Extracting Top Quark CP Violating Dipole Couplings via $t\bar{t}\gamma$ and $t\bar{t}Z$ Productions at the LHC

Hong-Yi Zhou

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

and

Institute of Modern Physics and Department of Physics, Tsinghua University, Beijing 100084, P.R. China

Abstract

We propose to extract the electric and weak dipole moments of the top quark via $t\bar{t}\gamma$ and $t\bar{t}Z$ productions at the CERN LHC. With the large numbers of events available at the LHC, these dipole moments can be measured to the accuracy of $10^{-18} e \text{ cm}$. 

$^a$Present mailing address
I. INTRODUCTION

In a recent work [1], we studied on the measurements of the top quark electric dipole moment (EDM) and weak dipole moment (WDM) via top quark pair production at the NLC. These dipole moments are CP violating. It was shown that these dipole moments can be measured to the accuracy of $10^{-18}$ cm at $\sqrt{s} = 500$ GeV $e^+e^-$ collider with $50fb^{-1}$ integrated luminosity by using optimal observables. The methods of measuring the top quark dipole moments via top quark pair production at $e^+e^-$ colliders have also been studied in [2]-[11]. In this letter, we propose to measure the top quark dipole moments at the LHC via $t\bar{t}V$ ($V = \gamma, Z$) production as complimentary measurements to those obtained from the NLC. At the LHC, one can obtain a detector accumulated integrated luminosity of about $600fb^{-1}$ which will result in large number of $t\bar{t}V$ events. So that it may be possible to obtain better limits on the top quark dipole moments at the LHC than at the 500 GeV NLC. The processes of $t\bar{t}V$ productions are somewhat similar to $e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-\gamma$ studied in Ref. [12] for measuring $\tau$ dipole moments. In Ref. [12], it is found that naive CP-odd observables are not good for extracting the CP violating effects and the optimal observables are more effective. Therefore, in this study, we shall apply only the optimal observables [13]-[15] to extract the CP violating effects.

II. CALCULATIONS

The couplings between the top quark and $\gamma, Z$ bosons take the form:

$$-ieg^V_t \gamma^\mu(1 + \alpha^V_t \gamma_5) + \gamma^\mu \gamma^\nu k^V_{\nu}(id^V_t/e)\gamma_5,$$

(1)

where $k^V$ is the outgoing momentum of $V = \gamma$ or $Z$. $d^V_t$ is the dipole moment which we assume to have imaginary part as well as real part. We denote $\hat{d}^V_t = d^V_t/e$. The other couplings are:

$$g^\gamma_t = 2/3, \quad \alpha^\gamma_t = 0,$$

$$g^Z_t = \frac{1 - \frac{8}{3}\sin^2 \theta_W}{4\sin \theta_W \cos \theta_W}, \quad \alpha^Z_t = -\frac{1}{1 - \frac{8}{3}\sin^2 \theta_W}.$$

(2)

(3)

At the LHC, the main production process of $t\bar{t}V$ is $gg \rightarrow t\bar{t}V$ which is shown in Fig. 1(the corresponding diagrams of Fig.(c)-(e) with the interchanging of the two gluons are not depicted). We shall assume the dipole moments are small enough that their quadratic contributions to the total cross section are negligible. Therefore the dipole
moments contribute only to the CP violating effects through their interference with the standard model (SM) contribution. This interference is linear in \( d_1' \). To observe the CP violating effects, one needs to know the spins of the top quarks which can be determined statistically from their decay products. We assume the SM decay of the top quark and apply the narrow width approximations of the top quark and W-boson propagators:

\[
\frac{1}{|q_X^2 - m_X^2 + i m_X \Gamma_X|^2} \rightarrow \frac{\pi}{m_X \Gamma_X} \delta(q_X^2 - m_X^2),
\]

where \( X \) stands for top quark and W-boson, \( \Gamma_X \) is the width of \( X \).

The cross section for reaction \( pp \rightarrow t\bar{t}V \rightarrow b\bar{l}_1^+l_2^-\bar{\nu}_{l_2}V \ (b\bar{q}_1q'_1\bar{q}_2q'_2V) \) can be written as

\[
d\sigma = \frac{f_p^g(x_1)f_p^g(x_2) \lambda_t |M_D|^2 d\Omega_d d\Omega_d'}{2\pi^2 m_t^2 m_W^2 \Gamma_t^2 \Gamma_W^2} d\Phi_{t\bar{t}V} d\Omega_{l_1^+} d\Omega_{l_2^-} d\Omega_{l_1^{'+}} d\Omega_{l_2'^-},
\]

where \( f_p^g(x) \) is the gluon distribution function in proton. \( \hat{s} \) and \( d\Phi_{t\bar{t}V} \) are the C.M. energy and phase space element of the subprocess \( gg \rightarrow t\bar{t}V \), respectively, and

\[
\lambda_t = (1 - \frac{(m_W + m_b)^2}{m_t^2})(1 - \frac{(m_W - m_b)^2}{m_t^2}) \approx (m^2_t - m^2_W)\gamma^2/m_t^4,
\]

\( d\Omega_{l_1^+}(d\Omega_{l_2^-}) \) is the solid angle element of \( W^+(W^-) \) in the rest frame of the (anti) top quark, \( d\Omega_{l_1^{'+}}(d\Omega_{l_2'^-}) \) denotes the solid angle element of \( l_1^+(l_2^-) \) in the rest frame of \( W^+(W^-) \), \( |M_D|^2 \) is the amplitude square excluding the top quark and W-boson propagators after the decays of the top quarks.

