Cross sections for hard exclusive electroproduction of $\pi^+$ mesons on a hydrogen target

A. Airapetian,15 N. Akopov,25 Z. Akopov,25 E.C. Aschenauer,6 W. Augustyniak,24 A. Avetissian,25 E. Avetissian,10 L. Barion,9 S. Belostotski,17 N. Bianchi,10 H.P. Blok,16,23 H. Böttcher,6 C. Bonomo,9 A. Borissov,13 V. Bryzgalov,18 J. Burns,13 M. Capiluppi,9 G.P. Capitani,10 E. Cisbani,20 G. Ciullo,9 M. Contalbrigo,9 P.F. Dalpiaz,9 W. Deconinck,15 R. De Leo,2 M. Demey,16 L. De Nardo,5,21 E. De Sanctis,10 M. Dieffenbacher,6 P. Di Nezza,10 J. Drescher,16 M. Dürer,12 M. Ehrenfried,12 G. Elbakian,25 F. Ellinghaus,4 R. Fabbi,9 A. Fantoni,10 S. Frullani,20 D. Gabbert,6 G. Gapinchenko,18 V. Gapinchenko,18 F. Garibaldi,20 G. Gavrilov,5,17,21 V. Gharibyan,25 F. Giordano,9 S. Gliske,15 H. Guler,6 C. Hadjidakis,10 D. Hasch,10 G. Hill,13 A. Hillebrand,8 M. Hoek,13 I. Hristova,6 A. Ilyichev*,7,14 V. Ivanov,18 H.E. Jackson,13 S. JoostVen,11 R. Kaiser,13 T. Keri,12 E. Kinney,4 A. Kisselev,14,17 M. Kopytin,6 V. Korotkov,18 P. Kravchenko,17 V.G. Krivokhijine,7 L. Lagamba,2 R. Lamb,14 L. Lapikás,16 I. Lehnmann,13 P. Lenisa,9 L.A. Linden-Levy,14 A. Lopez Ruiz,11 W. Lorenzon,15 S. Lu,12 X. Lu,22 D. Mahon,13 N.C.R. Makins,14 B. Mariantoni,24 H. Marukyan,25 C.A. Miller,21 Y. Miyachi,22 V. Muccifora,10 M. Murray,13 A. Mussgiller,8 E. Nappi,2 Y. Naryshkin,17 A. Nass,8 M. Negodaev,6 W.-D. Nowak,6,19 L.L. Pappalardo,9 R. Perez-Benito,12 N. Pickert,8 M. Raithel,8 P.E. Reimer,1 A.R. Reolon,10 C. Riedl,10 K. Rith,8 S.E. Rock,5 G. Rosner,13 A. Rostomyan,5 L. Rubacek,12 J. Rubin,14 D. Ryckbosch,11 Y. Salomatín,13 A. Schäfer,19 G. Schnell,11 K.P. Schiller,5 B. Seitz,13 C. Shearer,13 T.-A. Shibata,22 V. Shuto,7 M. Stancioli,9 M. Statera,9 J.J.M. Steijger,16 H. Stenzel,12 J. Stewart,6 F. Stinzinger,8 J. Streit,12 S. Taroian,25 E. Thomas,10 A. Trzciński,24 M. Tytgat,11 A. VandenBroucke,11 P.B. van der Nat,16 G. van der Steenhoven,16 Y. van Haarlem,11 C. van Hulse,11 M. Varanda,5 D. Veretennikov,17 V. Vikhrov,17 I. Vilardi,2 C. Vogel,8 S. Wang,3 S. Yaschenko,8 H. Ye,3 Z. Ye,5 S. Yen,21 W. Yu,12 D. Zeiler,8 B. Zihlmann,11 and P. ZuPanski24
(The HERMES Collaboration)

1Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843, USA
2Istituto Nazionale di Fisica Nucleare, Sezione di Bari, 70124 Bari, Italy
3School of Physics, Peking University, Beijing 100871, China
4Nuclear Physics Laboratory, University of Colorado, Boulder, Colorado 80309-0390, USA
5DESY, 22603 Hamburg, Germany
6DESY, 15738 Zeuthen, Germany
7Joint Institute for Nuclear Research, 141980 Dubna, Russia
8Physikalisches Institut, Universität Erlangen-Nürnberg, 91058 Erlangen, Germany
9Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara and Dipartimento di Fisica, Università di Ferrara, 44100 Ferrara, Italy
10Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, 00044 Frascati, Italy
11Department of Subatomic and Radiation Physics, University of Gent, 9000 Gent, Belgium
12Physikalisches Institut, Universität Gießen, 35392 Gießen, Germany
13Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom
14Department of Physics, University of Illinois, Urbana, Illinois 61801-3080, USA
15Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA
16National Institute for Subatomic Physics (Nikhef), 1009 DB Amsterdam, The Netherlands
17Petersburg Nuclear Physics Institute, St. Petersburg, Gatchina, 188350 Russia
18Institute for High Energy Physics, Protvino, Moscow region, 142281 Russia
19Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany
20Istituto Nazionale di Fisica Nucleare, Sezione Roma I, Gruppo Sanità and Physics Laboratory, Istituto Superiore di Sanità, 00161 Roma, Italy
21TRIUMF, Vancouver, British Columbia V6T 2A3, Canada
22Department of Physics, Tokyo Institute of Technology, Tokyo 152, Japan
23Department of Physics and Astronomy, Vrije Universiteit, 1081 HV Amsterdam, The Netherlands
24Andrzej Sołtan Institute for Nuclear Studies, 00-689 Warsaw, Poland
25Yerevan Physics Institute, 375036 Yerevan, Armenia

(Dated: February 1, 2008)

The exclusive electroproduction of $\pi^+$ mesons was studied with the HERMES spectrometer at the DESY laboratory by scattering 27.6 GeV positron and electron beams off an internal hydrogen gas target. The virtual-photon cross sections were measured as a function of the Mandelstam variable $t$ and the squared four momentum $-Q^2$ of the exchanged virtual photon. A model calculation based on Generalized Parton Distributions is in fair agreement with the data at low values of $|t|$ if power corrections are included. A model calculation based on the Regge formalism gives a good description of the magnitude and the $t$ and $Q^2$ dependences of the cross section.
The interest in hard exclusive processes has grown since a Quantum Chromodynamics (QCD) factorization theorem was proven for the hard electroproduction of mesons by longitudinal photons \[1\]. It was shown that at leading order in the strong-coupling constant \(\alpha_S\) and in the fine-structure constant \(\alpha\) the amplitude for such reactions can be factorized into three parts (see left panel of Fig. 1): a hard lepton-scattering part, which can be calculated in Quantum Electrodynamics and in perturbative QCD (pQCD), and two soft parts that parametrize the structure of the target nucleon by Generalized Parton Distributions (GPDs) \[2, 3, 4\] and the structure of the produced meson by a distribution amplitude. The GPDs offer the possibility to reveal a three-dimensional representation of the structure of hadrons at the partonic level, correlating the longitudinal momentum fraction to transverse spatial coordinates \[5, 6, 7, 8, 9\]. For recent theoretical reviews, see Refs. \[10, 11, 12\].

The amplitude for exclusive electroproduction of mesons with specific quantum numbers is described by a particular combination of GPDs. At leading twist, exclusive vector-meson production is sensitive to only unpolarized GPDs (\(H\) and \(E\)), while pseudoscalar-meson production is sensitive to polarized GPDs (\(\tilde{H}\) and \(\tilde{E}\)) without the need for a polarized target or beam. Moreover, for \(\pi^+\) production, the pseudoscalar contribution involving \(\tilde{E}\) dominates at small momentum transfer to the target as it contains the \(t\)–channel pion-pole contribution. The complete amplitude of that contribution to \(\pi^+\) production also contains the pion charge form factor \(\frac{1}{\sqrt{t}}\) \[13, 14\].

At moderate virtuality of the exchanged photon, next-to-leading-order (NLO) in \(\alpha_S\) and higher-twist corrections to the pQCD leading-order amplitude can contribute. Two different types of higher-twist corrections, jointly denoted by the term “power corrections”, have been estimated in the case of \(\pi^+\) production \[15\]: one arising from the intrinsic transverse momentum of the partons, and the other resulting from the soft-overlap diagram. In contrast to the leading-order perturbative mechanism, the latter, shown on the right panel of Fig. 1, does not proceed through one-gluon exchange. Although significant NLO corrections have been calculated \[16, 17\], the higher-twist corrections dominate for the virtuality of the exchanged photon of the present data.

