$B - \bar{B}$ Mixings and Rare $B$ Decays in the Topflavour Model

Kang Young Lee∗ and Jong Chul Lee†

Department of Physics, Korea Advanced Institute of Science and Technology
Taejon 305 – 701, Korea
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Abstract

We explore the impact of topflavour model to $B - \bar{B}$ mixings and rare decays of $B$ mesons. By the flavour–changing neutral current interactions of this model, the $B - \bar{B}$ mixing amplitudes are substantially affected while it is hard to investigate the effects on rare decays. Violation of the unitarity of the CKM matrix is also discussed. We find that the bound on $|V_{td}|$ can be stronger in this model by combining the experimental bound of the $B - \bar{B}$ mixing and the bound from violated unitarity.
I. INTRODUCTION

After the observation of a nonvanishing amount of $B_d - \bar{B}_d$ mixing \[1,2\], it has been one of the most interesting phenomena of the $B$ meson system. As being an flavour-changing neutral current (FCNC) process, the $B - \bar{B}$ mixing involves top quark in its loops and consequently it is sensitive to the flavour dynamics of the third generations as well as being a source of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements for the top quark and top quark mass. The strengths of $B_q - \bar{B}_q$ mixings are measured by the mass difference

$$\Delta M_q = 2|M_{12}^{(q)}|, \quad (1)$$

where $M_{12}^{(q)}$ is the off–diagonal term of the mass matrix of neutral $B^0_q$ mesons. The present world average for measured $\Delta M_d$ and the best limit for $\Delta M_s$ are reported by \[3\]

$$\Delta M_d = 0.464 \pm 0.018 \, \text{ps}^{-1}, \quad \Delta M_s > 7.8 \, \text{ps}^{-1}, \quad (2)$$

which are consistent with the Standard Model (SM) predictions and constrain the CKM parameters $V_{td}$ and $V_{ts}$:

$$0.15 < \frac{|V_{td}|}{|V_{cb}|} < 0.34, \quad \frac{|V_{ts}|}{|V_{cb}|} > 0.6 . \quad (3)$$

Please see Ref. \[4\] to understand the experimental errors and theoretical uncertainties for these constraints. This bound of $\Delta M_d$ is better than that obtained from unitarity of the CKM matrix alone, which gives

$$0.11 < \frac{|V_{td}|}{|V_{cb}|} < 0.33 . \quad (4)$$

Many models of new physics beyond the SM provide new contributions to $B - \bar{B}$ mixings which could result in altering the constraints on $V_{td}$ and $V_{ts}$ given in Eq. (3). In order to be detectable, however, new contributions to $B - \bar{B}$ mixings should be at least comparable to those of the SM. This paper focuses on the model that include the separate SU(2) group for the third generations, so–called topflavour model. This model alters the mass difference $\Delta M_q$ in two distinct ways: the additional contributions to the box diagrams and the FCNC
interactions to $Z$ and $Z'$ bosons. Since the couplings of the left–handed third generation fermions are different from those of the first and second generations, the FCNC interactions appear at tree level and it renders one of the characteristic features of the topflavour model. We show that the new FCNC effects can be as large as that of the box diagram of the SM. When the $B - \bar{B}$ mixings are affected, generically rare decay modes of $B$ mesons via penguin diagrams are also affected by this new physics. Thus it is demanded to investigate the flavour–changing rare decays in the topflavour model. The new contributions are also due to the FCNC interactions at tree level as well as the additional contributions to the penguin diagrams.

This model was constrained by the LEP data in Ref. [5–7] and the lower bound of the mass of the additional $Z'$ boson is predicted to be about 1.1 TeV. We expect that the model could be also examined by the current data of $B - \bar{B}$ mixings and rare decay here. In this model, the CKM matrix is in general not uniquely defined but depends upon each bases of $U$–type and $D$–type quarks. Accordingly an additional phase can be introduced in the model to produce the new CP violating phenomena and more parameters are needed to represent quark mixings. In this paper, however, we choose the case that the weak eigenstates of $U$–type quarks are identified as mass eigenstates for simplicity: $V_U = I$ and $V_D = V_{CKM}^\theta$ respectively, where $V_U(D)$ represents the unitary transformation diagonalizing $U(D)$–type quark mass matrix. Then we have no more phases and no additional CP violating phenomena. Even in this case the CKM matrix is no more unitary.

