Algebraic expressions for estimating the impact depths of a surface barrier over a homogeneous soil

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Abstract

Engineered surface barriers are used to isolate subsurface contaminants for effective long-term containment of waste of a variety of types. The impact depths of a surface barrier are affected by the pre-barrier recharge rate and the properties of the soil beneath the barrier. In this paper, algebraic expressions are developed to estimate drainage velocities and barrier impact depths after the emplacement of a surface barrier. Four impact depth terms are used to convey barrier impact: the near-zero-, 50%, average-, and full-impact depths. The algebraic expressions show that the average-impact depth is no more than one-third of the near-zero-impact depth, whereas the 50% impact depth is slightly larger than one-half of the near-zero-impact depth. The full-impact depth, depending on the final recharge rate from the surface barrier, is usually much smaller than the other impact depths. These differences lead to a very large transition zone beneath a surface barrier. A field drainage experiment and a series of numerical simulations were used to test the algebraic expressions. The experimental data and numerical results corroborated the analytical models by predicting very similar water content profiles and/or near-zero-, 50%, average-, and full-impact depths during the drainage process. The algebraic expressions provided are useful for quickly identifying sites where the depth of the existing contaminants could be beyond the protection of a surface barrier.

1 | INTRODUCTION

Engineered surface barriers are used to isolate subsurface contaminants for safe long-term containment of municipal solid waste, other nonhazardous solid and liquid waste, hazardous and toxic wastes, and radioactive waste. They have been used above landfills (e.g., Albright et al., 2004; Apiwantragoon, Benson, & Albright, 2015; Barnswell & Dwyer, 2011; Benson, Abichou, Albright, Gee, & Roesler, 2001; Benson, Sawangsuriya, Trzebiatowski, & Albright, 2007; Santini & Fey, 2014; Scanlon, Reedy, Stonestrom, Prudic, & Dennehy, 2005), waste sites (e.g., Bowerman & Redente, 1998; Gee, Ward, & Fayer, 1997; Link, Wing, & Gee, 1995; Pettit, Ridenour, Walker, & Saugier, 1994; Rutqvist et al., 2011; Scanlon, Mullican, Reedy, & Angle, 1997; Waugh, Benson, & Albright, 2010; Waugh et al., 2007; Wing & Gee, 1994; Zhang, 2015), and mine lands (e.g., Barber, Ayres, & Schmid, 2015; Meiers et al., 2015; O’Kane & Wels, 2003; Zhan et al., 2006). A primary function of the surface barrier is to minimize infiltration of meteoric water into the underlying buried waste in order to reduce the mobility of the contaminants.
With limited or near-zero water infiltrating through the barrier, water flow in the soil beneath a surface barrier is generally a drainage process, and it can take a considerable amount of time for the soil water above the waste zone to drain. As a result, it will take some time for a surface barrier to start taking effect in reducing the mobility of the contaminants. The duration of this process is dependent on the recharge rate before barrier deployment, the hydraulic properties of the soil, the depth of the waste zone, and the thickness of the vadose zone.

At a given time, the impact of a surface barrier can be quantified by one or more depths at which the flow rates have been reduced. Because the zone of drainage may be characterized with a few depths such as the depth of the drainage front, tail, the average drainage, and the depth that the flux rate has been reduced by a desired amount (e.g., 50%), these depths of drainage can be considered as the impact depths of a surface barrier. Knowing the impact depths is important in predicting the length of time before the effects of the surface barrier reach the waste zone. This time must be less than the time needed for the contaminants to arrive at the groundwater so that the surface barrier is effective in minimizing the transport of the contaminants. The knowledge of the advancement velocity of the impact depths is useful for identifying sites where the depth of the existing contaminants exceeds the protection of a surface barrier.

The impact depths may be investigated with numerical simulations or analytical expressions. A drawback of the numerical approach is that the mathematical relationship between inputs and outputs is given in complex differential equations due to the nature of numerical simulation. Hence, based on the results of numerical simulation, it is usually difficult to tell which characteristics are common for all sites and which are site specific. Consequently, the findings from numerical simulations for one site may not be used for sites with different conditions such as the recharge rates and soil hydraulic properties. Alternatively, if the impact depths can be estimated with algebraic expressions, the common characteristics can be easily revealed, as will be shown in this investigation, and the impact depths can be calculated easily and quickly for the sites of interest. The drawback of algebraic expressions, if available, is their validity under specific conditions (e.g., uniform soil and constant pre- and post-barrier recharge rates). Despite their limitations, algebraic expressions can be used for screening of sites at which a surface barrier may be effective in reducing the mobility of the underlying contaminants. Currently, I am not aware of the availability of such algebraic expressions for estimating barrier impact depths.

