Dark Matter, Muon g-2 and Other SUSY Constraints

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Abstract. Recent developments constraining the SUSY parameter space are reviewed within the framework of SUGRA GUT models. The WMAP data is seen to reduce the error in the density of cold dark matter by about a factor of four, implying that the lightest stau is only 5-10 GeV heavier than the lightest neutralino when $m_0, m_{1/2} < 1$ TeV. The CMD-2 re-analysis of their data has reduced the disagreement between the Standard Model prediction and the Brookhaven measurement of the muon magnetic moment to 1.9 $\sigma$, while using the tau decay data plus CVC, the disagreement is 0.7 $\sigma$. (However, the two sets of data remain inconsistent at the 2.9 $\sigma$ level.) The recent Belle and BABAR measurements of the $B \to \phi K$ CP violating parameters and branching ratios are discussed. They are analyzed theoretically within the BBNS improved factorization method. The CP parameters are in disagreement with the Standard Model at the 2.7 $\sigma$ level, and the branching ratios are low by a factor of two or more over most of the parameter space. It is shown that both anomalies can naturally be accounted for by adding a non-universal cubic soft breaking term at $M_G$ mixing the second and third generations.

1 Introduction

While SUSY particles are yet to be discovered, a wide range of data has begun to limit the allowed SUSY parameter space. We review here what has happened over the past year to further restrict SUSY models of particle physics. A number of new experimental and theoretical analyses have occurred:

- The current experiments that most strongly restrict the SUSY parameter space are the following:
  - WMAP data has greatly constrained the basic cosmological parameters
  - While the analysis of $\mu^-$ data for the muon anomalous magnetic moment has not yet been completed, there has been further experimental results and theoretical analysis that have modified the theoretical Standard Model (SM) prediction of $g_\mu - 2$.
  - The B-factories, BABAR and Belle measurements of the CP violating B decays, particularly $B^0 \to J/\psi K_s$ and $B^0 \to \phi K_s$, $B^\pm \to \phi K^\pm$ impose new constraints on any new theory of CP violation when they are combined with theoretical advances that have occurred in calculating these decays.

In addition one must continue to impose the previously known constraints on the SUSY parameter space. The most important of these are:
• The light Higgs mass bound \( m_h > 114.1 \) GeV \[1\]
• The \( b \to s\gamma \) branching ratio constraint \[2\]
• The light chargino mass bound \( m_{\tilde{\chi}^\pm_1} > 103 \) GeV \[3\]
• The electron and neutron electric dipole moments bounds \[4\]

In order to analyze these phenomena it is necessary to chose the SUSY model. At one extreme one has the MSSM with over 100 free parameters (with 43 CP violating phases). At the other one has mSUGRA with four parameters and one sign (often augmented with two or four additional CP violating phases). The large number of free parameters in the MSSM generally allows one to fit each experimental constraint separately by tuning one or another parameter. For mSUGRA, where the parameters are specified at the GUT scale, \( M_G \approx 2 \times 10^{16} \) GeV, the situation is far more constrained. One can ask here if all the data taken together is still consistent with mSUGRA (or indeed with the Standard Model). The new B-factory data appears to be putting the greatest strain on mSUGRA, and if the current data is confirmed, it may be that one is seeing a breakdown of mSUGRA (and also the SM) for the first time and one may need a modification of the universal soft breaking assumptions of mSUGRA.

2 mSUGRA Model

We begin by reviewing the status of mSUGRA models with R-parity invariance as of a year ago. Recall that mSUGRA depends on the following parameters: \( m_0 \), the universal soft breaking mass at \( M_G \); \( m_{1/2} \), the universal gaugino mass at \( M_G \); \( \tan \beta = \langle H_2 \rangle / \langle H_1 \rangle \) where \( \langle H_{1,2} \rangle \) gives rise to \( (d, u) \) quark masses. In addition, the model allows \( A_0 \) and the \( \mu \) parameter to be complex (where \( \mu \) is the Higgs mixing parameter in the superpotential term \( \mu H_1 H_2 \)). The renormalization group equations (RGE) leading to electroweak symmetry breaking at the electroweak scale determine \( |\mu| \) and so there can be two CP violating phases, \( \mu = |\mu| e^{i\alpha} \) and \( A_0 = |A_0| e^{i\phi} \) (in addition to the CKM CP violating phase). To accommodate the electron and neutron electric dipole moments we will also allow the three gaugino masses at \( M_G \) to have phases: \( \tilde{m}_i = m_{i/2} e^{i\phi_i} \) with the convention \( \phi_2 = 0 \).

