The Origin of the Heavy-element Content Trend in Giant Planets via Core Accretion

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Abstract

We explore the origin of the trend of heavy elements in observed massive exoplanets. Coupling of better measurements of the mass ($M_p$) and radius of exoplanets with planet structure models enables estimating the total heavy-element mass ($M_{Z}$) in these planets. The corresponding relation is characterized by a power-law profile, $M_{Z} \propto M_p^{4/5}$. We develop a simplified but physically motivated analysis to investigate how the power-law profile can be produced under the current picture of planet formation. Making use of the existing semi-analytical formulae of accretion rates of pebbles and planetesimals, our analysis shows that the relation can be reproduced well if it traces the final stage of planet formation. In the stage, planets accrete solids from gapped planetesimal disks, and gas accretion is limited by disk evolution. We find that dust accretion accompanying gas accretion does not contribute to $M_{Z}$ for planets with $M_p < 10^3 M_J$. Our findings are broadly consistent with those of previous studies, yet we explicitly demonstrate how planetesimal dynamics is crucial for better understanding the relation. While our approach is simple, we can reproduce the trend of a correlation between planet metallicity and $M_p$ that is obtained by detailed population synthesis calculations when the same assumption is adopted. Our analysis suggests that pebble accretion would not play a direct role at the final stage of planet formation, whereas radial drift of pebbles might be important indirectly for metal enrichment of planets. Detailed numerical simulations and more observational data are required for confirming our analysis.

Key words: methods: analytical – planets and satellites: composition – planets and satellites: formation – planets and satellites: gaseous planets – protoplanetary disks

1. Introduction

The detection of a large number (>3000) of confirmed exoplanets has rapidly filled out a greater area in the mass–semimajor axis diagram (e.g., Borucki et al. 2011; Mayor et al. 2011, 2014; Twicken et al. 2016). These observations unveil a huge diversity of exoplanetary systems that gives a number of challenges to the current theory of planet formation. These include the presence of hot Jupiters that were first discovered by the radial velocity technique (Mayor & Queloz 1995), the rich population of close-in super-Earths that is confirmed by both Doppler and transit methods (e.g., Howard et al. 2010; Mayor et al. 2011), the existence of distant giant planetary systems that is revealed by direct imaging (e.g., Marois et al. 2010), and the prediction of a significant population of free-floating planets made by microlensing observations (e.g., Sumi et al. 2011; Mróz et al. 2017).

A number of improvements have been made so far for better understanding observed exoplanetary systems and eventually developing a complete picture of planet formation. One of the biggest leaps achieved was planetary migration (e.g., Goldreich & Tremaine 1980). This process arises from gravitational, tidal resonant interaction between planets and their gas disks and was initially invoked for explaining the presence of hot Jupiters (Lin et al. 1996). However, subsequent studies showed that migration is inevitable for planets in a wide mass range ($M_p > 1 M_J$), and that the migration rate is generally much faster than the growth rate of planets (e.g., Ward 1997; Nelson et al. 2000; Masset 2001; Tanaka et al. 2002; Paardekooper et al. 2010; Hasegawa & Pudritz 2011a). As a result, whereas some mechanisms for slowing down or even stopping migration have been proposed (e.g., Masset et al. 2006; Hasegawa & Pudritz 2011b; Kretke & Lin 2012; Dittkrist et al. 2014), the fundamental role of planetary migration is still unclear (see Kley & Nelson 2012 for a review). The general consensus in the community is that planet-forming materials move through protoplanetary disks, and hence planet formation is a global process involved with the entire region of the disks, rather than a local process.

Characterization of exoplanets is crucial for making further progress. For instance, the influence of the host stellar metallicity on the occurrence rate of planets has been explored to specify the formation mechanism of observed exoplanets (e.g., Santos et al. 2004; Fischer & Valenti 2005; Buchhave et al. 2014). Mass measurements by radial velocity coupled with radius measurements by transit allow one to estimate the bulk density of exoplanets (e.g., Weiss & Marcy 2014; Gettel et al. 2016; Jontof-Hutter et al. 2016). More recently, observations of exoplanets’ atmospheres have become feasible, and one can now detect some molecules in the atmospheres (e.g., Tinetti et al. 2007; Swain et al. 2008; Madhusudhan et al. 2011; Kreidberg et al. 2014; Wakeford et al. 2018). Accompanying such observations, theoretical studies have been undertaken for making a link with the observations and obtaining insights into the formation and migration histories of planets (e.g., Ida & Lin 2004b; Mordasini et al. 2012, 2016; Hasegawa & Pudritz 2014; Madhusudhan et al. 2014, 2017). For example, Guillot et al. (2006) directly computed the total heavy-element mass in planets using mass and radius measurements of observed hot Jupiters (also see Miller & Fortney 2011).

In this paper, we develop a consistent view of how accretion of gas and solids takes place onto growing planets in protoplanetary disks. We focus on the total heavy-element mass ($M_{Z}$) in observed exoplanets that is calculated by Thorngren et al. (2016, hereafter T16). In their study, the
radius evolution of warm Jupiters is computed utilizing their thermal evolution models of planets (see Section 4.1 for details). By comparing their computed planet radii with observed ones, they specify the value of $M_Z$ for warm Jupiters and derive correlations between $M_Z$ and $M_p$ and between $M_p$ and the planet metallicity ($Z_p = M_Z/M_p$; see Figure 1). Hereafter, these two correlations are referred to as the $M_Z-M_p$ and $Z_p-M_p$ relations. In this work, we examine both planetesimal and pebble accretion within a single framework to account for the results of T16. More specifically, we make use of the existing semi-analytical formulae for the accretion rates of planetesimals and pebbles and compute the power-law indices for the $M_Z-M_p$ and $Z_p-M_p$ relations. As clearly demonstrated below, we find that the subsequent planetesimal accretion after core formation is the most plausible case for better reproducing the relations. This is consistent with the results of previous studies (e.g., Pollack et al. 1996; Mordasini et al. 2014, 2016). Yet our follow-up work pins down the importance of planetesimal dynamics on the $M_Z-M_p$ relation.

The plan of this paper is as follows. In Section 2, we describe the core accretion scenario and summarize some key quantities and equations. In Section 3, we develop a framework to investigate how both gas and solid accretion onto growing planets determines the power-law indices of the $M_Z-M_p$ and $Z_p-M_p$ relations in the core accretion picture. We treat core formation, planetesimal accretion, pebble accretion, and the effect of gas accretion separately and examine their contributions to these two relations individually. In Section 4, we introduce the results of T16 and reanalyze them. We also compare the results of our theoretical analysis with those of T16. In Section 5, we summarize the limitation of our analysis. We also discuss other physical processes that are not included in our analysis and compare our findings with those of previous studies. We propose a classification of observed exoplanets. We finally list the potential roles of the current and future observations. A brief summary and conclusions of this work are presented in Section 6.

## 2. Planet Formation via Core Accretion

Here we consider the basic picture of core accretion. The key quantities of this work are summarized in Table 1.

### Table 1. List of Key Quantities

| Name                                      | Symbol | Related Process                                      |
|-------------------------------------------|--------|------------------------------------------------------|
| Host stellar metallicity                  | $Z_s$  | ...                                                  |
| Total planet mass                         | $M_p$  | ...                                                  |
| Radius of planets                         | $R_p$  | ...                                                  |
| Total envelope mass in planets            | $M_{XY}$ | Gas accretion                                         |
| Gas accretion timescales                  | $\tau_{g_{\text{acc}}}(\propto M_p^{3/4})$ | ...                                                  |
| Kelvin–Helmholtz timescales               | $\tau_{\text{KH}}(\propto M_p^{1/3})$ | Disk evolution $(d = 4)$                              |
| Upper limit of $\tau_{g_{\text{acc}}}$   | $\tau_{\text{g_{hydro}}}(\propto M_p^{-1})$ | ...                                                  |
| Total heavy-element mass in planets       | $M_Z$  | Solid accretion                                       |
| Planet metallicity                        | $Z_p (\propto M_Z/M_p)$                     | Accretion of dust via gas accretion                   |
| Heavy-element mass via gas accretion      | $M_{Z_{\text{gas}}}(\propto Z_p M_{XY})$ | Accretion of pebbles and planetesimals               |
| Heavy-element mass due to solid accretion | $M_{Z_{\text{solid}}}$                     | Accretion of pebbles and planetesimals               |
| Core mass of planets                      | $M_{\text{core}}$                         | ...                                                  |
| Heavy-element mass via planetesimal accretion | $M_{Z_{\text{pl}}}$                     | ...                                                  |
| Heavy-element mass via pebble accretion   | $M_{Z_{\text{pe}}}$                       | ...                                                  |
2.1. Core Formation and Gas Accretion

The core accretion scenario is the widely accepted picture of how planets form in protoplanetary disks (e.g., Ida & Lin 2004a; Mordasini et al. 2009; Benz et al. 2014). In this scenario, planetary cores form first, and then gas accretion onto the cores proceeds with simultaneous accretion of nonnegligible amounts of solids (e.g., Pollack et al. 1996). Currently, two scenarios of core formation are actively investigated: one is runaway and oligarchic growth, and the other is pebble accretion. For the former, planetesimals are the dominant form of solids to build planetary cores, and their size is generally considered as a few hundred km (e.g., Wetherill & Stewart 1989; Kokubo & Ida 1998). In this scenario, core formation is terminated when cores accrete all the planetesimals in their feeding zone and achieve the so-called isolation mass that is a function only of the solid surface density. For the latter, pebble-sized (∼cm–m) particles that are weakly coupled with the disk gas provide the main contribution to core formation through the radial drift of such particles (e.g., Ormel & Klahr 2010; Lambrechts & Johansen 2012). In this case, mass growth of planetary cores shuts off when the cores become massive enough to open up a gap in their gas disks (e.g., Lambrechts et al. 2014; Bitsch et al. 2018). In other words, the cores are not exposed to the pebble flux anymore due to blocking out of pebbles by a gas gap formed around the cores. Both scenarios therefore lead to the final core mass that is a function only of disk parameters.

