When is the spatial confounding a real problem? A fast Gaussian Markov random fields alternative to alleviate spatial confounding

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Abstract

Spatial generalized linear mixed models for areal data analysis have become a common model strategy in recent years with the development of statistical methods and with the availability of spatially referenced data sets. Recently, spatial confounding between the spatial random effects and the fixed effects were discovered and studied showing that this confounding may bring misleading interpretation to the model results. In this paper we present a novel fast alternative to alleviate spatial confounding. We propose an alternative way to define the dependence structure of the model, and in doing so, alleviate the

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spatial confounding reducing the restriction in modeling from previous solutions in the literature and increasing drastically the computational efficiency to adjust the model. A simulation study and a real data analysis is presented to better understand the advantages of the new method in comparison with the existing ones.

1 Introduction

Spatial generalized linear mixed models (SGLMM) for areal data analysis have become a common model strategy in recent years with the development of statistical methods and with the availability of spatially referenced data sets. Besag and Kooperberg (1995) introduced a hierarchical modeling in which the spatial dependence is captured by a latent Gaussian Markov Random Field (GMRF). In the past 20 years Besag and Kooperberg (1995) model (hereafter ICAR) have become the most popular areal model, mainly for the availability of the WinBUGS software (Lunn et al., 2000) that made accessible to anyone the use of such model.

In linear models it is often observed multicolinearity between explanatory variables. This problem occurs when explanatory variables, assumed independent, are correlated, generating a variance inflation in the estimates of the fixed effects. Although SGLMM have been widely used in applications, Clayton et al. (1993) and more recently Reich et al. (2006) identified the existence of confounding between the fixed and random effects. In their work, Reich et al. (2006) show that explanatory variables with spatial pattern may confound with the random effects, providing estimates for the fixed effects that are counter intuitive from a practical point of view. Thus, they proposed an alternative model to alleviate the confounding, hereafter RHZ model. The proposed alternative relies in constructing and including random effects that are orthogonal to the space spanned by the explanatory variables (for more details see Reich et al., 2006). From a geostatistical perspective Paciorek (2010) study and discuss the effects of confounding between the spatial random effects and possible explanatory variables.

Another well known short-come of SGLMM is the computational burden when dealing with high dimensional latent effects. Recently, Hughes and Haran (2013) introduces an alternative model (hereafter JM model), that alleviate spatial confounding and reduces the dimension of the random effects making the computational effort smaller for high dimensional random effects.
Hughes and Haran (2013) uses an alternative operator that retains the spatial properties but reduces the dimension of the random effects, they coined the new operator as Moran operator. With the reduction of the dimension of the random effects by the use of the Moran operator they are able to reduce the computational burden of their proposed model (for more details see Hughes and Haran, 2013).

A drawback from Reich et al. (2006) and Hughes and Haran (2013) methods is that neither of them allow for any parameters in the precision matrix of the latent field. In this paper we introduce a fast alternative for SGLMM that have no such restriction. Moreover, Rue and Held (2005) show that the use of GMRF can lead to computationally efficient models due to the sparsity of the precision matrix of GMRF’s in the latent field. Based on such property our method takes advantage of GMRF’s to considerably reduce computational burden controlling for spatial confounding. In summary, the method proposed in Section 3 have the following main advantages: 1) alleviate spatial confounding; 2) because of the Markov property is extremely efficient for any dimension of random effects; 3) there is no restriction in the existence of parameters for the precision matrix and 4) is simple to program, being able to be executed in a variety statistical software, e.g., WinBUGS and R-INLA (Rue et al., 2009).

Reich et al. (2006) showed that under some characteristics spatial confounding may provide counter intuitive results. However, when does that occur? It is always necessary to “correct” for spatial confounding? If not, when to do so? In our simulation study, Section 4 we revisit this questions and try to better understand what are the consequences of correcting for spatial confounding and when it is adequate to do so.

