Integrating Mathematical and Simulation Approach for Optimizing Production and Distribution Planning With Lateral Transshipment in a Supply Chain

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ABSTRACT

Supply chain planning aims to maximize the chain’s profit and find an effective way to integrate production and distribution. Mathematical and simulation-based optimizations are two common disciplines. This study integrates both of them together to consolidate their advantages. A mathematical model is formulated to find an optimal production-distribution plan. Then, the result is fed into a simulation model operating under uncertainty to verify the feasibility of the plan. The integrated approach tries to find a feasible plan that satisfies both required customer service level and makespan limitation where safety stock is used to hedge against uncertainties, and lateral transshipment is used for emergency measures against excessive fluctuation of customer demand. A case study that optimizes the profit of an entire chain is used to demonstrate the algorithm. The outcomes of the study show that the proposed approach can yield feasible results (with near or even optimal solution) with much faster computational time as compared to the traditional simulation-based optimization.

KEYWORDS

Integrated Mathematical-Simulation Approach, Lateral Transshipment, Production-Distribution Planning, Safety Stock

1. INTRODUCTION

A supply chain’s activities involve the production and distribution of finished goods to the hands of end-customers. They generally contain two parts: production system and distribution system. Production system involves in planning, managing, and operating for the whole manufacturing activities (e.g., manufacturing itself, part handling, sequencing, and inventory level controlling. Distribution system is the set of process that conducts since the stage of picking the right finished goods from supplier, manufacturer, or warehouse until delivering it to customer. Traditionally, production and distribution planning are planned independently. With an increase of information sharing within the company or between the company and logistics providers, integration of production-distribution planning becomes more critical for solution improvement. However, this integration comes with greater complexity,
such as an increase in the size of the models and number of decision variables. An efficient method for solving these complex problems is needed that can provide a practical solution with reasonable computational time.

Disruption in the supply chain negatively affects a company’s finance and reputation (Filbeck and Zhao, 2020). Shrivastava et al. (2019) explored financial and quality problems resulting from uncertainty in the supply chain. As an aim to optimize the profit of the supply chain operations, the network design of the chain including the flow of materials among members and their locations as well as their production and inventory plans need to be optimized. Conventionally, there are two common optimization approaches, namely mathematical or analytical and simulation-based approaches. Mathematical approach can give exact optimal results and can quickly solve problems but it provides static results and has a drawback to incorporate uncertainties. In addition, when the problems become bigger and more complex, it would be too complex to model by using the mathematical formulation. Rather, simulation-based optimization can be used to model and optimize more complex and realistic problems but it cannot guarantee an optimal solution and tends to take a long running time. As a result, this study proposes an integrated mathematical-simulation method, which is an effective way to both provide realistic results and reduce the computational time.

The problem with production or distribution planning is once they are planned, it is difficult to change. For example, once the actual orders are realized, it will be too late to change the production plan, which can result in shortages or excess inventories. One way to tackle this problem is an introduction of lateral transshipment. Lateral transshipment is defined as stock movements among members in an echelon (Peterson et al., 2012). This can increase the supply chain profit by balancing inventories among retailers and reducing possible shortages without changing the original production-distribution plan of the supply chain.

The objectives of this study are:

- Propose an integrated mathematical and simulation method for optimizing the production and distribution planning in a supply chain under uncertainty environments.
- Develop near or possibly optimal and feasible solution with relatively fast computational time, as compared to traditional simulation-based optimization methods.
- Introduce lateral transshipment among retailers to further increase the performance of the solution.

The rest of this paper is organized as follows. The literature review section summarizes the relevant literature as well as identifies possible research gaps in the field. The case study section defines the problem to be solved in this study. The methodology section explains the proposed integrated approach. Then, the overall outcomes are discussed. Lastly, the study is finalized in the conclusion section.

2. LITERATURE REVIEW

As literature in supply chain optimization increased in the last decade, only the literature related to the study regarding the production-distribution supply chain planning, lateral transshipment, as well as mathematical and simulation-based optimization are reviewed here.

2.1 Production-Distribution Supply Chain Planning

The modelling of production-distribution systems in supply chains has been discussed in many studies over the years. Many studies have proposed different methods in finding the optimal production-distribution planning under these complexities. Fahimnia et al. (2013) carried out a comprehensive review and critique on production-distribution planning and optimization literature. They also defined the production-distribution planning problem as the problem of simultaneously optimizing the decision variables from different functions that have traditionally been optimized sequentially. Senoussi et
al. (2018) introduced a capacitated multi vehicle production-distribution problem. Their objective was to minimize the sum of production and distribution costs at the production facility and at the retailers. The problem was solved by five different genetic algorithm-based heuristics with three of them including the resolution of a Mixed Integer Linear Programming (MILP) as a sub-problem to generate new individuals in the population. An integrated production-distribution planning of a dairy industry was discussed in Ghosh and Mondal (2018). In this study, the used model included both production and distribution scheduling, which maximized the business’s overall profit contribution. MILP was used to formulate the problem, which was solved by CPLEX, an optimization software. From the mentioned studies above, the production and distribution plans need to be integrated together to realize the synergy benefit, which is the aggregated profit of the whole business.

2.2 Lateral Transshipment and Safety Stock

In this study, lateral transshipment and safety stock are introduced to manage an excess or shortage of inventory among retailers. Safety stock is an additional inventory that can mitigate risk of stock outs. It helps to accommodate supply or demand uncertainties Pham et al. (2020) demonstrated the benefit of safety stock in a supply chain with uncertainty. Zhou et al. (2020) formulated a (s,S) inventory model of Automated Teller Machines (ATMs) with two safety stocks corresponding to out-of-stock and full-of-stock risks. Experiments with real data showed that the model can significantly reduce costs and improve the overall service level. Bahroun and Belgacem (2019) studied the determination of dynamic safety stock levels under cyclic production schedules using the simulation. Their outcome showed the benefits of implementing such dynamic approaches including minimizing holding costs, improving the service level and lowering the probability of stock outs.

Lateral transshipment is defined as stock movements among the members of the supply chain in the same echelon (Paterson et al., 2011). Davis (1998) addressed that such transshipments can be performed by a retailer with excess inventory to provide items to other retailers that are out of stock. As shipments from central depots requires more handling procedures, lateral shipments are assumed to be faster. Tiacci and Saetta (2011) supported using lateral transshipment to compensate for order variability in a supply system without adding new or unnecessary items into the system. Liao et al. (2014) also confirmed that lateral transshipment can be cheaper and faster than placing emergency orders, which are more expensive and require longer time to process.

Emergency Lateral Transshipment (ELT) and Preventive Lateral Transshipment (PLT) are two opposite categories of transshipment policies. ELT is an emergency redistribution from a retailer that has reached a stock out level (Lee, 1987), while PLT redistributes stock between retailers to anticipate a stock out before actual customer order is realized (Tagaras, 1999). ELT is applicable after stock outs have occurred, whereas PLT reduces the risk of future stock outs. As a result, PLT takes place before the order is realized but after the order distribution has been updated (Li et al., 2013). A PLT process can be arranged in advance, to organize and manage inventory and minimize handling costs (Yousuk and Luong, 2013). Paterson et al. (2012) claimed that both ELT and PLT provide monetary benefits, although the benefits of PLT have only been used in a periodic review setting. In this study, ELT is used as the production-distribution plan is developed with forecasted customer demand and cannot be changed. So, actual customer demand is realized prior to applying the transshipment policy. An ELT algorithm is then used to determine the transshipment amount among retailers as a corrective policy to further maximize the whole supply chain’s profit.

2.3 Different Types of Optimization Methods

Optimization techniques are varying to be introduced for solving supply chain problems. Beamon (1998) carried out a review of the research in multi-stage and multi-shop supply chain modeling and identified four categories of models: deterministic analytic, stochastic analytic, economic, and simulation. Deterministic analytic can be modeled simply and solved efficiently with mathematical formulation. However, it does not include uncertainty. The stochastic analytic can consider uncertainty
while the economic model is used as a framework for modeling the buyer-supplier relationship in a supply chain. Lastly, simulation is the use of software to imitate the behavior of a system that would otherwise be difficult to analyze in reality. In this study, the deterministic analytic and simulation modelling will be focused on.

Deterministic mathematical optimization is a classic method of optimization. It is an ideal choice for optimization because of its capability of obtaining a globally optimal solution. Nevertheless, it is hard to mimic real-world situations because of its incapability of incorporating the attributes of uncertainty. An example of mathematical optimization is presented by Susarla and Karimi (2012) where an optimal production and distribution plan was solved by maximizing the profit of a pharmaceutical supply chain.

