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Analogue simulation of two-body quantum dynamics with classical setup

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Abstract. One-dimensional Bose-Hubbard models provide a variety of intriguing predictions including the formation of repulsively bound two-boson pairs, their collapse and revival and the realization of the two-boson topological states. While probing such systems in quantum regime is quite a challenging task, there is another appealing option to explore the properties of repulsively bound bosons based on mapping of the one-dimensional interacting system onto the two-dimensional classical model described by the standard coupled-mode equations. Adopting the latter approach, we develop the procedure to probe the energy of bound boson pairs as well as to perform the tomography of the bound pair quantum state in realistic dissipative systems.

1. Introduction
Since the theoretical prediction and experimental observation of repulsively bound boson pairs (doublons) in one-dimensional systems described by the Bose-Hubbard model [1] there is an immense interest in such strongly correlated two-boson states. Further studies revealed a number of their exciting properties including coherent destruction (collapse) [2], emergence of topological two-boson states [3, 4], interaction-induced topological order in two-dimensional systems [5] and rich interplay of local and non-local interaction mechanisms in extended Hubbard models [6, 7].

While the experimental studies of bound boson pairs remain quite challenging, there exists an alternative strategy based on the possibility to map a one-dimensional interacting system described by the Bose-Hubbard Hamiltonian onto a two-dimensional classical system obeying the usual coupled-mode equations [8]. Such two-dimensional systems have recently been realized in waveguide geometries thus paving a way to quantum simulation of few-body physics with photonic systems [9].

In this work, we analyze how one can extract doublon energies and a specific type of the doublon state (bulk or edge) from the experiments with such analogue quantum simulators.

2. Emulating bound boson pairs
We start by considering the dynamics of the two interacting bosons in the Su-Schrieffer-Heeger model illustrated schematically in Fig. 1(a). This system is described by the Bose-Hubbard
Figure 1. Illustration of the analogue simulation of quantum system with a classical setup. (a) One-dimensional two-body system described by the Bose-Hubbard Hamiltonian. The array starts from a weak link edge and finishes with a strong link edge. (b) Two-dimensional nearest-neighbor coupled setup emulating quantum two-body problem. (c) Spectrum of various two-body states possible in the system: green, blue and red bands designate scattering states, single-boson edge states and bulk doublon states, respectively. Horizontal black and magenta lines indicate doublon edge states localized at strong link and weak link edges, respectively. (d) Field intensity from the sites at the diagonals $m = n$ and $m = n \pm 1$. Light red regions show the boundaries of the doublon bands. Blue and green curves correspond to the excitation of the system from (3,3) and (1,1) sites, respectively. Insets illustrate the excitation protocol.

Hamiltonian

$$\hat{H} = \omega_0 \sum_m \hat{n}_m + U \sum_m \hat{n}_m \left( \hat{n}_m - 1 \right) - \sum_m \left( J_1 \hat{a}^\dagger_{2m-1} \hat{a}_{2m} + J_2 \hat{a}^\dagger_{2m} \hat{a}_{2m+1} + \text{H.c.} \right)$$

where $\hat{a}^\dagger_m$ and $\hat{a}_m$ are photon creation and annihilation operators for the $m$-th cavity, $\hat{n}_m = \hat{a}^\dagger_m \hat{a}_m$ is the photon number operator, $J_{1,2}$ are the tunneling constants, $U$ is the interaction strength, $\omega_0$ is the cavity eigenfrequency ($\hbar = 1$), and only one photon polarization state is considered.

The analysis of the system [3, 4] demonstrates that this model is equivalent to a two-dimensional classical system [Fig. 1(b)] with nearest-neighbor coupling. The latter two-dimensional system can be realized using various technological platforms, for instance, arrays of polariton pillars, while its properties can be probed by exciting one or several pillars and measuring the intensity distribution in the array.

To detect doublon states, we compute the population of the three diagonal lines $m = n$ and $m = n \pm 1$ as a function of excitation frequency. The results of numerical calculation based on coupled mode theory are presented in Fig. 1(d). The excitation spectrum features several peaks with an amplitude that strongly depends on the choice of the excitation point. The intensity from the diagonal pillars $I_{\text{diag}}$ quantifies the probability of two bosons co-localization in the same cavity in 1D quantum problem. Hence, we associate the peaks in the excitation spectrum of a 2D system with the doublon modes. However, a single excitation spectrum does not allow one to distinguish doublon bulk and edge states.

To draw such a distinction, we calculate the analogous excitation spectrum for a different choice of the excitation point, further away from the (1,1) corner of the system. Since doublon
edge states decay away from the array edges, we expect that the corresponding peaks in the spectrum will be suppressed dramatically, while the amplitude of the peaks corresponding to bulk doublon modes will stay roughly the same. Based on this interpretation, we are able now to make a distinction between bulk doublon states associated with the peaks (1), (3), (5) and edge doublon states represented by the peaks (2) and (4). Comparing the obtained results with numerical calculation of the system eigenmodes, we verify the validity of our technique.

To summarize, our technique provides an experimentally feasible way to probe doublon bands and doublon edge states using simple photonic systems which uncovers interesting perspectives in analogue simulation of strongly correlated quantum states.

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