Simulation of ultrasonic wave propagation in welds using ray-based methods

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Abstract. Austenitic or bimetallic welds are particularly difficult to control due to their anisotropic and inhomogeneous properties. In this paper, we present a ray-based method to simulate the propagation of ultrasonic waves in such structures, taking into account their internal properties. This method is applied on a smooth representation of the orientation of the grain in the weld. The propagation model consists in solving the eikonal and transport equations in an inhomogeneous anisotropic medium. Simulation results are presented and compared to finite elements for a distribution of grain orientation expressed in a closed-form.

1. Introduction

Ultrasonic Non Destructive Testing (NDT) is a major technique used in various industrial sectors such as nuclear, aerospace, petrochemical for the inspection of welds. Because of their polycrystalline structure, the detection and characterization of defects located inside or in the vicinity of welds can be complicated. Some disturbances of the beam such as splitting and skewing can be observed on experimental structural echoes [1]. In this context, the simulation of ultrasonic inspection is helpful to have a better understanding of the physical phenomena in complex media. Numerical methods such as finite element [2] may be used to simulate the propagation of ultrasonic waves in welds. Alternatively, a semi-analytic propagation model based on ray theory has been implemented in the CIVA software [3] [4]. This model has to be applied on a piecewise description of the weld. A set of several homogeneous domains with a constant crystallographic orientation is considered. In this case, the rays propagate along a constant direction of energy for a given slowness vector and the reflection and refraction are calculated at each interface. In this paper, we propose to extend the ray model implemented in CIVA to take account of a continuously variable description of the grain orientation in the weld.

2. Context: Ultrasonic simulation of wave propagation in welds

A weld is an anisotropic and inhomogeneous medium with an internal grain organization that depends on the steel shade and the parameters of the welding process such as the velocity, the electrode diameter or the welding technique. The difficulties of on-site inspection are due to this polycrystalline structure possibly responsible for beam splitting and skewing. The Dynamic Ray Tracing method (DRT) usually applied in Geophysics [5] will be used to simulate the ultrasonic propagation in welds. The geometry of the weld, the elasticity constants and the attenuation
of the welding materials and the crystallographic orientation of the grain at any position in the weld are required as input data for DRT. These data may be obtained either in a closed-form [6] or thanks to an image processing technique [7] applied on the macrograph as shown in Figure 1. It is important to note that, as the DRT relies on a high frequency approximation, it has to be applied on a slowly continuously varying description of the orientation of the weld.

Figure 1. Possible descriptions of the crystallographic orientation of a weld: (a) Closed-form expression proposed by Ogilvy [6] for a V-shaped weld, (b) Description obtained from a macrograph for a bimetallic weld [7].

To evaluate the ray-paths and the travel time, we have to solve the eikonal equation (1).

\[ \nabla^2 T = \frac{1}{c(x)^2} \]  

The eikonal equation represents a non linear partial differential equation of the first order for the travel time \( T(x) \). It describes the propagation of the wavefront \( T(x) \) of the considered wave and depends on the phase velocity \( c(x) \) of this wave.

A second equation called the transport equation (2) represents a linear partial differential equation of the first order in \( A(x) \), the amplitude function. It is solved along a ray \( \Omega \) and describes the conservation of the energy inside a ray tube.

\[ A(x) \nabla^2 T(x) + 2 \nabla A(x) \cdot \nabla T(x) = 0 \]  

3. Dynamic Ray Tracing Model for smooth description of weld

3.1. Ray theory
To obtain the ray trajectories, a differential ray tracing system is derived from the eikonal equation [5] and written in Hamiltonian form:

\[
\begin{align*}
\frac{dx_i}{dT} &= \partial_{a_{ijkl}} p_l g_j^{(m)} g_k^{(m)} = V_i^e, \\
\frac{dp_i}{dT} &= -1 \partial_{a_{ijkl}} p_k p_l g_j^{(m)} g_i^{(m)}, \\
\end{align*}
\]
where $T$ is the travel time, $a_{ijkl}$ the elasticity constants of the medium at position $x_i$ normalized by the density $\rho$. $p_i$ represents the components of the slowness vector, $g_j^{(m)}$ the eigenvectors of the Christoffel tensor [5] are the components of the polarization vector and $V_i^e$ is the energy velocity. This system called axial ray system is composed of two coupled ordinary differential equations. The first one describes the variations of the position with respect to the travel time and the second one the variations of the slowness.

