Multi-field cold and warm inflation and the de Sitter swampland conjectures

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Abstract

We discuss under which conditions multi-field cold and warm inflationary models with canonical kinetic energy terms are compatible with the swampland conjectures about the emergence of de Sitter solutions in string theory. We find that under quite general conditions the slow-roll conditions for multi-field cold inflation are at odds with the swampland conjectures for an arbitrary number of scalar fields driving inflation. However, slow-roll conditions can be reconciled with the swampland conjectures in the strong dissipative regime of warm inflation.

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1 Introduction

Despite its impressive capability to comprise the most desirable features of a fundamental theory of quantum gravity, string theory has failed, among other shortcomings, to allow for a natural scenario for inflation. This situation arises as the fundamental scalar field of the theory, the dilaton, does not acquire any potential in any order of perturbation theory and non-perturbative contributions tend to lead to very shallow and phenomenologically untenable models. In fact, de Sitter solutions seem to be hard to obtain in string theory, even though possible routes have been proposed [1]. This is sharply contrasting with the situation of $N = 1$ supergravity in the context of which successful inflationary models can be rather easily achieved [2]. This is somewhat surprising as, in fact, it is thanks to inflation that, for instance, some specific issues of superstring models with intermediate scale can be circumvented [3]. Of course, beyond single-field inflation, proposals for two-field inflation have been put forward (see e.g. [4, 5]), but to our knowledge these have not been systematically explored in the context of string theory.

The recent set of conjectures that de Sitter solutions in string theory cannot be found in its landscape, but lie actually in a “swampland”, i.e. the set of consistent-looking theories that do not admit a suitable ultraviolet completion in string theory, has led to an intense discussion whether the resulting solutions are consistent with the conditions required by inflation. These set of conjectures comprise quite general criteria: charge symmetries are supposed to be local gauge symmetries so that at least one particle must have a mass in Planck units less than the gauge coupling strength so to ensure that gravity is weak; the sign of some higher-order terms in the effective action is constrained to warrant the absence of superluminal propagation; there are a finite number of massless particles (see Ref. [6] for a review). More specifically, in what concerns inflation, these conjectures can be expressed in terms of constraints on scalar fields, generically denoted by $\phi$ in the field space [7, 8]:

$$\frac{\Delta \phi}{M_P} < c_1, \quad (1)$$

$$M_P \left| \frac{\partial \phi V}{V} \right| > c_2, \quad (2)$$

where $\Delta \phi$ is the range of variation of the field, $M_P \equiv M_{Pl}/\sqrt{8\pi}$ is the Reduced Planck’s mass, $V(\phi)$ is the scalar field potential, $\partial \phi V \equiv \partial V/\partial \phi$, and $c_1$ and $c_2$ are $O(1)$ constants. It is argued that one should also consider a more refined condition [9, 10, 11]

$$M_P^2 \frac{\partial^2 \phi V}{V} < -c_3, \quad (3)$$

where $\partial^2 \phi V \equiv \partial^2 V/\partial \phi^2$ and, likewise $c_1$ and $c_2$, the constant $c_3$ is of order one.

Conditions given by Eqs. (2) and (3) are somewhat at odds with the onset conditions of single-field inflation which require that the parameters for the inflaton field [12],

$$\epsilon_\phi = \frac{M_P^2}{2} \left( \frac{\partial \phi V}{V} \right)^2 \quad (4)$$

2
and

$$\eta_\phi = M_P^2 \frac{\partial^2 \phi V}{\partial \phi^2},$$

(5)

satisfy the slow-roll requirements, $\epsilon_\phi \ll 1$ and $|\eta_\phi| \ll 1$ at the onset of inflation, so that at the end of inflation $\epsilon_\phi \sim |\eta_\phi| \sim 1$. In order to successfully solve the initial conditions problems of standard cosmology, the number of e-foldings of inflation must satisfy the condition $N_e > 65$, where

$$N_e \equiv \ln \left( \frac{a_e}{a_i} \right) \simeq -\frac{1}{M_P^2} \int_{\phi_i}^{\phi_e} \frac{V}{\partial \phi V} d\phi = -\frac{1}{\sqrt{2} M_P} \int_{\phi_i}^{\phi_e} \frac{1}{\sqrt{\epsilon_\phi}} d\phi,$$

(6)

with $\phi_i$ and $\phi_e$ corresponding to the initial and final values of the inflaton field. Since during inflation the slow-roll parameters are approximately constant, $d\epsilon_\phi/dN \simeq \mathcal{O}(\epsilon_\phi^2)$, hence

$$N_e \simeq \frac{1}{\sqrt{2\epsilon_\phi} M_P} \Delta \phi,$$

(7)

and thus for $\epsilon_\phi \lesssim 10^{-4}$ one obtains $\Delta \phi \sim M_P$, which is consistent with the constraint given by Eq. (1).

