$Z_3$ orbifolds of the $SO(32)$ heterotic string: 1-Wilson-line embeddings

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Abstract

We consider compactification of the $SO(32)$ heterotic string on a 6-dimensional $Z_3$ orbifold with one discrete Wilson line. A complete set of all possible embeddings is given, 159 in all. The unbroken subgroups of $SO(32)$ are tabulated. The extended gauge symmetry $SU(3)^3$, recently discussed by J. E. Kim [hep-th/0301177] for semi-realistic $E_8 \times E_8$ heterotic string models, occurs for several embeddings, as well as other groups that may be of interest in unified string models. The extent to which extra gauge group factors can be hidden is discussed and compared to the $E_8 \times E_8$ case. Along flat directions where an effective hidden sector exists, the embeddings described here provide for hidden gauge groups that are not possible in the $E_8 \times E_8$ heterotic string.

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1 Introduction

One of the main motivations for starting with the $E_8 \times E_8$ heterotic string in semi-realistic applications is that the second $E_8$ factor would appear to provide a natural source of a hidden sector in which to break supersymmetry by, say, gaugino condensation. Indeed, in a field theoretic dimensional reduction of the 10-dimensional theory to 4 dimensions, one finds only Planck mass suppressed operators communicating between the sets of gauge-charged fields coming from the two $E_8$’s.

However, when the underlying string theory is formulated with 6 dimensions compactified on an orbifold, quantum consistency of the 2-dimensional conformal field theory necessitates the addition of twisted sector states. These are states that would not be present in a field theoretic dimensional reduction of the original 10-dimensional theory. Quite generally, these twisted states are simultaneously charged under subgroups of both $E_8$ factors. Thus these states can mediate supersymmetry breaking through gauge interactions and the so-called hidden sector is no longer hidden. Typically one overcomes this difficulty by breaking the gauge interactions (most often extra $U(1)$’s and $SU(2)$’s) at or near the string scale so that their interactions are more or less Planck scale suppressed. This requires a careful choice of flat direction; for examples see [6].

Against this backdrop it is worth reconsidering the aversion to the $SO(32)$ heterotic string, since: (i) with an appropriate orbifold embedding chiral representations occur, and (ii) extra gauge symmetries can be hidden the same way that they are hidden in $E_8 \times E_8$ heterotic orbifolds. It will be seen from the gauge groups obtained below that most embeddings give rise to a product of nonabelian simple groups and some $U(1)$’s. Generically representations will arise that “feel” any pair of factors. However, if we can choose a flat direction that renders these states supermassive (say, near the string scale), then it would seem that we can manufacture an effectively hidden sector that is comparable to what may be obtained from a 4-dimensional $E_8 \times E_8$ heterotic string construction via orbifold compactification. Indeed, it is generic in such constructions that many states do decouple near the string scale, due to the presence of an anomalous $U(1)$, since the anomaly is cancelled by a counterterm that induces a Fayet-Iliopoulos term. The point is that in either case, $E_8 \times E_8$ or $SO(32)$, there exist difficulties hiding the hidden sector and specific assumptions regarding flat directions must be made.

It should be remarked however, that we may not want to hide extra gauge groups. Rather, one may have in mind gauge mediation scenarios (see [9] for a review, and the extensive references therein) with exotic messenger states that feel both the strong dynamics of the sector that breaks supersymmetry and the ordinary gauge symmetries of the observable world. In this case the larger messenger representations that are possible in the $SO(32)$ string could be advantageous.\footnote{In the $E_8 \times E_8$ heterotic string only twisted states can “talk” between the two $E_8$’s. The twisted states must have smaller $E_8 \times E_8$ root torus winding numbers and hence tend to be in smaller representations of the gauge group.}

In either event, model builders might wish to enlarge their vistas by considering orbifolds of the $SO(32)$ heterotic string. In this regard it is useful to have some idea for a good starting
point. Selecting the string scale gauge symmetry is certainly an important first step, and it is therefore worth knowing what is possible and how to get it. With this goal in mind, in the present article we enumerate all inequivalent embeddings with 1 discrete Wilson line for the 6-dimensional $Z_3$ orbifold of the $SO(32)$ heterotic string. Thus we perform the analogue of previous calculations in the $E_8 \times E_8$ heterotic string; most notably those in [10, 11, 12].

Models that contain unbroken gauge groups such as $SU(5)^2$ are starting points for higher affine level models. It has been described in [13] for instance how the presence of $(5, \bar{5}) + (\bar{5}, 5)$ representations can be used to break to the diagonal subgroup, which is realized at level 2, with the requisite Higgses to break further to the Standard Model gauge group. However, the authors of [13] have also concluded that for the $Z_3$ orbifolds it is not possible to get a chiral spectrum for the resulting level $k = 2$ $SU(5)$ GUT. They argue that this is because the net number of fermion generations vanishes in constructions with the requisite $(5, \bar{5}) + (\bar{5}, 5)$ representations. After some elementary calculations, we have reached the same conclusion. Therefore, although some embeddings here do have $SU(5)^2$ and the requisite representations to obtain a level $k = 2$ diagonal subgroup with adjoint Higgses, they are not viable routes to semi-realistic string GUTs, and one should look elsewhere.

Another sort of “unified” model has recently been studied by J. E. Kim for the $E_8 \times E_8$ heterotic $Z_3$ orbifold [17]. This is the group $SU(3)^3$. The advantages of this approach over standard-like constructions are notable (see [17] for further details). We would like to point out that several of the embeddings listed here also give $SU(3)^3$ in the surviving group; we leave as a topic for further investigation the phenomenology of these models, which should share in the advantages noted in [17].

