Residual Kondo effect in quantum dot coupled to half-metallic ferromagnets

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Abstract
We study the Kondo effect in a quantum dot coupled to half-metallic ferromagnetic electrodes in the regime of strong on-dot correlations. Using the equation of motion technique for non-equilibrium Green functions in the slave boson representation, we show that the Kondo effect is not completely suppressed for anti-parallel magnetization of the leads. In the parallel configuration, there is no Kondo effect but there is an effect associated with elastic co-tunnelling which, in turn, leads to similar behaviour of the local (on-dot) density of states (LDOS) as the usual Kondo effect. Namely, the LDOS shows the temperature-dependent resonance at the Fermi energy, which splits with the bias voltage and the magnetic field. Moreover, unlike for non-magnetic or not fully polarized ferromagnetic leads, only the minority spin electrons can form such resonance in the density of states. However, this resonance cannot be observed directly in the transport measurements, and we give some clues how to identify the effect in such systems.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Nowadays, spin-dependent phenomena play an important role in the mesoscopic systems as they lead to potential applications in nanotechnology (spintronics) [1, 2] and quantum computing [2, 3]. Moreover, new transport and thermodynamic phenomena can be observed in spintronic devices which are associated with the spin of the electron rather than the charge. These include tunnel magnetoresistance (TMR) in magnetic tunnel junctions [4], spin-dependent Andreev reflections [5], non-monotonic behaviour of the superconducting transition temperature [6] and spontaneous currents in ferromagnet–superconductor proximity systems [7] or the Kondo effect [8] in quantum dots (QD) coupled to the ferromagnetic leads [9–27].
The Kondo effect is a prime example of the many-body physics in quantum dot systems, i.e. the formation of the many-body singlet state by the on-dot spin and the conduction electron spins. This state gives rise to the resonance at the Fermi energy in the QD density of states and zero-bias maximum in differential conductance. The Kondo effect was predicted a long time ago [28–30], extensively studied theoretically [31–34] and confirmed in a series of beautiful experiments [35–39] on the QD coupled to normal (non-magnetic) leads.

If the normal leads are replaced by the ferromagnetic leads, the spin degrees of freedom start to play a significant role in the transport and thermodynamic properties of the system, eventually leading to new phenomena. One of these such new effects is the splitting of the Kondo resonance [12, 18–20, 25, 26] due to the spin-dependent quantum charge fluctuations induced by the tunnelling between the QD and spin-polarized leads. Remarkably, it is possible to recover the full Kondo effect (no splitting) by applying the external magnetic field [12, 19, 25, 26]. The splitting of the Kondo resonance strongly depends on the alignment as well as on the magnitude of the lead magnetizations. In particular, when the magnetizations in both leads point in opposite directions (anti-parallel alignment), the full equilibrium Kondo effect survives for all values of polarizations and there is no splitting of the zero-energy resonance. In the differential conductance, however, the zero-bias resonance gets smaller and smaller as the leads become more polarized, finally leading to complete disappearance of the Kondo anomaly. On the other hand, in parallel configuration (magnetizations in both leads are parallel to each other) the Kondo resonance is split and gets suppressed when the magnitude of the polarization is being increased.

The presence of ferromagnetism in the electrodes can also lead to the quantum critical point with non-Fermi liquid behaviour. It was shown recently [40] that the competition of spin waves (collective low-energy excitations in a ferromagnet) and the Kondo effect is responsible for such behaviour. In this case, the critical Kondo effect manifests itself in fractional power-law dependences of the conductance on temperature and of the ac conductance and thermal noise on frequency $\omega$. Thus the QD system with ferromagnetic electrodes can help us to understand quantum critical phenomena in heavy fermions and other correlated electron systems.

In the present paper, we show that if the leads are fully polarized, i.e. the density of states in the leads is non-zero for one electron spin direction only, there is the Kondo effect, provided that the leads are in an anti-parallel magnetization configuration. On the other hand, in the parallel configuration there is no usual Kondo effect but there is an effect that is associated with elastic co-tunnelling, which leads to a similar behaviour of the density of states as in the usual Kondo effect. Moreover, this effect occurs only in the minority spin electron channel when there is an unpaired spin on the dot and strong on-dot Coulomb interactions, i.e. when the QD is in the Coulomb blockade regime.