In this study, the drainage process in a soil below a surface barrier is characterized with the depths of drainage front, drainage tail, and the average drainage, and the depth that water flux rate is reduced by 50%. These are considered as the near-zero-, full-, average-, and 50% impact depths of a surface barrier, respectively. Recognizing that soil water moves faster in larger pores, the pore-size-specific (PSS) water velocity is defined, and an algebraic expression of PSS velocity is derived based on the Brooks and Corey (1966) water retention model and the Burdine (1953) relative permeability model. Algebraic expressions are developed to estimate water content distribution and barrier impact depths after the emplacement of a surface barrier. The expressions for water content, PSS velocities, and impact depths are tested against a field drainage experiment and/or numerical simulations for four types of soils under two initial recharge rates.

2 | THEORY

2.1 | The drainage process below a surface barrier

Before barrier emplacement, a fraction of precipitation enters the deep soil as recharge ($q_i$), which is assumed to be (and usually is) less than the saturated hydraulic conductivity. Hence, a fraction of the relatively smaller soil pores is filled with water (residual or mobile, with an initial water content, $\theta_i$), and the remaining larger pores are filled with air (Figure 1a). After the emplacement of a surface barrier, the mobile soil water starts draining. Soil water in larger pores moves faster than it does in smaller pores during the drainage process. As a result, the soil-water-content profile during drainage can be divided roughly into three zones (Figure 1b): an “unaffected zone” in the deep soil with pre-barrier water content and flow rate, an “equilibrated zone” in the shallow soil that has the final water content ($\theta_f$) corresponding to the water flux rate through the barrier ($q_f$), and a “transition zone” in the middle with a water content between the pre-barrier and final conditions.

For convenience, the lower end of the transition zone is referred to as the drainage front, and the upper end is referred to as the drainage tail. The depths of the drainage front and tail are considered as the near-zero- ($z_0$) and full-impact ($z_f$) depths of the surface barrier. As time ($t$) goes on, these depths increase because of drainage. However, at any time, the water
2.2 Drainage velocities and water content distribution

This section defines four drainage velocities and derives the algebraic expressions for them. These velocity expressions will be used for deriving the four corresponding impact depths in Section 2.3. The expression for the distribution of water content at any time and depth is also derived.

2.2.1 The pore-size-specific water velocity

Based on the STB model and the unit-gradient assumption, the soil water flux, $q$, is equal to the unsaturated hydraulic conductivity, $K$ [i.e., $q(\theta) = K(\theta)$], during the drainage process. Hence, based on Equation A2,

$$q = K_s \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{3+2/\lambda}$$  \hspace{1cm} (1)

where $\theta_s$ and $\theta_r$ are saturated and residual water content, respectively, $K_s$ is the saturated hydraulic conductivity, and $\lambda$ is the pore-size distribution parameter.

The pore water velocity, defined as the $q/\theta$ (Warrick, 2002), is usually used to describe the average movement of all the water. Here, the PSS water velocity is defined for water in stream tubes of specific size (which corresponds to a specific water content). As shown in Figure 1, water flow is at steady state in the equilibrium and unaffected zones but varies with time in the transition zone. In the transition zone, when one or more stream tubes are emptied, it will cause $\theta$ to change by a small amount ($\Delta\theta$) and $q$ to change by $\Delta q$. In other words, the water content change of $\Delta\theta$ in these stream tubes contributes to the water flux change of $\Delta q$. Here, the PSS
water velocity \( (v) \) is the derivative \( dq/d\theta \) when \( \Delta \theta \) approaches zero:

\[
v = dq/d\theta
\] (2)