In Section 2 we describe the embeddings of the orbifold action into the sixteen internal bosons responsible for gauge symmetry in the low-energy theory. In Section 3 we discuss equivalence relations that are used to reduce the number of embeddings. In Section 4 we summarize the key elements used in constructing all consistent embeddings. In Section 5 we address the various possible Wilson lines and our results. In Section 6 we state our conclusions. In Appendix A we discuss an important set of equivalence relations. In Appendix B we present tables of the 159 embeddings found here.

2 Abelian $Z_3$ embeddings

The sixteen internal left-moving bosons $X^I(\sigma_+)$—the gauge degrees of freedom—are compactified on the spin(32)/$Z_2$ lattice, which we will denote by $\Lambda$. This lattice consists of all 16-vectors of the form

$$(n_1, \ldots, n_{16}), \quad (n_1 + \frac{1}{2}, \ldots, n_{16} + \frac{1}{2}),$$

subject to the constraints $n_I \in \mathbb{Z}$ and $\sum_I n_I = 0 \mod 2$. We remind the reader that spin(32) is the covering group for $SO(32)$. The $SO(32)$ roots are

$$(\pm 1, \pm 1, 0^{14}).$$

2 For other works on how higher affine level models can be constructed and how they might afford semi-realistic string GUTs, see Refs. [14, 15, 16].
Here, signs are not correlated. Underlining indicates all permutations are to be taken. The “exponent” indicates that the entry is repeated 14 times. Analogous notations will be used below.

The $Z_3$ orbifold is obtained as the quotient of the 6-dimensional $SU(3)^3$ root torus by simultaneous $2\pi/3$ rotations in each of the three complex planes labeled by complex coordinates $z^i (i = 1, 3, 5)$. This twist in the 6-dimensional compact space is embedded into internal gauge degrees of freedom in an abelian manner—through a shift:

$$z^i \rightarrow e^{2\pi i/3}z^i \Rightarrow X^I(\sigma_+) \rightarrow X^I(\sigma_+) + \pi V^I.$$  

In addition we allow for the possibility of discrete Wilson lines $a_i (i = 1, 3, 5)$ which embed translations in the 6-dimensional compact space. It suffices to specify this embedding for three such shifts, due to constraints arising from the torus construction:

$$z^i \rightarrow z^i + 1 \Rightarrow X^I(\sigma_+) \rightarrow X^I(\sigma_+) + \pi a_i \quad \forall \ i = 1, 3, 5.$$  

The four embedding vectors are subject to consistency relations that follow from world-sheet modular invariance:

$$3V, 3a_i \in \Lambda, \quad 3V^2 \in \mathbb{Z}, \quad 3a_i^2 \in \mathbb{Z}, \quad 3V \cdot a_i \in \mathbb{Z}, \quad 3a_i \cdot a_j \in \mathbb{Z}.$$  

An infinite number of solutions to these conditions exist. Fortunately only a finite number of inequivalent possibilities are contained in this set, due to equivalence relations that we discuss in Section 3.

The massless gauge-charged states are characterized by 16-dimensional winding vectors. In the untwisted sector we have for states with nontrivial weights with respect to the Cartan subalgebra:

$$K \in \Lambda, \quad K^2 = 2.$$  

The Wilson lines enforce a projection on these states. Only those that satisfy

$$a_i \cdot K \in \mathbb{Z} \quad \forall \ i = 1, 3, 5$$

survive. Those that do survive fall into three categories:

$$3V \cdot K = \begin{cases} 0 \mod 3 & \text{gauge} \\ 1 \mod 3 & \text{matter} \\ -1 \mod 3 & \text{antimatter} \end{cases}$$  

In truth this is a further projection onto states with differing right-moving quantum numbers.

For the twisted states, corresponding to string states with nontrivial monodromy, we have weights $\tilde{K}$ which satisfy

$$\tilde{K}^2 = \frac{4}{3} - 2N_L, \quad \tilde{K} = K + V + \sum_{i=1,3,5} n_i a_i, \quad K \in \Lambda.$$  

If left-moving oscillators are excited in the 6-dimensional compact space, we can have $N_L = 1/3$ or $2/3$. (However, $N_L = 2/3$ only has a solution to (2.9) if the embedding is equivalent to the trivial one; i.e., $V = a_i = 0$.) The integers $n_i = 0, \pm 1$ label fixed point locations in each of the 3 complex planes. Each twisted state is labeled by a triple $(n_1, n_3, n_5)$. Note that (2.8) implies $3\tilde{K} \in \Lambda$. 

3
3 Equivalence relations

The equivalence relations are essentially those already alluded to in [3] and discussed
in detail in [11] for $Z_3$ orbifolds of the $E_8 \times E_8$ heterotic string.

Lattice group equivalence. This merely states that $V \rightarrow V + K$ and $a_i \rightarrow a_i + K_i$ yield
equivalent embeddings for any choice $K, K_i \in \Lambda$.

Weyl group equivalence. This states that for any $SO(32)$ root $e$ (given in (2.2)) the
simultaneous Weyl reflection

$$V \rightarrow V - (V \cdot e)e, \quad a_1 \rightarrow a_1 - (a_i \cdot e)e,$$

(3.1)
gives an equivalent embedding. It is easy to check that the Weyl group here consists of
permutation of entries, pair-wise sign flips, and compositions of these operations. This is a
considerable simplification over the $E_8 \times E_8$ case where half-integral roots exist that lead to
a more complicated Weyl group. (The half-integral $SO(32)$ weights are not roots.)

