij}^{D+} Q_{iL} \phi_2 D_{jR} + \tilde{\rho}_{ij}^{U+} Q_{iL} \tilde{\phi}_3 U_{jR} + \rho_{ij}^D Q_{iL} \phi_3 D_{jR} + h.c.,
\]

4
where, the charged couplings $\xi_{ch}^{U,D}$ and $\rho_{ch}^{U,D}$ are

\[
\begin{align*}
\xi_{ch}^{U} &= \xi_N V_{CKM}, \\
\xi_{ch}^{D} &= \xi_N V_{CKM}, \\
\rho_{ch}^{U} &= \rho_N V_{CKM}, \\
\rho_{ch}^{D} &= \rho_N V_{CKM},
\end{align*}
\]

and

\[
\begin{align*}
\xi_{N}^{U(D)} &= (V_{R(L)}^{U(D)})^{-1} \xi_{N}^{U(D)} V_{L(R)}^{U(D)}, \\
\rho_{N}^{U(D)} &= (V_{R(L)}^{U(D)})^{-1} \rho_{N}^{U(D)} V_{L(R)}^{U(D)}. \\
\end{align*}
\]

(18)

In the 3HDM, there are additional charged Higgs particles, $F^\pm$, and neutral Higgs bosons $h'^0$, $A'^0$ (see [14]). These particles bring new part to EDM of $b$-quark and by taking only couplings $\tilde{\xi}_{N,tt}^{U}, \tilde{\xi}_{N,bb}^{D}$ and $\tilde{\rho}_{N,bb}^{D}$ into account, it can be written as

\[
d = d^{H^\pm} + d^{h'^0} + d^{A'^0} + d^{F^\pm} + d^{h'^0} + d^{A'^0},
\]

(20)

where $d^{F^\pm}$, $d^{h'^0}$ and $d^{A'^0}$ are

\[
d^{F^\pm} = \frac{4 G_F}{\sqrt{2}} \frac{e}{32 \pi^2} \frac{1}{m_t} \tilde{\rho}_{N,tt}^{D} m_t \text{Im}(\tilde{\rho}_{N,bb}^{D}) |V_{tb}|^2 y ((-1 + Q_t (-3 + y') - y') (y' - 1) + 2 (Q_t + y') \ln y')) \frac{(y' - 1)^3}{y'},
\]

\[
d^{h'^0} = -\frac{4 G_F}{\sqrt{2}} \frac{e}{16 \pi^2} \frac{Q_b}{m_b} \text{Im}(\tilde{\rho}_{N,bb}^{D}) \text{Re}(\tilde{\rho}_{N,bb}^{D}) (1 - \frac{r_1' (r_1' - 2)}{\sqrt{r_1' (r_1' - 4)}} \ln \frac{\sqrt{r_1' - r_1'} - \sqrt{r_1'} - 4}{2} - \frac{1}{2} r_1' \ln r_1'),
\]

\[
d^{A'^0} = -d^{h'^0}(r_1' \to r_2'),
\]

(21)

with $r_1' = m_{h'^0}/m_b$, $r_2' = m_{A'^0}/m_b$ and $y' = \frac{m_t^2}{m_{F^\pm}^2}$. The expressions for $d^{H^\pm}$, $d^{h'^0}$ and $d^{A'^0}$ are defined in eq. [11]. Further, by introducing $O(2)$ symmetry in the Higgs sector, the masses of new Higgs bosons $F^\pm$, $h'^0$ and $A'^0$ become the same as the masses of model III Higgs bosons, $H^\pm$, $h^0$ and $A^0$ respectively (see [14]). Therefore, the final result for EDM of $b$-quark in the $3HDM(O_2)$ is

\[
d = \frac{4 G_F}{\sqrt{2}} \frac{e}{32 \pi^2} \frac{1}{m_t} (\xi_{N,tt}^{U} \text{Im}(\tilde{\xi}_{N,bb}^{D}) + \tilde{\xi}_{N,tt}^{D} \text{Im}(\tilde{\rho}_{N,bb}^{D})) |V_{tb}|^2
\]

\[
\frac{y ((-1 + Q_t (-3 + y') - y') (y' - 1) + 2 (Q_t + y') \ln y')) \frac{(y' - 1)^3}{y'} + \frac{2 Q_b}{m_b} (\text{Im}(\tilde{\xi}_{N,bb}^{D}) \text{Re}(\tilde{\xi}_{N,bb}^{D}) + \text{Im}(\tilde{\rho}_{N,bb}^{D}) \text{Re}(\tilde{\rho}_{N,bb}^{D}))}
\]

\[
\left( \frac{r_1' (r_1' - 2)}{\sqrt{r_1' (r_1' - 4)}} \ln \frac{\sqrt{r_1' - r_1'} - \sqrt{r_1'} - 4}{2} + \frac{1}{2} r_1' \ln r_1'
\]

\[
- \frac{r_2' (r_2' - 2)}{\sqrt{r_2' (r_2' - 4)}} \ln \frac{\sqrt{r_2' - r_2'} - \sqrt{r_2'} - 4}{2} - \frac{1}{2} r_2' \ln r_2' \right) \right).}
\]