Hence, \( v \) is the slope of the \( q \) vs. \( \theta \) curve. Substituting Equation 1 into Equation 2 and conducting differentiation yields the PSS velocity in the largest water-filled stream tubes corresponding to \( \theta \):

\[
v = \frac{K_s(3 + 2/\lambda)}{\theta_s - \theta_t}(\theta - \theta_t) (\theta_s - \theta_t)^{2+2/\lambda}.
\] (3)

Equation 3 shows that \( v \) is a function of \( \theta \) and increases with \( \theta \), indicating larger \( v \) in larger pores. The PSS \( v \) at a given \( \theta \) is independent of the pre- or post-barrier soil water content. Furthermore, \( v \) is independent of \( z \) and \( t \), meaning the PSS water velocity in water-filled pores of a specific size is a constant for a given soil during the drainage process. Based on Equation A5, \( v \) in Equation 3 can be written in terms of the pre-barrier conditions (i.e., \( q_i \) and \( \theta_i \)) as

\[
v = \frac{q_i(3 + 2/\lambda)}{\theta_i - \theta_t}(\theta - \theta_t) (\theta_s - \theta_t)^{2+2/\lambda}.
\] (4)

Equation 4 can also be expressed as a function of \( q \). Combining Equations 1 and 4 and rearranging produce

\[
v = \frac{q_i(3 + 2/\lambda)}{\theta_i - \theta_t}(\frac{q}{q_i})^{(2\lambda+2)/(3\lambda+2)}.
\] (5)

The PSS velocity at the drainage front and the unaffected zone (\( v_0 \), Figure 1b) has the largest value corresponding to the initial recharge rate of \( q = q_i \) (and water content of \( \theta = \theta_i \)):

\[
v_0 = \frac{q_i(3 + 2/\lambda)}{\theta_i - \theta_t}.
\] (6)

Combining Equations 4, 5, and 6 yields PSS water velocity in the transition zone as

\[
v = v_0 \left(\frac{\theta - \theta_t}{\theta_i - \theta_t}\right)^{2+2/\lambda} = \frac{v_0}{\frac{q_i}{q}} \left(\frac{q}{q_i}\right)^{(2\lambda+2)/(3\lambda+2)}.
\] (7)

### 2.2.2 Four characteristic drainage velocities

The drainage velocity varies between the minimum and maximum values. Four characteristic drainage velocities are defined here. The drainage front velocity, which is the maximum value \( v_0 \), has been given in Equation 6. In the same way as used in deriving Equation 5, the drainage tail velocity \( (v_t, \text{Figure 1b}) \) has the minimum \( v \) value corresponding to the drainage rate from the surface barrier \( q = q_i \):

\[
v_t = \frac{q_i(3 + 2/\lambda)}{\theta_i - \theta_t}.
\] (8)

where \( \theta_t \) is the water content corresponding to \( q_i \).

The average drainage velocity \( (\bar{v}) \) is defined by assuming all the draining water in the transition zone flows at the same velocity (Figure 1b) and is calculated by

\[
\bar{v} = \frac{q_i - q_t}{\theta_i - \theta_t}.
\] (9)

At the average drainage velocity, according to Equation 7, the ratio \( (R_t) \) of the flux rate to the initial value is

\[
R_t = \frac{q}{q_i} \left(\frac{\bar{v}}{v_0}\right)^{(3+2)/(2+2\lambda)}.
\] (10)

Comparing Equations 9 and 6 shows that \( \bar{v} \) is much smaller than \( v_0 \). When \( q_i \ll q_t, \theta_t \) approaches \( \theta_i \), the \( \bar{v}/v_0 \) ratio approaches \( 1/(3 + 2\lambda) \), which has a maximum value of 1/3. For soils with \( \lambda \) varying from 0.2 to 2 (which covers the range for most soils), \( \bar{v}/v_0 \) ratio varies between 0.08 and 0.25, and \( R_t \) varies between 0.06 and 0.16.

The 50\% soil water velocity \( (v_{0.5}) \) is the velocity at which soil water flux rate is reduced by 50\% (i.e., \( q = 0.5q_i \); Figure 1b) and can be obtained from Equation 7 by setting \( q/q_i = 0.5 \):

\[
v_{0.5} = \frac{v_0}{2^{(2\lambda+2)/(3\lambda+2)}} = \beta_{0.5}v_0
\] (11)

where \( \beta_{0.5} \) ranges between 0.5 and 0.63 for soils with \( \lambda \) varying from 0.2 to 2. These velocity expressions will be used to derive the four impact depths below.

