Conformalons and Trans-Planckian problem

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In any conformally invariant gravitational theory, the space of exact solutions is greatly enlarged. The Weyl’s conformal invariance can then be spontaneously broken to spherically symmetric vacuum solutions that exclude the spacetime region inside the black hole’s event horizon from our Universe. We baptize these solutions conformalons. It turns out that for all such spacetimes nothing can reach the Schwarzschild event horizon in a finite amount of proper time for conformally coupled “massive” particles, or finite values of the affine parameter for massless particles. Therefore, for such vacuum solutions the event horizon is an asymptotic region of the Universe. As a general feature, all conformalons show a gravitational blueshift instead of a gravitational redshift at the unattainable event horizon, hence avoiding the Trans-Planckian problem in the Hawking evaporation process. The latter happens as usual near the event horizon, but now the annihilation process between the matter and Hawking’s negative energy particles takes place outside of the event horizon. Indeed, in these solutions the gravitational collapse consists of matter that falls down forever towards the horizon without ever reaching it. According to a previous work that has been faithfully adapted to the conformalons’ scenario, the information is not lost in the whole process of singularity-free collapse and evaporation, as evident from the Penrose diagram.

I. INTRODUCTION

In Einstein’s conformal gravity (ECG), which we will shortly remind the reader of, as well as in any conformally invariant theory solved by Ricci-flat spacetimes, a general solution takes the following form,

\[ ds^2 = \hat{g}^*_\mu_\nu(x)dx^\mu dx^\nu = S(x)\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu, \]  

where the rescaling factor \( S(r) \) is the vacuum selected through the spontaneous symmetry breaking of conformal invariance. Namely, a geodesically complete vacuum is naturally selected without explicitly breaking the conformal symmetry. Indeed, in the conformal phase there is no singularity issue as long as the singular spacetimes are conformal to those that are singularity free. On the other hand, the vacuum in the spontaneously broken phase is consistently selected in order to interpolate with the singularity-free spacetime in the conformal phase.

From now on, we will focus on ECG, but most of the results can be extended to Weyl conformal gravity and all the other theories invariant under the following conformal symmetry

\[ \hat{g}_{\mu\nu}(x) = \Omega^2(x)\hat{g}_{\mu\nu}(x), \]
\[ \phi'(x) = \Omega^{-1}(x)\phi(x), \]  

where \( \phi \) is the dilaton field that guarantees the Weyl invariance of the action.

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1 See, for example, the theories proposed in [1–5], which are consistent with quantum super-renormalizability or finiteness, linear and non linear stability [6, 7], and perturbative unitarity [8, 9]. In particular, the theory in [3] in odd dimension and the generalization in even dimension [5] are both finite at quantum level, a property that turns out to be crucial in order to support the main statement in this paper. Indeed, here we assume that conformal symmetry is broken spontaneously and not by the quantum Weyl anomaly. Therefore, the gravitational theory has to be finite at quantum level [10,11].
The action for ECG in a $D$–dimensional spacetime is obtained replacing
\[ g_{\mu\nu} = \left( \phi^2 \kappa_D^2 \right)^{-2} \hat{g}_{\mu\nu}, \] (3)
in the Einstein-Hilbert gravitational action
\[ S_{EH} = \frac{2}{\kappa_D^2} \int d^D x \sqrt{|g|} R(g), \] (4)
where $2/\kappa_D^2 = 1/16\pi$. The resulting Lagrangian reads:
\[ \mathcal{L} = 2 \sqrt{|\hat{g}|} \left[ \phi^2 \hat{R}(\hat{g}) + \frac{4(D - 1)}{D - 2} \hat{g}^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi) \right]. \] (5)

If the conformal symmetry is spontaneously broken to a constant value for the dilaton field, namely $\phi = 1/\kappa_D$, the action in (5) turns into the Einstein-Hilbert one in (4). Notice that there is no propagating degree of freedom related to the dilaton field because it can be always gauge fixed to zero making use of the conformal invariance.

