Spin Accumulation without the Spin Hall Effect

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The spin Hall (SH) effect is a phenomenon in which the spin current flows perpendicular to an applied electric field and causes the spin accumulation at the boundaries. However, in the presence of spin-orbit couplings, the spin current is not well defined. Here, we calculate the spin response to an electric field gradient, which naturally appears at the boundaries. We derive a generic formula using the Bloch wave functions and the phenomenological relaxation time. We also calculate the response for the Rashba model with δ-function nonmagnetic disorder within the first-order Born approximation and corresponding vertex corrections. We find the nonzero spin accumulation, although the SH conductivity exactly vanishes. This result is a counterexample to the above scenario of the SH effect, where the spin current plays a major role.

Introduction. Spintronics is an active research field in condensed matter physics to make use of the spin degree of freedom of electrons. Key steps are creation, transportation, and detection of spins, and hence the spin current has been believed to play an important role. Such a current can be generated via spin-orbit (SO) couplings perpendicular to an applied electric field. This phenomenon, proposed by D’yakonov and Perel’ [1] and later by Hirsch [2], is called the spin Hall (SH) effect [3]. It has attracted renewed interest since the theoretical proposals [4, 5] and experimental observations in semiconductors [6, 7].

Experimentally, the spin current cannot be directly observed. What is observed using the inverse SH effect is the charge current [8]. In Refs. [6, 7], the spin accumulation at the boundary was detected optically and attributed to the SH effect: The spin current is generated via the SH effect and then turns into spin at the boundaries via the spin relaxation, as depicted in Fig. 1(a). Hence, spin, rather than the spin current, is the primary physical object in order to describe these experimental results.

In the presence of SO couplings, spin is not conserved, and the spin current is not well defined. When a spin current density \( J_{ax}^s(t, x) \) is given, there exists the corresponding spin torque density \( \tau_a(t, x) \), and the spin continuity equation is expressed as \( \partial_t \hat{s}_a(t, x) + \partial_x \cdot \hat{J}_{ax}^s(t, x) = \tau_a(t, x) \). Widely used is the conventional definition, \( \hat{J}_{ax}^s(k) = \{ \hat{s}_a, \hat{v}^i(k) \} / 2 \), where \( \hat{s}_a \) and \( \hat{v}^i(k) \) are the spin and velocity operators, respectively. However, this definition is unphysical in the sense that its equilibrium expectation value is nonzero in noncentrosymmetric systems such as the Rashba and Dresselhaus models [9]. Another definition is the so-called conserved spin current [10, 11]. If the spin torque vanishes in average over the whole system, we can define the spin torque dipole density as \( \tau_a(t, x) = -\partial_x \cdot \hat{P}_{ra}^i(t, x) \), and \( \hat{J}_{ax}^s(t, x) = \hat{J}_{ax}^s(t, x) + \hat{P}_{ra}^i(t, x) \) is conserved in average. This definition has interesting properties such as the Streda formula between the SH conductivity and the SO magnetic susceptibility [12] and the Mott relation between the SH and spin Nernst conductivities [13, 14]. Whatever definition we choose, however, we need to consider the corresponding spin torque density to evaluate the observable spin density. So far, the spin torque density has been taken into account only by hand or classically with the help of the spin diffusion equation, and theoretical understanding of the spin density is still lacking.

In the case of the Rashba model that describes n-type semiconductors, the SH conductivity of the conventional spin current exactly vanishes when the vertex corrections are taken into account [15–21]. This cancellation is owing to the special property that the conventional spin current operator is proportional to the time derivative of the spin operator [18–20]. The SH conductivity of the conserved spin current also vanishes [22]. Following the traditional

FIG. 1. (a) Traditional scenario: The spin current is generated by a uniform electric field via the SH effect and then turns into spin at the boundaries via the spin relaxation. In the Rashba model, the SH conductivity vanishes, and no spin accumulation is expected. (b) Our scenario: Spin is induced by the electric field gradient at the boundaries. The spin accumulation may occur even when the SH conductivity vanishes. Our theory is free from the ambiguity regarding the definition of the spin current and spin torque.
scenario in Fig. 1(a), the spin accumulation would be zero, but in fact observed experimentally [7]. This inconsistency between theories and experiments clearly indicates insufficiency of theories focusing on the SH conductivity only.

