Exclusive pion electroproduction and transversity

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In this talk it is reported on an analysis of hard exclusive \( \pi^+ \) electroproduction within the handbag approach. Particular emphasis is laid on single-spin asymmetries. It is argued that a recent HERMES measurement of asymmetries measured with a transversely polarized target clearly indicate the occurrence of strong contributions from transversely polarized photons. Within the handbag approach such \( \gamma^*_T \to \pi \) transitions are described by the transversity GPDs accompanied by a twist-3 pion wave function. It is shown that this approach leads to results on cross sections and single-spin asymmetries in fair agreement with experiment.

Keywords: Handbag factorization, generalized parton distributions, transversity, electroproduction

1. Introduction

In this article it will be reported upon an analysis of hard exclusive electroproduction of positively charged pions\(^1\) within the framework of the so-called handbag approach which offers a partonic description of meson electroproduction provided the virtuality of the exchanged photon, \( Q^2 \), is sufficiently large. The theoretical basis of the handbag approach is the factorization of the process amplitudes in hard partonic subprocesses and soft hadronic matrix elements, parameterized as generalized parton distributions (GPDs), as well as wave functions for the produced mesons, see Fig. 1. In collinear approximation factorization has been shown\(^2,3\) to hold rigorously for exclusive meson electroproduction in the limit \( Q^2 \to \infty \). It has also been shown that the transitions from a longitudinally polarized photon to the pion, \( \gamma^*_L \to \pi \), dominates at large \( Q^2 \). Transitions from transversely polarized photons to pions, \( \gamma^*_T \to \pi \), are suppressed by inverse powers of the hard scale.

However, as has been argued in Ref. 1, \( \gamma^*_T \to \pi \) transitions are quite
large at experimentally accessible values of $Q^2$ which are typically of the order of a few GeV^2. This follows from data of asymmetries measured with a transversely polarized target and is also seen in the transverse cross section measured by the $F_\pi - 2$ collaboration. It is demonstrated in Ref. 1 that within the handbag approach, these $\gamma_T^* \rightarrow \pi$ transitions can be calculated as a twist-3 effect consisting of the leading-twist helicity-flip GPDs combined with the twist-3 pion distribution amplitude. In the following the main ideas of the approach advocated for in Ref. 1 will be briefly described and some of the results will be discussed and compared to experiment.

2. The handbag approach

Within the handbag approach the amplitudes for pion electroproduction through longitudinally polarized photons read

$$M_{0+,0+}^+ = \sqrt{1 - \xi^2} \frac{e_0}{Q} \left[ (\tilde{H}^{(3)}) - \frac{\xi^2}{1 - \xi^2} (\tilde{E}^{(3)}) - \frac{2\xi m Q^2}{1 + \xi^2} \frac{\rho_\pi}{t - m_\pi^2} \right],$$

$$M_{0-,0+}^+ = \frac{e_0}{Q} \left[ \xi (\tilde{E}^{(3)}) + 2m Q^2 \frac{\rho_\pi}{t - m_\pi^2} \right].$$

(1)

Here, the usual abbreviation $t' = t - t_0$ is employed where $t_0 = -4m^2\xi^2/(1 - \xi^2)$ is the minimal value of $t$ corresponding to forward scattering. The mass of the nucleon is denoted by $m$ and the skewness parameter, $\xi$, is related to Bjorken-$x$ by

$$\xi = \frac{x_{Bj}}{2 - x_{Bj}}. \quad (2)$$

Helicity flips at the baryon vertex are taken into account since they are only suppressed by $\sqrt{-t'}/m$. In contrast to this, effects of order $\sqrt{-t'}/Q$ are neglected. The last term in each of the above amplitudes is the contribution from the pion pole (see Fig. 1). Its residue reads

$$\rho_\pi = \sqrt{2g_{\pi NN}} F_\pi(Q^2) F_{\pi NN}(t'),$$

(3)

where $g_{\pi NN}$ is the familiar pion-nucleon coupling constant. The structure of the pion and the nucleon is taken into account by form factors, the electromagnetic one for the pion, $F_\pi(Q^2)$, whereby the small virtuality of the exchanged pion is as usual ignored, and $F_{\pi NN}(t)$ for the $\pi$-nucleon vertex. The pion-pole term has been used in essentially this form in the measurement of the pion form factor. In contrast to other work on hard exclusive pion electroproduction (an exception is Ref. 9) the full pion form...
factor is taken into account this way and not only its so-called perturbative contribution which only amounts to about a third of its experimental value. It is to be stressed that the pion pole also contributes to the amplitudes for transversely polarized photons. However, these contributions are very small.\(^1\)

