Evolution of Cool Close Binaries - Approach to Contact

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ABSTRACT

As a part of a larger project, a set of 27 evolutionary models of cool close binaries was computed under the assumption that their evolution is influenced by the magnetized winds blowing from both components. Short period binaries with the initial periods of 1.5, 2.0 and 2.5 d were considered. For each period three values of 1.3, 1.1 and 0.9 $M_\odot$ were taken as the initial masses of the more massive components. The initial masses of the less massive components were adjusted to avoid extreme mass ratios.

Here the results of the computations of the first evolutionary phase are presented, which starts from the initial conditions and ends when the more massive component reaches its critical Roche lobe. In all considered cases this phase lasts for several Gyr. For binaries with the higher total mass and/or longer initial periods this time is equal to, or longer than the main sequence life time of the more massive component. For the remaining binaries it amounts to a substantial fraction of this life time.

From the statistical analysis of models, the predicted period distribution of detached binaries with periods shorter than 2 d was obtained and compared to the observed distribution from the ASAS data. An excellent agreement was obtained under the assumption that the period distribution in this range is determined solely by magnetic braking (MB) i.e., the mass and angular momentum loss due to the magnetized winds, as considered in the present paper. This result indicates, in particular, that virtually all cool detached binaries with periods of a few tenths of a day, believed to be the immediate progenitors of W UMa-type stars, were formed from young detached systems with periods around 2-3 d. MB is the dominant formation mechanism of cool contact binaries. It operates on the time scale of several Gyr rendering them rather old, with age of 6-10 Gyr.

The results of the present analysis will be used as input data to investigate the subsequent evolution of the binaries, through the mass exchange phase and contact or semi-detached configuration till the ultimate merging of the components.

Stars: activity – binaries: close – Stars: evolution – Stars: late-type – Stars: rotation

1 Introduction

W UMa-type binaries are contact binaries consisting of two cool stars surrounded by a common envelope lying between the inner and outer Lagrangian zero velocity equipotential surfaces, called also the Roche lobes (Mochnacki 1981). In spite of different component masses they possess nearly identical mean surface brightnesses. The more massive, primary component is a main sequence (MS) star whereas the secondary is oversized compared to its expected MS radius.

Kuiper (1941) noted that contact binaries with unequal zero-age components cannot exist in equilibrium because radii of the components must fulfill two, mutually contradictory conditions: one resulting from the mass-radius relation for zero-age stars and the other relating sizes of the Roche lobes, identical in this case with stellar sizes, to stellar masses. The fact that contact binaries are nevertheless observed is known as “Kuiper paradox”. Lucy (1968) noted that the Kuiper paradox can be solved by assuming that both zero-age components (each in the thermal equilibrium) have a common convective envelope with the same adiabatic constant. However, detailed calculations showed that realistic models can exist only within a narrow range of component masses, contrary to what was observed. Later Lucy (1976) considered a binary configuration which is not in equilibrium. A similar model was developed by Flannery (1976). The
model is known as Thermal Relaxation Oscillation (TRO) model. According to it each component of the binary is out of thermal equilibrium and its size oscillates around the inner Roche lobe but the whole binary is in the global thermal equilibrium. The energy is transported from primary to secondary via a turbulent convection which results in equal entropies of both convective envelopes, hence equal surface brightnesses.

TRO model explains in a very elegant way two basic observational facts about W UMa-type stars: the Kuiper paradox, i.e. the geometry of the binary in which primary is an ordinary MS star and secondary is also a MS star but swollen to the size of its Roche lobe by the energy transfer, and equal apparent effective temperatures of both components resulting in a characteristic light curve with two equal minima. It encounters, however, several difficulties. In particular, some of its basic assumptions seem to be incorrect and some of its predictions are at odds with the observations.

TRO model predicts that a binary oscillates between two states: contact – when both stars fill their respective Roche lobes and there is a net mass flow from the secondary to the primary, and semidetached – when the primary still fills its Roche lobe but the secondary is within its Roche lobe and the mass flows from the primary to the secondary (Lucy 1976). Robertson and Eggleton (1977) argued that the time scales of both states should be close to the thermal time scales of both components. For most W UMa-type stars the mass ratio is rather moderate, about 0.5, so the time scales should be comparable to one another. This is at odds with observations showing that contact (or near-contact) binaries within the same parameter range as genuine contact binaries but with distinctly different depths of minima are quite rare (Rucinski 1998).

Additional effects, like stellar evolution and/or angular momentum loss (AML) can influence the ratio of both time scales reducing the duration of the semidetached state (Robertson and Eggleton 1977, Rahnen 1983, Yakut and Eggleton 2005, Li, Han and Zhang 2005). In fact, a binary can remain in the contact state all the time if AML rate is high enough but then its lifetime as a contact binary must be as short as $\sim 10^8$ years. Such binaries would be rare in space, contrary to observations (Webbink 2003).

A recent photometric sky survey ASAS (Pojmanski 2002) detected several thousand eclipsing binaries with periods shorter than 1 day (Paczyński et al. 2006). Their analysis showed a significant proportion of semi-detached systems. This would seemingly solve the problem and support TRO model. However, a closer look at the sample shows that among stars with periods shorter than 0.45 day there is very few semi-detached binaries (Pilecki, 2010) whereas a great majority of contact stars has periods within this range (Rucinski 2007). On the other hand, semi-detached binaries are commonly observed among binaries with periods between 0.7 and 1 day where fewer contact systems occur. In addition, one should keep in mind that, owing to the larger expected light variation amplitude of semi-detached binaries, they are preferentially detected over small mass ratio contact binaries with shallow minima (Rucinski 2006). So, the problem still remains.