Fixed point label equivalences. First, we have $a_i \rightarrow -a_i$ for any of the Wilson lines.
This is just a relabeling $n_i \rightarrow -n_i$ of the fixed points. Second, we have $V \rightarrow V \pm a_i$
for any choice $i = 1, 3, 5$. This is a relabeling $n_i \rightarrow n_i \mp 1$ of the fixed points. For instance the
twisted sector winding vector is rewritten

$$\tilde{K} = K + V + \sum_j n_j a_j = K + (V \pm a_i) + (n_j \mp 1)a_i + \sum_{j \neq i} n_j a_j$$

(3.2)
to display that this is nothing but a relabeling of fixed points, keeping in mind $n_i \simeq n_i$ mod 3.
The complete twisted sector spectrum will be the same; only the labeling will be different.
It is easy to check that the projections (2.8) in the untwisted sector are unchanged, due
to (2.7).

4 Building blocks

Recall from (2.5) that the embedding vectors $V, a_i$ must satisfy $3V \in \Lambda, 3a_i \in \Lambda$. Taking
into account (2.4), the entries of $3V, 3a_i$ are either all integral or all half-integral. We easily
restrict to integral weight lattice vectors $3V, 3a_i$ using the lattice group equivalence; simply
add $3K$ where $K$ is any half-integral lattice vector. Repeatedly adding $3e_j$’s, where $e_j$’s are
the roots in (2.2), allows us to lower the magnitudes of entries of $3V, 3a_i$ until no entry has
magnitude greater than 2. We can restrict to no more than one $\pm 2$ appearing in $3V, 3a_i$
using the lattice group equivalence: addition of some $3e_j$, where $e_j$ is one of the roots (2.2),
can be used to eliminate any pair of $\pm 2$’s. The self-consistency constraints in (2.5), which
we find it convenient to write

$$(3V)^2 = 0 \bmod 3, \quad (3a_i)^2 = 0 \bmod 3,$$

(4.1)
provide a further restriction, and since $K \in \Lambda$ implies $K^2$ even, we only get even multiples
of 3. Thus,

$$(3V)^2 = 0 \bmod 6, \quad (3a_i)^2 = 0 \bmod 6.$$

(4.2)
Therefore we find that $3V, 3a_i$ can only belong to the set

$$\{(0^{16}), (1^6, 0^{10}), (1^{12}, 0^4), (2, 1^2, 0^{13}), (2, 1^8, 0^7), (2, 1^{14}, 0)\}, \quad (4.3)$$

up to ordering and sign permutations. It follows that these 6 vectors form the basis of all subsequent analysis.

We fix ordering and signs for the twist embeddings using the Weyl group equivalence. Then the inequivalent twist embeddings $3V$ together with their unbroken subgroups of $SO(32)$ and untwisted matter are given in Table 1. We find it convenient in what follows to concentrate on how Wilson lines further break the gauge group in the two distinct parts of the 16-dimensional space, emphasized by the placement of a semicolon in the entries for $3V$ in Table 1. The first subspace, where $3V^I \neq 0$, we will refer to as the nonzero sector. The second subspace, where $3V^I = 0$, we will refer to as the zero sector. These are not to be confused with the usual sectors of the Hilbert space of the underlying conformal field theory.

### Table 1: Inequivalent twist embeddings.

| $3V$                     | $G$                              | Untw. matter                  |
|--------------------------|----------------------------------|-------------------------------|
| $(0^{16})$               | $SO(32)$                         | none                          |
| $(1^6, 0^{10})$          | $SU(6) \times SO(20) \times U(1)$ | $3[[T5, 1] + (6, 20)]$        |
| $(1^{12}, 0^4)$          | $SU(12) \times SO(8) \times U(1)$ | $3[[66, 1] + (12, 8_0)]$      |
| $(-2, 1^2, 0^{13})$      | $SU(3) \times SO(26) \times U(1)$ | $3[[3, 1] + (3, 26)]$        |
| $(-2, 1^8, 0^7)$         | $SU(9) \times SO(14) \times U(1)$ | $3[[36, 1] + (9, 14)]$      |
| $(-2, 1^{14}, 0)$        | $SU(15) \times U(1)^2$          | $3[[105] + 2(15)]$           |

Therefore we find that $3V, 3a_i$ can only belong to the set

$$\{(0^{16}), (1^6, 0^{10}), (1^{12}, 0^4), (2, 1^2, 0^{13}), (2, 1^8, 0^7), (2, 1^{14}, 0)\}, \quad (4.3)$$

5 Wilson line embeddings

Inequivalent Wilson lines $3a_1$ are obtained by taking sign and ordering permutations of the six vectors $3V$ subject to the $V \cdot a_1$ constraint in (2.5), which we find it convenient to write

$$3V \cdot 3a_1 = 0 \mod 3. \quad (5.1)$$

In the case of $V = 0$ we have only the six possibilities listed in Table 4.

For the effects of Wilson lines when $V \neq 0$ it is convenient to focus on the nonzero sector and the zero sector separately. Condition (5.1) constrains the entries of $a_1$ in the nonzero sector, whereas it places no constraint on the entries of $a_1$ in the zero sector. As an example we consider $3V = (1^6, 0^{10})$ in some detail. For the other cases we merely state our results, except as regards some none-too-apparent equivalences.