In contrast to the spin current, spin is well defined. Recently, one of the authors considered the spin response to an electric field gradient [23]. Such a gradient naturally appears at the boundaries. Hence, this theory directly explains the spin accumulation, as depicted in Fig. 1(b). The theory also explains generation of spin current using the SH effect or the spin pumping and detection using the inverse SH effect in terms of the nonlocal spin fluctuation.

In this Letter, we study the spin response to the electric field gradient with the quantum-mechanical linear response theory. First, we derive a generic formula expressed by the Bloch wave functions. Although disorder effects are taken into account via a phenomenological relaxation time, the formula can be applied to any Hamiltonian as far as the system is clean and noninteracting. Second, we calculate the spin response with the Green’s functions. We consider the Rashba model with δ-function nonmagnetic disorder within the first Born approximation and corresponding vertex corrections, which results in the vanishing SH conductivity [15–21]. Nonetheless, we find the nonzero spin accumulation, which is consistent with the experimental result [7]. This result is a counterexample to the traditional scenario of the SH effect.

Bloch formulas. First, we calculate the spin–charge-current correlation function that characterizes \( \langle \Delta \hat{s}_a \rangle(\omega, q) = \chi^{\text{R}}_{\hat{s}_a, j}(\omega, q)A_j(\omega, q) \). We assume that the Hamiltonian \( \hat{H}(k) \) is clean and noninteracting. Using the Bloch wave functions \( |u_n(k)\rangle \), the correlation function is expressed as

\[
\chi^{\text{SO}}_{\hat{s}_a, j}(\omega, q) = -\frac{q}{\hbar} \sum_n \int \frac{d^d k}{(2\pi)^d} \left[ \langle u_n(k - \frac{q}{2})|\hat{s}_a|u_m(k + \frac{q}{2}) \rangle \right. \\
\times \langle u_m(k + \frac{q}{2})|\hat{v}^j(k; q)|u_n(k - \frac{q}{2}) \rangle \right. \\
\left. \times \frac{f(\epsilon_n(k - \frac{q}{2}) - f(\epsilon_m(k + \frac{q}{2}))}{\hbar \omega + \epsilon_n(k - \frac{q}{2}) - \epsilon_m(k + \frac{q}{2}) + i\eta} \right] \\
= \chi_{\hat{s}_a, j}(0, q) + (i\omega)e^{\hat{\mathbf{R}}} \chi_{\hat{s}_a, j}(\omega, q),
\]

where \( q \) is the electron charge, \( d \) is the spatial dimension, \( \eta \to 0 \) is the convergence factor, \( \hat{v}^j(k; q) = [\hat{v}^j(k + q/2) + \hat{v}^j(k - q/2)]/2 \), and \( f(\epsilon) = [e^{(\epsilon - \mu)/T} + 1]^{-1} \) is the Fermi distribution function. We expand Eq. (1) up to the first order with respect to \( q \) with keeping \( \omega \) nonzero. The first term of Eq. (1) takes the form of \( \chi_{\hat{s}_a, j}(0, q) = e^{ij\hat{\mathbf{R}}(\eta_0)}\chi_{\hat{a}k} \), and we reproduce the SO magnetic susceptibility [24, 25],

\[
\chi^{\text{SO}}_{\hat{a}k} = -\frac{q}{\hbar} \sum_n \int \frac{d^d k}{(2\pi)^d} \left[ \langle \epsilon_{ij k} s_{na} \partial_k \epsilon_n + s_{na} m_{nk} \rangle f'(\epsilon_n) \\
+ b_{nak} f(\epsilon_n) \right].
\]

