Inflation in minimal left-right symmetric model with spontaneous $D$-parity breaking

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We present a simplest inflationary scenario in the minimal left-right symmetric model with spontaneous $D$-parity breaking, which is a well motivated particle physics model for neutrino masses. This leads us to connect the observed anisotropies in the cosmic microwave background to the sub-eV neutrino masses. The baryon asymmetry via the leptogenesis route is also discussed briefly.

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It is now widely believed that the Universe has gone through a period of inflation [1] at the earliest moment of its history. Inflation is required to explain finely tuned initial conditions of the standard hot big bang cosmology, as well as to solve many cosmological problems such as homogeneity, isotropy and flatness of the observable Universe. Moreover, it is predicted that during inflation primordial density perturbations, necessary for large scale structure in the Universe and isotropy and flatness of the observable Universe. Moreover, it is naturally explained in the minimal left-right symmetric model where the parity is broken by a singlet field $\Delta_{R}(1,3,2)$. Through the Majorana Yukawa coupling $\Delta_{R}$ gives masses to the right-handed neutrinos which anchor the canonical seesaw mechanism [8] to give small Majorana masses to the left-handed physical neutrinos. The left-right gauge symmetry requires another triplet $\Delta_{L}(3,1,2)$ whose vacuum expectation value (VEV) gives masses to the physical left handed neutrinos through the triplet see-saw [9]. Finally $SU(2)_{L} \times U(1)_{Y}$ is broken to $U(1)_{em}$ by a bidoublet $\Phi(2,2,0)$ which essentially contains two copies of $SU(2)$ doublets with opposite hypercharge. This gives masses to all the SM fields. Under the left-right parity the scalars transform as

$$\sigma \leftrightarrow -\sigma, \quad \Delta_{R} \leftrightarrow \Delta_{L} \quad \text{and} \quad \Phi \leftrightarrow \Phi^{\dagger}. \quad (1)$$

On the other hand, the fermion doublets $\Psi_{L}(2,1,-1) \equiv (\nu_{L}, e_{L})$ and $\Psi_{R}(1,2,-1) \equiv (\nu_{R}, e_{R})$ under the left-right parity transform as $\Psi_{L} \leftrightarrow \Psi_{R}$. 

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Since $\sigma$ is a singlet field under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ it may dominate the energy density of the Universe for some duration and hence can play the role of the inflaton field \([10]\). As we will see soon, inflation occurs while $\sigma$ is slowly rolling on its potential towards the minimum. As soon as $\sigma$ acquires a VEV parity is broken. Therefore, $\sigma$ plays a dual role in this model. However, it does not affect the gauge symmetry of the group, since as mentioned above it is a singlet under the remaining gauge group. A bonus point in this model is that inflation solves the generic domain wall problem by sweeping them away.

We now write down the potential involving the scalar fields $\Delta_R$, $\Delta_L$, $\Phi$ and $\sigma$. The relevant potential for the rest of our discussion is given by

$$V = V_\sigma + V_\Phi + V_\Delta + V_{\sigma\Delta} + V_{\sigma\Phi} + V_{\phi\Delta}, \quad (2)$$

where

$$V_\sigma = -\frac{1}{2}\mu^2\sigma^2 + \frac{1}{4}\lambda\sigma^4 + V_0,$$

$$V_\Delta = -\mu_\lambda^2 \left[ Tr\left( \Delta_R^2 \right) + Tr\left( \Delta_L^2 \right) \right] + \text{quartic terms},$$

$$V_{\sigma\Delta} = \lambda[ Tr(\Delta_R^2) - Tr(\Delta_L^2) ],$$

$$V_{\sigma\Phi} = \lambda[ Tr(\Phi^\dagger \Delta_R^2) + Tr(\Phi^\dagger \Delta_L^2) ],$$

$$V_{\phi\Delta} = \beta[ Tr(\Phi^\dagger \Phi^\dagger \Delta_R^2) + Tr(\Phi^\dagger \Delta_L^2 \Phi^\dagger ) ],$$

where $\mu$ and all $\mu_{ai}$, with $a$ denoting $\Delta$, $\Phi$, and $\Phi^*$, are positive. $V_\Phi$ and $V_{\sigma\Phi}$ are chosen in such a way that $\Phi$ acquires a VEV and hence breaks the gauge symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_em$. In $V_\sigma$, $V_0$ is a constant and properly chosen so that the minimum of the potential $V_\sigma$ settles at zero.

