A sound spatio-temporal Hoare logic for the verification of structured interactive programs with registers and voices*

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Abstract. Interactive systems with registers and voices (shortly, rv-systems) are a model for interactive computing obtained closing register machines with respect to a space-time duality transformation (“voices” are the time-dual counterparts of “registers”). In the same vain, AGAPIA v0.1, a structured programming language for rv-systems, is the space-time dual closure of classical while programs (over a specific type of data). Typical AGAPIA programs describe open processes located at various sites and having their temporal windows of adequate reaction to the environment. The language naturally supports process migration, structured interaction, and deployment of components on heterogeneous machines.

In this paper a sound Hoare-like spatio-temporal logic for the verification of AGAPIA v0.1 programs is introduced. As a case study, a formal verification proof of a popular distributed termination detection protocol is presented.

1 Introduction

Verification, a pillar of the development of reliable software, is notoriously difficult. Full verification was a never reach goal even for sequential programs. (However, currently Floyd-Hoare verification style regains popularity, being part of modern programming development platforms where users interactively develop and verify complex software systems.) For concurrent, parallel, or distributed programs, where the tasks are much more complex, the approach was partially replaced by lighter verification methods (for instance model-checking, run-time verification, testing, etc.), where specific system properties are verified. The downside of these propriety-based verification methods is the partial coverage of

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systems, either due to theoretical limitations of the methods themselves or due to practical consideration (as the proprietary rights of certain running platforms).

Interactive computing [14] is a step forward on system modularization. The approach allows to describe parts of the systems and verify them in an open environment. A model for interactive computing systems (consisting of interactive systems with registers and voices - \textit{rv-systems}) and a core programming language (for developing \textit{rv-programs}) have been proposed in [26] based on register machines and a space-time duality transformation. Later on, structured programming techniques for \textit{rv-systems} and a kernel programming language AGAPIA have been introduced, with a particular emphasis on developing a structural spatial programming discipline, see [9, 10].

Structured process interaction greatly simplifies the construction and the analysis of interactive programs. For instance, method invocation in current OO-programming techniques may produce unstructured interaction patterns, with free \texttt{goto}'s from a process to another and should be avoided. Compared with other interaction or coordination calculi (e.g., \(\pi\)-calculus [20], actor models [1], REO [2], Orc [21], etc.), the \textit{rv-systems} approach paves the way towards a name-free calculus and facilitates the development of modular reasoning with good expectations for proof scalability to systems with thousands of processes.

A new and key element of our structured interaction model is the extension of temporal data types used on interaction interfaces. These new temporal data types (including voices as a time-dual version of registers) may be implemented on top of streams similar to the implementation of usual data types on top of Turing tapes.

AGAPIA [9, 22] is a kernel high-level massively parallel programming language for interactive computation. It can be seen as a coordination language on top of imperative or functional programming languages as C++, Java, Scheme, etc. Typical AGAPIA programs describe open processes located at various sites and having their temporal windows of adequate reaction to the environment. The language naturally supports process migration, structured interaction, and deployment of components on heterogeneous machines. Nonetheless, the language has simple denotational and operational semantics based on scenarios (scenarios are two-dimensional running patterns; they can be seen as the closure with respect to the space-time duality transformation of the running paths used to define operational semantics of sequential programs).

The backbone of our approach to interactive systems is the emphasized space-time duality principle: not only the model of (structured) \textit{rv-systems}, but also most of its features or extensions developed so far are all space-time invariant. For the verification tasks, this duality is again our guiding light towards the development of Hoare-like spatio-temporal logics for structured \textit{rv-programs}. We present a rich set of sound rules \textit{STHlog} for verifying structured \textit{rv-programs} (no claim on their completeness is included). As a case study, we present an

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1 The term “interactive computation” often refers to interactive systems where one participant is human, dealing with development of powerful human-computer interfaces. In our approach, all “participants” are “computing components”. 
implementation and a detailed formal verification in $STHlog_0$ of a popular distributed termination detection protocol. The method may be applied to many other sophisticated distributed protocols. A short description of the method follows.

For verification of sequential programs we have to find assertions in a few key points of the tested program and to prove certain invariance conditions, see, e.g., [16]. For rv-programs cut-points become contours, surrounding finite scenarios. The verification procedure [27] consists of the following three steps: (i) find an appropriate set of contours and assertions; (ii) fill in the contours with all possible scenarios; and (iii) prove these scenarios respect the border assertions. Except for the guess of assertions, the proof is finite and can be fully automatized.

The verification of structured rv-programs follows the same pattern. However, structured rv-programs have a more restricted way to construct scenarios, hence the procedure is more regular: (1) provide assertions for each basic statement and (2) lift assertions to larger and larger programs applying $STHlog_0$ inference rules.

The paper is organized as follows. We start with a brief presentation of scenarios and spatio-temporal specifications. Next, structured rv-programs and a scenario-based operational semantics are presented. A short section describes the syntax of AGAPIA v0.1 language. Then, our approach for developing Hoare-like spatio-temporal verification logics is presented. Finally, a detailed proof of the correctness of a termination detection protocol is included. A brief section on related works conclude the paper.

2 Scenarios

In this section temporal data, spatio-temporal specifications, grids, scenarios, and operations on scenarios are briefly presented.

*Spatio-temporal specifications.* What we call “spatial data” are just the usual data occurring in imperative programming. For them, common data structures and the usual memory representation may be used. On the other hand, “temporal data” is a name we use for a new kind of (high-level) temporal data implemented on streams. A *stream* [3] is a sequence of data ordered in time, denoted as $a_0 \sim a_1 \sim \ldots$ where $a_0, a_1, \ldots$ are its elements at time clocks 0, 1, \ldots, respectively. Typically, a stream results by observing data transmitted along a channel: it exhibits a datum (corresponding to the channel type) at each clock cycle.

