Spectrum of Mesons and Baryons with \(b\) Quarks

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We present highlights of the spectrum of mesons and baryons calculated using NRQCD for heavy quarks and tadpole improved clover action for the light quarks.

1. HEAVY-LIGHT MESONS

The results for heavy-light mesons are summarized in Table 1. The results presented here are based on the same statistical sample, consisting of 102 quenched configurations at \(\beta = 6.0\) with lattice size \(16^3 \times 48\), as used in our study of \(f_B\) and \(f_{B_s}\). The details of the NRQCD action, the evolution equation for calculating the heavy quark propagator, the method used for setting the lattice scale, and the fixing of light and strange quark masses are given in [1]. The lattice scale, determined from \(M_{\rho}\), is 1.92(7) GeV, and we consider the range 1.8 – 2.0 GeV to determine the associated uncertainty. The bare \(b\) quark mass used in the heavy-light analyses is \(aM_b^0 = 2.31(12)\). From this we estimate \(m_{b_{\overline{MS}}}(m_{b_{\overline{MS}}}) = 4.35(10)(-3+2)(10)\) GeV. For hadrons containing strange quarks, the central values correspond to fixing \(m_s\) using \(M_K\), and for errors we use the difference between it and using \(m_s(M_K\)). Details of the analyses of the spectrum will appear soon in [2].

Overall, our estimates are in rough agreement with experimental data, the one exception being the hyperfine splittings as discussed below. To understand the various mass splittings, we use the following qualitative picture in which the mass of a heavy-light hadron is considered to be a sum of:

- the pole mass of the heavy quark \((M_h)\) which is \(\sim 1.5\) GeV for the \(c\) quark and \(\sim 5.0\) GeV for the \(b\);

\[\begin{array}{|c|c|c|}
\hline
\text{state} \ (nJ^P) & \text{Lattice} & \text{Expt.} \\
\hline
\hline
\text{heavy-light mesons} & & \\
B & 1(0^-) & 5296(04)(-3) & 5279 \\
 & 2(0^-) & 5895(116)(+20)(-32) & 5860(*) \\
B^* & 1(1^-) & 5319(02)(+5) & 5325(1) \\
B_6^* & 1(0^+) & 5670(37)(+16) & \\
B_1 & 1(1^+) & 5726(38)(+20) & 5698(12) \\
B_2^* & 1(2^+) & 5822(45)(+27)(+35) & 5779(*) \\
\hline
\text{heavy-strange mesons} & & \\
B_s & 1(0^-) & 5385(15)(+7)(+20) & 5375(6) \\
 & 2(0^-) & 5935(57)(+27)(+9) & \\
B_s^* & 1(1^-) & 5412(14)(+4)(+20)(-0) & 5422(6) \\
B_{s0}^* & 1(0^+) & 5742(27)(+14)(+10) & \\
\hline
B_{s1} & 1(1^+) & 5804(31)(+17)(+16)(-0) & 5853(15) \\
B_{s2}^* & 1(2^+) & 5878(26)(+23)(-33)(-1) & \\
\hline
\end{array}\]

Table 1

Mass estimates in MeV for various meson states. The \(b\) quark mass is fixed using the spin-averaged \(\overline{B}(1S)\). The first error of the lattice data is statistical (this bootstrap estimate includes uncertainties due to extrapolations in quark masses), the second represents the scale uncertainty due to varying \(a^{-1}\) between 1.8 and 2.0 GeV, and for the strange mesons we quote a third error associated with the uncertainty in fixing the strange quark mass. Preliminary experimental values are denoted by asterisks. The experimental numbers quoted against the \(1^+\) states correspond to broad resonances that are possibly a mixture of the different \(P\) states. The corresponding lattice estimates are an unresolved combination of the \(1^+\) and \(1^{++}\) states.

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• the constituent mass of the light quarks which is approximately 300 MeV for the u, d and 450 MeV for the s quark as inferred from the octet and decuplet light baryons.

• for orbitally and radially excited states, an excitation energy of the light quark, which we expect to be of the order of \( \Lambda_{QCD} \);

• the \( O(\Lambda_{QCD}^2/M_b) \) contributions are the kinetic energy of the heavy quark and the heavy-light hyperfine energy \( E_{\sigma_H-\sigma_l} \approx 46 \) MeV, inferred from the experimental \( B^* - B \) splitting;

• and a residual binding energy \( E_{\text{BD}} \) encapsulating the remaining interactions which we expect to be small \( (O(\Lambda_{QCD}^2/M_b^2)) \).

To isolate individual terms and estimate their size and dependence on the quark masses we construct different linear combinations of meson and baryon masses.

The spin-averaged splitting \( \overline{m}_{B_s} - \overline{m}_{B_s} \) should be dominated by the difference of the strange and light quark masses. We find 90(9)(\( ^{+0}_5 \))(\( ^{-2}_0 \)) MeV (the first error includes statistical and extrapolation in quark masses, the second is due to the scale uncertainty, and the third is the variation if \( m_s \) is set using \( M_K \) instead of \( M_K \)) whereas the experimental value is 96(6) MeV. The splitting shows no significant dependence on the heavy quark mass, consistent with the expectation that the change in the difference in the kinetic energy of the heavy quark is small.

By a judicious combination of operators and sources for quark propagators we are able to obtain a signal for P-wave states. The estimates for \( ^3P_2 \) and \( ^3P_0 \) are unambiguous as the operators used to probe these states do not mix with other states. Thus, their masses and the splittings \( B_2 - B_0^* = 155(32)(^{+9}_{-13}) \) MeV and \( B_2^* - B_0^* = 136(23)(^{+10}_{-13})(^{+9}_{-4}) \) MeV, are predictions since the \( P \) states have not been resolved experimentally. We have used \( ^3P_1 \) and \( ^1P_1 \) operators in the \( LS \) coupling scheme to probe the physical \( 1^+ \) and \( 1^{++} \) states. In this case each operator has a non-zero overlap with the two physical states; consequently to get the physical masses it is essential to get a signal in both the direct and mixed correlators. Since the data show no reliable signal in the mixed correlators, we do not have predictions for the masses of the physical states.

In Table 1 we quote the result from the \( ^3P_1 \) correlator as a rough estimate for the \( 1^+ \) state.

Radial and orbital splittings are expected to be dominated by the difference in kinetic energies of the heavy and light quarks. Of these, the light quark contributes the most \( O(\Lambda_{QCD}^2/m_{\text{constituent}}) \sim O(\Lambda_{QCD}) \). We find 602(86)(\( ^{+25}_{-35} \)) and 559(55)(\( ^{+31}_{-38} \)) MeV for the \( 2^1S_0 - 1^1S_0 \) splitting in the \( B \) and \( B_s \) systems respectively. The preliminary experimental value for the \( B \) is 581 MeV \( ^3 \). For the spin-averaged \( 1P - 1S \) splitting we find 457(31)(\( ^{+24}_{-35} \)) MeV for the \( B \), and 428(27)(\( ^{+27}_{-41} \))(\( ^0_2 \)) MeV for the \( B_s \).

The results for the hyperfine splittings are not as encouraging. We find \( \Delta E(B_2^* - B_3) = 24(5)(^{+0}_{+3}) \) MeV and \( \Delta E(B_3^* - B_s) = 27(3)(^{+0}_{+5})(^{+0}_{-1}) \) MeV, i.e. roughly half the experimental values, 46 and 47 MeV respectively. Further work is required to clarify whether this discrepancy is due to the quenched approximation or due to an underestimate of the \( \sigma \cdot B \) term (the clover term) in the action.

2. BARYONS

Our results for baryons are summarized in Table 2. The first splitting we comment on is \( M_{\Lambda_b} - (M_H + 3M_{H^*})/4 \). In this there is no contribution from the heavy quark mass and hyperfine interaction \( E_{\sigma_H-\sigma_l} \), so it should be dominated by mass of the extra light quark in \( \Lambda \). The experimental values, 311(10) and 310(2) MeV respectively. The lattice estimates are \( \Lambda_b - \overline{\Xi}_b = 370(67)(^{+14}_{-20}) \) MeV and \( \Xi_b - \overline{\Xi}_b = 392(50)(^{+15}_{-0}) \) MeV. Also, the data show little dependence on the heavy quark mass.