Alternatively, exclusive processes can be described at the hadronic level within the Regge formalism (e.g. \[18\]) where the interaction between the virtual photon and the nucleon is described in terms of Regge-trajectory exchanges in the \(t\)-channel. This approach can describe photoproduction \[19\] and electroproduction \[20\] of pseudoscalar mesons above the resonance region. In the latter case, meson production by both transverse and longitudinal photons contribute and can be calculated within the Regge formalism.

This letter reports a measurement of the virtual-photon cross section for the hard exclusive reaction \(ep \rightarrow e'n\pi^+\) on a hydrogen target. The relevant kinematic variables of the process are the squared four-momentum \(-Q^2\) of the exchanged virtual photon, either the Bjorken variable \(x_B = Q^2/2M_p\mu\) (where \(M_p\) is the proton mass and \(\mu\) is the energy of the virtual photon in the target rest frame) or the squared invariant mass of the photon-nucleon system \(W^2\), the Mandelstam variable \(t\), and the azimuthal angle \(\phi\) of the pion around the virtual photon momentum relative to the lepton scattering plane. Instead of \(t\), the quantity \(t' = t - t_0\) was used in the analysis, where \(-t_0\) represents the minimum value of \(-t\) for a given value of \(Q^2\) and \(x_B\).

The virtual-photon cross section has been previously measured above the resonance region (for \(W^2 > 4\text{ GeV}^2\)) at CEA \[21\], Cornell \[22\], DESY \[23\], and more recently at JLAB \[24, 25\]. The present measurement extends the kinematic region to higher values of \(W\) and higher values of \(-t'\).

The data were collected with the HERMES spectrometer \[26\] during the period 1996-2005. The 27.6 GeV HERA electron or positron beam at DESY scattered off polarized and unpolarized proton targets. Events were selected in which only one lepton and one positively charged hadron were detected and in which no additional cluster was recorded by the electromagnetic calorimeter. The HERMES geometrical acceptance of \(\pm 170\) mrad horizontally and \(\pm (40 - 140)\) mrad vertically results in detected scattering angles ranging from 40 to 220 mrad. Leptons were distinguished from hadrons with an average efficiency of 98% and a hadron contamination less than 1% by using an electromagnetic calorimeter, a transition-radiation detector, a preshower scintillation counter, and

\[\text{FIG. 1: Leading-order (left panel) and soft-overlap (right panel) diagram for the exclusive } \pi^+ \text{ electroproduction amplitude.}\]

---

*Present address: National Center of Particle and High Energy Physics, Belarusian State University, 22040 Minsk, Belarus

*Present address: CERN, Geneva, Switzerland
a threshold gas Čerenkov counter. In 1998 the threshold gas Čerenkov counter was replaced by a Ring Imaging Čerenkov detector \[27\]. The threshold gas Čerenkov counter (Ring Imaging Čerenkov detector) provided pion identification in the momentum range \(4.9 \text{ GeV} < p < 15 \text{ GeV} \) (\(1 \text{ GeV} < p < 15 \text{ GeV} \)). For the exclusive data sample, the pion momentum was required to be \(7 \text{ GeV} < p < 15 \text{ GeV} \). For this momentum range, the pion identification efficiency is on average 97\% (99\%) and the contamination from other hadrons less than 3\% (2\%) for the threshold gas Čerenkov counter (Ring Imaging Čerenkov detector).

The kinematic requirement \(Q^2 > 1 \text{ GeV}^2 \) was imposed on the scattered lepton in order to select the hard scattering regime. The value of \(W^2\) was required to be higher than \(10 \text{ GeV}^2\) to avoid the low-acceptance region for the hadron defined by the spectrometer upper angular limit of 220 mrad. The resulting kinematic range is \(1 \text{ GeV}^2 < Q^2 < 11 \text{ GeV}^2\) and \(0.02 < x_B < 0.55\). The mean \(W^2\) value of the data is \(16 \text{ GeV}^2\).

