The phase diagram in $T-\mu-N_c$ space

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Outline

1. Introduction
2. The percolation model
3. The deconfinement line in $(\rho, N_c)$
4. Final remarks
QCD phase diagram

- QCD in the low-temperature, high-density ($\mu_B \sim \Lambda_{QCD}$) region is poorly understood:
  - lattice QCD of little help there;
  - perturbation theory not yet available.

- Crucial ingredient for: high-density matter, HIC, neutron stars, . . .

- Valuable insight from the large-$N_c$ limit!
Large-$N_c$ approach

Idea ['t Hooft; Witten, ’70s]: assume there is a confining limit

$$N_c \to \infty , \quad g \to 0 \quad \text{such that} \quad g^2 N_c = \lambda \quad (\Lambda_{\text{QCD}} \sim N_c^0)$$

Simpler calculations (planar diagrams only):
- mesons $\to$ stable and noninteracting;
- baryons $\to$ states of $M_B \sim N_c$ and size $\sim \Lambda_{\text{QCD}}^{-1}$

Corrections appear as $1/N_c, 1/N_c^2$ (in real life: $1/3, 1/9 \ldots$)

Large-$N_c$ phase diagram [McLerran, Pisarski ’07]:

![Diagram showing the phase diagram with deconfined and hadronic (confined) regions]
Possible objections

$N_c = 3$ is not “large” . . .
{“Large $N_c$” is such that deconfinement is first order for $N_f = 2$ [P. Petreczky] }

Most important: if there is a discontinuity, how reliable are results extrapolated from $N_c = \infty$ to our world?

Compare: the “Skyrme crystal” of large-$N_c$ dense baryons $\Leftrightarrow$ the $N_c = 3$ “nuclear liquid” . . .
Large $N_c$ and deconfinement

We expect deconfinement when quark-hole (screening) is $\sim$ gluon loop (anti-screening):

$\sum_{\text{color}} \sim \sum_{\text{hole}}$

$N_c^2 \sim \mu_q^2 N_c N_f \implies \mu_q = \sqrt{N_c/N_f} \Lambda_{\text{QCD}}$

On the other hand, quarks are as close as $\sim N_c^{-1/3}$:
- at large $N_c$, they feel “asymptotically free” to reach each other.

$\implies$ Apparent paradox starting at $\mu_q \sim \Lambda_{\text{QCD}}$ . . .

Quark DOFs inside, baryonic DOFs on the surface . . .
“Quarkyonic phase”

“Quarkyonic phase” suggested by [McLerran, Pisarski ’07] for $T < T_d$ and $\mu$ high enough, that is, dense matter

- Quarks below the Fermi surface are quasi-free, while on the surface we have baryonic excitations.
  $\Rightarrow$ Pressure and entropy density scale as $\sim N_c$
  (unlike confined: $N_c^0$, or deconfined: $N_c^2$).
- “Still confined but already chirally restored”.
- It is an educated guess, quite hard to test rigorously
  (e.g. standard AdS/CFT has $N_c = \infty$ . . . ).
Goal of the model

- Suppose close-by baryons “exchange” (energy, momentum)
- *Confined* system, but net effect may be *large-scale* exchanges (on a short timescale $\sim 1/\Lambda_{\text{QCD}}$).

⇒ Identify this to **quarkyonic phase**?

- Formal framework is *(bond-)* percolation theory: call $p$ the probability of neighbours “speaking”: then $p = p_c$ is a second-order point: $p_c \simeq 0.2488$ for

- Build a model for confined baryons $\rightarrow$ find a $p(N_c, \rho, \ldots)$ $\rightarrow$ get a curve for critical $N_c^*(\rho)$:

$$N_c^* \rho \text{ percolating phase} \quad \Longrightarrow \quad \text{compare to deconfinement} \ldots$$
Model for baryonic matter

*Matter* is a cubic lattice of baryons.

A *baryon* is a hard-sphere with $N_c$ randomly-distributed quarks:

\[ f(x) \sim \theta\left(1 - \Lambda_{\text{QCD}} |x - x_{\text{centre}}| \right). \]

*Density* ↔ lattice spacing $2\epsilon\Lambda_{\text{QCD}}^{-1}$:

\[ \bar{\rho} = \frac{\bar{\rho}_0}{\epsilon^3} = \left(\frac{\Lambda_{\text{QCD}}^3}{8}\right) \frac{1}{\epsilon^3}. \]
From “interquark” to “interbaryon”

quark-quark \textit{squared propagator} \Rightarrow \text{b-b “exchange” probability}

This is done using

$$p(N_c) = 1 - \left[ \int f_A(x_A) d^3x_A \int f_B(x_B) d^3x_B \left( 1 - F(|x_A - x_B|) \right) \right]^{N_c^2}$$
Choices for “squared propagator”

strength $\lambda/N_c$, range $\propto r_T/\Lambda_{QCD}$.