![Figure 2. Principle of an iterative solving of a ray system.](image)

In order to obtain the trajectories of the ray, the axial ray system has to be solved. Taking into account the initial conditions of the system, we can use classic iterative techniques (Figure 2) such as the Euler or Runge-Kutta methods [8].

The amplitude of the ray is evaluated by solving the transport equation (2) in inhomogeneous anisotropic media. By deriving the axial ray system with respect to an initial parameter $\gamma$, we obtain a system of linear ordinary differential equations of the first order for $Q_i(x)$ and $P_i(x)$ (4). These quantities represent respectively the first partial derivatives of $x_i$ and $p_i$ with respect to $\gamma$. The parameter $\gamma$ may represent any parameter of the ray $\Omega$. For example, it can be chosen as the take-off angle between the axial ray and a paraxial one.

$$\begin{cases}
\frac{d}{dT} \left( \frac{dx_i}{d\gamma} \right) &= \frac{dQ_i^{(x)}}{dT} = \frac{1}{2} \frac{\partial^2 G^{(x)}}{\partial p_i^{(x)} \partial \gamma} = A_{ij}^{(x)} Q_j^{(x)} + B_{ij}^{(x)} P_j^{(x)}, \\
\frac{d}{dT} \left( \frac{dp_i^{(x)}}{d\gamma} \right) &= \frac{dP_i^{(x)}}{dT} = -\frac{1}{2} \frac{\partial^2 G^{(x)}}{\partial x_i \partial \gamma} = -C_{ij}^{(x)} Q_j^{(x)} - D_{ij}^{(x)} P_j^{(x)}.
\end{cases} \quad (4)$$

$G$ represents any of the three eigenvalues of the Christoffel tensor and is expressed as $G_m^{(x)} = a_{ijkl} p_j^{(x)} p_l^{(x)} g_i^{(m)} g_k^{(m)}$. Three eigenvalues are evaluated, associated to three eigenvectors $g_i^{(m)}$ of the Christoffel tensor representing the three plane waves that propagate in the medium.

The paraxial ray system is solved simultaneously with the axial ray system by using a numerical iterative technique. This paraxial ray system detailed in [5] can be written with a matrix formulation (5). The equation (4) is the paraxial ray system expressed in a Cartesian regular coordinates system $x_i$ where the indexes $i$ and $j$ are equal to 1, 2 or 3. Indeed the paraxial ray system in this coordinate system consists of six scalar linear ordinary differential equations. It may be more convenient to express the paraxial ray system in a coordinate system that allows to decrease the number of equations. The expression (5) also represents the paraxial ray system in a wavefront orthonormal coordinates system $y_i$ where the indexes $M$ and $N$ are equal to 1 or 2. In this coordinate system, the origin moves along the central ray with the propagating wavefront. The $y_3$ axis is oriented along the slowness vector $\vec{p}$ at the origin point, and axes $y_1$ and $y_2$ are in the plane tangent to the wavefront at the origin point and are mutually perpendicular. The system consists then of a four scalar linear differential equations.
Mathematically, a paraxial ray of the tube is described as a 4x1 vector composed of the paraxial quantities [5]. $Q_1$ and $Q_2$ represent the spatial deviation of the paraxial ray from the axial ray respectively in the directions $x_1$ and $x_2$ and $P_1$ and $P_2$ the slowness deviation of the paraxial ray from the axial one in the same directions as shown in Figure (3).

\[
\begin{pmatrix}
Q^{(y)(r+1)}_M \\
M^{(y)(r+1)}_M
\end{pmatrix} =
\begin{pmatrix}
\Pi^{(y)}_{11} & \Pi^{(y)}_{12} \\
\Pi^{(y)}_{21} & \Pi^{(y)}_{22}
\end{pmatrix}
\begin{pmatrix}
Q^{(y)(r)}_N \\
P^{(y)(r)}_N
\end{pmatrix} =
\begin{pmatrix}
1 + A^{(y)}_{MN}\Delta T & B^{(y)}_{MN}\Delta T \\
-C^{(y)}_{MN}\Delta T & 1 - D^{(y)}_{MN}\Delta T
\end{pmatrix}
\begin{pmatrix}
Q^{(y)(r)}_N \\
P^{(y)(r)}_N
\end{pmatrix}.
\]

(5)

At each time step, the eikonal and transport equations are solved. This means that the matrices $A^{(y)}_{MN}, B^{(y)}_{MN}, C^{(y)}_{MN}$ and $D^{(y)}_{MN}$ are re-evaluated at each step. The propagation matrix is then updated as shown in Figure 4 and the position and the slowness vector evaluated from the axial ray system.

