Quantum electronic noise, the random current fluctuations generated by electrons crossing a quantum conductor, has been thoroughly studied from the electron point of view. Recent experiments have shown that such noise may have striking properties in terms of the quantum electromagnetic field it generates. Both the existence of vacuum squeezing \cite{1} and generation of correlated photon pairs \cite{2} have been observed. Here we demonstrate that this electromagnetic field also violates Bell-like inequalities, thereby proving its true quantum nature. This is achieved by the observation of two-mode quadrature squeezing of vacuum: the electromagnetic field quadratures at two different frequencies can be so correlated that their difference goes below vacuum fluctuations. Thus, electronic noise contains entangled photon pairs, a key ingredient for quantum computing and communication.

Electrical current flowing in a conductor always fluctuates in time, a phenomenon usually referred to as “electrical noise”. At room temperature, one way this can be described is using a time-dependent current $I(t)$. While the dc current corresponds to the average $\langle I(t) \rangle$, current fluctuations are characterized by their statistical properties such as their second order correlator $\langle I(t)I(t') \rangle$, where the brackets $\langle \rangle$ represent the statistical average. An alternate approach is to consider that this time-dependent current generates a random electromagnetic field that propagates along the electrical wires. Both these descriptions are equivalent. For example, the equilibrium current fluctuations (Johnson-Nyquist noise \cite{3,4}) correspond to the blackbody radiation in one dimension \cite{5}. More precisely, the power radiated by a sample at frequency $f$ in a cable is proportional to the spectral density $S(f)$ of current fluctuation which, at high temperature and at equilibrium (i.e., with no bias), is given by $S_0(hf \ll k_B T) = 2k_B T/R$ where $T$ is the temperature and $R$ the electrical resistance of the sample \cite{3}.

In short samples at very low temperatures, electrons obey quantum mechanics. Thus, electron transport can no longer be modeled by a time-dependent, classical number $I(t)$, but needs to be described by an operator $\hat{I}(t)$. Current fluctuations are characterized by correlators such as $\langle \hat{I}(t)\hat{I}(t') \rangle$. Quantum predictions differ from classical ones only when the energy $hf$ associated with the electromagnetic field is comparable with energies associated with the temperature $k_B T$ and the voltage $eV$. Hence for $hf \gg k_B T, eV$, the thermal energy $k_B T$ in the expression of $S_0(f)$ has to be replaced by that of vacuum fluctuations, $hf/2$. Some general link between the statistics of current fluctuations and that of the detected electromagnetic field is required beyond the correspondence between spectral density of current fluctuations and radiated power \cite{7-10}. In particular, since the statistics of current fluctuations can be tailored by engineering the shape of the time-dependent bias voltage \cite{11}, it may be possible to induce non-classical correlations in the electromagnetic field generated by a quantum conductor. For example, an ac bias at frequency $f_0$ generates correlations between current fluctuations at frequencies $f_1$ and $f_2$, i.e. $\langle \hat{I}(f_1)\hat{I}(f_2) \rangle \neq 0$, if $f_1 \pm f_2 = n f_0$ with $n$ integer \cite{12,13}. This is responsible for the existence of correlated power fluctuations \cite{15} and for the emission of photon pairs \cite{2} recently observed. For $f_1 = f_2$, $\langle \hat{I}^2(f_1) \rangle \neq 0$ leads to vacuum squeezing \cite{1}.

In this article, we report measurements of the correlations between electromagnetic field quadratures at frequencies $f_1 = 7$ GHz and $f_2 = 7.5$ GHz when the sample, a tunnel junction placed at very low temperature, is biased by a dc voltage and irradiated at frequency $f_0 = f_1 + f_2$. The experimental setup is depicted in Fig. \cite{14}, see also Methods section. By analyzing these correlations, we show that the electromagnetic field produced by electronic shot noise can be described in a way similar to an Einstein-Podolski-Rosen (EPR) pair: when measuring fluctuations at only one frequency, i.e. one mode of the electromagnetic field, no quadrature is preferred. But when measuring two modes, we observe strong correlations between identical quadratures. These correlations are stronger than what is allowed by classical mechanics.

