The Generalized Gerasimov-Drell-Hearn Integral
and the Spin Structure of the Nucleon

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Abstract

The spin structure functions $g_1$ and $g_2$ have been calculated in the resonance region and for small and intermediate momentum transfer. The calculation is based on a gauge-invariant and unitary model for one-pion photo- and electroproduction. The predictions of the model agree with the asymmetries and the spin structure functions recently measured at SLAC, and the first moments of the calculated spin structure functions fulfil the Gerasimov-Drell-Hearn and Burkhardt-Cottingham sum rules within an error of typically 5-10 %.

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I. INTRODUCTION

The spin structure of the nucleon has been studied by increasingly accurate measurements since the end of the 70’s. By scattering polarized lepton beams off polarized targets, it has become possible to determine the spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$. These functions depend on the fractional momentum of the proton, $x = Q^2 / 2m\nu$, where $-Q^2$ is the square of four-momentum transfer, $\nu$ the energy transfer in the lab frame, and $m$ the nucleon mass. The results of the first experiments at CERN [1] and SLAC [2] sparked considerable interest in the community, because the first moment of $g_1$, $\Gamma_1 = \int_0^1 g_1(x) dx$, was found to be substantially smaller than expected from the quark model, in particular from the Ellis-Jaffe sum rule [3]. This “spin crisis” of the nucleon led to the conclusion that only a small fraction of the nucleon’s spin is carried by the valence quarks, and that the main spin contributions should be due to sea quarks, quark orbital momentum and gluons. However, the difference of the proton and neutron moments, $\Gamma_p^1 - \Gamma_n^1$, was found to be well described by Bjorken’s sum rule [4], which is a strict QCD prediction.

Recent improvements in polarized beam and target techniques have made it possible to determine both spin structure functions over an increased range of kinematical values. The reader is referred to the publications of the collaborations at CERN [5], SLAC [6] and DESY [7] and to the references on earlier work quoted therein. While most of these investigations have concentrated on the high $Q^2$ region, the recent measurements of the E143 Collaboration at SLAC [8] have been extended to the range of $0.6 (\text{GeV/c})^2 \leq Q^2 \leq 1.2 (\text{GeV/c})^2$, which bridges the gap between low-$Q^2$ nonperturbative resonance phenomena and high-$Q^2$ perturbative QCD.

It is the aim of our contribution to present the results of the recently developed Unitary Isobar Model (UIM, Ref. [9]) for the spin asymmetries, structure functions and relevant sum rules in the resonance region. This model describes the presently available data for single-pion photo- and electroproduction up to a total cm energy $W_{\text{max}} \approx 2 \text{ GeV}$ and for $Q^2 \leq 2 (\text{GeV/c})^2$. It is based on effective Lagrangians for Born terms (background) and
resonance contributions, and the respective multipoles are constructed in a gauge-invariant and unitary way for each partial wave. The eta production is included in a similar way \[10\], while the contribution of more-pion and higher channels is modeled by comparison with the total cross sections and simple phenomenological assumptions.

The spin structure in the resonance region has intrigued many authors because of an expected rapid change from the physics of resonance-driven coherent processes to incoherent scattering off the constituents. In particular the first moment $\Gamma_1$ is constrained, in the limit of $Q^2 \to 0$ (real photons), by the famous Gerasimov-Drell-Hearn sum rule (GDH, Ref. \[11\]),

$$\Gamma_1 \to -Q^2 \kappa^2/8m^2,$$

where $\kappa$ is the anomalous magnetic moment of the nucleon. The reader should note that here and in the following we have included the inelastic contribution to $\Gamma_1$ only. As has been pointed out by Ji and Melnitchouk \[12\], the elastic contribution is in fact the dominant one at small $Q^2$ and has to be taken into account in comparing with twist expansions about the deep-inelastic limit at those values of $Q^2$.

In the case of the proton, the GDH sum rule predicts $\Gamma_1 < 0$ for small $Q^2$, while all experiments for $Q^2 > 1$ (GeV/c)$^2$ yield positive values. Clearly, the value of the sum rule has to change rapidly at low $Q^2$, with some zero-crossing at $Q_0^2 < 1$ (GeV/c)$^2$. The evolution of the sum rule was first described by Anselmino et al. \[13\] in terms of a parametrization based on vector meson dominance. Burkert, Ioffe and others \[14,15\] refined this model considerably by treating the contributions of the resonances explicitly. Soffer and Teryaev \[16\] suggested that the rapid fluctuation of $\Gamma_1$ should be analyzed in conjunction with $\Gamma_2$, the first moment of the second (transverse) spin function. The latter is constrained by the less familiar Burkhardt-Cottingham sum rule (BC, Ref. \[17\]) at all values of $Q^2$. Therefore the sum of the two moments, $\Gamma_1 + \Gamma_2$, is known for both $Q^2 = 0$ and $Q^2 \to \infty$. Though this sum is related to the practically unknown longitudinal-transverse interference cross section $\sigma'_{LT}$ and therefore not yet determined directly, it can be extrapolated smoothly between the two limiting values of $Q^2$. The rapid fluctuation of $\Gamma_1$ then follows by subtraction of the BC value of $\Gamma_2$. The importance of $\sigma'_{LT}$ for a complete understanding of the spin structure has also been stressed by Li and Li \[18\].
The GDH sum rule is obtained by integrating the quantity \((σ_{3/2} − σ_{1/2})/ν\) over the photon energy, where \(σ_{3/2}\) and \(σ_{1/2}\) are the cross sections for virtual photon and target spins in parallel and antiparallel, respectively. Several authors have pointed out that serious discrepancies exist between the sum rule and theoretical analysis based on the existing unpolarized data. The small momentum evolution of the extended GDH sum rule was also investigated in the framework of heavy baryon ChPT [19]. The authors predicted a positive slope of the GDH integral for \(Q^2 = 0\), while the phenomenological analysis of Burkert et al. [14] indicated a negative slope. However, the integrands of all spin-related sum rules are fluctuating functions of energy with both positive and negative contributions. Therefore, even seemingly small uncertainties may give rise to large differences (see Ref. [20] and references quoted therein).

The first direct experimental data have been recently taken at MAMI [21] in the energy range \(200 \text{ MeV} < ν < 800 \text{ MeV}\), and data at the higher energies are expected from ELSA within short. Concerning electroproduction in the resonance region, the \(Δ(1232)\) is being investigated at ELSA, Jefferson Lab, MAMI and MIT/Bates [22]. The high-duty factor of these modern electron accelerators makes it possible to measure not only the (total) helicity cross sections but also the individual decay channels, thus providing a much more stringent test of model descriptions. The spin structure of the nucleon will be studied at the Jefferson Lab by a series of experiments [23], which are expected to cover the entire resonance region and momentum transfers up to \(Q^2 ≈ 2 (\text{GeV}/c)^2\). The outcome of these experiments should answer the open questions about the helicity structure at low and intermediate energies, the validity of the GDH sum rule, the size and role of the transverse-longitudinal contribution in connection with the BC sum rule and the generalized GDH integral, the evolution of these integrals with momentum transfer, and the contribution of individual decay channels to the spin structure functions.

In the following we review the formalism of spin structure functions and sum rules in sect. 2. We then present the predictions of the UIM in sect. 3, and close by a brief summary of our findings in sect. 4.
II. FORMALISM

We consider the scattering of polarized electrons off polarized target nucleons. The \textit{lab} energies of the electrons in the initial and final states are denoted with $E$ and $E'$, respectively. The incoming electrons carry the (longitudinal) polarization $h = \pm 1$, and the two relevant polarization components of the target are $P_z$ (parallel to the \textit{lab} momentum $\vec{k}$ of the virtual photon) and $P_x$ (perpendicular to $\vec{k}$ in the scattering plane of the electron and in the half-plane of the outgoing electron). The differential cross section for exclusive electroproduction can then be expressed in terms of four “virtual photoabsorption cross sections” $\sigma_i(\nu, Q^2)$ by \cite{24}

$$\frac{d\sigma}{d\Omega \ dE'} = \Gamma \sigma(\nu, Q^2),$$

(1)

$$\sigma = \sigma_T + \epsilon \sigma_L + h P_x \sqrt{2\epsilon(1 - \epsilon)} \sigma'_{LT} + h P_z \sqrt{1 - \epsilon^2} \sigma'_{TT},$$

(2)

with

$$\Gamma = \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{K}{Q^2} \frac{1}{1 - \epsilon},$$

(3)

the flux of the virtual photon field and $\epsilon$ its transverse polarization, $\nu = E - E'$ the virtual photon energy in the \textit{lab} frame and $Q^2 > 0$ describing the square of the virtual photon four-momentum. In accordance with our previous notation \cite{9} we shall define the flux with the “photon equivalent energy” $K = k_\gamma = (W^2 - m^2)/2m$, where $W$ is the total \textit{cm} energy and $m$ the mass of the target nucleon.

The quantities $\sigma_T$ and $\sigma'_{TT}$ can be expressed in terms of the total cross sections $\sigma_{3/2}$ and $\sigma_{1/2}$, corresponding to excitation of hadronic states with spin projections $3/2$ and $1/2$, respectively,

$$\sigma_T = \frac{1}{2}(\sigma_{3/2} + \sigma_{1/2}) \quad \text{and} \quad \sigma'_{TT} = \frac{1}{2}(\sigma_{3/2} - \sigma_{1/2}).$$

(4)

The virtual photoabsorption cross sections in Eq. (2) are related to the quark structure functions,
\[ \sigma_T = \frac{4\pi^2\alpha}{mK} F_1, \]
\[ \sigma_L = \frac{4\pi^2\alpha}{K} \left[ \frac{F_2}{\nu} (1 + \gamma^2) - \frac{F_1}{m} \right], \]
\[ \sigma'_{LT} = -\frac{4\pi^2\alpha}{mK} \gamma (g_1 + g_2), \]
\[ \sigma'_{TT} = -\frac{4\pi^2\alpha}{mK} (g_1 - \gamma^2 g_2), \]

(5)

with \( \gamma = Q/\nu \), and the quark structure functions \( F_1, F_2, g_1, \) and \( g_2 \) depending on \( \nu \) and \( Q^2 \).

In comparing with the standard nomenclature of DIS \[8\] we note that \( \sigma'_{LT} = -\sigma_{LT}(DIS) \) and \( \sigma'_{TT} = -\sigma_{TT}(DIS) \).

We generalize the GDH sum rule by introducing the \( Q^2 \)-dependent integral

\[ I_1(Q^2) = \frac{2m^2}{Q^2} \int_0^{x_0} g_1(x, Q^2) \, dx \rightarrow \begin{cases} -\frac{1}{4} \kappa_N^2 & \text{for } Q^2 \rightarrow 0 \\ \frac{2m^2}{Q^2} \Gamma_1 + \mathcal{O}(Q^{-4}) & \text{for } Q^2 \rightarrow \infty \end{cases}, \]

(6)

where \( x = Q^2/2m\nu \) is the Bjorken scaling variable and \( x_0 = Q^2/(2mm_\pi + m_\pi^2 + Q^2) \) refers to the inelastic threshold of one-pion production. In the scaling regime the structure functions should depend on \( x \) only, and \( \Gamma_1 = \int g_1(x)dx = \text{const.} \) We note that several other generalizations of the GDH integral have been proposed by introducing admixtures of the second spin structure functions \( g_2 \) and/or factors depending on (different) definitions of the photon flux. While all generalizations have the same limits as in Eq. (6), they may differ substantially at intermediate \( Q^2 \) \[25\].

For the second spin structure function the Burkhardt-Cottingham (BC) sum rule asserts that the integral over \( g_2 \) vanishes if integrated over both elastic and inelastic contributions \[17\]. As a consequence one finds

\[ I_2(Q^2) = \frac{2m^2}{Q^2} \int_0^{x_0} g_2(x, Q^2) \, dx = \frac{1}{4} \frac{G_M(Q^2) - G_E(Q^2)}{1 + Q^2/4m^2} G_M(Q^2), \]

(7)

i.e. the inelastic contribution for \( 0 < x < x_0 \) equals the negative value of the elastic contribution given by the rhs of Eq. (7), which is parametrized by the magnetic and electric Sachs form factors \( G_M \) and \( G_E \), respectively. These form factors are normalized by the nucleon’s magnetic moment \( \mu_N = \kappa_N + e_N \) and its charge \( e_N \) in the limit of \( Q^2 \rightarrow 0 \), \( G_M(0) = \mu_N \) and \( G_E(0) = e_N \). The BC sum rule has the limiting cases
\[ I_2(Q^2) \rightarrow \begin{cases} \frac{1}{4}\mu_N\kappa_N & \text{for } Q^2 \rightarrow 0 \\ \mathcal{O}(Q^{-10}) & \text{for } Q^2 \rightarrow \infty \end{cases} \] (8)

We have calculated the integrals \( I_1 \) and \( I_2 \) in terms of the virtual photon cross sections,

\[ I_1(Q^2) = \frac{m^2}{8\pi^2\alpha} \int_{\nu_0}^{\infty} \frac{1-x}{1+\gamma^2} \left( \sigma_{1/2} - \sigma_{3/2} - 2\gamma \sigma'_{LT} \right) \frac{d\nu}{\nu}, \]

(9)

\[ I_2(Q^2) = \frac{m^2}{8\pi^2\alpha} \int_{\nu_0}^{\infty} \frac{1-x}{1+\gamma^2} \left( \sigma_{3/2} - \sigma_{1/2} - 2\gamma \sigma'_{LT} \right) \frac{d\nu}{\nu}, \]

(10)

where \( \nu_0 = m_\pi + (m_\pi + Q^2)/2m \) is the threshold lab energy of one-pion production.

Since \( \gamma \sigma'_{LT} = \mathcal{O}(Q^2) \), the longitudinal-transverse term does not contribute to the integral \( I_1 \) in the real photon limit. However, the ratio \( \sigma'_{LT}/\gamma \) remains constant in that limit and hence contributes to \( I_2 \). As a result we find

\[ I_2(0) = \frac{1}{4}\kappa_N^2 + \frac{1}{4}e_N\kappa_N, \] (11)

with the two terms on the rhs corresponding to the contributions of \( \sigma_{3/2} - \sigma_{1/2} \) and \( \sigma'_{LT} \), respectively. Equation (11) provides an interesting model-independent constraint for the two spin-dependent virtual photon cross sections in the real photon limit.

Finally, in order to relate our calculations with the experimental data, we express the virtual photon asymmetries \( A_1 \) and \( A_2 \) defined in Ref. [8] by the virtual photon cross sections,

\[ A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = -\frac{\sigma'_{TT}}{\sigma_T}, \quad A_2 = -\frac{\sigma'_{LT}}{\sigma_T}. \] (12)

From the measured values for the asymmetries \( A_1 \) and \( A_2 \) and from the total transverse cross section \( \sigma_T \) we then construct the quark structure functions \( g_1 \) and \( g_2 \) by use of Eqs. (4), (5) and (12),

\[ g_1 = \frac{F_1}{1+\gamma^2}(A_1 + \gamma A_2), \quad g_2 = \frac{F_1}{1+\gamma^2} \left( -A_1 + \frac{1}{\gamma} A_2 \right). \] (13)

