QCD-IMPROVED FACTORIZATION IN NONLEPTONIC $B$ DECAYS

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I consider nonleptonic decays of $B$ mesons into two light mesons using the light-cone wave functions for the mesons. In the heavy quark limit, nonfactorizable contributions are calculable from first principles in some decay modes. I review the idea of the QCD-improved factorization method and discuss the implications in phenomenology.

1 Introduction

Nonleptonic decays of $B$ mesons have attracted a lot of attention recently since they were observed experimentally in CLEO, BaBar and BELLE. These decays are important in extracting the information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and CP violation. On the theoretical side, it is the least understood area. Since nonleptonic decays involve nonperturbative effects such as final-state interactions, it is difficult to obtain a quantitative theoretical prediction. However, it has been recently found that nonperturbative effects such as the nonfactorizable contribution in nonleptonic decays could be systematically understood in the heavy quark limit with $m_b \to \infty$.

Here I will review the current status of understanding on nonleptonic $B$ decays very schematically. I will focus on the underlying ideas omitting technical complication. I hope that this talk will give a clear sketch of what is being studied currently. It has been found that nonfactorizable contributions in nonleptonic $B$ decays into two light mesons could be calculated using perturbation theory in the heavy quark limit $m_b \to \infty$. I will explain in detail how we treat nonleptonic $B$ decays in the heavy quark limit and discuss some phenomenological aspects.

The theoretical framework for $B$ decays is to use the effective weak Hamiltonian at the renormalization scale $\mu \approx m_b$, which is schematically given as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_q V_{qb} V_{qd}^* C_i(\mu) Q^Q_i(\mu),$$  \hspace{1cm} (1)

where $V_{ij}$ are the CKM matrix elements, $C_i$ are the Wilson coefficients and $Q^Q_i$ are the effective four-quark operators. The Wilson coefficients are calculable order by order in perturbation theory, and the main issue in analyzing
nonleptonic decays is how to evaluate the matrix elements of the four-quark operators. For example, if $\bar{B}$ decays into the final two mesons $M_1$ and $M_2$, how do we evaluate $\langle M_1 M_2 | O_i^q | \bar{B} \rangle$?

First we can use the naive factorization in which we neglect the strong interaction effects between mesons and separate the operators into a current-current form, and calculate their matrix elements. Schematically this process can be expressed as

$$\langle M_1 M_2 | O_i^q | B \rangle \approx \langle M_1 | J_\mu | 0 \rangle \langle M_2 | J^\mu | B \rangle.$$  

(2)

Here $J_\mu$ is the current operator in the effective Hamiltonian. Each matrix element is parameterized by a decay constant or a form factor, which describe intrinsic nonperturbative effects. The decay constant and the form factors can be obtained from either experiment or other theoretical techniques such as QCD sum rules.

Unfortunately this naive factorization is unsatisfactory. First of all, there is no justification in neglecting the final-state interactions between mesons. The argument of color transparency can be applied in the case of two final light mesons, but it should be proved explicitly if the matrix elements can be truly factorized including final-state interactions. Secondly, theoretically the naive factorization gives an unphysical result. Decay constants and form factors are independent of the renormalization scale $\mu$, and the decay amplitude has an arbitrary $\mu$ dependence through the Wilson coefficients $C_i(\mu)$. This is because we replace the matrix elements, which depend on $\mu$, by the decay constants and the form factors, which are independent of $\mu$. Therefore the resulting amplitude is unphysical.

As a remedy to this problem, Ali and Greub suggested to calculate the radiative corrections of the operators before taking the matrix elements. Following this procedure, we can write the matrix element as

$$\langle M_1 M_2 | O_i^q | B \rangle \approx F(\mu, \alpha_s) \left[ \langle M_1 | J_\mu | 0 \rangle \langle M_2 | J^\mu | B \rangle \right]_{\text{tree}}.$$  

(3)

It turns out that the $\mu$ dependence in $F(\mu, \alpha_s)$ arising from the radiative corrections cancel the $\mu$ dependence in the corresponding Wilson coefficients at any given order, hence the decay amplitude does not depend on the renormalization scale. I call this procedure the "improved factorization".

