Right-handed Electrons in Radiative Muon Decay

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Abstract

Electrons emitted in the radiative decay $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$ have a significant probability of being right-handed, even in the limit $m_e \rightarrow 0$. Such “wrong-helicity” electrons, arising from helicity-flip bremsstrahlung, contribute an amount $\alpha^4/4\pi \Gamma_0$ to the muon decay width ($\Gamma_0 \equiv G_F^2 m_\mu^5/(192\pi^3)$). We use the helicity-flip splitting function $D_{hf}(z)$ of Falk and Sehgal (Phys. Lett. B 325, 509 (1994)) to obtain the spectrum of the right-handed electrons and the photons that accompany them. For a minimum photon energy $E_\gamma = 10 \text{MeV}$ ($20 \text{MeV}$), approximately 4% (7%) of electrons in radiative $\mu$-decay are right-handed.

It is usually thought that in V-A theory, electrons emitted in muon decay are purely left-handed, in the limit $m_e \rightarrow 0$. This statement, however, is not true for electrons in the radiative decay $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$, where the photon is the result of inner bremsstrahlung. We show in this Letter that radiative muon decay contains a well-defined constituency of right-handed electrons, contributing an amount $\alpha^4/4\pi \Gamma_0$ ($\Gamma_0 \equiv G_F^2 m_\mu^5/(192\pi^3)$) to the decay width. We calculate the spectrum of these “wrong-helicity” electrons, and of the photons that accompany them. These spectra are compared with the unpolarized spectra, summed over electron helicities. This comparison provides a quantitative measure of the right-handed fraction and its distribution in phase space.

The appearance of “wrong-helicity” electrons in the decay $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$, even in the limit $m_e \rightarrow 0$, is a consequence of helicity-flip bremsstrahlung in quantum electrodynamics, a feature first noted by Lee and Nauenberg [1]. It was found that in the radiative scattering of electrons by a Coulomb field, the probability of helicity-flip (i.e. $e_L \rightarrow e_R$ or $e_R \rightarrow e_L$) did not vanish in the limit $m_e \rightarrow 0$. This unexpected result, in apparent contradiction to the naive expectation of helicity conservation in the $m_e \rightarrow 0$ limit, arises from the fact, that the helicity-flip cross section for bremsstrahlung at small angles has the
form

\[
\frac{d\sigma}{d\theta^2} \sim \left( \frac{m_e}{E_e} \right)^2 \frac{4}{\left( \frac{m_e}{E_e} \right)^2 + \theta^2},
\]

(1)

which, when integrated over angles, gives a finite non-zero answer in the limit \( m_e \to 0 \).

In Ref. [2], Falk and Sehgal examined the helicity structure of the bremsstrahlung process in an equivalent particle approach, and showed that helicity-flip radiation \( e_L^- \to e_R^- + \gamma(z) \), in the limit \( m_e \to 0 \), can be described by a simple and universal splitting (or fragmentation) function

\[
D_{hf}(z) = \frac{\alpha}{2\pi} z,
\]

(2)

where \( z = E_\gamma/E_e \) is the ratio of the photon energy to the energy of the radiating electron. This function is analogous to the familiar Weizsäcker-Williams function describing helicity-conserving (non-flip) bremsstrahlung

\[
D_{nf}(z) = \frac{\alpha}{\pi} \frac{1 + (1 - z)^2}{z} \log \left( \frac{E_e}{m_e} \right).
\]

(3)

Several applications of the helicity-flip function \( D_{hf}(z) \) were considered in [2], including the process \( e_L^- + p \to \nu_L + \gamma + X \) (“fake right-handed currents”) and \( e^- e^+ \to f f' \gamma \) (wrong-helicity \( e^+ e^- \) annihilation). It was shown that the splitting function approach reproduced the results of the usual bremsstrahlung calculation in which the limit \( m_e \to 0 \) was taken at the end [3,4]. Subsequently, the equivalent-particle technique has been successfully applied to other helicity-flip processes such as \( \pi^- \to e_L^- \bar{\nu}_e \gamma \) and \( Z^0 \to e_L^- e_L^+ \gamma \) [5].

