Sphalerons in the Standard Model with a real Higgs singlet

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Abstract

Sphaleron energies within the standard model with a real Higgs singlet added on are calculated. The coupled non-linear equations of motion are numerically solved and the sphaleron energy evaluated for a set of parameters in the Higgs potential. I find a small difference in the sphaleron energy compared to the standard model. A slightly stronger constraint on the strength of the first order phase transition thus results for this model.

1 Introduction

If the baryon asymmetry of the universe was generated at the electroweak phase transition, as many models are now speculating [1], it is important that the rate of baryon violation be determined more accurately. This rate in the broken symmetry phase is directly related to the energy of the sphaleron configuration in the model, because the sphaleron energy sets the height of the energy barrier between the topologically inequivalent vacua, and baryon violation occurs whenever such a vacuum transition takes place. To preserve any baryon asymmetry created at the phase transition the baryon violation rate must be suppressed in the broken phase and this imposes a constraint on the sphaleron energy at the phase transition. Hence it is important to calculate the sphaleron energy within each model.
being considered. It is in any case interesting to know the explicit form of the sphaleron configuration in extensions of the Standard Model.

Manton et al. calculated the sphaleron energy within the Standard Model (SM) using a radially symmetric ansatz for the gauge and Higgs fields. They obtained

\[ E_{\text{sph}} = \frac{4\pi v}{g} B \left( \frac{\lambda}{g^2} \right) \]  

(1)

where \( v \) is the Higgs vacuum expectation value (VEV), \( g \) is the weak gauge coupling and \( \lambda \) is the quartic self coupling of the Higgs boson. Their calculations showed \( 1.5 \leq B \leq 2.7 \) for \( \lambda/g^2 \) between 0 and \( \infty \). Since the baryon violation rate in the broken phase is proportional to \( e^{-E_{\text{sph}}/T} \), suppression of the baryon violation rate in the broken phase requires

\[ \frac{E(T_c)}{T_c} \gtrsim 45, \]  

(2)

independent of the model, where \( T_c \) is the critical temperature for the phase transition. Hence in the SM, one obtains

\[ \frac{v}{T_c} \gtrsim 1.4 \]  

(3)

as the condition on the strength of the first order phase transition required.

Because this constraint seems so restrictive in the SM [apart from other possible inadequacies like the strength of Kobayashi-Maskawa (KM) CP violation] it seems likely that an extension of the SM is required to generate sufficient baryon asymmetry at the electroweak phase transition. Hence it is important that the sphaleron energy be calculated in these models. In this paper I consider the sphaleron solutions in the minimal SM augmented by a real singlet Higgs field. This model was shown to be capable of satisfying at least one of the several conditions needed for electroweak baryogenesis, namely the condition that any baryon asymmetry created at a first order phase transition does not get washed away by subsequent sphaleron processes. This model is interesting due to the existence of tree-level trilinear Higgs couplings which seem to enhance the possibility of satisfying this condition (I also point out other possible interest this model has in this regard in the discussions later). Kastening and Zhang have investigated sphaleron solutions when a complex Higgs singlet is added to the SM (their paper also applies to
the real Higgs singlet case but with the trilinear coupling terms missing) and found non-zero differences in $E_{\text{sph}}$ compared to the SM case. I too want to compare the sphaleron energy of this model with the SM. If the calculations here show a marked increase in the sphaleron energy compared to the SM value then one can expect a greater suppression of the sphaleron rate in the broken phase, relaxing the constraints on the Higgs masses somewhat. If on the other hand a considerable decrease in the sphaleron energy is seen in this model, then the first order phase transition will need to be even stronger and consequently less parameter space will be available to prevent a washout of the baryon asymmetry of the universe. The present work is thus a refinement of Ref.[4], since the minimal SM sphaleron configuration was used there as an approximation.

2 The equations of motion

A real singlet Higgs field $S$ is added to the minimal SM with its doublet Higgs field $\phi$. The Lagrangian of the gauge and the Higgs sector of the model is then:

$$\mathcal{L} = -\frac{1}{4}F_{\mu \nu}^{\mu \nu} + (D_{\mu} \phi)^\dagger (D_{\mu} \phi) + \frac{1}{2} \partial_{\mu} S \partial_{\mu} S - V(\phi, S), \quad (4)$$

where

$$F_{\mu \nu}^{\mu \nu} = \partial_{\mu} W_{\nu}^{\mu} - \partial_{\nu} W_{\mu}^{\mu} + g \epsilon_{abc} W_{b}^{\mu} W_{c}^{\nu} \quad (5)$$

$$V(\phi, S) = \lambda_{\phi} (\phi^\dagger \phi)^2 - \mu_{\phi}^2 \phi^\dagger \phi + \frac{\lambda_{S}}{2} S^4 - \frac{\mu_{S}^2}{2} S^2 - \alpha \frac{S^3}{3} + 2 \lambda (\phi^\dagger \phi) S^2 - \frac{\sigma}{2} (\phi^\dagger \phi) S. \quad (6)$$

$$D_{\mu} \phi = (\partial_{\mu} - \frac{1}{2} i g \tau^{a} W_{\mu}^{a}) \phi. \quad (7)$$

(For simplicity, the fermion fields will be set to zero as well as the $U_Y(1)$ gauge field by setting the weak mixing angle to zero.)