Further contact with observations can be established through the amplitude of the inflaton quantum fluctuations and its imprint of the Cosmic Microwave Background Radiation (CMB) through curvature and density fluctuations,

$$\Delta^2_R = A_s \left( \frac{k}{k_s} \right)^{n_s-1},$$

(8)

where $k_s$ is a chosen scale measured on the CMB, the scalar spectral index is

$$n_s \simeq 1 - 6 \epsilon_\phi + 2 \eta_\phi,$$

(9)

and the amplitude of scalar fluctuations is given by

$$A_s \simeq \frac{1}{24\pi^2} \left( \frac{V(\phi_H)}{M_P^4} \right) \frac{1}{\epsilon_\phi(k_H)},$$

(10)

with $\phi_H$ and $\epsilon_\phi(k_H)$ corresponding to a scale when the fluctuating modes cross the horizon, that is between 50 and 60 e-foldings before the end of inflation.

Furthermore, from the spectrum of tensor modes generated by the inflaton quantum fluctuations,

$$\Delta^2_T = A_t \left( \frac{k}{k_s} \right)^{n_t},$$

(11)

where

$$A_t \simeq \frac{2}{3\pi^2} \left( \frac{V(\phi_H)}{M_P} \right),$$

(12)

and

$$n_t \simeq -2 \epsilon_\phi,$$

(13)
it is found that the tensor-to-scalar ratio is given by

\[ r = \frac{\Delta^2_T}{\Delta^2_R} \simeq 16\epsilon_\phi. \tag{14} \]

Planck 2018 temperature and polarisation data indicate that for \(0.008\ h^{-1}\text{Mpc}^{-1} \leq k \leq 0.1\ h^{-1}\text{Mpc}^{-1}\), the expansion rate \(H = (67.4 \pm 0.5)\ \text{km s}^{-1}\text{Mpc}^{-1}\), and absence of running, the scalar spectral index is

\[ n_s = 0.9649 \pm 0.0042, \tag{15} \]

while BICEP2/Keck Array, Planck and other data place an upper bound on the tensor-to-scalar ratio at \(k = 0.002\ \text{Mpc}^{-1}\), namely,

\[ r < 0.06. \tag{16} \]

From which follows, dropping higher-order slow-roll parameters, that

\[ \epsilon_\phi < 0.0044 \tag{17} \]

and

\[ \eta_\phi = -0.015 \pm 0.006, \tag{18} \]

which do not match \(c_2\) and \(c_3\).

In what follows we shall consider the situation in the context of multi-field cold inflationary models with canonical kinetic energy terms, initially for two and then for an arbitrary number of fields (section 2.1). We shall then address in section 2.2 the problem in the context of warm inflationary models which exhibit dissipation. We shall see that the swampland conjectures and the slow-roll conditions can be reconciled in the strong dissipative regime of warm inflation for one, two, and multi-field models. Conclusions will be drawn in section 3.

2 Beyond Single Field inflation

As seen above, the de Sitter swampland conjectures and the slow-roll conditions cannot be matched for single-field cold inflation. As discussed, the purpose of the swampland conjectures is to ensure that a suitable effective field theory in a de Sitter background arises from string theory. It is further assumed that these conjectures also allow for the emergence of a classical theory of gravity, which then drives inflation. The connection between curvature and matter-energy as established by General Relativity (GR) is well supported experimentally and observationally (see e.g. [13, 14] for discussions). Indeed, theoretical and experimental evidence suggest that GR reflects a provisional stage, although highly relevant, towards the ultimate description of gravity. Well-known theoretical difficulties concern the existence of spacetime singularities and the cosmological constant problem, both related to the key issue of making quantum mechanics and GR compatible with each other, string theory being, of course, quite relevant in this respect.

