The boundary value problem for the heat transfer task between a human and the environment

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Temperature is one of the most important indicators of human health. Therefore, the study of the effect of temperature on the human body is necessary. The approach of modeling heat transfer between a human and the environment with dispersed parameters is presented in this article. This approach is the development of a model heat transfer between a human and the environment with lumped parameters. In the process of modeling, the Fourier law of thermal conductivity, Newton Richman’s law and mathematical apparatus of partial differential equations are used.

Keywords: the mathematical model of heat transfer between a human and the environment, compartmental model, air temperature, air humidity, air velocity

Introduction. The vital activity of any organism is built on processes, the totality of which is reduced to the redistribution of heat between it and the environment. Air temperature has a great influence on the physiological state of a person. A change in average daily air temperature by 1-2°C is considered weak, by 3-4°C — moderate, more than 4°C — sharp. With any change in air temperature, a redistribution of human body temperature occurs. It affects his health.

Climatic comfort — environmental conditions in which a person resides and in which his psychological and physiological health is maintained. One of the conditions of climatic comfort is such temperature characteristics of the environment under which there is no human hypothermia or overheating. The subjective feeling of climatic comfort is associated with the level of human activity, radiation temperature, etc. In addition, the effect of air temperature on the human body also depends on air humidity. At the same temperature, a change in the content of water vapor in the surface layer of the Earth can have a significant effect on the state of the body. Compared to natural fluctuations in air temperature, the temperature range in which the human body feels
comfortable is significantly narrower. At a body temperature that goes beyond 26-
40°C, irreversible processes in the body are possible. With increasing air humidity,
which prevents evaporation from the surface of the human body, heat is poorly
tolerated and the effect of cold intensifies. Perception of temperature individually.
Some people feel comfortable in cold, frosty weather, while others feel comfortable in
warm and dry weather. It depends on the physiological and psychological
characteristics of a person, as well as the emotional perception of the climate in which
childhood passed.
The optimal meteorological conditions for humans are air temperatures of 18-30°C
with a relative humidity of 40-60 and an air velocity of 0.5-1.0 m/s. Human tolerance
to temperature, as well as its heat perception, largely depends on humidity and the
speed of the surrounding air. With increased humidity and high air temperature, when
evaporation is difficult, acute overheating of the body most often occurs. Such
conditions often arise when working in tight, non-ventilated clothing. A number of
other factors also contribute to overheating of the body: high physical activity,
inadequate drinking water, etc. High temperatures have a negative effect on human
health.
Work at high temperatures is accompanied by intense sweating, which leads to
dehydration, loss of mineral salts and water-soluble vitamins. This causes serious and
persistent changes in the activity of the cardiovascular system, increases the respiratory
rate, and affects the functioning of other organs and systems. The following symptoms
are observed: weakened attention, coordination of movements worsens, reactions slow
down, etc. During hyperthermia and due to heat stroke, headaches, dizziness, general
weakness, distortion of color perception, dry mouth, nausea, vomiting, and profuse
sweating appear. Pulse and breathing become more frequent, the content of nitrogen
and lactic acid in the blood increases. In this case, pallor is observed; the pupils are
dilated, convulsions, loss of consciousness sometimes occur. Sunstroke is a special
form of overheating due to direct local exposure to sunlight on an unprotected head. In
this case, a general overheating of the body may not be observed. General weakness, a
feeling of malaise, headache, dizziness, tinnitus, sometimes nosebleeds, nausea and
vomiting appear. Skin turns red, perspiration intensifies. In severe cases, serious
disturbances occur on the part of the central nervous system: darkening of
consciousness, sharp excitement, convulsions, involuntary movements, hallucinations,
delirium.
Therefore, the study of the effect of temperature on the body is necessary for every
person who wants to have control over both the level of productivity and the state of
their own health.
1. Overview

The problem of mathematical modeling of the processes of heat exchange in the human
body was considered in the works devoted to the normalization, prediction and
regulation of the thermal state of human [1]. Some of the works are based on the
principles laid down in [2]. These works are based on the representation of the human
body as a branch of geometric elements. Most of these elements are cylinders. The
The human body is regarded as a system of interacting components of the whole. Such components are the layers of the selected cylinders: core, muscles, fat and skin. These layers form the elements of the system that is part of the human body, called the compartments. In most works, such compartment and Allocate: head, torso, arm, palm, leg, foot.

