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Sensors Faults: A YJBK Approach for Compensation

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Abstract: The focus in this paper is on compensation for sensor faults. Virtual sensors or compensation using the YJBK controller architecture (after Youla, Jabr, Bongiorno and Kucera) are two ways to compensate sensor faults without changing the nominal controller directly. Design of virtual sensors is quite simple due to the separation between the nominal controller and the virtual sensor. A short analysis of the virtual sensor in connection with the YJBK controller architecture is given.

Further, a reformulation of the YJBK controller architecture is given and the connection with the Bezout equation is described. It is shown that this new formulation makes it easy to change the sensors in the system without changing the nominal controller, but only extend the included system matrices. System uncertainties in connection with sensor faults will also be considered in connection with the YJBK controller architecture.

Virtual sensors is one of the methods to handle sensor faults in control systems, Blanke et al. [2006], Richter [2011], Richter et al. [2011], Seron et al. [2012]. As an alternative to this approach, the YJBK approach developed by Youla et al., Youla et al. [1976a,b], Kucera [1975], can be applied.

In the virtual sensor approach, the faulty sensor signals are estimated by using observers. This architecture modifies the outputs from the system such that the nominal controller does not need to be modified. This means that the reconfiguration block will depend directly on the type of faults appearing in the system.

With this construction of the virtual sensor using an observer, it is possible to obtain a separation between the nominal controller and the observer. This simplifies the design of the virtual sensor and also virtual actuators intuitively and reasonable simple.

An alternative to the virtual sensor concept for handling sensor faults is to apply the YJBK concept. This controller architecture is based on the YJBK parameterization of all controllers stabilizing a given system, Tay et al. [1997], Zhou et al. [1995]. The architecture has been developed in connection with fault tolerant control in Stonstrup and Niemann [2001] and later used in Niemann and Stonstrup [2002, 2005]. An equivalent FTC setup has been developed in Zhou and Ren [2001].

In contrast with the virtual sensor setup, the nominal controller is applied directly to the faulty system. In this case, the controller is modified instead by including an additional feedback loop around the nominal controller.

The modification block is a general block for which the structure does not depend on the fault case and is therefore not limited to sensor faults. The generality gives an architecture that can be used for different types of faults with the cost that the modification block is a general dynamic feedback system in contrast to the virtual sensor setup where the modification block is an estimate of the faulty measurement signals, i.e. a virtual sensor.

The focus in this paper is first shortly to consider the virtual sensor architecture in connection with the YJBK architecture. The rest of the paper is related to the YJBK architecture in connection with sensor faults. The stability issue is considered. This includes also a description of system uncertainties in connection with sensor faults. The problem can be formulated as a robust controller design problem. Further, a new formulation of the YJBK architecture is given, where it is also possible to integrate new sensors in relation with faults in a systematic way.

2. SYSTEM SETUP

Consider the following generalized nominal system \( \Sigma \),

\[
\dot{x} = Ax + Bu
\]

or given as transfer functions

\[
\Sigma : \{ y = G_{yu}(s)u \}
\]

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) the control input signal vector, and \( y \in \mathbb{R}^p \) is the measurement vector.

Note, that it is assumed that there is no direct term from control input \( u \) to the measurement vector \( y \). This can be done without loss of generality.

Further, let the system be controlled by a stabilizing feedback controller given by:

\[
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3. THE YJBK ARCHITECTURE

The YJBK parameterization was first derived by Youla et al. [1976a,b] and independently by Kucera [1975]. The coprime factorization of the nominal system \( G_{yu}(s) \) from (2) and the stabilizing controller \( K(s) \) from (3) are given by:

\[
G_{yu}(s) = NM^{-1} = \hat{M}^{-1} \hat{N}, \quad N, M, \hat{N}, \hat{M} \in \mathcal{RH}_\infty
\]

\[
K(s) = UV^{-1} = \hat{V}^{-1} \hat{U}, \quad U, V, \hat{U}, \hat{V} \in \mathcal{RH}_\infty
\]

where the eight matrices in (4) must satisfy the double Bezout equation given by, see Zhou et al. [1995]:

\[
I = \begin{pmatrix} \hat{V} & -\hat{U} \\ -\hat{N} & M \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} = \begin{pmatrix} M & U \\ N & V \end{pmatrix} \begin{pmatrix} \hat{V} & -\hat{U} \\ -\hat{N} & M \end{pmatrix}
\]

