Circuit QED: cross-Kerr effect induced by a superconducting qutrit without classical pulses

Tong Liu · Yang Zhang · Bao-Qing Guo · Chang-Shui Yu · Wei-Ning Zhang

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Abstract The realization of cross-Kerr nonlinearity is an important task for many applications in quantum information processing. In this work, we propose a method for realizing cross-Kerr nonlinearity interaction between two superconducting coplanar waveguide resonators coupled by a three-level superconducting flux qutrit (coupler). Because the resonator photons are virtually excited and the coupler is unexcited for the entire process, the effect of resonator decay and the coupler decoherence are greatly minimized. More importantly, compared with the previous proposals, our proposal does not require classical pulses. Furthermore, due to use of only a three-level qutrit, the experimental setup is much simplified when compared with previous proposals requiring a four-level artificial atomic systems. In addition, we implement a two-resonator qubits controlled-phase gate and generate a two-resonator entangled coherent state. Numerical simulation shows that the high-fidelity implementation of the phase gate and creation of the entangled coherent state are feasible with current circuit QED technology.

Keywords Circuit QED · Cross-Kerr nonlinearity · Controlled-phase gate · Entangled coherent state

1 Introduction

Circuit quantum electrodynamics (QED), consisting of microwave resonators and superconducting (SC) qubits, is an analogue of cavity QED and a well-established...
platform for the investigation of light-matter interaction at the quantum level [1–4]. In the context of cavity/circuit QED, many theoretical proposals have been presented for realizing quantum state generation and quantum gates [1,5–16]. SC qubits are promising candidates to achieve scalable quantum computing and quantum information processing (QIP), especially with continuing improvements to coherence times [17–26]. Furthermore, improving the quality factor of SC resonators is a key development for QIP. For example, a SC coplanar waveguide resonators with a loaded quality factor $Q \sim 10^6$ [27,28] or with internal quality factors above one million ($Q > 10^7$) have been previously reported [29]. Recently, a SC microwave resonator with a loaded quality factor $Q \sim 3.5 \times 10^7$ has been demonstrated in experiments [30]. Combined with the large electric dipole moment of SC qubit, the strong-coupling or ultrastrong-coupling regime of a SC qubit with a resonator has been reported in experiments [31–34].

The microwave resonators have been considered as good memory elements in QIP [30,35]. Based on circuit QED, there is much interest in large-scale QIP which usually involves two or more microwave resonators/cavities. Recently, a number of proposals have been presented for synthesizing entangled states (e.g., Bell states, GHZ states, NOON states, and entangled coherent states) of two or more than two resonators [36–48], and realizing two-qubit or multiqubit quantum gates with microwave photons distributed in two resonators [49,50]. In addition, a great deal of quantum effects and operations involved multiple microwave resonators have been experimentally demonstrated in circuit QED. For instance, Ref. [35] experimentally implemented the quantum von Neumann architecture with SC circuits, Ref. [51] generated photon NOON states of two SC resonators, Ref. [52] realized full deterministic quantum teleportation with feed-forward, and Ref. [53] created a two-mode cat state of microwave fields in two SC cavities, respectively. Those progresses in circuit QED provide a promising perspective of microwave photons as resource for quantum communication and computation.

Cross-Kerr nonlinearity, known as cross phase modulation, is one of the most promising tool for quantum computation and QIP. In quantum optics, when two light beams are simultaneously input into a nonlinear medium, the refractive index of a beam is changed with respect to the square amplitude of a second one [54]. During the last several decades, many QIP tasks have been proposed in optical systems based on cross-Kerr nonlinearity, including construction of quantum phase gates [55,56], generation of macroscopic quantum superposition states [57,58], completion of quantum teleportation [59], realization of the quantum-nondemolition (QND) measurements [60,61], and implementation of entanglement purification [62,63].