A *voice* is defined as the time-dual of a register: *It is is a temporal data structure that holds a natural number. It can be used (“heard”) at various locations. At each location it displays a particular value.*

This formulation may be difficult to understand at a first sight (the reader is invited to come back here after the reading of the section on rv-programs and their scenario semantics). In a different formulation, this means high-level temporal data structures on streams (including voices) may be common to multiple processes, each process having particular values for these data structures.
Voices may be implemented on top of a stream in a similar way registers are implemented on top of a Turing tape, for instance specifying their starting time and their length. Most of usual data structures have natural temporal representations. Examples include timed booleans, timed integers, timed arrays of timed integers, etc.

A spatio-temporal specification \( S : (m, p) \rightarrow (n, q) \) (using registers and voices only) is a relation \( S \subseteq (\mathbb{N}^m \times \mathbb{N}^p) \times (\mathbb{N}^n \times \mathbb{N}^q) \), where \( m \) (resp. \( p \)) is the number of input voices (resp. registers) and \( n \) (resp. \( q \)) is the number of output voices (resp. registers). The associated relation \( S \) is often functional, sometimes written as \( \langle v | r \rangle \mapsto \langle v' | r' \rangle \), where \( v, v' \) (resp. \( r, r' \)) are tuples of voices (resp. registers).

Specifications may be composed horizontally and vertically, as long as their types agree; e.g., for two specifications \( S_1 : (m_1, p_1) \rightarrow (n_1, q_1) \) and \( S_2 : (m_2, p_2) \rightarrow (n_2, q_2) \) the horizontal composition \( S_1 \bowtie S_2 \) is defined only if \( n_1 = m_2 \) and the type of \( S_1 \bowtie S_2 \) is \( (m_1, p_1 + p_2) \rightarrow (n_2, q_1 + q_2) \); the result is as expected: it consists in tuples \( ((v_1, r_1), (v'_1, r'_1)) \) such that there exists \( v' \) with \( ((v, r_1), (v', r'_1)) \in f_1 \) and \( ((v', r_2), (v''_2, r''_2)) \in f_2 \).

Grids and scenarios. A grid is a rectangular two-dimensional array containing letters in a given alphabet. A grid example is presented in Fig. 1(a). Our default interpretation is that columns correspond to processes, the top-to-bottom order describing their progress in time. The left-to-right order corresponds to process interaction in a nonblocking message passing discipline: a process sends a message to the right, then it resumes its execution.

A scenario\(^2\) is a grid enriched with data around each letter. The data may be given in an abstract form as in Fig. 1(b), or in a more detailed form as in Fig. 1(c).

The type of a scenario interface is represented as \( t_1; t_2; \ldots; t_k \), where each \( t_k \) is a tuple of simple types used at the borders of scenario cells.\(^3\) The empty tuple is also written 0 or nil and can be freely inserted to or omitted from such descriptions. The type of a scenario is specified as \( f : \langle w|n \rangle \rightarrow \langle e|s \rangle \), where

\(^2\) See [15] for less shape-constrained scenarios.
\(^3\) If only registers and voices are used, then each tuple may by simply replace by a number (counting of its components).
$w, n, e, s$ are the types for its west, north, east, south interfaces. For example, the type of the scenario in Fig. 1(c) is $\langle \text{nil}; \text{nil}|\text{sn}; \text{nil}; \text{nil} \rangle \rightarrow \langle \text{nil}; \text{nil}|\text{sn}; \text{sn}; \text{sn} \rangle$, where $\text{sn}$ denotes a spatial integer type.

Operations with scenarios. We say two scenario interfaces $t = t_1; t_2; \ldots; t_k$ and $t' = t'_1; t'_2; \ldots; t'_k$ are equal, written $t = t'$, if $k = k'$ and the types and the values of each pair $t_i, t'_i$ are equal. Two interfaces are equal up to the insertion of $\text{nil}$ elements, written $t = n t'$, if they become equal by appropriate insertions of $\text{nil}$ elements.

Let $\text{Id}_{m,p} : \langle m|p \rangle \rightarrow \langle m|p \rangle$ denote the constant cells whose temporal and spatial outputs are the same with their temporal and spatial inputs, respectively; an example is the center cell in Fig. 2(c), namely $\text{Id}_{1,2}$.

Horizontal composition: Let $f_i : \langle w_i|n_i \rangle \rightarrow \langle e_i|s_i \rangle$, $i = 1, 2$ be two scenarios. Their horizontal composition $f_1 \triangleright f_2$ is defined only if $e_1 = n w_2$. For each inserted $\text{nil}$ element in an interface (to make the interfaces $e_1$ and $w_2$ equal), a dummy row is inserted in the corresponding scenario, resulting a scenario $f_1\triangleright f_2$. The result $f_1 \triangleright f_2$ is obtained putting $f_1\triangleright f_2$ on left of $f_2$. The operation is briefly illustrated Fig. 2(b) and in more details in Fig. 3. The result is unique up to insertion or deletion of dummy rows. Its identities are $\text{Id}_{m,0}, m \geq 0$.

Vertical composition: The definition of vertical composition $f_1 \cdot f_2$ (see Fig. 2(a)) is similar, but now $s_1 = n n_2$. For each inserted $\text{nil}$ element (to make $s_1$ equal to $n_2$), a dummy column is inserted in the corresponding scenario, resulting a scenario $f_1\triangleright f_2$. The result $f_1 \cdot f_2$ is obtained putting $f_1\triangleright f_2$ on top of $f_2$. Its identities are $\text{Id}_{0,m}, m \geq 0$.

Diagonal composition: This is a derived operation. The diagonal composition $f_1 \bullet f_2$ (see Fig. 2(c)) is defined only if $e_1 = n w_2$ and $s_1 = n n_2$. The result is defined by the formula

$$f_1 \bullet f_2 = (f_1 \triangleright R_1 \triangleright A) \cdot (S_2 \triangleright \text{Id} \triangleright R_2) \cdot (A \triangleright S_1 \triangleright f_2),$$

for appropriate constants $R, S, \text{Id}, A$. Its identities are $\text{Id}_{m,n}, m, n \geq 0$. (The involved constants $R, S, \text{Id}, A$ are described below.)

Constants: Except for the defined identities, we use a few more constants. Most of them may be found in Fig. 2(c): a recorder $R$ (2nd cell in the 1st row), a speaker $S$ (1st cell in the 2nd row), an empty cell $A$ (3rd cell in the 1st row). Other constants of interest are: transformed recorders Fig. 2(e) and transformed speakers Fig. 2(g).

