QCD CALCULATION OF HEAVY MESON SEMILEPTONIC TRANSITIONS

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ABSTRACT

I review the QCD sum rules calculations of the form factors governing the semileptonic decays of charmed and beauty mesons. In particular, I discuss the predicted dependence of the various form factors on $q^2$ and how it can be obtained for the form factors of vector and axial currents. In some cases the $q^2$ dependence, computed by QCD sum rules, is different from the outcome of lattice QCD. A Tau-Charm factory could be an efficient tool to study this aspect of the charmed meson transitions.

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QCD CALCULATION OF HEAVY MESON SEMILEPTONIC TRANSITIONS

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Abstract

I review the QCD sum rules calculations of the form factors governing the semileptonic decays of charmed and beauty mesons. In particular, I discuss the predicted dependence of the various form factors on \( q^2 \) and how it can be obtained for the form factors of vector and axial currents. In some cases the \( q^2 \) dependence, computed by QCD sum rules, is different from the outcome of lattice QCD. A Tau-Charm factory could be an efficient tool to study this aspect of the charmed meson transitions.

In this review I briefly describe how QCD sum rules can be used to evaluate the hadronic matrix elements governing the semileptonic decays of charmed and beauty mesons. These decays can be used to determine some elements of the Cabibbo-Kobayashi-Maskawa matrix, which are fundamental parameters of the Standard Model, and to check the V-A structure of the weak interactions. Moreover, they allow us to investigate the non perturbative structure of the strong dynamics, since the hadronic matrix elements can be measured and then compared to the predictions of the various theoretical approaches used to handle the difficult problem of describing the confining interactions.

QCD sum rules [1], as well as lattice QCD [2], are rooted in the QCD framework of the strong interactions and provide predictions from first principles; in this respect they differ from the other approaches to the hadronic dynamics, e.g. potential models, where the reference to QCD is diluted in "ad hoc" assumptions. On the other hand, QCD sum rules are different from lattice QCD since they use analytic methods, whereas lattice QCD heavily relies on numerical calculations. A general opinion is that the two approaches are complementary to each other; as a matter of fact the predictions of the first one, as we shall see in the following, cannot be improved to an arbitrary accuracy, whereas lattice QCD is restricted by the present computer facilities.

In order to summarize the main results obtained by different QCD sum rules calculations for the semileptonic transitions \( D \to (K, K^*)\ell\nu \) and \( D \to (\pi, \rho)\ell\nu \).
(which mainly concern a Tau-Charm factory) let us fix the notations. In the Bauer-Stech-Wirbel parameterization [3] the hadronic matrix elements involved in $D \to (K, K^*)\ell\nu$ can be written as follows:

$$\langle K(p')|J_\mu|D(p)\rangle = F_1(q^2) (p + p')_\mu + \frac{m_D - m_K^2}{q^2} q_\mu [F_0(q^2) - F_1(q^2)] \quad (1)$$

$$\langle K^*(p')|J_\mu|D(p)\rangle = \frac{2V(q^2)}{m_D + m_{K^*}} \epsilon_{\mu\alpha\beta\sigma} \epsilon^{*\alpha} (p^\beta p^\sigma) -$$

$$- i \left[ (m_D + m_{K^*}) A_1(q^2) \epsilon^{*\mu} - \frac{A_2(q^2)}{m_D + m_{K^*}} (\epsilon^* \cdot p)(p + p')_\mu -$$

$$- (\epsilon^* \cdot p_D) \frac{2m_{K^*}}{q^2} q_\mu (A_3(q^2) - A_0(q^2)) \right] \quad (2)$$

where $q^2 = (p - p')^2$ and $J_\mu = \bar{s} \gamma_\mu (1 - \gamma_5) c$ : $\epsilon$ is the $K^*$ meson polarization vector. To avoid unphysical poles at $q^2 = 0$ the conditions $F_1(0) = F_0(0)$ and $A_3(0) = A_0(0)$ must be implemented; $A_3$ can be expressed in terms of $A_1$ and $A_2$:

$$A_3(q^2) = \frac{m_D + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_D - m_{K^*}}{2m_{K^*}} A_2(q^2). \quad (3)$$

In the limit of massless charged leptons the relevant form factors are $F_1, V, A_1$ and $A_2$. The matrix elements of the transitions $D \to (\pi, \rho)\ell\nu$ and of the $B$ meson semileptonic decays can be written as in Eqs.(1,2) with obvious changes.

