Path Planning Algorithm for Fatigue Life Prediction of SiC/AL MMCs Considering Residual Stress

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Abstract

To explore the influence of path deflection on crack propagation, a path planning algorithm is presented to calculate the crack growth length. The fatigue crack growth life of metal matrix composites (MMCs) is estimated based on an improved Paris formula. Considering the different expansion coefficient of different materials, the unequal shrinkage will lead to residual stress when the composite is molded and cooled. The crack growth model is improved by the modified stress ratio based on residual stress. Although the data obtained by the new model are not consistent with the experimental data very well, it can still prove that the idea of establishing the model is effective to a certain extent.

Keyword: Fatigue life estimation; Path planning; Residual temperature stress; Composite material

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Nomenclature

\( K_{IC} \) fracture toughness

\( F \) geometric factor

\( a \) crack size

\( a_0 \) initial crack length

\( a_c \) crack size at failure

\( R \) stress ratio

\( \sigma_a \) stress change related to residual stress

\( \Delta \alpha \) difference of expansion coefficient

\( \Delta t \) temperature difference

\( K_m \) elastic modulus of matrix

\( K_i \) elastic modulus of reinforcement

\( \sigma_{min} \) minimum stress

\( \sigma_{max} \) maximum stress

\( \Delta \sigma \) stress amplitude

\( \sigma_b \) tensile strength

\( \Delta \sigma_{egv} \) equivalent stress

\( N_i \) crack initiation life

\( N_f \) crack propagation life

\( \Delta \varepsilon_c \) threshold of strain range

\( \varepsilon_f \) fracture fatigue strain
\[\sigma_f\] fracture fatigue strength

\[E\] elastic modulus

\[K\] intensity coefficient

\[n\] strain hardening exponent

\[K_t\] theoretical stress concentration factor

\[\tau_{-4}\] torsional fatigue limit

\[\psi\] reduction of area

\[\Delta K\] stress intensity factor range

\[C, m\] material parameters of Paris formula

\[\theta\] deflection angle of crack

\[P\] coefficient of expansion
**Introduction**

Composite have been widely used in aerospace, machinery, shipbuilding and other fields [1]. Carbon fiber composites have good mechanical properties in various aspects, it is one of the most popular composite materials at present, but it need a more stringent working environment temperature compare to other composite materials. Excessive temperature will have a destructive effect on the material. The current manufacturing cost is too high to mass-produce civilians. Particle-reinforced metal materials are isotropy which similar to metal materials, it has been mass-produced with different smelting technologies.

Mechanical Components often suddenly failure when subjected to cyclic stress far below the yield limit, which is defined as fatigue failure. In engineering applications, most failures are caused by fatigue failures, fatigue life prediction play a significant role for reliability evaluation [2]. It is dangerous to staff in the working environment if parts of the machine are suddenly damaged during operation. In addition, it will also cause unnecessary economic losses to businesses and society. Therefore, accurately predicting the fatigue life of component is of great significance for production safety and product design [3-5]. The fatigue life prediction of traditional metal materials includes strength degradation model, damage accumulation model, energy method, S-N curve and other methods [6-8]. The fatigue life of materials is affected by loading strength, load-stress ratio, and different load loading sequences [9-13]. In addition, the surface roughness and operating temperature of materials will affect the fatigue life of component [14-16]. Many scholars and researchers have modified the model based on above all these factors, and propose more prediction methods suitable for material properties and working environment [17,18]. More and more scholars analyze the fatigue life by establishing numerical models [19-23]. Based on the fatigue research results and experience of metal materials, many scholars have carried out fatigue life analysis of metal matrix composites [24, 25]. Metal matrix composites perform better than metal alloys in terms of fatigue performance [27-29]. Researchers believe that it is caused by multiple reasons, such as reinforcements that increase crack closure. This paper
interprets the increase in fatigue life of metal matrix composites as the increase in fatigue crack length, and the reduction in the effective driving force for crack propagation caused by the deflection angle.