![Fig. 2. Operations on scenarios](image-url)
3 Structured rv-programs

Rv-programs [26] resemble flowcharts and assembly languages: one can freely uses go-to statements with both, temporal labels (free jumps in a process from a statement to another) and spatial labels (free jumps in a macro-step from a process to another). The original approach of the authors in 2006 was to introduce structured programming techniques on top of rv-programs. However, the resulted structured rv-programs and their scenario based semantics may be described directly, from scratch. The lower level of rv-programs is still useful (and important!), as it may be used as a target language for compiling and is more appropriate for running programs on (possible multicore/manycore) computer architectures. Below, we restrict ourself to structured rv-programs. The Hoare logics for structured rv-programs, to be presented later in the paper, has its roots in a Floyd logics developed for unstructured rv-programs [27].

The syntax of structured rv-programs. The basic blocks for constructing structured rv-programs are modules. A module gets input data from its west and north interfaces, process them (applying the module’s code), and delivers the computed outputs at its east and south interfaces. While one can argue for the use of nondeterministic behaviors for processes associated to modules, we afraid of doing so: in all the example we have developed so far the basis modules (and the resulting structured rv-programs) have deterministic behavior. Nondeterministic behaviors naturally occur at more abstract levels when lot of details on particular low-level temporal or spatial data are hidden.

On top of modules, structured rv-programs are built up using “if” and both, composition and iterated composition statements for the vertical, the horizontal, and the diagonal directions. The composition statements capture at the program level the corresponding operations on scenarios. The iteration statements are also called the temporal, the spatial, and the spatio-temporal while statements - their scenario meaning is described below.

The syntax for structured rv-programs is given by the following BNF grammar

\[
P ::= X \mid \text{if}(C)\text{then}\{P\}\text{else}\{P\} \mid P\%P \mid P\#P \mid P\$P \\
\quad \mid \text{while}_{t}(C)\{P\} \mid \text{while}_{s}(C)\{P\} \mid \text{while}_{st}(C)\{P\}
\]
\( X ::= \text{module}\{\text{listen } t\text{\_vars}\}{\text{read } s\text{\_vars}} \)
\{\text{code;}\}{\text{speak } t\text{\_vars}}{\text{write } s\text{\_vars}} \)

This is a core definition of structured rv-programs, as no data types or language for module’s code is specified. On the other hand, Agapia, to be shortly presented, is a concrete incarnation of structured rv-programs into a fully running environment.

Notice that we use a different notation for the composition operators on scenarios \(\cdot, \triangleright, \triangleright\) and on programs \(\%", "\%\). Moreover, to avoid confusion, the extension of the usual program composition operator ‘;’ to structured rv-programs (i.e., the vertical composition) is denoted by a different symbol “\%”.

**Operational semantics.** The operational semantics

\[ | | : \text{Structured rv-programs} \rightarrow \text{Scenarios} \]

associates to each program the set of its running scenarios.

The type of a program \(P\) is denoted \(P : \langle w(P) | n(P) \rangle \rightarrow \langle e(P) | s(P) \rangle\), where \(w(P) / n(P) / e(P) / s(P)\) indicate its types at the west/north/east/south borders. On each border, the type may be quite complex (see AGAPIA interface types in Sec. 4). The convention is to separate by “,” the data from within a module and by “;” the data coming from different modules. This convention refers to both spatial and temporal data.

The type associated to a program may include different types for the interfaces of its running scenarios. For instance, a temporal while statement may have running scenarios with different numbers of rows which may exhibit different interfaces at their west/east borders. With this explanation, the definition below makes sense. We say, two interface types match if they have a nonempty intersection.

**Modules.** Modules are the starting blocks for building structured rv-programs. The \text{listen} (\text{read}) instruction is used to get the temporal (spatial) input and the \text{speak} (\text{write}) instruction to return the temporal (spatial) output. The \text{code} consists in simple instructions as in the C code. No distinction between temporal and spatial variables is made within a module.

A scenario for a module consists of a unique cell, with concrete data on the borders, and such that the output data are obtained from the input data applying the module’s code.

**Composition.** Programs may be composed “horizontally” and “vertically” as long as their types on the connecting interfaces agree. They can also be composed “diagonally” by mixing the horizontal and vertical compositions.

For two programs \(P_i : \langle w_i | n_i \rangle \rightarrow \langle e_i | s_i \rangle, i = 1, 2\) we define the following composition operators.

**Horizontal composition:** \(P_1 \% P_2\) is defined if the interfaces \(e_1\) and \(w_2\) match, see Fig. 4(left). The type of the composite is \(\langle w_1 | n_1; n_2 \rangle \rightarrow \langle e_2 | s_1; s_2 \rangle\). A scenario for \(P_1 \% P_2\) is a horizontal composition of a scenario in \(P_1\) and a scenario in \(P_2\).
If. For two programs $P_i : \langle w_i | n_i \rangle \rightarrow \langle e_i | s_i \rangle$, $i = 1, 2$, a new program $Q = \text{if } (C) \text{ then } P_1 \text{ else } P_2$ is constructed, where $C$ is a condition involving both, the temporal variables in $w_1 \cap w_2$ and the spatial variables in $n_1 \cap n_2$, see Fig. 4(right). The type of the result is $Q : \langle w_1 \cup w_2 | n_1 \cup n_2 \rangle \rightarrow \langle e_1 \cup e_2 | s_1 \cup s_2 \rangle$.

A scenario for $Q$ is a scenario of $P_1$ when the data on the west and the north borders of the scenario satisfy condition $C$, otherwise is a scenario of $P_2$ (with these data on the borders).

While. Three types of while statements are used for defining structured rv-programs, each being the iteration of a corresponding composition operation.