As the recoiling neutron was not detected, exclusive meson production was selected by requiring that the squared missing mass \(M_X^2\) of the reaction \(e p \rightarrow e \pi^+ X\) corresponds to the squared neutron mass. Due to the limited experimental resolution, the exclusive \(\pi^+\) channel cannot be separated from the neighboring channels (defined as background channels) with final states such as \(\pi^+ + (N\pi)\) and \(\pi^+ + (N\pi\pi)\), as their \(M_X^2\) values can be smeared into the region corresponding to exclusive \(\pi^+\) channel. In order to subtract the background channels, which cannot be fully described by existing Monte Carlo simulations, a two-step procedure was developed. In the first step, the yield difference between \(\pi^+\) and \(\pi^-\) was used. In this yield difference, exclusive \(\pi^+\) events remain as the exclusive production of \(\pi^-\) on a hydrogen target with a recoiling nucleon in the final state is forbidden by charge conservation. Background events arising from the production of neutral vector mesons cancel as they contribute equally to \(\pi^+\) and \(\pi^-\) production. In the second step, the PYTHIA Monte Carlo generator \[25\] was used in conjunction with a special set of JETSET \[29\] fragmentation parameters tuned to provide an accurate description of deep inelastic hadron production in the HERMES kinematic domain. The difference \(N_{\pi+} - N_{\pi-}\), where \(N\) represents the yield normalized by the integrated luminosity \(L\), is well described by PYTHIA if pion momenta larger than \(7 \text{ GeV}\) are required. The latter constraint removes mainly background events for which \(Q^2 < 3 \text{ GeV}^2\). The upper panel of Fig. 2 shows the \(M_X^2\) dependence of the normalized-yield difference \(N_{\pi+} - N_{\pi-}\) for the data and for the PYTHIA Monte Carlo simulation. The Monte Carlo sample describes the data for values of the squared missing mass higher than \(2 \text{ GeV}^2\), i.e., outside the region corresponding to exclusive \(\pi^+\) production. Finally, the exclusive \(\pi^+\) yield was obtained by subtracting the normalized-yield difference of the PYTHIA Monte Carlo from that of the data: \(N_{\pi+}^{\text{excl}} = (N_{\pi+} - N_{\pi-})_{\text{data}} - (N_{\pi+} - N_{\pi-})_{\text{PYTHIA}}\). The lower panel of Fig. 2 shows the peak corresponding to exclusive \(\pi^+\) production resulting from this double difference. The peak is centered at the squared neutron mass and its width of \(0.67 \text{ GeV}^2\) is consistent with that of a Monte Carlo sample for exclusive \(\pi^+\) production normalized to the data. The Monte Carlo, denoted as exclusive Monte Carlo, is based on a GPD model \[13\] and will be described below. As PYTHIA does not simulate nucleon resonance production, the agreement of the exclusive Monte Carlo sample with the data for the double difference is an indication that there is very little contribution from resonant channels (\(\pi^+ + \Delta^0\) for \(\pi^+\) and \(\pi^- + \Delta^{++}\) for \(\pi^-\)) to the normalized-yield difference \(N_{\pi+} - N_{\pi-}\). In order to estimate the systematic uncertainty of the background subtraction, the PYTHIA Monte Carlo normalized-yield difference \((N_{\pi+} - N_{\pi-})_{\text{PYTHIA}}\) was changed by the discrepancy between that normalized-yield and the data in the region \(3 < M_X^2 < 7 \text{ GeV}^2\). The discrepancy amounts to between 20\% and 50\%, depending on the specific kine-

![FIG. 2: Upper panel: squared missing-mass dependence of the normalized-yield difference \(N_{\pi+} - N_{\pi-}\) for data (filled points) and PYTHIA Monte Carlo (histogram). The error bars represent the statistical uncertainty. Lower panel: squared missing-mass dependence of the normalized-yield after background subtraction procedure. The data (filled points) are compared to a Monte Carlo sample for exclusive \(\pi^+\) production (histogram) normalized to the data. The inner error bars represent the statistical uncertainties and the outer error bars represent the quadratic sum of statistical and systematic uncertainties. The latter originate from the background subtraction procedure. The dashed vertical line indicates the squared neutron mass.](image-url)
matic bin in $Q^2$, $x_B$, or $t'$. The largest discrepancies correspond to the least populated bins ($Q^2 > 4 \text{ GeV}^2$ and $-t' > 0.3 \text{ GeV}^2$). An upper limit on $M_X^2$ of 1.2 $\text{ GeV}^2$ was chosen in order to optimize the combined statistical and systematic uncertainties for the number of exclusive events. The resulting relative systematic uncertainty for the number of exclusive events ranges from 5% to 20%. Within the $M_X^2$ limit, the fraction of background events in the normalized-yield difference $N_{π^+} - N_{π^-}$ estimated with PYTHIA amounts to 20%. The number of $π^+$ events after background subtraction is 4510.