The allowed parameter space for \( m_0, m_{1/2} < 1 \) TeV is shown in Figs. 1, 2, 3 for \( \tan \beta = 10, 40, 50 \) with \( A_0 = 0, \mu > 0 \) (\( \mu \) real)[5]. Here the lightest neutralino \( \tilde{\chi}^0_1 \) is the dark matter candidate, and the narrow rising (pink) band is the region of parameter space where the predicted amount of relic dark matter left over after the Big Bang is in agreement with the CMB measurements of \( \Omega_{DM} = \rho_{DM}/\rho_c \) as of a year ago from the various balloon flights. Here \( \rho_{DM} \) is the dark matter (DM) mass density, and \( \rho_c \) is the critical density to close the universe (\( \rho_c = 3H_0^2/8\pi G_N \), \( H_0 \)=Hubble constant, \( G_N \)=Newton constant).

It is important to realize that the narrowness of the dark matter allowed band is \textit{not} a fine tuning but rather a consequence of the co-annihilation effect for \( \tilde{\chi}^0_1 \) and the light \( \tilde{\tau}_1 \) in the early universe. This arises naturally in mSUGRA (and are generic features for many GUT models) due to the near accidental degeneracies between the \( \tilde{\tau}_1 \) and the \( \tilde{\chi}^0_1 \) and the fact that the Boltzman exponential
Fig. 1. Allowed region in the $m_0 - m_{1/2}$ plane from the relic density constraint for $\tan \beta = 10$, $A_0 = 0$ and $\mu > 0$ [5]. The red region was allowed by the older balloon data, and the narrow blue band by the new WMAP data. The dotted red vertical lines are different Higgs masses, and the current LEP bound produces the lower bound on $m_{1/2}$. The light blue region is excluded if $\delta a_\mu > 11 \times 10^{-10}$. (Other lines are discussed in reference [5].)

Factors in the early universe annihilation analysis produces sharp cut offs in the relic density. Thus the bottom of the allowed band is at the experimental lower bound on $\Omega_{DM}$ (where $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ takes its minimum allowed value) and the top corresponds to the experimental upper bound on $\Omega_{DM}$ (above which $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ is too large to get efficient early universe annihilation).

If one allows $m_0$ and $m_{1/2}$ to be in the multi-TeV region, two additional region occur with acceptable relic density [6]: (1) the “focus point” region where $m_0 \sim 1$ TeV and $m_{1/2} \sim 400$ GeV and (2) the funnel region where $m_0 \simeq m_{1/2} \gtrsim 1$ TeV and $\tan \beta$ is large. These have been studied by a number of authors (e.g. [6–14]). If the $g_\mu - 2$ data eventually confirms a deviation from the SM, would be eliminated (as shown in the blue regions of Figs. 1-3) but this is still in doubt
Fig. 2. Same as Fig. 1 for $\tan \beta = 40$, $A_0 = 0$, $\mu > 0$ except that now that the $b \to s\gamma$ constraint (green region) produces the lower bound on $m_{1/2}$ [5].

as we discuss below. However, aside from $g_\mu - 2$, these are regions of relatively high fine tuning. Thus one can define the fine tuning parameter

$$\Delta \Omega \equiv \left[ \sum \frac{\partial \ln \Omega_{\chi_i}}{\partial \ln a_i} h^2 \right]^{1/2}; \quad a_i = m_0, m_{1/2}, \text{etc.} \quad (1)$$

and $H_0 = h(100 \text{ km/sec Mpc})$. Large $\Delta \Omega$ implies significant fine tuning. Figs 4 and 5 [15] show the values for $\Delta \Omega$ for the focus-point region ($\tan \beta = 10$) and the funnel region $\tan \beta = 50$. One sees that in the co-annihilation region $\Delta \Omega \simeq 1 - 10$, while in the focus point or funnel regions $\Delta \Omega \simeq 100 - 1000$. Whether this is an argument to exclude these high fine tuning regions is a matter of taste.

3 The WMAP data

The Wilkinson Microwave Anisotropy Probe (WMAP) has determined the basic cosmological parameters with remarkable precession. What is measured is the
Fig. 3. Same as Fig. 2 for $\tan \beta = 50$, $A_0 = 0$, $\mu > 0$. Note that the large bulge at lower $m_{1/2}$ allowed by the older balloon data is now mostly excluded by the WMAP data [5].

baryon density $\Omega_m$, the total density $\Omega_{tot}$, the Hubble constant $h = H/100(\text{km/secMpc})$, as well as many other cosmological parameters. Characterizing the difference between $\Omega_{tot}$ and $\Omega_m$ by a cosmological constant $\Lambda$, WMAP finds $\Omega_b = 0.044 \pm 0.004$, $\Omega_m = 0.27 \pm 0.04$, $\Omega_A = 0.73 \pm 0.04$, $h = 0.71^{+0.04}_{-0.03}$. $\Omega_A$ is in good agreement with the direct measurement of $\Lambda$ using type IA supernovae [16] and $h$ agrees very well with the Hubble Key Project’s direct measurement [17]. $\Omega_{CDM}$ is then given by $\Omega_m - \Omega_b$, and there is a general concordance with a cold dark matter universe with a cosmological constant. (The distinction between a cosmological constant and a quintessence model is not yet measurable). The WMAP 2$\sigma$ range for $\Omega_{CDM}$ is

$$0.094 < \Omega_{CDM} h^2 < 0.13$$

(2)