The paper is organized as follows. In section 2, a theoretical description of the QD coupled to the external leads is presented. Section 3 is devoted to various elastic and inelastic co-tunnelling processes in the case of half-metallic leads. In the rest of the paper, the numerical results concerning the density of states (section 4), applying the external magnetic field (section 5) and the transport properties (section 6), are discussed, and finally some conclusions are presented in section 7.

2. The model

Our system under consideration is represented by the singule-impurity Anderson model Hamiltonian in the limit of strong on-dot Coulomb interaction ($U \rightarrow \infty$) in the slave boson representation, where the real on-dot electron operator $d_\sigma$ is replaced by the product of the
Residual Kondo effect in quantum dot coupled to half-metallic ferromagnets 6925

... boson $b$ and the fermion $f$ operators ($d = b^+ f$) [41, 42]:

$$H = \sum_{\lambda k \sigma} \varepsilon_{\lambda k \sigma} c_{\lambda k \sigma}^+ c_{\lambda k \sigma} + \sum_{\sigma} \tilde{\varepsilon}_{\sigma} f_{\sigma}^+ f_{\sigma} + \sum_{\lambda k} (V_{\lambda k} c_{\lambda k \sigma}^+ b^+ f_{\sigma} + H.c.), \tag{1}$$

where $c_{\lambda k \sigma}$ stands for electrons with the single particle energy $\varepsilon_{\lambda k \sigma}$, the wavevector is $k$, and the spin $\sigma$ in the lead $\lambda = L, R$. $\varepsilon_{\sigma}$ denotes the dot energy level and $V_{\lambda k}$ is the hybridization matrix element between the electrons on the dot and those in the leads.

Within the Keldysh formalism [43], the total current $I = \sum_{\sigma} I_{\sigma}$ flowing through the quantum dot is given in the form:

$$I = \frac{e}{\hbar} \sum_{\sigma} \int d\omega \frac{\Gamma_{L\sigma}(\omega)\Gamma_{R\sigma}(\omega)}{\Gamma_{L\sigma}(\omega) + \Gamma_{R\sigma}(\omega)} \left[ f_L(\omega) - f_R(\omega) \right] \rho_{\sigma}(\omega), \tag{2}$$

where we have introduced the elastic rate $\Gamma_{\lambda\sigma}(\omega) = \sum_{k} |V_{\lambda k}|^2 \delta(\omega - \varepsilon_{\lambda k \sigma})$, and $\rho_{\sigma}(\omega)$ is the spectral function of the dot retarded Green’s function (GF) $G_{\sigma}^r(\omega)$, calculated within the equation of motion technique (EOM) in the slave boson representation [34, 44].

As is well known, the EOM technique is reliable in the high-temperature regime, however it also qualitatively captures the Kondo physics [34]. Moreover, the EOM is one of very few techniques that allows us to study non-equilibrium properties of the spin-polarized QD system.

Within this approach, the dot retarded Green’s function reads:

$$G_{\sigma}^r(\omega) = \frac{1}{\omega - \varepsilon_{\sigma} - \Sigma_{0\sigma}(\omega) - \Sigma_{f\sigma}(\omega) + i0^+}, \tag{3}$$

with non-interacting ($U = 0$) 

$$\Sigma_{0\sigma}(\omega) = \sum_{\lambda k} \frac{|V_{\lambda k}|^2}{\omega - \varepsilon_{\lambda k \sigma}}, \tag{4}$$

and interacting self-energy

$$\Sigma_{f\sigma}(\omega) = \sum_{\lambda k} \frac{|V_{\lambda k}|^2 f_{\lambda k \sigma} (\varepsilon_{\lambda k \sigma})}{\omega - \varepsilon_{\lambda k \sigma} - \varepsilon_{\sigma} + \varepsilon_{\sigma}}, \tag{5}$$

which is responsible for the generation of the Kondo effect. $\langle n_{\sigma} \rangle$ is the average occupation on the QD, calculated under non-equilibrium within the standard scheme [34, 44].