One of the key quantities in core accretion is the critical core mass that regulates the onset of efficient gas accretion onto planetary cores (e.g., Mizuno 1980; Bodenheimer & Pollack 1986; Ikoma et al. 2000). The critical core mass is defined such that gaseous envelopes around the cores cannot maintain a hydrostatic equilibrium and runaway gas accretion takes place. Under the assumption that the grain opacity of the envelopes is comparable to the interstellar medium (ISM) value, the canonical value of ∼10 M⊕ has been widely adopted in the literature (e.g., Pollack et al. 1996; Ikoma et al. 2000; Ida & Lin 2004a; Mordasini et al. 2009). Recent studies, however, show that when dust grain growth in planetary envelopes is properly taken into account, the value of the critical core mass tends to decrease considerably. This arises from a lower value of the grain opacity in planetary envelopes, which leads to rapid cooling of the envelopes and their resulting efficient contraction (e.g., Movshovitz & Podolak 2008; Hori & Ikoma 2010; Movshovitz et al. 2010; Ormel 2014). It is interesting that a lower value (≤5–10 M⊕) of the critical core mass is in favor of theoretically reproducing the trends of the observed exoplanet population (e.g., Hasegawa & Podolak 2014; Mordasini et al. 2014). This can be readily seen by considering gas accretion onto planetary cores (see below). Another interesting feature of the critical core mass is that it may be used as one of the tracers to differentiate the origin of super-Earths from that of gas giants. Given that one clear difference between these two types of planets is the envelope mass and that the formation mechanism(s) of super-Earths is still unclear (e.g., Chiang & Laughlin 2013; Hansen & Murray 2013; Hasegawa 2016), it is of fundamental importance to identify the value of the critical core mass using the observational data of exoplanets.

Gas accretion onto planets begins once planetary cores become massive enough. In principle, the gas accretion process can be modeled as the Kelvin–Helmholtz timescale (τKH,acc). This timescale is written as (e.g., Ikoma et al. 2000; Ida & Lin 2004a; Hasegawa & Podolak 2012)

$$\tau_{\text{g,KH}} = \frac{10^{12} f_{\text{grain}}}{M_p (10 M_\oplus )^{d}} \text{yr},$$

where $f_{\text{grain}} \ll 1$ is the acceleration factor due to the reduction of grain opacity in planetary envelopes resulting from grain growth there. In this paper, we adopt $c = 7$ and $d = 4$, following Tajima & Nakagawa (1997; see their Equation (26)). As clearly seen in Equation (1), $\tau_{\text{g,KH}}$ becomes much shorter than the typical disk lifetime of a few 106 yr (e.g., Williams & Cieza 2011) when the initial core mass exceeds ∼10 M⊕. This is one of the reasons why smaller core masses are preferred for reproducing the observed population of exoplanets.

One would notice that $\tau_{\text{g,KH}}$ keeps decreasing as $M_p$ increases (see Equation (1)). This can eventually lead to an unrealistically high value of the gas accretion rate ($dM_p/dt$) for massive planets ($\geq 100 M_\oplus$). Accordingly, an upper limit is generally imposed for limiting $dM_p/dt$. In this paper, we adopt the results of Tanigawa & Watanabe (2002). In their work, 2D hydrodynamical simulations are performed, and gas accretion flow onto planets from protoplanetary disks and the fine structure of circumplanetary disks are resolved with high spatial resolution simulations. They find that the upper limit of the gas accretion rate is given as (Tanigawa & Watanabe 2002; see their Equation (20))

$$\tau_{\text{g,hydro}} \approx 1.1 \times 10^3 \sigma_{p,\text{acc}}^{-1} \left( \frac{d_p}{5 \text{au}} \right)^{1.5} \left( \frac{M_p}{10 M_\oplus} \right)^{-d'} \text{yr},$$

where $\sigma_{p,\text{acc}}$ is the normalized surface density of gas that participates in gas accretion, $d_p$ is the semimajor axis of the planet, and $d' = 1/3$. Note that when a gap is opened up in gas disks due to disk–planet interaction (e.g., Nelson et al. 2000; Crida et al. 2006; Kley & Nelson 2012), $\sigma_{p,\text{acc}}$ becomes a function of $M_p$ (Tanigawa & Ikoma 2007).

In summary, the mass growth rate of planets via gas accretion can be written as

$$\frac{dM_p}{dt} \approx \frac{dM_p}{dt} = \frac{M_p}{\tau_{\text{g,acc}}},$$

where

$$\tau_{\text{g,acc}} = \max \{ \tau_{\text{g,KH}}, \tau_{\text{g,hydro}} \}.$$

The above equations are valid mainly at the final stages of planet formation, in which core formation nearly ends and solid accretion onto planets is insignificant compared with gas accretion.

2.2. Additional Solid Accretion

It has been suggested for a long time that additional solid accretion is essential for fully understanding the total heavy-element mass of gas-giant planets (e.g., Pollack et al. 1986; Podolak et al. 1988). For instance, the enhanced metallicity in
the atmosphere of Jupiter and Saturn claims the need of additional solid accretion during the process of forming (e.g., Pollack et al. 1996; Saumon & Guillot 2004). As another example, Mordasini et al. (2014) showed that planetesimal accretion after core formation completes is important for reproducing the $Z_p-M_p$ relation of observed exoplanets (see also Mordasini et al. 2016).

In order to examine at what stage and how solid accretion occurs for growing planets in protoplanetary disks, we explore the mass contribution ($M_{Z,\text{solid}}$) arising from solid accretion by decomposing it into three components,

$$M_{Z,\text{solid}} = M_{\text{core}} + M_{\text{pl}} + M_{\text{pe}},$$

where $M_{\text{core}}$ is the initial seed core mass of a protoplanet at which the subsequent gas accretion begins, $M_{\text{pl}}$ is the total heavy-element mass that is obtained via planetesimal accretion, and $M_{\text{pe}}$ is the total heavy-element mass that is gained during accretion of small bodies such as pebbles (see Table 1). Accretion of both planetesimals and pebbles onto (proto) planets would be possible during the gas accretion stage (e.g., Pollack et al. 1996; Rafikov 2004; Alibert et al. 2005; Tanigawa et al. 2014). As described in Equation (5), we treat them separately in this paper.

### 2.3. Mass Budget in Planets

Finally, the mass budget of a planet can be written as

$$M_p = M_{\text{XY}} + M_Z,$$

$$M_Z = M_{Z,\text{solid}} + M_{Z,\text{gas}},$$

where $M_{\text{XY}}$ is the total envelope mass of the planet, $M_Z$ is the total heavy-element mass of the planet, $M_{Z,\text{solid}}$ is the total heavy-element mass that is accumulated in the planet through accretion of solids such as pebbles and planetesimals (see Equation (5)), and $M_{Z,\text{gas}}$ is the total heavy-element mass that is accreted through gas accretion (see Table 1). Note that the disk gas accreted onto planets contains small (~$\mu$m–mm) dust particles. Such solids are well coupled with the disk gas and hence follow the gas motion. Accordingly, these solids are also accumulated in planets as the gas is accreted onto the planets. We take this contribution into account by including the term of $M_{Z,\text{gas}}$.

### 3. Theoretical Analysis

We develop a simplified but physically motivated analysis to understand how accretion of gas and solids takes place onto growing protoplanets in protoplanetary disks. We make use of the equations in the above section.

#### 3.1. Basic Formulation

We first formulate the basic equation exploring the $M_Z-M_p$ relation.

As discussed in Section 2.2, additional solid accretion would be plausible during the gas accretion stage. It is nonetheless important to point out that the actual efficiency is currently under active investigation (e.g., Zhou & Lin 2007; Johansen & Lambrechts 2017) and is most likely determined by disk parameters. In order to shed light on the underlying physics, we focus only on the power index of the $M_Z-M_p$ relation in this paper. While this simplification provides some limitations for our analysis (see Section 5.1), we then only need to care about the $M_p$ dependence on each variable.