(5)

Based on the above coprime factorization, a parameterization of all controllers that stabilize the system in terms of a stable transfer function \( Q \), i.e. all stabilizing controllers are given by using a right factored form Tay et al. [1997]:

\[
K(Q) = (U + MQ)(V + NQ)^{-1}, \quad Q \in \mathcal{RH}_\infty
\]

(6)

or by using a left factored form Tay et al. [1997]:

\[
K(Q) = (\hat{V} + Q\hat{N})^{-1}(\hat{U} + Q\hat{M}), \quad Q \in \mathcal{RH}_\infty
\]

(7)

Using the Bezout equation, the controller given either by (6) or by (7) can be realized as an LFT (linear fractional transformation) in the parameter \( Q \):

\[
K(Q) = \mathcal{F}_I \left( \begin{pmatrix} UV^{-1} & \hat{V}^{-1} \\ V^{-1} & -\hat{V}^{-1} \hat{N} \end{pmatrix}, Q \right) = \mathcal{F}_I(J_K, Q)
\]

(8)

\( K(Q) \) in (8) is the same for both the right form given in (6) or the left form given in (7).

The YJBK controller architecture is shown in Fig. 1.

Fig. 1. The YJBK parameterization of all stabilizing controllers \( K(Q) \) for a given system \( \Sigma \).

Based on the YJBK parameterization shown above, it is possible to describe the closed-loop matrix transfer function from external input to external output as an affine function of the YJBK matrix transfer function \( Q \), Tay et al. [1997].

It is also possible to derive a parameterization of all systems that are stabilized by one controller in terms of a stable matrix transfer function \( S \), i.e. the dual YJBK parameterization. The dual YJBK parameterization is given by Niemann [2003], Tay et al. [1997]:

\[
G_{yu}(S) = (M + S\hat{U})^{-1}(\hat{N} + S\hat{V}), \quad S \in \mathcal{RH}_\infty
\]

(9)

using the left form. There also exists a right form for the dual YJBK parameterization.

The dual YJBK parameterization can also be represented in an LFT form. This is given by:

\[
G_{yu}(S) = \mathcal{F}_I \left( \begin{pmatrix} NM^{-1} & \hat{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix}, S \right)
\]

\[
= \mathcal{F}_I(J_G, S)
\]

(10)

Further, \( S \) is given as an upper LFT by, Tay et al. [1997]:

\[
S = \mathcal{F}_u(J_K, G_{yu}(S))
\]

(11)

or

\[
\varepsilon = S\eta
\]

(12)

(see Fig. 1).

The matrix transfer function \( S \) is a function of the difference between the nominal model and the real system. In the case of perfect model, \( S \) is given by, Niemann [2003]:

\[
S(\Delta) = 0, \quad \Delta = 0
\]

Note that the dual YJBK matrix transfer function \( S \) can be applied in different connections. From Niemann [2003], Tay et al. [1997], the closed-loop uncertain system is stable if the nominal closed-loop system is stable and the resulting \( S \) is stable. This result will be used in the following in connection with closed-loop stability analysis as well as in connection with design of YJBK matrix transfer function \( Q \) for compensation of sensor faults.

Applying the above results based on the dual YJBK parameterization in connection with sensor faults, it is important to point out that the results cannot be applied directly without require that the faulty system is detectable. It is a requirement for the existing of stabilizing controllers that the unstable poles in the system is observable in the output. This condition cannot be seen from the dual YJBK matrix transfer function \( S \). This requirement is not pointed out very clearly in connection with model uncertainty. The condition is included indirectly through the description of the uncertainties, where it is assumed that the uncertainties will not change the detectability and the stability of the system.