The calculation of the form factors in (1,2) has been first performed using potential models [3, 4, 5, 6], [7]. In this approach only few kinematical configurations can be treated analytically; as a matter of fact, in order to evaluate a hadronic matrix element as in (1,2) in a range of $q^2$, typically one has to compute an overlap integral involving the wave functions of mesons of arbitrary momentum. However, in potential models such wave functions are determined for mesons in particular kinematical configurations (e.g. for mesons at rest, or in the infinite momentum frame), and cannot be given for states of arbitrary momentum since the problem of performing their relativistic boost is not solved on general grounds. For this reason potential models provide the value of form factors at the maximum recoil point $q^2 = 0$ [3, 4] or at the zero recoil point $q^2 = q^2_{\max} = (m_D - m_{K,K^*})^2$ [3, 6, 8], and the functional $q^2$ dependence (polar, multipolar, exponential) is assumed invoking nearest pole dominance, QCD counting rules, etc. For example, in the BWS model [3] all the form factors are assumed to have a polar dependence:

$$F_i(q^2) = \frac{F_i(0)}{1 - q^2/m_{pole}^2} \quad (4)$$

with the pole given by the nearest resonance in the t-channel [4]. It is worth reminding that the $t-$dependence of the heavy meson semileptonic form factors
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$F^{D\rightarrow K}_1(0)$ & $V^{D\rightarrow K^*}(0)$ & $A^{D\rightarrow K}_1(0)$ & $A^{D\rightarrow K^*}_1(0)$ & Ref. \\
\hline
0.6 \pm 0.1 & - & - & - & [10] \\
0.8 \pm 0.2 & 1.6 \pm 0.5 & 0.9 \pm 0.2 & 0.8 \pm 0.3 & [11] \\
0.6 \pm 0.1 & - & - & - & [12] \\
0.6_{-0.1}^{+0.10} & 1.1 \pm 0.25 & 0.5 \pm 0.15 & 0.6 \pm 0.15 & [13] \\
\hline
$F^{D\rightarrow \pi}_1(0)$ & $V^{D\rightarrow \rho}(0)$ & $A^{D\rightarrow \rho}_1(0)$ & $A^{D\rightarrow \rho}_2(0)$ & Ref. \\
\hline
0.7 \pm 0.2 & - & - & - & [11] \\
0.75 \pm 0.05 & - & - & - & [19] \\
0.5 \pm 0.1 & 1.0 \pm 0.2 & 0.5 \pm 0.2 & 0.4 \pm 0.1 & [23] \\
\hline
\end{tabular}
\caption{QCD sum rules estimates of semileptonic form factors for $D \rightarrow K, K^*$ and for $D \rightarrow \pi, \rho$.}
\end{table}

is an important information employed e.g. in the framework of the heavy quark effective theory coupled to chiral symmetry, when $B \rightarrow \pi \ell \nu$ and $D \rightarrow \pi \ell \nu$ are related: in this case an extrapolation is needed from zero recoil, where predictions can be derived, to the maximum recoil point where experimental data are available [7, 9].

Using three-point QCD sum rules the form factors of the transition $D \rightarrow (K, K^*)\ell \nu$ at $q^2 = 0$ have been first computed in [10, 11] and then in [12, 13]. Since the method is general, also the form factors of $D \rightarrow (\pi, \rho)\ell \nu$ and of the semileptonic $B$ decays to negative and positive charmed and non-charmed states have been computed (at $q^2 = 0$) [12, 14, 15, 16, 17, 18]. In [19] the calculation for $D \rightarrow K\ell \nu$ has been performed by two point QCD sum rules.