To proceed, we adopt the approach originally developed by Shiraiishi & Ida (2008). In this approach, the derivative of $M_Z$ is examined, which is given as

$$\frac{dM_Z}{dt} = \frac{dM_Z}{dt} \frac{\tau_{g,\text{acc}}}{M_p} \propto M_p^{\nu'},$$

(8)

where it is assumed that the mass growth ($dM_p/dt$) of planets is dominated by gas accretion ($dM_{XY}/dt$; see also Equation (3)). This assumption would be valid at the final stages of planet formation.

Then, we simplify the gas accretion timescale ($\tau_{g,\text{acc}}$; see Equation (4)) as

$$\tau_{g,\text{acc}} = \max\{\tau_{g,\text{KH}}, \tau_{g,\text{hydro}}\} \propto M_p^D,$$

(9)

where $D = -d = -4$ when $\tau_{g,\text{KH}} > \tau_{g,\text{hydro}}$, and $D = -d' = -1/3$ when $\tau_{g,\text{KH}} < \tau_{g,\text{hydro}}$. Note that we only pay attention to the $M_p$ dependence in this analysis. Also, we neglect the effect of gas gaps that can be opened up by disk–planet interaction. We discuss this effect in Section 5.2 and the Appendix.

In the following, we utilize Equation (8) and investigate how the power index of the $M_Z-M_p$ relation changes as a function of the forms (planetesimals versus pebbles) of solids that are accreted onto planets.

#### 3.2. Contribution from Planetesimal Accretion

In this section, we consider the contribution arising from $M_{\text{pl}}$ to $M_{Z,\text{solid}}$, that is, how planetesimal accretion proceeds in a post-stage of (initial) core formation. Equivalently (see Equations (5), (7), and (8)),

$$M_Z \approx M_{Z,\text{solid}} \approx M_{\text{pl}},$$

(10)

$$\frac{dM_Z}{dt} \approx \frac{dM_{\text{pl}}}{dt} \frac{\tau_{g,\text{acc}}}{M_p} \propto M_p^{\nu'}. $$

(11)

The remarkable recognition that continuous accretion of planetesimals is important for planet formation is made by the milestone work of Pollack et al. (1996). In this study, it is assumed that such accretion originates from the expansion of planets’ feeding zones as the planets grow in mass and their Hill radius increases. Adopting the most efficient accretion rate of planetesimals, they can reproduce the trend of the enhanced atmospheric metallicity of Jovian planets in the solar system such as Jupiter and Saturn. Such efficient accretion of planetesimals leads to the emergence of the so-called “phase 2,” where the planetesimal accretion rate is so high (~$10^{-6} M_\odot$ yr$^{-1}$) that the onset of runaway gas accretion is postponed for ~a few Myr. Despite the success achieved by their model, a number of follow-up studies pose a question about their assumption that the most efficient planetesimal accretion would be realized and continue for a long (~Myr) time (e.g., Fortier et al. 2007; Zhou & Lin 2007; Shiraiishi & Ida 2008; Hasegawa & Pudritz 2014). This is because, following the mass growth of planets, the planetesimals in their feeding zone will be used up, and some of them will even be scattered out of the zone due to gravitational interaction with the planets. Coupled with eccentricity damping by the disk gas, this scattering process can end up with the creation of a gap in planetesimal disks around planets. In fact, the common conclusion of these studies is that when both the dynamics of planetesimals in
gas disks and the effect of planetary growth are considered realistically, efficient planetesimal accretion cannot be established.

To appropriately take into account the dynamics of planetesimals around a growing planet in a gas disk and reliably derive the power-law index \( \Gamma_{pl} \) of \( \dot{M}_g/\dot{M}_p \) (see Equation (11)), we make use of the results of Shiraishi & Ida (2008). In their study, a number of N-body simulations are carried out to investigate how planetesimal accretion takes place for planets that undergo gas accretion and to derive a semi-analytical accretion rate of planetesimals \( \dot{M}_{pl}/dt \).

Based on their results, \( \dot{M}_{pl}/dt \) is determined by the interplay among excitation of planetesimals’ eccentricity by a growing planet, dumping of their eccentricity by the disk gas, and expansion of the Hill radius of the planet. When the dumping efficiency of the planetesimals’ eccentricity by the disk gas is less than the expansion rate of the Hill radius of a growing planet, the growth rate of the planet is so fast that the planet can keep accreting planetesimals in its expanding feeding zone. In other words, a gap is not generated in the planetesimal disk. For this case, the planetesimal accretion rate is given as (see Equations (22) and (24) in Shiraishi & Ida 2008)

\[
\frac{dM_{pl}}{dt}_{\text{nogap}} \propto R_p^2 M_p^{\alpha/3} \tau_{g, acc}^{-\alpha} \propto M_p^{(2-\alpha)/3} \tau_{g, acc}^{-\alpha},
\]

where \( \alpha \approx 4/5 \). Note that \( dM_{pl}/dt \) is a function of \( \tau_{g, acc} \). This originates from the assumption that planetary growth is regulated mainly by gas accretion. On the other hand, when the eccentricity dumping of scattered planetesimals by the disk gas is more significant than the Hill radius expansion, planetary growth is slow enough that planetesimals can leave the feeding zone of a planet before they will be accreted. Equivalently, a gap can open up in planetesimal disks. In this situation, the accretion rate of planetesimals is written as (see Equations (23) and (25) in Shiraishi & Ida 2008)

\[
\frac{dM_{pl}}{dt}_{\text{gap}} \propto R_p^2 M_p^{\alpha'/6} \tau_{g, acc}^{-\alpha'} \propto M_p^{(4-\alpha')/6} \tau_{g, acc}^{-\alpha'},
\]

where \( \alpha' \approx 7/5 \). Again, \( dM_{pl}/dt \) is related to \( \tau_{g, acc} \). Thus, the planetesimal accretion rate is a function of both \( M_p \) and \( \tau_{g, acc} \), and the functional forms of \( dM_{pl}/dt \) are different, depending on the creation of a gap in the planetesimal disks.

We are now in a position to derive the power-law index of \( dM_g/dM_p \), which is given as (with Equation (9))

\[
\Gamma_{pl} = -\frac{1 + \alpha}{3} + D(1 - \alpha) = \frac{D - 3}{5}
\]

without planetesimal gaps and

\[
\Gamma_{pl} = -\frac{2 + \alpha'}{6} + D(1 - \alpha') = -\frac{12D + 17}{30}
\]

with planetesimal gaps. Given that there are two modes in gas accretion (see Equation (4)), one of which is regulated by the Kelvin–Helmholtz timescale and the other of which is limited by disk evolution, the corresponding power-law indices are summarized in Table 2. By integrating \( dM_g/dM_p \), we find the resulting power-law indices of \( M_g(\propto M_p^{\Gamma_{pl}'}) \) for planetesimal accretion (see Table 3):

\[
\Gamma_{pl'} = \begin{cases} 
-2/5 \text{ with no gap and } \tau_{g, acc} = \tau_{g, KH} \\
1/3 \text{ with no gap and } \tau_{g, acc} = \tau_{g, hydro} \\
2 \text{ with a gap and } \tau_{g, acc} = \tau_{g, KH} \\
3/5 \text{ with a gap and } \tau_{g, acc} = \tau_{g, hydro} 
\end{cases}
\]

Based on the above analysis, the power-law index of \( Z_p(\propto M_p^{\beta_{pl}}) \) is the same as \( \Gamma_{pl} \) and is given as (also see Table 2)

\[
\beta_{pl} = \begin{cases} 
-7/5 \text{ with no gap and } \tau_{g, acc} = \tau_{g, KH} \\
-2/3 \text{ with no gap and } \tau_{g, acc} = \tau_{g, hydro} \\
1 \text{ with a gap and } \tau_{g, acc} = \tau_{g, KH} \\
-2/5 \text{ with a gap and } \tau_{g, acc} = \tau_{g, hydro} 
\end{cases}
\]

It is interesting that our analysis predicts that \( \beta_{pl} = 1 \) for the case with planetesimal gaps and \( \tau_{g, acc} = \tau_{g, KH} \), which is inconsistent with the current trend of observed exoplanets. We consider that this inconsistency suggests that such a case never occurs in planet formation. In fact, it can be readily expected that if planets are massive enough to open up a gap in planetesimal disks, the corresponding \( \tau_{g, KH} \) should be smaller than \( \tau_{g, hydro} \) (see Equation (9)). Our case study therefore would be useful for specifying the mass growth path of planets without any detailed calculations.

Thus, we find that the \( M_g-M_p \) relation has different slopes, depending on the planetesimal distribution around planets and their gas accretion rates.