It is quite logical that de Sitter swampland conjectures have attracted great interest of the cosmological community given that they impose conditions on the field spacing and on the
first two derivatives of the potential of the background effective field theory (see, for instance, Ref. [11] for a partial list), which can be accessed in single-field inflation with features of the CMB as discussed above.

Clearly, the conjectures (2) and (3) are at odds with constraints arising from the CMB data, Eqs. (17) and (18). This conflict has been pointed out, for instance, in Ref. [15], even though it has been argued that definite conclusions about an actual tension depend on the knowledge about the origin of the adiabatic curvature perturbations, within the slow-roll single-field models of inflation [16]. In fact, the incompatibility of the swampland conjectures with the observations has been an object of critique from the authors of Ref. [17]. In any case, the swampland conjectures have given origin to many ideas and sparked interesting proposals [18, 19, 20, 21, 22].

Even though the conflict is still depending on the nature of the perturbations, a natural way to avoid this tension is to consider multi-field inflationary models. Most of these models are known to show no contradiction with the CMB features [23]. In fact, multi-field models open interesting perspectives, for instance, for unification with dark matter and dark energy [24, 25]. Two-field inflationary models were first considered in the context of $N = 1$ supergravity [26] and their dynamics was scrutinized in Refs. [27, 28] for a broader class of models. In a broad context and in string theory, two-field inflationary with different mass scales and an interaction term were considered in Refs. [4, 5, 29]. In the context of the swampland conjectures, two-field inflationary models were discussed in Refs. [30, 31], where in Ref. [30] non-canonical kinetic energy terms have been considered. However, in what follows, we shall present quite general arguments which seem to rule out a putative conciliation using multi-field cold inflation with canonical kinetic energy terms (see section 2.1). We shall consider first a two-scalar cold inflationary model and then generalize the argument for multi-field inflationary models. In section 2.2 we turn to the case of warm inflation, for which, in the strong dissipative regime, slow-roll conditions can be reconciled with the swampland conjectures.

### 2.1 Cold inflation

Quite generically, inflationary models driven by two scalar fields $\phi$ and $\chi$ can be described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\phi, \chi),$$

(19)

where the potential $U(\phi, \chi)$ contains the potentials of the fields $\phi$ and $\chi$ and an interaction term between these fields. This potential is assumed to be designed to yield a healthy period of inflation. Let us also assume that it satisfies the conjecture (1), even though it might be trickier to achieve this condition for many fields. But, as we shall see, if conjecture (1) can, at least in principle, be satisfied, the conjecture (2) cannot be met in the context of quasi-exponential cold inflation.

Coupling these fields to gravity in an homogeneous, isotropic and flat cosmological space-time background and assuming that the scalar fields can decay to radiation leads to the field
\[
\ddot{\phi} + 3H \dot{\phi} + \partial_\phi U = -\Gamma_{\phi} \dot{\phi},
\]
\[
\ddot{\chi} + 3H \dot{\chi} + \partial_\chi U = -\Gamma_{\chi} \dot{\chi},
\]
\[
\dot{\rho}_R + 4H \rho_R = \Gamma_{\phi} \dot{\phi}^2 + \Gamma_{\chi} \dot{\chi}^2,
\]
\[
H^2 = \frac{1}{3M_p^2} \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 + U + \rho_R \right),
\]
\[
\dot{H} = -\frac{1}{2M_p^2} \left( \dot{\phi}^2 + \dot{\chi}^2 + \frac{4}{3} \rho_R \right),
\]

where, in general terms, \( \Gamma_{\phi} = \Gamma_{\phi}(\phi) \) and \( \Gamma_{\chi} = \Gamma_{\chi}(\chi) \) denote the decay widths of these fields to radiation, \( \rho_R \) is the energy density of the radiation fluid, \( H = \dot{a}/a \) is the expansion rate, and \( a \) the scale factor of the Friedman–Lemaître–Robertson–Walker metric. In the above equations, an overdot denotes a derivative with respect to the cosmic time \( t \).

In this subsection we shall consider that \( \Gamma_{\phi} = \Gamma_{\chi} = 0 \) and neglect \( \rho_R \), meaning that inflation will supercool the Universe and a reheating process must be considered separately. Non-vanishing dissipative terms and a non-negligible energy density of radiation will be considered in section 2.2.

