Fractal Analysis of Black Carbon in the Coal Mine Regions of India

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Abstract. Time series display irregular patterns and self-similar microstructures usually termed as fractals over a period of time, depending on several factors like climate, season, market fluctuations, growth cycles etc. Various methods are developed in mathematical modeling to investigate such trends in the time series. Black carbon is one of the major air pollutants released during several mining activities having dangerous effect on climate and human health. In this paper we aim to study the behavioral pattern and trends in the time series of black carbon concentration over the three major coal mines of India namely Raniganj, Jharia and Bokaro considering a long term time series data of past 38 years using regression and fractal dimensional analysis of black carbon between the major coal mines with the help statistical tools like Hurst exponent, predictability index and trend percent. In this comparative study it is observed that the time series display anti persistent behavior with positive and large variation then the past time. The results and comparisons obtained will be helpful in studying the behavior and trends in the concentration of black carbon over the coal mines regions. The significance of the study will be helpful to gather the interest of researcher’s, NGO’s and government towards the black carbon concentration over the coal mines so that preventive measures and new policies be framed to keep a check on black carbon concentration in these areas.

Keywords: Black carbon, Mathematical modeling, Fractals, Hurst exponent, Trend.

1. Introduction

Estimation and interpretation of climatic changes is one of the main areas of research nowadays. These changes are often found to have complex patterns and irregular shapes termed as fractals. Fractals were introduced by Benoit Mandelbrot in 1960’s [1-4] in a series of papers to capture the idea of fragmentation, for modeling irregular patterns which may sometimes be difficult to describe by geometrical scales and that tend to repeat each other as we zoom in, commonly occurring in the science, economy cycles and nature such as, galaxy formation, eroded coastlines, cauliflower etc. Fractals are also used nowadays to study the behavior of time series, various studies have shown that time series display a wide variety of fluctuations which had been studied using fractals dimensional analysis for geophysical time series [5-7]. It has been used to observe changes due to climate, market
fluctuations, recovery, recession and time period at local, regional, state, national and global level and thus study their fluctuations to analyze the variability and trends for a given time series [8-12].

Black carbon is a major component released during combustion activities in indoor as well as outdoor conditions. In indoors it is mainly released due to combustion of fossil fuels, wood, animal manure etc. while in outdoor it is released from automobiles, smoke from steel plants, oil refineries, coal industries along with various other [13]. Black carbon (BC) has severe harmful effects on climatic conditions, human health, rainfall patterns and ecological cycle. Studies related to the time series behavior of such pollutants contributing to air, water, and soil pollution are the major focus areas for researchers in this decade [14-19]. Coal is the major sources of energy for developing country like India, so coal mining industries has flourished in the states of India to meet the need ever increasing demand for coal. This has led to a vital growth in the black carbon concentration and air pollution index which is leading to adverse effect on the climate and health of the nation [20-21]. Time series is a set of data points arranged in sequential order over a period of time, mathematical modeling has been used to study the behavior of time series involving air, water and soil pollutants data to make future forecast and study the fluctuations in it over a period of time using several time series models [22-27].

The present study aims to develop a fractal analysis for black carbon concentration in the major coal mines of India considering a long term time series data of past 38 years so as to study the behavior of our time series using Hurst exponent, predictability index, fractal dimension and trend percent [28-36].

2. Research methodology

2.1 R square

R square is used to measure the amount of total variation in the series. Both R-squared value and stationary R-squared value are used as goodness of fit measures for a time series model also termed as the coefficient of determination for a model. The value of stationary R-squared varies from -∞ to 1 and it is used to compare the stationary part of the model to a simple mean model while the value of R square varies from 0 to 1, closer the value to 1 in both indicates that the model under consideration is better than the baseline model.

2.2 Regression analysis

This technique is used for predicting the value of the dependent variable (parameter) by using the values of the independent variable required and by holding the other variable as fixed after developing the equation of the regression line as described below, using this for modeling we can predict future or missing values and analyze the present variables of the sample. It helps in understanding the variation in value of the dependent variable due to change in the independent variables. Regression equation of line with dependent variable Y in terms of the independent variable X is defined as,

\[ Y = b_{yx}X + C \]  

(1)

Where C is the constant of integration. \( b_{yx} \) is the regression coefficient = \( r \times \frac{\sigma_Y}{\sigma_X} \).

