Compact quantum distance-based binary classifier

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Quantum computing opens exciting opportunities for kernel-based machine learning methods, which have broad applications in data analysis. Recent works show that quantum computers can efficiently construct a model of a classifier by engineering the quantum interference effect to carry out the kernel evaluation in parallel. For practical applications of these quantum machine learning methods, an important issue is to minimize the size of quantum circuits. We present the simplest quantum circuit for constructing a kernel-based binary classifier. This is achieved by generalizing the interference circuit to encode data labels in the relative phases of the quantum state and by introducing compact amplitude encoding, which encodes two training data vectors into one quantum register. When compared to the simplest known quantum binary classifier, the number of qubits is reduced by two and the number of steps is reduced linearly with respect to the number of training data. The two-qubit measurement with post-selection required in the previous method is simplified to single-qubit measurement. Furthermore, the final quantum state has a smaller amount of entanglement than that of the previous method, which advocates the cost-effectiveness of our method. Our design also provides a straightforward way to handle an imbalanced data set, which is often encountered in many machine learning problems.

I. INTRODUCTION

As the quest for fault-tolerant quantum computers continues, noisy intermediate-scale quantum (NISQ) computers are expected to be available in the near future [1], supported by the recent technological advances in quantum computing [2–9]. Although the size of quantum circuits that NISQ devices can execute reliably is limited, the size of the quantum state space efficiently manipulated by them is much beyond what classical computers can handle. As such an interesting era is within reach, an important task in the quantum computing community is to find problems and applications for which the NISQ technology can provide practical quantum advantage.

Machine learning has been considered as a promising domain for which quantum computing can shine [10–15]. Quantum advantages in machine learning are expected, since quantum computers can in principle store and manipulate the amount of classical information that scales exponentially with the number of qubits [16–18]. Moreover, quantum computers can reduce the computational cost exponentially for solving certain basic linear algebra problems [19,20] that often appear as basic subroutines in machine learning tasks, such as in support vector machine [21] and principal component analysis [22]. However, the size of quantum circuits required for implementing basic linear algebra subroutines on a quantum computer is too large for near-term quantum devices.

Several quantum machine learning algorithms have been proposed to perform the kernel-based classification by exploiting the ability of quantum computers to efficiently evaluate inner products in an exponentially-large Hilbert space [15], and without relying on expensive subroutines [21,23–27]. In particular, Refs. [25,26] proposed the most simple quantum circuit for utilizing the quantum interference effect now known as the Hadamard-test classifier (HTC) and show that it can be used as a simple model of a kernel-based classifier for real-valued data. If an efficient state preparation routine is known, the algorithm achieves the logarithmic scaling in the dimension and number of the input data with a simple setup. Namely, given a quantum state that encodes the classical data in a specific form, the algorithm only uses a Hadamard gate and the expectation measurement of a two-qubit observable to complete the labeling task. Furthermore, the algorithm is agnostic to the quantum data encoding method, such as amplitude encoding [12] and quantum feature mapping [21].

In this work, we present a kernel-based quantum binary classifier that is even simpler than HTC by introducing compact amplitude encoding (CAE) of real-valued data, which reduces the number of training steps linearly and the number of qubits by two. In CAE, training data belonging to one class is encoded as the real part of the probability amplitudes, while the remaining training data is encoded as the imaginary part. In order to utilize CAE, we show that the single-qubit interfering circuit of the HTC can be generalized to take the imaginary part of the quantum state into account. In this way, the label information encoding is not explicitly executed on a quantum circuit and two sets of data are encoded in
a single quantum register, thereby eliminating the state preparation subroutine for preparing the label registers and reducing the number of index qubits. Furthermore, our classifier provides a simple method for assigning arbitrary weights to two training data sets with different labels, which broadens the application of our method to imbalanced data sets in which the number of training data points in two classes are unequal. The CAE also localizes data so that the entanglement is reduced compared to the HTC. Although this finding is only on a numerical basis, we suspect that this is a general feature. This could lead to better performance of the classifier in the NISQ era since less entanglement implies reduced circuit complexity [28–31].

