A flexible fiber reveals the secrets of turbulence

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We study the dynamics of a flexible fiber freely moving in a three-dimensional fully-developed turbulent field and present a phenomenological theory to describe the interaction between the fiber elasticity and the turbulent flow. This theory leads to the identification of two distinct regimes of flapping, which we validate against Direct Numerical Simulations (DNS) fully resolving the fiber dynamics. The main result of our analysis is the identification of a flapping regime where the fiber, despite its elasticity, is slaved to the turbulent fluctuations. In this regime the fiber can be used to measure two-point statistical observables of turbulence, including scaling exponents of velocity structure functions, the sign of the energy cascade and the energy flux of turbulence, as well as the characteristic times of the eddies within the inertial range of scales. Our results are expected to have a deep impact on the experimental turbulence research as a new way, accurate and efficient, to measure two-point statistics of turbulence.

Understanding how elastic structures interact with a flow is a problem attracting a great deal of attention in different fields of science and technology. Numerous observations \cite{1} suggest that aerial and aquatic animals control the flow around their bodies and reduce drag by pelage, scales, feathers and other elastic appendages \cite{2–4}, with a wide range of textures active under different flow conditions. In many cases the functional role of elastic appendages is not fully understood yet, as their complex motion varies greatly depending on the task.

With the advent of efficient Wireless Communication Technologies (WCT), the instabilities arising from the interactions between elastic structures (e.g. elastically-mounted flapping wings \cite{5, 6}) and a flow are being exploited to harvest energy from the environment \cite{5–8}. This strategy is promising to overcome the bottleneck of battery costs and duration. From a technological point of view, a deeper understanding of the basic mechanisms of interaction between fluid flows and elasticity is therefore a crucial requirement towards an all-pervasive spread of a low-cost Internet of Things.

The study in Ref. \cite{9} allowed a huge step forward in the understanding of the coupling between laminar flows and structure elasticity. The breakthrough was possible thanks to the combined choice of a simple flow configuration and a simple elastic structure (a flexible fiber). As far as the first is concerned, a soap film was considered as a convenient experimental system for laminar two-dimensional flows \cite{4, 10–12}. The elastic object was a flexible fiber of given rigidity and inertia. Even in the (apparently simple) case of laminar flows and simple elastic objects the coupling between fluid and structure gives rise to a nontrivial rich phenomenology. Once the underlying mechanisms have been understood in the laminar case, a new Pandora’s box is opening, containing questions to be answered on a fiber freely-moving in a three-dimensional turbulent environment (see Fig. 1). In which way a flexible fiber interacts with a turbulent flow? Under which conditions flapping motion will appear? How many states of flapping are possible? Can we control the amplitude/frequency of the resulting flapping states? Can the fiber be used to reveal the two-point statistics of turbulence?

We answer these unsolved questions exploiting, in synergy, a phenomenological theory we propose here, and
DNS of a flexible fiber in three-dimensional homogeneous isotropic stationary turbulence.

First, we present a simple model coupling the fiber dynamics to the flow. The fluid flow, \( \mathbf{u}(x, t) \), and fiber position \( \mathbf{X}(s, t) \) are ruled by the mass and momentum conservation equations, written in the immersed boundary form [13–15],

\[
\begin{align*}
\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla p/\rho_0 + \nu \nabla^2 \mathbf{u} + \mathbf{f}, \\
\nabla \cdot \mathbf{u} &= 0, \\
\rho_1 \ddot{\mathbf{X}} &= \partial_s (T \partial_s (\mathbf{X})) - \gamma \partial_s^4 \mathbf{X} + \mathbf{F}.
\end{align*}
\]