Let us assume that inflation is almost exponential, namely,

\[
\frac{|\dot{H}|}{H^2} \ll 1.
\]

From this expression, using Eqs. (23) and (24), follows that \( \dot{\phi}^2 + \dot{\chi}^2 \ll U \), which implies that Eq. (23) can be written as

\[
U \simeq 3M_P^2 H^2.
\]

In the above expression and in what follows, the symbol \( \simeq \) means “equal within the slow-roll approximation”.

Taking the time derivative of Eq. (26) and using Eq. (24) we obtain

\[
\dot{\phi} \partial_\phi U + \dot{\chi} \partial_\chi U \simeq -3H(\dot{\phi}^2 + \dot{\chi}^2),
\]

which can be written as

\[
\partial_\phi U + 3H \dot{\phi} \left(1 + \frac{\dot{\chi}^2}{\dot{\phi}^2} \right) \left(1 + \frac{\dot{\chi} \partial_\chi U}{\dot{\phi} \partial_\phi U} \right)^{-1} \simeq 0.
\]

Imposing the condition

\[
\left(1 + \frac{\dot{\chi}^2}{\dot{\phi}^2} \right) \left(1 + \frac{\dot{\chi} \partial_\chi U}{\dot{\phi} \partial_\phi U} \right)^{-1} \simeq 1,
\]

we obtain

\[
\partial_\phi U + 3H \dot{\phi} \simeq 0,
\]
which is equivalent to neglecting $\ddot{\phi}$ in Eq. (20).

Now, using Eq. (30), the condition given by Eq. (29) can be simplified to yield

$$\partial \chi U + 3H \dot{\chi} \simeq 0,$$

which is equivalent to neglecting $\ddot{\chi}$ in Eq. (21).

Following Ref. [32], the slow-roll parameter $\epsilon$ is defined as

$$\epsilon = \frac{1}{2} M_P^2 \frac{\nabla U \cdot \nabla U}{U^2},$$

which is a straightforward generalization of the case with a single scalar field [see Eq. (4)]. Using Eqs. (24)–(26), (30), and (31), we obtain

$$\epsilon \simeq \frac{\dot{H}}{H^2} \ll 1.$$

The de Sitter swampland conjecture [see Eq. (2)]

$$M_P \frac{\partial \phi U}{U} > c_2$$

can be written as

$$c_2^2 \lesssim 2 \frac{\dot{H}}{H^2} f,$$

where

$$f(\dot{\phi}, \dot{\chi}) \equiv \frac{\dot{\phi}^2}{\dot{\phi}^2 + \dot{\chi}^2}.$$

Since the above function satisfies the condition $0 < f(\dot{\phi}, \dot{\chi}) \leq 1$, hence we conclude that $c_2^2 \ll 1$.

The main issue is that, if we assume that inflation is quasi-exponential, i.e., $|\dot{H}|/H^2 \ll 1$, then both $\epsilon$ and $c_2$ are much smaller than one.

These results can be straightforwardly generalized to the case of an arbitrary number of scalar fields. Indeed, let us consider the scalar fields $\phi$ and $\chi_i$ ($i = 1, \ldots, N$) with potential $U = U(\phi, \chi_1, \ldots, \chi_N)$.

Once again, assuming quasi-exponential inflation and using the evolution equations — generalized to the case of the above $N + 1$ scalar fields — we obtain

$$\sum_{i=1}^{N} (\dot{\chi}_i \partial \chi_i U + 3H \dot{\chi}_i^2) \simeq 0.$$

Since this equation does not imply

$$\partial \chi_i U + 3H \dot{\chi}_i \simeq 0, \quad (i = 1, \ldots, N),$$

7
these latter expressions must be assumed (as in Ref. [32]), instead of being derived from Eq. (26) as in the two-scalar field case. The parameter \( \epsilon \) satisfies the condition given by Eq. (33), i.e., it is much smaller than unity during the inflationary period, and \( c_2^2 \) is given by Eq. (35), but now

\[
f(\dot{\phi}, \dot{\chi}_1, \ldots, \dot{\chi}_N) \equiv \frac{\dot{\phi}^2}{\dot{\phi}^2 + \sum_{i=1}^{N} \dot{\chi}_i^2}.
\]

Since the above function satisfies the condition \( 0 < f(\dot{\phi}, \dot{\chi}_1, \ldots, \dot{\chi}_N) \leq 1 \), it then follows that \( c_2^2 \ll 1 \), as in the two-scalar-field case.