In the nonzero sector the possibilities that exist for the corresponding 6 entries of $3a_1$ are given in Table 2. We have indicated how the $SU(6)$ factor surviving $V$ is broken by the Wilson line. Equivalence relations have been used to eliminate obvious redundancies. In an
appendix we show that embeddings with \((-2, (-1)^4, 0)\) are equivalent to embeddings with \((-2,1^3, -1, 0)\). This is why we have dropped the former possibility. For the zero sector we have the set

\[
(1^9, -1) \quad \text{and} \quad (1^n, 0^{10-n}) \quad n = 0, 1, \ldots, 10. \tag{5.2}
\]

Note that the sign in the first vector cannot be removed by the pairwise sign flips without disturbing \(V\). In all other cases signs can be removed in the zero sector. In the case where we have for \(a_1\) entries \((0^6)\) in the nonzero sector of \(V\), we must also include the possibilities of

\[
(-2, 1^2, 0^7) \quad \text{and} \quad (-2, 1^8, 0) \tag{5.3}
\]

in the zero sector of \(a_1\) since, lacking \(\pm 1\)'s in the nonzero sector, we cannot push the \(-2\) into the nonzero sector using lattice group equivalence. That is, in the other cases we can eliminate an entry of \(-2\) from the zero sector using the lattice group equivalence \(3a_1 \rightarrow 3a_1 + 3K\) where \(3K = (\pm 3, 0^5, 3, 0^9)\).

It is simple to determine the possibilities that are consistent with a given choice for the nonzero sector, by comparing to the vectors in (4.3) and requiring that the total \(3a_1\) be one of these up to sign and ordering permutations. For example if we choose \((1, -1, 0^4)\) in the nonzero sector then we have only

\[
(1, -1, 0^4) \oplus \{(1^4, 0^6), (1^9, \pm 1)\}, \tag{5.4}
\]

corresponding the second and third vectors in (4.3). These are Embeddings 2.5-2.7 of Table 5. As another example if we choose \((-2, 1^2, (-1)^3)\) in the nonzero sector then we have only

\[
(-2, 1^2, (-1)^3) \oplus \{(1^3, 0^7), (1^9, 0)\}, \tag{5.5}
\]

corresponding to the last two vectors in (4.3). These are Embeddings 2.21 and 2.22 of Table 5.

The possibilities for the zero sector lead to breakings of the \(SO(20)\) that survives \(V\). These are given in Table 6. Of course they are correlated to what occurs in the nonzero sector. Thus we obtain as a complete list of consistent embeddings for \(3V = (1^6; 0^{10})\) the entries of Table 6. We also list the nonabelian part \(G_{NA}\) of the surviving gauge group \(G\). One should add as many \(U(1)'s\) as are needed to have \(G\) a rank 16 group.

Including the cases with \(a_1 = 0\), it can be seen from Tables 4-9 that we have a total of 159 different embeddings when one discrete Wilson line is permitted. Although it is plausible

| \(3(a_1^1, \ldots, a_6^6)\) | SU(6) Subgroup |
|-----------------|----------------|
| \((0^6), (1^5), (-2, 1^5)\) | \(SU(6)\) |
| \((1, -1, 0^4), (1^4, -1, 0), (-2, -1, 0^4), (-2, 1^3, -1, 0)\) | \(SU(4) \times U(1)^2\) |
| \((1^3, 0^3), (1^3, (-1)^3), (-2, 1^2, (-1)^3), (-2, 1^2, 0^3)\) | \(SU(3)^2 \times U(1)^1\) |
| \((1^2, (-1)^2, 0^2), (-2, 1, (-1)^2, 0^2)\) | \(SU(2)^3 \times U(1)^2\) |

Table 2: Entries in the nonzero sector for \(3V = (1^6; 0^{10})\).
Table 3: Entries in the zero sector for $3V = (1^6; 0^{10})$.

| $3(a_7^i, \ldots, a_{16}^i)$ | $SO(20)$ Subgroup |
|-----------------------------|--------------------|
| $(0^{10})$                  | $SO(20)$          |
| $(1, 0^9)$                  | $SO(18) \times U(1)$ |
| $(1^{10-n}, 0^n)$ $n = 2, \ldots, 8$ | $SU(10 - n) \times SO(2n) \times U(1)$ |
| $(1^{9}, 0), (2, (-1)^8, 0)$ | $SU(9) \times U(1)^2$ |
| $(1^9, \pm 1)$              | $SU(10) \times U(1)$ |
| $(-2, 1^2, 0^7)$            | $SU(3) \times SO(14) \times U(1)$ |

some redundancy may yet exist, most of it has been removed using the equivalence relations described above. Certainly the list is complete, meaning that any consistent embedding with one discrete Wilson line is equivalent to one of the embeddings presented here.

6 Conclusions

We have argued that the $SO(32)$ heterotic string provides an interesting starting point for semi-realistic string phenomenology. We have worked out a complete list of all embeddings with one discrete Wilson line for the symmetric $Z_3$ orbifold. To our knowledge this calculation has not been presented previously in the literature. We have addressed most of the equivalences that relate embeddings. Some of the less obvious equivalences have been described in detail in an appendix.

We have commented briefly on the prospects for obtaining a hidden sector and for softly broken supersymmetry via gaugino condensation in this sector. It is our conclusion that there is every reason to believe that some of the models described here will be viable in this respect subject to an appropriate choice of flat direction. We leave explicit explorations of this conjecture for future investigation.

Many avenues for future research present themselves. One interesting possibility has to do with the phenomenology of the $SU(3)^3$ embeddings, along the lines of [17]. Research in this direction is in progress and we hope to report on it shortly. Another issue worth exploration is flat directions that might lead to a level $k = 2$ $SU(5)$ GUT for the embeddings that contain $SU(5)^2$ as a proper subgroup, such as those embeddings with $SU(6)^2$. In this case, an important question is whether or not it is possible to obtain a chiral spectrum with respect to the diagonal subgroup. A final issue we would like to mention is whether or not gauge mediation is viable for any of these models.

