Relativistic velocity singularities

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Relativistic and non-relativistic fluid equations can exhibit finite time singular solutions including density singularities appearing in collapse or compression systems and gradient singularities in shock waves. However, only the non-relativistic fluid equations have been shown to exhibit finite-time velocity singularities, \( v \to \infty \) in this regime. As this limit violates the cosmic speed limit \( v < c \), where \( c \) is the speed of light, it is unclear how velocity singularity source terms in the non-relativistic equations are modified in the relativistic equations. This paper uses a driven system to demonstrate that singular points within the relativistic fluid equations can generate finite time relativistic velocity singularities \( v \to c \).

I. INTRODUCTION

The relativistic and non-relativistic fluid equations must necessarily provide identical descriptions of fluid motion in the non-relativistic regime suggesting that density and gradient singularities present in the non-relativistic regime must also appear in the relativistic equations. In particular, density singularities \( \rho \to \infty \) feature in the relativistic gravitationally collapse of a non-rotating zero pressure dust cloud [1], as well as in non-relativistic compressive systems such as shock tunnels where growing pressures burst thick metal diaphragms [2], in the hand-held Diamond-Anvil generating 2 million atmospheres of pressure [3], in cavitation generating energy densities sufficient to pit steel [4], in sonoluminescence generating temperatures in excess of 100,000 degrees [5, 6, 7], in explosive compression of magnetic fields generating fields up to 1,000T [8, 9] and in supernovae collapse experiments where inertial confinement generates temperatures in excess of 100 million degrees [10]. Of course, in the non-relativistic regime fluid parameters do not literally become singular—the approximations deriving from the non-relativistic equations break down long before that point.

Similarly, gradient singularities appear within both the relativistic and non-relativistic fluid equations to generate shock waves within relativistic jets [11, 12], in non-relativistic supersonic flows [13, 14, 15], and interestingly, within trombones [16].

Singular points present within the fluid equations can potentially drive not only density and gradient singular solutions, but also finite time velocity singularities. In the non-relativistic regime, velocity singularities \( v \to \infty \) have been exhibited in fissioning fluid drops [17, 18, 19, 20], thin-films evaporating or pinched to zero thickness [21, 22], and curvature collapse and jet eruption from fluid surfaces [23]. These systems typically feature a free fluid surface of microscopic scale in one dimension and macroscopic scale in other dimensions. This scale disparity allows surface tension to dominate other forces in the microscopic dimensions allowing self-similar solutions where essentially, fluid is evacuated across some macroscopic distance on infinitesimal timescales generating a singular velocity solution. These systems can be considerably simplified and scaled up in size by replacing the free fluid surface by a flat moving boundary which forces fluid evacuation over macroscopic distances on infinitesimal timescales to again generate singular velocities. The validity of this approach was experimentally confirmed in Ref. [24].

More generally, it is possible that similar singular source terms might support finite time current singularities in magnetohydrodynamic models of magnetic reconnection [25], and that unexplained car tire noise might result from nonlinearities present in such driven collapse systems [26]. Additionally, velocity singularities appear at boundary layer separation points [5], while examinations of non-relativistic fluid singularities and the hydrodynamic blow-up problem appear in [27, 28, 29, 30, 31, 32, 33, 34].

To date, there has been no examination of the potential for singular points within the relativistic equations to drive “relativistic velocity singularities” \( v \to c \), where here, \( c \) is the speed of light recently termed “Einstein’s constant” [12]. Existing schemes to accelerate particles to near light speed do not exploit velocity singularities and typically rely on density singularities where essentially, “fireballs” accelerate particles as when coalescing neutron stars source gamma ray bursts [35], asymmetric (or turbulent) supernovae collapse driving bullets of core material through overlying stellar layers [36, 37], and high intensity table-top lasers drive particle acceleration [38]. Other proposed mechanisms employ gradual particle acceleration over macroscopic distances and include shock wave surfing mechanisms generating cosmic ray particle energies up to \( 10^{20}\text{eV} \) [39], and magnetohydrodynamic acceleration of polar jets from accretion disks [40, 41].

