Comment on ”Single-parameter quantum charge and spin pumping in armchair graphene nanoribbons” (arXiv:1206.3435v1, by Zhou and Wu)

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In their recent submission arXiv:1206.3435v1 Zhou and Wu addressed the charge and spin pumping in an armchair graphene nanoribbon connected to magnetic and/or nonmagnetic leads at zero bias. They used thereby a pumping current formula based on time-reversal symmetry. In their appendix they made, based on their formulas, a number of claims concerning our recent works on the laser modification of the spin-dependent current in an extended graphene monolayer contacted to two magnetic leads under a finite bias. Below we show in detail that the physical system considered by us is conceptually different from that of Zhou and Wu (arXiv:1206.3435v1) and cannot be treated with their theory for fundamental reasons (e.g., the time-reversal symmetry is inherently broken in our case due to the presence of the magnetic leads and the dc current generated by the finite bias). Hence, the statements of Zhou and Wu based on a comparison of their results with ours are scientifically groundless.

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In the submission arXiv:1206.3435v1 Zhou and Wu presented their views on the spin pumping in an armchair graphene nanoribbon under an ac gate voltage while the nano ribbon is being connected to nonmagnetic/ferromagnetic leads. Such an approach cannot make any profound statements on the laser-induced modulations of the dc current in an extended graphene sheet contacted to two biased magnetic leads, a case which we studied in [2,3]. Hence, their claims and statements on our calculated current as a function of the applied bias lack a scientific basis.

Here we only focus on the difference between our case and the case studied by Zhou and Wu. Their starting point is eq.(17) in arXiv:1206.3435v1 for the pumping current in the spin channel \( \sigma \)

\[
I^\sigma_{\text{pump}} = \frac{e}{h} \sum_n \int_{-\infty}^{E_F} d\epsilon \left[ T_{nLR\sigma}^n(\epsilon) - T_{nRL\sigma}^n(\epsilon) \right].
\]

(1)

\( T_s \) are the transmission probabilities in standard notations, \( E_F \) is the Fermi energy, and \( e \) is the electron charge. As stated in their paper, they use a time-reversal symmetry (TRS) to write \( T_{nLR\sigma}^n(\epsilon) = T_{nLR\sigma}^{n}\left(\epsilon + n\Omega\right) \) where \( \Omega \) is the frequency of the ac field applied to the graphene nano ribbon. With this they proclaim (Eq. (18) arXiv:1206.3435v1) that

\[
I^\sigma_{\text{pump}} = \frac{e}{h} \sum_{n>0} \int_{E_F-n\Omega}^{E_F} d\epsilon \left[ T_{nLR\sigma}^{n}(\epsilon) - T_{nRL\sigma}^{n}(\epsilon) \right].
\]

(2)

is an exact formula valid without the need to introduce an energy cutoff because the upper limit is \( E_F \) and the lower limit is \( E_F - n\Omega \). On the basis of this formula they show in their Fig. 5(a) two sets of curves: one set is generated by Eq. (1) and one set is generated by introducing a theta function \( \Theta(E_{\text{cut}} - |\epsilon + n\Omega|) \) in the Green function that enters, e.g. \( T_s \) (here \( E_{\text{cut}} \) is an ad hoc energy cutoff). In Fig. 5(b), they plot the pumping current vs. \( V_{ac} \) by using the two ways. By the latter (cutoff) way, they notice that the current at first increases and then slowly decreases. They claimed that these curves generated by the latter way coincide our results and further claim that the current decrease they observe in their calculations is just a consequence of an introduced cutoff. In fact, they compare their own calculations generated by their two different equations. A comparison with our results has absolutely no scientific ground. Their approach does not apply to our system (as a matter of fact nor to any system with a broken time-reversal symmetry, e.g. when magnetic leads or dc currents are involved) and in particular, their scheme does not reproduce our formulas.

For clarity we note the following:

1. In our work, we did not study pumping effects. There is always a dc bias between the two magnetic leads in addition to the ac field acting on the graphene sheet. This dc bias gives rise to a background current. We investigated the effect of the ac field on the background current. While, Zhou and Wu’s work is about the charge pumping and spin pumping. There is no such a dc bias through the system. Their formulas are not applicable to our system. This is why they call the calculated current in Fig. 5 pumping current. Therefore, their demonstration and conclusions have nothing to do with ours.
2. They claim their formula, Eq. (18) for the pumping current has the benefit that no cutoff is needed. They derive this formula by using the crucial relation

\[ T_{LR}^n(\varepsilon) = T_{RL}^{-n}(\varepsilon + n\Omega). \]  

They claimed that this is due to the time-reversal symmetry (TRS). We should point out that in our study we considered a system with ferromagnetic (FM) leads where spin splitting is present. In the presence of these FM leads, there is no TRS. One can define a time reversal operator and try to calculate the commutator with the Hamilton operator including the FM leads. The finding is that these operators do not commute. Therefore, there is no TRS. The current formula, i.e. Eq. (18) in their paper (i.e. Eq. 2 here), needs to be justified fundamentally. Their demonstrations for the spin dependent pumping currents should thus be taken with care, but anyway they do not compare with our results because we are dealing with a different physical problem.

3. The TRS consideration, i.e. the Eq. (3), is not straightforward. Usually, the TRS is verified by constructing a time reversal operator and considering its commutation properties with the Hamiltonian. The demonstration of TRS can not be done based on a consideration of a calculated statistical physical quantity. For example, when a dc bias is present (as in our study), the system is in a nonequilibrium situation and a charge current is flowing through the system (the leads are expected to act as particle baths). TRS does not exist in this case even for systems with nonmagnetic leads.

4. Zhou and Wu studied the time-averaged current through a lead/graphene nanoribbon/lead system in which an ac field is applied to the graphene region. Under such an ac field, the graphene part is actually in a nonequilibrium state. To calculate the current, we have to start from the charge conservation law

\[ e \frac{dN_G}{dt} = J_R(t) + J_L(t), \]  

where \( N_G \) is the occupation of graphene, \( J_L(t) \) and \( J_R(t) \) are the current flowing through the left and the right leads. The left hand side of the equal sign is just the displacement current reflecting the charge accumulation in the central region and related to the leads currents. We can use the standard definition of the time-averaged current

\[ \langle F(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt F(t). \]  

The displacement current tends to zero after this time average. Only in this case, will the charges flowing from the left lead, flow into the right lead completely. There is no charge accumulation in the central region in this case. However, Zhou and Wu used another definition, namely

\[ \bar{I} = \frac{1}{T_0} \int_{0}^{T_0} I, \]  

where \( T_0 = 2\pi/\Omega \). This is an average in one period of the ac field. In this time period, the displacement current is in general finite and there is a charge accumulation in the central region. However, they ignore this term in their Eq. (6). A serious problem is that the charge conservation is violated in the formula of the current. Therefore, we conclude that the equations of the current and the calculations based upon these equations need to be reexamined.

In summary, the remarks and statements by Zhou and Wu’s based on a comparison of their calculations in arXiv:1206.3435v1 with our work\(^2\)\(^3\) have no scientific basis.

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