### 2.2.3 Water content distribution

Soil water content at any \( z \) and \( t \) can be derived using Equation 7 by setting \( v = z/t \):

\[
\theta(z, t) = \begin{cases} 
\theta_i, & \text{if } z > v_0t \\
\frac{\theta_i}{\theta_t}, & \text{if } z < v_1t \\
\theta_i + (\theta_t - \theta_i)(\frac{z}{v_{0.5}t})^\lambda, & \text{otherwise}
\end{cases}
\] (12)

The third expression of Equation 12 is the same as the formula in the Table 1 of Sisson, Ferguson, and van Genuchten (1980) for the Brooks and Corey (1966) model, confirming the validity of the above derivation.
### Table 1 Formulas for calculating the near-zero-impact, full-impact, average-impact, and 50% impact depths of a surface barrier

| Depth                                | Formula                                                                 | Equation number |
|--------------------------------------|-------------------------------------------------------------------------|-----------------|
| Near-zero-impact depth (drainage front) | \( z_0 = \frac{q_0 (3 + 2\beta) t}{\theta_i - \theta_s} \)         | 13              |
| Full-impact depth (drainage tail)    | \( z_1 = \frac{q_1 (3 + 2\beta) t}{\theta_i - \theta_s} \)         | 14              |
| Average-impact depth                 | \( \bar{z} = \frac{(q_1 - q_i) t}{\theta_i - \theta_s} \)         | 15              |
| 50% impact depth                     | \( z_{0.5} = \frac{q_i (3 + 2\beta) t}{(\theta_i - \theta_s) 2^{(2\lambda + 2)/(3\lambda + 2)}} \) | 16              |

### 2.3 Barrier impact depths

With the unit gradient assumption, the velocity of water, and thus of the air–water interface, in a stream tube is invariant with time. The part of the tube above the air–water interface is air filled and the part below is water filled. The air–water interface is considered as the drainage depth in the tube. Hence, the travel distance, \( z \), of the air–water interface in a stream tube at time \( t \) can be calculated by \( z = vt \). Corresponding to the four characteristic drainage velocities described above, four barrier impact depths, which are illustrated in Figure 1b, are defined here:

- The **near-zero-impact depth** \( (z_0) \) corresponds to the depth of the drainage front. It is the largest depth at which the surface barrier starts taking effect and is calculated by \( z_0 = v_0 t \).
- The **full-impact depth** \( (z_1) \) corresponds to the drainage tail depth, above which the surface barrier effect has reached its maximum. In other words, the water content has reached the final equilibrium water content. It is calculated by \( z_1 = v_1 t \).
- The **average-impact depth** \( (\bar{z}) \) is obtained by assuming all the draining water flows at the average velocity, \( \bar{v} \) (i.e., \( \bar{z} = \bar{v} t \)).
- The **50% impact depth** \( (z_{0.5}) \) is the depth at which the maximum PSS water velocity is half of the initial maximum velocity (i.e., \( z_{0.5} = v_{0.5} t \)).

The formulas for calculating these impact depths are summarized in Table 1 (Equations 13–16). The results indicate that the barrier impact varies from 0 (no effect) to 100% (full effect) with a large depth range between \( z_1 \) and \( z_0 \) as shown in Figure 1. The above derivation produces the following observations:

- All the barrier impact depths have an initial value of zero at the time of barrier emplacement and then linearly increase with time. At a given time, \( z_1 < \bar{z} < z_{0.5} < z_0 \).
- Except for the full-impact depth, all the other impact depths of a surface barrier are linearly proportional to the initial recharge rate according to Equations 13, 14, and 16.
- The full-impact depth is linearly proportional to \( q_i \). When \( q_i = 0 \), \( z_1 \) is always zero because theoretically it takes an infinitely long time for the water flux to reach zero.
- Finer soils tend to have larger \( \theta_i - \theta_s \) and hence smaller impact depths for given \( q_i \) and \( t \).
- \( z_{0.5}/z_0 = 2^{-\frac{2\lambda + 2}{3\lambda + 2}} \), which ranges between 0.5 (when \( \lambda \) approaches to zero) and 0.63 \( (= 2^{-2/3}) \) when \( \lambda \) approaches infinity), indicating that \( z_{0.5} \) is slightly larger than half of \( z_0 \).
- \( \bar{z}/z_0 \approx 1/(3 + 2\beta) \), which has the maximum of 1/3 (when \( \lambda \) approaches infinity) for all the soils, indicating that \( \bar{z} \) is no more than 1/3 of \( z_0 \).

