Role of electron correlations in transport through domain walls in magnetic nanowires

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The transmission of correlated electrons through a domain wall in ferromagnetic quasi-one-dimensional systems is studied theoretically in the case when the domain wall width is comparable with the Fermi wavelength of the charge carriers. The wall gives rise to both potential and spin dependent scattering. Using a poor man’s renormalization group approach, we obtain scaling equations for the scattering amplitudes. For repulsive interactions, the wall is shown to reflect all incident electrons at the zero temperature fixed points. In one of the fixed points the wall additionally flips the spin of all incident electrons, generating a finite spin current without associated charge current.

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Introduction Electronic properties of magnetic wires with domain walls (DWs) attract much interest because of possible applications of the associated magnetoresistance effect [1]. So far, the puzzlingly huge magnetoresistance up to thousands of percents in Ni wires and structured magnetic semiconductors is not well understood [2, 3, 4]. Most of the existing theories of the DW resistance do not take into account the electron correlations [2, 3, 4]. However, in effectively one-dimensional (1D) structures one should take into account the tendency to the non-Fermi-liquid behavior [5, 6, 7, 8, 9], except for the case of a wide DW (adiabatic regime) [10]. Moreover, non-adiabatic DWs (smaller or comparable to the Fermi wavelength) can be achieved in ferromagnetic semiconductor wires with constrictions.

In this Letter we study the effect of electron correlations on the resistance of a ferromagnetic wire with a non-adiabatic DW. Since the DW in a 1D wire acts as a localized spin-dependent scattering center, a strong influence of electron correlations is expected [12]. We apply the renormalization method [14, 15] for the logarithmically diverging terms in the perturbation theory of the electron interactions. We find the zero-temperature fixed points for repulsive interactions, in which the wall reflects all incident electrons. This might explain the giant magnetoresistances observed in magnetic nanowires. Moreover, we show that for one fixed point the electron spin is reversed at the reflection. This leads to a nonzero spin current with no charge current, which is of high interest for applications in spintronics devices.

Model and method We consider a magnetic 1D system with a local exchange coupling between conduction electrons and a spatially varying magnetization $M(r)$. The wire itself defines the easy axis ($z$-axis), and a DW centered at $z = 0$ separates two regions with opposite magnetizations: $M_{z}(z \to \pm \infty) = \pm M_{0}$. Assuming $M(r)$ to lie in the $xz$ plane and the DW width to be smaller than the Fermi wavelength, one can write the single-particle Hamiltonian as ($J > 0$):

$$\hat{H}_{0} = -\frac{\hbar^{2}}{2m} \frac{d^{2}}{dz^{2}} + \hbar V \delta(z) + JM_{z}(z)\hat{\sigma}_{z} + \hbar \lambda \delta(z)\hat{\sigma}_{x},$$

where the term $\hbar \lambda \delta(z)\hat{\sigma}_{x}$ describes spin dependent scattering due to the $M_{z}(z)$ component, $\hbar \lambda = J \int_{-\infty}^{\infty} M_{z}(z)dz$, and $V$ is a potential (spin independent) scattering term. We assume that spin-$\uparrow$ electrons are spin majority ones for $z < 0$ and spin-minority for $z > 0$.

An incident electron from the left with momentum $k$ and spin $\uparrow$ (or $\downarrow$) can be transmitted to the $z > 0$ region preserving its spin, but changing momentum from $k$ to $k^{-}$ (or $k^{+}$), given by $k^{\pm} = \sqrt{k^{2} \pm 4JM_{0}m/\hbar^{2}}$. If the transmission occurs with spin reversal, the momentum $k$ remains unchanged. The reflection amplitudes for spin-$\sigma$ electrons with or without spin reversal are denoted by $r_{\sigma}$ and $r_{\sigma}^{\prime}$, respectively. The same convention applies to the transmission amplitudes $t_{\sigma}$ and $t_{\sigma}^{\prime}$. The scattering amplitudes are given by:

$$t_{\uparrow(\downarrow)}(k) = \frac{2(v + v^{\mp} + 2V)v}{(v + v^{\mp} + 2V)^{2} + 4\lambda^{2}} = r_{\uparrow(\downarrow)}(k) + 1, \quad (2)$$

$$t_{\uparrow(\downarrow)}^{\prime}(k) = \frac{4i\lambda v}{(v + v^{\mp} + 2V)^{2} + 4\lambda^{2}} = r_{\uparrow(\downarrow)}^{\prime}(k), \quad (3)$$

