Efimov states in Li-Cs mixtures within a minimal model

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We use a minimal zero-range model for describing the bound state spectrum of three-body states consisting of two Cesium and one Lithium atom. Using a broad Feshbach resonance model for the two-body interactions, we show that recent experimental data from the Chin group can be described surprisingly well for particular values of the three-body parameter that governs the short-range behavior of the atomic potentials and is outside the scope of the zero-range model. Studying the spectrum as a function of the three-body parameter suggests that the lowest state seen in experiment could be influenced by finite range corrections. We also consider the question of Fermi degeneracy and corresponding Pauli blocking of the Lithium atoms on the Efimov states.

Introduction. The ability to study few-body states in the low-energy regime with cold atoms has allowed the observation of Efimov three-body states [1, 2] and also resonances associated with bound states of four or more particles [3, 22]. The key property of these low-energy bound states is a discrete scaling symmetry [23, 24] that has been difficult to observe in experiments (a very recent paper presents evidence for the observation of a second Efimov resonance in Cesium [25]). Theory shows that systems with a large mass imbalance between the bound state constituents are highly suited for studying the discrete scaling since the scaling factor becomes small and more states should be observable within experimental constraints.

This search has prompted different groups to pursue cold atomic experiments with mixtures of different atoms. In particular, recent experiments by the Weidemüller group in Heidelberg [26] and the Chin group in Chicago [27] have pursued mixtures of $^6$Li and $^{133}$Cs which has a suitably low scaling factor of around 4.8 (depending on the number of resonant two-body subsystems 23). The first experimental results have now been presented by the Chicago group [28]. They find three consecutive Efimov peaks that are attributed to the Cs-Cs-Li system when working with a Feshbach resonance located at a magnetic field of 842.75 Gauss (G). Since the Cs-Cs scattering length at these fields is negative there are no molecular thresholds to worry about and thus the system is ideal for studying the formation of Efimov trimers from the three-atom continuum. The observations are done by analyzing atom loss peaks associated with the threshold. In this short paper, we consider a minimal model for describing the experimental data that uses zero-range interactions and the simplest model for describing the two-body Feshbach resonances. Our results are in good agreement with the experiments although we do see some differences for the most bound trimer that may be associated with finite range corrections. This system could be useful for studying the effects of a many-body background on the few-body physics 29. We therefore consider the influence of the Fermi degeneracy in the Lithium component on the Efimov states by introducing a Pauli blocking in the light constituent. For Lithium densities in the range $10^9$ to $10^{12}$ cm$^{-3}$ we find virtually no changes in the three-body thresholds for Efimov trimer formation.

Model. We use a zero-range momentum-space method similar to the one introduced by Skorniakov and Ter-Martirosian [30] but suitably regularized by the method of Danilov [31] (see the discussion in 32). The zero-range interactions are parameterized by the two-body scattering lengths and in the general case of non-identical masses and scattering lengths this yields a set of coupled integral equations that can be solved by discretization (see the appendix of Ref. 33 for details on the formalism). A necessity of a zero-range model is a three-body parameter, $\kappa$, that is used in the regularization of the equations. It can be physically interpreted as a short-range repulsive force that is due to the hard-core nature of the inter-atomic potentials at short distance. We will work at zero temperature and without any explicit decay channels in the model. The latter implies that we are making the assumption that there is a one-to-one correspondence between the threshold at which an Efimov state appears out of the three-atom continuum and the loss peak in the measurement. This is a simplified picture yet it is our express goal to explore the capabilities of a minimal model for describing the data.

