In-medium $\Sigma^0 - \Lambda$ Mixing in QCD Sum Rules

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Abstract

The $\Sigma^0 - \Lambda$ mixing angle in isospin-asymmetric nuclear medium is investigated by using QCD sum rules. From the general consideration of the in-medium baryonic correlations, in-medium baryon mixings are shown to have several Lorentz structures such as the scalar mixing angle $\theta^S$ and the vector mixing angle $\theta^V$. This causes a difference between the particle mixing $\theta (= \theta^S + \theta^V)$ and the anti-particle mixing $\bar{\theta} (= \theta^S - \theta^V)$. From the finite energy sum rules for the $\Sigma^0 - \Lambda$ mixing, we find that the in-medium part of the mixing angle has a relation $\theta_{\text{Med}}^S \sim -\theta_{\text{Med}}^V$ in the isospin-asymmetric medium. This implies that the medium affects mainly the anti-particle mixing. From the Borel sum rules, we obtain $|\bar{\theta} - \theta_0| \simeq 0.39 |(\rho_n - \rho_p)|/\rho_0$ with $\theta_0$, $\rho_n$, $\rho_p$ and $\rho_0$ being the vacuum mixing angle, the neutron density, the proton density and the normal nuclear matter density respectively.

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1 INTRODUCTION

The $SU(2)$ isospin symmetry is slightly broken in the hadronic world. Examples of this symmetry breaking are the mass splittings within a same isospin multiplet ($p - n$, $\Sigma^\pm - \Sigma^0$ and $\pi^\pm - \pi^0$), the particle mixing among different isospin multiplets ($\pi^0 - \eta$, $\pi^0 - \eta'$, $\rho^0 - \omega$ and $\Sigma^0 - \Lambda$), and the nuclear force in the $^1S_0$ channel ($V_{pp} \neq V_{pn} \neq V_{nn}$) \cite{1,2}.

The isospin symmetry breaking has two different sources: (i) the electromagnetic (EM) effect due to the electric-charge difference between $u$ and $d$ ($e_u \neq e_d$), and (ii) the quark-mass difference between $u$ and $d$ ($m_u \neq m_d$). The latter effect can be evaluated from the mass term $H_{\text{QCD mass}}$ of the QCD Hamiltonian density for light flavors,

$$H_{\text{QCD mass}} = \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) + \frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u) + m_s\bar{s}s,$$

where $(m_d - m_u)/(m_u + m_d) \sim 0.29$ \cite{3}. $H_{\text{QCD mass}}$ is known to be more important than the EM effect for the $p - n$ mass difference, the $\rho^0 - \omega$ mixing, and the $\Sigma^0 - \Lambda$ mixing \cite{4}.

The QCD sum rule \cite{5} is a useful method to evaluate the magnitude of the isospin symmetry breaking with non-perturbative QCD dynamics. It has been applied for the isospin mass splittings in octet baryons \cite{6,7,8}, the $\rho^0 - \omega$ mixing \cite{5,9}, the $\pi^0 - \eta$ mixing \cite{10} and the $\Sigma^0 - \Lambda$ mixing \cite{11,12}. For example, the $\Sigma^0 - \Lambda$ mixing angle in the vacuum defined by

$$\theta_0 = -\frac{\langle \Lambda | H_{\text{QCD mass}} | \Sigma^0 \rangle}{M_{\Sigma^0} - M_{\Lambda}}$$

is evaluated as $|\theta_0| = 1.4 \times 10^{-3}$ \cite{12} and $7 \times 10^{-3}$ \cite{11} in QCD sum rules. This value is comparable to the other estimate $|\theta_0| \simeq 1 \times 10^{-2}$ in the naive quark model \cite{3} and in the chiral perturbation theory \cite{3,4}.

In this paper, we will consider the isospin-asymmetric nuclear medium where the difference between the neutron density ($\rho_n$) and the proton density ($\rho_p$) becomes an extra source of the isospin symmetry breaking. In particular, we study how this new source affects the $\Sigma^0 - \Lambda$ mixing. The in-medium QCD sum rule \cite{14,15,16} is a suitable method for this purpose, since we can treat the isospin-asymmetric medium as a background field acting on the $\Sigma^0 - \Lambda$ correlation through the operator product expansion. Also, it allows us to investigate the response of the mixing angle under the variation of the magnitude of isospin-asymmetry.

We should mention here that the $\rho^0 - \omega$ mixing in isospin-asymmetric medium has been recently studied in \cite{17}. A major difference between the meson-mixing
treated in [17] and the baryon-mixing in the present paper lies in the fact that the latter can have several Lorentz structures (such as scalar and vector mixing angles) in the medium because of the spinor structure of the baryon fields. This will cause an interesting difference between the particle mixing ($\Sigma^0 - \Lambda$) and the anti-particle mixing ($\Sigma^0 - \bar{\Lambda}$). In the former (latter), the scalar mixing and the vector mixing act in destructive (constructive) manner. A close analogy of this phenomenon is the scalar and vector self-energies of the nucleon (anti-nucleon) in the symmetric nuclear medium, where scalar and vector act in destructive (constructive) way [18].

The organization of this paper is as follows. In Sec. 2, we analyze the general structure of in-medium correlation functions of spin $\frac{1}{2}$ baryonic currents with and without the mixing. Dispersion relations satisfied by the correlation functions are also written down after decomposing them into even and odd parts with respects to the frequency $\omega$ of the currents. In Sec. 3, we introduce a generalized mixing matrix in the spinor space and make physical interpretation of the scalar and vector mixing angles. In Sec. 4, we carry out the operator product expansion (OPE) of the mixed correlation function in the $\Sigma^0 - \Lambda$ channel. The Lorentz-tensor and isospin-asymmetric operators are kept in OPE since they have non-vanishing expectation values in the isospin-asymmetric medium. The in-medium condensates which appear in OPE are also evaluated in the low density expansion in this section. Since the in-medium expectation values of the isospin-asymmetric operators beyond dimension 4 are hard to be determined at present, we limit ourselves to the OPE up to dimension 4. In Sec. 5, we construct the finite energy sum rules [19] and the Borel sum rules [5] using the results in previous sections. Then we extract a qualitative result from the finite energy sum rule. In Sec. 6, to reduce the uncertainties due to the absence of higher dimensional operators in OPE, we examine the reliability of the sum rules constructed in Sec. 5 from the point of view of the consistency among different sum rules. Then we evaluate the $\Sigma^0 - \Lambda$ mixing angles numerically. Sec. 7 is devoted to summary and concluding remarks.

2 GENERAL PROPERTIES OF THE CORRELATION FUNCTIONS

In this section we examine the spinor structures of the diagonal and off-diagonal correlation functions of spin $\frac{1}{2}$ baryonic currents. We will also derive the dispersion relation for each spinor component of the correlation functions.
2.1 Spinor structure

Let us start with the following two-point functions:

\[
\Pi_T(p \mid q) = i \int d^4 x e^{ipx} \langle q \mid T [\eta_A(x) \pi_B(0)] \rangle q, \tag{2.1}
\]

\[
\Pi_R(p \mid q) = i \int d^4 x e^{ipx} \langle q \mid R [\eta_A(x) \pi_B(0)] \rangle q, \tag{2.2}
\]

where \(T\) and \(R\) denote time-ordered and retarded products respectively, and \(\eta_{A(B)}(x)\) is an interpolating operator for the baryon \(A(B)\). If \(A\) is different from \(B\), the correlations describe the particle mixings. \(\langle q \rangle\) is a state vector with four-momentum \(q^\mu\). Later, this state will be identified with the isospin-asymmetric nuclear medium to investigate the \(\Sigma^0 - \Lambda\) mixing with \(A = \Lambda\) and \(B = \Sigma^0\).

These correlation functions have the following spectral representations,

\[
\Pi_{T\alpha\beta}(p \mid q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp^0 \rho_{\alpha\beta}(p^0 \mid q) + \tilde{\rho}_{\alpha\beta}(p^0 \mid q), \tag{2.3}
\]

\[
\Pi_{R\alpha\beta}(p \mid q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp^0 \rho_{\alpha\beta}(p^0 \mid q) + \tilde{\rho}_{\alpha\beta}(p^0 \mid q), \tag{2.4}
\]

with \(p^\mu = (p^0, \mathbf{p})\) and \(p'^\mu = (p'^0, \mathbf{p'})\). \(\rho_{\alpha\beta}(p \mid q)\) and \(\tilde{\rho}_{\alpha\beta}(p \mid q)\) are the spectral functions defined by

\[
\rho_{\alpha\beta}(p \mid q) = \int d^4 x e^{ipx} \langle q \mid \eta_A(x) \rangle \{ \pi_B(0) \}_\alpha \{ \eta_A(x) \}_\beta \rangle, \tag{2.5}
\]

\[
\tilde{\rho}_{\alpha\beta}(p \mid q) = \int d^4 x e^{ipx} \langle q \mid \pi_B(0) \rangle \{ \eta_A(x) \}_\alpha \{ \eta_A(x) \}_\beta \rangle, \tag{2.6}
\]

where spinor indices \((\alpha, \beta)\) are explicitly written.

To make the following discussion concise, let us introduce a linear combination of the spectral functions with real parameters \(a\) and \(b\) as

\[
\Delta(p \mid q) \equiv a \rho(p \mid q) + b \tilde{\rho}(p \mid q) \tag{2.7}
\]

and define \(\Pi_{\pm}\) as

\[
\Pi_{\pm}(p \mid q) \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} dp^0 \Delta(p^0 \mid q) \tag{2.8}
\]

Then the time-ordered and retarded correlations can be written as

\[
\Pi_T(p \mid q) = \Pi_-(p \mid q)\big|_{a=b=1/2} + \Pi_+(p \mid q)\big|_{a=0, b=1/2}, \tag{2.9}
\]

\[
\Pi_R(p \mid q) = \Pi_-(p \mid q)\big|_{a=b=1/2}. \tag{2.10}
\]

Let us first consider the spinor structure of the spectral function \((2.7)\). \(\Delta\) has a \(4 \times 4\) spinor structure and can be expanded in terms of a complete set of Dirac
matrices. The Lorentz covariance restricts the general form of $\Delta$ as

$$\Delta(p \mid q) = \Delta^S + \Delta^P \gamma^5 + \Delta^V_1 p + \Delta^V_2 q + \Delta^A_1 p \gamma^5 + \Delta^A_2 q \gamma^5 + \Delta^T_1 (i \sigma_{\mu \nu} p^{\mu} q^{\nu}) + \Delta^T_2 (\sigma_{\mu \nu} p^{\mu} q^{\nu}), \quad (2.11)$$

where we define the coefficients $\Delta^l = \Delta^l(p^2, p \cdot q, q^2)$ for $l = S, P, V_1, V_2, A_1, A_2, T_1, T_2$ and $\sigma_{\mu \nu} \equiv \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} \sigma^{\alpha \beta} = -i \sigma_{\mu \nu} \gamma^5$ with a convention $\epsilon^{0123} = 1$.