On the other hand, many theories and experiments involved cross-Kerr effect have been discussed in cavity QED [64–67] and circuit QED [68–77]. Experimentally, based on the cross-Kerr nonlinearity, the observation of quantum state collapse and revival [72], the investigation of the feasibility of microwave photon counting [73], and the realization of the giant cross-Kerr effect for propagating microwaves [74] have been reported. In recent years, the cross-Kerr nonlinearity between two microwave resonators has been extensively researched in circuit QED [75–77]. For example, Refs. [75,76] proposed a scheme for implementing cross-Kerr nonlinearity between two SC resonators via an $N$-type SC artificial atomic systems, and Ref. [77] experi-
mentally demonstrated a state dependent shift between two microwave cavities via a cross-Kerr effect. The $N$-type four-level nature atoms was studied in Refs. [78–80], and the Refs. [75,76] extended it to the SC circuits which consists of two SC transmon qubits coupled by a SQUID (quantum interference device). The Hamiltonian for a cross-Kerr interaction between two resonators is given by (in units of $\hbar = 1$)

$$H_{\chi} = -\chi_{ab} \hat{n}_a \hat{n}_b,$$  \hspace{1cm} (1)

where $\chi_{ab}$ is the coupling strength and $\hat{n}_a$ ($\hat{n}_b$) is the photon number operator of resonator $a$ ($b$).

In this paper, we propose a method to realize a cross-Kerr nonlinearity interaction between two microwave resonators by coupling a three-level SC artificial atom (Fig. 1a). This proposal has the following features and advantages: (i) Different from the previous works [75,76], in our scheme only one operational step is needed, only one single three-level qutrit is used, and no need to use classical pulses. Thus the operation and experimental setup are greatly simplified. (ii) Because resonator photons are virtually excited and the qutrit is unexcited during the entire process, the effect of resonator decay, the unwanted inter-resonator crosstalk, and the qutrit decoherence are greatly minimized. (iii) This proposal can be applied to accomplish the same task with various SC qutrits (e.g., SC charge qutrits, transmon qutrits, Xmon qutrits, phase qutrits) coupled to two 1D resonators or two 3D cavities. (iv) Numerical simulation shows that our cross-Kerr interaction Hamiltonian can be used to high-fidelity realize a two-resonator qubits controlled-phase gate and generate a two-resonator entangled coherent state.

This paper is organized as follows. In Sect. 2, we show how to realize a cross-Kerr interaction effect between two SC resonators in circuit QED. In Sect. 3, we show how to use our effective Hamiltonian to construct a controlled-phase gate on two resonators and then discuss how to create a macroscopic entangled coherent state of two resonators. In Sect. 4, we discuss the possible experimental implementation of our proposal and numerically calculate the operational fidelity for realizing a controlled-phase gate and generating an entangled coherent state. A concluding summary is given in Sect. 5.

## 2 Cross-Kerr nonlinearity effect in circuit QED

Consider a system consists of two SC coplanar waveguide resonators connected by a ladder-type SC flux qutrit (coupler) (Fig. 1a). As shown in Fig. 1a, b, resonator $a$ ($b$) is off-resonantly coupled to the $|g\rangle \leftrightarrow |e\rangle$ ($|e\rangle \leftrightarrow |f\rangle$) transition of qutrit with a coupling constant $g$ ($\mu$). In the interaction picture, the Hamiltonian of the whole system can be written as (in units of $\hbar = 1$)

$$H_I = g \left( e^{i\delta_a t} a \sigma_{eg}^+ + h.c. \right) + \mu \left( e^{i\delta_b t} b \sigma_{fe}^+ + h.c. \right),$$  \hspace{1cm} (2)

where $\sigma_{eg}^+$ = $|e\rangle \langle g|$ and $\sigma_{fe}^+$ = $|f\rangle \langle e|$, $\delta_a$ ($\delta_a = \omega_{eg} - \omega_e < 0$) is a negative detuning but $\delta_b$ ($\delta_b = \omega_{fe} - \omega_f > 0$) is a positive detuning. Here, $\omega_{fe}$ ($\omega_{eg}$) is the $|e\rangle \leftrightarrow |f\rangle$
Fig. 1  a Setup of two SC resonators coupled to a qutrit (coupler) via capacitances $C_a$ and $C_b$. The flux qutrits can be other types of solid-state qutrit, such as a quantum dot, a SC phase qutrit, a charge qutrit or a transmon qutrit. b Resonator $a$ is far-off resonant with $|g\rangle \leftrightarrow |e\rangle$ transition of coupler with coupling strength $g$ and detuning $\delta_a$, while resonator $b$ is far-off resonant with $|e\rangle \leftrightarrow |f\rangle$ transition of coupler with coupling strength $\mu$ and detuning $\delta_b$. Here, $\delta_a (\delta_a = \omega_{eg} - \omega_a < 0)$ is a negative detuning but $\delta_b (\delta_b = \omega_{fe} - \omega_b > 0)$ is a positive detuning.