It is our hope that we have convinced the reader that the $SO(32)$ heterotic string can provide intriguing unified models. Further research on the phenomenological possibilities when this is taken as the starting point would certainly be a welcome supplement to what is a comparatively sparse examination in the existing literature.
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Appendices

A Technical aspects of embedding equivalences

In this appendix we address some of the more technical details in uncovering the equivalence between different embeddings.

One type of equivalence is particularly important, since it is easy to overlook, but removes a good deal of redundancy. It is used on the nonzero sector embeddings with a $-2$. We show the intermediate step which sketches out the proof of the equivalence:

\[
(-2, 1^{m+2}, (-1)^{3n+m}) \simeq (1^{m+3}, 2, (-1)^{3n+m-1}) \simeq (-2, 1^{3n+m-1}, (-1)^{m+3}). \tag{A.1}
\]

In the first step we add a pair of 3’s using lattice equivalence. In the second step we send $3a_1 \rightarrow -3a_1$. Whatever signs we had in the zero sector are reversed when this is done. In many cases the original signs in the zero sector can be restored by pairwise sign flips. In those cases where this is not true, we can always obtain $(1^p, \pm 1)$ in the nonzero sector, again by pairwise sign flips. But both possibilities are included in our tables, so there is nothing lost by using the equivalence (A.1).

For example, an equivalence of the form (A.1) exists in the embeddings with $3V = (1^6; 0^{10})$. In the nonzero sector we have:

\[
(-2, (-1)^4, 0) \simeq (-2, 1^3, -1, 0). \tag{A.2}
\]

It is for this reason that $(-2, (-1)^4, 0)$ does not appear in our tables.

As a second example, consider the embeddings with $3V = (1^{12}; 0^4)$. In this case when there is a $-2$ present in the nonzero sector part of $3a_1$, we have the range $3V \cdot 3a_1 = -12, -9, \ldots, 9$. However, equivalences of type (A.1) need to be accounted for. It is not too hard to check that:

(i) all cases of $3V \cdot 3a_1 = -12$ are equivalent to one of the cases of $3V \cdot 3a_1 = 6$;

(ii) all cases of $3V \cdot 3a_1 = -9$ are equivalent to one of the cases of $3V \cdot 3a_1 = 3$;

(iii) all cases of $3V \cdot 3a_1 = -6$ are equivalent to one of the cases of $3V \cdot 3a_1 = 0$;

Then we can restrict to $3V \cdot 3a_1 = -3, 0, \ldots, 9$. It is easy to check that the possibilities are those listed in Table 6.
### B Embedding Tables

#### Table 4: $3V = (0^{16})$

| No. | $3a_1$          | $G_{NA}$         |
|-----|-----------------|-----------------|
| 1.1 | $(0^{16})$      | $SO(32)$        |
| 1.2 | $(1^6, 0^{10})$ | $SU(6) \times SO(20)$ |
| 1.3 | $(1^{12}, 0^4)$ | $SU(12) \times SO(8)$ |
| 1.4 | $(-2, 1^2, 0^{13})$ | $SU(3) \times SO(26)$ |
| 1.5 | $(-2, 1^8, 0^7)$ | $SU(9) \times SO(14)$ |
| 1.6 | $(-2, 1^{14}, 0)$ | $SU(15)$ |

#### Table 5: $3V = (1^6; 0^{10})$

| No. | $3a_1$          | $G_{NA}$         |
|-----|-----------------|-----------------|
| 2.1 | $(0^6; 0^{10})$ | $SO(20) \times SU(6)$ |
| 2.2 | $(0^6; 1^6, 0^4)$ | $SO(8) \times SU(6)^2$ |
| 2.3 | $(0^6; -2, 1^2, 0^7)$ | $SO(14) \times SU(6) \times SU(3)$ |
| 2.4 | $(0^6; -2, 1^8, 0)$ | $SU(9) \times SU(6)$ |
| 2.5 | $(1, -1, 0^4; 1^4, 0^6)$ | $SO(12) \times SU(4)^2$ |
| 2.6, 2.7 | $(1, -1, 0^4; 1^9, \pm 1)$ | $SU(10) \times SU(4)$ |
| 2.8 | $(1^2, (-1)^2, 0^2; 1^2, 0^8)$ | $SO(16) \times SU(2)^4$ |
| 2.9 | $(1^2, (-1)^2, 0^2; 1^8, 0^2)$ | $SU(8) \times SU(2)^5$ |
| 2.10 | $(1^3, (-1)^3; 0^{10})$ | $SO(20) \times SU(3)^2$ |
| 2.11 | $(1^3, (-1)^3; 1^6, 0^4)$ | $SU(6) \times SO(8) \times SU(3)^2$ |
| 2.12 | $(1^3, 0^3; 1^3, 0^7)$ | $SO(14) \times SU(3)^3$ |
| 2.13 | $(1^3, 0^3; 1^9, 0)$ | $SU(9) \times SU(3)^2$ |
| 2.14 | $(1^4, -1, 0; 1^0)$ | $SO(18) \times SU(4)$ |
| 2.15 | $(1^4, -1, 0; 1^7, 0^3)$ | $SU(7) \times SU(4)^2$ |
| 2.16 | $(1^6, 0^{10})$ | $SO(20) \times SU(6)$ |
| 2.17 | $(1^6; 1^6, 0^4)$ | $SU(6)^2 \times SO(8)$ |
| 2.18 | $(-2, -1, 0^4; 1, 0^9)$ | $SO(18) \times SU(4)$ |
| 2.19 | $(-2, -1, 0^4; 1^7, 0^3)$ | $SU(7) \times SU(4)^2$ |
Table 5: $3V = (1^6; 0^{10})$ (cont.)

