The fermion dynamical symmetry model for the even–even and even–odd nuclei in the Xe–Ba region

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Abstract

The even–even and even–odd nuclei $^{126}$Xe–$^{132}$Xe and $^{131}$Ba–$^{137}$Ba are shown to have a well-realized $SO_8 \supset SO_6 \supset SO_3$ fermion dynamical symmetry. Their low-lying energy levels can be described by a unified analytical expression with two (three) adjustable parameters for even–odd (even–even) nuclei that is derived from the fermion dynamical symmetry model. Analytical expressions are given for wavefunctions and for $E2$ transition rates that agree well with data. The distinction between the FDSM and IBM $SO_6$ limits is discussed. The experimentally observed suppression of the the energy levels with increasing $SO_5$ quantum number $\tau$ can be explained as a perturbation of the pairing interaction on the $SO_6$ symmetry, which leads to an $SO_5$ Pairing effect for $SO_6$ nuclei.

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I. INTRODUCTION

In the past few decades, extremely rich experimental data have accumulated for low-lying nuclear spectroscopy. The observed levels are interwoven in a rich and complicated manner, and understanding them is a challenging problem. The low-energy spectroscopy of even–even, medium-heavy and heavy nuclei can now be explained rather well by the interacting boson model (IBM) \[1\]. The first attempt to describe on the same footing the spectroscopy of both even–even and even–odd nuclei in an analytical way is due to Iachello \[2\], through the concept of supersymmetry.

In this paper, we shall use the terminology NSUSY to denote nuclear dynamical supersymmetry. NSUSY is an outgrowth of the phenomenological IBM that treats fermions and bosons as basic building blocks and identifies the even–even and even–odd collective nuclear states as multiplets of a higher symmetry described by a supergroup $U(6/4)$ or $U(6/20)$ \[2–4\]. The decomposition of $U(6/4)$ ($U(6/20)$) contains $U^B(6) \times U^F(4)$ ($U^B(6) \times U^F(20)$) with $U^B(6)$ referring to the six bosons and $U^F(4)$ ($U^F(20)$) to the odd fermion moving in a single-j level of $j = \frac{3}{2}$ (multi-j levels with $j = \frac{1}{2}, j = \frac{3}{2}, j = \frac{5}{2}$ and $j = \frac{7}{2}$). Later, Jolie et al. \[5\] proposed a new reduction scheme for the supergroup $U(6/20)$ and applied it to the $A = 130$ mass region. They considered the single-particle orbits $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ and $\frac{7}{2}$ resulting from the coupling of a pseudo orbital $l = 2$ part and a pseudo spin $s = \frac{3}{2}$. Instead of the group chain $U(6/20) \supset U^B(6) \times U^F(4)$ \[4\], they suggested use of the group chain

$$
U(6/20) \supset U^B(6) \times U^F(20) \supset O^B(6) \times U^F(4) \times U^F(5) \supset Spin(6) \times U^F(5) \\
\supset Spin(5) \times O^F(5) \supset Spin(5) \supset Spin(3).
$$

NSUSY has had some successes (generally up to 15–30% accuracy for spectroscopic fitting for a few nuclei). But the basic building blocks of nuclei are fermions, and the $s$ and $d$ bosons in the IBM are supposedly simulations of coherent nucleon pairs with angular momenta 0 and 2. Therefore, it is a simplification of the real situation to introduce $U^B(6)$ and $U^F(4)$ (or $U^F(5)$, and $U^F(20)$) as a direct product $U^B(6) \times U^F(4)$ (or $U^B(6) \times U^F(20)$, or $U^B(6) \times U^F(4) \times U^F(5)$).

The Fermion Dynamical Symmetry Model (FDSM) \[6,7\] is defined by a fermionic Lie algebra. It has symmetry limits analogous to all the IBM limits and takes Pauli principle
into account \[8,9\]. Furthermore, the states for even and odd systems in the FDSM belong to vector and spinor representations, respectively, of $SO_8$ or $Sp_6$; thus, one can describe even–even and even–odd nuclei in the FDSM without additional degrees of freedom.

Two general regions are though to exhibit some level of supersymmetry in the properties of low-lying nuclear states: the Pt region \[4,10\] and the $A = 130$ (Xe–Ba) region \[11\]. In the Pt region, the normal-parity valence protons and neutrons are in the 6th and 7th shells respectively, and according to the FDSM \[7\] they have $SO_8^\pi \times Sp_6^\nu$ symmetry, which does not permit an analytical solution for the proton–neutron coupled system. However it could have effective $SO_6$-like symmetry as we have shown in \[12\]. On the other hand, nuclei in the Xe–Ba region have both their neutrons and their protons in the 6th shell with FDSM pseudo-orbital angular momentum $k = 2$ and pseudo-spin $i = \frac{3}{2}$ for the normal-parity levels. Thus they are expected to possess $SO_8^\pi \times SO_8^\nu$ symmetry, which contains coupled $SO_8$ symmetry (the FDSM analog of IBM-1), and have analytic solutions for the $SO_5 \times SU_2, SO_6$ and $SO_7$ dynamical symmetries. In fact, there is now empirical evidence \[11\] that nuclei in the $A = 130$ region are better portrayed by an $SO_6$ limit than the Pt region. For these reasons, we have chosen the Xe–Ba region to discuss the possibility of a simplified and unified description for even–even and even–odd nuclei by the FDSM as an alternative to the IBM and to NSUSY.

The organization of the paper is as follows: energy formulas for both even–even and even–odd nuclei and a comparison with data are given in Sec. II, the respective wavefunctions are constructed in Sec. III, the electromagnetic transitions are discussed in Sec. IV, and conclusions are presented in Sec. V.

**II. THE ENERGY SPECTRA**

In the simplest version of the FDSM, the numbers of nucleons in the normal and abnormal orbits are fixed for a given nucleus and therefore the quasi-spin group $SU_2$ for the abnormal levels plays no explicit dynamical role for low-lying states (it enters implicitly through the conservation of particle number and through effective interaction parameters). The wavefunctions for both even and odd nuclei are given by the following group chain

\[
( SO_8^i \supset SO_6^i \supset SO_5^i ) \times SO_5^k \supset SO_5^{i+k} \supset SO_5^{i+k+i}
\]  

(2)
where $[l_1 l_2 l_3 l_4]$, $[\sigma_1 \sigma_2 \sigma_3]$ and $[\tau_1 \tau_2]$ are the Cartan–Weyl labels for the groups $SO_8$, $SO_6$ and $SO_5$, respectively, $\tau = 0(1)$ for even (odd) nuclei, and $k$ and $i$ indicate pseudo-orbital and pseudo-spin parts of the groups respectively. We note the resemblance between Eq. (2) and the NSUSY group chain Eq. (1). The FDSM Hamiltonian is

$$H_{FDSM} = \varepsilon_1 n_1 + G_0 S^\dagger S + G_2 D^\dagger \cdot D + \sum_{r=1}^3 B_r P^r(i) \cdot P^r(i) + \sum_{r=1,3} [B_r P^r(k) \cdot P^r(k) + 2 b_r P^r(i) \cdot P^r(k)],$$

where $\varepsilon_1$ is the energy for the normal parity orbits (assumed degenerate) and $n_1$ is the number of nucleons in the normal parity orbits,

$$S^\dagger = A^0^\dagger, \quad D^\dagger = A^2^\dagger, \quad A^r_\mu = \sqrt{\Omega_1/2} \left[ b^\dagger_{ki} b^\dagger_{ki} \right]^{0r}_{0\mu}, \quad r = 0, 2,$$

where $k = 2$, $i = \frac{3}{2}$ and $\Omega_1 \equiv \Omega_{ki} = (2k + 1)(2i + 1)/2$. Similarly,

$$P^r_\mu(i) = \sqrt{\Omega_1/2} \left[ b^\dagger_{ki} b^\dagger_{ki} \right]^{0r}_{0\mu}, \quad r = 0, 1, 2, 3,$$

$$\bar{P}^r_\mu(k) = (-)\left[\frac{7}{2}\right] \sqrt{8/5} P^r_\mu(k), \quad P^r_\mu(k) = \sqrt{\Omega_1/2} \left[ b^\dagger_{ki} b^\dagger_{ki} \right]^{r0}_{\mu0}, \quad r = 0, 1, 2, 3,$$

where $\left[\frac{7}{2}\right]$ is the integer part of $\frac{7}{2}$. The operators $P^r_\mu(i)$ and $\bar{P}^r_\mu(k)$ for $r = 1$ and $3$ form the Lie algebras $SO_5^k$ and $SO_5^i$, respectively. The commutators among the $P^r_\mu(i)$ are given by Eq. (3.12) of ref. [3] for the $i$-active case and those for $\sqrt{\Omega_1/2} [b^\dagger_{ki} b^\dagger_{ki}]^{r0}_{\mu0}$ can be obtained from Eq. (3.12) of ref. [3] for the $k$-active case with

$$\sqrt{3} \left\{ \begin{array}{ccc} r & s & t \\ 1 & 1 & 1 \end{array} \right\} \rightarrow \sqrt{5} \left\{ \begin{array}{ccc} r & s & t \\ 2 & 2 & 2 \end{array} \right\}.$$

In Eq. (3) we have renormalized the multipole operators $\bar{P}^r_\mu(k)$ so that they are isomorphic with $P^r_\mu(i)$. Furthermore, $P^1_\mu(i)$ and $\bar{P}^1_\mu(k)$ are related to the total pseudo-orbital angular momentum and pseudo spin by

$$P^1_\mu(i) = \frac{1}{\sqrt{5}} I_\mu, \quad \bar{P}^1_\mu(k) = \frac{1}{\sqrt{5}} L_\mu.$$
By using the Casimir operators of $SO_8$, $SO_6$ and $SO_5$, the Hamiltonian of Eq. (3) can be rewritten as

$$H_{FDSM} = H_0 + \epsilon_1 n_1 + g_S S^i \cdot S + g_b C_{SO_6} + g_5 C_{SO_5}$$

$$+ g_5^i C_{SO_5}^i + g_5^k C_{SO_5}^k + g_1 I^2 + g_1 L^2 + g_J J^2;$$

where the total angular momentum is $J = I + L$, and

$$H_0 = \frac{1}{4}(n_1)^2 + G_2 [C_{SO_8} - S_0(S_0 - 6)], \quad S_0 = \frac{1}{2}(n_1 - \Omega_1),$$

$$g_S = (G_0 - G_2), \quad g_6 = (B_2 - G_2), \quad g_5 = b_3,$$

$$g_5^i = g_5^i - g_5, \quad g_5^i = B_3 - B_2, \quad g_5^k = B_3 - b_3,$$

$$g_I = g_I - g_J, \quad g_I^L = \frac{1}{5}(B_1 - B_3),$$

$$g_J = \frac{1}{5}(b_1 + b_3), \quad g_L = \frac{1}{5}(B_1 - B_3) - \frac{1}{5}g_J.$$

