Meteor Streams and Parent Bodies

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Abstract. Problem of meteor orbit determination for a given parent body is discussed. Some of the published methods for obtaining meteoroid’s orbital elements at the moment of intersecting Earth’s orbit on the basis of geometrical variation of parent body’s orbital elements are discussed. The main result concerns the following two facts: i) in real situations physical quantities for the change of orbital elements of the parent body must be used, and, ii) the usage of Southworth and Hawkins (1963) D-criterion yields results not corresponding to observations.

Key words: interplanetary medium - meteoroids – meteor streams

1. Introduction

In searching for the relation between parent body (comet, asteroid) and its possible meteor stream, one needs to calculate theoretical orbit of a meteoroid at the time when it may become a meteor. Several methods have been worked out during the last several tens of years. Their summary may be found in Svořeň et al. (1993). Our aim is to discuss some of the published methods and improve them. We also suggest physical method.

Svořeň et al. (1993), Neslušan et al. (1994), as examples, use D-criterion of Southworth and Hawkins (1963) in many of the methods. Moreover, in final comparison between theoretical and observational result, this D-criterion is considered to be a decisive quantity. However, it seems to be not adequate to use this criterion due to its many “curious” properties (Klačka 1995).
We will use standard orbital elements in the paper: $q$ – perihelion distance, $e$ – eccentricity, $i$ – inclination, $\Omega$ – longitude of the ascending node, $\omega$ – argument of perihelion. Moreover, subscript “$P$” refers to parent body, subscript “$M$” to meteoroid which intersects the orbit of the Earth. The Earth’s orbit is supposed to be circular.

2. General considerations

If the meteoroid intersects the Earth’s orbit, then one node of the meteoroid’s orbit is at distance $r_E = 1$ AU from the Sun. Thus, we have ($f$ – true anomaly)

$$\sin(\omega_M + f_M) = 0 \quad \text{and} \quad r_E = \frac{q_M (1 + e_M)}{1 + e_M \cos f_M}. \quad (1)$$

Thus, if the ascending node ($\varepsilon > 0 \Rightarrow \sin(\omega_M + f_M + \varepsilon) > 0$ – this corresponds to $\omega_M + f_M = 2 \pi$) intersects the orbit of the Earth, then

$$1 \ AU \equiv r_{AE} = \frac{q_M (1 + e_M)}{1 + e_M \cos \omega_M}. \quad (2)$$

If the descending node ($\varepsilon > 0 \Rightarrow \sin(\omega_M + f_M + \varepsilon) < 0$ – this corresponds to $\omega_M + f_M = \pi$) intersects the orbit of the Earth, then

$$1 \ AU \equiv r_{DE} = \frac{q_M (1 + e_M)}{1 - e_M \cos \omega_M}. \quad (3)$$

3. Methods of variation of the orbit in the orbital plane

Three orbital elements may vary: $q$, $e$, $\omega$. If $\omega_M = \omega_P$ and $q_M = q_P$, or, $\omega_M = \omega_P$ and $e_M = e_P$, the situation is trivial – in both cases only one orbital element ($e$, resp. $q$) is varied in order to Eq. (2) or Eq. (3) be fulfilled.

3.1. The argument of perihelion is varied

Let us suppose that it is sufficient to make a change of $\omega_P$, only, for the purpose of that a meteoroid may intersect the Earth’s orbit: $q_P = q_M$, $e_P = e_M$. Eq. (2) yields then

$$\omega_{M1} = \arccos \left\{ \frac{1}{e_P} \left[ \frac{q_P (1 + e_P)}{r_{AE}} - 1 \right] \right\},$$
$$\omega_{M2} = 360^\circ - \omega_{M1}. \quad (4)$$

Eq. (3) yields

$$\omega_{M3} = \arccos \left\{ \frac{1}{e_P} \left[ 1 - \frac{q_P (1 + e_P)}{r_{DE}} \right] \right\},$$
$$\omega_{M4} = 360^\circ - \omega_{M3}. \quad (5)$$

If $\omega_M$ does not exist, then the assumption that the change of only one element $\omega_P$ is sufficient in obtaining meteor, is incorrect.
3.2. Perihelion distance and eccentricity are simultaneously varied