The convolutions \(\langle F \rangle\) in (1) have been worked out in Ref. 1 with subprocess amplitudes calculated within the modified perturbative approach.\(^10\) In this approach the quark transverse momenta are retained in the subprocess and Sudakov suppressions are taken into account. The partons are still emitted and re-absorbed by the nucleon collinearly, i.e. we still have collinear factorization in GPDs and hard subprocess amplitudes. It has been shown\(^11\) that within this variant of the handbag approach the data on cross sections and spin density matrix elements for vector-meson production are well fitted for small values of skewness (\(\xi \simeq x_B/2 \lesssim 0.1\)).

3. \(\gamma^*_T \to \pi\) transitions

The electroproduction cross sections measured with a transversely or longitudinally polarized target consist of many terms, each can be projected out by \(\sin \phi\) or \(\cos \phi\) moments where \(\phi\) is a specific linear combination of \(\phi_s\), the azimuthal angle between the lepton and the hadron plane and \(\phi_s\), the orientation of the target spin vector. A number of these moments have been measured recently.\(^4,12\) A particularly striking result is the \(\sin \phi_s\) moment. The data on it, displayed in Fig. 2, exhibit a mild \(t\)-dependence and do not show any indication for a turnover towards zero for \(t' \to 0\). This behavior of \(A_{UT}^{T(0)}\) at small \(-t'\) can only be produced by an interference term between the two helicity non-flip amplitudes \(M_{0+,0+}\) and \(M_{0-,++}\) which are not forced to vanish in the forward direction by angular momentum conserva-

![Fig. 1. The pion exchange graph and a typical lowest order Feynman graph for pion electroproduction. The signs indicate helicity labels for the contribution from transversity GPDs to the amplitude \(M_{0-,++}\), see text.](image)
Table 1. Features of the asymmetries for a transversally and longitudinally polarized target. The photon polarization is denoted by L (longitudinal) and T (transversal). Asymmetries under control of TT interference terms are not shown in the table; they are very small. 1: There is a second contribution for which the helicities of the outgoing proton are interchanged.

| observable | dominant interf. term | amplitudes | low t’ behavior |
|------------|-----------------------|------------|-----------------|
| $A_{UT}^{\sin(\phi - \phi_s)}$ | LL | $\text{Im}[M_{0}^{\pi} - \pi, 0+, M_{0}^{\pi} +, 0+]$ | $\propto \sqrt{-t'}$ |
| $A_{UT}^{\sin(\phi)}$ | LT | $\text{Im}[M_{0}^{\pi} - \pi, 0+, M_{0}^{\pi} +, 0+]$ | const. |
| $A_{LT}^{\sin(2\phi - \phi_s)}$ | LT | $\text{Im}[M_{0}^{\pi} - \pi, 0+, M_{0}^{\pi} +, 0+]$ | $\propto t'$ |
| $A_{LU}^{\sin(\phi)}$ | LT | $\text{Im}[M_{0}^{\pi} - \pi, 0+, M_{0}^{\pi} +, 0+]$ | $\propto \sqrt{-t'}$ |
| $A_{LL}^{\cos(\phi)}$ | LT | $\text{Re}[M_{0}^{\pi} - \pi, 0+, M_{0}^{\pi} +, 0+]$ | $\propto \sqrt{-t'}$ |

The amplitude $M_{0\pi}^{\pi} - \pi$ has to be sizeable because of the large size of the $\sin \phi_s$ moment. We therefore have to conclude that there are strong contributions from $\gamma^*_T \rightarrow \pi$ transitions at moderately large values of $Q^2$.