### 3.3. Contribution from Pebble Accretion

Here we examine the case of pebble accretion. Equivalently, we consider the following case (see Equations (5), (7), and (8)):

\[
M_g \approx M_{g, solid} \approx M_{pe},
\]

\[
\frac{dM_g}{dM_p} \approx \frac{dM_{pe}}{dt} \tau_{g, acc} \propto M_p^{\Gamma_{pl}'}.\]

Substantial progress is currently being made for pebble accretion since the first realization of its importance in planet formation (see Johansen & Lambrechts 2017 for the most recent review). For the completeness of this paper, we will utilize the most recent results of pebble accretion and develop a formulation that is similar to that of planetesimal accretion (see Section 3.2). It is nonetheless fair to mention that pebble accretion is not explored at the final stages of gas-giant formation very much, compared with planetesimal accretion. In fact, even in the most recent studies, the primary target is the
role of pebble accretion on core formation (e.g., Bitsch et al. 2015; Madhusudhan et al. 2017). Furthermore, these studies essentially treat accretion of gas and pebbles onto planets separately. In other words, the adopted pebble accretion rate \( \dot{d} \) is independent of the gas accretion rate. The following analysis, therefore, should be viewed as a reference, rather than the final results. Once the similar level of complexity is included in numerical simulations of pebble accretion, one can undertake a more comprehensive calculation to examine the importance of pebble accretion on the \( M_Z-M_p \) and \( Z_p-M_p \) relations more realistically.

Keeping this caveat in mind, we discuss the accretion rate of pebbles onto growing planets. In practice, \( \dot{d} \) is written as (see Equation (34) in Johansen & Lambrechts 2017)

\[
\frac{dM_{\text{pe}}}{dt} \propto M_p^{2/3}, \tag{20}
\]

where the so-called Hill regime is considered. This is because our analysis assumes that (initial) core formation is almost completed and the core mass should be relatively large \((\gtrsim 1-5 M_\oplus)\). For this case, the growth mode is regulated by the relative velocity of Keplerian shear, rather than the azimuthal drift (e.g., Ormel & Klahr 2010; Lambrechts & Johansen 2012; Ida et al. 2016). Then, the power-law index of \( dM_{\text{pe}}/dM_p (\propto M_p^{\Gamma_{\text{pe}}}) \) can be calculated as

\[
\Gamma_{\text{pe}} = D - \frac{1}{3}. \tag{21}
\]

Table 2 summarizes the results for both cases of gas accretion \((\tau_{g,\text{acc}} = \tau_{g,\text{KH}} \text{ and } \tau_{g,\text{acc}} = \tau_{g,\text{hydro}})\). When integrating the above equation, we obtain the power-law index of \( M_Z(\propto M_p^{\Gamma_Z}) \) for pebble accretion, which is given as (see Table 3)

\[
\Gamma_Z = \begin{cases} 
-10/3 \text{ with } \tau_{g,\text{acc}} = \tau_{g,\text{KH}} \\
1/3 \text{ with } \tau_{g,\text{acc}} = \tau_{g,\text{hydro}}.
\end{cases} \tag{22}
\]

Also, the power-law index of \( Z_p(\propto M_p^{\beta_p}) \) is written as

\[
\beta_p = \begin{cases} 
-13/3 \text{ with } \tau_{g,\text{acc}} = \tau_{g,\text{KH}} \\
-2/3 \text{ with } \tau_{g,\text{acc}} = \tau_{g,\text{hydro}}.
\end{cases} \tag{23}
\]

As is the case with planetesimal accretion, the \( M_Z-M_p \) relation has different slopes for different gas accretion recipes.

### 3.4. Contribution from Planetary Cores

In this section, we focus on the contribution of \( M_{Z,\text{solid}} \) arising from core formation (see Equation (5)):

\[
M_Z \approx M_{Z,\text{solid}} \approx M_{\text{core}}. \tag{24}
\]

As discussed in Section 2.1, both the oligarchic growth and pebble accretion scenarios lead to the core mass that is independent of \( M_p \). Then, the power-law indices of \( M_Z(\propto M_p^{\Gamma}) \) and \( Z_p(\propto M_p^{\beta}) \) are readily computed as

\[
\Gamma_{\text{core}} = 0, \tag{25}
\]

\[
\beta_{\text{core}} = -1. \tag{26}
\]

It is interesting that these profiles are inconsistent with the trend of observed exoplanets (see Figure 1; also see Section 4).

### 3.5. Contribution Arising from Gas Accretion

Finally, we examine the contribution \( (M_{Z,\text{gas}}) \) originating from gas accretion (see Equation (7)). For this case, we can directly compute the total amount of \( M_{Z,\text{gas}} \). Assuming that the dust abundance in the gas accreted onto planets is comparable to \( Z_\oplus \), the value of \( M_{Z,\text{gas}} \) can be given as (using Equation (6))

\[
M_{Z,\text{gas}} = Z_\oplus M_\ast \sim Z_\oplus (M_p - M_\ast). \tag{27}
\]

Given that \( M_\ast \gg M_Z \) for gas-giant planets, the contribution of \( M_{Z,\text{gas}} \) is only about 1% \((\sim Z_\oplus)\) of the total planet mass. We thus can conclude that dust accretion accompanying gas accretion is not significant to \( M_Z \) for planets with a mass of \( M_p < 10^3 M_\oplus \).

As shown below (see Section 4.4), this conclusion is justified for observed massive exoplanets.

### 4. Reanalysis of the Results of T16

Here we turn our attention to the results obtained by T16. We reanalyze their computed values of the total heavy-element mass in observed exoplanets and investigate how they are useful for developing a better understanding of planet formation.

#### 4.1. The Results of T16

We first introduce the results of T16 (see Figure 1; also see Miller & Fortney 2011).

In the study, observed exoplanets that have better measurements of mass and radius are chosen from the Extrasolar Planets Encyclopedia (exoplanets.eu; Schneider et al. 2011) and the NASA Exoplanet Archive (Akeson et al. 2013). Specifically, 47 exoplanets are selected from larger samples.
based on the criterion of a relatively low value of stellar insolation \( (F_\text{b} < 2 \times 10^6 \text{erg s}^{-1} \text{cm}^{-2}) \). This criterion is adopted in order to filter out potentially inflated hot Jupiters, the origin of which is still unknown (see Burrows et al. 2007 for a potential explanation).

Through the careful examination of the data from both the original sources and the websites, they obtained the values of the planet mass \( (M_p) \) and radius \( (R_p) \) and the host star age and metallicity \( (Z_\ast) \). They made use of these values to combine their planet structure model and compute the thermal evolution of planets. Such computations allow one to trace the radius evolution of planets. More specifically, they adopted 1D planet structure models that are composed of an inert core (a 50/50 rock-ice mixture), a homogeneous convective envelope (an H/He-rock-ice mixture), and a radiative atmosphere as the upper boundary condition. For the atmosphere model, the solar metallicity grids are interpolated from Fortney et al. (2007). Their calculations employ a number of assumptions and simplifications. A more detailed model should include a self-consistent treatment of atmospheres, the composition of heavy elements, and the treatment of the thermal properties of cores (see Section 3 of T16). However, they found that uncertainties from observations (mass, radius, and host star age) are still dominant over model uncertainties (see Section 5.1). By comparing the computed radius of planets with the observational data, they identified the values of \( M_Z \) in the planets that can distribute in both their cores and envelopes.

Here we simply summarize their derived \( M_Z - M_p \) and \( Z_p - M_p \) relations (also see their Figures 7 and 11),

\[
M_Z \propto M_p^{1,16}, \\
Z_p \propto Z_\ast^{0.61} M_p^{0.95},
\]

where \( \Gamma_{16} \approx 0.61 \pm 0.08 \) and \( \beta_{16} \approx -0.45 \pm 0.09 \). In this paper, we adopt \( \Gamma_{16} \approx 3/5 \) and \( \beta_{16} \approx -2/5 \), respectively. For clear presentation, we do not show error bars in the figures in this and the following sections. It is interesting that \( \beta_{16} \approx \Gamma_{16} - 1 \). This suggests that both \( M_Z \) and \( M_p \) are almost independent of or only very weakly dependent on \( Z_\ast \) for observed exoplanets. In fact, exoplanet observations confirm that while the occurrence rate of exoplanets is correlated with stellar metallicity (e.g., Fischer & Valenti 2005; Buchhave et al. 2014; Hasegawa & Pudritz 2014), the maximum mass of planets is not related to \( Z_\ast \). Note that T16 found that the \( Z_p - M_p \) relation becomes clearer when the planet metallicity is normalized by the host stellar metallicity (see their Figures 10 and 11). Accordingly, we adopt the same convention.

In the following, we reanalyze the results of T16 in order to derive some constraints on planet formation and examine how the \( M_Z - M_p \) relation can be reproduced.

### 4.2. The Envelope Mass and Critical Core Mass

We begin with computing the envelope mass \( (M_{XY}; \text{see Equation (6)}) \) and considering the critical core mass.

