UNIFICATION OF THE NEARBY AND PHOTOMETRIC
STEellar LUMINOSITY FUNCTIONS

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Abstract

We introduce a model Galactic field low-mass stellar population that has a proportion of binary systems as observed, with a mass ratio distribution consistent with observational constraints. The model single star and system luminosity function agrees with the nearby and the Malmquist corrected photometric luminosity function, respectively. We tabulate the model luminosity functions in the photometric V-, I- and K-bands, and in bolometric magnitudes. Unresolved binary systems are thus a natural explanation for the difference between the nearby and photometric luminosity functions. A local overdensity of faint stars needs not be postulated to account for the difference, and is very unlikely. We stress that the nearby luminosity function can only be used to weakly constrain the stellar mass function below 0.5 $M_\odot$, because of the small sample size. The photometric luminosity function can only be used to put lower limits on the stellar mass function because most binary systems are not resolved. However, taken together the nearby and photometric stellar luminosity function data do not imply a mass function with a peak at a mass of 0.2–0.3 $M_\odot$. Instead, the data are consistent with a power-law mass function between 0.08 $M_\odot$ and 0.5 $M_\odot$. We urge researchers to only use star count data that are properly corrected for all contributions to cosmic scatter, photometric uncertainties, and unresolved binaries, and to be aware of the severe limitations of theoretical mass–luminosity relations for low mass stars, when drawing conclusions about structure in the stellar mass function.

Subject headings: stars: low-mass – stars: luminosity function, mass function, mass–luminosity relation
1 INTRODUCTION

The astrophysical importance of the stellar mass function for stars less massive than one solar mass has been stressed many times and much effort has gone into constraining its shape. Interest is inspired from the cosmological and Galactic dynamics fields (how much baryonic mass is stored in faint low mass stars?) as well as from the star formation field (does the initial mass function vary among star formation regions and is there a lower mass limit below which no ‘stars’ form?). The stellar mass function cannot be observed directly but can be estimated from the stellar luminosity function. Unfortunately, observational attempts at constraining the low mass stellar luminosity function have led to rather discordant results.

The ‘nearby’ luminosity function, $\Psi_{\text{near}}$, constructed from stars within 5–20 pc distance of the Sun using trigonometric parallax measurements to estimate stellar number densities has a significantly larger stellar number density at $M_V > 13$ than stellar luminosity functions estimated from low spatial resolution but deep (100–200 pc distance) magnitude limited surveys (Kroupa 1995a). We refer to these as ‘photometric’ luminosity functions, $\Psi_{\text{phot}}$, because distances are estimated using photometric parallax. Photometric luminosity functions need to be carefully corrected for systematic bias which results because stars of a given colour do not all have the same absolute magnitude owing to different metal abundances, unresolved binary components and different ages. The resulting dispersion of absolute magnitude at a particular colour in the colour–magnitude diagram is referred to as cosmic scatter. The imposed magnitude limit leads to an apparently larger stellar number density and an apparently brighter stellar sample. This effect is referred to as Malmquist bias. In Kroupa (1995a) we discuss this issue in greater detail and we combine four independent photometric luminosity functions to estimate the parent Malmquist corrected photometric luminosity function which is based on a sample of 448 stars. Applying a number of different statistical tests we establish that $\Psi_{\text{near}}$ and our estimate of the true Malmquist corrected photometric luminosity function, $\overline{\Psi}_{\text{phot}}$, are significantly different at $M_V > 13$. Both luminosity functions are tabulated in table 2 of Kroupa (1995a).

The difference has been claimed by Kroupa, Tout & Gilmore (1991, 1993) to result from unresolved binary systems in the low-spatial resolution photographic surveys used to estimate $\Psi_{\text{phot}}$. They find that the nearby and photometric luminosity functions can be understood to stem from the same underlying stellar mass function provided it is approximated by two power-law segments between $0.08$ and $1.1 M_\odot$. However, Reid (1991) concludes that unresolved binary systems cannot account for the difference. Furthermore, Tinney (1993) uses the estimate of the photometric luminosity function from the recent precise large-scale photographic survey of Tinney, Reid & Mould (1993) to address this issue. Tinney (1993) and Reid (1994) conclude (i) that the nearby and Tinney’s photometric luminosity function do not differ and (ii) that the stellar mass function has a significant maximum at a mass of about $0.2 – 0.3 M_\odot$, thus contradicting the results of Kroupa et al. (1991, 1993).

In Kroupa (1995a) we resolve point (i) by noting that the conclusion by Tinney (1993) and Reid (1994) rests upon the comparison of the nearby luminosity function with the photometric luminosity function in bolometric magnitudes which has not been corrected for Malmquist bias. Malmquist bias in Tinney’s photometric luminosity function leads to significant overestimation of stellar number densities. Comparing Tinney’s photometric luminosity function corrected to first order for Malmquist bias by Tinney et al. (1993) with $\overline{\Psi}_{\text{phot}}$ we find good agreement between the two. However, point (ii) remains unresolved, and we take the suggestion by Tinney (1993) and Reid (1994) seriously, because the survey of Tinney et al. (1993) significantly improves our estimate of the photometric luminosity function being based on about 3500 stars.

In this paper we study the possible reasons for the difference between the nearby and Malmquist corrected photometric luminosity functions. We also critically investigate the evidence for structure in the Galactic field stellar mass function claimed by Tinney (1993) and Reid (1994).

In Section 2 we contemplate whether a local stellar overdensity may be responsible for the significant difference between $\Psi_{\text{near}}$ and $\overline{\Psi}_{\text{phot}}$. In Section 3 we critically examine the models of Reid (1991) and Kroupa et al. (1993), and in Section 4 we introduce a realistic model of the dynamical properties of stellar systems in the Galactic disc and compare the model with the observed luminosity functions. In Section 5 we discuss the stellar mass function, and Section 6 contains our conclusions.
2 A LOCAL STELLAR OVERDENSITY?

Perhaps near the Sun there are significantly more stars with $M_V > 13$ than at a distance of 100 – 200 pc (see section 4.3 in Kroupa 1995a). Photometric luminosity functions in directions towards the north and south Galactic poles, and in directions not perpendicular to the Galactic plane, show that the shape of the distribution of stars with luminosity does not vary significantly with direction for $M_V < 16.5$ (fig. 1 in Kroupa 1995a; fig. 5 in Stobie et al. 1989; fig. 20 in Tinney et al. 1993). The different normalisation can readily be accounted for by correcting for a density gradient perpendicular to the Galactic midplane. This hypothesis would imply that the sun is situated in a bubble containing significantly more faint stars than the Galactic field.

The velocity dispersion of the old Galactic disc population is about 50 km sec$^{-1}$, and within 4 Myrs such a population would have dispersed over a distance of 200 pc. The nearby stars with $13 < M_V < 16.5$ that define $\Psi_{near}$ have kinematics typical of an old population (section 10.3 in Kroupa et al. 1993). A significant local population of young faint stars therefore cannot account for the difference between the nearby and photometric luminosity functions, as also stressed by Reid (1991) in his section 4. Thus, in order to explain the difference between the nearby and photometric luminosity functions with the “overdensity hypothesis” we need to postulate a very special (i.e. only faint old stars) and an extremely short lived constellation of stars.

Clearly this is not a satisfying solution to the problem.

3 BINARY STARS

If in the galactic disk 50-60 per cent of all “stars” are binary systems and if the mass ratio distribution is biased towards non-equal masses, then straightforward logics implies that the system luminosity function decreases significantly with decreasing luminosity when compared to the single star luminosity function. This has been shown by Kroupa et al. (1991), who have also demonstrated that one mass function can ‘unify’ both the nearby and photometric luminosity functions. This approach increases the statistical weight of the low-mass data.