Let us consider now that $\omega_M = \omega_P$ and that $q$ and $e$ may change. If we use also D-criterion of Southworth and Hawkins (1963), as it is done in Svorěn et al. (1993) (only $q$ and $e$ are different for parent body and meteoroid), the minimum simultaneously change of $q$ and $e$ is given by the condition that the function

$$D_{SH}^2 \equiv (q_M - q_P)^2 + (e_M - e_P)^2 \tag{6}$$

gains its minimum value. Besides this condition, Eqs. (2)-(3) also hold ($\omega_M = \omega_P$). If we substitute $q_M$ from Eq. (2) into Eq. (6), we obtain $D_{SH}$ as a function of $e_M$ only and its extremal – minimum value can be easily found. The same procedure must be replied for Eqs. (3) and (6). In both cases it can be analytically proved that only one solution corresponds to $e_M > 0$. For some parent bodies the minimum value of $e_M$ corresponds to $e_M \in (0, 1)$, for the others $e_M \in [1, 9/4]$ (the last value is approximate). In any case, the value of $D_{SH}$ obtains the value, which is even in the interval $e_M \in (0, 1]$ less than it would be if only one of the elements $q_M$ and $e_M$ is varied.

These analytical results are in contradiction with the results presented in Svorěn et al. (1993 – see Tables 1, 3, 4, 6).

Another important result shows Table 5 in Svorěn et al. (1993): the simultaneous optimal change of $q_M$ and $e_M$ yields value of $D_{SH}$ which is less than the value of $D_{SH}$ if only $q_M$ is changing, but the concidence between theoretical calculations and observed data is better in the latter case. This result confirms the fact that D-criterion of Southworth and Hawkins is not good approximation to the real processes, as it is discussed in Klačka (1995) from the general point of view.

4. Methods of variation and rotation of the orbit in space

We will discuss only one method here, in order to stress again that we do not need any type of D-criterion.

4.1. Adjustment of the orbit by rotation around the line of apsides

We will suppose $q_M = q_P$, $e_M = e_P$.

Let the coordinate system $S'$ be created in the form that the orbital plane of the parent body is characterized by the condition $z' = 0$ and let the perihelion lies on the positive part of the $x'$–axis: perihelion is characterized by the unit vector $x'_p = 1$, $y'_p = z'_p = 0$. This unit vector has coordinates $x$, $y$, $z$ in the original coordinate system $S$ (ecliptical
Let the unit vector normal to the orbital plane of the parent body has coordinates:

\[ x_n' = 0, \quad y_n' = 0, \quad z_n' = 1. \]

This normal unit vector is characterized by the following coordinates in the system S:

\[ x_{nP} = \sin \Omega P \sin i_P \]
\[ y_{nP} = - \cos \Omega P \sin i_P \]
\[ z_{nP} = \cos i_P . \]  \hspace{1cm} (8)

Now, we make a rotation around the \( x' \)-axis in an angle \( \Phi \). As a result we obtain a new orbit – orbital plane of the meteoroid – characterized by new normal unit vector, which has coordinates in the system S:

\[ x_{nM} = \sin \Omega M \sin i_M = \]
\[ = (\cos \Omega P \sin \omega_P + \sin \Omega P \cos \omega_P \cos i_P) \sin \Phi + \]
\[ + (\sin \Omega P \sin i_P) \cos \Phi \]
\[ y_{nM} = - \cos \Omega M \sin i_M = \]
\[ = (\sin \Omega P \sin \omega_P - \cos \Omega P \cos \omega_P \cos i_P) \sin \Phi - \]
\[ - (\cos \Omega P \sin i_P) \cos \Phi \]
\[ z_{nM} = \cos i_M = \]
\[ = - (\cos \omega_P \sin i_P) \sin \Phi + (\cos i_P) \cos \Phi . \]  \hspace{1cm} (9)

Unit vector of the perihelion of the meteoroid’s orbit is:

\[ x_{pM} = \cos \Omega M \cos \omega_M - \sin \Omega M \sin \omega_M \cos i_M \]
\[ y_{pM} = \sin \Omega M \cos \omega_M + \cos \Omega M \sin \omega_M \cos i_M \]
\[ z_{pM} = \sin \omega_M \sin i_M , \]  \hspace{1cm} (10)

and it is the same unit vector as the unit perihelion vector of the parent body (\( x' \)-axis – the line of apsides – is fixed during the rotation).