Temporal while: For a program $P : \langle w | n \rangle \rightarrow \langle e | s \rangle$, the statement \text{while } t(C) \{P\}$ is defined if the interfaces $n$ and $s$ match and $C$ is a condition on the spatial variables in $n \cap s$. The type of the result is $\langle (w;)^* | n \cup s \rangle \rightarrow \langle (e;)^* | n \cup s \rangle$. A scenario for \text{while } t(C) \{P\} is either an identity, or a repeated vertical composition $f_1 \cdot f_2 \cdot \ldots \cdot f_k$ of scenarios for $P$ such that: (1) the north border of each $f_i$ satisfies $C$ and (2) the south border of $f_k$ does not satisfy $C$.

Spatial while: \text{while } s(C) \{P\}$ is similar.

Spatio-temporal while: For $P : \langle w | n \rangle \rightarrow \langle e | s \rangle$, the statement \text{while } st(C) \{P\}$ is defined if $w$ matches $e$ and $n$ matches $s$ and, moreover, $C$ is a condition on the temporal variables in $w \cap e$ and the spatial variables in $n \cap s$. The type of the result is $\langle w \cup e | n \cup s \rangle \rightarrow \langle w \cup e | n \cup s \rangle$. A scenario for \text{while } st(C) \{P\} is either an identity, or a repeated diagonal composition $f_1 \cdot f_2 \cdot \ldots \cdot f_k$ of scenarios for $P$ such that: (1) the west and north border of each $f_i$ satisfies $C$ and (2) the east and south border of $f_k$ does not satisfy $C$.

A few particular cases of while statement may be easier to understand and use. For instance, when the body program $P$ of a temporal while statement has dummy temporal interfaces, the temporal while coincides with the while from imperative programming languages.
4 The AGAPIA v0.1 programming language

To develop and verify structured rv-programs for concrete computation tasks we need at least a couple of basic data types. The AGAPIA v0.1 programming language [9], to be be shortly introduced, forms a minimal languages: it describes what is obtained allowing for spatial and temporal integer and boolean types and applying structured rv-programming statements.

The syntax for AGAPIA v0.1 programs is presented in Fig. 5. The v0.1 version is intentionally kept simple to illustrate the key features of the approach (see [22] for v0.2 extension, including high-level structured rv-programs). The language is space-time invariant\(^4\) and has global scoping within modules and local scoping outside.

The types for spatial interfaces are built up starting with integer and boolean \(sn, sb\) types, applying the rules for \(\cup, \cdot, (\cdot)^*\) to get process interfaces, then the rules for \(\cup, \cdot, (\cdot)^*\) to get system interfaces. The temporal types are similarly introduced. Given a type \(V\), the notations \(V(k), V.k, V[k], V@@k, V@[k]\) are used to access its components.\(^5\) Expressions, usual while programs, modules, and programs are then naturally introduced. Notice that AGAPIA v0.1 has a strongly restricted format: module and program statements are not mixed (in the new v0.2 version of AGAPIA [22] programs have no longer this restriction - modules and programs can be freely combined).

An useful derived statement, to be used in the next sections, is a spatial “for” statement \(\text{for}_{s}(i=a;i<b;i++)\{R\}\). This is a macro stating for \(i=a\#\text{while}_{s}(i)b\{R\#i++\}\#\), where \(i=a\) and \(i++\) denote modules with such code, with empty spatial interfaces, and whose temporal interfaces are equal to the temporal interface of \(R\) (where \(i\) is included).

\(^4\) This means, we can formally define a space-time duality operator which maps an AGAPIA v0.1 program \(P\) to another AGAPIA v0.1 program \(P^\dagger\) such that \(P = P^\dagger\dagger\).

\(^5\) See [9] for details - we will not use this notation in the present paper.
5 Towards a Hoare-like logic for structured rv-programs

This section describes an approach for developing verification logics for structured rv-programs. The presentation starts with a few words on the verification of unstructured rv-programs (more details on developing Floyd logics for unstructured rv-programs may be found in [27]).

The semantics of (structured) rv-programs uses scenarios, a two-dimensional version of running paths used in sequential programs. The lifting of Floyd verification method to rv-programs is essentially a two-dimensional extension where cut-points with assertions become contours (borders of certain scenarios) with appropriate assertions.

The method. The Floyd method for unstructured sequential programs, requires to find assertions in a few key points of the programs and to prove appropriate invariance conditions. It should be at least one cut-point along each loop. The set of cut-points ensures that: (1) each syntactically possible path from input to output is decomposed into a sequence of small paths $p_1p_2\ldots p_k$, each $p_i$ starting and ending with cut-points and containing no cut-point inside and (2) the set of all these $p_i$ forms a finite set $K$. The proof finally reduces to the verification of the invariance conditions for the paths in $K$.

For rv-programs, cut-points becomes contours surrounding finite scenarios. Their set must be finite. The condition to “break all loops” becomes “each syntactically possible scenario can be decomposed in pieces corresponding to these contours”. To conclude, the verification procedure for rv-programs consists of the following three steps: (i) find an appropriate set of contours and assertions; (ii) fill in the contours with all possible scenarios; and (iii) prove these scenarios respect the border assertions. Notice that, except for the guess of good assertions, the proof is finite and can be fully automatized.

Structured rv-programs have a more restricted way to construct scenarios, hence the procedure is expected to be more regular: (1) provide assertions for each basic statement and (2) use appropriate inference rules to lift the assertions to larger programs.

Hoare-assertions. As we said, structured rv-programs have a restricted way to form scenarios, hence one expects the assertion format may be somehow simplified. However, notice that we are working in an open environment, hence the local application of a rule is to be integrated into a larger context, including assertions on parts of the contour that may look irrelevant to the current piece of code\(^6\), but are needed to infer the correctness of the behavior of the full system.

An assertion (for structured rv-programs) is defined using a rectangular contour surrounding a piece of structured rv-program and extended with dummy contours from its top-right and bottom-left corners, loosely along the 2nd diagonal. (Contours and assertions, to be shortly defined, are illustrated in Fig. 6.)