Hence, it has been shown that, under the quite general conditions of cold exponential inflation, the swampland conjectures cannot be met for any number of scalar fields.

### 2.2 Warm Inflation

In the warm inflation scenario, dissipation plays a crucial role in slowing down the inflaton, \( \phi \), as it rolls down the potential \( V(\phi) \) [33]. Within a strong dissipative regime, it is known that the swampland conjectures can be made compatible with single-field warm inflation [34, 35]. In what follows, we show that this result extends to multi-field warm inflationary models.

To characterize the slow-roll regime for a single field scenario, in addition to the parameters \( \epsilon_\phi \) and \( \eta_\phi \), defined by Eqs. (4) and (5), an additional parameter \( \beta_\phi \) should be introduced,

\[
\beta_\phi = M_P^2 \frac{\partial_\phi \Gamma_\phi}{\Gamma_\phi} \frac{\partial_\phi V}{V},
\]

where \( \Gamma_\phi = \Gamma_\phi(\phi) \) is the so-called dissipation coefficient [cf. Eq. (20)], responsible for a continuous energy transfer from the inflaton field \( \phi \) to a radiation bath with energy density \( \rho_R \).

Assuming quasi-exponential inflation, it follows [36]

\[
\frac{\dot{H}}{H^2} \approx -\frac{\epsilon_\phi}{1 + Q},
\]

\[
\frac{\ddot{\phi}}{H \dot{\phi}} \approx -\frac{1}{1 + Q} \left( \eta_\phi - \beta_\phi + \frac{\beta_\phi - \epsilon_\phi}{1 + Q} \right),
\]

\[
\frac{\dot{\rho}_R}{H \rho_R} \approx -\frac{1}{1 + Q} \left( 2 \eta_\phi - \beta_\phi - \epsilon_\phi + 2 \frac{\beta_\phi - \epsilon_\phi}{1 + Q} \right),
\]

where the dissipation ratio \( Q \) is defined as

\[
Q \equiv \frac{\Gamma_\phi}{3H}.
\]

Taking into account that \( |\dot{H}|/H^2 \ll 1 \) and that the slow-roll approximation requires \( |\ddot{\phi}| \ll |H \dot{\phi}| \) and \( |\dot{\rho}_R| \ll |H \rho_R| \), from the above equations we conclude that

\[
\epsilon_\phi \ll 1 + Q, \quad |\eta_\phi| \ll 1 + Q, \quad |\beta_\phi| \ll 1 + Q.
\]

8
This contrasts with the case of cold inflation, for which $\epsilon_\phi \ll 1$ and $|\eta_\phi| \ll 1$.

The constants $c_2$ and $c_3$, arising within the de Sitter swampland conjectures, Eqs. (2) and (3), are related to the slow-roll parameters $\epsilon_\phi$ and $\eta_\phi$ as $c_2^2 < 2\epsilon_\phi$ and $c_3 < |\eta_\phi|$, respectively, implying that

$$c_2^2 \ll 1 + Q, \quad c_3 \ll 1 + Q.$$  \hfill (46)

Thus, in the strong dissipative regime of warm inflation, for which $Q \gg 1$, both $c_2$ and $c_3$ can be of order unity, even if the expansion is quasi-exponential, $|\dot{H}|/H^2 \ll 1$. This behavior contrasts with the situation in cold inflation, for which both $c_2$ and $c_3$ are much smaller than unity during a quasi-exponential inflationary period.

We now turn to the case of multi-field warm inflation.