In contrast to these approaches, this paper examines whether singular points within the relativistic fluid equations can be exploited to generate near relativistic flows. The intractable nature of the problem requires excessive simplification and, as in the non-relativistic case [24], we...
exploit driven boundaries to force the relativistic fluid evacuation over macroscopic distances on infinitesimal
timescales, and demonstrate in the appropriate limit that
singular points within the relativistic fluid equations can
drive fluid velocities to the limit $v \to c$.

(a) A schematic of a near relativistic column of perfect fluid impacting an incompressible surface, and (b) modelling the column as a piston compressing underlying layers of gas.

II. RELATIVISTIC VELOCITY SINGULARITIES

Our initial examination of relativistic velocity singularities requires scaling up non-relativistic fluid evacuation systems to relativistic speeds. To this end, consider a perfect fluid column travelling at near relativistic speeds hitting an incompressible surface and generating an outflow as shown in Fig. 1(a). The impacting fluid column acts as a compressive piston as shown in Fig. 1(b) so that as the piston closes, entrapped fluid must evacuate over macroscopic distances on ever shrinking timescales (assuming the fluid remains relatively uncompressed.) This crude model suffices to allow the approximate solution of the fluid expulsion speed in horizontal directions.

For simplicity we ignore gravity so fluid motion is entirely governed by the relevant conservation equations, and consider a flat space-time with metric $\eta_{\alpha\beta} = \eta^{\alpha\beta} = \text{diag}(-1,1,1,1)$ where primed coordinates $x^\alpha = (t', x', y', z')$ give the contravariant fluid velocity vector as $U^\alpha = (\gamma, \gamma v')$ with $v' = (v_x', v_y', v_z')$ and $\gamma = 1/\sqrt{1 - v'^2}$ in geometrized units setting the speed of light to unity. (Greek indices run over values $0 \to 4$ while Latin indices vary from $1 \to 3$.)

A perfect gas has particle flux and energy-momentum tensor

$$N^\alpha = n U^\alpha$$
$$T^\alpha{}^\beta = \eta^{\alpha\beta} p + (p + \rho) U^\alpha U^\beta$$

where $n$, $p$ and $\rho$ are the momentarily comoving frame particle number density, pressure and energy density respectively. Conservation equations are $N^\alpha_{;\mu} = 0$ and $T^\mu{}_{\nu ;\nu} = 0$ giving

$$D' (\gamma n) = 0$$
$$D' (\gamma^2 (p + \rho)) - \partial_t p = 0$$
$$D' (\gamma^2 (p + \rho)v') + \nabla' p = 0,$$

with $D' = \partial_t + \nabla' v'$ and $\nabla' = (\partial_{x'}, \partial_{y'}, \partial_{z'})$. These equations give respectively, particle number conservation, mass continuity and momentum conservation.

The presence of a moving boundary (the descending piston) complicates analysis and it is useful to use time dependent computational space coordinates in which the descending piston is rendered stationary. If the piston is at height $d_0$ at time $t = 0$, then its subsequent height can be written $y'(t') = f(t')d_0$ in terms of a monotonically decreasing function $f(t')$ with $f(0) = 1$. Full closure occurs in the limit $f \to 0$, and the piston’s vertical velocity is $v_y(t') = f(t')d_0$. The transformation to unprimed computational space coordinates is then

$$x^\alpha = \begin{pmatrix} t \\ x \\ \chi \\ z \end{pmatrix} = \begin{pmatrix} t' \\ x' \\ y'/f(t') \\ z' \end{pmatrix},$$

so effectively, vertical compression in physical space $\Delta y' \to 0$ corresponds to a constant separation in computational coordinates, $\Delta \chi$ constant. The vertical compression is fully captured in the function $f(t) = f(t')$, and its effects can be assessed by tracking this function in governing equations.

The new coordinates determine a new metric $g_{\mu\nu}$ and proper time $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$d\tau^2 = \left[ \left( f' \right)^2 - 1 \right] dt^2 + 2f f' dt d\chi + dx^2 + f^2 d\chi^2 + dz^2,$$

while the non-zero affine connections are

$$\Gamma^\chi_{tt} = \frac{f'}{f}, \quad \Gamma^\chi_{\chi t} = \Gamma^\chi_{\chi \chi} = \frac{f'}{f}.$$

As required for a flat spacetime, the metric tensor $g_{\mu\nu}$ has three positive and one negative eigenvalue and the Ricci tensor is zero everywhere. The transformed contravariant velocity vector is $U'^\alpha = [\gamma, \gamma (v_x, v_y, v_z)]$ with

$$v_x = \frac{(v_y' - f' \chi)}{f}.$$
Then, as required, the piston velocity is \( v_\lambda = 0 \) as \( v_\nu = \dot{f} \gamma (t = \dot{f} \delta_0) \).