The Higgs potential $V(\phi, S)$ breaks the $SU_L(2) \otimes U_Y(1)$ symmetry to $U_Q(1)$ when the Higgs fields take on the VEV’s at:

$$\langle \phi(x) \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u \end{pmatrix}, \quad \text{and} \quad \langle S(x) \rangle = \frac{1}{\sqrt{2}} v, \quad (8)$$
where \( u = 246 \text{ GeV} \) and \( v \) is kept non-zero for full generality. The minimum of the potential (1) is given by \((u, v)\) where \( u, v \) are solutions to

\[
\begin{align*}
(-\mu_\phi^2 + \lambda_\phi u^2 + \lambda v^2 - \sigma v/2)u &= 0 \\
-\mu_S^2 v + \lambda_S v^3 + \lambda vu^2 - \sigma u^2/4 - \alpha v^2 &= 0.
\end{align*}
\]  
\tag{9}

I eliminate the parameters \( \mu_\phi^2 \) and \( \mu_S^2 \) in favour of one of the \((u, v)\) solutions to Eq.(9):

\[
\begin{align*}
\mu_\phi^2 &= \lambda_\phi u^2 + \lambda v^2 - \sigma v/2 \\
\mu_S^2 &= \lambda_S v^2 + \lambda u^2 - \sigma u^2/4 - \alpha v.
\end{align*}
\tag{10}

Now for static classical fields, one has

\[
\partial_0 F_\alpha^{\mu\nu} = \partial_0 \phi = \partial_0 S = 0 \quad \text{and} \quad W_0^\alpha = 0.
\tag{11}
\]

Then the equations of motion derived from \( \mathcal{L} \) are:

\[
\begin{align*}
(D_j F_{ij})^a &= -\frac{i}{2} g [\phi^{\dagger} \tau^a (D_i \phi) - (D_i \phi)^{\dagger} \tau^a \phi] \\
D_i D_i \phi &= [2 \lambda_\phi (\phi^{\dagger} \phi) - \mu_\phi^2] \phi + 2 \lambda S^2 \phi - \frac{\sigma}{2} S \phi \\
\partial_i \partial_i S &= [8 \lambda_\phi (\phi^{\dagger} \phi) - 2 \mu_S^2] S + 4 \lambda_S S^3 - \sigma \phi^{\dagger} \phi - 2 \alpha S^2.
\end{align*}
\tag{12-14}
\]

And the corresponding energy functional is:

\[
E = \int d^3x \left[ \frac{1}{4} F_\alpha^{ij} F_\alpha^{ij} + (D_i \phi)^{\dagger} (D_i \phi) + \frac{1}{2} (\partial_i S)(\partial_i S) + V(\phi, S) \right].
\tag{15}
\]

Now I use the spherically symmetric ansatz of the form:

\[
W_\alpha^a \tau^a dx^i = -\frac{2i}{g} f(gVr) dU^\infty (U^\infty)^{-1}
\tag{16}
\]

\[
\begin{align*}
\phi &= \frac{u}{\sqrt{2}} h(gVr) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
S &= \frac{v}{\sqrt{2}} p(gVr),
\end{align*}
\tag{17-18}
\]

where \( U^\infty = \frac{1}{r} \begin{pmatrix} z & x + iy \\ -x + iy & z \end{pmatrix} \) and \( V = \sqrt{u^2 + v^2} \).
Then the equations of motion become:

\[
\xi^2 \frac{d^2 f}{d\xi^2} = 2f(1-f)(1-2f) - \frac{\xi^2 u^2}{4v^2} h^2 (1-f)
\]

\[
\frac{d}{d\xi} \left( \xi^2 \frac{dh}{d\xi} \right) = 2h(1-f)^2 + \frac{\xi^2}{g^2 v^2} \left( \lambda_\phi u^2 h^3 - \mu_\phi^2 h + \lambda v^2 p^2 h - \frac{\sigma v}{2\sqrt{2}} ph \right)
\]