Let us start with two scalar fields $\phi$ and $\chi$ with potential $U(\phi, \chi)$ and the evolution Eqs. (20)–(24). From the condition that inflation is almost exponential [see Eq. (25)] and using Eqs. (23) and (24), follows that $\dot{\phi}^2 + \dot{\chi}^2 + \rho_R \ll U$, which implies that Eq. (23) can be written as

$$U \simeq 3M_P^2 H^2.$$ \hfill (47)

Taking the time derivative of the latter expression, and using Eq. (24), we obtain

$$\dot{\phi} \partial_\phi U + \dot{\chi} \partial_\chi U \simeq -3H \left( \dot{\phi}^2 + \dot{\chi}^2 + \frac{4}{3} \rho_R \right),$$ \hfill (48)

which can be written as

$$3H \dot{\phi} \left( 1 + \frac{4\rho_R}{3\dot{\phi}^2} \right) + \partial_\phi U \left( 1 + \frac{\dot{\chi}}{\dot{\phi}} \frac{\partial_\phi U}{\partial_\phi U} + \frac{3H \dot{\phi}}{\partial_\phi U} \frac{\dot{\chi}^2}{\dot{\phi}^2} \right) \simeq 0.$$ \hfill (49)

Comparing with Eq. (20), this is equivalent to neglecting $\ddot{\phi}$ and assuming

$$\dot{\chi} \frac{\partial_\phi U}{\partial_\phi U} + \frac{3H \dot{\phi}}{\partial_\phi U} \frac{\dot{\chi}^2}{\dot{\phi}^2} \simeq 0$$ \hfill (50)

and

$$\frac{4\rho_R}{3\dot{\phi}^2} \simeq \frac{\Gamma_\phi}{3H}, $$ \hfill (51)

yielding

$$\partial_\phi U \simeq -3H \dot{\phi} (1 + Q).$$ \hfill (52)

Note that Eq. (50) can be written as

$$\partial_\chi U \simeq -3H \dot{\chi}.$$ \hfill (53)

which is equivalent to neglecting $\ddot{\chi}$ and $\Gamma_\chi \ddot{\chi}$ in Eq. (21), while Eq. (51) can be written as

$$4H \rho_R \simeq \Gamma_\phi \dot{\phi}^2,$$ \hfill (54)
which is equivalent to neglecting $\dot{\rho}_R$ and $\Gamma_\chi \chi^2$ in Eq. (22).

Now, taking the time derivative of Eqs. (52)–(54), we obtain

$$\dot{\phi} \partial_{\phi} \Gamma_\phi \simeq \left(3\dot{H} - 6H^2 + 3H \frac{\ddot{\phi}}{\dot{\phi}} \rho_R \right) Q, \quad (55)$$

$$\partial_{\phi \phi}^2 U + \frac{\dot{\chi}}{\chi} \partial_{\phi \chi}^2 U \simeq -3\dot{H}(1 + Q) - 3H \frac{\ddot{\phi}}{\dot{\phi}}(1 - Q) - 3H \frac{\ddot{\rho}_R}{\dot{\rho}_R} Q, \quad (56)$$

$$\partial_{\chi \chi}^2 U + \frac{\dot{\phi}}{\chi} \partial_{\phi \chi}^2 U \simeq -3\dot{H} - 3H \frac{\ddot{\chi}}{\dot{\chi}}. \quad (57)$$

Following Ref. [32], we define the slow-roll parameters $\epsilon$ and $\eta_{ij}$ as

$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{(\partial_\phi U)^2 + (\partial_\chi U)^2}{U^2}\right), \quad (58)$$

$$\eta_{\phi \phi} = M_P^2 \frac{\partial_{\phi \phi}^2 U}{U}, \quad \eta_{\phi \chi} = M_P^2 \frac{\partial_{\phi \chi}^2 U}{U}, \quad \eta_{\chi \chi} = M_P^2 \frac{\partial_{\chi \chi}^2 U}{U}, \quad (59)$$

while for $\beta$ we adopt the definition [see Eq. (40)]

$$\beta = M_P^2 \frac{\partial_{\phi} \Gamma_\phi \partial_\phi U}{U}. \quad (60)$$

Using Eqs. (47), (52)–(57), it is straightforward to obtain

$$\frac{\dot{H}}{H^2} \simeq -\frac{\epsilon}{F}, \quad (61)$$

$$\frac{\ddot{\phi}}{H \dot{\phi}} \simeq -\frac{1}{1 + Q} \left(\eta_{\phi \phi} + \frac{\dot{\chi}}{\chi} \eta_{\phi \chi} - \beta + \frac{\beta}{1 + Q} - \frac{\epsilon}{F}\right), \quad (62)$$

$$\frac{\ddot{\chi}}{H \dot{\chi}} \simeq \frac{\epsilon}{F} - \eta_{\chi \chi} - \frac{\dot{\phi}}{\chi} \eta_{\phi \chi}, \quad (63)$$

