Investigation on terminal velocity and drag coefficient of particles with different shapes

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Abstract. Multiphase flow are involved in many fields in natural phenomenon as well as industrial applications. Drag coefficient is an important parameter in the modelling and simulation of multiphase flow. Non-sphere particles are usually simplified to sphere particles in multiphase flow simulation. However, non-sphere particles are more common in practice. Hence, it is significant to investigate shape effects on the drag coefficient of non-sphere particles. This article selects several typical shapes of particles as research objects. 3-D printers are used to build particle models. And high speed cameras are used in record the whole process of particle settling. Detailed information of particle path and speed are gotten through image processing of high speed cameras.

1. Introduction
Fluid-solid system exists widely in natural world and human’s life such as sand storm, haze, sediment transportation etc.[1] Sand storm and haze are increasing in recent years. And particle transportation and catalytic reaction propose higher requirement for the research of multiphase flow in industrial application. Because of force analysis is the base of multiphase flow predictions, many scientists have done a lot of researches on frag force of particles in the fluid.

The earliest research on drag coefficient may trace back to 19th century. In 1851, Stokes[2] solved N-S equation in creeping flow (Re<<1) lose sight of inertia part. He gave the formula for calculating drag of smooth sphere particle as:

\[ F = 3 \mu d u \]  

(1)

Where \( \mu \) is dynamic coefficient of viscosity, \( d \) is diameter of sphere, \( u \) is the relative speed of particle and fluid. According to historical document, drag coefficient are defined as equation.(2):

\[ C_D = \frac{F}{2 \rho u^2 A} \]  

(2)

And can be formed as equation.(3):

\[ C_D = \frac{24}{Re} \]  

(3)

Where Re is Reynolds number defined as \( Re = \rho ud/\mu \). This formula is appropriate when Re<<1; as Re grows, equation.(3) shows larger deviation compared to experiment.

Oseen[3] reserved the linear part of the inertia part, and he get an approximate solution:
\[ C_D = \frac{24}{Re} \left( 1 + \frac{3}{16} Re \right) \quad (4) \]

As \( Re \) grows, the inertia part turned non-ignorable, and we can barely get the analytical solution. Nevertheless, it is still necessary to use experiment data to get fitted curve. The existing literature gave several formulas based on mass experiment data of free fall experiment and fluidized bed.

Schiller and Naumann[4] gave an formula based on experiment:

\[ C_D = \frac{24}{Re} (1 + 0.150 Re^{0.687}) \quad \text{for} \ Re < 800 \quad (5) \]

White[5] has given a similarity fitted curve formula:

\[ C_D = \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} + 0.4 \quad \text{for} \ Re < 2 \times 10^5 \quad (6) \]

Clift and Gauvin[6] gave a more accurate formula beneath the critical Reynolds’s number:

\[ C_D = \frac{24}{Re} (1 + 0.15 Re^{0.687}) + \frac{0.42}{1+42500 Re^{1.16}} \quad \text{for} \ Re > 3 \times 10^5 \quad (7) \]

Concerning non-sphere particle, many researchers proposed their shape factor to amend the formula for non-sphere[7][8][9][10][11]. Some frequently used are as followed:

Ganser:

\[ C_D = \frac{24}{Re K_1 K_2} \left( 1 + 0.1118 \left( Re K_1 K_2 \right)^{-0.6567} \right) + 0.43056 \left( \frac{Re}{K_1 K_2} \right)^{-1.3305} \quad (8) \]

Chien:

\[ C_D = \frac{30}{Re} + 67.289 \exp(-5.03\psi) \quad (9) \]

Tran-Cong:

\[ C_D = \frac{24 d_A}{Re \, d_n} \left[ 1 + 0.15 \sqrt{c} \left( \frac{d_A}{d_n} Re \right)^{0.687} \right] + 0.42 \left( \frac{d_A}{d_n} \right)^2 \sqrt{c} \left( 1 + 4250 \left( \frac{d_A}{d_n} Re \right)^{-1.16} \right) \quad (10) \]

Haider:

\[ C_D = \frac{24}{Re} \left[ 1 + \exp \left( 2.3288 - 6.458 \psi + 2.4486 \psi^2 \right) \right] Re^{(0.0964 + 0.5565 \psi)} \]

\[ \frac{Re \exp(4.905 - 13.8944 \psi + 18.4222 \psi^2 - 10.2599 \psi^3)}{Re + \exp(1.4681 + 12.2584 \psi - 20.7325 \psi^2 + 15.8855 \psi^3)} \quad (11) \]

All these formulae have improved the departure of drag coefficient for sphere particles when it is used to calculate drag coefficient of non-sphere particles. But the accuracy of these formulas are relatively low because of the limited experimental technique. Due to the development of experiment technique, some new equipment could be used in the experiment to get some more detailed information, and here in this paper we bring in high speed camera and 3-D printer techniques to do some investigations on the movement of non-sphere particles.

