Equilibrium sequences of non-rotating and rapidly rotating crystalline color-superconducting hybrid stars

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The three-flavor crystalline color-superconducting (CCS) phase of quantum chromodynamics (QCD) is a candidate phase for the ground state of cold matter at moderate densities above the density of the deconfinement phase transition. Apart from being a superfluid, the CCS phase has properties of a solid, such as a lattice structure and a shear modulus, and hence the ability to sustain multipolar deformations in gravitational equilibrium. We construct equilibrium configurations of hybrid stars composed of nuclear matter at low, and CCS quark matter at high, densities. Phase equilibrium between these phases is possible only for rather stiff equations of state of nuclear matter and large couplings in the effective Nambu–Jona-Lasinio Lagrangian describing the CCS state. We identify a new branch of stable CCS hybrid stars within a broad range of central densities which, depending on the details of the equations of state, either bifurcate from the nuclear sequence of stars when the central density exceeds that of the deconfinement phase transition or form a new family of configurations separated from the purely nuclear sequence by an instability region. The maximum masses of our non-rotating hybrid configurations are consistent with the presently available astronomical bounds. The sequences of hybrid configurations that rotate near the mass-shedding limit are found to be more compact and thus support substantially larger spins than their same mass nuclear counterparts.

I. INTRODUCTION

Matter in the interiors of neutron stars is compressed by gravity to densities which are by factors 5 to 10 larger than the density of an ordinary nucleus. At such densities baryons are likely to lose their identity and dissolve into their constituents - deconfined quarks. It has been suggested long ago that quark matter may exist in the interiors of compact objects. Within the deconfined phase unbound quarks form Fermi seas and attractive interactions bind them into Cooper pairs thus giving rise to quark superconductivity. Along with the deconfinement phase transition the dense matter undergoes a second transition related to the restoration of chiral symmetry (the dynamical symmetry of the strong interaction). This symmetry may remain broken in some color-superconducting phases, such as the color-flavor-locked phase.

If compact (hybrid) stars featuring quark cores surrounded by a nuclear mantle exist in nature, they could provide a unique window on the properties of QCD at high baryon densities. Even if the relevant densities and sufficiently low temperatures could be reached in a laboratory in the future, the conditions prevailing in compact stars are different from those produced in accelerators: the matter is long-lived, charge neutral and in \( \beta \)-equilibrium with respect to weak interactions.

A central question of the theory of compact stars, which we will address in this work, is whether the equation of state of matter at high densities admits stable configurations of self-gravitating objects in General Relativity featuring deconfined quark matter, and if so, are the gross parameters of these objects, like their mass and radius, compatible with the known astronomical bounds? The deconfinement phase transition from baryonic to quark matter leads to a softening of the equation of state which could lead to an instability towards a collapse into a black hole. The details of a stability analysis may depend on the theoretical model of deconfined, color-superconducting quarks. The Nambu–Jona-Lasinio (NJL) model is a non-perturbative low-energy approximation to QCD, which is anchored in the low-energy phenomenology of the hadronic spectrum. While dynamical symmetry breaking, by which quarks acquire mass, is incorporated in this model, it lacks confinement. Previous studies of hybrid color-superconducting stars within the three-flavor NJL model displayed a general instability of hybrid stars towards collapse into a black hole. Stable stars featuring two-flavor superconducting (2SC) phases were obtained with typical maximum masses \( M \sim 1.7M_\odot \) under a favorable choice of constituent quark masses (which are related to the value of the baryo-chemical potential at zero pressure) or by replacing the hard NJL cut-offs by soft form factors with parameters fitted to the same set of data. Heavier objects are obtained if a repulsive vector interaction is introduced in the NJL Lagrangian.

More phenomenological studies of hybrid stars based on the MIT bag model where carried out using a generic parameterization of the quark matter equation.

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of state. Non-perturbative QCD corrections to the equation of state of the Fermi gas were accounted through the parameter \( a_s = 1 - c \), where \( c \simeq 0.3 \) is a reasonable estimate. For this value of the correction stable hybrid stars containing color-flavor-locked superconducting quark matter exist with maximum mass 1.9 \( M_\odot \).