4. SENSOR FAULTS

Let us consider the system \( \Sigma \) in the case of sensor faults. Sensor faults are modelled as a change in the output matrix \( C_y \). In many cases, a sensor fault will result in a reduction of the number of measurement signals. In case of sensor faults, the system \( \Sigma \) is given by:

\[
\Sigma_{sen} : \begin{cases} y_f = G_{yu,sen}(s)u \\ \text{or in state space form} \\ \Sigma_{sen} : \begin{cases} \dot{x} = A\bar{x} + B_\text{u}u \\ y_f = C_yf_j \\ y_f = (1 + \theta_i)G_{yu,s}(s)u, \quad i = 1, \ldots, p
\end{cases}
\end{cases}
\]

(13)

(14)

where \( y_f \in \mathcal{R}^p \), i.e. the system include \( k \) sensor faults. Let the \( i' \)th measurement signal be given by:

\[
y_i = (1 + \theta_i)G_{yu,s}(s)u_i, \quad i = 1, \ldots, p
\]

(15)
where \( \theta_i = \{0, -1\}, \ i = 1, \cdots, p \) is a parametric description of the sensor faults. \( \theta_i = 0 \) describe a fault free \( i \)'th sensor and \( \theta_i = -1 \) describe a complete loss of the \( i \)'th sensor. Using a matrix notation for the sensor faults, the faulty output vector \( y_f \) is given by
\[
y_f = (I + \theta)G_{yu}(s)u
\] (16)
or using the state space description
\[
C_{y,f} = (I + \theta)C_y
\] (17)
where \( \theta = \text{diag}(\theta_1, \cdots, \theta_p) \).

It is assumed in this paper that a sensor fault result in a complete loss of the faulty sensor, i.e. \( \theta_i = -1 \). In the case where the faulty sensor is working partly, it will be assumed that the sensor is decoupled.

A more explicit description of the system setup for systems with parameter faults can be given by including an extra input and output vector in the system. The above system is then given by
\[
\Sigma_{\theta} : \begin{cases}
z_{\theta} = G_{zu,s}(s)w_\theta + G_{zu,s}(s)u \\
y = G_{yus}(s)w_\theta + G_{yu}(s)u
\end{cases}
\] (18)
where the connection between the two external vectors \( w_\theta \) and \( z_\theta \) is given by
\[
w_\theta = \theta z_\theta
\] (19)
This description is equivalent with the general description of system with model uncertainties, see e.g. Zhou et al. [1995].

The general system in (18) is given by
\[
\Sigma_{\text{sensor}} : \begin{cases}
z_\theta = G_{yu}(s)u \\
y = Iw_\theta + G_{yu}(s)u
\end{cases}
\] (20)
in the case of sensor faults.

4.1 Virtual Sensors

The concept of virtual sensors is described in a number of papers, see e.g. Richter [2011], Richter et al. [2011], Seron et al. [2012]. A virtual sensor is an observer that estimates the faulty sensor signal. Consider the system with a sensor fault \( \Sigma_{\text{sen}} \) given by (14). A virtual sensor for this faulty system is then given by:
\[
\Sigma_{\text{virtual}} : \begin{cases}
\dot{x}_v = (A - LC_{y,f})x_v + Ly_f + B_u u \\
y = (C_y - PC_{y,f})x_v + Py_f
\end{cases}
\] (21)
where \( x_v \) is the state in the observer, \( P \) and \( L \) are matrices that can be chosen freely.

The virtual sensor can also be described by matrix transfer functions given by:
\[
\Sigma_{\text{virtual}} : \{ y = H_y(s)y_f + H_u(s)u \}
\] (22)
where
\[
H_y = (C_y - PC_{y,f})(sI - A + LC_{y,f})^{-1}L + P \\
H_u = (C_y - PC_{y,f})(sI - A + LC_{y,f})^{-1}B_u
\]

Now, let the virtual sensor be included in the YJBK setup in Fig. 1 together with the faulty system given by (20). The complete setup is shown in Fig 2, where \( J_V \) is given by:
\[
J_V = \begin{pmatrix} 0 & I \\ H_y(s) & H_u(s) \end{pmatrix}
\] (23)
\[ J_{VK}^{\Sigma} = \Sigma_{\text{sensor}} \ast J_V \ast J_K \]

\[
\begin{pmatrix}
N \hat{U} & N \\
M & 0
\end{pmatrix}
\]

(26)

with \( J_{22} = 0 \).

