Flux pattern instability in a strongly anisotropic type-II superconducting slab.

B.Ya. Shapiro, I. Shapiro and E.E. Dvash

Abstract. The thermomagnetic stability of the Bean critical state in an anisotropic type-II superconducting slab is studied theoretically. It is predicted that in a strongly anisotropic superconducting slab the non-uniform, fingering type instability appears as readily as in superconducting films. In particular, the fingering instability emerges for extremely small rate of the external magnetic field ramp.

Department of Physics, Institute of Superconductivity, Bar-Ilan University, Ramat-Gan 52900, Israel
E-mail: shapib@mail.biu.ac.il

1. Introduction
The dynamics of magnetic flux penetration into type II superconductors and its instabilities has been studied by a variety of techniques over the years, (see [1] and references therein). In most of the experimental situations, when the superconducting sample is subjected to an external magnetic field growing with time, a thermomagnetic instability arises. It appears due to local perturbations of the vortex matter by the heat released by a moving vortex. This process leads to the thermal softening of the vortex system which in turn is responsible for the instability. In this case the instability develops around a well defined thermodynamically stable Bean critical state. The conventional theory of the thermomagnetic instability predicts “uniform” flux jumps, where the flux front is essentially flat. This prediction holds for many experimental conditions, but not for all. In particular, magneto-optical techniques have revealed a wide class of spatial magnetic flux instabilities. The spatially non-uniform instabilities of the magnetic flux in the films are observed both in anisotropic HTSC [2] and in an isotropic conventional superconducting material like Nb [3]. In an isotropic slab, where the non-uniform instability arises under a high rate of increase of the external magnetic field. In this case the rate of the magnetic field growth must exceed some threshold [4].

In the present paper the thermomagnetic instability in an anisotropic bulk superconductor is studied. We show that the anisotropy has an important effect on the development of a spatially non-uniform instability even in the bulk. In particular, the finger pattern instability emerges with a negligibly small threshold of the external magnetic field rate.

2. Model and Basic Equations
We study the instability in the bulk geometry, where an anisotropic layered superconducting slab consisting on wide superconducting layers devided by thin dielectric strips fills the semispace $x > 0$, and the external magnetic field $H$ is parallel to the $z$-axis so that the screening current $J$ flows along the $y$ axis. The $a - b$ axis of the layered structure are placed in the $y - z$ plane,
while the $c$ axis is parallel to the $x$ direction. The magnetic flux penetrates the sample in depth $l = H/J_c$ ($J_c$ is the macroscopic critical current) forming the Bean steady state. The flux dynamics is caused by a constant ramp of the magnetic. This process is considered as a quasistatic with characteristic time smaller than the instability time.

The current, magnetic field and the temperature distributions in the sample are determined by the Maxwell equations,

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad \mathbf{B} \big|_{x=0} = \mu_0 \mathbf{H}_c$$

(1)

These equations should be supplemented by a current-voltage characteristic which for the anisotropic case, when the anisotropy is determined by the symmetry axes of the layered structure, is also anisotropic and has the form:

$$J_x = \begin{cases} 0, & J_x < J_c \\ \frac{1}{2} J (T,B,E_x) \frac{E_x}{E}, & J_x > J_c \end{cases}, \quad J_y = J (T,B,E_y) \frac{E_y}{E},$$

(2)

where $J (T,B,E)$ dependence is strongly nonlinear and describes the in-plane current transport, $\gamma$ is the parameter of the electric anisotropy.

The exact form of the current-voltage curve is not crucially important. The only important point is that the $E - J$ curve is very steep, and anisotropic. Its logarithmic derivative is large:

$$n = \left( \frac{\partial \ln E}{\partial \ln J} \right)|_{J=J_c} \approx \sigma_y / \sigma_x E \gg 1,$$

where $\sigma_y = (\partial J / \partial E)_{J=J_c}$ is the differential conductivity.

At low electric fields, the $E(J)$ curve is often approximated by a power law $E \propto J^n$ where $n >> 1$ is large and is assumed independent of the electric field.

The set of the Maxwell equations has to be completed by the corresponding anisotropic thermal conductivity equation

$$C \frac{\partial T}{\partial t} = \kappa_\perp \nabla^2 T + \kappa_\parallel \nabla^2 y T + \mathbf{J} \cdot \mathbf{E} - r (T - T_0)$$

(3)

where $C$ is the specific heat, $\kappa_\perp, \kappa_\parallel$ are the thermal diffusion coefficients, $r$ is the coefficient of heat transfer to the coolant liquid which is held at temperature $T_0$. In our case the thermal conductivity anisotropy parameter $\epsilon = (\kappa_\perp / \kappa_\parallel)$ is assumed to be small.

Solving the Eqs.(1)-(3) one obtains to leading order (see [1])

$$T = T_0, \quad E = (0, E_{0y}), \quad E_{0y} \approx l \left( \frac{\partial H_{ext}}{\partial t} \right)$$

(4)

This is appropriate for the experimentally realistic situation when the nonuniformity of $E$ is induced by ramping of the external magnetic field. From the symmetry of the problem $E_x = 0$.