To model the two-body interactions that are tuned by Feshbach resonances in experiments, we note that the resonances in the Cs-Cs-Li case are all broad. Therefore we use the simplest model where the scattering length has the parameterization 34

$$a(B) = a_0 \left(1 + \frac{\Delta}{B - B_0}\right). \quad (1)$$
Here $a_{bg}$ is the background scattering length away from the resonance, $\Delta$ is the resonance width, and $B_0$ is the position. The Cs-Cs system has a very wide resonance around 787 G \[35\]. The Efimov states occur for fields around 840-850 G and in this regime the Cs-Cs systems has a large negative scattering length. We will therefore keep the parameters of this heavy-heavy subsystem fixed in our model with $B_0 = 787$ G, $\Delta = 87.5$ G, and $a_{bg} = -1940a_0$, where $a_0$ is the Bohr radius. We have checked that changes in $\Delta$ and $a_{bg}$ for the Cs-Cs channel within the ranges that are given in the literature \[34\-37\] have no noticeable effect on the results discussed here. This is connected to the fact that this resonance is very broad. On the other hand, for the Li-Cs subsystems we work with two parameter sets below to reflect the different

\[\text{Results.}\] We first consider the Li-Cs Feshbach resonance data given in Ref. \[27\] (and updated in Ref. \[28\]). The reported best fit value for the position is $B_0 = 842.75$ G, while the background scattering length is $a_{bg} = -22a_0$. We use $\Delta = 62$ G from Ref. \[27\]. This specifies everything in our model except for the three-body parameter, $\kappa$, which we can then vary. In Fig. 1 we show the calculated magnetic field positions as a function of $\kappa$ along with the experimental data. The thick solid lines denote the position at which the three-body Efimov trimer hits the three-atom continuum. Each line terminates at a given value of $\kappa$ as the spectrum is pushed upwards by decreasing $\kappa$ and thus less trimers are present.

The dotted line in Fig. 1 is a guide to the eye that goes through the points at which our minimal model calculation and the experimental data coincide. Ideally this should be a horizontal line since we typically associate $\kappa$ with short-range physics that does not change from state to state. On the other hand, the deeper Efimov states are more susceptible to finite-range corrections \[38\]. This can cause the discrepancy and thus slope of the dotted line. The difference in $\kappa$ between the two states at the smaller magnetic fields (closer to the Feshbach resonance) is seen to be much smaller as expected. The need for variation in $\kappa$ with magnetic field to describe experimental data has been noted also for equal mass Efimov studies in three-component $^6\text{Li}$ systems and interpreted as an energy-dependence of $\kappa$ \[13\]. Finite-range corrections can be one cause of such an effect. In a model where one also includes the effective range, $r_e$, the threshold for the most bound Efimov trimer will move depending on the sign of $r_e$ \[38\]. The trimer line on the far right in Fig. 1 would move to smaller magnetic field values for $r_e > 0$ according to the results of Ref. \[38\]. However, other models have found different behaviour \[39\] and a more precise model such as a multi-channel framework \[40\] seems necessary to explore this issue.

Overall, we would describe the agreement as very good given the simplicity of the model. Moreover, it is interesting to note that our zero temperature model so closely reproduces the data for the two states closest to the Feshbach resonance. Also, the experiment finds very little change with temperature of the Efimov resonance closest to resonance which should be the most sensitive one. A zero temperature formalism seems therefore to work well.

We have also studied a second set of parameters that are taken from Ref. \[26\]. The resonance position is slightly changed to $B_0 = 843.5$ G, while $\Delta = 60.4$ G and $a_{bg} = -28.5a_0$. We do this to test the overall features and their robustness while still keeping close contact with experimentally realistic numbers. However, since the qualitative picture is identical to Fig. 1 we omit a figure. The only change is quantitative as the Efimov features move with the Feshbach resonance position. This implies that the scenario seen in the Chin experiment is quite robust and does not depend much on the specifics of the two-body resonances. We conclude that the Cs-Cs-Li system is extremely well-suited for the study of few-body physics.

Recent data \[14\] and a number of theoretical works \[29\-47\] have suggested a new type of universality in the Efimov three-body problem where the three-body parameter, $\kappa$, is related to the two-body van der Waals length of the inter-atomic potential in a simple manner. In Fig. 4 we see that a value of $\kappa \sim 0.01a_0^{-1}$ is needed to reproduce the experimental data in our model. Translating this momentum scale into length yields $L = \kappa^{-1} = 100a_0$ which is almost exactly equal to the van der Waals length of the Cs-Cs system ($101a_0$). This can be understood from a Born-Oppenheimer point
of view where the light Li atom creates an effective potential for the two heavy Cs-Cs atoms where in the bound states can be calculated. The natural cut-off on the universal part in this picture is the short-range Cs-Cs van der Waals length. Here we have used a momentum space approach so a more precise translation from $\kappa$ to length scale is necessary to put this on a more firm footing.