In practice it is difficult to measure the two (unpolarized) cross sections \( \sigma_T \) and \( \sigma_L \) separately by means of a Rosenbluth plot. The structure function \( F_1 \) is then extracted from the total (unpolarized) cross section, \( \sigma_{tot} = \sigma_T + \epsilon \sigma_L \), by use of an “educated guess” for the ratio
\[ R = \frac{\sigma_L}{\sigma_T}. \] Moreover, if only the longitudinal target polarization can be measured, only the combination \( A_1 + \eta A_2 \) can be determined, where \( \eta = \epsilon Q/(E - \epsilon E') \). In this case the structure function \( g_1 \) has to be extracted by use of additional assumptions about the second spin structure function \( g_2 \).

**III. RESULTS AND DISCUSSIONS**

**A. Total cross sections**

The recently developed Unitary Isobar Model (UIM) \[9\] describes the single-pion electroproduction channel quite adequately. However, for energies \( W > 1.3 \text{GeV} \) the contributions from other channels, in particular from multipion production, become increasingly important. At present several theoretical models are available for the two-pion photoproduction channels \[26-28\], but little information exists on multipion electroproduction. In this paper we extract the necessary information from the available data for the total cross sections of real and virtual photon absorption on the proton \[29,30\], assuming that the single-pion channel is well determined by the UIM. In addition we account for the well investigated eta production channel by using the parametrization for the corresponding total cross section \( \sigma_T^{(\eta)}(W,0) \) at the photon point as suggested in Ref. \[10\]. We extend this parametrization for the case of virtual photons using the helicity amplitude \( A_{1/2}^{(S_{11})} \) for the \( S_{11}(1535) \) resonance obtained within the UIM, i.e.

\[
\sigma_T^{(\eta)}(W,Q^2) = \sigma_T^{(\eta)}(W,0) \left[ A_{1/2}^{(S_{11})}(Q^2)/A_{1/2}^{(S_{11})}(0) \right]^2. 
\]}

(14)

The transverse part of the total cross section for multipion production is now obtained by

\[
\sigma_T^{(n\pi)}(W,Q^2) = F^2(Q^2) \left[ \sigma_T^{(\text{exp})}(W,0) - \sigma_T^{(1\pi)}(W,0) - \sigma_T^{(\eta)}(W,0) \right] 
\]}

(15)

with a monopole form factor \( F(Q^2) = 1/(1 + \alpha_{n\pi} Q^2) \) describing the \( Q^2 \) dependence. A value of \( \alpha_{n\pi} = 1.05 (c/\text{GeV})^2 \) provides the best fit to the total inelastic cross section up to
the third resonance region and for $Q^2 < 1 \,(GeV/c)^2$. In Fig. 1 we present our results for the total cross sections at $Q^2 = 0$ and $0.5 \,(GeV/c)^2$.

We have not yet included the contributions of the eta and multipion channels to the longitudinal part of the cross section. Since the ratio $R$ is small in the resonance region, $R = \sigma_L/\sigma_T \approx 0.06 \,[30,31]$, these contributions should not be of major importance anyway. On the other hand, we find a substantial longitudinal strength for single-pion production at the higher energies, $R^{(1\pi)} = \sigma_L^{(1\pi)}/\sigma_T^{(1\pi)} > 0.5$ at $W > 1.8 \,GeV$ and $Q^2 > 0.3 \,(GeV/c)^2$. Hence we are led to the conclusion that the (small) longitudinal contribution is dominated by single-pion production.