(22)
New $O(2)$ symmetry also permits us to parametrize the Yukawa matrices $\bar{\xi}^{U(D)}$ and $\bar{\rho}^{U(D)}$ as

$$\bar{\xi}^{U(D)} = \epsilon^{U(D)} \cos \theta,$$

$$\bar{\rho}^{U} = \epsilon^{U} \sin \theta,$$

$$\bar{\rho}^{D} = i \epsilon^{D} \sin \theta,$$ (23)

where $\epsilon^{U(D)}$ are real matrices satisfy the equation

$$(\bar{\xi}^{U(D)})^T + \bar{\xi}^{U(D)} + (\bar{\rho}^{U(D)})^T + \bar{\rho}^{U(D)} = (\epsilon^{U(D)})^T \epsilon^{U(D)}$$ (24)

Here $T$ denotes transpose operation. In eq. (23), we take $\bar{\rho}^D$ complex to carry all CP violating effects on the third Higgs scalar. Using the parametrization in eq. (23), it is easy to see that only charged Higgs bosons contribute to EDM of $b$-quark but not the neutral Higgs ones, since $\bar{\rho}^{N,bb}_D$ is a pure imaginary number. Finally, $b$-quark EDM reads as

$$d = \frac{4G_F}{\sqrt{2}} \frac{e}{32\pi^2 m_t} |V_{tb}|^2 \epsilon^{U}_{N,tt} \epsilon^{D}_{N,bb} \sin^2 \theta \frac{y((-1 + Q_t (-3 + y) - y)(y - 1) + 2 (Q_t + y) \ln y)}{(y - 1)^3}$$ (25)

3 Discussion

In this section, we study dependencies of $b$-quark EDM on the masses of charged and neutral Higgs bosons and the CP violating parameter $\theta$. In the analysis, we use the CLEO measurement (26)

$$Br(B \to X_s \gamma) = (3.15 \pm 0.35 \pm 0.32) \times 10^{-4}.$$ (26)

to find a constraint region for our free parameters. The procedure is to restrict the Wilson coefficient $C_7^{\text{eff}}$, which is the effective coefficient of the operator $O_7 = \frac{e}{16\sqrt{2} s_w s_t} \sigma_{\mu\nu}(m_b R + m_s L) b_{\alpha} F^{\mu\nu}$ (see [18] and references therein), in the region $0.257 \leq |C_7^{\text{eff}}| \leq 0.439$. Here upper and lower limits were calculated using the CLEO measurement and all possible uncertainties in the calculation of $C_7^{\text{eff}}$ [18]. This restriction allows us to find a region for the parameters $\bar{\xi}^{U}_{N,tt}$, $\bar{\xi}^{D}_{N,bb}$, $\theta$ in the model III and $\bar{\xi}^{U}_{N,tt}$, $\bar{\xi}^{D}_{N,bb}$, $\theta$ in the general $3HDM(O_2)$. In our numerical calculations, we also respect the constraint for the angle $\theta$ due to the experimental upper limit of neutron electric dipole moment, $d_n < 10^{-25} e\cdot cm$, which leads to $\frac{1}{m_{cm}} \text{Im}(\xi_{N,tt}^{U} \xi_{N,bb}^{D}) < 1.0$ for $M_{H^\pm} \approx 200 \text{ GeV}$ [19] in the model III and $\frac{1}{m_{cm}} (\bar{\xi}_{N,tt}^{U} \xi_{N,bb}^{D}) \sin^2 \theta < 1.0$ in $3HDM(O_2)$. Further, we take only $\xi_{N,tt}^{U}$ and $\xi_{N,bb}^{D}$ and $\bar{\xi}_{N,tt}^{U}$ and $\bar{\xi}_{N,bb}^{D}$ nonzero in the model III $(3HDM(O_2))$ and neglect all other couplings. Note that $h^0$ is assumed as the highest Higgs boson in our calculations.
In Fig. 3 we plot EDM "d" with respect to the ratio $R_{\text{neutr}} = \frac{m_{h^0}}{m_{A^0}}$ for $\sin \theta = 0.5$, $m_{A^0} = 80 \text{ GeV}$, $m_{H^\pm} = 400 \text{ GeV}$, $\xi_{N,bb}^D = 40 m_b$ and $|r_{tb}| = |\xi_{N,tt}^U| < 1$ in the model III. Here "d" lies in the region bounded by solid lines for $C_{7}^{eff} > 0$ and by dashed lines for $C_{7}^{eff} < 0$. It is observed that $b$-quark EDM is strongly sensitive to the ratio $R_{\text{neutr}}$ and this dependence increases with decreasing masses of neutral Higgs bosons. If the ratio $R_{\text{neutr}}$ becomes small the considerable enhancement of $d$ will be obtained and therefore the mass difference of $h^0$ and $A^0$ should not be large. In the case of degenerate masses of $h^0$ and $A^0$, the contribution of the neutral Higgs sector to $b$-quark EDM vanishes.