Path planning was originally part of a robot behavior algorithm. Its definition refers to finding a collision-free path from the starting position to the target position according to certain evaluation criteria in an environment with obstacles [30, 31]. The different distribution of obstacles in the environment directly affects the planned path. So path planning can be defined as an active behavior that allows a robot or mechanical device to choose a path around an obstacle based on the environment. Path planning algorithms include Dijkstra algorithm, genetic algorithm, neural network algorithm, ant colony algorithm and so on. In studying the crack growth of particle-reinforced metal matrix composites, it is found that when the reinforcement particles are distributed uniformly and disorderly in the material, fatigue crack growth will deflect or break through the reinforcement particles when they encounter the reinforcement particles. It is precisely because the presence of reinforcing particles hinders the growth of fatigue cracks that the composite material exhibits a higher fatigue life in the macro. Whether the crack passes through the reinforcing particles depends on the current crack tip stress. In an ideal condition, all the reinforcing particles will not be penetrated, which could be hinder crack growth and propagation to the greatest extent. The behavior of cracks bypassing all reinforcing particles can be similar to robot path planning, but crack growth is a passive behavior which is different with path planning. It is possible to predict the crack path by path planning algorithm. The fatigue behavior of composite materials is quite different from that of traditional metals. By combining crack length and path planning, the composite materials fatigue life prediction can be more accurate.

1 statement of the problem

The fatigue life prediction of components is of great significance to product design and production safety. Composite materials have superior performance than traditional materials, which are replacing traditional materials in engineering in a wide range. As far as composite parts are concerned, the currently fatigue life estimation
models are only applicable within a certain range. In order to predict the life of composite parts more accurately, the following issues need to be addressed urgently:

(1) As a new type of material, composite materials are far less mature than traditional materials. So that the test data of composite materials are less than those of common metal materials at present. Besides that, parameters of traditional metal materials, such as strength, Poisson's ratio, and section shrinkage, are easy to be collected by a great number of experiments, but there are more factors affecting the performance of composites. For example, ceramic particle-reinforced metal-based composite materials, its reinforcing matrix volume fraction, metal-based alloy properties, and even the material manufacturing process will affect the material properties, so the experimental data is difficult to reproduce.

(2) Composite materials have a different structure from traditional metal materials. Metals usually have metal atoms arranged in a certain shape, and therefore have certain properties. There are many forms of composite materials. The ceramic particle reinforced metal matrix composites discussed in this paper include silicon carbide particles and alloys. Different materials are connected through the interface. Due to different microstructure, the internal stress distribution and fatigue crack growth of composite materials are different from traditional metal materials.

(3) The fatigue failure mechanism of composite materials is different from that of traditional materials. Most of the current fatigue prediction methods for composites use traditional fatigue prediction models. Material performance parameters are replaced during prediction to obtain rough estimates of fatigue life. For example, when it is necessary to estimates the crack fracture length, the calculation process is shown as follows:

\[
a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{F\sigma} \right)^2
\]  

(1)

When it is necessary to calculate the crack fracture length of SiC/Al, the current general processing method is to use the fracture toughness of SiC/Al instead of AL to obtain the result. The force change caused by the SiC reinforcing particles inside the material and the influence on the calculation results are not considered. This method
can meet engineering applications within a certain range, but it cannot reflect the failure mechanism of the composite material. By analyzing the failure process of composite materials, a model to improve the accuracy of life prediction is established.

2 Crack propagation path model

2.1 Dijkstra algorithm

Dijkstra algorithm is used to simulate the path planning of random paths. This algorithm solves the single-source shortest path problem of weighted directed graphs by breadth-first search. The principle is to go back from the end point, find the last shortest path, and keep repeating until the start point.

Fig. 1 Schematic diagram of local shortest path

As shown in Figure 1, the purpose of the algorithm is to find the shortest path from the starting point A to the ending point F. There are two arrays U and V. U indicates points that have been backtracked, V indicates points that have not been backtracked, and the number in parentheses is the shortest distance from point F. The process is as follows:

Step 1: U=[F(0)];
V=[D(5),E(4),C(∞),B(∞),A(∞)];
Step 2: U=[F(0),E(4)];
V=[D(5),C(13),B(11),A(∞)];
Step 3: U=[F(0),E(4),D(5)];
V=[ C(12),B(11),A(∞)];
Step 4: U=[F(0),E(4),D(5),B(11)];
V=[ C(12),A(17)];
Step 5: \( U = [ F(0), E(4), D(5), B(11), C(12)] \);
\( V = [A(16)] \);

Step 6: \( U = [ F(0), E(4), D(5), B(11), C(12), A(16)] \);
\( V = \emptyset \);

As shown above, in step two, the shortest distance between point C and point F is 13, and the path at this moment is C, E, F; after an iteration, point D is added to the U array as the shortest path point. The shortest distance between point C and point F is 12, and the path at this time is C, D, F. After continuous iteration, the shortest path between any two points can be obtained.

