Obervation of non-linear stationary spin waves in superfluid $^3$He-B

A. S. Chen, Yu. M. Bunkov, H. Godfrin, R. Schanen, F. Scheffler.

Centre de Recherches sur les Très Basses Températures,
Centre National de la Recherche Scientifique,
BP 166, 38042 Grenoble, Cedex 9, France.

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Due to its broken spin and orbit rotation symmetries, superfluid $^3$He plays a unique role for testing rotational quantum properties on a macroscopic scale. In this system the orbital momentum forms textures that provide an effective potential well for the creation of stationary spin waves. In the limit of the lowest temperatures presently attainable, we observe by NMR techniques a profound change in the spin dynamics. The NMR line shape becomes asymmetric, strongly hysteretic and displays substantial frequency shifts. This behavior, quantitatively described by an anharmonic oscillator model, indicates that the parameters of the potential well depend on the spin waves amplitude, and therefore that the orbital motion is not damped in this new regime, not considered by the standard Leggett-Takagi theory. This regime of non-linear stationary spin waves is shown to give rise to the pulsed NMR "Persistent Signals" reported recently.

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The ground state of superfluid $^3$He is formed by Cooper pairs of $^3$He atoms in a triplet state with Spin and Orbital momentum quantum numbers equal to unity. In consequence superfluid $^3$He shows not only the usual superfluid properties arising from the broken gauge symmetry, but also displays macroscopic quantum rotation phenomena related to the broken spin and orbit rotation symmetries. The spin concerned is that of the nuclear rotation and is thus associated with a nuclear magnetic moment. The orbital motion consists of the mutual orbiting of two neutral $^3$He atoms and is thus not associated with a magnetic moment. These two motions are not independent, since the dipole-dipole interaction of two nuclear magnetic moments depends on their relative orientation.

In the $^3$He-B phase, at zero field, the magnetization and the average orbital momentum are equal to zero. Applying an external magnetic field induces a magnetization $\vec{M}$ and, via the spin-orbit symmetry of the wave function, an orbital momentum $\vec{L}$. This symmetry is usually characterized by the rotation matrix $\hat{R}$ of angle $\Theta$ around an axis $\vec{n}$. The orbital momentum is therefore $\vec{L} = \hat{R}(\vec{n}, \Theta)\vec{M}$.

The early investigations of superfluid $^3$He have been performed at relatively high temperatures of order $0.4 \, T_C$ and above. In these conditions, first, the orbital motion is blocked due to an effective interaction with quasiparticles. Consequently, the NMR of the spin system considered up to now only involves quasi-stationary orbital dynamics. Second, there is an effective magnetic interaction between the superfluid and normal components of $^3$He, which is characterized by an effective quasiparticle lifetime $\tau$. One has to distinguish two different regimes of spin dynamics, hydrodynamic and non hydrodynamic, for $\omega \tau$ smaller or larger than unity (here $\omega$ is the NMR frequency). The equations for the spin precession derived by Leggett and Takagi (L-T) [1], written down for hydrodynamic conditions, describe well the high temperature experimental results. The superfluid component is responsible for spin supercurrents, the order parameter stiffness and the dipole frequency shift, while the normal component determines magnetic relaxation and spin diffusion (a review is given in ref. [2]).

It is usually believed that these equations remain valid in non-hydrodynamic conditions, with some sort of renormalisation of the relaxation parameters. This renormalisation was tested in NMR experiments in the temperature region above about $0.4 \, T_C$ and found to be in good agreement with the theoretical predictions [3]. The effective value of the magnetic interaction decreases very rapidly in the non-hydrodynamic region, as $1/(\omega \tau)$.

In the present work, performed at much lower temperatures, we are able to reach a new condition where the spin and orbit dynamics of the superfluid component are no longer damped by the interaction with the normal component, where we can study true superfluid properties of rotation quantum dynamics. We describe in this article the observation of a new non-linear phenomenon at temperatures of about $0.2 \, T_C$, now accessible to experimental investigation.

Let us first remind the properties of spin dynamics at temperatures above $0.4 \, T_C$. First, the equilibrium local orientation of the orbital momentum $\vec{L}$ with respect to the external magnetic field supplies an additional potential for NMR and leads to a NMR frequency shift. Usually the orbital momentum orientation is determined by boundary conditions at the walls of the cell, magnetic field anisotropy and bending energy. Since the angle $\Theta$ is fixed at about $104^\circ$ by dipole-dipole interactions, the relative orientation of $\vec{M}$ is then determined by $\vec{n}$ which
can form a complicate texture. The local NMR frequency at relatively high field is:

$$\omega^2 = (\gamma H)^2 + n_\perp^2 \Omega_D^2$$  \hspace{1cm} (1)$$

where $\Omega_D$ characterizes the dipole-dipole interaction and $n_\perp$ is the component of $\vec{n}$ normal to the magnetic field. The distribution of the orientation of $\vec{L}$ gives rise to a broadening of the NMR line by about $\frac{\Omega_D^2}{\gamma H}$, which can be on the order of tens of kHz.