In [2] presented a thermodynamic model of a human body - arm, whose shape resembled a cylinder. A heat balance equation is also given to calculate the static temperature of this compartment. This model considers the conduction of heat in the radial direction of the cylinder, the metabolic formation of heat in the tissues, the convection of heat with the blood, the loss of heat from the skin surface by convection, radiation, and evaporation. An important achievement of this model is that it can be applied to any part of the human body.

Based on the presented model for one compartment, a model of the thermoregulation system of the entire human body was proposed in [3]. It was believed that each cylindrical element corresponding to the human body was isotropic. The main disadvantage of this model is the lack of regulation of blood flow due to narrowing and expansion of blood vessels in the skin.

Also in [3], physical and physiological factors are described that the model must take into account, namely: the temperature differences of the local parts of the human body, conductive heat transfer due to temperature differences, convective heat transfer by the bloodstream, the representation of the human body is compartmentalized, the thermal conductivity of the layers of fat and skin human, heat flows in arteries and veins, heat dissipation by evaporation from the respiratory tract, heat dissipation by evaporation from the surface of the skin, heat preservation inside the human body, environmental parameters.

Most modern compartmental models of human thermoregulation are built on the basis of the thermal model [4-6]. This is the first mathematical model in which all the factors described in [3] were taken into account. The regulation concept in this model is developed according to the automatic control theory. The human thermoregulation system is considered as a closed loop with negative feedback [7], consisting of two parts - passive (controlled) and active (control). Heat transfer processes occurring inside the human body or heat transfer between the human body and the environment affect the passive part. The active part controls the passive part in order to stabilize the temperature of the human body, with the help of regulatory reactions of the body, sweating and trembling, narrowing or expansion of blood vessels. For this, feedback signals are used to change the parameters of the passive part. The model takes into account that the body is symmetrical to reduce the number of calculations, that is, a pair of arms or a pair of legs is represented by one cylinder. An additional central compartment is isolated blood flowing in the large arteries and veins and exchanges heat in a conductive way with all other compartments.

In [4-6], it was shown for the first time that a mathematical model based on the principles of automatic control theory and using the apparatus of ordinary differential equations [8-9] allows one to adequately describe the processes of thermoregulation and heat transfer of a person with the environment. The mathematical model [4-6]
predicts the temperature of all parts of the human body under given air conditions. The model does not take into account human clothing, the input conditions are constant (do not change over time), the rate of change of skin temperature for thermoregulatory reactions is not taken into account, and moisturizing of the skin surface is not taken into account when sweat evaporation is limited by environmental conditions. The model is suitable for predicting body temperatures in homogeneous conditions with high and moderate air temperatures, but drawbacks of its use for cold conditions have been discovered [10].

In [11], a model was developed for predicting human physiological responses to cold. The compartmental representation of the human body consists of 14 segments: forehead, face, skull, neck, chest, abdomen, arms, hands, legs and feet. Segments have a cylindrical or spherical shape. The separation of the human head into segments allows you to more accurately take into account heat transfer through the upper respiratory tract from cold environmental conditions. The control system is based on the temperature of the brain and skin, but with the additional calculation of the heat flux through the skin. That is, the number of thermoreceptors per unit surface area of each segment (body part) is taken into account. The active part also includes additional input signals to control vasoconstriction of the skin and trembling of the muscles of the human body. The model is built only to predict human cooling, so it does not take into account sweating.