($|g\rangle \leftrightarrow |e\rangle$) transition frequency of qutrit and $\omega_a$ ($\omega_b$) is the frequency of resonator $a$ ($b$).

Consider the large-detuning conditions $\delta_a \gg g$ and $\delta_b \gg \mu$, the Hamiltonian (2) becomes [81]

$$H_e = -\frac{g^2}{\delta_a} \left( a^\dagger a|g\rangle\langle g| - aa^\dagger |e\rangle\langle e| \right) - \frac{\mu^2}{\delta_b} \left( b^\dagger b|e\rangle\langle e| - bb^\dagger |f\rangle\langle f| \right) + \lambda \left( e^{-i\Delta t} a^\dagger b^\dagger \sigma_{fg}^- + h.c. \right),$$  

(3)

where $\lambda = \frac{g\mu}{2} \left( \frac{1}{|\delta_a|} + \frac{1}{\delta_b} \right)$, $\Delta = \delta_b - |\delta_a|$, and $\sigma_{fg}^- = |g\rangle\langle f|$. Under the large-detuning conditions $\Delta \geq \{g^2/|\delta_a|, \mu^2/\delta_b, \lambda \}$ and $\delta_b > |\delta_a|$, the effective Hamiltonian $H_e$ changes to

$$H_e = -\frac{g^2}{\delta_a} \left( a^\dagger a|g\rangle\langle g| - aa^\dagger |e\rangle\langle e| \right) - \frac{\mu^2}{\delta_b} \left( b^\dagger b|e\rangle\langle e| - bb^\dagger |f\rangle\langle f| \right) + \chi \left( aa^\dagger bb^\dagger |f\rangle\langle f| - a^\dagger a b^\dagger b|g\rangle\langle g| \right),$$  

(4)

where $\chi (\chi = \lambda^2/\Delta)$ is the cross-Kerr interaction coefficient. When the levels $|e\rangle$ and $|f\rangle$ are not occupied, the effective Hamiltonian (4) reduces to

$$H_e = H_0 + H_i$$  

(5)
with

\[ H_0 = -\frac{g^2}{\delta_a} a^\dagger a |g\rangle \langle g| = -\frac{g^2}{\delta_a} \hat{n}_a |g\rangle \langle g|, \]

\[ H_i = -\chi a^\dagger a b^\dagger b |g\rangle \langle g| = -\chi \hat{n}_a \hat{n}_b |g\rangle \langle g|, \]

where \( \hat{n}_a \) and \( \hat{n}_b \) are the photon number operators for resonators \( a \) and \( b \). When the qutrit is in the state \( |g\rangle \), the Hamiltonian \( H_0 \) describes the photon-number-dependent shift of the resonator \( a \), the Hamiltonian \( H_i \) describes the cross-Kerr photon-photon interaction between the resonators \( a \) and \( b \).

From Eq. (6), one can see that \([H_0, H_i] = 0\), thus, we obtain the effective cross-Kerr interaction Hamiltonian

\[ \tilde{H}_e = e^{iH_0 t} H_i e^{-iH_0 t} = -\chi \hat{n}_a \hat{n}_b |g\rangle \langle g|. \]

(7)

It should be noted that the Hamiltonian (7) is different from the well-known cross-Kerr Hamiltonian \( H_\chi = -\chi_{ab} \hat{n}_a \hat{n}_b \) describing the cross-Kerr interaction of two resonators with coefficient \( \chi_{ab} \). It is because that the Hamiltonian (7) contains a coupler operator \( |g\rangle \langle g| \) which is not involved in \( H_\chi \). In the next section, we first show how to use the effective Hamiltonian (7) to construct a controlled-phase gate on two resonators and then discuss how to use the effective Hamiltonian (7) to create a macroscopic entangled coherent state of two resonators.

3 Controlled-phase gate implementation and entangled-state preparation

Assume that the resonator \( a \) (\( b \)) is in an arbitrary pure state \( |\varphi\rangle_a \) (\( |\varphi\rangle_b \)) and the coupler is in the state \( |g\rangle \). In this section, we first show how to use the Hamiltonian (5) or (7) to construct a two-qubit controlled-phase gate of two resonators \( a \) and \( b \). We then discuss how to generate a macroscopic entangled coherent state for two resonators with Hamiltonian (5) or (7).