The remainder of the paper is organized as follows. Section II provides theoretical backgrounds for this study, such as the description of the classification problem and the existing kernel-based quantum classifier that has been known to be simplest. Section III explains the main classification algorithm proposed in this work. In Sec. IV we carry out the entanglement analysis with numerical simulations of classification on Iris and Wine data sets as examples. The simulation results show that the classifier proposed in this work is more compact than the HTC with respect to the amount of entanglement the circuit produces. Section V concludes and discusses future research directions.

II. PRELIMINARIES

A. Binary classification

Classification is a fundamental problem in machine learning. The goal of $L$-class classification is to infer the class label of an unseen data point $\hat{x} \in \mathbb{R}^N$, given a labelled data set

$$\mathcal{D} = \{(x_0, y_0), \ldots , (x_{M-1}, y_{M-1})\} \subset \mathbb{R}^N \times \{0, 1, \ldots , L-1\}. \label{eq:D}$$

In this work, we focus on real-valued data as is common in usual machine learning tasks. One famous way for encoding classical information as a quantum state is the amplitude encoding which represents a classical vector $x_j = (x_{0j}, \ldots, x_{(N-1)j})^T \in \mathbb{R}^N$ as a quantum state in the following form,

$$|x_j\rangle := \sum_{i=0}^{N-1} x_{ij} |i\rangle, \label{eq:xj}$$

using $\lceil \log_2(N) \rceil$ qubits, where the input vector is normalized and have unit length, i.e. $\|x_j\| = 1$. Similarly, a set of $M$ data points $x_1, \ldots, x_M$ can be encoded in $\lceil \log_2(NM) \rceil$ qubits as

$$\frac{1}{\sqrt{M}} \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} x_{ij} |i\rangle \otimes |j\rangle. \label{eq:xM}$$

Hereinafter we focus on binary classification (i.e. $L = 2$) like majority of works on quantum kernel-based classifiers since a multi-class classification can be constructed with binary classifiers by one versus all or one versus one scheme [22]. Moreover, we use $\pm 1$ to label two classes. Finally, we will omit the Kronecker product symbol ($\otimes$) whenever the meaning is clear (e.g. $|i\rangle \otimes |j\rangle = |ij\rangle$).

B. Hadamard-test classifier

This work focuses on generalizing and improving the kernel-based quantum classifier presented in Ref. [25], since it establishes a simple model of quantum classifier for real-valued data. This algorithm is referred to as Hadamard-test Classifier (HTC). Construction of this algorithm can be broken into two parts: preparation of a quantum state that encodes data in a specific form and expectation value measurement. These are explained in more detail below.

The Hadamard-test classifier encodes the data set $\mathcal{D}$ in a quantum state as

$$|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{j=0}^{M-1} \sqrt{a_j} (|0\rangle |x_j\rangle + |1\rangle |\bar{x}\rangle) |y_j\rangle |j\rangle, \label{eq:psi}$$

where $|x_j\rangle$ and $|\bar{x}\rangle$ encodes classical training and test data vectors via an encoding of choice, and the label $y_j \in \{-1, +1 \}$ is represented by the computational basis of the label qubit as $y_j \rightarrow \{(1 - y_j)/2 \} \in \{|0\rangle, |1\rangle\}$. In Ref. [25], the weights are uniform, i.e. $a_j = 1/M \forall j$, but it can be left as a variable to be optimized, similar to the treatment in support vector machines [27]. The measurement scheme utilizes a Hadamard-test, which applies a Hadamard gate on the ancilla qubit to interfere expectation value of a two-qubit observable $\sigma_z^{(a)} \sigma_z^{(l)}$ on the ancilla qubit and the label qubit, one obtains

$$\langle \psi|H^{(a)} \sigma_z^{(a)} \sigma_z^{(l)} H^{(a)} |\psi\rangle = \sum_{j=0}^{M-1} a_j y_j \langle \bar{x}|x_j\rangle, \label{eq:score}$$

where the superscripts $a$ and $l$ indicate that the corresponding operator is acting on the ancilla qubit and the label qubit, respectively. From this equation, one can see that the kernel function in HTC is $k(x_j, \bar{x}) = \langle x_j|\bar{x}\rangle$, and the right hand side of Eq. (1) defines the classification score, which we denote by $f$, in an HTC. The HTC assigns a new label to the test data as

$$\hat{y} = \text{sgn} \left( \sum_{j=0}^{M-1} a_j y_j \langle x_j|\bar{x}\rangle \right). \label{eq:yhat}$$
III. COMPACT CLASSIFIER