Section 2 contains a short description of the essentials of a new model of the origin of cool contact binaries, suggested by the present author. In Section 3 the assumptions, equations and other details of the model are given and Section 4 presents the initial parameters of the considered binaries together with the results of the model calculations. In Section 5 a comparison of the model data with observations is given and the most important uncertainties of the model are discussed. Section 6 contains the main conclusions of the paper and plans for continuation of this investigation.
2 Essentials of a New Model

The original TRO model has been developed for zero-age stars. However, the numerical simulations of the binary formation favored an early fragmentation of a protostellar cloud (Boss 1993, Bonnel 2001, Machida et al. 2008) rather than fission. The observations support this conclusion. We do not observe binaries with orbital periods shorter than one day in young stellar clusters or among T Tauri-type stars but they are very numerous in old open clusters with an age exceeding 4.5 Gyr and in globular clusters (Mathieu 1994, Kaluzny and Rucinski 1993, Rucinski 1998, 2000). Large numbers of W UMa-type stars have been observed in the galactic bulge (Szymański, Kubiak and Udalski 2001). The analysis of kinematics of these stars in the solar vicinity showed that they are several Gyr old (Guinan and Bradstreet 1988, Bilir et al. 2005). The time interval of 5 to 13 Gyr is long enough for a primary with mass between 1.3 and 0.9 $M_\odot$ to complete its MS evolution. We should thus ubiquitously observe W UMa-type stars with components possessing hydrogen-depleted cores. Such a star expands rapidly when moving towards the red giant branch and its evolution must very strongly influence the fate of the whole binary.

Based on these facts a new model of a cool contact binary has been developed by the present author (Stepień 2004, 2006a, 2006b, 2009). The model assumes that contact binaries originate from young cool close binaries with initial orbital periods close to a couple of days. Both components possess subphotospheric convective zones necessary for generating surface magnetic fields. Such stars show magnetic activity with intensity increasing with the increase of rotational angular velocity. Assuming the synchronization of rotation with orbital period we expect the activity level of each component to be at the very high, so called “saturation” level. The magnetic activity drives stellar winds carrying away the spin angular momentum (AM). Because of synchronization, this loss results in a decrease of the orbital AM. The orbit tightens and the components approach each other until the more massive component fills the critical Roche surface.

Because of the ambiguity of the term “primary” which is sometimes used to describe the more massive component and sometimes the component with the higher surface brightness we will identify the components as star “1” and star “2” according to their initial mass: $M_1$ denotes initially the more massive component and $M_2$ initially the less massive component, so that initial mass ratio $q_{\text{init}} = M_1/M_2 \geq 1$ whereas after mass ratio reversal $q \leq 1$. For $M_{1,\text{init}} \approx 1.3M_\odot$ and the initial period of 2 d the time scale for AML is close to the lifetime of the star “1” on MS so the Roche lobe overflows (RLOF) begins when the star is close to, or beyond the terminal age main sequence (TAMS) on the Hertzsprung-Russell (HR) diagram and its radius increases quickly on the way to red giant region. RLOF by star “1” results in the mass transfers to the component “2”. The model assumes that the mass transfer proceeds in the same way as in Algols, i.e., after a possible phase of a rapid mass exchange resulting in a loss of thermal equilibrium of both components and a transient formation of the common envelope, both stars reach the thermal equilibrium within a configuration with the mass ratio reversed. In binaries with the mass ratio not far from unity star “2” approaches zero age main sequence (ZAMS) because it has been fed with hydrogen rich matter and in binaries with the low mass ratio it simply stays close to ZAMS. Star “1” is evolutionary over sized because its core is hydrogen depleted. After the phase of the rapid mass exchange is over, star “1” continues expansion due to increase of its helium core which results in a further mass transfer although at a much reduced rate. The mass transfer rate in this evolutionary phase depends on two effects acting in opposite directions: mass transfer results in the orbit expansion whereas AML tightens it. Interplay between these two processes determines the rate of evolution of a contact binary to-
wards the extreme mass-ratio configuration which should ultimately end up in merging of both components. In some binaries star “2” may become massive enough that its evolutionary expansion may become important. In particular, if it reaches TAMS, the common envelope will overflow the outer critical Roche surface and a violent coalescence of both components is expected.

Models published hitherto by the present author were calculated for a few selected component masses and the initial orbital period of 2 days. Now the results of calculations of 27 models with different component masses, covering the broad range of mass ratios and with three different initial periods: 1.5, 2.0 and 2.5 d will be discussed. Analysis of their evolution provides data which can be compared with observations of close binaries – both with individual systems and with statistical data. The present paper discusses in detail the first evolutionary phase of the modeled binaries: the approach to contact.

3 Assumptions, equations and details of the model

According to our assumptions cool contact binaries are formed from close detached binaries which lose AM via the magnetized wind. AML loss is sufficiently efficient to form a contact binary within several Gyr if both components possess magnetized winds. Therefore we consider only binaries with initial component masses less than the limit for the occurrence of the subphotospheric convective zone. Let us fix this mass at 1.3 $M_\odot$ so that the total initial mass of a binary is less than 2.6 $M_\odot$. We apply the Roche model for the description of the orbital parameters of the binary. The first model equation is the third Kepler law

$$P = 0.1159a^{3/2}M^{-1/2},$$

where $M = M_1 + M_2$ is the total mass, $a$ – semi axis and $P$ – orbital period. Here masses and $a$ are expressed in solar units, and period in days. We also need an expression for the total angular momentum

$$H_{\text{tot}} = H_{\text{spin}} + H_{\text{orb}},$$

where

$$H_{\text{spin}} = 7.13 \times 10^{50} (k_1^2 M_1 R_1^2 + k_2^2 M_2 R_2^2) P^{-1},$$

$$H_{\text{orb}} = 1.24 \times 10^{52} M^{5/3} P^{1/3} q (1 + q)^{-2},$$

where $k_1^2$ and $k_2^2$ are gyration radii of both components and $H_{\text{spin}}$ and $H_{\text{orb}}$ are in cgs units. The approximations for critical Roche lobe sizes, $r_1$ and $r_2$, given by Eggleton (1983) are adopted

$$\frac{r_1}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln (1 + q^{1/3})},$$

$$\frac{r_2}{a} = \frac{0.49 q^{-2/3}}{0.6 q^{-2/3} + \ln (1 + q^{-1/3})},$$

where $q = M_1/M_2$ is the mass ratio.