3.1 Controlled-phase gate on two resonators

Suppose that the resonators \( a \) and \( b \) are in an arbitrary superposition state, respectively. Assume that \( |\varphi\rangle_a = \alpha |0\rangle_a + \beta |1\rangle_a \) and \( |\varphi\rangle_b = \gamma |0\rangle_b + \delta |1\rangle_b \). Here, \( \alpha, \beta, \gamma, \text{ and } \delta \) are the normalized complex numbers; \( |0\rangle_a \text{ or } |1\rangle_a \) (\( |0\rangle_b \text{ or } |1\rangle_b \)) represents the vacuum state or the single photon state of resonator \( a \) (\( b \)). The time-evolution operator for the Hamiltonian (5) is defined as \( U = \exp \{i(g^2/\delta_a) \hat{n}_a t\} \otimes \exp(i \chi \hat{n}_a \hat{n}_b t) \). Therefore, under the Hamiltonian (5), the resonator-system state \( |\varphi\rangle_a |\varphi\rangle_b \) evolves into

\[ U |\varphi\rangle_a |\varphi\rangle_b = e^{i(g^2/\delta_a) \hat{n}_a t} e^{i \chi \hat{n}_a \hat{n}_b t} |\varphi\rangle_a |\varphi\rangle_b \]
Assume that the resonators phase gate can be achieved only using a single-step operation and no pulses are needed. Compared with Refs. [49,50], our proposal is much improved because our systems. In circuit QED, a two SC qubits controlled-phase gate has already been presented for realizing two-qubit controlled-phase gate in many physical universal gates can be constructed. Hitherto, a great deal of theoretical proposals information and quantum computation. By using it with single-qubit gates, a set of

where \( \theta = \chi + (g^2/\delta_a) \). If we choose the interaction time \( t = \pi/|\theta| \), one can obtain a two-resonator qubits controlled-phase gate

where we have set \((g^2/|\delta_a|)t = 2k\pi \) (k is a positive integer). For \( t = \pi/|\theta| \) and \((g^2/|\delta_a|)t = 2k\pi \), one has the following relationship between the parameters

Notice that the effective coupling strength \( \lambda \) and the coupling strength \( \mu \) can be adjusted by varying the detuning \( \delta_a \) or \( \delta_b \). The level spacings of the SC “artificial atom” can be rapidly adjusted by varying external control parameters (e.g., magnetic flux applied to the superconducting loop of a superconducting phase, transmon, Xmon or flux qubit; see, e.g., [19,82–84]).

The controlled-phase gate of Eq. (9) is one of the paradigmatic gates for quantum information and quantum computation. By using it with single-qubit gates, a set of universal gates can be constructed. Hitherto, a great deal of theoretical proposals have been presented for realizing two-qubit controlled-phase gate in many physical systems. In circuit QED, a two SC qubits controlled-phase gate has already been experimentally demonstrated [85–88]. In addition, the controlled-phase gate with two microwave-photon-resonator qubits has been previously proposed in Refs. [49,50]. The proposals [49,50] require several operational steps and the application of classical pulses. Compared with Refs. [49,50], our proposal is much improved because our phase gate can be achieved only using a single-step operation and no pulses are needed.

### 3.2 Creation of a two-resonator macroscopic entangled coherent state

Assume that the resonators \( a \) and \( b \) are in coherent states \(|\varphi\>_a = |\alpha_a\>_a\) and \(|\varphi\>_b = |\beta_b\>_b\), respectively. Under the Hamiltonian (7), the joint state \(|\varphi\>_a|\varphi\>_b\) of the resonators evolves into

\[
|\psi\> = e^{i\chi\hat{n}_a\hat{n}_b}\cdot e^{\chi\hat{n}_a\hat{n}_b}\cdot |\varphi\>_a|\varphi\>_b
\]