As the recoiling neutron was not detected, $t'$ was determined from the measurement of the four-momenta of the scattered lepton and the produced pion. In order to improve the resolution for exclusive events, $t'$ was calculated by setting $M_X = M_n$. This offered the possibility to discard the lepton energy, the quantity subject to the largest uncertainty for the present data sample. This procedure results in a factor of two improvement in the $t'$ resolution. The same calculation was applied to the PYTHIA Monte Carlo events that were used to subtract the background.

The differential cross section for exclusive $π^+$ production by virtual photons can be written as

$$
\frac{d\sigma^{γ^*p→π^+}(x_B,Q^2,t',φ)}{dt'dφ} = \frac{1}{Γ_V(x_B,Q^2)} \frac{dσ^{π^+p→π^+}(x_B,Q^2,t',φ)}{dx_BdQ^2dt'dφ},
$$

where $Γ_V$ is the virtual-photon flux factor. Within the Hand convention [30], this flux factor is

$$
Γ_V(x_B,Q^2) = \frac{α}{8π M_pE^2} \frac{Q^2}{x_B} \left(1 - \frac{1}{1 - ε}\right),
$$

where $E$ is the beam energy and $ε$ is the virtual-photon polarization parameter. The $t'$ dependence of the photon-nucleon differential cross section integrated over φ were extracted from the data as follows:

$$
\frac{dσ^{γ^*p→π^+}(x_B,Q^2,t')}{dt'} = \frac{1}{Γ_V} \frac{N_{π^+}^{\text{excl}}}{ΔX_B ΔQ^2 Δt' η},
$$

where $N_{π^+}^{\text{excl}}$ is the number of $π^+$ events after background subtraction, $κ$ is the probability to detect the scattered lepton and the produced $π^+$ within the HERMES spectrometer acceptance, and $η$ is the radiative correction factor. These quantities were determined for each kinematic bin. The symbols $ΔX_B$, $ΔQ^2$, and $Δt'$ denote the bin size, which was chosen according to the statistical precision, instrumental resolution, and kinematic smearing affecting each individual bin. The $t'$ dependence of the differential cross section $\frac{dσ^{π^+}(x_B,Q^2,t')}{dt'}$ was determined for four $Q^2$ bins and the $Q^2$ dependence of the cross section integrated over $t'$, $σ(x_B,Q^2)$, for three $x_B$ bins.

The detection probability $κ$ was determined using an exclusive $π^+$ Monte Carlo simulation based on a GPD model [15] and a GRANT simulation [31] of the detector. The variable $κ$ is the ratio between the number of simulated events reconstructed in the HERMES spectrometer and the number of generated events. These numbers were evaluated at the reconstructed and generated kinematics respectively, and were integrated over the bin size and over $φ$ and $t'$ for the determination of $σ(x_B,Q^2)$. The variable $κ$ represents the combined effect of the spectrometer acceptance, the analysis constraints (such as the $M_X^2$ constraint) and the detector efficiencies. For the GPD model [15], the power corrections were included and the factorized ansatz was used for the $t'$ dependence. However, the $t'$ and $φ$ dependences were modified in order to describe the data. Radiative events were included in the Monte Carlo simulation according to the code RADGEN [32] adapted to exclusive meson production using the GPD model [15]. Fig. 3 shows good agreement between the kinematic distributions of the data and of the exclusive Monte Carlo sample (which is normalized to the data by a constant factor) with the exception of yields at high values of $Q^2$, $x$ and especially $-t'$.