with $v = \hbar k/m$, $v^{\pm} = \hbar k^{\pm}/m$, where the upper (lower) sign refers to $\uparrow$ ($\downarrow$). We shall henceforth denote by
where \( \epsilon(p, \sigma) \) is the energy of a scattering state with momentum \( p \) (or \( -p \)) and spin \( \sigma \), incident from the left (or right). The scattering amplitudes satisfy general relations that can be found from a generalization of the Wronskian theorem \(^{12, 17}\) to spinor wave-functions: the Wronskian of any two states having the same energy is a constant,

\[
W(\psi_1, \psi_2) \equiv \psi^t_1(z) \frac{d\psi_2}{dz} - \frac{d\psi_1}{dz} \psi_2(z) = \text{const}, \tag{4}
\]

where \( \psi^t \) denotes the transpose of the spinor \( \psi \).

In order to deal with the electron interactions, it is convenient to rewrite the scattering states in second quantization form, making use of right \( (a_{\sigma}) \) and left \( (b_{\sigma}) \) moving plane wave states. We introduce operators \( c_{k, \sigma} \) and \( \hat{a}_{k, \sigma} \) for the eigenstates corresponding to electrons incident from the left and right, respectively. The plane wave operators are linear combinations of the operators of scattering states. Electron interactions are then introduced through the Hamiltonian

\[
\hat{H}_{\text{int}} = g_1,\alpha,\beta \int \frac{dk_1 dq}{(2\pi)^2} \hat{a}^\dagger_{k_1,\alpha} \hat{b}_{k_2,\beta} \hat{a}_{k_2+q,\gamma} \hat{b}_{k_1-q,\alpha} + g_2,\alpha,\beta \int \frac{dk_1 dq}{(2\pi)^2} \hat{a}^\dagger_{k_1,\alpha} \hat{b}^\dagger_{k_2,\beta} \hat{b}_{k_2+q,\gamma} \hat{a}_{k_1-q,\alpha}, \tag{5}
\]

where the Greek letters denote spin indices, and the summation convention over repeated indices is used. The coupling constants \( g_1 \) and \( g_2 \) describe back and forward scattering processes between electrons moving in opposite directions, respectively. Because the Fermi momentum is spin dependent, we distinguish between \( g_1(b_{2\uparrow}) \), which describe interaction between spin-majority particles (that is spin-\( \uparrow \) on the left and spin-\( \downarrow \) on the right of the barrier) and \( g_1(2\downarrow) \), which describe interaction between spin-minority particles. We use \( g_1(2\perp) \) to denote interaction between particles with opposite spin. The forward scattering process between particles which move in the same direction will not affect the transmission amplitudes, although it will renormalize the Fermi velocity \(^{11, 12}\). This effect is equivalent to an effective mass renormalization and the electrons with different spin orientations may turn out to have different effective masses, in which case our calculations remain valid \(^{13}\).

Following Ref. \(^{14}\), the corrections to the transmission amplitudes are calculated to first order in the perturbation \( \hat{H}_{\text{int}} \). Such corrections diverge logarithmically near the Fermi level and will be dealt with in a poor man’s renormalization approach. The perturbative correction to \( t_\sigma(p') \) can be obtained from the perturbation expansion for the Matsubara propagator

\[
G_\sigma(\tau) = -(T_\tau e^{-\int \hat{H}_{\text{int}}(\tau')d\tau'}) \hat{a}_{p,\sigma}(\tau) \hat{c}_{p',\sigma}^\dagger(0), \tag{6}
\]

where \( \langle ... \rangle_0 \) denotes the average in the Fermi sea of scattering states. The zero-order propagator for \( \sigma = \uparrow \) is:

\[
G^{(0)}_\uparrow(it_\sigma(p')) = \frac{1}{it_\sigma(p') - \epsilon(p', \uparrow)} = \frac{i}{p-p'+i0} - \frac{it_\uparrow(p')}{p-p'-i0}, \tag{7}
\]

where 0 denotes a positive infinitesimal. The poles in the denominators identify the semi-axis on which the electron behaves as a right moving plane wave. The transmission amplitude appears associated with the denominator \( p-p'-i0 \), which, for the variable \( p \), gives a pole in the upper half plane. The meaning of this pole is that the transmitted particle is right-moving in the \( z > 0 \) half-axis. Our strategy is to calculate the first order correction term to \( G \), in which a pole in \( p - p' - i0 \) will appear with the residue \(-i\delta t_\uparrow(p')\), which is the transmission amplitude correction.