\[
= e^{-\frac{1}{2}|\alpha_a|^2}e^{-\frac{1}{2}|\beta_b|^2} \sum_{n_a,n_b=0}^{\infty} \frac{\alpha_a^{n_a}\beta_b^{n_b}e^{i\chi n_a n_b}}{\sqrt{n_a!n_b!}} |n_a\>_a|n_b\>_b.
\]
When the evolution time is equal to $t = \pi/\chi$, one has $\exp(i \chi n_a n_b t) = (-1)^{n_a n_b}$. We divide the sum of Eq. (11) into a part with $n_a$ even and another with $n_a$ odd. It is apparent that an even/odd coherent state of $|\alpha_a\rangle$ can be expressed as

$$e^{-\frac{1}{2} |\alpha_a|^2} \sum_{n_a=\text{even}}^{\infty} \frac{\alpha_{na}^n}{\sqrt{n_a!}} |n_a\rangle = e^{-\frac{1}{2} |\alpha_a|^2} \left( |0\rangle + \frac{\alpha_a^2}{\sqrt{2}} |2\rangle + \cdots + \frac{\alpha_{na}^n}{\sqrt{n_a!}} |n_a\rangle \right)$$

$$= \frac{1}{2} (|\alpha_a\rangle + |\alpha_a\rangle),$$

(12)

$$e^{-\frac{1}{2} |\alpha_a|^2} \sum_{n_a=\text{odd}}^{\infty} \frac{\alpha_{na}^n}{\sqrt{n_a!}} |n_a\rangle = e^{-\frac{1}{2} |\alpha_a|^2} \left( |\alpha_a\rangle + \frac{\alpha_a^3}{3 \sqrt{3!}} |3\rangle + \cdots + \frac{\alpha_{na}^n}{\sqrt{n_a!}} |n_a\rangle \right)$$

$$= \frac{1}{2} (|\alpha_a\rangle - |\alpha_a\rangle).$$

(13)

One finds that for the interaction time $t = \pi/\chi$, two cat states ($|\alpha_a\rangle \pm |\alpha_a\rangle$) of the resonator $a$ are prepared. On substituting Eqs. (12) and (13) into Eq. (11), one obtains a macroscopic entangled coherent state

$$|\psi\rangle = e^{-\frac{1}{2} |\alpha_a|^2} e^{-\frac{1}{2} |\beta_b|^2} \left\{ \sum_{n_a=\text{even}}^{\infty} \frac{\alpha_{na}^n}{\sqrt{n_a!}} |n_a\rangle \sum_{n_b=0}^{\infty} \frac{\beta_{nb}^n}{\sqrt{n_b!}} |n_b\rangle + \sum_{n_a=\text{odd}}^{\infty} \frac{\alpha_{na}^n}{\sqrt{n_a!}} |n_a\rangle \sum_{n_b=0}^{\infty} \frac{(-\beta_{nb})^n}{\sqrt{n_b!}} |n_b\rangle \right\}$$

$$= \frac{1}{2} (|\alpha_a\rangle + |\alpha_a\rangle) |\beta_b\rangle + \frac{1}{2} (|\alpha_a\rangle - |\alpha_a\rangle) |\beta_b\rangle$$

$$= \frac{1}{2} (|\alpha_a\rangle |\beta_b\rangle + |\alpha_a\rangle |\beta_b\rangle + |\alpha_a\rangle |\beta_b\rangle - |\alpha_a\rangle |\beta_b\rangle).$$

(14)

The minus sign in the last term of Eq. (14) makes the state entangled which crucially depend on the phase accumulation by the resonators with the particular phase shift $\chi t = \pi$. Thus a macroscopic entangled coherent state of two resonators is generated by the cross-Kerr interaction under the given condition ($\chi t = \pi$). After returning to the original interaction picture by performing a unitary transformation $U = \exp [i (g^2/\delta_a) \hat{n}_a t]$, the entangled coherent state (14) becomes

$$|\varphi\rangle = \frac{1}{2} (|\beta_b\rangle |\beta_b\rangle + |\alpha_a\rangle |\beta_b\rangle + |\beta_a\rangle |\beta_b\rangle - |\alpha_a\rangle |\beta_b\rangle),$$

(15)

where $\beta_a = \alpha_a \exp [i (g^2/\delta_a) t]$. It is obvious that the coherent state ($|\beta_b\rangle$ or $|\beta_b\rangle$) of the resonator $b$ is unaffected by the unitary transformation but the coherent state ($|\alpha_a\rangle$ or $|\alpha_a\rangle$) of the resonator $a$ picks up a phase shift $(g^2 t/\delta_a)$. Note that the entangled coherent states have many applications in the field of quantum information. For instance, they can be used to construct quantum gates [89] (using coherent states as the logical qubits [90]), implement quantum key distribution [91], build quantum
repeaters [92], test violation of Bell inequalities [93, 94], and applicant in quantum metrology [95].