In [12], a model was proposed where the thermoregulation of a person is considered as a function of time and three-dimensional coordinates in the human body. Thus, the geometry and anatomy of the body must be considered in great detail in order to obtain a useful prognosis from this model. The dynamic temperature forecast provided by this model is difficult to verify.

In [13], a model was developed for predicting human thermal reactions indoors and outdoors. This model can be used for a wide temperature range of the environment (cold, cool, neutral, warm and hot conditions). The passive part of the model is a multi-segment multi-layer multi-compartment representation, describes in detail the anatomical, thermophysical and thermophysiological properties of the human body. The equations of the passive part describe heat transfer inside the body (blood flow, metabolism, conduction and accumulation of heat) and on its surface (free and forced convection, radiation, evaporation, diffusion and accumulation). The active part simulates the reactions of the human thermoregulatory system, that is, the narrowing and expansion of blood vessels in the skin, sweating and cold tremors of skeletal muscles. The active part of the system was developed using statistical regression analysis using measured data obtained under various environmental conditions (from 5°C to 48°C) at various intensities of physical activity (from 0.8 to 8 MET). The model predicts the response of a standard healthy person without acclimatization and allows to predict the thermal comfort or discomfort that occurs in a person in a stable and transient mode [14-15].

In [15–16], based on the model [4], a thermoregulatory model THERMODE 193 was developed for a heterogeneous human environment. Later in [17], the THERMODE model was improved: the active and passive parts were significantly
improved and the correction for clothes was taken into account according to the effect of human body movements. In [18] developed a two-dimensional model of heat distribution in the human eye.

2. Compartmental model

2.1. Formulation of the problem. The main types of thermoregulation, as is known, are heat production and heat transfer. Heat production in the body is carried out chemically. Heat transfer occurs physically: by radiation, heat conduction and evaporation.

Air temperature, radiation, humidity and air movement are the four major environmental variables that affect human response to the environment. Many models have been developed to predict human thermophysiological responses to environmental variables, but most of them are focused on a narrow range of tasks and do not take into account the combination of major environmental variables.

The mathematical model of heat transfer between humans and the environment is built on the basis of a multi-compartment model of thermoregulation and heat transfer and the method of I.I. Yermakova.

According to the definition given in [1], compartment is a certain amount of substance released in the biological system and having the property of unity, so in the processes of transport and chemical transformations it can be considered as one. Models in which the system under study is represented as a set of compartments are called multicomponent.

The compartment has an energy source, the ability to receive and transfer heat to neighboring compartments and exchange energy with the environment.

According to the method of I.I. Yermakova approximation of the human body can be carried out on an arbitrary number of geometric parts. For this model, 13 cylinders and sphere approximate the human body: head, torso, left and right hand (arms, forearms, and hands), left and right legs (hips, legs, and feet) (Fig.1).

Fig.1 Approximation of the human body
Each compartment corresponds to a specific organ, part of the body or tissue of the body. Models take into account human anatomical and physiological parameters: weight, height, surface area of the body, biophysical characteristics of tissues and organs, metabolism, oxygen consumption, blood flow, pulmonary ventilation, heat transfer and heat transfer coefficients, cardiac output, sweating, etc. [2].

The heat produced by the body of a healthy person is released into the environment by the surface of the body, so the temperature of parts of the human body near its skin should be below the temperature of its central parts. Fluctuations in body temperature caused by changes in external temperature are most pronounced near the surface of the body and on the limbs. The internal temperature of the body is not constant neither in relation to space, nor in relation to time. In thermo-neutral conditions, the temperature differences in the inner regions of the body are 0.2-1.2 °C; even in the brain, the temperature difference between the central and outer parts reaches more than 1 °C. The highest temperature is noted in the internal organs. Therefore, each cylinder approximating a part of the human body has nested cylinders that describe the properties of the corresponding tissues of the human body.