The eigenvalue of $C_{SO_8}$ is

$$C_{SO_8} = \sum_{i=1}^{4} l_i (l_i + 8 - 2i).$$

and the condition for the realization of the symmetry is $g_S = 0$, implying that

$$G_0 = G_2.$$

The low-lying states of even–even nuclei belong to the $SO_8$ irrep $[\Omega_1^4 \Omega_1^4 \Omega_1^4 \Omega_1^4 \Omega_1^4 \Omega_1^4 \Omega_1^4 - 1]$, i.e., the irrep with heritage $u = 0$, $u$ being the number of valence nucleons not contained in $S$ and $D$ pairs [7,8]. By letting $G_0 = G_2$ and $B_i = b_i = 0$ for $i = 1, 3$, from Eq. (8, 9, 10) we have

$$H^{even} = E_0^{(e)} + g_b C_{SO_6} + g_5^i C_{SO_5}^i + g_1^L L^2,$$

where

$$E_0^{(e)} = \frac{1}{4}(1 - G_2)(n_1)^2 + \left[\frac{1}{2}G_2(\Omega_1 + 6) + \epsilon_1\right] n_1,$$

For odd nuclei the low-lying states are expected to belong to the $SO_8$ irrep $[\Omega_1^4 \Omega_1^4 \Omega_1^4 \Omega_1^4 \Omega_1^4 - 1]$, corresponding to heritage $u = 1$. The conditions for realizing the symmetry of Eq. (2) are Eq. (11) and $g_I = 0$, which implies that

$$B_1 - B_3 = b_1 + b_3.$$

Under the conditions of Eqs. (11) and (14), and noting that for \( u = 1 \) the pseudo-orbital angular momentum is \( L^2 = 2(2 + 1) = 6 \), we have

\[
H^{\text{odd}} = E^{(o)}_0 + g_6 C_{SO_6} + (g_5' - g_5) C_{SO_5} + g_5 C_{SO_5} + g_J J^2;
\]

\[
E^{(o)}_0 = E^{(e)}_0 + 4g_5^k + 6g_L + \frac{1}{2}G_2(2 - \Omega_1).
\]

Using the eigenvalue formulas for the \( SO_6 \) and \( SO_5 \) Casimir operators, the energies for even and odd systems are

\[
E^{\text{even}} = E^{(e)}_0 + g_6\sigma(\sigma + 4) + g_5'\tau(\tau + 3) + g_J J(J + 1),
\]

\[
E^{\text{odd}} = E^{(o)}_0 + g_6[\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2) + (\sigma_3)^2] + g_J J(J + 1)
\]

\[
+(g_5' - g_5)[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + g_5[\omega_1(\omega_1 + 3) + \omega_2(\omega_2 + 1)].
\]

The reduction rules are as follows [2]:

**even system**: \( SO_8 \supset SO_6 \supset SO_5 \supset SO_3 \)

\[
[www] \quad [\sigma 00] \quad [\tau 0] \quad J
\]

\[
w = \frac{N_1}{4}, \quad \sigma = N_1, N_1 - 2, ..., 1 \text{ or } 0,
\]

\[
\tau = \sigma, \sigma - 1, ..., 0, \quad \tau = 3n_\Delta + \lambda,
\]

\[
n_\Delta = 0, 1, 2, ..., \quad J = \lambda, \lambda + 1, ..., 2\lambda - 2, 2\lambda,
\]

with \( N_1 = \frac{n_1}{2} \), where \( n_1 \) is the number of the nucleons in the normal parity levels, and

**odd system**: \((SO_8^i \supset SO_6^i \supset SO_5^i \times SO_5^k \supset SO_5^{k+i} \supset SO_3^{k+i})\)

\[
[www, w - 1] \quad [\sigma + \frac{1}{2}, \frac{1}{2}] \quad [\tau + \frac{1}{2}, \frac{1}{2}] \quad [10] \quad [\omega_1 \omega_2] \quad J
\]

\[
\sigma = N_1, N_1 - 1, ..., 1, 0; \quad \tau = \sigma, \sigma - 1, ..., 0,
\]

where \( N_1 = \left\lfloor \frac{n_1}{2} \right\rfloor \). The relevant Clebsch–Gordan series for \( SO_5 \) are [3]
\[ [\tau_1^{1/2}] \times [10] = [\tau_1 + 1, 1/2] + [\tau_1^{3/2}] + [\tau_1^{1/2}] + [\tau_1 - 1, 1/2], \]
\[ [\frac{1}{2}^{1/2}] \times [10] = [\frac{3}{2}^{1/2}] + [\frac{1}{2}^{1/2}]. \]  
(22)

and the \( SO_3 \) content of the \( SO_5 \) irreps \([\omega_1 \omega_2]\) is \([2]\)

\[ J = [2(\omega_1 - \omega_2) - 6\nu_\Delta + \frac{3}{2}], [2(\omega_1 - \omega_2) - 6\nu_\Delta + \frac{1}{2}], ..., \]
\[ [(\omega_1 - \omega_2) - 3\nu_\Delta - \frac{1}{4}[1 - (-)^{2\nu_\Delta} + \frac{3}{2}], \]  
(23)

\[ \nu_\Delta = 0, \frac{1}{2}, 1, \frac{3}{2}, .... \]  
(24)

The above discussions adopt as a simplification that the numbers of valence nucleon pairs in the normal and abnormal parity levels, \( N_1 \) and \( N_0 \), are fixed. In reality \( N_1 \) or \( N_0 \) have a distribution and the semi-empirical formula of ref. [7] has been used to obtain \( N_1 \), which may generally take non-integer values to simulate an average behavior of the nuclear states with different values of \( N_1 \). For computing the spectra and the \( B(E2) \) values, we have taken the nearest integer to the non-integer number.

The low-lying energy spectra for \( ^{120-132}Xe \) isotopes predicted by Eq. (17) are compared with data in Fig. 1 and Fig. 2, and the parameters used in the calculations are given in Table I. The experimental spectra indicate that the \( SO_3 \) parameter \( g'_I \) is not sensitive to the neutron number in fitting the spectra of a chain of isotopes (including both even–even and even–odd nuclei). Therefore, in fitting the even–even nuclear spectra we fix the parameter \( g'_I \) to be 11.9 keV, and the adjustable parameters were taken to be \( g_6 \) and \( g'_5 \), which will be used for the neighbouring even-odd isotopes as well.

From Table I, we find that \(-g_6\) and \(g'_5\) are nearly equal. It is interesting to note that if the quadrupole–quadrupole interaction is dominant over the pairing, \(|B_2| >> |G_0|\) (\(= |G_2| \) in the symmetry limit), and \(|B_2| >> |B_3|\) (cf. Eq. (5.3c) in ref. [14]), from Eq. (3) we obtain the relation

\[ g'_5 \approx -g_6, \]  
(25)

which is precisely the empirical relation \( \frac{4}{3} \approx B \) for the parameters in the IBM \( SO_6 \) limit [11]. The parameter \( g_6 \) can be determined through the \( 0^+_3 \) \((i.e., \sigma = N_1 - 2)\) level. The
good agreement of $2^+_1 (\sigma = N_1 - 2, \tau = 1)$ with the experimental results indicates that the parameters $g_0$ in Table I are reasonable.

Comparing Eq. (17) with Eqs. (6.2) and (6.3) of ref. [3], or Eqs. (2.9) of ref. [5], we see that the spectral formulas in the $U(6/4)$ or $U(6/20)$ NSUSY and in the FDSM are essentially identical (except for the replacement of $N$ by $N_1$ ) for even nuclei, while for the odd nuclei they are similar in appearance but differ in two ways: (1) the parameters of the $SO(3)$ group for the even and odd systems are different here, but the same in $U(6/4)$ or $U(6/20)$ NSUSY. This difference comes from the Coriolis-like term $\mathbf{I} \cdot \mathbf{L}$ in our case. (2) There are one $SO_6$ Casimir operator and two $SO_5$ Casimir operators for the FDSM, in contrast to two $SO_6$ Casimir operators and one $SO_5$ Casimir operator in the $U(6/4)$ NSUSY. These differences have a significant effect on the spectrum. In the $U(6/4)$ NSUSY, the five lowest-energy irreps of $SO_5$ are $[1 \frac{1}{2} \frac{1}{2} \frac{1}{2}], [3 \frac{1}{2} \frac{1}{2} \frac{1}{2}], [\frac{5}{2} \frac{1}{2} \frac{1}{2}]$ and $[\frac{9}{2} \frac{1}{2} \frac{1}{2}]$; therefore the states $\frac{3}{2}, 5\frac{1}{2}, \frac{7}{2}, \frac{9}{2}$ and $\frac{11}{2}$ belonging to the irrep $[\frac{5}{2} \frac{1}{2} \frac{1}{2}]$ of $SO_5$ lie quite low in energy, while in the FDSM the lowest six $SO_5$ irreps are $[1 \frac{1}{2}]^2, [3 \frac{1}{2}]^2, [\frac{5}{2}]^2$ and $[\frac{9}{2}]^2$. Consequently, for the FDSM in the low-energy region there are more low-spin states (there are two $\frac{1}{2}$’s and four $\frac{3}{2}$’s, while the $\frac{7}{2}$, $\frac{9}{2}$, and $\frac{11}{2}$ states are pushed up).