Here \( r \) represents the coefficient of correlation and it is given by,

\[ \rho_{xy} = r = Correlation \ coefficient = \frac{\text{cov}(XY)}{\sigma_X \sigma_Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{(E(X)^2) - (E(X))^2)(E(Y)^2) - (E(Y))^2)}} \]  

(2)

\( E(X) \) denotes the expected value of the independent variable X, while \( E(Y) \) and \( E(XY) \) represents the expected value of the dependent variable Y and XY. \( \sigma_X \), \( \sigma_Y \) denotes the standard deviation of the variables X and Y respectively [28].
2.3 Hurst exponent (H)

It is a statistical measure used in time series for predictability; it determines the relative direction for a time series. The value of H varies from 0 to 1; depending on this value of H we determine the pattern of trend in the time series. If H = 0.5 then the series is random and also termed as a Brownian time series, in this there is no relation between the past and the future values and the series is hard to predict. If H > 0.5 then it indicates a trend in the reinforcing series and the series exhibits a ‘persistent behavior’ along with positive auto correlation in the time series, while if H < 0.5 then the trend indicates an ‘anti-persistent behavior’ also termed as ‘mean reversion’ along with negative auto correlation values, in this the trend in the series is subsequent such as, a rise followed by a fall and then again by a rise. The closer the value of H to 0 or 1 indicates the greater strength of the trend in the series and thus the model leads to more accurate forecast of the time series [29].

Mathematically it is calculated as,

\[ H = \frac{\log \sigma - \log \sigma_0}{\log \log \sigma} \]  

(3)

Also Hurst exponent can be derived by the power law of decay as,

\[ p(k) = Ck^{-\alpha} \]

Where \( p(k) \) is the auto correlation function at lag \( k \) related to \( \alpha \) by the relation,

\[ H = 1 - \frac{\alpha}{2} \]

2.4 Fractal dimension

Fractal dimension (D) is a statistical measure used to find the appearance of fractals in empty spaces as we further zoom into finer scales. If D > 1.5 then anti persistent behavior is observed while for D < 1.5 persistent behavior exists. For D = 1.5 Brownian motion is observed and thus leading to unpredictability. It is calculated as [29],

\[ D = 2 - H \]  

(4)

2.5 Predictability index

The ability to make accurate and timely forecast is termed as predictability index. It is used to reduce the gap between actual and predicted results thus enabling to determine the future performance of a time series; its value depends on the value of fractal dimension (D) and is calculated as [29],

\[ PI = 2 \left| D - \frac{3}{2} \right| \]

(5)

2.6 Trend percent

The amount of variation in the parameter over an extended period of time is termed as trend percent; it is used to determine the future movement of the time series considering the past observations. Trends can be both upward and downward and trend percent can be obtained by [29],

\[ Trend\% = \frac{N \cdot \alpha \cdot 100}{\bar{x}} \]  

(6)

Where \( \bar{x} \) denotes the average of the time series, \( N \) is the number of data values (monthly), \( \alpha \) represents the slope of the regression line.

3. Results and Discussion

Study sites: Raniganj (23°40’N 87°05’E) in West Bengal, Jharia (23°50’N 86°33’E) and Bokaro (23°46’N 85°55’E) in Jharkhand which are among the major coal mines of India are the sample sites for the current study. A long term time series data is collected and processed by NASA (http://nasa.gov/) for the amount of black carbon concentration of past 38 years from Jan 1980 to May
2018 have been considered at these three sites. The time series for the three study locations namely Raniganj, Jharia and Bokaro are shown in Fig.1 below, the unit for the magnitude of black carbon concentration in the time series expressed is in volume as $e^{-11} \text{kg m}^{-3}$ units.