The results for the $D$ meson transitions are collected in Table 1; there is an overall agreement among the different estimates (only the central value of the form factors computed in [11] are larger than in the other calculations). Moreover, the comparison of the results for $D \rightarrow K, K^*$ and $D \rightarrow \pi, \rho$ shows that the $SU(3)_F$ breaking effects in the two channels are of the order of $10 - 20\%$. A comparison with other theoretical approaches and with the experimental results can be found in these proceedings [7].

As for the $q^2$ dependence, already from the early investigations of the pion electromagnetic form factor [20] QCD sum rules have been proven to be successful in describing the dependence of hadronic matrix elements on intermediate (space-like) values of the transferred momentum $t$. Also in this respect QCD sum rules are analogous to lattice QCD, although this last approach has been limited, so far, by statistics and by small lattice sizes which compel an extrapolation to $q^2 = 0$ [21]. In [13] the $q^2$ dependence has been explicitly studied for the form factors governing $D \rightarrow K^*\ell \nu$, whereas $B$ decays have been considered in [22, 23]. In [13, 24] the $q^2$ dependence has also been studied by light-cone QCD sum rules. The result of these investigations is that the form factors $F_1$ and $V$ of the vector current in $D \rightarrow K, K^*$, $D \rightarrow \pi, \rho$ have a polar $t$–dependence as in Eq.(4),
with a pole mass in some agreement with the mass of the first resonance in the $t$ channel. The fitted masses are: $m_{pole} = 1.81 \pm 0.10 \text{ GeV}$ for $D \to K$ and $m_{pole} = 1.95 \pm 0.10 \text{ GeV}$ for $D \to K^*$, to be compared to the measured mass of $D^*_s$: $m_{D^*_s} = 2.11 \text{ GeV}$; $m_{pole} = 1.95 \pm 0.10 \text{ GeV}$ for $D \to \pi$ and $m_{pole} = 2.5 \pm 0.2 \text{ GeV}$ for $D \to \rho$, to be compared to $m_{B^*} = 2.01 \text{ GeV}$. Also the form factors of the vector current in $B \to \pi, \rho$ transitions have such behaviour; in this case $m_{pole} = 5.25 \pm 0.10 \text{ GeV}$ for $B \to \pi$ and $m_{pole} = 6.6 \pm 0.6 \text{ GeV}$ for $B \to \rho$, to be compared to $m_{B^*} = 5.33 \text{ GeV}$ [13, 23]. This is in agreement with the usual assumption made in BWS and in other potential models, and with the outcome of lattice QCD [21].

For the form factors $A_1$ and $A_2$ of the axial current in $D \to K^*, \rho$ and in $B \to \rho$ transitions, QCD sum rules show the absence of the polar dependence; the form factors are nearly independent of $q^2$, at odds with the outcome e.g. of lattice simulations [24]. A confirmation of this result comes from an analysis of the scaling properties of the $B \to \pi, \rho, K^*$ form factors in the limit $m_b \to \infty$ [26].

This result requires a careful investigation: from the experimental point of view, there is evidence that $F^{D \to K}$ is polar [25], whereas no information is available on the $t$ dependence of the form factors of $D \to K^*$. A description of the difficulties of such analysis and of the potentialities of a Tau-Charm factory can be found in these proceedings [7, 27]. From the theoretical point of view this behaviour has not found an explanation, yet.

