Euclidean Auto Calibration of Camera Networks: Baseline Constraint Removes Scale Ambiguity
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Abstract—Metric auto calibration of a camera network from multiple views has been reported by several authors. Resulting 3D reconstruction recovers shape faithfully, but not scale. However, preservation of scale becomes critical in applications, such as multi-party telepresence, where multiple 3D scenes need to be fused into a single coordinate system. In this context, we propose a camera network configuration that includes a stereo pair with known baseline separation, and analytically demonstrate Euclidean auto calibration of such network under mild conditions. Further, we experimentally validate our theory using a four-camera network. Importantly, our method not only recovers scale, but also compares favorably with the well known Zhang and Pollefeys methods in terms of shape recovery.

Index Terms—Camera network, stereo camera pair, baseline separation, auto calibration, point cloud.

I. INTRODUCTION

In a variety of applications, including telepresence [1], remote surgery [2], and virtual education [3], [4], one aspires to reconstruct the three-dimensional (3D) likeness of objects at a remote location. This is often attempted based on views captured by a multitude of cameras [5], [6]. As a first step, the cameras are calibrated, i.e., the intrinsic parameters such as a focal length, as well as the extrinsic parameters such as camera position and orientation are estimated. Traditionally, extraneous objects, such as chessboard patterns, checkered cubes and laser pointer, have been used for calibrating both single camera [7], [8], and network of cameras [1], [9], [10].

In contrast, scene-based auto calibration appears attractive in various scenarios, such as telepresence and remote classroom, where an adaptive camera network dynamically maintains the image quality, pre-calibrated cameras could be restrictive, and extraneous objects may not be introduced [11]. Further, applications such as multi-party telepresence, requiring fusion of multiple scenes, additionally demand preservation of scale. In this backdrop, we propose a Euclidean auto calibration method enabling faithful 3D reconstruction accurate to scale.

Traditionally, a multicamera network, depicted in Fig. 1a, consists of a number of monocular cameras. For such networks, various authors report metric auto calibration, which allows faithful recovery of shape, but not scale [5], [6], [11], [12]. Such a technique would typically obtain point correspondences from the multiple views at hand (practically, often using SIFT or SURF algorithms [13], [14], while removing false correspondences using RANSAC algorithm [5]), and then make use of either a factorization method [16], or Kruppa equations [17]. Adopting the former approach, Han and Kanade exploited inherent structural constraints, and faithfully recovered shape up to scale ambiguity [18], [19]. On the contrary, Kruppa equations, relating intrinsic parameters to the absolute conic, yield those parameters for a pair of cameras up to a scale, thereby introducing projective ambiguity in 3D reconstruction [20]. By fixing the plane at infinity, metric calibration was obtained for invariant and later for slowly varying intrinsic parameters [21], [22], [23]. Metric calibration has also been obtained by imposing other related constraints [24]–[31]. Further, attempts have been made at combining factorization and Kruppa equations towards distributed auto calibration [32]. Subsequently, making use of efficient sparse bundle adjustment (SBA) [33], Furukawa and Ponce proposed a method for auto calibration and dense 3D reconstruction [34]. At the same time, metric auto calibration has been obtained in the stochastic framework [35], [36], [37].

In comparison, significantly less effort has been directed towards Euclidean auto calibration, required for recovery of both shape and scale. Lerma et al. considered a three-camera configuration, imposed baseline constraint on each of the three camera pairs (resulting in an inflexible structure), and reported photogrammetrically improved auto calibration [38]. However, questions on uniqueness of the solution and scalability to larger networks were not addressed. In contrast, building on our earlier work [39], we propose a configurable camera network including a stereo pair with known baseline separation, as depicted in Fig. 1b, and analytically establish its Euclidean auto calibration accurate to scale. Clearly, our analysis applies to the three-camera network of Lerma et al., and hence, just one of their three baseline constraints suffices

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Fig. 1: Camera network: (a) classical; (b) proposed network including a stereo pair with known baseline length.
Fig. 2: (a) Proposed four-camera network including a stereo pair and two monocular cameras; (b), (c), (d), (e) respective images, with highlighted feature points, taken by left monocular, left stereo, right stereo, and right monocular cameras.

for Euclidean auto calibration. Subsequently, we validate our theory experimentally using an arbitrary four-camera network including a stereo pair. In particular, we demonstrate an accurate recovery of the scale, and the shape by directly comparing with the original object using a state-of-the-art lightfield display [40]. Significantly, the proposed method recovers scale from the mild baseline constraint, while still achieving reprojection errors comparable to that achieved by the well-known Zhang and Pollefeys methods [8],[23].