The paper proceeds as follows. In Section 2 we review the traditional SGLMMM model and present in summary RHZ and JM methods. Section 3 the proposed method is introduced and its characteristics are discussed. In Section 4 a simulation study is performed to compare the proposed method against RHZ and JM methods in terms of precision of estimates and time. Moreover, it provide insights on when to correct for spatial confounding and what are the consequences of it. In Section 5 we revisit the Slovenia data (Zadnik and Reich, 2006) to illustrate the conclusion obtained by the 3 models and its computational efficiency. Finally, the paper concludes with a discussion in Section 6.
2 Existing Methods

2.1 SGLMM

Spatial Generalized Linear Mixed Models (SGLMM) is a wide class of models that accommodates spatial dependence through a random effect term. Let \( Y_i \) be the observations of an area \( i, i = 1, \ldots, n \) with likelihood defined by \( \pi(y_i|\mu_i, \delta, X_i) \) where \( \mu_i = E(Y_i|X_i) \) and \( \delta \) is a vector of hyperparameters of the distribution. Therefore, SGLMM can be represented by a hierarchical structure of the form:

\[
Y_i \sim \pi(y_i|\mu_i, \delta, X_i, \beta) \\
g(\mu_i) = X_i \beta + \theta_i,
\]

where \( \beta \) are the regression coefficients, \( X \) is a design matrix, \( g \) is an appropriate link function and \( \theta \) represents a spatially structured effect that will capture the spatial patterns shared by the areas in study.

Without loss of generality, let us simplify the problem for the moment. The results that follows can be carried to any model under the SGLMM framework. Suppose that

\[
Y_i|\mu_i, \tau_e \sim N(\mu_i, \tau_e I) \\
\mu_i = X_i \beta + \theta_i.
\]  (1)

Traditionally the spatial random effect \( \theta^\top = (\theta_1, \ldots, \theta_n) \) is defined as an intrinsic conditional autoregressive (ICAR) models introduced by [Besag and Kooperberg (1995)]. The prior specification of the ICAR model is given by

\[
\pi(\theta|\tau_\theta) \propto \tau_\theta^{r(Q)/2} \exp\left(\frac{-\tau_\theta}{2} \theta^\top Q \theta\right)
\]

where \( Q \) is the precision matrix, \( r(Q) \) is the rank of the \( Q \) matrix and \( \tau_\theta \) is the spatial precision. [Rue and Held (2005)] showed that the non-zero pattern of the \( Q \) defines the dependence structure of the graph under studies. In other words, with the underlying undirected graph in analysis it is possible to define the non-zero pattern of \( Q \) and vice-versa. Moreover, [Rue and Held (2005)] argues that the precision matrix \( Q \) is commonly sparse and because of that is possible to take advantage of sparse computational methods to improve speed drastically in model fitting.
2.2 Non-Confounding SGLMM

Clayton et al. (1993) introduced the concept of spatial confounding between the fixed effects estimates and the spatial random effects in SGLMM. However, only recently, Reich et al. (2006) revisited the problem and proposed an alternative method to alleviate the confounding. In general, the idea proposed is to include in the model a random effect that belong to the orthogonal space of the fixed effects predictors. Reich et al. (2006) noted that Eq (1) can be re-expressed as

\[ Y_i | \mu_i, \tau_i \sim N(\mu_i, \tau_i I) \]
\[ \mu_i = X_i \beta + K_i \theta_1 + L_i \theta_2, \]

where \( K \) have the same span as \( X \) and \( L \) lies in the orthogonal space of \( X \). Using this parametrization it was shown that \( K \) causes a confounding in the estimates of \( \beta \). Finally, it was suggested to remove the \( K \) component letting to the RHZ model:

\[ Y_i | \mu_i, \tau_i \sim N(\mu_i, \tau_i I) \]
\[ \mu_i = X_i \beta + L_i \theta_2 \]
\[ \theta_2 \sim N(0, LQL^\top). \] (2)