4 Possible experimental implementation

When the inter-resonator crosstalk is taken into account, the Hamiltonian (2) becomes 
\[ \tilde{H}_I = H_I + \epsilon, \]
where \( \epsilon \) describes the unwanted inter-resonator crosstalk, given by 
\[ \epsilon = g_{ab} (e^{i \Delta_{ab} t} a b^\dagger + \text{h.c.}), \]
with the inter-resonator crosstalk coupling strength \( g_{ab} \) and the two-resonator frequency detuning \( \Delta_{ab} = \omega_b - \omega_a \).

Including the dissipation and the dephasing, the dynamics of the lossy system is determined by the following master equation

\[ \frac{d\rho}{dt} = -i(\tilde{H}_I, \rho) + \kappa_a \mathcal{L}[a] + \kappa_b \mathcal{L}[b] 
+ \gamma_{eg} \mathcal{L}[\sigma_{eg}^-] + \gamma_{fe} \mathcal{L}[\sigma_{fe}^-] + \gamma_{fg} \mathcal{L}[\sigma_{fg}^-]
+ \sum_{j=e,f} \{ \gamma_{\phi j} (\sigma_{jj} \rho \sigma_{jj} - \sigma_{jj} \rho / 2 - \rho \sigma_{jj} / 2) \}, \] (16)

where \( \tilde{H}_I \) is given above, \( \sigma_{eg}^- = |g\rangle \langle e|, \sigma_{fe}^- = |e\rangle \langle f|, \sigma_{fg}^- = |f\rangle \langle g|, \sigma_{jj} = |j\rangle \langle j| (j = e, f) \); and \( \mathcal{L}[\Lambda] = \Lambda \rho \Lambda^+ - \Lambda^+ \Lambda \rho / 2 - \rho \Lambda^+ \Lambda / 2, \) with \( \Lambda = a, b, \sigma_{eg}, \sigma_{fe}, \sigma_{fg} \). Here, \( \kappa_a (\kappa_b) \) is the photon decay rate of resonator \( a (b) \). In addition, \( \gamma_{eg} \) is the energy relaxation rate of the level \( |e\rangle \) of qutrit, \( \gamma_{fe} (\gamma_{fg}) \) is the energy relaxation rate of the level \( |f\rangle \) of qutrit for the decay path \( |f\rangle \rightarrow |e\rangle (|g\rangle) \), and \( \gamma_{\phi j} \) is the dephasing rate of the level \( |j\rangle \) of qutrit \( (j = e, f) \).

The fidelity of the operation is given by
\[ F = \sqrt{\langle \psi_{id}| \rho |\psi_{id} \rangle}, \] (17)

where \( |\psi_{id} \rangle \) is the output state of an ideal system (i.e., without dissipation, dephasing, and crosstalk), while \( \rho \) is the final density operator of the system when the operation is performed in a realistic situation.

4.1 Fidelity for the two-resonator qubits controlled-phase gate

As an example, we will consider the case of \( \alpha = \beta = \gamma = \delta = 1/\sqrt{2} \). In this case, we have the qutrit–resonator–system initially state \( |\varphi\rangle_a |\varphi\rangle_b \otimes |g\rangle = (1/2)(|0\rangle_a + |1\rangle_a)(|0\rangle_b + |1\rangle_b) \otimes |g\rangle \) and the output state \( |\psi_{id} \rangle = |\phi_{id})_{ab} \otimes |g\rangle \). Here, \( |\phi_{id})_{ab} \) is given by \( |\phi_{id})_{ab} = (1/2)(|0\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |1\rangle_a|0\rangle_b - |1\rangle_a|1\rangle_b) \), which is obtained based on Eq. (8) for \( |\theta| t = \pi \).