Results. We begin our analysis by illustrating the effect of microwave excitation and dc bias on the current fluctuations. Using $A, B$ to represent any of the $X, P$ quadratures at frequencies $f_1 = 7$ GHz and $f_2 = 7.5$ GHz, we show on Fig. \cite{14} the difference $\Delta P$ between the 2-D probability distribution $P(A, B)$ in the presence of $V_{dc} = 29.4 \mu V$ and $V_{ac} = 37 \mu V$ bias and $P(A, B)$ for $V_{dc} = V_{ac} = 0$. Fig. \cite{14}a) and (d), which show respectively $\Delta P(X_1, X_2)$ and $\Delta P(P_1, P_2)$, clearly indicate the existence of correlations between identical quadratures at different frequencies. In contrast, Fig. \cite{14}b) and (c), which correspond to $\Delta P(X_1, P_2)$ and $\Delta P(P_1, X_2)$, seem to indicate the absence of correlations between opposite quadratures.

To be more quantitative, we show on Fig. \cite{14} the
(AB) correlators as a function of the dc bias voltage for a fixed $V_{ac}$. Clearly, $\langle X_1 P_2 \rangle = \langle P_1 X_2 \rangle = 0$ while $\langle X_1 X_2 \rangle = - \langle P_1 P_2 \rangle$ is non-zero for $V_{dc} \neq 0$. These results are presented in temperature units (K), using the usual unit conversion $T_{noise} = RS/2k_B$ for the measured noise spectral density $S$ of a conductor of resistance $R$.

**Theory.** We now compare our experimental results with theoretical predictions. In order to link the measured quantities to electronic properties, we will first define the quadrature operators, following Ref. 10.

$$\hat{X}_{1,2} = \frac{i(f_{1,2}) - i(-f_{1,2})}{\sqrt{2}}, \quad \hat{P}_{1,2} = \frac{i(f_{1,2}) + i(-f_{1,2})}{i\sqrt{2}}$$

where $i(f)$ is the current operator at frequency $f$. In the absence of RF excitation, the currents observed at two different frequencies are uncorrelated, $\langle i(f_1) i(f_2) \rangle = 0$. The excitation at frequency $f_0 = f_1 + f_2$ induces correlations so that $\langle i(f_1) i(f_2) \rangle = \langle i(-f_1) i(-f_2) \rangle \neq 0$.

More precisely, one has 12–14:

$$\langle i(f) i(f_0 - f) \rangle = \sum_n \alpha_n [S_0 (f_{n+}) - S_0 (f_{n-})] \quad (1)$$

where $f_{n\pm} = f + nf_0 \pm eV_{dc}/h$ and $\alpha_n = J_n (eV_{ac}/h f_0) J_{n+1} (eV_{ac}/h f_0)$, with $J_n$, the Bessel func-
tions of the first kind. From this we can calculate the theoretical predictions for all the correlators, which are represented as black lines on Fig. 3, showing a very good agreement between theory and experiment. The optimal squeezing observed corresponds to $\langle \hat{a}^2 \rangle = 0.31 \pm 0.05$, i.e. 2.1 dB below vacuum, versus the theoretical expectation of $\langle \hat{a}^2 \rangle = 0.33$. This minimum is observed at $V_{dc} \approx 30 \mu V \approx h f_{1,2}/e$. All data are in good agreement with theoretical predictions, plotted as full black lines on Fig. 4 with $\langle \hat{x}_k^2 \rangle = \langle \hat{p}_k^2 \rangle = S(f_k)$, the photoassisted shot noise given by $S(f) = \frac{1}{2} \sum n_i^2 (e V_{ac}/h f_0) \left| S_0 (f_{n+}) + S_0 (f_{n-}) \right|$. Curves for $\langle \hat{x}_1 + \hat{x}_2 \rangle^2/2$ and $\langle \hat{p}_1 - \hat{p}_2 \rangle^2/2$ follow the same behaviour with reversed dc bias, showing minima of $0.35 \pm 0.05$ and $0.41 \pm 0.05$ or 1.6 dB at $V_{dc} \approx -30 \mu V \approx -h f_{1,2}/e$. The latter data was omitted from Fig. 4 in order to simplify it.

**Entanglement.** While the presence of two-mode squeezing shows the existence of strong correlations between quadratures of the electromagnetic field at different frequencies, this is not enough to prove the existence of entanglement. A criterion certifying the inseparability of the two modes, and thus entanglement between them, is given in terms of the quantity $\delta = \langle \hat{a}^2 \rangle + \langle \hat{a}^2 \rangle$. In the case of a classical field, this must obey $\delta > 1[17]$. This is equivalent to a Bell-like inequality for continuous variables. As we reported in Fig. 4, we observe $\delta = 0.6 \pm 0.1$. Thus, photons emitted at frequencies $f_1$ and $f_2$ are not only correlated but also form EPR pairs suitable for quantum information processing with continuous variables [18].