Because of \( \beta \) equilibrium in the light quark sector the chemical potentials of \( u \) and \( d \) quarks obey the constraint \( \mu_d \simeq \mu_u + \mu_e \) \cite{12}, from which it is evident that the Fermi surfaces of the light flavor quark flavor are shifted by an amount corresponding to the chemical potential of electrons. Furthermore, the Fermi surface of strange quarks is mismatched with the Fermi surfaces of light \( u \) and \( d \) quarks because of the large strange quark mass. A generic feature of fermionic systems with mismatched Fermi surfaces is the existence of phases with broken spatial symmetries. A particular realization is the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) phase \cite{13, 14}, which is theoretically predicted in different fermion systems ranging from dilute atoms \cite{13}, nuclear matter \cite{16} to dense quark matter \cite{3, 17, 18, 19, 20, 21, 22, 23, 24}.

The rotational and translation symmetries are broken by the superconducting LOFF phase, since the condensate carries a non-zero momentum \cite{13, 14}. The study of crystalline color superconductivity (CCS) in the context of QCD reveals that this phase could be the true ground state of deconfined quark matter at intermediate densities \cite{3, 17, 18, 19, 20, 21, 22}. Later on we shall describe a particular realization of the three-flavor LOFF phase, which is used in our search for stable sequences of hybrid stars \cite{18, 20}.

The nuclear equation of state, as is well known, can be constructed starting from a number of different principles \cite{20, 27}. We tested a large number of equations of state to construct hybrid star configurations. These belong to the classes of (i) non-relativistic variational and Bruckner-Hartree-Fock theories which use as an input a non-relativistic potential fitted to the elastic nucleon-nucleon scattering data; (ii) relativistic mean-field models which are fitted to the bulk properties of nuclear matter, and (iii) relativistic models which include correlations at the level of the covariant scattering amplitude (Dirac-Bruckner-Hartree-Fock theories). For present purposes, it is sufficient to characterize these equations of states by their stiffness; as we shall see, only the stiffest equations of state are admissible for phase equilibrium between nuclear and quark matter.

The paper is organized as follows. In Sec. II we discuss the input equations of state with the emphasis on the CCS phase of quark matter. Section III contains our results for the sequences of non-rotating and rapidly rotating hybrid stars. Our conclusions are collected in Sec. IV.

II. PHASE EQUILIBRIUM

A. The three-flavor crystalline phase of QCD

The LOFF phase arises naturally in three-flavor QCD as a generic feature of a fermionic systems with mismatched Fermi surfaces \cite{3, 17, 18, 19, 20, 21, 22}. The simplest possible realization of the LOFF phase invokes a single plane-wave modulation of the order parameter, related to a single value of the center-of-mass momentum of the Cooper pairs. More complex Ansätze for the order parameter maximize the free energy gain of the superconducting phase for periodic three-dimensional lattice structures (e.g. face-centered-cubic lattice \cite{13} or body-centered-cubic lattice \cite{10}). The studies of complex structures can be carried out in the Ginzburg-Landau regime, where the order parameter is small, as is the case near the critical temperature of the phase transition or the critical mismatch for unpairing. Independent of the form of the lattice structure, the resulting CCS state breaks the translational and rotational symmetries since Cooper pairs always carry non-zero center-of-mass momentum with respect to some fixed reference frame. Some of the characteristic features of the CCS state such as a non-zero shear modulus and gapless excitations imply that it could play a distinctive role in the phenomenology of compact stars \cite{17, 18, 19, 22}.

In the three-flavor case the quark condensate is written as a superposition of plane waves

\[
\langle \psi_{\alpha I} C \gamma_5 \psi_{\beta J} \rangle \propto \sum_{I=1} \epsilon_{\alpha \beta I} \epsilon_{ijI} \Delta_I \sum_{m=1} P_I \exp (2i \mathbf{q}_m^I \cdot \mathbf{r}) \ . \tag{1}
\]

Here Greek (Latin) indices correspond to color (flavor); \( P_I \) is the number of plane waves of the particular crystalline structure considered; \( \Delta_1, \Delta_2, \Delta_3 \) refer to \( ds, us, ud \) pairing respectively, and \( 2q_m^I \) is the quark pair momentum. The magnitude of \( |q_m^I| \) and of the gap parameters \( \Delta_I \) are determined by the minimization of the thermodynamic potential \( \Omega \).