To get the splitting of the Kondo resonance in the presence of the ferromagnetic leads, we follow reference [12] and replace $\varepsilon_{\sigma}$ on the right-hand side of equation (5) by $\tilde{\varepsilon}_{\sigma}$, which is found from the self-consistency relation

$$\tilde{\varepsilon}_{\sigma} = \varepsilon_{\sigma} + \text{Re} \left[ \Sigma_{0\sigma}(\tilde{\varepsilon}_{\sigma}) + \Sigma_{f\sigma}(\tilde{\varepsilon}_{\sigma}) \right]. \tag{6}$$

In numerical calculations, we have chosen $\Gamma = \sum_{\lambda, \sigma} \Gamma_{\lambda\sigma} = 1$ as an energy unit. The magnetization in the lead $\lambda$ is defined as $p_{\lambda} = \frac{\Gamma_{\lambda\uparrow} - \Gamma_{\lambda\downarrow}}{\Gamma_{\lambda\uparrow} + \Gamma_{\lambda\downarrow}}$. For half-metallic leads (HM) we have $\Gamma_{L\uparrow} = \Gamma_{R\uparrow} = 0.5$ and $\Gamma_{L\downarrow} = \Gamma_{R\downarrow} = 0$ in the parallel configuration, while $\Gamma_{L\uparrow} = \Gamma_{R\downarrow} = 0.5$ and $\Gamma_{L\downarrow} = \Gamma_{R\uparrow} = 0$ in the anti-parallel configuration.

3. Tunnelling processes

Before the presentation of the numerical results, let us discuss various tunnelling processes, associated with the second generation of Green functions obtained in the EOM procedure. They are the second-order processes in the hybridization and describe elastic and inelastic co-tunnelling. The co-tunnelling is a process which leaves the charge on the dot unchanged. Moreover, elastic co-tunnelling does not change spin on the dot either, thus it leaves the dot in its ground state. On the other hand, the inelastic co-tunnelling changes the ground state [45].
Figure 1. Various tunnelling processes associated with second-generation Green’s functions in the present EOM approach. For simplicity, we have shown only processes starting from the left electrode. The remaining ones can be obtained by replacing $L \rightarrow R$ and $\sigma \rightarrow -\sigma$ (see discussion in the text). Left panels show single-barrier co-tunnelling and right panels show double-barrier co-tunnelling events. Panels (a) and (b) show usual inelastic co-tunnelling, leading to the Kondo effect, changing the spin on the dot and in either one or both of the electrodes. (c) and (d) are similar to (a) and (b), with additional annihilation and creation of the spin on the dot. All the above processes change the spin on the dot and in the lead(s). The processes displayed in panels (e) and (f) lead to the same final spin state on the dot (elastic co-tunnelling). In general, the only processes contributing to the current across the dot are associated with double-barrier co-tunnelling, displayed in panels (b), (d) and (f). In the case of half-metallic ferromagnetic leads in the anti-parallel configuration, the allowed processes are those in (b), (d) and (e), but they do not contribute to the current, while in the parallel configuration they are those displayed in (e) and (f) with only (f) giving a contribution to the current.

As is well known, the EOM approach is a non-perturbative technique, and one cannot assume that those processes are fully taken into account. Rather, they are incorporated in the calculations only qualitatively. It should be stressed that they are likely to be not the only processes, as one may get other processes of the same order or contributions to those discussed here, while going further in the EOM procedure, i.e. calculating higher-generation GFs. Unfortunately, an exact solution of the model is not known and, for the present purposes, it is enough to consider those shown in figure 1.

In figure 1 we show such elastic and inelastic single-barrier (left panel) and double-barrier (right panel) co-tunnelling processes, which start from the spin-up electron in the lead L and the spin-down electron on the dot (Coulomb blockade regime). There are also processes which start from lead R and can be obtained by replacing $L \rightarrow R$. In general, for leads that are not fully polarized, there are processes for opposite spins ($\sigma \rightarrow -\sigma$) but, in the case of half-metallic ferromagnetic leads in the parallel configuration, they are not allowed due to the lack...
of the density of states for minority spin electrons in the leads. We do not show the processes with the same spins on the dot and in the lead, as well as those with an empty dot state.

