Surface electromagnetic actuator in rarefied hypersonic flow

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Abstract. Hypersonic flow past the surface with sharp edge is investigated. This surface forms an obtuse angle therefore the shock wave generated by the leading edge interacts with the surface. The effect of influence of the surface direct current discharge and a transverse magnetic field on the gas dynamic characteristics is investigated. To solve this problem a numerical simulation is used. The calculation model includes the set of the Navier–Stokes and energy conservation equations, as well as equations of electrodynamics in the ambipolar approximation, and the Poisson equation.

Results of numerical modelling the gas dynamics and electrodynamics of gas discharge with magnetic field show the change in the structure of the shock-wave interaction with surface far from location of the gas discharge. It is shown that the low-current glow discharge can be used as an electromagnetic actuator in hypersonic flows.

1. Introduction

In the late 90th of the last century, a series of experimental studies to provide data on the mass and heat fluxes for aerodynamic configurations typical for hypersonic vehicles [1] were carried out. The results of these experiments were thoroughly documented that allowed different groups to perform a validation of computational studies for author's computer codes developed by them. It is important that the results of the validation of these calculations were analyzed in detail by the group of highly qualified specialists in the field of computational aerodynamics (CFD), and the results of this analysis were published in the proceedings of a number of AIAA conferences. A review of these papers is presented in [1]. Among the studied aerodynamic configurations, special attention was focused on

- Blunt and sharp cones at Mach number \( M = 8 \),
- Blunt cone with flare at \( M = 8 \div 10 \),
- Hollow sharp cylinder with flare at \( M = 9 \div 11 \),
- Interaction of shock waves at Mach~10 (the fourth type of interaction in the Edney classification),
- Reflection of stationary shock waves with forming the regular configuration or the Mach configuration, as well as the hysteresis phenomena.

A series of experimental and theoretical works was devoted to the interaction of shock waves with the turbulent boundary layers in various configurations of the flow. Each of these configurations is of great interest for the study of hypersonic flows. In accordance of the purpose of
In this paper we draw attention to one of these tasks - hypersonic flow formed near surface which forms an obtuse angle with a sharp front edge. The detailed analysis of this problem was done in the papers [1–6], where the hollow-cylinder-flare flows were studied. Baseline data for one of the selected experiments (CUBRC Run # 14) are shown in table 1 (data are taken from table 1 [3]).

Among other above mentioned works we highlight the work [3], where not only the results of calculations using two computer codes (WIND [7] and DPLR [8]), but also accomplished study of the properties obtained from numerical solutions on different meshes are presented. It was found that the quality of computational meshes and the quality of the calculation method applied is a problem of particular importance. The good agreement between calculated and experimental data is obtained only on very detailed grids.

| Freestream Conditions | Extended Hollow Cylinder Flare |
|-----------------------|--------------------------------|
| CUBRC Run             | 14                             |
| Test gas              | N₂                             |
| Mach Number           | 10.3                           |
| V₀, cm/s              | 2.304 × 10⁵                   |
| p₀, erg/cm³           | 315.2                          |
| T₀, K                 | 120.4                          |
| ρ₀, g/cm³             | 8.810 × 10⁷                   |
| T₀, K                 | 295.2                          |

In this paper by the example of the problem of streamline of the sharp edge and surface, forming an obtuse angle, the impact of the glow discharge placed on the surface on the structure of the flow is examined. Schematic picture of the problem is shown in figure 1.

General formulation of the problem of creating plasma actuators was discussed in [9–12]. Earlier, the glow discharges localized on surface in the hypersonic flow was numerically studied in [13–15]. A detailed presentation of the numerical methods used to solve the problem of plasma actuators on the basis of the surface glow discharge of low power is given in [16, 17].

In the present paper an electromagnetic actuator formed by two electrodes made in the form of strips arranged transverse to the flow on the surface, as shown in figure 1. The subject of a numerical study is to identify perturbations in the pressure and heat flux distributions along the surface of the obtuse angle. As an additional control parameter of the plasma actuator a transverse magnetic field, which the polarization is shown in figure 1, is considered.

2. Gas dynamic model
We consider streamline of a sharp leading edge by a viscous, heat-conducting and partially ionized gas. The surface forms an obtuse angle before which the glow discharge is lighted, and magnetic field is applied in the cross-flow direction. The glow discharge is organized between two rectangular sections of the electrode at the surface, as shown in figure 1. An external electric circuit is taken into account. It consists of a power source and an ohmic resistance. In [9–12] it has been shown that the order in arrangement of electrodes in the flow is important. In present paper, it is assumed that the cathode is located upstream (ground electrode in figure 1).

The set of governing equations of the dynamics of a viscous, heat-conducting, and partially ionized gas takes into account the current flowing through that as well as the azimuthal magnetic field. It looks as follows:
\[
\frac{\partial \rho}{\partial t} + \text{div} (\rho \mathbf{V}) = 0 , \tag{1}
\]
\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \text{div} (\rho \mathbf{u} \mathbf{V}) = - \frac{\partial p}{\partial x} - \frac{2}{3} \frac{\partial}{\partial x} \left( \mu \text{div} \mathbf{V} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + 2 \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + f_{M,x}, \tag{2}
\]
\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \text{div} (\rho \mathbf{v} \mathbf{V}) = - \frac{\partial p}{\partial y} - \frac{2}{3} \frac{\partial}{\partial y} \left( \mu \text{div} \mathbf{V} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2 \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right) + f_{M,y}, \tag{3}
\]
\[
\rho c_p \frac{\partial T}{\partial t} + \rho c_p \mathbf{V} \text{grad} T = \frac{\partial \rho u}{\partial t} + \text{div} \left( \lambda \text{grad} T \right) + \mathbf{V} \text{grad} (p) + Q_\mu + Q_J, \tag{4}
\]
\[
\frac{\partial n}{\partial t} + \text{div} (\mathbf{V} n) = \text{div} \left( \mu_i \mathbf{D}_a \text{grad} n \right) + \dot{\omega}_e, \tag{5}
\]
\[
\text{div} \left[ \left( \mu_i + \mu_e \right) n \mathbf{E} + (\mathbf{D}_e - \mathbf{D}_i) \text{grad} n \right] = 0. \tag{6}
\]

where \( x \) and \( y \) are the Cartesian coordinates connected with unit vectors \( \mathbf{i} \) and \( \mathbf{j} \); \( \mathbf{V} = (u,v) \) is the velocity vector and its projections on axis \( x \) and \( y \); \( \rho \) and \( p \) are the density and pressure; \( \mu \) is the dynamic viscosity coefficient; \( c_p \) is the heat capacity at constant pressure; \( T \) is the temperature of the gas; \( \lambda \) is the heat conductivity coefficient; \( f_{M,x}, f_{M,y} \) are the components of the ponderomotive force; \( Q_J \) is the heat release power due to the electric current; \( Q_\mu \) is the dissipative function:
\[
Q_\mu = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 + 2 \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right], \tag{7}
\]
\[
\dot{\omega}_e = \alpha(E) |\Gamma_i| - \beta_e n^2 \] is the source term related to the ionization (the first summand) and to the recombination (the second summand); \( \alpha(E) \) is the coefficient of ionization (the \( 1^{st} \) Townsend coefficient); \( \beta_e \) is the recombination coefficient; \( |\Gamma_i| \) is the module of the vector of electron flux; \( E \) is the module of the electric field strength; \( D_a \) is the ambipolar diffusivity; \( \mathbf{j} = e (\Gamma_i - \Gamma_e) \) is the vector of current density; \( \Gamma_i \) and \( \Gamma_e \) are the vectors of the ion and electron flux densities.

An approximation of the quasineutral plasma is used, therefore \( n = n_i = n_e \), where \( n_i, n_e \) are the concentrations of ions and electrons.
\[ \tilde{\mu}_e = \frac{\mu_e}{1 + b_e^2}, \quad \mu_e' = \frac{\mu_e + \mu_i}{(1 + b_e^2)(\mu_i + \mu_e)}, \quad \tilde{D}_e = \frac{D_e}{1 + b_e^2}, \]

\( b_e = \frac{\mu_i B_e}{c} \) is the Hall parameter, \( B_e \) is the magnetic field induction, \( c \) is the speed of light.

When one takes into account the magnetic field the term in the right hand side in equation (5) should be modified as follows:

\[ \dot{\omega}_e = \alpha(E)\tilde{\mu}_e E - \beta n^2. \]

Equation (6) is the Maxwell equation for conservation of electric current under steady-state conditions. Equation (5) expresses the continuity of the charged components of the partially ionized gas [16,17]. To obtain this equation, the following assumptions were used:

- velocities of the charged and neutral particles in partially ionized gas are equal, \( V = V_e = V_i \) (\( V \) – is the mean velocity of neutral particles, \( V_e, V_i \) are the mean velocities of electrons and ions),
- the concentrations of ions and electrons are equal \((n = n_e = n_i)\),
- charged particles diffuse with a factor of ambipolar diffusivity \( D_a \).

Taking into account that the electric field strength is connected with the electric potential \( \varphi \)

\[ E = -\nabla \varphi, \]

instead of equation (6) it is more preferable to use the following equation for potential:

\[
\nabla \varphi \left( \frac{\partial b_e}{\partial x} \right) - \nabla \varphi \left( \frac{\partial b_i}{\partial y} \right) + \nabla \left( \mu_i \tilde{\mu}_e n \frac{\partial \varphi}{\partial x} \right) + \nabla \left( \mu_i \tilde{\mu}_e n \frac{\partial \varphi}{\partial y} \right) - \nabla \left( \tilde{D}_e - D_e \right) \frac{\partial n}{\partial x} + \nabla \left( \tilde{D}_e - D_e \right) \frac{\partial n}{\partial y} - \nabla \left( b_e \tilde{D}_e \frac{\partial n}{\partial x} \right) - \nabla \left( b_e \tilde{D}_e \frac{\partial n}{\partial y} \right) = 0. \tag{8}
\]

Equations (1)–(6) are recorded in the Cartesian coordinate system. To calculate the structure of gas flow the new variables in the curved geometry are introduced \( \xi = \xi(x, y), \eta = \eta(x, y) \). These variables are connected with Cartesian coordinates by the one-to-one transformations. Now, the coordinate \( \xi \) is directed along surface, and the \( \eta \)-coordinate is directed normally to surface.

The boundary conditions for system (1)–(6) are presented in the following form (see figure 1):

\( \xi = 0: \quad u = V_e, \quad v = 0, \quad T = T_e, \quad p = p_e, \quad \rho = \rho_e, \quad n = n_e, \quad \frac{\partial \varphi}{\partial \xi} = 0; \)

\( \xi = L: \quad \frac{\partial u}{\partial \xi} = \frac{\partial v}{\partial \xi} = \frac{\partial T}{\partial \xi} = \frac{\partial n}{\partial \xi} = \frac{\partial \rho}{\partial \xi} = \frac{\partial \varphi}{\partial \xi} = 0 \); \( \eta = 0, \quad \xi > \xi_{\text{edge}}, \quad u = v = 0, \quad \frac{\partial \rho}{\partial \eta} = 0 \); \( \eta = 0, \quad \xi_c \leq \xi \leq \xi_c \) (the cathode section):

\( \frac{\partial n}{\partial \eta} = 0, \quad \varphi = 0; \)

\( \eta = 0, \quad \xi_A \leq \xi \leq \xi_A \) (the anode section): \( \frac{\partial n}{\partial \eta} = 0, \quad \varphi = V_e \);

\( \eta = 0, \quad \xi_{\text{edge}} < \xi < \xi_c, \quad \xi_c < \xi < \xi_A, \quad \xi > \xi_A \) (the dielectric surface):
\[ n = 10^{-5} n_0, \quad \frac{\partial \varphi}{\partial \eta} = 0 ; \]
\[ y = H, \quad x > 0: \quad u = V_n, \quad v = 0, \quad T = T_n, \quad p = p_n, \quad \rho = \rho_n, \quad n = n_n, \quad \frac{\partial \varphi}{\partial \eta} = 0 . \]

Here: \( \xi_{\text{edge}} \) is the longitudinal coordinate of the leading sharp edge; \( \xi_{C}, \xi_{C_1}, \xi_{A}, \xi_{A_1} \) are the longitudinal coordinates of the cathode and anode boundaries; \( n_0 \) is the typical concentration of charged particles in glow discharge (\( n_0 \sim 10^{10} \text{ cm}^{-3} \)); \( V_n \) is the voltage drop between cathode and anode. Surface temperature \( T_n = 300 \) K is assumed as constant for all calculations.

The electro-technical equation for external electric circuit is used in the following form:
\[ IR_0 + V_c = E , \quad (9) \]

from which the voltage drop \( V_c \) can be found, \( R_0 \) is the ohmic resistance, \( I \) is the total current in electric circuit, which is determined by integral over the surfaces of electrodes:
\[ I = \int_{C} (j \cdot n) \, dx = \int_{A} (j \cdot n) \, dx , \]

where \( n \) is the unit vector normal to cathode (\( C \)) and anode (\( A \)) surfaces; \( j \) is the vector of current density. All dimensions in the task are related to the lengths of the electrodes in \( z \)-direction.

Gas dynamic profiles without electric discharge, as well as a plasma cloud with charged particles concentration of above cathode sections were used as initial conditions for the calculations. The time-asymptotic method was used for integration of governing equations therefore the precise type of initial conditions was not very significant. Nevertheless, due to nonlinear character of the problem under consideration the initial conditions should be quite accurate.

In accordance with [16] the hypersonic flow of molecular nitrogen is considered. The constitutive relationships for \( \text{N}_2 \) are based on the molecular-kinetic theory [18]:
\[ \mu_\sigma = 2.67 \cdot 10^{-5} \sqrt{M_A T}, \quad 1 \left[ \frac{1}{\sigma^{2.2 \rho}} \right], \quad \text{g/(cm·s)}, \]

is the dynamic viscosity,
\[ \lambda = 8.334 \times 10^{-4} \left[ \frac{T}{M_A} \right] \left( \frac{0.115 + 0.354 \sigma}{\sigma^2 \Omega^{(2,2 \rho)}} \right), \quad \text{W/(cm·K)}, \]

is the heat conductivity,
\[ c_p = 8.317 \times 10^{7} \frac{7}{2 M_A}, \quad \rho = \frac{p M_A}{T R_0}, \quad M_A = 28 \text{ g/mole}, \]
\[ R_0 = 8.314 \times 10^{7} \text{ erg/(mole·K)}, \quad \sigma = 3.68 \text{ Å}, \quad \Omega^{(2,2 \rho)} = 1.157 \left( T^* \right)^{-0.1472} , \]
\[ T^* = \frac{T}{(\varepsilon/k)}, \quad (\varepsilon/k) = 71.4 . \]

Coefficient of ionization was calculated as follows [16]:
\[ v_i = \left( \frac{\alpha}{p} \right)^* E \mu_i \left( \frac{p^*}{p^*} \right), \quad \left( \frac{\alpha}{p} \right)^* = A \exp \left[ -\frac{B}{(E/p^*)} \right], \quad \text{(cm-Torr)}^{-1}, \]

where:
\[ p^* = p \frac{293}{T}, \quad D_e = \mu_i \left( \frac{p^*}{p^*} \right) T_e, \quad D_i = \mu_i \left( \frac{p^*}{p^*} \right) T_e, \quad \text{cm}^2/\text{s}, \]
\[ \mu_i \left( p^* \right) = 4.2 \times 10^7 \frac{1}{p^*}, \quad \mu_i \left( p^* \right) = 1450 \frac{1}{p^*}, \quad \text{cm}^2/(\text{s-V}), \]
\[ \beta = 2 \times 10^{-7} \quad \text{cm}^3/\text{c}, \quad T_e = 11610 \quad \text{K}, \quad A = 12 \quad \text{(cm-Torr)}^{-1}, \quad B = 342 \quad \text{V/(cm-Torr)}, \]
\[ \mu_i \] is the drift of ions.

Heat release due to the electric current (the Joule heat) is calculated by using the calculated current density and electric field strength:
\[ Q_i = \eta \left( \mathbf{j} \cdot \mathbf{E} \right) = 1.6 \times 10^{-19} \eta \left[ n E^2 \left( \mu_e + \bar{\mu}_e \right) + (\bar{D}_e - D_e) \mathbf{E} \cdot \nabla n \right], \quad (10) \]

where \( \eta \) is the coefficient of efficiency of gas heating. Setting this ratio in the range \( \eta \sim 0.1 \div 0.3 \) allows partially take into account the shortcomings of the model used in the description of the kinetics of physical and chemical processes, as well as the processes of vibrational relaxation.

An additional force acted on gas due to the presence of magnetic field has two components:
\[ \mathbf{F}_M = f_{M_x} \mathbf{i} + f_{M_y} \mathbf{j} = c^{-1} [\mathbf{j} \mathbf{B}], \quad (11) \]

These two projections of the magnetic field are taken into account in equations (2) and (3). The set of equations (1) – (6) describing a motion of a viscous, heat-conducting, and partially ionized gas along the surface with glow discharge at the presence of external magnetic field was solved by the type-asymptotic method up to achieving a numerical convergence with a relative error of \( 10^{-5} \), which is calculated over the entire field of flow for functions \( u, v, p, T, n, \varphi \). At each step the following equations were integrated numerically:
- the continuity equation and the Navier-Stokes equations,
- the energy conservation equation,
- the set of electrodynamic equations.

The AUSM method was used for integrating the gas dynamic equations. The energy conservation equation was integrated with the use of the implicit finite-difference method of the second order accuracy in time and in space. The same numerical approach was used for integration of the electrodynamic equations. This part of the numerical simulation procedure was the most time-consuming due to exponential character of the source of ionization.

Second internal iterative process (at each time step) consisted in achieving the mutual convergence of functions obtained at solving the Fourier-Kirchhoff equations (the energy conservation equation) and the set of electro-dynamic equations. The need in this iterative process was caused by the fact that the gas temperature is very strongly influenced by its ionization rate, which, in turn, produces strong perturbations in the right-hand side of equation (5).

3. Results of numerical simulation

Two configurations of mutual location of electrodes were investigated. Coordinates of electrodes for these configurations are shown in table 2. The choice of these two configurations follows from the obvious fact that creation of an artificial disturbance to the leading edge leads to significant consequences for the entire flowfield.
Figure 2 shows the distribution of the gas-dynamic parameters of the streamlined surface without an electrical discharge. It is assumed that the edge of the plate is located at a distance of 1 cm from the computational domain boundaries.

Figure 2. Gas dynamic parameters without gas discharge: $a$ – Mach numbers; $b$ – pressure ($p/p_\infty$); $c$ – density ($\rho/\rho_\infty$); $d$ – temperature (in K); $e$ – longitudinal velocity ($u/V_\infty$); $f$ – transversal velocity ($v/V_\infty$).

Distributions of Mach number (figure 2a) and the longitudinal velocity (figure 2f) shows that near the sharp bend of the streamlined surface (cm) low speeds are observed. Calculations on detailed computational grids with high accuracy methods show that there is a fairly extensive area
of the vortex motion of the gas. In present case, the length of this zone does not exceed of about 1 cm.

From figure 2f one can see that two areas of the flow spread are in the positive direction. The first area is from the leading edge, and the second area is from the sharp turn of the surface. The most high pressure is observed on the surface in place of the fall of the bow shock. In the vicinity of this place there is a noticeable increase in temperature (figure 2d) and, as a consequence, the increasing of the heat flux to the surface. Here there is also a natural increase in density (figure 2c).

The formulation of present task includes an examination of the possibility to modify the gas dynamic functions by the use of the surface glow discharge of low power. Figure 3 shows the distribution of the gas-dynamic parameters when the glow discharge is used for the first electrode configuration

| Table 2. | Configuration # 1 | Configuration # 2 |
|----------|-------------------|-------------------|
| $x_{c_1}$, cm | 2 | 3 |
| $x_{c_2}$, cm | 3 | 4 |
| $x_{a_1}$, cm | 4 | 5 |
| $x_{a_2}$, cm | 5 | 6 |

The first configuration of the electrodes is considered with e.m.f. $E = 200 \text{ V}$ and the ohmic resistance of $R_0 = 300\text{k}\Omega$. In the place of the gas discharge plasma location one can observe heating of the gas up to $T \sim 1100 \text{ K}$. This temperature is commensurable with the temperature near place of the shock wave interaction with surface. Perturbations in pressure and density distributions are insignificant, whereas changes in the longitudinal and transversal velocity are visible.

In the case of the second configuration of electrodes with e.m.f. $E = 200 \text{ V}$ and $R_0 = 300\text{k}\Omega$ the similar perturbation in distributions of gas dynamic functions close to the new arrangement of the electrodes are observed. In this case the results of numerical simulation are shown in figure 4.

Significantly more noticeable perturbations of gas-dynamic parameters are observed in the magnetic field. Corresponding numerical results are shown in figures 5 and 6.

A polarization of the magnetic field produces a force component $f_{M,y}$ towards the surface, i.e. pushing the gas to the streamlined surface.

Figures 5 and 6 show the distribution of the gas-dynamic parameters for the 1st and 2nd electrode configurations respectively.

Before analyzing the characteristics of gas-dynamic perturbations associated with the interaction between the surface glow discharge and external magnetic field, we consider the appropriate distribution of the electro-dynamic parameters shown in figures 7 and 8. Figures 7a and 8a show the distribution of electric potential, which clearly shows the location of the anode and cathode (the largest potential).

Figures 7b and 8b show axial component of the ponderomotive force. Near the cathode (or rather, near its right border) it is positive, and at the left border of the anode it is negative, which leads to a marked braking of gas flow. The normal component of the ponderomotive force is directed to the surface. The highest value of the heat release power occurs to be between the cathode and the anode (figures 7d and 8d). In the same area the greatest concentration of charged particles (figures 7e and 8e) is observed.
Figure 3. No magnetic field. Gas dynamic parameters with gas discharge of the first configuration: a – Mach numbers; b – pressure \( (p/p_\infty) \); c – density \( (\rho/\rho_\infty) \); d – temperature (in K); e – longitudinal velocity \( (u/V_\infty) \), f – transversal velocity \( (v/V_\infty) \).

Perturbations of the gas-dynamic parameters shown in figures 5 and 6 are conditioned by the presents of glow discharge with magnetic field which general characteristics are shown in figures 7 and 8.

The localized domain of deceleration of the gas flow between the cathode and the anode is visible in figures 5a,e and 6a,e. In this domain the normal velocity component dramatically increases from the surface (figures 5f and 6f), as well as the local domain of high pressure and density are seen (figures 5b and 6b, figures 5c and 6c). It is noteworthy that in this area the gas is
heated to a temperature of ~2300 K, which is higher than that in the discharge without magnetic field.

Figure 4. No magnetic field. Gas dynamic parameters with gas discharge of the second configuration: a – