Spacetime Singularities vs. Topologies in the Zeeman-Göbel Class

Kyriakos Papadopoulos\textsuperscript{1}, Basil K. Papadopoulos\textsuperscript{2}

\textsuperscript{1} Department of Mathematics, Kuwait University, PO Box 5969, Safat 13060, Kuwait
\textsuperscript{2} Department of Civil Engineering, Democritus University of Thrace, Greece

Abstract

In this article we first observe that the Path topology of Hawking, King and McCarthy is an analogue, in curved spacetimes, of a topology that was suggested by Zeeman as an alternative topology to his so-called Fine topology in Minkowski spacetime. We then review a result of a recent paper on spaces of paths and the Path topology, and see that there are at least five more topologies in the class $Z^{-}\mathcal{G}$ of Zeeman-Göbel topologies which admit a countable basis, incorporate the causal and conformal structures, but the Limit Curve Theorem fails to hold. The ”problem” that L.C.T. does not hold can be resolved by ”adding back” the light-cones in the basic-open sets of these topologies, and create new basic open sets for new topologies. But, the main question is: do we really need the L.C.T. to hold, and why? Why is the manifold topology, under which the group of homeomorphisms of a spacetime is vast and of no physical significance (Zeeman), more preferable from an appropriate topology in the class $Z^{-}\mathcal{G}$ under which a homeomorphism is an isometry (Göbel)? Since topological conditions that come as a result of a causality requirement are key in the existence of singularities in general relativity, the global topological conditions that one will supply the spacetime manifold might play an important role in describing the transition from the quantum non-local theory to a classical local theory.
1 Motivation.

In paper [10] the authors talked about the causal structure of the “ambient boundary”, which is the conformal infinity of a five dimensional “ambient space”. The whole deal of this construction was to put relativity theory into a wider frame, in order to understand better the nature of the spacetime singularities. Actually, there is an unexplored ocean in the literature of singularities: do they belong to the spacetime or not? Or, better, are there singularities which do not belong to the spacetime and others which do belong to? Of course, as soon as there is spacetime, there are events and, for every event, there is a null cone and a Lorentzian metric. Having this in mind, Zeeman, Göbel and Hawking-King-McCarthy built natural topologies, which incorporate the causal, differential and conformal structures, against the manifold topology which is alien to the most fundamental properties of spacetime, for example the structure of the null cone. From a mathematical point of view, a spacetime is not a complete model, if it is not equipped with an appropriate “natural” topology. By natural we mean a topology compatible with its most fundamental structures. Having in mind that the manifold topology is not natural, one can easily ask the question: why do we use such a topology to prove the validity of theorems like the Limit Curve Theorem, while other, much more natural topologies, are ignored.

2 Preliminaries.

Throughout the text, unless otherwise stated, by $M$ we will denote any spacetime manifold and not, restrictively, the Minkowski space. In [7] the null-cones where viewed in a topological context, so when one constructs a topology in a spacetime incorporating its causal structure, one will consider the time-cone of an event (interior of the null-cone), the light-cone (the boundary), the causal-cone (interior union boundary) and the complement of the causal-cone (exterior); Zeeman calls this exterior “space-cone” and he proposes (in [7]) an alternative topology based on such space-cones, in its construction. Since the results of Zeeman have been proven to extend to any curved spacetime by Göbel (see [8]), the construction of topologies which incorporate the causal structure is independent on whether the null-cones are affected by the curvature of the spacetime. As soon as there is spacetime there are events and as soon there is an event there is a null-cone; it is its interior, boundary and exterior
which will play a role in a topological construction, and not the linear structure of a spacetime like the Minkowski space or the curvature of a spacetime manifold.

2.1 Causality.

In spacetime geometry, one can introduce three causal relations, namely, the chronological order \( \ll \), the causal order \( \prec \) and the relation horismos \( \rightarrow \), and these can be meaningful extended to any event-set, a set \((X, \ll, \prec, \rightarrow)\) equipped with all three of these orders having no metric \([1, 2]\). In this context we say that the event \( x \) chronologically precedes an event \( y \), written \( x \ll y \) if \( y \) lies inside the future null cone of \( x \), \( x \) causally precedes \( y \), \( x \prec y \), if \( y \) lies inside or on the future null cone of \( x \) and \( x \) is at horismos with \( y \), written \( x \rightarrow y \), if \( y \) lies on the future null cone of \( x \). The chronological order is irreflexive, while the causal order and horismos are reflexive. Then, the notations \( I^+(x) = \{ y \in M : x \ll y \} \), \( J^+(x) = \{ y \in M : x \prec y \} \) will be used for the chronological and the causal futures of \( x \) respectively (and with a minus instead of a plus sign for the pasts), while the future null cone of \( x \) will be denoted by \( N^+(x) \equiv \partial J^+(x) = \{ y \in M : x \rightarrow y \} \), and dually for the null past of \( x \), cf. \([2]\).

The above definitions of futures and pasts of a set can be trivially extended to the situation of any partially ordered set \((X, \prec)\). In a purely topological context this is usually done by passing to the so-called upper (i.e. future) and lower (i.e. past) sets which in turn lead to the future and past topologies (see \([3]\), for the special case of a lattice; here our topologies are constructed in a similar manner, but are weaker, since they do not depend on lattice orders, but on weaker relations). A subset \( A \subset X \) is a past set if \( A = I^-(A) \) and dually for the future. Then, the future topology \( T^+ \) is generated by the subbase \( S^+ = \{ X \setminus I^- (x) : x \in X \} \) and the past topology \( T^- \) by \( S^- = \{ X \setminus I^+ (x) : x \in X \} \). The interval topology \( T_{in} \) on \( X \) then consists of basic sets which are finite intersections of subbasic sets of the past and the future topologies. This is in fact the topology that fully characterises a given order of the poset \( X \). Here we clarify that the names “future topology” and “past topology” are the best possible inspirations for names that came in the mind of the authors, but are new to the literature. The motivation was that they are generated by complements of past and future sets, respectively (i.e. closures of future and past sets, respectively). Also, the authors wish to highlight the distinction between the interval topology \( T_{in} \) which appears to be of an important significance in lattice theory, from the “interval topology” of A.P. Alexandrov
(see [2], page 29). \( \mathcal{T}_m \) is of a more general nature, and it can be defined via any relation, while the Alexandrov topology is restricted to the chronological order. These two topologies are different in nature, as well as in definition, so we propose the use of “interval topology” for \( \mathcal{T}_m \) exclusively, and not for the Alexandrov topology. It is worth mentioning that the Alexandrov topology being Hausforff is equivalent to the Alexandrov topology being equal to the manifold topology which is equivalent to the spacetime being strongly causal; the topologies that we mention is this paper are not equal to the manifold topology.

The so-called orderability problem is concerned with the conditions under which the topology \( \mathcal{T}_< \) induced by the order \( < \) is equal to some given topology \( T \) on \( X \) ([3], [4], [16] and [17]). In [15] and [14] the authors found specific solutions of the orderability problem for six distinct topologies in the class of Zeeman-Göbel. In these article, we consider interval topologies which, together with the manifold topology, produce intersection topologies that belong to the class of Zeeman-Göbel and under which the Limit Curve Theorm fails. Here we remind the definition of intersection topology.

**Definition 2.1.** If \( T_1 \) and \( T_2 \) are two distinct topologies on a set \( X \), then the intersection topology \( T_{\text{int}} \) with respect to \( T_1 \) and \( T_2 \), is the topology on \( X \) such that the set \( \{ U_1 \cap U_2 : U_1 \in T_1, U_2 \in T_2 \} \) forms a base for \( (X, T) \).

and the following useful lemma (see [14]):

**Lemma 2.1.** Let \( T_1 \) and \( T_2 \) be two topologies on a set \( X \), with bases \( B_1 \) and \( B_2 \) respectively and let \( T_{\text{int}} \) be their intersection topology, provided that it exists. Then, the following two hold.

1. The collection \( B_{\text{int}} = \{ B_1 \cap B_2 : B_1 \in B_1, B_2 \in B_2 \} \) forms a base for \( T_{\text{int}} \).

2. If \( B_{\text{int}} \) is a base for a topology, then this topology is \( T_{\text{int}} \).

It is rather surprising, as we shall see, that the interval topology, generated from either the chronological order, the relation horismos or a spacelike non-causal order that we will define shortly, is one of the two constituents for several Zeeman-Göbel topologies which are actually intersection topologies, the other constituent being the manifold topology.
2.2 The Class of Zeeman-Göbel Topologies.

The class $\mathcal{Z}$, of Zeeman topologies, is the class of topologies on the Minkowski space $M$ strictly finer than the Euclidean topology and strictly coarser than the discrete topology, which have the property that they induce the 1-dimensional Euclidean topology on every time axis and the 3-dimensional Euclidean topology on every space axis.

Zeeman (see [9] and [7]) showed that the causal structure of the null cones on the Minkowski space determines its linear structure. After initiating the question on whether a topology on Minkowski space, which depends on the light cones, implies its linear structure as well, he constructed the Fine Topology, $F$, which is defined as the finest topology for $M$ in the class $\mathcal{Z}$.

$F$ satisfies, among other properties, the following two theorems:

**Theorem 2.1.** Let $f : I \to M$ be a continuous map of the unit interval $I$ into $M$. If $f$ is strictly $\ll$-preserving, then the image $f(I)$ is a piecewise linear path, consisting of a finite number of intervals along time axes.

Let $G$ be the group of automorphisms of $M$, given by the Lorentz group, translations and dilatations.

**Theorem 2.2.** The group of homeomorphisms of the Minkowski space under $F$ is $G$.

Göbel (see [8]) showed that the results of Zeeman are valid without any restrictions on the spacetime, showing in particular that the group of homeomorphisms of a spacetime $S$, with respect to the general relativistic analogue of $F$, is the group of all homothetic transformations of $S$.

Göbel defined Zeeman topologies in curved spacetimes as follows. Let $T_M$ be the manifold topology, let $M$ be a spacetime manifold and let $S$ be a set of subsets of $M$. A set $A \subset M$ is open in $Z(S, T_M)$, a topology in class $\mathcal{Z} - \mathcal{G}$ of Zeeman-Göbel, if $A \cap B$ is open in $(B, T_M|B)$ (the subspace topology of the manifold topology $(M, T_M)$ with respect to $(B, T_M)$), for all $B \in S$. 

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3 The Path Topology of Hawking-King-McCarthy is in $\mathfrak{Z} - \mathfrak{G}$.

Zeeman in [7], last section, introduced three alternatives to his Fine topology, one of which was described as follows:

**Definition 3.1.** $Z^T$ is the finest topology that induces the 1-dimensional Euclidean topology on every time axis; an open neighbourhood of $x \in M$, in this topology, is given by $B_\epsilon(x) \cap C^T(x)$.

Even the fact that there is no clear reference in [13] (or in later papers, like [18]), that the path topology $\mathcal{P}$ is the analogue of $Z^T$ for curved spacetimes, this is trivially deduced within the frame of point-set topology. Here we adopt the much clearer notation of Low in [6]: for each $x \in M$ and each open neighbourhood $U$ of $x$, let $I(p,U)$ denote the set of points connected to $p$ by a timelike path lying in $U$ and by $K(p,U)$ the set $I(p,U) \cup \{x\}$. By choosing an arbitrary Riemannian metric $h$ on $M$, let $B_\epsilon(x)$ denote an open ball centered at $x$ with radius $\epsilon > 0$, with respect to $h$.

**Definition 3.2.** The path topology, $\mathcal{P}$, is defined to be the finest topology so that the induced topology on every timelike curve coincides with the topology induced from the manifold topology.

For a proof of the following theorem see [13]:

**Theorem 3.1.** Sets of the form $K(p,U) \cap B_\epsilon(x)$ form a basis for the topology $\mathcal{P}$.

In Section 1.3, from [15], we introduced a way to partition the space cone $S(x)$, for an event $x$ in $M$, into two symmetrical subcones $S^+(x)$ and $S^-(x)$, and constructed a space-like (non-causal) order $<$, such that $x < y$ if $y \in S^+(x)$ and $x > y$ if $y \in S^-(x)$. By $x \leq y$ we meant that either $y \in S^+(x)$ or $x \to y$ and, respectively, $y \leq x$ if either $y \in S^-(x)$ or $x \to y$, where $\to$ denoted the irreflexive version of horismos. Using this material, we introduced the following theorem:

**Theorem 3.2.** The order $\leq$ induces a topology $T^\leq_{in}$ in $\mathfrak{Z}$, which is an interval topology generated by the spacelike order $\leq$ and, furthermore, $Z^T$ is the intersection topology of $T^\leq_{in}$ in $\mathfrak{Z}$. 
and the topology $T_{R^4}$ on the Minkowski space $M$. In addition, $T_{in}^{\leq}$ (and, consequently, $Z^T$) is defined invariantly of the choice of the partition of the null cone into two, for defining $\leq$.

We summarise our observations in the following theorem:

Theorem 3.3. The analogue of $Z^T$, in curved spacetimes, is the path topology $\mathcal{P}$. In particular, the order $\leq$ induces a topology $T_{in}^{\leq}$ in $\mathfrak{G} - \mathcal{G}$, which is an interval topology generated by the spacelike order $\leq$ and, furthermore, $\mathcal{P}$ is the intersection topology of $T_{in}^{\leq}$ and the topology $T_M$ on the spacetime manifold $M$. In addition, $T_{in}^{\leq}$ (and, consequently, $\mathcal{P}$) is defined invariantly of the choice of the partition of the null cone into two, for defining $\leq$.

From Theorem 3.3 we deduce that the path topology $\mathcal{P}$ is locally an order topology, whose order is non causal and spacelike. It is interesting that this spacelike order generates open sets that are timelike; there is, somehow, a link (a duality perhaps) between spacelike and timelike that could be further explored.

4 The L.C.T. fails under a $\mathcal{P}$ environment.

R.J. Low showed (see [6]) that if in a curved spacetime we substitute the manifold topology with $\mathcal{P}$, then the Limit Curve Theorem (L.C.T.) fails to hold. We read, in the conclusion of this article, that this suggests that although the path topology is of great interest from the point of view of encapsulating the differential and causal structure of spacetime, it is nevertheless inappropriate for at least some important aspects of the study of the causal structure, where the manifold topology remains both technically easier to work with and fruitful. A natural question though could be the following: why is it not fruitful that the L.C.T. fails, while it holds for a topology (the manifold one) which, as one can read in a list of arguments in [7] and [8], misses important elements of the spacetime?

We will give next a list of topologies, all in the class $\mathfrak{G} - \mathcal{G}$, where the L.C.T. fails. These topologies have a common characteristic: they all admit a countable basis and their basic-open sets depend on the causal structure of the spacetime.

5 Six Spacetime Topologies where the L.C.T. fails.

Before we proceed, we find it important to mention the version of L.C.T. given in [6] (for a further reading see [19]):
If $\gamma_n$ is a sequence of causal paths and $x_n \in \gamma_n$ with $x$ a limit point of $\{x_n\}$, then there is an endless causal path $\gamma$ through $x$, which is a limit curve of $\{\gamma_n\}$, all within the frame of the manifold topology.

We should also mention that by the relation $\ll$ we denote the irreflexive causal order $\prec$, i.e. $x \ll y$ if either $y$ lies on the future timecone of $x$ or future lightcone of $x$, but $y$ cannot be equal to $x$. By $\rightarrow_{irr}$ we define irreflexive horismos, that is, $x \rightarrow_{irr} y$, iff $y$ lies on the future lightcone of $x$ but $y$ cannot be equal to $x$.

**Theorem 5.1.** There are six topologies, in a spacetime manifold, which admit a countable basis, they incorporate the causal structure and the L.C.T. fails with each one of them respectively. These are the intersection topologies $Z$, $Z^T$, $Z^S$ and the interval topologies $T_{in}^{-irr}$, $T_{in}^{\leq}$ and $T_{in}^{<}$, which are all in the class $\mathcal{G}$.

**Proof.** The construction of the topologies $Z$ and $T_{in}^{-}$ is described analytically in [14] and of the rest four in [15]. By looking at the arguments of R.J. Low in [6] (section V., straight after Proposition 6), for each of the six topologies individually, we see that in each case, if $U$ is an open set in the assigned topology, not containing the origin, and $p$ is an event in $\gamma \cap U$, then $\gamma$ is not a limit curve of $\{\gamma_n\}$, under the assigned topology. The uniqueness of $\gamma$ implies the failure of the L.C.T. \hfill \square

In particular, $Z^T = \mathcal{P}$ is the intersection topology between $T_{in}^{\leq}$ and the manifold topology $T_M$; $Z^T$ is the intersection topology between $T_{in}^{<}$ and $T_M$, while $Z$ is the intersection topology between $T_{in}^{-irr}$ and $T_M$. All these topologies can be constructed in a spacetime independently from whether we are talking about Minkowski space or a curved spacetime (since they depend on topological characteristics of the nullcone), so we do not consider it important to use deferent notations for the general relativistic against the special relativistic analogues.

The basic-open sets of $T_{in}^{\leq}$ are timecones and of $T_{in}^{<}$ spacecones, so there is a duality between the topologies $\mathcal{P}$ and $Z^S$. Also, $T_{in}^{<}$ is generated by an irreflexive causal relation, so we again see another possible duality; a causal order generates a topology whose basic-open sets are spacecones, a similar pattern that we mentioned about $\mathcal{P}$. 

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6 Four Candidate Spacetime Topologies passing the L.C.T.