Recently, in an analysis of radiative corrections to the electron spectrum in muon decay, \( \mu^- \to e^- \bar{\nu}_e \nu_\mu \), Fischer et al. [6] have noted that the radiative correction to the helicity of the electron, calculated in an early paper by Fischer and Scheck [7], can be reproduced in a simple way using the helicity-flip function \( D_{hf}(z) \). This has motivated us to examine the helicity-dependence of the radiative decay \( \mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma \), to determine the incidence and spectrum of wrong-helicity (right-handed) electrons in this channel.

The electron spectrum in ordinary (non-radiative) muon decay \( \mu^- \to e^- \bar{\nu}_e \nu_\mu \) has, in Born approximation, the well-known form

\[
\left( \frac{d\Gamma}{dx d\cos \theta_e} \right)_{\text{non-rad}} = \Gamma_0 \left[ x^2(3 - 2x) + x^2(1 - 2x) \cos(\theta_e) \right],
\]

(4)
where \( x = 2E_e/m_\mu \) and \( \theta_e \) is the angle of the electron relative to the spin of the muon. We can obtain from this the spectrum of the radiative channel \( \mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma \), using the splitting functions \( D_{hf} \) or \( D_{nf} \). In the specific case of right-handed electrons in the final state, the spectrum, in the limit \( m_e \to 0 \) (collinear bremsstrahlung), is given by

\[
\left( \frac{d\Gamma}{dx_e d\cos \theta_e} \right)_{e_R}^{\text{rad}} = \\
\int_0^1 dx \int_0^1 dz \left( \frac{d\Gamma}{dx d\cos \theta_e} \right)_{\text{non-rad}}^{\text{rad}} D_{hf}(z) \delta(x_e - x(1 - z)) \theta(xz - x_{\gamma 0}),
\]

(5)

where the \( \theta \)-function in the integrand has been inserted to allow for a minimum energy cut on the photon:

\[ x_{\gamma} \equiv \frac{2E_\gamma}{m_\mu} \geq x_{\gamma 0}. \]

(6)

The result of the integration is

\[
\left( \frac{d\Gamma}{dx_e d\cos \theta_e} \right)_{e_R}^{\text{rad}} = \Gamma_0 \frac{\alpha}{2\pi} \left[ A(x_e, x_{\gamma 0}) + \cos(\theta_e)B(x_e, x_{\gamma 0}) \right],
\]

(7)

where

\[
A(x_e, x_{\gamma 0}) = -\frac{2}{3}[1 - (x_e + x_{\gamma 0})^3] + \frac{1}{2}[1 - (x_e + x_{\gamma 0})^2](2x_e + 3) - 3x_e[1 - (x_e + x_{\gamma 0})],
\]

\[
B(x_e, x_{\gamma 0}) = -\frac{2}{3}[1 - (x_e + x_{\gamma 0})^3] + \frac{1}{2}(1 + 2x_e)[1 - (x_e + x_{\gamma 0})^2] - x_e[1 - (x_e + x_{\gamma 0})].
\]

(8)

Integrating over \( \cos \theta_e \) and \( x_e \) (0 \( \leq \) \( x_e \) \( \leq \) 1 - \( x_{\gamma 0} \)), we obtain

\[
\Gamma_{e_R}^{\text{rad}}(x_{\gamma 0}) = \Gamma_0 \frac{\alpha}{\pi} \left[ \frac{1}{4} - x_{\gamma 0}^2 + x_{\gamma 0}^3 - \frac{1}{4} x_{\gamma 0}^4 \right].
\]

(9)

If no cut is imposed on the photon energy (i.e. \( x_{\gamma 0} = 0 \)) the spectrum of right-handed electrons given in Eq.(7) reduces to

\[
\left( \frac{d\Gamma}{dx_e d\cos \theta_e} \right)_{e_R}^{\text{rad}} = \Gamma_0 \frac{\alpha}{2\pi} \frac{1}{6}(1 - x_e)^2[(5 - 2x_e) - \cos(\theta_e)(2x_e + 1)],
\]

(10)
which coincides with the result obtained by Fischer and Scheck [7].

