Efimov states in atom-molecular collisions

M.A. Efremov,1 L. Plimak,1 B. Berg,1 M.Yu. Ivanov,2 and W.P. Schleich1
1Institut für Quantenphysik, Universität Ulm, 89069 Ulm, Germany
2Stecie Institute for Molecular Sciences, NRC Canada, 100 Sussex Drive, ON Ottawa, K1A 0R6 Canada
(Dated: May 25, 2009)

We analyse scattering of a heavy atom off a weakly bound molecule comprising an identical heavy and a light atom in the Born-Oppenheimer approximation. We focus on the situation where the heavy atoms are bosons, which was realized in several experiments. The elastic and inelastic cross sections for the atom-molecular scattering exhibit a series of resonances corresponding to three-body Efimov states. Resonances in elastic collisions are accessible experimentally through thermalization rates, and thus constitute an alternative way of observing Efimov states.

Introduction.— The Efimov effect [1] is the emergence of a large number of weakly bound three-body states if at least two of the three two-body subsystems exhibit a weakly bound state or resonance. This implies that the two-body scattering length $a_0$ is much larger than the characteristic radius of the two-body interaction $r_0$. The number of three-body states is proportional to $\ln(|a_0|/r_0)$. In the resonant limit $|a_0| \to \infty$, the energies of the three-body states form a geometric sequence, with the common ratio determined by the exponent $\exp(2\pi/s_0)$. The parameter $s_0$ depends on the masses of the particles and the number of “participating” resonant two-body interactions (two or three) [1, 2, 3].

First candidates for the Efimov effect were halo nuclei [4] and the helium trimer [5]. In these systems the scattering length is exceedingly large by nature and fixed. However, it is experimentally beneficial to have control of the scattering length so as to observe Efimov states emerging with changing $a_0$. This opportunity is provided by ultracold atomic gases which are now regarded the most promising candidates. Tuning the scattering length in an external magnetic field near a Feshbach resonance was used to observe a Efimov resonance in an ultracold gas of Caesium atoms for the negative scattering length [6]. In this experiment, resonant three-body recombination losses were observed when the strength of the two-body interaction $a_0$ was varied. The resonance was attributed to a Efimov state. More recently [7], an atom-dimer-scattering Efimov resonance for positive scattering length was observed in a mixture of atoms and halo dimers in an optically trapped gas of Caesium atoms. Efimov resonances were also observed in a mixture of potassium and rubidium atoms for positive and negative scattering lengths [8]. However, despite all the experimental effort, the most convincing signature of Efimov physics, namely, equally spaced resonances in three-body observables on the ln($|a_0|$) scale, is yet to be seen.

There exists a large body of theoretical work on different aspects of the Efimov physics (see [2, 4] and references therein). Majority of the theoretical effort was directed at three-body recombination processes [9]. Collisions of an atom with a weakly bound molecule comprising identical (fermionic) atoms were considered in Ref. [10].

In all experiments known to us [6, 7, 8], Efimov states show up as resonances in the dependence of the loss rate on the magnetic field. The goal of this paper is to point to another possibility: thermalization rate for cold atomic mixtures should exhibit a similar resonant behavior. There may well exist cases when resonant losses are unobservable due to unfavorable three-body parameters [1, 2, 3], and resonant thermalization becomes a natural means of “catching” the Efimov states. In this paper, we consider scattering of a heavy atom off a molecule comprising a light and an identical heavy atom and show that resonances due to intermediate Efimov states are equally present in the elastic and inelastic cross sections.

We focus on the situation where the heavy atoms are bosons, and the molecules exist in a cold atomic mixture due to an interspecies Feshbach resonance [11, 12]. The atom-molecular cross sections are calculated in the Born-Oppenheimer approximation. In connection with the Efimov states the Born-Oppenheimer approximation was firstly discussed in paper [13]. Applying this approximation to the scattering problem we express the cross section of three-body collisions in terms of the scattering amplitudes corresponding to the molecular terms (potentials) in which the three-body complex moves. Another simplification is the use of $s$-wave scattering approximation for the said molecular terms as well as for the two-body interactions. Such approximation is justified for a slow motion of the incident atom, characteristic of ultracold collisions.