The unphysical $\mu$ dependence is absent in the improved factorization, but it has other significant problems. First of all, in order to avoid infrared divergence, $F(\mu, \alpha_s)$ is calculated with off-shell external quarks. The external momenta $p^2$ play a role of the infrared cutoff. However, because of this, the decay amplitudes depend on the choice of the gauge, and the employed reg-
ularization schemes. Therefore the decay amplitude depends on arbitrary gauges and the schemes, hence also unphysical.

If we put external quarks on their mass shell, the gauge dependence and the scheme dependence go away, but, in this case, the amplitude becomes infrared divergent and gives also an unphysical result. Therefore we need a consistent scheme which solves all the problems I mentioned above. Before I explain the main idea of the QCD-improved factorization, note that the Feynman diagrams from which infrared divergence appears are those with vertex corrections of the weak currents.

2 QCD-improved factorization

The source of infrared divergence in the radiative corrections of the effective weak Hamiltonian, as mentioned above, suggests an interesting idea. The Feynman diagrams which cause infrared divergence are the radiative corrections of vertices. In other words, the infrared divergence comes from the radiative corrections for the decay constant of a light meson and form factors for $B \rightarrow M_2$.

It reminds us of the hadron-hadron scattering, in which the infrared divergence of the scattering amplitude is absorbed in the redefinition of the parton distribution functions. The remaining hard scattering amplitude and the parton distribution functions are factorized. We can apply the same idea to $B$ decays. The infrared divergences can be attributed to the renormalization of the decay constant and the form factors, and other radiative corrections constitute nonfactorizable contributions. That is, the infrared divergence is absorbed in the definition of the light-cone meson wave function or the form factors. This is first observed in Ref. 7.

Recently Beneke et al. 8 formulated this problem in the heavy quark limit and extensively studied nonleptonic $B$ decays into two final-state mesons. The idea can be summarized as follows: We first take the heavy quark limit $m_b \rightarrow \infty$. In this limit we can calculate nonfactorizable contributions systematically in perturbative QCD. Also we can obtain corrections to the perturbative results by expanding in powers of $\Lambda_{\text{QCD}}/m_b$.

The next step is to arrange external quarks using Fierz transformation such that the quark-antiquark pair, which forms a meson, is included in a single current. And we use the light-cone wave function for the mesons. If we can use the operators in the effective Hamiltonian as they are in arranging quarks, we call that configuration of quarks as “charge-retention configuration”. If we have to switch some quarks using Fierz transformation, we call that configuration as “charge-changing configuration”. This arrangement is important
since it determines which radiative corrections correspond to the corrections to the decay constants or the form factor. Since the decay amplitude in each process depends on how we arrange quarks, the scattering amplitude becomes process-dependent.

The infrared divergence is attributed to the renormalization of the wave function or the form factors. And we calculate all the nonfactorizable contributions along with the spectator contribution and the annihilation channels. This procedure is called the “QCD-improved factorization”. If all the nonfactorizable contributions are infrared finite, and suppressed as $m_b$ goes to infinity, these processes can be treated in the QCD-improved factorization. In this case, we have a theoretical method to analyze nonleptonic $B$ decays from first principles. In the QCD-improved factorization method, we can formally write the matrix element as

$$\langle M_1 M_2 | O_i | B \rangle \approx \langle M_1 | J_\mu | 0 \rangle \langle M_2 | J^\mu | B \rangle \left[ 1 + O(\alpha_s) + O(\Lambda_{\text{QCD}}/m_b) \right].$$

(4)

If we neglect the radiative corrections and the $1/m_b$ corrections, we restore the result obtained in the naive factorization. The matrix element can be written as a product of a decay constant and a form factor. And we can systematically calculate the corrections to the result in the naive factorization.