### 3 Methods of Verification

Field experiment data from literature and numerical simulations were used to verify the algebraic expressions for calculating water content distribution and/or the barrier impact depths.

#### 3.1 Field experiment

A drainage experiment was conducted in a steel caisson at the U.S. Department of Energy’s Hanford Site, near Richland, WA (Rockhold, Fayer, & Gee, 1988). The caisson was 7.6 m deep and 2.7 m in diameter. It was filled with a relatively uniform soil, consisting of approximately 97% sand, 2% silt, and 1% clay. Soil water contents were measured using a neutron probe at different times every 0.3 m down to 2.4-m depth, with an additional reading at 0.45-m depth. The hydraulic parameters of this soil were estimated using the inverse technique and reported in Zhang, Ward, and Gee (2003) as \( \theta_s = 0.310 \text{ m}^3 \text{ m}^{-3} \), \( \theta_i = 0.096 \text{ m}^3 \text{ m}^{-3} \), \( \alpha = 8.05 \text{ m}^{-1} \), \( n = 4.81 \), and \( K_s = 2.92 \times 10^{-4} \text{ m s}^{-1} \), where \( \alpha \) is the inverse of the air-entry pressure, and \( n \) is the pore-size distribution parameter.
The $\lambda$ parameter in Equation 12 was approximated by $\lambda = n - 1 = 3.81$.

### 3.2 Numerical simulations

Numerical simulation of flow beneath a surface barrier for hypothetical scenarios was conducted in soils typical of the Hanford site, where demonstrative surface barriers have been tested (DOE-RL, 2016) and interim surface barriers have been deployed (Henderson, 2011; Zhang, Strickland, Field, & Parker, 2010) to contain the underground waste. It was assumed that the vadose zone is homogeneous and the water table is beyond the maximum depth of investigation (i.e., 100 m). The flow in the soil before barrier emplacement was assumed to be at steady state under constant recharge rates of 30 or 100 mm yr$^{-1}$. The drainage rate from the barrier was assumed to be 0.5 mm yr$^{-1}$, which was the criteria for the 1,000-year prototype Hanford barrier over a nuclear waste site (Zhang, Strickland, & Link, 2017). Because a number of surface barriers are expected to be constructed at the Hanford site (DOE-RL, 2013), four types of sediments (Table 2) typical at the site were selected for the simulation: backfill (Bf), Hanford silt sand (Hss), Hanford fine sand (Hfs), and Hanford coarse sand (Hcs).

The simulation domain was 100 m deep and discretized uniformly into 1,000 cells. The initial condition was the steady-state condition corresponding to the pre-barrier recharge rate (i.e., 30 or 100 mm yr$^{-1}$) for the soil. The simulations started at the time of barrier emplacement ($t = 0$). After barrier emplacement, the top boundary condition was the post-barrier recharge rate of 0.5 mm yr$^{-1}$, and the bottom boundary condition was unit gradient to mimic a semi-infinite medium. The drainage process of 50 yr after barrier emplacement was simulated using the Subsurface Transport Over Multiple Phase (STOMP) simulator (White et al., 2015).

The numerical simulation results show diffused drainage front. Because there is no official definition for how to determine the location of the drainage front, the depth of the drainage front for the simulated results is defined here as the depth at which soil water content is lower than the initial value by a certain percentage $\beta_h$. For example, for the Hss sediment with a 30-mm yr$^{-1}$ pre-barrier recharge rate, $\theta_t = 0.1720$ m$^3$ m$^{-3}$. If $\beta_h = 2\%$, the water content at the barrier drainage front is $\theta = (1 - \beta_h)\theta_t = 0.1686$ m$^3$ m$^{-3}$. Similarly, the depth of the drainage tail is defined as the depth at which soil water content higher than the final value by a certain percentage $\beta_t$. For example, for the Hss sediment with a 0.5-mm yr$^{-1}$ post-barrier recharge rate, $\theta_t = 0.1180$ m$^3$ m$^{-3}$. If $\beta_t = 5\%$, the water content at the barrier drainage tail is $\theta = (1 + \beta_t)\theta_t = 0.1239$ m$^3$ m$^{-3}$. The average depth of the drainage zone from numerical simulation ($z_{num}$) was determined as if all the drainage water would have moved like a piston:

$$z_{num} = \frac{1}{\theta_t - \theta_i} \int_0^{z_{max}} (\theta_i - \theta) \, dz \quad (17)$$

where $z_{max}$ is the maximum depth of the simulation domain (= 100 m for the simulations conducted). Equation 17 requires that the drainage front not pass the bottom of the simulation domain in order to calculate $z_{num}$. The $z_{num}$ value cannot be determined once the drainage front passes the bottom of the simulation domain.

### 4 RESULTS AND DISCUSSION

The following compares the results from the algebraic expressions of water content and barrier impact depths against those from the drainage experiment and/or the numerical simulations as tests of the developed expressions. General characteristics of the drainage velocities and barrier impact depths and their implications on site closure with surface barriers are discussed. The simulation results for only two of the four sediments are shown in figures in the main text for the purpose of demonstration and conciseness, whereas the figures for the other two sediments are provided in the supplemental material.

### 4.1 Water content profile

Figure 2 compares the measured and predicted water content of the drainage experiment in the steel caisson at different times. At 8 min from the start of the drainage process, the model (Equation 12) predicted water content at shallow depths ($\leq 0.6$ m) compares well with the observations. The observations indicated that the soil at 0.9-m depth and deeper was still saturated, whereas the analytical model predicted...
a transition zone of drainage from ground surface to 2.2-m depth. The difference between observation and the analytical model prediction in water content distribution at depths between 0.9 and 2.2 m could be due to the error in time recording during experiment because it took about 5 min to complete one set of neutron logging. Additionally, the soil surface was ponded with water before drainage and hence the exact time at which the drainage process started was subject to error. At longer times (i.e., 34 min, 4 h, and 76 h) of drainage, the predicted water content matched the observations reasonably well except the bottom-most observation at 34 min and 4 h.

Figure 3 compares the water content profiles at different times after barrier emplacement between the analytical model (Equation 12) and the numerical model. The soils received a recharge of 30 or 100 mm yr⁻¹ before barrier emplacement. Overall, the analytical and numerical models predicted very similar results. For both analytical and numerical results, there was a clear transition zone for all the soils at a given time. The water content in the transition zone increased monotonically with depth and the drainage front extended to larger depth with time. An unaffected zone was present before the drainage front passed the maximum depth of the simulation domain (i.e., 100 m), and the soil water contents in it were the same as the corresponding initial values, as expected. For example, for the Hss sediment, when the pre-barrier recharge rate was 30 mm yr⁻¹ (Figure 3a), the water content in the transition zone ranged between the final water content (0.118 m³ m⁻³) and the initial water content (0.199 m³ m⁻³).

There were also some differences between results from the analytical and numerical models. For all cases, the numerical model predicted diffused drainage fronts, whereas the analytical model predicted sharp drainage fronts (Figure 3). As a result, the analytical model predicted slightly higher soil water contents near the drainage fronts. This difference could be due to the numerical dispersion from the numerical model. In the shallow depth, the analytical model predicted lower soil water content than the numerical model. Additionally, the analytical model did not predict clear equilibrated zones at the end of simulation time of 50 yr. A possible reason for the differences in the small depths is that unit gradient was assumed in the development of the analytical model, but capillary effect may cause the hydraulic gradient to be less than the unit gradient.

4.2 | Barrier impact depths

The barrier impact depths were quantified by the near-zero-, 50%, average-, and full-impact depths. For the numerical simulations, $z_0$ was determined by setting $\beta_h = 2\%$ and $z_1$ was determined by setting $\beta_l = 5\%$. These $\beta_h$ and $\beta_l$ values were selected because they are reasonably small, and the results between the analytical and numerical models compared relatively well.

