Multi-Terminal Coulomb-Majorana Junction

Alexander Altland\textsuperscript{1} and Reinhold Egger\textsuperscript{2}

\textsuperscript{1}Institut für Theoretische Physik, Universität zu Köln, Zülpicher Str. 77, D-50937 Köln, Germany
\textsuperscript{2}Institut für Theoretische Physik, Heinrich-Heine-Universität, D-40225 Düsseldorf, Germany

(Dated: May 6, 2014)

We study multiple helical nanowires in proximity to a common mesoscopic superconducting island, where Majorana fermion bound states are formed. We show that a weak finite charging energy of the center island may dramatically affect the low-energy behavior of the system. While for strong charging interactions, the junction decouples the connecting wires, interactions lower than a non-universal threshold may trigger the flow towards an exotic Kondo fixed point. In either case, the ideally Andreev reflecting fixed point characteristic for infinite capacitance (grounded) devices gets destabilized by interactions.

PACS numbers: 71.10.Pm, 73.23.-b, 74.50.+r

Introduction.—Electronic transport through topological insulators \cite{shindou09} or superconductors \cite{west09} has come into the limelight of condensed-matter physics over the past few years. In particular, understanding the physics of localized Majorana bound states, generically expected near the ends of one-dimensional (1D) topological superconductor wires \cite{km12,mourik12}, is crucial for exploiting the non-Abelian statistics of Majorana fermions in topological quantum computation schemes \cite{fukui09,pan09}. When a grounded Majorana nanowire — such as InSb and InAs wires subject to a Zeeman field and proximity coupling to an s-wave superconductor \cite{alicea10,alicea11} — is weakly contacted by a normal metal, the presence of a Majorana state should reflect in a conductance peak \cite{alicea10,alicea11}. When signatures of this type were indeed observed experimentally \cite{alicea12,alicea13}, Coulomb interactions affect the system in two distinct ways: (i) The combination of repulsive interactions, Coulomb interactions in devices comprising Majorana wires contacted to leads. Our setup is sketched in Fig. 1. For \( N \) nanowires proximity-coupled to the same mesoscopic superconducting island (‘dot’), there are \( 2N \) Majorana fermions, \( \gamma_j = \gamma_j^\dagger \), with anticommutator \( \{ \gamma_j, \gamma_k \} = \delta_{jk} \). The island connects to the \( j \)th lead by tunneling through the Majorana state \( \gamma_j \) with coupling strength \( t_j \); all other coupling mechanisms are irrelevant \cite{landshoff80}. Coulomb interactions affect the system in two distinct ways: (i) The combination of repulsive interactions, spin-orbit coupling and Zeeman field is expected to turn each of the \( M \leq 2N \) connecting leads into a spinless (helical) Luttinger liquid (LL) characterized by an interaction constant \( g < 1 \) \cite{altland97,shi09}. (ii) Weak intra-island interactions do not compromise the integrity of individual Majoranas \cite{altland97,altland10}. However, they introduce correlations between these states, and thereby correlations between the connecting wires. It stands to reason that the option to generate inter-wire coupling on mesoscopic scales, i.e., independently of microscopic single-particle tunneling between distinct terminals \cite{xi09,ying09}, offers new perspectives for quantum computation (and other) applications.

One of our main observations is that the charging energy of the island, \( E_c \), no matter how weak, always destabilizes the fully Andreev reflecting fixed point characterizing the grounded system. The ensuing consequences can be conveniently described in a picture wherein tunneling events from and into lead \( j \) correspond to particles and antiparticles carrying a flavor index \( j \). At time scales \( \tau \ll E_c^{-1} \), these ‘particles’ are asymptotically free and the corresponding tunnel amplitudes, \( t_j \), scale up. At scales \( \tau \sim E_c^{-1} \), this upward renormalization towards a fully Andreev reflecting fixed point is stopped by the formation of a strong ‘confinement force’ between particles and antiparticles, which enforces electro-neutrality of the dot. At larger scales, the effective degrees of freedom are charge dipoles, describing near instantaneous (at time scales \( \sim E_c^{-1} \)) transmission of charge from lead \( j \) to lead \( k \neq j \). The effective dipole coupling strengths, \( \lambda_{jk} \), are subject to a competition of downward renormalization due to fluctuations of individual dipoles, and upward renormalization due to dipole-dipole interactions. The balance of these two mechanisms defines an isotropic repulsive fixed point, \( \lambda^* \), separating a flow towards the decoupled dot (\( \lambda \to 0 \)) from a flow towards an exotic Kondo regime (\( \lambda \to \infty \)), generalizing the \( M = 3 \) topological Kondo effect discussed in Ref. \cite{egger09}. Surprisingly, our analysis below shows that for sufficiently weak charging energy, the system always flows towards the strong-coupling regime. We finally note that in supporting a novel type of Kondo flow, the present system differs strongly from the \( M = 2 \) Majorana single-charge transistor \cite{vincent12,altland13} as well as from other types of previously studied LL junctions \cite{altland14,altland15}.