The studies of the crystalline structure of the LOFF phase revealed that there exists a window of the parameter \( M_2^2/\mu \), where \( M_2 \) is the in-medium strange quark mass and \( \mu \) the mean quark chemical potential, where two crystalline phases, called CubeX and 2Cube45z, could compete for the ground state of three-flavor quark matter \cite{19}. For both of the structures \( \Delta_1 = 0, \Delta_2 = \Delta_3 \), \( P_2 = P_3 \equiv P \) and \( |q_2| = |q_3| \). In the CubeX phase \( P = 4 \); for each pairing channel \( I = 2, 3 \) the wave vectors \( \{ \mathbf{q} \} \) point to the vertices of a square, and the two squares are arranged in such a way that they point to the vertices of a cube. On the other hand, in the 2Cube25z one has \( P = 8 \); each wave vector set \( \{ \mathbf{q} \} \) forms a cube, and the two cubes are rotated by 45 degrees around an axis perpendicular to one of the faces of the cube. The analysis of Ref. \cite{19} has been extended in Ref. \cite{20} by evaluating self-consistently the strange quark mass for each value of \( \mu \), therefore translating the window of \( M_2^2/\mu \) into a window.
of $\mu$. The LOFF phases then exist within the chemical potential range $442 \text{ MeV} \leq \mu \leq 515 \text{ MeV}$ [20]. Thus, having the dependence of the pressure on the chemical potential, we are in a position to construct phase equilibrium and obtain the pressure as a function of the energy density.

A straightforward normalization of the quark pressure in the NJL model requires that the pressure vanishes at zero density and temperature [28, 29]. In the terminology of the MIT bag model, this is equivalent to subtraction of a bag constant from the thermodynamic potential [10]. Since the value of the bag constant is related to confinement which is absent in the NJL model, it appears reasonable to consider changes in its value, and hence in the normalization of the pressure. We shall consider the simple case of a constant shift in the asymptotic value of the pressure; alternatives include the use of form factors for the bag constant [20] or the Polyakov loop at nonzero density [30, 31].

In Ref. [20] the calculation of masses and condensates was carried out for $\eta = G_0/G_S = 0.75$, where $G_0$ and $G_S$ are, respectively, the diquark and the quark-antiquark coupling constants, and the proportionality factor $\eta$ is obtained via a Fierz rearrangement of the interaction term in the Lagrangian. This regime is usually referred to as “intermediate coupling”. Here we shall adopt a “strong coupling” regime with $\eta = 1$ [21, 22], for only in the latter case the matching to the nuclear equations state can be performed without variations in the bag constant (we will discuss this point in more detail in the following subsection). Clearly, a more complete analysis will require a (re)computation of the effect of the strong coupling on the competition among the various superconducting phases such as the CFL, 2SC and 2-flavor LOFF phases; the possibility of multiple transitions between these phases is not considered here.

For completeness we reproduce here the parameters of the NJL model of Ref. [21] used in our study: the values of the bare $u$, $d$, and $s$ quark masses are $m_u = m_d = 5.5 \text{ MeV}$ and $m_s = 135.4 \text{ MeV}$, the coupling constant for the 4-fermion term of the NJL Lagrangian is $G_S = 1.81/\Lambda^2$, the coupling constant for the 't Hooft term is $K = 8.80/\Lambda^5$, where the ultraviolet cutoff is $\Lambda = 643 \text{ MeV}$. With these parameters the following observables of QCD are reproduced: the pion decay constant $f_\pi = 93 \text{ MeV}$, the pion mass $m_\pi = 135 \text{ MeV}$, the kaon mass $m_K = 497 \text{ MeV}$, and the $\eta'$ mass $m_{\eta'} = 924 \text{ MeV}$.