Figure 4 shows the $z_0$, $z_{0.5}$, $\bar{z}$, and $z_1$ values determined with numerical simulations and the algebraic expressions for the two selected sediments that received recharge rates of 30 and 100 mm yr⁻¹ before barrier emplacement. For all the cases, all the four impact depths from the numerical simulations and the analytical methods compared fairly well, indicating the algebraic expressions can be used to predict these impact depths of a surface barrier. For all the cases, the impact depths increased nearly linearly with time, as indicated by Equations 13–16. Note that some of the $\bar{z}$ and/or $z_0$ values from numerical simulation could not be determined after the drainage front passed the bottom of the simulation domain (i.e., 100 m). For example, for soil Hss with a pre-barrier recharge rate of 100 mm yr⁻¹ (Figure 4c), $z_0 = 95$ m at 20 yr for the numerical model. At the next output time of 25 yr, $z_0$ was larger than 100 m. Hence, neither $z_0$ nor $\bar{z}$ could be determined from numerical results after 20 yr for this case.

For any of the drainage scenarios, the four impact depths differed substantially because of the diffused but wide transition zone below a surface barrier. For example, for soil Hss with a pre-barrier recharge rate of 30 mm yr⁻¹ (Figure 4a), the four impact depths of the barrier predicted by the analytical
FIGURE 3 The water content profiles determined with numerical simulation and the algebraic expression (Equation 12) for the two selected sediments (Hanford silt sand [Hss] and Hanford coarse sand [Hcs]) that received recharge rates of 30 and 100 mm yr\(^{-1}\) before barrier emplacement. \(q_i\) = initial water flux rate

model were \(z_1 = 2.8\) m, \(\bar{z} = 27.4\), \(z_{0.5} = 44.3\), and \(z_0 = 77.5\) m at \(t = 50\) yr. They vary by a factor of 28.

The velocities of the drainage front and tail, the average and 50% drainage velocities determined by the analytical model, are summarized in Table 3. The near-zero-impact depth for a given soil is a function of the drainage front velocity (Table 3) in the soil beneath the barrier, which is dependent on the initial recharge rate and the hydraulic properties of the soil. For the four sediments, the drainage front velocity varied between 1.55 to 3.54 m yr\(^{-1}\) if \(q_i\) was 30 mm yr\(^{-1}\) and between 4.09 and 9.25 m yr\(^{-1}\) if \(q_i\) was 100 mm yr\(^{-1}\). Very roughly, for the cases investigated, \(v_{0.5}\) is one-half of \(v_0\) and \(\bar{v}\) is one-third of \(v_0\), as indicated by the algebraic expressions.

The above results demonstrate that it may take considerable time for a surface barrier to become effective at relatively large depth. The full impact may never be reached in the waste zone during the design life of a surface barrier because of the very small \(v_1\) values (i.e., between 0.06 to 0.14 m yr\(^{-1}\)). For the cases investigated, the drainage front moves at a velocity of a few meters per year. For a vadose zone as thick as 100 m, it would take >10 or tens of years for the drainage front to arrive at the groundwater. For the contaminants residing in the deep vadose zone, it would take some time (depending on the residence depth of contaminants) for the drainage front to arrive at the residence depth of contaminants and slow down the contaminant transport. Before the arrival of the drainage front, the contaminant is expected to migrate at the same velocity as it would if there was no surface barrier.

Although the surface barrier technology is expected to apply to many of the waste sites for post-closure, a surface barrier will not be effective for sites where contaminants reside very close to the groundwater. If the contaminant is very close to the groundwater, it will arrive at the groundwater before the drainage front. The algebraic expressions provided in this paper are useful for quickly identifying sites where the depth of the existing contaminants could be beyond the protection of a surface barrier. In those cases, the identified sites should be examined more closely relative to the assumptions of homogeneity, infinite barrier size, groundwater depth, contaminant behavior (e.g., sorption, decay), and biological interactions, as well as the depth distribution of the contaminants. In addition, such sites might become candidates for remediation with other technologies.
FIGURE 4  The near-zero- ($z_0$), 50% ($z_{0.5}$), average- ($z_{\text{avg}}$), and full-impact ($z_1$) depths of a surface barrier determined with numerical simulations and the algebraic expressions (Equations 13–16) for the two selected sediments (Hanford silt sand [Hss] and Hanford coarse sand [Hcs]) that received recharge rates of 30 and 100 mm yr$^{-1}$ before barrier emplacement. $q_i =$ initial water flux rate