Then, the geometrical spreading of the ray tube between the iterations (0) and (r + 1) can be evaluated thanks to the $\Pi_{12tot}$ matrix of the propagation matrix between the source and the receiver points (7):

\[
L(r + 1, 0) = |det\Pi_{12tot}(r + 1, 0)|^{1/2}.
\]

(7)

In order to apply this method, we have made an approximation that consists in considering the medium locally homogeneous between each time step. This means that the matrices $\Pi_{11tot}$ and $\Pi_{22tot}$ are equal to the identity matrix and the matrix $\Pi_{21tot}$ is equal to zero at each time step. The matrix $\Pi_{12tot}$ is the only one re-evaluated at each time step.
3.2. Application and validation of the model

We have applied the dynamic ray tracing model using a closed-form expression to describe the distribution of grain orientation proposed by Ogilvy [6]:

\[
\theta = \begin{cases} 
\arctan \left( \frac{T(D + z \tan \alpha)}{x^n} \right), & \text{for } x \geq 0, \\
-\arctan \left( \frac{T(D + z \tan \alpha)}{(-x)^n} \right), & \text{for } x < 0.
\end{cases}
\]  

(8)

This mathematical description is adapted for V-shaped weld composed of a transversely isotropic medium. Parameters \( D \) and \( \alpha \) are geometrical: \( D \) is the half-width of the bottom of the weld and \( \alpha \) is the angle between the z-axis and the beveled edge as shown in Figure 5(a). The quantity \( \eta \) describes the variation of the orientation in the \( x \) direction and \( T \) is proportional to the tangent of the grain axis at the beveled edge. The ray trajectories obtained with the dynamic ray tracing model have been compared to the results of Connolly’s PhD thesis [9]. Connolly also used a ray-based model to simulate the propagation of ultrasonic waves in welds but at each time step, virtual interfaces are created so that the reflection and transmission coefficients can be evaluated. The parameters for (8) are: \( D = 2 \text{mm}, \alpha = 21.80^\circ, T = 1 \) and \( \eta = 1 \). In Figure 5(b) we can observe a very good agreement between both models for a ray propagating through the V-weld.

A hybrid finite element code developed by EDF and CEA is used as a validation tool [10] of the computed wave field. In this model, a finite element computation made by ATHENA is performed in a box containing the weld and takes an incident field computed by CIVA as input data. A comparison of the wave field is shown in Figure 6 for the same V-weld described with (8). A good agreement is observed on the wave field evaluated by both models. But we can observe slight discrepancies in the middle of the weld where the dynamic ray tracing model seems to overestimate the amplitude. We suppose that these differences between the two models can be explained by the approximation considering the medium locally homogeneous between each time step. Indeed, the paraxial quantities are not rigorously evaluated so the geometrical spreading is approximated.

\[\begin{pmatrix} Q(0) \\ P(0) \end{pmatrix}, \begin{pmatrix} Q(1) \\ P(1) \end{pmatrix}, \begin{pmatrix} Q(2) \\ P(2) \end{pmatrix}, \begin{pmatrix} Q(r) \\ P(r) \end{pmatrix}, \begin{pmatrix} Q(r+1) \\ P(r+1) \end{pmatrix}\]

\(L_0, L_1, L_2, L_3, L_{r-1}, L_r\)

\(S\)

\(M\)
Figure 5. (a) Parameters of a closed-form description of the crystallographic orientation of the weld proposed by Ogilvy [6]. (b) Representation of the ray trajectories in a bimetallic weld described by this law - (oo) Previously published results [9] and (−) Dynamic ray tracing results.

Figure 6. Comparison of the maximum particle velocity field for L60° inspection at 2, 25 MHz obtained with (a) (−) the dynamic ray tracing model implemented in the CIVA software and (b) (−) the hybrid finite element model (CIVA/ATHENA).

4. Conclusion and perspectives
A ray-based model has been applied on a continuously varying representation of the orientation of the grains in a weld. This method has been validated with the literature for a V-weld described by a closed-form expression. But some discrepancies most likely due to the approximated evaluation of the paraxial quantities and the geometrical spreading are observed in the computed wave field obtained with a hybrid finite element model and the ray-based model that has been developed.

In order to evaluate more precisely the wave field we are working on a rigorous computation of the paraxial quantities (5) and therefore the geometrical spreading. We intend to validate it numerically and experimentally on a realistic continuously varying description obtained from a macrograph of a weld thanks to an image processing technique.
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