This is to be compared with the previous balloon flight measurements of $0.07 < \Omega_{CDM} h^2 < 0.21$. Thus WMAP has reduced the uncertainty in the amount of cold dark matter by nearly a factor of four! This can be seen in Figs.1-3, where
the WMAP allowed region is the narrow blue band. Particularly striking is the reduction of parameter space for $\tan \beta > \sim 45$ as seen in Fig. 3. In addition an upper bound $m_{1/2} \lesssim 1 \text{ TeV}$ is found in [12]. As the data becomes more and more accurate the CMB measurements will effectively determine one of the mSUGRA parameters i.e. determine a relation of the form (neglecting CP violating phases)

$$m_0 = m_0(m_{1/2}, A_0, \tan \beta, \mu/|\mu|)$$  

(3)

4 Update on $g_\mu - 2$

While there has been no new data this year on the muon magnetic moment anomaly $a_\mu = 1/2(g_\mu - 2)$, there has been a re-evaluation of some of the old data used to calculate the SM contribution, which has reduced the significance of the effect. We begin first by reviewing where things stood last year. First, the current experimental values of the electron [18] and muon [19] anomalies are

$$a_e^{\text{exp}} = 1159652188.3(4.2) \times 10^{-12}$$  

(4)

$$a_\mu^{\text{exp}} = 11659203(8) \times 10^{-10}$$  

(5)

The theoretical values of $a_e$ can be expressed as a power series in $\alpha$ [20]

$$a_e^{\text{th}} = \sum_1^4 S_n^e \frac{\alpha^n}{n} + 1.70(3) \times 10^{-12}$$  

(6)
Fig. 5. Contours of total sensitivity $\Delta^2$, $\tan \beta = 50$, $A_0 = 0$, $\mu > 0$, $m_t = 175$ GeV [15].

where $S_n^c$ begins as $S_1^c = 1/2$ (the Schwinger term) and the last term in Eq.(6) are the electroweak and hadronic contributions. One equates Eqs.(4) and (6) and solves for $\alpha$. This is currently the most accurate determination of the fine structure constant at zero energy:

$$\alpha^{-1} = 137.03599875(52)$$

This number is about 1.6$\sigma$ down from the previously given values due to the correction of an error recently found in the $\alpha^4$ term [21].

The SM prediction for $a_\mu$ can be divided into QED, electroweak and hadronic parts:

$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{had}}$$

The QED part has been calculated through order $\alpha^5$. Fortunately, the new value of $\alpha$ does not change this significantly [21]:

$$a_{\mu}^{\text{QED}} = 11658470.35(28) \times 10^{-10}$$

The electroweak contribution has been calculated to two loop order (the two loop part being surprisingly large) by two groups[22,23] with good agreement yielding an average

$$a_{\mu}^{\text{EW}} = 15.3(0.2) \times 10^{-10}$$

The hadronic contribution can be divided into three parts, a leading order, a higher order and a scattering of light by light (LbL) part. The latter two have
been done by several groups, and there is now reasonable agreement in the LbL part:

\[
a_{\mu}^{\text{hadHO}} = -10.0(0.6) \times 10^{-10}; \quad a_{\mu}^{\text{LbL}} = 8(4) \times 10^{-10}
\] (11)

The term which is most unclear is the leading order hadronic contribution. This can be calculated by using a dispersion relation:

\[
a_{\mu}^{\text{LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_e^2}^{\infty} \frac{ds}{s} K(s) \frac{\sigma^{(0)}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}
\] (12)

In principle, one uses the experimental \(e^+e^- \rightarrow \text{hadrons}\) cross section to calculate the integral which is dominated by the low energy part. Note that \(\sigma^{(0)}\) means the experimental cross section corrected for initial radiation, electron vertex and photon vacuum polarization (so one does not count the higher order contributions).

A large number of experimental groups have contributed to the determination of \(\sigma^{(0)}\) and two independent ways have been used:

(i) Direct measurement of \(\sigma(e^+e^- \rightarrow \text{hadrons})\) with the above mentioned radiative corrections made. This data has been dominated by the very accurate CMD-2 experiment.

(ii) One uses the \(2\pi\) decays of the \(\tau\), which goes through the vector (V) interaction, and then uses CVC to determine \(\sigma_{e^+e^-}\) for \(\sqrt{s} \leq 1.7\) GeV (the kinematic reach of the \(\tau\) decay). Here one must make corrections to account for the breaking of CVC. This data is dominated by ALEPH.

