Transplanckian radiation in theories with extra dimensions

D.V. Gal’tsov

Department of Theoretical Physics, Moscow State University, 119899, Moscow, Russia

Email: galtsov@phys.msu.ru

Abstract

We discuss whether particles undergoing transplanckian collisions in TeV-scale gravity can deplete most of their energy on bremsstrahlung for impact parameters much larger than the gravitational radius of the presumably created black hole.

1 Introduction

The models of TeV-scale Quantum Gravity (TQG) proposed as an attempt to solve the hierarchy problem open the possibility to study physics beyond the Planck’s scale. A conceptually and technically simplest TQG model suggested by Arkani-Hamed, Dimopoulos and Dvali (ADD) [1] and further elaborated in [2] assumes that the standard model particles reside in the four-dimensional sector of space-time $\mathcal{M}_4$, while gravity propagates in the $D$-dimensional bulk with the $d = D - 4$ flat dimensions compactified on a torus $T^d$. $D$-dimensional gravity is strong, the corresponding Planck mass $M_*$ being of the order of TeV. The Planck length $l_*$ and the gravitational radius $r_s$ associated with the center-of-mass energy $\sqrt{s}$ in the ADD model are

$$l_* = \left( \frac{\hbar G_D}{c^3} \right)^{\frac{1}{d+2}} \sim \frac{\hbar}{M_* c}, \quad r_s = \left( \frac{G_D \sqrt{s}}{c^4} \right)^{\frac{1}{d+1}}.$$  \hspace{1cm} (1)

In the transplanckian (TP) particle collisions gravity not only becomes the dominant force, but it partly restores its classical character [3]. Since the gravitational radius $r_s$ corresponding to the energy in the center of mass frame grows with $s$, while the de Broglie length of the particles $\lambda_B = \hbar c/\sqrt{s}$ decreases, therefore for $\sqrt{s} > M_*$ the classicality condition is satisfied

$$\lambda_B \ll l_* \ll r_s.$$ \hspace{1cm} (2)

In the ADD model with $d \neq 1$ the classical TP region is also restricted from above [3] by the impact parameter
We have therefore the following three TP sectors: 

1) \( b > b_c \). This is the quantum sector where the scattering is dominated by the one-graviton exchange.

2) \( r_s < b < b_c \). This is the eikonal sector, where the scattering amplitude is dominated by the sum of ladder and cross-ladder diagrams whose summation gives the eikonal amplitude \([5]\). The stationary phase approximation of the latter coincides with the classical amplitude \([6]\). It is also known that the semiclassical calculation of TP elastic scattering cross section in four-dimensional space-time \([7]\) agrees with the string theory result \([8]\). Gravitational radiation in this region is expected to be well-described by the classical theory.

3) \( b < r_s \). This is the region of strong gravity, where the main process is the formation of the black hole \([9]\). This was checked within an approach based on the picture of colliding waves representing ultrarelativistic particles \([10]\). However this approximation is susceptible to radiation reaction \([11]\), which is still not well understood. Various more sophisticated approaches to test the conjecture of black hole creation at colliders were suggested (see a recent review \([12]\)), which gave additional arguments of its validity. Numerical work performed in this direction is reviewed in \([13]\).

Here we discuss whether gravitational radiation in TP collisions can be large enough in the region 2). If so, this could substantially modify the predicted cross-section of the formation of a black hole. Note, that for a head-on collision of black holes the upper bound was given by Eardley and Giddings \([10]\) generalizing Penrose limit:

\[
\epsilon \leq 1 - \frac{1}{2} \left( \frac{(D - 2) \Omega_{D-2}}{2 \Omega_{D-3}} \right)^{1/D}, \tag{4}
\]

which gives the bound about 41.9% for \( D = 11 \). A recent calculation \([14]\) using the approach of D’Eath and Payne \([15]\) gave in the first perturbative order the result slightly below this bound (the second order and numerical calculations give smaller values). However, it is known that radiation from particles plunging into the black hole grow with the impact parameter. Moreover, the above calculations are based on the picture of colliding waves, which itself is appropriate only if radiation losses are small, while the above values can not be considered small.