There are three cylinders in the cylinder that approximate the human body: internal organs, muscles, fat. The cylinder in which the cylinders are inscribed represents the skin of the torso (Fig.2).

![Fig.2 Torso compartments](image)

Three cylinders that fit into each other represent the arms, forearms and legs in the model: skin, fat, muscles (Fig.3).

![Fig.3 Arms and legs compartments](image)

Two cylinders approximate each foot and each hand: skin and muscle (Fig.4).
Fig. 4 Hands and feet compartments

Two embedded spheres that represent the properties of skin and brain tissue approximate the human head (Fig. 5).

Fig. 5 The compartments of the human head

2.2. Heat transfer equations. The temperature of the human body is not the same; it is different in different areas. The temperature field $T(x, y, z, t)$ is the set of temperature values at all points in the human body and their change over time.

If we apply the Fourier law to each compartment and to each of its layers, we get a system of equations:

$$c_{ij}m_{ij} \frac{\partial T_{ij}}{\partial t} - \lambda_{ij} \Delta T_{ij} = M'_{ij} + Q_{ij}^R - Q_{ij}^C - Q_{ij}^b - Q_{ij}^E - Q_{ij}^{str} \quad (1)$$

where $i = 0, 1, \ldots, N-1$, $N$ is the number of layers; $j = 0, 1, \ldots, L-1$, $L$ is the number of compartments; $T$ - temperature, °C; $t$ - time, hour.; $c$ - specific heat, kcal/(kg·°C); $m$ - mass, kg; $M$ - metabolism, kcal/hour; $Q^R$ - heat loss by radiation, °C; $Q^C$ - heat loss by convection, °C; $Q^K$ - heat loss by conduction; $Q^b$ - blood heat transfer, °C; $Q^E$ - the loss of heat by evaporation, °C.

The heat exchange between the compartments, which model the human body, occurs convectively with the circulation of blood between the compartments of all segments and the conductive path between adjacent compartments of one segment. Conduction in homogeneous opaque solids is the transfer of heat from one part to another under the influence of temperature changes without noticeable movement of particles. Fourier law determines conductivity [3]:

$$Q_{ij}^K = a_{ij} \lambda_{ij} (T_{ij} - T_{ij+1})$$
where $a$ — the thickness of the compartment, m; $\lambda$ - the coefficient of thermal conductivity, kcal/(m·°C·h).

Thermal exchange with the environment is carried out through human skin. At the same time, it consists of heat exchange of radiation, convection and evaporation [4]:

$$Q_{i,sk}^{air} = Q_{i,sk}^{C} + Q_{i,sk}^{R} + Q_{i,sk}^{E}$$

(2)

where are the indices: sk - skin; C - convection; R is radiation; E - evaporation.

In equation (2), the first component is that convection. Convection is a heat exchange that occurs only in moving media (gases or liquids), with heat transfer being carried out by the transfer of the volume of the medium. The basic law of convective heat transfer is Newton Richman's law:

$$Q^{C} = h^{C}a(T_{sk} - T_{air})$$

(3)

where $h^{C}$ is the coefficient of heat transfer by convection, W/(m$^2$·°C).

The heat loss due to the evaporation of moisture from the skin surface is determined by the difference in vapor pressures directly above the moist skin and in the surrounding air:

$$Q_{i,sk}^{E} = h_{i,sk}^{E}a_{i,sk}(P_{i,sk} - \frac{\varphi_{i}^{air}P_{i}^{air}}{100\%})$$

(4)

where $h_{i,sk}^{E}$ - the coefficient of heat transfer by evaporation, kcal/(m$^2$·kPa·h); $\varphi$ - relative humidity (the ratio of absolute humidity to its maximum value at a given ambient temperature),%; $P$ - saturated vapor pressure, kPa.