An interesting problem is to investigate if such anomalous $t$-dependence is common to all the matrix elements of the axial current or if it is peculiar of the $0^- \to 1^-$ transitions. To study this problem, and to show in detail how QCD sum rules can be used to evaluate semileptonic form factors I consider the decays:

\begin{align}
D^0 & \to \pi^- \ell^+ \nu_\ell \quad (5) \\
D^0 & \to a_0^- \ell^+ \nu_\ell \quad (6)
\end{align}

where $a_0$ is the $J^P = 0^+$ orbital excitation of the pion system which can be identified with $a_0(980)$. The decay (5) is induced by a vector weak current and (6) by an axial current [2]. In terms of form factors the hadronic matrix elements of (5) and (6), keeping only the terms that contribute for massless charged leptons, can be written as follows:

\begin{align}
< \pi(p')|V_\mu|D(p)> &= F^{D \to \pi}_1(q^2) (p+p')_\mu + \ldots \quad (7) \\
i < a_0(p')|A_\mu|D(p)> &= F^{D \to a_0}_1(q^2) (p+p')_\mu + \ldots \quad (8)
\end{align}

The starting point to compute $F^{D \to \pi}_1(q^2)$ is the three-point function correlator

$$T^{D \to \pi}_{\mu\nu}(p, p', q) = i^2 \int dx \; dy \; e^{i(p'-p-y)} <0|T\{j_{\nu}(x)V_\mu(0)j^{\dagger}_3(y)\}|0> \quad (9)$$

\textsuperscript{2}The following analysis of $D \to \pi \ell \nu$ is similar to [10, 13, 24]: the results for $D \to a_0 \ell \nu$ are new.
where $j_\mu$ and $j_5$ are local currents of quark fields with the same quantum numbers of a pion and of a $D$ meson: $j_\mu(x) = \bar{u}(x)\gamma_\mu\gamma_5d(x)$, $j_5(y) = \bar{u}(y)i\gamma_5c(y)$; $V_\mu$ is the vector current inducing the transition (5): $V_\mu(0) = \bar{d}(0)\gamma_\mu c(0)$. In analogous way the calculation of $F_1^{D\rightarrow\pi_0}(q^2)$ starts from:

$$T^{D\rightarrow\pi_0}_\mu(p, p', q) = i^2 \int dx \, dy \, e^{i(p'-x-p'y)} <0|T\{j_\mu(x)A_\mu(0)j^\dagger_5(y)\}|0> \tag{10}$$

where $j_\mu(y) = \bar{u}(y)d(y)$ and $A_\mu(0) = \bar{d}(0)\gamma_\mu c(0)$ is the axial current which induces the transition (5). It is immediate to generalize the method by changing the currents in order to compute different charmed or beauty mesons decays to vector or axial states.

After a decomposition in Lorentz invariant structures:

$$T^{D\rightarrow\pi}_\mu(p, p', q) = i T^{D\rightarrow\pi}(p^2, p'^2, q^2) (p + p')_\mu p'_\nu + .... \tag{11}$$

$$T^{D\rightarrow\pi_0}_\mu(p, p', q) = i T^{D\rightarrow\pi_0}(p^2, p'^2, q^2) (p + p')_\mu + .... \tag{12}$$

the strategy of QCD sum rules is to evaluate $T^{D\rightarrow\pi}(p^2, p'^2, q^2)$ and $T^{D\rightarrow\pi_0}(p^2, p'^2, q^2)$ in two independent ways. First, taking into account the analytical properties of $T^{D\rightarrow\pi}(p^2, p'^2, q^2)$ and $T^{D\rightarrow\pi_0}(p^2, p'^2, q^2)$ in the variables $p^2$ and $p'^2$, a double dispersion relation is written:

$$T(p, p', q) = \frac{1}{(2\pi)^2} \int ds \, ds' \frac{\rho(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtractions.} \tag{13}$$

with the spectral function $\rho$ getting contributions from hadronic intermediate states; in terms of the lowest lying resonances the physical spectral functions read:

$$\rho^{D\rightarrow\pi}(s, s', q^2) = (2\pi)^2 f_\pi f_D \frac{m_D^2}{m_c} F_1^{D\rightarrow\pi}(q^2) \delta(s - m_D^2)\delta(s' - m_\pi^2) + \rho^{D\rightarrow\pi}_{\text{cont}}(s, s', q^2) \tag{14}$$

