Sparticle And Higgs Masses Within Minimal String Unification

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\textbf{ABSTRACT}

We consider the sparticle and higgs spectroscopy in a class of superstring inspired models in which the string threshold corrections ensure the consistency of the string unification scale with the low energy data. The lightest neutralino is almost a pure bino and it is predicted to be the lightest sparticle (LSP). Requiring that $\Omega_{LSP} \leq 0.9$, we find an upper bound on its mass which, in the case of dilaton supersymmetry breaking, turns out to be 160 GeV. The LEP 1.5 experimental lower bound on the chargino mass, $m_{\chi^\pm} > 65$ GeV, implies that the lower bound on the LSP mass is $m_{LSP} > 32(45)$ GeV, corresponding to $\mu < (>0).$ We also determine the lower and upper bounds on the higgs and other sparticle masses. For instance, the lightest higgs lies between 65 and 115 GeV, while the mass of the lightest charged sparticle satisfies $47 \text{ GeV} < m_{\tilde{e}_R} < 325$ GeV. With only the top Yukawa coupling of order unity we find that $1.5 \leq \tan \beta \leq 3.5.$
By incorporating gravity from the outset, superstring theories appear to provide a ‘more complete’ unification of the fundamental forces than standard supersymmetric grand unification. It is natural to suspect that the string unification scale $M_c$ is comparable to the (reduced) Planck Mass $M \sim 2.4 \times 10^{18}$ GeV. More concrete calculations suggest that $M_c$ is about an order of magnitude below $M$. This still poses difficulties for the theory since it would seem to be inconsistent with the scale $M_{GUT} \sim 2 \times 10^{16}$ GeV deduced from considerations of the low energy data.

A number of ideas have been put forward to tackle this problem, including the existence of a grand unified group, the presence of new particles at an intermediate scale or, more recently, the possibility that $M_c$ can indeed be lowered to coincide with $M_{GUT}$. One approach for resolving this discrepancy has to do with the potential string threshold corrections. This can be realized in certain orbifold compactification schemes in which the consistency of the string unification scale $M_c$ with the low energy data is achieved by an appropriate choice of the modular weights of the matter fields \[1\]. Following these authors, we refer to them as "minimal string unification". Moreover, in a special class of such models \[2\] on which we will focus here, the supersymmetry breaking soft terms are characterized in terms of the gravitino mass $m_3/2$, the modular weights $n_i$ and a mixing angle $\theta$ defined by $\tan \theta = \langle F^S \rangle / \langle F^T \rangle$. Here $\langle F^S \rangle$ and $\langle F^T \rangle$ denote the magnitude of the vacuum expectation values (vevs) of the F-terms of the dilaton and modulus supermultiplets $S$ and $T$. For simplicity we will assume the presence of a single dominant modulus $T$.

An attractive feature of this approach, which was recently emphasized, is that it leads to a number of ‘low energy’ predictions which can be tested at LEPII, Tevatron and, of course, the LHC \[3\]. In particular, within the framework of radiative electroweak breaking it was shown that in the small $\tan \beta$ region, the lightest charged sparticles include a chargino and the (right) sleptons, and that interesting correlations occur between their masses and with the mass of the lightest higgs \[3\]. These should provide important tests of the particular superstring inspired scheme.

In this paper we wish to pursue this investigation further by focussing in particular on
the composition and mass of the LSP. Remarkably, the lightest neutralino turns out to be the LSP (without imposing additional requirements), and it is very nearly a pure bino for much of the parameter range. It annihilates predominantly into lepton-antilepton pairs via the ‘right’ slepton exchange. These considerations, and in particular the requirement that $\Omega_{LSP} \leq 0.9$ (with $\Omega_{total} = 1$ from inflation) lead us to a fairly stringent upper bound on the LSP mass which depends on the “mixing angle” $\theta$. For $\theta = \pi/2$ i.e.in the pure dilaton case, for instance, the upper bound is 160 GeV, with the maximum value of 300 GeV occurring for $\theta$ close to 0.98 radian. Furthermore, this enables us to provide upper bounds on the sparticle masses, as well as a lower bound (of $\sim 65$ GeV) on the mass of the lightest ($CP$ even) neutral higgs. Note that the lower bound of 65 GeV by LEP 1.5 on the lightest chargino mass \(^1\) implies a lower bound on the LSP mass of 32 GeV which is rather insensitive to the $\theta$ parameter. Our analysis in the context of minimal string unification agrees with some previous results obtained by De Carlos and Kraniotis in effective supergravities.