Model.—We start by deriving the low-energy effective theory for the setup in Fig. 1. The Hamiltonian, \( H = H_c + H_t + H_I \), contains a piece \( H_c \), describing the island, the tunnel Hamiltonian \( H_t \), and \( H_I \) for the LL leads (we put \( h = k_B = 1 \) below). For each nanowire (\( \alpha = 1, \ldots, N \)), we have two spatially well separated Majorana fermions, \( \gamma_{2\alpha-1} \) and \( \gamma_{2\alpha} \). It is useful to define the nonlocal auxiliary fermion operators \( d_\alpha = (\gamma_{2\alpha-1} + \imath \gamma_{2\alpha})/\sqrt{2} \), where the total Majorana occupation number operator is \( \hat{n} = \sum_\alpha d_\alpha^\dagger d_\alpha \). Assuming that the proximity gap ex-
The creation of a lead electron lowers the occupation of the dot either by annihilation of a $d$-fermion (first term), or by creation of a $d$-fermion along with annihilation of a Cooper pair (the second term.)

**Effective phase action.**— We next derive an effective action, $S$, describing the system in terms of phase-like degree of freedoms. Integrating over the harmonic LL bosons away from $x = 0$, we obtain a contribution $S_l = \frac{Tg}{2\pi} \sum_{j,n} |\omega_n| |\Phi_j(\omega_n)|^2$, describing how fluctuations of $\Phi_j(\tau) \equiv \phi_j(0, \tau)$ dissipate into lead excitations ($\omega_n = 2\pi n T$ are Matsubara frequencies and $T$ is temperature.) Next, by virtue of a Hubbard-Stratonovich transformation to the condensate phase field $\tilde{\varphi}(\tau)$, the charging term in Eq. (1) leads to the contribution

$$S_c = \int_0^{1/T} d\tau \left( \frac{\tilde{\varphi}^2}{4E_c} - i\tilde{\varphi} \dot{\tilde{\varphi}} \right).$$

Noting that there also is a free fermion piece, $S_f = \sum_q \int dx d\alpha d\alpha + \frac{i}{2} \sum_q \int d\eta \eta \eta$, we eliminate the $\tilde{\varphi}$ term in Eq. (4) by a gauge transformation, $d\alpha \rightarrow e^{-i\varphi_0(\tau)} d\alpha(\tau)$. Switching back to the language of Majoranas, $\gamma_{2a-1} = (d_a + d^\dagger_a)/\sqrt{2}$ and $\gamma_{2a} = -i(d_a - d^\dagger_a)/\sqrt{2}$, the tunnel action now assumes the form

$$S_t = \sum_j t_j \int d\tau (-2i\eta_j \gamma_j) \sin(\Phi_j + \varphi).$$

Crucially, the Klein factor $\eta$ and the Majorana $\gamma$ can be fused to form an auxiliary fermion $f_j = (\eta_j - \gamma_j)/\sqrt{2}$, where $-2i\eta_j \gamma_j = 2f_j^\dagger f_j - 1 \equiv \sigma_j = \pm$. The resulting fermion Hamiltonian conserves $f_j^\dagger f_j$, which means that the functional integral over the $f_j$-fermions reduces to a sum over the static quantum number $\sigma_j = \pm$. The sole effect of this summation is to eliminate contributions of odd order in the tunneling amplitudes $t_j$ to the functional integral, while even-order contributions remain unaffected. Keeping only even orders, we will drop the then immaterial presence of the $\sigma_j$'s throughout. This Klein-Majorana fusion procedure implies an enormous technical simplification, since we are now left with an effective phase action only. Before writing down this action, we shift $\Phi_j \rightarrow \Phi_j - \varphi$, thus removing $\varphi$ from the tunnel action (5). Performing the remaining Gaussian functional integral over $\varphi$, after unitary transformation to the discrete Fourier basis ($q = 0, \ldots, M - 1$),