Figure 2 depicts the computed value of \( M_{XY} = M_p - M_Z \) and the mass fraction \( (M_{XY}/M_p) \) as a function of \( M_p \) in the left and right panels, respectively. Our simple calculations show that the envelope mass becomes comparable to the total mass of the planets when they are more massive than \( \sim 100 \, M_\oplus \) (green dots in the left panel). This suggests that efficient gas accretion occurred for all of the observed exoplanets that have masses larger than \( \sim 100 \, M_\oplus \), which is also confirmed by the mass fraction of \( M_{XY} \) (right panel). Importantly, we find that some of the planets in the mass range of \( 20 \, M_\oplus < M_p < 100 \, M_\oplus \) did not experience efficient gas accretion. Given that previous studies demonstrate that the critical core mass is about 5–10 \( M_\oplus \) (see Section 2.1), our computations indicate that some mechanisms would be needed to postpone the onset of efficient gas accretion for some exoplanets until their masses reach \( \sim 20–100 \, M_\oplus \). Note that the upper value of \( M_p (\sim 100 \, M_\oplus) \) comes from only two points (see Figure 2). This critical value may change when more and improved results of planet structure models become available.

#### 4.3. The Effect of Solid Accretion

Here we examine the effect of solid accretion on the \( M_Z - M_p \) and \( Z_p - M_p \) relations. Given that solid accretion can divide into the core formation stage \( (M_{\text{core}}) \) and post-core formation stage \( (M_{pl} \text{ and } M_{pel}; \text{see Equation (5)}) \), we subtract \( M_{\text{core}} \) from \( M_Z \) and explore the resulting behavior of the heavy-element mass \( (=M_Z - M_{\text{core}}) \). Note that, as discussed in Section 3.5, the contribution of \( M_{Z \text{,gas}} \) is negligible (also see Section 4.4).

Figure 3 shows the results of our analysis. Since it is unknown what the initial core mass for these planets is, we adopt a parameterized approach. In this approach, three plausible values \( (1, 5, \text{and } 10 \, M_\oplus) \) of the core mass are subtracted. We find that as the subtracted core mass increases (from the top to bottom panels in Figure 3), the slope of the heavy-element mass becomes steeper, especially at the less massive \( (M_p < 10^3 \, M_\oplus) \) region (red dots). This is simply because when planets are not so massive, the total heavy-element mass is also relatively small. If a certain value of the core mass is removed from \( M_Z \), then the reduction in \( M_Z \) becomes more enhanced for lower-mass planets than massive ones. Thus, our analysis indicates that the slope tends to be steeper \((>3/5)\) for planets with a mass of \( \lesssim 10^3 \, M_\oplus \) and shallower \((\sim 3/5)\) for more massive planets when the core mass is subtracted from the total heavy-element mass \( (M_Z) \).

We now turn our attention to the \( Z_p - M_p \) relation. For this case, we utilize the results of our analysis to develop an interpretation that is different from that above. More specifically, we assume that the metallicity computed from \( M_Z - M_{\text{core}} \) represents the envelope metallicity. This assumption will be valid if planetary cores do not dissolve into their envelopes and if solids accreted in the post-core formation stage fully dissolve into the envelopes due to thermal ablation.

Figure 4 shows the results. We have adopted the same parameterized approach as above. From top to bottom, the assumed core mass that is removed from \( M_Z \) is altered from 1, 5, and 10 \( M_\oplus \). Our analysis shows that subtraction of the core mass from \( M_Z \) tends to wash out the \( Z_p - M_p \) relation, especially for planets that have masses of \( <20–100 \, M_\oplus \) (red dots). This occurs simply because the value of planetary metallicity \( (Z_p) \) is more affected for lower-mass planets, as discussed above. If the above assumption is reasonable for observed exoplanets and envelope metallicity is determined only by the subsequent solid accretion, then our results can be interpreted as indicating that a correlation between envelope metallicity and planet mass should have a shallower slope than that of the \( Z_p - M_p \) relation.

---

4 T16 treated the core mass as a free parameter with an upper limit of 10 \( M_\oplus \), and their best-fit values are not provided in their paper.
Also, there should be a transition in envelope metallicity as the planet mass increases. This transition would be related to the core mass. In Section 5.3, we will discuss more about how these interpretations are related to the current observations of exoplanets’ atmospheres.

4.4. The Effect of Gas Accretion

Here we consider the effect of gas accretion ($M_{Z\text{gas}}$) on the total heavy-element mass ($M_Z$) and planet metallicity ($Z_p$).

As already shown in Section 3.5, the contribution of $M_{Z\text{gas}}$ is readily computed for given values of $M_Z$, $M_p$, and $Z_p$ (see Equation (27)). Figure 5 shows the resulting values (blue dots). Our analysis confirms that dust accretion accompanying gas accretion is not crucial for understanding the total heavy-element mass of observed planets (left panel). We also find that the contribution of dust accretion is an order of unity for massive ($>100 M_\oplus$) planets (right panel). This can be viewed as a verification of the assumption that the dust abundance in the accreted gas is about $Z_e$.

4.5. Comparison with Our Theoretical Analysis

We now compare our theoretical results (see Section 3) with those of T16 (see Figure 1). To proceed, we summarize our results in Table 3.

We find that if the core mass of observed exoplanets is relatively small ($\lesssim 1 M_\oplus$), the best fit is achieved for the case where planetesimal accretion is slowed down due to gap formation and gas accretion is also limited by disk evolution (see Figure 3 and Table 3). This implies that the $M_{Z\text{gas}}$–$M_p$ relation would be determined predominantly by the final stage of planet formation. Even if the core mass of these planets would be relatively large ($\lesssim 5–10 M_\oplus$), the trend for observed exoplanets can be reproduced well only in the case of gap formation in planetesimal disks (see Section 4.3): as the value of $M_p$ increases, the slope for the correlation between heavy-element mass and $M_p$ becomes shallower with increasing planet mass. Mathematically, the power-law index changes from 2 to 3/5 for this case (see Table 3).

As a conclusion, our analysis suggests that the trend found by T16 will be understood well if it traces the final stage of planet formation. Planets are already massive enough to generate a gap in their surrounding planetesimal disks, and the gas accretion rate onto the planets is considerably reduced and mainly regulated by disk evolution.

5. Discussion

We first list the limitations of our analysis. We then discuss other physical processes that are not considered in the above analysis and examine their effects on our conclusions. Also, we summarize previous studies that are directly related to this work and compare them with our findings. We provide a comprehensive picture of planet formation that is derived from our analysis and finally discuss some implications for current and future observations of exoplanets and their atmospheres.

5.1. Limitation of Our Analysis

Here we discuss the limitations of our analysis.

The first limitation is that the trend discovered by T16 is based on only 47 exoplanets (see Equations (28) and (29)). It is well known that while hot Jupiters are statistically rare, observed planets are actually not rare, since they are readily observed by both radial velocity and transit methods (e.g., Winn & Fabrycky 2015; Dawson & Johnson 2018). This limitation indeed originates from an incomplete understanding of inflation mechanisms of hot Jupiters (T16). Once the dominant mechanism is identified, a similar analysis will be carried out for such hot Jupiters. Furthermore, current and future observations will attempt to improve measurements of both the mass and radius of detected exoplanets. Such better data will make it possible to apply a similar analysis not only to hot/warm Jupiters but also to smaller-sized planets. Thus, it is not currently obvious that the $M_{Z\text{gas}}$–$M_p$ and $Z_p$–$M_p$ relations derived by T16 are universal for various types of planets, which remains to be explored in future work.

The second limitation is that our analysis relies heavily on the computed value of $M_Z$. As discussed in Section 4.1, both
Figure 3. Heavy-element mass as a function of $M_p$ for observed exoplanets. As in Figure 1 (left), the computed values and the best fit derived by T16 are denoted by the black dots and solid line, respectively, in each panel. We also plot the straight line of $M_Z = M_p$ for reference (green dashed line). From top to bottom, the assumed core mass (1, 5, and $10 M_{\odot}$) is subtracted from $M_Z$. This parameterized approach shows that the power-law index for the $M_Z - M_p$ relation tends to be smaller with increasing $M_p$. This trend is well reproduced when observed exoplanets formed under the condition that gap formation is achieved in planetesimal disks around the planets and gas accretion onto the planets is controlled by disk evolution ($\tau_{\text{acc}} = \tau_{\text{hydro}}$; see Table 3). Our analysis therefore implies that the relationship discovered by T16 provides useful information for the final stage of planet formation.