\(^6\) An example is P2, the invariant used in the verification of the termination detection protocol in the next section.
Formally, a Hoare-assertion contour is defined by a pair of lines starting from the same point
\[ \tau N^k E^l \sigma, \tau E^l N^k \sigma \]
where \( N \) and \( E \) denote unit lines towards the north and the east directions, and \( \tau \) and \( \sigma \) are sequences of lines of the following types:

- \( N^{a_1} E^{b_1} \ldots N^{a_k} E^{b_k} \)
- \( E^{b_1} \ldots N^{a_k} E^{b_k} \)
- \( N^{a_1} E^{b_1} \ldots N^{a_k} E^{b_k} \)

where all \( a_i, b_j \geq 1 \). Assertions use variables on the contour border. A border unit line is either horizontal and has an index from left, or vertical and has an index from top. The variables for the unit border lines are refereed to using these indices.

A Hoare assertion (see Fig. 6(left)) is a formula

\[ \{ \tau(N^k E^l)\sigma; C \} P \{ \tau(E^l N^k)\sigma; C' \} \]

where \( C \) is a condition on the west-north part \( \tau(N^k E^l)\sigma \) of the contour, \( P \) is a structured rv-program, and \( C' \) is a condition on the south-east part \( \tau(E^l N^k)\sigma \) of the contour. The conditions \( C, C' \) are first-order formulas described using contour variables. The pair of parentheses (..) locates the part of the contour where program \( P \) is used.

**Fig. 6.** Illustrations for the “Basic Rule” and the “Rule for horizontal composition”

**Inference rules.** We consider the following set of proof rules for structured rv-programs:

**Basic rule (see Fig. 6(left)):** The validity of an assertion \( \{ \tau(NE)\sigma; C \} M \{ \tau(EN)\sigma; C' \} \)

for a module \( M \) is reduced to the validity of the assertion \( \{ C \} M \{ C' \} \) in the setting of usual while programs (enriched with equalities showing that the variables in \( \tau \) and \( \sigma \) does not change by passing form \( C \) to \( C' \)).

**Rule for horizontal composition (see Fig. 6(right)):** If

\[ \{ \tau(N^k E^l)E^m \sigma; C \} P_1 \{ \tau(E^l N^k)E^m \sigma; C_1 \} \]
\[ \{ \tau(E^l N^k)E^m \sigma; C_1 \} P_2 \{ \tau(E^l E^m N^k)\sigma; C_2 \} \]

then

\[ \{ \tau(N^k E^l+m)\sigma; C \} P_1\#P_2 \{ \tau(E^l+m N^k)\sigma; C_2 \} \].

\[^7\text{Notice that the index is not changed when a program is applied. However, it may be changed when one insert or delete lines with nil type on the appropriate borders to handle program compositions.}\]
Rule for vertical composition: similar

Rule for diagonal composition: If
\{ \tau(N^kE')\sigma:C \} \ P \ \{ \tau(E^iN^k)\sigma:C1 \} \text{ and }
\{ \tau(N^kE')\sigma:C1 \} \ P \ \{ \tau(E^iN^k)\sigma:C2 \},
\{ \tau(N^kE')\sigma:C \} \ P \ \{ \tau(E^iN^k)\sigma:C2 \}.

(Notice that C1 is used on two different contour lines. The above convention on
variable indices ensures the rule is sound.)

Rule for “if”: For \( Q = i f(\text{Cond}) \{ P1 \} \text{ else } \{ P2 \} \), if
\{ \tau(N^kE')\sigma:C \land \text{Cond} \} \ P \ \{ \tau(E^iN^k)\sigma:C' \} \text{ and }
\{ \tau(N^kE')\sigma:C \land \neg\text{Cond} \} \ P \ \{ \tau(E^iN^k)\sigma:C' \},
\{ \tau(N^kE')\sigma:C \} \ Q \ \{ \tau(E^iN^k)\sigma:C' \}.

Rule for autonomous temporal or spatial “while”: For a temporal while with dummy
temporal interfaces (i.e., the west and east interfaces have a nil type), the classical
while rule may be used. By space-time duality, a similar rule applies to a spatial
while with dummy spatial interfaces.

Rule for spatio-temporal “while”: If an invariant Inv may be found such that
\{ \tau(N^kE')\sigma:Inv \land \text{Cond} \} \ P \ \{ \tau(E^iN^k)\sigma:C' \} \text{ and } C' \rightarrow Inv, then,
\{ \tau(N^kE')\sigma:Inv \} \ \text{while}_{\text{st}}(\text{Cond})(P) \ \{ \tau(E^iN^k)\sigma:Inv \land \neg\text{Cond} \}.

(See the comment on the diagonal composition to clarify the use of the assertions
Cond and Inv on different contour lines.)

Rule for a simple “for”: If \( i \) is not changed by \( R \) in a statement
\( Q = \text{for} \_ \_ s(i=0; i<a; i++) \{ R \} \),
then the following rule applies: if
\{ \tau(E^j)(N^kE')(E^a-j-1)\sigma:C_j \} \ R \ \{ \tau(E^j)(E^iN^k)(E^a-j-1)\sigma:C_{j+1} \},
for all \( j < a \), then
\{ \tau(N^k(E^a-1)\sigma:C_0 \} \ Q \ \{ \tau((E^a-1)N^k)\sigma:C_{a-1} \}.

Rule for implication: For a Hoare assertion
\{ \tau(N^kE')\sigma:C \} \ P \ \{ \tau(E^iN^k)\sigma:C' \}, if \( D \rightarrow C \), \( C' \rightarrow D' \),
then \( \{ \tau(N^kE')\sigma:D \} \ P \ \{ \tau(E^iN^k)\sigma:D' \} \).

Theorem 1. The inference rules are sound, i.e., if an assertion
\{ \tau(N^kE')\sigma:C \} \ P \ \{ \tau(E^iN^k)\sigma:C' \}
is proved, then all scenarios of \( P \) satisfying the input condition satisfy the output
condition, too. \( \square \)

6 A case study: The verification of a distributed
termination detection protocol

As a case study, we verify the correctness of a termination detection protocol.
The activity of processes and their interactions are all described using structured
rv-programs.