For the considered binaries spin AM is nearly two orders of magnitude lower than orbital AM and can be neglected.
The empirical AML rate of single, solar-type stars was found by Stepień (1995), based on the observations of the rotation of stars slightly hotter than the Sun (i.e., stars with $<B-V>=0.60$) in stellar clusters of different age. Assuming that each component of a binary loses AM as a single star, the AML rate of a contact binary was found. The resulting formula was parameter-free, i.e., the rate depends solely on the binary parameters. Later, observational data on spin down of stars of different mass became available which permitted to generalize the expression for AML rate of a close synchronized binary with arbitrary mass and radius (Stepień 2006b, Gazeas and Stepień 2008). The formula reads

$$\frac{dH_{\text{orb}}}{dt} = -4.9 \times 10^{41} \left( R_1^2 M_1 + R_2^2 M_2 \right) / P .$$

Note that the above formula does not contain the exponential term appearing in the expression for AML of a single star because its value is close to 1 for rapid rotation (Stepień 2006b). In such a limit the AML rate depends solely on the orbital period, stellar masses and radii, as Eq. (7) shows. The uncertainty of the numerical coefficient is of the order of 50%. For a binary with two solar components the formula reduces to the same expression as derived in Stepień (1995).

The observations of X-ray flux of single rapid rotators show a decrease of the flux for rotation periods shorter than 0.4 d (Randich et al. 1996, Stepień et al. 2001). This phenomenon is called supersaturation. Assuming that supersaturation affects also the magnetized wind, it was conservatively adopted that AML levels off for rotation periods shorter than 0.4 days.

The magnetized wind carries away not only AM but also mass. Wood et al. (2002) and Wood et al. (2005) determined mass loss rates of several single, active stars of the solar type. A fit to their data gives in the saturation limit (Stepień 2011)

$$\dot{M}_{1,2} = -10^{-11} R_{1,2}^2 ,$$

where mass loss rates are in $M_{\odot}/\text{year}$ and radii in solar units. The relation applies for stars with $M_{1,2} \leq 1M_{\odot}$. For more massive stars $R_{1,2} \equiv 1$ is adopted. Note that the above formula is also parameter-free. The numerical coefficient is uncertain within the factor of two.

The above set of equations was applied to a series of models with different initial parameters listed in Table 1. The consecutive columns give initial masses of both components in solar units, initial mass ratio, semi-axis $a$ in solar radii, orbital AM in units of $10^{51}\text{gcm}^2\text{s}^{-1}$ and the fractional radii of both components relative to the sizes of their Roche lobes.

As it was demonstrated earlier, the time scale for AML of a binary with the initial period of 2 d is roughly of the same order as the lifetime of star “1” on MS (Stepień 2006a, 2006b). It was argued by Stepień (1995) that 2 d is the expected low limit to orbital period of a close binary on ZAMS if the binaries are formed in the early fragmentation process (Boss 1993) accompanied by some AML in the pre-MS phase. The resulting expected period distribution of young close binaries would be given by the Duquennoy and Mayor (1991) distribution with the cut-off at 2 d. However, as comparison to observations showed, there is an indication of an excess of short-period young binaries compared to this prediction (Stepień 1995). The excess is even higher for binaries in Hyades (Griffin 1985, Stepień 1995). No discrepancy was noticed, however, for the cut-off period in this cluster. The excess of binaries with periods of a few days can be explained if one notes that majority of them possess a distant companion. Tokovinin et al. (2006) detected a third component in 96% of field solar-type binaries with orbital periods shorter than 3 d. A third component can effectively shorten the close binary period via Kozai cycles accompanied by the tidal friction (KCTF) if its orbit inclination...
Table 1: Initial parameters of the computed models