Panels (a) and (b) show the processes in which the electron with spin-up tunnels from the lead L into the dot and the electron with spin-down tunnels from the dot into the leads L (a) or R (b), describing inelastic co-tunnelling and leading to the usual Kondo effect, as the spin on the dot is changed. A similar situation is displayed in (c) and (d), namely, the dot starts with spin-down and ends with spin-up. However, during this tunnelling event, additionally, the spin-up electron on the dot is annihilated and created. These are renormalized inelastic co-tunnelling processes. In all the above processes, the spin on the dot and in the lead(s) is flipped, so these are the Kondo-related processes. There are two more processes, shown in (e) and (f), in which there is no spin flip. The process starts with spin-up in the lead L and ends with the same spin in leads L (e) or R (f). Similarly, the initial and the final spin states on the dot remains the same. Thus they also describe renormalized but elastic co-tunnelling, as the ground state remains unchanged. Both processes lead, as we shall see later on, to similar main features of the dot density of states as in the usual Kondo effect.

As one can read off from figure 1, all the processes are allowed only if the leads are not polarized (paramagnetic) or not fully polarized. Moreover, the only processes contributing to the current across the dot are double-barrier co-tunnelling processes, shown in (b), (d) and (f).

In the case of the half-metallic ferromagnetic leads in anti-parallel (AP) configuration, the allowed processes are those shown in (b), (d) and (e). However, they do not give any contribution to the current. In fact, processes (b) and (d), as they describe double-barrier co-tunnelling, allow for the tunnelling, but once the electron with spin-down tunnels off the dot into the lead R, the spin-up electron can tunnel from the lead L into the dot and further tunnelling is blocked. The only possibility is the opposite process, namely, the spin-up electron tunnels from the dot into the lead L and the spin-down electron from the lead R tunnels into the dot. Thus, in this case the electron transport is completely blocked. The process shown in (e) also does not contribute to the current, as in this case the electron starts and ends in the same lead (single-barrier co-tunnelling). As a result, in the case of the anti-parallel (AP) configuration, there is no current through the quantum dot.

On the other hand, in the case of the half-metallic leads in the parallel (P) configuration, there is one process giving a contribution to the current. This is the process shown in figure 1(f), coming from the Coulomb interaction. In this case, the process starts with spin-up in the lead L and ends with the same spin in the lead R, thus not changing the ground state (double-barrier elastic co-tunnelling). Note that the inelastic co-tunnelling is not allowed in this case. This process at zero temperature has a finite probability in the Coulomb blockade only, i.e. when the dot energy level is below the Fermi energy of the electrodes, and there is strong on-dot Coulomb interaction. Moreover, this process is allowed only when the dot is occupied by a spin-down electron. It gives a non-zero contribution to the dot density of states below the Fermi level (shifted by $\varepsilon_1 - \varepsilon_\downarrow$ in the parallel configuration) only (see figure 2 (bottom panel) and figure 4 (top panel)). It is shown in the next section that this process leads to similar main behaviour of the density of states of the dot as the usual Kondo effect.

4. Density of states

In figure 2 we show the spin-resolved non-equilibrium ($\mu_R = -\mu_L = 0.2$) density of states (DOS) of the quantum dot coupled to the external leads. Top panel shows the usual DOS of the QD with non-magnetic (NM) leads where two Abrikosov–Suhl resonances located at the chemical potentials of the leads can be observed. The middle panel displays the DOS of the QD coupled to the half-metallic electrodes in AP configuration ($p_L = -p_R = 1$). It is worthwhile
Figure 2. Non-equilibrium ($\mu_R = -\mu_L = 0.2$) density of states of the quantum dot coupled to non-magnetic (top panel), half-metallic leads in anti-parallel configuration (middle panel), and in parallel configuration (bottom panel). The solid (dashed) line shows the spin-up (spin-down) electron DOS. The model parameters are: $\varepsilon_\uparrow = \varepsilon_\downarrow = -2$, $T = 10^{-3}$. All energies are measured in units of $1/\Gamma_1$.

noting that we now have spin-up Kondo resonance at $\omega = \mu_R$ (solid line) and no resonance at $\omega = \mu_L$. In this case, the spin-up on the dot is screened by the spins-down in the lead R ($\Gamma_{R\downarrow} \neq 0$). For spin-down electrons, the situation is the opposite; the spin-down on the dot is screened by the spins-up in the lead L ($\Gamma_{L\uparrow} \neq 0$) and therefore there is a resonance for $\omega = \mu_L$ and a lack of it for $\omega = \mu_R$. This is different from general case of $p < 1$, where two resonances at both chemical potentials are present (similarly to the non-magnetic case) [12].