Second, owing to the stiffness of the order parameter, the spin system exhibits non-local resonant modes - Stationary Spin-Waves modes (SSW) trapped by a texture. These modes were observed in the pioneering work by Osheroff \cite{4}. Later SSW were used for studying spin diffusion \cite{5}, textures and vortices \cite{6}. The SSW can be created in a potential well determined by the walls of the cell and the field gradient \cite{7}, by the wall and a texture \cite{8} or only by a texture, for example the "Flared-out" texture \cite{9}. The equation of motion for the transverse component of the magnetization $M^+$ in the texture potential well $\beta_L(r)$ has the form of the Schrödinger equation \cite{10}:

$$(\omega-\omega(z))M^+ = \frac{\Omega_D^2}{2\omega(z)} \left( \frac{2}{3} \sin^2 \beta_L(r) - \frac{24}{65} \xi^2 \nabla^2 \right) M^+. \hspace{1cm} (2)$$

where $\beta_L$ is the local deflection of the orbital momentum from the direction of magnetic field and $\xi$ the dipole coherent length. The SSW modes have been studied at relatively high temperatures, above 0.4 T$_C$, and found to be in a good agreement with theory.

The new Grenoble refrigerator allows us to cool $^3$He to significantly lower temperatures. Owing to the exponential dependence of the number of quasiparticles on temperature, we are able to decrease their density by many orders of magnitude, thus suppressing the magnetic and orbit friction. In this article we report the observation of a dramatic change in the spin and orbit dynamics, observed in a CW-NMR experiment in this ultra-low temperature region.

The experiments were performed in a cylindrical experimental cell of diameter 6 mm and height 5 mm, at pressures ranging from 0 to 6 bars. The magnetic field, corresponds to a NMR frequency of about 500 kHz and was directed parallel to the axis of the cell. In addition, we were able to change the magnetic field gradient in order to determine the localisation of NMR signals within the cell. In these conditions a "flared out" texture should be formed. This is shown schematically in the inset of Fig.2: at the top and the bottom of the cell, the orbital momentum is oriented parallel to the field, while the vertical walls orient $\vec{L}$ perpendicular to the field. This textural configuration gives a region of minima of energy for NMR of SSW near the top and the bottom of the cell.

We observed SSW signals at temperatures down to 0.22 T$_C$, i.e. significantly lower than in previous investigations. We report here two new features:

a) we observe an enormous number of SSW new modes, b) the SSW modes exhibit strong non-linear behavior.

As shown in Fig.1 we observe at this temperature an enormous number of NMR peaks that cover practically all the NMR line. By changing the magnetic field gradient, we first determine the positions within the cell that generate these SSW modes. We performed a calibration in normal $^3$He and then, at 0.22 T$_C$, we studied the frequency vs. magnetic field gradient dependence of each peak in the limit of small excitation level. This allows us to show that the two largest peaks correspond to the top and to the bottom of the cell (as was expected in this textural configuration, and confirmed by the same study on SSW at 0.44 T$_C$), with dipole shifts from the Larmor frequency on the order of 150 Hz. Finally all the peaks correspond either to the fundamental state at different places within the cell, or to harmonics.

We then fixed the magnetic field gradient at a value of 5 $10^{-4}$ Tesla.m$^{-1}$ and studied the behavior of several low-frequency-shift SSW modes, when changing the excitation level and the frequency sweep direction. In Fig.2 we show the signals of a few low-frequency-shift SSW modes (absorption and modulus) at 0.44 T$_C$ and 0.22 T$_C$, for different excitation levels H$_1$ (note that our sweep is given in units of frequency, which is recalculated from our real field sweep. Sweeping the field up corresponds therefore to sweeping the frequency down,). Even if these signals are radiated by the same SSW modes at both temperatures, their properties are very different: at low temperatures, the broadening of the signals does not correspond to a relaxation process, but to a non-linear frequency shift with excitation. Let us look at signals for a relatively large rf field (H$_1=1.8$ $10^{-6}$ Tesla). If we sweep the field up (frequency down), the low-temperature-signals appear at the same frequency as
at high temperature, but they grow rapidly even well below the high temperature resonance frequency. At some critical point, the signal disappears. In the sweep back we observe a strong hysteresis since the signal appears only around the high temperature resonance frequency. This behavior is typical of a non-linear oscillator, where the frequency strongly depends on the excitation level. The bigger the excitation, the bigger frequency shift that can be achieved.

Following Landau and Lifchitz [7], we consider an anharmonic oscillator with a third order non-linearity:

\[ \ddot{M}^+ + 2\lambda \dot{M}^+ + \omega_0^2 M^+ = F \cos(\omega t) - \alpha M^{+2} - \beta M^{+3} \quad (3) \]