|       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|
| $M_1$ | $M_2$ | $q$   | $a$   | $H_{\text{orb}}$ | $R_1/r_1$ | $R_2/r_2$ |
| (M$_\odot$) | (M$_\odot$) | ($M_1/M_2$) | ($R_\odot$) | ($\times 10^{51}$) | (percent) | (percent) |
|-------|-------|-------|-------|-------|-------|-------|
| $P_{\text{init}} = 2.5$ d | | | | | | |
| 1.3  | 1.1  | 1.18  | 10.37 | 17.97 | 30    | 28    |
| 1.3  | 0.9  | 1.44  | 10.08 | 15.14 | 29    | 25    |
| 1.3  | 0.7  | 1.86  | 9.76  | 12.15 | 28    | 21    |
| 1.1  | 0.9  | 1.22  | 9.76  | 13.22 | 26    | 23    |
| 1.1  | 0.7  | 1.57  | 9.43  | 10.65 | 25    | 20    |
| 1.1  | 0.5  | 2.20  | 9.06  | 7.91  | 25    | 16    |
| 0.9  | 0.7  | 1.29  | 9.06  | 9.06  | 21    | 19    |
| 0.9  | 0.5  | 1.80  | 8.67  | 6.77  | 21    | 16    |
| 0.9  | 0.3  | 3.00  | 8.23  | 4.28  | 20    | 11    |
|-------|-------|-------|-------|-------|-------|-------|
| $P_{\text{init}} = 2.0$ d | | | | | | |
| 1.3  | 1.1  | 1.18  | 8.94  | 16.68 | 35    | 32    |
| 1.3  | 0.9  | 1.44  | 8.69  | 14.05 | 34    | 28    |
| 1.3  | 0.7  | 1.86  | 8.41  | 11.28 | 33    | 24    |
| 1.1  | 0.9  | 1.22  | 8.41  | 12.27 | 30    | 27    |
| 1.1  | 0.7  | 1.57  | 8.12  | 9.89  | 29    | 23    |
| 1.1  | 0.5  | 2.20  | 7.81  | 7.35  | 29    | 18    |
| 0.9  | 0.7  | 1.29  | 7.81  | 8.41  | 25    | 23    |
| 0.9  | 0.5  | 1.80  | 7.47  | 6.28  | 24    | 18    |
| 0.9  | 0.3  | 3.00  | 7.10  | 3.97  | 23    | 13    |
|-------|-------|-------|-------|-------|-------|-------|
| $P_{\text{init}} = 1.5$ d | | | | | | |
| 1.3  | 1.1  | 1.18  | 7.38  | 15.16 | 42    | 39    |
| 1.3  | 0.9  | 1.44  | 7.17  | 12.77 | 41    | 35    |
| 1.3  | 0.7  | 1.86  | 6.95  | 10.25 | 41    | 30    |
| 1.1  | 0.9  | 1.22  | 6.95  | 11.15 | 36    | 32    |
| 1.1  | 0.7  | 1.57  | 6.71  | 8.98  | 36    | 28    |
| 1.1  | 0.5  | 2.20  | 6.45  | 6.67  | 35    | 22    |
| 0.9  | 0.7  | 1.29  | 6.45  | 7.65  | 30    | 27    |
| 0.9  | 0.5  | 1.80  | 6.17  | 5.71  | 29    | 22    |
| 0.9  | 0.3  | 3.00  | 5.86  | 3.61  | 28    | 16    |
exceeds 40°, relative to the orbit of the inner binary (Eggleton and Kisseleva-Eggleton 2006). After only about 50 Myr the inner orbit can be circularized in such a case at a period of just a few days, but not less than 2.5 d. For shorter periods the mechanism becomes ineffective (see Fig. 2 in Eggleton and Kisseleva-Eggleton 2006). Similar calculations done by Fabrycky and Tremaine (2007) showed that KCTF produces predominantly binaries with \( P \approx 2 – 3 \) days. Shorter periods can also be attained by that mechanism but they need very special conditions and a longer time. We can therefore consider the orbital evolution of a typical close binary in a simplified way as consisting of two separated phases. Binaries with longer orbital periods (from several up to a few dozen days), which possess the suitably placed tertiary component, evolve towards shorter periods under the influence of KCTF on a time scale much shorter than the evolutionary time scale. AML via magnetized wind, called also magnetic braking (MB), is negligible for such long periods. When the orbital period reaches a value of 2-3 days, KCTF mechanism becomes ineffective but MB increases substantially. The shortening of the period is therefore continued under the influence of MB until RLOF and formation of a contact binary. Such a sequence of both processes in formation of contact binaries is supported by the ubiquitous presence of third companions to many W UMa-type stars observed by Rucinski et al. (2007).

Here we are interested in the orbital evolution of close binaries under the influence of MB so we restrict attention to binaries with short initial periods. The upper limit is set at 2.5 days. Exploratory model calculations of binaries with the initial period equal to 3 d show that they reach RLOF when star “1” is already close to the red giant branch that results in formation of an Algol rather a W UMa-type star. The lower limit is set at 1.5 days, assuming that a specific combination of higher than average AML in pre-MS phase and/or special configuration of a tertiary component may result in such a short initial orbital period. Binaries with the initial period of 1.5 d will reach RLOF in a shorter time than binaries with the longer initial periods and form contact binaries with a relatively young age. We know at least one W UMa-type star with the age substantially lower than the limiting age of 4-5 Gyr found by Kaluzny and Rucinski (1993), and Rucinski (1998, 2000). It is TX Cnc – a member of an intermediate-age open cluster Praesepe with the age of only 600 Myr (Zhang, Deng and Lu 2009). A few more W UMa-type stars show kinematically young age (Bilir et al. 2005). Dynamical properties of an individual star do not describe directly its true age but some of the kinematically young stars may in fact be also evolutionary young. We may conclude that young contact stars are observed although they are quite rare. The initial masses of star “1” are taken from the range between 1.3 \( M_\odot \) and 0.9 \( M_\odot \) to cover the range from the adopted upper limit for the existence of the subphotospheric convection zone, down to the mass for which lifetime on MS is shorter than the Hubble age. The initial masses of star “2” are fitted to avoid extreme mass ratios i.e., twin binaries, or very low mass ratio binaries in which the rapid mass transfer following RLOF is nonconservative. The considered initial mass ratios are between 1.18 and 3.0. The initial orbital AM range extends from 3.6 to 17 in units of \( 10^{51} \) gcm\(^2\)s\(^{-1}\). As we see from the two last columns of Table 1, the modeled binaries are initially well detached, with the radii of star “1” between 20% and 42%, and the radii of star “2” between 11% and 39% of the size of their respective Roche lobes. All binaries are assumed to have zero eccentricity and fully synchronized rotation of both components with the orbital period.
Table 2: The final parameters of the computed models