The helicity-flip fragmentation function also gives a simple way of calculating the spectrum of photons accompanying right-handed electrons in $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$. In the collinear limit ($m_e \rightarrow 0$), we have

$$
\left( \frac{d\Gamma_{\nu_R}}{dx, d\cos \theta_\gamma} \right)_{\nu_R} = \int_0^1 dx \int_0^1 dz \left( \frac{d\Gamma_{\nu_{\bar{R}}}}{dx, d\cos \theta_\gamma} \right)_{\nu_{\bar{R}}=\theta_\gamma} D_{hf}(z) \delta(x_\gamma - xz) \\
= \Gamma_0 \frac{\alpha}{2\pi} x_\gamma (1 - x_\gamma) [(2 - x_\gamma) - x_\gamma \cos(\theta_\gamma)].
$$

(11)

Integrating over all photon energies and over $\cos \theta_\gamma$ we get $\Gamma_{e^- R} = \frac{\alpha}{4\pi} \Gamma_0$, which is the same as Eq.(9) for $x_\gamma \rightarrow 0$.

The decay width into right-handed electrons, for a given minimum energy $x_{\gamma 0}$ (Eq.(9)), can be compared with the width summed over electron helicities. The helicity-summed photon spectrum in $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$ was calculated by Kinoshita and Sirlin [8] and Eckstein and Pratt [9], and the integrated width, for $x_\gamma > x_{\gamma 0}$ is [9]

$$
\Gamma_{e^- R}^{rad}(x_{\gamma 0}) = \Gamma_0 \frac{2\alpha}{\pi} \left\{ \left( -\frac{17}{12} + \log(\frac{m_\mu}{m_e}) \right) \log(\frac{1}{x_{\gamma 0}}) \\
- \frac{1}{2} \left( 1 - x_{\gamma 0} \right) \left[ \frac{1}{6} (1 - x_{\gamma 0})^3 + 1 \right] \log \left[ \frac{m_\mu^2}{m_e^2} (1 - x_{\gamma 0}) \right] \\
+ \frac{1}{288} (601 - 159 x_{\gamma 0} + 171 x_{\gamma 0}^2 - 61 x_{\gamma 0}^3) \\
- \frac{\pi^2}{12} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(x_{\gamma 0})^n}{n^2} \right\}.
$$

(12)

This function is plotted in Fig.(1) and compared with the right-handed width $\Gamma_{e^- R}(x_{\gamma 0})$ calculated in Eq.(9). The right-handed fraction $\Gamma_{e^- R}^{rad}/\Gamma_{e^- R+e^- L}^{rad}$ is shown in Fig.(2), as a function of $x_{\gamma 0}$. For a photon energy cut $E_\gamma > 10\, MeV$ (20 $MeV$), this fraction is approximately 4% (7%). [It may be noted here that the branching ratio of $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$, summed over electron spins, with a photon energy cut $E_\gamma > 10\, MeV$, was measured in Ref. [10] to be (1.4 ± 0.4)%]. The theoretical expression Eq.(12) yields for this quantity the value 1.3%.

A complete analysis of the channel $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$ involves a study of the decay intensity in all kinematical variables. A variable of particular interest is the angle $\theta_{e\gamma}$ between the electron and the photon. For helicity-flip radiation,
the characteristic angular distribution is [2]

$$\frac{dD_{h}(z, \theta^2)}{d\theta^2} \approx \frac{\alpha}{2\pi} \frac{1}{z} \left( \frac{m_e}{E_e} \right)^2 \frac{\left( \theta^2 + \left( \frac{m_e}{E_e} \right)^2 \right)^2}{\left( \theta^2 + \left( \frac{m_e}{E_e} \right)^2 \right)^2},$$

