Generalized multiparametric models for predicting the physical and mechanical properties of composites when exposed to extreme environmental factors

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Abstract. Based on the formulated principle of model multiplicity forecasting and the introduced concept of optimal forecasting models difficulties in the framework of variational statements the questions of development are investigated effective refined forecasting methods that determine the physical-mechanical characteristics of structures made of composite materials. A system of additional ratios has been developed, including them in the structure of the initial tasks allowed us to significantly reduce the dimension the original variational statements of the inverse problems of forecasting and significantly narrow down the original set of valid model options predictions.

1. Introduction
In recent decades, composite materials, structures, and coatings have been used in the modern fields of physics, rocket and space technology, and aviation, as well as in the oil and gas industry and shipbuilding. The role of composite materials, structures, and composite coatings in the design of wave energy converters for various purposes for solving problems of effective management of energy spectra of radiation in such areas of physics and instrument engineering as Radioelectronics, quantum physics, electrodynamics and acoustics, construction physics, optics, and others. [1-11]. One of the effective approaches for creating mathematical models for predicting the durability of composite materials and structures made of them is variational methods [12-15].

To maintain high performance of such devices for a long time under the influence of extreme environmental factors, the problem of effective forecasting of residual life, durability, strength, reliability and physical and mechanical properties of materials becomes important.
The issues of developing effective refined methods for predicting the defining characteristics of structures made of composite materials based on modern achievements in the field of mathematical and computer modeling are investigated. On the basis of the principle of multiplicity of forecasting models, generalized multiparametric models for predicting the optimal physical and mechanical structure have been developed, describing the impact of several extreme destabilizing factors of different physical nature on composite materials and structures. In the framework of the developed multivariate models to forecasting of a residual resource, physical and mechanical properties of materials when exposed to an indefinite number of extreme factors, so the impact of strictly defined pre-known factors.

2. Problem statement

An effective solution to the problem of long-term forecasting of the defining characteristics of composites under the influence of extreme environmental factors is possible if the results of short-term physical experiments can identify stable qualitative patterns of behavior of composites under these influences. Such stable qualitative regularities are determined by the micro- and macrostructure features of specific types of composites.

The definition and research of such stable qualitative regularities is the basis of a new approach to improving the efficiency of forecasting methods for determining the characteristics of composite materials and structures. The proposed approach is based on more accurate statements of forecasting problems and is developed within the framework of variational principles for solving inverse problems, which for the first time included an assessment of the accuracy of predicted solutions, which allowed us to obtain new scientifically based results and meet the required accuracy of the forecast.

In accordance with this, the problem of developing effective refined methods for predicting physical and mechanical properties, resource, reliability, strength, and durability of structures made of polymer composite materials based on modern achievements in the field of mathematical and computer modeling is relevant.

We introduce a generalized model $R$ that describes the impact on the composite material simultaneously several different factors $F_j$, associated with the hardening processes, as well as the impact of extreme environmental factors, such as

$$R = R_0 + \sum_{j=1}^{p} F_j(u_{j,1}, u_{j,2}, ..., u_{j,l_j}; t).$$

The functional dependence describing the effect of the j-factor on the composite $F_j(u_{j,1}, u_{j,2}, ..., u_{j,l_j}; t)$ can be represented as a series expansion for a certain system of basic functions $\psi_{kj}(t)$, $(k=1,2,3,...)$. This system of basic functions $\psi_{kj}(t)$ most fully characterizes the features of the process of increasing the damage of the composite material under the influence of extreme environmental factors. In these notations $u_{j,1}, u_{j,2}, ..., u_{j,l_j}$ is a system of undefined model parameters describing the impact of the j-factor.

The general formulation of the forecasting problem based on the introduced generalized forecasting model can be formulated as follows: on the basis of short-term tests carried out on the time interval $[0, T_{\text{min}}]$, i.e. on the basis of information about the values of the defining property $R = R_1, R_2, ..., R_m$, measured at times $t_1, t_2, ..., t_m$, predict the change in the defining property of the composite structure on the interval $[T_{\text{min}}, T_{\text{max}}]$, with an error not exceeding the predetermined maximum permissible accuracy of the forecast.
3. Evaluation of the impact of measurement errors in the source data on the accuracy of the forecast

Denote by \( \tilde{R}_i (i=1,\ldots,m) \) the values of the defining property at times \( t_1, t_2, \ldots, t_m \), measured with some errors \( Q_i \):

\[
\tilde{R}_i = R^*_i + Q_i, \quad (i=1,\ldots,m).
\]

We will consider the set of errors \( Q_1, Q_2, \ldots, Q_m \) as a set of continuous random variables. In this case, the total error \( \tilde{S} \) will also be a random value that depends, in addition to the parameter vector \( u \), additionally on random factors \( Q_1, Q_2, \ldots, Q_m \) associated with measurement errors:

\[
\tilde{S} = S(u; Q) = \frac{1}{m} \sum_{i=1}^{m} [R(Q) - R(u_1, u_2, \ldots, u_n; t_i)]^2 \Rightarrow \min_{u}. \quad (3)
\]

