Construction of Small Worlds
in the Physical Topology of Wireless Networks

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Abstract. The concept of small worlds is introduced into the physical
topology of wireless networks in this work. A. Helmy provided two con-
struction schemes of small worlds for the wireless networks, link rewiring
and link addition, but he mainly focused on the virtual topology. Based
on the broadcasting nature of the radio transmission, we propose a con-
struction scheme of small worlds for the physical topology of Multiple-
Input Multiple-Output (MIMO) wireless networks. Besides the topology-
related topics, we also evaluate the reduction of the power required by a request.

Key words: Small worlds, wireless networks, multiple-input multiple-
output, physical topology.

1 Introduction

The small world phenomenon was first discussed by S. Milgram et al. [1, 2] (also
known as six degrees of separation [3]). D.J. Watts et al. considered it in some
real world situations, such as the electrical power grids, the epidemic models
of infectious diseases, and the collaboration relations of actors, etc [4, 5], which
were called small world networks. Much more research work has been stimulated
in the literature [6–13].

The regular networks have large clustering coefficient 1 and large characteristic path hop 2, while the random networks with the same size and average node
degree have much smaller clustering coefficient and characteristic path hop. With
introducing some "short-cuts" into the regular networks by rewiring each edge
with probability p, D.J. Watts et al. constructed the small world networks and
observed that the characteristic path hop decreases dramatically as p increases,
but the clustering coefficient decreases slowly.

The multi-hop wireless communication networks own high clustering due to
their broadcasting nature, which leads to large characteristic path hop compared
to the random networks. Ahmed Helmy et al. proposed two construction schemes

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1 fraction of nodes’ neighbors that are also neighbors of each other
2 average hops of the shortest paths between nodes
of small worlds in such networks: link rewiring and link addition [13], and studied the concept in the virtual topology [14, 15].

Two virtual "shot-cut" links may share a physical link between two nodes, which makes them interfering with each other. Due to the broadcasting nature of the wireless networks, there may be many such interferences for the above construction schemes of small worlds in the virtual topology. In order to throw off these shortcomings, and still retain the short characteristic path hop, we apply the small world concept into the physical topology of multi-hop wireless networks in a multiple-input multiple-output (MIMO) manner. With \( k \) pairs of transmitting and receiving antennas at each node, the bandwidth of such MIMO wireless networks is equivalent to the sum of the capacity of \( k \) parallel single-input single-output (SISO) channels [16]. So the radio spectrum is divided into \( k \) channels with equal bandwidth. One of the channels acts as the normal data channel, called the normal channel, while the others are dedicated to the "short-cut" communications, called the short-cut channels. We construct the short-cuts over the short-cut channels, and evaluate some practical objectives in such MIMO wireless networks.

The remainder of this paper is organized as follows. Section 2 gives the problem model and some definitions. A construction scheme of the short-cut channels is presented in Section 3. The performance evaluation with some numerical results are given in Section 4. Section 5 concludes.

2 Preliminary

Since there must be some ACK packets between the two terminals of a request in most wireless protocols, we use an undirected weighted graph \( G = (V, E) \) to represent the MIMO wireless network.

The power that node \( a \) needs to transmit at the radio range \( R_a \) is proportional to \( R_a^\alpha \), where the power constant \( \alpha \) is a parameter ranging between 1 and 4, depending on the communication environment. Without loss of generality, we set the normalizing constant to 1. Let the transmitting ranges of node \( a \) and \( b \) be \( R_a \) and \( R_b \) respectively, and let the distance between \( a \) and \( b \) be \( R(a, b) \). If \( R_a \geq R(a, b) \) and \( R_b \geq R(a, b) \), there exists an edge \((a, b) \in E\), and the power that the edge needs is: \( p_{(a, b)} = \max \{ R_a^\alpha, R_b^\alpha \} \), which is denoted as the weight of the edge.

**Definition 1 (Characteristic Path Hop and Path Length).** In the MIMO wireless network, the number of edges along the shortest path between two nodes \( a \) and \( b \) is called the path hop between \( a \) and \( b \), denoted as \( H(a, b) \), and the sum of the weight of these edges is called the path length between \( a \) and \( b \), denoted as \( L(a, b) \). The characteristic path hop and characteristic path length of the network are defined as the average path hop and path length over all connected pairs of nodes respectively.
The characteristic path hop shows the separation of a network. There is an interesting discovery that the characteristic path hop of most real world networks is relatively small, even when these kinds of networks have many fewer edges than a typical globally coupled network with the same number of nodes. This observation stimulates the consideration of applying the small world concept into the communication networks. The characteristic path length represents the average power that a request needs.