Parity and time-reversal properties further restrict the spinor structure of $\Delta$. Since the baryonic currents $\eta_A(x)$ and $\eta_B(x)$ have the same transformation properties with elementary Dirac fields under parity ($P$) and time-reversal ($T$) transformations, we have

$$P \eta_A(x) \mathcal{P}^{-1} = P \eta_A(\bar{x}), \quad P \pi_A(x) \mathcal{P}^{-1} = \pi_A(\bar{x}) \mathcal{P}^{-1}, \quad (2.12)$$

$$T \eta_A(x) \mathcal{T}^{-1} = T \eta_A(-\bar{x}), \quad T \pi_A(x) \mathcal{T}^{-1} = \pi_A(-\bar{x}) \mathcal{T}^{-1}, \quad (2.13)$$

where $x^\mu = (x^0, \mathbf{x})$ and $\bar{x}^\mu \equiv (x^0, -\mathbf{x})$. $P$ ($T$) is a $4 \times 4$ matrix in the spinor space for the parity (time-reversal) transformation. We assume the same transformation matrices $P$ and $T$ for $\eta_B(x)$. Under the Hermitian conjugate, $P$ and $T$, the Dirac matrices transform as

$$\gamma^0 \left[ \{1, \gamma^5, \gamma_\mu \gamma^5, i \sigma_{\mu \nu}, \sigma_{\mu \nu}^5 \} \right]^* \gamma^0 = \{1, -\gamma^5, \gamma_\mu \gamma^5, -i \sigma_{\mu \nu}, \sigma_{\mu \nu}^5 \}, \quad (2.14)$$

$$P \left[ \{1, \gamma^5, \gamma_\mu \gamma^5, i \sigma_{\mu \nu}, \sigma_{\mu \nu}^5 \} P^{-1} \right] = \{1, -\gamma^5, \gamma_\mu \gamma^5, -i \sigma_{\mu \nu}, -\sigma_{\mu \nu}^5 \}, \quad (2.15)$$

$$\left[ T \left[ \{1, \gamma^5, \gamma_\mu \gamma^5, i \sigma_{\mu \nu}, \sigma_{\mu \nu}^5 \} T^{-1} \right] \right]^* = \{1, \gamma^5, \gamma_\mu \gamma^5, i \sigma_{\mu \nu}, \sigma_{\mu \nu}^5 \}, \quad (2.16)$$

where $\gamma^\mu = (\gamma^0, \gamma), \gamma_\mu \equiv (\gamma^0, -\gamma), \sigma_{\mu \nu} \equiv \frac{1}{2} [\gamma_\mu, \gamma_\nu]$ and $\sigma_{\mu \nu}^5 \equiv -i \sigma_{\mu \nu} \gamma^5$. The state vector $|q\rangle$ is assumed to have the property,

$$P |q\rangle = |\bar{q}\rangle, \quad T |q\rangle = |\bar{q}\rangle, \quad (2.17)$$

with $q^\mu = (q^0, \mathbf{q})$ and $\bar{q}^\mu \equiv (q^0, -\mathbf{q})$.

Because of the transformation properties of the baryonic current and the state vector shown above, the spectral function satisfies the following relations

$$\Delta(p \mid q)|_{A \leftrightarrow B} = \gamma^0 \{\Delta(p \mid q)\}^* \gamma^0,$$

$$\Delta(p \mid q) = P \Delta(\bar{p} \mid \bar{q}) P^{-1}, \quad (2.18)$$

where $A \leftrightarrow B$ stands for the exchange of $A$ and $B$.

Owing to Eq.(2.18) and Eqs.(2.14-2.16), eight independent functions in Eq.(2.11) reduces to four functions such as

$$\Delta(p \mid q) = \Delta^S + \Delta^V_1 p + \Delta^V_2 q + \Delta^T_1 (i \sigma_{\mu \nu} p^{\mu} q^{\nu}), \quad (2.19)$$
where
\[
\Delta^l \big|_{A \leftrightarrow B} = \Delta^l \ (l = S, V_1, V_2), \quad \Delta^{T_1} \big|_{A \leftrightarrow B} = -\Delta^{T_1},
\]
\[
\Delta^l = \Delta^l \ (l = S, V_1, V_2, T_1). \tag{2.20}
\]

Eqs. (2.19) and (2.20) also imply that the correlation function \( \Pi_\mp \) defined in Eq. (2.8) has a form
\[
\Pi_\mp (p \mid q) = \Pi_\mp^S + \Pi_\mp^{V_1} q + \Pi_\mp^{T_1} (i\sigma_{\mu \nu} p^\mu q^\nu) \tag{2.21}
\]
where \( \Pi_\mp^l = \Pi_\mp^l (p^2, p \cdot q, q^2) \) for \( l = S, V_1, V_2, T_1 \) and
\[
\Pi_\mp^l \big|_{A \leftrightarrow B} = \Pi_\mp^l \ (l = S, V_1, V_2), \quad \Pi_\mp^{T_1} \big|_{A \leftrightarrow B} = -\Pi_\mp^{T_1}. \tag{2.22}
\]

Because of Eqs. (2.9) and (2.10), the time-ordered and retarded correlation functions have the same decomposition as Eq. (2.21).

Note that, for \( A = B \), our results are fully consistent with the previous analysis in Ref. [20]. In particular, the tensor terms \( \Delta^{T_1} \) and \( \Pi_\mp^{T_1} \) vanish in this case. For \( A \neq B \), our results are new. The tensor terms do not vanish in this case unless \( p^\mu \) and \( q^\mu \) satisfy special conditions.

### 2.2 Dispersion relations

Eq. (2.21) enables us to decompose the dispersion relation (2.8) into independent structures. In this subsection we will work in the rest frame of the state vector \(|q\rangle\) (\( q^\mu = (q^0, 0) \)), since it is sufficient for later applications. For notational simplicity, we will omit the argument \( q \) in \( \Pi_\mp \), whenever we consider the rest frame of \(|q\rangle\).

Under this simplification, Eq. (2.21) becomes
\[
\Pi_\mp (p) = \Pi_\mp^S + \Pi_\mp^{V_1} p + \Pi_\mp^{T_1} (i\sigma_{\mu \nu} p^\mu q^\nu) (p \cdot \gamma), \tag{2.23}
\]
where we have introduced
\[
\Pi_\mp^{V_1} \equiv \Pi_\mp^{V_1} p^0 + \Pi_\mp^{V_2} q^0. \tag{2.24}
\]

Then the dispersion relation for \( l = S, V \) reads
\[
\text{Re} \Pi_\mp^l (\omega, |p|) = \pm \frac{1}{i} \int_{-\infty}^{\infty} d\omega' \frac{\text{Im} \Pi_\mp^l (\omega', |p|)}{\omega' - \omega}, \tag{2.25}
\]
where
\[
\text{Im} \Pi_\mp^l (\omega, p) = \pm \frac{1}{2i} \lim_{\epsilon \to 0} \left[ \text{Re} \Pi_\mp^l (\omega + i\epsilon, |p|) - \text{Re} \Pi_\mp^l (\omega - i\epsilon, |p|) \right], \tag{2.26}
\]
with \( \omega \equiv p^0 \) and \( P \) stands for the principal value integral. As far as \( p \neq 0 \), the same dispersion relation holds for \( l = V_1, T_1 \).
Next let us decompose the correlation functions to even and odd parts under the transformation $\omega \leftrightarrow -\omega$ [21]:

$$
\Pi^l_\pm (\omega, |p|) = \Pi^{l(E)}_\pm (\omega^2, |p|) + \omega \Pi^{l(O)}_\pm (\omega^2, |p|).
$$

(2.27)

Then Eq.(2.27) reduces to a formula which relates the even (odd) part of Re$\Pi^l_\pm$ with the odd (even) part of Im$\Pi^l_\pm$:

$$
\text{Re}\Pi^{l(E)}_\pm (s, |p|) = \pm \frac{1}{\pi} \int_0^\infty ds' \frac{\text{Im}\Pi^{l(O)}_\pm (s', |p|) \sqrt{s'}}{s' - s},
$$

(2.28)

$$
\text{Re}\Pi^{l(O)}_\pm (s, |p|) = \pm \frac{1}{\pi} \int_0^\infty ds' \frac{\text{Im}\Pi^{l(E)}_\pm (s', |p|) / \sqrt{s'}}{s' - s}.
$$

(2.29)

Also, Eq.(2.26) reduces to

$$
\text{Im}\Pi^{l(O)}_\pm (s, |p|) = \pm \frac{1}{2\pi i} \lim_{\epsilon \to +0} \left[ \text{Re}\Pi^{l(E)}_\pm (s + \epsilon, |p|) - \text{Re}\Pi^{l(E)}_\pm (s - \epsilon, |p|) \right] / \sqrt{s},
$$

(2.30)

$$
\text{Im}\Pi^{l(E)}_\pm (s, |p|) = \pm \frac{1}{2\pi i} \lim_{\epsilon \to +0} \left[ \text{Re}\Pi^{l(O)}_\pm (s + \epsilon, |p|) - \text{Re}\Pi^{l(O)}_\pm (s - \epsilon, |p|) \right] \sqrt{s},
$$

(2.31)

for $l = S, V, V_1, T_1$ where $s \equiv \omega^2$.

**2.3 Dispersion relations between Re$\Pi_T$ and Im$\Pi_R$**

The retarded correlation function defined in Eq.(2.10) satisfies the same dispersion relations (2.25), (2.28) and (2.29). Also, Eqs.(2.9) and (2.20) imply that the real part of $\Pi_T$ and that of $\Pi_R$ are equal for each spinor component:

$$
\text{Re}\Pi^l_T = \text{Re}\Pi^l_R \quad (l = S, V_1, V_2, T_1, V).
$$

(2.32)

Therefore, the dispersion relations in the rest frame of the state vector $|q\rangle$ reads

$$
\text{Re}\Pi^l_T (\omega, |p|) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\text{Im}\Pi^l_R (\omega', |p|)}{\omega' - \omega},
$$

(2.33)

$$
\text{Im}\Pi^l_R (\omega, |p|) = \frac{1}{2\pi i} \lim_{\epsilon \to +0} \left[ \text{Re}\Pi^l_T (\omega + \epsilon, |p|) - \text{Re}\Pi^l_T (\omega - \epsilon, |p|) \right].
$$

(2.34)

By decomposing the above to the even and odd parts, one finds

$$
\text{Re}\Pi^{l(E)}_T (s, |p|) = \frac{1}{\pi} \int_0^\infty ds' \frac{\text{Im}\Pi^{l(O)}_R (s', |p|) \sqrt{s'}}{s' - s},
$$

(2.35)

$$
\text{Re}\Pi^{l(O)}_T (s, |p|) = \frac{1}{\pi} \int_0^\infty ds' \frac{\text{Im}\Pi^{l(E)}_R (s', |p|) / \sqrt{s'}}{s' - s},
$$

(2.36)
and
\[
\text{Im}\Pi^I_R(s, |\mathcal{P}|) = \frac{1}{2i} \lim_{\epsilon \to 0} \left[ \text{Re}\Pi^I_T(s+i\epsilon, |\mathcal{P}|) - \text{Re}\Pi^I_T(s-i\epsilon, |\mathcal{P}|) \right] / \sqrt{s},
\]
(2.37)
\[
\text{Im}\Pi^I_R(s, |\mathcal{P}|) = \frac{1}{2i} \lim_{\epsilon \to 0} \left[ \text{Re}\Pi^I_T(s+i\epsilon, |\mathcal{P}|) - \text{Re}\Pi^I_T(s-i\epsilon, |\mathcal{P}|) \right] \sqrt{s},
\]
(2.38)
l = S, V, V_1, T_1 with \(\Pi^V_{T,R} \equiv \Pi^V_{T,R} F^0 + \Pi^V_{T,R} q^0\).