**B. Matching the equations of state**

Physically, the true nuclear equation state must go over to some sort of quark equation of state at some density if deconfinement takes place in nature. Since we have only models of deconfined matter and nuclear matter, this transition is modeled by requiring that there exists a baryo-chemical potential at which the pressures of these phases are equal. This is equivalent to the condition that pressure, $P$, vs. chemical potential, $\mu$, curves for these phases cross (matching). If the $P(\mu)$ curves for chosen equations of state of nuclear and quark matter do not cross, the models are incompatible in the sense that they cannot describe the desired transition between nuclear and quark matter. The low-density equation of state of nuclear matter and the high-density equation of state of CCS matter are matched at an interface via the Maxwell construction. The phase with largest pressure is the one that is realized at a given chemical potential. Thus, according to the Maxwell construction of the deconfinement phase transition, there is a jump in the density at constant pressure as illustrated in Fig. 1. The high-density regime contains two equations of states for crystalline color superconductivity which differ by the normalization of pressure at zero density (or, equivalently, the value of the bag constant). The model A1 is normalized such that the pressure vanishes at zero-density. For the models A and B the zero density pressure is shifted by an amount $\delta P = 10 \text{ MeV/fm}^3$. As discussed above, this is equivalent to a variation of the bag constant whose value is related to confinement, is unspecified in the NJL model, and is uncertain in general. We are aware of the arbitrariness of the latter procedure, the sole practical purpose of which is to produce an equation of state which can be matched to a particular nuclear equation of state. Yet another possibility is to set $\delta P = 0$, but vary the value.

![FIG. 1: Pressure versus density for the models A (heavy, black online), A1 (medium-light, red online), and B (light, blue online). The nuclear equations of state are shown by dashed lines (dark blue online). For the models A and A1 the nuclear (low density) equation of state is the same; for the models A and B the quark (high density) equation of state is the same. At the deconfination phase transition there is a jump in the density at constant pressure.](image-url)
of the constituent masses of the light quarks in the fit of the parameters of the NJL model; for small values of the light quark masses the matching between quark and nuclear equations of state is facilitated. Furthermore, we need to adopt a large value for the ratio \( \eta \), e. g. \( \eta = 1 \), since for \( \eta = 0.75 \) – the conventional value of this ratio – no matching occurs between the quark and the nuclear equations of state.

A set of nuclear equations of states were tested for matching with the models above; it included about dozen equations of state, listed in Refs. [26, 27]. Only two equations of state based on the Dirac-Brueckner-Hartree-Fock approach [26] are suitable to match with the quark equations of state presented above. These are shown in Fig. [1]. The selected equations of state are the two most hard equations of state in the collection. It should be noted that the matching with softer equations of state can be enforced by varying \( \delta p \) by larger amounts than quoted above. For our purposes, however, the three equations of state that follow upon matching are sufficient. The values of the chemical potential (in MeV) and pressure (in MeV/fm\(^3\)) at the quark-nuclear matter interface are \((\mu = 1234.2; p = 96)\) for the model A1, \((1230.0; 95)\) for the model A and \((1234.8; 108)\) for the model B.