A comparison of black carbon concentration among the major coal mines of India namely Raniganj, Jharia and Bokaro is done using fractal analysis. As shown in Table 1 below regression equation of lines are obtained for these locations comparatively using eq. (1). $R$ square also termed as coefficient of determination is usually used in trend analysis to measure the amount of variability between two variables or time series. Its value ranges between 0 and 1, closer the value to 1 in all the cases, indicates that the dependent series can be accurately predicted from the independent series with least prediction error and can be used as good tool for predicting future outcomes. The value of Hurst exponent $H$ obtained using eq. (3) is less than 0.5 and that of fractal dimension $D$ obtained using eq. (4) is greater than 1.5 in all the comparisons indicating that the time series display anti-persistent behavior also called as mean reversion. This indicates that the trend pattern in the time series is anti-persistent in which a rise is followed by a fall and then gain by a rise for a period and this behavior repeats itself regularly. The value of predictability index obtained using eq. (5) is close to 1 in all the cases indicating that the prediction model generated is very good and can lead to accurate forecasts.
Table 1. Fractal analysis of black carbon between the major coal mines.

| Site wise comparison | Regression equation \( (R^2) \) | Regression coefficient \( (b_{yx}) \) | Hurst exponent \( H(\text{abs}) \) | Fractal \( (D) \) | Predictability index \( (PI) \) |
|----------------------|----------------------------------|----------------------------------|----------------------------------|-----------------|------------------|
| Raniganj with Jharia | \( y = 0.9742x + 0.096 \) \( R^2 = 0.9206 \) | 0.9742 | 0.0129 | 1.9871 | 0.9742 |
| Jharia with Raniganj | \( y = 0.945x + 0.0833 \) \( R^2 = 0.9206 \) | 0.945 | 0.0275 | 1.9725 | 0.945 |
| Jharia with Bokaro   | \( y = 0.9628x + 0.1203 \) \( R^2 = 0.9086 \) | 0.9628 | 0.0186 | 1.9814 | 0.9628 |
| Bokaro with Jharia   | \( y = 0.8988x + 0.2188 \) \( R^2 = 0.8652 \) | 0.8988 | 0.0506 | 1.9494 | 0.8988 |
| Bokaro with Raniganj | \( y = 0.9443x + 0.0496 \) \( R^2 = 0.9379 \) | 0.9443 | 0.02785 | 1.97215 | 0.9443 |
| Raniganj with Bokaro | \( y = 0.9912x + 0.0667 \) \( R^2 = 0.8899 \) | 0.9912 | 0.0044 | 1.9956 | 0.9912 |

As shown in Table 2. The monthly trend percentage for Raniganj, Jharia and Bokaro are 10.5%, 14.45% and 18.28% respectively. This shows that the trend in the time series for all three locations is positive with sufficient amount of variation compared to the previous time at all the three sample sites and the highest is observed at Bokaro.

Table 2. Trend percent analysis of black carbon between the major coal mines.

| Site | Trend% |
|------|--------|
| Raniganj | 10.5113 |
| Jharia   | 14.45501 |
| Bokaro   | 18.28012 |

4. Conclusion

Mathematical parameters like regression equation of line, coefficient of determination, fractals, Hurst exponent, predictability Index (PI), and trend percent have been used to analyze trends and the behavior of time series of black carbon concentration in the major coal mine regions of India. The results are compared and tested at 95% confidence intervals. It is observed that the values of Hurst exponent are very near to 0 in all the cases thus the time series at all the three locations display anti-persistent behavior due to which the time series have a higher tendency to revert towards its long term mean value due to which a rise is followed by a fall for a period of time and this behavior continues in future. Since the time series does not display Brownian motion therefore different time series models can be applied over these time series to obtain future forecasts. The monthly trend percentage at all the three locations namely, Raniganj, Jharia and Bokaro are 10.5%, 14.45% and 18.28% respectively. Thus the time series exhibit positive trends at all the locations with sufficient amount of variation compared to the previous time and the highest trend value is observed at Bokaro. The results obtained will help us to develop an insight of the behavior of the time series over the coal mines of India by studying the irregular patterns in it with the help of fractals, enabling us to forecast and understand their future behavior.
Acknowledgement

The authors are thankful to CSIR-National Physical Laboratory, New Delhi for providing the data for research. We also grateful to Lovely Professional University Punjab, I.K. Gujral Punjab Technical University Jalandhar for providing facilities for research work. Author would also like to thank NASA website (http://nasa.gov/) for processing of data.

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