| Pre-barrier recharge rate mm yr$^{-1}$ | Soil class$^b$ | Variable$^a$ | Hss | Hfs | Hcs |
|-------------------------------------|----------------|-------------|-----|-----|-----|
| 30                                  |                | $v_0$       | 3.09| 1.55| 2.36| 3.54|
|                                     |                | $v_{0.5}$   | 1.68| 0.89| 1.37| 2.04|
|                                     |                | $\bar{v}$   | 0.95| 0.54| 0.85| 1.25|
|                                     |                | $v_1$       | 0.09| 0.06| 0.09| 0.14|
| 100                                 |                | $v_0$       | 8.85| 4.09| 6.09| 9.25|
|                                     |                | $v_{0.5}$   | 4.83| 2.34| 3.53| 5.32|
|                                     |                | $\bar{v}$   | 2.27| 1.23| 1.91| 2.83|
|                                     |                | $v_1$       | 0.09| 0.06| 0.09| 0.14|

$^a$ $v_0$, pore-size-specific velocity at the drainage front and the unaffected zone; $v_{0.5}$, 50% soil water velocity; $\bar{v}$, average drainage velocity; $v_1$, drainage tail velocity.

$^b$ Bf, backfill; Hss, Hanford silt sand; Hfs, Hanford fine sand; Hcs, Hanford coarse sand.

5 | SUMMARY

The impact depths of a surface barrier are characterized with the depths of drainage front, 50% flux reduction, average drainage, and drainage tail. These are termed as the near-zero-, 50%, average-, and full-impact depths, respectively. Algebraic expressions are developed to estimate drainage velocities and barrier impact depths based on the STB concept. The algebraic expressions for drainage velocities show that the drainage velocities at the impact depths are invariable with time in a uniform soil with fixed initial and final recharge rates. Hence, all four impact depths increase linearly with time. The barrier impact depths for a given soil are dependent on the hydraulic properties of the soil beneath the barrier and the initial and/or final recharge rates. The average-impact depth is no more than one-third of the near-zero-impact depth, whereas the 50% impact depth is slightly larger than one-half of the near-zero-impact depth. The full-impact depth, depending on the final recharge rate from the surface barrier, is usually much smaller than the other impact depths. These differences lead to a drainage zone of very large depth range during the drainage process below a surface barrier. The test against results from a drainage experiment and a series of numerical simulation showed that the analytical models predicted very similar results of water content profile and the near-zero-,
50%, average-, and full-impact depths of a surface barrier to the counterparts from the experiment and/or the numerical model.

CONFLICT OF INTEREST
The authors declare no conflict of interest.

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Global Change Biology is the saturated hydraulic conductivity. \( K \)

\[ K = \theta - \theta_i (\alpha \psi)^{-\lambda} \]  

\[ K = K_s (\theta - \theta_i) (\theta_s - \theta_i)^{3+2/\lambda} \]  

where \( \psi \) is the pressure head (with a negative value under an unsaturated condition) and \( \alpha \) is the inverse of the air-entry pressure, \( \lambda \) is the pore-size distribution parameter, \( \theta \) is water content, \( \theta_s \) and \( \theta_i \) are saturated and residual water contents, respectively, and \( K_s \) is the saturated hydraulic conductivity. The hydraulic conductivity at the initial (\( K_i \)) and the final (\( K_f \)) steady-state are

\[ K_i = K_s (\theta_i - \theta_i) (\theta_s - \theta_i)^{3+2/\lambda} \]  

\[ K_f = K_s (\theta_i - \theta_i) (\theta_s - \theta_i)^{3+2/\lambda} \]  

where \( \theta_i \) is the initial water content and \( \theta_i \) is the final steady-state water content. Combining Equations A2, A3, and A4 produces

\[ K = K_i (\theta_i - \theta_i)^{3+2/\lambda} = K_f (\theta_i - \theta_i)^{3+2/\lambda} \]  

\[ K = K_i (\theta - \theta_i)^{3+2/\lambda} = K_f (\theta - \theta_i)^{3+2/\lambda} \]  

**APPENDIX**

**Hydraulic properties of porous media**

The Brooks and Corey (1966) water retention model and the Burdine (1953) relative permeability model are used to describe the hydraulic properties:

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