$$\frac{\dot{\rho}_R}{H \rho_R} \simeq -\frac{1}{1 + Q} \left(2\eta_{\phi \phi} + 2 \frac{\dot{\chi}}{\chi} \eta_{\phi \chi} - \beta - \frac{1 + Q}{F} + \frac{2\beta}{1 + Q} - \frac{2\epsilon}{F}\right), \quad (64)$$

where the following notation was introduced,

$$F(Q, \dot{\phi}, \dot{\chi}) \equiv \frac{(1 + Q)^2 \dot{\phi}^2 + \dot{\chi}^2}{(1 + Q) \phi^2 + \chi^2}. \quad (65)$$

Now, taking into account that $|\ddot{H}|/H^2 \ll 1$ and that the slow-roll approximation requires $|\ddot{\phi}| \ll |H \dot{\phi}|$, $|\ddot{\chi}| \ll |H \dot{\chi}|$, and $|\dot{\rho}_R| \ll |H \rho_R|$, we obtain

$$\epsilon \ll F, \quad |\beta| \ll 1 + Q, \quad |\eta_{\phi \phi}| \ll 1 + Q, \quad |\eta_{\chi \chi}| \ll 1, \quad \left|\frac{\dot{\chi}}{\chi} \eta_{\phi \chi}\right| \ll (1 + Q), \quad \left|\frac{\ddot{\phi}}{\phi} \eta_{\phi \chi}\right| \ll 1. \quad (66)$$
From these expressions we conclude that, if $\chi^2/\dot{\phi}^2 \ll 1 + Q$, then $F \simeq 1 + Q$, and, in the strong dissipative regime of warm inflation, for which $Q \gg 1$, the slow-roll parameters $\epsilon$, $\beta$, and $\eta_{\phi\phi}$ can be of order unity.

On the other hand, the de Sitter swampland conjecture given by Eq. (34), can be written as

$$c_2^2 \lesssim \frac{2|H|}{H^2} G,$$

where

$$G(Q, \dot{\phi}, \chi) \equiv \frac{(1 + Q)^2 \dot{\phi}^2}{(1 + Q)\dot{\phi}^2 + \chi^2}.$$  

If $\chi^2/\dot{\phi}^2 \ll 1 + Q$, then $G \simeq 1 + Q$ and, consequently, $c_2^2 \ll 1 + Q$. Therefore, we conclude that, in the strong dissipative regime of warm inflation, $c_2$ can be of order unity. If, however, $\chi^2/\dot{\phi}^2 \simeq (1 + Q)^2$, then $G \simeq 1$ and, consequently, $c_2^2 \ll 1$.

The de Sitter swampland conjecture [see Eq. (3)]

$$M_P^2 \partial^2_{\phi\phi} U < -c_3$$

can be written in terms of the slow-roll parameter $\eta_{\phi\phi}$ as $c_3 < |\eta_{\phi\phi}|$, meaning that $c_3 \ll 1 + Q$. For $Q \gg 1$, corresponding to the strong dissipative regime of warm inflation, $c_3$ can be of order unity.

As in the case of cold inflation, these results can be generalized for an arbitrary number of scalar fields, $\phi$ and $\chi_i$ ($i = 1, \ldots, N$), with potential $U(\phi, \chi_1, \ldots, \chi_N)$. As before, the evolution equations yield Eq. (37), which, again, does not imply $\partial_\chi U + 3H\dot{\chi} \simeq 0$; these latter expressions must be assumed, instead of being derived from Eq. (47) as in the two-scalar field case. With this assumption and taking into account that $|\dot{H}|/H^2 \ll 1$ and that the slow-roll approximation requires $|\ddot{\phi}| \ll |H\dot{\phi}|$, $|\ddot{\chi}_i| \ll |H\dot{\chi}_i|$, and $|\ddot{\rho}_R| \ll |H\rho_R|$, we obtain

$$\epsilon \ll F, \quad |\beta| \ll 1 + Q, \quad |\eta_{\phi\phi}| \ll 1 + Q, \quad \left| \frac{1}{\chi_j} \sum_{i=1}^N \eta_{\chi_i \chi_j} \dot{\chi}_i \right| \ll 1,$$