$$\rho^{D\rightarrow\pi_0}(s, s', q^2) = (2\pi)^2 f_{a_0} f_D \frac{m_D^2}{m_c} F_1^{D\rightarrow\pi_0}(q^2) \delta(s - m_D^2)\delta(s' - m_{a_0}^2) + \rho^{D\rightarrow\pi_0}_{\text{cont}}(s, s', q^2) \tag{15}$$

where $f_\pi = 132 \, \text{MeV}$ and $f_{a_0}$ is defined by the matrix element $f_{a_0} = \langle 0|\bar{d}u|a_0\rangle$.

On the other hand, the correlators (11,13,14) are computed in QCD by an operator product expansion (OPE) at $p^2, p'^2, q^2$ spacelike and large: in this expansion not only the perturbative term is taken into account, but also higher order corrections given in terms of vacuum expectation values of quark and gluon fields.
(condensates) ordered by dimension and divided by powers of $p^2$ and $p'^2$. The result for Eq. (11), neglecting the light quark masses, is:

$$T_{\text{QCD}}^{D \to \pi}(p^2, p'^2, q^2) = \frac{1}{(2\pi)^2} \int ds \, ds' \rho_{\text{QCD}}^{D \to \pi}(s, s', q^2)$$

$$- \frac{<\bar{q}q>}{2} \left[ \frac{1}{rr'} - \frac{m_0^2}{6} \left( \frac{3m_c^2}{r^3r'} + \frac{2}{r^2r'} - \frac{2q^2}{r^2r'^2} \right) \right]$$

(16)

where $r = p^2 - m_c^2$, $r' = p'^2$. The spectral integral represents the perturbative contribution to the OPE at the lowest order in $\alpha_s$:

$$\rho_{\text{QCD}}^{D \to \pi}(s, s', q^2) = \frac{3m_c}{2\lambda^{3/2}} \left[ 2\Delta(u - s') + s'(u - 4s) \right]$$

$$- \frac{2m_c}{\lambda} \left( \Delta^2(u^2 - 3us' + 2ss') \right)$$

$$+ 2\Delta s'(u^2 - 3us + 2ss') + 3ss'^2(2s - u) \right]$$

(17)

($\Delta = s - m_c^2$, $u = s + s' - q^2$ and $\lambda = u^2 - 4ss'$); in principle, higher order $\alpha_s$ corrections can also be included, although their explicit calculation is not available, yet. The terms proportional to $<\bar{q}q>$ and to $m_0^2 <\bar{q}q>$ (using the notation $<\bar{q}g\sigma Gq> = m_0^2 <\bar{q}q>$) are the first two power corrections (in terms of operators of dimension $D = 3$ and $D = 5$, respectively) which parameterize the deviations from the asymptotically free behaviour. In QCD sum rules these condensates are universal parameters: they are independent of the channel, can be fixed from low energy phenomenology and compared to the evaluation by lattice QCD. Their value is known only for low dimensional operators, whereas the higher dimensional condensates are generally estimated by using factorization.

In practical cases the number of the power corrections that can be included in the expansion is limited; for example, in Eq. (17) the $D = 6$ term is known and gives a negligible contribution, whereas the contribution of $D = 4$ operator has been estimated for the Borel transformed sum rule, only [23]. The truncation in the series of the non-perturbative corrections is a limitation of the QCD sum rules approach. The heavy quark masses are parameters to be fixed; we use $m_c = 1.35 GeV$ and $m_b = 4.6 GeV$.