In the sum rule analysis in later sections, \(\Pi^I_T\) is evaluated by the operator product expansion, while phenomenological ansatzes are made for \(\text{Im}\Pi^I_R\).

3 STRUCTURE OF BARYON MIXING

In this section, we discuss the general structure of the baryon mixing.

3.1 Definition of the mixing angle

Let us consider a mass matrix for particles \(A\) and \(B\) which have definite quantum numbers but have different masses,
\[
M = \left(\begin{array}{cc}
\langle A | & | B \rangle \\
\langle B | & | B \rangle
\end{array}\right) H \left(\begin{array}{c}
| A \rangle \\
| B \rangle
\end{array}\right) = \begin{pmatrix}
M_A & \langle A | H | B \rangle \\
\langle H | B \rangle & M_B
\end{pmatrix},
\]
(3.1)
where \(H\) is the Hamiltonian of the system and \(| A \rangle\) and \(| B \rangle\) are one-particle states normalized as \(\langle A | A \rangle = \langle B | B \rangle = 1\). The diagonal matrix element is equal to the ‘mass’ of the symmetric state, namely \(M_A = \langle A | H | A \rangle\) and \(M_B = \langle B | H | B \rangle\). We choose the relative phase of the states \(| A \rangle\) and \(| B \rangle\) to be \(\langle B | H | A \rangle = \langle B | H | A \rangle^* = \langle A | H | B \rangle\).

The physical states \(| A \rangle_{\text{phys}}\) and \(| B \rangle_{\text{phys}}\) are represented as a linear combination of \(| A \rangle\) and \(| B \rangle\) by using the mixing angle \(\theta\),
\[
| A \rangle_{\text{phys}} = | A \rangle \cos \theta + | B \rangle \sin \theta,
\]
(3.2)
\[
| B \rangle_{\text{phys}} = | B \rangle \cos \theta - | A \rangle \sin \theta.
\]
(3.3)

The mass matrix in terms of the physical states reads
\[
M_{\text{phys}} = \begin{pmatrix}
\tilde{M} - \Delta M' / 2 & \langle A | H | B \rangle \cos 2\theta + \Delta M \sin 2\theta / 2 \\
\langle A | H | B \rangle \cos 2\theta + \Delta M \sin 2\theta / 2 & \tilde{M} + \Delta M' / 2
\end{pmatrix},
\]
(3.4)
where \(\tilde{M} = (M_A + M_B) / 2\), \(\Delta M = M_B - M_A\) and \(\Delta M' = \Delta M \cos 2\theta - 2 \langle A | H | B \rangle \sin 2\theta\).

Thus, for the weak mixing, \(\theta\) is written as
\[
\theta \simeq - \frac{\langle A | H | B \rangle}{\Delta M} = - \frac{\langle A | H_{\text{int}} | B \rangle}{\Delta M},
\]
(3.5)
where \(H_{\text{int}}\) is a part of \(H\) which mixes the states \(| A \rangle\) and \(| B \rangle\). When we consider baryon mixings, \(\theta\) acquires spinor structures as will be discussed in the next subsection.


### 3.2 Phenomenological ansatz

Let us consider the case that $A$ and $B$ are baryons with different flavor quantum-numbers and different masses. We further assume that the interaction which induces the mixing is small and can be treated in the 1st order perturbation. Then the correlation function Eq.(2.1) near the mass-shell of baryons $A$ and $B$ may be written as

$$\Pi_T (p \mid q) = \lambda_A \lambda_B \frac{1}{p - M_A + i \epsilon} (\Theta \Delta M) \frac{1}{p - M_B + i \epsilon}. \quad (3.6)$$

where we omit the single pole contributions. Here $\lambda_A$ is a coupling strength defined by $\langle 0 \mid \eta_A(0) \mid A(p, s) \rangle = \lambda_A u_A(p, s)$, where $\overline{\eta}_A(p, r) u_A(p, s) = 2M_A \delta_{rs}$ and $|A(p, s)\rangle$ satisfies the covariant normalization. The phase of the state $|A(p, s)\rangle$ (|$B(p, s)\rangle$) is chosen so that $\lambda_A$ ($\lambda_B$) becomes real. $\lambda_A$, $\lambda_B$, $M_A$, $M_B$ and $\Delta M = M_B - M_A$ take their unperturbed value in the 1st order of the mixing parameter $\Theta$. A diagrammatic illustration of Eq.(3.6) is shown in Fig.1. The state vector $|q\rangle$, which will be later identified with the isospin-asymmetric nuclear medium, is the major physical source of the mixing. In general, $\Theta$ is a $4 \times 4$ matrix in the spinor space. According to the discussion in Sec.2, the correlation function (3.6) must have a form

$$\Pi_T (p \mid q) = \Pi_T^S + \Pi_T^V p + \Pi_T^V q + \Pi_T^{T_1} (i \sigma_{\mu \nu} p^\mu q^\nu), \quad (3.7)$$

where $\Pi_T^l = \Pi_T (p^2, p \cdot q, q^2)$ and

$$\Pi_T^l \bigg|_{A \leftrightarrow B} = \Pi_T^l, \quad (l = S, V_1, V_2), \quad \Pi_T^{T_1} \bigg|_{A \leftrightarrow B} = - \Pi_T^{T_1}. \quad (3.8)$$

Thus the Eq.(3.7) restricts the structure of $\Theta$. In fact, it cannot contain $\gamma^5$, $\gamma_\mu \gamma^5$ and $\sigma_{\mu \nu}^5$, and one obtains

$$\Theta = \theta^S + \theta^V p + \theta^V q + \theta^{T_1} (i \sigma_{\mu \nu} p^\mu q^\nu), \quad (3.9)$$

where the parameters $\theta^l$ ($l = S, V_1, V_2, T_1$) are real.

Now, let us consider an effective Lagrangian $\mathcal{L}_{\text{int}}$ which describes Eq.(3.6),

$$\mathcal{L}_{\text{int}}(x) = \left( \overline{\Psi}_A(x) \Theta \Psi_B(x) + \overline{\Psi}_B(x) \Theta \Psi_A(x) \right) \Delta M. \quad (3.10)$$

Here $\overline{\Theta} = \gamma^0 \Theta^\dagger \gamma^0$ and $\Psi_A(\Psi_B)$ is the field that describes the particle $A$ ($B$). By the parity and time-reversal invariance of $\mathcal{L}_{\text{int}}$, $\Psi_A$ and $\Psi_B$ have the same transformation matrices $P$ and $T$ under the parity $(P)$ and time-reversal $(T)$ transformations. This is the same constraint for the interpolating operators $\eta_A$ and $\eta_B$ in Sec.2.1. Therefore, the fields $\Psi_A$ and $\Psi_B$ can simultaneously satisfy the relations $\langle 0 \mid \Psi_A(0) \mid A(p, s) \rangle = u_A(p, s)$ and $\langle 0 \mid \Psi_B(0) \mid B(p, s) \rangle = u_B(p, s)$ with the states

Fig.1: A diagrammatic illustration of Eq.(3.6). The baryon $A$ ($B$) couples to the interpolating operator $\eta_A$ ($\eta_B$) with the coupling strength $\lambda_A$ ($\lambda_B$). The baryon mixing $\Theta \Delta M$ is induced by the state with four-momentum $q^\mu$ which will be later identified with the isospin-asymmetric nuclear medium.

$|A(p,s)\rangle$ and $|B(p,s)\rangle$ defined below Eq.(3.6). Then, $\langle B | H_{\text{int}} | A \rangle = \langle A | H_{\text{int}} | B \rangle$ and $\langle B | H_{\text{int}} | A \rangle = \langle A | H_{\text{int}} | B \rangle$ hold.

We have implicitly assumed that the interaction does not contain any derivatives, which is equivalent to the assumption that $\Theta$ depends only on $q^\mu$ and not on $p^\mu$. Then, $\Theta$ reduces to a simple form,

$$\Theta = \theta^S + \theta^V q^0 \tag{3.11}$$

For later convenience, we define a dimensionless parameter $\theta^V$ as

$$\theta^V = \theta^V q^0 \tag{3.12}$$

We can make physical interpretation of the mixing angles $\theta^S$ and $\theta^V$ as follows. Consider the particle at rest in the rest frame of the medium ($p = q = 0$). Then the off-diagonal matrix elements of the Hamiltonian $H_{\text{int}} (= - \int d^3x \mathcal{L}_{\text{int}}(t, x))$ read

$$\langle A | H_{\text{int}} | B \rangle = - \left( \theta^S + \theta^V \right) \Delta M, \tag{3.13}$$

$$\langle A | H_{\text{int}} | B \rangle = - \left( \theta^S - \theta^V \right) \Delta M. \tag{3.14}$$

where $|A\rangle$ and $|B\rangle$ are normalized as $\langle A | A \rangle = \langle B | B \rangle = 1$. Together with Eq.(3.3), we thus find that $\theta$ ($\overline{\theta}$) defined below corresponds to the mixing angle in the particle (anti-particle) channel:

$$\theta = \theta^S + \theta^V, \tag{3.15}$$

$$\overline{\theta} = \theta^S - \theta^V. \tag{3.16}$$

$\theta^S$ and $\theta^V$ have formal analogy with the scalar and vector self-energies ($\Sigma^S$ and $\Sigma^V$) of the nucleon in the nuclear medium. The nucleon and the anti-nucleon feel an optical potential $\Sigma^S + \Sigma^V$ and $\Sigma^S - \Sigma^V$ respectively [18].
Let us rewrite the correlation functions in terms of the mixing angles $\theta^S$ and $\theta^V$ defined above. Since $\text{Re}\Pi_T^l$ has a form deduced from Eq.(3.6) and Eq.(3.11), we obtain

$$\text{Re}\Pi_T^S = \lambda_A \lambda_B \left[ \frac{\theta^SM_A + \theta^V(p\cdot q)}{M_A^2 - p^2} - \frac{\theta^SM_B + \theta^V(p\cdot q)}{M_B^2 - p^2} \right], \quad (3.17)$$

$$\text{Re}\Pi_T^V_1 = \lambda_A \lambda_B \left( \theta^S + \theta^V(p\cdot q) \right) \left[ \frac{1}{M_A^2 - p^2} - \frac{1}{M_B^2 - p^2} \right], \quad (3.18)$$

$$\text{Re}\Pi_T^V_2 = \lambda_A \lambda_B \left( \frac{\theta^V \Delta M^2}{2M} \right) \left[ \frac{M_A}{M_A^2 - p^2} + \frac{M_B}{M_B^2 - p^2} \right], \quad (3.19)$$

$$\text{Re}\Pi_T^T_1 = \lambda_A \lambda_B \left( -\theta^V \Delta M^2 \right) \left[ \frac{1}{M_A^2 - p^2} - \frac{1}{M_B^2 - p^2} \right]. \quad (3.20)$$

We mention here that $\text{Re}\Pi_T^l$ given above satisfy Eq.(3.8), since the mixing angles have the property

$$\theta^S,V \mid_{A \leftrightarrow B} = -\theta^S,V,$$

which is confirmed by Eqs.(3.13) and (3.14).