The calculated spectrometer acceptance ranges from 0.1 to 0.7 depending on the kinematic bin, while the efficiency of the detectors and analysis constraints together amounts to about 0.4–0.5. The combination of both leads to $κ$ values ranging from 0.04 to 0.28. The model dependence of the determination of $κ$ was estimated by using the GPD parametrization [15] with different $t'$ and $φ$ dependences and by using a different GPD parametrization [33] for the $Q^2$ and $x_B$ dependences. It was further studied to what degree the relevant kinematic variables $Q^2$, $t'$ and $φ$ are correlated by the detector acceptance. These model dependences amount to a relative systematic uncertainty in $κ$ of less than 15%.

Events can be smeared from one bin to another by multiple scattering and bremsstrahlung. This effect was included in the determination of the detection probability $κ$. With the selected bin sizes, the fraction of events that migrate out of (into) a certain kinematic bin is on average $0.1$ to $0.7$ depending on the kinematic bin, while the efficiency $κ$ of radiative events decreases from $0.12$ (0.15) to $0.04$ (0.07) according to the exclusive Monte Carlo simulation.

The Born cross section was extracted using the radiative correction factor $η$, which was determined from the ratio of Monte Carlo samples with and without radiative effects. The value of $η$ was found to be $0.77$ with little variation (less than $3\%$) as a function of $Q^2$, $x_B$ or $t'$ with the constraints applied to the data to select exclusive $π^+$ production.

The mean values for $x_B$, $Q^2$, and $t'$ for each kinematic bin were estimated from distributions generated at Born level by the exclusive Monte Carlo simulation. The flux factor in Eq. 3 was determined for these mean values of $Q^2$ and $x_B$. The resulting cross sections were corrected for bin-averaging effects to take into account the nonlin-
ear dependence of the cross section within each bin (in Eq. \[3\] a linear dependence inside a bin is implicitly assumed). These corrections were obtained from the exclusive Monte Carlo simulation by taking the ratio between the cross section evaluated at fixed kinematic values and the cross section integrated over the kinematic bins following Eq. \[3\]. The error arising from the evaluation of the flux factor at the mean values is then also corrected. The bin-averaging correction factor amounts on average to 1.08 and does not exceed 1.2 except for the highest-\(Q^2\) bin, where it reaches 2.2 at some \(t'\) values.

The integrated luminosity \(\mathcal{L}\) was determined by comparing the number of inclusive deep-inelastic scattering events in the data sample to the yield generated by a Monte Carlo sample for inclusive scattering based on world data [34, 35]. The HERMES luminosity detector provided another measurement of \(\mathcal{L}\), which was used to estimate the systematic uncertainty. The integrated luminosity \(\mathcal{L}\) amounts to 0.4 fb\(^{-1}\) with a 5\% systematic uncertainty.

The total systematic uncertainty of the cross section is dominated by the uncertainty of the background subtraction and of the detection probability. The latter takes into account the uncertainties due to the model dependence of its determination and the different detector resolutions in the different data taking periods.

The measured differential cross section integrated over the angle \(\phi\) can be written as 
\[
\sigma_{\tau} = \sigma_{\tau L} + \sigma_{\tau T},
\]
where \(\sigma_{\tau T}\) and \(\sigma_{\tau L}\) are, respectively, the contributions of transversely and longitudinally polarized virtual photons. At HERMES the separation of the transverse and longitudinal components of the cross section is not feasible. However, as the transverse contribution is predicted to be suppressed by \(1/Q^2\) with respect to the longitudinal contribution [1], the data at larger \(Q^2\) are expected to be dominated by the longitudinal part. Moreover, with the presence of the pion pole at low \(-t'\), the longitudinal part of the cross section is expected to dominate in this region. In Fig. 4 and 5, described below, the data are compared to calculations for the longitudinal part of the cross section computed using a GPD model [15] and to calculations for the total cross section computed using a Regge model [50]. The cross sections are calculated for the mean values of \(x_B\), \(Q^2\), and \(t'\) of the experimental data in each bin.

