Dynamical disappearance of superposition states in the thermodynamic limit

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April 1, 2022

Abstract

It is shown that a macroscopic superposition state of radiation, strongly interacting with an ensemble of two-level atoms, is removed generating a coherent state describing a classical radiation field, when the thermodynamic limit is taken on the unitary evolution obtained by the Schrödinger equation. Decoherence appears as a dynamical effect in agreement with a recent proposal [M. Frasca, Phys. Lett. A 283, 271 (2001)]. To prove that this effect is quite general, we show that this same behavior appears when a superposition of two Fock number states is also considered. Higher order corrections are computed showing that this result tends to become exact in the thermodynamic limit. It appears as a genuine example of intrinsic collapse of the wave function.

PACS: 42.50.Ct, 42.50.Hz, 03.65.Yz, 42.50.Lc

1 Introduction

Recent experimental findings in mesoscopic devices [1] seem to suggest that, at very low temperatures, some kind of mechanism is producing decoherence [2]. These same experiments seems to prove that this mechanism is intrinsic to the device. So, it is become demanding to understand a way to describe decoherence at zero temperature [3].
Zero temperature decoherence in the framework of dissipative quantum systems has been devised in [4]. A general description of these kind of quantum systems can be found in [5]. In this paper we want to describe another approach to decoherence, that is, decoherence produced by the unitary evolution in the thermodynamic limit [6]. This is non dissipative decoherence, a first theory of which has been given in [7] where time is considered as a stochastic variable. Besides, in mesoscopic physics, some examples of non dissipative decoherence were also given in [8, 9].

Some recent approaches rely on vacuum fluctuations to generate such kind of decoherence [10] and, in mesoscopic devices it appears that electron-electron interaction plays the dominant role. But, again these approaches can be leaded back to dissipative quantum systems.

Non-dissipative decoherence can play a major role as it can make quantum theory self-contained without requiring arbitrary separation between a bath and a system. Here we demand that some systems, in the thermodynamics limit, develop classical behavior. When such systems interact with other quantum systems, decoherence indeed appears. The evolution is anyhow unitary.

The foundation for this kind of view of decoherence is given by the formal analogy between the time evolution operator of quantum mechanics written as $\exp\left(-\frac{iHt}{\hbar}\right)$ and the density matrix of a system being at equilibrium $\exp\left(-\beta H\right)$. In the latter case the thermodynamic limit has a precise meaning recovering standard thermodynamic [11]. Here we will show that, in the case of quantum evolution, one can get classical states when the system is properly prepared at the initial time [6]. Then, when such a system interacts with another quantum system, decoherence can develop. Such classical states are characterized by having the system following exactly the classical equations of motion, for given observables, without any significant deviation due to quantum fluctuations, in agreement with the Ehrenfest’s theorem.

It is interesting to note that a recent experiment by Haroche and coworkers [12], aimed at realizing a conceptual experiment on complementarity due to Bohr, proves the appearance of classicality in the limit of increasing photons in a cavity. The states they have produced are those theoretically predicted, in a pioneering work on the Jaynes-Cummings model in the limit of a large number of photons, by Gea-Banacloche [13]. In this latter work, the question of the collapse of the wave function is properly discussed in a similar context as ours,
appearing as a first hint toward the appearance of decoherence in the thermodynamic limit.

In this paper we want to limit our analysis to the particular case of a single radiation mode interacting with \( N \) two-level systems. So, by studying the thermodynamic limit, we prove that, in the strong coupling limit and with the two-level systems properly prepared, decoherence develops destroying superposition states that are taken initially for the field mode, driving the system toward a coherent state describing a classical field. Then, by computing higher order corrections, using perturbation theory, we will be able to show how these terms are negligible small in the thermodynamic limit, so that, the leading order term tends to become an exact solution in the same limit. This seems a clear example of dynamical collapse of the wave function obtained solving perturbatively the Schrödinger equation.

The paper has the following structure. In sec 2 we introduce the quantum model that we want to analyze. In sec.3 we show how the appearance of classical states indeed happens for an ensemble of two-level systems. In sec.4 we give the leading order solution of the model in the strong coupling regime proving that, given a superposition of states, being Fock or coherent states, the system is driven toward a coherent state describing a classical radiation field. In sec.5 we evaluate the higher order corrections to the leading order solution to show that, in the thermodynamic limit, our result does not change but rather tends to be an exact solution. Finally, in sec.6 we give the conclusions with a brief discussion.