The argument of \( \mathbf{k} \) is omitted for simplicity. We have introduced \( s_{na} = \langle u_n|\hat{\mathbf{R}}|u_n \rangle \), the magnetic moment \( m_{nk} [26–28] \), spin magnetic quadrupole moment \( s_{na} \varepsilon [29, 30] \), and spin Berry curvature \( b_{nak} \) as

\[
e^{ij k} m_{nk} = 3\{\langle \partial_k u_n (\epsilon_n - \hat{\mathbf{R}}) \partial_k u_n \rangle, \tag{3a}\]
\[
s_{na} = 3\{\langle \partial_k u_n \hat{Q}_n \partial_k u_n \rangle, \tag{3b}\]
\[
e^{ij k} b_{nak} = -3\{\langle \partial_k u_n \hat{Q}_n (s_{na} + \hat{s}_a) \hat{Q}_n \partial_k u_n \rangle + \sum_{m(\ne n)} \times \langle \partial_k u_n \hat{s}_a | u_m \rangle/\langle \partial_k \epsilon_n + \hbar \hat{v} \rangle \hat{Q}_n \partial_k u_n \rangle \}
\]

\[
\epsilon_n - \epsilon_m - (i \leftrightarrow j), \tag{3c}\]

with \( \hat{Q}_n = 1 - |u_n\rangle\langle u_n | \) being the antiprojection operator.

Equation (3c) is a spin analog of the Berry curvature, because it is reduced to the Berry curvature when \( \hat{s}_a \) is replaced by \( \hat{I} \) and totally antisymmetric with respect to \( \partial_k, \partial_j \), and \( \partial_{x} \). Here, \( \hat{B}^a \) is the Zeeman field conjugate to \( \hat{s}_a \).

The second term of Eq. (1) takes the form of [25]

\[
\alpha^{\text{R}}_{\hat{s}_a, j}(\omega, q) = \chi^{\text{R}}_{\hat{s}_a, j}(0, q) f'(\epsilon_n),
\]

\[
= \frac{i\hbar}{\hbar \omega + i\eta} \alpha^j \gamma_{ij(1)},
\]

where

\[
\alpha^j = -\frac{q}{\hbar} \sum_n \int \frac{d^d k}{(2\pi)^d} s_{na} \partial_k \epsilon_n f'(\epsilon_n), \tag{5a}\]
\[
\gamma_{ij(1)} = -\frac{q}{\hbar^2} \sum_n \int \frac{d^d k}{(2\pi)^d} s_{na} \partial_k \partial_k \epsilon_n f'(\epsilon_n), \tag{5b}\]
\[
\gamma_{ij(1)} = -\frac{q}{\hbar^2} \sum_n \int \frac{d^d k}{(2\pi)^d} \left[ s_{na} \partial_k \epsilon_n - s_{na} \partial_k \partial_k \epsilon_n \right] f'(\epsilon_n). \tag{5c}\]

Equation (5a) describes the Edelstein effect [31], while Eqs. (5b) and (5c) describe the spin accumulation induced by the electric field gradient. Note that we drop the interband Fermi sea term because it breaks the time-reversal symmetry.

Combining Eqs. (2) and (4), the spin density is induced by electromagnetic fields as

\[
\langle \Delta \hat{s}_a \rangle(\omega, q) = \frac{i\hbar}{\hbar \omega + i\eta} \alpha^j E_j(\omega, q).
\]
Taking the limit of \( \omega \to 0 \) and introducing the phenomenological relaxation time \( \hbar/\eta \), we arrive at one of our main results,

\[
\langle \Delta \hat{s}_a \rangle (0, q) = \frac{\hbar}{\eta} \alpha_a^A E_j(0, q) - \frac{\hbar^2}{\eta^2} \gamma_{ia}^{(I)} \left( \hat{\mathcal{H}}(k) + \hbar \gamma_a^{(II)} \right) \times (iq_1) E_j(0, q).
\]

(6)

Let us apply the above formulas to the Rashba model,

\[
\hat{\mathcal{H}}(k) = \frac{\hbar^2 k^2}{2m} + \hbar \alpha (k_y \sigma_x - k_x \sigma_y),
\]

(8)

where \( \sigma \) is the Pauli matrix corresponding to the spin operator \( \hat{s} = (\hbar/2)\sigma \). The eigenvalues are \( \epsilon_\sigma(k) = \hbar^2 k^2/2m + \sigma \hbar \alpha k \). At \( T = 0 \), we obtain the SO magnetic susceptibility (2) and spin accumulation (9b) and spin accumulation (5c) as [25]

\[
\chi_z^{(so)} = -\frac{q}{4\pi} \left\{ \begin{array}{ll} 0 & (\mu > 0) \\ \sqrt{1 + 2\mu / \alpha m^2} & (\mu < 0) \end{array} \right.,
\]