In several papers it has been proved that the singularity issue can be successfully addressed in any conformally invariant theory having Ricci flat spacetimes as exact solutions [16–24]. Indeed, by proper rescalings, we can obtain regular black holes’ solutions starting from the Schwarzschild one. In this case the conformal symmetry is spontaneously broken to a non trivial profile for the dilaton instead of the constant one discussed above, in which conformal gravity turns into Einstein’s gravity. For all such singularity-free spacetimes, it turns out that nothing can reach the singularity in a finite proper time (massive particles) or for a finite value of the affine parameter (photons or massless particles). Therefore, the spacetime point $r = 0$ is outside of our Universe. Conformal symmetry can also play a crucial role in explaining the galactic rotation curves as explicitly shown in [25].

In this paper, motivated by the above results, we fully remove the whole black hole interior from our Universe by the means of a conformal rescaling that is singular at the event horizon. For the sake of simplicity, in the rest of the paper we will assume to be in $D = 4$, but it is straightforward to generalize all the results in this paper to any spacetime dimension $D$.

II. CONFORMALONS

As a consequence of the Weyl invariance (2), in ECG the Schwarzschild solution and all its rescalings are exact solutions of the theory (see equation (1)). Therefore, we hereby assume the Weyl conformal symmetry to be spontaneously broken to the following metric that exactly solve the equations of motion for the theory (5),
\[ ds^2 = S(r) \left[ - \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2 \right], \]
where for the rescaling $S(r)$, we consider multiple choices, namely,
\[ S(r) = \frac{r}{r - 2M}; \] (7)
\[ S(r) = \left( \frac{r}{r - 2M} \right)^2; \] (8)
\[ S(r) = \left( \frac{r^{20}}{(r^{20} - (2M)^{20})} \right)^2; \] (9)
\[ S(r) = \left( \frac{r^n}{r^n - (2M)^n} \right)^2, \quad n \in \mathbb{N}; \] (10)
\[ S(r) = \left( \frac{r^n}{r^n - (2M)^n} \right)^{m}, \quad n, m \in \mathbb{N}, \ m > 2. \] (11)

We name these solutions “conformalons” because, as it will be clear in the next section, the spacetime is defined only for $r > 2M$ (similar to gravitational instantons), while the interior of the black hole is an unreachable region for any massive or massless particle. In other words, conformalons are asymptotically unattainable regions of the Universe.
Since the rescaling excludes the spacetime region inside the event horizon ($S(r)$ is only defined for $r > 2M$), as a first check of the regularity of the spacetime we consider the Kretschmann invariant for $r \geq 2M$. The Schwarzschild spacetime $\hat{g}_{\mu\nu}$ is Ricci flat, hence, as it is well known, before the rescaling the Kretschmann scalar is:

$$\hat{K} := \hat{R}_{\alpha\beta\gamma\delta} \hat{R}^{\alpha\beta\gamma\delta} = \hat{C}_{\alpha\beta\gamma\delta} \hat{C}^{\alpha\beta\gamma\delta} = \frac{48M^2}{r^6},$$

(12)

where in the last equality we used that $\hat{R}_{\alpha\beta} = 0$ and introduced the Weyl tensor $\hat{C}_{\alpha\beta\gamma\delta}$. Under the Weyl rescaling the Weyl tensor is invariant, i.e.

$$\hat{C}^{\alpha\beta\gamma\delta} = \hat{C}^{\alpha\beta\gamma\delta} \hat{C}_{\alpha\beta\gamma\delta} = \hat{C}^{\alpha\beta\gamma\delta},$$

(13)

while the Kretschmann scalar turns into

$$\hat{K}' = \hat{K}^* = \frac{\hat{K}}{S^2(r)} = \frac{48M^2}{r^6} \left( \frac{r - 2M}{r} \right)^4,$$

(14)

which is zero for $r = 2M$. Therefore, the event horizon is an asymptotic region of zero curvature (the reason of using the terminology “asymptotic region” will be clear in the next section when we will study the geodesic completion). Certainly, it is not sufficient to look at the Kretschmann invariant to claim the regularity of the spacetime, but it is a useful tool in order to determine the right rescaling consistently with the spacetime geodesic completion. It deserves to be noticed that $\hat{K}'$ is zero at the horizon also for the rescaling, but as we will see later the spacetime is not geodesically complete.