The inseparability criterion derived in Ref. [17] is, in principle, only valid for Gaussian states, which is not a perfect representation for electronic shot noise. Specifically, a more general form of Bell-like inequality should read $\delta > 1 - \epsilon$ with $\epsilon$ involving higher order cumulants of the fluctuating current such as $\langle \langle \hat{x}_{1,2}^4 \rangle \rangle$. An example is given in Ref. [19], where a necessary but not sufficient condition for inseparability is written in terms of order 4 and lower correlators. Using the data reported in Fig. 2 for the residual signal in absence of RF excitation, an estimate yields $\langle \langle \hat{x}_{1,2}^4 \rangle \rangle \sim 10^{-3}$. Thus it is very likely that $\epsilon$ would be very small and that the present observation corresponds indeed to true entanglement. Note that taking into account moments of order 4 or higher is only mandatory to prove the negativity of the Wigner function[20]. It is not required for demonstrating the existence of entanglement.

Entanglement between radiations at two different frequencies has already been observed in the microwave domain with superconducting devices [21][22]. Two-mode quadrature-squeezed states are usually characterized by their covariance matrix. Following the notations of [21], our experiment corresponds to $n = 2 \langle \hat{x}_{1,2}^2 \rangle \simeq 2 \langle \hat{p}_{1,2}^2 \rangle$, $k = 2 \langle \hat{x}_1 \hat{x}_2 \rangle \simeq -2 \langle \hat{p}_1 \hat{p}_2 \rangle$ so that $\delta = n - k$. Equilibrium at $T = 0$ corresponds to $n = 1$ and $k = 0$. Our observed optimal squeezing corresponds to $n = 1.3 \pm 0.1$ and $k = 0.52 \pm 0.05$. From these numbers, one can calculate all the statistical properties that characterize the electromagnetic field generated by the junction. In particular,
we find an entanglement of formation of $E_F \simeq 0.2$ (as defined in Ref. [24]) and a purity of $\mu = 0.82$ (as defined in Ref. [25]). While in our experiment, the entangled photons are not spatially separated, this could easily be achieved using a diplexer, which can separate frequency bands without dissipation.

Methods. We used a two-channel digitizer with a rate of 400 MS/s to record the output of the IQ mixers, thus providing the in-phase $X_{1,2}$ and quadrature $P_{1,2}$ parts relative to their reference frequencies. Any two quantities $A, B$ among $X_1, X_2, P_1$ and $P_2$ can be digitized simultaneously by the diplexer, yielding a 2D probability map $P(A, B)$. From $P(A, B)$, one can calculate any statistical quantity, in particular the variances $\langle \Delta^2 \rangle$, $\langle B^2 \rangle$, as well as the correlators $\langle AB \rangle$. We chose to work at $f_0 = 14.5$ GHz, $f_1 = 7$ GHz $\Rightarrow f_2 = 7.5$ GHz so that $f_1$ and $f_2$ are far enough to suppress any overlap between the frequency bands.

The four detection channels must be calibrated separately. This is achieved by measuring the variances $\langle X_{1,2}^2 \rangle$, $\langle P_{1,2}^2 \rangle$ in the absence of RF excitation. These should all be proportional to the noise spectral density of a tunnel junction at frequency $f_{1,2}$, given by $S(f, V) = (S_0 (f + eV/h) + S_0 (f - eV/h))/2$ where $S_0 (f) = (hf/R) \coth (hf/2k_B T)$ is the equilibrium noise spectral density at frequency $f$ in a tunnel junction of resistance $R$. By fitting the measurements with this formula, we find an electron temperature of $T = 18$ mK and an amplifier noise temperature of $\sim 3$ K, which are identical for all four channels. The small channel cross-talk was eliminated using the fact that $\langle A_1 B_2 \rangle = 0$ when no microwave excitation is present.