How can this amplitude be modeled in the framework of the handbag approach? From Fig. 1 where the helicity configuration of the amplitude $M_{0\pi}^{\pi} - \pi$ is shown, it is clear that contributions from the usual helicity non-flip GPDs, $\tilde{H}$ and $\tilde{E}$, to this amplitude do not have the properties required by the data on the $\sin \phi_s$ moment. For these GPDs the emitted and reabsorbed partons from the nucleon have the same helicity. Consequently, there are net helicity flips of one unit at both the parton-nucleon vertex and the subprocess. Angular momentum conservation therefore forces both parts to vanish as $\sqrt{-t'}$. Thus, a contribution from the ordinary GPDs to $M_{0\pi}^{\pi} - \pi$ vanishes $\propto t'$. There is a second set of leading-twist GPDs, the helicity-flip or transversity ones $H_T, E_T, \ldots$ for which the emitted and reabsorbed partons have opposite helicities. As an inspection of Fig. 1 reveals the parton-nucleon vertex as well as the subprocess amplitude $H_{0\pi}^{\pi}$ are now of helicity non-flip nature and are therefore not forced to vanish in the forward direction. The prize to pay is that quark and anti-quark forming the pion have the same helicity. Therefore, the twist-3 pion wave function is needed instead of the familiar twist-2 one. The dynamical mechanism building up the amplitude $M_{0\pi}^{\pi} - \pi$ is so of twist-3 accuracy. It has been first proposed in Ref. 13 for wide-angle photo- and electroproduction of mesons where $-t$ is considered to be the large scale.
Fig. 2. The sin φs moment for a transversely polarized target at $Q^2 \simeq 2.45$ GeV$^2$ and $W = 3.99$ GeV for π$^+$ production. The predictions from the handbag approach$^1$ are shown as a solid line. The dashed line is obtained under neglect of the twist-3 contribution. Data are taken from Ref. 4.

Fig. 3. The axial-vector form factor. The green band represents the dipole fit to the experimental data,$^{15}$ the solid circles the lattice results$^{16}$ for $m_\pi = 354$ MeV. The thick (blue) curve is the form factor evaluated from $\tilde{H}$ with the profile function of Ref. 20.

Allowing only for $H_T$ as the only transversity GPD in an admittedly rough approximation the twist-3 mechanism only contributes to the amplitude

$$M^{\text{twist}_-3} = c_0 \int_{-1}^{1} dx H_T^{(3)}(x, \xi, t) \mathcal{H}_{0-,++}$$

For the calculation of the subprocess amplitude $\mathcal{H}_{0-,++}$ the twist-3 pion wave function is taken from Ref. 8 with the three-particle Fock component neglected.$^1$ This wave function contains a pseudo-scalar and a tensor component. The latter one provides a contribution to $\mathcal{M}_{0-,++}$ which is proportional to $t'/Q^2$ and, hence, neglected. The contribution from the pseudo-scalar component to $\mathcal{M}_{0-,+}$ has the required properties. It is proportional to the parameter $\mu_\pi = m^2_\pi/(m_u + m_d)$ which appears as a consequence of the divergency of the axial-vector current. Since $m_u$ and $m_d$ are current quark masses $\mu_\pi$ is large, actually $\simeq 2$ GeV at the scale of 2 GeV. Thus, although parametrically suppressed by $\mu_\pi/Q$ as compared to the longitudinal amplitudes, the twist-3 effect is sizeable for $Q$ of the order of a few GeV.

4. The GPDs at small skewness

For π$^+$ electroproduction the GPDs, namely $\tilde{H}$, the non-pole part of $\tilde{E}$ and the most important one of the transversity GPDs, $H_T$, contribute in the
isovector combination

\[ F_i^{(3)} = F_i^u - F_i^d. \]  

(5)

The GPDs are constructed with the help of double distributions ansatz consisting of the product of the zero-skewness GPDs and an appropriate weight function, actually parameterized as a power of the valence Fock state meson distribution amplitude. This weight function generates the skewness dependence of the GPD. It is important to note that other methods to generate the skewness dependence, namely the Shuvaev transform or the dual parameterization lead to very similar results for the GPDs at small skewness. The zero-skewness GPDs in the double distribution ansatz are assumed to be given by products of their respective forward limits and Regge-like \( t \) dependences, \( \exp[f_i(x,t)t] \), with profile functions that read

\[ f_i(x,t) = b_i - \alpha'_i \ln x \]  