## 2 Eikonal and bremsstrahlung

The eikonalized elastic scattering amplitude can be presented as the integral

\[
M_{\text{eik}}(s, t) = 2is \int e^{i\mathbf{q} \cdot \mathbf{b}} \left( 1 - e^{i\chi(s, b)} \right) d^2b, \tag{5}
\]

where the two-dimensional vectors \( \mathbf{q}, \mathbf{b} \) lie in the transverse plane, with \( \mathbf{b} \) playing the role of the impact parameter vector. The transverse component \( \mathbf{q} \) of the momentum transfer in this approximation satisfies \( q^2 \approx -\eta^\mu q_\mu \), so that
\( t \simeq -q^2 \). This expression in the usual four-dimensional theory corresponds to summation of the ladder and crossed-ladder diagrams. The eikonal phase \( \chi(s, b) \) can be obtained expanding the exponential and equating the leading term to the Born amplitude

\[
\chi(s, b) = \frac{1}{2s} \int e^{-iq \cdot b} M_{\text{Born}}(s, t) \frac{d^2q}{(2\pi)^2}.
\] (6)

The computation gives

\[
\chi(s, b) = \left( \frac{b_c}{b} \right)^d,
\] (7)

where \( b_c \) is given by (3). Substituting this into (5) and calculating the integral in the stationary phase approximation around the point

\[
b_s = \left( \frac{db_c^2}{q} \right)^{1/(d+1)},
\] (8)

one recovers the purely classical result:

\[
M_{\text{cl}}(s, t) = \frac{4\sqrt{\pi} s^{3/2 - \pi/2}}{q^\frac{d}{d+1}} \left( \frac{2\sqrt{\pi} s \Gamma(d/2 + 1)}{M^2_{c,\text{cl}} q^{d+2}} \right)^{\frac{\pi}{d+1}},
\] (9)

which was obtained classically for the small angle scattering of ultrarelativistic point particles [6].

Now consider gravitational bremsstrahlung, which is the main inelastic process in the eikonal region of TP scattering. In the eikonal approach radiation manifests itself as the imaginary part \( \chi(s, b) \) which can be found taking into account the leading corrections to the ladder approximation from the called H-diagrams [16]. According to [3],

\[
\text{Im} \chi \sim \left( \frac{b_r}{b} \right)^{3d+2},
\] (10)

where one more length parameter \( b_r \) is introduced, satisfying the relation

\[
b_r \sim \left( \frac{b_c}{r_s} \right)^{\frac{d}{d+2}}.
\] (11)

In the situation when the eikonal sector of TP scattering is wide enough, \( r_s \ll b_c \), one has \( b_r \gg r_s \) and the impact parameter \( b_c \) can lie in the interval

\[
r_s \ll b \ll b_r,
\] (12)

in which case \( \text{Im} \chi \) is not small. If the imaginary part of the eikonal is interpreted as the number of emitted gravitons whose frequency is much higher than \( \omega_b = 1/b \), this would lead to the conclusion that the colliding particles can deplete all the energy for impact parameters much larger than \( r_s \). Meanwhile, extraction of the eikonal imaginary part from the H-diagram probably is only consistent in the deep infrared region. Assuming that the spectrum is dominated by the frequencies \( \omega < \omega_b \), and the number of soft gravitons is \( N = \text{Im} \chi \), one finds the following estimate for the radiation efficiency [3]

\[
\epsilon = \frac{\Delta E}{E} \sim \left( \frac{r_s}{b} \right)^{\frac{d+2}{d}},
\] (13)
which is not catastrophical for \( b > r_s \). However, classical considerations \[18\] indicate that the dominant region of the bremsstrahlung spectrum is \( \omega \gg \omega_b \). Thus, one needs other methods to answer the question whether \( \epsilon \) may become of the order of unity for \( b > r_s \). An interesting approach to TP bremsstrahlung problem using the eikonal approximation in the spirit of ’t Hooft was suggested by Lodone and Rychkov \[17\], but so far it was applied only to gluons.

