The universal algorithm for calculating SPICE models’ parameters of semiconductor diodes

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Abstract. The methods and procedures for calculating SPICE models' parameters of semiconductor diodes for the forward and reverse branches of the VAC are described. Recommendations for planning the experiment and selecting initial approximations of parameters for forward and reverse switching of the diode are given. Existing methods for calculating SPICE models' parameters do not allow obtaining their statistical characteristics, and for some parameters, methods for estimating them from experimental data have not been developed at all. This article offers a universal method for evaluating all the necessary parameters of SPICE models of semiconductor diodes and their statistical characteristics.

1. Introduction

The identification task is to determine the parameters of the diode model based on the results of measurements of the forward and reverse branches of the VAC diode, respectively. Most publications devoted to methods of parameter identification [1-5] focus on nonlinear optimization algorithms based on a large number of measurements.

The complexity of the problem of estimating parameters from experimental data, as a problem of nonlinear optimization, is due to the following factors:

- large dimension of the optimization space;
- the complexity of setting the initial approximation vector for the general case (some parameters, such as saturation currents and recombination current parameters, may differ by several orders of magnitude for different devices);
- the need for a large number of measurements;
- difficulty in choosing the most stable and fast-acting optimization method.

The main disadvantages of the known methods for estimating parameters of SPICE models are that they are not universal, do not have sufficient stability, do not offer methods that reduce the number of measurements and, at the same time, increase the accuracy of parameter estimation, and do not provide statistical characteristics of the obtained parameter estimates. In addition, some methods for estimating the parameters of SPICE models use various assumptions to simplify calculations, for example, dividing the VAC into sections within which some parameters can be ignored. Such assumptions are often insufficiently justified, and in some situations this approach may lead to large errors in the estimation of parameters.

The absence of statistical characteristics of the obtained estimates actually makes it impossible to compare the values of the parameters of individual devices and does not allow comparing the average
parameters for batches of devices. Therefore, the identification and investigation of statistical properties of the obtained parameter estimates is extremely important.

2. Calculating SPICE models' parameters based on experimental data

2.1. Measurement errors

In this paper, the parameters are calculated based on the full SPICE model of the diode, taking into account the additive measurement errors in the form of the following expression for the VAC.

\[
I = F(V_{p-n}, B) + e,
\]

where \( F(\cdot) \) – the nonlinear function of the voltage on p-n junction \( V_{p-n} \) coefficient vector \( B, B=\{IS, N, ISR, NR, IKF, VJ, M, RS\}^T \) is a vector of model parameters representing a set of parameters for a SPICE model of the diode for the direct branch and \( B=\{IBV, NBV, BV, NBVL, IBVL, RS\}^T \) for the backward branch, \( e \) is the absolute measurement error of the reverse current.

The absolute measurement error of current is expressed in terms of the relative measurement error \( E \) as

\[
e = E \cdot I
\]

Therefore, in model (1), the measurement error is not consistent, since it is proportional to the measured current. To reduce the measurement error to a consistence, we logarithm the value of the diode current and as a result for small relative errors we get

\[
\ln(I + EI) = \ln(I) + \ln(1 + E) \approx E.
\]

Therefore, for logarithmic current values, the absolute measurement error is equal to the relative measurement error and does not depend on the current value of the diode. Then equation (1), taking into account the expression (2), takes the form

\[
\ln(I) = \ln[F(V_{p-n}, B)] + E,
\]

where \( E \) – observation error in logarithmic current values with \( \sigma_E^2 \) variance, which does not depend on the current value.

2.2. Parameter calculation

The regression analysis procedure [6] can now be used to estimate the parameters of model \( B \) (3). The most plausible estimates of parameters \( B \) can be obtained using an iterative procedure as in:

\[
\hat{B}^s = \hat{B}^{s-1} + \rho^{s-1} \left( \sum_{j=1}^{n} P^T(V_{p-n,j}) P(V_{p-n,j}) \right)^{-1} \times
\]

\[
\times \sum_{j=1}^{n} P(V_{p-n,j}) \left[ \ln(I_j) - \ln[F(\hat{B}^{s-1}, V_{p-n,j})] \right],
\]

where \( s \) – iteration index, \( \rho \) – coefficient chosen for the best convergence of the procedure, \( P(V_{p-n}) \) – a vector function defined by derivatives of the \( F(\cdot) \) function as follows:

\[
P(V_{p-n}) = \left\{ \frac{1}{F(B, V_{p-n})} \left[ \frac{\partial F(B, V_{p-n})}{\partial B} \right] \right\} | B = \hat{B}.
\]