In ref \[1\], following minimal string unification, we studied the phenomenological implications of effective supergravity theories derived from string vacua with N=1 supersymmetry spontaneously broken by the dilaton and moduli $F$-terms \[2\]. The mass spectrum in this approach is determined in terms of two independent parameters, the dilaton-modulus mixing angle $\theta$ and the gravitino mass $m_{3/2}$. The various soft supersymmetry breaking terms at the compactification scale are characterized as follows. The scalar masses are

$$m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta),$$

where $n_i$ are the various modular weights given in ref \[2\] to obtain minimal string unification:

$$n_{Q_L} = n_{D_R} = -1, \quad n_{u_R} = -2, \quad n_{L_L} = n_{E_R} = -3, \quad n_{H_1} = -2, \quad n_{H_2} = -3$$

The above modular weights are taken to be family-independent. The asymptotic gaugino masses are

$$M_1 = \sqrt{3} m_{3/2} (\sin \theta - 0.02 \cos \theta)$$

\(^1\)This bound is increasing with the higher available energy at LEPII. Presently it has reached approximately 84 GeV.
\[ M_2 = \sqrt{3}m_{3/2} \left( \sin \theta + 0.06 \cos \theta \right) \tag{3} \]
\[ M_3 = \sqrt{3}m_{3/2} \left( \sin \theta + 0.12 \cos \theta \right) \tag{4} \]

In the soft sector of the trilinear scalar couplings we focus only on the A-term which is related to the top quark Yukawa coupling \( A_t \). It is given by:

\[ A_t = -m_{3/2}(\sqrt{3} \sin \theta - 3 \cos \theta) \tag{5} \]

The bilinear soft breaking term \( B \mu H_1 H_2 \) (where \( H_1 \) and \( H_2 \) denote the scalar doublets) depends on the origin of the \( \mu \)-term in the superpotential. If \( \mu \) arises solely from the \( S \) and \( T \) sector then, as pointed out in ref [2], \( B \) takes the form:

\[ B = m_{3/2}(-1 - \sqrt{3} \sin \theta + 2 \cos \theta) \tag{6} \]

Given the boundary conditions in equations (1) to (3) at the compactification scale, we determine the evolution of the ‘couplings’ according to their one loop renormalization group equations (RGE) in order to estimate the mass spectrum of the supersymmetric particles at the weak scale. The radiative electroweak symmetry breaking scenario imposes the following well known conditions among the renormalized quantities:

\[ m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 > 2B\mu, \tag{7} \]
\[ (m_{H_1}^2 + \mu^2)(m_{H_2}^2 + \mu^2) < (B\mu)^2 \tag{8} \]

and

\[ \mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \tag{9} \]
\[ \sin 2\beta = \frac{-2B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2}, \tag{10} \]

where \( \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle \) is the ratio of the two higgs vevs that give masses to the up and down type quarks and \( m_{H_1}, m_{H_2} \) are the two higgs masses at the electroweak scale.

Using equations (11)-(14) we find that \( \mu \) and \( \tan \beta \) are specified in terms of the goldstino angle \( \theta \) and the gravitino mass \( m_{3/2} \). As is the case in MSSM, in our scheme too the results are in general sensitive to the sign of \( \mu \). We find that the choice of negative \( \mu \) allows for a lighter sparticle spectrum, in particular the chargino and the right selectron.