Using Table I and Eq. (18), the spectra of the neighboring even–odd Xe isotopes can be calculated. For the odd-mass Xe and Ba isotopes, we take $g_J$ to be 35.3 keV except for $^{127}$Xe. The difference between $g_J$ and $g'_J$ comes from the coupling of $SO_3^i$ and $SO_3^k$, and physically is related to the Coriolis force. We present the calculated and experimental results for $^{127-133}$Xe in Figs. 3–4, and for $^{131-135}$Ba in Fig. 5, with the parameters given in Table II. Apart from the constant term, the formula for $E_{even}$ contains three parameters, and that for $E_{odd}$ contains two parameters beyond the three parameters that are determined by fitting the spectra of neighboring even–even nuclei. With two extra parameters ($g_J$ is kept constant in the region discussed, except for $^{127}$Xe), Eq. (18) reproduces the spectral patterns for the nuclei $^{127-133}$Xe and $^{131-135}$Ba with about 15 levels each.

In the $U(6/4)$ NSUSY, the ground-state representation is $[\frac{11}{2}\frac{1}{2}]$, and thus the ground-state spin is always $\frac{3}{2}$. However, experimentally the odd nuclei in this region have both $\frac{1}{2}$ and $\frac{3}{2}$ as the ground-state spin. The second $SO_5$ Casimir operator in Eq. (18) provides this possibility. For alternative signs of the parameter $g_5$, the ground state spin can take the values $\frac{1}{2}$ or $\frac{3}{2}$. What is more, by allowing $g_5$ to change smoothly from positive to negative,
we can reproduce the systematic shift of the ground band of the Xe and Ba isotopes, as
shown in Fig. 6 and Fig. 7. From Figs. 3–5, we see that just as for $U(6/20)$ NSUSY \[3\], for
$^{129}$Xe, $^{131}$Ba and $^{133}$Ba, the FDSM predicts a natural occurrence of $\frac{1}{2}^+$ as the ground state
and four low-energy $\frac{3}{2}^+$ states as the excited states. We note that the major aim of this
present FDSM description is to give a simple and unified description of spectral pattern of
even and even–odd nuclei. For more quantitative agreement, additional physics should be
taken into account; for instance, the mixing of particles in normal and unique parity levels
and the single-particle energy contribution.

Due to scarcity of data, we are constrained to discuss only the low-lying levels ($\sigma_1 =
N_1 + \frac{1}{2}$). We could in principle compare them with levels belonging to $\sigma_1 = N_1 - \frac{3}{2}$, If more
data for higher levels were available, This would allow us to determine the parameter $g_6$ for
even–odd nuclei. Clearly, a comparison of the $g_6$ values obtained from fitting even and odd
nuclear spectra is a meaningful test for the validity of the $SO_6$ symmetry. Here we have only
considered fitting the levels with $\tau = 1, 2, 3$. If the levels with $\tau > 3$ were taken into account,
the $g_6'$ value would have to be smaller in order to fit the high-lying states; as a price, the
unified good fit of the low-$[\tau_1 \tau_2]$ states for the even–even nuclei and even–odd nuclei would
be spoiled.

Comparison of pure $SO_6$ spectra with the data for $^{120-132}$Xe (see Figs. 1–2) gives rea-
sonable agreement for $\tau \leq 3$ states for $^{120-126}$Xe. However, the experimental energies for
the high $\tau$ values in the nuclei $^{128-132}$Xe are much lower than predicted, with the discrepan-
cies increasing for the higher $\tau$ values. This is strongly reminiscent of the stretching effect
in nuclear rotational spectra. However, a more careful comparison shows that the energy
levels within the same $SO_5$ irrep (same $\tau$) follow the $J(J + 1)$ rule rather well. Therefore
the aforementioned discrepancy cannot be due to the usual stretching effect in which the
deforation should increase with angular momentum. In ref. \[15\], it was pointed out that
it is in fact an $SO_5$ $\tau$-compression effect. The driving force for this effect is the reduction
of pairing correlation with increasing $\tau$. Allowing for $g_S = G_0 - G_2 \neq 0$, thereby deviating
from the $SO_6$ limit, and treating the $g_S S\dagger \cdot S$ term as a perturbation, leads to the following
energy formula,

\[
E^{even} \cong E_0^{(e)} + g_6 \sigma (\sigma + 4) + A' \tau (\tau + 3) - B' [\tau (\tau + 3)]^2 + g_1' J(J + 1).
\] (26)
Fig. 8 shows the spectrum for $^{126}$Xe predicted by Eqs. (17) and (26), respectively. Inclusion of the $SO_5$ stretching effect improves the agreement significantly. More examples can be found in ref. [13].

For even–odd nuclei, the levels calculated in this investigation are limited to those for $[\tau_1 \tau_1] = [\frac{1}{2} \frac{1}{2}],[\frac{3}{2} \frac{3}{2}]$ bands, which corresponds to the $\tau = 0, 1$ states of the even–even core. Experimental energy levels for even–even nuclei show that the $\tau$-compression effect is negligible for $\tau \leq 2$ states. Therefore, the effect in the even–odd nuclei is not so conspicuous as in the even–even nuclei.

III. WAVEFUNCTIONS

A. Even–even nuclei

According to Eq. (4.3a) in ref. [14] the FDSM wavefunctions in the $SO_6$ limit for $u = 0$ is

$$|N_1 \sigma \tau n_\Delta IM\rangle = \mathcal{P}_{N_1 \sigma \tau} |N_1 \sigma \tau n_\Delta IM\rangle_{b \rightarrow f}^{IBM},$$

(27)

where $\mathcal{P}_{N_1 \sigma \tau}$ is a Pauli factor,

$$\mathcal{P}_{N_1 \sigma \tau} = \left[\frac{(\Omega_1 - N_1 - \sigma)!!(\Omega_1 - N_1 + \sigma + 4)!!}{\Omega_1!!(\Omega_1 + 4)!!}\right]^\frac{1}{2},$$

(28)

and $|N_1 \sigma \tau n_\Delta IM\rangle_{b \rightarrow f}^{IBM}$ denotes a wave function resulting from replacing the boson operators $s^\dagger$ and $d^\dagger_\mu$ by the fermion operators $S^\dagger$ and $D^\dagger_\mu$, in the $U_6 \supset O_6 \supset O_5 \supset O_3$ IBM wave function $|N_1 \sigma \tau n_\Delta IM\rangle^{IBM}$,

$$|N_1 \sigma \tau n_\Delta IM\rangle_{b \rightarrow f}^{IBM} = \xi_{N_1 \sigma}(I^\dagger)^{N_1-\sigma} f_{\sigma \tau}(S^\dagger, I^\dagger)|\tau \tau n_\Delta IM\rangle_{b \rightarrow f}^{IBM},$$

(29)

where $|\tau \tau n_\Delta IM\rangle^{IBM}$ denotes the IBM $U_6 \supset U_5 \supset SO_5 \supset SO_3$ wave function and

$$\xi_{N_1 \sigma} = \left[\frac{(2\sigma + 4)!!}{(N + \sigma + 4)!!(N - \sigma)!!}\right]^\frac{1}{2},$$

(30)

$$I^\dagger = D^\dagger \cdot D^\dagger - S^\dagger \cdot S^\dagger,$$

(31)

where $I^\dagger$ is a generalized pair and $f_{\sigma \tau}(S^\dagger, I^\dagger)$ is a polynomial in $S^\dagger$ and $D^\dagger$ of order $(\sigma - \tau)$,
\[ f_{\sigma\tau}(S^I, I^\dagger) = \sum_{p=0}^{[\sigma-\tau]} D_p(\sigma\tau)(S^I)^{\sigma-\tau-2p}(I^\dagger)^p, \tag{32} \]

\[ D_p(\sigma\tau) = \left[ \frac{2^{\sigma+1}(\sigma-\tau)!(2\tau+3)!!}{(\sigma+1)!(\sigma+\tau+3)!} \right]^{\frac{1}{2}} \frac{(\sigma+1-p)!}{4^p(\sigma-\tau-2p)!} p!. \tag{33} \]

Here the notation for the wavefunctions is the same as in ref. [14].

**B. Even–odd nuclei**

In this section we construct the wavefunctions for states in odd-mass nuclei that corresponds to heritage \( u = 1 \). According to the vector coherent state technique [16], the wavefunction for the \( i \)-active part can be constructed by coupling the “collective wave function” \( |N_1\sigma\tau n_{\Delta}I'M'\rangle_{b\rightarrow f}^{BM} \) and the “intrinsic state”, now the one-fermion state \(|u = 1\rangle\), by means of the SO\(_6 \supset SO_5 \supset SO_3\) CG coefficients,

\[ |2N_1 + 1, [\sigma_1\sigma_2\sigma_3] |\tau_1\tau_2\rangle n_{\Delta}IM, |10\rangle k = 2, m \rangle = \frac{1}{\sqrt{2N_1}} K^{-1}_I(N_1, \langle \sigma + \frac{1}{2} \rangle) \sum_{\tau' \tau'_{n_{\Delta}}} \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} \left[ b_{2m,3/2}^\dagger \left| N_1\sigma\tau n_{\Delta}I' \right\rangle_{b\rightarrow f}^{BM} \right]_M^{I'}, \tag{35} \]

where \( N_1 \) is the total number of \( S, D \) pairs in the “collective wave function”, \( l \) is the number of generalized pairs \( I^\dagger, N_1 = \sigma + 2l \), and \( K^{-1}_I(N_1, [\sigma_1\sigma_2\sigma_3]) \) is the diagonal matrix element of the inverse of the \( K \) matrix [16].

