Flow Stress Modeling using the Material Constitutive Model of Modified Zerilli-Armstrong

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Abstract—The high temperature compression tests of 2Cr13 martensitic stainless steel under the deformation conditions at the temperatures between 1000°C and 1150°C with strain rates between 0.01s⁻¹ and 10s⁻¹ were carried out by using the Gleeble simulator. A new material constitutive model of modified Zerilli-Armstrong to predict 2Cr13 martensitic stainless steel flow stress behavior was developed.

1. INTRODUCTION
Material constitutive model is commonly used to express the most basic functional relationship between material flow stress and thermal process parameters. A reasonable material constitutive model could be used to describe metal materials behavior, such as temperature, strain, strain rate and work hardening. However, it is extremely difficult or even impossible to fully describe all these phenomena [1]. In engineering application, in order to meet the actual needs, the constitutive model is usually simplified and some assumptions are made. Constitutive models are usually divided into empirical or semi-empirical models and physical mechanism based models [1-3]. Because of the convenience and accuracy of empirical or semi-empirical flow stress constitutive model, it has been widely used in the finite element calculation of metal hot forming process.

At present, many researchers have put forward many kinds of constitutive models. Lee et al. [4] established the constitutive model of TC4 titanium alloy by expressing material parameters as strain polynomials based on the Arrhenius equation. Investigation results show that this constitutive model could predict the acceptable flow stress within a reasonable error range. Based on the voce constitutive equation, Kim et al. [5] proposed a constitutive model considering the effect of dynamic recrystallization on the stress of AISI4140 steel. The results show that for AISI4140 steel, this developed equation is more accurate than other flow stress models, such as Misaka equation and Shida equation. The high temperature constitutive model of FVS0812 aluminum alloy was established by the polynomial method of strain compensation by Zhan et al. [6]. Investigation results show that the predicted values are in good agreement with the test data. Slooff et al. [7] proposed a constitutive model to predict the high temperature flow behavior of a magnesium alloy by considering the strain parameters into the hyperbolic sinusoidal constitutive model. The results show that the high temperature flow behavior predicted by the constitutive model is in good agreement with the measured flow stress. Lin et al. [8] developed a constitutive model for 42CrMo steel considering strain
compensation based on Arrhenius equation. The results show that the predicted high temperature flow behavior of the studied steel is consistent with the high temperature compression test data. Mandal et al. [9] developed a constitutive model for AISI316 steel by using the fourth order polynomial to fit the strain effect.

In this research, the high temperature flow behavior of 2Cr13 steel under different deformation parameters was analyzed using the modified Zerilli-Armstrong (MZA) model.

2. MATERIAL AND EXPERIMENTAL

Table I shows the chemical compositions of 2Cr13 stainless steel. The cylindrical specimen used for the hot compression test is 15 mm in height and 10 mm in diameter. The high temperature compression tests of 2Cr13 martensitic stainless steel were carried out at 1000°C, 1050°C, 1100°C and 1150°C and strain rates between 0.01s⁻¹ and 10s⁻¹ using the Gleeble-1500D simulator.

| TABLE I. CHEMICAL COMPOSITIONS OF 2Cr13 STEEL (WT.%) |
|------------------|---------|---------|---------|---------|---------|---------|---------|
| Element | C       | Cr      | Si      | Mn      | P       | S       | Fe      |
| Amount   | 0.19    | 0.131   | 0.18    | 0.16    | 0.02    | 0.004   | Bal.    |

3. RESULTS AND DISCUSSIONS

3.1 Experimental curves

The experimental curves of 2Cr13 steel under high temperature compression test at different temperatures from 1000°C to 1150°C with strain rates between 0.01s⁻¹ and 10s⁻¹ are shown in Fig. 1. It can be seen from the figure that if the deformation temperature decreases, the flow stress will increase. Similarly, if the strain rate increases, the flow stress will also increase. The experimental results show that the high temperature flow behavior of metal materials is closely related to the deformation temperature and strain rate.

![Figure 1](image-url)
3.2 Modified Zerilli-Armstrong constitutive model

The modified Zerilli-Armstrong equation to predict the flow stress property for metals could be expressed as below [10]:

\[
\sigma = \left(C_1 + C_2 \varepsilon^*\right) \exp\left\{-\left(C_3 + C_4 \varepsilon\right)T^* + \left(C_5 + C_6 T^*\right) \ln \dot{\varepsilon}^*\right\} \quad (1)
\]

Where \(\sigma\) and \(\varepsilon\) represent the flow stress and the equivalent plastic strain, respectively. \(\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0\) is the dimensionless strain rate while \(\dot{\varepsilon}_0\) is the reference strain rate. \(T^* = (T - T_{ref})\) with \(T\) being the current temperature and \(T_{ref}\) being the reference temperature. \(C_1, C_2, C_3, C_4, C_5, C_6\) and \(n\) are constants. In the process of constitutive model building, 1000°C is taken as the reference temperature and 1s\(^{-1}\) is taken as the reference strain rate.