From the above calculation of the dual YJBK matrix transfer function, it is not straightforward to show that the virtual sensor guarantee closed loop stability of the system by showing that \( S(\theta) \) given by (25) is stable in the general case. The decoupling approach applied for the design of the virtual sensor cannot be seen directly in \( S(\theta) \). It is therefore not relevant to consider virtual sensors in the YJBK controller architecture. Instead, the compensation can be obtained directly by designing the YJBK matrix transfer function \( Q \). This is considered in the following.

### 4.2 The YJBK Architecture

As a direct result of the theory for the YJBK parameterization given in Section 3, there is a separation between the nominal closed-loop system and the closed-loop consisting of the fault dynamic system given by the dual YJBK matrix transfer function \( S \) and the YJBK matrix transfer function \( Q \), the result of (11). This result is general and not restricted to sensor faults. This gives very simple way to analyze for closed-loop stability for sensor faults as well as an easy method for designing a stabilizing controller for the faulty system.

For sensor faults, the dual YJBK matrix transfer function is given by, Niemann [2003] [can also be seen directly from (26)]:

\[ S(\theta) = \hat{M}\theta(I - N\hat{U}\theta)^{-1}N \]

(27)

Assume sensor \( i \) is faulty, i.e. \( \theta_i = -1 \) (complete loss of sensor \( i \)) and \( \theta_j = 0 \), for \( j \neq i \). The faulty closed loop system is then stable if \( S(\theta) \) is stable and unstable if \( S(\theta) \) is unstable. Further, if the faulty closed-loop system is unstable, it is possible to stabilize it by designing a YJBK matrix transfer function \( Q_i \) that will stabilize \( S(\theta) \). If the faulty system can be stabilized by a feedback controller, \( S(\theta) \) can also be stabilized by a \( Q_i \). Including \( Q_i \) in the controller, we get:

\[ S(Q_i, \theta) = S(\theta)(I - Q_i S(\theta))^{-1} \]

(28)

and with \( S(\theta) \) from (27) in (28) gives:

\[ S(Q_i, \theta) = \hat{M}\theta(I - N\hat{U}\theta - NQ_i\hat{M}\theta)^{-1}N \]

(29)

When the YJBK controller architecture is applied directly to handle sensor faults, then we will also get a direct separation between the nominal closed loop system and a closed-loop only related to the faulty sensor.

### 4.3 Uncertain Systems

The results given above are for systems without uncertainties. In real applications, the systems will also include uncertainties. Using the YJBK controller approach for compensation of sensor faults, it is possible to include system uncertainties in connection with the design of the YJBK matrix transfer function \( Q \).

Let the uncertain system be described by the following general description (see Skogestad and Postlethwaite [2005]):

\[ G_{\text{unc}}(s, \Delta) = G_{\text{nom}}(s) + G_{\text{unc}}(s) \Delta(I - G_{zw}(s)\Delta)^{-1}G_{zu}(s) \]

(30)

where \( \Delta \) describe the uncertainty in the system. It can be a full uncertain complex block or it can be structured. It is further assumed that \( \Delta \) is scaled such that \( \|\Delta\| \leq 1, \forall \omega \).

Now, let the uncertain system \( \Sigma_{\text{uncertainty}} \) in (30) be included in the system description with sensor faults given by \( \Sigma_{\text{sensor}} \) in (20). Let \( \Sigma_{\text{sensor}} \) be extended with an additional input \( w_{\Delta} \) and output \( z_{\Delta} \) for including the uncertainties in the system. The extended system is the given by:

\[
\Sigma_{\text{unc}} : \begin{cases}
\begin{pmatrix}
z_{\Delta} \\
z_0
\end{pmatrix} = \begin{pmatrix}
G_{zu}(s) & 0 \\
G_{yw}(s) & 0
\end{pmatrix} \begin{pmatrix}
w_{\Delta} \\
w_{\theta}
\end{pmatrix} + \begin{pmatrix}
G_{zu}(s) \\
G_{yu}(s)
\end{pmatrix} u \\
y = \begin{pmatrix}
w_{\Delta} \\
w_{\theta}
\end{pmatrix} + G_{yu}(s)u
\end{cases}
\]