Although it resolve the confounding between the spatial random effects and the estimates of the fixed effects, Hughes and Haran (2013) noticed that this correction is computationally inefficient because the new precision matrix generated by Eq (2) is not sparse and dimension of order \( n - p \) where \( p \) is the number of explanatory variables. To reduce the computational demand of RHZ model they proposed an alternative model, named as a sparse reparametrization of the RHZ model. The new model uses the Moran operator \( P^\perp AP^\perp \) since it appear in the generalized Moran index

\[ I_X(A) = \frac{n}{1^\top A 1} \frac{Y^\top P^\perp A P^\perp Y}{Y^\top P^\perp Y} \]

where \( A \) is the graph zero/one adjacency matrix and \( P^\perp = I - X(X^\top X)^{-1}X^\top \) is the projection matrix into the orthogonal space of the span of \( X \). They showed that the Moran operator retain the spatial patterns of the data and it is only necessary to select the \( q \) higher eigenvalues of the spectrum of the Moran operator. This way, they were able to reduce the dimension of...
the random effect retaining the information necessary in model estimation. Finally, the JM model is defined as:

\[ Y_i | \mu_i, \tau_\epsilon \sim N(\mu_i, \tau_\epsilon I) \]
\[ \mu_i = X_i \beta + M_i \theta_2 \]
\[ \theta_2 \sim N(0, MQM^T), \]

where \( M \) contains the first \( q \ll n \) eigenvectors of the Moran operator.

3 Fast Non-Confounding SGLMM

Although Reich et al. (2006) and Hughes and Haran (2013) were successful in alleviating the confounding and reducing dimension of the spatial random effects as presented in Section 2.2, their approach have two main drawbacks: 1) the RHZ and JM models do not accept parameters in the precision matrix \( Q \) (e.g., Besag, 1974; Leroux et al., 1999; Rodrigues and Assunção, 2012); 2) none of them took advantage of the Markov property, and therefore, sparsity in the original SGLMM model (Section 2.1). In order to propose a fast SGLMM that alleviate the confounding problem, we present a novel approach to the problem capable of maintaining the Markov properties of the precision matrix as well as without any restriction in the number of parameters in matrix \( Q \).

Instead of reparametrizing the random effects separating it into two different spaces, we define a projected image of the original graph into the orthogonal space of the covariate matrix \( X \). This projected image of the original graph (hereafter projected graph), allows us to keep the sparsity of the precision matrix \( Q \) and the Markov properties of random effects, making estimation of the fast non-confounding GMRF (NCGMRF) much more efficient (Rue and Held, 2005). Moreover, after defining the non-zero pattern of \( Q \) by the project graph it allows to adjust a variety of the spatial structure available (Besag, 1974; Leroux et al., 1999; Rodrigues and Assunção, 2012, and others).

To create the projected graph, first, we represent the spatial arrangement of the areas by their centroids. The two coordinate vectors are, then, projected in the orthogonal space to matrix \( X \) multiplying by \( P^\perp \). The new set of coordinates are assumed to be the centroids of the areas under the orthogonal space of \( X \).
After projecting the centroids, we need to define a new neighborhood matrix. To reconstruct the neighborhood pattern we consider two alternatives: 1) fix the number of neighbors to be the same as in the original graph (Knn); 2) use the delaunay triangulation to automatically define the number of neighbor. In the first method, the number of neighbors of each area in this new structure to be the same as the number of neighbors $k_i$ in the original map according to the adjacency criterion. To do so, we compute the distance matrix between the projected vectors and define the set of neighbors of area $i$ to be the $k_i$ closest areas in the distance matrix. For example, if an area $i$ had $k_i$ adjacent areas in the original map, its neighbors in the projected map will be the $k_i$ projected centroids that are the closest ones based on the created distance measure. In the second method, a delaunay triangulation is performed over the centroids and the vertices connected by the triangles are considered neighbors. Although the second method have no pre-specified restriction related to the number of neighbors, its performance is not satisfactory to reconstruct the original map when the map is oddly shaped. To fix this problem we tried to use a constrained triangulation, this provides much better results to rebuild the original graph. However, there is no reason to believe that the projected graph have the same shape as the original in the space generated by $P^\perp$, being a stronger restriction than fixing the number of neighbors. Therefore, all presented analysis in this manuscript is performed using the Knn method.

The Knn method assumes that close centroids tend to share border. Although, this might not be true in all cases it is a reasonable assumption that we will use to reconstruct our dependence matrix $Q$. The new matrix generated by the Knn method will now replace the original adjacency matrix that is used in ICAR model. The model will be fitted using the INLA (Rue et al. 2009) algorithm in order to take advantages of the Markov properties of the new matrix $Q$ generated by the projected graph and to avoid traditional MCMC convergence problems. Although, we use INLA to perform our analysis this is not a restriction and other software as WinBUGS, spBayes (Finley et al. 2007), OpenBUGS (Lunn et al. 2009), CarBayes (Lee 2013) can be used to adjust our methodology with the ICAR or any other prior model for the spatial random effects available.