A. Generalization

The Hadamard-test classifier can be generalized by using arbitrary single qubit rotation gates instead of the Hadamard gates on the ancilla qubit for creating superposition in the beginning and interference at the end. In this case, Eq. (3) becomes

\[
|\psi\rangle = \sum_{j=0}^{M-1} \sqrt{a_j} \left( \cos \left( \frac{\theta_j}{2} \right) |0\rangle + \sin \left( \frac{\theta_j}{2} \right) e^{-i\phi} |1\rangle \right) |y_j\rangle |j\rangle.
\]

(6)

The Hadamard gate at the end of the circuit for interfering two subspaces spanned by the computational basis of the ancilla qubit is also replaced with an arbitrary rotation around the y-axis of the Bloch sphere \( R_y(\theta_1) = \cos(\theta_1/2)I - i \sin(\theta_1/2)\sigma_y \). This gate can be followed by an arbitrary rotation around the z-axis, but since we are also measuring in the \( \sigma_z \)-basis, we can neglect it as it does not alter the measurement result. Then the two-qubit expectation value measurement gives

\[
\langle \sigma_z^{(a)} \sigma_z^{(l)} \rangle = \sum_{j=0}^{M-1} a_j y_j \left( \cos(\theta_0) \cos(\theta_1) - \sin(\theta_0) \sin(\theta_1) \right) \\
\times \left( \cos(\phi) \text{Re} \langle \tilde{x} | \tilde{x} \rangle - \sin(\phi) \text{Im} \langle \tilde{x} | \tilde{x} \rangle \right).
\]

(7)

This equation shows that the imaginary part of the state overlap (\( \tilde{x} | \tilde{x} \rangle \)) can also contribute to the classification result, unlike in the original HTC. Since the goal of this paper is to utilize the imaginary part, we set \( \theta_0 = \pi/2 \) and \( \theta_1 = -\pi/2 \) for simplicity. In this case, Eq. (7) becomes

\[
\langle \sigma_z^{(a)} \sigma_z^{(l)} \rangle = \sum_{j=0}^{M-1} a_j y_j \left( \cos(\phi) \text{Re} \langle \tilde{x} | \tilde{x} \rangle - \sin(\phi) \text{Im} \langle \tilde{x} | \tilde{x} \rangle \right).
\]

(8)

The above result can also be obtained by applying a single-qubit rotation gate \( R_z(\phi) = \cos(\phi/2)I - i \sin(\phi/2)\sigma_z \) to the ancilla qubit of the HTC in Eq. (3).

B. Compact amplitude encoding

The main idea in this work is based on the observation that if \( \cos(\phi) \) and \( \sin(\phi) \) in Eq. (8) have the same sign, then the real and imaginary parts of the state overlap contributes with opposite sign. Thus by encoding the training data with label +1 (-1) to real (imaginary) part of the probability amplitudes of the quantum state, the binary classification can be done without explicitly preparing the label register. In fact, in the latter case, the label \( y_j \) must have the same sign for all \( j \). This means that \( |y_j\rangle = |0\rangle \) or \( |1\rangle \) for all \( j \), and hence it can be factorized in Eq. (9).

With this background, the compact amplitude encoding (CAE) is introduced to utilize the imaginary part as follows. In CAE, two \( N \)-dimensional real vectors \( x_k^+ = (x_{0k}^+, \ldots, x_{(N-1)k}^+) \) and \( x_k^- = (x_{0k}^-, \ldots, x_{(N-1)k}^-) \) with the corresponding labels indicated by the superscript \( (\pm) \) are loaded in a quantum state in the following form,

\[
|x_k\rangle_c := \sum_{j=0}^{N-1} (x_{jk}^+ + ix_{jk}^-) |j\rangle,
\]