In view of our special interest in potential discoveries at LEP II, we adopt this choice for
the sign of $\mu$ in the present analysis. Also, for $\tan \beta$ we prefer to work in the low $\tan \beta$ regime, \textit{i.e.} in the RGE evolution we consider that all Yukawa couplings, except $h_t$, are much smaller than unity. Interestingly enough, it turns out that only a rather narrow interval is allowed, namely $1.5 \leq \tan \beta \leq 3.5$. As explained in \cite{3}, an important constraint on $\theta$ arises from the requirement of colour and especially electric charge conservations, namely $0.98 \, \text{rad.} < \theta < 2 \, \text{rad}$ (the upper bound is not sensitive to the sign of $\mu$).

The neutralinos $\chi_0^i$ ($i=1,2,3,4$) are the physical (mass) superpositions of the Higgsinos $\tilde{H}_1^0$, $\tilde{H}_2^0$, and the two neutral gauginos $\tilde{B}^0$ (bino) and $\tilde{W}_3^0$ (wino). The neutralino mass matrix is given by \cite{4}

$$
M_N = \begin{pmatrix}
M_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\
0 & M_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\
-M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & 0 & \mu \\
M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & \mu & 0
\end{pmatrix}
$$

(11)

where $M_1$ and $M_2$ now refer to ‘low energy’ quantities whose asymptotic values are given in equations (2) and (3). The lightest eigenstates $\tilde{\chi}_1^0$ is a linear combination of the original fields:

$$
\tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}^3 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0,
$$

(12)

where the unitary matrix $N_{ij}$ relates the $\tilde{\chi}_i^0$ fields to the original ones. The entries of this matrix depend on $\tan \beta$, $M_2$ and $\mu$ which, as previously mentioned, are determined in terms of $\theta$ and $m_{3/2}$. The dependence of the $\tilde{\chi}_1^0$ (LSP) mass on $m_{3/2}$ is shown in figure 1. A useful parameter for describing the neutralino composition is the gaugino “purity” function \cite{5}

$$
f_g = |N_{11}|^2 + |N_{12}|^2
$$

(13)

We plot this function versus $m_{3/2}$ in figure (2) which clearly shows that the LSP is essentially a pure bino.

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\footnote{We impose the conservative constraint of the absence of electric charge or colour breaking local minima. Hence we do not take into account the possibility of existence of other nearby minima which conserve electric charge and colour and are reachable by tunnelling in a cosmologically short enough time.}
Given the LSP mass as a function of $m_{3/2}$ and $\theta$ and that it is bino like, we find that the annihilation is predominantly into leptons, with the other channels either closed or suppressed. The annihilation process is dominated by the exchange of the three slepton families ($\tilde{e}_R, \tilde{e}_L$, etc). The squark exchanges are suppressed due to their large mass, while the Z-boson contribution is suppressed, except for $m_\chi \sim m_Z/2$, due to the small $Z\chi\chi$ coupling $(ig/2 \cos \theta_W)(N_{13}^2 - N_{14}^2)\gamma^\mu \gamma_5$.

For the computation of the lightest neutralino relic abundance we follow the standard procedure \[6\]. First we need to determine the thermally averaged cross section $\langle \sigma_A v \rangle \sim a + bv \[7\]$. Since the lepton masses are small compared to $m_{LSP}$, we find that $a \sim 0$, while $b$ is given by:

$$b = \sum \frac{4}{\pi} G_F^2 m_\chi^2 y'^2 (u'^2 + v'^2) \left( \frac{2}{3} + r_1 \right) + \frac{1}{4} (N_{13}^2 - N_{14}^2)^2 x'^2 (c_L^2 + c_R^2)$$

$$+ \frac{2}{3} (N_{13}^2 - N_{14}^2) x'^4 (u'^2 + v'^2) \left( (u'_c R - u'_c L) - r (u'_c L + v'_c R) \right)$$

