MICROPHYSICS OF SHOCK ACCELERATION FROM OBSERVATIONS OF X-RAY SYNCHROTRON EMISSION FROM SUPERNOVA REMNANTS

S.P. Reynolds

1Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138
and
Physics Department, North Carolina State University, Raleigh NC 27695-8202

ABSTRACT

Recent observations of non-thermal X-rays from supernova remnants have been attributed to synchrotron radiation from the loss-steepened tail of a non-thermal distribution of electrons accelerated at the remnant blast wave. In the test-particle limit of diffusive shock acceleration, in which the energy in shock-accelerated particles is unimportant, the slope of a shock-accelerated power-law is independent of the diffusion coefficient $\kappa$, and on how $\kappa$ depends on particle energy. However, the maximum energy to which particles can be accelerated depends on the rate of acceleration, and that does depend on the energy-dependence of the diffusion coefficient. If the time to accelerate an electron from thermal energies to energy $E \gg m_e c^2$ is $\tau(E)$, and if $\kappa \propto E^\beta$, then $\tau(E) \propto E^\beta$ in parallel shocks, and $\tau \propto E^{2-\beta}$ in perpendicular shocks. Most work on shock acceleration has made the plausible assumption that $\kappa \propto r_g$ (where $r_g$ is the particle gyroradius), so that $\beta = 1$ at relativistic energies, implying a particular (wavelength-independent) spectrum of MHD turbulence, where Kolmogorov or Kraichnan spectra might be more physically plausible. I derive the $\beta$-dependence of the maximum electron energy resulting from limitations due to radiative (synchrotron and inverse-Compton) losses and to finite remnant age (or size). I then exhibit calculations of synchrotron X-ray spectra, and model images, for supernova remnants as a function of $\beta$ and compare to earlier $\beta = 1$ results. Spectra can be considerably altered for $\beta < 1$, and images are dramatically different for values of $\beta$ corresponding to Kolmogorov or Kraichnan spectra of turbulence. The predicted images are quite unlike observed remnants, suggesting that the turbulence near SNRs is generated by the high-energy particles themselves.

MICROPHYSICAL PARAMETERS OF STANDARD SHOCK-ACCELERATION THEORY

Diffusive (first-order Fermi) shock acceleration is presumed to provide the relativistic-electron distributions demanded by radio and, in a few cases, X-ray observations of supernova remnants (SNRs). In the test-particle limit in which the fast particles exert no influence on the shock, the spectrum is independent of the value and energy-dependence of the diffusion coefficient $\kappa$. When the energy in accelerated particles becomes non-negligible, however, various nonlinear effects appear. Most directly, the preshock gas is pre-accelerated by a shock precursor whose extent depends on the diffusion distance $r_D = \kappa/u_{\text{shock}}$ which does depend on particle energies. One observable effect is concave spectral curvature if $\kappa(E)$ increases with $E$ (Ellison and Reynolds 1991; Reynolds and Ellison 1992). For both test-particle and nonlinear acceleration, the acceleration rate for relativistic electrons depends on the diffusion coefficient.

Macrophysical parameters, those required for hydrodynamic and thermal-shock modeling of SNRs, include the remnant age $t$, shock velocity $u_{\text{sh}}$ (ideally $u_{\text{sh}}(t)$), preshock density $n_0$, and compression ratio $r$. (A piece of important thermal-shock microphysics concerns non-Coulomb electron heating at shock, in the absence of which $T_e \ll T_i$ initially).
For standard shock-acceleration theory, we need in addition the upstream magnetic field $B_1$, the obliquity angle $\theta_{\text{Boh}}$ between the shock normal and $B_1$, and the diffusion coefficient upstream and downstream, $k_1$ and $k_2$ (in general $k(E, r, \eta)$). In the standard picture (e.g., Blandford and Eichler 1987), in strong shocks ($M_A, M_s \gtrsim 10$), once particles have energies well above thermal, the primary scattering is from resonant MHD waves (wavenumber $k \propto \text{particle gyrofrequency } \Omega$: $k_R = \Omega/c$). Quasi-linear theory allows the inference of a diffusion coefficient. Assume a wave spectrum $I(k) = Ak^{-\eta} \text{ erg cm}^{-3} (\text{cm}^{-1})^{-1}$ (so the energy density at $k \sim kI(k)$). Then

$$k_\parallel = \frac{1}{3} \lambda_{\text{mfp}} v = \frac{c}{3} r_g \left( \frac{k_R I(k_R)}{B^2/8\pi} \right)^{-1} \equiv \frac{c}{3} \eta(E) r_g$$

where $\eta(E)$ is the (energy-dependent) “gyrofactor,” $\lambda_{\text{mfp}} = \eta r_g$, and $\eta^{-1}$ is the fractional magnetic energy in waves resonant with particles of that energy. In the so-called “Bohm limit”, normally taken to give the minimum physically possible diffusion coefficient, $\eta = 1$.