The diagrammatic representation of \( g^{(1)}_\uparrow \) is shown in Fig. 1. The horizontal lines represent the electron scattered by the Hartree-Fock potential of the Fermi sea. Only processes where the electron is back-scattered by the Fermi sea are logarithmically large \(^{14}\). Consider, for instance, the upper left diagram – an electron, initially in state \( c_{0\uparrow} \) close to the Fermi level passes through the DW as a right-moving \( (\hat{a}) \) particle. Then, it is reflected (from \( \hat{a} \) to \( \hat{b} \) particle) while exchanging momentum \( q \) with the Fermi sea on the \( z > 0 \) semi-axis. Finally, it is reflected by the DW again, becoming a spin-up right moving particle of momentum \( p \). A logarithmic divergence occurs if the polarized Fermi sea can provide exactly the momentum that is required to keep the electron always near the Fermi level in the intermediate virtual steps. According to the physical interpretation of the diagrams, we always know on which side of the DW the interaction with the Fermi sea (closed loop in the diagram) is taking place.

\[
FIG. 1: \text{Feynman diagrams for the first order contribution} \ G^{(1)}_\uparrow \text{ to the propagator} \ G. \text{ The scattering state is represented by a double line, the} \ \hat{a} \text{ (b)} \text{ particle is represented by a continuous (dashed) line. The loop represents the Hartree-Fock potential of the Fermi sea. The scattered electron exchanges momentum} \ q \text{ with the Fermi sea.}
\]

We use Fermi level velocities \( v_{\pm} \) and wavevectors \( k_{F, \pm} \) for majority or minority spin particles, henceforth. It can be seen that \( g_{1\perp} \) terms are proportional to \( \log |k_{F, +} - k_{F, -}| \), so they do not diverge. Logarithmic divergence
would be restored in a spin degenerate system ($k_{F+} = k_{F-}$). This can be understood from the diagrams in Fig. 1 as follows: the electron with spin $\alpha$ is reflected by a polarized Fermi sea with spin $-\alpha$. The momentum provided by the Fermi sea is $2k_{F-\alpha}$, while the momentum required by the electron is $2k_{F\alpha}$. The $g_{2\perp}$ terms produce logarithmic divergences that would not exist in the absence of spin-flip scattering ($t' = r' = 0$). For the calculation of $\delta t'_t(p')$ the propagators we need to consider are $-(T_r \delta p_{-\sigma}(r)/v)|^{p'}_{p'}$.

The expression for $\delta t_\perp(p')$ is directly proportional to $\log(|\epsilon'|/D)$, where $\epsilon'$ denotes the energy of the scattered electron and $D$ is an energy scale near the Fermi level where the electron dispersion can be linearized $[\delta t_\perp(p')/\log(|\epsilon'|/D)$ is given by the right-hand side of equation (8) below]. The logarithmically divergent perturbation can be dealt with using a renormalization procedure [12]: reducing step by step the bandwidth $D$ and removing states near the band edge is compensated by renormalization of $t_\perp$. Applying this procedure and noting that $t_\perp + \delta t_\perp$ remains invariant as $D$ is reduced, one finds the following differential equation:

$$dt_\perp + \frac{\partial t_\perp}{\partial D} D = 0.$$ 

Now, we introduce the variable $\xi = \log(D/D_0)$, which is integrated from 0 to $\log(|\epsilon'|/D_0)$, corresponding to the fact that the bandwidth is progressively reduced from $D_0$ to $|\epsilon'|$ (which will be taken as temperature: $|\epsilon'| = T$). It is convenient to rewrite the interaction parameters as $g_{\tau(\perp)} = (g_{2\tau(\perp)} - g_{1\tau(\perp)})/4\hbar v_{+(-)}$. $g_\perp = g_{2\perp}/2h(v_+ + v_-)$. The scaling differential equations for the transmission amplitudes are

$$\frac{dt_\perp}{d\xi} = g_\perp \left[ r_\perp^* r_\perp t_\perp + r_\perp^* r_\perp t_\perp \right] + g_\perp \left[ r_\perp^* r_\perp t_\perp + r_\perp^* r_\perp t_\perp \right],$$

$$\frac{dt'_\perp}{d\xi} = 2g_\perp r_\perp^* r_\perp t_\perp + 2g_\perp r_\perp^* r_\perp t_\perp + 2g_\perp \left[ r_\perp^* r_\perp t_\perp + r_\perp^* r_\perp t_\perp \right].$$