In our calculations, \( |M_D|^2 \) is easily obtained from the amplitude of \( gg \rightarrow t\bar{t}V \) by the following substitutions:

\[
\bar{u}(p_t) \rightarrow \frac{g^2}{8} \bar{u}_b \gamma_\mu(1 - \gamma_5)(\not{p}_t + m_t)\bar{u}_{l_1} \gamma^\mu(1 - \gamma_5)v_{l_1},
\]

\[
v(p_t) \rightarrow \frac{g^2}{8} \bar{u}_{l_2} \gamma_\mu(1 - \gamma_5)v_{l_2}(\not{p}_t - m_t)\gamma^\mu(1 - \gamma_5)v_b,
\]

where \( g \) is the weak \( SU(2) \) coupling constant and \( \bar{u}, v \) are Fermion wave functions. The above expressions are calculated numerically. Denoting \( g_1' = Re(d_1') \) and \( g_2' = Im(d_1') \), we can write \( |M_D|^2 \) as

\[
|M_D|^2 = \Sigma_0 + g_1' \Sigma_1 + g_2' \Sigma_2,
\]

where \( \Sigma_0 \) is the SM amplitude square. \( \Sigma_1 \) and \( \Sigma_2 \) are CP-odd amplitude terms which do not contribute to total cross section.
The optimized CP-odd observables in the full final state phase space are defined by
\[ O_{11} = \frac{\Sigma_1}{\Sigma_0}, \quad O_{12} = \frac{\Sigma_2}{\Sigma_0}. \] (9)

When the top quark decays hadronically, we can not distinguish quark and anti-quark jet. For hadronic-leptonic events, the missing neutrino momenta can be fully reconstructed using energy momentum conservation equations, so that we are left with two fold ambiguity of the jet momenta. For purely hadronic events, we have four fold ambiguity. Considering this ambiguity, one can define alternatively the optimal observables:
\[ O_{2i} = \frac{\sum_j \Sigma_i}{\sum_j \Sigma_0}, \quad O_{4i} = \frac{\sum_{j'} \Sigma_i}{\sum_{j'} \Sigma_0}. \] (10)

where \( i = 1, 2 \) and the sum \( j \) is over the two possible assignments of the jet momenta to the quark and antiquark in hadronic-leptonic events. \( j' \) is over the possible assignments of the jet momenta to the quark and antiquark in purely hadronic events. Because the number of events for two top quarks decay semi-leptonically is small, we shall not consider this case.

The mean value of the observable \( O_{2i} \) is defined as
\[ \langle O_{2i} \rangle = \frac{\int d\sigma^+ O_{2i}^+ + \int d\sigma^- O_{2i}^-}{\int d\sigma^+ + \int d\sigma^-}, \] (11)

where the superscript \(*(+, -)\) mean that the integrations are over \( b\bar{l}_1 \nu_1 \bar{b}q_2 \bar{q}'_2 V \) and \( b\bar{q}_1 q'_1 b\bar{q}_2 \bar{\nu}_2 V \) final states, respectively. The mean value of the observable \( O_{4i} \) is simply
\[ \langle O_{4i} \rangle = \frac{\int d\sigma O_{4i}}{\int d\sigma}. \] (12)

We can express the mean values by
\[ \langle O_{ni} \rangle = c_{ni} g_i^V, \] (13)

where \( n = 2, 4 \) and \( c_{ni} = \langle O_{ni} \rangle^2 \). The 1\( \sigma \) level(68% C.L.) statistical error of \( g_i^V \) is given by
\[ \Delta g_{ni}^V = \frac{1}{\sqrt{N c_{ni}}}, \] (14)

where \( N \) is the number of events. To reduce the statistical errors, one can combine the measurements of \( O_{2i} \) and \( O_{4i} \) to get a combined error \( \Delta g_{ci}^V \) [16]:
\[ \frac{1}{(\Delta g_{ci}^V)^2} = \frac{1}{(\Delta g_{2i}^V)^2} + \frac{1}{(\Delta g_{4i}^V)^2}, \] (15)

where \( \Delta g_{2i}^V \) and \( \Delta g_{4i}^V \) are the errors of the measurements using \( O_{2i} \) and \( O_{4i} \), respectively.
III. RESULTS

To calculate the number of events, we have used the following parameters in our calculations: (1) the overall detection efficiency $\epsilon = 0.1$; (2) the integrated luminosity $\mathcal{L} = 600 fb^{-1}$; (3) the branching ratio of hadronic-leptonic top quark decays $B_{ij}^l = 0.29 (l = e, \mu)$; the branching ratio of the purely hadronic top quark decays $B_{jj}^l = 0.46$; (4) the branching ratio of leptonic Z decay $B_{ll}^Z = 0.067$. We consider only the leptonic Z decay channel in the following calculations. The number of events is given by

$$N = \epsilon \mathcal{L} \sigma B^V,$$

where $\sigma$ is the total $t\bar{t}V$ production cross section, $B^l = B_{ij}^l$ or $B_{jj}^l$, $B^\gamma = 1$, $B^Z = B_{ll}^Z$.