The step to calculate 2Cr13 martensitic stainless steel constants under the test conditions in this study is as below.

According to equation (1), equation (2) could be obtained when \(\dot{\varepsilon}^* = 1\):

\[
\sigma = \left(C_1 + C_2 \varepsilon^*\right) \exp\left[-\left(C_3 + C_4 \varepsilon\right)T^*\right] \quad (2)
\]

The natural logarithm of both sides of equation (2) can be expressed as follows:

\[
\ln \sigma = \ln \left(C_1 + C_2 \varepsilon^*\right) - (C_3 + C_4 \varepsilon)T^* \quad (3)
\]

The plot of \(\ln \sigma\) vs. \(T^*\) obtained by substituting the experimental flow stress data at \(\dot{\varepsilon}^* = 1\) is shown in Fig. 2(a).

Thus the value of \(\ln \left(C_1 + C_2 \varepsilon^*\right)\) is obtained from the intercept \(I_1\) while the value of \(-(C_3 + C_4 \varepsilon)\) is obtained from the slope \(S_1\). Then,

\[
I_1 = \ln \left(C_1 + C_2 \varepsilon^*\right) \quad (4)
\]

Equation (4) could be rearranged as:

\[
\ln \left(\exp I_1 - C_1\right) = \ln C_2 + n \ln \varepsilon \quad (5)
\]

The \(C_1\) parameter is determined by the yield stress of the stress-strain curve when the reference strain rate is 1s\(^{-1}\) and the reference temperature is 1000°C. The relationship between \(\ln \left(\exp I_1 - C_1\right)\) and \(\ln \varepsilon\) obtained by substituting \(C_1\) into equation (5) is shown in Fig. 2(b). Then, the values of \(\ln C_2\) and \(n\) could be gained from the relationship between \(\ln \left(\exp I_1 - C_1\right)\) and \(\ln \varepsilon\), respectively.
The slope of the line in equation (3) could be expressed as:

$$S_3 = -(C_3 + C_4 \varepsilon)$$  \hspace{1cm} (6)

$C_3$ and $C_4$ could be obtained from the relationship between $S_3$ and $\varepsilon$ as shown in Fig. 3(a).

Taking natural logarithm on both sides of equation (1), the following expression can be obtained:

$$\ln \sigma = \ln \left( C_1 + C_2 \varepsilon \right) + \left( C_4 + C_5 T^* \right) \ln \varepsilon$$  \hspace{1cm} (7)

According to the test data of four different temperatures, four different $S_2$ values can be obtained under certain strain. The expression of slope is as follows:

$$S_2 = C_5 + C_6 T^*$$  \hspace{1cm} (8)

As shown in Fig. 3 (b), values $C_5$ and $C_6$ can be obtained from the curve of $S_2$ versus $T^*$. 

![Figure 2](image1.png)

**Figure 2.** Relationship between (a) $\ln \sigma$ and $T^*$ and (b) $\ln (\exp I - C_1)$ and $\ln \varepsilon$.

![Figure 3](image2.png)

**Figure 3.** Plot of (a) $S_1$ vs. $\varepsilon$ and (b) $S_2$ vs. $T^*$. 

![Figure 3](image3.png)
The above mentioned model could be optimized by the mean absolute relative error of standard statistical parameters [11]. When $C_5$ equals to 0.09632 and $C_6$ equals to 0.000351, the absolute value of the minimum average relative error is about 8.72%.

Therefore, the modified Zerilli-Armstrong model can be obtained as follows:

$$\sigma = (98.64 + 138.35 e^{0.58 t}) \exp\left(-(0.0092 + 0.0009957 t^2 + (0.09632 + 0.000351 t^2) \ln \varepsilon^*)\right)$$

(9)

4. CONCLUSION

Different material constants of 2Cr13 martensitic stainless steel used in the developing of modified Zerilli-Armstrong constitutive model were calculated based on the test data. The material constitutive model of modified Zerilli-Armstrong to predict 2Cr13 steel flow stress behavior is given as:

$$\sigma = (98.64 + 138.35 e^{0.58 t}) \exp\left(-(0.0092 + 0.0009957 t^2 + (0.09632 + 0.000351 t^2) \ln \varepsilon^*)\right)$$

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