| No.    | $3a_1$                                      | $G_{\text{NA}}$                        |
|--------|---------------------------------------------|----------------------------------------|
| 2.20   | $(-2, 1, (-1)^2, 0^2; 1^5, 0^5)$            | $SO(10) \times SU(5) \times SU(2)^3$  |
| 2.21   | $(-2, 1^2, (-1)^3; 1^3, 0^7)$               | $SO(14) \times SU(3)^3$               |
| 2.22   | $(-2, 1^2, (-1)^3; 1^9, 0)$                 | $SU(9) \times SU(3)^2$                |
| 2.23   | $(-2, 1^2, 0^3; 0^{10})$                    | $SO(20) \times SU(3)^2$               |
| 2.24   | $(-2, 1^2, 0^3; 1^6, 0^4)$                  | $SU(6) \times SO(8) \times SU(3)^2$   |
| 2.25   | $(-2, 1^3, -1, 0; 1^4, 0^6)$                | $SO(12) \times SU(4)^2$               |
| 2.26, 2.27 | $(-2, 1^3, -1, 0; 1^9, \pm 1)$            | $SU(10) \times SU(4)$                 |
| 2.28   | $(-2, 1^5; 1^3, 0^7)$                       | $SO(14) \times SU(6) \times SU(3)$    |
| 2.29   | $(-2, 1^5; 1^9, 0)$                         | $SU(9) \times SU(6)$                  |

Table 6: $3V = (1^{12}; 0^4)$

| No.    | $3a_1$                                      | $G_{\text{NA}}$                        |
|--------|---------------------------------------------|----------------------------------------|
| 3.1    | $(0^{12}; 0^4)$                             | $SU(12) \times SO(8)$                  |
| 3.2    | $(0^{12}; -2, 1^2, 0)$                      | $SU(12) \times SU(3)$                  |
| 3.3, 3.4 | $(1, -1, 0^{10}; 1^3, \pm 1)$             | $SU(10) \times SU(4)$                  |
| 3.5    | $(1^2, (-1)^2; 0^8; 1^2, 0^2)$              | $SU(8) \times SU(2)^5$                 |
| 3.6    | $(1^3, (-1)^3; 0^6; 0^4)$                   | $SU(6) \times SO(8) \times SU(3)^2$   |
| 3.7, 3.8 | $(1^4, (-1)^4; 0^4; 1^3, \pm 1)$         | $SU(4)^4$                              |
| 3.9    | $(1^5; (-1)^5; 0^2; 1^2, 0^2)$              | $SU(5)^2 \times SU(2)^4$               |
| 3.10   | $(1^6; (-1)^6; 0^4)$                        | $SU(6)^2 \times SO(8)$                 |
| 3.11   | $(1^3, 0^9; 1^3, 0)$                        | $SU(9) \times SU(3)^2$                 |
| 3.12   | $(1^4, -1, 0^7; 1, 0^3)$                    | $SU(7) \times SU(4)^2$                 |
| 3.13   | $(1^6; (-1)^3; 0^3; 1^3, 0)$                | $SU(6) \times SU(3)^3$                 |
| 3.14   | $(1^7; (-1)^4, 0; 1, 0^3)$                  | $SU(7) \times SU(4)^2$                 |
| 3.15   | $(1^6, 0^6; 0^4)$                           | $SU(6)^2 \times SO(8)$                 |
| 3.16, 3.17 | $(1^7, -1, 0^4; 1^3, \pm 1)$            | $SU(7) \times SU(4)^2$                 |
| 3.18   | $(1^8; (-1)^2; 0^2; 1^2, 0^2)$              | $SU(8) \times SU(2)^5$                 |
| 3.19   | $(1^9; (-1)^3; 0^4)$                        | $SU(9) \times SO(8) \times SU(3)$     |
| 3.20   | $(1^9, 0^3; 1^3, 0)$                        | $SU(9) \times SU(3)^2$                 |
### Table 6: $3V = (1^{12}; 0^4)$ (cont.)

| No. | $3a_1$ | $G_{\text{NA}}$ |
|-----|--------|-----------------|
| 3.21 | $(1^{10}, -1, 0; 1, 0^3)$ | $SU(10) \times SU(4)$ |
| 3.22 | $(1^{12}; 0^4)$ | $SU(12) \times SO(8)$ |
| 3.23 | $(-2, -1, 0^{10}; 1, 0^3)$ | $SU(10) \times SU(4)$ |
| 3.24 | $(-2, 1^2, (-1)^3, 0^6; 1^3, 0)$ | $SU(6) \times SU(3)^3$ |
| 3.25 | $(-2, 1^3, (-1)^4, 0^4; 1, 0^3)$ | $SU(4)^4$ |
| 3.26 | $(-2, 1^5, (-1)^6; 1^3, 0)$ | $SU(6)^2 \times SU(3)$ |
| 3.27 | $(-2, 1^2, 0^9; 0^4)$ | $SU(9) \times SO(8) \times SU(3)$ |
| 3.28, 3.29 | $(-2, 1^3, -1, 0^7; 1^3, \pm 1)$ | $SU(7) \times SU(4)^2$ |
| 3.30 | $(-2, 1^4, (-1)^2, 0^5; 1^2, 0^2)$ | $SU(5)^2 \times SU(2)^4$ |
| 3.31 | $(-2, 1^5, (-1)^3, 0^3; 0^4)$ | $SU(6) \times SO(8) \times SU(3)^2$ |
| 3.32, 3.33 | $(-2, 1^6, (-1)^4, 0; 1^3, \pm 1)$ | $SU(7) \times SU(4)^2$ |
| 3.34 | $(-2, 1^5, 0^6; 1^3, 0)$ | $SU(6)^2 \times SU(3)$ |
| 3.35 | $(-2, 1^6, -1, 0^4; 1, 0^3)$ | $SU(7) \times SU(4)^2$ |
| 3.36 | $(-2, 1^8, (-1)^3; 1^3, 0)$ | $SU(9) \times SU(3)^2$ |
| 3.37 | $(-2, 1^8, 0^3; 0^4)$ | $SU(9) \times SO(8) \times SU(3)$ |
| 3.38, 3.39 | $(-2, 1^9, -1, 0; 1^3, \pm 1)$ | $SU(10) \times SU(4)$ |
| 3.40 | $(-2, 1^{11}; 1^3, 0)$ | $SU(12) \times SU(3)$ |