The situation is quite different in the parallel (P) configuration ($p_L = p_R = 1$) where the spin-up DOS (the solid line in the bottom panel of figure 2) shows no signatures of the Kondo effect as the spin-up on the dot cannot be screened by spins-down in either lead ($\Gamma_{L\downarrow} = \Gamma_{R\downarrow} = 0$). However, in the spin-down channel the residual Kondo-like state can be produced due to the processes shown in figures 1(e) and (f). This manifests itself in two resonances in the DOS located at $\omega = \Delta\varepsilon + \mu_L$ (see the dashed line in the bottom panel of figure 2), where $\Delta\varepsilon = \varepsilon_\uparrow - \varepsilon_\downarrow$ is the splitting due to the ferromagnetic leads. Such a splitting has been also observed in the general case of FM ($p < 1$) leads, where the charge fluctuations play a significant role, i.e. when $2|\varepsilon_d| \neq U$ [12, 19, 20]. Note that, in the AP configuration, there is no such splitting, again, in agreement with the general case of $p < 1$ polarization [12, 19, 20].
Another important effect is a cutoff of one of the resonances at the energy \( \omega = \Delta \varepsilon + \mu_R \). This indicates that there is no direct tunnelling of the spin-down electrons and the resulting density of states comes form the virtual processes shown in figures 1(e) and (f) only, as discussed in section 3.

Figure 3 shows the low-temperature dependence of the full width at half maximum (FWHM) of the narrow (Kondo) resonance in the density of states near \( \omega_{K} \), which equals the Fermi energy \( \mu_L = \mu_R = 0 \) when QD is coupled to the non-magnetic or half-metallic leads in the AP configuration and \( \omega_{K} = \Delta \varepsilon \) for half-metallic leads in the P configuration. The points with error bars show numerically found values of the FWHM. The Kondo temperatures in all cases have been found by fitting the function \( \text{FWHM} = \sqrt{T_K^2 + aT^2} \) to those points. In the NM case it gives \( T_K = 1.36 \times 10^{-3} \), while in the AP configuration, \( T_K = 1.07 \times 10^{-3} \). The lower \( T_K \) in the AP configuration stems from the fact that the Kondo state in this case is formed by the electron spins in one lead only for a given direction of the spin on the dot. In the P configuration, \( T_K \) is equal to zero, as in this case there is no Kondo effect. Moreover, in the P configuration the FWHM significantly deviates from the linear behaviour for higher temperatures. On the other hand, in the NM and AP configurations it is still linear above \( T = 10^{-2} \) (not shown in figure 3).

5. Compensation by the external magnetic field

Now, the question arises if the residual Kondo-like effect in the P configuration can be compensated by the external magnetic field \( B \). Compensation in this case means no splitting of the dot energy levels, \( \Delta \varepsilon = 0 \). In figure 4 we show the equilibrium \( (\mu_L = \mu_R = 0) \) spin-down (top panel) and spin-up (bottom panel) density of states for different values of field \( B \). As is evident, the magnetic field shifts the spin-down Kondo resonance, and at \( B = B_{\text{comp}} = 0.385 \) it reaches the Fermi energy. No splitting, \( \Delta \varepsilon \), is observed in this case. Note that the Kondo
Figure 4. Equilibrium ($\mu_L = \mu_R = 0$) spin-down (top panel) and spin-up (bottom panel) density of states for various values of the external magnetic field indicated in the figure. At $B = B_{\text{comp}} = 0.385$ there is no splitting of the dot energy level, thus the Kondo effect is compensated. Note the strong suppression of the spin-up density of states for $B \geq B_{\text{comp}}$.