This paper is organized as follows. In section II, we briefly review the topflavour model. The unitarity violation of the CKM matrix is discussed and the FCNC is described. The contributions to the $B - \bar{B}$ mixings are studied in section III and the contributions to the flavour–changing penguin decays are examined in section IV. We conclude in section V.
We study the topflavour model with the extended electroweak gauge group $SU(2)_l \times SU(2)_h \times U(1)_Y$. The first and the second generations couple to $SU(2)_l$ and the third generation couples to $SU(2)_h$. This gauge group can arise as the theory at the intermediate scale in the pattern of gauge symmetry breaking of noncommuting extended technicolor (ETC) models, in which the gauge groups for extended technicolor and for the weak interactions do not commute. The left-handed quarks and leptons in the first and second generations transform as $(2,1,1/3), (2,1,-1)$ under $SU(2)_l \times SU(2)_h \times U(1)_Y$, and those in the third generation as $(1,2,1/3), (1,2,-1)$ while right-handed quarks and leptons transform as $(1,1,2Q)$ where $Q$ is the electric charge of fermions.

The covariant derivative is given by

$$D^\mu = \partial^\mu + i g_l T^a_l W^\mu_{la} + i g_h T^a_h W^\mu_{ha} + ig' Y B^\mu,$$

(5)

where $T^a_l$ and $T^a_h$ denote the $SU(2)_{l,h}$ generators and $Y$ is the $U(1)$ hypercharge generator. Corresponding gauge bosons are $W^\mu_{la}, W^\mu_{ha}$ and $B^\mu$ with the coupling constants $g_l, g_h$ and $g'$ respectively. The gauge couplings may be written as

$$g_l = \frac{e}{\sin \theta \cos \phi}, \quad g_h = \frac{e}{\sin \theta \sin \phi}, \quad g' = \frac{e}{\cos \theta}$$

(6)

in terms of the weak mixing angle $\theta$ and the new mixing angle $\phi$ between $SU(2)_l$ and $SU(2)_h$ defined in eq. (3) below.

The symmetry breaking is accomplished by the vacuum expectation values (VEV) of two scalar fields $\Sigma$ and $\Phi$: $\langle \Phi \rangle = (0, v/\sqrt{2})^\dagger$, $\langle \Sigma \rangle = uI$ where $I$ is $2 \times 2$ identity matrix. The scalar field $\Sigma$ transforms as $(2,2,0)$ under $SU(2)_l \times SU(2)_h \times U(1)_Y$ and we choose that $\Phi$ as $(2,1,1)$ corresponding to the SM Higgs field. In the first stage, the scalar field $\Sigma$ gets the vacuum expectation value and breaks $SU(2)_l \times SU(2)_h \times U(1)_Y$ down to $SU(2)_{l+h} \times U(1)_Y$ at the scale $\sim u$. The remaining symmetry is broken down to $U(1)_{em}$ by the the VEV of $\Phi$ at the electroweak scale. Since the third generation fermions do not couple to Higgs fields
with this particle contents, they should get masses via higher dimensional operators. The
different mechanism of mass generation could be the origin of the heavy masses of the third
generation. Or it is possible to explain it by introducing another Higgs doublet coupled to
the third generations as in Ref. [6].

We demand that both SU(2) interactions are perturbative so that the value of the mixing
angle \( \sin \phi \) is constrained \( g^2_{(l,h)}/4\pi < 1 \), which results in \( 0.03 < \sin^2 \phi < 0.96 \). In fact, we
are interested in the region \( \phi < \pi/2 \) leading to \( g_h > g_l \) where the third generation coupling
is stronger than those of the first and second generations. We also assume that the first
symmetry breaking scale is much higher than the electroweak scale, \( v^2/u^2 \equiv \lambda \ll 1 \).