2 The model: two-level atoms interacting with a single radiation mode

Our aim is to discuss how decoherence can appear as a dynamical effect, in the limit of a very large number of systems (thermodynamic limit). It is a well-known matter [14] that a single two-level atom under the effect of an increasing number of radiation modes changes its behavior from coherent Rabi oscillations to spontaneous emission, a typical decoherent effect.

Here, we reverse the situation by leaving a single radiation mode interacting with an ensemble of two-level atoms. The Hamiltonian of a single radiation mode interacting with a two-level atom is given by
being $\omega$ the frequency of the radiation mode, $\Delta$ the separation between the levels of the atom, $g$ the coupling between the radiation and the matter, $\sigma_x$ and $\sigma_z$ the Pauli matrices and $a$ and $a^\dagger$ the annihilation and creation operators of the radiation mode. In a recent experiment on a small Josephson-junction circuit irradiated with microwaves[15], the strong coupling regime for this Hamiltonian has been obtained describing Rabi oscillations as theoretically described in [16]. So, the strong coupling regime has been practically realized in this physical situation.

This model can be easily generalized to an ensemble of $N$ two-level atoms as

$$H = \omega a^\dagger a + \frac{\Delta}{2} \sigma_z + g \sigma_x (a + a^\dagger)$$

and again we consider the strong coupling regime as, in this case, decoherence is produced dynamically by unitary evolution in the thermodynamic limit. This model has been recently studied, in a similar context, in Ref.[17]. It can be realized by an array of Josephson-junction circuits, irradiated by microwaves, working as qubits for quantum computer.

Let us note, at this point, that having our Hamiltonian the form

$$H = H_{\text{system}} + H_{\text{bath}} + H_{\text{system-bath}},$$

we are neglecting $H_{\text{bath}}$, that is, a dual situation with respect to the one considered by Haake and coworkers [18] that neglect $H_{\text{system}}$. But, while they describe dissipative decoherence, we aim to show, using perturbation theory, that decoherence is produced dynamically in the thermodynamic limit.
3 Classical states by unitary evolution and thermodynamic limit

In this section we want to limit our study to the Hamiltonian of the ensemble of two-level atoms

\[ H_0 = \sum_{i=1}^{N} \frac{\Delta}{2} \sigma_{zi} \]  

and we prove the following result [6]:

For a proper set of initial conditions, the ensemble of two-level atoms evolves in time classically with respect to the variables \( \Sigma_x = \sum_{i=1}^{N} \sigma_{xi}, \Sigma_y = \sum_{i=1}^{N} \sigma_{yi}, \Sigma_z = \sum_{i=1}^{N} \sigma_{zi}, \) in the thermodynamic limit.

We want to emphasize that the system does not decohere but evolves in time washing out the quantum fluctuations in the thermodynamic limit. Besides, the system must be properly prepared as we cannot claim that when the system is in an eigenstate of \( H_0, \) behaves classically.

To prove our result, we consider the following initial state

\[ |\psi(0)\rangle = \prod_{i=1}^{N} (a_i | \downarrow \rangle_i + b_i | \uparrow \rangle_i) \]  

with \( |a_i|^2 + |b_i|^2 = 1. \) The set of coefficients \( a_i \) and \( b_i \) must be chosen in such a way that \(|\psi(0)\rangle\) is not in an eigenstate of \( H_0. \) The time evolution is determined through

\[ U_0(t) = \prod_{i=1}^{N} [e^{it \frac{\Delta}{2}} | \downarrow \rangle_i \langle \downarrow | + e^{-it \frac{\Delta}{2}} | \uparrow \rangle_i \langle \uparrow | ] \]  

giving
\[ |\psi(t)\rangle = \prod_{i=1}^{N} (a_i e^{it \frac{\Delta}{2}} | \downarrow \rangle_i + b_i e^{-it \frac{\Delta}{2}} | \uparrow \rangle_i). \]  

The average values are given by

\[ \langle \Sigma_x \rangle = \langle \psi(t)|\Sigma_x|\psi(t)\rangle = N(\xi_H \cos(\Delta t) + \xi'_H \sin(\Delta t)) \]  

being \( \xi_H = \sum_{i=1}^{N} (a_i^* b_i + a_i b_i^*)/N \) and \( \xi'_H = i \sum_{i=1}^{N} (a_i^* b_i - a_i b_i^*)/N, \) numbers of order of unity. In the same way one has

\[ \langle \Sigma_x^2 \rangle = \langle \psi(t)|\Sigma_x^2|\psi(t)\rangle = N \left[ 1 - \frac{1}{N} \sum_{i=1}^{N} (a_i^* b_i e^{i\Delta t} + b_i^* a_i e^{-i\Delta t})^2 \right] + N^2 (\xi_H \cos(\Delta t) + \xi'_H \sin(\Delta t))^2 \]  