### 3. Stability analysis

The linearization of Eqs.(1)-(3) for the temperature and electric field in the form

$$T = T_0 + \delta T (x,y,t), \quad \mathbf{E} = E_{0y} + \delta \mathbf{E}_x (x,y,t), \quad \alpha = x, y$$

(5)

reads for the current densities

$$\delta \mathbf{J}_x = \frac{1}{\gamma} J (T,E_x) \frac{\delta E_x}{E}, \quad \delta \mathbf{J}_y = \left[ \frac{\partial J}{\partial T} \delta T + \sigma_y \delta E_y \right],$$

(6)

and for the temperature fluctuations

$$C \frac{\partial \delta T}{\partial t} = \kappa_\parallel \nabla^2 y \delta T + \kappa_\perp \nabla^2 \delta T + \mathbf{E}_y \delta \mathbf{J}_y + \mathbf{J}_y \delta \mathbf{E}_y - r \delta T$$

(7)
Looking for a solution in the form

\[
\begin{pmatrix}
\delta T \\
\delta E_{x,y}
\end{pmatrix} = \begin{pmatrix}
T^* \theta \\
E_{\varepsilon x,y}
\end{pmatrix} \exp \left( \frac{\lambda t}{t_0} + ik_y \eta + ik_x \xi \right)
\]

(8)

where

\[
\alpha = \frac{\sigma_y r T^*}{J_c^2}; T^* = \left[ \frac{1}{J} \frac{\partial J}{\partial T} \right]_{J=J_c}^{-1}; \eta = y/w; \xi = x/w; t_0 = \mu_0 \sigma_y w^2; w^2 = \frac{C T^*}{\mu_0 J_c^2};
\]

and taking into account the Maxwell relation for fluctuations

\[
\nabla \times \nabla \times \delta E = -\mu_0 \frac{\partial \delta J}{\partial t}
\]

(9)

one obtains the dispersion law equation for the rate growth \( \lambda \) in the form

\[
\lambda^2 + P \lambda + S = 0
\]

(10)

where

\[
P = k_x^2 + \frac{\gamma k_y^2}{n} + \tau \left( k_y^2 + \epsilon k_x^2 \right) - 1
\]

(11)

\[
S = \frac{\left[ \tau n \left( k_y^2 + \epsilon k_x^2 \right) + \alpha n \right]}{n^2} \left( k_x^2 n + \gamma k_y^2 \right) + k_x^2 n - \gamma nk_y^2
\]

(12)

This equation has to be supplemented by the boundary conditions for fluctuations at the sample surface \((x = 0)\) and at the flux front \((x = l)\)

\[
\delta E_x(x = 0) = 0, \quad \left( \frac{\partial \delta E_x}{\partial x} \right)_{x=0} = 0, \quad \delta E_y(x = l) = 0
\]

(13)

which are satisfied for \(\delta E_y \propto \cos (k_x \xi)\) with \(k_x = \pi w/2l\).

The instability appears when \(\text{Re} \lambda > 0\). Despite the fact that the solution can be obtained for any anisotropy parameter \(\gamma\) we consider here analytically the mostly interesting cases of extremely anisotropic superconductors.

4. Instability and the Spatial flux pattern

A spatially nonuniform domain (pattern) of unstable magnetic flux appears when the maximal value of \(\text{Re} \lambda (k_x^*, k_y^*) > 0\). To find these \(k_x^*, k_y^*\) one needs two conditions from the Eqs.(10) - (12):

\[
S (k_x^*, k_y^*) = 0, \quad \Delta S (k_x^*, k_y^*) = 0
\]

(14)

where \(\Delta S (k_x^*, k_y^*)\) represents the \(S\) discriminant.

The boundary between uniform and nonuniform instabilities can be obtained from

\[
\text{Re} \lambda (k_x^*, k_y^*) = 0
\]

(15)

Restricting ourself to the case of a small heat transfer coefficient \(\alpha \ll 1\) we obtain the critical magnetic field \(H_f\), when the nonuniform instability is emerged, the characteristics size of the nonuniform flux pattern \(d_y = w/k_y^*\) and the critical ramp of the external magnetic field to
produce the unstable pattern \((\partial H_{ext}/\partial t) \simeq E_c/1\) appears, for different relations between the anisotropy parameters.

\[
\begin{align*}
\gamma \gg n \gg 1, \epsilon = 1 & \quad E_c = E_c^i/\gamma \quad H_f = H_f^i n^{-1/2} \quad d_y = d_y^i (2\gamma/n^2)^{1/4} \\
\gamma \gg \gamma \epsilon \gg n \gg 1 & \quad E_c = E_c^i \sqrt{4\epsilon/\gamma n} \quad H_f = H_f^i \sqrt{\epsilon/n} \quad d_y = d_y^i (8\gamma\epsilon/n^2)^{1/4} \\
\gamma \gg n \gg 1, \gamma \epsilon = 1 & \quad E_c = E_c^i/\gamma \quad H_f = H_f^i \gamma^{-1/2} \quad d_y = d_y^i \\
n \gg 1, \gamma = \epsilon = 1 & \quad E_c = E_c^i \quad H_f = H_f^i \quad d_y = d_y^i
\end{align*}
\]

Here

\[
E_c^i = \frac{\mu_0 \kappa \parallel J_c}{C}, \quad H_f^i = \frac{\pi}{2} \sqrt{\frac{T^* \kappa \parallel J_c}{E}}, \quad d_y^i = \left( \frac{1}{2n} \right)^{1/4} \left( \frac{T^* \kappa \parallel}{E J_c} \right)^{1/2}
\]

have been obtained for isotropic model (see Ref.[4]).

In particular for a superconductor with parameters \(\gamma = 1000, \epsilon = .1, n = 10\), the critical electric field for the instability to emerge is \(10^2\) times smaller than that in the isotropic case.

References

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