Thus, we have one set of three equations given by the equality of the right-hand-sides of Eqs. (7) and (10) and another set of three equations given by Eq. (9). For a given angle...
Eq. (9) determines quantities \( i_M \) and \( \Omega_M \), and, then, Eqs. (7) and (10) determine the angle \( \omega_M \). The angle \( \Phi \), for which Eqs. (2) or (3) hold, may be easily found.

We have found the angle of rotation \( \Phi \) around the line of apsids which determines the orbit of a meteoroid to become a crosser of the Earth’s orbit. Thus, we have found all the required elements: \( i_M, \Omega_M \) and \( \omega_M \). We do not need any knowledge about the D-criterion of Southworth and Hawkins as it is in the method presented in Svoreň et al. (1993).

5. Physical Methods

Let us consider physical access to the problem.

5.1. Energy-\(H_z-\pi\) Method

By this method we consider simple model in which the following quantities of the meteoroids are equivalent to those of the parent body: semimajor-axis \( a \), z-component of angular momentum \( \sqrt{a (1 - e^2) \cos i} \), \( \pi = \omega + \Omega \). Considering Eqs. (2) and (3), the minimization of the magnitude of the difference of the angular momenta vectors between meteoroid and its parent body \( |\Delta H| \equiv |H - H_c| \) (global and local minima) yields several theoretical radiants – those with the smallest values of \( |\Delta H| \) should be realized.

5.2. Energy-\(C_1-C_2-C_3\) Method

By this method we consider the model of Babadzhanov and Obrubov (1987) in which the four quantities of the meteoroids are equivalent to those of parent body: semimajor-axis \( a \), z-component of angular momentum \( \sqrt{a (1 - e^2) \cos i} \equiv C_1 \), \( C_2 = e^2 (0.4 - (\sin i)^2 (\sin \omega)^2) \), \( C_3 \equiv \pi = \omega + \Omega \). Possible orbits of meteoroid and the corresponding radiants are found on the basis of these four conservation quantities. The physical idea of the method yields that all the obtained radiants are equivalently significant and they all should exist in reality.

6. Application of the Physical Methods and Method of the Section 4.1

As an application of the mentioned physical methods we present theoretical orbital elements and radiants of meteoroids of the comet 73P/Schwassmann-Wachmann 3 for the year 2006. Orbital elements of the comet, considered in our calculations, are: \( q_c = 0.93973 \) AU, \( e_c = 0.69331 \), \( i_c = 11.398^\circ \), \( \omega_c = 198.824^\circ \), \( \Omega_c = 69.883^\circ \); these elements were obtained by the authors of the paper Gajdoš et al. (1998), where also theoretical radiants by other methods (sections 3 and 4 of this paper) are presented.
The Energy-H$_2$-π method yields, in principle, four theoretical radiants. The values of the last column of the Table 1 suggest, however, that only one radiant should correspond to reality – that given by the first row in Table 1.

**Table 1. Energy-H$_2$-π Method.**

| $q$(AU) | $e$  | $i$[°] | $\omega$[°] | $\Omega$[°] | $\lambda_0$[°] | $\alpha$[°] | $\delta$[°] | $|\Delta H|^2$ |
|--------|-----|--------|------------|-----------|---------------|-----------|---------|-----------|
| 0.9470 | 0.6910 | 12.3 | 209.5 | 59.2 | 59.2 | 211.4 | 26.9 | 0.0009 |
| 0.9176 | 0.7001 | 8.1 | 143.1 | 125.6 | 125.6 | 172.0 | 28.9 | 0.0143 |
| 0.8954 | 0.7078 | 0.1 | 318.4 | 310.3 | 130.3 | 163.5 | 6.7 | 0.0205 |
| 0.8954 | 0.7078 | 0.1 | 41.6 | 227.1 | 47.1 | 193.9 | −6.2 | 0.0207 |

The Energy-C$_1$-C$_2$-C$_3$ method yields also four radiants. The physical idea of the method predicts the real existence of all the four radiants (see Table 2). (We mention that no meteor stream existed for the year 1930 according to Energy-C$_1$-C$_2$-C$_3$ method.)