In terms of mass eigenstates of gauge bosons the covariant derivative can be rewritten
as
\[
D_\mu = \partial_\mu + \frac{ie}{\sin \theta} \left( T^\pm_h + T^\pm_l + \lambda \sin^2 \phi \left( \cos^2 \phi T^\pm_h - \sin^2 \phi T^\pm_l \right) \right) W^\mu_\pm \\
+ \frac{ie}{\sin \theta} \left( \cos \phi \sin \phi T^\pm_h - \sin \phi \cos \phi T^\pm_l - \lambda \sin^2 \phi \cos \phi \left( T^\pm_h + T^\pm_l \right) \right) W'^\mu_\pm \\
+ \frac{ie}{\sin \theta} \left( \cos \phi T_{3h} + \sin \phi T_{3l} - Q \sin^2 \theta + \lambda \sin^2 \phi \left( \cos^2 \phi T_{3h} - \sin^2 \phi T_{3l} \right) \right) Z_\mu \\
+ \frac{ie}{\sin \theta} \left( \cos \phi T_{3h} - \sin \phi \cos \phi T_{3l} - \lambda \sin^3 \phi \cos \phi \cos^2 \theta \left( T_{3h} + T_{3l} - Q \sin^2 \theta \right) \right) Z'_\mu \\
+ ieQA_\mu \tag{7}
\]

where \( Q \) is the electric charge operator, \( Q = T_{3l} + T_{3h} + Y/2 \). The additional gauge bosons
get masses such as
\[
m^2_{W'^\pm} = m^2_{Z'} = m^2_0 \left( \frac{1}{\lambda \sin^2 \phi \cos^2 \phi} + \tan^2 \phi \right) , \tag{8}
\]
while the ordinary gauge boson masses are given by \( m^2_{W^\pm} = m^2_0 (1 - \lambda \sin^4 \phi) = m^2_0 \cos^2 \theta \)
where \( m_0 = ev/(2\sin \theta) \).

The couplings to the gauge bosons for the third generations are different from those of
the first and second generations and we separate the nonuniversal part from the universal
part. First we consider the charged current:
\[
\mathcal{L}^{(cc)} = \mathcal{L}^{(cc)}_I + \mathcal{L}^{(cc)}_A , \tag{9}
\]
where $L_{I}^{(cc)}$ denotes the universal part and $L_{3}^{(cc)}$ the nonuniversal part. We consider the unitary matrices $V_U$ and $V_D$ diagonalizing $U$–type and $D$–type quark mass matrices respectively. The universal part is given by

$$L_{I} = \bar{U}_{L} \gamma_{\mu} [G_{L} W_{\mu} + G'_{L} W'_{\mu}] (V_{U}^{T} V_{D}) D_{L} + \text{H.c.} \tag{10}$$

where

$$G_{L} = -\frac{g}{\sqrt{2}} \left( 1 - \lambda \sin^{4}\phi \right) I$$

$$G'_{L} = \frac{g}{\sqrt{2}} \left( \tan \phi + \lambda \sin^{3}\phi \cos \phi \right) I \tag{11}$$

with the $3 \times 3$ identity matrix $I$ and $U = (u, c, t)^{T}$ and $D = (d, s, b)^{T}$. We define the unitary matrix $V_{CKM}^{0} \equiv V_{U}^{T} V_{D}$ which is corresponding to the CKM matrix of the SM. The nonuniversal part is written in terms of mass eigenstates as:

$$L_{3}^{(cc)} = (V_{U}^{*} \bar{u}_{L} + V_{U32}^{*} \bar{c}_{L} + V_{U33}^{*} \bar{t}_{L})$$

$$\times \gamma_{\mu} (X_{L} W_{\mu}^{+} + X'_{L} W'_{\mu}^{+})(V_{D31} d_{L} + V_{D32} s_{L} + V_{D33} b_{L}) \tag{12}$$

where

$$X_{L} = -\frac{g}{\sqrt{2}} \lambda \sin^{2}\phi .$$

$$X'_{L} = -\frac{g}{\sqrt{2}} \left( \frac{1}{\sin \phi \cos \phi} \right). \tag{13}$$

Because of the existence of $L_{3}$, the quark mixing matrix is no more unitary. Moreover it cannot be uniquely defined but depends upon the elements of each matrices $V_U$ and $V_D : \{V_{U3j}, V_{D3k}\}$. Hence the number of parameters are doubled. With this property, it would be very interesting to study the model since we expect lots of new phenomena, e.g. additional CP violating phases. At the same time, however, it is too complex and exhausting to analyze them fully. We choose the simplest bases here: $V_{U} = I$ and $V_{D} = V_{CKM}^{0}$ in order to avoid so
tedious analysis because we just intend to investigate the impact of the topflavour model. As a matter of fact, our choice is not so arbitrary but has a few reason in its own way. In this basis, only three elements of $V_{td}, V_{ts}, V_{tb}$ are altered which are not experimentally examined yet. Measured values of other six elements almost show unitarity at present. And our basis provides the economical extension which means that no new parameters do not enter the theory for the quark mixing matrix. As we see below, additional contributions to $B - \bar{B}$ mixings and rare decays contain the same CKM factors as appear in the SM. Then we have the modified CKM matrix in the lagrangian:

$$V_{CKM} = V_{0}^{0}{\mathcal{C}K}{M} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_{td}^{0} & V_{ts}^{0} & V_{tb}^{0} \end{pmatrix} \cdot \lambda \sin^{2} \phi ,$$ (14)

which describes the quark mixings for the charged currents coupled to the $W^{\pm}$ bosons. The mixing matrix for $W'^{\pm}$ bosons has the same structure as above matrix while the model parameter $\lambda \sin^{2} \phi$ is changed to $1/\sin \phi \cos \phi$.

Considering the neutral current interaction terms, we face up to more interesting outcomes. The nonuniversal terms bring forth the flavour–changing neutral current interactions at tree level which does not exist in the SM. We discussed the FCNC effects in Ref. [7], especially stressed on the lepton number violating processes. For the quark sector, the FCNC interaction terms evolve with the basis used above as follows

$$\mathcal{L}^{(nc)}_3 = (V_{D31}^{*} d_{L} + V_{D32}^{*} s_{L} + V_{D33}^{*} b_{L}) ,$$

$$\times \gamma^{\mu}(Y_{L} Z_{\mu}^{0} + Y_{L}' Z_{\mu}^{0})(V_{D31} d_{L} + V_{D32} s_{L} + V_{D33} b_{L}) ,$$ (15)

where

$$Y_{L} = \frac{g}{2 \cos \theta} \cdot \lambda \sin^{2} \phi$$

$$Y_{L}' = \frac{g}{2} \cdot \frac{1}{\sin \phi \cos \phi} .$$ (16)

These would yield the main additional contributions to the $B - \bar{B}$ mixings and rare decays via $Z$ and $Z'$ exchange diagrams, which will be discussed in the subsequent sections.
III. B – \bar{B} MIXING

The $B_q - \bar{B}_q$ mixings in SM are described by the off-diagonal term $M_{12}^{(q)}$ of the mass matrix of $B_q^0$ [2]:

$$M_{12}^{(q)} = \frac{G_F m_{B_q} \eta_{B_q} m^2_w}{2 \pi^2} f^2_{B_q} B_{B_q} x_t f_2(x_t) (V_{tq} V^*_{tb})^2,$$  \hspace{1cm} (17)

where $x_t = m^2_t/m^2_w$ and

$$f_2(x) = \left[\frac{1}{4} + 9 \frac{1}{4 (1 - x)} - \frac{3}{2 (1 - x)^2} - \frac{3}{2 (1 - x)^3}\right]$$

and $q = d, s$. $\eta_{B_q}$ denotes the QCD correction and $f^2_{B_q} B_{B_q}$ represents our lack of knowledge of the hadronic matrix elements. Their values are quoted in Ref. [4]: $\eta_{B_q} = 0.55$ and $f_{B_d} \sqrt{B_{B_d}} = 200 \pm 40$.

There are three kinds of additional contributions in topflavour model to $B – \bar{B}$ mixing amplitude as well as $W$–mediated box diagram of the SM. One of them comes from another box diagrams containing $W'$ and others from tree level diagrams with FCNC couplings involving $Z$ and $Z'$ bosons. The relevant couplings to $W'$ bosons yielding leading contributions in the order of $\lambda$ are given by

$$L_{W'} = \frac{g \cot \phi}{\sqrt{2}} t_L \gamma^\mu (V_{td} d_L + V_{ts} s_L + V_{tb} b_L) W'_\mu \gamma^\nu + H.c. ,$$  \hspace{1cm} (19)

while the relevant FCNC couplings given by

$$L^Z_{FCNC} = \frac{g}{2 \cos \theta} \lambda \sin^2 \phi \left[V_{ts}^* V_{tb} s_L \gamma^\mu b_L + V_{td}^* V_{tb} d_L \gamma^\mu b_L \right] Z_\mu + H.c.$$  \hspace{1cm} (20)

$$L^{Z'}_{FCNC} = \frac{g}{2 \sin \phi \cos \phi} \left[V_{ts}^* V_{tb} s_L \gamma^\mu b_L + V_{td}^* V_{tb} d_L \gamma^\mu b_L \right] Z'_\mu + H.c.$$  \hspace{1cm} (20)