So $\eta \propto (k_R I(k_R))^{-1} \propto k_R^{n-1}$. Since $\Omega = eBc/E$ for extreme-relativistic particles, $\eta \propto E^{1-n}$ and $k_\parallel \propto \eta r_g \propto E^{2-n} \equiv E^\beta$. Then for Kolmogorov turbulence we have $n = 5/3 \Rightarrow \beta = 1/3$, while for a Kraichnan turbulent spectrum, $n = 3/2 \Rightarrow \beta = 1/2$. Note that for constant gyrofactor (as is often assumed), $\beta = 1 \Rightarrow n = 1$, i.e., a “white-noise” spectrum with equal energy per decade.

This describes diffusion along magnetic-field lines. Perpendicular (cross-field) diffusion is generally assumed to result from a cross-field displacement of one gyroradius for every scattering along the field:

$$\kappa_\perp = \frac{k_\parallel^2}{1 + \left( \frac{\theta_{\text{Boh}}}{\eta} \right)^2}$$

Then

$$\kappa = k_\parallel \cos^2 \theta_{\text{Boh}} + \kappa_\perp \sin^2 \theta_{\text{Boh}} = k_\parallel \left( \cos^2 \theta_{\text{Boh}} + \frac{\sin^2 \theta_{\text{Boh}}}{1 + \eta^2} \right).$$

For quasi-perpendicular shocks ($\theta_{\text{Boh}} \approx 90^\circ$) and large $\eta, \kappa \ll k_\parallel$ and acceleration can be much faster than for parallel shocks (Jokipii 1987). Even if $\eta = 1$ (Bohm limit), there is some obliquity dependence, mainly from field compression where $\theta_{\text{Boh}} \approx 90^\circ$. But for large $\eta$, we have faster acceleration where the shock is perpendicular but slower where $\theta_{\text{Boh}}$ drops well below $90^\circ$, giving larger azimuthal brightness variations. Large $\eta$ corresponds to weaker turbulence, so that $\theta_{\text{Boh}}$ is well-defined. When $\eta$ is near 1, the obliquity-dependence is smaller anyway.

Radio observations of SNRs provide few constraints on the microphysical parameters, because the power-law part of the spectrum contains relatively little information: the diffusion-coefficient dependence is only second order, through nonlinear effects (e.g., a slight spectral hardening with energy; Ellison and Reynolds 1991). But the maximum energies to which particles can be accelerated depend on acceleration rates which strongly reflect the diffusion coefficient. Now all known shell SNRs are fainter in X-rays (thermal or not) than the extrapolation of their radio spectrum (14 Galactic remnants, Reynolds and Keohane 1999; 11 LMC remnants, Hendrick and Reynolds 2001). So any non-thermal emission results from the cutting-off tail of the electron distribution, giving direct information on the maximum energies. Such synchrotron X-ray emission has now been identified in several Galactic remnants, inferred from lineless spectra that cannot be produced by thermal processes (SN 1006: Koyama et al. 1995; G347.5-0.3, Slane et al. 1999; and several others).

These remnants can then be used to study various microphysical quantities. The magnitude of the diffusion coefficient is constrained by the maximum electron energies observed, and its upstream value in particular is related to possible “halo” emission from upstream electrons diffusing ahead of the shock. The energy-dependence of the diffusion coefficient can be constrained by studying remnant morphology and detailed spectral shape, compared with model calculations such as those to be summarized below. The obliquity dependence of acceleration affects the azimuthal variation of X-rays around the shell. The overall compression ratio can be constrained by the halo visibility (and thermal modeling). Finally, observed maximum energies also constrain the shock speed and other hydrodynamic parameters.