Observational evidence indicates that the above proportion of binary systems is about correct, and that there are more main sequence binaries with low-mass secondaries than with high mass secondaries. Photographic surveys have a resolution of 3–6 arc sec (Reid 1987) which, at a distance of 100 pc, corresponds to 300–600 AU. For a binary system with total mass of 1.3 $M_\odot$ this corresponds to $\log_{10} P = 6.2–6.7$, where $P$ is the orbital period of the binary system in days. This range of orbital periods lies well beyond the maximum at $\log_{10} P \approx 5$ in the period distribution of late-type binaries (see Kroupa 1995b and references therein). Thus virtually no binary systems can be resolved by photographic techniques.

3.1 They don’t matter?

Reid (1991) adopts a similar approach to that of Kroupa et al. (1991) but instead of drawing stars from a mass function he draws single stars from an adopted model of the nearby luminosity function. He redistributes stars in the observed nearby luminosity function for ‘aesthetic reasons’ to give his preferred model which is flat for $M_V > 12$ and on which his models A–J and M,N are based. He also considers a model single star luminosity function which has a pronounced maximum at $M_V \approx 12$ (his models K and L taken from Kroupa et al. 1991).

Some of the stars Reid (1991) combines to binary systems and changes the photometric properties of the resulting population to account for Gaussian cosmic scatter (his adopted model single star luminosity function is thus an ideal model without measurement errors of a single metallicity and single age stellar population). The resulting model photometric luminosity function he compares with his fit, $\Psi_{\text{Reid, phot}}$, to the logarithmic data presented in fig. 5 in Stobie et al. (1989) which are not corrected for Malmquist bias. We note that his adopted observed photometric luminosity function appears in his fig. 9 stretched for unspecified reasons (thus enhancing the decay at $M_V > 12$). For example, in his table 1 we find $\Psi_{\text{Reid, phot}} = 2.0 \times 10^{-3}$ stars pc$^{-3}$ mag$^{-1}$ at $M_V = 16$. In fig. 9 ‘D’ this datum appears as $\Psi_{\text{phot}} = 1500$ stars mag$^{-1}$ which implies a scaling factor of 750. Thus we would expect $\Psi_{\text{Reid, phot}} = 12.50 \times 10^{-3}$ stars pc$^{-3}$ mag$^{-1}$ at $M_V = 12$ (his table 1) to appear as $\Psi_{\text{phot}} = 9375$ stars mag$^{-1}$ in his fig. 9 ‘D’. Instead we find $\Psi_{\text{phot}} = 11300$ stars
mag$^{-1}$. It is important to bear this in mind because his conclusions are based on an eye-ball comparison of the shapes of the luminosity functions.

Reid adopts a number of models for the binary star population based on stellar data within a distance of 5–10 pc. He assumes his binary population consists of ‘wide’ binaries (components chosen at random from his adopted model single-star luminosity function) and ‘equal-mass’ binaries (components restricted to lie within 2 mag of the luminosity of the primary – note that for stars with $5 \leq M_V \leq 12$ this corresponds to a mass range of $0.17 M_\odot$ using his adopted linear mass–$M_V$ relation and to a mass range of about 0.1 $M_\odot$ or less for fainter stars).

His models assume a uniform spatial distribution of stars and a Gaussian cosmic scatter independent of absolute magnitude.

From his fig. 9 Reid (1991) concludes that his model photometric luminosity functions make a poor eye-ball comparison with his adopted observational photometric luminosity function. He rejects the hypothesis that unresolved binary systems account for the difference between the nearby and photometric luminosity functions, putting greatest weight on his ‘favorite’ models F, G and H (30–50 per cent binary population, of which 50 per cent are wide binaries and 50 per cent have equal-mass components).

His conclusion does not rest on a statistical analysis, and so its confidence cannot be assessed. Below we show that observational constraints contradict these models.

It is useful at this point to consider why Stobie et al. (1989) also conclude that binary systems cannot explain the difference between the nearby and photometric luminosity functions. They use the sample of 60 stars within 5.2 pc of the sun. This sample consists of 45 systems of which 32 are single stars. From this sample they obtain the single star and system luminosity function and find no eye-ball difference between the two. In this case, only 5–7 stars per magnitude bin are in the single star luminosity function. Thus, even a reduction of the number of stars per magnitude bin by a factor of two leads to no significantly different distribution, and the hypothesis that binaries cannot account for the difference cannot be ruled out with any reasonable confidence. Henry & McCarthy (1990) arrive at the opposite conclusion to Stobie et al. (1989) using a similar stellar sample.

The lesson to be learned here is that conclusions with useful confidence are not possible if the investigation is restricted to the nearby sample only. Extension of the analysis to contain photographic star count data, and using extensive numerical modeling, however, leads to statistically significant conclusions (Kroupa et al. 1991, 1993).

3.2 They do matter!

A detailed and consistent model of star count data is studied by Kroupa et al. (1993). They construct physical models of all contributions to cosmic scatter and measurement uncertainties in photometric and trigonometric parallax and vary the proportion of unresolved binary systems which they assume have component masses paired randomly from one stellar mass function (i.e. an uncorrelated mass ratio distribution). The resulting non-Gaussian model cosmic scatter agrees with the observed value and is a function of absolute magnitude. The spatial distribution of stars is assumed to follow an exponential density law perpendicular to the Galactic plane. Realistic modelling of the spatial distribution of stars is important in their analysis because raw star count data (i.e. not corrected for Malmquist bias nor a vertical density gradient) are modelled.

Kroupa et al. (1993) show that (i) binary systems lead to a significant underestimation of faint star number densities in photographic surveys even if only 50 per cent of stars on the photographic plates are unresolved binaries (fig. 21 in Kroupa et al. 1993) and (ii) one single stellar mass function can ‘unify’ both the nearby and photographic star counts. They find that the initial mass function can be approximated conveniently by the KTG($\alpha_1$) mass function with $0.70 < \alpha_1 < 1.85$ (equation 4 below).

3.3 Why do the conclusions of Reid (1991) and Kroupa et al. (1993) differ?

The models (A, B, C, E, F, G, H, I, L, M, N, O) computed by Reid (1991) which have binaries of which either half or all have equal-mass components are not consistent with observational constraints:

The most thoroughly determined mass-ratio distributions by Duquennoy & Mayor (1991) and Mazeh et al. (1992) are for long- and short-period G-dwarf binary systems, respectively. The long-period ($P > 10^3$ days) systems mostly have low-mass companions, whereas the short-period systems ($P < 10^3$ days) may
show some weak bias towards equal mass systems. However, of all G-dwarf binaries, only about 10 per cent have $P < 10^{4}$ days, of which only about 25 per cent have a mass ratio $q > 0.8$ (see also figs. 1 and 2 in Kroupa 1995b), where $q = m_2 / m_1 \leq 1$, where $m_1$ and $m_2$ are the masses of the primary ($m_1 \approx 1 M_\odot$) and secondary component, respectively. About 8 per cent of all long-period G-dwarf binaries have $q > 0.8$, so that of all G-dwarf binaries only about 10 per cent have $q > 0.8$, and the rest have $q < 0.8$ with about 55 per cent of all G-dwarf binaries having $q < 0.4$. Similar conclusions are arrived at from the preliminary results of the extensive radial velocity study of K-dwarf binaries (Mayor et al. 1992). For M-dwarf binaries Fisher & Marcy (1992) conclude that the mass function of companion masses shows no significant bias towards $q > 0.8$.

The assumption made by Reid (1991) in his models A, B, C, E, F, G, H, I, L, M, N and O that half or all binaries have equal component masses is thus inconsistent with the observational constraints. We therefore concentrate only on those remaining models which are most consistent with observational constraints. These are his models D (50 per cent wide binaries), J and K (100 per cent wide binaries). Models D and J are based on a flat model single star luminosity function, and model K is based on a model single star luminosity function which has a maximum at about $M_V \approx 12$.