| Model          | age at RLOF | age/age TAMS | $H_{\text{fin}}$ ($\times 10^{51}$) | $H_{\text{fin}}/H_{\text{init}}$ | $P_{\text{fin}}$ | days |
|----------------|-------------|--------------|-------------------------------------|----------------------------------|----------------|------|
| $M_1 + M_2 (P_{\text{init}})$ | Gyr         |             |                                     |                                  |                |      |
| 1.3+1.1(2.5)   | 4.2         | 1.0         | 14.6                                | 0.81                             | 1.61           |      |
| 1.3+0.9(2.5)   | 4.2         | 1.0         | 12.3                                | 0.81                             | 1.61           |      |
| 1.3+0.7(2.5)   | 4.2         | 1.0         | 9.6                                 | 0.79                             | 1.43           |      |
| 1.1+0.9(2.5)   | 6.8         | 1.0         | 10.1                                | 0.76                             | 1.53           |      |
| 1.1+0.7(2.5)   | 6.8         | 1.0         | 8.1                                 | 0.76                             | 1.43           |      |
| 1.1+0.5(2.5)   | 6.8         | 1.0         | 5.3                                 | 0.67                             | 0.94           |      |
| 0.9+0.7(2.5)   | 12.9        | 1.0         | 5.9                                 | 0.65                             | 1.11           |      |
| 0.9+0.5(2.5)   | 12.9        | 1.0         | 3.8                                 | 0.56                             | 0.67           |      |
| 0.9+0.3(2.5)   | 8.9         | 0.6         | 2.1                                 | 0.49                             | 0.35           |      |
| 1.3+1.1(2.0)   | 4.2         | 1.0         | 11.7                                | 0.70                             | 0.83           |      |
| 1.3+0.9(2.0)   | 4.2         | 1.0         | 9.9                                 | 0.68                             | 0.83           |      |
| 1.3+0.7(2.0)   | 4.2         | 1.0         | 7.3                                 | 0.65                             | 0.63           |      |
| 1.1+0.9(2.0)   | 6.8         | 1.0         | 7.3                                 | 0.59                             | 0.57           |      |
| 1.1+0.7(2.0)   | 6.6         | 1.0         | 5.7                                 | 0.58                             | 0.52           |      |
| 1.1+0.5(2.0)   | 5.4         | 0.8         | 4.3                                 | 0.59                             | 0.48           |      |
| 0.9+0.7(2.0)   | 10.8        | 0.8         | 4.5                                 | 0.54                             | 0.44           |      |
| 0.9+0.5(2.0)   | 9.1         | 0.6         | 3.5                                 | 0.56                             | 0.42           |      |
| 0.9+0.3(2.0)   | 6.2         | 0.5         | 2.0                                 | 0.50                             | 0.31           |      |
| 1.3+1.1(1.5)   | 2.7         | 0.7         | 10.8                                | 0.71                             | 0.62           |      |
| 1.3+0.9(1.5)   | 2.9         | 0.7         | 8.7                                 | 0.68                             | 0.56           |      |
| 1.3+0.7(1.5)   | 2.6         | 0.7         | 7.1                                 | 0.69                             | 0.55           |      |
| 1.1+0.9(1.5)   | 4.1         | 0.6         | 7.1                                 | 0.64                             | 0.47           |      |
| 1.1+0.7(1.5)   | 4.0         | 0.6         | 5.7                                 | 0.63                             | 0.45           |      |
| 1.1+0.5(1.5)   | 3.3         | 0.5         | 4.2                                 | 0.63                             | 0.42           |      |
| 0.9+0.7(1.5)   | 6.3         | 0.5         | 4.5                                 | 0.59                             | 0.38           |      |
| 0.9+0.5(1.5)   | 5.8         | 0.5         | 3.2                                 | 0.56                             | 0.33           |      |
| 0.9+0.3(1.5)   | 3.9         | 0.3         | 2.0                                 | 0.55                             | 0.27           |      |
Fig. 1. Time variations of geometrical parameters of the binary with initial masses $1.3+1.1\,M_\odot$ and initial period of 2.5 d. Star “1” reaches TAMS when the binary is still detached.

4 Results of the modeling

4.1 Time evolution of the orbital period

Eqs. (7-8) are the basic equations governing the evolution of geometrical parameters of a close binary in the detached phase. They are integrated in time, starting from initial conditions. The amount of mass and orbital AM lost at each time step is calculated and subtracted from the corresponding quantities. Radii of components are calculated from a grid of evolutionary models of single stars with the solar composition by simple interpolation formulas. Models obtained by Schaller et al. (1992), Girardi et al. (2000) and models of very low mass stars obtained by Dr. R. Sienkiewicz for the present author were used. New values of masses, radii and $H_{\text{orb}}$ are used to calculate all other binary parameters at each time step. The calculations stop when star “1” fills its in-

*A description of the latter models is given in Stepien (2006a) and the details of the modeling program are given in Paczynski et al. (2007).*
Fig. 2. Time variations of geometrical parameters of the binary with initial masses 0.9+0.3 $M_\odot$ and initial period of 2.5 d. Here star “1” reaches the critical Roche lobe when it is still on MS.

Fig. 2. Time variations of geometrical parameters of the binary with initial masses 0.9+0.3 $M_\odot$ and initial period of 2.5 d. Here star “1” reaches the critical Roche lobe when it is still on MS.

ner critical surface. Depending on initial conditions, it happens when the star is still evolving across MS, when it is at terminal age MS (TAMS) or beyond it. Following RLOF mass exchange between the components takes place in case A, AB or B, respectively. The subsequent evolution of the binary depends sensitively on the evolutionary advancement of star “1” at the time of RLOF. This evolution will be the subject of the future investigation. At present we concentrate on the early stages of evolution, when the binary is still detached.

