A New Code for Fourier-Legendre Analysis of Large Datasets – First Results and a Comparison with Ring-Diagram Analysis

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Fourier-Legendre decomposition (FLD) of solar Doppler imaging data is a promising method to estimate the sub-surface solar meridional flow. FLD is sensitive to low-degree oscillation modes and thus has the potential to probe the deep meridional flow. We present a newly developed code to be used for large scale FLD analysis of helioseismic data as provided by the Global Oscillation Network Group (GONG), the Michelson Doppler Imager (MDI) instrument, and the upcoming Helioseismic and Magnetic Imager (HMI) instrument. First results obtained with the new code are qualitatively comparable to those obtained from ring-diagram analysis of the same time series.

1 Introduction

From summer 2010 on, the Helioseismic and Magnetic Imager (HMI) instrument aboard the Solar Dynamics Observatory (SDO) satellite is expected to deliver continuous high resolution (4 k × 4 k) full-disk Doppler images at a cadence of 45 seconds. The availability of this data will surely mark a new era for helioseismology, but the huge amount of data that HMI will produce also poses a challenge for helioseismic data processing pipelines. With the given resolution and cadence, the HMI full-disk Doppler images alone will sum up to roughly 2 terabytes per month.

The current major sources of solar full-disk Doppler data are the Global Oscillation Network Group (GONG; [Harvey et al. 1996]) and the Michelson Doppler Imager (MDI; [Scherrer et al. 1995]) instrument aboard the Solar and Heliospheric Observatory (SOHO) spacecraft. They deliver Dopplergrams with a maximum volume of about 100 gigabytes per month, each. Both GONG and MDI have been operating for more than one decade and it is exciting that one can now apply helioseismic techniques to data spanning about one solar cycle from two independent observatories.

The characteristics of the solar meridional circulation are an important ingredient to some solar dynamo models, see e.g. the review by [Dikpati & Gilman 2009] and references therein. A poleward flow of the order of 10 to 20 m/s can be derived at the surface and in the near-surface layers by a variety of techniques (e.g. [Duvall 1979]; [Haber et al. 2002]; [Hathaway 1996]; [Komm et al. 1993]; [Wöhl & Brajša 2001]) but no significant evidence for a return flow in deeper layers has been detected so far.

Fourier-Legendre spectral decomposition (FLD) is sensitive to low-degree modes and thus has the potential to detect flows in deep layers. From encouraging results of previous research ([Braun & Fan 1998]; [Krieger et al. 2007]) and the progress of our ongoing work, we are confident that the Fourier-Legendre analysis can become a standard tool for meridional flow measurements below the surface along with ring-diagram analysis ([Hill 1988]) and time-distance helioseismology ([Duvall et al. 1993]).

However, this requires a data processing pipeline that is able to handle efficiently the available and upcoming helioseismic data. In this paper, we present a newly developed code for Fourier-Legendre analysis that fits these requirements. The code has been subject to several tests. Here we present first results obtained with this code and compare them to results from ring-diagram analysis of the same time series. It is our intention making the code available to the helioseismology community as part of the European Helio- and Asteroseismology Network (HELAS).

2 Methods

2.1 The Fourier-Legendre spectral decomposition

The “Fourier-Hankel spectral decomposition” technique has been used to study p-mode scattering by sunspots ([Bogdan et al. 1993]; [Braun et al. 1987]; [1992]). Later the method was also applied for sub-surface meridional flow measurements by [Braun & Fan 1998] and [Krieger et al. 2007].

In plane geometry, Hankel functions are a good approximation to Legendre functions and in some of the publications mentioned above they were used instead of Legendre functions because they are numerically much easier to compute. In this work we do not use Hankel functions, and to
avoid confusion we prefer to use the more descriptive notation “Fourier-Legendre decomposition” for the remainder of this paper.

Following Braun et al. (1988) and Braun & Fan (1998), the surface oscillation signal $\delta V(\theta, \phi, t)$ within an annular region around a point of interest can be represented as a superposition of inward and outward traveling waves of the form

$$\delta V(\theta, \phi, t) = \sum_{l m \nu} \left[ A_{l m \nu} X_l^m(\theta) + B_{l m \nu} \left( X_l^m(\theta)^* \right) e^{i(m \phi + 2\pi \nu t)} \right],$$

(1)

where $A_{l m \nu}$ and $B_{l m \nu}$ are the complex amplitudes of the oscillation modes of the temporal frequency $\nu$, the harmonic degree $l$ and azimuthal order $m$. The angle $\theta$ equals zero in the center of the region of interest and increases with radial distance from the center. The basis functions and its complex conjugate $X_l^m(\theta)^*$ are superpositions of the associated Legendre functions of first and second kind $P_l^m(\cos \theta)$ and $Q_l^m(\cos \theta)$, and $N_l^m = (-1)^m \frac{(l-m)!}{(l+m)!}$ is a normalization factor.