Concerning his models D and J, we stress that the nearby star-count data do not require the nearby LF to be flat (figs. 1 and 3 in Kroupa 1995a). The single star, single metallicity and single age luminosity function is not likely to be flat for $M_V > 12$ because changes in the first derivative of the mass–absolute magnitude relation imply pronounced universal structure at $M_V \approx 7$ (the ‘H’–plateau’) and at $M_V \approx 12$ (the ‘H2–convection peak’) (Kroupa et al. 1990, 1993). It is necessary to note that while Malmquist-type bias does not significantly affect the stellar space densities in the nearby luminosity function, its shape is smoothed because of uncertainties in parallax measurements and the spread in metallicities (an age spread does not significantly contribute to cosmic scatter for low-mass stars with $V - I > 2.5$ – see fig. 4 in Kroupa et al. 1993). Comparison of figs. 1 and 20 in Kroupa et al. (1993) demonstrates that the measured nearby luminosity function appears much flatter than the true single metallicity, single age luminosity function actually may be. Reid’s figs. 9 ‘D’ and ‘J’ show that the model photometric luminosity functions do not decay as steeply with increasing $M_V > 12$ as his adopted observed photometric luminosity function. We take this as evidence that the single star luminosity function cannot be flat.

Concerning his model K, we find acceptable agreement in his fig. 9 ‘K’ between model photometric luminosity function and adopted observed photometric luminosity function. This model most closely resembles the models found by Kroupa et al. (1993) to be consistent with the nearby and photographic star count data.

An important reason why Kroupa et al. (1993) find that the nearby and photometric luminosity functions are manifestations of a single Galactic field stellar mass function whereas Reid (1991) does not is because Kroupa et al. model a realistic population of stellar systems and apply on this model population the same observational criteria as are used in the definition of the observed nearby and photographic stellar samples. Reid’s models lack the non-Gaussian nature of cosmic scatter and its dependence on absolute magnitude. Contrary to his assertion the reduction of stellar number densities at distances between 100–300 pc from the Galactic plane has to be taken into account when modelling raw star count data. Consider a maximal case and a photographic survey perpendicular to the Galactic plane with a photometric distance limit of $d_p = 130$ pc (Stobie et al. 1989): An unresolved binary system with equal mass components will be included if its distance is $\sqrt{2}d_p = 184$ pc (Kroupa et al. 1991). If in addition the combined absolute magnitude is $3 \sigma_{\text{tot}} = 1.5$ mag too bright for its colour then this system will be included in the survey if its true distance is as large as 367 pc. At this distance the stellar number density has fallen to 0.3 of its value at the Galactic midplane assuming an exponential Galactic disk scale height $h = 300$ pc. Clearly, a uniform spatial distribution of stars to such distances is not an adequate approximation. Taking account of this reduction of stellar number density with distance from the Galactic midplane allows Kroupa et al. (1993) to account for the smaller number densities in the observed photometric luminosity function (their fig. 20).

Summarising, we interpret Reid’s modelling as follows: he investigates a region of parameter space not considered by Kroupa et al. (1993) by studying other than uncorrelated mass ratio distributions in the binary star model populations. His computational results lead to rejection of the hypothesis that binary stars explain the difference between the nearby and photometric luminosity functions, unless the mass ratio distribution is approximately uncorrelated. This result is consistent with the observed mass ratio distributions of G, K and
M dwarf binaries. He finds that binaries can account for the difference if the single star single metallicity and age luminosity function has a maximum at $M_V \approx 12$ (his model K). This finding verifies that of Kroupa et al. (1991). Kroupa et al. (1993), on the other hand, scan mass function power-law index, $\alpha_1$, and disc scale height, $h$, space under the assumption of an uncorrelated mass ratio distribution (a good first approximation) which consistently models the complex interdependence of normalisation and shape of the model Malmquist photometric luminosity function. They find binaries can account for the difference between the nearby and photometric luminosity functions if $0.70 < \alpha_1 < 1.85$.

The assumption by Kroupa et al. (1993) that the component masses in binary systems are uncorrelated (note that this implies that the mass function of the secondaries is steeper than the mass function of the primaries) is a reasonable first approximation. For example, Tout (1991) shows that simple selection effects can account for the apparent correlation of component masses seen in single- and double-lined spectroscopic binaries. However, some correlation of component masses is present. Kroupa & Tout (1992) point out that the G-dwarf binary systems (Duquennoy & Mayor 1991) have too few low-mass companions with respect to the KTG($\alpha_1$) mass function if $\alpha_1 > 1$ approximately.

4 A REALISTIC MODEL

If it is assumed that most stars form in embedded clusters, as suggested by Lada & Lada (1991), then most stars must also form in binary systems. Conversely, if it is assumed that most stars form in binary systems (see e.g. Mathieu 1994 and references therein) then the dominant mode of star formation must be clustered star formation. Both conclusions are based on the observed dynamical properties of Galactic field stars (Kroupa 1995b).

The dynamical properties of a stellar population are the mass function, the proportion of binary systems and their distribution of orbital elements. In Kroupa (1995b) we assume that all stars form in binary systems which have component masses picked at random from the KTG(1.3) mass function (equation 4 below). This assumption and our adopted initial distribution of periods are consistent with pre-main sequence binary star observations. Our N-body simulations in Kroupa (1995b) of a range of initial stellar aggregate sizes shows that in a stellar aggregate consisting initially of a few hundred binaries with half mass radius of about 0.8 pc the initial dynamical properties evolve to the stellar dynamical properties observed in the Galactic field. This is shown in detail in Kroupa (1995c), where we perform 20 N-body simulations of this ‘dominant mode cluster’. Specifically, the ionisation of binaries in the stellar aggregate leads to a final binary proportion of 48 per cent and the observed mass-ratio distribution of G-dwarf binaries is obtained thus solving the discrepancy noted by Kroupa & Tout (1992), and thus also correctly reproducing the deviation from an uncorrelated mass-ratio distribution.

We need to test whether the depletion of the mass ratio distribution at small values invalidates the results of Kroupa et al. (1993). Using the extensive data from our N-body simulations in Kroupa (1995c) we compute the single star luminosity function and the system luminosity function (merging the components of a binary system to an unresolved system and counting stars not in binary systems as separate systems) in the photometric V-, I- and K-bands and in bolometric magnitudes and tabulate these in Appendix 1. We restrict stellar masses to lie in the range $0.1 - 1.1 M_\odot$.

Kroupa et al. (1993) tabulate a mass–$M_V$ relation obtained by combining theoretical and observational (compiled by Popper 1980) constraints. This relation is an excellent fit to the new mass–luminosity data compiled by Henry & McCarthy (1993). This is shown by Kroupa & Gilmore (1994) by comparing the mass–$M_V$ data of Henry & McCarthy (1993) that are not listed by Popper (1980) with the model mass–$M_V$ relation of Kroupa et al. (1993). This model mass–$M_V$ relation may prove useful in constraining opacity and mixing-length theory.

Transformation to the K-band is obtained using the $V - K, M_K$ relation from Henry & McCarthy (1993) (their equation 1a) and to the I-band using the $M_V, V - I$ relation from Stobie et al. (1989) (their equation 1) which provides a good fit to the data of Monet et al. (1992) (we note that a slight correction of this relation owing to systematic bias from unresolved binary stars is discussed by Kroupa et al. 1993). To obtain bolometric absolute magnitudes we use equation 6b in Kroupa (1995a).