$$\tilde{\Phi}_q(\tau) = \frac{1}{\sqrt{M}} \sum_{j=1}^M e^{i2\pi q j/M} \Phi_j(\tau),$$

we arrive at the effective phase action

$$S = \frac{Tg}{2\pi} \sum_{q,n} \frac{\omega_n}{1 + \delta_{q,0}} |\tilde{\Phi}_q(\omega_n)|^2$$

+ $\sum_j t_j \int d\tau \sin(\Phi_j(\tau)).$
with the energy scale $\epsilon \equiv 2gME_c/\pi$. Notice that the charging energy only affects the zero mode $\Phi_0$, which effectively becomes free at low frequencies, $|\omega_n| \ll \epsilon$.

**Particle analogy and renormalization.**—We next discuss the expansion in the tunnel couplings $t_j$. A very instructive analogy follows by interpreting $O_j^+(\tau) = e^{i\Phi_j(\tau)}$ as particles (‘quarks’) and antiparticles living on the imaginary-time axis. Our particles carry a ‘flavor’ index ($j = 1, \ldots, M$) and an effective charge $-it_j/2$.

We study the properties of the ensuing interacting particle gas by standard renormalization group (RG) methods \textsuperscript{[23]} \textsuperscript{[37]}. In an RG step, we integrate over fast $\Phi_j$ modes with frequencies in the shell $\Lambda/b < |\omega_n| < \Lambda$, with rescaling parameter $b > 1$ and a high-energy cutoff $\Lambda$ of the order of the proximity gap. The RG step is completed by rescaling all frequencies, $\omega \to b\omega$, such that $\Lambda$ stays invariant. For small $t_j$, the particle density is low and different charges (i.e., the $t_j$) renormalize independently. We first RG-integrate out modes in the shell $\epsilon < |\omega_n| < \Lambda$, where the zero mode $\Phi_0$ is not significantly affected by $E_c$, see Eq. (7). Some algebra yields the net scaling dimension $1 - 1/(2g)$ for $t_j$, which are therefore RG-relevant couplings for $g > 1/2$. This signals a flow towards the resonant Andreev reflection fixed point \textsuperscript{[20]}. In the lead-non-interacting case, $g = 1$, the scaling dimension $1/2$ represents the naive dimension of a fermion-Majorana scattering operator. After the RG integration over the shell $\epsilon < |\omega_n| < \Lambda$, the renormalized couplings are then given by $t_j^{(1)} = t_j(\Lambda/e)^{-1/(2g)}$.

Proceeding to lower frequency scales, $|\omega| < \epsilon$, the charging energy renders the zero-mode action non-dissipative, $S[\Phi_0] \simeq \frac{d}{2\pi} \sum_n \omega_n^2|\Phi_0(\omega_n)|^2$, see Eq. (7). The integration over $\Phi_0$ generates a strong ‘confinement’ potential linear in the time separation between particle creation, $O_j^+(\tau) \sim e^{i\Phi_0(\tau)/\sqrt{M}}$, and particle annihilation events, $O_j^-(\tau') \sim e^{-i\Phi_0(\tau')/\sqrt{M}}$ of arbitrary flavor index $k \neq j$. (For $k = j$, particle-anti-particle annihilation occurs – the in consequence of virtual tunneling of a particle to and fro the same lead, cf. Fig. 2 upper inset.) At time resolutions lower than $E_c^{-1}$, the relevant excitations of the theory then are quark-antiquark pairs (‘mesons’). \textsuperscript{[23]}

$$\langle O_j^+(\tau)O_k^-(\tau') \rangle_0 \simeq e^{-\frac{2\omega_{jk}}{\sqrt{M}}|\tau - \tau'|} O_{jk}(\tau), \quad (8)$$

$$O_{jk}(\tau) = e^{i\Phi_k(\tau)} e^{-i\Phi_k(\tau)}. \quad (9)$$

**RG equations in dipole regime.**—We now consider the effective low-energy theory at resolutions $E_c \tau > 1$, where we are dealing with a gas of dipoles, $O_{jk}$, with couplings $\lambda_{jk}, j \neq k$. The ‘bare value’ of these couplings is given by $\lambda_{jk}^{(1)} \approx t_j^{(1)} t_k^{(1)} / E_c > 0$, where the factor $E_c^{-1}$ is due to a time integration over the distance between particle and antiparticle. Physically, these couplings bundle the effect of in-tunneling from lead $j$ into a virtual on-dot state of longevity $\sim E_c^{-1}$, followed by out-tunneling into lead $k$.

