Strangeness -2 hypertriton

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We have solved for the first time the Faddeev equations for the bound state problem of the coupled ΛΛN - ΞNN system to study whether an hypertriton with strangeness -2 may exist or not. We make use of the interactions obtained from a chiral quark model describing the low-energy observables of the two-baryon systems with strangeness 0, -1, and -2 and three-baryon systems with strangeness 0 and -1. The ΛΛN system alone is unbound. However, when the full coupling to ΞNN is considered, the strangeness -2 three-baryon system with quantum numbers \((I, J^P) = (\frac{1}{2}, \frac{3}{2}^+)\) becomes bound, with a binding energy of about 0.5 MeV. This result is compatible with the non-existence of a stable \(3\lambda\)H with isospin one.

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The strangeness \(\hat{S} = -2\) sector has become an important issue for theoretical and experimental studies of the strangeness nuclear physics. The \(\Xi N - \Lambda\Lambda\) interaction accounts for the existence of doubly strange hypernuclei, which is a gateway to strange hadronic matter. The \((K^-, K^+)\) reaction is one of the most promising ways of studying doubly strange systems. \(\Lambda\Lambda\) hypernuclei can be produced through the reaction \(K^- p \to K^+ \Xi^-\) followed by \(\Xi^- p \to \Lambda\Lambda\). Strangeness -2 baryon-baryon interactions also account for a possible six-quark \(\Lambda\Lambda\) dibaryon, which has yet to be experimentally observed. The future E07 experiment from J–PARC \([1, 2]\) is expected to improve our knowledge on the \(\hat{S} = -2\) sector, giving ten times more emulsions events for double–Λ hypernuclei.

On the experimental side, there are very few data in the \(\hat{S} = -2\) sector coming from the inelastic \(\Xi^- p \to \Lambda\Lambda\) cross section at a lab momentum of around 500 MeV/c, and from the elastic \(\Xi^- p \to \Xi^- p\) and inelastic \(\Xi^- p \to \Xi^0 n\) cross sections for lab momenta in the range of 500 – 600 MeV/c \([3, 4]\). The relevant information we have is indirect and comes from double–Λ hypernuclei. Their binding energies, \(B_{\Lambda\Lambda}\), provide upper limits for that of the \(\Lambda\Lambda\) dibaryon, i.e., \(B_H < B_{\Lambda\Lambda}\). The first hypernuclear events are quite old and admit several interpretations \([5, 6]\). In 2001 it was reported the so–called Nagara event \([7]\), interpreted uniquely as the sequential decay of \(^5\Lambda\Lambda\)He emitted from a \(\Xi^-\)–hyperon nuclear capture at rest. The mass and the values of \(B_{\Lambda\Lambda}\) and of the \(\Lambda\Lambda\) interaction energy, \(\Delta B_{\Lambda\Lambda}\), were determined without ambiguities. It gave the most stringent constraint to the mass of the \(\Lambda\Lambda\) dibaryon to date, i.e., \(M_H > 2223.7\) MeV at a 90% confidence level. It took almost one decade, but four more double–Λ hypernuclear events were reported, from KEK E176 and E373 experiments \([8]\), still with preliminary results. All the details are summarized in Table I.

Besides the double-Λ hypernuclei quoted in Table I, there is a general consensus that the mirror \(\Lambda\Lambda\) hypernuclei \(^5\Lambda\Lambda\)H, \(^5\Lambda\Lambda\)He are particle stable \([9]\). The existence of a \(^4\Lambda\Lambda\)H bound state has been claimed by the AGS-E906 experiment \([10]\), from correlated weak-decay pions emitted sequentially by \(\Lambda\Lambda\) hypernuclei produced in a \((K^-, K^+)\) reaction on \(^9\text{Be}\). The stability of the \(\Lambda\Lambda\)N system was discarded long ago \([11]\) by using symmetry considerations with respect to the \(3\lambda H\) and therefore without considering the important coupled channel effect due to the existence of the \(\Xi NN\) system.