Written out explicitly, Eq. (34) becomes

\[ |2N_1 + 1, [\sigma_1 + \frac{1}{2}, \tau_1] [\tau_1, \frac{1}{2}] n_{\Delta}IM, |10\rangle k = 2, m \rangle = \frac{1}{\sqrt{2N_1}} K^{-1}_I(N_1, \langle \sigma + \frac{1}{2} \rangle) \sum_{\tau' \tau'_{n_{\Delta}}} \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} \left[ b_{2m,3/2}^\dagger \left| N_1\sigma\tau n_{\Delta}I' \right\rangle_{b\rightarrow f}^{BM} \right]_M^{I'}, \tag{35} \]

where the factor \( \frac{1}{\sqrt{2N_1}} \) is due to the different definitions of \( A^I_{LM} \) in refs. [7,10], the shorthand notation stands for

\[ \left\{ \sigma + \frac{1}{2}, \tau_1 \right\} \equiv \left\{ \langle \sigma + \frac{1}{2} \rangle, [\tau_1, \frac{1}{2}] \right\}, \quad \langle \sigma + \frac{1}{2} \rangle \equiv \left[ \sigma + \frac{1}{2}, \frac{11}{2}, \frac{11}{2} \right], \tag{36} \]

and \( \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} \) is the isoscalar factor for the group chain SO\(_6 \supset SO_5 \supset SO_3\) [17], which is a product of the SO\(_6 \supset SO_5\) and SO\(_5 \supset SO_3\) isoscalar factors,

\[ \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} = \begin{pmatrix} \langle \sigma 0 0 \rangle_{\frac{1}{2}}^{\frac{1}{2}} & \langle \sigma + \frac{1}{2} \rangle_{\frac{1}{2}}^{\frac{1}{2}} & \langle \sigma 0 0 \rangle_{\frac{1}{2}}^{\frac{1}{2}} & \langle \sigma + \frac{1}{2} \rangle_{\frac{1}{2}}^{\frac{1}{2}} \\ \langle \tau 0 \rangle_{\frac{1}{2}}^{\frac{1}{2}} & \langle \tau 0 \rangle_{\frac{1}{2}}^{\frac{1}{2}} & \langle \tau 0 \rangle_{\frac{1}{2}}^{\frac{1}{2}} & \langle \tau 0 \rangle_{\frac{1}{2}}^{\frac{1}{2}} \\ [\tau 0]_{\frac{1}{2}} & [\tau 0]_{\frac{1}{2}} & [\tau 0]_{\frac{1}{2}} & [\tau 0]_{\frac{1}{2}} \\ [\tau 0]_{\frac{1}{2}} & [\tau 0]_{\frac{1}{2}} & [\tau 0]_{\frac{1}{2}} & [\tau 0]_{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} \\ \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} \\ \xi^{\sigma\tau\tau'_{n_{\Delta}}}_{\sigma+\frac{1}{2} \tau'_{n_{\Delta}}} \end{pmatrix}. \tag{37} \]
By following the steps given in ref. \cite{16}, the matrix $K_i^{-1}$ is found to be
\[
K_i^{-1}(N_1, \langle \sigma + \frac{1}{2} \rangle) = \left[ \frac{(\Omega_1 - N_1 + \sigma + 4)!!(\Omega_1 - N_1 - \sigma - 2)!!}{2^{-N_1}(\Omega_1 - 2)!!(\Omega_1 + 4)!!} \right]^{\frac{1}{2}}
\]
\[
= \left[ \frac{2^{-N_1}}{\Omega_1 - N_1 - \sigma} \right]^{\frac{1}{2}} \mathcal{P}_{N_1\sigma\tau}.
\] (38)
Inserting (38) into (35)
\[
\langle |2N_1 + 1, \{\sigma + \frac{1}{2}, \tau_1\}n_\Delta IM, [10]|k = 2, m \rangle = \left[ \frac{\Omega_1}{\Omega_1 - N_1 + \sigma} \right]^{\frac{1}{2}} \zeta_{\sigma\tau n\prime} \xi_{\sigma + \frac{1}{2}\tau_1 n_\Delta I} \left[ b_{2n_\frac{1}{2}}^{|N_1\sigma\tau n\prime_1 I\prime}\rangle \right]_M. \] (39)
Coupling the $i$-active and $k$-active parts gives the total wavefunction of the $SO_6$ FDSM for even–odd nuclei
\[
\langle |2N_1 + 1, \{\sigma + \frac{1}{2}, \tau_1\}[\omega_1\omega_2]n_\Delta'' JM \rangle = \sum_{l_\Delta n_\Delta} \left( \frac{\{T_\frac{1}{2}\}[10]}{n_\Delta I} \right) \left( \frac{[\omega_1\omega_2]}{n_\Delta'' J} \right) \left[ |2N_1 + 1, \{\sigma + \frac{1}{2}, \tau_1\}n_\Delta IM, [10]|k = 2 \rangle \right]_M. \] (40)
Combining (34)–(40) we have
\[
\langle |2N_1 + 1, \{\sigma + \frac{1}{2}, \tau_1\}[\omega_1\omega_2]n_\Delta'' JM \rangle = \sum_{l_\Delta \tau n\prime_1 I' n_\Delta ' I''} \sum_{n_\Delta I} \left( \frac{\{T_\frac{1}{2}\}[10]}{n_\Delta I} \right) \left( \frac{[\omega_1\omega_2]}{n_\Delta'' J} \right) \left( \frac{[\sigma00][\frac{1}{2}]}{\tau0} \right) \left( \frac{\{\frac{1}{2}\frac{1}{2}\}}{\frac{1}{2}} \right) \left( \frac{[\tau1\frac{1}{2}]}{\frac{1}{2}} \right) \left( \frac{[\frac{1}{2}\frac{1}{2}]}{n_\Delta I} \right) \sum_j (-)^{l'' + \frac{3}{2} + j} \left( I' \frac{3}{2} J \right) \left( j \frac{1}{2} J \right) \hat{J} \left( \frac{N_1\sigma\tau n\prime_1 I'}{a_j} \right)_M, \] (41)
where $\hat{J} = \sqrt{2J + 1}$. It is interesting to note that when the fermion state for the core, $|N_1\sigma\tau n\prime_1 I\rangle$, is replaced by the boson state $|N_1\sigma\tau n\prime_1 I\rangle$ and the Pauli factor is ignored, Eq. (41) goes over to the $U(6/20)$ NSUSY wave function (2.14) of ref. \cite{16}. Thus, NSUSY can be obtained as an approximation to the FDSM for odd-mass $SO_6$ nuclei.

**IV. ELECTROMAGNETIC TRANSITIONS**

**A. The $E2$ transition rate for even–even nuclei**
In ref. [7], the $E2$ transition operator in the FDSM is defined as

$$T(E2)_{\mu}^2 = qP_\mu^2(i)$$  \hspace{1cm} (42)$$

while the $E2$ transition operator for the IBM $SO_6$ limit is [7]

$$T(E2)_{\mu}^2 = qB_{\mu}^2, \quad B_{\mu}^2 = (d^\dagger s + s^\dagger d)^2_{\mu}. \hspace{1cm} (43)$$

Owing to the isomorphism between the commutators for the FDSM and IBM:

$$[P_\mu^2(i), S^\dagger] \leftrightarrow [B_{\mu}^2, s^\dagger],$$  \hspace{1cm} (44)$$

$$[P_\mu^2(i), D^\dagger_{\nu}] \leftrightarrow [B_{\mu}^2, d^\dagger_{\nu}],$$  \hspace{1cm} (45)$$

the formula for the reduced matrix elements of the $E2$ transition operator in the FDSM is identical to that in the IBM,

$$\langle N\sigma\tau n_{\Delta} J' \parallel P_{\mu}^2(i) \parallel N\sigma\tau n_{\Delta} J \rangle^{FDSM} = \langle N\sigma\tau n_{\Delta} J' \parallel B_{\mu}^2 \parallel N\sigma\tau n_{\Delta} J \rangle^{IBM}. \hspace{1cm} (46)$$

Although it is well known now, it is nevertheless a remarkable fact that the FDSM and IBM have the same selection rules, $\Delta \sigma = 0$ and $\Delta \tau = \pm 1$, and the same closed expression for the $E2$ transition rates [1]. The commonly needed results are given in Table III. It should be noted that the $O_6$ limit of the IBM and the $SO_6$ limit of the FDSM share the same analytical form for the spectra and the $E2$ transitions, but the accounting of the collective pair number is different in these two models: the collective pair number is half of the total valence nucleons ($N$) in the IBM, whereas in FDSM it is taken as half of the total valence nucleons in the normal parity levels ($N_1$).

As pointed out in ref. [18], when we define the $E2$ transition operators as Eq. (12) or Eq. (13), the $\Delta \sigma = 0$ and $\Delta \tau = \pm 1$ selection rules prohibit some transitions that are observed in many nuclei. As a particular case of these selection rules, the quadrupole moments are predicted to be zero in the $SO_6$ limit, but most of the observed quadrupole moments of the transitional nuclei differ from zero. This deviation from zero may be due to two causes: one is the breaking of the $SO_6$ symmetry; the other is that the $E2$ transition operator may require a more general definition. We have chosen the latter to study this problem. It is straightforward to define a new $E2$ transition operator that relaxes the selection rule while
assuming that the wavefunctions still has good $SO_6 \supset SO_5$ symmetry. The new operator takes the form:

$$T(E2)_{\mu}^2 = qP_{\mu}^2(i) + q'(D^\dagger D)_{\mu}^2.$$  \hspace{1cm} (47)

The $(D^\dagger D)_{\mu}^2$ term makes the following transition possible,

$$\Delta \sigma = \pm 2, \quad \Delta \tau = 0, \pm 2.$$  \hspace{1cm} (48)

The reduced matrix element of $(D^\dagger D)_{\mu}^2$ can be calculated by inserting a complete set of intermediate states (the reduced matrix elements used here are defined according to the Rose convention)

\begin{equation}
\langle N_1 \sigma n\Delta L \parallel (D^\dagger D)_{\mu}^2 \parallel N_1 \sigma n\Delta L \rangle
= \sqrt{5(2L+1)} \sum_{\sigma'' \tau'' L''} (-)^{L''+L'} \langle N_1 \sigma' \tau' n\Delta L' \parallel D^\dagger \parallel N_1 - 1\sigma'' \tau'' n\Delta L'' \rangle
\times \langle N_1 \sigma n\Delta L \parallel D^\dagger \parallel N_1 - 1\sigma'' \tau'' n\Delta L'' \rangle \begin{pmatrix} 2 & 2 & 2 \\ L' & L'' \end{pmatrix},
\end{equation}

\hspace{1cm} (49)

The $SO_3$ reduced matrix element of $D^\dagger$ in Eq. (49) is

$$\langle N_1 + 1\sigma' \tau' n\Delta L' \parallel D^\dagger \parallel N_1 \sigma n\Delta L \rangle
= \langle N_1 + 1\sigma' \parallel D^\dagger \parallel N_1 \sigma \rangle \begin{pmatrix} \sigma & 1 \\ \tau & 1 \end{pmatrix} \begin{pmatrix} \sigma' \\ \tau' \end{pmatrix} \begin{pmatrix} \tau & 1 \\ L & 2 \end{pmatrix} \begin{pmatrix} \tau' \\ L' \end{pmatrix},$$

\hspace{1cm} (50)

where $\langle N_1 + 1\sigma' \parallel D^\dagger \parallel N_1 \sigma \rangle$ is the $SO_6$ reduced matrix element and the last two factors are the $SO_6 \supset SO_5$ and $SO_5 \supset SO_3$ isoscalar factors, respectively, which have been given in ref. [21] for some simple cases. The $SO_5$ reduced matrix elements of $S^\dagger$ and $D^\dagger$ are given in the Appendix.