(13)

which is maximum at $\theta = 0$ (forward direction). By contrast, the helicity-conserving bremsstrahlung has the spectrum [2]

$$\frac{dD_{n}(z, \theta^2)}{d\theta^2} \approx \frac{\alpha}{2\pi} \frac{1}{z} \frac{(1 - z)^2}{\left( \theta^2 + \left( \frac{m_e}{E_e} \right)^2 \right)^2},$$

(14)

which peaks at $\theta \approx m_e^2/E_e^2$. This suggests that in the decay $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu\gamma$, the distribution in the angle $\theta_{e\gamma}$ between the electron and photon could be a useful discriminant in separating the two electron helicities. A full analysis of the helicity-dependent decay spectrum in different kinematical variables will be reported elsewhere.

Summary:

(i) The decay $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu\gamma$ contains in the final state a constituency of right-handed electrons, which contribute an amount $\frac{\alpha}{4\pi} \Gamma_0$ to the decay width, in the limit $m_e \rightarrow 0$.

(ii) The spectrum of the right-handed electrons is given by Eq.(7), and reduces to Eq.(10) if no cut on photon energy is imposed. The latter differs in a characteristic way from the spectrum of left-handed electrons, which (on account of the soft $1/x_\gamma$ nature of helicity-conserving bremsstrahlung) tends to follow the non-radiative pattern Eq.(4). Thus the energy spectra, integrated over angles are $(d\Gamma/dx_e)_R \sim (1 - x_e)^2(5 - 2x_e)$, $(d\Gamma/dx_e)_L \sim x_e^2(3 - 2x_e)$, while the angular distribution, integrated over energies, is $(d\Gamma/d\cos\theta_e)_L,R \sim (1 - \frac{1}{3}\cos\theta_e)$, the same for $e_e^-L$ and $e_e^-R$.

(iii) The photon spectrum associated with right-handed electrons is $(d\Gamma/dx_\gamma)_R \sim x_\gamma (1 - x_\gamma)(2 - x_\gamma)$, and is hard compared to that accompanying left-handed electrons $(d\Gamma/dx_\gamma)_L \sim 1/x_\gamma$.

(iv) The right-handed fraction $\Gamma_{e_e^-R}/(\Gamma_{e_e^-R} + \Gamma_{e_e^-L})$ has been calculated as a function of the photon energy cut $x_{\gamma0}$, and amounts to 4% (7%) for $E_\gamma > 10 \text{MeV}$ (20 MeV).

(v) The radiatively corrected decay width of the muon, usually written as

$$\Gamma_\mu = \Gamma_0 \left[ 1 + \frac{\alpha}{\pi} \left( \frac{25}{8} - \frac{\pi^2}{2} \right) \right]$$

(15)

can be regarded as a sum of two mutually exclusive helicity contributions [6]
\[ \Gamma_\mu = \Gamma_\mu(e^-_L) + \Gamma_\mu(e^-_R) \]

where

\[ \Gamma_\mu(e^-_L) = \Gamma_0 \left[ 1 + \frac{\alpha}{\pi} \left( \frac{23}{8} - \frac{\pi^2}{2} \right) \right] \]

\[ \Gamma_\mu(e^-_R) = \Gamma_0 \frac{1}{\pi} \frac{\alpha}{4}. \]

(vi) A full analysis of \( \mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma \), aimed at finding regions of phase space with enhanced concentration of right-handed electrons will be reported elsewhere.

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Fig. 1. Radiative decay width $\Gamma_{eL+eR}^{\text{rad}}(x_{\gamma 0})$ (full line), compared with the right-handed decay width $\Gamma_{eR}^{\text{rad}}(x_{\gamma 0})$ (multiplied by factor 10, dashed line) as function of minimum photon energy $x_{\gamma 0}$. Decay widths in units of $\Gamma_0$. 

$R \times 10$

$L + R$
Fig. 2. Right-handed fraction \( \frac{\Gamma_{rad}^R}{\Gamma_{rad}^R + \Gamma_{rad}^L} \) as function of minimum photon energy \( x_{\gamma 0} \).