Isgur-Wise Function on the Lattice

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We review our method and numerical results for calculation of the Isgur-Wise function on the lattice. We present a discussion of the systematic errors. Using recent experimental results, we find $V_{cb} = 0.044 \pm 0.005 \pm 0.007$.

1. Method

A very active subfield in high energy physics recently is the study of hadrons with heavy-light quark content [1]. A major effort has been spent in calculating the Isgur-Wise function, which, once it is determined, can be widely used in calculations of heavy meson decay ($b \to c$) processes. After an initial exploration [2], calculations of the Isgur-Wise function on the lattice [3–5] have quickly obtained interesting results which can be directly compared with experimental data and can be used to determine one of the elements of the CKM matrix, $V_{cb}$, in the Standard Model.

For calculating the Isgur-Wise function, $\xi(v \cdot v')$, on the lattice, we have proposed [2] to use the flavor symmetry of the heavy quark effective theory (HQEFT) [6] and measure the $D \to D$ elastic scattering matrix element

$$< D_v | \bar{c} \gamma_\nu c | D_v > = m_D C_{cc}(\mu) \xi(v \cdot v'; \mu)(v + v')_\nu ,$$

where $m_D$ is D meson mass, $v$ and $v'$ are four-velocities of the initial and final D mesons. The constant $C_{cc}(\mu)$ represents the QCD renormalization effect from the heavy quark scale to a light scale $\mu$. The calculation was performed in the quenched approximation using Wilson fermions. Both light and heavy quarks are treated as propagating. For details of the numerical simulation, refer to Refs. [2,3].

From the lattice point of view, calculating the elastic scattering matrix element has significant advantages. In comparison to the $B \to D$ process, the elastic process on the lattice has much less noise and therefore has smaller statistical errors. Furthermore, because of the exactly known value

$$< D_v | \bar{c} \gamma_\nu c | D_v > = 2m_D ,$$

at the “zero recoil” point $v' = v$, the lattice artifacts that are independent of momentum can be removed without ambiguity using Eq. (2) as normalization condition for lattice data [3,4]. A similar strategy for $B \to D$ decay would have introduced an extra (unknown) $O(1/m_Q^2)$ correction. Therefore, not surprisingly, the most accurate data obtained so far on the lattice are from $D \to D$ elastic scattering [3,4]. However, inelastic processes, such as $B \to D$ and $B \to D^*$, can be valuable consistency checks [4,5].

2. Systematic Errors

For analysis of the systematic errors, let us consider the slope, $\rho^2$, of the Isgur-Wise function at $y \equiv v \cdot v' = 0$. A fit of the lattice data [3] to the relativistic harmonic oscillator model [12]

$$\xi(y) = \frac{2}{y + 1} \exp \left( -2\rho^2_{NR} - \frac{y - 1}{y + 1} \right) ,$$

gives $\rho^2_{NR} = 1.41(19)$. For a model independent determination of $\rho^2$, one may choose to fit $\xi(y)$ near $y = 1$

$$\xi(y) = 1 - \rho^2(y - 1) ,$$

where $\rho^2$ is the slope most relevant to the calculation.
and obtain $\rho^2 = 1.24(26)$. All the fits have taken account of the correlations between data points using covariance matrices. There are several potential sources of systematic corrections: quenching, scaling violation, light quark mass $m_q$ dependence, finite volume effect, heavy quark mass $m_Q$ dependence.

Quenching. The error due to quenching is the most difficult to quantify. Although the effect is expected to be small if a scale such as $f_\pi$ is set to the physical value (we use Ref. 1 to set the scale with $f_\pi$), a systematic numerical study is still lacking. We will not give an assessment on the quenching effect here.

Scaling violation. Since by using the normalization condition Eq. (2) all the momentum independent lattice artifacts are removed and the remaining scaling violations are proportional to $y - 1$. Therefore, we expect the residual scaling violations to be small. A fit to data at $\beta = 6.3$ and $\beta = 6.0$ found a difference of 13% for $\rho^2_{NR}$.

A direct check on the Euclidean invariance on the lattice is to measure the ratio of the form factors $f_- / f_+$. This ratio was found small and consistent with zero within large errors 23.45.

Light quark mass ($m_q$) dependence. Our lattice data for $\xi(y)$ are presented with $m_q$ set to the strange quark mass, $m_q = m_s$. These data are directly relevant to processes such as $B_s \to D_s$, $B_s \to D_s^*$. For $B \to D$, they have to be extrapolated to the “chiral limit” $m_q = m_{u,d}$. An inspection shows that the linear size of the physical volume is in the range of $(100 \text{MeV})^{-1}$. Therefore, at $m_q < m_s$ the finite size effect becomes important and contaminates the $m_q$ dependence.