(9)

where \( \|x_k^+\|^2 + \|x_k^-\|^2 = 1 \) to satisfy the normalization condition. Note that various scaling methods can be employed to satisfy this condition, and one of the natural ways is to scale each vector so that \( \|x_k^+\| = 1/\sqrt{2} \). We implicitly assume this way of normalizing vectors unless stated otherwise. The above equation shows that two \( N \)-dimensional vectors are encoded in [\( \log_2(N) \)] qubits. The subscript \( c \) in Eq. (9) distinguishes the quantum state from amplitude encoding, and indicates that the classical vector is encoded via CAE. In addition, we define two more states as

\[
|x_k^\pm\rangle_c := \frac{1}{\|x_k^\pm\|} \sum_{j=0}^{N-1} x_{jk}^\pm |j\rangle.
\]

Then the state overlap between two quantum registers, one encodes an \( N \)-dimensional real vector \( \tilde{x} \) via amplitude encoding (Eq. (1)) and the other encodes two \( N \)-dimensional real vectors \( x_k^+ \) and \( x_k^- \) via CAE (Eq. (9)) is

\[
\langle \tilde{x} | x_k \rangle_c = \frac{1}{\sqrt{2}} \left( \langle \tilde{x} | x_k^+ \rangle + i \langle \tilde{x} | x_k^- \rangle \right).
\]

(11)

C. Compact quantum binary classifier

The compact quantum machine learning algorithm that implements the binary classifier from Eq. (3) is constructed as follows. We first prepare an initial state that encodes the data set \( \mathcal{D} \) as

\[
|\psi_i\rangle = \frac{1}{\sqrt{2}} \sum_{j=0}^{M-1} \sqrt{a_j} \left( |0\rangle |x_j\rangle_c + e^{-i\phi} |1\rangle |\tilde{x}\rangle \right) |j\rangle,
\]

(12)

where the subscript \( i \) indicates that the state is prepared via CAE. It is important to note that \( \sum_{j=1}^{M/2} b_j = 1 \), and hence the set of weights is different from that of the Hadamard-test classifier which satisfies \( \sum_{j=0}^{M-1} a_j = 1 \). For example, for a set of uniform weights, \( b_j = 2/M \) and \( a_j = 1/M \). For simplicity, we assume that the number of data with label +1 denoted by \( N_+ \) is equal to the number of data with label -1 denoted by \( N_- \). The state above is easier to prepare than the state required in HTC shown in Eq. (3) since the label information is not explicitly encoded and the relative phase \( e^{-i\phi} \) can be
added by applying a single-qubit rotation gate $R_x(\phi)$ on the ancilla qubit. Moreover, since two training data are encoded in one quantum register, the number of terms in the summation is decreased by a factor of 2, meaning that the dimension of the index register is also decreased by a factor of 2. After the state preparation, the rest of the algorithm only requires a Hadamard gate and the measurement of the ancilla qubit in the $\sigma_z$ basis. The Hadamard gate interferes the copies of the new input and the training inputs to produce a state

$$|\psi_f\rangle = \frac{1}{\sqrt{2}} \sum_{j=0}^{M-1} \sqrt{b_j} \left[ |0\rangle (|x_j\rangle_c + e^{-i\phi} |\bar{x}\rangle) + |1\rangle (|x_j\rangle_c - e^{-i\phi} |\bar{x}\rangle) \right]|j\rangle. \quad (13)$$

The probability to measure the ancilla qubit in $|0\rangle$ is given as

$$\Pr(0) = \frac{1}{4} \sum_{j=0}^{2^M-1} b_j \left( 2 + e^{-i\phi} c_\langle \bar{x} | x_j \rangle + e^{i\phi} \langle \bar{x} | x_j \rangle_c \right).$$

By denoting $\kappa_j = \langle \bar{x} | x_j \rangle_c$, the above equation can be written as

$$\Pr(0) = \frac{1}{2} \sum_{j=0}^{2^M-1} b_j \left( 1 + \cos(\phi) \text{Re}(\kappa_j) - \sin(\phi) \text{Im}(\kappa_j) \right). \quad (14)$$

Similarly, the probability to measure the ancilla qubit in $|1\rangle$ can be calculated as

$$\Pr(1) = \frac{1}{2} \sum_{j=0}^{2^M-1} b_j \left( 1 - \cos(\phi) \text{Re}(\kappa_j) + \sin(\phi) \text{Im}(\kappa_j) \right). \quad (15)$$