With increasing computational power during the last decade, heuristic optimization methods under the simulation-based optimization model has become more widely known. Singh et al. (2019) presented genetic algorithm-based approaches for solving a Vehicle Routing Problem. Fu (2002) indicated that current commercial software mainly integrates simulation and heuristics optimization algorithms. The satisfactory solution can be obtained from working with a family of solutions. However, a significant problem, is that a system is unaware of its stochastic nature. Hence, it does not make good use of the efficiency of computing resources. Variance reduction techniques are suggested in order to improve the convergence rate. Glover et al. (1999) presented a simulation-based optimization model using a practical software system called OptQuest, which combines three metaheuristics to optimize decisions. Layeb et al. (2018) also used OptQuest to solve a scheduling problem in freight transportation. However, its long computational solving time has led to various modifications to lower the computational time.

2.4 Hybrid Mathematical-Simulation Methods

To reduce the computational time and increase the solving ability with a reasonable realistic solution, many researchers started combining different metaheuristic methods together. Tavakkoli-Moghaddam et al. (2012) combined simulated annealing with genetic algorithm to minimize the traveled distance, total traveling time, number of vehicles and the cost function of transportation in a vehicle routing problem. Kumar et al. (2012) proposed a meta-hybrid heuristic technique based on genetic algorithm and particle swarm optimization to minimize system unbalance and maximize throughput in a production planning problem. These problems require metaheuristic methods because of the difficulty in obtaining satisfactory solutions using traditional optimization techniques due to the problems being non-deterministic polynomial (NP)-hard problems. There have been some attempts to integrate the mathematical model with the simulation model to benefit the shorter computational time. For instance, Acar et al. (2009) introduced a hybrid approach that incorporate the technique of optimization and simulation. First, the mathematical optimization model was used to obtain an optimal solution. Then, the result from the mathematical model was applied as an input in the simulation model to determine the influence of uncertainty on the value of objective function. Thammatadatrakul and Chidamrong (2017) further experimented with a modified hybrid mathematical-simulation approach for finding the optimum policy of controlling inventory in a hybrid manufacturing–remanufacturing system. Then, Chidamrong and Piyathanavong (2017) developed a hybrid mathematical-simulation approach based on iterative procedures to optimize the design of supply chain networks. The process would stop when the difference between subsequent solutions satisfies the pre-determined termination criteria.

After the hybrid mathematical and simulation methods created interest in the research area of optimization, many researchers started to apply this concept to various techniques of optimization. Some studies have incorporated the utilization and capacities of the system as one of the criteria in determining the optimal solution. Ko et al. (2006) proposed a hybrid optimization and simulation modeling approach for the design of a distribution network. Genetic Algorithm-based heuristics were used to determine the dynamic distribution network. Then, the simulation model determined the best capacities for warehouses based on the level of service time. Suyabatmaz et al. (2014) used the Ko et
al. (2006) and Acar et al. (2009) approach for solving the problem where the performance measures are related to the capacity utilization. Byrne and Bakir (1999) presented a hybrid mathematical-simulation approach for optimizing the effective production planning in a multi-period and multiple finished goods problem. First, they used Linear Programming (LP) for finding the optimal level of production. Then, the obtained result from the LP model was sent into a manufacturing simulation model and checked for the satisfaction of capacity. When the capacity is satisfied, the solving process will stop. Lee and Kim (2002) further extended the study of Byrne and Bakir (1999) by growing the complication of the supply chain and changing the method for capacity adjustment. An algorithm combining the analytic method with the simulation method was developed to solve production-distribution problems in a supply chain. Table 1 summarizes different methods of optimization and various approaches of hybrid mathematical-simulation approach.

From the review, both mathematical and simulation-based optimization methods have their pros and cons. The mathematical optimization method can provide a global optimal solution with fast computation speed, but is difficult to find dynamic variables such as makespan as it cannot include queuing and most uncertainties in the model. Rather, the simulation-based optimization can include various uncertainties to solve for dynamic variables but cannot provide a global optimal solution and the computational time is significantly longer than other optimization methods. As also seen from most reviewed literature, a hybrid between the mathematical model with the simulation model can improve both the obtained solution and computational time. However, the integrated mechanisms and imposed constraints to set an appropriate level of hybridization in each study are different depending on their different natures of the system. In our study, we focus on the feasibility of a plan, which consists of the limitation of working time for an allowable limited makespan and a possible required service

Table 1. Literature review of different methods of optimization

| Authors                  | Area of interest                                      | Authors                  | Area of interest                                      | Authors                  | Area of interest                                      |
|--------------------------|-------------------------------------------------------|--------------------------|-------------------------------------------------------|--------------------------|-------------------------------------------------------|
| Susarla & Kurmi (2012)   | Optimal production and distribution plan of           | Fu (2002)                | Overview of simulation based optimization.            | Acar et al. (2009)       | Hybrid mathematical-simulation method by determining the impact of uncertainty on the objective function. |
|                          | pharmaceutical supply chain.                         |                          |                                                       |                          |                                                       |
| Glover et al. (1999)     | Introduction of simulation based-optimization tool    | Thammanatarrakul and    | Optimal inventory control policy of a hybrid          |                          |                                                       |
|                          | called OptQuest.                                      | Chidambrong and          | manufacturing-remanufacturing system.                |                          |                                                       |
|                          |                                                        | Piyathanasawong (2017)   |                                                        |                          |                                                       |
| Layeb et al. (2018)      | Simulation based optimization for scheduling freight  | Chidambrong and          | Hybrid mathematical-simulation approach in           |                          |                                                       |
|                          | transportation.                                       | Piyathanasawong (2017)   | supply chain network design.                         |                          |                                                       |
| Ko et al. (2006)         |                                                       |                          |                                                       |                          |                                                       |
| Sayahatraz et al. (2014) |                                                       |                          |                                                       |                          |                                                       |
| Byrne & Bakir (1999)     |                                                       |                          |                                                       |                          |                                                       |
| Lee & Kim (2002)         |                                                       |                          |                                                       |                          |                                                       |
| Tavakkoli-Moghaddam et al.|                                                       |                          |                                                       |                          |                                                       |
|                          |                                                        |                          |                                                       |                          |                                                       |
| Kurnur et al. (2012)     |                                                       |                          |                                                       |                          |                                                       |
level. To the best of our knowledge, the makespan has not been sufficiently taken into consideration in most studies and thus, is considered in this study. In addition, many studies also show that lateral transshipment can improve the solution without affecting the original production-distribution plan, therefore is also included in this study to further improve the solution.

The main contribution of this study is to propose an integration of production and distribution allocation in a supply chain problem with lateral transshipment. Taking into the account of these matters, we aim to develop an integrated approach that is able to solve for the near or possibly optimal (and feasible) production-distribution plan considering lateral transshipment that not only maximizes the profit of the whole supply chain but also meets the required service level and makespan requirement in a reasonable computational time under uncertain environments.

3. CASE STUDY

In this paper, a model of supply chain with different structures, and activities are examined. Specifically, there are two main activities, which are production and distribution, are considered. The production activities are associated with transforming materials into components, sub-assemblies, or finished goods at a production facility while distribution activities are responsible for keeping and transporting final products among various locations in the supply chain, including the production facility, warehouses, and the retailers. It is assumed that safety stock is hold at the retailer as counter measure for unexpected order. In addition, lateral transshipments among retailers are allowed within the same period as an emergency measure against the excessive fluctuation of order. It should be noted that when the actual order is realized, the production-distribution plan cannot be changed at this stage, and the retailers are assumed to be in close proximity so that lateral transshipment can be achieved instantly within the same period.

3.1 Supply Chain Model

This hypothetical case study is a multi-echelon, multi-period, and multi-product problem. It was modified from Lee and Kim (2002) to illustrate the proposed integrated mathematical and simulation approach in the study. Figure 1 presents the flow of the supply chain in this study. The production part has two production lines, where production line 1 (shop 1) fabricates $N$ components to be used to produce $M$ finished goods in production line 2 (shop 2). Each production line contains 3 machines (MCs), arranged in a flow line. Unprocessed materials are the inputs for the first MC of each production line, which are MCs 1.1 and 2.1. In Figure 1, the arrows indicate the manufacturing
flow, which starts at MC 1.1 and ends at MC 2.3. The determination of production quantity is done by the retailers, where the finished goods are sold and revenues are generated. The finished goods are assumed to be sold at the end of each period. However, the unsold finished goods is left for the following period and thus results in an inventory holding cost.

There is a stack point, warehouses, and retailers in the part of distribution system. Before the finished goods are distributed to warehouses or retailers, they are initially gathered and held at the stack point. The finished goods at the stack point then are transferred and stored at warehouses before distributing to retailers, or they can be directly distributed to the retailers. It should be noticed that the storage costs in warehouse are generally lower than other locations in the supply chain. Unprocessed materials and components are only kept in the production plant. As backlogging is not allowed, the unsatisfied orders in the current period cannot be fulfilled in the next period resulting in lost sales and incurring shortage cost. It is also assumed that there is no initial inventory at every location. The aim is to maximize the profit of the whole chain, subject to imposed resource constraints. For a demonstration, this study considers three periods. Each period is equivalent to a working month. In other words, a period has 21,600 minutes of working time assuming that there are 30 days of 12 working hours a day in every working month. As a result, the time until all finished goods are fabricated and distributed, also called makespan limitation, is restricted under 21,600 minutes every period. In other circumstances, the makespan limitation is determined differently, such as the working hour based on a machine’s capacity or on regulation. The service level (SL) in Equation (1) is set at 90% for all finished goods, retailers, and periods:

$$SL = \frac{\text{finished goods sold}}{\text{Demand}}$$  \hspace{1cm} (1)

### 3.2 Data in Case Study

In each period, the demand for each finished goods at each retailer is considered to be normally distributed with a mean and a standard deviation as presented in Table 2. The number of units of unprocessed materials or components required to fabricate a unit of finished goods or components is shown in Table 3. The Bill of Materials of both finished goods is shown in Figure 2. All monetary units are in the dollar ($).