The effective action describing the system in the dipole regime reads

$$S = \frac{T g}{2\pi} \sum_n |\omega_n| |\Phi(\omega_n)|^2 - \sum_{j \neq k} \lambda_{jk} \int d\tau e^{i(\epsilon_j - \epsilon_k)\cdot\Phi(\tau)}, \quad (10)$$

where $\Phi = (\Phi_1, \ldots, \Phi_M)$, and $\epsilon_k$ is a unit vector in $k$-direction. RG analysis of this action obtains the flow equations

$$\frac{d\lambda_{jk}}{d\ln b} = (1 - g^{-1}) \lambda_{jk} + \frac{\kappa}{E_c} \sum_{m \neq (j, k)} \lambda_{jm} \lambda_{mk}, \quad (11)$$

where $\kappa$ is a non-universal constant of order unity. The first term describes the standard power-law suppression of the tunneling density of states in/out of the LL leads \textsuperscript{[21]} and implies a suppression of the $\lambda_{jk}$. This term is fought by the positive second contribution, which describes the effect of dipole fusion $[\langle j, m \rangle + (m, k) \to (j, k)]$ by particle/anti-particle annihilation (cf. Fig. 2 lower inset.) Equation (10) suggests the existence of an RG-unstable fixed point with isotropic couplings,

$$\lambda_{jk} = \lambda^* = \frac{g^{-1} - 1}{\kappa(M - 2)} E_c. \quad (12)$$

When decreasing the effective frequency scale $\omega$, the couplings, $t$ and $\lambda$, resp., show the RG flow schematically illustrated in Fig. 2. During the initial RG flow (down to $\omega \approx E_c$), particles are asymptotically free, implying a power-law increase of a typical tunnel coupling, $t \propto \omega^{-1}$, with the time-like variable $\omega^{-1}$. We now compare the value $\lambda^{(1)} = (t(1))^{2}/E_c \sim E_c^{-3+1/g}$, reached at the end of the asymptotic freedom phase ($\omega \approx E_c$), to the fixed-point value $\lambda^* \sim E_c$ in Eq. (11). For sufficiently large $E_c$, \textsuperscript{[39]} the fixed point cannot be reached, and therefore the $\omega \to 0$ stable fixed point describes a completely decoupled LL junction. For sufficiently small $E_c$, however, the fluctuations generated during the asymptotic freedom phase tip the balance, $\lambda^{(1)} > \lambda^*$. A straightforward expansion of Eq. (10) near $\lambda = \lambda^*$ then shows that the isotropic baseline $\lambda > \lambda^*$ of the couplings flows to large values with dimension $g^{-1} - 1 > 0$, while deviations between the couplings are RG irrelevant, i.e., the flow is towards an isotropic configuration $\lambda_{jk} = \lambda$. At a characteristic ‘Kondo temperature’, $T_K \sim E_c \exp(-\frac{E_c}{\lambda(M - 2)}$, the coupling $\lambda$ begins to diverge, and our perturbative expansion is no longer applicable.

**Strong-coupling fixed point.**—At low frequencies $\omega \ll T_K$ we are met with a dual picture, where the field vector $\Phi$ is pinned to stay close to the minima of the potential identified by the second term of Eq. (9) at $\lambda_{jk} \approx \lambda \gg 1$. (Notice, however, that the zero-mode component $\Phi_0$ defined through Eq. (6) remains free.) Literally the same problem was studied in Refs. \textsuperscript{[40]} \textsuperscript{[41]}, where it was shown...
that fluctuations near the limit $\lambda \to \infty$ – corresponding to occasional tunneling events between nearest-neighbor minima – carry scaling dimension $2q(M-1)/M$ [11]. The fixed point $\lambda \to \infty$ ensuing for $g > M/2(M-1)$ was identified with a multi-channel Kondo fixed point. However, unlike with previous proposals, where anisotropy is a relevant perturbation and easily destabilizes the Kondo fixed point [21], the present system is robust in that it flows towards an isotropic configuration.