Before we discuss the details of the QCD-improved factorization, I would like to explain another approach to study nonleptonic $B$ decays. Keum et al. have also considered nonleptonic $B$ decays using light-cone wave functions, but they concentrated on the calculation of the form factors instead of nonfactorizable contributions. They calculated a single gluon exchange which is responsible for the correction to the form factor. There also appears infrared divergence in the form factor, but they introduce the Sudakov factor which smears the endpoint region such that there is no infrared divergence. And the origin of imaginary parts in their calculation comes from the modification of the propagator with transverse momentum, which is totally different from the source of the imaginary part in the QCD-improved factorization.

They do not consider nonfactorizable contributions at the moment, and hence the leading-order Wilson coefficients are employed. Because what is calculated is totally different in the two approaches, we have to be careful when we try to compare the results in both approaches.

3 Nonfactorizable contribution

Let us go into the detail of how to calculate nonfactorizable contributions. When we use the light-cone meson wave functions for exclusive decays, the
transition amplitude of an operator $O_i$ in the weak effective Hamiltonian is

given by

$$\langle M_1 M_2 | O_i | \mathcal{B} \rangle = \sum_j F_j^{B \rightarrow M_2} \int_0^1 dx T_{ij}^I(x) \phi_{M_i}(x)$$

$$+ \int_0^1 d\xi du T_{ii}^{II}(\xi, x, u) \phi_B(\xi) \phi_{M_i}(x) \phi_{M_2}(u), \quad (5)$$

where $F_j^{B \rightarrow M_2}$ are the form factors for $B \rightarrow M_2$, and $\phi_{M_i}(x)$ is the light-cone wave function for the meson $M_i$. $T_{ij}^I(x)$ and $T_{ii}^{II}(\xi, x, u)$ are hard-scattering amplitudes, which are perturbatively calculable. The second term in Eq. (5) represents spectator contributions.

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Figure 1. Feynman diagrams for nonfactorizable contribution at order $\alpha_s$. The dots represents the operators $O_i$.

The relevant Feynman diagrams for $T_{ij}^I$ are shown in Fig. 1. Each Feynman diagram has an infrared divergence. But if we sum over all the Feynman diagrams and symmetrize with respect to $x \leftrightarrow 1-x$, where $x$ is the momentum fraction of the outgoing meson, we have an infrared-finite result. Another feature of the nonfactorizable contribution is that there appears an imaginary part due to the final-state interactions. This plays an important role in studying CP violation in nonleptonic decays. The strong phase is calculable in perturbation theory. And since we are working at next-to-leading order accuracy, the dependence on $\mu$ becomes very mild.

There are other nonfactorizable contributions such as the spectator contributions. The Feynman diagrams for $T_{ij}^{II}$ are shown in Fig. 2. In calculating nonfactorizable contributions, we use the light-cone wave functions. The projection of the quark bilinears to each pseudoscalar light meson wave function to the order of twist three can be written as

$$\langle P(p) | \bar{\tau}_\alpha(y) q_\beta'(x) | 0 \rangle = \frac{i f_P}{4} \int_0^1 du e^{i (u p y + (1-u)p \cdot x)}$$
Figure 2. Feynman diagrams for the spectator contribution.

\[ \times \left\{ \slashed{p} \gamma_5 \phi(u) - \mu_p \gamma_5 \left( \phi_p(u) - \sigma_{\mu \nu} p^\mu (y - x)^\nu \right) \frac{\phi_\sigma(u)}{6} \right\}. \tag{6} \]

For the \( B \) meson, we can write the projection as

\[ \langle 0 | \gamma_\alpha b | B \rangle = -i \frac{f_B}{4} \phi_B(\xi) \left\{ \left( \phi_B + m_B \right) \gamma_5 \right\}_\beta \alpha. \tag{7} \]

Here \( \phi \) is the leading-twist wave function, and \( \phi_p \) and \( \phi_\sigma \) are twist-three wave functions for the pseudoscalar and the tensor currents respectively. For the \( B \) wave function, we take the leading-twist wave function only. Since we expand in powers of \( 1/m_b \), at leading order \( \phi_B(\xi) \sim \delta(\xi - \Lambda_{QCD}/m_b) \). In calculating the spectator contribution, we have the integral of the form

\[ \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \approx \frac{m_B}{\Lambda_{QCD}}, \tag{8} \]

which is enhanced.