After receiving parameter estimates as an estimate of the variance of the observation error \( \sigma_E^2 \), the \( s_E^2 \) residual variance is calculated as in:

\[
s_E^2 = \sum_{j=1}^{n} \left[ \ln(I_j) - \ln[F(\hat{B}, V_{p-n,j})] \right]^2.
\]

The covariance matrix of parameter estimates is calculated using the formula
\[ V_B \approx s_E^2 \{ \sum_{j=1}^{n} P^T(V_{p-n,j}, \hat{B})P(V_{p-n,j}, \hat{B}) \}^{-1}. \] (5)

The presence of a covariance matrix (5) of parameter estimates allows us to obtain confidence intervals for each parameter and compare the values of the corresponding parameters for different diodes in the batch. If some parameters for different diodes are statistically indistinguishable, this means that they are not informative in terms of, for example, their use in solving diagnostic problems.

2.3. Initial approximations

Procedure (4) is very sensitive to initial approximations of parameters and is divergent if the initial approximation is not selected correctly. Therefore, to obtain estimates of the initial approximation, a statistical modeling method was used, including the following set of steps.

1) for each parameter, the interval for its change is defined.
2) a random vector of B parameters, whose elements are in the allowed ranges, is generated.
3) for the obtained values of B parameters, the sum of deviation squares from the model is calculated as in
\[ \Delta(B) = \sum_{j=1}^{n} \{ \ln(I_j) - \ln[F(B, V_{p-n,j})] \}^2. \] (6)

4) Steps 2–3 are repeated a specified number of times, and the \( B^0 \) initial approximation of the parameters is determined from the condition
\[ \Delta(B^0) = m_n \Delta(B). \] (7)

To obtain a sufficiently reliable initial approximation, the number of statistical tests must amount to at least 106–107.

When selecting the initial parameter values for the reverse branch of the VAC, it is necessary to determine the breakdown voltage with an accuracy of at least \( \pm 0.001\% \) using experimental methods, since when the value of the parameter BV differs from the true value by more than 0.001\%, the phenomenon of overflow appears when calculating the reverse current of the diode.

2.4. Diode current calculation

The calculation of parameter estimates by formula (4) assumes at each iteration s the calculation of the diode current in accordance with equation (1) \( I = F(V_{p-n}, B, I) \), the right part of which depends on the current value \( I \). The most stable solution of this equation is provided by an iterative procedure as in
\[ I' = I'^{-1} + k[I'^{-1} - F(\hat{B}, V_{p-n}, I'^{-1})], \] (8)
where \( r \) – iteration index.

The value of the \( k \) coefficient affects the stability of the iterative procedure (8), which is especially important in this case, since this procedure is included as an internal loop in the procedure (4). Studies of the procedure stability (8) for various parameter values have shown that this iterative procedure may diverge or loop in some situations. According to the results of the conducted research, the \( k \) coefficient was chosen to be equal to \( \pi/12 \) in order to ensure absolute convergence and eliminate looping of the procedure (8). As an initial approximation, the current value was taken as
\[ I^0 = F(\hat{B}, V_{p-n}) \] \( R_5=0 \).

3. Experiment plan for estimating parameters

Usually, the minimum number of measurements was assumed to be 30–40. The location of the measurement points (the experiment plan) \( VD, j (j=1, ..., n) \) in the measurement range is very critical. Matrix F
\[ \Phi = \sum_{j=1}^{n} P^T \left( V_{p-n,j} \right) P \left( V_{p-n,j} \right) \]

It is called the information matrix of the experiment and actually determines the value of the covariance matrix of parameter estimates (5). If the measurement points are not located correctly, the information matrix may be poorly conditioned, which is often the reason for failures when trying to calculate parameter estimates using the formula (4). Obviously, the wider the measurement range (planning area), the more accurately the parameters are estimated, so the maximum value of the VD voltage was determined in accordance with the permissible power dissipation of the diode. The experiment plan in this case is a set of control points \( O = \{ V_{D,1}, V_{D,2}, ..., V_{D,n} \} \), where \( n \) – the number of measurements.