(31)

where the connection between the two external vectors \( w_{\Delta} \in \mathbb{R}^r \) and \( z_{\Delta} \in \mathbb{R}^r \) is connected through the uncertain block \( \Delta \), i.e.

\[ w_{\Delta} = \Delta z_{\Delta} \]

(32)

Including both the uncertain block \( \Delta \) and the sensor faults \( \theta \) in the feedback loop from \( z \) to \( w \), we get the following description:

\[ w = \begin{pmatrix}
w_{\Delta} \\
w_{\theta}
\end{pmatrix} = \begin{pmatrix}
\Delta & 0 \\
0 & \theta
\end{pmatrix} \begin{pmatrix}
z_{\Delta} \\
z_0
\end{pmatrix} = \Delta z \]

(33)

Based on the uncertain system description given by (31), the dual YJBK matrix transfer function is then given by (using the general result from Niemann [2003]):

\[ S_{\text{unc}}(\Delta) = S_{21} \Delta(I - S_{11}\Delta)^{-1}S_{12} \]

(34)

where

\[ S_{11} = \begin{pmatrix} G_{zu} & 0 \\ G_{yw} & 0 \end{pmatrix} + \begin{pmatrix} G_{zu} \\ G_{yu} \end{pmatrix} U\hat{M} \begin{pmatrix} G_{yu} & I \end{pmatrix} \]

\[ = \begin{pmatrix} G_{zu} + G_{zu}U\hat{M}G_{yu} & G_{zu}U\hat{M} \\ G_{yw} + G_{yu}U\hat{M}G_{yu} & N\hat{U} \end{pmatrix} \]

\[ S_{12} = \begin{pmatrix} G_{zu} \\ G_{yu} \end{pmatrix} M \]

\[ = \begin{pmatrix} G_{zu}M \\ G_{yu} \end{pmatrix} \]

\[ S_{21} = \hat{M} \begin{pmatrix} G_{yu} & I \end{pmatrix} \]

\[ = \begin{pmatrix} \hat{M}G_{yu} & \hat{M} \end{pmatrix} \]

The closed-loop system is unstable if \( S_{\text{unc}}(\Delta) \) given by (34) is unstable for a given faults with \( \Delta \) satisfying \( \|\Delta\| \leq 1, \forall \omega \). If the closed-loop system is unstable, it might be possible to design a YJBK matrix transfer function \( Q \) such the closed-loop system given by:

\[ S_{\text{unc}}(Q, \Delta) = S_{\text{unc}}(\Delta)(I - QS_{\text{unc}}(\Delta))^{-1} \]

(35)

is stable for \( \|\Delta\| \leq 1, \forall \omega \). For a given sensor fault, this is a standard robust control design problem and standard design methods can be applied, see e.g. methods described in Skogestad and Postlethwaite [2005], Zhou et al. [1995].
5. REPRESENTATION OF THE YJBK ARCHITECTURE

The representation of the YJBK controller showed in Fig. 1 is one of the standard implementations shown in many publications. Equivalent, there exist also an implementation based on the right factored form.

The implementation of the controller is important. Implementing the YJBK architecture as showed in Fig. 1 directly might result in a very high order controller. Instead, implementation based on observer based controllers can be used where it is possible to implement the YJBK feedback controller as an nth-order controller + the order of Q. It will normally be required a model-based feedback controller to be able to make a controller implementation of the same order as the system.

5.1 Controller Architecture

First, let’s look at the YJBK controller architecture. The architecture from Fig. 1 for the YJBK controller is just one way for the implementation. Another way is to use an architecture based directly on the description of the controller given by (7) or (6). Applying an observer-based controller, $J_k$ in (8) is reduced to the same order as the system. The architecture showed in Fig. 1 includes the invers of a matrix transfer function.