For meridional flow measurements, the center of the annular region is identical to either the northern or southern pole, so that $A_{l m \nu}$ and $B_{l m \nu}$ become the poleward and equatorward waves, respectively.

Using equation (1), the mode amplitudes can be extracted from the measured Doppler signal by

$$A_{l m \nu} = \frac{C_l}{2\pi T} \int_0^{\pi} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \delta V(\theta, \phi, t) e^{-i(m \phi + 2\pi \nu t)} d\theta d\phi dt,$$

$$B_{l m \nu} = \frac{C_l}{2\pi T} \int_0^{\pi} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \delta V(\theta, \phi, t) e^{-i(m \phi + 2\pi \nu t)} X_l^m(\theta)^* d\theta d\phi dt,$$

(3)

(4)

where $C_l$ is a normalization factor which is approximately given by $\pi \sqrt{l(l+1)/2(\theta_{\text{max}} - \theta_{\text{min}})}$, $\theta_{\text{max}}$ and $\theta_{\text{min}}$ mark the latitudinal extent of the annular region, and $T$ is the total length of the observed time series.

3 The numerical code

The decomposition code is written in the C language for efficiency and portability. It is parallelized using the Message Passing Interface (MPI) standard. Further steps in the post-processing pipeline such as peak fitting and the inversion are currently implemented as prototype code in IDL (Interactive Data Language).

The design goals for the code were: 1) to address the most relevant issues and open questions that arise from previous publications, 2) to provide the flexibility to easily use data from common sources such as MDI, GONG and the upcoming HMI, 3) to write a code that is fast enough to process all available data in reasonable time, and 4) to make the code flexible enough to not only use it for meridional flow measurements, but also to measure p-mode absorption in sunspots as demonstrated by Braun et al. (1988).

We addressed the issues as discussed by Braun & Fan (1998) and Krieger et al. (2007). This includes proper handling of the spherical geometry by using Legendre functions instead of Hankel functions (this was already done by Braun & Fan, though) and an inversion for the flow-profile.

Requirement 2) is implemented by following a modular concept: for each data source, a plug-in module is provided that knows how to read data from the respective source and forwards the Doppler images to the decomposition module in a standardized format. Goal 4) will be implemented in the future.

On a modern quad-core processor, one month of GONG data can be processed in less than one day with a setup as we use it in this paper (see below). The decomposition into the mode coefficients only takes a minor fraction of the total runtime and the further steps are not optimized for speed yet, so that we expect future versions of the code to run much faster. The code also scales well up to the amounts of data we expect from the HMI instrument later this year.

3.1 Data preparation

The Doppler images are processed in chunks. Each chunk consists of a configurable number of Dopplergrams which usually equals to 60 for the one-minute cadence of GONG and MDI.

The Dopplergrams are interpolated to an equidistant heliocentric $\theta, \phi$-grid. Effects of the solar rotation and relative movement between instrument and Sun are removed by subtracting the mean Dopplergram of one chunk from each Dopplergram in the chunk. Each Dopplergram is then apodized using a Hann window function to avoid spatial aliasing in the resulting power spectra. Bad or missing Dopplergrams are detected and the mode amplitudes of the corresponding timestep are set to zero. A binary mask is stored along with the time series that allows to properly detect and account for gaps in the time series.

3.2 Decomposition & frequency fitting

For the decomposition of the mode coefficients, we follow an approach that was first used by Brown (1985) for fast Spherical Harmonic decomposition of solar oscillation data. The integration in longitude in Eqs. (3) and (4) is implemented as a Fast Fourier Transform (FFT). Compared to a straightforward numerical evaluation of the integrals, this results in a greatly improved performance of the overall computation.
Because the associated Legendre functions $P_{l}^{m}$ and $Q_{l}^{m}$ can only be calculated reliably using a recursion relation (Zhang 1996), we precompute the basis functions $X_{l}^{m}$ for all combinations of $l$, $m$ and $\theta$. Since we are only interested in low azimuthal orders $m = -25 \ldots +25$, the lookup tables consume a few hundred megabytes and fit easily into main memory.

The resulting time series for the complex amplitudes $A_{lm}$ and $B_{lm}$ are stored as FITS (Flexible Image Transport System) binary tables for the further steps in the processing pipeline. From the power spectra of $A_{lm}$ and $B_{lm}$ the peak frequencies are determined by fitting asymmetric Lorentzian profiles. From these fits the frequency differences between the poleward and equatorward propagating waves are determined.