**Definition 2 (Clustering Coefficient).** In the MIMO wireless network, node a ∈ V has k neighbors. The ratio between the number $E_a$ of edges actually existing among node a’s neighbors and the total possible number $k(k - 1)/2$ is called the clustering coefficient $C_a$ of node a. The clustering coefficient $C$ of the network is the average of $C_a$ over all the nodes in V.

In this work, we focus on the Media Access Control (MAC) protocols in the physical topology of MIMO wireless networks, in which no node mobility is considered, and the channel condition remains unchanged.

### 3 Construction of Small worlds

Without loss of generality, we conduct our simulations with 1000 nodes over a 1km × 1km area. We investigate four node distributions, including random, normal, skewed, and grid, and several broadcasting ranges to represent different network layouts.

Ahmed Helmy [13] proposed two short-cut construction schemes, link rewiring and link addition, for the virtual topology of wireless networks. For link rewiring, a node is randomly chosen, and then a link to one of its neighbors is removed and relinked to a random node. For link addition, a pair of nodes are randomly chosen, and connected with a link.

In the physical topology of wireless networks without mobility, the established links may not be removed or rewired, while the addition of a "long-distance" link may introduce a large amount of interference into the wireless networks. So we construct the short-cuts over the short-cut channels in the MIMO wireless network. According to I.E. Telatar’s work [16], we can reasonably anticipate the performance improvement over the widely-deployed wireless networks with SISO topologies, such as the IEEE 802.11 protocols.

Below we show how to construct the short-cuts in the above MIMO wireless networks.

The MIMO wireless network is denoted as an undirected weighted graph $G = (V, E)$. The location of nodes in V is decided according to one of the four node distributions: random, normal, skewed, and grid, as illustrated in Fig. 1. There are $(k + 1)$ communication channels: one normal channel $C_0$ and $k$ short-cut channels $C_1$, $C_2$, · · · , $C_k$ ($k ≥ 0$). For a pair of nodes a and b with radio

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3 This concept is known as *characteristic path length* in most previous papers. In this paper, we use *characteristic path hop* instead, and redefine *path length* to represent the power requirement of a request.
Fig. 1. Four node distributions with 1000 nodes over a $1km \times 1km$ area
range $R_a$ and $R_b$ respectively over the channel $C_i$ ($0 \leq i \leq k$), if $R(a, b) \leq R_a$, we say that node $b$ is covered by node $a$ over the channel $C_i$. If node $a$ and $b$ are covered by each other over the channel $C_i$, there exists an edge $(a, b)$ in the set of edges $E_i$ ($0 \leq i \leq k$). For node $c \in V$, if $c$ is covered by $a$ or $b$, we say that node $c$ is covered by the edge $(a, b)$. So we have: $E = E_0 \cup E_1 \cup \cdots \cup E_k$.

As in most wireless protocols, the radio range over the normal channel $C_0$ is a fixed constant $R_0$. So there exists an edge $(a, b) \in E_0$, if the distance between node $a$ and $b$ satisfies: $R(a, b) \leq R_0$. Such an edge is called a normal edge, of which the weight is $R_0 \times \text{RadiiRatio}$.

**Procedure 1 BuildSCChannel**

**Input:**
The nodes: $Nodes$,
the edges in the established channels: $Edges$,
and the ratio over $R_0$ of the upper bound of the radio range: $\text{RadiiRatio}$.

$SC\_Edges = \text{Nil}$;

do {
  Randomly choose a pair of nodes $a$ and $b$ satisfying:
  1. Edge $(a, b)$ does not exist in $Edges$;
  2. in $SC\_Edges \cup \{(a, b)\}$, there doesn’t exist an edge whose terminals are covered by other edges;
  3. the distance between $a$ and $b$ $R(a, b)$ satisfying: $R_0 < R(a, b) \leq R_0 \times \text{RadiiRatio}$.
   $Edges = Edges \cup \{(a, b)\}$;
   $SC\_Edges = SC\_Edges \cup \{(a, b)\}$;
} while (There exists such an edge);
return $SC\_Edges$;

Different from the previous small world networks, in the MIMO wireless networks, some edges may interfere with each other, and cannot act as transmitting links simultaneously. To avoid such interference, which will greatly increase the complexity of the wireless routing protocol, we construct a short-cut channel with edges whose terminals are not covered by other edges over the current channel, as described in the procedure BuildSCChannel. To increase the minimum lifetime of nodes in $V$, we prevent the construction of parallel links. We also limit the radio range of the short-cuts by the upper bound $R_0 \times \text{RadiiRatio}$.