The results of these calculation, as of last year were very puzzling. Three groups did the analysis using method (i) getting almost identical answers[24–26]. An average value was \(a_{\mu}^{\text{hadLO}}(e^+e^-) = (683.8 \pm 7.0) \times 10^{-10}\). However, using the \(\tau\) data (augmented by the \(e^+e^-\) data at higher energy), Ref.[24] found \(a_{\mu}^{\text{hadLO}}(e^+e^-) = (709.0 \pm 5.9) \times 10^{-10}\). (In fact, the \(e^+e^-\) and \(\tau\) data were inconsistent with each other at the level of 4.6\(\sigma\)! The \(e^+e^-\) data gave rise to a 3.0\(\sigma\) disagreement between experiment and the SM while the \(\tau\) gave a smaller 1.0\(\sigma\) disagreement.

Recently, however there have been new results that have significantly reduced the discrepancy between the two approaches and between experiment and the SM. CMD-2 found an error in its analysis of the radiative correction to be made to their \(e^+e^- \rightarrow \text{had}\) data (a lepton vacuum polarization diagram was omitted). A full re-analysis of the SM value of \(a_{\mu}^{\text{hadLO}}\) was then carried out in [27] with the results now of \(a_{\mu}^{\text{hadLO}}(e^+e^-) = (696.3 \pm 7.2) \times 10^{-10}\) and \(a_{\mu}^{\text{hadLO}}(\tau\text{based}) = (711.0 \pm 5.8) \times 10^{-10}\). The discrepancy with the SM now becomes

\[
a_{\mu}^{\exp} - a_{\mu}^{\text{SM}}(e^+e^-) = (22.1 \pm 11.3) \times 10^{-10}
\] (13)

\[
a_{\mu}^{\exp} - a_{\mu}^{\text{SM}}(\tau) = (7.4 \pm 10.5) \times 10^{-10}
\] (14)

corresponding to 1.9\(\sigma\) and 0.7 \(\sigma\) discrepancy respectively. However the \(\tau\) data still disagrees with the \(e^+e^-\) data at a 2.9\(\sigma\) level (the disagreement occurring particularly at \(\sqrt{s} > 850\) MeV) and so a puzzle still remains.
One may expect some further clarification in the not too distant future. Thus the Brookhaven analysis of their $\mu^-$ data should reduce the experimental error on $a_\mu^{\exp}$. Further, new data from KLOE and BABAR can check the CMD-2 results. In Figs.(1-3), we have shown the excluded region in the parameter space (blue) if the discrepancy with the SM is $11 \times 10^{-10}$ (i.e. about $1 \sigma$ from zero). A large amount of parameter space is eliminated.

5 CP Violating B decays

BABAR and Belle have now measured with increasing accuracy a number of CP violating B decays. This has opened up new tests of the Standard Model and new ways to search for new physics.

Simultaneously, improved techniques for calculating these decays have been developed over the past two years by Beneke, Buchalla, Neubert and Sachrajda [28] (BBNS) and these procedures have been further discussed by Du et al. [29]. In previous analyses using the so-called “naive factorization”, decay amplitudes for $B \rightarrow M_1 + M_2$ depend on the matrix elements of operators $O_i$, $\langle M_1 M_2 | O_i | B \rangle$, and these matrix elements were factorized to calculate them. In the BBNS scheme, “non-factorizable” contributions can be calculated allowing the calculations of the strong phase, which is needed to discuss direct CP violation.

We consider here the decays $B^0 \rightarrow J/\Psi K_s$, $B^0 \rightarrow \phi K_s$ and $B^\pm \rightarrow \phi K^\pm$. The results below are presented using the BBNS analysis and we discuss here the decays for SUGRA models. We require, of course, the simultaneous satisfaction of the dark matter, the $b \rightarrow s\gamma$ branching ratio, the electron and neutron EDM constraints etc.[30].

The B-factories measure a number of parameters of the B decays that relate to CP violation. Thus the time dependent CP asymmetry for $B^0$ decaying into a final state $f$ is given by:

$$A_f(t) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)}$$ (15)

which is parametrized by

$$A_f(t) = -C_f \cos(\Delta m_B t) + S_f \sin(\Delta m_B t)$$ (16)

with

$$S_f = \sin 2\beta_f , \quad C_f = \frac{1 - |A(B^0(t) \rightarrow f)/A(B^0(t) \rightarrow f)|^2}{1 + |A(B^0(t) \rightarrow f)/A(B^0(t) \rightarrow f)|^2}$$ (17)

and $\Delta m_B$ is the $B_d$ mass difference. For charged modes one has

$$A_{CP} = \frac{\Gamma(B^- \rightarrow f^-) - \Gamma(B^+ \rightarrow f^+)}{\Gamma(B^- \rightarrow f^-) + \Gamma(B^+ \rightarrow f^+)}$$ (18)

These B decays have been previously examined in SUSY models by a number of different authors [30] in low energy MSSM using the mass insertion method [31] and/or without taking into full account of the BBNS analysis.
$C_f$ and $A_{CP}$ give a measure of direct CP violation. In addition the branching ratios have been well measured.