Theoretically, the \(\hat{S} = -2\) sector was recently put back on the agenda by lattice QCD calculations of different collaborations, NPLQCD \([13]\) and HAL QCD \([14]\), providing evidence for a \(\Lambda\Lambda\) bound state for non–physical values of the pion mass \((m_\pi = 389\text{ MeV} \text{ and } m_\pi = 673 \to 1010\text{ MeV}, \text{ respectively})\). When performing quadratic and linear extrapolations to the physical point \([15]\), a bound dibaryon (around 7 MeV) and a \(H\) at threshold, respectively, are predicted. Ref. \([15]\) presents preliminary results for \(m_\pi = 230\text{ MeV}, \text{ much closer to the physical pion mass, pointing to a H dibaryon at threshold, as also experimentally suggested by the enhancement of the \(\Lambda\Lambda\) dibaryon production at high energies.}"

### Table I. Double Λ hypernuclear events.

| Event     | Nuclide | \(B_{\Lambda\Lambda}\) (MeV) | \(\Delta B_{\Lambda\Lambda}\) (MeV) |
|-----------|---------|-------------------------------|-----------------------------------|
| 1963      | \(^{10}\Lambda\Lambda\)Be | 17.7 ± 0.4                  | 4.3 ± 0.4                        |
| 1966      | \(^{9}\Lambda\Lambda\)He | 10.9 ± 0.5                  | 4.7 ± 1.0                        |
| 1991      | \(^{12}\Lambda\Lambda\)B | 27.5 ± 0.7                  | 4.8 ± 0.7                        |
| NAGARA    | \(^{6}\Lambda\Lambda\)He | 7.13 ± 0.87                 | 1.0 ± 0.2                        |
| MIKAGE    | \(^{6}\Lambda\Lambda\)He | 10.06 ± 1.72                | 3.82 ± 1.72                      |
| DEMACHIYANAGI | \(^{10}\Lambda\Lambda\)Be | 11.90 ± 0.13                | -1.52 ± 0.15                     |
| HIDA      | \(^{11}\Lambda\Lambda\)Be | 20.49 ± 1.15                | 2.27 ± 1.23                      |
| E176      | \(^{13}\Lambda\Lambda\)Be | 22.23 ± 1.15                |                                 |
| E373      | \(^{13}\Lambda\Lambda\)Be | 23.3 ± 0.7                  | 0.6 ± 0.8                        |
production near threshold found in Ref. [16].

The purpose of this letter is twofold. On the one hand we present the solution of the Faddeev equations for the bound state problem of the coupled $\Lambda\Lambda-\Xi NN$ system. The system has been formally studied and its Faddeev equations written down [17, 18], although they have never been applied in a numerical calculation with realistic two-body interactions. This is basically due to the fact that one requires a model of the baryon-baryon interaction which should be able to simultaneously describe two-baryon states with strangeness 0, −1, and −2 within a single consistent theoretical framework. Afterwards, we will apply the formalism by means of the inter-

actions obtained from a chiral quark model describing the low-energy observables of the two-baryon systems with strangeness 0, −1, and −2 and also three-baryon systems with strangeness 0 and −1, trying to elucidate the nature of the three-baryon system with strangeness −2.

The coupled $\Lambda\Lambda-\Xi NN$ system is peculiar because it has two identical particles in each of its two components although they are of different type, which complicates considerably its analysis. The Faddeev equations for the bound-state problem of the coupled $\Lambda\Lambda-\Xi NN$ system have been derived in Ref. [18]. We have obtained these same equations by an independent method [19]. They read:

$$
\begin{align*}
T_{\Sigma NN}^{NNN} &= t_{NNN}^N (1 - P_{23}) P_{13} P_{23} G_0^{\Sigma NNN}, \\
T_{\Sigma NN}^{NNN} &= t_{NNN}^N P_{12} P_{23} G_0^{\Sigma NNN} + t_{NNN}^N P_{13} G_0^{\Sigma NNN}, \\
T_{\Sigma NN}^{N\Lambda} &= t_{NNN}^\Lambda P_{12} P_{23} G_0^{\Sigma NN} + t_{NNN}^\Lambda P_{13} G_0^{\Sigma NN}, \\
T_{\Sigma NN}^{\Lambda\Lambda} &= t_{NNN}^{\Lambda\Lambda} P_{12} P_{23} G_0^{\Lambda\Lambda NN} + t_{NNN}^{\Lambda\Lambda} P_{13} G_0^{\Lambda\Lambda NN}.
\end{align*}
$$