The production, holding, shortage, and distribution costs are shown in Tables 4 – 8. The transportation time from each origin to the destination is shown in Table 7. For example, the transportation time from the stack point to retailer 2 is 40 minutes and the cost is $25. The production time of each component and finished goods is shown in Table 9. The holding capacity for each location

| Table 2. Mean and standard deviation of finished goods j’s demand at retailer q in period t (units) |
|-----------------------------------------------|
| **Retailer (q)** | **1** | **2** | **1** | **2** | **1** | **2** |
| **Finished Goods (j)** | **Mean Order (D_{jq})** | **Period (t)** | **1** | **14** | **12** | **14** | **16** | **14** | **12** |
| | **2** | **16** | **10** | **14** | **14** | **10** | **14** |
| | **3** | **14** | **14** | **14** | **14** | **12** | **12** |
| **Standard Deviation (sd_{jq})** | **Period (t)** | **1** | **3** | **2** | **3** | **3** | **2** |
| | **2** | **3** | **2** | **3** | **3** | **2** | **3** |
| | **3** | **2** | **3** | **3** | **3** | **2** | **3** |
Table 3. Number of units of unprocessed materials or components required to fabricate one unit of finished goods or components (units)

| Component (i) | 1 | 2 | Material (r) | 1 | 2 | Component (j) | 1 | 2 | Component (i) | 1 | 2 |
|---------------|---|---|-------------|---|---|--------------|---|---|--------------|---|---|
| Material (k)  | 1 | 2 | 4           | 1 | 2 | 3            | 1 | 2 | 3            | 1 | 2 |
|               | 2 | 3 | 2           | 2 | 2 | 2            | 2 | 3 | 4            |   |   |

Figure 2. Bill of Materials for finished goods 1 and 2

Table 4. Manufacturing cost ($) of component i, finished goods j, and unprocessed materials k and r in each period t

| Component (i) | 1 | 2 | Finished goods (j) | 1 | 2 | Unprocessed material (k) | 1 | 2 | Unprocessed material (r) | 1 | 2 |
|---------------|---|---|-------------------|---|---|--------------------------|---|---|--------------------------|---|---|
| Period (t)    |   |   |                    |   |   |                          |   |   |                          |   |   |
| 1             | 15| 10| 30                | 30| 30| 4                        | 3 | 4 | 4                        | 5 | 5 |
| 2             | 15| 10| 30                | 30| 30| 5                        | 4 | 6 | 6                        |   |   |
| 3             | 15| 10| 30                | 30| 30| 7                        | 5 | 7 | 8                        |   |   |

Table 5. Holding cost ($) of component i and unprocessed material k and r in each period t in the production plant

| Component (i) | 1 | 2 | Unprocessed material (k) | 1 | 2 | Unprocessed material (r) | 1 | 2 |
|---------------|---|---|--------------------------|---|---|--------------------------|---|---|
| Period (t)    |   |   |                          |   |   |                          |   |   |
| 1             | 12| 10| 5                        | 5 | 5 | 5                        | 5 | 5 |
| 2             | 12| 10| 5                        | 5 | 5 | 5                        | 5 | 5 |
| 3             | 12| 10| 5                        | 5 | 5 | 5                        | 5 | 5 |

Table 6. Holding cost ($) of finished goods j at the stack point, warehouse p, and retailer q

| Finished Goods (j) | Stack Point | Warehouse (p) | Retailer (q) |
|--------------------|-------------|---------------|--------------|
|                    | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| 1                  |   |   | 1 | 10| 20| 15| 30| 20| 30|
|                    |   |   | 2 | 10| 20| 15| 30| 20| 30|
|                    |   |   | 3 | 10| 20| 15| 30| 20| 30|
| 2                  |   |   | 1 | 10| 15| 20| 40| 50| 40|
|                    |   |   | 2 | 10| 15| 20| 40| 50| 40|
|                    |   |   | 3 | 10| 15| 20| 40| 50| 40|
in the supply chain is shown in Table 10. The transshipment costs among retailers are shown in Table 11. It is assumed to be the same in every period. Big M (very high value) is introduced to discourage the algorithm from choosing to transship to and from the same place. These cost parameters were partly assumed by the authors based on the guideline described in Lee and Kim (2002).

### Table 7. Transportation cost ($) and time (min) for all finished goods

| Warehouse (\(p\)) | 1  | 2  | Retailer (\(q\)) | 1  | 2  | 3  | Retailer (\(p\)) | 1  | 2  | 3  |
|-------------------|----|----|-------------------|----|----|----|-------------------|----|----|----|
| Transportation cost ($) | Stack | 10 | 15 | Stack | 20 | 25 | 20 | 1   | 20 | 20 | 15 |
| Transportation time (min) | Stack | 80 | 90 | Stack | 50 | 40 | 50 | 1   | 90 | 60 | 80 |

### Table 8. Shortage cost ($) for each finished goods \(j\) at retailer \(q\) in period \(t\)

| Retailer (\(q\)) | 1  | 2  | 3  | 2  | 3  | 3  | 2  |
|-------------------|----|----|----|----|----|----|----|
| Finished Goods (\(j\)) | 1  | 2  | 1  | 2  | 1  | 2  |
| Period (\(t\))   | 1  | 550| 700| 600| 750| 600| 700|
|                   | 2  | 550| 700| 600| 750| 600| 700|
|                   | 3  | 550| 700| 600| 750| 600| 700|

### Table 9. Production time (min)

| Production line 1 | Production line 2 |
|-------------------|-------------------|
| Machine (\(u\))  | Machine (\(v\))  |
| 1  | 2  | 3  | 1  | 2  | 3  |
| Component (\(i\)) | 1  | 15 | 10 | 10 | 1  | 30 | 20 | 30 |
|     | 2  | 15 | 15 | 5  | 2  | 30 | 30 | 20 |

### Table 10. Holding capacity of each location in the distribution system (units)

| Stack Point | Warehouse (\(p\)) | Retailer (\(q\)) |
|-------------|-------------------|-------------------|
|             | 1  | 2  | 1  | 2  | 3  | 1  | 2  | 3  |
| Period (\(t\)) | 1  | 10 | 1,000| 1,000| 20 | 20 | 20 | 20 |
|              | 2  | 10 | 1,000| 1,000| 20 | 20 | 20 | 20 |
|              | 3  | 10 | 1,000| 1,000| 20 | 20 | 20 | 20 |
Indices:

- $t$ period ($t = 1, 2, \ldots, T$)
- $i$ component in production line 1 ($i = 1, 2, \ldots, I$)
- $j$ finished goods in production line 2 ($j = 1, 2, \ldots, J$)
- $u$ machine in production line 1 ($u = 1, 2, \ldots, U$)
- $v$ Machine in production line 2 ($v = 1, 2, \ldots, V$)
- $k$ unprocessed material used for production line 1 ($k = 1, 2, \ldots, K$)
- $r$ unprocessed material used for production line 2 ($r = 1, 2, \ldots, R$)
- $p$ warehouse ($p = 1, 2, \ldots, P$)
- $q$ retailer ($q = 1, 2, \ldots, Q$)

Parameters:

- $D_{jqt}$ demand for finished goods $j$ at retailer $q$ in period $t$ (units)
- $a_{ij}$ numbers of component $i$ needed to fabricate a unit of finished goods $j$ (units)
- $d_{ki}$ numbers of unprocessed material $k$ needed to fabricate a unit of component $i$ (units)
- $g_{ijr}$ numbers of unprocessed material $r$ needed to fabricate a unit of finished goods $j$ (units)
- $c_{it}$ manufacturing cost of component $i$ in period $t$ ($/unit$)
- $c_{jt}$ manufacturing cost of finished goods $j$ in period $t$ ($/unit$)
- $c_{kt}$ ordering cost of unprocessed material $k$ in period $t$ ($/unit$)
- $c_{rt}$ ordering cost of unprocessed material $r$ in period $t$ ($/unit$)
- $h_{ii}$ holding cost of component $i$ in period $t$ ($/unit$)
- $h{k}_t$ holding cost of unprocessed material $k$ in period $t$ ($/unit$)
- $h{r}_t$ holding cost of unprocessed material $r$ in period $t$ ($/unit$)
- $HL_{jt}$ holding cost of finished goods $j$ at stack point in period $t$ ($/unit$)
- $HP_{jt}$ holding cost of finished goods $j$ at warehouse $p$ in period $t$ ($/unit$)
- $HQ_{jt}$ holding cost of finished goods $j$ at retailer $q$ in period $t$ ($/unit$)
- $SO_{jq}$ shortage cost of finished goods $j$ at retailer $q$ in period $t$ ($/unit$)
- $LPC_{pt}$ transportation cost of finished goods from stack point to warehouse $p$ ($/unit$)
- $LQC_{qt}$ transportation cost of finished goods from stack point to retailer $q$ ($/unit$)
- $PQC_{pq}$ transportation cost of finished goods from warehouse $p$ to retailer $q$ ($/unit$)
- $LC_{t}$ holding capacity of stack point for all finished goods in period $t$ (units)
- $PC_{pt}$ holding capacity of warehouse $p$ for all finished goods in period $t$ (units)
- $QC_{qt}$ holding capacity of all finished goods at retailer $q$ in period $t$ (units)
- $a_{iu}$ operating time to fabricate a unit of component $i$ on machine $u$ (min)
- $a_{jv}$ operating time to fabricate a unit of finished goods $j$ on machine $v$ (min)
- $A_{pt}$ transportation lead time for finished goods from stack point to warehouse $p$ (min)
- $B_{pt}$ transportation lead time for finished goods from stack point to retailer $q$ (min)
- $C_{pq}$ transportation lead time for finished goods from warehouse $p$ to retailer $q$ (min)
- $WLC_{t}$ workload capacity in period $t$ (min)
Price\(_j\)_ price of finished goods \(j\) ($/unit)

\(SS_{jq}\)_ safety stock of finished goods \(j\) at retailer \(q\) (units)

\(TC_{jq1q2t}\)_ transshipment cost of finished goods \(j\) from retailer \(q1\) to retailer \(q2\) in period \(t\) ($/unit)

Decision variables:

\(X_{it}\)_ numbers of component \(i\) fabricated in production line 1 in period \(t\) (units)

\(Y_{jt}\)_ numbers of finished goods \(j\) fabricated in production line 2 in period \(t\) (units)

\(I_{it}\)_ numbers of end of period inventory of component \(i\) kept in period \(t\) (units)

\(E_{kt}\)_ numbers of unprocessed material \(k\) purchased in period \(t\) (units)

\(F_{rt}\)_ numbers of unprocessed material \(r\) purchased in period \(t\) (units)

\(I_{kt}\)_ numbers of inventory of unprocessed material \(k\) in period \(t\) (units)

\(L_{rt}\)_ numbers of inventory of unprocessed material \(r\) in period \(t\) (units)

\(LP_{jpt}\)_ numbers of finished goods \(j\) distributed from stack point to warehouse \(p\) in period \(t\) (units)

\(LQ_{jqt}\)_ numbers of finished goods \(j\) distributed from stack point to retailer \(q\) in period \(t\) (units)

\(PQ_{jpt}\)_ numbers of finished goods \(j\) distributed from warehouse \(p\) to retailer \(q\) in period \(t\) (units)

\(L_{jt}\)_ numbers of finished goods \(j\) kept at stack point in period \(t\) (units)

\(P_{jpt}\)_ numbers of finished goods \(j\) kept at warehouse \(p\) in period \(t\) (units)

\(Q_{jqt}\)_ numbers of finished goods \(j\) kept at retailer \(q\) in period \(t\) (units)

\(sold_{jpt}\)_ numbers of finished goods \(j\) sold at retailer \(q\) in period \(t\) (units)

\(WL_{t}\)_ workload of system in period \(t\) (min)

\(trans_{jq1q2t}\)_ transshipment of finished goods \(j\) from retailer \(q1\) to retailer \(q2\) in period \(t\) (units)

Objective Function

\[
\text{Max } Z = \text{Profit of the whole chin} = \text{Revenue} - \left( \text{Manufacturing Cost} + \text{Holding Cost} + \text{Transportation Cost} + \text{Shortage Cost} \right)
\]

\[
\text{Profit} = \text{Revenue} - \sum_{i} \sum_{j} \sum_{p} \sum_{q} price_j \cdot sold_{jpt} - \sum_{t} \left[ \sum_{i} (c_{ij}X_{it} + h_i + \frac{L_{it} + I_{it_{t+1}}}{2}) + \sum_{j} Q_{jpt} + \sum_{k} (c_{jk}E_{kt} + h_k + \frac{L_{kt} + I_{kt_{t+1}}}{2}) + \sum_{r} h_r + h_r + \frac{L_{rt} + I_{rt_{t+1}}}{2} + \sum_{p} \sum_{q} H_{ij} \cdot \frac{P_{jpt} + P_{jpt_{t+1}}}{2} + \sum_{q} \sum_{p} H_{ij} \cdot \frac{Q_{jpt} + Q_{jpt_{t+1}}}{2} + \sum_{q} \sum_{p} L_{ij} \cdot \frac{LP_{jpt} + LP_{jpt_{t+1}}}{2} + \sum_{q} \sum_{p} L_{ij} \cdot \frac{LQ_{jqt} + LQ_{jqt_{t+1}}}{2} + \sum_{q} \sum_{p} PQ_{jpt} + PQ_{jpt_{t+1}} + \sum_{q} \sum_{p} \sum_{t} (SQQ_{iqt} + \frac{SQQ_{iqt_{t+1}}}{2}) + \sum_{q} \sum_{p} \sum_{t} TC_{jq1q2t} + tran_{jq1q2t} \right]
\]
subject to:

\[ I_{it} = I_{it-1} + X_{it} - \sum_{j} q_{ij} Y_{jt}, \forall i, t \]  \hspace{1cm} (3)

\[ I_{rt} = I_{rt-1} + F_{rt} - \sum_{j} q_{jr} Y_{jt}, \forall r, t \]  \hspace{1cm} (4)

\[ I_{kt} = I_{kt-1} + E_{kt} - \sum_{i} d_{ik} X_{it}, \forall k, t \]  \hspace{1cm} (5)

\[ L_{jt} = L_{jt-1} + Y_{jt} - \sum_{p} P_{jpt} - \sum_{q} L Q_{jqqt}, \forall j, t \]  \hspace{1cm} (6)

\[ P_{jpt} = P_{jpt-1} + L P_{jpt} - \sum_{q} P Q_{jpt}, \forall j, p, t \]  \hspace{1cm} (7)

\[ Q_{jqt} = Q_{jqt-1} + \sum_{p} P Q_{jpt} + L Q_{jqqt} - sold_{jqt}, \forall j, q, t \]  \hspace{1cm} (8)

\[ \sum_{j} L_{jt} \leq LC_{t}, \forall t \]  \hspace{1cm} (9)

\[ \sum_{j} P_{jpt} \leq PC_{pt}, \forall p, t \]  \hspace{1cm} (10)

\[ \sum_{j} Q_{jqt} \leq QC_{qt}, \forall q, t \]  \hspace{1cm} (11)
s_{jqt} \leq D_{jqt}, \forall j, q, t \tag{12}

\begin{align*}
WL_t = & \sum_{i}^{U} \sum_{a}^{V} a_{it} X_{ai} + \sum_{j}^{J} \sum_{q}^{Q} a_{jqt} Y_{jqt} + \sum_{p}^{P} A_{p} LP_{pqt} \\
& + \sum_{j}^{J} \sum_{q}^{Q} B_{pq} LQ_{jqt} + \sum_{j}^{J} \sum_{p}^{P} \sum_{q}^{Q} C_{pq} PQ_{jpat}, \forall j, q, t \tag{13}
\end{align*}

WL_t \leq WLC_t, \forall t \tag{14}

Q_{jqt} \geq SS_{jqt}, \forall j, q, t \tag{15}

X_{ai}, I_{it} \geq 0, \forall i, t \tag{16}

Y_{jqt} \geq 0, \forall j, t \tag{17}

E_{kt}, I_{kt} \geq 0, \forall k, t \tag{18}

F_{rt}, I_{rt} \geq 0, \forall r, t \tag{19}

L_{jqt} \geq 0, \forall j, t \tag{20}

P_{jpt}, LP_{jpt} \geq 0, \forall j, p, t \tag{21}

Q_{jqt}, LQ_{jqt} \geq 0, \forall j, q, t \tag{22}

PQ_{jpat} \geq 0, \forall j, p, q, t \tag{23}

s_{jqt} \geq 0, \forall j, q, t \tag{24}

I_{i0}, L_{k0}, I_{r0}, L_{j0}, P_{j0}, Q_{j0} = 0, \forall i, j, r, k, p, q \tag{25}
An ideal and optimal production and distribution plan under no uncertainty is built to find the maximum total supply chain profit with the mathematical model. It is the profit from the total revenue of all retailers from selling products deducted by the overall costs in the supply chain for all periods as presented in Equation (2). The overall costs include manufacturing and inventory cost for all components, finished goods, and unprocessed materials, transshipment cost, transportation cost, and shortage cost. Equations (3) to (8) are the constraints related in balancing inventory of their corresponding units of interest. For example, under units of finished goods, the respective constraint ensures that the numbers of finished goods at the end of period inventory must be equal to the sum of the numbers of finished goods received in the present period and the numbers of their inventory in the previous period minus the finished goods leaving. Equations (9) to (11) are the constraints related to the warehouse capacity at the stack point, warehouses, and retailers, respectively. Equation (12) shows that the numbers of each finished goods in each period cannot be sold over the number of their customer need. Equation (13) corresponds to the total workload of the production-distribution system in each period. With a specified value of workload, this equation will determine the production and transportation quantity of the solution. The workload capacity constraint as presented in Equation (14) will be further discussed in the next section. Equation (15) is related to safety stock constraint. This constraint guarantees that the holding inventory of finished goods \( j \) need to be larger than the safety stock at retailer \( q \), which will also be further explained in the next section and assumed to be zero at first. Equations (16) to (24) show the non-negativity restriction on the decision variables. Equation (25) sets the initial inventory in the system to be zero.