(14)

where $G_F$ is the Fermi constant, $r_1 = \frac{\sqrt{3}}{4} (-4 + 4 r)$, with

$$r = \frac{m_\chi^2}{M_i^2 + m_\chi^2},$$

$$y'^2 = \frac{m_W^2}{M_i^2 + m_\chi^2},$$

and

$$x'^2 = \frac{m_Z^2}{(m_Z^2 - s)^2 + \Gamma_Z^2 m_Z^2}^{1/2}$$

is the Z pole factor with $\Gamma_Z$ the Z decay width. Finally,

$$u' = (T_{3L} N_{12} - \tan \theta_W (T_{3L} - e_l) N_{11})^2, \quad v' = (\tan \theta_W e_l N_{11})^2.$$}

Here $T_{3L}$ is the weak isospin, $e_l$ is the lepton charge, $\sin^2 \theta_W = 0.23$, $c_L = T_{3L} - e_l \sin^2 \theta_W$ and $c_R = -e_l \sin^2 \theta_W$.

Given $a$ and $b$ we can determine the freeze-out temperature $T_F$, below which the $\chi\chi$ annihilation rate is smaller than the expansion rate of the universe. Following refs(\[8\], \[9\] and \[10\]) we can iteratively compute the freeze-out temperature from

$$x_F = \ln \frac{0.0764 M_P (a + 6b/x_F) c (2 + c) m_\chi}{\sqrt{g_* x_F}}$$

(15)
Here \( x_F = m_\chi / T_F \), \( M_P = 1.22 \times 10^{19} \) GeV is the Planck mass and \( g_* \) (8 \( \leq \sqrt{g_*} \leq 10 \)) is the effective number of relativistic degrees of freedom at \( T_F \). Also \( c = 1/2 \) as explained in Ref [2].

The relic LSP density is given by

\[
\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_c / h^2} = 2.82 \times 10^8 Y_\infty (m_\chi / \text{GeV}),
\]

where

\[
Y_\infty^{-1} = 0.264 \ g_*/g_{10} \ M_P \ m_\chi \ (a / x_F + 3b / x_F),
\]

\( h \) is the well known Hubble parameter, \( 0.4 \leq h \leq 0.8 \), and \( \rho_c \sim 2 \times 10^{-29} h^2 \) is the critical density of the universe. Motivated by the inflationary scenario we will assume that the total density parameter \( \Omega_{TOT} = 1 \).

Figure 3 shows the relic abundance of the lightest neutralino \( \Omega_\chi h^2 \) as function of the gravitino mass in the case of dilaton supersymmetry breaking, i.e. \( \theta = \pi/2 \). We require the neutralino relic density to be \( 0.1 \leq \Omega_{LSP} \leq 0.9 \), with \( 0.4 \leq h \leq 0.8 \). We find that there is no point in the parameter space \( (m_{3/2}, \theta) \) that leads to \( \Omega_\chi h^2 \) less than the minimum value \( 0.014 \), while the maximum value \( 0.576 \) imposes an upper bound on \( m_{3/2} \) which is very sensitive to \( \theta \), as shown in figure 4. In turn, this leads to (a fairly stringent) upper bound on the LSP mass of about 160 GeV. For \( \theta \simeq 0.98 \text{rad.} \), which represents the maximum moduli contribution to SUSY breaking, i.e. maximal non-universal soft SUSY breaking terms, this bound approaches 300 GeV.

The bounds on the gravitino mass can be translated into bounds on the sparticle masses. We are particularly interested in the particles which possibly can be seen at LEP II, Tevatron or the LHC. These include the lightest higgs scalar \( (h^0) \) as well as the "right" selectron and the lightest chargino. At tree level, the mass of \( h^0 \) is determined by

\[
m_{h^0}^2 = \frac{1}{2} \left( m_A^2 + m_Z^2 - \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2m_A^2 \cos^2 2\beta} \right)
\]

where \( m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \). The leading radiative corrections to \( m_{h^0}^2 \) depend on the fourth power of the top mass as well as on \( m_{t_{1,2}}, A_t, \mu \) and \( \tan \beta \). The expression for the lightest higgs mass which we used in our calculation has the form [10]