Statement of the problem.— We analyze a collision of a heavy atom $A$ with a molecule $BL$ formed by one heavy atom $B$ and one light atom $L$. Kinematics of the problem is illustrated in Fig. 1. To start with, we disregard the fact that $A$ and $B$ are identical bosons. We can then distinguish “straight” and rearrangement collisions,

$$A + \{BL\} \to A + \{BL\}, \quad A + \{BL\} \to B + \{AL\}.$$  

We assume that the energy of the incident atom $A$ is insufficient to break the molecule, so that the channel where all three particles break free is closed. In a real
experiment such channel is open due to four-body collisions, but we assume that it can be neglected.

The Schrödinger equation for a particle \( L \) with mass \( m \), interacting with two particles \( A \) and \( B \) of mass \( M \) reads

\[
\left[ \frac{\hbar^2}{M} \frac{\partial^2}{\partial \mathbf{R}^2} - \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial \mathbf{r}^2} + U_0(\mathbf{r}_{LA}) + U_0(\mathbf{r}_{LB}) \right] \Psi = E \Psi.
\]  

(1)

Here, \( \mathbf{R} \) is the distance between the heavy particles, \( \mathbf{r} \) is the position of the light particle relative to the center of mass of \( A \) and \( B \), \( \mathbf{r}_{LA} = \mathbf{r}_L - \mathbf{r}_A = \mathbf{r} + \mathbf{R}/2 \) and \( \mathbf{r}_{LB} = \mathbf{r}_L - \mathbf{r}_B = \mathbf{r} - \mathbf{R}/2 \) are the positions of the light particle relative to the heavy ones, \( \mu = 2Mm/(2M + m) \approx m \) is the reduced mass of the light particle, and \( E \) is the total energy of system. The choice of coordinates is illustrated in Fig. 1.

Equation (1) applies if \( R = |\mathbf{R}| \ll R_0 \), where \( R_0 \) is the range of direct heavy-heavy interactions omitted in (1). The light-heavy potential \( U_0 \) is characterised by the two-body scattering length \( a_0 \) in the zero-range approximation: \( a_0 \gg r_0 \), where \( r_0 \) is the range of \( U_0 \). We assume that \( a_0 \) is positive, i.e., that there exists a weakly bound state of the light and heavy atoms. For overall consistency we should also assume that (cf. (1) (3))

\[
r_0 \leq R_0 \ll a_0.
\]  

(2)

The Born-Oppenheimer approximation. In this approximation \( \mathbf{L} \), the light particle, described by the wave function \( \chi(\mathbf{r}; \mathbf{R}) \), moves in a two-well potential,

\[
\left[ -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial \mathbf{r}^2} + U_0 \left( \mathbf{r} + \frac{\mathbf{R}}{2} \right) + U_0 \left( \mathbf{r} - \frac{\mathbf{R}}{2} \right) \right] \chi(\mathbf{r}; \mathbf{R}) = \epsilon(\mathbf{R}) \chi(\mathbf{r}; \mathbf{R}),
\]  

(3)

where \( \mathbf{R} \) is regarded a parameter. In the zero-range approximation for \( U_0 \) the symmetric and antisymmetric solutions \( \chi^{(\pm)}(\mathbf{r}; \mathbf{R}) \) read \( \chi^{(\pm)}(\mathbf{r}; \mathbf{R}) = \frac{1}{\sqrt{2}} \left[ \psi_{\kappa_+}(\mathbf{r} - \mathbf{R}/2) \pm \psi_{\kappa_+}(\mathbf{r} + \mathbf{R}/2) \right] \),

(4)

where \( \psi_{\kappa_+}(\mathbf{r}) = \sqrt{\kappa_+/(2\pi) \exp(-\kappa_+|\mathbf{r}|)/\kappa_+ \), and \( \kappa_\pm = \kappa_\pm(\mathbf{R}) \) are related to the bound state energies as \( \epsilon(\pm) = -\hbar^2 \kappa_\pm^2/2\mu \). Their dependence on \( \mathbf{R} \) follows from the equations

\[
\pm e^{-\kappa_\pm R} = \kappa_\pm R - \mathbf{R}/a_0.
\]  

(5)

We now look for a solution of Eq. (1) in the form

\[
\Psi(\mathbf{r}, \mathbf{R}) = F^{(\pm)}(\mathbf{R}) \chi^{(\pm)}(\mathbf{r}; \mathbf{R}) + F^{(-)}(\mathbf{R}) \chi^{(-)}(\mathbf{r}; \mathbf{R}).
\]  

(6)

Substituting this Ansatz in Eq. (1) gives rise to two independent equations for the functions \( F^{(\pm)}(\mathbf{R}) \) \( [18] \)

\[
\left[ -\frac{\hbar^2}{M} \frac{\partial^2}{\partial \mathbf{R}^2} + \epsilon^{(\pm)}(\mathbf{R}) \right] F^{(\pm)}(\mathbf{R}) = EF^{(\pm)}(\mathbf{R}).
\]  

(7)

Elastic cross section. We look for solutions of (7) with the standard scattering behavior for large \( R \) \( [14] [18] \)

\[
F^{(\pm)}(\mathbf{R})|_{R \to \infty} = e^{i \mathbf{k} \mathbf{R}} + \frac{f^{(\pm)}(\vartheta)}{R} e^{i \mathbf{k} \mathbf{R}},
\]  

(8)

where \( \vartheta \) is the angle between vectors \( \mathbf{k} \) and \( \mathbf{R} \). This solution corresponds to the total energy \( E = \epsilon_0 + \hbar^2 k^2/2M \), consisting of the binding energy of the light particle \( \epsilon_0 = -\hbar^2/2ma_0^2 \), and of the energy of relative motion of the incident heavy atom and the molecule. Using solutions \( F^{(\pm)}(\mathbf{R}) \) as building blocks, one can construct a properly symmetrized three-body wave function for identical heavy atoms \( [18] \). Of importance to us is its asymptotic form for large \( R \),

\[
\Psi(\mathbf{r}, \mathbf{R}) = e^{-i \mathbf{k} \mathbf{R}} \psi(\mathbf{r}_{LB}) + e^{i \mathbf{k} \mathbf{R}} \psi(\mathbf{r}_{LA})
\]  

\[
+ \frac{e^{i \mathbf{k} \mathbf{R}}}{2r} \left[ f^{(A)}(\vartheta) \psi(\mathbf{r}_{LA}) + f^{(B)}(\vartheta) \psi(\mathbf{r}_{LB}) \right].
\]  

(9)

where

\[
f^{(A)} = f^{(+)}(\vartheta) + f^{(+)}(\pi - \vartheta) + f^{(-)}(\vartheta) - f^{(-)}(\pi - \vartheta),
\]

\[
f^{(B)} = f^{(+)}(\vartheta) + f^{(+)}(\pi - \vartheta) - f^{(-)}(\vartheta) + f^{(-)}(\pi - \vartheta).
\]  

(10)

In deriving this we used approximations for the reduced masses, \( Mm/(M + m) \approx m \) and \( (m + M)/(m + 2M) \approx M/2 \), valid for \( m \ll M \). We also neglected the distinction between the positions of heavy atoms and centers of mass of the corresponding molecules, cf. Fig. 1. This is justified so far as the additional phase factor in the wave function is very small, \( k(m/M)r_{LB} \sim k(m/M)r_{LA} \sim (m/M)ka_0 \ll 1 \). This inequality coincides with the validity criterion of the Born-Oppenheimer approximation. The latter is applicable if the velocity of the relative motion \( v \sim \hbar k/M \) is small compared to that of the light atom bound to the heavy atom, \( v_L \sim \hbar/ma_0 \), i.e., \( (m/M)(ka_0) \ll 1 \).

To calculate the elastic cross section we note that, as \( R \to \infty \), \( \psi(\mathbf{r}_{LB}) \) and \( \psi(\mathbf{r}_{LA}) \) become orthogonal as
functions of \( r \). Hence the incident flow of heavy atoms is determined by two independent contributions both equaling \( 2\hbar k/M \). By the same reason the flow of scattered atoms is equal to \( (\hbar k/2M)[|f^{(A)}|^2 + |f^{(B)}|^2] \), resulting in the elastic cross section

\[
\sigma = \frac{\pi}{4} \int \left( |f^{(A)}(\theta)|^2 + |f^{(B)}(\theta)|^2 \right) \sin \theta d\theta. \tag{11}
\]

**Three-body scattering length.**— A closer inspection of Eq. \( 5 \) shows that for \( R > a_0 \) both molecular terms \( \varepsilon^{(\pm)}(R) \) exponentially approach \( \varepsilon_0 \), i.e., the range of the atom-molecular interaction is of the order of \( a_0 \). Assuming that \( ka_0 \ll 1 \), the s-wave scattering approximation is also applicable to the atom-molecular interactions. We are interested in a double-resonant situation, when not only the light-heavy interaction is resonant, but also the atom-molecular interaction becomes resonant due to an emerging Efimov state. This limits our analysis to vicinity of scattering resonances related to Efimov states. In this case only the s-wave amplitudes matter, which are spherically symmetric. As a result \( f^{-1}(\theta) \) cancels in Eqs. \( 10 \), and we find

\[
f^{(A)} = f^{(B)} = -\frac{2}{1/a_0^{(+)} + ik}, \tag{12}
\]

where \( a_0^{(+)} \) is the s-wave scattering length for the molecular term \( \varepsilon^{(+)}(R) - \varepsilon_0 \). Under the Born-Oppenheimer approximation, we have thus effectively reduced the three-body problem to a two-body one.

**Radial law.**— It is convenient to work with a dimensionless form of \( 7 \) for the “plus” term,

\[
\frac{d^2}{d\rho^2} u(\rho) - V(\rho) u = 0. \tag{13}
\]

Here, \( \rho = R/a_0 \), \( u(\rho) = F(a_0 \rho) \phi \rho \), and

\[
V(\rho) = -\frac{M}{2m} \left[ \frac{G^2(\rho)}{\rho^2} - 1 \right], \tag{14}
\]

where \( G(\rho) = \rho \kappa_+ (\rho) a_0 \). Eq. \( 13 \) then reads \( G(\rho) = \rho \kappa(\rho) a_0 \). Note that Eq. \( 13 \) is written for \( k = 0 \), or \( E = \varepsilon_0 \) \( 12 \). For large \( \rho \) the solution of Eq. \( 13 \) behaves as \( u(\rho) \propto 1 - \rho a_0/a_0^{(+)} \) \( 10 \), so that

\[
\frac{a_0^{(+)}}{a_0} = \lim_{\rho \to \infty} \rho - \frac{u(\rho)}{du/d\rho}. \tag{15}
\]

For \( \rho \ll 1 \), the potential \( V(\rho) \) in Eq. \( 14 \) behaves as \(- (s_0^2 + 1/4)/\rho^2 \), where

\[
s_0 = \sqrt{G^2(0)M/2m - 1/4}, \tag{16}
\]

and \( G(0) \approx 0.5671 \). Consequently, the general solution to Eq. \( 13 \) for \( R_0/a_0 \ll \rho \ll 1 \) reads,

\[
u(\rho \to 0) \sim \sqrt{\rho} \sin(s_0 \ln(\Lambda_0 a_0 \rho)), \tag{17}
\]

where \( \Lambda_0 \) is a constant. It is determined by the boundary condition at \( \rho \to R_0/a_0 \), and plays the role of the so-called three-body parameter, containing all the necessary information about the short-range interactions. Importantly, \( \Lambda_0 \) does not depend on \( a_0 \). Indeed, if \( \rho \ll 1, \)

\[
a_0\rho = R, \quad a_0 \quad \text{only occurs in the overall coefficient. Furthermore, by allowing \( \Lambda_0 \) to be complex, we can also include the information about the losses due to transitions from the weakly bound heavy-light molecular state into deep diatomic ones.} \quad \text{The range} \quad R_0 \quad \text{then characterizes the “black box” within which the whole of short-range physics is contained.} \quad \text{Condition} \quad \text{ensures consistency of the whole viewpoint, cf.} \quad [1 \ 2].
\]

The ratio \( a_0^{(+)}/a_0 \) is a periodic function of \( \ln a_0 \), which is a particular case of the “radial law” \( 1 \ 3 \). Indeed, let \( u_{1,2} \) be two linearly independent solutions of Eq. \( 13 \), such that \( u_1(\rho) = \sqrt{\rho} \cos(s_0 \ln \rho) \) and \( u_2(\rho) = \sqrt{\rho} \sin(s_0 \ln \rho) \) for \( \rho \ll 1 \). The solution coinciding for \( \rho \ll 1 \) with \( 17 \) reads

\[
u(\rho) \sim \sin(s_0 \ln \Lambda_0 a_0) u_1(\rho) + \cos(s_0 \ln \Lambda_0 a_0) u_2(\rho).
\]

For \( \rho \to \infty \), \( u_{1,2}(\rho) = \alpha_{1,2} + \beta_{1,2} \rho \), where the coefficients \( \alpha_{1,2} \) and \( \beta_{1,2} \) are determined by the potential \( V(\rho) \); they depend only on the mass ratio \( M/m \). By direct calculation with Eq. \( 15 \) we have,

\[
\frac{a_0^{(+)}}{a_0} = \alpha + \beta \cot(s_0 \ln(a_0/a_* + i \eta_*), \tag{18}
\]

where \( \alpha = -\alpha_1 \beta_1 + \alpha_2 \beta_2)/(\beta_1^2 + \beta_2^2) \) and \( \beta = (\alpha_1 \beta_2 - \alpha_2 \beta_1)/(\beta_1^2 + \beta_2^2) \). Instead of one complex parameter \( \Lambda_0 \) we have introduced two real parameters \( a_* \) and \( \eta_* \) by the equation, \( s_0 \ln(a_0/a_*) = -\arctan(\beta_2/\beta_1) + i \eta_* \).

**Results and discussion.**— With losses Eq. \( 11 \) applies to the elastic cross section \( \sigma_e \), while the inelastic \( \sigma_i \) is found as a disbalance between the incoming and outgoing waves. In the limit \( k|\ln a_0^{(+)}| \ll 1 \), \( \sigma_e = 4\pi|a_0^{(+)}|^2 \) and

\[
\sigma_e = 4\pi a_0^2 \beta^2 \frac{\sin^2(s_0 \ln(a_0/a_*) + \theta_0) + \sinh^2(\eta_*)}{\sin^2(s_0 \ln(a_0/a_*) + \sinh^2(\eta_*)}, \tag{19}
\]

where \( \theta_0 = \arctan(\beta/\alpha) \). The parameters \( \alpha, \beta, s_0 \) and \( \theta_0 \) in \( 19 \) are known functions of the mass ratio \( M/m \), while \( a_* \) and \( \eta_* \) are in essence fitting parameters: \( a_* \) is the value of the scattering length for which the atom-molecular cross section has a Efimov resonance, while \( \eta_* \) determines its width. The experimentally controllable parameter is the two-body scattering length \( a_0 \).

In Fig. \( 2 \) \( \sigma_e \) and \( \sigma_i \) are plotted as functions of \( a_0/a_* \) (on a logarithmic scale) for a mixture of \( ^8\text{Rb} \) and
\[ \sigma_\text{el} = 4\pi a_0^2 \] (solid line), and the inelastic one \[ \sigma_i = 4\pi a_0^2 \] (dashed line), as functions of \[ a_0/a_\ast \] for the \(^{87}\text{Rb}\)–\(^{7}\text{Li}\) mixture, with \[ ka_0 = \eta_\ast = 0.1. \]

\( ^{7}\text{Li} \) \((M/m \approx 12.43)^\text{[12]} \). For this system, \[ s_0 = 1.322, \alpha = 2.17, \beta = 2.55, \text{ and } \theta_0 = 0.87. \] As functions of \[ \ln(a_0/a_\ast) \], \[ \sigma_\text{el} \] and \[ \sigma_i \] are periodic with the period \[ \exp(\pi/s_0) \approx 10.8. \] The graphs exhibit a typical series of equidistant resonances. The losses are maximal at \[ s_0 \ln(a_0/a_\ast) = \pi n, n = 0, \pm 1, \pm 2, \ldots \] while maxima of \[ \sigma_\text{el} \] are somewhat shifted.

The \(^{7}\text{Li} – ^{87}\text{Rb}\) mixture appears to be a good candidate for observing multiple Efimov resonances. Firstly, this mixture has a large mass ratio, and, consequently, a relatively small separation between Efimov resonances. Secondly, this mixture exhibits a sufficiently wide \((\Delta B = 175 \text{G})\) Feshbach resonance near the magnetic field \[ B_0 = 649 \text{G} \text{[12]} \].

While the inelastic cross section determines resonant losses, the elastic one manifests itself through, e.g., the resonant dependence of the thermalization rate \[ \gamma \] for the atom–molecule mixture, \[ \gamma \propto \sigma_\text{el} \text{[20]} \]. For the \(^{7}\text{Li} – ^{87}\text{Rb}\) mixture, the maxima of the elastic and inelastic cross sections are connected by the formula,

\[ \frac{\sigma_\text{el}^\text{max}}{\sigma_i^\text{max}} = 2.6 \frac{ka_0}{\eta_\ast}. \]

The Efimov resonances thus manifest themselves either as increased losses or accelerated thermalization; these two ways of observing Efimov resonances are complementary.

Acknowledgements.— The authors are deeply indebted to G. Shlyapnikov and D. Petrov for numerous enlightening discussions and comments on the manuscript, to C. Marzok for a discussion of experimental techniques, and to A. Wolf and A. Zhukov for discussions and technical assistance. LIP and MAE are grateful to Laboratoire de Physique Théorique et Modèles Statistiques, CNRS, Université Paris Sud, for generous hospitality. WPS, MAE and MYI acknowledge support of the Alexander von Humboldt Stiftung, BB of the scholarship “Mathematical Analysis of Evolution, Information and Complexity” at Ulm University. WPS also acknowledges support of the Max Planck Society. This work was supported in part by a grant from the Ministry of Science, Research and Arts of Baden-Württemberg.

* Electronic address: max.efremov@gmail.com

[1] V. Efimov, Sov. J. Nucl. Phys. 12, 589 (1971); Phys. Lett. 33B, 563 (1970); Nucl. Phys. A 210, 157 (1973)
[2] J.P. D’Incao and B.D. Esry, Phys. Rev. A 73, 030702 (2006)
[3] E. Braaten and H.-W. Hammer, Phys. Rep. 428, 259 (2006)
[4] A.S. Jensen et al., Rev. Mod. Phys. 76, 215 (2004)
[5] R. Brühl et al., Phys. Rev. Lett. 95, 063002 (2005)
[6] T. Kraemer et al., Nature 440, 315 (2006)
[7] S. Knoop et al., Nature Physics 5, 227 (2009)
[8] G. Barontini et al., arXiv:0901.3584
[9] P.O. Fedichev, M.W. Reynolds, and G.V. Shlyapnikov, Phys. Rev. Lett. 77, 2921 (1996); E. Nielsen and J.H. Macek, Phys. Rev. Lett. 83, 1566 (1999); B.D. Esry, C.H. Greene, and J.P. Burke, Jr., Phys. Rev. Lett. 85, 908 (2000); E. Braaten and H.-W. Hammer, Phys. Rev. Lett. 87, 160407 (2001)
[10] J.P. D’Incao, B.D. Esry, and C.H. Greene, Phys. Rev. A 77, 052709 (2008)
[11] C.A. Stan at al., Phys. Rev. Lett. 93, 143001 (2004); S. Inouye et al., Phys. Rev. Lett. 93, 183201 (2004)
[12] C. Marzok et al., Phys. Rev. A 79, 012717 (2009)
[13] A.C. Fonseca, E.F. Redish, and P.E. Shanley, Nuclear Physics A 320, 273 (1979)
[14] L.D. Landau and E.M. Lifshitz, Quantum Mechanics (Pergamon Press, Oxford, 1977)
[15] A.I. Baz, Y.B. Zeldovich, and A.M. Perelomov, Scattering, Reactions and Decay in Nonrelativistic Quantum Mechanics (Nauka, Moscow, 1971; Israel Program for Scientific Translations, Jerusalem, 1969)
[16] Yu. N. Demkov and V. I. Ostrovskii, Zero-range potentials and their applications in atomic physics (Plenum Press, New York, 1988)
[17] D.S. Petrov, C. Solomon, and G.V. Shlyapnikov, J. Phys. B 38, S645 (2005)
[18] N.F. Mott and H.S.W. Massey, Theory of Atomic Collisions, (third edition, Oxford, 1971)
[19] S. Flügge, Practical Quantum Mechanics I (Springer-Verlag, 1971)
[20] C. Marzok et al., Phys. Rev. A 76, 052704 (2007); A. Mosk et al., Appl. Phys. B 73, 791 (2001); M. Anderlini and D. Guery-Odelin, Phys. Rev. A 73, 032706 (2006)