Outdoors, long-wave radiation from a human body can be absorbed by a cloudless sky. If the surface temperature of the body and the environment is less than 20 °C, the radiation flux can be calculated linearly. Therefore, the heat exchange radiation between the human body and the environment in the model is calculated by the empirical formula:

$$Q_{i,sk}^{R} = h_{i,sk}^{R}a_{i,sk}(T_{i,sk} - T_{i,air})$$

(5)

where $h_{i,sk}^{R}$ is the heat transfer coefficient of radiation, which is determined by the Stefan-Boltzmann constant, the skin's emissivity and the factor of thermal propagation, kcal / (m$^2$·°C·h).

2.3. Blood flow convection. Convection is a heat transfer process carried out by the transfer of energy by fluid or gas flows. Sequential transfer of heat from places of its formation in deep parts of the body to its surface carries out heat transfer to the external environment. This heat transfer within the body is carried out by conduction and circulatory convection.

Conduction provides the transfer of heat through the tissues in contact with each other, the body along the temperature gradient - from the deeper parts of the body to its surface.

Body tissues are poor conductors of heat, so a very large temperature gradient is required to remove heat generated by metabolic reactions in the deep parts of the body. The main path of heat transfer inside the human body is related to the movement of fluids, and mainly to the circulation of blood in the body - circulatory convection. The
blood has a high heat capacity of 0.92 kcal / l / °C, so it can withstand large amounts of heat.

The circulatory system, together with the thermoregulation system, carries heat from the internal organs to the surface of the human body. The heat flow carried by the circulation of blood is equal to the difference of the sum of heat fluxes, which is ensured by the circulation of the blood of each compartment, the heat flow, when mixing venous and arterial blood, and heat loss with respiration:

\[
Q_b = \sum_{i=0}^{N-1} \sum_{j=0}^{L-1} Q_{ij} - Q_{H} - Q_{RS}
\]  

(6)

where \(Q_b\) is the heat flow carried by the blood circulation; \(Q_{ij}\) is the heat flow arising from the circulation of blood in the compartment; \(Q_{H}\) - loss of heat when mixing venous and arterial blood; \(Q_{RS}\) - heat loss with breathing.

Alternatively:

\[
V_b \rho_b c_b \frac{\partial T_b}{\partial t} = \lambda_b \Delta T_b = \sum_{i=0}^{N-1} \sum_{j=0}^{L-1} W_{ij} \rho_b c_b T_{ij} - W \rho_b c_b T_b - Q_{RS}
\]  

(7)

Equation (7) is the 39th equation in the human thermal balance system (1) in this model.

3. Boundary conditions and initial conditions

Initial and boundary conditions are required to solve system of second-order partial differential equations.

3.1 Initial conditions. At the beginning of time, temperatures, metabolism, blood flow, etc. are set for each compartment.

if \(t = 0\): \(T_{ij} = T_{0_{ij}}\), for each i,j when \(0 \leq i \leq N-1, 0 \leq j \leq L-1\)

3.2 Boundary conditions. To solve this system, boundary conditions of the fourth kind are used. For neighboring compartments, conditions must be met at the boundary between them.

If \(x \in G_x^{ij}\):

\[
\lambda_{ij} \frac{\partial T_{ij}}{\partial x} = \lambda_{i,j-1} \frac{\partial T_{ij-1}}{\partial x}, \forall t \geq 0, j \neq sk
\]

\[
\lambda_{b} \frac{\partial T_{b}}{\partial x} = \lambda_{i,j} \frac{\partial T_{ij}}{\partial x}, \forall t \geq 0, j \neq sk
\]

If \(y \in G_y^{ij}\):

\[
\lambda_{ij} \frac{\partial T_{ij}}{\partial y} = \lambda_{i,j-1} \frac{\partial T_{ij-1}}{\partial y}, \forall t \geq 0, j \neq sk
\]

\[
\lambda_{b} \frac{\partial T_{b}}{\partial y} = \lambda_{i,j} \frac{\partial T_{ij}}{\partial y}, \forall t \geq 0, j \neq sk
\]