For $p^\mu = (\omega, 0)$ and $q^\mu = (q^\omega, 0)$, the correlation function (3.7) is simplified to

$$\Pi_T(\omega) = \Pi_T^S(\omega) + \Pi_T^V(\omega) \gamma^0, \quad (3.21)$$

and its real parts become

$$\text{Re}\Pi_T^S(\omega) = \lambda_A \lambda_B \left[ \frac{\theta^SM_A + \theta^V\omega}{M_A^2 - \omega^2} - \frac{\theta^SM_B + \theta^V\omega}{M_B^2 - \omega^2} \right], \quad (3.22)$$

$$\text{Re}\Pi_T^V(\omega) = \lambda_A \lambda_B \left[ \frac{\theta^S\omega + \theta^VM_A}{M_A^2 - \omega^2} - \frac{\theta^S\omega + \theta^VM_B}{M_B^2 - \omega^2} \right]. \quad (3.23)$$

We decompose $\Pi_T^{S,V}(\omega)$ further into even and odd parts,

$$\Pi_T^l(\omega) = \Pi_T^l(E)(\omega^2) + \omega \Pi_T^l(O)(\omega^2), \quad (3.24)$$

$l = S, V$. Then we obtain a simple formula to be used later

$$\text{Re}\Pi_T^l(E)(s) = \lambda_A \lambda_B \theta^l \left[ \frac{M_A}{M_A^2 - s} - \frac{M_B}{M_B^2 - s} \right], \quad (3.25)$$

$$\text{Re}\Pi_T^l(O)(s) = \lambda_A \lambda_B \theta^l \left[ \frac{1}{M_A^2 - s} - \frac{1}{M_B^2 - s} \right], \quad (3.26)$$

where $s \equiv \omega^2$ and $l = S, V$. 

11
4 OPERATOR PRODUCT EXPANSION

In this section, we carry out the operator product expansion (OPE) of \( \Pi_T \) up to dimension 4 and evaluate the in-medium matrix elements of local operators. As is well-known, the operators with Lorentz indices should be retained since they do not vanish in the medium \([14]\). Furthermore, we need to keep not only the iso-scalar operators but also the iso-vector ones to take into account the isospin asymmetry in the medium.

4.1 OPE for \( \Sigma^0 - \Lambda \) mixed correlation function

Taking \( A = \Lambda \) and \( B = \Sigma^0 \), the retarded correlation function \((2.2)\) reads

\[
\Pi_R (p | q) = i \int \! d^4 x \, e^{ipx} \langle q | R[\eta_\Lambda(x) \mathbf{\Pi}_{\Sigma^0}(0)] | q \rangle ,
\]

which satisfies the dispersion relations \((2.33), (2.35)\) and \((2.36)\). On the other hand, the time-ordered correlation function \((2.1)\) reads

\[
\Pi_T (p | q) = i \int \! d^4 x \, e^{ipx} \langle q | T[\eta_\Lambda(x) \mathbf{\Pi}_{\Sigma^0}(0)] | q \rangle ,
\]

which is useful for making OPE. \( |q\rangle \) is the state vector corresponding to the nuclear medium with total four-momentum \( q^\mu \).

For the interpolating operators \( \eta_\Lambda(x) \) and \( \eta_{\Sigma^0}(x) \), we adopt the Ioffe’s current \([22]\),

\[
\eta_{\psi_1\psi_2\psi_3}(x) = \varepsilon_{abc} \left( \psi_1^T(x) C \gamma_\mu \psi_2(x) \right) \gamma^5 \gamma_\mu \psi_3(x)
\]

where \( \psi(x) \) is the quark field with flavor \( \psi \), \( C \) denotes the charge conjugation matrix and \( a, b, c \) are color indices. This current is symmetric under the exchange of \( \psi_1 \) and \( \psi_2 \), i.e., \( \eta_{\psi_1\psi_2\psi_3}(x) = \eta_{\psi_2\psi_1\psi_3}(x) \). Thus, \( \eta_\Lambda(x) \) and \( \eta_{\Sigma^0}(x) \) may be written as

\[
\eta_\Lambda(x) = i \sqrt{\frac{2}{3}} \left\{ \eta_{usd}(x) - \eta_{dus}(x) \right\} , \quad \eta_{\Sigma^0}(x) = i \sqrt{2} \eta_{uds}(x) .
\]

These are the same interpolating operators used in the analysis of \( \Sigma^0 - \Lambda \) mixing in the vacuum \([12, 11]\). Under the time-reversal, \( \eta_\Lambda(x) \) and \( \eta_{\Sigma^0}(x) \) transform in the same way as the quark field \( \psi(x) \).

Using Eq.(4.4) and the above mentioned exchange property, the mixed correlation \((4.2)\) becomes

\[
\Pi_T (p | q) = \frac{2}{\sqrt{3}} \left[ i \int \! d^4 x \, e^{ipx} \langle q | T[\eta_{usd}(x) \mathbf{\Pi}_{uds}(0)] | q \rangle - \{ u \leftrightarrow d \} \right] .
\]

We carry out OPE of this correlation up to dimension 4 operators. The quark masses are kept up to the 1st order. Since Eq.(4.5) is anti-symmetric under the exchange of
and $d$, $u - d$ symmetric terms such as the gluon condensate $\langle \alpha_s G^2 \rangle$ do not appear. All the diagrams contributing up to the order we consider are drawn in Fig. 2. The explicit forms of OPE may be summarized as follows:

$$\Pi_T (p | q) = \Pi_T^S + \Pi_T^{V_1} p + \Pi_T^{V_2} q + \Pi_T^T (i \sigma_{\mu \nu} p^\mu q^\nu).$$

(4.6)

$$\text{Re} \Pi_T^S = \frac{2}{\sqrt{3}} \left[ \frac{1}{64 \pi^2} (m_d - m_u) p^4 \log(-p^2) - \frac{1}{8 \pi^2} \langle dd - \pi u \rangle p^2 \log(-p^2) - \frac{1}{8 \pi^2} \left\{ (2m_u + m_d) \langle \bar{u} u \rangle 
- (m_u + 2m_d) \langle d \eta d \rangle + (m_d - m_u) \langle \bar{s} \eta s \rangle \right\} (n \cdot p) \log(-p^2) \right],$$

(4.7)

$$\text{Re} \Pi_T^{V_1} = \frac{2}{\sqrt{3}} \frac{1}{24 \pi^2} \langle dd - \pi u \rangle (n \cdot p) \log(-p^2) + \frac{1}{8 \pi^2} \left\{ m_s \langle dd - \pi u \rangle + (m_d - m_u) \langle \bar{s} \eta s \rangle \right\} \log(-p^2) - \frac{1}{72 \pi^2} \left\{ (m_d \langle dd \rangle - m_u \langle \bar{u} u \rangle 
- 4 \langle \bar{d} (n \cdot i \not{D}) u \rangle \frac{1}{\sqrt{-q^2}} \right\} \left( \log(-p^2) + \frac{2(n \cdot p)^2}{p^2} \right),$$

(4.8)

$$\text{Re} \Pi_T^{V_2} = \frac{2}{\sqrt{3}} \left[ \frac{5}{48 \pi^2} \langle \bar{d} \eta d - \pi \eta u \rangle p^2 \log(-p^2) + \frac{1}{18 \pi^2} \left\{ (m_d \langle dd \rangle - m_u \langle \bar{u} u \rangle 
- 4 \langle \bar{d} (n \cdot i \not{D}) u \rangle \frac{1}{\sqrt{-q^2}} \right\} (n \cdot p) \log(-p^2) \right] / \sqrt{q^2},$$

(4.9)

$$\text{Re} \Pi_T^T = \frac{2}{\sqrt{3}} \left[ \frac{1}{8 \pi^2} \left\{ (m_s - m_d) \langle \bar{s} \eta u \rangle - (m_u - m_s) \langle \bar{d} \eta d \rangle 
+ (m_d - m_u) \langle \bar{u} \eta s \rangle \right\} \log(-p^2) \right] / \sqrt{q^2},$$

(4.10)

where we have defined a normal vector $n^\mu \equiv q^\mu / \sqrt{q^2}$ characterizing the nuclear medium. Also we have replaced the in-medium matrix elements $\langle q | \cdots | q \rangle$ by $\langle \cdots \rangle$ for simplicity.

\[ \begin{array}{cccc}
\text{Fig.2:} & \text{Diagrams of OPE up to dimension 4 for the } \Sigma^0 - \Lambda \text{ mixed correlation function.}
\end{array} \]

(m stands for the $u, d, s$ quark masses and $\nabla$ for the covariant derivative.)

The in-medium expectation values of isospin-asymmetric operators beyond dimension 4 have large uncertainties. For example, $\langle d^\dagger (\sigma \cdot G) d - u^\dagger (\sigma \cdot G) u \rangle$, which may
give a major contribution to the OPE at dimension 5, is not known. In fact, even its isospin-symmetric partner \( \langle q^\dagger (\sigma \cdot G)q \rangle \) has large error \((-0.33\text{GeV}^2 \sim +0.66\text{GeV}^2) \cdot \rho_n \) with \( \rho_n \) being total nuclear medium density \([10]\). Therefore, we limit ourselves to the isospin-asymmetric operators up to dimension 4 in this paper.