4.2. Simulation Model

Actual makespan, representing by the total simulation run time, will be calculated by the simulation model. This is the time from the production of the first component till the last finished goods shipped to the retailer. In reality, it is not possible to calculate mathematically as there are a number of uncertainties, queueing and complex material flow inside the production processes. However, it can easily be carried out by the simulation model. In contrast, the workload, which is determined by multiplying the processing time by the total number of units, shows the total operation time of production-distribution system. Once the system does not have any simultaneous work or queues, the makespan can be equal to the workload. Regarding the uncertainties in the problem, demand uncertainty is shown in Table 2, following a normal distribution. Also, machine breakdown can create such uncertainties by causing a breakdown for a certain time. Each machine is estimated to have its availability around 90%. As a result, each machine has an uptime and a downtime. The uptime follows a normal distribution that has a mean is 100 minutes and a standard deviation is 20 minutes while the downtime follows a normal distribution that has a mean is 10 minutes and a standard deviation is 2 minutes. Under this terminating system, our simulation model is experimented with 10 replications. It has also been determined that the 95% confidence interval of the objective value (supply chain profit) has a width of less than 5% of the mean under this setting of the experiment.

4.3 An Integrated Mathematical-Simulation Approach

Our proposed approach is separated into two consecutive phases. Phase I produces a feasible plan, which does not surpass the limitation of makespan as well as satisfies the required service level requirement. Phase II is then further determined for the best amount of transshipments between retailers, resulting in the near or possibly optimal solution.

4.3.1 Phase I

Figure 3 illustrates the flowchart of Phase I where the initial safety stock is calculated by the mathematical model. This is an optimal ideal solution without concerning any uncertainties and
limitation of the makespan. With the safety stock to prevent any shortage possibility, this plan is considered to be feasible with regard to the imposed service level requirement. Under this feasible production-distribution plan, the plan will then be simulated in the simulation model to determine the actual realistic makespan that would otherwise have been hard to obtain from the pure mathematical model. In addition, demand variation and machine breakdowns, which are uncertainties in the system can be incorporated into the simulation model.

Having obtained the actual makespan of each period, it needs to be checked whether or not it is beyond the limitation of the available makespan. If any period exceeds this limitation, either its production level or the mode of transportation has to be cut down or changed to reduce the time, resulting in a higher cost. As a result, the workload capacity, denoted by $WLC_t$, in each period with a makespan over the limit needs to be recomputed based on Equation (26). Then, Equation (14) will be updated in the mathematical model. The mechanism and the workload adjustment integrating the mathematical and simulation model can be carried out with the formulation as follows:

$$WLC_t = WLC_t \times AF_t$$

(26)

$$AF_t = \frac{MSL_t}{MS_t}$$

(27)

where $t = 1, 2, \ldots, T$ is the period. $MSL_t$ is the makespan limitation in period $t$, which is set to 21,600 minutes (1 month) for all periods. $AF_t$ is the adjusting factor in period $t$ that surpasses the $MSL_t$ in each period, determined by Equation (27). The determined $AF_t$ will then be used for calculating the $WLC_t$ in the next iteration, as shown in Equation (26). This adjustment mechanism of the workload capacity allows faster convergence to find a near or possibly optimal and feasible solution. $MS_t$ is the actual makespan for each period $t$ obtained by the simulation model. The initial $WLC_t$ should be arbitrarily set to be large enough to allow the mathematical model to yield an optimal solution. As a
result, the initial $W_{LCT}$ is arbitrarily set to 50,000 minutes. Periods with their makespan not exceeding the limitation undergo no adjustment in their $W_{LCT}$. With 10 independent replications and selecting the upper 95% confidence interval for the average makespan of each period, an actual makespan in each period can be determined. This is to make sure that a bad situation in which the makespan could possibly be larger than usual is avoided. Each iteration is then repeated until the makespan limitation is satisfied for all periods.

Furthermore, the required service level must be greater than 90%. Otherwise, it is necessary to add safety stock ($SS_{jq}$) to the mathematical model so that the service level requirement of at least 90% can be satisfied. When the demand uncertainty is not considered, the mathematical model, added by safety stock, leads to a decrease in the profit. This is because an additional holding cost would incur. Nevertheless, as the demand uncertainty is present in the simulation model, the profit can be increased by increasing safety stock ($SS_{jq}$). This is because the additional safety stock can help reduce the shortage cost.

Based on Equations (28) – (30), the initial safety stock ($SS_{jq}$) is determined. Equation (28), which is the density function of the $t$-distribution from Chen (2018), was used to calculate $y$. However, $y$ is a standardized value. Hence, according to Equation (29), $y$ is converted into an observational value, $x$. $SS_{jq}$ is the additional stock required to exceed the average demand and can be determined by Equation (30):

$$
\alpha = F(y \mid v) = \alpha = \int_{-\infty}^{y} \frac{1}{\sqrt{\nu \pi}} \frac{1}{\frac{\nu + 1}{2} \left(1 + \frac{v^2}{\nu}\right)^{\frac{\nu + 1}{2}}} d\nu
$$

(28)

where:

$\alpha = 0.90$ - required service level
$v = n - 1$ - degrees of freedom
$n = 10$ - number of replications

then:

$$
y = \frac{x - \mu}{\sigma} \text{ test statistic}
$$

(29)

$$
SS = x - \mu \text{ safety stock}
$$

(30)

$x$ - total stock required to achieve the 90% service level

$$
\mu_{jq} = \frac{\sum_{t} T_{avg_{jq}}}{T} \text{ - average customer demand of each finished goods } j \text{ at retailer } q \text{ for all periods}
$$
\[ \sigma_{jq} = \frac{\sqrt{\sum_{t}^{T} sd_{jqt}^2}}{T} \] - average standard deviation of customer demand of each finished good

\( D_{avgjqt} \) and \( sd_{jqt} \) are from historical records before the actual demand is realized. As the number of safety stock units at each retailer is the same for all periods, the average demand of all periods (\( \mu \)) is used for the calculation. In this case, the data of demand is assumed to be collected 10 times (i.e., 10 previous years) for each retailer in each period. Therefore, \( n \) is equal to 10 and \( v \) is successively equal to 9. The aim is to find \( SS_{jq} \) that fulfills the 90% service level, so \( \alpha \) is equal to 0.90. From Table 2, the finished goods 1 at retailer 1 has an average demand from period 1 to period 3 are 14, 16, 14 units, respectively. The standard deviation of the demand (\( sd_{jqt} \)) from period 1 to period 3 is all 3 units. The average demand of all periods (\( \mu \)) is thus 14.67 units \( \left( \frac{14 + 16 + 14}{3} \right) \). The average standard deviation (\( \sigma \)) is 1.73 units \( \left( \frac{\sqrt{3^2 + 3^2 + 3^2}}{3} \right) \). From Equation (28), \( y \) is equal to 1.38. Then, \( y \) is converted to observational units (\( x \)) by Equation (29), which is equal to 17.06. This is rounded up to 18 units, which is the amount of stock required to satisfy the 90% service level. The mathematical model produces an average demand of 14.67 units. Hence, according to Equation (30), the \( SS_{jq} \) is equal to 3.33 units and rounded up to 4 units. With the calculated initial \( SS_{jq} \), the solution is now feasible in both makespan and service level requirement.

4.3.2 Phase II

Even though, the obtained plan from Phase I is feasible, it may not yet be good enough as some retailers may have excessive inventory that can be shared to other retailers that experience shortages. In this phase, lateral transshipment is introduced, to improve the solution without changing the production plans obtained from the previous phase. This is logical since once the actual orders are realized, it will be too late to change the production plan. There are two scenarios that showcase the approach of lateral transshipment. In scenario A, it is assumed that the actual orders for all periods are known. Once the actual orders for all periods are known, the algorithm searches for the best amount of transshipment between retailers in all three periods. In the algorithm, one unit of the order is transshipped at a time because determining the suitable amount of transshipment per iteration is difficult in this scenario. If more than one unit is transshipped per iteration, this may lead to a retailer that has inventory in one period to transship to other retailers and then experience shortages in the period after when order is higher than expected, incurring shortages and missing the best solution. Therefore, one unit of finished goods is transshipped at a time per iteration for a more thorough search until the near or possibly optimal solution is found. The procedure of Phase II in scenario A is illustrated in Figure 4.