(9a)

\[
\gamma_z^{(xy)(II)} = -\frac{q}{8\pi} \left\{ \begin{array}{ll} 1 & (\mu > 0) \\ 0 & (\mu < 0) \end{array} \right..
\]

(9b)

Note that Eq. (9a) is consistent with the previous result [32]. The spin accumulation (9b) is nonzero when the chemical potential is above the Rashba crossing. However, it is natural to ask if Eq. (9b) survives when the vertex corrections are taken into account.

**Green’s functions.** The above results are phenomenological in the sense that \( \hbar/\eta \) is interpreted as the relaxation time. Here, we consider \( \delta \)-function nonmagnetic disorder within the first Born approximation and take into account the corresponding ladder-type vertex corrections for the Rashba model (8). The bare retarded Green’s function is expressed as

\[
g^R(\epsilon, k) = \frac{1}{\epsilon + i\eta - \hat{\mathcal{H}}(k)} = \frac{1}{2} [g_+^R(\epsilon, k) + g_-^R(\epsilon, k)] + \frac{1}{2} [g_+^R(\epsilon, k) - g_-^R(\epsilon, k)]
\]

with \( g_\sigma^R(\epsilon, k) = (\epsilon + i\eta - \epsilon_\sigma(k))^{-1} \) being the diagonalized one. The imaginary part of the self-energy is then expressed as

\[
\hat{\Gamma}(\epsilon) = -\Im \left\{ \begin{array}{ll} 0 & (\epsilon > 0) \\ \frac{1}{2} \sqrt{1 + 2\epsilon / \alpha m^2} & (\epsilon < 0) \end{array} \right..
\]

(11)

with \( \Gamma_0 = mn v_t^2/\hbar^2 \). Below, we denote \( \hat{\Gamma}(\epsilon) \) as \( \hat{\Gamma}(\epsilon) \). The renormalized retarded Green’s function is expressed as

\[
\hat{G}^R(\epsilon, k) = \frac{1}{\epsilon + i\hat{\Gamma}(\epsilon) - \hat{\mathcal{H}}(k)} = \frac{1}{2} [G_+^R(\epsilon, k) + G_-^R(\epsilon, k)]
\]

\[
+ \frac{1}{2} [G_+^R(\epsilon, k) - G_-^R(\epsilon, k)] \times (\sigma_x \sin \phi - \sigma_y \cos \phi),
\]

(12)

For a given bare vertex \( \hat{\xi}(k) \), the renormalized vertex \( \hat{\xi}(\epsilon, k) \) at \( \omega = 0 \) and \( q = 0 \) is obtained by solving

\[
\hat{\xi}(\epsilon, k) = \hat{\xi}(k) + n_i v_t^2 \int \frac{d^2k'}{(2\pi)^2} \times \hat{G}^A(\epsilon, k') \hat{\xi}(\epsilon, k') \hat{G}^R(\epsilon, k').
\]

(13)

For the bare velocity vertex \( \hat{v}^y(k) = \hbar k_y/m + \alpha \sigma_x \) and spin vertex \( \hat{s}_z = (\hbar/2)\sigma_z \), the renormalized vertices are \( \hat{v}^y(\epsilon, k) = \hbar k_y/m + \alpha V^y(\epsilon) \sigma_x \) and \( \hat{s}_z(\epsilon, k) = (\hbar/2) \hat{S}_z^z(\epsilon) \sigma_z \), respectively. In the limit of \( \Gamma_0 \to 0 \), we reproduce [15, 25]

\[
V^y(\epsilon) = \left\{ \begin{array}{ll} 0 & (\epsilon > 0) \\ -2\epsilon / \alpha m^2 & (\epsilon < 0) \end{array} \right.,
\]

(14)

and \( \hat{S}_z^z(\epsilon) = 1 \) [31].