Fig. 4 shows the \(t'\) dependence of the differential cross section for four \(Q^2\) bins. As \(Q^2\) is closely related to \(x_B\) (due to the HERMES acceptance as well as the upper limit on the pion momentum) low values of \(Q^2\) correspond to low values of \(x_B\). The dashed-dotted lines in Fig. 4 show the leading-order calculations of the longitudinal part computed using the GPD model [15]. The GPD \(\tilde{E}\) is considered to be dominated by the \(t\)-channel pion-pole and the pseudoscalar contribution to the cross section is parametrized in terms of the pion electromagnetic form factor \(F_\gamma\). A Regge-inspired \(t'\) dependence is used for \(\tilde{E}\). The GPD \(H\) is neglected here as \(\tilde{E}\) is expected to dominate the cross section at low \(-t'\). The solid lines include the power corrections due to the intrinsic transverse momentum of the partons and due to the soft-overlap contribution, the latter being dominant. While the leading-order calculation strongly underestimates the data, the calculations including power corrections agree with the data for \(-t' < 0.3\) GeV\(^2\) for the four \(Q^2\) bins. As the GPD model requires \(-t'\) to be much smaller than \(Q^2\), the calculations are not expected to describe the full \(t'\) range. Furthermore, at larger \(-t'\), the data may receive a significant contribution from the transverse part of the cross section, which is not described by the GPD model.

Fig. 5 shows the \(Q^2\) dependence of the cross section integrated over \(t'\) for three \(x_B\) bins. The \(Q^2\) dependence of the data is in general well described by the calculations.
The available part of the cross section is expected to dominate, where systematic uncertainties. The curves represent calculations based on a GPD model \[15\] for the transverse part of the cross section and a Regge model \[36\] for the longitudinal part.

The inner error bars represent the statistical uncertainties and the outer error bars represent the quadratic sum of statistical and systematic uncertainties. The curves represent calculations based on a GPD model \[15\] at low $Q^2$ and a Regge model \[36\] for $\frac{d^2 \sigma}{dt \, dW}$ (dashed lines) and $\frac{d^2 \sigma}{dt \, dW}$ (dotted lines).

The thin, medium and thick lines correspond to the low, medium and high $x_B$ values, respectively.

Both the transverse and longitudinal parts of the cross section were computed using a Regge model \[36\], where pion production is described by the exchange of $\pi$ and $\rho$ Regge trajectories. In this formalism, the meson-nucleon coupling constants are fixed by pion photoproduction data. In the original version \[20\], the $\pi\pi\gamma$ form factor is fixed by pion form factor measurements, while the $\pi p\gamma$ transition form factor is unconstrained. In the version used here \[36\], both form factors are taken to be both $Q^2$- and $t'$-dependent. The dashed (dotted) lines in Fig. 4 show the total (longitudinal) cross section computed using the Regge model. In this model the transverse part of the cross section is estimated to represent from 6% to 8% of the total cross section at $-t' = 0.07$ GeV$^2$ and from 15% to 25% of the total cross section integrated over $t'$, confirming the expected suppression of the transverse to the longitudinal part of the cross section. However data from JLAB \[25\] at lower center of mass energy ($W^2 = 4.9$ GeV$^2$) show that the transverse part of the cross section is underestimated by the Regge model by a factor of three to four. It is not clear if this also holds at the higher $W^2$ and $-t'$ values of the HERMES data. Compared to the calculations for the longitudinal cross section from the Regge model (dotted lines), the $t'$ dependence of the GPD model \[15\] (solid lines) in Fig. 4 appears too steep. The total cross section computed by the Regge model describes well the $t'$ dependence of the differential cross section (dashed line on Fig. 4) and the $Q^2$ dependence of the cross section integrated over $t'$ (dashed lines on Fig. 5). The model calculations give also a good description of the magnitude of the data except at low $-t'$ for $Q^2 < 3$ GeV$^2$, where the calculations overestimate the data up to 70%.

In conclusion, the cross section was measured for exclusive electroproduction of $\pi^+$ mesons from a hydrogen target as a function of $-t'$ for four $Q^2$ bins and as a function of $Q^2$ for three $x_B$ bins. A model calculation for the longitudinal part of the virtual-photon cross section...
based on the Generalized Parton Distributions \[15\] does not describe the data at the leading order in the present \(Q^2\) range. However, if power corrections are included, the model is in fair agreement with the magnitude of the data at low values of \(-t'\). In this kinematic region, where \(\pi^+\) production by longitudinal photons is expected to dominate, the data support the order of magnitude of the power corrections. A model calculation based on the Regge formalism for both the longitudinal and the transverse part of the cross section \[36\] provides a good description of the magnitude and the \(t'\) and the \(Q^2\) dependences of the data.