Figure 4. Metallicity as a function of $M_p$. As in Figure 1 (right), the computed values of $Z_p$ and the best fit of T16 are plotted as the black dots and solid line, respectively, in each panel. We again adopt the parameterized approach for the core mass in order to examine how subtraction of possible values (1, 5, and $10 M_{\odot}$) of the core mass affects the $Z_p - M_p$ relation from the top to the bottom panels (as in Figure 3). Under the assumption that the envelope metallicity of the planets is purely determined by solid accretion in the post-core formation stage, our results can be viewed as indicating that a correlation between envelope metallicity and planet mass should be characterized by a shallower slope. Also, some transition in envelope metallicity should be present at the mass range of $10 M_{\oplus} \lesssim M_p \lesssim 100 M_{\oplus}$, which may be related to the core mass.
better observational data and modeling are needed to tightly constrain the value of $M_Z$. It was pointed out by T16 that the present observational data are still not good enough (see their Section 4.1). As a result, the error bars of $M_Z$ are currently determined mainly by uncertainties in mass and radius measurements of observed exoplanets. Even if the observational data become better, uncertainties in model parameters cannot be fully removed.

The third limitation is involved with our approach. In this approach, we focus only on the power-law indices of the $M_Z - M_p$ and $Z_p - M_p$ relations in order to elucidate the underlying physics. This simplification needs to be examined carefully by detailed numerical simulations. In particular, recent studies show that the gas accretion process behaves differently with different assumptions and numerical setups (e.g., Machida et al. 2010; D’Angelo & Bodenheimer 2013; Venturini et al. 2016; Lambrechts & Lega 2017). However, we emphasize that our adopted formula fits well the results of numerical simulations that were performed by different groups, such as Tanigawa & Watanabe (2002), D’Angelo et al. (2003), and Machida et al. (2010). As clearly shown in Figure 1 of Tanigawa & Tanaka (2016), the formula works well for planets with a mass range of $10^{-1} M_\odot \lesssim M_p \lesssim 30 M_\odot$ with a specific disk model that has a gas surface density of 140 g cm$^{-2}$, an aspect ratio of 0.05, and a turbulent parameter $\alpha$ of $4 \times 10^{-3}$ (Shakura & Sunyaev 1973) at the planet position of $r = 5.2$ au. This implies that once gas accretion is regulated by disk evolution (see Equation (2)), the formula will become reasonable until a (clear) gap is carved in the gas disks. Note that Lissauer et al. (2009) investigated gas accretion onto planetary cores, taking into account disk–planet interaction. While they derived a different form of the gas accretion recipe (see their Equation (2)), they adopted the simulations of D’Angelo et al. (2003). Thus, our formula should be broadly consistent with theirs. A more severe limitation of our approach is that we cannot compute the absolute value of $M_Z$ directly. The value would be determined by the combination of model and disk parameters. We will leave such a detailed study for future work.

Another limitation contained in our approach is that we have implicitly assumed that the physical radius of (proto)planets grows as $M_p^{1/3}$ when the $R_p$ dependence is converted to the $M_p$ dependence in Equations (12) and (13). Equivalently, gas accretion onto planets is characterized by the Hill radius, or the mean density of planets remains constant during accretion. While this assumption is reasonable when $\tau_{g,acc} = \tau_{g,KH}$, it is not justified for the case that $\tau_{g,acc} = \tau_{g,hydro}$. In fact, a number of 1D simulations show that if gas accretion is limited by disk evolution, the planet radius shrinks rapidly even as the planet mass increases (e.g., see phase D in Figure 2 of Mordasini et al. 2012). If this is the case, our assumption breaks down. We note, however, that the results of 1D simulations are obtained under a number of assumptions (see Section 2.1.1 of Mordasini 2013). For instance, 1D simulations assume that gas accretion occurs in a spherically symmetric manner and that when $\tau_{g,acc} = \tau_{g,KH}$, the disk gas is accreted onto planets at near freefall velocities (e.g., Bodenheimer et al. 2000; Lissauer et al. 2009; Mordasini 2013). As shown by 3D hydrodynamical simulations, the dynamics and morphology of accreting gas onto massive planets are much more complicated than those considered in 1D simulations. This is because circumplanetary disks are formed around planets and/or gas accretion proceeds from the polar direction rather than through circumplanetary disks (e.g., D’Angelo et al. 2003; Machida et al. 2010; Tanigawa et al. 2012). A more complete study of the mass and radius evolution of planets is required for determining the relationship between $M_p$ and $R_p$ for the case that $\tau_{g,acc} = \tau_{g,hydro}$.

5.2. Other Physical Processes

In this section, we consider the effect of other physical processes that are not included in our analyses. These include orbital evolution due to planetary migration, gas gap formation by the migration, and the effect of nearby forming planets.

First, we point out that our analyses do not take into account the orbital evolution of planets by planetary migration (e.g., Kley & Nelson 2012). It is expected that planetary migration...
allows protoplanets to replenish planetesimals in their feeding zones. This is because the protoplanets can sweep up a new region of their planetesimal disks. In fact, Alibert et al. (2005) showed that migrating protoplanets have more chances to accrete a larger number of planetesimals in the disks, which speeds up core formation. It is, however, important to emphasize that more detailed simulations with a direct $N$-body integrator suggest that the planetesimal accretion rate and gap formation in planetesimal disks depend on the migration speed, which is a function of planet mass (Tanaka & Ida 1999). A more self-consistent simulation is needed to investigate how gaps form around growing, migrating planets in their planetesimal disks and how semi-analytical formulae can be affected due to planetary migration (see Equations (12) and (13)).

Second, we discuss gap formation in gas disks that is the inevitable outcome of migration, especially for massive planets (e.g., Nelson et al. 2000; Crida et al. 2006; Hasegawa & Ida 2013; Dürmann & Kley 2015). As described in Section 3.1, the $M_z$–$M_p$ relation is determined not only by solid accretion but also by gas accretion onto planets (see Equation (8)). In the above analyses, the effect of gas gaps has not been considered explicitly. This is because our analyses rely heavily on the results of Shiraiishi & Ida (2008), and their results were obtained under the assumption of no gap formation in gas disks for simplicity. One might consider that the presence of gas gaps will affect our conclusion very much, since the gas surface density can now become a function of planet mass (see $\sigma_{p,\text{acc}}$ in Equation (2)). In order to address this point, we develop a similar analysis in the Appendix. Here we briefly summarize the results. We find that the trend found by T16 can be reproduced only when gaps are present in gas disks but there are no gaps in planetesimal disks (see Table 5). We argue that this situation is very unlikely to be achieved. This is because gap formation takes place more readily in planetesimal disks than gas disks due to the lack of the pressure term. Furthermore, even if planets accrete gas and solids from gapped gas disks, the total amounts of accreted gas and solids will not be significant, compared with those accreted from gas disks without any gap (e.g., Tanigawa & Ikoma 2007; Tanigawa & Tanaka 2016). Accordingly, it would be reasonable to consider that the trend of $M_z$ is determined predominantly before gap formation takes place in gas disks, and such a trend does not change very much after gas gap formation. Thus, our conclusion will be maintained even if gap formation in gas disks is properly taken into account, while a more self-consistent simulation is needed to fully justify this consideration.

Third, we have so far implicitly assumed that planet formation proceeds in an isolated region; that is, we consider formation of single planets. We must admit that this is a highly idealized situation. In reality, multiple planets form in single disks at the same time, and the gravitational interaction arising from nearby growing planets affects the dynamics of planetesimals there. This can change the spatial distribution of planetesimals and hence the condition of gap formation in planetesimal disks. It is interesting to investigate how the $M_z$–$M_p$ relation can be altered when formation of multiple planets is considered appropriately.

Thus, while some improvements will be required in our analyses for developing a more complete picture of planet formation, our present results are still useful for understanding a number of the currently known observational trends.

5.3. Comparison with Previous Studies

In this section, we touch on recent studies that are relevant to this work and compare their findings with ours.

One of the most advanced models that computes the total heavy-element mass in planets is that of Mordasini et al. (2014, 2016). In this model, the standard core accretion picture is adopted to trace the mass growth of planets. By coupling with planetary migration, they also made use of an enhanced planetesimal accretion rate, following the approach of Alibert et al. (2005). While they did not treat dust physics in planetary envelopes self-consistently, they mimicked this effect by artificially reducing the grain opacity there (Mordasini et al. 2014). Covering a large parameter space and performing population synthesis calculations, they found that the $Z_p$–$M_p$ relation is given as (see Table 7 in Mordasini et al. 2016)

$$\left( \frac{Z_p}{Z_{p,\text{M14}}} \right) = 7.2 \left( \frac{M_p}{M_{p,\text{M14}}} \right)^{-0.68},$$

where $M_p$ is the Jupiter mass. Note that this relationship is derived from the total heavy-element mass ($M_z$; see Mordasini et al. 2014). It is interesting that this slope is steeper than the results of T16 (see Figure 6 and Table 3). As discussed in Section 3.2, the slope is regulated by both planetesimal dynamics and gas accretion onto planets. In their model, the disk-limited gas accretion ($\tau_{\text{acc}} = \tau_{\text{hydro}}$) is taken into account, but the effect of gap formation in planetesimal disks is not. As a result, their simulations lead to a steeper slope. In
fact, our analysis predicts the value of their slope, which is about $-2/3$ (see the case of no planetesimal gap in Table 3). Thus, Mordasini et al. (2014, 2016) undertook a pioneering work and indicated the importance of planetesimal accretion for understanding the $Z_p$–$M_p$ relation. And our follow-up work reproduces the results of T16 better and derives a clearer view of how the $Z_p$–$M_p$ relation can be used for obtaining a better understanding of planet formation.