### III. GEODESIC COMPLETION

In order to prove the geodesic completion of (6) and to understand the global structure of the Universe, we can study the motion of conformally coupled particles and of massless particles in the metric (7). It will turn out that for the rescalings (8) - (11) nothing is able to reach the event horizon in a finite amount of proper time, or for a finite value of the affine parameter in the case of massless particles. We also could study the propagation of massive particles in the geometry (6), but such analysis is not very interesting and purely academic since we do not want to explicitly break the Weyl conformal symmetry.

For the sake of simplicity, from now on we make the following redefinitions:

$$\hat{g}_{\mu\nu} \rightarrow \hat{g}_{\mu\nu}, \quad \phi^* \rightarrow \phi.$$

(16)

#### A. Geodesics for conformally coupled particles

We begin studying the geodesic completion of the spacetime (6) considering a conformally coupled particle whose action reads:

$$S_{\text{cp}} = - \int \sqrt{-\hat{g}^2} \hat{g}_{\mu
u} dx^\mu dx^\nu = - \int \sqrt{-\hat{g}^2} \hat{g}_{\mu
u} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda,$$

(17)

where $f$ is a positive constant coupling strength, $\lambda$ is a parameter, and $x^\nu(\lambda)$ is the trajectory of the particle. Notice that in the unitary gauge $\phi = \kappa^{-1}_4$ the action (17) turns into the usual one for a particle with mass $m = f \kappa_4^{-1}$ ($f > 0$). The Lagrangian reads

$$L_{\text{cp}} = - \sqrt{-f^2 \phi^2 \hat{g}_{\mu
u} \hat{x}^\mu \hat{x}^\nu},$$

(18)

and the translation invariance in the time-like coordinate $t$ implies

$$\frac{\partial L_{\text{cp}}}{\partial t} = - \frac{f^2 \phi^2 \hat{g}_{\mu\nu} \hat{x}^\mu \hat{x}^\nu}{L_{\text{cp}}} = \text{const.} = -E \quad \Rightarrow \quad i = \frac{L_{\text{cp}}E}{f^2 \phi^2 \hat{g}_{tt}},$$

(19)
Since we are interested in evaluating the proper time that the particle takes to reach the event horizon located at $r = 2M$, we must choose the proper time gauge, namely $\lambda = \tau$. In this case, $E$ is the energy of the test-particle and $E^2 \kappa_4^2 / f^2 = -1$.

Replacing (19) in $\hat{g}_{\mu \nu} \dot{x}^\mu \dot{x}^\nu = -1$ and using the solution of the EOM for $\phi$, namely $\phi = S^{-1/2} \kappa_4^{-1}$, we end up with

$$S(r)^2 \dot{r}^2 + S(r) \left( 1 - \frac{2M}{r} \right) - \frac{E^2 \kappa_4^2}{f^2} S(r) = 0,$$

or

$$S(r)^2 + \left( 1 - \frac{2M}{r} \right) - \frac{E^2 \kappa_4^2}{f^2} = 0.$$

Introducing the definition $E^2 \kappa_4^2 / f^2 \equiv e^2 > 0$ we finally get:

$$S(r)^2 \dot{r}^2 = e^2 - 1 + \frac{2M}{r}.$$

For $r \gg 2M$, the latter equation simplifies to

$$S(r)^2 \dot{r}^2 \sim e^2 \implies \sqrt{S(r)|\dot{r}|} \sim |e|.$$

For the rescaling (8), and $r \approx 2M$, the above equation can be easily integrated,