The D-optimality criterion assumes such an arrangement of measurement points in the \( \Sigma = (V_{D,min}, V_{D,max}) \) planning area that the determinant of the \( \Phi \) matrix is minimal, that is, the D-optimal plan minimizes the volume of the ellipsoid dispersing of coefficient estimates as in \( D = det[\Phi] \).

Thus, the D-optimal plan is expressed as follows:

\[ det[V_B(O^o)] = \min_{B \in \Sigma} det[V_B(B)] = \min_{B \in \Sigma} det(\Phi^{-1}) \]

where the F value is determined by the expression (9), and \( O^o = \{ V_{D,1}^o, V_{D,2}^o, ..., V_{D,n}^o \} \) – optimal plan in terms of the D-optimality criterion.

In this paper, the optimal experimental plans were obtained by numerical methods, by minimizing the average parameters determinant of the covariance matrix parameters.

Taking into account the large dimension of the optimization problem (30–40 variables), the following algorithm for finding the optimal plan of the experiment is proposed.

1) Set the number of control points \( n \) and define an arbitrary initial plan of the experiment \( O^1 = \{ V_{D,1}^1, V_{D,2}^1, ..., V_{D,n}^1 \} \).

2) Select the first control point to search for a particular extremum \( V_{D,i}, (i = 1, ..., n) \).

3) Solve a particular extreme problem for one variable point of the \( V_{D,i} \) control and for other fixed points, design a plan \( O^2 \) as in

\[ O^2 = \{ V_{D,1}^2, V_{D,2}^2, ..., V_{D,i}^2, V_{D,n}^2 \} = \min_{V_{D,i} \in \Sigma} \frac{det(\Phi)}{\det} \]

4) Repeat the procedure in item 4 for all points of the plan.

5) Repeat items 3–4 until the specified calculation accuracy is reached, the specified number of cycles is obtained, or the specified counting time expires.

The calculations performed for the construction of optimal experimental plans showed that their use increases the accuracy of estimating the parameters of the direct branch of the VAC by an average of 2–3 times compared to the uniform location of control points and with the same number of measurements.

Optimal planning is particularly effective when estimating the parameters of the reverse branch. In this case, as a rule, about 10% of the control points are located on the edges of the planning area, and the vast majority of the remaining plan points are grouped in a very short area with a length of approximately (0.2–0.5)% of the entire planning area adjacent to the \( V_{D,max} \) point. The \( V_{D,max} \) point itself must exceed the \( V_B \) parameter by (10–15)% , which is determined by the permissible dispersing power of the diode, otherwise this parameter will not be able to be determined accurately.

Thus, using a uniform plan with a number of measurement points for the reverse branch, even 200–300, often makes it impossible to calculate all the parameters of the SPICE model, since very few or even none of the measurement points fall into the critical planning area. At the same time, using the optimal plan in this situation allows you to calculate all the parameters with accuracy (1–10) %, even for 30–40 measurement points.
4. Measurement and calculation methods for the SPICE-model's parameters of the diode

Based on the results obtained, the methodology for measuring and calculating the parameters of the SPICE model of the diode can be presented in a fairly general form by the following sequence of operations.

1) Determination of possible changes' range in the parameters of the SPICE model for the forward and reverse branches of the VAC for a given type of diode based on a priori information.

2) Determination of the optimal experimental plans for the forward and reverse branches of the VAC based on the expression (10) and the described procedure (11). The designed experiment plan will be used for all diodes of this type.

3) Performance of measurements of the current of a specific diode at specified voltages in accordance with the designed experimental plan.

4) Calculation of the initial approximation of parameters by statistical modeling in accordance with expressions (6) and (7). At this stage, you may need to adjust the parameter ranges.

5) Calculation of parameter estimates by formula (4) and covariance matrix of parameters by formula (5). At this stage, you may need to adjust the initial approximation of the parameters [7].

5. Conclusion

The measurement and calculation methods of the SPICE-model's parameters of the diode described in this paper were used to calculate the parameters of the SPICE-model of three types of diodes: KD221, KD411 and KD213. The experimental plans were designed for each type of diodes (thus, three such plans were obtained in total). All parameters of the SPICE model for the forward and reverse branches of the VAC and their covariance matrices were calculated for each diode. The accuracy of parameter estimation ranged from 0.05% to 1%, depending on the type of a parameter. Thus, the proposed method provides a fairly accurate estimation of all parameters of the SPICE model and their covariance matrix for different types of diodes, which confirms its practical significance.

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