Table 2 summarizes the results of the model calculations. All columns are self explanatory. Two characteristic evolutionary models are presented in Figs. 1 and 2. Fig. 1 shows the time variations of geometrical parameters of the most massive binary with the initial masses 1.3+1.1 $M_\odot$ and the longest initial period of 2.5 d. The binary has also the highest initial AM (see Table 1). After about 4 Gyr star “1” approaches TAMS when its radius is still significantly smaller than the size of the Roche lobe. The period has shortened to the value of 1.6 d during this time. When hydrogen is exhausted in the center, the star expands quickly and fills its Roche lobe. Fig. 2 shows the same
data for the least massive binary from Table 2 with initial masses $0.9\pm0.3 M_\odot$ and the initial period of 2.5 d. Here we see that the Roche lobe descends onto the surface of star “1” when it is still on MS. The binary has a period of only 0.35 d at that time.

Fig. 3. Time variations of orbital periods of all the considered binaries. The initial and end coordinates of the plotted curves are given in Table 1 and Table 2, respectively.

The period variations of all the considered models are presented in Fig. 3. All the plotted curves show a bend increasing with age. This is a consequence of the period dependence of the AML rate (see Eq. 7 above) - the shorter the period, the higher AML rate, hence the faster period shortening. The vertical broken line marks the limiting age of 4 Gyr beyond which stellar clusters contain high numbers of W UMa-type variables. Clusters younger than that age contain no, or very few such stars. Almost all the plotted curves reach that line and many go beyond. This indicates that very few binaries will lose enough AM for RLOF to occur in time shorter than 4 Gyr. As the data in Table 2 show, this happens only for binaries with the initial orbital period equal to 1.5 d and mass of star “1” equal to $1.3 M_\odot$. The mass of star “2” does not matter in this case. For the mass of star “1” equal to $1.1 M_\odot$ RLOF occurs before 4 Gyr only when the initial mass of star “2” is equal to $0.5 M_\odot$ (see Table 2). However, all binaries with $P_{\text{init}} =
1.5 d reach RLOF when star “1” is only about halfway to TAMS. Star “2” is even less evolutionary advanced. After the mass exchange, many such binaries may not fulfill the geometrical condition of the Roche model required to form a contact binary. Instead, a near contact binary will be formed in such a case, with star “1” filling its critical Roche surface and slowly transferring matter to star “2” which is still within its Roche lobe. The results of the calculations also show that the initial period of TX Cnc must have been even shorter – of the order of 1 d. Binaries with very short periods can be formed in stellar clusters by the interaction with other cluster members (e.g., Hut 1983) although paucity of such binaries in young and intermediate age clusters indicates that such events are not frequent.

The initial period of 2.5 d is long enough for nearly all stars “1” in the considered binaries to reach TAMS before MB leads to RLOF. In other words, both components are still well within their respective Roche lobes when star “1” completes its MS evolution and accelerates expansion on its way to the red giant branch. There is only one exception from this: the least massive model 0.9+0.3(2.5). Here the Roche lobe descends onto the surface of star “1” after about 9 Gyr (see Table 2) when it is still on MS. In models with $P_{\text{init}} = 3$ d the AML rate is even lower than in models with $P_{\text{init}} = 2.5$ d. As a result, in all cases, star “1” reaches TAMS when the period is barely shortened to values close to 2 d. Mass exchange in such binaries takes place in case B, resulting in Algols with periods of a few days which are beyond the scope of the present paper.

4.2 Period distribution

It was argued in Section 2 that the KCTF mechanism can effectively produce close, detached binaries with periods of a few days, in a time much shorter than the evolutionary time scale (Eggleton and Kiseleva-Eggleton 2006, Fabrycky and Tremaine 2007). As
a result, an excess of close binaries with such periods should occur among young stars, compared to expectations based on the standard period distribution (Duquennoy and Mayor 1991). The observations of Hyades show indeed an existence of such an excess (Griffin 1985, Stepień 1995). The crucial role of the KCTF mechanism in producing this excess is strongly suggested by the recent analysis of field binaries by Tokovinin et al. (2006) which showed that nearly all binaries with periods shorter than 3 d possess a distant companion. The effectiveness of the KCTF mechanism drops, however, rapidly for periods shorter than 2-2.5 d. The resulting period distribution produced solely by KCTF should show a cut-off at this value. A similar cut-off period of 2 d is expected for “pure” close binaries formed in the early fragmentation process (for a discussion see Stepień 1995). The cumulative initial period distribution of young cool binaries is therefore expected to have a cut-off near 2 d. Due to the magnetized winds, orbits of such binaries evolve further towards shorter periods but this evolution is now determined by MB. The period distribution predicted by model calculations can be determined and compared with the observed distribution.

To obtain the observed period distribution, the data from Pilecki (2010) were used. He selected a sample of short-period eclipsing binaries from the ASAS survey (for information see [http://www.astrouw.edu.pl/asas] brighter than 11^{m}.5 and frequently observed in two (V and I) colors. Then he solved their light curves with the Wilson-Devinney code and obtained the basic binary parameters. We selected binaries marked as detached by Pilecki, with periods shorter than 2 d, and with temperatures from the interval 5200-6600 K, corresponding approximately to the mass of the more massive component between 0.9 and 1.3 $M_\odot$. The total number of such binaries is 421. The plot of the actual numbers in bins with the width of 0.2 d is given in Fig. 4 as a solid line. The distribution based on eclipsing binaries needs a correction for the bias connected with the orbit inclination. A simple estimate of the correction factor is derived by Maceroni & Rucinski (1999). According to them, the number of stars observed at each period should be multiplied by $P^{-1/3}$. Thus corrected distribution, normalized to the total observed number of binaries, is plotted in Fig. 4 as a broken line. The latter distribution will be compared to the predicted distribution resulting from the model computations.