B. Asymmetries and structure functions

One of the basic ingredients for the asymmetries $A_{1,2}$ and structure functions $g_{1,2}$ is the difference of the two helicity cross sections,

$$\Delta \sigma = \sigma_{3/2} - \sigma_{1/2} = \Delta \sigma^{(1\pi)} + \Delta \sigma^{(\eta)} + \Delta \sigma^{(n\pi)}.$$  \hspace{1cm} (16)

The single-pion contribution to this difference has been calculated within the UIM approach. Since the dominant mechanism of eta electroproduction is due to the excitation of the $S_{11}(1535)$ resonance, the contribution of the eta channel to Eq. (16) may be approximated by $\Delta \sigma^{(\eta)} \approx -2\sigma_T^{(\eta)}$. The description of the multipion channels is the most delicate part in our calculation of the sum rules, because the mentioned theoretical models [26–28] for two-pion photoproduction were tuned to the existing data for total cross sections. The simplest estimate for $\Delta \sigma^{(n\pi)}$ is based on the assumption that the two-pion contribution is generated by resonances, and that its helicity structure follows the known behavior of the one-pion contribution $\Delta \sigma^{(1\pi)}$ [31,32], i.e. $\Delta \sigma^{(n\pi)}/\Delta \sigma^{(1\pi)} = const.$ In the present paper we first account for the one-pion and eta channels explicitly, and then use the simple prescription

$$\Delta \sigma^{(n\pi)} = \frac{\sigma_T^{(n\pi)}}{\sigma_T} \left( \Delta \sigma^{(1\pi)} + \Delta \sigma^{(\eta)} \right),$$  \hspace{1cm} (17)
with the factor $\sigma_T^{(n\pi)}$ in the numerator providing the correct threshold behaviour for the two-pion contribution.

In the upper part of Fig. 2, our results are compared to the asymmetry $A_1 + \eta A_2$ measured at SLAC [6]. As may be seen, the prescription of Eq. (17) is in reasonable agreement with the data up to $W = 2 \text{GeV}$. We also note that the contribution of the $\eta$ channel (dotted curves) leads to a substantial increase of the asymmetry over a wide energy region.

Having fixed all the ingredients we can now calculate the structure functions and generalized GDH integrals. Our results for the structure function $g_1$ are presented in the lower part of Fig. 2. Up to a value of $W^2 = 2 \text{GeV}^2$ (corresponding to $x = 0.31$ and $x = 0.52$ at $Q^2=0.5$ and $1.2 \text{(GeV/c)}^2$, respectively), the main contribution to $g_1$ is due to single-pion production. The negative structure above threshold is related to excitation of the $\Delta(1232)$ resonance. In the second and third resonance regions the contributions from the eta and multipion channels become increasingly important. Note that with increasing value of $Q^2$ the relative importance of the eta channel increases. This phenomenon is connected with the well known peculiarity that the helicity amplitude of the $S_{11}(1535)$ resonance decreases much slower with $Q^2$ than for any other resonance [34,35].

C. Integrals $I_1$ and $I_2$

In Fig. 3 we give our predictions for the integrals $I_1(Q^2)$ and $I_2(Q^2)$ in the resonance region, i.e. integrated up to $W_{max} = 2 \text{GeV}$. The striking feature of the integral $I_1$ is the evolution from large negative to small positive values in order to interpolate from the GDH prediction for real photons to the data obtained from DIS. As can be seen from the top of Fig. 3, our model is able to generate the dramatic change in the helicity structure quite well. While this effect is basically due to the single-pion component predicted by the UIM, the eta and multipion channels are quite essential to shift the zero-crossing of $I_1$ from $Q^2 = 0.75 \text{(GeV/c)}^2$ to $0.52 \text{(GeV/c)}^2$ and $0.45 \text{(GeV/c)}^2$, respectively. This improves the agreement with the SLAC data [8]. However, some differences remain. Due to a lack of data
in the Δ region (see lower part of Fig. 2), the SLAC data are likely to underestimate the Δ contribution. Since this contribution is negative, a numerical integration of the data points could overestimate the $I_1$ integral or the corresponding first moment $Γ_1$. A few more data points in the Δ region would be very useful in order to clarify the situation.