Termination detection is quite a popular research topic in distributed sys-
tems. The aim is to find when a set of distributed processes have terminated.
The problem is particularly complicate as one has to combine a local termina-
tion condition (each process has finished its current jobs) with a global condition
(no messages are in transit, as such messages may reactivate already terminated
processes). There are many termination detection protocols - we study a popular
dual-pass) ring termination detection protocol (see, e.g., [8]).
Ring termination detection. The dual-pass ring termination detection protocol is used to detect the termination of a pool of distributed processes logically organized as a ring. The protocol can handle the case when processes may be reactivated after their local termination. To this end, it uses colored (i.e., black or white) tokens. Processes are also colored: a black color means global termination may have not occurred. The algorithm works as follows:

1. The root process \( P_0 \) becomes white when it has terminated and it generates a white token that is passed to \( P_1 \).
2. The token is passed through the ring from one process \( P_i \) to the next when \( P_i \) has terminated. However, the color of the token may changed. If a process \( P_i \) passes a task to a process \( P_j \) with \( j < i \), then it becomes a black process; otherwise it is a white process. A black process will pass on a black token, while a white process will pass on the token in its original color. After \( P_i \) has passed on a token, it becomes a white process.
3. When \( P_0 \) receives a black token, it passes on a white token; if it receives a white token, all processes have terminated.

6.1 Implementation

Suppose there are \( n \) processes, denoted \( 0, \ldots, n-1 \). Besides the input \( n \), the program uses the spatial variables \( \text{id} : \text{sInt}, \text{c} : \{\text{white, black}\}, \text{active} : \text{sBool} \) and the temporal variables \( \text{tn}, \text{tid} : \text{tInt}, \text{msg} : \text{tIntSet[]} \). Their role is described below.

A run (for termination detection program)

![Diagram of ring termination detection](image)

Fig. 7. Vertical and horizontal compositions and “if” statements

Our structured rv-program \( P \) implementing the dual-pass ring termination protocol is the diagonal composition of an initialization program \( I \) and a core program \( Q \),

\[
P = I \$ Q
\]

where

\[
I = \ I1# \text{for}(\text{tid}=0; \text{tid}<\text{tn}; \text{tid}++)\{I2\}#
\]
**I1** = module{listen nil}{read n}{
  tn=n; token.col=black; token.pos=0;
  }{speak tn,tid,msg[ ],token(col,pos)}{write nil}

**I2** = module{listen tn,tid,msg[ ],token(col,pos)}{read nil}{
  id=tid; c=white; active=true; msg[id]=null;
  }{speak tn,tid,msg[ ],token(col,pos)}{write id,c,active}

**Q** = while(not(token.col==white & token.pos==0)){
  for s(tid=0;tid<tn;tid++){R}
}

**R** = module{listen tn,tid,msg[ ],token(col,pos)}{read id,c,active}{
  for(j=0;j<tn;j++){ //take my jobs
    if(msg[j] contains id){
      msg[j]=msg[j]-{id};
      active=true;
    }
  }
  if(active){ //execute code, send jobs, update color
    delay(random time);
    r=random(tn-1);
    for(i=0;i<r;i++) {
      k=random(tn-1);
      if(k!=id){msg[id]=msg[id]U{k}};
      if(k<id){c=black};
      active=random(true,false);
    }
  if(active & token.pos==id){ //termination
    if(id==0)token.col=white;
    if(id!=0 & c==black){
      token.col=black;c=white;
    }
    token.pos=token.pos+1[mod tn];
  }
  }{speak tn,tid,msg[ ],token(col,pos)}{write id,c,active}

Notice that, except for the operations on sets (for which AGAPIA programs have to be provided), the code represent a valid AGAPIA v0.1 program.

**Comments.** The spatial variables id, c, active represent the process identity, its color, and its active/passive status. The temporal variables used in this program are: (i) tn, tid - temporal versions of n, id; (ii) msg[ ] - an array of sets, where msg[k] contains the id of the destination processes for the pending messages sent by process k; (iii) token.col - an element of {white, black} representing the color of the token; and (iv) token.pos - the number of the process that has the token.

The program starts with the initialization of the network (program I) by activating all the processes (and setting the fields id, c, active). Initially, msg[i] = 0, for all 0 ≤ i < n, because no jobs were sent and the default color/position of the token is black/0.

After the initialization part and until the first process receives a white token back, each process executes its code. If one process has the token and terminates, it passes the token to the next process (only the first process has the right to change the color of the token into white once it terminates).

When a process executes the code R, whether active or passive, it checks if new jobs were assigned to it; if the answer is positive, it collects its jobs from
the jobs lists and stays/becomes active. When it is active, it executes some code, sends new jobs to other processes, and randomly goes to an active or passive state. If it has the token, it keeps it until it reaches termination and afterward it passes it. A white process will pass the token with the same color as it was received and a black process will pass a black token (after passing the token, the process becomes white).

6.2 Verification

The program $P$ is the diagonal composition of the initialization block and the repeated diagonal compositions given by the while \texttt{st} statement. A typical run is presented in Fig. 7. In each case, the temporal/spatial output of a block becomes the temporal/spatial input of the next block.

For $I$, the input is a spatial variable $n$. The output satisfies the condition:

$$\forall k \in [0, n) : (id, c, active)[k] = (k, white, true) \land tn = n \land token = (black, 0) \land \forall k \in [0, n) : msg[k] = \emptyset.$$ 

Notice that the spatial interface is expanded on $n$ processes $0, 1, \ldots, n - 1$. The notation $(id, c, active)[k]$ refers to the the values of variables $(id, c, active)$ in process $k$.

The invariant $Inv$. For $Q$ we need to find appropriate invariant properties. We define the following properties and prove they are satisfied by the program

$$\textsf{for} s (\textsf{tid}=0; \textsf{tid}<\textsf{tn}; \textsf{tid}++)\{\texttt{R}\}:$$

- **P1**: $token = (white, i) \rightarrow ([\forall r \in [0, i - 1] : active[r] = false \land msg[r] = \emptyset) \lor (\exists k > i - 1 : c[k] = black)$

  where the value $i - 1$ is interpreted as $tn - 1$ for $i = 0$. In words, if the token is white and reached process $i$, then all processes with smaller $id$ terminate and have no pending messages sent\(^8\) or a process with a larger $id$ is black.

