Voltage-flux-characteristics of asymmetric dc SQUIDs

Jochen Müller, Stefan Weiss, Rudolf Gross, Reinhold Kleiner, Dieter Koelle

Abstract — We present a detailed analysis of voltage-flux $V(\Phi)$-characteristics for asymmetric dc SQUIDs with various kinds of asymmetries. For finite asymmetry $\alpha_I$ in the critical currents of the two Josephson junctions, the minima in the $V(\Phi)$-characteristics for bias currents of opposite polarity are shifted along the flux axis by $\Delta\Phi = \alpha_I \beta_L$ relative to each other; $\beta_L$ is the screening parameter. This simple relation allows the determination of $\alpha_I$ in our experiments on YBa$_2$Cu$_3$O$_{7-\delta}$ dc SQUIDs and comparison with theory. Extensive numerical simulations within a wide range of $\beta_L$ and noise parameter $\Gamma$ reveal a systematic dependence of the transfer function $V_\Phi$ on $\alpha_I$ and $\alpha_R$ (junction resistance asymmetry). As for the symmetric dc SQUID, $V_\Phi$ factorizes into $g(\beta_L) \cdot f(\alpha_I, \beta_L)$, where now $f$ also depends on $\alpha_I$.

For $\beta_L \ll 2$ we find mostly a decrease of $V_\Phi$ with increasing $\alpha_I$, which however can only partially account for the frequently observed discrepancy in $V_\Phi$ between theory and experiment for high-$T_c$ dc SQUIDs.

Keywords — High-temperature superconductors, SQUIDs, superconducting devices.

I. INTRODUCTION

The observation of a significant discrepancy between numerical simulations and experimental results obtained for direct current (dc) superconducting quantum interference devices (SQUIDs) based on high-transition-temperature superconductors (HTS) is one of the most important unsolved problems for HTS dc SQUIDs which seriously hinders their optimization for applications. HTS dc SQUIDs show frequently asymmetric behavior which may be attributed to the large spread in the critical current $I_0$ and normal resistance $R$ of HTS Josephson junctions. This may lead to asymmetric critical current $I_c$ or voltage $V$ vs. external flux $\Phi$ characteristics of the dc SQUID [3] and can affect the transfer function $V_\Phi \equiv |dV/d\Phi|_{\text{max}}$ which is defined as the maximum slope of the $V(\Phi)$-curves.

II. ASYMMETRIC DC SQUID

However, such an asymmetry has been usually neglected in numerical simulations of $V_\Phi$ for HTS dc SQUIDs.

In this paper we present a detailed study of the impact of asymmetry on the $V(\Phi)$-characteristics and in particular on the transfer function of dc SQUIDs. We first introduce the main parameters which define the asymmetric dc SQUID (Sec.I). Then we show that an asymmetry in $I_0$ of the two Josephson junctions leads to a shift of the $I_c(\Phi)$- and in the $V(\Phi)$-characteristics, which can be used to determine the critical current asymmetry experimentally, as demonstrated on dc SQUIDs with YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) bicrystal grain boundary Josephson junctions [4] (Sec.III). We present numerical simulation results for $V_\Phi$ which we obtained within a wide range of parameters including the limit of large thermal fluctuations which are important for HTS dc SQUIDs (Sec.IV), and we compare those results with experimental data on HTS dc SQUIDs (Sec.IV).

Fig. 1

THE ASYMMETRIC DC SQUID

I. Introduction

THE observation of a significant discrepancy between numerical simulations and experimental results obtained for direct current (dc) superconducting quantum interference devices (SQUIDs) based on high-transition-temperature superconductors (HTS) is one of the most important unsolved problems for HTS dc SQUIDs which seriously hinders their optimization for applications. HTS dc SQUIDs show frequently asymmetric behavior which may be attributed to the large spread in the critical current $I_0$ and normal resistance $R$ of HTS Josephson junctions. This may lead to asymmetric critical current $I_c$ or voltage $V$ vs. external flux $\Phi$ characteristics of the dc SQUID [3] and can affect the transfer function $V_\Phi \equiv |dV/d\Phi|_{\text{max}}$ which is defined as the maximum slope of the $V(\Phi)$-curves.