Therefore, the expectation value of the $\sigma_z$ operator measured on the ancilla qubit is

$$\langle \sigma_z^{(a)} \rangle = \sum_{j=0}^{2^M-1} b_j \left( \cos(\phi) \text{Re}(\kappa_j) - \sin(\phi) \text{Im}(\kappa_j) \right). \quad (16)$$

By setting $\cos(\phi) = \sin(\phi) = 1/\sqrt{2}$ and using Eq. (11), we arrive at

$$\langle \sigma_z^{(a)} \rangle = \frac{1}{2} \sum_{j=0}^{2^M-1} b_j \left( \langle \bar{x} | x_j^+ \rangle - \langle \bar{x} | x_j^- \rangle \right). \quad (17)$$

Since the state overlap for the training data in class $+1$ ($-1$) always have the positive (negative) sign, the above equation can be written in its final form as

$$\langle \sigma_z^{(a)} \rangle = \frac{1}{2} \sum_{j=0}^{M-1} b'_j y_j \langle \bar{x} | x_j \rangle,$$  

where $b'_m = b'_{m+M/2} = b_m$ for $m = 0, \ldots, M/2 - 1$ and $\sum_{j=0}^{M-1} b'_j = 2$. This outcome is very similar to the classification score in the Hadamard classifier obtained by the two-qubit measurement. The constant factor of 1/2 is attributed to the different normalization conditions between the set of weights $b_j$ and $a_j$ as described below Eq. (12). We refer to this classifier as compact Hadamard classifier (CHC). The CHC is obtained with a single-qubit measurement implying that one can bypass the standardization of the training data set required in HTC for increasing the post-selection probability. The comparison of quantum circuits for implementing HTC and CHC is depicted in Fig. 1.

The reduction of quantum circuit sizes, which is of critical importance in practice, achieved with CHC is as follows. By introducing a controllable relative phase between two computational basis states of the ancilla qubit and using compact amplitude encoding, the number of qubits is reduced by two, one for the label register and another in the index register. Then the quantum operations for encoding the training data set is reduced from $M$ gates controlled by $\log_2(M)$ index qubits to $M/2$ gates controlled by $\log_2(M) - 1$ index qubits. Having one fewer controlled qubit can further reduce the quantum circuit depth by a factor of two $[33]$. Therefore, the number of operations for encoding the training data set $x_j$ is reduced by a factor of four. The number of operations in CHC is also reduced due to the removal of operations for explicitly encoding the label information in a separate register. The number of gates needed for preparing the label register in HTC is 1 when the number of data belonging to each class is equal since one controlled-NOT (CNOT) gate controlled by one of the index qubits applied to the label qubit can split the Hilbert space into two subspaces with an equal number of 0’s and 1’s in the label qubit. But the number of operation increases linearly with the difference in the number of data with different labels. For example, if $\alpha M$ and $\beta M$ data belong to class $+1$ and $-1$, respectively, where $\alpha + \beta = 1$, then the number of operations necessary to encode label information grows as $|\alpha - \beta|M$. Therefore, in total, our compact classifier can reduce the number of operations at least by a factor of four, while the reduction can be larger depending on the label distribution of the given training data set. Furthermore, the two-qubit measurement scheme used in HTC is reduced to single-qubit measurement.

As elucidated above, the number of training data vectors in two classes may be different. In this case, the real or imaginary part of the quantum state is simply zero for the missing data. The difference in the number of training data vectors in two classes can later be compensated by controlling the weights between the real and imaginary parts of the state overlap by finding $\phi$ that satisfies

$$\frac{\sin(\phi)}{\cos(\phi)} = \frac{N_-}{N_+},$$

where $N_{\pm}$ is the number of training data points in $\pm 1$ classes.
D. Smallest quantum binary classifier

Let us denote \(|\Psi(x_j, \tilde{x})\rangle = (|0\rangle \langle x_j | + e^{-i\phi} |1\rangle \langle \tilde{x} |)/\sqrt{2}.
By allowing classical sampling from an ensemble \(\{a_j, |\Psi(x_j, \tilde{x})\rangle\}\), where \(a_j\) is the probability to choose \(j\)th state, we can have the mixed state
\[
\frac{M-1}{2} \sum_{j=0}^{M-1} a_j |\Psi(x_j, \tilde{x})\rangle\langle\Psi(x_j, \tilde{x})|.
\]
(19)