We thank M. Guidal, J.M. Laget, and M. Vanderhaeghen for the theoretical calculations as well as M. Diehl for many interesting and useful discussions. We gratefully acknowledge the DESY management for its support, the staff at DESY and the collaborating institutions for their significant effort, and our national funding agencies and the EU RII3-CT-2004-506078 program for financial support.

\[1\] J.C. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. D 56 (1997) 2982.
\[2\] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, and J. Horejsi, Fortschr. Phys. 42 (1994) 101.
\[3\] A.V. Radyushkin, Phys. Lett. B 380 (1996) 417.
\[4\] X. Ji, Phys. Rev. Lett. 78 (1997) 610.
\[5\] M. Burkardt, Phys. Rev. D 62 (2000) 071503; Erratum–ibid. D 66 (2002) 119903.
\[6\] M. Diehl, Eur. Phys. J. C 25 (2002) 223; Erratum-ibid. C31 (2003) 277.
\[7\] J.P. Ralston and B. Pire, Phys. Rev. D 66 (2002) 111501.
\[8\] A.V. Belitsky and D. Müller, Nucl. Phys. A 711 (2002) 118.
\[9\] M. Burkardt, Int. J. Mod. Phys. A 18 (2003) 173.
\[10\] K. Gebeke, M.V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401.
\[11\] M. Diehl, Phys. Rept. 388 (2003) 41.
\[12\] A.V. Belitsky and A.V. Radyushkin, Phys. Rept. 418 (2005) 1.
\[13\] W. R. Frazer, Phys. Rept. 115 (1985) 1763.
\[14\] J. D. Sullivan, Phys. Lett. B 33 (1970) 179.
\[15\] M. Vanderhaeghen, P.A.M Guichon, and M. Guidal, Phys. Rev. D 60 (1999) 094017.
\[16\] A.V. Belitsky and D. Müller, Phys. Lett. B 513 (2001) 349.
\[17\] M. Diehl and W. Kugler, arXiv:0708.1121 [hep-ph] (2007).
\[18\] P.D.B. Collins, Cambridge University Press, Cambridge (1977).
\[19\] M. Guidal, J.M. Laget, and M. Vanderhaeghen, Phys. Lett. B 400 (1997) 6.
\[20\] M. Vanderhaeghen, M. Guidal, and J.M. Laget, Phys. Rev. C 57 (1998) 1454.
\[21\] C.N. Brown et al., Phys. Rev. D 8 (1973) 92.
\[22\] C.J. Bebek et al., Phys. Rev. D 9 (1974) 1229; Phys. Rev. D 13 (1976) 25; Phys. Rev. D 17 (1978) 1693.
\[23\] P. Brauel et al., Phys. Lett. B 65 (1976) 184; Phys. Lett. B 69 (1977) 253; Zeit. Phys. C 3 (1979) 101.
\[24\] J. Volmer et al., Phys. Rev. Lett. 86 (2001) 1713.
\[25\] T. Horn et al., Phys. Rev. Lett. 97 (2006) 192001.
\[26\] K. Ackerstaff et al., Nucl. Instrum. Methods A417 (1998) 230.
\[27\] N. Akopov et al., Nucl. Instrum. Methods A479 (2002) 511.
\[28\] T. Sjöstrand, L. Lonnblad, and S. Mrenna, Comput. Phys. Commun. 135 (2001) 238; P. Liebing, DESY-THESIS-2004-036.
\[29\] T. Sjöstrand, Comput. Phys. Commun. 82 (1994) 74.
\[30\] I.N. Hand, Phys. Rev. 129 (1963) 1834.
\[31\] R. Brun, R. Hagelberg, M. Hansroul, and J. Lassalle, CERN Report CERN-DD-78-2-REV (1978).
\[32\] I. Akushevich, H. Böttcher, and D. Ryckbosch, hep-ph/9906408 (1998).
\[33\] L. Mankiewicz, G. Piller, and A. Radyushkin, Eur. Phys. J. C 10 (1999) 307.
\[34\] H. Abramowicz and A. Levy, hep-ph/9712415 (1997).
\[35\] L. Whitlow, S. Rock, A. Bodek, E. Riordan, and S. Dasu, Phys. Lett. B 250 (1990) 193.
\[36\] J.M. Laget, Phys. Rev. D 70 (2004) 054023.