Using vector coherent state techniques \[16\] for the $u = 0$ case, the $SO_6$ reduced matrix element of $D^\dagger$ can be expressed as

$$\langle p + 1\sigma' \parallel D^\dagger \parallel p\sigma \rangle = \sqrt{2}(\Lambda_{p+1\sigma'} - \Lambda_{p\sigma})^{1/2} \langle p + 1\sigma' \parallel z \parallel p\sigma \rangle,$$  \hspace{1cm} (51)

where $\Lambda_{p\sigma}$ is the eigenvalue of the Toronto auxiliary operator

$$\Lambda_{p\sigma} = -\frac{1}{4} [p(p - 2\Omega_1 - 6) - \sigma(\sigma + 4)],$$

\hspace{1cm} (52)
\[ \langle p + 1\sigma + 1 \| z \| p\sigma \rangle = \left[ \frac{(\sigma + 1)(l + \sigma + 3)}{(\sigma + 3)} \right]^{\frac{1}{2}}, \]  
\( (53) \)

\[ \langle p + 1\sigma - 1 \| z \| p\sigma \rangle = \left[ \frac{(l + 1)(\sigma + 3)}{(\sigma + 3)} \right]^{\frac{1}{2}}, \]  
\( (54) \)

with \( p = N_1 = \sigma + 2l = \frac{1}{2}(n_1 - 1) \). For the special cases that will be used below, we have

\[ \langle p + 1\sigma + 1 \| D^\dagger \| p\sigma \rangle = \left[ \frac{(\Omega_1 - 2\sigma - 2l)(\sigma + 1)(l + \sigma + 3)}{(\sigma + 3)} \right]^{\frac{1}{2}}, \]  
\( (55) \)

\[ \langle p + 1\sigma - 1 \| D^\dagger \| p\sigma \rangle = \left[ \frac{(\Omega_1 + 4 - 2l)(l + 1)(\sigma + 3)}{(\sigma + 1)} \right]^{\frac{1}{2}}. \]  
\( (56) \)

In Table IV we summarize some expressions for \( B(E2) \) values and quadrupole moments in the FDSM \( SO_6 \) limit for transitions with \( \Delta \tau = 0, \pm 2 \) and \( \Delta \sigma = \pm 2 \). The contribution to the \( B(E2) \) from the second term \( q'(D^\dagger \tilde{D})^2_\mu \) differs from the corresponding term \( q'(d^\dagger \tilde{d})^2_\mu \) in the IBM \[18\] by the Pauli factors.

### B. The transition rates for even–odd nuclei

For the \( u = 1 \) case, the \( E2 \) transition operator can be defined as

\[ T(E2)^2_\mu = qP^2_\mu(i) + q''P^2_\mu(k). \]  
\( (57) \)

The reduced matrix element is

\[ \langle \{N_1 + \frac{1}{2}, \tau_1'\}[\omega_1'[\omega_2']J' \| qP^2(i) + q''P^2(k) \| \{N_1 + \frac{1}{2}, \tau_1\}[\omega_1[\omega_2]J \rangle \]

\[ = \sum_{I'J'} \begin{pmatrix} [\tau_1'\frac{1}{2}] & [10] & [\omega_1'[\omega_2']J' \\ I' & 2 & J' \end{pmatrix} \begin{pmatrix} [\tau_1\frac{1}{2}] & [10] & [\omega_1\omega_2]J \\ I & 2 & J \end{pmatrix} M, \]  
\( (58) \)

\[ M = \langle \{N_1 + \frac{1}{2}, \tau_1'\}(I', [10]k = 2)J' \| qP^2(i) + q''P^2(k) \| \{N_1 + \frac{1}{2}, \tau_1\}(I, [10]k = 2)J \rangle. \]  
\( (59) \)

According Eq. (6.8) in Judd \[19\], we have

\[ [b_{ki}^l b_{ki}]_{\mu 0}^{K 0} = \frac{1}{\sqrt{2l + 1}} \sum_{p=1}^{n}[b_p^l(p)b_p^l(p)]_{\mu}^K. \]  
\( (60) \)

Therefore, in computing the matrix elements of \( P^2_\mu(k) \) the operator can be replaced by
\[ P^2_\mu(k) = \sqrt{\frac{\Omega_1}{8}} \sum_{p=1}^{n} [b^*_k(p)b_k(p)]^2_\mu. \] (61)

Using (61) we have

\[
M = \hat{J}\hat{J}'(-)^{I'+J} \left\{ \begin{array}{c} J' \ J \\ I \ \ I' \end{array} \right\} \langle \{N_1 + \frac{1}{2}, \tau'_1\} I' \parallel qP^2(i) \parallel \{N + \frac{1}{2}, \tau_1\} I \rangle \\
+ q''\delta_{\tau_1}\delta_{\tau'_1}(-)^{I'+J'} \hat{J}\hat{J}' \sqrt{\frac{5\Omega_1}{2}} \frac{i}{(\Omega_1 - 2N_1 - 1)} \left\{ \begin{array}{c} J' \ J \\ 2 \ 2 \ I' \end{array} \right\} . \] (62)

Now only the matrix elements of \( P^2_\mu(i) \) remain to be calculated. The generators of \( \text{Spin}(6) \) for the IBFM are

\[ C^2_\mu = B^2_\mu + F^2_\mu, \quad F^2_\mu = (a^{\dagger}_{\frac{1}{2}}\tilde{a}_{\frac{1}{2}})^2_\mu \] (63)

corresponding to the commutator for the IBFM

\[ [F^2_\mu, a^{\dagger}_{\frac{1}{2}m_i}] = (-1)^{\frac{3}{2} - m_i} \langle im_i + \mu, i - m_i|2\mu\rangle a^{\dagger}_{\frac{1}{2}m_i + \mu}. \] (64)

There is a similar commutator in the FDSM

\[ [P^2_\mu(i), b^{\dagger}_{22\frac{1}{2}m_i}] = \sqrt{\frac{\Omega_1}{2(2k+1)}}(-1)^{\frac{3}{2} - m_i} \langle im_i + \mu, i - m_i|2\mu\rangle b^{\dagger}_{22\frac{1}{2}m_i + \mu}, \] (65)

where \( \sigma = \pi \) or \( \nu \), and the factor \( \sqrt{\frac{\Omega_1}{2(2k+1)}} \) is always equal to 1 for the 6th shell.

Because of (44, 45) and (64, 65), we have the following isomorphism between the commutators in the FDSM and IBFM,

\[ [P^2_\mu, S^{\dagger}] \leftrightarrow [B^2_\mu, s^{\dagger}] = [G^2_\mu, s^{\dagger}], \] (66)

\[ [P^2_\mu(i), D^{\dagger}_\nu] \leftrightarrow [B^2_\mu, d^{\dagger}_\nu] = [G^2_\mu, d^{\dagger}_\nu], \] (67)

\[ [P^2_\mu(i), b^{\dagger}_{22\frac{1}{2}m_i}] \leftrightarrow [F^2_\mu, a^{\dagger}_{\frac{1}{2}m_i}] = [G^2_\mu, a^{\dagger}_{\frac{1}{2}m_i}]. \] (68)

Therefore we establish the following identity:

\[
\langle \{N + \frac{1}{2}, \tau'_1\} I' \parallel P^2(i) \parallel \{N + \frac{1}{2}, \tau_1\} I \rangle^{\text{FDSM}} = \langle \{N + \frac{1}{2}, \tau'_1\} I' \parallel G^2 \parallel \{N + \frac{1}{2}, \tau_1\} I \rangle^{\text{IBFM}} . \] (69)
The reduced matrix element of $G^2_{\mu}$ is derived in ref. [17]. With these results we can calculate the $B(E2)$ values and the quadrupole moments for odd-mass nuclei. The selection rules for U(6/4) are [17]

$$
\Delta \tau_1 = 0, \pm 1, \quad \Delta \tau_2 = 0. \tag{70}
$$

For the $u = 1$ case in the FDSM, owing to the Kronecker product (22) the corresponding selection rules are

$$
\Delta \omega_1 = 0, \pm 1, \pm 2, \pm 3, \quad \Delta \omega_2 = 0, \pm 1, \tag{71}
$$

With these rules, the restrictions for the $B(E2)$ values will be less severe than that of the IBFM [17]. This enables us to explain some data that cannot be explained by the U(6/4) IBFM.

From Table III and Table IV, we can calculate the $B(E2)$ values for the transitions $\Delta \omega_1 = 0, \pm 1, \pm 2$. In order to make a direct comparison between the calculated and experimental results without a knowledge of $q$ and $q'$, we compute the relative $B(E2)$ value rather than the absolute values. The FDSM prediction and the experimental results [11,20] for Xe and Ba isotopes are listed in Tables V and VI. For the transitions with $\Delta \sigma = 0$ and $\Delta \tau = \pm 1$, the formulas for the $B(E2)$ in the IBM and FDSM are of the same form, but the numerical values differ for a given nucleus because in the IBM the $B(E2)$ is a function of $N$, while in the FDSM it is a function of $N_1$. For the transitions with $\Delta \sigma = \pm 2$ and $\Delta \tau = 0, \pm 2$, they also differ by the Pauli factors $(\Omega_1 - 2N_1 + 2)^2$ or $(\Omega_1 + 4)(\Omega_1 - 2N_1 + 2)$, as shown in Table IV.