2.2 Random barriers

SiC / AL composite material is made of aluminum alloy as the metal matrix, and SiC particles are evenly distributed in the metal matrix as a reinforcement. SiC particles are randomly distributed within a certain range on a microscopic scale, and their shapes are mostly random convex quadrilaterals. According to the above characteristics, a model for simulating the distribution of random SiC particles is established.

The crack propagation path in the composite material will deflect when it expands to the position of the reinforcing base, which is different from metal materials, and the crack propagation path will be Lengthen. It is easy to find the propagation path of cracks in composites, the law of crack propagation is similar to robot path planning. The crack bypasses the reinforcement from the beginning to the end, and the crack cracking rule is obtained by simulating the crack propagation path, and the relationship between the crack and the fatigue life is obtained.

By establishing the position of the random reinforcement base, the reinforcement base is an irregular quadrangle and is randomly distributed in the metal matrix. In the program, by using the square interval where each quadrilateral is located as a cell, if four random points are taken in a cell to represent the endpoints of the polygon, the random quadrilateral formed may have a "concave polygon". However, in actual engineering, the "concave polygon" structure is prone to stress concentration at the
shortest diagonal, and the reinforcing base particles often exist in the shape of "convex polygon". In the modeling process of this paper, the strengthening basis is treated as a random quadrilateral, and the four endpoints fall into four square sub-regions that bisect the cell. Each sub-region is a grid, and points are randomly taken in each grid to form a random quadrilateral representing the position and shape of the reinforcing base. In composite materials, the reinforcing matrix is uniformly distributed in the metal matrix on a macroscopic scale, but on microscopic observations, the spacing between particles is randomly distributed within a certain order of magnitude. During the modeling process, a random number is added to the cell distance. The size of the random number determines the average distance between the reinforcing bases, and it can also reflect the volume fraction of the reinforcing bases in the composite. The coordinates of each endpoint of the random quadrilateral need to be stored in matrix $A$ for subsequent calculations. The mechanical properties of particle-reinforced composites are affected by factors such as the volume fraction of the reinforcing matrix and the particle size of the reinforcing matrix, so the simulation results should be considered to match the actual material during the simulation.

It is observed from Fig. 2 that there is very little contact or even superposition between the reinforcing groups, and we can consider it as the phenomenon of reinforcing group polymerization in the timing material. Its endpoint coordinates are shown in table 1.
Table 1 Obstacle Endpoint Matrix