$$\int \sqrt{\frac{S(r)}{e^2 - 1 + \frac{2M}{r}}} dr = -\tau + \text{const}..$$

The solutions of (26) for the rescalings (7) and (8), assuming $e = 1$, are, respectively,

$$\tau = \frac{2}{3} \left[ - \frac{r + 2r_g}{\sqrt{r_g} - r_g} + \frac{r_0 + 2r_g}{\sqrt{r_0} - r_g} \right],$$

$$\tau = \frac{2}{3} \left[ - \sqrt{\frac{r_g}{r_g}} (r + 3r_g) + 3r_g \tanh^{-1} \left( \frac{r}{r_g} \right) + \sqrt{\frac{r_0}{r_g}} (r_0 + 3r_g) - 3r_g \tanh^{-1} \left( \frac{r_0}{r_g} \right) \right],$$

where $r_g = 2M$ is the Schwarzschild gravitational radius and $r_0$ is the initial condition, namely the point from which the particle starts its motion towards $r_g$. Near $r_g$, the solution (28) is approximated by:

$$\tau(r) \approx \frac{r_g}{e} \log \left( \frac{r_0 - r_g}{r - r_g} \right),$$

which is in agreement with (25).

For the case (7), the proper time to reach the event horizon is finite and the spacetime turns out to be incomplete because the radial geodesic equation (23) is only defined for $r > 2M$. On the other hand, for (8) the proper time to reach the horizon is infinity and, therefore, the spacetime is geodesically complete.

Looking at more general $S(r)$, it turns out that (8), (9), and (10) imply a logarithmic behavior of the proper time in the near horizon limit, while for (11) the behavior is the inverse of a polynomial.
B. Geodesics for light rays or general massless particles

For massless particles, the correct action, which is invariant under reparametrization of the world line, \( p' = f(p) \), is:

\[
S_\gamma = \int L_\gamma d\lambda = \int e(p)^{-1} \phi^2 \tilde{g}_{\mu\nu} \frac{dx^\mu}{dp} \frac{dx^\nu}{dp} dp,
\]

(30)

where \( e(p) \) is an auxiliary field that transforms as \( e'(p')^{-1} = e(p)^{-1}(dp'/dp) \) in order to guarantee the invariance of the action. The action \( S_\gamma \) is not only invariant under general coordinate transformations, but also under the Weyl conformal rescaling \( \hat{g}_{\mu\nu} \) because of the presence of the dilaton field.

The variation respect to \( e \) gives:

\[
\frac{\delta S_\gamma}{\delta e} = 0 \quad \implies \quad -\int dp \frac{\delta e}{e^2} \tilde{g}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0 \quad \implies \quad \dot{s}^2 = \dot{\tilde{g}}_{\mu\nu} dx^\mu dx^\nu = 0,
\]

(31)

consistently with the main property of massless particles or the equivalence principle.

The variation with respect to \( x^\mu \) gives the geodesic equation in the presence of the dilaton field, namely (in the gauge \( e(p) = \text{const.} \))

\[
D^2(\dot{g} = \phi^2 \dot{x}^\lambda) = \frac{D^2(\dot{g}) x^\lambda}{dp^2} + 2 \frac{\partial \phi}{\phi} \frac{dx^\mu}{dp} \frac{dx^\lambda}{dp} = 0,
\]

(32)

where \( D^2(\dot{g}) \) is the covariant derivative with respect to the metric \( \dot{g}_{\mu\nu} \).

