We propose to employ the circular polarization of gravitational waves emitted by core-collapse supernovae as an unequivocal indication of rapid rotation deep in their cores. It has been demonstrated by three dimensional simulations that non-axisymmetric accretion flows may develop spontaneously via hydrodynamical instabilities in the post-bounce cores. It is not surprising then that the gravitational waves emitted by such fluid motions are circularly polarized. We show in this letter that a network of the second generation detectors of gravitational waves worldwide may be able to detect such polarizations up to the opposite side of Galaxy as long as the rotation period is shorter than a few seconds prior to collapse.
to probe a rapid rotation in the core with GW signals alone once a detection of GW has been done on a network of detectors during their operations with stable outputs of Gaussian noise. In contrast to the previous studies [11], which focused on the waveforms of GW, particularly the amplitudes and characteristic frequencies, we pay attention here to circular polarizations of the GW emitted after core bounce. According to some recent three-dimensional simulations, collapse of rapidly rotating cores of massive stars will lead to the formation of non-axisymmetric, spiral patterns in the post-shock accretion flows [12,13]. From an analogy to the GW from binaries, it is expected that the GW emitted in the direction of rotation axis will have circular polarizations at the frequency just twice the rotational frequency. Although this seems to be rather obvious, the issue is, of course, whether such polarizations, if any, are observable on the terrestrial GW detectors or not. In the following, we demonstrate that the answer is positive indeed for Galactic supernovae if rotation is rapid enough. Employing the noise-free GW waveforms calculated theoretically in one of the latest 3D simulations of rapidly rotating CCSNe by Kuroda et al. [12], in their numerical models, the authors added by hand almost uniform rotations with periods of \( \sim 1.6 \times 10^{16} \text{cm}^2/\text{s} \) at the edge of uniformly rotating core with a size of \( R = 10^8 \text{ cm} \) (or \( M(R) \sim 1.3 M_\odot \)) to a 15M_\odot non-rotating progenitor model [19] and computed their evolutions up to \( \sim 50 \text{ ms} \) after core bounce. Since, [20] and [21] reported that the central angular velocity can range from \( \sim 0.15 \) to \( \sim 3 \text{ rad/s} \) for a non-magnetized 15M_\odot star at pre-collapse stage, the most rapidly rotating model with \( \pi \text{ rad/s} \) is consistent with those stellar evolution calculations. In addition, the specific angular momentum reaches \( \sim 1.6 \times 10^{16} \text{ cm}^2/\text{s} \) at the edge of uniformly rotating core with a size of \( R = 10^8 \text{ cm} \) (or \( M(R) \sim 1.315 M_\odot \)). The value is again in good agreement with that of a non-magnetized 15M_\odot model in [20].

The GWs were evaluated with the conventional quadrupole formula. They found in their fastest-rotation model that a one-armed spiral motion was formed at \( \sim 18 \text{ ms} \) post-bounce and continued to exist till the end of the simulation (see Fig. 1).

Using the coherent network analysis, we perform Monte Carlo simulations and following [16]. The noise spectrum densities for the four detectors H, L, V, and K are taken from [22,24], which the noise used for LIGO, Virgo and KAGRA might correspond to the year 2018. The actual locations and orientations of the detectors are adopted in the simulations. A Gaussian, stationary noise is produced by first generating four independent realizations of white noise with the sampling frequency of 2048Hz and then passing them through the finite impulse response filters having transfer functions that approximately match the design curves. Supernova signals are added to the simulated noises at regular intervals. The location in the sky is assumed to have the right ascension and declination of \( (0,0) \), which is not a special point in Galaxy. In fact, at this sky position and the GPS time of 1045569616, the mean-squared values of the antenna pattern \( (F_+^2 + F_\times^2)/2 \)^{1/2} for \( h_+ \) and \( h_\times \) are 0.21, 0.46, 0.41 and 0.49 for K, H, L and V, respectively. Since the average over the sky is 0.47, the position considered in this paper is not special indeed. The length of data segments is 100ms.

Figure 1 summarizes the results for the case, in which the GW source is assumed to be located at 20 kpc from the earth and observed from the rotation axis. A series of short-time (20ms) Fourier transforms are calculated to obtain the V parameter from the reconstructed waveforms at different times, to which we refer as the spectrum here. The interval chosen because the frequencies of the GWs at this early post-bounce phase of post-bounce...
FIG. 1. The original and reconstructed GWs from one-armed spiral motions in the rapidly rotating supernova core. The top left panel shows the color map of the GW emissivities for the fastest rotating model of [12]. The top right and the middle panels present the original and reconstructed \( h_+ \) and \( h_\times \). The bottom panel is the spectrogram for the Stokes V parameter. The observer is assumed to be located at 20kpc from the source and sitting on the rotation axis of the core.

are demonstrated to be higher than \( \sim \) 100Hz by previous researches (e.g. [7]) We adopt the initial time of each integration as a representative time for the interval. The origin of the time is set to the core bounce.