We construct the MIMO wireless networks with one compound channel, which consists of one normal channel and several short-cut channels. It can be reasonably anticipated that such networks will outperform the widely-deployed wireless networks with SISO topologies, as indicated in the following experiments on some topology-related topics and the power efficiency. Observing from the above construction scheme, in the MIMO wireless network with one compound chan-
nel, the short-cut edges can act as transmitting links simultaneously, and they
don’t interfere with the normal edges, too. Due to the broadcasting nature of the
wireless networks, and the distributed detection of interference, the short-cuts
can be constructed distributedly, which will improve the system performance of
such MIMO wireless networks greatly.

**Table 1.** The clustering coefficient, characteristic path hop, maximum path hop, characteristic path length, and maximum path length for the investigated topologies without short-cut channels ($k = 0$).

| Topology       | Range (m) | Links | $C(0)$  | $H(0)$ | $M(0)$ | $L(0)$ | $m(0)$ |
|----------------|-----------|-------|---------|--------|--------|--------|--------|
| Random Graph [13] | -         | -     | 0.009   | 3.3    | 5      | -      | -      |
| Random-40      | 40        | 2305  | 0.550   | 24.956 | 57     | 998.253| 2280   |
| Random-50      | 50        | 3645  | 0.576   | 15.564 | 39     | 778.208| 1950   |
| Random-60      | 60        | 5265  | 0.589   | 11.907 | 30     | 714.414| 1800   |
| Normal-60      | 60        | 8837  | 0.582   | 7.993  | 26     | 479.604| 1560   |
| Skewed-50      | 50        | 70752 | 0.729   | 7.585  | 45     | 379.235| 2250   |
| Grid-35        | 35        | 1936  | 0.000   | 21.121 | 62     | 739.222| 2170   |
| Grid-60        | 60        | 3811  | 0.451   | 14.783 | 31     | 886.986| 1860   |

**4 Numerical Results and Discussion**

In the above MIMO wireless environment, *Random-40* represents the instance with the random node distribution and the radio range 40m, and *Random-40* with $k$ short-cut channels is denoted as *Random-40 ($k$)} ($k \geq 0$). For instance *Random-40 ($k$),* the clustering coefficient, characteristic path hop, maximum path hop, characteristic path length, and the maximum path length are denoted as $C(k)$, $H(k)$, $M(k)$, $L(k)$, and $m(k)$ ($k \geq 0$) respectively. The power constant $\alpha$ is set to 1, and similar results can be got for the other value of $\alpha$. The topologies investigated in this work and some related information are illustrated in Table 1.

As in Table 1, the clustering coefficient and path hop of the topologies without short-cut channels, which are investigated in this work, are much higher than those of the random graph, except the Grid-35 instance, where $C(0) = 0$. This is due to the broadcasting nature of the wireless networks, which greatly increase the number of a node’s neighbors, which are also neighbors.

In the MIMO wireless network with one compound channel, we firstly limit the distance between the two terminals of a short-cut by an upper bound of $\text{RadiiRatio} \times R_0 = 5 \times R_0$, and evaluate the performance as the number of the
Fig. 2. Performance improvement vs. the number of short-cut channels
Table 2. Edges added in the short-cut channels. (The number of edges over the normal channel is $|E_0|$.)

(a) Random-40 ($|E_0| = 2520$)

| $i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|----|----|----|----|----|----|----|----|----|
| $|E_i|$ | 29 | 32 | 44 | 24 | 44 | 21 | 37 | 35 | 38 |
| $|SC_i|/|E_0|$ (%) | 1.15 | 2.42 | 4.17 | 5.12 | 6.87 | 7.70 | 9.17 | 10.56 | 12.06 |

(b) Random-50 ($|E_0| = 3888$)

| $i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|----|----|----|----|----|----|----|----|----|
| $|E_i|$ | 23 | 17 | 22 | 23 | 21 | 27 | 24 | 26 | 20 |
| $|SC_i|/|E_0|$ (%) | 0.59 | 1.03 | 1.59 | 2.18 | 2.73 | 3.42 | 4.04 | 4.71 | 5.22 |

(c) Random-60 ($|E_0| = 5505$)

| $i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|----|----|----|----|----|----|----|----|----|
| $|E_i|$ | 15 | 23 | 15 | 15 | 21 | 11 | 23 | 18 | 27 |
| $|SC_i|/|E_0|$ (%) | 0.27 | 0.60 | 0.90 | 1.24 | 1.62 | 1.82 | 2.23 | 2.56 | 3.05 |

short-cut channels $k$ changes \(^4\). Let $E_i$ be the set of edges over the $i^{th}$ short-cut channel, and $SC_i = E_1 \cup \cdots \cup E_i$, where $i \geq 1$.

The number of edges added over each short-cut channel is shown in Table 2. As in Fig. 2, an interesting trend exists for all the instances. With only a few edges added (about 20 – 30 edges for each short-cut channel), the path hop is reduced drastically. The characteristic path hop of the instance Random-60 with only 9 short-cut channels is reduced by 16%, and even better improvement can be got for the instances with smaller radio range, e.g. 25% and 41% for the instances Random-50 and Random-40 respectively. Similar trends exist for the maximum path hop. Besides the improvement of the topology-related topics, the power required by a request is also reduced greatly. As illustrated in Fig. 2, the characteristic path lengths are reduced by 24%, 11%, and 6% for Random-40, Random-50, and Random-60 respectively. The reduction curves of maximum path lengths are similar.

The above observations are different from those of the previous small-world networks, including Ahmed Helmy’s virtual topology model on the wireless networks, in which the path hops can be greatly reduced with only a few edges added, and further adding doesn’t contribute much. This is due to the construction scheme of the short-cuts, which makes that there is no interference between any two short-cuts, or between a short-cut and a normal edge. Table 2 and Fig. 2 also suggest that by introducing only a few short-cut channels into the MIMO

\(^4\) We mainly focus on the instances with random node distribution in this work, since they are more practical. Similar conclusion exists for the other instances.
wireless networks, the path hop and path length may be greatly reduced, and these short-cut channels are especially beneficial for those instances with small radio range, e.g. the wireless sensor networks [17].

Fig. 3. Performance improvement vs. RadiiRatio.

Besides the number of the short-cut channels, we are also interested in how the upper bound of short-cuts will reduce the evaluated topics. We conduct this set of experiments on the three instances, Random-40(9), Random-50(9), and Random-60(9). In the area of 1km × 1km, the distance between two nodes is at most 1000√2m. So we only need to consider \( \text{RadiiRatio} \leq \frac{(1000\sqrt{2})}{R_0} \), which are 36, 29, and 24 for the three instances respectively. Besides the topics considered in the above, we also study how the number of short-cuts added is affected by \( \text{RadiiRatio} \), which is denoted as \(|SC_9|/|E_0|\).

As in Fig. 3, there exists an interesting observation for the three instances: all the curves reach their minimum or near-minimum value at \( \text{RadiiRatio} = 5 \), except the curves of \(|SC_9|/|E_0|\), and there is almost no further contribution when \( \text{RadiiRatio} > 5 \). Let the value of the curves at \( \text{RadiiRatio} = 5 \) be \( SC' \),
Table 3. Value of the curves at $R_{ai}R_{atio} = 5$ vs. the minimum value.

|                        | Random-40 | Random-50 | Random-60 |
|------------------------|-----------|-----------|-----------|
| $SC'_{/ \min \{|SC_9|/E_0\}}$ | 7.79      | 3.98      | 3.90      |
| $C'_{/ \min \{C(9)/C(0)\}}$    | 1.04      | 1.01      | 1.00      |
| $H'_{/ \min \{H(9)/H(0)\}}$    | 1.01      | 1.00      | 1.00      |
| $M'_{/ \min \{M(9)/M(0)\}}$    | 1.00      | 1.03      | 1.04      |
| $L'_{/ \min \{L(9)/L(0)\}}$    | 1.03      | 1.03      | 1.02      |
| $m'_{/ \min \{m(9)/m(0)\}}$    | 1.03      | 1.00      | 1.05      |

$C'$, $H'$, $M'$, $L'$, and $m'$ respectively. Table 3 illustrates how close these values and the minimum ones are. The difference between them are less than 5% for all the topics except $SC'_{/ \min \{|SC_9|/E_0\}}$. The ratio of the number of short-cuts $SC'_{/ \min \{|SC_9|/E_0\}}$ is a bit higher than the other evaluated topics. But according to the above discussion, the short-cuts can be constructed distributedly. So the high ratio of the number of short-cuts will not be a great holdback for the global performance of the MIMO wireless networks.

The above discussion suggests that by limiting the distance between the two terminals of a short-cut with an upper bound $R_{adiiR_{atio}} \times R_0$ for a certain value of $R_{adiiR_{atio}}$, e.g. $R_{adiiR_{atio}} \sim 5$, the clustering, path hop and path length of the MIMO wireless networks can be reduced to the minimum or near-minimum value.

5 Conclusions and Future Work

In this work, the small world concept is introduced into the physical topology of MIMO wireless networks. By implementing several short-cut channels over the radio spectrum, the clustering, path hop and path length of the wireless networks can be greatly reduced, especially for those networks with small radio range. With the limited number of short-cut channels and the limited upper bound of the distance between the two terminals of a short-cut, the small world structure can be easily constructed in a distributed manner. Incorporating the small world structure into the practical wireless protocols will be our future work.

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