### III. RESULTS

#### A. Non-rotating configurations

In this subsection we consider equilibrium and stability of cold hybrid stars with CCS cores; rotating configurations are discussed in the next subsection. Each equation of state defines a sequence of equilibrium, non-rotating stellar configurations in General Relativity, which can be parameterized in terms of the central density \( \rho_c \) of the configuration. It is assumed that the configurations are cold, so that the temperature (or entropy) does not play any significant role in determining the stellar structure. The spherically symmetric solutions of Einstein’s equations for self-gravitating fluids are given by the well-known Tolman-Oppenheimer-Volkoff equations [32]. A generic feature of these solutions is the existence of a maximum mass for any equation of state; as the central density is increased beyond the value corresponding to the maximum mass, the stars become unstable towards collapse to a black hole. One criterion for the stability of a sequence of configurations is the requirement that the derivative \( dM/d\rho_c \) should be positive (the mass should be an increasing function of the central density). At the point of instability the fundamental (pulsation) modes become unstable. If stability is regained at higher central densities, the modes by which the stars become unstable towards the eventual collapse belong to higher-order harmonics. For configurations constructed from a purely nuclear equation of state the stable sequence extends up to a maximum mass of the order 2 \( M_\odot \) (Fig. [2]); the value of the maximum mass is large, since our chosen equations of state are rather hard. The hybrid configurations branch off from the nuclear configurations when the central density reaches that of the deconfinement phase transition. The jump in the density at constant pressure causes a plateau of marginal stability beyond the point where the hybrid stars bifurcate. This is followed by an unstable branch \((dM/d\rho_c < 0)\). Most importantly, the stability is regained at larger central densities: a stable branch of hybrid stars emerges in the range of central densities \( 1.3 \leq \rho_c \leq 2.5 \times 10^{15} \text{ g cm}^{-3} \). The models A and B feature the same high-density quark matter, whereas the models A and A1 the same nuclear equation of state. It is seen that the effect of having different nuclear equations of state (the models A and B) at intermediate densities is substantial (at densities below \( 10^{13} \text{ g cm}^{-3} \) all models are matched to the same equation of state). At the same time, the small shift \( \delta p \) by which the models A and A1 differ does not influence the masses of stable hybrid stars, although it is necessary for matching of nuclear and quark EOS in the models A and B. It is evident that there will exist purely nuclear and hybrid configurations with different central densities but the same masses. This is reminiscent of the situation encountered in non-superconducting hybrid stars [34]; the second branch of hybrid stars was called twins, since for each hybrid star there always exists a counterpart with the same mass composed entirely of nuclear matter.

The hybrid configurations are more compact than their nuclear counterparts, i. e. they have smaller radii (Fig. [3]). An exception are those configurations which are close to the bifurcation point. However, these belong

![FIG. 2: Dependence of masses of hybrid, non-rotating compact stars on their central density for the models A, A1, and B. The dashed lines show the same for the associated nuclear equations of states.](image-url)
to the metastable branch. Contrary to the case of self-bound quark stars, whose radii could be much smaller than the radii of purely nuclear stars, the differences between the radii of hybrid and nuclear stars are insignificant (≤ 1 km) and cannot be used to distinguish these two classes by means of current astronomical observations.

Figure 3 displays the astronomical bounds on the masses and radii of compact stars along with the “tracks” for our models on the mass-radius diagram. All bounds are quoted at the 1σ level. The bound inferred from EXO0748-676, which combines information from redshifted O and F lines, the emitting area of X-ray radiation, and the Eddington luminosity, constrain the mass and the radius of a compact star to lie on a straight line shown in Fig. 4 [35]. Note that multiwavelength observations of EXO0748-676 have led to an alternative interpretation of data which suggest a 1.35 M⊙ neutron star with a main-sequence companion [36, 37]. Both the hybrid stars and their nuclear counterparts have their masses and radii within these bounds. Our sequences differ from those described in Refs. [7, 38]. Instead of having a narrow region of stable hybrid stars, branching off at the bifurcation point from the purely nuclear sequence, we obtain configurations which cover a broad range of central densities, comparable to those covered by purely nuclear equations of state. For sequences constructed from models A and A1 there is a range of masses and radii that correspond to the new family of stars discussed in relation with Fig. 2. The stars belonging to the new family lie to the left from the sharp kink at the point R ≃ 13.5 km and M ≃ 1.9M⊙. The stable branch of this new family of stars is separated from the stable nuclear sequence by an instability region. The sequences corresponding to model B do not show such an instability region. Note that in some cases the quark equation of state can be indistinguishable from the nuclear equation of state (see, e. g., Ref. [10]) in which case quark matter can nucleate at densities that are small compared to those where the purely nuclear and hybrid configurations separate.