In the new coordinates, the particle flux and energy-momentum tensors become

\[
N^\alpha = nU^\alpha \\
T^{\alpha\beta} = g^{\alpha\beta} p + (p + \rho) U^\alpha U^\beta
\]

so the conservation equations \( N^\mu_{;\mu} = 0 \) and \( T^{\mu\nu}_{;\mu} = 0 \) give

\[
D (f \gamma n) = 0 \\
D (f \gamma^2 (p + \rho)) - \partial_t (fp) = -\dot{f} \partial_x (\chi p) \\
D (f \gamma^2 (p + \rho) v) + \nabla (fp) = \begin{pmatrix} 0 \\ A \end{pmatrix},
\]

with

\[
A = -\dot{f} \gamma T_{tt} - 2 \dot{f} T_{tx} - \partial_t \left( \dot{f} \gamma \rho \right) \\
- \partial_x \left[ f \rho \left( \frac{1}{f^2} - \frac{\dot{f}^2 x^2}{f^2 - 1} \right) \right],
\]

and \( D = \partial_t + \nabla \cdot v, \quad \nabla = (\partial_x, \partial_y, \partial_z) \), and

\[
T_{tt} = \gamma^2 (p + \rho) - p \\
T_{tx} = \gamma^2 (p + \rho) v_x + \dot{f} \chi p.
\]

These equations give respectively, particle number conservation, mass continuity and momentum conservation as can be seen by taking the limit \( f = 1 \) and \( \dot{f} = 0 \).

An understanding of the physics embodied in Eqs. (8) and (9) can be obtained by considering solutions in two exactly solvable physical limits. The first considers a stationary piston with the fluid in equilibrium. In this case, Eq. (8) is solved by setting fluid velocity \( v = 0 \), and number density, pressure and density everywhere constant. Eq. (8) reduces to Eq. (2) in the case \( f = 1 \), while for \( f < 1 \), Eq. (8) is solved by setting \( v = (0, -\dot{f} / f, 0) \), and again, uniform number density, pressure and density. The second case considers a collisionless relativistic dust with \( p = 0 \) (as some equation of state must be specified), and treats a closed piston lacking horizontal flows \( v_x = v_y = 0 \) with the piston descending at a constant velocity \( (\dot{f} = f) \). The dust at height \( y \) has initial vertical velocity at time \( t = 0 \) of \( v_\nu = \dot{f} y \) and these dust elements preserve this velocity for all times as pressure is zero. For later times \( t \) at fixed height \( y \), the fluid velocity increases as \( v_\nu = \dot{f} y / f \) and Eq. (8) is then solved by \( v = (0, \dot{f} y / f, 0) \) and everywhere uniformly increasing number density \( \gamma n(t) = c_1 / f \) and fluid density \( \gamma^2 \rho(t) = c_2 / f \) for constants \( c_1 \) and \( c_2 \). Conversely, the moving coordinate system tracks the initial dust elements leading to a constant velocity \( v = (0, 0, 0) \) which together with uniformly increasing number density \( \gamma n(t) = c_1 / f \) and fluid density \( \gamma^2 \rho(t) = c_2 / f \) solves Eq. (8).

We now turn to consider the role of the squeezing parameter \( f(t) \) in Eq. (8) which is linked everywhere on the left-hand-side (LHS) with \( \gamma \). As is well known, the singularity \( \gamma \to \infty \) ensures that all fluid velocities remain less than the speed of light, \( v < c \). Then, the countervailing linkage between this singularity and the zero asymptote of the squeezing parameter \( f \to 0 \) suggests a minimization of the effects of the combined terms \( f \gamma^2 \).