The new contribution by the box diagrams with one $W'$ boson and with one $W$ boson is given by

$$M_{12}^{W'} = \frac{G_F m_{B_q} \eta_{B_q} m^2_w}{3 \pi^2} f^2_{B_q} B_{B_q} x_t f'_2(x_t, x_{w'}) (V_{tq} V^*_{tb})^2 \cdot \cot^2 \phi ,$$  \hspace{1cm} (21)

where $x_{w'} = m^2_{w'}/m^2_w$ and the function $f'_2(x)$ is given by
\[ f'_2(x, y) = \frac{1}{4y(x-y)^2(1-x)^2} \left[ (1-x)(4x^2 + 4y^2 + 5x^2y - 8xy - 4xy^2 - x^2) 
- 3x^2(x-2y+xy) \log \left( \frac{y}{x} \right) - 3x(x-y)^2 \log \left( \frac{y}{y-1} \right) \right]. \quad (22) \]

We can see that this function goes to \( f_2(x) \) in Eq. (18) when \( y \to 1 \). Contributions from the \( Z^- \) and \( Z'^- \)-mediated FCNC interactions are as follows:

\[
M_{12}^Z = \frac{\sqrt{2}G_F m_B q f^2_{Bq} \eta_{Bq} \cdot \lambda^2 \sin^4 \phi (V^*_{tq}V_{tb})^2}{12} \]

\[
M_{12}^{Z'} = \frac{\sqrt{2}G_F m_B q f^2_{Bq} \eta_{Bq} \cdot \lambda (V^*_{tq}V_{tb})^2}{12}. \quad (23) \]

Note that the contribution from \( Z'^- \)-mediated FCNC is of order of \( \lambda \) while the other contribution from \( Z \) boson are of order of \( \lambda^2 \). Thus we expect that the \( Z'^- \)-mediated FCNC diagrams provide the dominant contribution among new physics effects. We also note that all new physics contributions to \( M_{12}^{(q)} \) involve the same CKM factors as those of the SM and has the same phase as the \( W^- \)-mediated box diagrams in our choice of the bases for quarks.

We estimate each contributions comparing to the mass difference of the SM,

\[
\frac{\Delta M_{Z}^{Z}}{\Delta M_{Z}^{W}} = \lambda^2 \sin^4 \phi \frac{\sqrt{2} \pi^2}{G_F m_w^2} \frac{1}{x_t f_2(x_t)} \approx 71.9 \lambda^2 \sin^4 \phi \sim 5 \times 10^{-4},
\]

\[
\frac{\Delta M_{Z}^{Z'}}{\Delta M_{Z}^{W}} = \lambda \frac{\sqrt{2} \pi^2}{G_F m_w^2} \frac{1}{x_t f_2(x_t)} \approx 71.9 \lambda \sim 0.86,
\]

\[
\frac{\Delta M_{Z}^{W'}}{\Delta M_{Z}^{W}} = 2 \cot^2 \phi \frac{f'_2(x_t, x_w')}{f_2(x_t)} \approx 2 \times 10^{-2} \quad (24)
\]

where we have taken \( |V_{tb}| = 1 \) and \( m_t = 175 \text{ Gev} \). To estimate numerical values, we use \( \lambda = 0.012 \) and \( \sin^2 \phi = 0.22 \) which give the central values of the measurement for the precision variables \( \epsilon_1 \) and \( \epsilon_b \) obtained in Ref. 7. According to above results, we find that mass differences normalized by the \( W^- \)-mediated box diagram are independent of the light quark type in the neutral \( B \) mesons. We also find that the \( Z'^- \)-mediated FCNC interactions dominates and is comparable to the usual box diagram contribution. The new physics contributions change the bounds of CKM matrix element \( V_{td} \) and \( V_{ts} \) as follows:

\[
0.08 < \left| \frac{V_{td}}{V_{cb}} \right| < 0.18, \quad \left| \frac{V_{ts}}{V_{cb}} \right| > 0.32. \quad (25)
\]
As explained in the previous section, the CKM matrix of this model is no more unitary. In the case of $q = d$, the size of the unitarity violation is measured by

$$V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb} = 2V_{td}^0V_{tb}^0 \cdot \lambda \sin^2 \phi$$