Now we evaluate the spin response to a vector potential,

\[
\langle \Delta \hat{s}_z \rangle (\omega, q) = i q A_y(\omega, q) \int \frac{d\epsilon}{2\pi} \int \frac{d^2k}{(2\pi)^2} \text{tr} [\hat{s}_z \hat{G}(\epsilon + \hbar \omega/2, k + q/2) \hat{v}^y(k; q) \hat{G}(\epsilon - \hbar \omega/2, k - q/2)]
\]

\[
= i q A_y(\omega, q) \int \frac{d\epsilon}{2\pi} \int \frac{d^2k}{(2\pi)^2} \times \text{tr} \left( -\hat{s}_z \hat{G}^R(\epsilon + \hbar \omega/2, k + q/2) \hat{v}^y(k; q) \hat{G}^A(\epsilon - \hbar \omega/2, k - q/2) [f(\epsilon + \hbar \omega/2) - f(\epsilon - \hbar \omega/2)]
\]

\[
+ \hat{s}_z \{ \hat{G}^A(\epsilon, k + q/2) \hat{v}^y(k; q) \hat{G}^A(\epsilon - \hbar \omega/2, k - q/2) - [\hat{G}^A(\epsilon, k) \to \hat{G}^R(\epsilon + \hbar \omega, k)] f(\epsilon) \right).
\]

(15)
up to the first order with respect to \( \omega \) and \( q_z \). The zeroth-order terms with respect to \( q_x \) vanish owing to the \( C_4 \) symmetry of the Rashba model. The first-order terms are decomposed into two; one is the zeroth-order Fermi-sea term with respect to \( \omega \) and describes the SO magnetic susceptibility, while the other is the first-order Fermi-surface term and describes the spin accumulation. These terms are expressed as [25]

\[
\langle \Delta \hat{s}_z \rangle^{(0,1,1)}(\omega, q) = i \frac{h a}{2} q_z A_y(\omega, q) \int \frac{d \epsilon}{2 \pi} f(\epsilon) \int \frac{d^2 k}{(2\pi)^2} \times \text{tr} \left\{ \hat{s}_z \left[ \hat{G}^A(\epsilon) \hat{v}^x \hat{G}^R(\epsilon) - \hat{G}^A(\epsilon) \hat{v}^x \hat{G}^A(\epsilon) \right] - [\hat{G}^A(\epsilon) \rightarrow \hat{G}^R(\epsilon)] \right\},
\]

(16a)

\[
\langle \Delta \hat{s}_z \rangle^{(1,1,1)}(\omega, q) = - i \frac{h a}{2} q_z A_y(\omega, q) \int \frac{d \epsilon}{2 \pi} f'(\epsilon) \int \frac{d^2 k}{(2\pi)^2} \times \text{tr} \left\{ \hat{S}_z(\epsilon) [\hat{G}^R(\epsilon) \hat{v}^y \hat{G}^R(\epsilon) - \hat{G}^R(\epsilon) \hat{v}^y \hat{G}^A(\epsilon)] \right\},
\]

(16b)

and diagrammatically represented in Fig. 2. The argument of \( k \) is omitted for simplicity.

In the limit of \( \Gamma_0 \to +0 \), Eq. (16a) reproduces the SO magnetic susceptibility \((9a)\) obtained by the Bloch formula. In the Fermi-surface term \((16b)\) that involves both the retarded and advanced Green’s functions, we have replaced \( \hat{v}^y(k) \) and \( \hat{S}_z(\epsilon) \) with \( \hat{V}^y(\epsilon, k) \) and \( \hat{S}_z(\epsilon) \), respectively. To neglect the vertex corrections, we only have to put \( \hat{V}^y(\epsilon) = \hat{S}_z(\epsilon) = 1 \), and Eq. (16b) reproduces Eq. (9b) by identifying \( \eta \leftrightarrow 2f(\epsilon) \). Once we take into account the vertex corrections, the correct spin accumulation at \( T = 0 \) becomes [25]

\[
\langle \Delta \hat{s}_z \rangle^{(1,1,1)}(\omega, q) = \frac{\hbar}{2T(\mu)} \times \frac{q}{8\pi} (iq_z) E_y(\omega, q) \times \begin{cases} \frac{1}{\sqrt{1 + 2\mu/\eta a^2}} & (\mu > 0) \\ (\mu < 0) \end{cases}.
\]

(17)

This equation is another main result. The spin accumulation is nonzero even in the case where the chemical potential is below the Rashba crossing. We emphasize again that the SH conductivity vanishes in our setup [15–21].

