Analysis of almost-periodic and almost-proportional characteristics of a representative sample local minima time series

A A Paramonov, V I Kuzmin and R I Dzerjinsky

MIREA – Russian Technological University, 78, Vernadsky pros., Moscow, 119454, Russia

Email: paramonov_a_a99@mail.ru

Abstract. The methods' problem for analyzing nonlinear oscillations is now a general scientific and fundamental one, especially in the information systems field with a data large amount. In this paper, the analysis is carried out on an exchange data test example to identify hidden connections, general patterns, trends and fluctuations. Based on the analysis results, it is expected to reveal the desired ergodic dependence, as well as the data behaviour distinctive features in the almost-periods and near-proportions identified areas.

1. Introduction

The time series specificity is that they represent the processes' interaction result occurring on fundamentally different time scales [1, 2]. This leads to the need to divide the original time series into components that characterize the so-called fast and slow movements. As a result, the fast movements' analysis makes it possible to determine the time intervals' hierarchy that are closest to periods (near-periods).

Knowing the near-periods allows to use them as smoothing intervals for the initial series to determine the trends' hierarchy corresponding to the near-periods obtained values.

Slow movements, as the main trends, are also analyzed directly, without excluding fast movements, based on nonlinear transformations (anamorphoses), which translate the initial data into piecewise linear dependencies. As a result, the time series general structure is determined by comparing the near-periods and trends characteristics.

Also, there is a near-proportions problem, which can be identified both with the algorithms help for finding near-proportions and can be reproduced through the near-periods proportions [3-5].

2. Methods

To analyze the data, methods will be used to determine almost-periodic and almost-proportional characteristics. To begin with, consider a method for finding characteristic near-periods in a time series.

A function period is such a positive number, which property is to repeat the function values through an independent variable change interval equal to the period, i.e. \( f(t + \tau) - f(t) = 0 \).

However, since real data have to deal with nonlinear fluctuations, and therefore, pure periods are quite rare. As a result, the task is to identify the values that are closest to the periods. Such values are
called near-periods. That is, \( \tau \) is an almost-period if the inequality holds for it, \( |f(t+\tau)-f(t)|<\epsilon \), where \( \epsilon > 0 \) is the offset.

Then, for the discrete case, if \( n \) is the function \( f(t) \) samples total number given by the experimental values, we obtain a function that is called the shift function and has the form:

\[
a(\tau) = \frac{1}{n-\tau} \sum_{t=1}^{n-\tau} |f(t+\tau) - f(t)|
\]

Then, revealing the almost-periods \( \tau \) of the function \( f(t) \) can be defined as a function \( a(\tau) \) local minima set. Moreover, the greater the depth of the minima on the shift function graph, the closer to the true period the time interval value corresponding to this minimum, and the higher its significance among all near-periods of this time series [6].

When analyzing data, you can often come across the ratio scales use. When constructing such scales, the problem is to determine such numbers that unifiedly determine the systems' structure. It is known that such numbers are associated, in particular, with the Neper's number \( e=2.71828 \ldots \), it is also the exponent, which is the natural logarithms' base.

In natural systems, an interacting structural levels hierarchy is implemented (figure 1), generated according to the following rule - each next higher level has members of the previous exponent in the Neper's number.

![Figure 1. Critical constants hierarchy determined by the Neper's number.](image)

The relationship between the 1st level uniform intervals and the 2nd level geometric progression members is determined by their boundaries synchronization under the ratio:

\[
X_k = \frac{2e}{e - 1} T_k
\]

Here \( T_k \) is uniform unit bars of the arithmetic progression (level 1), \( X_k \) is a geometric progression boundary with the module \( e \) (level 2). This relationship is called the development cell (figure 2) [7].

Accordingly, as the values of \( T_k \) we will take the values identified during the shift function analysis.

As you can see, the arithmetic progressions rhythms influence is accompanied by the geometric progressions rhythms on the various systems functioning.

To identify geometric progression rhythms, we will rely on the fact that they satisfy the relation \( f(t*^k) - f(t) = 0 \), where \( f(t) \) is the series value under study at time \( t \), \( k \) is the geometric progression modulus.
Then, by near-proportion, we will call such a number \( k \) if the inequality \(|f(t \cdot k) - f(t)| < \epsilon\) holds for it, where \( \epsilon > 0 \) is the shift. For the discrete case, if \( N \) is the function \( f(t) \) samples the total number given by experimental values, we obtain a function for determining near-proportions, which has the form:

\[
b(k) = \frac{1}{N/k} \sum_{t=1}^{N/k} |f(t \cdot k) - f(t)|.
\]

To identify a geometric progression, it is necessary to know the position of the zero point \( t_0 \), which can be inside or outside the studied data. Therefore, taking into account the zero point definition, the formula for finding near-proportions takes the following form:

\[
b(k, t_0) = \frac{1}{N/k} \sum_{t=1}^{N/k} |f(t \cdot k + t_0) - f(t + t_0)|.
\] (1)

In this case, the function \( f(t) \) near-proportions \( k \) system can be defined as a set of presented function local minima. At the same time, as in the near-periods case, the deeper the minima on the function graph, the closer to the time interval geometric progression module true value corresponding to this minimum, and the higher its significance in this time series [1].

### 3. Results and discussion

As an object for finding and confirming the solution to the problem, we will take the AMD shares closing value weekly data from its listing beginning on the stock exchange (March 1980) to September 2019. The data source is the site https://finance.yahoo.com/. The share price is calculated in dollars.

The Advanced Micro Devices (AMD) is an integrated microcircuit electronics manufacturer. The CPU, GPU and motherboard chipsets largest manufacturers one.

The initial data are shown in figure 3.

As you can see, the data have a different order, to conduct a qualitative assessment, it will be necessary to reduce the data to a scale where the stock prices large and small values data could be evaluated in the same way. For this, we will use the form \( \ln(y) - t \) anamorphism, where \( y \) is the initial data, \( t \) is time. Such coordinates are also called semi-logarithmic.
Figure 3. The AMD stock price data.

Figure 4 shows the data on a semi-log scale. It can be seen that such a scale stretches the small course values range and compresses large values, which fundamentally changes the analyzed data perception.

Figure 4. The AMD stock value in $ln(y) \sim t$ coordinates data.

Now, let's proceed directly to our data series analysis. Acting with the shift function on the investigated series (figure 5), we obtain the near-periods values equal to 570 and 950. The near-period data presence in the analyzed data is shown in figure 6 and figure 7, respectively.
Using the obtained near-periods values, we will take them as a basis for constructing a development cell.
Figures 8 and 9 show development cells with $T_{570} = 570$ and $T_{950} = 950$, respectively. As you can see, in the selected boundaries, you can change the trends in stock prices.

At the same time, for $T_k = 950$, find and clearly show the ticks one level less presence. We can find the tact $T_{k-1}$ value by dividing the previous level's tact-950 value by $e$. Having performed the appropriate calculations, we find that $T_{k-1} = 350$. These tacts arrangement, under the previous level development cell, is shown in figure 9 at the chart bottom. It can be seen that in each of the tacts, the system has individual behaviour.
Let us apply our last method for revealing near-proportions to the AMD stock value analyzed data. The processing the data result in figure 3 by function (1) is shown in figure 10.

Here, local minima are well manifested at values of $t_0=130$ and $k=30-31$, and at a value of $t_0=270$, $k = 10-11$. It is worth noting that the obtained numbers K differ by approximately $e$ times. That says about a geometric progression obtained denominators relationship.

**Figure 9.** Development cell by near period $Tk=950$.

**Figure 10.** The processing the data result in figure 2 by function (1).
Consider the conditions for arithmetic and geometric progression synchronization. We denote $k$ - a geometric progression modulus, $\tau$ - the near-period value, $t_1$, $t_2$, $t_3$ - numbers that are simultaneously arithmetic and geometric progressions members, $n$ - the number of $\tau$ between $t_2$ and $t_1$, $m$ is $\tau$ number between $t_3$ and $t_2$ (figure 11). Then the equality holds:

$$\frac{t_1 \cdot k^{p+q} - t_1 \cdot k^q}{t_1 \cdot k^p - t_1} = \frac{n \cdot \tau}{m \cdot \tau}$$

The same as:

$$\frac{t_3 - t_2}{t_2 - t_1} = \frac{n \cdot \tau}{m \cdot \tau}$$

(2)

Figure 11. The arithmetic and geometric progressions synchronization.

Hence, we find that the period value can be determined by the known geometric progression modulus and vice versa.

As the geometric progression denominators, we take the found value of $k$, equal to 11, as well as a value that is $e$ times less than 11 - $k = 4$. Take $t_0 = 130$ as the zero points.

Substituting one by one into formula (2) the progressions' synchronization possible values of $t_1$, $t_2$, $t_3$, taking into account the value of $t_0$, we obtain the following near periods values.

For $k=11$, the values of the revealed near-periods $T_k$: 11, 121.

For $k=4$, the revealed near-periods values $T_k$: 4, 16, 64, 256.

Thus, we obtained almost-periods that are geometric progression corresponding to denominators multiples. Also, it can be noted that the obtained $T_k$, equal to 4 and 11, differ from the previously found almost-period 570 by $e^4$ and $e^5$ times, respectively.

4. Conclusion

Summing up, the following can be noted. For the AMD stock prices analyzed data, the near-periods characteristic values were identified using the shift function, as well as the near-proportions values using formula (1). The found values manifestation was demonstrated in the studied data. The resulting system for estimating near-periods for processes with nonlinear fluctuations with a trend using the stock data example shows that a process slow and fast characteristics' separation allows to effectively determine its periodic components.

References

[1] Kuzmin V I and Gadzaov A F 2015 Technical Analysis (Moscow, Russia: MGTU MIREA)
[2] Dzerjinsky R I, Pronina E N and Dzerzhinskaya M R 2020 The Structural Analysis of the World Gold Prices Dynamics AISC 1225 352-65
[3] Kuzmin V I and Samokhin A B 2015 Almost periodical functions with trend Bulletin MGTU MIREA 4-2(9) 105-7
[4] Kuzmin V I and Gadzaov A F 2012 Methods for Constructing Models from Empirical Data (Moscow, Russia: MIREA)
[5] Dzerzhinsky R I, Samokhin A B and Cherdyntsev V V 2018 Computational Mathematics (Electronic Resource) (Moscow, Russia: MIREA) Electron, opt. disk (ISO)
[6] Kuzmin V I and Gadzaov A F 2015 Mathematical methods of periodic components of nonlinear processes and predict the dynamics of the limited growth based on them Bulletin MGTU MIREA 4-2(9) 94-104
[7] Kuzmin V I and Gadzaov A F 2016 Scientific and Technical Forecasting Models and Methods (Moscow, Russia: MIREA)