The $D_{sJ}^+(2317)$: what can the Lattice say?

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We present lattice results on the scalar $D_s$ meson and comment on the $D_{sJ}^+(2317)$ state recently discovered by BaBar and confirmed by CLEO, in view of a series of theoretical claims and counter claims. Lattice predictions in the static limit indicate larger masses than observed for a scalar quark model state. Finite $c$ quark mass corrections seem to further enlarge this discrepancy, in support of a non quark-antiquark-state interpretation of experiment.

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I. INTRODUCTION

Recently the BaBar Collaboration announced the discovery of a $D_s^+$ positive parity meson at 2317\(\pm\)3 MeV and a width smaller than 10 MeV in the $D_s^+(1969)\pi^0$ channel [1]. No spin assignment has been made as yet but in view of the low mass $J = 0$ appears likely. For simplicity we shall refer to it as the scalar or the $0^+ D_s$ meson. This state was subsequently confirmed by the CLEO Collaboration [2]. CLEO also reports the observation of another state near 2460 MeV in the $D_s^+(2112)\pi^0$ channel which is consistent with having $J^{PC} = 1^+$. These discoveries triggered a series of articles with different claims. In this note we discuss the scalar state in view of recent lattice results, after briefly summarising the different interpretations.

II. QUARK MODEL OR NOT?

If one treats the charm quark as a heavy spectator, the spin and angular momentum of the light antiquark can either couple to $j = \frac{1}{2}^-$ ($l = 0$) or to $j = \frac{3}{2}^+$ and $j = \frac{1}{2}^+$ ($l = 1$). The interaction with the spectator spin will then result in a pseudoscalar-vector mass splitting for $j = \frac{1}{2}^-$, in $0^+$ and $1^+$ states for $j = \frac{3}{2}^-$ and in $1^+$ and $2^+$ states for $j = \frac{3}{2}^+$. The two $1^+$ states can undergo mixing. The pseudoscalar and vector $D_s$ states have been identified as $D_{sJ}^+(1969)$ and $D_{sJ}^+(2112)$, respectively. Then there is a $D_{sJ}^+(2536)$ state and a $D_{sJ}^+(2573)$ which, with the likely spin assignment $j = 2$ in the latter case, form the $j = \frac{3}{2}^-$ doublet. The $j = \frac{1}{2}^+$ states can strongly decay into $DK$ and $D^*K$ and are expected to be broad resonances. The new $D_{sJ}^+(2317)$ and the state at 2.46 GeV might constitute the missing doublet, where at least the former state, which lies almost 40 MeV below the $DK$ threshold, is narrow. Cahn and Jackson [3] interpret experiment in this way, in the context of a potential model.

Barnes and collaborators [4] in contrast argue that this state is most likely a $DK$ molecule since its mass is 160 MeV lighter than other potential model predictions which result in a mass around 2.48 GeV [5] for the scalar $P$-wave $D_s$. Their argument is supported by the proximity to the $DK$ threshold and they interpret this system as a generalisation of an $a_0/f_0(980)$ $KK$ molecule. A four-quark interpretation is also shared by Cheng and Hou [6]. Szczepaniak [7] argues in favour of a strong $D_s\pi$ atomic contribution.

Van Beveren and Rupp [8] also liken this state with the $a_0/f_0(980)$ but interpret it as a quark model state. In their view the $a_0/f_0$ states are part of a low lying scalar quark-antiquark nonet, together with a $\sigma(600)$ and a $\kappa(800)$. Consequently, they postulate additional scalar $D$ mesons. According to them, in both the $a_0/f_0$ and the new $D_{sJ}^+$ systems, due to mixing with the $KK$ or the $DK$ continuum, respectively, the lowest scalar nonet is artificially lowered with respect to the quark model expectation.

Bardeen et al. [9] discuss the heavy quark limit. They then follow Refs. [10,11] and interpret the $0^−−0^+$ splitting in terms of chiral symmetry. The symmetry breaking scale corresponds in leading order to the constituent quark mass in the chiral limit [10] and has been estimated to be $\Delta M \approx 338$ MeV, a value that is very close to the experimental splitting of $\approx 349$ MeV. Chiral loops however will somewhat reduce the former expectation [10]. Colangelo et al. [12] share this picture and Godfrey [13] investigates the decays that one would expect in the case of a quark-antiquark interpretation.

One should note that the vector-scalar splitting, which vanishes in the heavy quark limit, is as large as 143 MeV in the $D_s$ system, indeed an $\mathcal{O}(\Lambda/m_c)$ correction to $\Delta M$. In view of this, we would not expect the static approximation to be quantitatively correct for $D$ systems. We also remark that the $0^+$ can be interpreted as a chiral partner of the $0^-$, independent of the quark model content, as long as isospin and strangeness agree. Unfortunately, most predicted decay rates in many of the above pictures seem to be more dictated by the mass and quantum numbers of the state than by its quark content. However, in the case of an interpretation as a molecule or as part of an additional low lying scalar nonet (or triplet), an extra quark-model scalar state should still exist above the $DK$ threshold. However, this might turn out to be

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a rather broad resonance \[\Xi\]. In contrast, in a straight \(D_s\) interpretation there is no room for extra states other than the \(D_{s1}^+(2460)\) and \(D_{s1}^+(2536)\) between a \(D_{sJ}^+(2573)\) \((J = 2)\) and the newly discovered \(D_{sJ}^+(2317)\) \((J = 0)\). The chiral heavy quark interpretation results in similar predictions for \(B\) systems \([4, 10, 11]\), which in principle can be checked experimentally.