which is of the linear order of $\lambda$. With the values of $\lambda$ and $\sin^2 \phi$ used above, we obtain the bound for $|V_{td}|$:

$$0.12 < \left| \frac{V_{td}}{V_{cb}} \right| < 0.33 \ ,$$

which is close to the unitary bound of the SM. It is because the size of unitary violation is indeed small, $|V_{td}^0V_{tb}^0 \cdot \lambda \sin^2 \phi| \sim 10^{-6}$. Combining the unitarity bound with the bound from $B - \bar{B}$ mixing, we could obtain the stronger bound on the value of $|V_{td}|$, $0.12 < |V_{td}/V_{cb}| < 0.18$, than that of the SM.

IV. RARE DECAYS OF $B$ MESONS

Let us now examine the contributions of $Z$– and $Z'$–mediated FCNC processes to rare decays of neutral B mesons in topflavour model. Considering the rare decays, we have to explore the uncertainties of the predictions as well as the actual size of the branching ratios [11]. New physics effects will be considered important in a particular penguin decay only if they change the branching ratio by quite a bit more than the uncertainty in the SM prediction.

We are not interested in $b \rightarrow q\gamma$, $q = d, s$ decays since this model does not alter the coupling with photon. The annihilations $B_q^0 \rightarrow l^+l^-$, $q = d, s$ are quite interesting. Although the decay rates of $B_q^0 \rightarrow l^+l^-$ have some hadronic uncertainties dependent upon the decay constant $f_{B_q}$, they can be calculated rather precisely since the renormalization–scale uncertainty is quite small by including the QCD corrections [12]. The branching ratios are as follow [11]:

$$Br(B_s^0 \rightarrow \tau^+\tau^-) = (7.4 \pm 2.1) \times 10^{-7} \left( \frac{f_{B_s}}{232 \text{ Mev}} \right)^2 ,$$
\[
Br(B_s^0 \to \mu^+\mu^-) = (3.5 \pm 1.0) \times 10^{-9} \left( \frac{f_{B_s}}{232 \text{ Mev}} \right)^2, \\
Br(B_d^0 \to \tau^+\tau^-) = (3.1 \pm 2.9) \times 10^{-8} \left( \frac{f_{B_d}}{200 \text{ Mev}} \right)^2, \\
Br(B_d^0 \to \mu^+\mu^-) = (1.5 \pm 1.4) \times 10^{-10} \left( \frac{f_{B_d}}{200 \text{ Mev}} \right)^2,
\]

in the SM. At present, the restrict experimental upper bounds are \(Br(B_s^0 \to \mu^-\mu^+) < 8.4 \times 10^{-6}\) and \(Br(B_d^0 \to \mu^-\mu^+) < 1.6 \times 10^{-6}\) [13] and roughly four orders of magnitude larger than the SM predictions.

In the topflavour model, the decay processes due to \(Z\)– and \(Z'\)–mediated FCNC couplings give the amplitudes:

\[
\mathcal{M}_Z = -\frac{g^2}{4m^2} \sin^2 \phi V_{td}^* V_{tb}^* \bar{q}^\mu \left[ g_L^\mu P_L + g_R^\mu P_R \right] l, \\
\mathcal{M}_{Z'} = -\frac{g^2}{4m^2} \sin \phi \cos \phi V_{td}^* V_{tb}^* \bar{q}^\mu \left[ g_L^\mu P_L + g_R^\mu P_R \right] l, \tag{29}
\]

where the shifted couplings are given by

\[
g_e^L = g_L^\mu = (-\frac{1}{2} + \sin^2 \theta) + \frac{\lambda}{2} \sin^4 \phi, \\
g^-_L = (-\frac{1}{2} + \sin^2 \theta) - \frac{\lambda}{2} \sin^2 \phi \cos^2 \phi, \\
g^L_R = \sin^2 \theta, \quad l = e, \mu, \tau,
\]

and the couplings to \(Z'\) are

\[
g^e_L = g^\mu_L = -\frac{\sin \phi \cos \phi}{\cos^2 \phi} - \frac{\lambda \sin^3 \phi \cos \phi}{\cos^2 \theta} (1 - 2 \sin^2 \theta), \\
g^-_L = \frac{\cos \phi}{2 \sin \phi} - \frac{\lambda \sin^3 \phi \cos \phi}{\cos^2 \theta} (1 - 2 \sin^2 \theta), \\
g^L_R = -\frac{\lambda \sin^3 \phi \cos \phi \tan^2 \theta}{\cos^2 \phi}, \quad l = e, \mu, \tau. \tag{31}
\]