Phase II of scenario A starts by finding the critical score \( \text{criticalscore}_{jqt} \) for each finished good \( j \) at retailer \( q \) in period \( t \) by Equation (31). Determining when and which finished goods \( j \) should be transshipped to which retailer \( q \) is important. The \( \text{criticalscore}_{jqt} \) determines which finished goods \( j \) at retailer \( q \) in period \( t \) has the highest potential to increase the profit if receiving more finished goods. A higher score represents a higher potential. If a retailer is experiencing shortages and the shortage cost is high, the \( \text{criticalscore}_{jqt} \) will also be high, showing that the retailer needs extra finished goods. A positive \( \text{criticalscore}_{jqt} \) means that the retailer needs more finished goods, and a negative
critical score \( q_i \) means that the retailer \( q \) should have its inventory removed as the inventory cost is high. \( transshipment_{j,q_1,q_2,t} \) is the amount of finished goods \( j \) transshipped from retailer \( q_1 \) to retailer \( q_2 \) in period \( t \). However, if a transshipment between two retailers occurs, retailer \( q_1 \) would lose a unit, which can lead to a possible shortage.

To select which \( transshipment_{j,q_1,q_2,t} \) should occur, \( transshipmentscore_{j,q_1,q_2,t} \) is calculated. The highest score has the most potential in increasing the profit if the transshipment occurs. The calculation of \( transshipmentscore_{j,q_1,q_2,t} \) is shown in Equation (32). If \( q_2 \) has a very high critical score (retailer \( q_2 \) desperately needs finished goods) and \( q_1 \) has a very low critical score (retailer \( q_1 \) wants to remove its finished goods), the \( transshipmentscore_{j,q_1,q_2,t} \) will be high. Therefore, a transshipment from \( q_1 \) to \( q_2 \) is very likely to increase the profit. The transshipment cost \( (TC_{j,q_1,q_2,t}) \) also affects the \( transshipmentscore_{j,q_1,q_2,t} \) with a higher cost resulting in a lower score. However, a high \( transshipmentscore_{j,q_1,q_2,t} \) does not always guarantee an increase in profit because the origin

---

**Figure 4. Phase II’s flowchart for scenario A**
retailer \((q_1)\) may not experience a shortage in the period when the transshipment occurs, but may experience a shortage in the subsequent period resulting from that particular transshipment. In this approach, a lateral transshipment does not increase production but only reorganizes the number of finished goods each retailer receives:

\[
\text{criticalscore}_{jqt} = (\text{shortage cost}_{jqt} \times \beta_{jqt}) - (\text{inventory cost}_{jqt} \times \gamma_{jqt}) \quad \forall \ j, q, t
\]  

(31)

\[\beta = 1 \text{ when shortage}_{jqt} > 0 \quad \beta \text{ is equal to 1 if any shortage occurs}
\]

\[\beta = 0 \text{ when shortage}_{jqt} = 0 \quad \beta \text{ is equal to 0 if no shortage occurs}
\]

\[\gamma = 1 \text{ when inventory}_{jqt} > 0 \quad \gamma \text{ is equal to 1 if any inventory exists}
\]

\[\gamma = 0 \text{ when inventory}_{jqt} = 0 \quad \gamma \text{ is equal to 0 if no inventory exists}
\]

\[
\text{transshipmentscore}_{j,q_1,q_2,t} = \text{criticalscore}_{j,q_2,t} - \text{criticalscore}_{j,q_1,t} - TC_{j,q_1,q_2,t} \quad \forall \ j, q_1, q_2, t
\]  

(32)

A transshipment with the highest \(\text{transshipmentscore}_{j,q_1,q_2,t}\) is considered first as a candidate. After the candidate \(\text{transshipment}_{j,q_1,q_2,t}\) has been chosen, it is simulated in the simulation model to test whether the profit \((Q_{\text{Simu}})\) is increased. If the profit is increased, the \(\text{transshipment}_{j,q_1,q_2,t}\) solution set and \(Q_{\text{Optimal}}\) are updated. Then all \(\text{transshipment}_{j,q_1,q_2,t}\) from the destination of the chosen \(\text{transshipment}_{j,q_1,q_2,t}\) \((q_2)\) are added to the \(\text{tabu list}\), to prevent the receiving retailer \((q_2)\) from sending finished goods to other retailers. The \(\text{transshipment}_{j,q_1,q_2,t}\) in the \(\text{tabu list}\) is never considered as a candidate. It would not make sense for a retailer that received finished goods to send its finished goods away to other retailers in the same period. However, if the profit is not increased, the \(\text{transshipment}_{j,q_1,q_2,t}\) solution set is not updated and the \(\text{transshipment}_{j,q_1,q_2,t}\) candidate is added to the \(\text{tabu list}\). Then, the first iteration stops here.

The subsequent iteration commences by recalculating the \(\text{transshipmentscore}_{j,q_1,q_2,t}\) if the \(\text{transshipment}_{j,q_1,q_2,t}\) solution set has been updated. Otherwise, the \(\text{transshipmentscore}_{j,q_1,q_2,t}\) of the previous iteration will be used. After a candidate \(\text{transshipment}_{j,q_1,q_2,t}\) is chosen, if the chosen \(\text{transshipment}_{j,q_1,q_2,t}\) is in the \(\text{tabu list}\), the \(\text{transshipment}_{j,q_1,q_2,t}\) with the next highest \(\text{transshipmentscore}_{j,q_1,q_2,t}\) is selected as the candidate. If all the \(\text{transshipment}_{j,q_1,q_2,t}\) have the \(\text{transshipmentscore}_{j,q_1,q_2,t}\) less than or equal to zero, the algorithm stops. It is unreasonable to select a \(\text{transshipment}_{j,q_1,q_2,t}\) with a zero or negative \(\text{transshipmentscore}_{j,q_1,q_2,t}\) because a negative \(\text{transshipmentscore}_{j,q_1,q_2,t}\) means that the origin retailer \((q_1)\) has more potential to run out of stock than the destination retailer \((q_2)\). For improving the computational time, the algorithm also stops if the profit does not increase within 5 iterations. This termination constraint can be determined and adjusted by the decision makers, depending on the computational time. When the algorithm stops, a near or possibly optimal and feasible solution is found.

The approach of choosing a \(\text{transshipment}_{j,q_1,q_2,t}\) candidate is demonstrated in Tables 12–13, as a demonstration. Assume that Phase I has ended and the \(\text{shortage}_{jqt}\) and \(\text{inventory}_{jqt}\) are found from the simulation model. The \(\text{criticalscore}_{jqt}\) is then calculated using Equation (31) and shown in Table 12.

| Retailer | \(\text{shortage cost}_{jqt}\) ($) | \(\text{shortage}_{jqt}\) (unit) | \(\text{inventory cost}_{jqt}\) ($) | \(\text{inventory}_{jqt}\) (unit) | \(\text{criticalscore}_{jqt}\) |
|----------|-------------------------------|-------------------|-------------------------------|-------------------|-------------------|
| 1        | 500                           | 2                 | 30                            | -                 | 500               |
| 2        | 600                           | -                 | 40                            | -                 | 0                 |
| 3        | 550                           | -                 | 20                            | -                 | -20               |

Table 12. Critical score (\(\text{criticalscore}_{jqt}\)) of each retailer \(q\)
The transshipmentscore for every possible combination of transshipments is then calculated from the criticalscore for each retailer using Equation (32), as shown in Table 13 (only 3 combinations are shown for the demonstration). The highest transshipmentscore is then considered as the candidate with the most potential to increase the profit. In this demonstration, transshipment1,3,1,1 is chosen to be the candidate. Transshipment of finished goods 1 from retailer 3 to retailer 1 in period 1 has the highest possibility to increase the profit.