Fig. 4 (5) is devoted to $\sin \theta$ dependence of "d" for $m_{h^0} = 70 \text{ GeV}$, $m_{A^0} = 80 \text{ GeV}$ ($m_{h^0} = m_{A^0} = 80 \text{ GeV}$), $m_{H^\pm} = 400 \text{ GeV}$, $\xi_{N,bb}^D = 40 m_b$ and $|r_{tb}| < 1$. If the value of $\sin \theta$ decreases, "d" becomes small as expected and the restricted region becomes narrower, for both $C_{7}^{eff} > 0$ and $C_{7}^{eff} < 0$. "d" is positive for $C_{7}^{eff} > 0$, however, for $C_{7}^{eff} < 0$, it can also take negative values when $m_{h^0}$ reaches to $m_{A^0}$. This is an interesting result which can be used in the determination of the sign of $C_{7}^{eff}$.

In Fig. 6, we plot "d" with respect to the charged Higgs mass $m_{H^\pm}$ for $\sin \theta = 0.5$, $m_{h^0} = 70 \text{ GeV}$, $m_{A^0} = 80 \text{ GeV}$, $\xi_{N,bb}^D = 40 m_b$ and $|r_{tb}| < 1$. This figure shows that "d" is weakly sensitive to the charged Higgs mass $m_{H^\pm}$ for $m_{H^\pm} \geq 400 \text{ GeV}$, especially in the case $C_{7}^{eff} < 0$. The restriction region becomes narrower with increasing $m_{H^\pm}$.

In $3HDM(O_2)$, the possible choice of the parametrization for Yukawa couplings (eq. (23)) causes to vanish the contribution of the neutral Higgs sector to the $b$-quark EDM (see eq. (25)) and therefore only the charged Higgs sector contributes.

Fig. 7 is devoted to $\sin \theta$ dependence of "d" for $m_{H^\pm} = 400 \text{ GeV}$, $\xi_{N,bb}^D = 40 m_b$ and $|r_{tb}| < 1$ in $3HDM(O_2)$. The behaviour of EDM is similar to the model III case with degenerate masses $m_{h^0}$ and $m_{A^0}$, however, "d" is more sensitive to $\sin \theta$, since it is proportional to $\sin^2 \theta$ (see eq. (23)). Further, "d" is not sensitive to the charged Higgs mass $m_{H^\pm}$ for $m_{H^\pm} \geq 400 \text{ GeV}$, especially in the case $C_{7}^{eff} < 0$, similar to the model III. (Fig. 8)

Now we would like to summarize the main points of our results:

- It is interesting that EDM is generated by the one loop diagrams. This is due to the freedom to choose the Yukawa couplings as complex numbers in the models under consideration. Further, even in the model with two Higgs doublets, there exist a non-vanishing contribution to EDM due to the charged sector.

- Since $b$-quark EDM "d" is strongly sensitive to the ratio $R_{\text{neutr}}$, in model III, the mass difference of $h^0$ and $A^0$ should not be large. For the 3HDM under consideration, there is
no need to restrict the neutral Higgs masses because they do not contribute to "d" for
the given parametrization of Yukawa couplings.

• If "d" is positive, $C_{7}^{eff}$ can have both signs. However, if it is negative, $C_{7}^{eff}$ must be
negative for both models. This is an important observation which is useful in the deter-
mination of the sign of $C_{7}^{eff}$.

• "d" is not sensitive to the mass of charged Higgs boson for its large values in both models.

• EDM of "b" quark is at the order of $\sim 10^{-20} e\,cm$ in both model III and $3HDM(O_{2})$
and its magnitude is larger compared to the results ($\sim 10^{-23} - 10^{-22} e\,cm$) existing in the
literature. The neutral Higgs boson effects are strong in the model III and there is an
enhancement, even one order ($d \sim (10^{-19}$), if the masses of neutral Higgs bosons, $m_{h^{0}}$
and $m_{A^{0}}$, are far from degeneracy.

Therefore, the experimental investigation of the b-quark EDM gives powerful informations
about the physics beyond the SM.