Equations for the reflection amplitudes $r_\sigma(p')$ and $r'_\sigma(p')$ can be obtained from the propagators $-(T_r b_{p,\pm\sigma}(r)|^{p'}_{p'})$. The equation for $r_\tau(p')$ is

$$\frac{dr_\tau}{d\xi} = g_\perp \left[ r_\perp^* r_\perp r_\perp + r_\perp^* r_\perp r_\perp \right] + g_\perp \left[ r_\perp^* r_\perp r_\perp + r_\perp^* r_\perp r_\perp \right] - g_\perp r_\perp,$$

and the equation for $r'_\tau(p')$ is

$$\frac{dr'_\tau}{d\xi} = g_\perp \left[ r_\perp^* r_\perp r_\perp + r_\perp^* r_\perp r_\perp \right] + g_\perp \left[ r_\perp^* r_\perp r_\perp + r_\perp^* r_\perp r_\perp \right] + g_\perp \left[ r_\perp^* r_\perp r_\perp + r_\perp^* r_\perp r_\perp \right] - g_\perp r_\perp.$$

The scaling equations for spin-$\downarrow$ amplitudes follow from the above by simply inverting the spin and velocity indices. Equations (8) - (11) are the central result of this paper. Theorem 1 gives $v_-/v_+ = t_\perp/t_\perp = r'_\perp/r_\perp$. A standard second-order renormalization group treatment shows that in a 1D magnetized system the coupling constants in equation (5) are not renormalized because the logarithmically divergent contributions cancel each other.

**Fixed points** We have performed a numerical analysis of the scaling equations using the DW model [11], with $V = 0$, for the initial parameters. We now analyze the nature of the fixed points predicted by the scaling equations. The parameters of the model which enter the scaling equations are $g_\perp, g_\parallel, g$, and the ratio $v_-/v_+$. For repulsive interactions ($g_\perp, g_\parallel, g > 0$) the system flows to insulator fixed points. For $\lambda/v_+$ larger than about 0.1, all transmission amplitudes vanish faster than any reflection amplitude as $T \to 0$. In this limit we may rewrite the scaling equations for $r_\tau, r'_\tau$ and the theorem 1 neglecting the small transmission amplitudes. Theorem 1 for $W(\psi_k^+ \psi_k^-, \psi_k^-, \psi_k^+)$ tells us that $r_\perp^* r_\perp^* + r_\perp^* r_\perp^* = 0$. The charge conservation condition is satisfied solely by the reflections,

$$1 = |r_\tau|^2 + \frac{v_-}{v_+} |r'_\tau|^2 = |r'_\tau|^2 + \frac{v_-}{v_+} |r_\tau|^2,$$

from which we conclude that $|r_\tau| = |r'_\tau|$ at the fixed point. Equations (10) and (11) now take the form

$$\frac{dr_\tau}{d\xi} = \frac{v_-}{v_+} \left( 2g_\perp - g_\perp - g_\parallel \right) \left( 1 - |r_\tau|^2 \right) r_\tau,$$

$$\frac{dr'_\tau}{d\xi} = \left( g_\perp + g_\parallel - 2g_\perp \right) \left( 1 - \frac{v_-}{v_+} |r'_\tau|^2 \right) r'_\tau.$$

In the derivation of (13) and (14) only the smallness of the transmissions amplitudes was assumed. The phases of the complex numbers $r_\tau, r'_\tau$ are unchanged during scaling. The two fixed points we may consider correspond to $r_\tau$ approaching 0, or $|r_\tau|$ approaching 1.

The situation $|r_\tau| \to 0$ requires $2g_\perp - g_\perp - g_\parallel > 0$ and, by charge conservation we have $|r'_\tau| \to \sqrt{v_+/v_-}$. Upon integrating (11) with $\xi$ going from 0 to $\log(T)/D_0$, the amplitude $r'_\perp$ will vary from $r'_\perp$ to $r'_\perp(T)$. Introducing the reflection coefficient $R'_\perp = (v_-/v_+)|r'_\perp|^2$, we obtain

$$R'_\perp(T) = \frac{R'_\perp(0) \left( \frac{T}{T_0} \right)^{2(2g_\perp + g_\parallel - 2g_\perp)}}{1 + \frac{R'_\perp(0) \left( \frac{T}{T_0} \right)^{2(2g_\perp + g_\parallel - 2g_\perp)}}{R_\perp(0)}}.$$

