High-lying Gamow-Teller excited states in the deformed nuclei, $^{76}\text{Ge}$, $^{82}\text{Se}$ and $N = 20$ nuclei in the island of inversion by the Deformed QRPA (DQRPA)

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Abstract. With the advent of high analysis technology in detecting the Gamow-Teller (GT) excited states beyond one nucleon emission threshold, the quenching of the GT strength to the Ikeda sum rule (ISR) seems to be recovered by the high-lying (HL) GT states. We address that these HL GT excited states result from the smearing of the Fermi surface by the increase of the chemical potential owing to the deformation within a framework of the deformed quasi-particle random phase approximation (DQRPA). Detailed mechanism leading to the smearing is discussed, and comparisons to the available experimental data on $^{76}\text{Ge}$, $^{82}\text{Se}$ and $N = 20$ nuclei are shown to explain the strong peaks on the HL GT excited states.

1. Introduction

The deformation in nuclei becomes more important than last decades with the recent development of rare isotope (RI) accelerator facilities, from which one may perform lots of challenging experiments related to the RI nuclei. They are usually produced in the successive nuclear capture reactions in the cosmos, i.e. slow- and rapid- process, presumed to be occurred, respectively, at the initial and explosive stage of stellar evolution. Although they decay fractions of a second, their existence are imprinted on the nuclear abundances of stars [1].

On the other hand, recent experimental GT data on the high-lying (HL) states deduced by the more energetic projectiles shed a new light on the GT states above one nucleon threshold, whose contributions are thought to enable us to explain the quenching problem with the HL GT excited states through the multi-particle and multi-hole configuration mixing [2]. Of course, the contributions from the ∆ excitation and the two-body current may contribute more or less to the quenching problem.

Conventional approach to understand the nuclear structure is based on the spherical symmetry. But, in order to describe neutron-rich nuclei and their relevant nuclear reactions occurred in the nuclear processes, one needs to develop theoretical formalism including explicitly the deformation [3, 4]. Ref. [3] exploited the Nilsson basis to the DQRPA. But the two-body interaction was derived from the effective separable force. Realistic two-body interaction is firstly considered at Ref. [4] only with neutron-neutron (nn) and proton-proton (pp) pairing correlations which have only isospin $T = 1$ and $J = 0$ interaction. But, to properly describe the deformed nuclei, the $T = 0$ and $J = 1$ pairing should be also taken into account because it may easily lead to the deformation by the $J = 1$ pairing through the $^3S_0$ tensor force. In this work,
we extend our previous QRPA based on the spherical symmetry [5], which has been exploited as a useful framework for describing the nuclear reactions sensitive on the nuclear structure of medium-heavy and heavy nuclei [6].

2. Theoretical frameworks

Our DQRPA formalism is fully discussed at Ref. [7]. We start from a deformed Wood Saxon potential [8] and transform a physical state given by the diagonalization of total Hamiltonian in the Nilsson basis into the spherical basis, in which one may perform more easily theoretical calculations. In a cylindrical coordinate, eigenfunctions of a single particle state and its time-reversed state denoted as and in the deformed Woods-Saxon potential are given as

\[ |\alpha \rho \sigma > = \sum_{Nn_z} [b_{Nn_z,\Lambda}^{(+)} |N, n_z, \Lambda, \Omega \sigma = \Lambda + 1 \geq 1 > + b_{Nn_z,\Lambda}^{(-)} |N, n_z, \Lambda + 1, \Omega \sigma = \Lambda + 1 - 1 \geq 1 > ] \tag{1} \]

where \( N = n_\rho + n_z \) (\( n_\rho = 2n_\rho + \Lambda_\sigma \)) is a major shell number, and \( n_z \) and \( n_\rho \) are numbers of nodes of the physical state on the deformed harmonic oscillator wave functions in \( z \) and \( \rho \) directions, respectively. \( \Lambda_\sigma (\Omega \sigma) \) is the projection of the orbital (total) angular momentum onto the nuclear symmetric axis \( z \). The coefficients \( b_{Nn_z,\Lambda}^{(\pm)} \) are obtained by the eigenvalue equation of the total Hamiltonian in the Nilsson basis. The deformed wave function, \( |NN_\rho \Lambda_\sigma \sum (= \pm 1/2) > \) in Eq. (1) can be expanded in terms of the spherical harmonic oscillator wave function \( |N_0 l \Lambda \sum > \)

\[ |NN_\rho \Lambda_\sigma \sum >= \sum_{N_\rho, l} A_{NN_\rho, \Lambda}^{N_\rho l, n_z = N_\rho - l} \sum_{j} C_{l\Lambda_\sigma}^{j\Omega_\sigma} |N_0 l j \Omega_\sigma > \tag{2} \]

with the Clebsch-Gordan coefficient \( C_{l\Lambda_\sigma}^{j\Omega_\sigma} \) and the spatial overlap integral \( A_{NN_\rho, \Lambda}^{N_\rho l, n_z = N_\rho - l} \) numerically calculated in the spherical coordinate system.