To better visualize the impact of different spatial trends in the projected graph we present a lattice example. Figure 1(a) presents a regular lattice with $15 \times 15$ centroids. Now, different spatial trends are created for one explanatory variable $X$. After, the original centroids are projected into the
space generated by $P^\perp$, the neighborhood structure is reconstructed. Figure 1(b) represent the projected graph when $X$ is the horizontal coordinate of the centroid. When $X$ has a paraboloidal spatial trend, with high values in the upper left and lower right extremes, the new pattern can be seen in Figure 1(c). Finally, Figure 1(d) present a circular pattern for $X$. From Figure 1 it is clear that different spatial trends generates different effects over the project graph. This is expected since the project graph should be “orthogonal” trend generated by the spam of $X$.

To better understand the effects of the new adjacency structure, four centroids are selected in the original lattice (Figure 2(a)). The new adjacency is presented connecting the selected centroid with its new neighbors using distinct colors. In Figure 2(b) we can see that the new neighbors of the of the centroids differs drastically from the original ones (presented as a filled dot) having a diagonal structure. In the paraboloidal structure, Figure 2(c) the red and yellow centroids suffer very little influence since they are not in the top left or bottom right where the spatial trend is large. However, the green and blue centroids are close to these regions and have a more drastic change in its neighboring structure. Finally, Figure 2(d) represents the new dependence structure for the circular pattern which is different from the previously observed. In this case we can see that the points that are close to the center of the circumference are the ones that suffer more impact.

We introduce the Slovenia map (further used in Section 5) as our baseline map for analysis and simulation. A similar example performed in the lattice is performed in the real map of Slovenia to see the impacts of the projected graph in a real map. In this example we use a explanatory variable without and with spatial effect to see how is the project graph. In Figure 3(b) we define the explanatory variable $X$ generated from a uniform random variable. While in Figure 3(c) the explanatory variable $X$ is the the horizontal coordinates of the centroids. Therefore, Figure 3(b) is very similar to the original neighbor structure since its explanatory variables have no spatial trend. An opposite results is visualized in Figure 3(c) where $X$ contains a strong spatial trend. It is clear that when the covariates have no spatial tendency the reconstructed graph is almost the same as the original, while it changes drastically when the explanatory variable have a spatial trend, as expected.

To better evaluate Figure 3(b) and understand the capability of the model to reconstruct the original graph we use the following metrics: 1) the overall agreement between each position of the matrices (original and projected);
Figure 1: (a) Original lattice. (b) Reconstructed adjacency matrix when $X$ is the horizontal coordinate. (c) Reconstructed adjacency matrix when $X$ has a paraboloid spatial trend. (d) Reconstructed adjacency matrix when $X$ has a circular trend.

2) since in our framework the matrices are sparse, it is very common for them to agree in positions with 0’s, therefore we check the agreement of only the 1’s pattern. Table 1 show the agreement percentage for each metrics. Clearly, the precision matrix is totally changed when $X$ contains spatial
Figure 2: (a) Original lattice. (b) Reconstructed adjacency matrix when $X$ is the horizontal coordinate. (c) Reconstructed adjacency matrix when $X$ has a paraboloid spatial trend. (d) Reconstructed adjacency matrix when $X$ has a circular trend.

However, when $X$ possesses no spatial trend the neighbor structure is fairly recovered. As will be presented in our simulation studies and real data analysis, Sections 4 and 5 respectively, the impact of the small differences presented in the precision matrix between the original graph and
the projected graph when there is no spatial trend have minimum effect for inference, while it changes drastically when $X$ have spatial trend.