In these notations, \( Q \) is the vector of measurement errors: \( Q = (Q_1, Q_2, \ldots, Q_m) \); \( R(Q) = \tilde{R}_i \) - measured values of the defining parameter \( R \) at time points \( t_1, t_2, \ldots, t_m \), taking into account measurement errors \( Q_1, Q_2, \ldots, Q_m \). Denote by \( \bar{u}^* = u^*(Q) \) - a vector of undefined parameters that delivers the global minimum of the total error \( \tilde{S} \) (3):

\[
S[u^*(Q); Q] = \min_{u} S(u; Q). \quad (4)
\]

Important for the solution of research problems impact on the accuracy of the forecast level of errors of measurement of residual resource is the construction of functional analytical relations between the parameters of prediction models, define their structure, with prediction accuracy values and errors of experimental measurements of the original data. To construct estimates from above on the permissible error values \( Q_i \) (\( i=1,\ldots,m \)) of the measurement of the values of the defining property \( R \) at time points \( t_1, t_2, \ldots, t_m \), we decompose the n-dimensional vector function \( u^*(Q) = (u_1^*(Q), u_2^*(Q), \ldots, u_n^*(Q)) \) into a series of degrees of the vector parameter \( Q = (Q_1, Q_2, \ldots, Q_m) \), given that \( u^*(0) = u^* = (u_1^*, u_2^*, \ldots, u_n^*) \).

\[
u_j(Q) = u_j^* + \sum_{i=1}^{m} \beta_{ji} Q_i + o(\|Q\|). \quad (5)
\]

In these notations

\[
\|Q\| = \left( \sum_{i=1}^{m} Q_i^2 \right)^{1/2}.
\]

At the same time,

\[
o(\|Q\|) \rightarrow 0, \quad npu \|Q\| \rightarrow 0.
\]

We decompose the predicted time dependence of the defining parameter \( R \) of the composite structure \( R(u^*(Q); t) \) taking into account the measurement errors of the defining property at time points \( t_1, t_2, \ldots, t_m \) in a series of degrees of the vector parameter \( Q \) on the studied predicted time interval \([T_{\min}, T_{\max}]\).

\[
R(u^*(Q); t) = R(u^*; t) + \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial R(u^*; t)}{\partial u_j} \beta_{ji} Q_i + o(\|Q\|; t). \quad (6)
\]
Based on constructive mathematical analysis of necessary and sufficient conditions of extremum in the refined variational statements of the inverse problems of forecasting with decomposition the basic equations of the problem in a series in powers of the vector parameter $Q$ that describes the distribution of measurement errors for time points $t_1, t_2, \ldots, t_m$ builds a functional analytical relations satisfied by the coefficients of the optimal models to predict the optimal structure $\beta_{js}$ ($j=1,2,\ldots,n; s=1,2,\ldots,m$).

4. Promising ways to reduce the dimension of variational statements of inverse prediction problems of defining characteristics of composites

On the basis of a constructive study of the structure of performance indicators in the studied variational statements of inverse problems for predicting the residual resource of composites, it is established that optimal solutions that deliver an absolute minimum to the corresponding performance indicators can additionally satisfy a certain system of ratios. The development of a methodology for allocating such systems of additional ratios can significantly reduce the dimension of the original variational statements of inverse problems for predicting the residual resource of composites and, accordingly, significantly narrow the original set of acceptable options to a narrow compact set $M^*$, on which the globally optimal solution corresponding to the absolute minimum of the efficiency indicator can be effectively found by a complete search of all acceptable options.

Functional relationships are established that can additionally satisfy the desired globally optimal solutions in variational statements of inverse residual resource prediction problems. With functional correlation, which additionally can satisfy the desired globally optimal solution in the variational statements of the inverse problems of forecasting of a residual resource in the general case can be written in the form:

$$g_j(u_1,u_2,\ldots,u_n) = 0, \quad j = 1,2,\ldots,q; q < n.$$

Having resolved this system with respect to the first $q$ parameters $u_1, u_2, \ldots, u_q$ we will have

$$u_i = \varphi_i(u_{q+1},u_{q+2},\ldots,u_n), \quad i = 1,2,\ldots,q.$$

After substituting the ratios (8) in the expression for the performance indicator, we will have

$$J(\varphi_1(u_{q+1},u_{q+2},\ldots,u_n),\ldots,\varphi_q(u_{q+1},u_{q+2},\ldots,u_n)) = \Phi(u_{q+1},u_{q+2},\ldots,u_n)$$

As a result, a multiparameter variational problem with $n$ parameters can be reduced to a variational problem of significantly smaller dimension with $r = n-q$ parameters. As a result the original variational problem of sufficiently large dimension $n$ can be reduced to a simpler variational problem of significantly smaller dimension of the form:

$$\Phi(u_{q+1},u_{q+2},\ldots,u_n) \Rightarrow \min, \quad (u_{q+1},u_{q+2},\ldots,u_n) \in R.$$
In this notation R is the set of valid options for parameters $u_{q+1}, u_{q+2}, ..., u_n$. The variational problem (10) will have a much smaller dimension and can be effectively solved in full on the basis of the application of full search methods or their possible effective modifications taking into account the structural features of the variational problem of the form (10).