It is easy to verify that the expectation measurement of \(\sigma_x\) operator (equivalent to application of a Hadamard gate followed by \(\sigma_x\) measurement) yields the same outcome as shown in Eq. (16). In this approach, the index register is unnecessary, and hence we further reduce the number of qubits by \([\log_2(M)]\) and the number of gates by \(O(\text{poly}(M))\). This the smallest kernel-based binary classifier; one only requires \([\log_2(N)]\) qubits for encoding the data set, and a qubit for measurement.

E. Connection to quantum feature mapping

Under certain restrictions, the compact encoding scheme can be applied to the quantum feature mapping framework introduced in Ref. [23] to store two training data in one quantum register. In principle, this can be done with the state preparation
\[
|\Phi(x^\pm_k)\rangle = \frac{|\Phi(x^+_k)\rangle + i|\Phi(x^-_k)\rangle}{\sqrt{2}},
\]
(20)
with a feature map \(U_\Phi(x) |0\rangle^\otimes L = |\Phi(x)\rangle\) that maps \(N\)-dimensional data to \(L = O(N)\) qubits. Of course, the above state must satisfy \(\langle \Phi(x^+_k)|\Phi(x^-_k)\rangle = 0\). The feature map also needs to satisfy \(\langle \Phi(x)|\Phi(x^+_k)\rangle \in \mathbb{R}\) for all \(k\).

One way to prepare the above state is to apply unitary transformation
\[
V = U_\Phi(x^+_k) + iU_\Phi(x^-_k)
\]
\[
\frac{1}{\sqrt{2}} \sum_{j=0}^{M-1} a_j (\cos(\phi) \langle \Phi(x)|\Phi(x^+_k)\rangle - \sin(\phi) \langle \Phi(x)|\Phi(x^-_k)\rangle),
\]
which is reduced to the classifier of Eq. (18) when \(\phi = \pi/4\).

IV. ENTANGLEMENT ANALYSIS

Understanding the fundamental source of the quantum advantage is of critical importance for establishing the ground for further developments of new ideas. The quantum resource we considered in this Section is the entanglement of the system. Besides the fundamental perspective, entanglement is also deeply connected to the quantum circuit complexity. More specifically, it has been
reported that lower amount of entanglement required in a quantum algorithm implies reduction in the number of entangling gates during state preparation \cite{28,31}.

The measure of entanglement we use in this work is the Meyer-Wallach measure \cite{36} which calculates a value linearly related to the mean single-qubit purity of the state \cite{37}:

$$Q(\psi) = 2\left(1 - \frac{1}{n} \sum_{k=1}^{n} \text{Tr}[\rho_k^2]\right)$$  \hspace{1cm} (21)

where $\rho_k$ is the single-qubit density matrix obtained by partitioning $|\psi\rangle$ into one qubit and $n-1$ qubits.

The numerical survey shows that the Meyer-Wallach entanglement in the CHC is always lower than that of the HTC for all sample size $2^n + 1$ and each data set. The results are shown in Fig. 2 and Table I. In particular, the minimum $\Delta$ of each set shows that the value is always positive, hence the CHC is more resource saving in terms of entanglement.

The analysis with Iris and Wine data sets asserts that the classifier presented in this work is compact in the sense that it requires less entanglement for binary classification. The observed reduced entanglement has useful consequences. As mentioned in the beginning of this section, the lower entanglement allows for the reduction in the number of entangling gates by exploiting the consequential low Schmidt-rank of bi-partitions \cite{30,31}. Furthermore, if entanglement is lower, then an approximation with even less entangling operations can be found. We suspect that this can be a useful trait for machine learning protocols, because approximation errors in the state preparation will likely only affect near-decision boundary classifications. Even though the methods described in Refs. \cite{30,31} are computationally expensive subroutines like singular value decomposition, the reduction in the state preparation complexity can be beneficial in the NISQ era, if the classification error due to the approximation is less than that due to the hardware imperfections and decoherence.