It is clear from the bottom panel of the figure that right-handed circular polarizations exist indeed at \( f \sim 200 \)Hz with a peak amplitude of \( \sim 5 \times 10^{-41} \) for the V parameter. The dominance of the right-handed polarization is due to the counter-clockwise rotation of the one-armed spiral pattern (see the top left panel). The signal-to-noise ratio (SNR) is roughly estimated from the so-called radial distance, which corresponds to a network SNR as a detection statistics[15]: The value of the detection statistics of 0.5 for the signal should be compared with the one of 0.05 for the noise in this model. The SNR is then estimated to be 10, which is, conservatively speaking, significant.

Kuroda et al. [12] argued that the spiral motions are induced by the propagation of acoustic waves Doppler-shifted by the rapid rotation and that the angular velocity of the spiral motion is given by the sum of the angular velocities of the proto-neutron star, \( \Omega_{rot} \), and of the acoustic waves, \( \Omega_{aco} \). If true, the detection of the circular polarization will provide us with the information on \( \Omega_{aco} \), since some numerical simulations demonstrated that rotation does not affect \( \Omega_{aco} \) very much and we have almost always \( \Omega_{aco} \sim 100 \)Hz in the early post-bounce phase [11][12].

The amplitude of the Stokes V parameter depends on the viewing angle. The observer on the rotation axis is certainly the most advantageous. The upper two panels of Figure 2 display the spectrogram for the GW seen at 45 degrees from the rotation axis. The circular polarization

is clearly seen at \( \sim 200 \)Hz also in this case. The SNR is \( \sim 6 \). The magnitude of the V parameter is \( -3.9 \times 10^{-41} \), and the reduction from the previous value, \( -5.5 \times 10^{-41} \), is mainly originated from the cosine of the viewing angle. For comparison we show the spectrogram of the GW seen from the equator in the lower panel. As expected, the circular polarization disappears almost completely although the root sum square of \( + \) and \( \times \) modes is almost unchanged from the value \( 1.0 \times 10^{-22}[\text{Hz}^{-1/2}] \) for other angles.

Discussions & Conclusion — We have demonstrated that the circular polarization of the GWs emitted by rapidly rotating supernova cores can be observed up to the opposite end of Galaxy. This is not surprising and can be expected from the back-of-the-envelope calculations: using the analogy to the GW from binary systems, which is 100% circularly polarized when observed on the orbital rotation axis, we can evaluate the V-parameter amplitude approximately based on the quadrupole formula as

\[
V^{1/2} \sim 2.1 \times 10^{-20} \left( \frac{M_{\text{eff}}}{0.5 M_\odot} \right) \left( \frac{R_{\text{eff}}}{50 \text{km}} \right)^2 \times \left( \frac{\omega}{200 \text{Hz}} \right)^2 \left( \frac{D}{10 \text{kpc}} \right)^{-1}.
\]

Here the observer is assumed to be located at a distance of D from the source on the rotation axis; \( M_{\text{eff}} \) and \( R_{\text{eff}} \) are the effective mass and radius of the matter rotating non-axisymmetrically. Plugging in appropriate values taken from the simulation, we obtain the right order of magnitude.

We have so far based our investigations of the GW circular polarization on the single model by Kuroda et al., which may be a concern. This is simply because available calculations are quite limited at present. It is true that 3D simulations of CCSN are becoming possible these days, but they are very costly and still in their in-
fancy [25][26]. The number of numerical simulations done so far is small, particularly for rapidly rotating models. Very recently Nakamura et al. [13] published their 3D Newtonian simulations of the collapse, bounce and explosion, if any, for rapidly rotating cores, employing the so-called light-bulb approximation.

We have also calculated circular polarizations for the GW from their rapidly rotating model [13], which is demonstrated in Fig. 3. It is evident from the second-from-the-top plot in the right column that the circular polarization appears weakly around 30ms and then reappears more strongly from 100ms to 200ms post bounce with short punctuations. The former corresponds to what we have discussed so far. As a matter of fact, this model has the same initial rotation velocity as the second fastest model in Kuroda et al. [12], which indeed produces weak circular polarizations at similar frequencies (\(\sim 100\text{Hz}\)). They are hence consistent with each other. What is more remarkable here is the circular polarizations observed at later times. They have higher characteristic frequencies (\(\sim 500\text{Hz}\)), possibly due to larger values of \(\Omega_{\text{aco}}\) at this phase. The stalled shock wave revolves and an explosion commences at \(\sim 240\text{ms}\) post bounce in this model and the polarization subsides quickly thereafter. We estimate that this circular polarization is detectable up to the distance of \(\sim 10\text{kpc}\) if it is observed from the rotation axis and this distance will be reduced to 7kpc if the observer is off axis by 45 degrees. As expected again, the V parameter is not seen in both Kuroda and Nakamura’s non-rotating models.

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\[\text{[1]} \text{“https://www.advancedligo.mit.edu/,”} \]
\[\text{[2]} \text{“http://wwwcascina.virgo.infn.it/advirgo,”} \]
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FIG. 3. Same as figure 2, but for the Kuroda’s second fastest rotating (upper two panels in the left column) and non-rotating (lower two panels in the left column) and the corresponding Nakamura’s models (the right column). The source is assumed to be located at $D = 10\text{kpc}$ and is observed from the pole.