**Table 2. Energy-C$_1$-C$_2$-C$_3$ Method.**

| $q$(AU) | $e$  | $i$[°] | $\omega$[°] | $\Omega$[°] | $\lambda_0$[°] | $\alpha$[°] | $\delta$[°] |
|--------|-----|--------|------------|-----------|---------------|-----------|---------|
| 0.9263 | 0.6977 | 9.6 | 214.9 | 53.9 | 53.9 | 206.7 | 18.8 |
| 0.9263 | 0.6977 | 9.6 | 145.2 | 123.6 | 123.6 | 173.9 | 33.1 |
| 0.9263 | 0.6977 | 9.6 | 325.2 | 303.6 | 123.6 | 151.1 | −17.9 |
| 0.9263 | 0.6977 | 9.6 | 34.9 | 233.9 | 53.9 | 183.7 | −32.1 |

Finally, we present also radiants for the year 2006 obtained by the method described in section 4.1. Again, also this method yields, in principle, four theoretical radiants. The values of the last column of the Table 3 suggest, however, that only one radiant should correspond to reality – that given by the first row in Table 3. (We mention that no meteor stream existed for the year 1930 according to this method – $q_c > 1.0$ AU.)

The first two rows of Table 2 correspond to the first two rows of Table 1. The third row of Table 2 corresponds to the second row of Table 3. However, the last columns of Tables 1 and 3 show that only the first row of Tables 1 and 3 can correspond to the reality. In other words, the real existence of all the four radiants given by the Energy-C$_1$-C$_2$-C$_3$ method seems to be improbable.
Table 3. Adjustment of the orbit by rotation around the line of apsides method – method described in section 4.1

| $q(AU)$ | $e$  | $i(\degree)$ | $\omega(\degree)$ | $\Omega(\degree)$ | $\lambda_\odot(\degree)$ | $\alpha(\degree)$ | $\delta(\degree)$ | $\Phi(\degree)$ |
|---------|------|--------------|-------------------|------------------|--------------------------|-----------------|-----------------|-----------------|
| 0.9397  | 0.6933 | 7.0          | 211.5             | 57.1             | 203.1                    | 14.0            | 4.8             |
| 0.9397  | 0.6933 | 7.0          | 328.5             | 299.7            | 119.7                    | 154.0           | -12.9           | 16.8            |
| 0.9397  | 0.6933 | 173.0        | 328.5             | 237.1            | 57.1                     | 337.7           | -13.5           | 184.8           |
| 0.9397  | 0.6933 | 173.0        | 211.5             | 119.7            | 119.7                    | 19.2            | 12.3            | 196.8           |

7. Discussion

We have discussed some of the published methods of calculating meteoroid orbits crossing the Earth’s orbit if the parent body’s orbit is known.

If the parent body’s orbit is changed in a trivial geometric way as it is often done, one should try to avoid to any further requirement, e.g., to use D-criterion of Southworth and Hawkins. The reason is that this criterion yields nonphysical results, as it is pointed out in the last paragraph of the section 3.2 and discussed in Klačka (1995).

The use of simple methods has one great advantage – calculations are very simple. However, since changes of orbital elements in such simple methods are only of geometrical character, the methods do not correspond to any real physical process – the methods do not use any physical basis. Since real physics is very complicated and we do not know exact perturbations on meteoroids (the importance of nongravitational effects), we have to make predictions for a new possible pairs “parent body – meteor stream” on the basis of a simple physics, or, on the basis of the known pairs.

Possible way is to find a function $F$ (containing five orbital elements in an independent way) which is a good approximation to reality and does not exhibit any inconsistencies. Suggestion of this type may be found in Klačka (1995), where also general method is presented – special case of this method is Energy-H$_2$-π method. The method in reality corresponds to the following set of equations:

$$
\beta_{iM} = f_i(\beta_{jP}) , \quad i, j = 1 \text{ to } 5 ,
$$

(11)

where $\beta$ – s represent orbital elements ($P$ – parent body, $M$ – meteoroid, meteor stream) and Eqs. (2) or (3) hold. The unknown functions must be found (approximated) on the basis of observational data – pairs “parent body – meteor stream”. The conditions $f_i = \beta_{iP}$ ($i = 1 \text{ to } 5$) hold if $\beta_{jP}$ fulfill Eqs. (2) or (3). Thus, the method presented in
Klačka (1995) is equivalent to the following sets of equations:

\[
\beta_i^M = \beta_i^P + \left\{ \frac{q_p (1 + e_P)}{1 + e_p \cos \omega_P} - 1 \right\} f_{1i}(\beta_j^P), \quad \text{or}
\]

\[
\beta_i^M = \beta_i^P + \left\{ \frac{q_p (1 + e_P)}{1 - e_p \cos \omega_P} - 1 \right\} f_{2i}(\beta_j^P),
\]

where \( f_{1i} \) and \( f_{2i} \) are finite, and, moreover, Eqs. (2) or (3) hold.

In any case, one must be aware of the fact that real meteoroid streams have various dispersions of orbital elements and that our methods of calculating new pairs “parent body – meteor stream” are only a first approximation.

In principle, no reasonable metric for various trajectories of parent bodies and their meteoroids exists. Gravitational and nongravitational forces acting on parent bodies and their meteoroids are different in various parts of orbital element’s phase space, and, no global metric can be constructed on the basis of the know pairs “parent body – meteor stream”. Useful methods for predicting new pairs “parent body – meteor stream” must be based on the known pairs “parent body – meteor stream” and inevitable weighting factors, functions of parent body’s orbital elements, present in “metric” disturb the basic property of metric – its symmetricity.

It is hard to make reasonable predictions for meteor streams corresponding to new parent bodies lying outside the zone of the known parent bodies for which we know the relation “parent body – meteor stream”. Only extrapolations can be made in this case. Much better predictions are obtained for the inner part of the zone of the phase space. Energy-H\(\_\)\(-\)\(\pi\) method seems to yield good results in any case.

8. Appendix

Derivation of equations of section 4.1 is presented here, on the request of the referee.

Let the Cartesian coordinate system S’ is created from the Cartesian coordinate system S by the rotation characterized with Eulerian angles \( \Omega, i \) and \( \omega \). If \( x, y, z \) are coordinates of a vector in the system S, then \( x', y', z' \) are coordinates of the same vector in the system S’. We have

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{pmatrix}
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix},
\]

and the transformation matrix \( C \) is of the form
\[
\begin{pmatrix}
\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i & -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i & \sin \Omega \sin i \\
\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i & -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i & -\cos \Omega \sin i \\
\sin \omega \sin i & \cos \omega \sin i & \cos i
\end{pmatrix}
\]

(Transformation is orthogonal and so inverse transformation can be easily obtained, using transposed matrix.)

Unit vectors of perihelia are defined by conditions \( x' = 1, y' = z' = 0 \), and so the last equation immediately yields Eqs. (7) and (10). Unit vector normal to the orbital plane is defined by conditions \( x' = y' = 0, z' = 1 \), and, again, the last equation immediately yields Eq. (8).

Moreover, if we make a rotation characterized by an angle \( \Phi \) around \( x' \)-axis, we obtain

\[
\begin{align*}
x'' &= x' \\
y'' &= y' \cos \Phi + z' \sin \Phi \\
z'' &= -y' \sin \Phi + z' \cos \Phi.
\end{align*}
\]

Now, if we put \( x'' = y'' = 0, z'' = 1 \), then \( x' = 0, y' = -\sin \Phi, z' = \cos \Phi \), and the first equation of the appendix yields results which are consistent with Eq. (9).

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References

Babadzhanov, P. B., Obrubov, Yu. V., 1987, in: Interplanetary Matter (10-th ERAM), eds. Z. Ceplecha and P. Pecina, Astronomical Institute of the Czechoslovak Academy of Sciences, Ondřejov, p. 141

Gajdoš, Š., Galád, A., Klačka J., Pittich, E. M., 1998, presented at the IAU Colloquium 173

Klačka J., 1995, Astron. Astrophys. (submitted)

Neslušan, L., Porubčan, V., Svoreň, J., 1994, Planet. Space Sci. 42, 669

Southworth, R. B., Hawkins, G. S., 1963, Smithson. Contrib. Astrophys. 7, 261

Svoreň, J., Neslušan, L., Porubčan, V., 1993, Contrib. Astron. Obs. Skalnaté Pleso 23,
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