|     | Column 1 | Column 2 | Column 3 | Column 4 | Column 5 | Column 6 |
|-----|----------|----------|----------|----------|----------|----------|
| Point 1 | 2.87     | 2.92     | 3.53     | 1.89     | 4.81     | 2.78     | 7.05 | 1.73 | 7.78 | 2.09 | 9.75 | 1.99 |
| Point 2 | 2.02     | 2.92     | 2.90     | 2.11     | 4.26     | 2.69     | 6.65 | 1.68 | 7.64 | 1.97 | 9.24 | 1.69 |
| Point 3 | 2.27     | 2.01     | 3.10     | 1.74     | 4.67     | 2.23     | 6.62 | 1.17 | 7.70 | 1.82 | 9.48 | 1.20 |
| Point 4 | 2.60     | 2.24     | 3.42     | 1.33     | 4.80     | 2.00     | 6.96 | 1.39 | 7.98 | 1.75 | 10.13 | 1.56 |
| Point 5 | 2.37     | 4.45     | 4.04     | 3.60     | 5.27     | 3.60     | 7.28 | 4.18 | 8.35 | 4.26 | 9.41 | 3.39 |
| Point 6 | 2.27     | 4.20     | 3.52     | 3.60     | 4.64     | 3.28     | 6.56 | 4.01 | 7.63 | 4.10 | 9.07 | 3.46 |
| Point 7 | 2.19     | 3.53     | 3.58     | 3.15     | 4.62     | 2.71     | 6.54 | 3.45 | 7.61 | 3.80 | 8.98 | 2.86 |
| Point 8 | 2.50     | 3.92     | 3.71     | 3.10     | 5.49     | 2.94     | 7.07 | 3.55 | 8.21 | 3.69 | 9.30 | 3.05 |
| Point 9 | 2.48     | 4.69     | 3.56     | 5.48     | 5.11     | 5.05     | 6.95 | 5.28 | 7.66 | 5.66 | 9.74 | 5.32 |
| Point 10 | 2.08    | 5.14     | 3.17     | 5.34     | 4.36     | 5.00     | 6.53 | 5.35 | 7.59 | 5.62 | 9.57 | 5.53 |
| Point 11 | 2.00    | 4.55     | 3.34     | 4.71     | 4.39     | 4.28     | 6.30 | 5.06 | 7.19 | 4.85 | 9.19 | 4.83 |
| Point 12 | 2.53    | 4.37     | 3.76     | 4.95     | 4.80     | 4.30     | 7.06 | 4.86 | 7.80 | 5.06 | 9.92 | 4.84 |
| Point 13 | 2.80    | 6.05     | 3.05     | 6.36     | 5.08     | 6.14     | 6.33 | 6.59 | 8.25 | 6.53 | 9.64 | 6.56 |
| Point 14 | 2.09    | 6.06     | 2.78     | 6.73     | 4.82     | 6.58     | 5.96 | 6.55 | 7.95 | 6.59 | 9.00 | 6.83 |
| Point 15 | 2.00    | 5.95     | 2.98     | 5.90     | 4.62     | 5.86     | 6.15 | 6.03 | 8.06 | 6.15 | 9.07 | 5.89 |
| Point 16 | 2.80    | 5.69     | 3.30     | 6.07     | 5.04     | 6.06     | 6.34 | 6.02 | 8.64 | 5.87 | 9.68 | 5.94 |
| Point 17 | 2.43    | 8.45     | 3.77     | 8.65     | 5.47     | 7.76     | 6.41 | 8.37 | 8.32 | 8.45 | 9.89 | 8.94 |
| Point 18 | 2.09    | 8.65     | 3.35     | 8.51     | 4.70     | 7.76     | 5.97 | 8.00 | 8.06 | 8.32 | 8.94 | 8.85 |
| Point 19 | 1.80    | 8.00     | 3.27     | 8.05     | 4.62     | 7.13     | 6.21 | 7.90 | 8.26 | 7.88 | 9.05 | 8.19 |
| Point 20 | 2.43    | 8.08     | 3.82     | 8.08     | 5.55     | 7.54     | 6.48 | 7.46 | 8.61 | 8.01 | 9.69 | 8.45 |
| Point 21 | 2.22    | 9.56     | 3.30     | 9.49     | 5.27     | 9.63     | 6.72 | 9.64 | 8.09 | 9.59 | 10.15 | 9.52 |
| Point 22 | 1.91    | 9.31     | 2.69     | 9.90     | 5.06     | 9.46     | 6.23 | 9.86 | 7.94 | 9.49 | 9.83 | 9.44 |
| Point 23 | 1.65    | 8.97     | 2.65     | 9.36     | 4.67     | 9.04     | 6.04 | 9.07 | 7.75 | 9.06 | 9.85 | 9.00 |
| Point 24 | 2.57    | 8.89     | 3.35     | 9.44     | 5.47     | 9.32     | 6.57 | 9.17 | 8.49 | 9.33 | 10.09 | 8.74 |

2.3 Propagation path model

After the strengthening basis is determined, the midpoint of the line connecting the two adjacent quadrilateral endpoints is taken as the passable point based on the position information of the random quadrilaterals. The line connecting these passable points is all possible paths for crack growth. The coordinates of the passable points can be calculated from the data in matrix \( A \), and the coordinates of each passable point are stored in matrix \( B \).

After determining the starting point and ending point of the crack, the shortest crack is selected by using the Dijkstra algorithm. Considering the ideal situation of the modeling, the stress intensity factors at the crack tip are not enough to break through the reinforcing base, so the path between the passing points must bypass the quadrilateral region. If the line connecting the passable points is called possible path,
that is, the possible path does not intersect any random quadrilateral edge. The rapid exclusion test and straddle experiment can be used to select the connecting lines. The principle is as follows:

Take a passable path and one side of the quadrilateral as an example, the line segment $P_1P_2$ is a passable path, and $Q_1Q_2$ is an edge of a quadrilateral. Suppose a rectangle with $P_1P_2$ as the diagonal, $Q_1Q_2$ is a rectangle made diagonally. When two rectangles do not intersect, the two line segments will not intersect. If the two rectangles do not intersect, they will not pass the fast rejection experiment. If they fail to pass the fast rejection experiment, the two line segments will inevitably disjoin. If rapid rejection experiment is passed, a straddle experiment is performed.