resonance rapidly grows as the $B$ field is increased, while the broad resonance around the dot energy level initially also grows, but for $B > B_{\text{comp}}$ it remains almost unchanged (only its position changes). The spin-up density of states (bottom panel) shows different behaviour. Namely, the broad charge fluctuation resonance starts to decrease with an increase in the $B$ field, and at $B = B_{\text{comp}}$ it is remarkably small. This effect is associated with the change of the average occupation for different spin directions on the dot ($\langle n_\sigma \rangle$). When the $B$ field increases, $\varepsilon_\uparrow$ moves towards the Fermi energy while $\varepsilon_\downarrow$ moves in the opposite direction and therefore $\langle n_\uparrow \rangle$ decreases while $\langle n_\downarrow \rangle$ increases its value. This is clearly seen in figure 5, where the occupations of the dot for both spin directions are shown. As one can see, the spin-up (down) occupation number decreases (increases) with an increase in the external magnetic field $B$. As soon as the $B$ field exceeds $B_{\text{comp}} = 0.385$, one can think in this case about the overcompensated residual Kondo-like effect, and both occupation numbers remain almost constant, strictly speaking; they change very slowly towards 1 in the spin-down channel (fully occupied state) and 0 in the spin-up channel (empty state).

The corresponding non-equilibrium density of states for different magnetic fields is shown in figure 6. While the behaviour is similar to that shown in figure 3, except the fact that there are two resonances now, one can see a larger spin-up density of states at $B = B_{\text{comp}}$. This is simply due to the smaller value of the spin-down electron occupation number.

As we have seen, the residual Kondo-like effect can be compensated by the external magnetic field. However, the compensation in this case means something different than in the general case of $p < 1$. First of all, we have to remember that there is only one resonance (for one spin direction only) in the DOS, which can be shifted to the Fermi energy by the external magnetic field. This is what we call the compensation. In the general case of $p < 1$, there are two resonances (for both spin directions) in the DOS, which can be moved
Figure 5. Evolution of the average spin-dependent occupation number with the external magnetic field $B$. Note that, with an increase in the $B$ field, the spin polarization (the difference between the occupation of the spin-up and the spin-down electrons) also increases and remains almost constant (changes very slowly) for $B > B_{\text{comp}} = 0.385$.

Figure 6. Non-equilibrium ($\mu_L = -\mu_R = -0.2$) spin-down (top panel) and spin-up (bottom panel) density of states for the same values of the external magnetic field as in figure 3.

to the Fermi energy. Moreover, as has been shown [12, 19], the occupations for both spin directions become equal at $B = B_{\text{comp}}$—there is no spin polarization for such an external magnetic field. In the case of HM ($p = 1$) leads, there is non-zero spin polarization,
\[ \langle n_t \rangle \neq \langle n_\uparrow \rangle \] (see figure 5). Consequently, the spin on the dot cannot be fully screened by the spins in the leads, and there is no Kondo effect in the common sense. This is the main and important difference between the compensation effect in the case of HM and FM \((p < 1)\) leads.

Finally, we would like to comment on the compensation of the Kondo effect in the general case of \(p < 1\) within the present approach and compare it to the other works known in the literature \([12, 19, 20]\). The results obtained and the conclusions are qualitatively the same as in the papers mentioned above. However, the values of \(B_{\text{comp}}\) in the present work differ from those in \([12]\) due to the fact that we have assumed spin-dependent bandwidths in electrodes in order to have the densities of states in the leads normalized to 1. For this reason, our approach gives values of \(B_{\text{comp}}\) smaller than those of \([12]\), also obtained within the EOM technique. Without this normalization requirement, we get perfect quantitative agreement with that approach.

All this shows clear evidence of the usual Kondo effect in the DOS of the quantum dot coupled to the half-metallic ferromagnets in the AP configuration and a similar co-tunnelling related effect in P configurations. In both cases, the density of states shows the splitting of the zero-energy resonance caused by the non-equilibrium conditions \((\mu_L \neq \mu_R)\) or the exchange field coming from the electrodes in the P configuration and finally the compensation of it by the external magnetic field. Unfortunately, it is not possible to measure directly the density of states in transport experiments and the problem arises of how to confirm this effect experimentally. In the AP configuration there is Kondo effect for both spin directions, but the tunneling current is zero in both cases due to the product of \(\Gamma_{1,\sigma}(\omega)\Gamma_{R,\sigma}(\omega)\) (see equation (2)), which vanishes (see also the discussion in the previous section). On the other hand, in the P configuration the co-tunnelling related effect is present for minority spin electrons, but there is no density of states in the electrodes for this spin direction. Hopefully, in this case the presence of the effect in the minority electron channel also modifies the transport properties in the other channel, therefore in general it is possible to get some information on this effect.