(9)
so that, one gets finally that \((\Delta \Sigma_x)^2 \propto N\) and the mean values are overwhelming large with respect to quantum fluctuations in the thermodynamic limit. A similar argument runs for \(\Sigma_y\) and \(\Sigma_z\) components. This means in turn that the classical equations of motion from the Ehrenfest theorem

\[
\begin{align*}
\langle \dot{\Sigma}_x \rangle &= -\Delta \langle \Sigma_y \rangle \\
\langle \dot{\Sigma}_y \rangle &= \Delta \langle \Sigma_x \rangle \\
\langle \dot{\Sigma}_z \rangle &= 0
\end{align*}
\]

are obeyed without any significant deviation due to quantum fluctuations in the limit of very large \(N\). This proves our assertion.

A relevant question that can be asked with this result is if such a model, with this kind of initial state, can produce decoherence interacting with other quantum systems. We will answer this question in the next sections.

### 4 Decoherence generated by unitary evolution: Leading order solution

We want to show as, an ensemble of two-level atoms, strongly interacting with a single radiation mode, produces decoherence dynamically in the thermodynamic limit. For our aim, we start initially with a macroscopic quantum superposition state (generally known in literature as a Schrödinger cat states) given by [19]

\[
|\psi(0)\rangle = N(|\alpha e^{i\phi}\rangle + |\alpha e^{-i\phi}\rangle)|\chi\rangle
\]

being \(|\chi\rangle\) the contribution of the two-level atoms to be specified further, \(\alpha\) a real number, \(\phi\) a phase and \(N\) a normalization factor given by

\[
N^2 = \frac{1}{2 + 2\cos[\alpha^2 \sin(2\phi)] \exp(-2\alpha^2 \sin^2 \phi)},
\]

accounting for the nonorthogonality of the two coherent states.

Then, we consider the strong coupling regime of the Hamiltonian (2) as done in Ref.[16] by assuming as an unperturbed Hamiltonian

\[
H_u = \omega a^\dagger a + g \sum_{i=1}^N \sigma_{xi}(a + a^\dagger).
\]
We can compute the unitary evolution operator by treating formally
\[ \Sigma_x = \sum_{i=1}^{N} \sigma_{xi} \] as a c-number obtaining

\[ U_F(t) = e^{i\hat{\xi}(t)e^{-i\omega a^\dagger a}} \exp[\hat{\beta}(t)a^\dagger - \hat{\beta}(t)^\dagger] \] (14)

being

\[ \hat{\xi}(t) = \frac{\Sigma_x g^2}{\omega^2}(\omega t - \sin(\omega t)) \] (15)
and

\[ \hat{\beta}(t) = \frac{\Sigma_x g}{\omega}(1 - e^{i\omega t}) \] (16)
operators for the ensemble of two-level atoms.

At this stage, we choose a simplified form of the initial state (5) by
simply taking the eigenstate \[ |1\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle) \] of \( \sigma_x \), with eigenvalue
1, for each atom in the ensemble, so

\[ |\chi\rangle = \prod_{i=1}^{N} |1\rangle_i. \] (17)

This kind of “ferromagnetic” state simplifies the computation giving
us at the leading order in the strong coupling perturbation series [20]

\[ |\psi(t)\rangle \approx U_F(t)|\psi(0)\rangle = e^{i\xi(t)}N(e^{i\phi_1(t)}|\beta(t)e^{-i\omega t} + \alpha e^{i\phi - i\omega t} + \beta(t)e^{-i\omega t} + \alpha e^{-i\phi - i\omega t})|\chi\rangle. \] (18)

We have used the property of the displacement operator for coherent
states so to yield

\[ \xi(t) = \frac{N^2 g^2}{\omega^2}(\omega t - \sin(\omega t)), \] (19)
\[ \beta(t) = \frac{Ng}{\omega}(1 - e^{i\omega t}) \] (20)

and

\[ \phi_1(t) = -i\frac{\alpha}{2}[\beta(t)e^{-i\phi} - \beta^*(t)e^{i\phi}], \] (21)
\[ \phi_2(t) = -i\frac{\alpha}{2}[\beta(t)e^{i\phi} - \beta^*(t)e^{-i\phi}], \] (22)

with the phases \( \phi_1(t) \) and \( \phi_2(t) \) generated by the multiplication of
two displacement operators. Then, it is straightforward to impose
the thermodynamic limit \( N \rightarrow \infty \), keeping \( \alpha \) fixed, to verify that
the macroscopic quantum superposition state is driven to the classical state \( |\beta(t) e^{-i\omega t}\rangle \), proving our assertion. The system decoheres removing the superposition of the states. This state describes a classical radiation field as we are in the thermodynamic limit and so, the quantum fluctuations can be neglected with respect to the mean value [21].