The principle of straddle experiment is that if a line segment $P_1P_2$ intersects with a line segment $Q_1Q_2$, then $P_1P_2$ is distributed at both ends of $Q_1Q_2$. To summarize, the following condition should be satisfied:

$$((P_1 - Q_1) \times (Q_2 - Q_1)) \times ((Q_2 - Q_1) \times (P_1 - Q_1)) > 0$$

(2)

where "×" is the symbol of vector products, and "\times" is the symbol of quantity products.

The logical relationship is as follows:
The connection relationship between the passable points is stored in the matrix $C$, which can be expressed as 1 and the unconnected is recorded as 0 for subsequent program calls, the connected matrix partial value are shown in table 2. The Dijkstra algorithm will traverse all the paths between the set start and end points and find the shortest path through backtracking. The program result is shown in the figure 4.

Table 2 Connected matrix partial value

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  |
| 3 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0  | 0  | 0  | 0  |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  |
| 5 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 0  | 0  | 0  |
| 6 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0  | 0  | 0  | 0  |
| 7 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0  | 0  | 0  | 0  |
| 8 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0  | 0  | 0  | 0  |
| 9 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0  | 0  | 0  | 0  |
| 10| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 1  | 1  | 1  |
| 11| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1  | 0  | 0  | 1  |
| 12| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1  | 0  | 0  | 1  |
| 13| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1  | 1  | 1  | 0  |

Fig .4 Shortest path of crack propagation in composites

As the Fig .4 shows that because of the crack needs to “bypass” the reinforcing
base, the crack will deflect without changing the overall direction, resulting in an increase in the total length of the crack, and this increase corresponds to the fatigue life of the composite material and the same metal. The path planning model can select the shortest path to avoid obstacles.

3 Fatigue life model

3.1 Residual stress model

The preparation temperature of the SiC / AL composite material is between 680 and 780 degrees, and the material is cooled to normal temperature after the preparation is completed. It will cause the material volume to change. Since the composite material is made of a mixture of metal and non-metal, the two materials have different expansion coefficients based on temperature, and residual stress will be generated during the cooling process. As shown in Figure 5, during the working process of the test piece, the residual stress will inevitably affect the stress ratio of the test piece.

![Fig. 5 The change of stress ratio related to residual stress](image)

The residual stress calculation equation is as follows [32]:

$$\sigma_n = \Delta \alpha \Delta t \frac{K_m K_i}{K_m + K_i}$$  \hspace{0.5cm} (3)

Among them, $\Delta \alpha$ is the difference between the expansion coefficients of the two materials, $\Delta t$ is the difference between the preparation temperature and loading temperature, which fluctuates around 650. $K$ is the elastic modulus, the subscripts $m$
and $i$ are the matrix and the reinforcement, respectively.

The existence of residual stress will affect the stress ratio of the load on the test piece. The stress ratio after considering the residual stress can be shown as follows:

$$R = \frac{\sigma_{\min} + \sigma_i}{\sigma_{\max} + \sigma_i}$$  \hspace{1cm} (4)

### 3.2 Initial crack model

The initial size of crack initiation should be determined first which is related to crack initiation life calculation. The equation for calculating crack initiation life and size can be described as [33]:

$$N_i = \frac{2}{\varepsilon_f} \left[ \frac{\Delta \varepsilon_{egv}}{E K} \right]^{\frac{1}{1+n}} - \frac{\Delta \varepsilon_c}{\varepsilon_f} \right]^{-2}$$

$$= \frac{1}{4} \left[ \frac{1}{E K \varepsilon_f^{1+n}} \right]^{\frac{1}{1+n}} \left[ \frac{\Delta \varepsilon_{egv}}{2 \varepsilon_f} \left( E \varepsilon_f \right)^{1+n} \right]^{-2}$$

$$= \frac{1}{4} \left[ \frac{1}{E \sigma_f \varepsilon_f} \right]^{\frac{2}{1+n}} \left[ \Delta \varepsilon_{egv} \Delta \varepsilon_c \left( E \varepsilon_f \right)^{1+n} \right]^{-2}$$