**SYNCHROTRON X-RAY OBSERVATIONS OF REMNANTS**

SN 1006, the prototype object for synchrotron X-rays, has a spectrum that is well fit by a model in which acceleration is limited by electron escape above some $E_{\text{max}}$ (Reynolds 1996). The fitted characteristic rolloff frequency
of $7 \times 10^{17}$ Hz (where the spectrum has dropped a factor $\sim 6$ below the lower-frequency extrapolation; Long et al. 2002), with the known mean magnetic field (from inverse-Compton TeV emission, $\langle B \rangle \sim 10$ µgauss; Tanimori et al. 1998; Dyer et al. 2001) implies $E_{\text{max}} = 53$ TeV. Furthermore, the known shock velocity implies that the observed electron spectral cutoff energy is too low to result from synchrotron losses in this magnetic field ($t_{1/2}(E_{\text{max}}) \geq 1200$ yr) or finite age ($t_{\text{accel}} \leq 800$ yr). This leaves only a change in diffusive properties for $\lambda_{\text{MHD}} \geq 2 \times 10^{17}$ cm to explain the observed rolloff.

The detailed cylindrically symmetric model of Reynolds (1996; 1998) fits the spectrum well (Dyer et al. 2001), but predicts an external halo with brightness $5 - 10\%$ of postshock peak just ahead of the shock, and an extent of many arcmin, the diffusion length of the high-energy electrons. However, no halo is seen in the deep, low-background Chandra image: any preshock emission $< \sim 1\%$ of the postshock maximum on the bright rims. Plane-shock models seen edge on predict a lower brightness, $\sim 3\%$, but still in conflict with the observations. While efficient acceleration of particles can increase the overall compression ratio (e.g., Berezhko, Elshin, et al., 1996), this occurs over the long diffusion lengthscale, while the immediate density jump at the small-scale thermal subshock actually decreases, making the problem worse. This problem is discussed at length in Long et al. (2002) where it is suggested that the post-shock magnetic field may actually increase by more than due to simple fluid compression, perhaps from some kind of cosmic-ray-driven instability (Lucek and Bell 2000).

In general, the halo can be made unobservable by making it thin rather than faint. For this option, the diffusion length $\kappa_1/\eta_{\text{sh}}$ must be less than the width of the sharp edge, $< 10''$ for the Chandra observations. This would require an unreasonably high upstream magnetic field, at least $20 \mu$G, especially given SN 1006’s height of 570 pc above the Galactic plane.

In the larger, older remnant RCW 86, weak X-ray lines in SW corner imply anomalous abundances if the continuum is thermal (Vink et al. 1997), while energetics problems (Rho et al. 2002) afflict an explanation of the continuum as non-thermal bremsstrahlung (Vink et al. 2002). However, if the harder continuum is dominated by synchrotron emission, reasonable abundances can be accommodated. (Borkowski et al. 2001; Rho et al. 2002). Chandra data imply rolloff frequencies $\nu_c \sim (7 - 10) \times 10^{16}$ Hz, consistent with loss-limited acceleration (shock velocities are known from Hα emission to be 600 – 900 km s$^{-1}$; Ghavamian et al. 2002). For radiative-loss-limited acceleration,

$$\nu_c = 5 \times 10^{16} \eta \frac{B_2^2}{4B_1} \left( \frac{u_{\text{sh}}}{1000 \text{ km s}^{-1}} \right)^2 \text{ Hz}$$

(4)
so we need $\eta > 1$ and/or $r > 4$ again, even for a perpendicular shock.

**CONSTRAINTS ON ENERGY DEPENDENCE OF THE DIFFUSION COEFFICIENT**

I have generalized the modeling code of Reynolds (1998) to include arbitrary energy-dependence of the diffusion coefficient. The models assume a spherical blast wave encountering uniform upstream material containing a uniform magnetic field whose sky-plane projection is vertical and which makes an angle $\phi$ with respect to the sky plane. The remnant dynamics are Sedov; each post-shock fluid element is endowed with an electron distribution $N(E) = KE^{-\gamma} \exp(-E/E_{\text{max}})$, where the maximum energy is in general a function of shock speed, remnant age, diffusion coefficient, and shock obliquity angle $\theta_{\text{obs}}$. This distribution is then evolved downstream subject to radiative and adiabatic losses; electrons are assumed to have short enough post-shock mean free paths that they stay confined to their original fluid element and share its expansion. Details are given in Reynolds (1998).