If \(z \in G_z^{ij}\):

\[
\lambda_{ij} \frac{\partial T_{ij}}{\partial z} = \lambda_{i,j-1} \frac{\partial T_{ij-1}}{\partial z}, \forall t \geq 0, j \neq sk
\]

\[
\lambda_{b} \frac{\partial T_{b}}{\partial z} = \lambda_{i,j} \frac{\partial T_{ij}}{\partial z}, \forall t \geq 0, j \neq sk
\]

For \(j = sk, x \in G_x^{i,sk}\):

\[
\frac{\partial T_{i,sk}}{\partial x} = a_{ij} \lambda_{i,sk} (T_{i,sk} - T_{air}), \forall t \geq 0,
\]

\[
\lambda_{b} \frac{\partial T_{b}}{\partial x} = \lambda_{i,sk} \frac{\partial T_{i,sk}}{\partial x}, \forall t \geq 0
\]
For $j = sk$, $z \in G_{ij}^{i,sk}$:
\[
\frac{\partial T_{i,sk}}{\partial y} = \alpha_{ij} \lambda_{i,sk} (T_{i,sk} - T^{air}), \quad \forall t \geq 0,
\]
\[
\lambda_b \frac{\partial T_{b}}{\partial y} = \lambda_{i,sk} \frac{\partial T_{i,sk}}{\partial y}, \quad \forall t \geq 0
\]

For $j = sk$, $x \in G_{z}^{i,sk}$:
\[
\frac{\partial T_{i,sk}}{\partial z} = \alpha_{ij} \lambda_{i,sk} (T_{i,sk} - T^{air}), \quad \forall t \geq 0,
\]
\[
\lambda_b \frac{\partial T_{b}}{\partial z} = \lambda_{i,sk} \frac{\partial T_{i,sk}}{\partial z}, \quad \forall t \geq 0
\]
where $G^{ij}$ surface of ij-compartment, $G_x$, $G_y$, $G_z$ - the projection of these surfaces on the coordinate axis.

To solve the system of heat transfer of a person and the environment using finite difference method [19].

4. Results

Using a system of equations (1)-(6), three computational experiments were carried out. The conditions for all three experiments were the same. It is required to determine the dynamics of temperature changes of a person who makes a jog (700 kcal/h) for one hour. Air temperature – 30 °C, air velocity - 0.1 m/s. For the first experiment, air humidity was 10%, for the second - 50%, and for the third - 90% (Fig. 6).
As can be seen from Fig. 2, we observe the highest mean temperatures of blood in the second experiment, when the air humidity is high. Heat transfer, especially at high air temperatures, can be carried out by evaporation of moisture from the surface of human skin. However, at high humidity, it does not work; but on the contrary, the whole body continues to heat up, which leads to the loss of water and useful trace elements.

High humidity in combination with elevated temperature can lead to cramps, muscle cramps, fainting, a drop in blood pressure, loss of fluid, and an increase in human body temperature. At risk can be people of any age, even young and physically healthy.

**Conclusions.** An analysis of the results of computational experiments that the mathematical model of heat transfer between a human and the environment can provide allows us to identify and evaluate possible risk factors that threaten human health.

A preliminary forecast obtained using the mathematical model of heat transfer between a human and the environment shows the time range of a person’s safe stay depending on the combination of power and duration of physical activity, environmental characteristics.

This model provides an opportunity to study the influence of the environment on a person, adaptation, physical activity of various capacities and durations.

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Крайова задача теплообміну між людиною та навколишнім середовищем

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У статті представлений підхід моделювання теплопередачі між людиною і навколишнім середовищем з дисперсними параметрами. Цей підхід є розробкою моделі теплообміну між людиною і навколишнім середовищем з зосередженими параметрами. В процесі моделювання використовуються закон теплопровідності Фур'є, закон НьютонаРіхмана і математичний апарат рівнянь в часткових похідних.

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