Concerning the integral $I_2$, our full results are in good agreement with the predictions of the BC sum rule (see Fig. 3, lower part). The remaining differences are of the order of 10 % and should be attributed to contributions beyond $W_{\text{max}} = 2 \text{ GeV}$ and the scarce experimental data for $σ'_{LT}$. With regard to the latter problem we note that the BC sum rule, Eqs. (7) and (11), contains very sizeable contributions from the longitudinal-transverse cross section. In Table 1 we list the contributions of the different ingredients of our model to the integrals $I_1$ and $I_2$ at the real photon point.

The convergence of the sum rules can not be given for granted. In fact Ioffe et al. have argued that the BC sum rule is valid only in the scaling region, while it is violated by higher twist terms at low $Q^2$. Therefore the good agreement of our model with the BC sum rule could be accidental and due to a particular model prediction for the essentially unknown longitudinal-transverse interference term. As can be seen from Fig. 3, the contribution of $σ_{LT'}$ is quite substantial for $I_2$ even at the real photon point due to the factor $γ^{-1}$ in Eq. 10. This contribution, however, is constrained by the positivity relation $|σ'_{LT}| \leq \sqrt{σ_Lσ_T} = \sqrt{R}σ_T$. The dash-dotted line shows the integral for the upper limit of this inequality and a similar effect would occur for the lower limit. This surprisingly large contribution can be understood in terms of multipoles. In a realistic description of the integrated cross section $σ_{LT'}$ the large $M_{1+}$ multipole can only interfere with the small $L_{1+}$ multipole. The upper and lower limits of the positivity relation overestimate the structure function considerably due to an unphysical “interference” between $s$- and $p$-waves.

In Fig. 4 we compare our result for the function $Γ_1$, defined in Eq. (6), with the calculation of Ioffe for the resonance contribution. This representation demonstrates more clearly the importance of the higher channels, e.g. $η$ and $2π$.

Fig. 5 shows our predictions for $Δσ = σ_{3/2} - σ_{1/2}$ at $Q^2=0$, 0.5 and 1.2 (GeV/c)$^2$. 
In particular we note the following peculiarity of the $Q^2$ dependence of the resonance and multipion contributions. With increasing value of $Q^2$ the positive contribution of the $\Delta$ resonance decreases rapidly due to the strong $N\Delta$ transition form factor. In the second and third resonance regions, however, $\Delta \sigma$ changes from positive to negative values of $Q^2$. The reasons for this zero-crossing are (I) the helicity structure of the $D_{13}$ and $F_{15}$ resonances, which changes from $\sigma_{3/2}$ to $\sigma_{1/2}$ dominance [33], and (II) the surprisingly slow fall-off of the $S_{11}$ transition form factor [34,9], which increases the negative contribution of this resonances relative to the others.

Both effects conspire to cancel the $\Delta$ contribution already at $Q^2 \approx 0.5\text{(GeV/c)}^2$ and lead to the observed positive value of $I_1$ at larger momentum transfer. The same cancellations are responsible for the rapid decrease of $I_2$ with $Q^2$. However, it should be kept in mind that this integral also receives strong contributions from $\sigma'_{LT}$. In view of our bad knowledge of the longitudinal-transverse response we find it actually quite surprising that the UIM “knows” about the BC sum rule and the predicted rapid fall-off with $Q^2$.

**IV. CONCLUSION**

We have studied the nucleon spin structure functions and the generalized GDH and BC integrals for small and moderate momentum transfer within the framework of the Unitary Isobar Model [9]. The contributions from eta and multipion channels in the second and third resonance regions have been modeled by accounting for the total photoproduction cross section.

We find that the role of the eta channel becomes more and more important with increasing values of $Q^2$. At the real photon point, this channel tends to cancel the contribution of multipion production. However, at finite $Q^2$ the contributions of both channels add coherently. As a result the zero-crossing of the integral $I_1$ shifts from $Q^2 = 0.75\text{(GeV/c)}^2$ (for the one-pion channel only) to $Q^2 = 0.45\text{(GeV/c)}^2$ (if eta and multipion channels are included).