One may ask if this is true decoherence. Actually, looking at the state (18) it appears as if we have just displaced the initial coherent state and it seems that the quantum effects are all there yet and we have not disposed of them as normally happens by a true decoherent model. We prove now, by computing the Wigner function of the state (18), that, in the thermodynamic limit, quantum effects are washed away and decoherence is recovered. Indeed, it is straightforward to obtain just for the interference term containing negativity and oscillations as

\[
W_{INT} = \frac{2}{\pi} \exp \left[ -\left( x + \frac{\sqrt{2}Ng}{\omega}(1 - \cos(\omega t)) - \sqrt{2}a \cos(\phi) \cos(\omega t) \right)^2 \right] \exp \left[ -\left( p + \frac{\sqrt{2}Ng}{\omega} \sin(\omega t) + \sqrt{2}a \cos(\phi) \sin(\omega t) \right)^2 \right] \times \cos \left[ 2\sqrt{2}a \sin(\phi) (p \sin(\omega t) - x \cos(\omega t)) + \alpha^2 \sin(2\phi) + 8\alpha \frac{Ng}{\omega} \sin(\phi)(1 - \cos(\omega t)) \right].
\]

It is easily realized that it depends also on time that appears in strongly oscillating terms like \( \cos \left( 8\alpha \frac{Ng}{\omega} (1 - \cos(\omega t)) \sin(\phi) \right) \) and \( \sin \left( 8\alpha \frac{Ng}{\omega} (1 - \cos(\omega t)) \sin(\phi) \right) \) and a sense should be attached to such terms in the thermodynamic limit \( N \to \infty \). Indeed, this is mathematically possible and such terms can be assumed to be zero in such a limit, e.g. in the sense of Abel or Euler [22]. A physical meaning should be attached to the oscillating part taken to be zero in the thermodynamic limit. Indeed, One can see that the time scale of variation of the oscillating part becomes even more small as \( N \) increases. This means that one could reach, in principle, oscillations on a time scale of the Planck time but one should expect that an average in time happens largely before this can happen. This is the question of the singular limits that are at the foundations of the theory of decoherence as pointed out by Berry [23]. So, the interference term can be neglected and true decoherence happens. This point of view has also been emphasized in Ref. [8] for a
spin interacting with a spin bath. So, we recognize commonality between these models and the limit in the Abel or Euler sense translates into an average in time.

We want to verify if this same effect happens for a superposition of number Fock states. So, we consider an initial state given by

$$\left|\psi(0)\right\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |k\rangle)|\chi\rangle$$  \hspace{1cm} (24)

being $k$ an integer, yielding

$$\left|\psi(t)\right\rangle \approx U_F(t)|\psi(0)\rangle = e^{i\xi(t)} \frac{1}{\sqrt{2}} \left( |\beta(t)e^{-i\omega t}\rangle + e^{-ik\omega t}|k,\beta(t)e^{-i\omega t}\rangle \right)|\chi\rangle$$  \hspace{1cm} (25)

being

$$|k,\beta(t)e^{-i\omega t}\rangle = \sum_{n=0}^{\infty} |n\rangle \langle n|D[\beta(t)e^{-i\omega t}]|k\rangle$$  \hspace{1cm} (26)

a displaced number state [24]. From Ref.[24] we derive the following formulas

$$\langle n|D[\alpha]|k\rangle = \binom{k!}{n!}^{\frac{1}{2}} \alpha^{n-k} e^{-|\alpha|^2} L_{k}^{(n-k)}(|\alpha|^2)$$  \hspace{1cm} (27)

for $n \geq k$, otherwise one has

$$\langle n|D[\alpha]|k\rangle = \binom{n!}{k!}^{\frac{1}{2}} (-\alpha^*)^{(k-n)} e^{-|\alpha|^2} L_{n}^{(k-n)}(|\alpha|^2)$$  \hspace{1cm} (28)