(1)

### Table II. S-wave two-body channels (i, j) of the various subsystems that contribute to the strangeness −2 $(I, J^P) = (\frac{1}{2}^+, \frac{1}{2}^-)$ three-body state.

| Subsystem | (i, j) channels |
|-----------|-----------------|
| $NN$      | (0,1),(1,0)     |
| $N\Lambda$| (\frac{1}{2},0), (\frac{1}{2},1) |
| $\Lambda\Lambda$ | (0,0) |
| $N\Xi$   | (0,0),(0,1),(1,0),(1,1) |

where $G_{ijk}^{\ell}$ is the propagator for three free particles $ijk$, $t_{ij,kl}$ is the two-body $t$-matrices for the different transitions $ij \rightarrow kl$, and $P_{ij}$ is the exchange operator for particles $i$ and $j$. The first superscript in the $T$–functions is the spectator and the other two are the interacting pair. We will solve these equations including all the $S$–wave configurations $\ell_i = \ell_i = 0$, where $\ell_i$ is the orbital angular momentum between particles $i$ and $j$, and $K_i$ is the orbital angular momentum between particle $i$ and the pair $jk$. Therefore, the total angular momentum $J = 1/2$ is equal to the total spin.

The set of Eqs. (1) are integral equations in two continuous variables which couple the nine two-body channels obtained from Table II. In order to solve these equations we use the method applied in our previous works [20, 21], where the two-body $t$–matrices are expanded in terms of Legendre polynomials leading to integral equations in only one continuous variable coupling the various Legendre components required for convergence.

In each of the two components of the coupled $\Lambda\Lambda-\Xi NN$ system we take particles 2 and 3 to be the two identical ones and particle 1 to be the different one. We will take the basis states 1 and 3 using a cyclic coupling scheme, i.e., $1 = (2 + 3) + 1$, and $3 = (1 + 2) + 3$, while for the basis state 2 we use the anticyclic scheme $2 = (1 + 3) + 2$. With these conventions, Eqs. (1) take the explicit form [19],

$$
\begin{align*}
T_{\alpha_m}^{\Sigma NN} (q_1) &= 2 \sum_{\alpha_j} \int_0^\infty q_3^2 dq_3 K_{mn,\alpha_1,\alpha_3;13}^{\Sigma NN} (q_1, q_3) T_{\alpha_3 ; \alpha_j}^{NNN} (q_3), \\
T_{\alpha_m}^{\Sigma NN} (q_3) &= \sum_{\alpha_1} \int_0^\infty q_2^2 dq_2 K_{mn,\alpha_2,\alpha_3;13}^{\Sigma NN} (q_2, q_3) T_{\alpha_1 ; \alpha_3}^{\Sigma NN} (q_3) - \sum_{\alpha_1} \int_0^\infty q_3^2 dq_3 K_{mn,\alpha_1,\alpha_3;23}^{\Sigma NN} (q_3, q_3') T_{\alpha_1 ; \alpha_3}^{\Sigma NN} (q_3'), \\
&\quad + 2 \sum_{\alpha_j} \int_0^\infty q_3^2 dq_3 K_{mn,\alpha_1,\alpha_3;13}^{\Sigma \Lambda\Lambda NN} (q_3, q_3') T_{\alpha_1 ; \alpha_j}^{\Lambda\Lambda NN} (q_3'), \\
T_{\alpha_m}^{\Lambda\Lambda} (q_1) &= \sum_{\alpha_1} \int_0^\infty q_1^2 dq_1 K_{mn,\alpha_2,\alpha_3;13}^{\Lambda\Lambda NN} (q_1, q_1') T_{\alpha_1 ; \alpha_3}^{\Sigma NN} (q_1') - \sum_{\alpha_3} \int_0^\infty q_3^2 dq_3 K_{mn,\alpha_1,\alpha_3;23}^{\Lambda\Lambda NN} (q_1, q_3) T_{\alpha_3 ; \alpha_3}^{\Sigma NN} (q_3).
\end{align*}
$$
\[ T_{α_{α_{m}}}(q_3) = \sum_{α_{m}} \int_{0}^{∞} q_3^2 dq_3 K^{α_{α_{m}}\Lambda\Lambda;\Lambda\Lambda}(q_3; q_3) T_{α_{α_{m}}}(q_3), \]