The rest of the paper is organized as follows. The proposed auto calibration problem is formulated as a multi-objective minimization in Sec. II, while the uniqueness condition is derived in Sec. III. Further, Sec. IV experimentally validates our theoretical results. Finally, Sec. V concludes the paper.

II. PROBLEM FORMULATION

Consider an M-camera network. Further, consider N feature points, with homogeneous coordinates $X_j = [X_j, Y_j, Z_j]^T$ (j = 1, 2, ..., N), each visible by all the M cameras [18]. Denote by $\bar{x}_{ij} = [x_{ij}, y_{ij}]^T$ the projection of $X_j$ on the i-th image plane (in local coordinates), i = 1, 2, ..., M. Auto calibration consists in estimating the intrinsic and the extrinsic parameters of the M cameras from $\bar{x}_{ij}$ (i = 1, 2, ..., M, j = 1, 2, ..., N). Assume that the first two cameras (i = 1, 2) form a stereo pair [41] with parallel optical axis and given baseline separation $l$. A typical deployment with M = 4 is shown in Fig. 2a. We shall show that unambiguous auto calibration is possible if $l$ is accurately known. This in turn enables true-to-scale recovery of 3D point cloud $\{X_j\}$.

A. Image formation

Ignoring lens distortion or assuming it is corrected already, image $\bar{x}_{ij}$ of $X_j$ (j = 1, 2, ..., N) by the i-th (i = 1, 2, ..., M) camera is given by [5]

$$
\begin{pmatrix}
\bar{x}_{ij} \\
\bar{y}_{ij} \\
1
\end{pmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
f_i & 0 & 0 & r_i^1 & r_i^2 & r_i^3 & t_{iX}^1 & t_{iY}^1 & t_{iZ}^1 \\
f_i & 0 & 0 & r_i^2 & r_i^3 & r_i^4 & t_{iX}^2 & t_{iY}^2 & t_{iZ}^2 \\
f_i & 0 & 0 & r_i^3 & r_i^4 & r_i^5 & t_{iX}^3 & t_{iY}^3 & t_{iZ}^3
\end{bmatrix}
\begin{pmatrix}
X_j \\
Y_j \\
Z_j
\end{pmatrix},
$$

(1)

where $s_{ij}$ denotes a scale factor. In the right hand side of (1), the first matrix of intrinsic parameters including focal length $f_i$ is diagonal under the assumption that the origin of the image plane lies at its intersection with the principal axis, and the skew factor is zero. The second matrix of extrinsic parameters consists of a $3 \times 3$ unitary rotation parameter matrix concatenated with a $3 \times 1$ translation parameter vector as shown. Our task is to determine all other quantities based on the MN images $\{\bar{x}_{ij}\}$ of the N feature points.

Recall that cameras '1' and '2' constitute the stereo pair, and the rest are indexed '3', '4', 'M'. Now fix the origin of the world coordinate system at the center of Camera 1, and take its principal axis as the Z-axis. Thus the center of Camera 2 has the location $[l 0 0]^T$. Further, for simplicity, assume that the sensor arrays in Cameras 1 and 2 are shifted versions of each other such that both rotation matrices are identity (see, e.g., [41]). Accordingly, for i = 1, 2, (1) yields

$$
X_j = \begin{bmatrix}
x_{ij}y_{ij}l \\
x_{ij}y_{ij}l - x_{ij}y_{ij} \\
x_{ij}y_{ij}l
\end{bmatrix},
$$

(2)

$$
f_2 = \frac{y_{ij}f_1}{x_{ij}},
$$

(3)

$$
Y_j = \frac{y_{ij}f_1}{x_{ij}},
$$

(4)

$$
Z_j = \frac{f_1X_j}{x_{ij}}.
$$

(5)

In view of (2)–(5), $f_1$ remains the only unknown quantity relating to the stereo pair (i = 1, 2). Here (4) forces the ratio $\frac{y_{ij}}{x_{ij}}$ to be the same for every $j = 1, 2, ..., N$. In practice, we shall take average ratio in view of possible errors.