However, when we contract equation (32) with the velocity \( dx_\lambda/dp \) and we use \( \dot{s}^2 = 0 \) obtained in (31), we get the following on-shell condition,

\[
\frac{dx_\lambda}{dp} D^2(\dot{g}) x^\lambda = f \frac{dx^\lambda}{dp} \quad (f = \text{const.})
\]

(34)

because the velocity is null on the light cone. Under a reparametrization of the world line \( q = q(p) \), eq. (34) becomes

\[
\frac{d^2 x^\lambda}{dq^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{dq} \frac{dx^\nu}{dq} = \frac{dx^\lambda}{dp} \left( \frac{dq}{dp} \right)^2 \left( \frac{dp}{dq} - \frac{d^2 q}{dp^2} \right).
\]

(35)

Choosing the dependence of \( q \) on \( p \) to make the right-hand side of (35) vanish, we end up with the geodesic equation in the affine parametrization. Hence, we can redefine \( q \to \lambda \) and, finally, we get the affinely parametrized geodesic equation for photons in the metric \( \dot{g}_{\mu\nu} \),

\[
D^2(\dot{g}) x^\lambda = 0.
\]

(36)

We can now investigate the conservation laws based on the symmetries of the metric. Let us consider the following scalar,

\[
\dot{\alpha} = \dot{g}_{\mu\nu} v^\mu \frac{dx^\nu}{d\lambda}.
\]

(37)

where \( v^\mu \) is a general vector. Taking the derivative of (37) with respect to \( \lambda \) and using the geodesic equation (36), we get

\[
\frac{d}{d\lambda} \dot{\alpha} = \frac{1}{2} v^\mu \partial_\mu \dot{g}_{\rho\nu} \frac{dx^\rho}{d\lambda} \frac{dx^\nu}{d\lambda} + \dot{g}_{\mu\nu} \partial^\mu v^\nu \frac{dx^\rho}{d\lambda} \frac{dx^\rho}{d\lambda} = \frac{1}{2} [\hat{L} v]_{\rho\nu} \frac{dx^\rho}{d\lambda} \frac{dx^\nu}{d\lambda},
\]

(38)
where \([\mathcal{L}_v \hat{g}]\) is the Lie derivative of \(\hat{g}_{\mu\nu}\) by a vector field \(v^\mu\). Thus, if \(v^\mu\) is a Killing vector field, namely \([\mathcal{L}_v \hat{g}] = 0\), \(\dot{\alpha}\) is conserved:

\[
\frac{d}{d\lambda} \left[ \hat{g}_{\mu\nu} v^\mu \frac{dx^\nu}{d\lambda} \right] = 0.
\]  

(39)

The metric (6) is time-independent and spherically symmetric (in particular, it is invariant under \(t \rightarrow t + \delta t\) and \(\varphi \rightarrow \varphi + \delta \varphi\)). Therefore, we have the following Killing vectors associated with the above symmetries

\[
\xi^\alpha = (1, 0, 0, 0), \quad \eta^\alpha = (0, 0, 0, 1).
\]  

(40)

Since the metric is independent of the \(t\)- and \(\varphi\)-coordinates, we can construct the following conserved quantities

\[
e = -\xi \cdot u = -\xi^\alpha u_\alpha \hat{g}_{\alpha\beta} = -\hat{g}_{\alpha\beta} u^\alpha = -\hat{g}_{\alpha\beta} u^\alpha = S(r) \left( 1 - \frac{2M}{r} \right) \frac{dt}{d\lambda} = S(r) \left( 1 - \frac{2M}{r} \right) \dot{t},
\]  

(41)

\[
\ell = \eta \cdot u = \eta^\alpha u_\alpha \hat{g}_{\alpha\beta} = \hat{g}_{\alpha\beta} u^\alpha = \hat{g}_{\phi\phi} u^\phi = S(r) r^2 \sin^2 \theta \dot{\varphi},
\]  

(42)

where the null vector

\[
u^\alpha = \frac{dx^\alpha}{d\lambda}
\]  

(43)

satisfies

\[
u \cdot u = \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0.
\]  

(44)