It can be easily seen that the L.C.T., in the topologies of the previous section, failed because, in all cases, the light cone was extracted when forming the basic open sets. What about if the light-cone was “replaced back”, and considered the causal-cone (time-cone union light-cone) for forming basic-open sets? In this case, $Z'$ (the topology which would have as basic-open sets those made of by basic-open sets of $Z$ adding the lightcone) would become the usual manifold topology with the Riemannian balls and $[T_{in}^\leq]'$ would be the indiscrete topology. We are not interested for these two cases, especially for the indiscrete one. Now, $[Z_T]'$ would have as basic open sets the bounded causal cones (time-cone union light cone, intersected with a ball from the manifold topology) and $[T_{in}^\leq]'$ would have as basic open sets the causal cones. Similarly for $[Z_S]'$: its basic open sets would be the bounded closure of space-cones), i.e. space-cone union light cone, intersected with a ball from the manifold topology. In $[T_{in}^\leq]'$, the basic open sets would be unbounded closures of space-cones.

**Theorem 6.1.** There are four topologies, in a spacetime manifold, which admit a countable basis, they incorporate the causal structures and the L.C.T. holds for each one of them respectively. These are the topologies $(Z_T)', (Z_S)', (T_{in}^\leq)'$ and $(T_{in}^{\leq^\rightarrow})'$.

**Proof.** All basic-open sets of the dashed topologies are containing the light cone, so a similar argument to the proof of Theorem 5.1 leads us to the conclusion that the L.C.T. holds for each one of the four topologies, individually.

A few more comments about the construction of the dashed topologies. Both $(T_{in}^\leq)'$ and $(T_{in}^{\leq^\rightarrow})'$ are interval topologies and $(Z_T)', (Z_S)'$ intersection topologies.

To see, for example, if $(Z_T)'$ is the intersection topology of the interval topology $(T_{in}^\leq)'$ and the manifold topology $T_M$ and, if so, to find the order from which this interval topology is induced, we first consider the complements of the sets:

$$S^+(x) = \{y \in M : y > x\}, \quad (1)$$

and dually for $M - S^-(x)$; these will be considered subbasic-open sets. Then, we observe that
the intersection of the subbasic sets $M-S^+(x)$ and $M-S^-(x)$ gives a $(T^\infty_{in})'$ open set (a causal cone, that is, the time cone at $x$ union the light cone) and, consequently, a $Z^T$ open set if we intersect it with a ball $N^M_\epsilon(x)$ of $T_M$, because for each $x$, $M-S^+(x) \cap M-S^-(x) \cap N^M_\epsilon(x)$ gives a neighbourhood $N^M_\epsilon(x)$ of some radius $\epsilon$, under a Riemannian metric, with the space cone removed, but $x$ is kept. The considerations for the remaining topologies are similar.

7 A Few Questions.

Conjecture. We conjecture that Theorem 5.1 exhausts all possible spacetime topologies which belong to the class $Z-G$, admit a countable basis, incorporate the causal and conformal structure L.C.T. fails at each one, respectively. This is due to the fact that these topologies have basic-open sets which depend on the structure of the null cone and, in each case, the light-cone is subtracted when forming the basic-open sets.

Question. How many alternative spacetime topologies are there, that admit a countable basis, they incorporate the causal structure and where the L.C.T. holds at each one, respectively? There is a number of recent articles, which define such topologies which do not have any obvious link to the topologies in class $Z-G$, such as in [20] and [21].

Question. Changing topologies in the class $Z-G$ looks like “dressing” the spacetime with different clothes, while the spacetime itself remains the same; or, maybe not? Topologies in $Z-G$ are compatible with the metric (see [8]); in some of them, the topologies in Theorem [9] lead to the conclusion that it may not be possible to formulate the same sufficient conditions for geodesic incompleteness as it is usually done using the manifold topology, while for some others (including the Fine topology, in $Z-G$) the L.C.T. holds. The question is: what is the physical interpretation of this phenomenon?

8 Implications in Quantum Gravity.

The implications of the above considerations for gravitational theories might be considerable. We only comment on an issue that has been raised in the causal set program for quantum gravity. There is a growing sense about an underlying non-locality of gravitational interactions at the quantum level. The question that has to be addressed is how one makes the transition from the quantum non-local theory to a classical local theory. This was eloquently formulated in [22]. Since locality is fundamentally a topological issue, it might be possible
to re-frame this question in topological terms as a “phase transition” between two different topologies one operating at the quantum and the other at the classical levels. How exactly this transition occurs, or if it occurs at all, can be the subject of future work.

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