### Table 7: $3V = (-2, 1^2; 0^{13})$

| No. | $3a_1$ | $G_{\text{NA}}$ |
|-----|--------|-----------------|
| 4.1 | $(0^3; 0^{13})$ | $SO(26) \times SU(3)$ |
| 4.2 | $(0^3; 1^6, 0^7)$ | $SO(14) \times SU(6) \times SU(3)$ |
| 4.3 | $(0^3; 1^{12}, 0)$ | $SU(12) \times SU(3)$ |
| 4.4 | $(0^3; -2, 1^2, 0^{10})$ | $SO(20) \times SU(3)^2$ |
| 4.5 | $(0^3; -2, 1^8, 0^4)$ | $SU(9) \times SO(8) \times SU(3)$ |
| 4.6 | $(1, -1, 0; 1^4, 0^9)$ | $SO(18) \times SU(4)$ |
| 4.7 | $(1, -1, 0; 1^{10}, 0^3)$ | $SU(10) \times SU(4)$ |
| 4.8 | $(1^3; 1^3, 0^{10})$ | $SO(20) \times SU(3)^2$ |
| 4.9 | $(1^3; 1^9, 0^4)$ | $SU(9) \times SO(8) \times SU(3)$ |
Table 7: $3V = (-2, 1^2; 0^{13})$ (cont.)

| No.    | $3a_1$                          | $G_{NA}$     |
|--------|---------------------------------|--------------|
| 4.10   | $(-2, -1, 0; 1, 0^{12})$        | $SO(24)$     |
| 4.11   | $(-2, -1, 0; 1^7, 0^6)$         | $SO(12) \times SU(7)$ |
| 4.12, 4.13 | $(-2, -1, 0; 1^{12}, \pm 1)$  | $SU(13)$     |
| 4.14   | $(-2, 1^2; 0^{13})$             | $SO(26) \times SU(3)$ |
| 4.15   | $(-2, 1^2; 1^6, 0^7)$           | $SO(14) \times SU(6) \times SU(3)$ |
| 4.16   | $(-2, 1^2; 1^{12}, 0)$          | $SU(12) \times SU(3)$ |

Table 8: $3V = (-2, 1^8; 0^7)$

| No.    | $3a_1$                          | $G_{NA}$     |
|--------|---------------------------------|--------------|
| 5.1    | $(0^9, 0^7)$                    | $SU(9) \times SO(14)$ |
| 5.2    | $(0^9; 1^6, 0)$                 | $SU(9) \times SU(6)$ |
| 5.3    | $(0^9; -2, 1^2, 0^4)$           | $SU(9) \times SO(8) \times SU(3)$ |
| 5.4    | $(1, -1, 0^7; 1^4, 0^3)$        | $SU(7) \times SU(4)^2$ |
| 5.5    | $(1^2, (-1)^2, 0^5, 1^{12}, 0^5)$ | $SO(10) \times SU(5) \times SU(2)^3$ |
| 5.6    | $(1^3, (-1)^3, 0^3; 0^7)$       | $SO(14) \times SU(3)^3$ |
| 5.7    | $(1^3, (-1)^3, 0^3; 1^6, 0)$    | $SU(6) \times SU(3)^3$ |
| 5.8    | $(1^4, (-1)^4, 0; 1^4, 0^3)$    | $SU(4)^4$     |
| 5.9    | $(1^3, 0^6; 1^3, 0^4)$          | $SU(6) \times SO(8) \times SU(3)^2$ |
| 5.10   | $(1^4, -1, 0^4; 1, 0^6)$        | $SO(12) \times SU(4)^2$ |
| 5.11, 5.12 | $(1^4, -1, 0^4; 1^6, \pm 1)$  | $SU(7) \times SU(4)^2$ |
| 5.13   | $(1^5, (-1)^2, 0^2; 1^5, 0^2)$  | $SU(5)^2 \times SU(2)^4$ |
| 5.14   | $(1^6, (-1)^3; 1^3, 0^4)$       | $SU(6) \times SO(8) \times SU(3)^2$ |
| 5.15   | $(1^6, 0^3, 0^7)$               | $SO(14) \times SU(6) \times SU(3)$ |
| 5.16   | $(1^6, 0^3; 1^6, 0)$            | $SU(6)^2 \times SU(3)$ |
| 5.17   | $(1^7, -1, 0; 1^4, 0^3)$        | $SU(7) \times SU(4)^2$ |
| 5.18   | $(1^9, 1^3, 0^4)$               | $SU(9) \times SO(8) \times SU(3)$ |
| 5.19   | $(-2, -1, 0^7; 1, 0^6)$         | $SO(12) \times SU(7)$ |
| 5.20, 5.21 | $(-2, -1, 0^7; 1^6, \pm 1)$   | $SU(7)^2$     |
| 5.22   | $(-2, 1, (-1)^2, 0^5; 1^5, 0^2)$ | $SU(5)^2 \times SU(2)^4$ |
Table 8: $3V = (−2, 1^8; 0^7)$ (cont.)