The physical meaning of the squeezing parameter \( f \) can be seen in more detail using the injection form of the relativistic fluid equations

\[
D (\gamma n) = -\dot{f} \gamma n / f \\
D \left[ \gamma^2 (p + \rho) \right] - \partial_t (fp) = -\dot{f} \partial_x (\chi p) \\
- \dot{f} \gamma (\gamma^2 (p + \rho) - p) \\
D \left[ \gamma^2 (p + \rho) v \right] + \nabla p = \begin{pmatrix} 0 \\ A / f \end{pmatrix} - \dot{f} \gamma^2 (p + \rho) / f \begin{pmatrix} v_x \\ v_x / v_z \end{pmatrix}.
\]

The left hand sides (LHS) here are identical to those of Eq. (2) while the right hand sides (RHS) represent sources injecting matter and momentum, including into the horizontal directions \( x \) and \( z \). (Similar equations are used to describe rockets employing real injection systems to generate high speed expelled flows [14].) The injection sources are dominated in the relativistic large \( \gamma \) regime by terms proportional to the ratio \( f / f \) and either \( \gamma \) or \( \gamma^2 \). Then, as long as \( \dot{f} \neq 0 \), the respective limits \( \gamma \to \infty \) and \( f \to 0 \) reinforce each other to inject (potentially) infinite mass and momentum in finite time to drive a horizontal relativistic velocity singularity.

A very approximate analysis of the velocity and energy of the horizontal expelled jet can be performed but solutions are suggestive only. Consider the case where again the moving coordinate system approximately tracks the descending fluid so \( v_x \approx 0 \), though now, the increasing pressure drives a near relativistic horizontal velocity \( v_x \) constituting an expelled jet. (Symmetry allows ignoring the \( z \) direction.) Fluid velocities are then \( (v_x, 0, 0) \). In the large \( \gamma \) limit each of the above conservation equations gives the time rate of change of the expansion velocity as

\[
\partial_t v_x = \frac{\nabla \cdot v \gamma n + \gamma \dot{n}}{n^2 v_x} - \frac{\dot{f}}{f \gamma^2 v_x} v_x \\
\partial_t v_x = \frac{\nabla \cdot v \gamma^2 R + \gamma^2 \dot{R}}{2 \gamma^2 R v_x} - \frac{\dot{f}(R - p)}{2 f \gamma^2 v_x^2} v_x \\
\partial_t v_x = \frac{\nabla \cdot v \gamma^2 R v_x + \partial_x p}{\gamma^2 R (1 + 2 \gamma^2 v_x^2)} - \frac{\dot{f} v_x}{f (1 + 2 \gamma^2 v_x^2)} (12)
\]

where \( R = \gamma^2 (p + \rho) \) and the top line results from the conservation of particle number, the second line from the conservation of mass and the third line stems from the conservation of momentum in the \( x \) direction. The
asymptotic form of each of these equations in the large $\gamma$ limit is approximately

$$\partial_t v_x \approx O \left( \frac{1}{\gamma^2} \right) - \frac{f}{f \gamma^2 v_x}$$  \hspace{1cm} (13)

and this form is used for further discussion. The first term on the RHS is independent of $f$ and, in the absence of squeezing ($f = 0$), necessarily goes to zero so that $\partial_t v_x \rightarrow 0$ as $\gamma \rightarrow \infty$ ensuring $v < c$. The second term reflects squeezing effects and is positive ($\dot{f} < 0$) and the linkage between the limits $f \rightarrow 0$ and $\gamma \rightarrow \infty$ offsets the effect of $\gamma$. This equation can be integrated by ignoring the small first term with initial conditions $f(t_0) = 1$ and $v_x(t_0) = v_0$ giving

$$v_x^2(t) = 1 - f^2 \left( 1 - v_0^2 \right),$$ \hspace{1cm} (14)

or equivalently

$$\gamma^2(t) = \frac{v_0^2}{f^2}.$$ \hspace{1cm} (15)

It is of interest to consider the non-physical mathematical limit $f \rightarrow 0$ for some finite time $T > t_0$ representing full closure so $f(T) = 0$. This limit gives

$$v_x(T) = 1,$$ \hspace{1cm} (16)

a light speed jet. It is of course impossible for any material fluid to reach light speed though squeezed system might be able to exploit relativistic velocity singularity source terms to approach this ideal using singular mass and momentum injection source terms proportional to $-f \gamma^2/f$.