- **P2**: $token.col = white \rightarrow (\forall k \in [0, n) : msg[k] \neq \emptyset \rightarrow c[k] = black)$

  In words, if a process has a job inserted in its pending message list, then its color is black.

We want to prove $Inv = P1 \land P2$ is really an invariant, i.e., the same assertion $Inv$, translated to the output values of the variables, holds at the end of the for\_s statement. Formally,

$$\{\{Inv\}\}\textsf{for} s (\textsf{tid}=0; \textsf{tid}<\textsf{tn}; \textsf{tid}++)\{\texttt{R}\}\{\{Inv\}\}.$$

Notice that due to the fact that the token is black, $Inv$ holds at the beginning of the spatio-temporal while.

\(^8\) The pending message lists are the lists of messages that have been inserted in and not removed from the message lists during a complete passing through the ring. Formally, they are $msg[r]$’s at the start of the for\_s statement.
Proof of the invariance of Inv. To simplify the presentation, we directly prove the invariance of Inv for $\forall s.(\text{tid}=0;\text{tid}<\text{tn};\text{tid}++)\{R\}$. A fully formal proof, including an appropriate new invariant for R itself, is included in Appendix A.

Suppose Inv holds at the start of the for_s statement. We want to prove that the property Inv' = P1' \land P2', where Inv' is Inv translated to the output values of the variables$^9$, holds at the end of the for_s statement.

First, we prove P1', where

\[ P1': \text{token}' = (\text{white}, i') \rightarrow (\forall r \in [0, i' - 1]: \text{active}'[r] = \text{false} \land \text{msg}'[r] = \emptyset) \]

Suppose token'.col = white; then token.col = white, too. Notice that $\forall r \in [i, i' - 1]: \text{active}'[r] = \text{false} \land \text{msg}'[r] = \emptyset$ holds because: (i) the token could not reach the process \(i'\) unless processes $i, \ldots, i' - 1$ hadn’t terminated and (2) token'.col hadn’t been white unless msg'[\(i\)], \ldots, msg'[\(i' - 1\)] are all empty.

As P1 holds and token = (white, i), either (i) or (ii) below applies, where:

(i) $\forall r \in [0, i - 1]: \text{active}[r] = \text{false} \land \text{msg}[r] = \emptyset$. In this case:

(a) If all processes $0, \ldots, i - 1$ stay passive, then by the above observation this situation is extended to $\forall r \in [0, i' - 1]: \text{active}'[r] = \text{false} \land \text{msg}'[r] = \emptyset$ and we are done.

(b) If one process $0, \ldots, i - 1$ becomes active, it may be reactivated only by a message from a process $k$ with $k > i - 1$ (indeed, msg[0], \ldots, msg[i - 1] are all empty). Then, by P2, c'[$k$] = black. Moreover $k > i' - 1$ (otherwise token'.col hadn’t been white), hence c'[\(k\)] = black and the second part is true.

(ii) $\exists k > i - 1: c[k] = \text{black}$. In this case, $k > i' - 1$ (otherwise token'.col hadn’t been white) and c'[\(k\)] = black, hence the implication holds.

Next, we prove P2', where

\[ P2': \text{token'.col} = \text{white} \rightarrow (\forall k \in [0, n]: \text{msg}'[k] \neq \emptyset \rightarrow c'[k] = \text{black}) \]

Notice that after the execution of R by the process $k$, msg'[\(k\)] consists in the processes that were contacted by $k$. The execution of R for tid = $k$ is followed by the execution of R for $k < \text{tid} < \text{tn}$. All these executions of R that follows, will discard all the messages sent to processes greater than $k$ from msg'[\(k\)] and consequently, by the end of the for_s, msg'[\(k\)] $\subseteq [0, k)$.

Hence, if msg'[\(k\)] $\neq \emptyset$, then the process $k$ had sent a message to a process $p$ with $p < k$ and the color of the process became black. Moreover, if token'.col = white, then the color of the process stayed black until the end of the for_s instruction, which implies c'[\(k\)] = black.

The final step. Applying the rule for the spatio-temporal while

\[ \{[\text{Inv}]\} \text{while}_{\text{st}}(\text{!(token=(white,0))}) \{Q'\} \{[\text{Inv} \land (\text{token} = (\text{white}, 0))])\} \]

$^9$ We use the standard “prim” notation, i.e., if $x$ is a variable, then $x'$ refers to the value of the variable $x$ at the end of the program. Here, the convention applies also to $i$ (which is not directly a variable of the program, but it actually denotes token.pos).
where $Q' = \text{for}_\text{tid=0;tid<tn;tid++}\{R\}$, it follows that

$$\forall i \in [0, tn - 1]: active[i] = false \land msg[i] = \emptyset$$

hence all process have terminated and there are no pending jobs/messages in the communication lists.

**Theorem 2.** The program for the dual-pass ring termination detection protocol is correct.

It would be interesting to compare our proof with proofs of the protocol using process algebra or other formal verification methods, if such proofs are available.

### 7 Related and future works

This is a brief section on related works, with emphasis on two-dimensional patterns and spatio-temporal logics. Our grids (scenarios without data around) are closely related to two-dimensional (or picture) languages [12, 17][10] - actually, finite interactive systems [25] (on which rv-systems are based), are equivalent to tile systems or to existential monadic second order logic [13]. Regarding scenarios, a worthwhile approach may be to use results on two-dimensional languages in combination with model-checking to (lightly) verify rv-programs.

Space and time are fundamental entities, so no surprise to find many proposals on developing space-time logics. Compared with [6], we use linear not branching space and time. Tile logic [4, 11] use similar two-dimensional patterns, but with emphasis on rewriting and declarative computation models. Other interesting space-time proposals on verifying mobile or open systems are presented in [19, 28].