II. Asymmetric DC SQUID

The asymmetric dc SQUID shown in Fig.I consists of a superconducting loop of inductance $L$ intersected by two Josephson junctions with average values of critical current $I_0$, resistance $R$ and capacitance $C$. The asymmetry in the junction parameters is described via the asymmetry parameters $\alpha_I$, $\alpha_R$ and $\alpha_C$, which are defined according to:

\[
I_{0,1} = I_0(1 - \alpha_I), \quad R_1 = R/(1 - \alpha_R), \quad C_1 = C(1 - \alpha_C), \\
I_{0,2} = I_0(1 + \alpha_I), \quad R_2 = R/(1 + \alpha_R), \quad C_2 = C(1 + \alpha_C),
\]

where the subscripts 1, 2 denote the parameters of the left and right junction, respectively.

Throughout this paper we consider two different origins of the junction asymmetry, noting, that in real devices we may have a combination of both: (i) geometric asymmetry and (ii) intrinsic asymmetry.

In the case of geometric asymmetry we assume identi-
cal values of critical current density \( j_0 \equiv I_0/A \), resistance times area\( \rho \equiv R \times A \), and specific capacitance \( C' = C/A \) for both junctions. Here, \( A = w \times l \) is the junction area with width \( w \) and length \( l \). We then introduce an asymmetry via different values for \( w \), assuming constant \( l \). In the case of bicrystal grain boundary junctions \( l \) equals the film thickness \( d \), which can be assumed to be constant in practical devices. The geometric asymmetry is then described by the asymmetry parameter \( \alpha_g \) according to
\[
\begin{align*}
\alpha_g = & \frac{w_1}{w_2} = (1 - \alpha_g)w, \\
& (w \equiv (w_1 + w_2)/2),
\end{align*}
\]
and we find the simple relation for the asymmetry parameters \( \alpha_g = \alpha_l = \alpha_R = \alpha_C \).

In the case of intrinsic asymmetry we assume \( \alpha_g = 0 \) and different values of \( j_0 \) and \( \rho \) for the two junctions which may reflect the natural spread in junction parameters. For simplicity we neglect the spread in \( j_0 \) and \( \rho \). The intrinsic asymmetry is then described by the asymmetry parameters \( \alpha_j \) and \( \alpha_\rho \) according to
\[
\begin{align*}
\alpha_j = & (1 - \alpha_j)/\alpha_j, \\
\alpha_\rho = & (1 + \alpha_\rho)/\alpha_\rho.
\end{align*}
\]
and we get \( \alpha_l = \alpha_j, \alpha_R = \alpha_\rho, \) and \( \alpha_C \approx 0 \).

If we assume the scaling relation \( I_0R \equiv j_0\rho \propto j_0^{1/2} \) as derived from the intrinsically shunted junction model [1], we derive the relations
\[
\alpha_\rho = [1 - (1 - \alpha_j^2)^{1/2}] / \alpha_j \quad \text{or} \quad \alpha_j = 2\alpha_\rho / (1 + \alpha_\rho^2). \tag{3}
\]

Finally, if we consider a combination of geometric and intrinsic asymmetry we can derive from the definitions of the asymmetry parameters given above the following relations between the asymmetry parameters
\[
\alpha_j = \frac{\alpha_g - \alpha_l}{\alpha_R \alpha_g - 1}, \quad \text{and} \quad \alpha_\rho = \frac{\alpha_g - \alpha_l}{\alpha_R \alpha_g - 1}. \tag{4}
\]