III. THE STATIC LIMIT

We will confront the new scalar state with lattice results in the static limit in the quenched approximation and for \(n_f = 2\), before discussing finite charm quark mass corrections.

In the static limit the \(j = \frac{1}{2}\) and \(j = \frac{3}{2}\) doublets will be exactly mass degenerate. We wish to calculate the \(\frac{1}{2}^-\) and \(\frac{3}{2}^+\) masses. These can be extracted from the asymptotic large \(t\) decay of the two Euclidean correlation functions,

\[ C_+(t) = U_{0,t} \text{Tr} \left( \frac{1 + \gamma_4}{2} M^{-1}_{0,t} \right), \]
\[ C_-(t) = U_{0,t} \text{Tr} \left( M^{-1}_{0,t} \gamma_4 \right), \]

respectively. We made use of the relation \(M^1 = \gamma_5 M \gamma_5\) for the Wilson-Dirac operator \(M\) and \(\gamma_4 \gamma_5 = -\gamma_5 \gamma_4\). \(U_{0,t}\) denotes the Wilson-Schwinger line, connecting the point \((x, t)\) with \((x, 0)\). The spatial coordinate \(x\) is suppressed in \(U\) as well as in \(M\) and the colour trace is implicit.

![FIG. 1](image_url)

**FIG. 1:** The \(\frac{1}{2}^- - \frac{3}{2}^+\) splitting in the static limit, as a function of the light quark mass \(\propto m_l^2/m_t^2\). Open symbols denote inter- and extrapolations to physical up/down and strange quark masses. Circles denote the \(n_f = 2\) case while squares denote the quenched approximation. The horizontal lines are the experimental values for the \(D_{sJ}^+(2317) - D_s^+(1969)\) splittings and the \(DK\) threshold.

This static-light splitting has been calculated in the quenched approximation by Michael and Peisa \([14]\) with Wilson action at \(\beta = 5.7\) and \(\beta = 6.0\). The results are in agreement with earlier references \([13, 16, 17]\) at additional lattice spacings and no significant lattice spacing dependence has been observed: the fine structure splittings, that strongly depend on short distance physics \([13]\), vanish by definition. The quenched results, extrapolated to up/down and strange quark masses are depicted in Figure 1 (open squares) where \(r_0^{-1} \approx 400\) MeV. The results are \(\Delta M_u = 384(50)\) MeV and \(\Delta M_s = 299(141)\) MeV, respectively. The splittings of the \(\frac{1}{2}^-\) states with respect to the \(\frac{3}{2}^+\) states are 434(5) MeV and 323(131) MeV for strange and up/down quarks, roughly 50 MeV larger. In contrast, the \(D_{sJ}^+(2536)\) is 120 MeV heavier than the scalar \(D_s\).

![TABLE I](image_url)

**TABLE I:** The static \(\frac{1}{2}^- - \frac{3}{2}^+\) mass splittings \(\Delta M\) for \(n_f = 2\) sea quarks \([13]\) for different hopping parameters \(\kappa\) at \(\beta = 5.6\). The numbers in the last column are subject to an additional 5% overall scale uncertainty.

| \(\kappa\) | \(r_0/a\) | \(m_p/m_Y\) | \(\Delta M_{r_0}\) | \(\Delta M/\text{MeV}\) |
| --- | --- | --- | --- | --- |
| 0.1560 | 5.11(3) | 0.834(3) | 1.16(9) | 465(35) |
| 0.1565 | 5.28(5) | 0.813(9) | 1.15(11) | 460(45) |
| 0.1570 | 5.48(7) | 0.763(6) | 1.10(13) | 440(50) |
| 0.1575 | 5.89(3) | 0.704(5) | 1.24(12) | 495(50) |
| 0.1580 | 6.23(6) | 0.574(13) | 1.08(24) | 430(110) |

Results with \(n_f = 2\) mass-degenerate flavours of Wilson sea quarks have been obtained by the SESAM Collaboration \([19]\) (circles) on slightly finer lattices. The data of Table II do not exhibit any visible light quark mass dependence. The interpolated values have been obtained from a linear fit in \(m_l^2\) (with tiny slope) and the errors of the interpolation, that have been conservatively estimated by varying fit range and functional form, are dominated by systematics. The lattice spacing obtained from the phenomenological value \(a/r_0 \approx a \times 400\) MeV is in agreement with the one obtained from \(m_p a\), within errors \([10]\). The experimental lines in the figure correspond to masses, relative to the pseudoscalar state, a somewhat arbitrary choice since vector and pseudoscalar will be degenerate in the static limit. The rationale behind this is that Ref. \([9]\) assumes the \(1^+ - 0^+\) splitting to be identical to the \(1^- - 0^-\) splitting. The inclusion of sea quarks seems to result in the slightly increased value, \(\Delta M = 468(43)(24)\) MeV for the \(s\) quark system. We do not expect a lattice spacing dependence of this number in excess of the statistical uncertainty, based on the quenched experience.