Further, using (5) in (1) for i ≥ 3, we obtain

$$
x_{ij} = X_j \frac{f_3r_i^1}{lZ_j} + Y_j \frac{f_3r_i^2}{lZ_j} + \frac{X_j f_3f_i^3}{x_{ij}} + \frac{f_3 f_i^3}{lZ_j},
$$

(6)

$$
y_{ij} = X_j \frac{f_3r_i^1}{lZ_j} + Y_j \frac{f_3r_i^2}{lZ_j} + \frac{X_j f_3f_i^3}{x_{ij}} + \frac{f_3 f_i^3}{lZ_j},
$$

(7)

which gives rise to multi-objective constrained optimization.

B. Multi-objective optimization

Stacking (6) and (7) (j = 1, 2, ..., N), we get respectively

$$
A_x \bar{x} = \bar{b}_x,
$$

(8)

$$
A_y \bar{y} = \bar{b}_y,
$$

(9)
Fig. 3: 3D visualization of reconstructed point cloud: (a) orthographic projection with feature point correspondence with 2D perspective view; (b), (c), (d) rendering on 3D lightfield display [40]; images taken from approximate perspectives of the left monocular, left stereo, and right monocular cameras, respectively.

Fig. 4: 3D point cloud comparison: (a) proposed method, (b) Zhang method with BA, (c) Pollefeys method with BA, and (d) proposed method degraded by BA. Grossly mislocated feature points are circled in red.

where \( \vec{b}_i^y = \begin{bmatrix} x_{i1} & \ldots & x_{iN} \end{bmatrix}^T \), \( \vec{b}_i^y = \begin{bmatrix} y_{i1} & \ldots & y_{iN} \end{bmatrix}^T \),

\[
\alpha_i = \begin{bmatrix} f_{x1} & f_{y1} & f_{x1} & f_{y1} \end{bmatrix}^T,
\]

\[
\beta_i = \begin{bmatrix} f_{x1} & f_{y1} & f_{x1} & f_{y1} \end{bmatrix}^T,
\]

\[
A_i^x = \begin{bmatrix} x_1 y_1 & 1 & 0 & -x_1 y_1 x_{11}^3 \end{bmatrix},
\]

\[
A_i^y = \begin{bmatrix} x_N y_N & 1 & 0 & -x_N y_N x_{11}^3 \end{bmatrix}
\]

and \( A_i^y \) takes the same form as \( A_i^x \) with each occurrence of \( x_{1j} \) replaced by \( y_{1j} \) (j = 1, 2, ..., N).

In practical error-prone scenarios, rather than (8) and (9), we propose to solve a multi-objective minimization problem:

\[
\min \sum_{i=1}^{M} \lambda_i||A_i^x \vec{a}^i - \vec{b}_i^x|| + (1 - \lambda_i)||A_i^y \vec{b}^i - \vec{b}_i^y||,
\]

where \( \lambda_i \in [0, 1] \) determines the relative weights of reprojection error along \( x \) and \( y \) axes, \( i = 1, 2, ..., M \).

III. EUCLIDEAN AUTO CALIBRATION

Since components of \( \vec{a}^i \) and \( \vec{b}^i \) are dependent (refer (10) and (11)), constituent least squares problems in (13) are nonlinear. Specifically, \( f_{x1} \) and \( \{f_{1x1}, f_{1y1}, f_{2x1}, f_{2y1}, f_{1x1}, f_{1y1}, f_{2x1}, f_{2y1}\}_{i=3}^{M} \) constitute a set of \( 7(M-2) + 1 \) independent variables. To see this, it is enough to verify that \( r_{x1}^i, r_{y1}^i \) and \( r_{x2}^i \) specify the \( 3 \times 3 \) unitary rotation matrix in (1). First, write \[7\]

\[
r_{x1}^i = \frac{-r_{2y1}^i r_{4x1}^i}{1 - r_{11}^2},
\]

\[
\pm \sqrt{1 - 2r_{11}^2 - r_{22}^2 - r_{33}^2 + r_{23}^2 r_{32}^2 + r_{12}^2 r_{21}^2 + r_{13}^2 r_{31}^2 + r_{12}^2 + r_{13}^2 + r_{23}^2}
\]

(14) (dropping index ‘i’ on the right hand side), and hence, using the unitarity property, obtain \( r_{12}^i, r_{13}^i, r_{21}^i, r_{23}^i, r_{31}^i \) and \( r_{32}^i \) up to respective signs. These sign ambiguities translate to eight options for the desired rotation matrix [39]. Thus, theoretically, one may need to solve up to a finite number \( 8^{M-2} \) of minimization problems. Practically, a divide-and-conquer method could reduce the number of problems to solve.