Conductance matrix.—The two-stage scenario outlined above, with its branching into either a strong-coupling or a decoupled low-frequency limit, will bear consequences for all physical observables. We here briefly discuss the resulting temperature dependence of the linear conductance $G_{jk}$ between different leads $j$ and $k$. For high $T > E_c$, asymptotic freedom causes the power-law scaling $G_{jk} \sim T^{-2+1/g}$, while for $T \to 0$, we either have a vanishing conductance, $G_{jk} \sim T^{-2+2/g}$, if the decoupled fixed point is approached, or the unitary limit of an isotropically hybridized junction, $G_{jk} = \frac{2\pi^2}{T/\lambda}$. (Transport in this case is carried by the fully isotropic zero mode $\tilde{\Phi}_0$, which is protected by gauge invariance.) The above result for the conductance tensor generalizes the teleportation scenario discussed for $M = 2$ by Fu [31] to arbitrary channel numbers. It is worth stressing that for both stable fixed points (strong coupling or decoupled), resonant Andreev reflection is unstable, and hence arbitrary $E_c$ destroy the corresponding fixed point.

Conclusions.—In this paper, we have formulated and studied the problem of junctions of Majorana wires meeting on a superconducting island with charging energy $E_c$. We also included correlations in the leads, since these typically are 1D nanowires themselves. Our RG analysis reveals that the physics of the system is determined by asymptotic freedom at short time scales ($\tau < E_c^{-1}$) and confinement at long time scales ($\tau > E_c^{-1}$) correspondingly, to independent dot-lead tunneling, and virtual co-tunneling. For sufficiently weak $E_c$, we find that a strong-coupling Kondo fixed point is approached, while otherwise the junction describes decoupled leads at $T = 0$. These predictions can be tested using the temperature dependence of the conductance.

We thank P. Sodano and A. Zazunov for discussions. This work was supported by the SFB TR 12 and SPP 1666 of the DFG. Note added: During the preparation of this manuscript, we learned of independent related work by Béri [42].

---

Figure 2. (Color online) Schematic RG flow of typical couplings, $t$ and $\lambda$, vs the time-like RG parameter $\omega^{-1}$, and illustration of the system excitations. At short times, individual in- and out-tunneling events into different leads dominate (indicated by dots and circles of different color). These ‘quarks’ are asymptotically free, and their fluctuations generate a power-law increase of $t$. For $\omega^{-1} \gtrsim E_c^{-1}$, confinement leads to bound quark-antiquark dipoles (mesons) for $j \neq k$ (indicated in the lower inset), or annihilation for $j = k$ (indicated by ‘X’ in the upper inset). The dipole coupling, $\lambda$, experiences a downward renormalization due to individual fluctuations and an upward renormalization due to dipole-dipole fusion events (cf. lower inset). For sufficiently strong charging, $E_c = E_{c1}$, $\lambda$ does not reach the unstable fixed point $\lambda^*$ in Eq. (1). Then the first mechanism dominates, and hence $\lambda$ flows to the decoupled fixed point, $\lambda = 0$, as $\omega^{-1} \to \infty$. For small $E_c = E_{c2}$, however, the increase of $t$ during the asymptotic freedom phase brings $\lambda$ beyond the balance point $\lambda^*$, and the RG flow approaches the strong-coupling fixed point.

---

[1] M.Z. Hasan and C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[2] X.L. Qi and S.C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[3] A.Yu. Kitaev, Phys. Usp. 44, 131 (2001).
[4] L. Fu and C.L. Kane, Phys. Rev. Lett. 100, 096407 (2008).
[5] M. Sato and S. Fujimoto, Phys. Rev. B 79, 094504 (2009).
[6] J. Alicea, Rep. Prog. Phys. 75, 076501 (2012).
[7] C.W.J. Beenakker, arXiv:1112.1950.
[8] M. Leijnse and K. Flensberg, Semicond. Sci. Techn. 27, 124003 (2012).
[9] R.M. Lutchyn, J.D. Sau, and S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).
[10] Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).
[11] C.J. Bolech and E. Demler, Phys. Rev. Lett. 98, 237002 (2007).
[12] J. Nilsson, A.R. Akhmerov, and C.W.J. Beenakker, Phys. Rev. Lett. 101, 120403 (2008).
[13] K.T. Law, P.A. Lee, and T.K. Ng, Phys. Rev. Lett. 103, 237001 (2009).
[14] K. Flensberg, Phys. Rev. B 82, 180516(R) (2010).
[15] M. Wimmer, A.R. Akhmerov, J.P. Dahlhaus, and C.W.J. Beenakker, New J. Phys. 13, 053016 (2011).
[16] V. Mourik, K. Zuo, S.M. Frolov, S.R. Plissard, E.P.A.M.
Bakkers, and L.P. Kouwenhoven, Science 336, 1003 (2012).