2. Experiment and method

We use five different shapes of rigid particles as research objects, which are sphere, ellipsoid, cylinder, hexahedron and octahedron. All particles were printed by 3-D printer, and the material is resin, the density is 1.16E3Kg/m^2, all the surfaces of particles are smooth. Each shape of non-sphere particles contains various sphericities.

The method used in this article is particle settling, which is widely used in literatures[9][12][13][14].ie. let the particle fall from the top surface of a water tank, when the particle reaches its fall velocity, we consider it attains balanced state. Under this circumstance, the value of drag force and gravity of particle are equal, and the drag coefficient can be calculated;
The setup of experiment is showed in figure 1, which consists of 3 parts, lighting system, settling particles and image capture system. Lighting system are formed by laser device, cylindrical lenses, mirror. Laser device emits a dot laser light source. Then a volume light is formed after the dot laser light source passes through two orthogonal column prisms. The reflected light of volume light by the mirror can supply fully light for water tank and particles. The high speed camera is placed vertical to the lighting direction of laser device. The high speed camera is opened as a particle releases in water tank. The liquid in this experiment is water, which is standing for at least 24 hours in room temperature. It ensures the gas dissolving in water discharge sufficiently so that it won’t attach on the particle and affect the experiment result. The frame rate are 250fps and 500fps. Any case that the particle fall too close to wall will be deleted.

2.1 Data processing
High-speed camera get digital image through CCD sensor, which is actually a two-dimensional array[15]. In this experiment, a series of gray level images with 1024*1024 pixel are obtained. The particle can be separated from background by picture processing. The velocity of a particle can be obtained through counting the distance of the particle positions in two adjoin images and time interval of the two adjoin images.

2.2 Settling behaviour and result
The experimental results of drag coefficients of 5 shapes of rigid particles are shown in figure 2, it is obvious that the experiment data of sphere coincide well to the well-known standard curve. As regard to drag coefficient of non-sphere particles, the experiment data show great difference, they all turn bigger than that of sphere.
To test the accuracy of the modified formulas for non-sphere particle, we compare the drag coefficients of these formulas with the experiment data. The result are shown in figure 3:

**Figure 2.** The experimental results of drag coefficients of 5 shapes of rigid particles

**Figure 3.** Comparison of modified formulas
As we can see from figure 3, each formula improves the accuracy to a certain extent; The mean errors are listed in table 1. It can be seen that the formula proposed by Tran-Cong in 2004 is the most accurate one.

| author         | A.Haider | Tran-Cong | Chien   | Ganser |
|----------------|----------|-----------|---------|--------|
| Mean error     | 16.72%   | 10.26%    | 18.69%  | 15.11% |

Through high speed camera, our experimental results successfully record the oscillation of particles when they fall in the tank. The images shows that non-sphere particles do oscillate regularly at high Reynolds number, which approve the phenomenon in previous literatures that sometimes particles will not fall straightly.

To dig detail information, the horizontal velocity and vertical velocity of hexahedron and Octahedron particles, are calculated. And the line graph of velocity-time are drawn as figure 4. The Reynold number of the particles are 55.4 (figure 4-1) and 139.6 (figure 4-2) respectively. The figures show that velocities in both horizontal and vertical directions are affected by the unsteady motion of particles. The horizontal motion of particles shows a regular forth and back movement with periodic velocity. The vertical velocities show fluctuate characteristics with periodicity in some extent. The velocity-time line graph shows that the fluctuation of velocities in two directions has some connections, it may result from the periodic drop of wake vortices from the particles. As we can see from figure 4, the extreme value of horizontal velocity can be up to 30%-40% higher than that of the mean vertical velocity, which means the oscillation motion may have sufficient influence on particle velocity, posture, as well as the shape of projective plane. Further it may lead to the wrong judgement of particle orientation, projected area and finally affect the calculation of drag coefficient.

Figure 4-1. the line graph of velocity-time for Re 55.4

Figure 4-2. the line graph of velocity-time for Re 139.6
3. Conclusion
This study uses 3-D printer and high-speed camera techniques to reproduce the particle settling experiment, and drag coefficients for 5 shapes of particles are calculated. The results of sphere fit well with existing formulas. A comparison between our experiment data of non-sphere particles and the modified formulas shows that each formula improves the accuracy to a certain extent. Among these formulas, the formula proposed by Tran-Cong is the most accurate one.

As for particle oscillation, we get the image of particle oscillation throughout its settling process. The results indicate that non-sphere particles do oscillate regularly at high Reynolds number, which may lead to the wrong judgement of stress state, particle orientation and projected area, and finally affect the calculation of drag coefficient.

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