As the Universe expands, the temperature falls so that below the critical temperature $T_c \equiv \sigma_P$, $\sigma$ acquires a VEV

$$\langle \sigma \rangle \equiv \sigma_P = \frac{\mu}{\sqrt{\lambda}}. \quad (4)$$

As a result, the effective masses of the triplets $\Delta_L$ and $\Delta_R$ are given by

$$M_{\Delta_L} = \sqrt{\mu_\lambda^2 - (\lambda \sigma_P^2 + \gamma \sigma_P^2)},$$

$$M_{\Delta_R} = \sqrt{\mu_\lambda^2 + (\lambda \sigma_P^2 - \gamma \sigma_P^2)}. \quad (5)$$

We now do a fine tuning to set $M_{\Delta_R} > 0$, so that it acquires a VEV

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 \\ v_R \\ 0 \end{pmatrix}. \quad (6)$$

At a few hundred GeV $\Phi$ and $\Phi^*$ will acquire VEVs

$$\langle \Phi \rangle = \begin{pmatrix} k_1 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \langle \Phi^* \rangle = \begin{pmatrix} 0 \\ k_2 \\ 0 \end{pmatrix}. \quad (7)$$

However, this induces a non-trivial VEV for the triplet $\Delta_L$ as

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 \\ v_L \\ 0 \end{pmatrix}. \quad (8)$$

This gives masses to neutrinos through type-II seesaw. Therefore, it is worth checking the order of magnitude of $v_L$. From $V_\Delta$, $V_{\sigma\Delta}$ and $V_{\Phi\Delta}$ of Eq. \((3)\) we get

$$v_L \frac{\partial V}{\partial v_L} - v_R \frac{\partial V}{\partial v_R} = v_L v_R |4 M \sigma_P| + 2 \beta k_1^2 (v_R^2 - v_L^2) = 0. \quad (9)$$

Observed phenomenology requires $v_L \ll k_2 < k_1 \ll v_R$. Thus the above equation gives

$$v_L \approx -\frac{\beta v_R^2 v_R}{2 M \sigma_P}, \quad (10)$$

where we have used $v = \sqrt{k_1^2 + k_2^2} \approx k_1 = 174$ GeV and $\beta$ is a coupling constant of order 1. Notice that in the above equation the smallness of the VEV of $\Delta_L$ is decided by the parity breaking scale, but not the $SU(2)_R$ breaking scale \([15]\). So there are no constraints on $v_R$ from the type-II seesaw point of view.

**Inflation by $\sigma$:** As mentioned before, since $\sigma$ is a singlet its energy density dominates the total energy density of the Universe and hence is able to drive inflation. From $V_\sigma$ of Eq. \((3)\) we can see that the choice $V_0 = \mu^4 / (4 \lambda)$ sets the minimum of the potential to be zero. We now write the slow-roll parameters in terms of $V(\sigma)$ as

$$\epsilon \equiv \frac{M_P^2}{16\pi} \left( \frac{V'}{V} \right)^2 \quad \text{and} \quad \eta \equiv \frac{M_P^2}{8\pi} \frac{V''}{V}, \quad (11)$$

where $M_P \equiv G^{-1/2} \approx 1.22 \times 10^{19}$ GeV is the Planck mass and the prime denotes a derivative with respect to $\sigma$. Inflation ends when the scale factor accelerates no more, and this happens when $\epsilon_{\text{end}} = 1$. This gives

$$\epsilon_{\text{end}} \approx \frac{\mu_1}{4 \lambda \left( \lambda M_P^2 / (4 \pi)^2 + \mu^2 \right)}. \quad (12)$$

Thus the number of $e$-folds from $\sigma$ to $\sigma_{\text{end}}$ can be estimated as

$$N(\sigma) = -\frac{8\pi}{M_P^2} \int_{\sigma_{\text{end}}}^{\sigma} \frac{V}{V'} d\sigma = \frac{\pi \mu^2}{\lambda M_P^2} \log \left[ \frac{\mu^4}{4 \lambda (\lambda M_P^2 / (4 \pi)^2 + \mu^2) \sigma^2} \right] - \frac{\pi}{M_P^2} \left[ \frac{\mu^4}{4 \lambda (\lambda M_P^2 / (4 \pi)^2 + \mu^2) \sigma^2} - \sigma^2 \right], \quad (13)$$

where we note that the contribution from the second term is much less than that from the first term. From the observed amplitude of the density perturbations on the COBE scale \([11]\)

$$\delta_H = \sqrt{\frac{1}{75 \pi^2 M_P^4 V_{\Phi}^2}} \approx 1.91 \times 10^{-5}, \quad (14)$$
we can find the corresponding value of \( \sigma \) as

\[
\sigma^2 \equiv \frac{8\pi^3 \lambda^6}{3\lambda A_H M_{Pl}^2},
\]

where \( A_H \equiv \sqrt{3} \pi \delta_H \approx 5.19 \times 10^{-4} \). Then we can easily estimate the spectral index at the COBE point as [12]

\[
n_s \approx 1 - \frac{\lambda M_{Pl}^2}{2\mu^2} + \frac{40\pi^3 \lambda^4}{\lambda A_H M_{Pl}^2},
\]