From Tables V and VI, we can see that the $B(E2)$ transitions for Xe and Ba isotopes exhibit an $SO_6$ symmetry, especially for the $\Delta \tau = \pm 1$ transitions. There are two reasons to expect less accuracy for the $\Delta \tau = \pm 2$ transitions: one is the definition of the new $T(E2)$ operator and the other is the fitting of the parameter $[\frac{2}{q}]^2$. In fact, the determination of $[\frac{2}{q}]^2$ from the rate $B(E2, 2^+_2 \rightarrow 0^+_1)/B(E2, 2^+_2 \rightarrow 2^+_1)$ is very inaccurate. A possible way to obtain $q$ and $q'$ is, as in ref. [17], through fitting the $B(E2, 2^+_1 \rightarrow 0^+_1)$ and the quadrupole moment $Q(2^+_1)$, respectively. For example, from $Q(2^+_1) = -0.16 \text{ eb}$ and $B(E2, 2^+_1 \rightarrow 0^+_1) = 0.146 \text{ (eb)}^2$ for $^{134}\text{Ba}$, we can determine $(\frac{2}{q})^2$ to be equal to 0.34, which in turn gives the $B(E2)$ value listed in the last column (theo.b) of Table VI. By comparing the last two columns in Table VI, we see that the last column gives a better fit. If more $Q(2^+_1)$ values
were available, it would be possible to obtain a better description of the relative $B(E2)$ for the $\Delta \tau = \pm 2$ transitions.

In ref. [21] the ratio $R_4$ between two $B(E2)$ values is introduced to distinguish the $SO_6$ limit from the $U_5$ limit of the IBM,

$$R_4 = \frac{B(E2, 4^+_1 \rightarrow 2^+_1)}{B(E2, 4^+_1 \rightarrow 0^+_1)}.$$  

(72)

The explicit expression for $R_4$ predicted by the IBM is

$$R_4 = \begin{cases} 
\frac{2(N - 1)}{N} & \text{for the } U_5 \text{ limit}, \\
\frac{10(N - 1)(N + 5)}{7N(N + 4)} & \text{for the } SO_6 \text{ limit}
\end{cases}$$  

(73)

where $N$ is the boson number. The $R_4$ value derived from the FDSM has the same form as above, but with $N$ replaced by $N_1$,

$$R_4 = \frac{10(N_1 - 1)(N_1 + 5)}{7N_1(N_1 + 4)}$$  

(74)

The $N_1$ values can be estimated from shell model configurations of protons and neutrons in the odd-A nuclei, and are shown in the Table 7.1 of ref. [22]. In Table VII we list the FDSM prediction for $R_4$ along with the experimental results of ref. [21]. It can be seen that the $SO_6$ limit of the FDSM seems to explain the experimental data better than the IBM. Alternatively, we note that if accurate $R_4$ values are available, we may be used to obtain the empirical $N_1$ value from Eq. (74).

It should be mentioned again that apart from the Pauli effect, the FDSM differs from the IBM [20] in the value of the number of the collective pairs ($N_1$ vs. $N$). The spectrum of the $\sigma = N_1$ band is not sensitive to the value of $N_1$, but the observation that the parameters $g_6$ and $g'_5$ in Table I change smoothly between nuclei, and that the experimental spectra for the even and odd nuclei can be fitted with the same $g_6$ and $g'_5$ values, suggest that the choice of $N_1$ taken in this paper is reasonable.

The difference between $N$ and $N_1$ does affect the energies for the bands with $\sigma = N_1 - 2, N_1 - 4 \ldots$ In ref. [11], it is pointed out that the parameters for the $SO_6$ nuclei in both the A=130 and Pt regions have a common characteristic that $g'_5 \approx -g_6$ (see Eq. (28)). With such an empirical relation, the following energy ratio has a simple form in both the IBM and the FDSM.
\[
\frac{E_{O_3}^+(\sigma = N - 2)}{E_{2_1}^+ - E_{2_1}^+} = \frac{2(N + 1)}{3},
\]
\[
\frac{E_{O_3}^+(\sigma = N_1 - 2)}{E_{2_1}^+ - E_{2_1}^+} = \frac{2(N_1 + 1)}{3},
\]

A comparison of the ratios calculated using Eqs. (75, 76) and the experimental data is shown in Table VIII. This example indicates also that the FDSM SO_6 model reproduces this ratio better than the IBM SO_6 for nuclei in the Xe–Ba region. This suggests that in this region an empirical effective boson number maybe needed to give a better agreement with data in IBM calculations.

The \(E2\) transition rate is generally more sensitive to the parameter \(N_1\) than the spectra. The reasonableness of the chosen \(N_1\) value can also be seen from the good agreement between the calculated and experimental values of the \(B(E2)\) values for the isotopes of Ba and Xe, as shown in Table V and VI. Finally, we reiterate that as \(N_1\) increases in the shell, the spins for the ground states of odd nuclei swap naturally between \(\frac{1}{2}\) and \(\frac{3}{2}\) (this transition occurs at \(^{131}\text{Xe}\) and \(^{135}\text{Ba}\) for the isotopes of Xe and Ba, respectively).

In Table IX, both experimental and theoretical \(B(E2)\) values for the even–odd nuclei of \(^{129}\text{Xe}\) and \(^{131}\text{Xe}\) are given, and compared with the calculated results of the NSUSY case. Here the effective charges (i.e., \(q\)) are the same as the neighbouring even-even nuclei, and determined by the experimental \(B(E2, 2^+ \rightarrow 0^+)\) values. While effective charges for \(k\)-active part (i.e., \(q''\)) are fitted by E2 transitions of even–odd nuclei. In this work, \((q, q'') = (0.129, 0.075)\) \(eb\), \((0.143, 0.093)\) \(eb\) for \(^{129}\text{Xe}\) and \(^{131}\text{Xe}\) respectively. The agreement with data is comparable in the two cases, although the NSUSY calculations give a somewhat better agreement of the weaker transitions. Finally, we reiterate that the group chain (2) is very similar to the NSUSY group chain (1). However, the pseudo orbital angular momentum 2 is introduced \(ad hoc\) in the NSUSY, while in the FDSM it is a natural result of the reclassification (in terms of the \(k-i\) basis) of the shell model single-particle states for the sixth shell.

\[V. \ CONCLUSIONS\]

In this work, we provide simple but unified analytic solutions of even and even–odd nuclei within the framework of the fermion dynamical symmetry model. The good agreement of
both level pattern and E2 transitions with our simplified solutions indicates a good SO(6) symmetry for both even and even-odd nuclei in A=130 region. We find that generally the FDSM results provide a unified description of the even and odd nuclei in this region that is comparable to or even somewhat better than IBM and NSUSY approaches, but to a lesser degree in phenomenology.

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[1] A. Arima and F. Iachello, *Ann. Rev. of Nucl. Part. Sci.* **31**, 75 (1981); *Ann. Phys. (N. Y.)* **99**, 253 (1976); **111**, 201 (1976); **115**, 325 (1978); **123**, 468 (1979).

[2] F. Iachello, *Phys. Rev. Lett.* **44**, 772 (1980).

[3] A.B. Balentekin, I. Bars and F. Iachello, *Nucl. Phys.* **A370**, 284 (1981).

[4] Y.S. Ling, M. Zhang, J.M. Wu, M. Vallieres, R. Gilmore and D.H. Feng, *Phys. Lett.* **B148**, 13 (1984).

[5] J. Jolie, K. Heyde, P. Van Isacker and A. Frank, *Nucl. Phys.* **A466**, 1 (1987).

[6] C.L. Wu, D.H. Feng, X.G. Chen, J.Q. Chen and M. Guidry, *Phys. Rev.* **C36**, 1157 (1987).

[7] C.-L. Wu, D. H. Feng, and M. W. Guidry, *Advances in Nuclear Physics*, **21**, 227 (1994).

[8] J.N. Ginocchio, *Ann. Phys.* **126**, 234 (1980).
[9] J.Q. Chen, D.H. Feng and C.L. Wu, Phys. Rev. C34, 2269 (1986).

[10] J.A. Cizewski, R.F. Casten, G.J. Smith, M.L. Stelts, and W.R. Kane, Phys. Rev. Lett., 40, 167 (1978).

[11] R.F. Casten and P. von Brentano, Phys. Lett. B152, 22 (1985). R.F. Casten, J.N. Ginocchio, D.H. Feng and C.L. Wu, Mod. Phys. Lett. A1, 161 (1986).

[12] D.H. Feng, et al., Phys. Rev. C48, R1488 (1993).

[13] G.R. Black, R.C. King and B.G. Wybourne, J. Phys. A16, 1555 (1983).

[14] Z.M. Lu, X.W. Pan, J.Q. Chen, X.G. Chen and D.H. Feng, Phys. Rev. C37, 3789 (1988).

[15] X. W. Pan, T. Otsuka, J. Q. Chen and A. Arima, Phys. Lett. B287, 1 (1992); X. W. Pan, D.H. Feng, J.Q. Chen and M. Guidry, Phys. Rev. C49, 2493 (1994).

[16] K.T. Hecht, Nucl. Phys. A475, 276 (1987).

[17] F. Iachello and S. Kuyucak, Ann. Phys. 136, 19 (1981).

[18] P.van Isacker, Nucl. Phys. A465, 497 (1987).

[19] B.R. Judd, Operator Techniques In Atomic Spectroscopy, (Mcgraw-Hill, 1963).

[20] R. Reinhardt, A. Dewald, A. Gelberg, W. Lieberz, K. Schiffer, K.P. Schmittgen, K.O. Zell and P.Von Brentano, Z.Phys. A329, 507 (1988).

[21] P. von Brentano, A. Gelberg, S. Harissopulos and R.F. Casten, Phys. Rev. C38, 2386 (1988).