It is also possible to make an implementation based directly on the matrix transfer functions from the Bezout equation. Rewriting the controllers given by (7), the YJBK parameterized controller can be implemented as shown in Fig. 4, where $\hat{Z}$ and $Q$ are given by:

$$\hat{Z} = \begin{pmatrix} \hat{V} - I - \hat{U} \\ -\hat{N} & \hat{M} \end{pmatrix}$$

$$Q = (-I \ Q)$$

and $\varepsilon_Q$ is given by

$$\varepsilon_Q = \begin{pmatrix} \varepsilon_u \\ \varepsilon \end{pmatrix} = \begin{pmatrix} (\hat{V} - I)u - \hat{U}y \\ \hat{M}y - \hat{N}u \end{pmatrix}$$

$\varepsilon$ is the standard residual vector as shown in the previous block diagrams.

![Fig. 4. A compact implementation of the YJBK parameterization based on the left factored form where $\hat{Z}$ and $Q$ are given by (36).](image)

The controller architecture showed in Fig. 4 does not include inversion of matrix transfer functions. Further, the main block in the architecture is directly connected to $\hat{Z}$ from the Bezout equation in (5).

Fig. 4 gives a direct relation between the Bezout equation and the YJBK controller architecture showed in Fig. 1.

This representation of the YJBK parameterization showed in Fig. 4 has been discussed in more details in Niemann [2018].

5.2 Sensor extension

The representation of the YJBK controller architecture shown above is interesting in connection with a number of different applications. As one application, let’s consider the case where the system is extended with extra sensors. Including extra sensors and actuators are considered in Niemann [2006] in connection with the YJBK parameterization. Here, it will only be shown how these results give a simple extension of the controller architecture shown above.

Let the system $\Sigma$ be extended with additional sensors, i.e. the system is now given by:

$$\Sigma_{ext} : \begin{cases} y_{ext} = y \\ y_s = G_{yu,s}(s) \end{cases}$$

where $y_s$ is the additional measurements from the extra sensors. From Niemann [2006] we have the coprime matrices for the extended system controlled with the nominal controller given in (3) are given by:

$$\begin{pmatrix} M_{ext} & U_{ext} \\ N_{ext} & V_{ext} \end{pmatrix} = \begin{pmatrix} M & (U \ 0) \\ N & (V \ 0) \end{pmatrix}$$

$$\begin{pmatrix} V_{ext} \\ -N_{ext} \end{pmatrix} \begin{pmatrix} M_{ext} \end{pmatrix} = \begin{pmatrix} \tilde{V} \\ -\tilde{N} \end{pmatrix} \begin{pmatrix} \tilde{M} \end{pmatrix}$$

A state space formulation of the extended coprime matrices in (39) can be found in Niemann [2006].

Based on the coprime matrices given in (39), it is easy to extend the implementation of the compact YJBK controller architecture shown in Fig. 4 to handle the case when additional sensors is included. In this case, $\hat{Z}$, $Q$ and $\varepsilon_Q$ are given by:

$$\hat{Z} = \begin{pmatrix} \hat{V} - I \\ -\hat{N}u + \hat{M}y \end{pmatrix}$$

$$Q = (-I \ Q)$$

$$\varepsilon_Q = \begin{pmatrix} \varepsilon_u \\ \varepsilon \end{pmatrix} = \begin{pmatrix} (\hat{V} - I)u - \hat{U}y \\ -\hat{N}u + \hat{M}y \end{pmatrix}$$

where $\varepsilon_s$ is the residual vector related to the additional sensors and is equivalent to the residual vector $\varepsilon$ for the original system. The additional measurement signals $y_s$ can directly be applied in the feedback controller through the additional matrix transfer function $Q_s$.

Based on the extended system in (38) and the associated coprime matrices, the dual YJBK matrix transfer function $S_{ext}$ can be calculated. $S_{ext}$ is given by (note that this result can also be calculated from the results given in Niemann [2006]):

$$S_{ext} = \begin{pmatrix} S \\ S_s \end{pmatrix} = \begin{pmatrix} \hat{M}\theta N \\ \hat{M}_s\theta N + \theta_s N_s \end{pmatrix} (I - \hat{U}\theta N)^{-1}$$

where $\theta_s$ is the description of the sensor fault for the additional sensors. It is clear that $S$ is the same as in...
that we are now using to other fault free sensors. Further, system uncertainties in with e.g. sensor faults, where it might be relevant to change the controller, the system extension can be included in the controller architecture. For a given sensor fault, the resulting design problem will be a standard robust controller design problem where standard methods can be applied.

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