As a sample set of values, let us take \( \mu = 2/\pi \times 10^{-6} M_{Pl} \approx 7.77 \times 10^{12} \text{GeV} \) and \( \lambda = 4/\pi^2 \times 10^{-12} \approx 4.11 \times 10^{-14} \). This set gives the minimum of the potential at \( \pi M_{Pl} \) with an inflationary energy scale \( \approx (10^{18}) \text{GeV} \). From Eqs. [13], [15] and [16], we obtain [16] \( N_H \approx 59.0 \) and \( n_s \approx 0.963 \). Also, due to the relatively high inflationary energy scale, we find a tensor-to-scalar ratio \( r \) very close to the observational sensitivity of near future experiments, \( r \approx 0.0163 \). In Fig. [1] we show the contour plots of both \( n_s \) and \( N_H \) on the \( \mu, \lambda \) plane.

After the end of inflation, \( \sigma \) eventually starts oscillation around its minimum \( \sigma / \sqrt{\lambda} \) and decays into light relativistic particles, reheating the universe to restore the gauge symmetry \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) with the reheating temperature being estimated as [13]

\[
T_{RH} \sim O(0.1) \sqrt{\pi \sigma M_{Pl}},
\]

where we have taken the number of relativistic degrees of freedom to be \( O(10^2 \sim 10^3) \).

**Neutrino masses and the CMB anisotropies:** The relevant Yukawa couplings that are giving masses to the three generations of leptons are given by

\[
-\mathcal{L}_{\text{Yukawa}} = h_{ij} \bar{\psi}_{ijL} \Phi \psi_{iR} + h_{ij} \bar{\psi}_{ijR} \Phi^c \psi_{iL} + h.c.
\]

\[
+ f_{ij} \left[ \bar{\psi}_{ijR} C \sigma_2 \Delta R \psi_{iL} + (R \leftrightarrow L) \right] + h.c.
\]

The discrete left-right symmetry ensures the Majorana Yukawa coupling \( f \) to be the same for both left and right-handed neutrinos. The breaking of the left-right symmetry down to \( U(1)_{em} \) results in the effective mass matrix of the physical left-handed neutrinos to be

\[
m_\nu = \frac{-\beta v^2}{2\lambda M_{Pl}} \hat{f} v^2 h f^{-1} h^T = m_\nu^L + m_\nu^R,
\]

where we have used Eq. [10] for type-II contribution and neglected \( O(k_2/k_1) \approx (m_\nu/m_\nu) \) terms in the type-I contribution. Assuming that \( h, f \) and \( \beta \) are \( O(1) \) couplings, the relative magnitudes of \( m_\nu^L \) and \( m_\nu^R \) depend on the parameter space of \( v_R, M, \) and \( \sigma \). In the following we assume that type-II term dominates. This is a viable assumption for \( M < v_R^2 / \sigma \). In what follows we will work in this regime and then we have

\[
\mathcal{H} \equiv m_\nu^L m_\nu^R \approx \left( \frac{-\beta v^2}{2\lambda M_{Pl}} \right)^2 f f^T \hat{f},
\]

where an appropriate choice of \( f \) will explain the leptonic mixing. \( \mathcal{H} \) can be diagonalised by using the \( U_{PMNS} \) matrix and then we will get the solar and atmospheric mass scales

\[
\Delta m^2_{\odot} = m^2_1 - m^2_2 = \frac{\beta v^2}{2\lambda M_{Pl}} \Delta f^2_{12},
\]

\[
\Delta m^2_{\text{atm}} = |m^2_2 - m^2_3| = \frac{\beta v^2}{2\lambda M_{Pl}} |\Delta f^2_{23}|,
\]

where \( \Delta f^2_{12} = f_2^2 - f_1^2 \) and \( \Delta f^2_{23} = f_3^2 - f_2^2 \). Using Eq. [15] in the above equation we get the solar and atmospheric mass scales to be

\[
\Delta m^2_{\odot} = \left( \frac{-\beta v^2}{2\lambda M_{Pl}} \right)^2 \left( \frac{8\pi^3 \lambda^2}{75\sigma v^2} \right)^{1/3} \Delta f^2_{12} \delta_H^{-2/3},
\]

\[
\Delta m^2_{\text{atm}} = \left( \frac{-\beta v^2}{2\lambda M_{Pl}} \right)^2 \left( \frac{8\pi^3 \lambda^2}{75\sigma v^2} \right)^{1/3} |\Delta f^2_{23}| \delta_H^{-2/3}.
\]

In the above equations \( \mu \) can be determined from the precise measurement of \( n_s \) in the future CMB experiments. Notice that Eqs. (22) and (23) give an important relation between the observed neutrino mass scales \( \Delta m^2_{\odot} \) and \( \Delta m^2_{\text{atm}} \) and the amplitude of perturbations on the CMB scale predicted by inflationary scenario in left-right symmetric models with spontaneous \( D \)-parity breaking. This is an important prediction of the theory.