where

\[ K^{α_{γ};\Lambda\Lambda;α_{λ}}(q_3; q_3) = \frac{2m + 1}{4} A^{α_{γ};α_{λ}} \int_{-1}^{1} d\cos θ \int_{-1}^{1} dx_i P_1(x_i) P_2(x_j) \]

\[ \times i^{α_{γ}δ}(p_i, p_j; E + ∆E - q_3^2/2ν_j)/ E + ∆E - q_3^2/2ν_j. \]

η_j and ν_j are the usual reduced masses and \( P_n(x) \) is a Legendre polynomial. \( p_i = b(1 + x_i)/(1 - x_i), \)
\( x_j = (p_j - b)/(p_j + b), \) and \( b \) is a scale parameter on which the solution does not depend. \( p_i = |q_i^2 + (η_i q_i/m_k)^2 + 2(η_i q_i/m_k)\cos θ_i|^{1/2}, \)
\( p_j = |q_j^2 + (η_j q_j/m_k)^2 + 2(η_j q_j/m_k)\cos θ_j|^{1/2}, \) and \( ∆E = 0 \) if the corresponding state (either \( i \) or \( j \)) belongs to the ΛΛ component, while \( ∆E = 2m_Λ - m_N - m_Ξ \) if the corresponding state belongs to the ΞNN component. Finally, \( A^{α_{γ};α_{λ}} \) are the usual spin-isospin transition coefficients \( [29] \), where \( δ_ζ \) is the interacting pair in the state \( i \) and \( λ_ρ \) is the interacting pair in the state \( j \).

For practical purposes, we took into account all the \( S \)-wave two-body amplitudes that contribute in Eqs. (2) as shown in Table III. Even though our calculation will include only two-body \( S \)-waves, the corresponding two-body amplitudes will be obtained from a full model, including \( D \) waves in spin-triplet channels and the coupling to higher mass states in those cases where the quantum numbers allow for it.

Once the method to solve the bound state problem of the ΛΛNN system has been designed, we apply it to the chiral quark model of the baryon-baryon interaction developed in Ref. [22]. The model is capable to describe the low-energy parameters of the two-nucleon system, the \( S \)-wave phase shifts, and the triton binding energy [23]. It reproduces the elastic and inelastic scattering cross sections of the \( S = -1 \) two-baryon systems and the hypertriton binding energy [20, 21]. As can be seen in Fig. 2 of Ref. [21], the isospin one ΛΝΝ system is unbound. Finally, the model provides parameter free predictions for the elastic and inelastic scattering cross sections of the \( S = -2 \) two-baryon systems [24] that are consistent with the scarce available data. In particular, the relevant Ξ \( p \rightarrow ΛΛ \) is correctly described (see Fig. 2 of Ref. [24]). Thus, we are confident that the interactions are realistic enough to allow for the study of the existence (or non-existence) of the strangeness \(-2 \) hypertriton.

The H dibaryon has strangeness \(-2 \), positive parity, and isospin and spin \( (i, j) = (0, 0) \). It appears in our model as a bound state of the coupled ΛΛ-ΝΞ-ΞΣ system with a binding energy of 6.928 MeV [24]. Therefore, the main configuration of the strangeness \(-2 \) hypertriton will be an H dibaryon as the interacting pair and a \( S \)-wave nucleon as spectator, which leads to total isospin and spin \( (I, J) = (\frac{1}{2}, \frac{1}{2}) \) and positive parity. This configuration is also favored by having a deuteron as interacting pair and a \( S \)-wave Ξ hyperon as spectator. We give in Table III all the \( S \)-wave two-body channels that contribute to the \( (I, J^P) = (\frac{1}{2}, \frac{1}{2}^+) \) three-body state.