3 TP bremsstrahlung at \( b \gg r_s \)

In a series of papers \[11, 18, 19\] we calculated ultrarelativistic bremsstrahlung in \( M_4 \times T^d \) using classical perturbation theory in momentum space, which was suggested long ago in the context of four-dimensional General Relativity \[20\]. Starting with the action

\[
S = -\sum_\text{M} \sqrt{g_{MN} \ddot{x}^M \ddot{x}^N} ds + S_\phi(\phi, g) + S_{\text{int}} [x(s), \phi, g] + S_g(g), \quad (14)
\]

for two point particles, mutually interacting with the set of non-gravitational fields \( \phi \) and the gravity described by bulk metric \( g_{MN} \). The metric is presented as \( g_{MN} = \eta_{MN} + \kappa_D h_{MN} \), and \( x(s) \), \( \phi \) and \( h_{MN} \) are further expanded in terms of the particle-field couplings \( f \) and the gravitational coupling \( \kappa_D \). In the zero order approximation particles move freely in the opposite directions at an impact parameter \( b \). Then we iterate the system of the particle equations of motion and the field equations up to the second order, in which radiation is manifest. This procedure presumably converges in the ultrarelativistic case when the scattering angle \( \theta' \) is small, though to get more precise limits of applicability on has to go to the next iteration order, which is quite non-trivial.

One is interested in computing the total radiation efficiency \( \epsilon \) and the spectral distribution under different assumptions about nature of the field dominating the interaction between the particles (mediator field), and nature of the radiation field. The set of \( \phi \) generically contains the brane \( \varphi \) and the bulk \( \Phi \) fields either of which can be mediator and/or radiated field. Gravity interacts with particles and with both fields \( \varphi, \Phi \), introducing non-linearity into the problem. Coupling constants of the particles with \( \varphi (f_0) \) and \( \Phi (f_d) \) have different dimensions and are related though the volume of the torus \( V_d \) as \( f_d^2 = f_0^2 V_d \). The corresponding classical length parameters \( r_d \) are introduced via the relation \( f_d^2 / r_d^{d+1} = m \), so \( r_d^{d+1} = r_0^2 V_d \). The bulk fields \( \Phi, h_{MN} \) depending on \( x^M = (x^\mu, y^i) \) are expanded in the Kaluza-Klein modes \( \Phi_n(x), h_{MN}^n(x), n \in \mathbb{Z}^d \) which have the masses \( \mu_n^2 = (2\pi n^2 / L^2) \).

The radiation efficiency and its spectrum depend on the nature of \( \phi \) and on whether gravity is the dominant mediator. If not, one deals with the flat space problem in which both mediating and emitted fields are linear. This is the case of the Maxwell-Lorentz theory, which was exhaustively explored both in classical and quantum electrodynamics. The only novel here feature is presence of extra dimensions. Note that if one deals only with bulk fields, the ADD problem in
$\mathcal{M}_4 \times T^d$ reduces to that in $D$-dimensional Minkowski space provided the large number of KK modes in involved.

The frequency of radiation depends on the emission angle with respect to the direction of the collision $\omega_{\text{cr}}(\theta) \simeq b^{-1}(\theta^2 + \gamma^2)^{-1}$, where $\gamma$ is the Lorentz factor in the rest frame of one of the particles. So most of the radiation is beamed inside the cone $\theta < 1/\gamma$ in any dimension $D$. The maximal frequency of radiation is

$$\omega_{\text{max}} \simeq \omega_{\text{cr}}(\theta = 0) \sim \frac{2\gamma^2}{b},$$

whose vicinity gives the dominant contribution. The total bremsstrahlung efficiency for different combinations of exchange modes and radiated modes reads [18] (omitting numerical coefficients):

1. brane modes exchanged and radiated: $\epsilon_{\phi\phi} \sim \gamma r_0^3/b^3$;
2. bulk modes exchanged brane modes radiated: $\epsilon_{\Phi\phi} \sim \gamma \left(\frac{r_0}{b}\right)^2(1+d)$;
3. brane modes exchanged, bulk modes radiated: $\epsilon_{\phi\Phi} \simeq \left(\frac{r_0}{b}\right)^2 \left(\gamma \frac{r_0}{b}\right)^{1+d}$;
4. bulk modes exchanged and radiated: $\epsilon_{\Phi\Phi} \simeq \left(\frac{r_0^3}{b^3}\right)^{1+d}$.