In the rest frame of the medium with \( n^\mu = (1,0) \), the decompositions to even and odd parts of \( \text{Re}\Pi_T^{S,V} \) are written as

\[
\text{Re}\Pi_T^{S(E)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{1}{4} \delta m p^4 \log(-p^2) + \frac{(-4\pi^2)}{2} \langle \delta(qq) \rangle p^2 \log(-p^2) \right], \quad (4.11)
\]

\[
\text{Re}\Pi_T^{V(E)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{4\pi^2}{4} \langle \delta(q^\dagger q) \rangle p^2 \log(-p^2) \right], \quad (4.12)
\]

\[
\text{Re}\Pi_T^{S(O)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{4\pi^2}{2} \left\{ \delta m \left( \langle q\bar{q} \rangle - \langle s\bar{s} \rangle \right) - \bar{m} \langle \delta(q^\dagger q) \rangle \right\} \log(-p^2) \right], \quad (4.13)
\]

\[
\text{Re}\Pi_T^{V(O)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{4\pi^2}{2} \left\{ (m_s \langle \delta(qq) \rangle + \delta m \langle \bar{s}s \rangle + X) \log(-p^2) \right. \right.
\]

\[
\left. \left. \left. - \frac{2}{3} X \left( \frac{s}{p^2} \right) \right\} \right\}, \quad (4.14)
\]

with

\[
X \equiv \frac{1}{3} \left\{ \langle \delta(q\bar{i}\gamma q) \rangle - 4 \langle \delta(q\bar{i}D^\alpha\gamma^\alpha q) \rangle \right\}, \quad (4.15)
\]

and \( p^2 = s - p^2 \). In the above formulas, \( \sim \) denotes the \( u - d \) average, namely \( \bar{m} = (m_u + m_d) / 2 \), \( q\bar{q} = \left( u^\dagger u + d^\dagger d \right) / 2 \) and \( \bar{q}q = (\bar{u}u + \bar{d}d) / 2 \), while \( \delta \) denotes \( d - u \) difference, namely \( \delta m = m_d - m_u \), \( \delta(q\bar{q}) = \bar{d}d - \bar{u}u \) and \( \delta(q^\dagger q) = d^\dagger d - u^\dagger u \).

### 4.2 In-medium condensates

In the previous subsection, we have encountered various \( u - d \) symmetric and \( u - d \) anti-symmetric condensates. In this subsection, we will evaluate those in a model independent way using the low density expansion.

First of all, the expectation value of the local operator \( \mathcal{O} \) has a vacuum part which is density independent and the medium part which is density dependent;

\[
\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_0 + \langle \mathcal{O} \rangle_{\text{Med}}. \quad (4.16)
\]

At low density, \( \langle \mathcal{O} \rangle_{\text{Med}} \) is expanded as

\[
\langle \mathcal{O} \rangle_{\text{Med}} = \langle \mathcal{O} \rangle_p \rho_p + \langle \mathcal{O} \rangle_n \rho_n + \cdots, \quad (4.17)
\]

where \( \rho_p \) (\( \rho_n \)) is the proton (neutron) density, and \( \langle \mathcal{O} \rangle_p \) (\( \langle \mathcal{O} \rangle_n \)) is the spin-averaged expectation value taken by the one particle state of the proton (neutron).

\[
\langle \mathcal{O} \rangle_N = \int d^3x \left\{ \langle \mathcal{N} | \mathcal{O} | N \rangle - \langle 0 | \mathcal{O} | 0 \rangle \right\}, \quad (4.18)
\]
where \( \langle N | N \rangle = 1 \) for \( N = p, n \).

Since the vector condensate \( \langle q^\dagger q \rangle \) is nothing but the quark number density, we have
\[
\langle q^\dagger q \rangle = \frac{3}{2} \rho_N, \quad \langle \delta(q^\dagger q) \rangle = \delta \rho_N, \quad \langle s^\dagger s \rangle = 0, \quad (4.19)
\]
where \( \rho_N (\equiv \rho_p + \rho_n) \) is the total nucleon density, \( \delta \rho_N (\equiv \rho_n - \rho_p) \) is the \( n - p \) asymmetry.

The scalar condensate \( \langle \overline{q} q \rangle \) is evaluated by the Feynman-Hellmann theorem [23]:
\[
\left\langle \frac{\partial \mathcal{H}_{QCD}^\text{med}}{\partial \lambda} \right\rangle_{\text{med}} = \frac{\partial \mathcal{E}}{\partial \lambda}, \quad (4.20)
\]
where \( \lambda \) is a parameter in the QCD Hamiltonian, and \( \mathcal{E} \) is the energy density of the nuclear medium. At low density,
\[
\mathcal{E} \equiv \left\langle \mathcal{H}_{QCD}^\text{med} \right\rangle_{\text{med}} \simeq M_p \rho_p + M_n \rho_n = \tilde{M}_N \rho_N + \delta M_N \delta \rho_N/2 \quad (4.21)
\]
where \( \tilde{M}_N = (M_p + M_n)/2 \) and \( \delta M_N = M_n - M_p \). Under the choice \( \lambda = 2\tilde{m} \) together with the mass term of the QCD Hamiltonian
\[
\mathcal{H}_{QCD}^\text{mass} = \tilde{m} (uu + dd) + \frac{\delta m}{2} (dd - uu) + m_s ss, \quad (4.22)
\]
one finds from Eq.(4.20)
\[
\left\langle \overline{q} q \right\rangle_{\text{med}} \simeq \frac{\partial \tilde{M}_N}{2 \partial \tilde{m}} \rho_N = \frac{\sigma_N}{2 \tilde{m}} \rho_N, \quad (4.23)
\]
where we have used a definition of the nucleon sigma-term \( \sigma_N = \tilde{m} \frac{\partial \tilde{M}_N}{\partial \tilde{m}} \).

On the other hand, the choice \( \lambda = \delta m/2 \), with Eq.(4.20) gives
\[
\left\langle \delta(\overline{q} q) \right\rangle_{\text{med}} \simeq \frac{\partial (\delta M_N)}{\partial (\delta m)} \delta \rho_N, \quad (4.24)
\]
which is valid up to the 1st order in \( \delta m \).

The strange-quark condensate up to the 1st order in \( \rho_N \) reads
\[
\left\langle \overline{s}s \right\rangle_{\text{med}} \simeq \frac{1}{2} \left( \left\langle \overline{s}s \right\rangle_p + \left\langle \overline{s}s \right\rangle_n \right) \rho_N \equiv \frac{y}{2} \left( \left\langle \overline{q} q \right\rangle_p + \left\langle \overline{q} q \right\rangle_n \right) \rho_N. \quad (4.25)
\]
Here \( y \) is a parameter characterizing the OZI violation in the nucleon [3].

Thus the quark condensates \( \langle \overline{q} q \rangle \) in isospin-asymmetric nuclear medium are summarized as follows:
\[
\langle \overline{q} q \rangle \simeq \langle \overline{q} q \rangle_0 + \frac{\sigma_N}{2 \tilde{m}} \rho_N, \quad (4.26)
\]
\[
\langle \delta(\overline{q} q) \rangle \simeq \langle \delta(\overline{q} q) \rangle_0 + \frac{\partial (\delta M_N)}{\partial (\delta m)} \delta \rho_N, \quad (4.27)
\]
\[
\langle \overline{s}s \rangle \simeq \langle \overline{s}s \rangle_0 + y \frac{\sigma_N}{2 \tilde{m}} \rho_N. \quad (4.28)
\]
By using the result of evaluation of $\langle \delta(\bar{q} iD^\alpha \gamma^\nu q) \rangle$ in Appendix, we obtain

$$X \equiv \frac{1}{3} \{ \langle \delta(\bar{q} iD q) \rangle - 4 \langle \delta(\bar{q} iD^\alpha \gamma^\nu q) \rangle \} \simeq \frac{1}{2} \delta(A_2^0(\mu^2)) \left( \hat{M}_N \delta \rho_N + \delta M_N \rho_N / 2 \right),$$

where $A_2^0(\mu^2)$ is the 2nd moment of the parton distribution function of the proton defined by Eq.(A.4) in Appendix.

### 4.3 Summary of OPE for Re$\Pi_T$

In the isospin-asymmetric nuclear medium, the OPE of the correlation function is finally expressed as

$$\text{Re}\Pi_T^{S(E)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{1}{4} \delta m p^4 \log(-p^2) + \frac{-4\pi^2}{2} \langle \delta(\bar{q}q) \rangle p^2 \log(-p^2) \right],$$

$$\text{Re}\Pi_T^{V(E)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{(4\pi^2)}{4} \langle \delta(q^4) \rangle p^2 \log(-p^2) \right],$$

$$\text{Re}\Pi_T^{S(O)}(s, |p|) = O(\hat{m} \delta \rho_N, \delta m \rho_N),$$

$$\text{Re}\Pi_T^{V(O)}(s, |p|) = \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left[ \frac{(4\pi^2)}{2} \left\{ \left( m_s \langle \delta(\bar{q}q) \rangle + \delta m \langle \bar{s}s \rangle \right) \log(-p^2) \right. \\
+ \frac{1}{2} \delta(A_2^0(\mu^2)) \left( \hat{M}_N \delta \rho_N + \delta M_N \rho_N / 2 \right) \left( \log(-p^2) - \frac{2}{3} \frac{s}{p^2} \right) \left\} \right],$$

where $p^2 = s - p^2$.

Above formulas are valid up to the 1st order in the quark masses and the baryon density. In other words, the terms such as $O(\delta \hat{m})$ and $O(\delta \rho_N)$ are kept, while the terms such as $O(m_{u,d,s} \rho_N), O(m_{u,d,s} \delta \rho_N)$ are neglected. Within this approximation, the scalar-odd correlation $\Pi_T^{S(O)}$ vanishes as shown above and cannot be used to construct sum rules. Since it is theoretically consistent to use the correlation functions with the same reflection symmetry under $\omega \leftrightarrow -\omega$, we will exclusively use the “even” correlations $\Pi_T^{S(E)}$ and $\Pi_T^{V(E)}$ in the following analyses.

### 5 QCD SUM RULES

In this section, we construct finite energy sum rules (FESR) [13] and Borel sum rules (BSR) [1] on the basis of the retarded correlation function (4.1) and the dispersion relations (2.35) and (2.36). For the phenomenological side, we use the ansatz given in Eqs.(3.25) and (3.26). The OPE side is given in Eqs.(4.11-4.14).
5.1 Finite Energy Sum Rules (FESR)

In FESR, we identify the integral of Im$\Pi_R$ extracted from OPE with that introduced phenomenologically:

$$
\int_0^{S_{0}^{l(O)}} ds \left\{ \text{Im} \Pi_{\text{Phen}}^{l(O)} (s, |p|) \sqrt{s} \right\} s^n = \int_0^{S_{0}^{l(E)}} ds \left\{ \text{Im} \Pi_{\text{OPE}}^{l(E)} (s, |p|) \sqrt{s} \right\} s^n, \quad (5.1)
$$

$$
\int_0^{S_{0}^{l(E)}} ds \left\{ \text{Im} \Pi_{\text{Phen}}^{l(E)} (s, |p|) \sqrt{s} \right\} s^n = \int_0^{S_{0}^{l(\Sigma)}} ds \left\{ \text{Im} \Pi_{\text{OPE}}^{l(\Sigma)} (s, |p|) \sqrt{s} \right\} s^n, \quad (5.2)
$$

where $s = \omega^2$. $S_{0}^{l(O)}$ and $S_{0}^{l(E)}$ are the continuum thresholds of Im$\Pi_{\text{Phen}}^{l(O)}$ and Im$\Pi_{\text{Phen}}^{l(E)}$ respectively. These phenomenological spectral functions at $p = 0$ in the left hand side of the sum rule are obtained as follows. We substitute Eqs.(4.11-4.14) together with Eq.(2.32) into Eqs.(2.37) and (2.38) to obtain,

$$
\text{Im} \Pi_{\text{Phen}}^{S} (\omega) = \frac{\pi}{2} \lambda_{\Lambda} \lambda_{\Sigma^0} \left\{ (\theta^S + \theta^V) \left\{ \delta (\omega - M_{\Lambda}) - \delta (\omega - M_{\Sigma^0}) \right\} - (\theta^S - \theta^V) \left\{ \delta (\omega + M_{\Lambda}) - \delta (\omega + M_{\Sigma^0}) \right\} \right\}, \quad (5.3)
$$

$$
\text{Im} \Pi_{\text{Phen}}^{V} (\omega) = \frac{\pi}{2} \lambda_{\Lambda} \lambda_{\Sigma^0} \left\{ (\theta^S + \theta^V) \left\{ \delta (\omega - M_{\Lambda}) - \delta (\omega - M_{\Sigma^0}) \right\} + (\theta^S - \theta^V) \left\{ \delta (\omega + M_{\Lambda}) - \delta (\omega + M_{\Sigma^0}) \right\} \right\}. \quad (5.4)
$$