To ensure unique solution, we desire the system to be over-determined. Since we have \( 2N \) equations for the \( i \)-th (\( i = 3, ..., M \)) camera in the form of (8) and (9), i.e., \( 2(M-2)N \) equations altogether, and \( 7(M-2) + 1 \) variables, we require

\[
(M-2)(2N-7) \geq 1.
\]

From (15), we find the following conditions to be both necessary and sufficient: (i) number of cameras \( M \geq 3 \), and (ii) number of feature points \( N \geq 4 \). Finally, upon finding the independent variables minimizing (13), the accurate-to-scale 3D point cloud \{\( X_j \)\} can be obtained from (2), (3) and (5).

IV. EXPERIMENTAL CORROBORATION

At this point, we set up an experimental four-camera network \( M = 4 \), as shown in Fig. 2a, using Basler Ace 1300-30gc GigE cameras [42], fitted with Goyo Optics GMDN24012C lenses [43]. The baseline length for the stereo camera pair \( i = 1, 2 \) is set at \( l = 1255mm \). Next we perform autocalibration as proposed, based on images of \( N = 57 \) feature points, indicated by white dots, in each of the four views shown in Figs. 2b–2e. We now demonstrate accurate Euclidean autocalibration, by establishing accurate recovery of the shape (geometry) and the scale of the reconstructed 3D cloud of the aforementioned feature points.
Next we compare the scales and shapes recovered using the proposed and competing algorithms. Our method yields a cloud, shown in Fig. 4a, that fits inside a rectangular parallelepiped of dimensions 98mm × 168mm × 75mm (X × Y × Z), which closely match physical measurements. The corresponding shape, authenticated visually in Figs. 3b, 3c and 3d, is now taken as a reference. In comparison, the Zhang method with bundle adjustment (BA) [8],[44], shown in Fig. 4b, finds the point cloud inside a volume 72mm × 124mm × 46mm, which considerably deviates from the original scale. This happens because the accurate scale information, obtained during calibration of individual cameras using a chessboard with known dimension, is lost during BA. Also, the shape, with clearly misplaced features, matches the original only roughly, showing performance limits of BA. Further, the Pollefeys method with BA [23], shown in Fig. 4c, clearly ignores the scale, as expected from the theory. However, this method reconstructs the shape reasonably well, which indicates its efficacy, because we have only four as opposed to the customary large number of views.

For the sake of completeness, we now turn to comparing reprojection errors. To this end, in Fig. 5, we plot such error for each feature point and each of the four cameras in three cases, the proposed method, the Zhang method with BA [8], and the Pollefeys method with BA [23]. Notice that the reprojection errors for these three methods are comparable for monocular cameras, while the proposed method, as expected, incurs negligible errors in the stereo cameras. At this point, an attentive reader may ask: Would additional BA improve the proposed method, as it generally does the Zhang and Pollefeys methods? Interestingly, the answer is no. In fact, if subjected to additional BA, our method would further require the fundamental baseline constraint, and as depicted in Fig. 4d, would accordingly cause the reconstructed point cloud to lose the scale and the shape with major mislocations. However, notice in Fig. 5 that the proposed method, upon degradation by additional BA as above, manages to obtain less reproduction error in general, by violating the fundamental baseline constraint. This clearly demonstrates that low reproduction error, by itself, is a poor indicator of calibration accuracy.

V. DISCUSSION

In this paper, we proposed a multicamera network that includes a stereo pair with known baseline separation, and demonstrated, both analytically and experimentally, Euclidean auto calibration of such networks. Our result provides a framework for true-to-scale 3D recovery of objects without interfering with those, thus opening up the possibility of adaptive and unobtrusive reconstruction of dynamic scenes. Of course, to realize a practical system, one would further require faithful surface reconstruction [34], and realistic rendering [45]. Here we hasten to add that ours remains one approach among several towards 3D reconstruction [46]–[49].
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