The constructive analysis of the studied variational statements of inverse residual resource forecasting problems allowed us to establish that the necessary conditions of the extremum for performance indicators for a number of parameters simultaneously coincide with sufficient conditions and these relations determine only globally optimal solutions, which are the desired solutions in the problems under consideration. Let's denote such parameters by $u_{j_1}, u_{j_2}, ..., u_{j_q}$. The necessary minimum conditions for this selected system of parameters $u_{j_1}, u_{j_2}, ..., u_{j_q}$ can be taken as a system of functional relations $g_j (j=1,2,...,q)$ (7), which can serve as a basis for the subsequent significant reduction in the dimension of the original forecasting problem and simplification of its structure. In accordance with this

$$g_i(u) = \frac{\partial J(u_1, u_2, ..., u_n)}{\partial u_{j_i}}, \ (i = 1, 2, ..., q).$$  \hspace{1cm} (11)

We calculate on the basis of solving a system of algebraic equations, which result in the necessary and sufficient minimum conditions parameters $u_{j_1}, u_{j_2}, ..., u_{j_q}$ through the remaining system of $r=n-q$ parameters $u_l \ (l = 1, 2, ..., n; \ l \neq u_{j_i}, i = 1, 2, ..., q)$:

$$u_{j_i} = \varphi_i\left(u_1, u_2, ..., u_{j_i-1}, u_{j_i+1}, ..., u_{n-1}, u_n\right), \ i = 1, 2, ..., q.$$  \hspace{1cm} (12)

As a result we get a variational problem of much smaller dimension and correspondingly simpler structure of the form

$$\Phi\left(u_{j_1}, u_{j_2}, ..., u_{j_q}\right) = \min,$$

$$\left(u_{j_1}, u_{j_2}, ..., u_{j_q}\right) \in R \subseteq E^r,$$

$$1 \leq u_{j_i} \leq n, \ k = 1, 2, ..., r,$$

$$\left(u_{j_i} \neq u_l, k = 1, 2, ..., r; \ i = 1, 2, ..., q\right).$$  \hspace{1cm} (13)

In these notations

$$\Phi\left(u_{j_1}, u_{j_2}, ..., u_{j_q}\right) =$$

$$= J\left(\varphi_1\left(u_{j_1}, u_{j_2}, ..., u_{j_q}\right), ..., \varphi_q\left(u_{j_1}, u_{j_2}, ..., u_{j_q}\right), u_1, u_2, ..., u_n\right).$$  \hspace{1cm} (14)

Let be $\left(u_{l_1}^*, u_{l_2}^*, ..., u_{l_r}^*\right)$ the optimal solution in the considered auxiliary variational problem (14). In accordance with this

$$\Phi\left(u_{l_1}^*, u_{l_2}^*, ..., u_{l_r}^*\right) = \min_{u_{j_1}, u_{j_2}, ..., u_{j_q}} = \Phi\left(u_{j_1}, u_{j_2}, ..., u_{j_q}\right).$$  \hspace{1cm} (15)
Then the rest of the optimal parameters are determined according to the relations (12):

\[
\varphi_i = \varphi\left(u_i^*, u_{i-1}^*, \ldots, u_{i-1}^*, u_{i+1}^*, \ldots, u_n^*, u_n^*, u_{n+1}^* \right),
\]

\[i = 1, 2, \ldots, q.\]  

Effective methods of nonlocal optimization have been developed for the formulated auxiliary variational problem (15), which has a significantly smaller dimension and simpler structure compared to the original variational problem, taking into account the established features of its structure.

Comparative computational experiments were performed using the developed methods based on the application of necessary and sufficient extremum conditions for effective constructive transformation of variational statements of inverse residual resource prediction problems to variational problems of smaller dimension. Comparative computational experiments have shown high efficiency of the developed methods for solving problems of predicting the residual resource of composites in variational statements, the structure of which takes into account the system of relations based on the necessary and sufficient conditions of the extremum.

Based on a constructive study of the structure of performance indicators in the studied variational statements of inverse problems for predicting the remaining resource of composites, a system of additional relations was developed, the inclusion of which in the structure of the original problems allowed to significantly reduce the dimension of the original variational statements of inverse forecasting problems and significantly narrow the initial set of acceptable variants of forecasting models.

The developed methods for solving inverse prediction problems in refined variational statements can be applied to solve a wide range of practical problems of effective prediction of residual resource, physical and mechanical properties of structures made of composite materials in modern fields of physics.

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