References

[1] K. F. Smith et.al, *Phys. Lett.* B234 (1990) 191, 2885; I. S. Altarev et. al, *Phys. Lett.* B276
(1992) 242.

[2] K. Abdullah et,al. *Phys. Rev.Lett.* 65 (1990) 2347.

[3] J. Bailey et al, *Journ. Phys.* G4 (1978) 345;

[4] E. P. Sahabalin, *Sov. J. Nucl. Phys.* 28 (1978) 75.

[5] J. F. Donoghue, *Phy. Rev.* D18 (1978) 1632.

[6] A. Czarnecki and B. Krause, *Phys. Rev. Lett.* 78 (1997) 4339.

[7] S, Weinberg, *Phys. Rev. Lett.* 58 (1976) 657; N. G. Deshpande and E. Ma, *Phys. Rev.* D
16 (1977) 1583; for a recent review, see H. Y. Cheng, *Int. J. Mod. Phys.* A 7 (1992) 1059.

[8] G. C. Branco and M. N. Rebelo,*Phys. Rev. Lett.* B 160 (1985) 117; J. Liu and L. Wolfen-
stein, *Nucl. Phys.*.B 289 (1987) 1.

[9] C. H. Albright, J. Smith and S. H. H. Tye, *Phys. Rev. D.* D 21 (1980) 711.
[10] Atwood. et. al., *Phys. Rev.* **D 51** (1995) 1034.

[11] Soni and Xu, *Phys. Rev. Letter* **69** (1992) 33.

[12] Y. Liao and X. Li, *Phys. Rev. D* **60** (1999) 073004.

[13] D. G. Dumm and G. A. G Sprinberg,, *Eur-Phys. J. C* **11** (1999) 293.

[14] E. Iltan *Phys. Rev. D* **61** (2000) 054001.

[15] D. Atwood, L. Reina and A. Soni, *Phys. Rev. D* **53** (1996) 119.

[16] E. Iltan, *Phys. Rev. D* **60** (1999) 034023.

[17] M. S. Alam Collaboration, to appear in ICHEP98 Conference (1998)

[18] T. M. Aliev, E. Iltan, *J. Phys. G**25**(1999) 989.

[19] D. B. Chao, K. Cheung and W. Y. Keung, *Phys. Rev. 59* (1999) 115006.
Figure 1: One loop diagrams contribute to EDM of $b$-quark due to $H^\pm$ in the 2HDM. Wavy lines represent the electromagnetic field and dashed lines the $H^\pm$ field.
Figure 2: The same as Fig. [1] but for neutral Higgs bosons $h^0$ and $A^0$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Diagram showing the process $\gamma \to b b \to d, s, b, A, h^0, A^0, h^0, \gamma$.}
\end{figure}
Figure 3: $b$-quark EDM "$d$" as a function of the ratio $R_{\text{neutr}} = \frac{m_{\nu \theta}}{m_{\theta \nu}}$, for $m_{H^\pm} = 400 GeV$, $sin \theta = 0.5$, $\xi_{N \bar{b}b}^D = 40 m_b$ and $|r_{tb}| = |\frac{\xi_{N \bar{b}b}^U}{\xi_{N \bar{b}b}^D}| < 1$, in the model III. Here $d$ is restricted in the region bounded by solid lines for $C_7^{\text{eff}} > 0$ and by dashed lines for $C_7^{\text{eff}} < 0$. 
Figure 4: $b$-quark EDM "d" as a function of $\sin \theta$ for $m_H^\pm = 400 \text{ GeV}$, $m_{h^0} = 70 \text{ GeV}$, $m_{A^0} = 80 \text{ GeV}$, $\xi_{N,bb} = 40 \text{ m}_b$ and $|r_{tb}| < 1$, in the model III. Here $d$ is restricted in the region bounded by solid lines for $C_{7}^{eff} > 0$ and by dashed lines for $C_{7}^{eff} < 0$.

Figure 5: The same as Fig.4 but for $m_{h^0} = m_{A^0} = 80 \text{ GeV}$. 

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Figure 6: $b$-quark EDM "$d$" as a function of $m_H^\pm$, for $\sin \theta = 0.5$, $m_{h^0} = 70 \text{ GeV}$, $m_{A^0} = 80 \text{ GeV}$, $\xi^D_{N,bb} = 40 \text{ m}_b$ and $|r_{tb}| < 1$, in the model III. Here $d$ is restricted in the region bounded by solid lines for $C_7^{\text{eff}} > 0$ and by dashed lines for $C_7^{\text{eff}} < 0$.

Figure 7: The same as Fig. 6, but in $3HDM(O_2)$. 
Figure 8: The same as Fig. 6, but in $3HDM(O_2)$. 