The amplitude of the transverse component of the magnetization, \( M^+ = A \cos(\omega t) \), is found by solving the equation

\[ 4\omega_0^2 A^2 (\omega - \omega_0 - \kappa A^2)^2 + \lambda^2 = F^2, \quad (4) \]

where \( \kappa \) describes the dynamic frequency shift. Equation (4) is of order six in \( A \), and we are only interested in real and positive solutions. The coefficients of equation (4) are obtained by fitting our experimental data. Fig.3 shows the comparison between the theoretical curve and the experimental signal amplitude from the top of the cell for a rf field level of 1.8 \( 10^{-6} \) Tesla. When sweeping the frequency \( \omega \) down, the amplitude of the transverse magnetization follows the part 1 – 2 and then drops down to zero. When the frequency increases, it stays equal to zero until point 3 and jumps to point 4, thus showing hysteretical behavior. We find that \( \lambda = 0.001 \) kHz is the damping factor giving the width of our resonant curve, while the anharmonic coefficient (\( \kappa = -1700 \) kHz, \( A^{-2} \)) relates the frequency to the amplitude of excitation. Here the amplitude \( A \) was measured in units of the full HPD signal (Homogeneously Precessing Domain: in all the sample, the magnetization is deflected by 104° and precesses homogeneously). For 0.44 \( T_c \) we find \( \lambda = 0.007 \) kHz, while the anharmonic coefficient is negligible.

![FIG. 2. CW NMR absorption (negative signals) and NMR amplitudes (that correspond to \( M^+ \), positive signals) for different temperatures and amplitudes of rf field, \( H_1 \). The arrows indicate the direction of the frequency sweep. The inset shows the \( \tilde{L} \) texture and the regions (labeled A and B) radiating the SSW signals A, B, and the higher harmonics, like B’.](image)

![FIG. 3. A non-linear SSW signal (points) and its fit by the non-linear oscillator equation (line).](image)

This anharmonic oscillator model corresponds to the fact that the potential well radiating the SSW becomes anharmonic when the transverse magnetization increases. This effect can be quantitatively described by incorporating a feedback correction into the potential term, \( \sin^2 \beta L \), of the Schrödinger equation (2): \( \beta L \) should no longer depends only on \( r \), but also on \( M^+ \). Indeed, the non-linear SSW are obtained by pumping the energy into the spin wave modes localized in the potential well formed by the orbital texture. When the density of a given spin wave increases, it influences the orbital degrees of freedom. As a result, the potential well should be modified and change the eigen-frequency of the spin-wave mode, which in turn affects the orbital texture. The frequency of this self-consistent precessing state thus depends on the excitation level and decreases with the magnetization deflection. Qualitatively, when the precessing magnetization is deflected, the texture becomes shallower, causing a decrease of the spin waves frequency shift.

The signal from the bottom of the cell, labeled A in Fig.2, has a more complicate behavior. Its amplitude increases faster than in the model considered here. Nevertheless, the main features are similar. Possibly, the difference originates from the spatial extension of the SSW mode at high levels of excitation.
Finally, we investigated the relation between the excitations reported here and Persistent Signals (PS). A Persistent Signal is a small but extremely long lived induction decay signal that was observed in previous pulsed NMR experiments [8]. Its frequency increases with time [9] (which is opposite to conventional HPD!), and it is radiated by a texture [10]. We made pulsed and CW NMR, in the same low temperature conditions. In CW NMR, we observed the non-linear SSW described before, while in pulsed NMR, we observed Persistent Signals. We show in Fig. 4 the time dependent spectrum of the signal after an NMR pulse. We find two PS whose frequencies increase with time, while their amplitudes decrease. The two PS appear at the same frequencies, and they have the same width and frequency vs. amplitude dependence as the two main non-linear CW NMR signals. So, they seem to be radiated from the same texture by the same mechanism. This identification is particularly informative on the nature of the PS and suggests a physical scenario for their creation. At low temperatures, the magnetization precession with high deflection angles is very unstable [11]. So, after the NMR pulse, the deflected magnetization decays quickly to the spin waves modes. However, since the orbital momentum has also some kind of flexibility, the PS signal arises as a combined solution for the magnetization and texture motion. This explanation is consistent with the feedback mechanism for SSW in a "flared out" texture proposed earlier to explain some of the PS properties [12]. In principle, it should be possible to develop a quantitative theory for PS as well as non-linear SSW by numerical simulations. These would be extremely helpful for guiding future experimental work.

In conclusion, we have shown that the spin dynamics in superfluid $^3$He are profoundly modified in the limit of ultra-low temperatures. In this new regime the stationary spin waves become extremely non-linear, displaying the typical behavior of a third order anharmonic oscillator. This effect can be ascribed to a softening of the texture potential for the spin waves, and modeled by a non-linear Schrödinger equation which governs the transverse magnetization dynamics. We also show that the origin of the Persistent Signals observed in pulsed NMR experiments is directly related to the non-linear dynamics of the system. Our new qualitative and quantitative results call for an extension of the "Standard Model of $^3$He" given by Leggett-Takagi in order to describe the ultra-low temperature regime, where the orbital motion plays an important role in the spin dynamics. Finally, we would like to mention that spin waves in superfluid $^3$He-B can serve as an interesting experimental model for Particle Physics systems described by the non-linear Schrödinger equation.

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\[ \text{FIG. 4. The time dependent spectrum of Persistent Signals observed after a NMR pulse. Inset: the two main CW NMR signals in the same conditions. (the base frequency of the pulse NMR spectrometer (467 kHz) was subtracted from both signals)} \]