Our analysis indicates that both the measured asymmetries and the theoretical models
have to be quite accurate in order to determine the integrals $I_1$ and $I_2$ in the resonance region. As far as theory is concerned, a more quantitative description of the multipion channels is absolutely necessary. On the side of the experiment, a measurement of all four virtual photon cross sections is needed to determine the quark spin structure functions in a model-independent way. Moreover, the oscillatory behaviour of the integrand in the resonance region will require small error bars and a relatively small energy binning in order to give precise values for the GDH and BC integrals. The proposed experiments at Jefferson Lab [23] carry the promise to obtain such data in the resonance region and we are eagerly waiting for the outcome of these investigations.
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TABLES

| $I_{1,2}$ | Born+Δ | $P_{11}, D_{13}, \ldots$ | $\eta$ | multipion | total  | sum rule |
|-----------|--------|--------------------------|-------|-----------|-------|----------|
| $I_1$     | -0.565 | -0.152                   | 0.059 | -0.088    | -0.746| -0.804   |
| $I_2$     | 1.246  | 0.063                    | -0.059| 0.088     | 1.338 | 1.252    |

TABLE I. Contributions of the different channels to the integrals $I_1$ and $I_2$ at the real photon point, $Q^2 = 0$. The column "$P_{11}, D_{13}, \ldots$" shows the contributions of all the resonances above the $\Delta$ that are included in Ref. [9]. Note that the column “total” lists the values for integration up to $W_{\text{max}} = 2 \text{ GeV}$ only, while “sum rule” is the prediction for the full integral.
FIG. 1. Total cross sections for photoabsorption and inelastic electron scattering on the proton for $\varepsilon = 0.9$ and $Q^2 = 0.5\,(GeV/c)^2$. Dashed, dotted and dash-dotted curves: contributions of single-pion, eta and multipion channels, respectively; solid curves: final result. Experimental data for the total cross sections from Refs. 30 (x) and 29 (○), for the two-pion production channels from Ref. 29 (△).
FIG. 2. The asymmetry $A_1 + \eta A_2$ (top) and the spin structure function $g_1$ (bottom) as function of the Bjorken scaling variable $x$ at $Q^2 = 0.5$ and $1.2 \ (GeV/c)^2$. Dashed, dotted and solid curves: calculations obtained with $1\pi$, $1\pi + \eta$, and $1\pi + \eta + n\pi$ contributions, respectively. Data from Refs. [8] (●) and [36] (○). The error bars give the statistical errors, the systematical errors are estimated to be of equal size.
FIG. 3. The integrals $I_1$ and $I_2$ defined by Eqs. (10) and (11) as functions of $Q^2$ in the resonance region, integrated up to $W_{\text{max}} = 2\text{GeV}$. Upper figure: Notation as in Fig. 2 and data from Ref. [8]. Lower figure: The full and dashed lines are our predictions with and without $\sigma'_{LT}$ (see Eq. (10)), the dash-dotted line is obtained for $\sigma'_{LT} = \sqrt{\sigma_L \sigma_T}$ (see text), and the dotted line is the sum rule prediction of Ref. [17]. All calculations for $I_2$ include $1\pi + \eta + n\pi$ contributions.
FIG. 4. The integral $\Gamma_1$ for the proton defined by Eq. (6) as function of $Q^2$. The solid and dashed lines show the integral up to $W_{\text{max}} = 2\,\text{GeV}$ for our full calculation and our calculation without the $\eta$ and $2\pi$ contributions, respectively. The dash-dotted line is obtained from Ref. [15] and contains only the resonance component. The data are from Ref. [8].
FIG. 5. Predictions for $\Delta\sigma = \sigma_{3/2} - \sigma_{1/2}$ at $Q^2=0$, 0.5 and 1.2 $(GeV/c)^2$. Notation as in Fig. 1.