- Coordinate-space step function

- Momentum-space step function

- Momentum-space rounded step

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High density: next-to-nearest neighbours important!

- Define \{p_i\} for various b-b distances, (1, 0, 0), (1, 1, 0), \ldots (2, 2, 2).
- Define \{p_i\} for various b-b distances, (1, 0, 0), (1, 1, 0), \ldots (2, 2, 2).
- With a blocking-out of step \(b = 3, \ldots 7\),
  a \(b^3\) cell becomes 1\(^3\) in a superlattice:

- Also, \(\{p_i\} \rightarrow p'\), to compare with \(p_c\) (the RG fixed point).
Getting results

Numerically get curves $p(N_c; F, \lambda, r_T, \epsilon)$.

Intersect with $p_c$, find $N^*_c(\bar{\rho}; F, \lambda, r_T)$:
Deconfinement line

Deconfinement line vs. percolation line in \((\rho, N_c)\):
a minimum \(N_c^\heartshape\) for percolation!

One such line for each \(T\), i.e. each point on:

\[ T_c \sim \frac{2}{3} \Lambda_{QCD} \]

\[ \mu_0 = N_c^{3/2} N_f^{-1/2} \Lambda_{QCD} \]

Find it with Boltzmann integrals (\(N_c\)-scaling, parametrisations . . .)

first at \(T = 0\), then at \(T > 0\) . . .
\[ T = 0 \text{ deconfinement line} \]

Implement

\[
\rho(T=0) = \frac{4\pi g_f g_s}{(2\pi)^3} \left[ \int \frac{p^2 dp}{1 + \exp \left( \frac{\sqrt{p^2 + m^2 - \mu_0}}{T} \right)} - \langle \mu_0 \leftrightarrow -\mu_0 \rangle \right]
\]

\(N_c\)-friendly change of variables

\[
\gamma = \frac{\sqrt{N_c}}{\mu_0} m ; \quad \alpha = \frac{\sqrt{N_c}}{\mu_0} \rho ; \quad \beta = \frac{\Lambda_{\text{QCD}}}{T} N_c \frac{N_c}{\sqrt{N_f}} ;
\]

At zero temperature, can be solved analytically: here for \(N_f = 1, 3\):

![Graph](image-url)
$T > 0$ deconfinement line

- Higher-spin states (a spin-flip costs $\sim \Lambda_{\text{QCD}}/N_c$)
- Antibaryons start to contribute
- Numerical computation

(some adjustment to convert the physical $\rho$ into the $\bar{\rho}$ of percolation . . . )

Analogous procedure for the energy density curve in $(e, N_c)$
(including mesons . . . )
Energy density $e$ ($N_f = 1, 3$)

$$e / \Lambda_{QCD}^4$$

$(\theta = T / T_c)$
Baryonic density $\bar{\rho}$ ($N_f = 1, 3$)

$(\theta = T / T_c )$

$\Rightarrow$ Non-trivial behaviour in $T$: temperature-dependent $N_c$ !
An analogy from condensed matter

- In the metal-insulator transition, electrons start to tunnel among the atoms’ potential wells.
- Large $N_c \leftrightarrow \text{“confined conductor” picture.}$
- Atoms remain well-defined objects; electrons delocalise: Bloch constraints $\rightarrow$ gaps in the spectral functions.

Gaps could be detected within dilepton spectral functions!

\[ \rho_q(\ M\ ) \]

- Hadronic resonance peaks, $M > 0.5$ GeV
- $(\eta, \omega, \rho, \phi, \ldots)$
- QGP Continuum
- Flavor excitations
- Color excitations
- Gap $\sim 0.2-0.4$ GeV

$M \sim \rho_B^{1/3}$
This quite rough model should be taken *cum grano salis*: it should mostly suggest a direction for more quantitative future works. (Convert the many “∼” appearing here into “=”)

If the “percolating” (quarkyonic?) phase is accessible in our $N_c = 3$ world, this happens at very low temperatures and *really high densities*. (maybe relevant for e.g. cold compact stars)