III. CRITICAL CURRENT VS. FLUX AND VOLTAGE VS. FLUX CHARACTERISTICS

We derive now a very simple expression for \( \alpha_l \), which can be used to determine its value experimentally without cutting the SQUID loop. Let us first consider the \( I_c(\Phi) \)-characteristics of the dc SQUID [c.f. Fig. 3] for \( \Gamma = 0 \): the maximum critical \( I_{c_{\text{max}}} = I_{0,1} + I_{0,2} = 2I_0 \) is maintained when \( I_1 = I_{0,1} \) and \( I_2 = I_{0,2} \). In this case the circulating current \( J \) is given as \( J(I_{c_{\text{max}}} = (I_{0,2} - I_{0,1})/2 = \alpha_l I_0 \), which is flowing in the SQUID loop if we apply an external flux \( \Phi^+ = \alpha_l I_0 L \). Hence, the maxima of the \( I_c(\Phi) \)-characteristics are shifted by \( \Phi^+ \) along the \( \Phi \)-axis, as compared to the symmetric SQUID with \( \alpha_l = 0 \), where \( I_c \) is maximum at \( \Phi = 0 \). The same argument leads to a negative shift \( \Phi^- = -\alpha_l I_0 L \) if the current direction is reversed. Hence, the maxima of the \( I_c(\Phi) \)-characteristics for opposite polarity of the current are shifted by \( \Delta \Phi \equiv \Phi^+ - \Phi^- = 2\alpha_l I_0 L \). Using the screening parameter \( \beta_L \equiv 2I_0 L/\Phi_0 \) we arrive at the very simple relation
\[
\Delta \Phi/\Phi_0 = \alpha_l \beta_L, \tag{5}
\]
which is also valid for finite values of \( \Gamma \) and for the shift of the minima of the \( V(\Phi) \)-characteristics measured at constant bias current \( I \approx 2I_0 \) and \( -I \approx -2I_0 \).

The clear-cut experimental determination of \( \Delta \Phi \) requires the measurement of \( V(\Phi) \)-curves at various values of \( \beta_L \), which can be varied by temperature. An example of such a measurement on one device at \( T = 77 \) and \( 63.7K \) is shown in Fig. 2. Plotting the measured flux shift, normalized by \( \alpha_l \Phi_0 \) vs. \( \beta_L \) should give according to Eq. (3) a linear dependence with slope 1, with the reasonable assumption that \( \alpha_l \) does not depend on \( T \). Fig. 3 shows the results of such measurements obtained for 6 different dc SQUIDs, where \( \alpha_l \) was obtained as a fitting parameter to give the expected slope of 1. For comparison, the results from simulated \( V(\Phi) \)-curves for various values of \( \alpha_l \) are also shown, which are in excellent agreement with Eq. (5).

Except for one device, these SQUIDs have intentionally been fabricated with a geometric asymmetry \( (\alpha_g \neq 0) \). From the known value of \( \alpha_g \) and the measured value of \( \alpha_l \) the asymmetry parameter \( \alpha_l \) can be calculated using Eq. (4). The results are listed in Table I. As a main result, we see that 3 SQUIDs show only a small asymmetry in \( j_0 \) with \( |\alpha_j| \leq 0.1 \). However, for the three other devices the asymmetry in \( j_0 \) is significant with values of \( |\alpha_j| \) up to 0.4, which demonstrates that the difference in critical current density for the two junctions can be quite large.
IV. TRANSFER FUNCTION: NUMERICAL SIMULATIONS

As already evident from Fig. 2 the asymmetry can induce distortions of the \(V(\Phi)\)-curves, which leads to different values of \(V^+_\Phi\) and \(V^-_\Phi\) for the maximum positive and negative slope of the \(V(\Phi)\)-curves, respectively. To understand the impact of the asymmetry on the transfer function we performed numerical simulations to solve the equations for the phase differences \(\delta_1(t)\) and \(\delta_2(t)\) of the two junctions [2].

Over a wide range of values for \(\beta\) and the noise parameter \(\Gamma \equiv 2\kappa L T / I_0\Phi_0\) for both, the geometric and intrinsic asymmetry. In the latter case we assume the correlation between \(\alpha_L\) and \(\alpha_R\) as given in Eq. (2).

Figure 3 shows \(V^+_\Phi\) and \(V^-_\Phi\) vs. \(\beta_L\) obtained for fixed \(\beta_L = L / \Phi_0\) = 0.2 for geometric and intrinsic asymmetry. For \(\alpha_L = 0\) we closely reproduced the results obtained in [1] for symmetric dc SQUIDs. In most cases we find for \(\beta_L \leq 5\) the asymmetric SQUID a reduction of \(V^\Phi\) as compared to the symmetric SQUID, which increases with decreasing \(\beta_L\) and increasing \(\alpha_L\). However, in the case of geometric asymmetry, we find for intermediate values of \(\beta_L \leq 5\) an increase in \(V^+\) for \(\alpha_L = 0\) \(\leq 0.5\), which is concomitant with a strong distortion of the \(V(\Phi)\)-characteristics. If we assume \(I_0 R \propto J_0^2 / 2\), we find that for geometric asymmetry \(\alpha_R\) is always larger than for intrinsic asymmetry (for given \(\alpha_L\)). This implies that the distortion in \(V(\Phi)\) is dominated by the asymmetry in the junction resistances which becomes important for \(\beta_L \geq 1\). At \(\beta_L \leq 0.2\) the reduction of \(V^+\) and \(V^-\) is similar for both types of asymmetry, indicating that for small \(\beta_L\) the asymmetry in the critical currents gives the main contribution to \(V^\Phi\).