Particle energy within the expelled jet is then

$$E \propto \gamma(t) \propto \frac{1}{\gamma} \rightarrow \infty,$$ \hspace{1cm} (17)

though this result is suggestive only.

III. NON-RELATIVISTIC VELOCITY SINGULARITIES

For completeness, details of the non-relativistic velocity singularity created by asymmetrically compressing a fluid are given. (See Ref. [24] for more details.) The conservative form of the inviscid and dimensionless Euler equations with zero conductivity are derived from the the relativistic equations (8) after taking the small velocity limit $\gamma \rightarrow 1$, $p \approx O(\rho v^2)$, and $p + \rho \rightarrow \rho$, so now $\rho$ and $p$ become the usual fluid density and pressure. The use of time-dependent coordinates [Eq. (3)] gives the Energy-Momentum tensor for a perfect gas as

$$T^{00} = \rho$$
$$T^{0j} = \rho v^j$$
$$T^{ij} = \rho v^i v^j + \text{diag} \left( p, p/f^2, p \right).$$ \hspace{1cm} (18)

Here, the diagonal pressure terms in the vertical direction are scaled by a factor $1/f^2$ over those in horizontal directions so that the limit $f \rightarrow 0$ creates the large pressure gradients driving the horizontal velocity singularity. Conservation of the energy-momentum tensor $T^{\mu \nu} \partial_\mu = 0$ then gives the modified Euler equations

$$\partial_t U + \partial_x F + \partial_\chi G = 0$$ \hspace{1cm} (19)

$$U = \left( \begin{array}{c}
\frac{f p}{f \rho v_x} \\
\frac{f \rho v_x}{f \rho v_y} \\
\frac{f \rho v_y}{f \rho v_x + \rho^\gamma} \\
\end{array} \right),$$

$$F = \left( \begin{array}{c}
\frac{f p v_x}{f \rho v_x} \\
\frac{f \rho v_y}{f \rho v_y} \\
\frac{f \rho v_x v_y}{f \rho v_x v_y + \rho^\gamma} \\
\end{array} \right),$$

$$G = \left( \begin{array}{c}
f \rho v_y v_x + \rho^\gamma \\
\frac{f \rho v_x}{f \rho v_y} \\
\frac{f \rho v_y}{f \rho v_x + \rho^\gamma} \\
\end{array} \right).$$

Here $v_\chi = (v_y - \dot{f})/f$ and we show mass continuity (top line) and momentum conservation in the $x$ and $y$ directions. (Symmetry constrains fluid motions to this plane.) Adiabatic perfect gases with $p = \rho^\gamma$ ($\gamma = 1.4$) are considered and all variables are dimensionless with dimensioned (primed) variables being given by $x' = xL$, $v' = v_0a_0$, $p' = pp_0$, $\rho' = \rho p_0$ and $t' = tL/a_0$ with $L$ being some convenient length parameter and $a_0^2 = \gamma p_0/\rho_0$ giving the local speed of sound.

Analytic solutions can be obtained in the low velocity limit $v_x \ll 1$ (allowing $v_0^2 \approx 0$), and by assuming linear squeezing $f(t) = 1 - t/T$ for some closure time $T$, with fluid elements possessing velocity $v_y = \dot{f} \chi$ tracked by the time dependent coordinates giving $v_\chi = 0$. The analytic solution for an incompressible fluid with $\rho = 1$ (to give the maximum expulsion velocity) is then

$$v_x(x,t) = -\frac{\dot{f} x}{f}$$ \hspace{1cm} (20)

displaying a velocity singularity as $f \rightarrow 0$. This solution is strictly valid only while $v_x < 0.3$ where an unconfined compressible fluid remains approximately uncompressed. Experiments demonstrate that this solution provides a reasonable approximation to the expulsion velocity into the supersonic regime [24].

IV. CONCLUSION

This paper uses a preliminary and necessarily crude analysis to establish that relativistic velocity singularities can appear in relativistic hydrodynamic equations. Velocity singularities provide such an elegant and simple means of sourcing near light speed relativistic jets that it would be surprising if this mechanism was not accessed in some astrophysical systems.
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