We consider first the decay $B^0 \rightarrow J/\Psi K_s$. The BABAR and ELLE measurements give [32].

$$
\sin 2\beta_{J/\Psi K_s} = 0.734 \pm 0.055, \ C_{J/\Psi K_s} = 0.052 \pm 0.047
$$

(19)

In the Standard Model for any decay $B^0 \rightarrow f$, $\sin 2\beta_f$ should be close to $\sin 2\beta$ of the CKM matrix and indeed an evaluation of the CKM $\beta$ (without using the B-factory data) gives [33]

$$
\sin 2\beta = 0.715^{+0.055}_{-0.045}
$$

(20)

in good agreement with Eq.(19). Since $B^0 \rightarrow J/\Psi K_s$ proceeds through the tree diagram in the SM while SUSY effects begin only at the loop level, one would expect only a small SUSY correction to $\sin 2\beta_{J/\Psi K_s}$, again in accord with Eqs. (19). Note also that the smallness of $C_{J/\Psi K_s}$ implies that there is very little direct CP violation.

We consider next the $B \rightarrow \phi K$ decays. These decays begin for both the SM and SUSY at the loop level and so deviations from the SM result might occur. Using the recent new data from Belle [34] and the preliminary analysis of new BABAR data [35] one has:

$$
\sin 2\beta_{\phi K} = -0.15 \pm 0.33, \ C_{\phi K} = -0.19 \pm 0.30
$$

(21)

We see that $\sin 2\beta_{\phi K}$ differs from the expected Standard Model result of $\sin 2\beta_{J/\phi K_s}$ (given in Eq.(19)) by $^2 2.7 \sigma$. In addition BABAR has measured [2] $A_{CP}(B^- \rightarrow \phi K^-) = (3.9 + 8.7\%)$.

Further suggestion that there may be a breakdown of the Standard Model comes from the BABAR and Belle measurements of the branching ratios [2]:

$$
\text{Br}(B^0 \rightarrow \phi K_s) = (8.0 \pm 1.3) \times 10^{-6}
$$

(22)

$$
\text{Br}(B^\pm \rightarrow \phi K^\pm) = (10.9 \pm 1.0) \times 10^{-6}
$$

(23)

While the BBNS analysis is a significant improvement over naive factorization, it is not a complete theory. The largest theoretical uncertainty comes from the weak annihilation diagram (where a gluon splits into two final state quarks). These diagrams have divergent end-point integrals $X_A$ (which are cut off at $A_{QCD}$) and parametrized by:

$$
X_A = (1 + \rho_A e^{i\phi_A}) \ln \frac{m_B}{A_{QCD}}, \ \rho_A \leq 1.
$$

(24)

$^2$ The preliminary analysis of the new BABAR data is about 1$\sigma$ higher than the published older data [36], are so if the latter were used the discrepancy would be even larger
Fig. 6. Branching ratio of $B^- \to \phi K^-$ at $\rho_A = 1$. The solid curve corresponds to $\mu = m_b$, dashed curve for $\mu = 2.5$ GeV with $m_s(2 \text{ GeV}) = 96$ MeV and the dot-dashed curve for $\mu = m_b$ with $m_s(2 \text{ GeV}) = 150$ MeV. The two straight lines correspond to the cases without weak annihilation [38].

Fig. 6 shows the dependence of $\text{Br}(B^\pm \to \phi K^\pm)$ on $\phi_A$ for $\rho_A = 1$, and Fig. 7 the dependence on $\rho_A$ for $\phi_A = 0$. We see that the branching ratio over the most of the parameter space is $\simeq 4 \times 10^{-6}$ and does not get large unless $\rho_A$ is near its maximum ($\rho_A \simeq 1$) and $\phi_A$ is near 0 (or $2\pi$). However, Figs 6 and 7 also show that when the branching ratio becomes large and is then in accord with the experimental value of Eq.(23), the weak annihilation diagram dominates the decay amplitude, and so the theory is least reliable. Thus in the region where the theory is most reliable the predicted SM branching ratio is too small and in order to obtain a SM value in accord with experiment, one must go to a region of parameters where the theory is least reliable. (A similar result holds for $\text{Br}(B^0 \to \phi K_s)$.) However, even if one were to do this, it would not resolve the $2.7\sigma$ discrepancy of $\sin 2\beta_{\phi K_s}$, since $\sin 2\beta_{\phi K_s}$ is insensitive to $\rho_A$ and $\phi_A$. Hence that discrepancy with the SM would still remain.

One can next ask if the SUSY corrections of mSUGRA can resolve the difficulties of the SM for the $B \to \phi K_s$ decays. The answer is no. One finds, as one varies $\tan \beta$, $A_0$, $m_{1/2}$ that $\sin 2\beta_{\phi K_s} \simeq 0.69 - 0.74$ which is essentially the same as the SM value and is in disagreement with the experimental value of Eq.(21)). Also, unless the weak annihilation diagrams are larger $\text{Br}(B \to \phi K_s) \simeq 4 \times 10^{-6}$ for mSUGRA and again is too small to account for the experimental values.
of Eqs. (22) and (23). The reason mSUGRA cannot account for a reduction of $\sin^2\beta_{\phi K}$ from the SM value is that in mSUGRA the only flavor violating source is in the CKM matrix which cannot provide enough flavor violation in the $b \to s$ transition of the $B \to \phi K$ decays.