Table 1: Percentage agreement in the zero-one pattern between the original graph precision matrix and the projected precision matrices without and with spatial trends.

|                      | Without Spatial Trend (%) | With Spatial Trend (%) |
|----------------------|---------------------------|------------------------|
| Overall Agreement    | 98.4                      | 94.8                   |
| Ones Agreement       | 82.0                      | 18.8                   |

### 4 Simulation

In this Section a simulation study is performed with two main goals: 1) compare the results of the proposed methodology in Section 3 (NCGMRF) with the existing alternatives RHZ and JM models, 2) understand and discuss what are the models assumptions where is necessary to correct for spatial confounding and what will you obtain from it. The following model is selected to generating the data:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \theta_i + e_i \quad i = 1, \ldots, n$$
$$e \sim N(0, \tau_e I)$$
where $\theta$ is the spatial effect, $n$ is the number of regions in the study and the selected coefficients are $\beta^T = (2, 1, -1)$.

To compare the models and better understand the implications of spatial confounding we propose 3 simulation scenarios:

1. (ICAR spatial) The spatial effect is generated using the ICAR structure, that is, $\theta \sim N(0, \tau_\theta Q)$ with $X_1$ generated from a standard normal and $X_2$ is the first coordinates of centroids the areas;

2. (RHZ) The spatial random effect is generated by the orthogonal RHZ proposal, basically, $\theta \sim N(0, \tau_\theta P^\perp Q (P^\perp)^\top)$. The explanatory variables are the same as the one in 1;

3. (ICAR non-spatial) Its is very similar to scenario 1. However, the explanatory variables, $X_1$ and $X_2$, were generated from a standard normal without any spatial information.

For each scenario 1000 datasets were generated with the following combination of the precision of the random effects $\tau_\theta$ and error term $\tau_e$:

1. $\tau_e = 0.2$ and $\tau_\theta = 1$;
2. $\tau_e = 1$ and $\tau_\theta = 1$;
3. $\tau_e = 1$ and $\tau_\theta = 0.2$,

to study its effects in modeling estimation and execution time. Finally, we have a total of 9 scenarios each of them with 1000 datasets. For each scenario, the posterior estimates of the following models were recorded: 1) NCGMRF, 2) linear model (LM), 3) ICAR, 4) RHZ, and 5) JM as well as their execution time. The R software (R Development Core Team, 2011) were used to fit all proposed models. For the RHZ model we used the R script freely available in [http://www4.stat.ncsu.edu/~reich/Code/](http://www4.stat.ncsu.edu/~reich/Code/); JM model was fitted using the R package ngspatial (Hughes and Cui, 2013, ver 1.02); and to fit the LM, ICAR as well as the NCGMRF the R-INLA package (Rue et al, 2009) was used.

The posterior estimates of the fixed effects are presented in Table 2. From Table 2 we can see that for all scenarios the NCGMRF provide very similar inference to the LM, RHZ and JM models. The ICAR model presents a slight different behavior. When the true generating model is the ICAR (scenario
1) the ICAR outperforms the other models as the random effects precision \((\tau_\theta)\) reduces. However, in scenario 2 (RHZ true generating model) the ICAR model clearly suffers from over inflation, specially with \(\tau_\theta = 0.2\). Finally, in scenario 3 with no spatial information in the explanatory variables all models seems to present similar behavior independent of variances differences.

Therefore, from Table 2 we can get two main conclusion. First, there is no major differences in the estimates provided by the NCGMRF and RHZ nor JM models, making it a competitive model. The fitting distance between the ICAR model and the other models is highlighted when the ratio between the spatial effect precision and the error precision is small \((\tau_\theta/\tau_e = 0.2)\). Moreover, we can see that the ICAR model behaves better in scenario 1 while the other models outperforms the ICAR in scenario 2.

Figure 4 present the results in Table 2 for the 3 scenarios with \(\tau_\theta = 0.2\) and \(\tau_e = 1\) fixed. The results presented in Figure 4 corroborates with the analysis in Table 2. Clearly, we can observe that the ICAR model presents a different behavior in \(\beta_1\) and \(\beta_2\) when compared to the other models. Moreover, it has better performance when is the true generating model and worse performance when data is generated from RHZ model. The other 4 models present very similar results under all scenarios.