While we focus mainly on the total heavy-element mass ($M_Z$) in this paper, it would be interesting to consider atmospheric metallicity, as done in Section 4.3 (see Figure 4). To proceed, we discuss a correlation between envelope/atmospheric metallicity and planet mass. As an example, we adopt the result of Kreidberg et al. (2014), which is given as (see Table 3; also see Table 7 of Mordasini et al. 2016)

$$\frac{Z_{\text{atm}}}{Z_s} = 2.75 \left( \frac{M_p}{M_f} \right)^{-1.1}. \quad (31)$$

In their work, the metallicity of a hot Jupiter’s atmosphere is estimated based on the precise determination of the water abundance in the atmosphere. Combining the data points of four giant planets in the solar system, they obtain the above trend (green dashed line in Figure 6). It is obvious that their slope is much steeper than that of T16. Since such a steep slope cannot be explained by removing the initial core mass (see Figure 4), we propose that the results of Kreidberg et al. (2014) are very likely to trace the metallicity evolution in exoplanet atmospheres. Dust grain growth and settling take place in planetary atmospheres, namely, in the top, thin layer of planetary envelopes, and their effects are more pronounced for massive planets. If this is the case, a comparison between the total heavy-element mass ($M_Z$) and atmospheric metallicity can be used as an indicator of how the atmospheric metallicity of planets evolves with time. Given that most of the heavy elements should be present in planetary envelopes for massive planets, not in their cores (see Figure 3), they would be stored in the inner region of these envelopes. It is interesting that numerical simulations already show that these processes operate efficiently in planetary envelopes even during the process of forming (e.g., Movshovitz & Podolak 2008; Movshovitz et al. 2010). Note that the primordial envelope of planets should be more tenuous than the present one due to larger sizes, which principally leads to inefficient dust growth and settling there.

Finally, we discuss pebble accretion. As already pointed out in Section 3.3, recent studies focus mainly on core formation (e.g., Bitsch et al. 2015; Johansen & Lambrechts 2017), and application of their results to the final stage of planet formation may not be reasonable. In fact, we find that the resulting power-law profile of the $M_Z$–$M_p$ relation is not consistent with the result of T16 (see Table 3.) It is nonetheless important to point out that there is significant potential that pebble accretion may play a role in understanding the $M_Z$–$M_p$ and $Z_p$–$M_p$ relations. For example, it can be anticipated that a large number of pebbles will accumulate at the outer edge of the gas gaps after core formation is nearly terminated due to gas gap formation. If this were the case, such an accumulation of pebbles would lead to planetesimal formation there. Then, it would be possible to trigger the subsequent planetesimal accretion onto planets, which can eventually achieve enrichment of heavy elements in the planets. In fact, a high abundance of heavy elements in observed exoplanets requires a large amount of supplies that can potentially be delivered to the feeding zone of planets via radial drift of pebbles. Thus, while new numerical simulations of pebble accretion are desired, pebble accretion might not play a direct role at the final stage of planet formation.

5.4. A Comprehensive Picture

In this section, we combine the analyses and discussions done in the above sections. Keeping the limitations of our analysis in mind (Section 5.1), we develop a comprehensive picture of how observed exoplanets likely formed and how our understanding of planet formation can be improved (see Figure 7 and Table 4).

We begin with the $M_Z$–$M_p$ relation (left panel of Figure 7). We first divide the mass range explored by T16 into two regimes based on the gas accretion process (blue dots and green and gray regions; see also Section 4.2). Our analysis suggests that the behavior of $M_Z$ for planets with a mass of $100 M_\oplus \lesssim M_p \lesssim 3 \times 10^4 M_\oplus$ can be understood well if the observed planets keep the formation histories at their final stages. It is interesting to point out that this region can be extended to the mass range of brown dwarfs, beyond which gas accretion plays a more important role in determining the value of $M_Z$ (yellow region). This implies that while their formation efficiency may not be high, some brown dwarfs may form via the same mechanisms of forming planets. As $M_p$ increases, the contribution ($M_{Z,\text{gas}}$) coming from gas accretion becomes more significant (blue line). When $M_p$ reaches $3 \times 10^4 M_\oplus (> 10^3 M_\oplus)$, $M_Z \approx M_{Z,\text{gas}}$ (yellow region). In this paper, we tentatively call objects residing in the yellow region “stars.” Note that the theoretical distinction between a star and a planet should come from formation mechanisms (e.g., Chabrier et al. 2014; Hatzes & Rauer 2015).

We now discuss the lower-mass region. We first mention that the interpretation that is developed for the gray region is still applicable to planets that have masses of $15 M_\oplus \lesssim M_p \lesssim 100 M_\oplus$ (green region). In fact, the best fit is obtained for exoplanets with a mass range of $20 M_\oplus \lesssim M_p \lesssim 3 \times 10^3 M_\oplus$ in the original analysis (T16). It is nonetheless important to emphasize that some planets in the green region did not undergo efficient gas accretion (blue dots). Additional explanation would be needed to fully understand these planets, which remain to be explored in future work.

Another interesting point in the left panel of Figure 7 is that the line of $M_Z \propto M_p^{3/2}$ and the straight line of $M_Z = M_p$ intersect at $M_p \approx 4 M_\oplus$ (red solid line). This indicates that planets with a mass of $>4 M_\oplus$ can essentially obtain gaseous envelopes from their natal protoplanetary disks. This in turn implies that the critical core mass for the onset of gas accretion is about $4 M_\oplus$. It should be noted that this value of the core mass is roughly consistent with previous studies, which show that exoplanets with a radius larger than $\approx 1.6$ Earth radii (the corresponding mass of $\approx 5-6 M_\oplus$) are unlikely to be purely rocky (e.g., Marcy et al. 2014; Rogers 2015; Hasegawa 2016). We can therefore suggest that planets less massive than $\approx 4 M_\oplus$ tend to be made mostly of rocky (or solid) materials. Thus, the $M_Z$–$M_p$ diagram is useful for developing a better understanding of planet formation.
Finally, we turn our attention to the $Z_p$–$M_p$ relation (right panel of Figure 7). Our analysis suggests that the evolution of atmospheric metallicity in (exo)planets can be explored in the gray region by comparing the total planet metallicity ($Z_p$, black solid line) with the atmospheric metallicity (green dashed line). In the entire gray region, gas accretion provides only a minor contribution to $Z_p$ (blue dots). As a result, in order to fully understand the composition of (exo)planet atmospheres and reliably make a link with planet formation, a number of physical processes should be taken into account self-consistently. These are not only gas and solid accretion onto growing planets but also the subsequent processes, such as planetesimal ablation in planetary envelopes (e.g., Podolak et al. 1988) and dust growth and settling there (e.g., Movshovitz et al. 2010; Mordasini et al. 2014; Ormel 2014). For the low-mass region, there is no clear difference between $Z_p$ and the atmospheric metallicity, which might imply that most masses of planetary cores would potentially dissolve into their envelopes. Further analysis and/or modeling are certainly required for carefully examining exoplanets in the gray region.

In summary, we propose that observed exoplanet populations can be classified into three categories, depending on their masses (see Table 4). When $M_p \lesssim 4 M_J$, planets are made predominantly of rocky (or solid) materials, and they can be regarded as (super-)Earths. When $4M_J \lesssim M_p \lesssim 100 M_J$, they contain some amount of gas, so they can be called gas-poor, subgiant planets like Neptune in the solar system. Note that some mechanisms and/or fine-tuning of the formation timing are necessary for fully reproducing these planets. This additional requirement leads to a prediction that the population of planets in this category may not be so common (e.g., Ida & Lin 2004a; Mordasini et al. 2009). Such a prediction would be effective only for massive ($\gtrsim 10 M_J$) planets, since a couple of formation mechanisms are proposed for mini-Neptune-mass planets (e.g., Chiang & Laughlin 2013; Hansen & Murray 2013). Finally, planets that have a mass of $0.4M_J \lesssim M_p \lesssim 10^2 M_J$ are viewed as gas-rich giant planets. It is important to realize that most of their masses originate from their gaseous disks, while most of the heavy elements in these planets are determined by solid accretion such as planetesimals in the last formation stage.

5.5. Potential Roles of Current and Future Observations

We finally discuss the potential roles of current and future observations of exoplanets and their atmospheres that can be deduced from this work.