These formulas can be interpreted in terms of the effective numbers of massive KK states contributing as exchange modes $N_{\text{ex}} \sim V_d/b^d$ and radiated modes $N_{\text{rad}} \sim V_d \gamma^d/b^d$, where the $\gamma$-enhancement factor in $N_{\text{rad}}$ accounts for modes with masses up to $\omega_{\text{max}}$ emitted inside the cone $\theta < 1/\gamma$. In the bulk, Extra dimensions contribute to radiation efficiency with the factor $N_{\text{ex}}^2$ for KK exchange modes and with the factor $N_{\text{rad}}$ for radiation modes, so, e.g., $\epsilon_{\Phi\Phi} \sim \epsilon_{\phi\phi} N_{\text{ex}}^2 N_{\text{rad}}$. This explains the origin of an extra factor $\gamma^d$ in $\epsilon_{\Phi\Phi}$. Therefore, classically, the bremsstrahlung efficiency grows with $\gamma$, and the $\gamma$-factor is $d$-dependent.

However, one has the quantum restriction on the frequency $\hbar \omega < m \gamma$, which for the maximal frequency $\omega = \omega_{\text{max}} = 2\gamma^2/b$ gives $b_{\text{min}} = \lambda c$, where $\lambda_c = 1/m$ is the Compton length. The boundary value of the radiation efficiency $\epsilon_{\Phi\Phi} \simeq (r_0^3/(\lambda_c^3\gamma^2))^{1+d}$ thus decreases with energy.

The situation becomes more complicated if gravity is the dominant mediator [19]. The main new feature is non-linearity of the problem due to $\phi\phi h$ vertex and the three-graviton coupling, which leads to non-locality of the source in the D’Alembert equation for radiation modes:

$$\Box \Phi_{\text{rad}}^n = j_n \equiv \rho_n + \sigma_n.$$  

Here $\rho_n$ has support on the particles world-lines, while $\sigma_n$ is given in terms of lower order field perturbations; it is extended in space (including extra dimensions). It turns out that $\rho_n$ and $\sigma_n$ compete with each other and the result depends on the number of extra dimensions. In the case $d = 0$, as it was shown long ago [20], these contributions mutually cancel in the frequency range

$$\omega'_{\text{max}} < \omega < \omega_{\text{max}}, \quad \omega'_{\text{max}} = \gamma/b = \omega_{\text{max}}/2\gamma,$$

5
so the frequency is bounded by \( \omega'_{\text{max}} = \gamma / b \). In this case the quantum boundary \( \omega'_{\text{max}} < m \gamma \) is energy-independent. The efficiency of the gravitational bremsstrahlung in the gravity mediated collision in \( D = 4 \) \(^{[20]}\) is

\[
\epsilon_{\text{hh}} \sim \gamma_{\text{cm}} r_s^3 / b^3,
\]

where \( r_s \) is given by \(^{[1]}\) with \( d = 0 \). The energy-dependent restriction on \( b \) arises from the condition of smallness of the gravitational potential energy of the fast particle in the rest frame of the other, with respect to the particle energy, equivalent to \( b \gg r_s \gamma_{\text{cm}} \). Thus, on the boundary \( \epsilon_{\text{hh}} \) remains small.