Some evidence for massive neutron stars with M ∼ 2M⊙ has been inferred from astronomical observations. A massive compact star may exist in LMXB 4U 1636-536 with 2.0 ± 0.1M⊙ [39]. Recent measurements on PSR B1516+02B in GC M5 gave M = 1.96 ± 0.1M⊙ [40]. The models A and A1 are consistent with these bounds. For the model B these bounds correspond to the stable configuration with the largest mass. For completeness, the lower bound on the neutron star mass 1.249±0.001 M⊙ is shown in Fig. 4, which is inferred from the millisecond binary J0737-3039 [41]. It is seen that the latter bound is relevant only to the low-central-density stars which are composed of nuclear matter. It should be noted that canonical 1.4 M⊙ mass pulsars will be purely nuclear if they are described well by
FIG. 5: Same as in Fig. 2 but for stars rotating at their Keplerian frequency.

the models A and A1, but will contain quark matter cores if model B is more appropriate.

B. Rapidly rotating configurations

Millisecond neutron stars can rotate at frequencies which are close to the limiting orbital Keplerian frequency at which mass shedding from the equatorial plane starts. The Keplerian frequency sets an upper limit on the rotation frequency, since other (less certain) mechanisms, such as gravitational radiation reaction instabilities, could impose lower limits for the rotation frequency. Rapidly rotating pulsars are potentially useful for placing bounds on the state of matter at high densities. Because the centrifugal potential counteracts the compressing stress exerted on matter by gravity, rotating configurations are more massive than their non-rotating counterparts. It is assumed that the stars are rotating uniformly: while nascent hybrid stars can form with large internal circulation, viscosity and magnetic stress will act to damp such motions, at least in the non-superfluid component [42]. The mass versus central-density dependence of compact stars rotating at the Keplerian frequency is similar to that for non-rotating stars (compare Figs. 2 and 5), with the scales for mass shifted to larger values [43]. The branch corresponding to hybrid stars is flattened for fast rotating configurations compared to the non-rotating ones, which implies that small variations in the stellar mass can drive the star unstable. The increase in the maximum mass for stable hybrid configurations (in solar mass units) is $2.052 \rightarrow 2.462$ for the model A, $2.017 \rightarrow 2.428$ and $2.4174$ for the model A1 (there are two maxima) and $1.981 \rightarrow 2.35$ for the model B.

Fast spinning radiopulsars can potentially limit the mass–radius relation of hybrid stars. The requirement that the maximum spin frequency cannot exceed the Keplerian frequency at the neutron star surface translates into the bound $\nu_{\text{max}} \leq 1045(M/M_\odot)^{1/2}R_{10}^{-3/2}$, where $R$ is the radius of non-rotating star in units of $10 \text{ km}$ [44, 45]. The frequency of the fastest radiopulsar observed to date, PSR J1748-2446ad, is 716 Hz [46]. The dependence of the orbital Keplerian frequencies of hybrid stars on their maximum mass is shown in Fig. 6. The configurations with masses below $1 M_\odot$ are excluded by the observed frequency 716 Hz. However, hybrid stars are much more massive - they bifurcate from the sequence of purely nuclear configurations for $M \geq 2 M_\odot$. Therefore this particular observation is not useful in limiting the properties of CCS hybrid stars.

Finally note that at the point of bifurcation the Keplerian frequency of configurations jumps to higher values, which is the consequence of the fact that hybrid stars, having the same mass as their nuclear counterparts, are more compact and thus can support larger rotation rates.