being $L_n^k(x)$ the associated Laguerre polynomials [25]. We put

$$\beta'(t) = \beta(t)e^{-i\omega t}$$  \hspace{1cm} (29)

and then rewrite eq.(25) as

$$\left|\psi(t)\right\rangle \approx e^{i\xi(t)} \frac{1}{\sqrt{2}} (|\beta'(t)\rangle +$$

$$e^{-ik\omega t} \sum_{n=0}^{k-1} |n\rangle \binom{n!}{k!}^{\frac{1}{2}} [-\beta'^*(t)]^{(k-n)} e^{-|\beta'(t)|^2} L_{n}^{(k-n)}(|\beta'(t)|^2) +$$

$$e^{-ik\omega t} \sum_{n=k}^{\infty} |n\rangle \binom{k!}{n!}^{\frac{1}{2}} [\beta'(t)]^{(n-k)} e^{-|\beta'(t)|^2} L_{k}^{(n-k)}(|\beta'(t)|^2) \rangle |\chi\rangle.$$
In the thermodynamic limit we can take $\beta'(t)$ to be increasingly large being proportional to $N$. So, we can approximate the associated Laguerre polynomials as $L_n^k(x) \approx \frac{(-x)^n}{n!}$ for $x \to \infty$ and substituting into eq.(31) one gets

$$|\psi(t)\rangle \approx e^{i\xi(t)} \frac{1}{\sqrt{2}} (|\beta'(t)\rangle + e^{-i\omega t} \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} [\beta'(t)]^n e^{-\frac{|\beta'(t)|^2}{2}}) |\chi\rangle$$

and it is not difficult to recognize the second term on the r.h.s. being again the coherent state $|\beta'(t)\rangle$. Then, the radiation field is driven again toward a coherent state describing a classical field, the initial superposition state being washed out in the thermodynamic limit.

In both cases we are left with corrections to the normalization factor due to the approximations we have introduced. They will not play any role when the average values are computed as we have applied a limit procedure on the wave function implying a recomputation of the normalization factor.

5 Decoherence generated by unitary evolution: Higher order corrections

In this section our aim will be to compute higher order corrections to the results in sec.4 to prove that indeed, these corrections do not modify our argument in the given approximations.

The dual Dyson series can be formally written as [20]

$$U(t) = U_F(t) T \exp \left( -i \int_{t_0}^{t} dt' U_F^{\dagger}(t') H_0 U_F(t') \right)$$

being $T$ the time ordering operator, giving at the first order

$$U^{(1)}(t) = -i U_F(t) \int_{t_0}^{t} dt' U_F^{\dagger}(t') H_0 U_F(t')$$

and, for the second order

$$U^{(2)}(t) = -U_F(t) \int_{t_0}^{t} dt' U_F^{\dagger}(t') H_0 U_F(t') \int_{t_0}^{t'} dt'' U_F^{\dagger}(t'') H_0 U_F(t'').$$
Our aim is to evaluate both the contributions. The first order solution has been put forward, firstly, in Ref.[16]. We introduce a quite general way to evaluate higher order corrections by introducing the reduced unitary evolution as

$$U_{R\chi}(t) = \langle \chi | U(t) | \chi \rangle.$$  \hspace{1cm} (35)

We consider only the transitions with the spin bath untouched, the most relevant for our analysis. We will justify a posteriori this definition. This describes an unitary evolution for the radiation field. As we have seen in Sec.4, the unitary evolution, at the leading order, produces the disappearance of superposition states in the thermodynamic limit. This effect, also true for the reduced unitary evolution, disappears for all the other orthogonal states of the spin bath that appear during the evolution.

Indeed, by eq.(33) the first order gives

$$-ie^{i(N-2)^2\omega^2/(\omega t - \sin(\omega t))} e^{-i\omega a^\dagger a t} e^{N \omega \Delta [\beta(t) a^\dagger - \beta(t)^* a]} \frac{\Delta}{2} \int_0^t dt' e^{4i(N-1)\omega^2/[\omega t' - \sin(\omega t')]} e^{a(t') a^\dagger - \alpha(t')^* a} |\chi\rangle.$$  \hspace{1cm} (36)

with

$$\alpha(t) = \frac{2\beta(t)}{N},$$  \hspace{1cm} (37)

not dependent on $N$. But the first order correction modifies the spin bath state as

$$|\chi\rangle = |1\rangle_1 |1\rangle_2 \cdots |1\rangle_N + |1\rangle_1 |1\rangle_2 \cdots |1\rangle_N + \cdots + |1\rangle_1 |1\rangle_2 \cdots |1\rangle_N,$$