The result for Eq. (12) is obtained in analogous way:

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The dependence of the QCD sum rules predictions for the leptonic constants on the heavy quark masses is discussed in [28].
\[ T^{D \to a_0}_{QCD}(p^2, p'^2, q^2) = \frac{1}{(2\pi)^2} \int ds \, ds' \frac{\rho^{D \to a_0}_{QCD}(s, s', q^2)}{(s - p^2)(s' - p'^2)} \]
\[ - \frac{m_c < \bar{q} q >}{2} \left[ \frac{1}{r r'} - \frac{m_0^2}{6} \frac{3m_c^2}{r^3 r'} + \frac{4}{r^2 r'} + \frac{2}{r' r'^2} \right] \]
\[ + \frac{2(m_c^2 - q^2)}{r^2 r'^2} \] (18)

with the spectral function:
\[ \rho^{D \to a_0}_{QCD}(s, s', q^2) = \frac{s'}{\lambda^{3/2}} \left[ m_c^2(s - s') - q^2(2s - m_c^2) \right] . \] (19)

Invoking duality, it is assumed that \( \rho_{\text{cont}} \) in (14), which includes the contribution of the higher resonances and of the continuum of states, is equal to \( \rho_{QCD} \) given in (17) for all values of \( s, s', q^2 \) but for a region where the lowest lying resonances dominate; a model for this region is:

\[ m_c^2 < s < s_0 \]
\[ 0 < s' < \min(s_0', \frac{(s - m_c^2)(m_c^2 - q^2)}{m_c^2}) . \] (20)

\( s_0 \) and \( s'_0 \) are thresholds which separate the domain of the resonance from continuum; their exact position is not known, although indications can be derived from the experimental or theoretical spectrum in a given channel. For example, for \( D \to \pi \) it can be assumed that \( s_0 \) is not larger than the mass squared of the first resonance above \( D \) coupled to the pseudoscalar current, \( s_0 = 6 - 7 \, \text{GeV}^2 \) and that \( s'_0 \) is around the \( \rho, \omega \) mass squared, \( s'_0 = 0.6 - 0.7 \, \text{GeV}^2 \). For \( a_0 \) we choose: \( s'_0 = 1.6 - 1.7 \, \text{GeV}^2 \), and for \( B \): \( s_0 = 33 - 36 \, \text{GeV}^2 \).

Assuming duality, a rule is obtained where the form factors in (14) are given in terms of the QCD quantities in (16) (quark masses, condensates, eventually \( \alpha_s \)) and of the leptonic constants \( f_\pi, f_D, f_{a_0} \). The subtraction terms (polynomials in \( p^2 \) or \( p'^2 \)), which can be present in (14), are removed by performing a double Borel transform in the variables \(-p^2, -p'^2\):

\[ \mathcal{B} = \frac{(-p^2)^n}{(n-1)!} \left( \frac{d}{dp^2} \right)^n \frac{(-p'^2)^m}{(m-1)!} \left( \frac{d}{dp'^2} \right)^m \] (21)
in the limit \(-p^2, -p'^2 \to \infty, n, m \to \infty\), keeping \(-p^2/n = M^2\) and \(-p'^2/m = M'^2\) fixed. This transformation has also the property of factorially suppressing the higher order power corrections in the operator product expansion, and of
enhancing the contribution of the low lying states in the perturbative term. The Borel transformed sum rules for $F_{1}^{D \to \pi}$ and $F_{1}^{D \to a_{0}}$ read:

$$f_{\pi} f_{D} \frac{m_{c}^{2}}{m_{c}} F_{1}^{D \to \pi}(q^{2}) e^{-\frac{m_{D}^{2}}{M^{2}}} = \frac{1}{(2\pi)^{2}} \int_{D} ds \, ds' \rho^{D \to \pi}(s, s', q^{2}) e^{-\frac{m_{D}^{2}}{M^{2}}} - \frac{<\bar{q}q>}{2} e^{-\frac{m_{b}^{2}}{M^{2}}} [1 - \frac{m_{b}^{2}}{6} (\frac{3m_{p}^{2}}{2M^{4}} - \frac{2}{M^{2}} - \frac{2q^{2}}{M^{2}M^{2}})]$$