The discovery of the double-pulsar system PSR J0737-3039 [41, 47] offers a unique opportunity to place further bounds on the gross parameters of compact stars by a measurement of the moment of inertia of star A, since the spin frequencies and the masses of both pulsars are ac-
accurately measured. Timing measurements over a period of years could provide information on spin-orbit coupling which could be revealed through an extra advancement of the periastron of the orbit above the standard post-Newtonian advance or in the precession of the orbital plane about the direction of the total angular momentum \[ \text{[48]} \]. The dependence of the moment of inertia, \( I \), of configurations on their mass (in the case of rotation at the Keplerian frequency) is shown in Fig. 7. Since the moment of inertia is independent of the rotation frequency, we have chosen to extract it for configurations rotating at the limiting frequency; note that the masses of configurations should be rescaled appropriately, if one is interested in the \( I(M) \) function for slowly rotating configurations (see ref. \[49\] for a computation of the moment of inertia in the slow-rotation approximation and a discussion of the moment of inertia in the context of PSR J0737-3039). It is seen that while an accurate measurement of the moment of inertia of pulsar A can discriminate between the two nuclear equations of state, it will not be useful for accessing the properties of hybrid stars since the measured masses of pulsars are too low: \( M/M_\odot = 1.337 \) for pulsar A and \( M/M_\odot = 1.250 \) for pulsar B; as apparent from Fig. 7 observations of heavier objects are needed to obtain useful bounds on the moment of inertia of hybrid configurations. The differences \( \sim 20\% \) in the moments of inertia of purely nuclear and hybrid configurations of the same mass (in the case of the models A and A1) are within the accuracy that can be achieved in measurements similar to PSR J0737-3039. Finally we note that the moment of inertia of the CCS phase is required to compute the strain amplitude of gravitational wave emission from an isolated spinning CCS hybrid star. Upper limits on the strain amplitude have been placed by the LIGO and GEO600 detectors \[50, 51\]. The key unknown quantity is the equatorial ellipticity of a hybrid star \( \epsilon = (I_{xx} - I_{yy})/I_{zz} \), where \( I_{ii} \) are the components of the moment of inertia. The latter quantity has been estimated for CCS stars at the qualitative level in Ref. \[52\] assuming uniform-density incompressible models and maximally strained core \[53\]. Our models of CCS stars can be utilized to obtain quantitative limits on the strain of gravitational wave emission that can be emitted by CCS hybrid stars.

IV. CONCLUSIONS

We constructed a set of equations of states of hybrid stars which are composed of color-superconducting quark matter at high, and purely nuclear matter at low densities. High-density quark matter is described in terms of the semi-microscopic NJL model which includes pair correlations that lead to the three-flavor LOff phase as the ground state of QCD at moderate densities. The low-density nuclear phase is described in terms of hard relativistic equations of state based on the Dirac-Bruckner-Hartree-Fock theory. The sequences of stellar configurations constructed on the basis of these equations of state reveal a new branch of stable hybrid configurations featuring CCS matter. These are analogous to the twin configurations obtained for non-superconducting quark matter in Refs. \[54\] and are in contrast to the previously reported NJL-model based sequences of non-rotating three-flavor color-superconducting hybrid stars, which were either entirely unstable or had a narrow range of stable configurations adjacent to the point where they bifurcate from the nuclear ones.

A further search in the parameter space is required in order to judge how generic our constructions are; a quite general conclusion of our analysis is that the nuclear equations of states need be hard to enable the matching to the quark equations of state. Furthermore, matching is only achieved if the ratio of the couplings in the diquark and quark-anti-quark channels is large (\( \eta = 1 \)) compared to the value obtained through the Fierz transformation (\( \eta = 0.75 \)). The lack of confinement in the NJL model leaves the low-density behavior of the quark matter equation of state somewhat uncertain. Small variations in the bag constant help in achieving a phase transition, but do not affect largely the global properties of hybrid stars (cf. our models A and A1). The maximum masses of our stable hybrid configurations are consistent with the presently available astronomical bounds on masses and the mass-radius relationship of compact stars.

We provided a first discussion of rapidly rotating configurations of CCS hybrid stars and mapped out their masses and orbital Keplerian frequencies at which they are destabilized by mass-shedding from the equator.
Rapidly rotating configurations support larger masses (as expected). The mass-central density relation for this class of objects is “flatter” than for their non-rotating counterparts, i.e., smaller variations of mass can drive the system unstable. The Keplerian frequencies of massive members of the sequences, in particular hybrid configurations, are by a factor of two larger than the rotation frequency 716 Hz of the fastest millisecond pulsar known to date. We find that the hybrid configurations, being more compact, can rotate at substantially larger rates than their nuclear counterparts of the same mass. The moment of inertia of a hybrid star is smaller than that of a nuclear star of the same mass. At the point of bifurcation it drops abruptly giving rise to a sequence of stable hybrid stars with almost the same masses but different moments of inertia; this implies that simultaneous mass and moment-of-inertia measurements are useful in distinguishing hybrid configurations from their nuclear counterparts.

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