By solving the master Eq. (16), the fidelity of the gate operation can be calculated based on Eq. (17). We now numerically calculate the fidelity for the operation. The frequency of SC resonators typically is 1–10 GHz. Thus, we choose \( \omega_a / 2\pi = 3.5 \) GHz and \( \omega_b / 2\pi = 6.5 \) GHz such that \( \Delta_{ab} = 3.0 \) GHz. We then choose \( \delta_{a} / 2\pi = -0.3 \) GHz and \( g / 2\pi = 50 \) MHz in our scheme. In the following, we set the inter-resonator...
crosstalk strength $g_{ab} = 0.1g$, which can be readily met in experiments [41]. Furthermore, we set $k=1$, $\gamma_{j,\varphi e}^{-1} = \gamma_{j,\varphi f}^{-1} = \gamma$, $\gamma_{eg}^{-1} = 3\gamma$, $\gamma_{fe}^{-1} = 2\gamma$, $\gamma_{fg}^{-1} = 10\gamma$, and $\kappa_{a}^{-1} = \kappa_{b}^{-1} = \eta$.

Figure 2 shows the fidelity versus $\delta_b/2\pi$. Green squares are plotted by choosing $\gamma = 10\,\mu s$ and $\eta = 20\,\mu s$, which correspond to the case that the systematic dissipation and dephasing are taken into account. Here we consider a rather conservative case for the decoherence times of flux qutrit because the decoherence times ranging from $85\,\mu s$ to $2\,ms$ have been reported for a flux qutrit [24–26]. It is noted that by designing the flux qutrit, the $|g\rangle \leftrightarrow |f\rangle$ dipole matrix element can be made much smaller than that of the $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions. Thus, $\gamma_{fg}^{-1} \gg \gamma_{eg}^{-1}, \gamma_{fe}^{-1}$. In addition, the leakage errors of a SC qutrit have been efficiently reduced in experiments [96].

From Fig. 2, one can see that for $\delta_b/2\pi = 0.7\,GHz$, a high fidelity $\sim 99.4\%$ is achievable for a two-resonator qubits controlled-phase gate. For $\delta_b/2\pi = 0.7\,GHz$, one has $\mu/2\pi \sim 342\,MHz$. The values of $g$ and $\mu$ are readily available in experiments [34]. In addition, one can obtain $\omega_{eg}/2\pi = 3.2\,GHz$ and $\omega_{fe}/2\pi = 7.2\,GHz$. The transition frequency between two neighbor levels of a SC flux qutrit is typically in the range of $1–30\,GHz$. With the above given parameters, we obtain the large cross-Kerr interaction coefficient $\chi \sim 4.2\,MHz$. Our numerical simulation indicates that the high-fidelity implementation of a controlled-phase gate on two resonators is feasible with current circuit QED technology. As in Fig. 2, the effect of the qutrit decoherence and resonator decay on the fidelity is very small with the current parameter values. To illustrate the effect of the dissipation and dephasing of the system, we employ shorter qutrit decoherence and resonator-decay times in Fig. 3.

Figure 3 displays the fidelity versus $\gamma$ and $\eta$, which is plotted by choosing $\delta_b/2\pi = 0.7\,GHz$. From Fig. 4, one can obtain $\{F, \gamma, \eta\}$: (i) $\{0.949, 0.1, 1.0\,\mu s\}$; (ii) $\{0.972, 0.3, 2.0\,\mu s\}$; (iii) $\{0.983, 0.5, 3.0\,\mu s\}$; (iv) $\{0.987, 0.7, 4.0\,\mu s\}$; and (v) $\{0.991, 1.0, 5.0\,\mu s\}$. Figure 3 shows that for $\gamma \in [0.1, 1\,\mu s]$ and $\eta \in [1, 5\,\mu s]$, the fidelity can be greater than 94%. This is because the qutrit is unexcited and resonator...
Fig. 3  Fidelity versus $\gamma$ and $\eta$, which is plotted by choosing $\delta_b/2\pi = 0.7$ GHz (Color figure online)

Fig. 4  Fidelity versus the normalized detuning $D = \delta_b/\mu$, which is plotted by $m = 4, 5, 6, 7$ (Color figure online)

photons are virtually excited during the entire process, qudit decoherence and resonator decay can be efficiently suppressed.