that is a non normalized state orthogonal to $|\chi\rangle$. We normalize this state by multiplying it by the factor $\frac{1}{\sqrt{N}}$ and call it $|\chi'\rangle$. The conclusion is that the first order term is zero and does not contribute to the reduced unitary evolution. But, if we want to know the reduced unitary evolution with respect to the state $|\chi'\rangle$ we can see that now is the leading order that does not contribute being zero, and we have

$$U_{R\chi'}^{(1)}(t) = \langle \chi' | U^{(1)}(t) | \chi' \rangle =$$

$$-ie^{i(N-2)^2\omega^2/(\omega t - \sin(\omega t))} e^{-i\omega a^\dagger a t} e^{N \omega \Delta [\beta(t) a^\dagger - \beta(t)^* a]} \sqrt{\frac{\Delta}{N}} \frac{\Delta}{2} \times$$

$$\int_0^t dt' e^{4i(N-1)\omega^2/[\omega t' - \sin(\omega t')]} e^{a(t') a^\dagger - \alpha(t')^* a}.$$
This term, in the thermodynamic limit $N \to \infty$, has the integral wildly oscillating and giving a contribution proportional to $\frac{1}{N}$. So, this term that modifies the unitary evolution as due to a modified state of the spin bath, can be safely neglected in the thermodynamic limit, being $O \left( \frac{1}{\sqrt{N}} \right)$. This proves a posteriori, as promised, our statement on the relevance of the reduced unitary evolution just with respect to the state $|\chi\rangle$. Other non orthogonal states appear at higher orders, as we will see, but their contribution is even more smaller in the thermodynamic limit.

We can now evaluate the second order contribution to the reduced unitary evolution that is given by

$$U^{(2)}_{R\chi}(t) = \langle \chi | U^{(2)}(t) | \chi \rangle =$$

$$e^{\frac{i \Delta^2}{2} (\omega t - \sin(\omega t))} e^{-i \omega a^\dagger a t} e^{\beta(t) a^\dagger - \beta(t)^* a} \times$$

$$\frac{N \Delta^2}{4} \int_0^t dt' e^{-4i(N-1) \frac{\omega^2}{2} (\omega t' - \sin(\omega t'))} e^{-\alpha(t') a^\dagger + \alpha(t')^* a} \times$$

$$\int_0^{t'} dt'' e^{4i(N-1) \frac{\omega^2}{2} (\omega t'' - \sin(\omega t''))} e^{\alpha(t'') a^\dagger - \alpha(t'')^* a}$$

and we have two wildly oscillating integrals. Again we can estimate this term, in the thermodynamic limit, as being $O \left( \frac{1}{N} \right)$ and then, it is even much faster in going to zero with respect to the first order term. This proves our assertion that, in the thermodynamic limit, the unitary evolution is able to wash out quantum superposition states, if the initial state of the spin bath is properly set. Besides, we point out that also the second order term of the Dyson series changes the state of the spin bath due to the product of $H_0$ terms. We do not report this modified state here as not essential for our discussion.

The main conclusion of this section is that, in the thermodynamic limit, the leading order unitary evolution tends to an exact solution of the Schrödinger equation. This solution, in the same limit, can produce decoherence washing out quantum superposition states.

6 Discussion and Conclusions

We have showed that a single radiation mode, interacting with an ensemble of two-level systems, can undergo decoherence during its unitary evolution. The proof is obtained using perturbation theory
and assuming strong coupling between the field and the ensemble of two-level systems. At the leading order, the unitary evolution washes out superposition states when the thermodynamic limit is taken and the ensemble of two-level systems is properly initially prepared. The proof is given both for a macroscopic quantum superposition state and for a superposition of Fock states. Higher order corrections till second order are then computed, proving that the leading order solution tends to be an exact one in the thermodynamic limit. So, the classical radiation field is an attracting solution when a radiation mode interacts with a very large ensemble of two-level atoms. It is interesting to note that this solution of our model seems to be a genuine example of dynamical collapse of the wave function.

This result opens up the possibility to prove a quite general result. That is, when $N$ non interacting particles can realize classical evolution of observables, one may ask if, interacting with such a system and assuming the proper initial state, such an effect of decoherence may be an ubiquitous one. Indeed, our approach proves to be a useful extension of the formal analogy between statistical physics and quantum mechanics.

**Acknowledgement**

I have to thank Kazuyuki Fujii for many comments that showed to be useful to improve the paper.

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