In our picture the hyperfine interaction between the light quarks should dominate the splitting \( (2\Sigma_b + 4\Sigma_b^*)/6 - \Lambda_b \). A calculation of the \( \sigma \cdot \sigma_t \) term, assuming a simple non-relativistic model, suggests that this splitting should be 2/3 of the Delta-Nucleon splitting which is 293 MeV. The experimental results, \( (2\Sigma_c + 4\Sigma_c^*)/6 - \Lambda_c = 212 \)
Our results

| baryon | expt. | Our results |
|--------|-------|-------------|
| $\Lambda_h$ (udb) | 5.624(9) | 5.679(11) |
| $\Xi_h$ (lsb) | 5.795(53) | 5.795(54) |
| $\Sigma_h$ (llb) | 5.797(8) | 5.887(49) |
| $\Xi_h'$ (lsb) | 5.968(39) | 5.968(39) |
| $\Omega_h$ (ssb) | 6.048(33) | 6.048(33) |
| $\Sigma_h^*$ (llb) | 5.853(8) | 5.909(47) |
| $\Xi_h^*$ (lsb) | 5.989(39) | 5.989(39) |
| $\Omega_h^*$ (ssb) | 6.069(34) | 6.069(34) |

Table 2
Summary of $b$ baryons masses in GeV ($l$ stands for a $u$ or $d$ quark).

MeV and $(2\Sigma_h + 4\Sigma_h^*)/6 - \Lambda_h = 210$ MeV (preliminary) are consistent with this and suggest negligible dependence on the heavy quark mass. We find $221(71)$ MeV at $M_b$ and no significant dependence on the heavy quark mass. The value decreases to $186(51)$ MeV on replacing $d$ with $s$, in qualitative agreement with the experimental results in the charmed sector.

The $\Sigma_h^* - \Sigma_h$ splitting is expected to be proportional to $(1/M_h)$ as it should depend only on the heavy-light hyperfine interaction $E_{\sigma\gamma,\gamma}$. Using this heavy quark scaling one expects a value $(B^* - B)(\Sigma_h^* - \Sigma_h)/(D^* - D) \approx 46 \times 66/140 \approx 22$ MeV. We find $19(7)$ MeV, whereas the preliminary experimental value is $56(8)$ MeV. However, the experimental identification of the states is still under debate.

3. HEAVY-HEAVY MESONS -- ONIA

The bottomonia spectrum has been successfully used for an independent determination of the lattice scale and $m_b$. Here we present preliminary results based on the same data set as for the heavy-light analyses presented above. To extract the lattice scale we assume that the splittings $(M_{\Upsilon} - M_T)$, $(M_{\Upsilon} - m_b)$, and $(M_{\Upsilon} - M_T)$ are independent of the heavy quark mass. We include $(M_{\Upsilon} - m_b)$ as it has the best signal and

| | $a^{-1}$ MeV | $M_0^b$ a | $M_0^b$ GeV |
|---|---|---|---|
| $2^3S_1 - 1^3S_1$ | 2313(99) | 1.76(7) | 4.07(24) |
| $2^1S_0 - 1^1S_0$ | 2413(83) | 1.69(5) | 4.08(19) |
| $3^3P_1 - 3^1S_1$ | 2424(133) | 1.69(8) | 4.10(30) |

Table 3
Estimates of lattice scale and $b$ quark mass from $\Upsilon$. The final column gives $M_0^b$ (difference of two fits).

we assume it is equal to $(M_T - M_T)$ on basis of the approximate equality in the charm system. Having fixed $1/a$ we determine $M_0^b$ by fixing $M_T$ using the dispersion relation. These results are summarized in table 3.

Typically, the results for mass-splittings with best control over statistical errors are obtained from fits to ratios of correlators. In our data we find marginally better signal from fits to individual as compared to ratio of correlators (a consequence of choosing a smearing in heavy quark propagator generation that optimizes signal for heavy-light states). For the hyperfine splitting $M_T - M_{\eta_b}$ we find 34.2(3.6) MeV (ratio fits) and 30.4(3.2) MeV (difference of mass fits). If one scales the $J/\psi - \eta_b$ splitting, 127 MeV, by $M_{J/\psi}/M_T$ then one expects $\sim 42$ MeV. Thus, once again the hyperfine splitting is underestimated though not by as much as for $M_{B*} - M_B$.

The determination of $P$ state splittings has larger errors. We find $1^3P_1 - 3^3P_1 = 0(8)$ where $3^3P_1 = (3P_0 + 3*3P_1 + 3*3P_2(T) + 2*3P_2(E))/9$; $3P_0 - 3^3P_1 = 42(23); 3P_1 - 3^3P_1 = 13(23); 3P_2(T) - 3^3P_1 = 28(14);$ and $3P_0(E) - 3^3P_1 = 40(23)$ MeV. It is disturbing to find the $T$ and $E$ cubic representations of $3P_2$ give such different estimates. The corresponding experimental values are $-40.2, -8.3,$ and 13 MeV.

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