Hodges and Reich (2011) discuss the interpretation of confounding in spatial regression and also link with multicolinearity problem in linear regression. By construction, the solution proposed by Reich et al. (2006) and more recently by Hughes and Haran (2013) attributes to \(X\) all the spatial variation that \(\theta\) and \(X\) were competing. From our simulation results, specifically in scenario 1, where \(X\) has a spatial trend and there is an extra source of spatial variability in the random effects, the results presented by the ICAR model seems to be better than the one presented by the models that alleviate confounding. In linear models with multicolinearity problem, say between two possible covariates \(X_1\) and \(X_2\) a possible solution for the multicolinearity is to regress \(X_1\) in \(X_2\) and use the residual \(X_1^*\) of this regression as a covariate instead of \(X_1\). Although this fixes the multicolinearity problem, the new covariate \(X_1^*\) is hard to interpret and this trick can be in practice hard to justify. Apparently, in the spatial setup, assigning all the spatial variation to \(X\) may not always provide the best solution as it can be seen in scenario 1 of our simulation. Paciorek (2010) argues that although the proposed methodology of making the spatial random effects orthogonal to the space of the spam of \(X\) alleviate the confounding problem, this is a strong assumption since it gives all source of spatial variation in the spam of
Table 2: Median summary of the posterior mean estimate from 1000 replicates in the simulation setting. The brackets contains the 2.5% and 97.5% posterior percentiles of fixed effects estimates in 1000 replicates.

| Model  | Scenario | ICAR Spatial | RZH | ICAR Non-spatial |
|--------|----------|--------------|-----|------------------|
|        |          | $\tau_e$, $\tau_\theta$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_0$ | $\beta_1$ | $\beta_2$ |
|        |          | 0.2 | 1.00 (0.66, 1.32) | -0.99 (-1.80, -0.12) | 2.00 (1.69, 2.31) | 1.00 (0.67, 1.31) | -1.00 (-1.30, -0.67) | 2.00 (1.69, 2.31) | 1.00 (0.66, 1.32) | -1.00 (-1.29, -0.62) |
|        |          | 1.1 | 1.00 (0.84, 1.16) | -1.01 (-1.89, -0.11) | 2.00 (1.69, 2.31) | 1.00 (0.67, 1.31) | -1.00 (-1.30, -0.67) | 2.00 (1.69, 2.31) | 1.00 (0.66, 1.32) | -1.00 (-1.29, -0.68) |
|        |          | 1.02 | 1.00 (0.84, 1.16) | -1.01 (-1.89, -0.11) | 2.00 (1.69, 2.31) | 1.00 (0.67, 1.31) | -1.00 (-1.30, -0.67) | 2.00 (1.69, 2.31) | 1.00 (0.66, 1.32) | -1.00 (-1.29, -0.68) |

ICAR: Intrinsic Conditional Autoregressive; RZH: Random Zero-Homoscedastic.
Figure 4: (a) Posterior estimates for $\beta_0$ under the different models. (b) Posterior estimates for $\beta_1$ under the different models. (c) Posterior estimates for $\beta_2$ under the different models.
\( \mathbf{X} \) to the exploratory variable which might not be always the case. Therefore, a better understanding of the effects of alleviating confounding must be investigated to propose guidelines to researchers in when it is more appropriate to use this type of model.

Table 3: Median execution time (in seconds) from 1000 replicates in scenario 2 (RHZ) setting and the execution standard deviation (sd) of the evaluation time of each method.

| \( \tau_e, \tau_\theta \) | Model | Time (sec) | sd |
|--------------------------|-------|------------|----|
| NCGMRF                   | 1.850 | 0.10       |
| RHZ                      | 157.006 | 0.92      |
| 0.2, 1                   | JM    | 19.157     | 3.38|
|                          | ICAR  | 0.597      | 0.09|
|                          | LM    | 0.257      | 0.02|
| NCGMRF                   | 1.872 | 0.09       |
| RHZ                      | 157.152 | 0.62      |
| 1, 1                     | JM    | 18.116     | 3.66|
|                          | ICAR  | 0.573      | 0.04|
|                          | LM    | 0.261      | 0.02|
| NCGMRF                   | 1.879 | 0.10       |
| RHZ                      | 158.254 | 1.43      |
| 1, 0.2                   | JM    | 20.898     | 3.67|
|                          | ICAR  | 0.582      | 0.04|
|                          | LM    | 0.259      | 0.02|