### 3.3 Inversion

The frequency shift between the poleward- and equatorward propagating waves is used to invert for the horizontal component of the sub-surface meridional flow. The frequency shift $\Delta \nu$ is related to the meridional flow by (Gough & Toomre 1983)

$$\Delta \nu_{nl} = \frac{l}{\pi R_{\odot}} \int_{0}^{\infty} \langle U_{mer}(r) \rangle K_{nl}(r) \, dr,$$

where $R_{\odot}$ is the solar radius, $\langle U_{mer}(r) \rangle$ is the mean meridional flow averaged over the patch, and $K_{nl}(r)$ are the energy density kernels, which are calculated from solar 'Model S' by Christensen-Dalsgaard et al. (1996). Based on this equation, inversions for the meridional flow are carried out by employing a SOLA technique (Subtractive Optimally Localized Averaging) as described in Pijpers & Thompson (1992, 1994). Taking measurement errors of the frequencies into account, the inverted solution has to be regularized. For simplicity, we used one regularization parameter for all positions in the Sun. We selected a rather large regularization parameter in order to give trust only to those modes where a precise frequency measurement was possible.

### 4 Results

For a first test, we use GONG data from January and February 2006. These data sets where chosen because at the beginning of this period the GONG duty cycle was comparably high (91%) and it is in the middle of the declining phase of the past solar cycle, so that a minimum of temporal change in the flow pattern can be estimated.

To estimate the meridional flow as a function of depth and latitude, the Dopplergrams are subdivided into smaller patches. All patches have a height of 16 degree in latitude while the longitudinal extent depends on the position on the disk in order to avoid foreshortened regions. The location and width of the patches we used is shown in Table 1.

| center latitude [deg] | longitudinal extent [deg] |
|-----------------------|---------------------------|
| ± 45.0                | -30.0 to +30.0            |
| ± 37.5                | -45.0 to +45.0            |
| ± 30.0                | -45.0 to +45.0            |
| ± 22.5                | -52.5 to +52.5            |
| ± 15.0                | -52.5 to +52.5            |
| ± 7.5                 | -52.5 to +52.5            |
| 0                     | -52.5 to +52.5            |

These are the same parameters that were used for positioning 16x16 degree square patches for the ring-diagram analysis. For each patch the Fourier-Legendre decomposition is carried out and the meridional flow is determined from the frequency shifts between pole- and equatorward propagating waves of harmonic degrees $l = 100 - 1000$ and radial order $n = 0 - 11$.

### 4.1 Comparison with ring-diagram analysis

Figure 1 shows cross sections of the flow profiles at depths of 3, 5 and 7 Mm obtained from Fourier-Legendre decomposition (left panel) and ring-diagram analysis (right panel). With both techniques the measurement error is lowest close to the surface and at the equator ($\pm 0.2 \text{ m/s}$ for FLD and ring-diagram analysis) and highest in the deep layers and at high latitudes ($\pm 0.5 \text{ m/s}$ for ring-diagram analysis and $0.8 \text{ m/s}$ for FLD).

Both methods reproduce the same qualitative features of the flow profile. The direction of the flow is mainly poleward in the order of $20 \text{ m/s}$. FLD and ring-diagram both favor a weak equator-crossing flow but with opposite sign. With both techniques the derived flow velocities increase with latitude. There is a clear asymmetry between both hemispheres visible in the velocity profiles from both methods. On the northern hemisphere the velocities tend to decrease with depth, whereas the curves for the different depth agree within their error marings down to -22.5 degree latitude. Interestingly, both methods show the same bend in the velocity profile at -7.5 degree latitude.

### 5 Conclusions & Discussion

Combined with helioseismic inversion techniques Fourier-Legendre decomposition of Doppler imaging data can be used to derive the sub-surface solar meridional flow. The method is sensible to low-degree modes and thus could greatly increase the range in depth that is currently accessible to other methods such as ring-diagram analysis.

We developed a new numerical code suitable for Fourier-Legendre analysis of large sets of Dopplergrams as provided by GONG, MDI and the upcoming HMI instrument. For a first test of the new code, we compared the near-surface flow velocities derived from FLD to those obtained from ring-diagram analysis of the same data sets.
Both methods result in qualitatively comparable flow velocities with small errors in the order of one percent. However, the absolute values for the velocities obtained with both methods are not in complete agreement. As for the inversion of the Fourier-Legendre data one regularization parameter was used for all positions in the Sun, the discrepancies between the two methods might be due to this systematic effect.

A careful analysis of possible sources of further systematic errors needs to be carried out in future. For the case of FLD this also includes the impact of leakage between modes of neighboring degree $l$ and order $m$ which is not taken into account in the present paper. Possible solutions are either to use only a limited set of values of $l$ for which orthogonality of the Legendre functions is guaranteed (Braun et al. 1988). Another possibility is to include the covariance matrix of the mode coefficients in the procedure for the frequency determination of the modes.

Inversions for the meridional flow in deeper layers of the Sun as well as long-term studies of the variability of the flow will be carried out when the open issues are resolved in the near future.

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