If $2g_\perp - g_\perp - g_\parallel > 0$ then $R'_\perp(T) \to 1$ as $T \to 0$. The DW reflects all incident electrons while additionally reversing their spin – it is a 100% “spin-flip reflector” at zero temperature, generating a finite net spin current but no charge current. In order to find the low $T$ behavior of transmissions we put $r_\perp = 0, |r'_\perp| = \sqrt{v_+/v_-}$ in the scaling equations for the transmission amplitudes and obtain $|t_\perp| \sim |t'_\perp| \sim T^{2g_\perp}$.
In the regime where \( g_1 + g_2 - 2g_1 > 0 \) we have \( \mathcal{R}'_i(T) \to 0 \), \( \mathcal{R}_i(T) \to 1 \). The DW reflects then all incident electrons while preserving their spin. The scaling equations for the transmission amplitudes yield \( |t_\uparrow| \sim T^{g_1 + g_2}, |t'_\sigma| \sim T^{2g_\sigma} \).

If \( g_1 + g_2 - 2g_1 = 0 \) then both \( r'_\sigma(T) \) and \( r_\sigma(T) \) tend to finite values. The scaling equations for \( t_\uparrow, \ t'_\sigma \), with constant reflection amplitudes, become a linear algebraic 3 by 3 system. The eigenvalues of the matrix give the transmission amplitudes on the right hand side. The behavior of \( r_\sigma \) at \( T \to 0 \) can be found from the charge conservation condition: \( 1 - |r_\sigma|^2 \sim T^{\min(2g_\uparrow+g_\downarrow), 4g_\sigma} \).

We may estimate the \( \lambda \) parameter in Eq. (11) by assuming that \( M(z) = M_0 \cos \theta(z) \hat{z} + M_0 \sin \theta(z) \hat{x} \) with \( \cos \theta(z) = \tanh(z/L) \), where \( L \) is the length of the DW. We then have \( \lambda = \pi M_0 L / \hbar \), implying that

\[
\frac{\lambda}{v_+} = \pi m \frac{J M_0}{\hbar^2 k_{F+}^2} (L k_{F+}).
\]  

The condition for the DW to be smaller than the Fermi wavelength is \( L k_{F+} < 2\pi \). The smaller Fermi wavelength is that of spin-majority electrons, \( 2\pi/k_{F+} \). The ratio \( v_-/v_+ \) depends on the degree of polarization of the electron system. In a 1D nonmagnetic system there is a single Fermi momentum, \( k_F \), for up and down electrons and a Fermi energy \( E_F = \hbar^2 k_F^2/(2m) \). Once the system becomes magnetized, there is a Zeeman energy shift of the bands, \( \Delta E / 2 = J M_0 \), and the two new Fermi momenta, \( k_{F\pm} \), satisfy

\[
\frac{k_{F\pm}}{k_F} = 1 \pm \frac{\Delta E}{4E_F}.
\]  

Inserting this result in Eq. (10) above, we obtain

\[
\frac{\lambda}{v_+} = \pi \frac{(\Delta E/4E_F)}{[1 + (\Delta E/4E_F)]^2} (L k_{F+}).
\]

In the full polarization limit \( k_{F_-} = 0, k_{F_+} = 2k_F \), and Eq. (15) gives \( \lambda/v_+ \approx 0.79 L k_{F+} \). Typical values for a non-fully polarized system are \( E_F = 90 \text{meV} \) and \( \Delta E = 30 \text{meV} \). In this case we have \( v_-/v_+ = 0.84 \) and Eq. (15) gives \( \lambda/v_+ \approx 0.22 L k_{F+} \). Therefore, if \( L k_{F+} \) is smaller than about \( 2\pi \), the system can flow to any of the fixed points described above.

Lateral quantization may produce several channels. The possibility of inter-channel scattering then arises due to two causes: (i) electron interactions (which would require a modification of our theory to allow for inter-channel scattering); (ii) impurity scattering. For the latter to be negligible the electron mean free path must be larger than the size of the constriction pinning the DW.

The above spin-flip reflector DW was not found in Ref. [10]. This is because the adiabatic DW considered in Ref. [11] is a poor spin-flip reflector at the noninteracting level already – as in the regime of small \( \lambda/v_+ \) above.

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\[\text{(11)}\]

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