After demonstrating that the NCGMRF model is capable of removing the spatial confounding and provides very similar results to RHZ and JM models, we can now evaluate the average time to run each method. Table 3 presents the median execution time of 1000 replicates of each model for scenario 2. From this Table we have that the LM method is significantly faster than the others. However, the LM method does not take into account the spatial variation that improves modeling. From the spatial models, clearly the NCGMRF outperforms the RHZ and JM methods in time consumption and it has comparable time with the ICAR model. The RHZ have the largest median time, while the JM method seems to perform around 10 times slower than the NCGMRF.
5 Slovenia Data

The proposed model was adjusted to the same database used in Reich’s application (Reich et al., 2006). The response variable, $Y_i$, is the number of cases of stomach cancer observed in the municipalities of Slovenia during the period of 1995-2001. A single explanatory variable is included in the model: standardized socioeconomic level of each area $i = 1, \ldots, 192$. Therefore, we have the following model:

$$Y_i | \lambda_i, \sim \text{Poisson}(\lambda)$$

$$\log(\lambda_i) = X_i \beta + \theta_i$$

with $\beta$ the fixed effect and $\theta$ the spatial effect.

Using a simple exploratory analysis, the authors noticed that the data show a negative relationship between the response and explanatory variable. Their first attempt was to fit the data with the traditional SGLMM to capture the spatial heterogeneity. However, from Figure 5(a) it is clear that the explanatory variable have a diagonal spatial trend presenting higher values in the southwest and smaller in the northeast. Thus, when the SGLMM model was fitted, the coefficient associated with socioeconomic level had a very wide confidence interval that covered even positive values, an indication of spatial confounding between the spatial effects and the socioeconomic level.

To better understand the confounding effect between the exploratory variable and the random effects, we look over the spatial residuals of the ICAR model (Figure 5(b)). From Figure 5(b) it is clear that both spatial effect and explanatory variable share a southwest to northeast trend and therefore are competing for the spatial variability contained in the response $Y_i$. Figure 5(c) show that after alleviating the confounding there is still some spatial trend left in the same direction that was not captured by the explanatory variable and it is visualized in the spatial effect. However, the spatial dependence now is weaker and the spatial effects are smoothed toward zero. This is expected since after alleviating the spatial confounding we assume that the exploratory variable carries most of the spatial information in $Y_i$. Although weaker, it is important to notice that the spatial random effect structure from the NCGMRF model is still coherent with the space under analysis.

Table 4 shows the posterior mean estimate and the credible intervals obtained applying all discussed models. The point estimate and credible interval obtained by NCGMRF is similar to the RHZ and JM models. How-
Figure 5: (a) Standardized socioeconomic level for the municipalities of Slovenia. (b) Posterior mean estimates for spatial random effects of the ICAR model. (c) Posterior mean estimates for spatial random effects of the NCGMRF model.

ever, as it can be seen in the last column of Table 4, time wise the NCGMRF drastically outperforms the non-confounding methods.

6 Conclusion

In this paper we successfully introduced an alternative way to alleviate spatial confounding. The main idea is to construct a graph capable of capturing the spatial dependence orthogonal to the space generated by the span of $X$. By doing so, the introduced method maintains the original sparsity of the precision matrix $Q$ and introduces no restriction in the spatial modeling.
Table 4: Posterior mean estimate and credible intervals of the coefficient associated with the variable socioeconomic (Se) level for the five fitted models.

| Model       | $\beta_{Se}$ | 95% Credible Interval       | Time (sec) |
|-------------|--------------|----------------------------|------------|
| NCGMRF      | $-0.1004$    | $(-0.1635, -0.0369)$        | 2.21       |
| LM          | $-0.0682$    | $(-0.1067, -0.0298)$        | 0.16       |
| ICAR        | $-0.0474$    | $(-0.1404, 0.0473)$         | 0.73       |
| RHZ         | $-0.1137$    | $(-0.1688, -0.0596)$        | 251.83     |
| JM          | $-0.1128$    | $(-0.1656, -0.0450)$        | 29.39      |

From our simulation study and real data example we were able to show that our approach provides similar results to the ones that alleviate confounding using the projection of the spatial random effects over the orthogonal space spanned by $X$ instead of directly projecting its original graph. Moreover, our method, in all simulated scenarios was at least 10 times faster than the existing methodologies. This advantage can make it more accessible to researchers in different areas.