To estimate $m_q$ dependence we therefore use data obtained on the largest physical volume ($24^3 \times 39$ lattice at $\beta = 6.0$). We first estimate the shift in $\rho^2_{NR}$ from $m_q$ to $m_q'$ with both $m_q, m_q'$ in the range of $m_s$. Then this shift in $\rho^2_{NR}$ is extrapolated to the chiral limit. Using this procedure, we find $\rho^2_{NR}$ decreases by 12% from $m_q = m_s$ to $m_q = m_{u,d}$. It is interesting to note that the sign of this shift is opposite to the chiral perturbation prediction 71 and in agreement to the bag model calculation 72. It is important to confirm this trend in the future with improved statistics.

Finite volume effect. To estimate the finite volume effect, we compare our data on $16^3 \times 39$ and $24^3 \times 39$ lattices at $\beta = 6.0, \kappa_q = .154 (m_q = m_s)$. There is a shift of 15% in $\rho^2_{NR}$. We expect that the finite size effect would be smaller at a heavier $m_q$. Indeed, the shift in $\rho^2_{NR}$ is reduced to 9% at $\kappa_q = .152$.

Heavy quark mass ($m_Q$) dependence. Recent lattice calculations indicated that the heavy quark symmetry begins to set in in the neighborhood of the charm mass. The leading $1/m_Q$ dependence agrees with the expectations of HQEFT. We refer to Ref. 13 for discussions of specific examples. Therefore, simulation results obtained at the charm mass range can be used and extrapolated to the heavy quark limit. For the Isgur-Wise function the leading order $1/m_Q$ correction should be $\sim (y - 1)\Lambda_{QCD}/m_Q$. It should be relatively small for current lattice calculations $y - 1 < 0.2$. Indeed, comparing $\rho^2_{NR}$ at $m_Q \sim 1.6 \text{GeV}$ and $2.3 \text{GeV}$, we find 15% shift.

Summary. Adding up the above items in quadrature, the total systematic correction becomes 29%. We have

$$\rho^2_{NR} = 1.41 \pm 0.19 \pm 0.41,$$

where the first error is statistical and the second is systematic error. For linear fit, we get

$$\rho^2 = 1.24 \pm 0.26 \pm 0.36,$$

We should point out that this 29% systematic error is probably an overestimate. Our fit in Eqs. (5) and (6) have been performed with data at all $\beta$, lattice size, heavy quark mass values. Therefore, the combined systematic error is unlikely to be much larger than the statistical error (.19). Indeed, though we use it as an indication of the systematic errors, the shift in $\rho^2$ due to each item discussed above is not statistically significant. To get a better analysis of the systematic errors, one needs more data points and better statistics. At this point, our discussion of the systematic errors should be taken primarily as a discussion on the methodology; the estimates obtained are only qualitative.

A comparison with continuum model calculations is given in Ref. 3. Clearly, lattice result
has reached similar, if not better, numerical accuracy as the continuum models for $\rho^2$.

Our result for $\rho_{NR}^2$ is also consistent with a recent lattice calculation by UKQCD Collaboration [4,5].

3. Extracting $V_{cb}$

Although $\rho^2$ is useful for comparison with continuum model calculations, it is less useful for getting $V_{cb}$ from the experimental data. Around $y = 1$, experimental data have the lowest statistical precision [7]. On the lattice, a model independent determination of $\rho^2$ also tends to have larger uncertainty because only a few data point close to $y = 1$ can be used.

In ARGUS (and CLEO) experiments, what has been measured is $|V_{cb}|\xi(y)$ with the most accurate data obtained in the range $1.1 < y < 1.5$. In the lattice calculation, we have obtained $\xi(y)$ in the range $1 < y < 1.2$. Therefore, at least in the range $1.1 < y < 1.2$ we can directly fit the experimental data with the lattice data with only one unknown parameter $V_{cb}$. One such fit is shown in Fig. 1. We obtain

$$|V_{cb}|\sqrt{\frac{\tau_B}{1.53 \text{ps}}} = 0.044 \pm .005 \pm .007,$$

where the first error is due to the statistical and systematic errors in the lattice calculation and the second error is from the experimental uncertainties. The errors on the lattice data essentially reflect the spread of $\xi$ over different $\beta$, lattice size, and heavy quark mass values.

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