and

$$f_{a_{0}} f_{D} \frac{m_{b}^{2}}{m_{b}} F_{1}^{D \to a_{0}}(q^{2}) e^{-\frac{m_{D}^{2}}{M^{2}}} = \frac{1}{(2\pi)^{2}} \int_{D} ds \, ds' \rho^{D \to a_{0}}(s, s', q^{2}) e^{-\frac{m_{D}^{2}}{M^{2}}} - \frac{<\bar{q}q>}{2} e^{-\frac{m_{b}^{2}}{M^{2}}} [1 - \frac{m_{b}^{2}}{6} (\frac{3m_{p}^{2}}{2M^{4}} - \frac{4}{M^{2}} - \frac{2}{M^{2} + \frac{2(q^{2} - m_{c}^{2})}{M^{2}M^{2}}})].$$

(22)

The last quantities to be fixed are the leptonic constants. Some of them, as $f_{\pi}$, come from experiment; others, as $f_{D}$, can be obtained from two-point QCD sum rules [23]. The analysis of Eqs. (22, 23) proceeds by looking for a range of parameters $M^{2}, M^{2}$ (duality window) where the values of the form factors do not depend on the Borel parameters and on the thresholds $s_{0}, s'_{0}$; in this window a hierarchy between the perturbative $D = 0$ and the non perturbative $D = 3, D = 5$ terms should be verified in order to be confident on the convergence of the series of the power corrections; moreover, the perturbative integral should be larger than the contribution of the continuum. Other conditions can be imposed to restrict the dependence of the predictions on the the parameters of the method; for example, one could check that the sum rule for the mass of the resonances, obtained by taking logarithmic derivatives with respect to the Borel parameters, gives a result in agreement with the experimental mass.

If all these requirements are fulfilled a prediction for the form factors in the deep Euclidean region $q^{2} \leq 0$ is obtained. Moreover, as discussed in [12], the analysis of the sum rule can also be done for positive values of $t$, since long distance effects in the $t$–channel can only be relevant near the threshold $t_{th} \simeq m_{c}^{2}$ (for $D \to K, K^{*}, \pi$) or $t_{th} \simeq m_{b}^{2}$ (for $B \to \pi, \rho$). In this way indications can be obtained about the $q^{2}$ dependence of the form factors in a quite large range of $q^{2}$.

The $t$ dependence of $F_{1}^{D \to \pi}$ and $F_{1}^{D \to a_{0}}$ is depicted in fig. 4. As already observed in [13, 23] $F_{1}^{D \to \pi}$ displays a $t$ dependence that can be fitted with a pole. The interesting point, here, is that also $F_{1}^{D \to a_{0}}$, which is related to an axial current, displays a polar behaviour with a fitted pole mass $m_{pole} \simeq 1.9$ GeV; however, this mass is smaller than the mass of $D^{**}(1^{+})$ which is the first resonance in the $t$ channel: $m_{D^{**}(1^{+})} = 2.42$ GeV. A similar result is obtained for $B \to \pi$ and $B \to a_{0}$ (fig. 4); in the second case the fitted pole mass is $m_{pole} \simeq 6.2$ GeV.
Figure 1: $q^2$ dependence of the form factors $F_{1}^{D\rightarrow\pi}$ (continuous line) and $F_{1}^{D\rightarrow a_0}$ (dashed line).

The conclusion is that there is no common behaviour for the form factors of the axial current (and, presumably, of the vector current); the $q^2$ dependence of the various form factors must be computed, since general arguments of dominance, universality, etc., could fail.

The experimental investigations on semileptonic $D$—meson decays are in their initial status. The predictions made by QCD sum rules, or by other theoretical approaches, have to be checked not only because of their own interest, but also in the light of their implications for the phenomenology of $B$ and other heavy mesons. A Tau-Charm factory, with its potentialities in collecting large samples of decaying charmed mesons, could be an ideal tool for such investigations.
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