(6)

where \( \alpha'_i \) is the slope of an appropriate Regge trajectory (pole or cut). These profile functions can be regarded as small-\( x \) approximations of more complicated versions used for the determination of the zero-skewness GPDs from the nucleon form factors

\[ f_i(x,t) = b_i(1-x)^3 - \alpha'_i(1-x)^3 \ln x + A_i x(1-x)^2 \]  

(7)

which hold at all \( x \). There is a strong correlation between \( t \) and \( x \) in this ansatz: the behavior of moments or convolutions of a GPD at small (large) \( -t \) is determined by the small (large) \( x \) behavior of this GPD. It is to be stressed that the analysis performed in Ref. 20 as well as recent results from lattice QCD clearly rule out a factorization of the zero-skewness GPDs in \( x \) and \( t \).

The forward limit of \( \tilde{H} \) is given by the polarized parton distributions \( \Delta q(x) \), that of \( H_T \) by the transversity distribution \( \delta(x) \) for which the results of an analysis of the asymmetries in semi-inclusive electroproduction have been taken. Finally, the forward limit of the non-pole part of \( \tilde{E} \) is parameterized as

\[ \tilde{e}^{(3)}(x) = \tilde{E}^{(3)}_{n,p}(x, \xi = t = 0) = \tilde{N}_{\tilde{e}}^{(3)} x^{-0.48} (1-x)^5, \]  

(8)

in analogy to the PDFs. The normalization \( \tilde{N}_{\tilde{e}}^{(3)} \) is fitted to experiment. The full set of parameters used in the analysis of \( \pi^+ \) electroproduction can be found in Ref. 1.
The full GPD $\tilde{E}^{(3)}$ is the sum of the pole and non-pole contribution where the first one reads

$$\tilde{E}_{\text{pole}}^u = -\tilde{E}_{\text{pole}}^d = \Theta(|x| \leq \xi) \frac{F^\text{pole}_P(t)}{4\xi} \Phi_\pi((x + \xi)/(2\xi)) \quad (9)$$

Here, $\Phi_\pi$ is the pion's distribution amplitude and $F_P$ is the pseudo-scalar form factor of the nucleon being related to $\tilde{E}^{(3)}$ by the sum rule

$$\int_{-1}^1 dx \tilde{E}^{(3)}(x, \xi, t) = F_P(t) \quad (10)$$

The evaluation of the pion electroproduction amplitude from the graph shown on the left hand side of Fig. 1 just using $\tilde{E}_{\text{pole}}$ leads to the pion-pole contribution as given in (1) but with only the perturbative contribution to the pion’s electromagnetic form factor occurring in the residue (3). Other graphs have to be considered in addition for the pion pole, e.g. the Feynman mechanism. In order to avoid this complication the pion-pole contribution is simply worked out from the graph shown on the right hand side of Fig. 1.

In summary: the GPDs used in Ref. 1 are valid at small skewness ($\xi \lesssim 0.1$) and are probed by experiment for $x \lesssim 0.6$. Due to the double distribution ansatz they satisfy polynomiality and the reduction formulas. It has also been checked numerically that the lowest moments of the GPDs $\tilde{H}$ and $\tilde{E}$ are in agreement with the data on the axial-vector and pseudo-scalar form factors of the nucleon (see Fig. 3) and respect various positivity bounds. Comparison with recent lattice QCD studies reveals that there is good agreement with the relative strength of moments and their relative $t$ dependences. At small $t$ even the absolute values of the moments agree quite well but the $t$ dependence of the moments obtained from lattice QCD are usually flatter than those from the GPDs and the form factor data. An exception is the lowest moment of $H_T$ for $u$ quarks for which we have a value that is about 25% smaller than the lattice result. Similar observation can be made for the GPDs $H$ and $E$ which have been constructed analogously and probed in vector meson electroproduction. As an example the axial-vector form factor obtained from $\tilde{H}$ as used in Ref. 1 is compared with experiment and with the lattice QCD results in Fig. 3.