[17] L. Rokhinson, X. Liu, and J. Furdyna, Nature Phys. 8, 795 (2012).

[18] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nature Phys. 8, 887 (2012).

[19] M.T. Deng, C.L. Yu, G.Y. Huang, M. Larsson, P. Caroff, and H.Q. Xu, Nano Lett. 12, 6414 (2012).

[20] L. Fidkowski, J. Alicea, N.H. Lindner, R.M. Lutchyn, and M.P.A. Fisher, Phys. Rev. B 85, 245121 (2012).

[21] A.O. Gogolin, A.A. Nersesyan, and A.M. Tsvelik, Bosonization and strongly correlated systems (Cambridge University Press, Cambridge, England, 1998).

[22] J. von Delft and H. Schoeller, Ann. Phys. (Leipzig) 7, 225 (1998).

[23] A. Altland and B. Simons, Condensed Matter Field Theory (Cambridge University Press, Cambridge, England, 2010), 2nd edition.

[24] S. Gangadharaiah, B. Braunecker, P. Simon, and D. Loss, Phys. Rev. Lett. 107, 036801 (2011).

[25] E.M. Stoudenmire, J. Alicea, O.A. Starykh, and M.P.A. Fisher, Phys. Rev. B 84, 014503 (2011).

[26] E. Sela, A. Altland, and A. Rosch, Phys. Rev. B 84, 085114 (2011).

[27] J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M.P.A. Fisher, Nature Phys. 7, 412 (2012).

[28] B.I. Halperin, Y. Oreg, A. Stern, G. Refael, J. Alicea, and F. von Oppen, Phys. Rev. B 85, 144501 (2012).

[29] B. Béri and N.R. Cooper, Phys. Rev. Lett. 109, 156803 (2012); A.M. Tsvelik, preprint arXiv:1211.3481.

[30] R. Hützen, A. Zazunov, B. Braunecker, A.L. Yeyati, and R. Egger, Phys. Rev. Lett. 109, 166403 (2012).

[31] L. Fu, Phys. Rev. Lett. 104, 056402 (2010).

[32] A. Zazunov, A.L. Yeyati, and R. Egger, Phys. Rev. B 84, 165440 (2011).

[33] C. Nayak, M.P.A. Fisher, A.W.W. Ludwig, and H.H. Lin, Phys. Rev. B 59, 15694 (1999).

[34] M. Oshikawa, C. Chamon, and I. Affleck, J. Stat. Mech. Theor. Exp. 2006, P02008 (2006).

[35] A. Altland, Y. Gefen, and B. Rosenow, Phys. Rev. Lett. 108, 136401 (2012).

[36] The $\sigma_j = \pm$ correspond to the eigenvalues of the Pauli matrix $\sigma_x$ in Ref. [20], which there was found through an elaborate Jordan-Wigner transformation.

[37] J. Kogut, Rev. Mod. Phys. 51, 659 (1979).

[38] In the effective action governing the meson ($O_{\mu}$) dynamics on time scales with $E_c \tau \gtrsim 1$, we can then replace the zero-mode part by the same dissipative action as for $\Phi_{\phi>0}$ without altering the physics. While the crossover scale in $S[\Phi_0]$ is $\epsilon = 2gME_c/\pi$, confinement sets in only for $|\omega| < E_c$. For large number of leads $M$, this allows for an intermediate frequency regime which, however, does not qualitatively change our conclusions.

[39] The threshold value for $E_c$ is non-universal and strongly depends on the bare tunnel couplings $t_j$.

[40] H. Yi and C.L. Kane, Phys. Rev. B 57, R5579 (1998).

[41] H. Yi, Phys. Rev. B 65, 195101 (2002).

[42] B. Béri, preprint arXiv:1212.4465.