where the fracture fatigue strength $\sigma_f$ and the fracture fatigue strain $\varepsilon_f$ are related to the reduction of area $\psi$, $\Delta \sigma_{egv}$ is equivalent stress, $\Delta \varepsilon_c$ is the threshold of strain range, they can be calculate as follows respectively:

$$\Delta \sigma_{egv} = \sqrt{\frac{1}{2(1-R)} K_c \Delta \sigma}$$ \hspace{1cm} (6)

$$\Delta \varepsilon_c = \frac{2 \tau_{-1}}{E} \frac{\varepsilon_f}{10^{15}}$$ \hspace{1cm} (7)

$$\varepsilon_f = - \ln(1-\psi)$$ \hspace{1cm} (8)

$$\sigma_f = \left( 1 + \ln \left( \frac{1}{1-\psi} \right) \right) \delta_b$$ \hspace{1cm} (9)
among the equation 6, $K_r$ is the stress concentration factor, taking $K_r = 1$ in this paper.

The initial crack size can be given by:

$$a_0 = \frac{(K_{ic} / \sigma_b)^2}{\pi} \quad (10)$$

where

$$K_{ic} = 0.032E\sqrt{\pi n} \quad (11)$$

3.3 Crack growth model

According to fracture mechanics theory, when a component is subjected to cyclic loading, fatigue cracks propagate until fatigue failure occurs in the component. The relationship between crack length and crack propagation life can be constructed according to the Paris formula [34]:

$$\frac{d a}{d N_f} = C(\Delta K)^m \quad (12)$$

where $a$ is the crack length, $n$ is the number of stress cycles, $C$ and $m$ are material coefficients.

$$\Delta K = K_{max} - K_{min} = \frac{F}{\pi a}(\delta_{max} - \delta_{min}) \quad (13)$$

In order to make the crack propagation life prediction more accurate, the researchers made other improvements to the Paris formula, such as considering the stress ratio [35]:

$$\frac{d a}{d N_f} = \frac{C(\Delta K)^m}{(1 - r)K_e - \Delta K} \quad (14)$$

Integrating from the formula gives:

$$N_f = \int_{a_0}^{a_e} \frac{\left[(1 - r)K_e - \Delta K\right]d a}{C(F\Delta \sigma \sqrt{\pi a})^m} \quad (15)$$

It can be rewritten as:
\[ N_f = \frac{(1-r)K_c}{C(\Delta\sigma\sqrt{\pi})^m\left(1 - \frac{m}{2}\right)} \left( a_{c0}^{\frac{1}{m}} - a_0^{\frac{1}{m}} \right) \]
\[ \quad - \frac{2}{C(\Delta\sigma\sqrt{\pi})^{m-1}\left(3 - m\right)} \left( a_{c0}^{\frac{3-m}{2}} - a_0^{\frac{3-m}{2}} \right) \]

(16)

where \(a_c\) is the crack length when the material breaks, \(a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{F\sigma} \right)^2\), \(K_{IC}\) is the fracture toughness, \(K_{IC} = 0.032E\sqrt{\pi n}\), \(n\) is the hardening coefficient, \(F\) is the shape factor.

The total life of the material is the sum of the initiation life and the extended life:
\[ N = N_i + N_f \]
(17)

When the crack path bends, the path direction deviates from the ideal crack direction, so the corresponding relationship between the crack length and the fatigue growth life also changes accordingly. Consider the following equation to modify it:
\[ \frac{d a_i}{d n} = a_i \left( \frac{d a_1}{d n} \right) + a_2 \left( \frac{d a_2}{d n} \right) + K a_i \left( \frac{d a_i}{d n} \right) \]
\[ = \sum a_i \left( \frac{d a_i}{d n} \right) \]
\[ \alpha = \frac{d \cos \theta + s}{d + s} \]
(18)
(19)

Among them, \(\theta\) is the deflection angle, \(d\) is the distance to extend in the deflection direction, and \(s\) is the extension distance in the direction in which the crack does not deflect. As the Fig. 6 shows:

![Fig. 6 Crack path deflection diagram](image)

Each possible path will get a deflection coefficient \(\alpha\), bring all the deflection coefficient into equation (19), the final crack propagation rate can be obtained.