**Lepton asymmetry:** Assuming a normal hierarchy in the right-handed neutrino sector, the decay of the lightest right-handed neutrino can give rise to a net lepton asymmetry through

\[
N_1 \rightarrow \left\{ e^\nu_R + \phi_1^L \right\},
\]

where \( N_1 = [v_{1R} + (v_{1R})^c] / \sqrt{2} \). The CP asymmetry in the above decay process is estimated to be

\[
\delta_{CP} \approx -\frac{1}{8\pi} \left( \frac{f_1}{f_2} \right)^3 \left( \frac{h^2 h}{(h^2 h)_{12}} \right)^2,
\]

where \( f_1 \) and \( f_2 \) are two of the eigenvalues of \( f \) matrix, and we have neglected \( O(k_2/k_1) \approx (m_\nu/m_\nu) \) terms. The lepton asymmetry is then transferred to the required baryon asymmetry through the electroweak sphaleron processes which conserve \( B-L \) but violate \( B+L \). A successful baryon asymmetry requires a lower bound on the mass scale of the lightest right-handed neutrino to be \( M_1 \gtrsim 4.8 \times 10^9 \text{GeV} \) [14].

**Conclusions and outlooks:** We have seen that within the left-right symmetric model inflation is possible only if the left-right parity and \( SU(2)_R \) gauge symmetry are broken at different scales. In particular, the left-right parity is broken at \( O(M_{Pl}) \), while leaving \( SU(2)_L \) gauge symmetry preserved until \( O(10^{14}) \text{GeV} \) or so. As a standard routine, after inflation the Universe is reheated to restore the left-right gauge symmetry \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). As a result a net baryon asymmetry, required for successful big bang nucleosynthesis,
could be generated through the leptogenesis route. An important prediction in this model is that the neutrino masses are connected to the anisotropies in the CMB predicted by inflation. We conjecture that this can be implemented in the $SO(10)$ model which, at present, is the most favorable scenario for neutrino masses and mixings. Since $\{210\}$ field contains a $SU(4)_{C} \times SU(2)_{L} \times SU(2)_{R}$ singlet it can play the role of $\sigma$ as in the present case. This is under consideration and will be reported separately.

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FIG. 1: The contour plots of (left) $n_{s}$ and (right) $N_{H}$. The horizontal and vertical axes are $\log_{10} \lambda$ and $\log_{10} (\mu/\text{M}_{\text{Pl}})$, respectively, for both graphs. In the contour plot of $n_{s}$, the contours denote 0.99, 0.97, 0.94, 0.90 and 0.85 from the innermost line. Likewise, we have set 1000, 500, 100, 10 and 1 in the $N_{H}$ plot. Note that in the right panel although we have $N_{H} \gg 1$ in the upper left region, the values of $\lambda$ and $\mu$ taken from here will place the minimum of potential far larger than $M_{\text{Pl}}$ and the form of the effective potential is apt to an appreciable modification, spoiling all the results we have estimated. Thus we disregard the values of $\lambda$ and $\mu$ within this region.
N. Sahu and U. Sarkar, Phys. Rev. D 74, 093002 (2006) [arXiv:hep-ph/0605007].

[15] If the parity and $SU(2)_R$ are broken at the same scale then the smallness of $v_L$ depends on the large value of $v_R$ through the seesaw relation $v_L v_R \approx v^2$, which implies $v_R \approx (10^{13} \sim 10^{14})$ GeV to obtain $v_L \approx O(1)$ eV.

[16] In fact, there exists some level of uncertainty on from which value of $\sigma$ inflation begins. Because of uncertainty principle, quantum fluctuations $\delta \sigma \approx H_i/(2\pi)$ are so strong near the origin and the classical downhill motion dominates only when $\sigma^2 \gtrsim \sigma_i^2 \approx 2\pi \mu^3/(3\lambda^3 M_{Pl})$. We can find the ratio $\sigma_H^2/\sigma_i^2 \sim 10^8$, i.e. $\sigma_H$ lies well within the regime of classical evolution of $\sigma$ and we need not worry about $\sigma_i$. 