From (44), we get the following equation

\[-\left( 1 - \frac{2M}{r} \right) \ell^2 + \frac{\dot{\ell}^2}{1 - \frac{2M}{r}} + r^2 \sin^2 \theta \dot{\varphi}^2 = 0.
\]  

(45)

Note that the rescaling of the metric cancels out in the above equation (45) for null geodesics, but \(S(r)\) will appear again when the conserved quantities (41) and (42) are taken into account. Let us solve (41) for \(\dot{\ell}\) and (42) for \(\dot{\varphi}\) and, afterwards, replace the results in (45). The outcome is:

\[-\frac{e^2}{S(r)^2} \left( 1 - \frac{2M}{r} \right) \ell^2 + \frac{\dot{\ell}^2}{1 - \frac{2M}{r}} + \frac{\ell^2}{S(r)^2} = 0.
\]  

(46)

Let us focus on the radial geodesics (i.e. \(\ell = 0\)), which will be sufficient to verify the geodesic completeness. Equation (46) simplifies, for \(r > 2M\), to

\[-\frac{e^2}{S(r)^2} + r^2 = 0 \implies S(r)|\dot{r}| = |e|.
\]  

(47)

For the sake of simplicity, from now on we assume \(e > 0\). The above (47) first order differential equation can be easily integrated for a photon trajectory approaching \(r_g = 2M\), namely for \(r < 0\).

\[
\lambda e = 2r_g \log \left( \frac{r_0 - r_g}{r - r_g} \right) + r \left( 1 - \frac{r_0 - r_g}{r_g} \right) - \frac{r_0 r_g}{r_0 - r_g} + r_0.
\]  

(48)

It turns out that photons cannot reach \(r = 2M\) for any finite value of the affine parameter \(\lambda\).

**IV. GRAVITATIONAL BLUESHIFT**

Let us look at the time measured by two observers located, respectively, at the radial coordinates \(r_1\) and \(r_2 > r_1\). In the limit of very large \(r_2\), we have

\[
\frac{\Delta r_2}{\Delta r_1} = \sqrt{\frac{S(r_2) \left( 1 - \frac{r_2}{r_1} \right)}{S(r_1) \left( 1 - \frac{r_2}{r_1} \right)}} \rightarrow \sqrt{\frac{1}{S(r_1) \left( 1 - \frac{r_2}{r_1} \right)}}.
\]  

(49)
FIG. 1: Left-plot: the affine parameter $\lambda(r)$ given in (48) for the rescaling $[8]$, $e = 1$, $r_0 = 3M$, $M = 5$. Right-plot: the affine parameter $\lambda(r)$ given in (48) for the rescaling $[9]$, $e = 1$, $r_0 = 3M$, $M = 5$.

The main difference with respect to the black hole case consists on a blueshift instead of a redshift as evident looking at the ratio (49). Indeed, for every rescaling except $[7]$ and $[19]$ vanishes near the horizon whereas in the black hole case it diverges. Notice that for $[2]$ there is neither gravitational redshift nor gravitational blueshift because $\frac{\Delta \tau_2}{\Delta \tau_1} = 1$. In Fig. [1] we compare the ratio $\Delta \tau_2/\Delta \tau_1$ evaluated for the rescalings $[8]$ and $[9]$.

Moving now to the frequency issue, since the frequency is the inverse of proper time, namely $\omega = 1/\tau$, we get

$$\omega_2 = \omega_1 \sqrt{S(r_1) \left(1 - \frac{r_g}{r_1}\right)} ,$$

where $\omega_1$ is the frequency measured in the proper time of the source emitting the light and $\omega_2$ the frequency measured by an observer at infinity. In the Schwarzschild case, when $r$ approaches $r_g$ the frequency seen by an observer at
infinity goes to zero, while in the metric \[9\] the frequency goes to infinity, except for \[7\].