6 SUGRA With Non-Universal A Terms

One can ask whether one can add any non-universal soft breaking terms to mSUGRA to try to account for the apparent disagreement between experiment and the SM in $\sin^2\beta_{\phi K}$. In a GUT model, at least the SM gauge group must hold at $M_G$ and so there are only two ways one can enhance mixing between the second and third generations. One can have non-universal squark masses, $m^2_{23}$, at the GUT scale or non-universal A terms in the u or d sectors. The first possibility gives rise to left-left or right-right couplings only, and it was shown in [37] that these produce only a small effect on the $B \to \phi K$ decays. We therefore consider instead $A^{U,D}_{23}$ terms which produce left-right couplings[38]. We write

$$A^{U,D} = A_0 Y^{U,D} + \Delta A^{U,D}$$  \hspace{1cm} (25)
Here $Y^{U,D}$ are the Yukawa matrices and so the first term in the universal contribution. We assume that $\Delta A_{ij}^{U,D}$ has non-zero elements only for $i = 2, j = 3$ or $i = 3, j = 2$, and write $\Delta A_{ij}^{U,D} = |\Delta A_{ij}^{U,D}| e^{\phi_{ij}}$. Tables 1 and 2, for $\tan \beta = 10$ and 40 show that there is a wide range of parameters that can accommodate the experimental results of Eq. (21) with $\Delta A_{23}^{D} \neq 0$. We assume here parameters such that the weak annihilation effects are small and so the theoretical uncertainty is reduced. In spite of this the large experimental branching ratio of $B \to \phi K$ are achieved as can be seen in Table 1. (Br($B^\pm \to \phi K^\mp$ is $\sim 10 \times 10^{-6}$ for all entries of Table 1). The parameter space giving satisfactory results for the case of $\Delta A_{23}^{D} \neq 0$ is more restrictive generally requiring large $\tan \beta$ and lower $m_{1/2}$ as can be seen in the examples in Table 3. However, it is still possible for the theory to be within $1\sigma$ of the experiment with reasonable choices of parameters.

| $|A_0|$ | 800 | 600 | 400 | 0 | $|\Delta A_{23}^{D}|$ |
|-------|-----|-----|-----|---|----------------|
| $m_{1/2} = 300$ | -0.50 | -0.49 | -0.47 | -0.43 | $\sim 50$ |
| $m_{1/2} = 400$ | -0.43 | -0.40 | -0.38 | -0.36 | $\sim 110$ |
| $m_{1/2} = 500$ | -0.46 | -0.46 | -0.44 | -0.34 | $\sim 200$ |
| $m_{1/2} = 600$ | -0.15 | -0.13 | -0.04 | 0.05 | $\sim 280$ |

Table 1. $S_{\phi K}$ at $\tan \beta = 10$ with non-zero $A_{23}^{D}$ and $A_{32}^{D}$ [38].

| $|A_0|$ | 800 | 600 | 400 | 0 |
|-------|-----|-----|-----|---|
| $m_{1/2} = 300$ | -0.40 | 10.0 | -0.38 | 10.0 |
| $m_{1/2} = 400$ | -0.11 | 8.0 | -0.05 | 8.0 |
| $m_{1/2} = 500$ | 0.07 | 6.0 | 0.16 | 6.1 |
| $m_{1/2} = 600$ | 0.37 | 6.2 | 0.44 | 6.2 |

Table 2. $S_{\phi K}$ (left) and Br($B^- \to \phi K^-$) $\times 10^6$ (right) at $\tan \beta = 40$ with non-zero $\Delta A_{23}^{D}$ and $\Delta A_{32}^{D}$ [38].

| $|A_0|$ | 800 | 600 | 400 | 0 | $|\Delta A_{23}^{D}|$ (GeV) |
|-------|-----|-----|-----|---|----------------|
| $m_{1/2} = 300$ | 0.03 | 8.4 | 0.04 | 9.0 | 0.01 | 8.0 | 0.17 | 8.0 | $\sim 300$ |
| $m_{1/2} = 400$ | 0.07 | 8.5 | -0.03 | 8.4 | 0.01 | 7.1 | 0.32 | 6.3 | $\sim 600$ |
| $m_{1/2} = 500$ | 0 | 6.5 | 0.07 | 6.4 | 0.18 | 6.0 | 0.44 | 6.1 | $\sim 800$ |
| $m_{1/2} = 600$ | 0.27 | 6.1 | 0.30 | 6.1 | 0.35 | 6.1 | 0.51 | 5.9 | $\sim 1000$ |

Table 3. $S_{\phi K}$ (left) and Br($B^- \to \phi K^-$) $\times 10^6$ (right) at $\tan \beta = 40$ with non-zero $\Delta A_{23}^{D}$ and $\Delta A_{32}^{D}$.