IV. FINITE MASS CORRECTIONS

Effects of the finite charm quark mass have only been investigated in the quenched approximation. In particular three studies exist: two using lattice NRQCD \([20, 21]\) to order \(1/m_t^2\) and \(1/m_l^2\), respectively (the leading cor-
rections are of order \( \alpha_s/m \) in both cases), and one using relativistic charm quarks \[^{22}\]. Both NRQCD results are consistent with each other. The study of Hein et al. \[^{20}\] has been performed at \( \beta = 5.7 \) and \( \beta = 6.2 \) for the \( B_s \) and \( B_d \) families and at \( \beta = 5.7 \) for the \( D_s \). The relativistic study has been made at \( \beta = 6.0 \) and \( \beta = 6.2 \). In the latter case we refrain from citing values for the \( B \) meson since the extrapolation of results obtained for heavy quark masses much lighter than the \( b \) is not fully under control. In none of these cases statistically significant lattice spacing effects have been observed and we display the results for the \( 0^+ - 0^- \) splittings in Table II.

TABLE II: The \( 0^+ - 0^- \) mass splitting in the heavy-light system for two sea quarks in the static limit and in the quenched approximation, for the \( B \) and \( D \) systems in NRQCD \[^{21}\] and for the \( D \) system with relativistic quarks \[^{22}\]. The errors do not include uncertainties in the overall scale which we estimate to be about 5% for \( n_f = 2 \). All numbers are in units of MeV.

| \( n_f = 2 \) | \( n_f = 0 \) |
|---|---|
| static | static | NRQCD | NRQCD | relativ. |
| \( h_B \) | 468(43) | 384(50) | 345(55) | 465(50) | 495(25) |
| \( h_D \) | 472(85) | 299(114) | 370(50) | — | 465(35) |

Note that while the NRQCD results for the \( B \) systems agree with the respective \( n_f = 0 \) static limits, the splitting is enhanced in the \( D \) system, in agreement with the fully relativistic calculation. The relativistic \( D_s \) splitting is bigger by as much as \( (29 \pm 16) \% \) with respect to the static limit. If we assume a similar increase for the case with sea quarks we would expect a splitting of 600(110) MeV for the \( D_s \) system, yielding the prediction \( m(D_s^+) = 2.57(11) \) GeV. The potential model of Ref. \[^{3}\] predicts 2.48 GeV, while the quenched results are 2.44(5) GeV (NRQCD) and 2.47(3) GeV (relativistic \( D \) quark), all significantly bigger than the candidate’s mass of 2.32 GeV. The quenched lattice results for the \( D_s \) system also indicate a tiny \( 1^{-} - 1^{+} \) splitting, suggesting that the \( 1^{+} \) state should be heavier than 2.46 GeV.

V. SUMMARY

We calculate a scalar-pseudoscalar splitting of \( \Delta M = 468(43)(24) \) MeV in the static limit for \( n_f = 2 \) sea quarks, significantly larger than the value 338 MeV suggested by a heavy quark constituent quark model \[^{11}\] and larger than the quenched QCD value \( \Delta M = 384(50)(20) \) MeV.

We also report a significant finite charm mass correction that casts doubt onto naïve generalisations to the \( B \) system. Lattice predictions on the masses are consistent with the quark model of Ref. \[^{3}\] and incompatible with the new state observed by BaBar and CLEO. We conclude that the \( D_s^+(2317) \) might receive a large \( DK \) component: the physics of this heavy scalar might indeed resemble elements of that governing the \( f_0(980) \) system. If this is the case then the masses of the up and down quarks will play a major rôle and simulations with non-mass-degenerate sea quarks are required.

Unfortunately, on the lattice the possibility of four-quark states has so far only been addressed in the static limit where attraction was reported in some channels \[^{23}\]. In view of the new experimental candidate quenched simulations of relativistic four quark molecules are urgent. To understand the exact nature of the new state not only the spectrum but also predictions of decay rates are required. While lattice calculations of strong decays are unfeasible, a study of electro-magnetic decay rates is a possibility.

Notes added in proof

The discovery of the two \( D_s \) mesons has also been confirmed by the Belle Collaboration \[^{24}\].

A new lattice study of \( D_s \) mesons by the UKQCD Collaboration has appeared recently \[^{25}\] and a paper by Terasaki \[^{24}\] on the new \( D_s \) mesons was submitted to the preprint server only one day after this article.

In view of the possibility of similar states in the \( B_s \) spectrum it appears worthwhile to mention that the static \( n_f = 2 \) lattice results presented here imply that the scalar quark model \( B_s \) meson should have a mass of 5837(43)(24) MeV, with additional \( 1/m \) corrections of order 40 MeV, possibly upwards, based on the \( D_s \) experience. This has to be compared with the \( B\bar{K} \) threshold of about 5775 MeV.

Acknowledgments

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