\[
\frac{d}{d\xi} \left( \xi^2 \frac{dp}{d\xi} \right) = \frac{4\xi^2}{g^2 v^2} \left[ (\lambda u^2 h^2 - \mu_\phi^2 / 2)p + \frac{\lambda}{2} v^2 p^3 - \frac{\alpha v}{2\sqrt{2}} p^2 - \frac{\sigma u^2}{4\sqrt{2}} h^2 \right],
\]

where \(\xi \equiv g V r\). And the energy functional becomes

\[
E = \frac{4\pi v}{g} \int_0^\infty d\xi \left\{ 4 \left( \frac{df}{d\xi} \right)^2 + \frac{8}{\pi^2} [f(1-f)]^2 + \frac{\xi^2 u^2}{2v^2} \left( \frac{dh}{d\xi} \right)^2 + \frac{u^2}{\sqrt{2}} [h(1-f)]^2 \\
+ \frac{\xi^2 v^2}{4g^2 v^2} \left[ \lambda_\phi (u^2 h^2 - \mu_\phi^2 / \lambda_\phi)^2 + (2\lambda u^2 h^2 - \mu_\phi^2) v^2 p^2 \right] + \frac{\lambda}{2} v^4 \right\},
\]

where \(\epsilon_\phi\) is the normalisation needed to set the minimum value of \(V(\phi, S)\) at zero.

Hence within the validity of the ansatz being used, the problem of finding the sphaleron energy in this model becomes equivalent to either minimizing the above energy functional (20) or solving the coupled non-linear differential equations (19). The latter method will be used in this paper, by numerically solving the equations and then calculating the value of \(E\) which corresponds to the solutions found.

Boundary conditions for the functions are required and to obtain these at \(\xi = 0\), the equations are Taylor expanded about \(\xi = 0\) and regularity is imposed there to obtain

\[
\xi \to 0 : \quad f \to a_s \xi^2, \quad h \to b_s \xi, \quad p \to c_s + c_2 \frac{\xi^2}{2},
\]

where

\[
c_2 = -\frac{c_s}{3g^2 v^2} \left[ -\frac{\sigma}{\sqrt{2}} u^2 + 4\lambda u^2 v - \sqrt{2}\alpha v^2(1 - c_s) + 2\lambda_\phi v^3(1 - c_3) \right],
\]

while at \(\xi = \infty\) the Higgs fields are made to take on their zero temperature VEV's by \(f, h, p \to 1\). The equations are actually singular at both boundaries, so for large \(\xi\) one looks for asymptotic forms of the functions. I find

\[
\xi \to \infty : \quad f \to 1 - a_t e^{-\alpha \xi}, \quad h \to 1 - b_t e^{-b \xi} / \xi, \quad p \to 1 - c_t e^{-c \xi} / \xi,
\]
where
\[ a = \sqrt{\frac{u^2}{4V^2}}, \quad b = \sqrt{\frac{2\lambda_s u^2}{g^2 V^2}}, \quad c = \sqrt{\frac{4}{g^2 V^2} \left[ -\frac{\alpha v}{\sqrt{2}} + 2\lambda_s v^2 + \frac{\sigma v^2}{2\sqrt{2}v} \right]} \].

(24)

There are three second order ODEs, so one needs to have six boundary conditions. The variables \((a_s, b_s, c_s, a_l, b_l, c_l)\) implicitly contain these boundary conditions so the solutions to the equations of motion and these variables need to be determined simultaneously.

3 Solutions and results

Because there is no analytical way of finding the solutions to the coupled non-linear ODEs of (19), numerical methods must be used. In order to do this, some values of the parameters in the potential need to be chosen, and I choose the same set here as was used in Ref.[4] for easy comparison. These sets of parameters then indicate the values for which the first order phase transition is strong enough while the mass of the smallest Higgs is above the experimental lower limit, where the minimal SM sphaleron energy is used as an approximation. Here I improve on this by using a corrected sphaleron energy.

I used the shooting method to solve the equations of motion. The results obtained for each set of parameters are tabulated in Table 1, and a typical solution is shown in Fig.1. Note that the values of \(E\) are given here in units of \(4\pi V/g\) where \(V = \sqrt{u^2 + v^2}\), unlike the SM values which are measured in units of \(4\pi u/g\). The results show that the sphaleron energies in this model are somewhat lower than the SM case as calculated by Manton et.al [2]. Similarly when compared to the SM + a complex Higgs singlet case considered by Kastening and Zhang [3], I find lower values of the sphaleron energy.