The even-odd decompositions of the above formula give Im$\Pi_{\text{Phen}}^{l(O)}$ and Im$\Pi_{\text{Phen}}^{l(E)}$. As we have discussed in Sec.3.2 and Sec.4.3, we consider only the first-order effect of the isospin-asymmetry on the mixed correlation function and neglect the effects of $O(m_u,d,s,\rho_N)$ and $O(m_u,d,s,\delta \rho_N)$. Therefore, the pole positions $M_{\Lambda}$ and $M_{\Sigma^0}$ take their vacuum value and only the pole residues are affected in a different way in even and odd spectral functions. The situation is the also same for excited states of $\Lambda$ and $\Sigma^0$. Therefore, in the present approximation, the continuum threshold also takes their vacuum value ($S_0 = S_{0}^{l(E)} = S_{0}^{l(O)}$) and only the height of the continuum is affected by the isospin asymmetry.

The OPE motivated spectral functions at $p = 0$ in the right hand side of the sum rule are obtained as follows. We substitute Eqs.(4.11-4.14) together with Eq.(2.32) into Eqs.(2.37) and (2.38) to obtain,

$$
\text{Im} \Pi_{\text{OPE}}^{S} (\omega) = -\pi \text{sgn}(\omega) \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left\{ \frac{1}{4} \delta m \omega^4 + \frac{(-4\pi^2)}{2} \langle \delta(q\bar{q}) \rangle \omega^2 \right\}
$$

$$
+ \frac{(4\pi^2)}{2} \left\{ \delta m \left( \langle q^2 \rangle - \langle s^4 \rangle \right) - \tilde{m} \langle \delta(q^4) \rangle \right\} \omega, \quad (5.5)
$$

$$
\text{Im} \Pi_{\text{OPE}}^{V} (\omega) = -\pi \text{sgn}(\omega) \left( \frac{1}{16\pi^4} \right) \frac{2}{\sqrt{3}} \left\{ \frac{(4\pi^2)}{4} \langle \delta(q^4) \rangle \omega^2 \right\}
$$

$$
+ \frac{(4\pi^2)}{2} \left\{ \left( m_s \langle \delta(q\bar{q}) \rangle + \delta m \langle \pi s \rangle \right) + \frac{1}{2} \delta(A_2^0(\mu^2)) \left\{ M_N \delta \rho_N + \delta M_N \rho_N / 2 \right\} \right\} \omega. \quad (5.6)
$$
Here, \( \text{sgn}(\omega) = \omega/|\omega| \) (\( \omega \neq 0 \)) and \( \text{sgn}(0) = 0 \). The even-odd decompositions of the above formula give \( \text{Im}\Pi_{\text{OPE}}^{(O)} \) and \( \text{Im}\Pi_{\text{OPE}}^{(E)} \). The isospin-asymmetric condensates affect the magnitude (height) of even and odd components in a different way.

As we have mentioned at the end of Sec.4, we use \( \text{Re}\Pi_{\text{OPE}}^{(E)} \) for the actual analysis, which corresponds to adopt the sum rule \( (5.1) \). Remember that the even part \( \text{Re}\Pi_{\text{OPE}}^{(E)} \) is related to the odd part \( \text{Im}\Pi_{\text{OPE}}^{(O)} \) through the dispersion relation \( (2.35) \). Resulting FESR for \( p = 0 \) reads

\[
\beta_{\lambda\Sigma^0} \left( M_{\Sigma^0}^{2n+1} - M_{\Lambda}^{2n+1} \right) \theta^S = \frac{2}{\sqrt{3}} \left[ \frac{1}{4} \delta m \frac{S_0^{n+3}}{n+3} + \frac{(-4\pi^2)}{2} \langle \delta(\bar{q}q) \rangle \frac{S_0^{n+2}}{n+2} \right], \tag{5.7}
\]

\[
\beta_{\lambda\Sigma^0} \left( M_{\Sigma^0}^{2n+1} - M_{\Lambda}^{2n+1} \right) \theta^V = \frac{2}{\sqrt{3}} \left[ \frac{(4\pi^2)}{4} \langle \delta(\bar{q}^i q) \rangle \frac{S_0^{n+2}}{n+2} \right]. \tag{5.8}
\]

where \( \beta_{\lambda\Sigma^0} = 16\pi^4\lambda_{\Lambda}\lambda_{\Sigma^0} \). As is evident from the right hand side of Eq.(5.8), \( \theta^V \) appears only in the nuclear medium, \( \theta^V = \theta^V_{\text{Med}} \). On the other hand, \( \theta^S \) has both vacuum part and in-medium part \( \theta^S = \theta^S_0 + \theta^S_{\text{Med}} \). We subtract out the vacuum part from the sum rule \( (5.7) \) to obtain

\[
\beta_{\lambda\Sigma^0} \left( M_{\Sigma^0}^{2n+1} - M_{\Lambda}^{2n+1} \right) \theta^S_{\text{Med}} = \frac{2}{\sqrt{3}} \left[ \frac{(-4\pi^2)}{2} \langle \delta(\bar{q}q) \rangle_{\text{Med}} \frac{S_0^{n+2}}{n+2} \right]. \tag{5.9}
\]

Combining Eqs.(5.8) and (5.9), one has a simple formula for the ratio \( \theta^V_{\text{Med}} / \theta^S_{\text{Med}} \):

\[
\frac{\theta^V_{\text{Med}}}{\theta^S_{\text{Med}}} = -\frac{1}{2} \langle \delta(\bar{q}^i q) \rangle_{\text{Med}} \sim -\left\{ 2 \frac{\partial(\delta M_N)}{\partial(\delta m)} \right\}^{-1}, \tag{5.10}
\]

where we have used Eqs.\((4.19)\) and \((4.24)\) for the last equality with \( \delta M_N = M_n - M_p \) and \( \delta m = m_d - m_u \).

Let us estimate the right hand side of Eq.(5.10). The \( n - p \) mass difference \( \delta M_N = 1.29 \text{ MeV} \) is known to be decomposed into two parts, \( \delta M_N \simeq \delta M_N^{\text{EM}} + \delta M_N^{\text{QCD}} \). Here \( \delta M_N^{\text{EM}} (\simeq -0.76 \text{ MeV}) \) originates from the electromagnetic interaction of \( O(\alpha) \), while \( \delta M_N^{\text{QCD}} (\simeq 2.04 \text{ MeV}) \) is due to the \( u - d \) quark mass difference of \( O(\delta m) \) \( [3] \). Therefore, in the leading order of \( \delta m \) and \( \alpha \), one finds

\[
\frac{\partial(\delta M_N)}{\partial(\delta m)} = \frac{\partial(\delta M_N^{\text{QCD}})}{\partial(\delta m)} = \frac{\delta M_N^{\text{QCD}}}{\delta m} = 0.52 \left( \frac{3.9 \text{ MeV}}{\delta m} \right), \tag{5.11}
\]

where we have used \( \delta m = 3.9 \text{ MeV} \) as a typical value at the renormalization \( \mu^2 = 1 \text{ GeV}^2 \) \( [3, 12] \) (See also Table I). Thus we find that the scalar and vector mixing angles induced by the nuclear medium have opposite sign and approximately equal in magnitude,

\[
\frac{\theta^V_{\text{Med}}}{\theta^S_{\text{Med}}} \sim -1. \tag{5.12}
\]
This together with the definition of the total mixing angles Eqs.\((3.13)\) and \((3.16)\)
implies that the medium modification of the particle mixing is largely cancelled between the scalar and vector, while the anti-particle mixing is enhanced in medium.
The magnitude of the mixing angles will be discussed in Sec.6.

5.2 Borel sum rules (BSR)

In BSR, we make a Borel transform of the dispersion relations \((2.35)\) and \((2.36)\) for the retarded correlation \((4.1)\) in the deep Euclidian region \(s = \omega^2 \rightarrow -\infty\):

\[
B \left[ \frac{1}{\pi} \int_0^\infty ds' \frac{\text{Im}\Pi^{(O)}_{\text{Phen}}(s', |p|) \sqrt{s'}}{s' - s} \right] = B \left[ \text{Re}\Pi^{(E)}_{\text{OPE}}(s, |p|) \right], \tag{5.13}
\]

\[
B \left[ \frac{1}{\pi} \int_0^\infty ds' \frac{\text{Im}\Pi^{(E)}_{\text{Phen}}(s', |p|) / \sqrt{s'}}{s' - s} \right] = B \left[ \text{Re}\Pi^{(O)}_{\text{OPE}}(s, |p|) \right], \tag{5.14}
\]

where the Borel transform \(B\) is defined as

\[
B[\Pi(s)] = \lim_{-s,n \rightarrow \infty} \left( \frac{-s}{n} \right)^n \frac{n!}{(n-1)!} \left( \frac{d}{ds} \right)^n \Pi(s), \tag{5.15}
\]

with \(M\) being the Borel mass.

The left hand side of the sum rule (the phenomenological side) is assumed to have the pole + continuum structure:

\[
\Pi^{(E,O)}_{\text{Phen}}(s, |p|) = \Pi_{\text{Phen(pole)}}^{(E,O)}(s, |p|) + \Pi_{\text{Phen(cont)}}^{(E,O)}(s, |p|), \tag{5.16}
\]

where the continuum part \(\Pi_{\text{Phen(cont)}}^{(E,O)}\) is extracted from \(\text{Im}\Pi^{(E)}_{\text{OPE}}\),

\[
\text{Im}\Pi^{(E,O)}_{\text{Phen(cont)}}(s, |p|) = \text{Im}\Pi^{(E,O)}_{\text{OPE}}(s, |p|) \theta \left( s - S^{(E,O)} \right), \tag{5.17}
\]

with \(\theta(x)\) being the step function. \(S^{(E)}_0\) and \(S^{(O)}_0\) are the continuum thresholds of \(\text{Im}\Pi^{(E)}_{\text{Phen}}\) and \(\text{Im}\Pi^{(O)}_{\text{Phen}}\) respectively. The pole part for \(p = 0\) has been already discussed in Eqs.(5.3) and (5.4). The right hand side of the sum rule (the OPE side) is derived from Eqs.(4.11-4.14). For the reason which we denoted in the previous subsection, we use universal threshold \(S_0 = S^{(E)}_0 = S^{(O)}_0\). However the height of the continuum is affected by the isospin-asymmetric nuclear medium, and \(\text{Im}\Pi^{(E,O)}_{\text{Phen(cont)}}(\omega)\) becomes asymmetric under \(\omega \leftrightarrow -\omega\). This is taken into account in Eq.(5.17) through the asymmetry of \(\text{Im}\Pi^{(E,O)}_{\text{OPE}}(\omega)\). Its explicit form is shown in Eqs.(5.5-5.6).