Results similar to those shown in Fig. 4 have been obtained over a wide range \(1 / 80 \leq \Gamma \beta_L \leq 0.5\) which corresponds to \(4 \Phi_0 \leq L \leq 160 \Phi_0\) for the SQUID inductance at \(T = 77 K\) where \(L_{dc} = 321 \Phi_0\). We note that for the symmetric dc SQUID it was shown in [1] that the normalized transfer function \(v^\Phi_\alpha \equiv V^\Phi_\Phi / I_0 R\) factorizes in \(v^\Phi_\alpha = g(\Gamma \beta_L) \cdot f(\beta_L)\) where \(f(\beta_L)\) is given as \(v^\Phi_\alpha(\beta_L; \Gamma \beta_L = 1/80)\).

As a main result of our simulations for the asymmetric dc SQUID we find a similar factorization, with \(f(\alpha_L, \beta_L)\) being now also dependent on \(\alpha_L\), while \(g(\Gamma \beta_L)\) shows no dependence on \(\alpha_L\). For \(V^+\) this is shown in Fig. 4(a) for geometric and in Fig. 4(b) for intrinsic asymmetry. The simulation data shown in Fig. 4 can be approximated as

\[
\begin{align*}
g^+_\text{geo}(\Gamma \beta_L) &= [(80 \Gamma \beta_L)^{0.4} + 0.35(4 \Gamma \beta_L)^{2.5}]^{-1} \\

\end{align*}
\]

TABLE I

| \# | \(\alpha_R\) | \(\alpha_L\) | \(\alpha_J\) |
|----|------------|------------|------------|
| 1  | 0         | 0.33       | 0.1        |
| 2  | 0.33      | 0.33       | -0.29      |
| 3  | 0.33      | 0.3        | -0.04      |
| 4  | 0.5       | 0.4        | 0.08       |
| 5  | 0.5       | 0.12       | -0.4       |
| 6  | 0.5       | 0.14       | -0.39      |

Fig. 4

Calculated normalized transfer function \(V^+_{\Phi}\) and \(V^-_{\Phi}\) vs. \(\beta_L\) for \(\Gamma \beta_L = 0.2\) with geometric asymmetry (a), (b) and intrinsic asymmetry (c), (d); \(\alpha_L = 0\) (d), 0.2 (c), 0.4 (a), 0.6 (f), 0.8 (g), 0.9 (h); solid lines are guide to the eye. For comparison, simulation data (f) together with the according fit-function (dotted line) from [1] for symmetric dc SQUIDs are also shown.
ing intrinsic asymmetry with large values of symmetric SQUIDs cannot be explained by asymmetry for observed deviations between experiment and simulation for experimental determination of at least asymmetry in the SQUID, although a large reduction of values covered by simulations which take into account asymmetry in the SQUID, although a large reduction of VΦ due to asymmetry, say by a factor of five requires a very large αI ≈ 0.9.

VI. CONCLUSIONS

We have analyzed the performance of asymmetric HTS dc SQUIDs both experimentally and by numerical simulation, with focus on transfer function. Our simulations show that strong critical current asymmetry which may arise from a large spread in critical currents in HTS Josephson junctions can significantly reduce VΦ for small βL < 2. This observation is important, since optimum performance requires the realization of small βL ≈ 1. We wish to stress that the asymmetry, which is most likely present in almost all HTS dc SQUIDs, may be one source for the previously found discrepancy in VΦ between experiments and simulations, however it is not likely that this asymmetry is the major source of this discrepancy.

ACKNOWLEDGMENT

We gratefully acknowledge valuable support from Knut Barthel and Alex I. Braginski.

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Fig. 7
COMPARISON OF CALCULATED AND MEASURED NORMALIZED TRANSFER FUNCTION PLOTTED VS. $\beta_L$.

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