We obtain the branching ratios of rare decays due to FCNC interactions of Eq. (29) as

\[
Br(B_s^0 \to \tau^+\tau^-)_{\text{FCNC}} \simeq 9.5 \times 10^{-7} \left( \frac{f_{B_s}^2}{232 \text{ Mev}} \right)^2, \\
Br(B_d^0 \to \mu^+\mu^-)_{\text{FCNC}} \simeq 1.2 \times 10^{-9} \left( \frac{f_{B_d}^2}{232 \text{ Mev}} \right)^2.
\]
\[ Br(B_d^0 \to \tau^+\tau^-)_{FCNC} \simeq 6.5 \times 10^{-8} \left( \frac{f_{B_d}^2}{200 \text{ Mev}} \right)^2, \tag{32} \]
\[ Br(B_d^0 \to \mu^+\mu^-)_{FCNC} \simeq 8.2 \times 10^{-11} \left( \frac{f_{B_d}^2}{200 \text{ Mev}} \right)^2, \]

where the values of \( \lambda \) and \( \sin^2 \phi \) are taken as used in the previous section. We find that the contributions to the decays into \( \mu \) pair is less than those of the SM predictions, while the contributions to the decays into \( \tau \) pair are comparable with the SM predictions. This is caused by the fact that the couplings of \( \tau \) pair to \( Z \) and \( Z' \) bosons are stronger than those of \( e \) and \( \mu \) pairs. The enhancement of the decay rate of \( B_s^0 \to \tau^+\tau^- \) can be observed since the total branching ratio will be four times of the theoretical uncertainty higher from the SM prediction. However the decay rates into \( B_d^0 \to \tau^+\tau^- \) are at most two times of the theoretical uncertainty higher and consequently this can only be considered marginal signals of new physics at best.

The gluon–mediated hadronic decays and \( b \to ql^+l^- \) processes have larger errors than the annihilation decays \[11\] and it is hard to expect the enhancement of the effects of the topflavour model by order of magnitudes since new contributions of this model always have the same CKM factors as those of the SM. Thus we hardly expect to find the new physics signals in the gluon–mediated hadronic decays and \( b \to ql^+l^- \) decays, and we do not show the explicit calculations for those processes here.

**V. DISCUSSIONS AND CONCLUSION**

We study the effects of the topflavour model on the \( B - \bar{B} \) mixings and rare decays. In the generic model containing the FCNC interactions at tree level \[14\], the rare decays are much enhanced by one or more order of magnitudes due to the FCNC interactions when the new physics effect on the \( B - \bar{B} \) mixings are observable \[15\]. However we find the sizable contributions of this model to the \( B_d - \bar{B}_d \) mixings while there is no ”smoking gun” signals in rare \( B_d^0 \) decays. In fact, the FCNC coupling of this model itself is too small to produce significant contributions to decay rates. As estimated in Ref. \[15\], the FCNC couplings have
to be of order of $10^{-3}$ to give substantial enhancements of branching ratios of rare decays enough for observation. In the topflavour model, the couplings are suppressed by the CKM matrix elements $|V_{td}| \sim 10^{-3}$ or $|V_{ts}| \sim 10^{-2}$ as well as the model parameters $\lambda \sim 10^{-2}$ and $\sin^2 \phi \sim 10^{-1}$. The coupling to $Z'$ boson is not suppressed by $\sin^2 \phi$ but still of order of $10^{-5}$. In the $B - \bar{B}$ mixing amplitude, the contribution of $Z'$ exchange diagram is rather large and even comparable to the SM contribution. Contribution of this diagram is of order of $\lambda$ while that of the $Z$ exchange diagram is of order of $\lambda^2$. And the FCNC coupling to $Z'$ is not suppressed by $\sin^2 \phi$. (Note that both contributions to the branching ratios of rare decays are always of order of $\lambda^2$.) Meanwhile the $W'$ mediated box diagrams is of order of $\lambda$, but suppressed by loop integrals.

We showed that the bound of $|V_{td}|$ from the $B_d - \bar{B}_d$ mixing can be considerably changed in the topflavour model with the value of $\lambda$ and $\sin \phi$ fitted by the LEP data. Therefore when we directly measure the very low value of $|V_{td}|$ in the future and rare decay rates of $B^0_d$ are still consistent with the SM predictions, it would be a critical test of the topflavour model.