4 Simulation Analyses
This paper proposes a life prediction model for metal matrix particle reinforced composites. In order to verify the accuracy of the model, the test data in the literature [36] is used for verification.

| Material | Si (mass: %) | Mg (mass: %) | Cu (mass: %) | Mg (mass: %) | Mn (mass: %) | Al (mass: %) |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|
| Al alloy | 20          | 0.3         | 0.116       | 0.3         | 0.116       | remain      |

Table 4 Material properties of SiC/AL

| Material | $E$ (GPa) | $\delta_{0.2}$ (MPa) | $\delta_{0}$ (MPa) | $\psi$ (%) | $P$ (C$^{-1}$) |
|----------|----------|---------------------|---------------------|-----------|----------------|
| Al-20Si  | 89.1     | 361                 | 309                 | 10        | 24.2           |
| SiC      | 450      | -                   | -                   | -         | 3.4            |

In the reference [36], by preparing a new type of aluminum alloy, and using this aluminum alloy as a matrix which reinforced by SiC particles, a SiC/AL composite material was prepared. Table 2 shows the basic properties of the metal matrix. Material parameters $C$ and $m$ are equal to $7.8 \times 10^{-4}$ and 0.09 respectively.

![Fig. 7 Comparison of experience data and fitting curve in AL alloy](image)

The S-N curve of AL-20Si metal is fitted by equation 18 as shown in the figure 7.
Material parameters $C$ is equal to $7.8 \times 10^{-5}$. It can be seen that most of the test value is close to the fitted curve. When the load is 140 MPa, there are some error between the test result and curve. Excluding experimental error factors, the possible cause is that the load does not reach the fatigue expansion threshold, resulting in the actual life being longer than the estimated life.

Figure 8 shows the relationship between the calculated life curve of the composite material in this paper and the test results. The curve can predict the life trend of experimental materials well. The tensile strength, elongation and other parameters of the metal matrix will change to varying degrees after adding the reinforcing base. Different from the regular fatigue life prediction method, crack deflection and residual stress are considered, and the change in material properties caused by the strengthening base is ignored. It can be seen from Figure 8, when the specimen is in a low stress environment, the new model data are not consistent with the experimental data very well, it can still prove that the idea of establishing the model is effective to a certain extent.
the error between the predicted data and the test data is the smallest. When the stress level is higher than 160MPa, the estimated life is higher than the test life, and the error increases with the increase of stress. Under high stress levels, cracks will more easily penetrate the reinforced base without deflection, and this process cannot be reflected by the life prediction model, so the test data will be larger than the test data. When the stress level is lower than 160MPa, the predicted data is smaller than the test data. The presence of the reinforcement hinders the slip of the metal crystals, thereby greatly increasing the fatigue initiation life under small loads.

5 Conclusions

Fatigue propagation characteristic of composite materials is studied in this paper, and a new model of the crack propagation path of composite materials is established according to the path planning algorithm. The differences between crack propagation length of composite materials and metal materials are estimated. The modified Paris model is used to estimate the fatigue life, and some conclusions are drawn as follows.

(1) The crack trajectory simulated by the path planning model is closer to the microscopic crack trajectory, and the fatigue life is more accurately calculated using the estimated crack length. During regular fatigue failure of composite materials, the crack path will expand around the reinforcement base. The Dijkstra algorithm can effectively avoid the simulation of cracks passing through the strengthening base, and the local path optimization of the Dijkstra algorithm can simulate the characteristics of crack "steps". This model can be extended to other similarly-structured composite materials to estimate crack growth length.

(2) Considering the residual stress cause by temperature changing, the fatigue crack propagation model is improved. The residual thermal stress in the material is fully considered in the improved fatigue crack growth model.

(3) Based on the propagation of microscopic cracks, a new idea for the life prediction of composite materials is presented. Combined with the crack length estimated by the path planning model, the fatigue crack propagation life is calculated. The experimental data show that although the new model still has some defects, it still has potential research value.
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Conflict of Interest

The authors declare that they have no conflict of interest.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author contributions

Conceptualization and methodology were performed by Liu Xintian and Shang Wenqian, data curation was performed by Shang Wenqian, supervision was performed by Liu Xintian and Wang Xiaolan, reviewing and editing were performed by Wang Xu. The first draft of the manuscript was written by Shang Wenqian and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.
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