In calculating the total cross sections and the mean values of the observables, we set $m_t = 176$ GeV, $\alpha_{em} = 1/128.8$ and $pp$ energy $\sqrt{s} = 14$ TeV. The parton distribution functions of MRS A’ are used with $Q^2 = m_t^2 \, [17]$. In order to give more realistic results, we also apply the following cuts on $t, \bar{t}, V$:

$$p_T(i) > 20 GeV, \quad |y(i)| < 2.5, \quad \Delta R(i, j) > 0.4,$$

where $i, j = t, \bar{t}, V$, $p_T$ is the transverse momentum, $y$ is the rapidity and $\Delta R = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$ is the solid angle separation of two particles.

With the above parameters and cut conditions, we get the following results for the total cross sections

$$\sigma(t\bar{t}\gamma) = 1.465 pb, \quad \sigma(t\bar{t}Z) = 0.614 pb.$$

In Table I., we present the 1σ statistical errors of the dipole couplings. The results for $\hat{d}_t^\gamma$ are even better than the 500 GeV NLC with 50 fb$^{-1}$ [1]. Due to the small branching ratio of Z decay to leptons, the results for $\hat{d}_t^Z$ are not as good as that in Ref. [1]. Our conclusion is that the limits on the top quark dipole couplings which can obtained from $t\bar{t}V$ production at the LHC with 600 fb$^{-1}$ are about $10^{-18}$ cm. These limits are comparable with those obtained from $e^+e^- \rightarrow t\bar{t}$ at the 500 GeV NLC.
TABLES

TABLE I.
1σ statistical errors of the coupling constants $g_i^Y$. Unit: $10^{-18} \text{ cm}$.

|       | $g_1^\gamma$ | $g_2^\gamma$ | $g_1^Z$ | $g_2^Z$ |
|-------|---------------|---------------|---------|---------|
| $O_{2i}$ | 3.4           | 1.1           | 10.4    | 6.0     |
| $O_{4i}$ | 3.5           | 1.2           | 10.6    | 5.0     |
| combined | 2.4           | 0.8           | 7.4     | 3.9     |

Acknowledgements

The author is financially supported by the Alexander von Humboldt Foundation of Germany.
REFERENCES

[1] H.Y. Zhou, hep-ph/9806239.
[2] For a review see, for example, C.-P. Yuan, Mod. Phys. Lett. A 10 (1995) 627; D.Atwood and A. Soni, hep-ph/9609183. Published in ICHEP 96;(1996) 1119.
[3] G.Kane, G.A.Ladinsky and C.-P. Yuan, Phys. Rev. D 45 (1992) 124.
[4] D.Atwood and A.Soni, Phys. Rev. D 45 (1992) 2405.
[5] W. Bernreuther, O.Nachtmann, P.Overmann, T. Schröder, Nucl. Phys. B 388 (1992) 53;(E) ibid B406 (1993) 516.
[6] B.Grzadkowski and W.Y.Keung, Phys. Lett. B 319 (1993) 526.
[7] W. Bernreuther and P. Overmann, Z. Phys. C 61 (1994) 599.
[8] W. Bernreuther and P. Overmann, Z. Phys. C 72 (1996) 461.
[9] P.Poulose and S.D. Rindani, Phys. Lett. B 349,379 (1995); F.Guypers and S.D. Rindani, Phys. Lett. B 343,333 (1995); P.Poulose and S.D. Rindani, Phys. Lett. B 383 (1996) 212; P.Poulose and S.D. Rindani, Phys. Rev. D 54 (1996) 4326.
[10] M.S. Baek, S.Y.Choi ans C.S.Kim, Phys. Rev. D 56 (1997) 6835.
[11] B.Grzadkowski and Z. Hioki, hep-ph/9805318, and references therein.
[12] D. Bruß, O.Nachtmann and P.Overmann, Eur. Phys. J. C 1(1998)191.
[13] M.Dovier, L.Duflot, F.Le Diberder, A.Rougé, Phys. Lett. B 306 (1993) 411.
[14] M.Diehl, O. Nachtmann, Z.Phys. C 62 (1994) 397; Eur. Phys. J. C 1(1998)177.
[15] J.F.Gunion, B.Grzadkowski and X.-G.He, Phys. Rev. Lett. 77 (1996) 5172.
[16] Review of Particle Properties, Phys. Rev. D 54 (1996) 1.
[17] A.D. Martin, W.J. Stirling and R.G. Roberts,Phys. Lett. B354, 155(1995).
FIG. 1. Feynman diagrams of $gg \rightarrow t\bar{t}V (V = \gamma, Z)$. 