The predicted period distribution is calculated under the following assumptions:

1. Stationary state prevails i.e., the volume density of binaries at each value of the orbital period is constant.

2. Initial period and component masses are not correlated, i.e., $n(P,M_1,q)dPdMdq = n_1(P)n_2(M_1)n_3(q)dPdMdq$, where $n$ is the number density of binaries with different periods and component masses whereas $n_1, n_2, n_3$ are the number densities depending on period, mass and mass ratio, respectively.

3. Mass variations of both components are neglected during the considered time so $n_2$ and $n_3$ are time-independent. The only time variable is $n_1$.

4. The initial period distribution $n_1(P_{\text{init}}) = \text{const.}$ down to the cut-off period. Two values of the cut-off period are considered: 1.5 d or 2 d.

5. The mass distribution is given by the initial mass function of Salpeter (1955): $d\rho_2(M_1) \propto M_1^{-2.35}dM_1$

6. The mass ratio distribution $n_3(q) = \text{const.}$

Let us consider the evolution of period along a given curve $P_i(t)$, see Fig. 3. In the stationary state, the number density $n_i$ of binaries evolving along that curve and having
Fig. 5. The observed period distribution of the detached binaries, based on the ASAS data and corrected for the detection bias as described in the text, is shown by the solid line. It is compared to the predicted distribution with the cut-off period equal to 1.5 d (dotted line) and to 2 d (broken line).

periods between \( P \) and \( P + \Delta P \) is proportional to the relative time spent by a binary within this period interval, i.e.,

\[
n_i(P, P + \Delta P) = \frac{1}{T_i} \int_{t_i}^{t_i + \Delta t} \frac{dt}{dP_i/dt},
\]

where \( T_i \) is the total evolutionary time (listed in column 2 of Table 2).

To obtain the final expected period distribution, \( n_1 \), a summation of all \( n_i \) for the respective period interval should be done, with weights resulting from assumptions (4)-(6) above

\[
n_1(P, P + \Delta P) = \frac{T_i w_i n_i(P, P + \Delta P)}{T},
\]

where \( w_i \) are the weights and \( T = \sum T_i \). Summation was done over all models listed in Tables 1 and 2. The resulting distributions with the cut-off period equal to 1.5 d (dotted line) and to 2 d (broken line) are compared in Fig. 5 to the observed distribution, corrected for the selection bias. Both predicted distributions were binned in the same way as the observed one. All three distributions are normalized to the unity area. It is immediately visible that the predicted distribution with the cut-off period of 2 d fits the observational data much better than the one with the cut-off period of 1.5 d. The results of the \( \chi^2 \)-test support this conclusion. The value of the reduced \( \chi^2 \) is equal to 1.04 in the first case and to 2.91 in the latter case. One more predicted distribution was also calculated, with a weight of 1/3 given to all binaries with initial period of 1.5 d, compared to unity given to binaries with 2 and 2.5 d (in the two previous cases this weight was equal to 1 and 0, respectively). That distribution is not plotted in Fig. 5 to avoid line crowding but the value of the reduced \( \chi^2 \) in this case is equal to 1.44 which indicates that the distribution is acceptably close to the observed one although
the agreement is worse than in case of the distribution with the cut-off period of 2 d. This result suggests that the pool of short period binaries evolving further under the influence of MB may also contain a small share of binaries with initial periods of 1.5 d.

4.3 Uncertainties of the model

The main uncertainties of the presented model are connected with inaccuracies of the numerical coefficients in Eqs. (7-8) describing the AML rate and the mass loss rate. These inaccuracies do not alter significantly the main qualitative conclusions, however, some of the quantitative results may change with a change of the coefficients. For example, if we increase the AML rate by the uncertainty limit \textit{i.e.}, 50 \%, the time till RLOF will shorten correspondingly. To obtain the final values similar to those listed in Table 2 the initial periods of all considered binaries would have to be increased by a few tenths of a day. A decrease of the AML rate would result in lengthening of the time till RLOF and an increased evolutionary advancement of the stars filling their Roche lobes. To obtain the values close to those listed in Table 2 the initial periods of all considered binaries would have to be \textit{decreased} by a few tenths of a day. Allowing for these uncertainties it is concluded that the detached cool binaries with initial periods between 1.5 and 3 d are the progenitors of short period semi-detached and contact binaries. An increase/decrease of the mass loss rate would increase/decrease the MS life times of the more massive components and decrease/increase the final component masses by a few percent when RLOF occurs, compared to the values given in the paper. This will modify to some degree the further evolutionary stages of the binaries.

Another source of uncertainties results from the use of single star models. A comparison of the existing grids of models reveals differences among them at the level of several percent in stellar radii, MS life times etc. They result from different input physics and computational methods. In addition, the observed radii of cool MS stars are systematically larger by about 10 \% than obtained from models (Torres et al. 2010) which is probably connected with the presence of magnetic fields in these stars. A similar uncertainty of several percent in stellar parameters is introduced by neglecting the influence of the component interaction on their evolution. It is rather difficult to precisely calculate the influence of model inaccuracies on the presented results but we expect that the main conclusions of the present study are not altered.

5 Main Conclusions

A set of 27 evolutionary models of cool close binaries with the initial periods of 1.5, 2.0 and 2.5 d was computed under the assumption that mass and AM loss due to the magnetized winds of both components determine the orbital evolution until the more massive component fills its critical Roche lobe. Following RLOF, mass transfer between the components occurs which results in formation of a semi-detached or contact binary with the mass ratio reversed. Nine models were computed for each period with different component masses (see Table 1). The present paper deals with the first phase of binary evolution - from the beginning till RLOF. The consecutive phases which include mass transfer following the RLOF and the further evolution towards contact or a classical Algol configuration will be the subject of the subsequent study.