| No.  | $3a_1$                          | $G_{NA}$          |
|------|---------------------------------|-------------------|
| 5.23 | $(-2, 1^2, (-1)^3, 0^3; 1^3, 0^4)$ | $SO(8) \times SU(3)^4$ |
| 5.24 | $(-2, 1^3, (-1)^4; 0; 1, 0^6)$ | $SO(12) \times SU(4)^2$ |
| 5.25, 5.26 | $(-2, 1^3, (-1)^4; 0; 1^6, ±1)$ | $SU(7) \times SU(4)^2$ |
| 5.27 | $(-2, 1^2, 0^6; 0^7)$ | $SO(14) \times SU(6) \times SU(3)$ |
| 5.28 | $(-2, 1^2, 0^6; 1^6, 0)$ | $SU(6)^2 \times SU(3)$ |
| 5.29 | $(-2, 1^3, -1, 0^4; 1^4, 0^3)$ | $SU(4)^4$ |
| 5.30 | $(-2, 1^4, (-1)^2, 0^2; 1^2, 0^5)$ | $SO(10) \times SU(5) \times SU(2)^3$ |
| 5.31 | $(-2, 1^5, (-1)^3; 0^7)$ | $SO(14) \times SU(6) \times SU(3)$ |
| 5.32 | $(-2, 1^5, (-1)^3; 1^6, 0)$ | $SU(6)^2 \times SU(3)$ |
| 5.33 | $(-2, 1^5, 0^3; 1^3, 0^4)$ | $SU(6) \times SO(8) \times SU(3)^2$ |
| 5.34 | $(-2, 1^6, -1, 0; 1, 0^6)$ | $SO(12) \times SU(7)$ |
| 5.35, 5.36 | $(-2, 1^6, -1, 0; 1^6, ±1)$ | $SU(7)^2$ |
| 5.37 | $(-2, 1^{14}; 0^7)$ | $SU(9) \times SO(14)$ |
| 5.38 | $(-2, 1^8; 1^6, 0)$ | $SU(9) \times SU(6)$ |

Table 9: $3V = (-2, 1^{14}, 0)$

| No.  | $3a_1$                          | $G_{NA}$          |
|------|---------------------------------|-------------------|
| 6.1  | $(0^{15}; 0)$ | $SU(15)$ |
| 6.2  | $(1^3, (-1)^3, 0^9; 0)$ | $SU(9) \times SU(3)^2$ |
| 6.3  | $(1^6, (-1)^6, 0^3; 0)$ | $SU(6)^2 \times SU(3)$ |
| 6.4, 6.5 | $(1^4, -1, 0^{10}; ±1)$ | $SU(10) \times SU(4)$ |
| 6.6, 6.7 | $(1^7, (-1)^4, 0^4; ±1)$ | $SU(7) \times SU(4)^2$ |
| 6.8  | $(1^6, 0^9; 0)$ | $SU(9) \times SU(6)$ |
| 6.9  | $(1^9, (-1)^3, 0^3; 0)$ | $SU(9) \times SU(3)^2$ |
| 6.10, 6.11 | $(1^{10}, -1, 0^4; ±1)$ | $SU(10) \times SU(4)$ |
| 6.12 | $(1^{12}, 0^3; 0)$ | $SU(12) \times SU(3)$ |
| 6.13, 6.14 | $(-2, -1, 0^{13}; ±1)$ | $SU(13)$ |
| 6.15, 6.16 | $(-2, 1^3, (-1)^4, 0^7; ±1)$ | $SU(7) \times SU(4)^2$ |
| 6.17, 6.18 | $(-2, 1^6, (-1)^7, 0; ±1)$ | $SU(7)^2$ |
Table 9: \(3V = (-2, 1^{14}, 0)\) (cont.)

| No.     | \(3a_1\)                  | \(G_{\text{NA}}\)         |
|---------|----------------------------|----------------------------|
| 6.19    | \((-2, 1^2, 0^{12}; 0)\)   | \(SU(12) \times SU(3)\)   |
| 6.20    | \((-2, 1^5, (-1)^3, 0^6; 0)\) | \(SU(6)^2 \times SU(3)\)   |
| 6.21    | \((-2, 1^8, (-1)^6; 0)\)   | \(SU(9) \times SU(6)\)    |
| 6.22, 6.23 | \((-2, 1^6, -1, 0^7; \pm 1)\) | \(SU(7)^2\)                |
| 6.24, 6.25 | \((-2, 1^9, (-1)^4, 0; \pm 1)\) | \(SU(10) \times SU(4)\)   |
| 6.26    | \((-2, 1^8, 0^6; 0)\)      | \(SU(9) \times SU(6)\)    |
| 6.27    | \((-2, 1^{11}, (-1)^3; 0)\) | \(SU(12) \times SU(3)\)   |
| 6.28, 6.29 | \((-2, 1^{12}, -1, 0; \pm 1)\) | \(SU(13)\)                |
| 6.30    | \((-2, 1^{14}; 0)\)        | \(SU(15)\)                |
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