Most of the experimental bounds for flavour-changing processes are not so strong that cannot further constrain this model. However, we can find interesting possibility. If we have the better experimental bound on $|V_{td}|$ from the $B_d - \bar{B}_d$ mixing, it is possible that the value of $|V_{td}|$ could not satisfy the $B_d - \bar{B}_d$ mixing bound and the unitarity bound at the same time in some region of $(\lambda, \sin^2 \phi)$ parameter space because the $B_d - \bar{B}_d$ mixing bound is shifted from that of the SM in this model. Then we will have an additional constraint on the model parameters to avoid such contradiction.

Finally we wish to remark the general treatment on the quark mixing matrix. In this paper, all couplings to the charged currents not including $t$ quark contain the common factor $(1 - \lambda \sin^4 \phi)$ of which effect is washed out by introducing the Fermi constant defined by $\mu$ decay. When we take general unitary matrices for $V_U$ and $V_D$, each charged current coupling is differently changed, which depends on the elements $V_{U_{ij}}$ and $V_{D_{ij}}$. If so, we have more undetermined parameters and we should restrict them by many low-energy processes such
as $\beta$-decay, $\pi$ decays, $K$ decays etc.. But it is still hard to define the CKM matrix in that case. More interestingly additional phase is introduced from $V_U$ in the general case. It provides many new possibilities of CP violating phenomena which is not tested yet.

In conclusion we considered the effects of the topflavour model in the $B_d - \bar{B}_d$ mixings and rare $B$ decays. We found that the $B_d - \bar{B}_d$ mixings are substantially affected but rare decays of $B_d$ are not because the effects on the $B_d - \bar{B}_d$ mixings are of order of $\lambda$ while those on the rare decays of $\lambda^2$. This may be a characteristic feature of this model while other new physics containing tree level FCNC interactions do not have.

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REFERENCES

[1] C. Albajar et al., UA1 collaboration, Phys. Lett. B 186, 247 (1987).

[2] H. Albrecht et al., ARGUS collaboration, Phys. Lett. B 192, 245 (1987).

[3] L. Gibbons (CLEO collaboration), Invited talk at the International Conference on High Energy Physics, Warsaw, Poland, ICHEP 96 (1996); ALEPH collaboration, Contributed paper to ICHEP 96 (1996).

[4] A. Ali and D. London, DESY Report No. DESY 96–140, hep-ph/9607392.

[5] E. Malkawi, T. Tait and C.-P. Yuan, Phys. Lett. B 385, 304 (1996).

[6] D. J. Muller and S. Nandi, Phys. Lett. B 383, 345 (1996).

[7] J. C. Lee, K. Y. Lee and J. K. Kim, Report No. KAIST-TH-97/19, hep-ph/9711509, to appear in Phys. Lett. B.

[8] R. S. Chivukula, E. H. Simmons and J. Terning, Phys. Lett. B 331, 383 (1994); Phys. Rev. D 53, 5258 (1996).

[9] J. Hagelin, Nucl. Phys. B 193, 123 (1981); T. Inami and C. S. Lim, Prog. Theo. Phys. 65, 297 (1981); 65, 772 (E) (1982); A. Buras, W. Slominski and H. Steger, Nucl. Phys. B 238, 529 (1984); B 245, 369 (1984).

[10] D. S. Kestenbaum, Report No. FERMILAB-CONF-97-016, presented at the 16th International Conference on Physics in Collision, Mexico (1996).

[11] For a review, see A. Ali, DESY Report No. DESY 96–106, hep-ph/9606324 and references therein.

[12] G. Buchala and A. J. Buras, Nucl. Phys. B 400, 225 (1993).

[13] F. Abe et al., CDF collaboration, Phys. Rev. Lett. 76, 4675 (1996).

[14] Y. Nir and D. Silverman, Phys. Rev. D 42, 1477 (1990); D. Silverman, ibid 45, 1800
(1992); G. C. Branco, T. Morozumi, P. A. Parada and M. N. Rebelo, *ibid* 48, 1167 (1993); V. Barger, M. S. Berger and R. J. N. Phillips, *ibid* 52, 1663 (1995).

[15] M. Gronau and D. London, Phys. Rev. D 55, 2845 (1997).