An important test for the correctness of the adopted assumptions and model calculations results from the comparison of the predicted period distribution to the observed distribution of detached binaries with the same masses. The comparison was restricted to periods shorter than 2 d where, according to the model assumptions, the period
evolution is determined by MB. An excellent agreement is obtained for the flat initial period distribution with a cut-off at 2 d. A flat distribution with a cut-off at 1.5 d differs significantly from the observations, however, a small admixture of such stars is not excluded. This result indicates that MB is a dominating mechanism populating the orbital period region below 2 d. Other mechanisms, like KCTF or N-body interactions play a minor role in producing binaries with such short periods.

Fig. 6. Comparison of the MS life time of the massive component (heavy solid line) with time needed to reach RLOF for binaries with different initial orbital periods (light solid lines). A broken line gives time needed for the occurrence of RLOF in a fictional situation when only MB operates and the star does not expand beyond its TAMS size.

The results show that it is the length of the initial orbital period that primarily controls the evolution of a cool close binary in phases following the rapid mass exchange. As it was mentioned earlier, RLOF occurs due to the operation of two, equally important processes: the evolutionary expansion of the massive component of the binary and the contraction of its Roche critical lobe resulting from MB. For the discussed binaries time scales of both processes are roughly of the same order. However, while the MS life time of a star does not depend on the orbital parameters, the MB time scale depends sensitively on the initial length of the orbital period (and also, to some degree, on the
initial mass ratio). This is illustrated in Fig. 6 where the MS life time of the massive component, plotted as a heavy solid line, is compared to the time needed for RLOF to occur in binaries with different initial orbital period. The latter times are shown as light solid lines labeled with a value of the initial period. MS life times refer to the Girardi et al. (2000) models with solar composition. The times to reach RLOF were taken from Table 2 for binaries with moderate mass ratios. Note that they cannot significantly exceed MS life time because the star expands beyond TAMS quickly until it fills the Roche lobe. A broken line describes the time needed for the occurrence of RLOF in binaries with the initial period of 2.5 d, assuming the fictional situation that the massive component does not expand beyond its TAMS size and the Roche lobe descends onto the stellar surface solely due to MB.

It is seen from Fig. 6 that the MB time scale for binaries with the initial period of 2.5 d or more is longer than the MS life time of the massive component that reaches TAMS being well inside its Roche lobe whereas the binary still possesses a substantial fraction of its initial AM. As a result, the star expands and fills the Roche lobe on its way to the red giant branch. Following the rapid mass exchange (in the so called case B) a short period Algol is formed in which the (slow) mass transfer prevails over MB and the orbital period increases. Only for binaries with the massive component of 0.9 \( M_\odot \) the time to reach RLOF is shorter than the MS life time but this is a result of the solar composition which was used in all calculations to preserve consistency. The 0.9 \( M_\odot \) model with low metallicity, characteristic of globular clusters would be more appropriate here. Its MS lifetime is around 10 Gyr as compared to about 15 Gyr for a solar composition star. The lifetime of 10 Gyr is shorter than the MB time scale of a binary with the initial period of 2.5 d and is close to the time for RLOF occurrence in a binary with the initial period of 2 d (see Fig. 6).

For binaries with the initial period of 2 d the MS life times are close to the time needed for RLOF to occur so the massive component fills the Roche lobe just when it is at, or close to TAMS and the case AB mass transfer follows. The orbital period shortens to only a fraction of a day at that point and the rapid mass exchange results in a formation of contact, or near contact binary which soon reaches full contact. During the further evolution the (slow) mass transfer approximately balances AML and the orbital period does no vary much when the mass ratio decreases to its extreme value of about 0.1.

For binaries with the initial period of 1.5 d or less the time to reach RLOF is shorter than the MS life time of the massive component. The star fills the Roche lobe during its MS evolution and the case A mass transfer follows. The orbital period is very short at RLOF - typically less than 0.5 d. After the mass exchange a contact or near contact binary is formed. Because MB prevails over the (slow) mass transfer, the orbital period shortens and the coalescence of both components occurs for a moderate rather than extreme mass ratio.

The present calculations cannot be directly applied to globular clusters as the stellar evolutionary models with low metallicity should be used in modeling evolution of cluster binaries. Nonetheless it is possible to sketch a qualitative picture regarding the presence of contact binaries in these clusters.

Members of a globular cluster reach TAMS when they are at the turn-off point. If such a star is also a member of a close binary with a lower mass component and a proper initial orbital period (around 2 d, although more accurate value will follow from a full scale evolutionary calculations), RLOF occurs, followed by the approximate mass ratio inversion. A contact binary is formed in which the presently more massive component has a mass close to the pre-RLOF massive component, \textit{i.e.,} close to the turn-off mass. The new-born contact binaries are therefore expected to appear in the turn-off region. During the subsequent evolution towards the extreme mass ratio the
Contact binaries move to the region of blue stragglers until the ultimate coalescence occurs. Contact systems can also be formed below the turn-off region from binaries with initial periods significantly shorter than those of contact binaries from the turn-off region. An apparent shortage of such systems in the observed clusters indicates that mechanisms producing cool binaries with very short initial periods are not very efficient in globular clusters, similarly as it is in open clusters or among field binaries.

The above picture is very simplified and approximate. It will vary in actual situations from one binary to another, depending on initial values of individual component masses, their chemical composition, orbital parameters and details of the evolutionary modeling.

The final values from Table 2 will be used as input data for calculating the subsequent evolution of the binaries. The results will be compared with the observations of W UMa-type contact binaries and related stars.

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