We compare our model luminosity functions with the observational data in Fig. 1. Our model luminosity function for individual stars, $\Psi_{\text{mod,sing}}$, is a good representation of the nearby data in the V-band and in bolometric magnitudes. If all stars are paired at random to form binary systems we obtain the initial system
luminosity function, $\Psi_{\text{mod,sys}}(t = 0)$ (‘initial’ is to be understood here only w.r.t. pairing of component masses – we do not model pre-main sequence stellar evolution). After the dominant mode cluster has disintegrated, $\Psi_{\text{mod,sys}}$ becomes the model system luminosity function of the Galactic field, $\Psi_{\text{mod,sys}}(t = 1 \text{Gyr})$. Our model system luminosity function is in good agreement with the photometric luminosity function in the V-band. Agreement in bolometric magnitudes is somewhat worse.

The residual discrepancies between $\Psi_{\text{mod,sys}}(t = 1 \text{Gyr})$ and $\Psi_{\text{phot}}$ can partly be attributed to remaining inadequacies of the model (mostly residual inaccuracies in the mass–luminosity relation and the mass function – it is probably not exactly a power law below $0.5 M_\odot$). We do not model triple, quadruple, etc. stellar systems. While the exact proportion of higher order systems is unknown, it may be as large as 10–20 per cent (Abt & Levy 1976, Duquennoy & Mayor 1991) of all binary systems. Taking account of these will cause additional depression of the model system luminosity function bringing it into better agreement with the photometric luminosity function at faint magnitudes. Better agreement is achieved in the V-band (upper panel in Fig. 1) than in bolometric magnitudes (lower panel in Fig. 1) which is not surprising because our model was ‘fine-tuned’ in the V-band, and because the transformation to bolometric magnitudes differs for the model data and the observational data. We care to point out that bolometric corrections remain rather uncertain for low-mass stars. A probably not negligible source of systematic uncertainty is treatment of Malmquist bias in the observed photometric luminosity functions. The best-estimate Malmquist corrected photometric luminosity function was obtained from Malmquist uncorrected photometric luminosity functions (Kroupa 1995a) with the assumption that cosmic scatter is Gaussian and independent of absolute magnitude, both of which are reasonable first assumptions but not strictly true (Kroupa et al. 1993). Also, at $M_V > 15$, $M_{\text{bol}} > 11$ photometric luminosity functions may underestimate stellar number densities because the $1/V_{\text{max}}$ method becomes unreliable if the underlying number of stars is too small (Jahreiss 1994 and references therein). Furthermore, photometric calibration of photographic plates is non trivial and uncertain for faint, very red stars. Given these considerations we warn against an over-interpretation of the residual discrepancies between model and data.

We do not exclude the possible enhancement at about $M_V > 16$ in stellar number densities close to the Galactic midplane (Kirkpatrick et al. 1994).

Fig. 1 thus demonstrates that the discrepancy between the nearby and photometric luminosity functions can be resolved by identifying the former with the single star luminosity function (smoothed by the metallicity and age dispersion and parallax and photometric measurement errors), and the latter with the system luminosity function. No additional hypothesis needs to be postulated to account for the discrepancy between the nearby and photometric luminosity functions.

5 THE STELLAR MASS FUNCTION

5.1 Is there a significant maximum at about $0.3 M_\odot$?

Although the photometric luminosity function used by Tinney (1993), $\Psi_{\text{TRM}}^*$, significantly overestimates the stellar number densities making it an incorrect estimator of the true stellar number density distribution with luminosity, its shape appears to be similar to our best-estimate Malmquist corrected photometric luminosity function, $\Psi_{\text{phot}}$ (fig. 4 in Kroupa 1995a). Thus we take seriously the suggestion by Tinney (1993) and Reid (1994) that the stellar mass function has a maximum at $m \approx 0.25 M_\odot$, and we investigate this matter more closely.

We note from our considerations in section 5 in Kroupa 1995a that Tinney’s survey contains nearly a factor two more stars at $M_{\text{bol}} = 9$ ($M_V \approx 10.5$), than the nearby star sample. The mass function estimated by Scalo (1986) assumes the stellar number density in the solar neighbourhood at this magnitude. We would thus expect the mass functions which Tinney (1993) derives from his luminosity function to lie above Scalo’s mass function at $m \approx 0.45 M_\odot$ by this same factor (the slope of the mass–$M_V$ relation is well defined at $M_V \approx 10.5$, see fig. 2 in Kroupa et al. 1993). However, we find in all panels of fig. 7 of Tinney (1993) that the “Scalo function seems to overpredict the number of stars in the solar neighbourhood . . . by about a factor of 2”. This results from an incorrect scaling of the Scalo mass function in his fig. 7 a–g (Tinney, private communication).

We focus attention on fig. 7g of Tinney (1993) and consider the mass–luminosity relation from Smith (1983) used by Tinney in fig. 7g as representing the empirical relation, adopt $M_{\text{bol, TRM}} = 4.72$, and obtain
\[ \log_{10} m = 1.119 - 0.177 M_{\text{bol}}, \]  \hspace{1cm} (1)

where \( m \) is the stellar mass in solar units. We follow Tinney (1993) and restrict this relation to \( 0.2 M_{\odot} \leq m \leq 0.4 M_{\odot} \). Thus, \( \frac{dM_{\text{bol}}}{dm} = -5.65/(m \ln 10) \), and the mass function in units of number of stars per \( pc^3 \) per solar mass is

\[ \xi_{\text{Tin}}(m) = 2.454 \frac{\Psi^*_{\text{TRM}}(M_{\text{bol}})}{m}, \]  \hspace{1cm} (2)

which we tabulate in Table 1.

It is important to emphasise that equation 1 is not a valid representation of the mass–luminosity data because it fails to account for the changes in stellar structure and photometric properties due to the onset of full convection and association of \( H_2 \) at about \( 0.3 M_{\odot} \). Structure in the mass function is inversely proportional to the first derivative of the mass–absolute magnitude relation which has a minimum at \( M_V \approx 7 \) and a pronounced maximum at \( M_V \approx 12 \) (fig. 2 in Kroupa et al. 1993). Although theory requires these features, Kroupa et al. 1990 and recently D’Antona & Mazzitelli (1994) warn against blind use of theoretical mass–luminosity relations, especially for stellar masses below \( 0.6 M_{\odot} \), because mixing-length theory and low temperature opacities are too uncertain. The derivative of the theoretical mass–luminosity relation is thus mostly useful for identification of structure and of the relevant physics, but cannot be used to obtain detailed quantitative results. These considerations should make it clear that any structure seen in the mass function which is derived from any mass–luminosity relation is very questionable.

In Table 1 Column 1 lists the absolute bolometric luminosity and Column 2 the corresponding stellar mass obtained from equation 5. Column 3 contains the photometric luminosity function used by Tinney (1993, his table 4), and Column 4 tabulates the resulting mass function (equation 8). We also add a short segment of the mass function derived by Scalo (1986, his table 4) by listing the absolute visual magnitude (Column 5), the stellar mass (Column 6) and the Scalo mass function which is obtained from his table IV by evaluating

\[ \xi_{\text{Sc}}(m) = \frac{\phi_{\text{ms}}(\log_{10} m)}{2h m \ln 10}, \]  \hspace{1cm} (3)

where \( \phi_{\text{ms}} \) is the number of stars per \( pc^2 \) per \( (\log_{10} m)^{-1} \). Following Scalo the vertical Galactic disk scale height is \( h = 325 \) pc (note that Tinney appears to adopt \( h = 350 \) pc but references Scalo).

In Fig. 2 we plot \( \xi_{\text{Tin}}(m) \) and \( \xi_{\text{Sc}}(m) \). In our figure the Scalo mass function lies below the mass function derived from Tinney’s luminosity function as expected.