The Deformed BCS equation is solved by using \textit{ab initio} Brueckner G-matrix calculated from the realistic Bonn CD potential for the nucleon-nucleon interaction. The \( \beta^\pm \) transition amplitudes from the ground state of an initial nucleus to the excited state are expressed by

\[ < 1(K), m | \beta^\pm_{K} | QRPA > = \sum_{\alpha \rho, \beta \rho \beta' \rho} N_{\alpha \sigma \rho, \beta \rho, \beta' \rho} \times \tag{3} \]

\[ < \alpha \rho' \sigma | \sigma_{K} | \beta \rho \beta' \rho > [u_{\rho \alpha \rho'} v_{\rho \beta \beta'} X_{\alpha \rho \beta \beta', K} + v_{\rho \alpha \rho'} u_{\rho \beta \beta'} Y_{\alpha \rho \beta \beta', K}], \]

where \( |QRPA> \) denotes the correlated QRPA ground state in the intrinsic frame with the normalization factor \( N_{\alpha \rho \beta \rho} \beta \rho J(J) \). To compare to the experimental data, the GT(\( \pm \)) strength functions

\[ B_{\pm \Omega}^\pm (m) = \sum_{K=0, \pm 1} | < 1(K), m | \beta^\pm_{K} | QRPA > |^2 \tag{4} \]

for \( ^{76}\text{Ge} \) and \( ^{82}\text{Se} \) are presented in this report. The single particle states are used up to \( 4\hbar \omega \) for two nuclei, in the spherical limit. Since the GT strength distribution turns out to rely on the deformation parameter \( \beta_2 \) [8], we exploited the deformation parameter, \( |\beta_2| \leq 0.35 \), as default values.

An interesting and important point to be noticed is the wide smearing of the Fermi surface in deformed nuclei, which makes the \( \omega \omega \) coefficients for the paring gaps small around the Fermi surface and enlarges the pairing strength to fit the empirical pairing gap. This tendency is explicitly revealed in the occupation probabilities of \( ^{82}\text{Se} \) and \( ^{76}\text{Ge} \) in Fig.1, which shows a wide smearing, \textit{i.e.} significant change of the occupation probabilities of the particles around the Fermi surface by the increase of the deformation parameter \( \beta_2 \).
Figure 1. (Color online) Occupation probabilities of the nucleons in $^{82}$Se and $^{76}$Ge as a function of the single particle energy given by Nilsson basis for two deformation parameters $\beta_2$. In left panel, with the increase of $\beta_2$ from 0.1 to 0.2, the Fermi energy $\lambda_n$ is increased to the wider smearing of the Fermi surface.

3. Results

By the $\beta_2$ deformation leading to the smearing, some states above the Fermi surface are reallocated below or around the surface, because the single particle state energies adopted from the deformed Woods Saxon potential depend on the parameter $\beta_2$ [7]. The deformation of nuclei may be conjectured to come from macroscopic phenomena, for example, the core polarization, the high spin states and so on. Microscopic reasons may be traced to the tensor force in the nucleon-nucleon interaction, which is known to account for the shell evolution according to the recent shell model calculations [9, 10]. Therefore, the deformation parameter adopted in this work may include implicitly and effectively such effects, because the single particle states from the deformed Wood Saxon potential show strong dependence on the $\beta_2$.

In Fig 2, we show the GT strength distributions on $^{82}$Se and $^{76}$Ge in terms of the $\beta_2$ parameter as a function of excited energy of parent nuclei, so that experimental data are presented by adding the empirical Q values from the measured data. The GT strength distributions are widely scattered due to deformation. With the redistribution of the GT strengths, the GT data around 12MeV are well reproduced at $\beta_2 = 0.2$ and 0.35, respectively. For a reference, the $\beta_2$ values from the relativistic mean field (RMF) are 0.133 and 0.157[12]. Ikeda sum rules are also satisfied within a few %. It means that these HL GT states are intimately associated with the smearing of some physical states around the Fermi surface in deformed nuclei. Similar results are for N=20 nuclei will be appeared elsewhere.

In summary, by taking the deformed axially symmetric Woods-Saxon potential, we performed

\begin{align*}
\text{(Color online) Occupation probabilities of the nucleons in $^{82}$Se and $^{76}$Ge as a function of the single particle energy given by Nilsson basis for two deformation parameters $\beta_2$. In left panel, with the increase of $\beta_2$ from 0.1 to 0.2, the Fermi energy $\lambda_n$ is increased to the wider smearing of the Fermi surface.}
\end{align*}
the deformed BCS and deformed QRPA with a realistic two-body interaction calculated by ab initio G-matrix based on Bonn potential. Results of the GT strength, $B(\text{GT})$, for $^{76}\text{Ge}$ and $^{82}\text{Se}$ show that the deformation effect leads to a fragmentation of the GT strength and reproduces the HL GT excitation deduced by higher energy projectiles, which turns out to result from the wide smearing by the increase of the Fermi surface energy due to the deformation. This work was supported by the National Research Foundation of Korea (2012R1A1A3009733, 2011-0015467).

References

1. Hayakawa T, Iwamoto N, Shimizu T, Kajino T, Umeda H and Nomoto K 2004 Phys. Rev. Lett. 93 161102
2. Yako K et al. 2005 Phys. Lett. B 615 193
3. Simkovic F, Pacearescu L and Faessler A 2004 Nucl. Phys. A 733 321
4. Yousef M, Rodin V, Faessler A and Simkovic F 2009 Phys. Rev. C 79 014314
5. Cheoun M K, Bolyk A, Faessler A, Simcovic F and Teneva G 1993 Nucl. Phys. A 561 74
6. Cheoun M K, Ha E, Kim K S and Kajino T 2010 J. Phys. G 37 055101
7. Ha E and Cheoun M K 2012 Preprint arXiv:1205.4561 [nucl-th] and arXiv:1206.2156[nucl-th]
8. Hamamoto I 2004 Phys. Rev. C 69 041306(R); 2007 Phys. Rev. C 76 054319
9. Otsuka T et al. 2005 Phys. Rev. Lett. 95 232502
10. Otsuka T et al. 2010 Phys. Rev. Lett. 104 012501
11. Madey R et al. 1989 Phys. Rev. C 40 540
12. Lalazissis G A, Raman S and Ring P 1999 At. Data and Nucl. Data tables 71 1