In all the above cases ($\Delta A^{U}$ or $\Delta A^{D}$) the direct CP violating effects are small i.e. $A_{\phi K} \sim -(2 - 3)\%$ and also we find $|A_{CP}(B \to X_s \gamma)| \simeq (1 - 5)\%$.
We see thus that a non-universal $A$ term can account for the B-factory results on $B \to \phi K$ decays.

7 Conclusions

We have surveyed here three developments of the past year that have further narrowed the parameter space of possible SUSY models: the new WMAP data, the status of the muon magnetic moment anomaly, and the Belle and BABAR data on CP violating B decays. We have examined these questions within the framework of SUGRA GUT models as these have fewer free parameters, and so are more constrained by the array of other data, the full set of all the data allowing one to see more clearly what the constraints on SUSY are.

The WMAP data has determined the basic cosmological parameters with great accuracy, and this new data has reduced the error in the values of the cold dark matter density by a factor of about four. In the region of parameter space $m_0$ and $m_{1/2} < 1$ TeV, it implies for mSUGRA (and many SUGRA GUT models) that the lightest stau is only 5 - 10 GeV heavier than the lightest neutralino (the dark matter candidate). As the data from WMAP (and later Planck) become more and more accurate, these measurements will effectively determine $m_0$ in terms of the other mSUGRA parameters, a result that could be tested later at high energy accelerators.

The CMD-2 reanalysis of their $e^+ - e^-$ data has reduced the disagreement between the Standard Model prediction of the muon magnetic moment and the Brookhaven measurements. Thus using this data one now finds a 1.9σ disagreement, but using instead the tau decay into $2\pi$ final states plus CVC, there is only a 0.7σ discrepancy. However, the matter is still not clear as the $e^+ - e^-$ data is still in disagreement with the tau data at the 2.9 σ level. Future precision data from KLOE and perhaps BaBar using the radiative return method should help clarify the issue as these experiments will be able to check the CMD-2 data. The Brookhaven analysis of their $\mu^-$ data will further reduce the experimental error on the muon magnetic moment.

Perhaps the most interesting new data this year is the Belle and BABAR measurements of the CP violating parameters and branching ratios of $B^0 \to \phi K_s$ and $B^{\pm} \to \phi K^{\pm}$. Current data for $S_{\phi K_s}$ is in disagreement with the Standard Model at the 2.7σ level, and further, SUSY corrections based on mSUGRA give essentially the same results as the Standard Model. In addition, over most of the parameter space, the Standard Model and mSUGRA predict branching ratios for these decays that are a factor of two or more too low. SUGRA models offer an essentially unique way of accommodating both anomalies by adding a non-universal contribution mixing the second and third generations in either the up or down quark sectors to the cubic $A$ soft breaking terms at $M_G$. If added to the down quark sector, this term would also lower the CP odd Higgs mass opening up a new region in the $m_0 - m_{1/2}$ plane satisfying the relic density constraint. (Dark matter detector signals would not be increased, however.) If added to the up quark sector it would lower the stop mass again giving a new region consistent
with dark matter, and making the $\tilde{t}_1$ more accessible to colliders. Thus if this new data is further confirmed, it would indicate the first breakdown of the Standard Model and perhaps point to the nature of physics at the GUT and Planck scales, as string models with this type of non-universality can be constructed.