Another interesting result obtained from the simulation study, is that, when the spatial covariate is not the only true generating source of spatial variability in its space, the traditional ICAR model had a better fitting performance than the methods that put all spatial variability into the spatial covariate. This observation lights up the necessity of further studies to better understand what characteristics of the data generating system are important to consider to justify the use of the traditional SGLMM or non-confounding methods. As explicitated by [Paciorek (2010)] the data generating system may have non-observed spatial confounder with the observed explanatory variables and setting all spatial variation to the observed covariates may not be appropriate.

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References

Besag, J. (1974), “Spatial Interaction and the Statistical Analysis of Lattice Data Systems (with discussion),” *Journal of the Royal Statistical Society, Series B*, 36, 192–225.

Besag, J. and Kooperberg, C. (1995), “On Conditional and Intrinsic Autoregressions,” *Biometrika*, 82, 733–746.

Clayton, D., Bernardinelli, L., and Montomoli, C. (1993), “Spatial correlation in ecological analysis,” *International Journal of Epidemiology*, 6, 1193–1202.

Finley, A. O., Banerjee, S., and Carlin, B. P. (2007), “spBayes: An R Package for Univariate and Multivariate Hierarchical Point-referenced Spatial Models,” *Journal of Statistical Software*, 19, 1–24.

Hodges, J. S. and Reich, B. J. (2011), “Adding Spatially-Correlated Errors Can Mess Up the Fixed Effect You Love,” *The American Statistician*, 64, 325–334.

Hughes, J. and Cui, X. (2013), *ngspatial: Classes for Spatial Data*, Minneapolis, MN, r package version 1.0-2.

Hughes, J. and Haran, M. (2013), “Dimension reduction and alleviation of confounding for spatial generalized linear mixed models,” *Journal of the Royal Statistical Society, Series B*, 75, 139–159.

Lee, D. (2013), “CARBayes: An R Package for Bayesian Spatial Modeling with Conditional Autoregressive Priors,” *Journal of Statistical Software*, 55, 1–24.

Leroux, B. G., Lei, X., and Breslow, N. (1999), “Estimation of Disease Rates in Small Areas: A New Mixed Model for Spatial Dependence,” in *In Statistical Models in Epidemiology; the Environment and Clinical Trials*, eds. Halloran, M. E. and Berry, D., New York: Springer–Verlag, pp. 179–192.

Lunn, D., Spiegelhalter, D., Thomas, A., and Best., N. (2009), “The BUGS project: Evolution, critique and future directions (with discussion),” *Statistics in Medicine*, 28, 3049–3082.
Lunn, D. J., Thomas, A., Best, N., and Spiegelhalter, D. (2000), “WinBUGS – a Bayesian modelling framework: Concepts, structure, and extensibility,” Statistics and Computing, 10, 325–337.

Paciorek, C. J. (2010), “The Importance of Scale for Spatial-Confounding Bias and Precision of Spatial Regression Estimators,” Statistical Science, 25, 107–125.

R Development Core Team (2011), R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0.

Reich, B. J., Hodges, J. S., and Zadnik, V. (2006), “Effects of Residual Smoothing on the Posterior of the Fixed Effects in Disease-Mapping Models,” Biometrics, 62, 1197–1206.

Rodrigues, E. C. and Assunção, R. (2012), “Bayesian spatial models with a mixture neighborhood structure,” Journal of Multivariate Analysis, 109, 88 – 102.

Rue, H. and Held, L. (2005), Gaussian Markov random fields: Theory and applications, Chapman & Hall.

Rue, H., Martino, S., and Chopin, N. (2009), “Approximate Bayesian Inference for Latent Gaussian Models Using Integrated Nested Laplace Approximations (with discussion),” Journal of the Royal Statistical Society, Series B, 71, 319–392.

Zadnik, V. and Reich, B. J. (2006), “Analysis of the relationship between socioeconomic factors and stomach cancer incidence in Slovenia,” Neoplasma, 53, 103–110.