The Role of Uncertainty in Controlling Climate Change

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Abstract

Integrated Assessment Models (IAMs) of the climate and economy aim to analyze the impact and efficacy of policies that aim to control climate change, such as carbon taxes and subsidies. A major characteristic of IAMs is that their geophysical sector determines the mean surface temperature increase over the preindustrial level, which in turn determines the damage function. Most of the existing IAMs are perfect-foresight forward-looking models, assuming that we know all of the future information. However, there are significant uncertainties in the climate and economic system, including parameter uncertainty, model uncertainty, climate tipping risks, economic risks, and ambiguity. For example, climate sensitivity, a well-known parameter that measures how much the equilibrium temperature will change if the atmospheric carbon concentration doubles, can range from one to ten in the literature. Climate damages are also uncertain: some researchers assume that climate damages are proportional to instantaneous output, while others assume that climate damages have a more persistent impact on economic growth. The spatial distribution of climate damages is also uncertain.

Climate tipping risks represent (nearly) irreversible climate events that may lead to

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significant changes in the climate system, such as the Greenland ice sheet collapse, while the conditions, probability of tipping, duration, and associated damage are also uncertain. Technological progress in carbon capture and storage, adaptation, renewable energy, and energy efficiency are uncertain too. Future international cooperation and implementation of international agreements in controlling climate change may vary over time, possibly due to economic risks, natural disasters, or social conflict. In the face of these uncertainties, policymakers have to provide a decision that considers important factors such as risk aversion, inequality aversion, and sustainability of the economy and ecosystem. Solving this problem may require richer and more realistic models than standard IAMs, and advanced computational methods. The recent literature has shown that these uncertainties can be incorporated into IAMs and may change optimal climate policies significantly.

Keywords: climate policy, parameter uncertainty, economic risk, climate risk, model uncertainty, scenario uncertainty, ambiguity, misspecification, policy uncertainty
1 Introduction

It has been widely recognized that anthropogenic greenhouse gas emissions have been distorting the planet’s energy balance system, resulting in global warming, sea level rise, and more frequent extreme weather, with industrial carbon emissions constituting the major component of greenhouse gas emissions. These emissions then influence economic well-being via a damage function. Integrated Assessment Models (IAMs) combine the climate and the economy, as well as the interactions between them, to analyze which policies are more efficient in controlling climate change. DICE (Nordhaus, 2008, 2017) is a representative IAM. Most existing IAMs assume that the climate and economic systems as well as interactions between them are deterministic, and that economic agents are myopic. However, there are significant uncertainties in these systems and their interactions, and these uncertainties may play an essential role. For example, the dismal theorem of Weitzman (2009) shows that the risk premium could be infinite for unboundedly distributed uncertainties. Pindyck (2013, 2017) criticize IAMs as being crucially flawed and fundamentally misleading, but Heal (2017) argues that “IAMs can play a role in providing qualitative understanding of how complex systems behave, but are not accurate enough to provide quantitative insights. Arguments in favor of action on climate issues have to be based on aversion to risk and ambiguity and the need to avoid a small but positive risk of a disastrous outcome.”

However, a policymaker often has to make a quantitative decision, such as the size of a carbon tax, in the face of this uncertainty. Moreover, the “wait and see” approach will not make sense, because carbon emissions have long-lasting impacts on temperature, and changes in the climate system may be irreversible. For example, Steffen et al. (2018) point out a risk that if the Earth System crosses a planetary threshold then continued warming could occur even as human emissions are reduced, preventing climate stabilization. Moreover, once an ice sheet collapses, it is irreversible for millennia (IPCC, 2013). Metcalf and Stock

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1 See Hsiang and Kopp (2018) for an introduction of the physical science of climate change for economists.

2 See Chari (2018) for a discussion about uncertainty and risk in climate change.
argue that policymakers need a numerical value for the social cost of carbon (SCC) for policy evaluation and implementation, and producing a credible numerical value requires sophisticated computer models, i.e. IAMs. Brock and Hansen (2018) stress: “Defenses for policies that combat climate damage externalities induced by human activity need not require precise knowledge of the magnitude or timing of the potential adverse impacts. ...Waiting for precise knowledge of the eventual consequences of continued or expanded human induced CO2 emissions could make mitigation or adaptation extremely costly.” Goulder (2020) calls for urgent and stronger policy action to address global climate change.

This review focuses on recent work about the role of uncertainty in controlling climate change. Here I use a broad perspective of uncertainty, which includes parameter uncertainty, risk, model uncertainty, scenario uncertainty, policy uncertainty, ambiguity, and misspecification. I focus on discussing the first five types of uncertainty and methods on making decisions in the face of these uncertainties (see Brock and Hansen (2018) for a more complete survey about ambiguity and misspecification). Moreover, I focus on reviewing discrete-time stochastic IAMs (see Brock and Xepapadeas (2018) for discussion of continuous-time IAMs).

The review is organized as follows. Section 2 discusses parameter uncertainty and how to deal with it, focusing on uncertainty in the discount rate, climate sensitivity, and damage function. Section 3 reviews economic and climate risks and methods for handling them. Section 4 discusses model uncertainty and scenario uncertainty. Section 5 reviews policy uncertainty. Section 6 briefly discusses ambiguity and misspecification. Section 7 concludes.

2 Parameter Uncertainty

Parameters in IAMs are estimated from historical data, expert opinions, survey data, and/or projections for future scenarios. Their values are often uncertain because no model can replicate the real world, historical data may have errors, expert opinions and survey data may be too subjective, or projections for future scenarios may be not close to what will
actually happen. These uncertain parameters are assumed to have fixed and unchanged values over time, but we do not know the exact values. In some cases, knowledge of the exact values can be expressed by some probability distributions, which are referred to as belief distributions.

The most well-known and debated uncertain parameters in IAMs include the discount rate, climate sensitivity (also known as “equilibrium climate sensitivity”), and parameters in climate damage functions. These parameters can change optimal solutions significantly but are still hard to pin down. Other uncertain parameters include economic growth, the intertemporal elasticity of substitution, and risk aversion.

### 2.1 Discount Rate

Economic analysis often uses two types of discount rates: a utility discount rate that represents the rate at which utility is discounted, and a consumption discount rate that represents the rate at which consumption is discounted. These two discount rates can be connected by the famous Ramsey rule. There are a large number of papers discussing these discount rates, see for example Weitzman (2001), Frederick, Loewenstein, and O’Donoghue (2002), Gollier (2012), Arrow et al. (2013, 2014), and Heal (2017).

IWG (2010) employs three consumption discount rates (2.5%, 3%, and 5%) to compute the SCC. Nordhaus (2008) uses a utility discount rate of 1.5%, which is calibrated together with the intertemporal elasticity of substitution to match the estimated growth of consumption. For ethical reasons, Stern (2007) sets a utility discount rate of 0.1% and finds that the SCC will be significantly higher.

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3See e.g. Gillingham et al. (2018); Cai, Judd, and Lontzek (2018); Cai and Lontzek (2019) for investigations on the impact of these uncertain parameters.

4It has many other names, including the social rate of time preference and the pure rate of time preference.
2.2 Climate Sensitivity and Transient Climate Response

Climate sensitivity is a parameter that measures the long-run increase in atmospheric temperature (in degrees Celsius) if the atmospheric carbon concentration doubles. A typical value of climate sensitivity used in the literature is 3°C, which is considered to be the median of the distribution of climate sensitivity. IPCC (2007) suggests the likely range (i.e. with a 66% probability) of climate sensitivity is [2.0, 4.5], but a later report IPCC (2013) expands the likely range to [1.5, 4.5] (the same as given by Jule Charney in 1979) in light of recent research, demonstrating that it is challenging to narrow the envelope of parameter uncertainty of climate sensitivity with additional research.

Instead of climate sensitivity, recently some climate scientists suggest using a more stable measurement, called “transient climate response to emissions” (TCRE), to simplify modeling the climate system. The TCRE scheme assumes that contemporaneous atmospheric temperature is nearly linearly proportional to cumulative carbon emissions. Therefore, we can use only one state variable, cumulative emissions, to obtain atmospheric temperature, which is then used in a typical economic damage function or hazard rate. In contrast, the climate system modelled using climate sensitivity is often complicated and requires many state variables. With the simplification of the TCRE scheme, economists have also employed it in their models, including Brock and Xepapadeas (2017), Anderson, Brock, and Sanstad (2018), and van der Ploeg (2018). Recently, Dietz and Venmans (2019) use a continuous-time IAM with the TCRE scheme and find that the optimal carbon price should start relatively high and grow relatively fast. However, the value of TCRE is still uncertain: MacDougall, Swart, and Knutti (2016) estimate the mean value of TCRE as 1.72°C, and its 5% to 95% percentile range as [0.88, 2.52], while IPCC (2013) reports its likely range as [0.8, 2.5].

For example, DICE uses five state variables for the climate system: carbon concentration in the atmosphere, carbon concentration in the upper ocean, carbon concentration in the deep ocean, atmospheric temperature, and oceanic temperature. See Matthews et al. (2009), MacDougall and Friedlingstein (2013), and Knutti, Rugenstein, and Heger (2017) for further details.
2.3 Damage Function

The specification of the damage function, which measures the damage from global warming, has been debated extensively. The most common damage function is a quadratic function of the temperature increase, specified by Professor William Nordhaus in his DICE/RICE models (Nordhaus, 2008, 2017, 2010). It assumes that temperature increases reduce instantaneous economic output in a ratio represented by the damage function. However, Weitzman (2012) points out that the quadratic function results in implausibly low damage at high temperatures, and thus suggests adding one high-exponent term to the damage function so that 50% of output is lost if the temperature increase is 6 °C. Dietz and Stern (2015) find that the modification of Weitzman (2012) leads to a much higher SCC. Diaz and Moore (2017) review and synthesize the limitations of the damage functions.

Sterner and Persson (2008) argue that “it is exactly the nonmarket effects of climate change that are the most worrisome”, which include “biodiversity and ecosystem loss; effects on human well-being (human amenity, loss of lives, and air pollution); impacts from natural disasters, such as extreme weather events, droughts, hurricanes or floods (Manne, Mendelsohn, and Richels, 1995); as well as socially contingent consequences, such as migration and risk for conflicts”, many of which have been presented in the Stern review (Stern, 2007). The nonmarket amenities and aggregate consumption can be combined with a utility function with a constant elasticity of substitution kernel, which leads to a much higher SCC (Sterner and Persson, 2008; Cai et al., 2015).

Besides the climate damage to instantaneous output and nonmarket goods, researchers also find that climate change can reduce economic growth. For example, Dell, Jones, and Olken (2012) find a reduction in economic growth of approximately 1.3% for a 1°C increase in global temperatures. In the face of the damage to economic growth, the SCC will be significantly higher (Moore and Diaz, 2015; Dietz and Stern, 2015). Recent empirical work in estimating climate impact includes Burke, Hsiang, and Miguel (2015a,b), Burke et al. (2015),

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6See Heal and Park (2016) for a review about the climate impact on output levels and growth rates.
2.4 Dealing with Parameter Uncertainty

The most common way to deal with parameter uncertainty is sensitivity analysis, that is, choosing a different value of the uncertain parameter and checking its impact on the solution to see if results are qualitatively robust. If there are multiple uncertain parameters, then doing sensitivity analysis over each uncertain parameter might not be enough, as a combination of multiple uncertain parameters may produce nontrivial results. Thus, a global sensitivity analysis—going through a number of combinations of multiple uncertain parameters—should be conducted. For example, Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) use global sensitivity analysis in a dynamic stochastic integrated framework of climate and economy (DSICE) to study the impact of the intertemporal elasticity of substitution (IES) and risk aversion on the SCC in the face of long-run economic growth risk. They find that a larger degree of risk aversion increases the SCC if the IES is small, but decreases the SCC if the IES is large, which can be reconciled using the impact of economic growth. However, in the face of climate tipping risk, a larger degree of risk aversion always increases the SCC regardless of the size of the IES. It may be too time-consuming to run global sensitivity analysis if tensor grids for uncertain parameters are used. Uncertainty quantification can be applied to address this issue: choose a small set of nodes (e.g., a sparse grid) for uncertain parameters, then apply an approximation method to estimate solutions over the whole domain of uncertain parameters (see Harenberg et al. (2019) for a detailed discussion).

Sensitivity analysis methods provide us with the lower and upper bounds of the solutions and also the qualitative robustness of solutions, but it is often challenging to provide one
specific quantitative value, e.g. the SCC, for use in decision making, as we do not know which values of uncertain parameters are correct. However, if we have a belief distribution for possible values of the uncertain parameters, then it is possible to give a quantitatively robust solution. For example, Cai and Sanstad (2016) use an expected cost minimization method to find a robust mitigation pathway in the face of R&D (research and development) technology uncertainty. Some researchers use a Monte Carlo method: they draw samples of the uncertain parameters from the belief distributions, solve the deterministic model with each sampled realization of the uncertain parameters, then use the average over the solutions as an approximate solution in the face of uncertainty. While this Monte Carlo analysis may be helpful in some cases, it may also lead to very biased solutions (see Crost and Traeger (2013), Lemoine and Rudik (2017) and Cai (2019) for detailed discussions).

If we have belief distributions of uncertain parameters, we may collect more data over time to update the belief distributions, by shrinking the range of uncertain parameters or reducing the variances of these distributions. This process is called Bayesian learning. Note that any sampled realization of the uncertain parameters in simulated solutions should be their fixed true values, so the true values have to be assumed before simulation in order to study the impact of learning under the assumed true values. Bayesian learning has been applied in climate change economics, for example in Kelly and Kolstad (1999), Keller, Bolker, and Bradford (2004), Leach (2007), Kelly and Tan (2015), Hwang, Reynes, and Tol (2017), Gerlagh and Liski (2018), and Rudik (2019). Since Bayesian learning depends on the frequency of new data collection, in dynamic models it depends on the size of the time steps: if we use annual time steps then we can update belief distributions every year, but if we use decadal time steps then we can only update belief distributions every decade. Thus, the size of the time steps should be chosen reasonably to be consistent with the frequency of real data collection. In addition, Bayesian learning ignores the possibility of active learning through R&D (see Goulder and Mathai 2000), so it may provide biased results.

7See e.g., New and Hulme (2000), Nordhaus (2008), Ackerman, Stanton, and Bueno (2010), and Anthoff and Tol (2013).
In the face of parameter uncertainty without belief distributions, decision makers may have to provide only one robust solution instead of many solutions from (global) sensitivity analysis. This robust decision making problem can be solved using a robust decision making method, such as the max-min method or the min-max regret method (Iverson, 2012, 2013; Anthoff and Tol, 2014; Cai and Sanstad, 2016). See Cai (2019) for a detailed discussion.

3 Risks

There are many risks in economic and climate systems, where risks refer to random variables with probability distributions that are known or dependent on state or control variables at each time period but could be time-varying. Unlike parameter uncertainty where the true value of an uncertain parameter is unchanged over time, risks are assumed to be realized with possibly different values over time. Risks in the economic system can happen in technology, productivity, health and mortality, research and development (R&D), and international cooperation and noncooperation, as well as in many other areas. Risks in the climate system include the frequency and damage of extreme weather, and regime switching of the climate system (often called climate tipping risks). In particular, fat-tailed uncertainty in catastrophic climate change is the most worrisome (Weitzman, 2011).

3.1 Economic Risks

The economic system of an IAM often assumes that output is proportional to total factor productivity (TFP), $A_t$, and business cycle models assume that $A_t$ is a stochastic process. For example, Fischer and Springborn (2011) use a dynamic stochastic general equilibrium (DSGE) model with $\ln(A_{t+1}) = \rho \ln(A_t) + \epsilon_t$, where $\epsilon_t$ is an i.i.d. shock which is normally distributed with mean zero, to compare the performance of three instruments in achieving an exogenous and fixed level of expected emissions reduction: a carbon tax, emissions cap-and-trade (i.e. an exogenous limit on aggregate emissions), and an emission intensity target (i.e.
an exogenous limit on emissions per unit of aggregate output). They find that the intensity target scheme produces higher mean values and lower welfare costs than the other two policies, and compared to the no-policy case, volatility of the main macroeconomic variables decreases under the carbon tax policy but increases under the cap-and-trade instrument. Heutel (2012) also uses the same form of $A_t$ in his DSGE model, but focuses on comparing the optimal emissions tax rate and the optimal emissions quota. He finds that they both decrease during recessions, and during economic expansions, a price effect from costlier abatement dominates an income effect of greater demand for clean air. Fischer and Heutel (2013) survey some related work using real business cycle models with environmental policy and induced technological progress.

Annicchiarico and Di Dio (2015) formulate a New-Keynesian-type DSGE model to study the role of different environmental policy regimes in economic fluctuations, in the presence of nominal rigidities and accounting for two additional sources of uncertainty: public consumption shocks and monetary policy shocks. They find that in the presence of nominal rigidities, a cap-and-trade scheme is likely to dampen the response of the main macroeconomic variables to shocks, an intensity target environmental policy makes macroeconomic variables more volatile, and a carbon tax policy tends to have slightly higher mean welfare and lower volatility than the cap-and-trade scheme, as long as the degree of price stickiness is not too high. However if prices adjust very slowly, the cap-and-trade policy will have higher mean welfare. Karydas and Xepapadeas (2019) include transition risks of climate policy (i.e. the risks associated with carbon-intensive assets, which may become stranded due to stringent climate policies) in their dynamic asset pricing framework with rare disasters related to climate change, and show that transition risks substantially lower the participation of carbon-intensive assets in the market portfolio.

Jensen and Traeger (2014), Cai, Judd, and Lontzek (2017), and Cai and Lontzek (2019) also build DSGE models assuming that TFP, $A_t$, is a Markov process but is extended to be a long-run risk (Bansal and Yaron, 2004) that includes persistency of shocks. That is, they
assume $A_t = \tilde{A}_t \zeta_t$ where $\tilde{A}_t$ is an exogenous deterministic trend of TFP from DICE, and $\zeta_t$ is a shock with

$$\log (\zeta_{t+1}) = \log (\zeta_t) + \chi_t + \varphi \omega_{\zeta,t}$$  \hspace{1cm} (1)

$$\chi_{t+1} = r \chi_t + \varsigma \omega_{\chi,t},$$  \hspace{1cm} (2)

where $\chi_t$ represents the persistence of the shock, $\omega_{\zeta,t}, \omega_{\chi,t} \sim i.i.d. N(0, 1)$, and $\varphi, r,$ and $\varsigma$ are parameters. Moreover, the papers incorporate a recursive utility function (Epstein and Zin, 1989) with two preference parameters: the intertemporal elasticity of substitution (IES), and relative risk aversion. However, the DSICE model of Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) is a stochastic extension of the full DICE model (Nordhaus, 2008), and its parameters for the long-run risk on growth are calibrated with historical consumption growth data, whereas Jensen and Traeger (2014) use a reduced form model without empirical validation for the calibration of their long-run risk (see Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) for a more detailed discussion). Moreover, Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) discretize $\zeta_t$ and $\chi_t$ to be Markov chains with a large number of time-varying values, while Jensen and Traeger (2014) do not. The discretization is to avoid existence issues caused by the unbounded normal distributions of $\omega_{\zeta,t}$ and $\omega_{\chi,t}$, and to avoid excessive dependence on extreme tail events (as mentioned in Section 1, the risk premium could be infinite for unboundedly distributed uncertainties (Weitzman, 2009)).

Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) find that in the presence of long-run risk on growth, the SCC itself is a stochastic process with a wide range of possible values, and the recursive utility’s preference parameters have a nontrivial impact on the initial-time SCC: if the IES is large (not less than 0.9 in their numerical examples), then a larger risk aversion implies a smaller SCC; if IES is small (not larger than 0.7 in their numerical examples), then a larger risk aversion implies a larger SCC; and if risk aversion is small (not larger than 5), then the SCC increases with the IES.

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8Asset pricing theory has also been applied to estimate the SCC in the face of risks. For example, Bansal, Ochoa, and Kiku (2018) and Daniel, Litterman, and Wagner (2018) explore the implications of risk
3.2 Climate Risks

One major type of climate risks is ‘climate tipping risks’, referring to risks that can qualitatively alter the state or development of the climate system, if a large-scale component of the Earth system (the ‘tipping element’) passes a critical threshold (the ‘tipping point’). A climate tipping risk is often represented by an irreversible climate process, called a tipping process, which is often a Markov chain.

Lemoine and Traeger (2014) study the impact of tipping points on climate policy, where their tipping point is an instantaneous, irreversible increase in climate sensitivity (from 3°C to 4, 5, or 6°C), or an instantaneous, irreversible weakening of carbon sinks (by 25, 50, or 75%). However, Lontzek et al. (2015) point out that such a ‘tipping point’ is not scientifically plausible, because “positive feedbacks are never instantaneously switched on – instead they may get progressively stronger as temperature increases – so an instantaneous ‘tipping’ formulation is qualitatively wrong.” Climate scientists tell us that the duration of a climate element tipping to a new state is unknown and can last from a decade to even millennia (Lenton and Ciscar, 2013); no one believes that the entire Greenland ice sheet will melt within one year and cause an instant global sea level rise of six meters.

Lemoine and Traeger (2014) assume that a tipping point cannot occur when the contemporaneous temperature is below last period’s temperature. However, this assumption violates the consensus in climate science and completely ignores recent findings that suggest climate tipping might already have occurred, despite the last two decades, in which global warming had a decreasing short-period trend. For example, there is already scientific evidence that major ice sheets are losing mass at an accelerating rate (Khan et al., 2014). The preferences for the SCC and optimal abatement policies. Bansal, Kiku, and Ochoa (2019) show that the long-run temperature elasticity of equity valuations is significantly negative and that long-run temperature fluctuations carry a positive risk premium in equity markets.

9 See e.g. Lenton et al. (2008), Kriegler et al. (2009), Scheffer et al. (2009), Ditlevsen and Johnsen (2010), and Ghil and Lucarini (2019) for a discussion about the physics and early warning of climate tipping points.

10 Footnote 5 of Lemoine and Traeger (2014) states: “In our climate application, the decision maker keeps track of the greatest historic temperature.” However, given their model equations and code, that statement is wrong. The decision maker keeps track of the last-period temperature.
Greenland ice sheet’s mass loss is estimated to be contributing $\sim 0.7$ mm/year to sea level rise (Csatho et al., 2014), and Joughin, Smith, and Medley (2014) argue that the collapse of the West Antarctic ice sheet is already underway. Moreover, a real temperature path always rises and falls frequently, so the tipping probabilities assumed by Lemoine and Traeger will also frequently fluctuate between nonzero and zero, which is implausible. Furthermore, Lemoine and Traeger (2014) assume that the tipping probability only depends on the positive change between this year’s and next year’s temperature. Hence, no matter how low the current year’s temperature is, as long as it increases, the tipping probability will be nonzero. Moreover, no matter how high the current year’s temperature is, if next year’s temperature does not increase, then the tipping probability is zero. For example, suppose global temperatures reach 4°C above preindustrial levels without a tipping point occurring. In such a case, the Lemoine-Traeger model suggests that the world will not be exposed to a tipping point as long as next year’s temperature does not rise above 4°C. Thus, the decision-maker’s mitigation efforts and optimal climate policy will be based on completely distorted incentives. To conduct any reasonable policy analysis, one wants to correctly model the response of the climate tipping process to temperature, as this matters for the incentives to control greenhouse gases.

In addition, the hazard rate in Lemoine and Traeger (2014) is for illustrative purposes only and is not based on any calibration. Climate scientists admit that they do not know when or at which level of global warming a tipping point will occur (Lenton et al., 2008; Kriegler et al., 2009), but Lemoine and Traeger (2014) claim that a tipping point will definitely occur if temperatures reach 4.27°C above preindustrial levels—an assumption that is not supported by any study in climate science. When one aggregates the implied probabilities from expert elicitation by Kriegler et al. (2009), one finds that tipping is more likely than not in a 4–8°C long-term warming scenario, but still not certain.

In contrast, Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) investigate the impact on the SCC from climate tipping risks based on calibration with scientific beliefs.
Their tipping point process incorporates basic features of how climate scientists think about climate tipping points, such as a stochastic formulation of the physical process of triggering the tipping point and a transition time of tipping impacts with uncertain duration and long-run impact size. Their hazard rate formulation treats the physical process of the tipping point as stochastic with tipping probabilities that depend on the contemporaneous temperature itself, so a higher temperature will always have a higher tipping probability. Their hazard rate is calibrated according to beliefs expressed in expert elicitation studies, where the experts based their probability statements partly on what we know about the Earth’s history, partly on fundamental understanding, and partly on future model projections (Lenton et al., 2008). Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) model their tipping process as a 5-stage sequential process with each stage having a stochastic duration. Thus, not only do they model the long transition of climate tipping, as postulated by climate scientists, but they also account for these scientists’ lack of knowledge and imperfect information regarding the length of the transition process. Their multiple-stage approach to modeling the representative tipping element can produce lengths of transition in accordance with scientific beliefs. The complex Markovian structure in their DSICE model can produce a transition as short as 10 years, such as the increased frequency of El Niño events that is assumed to happen within one or two decades, but it could also result in a transition as long as several centuries, such as the melting of the ice sheets. Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) also account for the additional uncertainty about the long-run impact of climate tipping on the economy, which according to climate scientists, is the biggest uncertainty.

Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) find that under recursive utility and climate tipping risks, a higher IES or risk aversion always implies a higher SCC at the initial time. If a tipping event has not happened, then the SCC is significantly higher than in a deterministic model, but it will jump down significantly and immediately once the tipping event happens, even though the post-tipping damage has just started to unfold and may take many years to reach its maximum level. This pattern appears because the
incentive to prevent or delay the tipping event has disappeared if it has been triggered.

In addition, Cai, Judd, and Lontzek (2017) and Cai and Lontzek (2019) investigate the impact of a combination of climate tipping risk and economic risk, and find that such a combination may lead to an initial SCC that is lower than in the case with the climate tipping risk only and higher than in the case with the economic risk only, while either risk leads to a higher initial SCC than in the deterministic model.

The DSICE framework has also been applied with various variants in the literature. Lontzek et al. (2015) use it to investigate the impact of a tipping risk with a continuous tipping damage path under separable utility, and find that today’s optimal carbon tax will increase by around 50% even with conservative assumptions about the rate and impacts of a stochastic tipping event. Moreover, the effective discount rate for the costs of stochastic climate tipping is much lower than the discount rate for deterministic climate damages. Cai et al. (2015) use the DSICE framework to study the impact of environmental tipping risk on market and nonmarket goods and services, and find that even if a tipping risk only has nonmarket impacts, it could substantially increase the present optimal carbon tax. Cai, Lenton, and Lontzek (2016) extend DSICE to incorporate five major interacting climate tipping risks (Atlantic meridional overturning circulation, disintegration of the Greenland ice sheet, collapse of the West Antarctic ice sheet, dieback of the Amazon rainforest, and shift to a more persistent El Niño regime) simultaneously in their model. They find that doing so increases the present SCC by nearly eightfold, and passing a tipping point may abruptly increase the SCC if it increases the likelihood of other tipping events (note that if there is only one tipping risk, then passing its tipping point will abruptly decrease the SCC, as shown in Lontzek et al. (2015), Cai et al. (2015), and Cai and Lontzek (2019)). Nordhaus (2019) finds that the risk of Greenland ice sheet disintegration makes a small contribution to the overall social cost of climate change. This finding is consistent with Cai, Lenton, and Lontzek (2016) in the case without interactions between tipping events (see Figure 3 of Cai, Lenton, and Lontzek (2016)), but Cai, Lenton, and Lontzek (2016) also
show that in the case with interactions between tipping events, if the Greenland ice sheet tips first, it leads to the most stringent emissions control because the likelihood of the presumably most-damaging event (Atlantic meridional overturning circulation collapse) significantly increases.

Cai et al. (2019) construct a numerical dynamic stochastic two-region model, DIRESCU, which separates the world into the North and the Tropic-South regions with their own economic and climate systems and includes interactions between regions. They also consider a global climate tipping risk, global sea level rise, and regional adaptation, and find that carbon taxes increase significantly in both regions to curb or delay the occurrence of climate tipping and sea level rise in both cooperative and non-cooperative worlds, while regional adaptation reduces carbon taxes significantly.

There is also theoretical analysis for pricing carbon in the face of climate tipping. For example, van der Ploeg and de Zeeuw (2016) build a simple stylized North-South model of the global economy to investigate how differences between regions in terms of their vulnerability to climate change and their stage of development affect cooperative and non-cooperative responses, both to curb the risk of a calamity and accumulate precautionary capital to facilitate consumption smoothing. van der Ploeg and de Zeeuw (2018) find that if the mean lag for the impact of the catastrophe from climate tipping is long enough, the saving response will be negative, because the precautionary return in the Keynes–Ramsey rule becomes negative.

### 3.3 Dealing with Risks

A typical dynamic stochastic IAM can be written as

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\max_{a_t \in D_t(x_t)} \mathbb{E} \left\{ \sum_{t=0}^{T-1} \beta^t u_t(x_t, a_t) + \beta^T V_T(x_T) \right\}
$$

s.t.

$$
x_{t+1} = f_t(x_t, a_t, \epsilon_t), \ t = 0, 1, \ldots, T - 1
$$

$$
x_0 \text{ given}
$$
where $E$ is the expectation operator over all random variables $\epsilon_t$ for all time $t$, $x_t$ is a vector of state variables (such as capital, oil stock, carbon concentration in the atmosphere, and global average atmospheric temperature), $a_t$ is a vector of control variables (also called action or decision variables, such as consumption and emission mitigation rates), $\beta$ is the discount factor, $T$ is the terminal time (could be infinite), $u_t$ is a utility function, $V_T$ is the terminal value function (when $T = \infty$, this term disappears), $f_t$ is a vector of functions representing transition laws of state variables, and $D_t(x_t)$ is a feasible set of the control variables and depends on the state variables at each time $t$. When the transition law of the $j$-th state variable is deterministic, $x_{t+1,j} = g_{t,j}(x_t, a_t)$, we still denote it as $x_{t+1,j} = f_{t,j}(x_t, a_t, \epsilon_t) = g_{t,j}(x_t, a_t) + 0 \cdot \epsilon_t$ for convenience. We can also transform (3) to the following Bellman equation (Bellman, 1957):

\[
V_t(x_t) = \max_{a_t \in D_t(x_t)} \left[ u_t(x_t, a_t) + \beta E_t \{ V_{t+1}(x_{t+1}) \} \right]
\]

s.t. $x_{t+1} = f_t(x_t, a_t, \epsilon_t)$

for $t = 0, 1, ..., T - 1$, where $E_t$ is the expectation operator over $\epsilon_t$ conditional on time-$t$ information $(x_t, a_t)$, and $V_t$ is the value function at time $t$.

In economics, consumption $c_t$ is often a decision variable (one element of $a_t$), and a typical utility function is a power function:

\[
u_t(x_t, a_t) = \frac{c_t^{1-\gamma}}{1-\gamma}
\]

for $\gamma > 0$ and $\gamma \neq 1$, where $\gamma = 1$ is the special case of logarithmic utility:

\[
\lim_{\gamma \to 1} \frac{c_t^{1-\gamma} - 1}{1-\gamma} = \ln(c_t).
\]

For a deterministic dynamic model, $\gamma$ is the inverse of intertemporal elasticity of substitution (IES). For a stochastic model, $\gamma$ is also the relative risk aversion parameter.
disentangle the IES from risk aversion, we can use recursive utility \cite{Epstein1989}, which has been employed in IAMs recently, for example, in \cite{Jensen2014, Cai2016, Cai2019}. The corresponding Bellman equation is

\[
V_t(x_t) = \max_{a_t \in D_t(x_t)} u_t(x_t, a_t) + \beta \mathcal{G}_t \{ V_{t+1}(x_{t+1}) \}
\]

\hspace{1cm} \text{s.t. } x_{t+1} = f_t(x_t, a_t, \epsilon_t)

where

\[
\mathcal{G}_t \{ V_{t+1}(x_{t+1}) \} \equiv \frac{1}{1 - 1/\psi} \left( \mathbb{E}_t \left\{ \left( (1 - 1/\psi) V_{t+1}(x_{t+1}) \right)^{1-\gamma/\psi} \right\} \right)^{1-1/\psi}
\]

with $\psi$ and $\gamma$ as the IES and the risk aversion parameter respectively.

The most common method to solve the (time-varying) Bellman equation is value function iteration (VFI). When some state variables are continuous, value functions $V_t$ have to be approximated. An efficient approximation method is to use complete Chebyshev polynomials (over multi-dimensional continuous state variables) and associated Chebyshev nodes. It is essential for approximation errors to be small, and we should always check the errors of the solution, otherwise the numerical solution could be far away from the true solution.

For example, assume that we employ VFI to solve an infinite-horizon stationary problem, where the true value function $V$ satisfies the Bellman equation $V = \Gamma(V)$, where $\Gamma$ is the Bellman operator. Starting with an initial guess $V_0$, VFI solves $V_t = \Gamma(V_{t-1})$ for $t = 1, 2, ...$ until it converges under a stopping criterion: $\|V_t - V_{t-1}\| < \varepsilon$, where $\varepsilon$ is a small positive number and $\|\cdot\|$ is a norm operator over functions. Since we cannot solve $V_t$ at all states if some state variables are continuous, numerically we solve $V_t$ at approximation nodes $x_j$ and then use values $V_t(x_j)$ to construct an approximation of $V_t$ at all states. Let the value function at iteration $t$ be approximated by a linear combination of basis functions, denoted $\hat{V}_t$, and numerical VFI converges under a stopping criterion: $\|\hat{V}_t - \hat{V}_{t-1}\| < \varepsilon$. This convergence does not guarantee that $\hat{V}_t$ is a good approximation to the true value.
function $V$, as numerical VFI computes $(\Gamma(\hat{V}_{t-1}))(x_j)$ and then uses the values $(\Gamma(\hat{V}_{t-1}))(x_j)$ to approximate $\Gamma(\hat{V}_{t-1})$ with $\hat{V}_t$; that is $\hat{V}_t \neq \Gamma(\hat{V}_{t-1})$. In fact, numerical VFI may converge under the stopping criterion with any degree of approximation (such as linear or quadratic polynomial approximation). Therefore, if the approximation error, $\|\hat{V}_t - \Gamma(\hat{V}_{t-1})\|$, is large, then the converged solution $\hat{V}_t$ may be far away from the true solution. In addition, even if a high-degree Chebyshev polynomial is used in approximation, a loose stopping criterion may also lead to large errors in the solution. For example, Cai (2019) points out that the stopping criterion of VFI used in Lemoine and Traeger (2014) is problematic, so the numerical solution of Lemoine and Traeger (2014) may have large errors.  

However, the Bellman equation (4) or (5) only applies to a social planner’s problem, where the social planner make all decisions including reallocating resources among different regions or countries (with some costs), while these regions/countries are completely cooperative. In the real world, the regions/countries may be non-cooperative, which leads to a (time-varying) dynamic stochastic game. Cai et al. (2019) introduce a new time-backward iterative method to solve a system of Bellman equations and then find a feedback Nash equilibrium numerically.

In many cases, it could be very challenging to employ VFI if there are kinks in value functions and/or the number of state variables is large. Cai, Judd, and Steinbuks (2017) introduce a nonlinear certainty equivalent approximation method to solve an infinite-horizon stationary problem in the form (3). Cai et al. (2020) then extend it to solve a (finite/infinite) nonstationary problem. See Cai (2019) for discussion about other computational methods.

4 Model Uncertainty and Scenario Uncertainty

No model can replicate the real world completely. For tractability, every model has to make some simplifying assumptions, particularly in mathematical representations of economic and climate systems. Different assumptions then lead to different models. In the words of Albert Cai (2019) for details about computational methods and error checking.
Einstein, “Everything should be made as simple as possible, but not simpler”. Economists often adopt the first half of the sentence, ignoring the second half, as an excuse for using oversimplified models, particularly for IAMs, as climate models are often too complicated to be applied in IAMs for economic analysis. Moreover, there is large uncertainty in future temperature projections from climate models, as well as in future economic systems.

There are many IAMs in the literature, such as DICE (Nordhaus, 2008, 2017), FUND (Anthoff and Tol, 2013), PAGE (Hope, 2011), WITCH (Bosetti et al., 2006), MERGE (Manne and Richels, 2005), RICE (Nordhaus, 2010), NICE (Dennig et al., 2015), DSICE (Cai, Judd, and Lontzek, 2017; Cai and Lontzek, 2019), and DIRESCU (Cai et al., 2019). These IAMs are DSGE models (some are deterministic perfect foresight models) that include a damage function mapping temperature increases to economic damages, allowing the optimal policy to be found using cost-benefit analysis, so they are also policy optimization models. For instance, DICE, FUND, and PAGE have been used by the US Interagency Working Group to calculate the SCC (IWG, 2010) under different consumption discount rates.

Since DSGE models can only use a simple climate system for computational tractability, some researchers developed policy evaluation models that assume that emissions or mitigation policies are exogenous and have no feedback to the economy. These policy evaluation models include GCAM (Calvin et al., 2019), IMAGE (Stehfest et al., 2014), MESSAGE (Huppmann et al., 2019), AIM/CGE (Fujimori, Masui, and Matsuoka, 2017), REMIND (Luderer et al., 2015), and IGSM (Chen et al., 2016).

IAMs have to assume some exogenous paths, such as population and technology paths, but these paths are often uncertain, so these models have scenario uncertainty. For example, policy evaluation models often rely on exogenous emission scenarios, but there are four representative concentration pathways (RCPs) of greenhouse gas concentrations (Meinshausen et al., 2011): RCP2.6, RCP4.5, RCP6, and RCP8.5, and they all have wide ranges in 2100.

12 There are also many reduced form IAMs (e.g. Golosov et al. 2014, Jaakkola and van der Ploeg 2019, Brock and Xepapadeas 2019), but they are mainly used for theoretical or qualitative analysis. This review focuses on quantitative analysis for climate policies.

13 In addition, agent-based models have also been employed in IAMs; see Farmer et al. (2015) for a review.
O’Neill et al. (2014) describe five Shared Socio-Economic Pathways (SSPs) covering wide ranges of the projected population, income, and temperature in 2100.

A different model or scenario provides a different optimal policy. Usually policymakers do not know which model or scenario is more reliable, and it is challenging to assign a belief distribution over the models or scenarios. But they often have to choose only one policy. One way is to do a multi-model or multi-scenario comparison and then find some robust results from the models. For example, Kim et al. (2017) analyze a set of simulations to assess the impact of climate change on global forests under two emissions scenarios: a business-as-usual reference scenario analogous to the RCP8.5 scenario, and a greenhouse gas mitigation scenario, which is in between the RCP2.6 and RCP4.5 scenarios. Gillingham et al. (2018) compare six models: DICE, FUND, GCAM, MERGE, IGSM, and WITCH. But in many cases, different models or scenarios may lead to significantly different solutions, so it is not possible to extract a robust policy from a multi-model or multi-scenario comparison. A robust decision-making method, such as the max-min method or the min-max regret method (Iverson, 2012, 2013; Anthoff and Tol, 2014; Cai and Sanstad, 2016), is a tool to solve this problem in the face of model uncertainty. For example, Cai, Golub, and Hertel (2017) apply the min-max regret method to study robust decisions of agricultural research and development in the face of uncertain SSP scenarios in population, income, and temperature. Rezai and van der Ploeg (2017) derive max-min, max-max, and min-max regret policies to deal with climate model uncertainty (among DICE, FUND, and PAGE) and climate skepticism. If we can give a distribution for model uncertainty, then we may incorporate model uncertainty aversion. For example, Berger, Emmerling, and Tavoni (2017) study the impact of model uncertainty aversion on optimal mitigation policy under catastrophic climate risks.
5 Policy Uncertainty

There are many climate policy instruments, including carbon taxation, cap-and-trade, intensity-based targets, subsidies (for renewable energy, research for new clean technology, emission reductions, etc.), solar geoengineering, carbon geoengineering (including afforestation, bio-energy with carbon capture and sequestration, carbon removal and storage), and adaptation. Each instrument has its advantages and disadvantages. For example, a carbon tax gives a direct price on carbon emissions so companies can adjust their emissions based on cost and benefit analysis, but there is uncertainty in its effect on total emissions. A cap-and-trade scheme issues a number of emission allowances for the market to auction and trade, so it provides direct control over future emissions and it would be more straightforward to control temperature increase under some threshold (e.g. 2 or 1.5 °C), but it is hard to estimate its economic cost. An intensity-based target scheme requires emissions per unit of economic activity (e.g. output) to not exceed given targets, so it may be appealing to developing economies, but there is uncertainty in aggregate emissions and economic costs. Carbon tax is the most popularly debated policy (its representative model is DICE), and it is often estimated to be equal to the SCC if it is not explicitly modeled (as in Baldwin, Cai, and Kuralbayeva (forthcoming)), and if emissions control has not reached its limit (Cai, Judd, and Lontzek, 2017; Cai and Lontzek, 2019). However, it could be challenging to politically pass a carbon tax policy in some countries (such as the US). Instead, the cap-and-trade scheme may be implemented at a regional level. For example, there are currently cap-and-trade programs like the European Union Emissions Trading Schedule, the Regional Greenhouse Gas Initiative, and the California cap-and-trade program, though these programs require careful design to make them effective.

There are a large number of research works in climate policy analysis, even just in recent years and after I exclude those cited in the previous sections. For example, Acemoglu et al. (2016) develop a microeconomic model of endogenous growth where clean and dirty technologies compete in production and innovation, and then characterize the optimal policy path.
that makes heavy use of research subsidies as well as carbon taxes. Barreca et al. (2016) examine the evolution of the temperature-mortality relationship in the US to identify potentially useful adaptations, and find that residential air conditioning contributes a substantial fraction of the welfare gains. Burke and Emerick (2016) exploit large variation in recent temperatures and precipitation trends to identify adaptation to climate change in US agriculture, and find that longer-run adaptations appear to have mitigated less than half—and more likely none—of the large negative short-run impacts of extreme heat on productivity. Heutel, Moreno-Cruz, and Shaveghi (2016, 2018) and Keith, Wagner, and Zabel (2017) investigate the use of solar geoengineering as a substitute for emissions abatement to reduce the atmospheric carbon burden. Favero, Mendelsohn, and Sohngen (2017) recommend using forests to mitigate greenhouse gases by storing carbon and supplying woody biomass for burning in power plants with carbon capture and storage. Griscom et al. (2017) estimate natural climate solutions including conservation, restoration, and improved land management actions that increase carbon storage and/or avoid greenhouse gas emissions across global forests, wetlands, grasslands, and agricultural lands. Economides and Xepapadeas (2018) build a New Keynesian DSGE model to explore how and to what extent monetary policy should be adjusted under conditions of climate change. Gopalakrishnan, Landry, and Smith (2018) analyze coastal climate change adaptation in the face of sea level rise, ocean warming and acidification, and increased storminess, and conclude that adaptation will require governance coordination across multiple levels. Massetti and Mendelsohn (2018) examine methods for measuring climate adaptation using the empirical evidence. Proctor et al. (2018) estimate the global agricultural effects of solar radiation management for managing global temperatures by scattering sunlight back to space. Fried, Novan, and Peterman (2018) and Cronin, Fullerton, and Sexton (2019) study redistributions from a carbon tax. Fried (2018) finds that a carbon tax induces large changes in innovation, and the innovation response increases the effectiveness of the policy at reducing emissions. Meng and Rode (2019) evaluate the social cost of lobbying over climate policy. Barrage (2020) characterizes and quantifies
optimal carbon taxes in a dynamic general equilibrium climate–economy model with distortionary fiscal policy, and finds that optimal carbon tax schedules are 8–24% lower when there are distortionary taxes, compared to the setting with lump-sum taxes considered in the literature. Hafstead and Williams (2020) examine the role for tax adjustment mechanisms, which automatically adjust the carbon tax rate based on the level of actual emissions relative to a legislated target, and the trade-offs of alternative designs. They show that tax adjustment mechanisms in carbon tax design can substantially reduce emissions uncertainty. Kalkuhl, Steckel, and Edenhofer (forthcoming) find that the time-consistent policy is the “all-or-nothing” policy with either a zero carbon tax or a prohibitive carbon tax that leads to zero fossil investments, and it is the lobbying power of owners of fixed factors (land and fossil resources), rather than fiscal revenue considerations or the lobbying power of renewable or fossil energy firms, that determines which of the two outcomes (all or nothing) is chosen. van der Ploeg and Rezai (forthcoming) allow for immediate or delayed carbon taxes and renewable subsidies that will cause discrete jumps in today’s valuation of physical and natural capital, and then investigate how the legislative “risk” of tipping into policy action affects when the fossil era ends, the profitability of existing capital, and green paradox effects (Sinn, 2008).

Policy comparisons have also been conducted in the literature. For example, Goulder and Parry (2008) review many instrument choices in climate policy with different evaluation criteria, including economic efficiency and cost-effectiveness, distribution of benefits or costs (across income groups, ethnic groups, regions, generations, etc.), ability to address uncertainties, and political feasibility. Fischer and Springborn (2011) compare carbon tax, cap and trade, and intensity-based targets in a DSGE model with stochastic productivity. Heutel (2012) compares the optimal emissions tax rate and the optimal emissions quota. Drouet, Bosetti, and Tavoni (2015) discuss selection of climate policies under uncertainties. Goulder, Hafstead, and Williams III (2016) argue that under plausible conditions a more conventional form of regulation, a clean energy standard, is more cost-effective than emissions
pricing such as carbon taxation or cap-and-trade. Meckling, Sterner, and Wagner (2017) investigate the combination and sequence of policies to avoid environmental, economic, and political dead-ends in decarbonizing energy systems. Gillingham and Stock (2018) review the costs of various technologies and actions aimed at reducing greenhouse gas emissions. Rozenberg, Vogt-Schilb, and Hallegatte (2018) compare the impact of mandates (for new power plants, buildings and appliances), feebates (programs that tax energy-inefficient equipment and subsidize energy-efficient equipment), energy efficiency standards, and carbon pricing in a simple model with clean and polluting capital, irreversible investment, and a climate constraint. They find that carbon prices are efficient but can cause stranded assets, while feebates and mandates do not create stranded assets. Baldwin, Cai, and Kuralbayeva (forthcoming) compare a carbon tax with a subsidy for renewable energy using a DSGE model, which is based on the full DICE model but adds renewable and non-renewable energy sectors as well as a government who decides the optimal dynamic carbon tax or subsidy. They find that a carbon tax is more efficient under a stringent climate target, while a subsidy is more efficient under a mild climate target.

6 Ambiguity and Misspecification

A stochastic IAM often assumes that the probability distribution functional form of a risk or shock is given with parameters estimated from historical data, future projections, survey data, or expert opinions. For example, researchers often assume that TFP is a lag-1 autoregression process and its shock has a normal distribution with mean zero and an estimated standard deviation. However, estimated parameters have standard errors, implying that we are uncertain about the true probability distribution. Sometimes even the functional form of the probability distribution may be misspecified. For example, researchers provide many different belief distributions for the climate sensitivity parameter (IPCC, 2013), but it is unclear which particular one should be applied in IAMs.
In the face of the probability ambiguity and misspecification, Hansen and Sargent (2008) introduce a robust control framework with risk and ambiguity aversion, which is applied by Athanassoglou and Xepapadeas (2012) for an analytical pollution control problem. Millner, Dietz, and Heal (2013) study climate mitigation policy with ambiguity aversion and find that the value of emissions abatement increases as ambiguity aversion increases. Anderson, Brock, and Sanstad (2018) conduct an empirically disciplined robustness analysis for the size of the set of perturbations from their baseline model of economic growth dynamics and climate dynamics. Rudik (2019) incorporates the robust control framework to include learning on climate damage. Barnett, Brock, and Hansen (forthcoming) investigate risk, ambiguity, and misspecification with continuous-time models and corresponding pricing methods to assess what sources of uncertainty matter the most for the SCC.

7 Conclusion

I have reviewed state-of-the-art work on different types of uncertainty in controlling climate change: parameter uncertainty, risk, model uncertainty, scenario uncertainty, policy uncertainty, ambiguity, and misspecification. Uncertainty often plays an essential role in models and changes results significantly. With advanced computational methods and hardware, it becomes possible to analyze policies in more complex and realistic IAMs with uncertainty.

With advances in understanding the physical science of climate change and the economic system, there are a number of potential future studies that can incorporate uncertainty in climate change economics. Burke et al. (2016) discuss some research opportunities in climate change economics, particularly in three areas: SCC refinement, policy evaluation, and evaluation of climate impact and policy choices in developing countries. Incorporating uncertainty into related research may generate interesting results. Other research opportunities include richer and more realistic representations of the economic and climate systems as

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14See Brock and Hansen (2018) for a review on research challenges in climate economics that focuses on three types of uncertainty: risk, ambiguity, and misspecification.
well as policies with uncertainty: spatial disaggregation as in Krusell and Smith (2017), disaggregation of intertemporal agents (overlapping generations) as in Kotlikoff et al. (2019), disaggregation of sectors (e.g. adding the green finance sectors), disaggregation of heterogeneous agents, integration with other systems (e.g. the water system), and more realistic international trade and international agreements in climate policies. It will be interesting to incorporate uncertainty in research work on IAMs with climate impact on income inequality, regional inequality, social conflict, human health and ecosystems, and migration. As suggested by Irwin, Gopalakrishnan, and Randall (2016), it will also be important for future work to evaluate sustainability and resilience in the face of uncertainty and climate change.

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