How to test this picture?
⇒ Our world has fixed $N_c$, but there is the lattice!
⇒ And, phenomenologically:
  • The $\sim N_c$ pressure jump should affect supernova explosions.
  • The percolation phase could alter the dilepton spectral function.
End of the talk.
Clues to a hidden phase transition

Take a Skyrme crystal of baryons at large $N_c$:

\[
\begin{align*}
\text{Mass} & \sim N_c \\
\text{Fermi motion energy} & \sim 1
\end{align*}
\]

\[\Rightarrow \text{Fermi momentum } \sim \sqrt{N_c}\]

Interbaryonic binding energy in the crystal \(\sim N_c\)

\[\Rightarrow \text{below some } N_c^* \text{ the crystal melts into a liquid}\]

Symmetry is changed: has a phase transition occurred? [suggested by Klebanov already in 1985...]

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Large-$N_c$ vs. $N_c = 3$ phase diagram

- $N_c$ large
- **III** Quark-Gluon plasma
- $P \sim N_c^2$
- $T_c \sim N_c^0$
- **II** Quarkyonic matter:
  - $P \sim N_c^0$
  - Nuclear matter: $P \sim N_c^0$
- **I** confined vacuum
  - $P \sim N_c^0$
  - $\mu_c \sim N_c^0$
  - $T_c \sim N_c^0$

$N_c = 3$?

Phase diagram in $T-\mu-N_c$.

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Baryon-baryon “exchange” probabilities

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Phase diagram in $T$-$\mu$-$N_c$
$N_c$-scaling of $\mu_0(T \approx 0)$

\[
\sum \text{color} \sim \sum \text{hole}
\]

\[
N_c^2 \sim \mu_q^2 N_c N_f
\]

\[
\Rightarrow \quad \mu_B(\text{deconfinement}) \sim N_c \mu_q \sim \frac{N_c^{3/2}}{\sqrt{N_f}} \Lambda_{\text{QCD}}
\]
\[ \rho_{B}^{\text{conf}} = \frac{4\pi g_{f}}{(2\pi)^{3}} \frac{N_{c}^{3}}{N_{f}^{3/2}} \Lambda_{\text{QCD}}^{3} \sum_{\eta=0,1,\ldots}^{Q} (2\eta + 2) \left\{ \int \frac{\alpha^{2} \, d\alpha}{1 + \exp \left[ \frac{3}{2} \frac{N_{c}}{\sqrt{N_{f}}} \frac{1}{\theta} \left( \sqrt{\alpha^{2} + N_{f}} + \eta \frac{\sqrt{N_{f}}}{N_{c}^{2}} - \sqrt{N_{c} \sqrt{1 - \theta^{2}}} \right) \right]} - \int \frac{\alpha^{2} \, d\alpha}{1 + \exp \left[ \frac{3}{2} \frac{N_{c}}{\sqrt{N_{f}}} \frac{1}{\theta} \left( \sqrt{\alpha^{2} + N_{f}} + \eta \frac{\sqrt{N_{f}}}{N_{c}^{2}} + \sqrt{N_{c} \sqrt{1 - \theta^{2}}} \right) \right]} \right\} \]
Energy density integral

\[ e^{\text{conf}} = N_f^2 \frac{\pi^2}{15} T^4 + e_B^{\text{conf}} \]

\[ e_B^{\text{conf}} = \frac{4\pi g_f}{(2\pi)^3} \frac{N_c^4}{N_f^2} \Lambda_{\text{QCD}}^4 \sum_\eta (2\eta + 2) \left\{ \right. \]

\[ \int \frac{\alpha^2 \left[ \sqrt{\alpha^2 + N_f} + \eta \frac{N_f}{N_c^2} \right] d\alpha}{1 + \exp \left[ \frac{3}{2} \frac{N_c}{N_f} \frac{1}{\theta} \left( \sqrt{\alpha^2 + N_f} + \eta \frac{N_f}{N_c^2} - \sqrt{N_c} \sqrt{1 - \theta^2} \right) \right]} \]

\[ + \int \frac{\alpha^2 \left[ \sqrt{\alpha^2 + N_f} + \eta \frac{N_f}{N_c^2} \right] d\alpha}{1 + \exp \left[ \frac{3}{2} \frac{N_c}{N_f} \frac{1}{\theta} \left( \sqrt{\alpha^2 + N_f} + \eta \frac{N_f}{N_c^2} + \sqrt{N_c} \sqrt{1 - \theta^2} \right) \right]} \left\} \]