In the last equation, \( T \) is the tension, which enforces the fiber inextensibility, \( \rho_0 \) and \( \nu \) are the fluid density and kinematic viscosity, \( \rho_1 \) the difference between the fiber density and fluid density, \( \gamma \) the fiber bending rigidity and \( c \) the fiber length. The fluid and the fiber are coupled at their interface by the no-slip condition \( \mathbf{X} = U(\mathbf{X}(s, t), t) \), with \( U(\mathbf{X}(s, t), t) = \int \mathbf{u}(x, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) \, dx \) the Lagrangian force density and \( f(x, t) = \int d\mathbf{F}(s, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) \) the Eulerian fluid-structure interaction force density and \( \mathbf{F}(s, t) \) the Lagrangian force density. Free-end conditions at \( s = 0 \) and \( s = c \) are used, \( c \) being the rest length of the fiber. An additional volume force is considered on the right hand side of the Navier–Stokes equations in (1) (not shown for the sake of brevity) to generate a fully-developed turbulent state with isotropic, homogeneous, and stationary statistics.

We focus our attention on the fiber equation in a given turbulent environment. Such an intrinsically passive way of thinking at the fiber dynamics has a analog in polymer physics [20, 21] where it was successful, e.g., to predict the statistics of polymer elongations in a turbulent flow [22]. Let us assume a viscous coupling of the form \( \mathbf{F} = -\mu \mathbf{X} - \mathbf{u} \), with \( \mu \) being the dynamic viscosity of the flow [23]. Different regimes are expected depending on the value of the damping ratio \( \zeta = \tau_\mu/\tau_\gamma = (\alpha c^2/\mu)/(2\rho_1^2/\gamma)^{1/2} \), where \( \tau_\mu = 2\rho_1/\mu \) is the viscous damping and \( \tau_\gamma = \alpha (\rho_1 c^4/\gamma)^{1/2} \) is the fiber elastic time [24].

For \( \zeta > 1 \) the fiber undergoes an over-damped regime where inertia effects are strongly inhibited. Conversely, inertia overcomes dissipation in the under-damped regime corresponding to \( 0 < \zeta < 1 \). In this case the elasticity is expected to affect the fiber dynamics.

Let us focus on the latter, dynamically richer, regime and start to analyze two opposite situations. For large elasticity, only large strains may appreciably deform the fiber. When these events occur the fiber, once deformed, rapidly reacts back trying to restore the straight position. Because of inertia, the relaxation process is expected to be dominated by rapid oscillations having the characteristic elastic time \( \sim \tau_\gamma \). In the opposite limit of small fiber elasticity, the fiber does not resist strains, even to the weakest, and is thus expected to be slaved to the turbulent fluctuations. We thus argue the existence of a critical value \( \gamma_{\text{crit}} \) of the fiber bending rigidity separating these two distinct behaviors. We claim here that \( \gamma_{\text{crit}} \) can be extracted from a resonance condition between the fiber elastic time \( \tau_\gamma \) and the eddy turnover time \( \tau_r \), with \( \epsilon \) being the turbulence dissipation rate of kinetic energy. The condition \( \tau_r \sim \tau_\gamma \) immediately gives:

\[
\alpha \left( \frac{c^4 \rho_1}{\gamma} \right)^{1/2} \sim \epsilon^{2/3} \gamma_{\text{crit}} \sim \epsilon^{8/3} \gamma^{2/3} \rho_1 \alpha^2.
\]

For \( \gamma/\gamma_{\text{crit}} < 1 \), the elastic fiber, in spite of its elasticity, is prone to capture the turbulent fluctuations carried by the turbulent eddies at the scale of the fiber length. In this dynamical regime, an elastic fiber is expected to reveal all relevant properties of turbulence two-point statistics.

The remaining of the present Letter is devoted to prove our conjectures exploiting accurate DNS coupled to an efficient IBM strategy to resolve the fully-coupled fiber-flow dynamics. Details on the numerical strategy are given in [17, 19]. Once the turbulent flow is at steady state, a fiber is left free to evolve and the resulting fully-coupled problem is numerically solved following the IBM strategy detailed in Ref. [13].