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Appendix A: On the termination detection protocol

A fully formal proof of $Inv$ invariance. Here we give a fully formal proof (in $STHlog_0$ for

$\{ |Inv| \}$ for

$s(tid=0;tid<tn;tid++)\{ R \} \{ |Inv| \}$

For the beginning, we identify a few more detailed assertions, which essentially depend on $tid$ and describe the effect of the computation within $R$:

Q1: (case $token.col = white \land tid < token.pos$)

$token'=token \land \forall k \neq tid: msg'[k] = msg[k] - \{tid\} \land$

$[ (active'[tid] = false \land \exists k, tid \in msg[k] \\
\quad \quad \quad \rightarrow active'[tid] = false \land msg'[tid] = \emptyset ) \\
\quad \quad \lor (active'[tid] = true \lor \exists k, tid \in msg[k] \\
\quad \quad \quad \rightarrow msg'[tid] \subseteq [0,tid) \cup [tid+1,n) \land \\
\quad \quad \quad \quad \quad \quad \quad \quad msg'[tid] \cap [0,tid) \neq \emptyset \rightarrow c'[tid] = black )]$

Q2: (case $token.col = white \land tid = token.pos$)

$\forall k \neq tid: msg'[k] = msg[k] - \{tid\} \land$

$[ (active'[tid] = true \\
\quad \rightarrow token' = token \land \\
\quad \quad \quad msg'[tid] \cap [0,tid) \neq \emptyset \rightarrow c'[tid] = black ) \\
\quad \lor (active'[tid] = false \\
\quad \rightarrow token'.pos = token.pos + 1 \land \\
\quad \quad \quad token'.col = white \rightarrow msg'[tid] \cap [0,tid) = \emptyset )]$

Q3: (case $token.col = white \land tid > token.pos$) - same as Q1.

To prove $Inv$, we use a more detailed version $Inv2$ satisfying the following properties: (i) If $Inv2$ holds “up-to” to an $tid$ and module $R$ is applied, then $Inv2$ holds up to $tid + 1$; (ii) $Inv$ follows from the fact that $Inv2$ holds for the last value of $tid$. Formally, we have to prove

$\{ |Inv2| \} R \{ |Inv2'| \};$

$Inv2 \land (tid = n) \rightarrow Inv$

The basic step is illustrated in the next figure (Fig. 8).

Fig. 8. The basic step for the application of Hoare method: a classical triple surrounding $R$ extended with an empty contour

The new invariant $Inv2$ is $P1d \land P2d$, where $P1d, P2d$ are the following slightly more detailed variations of the previous properties $P1, P2$: 
∀ (i) or (ii) holds, where, as before, the value \( i - 1 \) is interpreted as \( tn - 1 \) for \( i = 0 \).

**Proof of P1d.** For \( \{P1d\} \) R \( \{P1d\} \), we have to prove that \( P1d \) and Q1-3 implies

\[ \text{P1d'} : token' = (white, i') \rightarrow 
\begin{align*}
&[[\forall r \in [0, i - 1]: active[r] = false \land msg[r] \subseteq \{max(tid, i), n\}] \\
&\lor (\exists k > i - 1: c[k] = black)]
\end{align*}
\]

Suppose \( token' = (white, i') \) and \( i' = i \), hence: \( tid \neq i \) or (\( tid = i \) and process \( tid \) doesn’t terminate). By P1d, either (i) or (ii) holds, where

(i) \( \exists k > i - 1: c[k] = black \): The property is preserved, i.e., \( c'[k] = black \). (Indeed, a black process may become white only if it terminates, which is not the case as \( i' = i \).)

(ii) \( \forall r \in [0, i - 1]: active[r] = false \land msg[r] \subseteq \{max(tid, i), n\} \land (\exists k > i - 1: c[k] = black) \): For the “active” part:

- If \( tid \geq i \) the property is outside of the action of R, hence still true.
- If \( tid < i \), the process \( tid \) cannot be activated by a processes \( r \) with \( r < i \) (there are no messages for \( tid \) there). On the other hand, if a process \( r \), with \( r \geq i \), activates process \( tid \), then by P2d its color \( c[r] \) is black and this contradicts this care premises.

For the second part, notice that a process \( r \) with \( r \neq tid \) has \( msg'[r] = msg[r] - \{tid\} \), while the process \( tid \) with \( tid < i \) is inactive, hence \( msg'[tid] = \emptyset \).

Suppose \( token' = (white, i') \) and \( i' = i + 1 \), hence: \( tid = i \), the process \( tid \) terminates, and \( tid \) was a white process before termination. Again by P1d, either (i) or (ii) holds, where

(i) \( \exists k > i - 1: c[k] = black \): As \( token' = (white, i') \), actually \( k > i \), hence the property is preserved.

(ii) \( \forall r \in [0, i - 1]: active[r] = false \land msg[r] \subseteq \{max(tid, i), n\} \land (\exists k > i - 1: c[k] = black) \): For \( r < i \) the proof is as before. For \( r = i \), by the previous observations, \( active'[i] = false \land msg'[i] \subseteq [i + 1, n] \), hence the property is preserved

Finally, notice that at the end of the for\( s \) statement \( tid = n \); moreover, clearly \( P1d \land (tid = n) \rightarrow P1 \).

\[ \text{11 Notice that } tid = i, \text{ hence } i + 1 = i' = tid' = max(tid', i'). \]
Proof of $P2d$. For $\{P2d\} R \{P2d\}$, we have to prove

$P2d'$: $\text{token'.col} = \text{white} \rightarrow$
\[
\forall k \in [0, tid') : \text{msg}'[k] \subseteq [0, k) \cup [tid', n) \land \\
\text{msg}'[k] \cap [0, k) \neq \emptyset \rightarrow c'[k] = \text{black}
\]

This directly follows from $P2d$ and $Q1$-$3^{12}$. Finally, after for $s$ statement $tid = n$ and $P2d \land (tid = n) \rightarrow P2$.

**Theorem 3.** The program for the dual-pass ring termination detection protocol is correct. Moreover, there is a fully formal proof of its correctness using the $STH\log_0$ inference rule defined in Sec. 5. \hfill \Box

---

$^{12}$ In the last implication, if $\text{msg}'[k] \cap [0, k) \neq \emptyset$ the process becomes black by the first part of the code of $R$. Its color may be changed to white by the last part of the code of $R$ only if the process has the token and terminates, but then the token will be black and this contradicts the premise $\text{token'.col} = \text{white}$. 