$$\left| \frac{1}{\phi} \sum_{i=1}^N \eta_{\phi \chi_i} \dot{\chi}_i \right| \ll 1 + Q, \quad \left| \frac{\ddot{\phi}}{\dot{\phi}} \right| \eta_{\phi \chi_i} \ll 1,$$

where $\eta_{\phi \chi_i}$ and $\eta_{\chi_i \chi_j}$ are slow-roll parameters defined as

$$\eta_{\phi \chi_i} = M_P^2 \partial_{\phi \chi_i}^2 U / U, \quad \eta_{\chi_i \chi_j} = M_P^2 \partial_{\chi_i \chi_j}^2 U / U,$$

and $F$ is given by

$$F(Q, \dot{\phi}, \dot{\chi}_1, \ldots, \dot{\chi}_N) \equiv \frac{(1 + Q)^2 \dot{\phi}^2 + \sum_{i=1}^N \dot{\chi}_i^2}{(1 + Q)\dot{\phi}^2 + \sum_{i=1}^N \dot{\chi}_i^2}.$$  

Furthermore, $c_2$ is given by Eq. (67) with

$$G(Q, \dot{\phi}, \dot{\chi}_1, \ldots, \dot{\chi}_N) \equiv \frac{(1 + Q)^2 \dot{\phi}^2}{(1 + Q)\dot{\phi}^2 + \sum_{i=1}^N \dot{\chi}_i^2}.$$  

11
while $c_3$ relates to the slow-roll parameter $\eta_{\phi\phi}$ as $c_3 < |\eta_{\phi\phi}|$. If $\sum_{i=1}^{N} \dot{\chi}_i^2/\dot{\phi}^2 \ll 1 + Q$, then $F \simeq 1 + Q$ and $G \simeq 1 + Q$, implying that, in the strong dissipative regime of warm inflation ($Q \gg 1$), both $c_2^2$ and $c_3$ can be of order unity.

## 3 Discussion and Conclusions

In this work we have examined, on quite general grounds, the possibility of matching the slow-roll conditions of inflation and the de Sitter swampland conjectures in the context of cold and warm inflationary models driven by more than one scalar field.

We have shown that irrespective of the number of scalar fields with canonical kinetic energy terms, quasi-exponential cold inflation and the swampland conjectures are incompatible up to issues related to the origin of adiabatic curvature perturbations.

The situation is different once dissipation is introduced. Indeed, in the context of single-field warm inflation, it is known that the slow-roll conditions and the swampland conjectures can be matched provided the dissipation ratio, $Q = \Gamma_\phi/(3H)$, satisfies the condition $Q \gg 1$, corresponding to a strong dissipative regime. Even though the situation of warm inflation driven by two scalar fields is more complex as it requires more parameters in order to fully characterize the slow-roll conditions, we have shown that quasi-exponential inflation and de Sitter swampland conditions can be reconciled provided a strong dissipative regime is ensured for one of the scalar fields, say $\phi$, and the other scalar field $\chi$ satisfies the condition $\dot{\chi}^2/\dot{\phi}^2 \ll 1$. For more than two scalar fields, as expected, the slow-roll conditions are richer, however, they can be made compatible with the swampland conditions, likewise in the cases of single- and two-field inflation, if the strong dissipative regime holds for one of the fields, say $\phi$, and if the condition $\sum_{i=1}^{N} \dot{\chi}_i^2/\dot{\phi}^2 \ll 1 + Q$ is satisfied by the remaining fields, $\chi_i$. However, differently from the two-scalar-field case, the conditions $\partial_{\chi_i} U + 3H \dot{\chi}_i \simeq 0$ should be imposed, since they do not follow from the slow-roll expression $U \simeq 3M_p^2 H^2$.

To close the discussion we emphasize that our work opens the possibility of reconciling the swampland conjectures with warm inflation, provided the single- or multi-field models admit, during the slow-roll, a strong dissipative regime and satisfy the conditions discussed above. These extra requirements might be relevant to narrow down the possible class of viable effective models that, although not in the landscape of string theory, are suitable from the point of view of inflation. It is intriguing that inflation with dissipative features is also an important ingredient in a recent proposal to address the cosmological constant problem \cite{37}. Moreover, it is quite interesting that dissipation, being a quite generic manifestation of the arrow of time at the macroscopic level, appears in a fundamental discussion about the viable emerging solutions of string theory.

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