To start our analysis, let us provide a justification for the term ‘resonance’ we have associated to the condition (2). In Fig. 2 we report the fiber elastic energy as a function of \( \gamma/\gamma_{\text{crit}} \) for different values of the fiber length. All cases fall in the under-damped regime, \( \zeta \) being always smaller than 1. The peaks at \( \gamma/\gamma_{\text{crit}} \sim 1 \) provide a first clue that \( \gamma_{\text{crit}} \) we have identified plays a dynamical role. To confirm the role of \( \gamma_{\text{crit}} \) we have analyzed long time-series (corresponding to \( \sim 20 \) large-eddy turnover times),
of the motion of 30 different fibers, corresponding to different combinations of 3 different densities $\rho_1$, 3 lengths $c$ and 9 bending rigidities $\gamma$. Also in the present situation, all analyzed cases belong to the under-damped case. Once the fiber evolution has been computed, we examine the Fourier transform of the time history of the end-to-end distance and extract the leading oscillation frequency. These oscillation frequencies, $f$, normalized by $f_{\text{turb}} = 1/\tau(c)$, are reported as a function of $\gamma/\gamma_{\text{crit}}$ in Fig. 3. The outcome is quite impressive and fully confirms our expectations. We indeed observe a beautiful data collapse pointing to the following four main conclusions. i) The value of $\gamma_{\text{crit}}$ we have identified separates two distinct regimes in the under-damped case. Surprisingly, the transition is very sharp. ii) The expression for $\gamma_{\text{crit}}$ extrapolated from Eq. (6) of Ref. [25] ($\gamma_{\text{crit}} \sim c^4 (\rho_0 L)^{1/2}$) seems to be ruled out from our data. The formulation provided in Ref. [25] does not account for any dependence on the fiber density which is however correctly captured by our model as shown in Fig. 3. A possible reason for the neat discrepancy between the two formulations is that the fibers considered in Ref. [25] are close to the integral scale of the flow while they are well within the inertial range of scales in the present case. iii) For $\gamma/\gamma_{\text{crit}} < 1$ the most energetic mode of oscillation of the fiber is at the turbulence frequency $1/\tau(c)$. iv) For $\gamma/\gamma_{\text{crit}} > 1$ the most energetic mode of oscillation is associated to the first fiber normal mode and has the frequency $f = 1/\tau_{\gamma}$.

The fact that for $\gamma/\gamma_{\text{crit}} < 1$ the fiber is locked to the frequency of the turbulent eddies with the same size of the fiber suggests the possibility that the fiber is able to reveal the turbulence velocity fluctuations. In plain words, we consider our fiber as a physical proxy of the celebrated turbulent eddies. This being the case, a massive fiber, which can be easily tracked in a turbulent flow, may reveal the features of eddies of different scales. To demonstrate that our conjecture is true, we compute the longitudinal structure functions $S_p(r)$, $p = 2, 3$ defined in terms of the fiber velocities at the fiber end points projected along the end-to-end fiber vector for $\gamma/\gamma_{\text{crit}} = 1/2$. We compute $S_p(r)$ from three different fibers, with different rest lengths belonging to the inertial range of scales. As far as the separation $r$ is concerned, instead of the fiber rest length, we use the time-average value of the fiber end-to-end distance, as this is a quantity more representative of the dynamical fiber length. All regimes analyzed fall in the under-damped case and each fiber has been tracked for 30 large-eddy turnover times. The results are presented in Fig. 4. Black bullets indicate the second and third-order velocity structure functions obtained following the standard Eulerian procedure, with averages both in space and in time exploiting the homogeneity and stationarity of turbulence statistics. Here we see the high quality of the resulting scaling behavior and also a convincing reproduction of the celebrated Kolmogorov 4/5th law for the third-order structure function. The value of $c$ used here has been determined independently from its definition. There are thus no free parameters.