\section{V. CONCLUSION}

This work proposes a compact quantum binary classifier whose quantum circuit size is smaller than that of the Hadamard-test classifier, which was previously considered to be the simplest kernel-based quantum classifier. Thus, our method is placed above existing kernel-based quantum classifiers as the potential application of NISQ computing. The compact quantum classifier is enabled by compact amplitude encoding we introduced in this work. This technique encodes one training data point from each class as the real and the imaginary part of the probability amplitude of a computational basis state. Since the label information of training data is implicitly encoded, there is no need to use a separate quantum register to explicitly encode the label information as required in previous methods. The removal of the label register naturally further reduces the two-qubit measurement scheme required in previous methods to the single-qubit measurement. Furthermore, the ease with which unbalanced data can be encoded and the applicability of feature maps is highlighted as good traits for applications. Using binary classification tasks with Iris and Wine data sets, we show that the quantum classifier proposed in this work is compact also in the sense that it requires less entanglement than the Hadamard-test classifier.

State preparation is responsible for the main cost of CHC, HTC, and many machine learning algorithms. With a reduction in entanglement, one can attempt to
Furthermore, analytical analysis of the entanglement discovery of quantum-inspired machine learning algorithms. This could also lead to the designing a compact version of existing quantum machine learning algorithms. In addition, understanding the reason behind the reduced amount of entanglement created in the CHC classifier. In order to reduce the required number of qubits without a reduction in the accuracy of a classifier, one could investigate how to reduce the required number of qubits without a reduction in the accuracy of a classifier. In addition, understanding the reason behind the reduced amount of entanglement created in the CHC algorithm compared to HTC remains an interesting open question. Answering this question could also help in designing a compact version of existing quantum machine learning algorithms. Furthermore, analytical analysis of the entanglement difference ($\Delta$) for the given size of the samples ($M$) and the features ($N$) remains to be done. Another interesting question is the condition on the unitary when using quantum feature maps to encode data, both in terms of expressibility and complexity.

### DATA AVAILABILITY

The source code and data used in this study are available from the corresponding author upon reasonable request.

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| (Iris, 1) | (Iris, 2) | (Iris, 4) | (Iris, 8) | (Iris, 16) | (Iris, 32) | (Wine, 1) | (Wine, 2) | (Wine, 4) | (Wine, 8) | (Wine, 16) | (Wine, 32) |
|----------|----------|----------|----------|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| mean     | 0.251    | 0.230    | 0.186    | 0.171     | 0.157     | 0.144    | 0.064    | 0.063    | 0.062    | 0.059     | 0.057     |
| std      | 0.083    | 0.061    | 0.037    | 0.026     | 0.017     | 0.016    | 0.028    | 0.020    | 0.016    | 0.011     | 0.010     |
| min      | 0.072    | 0.105    | 0.111    | 0.110     | 0.114     | 0.105    | 0.020    | 0.031    | 0.038    | 0.040     | 0.045     |
| 25%      | 0.190    | 0.188    | 0.159    | 0.151     | 0.146     | 0.132    | 0.043    | 0.048    | 0.050    | 0.053     | 0.052     |
| 50%      | 0.256    | 0.230    | 0.180    | 0.172     | 0.160     | 0.147    | 0.060    | 0.060    | 0.060    | 0.057     | 0.056     |
| 75%      | 0.312    | 0.278    | 0.211    | 0.185     | 0.167     | 0.157    | 0.078    | 0.072    | 0.071    | 0.070     | 0.065     |
| max      | 0.472    | 0.420    | 0.269    | 0.237     | 0.191     | 0.168    | 0.132    | 0.129    | 0.125    | 0.088     | 0.089     |

TABLE I. Data of the difference between the entanglement (MW) of the HTC and the compact classifier proposed here from a statistical description of the distribution, i.e. $\Delta = Q_{HTC}(|\psi_f\rangle) - Q_{CQBC}(|\psi_f\rangle)$. Each value is for a data type and a number of training data that are indicated in the first row as (data type, number of data). It can be seen that the minimum is always greater than zero. The Iris data set has 4 features, while the Wine data has 13 features. The table shows the statistical description using the mean, standard deviation, the minimum and maximum and finally the 25, 50, 75 percentiles.

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