For the scalar mixing angle, resulting BSRs from Eq.(5.13) at \(p = 0\) become

\[
\beta_{\Lambda\Sigma^0} \left( M_{\Sigma^0} e^{-M_{\Sigma^0}^2/M^2} - M_{\Lambda} e^{-M_{\Lambda}^2/M^2} \right) \theta \left( s - S^{(E)}_0 \right) = f^{S}(M^2, S_0), \tag{5.18}
\]
with

\[ f^S(M^2, S_o) \equiv \frac{2}{\sqrt{3}} \left\{ \frac{1}{2} \left[ \delta m M^6 E_2 \left( \frac{S_o}{M^2} \right) + (-4\pi^2) \langle \delta(\bar{q}q) \rangle M^4 E_1 \left( \frac{S_o}{M^2} \right) \right] \right\}, \]

and for vector mixing angle

\[ \beta_{\Lambda \Sigma^0} \left( M_{\Sigma^0} e^{-M_{\Sigma^0}/M^2} - M_\Lambda e^{-M_\Lambda^2/M^2} \right) \theta^V = f^V(M^2, S_o), \quad (5.19) \]

with

\[ f^V(M^2, S_o) \equiv \frac{2}{\sqrt{3}} \left[ \frac{(4\pi^2)}{4} \langle \delta(q\bar{q}) \rangle M^4 E_1 \left( \frac{S_o}{M^2} \right) \right]. \]

Here \( E_n(x) = 1 - e^{-x} \sum^n_{r=0} x^r / r! \).

One can simplify the above sum rules without loss of generality by expanding the left hand side in terms of a small parameter \( \Delta M/M = (M_{\Sigma^0} - M_\Lambda)/M \sim (M_{\Sigma^0} - M_\Lambda)/(M_{\Sigma^0} + M_\Lambda)/2 \sim 77.0 \text{ MeV}/1.15 \text{ GeV} \sim 0.07 \). Using this expansion and neglecting \( O((\Delta M)^2) \) contribution, Eq.(5.18) becomes

\[ (M^2 - 2\tilde{M}^2) \theta^S_I = F^S(M^2, S_o), \quad (5.20) \]

where \( F^S(M^2, S_o) \equiv f^S(M^2, S_o) M^2 e^{S^2/M^2} / (\beta_{\Lambda \Sigma^0} \Delta M) \) and we put the suffix I to the mixing angle for later convenience. One may alternatively take derivative of Eq.(5.18) with respect to \( M^2 \) to enhance the lower dimensional operator in OPE and then expand the result by \( \Delta M/M \). Then we obtain the second sum rule for the mixing angle

\[ (3M^2 - 2\tilde{M}^2) \theta^S_I = \tilde{F}^S(M^2, S_o), \quad (5.21) \]

where \( \tilde{F}^S(M^2, S_o) \equiv (dM^2 f^S(M^2, S_o)/dM^2)(M^6/\tilde{M}^2) e^{S^2/M^2} / (\beta_{\Lambda \Sigma^0} \Delta M) \).

Extracting a term which is proportional to \( M^2 \) and a term which is \( M^2 \)-independent in Eqs.(5.20) and (5.21), one finally arrive at four types of sum rules, which we call Type Ia,Ib,IIa and IIb.

\[ \theta^S_{\text{Ia}} = \frac{d}{dM^2} F^S(M^2, S_o), \quad (5.22) \]

\[ \theta^S_{\text{Ib}} = \left\{ M^2 \frac{d}{dM^2} F^S(M^2, S_o) - F^S(M^2, S_o) \right\} / (2\tilde{M}^2), \quad (5.23) \]

\[ \theta^S_{\text{IIa}} = \frac{1}{3} \frac{d}{dM^2} \tilde{F}^S(M^2, S_o), \quad (5.24) \]

and

\[ \theta^S_{\text{Iib}} = \left\{ M^2 \frac{d}{dM^2} \tilde{F}^S(M^2, S_o) - \tilde{F}^S(M^2, S_o) \right\} / (2\tilde{M}^2). \quad (5.25) \]

The Borel sum rules for the mixing angle \( \theta^V \) can be formulated exactly in the same manner starting from Eq.(5.19). The results are obtained by replacing the suffix \( S \) by \( V \) in Eqs.(5.22), (5.23), (5.24) and (5.25).
6 NUMERICAL RESULTS

In this section, we evaluate the absolute value of the mixing angle with the use of the BSRs, Type Ia (5.22), Ib (5.23), IIA (5.24) and IIB (5.25), supplemented with the FESRs (5.7) and (1.8). To extract the mixing angles from the sum rules, we need to know various QCD parameters (vacuum condensates and quark masses), the coupling strength $|\beta_{\Lambda \Sigma^0}|$ and also the Borel window in which Borel analysis is made. They are determined by the following procedures.

1. QCD parameters and $\beta_{\Lambda \Sigma^0}$ in the vacuum
   In Table 1, the QCD parameters which we use in our analysis are summarized. These parameters reproduce the mass spectrum of octet baryons in QCD sum rules within 10% [12]. (OPE up to dimension 7 and quark masses up to the 2nd order have been taken into account in this analysis.) $\beta_{\Lambda \Sigma^0}(= 16\pi^4\lambda_{\Lambda}\lambda_{\Sigma^0})$ has been determined by the BSR in the vacuum for “diagonal” correlations ($A = B = \Lambda$ and $A = B = \Sigma^0$). Using the parameters in Table 1, we obtain $|\beta_{\Lambda \Sigma^0}| = 2.5 \text{ GeV}^6$ from the scalar-even sum rule [12]. (Note that the sum rules for the diagonal correlations provide only the absolute values of $\lambda_{\Lambda}$ and $\lambda_{\Sigma^0}$.)
   The optimum threshold $S_0$ turns out to be 3.2 GeV$^2$ from the Borel stability. This number is consistent with the position of the second resonances of $\Lambda$ and $\Sigma^0$.

2. Mixing angle in the vacuum
   The scalar-even BSR for $\Sigma^0 - \Lambda$ mixing angle in the vacuum $\theta_0$ (in which OPE up to dimension 7 and quark mass up to 2nd order have been taken into account) provides $|\theta_0| = 1.4 \times 10^{-3} \left(\frac{2.5 \text{ GeV}^6}{|\beta_{\Lambda \Sigma^0}|}\right)$ with the Borel window $1.4 \text{ GeV}^2 \leq M^2 \leq 2.6 \text{ GeV}^2$ and the threshold $S_0 \simeq 3.2 \text{ GeV}^2$ [12]. Zhu et al. [11] obtained $|\theta_0| \simeq 7 \times 10^{-3}$ for different QCD parameters with $|\beta_{\Lambda \Sigma^0}| = 1.76 \text{ GeV}^6$. We will discuss the effect of this difference to the in-medium mixing angle at the end of this section.

3. QCD parameters and Borel window for in-medium mixing angle
   The QCD parameter essential for obtaining the in-medium mixing angles is Eq.(5.11):
   \[ \frac{\partial(\delta M_N)}{\partial(\delta m)} = 0.52. \]
   In the standard Borel analysis, the Borel window is chosen such that the higher orders in OPE and the continuum contribution are well suppressed. The Borel window satisfying these conditions for the $\Sigma^0 - \Lambda$ mixing angle in the vacuum is $1.4 \text{ GeV}^2 \leq M^2 \leq 2.6 \text{ GeV}^2$ as shown above where OPE up to dimension 7
has been taken into account [12]. For the mixing angle in the medium, we have OPE only up to dimension 4 in which the medium effects appear only in the highest dimensional operators and are dominant. Therefore, it is difficult to find the Borel window and to reach similar level of Borel stability. This is shown in Fig.3 where in-medium mixing angle $\theta_{\text{IIa}}^{S}$ as a function of the Borel mass for different values of $S_0$ is plotted. Since the Borel curve is not enough stable in the medium, we simply adopt the Borel window determined in the vacuum and extract the in-medium mixing angle by making average over the Borel window.

$$\hat{m} = (m_u + m_d)/2$$

$$\delta m/\hat{m} = (m_d - m_u)/\hat{m}$$

$$m_s/\hat{m}$$

| $\langle \bar{q}q \rangle_0 = (\langle \bar{u}u + \bar{d}d \rangle_0/2)$ | $(-275 \text{ MeV})^3$ |
| $\gamma = (\langle \bar{d}d \rangle_0/\langle \bar{u}u \rangle_0 - 1)$ | $-5.6 \times 10^{-3}$ |
| $\beta = (\langle \bar{s}s \rangle_0/\langle \bar{u}u \rangle_0 - 1)$ | $-0.22$ |
| $m_0^2 = -g_\Lambda \langle (\sigma \cdot G)q \rangle_0/\langle \bar{q}q \rangle_0$ | $0.91 \text{ GeV}^2$ |

Table 1: QCD parameters in the vacuum at the renormalization scale 1 GeV$^2$. Those are determined to reproduce the octet baryon spectrum [12].

Fig.3: The scalar angle in the Type IIa sum rule is shown as a function of the Borel mass $M^2$ for different values of the threshold $S_0$ [GeV$^2$]. The straight horizontal lines imply the Borel window and the averaged value in the Window. $\beta_{\Lambda\Sigma^0} = 2.5 \text{ GeV}^6$ and $\delta m = 3.9 \text{ MeV}$ are used.
Adopting the procedure described above, we obtain the in-medium mixing angles $\theta^{S,V}$ as a function of the continuum threshold $S_0$ for four different types of BSR; Type Ia, Ib, IIa and IIb. They are shown in Fig. 4 and in Fig. 5 at nuclear saturation density $\rho_N = 0.16\text{fm}^{-3} \equiv \rho_0$ and at typical value of the isospin-asymmetry for heavy nuclei such as Pb, $\delta\rho_N/\rho_N = 0.21$.

Fig.4: The scalar angle is shown as a function of the continuum threshold $S_0$ [GeV$^2$]. The left panel is for BSR Type I and the right panel is for BSR Type II with corresponding $n$-th order FESR. $\beta_{\Lambda\Sigma^0} = 2.5\text{GeV}^6$ and $\delta m = 3.9\text{MeV}$ are used.