The mass function, \( \xi_{\text{Tin}}(m) \), which we obtain from Tinney’s luminosity function, has a maximum at a mass of approximately \( 0.23 M_{\odot} \). If we were to extend the mass–luminosity relation (equation 1) to \( m < 0.2 M_{\odot} \) then the mass function would decrease further, making the maximum more apparent. Rather than recomputing \( \xi_{\text{Tin}} \) for all values of \( m \) we are satisfied that in his fig. 7g Tinney correctly plots his mass function (compare with Table 1). Tinney (1993) also finds the maximum for all theoretical mass–luminosity relations that he considers, and he and Reid (1994) conclude that the mass function for low mass stars is not a power law but shows significant structure with a maximum at \( 0.25 M_{\odot} \).

Reid (1994) argues that this “turnover in number density is real - the only surveys that find a continuously increasing mass function are those deriving masses from \( M_V \), a notoriously inaccurate mass indicator for very low mass stars”. We strictly contradict this statement for stellar masses \( m > 0.1 M_{\odot} \). While it is true that precise estimation of the true luminosity of low mass stars is difficult in the V-band because most of the radiation energy is emitted in the near infrared, the accuracy of V-band photometry cannot be questioned for very low mass stars”. Their data show that for \( 0.08 M_{\odot} < m < 0.18 M_{\odot} \) the dispersion in \( \log_{10} m \) for the mass–\( M_V \) relation fit is 0.060, for the mass–\( M_J \) relation fit it is 0.066, for the mass–\( M_H \) relation fit it is 0.054 and for the mass–\( M_K \) relation fit it is 0.067 (Henry & McCarthy 1993). Kroupa & Gilmore (1994) compare the mass–\( M_V \) relation derived by Kroupa et al. (1993) using V-band photometry with these data (not listed by Popper 1980),
which were derived at infrared wavelengths. The agreement is very good. For \( m < 0.1 \, M_\odot \) any photometric band provides a poor mass estimator because of the long contraction time scales for such stars (see e.g. D’Antona & Mazzitelli 1994). In any case, a rising mass function was also found by Henry & McCarthy (1990) based on \( M_K \).

In fig. 7 of Tinney (1993) only two panels show a difference between the alleged peak at about 0.22 \( M_\odot \) and the minimum at about 0.12 \( M_\odot \) amounting to a factor of approximately 2.1 (fig. 7c and 7d). Both mass functions were derived using the mass–luminosity relation model D of Burrows, Hubbard & Lunine (1989) evaluated at two ages. Figs 7a and 7b, based on their model B mass–luminosity relation at two ages, show a difference between maximum and minimum of a factor of 1.2, and in figs 7e and 7f (based on mass–luminosity relations from D’Antonna & Mazzitelli 1986) the mass function merely flattens in the range 0.15 \( M_\odot < m < 0.3 \, M_\odot \). The mass function in Tinney’s fig. 7g is based on the linear mass–luminosity relation, and shows a difference between peak and minimum of a factor of 1.6.

Thus, of the seven mass functions Tinney computes from his uncorrected luminosity function, only three show structure which amounts to a difference between alleged peak and minimum of more than 50 per cent. One of these is based on an inadequate linear mass–luminosity relation, and the other two are based on one theoretical mass–luminosity relation. The remaining four mass functions show structure amounting to less than a 20 per cent difference between peak and minimum and are based on only two theoretical mass–luminosity relations. It is important to bear in mind that the theoretical mass–luminosity relations used by Tinney (1993) are based on the same opacity tables and use similar equations of state, so that the physical parameter space scanned by Tinney (1993) is limited.

Having discussed the limited evidence for structure in the stellar mass function we must, however, keep in mind that Tinney (1993) computes the mass functions using the Malmquist uncorrected bolometric photometric luminosity function. Correcting for Malmquist bias would enhance the alleged maximum in the mass function.

The observed stellar luminosity function has the kind of structure we see in the first derivative of the theoretical mass–absolute magnitude relation (Kroupa et al. 1990, 1993). By Occam’s Razor we thus argue that the plateau at \( M_V \approx 7 \) and the maximum at \( M_V \approx 12 \) in the luminosity function reflect the changes in the slope of the mass–luminosity relation. While this does not disprove a maximum in the stellar mass function, it would appear a remarkable coincidence if main sequence stellar structure and star formation both lead to the same structure in the stellar luminosity function. Our assumption leads to a model consistent with star count data and can be verified observationally because luminosity functions of all stellar populations must show this same structure, independent of the stellar mass function (Kroupa et al. 1993).

In view of the uncertainties in stellar physics below 0.6 \( M_\odot \) (Kroupa et al. 1990, D’Antonna & Mazzitelli 1994), the presence of a plateau and maximum in the luminosity function at just the correct absolute magnitudes where theory suggests the same features in the slope of the mass–absolute magnitude relation, the lack by Tinney to account for unresolved binary stars in his sample (which can depress the photometric luminosity function at \( M_V \approx 15 \) by a factor of more than 2.5 – see fig. 21 in Kroupa et al. 1993) and the discrepancy in stellar number densities in his luminosity function (corrected and uncorrected) at \( M_{Bol} < 9.5 \) (Kroupa 1995a), we argue that the structure in the mass function found by Tinney is not significant. All these effects distort the shape of the luminosity function making any structure seen in the mass function derived from a photometric luminosity function using the (incorrect) ‘standard technique’ [(photometric luminosity function)/(slope of mass–absolute magnitude relation)] very questionable indeed.

Similar reservations apply to the result obtained by Strom, Strom & Merrill (1993) that the mass function for the young stellar population associated with the L1641 molecular cloud (at a distance of 480 pc) peaks near 0.3 \( M_\odot \). They base their conclusion on a theoretical mass–luminosity relation corrected for pre-main sequence brightening, which significantly adds to the uncertainties discussed above. Furthermore, at a distance of 480 pc virtually all binary systems remain unresolved. Strom et al. (1993) neglect to correct for these which can lead to serious underestimation of the number of faint stars because the proportion of pre-main sequence binary systems must be assumed to be higher than in the Galactic field (see Mathieu 1994 and references therein, Kroupa 1995b).

Finally, we emphasise that in their section 9 Kroupa et al. (1993) explicitly test a model of star count data based on a mass function with a maximum at about 0.3 \( M_\odot \) and find poor agreement with observed star counts. It is also worth noting that Kroupa et al. (1991) mention that they experienced difficulty unifying
both the nearby and photometric luminosity functions using other than a power law mass function below $0.5M_\odot$.

5.2 No significant maximum!

Consistent treatment of photographic and nearby star count data and of the mass–luminosity relation with detailed modelling of cosmic scatter (see also Section 3.2) provides good solutions to raw star count data if the number of stars at birth, $\xi(m)$, in the mass interval $m$ to $m + dm$ (in solar units) is approximated by

$$\xi(m) = k_\xi \begin{cases} 
0.5^{\alpha_1} m^{-\alpha_1}, & \text{if } 0.08 \leq m < 0.5; \\
0.5^{2.2} m^{-2.2}, & \text{if } 0.5 \leq m < 1.0; \\
0.5^{2.7} m^{-2.7}, & \text{if } 1.0 \leq m < \infty,
\end{cases}$$

(4)

where $k_\xi = 0.0873$ stars pc$^{-3}$ $M_\odot^{-1}$ if $\alpha_1 = 1.3$ scales to the observed stellar number density. We refer to equation 4 as the KTG($\alpha_1$) initial mass function. Allowing for unresolved binary stars and a possible metallicity gradient perpendicular to the Galactic disc Kroupa et al. (1993) constrain $\alpha_1$ to lie in the 95 per cent confidence interval 0.70–1.85.

The nearby data place poorer constraints on the stellar mass function. Considering the nearby star count data alone and applying the Kolmogorov-Smirnov test on a model nearby sample which accounts for trigonometric parallax and photometric measurement uncertainties and a detailed physical model of the dispersion in stellar luminosities due to a spread in metallicities and ages, the 95 per cent confidence interval for $\alpha_1$ is 0.6–2.4 (fig. 16 in Kroupa et al. 1993). Fitting a power-law mass function to the nearby luminosity function Haywood (1994) finds $1.3 < \alpha < 1.9$ (confidence interval is unspecified) for $m < 0.35M_\odot$, but notes that a change in $\alpha$ at about 0.35$M_\odot$ is not required by the data. His result is based on a theoretical mass–luminosity relation and does not account for the dispersion in metallicities and ages, nor for measurement errors. Using the mass–$M_V$ relation of Kroupa et al. (1993) and the KTG($\alpha_1$) mass function without modelling the metallicity and age dispersion and measurement errors we compute model single star luminosity functions, which we scale to the nearby luminosity function at $5.5 \leq M_V \leq 8.5$ (0.0106 stars pc$^{-3}$, table 2 in Kroupa 1995a). A $\chi^2$ test on the nearby luminosity function at $12.5 \leq M_V \leq 16.5$ (observed value: $0.051 \pm 0.011$ stars pc$^{-3}$) constrains $\alpha_1$ to lie in the range 0.66–1.44 with 95 per cent confidence. The corresponding ideal single star model luminosity functions are plotted in Fig. 3.

Comparison with the above result by Kroupa et al. (1993) illustrates that different statistical tests together with proper modelling of cosmic scatter and measurement errors can yield different results. It is always advisable to use as many different tests as possible, and we note that consistent modelling of cosmic scatter and measurement errors together with the Kolmogorov-Smirnov test yields more conservative (i.e. wider) bounds on $\alpha_1$ than the more constraining $\chi^2$ test. We stress however, that our fitting here (0.66 < $\alpha_1$ < 1.44), that of Haywood (1994) (1.3 < $\alpha$ < 1.9), and Kroupa et al. (1993) (0.70 < $\alpha_1$ < 1.85) are all consistent with $\alpha_1$ ≈ 1.3.

We emphasise that in our model the maximum in the photometric luminosity function at $M_V \approx 12$, $M_{\text{Bol}} \approx 9.7$ is reproduced despite the absence of any such structure in the power-law mass function. It results from a maximum in the first derivative of the mass–absolute magnitude relation because of changes in the stellar constitution at about 0.33$M_\odot$ owing to the onset of association of $H_2$ and the onset of a fully convective interior. This feature in the stellar luminosity function is thus universal, i.e. independent of the stellar mass function (Kroupa et al. 1993).

The KTG(1.3) mass function of the Galactic field star population is plotted as the thick solid line in Fig. 2. It has the same stellar number densities per mass interval as Scalo’s mass function for $m > 0.25M_\odot$ and the latter does not overpredict the number of stars as suggested by Tinney (1993). At lower masses, $\xi_{SC}(m)$ underestimates the number densities of stars because Scalo (1986) does not allow for unresolved binaries in his adopted luminosity function. Also, $\xi_{SC}$ follows the shape of the luminosity function adopted by Scalo (1986) because he uses an empirical, approximately linear, log(mass)–$M_V$ relation. The KTG($\alpha_1$) mass function is plotted and compared with $\xi_{SC}(m)$ for all stellar masses in fig. 22 of Kroupa et al. (1993).

It is of interest to note that our inverse dynamical population synthesis (Kroupa 1995b) suggests that there may be a relationship between $\alpha_1$ and the star formation mode. The dynamical properties of Galactic
field systems are best reproduced if most stars form in the dominant mode cluster (see Section 4) and if $\alpha_1 \approx 1.3$.

6 CONCLUSIONS

The highly significant difference between the nearby and Malmquist corrected photometric luminosity functions (Kroupa 1995a) can be resolved naturally if the nearby luminosity function is identified with the single star luminosity function, and if the Malmquist corrected photometric luminosity function is identified with the system luminosity function (Fig. 1). To this end we introduce in Section 4 a model of the dynamical properties of stellar systems which is consistent with all presently available observational constraints. The model luminosity functions are tabulated in the photometric V-, I- and K-bands and in bolometric magnitudes in Appendix 1.

It is important to emphasise that our model system luminosity function is a reasonable fit to the Malmquist corrected photometric luminosity function because cosmic scatter and photometric errors have been removed from the photometric luminosity function. These lead to Malmquist bias which, among other effects, broadens or smooths structure in the photometric luminosity function. The nearby luminosity function, however, has not been corrected for the metallicity and age spread nor for photometric and trigonometric parallax measurement errors. The pronounced ‘H$\alpha$–convection maximum’ at $M_V \approx 12$, which we expect on the basis of our ideal (i.e. single metallicity and age, no measurement errors) single-star model is thus smoothed out and swamped by the large statistical uncertainties.

Our result that binaries account for the difference agrees with the conclusions by Kroupa et al. (1993) who modelled raw star count data. No additional hypothesis to account for the difference is required.

The objections by Reid (1991) that binaries cannot account for the difference are valid only for the particular (‘favorite’) model parameters he considers. These, however, are inconsistent with observational mass-ratio distributions and with evidence from theoretical stellar models that the single star luminosity function is likely to have a pronounced maximum. Rather than concluding from his investigation that unresolved binary systems cannot account for the difference, the correct interpretation of his modelling is that it shows that his ‘favorite’ models are inconsistent with star count data.

The stellar mass function, which ‘unifies’ both the nearby and photometric luminosity functions, can be approximated by the KTG(1.3) mass function (equation 4).

The nearby star count data alone constrain the stellar mass function poorly. To significantly improve the statistical uncertainties of the single star luminosity function the stellar sample in which all binaries are resolved and which is complete to the faintest stellar luminosities has to be increased by at least an order of magnitude. This requires significant future effort in large-scale trigonometric parallax surveys to identify all faint stars in close proximity (e.g. within about 5 pc in the southern hemisphere and/or to distances larger than the 5.2 pc distance limit), and infrared speckle observations to identify all binary systems in the parallax survey.

There is no evidence for structure in the stellar mass function for low mass stars beyond the change in $\alpha_i$ at $m = 0.5 \, M_\odot$ (equation 4). A maximum in the mass function at a mass of approximately $0.25 \, M_\odot$ is found by researchers who (i) restrict their analysis to the photometric luminosity function (or a stellar luminosity function for a population of stars at sufficiently large distance so that the observational apparatus cannot resolve the majority of binary systems), (ii) do not correct for unresolved binaries, and (iii) use inadequate mass–luminosity relations (e.g. a linear log(mass)–absolute magnitude relation). In their section 9 Kroupa et al. (1993) show that a mass function with a maximum at about $0.3 \, M_\odot$ is a poor fit to the nearby and photometric luminosity functions.

Robust evidence in support of the proposition that there exists structure in the mass function beyond the flattening at $0.5 \, M_\odot$ does not presently exist. The extensive star count data obtained by Tinney et al. (1993) may shed light on this problem, but only after Malmquist bias has been taken care of very carefully, after the reason for the significant deviation of stellar number density at $M_{bol} < 9.8$ in their photometric luminosity function from the density in the other photometric luminosity functions (figs. 4 and 5 in Kroupa 1995a) has been resolved, and after unresolved binary systems have been accounted for. To this end a
detailed study of cosmic scatter for faint stars must be performed in the V-, R-, and I-bands. However, even then the residual uncertainties in the mass–luminosity relation will make conclusions concerning structure in the mass function beyond a flattening questionable.

Because cosmic scatter and the effects of binaries lead to a non-Gaussian dispersion of stellar absolute magnitudes which is also absolute magnitude dependent, we strongly urge researchers studying the stellar mass function to proceed along the lines of Kroupa et al. (1993), who model cosmic scatter and raw star count data (which are filtered to exclude galaxies, white dwarfs, and giant stars), rather than using the incorrect ‘standard technique’ of deriving a mass function directly from the photometric (or low spatial resolution) luminosity function.

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Table 1. The stellar mass function from $\Psi^*_\text{TRM}(M_{\text{Bol}})$ and from Scalo (1986).

| $M_{\text{Bol}}$ | $m$   | $\Psi^*_\text{TRM}(M_{\text{Bol}})$ | $\xi_{\text{TRM}}(m)$ | $M_V$ | $m$   | $\xi_{\text{Sc}}(m)$ |
|------------------|-------|-----------------------------------|------------------------|-------|-------|---------------------|
| 9.00             | 0.336 | 27.007                            | 0.197                  | 10    | 0.447 | 0.101               |
| 9.25             | 0.303 | 25.946                            | 0.210                  | 11    | 0.363 | 0.140               |
| 9.50             | 0.274 | 27.622                            | 0.247                  | 12    | 0.288 | 0.176               |
| 9.75             | 0.247 | 32.074                            | 0.318                  | 13    | 0.224 | 0.226               |
| 10.00            | 0.223 | 30.879                            | 0.340                  | 14    | 0.178 | 0.266               |
| 10.25            | 0.202 | 24.944                            | 0.303                  | 15    | 0.141 | 0.211               |
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### APPENDIX 1: The Stellar Luminosity Function (Section 4)

| $M_V$ | $\Psi_{\text{mod,sing}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ |
|-------|--------------------------|---------------|--------------------------|---------------|--------------------------|---------------|
|       |                          | $t = 0$       |                          | $t = 1 \text{ Gyr}$ |                          |               |
| 1.0   | 0.00                     | 0.00          | 0.00                     | 0.00          | 0.00                     | 0.00          |
| 1.5   | 0.05                     | 0.05          | 0.05                     | 0.05          | 0.10                     | 0.07          |
| 2.0   | 0.30                     | 0.12          | 0.30                     | 0.12          | 0.35                     | 0.13          |
| 2.5   | 0.25                     | 0.11          | 0.25                     | 0.11          | 0.50                     | 0.16          |
| 3.0   | 0.15                     | 0.08          | 0.15                     | 0.08          | 0.45                     | 0.15          |
| 3.5   | 0.25                     | 0.11          | 0.80                     | 0.20          | 0.65                     | 0.18          |
| 4.0   | 1.10                     | 0.24          | 0.75                     | 0.19          | 1.20                     | 0.25          |
| 4.5   | 3.40                     | 0.42          | 3.45                     | 0.42          | 3.35                     | 0.41          |
| 5.0   | 5.40                     | 0.53          | 4.95                     | 0.51          | 4.45                     | 0.48          |
| 5.5   | 3.25                     | 0.41          | 2.40                     | 0.35          | 2.65                     | 0.37          |
| 6.0   | 5.20                     | 0.52          | 5.35                     | 0.53          | 4.75                     | 0.50          |
| 6.5   | 5.40                     | 0.53          | 6.85                     | 0.60          | 5.45                     | 0.53          |
| 7.0   | 9.30                     | 0.69          | 5.00                     | 0.51          | 6.65                     | 0.59          |
| 7.5   | 3.95                     | 0.45          | 3.25                     | 0.41          | 3.80                     | 0.44          |
| 8.0   | 2.80                     | 0.38          | 2.30                     | 0.34          | 2.50                     | 0.36          |
| 8.5   | 8.45                     | 0.66          | 8.65                     | 0.67          | 8.20                     | 0.65          |
| 9.0   | 7.35                     | 0.62          | 6.80                     | 0.59          | 6.95                     | 0.60          |
| 9.5   | 12.75                    | 0.81          | 9.55                     | 0.70          | 10.55                    | 0.74          |
| 10.0  | 10.95                    | 0.75          | 12.00                    | 0.79          | 11.90                    | 0.79          |
| 10.5  | 19.95                    | 1.02          | 19.85                    | 1.02          | 19.85                    | 1.02          |
| 11.0  | 25.35                    | 1.15          | 21.20                    | 1.05          | 22.90                    | 1.09          |
| 11.5  | 34.80                    | 1.35          | 26.60                    | 1.18          | 28.15                    | 1.21          |
| 12.0  | 43.35                    | 1.51          | 21.85                    | 1.07          | 29.55                    | 1.24          |
| 12.5  | 41.70                    | 1.48          | 8.85                     | 0.68          | 18.95                    | 0.99          |
| 13.0  | 27.75                    | 1.20          | 10.15                    | 0.73          | 15.10                    | 0.89          |
| 13.5  | 23.40                    | 1.10          | 9.15                     | 0.69          | 13.55                    | 0.84          |
| 14.0  | 24.45                    | 1.13          | 3.85                     | 0.45          | 10.80                    | 0.75          |
| 14.5  | 19.80                    | 1.02          | 2.10                     | 0.33          | 8.55                     | 0.67          |
| 15.0  | 17.70                    | 0.96          | 3.40                     | 0.42          | 9.05                     | 0.69          |
| 15.5  | 23.75                    | 1.11          | 0.00                     | 0.00          | 9.10                     | 0.69          |
| 16.0  | 11.50                    | 0.77          | 0.00                     | 0.00          | 4.50                     | 0.48          |
| 16.5  | 0.00                     | 0.00          | 0.00                     | 0.00          | 0.00                     | 0.00          |
| $M_K$ | $\Psi_{\text{mod,sing}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ |
|---|---|---|---|---|---|---|
|   | $t = 0$ | $t = 1 \text{ Gyr}$ |   |   |   |   |
| 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 |
| 1.0 | 0.20 | 0.10 | 0.25 | 0.11 | 1.00 | 0.22 |
| 1.5 | 0.55 | 0.17 | 0.55 | 0.17 | 1.00 | 0.22 |
| 2.0 | 0.25 | 0.11 | 1.00 | 0.22 | 1.00 | 0.22 |
| 3.0 | 4.00 | 0.45 | 4.95 | 0.51 | 4.70 | 0.49 |
| 3.5 | 5.95 | 0.55 | 7.65 | 0.63 | 6.15 | 0.56 |
| 4.0 | 17.75 | 0.96 | 14.10 | 0.86 | 14.00 | 0.85 |
| 4.5 | 11.20 | 0.76 | 8.85 | 0.68 | 9.95 | 0.72 |
| 5.0 | 11.55 | 0.77 | 14.85 | 0.88 | 14.15 | 0.86 |
| 5.5 | 20.05 | 1.02 | 18.50 | 0.98 | 18.80 | 0.99 |
| 6.0 | 23.80 | 1.11 | 27.20 | 1.19 | 25.40 | 1.15 |
| 6.5 | 36.80 | 1.39 | 41.15 | 1.47 | 40.70 | 1.46 |
| 7.0 | 57.20 | 1.73 | 29.15 | 1.23 | 38.70 | 1.42 |
| 7.5 | 61.80 | 1.80 | 18.30 | 0.98 | 31.15 | 1.28 |
| 8.0 | 50.40 | 1.62 | 9.70 | 0.71 | 22.20 | 1.08 |
| 8.5 | 43.80 | 1.51 | 3.65 | 0.43 | 18.05 | 0.97 |
| 9.0 | 48.50 | 1.59 | 0.00 | 0.00 | 18.05 | 0.97 |
| 9.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| $M_I$ | $\Psi_{\text{mod,sing}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ |
|---|---|---|---|---|---|---|
|   | $t = 0$ | $t = 1 \text{ Gyr}$ |   |   |   |   |
| 0.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 |
| 1.5 | 0.20 | 0.10 | 0.20 | 0.10 | 0.15 | 0.08 |
| 2.0 | 0.25 | 0.11 | 0.25 | 0.11 | 0.60 | 0.17 |
| 2.5 | 0.30 | 0.12 | 0.30 | 0.12 | 0.60 | 0.17 |
| 3.0 | 0.25 | 0.11 | 0.80 | 0.20 | 0.70 | 0.19 |
| 3.5 | 2.05 | 0.32 | 2.05 | 0.32 | 2.35 | 0.35 |
| 4.0 | 4.60 | 0.49 | 4.35 | 0.47 | 4.30 | 0.47 |
| 4.5 | 5.55 | 0.54 | 4.65 | 0.49 | 4.35 | 0.47 |
| 5.0 | 6.20 | 0.57 | 7.60 | 0.63 | 6.20 | 0.57 |
| 5.5 | 11.10 | 0.76 | 8.30 | 0.66 | 8.65 | 0.67 |
| 6.0 | 6.45 | 0.58 | 5.20 | 0.52 | 5.80 | 0.55 |
| 6.5 | 5.85 | 0.55 | 6.70 | 0.59 | 6.50 | 0.58 |
| 7.0 | 11.95 | 0.79 | 11.65 | 0.78 | 11.95 | 0.79 |
| 7.5 | 16.60 | 0.93 | 15.20 | 0.89 | 15.35 | 0.89 |
| 8.0 | 18.25 | 0.98 | 24.15 | 1.12 | 21.75 | 1.06 |
| 8.5 | 36.05 | 1.37 | 36.10 | 1.37 | 37.30 | 1.40 |
| 9.0 | 52.30 | 1.65 | 32.80 | 1.31 | 39.15 | 1.43 |
| 9.5 | 63.75 | 1.83 | 14.80 | 0.88 | 29.20 | 1.23 |
| 10.0 | 40.40 | 1.45 | 14.45 | 0.87 | 22.45 | 1.08 |
| 10.5 | 31.85 | 1.29 | 5.00 | 0.51 | 13.65 | 0.84 |
| 11.0 | 29.35 | 1.24 | 4.85 | 0.50 | 13.90 | 0.85 |
| 11.5 | 31.45 | 1.28 | 0.45 | 0.15 | 12.05 | 0.79 |
| 12.0 | 19.05 | 1.00 | 0.00 | 0.00 | 7.50 | 0.62 |
| 12.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
These data are a model of an equal metallicity and age population of stars and binaries for which ideal photometry and distance information are available. It is based on the mass–$M_V$ relation tabulated by Kroupa et al. (1993) and the KTG(1.3) mass function. We restrict stellar masses to lie in the range $0.1 - 1.1 M_\odot$. This model stellar population results if the stars form in aggregates that are dynamically equivalent to the dominant mode cluster (Kroupa 1995c). The above tables list the luminosity functions in the V-, K- and I-bands, and also the bolometric luminosity functions. For each photometric band column 1 contains the absolute magnitude. Columns 2 and 3 contain the luminosity function obtained if all stars are counted individually and the Poisson uncertainty, respectively. Columns 4 and 5 list the initial system luminosity function (pre-main sequence brightening is not included) and the uncertainty, respectively. This luminosity function is obtained if all stars are paired at random from the KTG(1.3) mass function giving a binary proportion of unity and an uncorrelated mass ratio distribution with slight adjustment for ‘feeding’ during ‘pre-main sequence eigenevolution’ (for details see Kroupa 1995c). Columns 6 and 7 list the final system luminosity function and the uncertainty, respectively. This luminosity function corresponds to a population of Galactic field systems of which 48 per cent are binaries with orbital parameters as discussed in Section 4, and has evolved from the luminosity function listed under Column 4 after aggregate disintegration. The luminosity functions and the uncertainties tabulated here are averaged from $N_{\text{run}} = 20$ N-body simulations, each of an aggregate consisting of 200 binary systems (table 1 in Kroupa 1995b). Thus $\Psi = \frac{1}{N_{\text{run}}} \sum_{i=1}^{N_{\text{run}}} N_{M_i}$ and the uncertainty, $\delta \Psi = \sqrt{\frac{\sum_{i=1}^{N_{\text{run}}} N_{M_i}}{(N_{\text{run}} - 1)^2}}$ (standard deviation of the mean), where $N_{M_i}$ is the number of stars or systems in absolute magnitude bin $M_i$ in simulation $i$. 

| $M_{\text{bol}}$ | $\Psi_{\text{mod,sing}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ | $\Psi_{\text{mod,sys}}$ | $\delta \Psi$ |
|------------------|-------------------------|----------------|-------------------------|----------------|-------------------------|----------------|
| 8.0              | 15.20                   | 0.89           | 14.90                   | 0.88           | 15.50                   | 0.90           |
| 8.5              | 17.20                   | 0.95           | 25.95                   | 1.16           | 21.55                   | 1.06           |
| 9.0              | 33.65                   | 1.33           | 29.55                   | 1.24           | 32.05                   | 1.29           |
| 9.5              | 40.80                   | 1.46           | 40.50                   | 1.46           | 39.50                   | 1.44           |
| 10.0             | 61.05                   | 1.79           | 27.65                   | 1.20           | 37.55                   | 1.40           |
| 10.5             | 45.00                   | 1.53           | 21.15                   | 1.05           | 27.15                   | 1.19           |
| 11.0             | 42.95                   | 1.50           | 10.20                   | 0.73           | 19.75                   | 1.01           |
| 11.5             | 41.00                   | 1.46           | 8.15                    | 0.65           | 19.10                   | 1.00           |
| 12.0             | 45.05                   | 1.53           | 3.80                    | 0.44           | 18.10                   | 0.97           |
Figure captions

Figure 1. Comparison of our ideal model (i.e. single-metallicity and age and no measurement errors) Galactic field luminosity functions (Appendix 1) with observations in the photometric V-band (upper panel) and in bolometric magnitudes (lower panel) (Section 4). The solid-line histogram represents the observed nearby stellar luminosity function, \( \Psi_{\text{near}} \), which is not corrected for Malmquist-type bias (tables 2 and 8 in Kroupa 1995a) and which is smoothed at the faint end as detailed in section 4 of Kroupa (1995a). The filled circles represent our best estimate Malmquist corrected photometric luminosity function, \( \Psi_{\text{phot}} \) (tables 2 and 8 in Kroupa 1995a). We scale the model single star luminosity function to the nearby luminosity function at \( M_V \approx 10, M_{\text{bol}} \approx 9 \), and plot \( k \Psi_{\text{mod,sing}}(t = 0) \) (dotted curve, without pre-main sequence brightening) and \( k \Psi_{\text{mod,sys}}(t = 1 \text{ Gyr}) \) (solid curve). We stress that the solid curves in both panels are luminosity functions for a realistic model of the Galactic field population of systems consisting of 48 per cent binaries which have a period distribution consistent with the empirical G-, K-, and M-dwarf period distributions, the mass ratio distributions for G-dwarf systems as observed (Duquennoy & Mayor 1991), and the overall mass-ratio distribution given in Kroupa (1995c). The underlying KTG(1.3) mass function is plotted in Fig. 2.

Figure 2. Open triangles are the stellar mass function we derive from the luminosity function of Tinney (1993, shown in fig. 4 of Kroupa 1995a) using the log(mass)–bolometric magnitude relation from Smith (1983) and neglecting binary stars, and the solid dots are the mass function derived by Scalo (1986). The thick solid line is the KTG(1.3) mass function (equation 4). It is the mass function of the model Galactic field star population shown in Fig. 1.

Figure 3. The ideal model single star luminosity function corresponding to the 95 per cent confidence range on the KTG(\( \alpha_1 \)) mass function index \( \alpha_1 (0.66 \leq \alpha_1 \leq 1.44) \) is compared with the observational nearby luminosity function (table 2 in Kroupa 1995a) shown as the histogram and smoothed at the faint end as detailed in section 4 of Kroupa (1995a).