Here we generalize by allowing an energy dependence of the diffusion coefficient, $\kappa \propto E^\beta$. For the relatively steep spectra of plasma waves predicted in simple homogeneous turbulence models, such as Kolmogorov turbulence, $\beta < 1$, so that $\lambda_\parallel$ rises more slowly with energy than $r_g$. At some high energy $E_h$, then, we will have $\lambda_\parallel = r_g$. Above this energy, we must have an effective $n = 1$ (constant “gyrofactor.”) So we confine our attention to $E < E_h$ and write

$$\kappa = \kappa_B \left( \frac{E}{E_h} \right)^\beta \quad \frac{\lambda_\parallel}{r_g} = \frac{E}{E_h}^{\beta-1} \quad E < E_h \quad (5)$$

where $\kappa_B = r_g(E_h)c/3 = E_hc/3eB$, the Bohm-limit value at $E = E_h$.

Then for a parallel shock, $\theta_{\text{Bn}} = 0^\circ$, the acceleration time is given by (generalizing from Reynolds 1998)

$$\tau_{\text{acc}} = \frac{r(r+1) c}{\beta(r-1) e u_1^2 B_1} \left( \frac{E}{E_h} \right)^\beta \quad (6)$$

whereas for a perpendicular shock, $\theta_{\text{Bn}} = 90^\circ$, we have

$$\tau_{\text{acc}} = \frac{2r}{(2-\beta)(r-1)} \frac{c}{e u_1^2 B_1} \left( \frac{E}{E_h} \right)^{2-\beta} \quad (7)$$

with $r$ the shock compression ratio. Note that $\tau(90^\circ)/\tau(0^\circ) \propto (E/E_h)^{(2-2\beta)}$ so that if $\beta < 1$, acceleration to energies $E \ll E_h$ is much faster at perpendicular shocks than parallel.

The result of these effects for supernova-remnant morphology is dramatic (see Figure 2). For a spherical remnant expanding into a uniform magnetic-field $B_1$, around the “equator” where $\theta_{\text{Bn}} \sim 90^\circ$ the electron spectrum will continue to much higher energies than near the “poles” ($\theta_{\text{Bn}} \sim 0^\circ$) if $\beta < 1$. However, the spectrum will not differ much from the $\beta = 1$ case because the emission is dominated by the bright equatorial “belt”.

**CONCLUSIONS AND FUTURE WORK**

Because it samples electron energies where the spectrum is cutting off, X-ray synchrotron emission from SNRs can provide significant information on the microphysics of shock acceleration (mainly on the diffusion coefficient), to a much greater extent than radio synchrotron emission. In the remnant of SN 1006, the shock is almost certainly perpendicular at the bright non-thermal rims. Simple predictions for the spectral break due to radiative losses give too high a break, as do predictions based on the finite remnant age (or size). For the spectrum to break where it is observed, the diffusive properties of the external medium probably have to change on scales above $\sim 0.1$ pc, so that electrons escape much more freely when their gyroradii are comparable to this scale. The absence of an exterior synchrotron “halo” probably requires a cross-shock jump in magnetic-field strength considerably greater than given by the fluid compression ratio there, suggesting the possibility of some kind of immediate post-shock turbulent or plasma amplification of magnetic field. In the considerably older remnant RCW 86, synchrotron emission is important, though not dominant; careful modeling of both thermal and non-thermal emission is required to extract information. The rather high observed break frequencies require rapid acceleration: perpendicular shocks with $\eta > 1$ and/or $r > 4$.

The modeling of shock acceleration for diffusion in steep wave spectra in the context of a spherical SNR shock has shown that quasi-perpendicular favoritism, already present in constant-gyrofactor models, is much more extreme
Fig. 2. Four simulations for different values of $\beta$. The magnetic field is assumed to have a vertical projection on the sky plane, and to make an angle $\phi = 30^\circ$ with that plane.
if $\beta < 1$ (wave spectra steeper than $kI_k \propto k^{-1}$). While predicted spectra aren’t much different from the $\beta = 1$ (constant-gyrofactor) case, predicted images are strikingly unlike real remnants for $\beta < 1$. A likely explanation is that the MHD waves causing diffusion are self-generated, with $k^{-1}$ spectrum. In general, however, it is clear that we need better thermal and non-thermal models for X-ray emission in SNRs. Such improved models ought to allow the extraction of a great deal of important and useful information about the microphysics of shock acceleration.