5. Results

It is shown in Ref. 1 that with the described GPDs, the $\pi^+$ cross sections as measured by HERMES are nicely fitted as well as the transverse target asymmetries. This can be seen for instance from Fig. 1 where $A_{UT}^{\sin \phi}$
is displayed. Also the $\sin(\phi - \phi_s)$ moment which is dominantly fed by an interference term of the two amplitudes for longitudinally polarized photons, is fairly well described as is obvious from Fig. 4. Very interesting is also the asymmetry for a longitudinally polarized target. It is dominated by an interference term between $M_{0-++}$ which comprises the twist-3 effect, and the nucleon helicity-flip amplitude for $\gamma^*_L \rightarrow \pi$ transition, $M_{0-0+}$. Results for $A^{\sin\phi}_{UL}$ are displayed and compared to the data in Fig. 5. In both the cases, $A^{\sin\phi}_{UT}$ and $A^{\sin\phi}_{UL}$, the prominent role of the twist-3 mechanism is clearly visible. Switching it off one obtains the dashed lines which are significantly at variance with experiment. In this case the transverse amplitudes are only fed by the pion-pole contribution.

Although the main purpose of the work presented in Ref. 1 is focused on the analysis of the HERMES data one may also be interested in comparing this approach to the Jefferson Lab data on the cross sections. With the GPDs $\tilde{H}$, $\tilde{E}$ and $H_T$ in their present form the agreement with these data is poor. I remind the reader that the approach advocated for in Refs. 1 and 11 is optimized for small skewness. At larger values of it the parameterizations of the GPDs are perhaps to simple and may require improvements as for instance the replacement of the profile function (6) by (7). As mentioned above the GPDs are probed by the HERMES data only for $x$ less than about 0.6. One may therefore change the GPDs at large $x$ to some extent without changing much the results for cross sections and asymmetries in the kinematical region of small skewness. For Jefferson Lab kinematics,
on the other hand, such changes of the GPDs may matter. Finally one should be aware that at larger values of skewness the other transversity GPDs may not be negligible. In a recent lattice study the moments of the combination \(2\tilde{H}_T + E_T\) have been found to be rather large in comparison to those of \(H_T\). Including this combination of GPDs into the analysis of pion electroproduction one would have

\[
M_{0+,\mu^+}^{\text{twist-3}} = -e_0 \frac{\sqrt{-t'}}{4m} \int_{-1}^{1} dx \left[2\tilde{H}_T^{(3)} + E_T^{(3)}\right] H_{0-;++} \tag{11}
\]

in addition to (4). Here, \(\mu (\pm 1)\) labels the photon helicity. The amplitude (11) holds up to corrections of order \(\xi\).

6. Summary and outlook

In summary, there is strong evidence for transversity in hard exclusive electroproduction of pions. A most striking effect is seen in the target asymmetry \(A_{UT}^{\sin \phi_s}\). The interpretation of this effect requires a large helicity non-flip amplitude \(M_{0-,++}\). Within the handbag approach this amplitude is generated by the helicity-flip or transversity GPDs in combination with a twist-3 pion wave function. This explanation establishes an interesting connection to transversity parton distributions measured in inclusive processes. Further studies of transversity in exclusive reactions are certainly demanded. Good data on \(\pi^0\) electroproduction would also be welcome. They would not only allow for further tests of the twist-3 mechanism but also give the opportunity to verify the model GPDs \(\tilde{H}\) and \(\tilde{E}\) as used in Ref. 1. An intriguing issue is whether or not the handbag approach in its present form for pion electroproduction works for the kinematics presently accessible at Jlab. It is known that it cannot accommodate the CLAS data on \(\rho^0\), \(\rho^+\) and \(\omega\) production.\(^{28}\) Other applications and tests of the handbag approach including the twist-3 mechanism are pion electroproduction measured at the upgraded Jlab accelerator or by the COMPASS collaboration and the measurement of the time-like process \(\pi^- p \rightarrow \mu^+ \mu^- n\).\(^{29}\) The extension of this approach to electroproduction of other pseudoscalar mesons, in particular the \(\eta\) and \(\eta'\), is also of interest. In principle this would give access to the GPDs for strange quarks. As has been shown in Ref. 30 there is no complication in the analysis of the electroproduction data due to the two-gluon Fock components of the \(\eta\) and \(\eta'\) since they are suppressed by \(t'/Q^2\).
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