We begin with listing these roles. Observations of exoplanets’ atmospheres will allow one to explore the evolution of atmospheric metallicity, as shown in the right panel of Figure 7 (gray region). Specifically, comparison between hot and warm Jupiters will be invaluable for investigating how dust growth and settling take place in exoplanetary atmospheres. This is because the atmospheres of hot Jupiters are considered fully radiative (e.g., Fortney et al. 2007), and atmospheric metallicity may have a steeper slope due to efficient dust growth and settling (green dashed line). On the contrary, warm Jupiters tend to have convective atmospheres because they are far away from their host stars. If this is the case, a larger amount of heavy elements may be able to stay in planetary atmospheres, and their trend in atmospheric metallicity may differ from that of hot Jupiters. In addition, observations taken toward brown dwarfs will be interesting for examining their formation origins. Finally, the dots in the green region are not large enough to fully understand why some
planets in this region did not undergo runaway gas accretion. More observations are obviously needed to investigate how understanding of planetesimal accretion can be extended to this green region (e.g., Mordasini et al. 2016; Espinoza et al. 2017) and/or how pebble accretion comes into play to develop a better understanding of planet formation (e.g., Madhusudhan et al. 2017).

How can we examine the effect of planetary migration through observations of exoplanets and their atmospheres? In order to address this issue, we consider the bulk density of observed exoplanets. Figure 8 shows the data points obtained from the NASA Exoplanet Archive (Akeson et al. 2013) with our classification of exoplanets (see Table 4). One interesting feature of this figure is that the data points in the green region have scatter. As demonstrated clearly in Figure 3, planets in this region are most sensitive to subtraction of the assumed core mass. This in turn suggests that the total heavy-element mass is regulated predominantly by the core mass itself. Given that the contribution of the planets’ atmospheres is not very significant to their total mass (see Figure 2), this scatter may be interpreted as a potential signature of planetary migration: when core formation takes place in various regions of protoplanetary disks, their bulk densities possess diversity. If the subsequent planetary migration delivers these cores in the current positions, the observed diversity can be used as a fossil record of where they formed in the disks. Another interpretation of Figure 8 is that while the contribution of H/He-dominated atmospheres is not substantial to the total mass, their contribution to the planet radius is crucial, leading to diversity in the bulk density of planets (e.g., Wolfgang & Lopez 2015).

6. Summary and Conclusions

We have investigated how accretion of gas and solids onto growing planets determines the trend of the total heavy-element mass ($M_f$) in observed exoplanets. This work is motivated by T16, which shows that observed exoplanets have the correlations of $M_f \propto M_p^{1/5}$ and $Z_p/Z_\ast \propto M_p^{-2/5}$ (see Figure 1 and Table 3). We have made use of the existing semi-analytical formulae that are derived from more detailed studies and explored how accretion of planetesimals and pebbles proceeds onto planets with simultaneous gas accretion (see Table 3). We have demonstrated that the $M_f$–$M_p$ relation discovered by T16 is understood well if the relation traces the final stage of planet formation. At that stage, planets accrete solids from their gapped planetesimal disks, and gas accretion is limited by disk evolution. We have also found that core formation and pebble accretion cannot reproduce the power-law index derived by T16. It is interesting that this work suggests that pebble accretion might not play a direct role in the final formation stage. Moreover, our analysis has shown that the contribution arising from gas accretion is negligible for the total heavy-element mass in planets (see Figure 5).

We have reanalyzed the results of T16 to consider how they can be used for deriving some insights about planet formation. We have found that the envelope mass becomes comparable to the total planet mass at $M_p > 100 M_{\oplus}$ (see Figure 2). It is interesting that some planets in the mass range of $20 M_{\oplus} \lesssim M_p \lesssim 100 M_{\oplus}$ have less massive envelopes, compared with the total mass. This indicates that they did not undergo runaway gas accretion. Some mechanisms and/or fine-tuning of formation timing are needed for postponing the onset of runaway gas accretion for these planets. Furthermore, we have applied the results of our analysis to the atmospheric metallicity of exoplanets. Our analysis has suggested that the

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### Table 4

Classification of Observed Exoplanets and the Key Physical Processes of Forming These Planets

| Name                  | Mass Range             | Color in Figure 7 | Key Process* |
|-----------------------|------------------------|-------------------|--------------|
| Rocky planets         | $M_p \lesssim 4 M_\oplus$ | Blue              | Significant solid accretion with an almost negligible amount of gas |}

Note. *Detailed numerical simulations and further modeling for the observations of exoplanets are obviously needed to confirm our prediction.
evolution of metallicity in exoplanets’ atmospheres can be examined by comparing the total heavy-element mass in planets and the heavy-element mass in their atmospheres (see Figure 6).

We have compared our results with those of previous studies (see Table 3). We have found that despite the simplicity of our analysis, we can reproduce the power-law index of the $Z_p$–$M_p$ relation that is obtained by Mordasini et al. (2014) when the same assumption is employed. Note that the power-law index of Mordasini et al. (2014) is different from that of T16. We can therefore conclude that our simplified but physically motivated framework provides a clearer view of under what conditions the correlations of $M_X \propto M_p^{3/5}$ and $Z_p \propto M_p^{-2/5}$ are generated in the course of planet formation.

We have listed the limitations of our analysis that should be examined by detailed numerical simulations and future observations. We have discussed other physical processes that are not included in our analysis, such as planetary migration and the effect of multiple planet formation. Combining all the analyses done in this paper, we have proposed a classification of observed exoplanets. Finally, we have summarized the potential roles of current and future observations of exoplanets. Thus, we conclude that investigation of the $M_X$–$M_p$ relation is very important for understanding the final stage of planet formation. Further detailed modeling, numerical simulations, and dealing with a larger number of observational data are required for confirming our results and drawing a more complete picture of planet formation.

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Appendix

Effect of Gas Gaps on the $M_X$–$M_p$ Relation

Here we briefly discuss how the presence of gas gaps will affect the $M_X$–$M_p$ relation.

To proceed, we make use of the results obtained by Tanigawa & Ikoma (2007). In this study, mass growth of planets via gas accretion is investigated. In order to reliably take into account the effect of gas gaps that are opened up by planets, they considered both tidal interaction arising from the planets and viscous diffusion of gas disks and computed the resulting value of $\sigma_{p,\text{acc}}$ that regulates gas accretion onto the planets (see Equation (2)). They found that the profile of $\sigma_{p,\text{acc}}$ can divide into two regions. Provided that $M_p$, $r_p$, $\Omega_p$, $h$, and $\nu$ are the stellar mass, position of a planet, angular frequency at $r = r_p$, disk pressure scale height, and disk viscosity, respectively, the Hill radius of the planet ($r_H$) and the characteristic lengths, $l$ and $x_m$, are given as

$$r_H = \left( \frac{M_p}{3M_\ast} \right)^{1/3} r_p \propto M_p^{1/3}, \quad (32)$$

$$l = \left[ \frac{8}{81 \pi} \left( \frac{\nu}{r_p \Omega_p} \right)^{-1} \left( \frac{M_p}{M_\ast} \right)^2 \right]^{1/3} r_p \propto M_p^{2/3}, \quad (33)$$

and

$$x_m = 12^{1/5} \left( \frac{h}{l} \right)^{2/5} l. \quad (34)$$

Then, the profile of $\sigma_{p,\text{acc}}$ is determined by the balance between tidal torque and viscous diffusion for the case of $2r_H > x_m$. For the case of $2r_H < x_m$, its profile becomes steep enough that the Rayleigh instability condition eventually regulates the behavior of $\sigma_{p,\text{acc}}$. Finally, the gas accretion timescales can be given as (see Equation (B3) in Tanigawa & Ikoma 2007)

$$\tau_{g,\text{acc}}^{\text{Gap}} \propto M_p^4 \text{ for } 2r_H > x_m \quad (35)$$

and (see Equation (B8) in Tanigawa & Ikoma 2007)

$$\tau_{g,\text{acc}}^{\text{Gap}} \propto \frac{1}{3} \left( \frac{2r_H}{h} \right)^{2} + \frac{11}{124/5} \left( \frac{2r_H}{h} \right) \left( \frac{l}{h} \right)^{3/5}$$

$$- \frac{8}{12^{7/5}} \left( \frac{l}{h} \right)^{6/5} \text{ for } 2r_H < x_m. \quad (36)$$

Combining these timescales with the solid accretion rate ($dM_{Z,\text{solid}}/dh$), we obtain the $M_X$–$M_p$ relation (see Equations (14) and (15)). Table 5 summarizes the results. One may wonder if the trend found by T16 can be reproduced if no gap is formed in planetesimal disks for the cases of both $2r_H > x_m$ and $2r_H < x_m$. We argue that if gas gaps are already formed in planetesimal disks, the Rayleigh instability condition eventually regulates the behavior of $\sigma_{p,\text{acc}}$. Thus, when gaps are present around planets in gas disks, it is reasonable to consider that planetesimal disks also have gaps. Under such a condition, the $M_X$–$M_p$ relation that is derived from observed exoplanets cannot be reproduced.