REFERENCES

Blandford, R. D., and D. Eichler, Particle acceleration at astrophysical shocks: a theory for cosmic ray origin, *Phys. Reports*, **154**, 1–75, 1987.

Berezhko, E. G., V. K. Elshin, and L. T. Ksenofontov, Numerical studies of the acceleration of cosmic rays in supernova remnants, *Astr. Reports*, **40**, 155–166, 1996.

Borkowski, K. J., J. Rho, S. P. Reynolds, and K. K. Dyer, Thermal and non-thermal X-ray emission in supernova remnant RCW 86, *Astrophys. J.*, **550**, 334–345, 2001.

Dyer, K. K., S. P. Reynolds, Borkowski, K. J., et al., Separating thermal and non-thermal X-rays in supernova remnants. I. Total fits to SN 1006 AD, *Astrophys. J.*, **551**, 439–453, 2001.

Ellison, D. C., and S. P. Reynolds, Electron acceleration in a nonlinear shock model with applications to supernova remnants, *Astrophys. J.*, **382**, 242–254, 1991.

Ghavamian, P., P. F. Winkler, J. C. Raymond, and K. S. Long, Balmer-dominated spectra of nonradiative shocks in the Cygnus Loop, RCW 86, and Tycho supernova remnants, *Astrophys. J.*, **572**, 888–896, 2002.

Hendrick, S. P., and S. P. Reynolds, Maximum energies of shock-accelerated electrons in Large Magellanic Cloud supernova remnants, *Astrophys. J.*, **559**, 903–908, 2001.

Jokipii, J. R., Rate of energy gain and maximum energy in diffusive shock acceleration, *Astrophys. J.*, **313**, 842–846, 1987.

Long, K. S., S. P. Reynolds, J. C. Raymond, et al., *Chandra* CCD imagery of the NW and NE limbs of SN 1006, *Astrophys. J.*, **586**, 1162–1178, 2003.

Lucek, E. G., and A. R. Bell, Non-linear amplification of a magnetic field driven by cosmic ray streaming, *Mon. Not. Royal Astr. Soc.*, **314**, 65–74, 2000.

Koyama, K., R. Petre, E. V. Gotthelf, et al., Evidence for shock acceleration of high-energy electrons in the supernova remnant SN 1006, *Nature*, **378**, 255–258, 1995.

Reynolds, S. P., Synchrotron models for X-rays from the supernova remnant SN 1006, *Astrophys. J.*, **459**, L13–L16, 1996.

Reynolds, S. P., Models of synchrotron X-rays from shell supernova remnants, *Astrophys. J.*, **493**, 375–396, 1998.

Reynolds, S. P., and D. C. Ellison, Electron acceleration in Tycho’s and Kepler’s supernova remnants - Spectral evidence of Fermi shock acceleration, *Astrophys. J.*, **399**, L75–L78, 1992.

Reynolds, S. P., and J. W. Keohane, Maximum energies of shock-accelerated electrons in young shell supernova remnants, *Astrophys. J.*, **525**, 368–374, 1999.

Rho, J., K. K. Dyer, K. J. Borkowski, et al., X-ray synchrotron-emitting Fe-rich ejecta in supernova remnant RCW 86, *Astrophys. J.*, **581**, 1116–1131, 2002.

Slane, P., B. M. Gaensler, T. M. Dame, et al., Non-thermal X-ray emission from the shell-type supernova remnant G347.3-0.5, *Astrophys. J.*, **525**, 357–367, 1999.

Tanimori, T., Y. Hayami, S. Kamei, et al., Discovery of TeV gamma rays from SN 1006: further evidence for the supernova remnant origin of cosmic rays, *Astrophys. J.*, **497**, L25–L28, 1998.

Vink, J., J. S. Kaastra, and J. A. M. Bleeker, X-ray spectroscopy of the supernova remnant RCW 86. A new challenge for modeling the emission from supernova remnants, *Astron. Astrophys.*, **328**, 628–633, 1997.

Vink, J., J. A. M. Bleeker, J. S. Kaastra, K. van der Heyden, and J. Dicke, XMM-Newton observations of MSH 14-63 (RCW 86), in *Neutron Stars in Supernova Remnants*, ASP Conference Series, Vol. 271, ed. P. O. Slane and B. M. Gaensler (San Francisco: ASP), 423–426, 2002.

e-mail: steve_reynolds@ncsu.edu