Markers with error bars indicate the structure functions obtained from the fibers as explained above. Error bars have been determined from the convergence profile of both structure functions (ordinates) and end-to-end fiber distances (abscissa). The agreement between the Eulerian measurements and those obtained from the fibers is within error bars.

A further confirmation comes from the inspection of
the probability density function (pdf) of longitudinal velocity increments. We have reported such a quantities in Fig. 5 for a separation corresponding to one of the three fibers reported in Fig. 4. Open circles reproduce the pdf built in the Eulerian frame while bullets the pdf built from the longitudinal velocity differences evaluated from the fiber velocities at the fiber end points. Similar agreements have been observed (not shown) for the other two fiber lengths considered in Fig. 4. We can thus conclude that the regime $\gamma/\gamma_{\text{crit}} < 1$ not only allows one to measure the eddy turnover time of turbulence at the fiber length-scale, but also to quantitatively access the statistical properties of the two-point statistics of turbulence.

There remain to discuss the over-damped regime, $\zeta > 1$. The fiber equation becomes in this case first-order in time: once deformed by strain, the fiber reacts exponentially with the typical time scale $\mu_c^4/\gamma$. No elastic oscillations occur in the present regime. For $\mu_c^4/\gamma \ll \tau(c)$ the relaxation process is faster than the eddy turnover time at the length-scale of the fiber. The opposite occurs for $\mu_c^4/\gamma \gg \tau(c)$. A critical value of the fiber bending rigidity separating the two regimes can thus be identified: $\gamma_{\text{crit}}^{od} \sim \mu_c^{10/3} \zeta^{1/3}$. For different reasons, we argue that the fiber undergoes oscillations with frequency $\sim 1/\tau(c)$ in both limits. For $\gamma/\gamma_{\text{crit}}^{od} < 1$ all points of the fiber are indeed expected to move with the fluid velocity under the constraint of fiber inextensibility. For $\gamma/\gamma_{\text{crit}}^{od} > 1$, as in the under-damped case, only large strains may deform the fiber. When it happens, the fiber rapidly reacts back trying to restore the straight position. The relaxation process takes place without oscillations because of the negligible role of inertia. We thus expect that, differently from the under-damped regime, oscillations have frequency $\sim 1/\tau(c)$. Our expectation has been verified numerically for $p_1 \sim 0$, corresponding to $\gamma/\gamma_{\text{crit}}^{od} \gg 1$ and the results (not shown) fully confirm our guess.

In conclusion, we have explored the dynamical properties of a single elastic fiber, free to evolve in a turbulent field. The size of the fibers considered is well within the inertial range of scales. The main result of our analysis has been the identification of a dynamical regime where the fiber, in spite of its elasticity, is slaved to turbulence thus becoming a material realization of the well-known concept of turbulent eddy. Our results extends to inertial range two-point statistics the idea of using deformable particles for single particle measurements of velocity gradient recently presented in Ref. [26]. Further pioneering extensions to multi-point statistics in turbulence seem to be realizable exploiting flexible membranes or other spatially-extended elastic objects.

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FIG. 5: The probability density function of velocity increments for a separation between points belonging to the inertial range of scales ($r = 0.14L$, corresponding to the second marker with error bars of Fig. 4).

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The Navier–Stokes equations are solved using a second order finite-difference scheme in space and third-order Runge-Kutta scheme in time. The pressure is obtained by solving the Poisson equation using Fourier transforms. We use a Cartesian uniform mesh in a rectangular triperiodic box of size $L = 2\pi$, with 128 grid points per side. The grid size is sufficient to obtain a clear inertial range of scale clearly displaying the expected $4/5$th Kolmogorov law. Doubling the resolution in all directions results in an insignificant change in the results. The turbulent dissipation rate $\epsilon$, made dimensionless with the cube of the velocity root-mean square divided by the size of the box, is $2.54$ and the Reynolds number at the Taylor microscale is $Re_{\lambda} = 92$. In order to sustain the turbulent field, we use the spectral forcing scheme described in Ref. [18].

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