[22] M.A. Preston and R.K. Bhaduri, Structure of the Nucleus, (Addison-Wesley Publishing Company, 1975).
### Table I. Parameters for the even Xe isotopes.

| Nuclei | $g_6$(keV) | $g'_5$(keV) | $g'_I$(keV) |
|--------|------------|-------------|-------------|
| $^{120}$Xe | -60 | 53 | 11.9 |
| $^{122}$Xe | -64 | 59 | 11.9 |
| $^{124}$Xe | -68.8 | 64 | 11.9 |
| $^{126}$Xe | -73.3 | 71 | 11.9 |
| $^{128}$Xe | -78.2 | 79 | 11.9 |
| $^{130}$Xe | -100.9 | 100 | 11.9 |
| $^{132}$Xe | -106.5 | 122 | 11.9 |

### Table II-a. Parameters for the odd Xe isotopes.

| Nuclei | $g_6$(keV) | $g'_5$(keV) | $g_5$(keV) | $g_I$(keV) |
|--------|------------|-------------|------------|------------|
| $^{127}$Xe | -73.3 | 71.0 | -38.0 | 25.0 |
| $^{129}$Xe | -78.2 | 79.0 | -18.0 | 35.3 |
| $^{131}$Xe | -100.9 | 100.0 | 30.0 | 35.3 |
| $^{133}$Xe | -106.5 | 122.0 | 50.5 | 35.3 |
| $^{135}$Xe | | 142.1 | 70.6 | 35.3 |

### Table II-b. Parameters for the odd Ba isotopes.

| Nuclei | $g'_5$(keV) | $g_5$(keV) | $g_I$(keV) |
|--------|------------|------------|------------|
| $^{131}$Ba | 72.5 | -20 | 35.3 |
| $^{133}$Ba | 105 | -15 | 35.3 |
| $^{135}$Ba | 90 | 50 | 35.3 |
| $^{137}$Ba | 72 | 70 | 35.3 |
Table III. $B(E2)$ formulas for even–even nuclei in the $SO_6$ limit.

| $|N_1\sigma\tau J_i\rangle$ | $\rightarrow$ | $|N_1\sigma'\tau' J_f\rangle$ | $B(E2; J_i \rightarrow J_f)$ |
|--------------------------|--------------|--------------------------|--------------------------|
| $|N_1N_1 1 + L/2 L + 2\rangle$ | $\rightarrow$ | $|N_1N_1 L/2 L\rangle$ | $q^2\frac{(L + 2)(2N_1 + L + 8)}{8(L + 5)}(2N_1 - L)$ |
| $|N_1N_1 1 2\rangle$ | $\rightarrow$ | $|N_1N_1 0 0\rangle$ | $\frac{1}{5}q^2N_1(N_1 + 4)$ |
| $|N_1N_1 2 2\rangle$ | $\rightarrow$ | $|N_1N_1 1 2\rangle$ | $\frac{2}{7}q^2(N_1 - 1)(N_1 + 5)$ |
| $|N_1N_1 3 4\rangle$ | $\rightarrow$ | $|N_1N_1 2 4\rangle$ | $\frac{10}{63}q^2(N_1 - 2)(N_1 + 6)$ |
| $|N_1N_1 3 3\rangle$ | $\rightarrow$ | $|N_1N_1 2 4\rangle$ | $\frac{2}{21}q^2(N_1 - 2)(N_1 + 6)$ |
| $|N_1N_1 3 3\rangle$ | $\rightarrow$ | $|N_1N_1 2 2\rangle$ | $\frac{5}{21}q^2(N_1 - 2)(N_1 + 6)$ |
| $|N_1N_1 3 4\rangle$ | $\rightarrow$ | $|N_1N_1 2 2\rangle$ | $\frac{11}{63}q^2(N_1 - 2)(N_1 + 6)$ |
| $|N_1N_1 3 0\rangle$ | $\rightarrow$ | $|N_1N_1 2 2\rangle$ | $\frac{7}{21}q^2(N_1 - 2)(N_1 + 6)$ |
Table IV. Some quadrupole moments and $B(E2)$ values for the transitions $\Delta \sigma = 0, \pm 2$, $\Delta \tau = 0, \pm 2$.

| Transition | Quadrupole Moment | $B(E2)$ Value |
|------------|-------------------|----------------|
| $B(E2; N_1N_1 2 2 \rightarrow N_1N_1 0 0)$ | $q^2 \frac{N_1(N_1 - 1)(N_1 + 4)(N_1 + 5)}{70(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)^2$ | |
| $B(E2; N_1N_1 3 3 \rightarrow N_1N_1 1 2)$ | $q^2 \frac{5(N_1 - 1)(N_1 - 2)(N_1 + 5)(N_1 + 6)}{294(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)^2$ | |
| $B(E2; N_1N_1 3 4 \rightarrow N_1N_1 1 2)$ | $q^2 \frac{11(N_1 - 1)(N_1 - 2)(N_1 + 5)(N_1 + 6)}{882(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)^2$ | |
| $B(E2; N_1N_1 3 4 \rightarrow N_1N_1 3 3)$ | $q^2 \frac{2(N_1^2 + 4N_1 + 23)^2}{231(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)^2$ | |
| $B(E2; N_1N_1 3 0 \rightarrow N_1N_1 1 2)$ | $q^2 \frac{(N_1 - 1)(N_1 - 2)(N_1 + 5)(N_1 + 6)}{42(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)^2$ | |
| $B(E2; N_1N_1 2 2 \rightarrow N_1N_1 2 4)$ | $q^2 \frac{16(N_1^2 + 4N_1 + 15)^2}{2205(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)^2$ | |
| $B(E2; N_1N_1 - 2 0 0 \rightarrow N_1N_1 1 2)$ | $q^2 \frac{(N_1 + 2)(N_1 + 3)(N_1 + 4)(N_1 + 5)}{14N_1(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)(\Omega_1 + 4)$ | |
| $B(E2; N_1N_1 - 2 1 2 \rightarrow N_1N_1 1 2)$ | $q^2 \frac{(N_1 - 1)(N_1 - 2)(N_1 + 3)(N_1 + 4)}{49N_1(N_1 + 1)^2} (\Omega_1 - 2N_1 + 2)(\Omega_1 + 4)$ | |
| $Q(N_1N_1 1 2)$ | $q^2 \frac{\sqrt{2\pi}4(N_1^2 + 4N_1 + 9)}{35(N_1 + 1)} (\Omega_1 - 2N_1 + 2)$ | |
Table V. Relative $B(E2)$ values for the even Xe isotopes.

|          | $^{120}$Xe | $^{124}$Xe | $^{126}$Xe | $^{128}$Xe | $^{130}$Xe |
|----------|------------|------------|------------|------------|------------|
| $J_i \rightarrow J_f$ | Exp. | Theo. | Exp. | Theo. | Exp. | Theo. | Exp. | Theo. | Exp. | Theo. | Exp. | Theo. | Exp. | Theo. |
| $2^+_2 \rightarrow 1^+_1$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 0^+_1$ | 5.6 | 5.6 | 3.9 | 3.9 | 1.5 | 1.4 | 1.2 | 1.2 | 0.6 | 0.6 |
| $3^+_1 \rightarrow 2^+_2$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 4^+_1$ | 50 | 40 | 46 | 40 | 34 | 40 | 37 | 40 |
| $\rightarrow 2^+_1$ | 2.7 | 7.1 | 1.6 | 4.9 | 2.0 | 1.85 | 1.0 | 1.5 | 1.4 | 0.72 |
| $4^+_2 \rightarrow 2^+_2$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 4^+_1$ | 62 | 91 | 91 | 91 | 76 | 91 | 133 | 91 |
| $\rightarrow 2^+_1$ | — | 7.11 | 0.4 | 4.91 | 1.0 | 1.83 | 1.7 | 1.49 | 3.2 | 0.97 |
| $0^+_2 \rightarrow 2^+_2$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 2^+_1$ | — | 7.11 | 1 | 4.91 | 7.7 | 1.83 | 14 | 1.49 | 26 | 0.97 |
Table VI. Relative $B(E2)$ values for the even Ba isotopes.

|       | $^{126}$Ba | $^{128}$Ba | $^{130}$Ba | $^{132}$Ba | $^{134}$Ba |
|-------|------------|------------|------------|------------|------------|
| $J_i \rightarrow J_f$ | Exp. Theo. | Exp. Theo. | Exp. Theo. | Exp. Theo. | Exp. Theo. | Exp. Theo.a | Exp. Theo.b |
| $2^+_2 \rightarrow 2^+_1$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 0^+_1$ | 11 | 11 | 9.2 | 9.2 | 5.7 | 5.7 | 0.2 | 0.2 | 1.0 | 1.1 | 3.06 |
| $3^+_1 \rightarrow 2^+_2$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 4^+_1$ | 13 | 40 | — | 40 | 30 | 40 | 73 | 40 | 40 | 40 | 40 |
| $\rightarrow 2^+_1$ | 5.8 | 14.4 | — | 12 | 1.5 | 7.46 | 0.2 | 0.24 | 1.0 | 1.26 | 3.89 |
| $4^+_2 \rightarrow 2^+_2$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 3^+_1$ | — | 26.1 | — | 21.8 | — | 13.5 | — | 0.44 | 14.5 | 3.06 | 9.45 |
| $\rightarrow 4^+_1$ | 28 | 91 | 42 | 91 | 89 | 91 | 75 | 91 | 77 | 91 | 91 |
| $\rightarrow 2^+_1$ | 1.1 | 14.4 | 1.7 | 12 | 3.9 | 7.46 | 2.2 | 0.24 | 2.5 | 1.26 | 3.89 |
| $0^+_2 \rightarrow 2^+_2$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| $\rightarrow 2^+_1$ | — | 14.4 | — | 12 | — | 7.46 | 0 | 0.24 | 4 | 1.26 | 3.89 |

Table VII. The value of $R_4$.

| Nuclei | $N$ | $N_1$ | $R_4^{exp}$ | $R_4^{FDSM}(SO_6)$ | $R_4^{IBM}(SO_6)$ | $R_4^{IBM}(U_5)$ |
|--------|----|-------|-------------|---------------------|-------------------|------------------|
| $^{120}$Xe | 10 | 7 | 1.46(20) | 1.34 | 1.38 | 1.80 |
| $^{124}$Xe | 8 | 6 | 1.29(15) | 1.31 | 1.35 | 1.75 |
| $^{126}$Ba | 9 | 6 | 1.12(20) | 1.34 | 1.37 | 1.75 |
| $^{128}$Ba | 8 | 6 | 1.03(20) | 1.31 | 1.35 | 1.75 |
| $^{130}$Ba | 7 | 5 | 0.90(13) | 1.27 | 1.34 | 1.71 |
| $^{130}$Xe | 5 | 4 | 1.35(18) | 1.21 | 1.27 | 1.60 |
Table VIII. The $E(0^+_3)/[E(2^+_2) – E(2^+_1)]$ ratio.