In scenario B, it is assumed that the actual orders are known only at the end of each period. Since actual orders of subsequent periods are not known, this scenario is more dynamic than scenario A as knowing actual orders as well as fixing them for all periods in advance is difficult in reality. However, without knowing the order in advance, there is a possibility that the transshipment amount among retailers calculated in each period may not be the best amount as compared to considering all periods. When one retailer has excess stock in one period, the same retailer may face a stockout due to a higher customer order in the next period. Then the algorithm might suggest to allow transshipment from the retailer that received finished goods in the previous period to transship the finished goods back to the retailer it previously received finished goods from. We call this action a repeated transshipment, when two retailers send their finished goods back and forth and incurring additional cost. In this instance, there should not be any transshipment while considering both periods. But when one period is executed at a time without looking ahead, there is a possibility of a repeated transshipment. Without knowing the order in advance, it is possible to calculate the amount of transshipment in each iteration instead of transshipping one unit per iteration. The amount of transshipment in calculated by Equation (33), which is the minimum between the inventory of the origin retailer and the amount of shortage of the destination retailer. It is not sensible to send a transshipment more than what the destination retailer needs or the origin retailer has. This method can also be used in scenario A but will not give a near or possibly optimal solution because of the possibility of a repeated transshipment. As future orders are unknown in this scenario, decision must be made that is best for the period in interest only. The approach of choosing the best amount of transshipment for this scenario is shown in Figure 5:

\[
\text{transunit}_{j,q_1,q_2} = \min (\text{inventory}_{j,q_1}, \text{shortage}_{j,q_2})
\]  

(33)

For this scenario, index \(t\) (period) will not be included in the algorithm as the calculation of transshipment is done in each period independently. The algorithm starts by first calculating the criticalscore and transshipmentscore, which is the same as scenario A. Then it is checked whether there is any inventory or not, as a transshipment would be impractical if all retailers have no inventory. If there is no inventory for all retailers, the algorithm stops. The transshipmentscore of all retailers are checked whether all of them have negative values, as allowing a transshipment with negative transshipmentscore will reduce the profit. If all transshipmentscore are negative, the algorithm stops. Each transshipment would be ranked according to their transshipmentscore. Transshipment with the highest transshipmentscore would be chosen first as a candidate, and the algorithm continues similarly to the algorithm for scenario A. When no retailers are experiencing any shortage, the algorithm stops. The near or possibly optimal amount of transshipment has been found for that particular period.
5. RESULTS AND DISCUSSIONS

The mathematical model is first formulated and solved through IBM ILOG CPLEX software. Then, the simulation model is constructed in ARENA simulation software. In addition, Visual Basic for Applications (VBA) is also built to connect and pass along the results between the two models iteratively.

5.1 Phase I’s Results

Tables 14-16 as well as Figure 6 present the workload capacity, production level, distribution plan and makespan of each iteration during Phase I.
The initial solution (iteration 1) of the mathematical model as presented in Figure 6 and Table 14 is an optimal solution but it is not feasible because the makespan in period 1 (26,768 min) surpasses the makespan limitation of 21,600 min, as displayed in circle A. The obtained solution recommends the production of everything in the first two periods due to the lower production cost, as displayed in Table 15 (circle C for finished goods 1). However, for the next two iterations, the workload constraint manages to cut down the level of production in periods 1 and 2 and postpone the production to period 3, as shown in circle D. This results in a more balanced production throughout the planning period.

### Table 14. Workload capacity (WLC) and makespan (MS)

| Period (i) | 1      | 2      | 3      |
|------------|--------|--------|--------|
| Workload capacity (WLC) min | 50,000 | 40,347 | 40,347 |
|       2   | 50,000 | 47,912 | 40,492 |
|       3   | 50,000 | 50,000 | 50,000 |
| Makespan (MC) min | 26,768 | 21,311 | 21,296 |
|       2   | 22,541 | 25,558 | 21,180 |
|       3   | 5,800  | 4,780  | 5,183  |
| Profit ($Q_{MILP}$) | 89,028 | 87,999 | 87,343 |
| Profit ($Q_{SIMU}$)   | 86,433 | 85,542 | 84,713 |

### Table 15. Production level (units) and profit

| Period (i) | Demand (units) | Iteration |
|------------|----------------|-----------|
| Component (j) (units) | | 1      | 2      | 3      |
| 1 1        | 200            | 376      | 305    | 305    |
| 2 2        | 204            | 291      | 344    | 300    |
| 3 3        | 196            | 0        | 18     | 62     |
| 1 2        | 280            | 520      | 426    | 425    |
| 2 2        | 286            | 413      | 480    | 415    |
| 3 3        | 274            | 0        | 27     | 93     |
| Finished Goods (j) (units) | | 1      | 2      | 3      |
| 1 1        | 40             | 56       | 55     | 55     |
| 2 2        | 42             | 75       | 67     | 45     |
| 3 3        | 38             | 0        | 9      | 31     |
| 1 2        | 40             | 88       | 65     | 65     |
| 2 2        | 40             | 47       | 70     | 70     |
| 3 3        | 40             | 0        | 0      | 0      |
| Profit ($Q_{MILP}$) | 89,028 | 87,999 | 87,343 |
| Profit ($Q_{SIMU}$)   | 86,433 | 85,542 | 84,713 |
### Table 16. Distribution plan (units) and profit

| Finished Goods (j) | Period (r) | Distribution routes | Iteration (s) |
|--------------------|------------|---------------------|---------------|
|                    |            |                     | 1   | 2   | 3   |
| 1                  | 1          | L – P               | 0   | 0   | 0   |
|                    |            | L – Q               | 55  | 55  | 55  |
|                    |            | P – Q               | 0   | 0   | 0   |
| 2                  | 1          | L – P               | 38  | 19  | 0   |
|                    |            | L – Q               | 38  | 48  | 45  |
|                    |            | P – Q               | 0   | 0   | 0   |
| 3                  | 1          | L – P               | 0   | 0   | 0   |
|                    |            | L – Q               | 0   | 9   | 31  |
|                    |            | P – Q               | 38  | 19  | 0   |
| 2                  | 1          | L – P               | 26  | 2   | 2   |
|                    |            | L – Q               | 53  | 53  | 53  |
|                    |            | P – Q               | 0   | 0   | 0   |
| 2                  | 2          | L – P               | 6   | 30  | 30  |
|                    |            | L – Q               | 40  | 40  | 40  |
|                    |            | P – Q               | 0   | 0   | 0   |
| 3                  | 1          | L – P               | 0   | 0   | 0   |
|                    |            | L – Q               | 10  | 10  | 10  |
|                    |            | P – Q               | 32  | 32  | 32  |

Profit (Q_MILP) $ $89,028 87,999 87,343 $86,433 85,542 84,713$

Remarks: L = Stack point  
P = Warehouse  
Q = Retailer

**Figure 6. Makespan of each period t (MSt)**
and the plan becomes feasible, as indicated in Table 14 circle B. For the distribution plan as shown in Table 16, firstly the model recommends distributing the finished goods to the warehouses, as the holding cost is lower (circle E). Later iterations suggest distributing the finished goods directly to retailers as the transportation time is shorter (circle F) despite a higher holding cost at the retailers.

From Equations (28) – (30) and the data from Table 2, the initial safety stock ($SS_{jq}$) can be determined and presented in Table 17, in which the comparative results of with and without the initial safety stock are carried out. Although additional production is needed for the safety stock, the makespan has not changed much because a faster mode of transportation is employed. By adding safety stock, it was found that the shortage cost is descended, hence leading to an increase in the profit ($p$-value < 0.001) and eventually the customer service level.

### 5.2 Phase II’s Results

Once the service level and makespan in all periods are feasible, the algorithm continues to check for further improvement by introducing lateral transshipments among retailers. After the initial safety stock is determined, a transshipment candidate with the highest transshipmentscore is selected and tested in the simulation model to check whether the profit has been improved or not. To compare the profit resulting from transshipment to the method without transshipment (Phase I), the same set of orders that was randomly generated in Phase I is used for the calculation of transshipment in Phase II. Ten sets of orders (1 set for each replication) are used for the calculation, and the average profit of all replications is used for comparison.

#### 5.2.1 Phase II Scenario A’s Results

The transshipment candidate selected in each iteration, based on the highest transshipmentscore, is shown in Table 18. The candidate is then tested in the simulation model to see if the profit ($Q_{Simu}$) is higher than the currently near or possibly optimal profit ($Q_{Optimal}$). $Q_{Optimal}$ for the first iteration is the profit after the safety stock is added, as shown in Table 18. Phase II stops after all transshipment candidates have their transshipmentscore less than or equal to zero or there is no improvement in $Q_{Optimal}$ within 5 consecutive iterations. From Table 18, it is found that the near or possibly optimal solution from iteration 4 has a profit of $95,773. This solution has the highest profit ($Q_{Simu} = Q_{Optimal}$) that can satisfy the required service level when there is no potential

| $SS_{jq}$ (units) | No safety stock | With initial safety stock |
|-------------------|-----------------|--------------------------|
| $SS_{11}$         | 0               | 4                        |
| $SS_{12}$         | 0               | 5                        |
| $SS_{13}$         | 0               | 4                        |
| $SS_{21}$         | 0               | 4                        |
| $SS_{22}$         | 0               | 5                        |
| $SS_{23}$         | 0               | 4                        |

| Makespan (min) | No safety stock | With initial safety stock |
|----------------|-----------------|--------------------------|
| Period 1       | 21,457          | 21,296                   |
| Period 2       | 21,404          | 21,180                   |
| Period 3       | 4,660           | 5,183                    |

95% lower bound service level: 78.12% vs. 90.61%

$Q_{Simu}$ ($): 77,210 vs. 84,713
transshipment that can improve the profit further. In the last iteration (iteration 6), all transshipment candidates have the \textit{transshipment score} less than or equal to zero, therefore the algorithm is terminated and the near or possibly optimal solution is found for this replication. For demonstration, only the solution of 1st replication is shown (showing all 10 replications would be redundant).