Let us now consider the black hole emission of Hawking’s particles that reach infinity with a particular (typically small) temperature \[28\]. When this temperature is traced back from infinity to the event horizon, it turns out to be Trans-Planckian at the Planck distance from \(r_g\). Indeed, nothing can reach the event horizon, but near the horizon their energy tends to zero, as evident from equation (52). Therefore, conformalons can easily solve the Trans-Planckian problem because of their geometrical conformal structure. Once again, it is simply classical geometry to solve an issue apparently traceable to quantum gravity effects.

\[\begin{align*}
T_\infty &= T_H = T_1 \sqrt{\left(1 - \frac{r_g}{r_1}\right)} \quad \Rightarrow \quad T_1 = \frac{T_H}{\sqrt{\left(1 - \frac{r_g}{r_1}\right)}} \to ^+\infty \quad \text{for} \quad r_1 \to r_g. \\

\text{In the past, this argument made people question the validity of Hawking’s original derivation that was based on a semi-classical approach } \[28\][34].

\text{However, for conformalons, the presence of the rescaling makes the temperature vanishing near } r_g,

\[T_\infty = T_H = T_1 \sqrt{S(r_1) \left(1 - \frac{r_g}{r_1}\right)} \quad \Rightarrow \quad T_1 = \frac{T_H}{\sqrt{S(r_1) \left(1 - \frac{r_g}{r_1}\right)}} \to 0 \quad \text{for} \quad r_1 \to r_g.\]

As we will show in the next section, these objects have the same Hawking’s temperature as a Schwarzschild black hole at infinity, but near the horizon their energy tends to zero, as evident from equation (52). Therefore, conformalons can easily solve the Trans-Planckian problem because of their geometrical conformal structure. Once again, it is simply classical geometry to solve an issue apparently traceable to quantum gravity effects.

V. HAWKING TEMPERATURE AND EVAPORATION TIME

The Hawking temperature is proportional to the surface gravity, which is invariant under a Weyl conformal transformation \[2\]. In order to prove the latter statement, we recall the definition of the surface gravity at the event horizon,

\[\kappa_H = \lim_{r \to 2M} \frac{1}{2} \frac{\partial_r g_{00}}{\sqrt{-g_{00} g_{11}}} = \lim_{r \to 2M} \frac{1}{2} \frac{\partial_r \left[S(r) \left(1 - \frac{2M}{r}\right)\right]}{\sqrt{S^2(r)}} = \lim_{r \to 2M} \frac{1}{2} \frac{(\partial_r S(r)) \left(1 - \frac{2M}{r}\right) + S(r) \frac{2M}{r}}{\sqrt{S^2(r)}} \equiv \frac{1}{4M}.\]

The above result is for any general rescaling \(S(r)\). Therefore, the temperature is conformally invariant and in natural units reads:

\[T_H = \frac{1}{8\pi M}.\]

On the other hand, the area of the event horizon is: \(A = 4\pi S(2M)^2\), which is singular for \(r = 2M\). Hence, according to the Boltzmann law, the evaporation time is obtained integrating the following equation,

\[-\frac{dM}{dt} = \sigma A_H T_H^4 = \sigma S[2M(1 + \delta)][2M(1 + \delta)]^2 \left(\frac{1}{8\pi M}\right)^4,\]

where the infinitesimal dimensionless parameter \(\delta\) has been introduced to regularize the event horizon’s area because most of the Hawking particles are created near – but not at – the horizon. Indeed, nothing can reach the event horizon in a finite amount time as proved in the previous section. As a particular example, eq. (55) for the rescaling \[8\] takes the form:

\[-\frac{dM}{dt} = \frac{\sigma}{1024\pi^4\delta^4} \frac{(M + \delta)^4}{M^2},\]

and the solution is:

\[M(t) = (M_0^3 - k t)^{1/3} \quad k = \frac{1024\pi^4\delta^4}{3(\delta + 1)^3} \equiv \frac{\delta^4}{(\delta + 1)^3} k_{\text{Sch.}}.\]

The evaporation time is finite like for the Schwarzschild case, but much longer because \(\delta \ll 1\). A similar result will turn out for the other rescalings. So far we have proved that the evaporation time of the conformalons is finite, but longer, than for the Schwarzschild black hole.