What does this mean for electroweak baryogenesis? Satisfying Eq.(2) in this model requires
\[ \frac{V}{T_c} \geq \frac{45g}{4\pi B}, \]
where \(B\) is now a function of the parameters in this model. Table II compares the values of \(45g/4\pi B\) obtained here with the \(V/T_c\) values obtained in Ref [4] at the phase transition, for each corresponding set of parameters. They show that the condition (25) is easily
still satisfied, because the first order phase transition is still too strong compared to the
decrease in the energy of the sphaleron found.

Of course one must keep in mind all the uncertainties involved in such an analysis.
The rate of sphaleron transitions in the broken phase involves prefactors to the Boltzmann
factor \[6\] which is difficult to calculate near the transition temperature due to infrared
divergences. I have also extrapolated my zero temperature sphaleron energy up to a
high temperature in making the comparison \([25]\) \([7]\) and have not taken any quantum
corrections into account \([8]\). Also, I have not considered any source of CP violation in
this model at all. There is still some debate on the question of whether or not the SM
can provide sufficiently strong CP violation in order to produce the observed baryon
asymmetry \([8]\). If it turns out that it can’t, this model being considered here will need a
new source of CP violation \([9]\). If however it turns out that the SM \textit{can} provide sufficiently
strong CP violation, then adding the real Higgs singlet will be an extremely economical
way of achieving electroweak baryogenesis as it can provide a first order phase transition
for a larger region of parameter space than can the minimal SM (Once again there is much
uncertainty in the calculations of the effective potential at finite temperature so hasty
conclusions cannot be made.) It also does not suffer from the possible problem faced by the
two Higgs doublet model which may not be able to produce sufficient baryon asymmetry
in the first place even though it can prevent a washout of any baryon asymmetry that
has been created \([10]\). This is because there are two VEV’s and the phase transition can
be in two stages. If after acquiring the first VEV the sphaleron rate is too suppressed
because the gauge symmetry is broken, not enough baryon asymmetry will be created
before the second phase transition which will preserve this asymmetry. Even though the
real Higgs singlet model considered in this paper also has a two stage phase transition,
because the gauge symmetry remains unbroken after the singlet acquires its VEV, the
baryon violating rate will not be suppressed in between the two phases.
Table 1: Representative parameter values and the corresponding sphaleron energies (in units of $4\pi V/g$). The parameters $\lambda_{\phi,S}$ and $\lambda$ are dimensionless while the other parameters have dimension GeV.

| $\lambda_{\phi}$ | $\lambda_{S}$ | $\lambda$ | $v$  | $\sigma$ | $\alpha$ | $E_{\text{sph}}$ |
|-----------------|---------------|-----------|------|----------|----------|----------------|
| 0.071           | 0.082         | -0.0234   | -125 | -12.81   | 12.20    | 1.40           |
| 0.063           | 0.073         | 0.0250    | -154 | -13.26   | 3.58     | 2.20           |
| 0.087           | 0.090         | 0.0114    | -245 | -9.37    | 1.24     | 0.82           |
| 0.096           | 0.082         | 0.0499    | -146 | -17.27   | 5.10     | 1.80           |
| 0.050           | 0.077         | 0.0168    | -117 | -12.43   | 8.34     | 1.91           |
| 0.057           | 0.0920        | 0.0026    | -171 | -16.39   | 4.64     | 0.55           |
| 0.057           | 0.0920        | 0.0026    | -107 | -16.54   | 16.76    | 0.91           |

4 Conclusion

I calculated the sphaleron energies in the standard model with a real Higgs singlet added on, for some representative sets of parameters. These show a decrease in the actual sphaleron energies compared to the values found in the Standard Model. However the decrease is not sufficient to constrain the strength of the possible first order phase transition in this model. If KM CP violation in the Standard Model is sufficiently strong for electroweak baryogenesis, then this model will be an extremely economical way of creating the baryon asymmetry of the universe at the electroweak phase transition.

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Table 2: Comparison of $V/T_c$ obtained from constraint (2) and $E_{\text{sph}}$ found with $V/T_c$ obtained at the first order electroweak phase transition.

| $45g/4\pi B$ | $m_{\phi,S}$ | $V_c/T_c$ |
|-------------|-------------|-----------|
| 1.61        | 61, 102     | 170/107   |
| 1.03        | 73, 88      | 390/93    |
| 2.75        | 103, 108    | 510/117   |
| 1.25        | 75, 109     | 430/97    |
| 1.18        | 67, 79      | 360/70    |
| 4.10        | 73, 95      | 360/85    |
| 2.48        | 68, 92      | 360/63    |

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Figure captions

Fig.1: A typical solution to the coupled non-linear ODE’s of Eq.(19). (The fourth line of parameters in Table 1 was used here.)
This figure "fig1-1.png" is available in "png" format from:

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