### 6. Transport properties

Figure 7 shows the temperature dependence of the linear conductance \(G = \frac{dI}{dV} \bigg|_{V \to 0}\) for different values of the external magnetic field \(B\). At zero magnetic field, the conductance of a QD coupled to the half-metallic leads is almost constant at low temperatures (panel (a)), similarly as for a QD with ferromagnetic leads where the polarization is \(p = 0.8\) (panel (b)). When the \(B\) field increases, \(G\) for HM starts to decrease, unlike for the FM system, where it grows, and finally the Kondo effect is fully compensated at \(B_{\text{comp}} = 0.165\). In the HM system at \(B = B_{\text{comp}} = 0.385\) (note that the values of \(B_{\text{comp}}\) are different in HM and FM systems, simply due to the different lead polarizations), the conductance decreases with \(T\) and at low temperatures is an order of magnitude smaller than at \(B = 0\). The decrease of \(G\) is related to the suppression of the DOS in the majority spin channel, as can be read off from figure 4. Such behaviour of the conductance remains in agreement with the results obtained within the numerical renormalization group technique (see figure 4(b)) of \([19]\), where \(G\) is plotted as a function of the polarization for \(B = B_{\text{comp}}\). For almost all values of the polarizations \(p\), the conductance is equal for both spin directions, except for \(p\) close to 1, where the \(G\) becomes spin polarized. In our case of \(p = 1\), the conductance is fully polarized, as only one spin channel contributes to the transport.

We have also calculated \(G\) in the HM (panel (c)) and FM (d) systems without elastic co-tunnelling and Kondo-like correlations taken into account, neglecting interacting self-energy \(\Sigma_{1,\sigma}(\omega)\) in equation (3), which corresponds to the skipping of all tunnelling processes shown in
Figure 7. The linear conductance as a function of temperature for the quantum dot coupled to the half-metallic (a) and ferromagnetic leads (b) with polarization $p = 0.8$. Note the different behaviour with respect to the external magnetic field. For ferromagnetic leads (b), it is possible to get the usual Kondo effect by tuning the magnetic field. For half-metallic leads, applying the magnetic field $B = B_{\text{comp}}$ suppresses the conductance. Panels (c) and (d) show the same as (a) and (b), respectively, neglecting co-tunnelling-like correlations.

It is clearly seen that the whole low-temperature contribution to the conductance of a QD with HM leads is due to the renormalized elastic co-tunnelling processes and therefore we can conclude that non-zero $G$ is a signature of this effect. The conductance of a QD coupled to FM leads without co-tunnelling correlations (panel (d)) shows similar behaviour to $G$ for HM
leads with those processes (panel (a)) at $B = 0$. However, a lack of $B$ dependence in this case can easily distinguish it from the general case of a QD coupled to HM leads. Therefore, such spectacular behaviour of the conductance of the QD with HM leads can be, in general, possible to observe in transport measurements.

Additional insight into the problem can be achieved from the behaviour of the differential conductance versus the bias voltage $eV = \mu_L - \mu_R$, displayed in figure 8. As one can see in the figure (panel (a)), there are temperature-dependent small kinks at $eV = \pm \Delta_1 \varepsilon$. For comparison, the differential conductance of a QD coupled to FM electrodes with $p = 0.8$ is also shown (panel (b)). As the temperature grows, those kinks become suppressed. This is an additional clue which can be verified experimentally. We have observed no such kinks, either in HM nor in FM systems without co-tunnelling-like correlations being taken into account.

7. Conclusions

In conclusion, we have studied the properties of a quantum dot coupled to half-metallic leads. In the case of a parallel configuration, the effect associated with elastic co-tunnelling, which leads to similar behaviour of the density of states, can be observed. The density of states shows the splitting of the zero-energy resonance caused by the non-equilibrium conditions ($\mu_L \neq \mu_R$) or the exchange field coming from the electrodes, and finally its compensation by the external magnetic field. However, the compensation means the shift of the resonance (in the DOS) to the Fermi energy only, without additional conditions of equal occupations and equal conductances for both spin directions, as in the case of $p < 1$. This effect can be observed experimentally by measuring the temperature dependence of the linear and differential conductances in the external magnetic field. On the other hand, in the case of the AP configuration, the DOS shows the usual Kondo effect but the transport is completely suppressed.
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