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References

1. P. Igo-Kemenes, LEPC meeting, 2000 (http://lepwww.web.cern.ch/LEPHIGGS/talks/index.html).
2. Heavy Flavor Averaging Group (http://www.slac.stanford.edu/xorg/hfag/index.html).
3. The ALEPH collaboration, ALEPH-CONF. 2001-009.
4. Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).
5. R. Arnowitt, B. Dutta, T. Kamon and V. Khotilovich, hep-ph/0308159.
6. K. L. Chan, U. Chattopadhyay and P. Nath, Phys. Rev. D 58, 096004 (1998) [hep-ph/9710473].
7. J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. D 61, 075005 (2000) [hep-ph/9909334]; Phys. Rev. Lett. 84, 2322 (2000) [hep-ph/9908309].
8. U. Chattopadhyay, A. Corsetti and P. Nath, Phys. Rev. D 68, 035005 (2003) [hep-ph/0303201].
9. H. Baer, C. Balazs, A. Belyaev, T. Krupovnickas and X. Tata, JHEP 0306, 054 (2003) [hep-ph/0304303].
10. H. Baer, C. Balazs, A. Belyaev and J. O’Farrill, hep-ph/0305191.
11. J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, hep-ph/0305212.
12. J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B 565, 176 (2003) [hep-ph/0303043].
13. H. Baer and C. Balazs, JCAP 0305, 006 (2003) [hep-ph/0303114].
14. A. B. Lahanas and D. V. Nanopoulos, Phys. Lett. B 568, 55 (2003) [hep-ph/0303130].
15. J. R. Ellis, K. A. Olive and Y. Santoso, New J. Phys. 4, 32 (2002) [hep-ph/0202110].
16. J. L. Tonry et al., Astrophys. J. 594, 1 (2003) [astro-ph/0305008]; R. A. Knop et al., astro-ph/0309368.
17. W. L. Freedman et al., Astrophys. J. 553, 47 (2001) [astro-ph/0012376].
18. P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. 72, 351 (2000).
19. G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 89, 101804 (2002) [Erratum-ibid. 89, 129903 (2002)] [hep-ex/0208001].
20. A. Nyffeler, hep-ph/0305135.
21. T. Kinoshiba and M. Nio, Phys. Rev. Lett. 90, 021803 (2003) [hep-ph/0210322].
22. M. Knecht, S. Peris, M. Perrottet and E. De Rafael, JHEP 0211, 003 (2002) [hep-ph/0205102].
23. A. Czarnecki, W. J. Marciano and A. Vainshtein, Phys. Rev. D 67, 073006 (2003) [hep-ph/021229].
24. M. Davier, S. Eidelman, A. Hoeker and Z. Zhang, Eur. Phys. J. C 27, 497 (2003) [hep-ph/0208177].
25. K. Hagiwara, A. D. Martin, D. Nomura and T. Teubner, Phys. Lett. B 557, 69 (2003) [hep-ph/0209187].
26. F. Jegerlehner, J. Phys. G 29, 101 (2003) [hep-ph/0104304].
27. M. Davier, S. Eidelman, A. Hocker and Z. Zhang, hep-ph/0308213.
28. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); [hep-ph/9905312]; Nucl. Phys. B 591, 313 (2000); [hep-ph/0006124]; Nucl. Phys. B 606, 245 (2001), [hep-ph/0104110].
29. D. s. Du, H. j. Gong, J. f. Sun, D. s. Yang and G. h. Zhu, Phys. Rev. D 65, 094025 (2002) [Erratum-ibid. D 66, 079904 (2002)] [hep-ph/0201253]; D. s. Du, J. f. Sun, D. s. Yang and G. h. Zhu, Phys. Rev. D 67, 014023 (2003). [hep-ph/0209233].
30. A. Kagan, hep-ph/9806266; talk at the 2nd International Workshop on B physics and CP Violation, Taipei, 2002; SLAC Summer Institute on Particle Physics, August 2002; G. Hiller, Phys. Rev. D 66, 071502 (2002) [hep-ph/0207356]; A. Datta, Phys. Rev. D 66, 071702 (2002) [hep-ph/0208016]; M. Ciuchini and L. Silvestrini, Phys. Rev. Lett. 89, 231802 (2002) [hep-ph/0208087]. B. Dutta, C. S. Kim and S. Oh, Phys. Rev. Lett. 90, 011801 (2003) [hep-ph/0208226]; S. Khalil and E. Kou, Phys. Rev. D 67, 055009 (2003) [hep-ph/0212023]; S. Baek, Phys. Rev. D 67 (2003) 096004 [hep-ph/0301269]; C. W. Chiang and J. L. Rosner, Phys. Rev. D 68, 014007 (2003) [hep-ph/0302094]; A. Kundu and T. Mitra, Phys. Rev. D 67, 116005 (2003) [hep-ph/0302123]; K. Agashe and C. D. Carone, Phys. Rev. D 68, 035017 (2003) [hep-ph/0304229]; G. L. Kane, P. Ko, H. b. Wang, C. Kolda, J. h. Park and L. T. Wang, Phys. Rev. Lett. 90 (2003) 141803 [hep-ph/0304239]; D. Chakraverty, E. Gabrielli, K. Huitu and S. Khalil, hep-ph/0306076; J. F. Cheng, C. S. Huang and X. h. Wu, hep-ph/0306086.
31. L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986); F. Gabrielli, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996). [hep-ph/9604387].
32. G. Hamel De Monchenault [BABAR Collaboration], hep-ex/0305055.
33. A. J. Buras, hep-ph/0210291.
34. K. Abe [Belle Collaboration], hep-ex/0308035.
35. T. Browder, talk given at XXI international symposium on lepton and photon interactions at high energies, Fermi National Accelerator Laboratory, Aug. 12, 2003 ( http://conferences.fnal.gov/lp2003/program/S5/browder_s05_ungarbled.pdf)
36. B. Aubert et al. [BABAR Collaboration], hep-ex/0207070.
37. G. L. Kane, P. Ko, H. b. Wang, C. Kolda, J. h. Park and L. T. Wang, Phys. Rev. Lett. 90 (2003) 141803 [hep-ph/0304239].
38. R. Arnowitt, B. Dutta and B. Hu, to appear in Phys. Rev. D [hep-ph/0307152].