Conclusions. Using a minimal zero-range three-body bound state calculation we have managed to reproduce experimental data on Efimov states in highly mass imbalanced systems quite well. The difference between the experimental data and the model was largest for the deepest bound Efimov state seen in experiment, yet it was still in fairly good agreement with the calculations. The lowest state is expected to have the largest finite-range corrections in a given spectrum and we speculate that this could be the cause. This is outside the scope of our minimal model. We also estimate the effect of the fermionic nature of the Lithium atoms by introducing a Pauli blocking effect. However, in a large range of experimentally relevant densities we find virtually no influence on the Efimov thresholds.

The current study does not take the decay of Efimov states into account. This can be done in different ways \cite{24, 53, 54}, but requires an input parameter to describe the decay which goes into deep bound two-body states as there are no Feshbach molecules on this side of the Feshbach resonance. Temperature is another effect that should be taken into account, particularly on the width of the Efimov resonances as discussed in Ref. \cite{28}. Furthermore, the potential effects of a condensate in the heavy Cesium component would be interesting to study. While the fermionic nature of the light component did not look important based on the current results, a condensate could still have an interesting effect. For instance, the linear dispersion relation of (interacting) condensed bosons can cause changes in three-body dissociation thresholds as the condensate coherence length changes. This can be shown most clearly using the Born-Oppenheimer approximation for the case where the light particle is condensed \cite{55}. For the current system it is the heavy component that could form a condensate and a different approach is needed.

Finally, we consider the question of whether the system could be suitable for investigating the effects of quantum degeneracy on Efimov physics could be difficult for this system.

FIG. 2: Magnetic field positions of the Efimov trimers for $\kappa = 0.01a_0^{-1}$ as function of the density of fermionic $^6$Li atoms, $n$. The horizontal axis is logarithmic and extends over six decades.

The density in Fig. 2 extends over six decades from $10^6$ cm$^{-3}$ to $10^{12}$ cm$^{-3}$ which is an experimentally relevant range. In spite of this large variation in density we find basically no effect on the thresholds. There is a small upturn on the third Efimov resonance closest to the Feshbach resonance for the largest densities. However, given the uncertainties in the numerical procedure we cannot conclude that this is a physical effect. These data suggest that studying the effect of Fermi degeneracy on Efimov physics could be difficult for this system.

During the calculation of the Efimov trimers we ask whether a finite density of fermionic $^6$Li atoms could be suitable for investigating the effects of quantum degeneracy on the three-body physics. More specifically, we ask whether a finite density of fermionic $^6$Li atoms can influence the loss peaks. We still assume zero temperature as above. The effects of a Fermi sea on bound state complexes have been the subject of a lot of recent work \cite{32, 48, 52}. Here we consider the simplest approximation where the Fermi sea is considered inert, and thus its effect is Pauli blocking of states for the Lithium atom constituent in the trimer. We again make the assumption that the loss peak can be mapped to the three-atom continuum threshold position which will generally shift due to the Fermi sea \cite{33}. In Fig. 2 we show the calculated loss peak positions as a function of the density of $^6$Li atoms. The three-body parameter was taken to be $\kappa = 0.01a_0^{-1}$. We have checked that the results in Fig. 2 are not sensitive to $\kappa$ as long as it is chosen in the range where three states are found as in Ref. \cite{11} ($\kappa > 0.0095a_0^{-1}$). The density in Fig. 2 extends over six decades from $10^6$ cm$^{-3}$ to $10^{12}$ cm$^{-3}$ which is an experimentally relevant range. In spite of this large variation in density we find basically no effect on the thresholds. There is a small upturn on the third Efimov resonance closest to the Feshbach resonance for the largest densities. However, given the uncertainties in the numerical procedure we cannot conclude that this is a physical effect. These data suggest that studying the effect of Fermi degeneracy on Efimov physics could be difficult for this system.

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