Fig.5: The vector angle is shown as a function of the continuum threshold $S_0$ [GeV$^2$]. The left panel is for BSR Type I and the right panel is for BSR Type II with corresponding $n$-th order FESR. $\beta_{\Lambda\Sigma^0} = 2.5\text{GeV}^6$ and $\delta m = 3.9\text{MeV}$ are used.

To reduce the uncertainties due to the absence of higher dimensional operators in OPE, we examine the reliability of each type of sum rules in the following ways. First of all, if the BSRs are consistent with each other, $\theta^{S,V}$ as a function of $S_0$ should
have a similar behavior between Type Ia and Type Ib and between Type IIa and Type IIb. Such comparison is shown in Fig.4 and Fig.5. The figures indicate that Type II sum rules are more reliable than Type I from this criterion.

One can make further selection of a reliable BSR by the comparison with FESR. Remember that the \( n \)-th order term in the \( 1/M^2 \)-expansion of the BSR is equivalent to the \( n \)-th order FESR. Therefore, if OPE is well behaved, BSR and corresponding FESR should give the same result. Such comparison is also shown in Fig.4 and Fig.5. From the right panels of Fig.4 and Fig.5, we conclude that the Type IIa is more reliable than Type IIb for reasonable range of the continuum threshold located around the second resonances of \( \Lambda \) and \( \Sigma^0 \), \( S_0 \simeq 3.2 \text{GeV}^2 \).

In Fig.6 we show the scalar angle \( \theta^S \) and the vector angle \( \theta^V \) in the Type IIa as a function of the continuum threshold \( S_0 \). The curves in the Fig.6 indicate the maximum (Max), the minimum (Min) and the average (Avg) value in the Borel window. Fig.6 shows the \( n-p \) asymmetry dependence of \( \theta^S \) and \( \theta^V \) for the total density \( \rho_N = 0.5 \rho_0 \), \( \rho_0 \) and 1.5 \( \rho_0 \).

Fig.6: The scalar and vector angles are shown as a function of the continuum threshold \( S_0 \, [\text{GeV}^2] \) for Type IIa. The curve with the label “Avg” is the average of the sum rule over the Borel window. The curve “Max” (“Min”) is the maximum (minimum) value of the sum rule in the Borel window.

Finally, by using the Type IIa sum rule, we obtain the scalar and vector mixing angles as follows;

\[
\theta^S_{\text{Med}} = [-(0.19 \pm 0.02) \alpha_{np} (\rho_N/\rho_0)] \left( \frac{2.5 \text{GeV}^6}{\beta_{\Lambda \Sigma^0}} \right) \left( \frac{3.9 \text{MeV}}{\delta m} \right), \tag{6.1}
\]

\[
\theta^V_{\text{Med}} = [(0.20 \pm 0.02) \alpha_{np} (\rho_N/\rho_0)] \left( \frac{2.5 \text{GeV}^6}{\beta_{\Lambda \Sigma^0}} \right), \tag{6.2}
\]
where the \( n-p \) asymmetry is defined as \( \alpha_{np} \equiv \delta \rho_N / \rho_N \). The error bars are determined by the minimum and maximum values of the sum rule for mixing angles in the Borel window. For typical values of the parameters, \( \delta m = 3.9 \text{ MeV} \) and \( |\beta_{\Lambda \Sigma^0}| = 2.5 \text{ GeV}^6 \), \( \rho_N = \rho_0 \) and \( \alpha_{np} = 0.21 \), \( \theta_{\text{med}}^S \) dominates over the vacuum mixing angle \( \theta_0 \). Also, the relation \( \theta_{\text{med}}^V / \theta_{\text{med}}^S \sim -1 \) discussed in Sec.2 is well satisfied.

![Diagram](image)

**Fig. 7**: The mixing angles (Type IIa) are shown as a function of the \( n-p \) asymmetry \( \alpha_{np} (= \delta \rho_N / \rho_N) \) for different values of the total density \( \rho_N \).

The particle and anti-particle mixing angles \( \theta \) and \( \bar{\theta} \) are obtained from Eq.(3.15) and (3.16) as

\[
\theta = \left[ (0.01 \pm 0.04) \alpha_{np} (\rho_N / \rho_0) \right] \text{sgn}(\beta_{\Lambda \Sigma^0}) + \theta_0, \quad (6.3)
\]

\[
\bar{\theta} = \left[ (-0.39 \pm 0.04) \alpha_{np} (\rho_N / \rho_0) \right] \text{sgn}(\beta_{\Lambda \Sigma^0}) + \theta_0, \quad (6.4)
\]

with \( \delta m = 3.9 \text{ MeV} \), \( |\beta_{\Lambda \Sigma^0}| = 2.5 \text{ GeV}^6 \).

In the analysis of the \( \Sigma^0 - \Lambda \) mixing in the vacuum by Zhu et al., they use different set of QCD parameters from Table 1 (in particular \( \delta m = 3.0 \text{ MeV} \) and \( \langle \overline{q}q \rangle_0 = (−241 \text{ MeV})^3 \)) and obtain \( |\beta_{\Lambda \Sigma^0}| = 1.76 \text{ GeV}^6 \) with the threshold \( S_0 = 3.4 \text{ GeV}^2 \) and the Borel window \( 1.3 \text{ GeV}^2 \leq M^2 \leq 2.5 \text{ GeV}^2 \). Substituting these values into Eqs.(3.1) and (3.2), one obtains

\[
\theta = \left[ (-0.12 \pm 0.08) \alpha_{np} (\rho_N / \rho_0) \right] \text{sgn}(\beta_{\Lambda \Sigma^0}) + \theta_0, \quad (6.5)
\]

\[
\bar{\theta} = \left[ (-0.75 \pm 0.09) \alpha_{np} (\rho_N / \rho_0) \right] \text{sgn}(\beta_{\Lambda \Sigma^0}) + \theta_0. \quad (6.6)
\]

which are qualitatively consistent with the result obtained using our parameter set.
7 CONCLUSIONS

In this paper, we have studied the $\Sigma^0 - \Lambda$ mixing angles in the isospin-asymmetric nuclear medium by using the QCD sum rules.

Firstly, we have discussed general properties of diagonal and off-diagonal correlation functions of the baryonic currents. We found that the off-diagonal (mixed) correlation function consists of scalar, vector and tensor terms. They are further decomposed into even and odd parts in terms of the reflection symmetry under $\omega \leftrightarrow -\omega$. Then we derived dispersion relations for each component.

Secondly, we examined the general structure of the mixing angle for baryons and introduced two independent mixing parameters $\theta^S$ and $\theta^V$ for the baryon at rest inside the medium. The sum (difference) of these parameters are shown to be the particle mixing angle $\theta$ (the anti-particles mixing angle $\overline{\theta}$). This situation is analogous to the self-energy of the nucleon and anti-nucleon in the relativistic mean-field theories.

Thirdly, we have carried out the OPE for the $\Sigma^0 - \Lambda$ mixed correlation function. Then we constructed sum rules for $\theta^S$ and $\theta^V$. From the finite energy sum rules, we found that $\theta^V_{\text{Med}}/\theta^S_{\text{Med}} \sim -1$. This implies that the particle mixing angle $\theta (= \theta^S + \theta^V)$ in the medium is nearly equal to the one in the vacuum, and the isospin-asymmetric medium affects mainly the anti-particle mixing $\overline{\theta} (= \theta^S - \theta^V)$. From the Borel sum rules, we evaluated the in-medium parts of $\theta$ and $\overline{\theta}$ numerically. The results are summarized in Eqs.(6.3-6.6) in Sec.6. As the baryon density and the isospin-asymmetry of the medium increase, the anti-particle mixing is enhanced, while the particle mixing remains less than 20% of the anti-particle mixing.

The strong correlation between $\theta^S_{\text{Med}}$ and $\theta^V_{\text{Med}}$ and the strong modification of the anti-particle mixing in the isospin-asymmetric medium shown in this paper are model independent consequence supported both by the finite energy sum rules and the Borel sum rules. On the other hand, the absolute magnitude of each mixing angle has uncertainties due to the absence of higher dimensional operators in OPE. Better evaluation of the matrix elements of isospin-asymmetric operators beyond dimension 4 is necessary for precise determination of the mixing angles. Also, it is an open but interesting problem to study whether one can measure the anti-particle mixing in nuclei in the laboratory experiments.

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APPENDIX

In this Appendix, we evaluate a dim.4 isospin anti-symmetric condensate $\langle \delta(qD^0\gamma^0q) \rangle$ following [17].

In the medium, the dim.4 quark condensate $\langle \bar{q}iD^0\gamma^0q \rangle$ is represented as

$$\langle \bar{q}iD^0\gamma^0q \rangle = \frac{1}{4} m_q \langle \bar{q}q \rangle + \langle \bar{q}i(D^0\gamma^0 - \not{D}/4)q \rangle.$$  \hspace{1cm} (A.1)

The vacuum part of the second term vanishes

$$\langle \bar{q}i(D^0\gamma^0 - \not{D}/4)q \rangle \approx \langle \bar{q}i(D^0\gamma^0 - \not{D}/4)q \rangle_p \rho_p + \langle \bar{q}i(D^0\gamma^0 - \not{D}/4)q \rangle_n \rho_n.$$  \hspace{1cm} (A.2)

The expectation value taken by the proton at rest is [14]

$$\langle \bar{q}i(D^0\gamma^0 - \not{D}/4)q \rangle_p = \frac{3}{8} M_p A_q^2(\mu^2).$$  \hspace{1cm} (A.3)

where $A_q^2(\mu^2)$ is the 2nd moment of the parton distribution function $q(x, \mu^2), \bar{q}(x, \mu^2)$ of the proton

$$A_q^2(\mu^2) = 2 \int_0^1 dx x^{n-1} \left\{ q(x, \mu^2) + (-1)^{n-1} \bar{q}(x, \mu^2) \right\}.$$  \hspace{1cm} (A.4)

For isospin anti-symmetric condensate $\langle \delta(qD^0\gamma^0q) \rangle$, we obtain

$$\langle \delta(qD^0\gamma^0 - \not{D}/4)q \rangle_p = \frac{3}{8} M_p \delta(A_q^2(\mu^2)).$$  \hspace{1cm} (A.5)

Then, the expectation value taken by the neutron at rest becomes

$$\langle \delta(qD^0\gamma^0 - \not{D}/4)q \rangle_n = -\frac{3}{8} M_n \delta(A_q^2(\mu^2))$$  \hspace{1cm} (A.6)

up to the 1st order in isospin-asymmetry. Thus we finally arrive at

$$\langle \delta(qD^0\gamma^0q) \rangle \simeq \frac{1}{4} \left\{ \hat{m} \langle \delta(qq) \rangle + \delta m \langle \bar{q}q \rangle \right\} - \frac{3}{8} \delta(A_q^2(\mu^2)) (\hat{M}_N \delta \rho_N + \delta M_N \rho_N/2).$$  \hspace{1cm} (A.7)

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