Discussion. First, let us discuss the directions of the spin and electric field. The second term of the Bloch formula \((5c)\) involves the spin \( s_{na} \) and the orbital magnetic moment \( m_{nak} \). Since these two are parallel to each other, the spin accumulation takes the form of \( \langle \Delta \hat{s}_n \rangle(0, q) \propto [iq \times \hat{E}(0, q)]_a \), more precisely, \( \langle \Delta \hat{s}_n \rangle(0, q) \propto [iq \times \langle \Delta \hat{J} \rangle(0, q)]_a \). Thus, the direction of the spin accumulation is consistent with the conventional scenario of the SH effect.

Second, we discuss a relation between our results and the previous results on the SH conductivity. By multiplying \( -i\omega \) to Eq. (6), we obtain the time derivative of the spin expectation value. If we take the limits of \( \omega \to 0 \) and \( \eta \to +0 \) in the arbitrary order, we may obtain

\[
-i\omega \langle \Delta \hat{s}_n \rangle(\omega, q) = \alpha_{so} E_y(\omega, q) \left[ \frac{\hbar}{\eta} \gamma_a \epsilon(\gamma_a, \epsilon(\gamma_a, \epsilon_0) + \epsilon^{ijk} \chi_{ak}) \right]
\]

and

\[
\times (iq_z) E_y(\omega, q).
\]

(18)

Here, we have used Faraday’s law, \( -i\omega \hat{B}(\omega, q) = - (iq \times \hat{E}(\omega, q)) \). Since the second term is the divergence, we can read the spin (Hall) conductivity as

\[
\sigma_{sa}^{ij} = \frac{\hbar}{\eta} \gamma_a \epsilon(\gamma_a, \epsilon(\gamma_a, \epsilon_0) + \epsilon^{ijk} \chi_{ak}) \frac{\hbar}{\eta} \gamma_a \epsilon(\gamma_a, \epsilon(\gamma_a, \epsilon_0) + \epsilon^{ijk} \chi_{ak}) \times [s_{na} \partial \epsilon_\eta f(\epsilon_\eta) + \epsilon^{ijk} b_{nak} f(\epsilon_\eta)],
\]

(19)

This formula is consistent with the SH conductivity of the conserved spin current [25] proposed in Refs. [10, 11]. Furthermore, the Streda formula between the SH conductivity and the SO magnetic susceptibility [12] is obvious in this formalism. However, this discussion is dangerous because the chemical potential is below the Rashba crossing. The response is nonzero only when \( \mu \) is nonuniform. Here, we have calculated the same response nonperturbatively and obtained the nonzero response for the uniform \( \mu \). In fact, Eq. (17) is universal, i.e., independent of \( \alpha \), apart from the imaginary part of the self-energy.

Summary. We have calculated the spin response to the electric field gradient for the Rashba model using the first-order perturbation theory with respect to the Rashba SO coupling \( \alpha \). It was found that the response is nonzero only when \( \alpha \) is nonuniform. Here, we have calculated the same response nonperturbatively and obtained the nonzero response for the uniform \( \alpha \). In fact, Eq. (17) is universal, i.e., independent of \( \alpha \), apart from the imaginary part of the self-energy.
SH conductivity vanishes [15–21], the spin response \((17)\) is nonzero. This result simply explains the experimental results on the spin accumulation in \(n\)-type semiconductors [7] and is a counterexample to the traditional scenario of the SH effect depicted in Fig. 1(a).

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![Feynman diagrams](image)

**FIG. 2.** Feynman diagrams for (a) the spin–charge-current correlation function of the first order with respect to \(q_x\), (b) the renormalized velocity vertex, and (c) the renormalized spin vertex. The filled squares, open squares, and open circles represent the bare vertices of \(\hat{v}_x\), \(\hat{v}_y\), and \(\hat{s}_z\), respectively.

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