The final solution set in iteration 6 contains \textit{transshipment}_2^2,1^2,2 and \textit{transshipment}_1^2,2^3,3. This means that the algorithm proposes the following transshipments:

1. One unit of finished goods 2 from retailer 2 to retailer 1 in period 2
2. Three units of finished goods 1 from retailer 2 to retailer 3 in period 3

The average profit for 10 replications of each phase of the integrated approach is summarized in Table 19. After introducing lateral transshipments into all replications and finding the average profit, the model suggests that the profit can be improved by $6,553 (7.74%), compared to the plan without lateral transshipments. A comparison has also been made between the proposed algorithm in Phase II and OptQuest, a built-in optimization tool in ARENA, and found that the proposed algorithm yielded higher profit than OptQuest by 2.18% and a faster computational time by 511%. It should be noted that this improvement could be a lot larger if the problem size is bigger.

5.2.2 Phase II Scenario B’s Results
The algorithm from Figure 5 is used in scenario B, which has the same data set as scenario A, but the actual requirement for each period is known only at the end of the period. Table 20 presents the

| Iteration | \textit{transshipment}_{q_1,q_2,t} candidate | Q_{SIMU} ($) | Q_{Optimal} ($) | Profit improves? | Solution set \textit{transshipment}_{q_1,q_2,t} |
|-----------|-----------------------------------------------|--------------|-----------------|-----------------|---------------------------------------------|
| 0         | –                                             | 90,703       | –               | ✓               | 2212 - 1 unit                               |
| 1         | \textit{transshipment}_{2212}                 | 92,233       | 90,703          | ✓               | 2212 - 1 unit                               |
| 2         | \textit{transshipment}_{2233}                 | 93,413       | 92,233          | ✓               | 1233 - 1 unit, 1233 - 1 unit                |
| 3         | \textit{transshipment}_{2233}                 | 94,593       | 93,413          | ✓               | 1233 - 1 unit, 1233 - 2 unit                |
| 4         | \textit{transshipment}_{2233}                 | 95,773       | 94,593          | ✓               | 1233 - 1 unit, 1233 - 3 units               |
| 5         | \textit{transshipment}_{2233}                 | 95,758       | 95,773          | ×               | 1233 - 1 unit, 1233 - 3 units               |
| 6         | \textit{transshipment}_{2212}                 | 95,763       | 95,773          | ×               | 1233 - 1 unit, 1233 - 3 units               |

Table 18. Result of Phase II scenario A (replication 1)

Table 19. Profit improvement in each phase

|                                         | Without safety stocks and transshipment | With safety stock but no transshipment (Phase I) | With safety stock and transshipment (Phase II) |
|-----------------------------------------|----------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Average profit of 10 replications ($)   | 77,210                                 | 84,713                                        | 91,266                                        |
| Improvement (%) from previous phase     | –                                      | 9.72                                          | 7.74                                          |
result of replication 1 in scenario B, which yields the same result as scenario A. Some replications yielded slightly lower profit because of the repeated transshipments.

5.3 Comparison of the Results

A comparison of the obtained outcome and computational time between the proposed integrated approach and the simulation-based optimization model by OptQuest for scenario A is shown in Table 21. The result from the mathematical method is not compared, as its result is not subjected to uncertainty. The benefits and drawbacks of OptQuest have been discussed in detail in Glover et al. (1999) and Fu (2002). For a fair comparison, both models were run on the same computer with an Intel Core i7-9750H CPU @2.60 GHz with 16GB Ram.

With 150 decision variables to be searched for the optimality, OptQuest takes a very long computational time and offers a statistically weaker solution than the integrated approach. Under 10 replications, the profits obtained from the integrated approach are significantly higher than the profits obtained from OptQuest under 95% confidence level ($p$-value < 0.01). Furthermore, the computational time will be much longer when the model is required to run for longer periods, since it requires a larger number of decision variables. Therefore, the integrated approach is shown to be far superior.

A comparison of the solution of each scenario is compared in Table 22. The profits of both scenarios are very close, but the improved method in scenario B has significantly faster computational time as it also computes transshipment quantity in each iteration. Scenario A has a higher profit because if the actual orders were known in advance, the algorithm can plan ahead to prevent retailers from transshipping to one retailer and receiving the same shipment next period (repeated transshipment), incurring additional transshipment cost. Scenario B cannot prevent this problem as the actual order for the next period is unknown. The profit difference between scenario A and B could be a lot larger if the problem size is larger.
5.4 Managerial Implication

The proposed approach is useful for finding a suitable production-distribution plan that aims to maximize the profit of the supply chain. The benefit of the proposed approach in Phase I is the capability to find the total time needed for the production-distribution plan (makespan) and ensures that the plan is completed within the makespan limit. In this Phase, the production-distribution plan is planned with forecasted order and calculate the amount stock before the actual order is known. This approach considers the service level and is solved for a suitable amount of safety stock that is required to satisfy the minimum service level of the customers. Too much safety stock can lead to excessive inventory holding cost while too low may not satisfy the minimum service level. Phase II further showcases the benefit of the approach with the introduction of an algorithm that searches for the best amount of transshipments between retailers and increasing the profit. In this Phase, the transshipment is used as a corrective measure since the production-distribution plan cannot be changed at this stage after the actual order is known. The solution set consists of the near or possibly optimal amount of production of each finished goods in each period, its distribution plan, and best amount of safety stock and transshipment. By combining mathematical and simulation models, it is possible to find a near or possibly optimal solution under uncertainty with a reasonable computational time, as compared to the simulation-based optimization alone. In the study, the profit has been improved by 18.20% (improvement from $77,210 without safety stocks nor transshipment to $91,266 with safety stocks and transshipment) with the introduction of safety stock and lateral transshipment. In practical use, the proposed approach is more suitable for a tactical-level planning than an operational-level planning, where it may require additional programming scripts to cope with the dynamism in the workplace. However, it can be achieved through developing a computer program by coding our developed cyclic procedures in each phase and linking them together to provide more friendliness to the users. For a large-scale business, this approach can be further applied to the business model to significantly increase the profit of a supply chain within a reasonable solving time, as required for market competition. The model may also require additional constraints and parameters depending on the complexity of the real working conditions.

6. CONCLUSION

An integrated mathematical-simulation approach for optimizing supply chain planning was demonstrated in this study. The objective is to determine a feasible plan, which is a near or possibly optimal production-distribution plan that can meet the limitation of makespan under the required service level and achieve in an acceptable computational time. The results showed that the mathematical model under uncertain environment alone cannot simply find the actual makespan, as uncertainties and queueing cannot be taken into the consideration in the model. Simulation model was required to simulate and incorporate any uncertainties that might incur in the operations. As, the service level is considered as the most crucial requirement from customers. Therefore, it is necessary to introduce safety stock for achieving the required service level. As a result, it helps to reduce possible penalty from demand shortages in the supply chain. Lateral transshipment was also introduced to further increase the profit without changing the production-distribution plan. With lateral transshipment, no additional production is needed as it requires only the transferring of finished goods between retailers. Retailers with excessive inventory can have their stock reduced. Additional finished goods can be provided to retailers that have possible shortages.

The proposed integrated mathematical-simulation approach is separated into two phases. The first phase determines a feasible plan that is not allowed to surpass the limitation of makespan and meets the required service level. The second phase is further determined for the best amount of transshipment among retailers to increases the overall profit. Two scenarios were discussed in this study, and the methods used to solve for the best transshipment for each of the scenarios were introduced. It was shown in this study that lateral transshipment can increase the profit of a supply
chain under uncertainty. In addition, the proposed approach had a better computational time (solving time is shorter), when compared to the traditional simulation-based optimization by OptQuest.

However, once the size of the problem is larger, the number of decision variables would thus become higher. Under this situation, our approach is still required for making some modifications to further shorten the computational time. For example, in scenario A, if the number of retailers become larger, the chance of missing the optimal solution will be higher with a termination constraint of only 5 iterations. However, the computational time can be longer with more iterations. Therefore, a suitable termination constraint should be calculated according to the size of the problem, to balance the computational time with the chance of missing the optimal solution. For further research, the proposed integrated approach can be applied to larger problems to verify the benefit of profit improvement and computational time reduction from this study. Different types of business-related problems can also use and adapt the proposed integrated approach to optimize their decision variables of interest. In addition, other types of uncertainty such as transportation delays from traffic may also be included in the model to better highlight the advantages of the hybrid optimization approach and represent realistic problems. The model could be modelled in a more dynamic way, such as when the actual customer demand is realized during the planning period in relation to its forecasted demand. Other methods and tools such as control theory, advanced simulation modelling, and artificial intelligence can also be applied to solve these dynamic behaviors.
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