We can consider corrections of order \( O(\Lambda_{QCD}/m_b) \). The most important contribution comes from the term proportional to \( \mu_p/m_b \) which is given by

\[ \mu_P = \frac{m_P^2}{m_1 + m_2}, \tag{9} \]

where \( m_1 \) and \( m_2 \) are the masses of the quarks which form a meson with mass \( m_P \). Compared to other \( O(\Lambda_{QCD}) \) terms, this is numerically large. For the case of \( \pi^+ \), for instance, it is \( \mu_{\pi^+} \sim 1.4 \text{ GeV} \). Therefore it has been of great interest to calculate higher-twist effects proportional to \( \mu_P \). However, the spectator contribution from \( O_1 \) and \( O_2 \) with higher-twist wave functions is infrared divergent \[ 4 \]. It has been suggested that we introduce some parameters to regulate the infrared divergence and regard the parameter as a theoretical uncertainty. But these contributions are numerically large and especially the extraction of the strong phase becomes too ambiguous to say anything quantitatively.
The annihilation topology poses another problem. When we calculate the spectator contribution with the operator $O_5$, the amplitude is also infrared divergent. Therefore the annihilation topology can give a significant power correction to the decay amplitude. The analysis on the annihilation channel with radiative correction is in progress.

One way to look at the infrared divergence is that it is not a serious problem. The divergence comes from the endpoint and it actually gives the logarithmic enhancement as

$$\int_{\Lambda_{QCD}/m_b}^{1} \frac{du}{u} \approx \ln \frac{m_b}{\Lambda_{QCD}}.$$ (10)

Since the cutoff $\Lambda_{QCD}$ is actually arbitrary, we can parameterize this contribution and regard this as a theoretical uncertainty. But its magnitude is numerically large, and thus it enlarges theoretical uncertainty.

Another view is a more conservative one. If there appears an infrared divergence in the hard scattering amplitude, the effect of soft gluon exchange is really significant and the QCD-improved factorization breaks down at this order. It remains to be seen if the QCD-improved factorization really breaks down, or there are some other contributions which cancel the infrared divergence rendering the final result infrared finite.

4 Conclusion

The understanding of nonleptonic $B$ decays into two mesons has acquired a new sophisticated level. In the heavy quark limit, nonfactorizable contributions are calculable using perturbative QCD for light final-state mesons. When one of the final-state meson is heavy, we can still use the QCD-improved factorization for the case in which the spectator quark in the $B$ meson goes to the heavy meson in the final state. If the spectator quark goes to a light meson, as in class II decays, the nonfactorizable contribution is infrared divergent, and the effect of soft gluon exchange is significant.

But the analysis of higher-twist effects is yet far from satisfactory. For example, the spectator contribution which is proportional to the twist-three contribution of the meson wave function is infrared divergent. And the annihilation topology also has the infrared divergence. The status of the QCD-improved factorization method for nonleptonic $B$ decays into two mesons is not complete until we disentangle the infrared divergence to give a quantitative prediction.

It will be an interesting project to combine the QCD-improved factorization with the calculation of the form factors using the light-cone wave
functions. Currently, in the QCD-improved factorization, we use the form factors extracted from experiment, as in semileptonic $B$ decays. On the other hand, in Ref. 9 they only consider the calculation of form factors. It will be interesting to see whether we can give a consistent theoretical description of nonleptonic $B$ decays combining these two approaches.

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