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[1] Gasse, G., Lupien, C. & Reulet, B. Observation of squeezing in the electron quantum shot noise of a tunnel junction. Phys. Rev. Lett. 111, 136601 (2013).
[2] Forgues, J.-C., Lupien, C. & Reulet, B. Emission of microwave photon pairs by a tunnel junction. Phys. Rev. Lett. 113, 043602 (2014). URL http://link.aps.org/doi/10.1103/PhysRevLett.113.043602
[3] Johnson, J. B. Thermal agitation of electricity in conductors. Physical Reviews 32, 97 (1928).
[4] Nyquist, H. Thermal agitation of electric charge in conductors. Phys. Rev. 32, 110–113 (1928). URL http://link.aps.org/doi/10.1103/PhysRev.32.110
[5] Oliver, B. M. Thermal and quantum noise. Proceedings of the IEEE 53, 436–454 (1965).
[6] Callen, H. B. & Welton, T. A. Irreversibility and generalized noise. Phys. Rev. 83, 34–40 (1951). URL http://link.aps.org/doi/10.1103/PhysRev.83.34
[7] Beenakker, C. W. J. & Schomerus, H. Counting statistics of photons produced by electronic shot noise. Phys. Rev. Lett. 86, 700–703 (2001). URL http://link.aps.org/doi/10.1103/PhysRevLett.86.700
[8] Beenakker, C. W. J. & Schomerus, H. Antibunched photons emitted by a quantum point contact out of equilibrium. Phys. Rev. Lett. 93, 096801 (2004). URL http://link.aps.org/doi/10.1103/PhysRevLett.93.096801
[9] Lebedev, A. V., Lesovik, G. B. & Blatter, G. Statistics of radiation emitted from a quantum point contact. Phys. Rev. B 81, 155421 (2010). URL http://link.aps.org/doi/10.1103/PhysRevB.81.155421
[10] Qassemi, F., Reulet, B. & Blais, A. Quantum optics theory of electronic noise in coherent conductors. (unpublished) (2014).
[11] Gabelli, J. & Reulet, B. Shaping a time-dependent excitation to minimize the shot noise in a tunnel junction. Phys. Rev. B 87, 075403 (2013). URL http://link.aps.org/doi/10.1103/PhysRevB.87.075403
[12] Gabelli, J. & Reulet, B. Dynamics of quantum noise in a tunnel junction under ac excitation. Phys. Rev. Lett. 100, 026601 (2008).
[13] Gabelli, J. & Reulet, B. The noise susceptibility of a photo-excited coherent conductor (2008). ArXiv:0801.1432.
[14] Gabelli, J. & Reulet, B. The noise susceptibility of a coherent conductor. Proc. SPIE, Fluctuations and Noise in Materials 6600-25, 66000T–66000T–12 (2007).
[15] Forgues, J.-C. et al. Noise intensity-intensity correlations and the fourth cumulant of photo-assisted shot noise. Sci. Rep. 3, 2869 (2013).
[16] Lesovik, G. B. & Levitov, L. S. Noise in an ac biased junction: Nonstationary aharonov-bohm effect. Phys. Rev. Lett. 72, 538–541 (1994).
[17] Duan, L.-M., Giedke, G., Cirac, J. I. & Zoller, P. Inseparability criterion for continuous variable systems. Phys. Rev. Lett. 84, 2722–2725 (2000). URL http://link.aps.org/doi/10.1103/PhysRevLett.84.2722
[18] Braunstein, S. L. & van Loock, P. Quantum information with continuous variables. Rev. Mod. Phys. 77, 513–577 (2005). URL http://link.aps.org/doi/10.1103/RevModPhys.77.513
[19] Simon, R. Peres-horodecki separability criterion for continuous variable systems. Phys. Rev. Lett. 84, 2726–2729 (2000). URL http://link.aps.org/doi/10.1103/PhysRevLett.84.2726
[20] Bednorz, A. & Belzig, W. Fourth moments reveal the negativity of the wigner function. Phys. Rev. A 83, 052113 (2011). URL http://link.aps.org/doi/10.1103/PhysRevA.83.052113
[21] Eichler, C. et al. Observation of two-mode squeezing in the microwave frequency domain.
[22] Flurin, E., Roch, N., Mallet, F., Devoret, M. H. & Huard, B. Generating entangled microwave radiation over two transmission lines. *Phys. Rev. Lett.* **109**, 183901 (2012).

[23] Nguyen, F., Zakka-Bajjani, E., Simmonds, R. W. & Aumentado, J. Quantum interference between two single photons of different microwave frequencies. *Phys. Rev. Lett.* **108**, 163602 (2012).

[24] Giedke, G., Wolf, M. M., Krüger, O., Werner, R. F. & Cirac, J. I. Entanglement of formation for symmetric gaussian states. *Phys. Rev. Lett.* **91**, 107901 (2003). URL http://link.aps.org/doi/10.1103/PhysRevLett.91.107901

[25] DiGuglielmo, J., Hage, B., Franzen, A., Fiurášek, J. & Schnabel, R. Experimental characterization of gaussian quantum-communication channels. *Phys. Rev. A* **76**, 012323 (2007). URL http://link.aps.org/doi/10.1103/PhysRevA.76.012323