In the spacetimes introduced in this paper, namely the conformalons, the Hawking process takes place outside of the event horizon where int-particles (of negative energy) and collapsing (entangled in Fig. \[3\]) matter interact near the event horizon, that is never achieved, till the complete exhaustion of the collapsing fuel. The whole process is explained in the caption of Fig. \[3\]. Hence, an initial pure state evolves into a final pure state according to the analysis explained in \[35].
FIG. 3: The Penrose diagram for an evaporating conformalon — This figure shows the transfer of entanglement from the collapsing matter (represented by the geodesic blue line for the pair of entangled particles shown on the surface \(\Sigma_{in}\)), which never forms the event horizon (dashed straight blue line), to the Hawking particles emitted from the conformalon. A Hawking pair is created on the Cauchy surface \(\Sigma_{a}\) and evolves to the surface \(\Sigma_{c}\) where we see two entangled pairs: the “int” (red) and “out” (black) Hawking particles on the right and two entangled black hole matter particles (green and blue). On \(\Sigma_{d}\) one of the matter particles (the blue one) and the “int” particle (red) interact and generate a new particle making a system of three entangled particles that evolve to \(\Sigma_{e}\). On \(\Sigma_{f}\) the remaining matter particles (green and blue) come very close to a new int-Hawking particle (red) created on \(\Sigma_{b}\) and in \(\Sigma_{g}\) they interact providing an entangled system of four particles: two very close the apparent horizon (AH), which never forms, and two far from the conformalon. Finally, assuming full annihilation of the green and blue particles near the AH we end up with two “out” entangled particles on \(\Sigma_{h}\) that evolve to \(\Sigma_{i}\) towards infinity. In this black hole geometry, particles take an infinite amount of time to reach the AH and interact with the int-Hawking particles outside. Thus, the interaction can happen smoothly, with no problems caused by any singularity.

conclusions

The solutions of a general class of conformally invariant theories could be proposed as an alternatives to black holes. The spacetime metric is obtained by means of a conformal rescaling the Schwarzschild metric, which is singular at the event horizon. The latter property rules out the black hole interior from our Universe, as proved looking at the dynamics of probe-particles in the resulting geometry. Indeed, neither massive nor massless particles can reach the event horizon.

In some respects conformalons remind gravitational instantons in Euclidean general relativity in which the Wick
rotation excludes the black hole interior from the spacetime. Therefore, conformalons are vacuum solutions that spontaneously break the Weyl symmetry, but preserve the general coordinate invariance. Indeed, all the observables will be invariant under diffeomorphisms. There are obviously multiple choices of this kind of vacuum solutions, but all of them with the same properties, that is: unreachable horizon, same Hawking’s temperature, same causal structure, singularity-free because there is no black hole’s interior.

A very peculiar property is the gravitational blueshift instead of a redshift. This is a remarkable difference with respect to the Schwarzschild spacetime. From the theoretical side, it provides a simple solution to the Trans-Planckian problem. Indeed, the frequency of the Hawking’s particles at infinity approaches zero when traced back to the event horizon, contrary to what happens in the Schwarzschild case where the energy diverges at the horizon. From the phenomenological point of view, it could be tested with observational data of astrophysical black holes. For example, the analysis of the reflection features commonly observed in the X-ray spectrum of accreting black holes can presumably rule out some choices of the rescaling \( S(r) \), as some of them may not be able to predict the very broadened iron lines observed in some sources. However, this would require a detailed analysis which is beyond the scope of the present paper and is postponed for the future. We can not easily measure the redshift/blueshift at a specific radial coordinate. From observations, we can only measure the spectrum of the radiation emitted from an extended region around the compact object.

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