| Nuclei | $N$ | $N_1$ | Exp. | IBM | FDSM |
|--------|-----|-------|------|-----|------|
| $^{118}$Xe | 9 | 6 | 2.912 | 6.67 | 4.67 |
| $^{122}$Xe | 9 | 6 | 3.68 | 6.67 | 4.67 |
| $^{124}$Xe | 8 | 6 | 3.44 | 6.00 | 4.67 |
| $^{126}$Xe | 7 | 5 | 3.58 | 5.33 | 4.00 |
| $^{128}$Xe | 6 | 5 | 3.56 | 4.67 | 4.00 |
| $^{130}$Xe | 5 | 4 | 3.44 | 4.00 | 3.33 |
| $^{134}$Ba | 5 | 5 | 3.84 | 4.00 | 4.00 |
| $^{136}$Ba | 4 | 4 | 2.918 | 3.33 | 3.33 |
| $^{138}$Ce | 5 | 5 | 3.25 | 4.00 | 4.00 |
Table IX. Transition probabilities in $^{129}\text{Xe}$ ($N_1 = 5$) and $^{131}\text{Xe}$ ($N_1 = 4$), $\Omega_1 = 20$.

|          | $^{129}\text{Xe}$ |          | $^{131}\text{Xe}$ |
|----------|-------------------|----------|-------------------|
|          | $B(E2)(e^2b^2)$   |          | $B(E2)(e^2b^2)$   |
| $J_i \rightarrow J_f$ | FDSM Exp. Ref.[5] | $J_i \rightarrow J_f$ | FDSM Exp. Ref.[5] |
| $\frac{3^+}{2_1} \rightarrow \frac{1^+}{2_1}$ | 0.036 0.007 | $\frac{1^+}{2_1} \rightarrow \frac{3^+}{2_1}$ | 0.0953 0.0039 0.0012 |
| $\frac{3^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.0186 <0.0005 0.013 | $\frac{5^+}{2_1} \rightarrow \frac{1^+}{2_1}$ | 0.075 0.030 0.016 |
| $\frac{5^+}{2_1} \rightarrow \frac{3^+}{2_1}$ | 0.084 0.12 0.12 | $\frac{3^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.004 0.10 0.10 |
| $\frac{5^+}{2_1} \rightarrow \frac{1^+}{2_1}$ | 0.011 0.22 0.10 | $\frac{3^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.053 0.057 0.058 |
| $\frac{1^+}{2_2} \rightarrow \frac{3^+}{2_2}$ | 0.028 | $\frac{3^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.000 |
| $\frac{1^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.14 0.044 0.12 | $\frac{7^+}{2_1} \rightarrow \frac{5^+}{2_1}$ | 0.028 0.005 0.0013 |
| $\frac{7^+}{2_1} \rightarrow \frac{5^+}{2_1}$ | 0.0000 | $\frac{3^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.043 0.081 0.082 |
| $\frac{5^+}{2_2} \rightarrow \frac{1^+}{2_1}$ | 0.004 0.057 0.071 | $\frac{3^+}{2_3} \rightarrow \frac{3^+}{2_1}$ | 0.025 0.027 0.017 |
| $\frac{3^+}{2_3} \rightarrow \frac{1^+}{2_1}$ | 0.056 0.0032 0.0056 | $\frac{5^+}{2_2} \rightarrow \frac{3^+}{2_2}$ | 0.004 <0.031 0.0011 |
| $\frac{3^+}{2_4} \rightarrow \frac{1^+}{2_1}$ | 0.0133 0.0030 0.0004 | $\frac{5^+}{2_2} \rightarrow \frac{1^+}{2_1}$ | 0.071 0.068 0.056 |
| $\frac{5^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.014 0.013 0.043 | $\frac{7^+}{2_2} \rightarrow \frac{3^+}{2_1}$ | 0.124 0.005 0.026 |
Appendices

The $SO_5$ reduced matrix elements

$$\langle N+1, \sigma + 1, \tau \| S^\dagger \| N\sigma\tau \rangle = \left[ \frac{(\Omega_1 - \sigma - N)(\sigma - \tau + 1)(\sigma + \tau + 4)(N + \sigma + 6)}{4(\sigma + 2)(\sigma + 3)} \right]^{\frac{1}{2}} \quad (A-1)$$

$$\langle N+1, \sigma - 1, \tau \| S^\dagger \| N\sigma\tau \rangle = -\left[ \frac{(\Omega_1 + \sigma - N + 4)(\sigma - \tau)(\sigma + \tau + 3)(N - \sigma + 2)}{4(\sigma + 1)(\sigma + 2)} \right]^{\frac{1}{2}} \quad (A-2)$$

$$\langle N+1, \sigma + 1, \tau' \| D^\dagger \| N\sigma\tau \rangle =$$

$$\left[ \frac{(\Omega_1 - \sigma - N)(N + \sigma + 6)}{4(\sigma + 2)(\sigma + 3)} \right]^{\frac{1}{2}} \times \left\{ \begin{array}{ll}
\left[ \frac{(\sigma + \tau + 4)(\sigma + \tau + 5)(\tau + 1)}{(2\tau + 5)} \right]^{\frac{1}{2}}, & \tau' = \tau + 1 \\
-\left[ \frac{(\sigma - \tau + 1)(\sigma - \tau + 2)(\tau + 2)}{(2\tau + 1)} \right]^{\frac{1}{2}}, & \tau' = \tau - 1
\end{array} \right\} \quad (A-3)$$

$$\langle N+1, \sigma - 1, \tau' \| D^\dagger \| N\sigma\tau \rangle =$$

$$\left[ \frac{(\Omega_1 + \sigma - N + 4)(N - \sigma + 2)}{4(\sigma + 1)(\sigma + 2)} \right]^{\frac{1}{2}} \times \left\{ \begin{array}{ll}
-\left[ \frac{(\sigma + \tau + 2)(\sigma + \tau + 3)(\tau + 2)}{(2\tau + 1)} \right]^{\frac{1}{2}}, & \tau' = \tau + 1 \\
\left[ \frac{(\sigma - \tau - 1)(\sigma - \tau)(\tau + 1)}{(2\tau + 5)} \right]^{\frac{1}{2}}, & \tau' = \tau - 1
\end{array} \right\} \quad (A-4)$$
The SO$_6 \supset$ SO$_5$ isoscalar factors

\[
\begin{pmatrix}
\sigma + \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 
\tau + \frac{1}{2}, \frac{1}{2}
\end{pmatrix}
\begin{bmatrix}
100 \\
10
\end{bmatrix}
\begin{pmatrix}
\sigma' + \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 
\tau + \frac{1}{2}, \frac{1}{2}
\end{pmatrix}
\]

\[
\sigma' = \sigma + 1 \\
\sigma' = \sigma - 1
\]

\[
\frac{(\sigma - \tau + 1)(\sigma + \tau + 5)}{2(\sigma + 1)(\sigma + 3)}\frac{1}{2} \\
\frac{(\sigma - \tau)(\sigma + \tau + 4)}{2(\sigma + 2)(\sigma + 4)}\frac{1}{2}
\]

\[
\begin{pmatrix}
\sigma + \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 
\tau + \frac{1}{2}, \frac{1}{2}
\end{pmatrix}
\begin{bmatrix}
100 \\
10
\end{bmatrix}
\begin{pmatrix}
\sigma' + \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 
\tau + \frac{1}{2}, \frac{1}{2}
\end{pmatrix}
\]

\[
\begin{array}{c|c|c}
[\tau_1 \tau_2] & \sigma' = \sigma + 1 & \sigma' = \sigma - 1 \\
\hline
[\tau + \frac{3}{2}, \frac{1}{2}] & \frac{(\sigma + \tau + 5)(\sigma + \tau + 6)(\tau + 1)}{2(\sigma + 1)(\sigma + 3)(2\tau + 5)}\frac{1}{2} & - \frac{(\sigma - \tau)(\sigma - \tau - 1)(\tau + 1)}{2(\sigma + 2)(\sigma + 4)(2\tau + 5)}\frac{1}{2} \\
[\tau + \frac{1}{2}, \frac{3}{2}] & 0 & 0 \\
[\tau + \frac{1}{2}, \frac{1}{2}] & -\frac{(\sigma + \tau + 5)(\sigma - \tau + 1)(\tau + 3)}{2(\sigma + 1)(\sigma + 3)(3\tau + 2)(2\tau + 5)}\frac{1}{2} & \frac{(\sigma + t + 4)(\sigma - \tau)(\tau + 2)}{2(\sigma + 2)(\sigma + 4)(3\tau + 2)(2\tau + 5)}\frac{1}{2} \\
[\tau - \frac{3}{2}, \frac{1}{2}] & -\frac{(\sigma - \tau + 2)(\sigma - \tau + 1)(\tau + 3)}{2(\sigma + 1)(\sigma + 3)(2\tau + 3)}\frac{1}{2} & \frac{(\sigma + \tau + 4)(\sigma + \tau + 3)(\tau + 3)}{2(\sigma + 2)(\sigma + 4)(2\tau + 3)}\frac{1}{2}
\end{array}
\]
Figure captions

Fig. 1. Comparison between calculated levels using eq. (17) and experimental energy levels for the even–even $^{120-126}$Xe isotopes.

Fig. 2. Comparison between calculated levels using eq. (17) and experimental energy levels for the even–even $^{128-132}$Xe isotopes.

Fig. 3. Comparison between calculated levels using eq. (18) and experimental energy levels for the even–odd $^{127-129}$Xe isotopes. The even–odd nuclei are constructed by coupling the neighboring even–even core to a valence neutron.

Fig. 4. Comparison between calculated levels using eq. (18) and experimental energy levels for the even–odd $^{131-133}$Xe isotopes. The even–odd nuclei are constructed by coupling the neighboring even–even core to a valence neutron.

Fig. 5. Comparison between calculated levels and experimental energy levels for the even–odd $^{131-135}$Ba isotopes. For the excited $[\tau_1\tau_2]$ states, the band-head states are compared with experimental ones.

Fig. 6. The systematic shift of the ground band as a function of mass number for Xe isotopes.

Fig. 7. The systematic shift of the ground band as a function of mass number for Ba isotopes.

Fig. 8. (a) The $SO_6$ spectrum calculated with the parameters are $g_6=-73.3$ (keV), $g_5=71$ (keV) and $g_I=11.9$ (keV); (b) The comparison of the experimental spectrum of $^{126}$Xe (lower numbers) and the spectrum of $SO(6)$ plus a perturbative pairing term (upper numbers) (i.e., eq. (26)). The parameters here are $g_6=-73.3$ (keV), $A'=80$ (keV), $B'=0.77$ (keV) and $g_I=11.9$ (keV).
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