For the resonator frequencies and the resonator-decay times used in Fig. 2, the required quality factors of the two resonators are $Q_a \sim 4.4 \times 10^5$ and $Q_b \sim 8.2 \times 10^5$, which are attainable in experiments because a quality factor $Q \sim 10^6$ for SC coplanar waveguide resonators has been experimentally demonstrated [27,28]. For Fig. 3, the required quality factors for the resonators are $Q_a \sim [2.2 \times 10^4, 1.1 \times 10^5]$ and $Q_b \sim [4.1 \times 10^4, 2.0 \times 10^5]$. Figure 3 shows that the phase-gate operation also can be high-fidelity performed assisted by the low-$Q$ resonators.

4.2 Fidelity for generation of a two-resonator entangled coherent state

The fidelity for the operation is calculated based on Eq. (17), where the ideal output state is $|\psi_{id}\rangle = |\varphi\rangle \otimes |g\rangle$ and $\rho$ is obtained by numerically solving the master Eq. (16)
for an initial input state $|\phi\rangle_a |\phi\rangle_b \otimes |g\rangle = |\alpha_a\rangle |\beta_b\rangle \otimes |g\rangle$. Here, state $|\phi\rangle$ is given by Eq. (15).

We choose $\omega_a/2\pi = 3.5 \text{ GHz}$, $\omega_b/2\pi = 6.5 \text{ GHz}$, and $\delta_a/2\pi = -1.0 \text{ GHz}$. We set $g/2\pi = 150 \text{ MHz}$ and $\mu/2\pi = 200 \text{ MHz}$ because the coupling strengths of values $g$ and $\mu$ are readily achievable in experiments [34]. In addition, we set $\gamma = 0.1 \mu$s and $\eta = 5 \mu$s. The qutrit decoherence and the resonator-decay times used here are referred to Fig. 3.

We now numerically calculate the fidelity for creation of the two-resonator entangled coherent state. In our numerical calculation, we consider the first $m$ terms in the expansions (i.e., we cut off the photons into the $m$-photon subspace) of coherent states $|\alpha_a\rangle (|\beta_b\rangle)$ and $|\beta_a\rangle (|\beta_b\rangle)$. Figure 4 shows the fidelity versus the normalized detuning $D = \delta_b/\mu$ with $|\alpha_a\rangle = 0.5$ and $|\beta_b\rangle = 1.0$, which is plotted for $m = 4, 5, 6, 7$. For $D = 8.48$, a high fidelity 96.75, 97.03, 97.08, 97.09% can be obtained for $m = 4, 5, 6, 7$, respectively. For $D = 8.48$, we have $\delta_b \sim 1.7 \text{ GHz}$ and $\chi \sim 0.83 \text{ MHz}$.

For resonators $a$ and $b$ with frequencies and dissipation times used in Fig. 4, the quality factors of the two resonators are $Q_a \sim 1.1 \times 10^5$ and $Q_b \sim 2.0 \times 10^5$. The numerical simulation indicates that the high-fidelity generation of a two-resonator entangled coherent state is feasible with current circuit QED technology.

5 Conclusions and discussion

We have proposed a method for realizing the cross-Kerr nonlinear interaction between two microwave resonators induced by a superconducting flux qutrit. Our present proposal differs from the previous protocols [75,76]. First, in our proposal only one qutrit is needed; thus, the circuit complexity is much reduced. Second, there is only need one operation step and unnecessary to employ classical pulse, so the operation procedure is greatly simplified. Finally, due to the resonator photons are virtually excited and the coupler is unexcited for the entire process, the effect of resonator decay, the unwanted inter-resonator crosstalk, and the coupler decoherence are greatly minimized.

Although we assume that the cross-Kerr nonlinearity effect is performed between two SC coplanar waveguide resonators, using the three-level flux qutrit, our proposal can in principle also be applied to other solid devices, for example, the schemes based on other kinds of SC qutrits (e.g., SC charge qutrits, transmon qutrits, Xmon qutrits, phase qutrits) coupled to two 1D resonators or two 3D SC cavities, or based on the two nitrogen-vacancy center ensembles (behaves as two bosonic modes) coupled to a flux qutrit.

Based on the cross-Kerr interaction Hamiltonian, we implement a two-resonator qubits controlled-phase gate and generate a two-resonator entangled coherent state. Numerical simulation shows that the high-fidelity implementation of the phase gate and creation of the entangled coherent state are feasible with state-of-the-art circuit QED technology. Our finding provides a new way for realizing the cross-Kerr nonlinearity interaction between two microwave resonators, and such cross-Kerr effect may find applications in quantum information processing.
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