| \( a_{1/2,1}^{ΛΛ} \) | 1.41 | 1.46 | 1.52 | 1.58 |
|-------|-------|-------|-------|-------|
| 2.33  | 0.416 | 0.455 | 0.495 | 0.542 |
| 2.39  | 0.424 | 0.463 | 0.504 | 0.551 |
| 2.48  | 0.447 | 0.487 | 0.528 | 0.577 |

one can see from this table, the strangeness \(-2 \) three-
baryon system with quantum numbers \((I, J^P) = (\frac{1}{2}, \frac{1}{2}^-)\) is bound, the binding energy varying between 0.4 and 0.6 MeV. However, as predicted in Ref. [12] due to the non-existence of an isospin one \(\Lambda\) bound state, the \(\Lambda N\) system alone is not bound. The bound state only appears when the coupling between the \(\Lambda N\) and \(\Xi NN\) components is considered, i.e., when the \((i, j) = (0, 0)\) two-body \(t^{\Lambda N, \Xi}\) amplitude is included in the calculation.

The relevance of the \(\Lambda N\)–\(\Xi N\) coupling for double-\(\Lambda\) hypernuclei has been emphasized for the case of the \(\Lambda^4H\) hypernucleus [25, 26]. If this system is studied with \(NN\), \(NA\) and \(\Lambda\Lambda\) interactions improved for the description of the \(\Lambda^4He\), it is found to be unbound. In the case of the \(\Lambda^6He\) the \(\Lambda N\)–\(\Xi N\) coupling plays a minor role, because the nucleon generated in the transition must occupy an excited \(p\)–shell, the lowest \(s\)–shell being forbidden by the Pauli principle. This is not the case of the \(\Lambda\Lambda^2H\), where the nucleon generated by the \(\Lambda N\)–\(\Xi N\) transition can occupy a hole in the lowest \(s\)–shell. This effect generates theoretical binding for the \(\Lambda\Lambda^2H\) [23] and it is also the responsible for generating binding in the strangeness \(−2\) three-baryon system with quantum numbers \((I, J^P) = (\frac{1}{2}, \frac{1}{2}^-)\). It is therefore important to obtain experimental information about the strength of the \(\Lambda N\)–\(\Xi N\) coupling. It could be derived from the measurement of the \(\Lambda\Lambda^2H\) binding energy. In the meantime, the only available experimental data is the inelastic cross section \(\Xi^0 p \to \Lambda N\), correctly described by the present model (see Fig. 2 of Ref. [24]).

The possible existence of a strangeness \(−2\) hypertriton will give a strong impact on forthcoming experimental projects as well as on-going theoretical studies. Experimentally, it could be measured in the J-PARC-E07 experiment, where more than 10\(^3\) \(\Lambda\Lambda\)–nuclei are expected to be detected by means of \(\Xi\)–capture reactions using different target nuclei: \(C, N,\) and \(O [23]\). Theoretically, Lattice QCD has evolved to the point where the calculation of the binding energy of light nuclei and hypernuclei with \(A \leq 4\) and \(S \leq 2\), at unphysically heavy light-quark masses, is possible [28]. Extrapolations to the physical light-quark masses have not been attempted because the quark mass dependences of the energy levels in the light nuclei are not known. Future calculations at smaller lattice spacings and at lighter quark masses will facilitate such extrapolations and, therefore, comparison with experiment and, thus, the analysis of the strangeness \(−2\) hypertriton.

In summary, we have solved for the first time the Faddeev equations for the bound state problem of the coupled \(\Lambda\Lambda N\)–\(\Xi NN\) system to study whether an hypertriton with strangeness \(−2\) may exist or not. We make use of the interactions obtained from a chiral quark model describing the low-energy observables of the two-baryon systems with strangeness 0, \(−1\), and \(−2\) and three-baryon systems with strangeness 0 and \(−1\). The \(\Xi NN\) system alone is unbound in agreement with the non-existence of an isospin one \(\Lambda\Lambda^2H\) bound state. However, when the full coupling to \(\Xi NN\) is considered through the \((i, j) = (0, 0)\) two-body \(t^{\Lambda N, \Xi}\) amplitude, the strangeness \(−2\) three-baryon system with quantum numbers \((I, J^P) = (\frac{1}{2}, \frac{1}{2}^-)\) becomes bound, with a binding energy of about 0.5 MeV.

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