For \( d \neq 0 \) cancelation of local and non-local contributions also takes place but less complete, so the spectral-angular distribution still contains higher frequencies \(^{[17]}\). The total efficiency in \( D = 5 \) scales as \( \epsilon_{\text{hh}} \sim \gamma_{\text{cm}} \ln \gamma_{\text{cm}} \cdot r_s^6 / b^6 \), while for \( d \geq 2 \) one obtains

\[
\epsilon_{\text{hh}} \sim (r_s / b)^{3(d+1)} / \gamma_{\text{cm}}^{2d-1}.
\]

Smallness of the scattering angle with respect to \( \gamma^{-1} \) (this restriction generalizes the one relevant for small angle bremsstrahlung calculation on the fixed background, in our two-body approach it may be overrestricting) implies \( b > r_s \gamma_{\text{cm}}^{1/d+1} \), so classically we get on the boundary \( \epsilon_{\text{hh}} \sim \gamma_{\text{cm}}^{2(d-2)} \). This means that for \( d \geq 3 \) the energy is depleted for \( b \gg r_s \). This result is still susceptible to quantum bounds, since for \( d \neq 0 \) the quantum frequency restriction on \( b \) is stronger than classical. This question is currently under investigation \(^{[21]}\). If confirmed, the above estimate means that classical radiation damping may be regarded as another classicalization mechanism additional to creation of the black hole.

**Acknowledgments**

The author thanks the Organizing Committee of ICGA-12 for an excellent conference. He is grateful to G.Kofinas, P.Spirin and Th.Tomaras for useful comments and fruitful collaboration. The work was supported by the RFBR grant 11-02-01371-a.

**References**

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) ; Phys. Rev. D 59, 086004 (1999) .

[2] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544, 3 (1999) ; T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D 59, 105006 (1999)

[3] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 630, 293 (2002).

[4] S. B. Giddings and M. Srednicki, Phys. Rev. D 77, 085025 (2008) .
[5] D. N. Kabat and M. Ortiz, Nucl. Phys. B 388, 570 (1992); D. Amati, M. Ciafaloni and G. Veneziano, Nucl. Phys. B 403, 707 (1993).

[6] D. V. Gal’tsov, G. Kofinas, P. Spirin and T. N. Tomaras, JHEP 0905, 074 (2009).

[7] G. ’t Hooft, Phys. Lett. B 198, 61 (1987).

[8] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B197 (1987) 81; Int. J. Mod. Phys. A 3 (1988) 1615.

[9] P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B 441, 96 (1998); T. Banks and W. Fischler, [arXiv:hep-th/9906038] S. B. Giddings and S. Thomas, Phys. Rev. D 65, 056010 (2002).

[10] D. M. Eardley and S. B. Giddings, Phys. Rev. D 66, 044011 (2002).

[11] D. V. Gal’tsov, G. Kofinas, P. Spirin and T. N. Tomaras, Phys. Lett. B 683, 331 (2010).

[12] S. C. Park, [arXiv:1203.4683] [hep-ph].

[13] V. Cardoso, L. Gualtieri, C. Herdeiro, U. Sperhake, P. M. Chesler, L. Lehner, S. C. Park and H. S. Reall et al., [arXiv:1201.5118] [hep-th].

[14] C. Herdeiro, M. O. P. Sampaio and C. Rebelo, JHEP 1107, 121 (2011); F. S. Coelho, C. Herdeiro and M. O. P. Sampaio, [arXiv:1203.5355] [hep-th].

[15] P. D. D’Eath and P. N. Payne, Phys. Rev. D 46, 658 (1992).

[16] D. Amati, M. Ciafaloni and G. Veneziano, Nucl. Phys. B 347, 550 (1990).

[17] P. Lodone and V. S. Rychkov, JHEP 0912, 036 (2009).

[18] D. V. Gal’tsov, G. Kofinas, P. Spirin and T. N. Tomaras, JHEP 1005 (2010) 055.

[19] Y. Constantinou, D. Gal’tsov, P. Spirin and T. N. Tomaras, JHEP 1111 (2011) 118.

[20] D.V. Gal’tsov, Yu.V. Grats, In “Modern problems of Theoretical Physics”, Moscow, Moscow State Univ. Publ., 1976, 258-273 (in Russian); D. V. Gal’tsov, Yu. V. Grats and A. A. Matyukhin, Sov. Phys. J. 23, 389 (1980); Lorentz-covariant perturbation theory for relativistic gravitational bremsstrahlung [arXiv:1012.3060] [hep-th].

[21] D. Gal’tsov, P. Spirin and T. N. Tomaras, JHEP01(2013)087 [arXiv:1210.6976] [hep-th].