\[
m_h^2 = m_{h^0}^2 + (\Delta m_h^2)_{1LL} + (\Delta m_h^2)_{\text{mix}}
\]
where
\[
(\Delta m_h^2)_{1LL} = \frac{3m_t^4}{4\pi^2 v^2} \ln\left(\frac{m_t m_{\tilde{t}_2}}{m^2_{\tilde{t}_1}}\right) \left[ 1 + O\left(\frac{m_W^2}{m_t^2}\right) \right]
\]
(20)
\[
(\Delta m_h^2)_{mix} = \frac{3m_t^4 \tilde{A}^2}{8\pi^2 v^2} \left[ 2h(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \tilde{A}^2 f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] \left[ 1 + O\left(\frac{m_W^2}{m_t^2}\right) \right]
\]
(21)
and \(\tilde{A}_t = A_t + \mu \cot \beta\), where the functions \(h\) and \(f\) are given by
\[
h(a, b) = \frac{1}{a - b} \ln\left(\frac{a}{b}\right) \quad \text{and} \quad f(a, b) = \frac{1}{(a - b)^2} \left[ 2 - \frac{a + b}{a - b} \ln\left(\frac{a}{b}\right) \right]
\]
(22)

This expression provides the upper bound on \(m_h\) for a given stop spectrum which is completely determined in terms of \(m_{3/2}\) and \(\theta\). However including two-loop effects remains necessary to obtain a correct estimate of the higgs mass. It was shown in Ref \[10\] that the two loop leading logarithmic contributions to \(m_h^2\) are incorporated by replacing \(m_t\) in equation (19) by the running top quark mass evaluated at the scale \(\mu_t\) which is given by \(\mu_t = \sqrt{M_t M_s}\) where \(M_t\) is the pole mass of the top quark and \(M_s = \sqrt{M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2}\). Then the lightest higgs mass is given by
\[
m_h^2 = m_{h^0}^2 + (\Delta m_h^2)_{1LL}(m_t(\mu_t)) + (\Delta m_h^2)_{mix}(m_t(M_s))
\]
(23)

Figure 5 shows how the higgs mass varies with \(m_{3/2}\) and \(\theta\), and we see that in the low \(\tan \beta\) regime and with \(\mu > 0\) (which gives maximum mixing in our convention), the mass \(m_h\) satisfies \(80 GeV \leq m_h \leq 115 GeV\). The lower bound reaches 65 Gev in the case of \(\mu < 0\) and \(\theta = 2\) rad.

We can similarly determine both the lower and upper bounds on all the supersymmetric particles since they are given in terms of the goldstino angle \(\theta\) and the gravitino mass \(m_{3/2}\). For instance, we find that the mass of the pseudoscalar A lies between 200 GeV and 2 TeV, while the gluino mass is between 400 GeV and 2.5 TeV.

In conclusion, we have discussed how the composition and cosmic abundance of the LSP in the so-called minimal superstring unification leads to important constraints on the underlying supersymmetry breaking parameters. In addition, one finds lower and upper bounds on the higgs and sparticle mass spectrum. For instance, the “Weinberg-Salam”

\[\text{We use the sign convention of } \mu \text{ opposite to that adopted in the Haber and Kane report} \]
higgs is estimated to lie in the mass range of 85-145 GeV. The squarks and gluinos turn out to be heavy but, depending on the parameters, a charged slepton or a chargino could still be found at LEPII.

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Figure Captions

Fig. 1 The lightest neutralino mass as function of the gravitino mass ($m_{3/2}$).

Fig. 2 The gaugino ‘purity’ function versus the gravitino mass.

Fig. 3 The neutralino relic abundance as a function of the gravitino mass in the pure dilaton case of supersymmetry breaking. The long- and short-dashed lines correspond to $\Omega_{LSP} = 0.1, h = 0.4$ and $\Omega_{LSP} = 0.4, h = 0.8$, respectively.

Fig. 4 The allowed region of the parameter space ($m_{3/2}, \theta$) corresponding to $0.1 \leq \Omega_{LSP} \leq 0.9$ and $m_{\chi^\pm} \geq 65 \text{GeV} (\mu < 0)$.

Fig. 5 The lightest (‘Weinberg-Salam’) higgs mass as function of the gravitino mass.
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The Allowed Range
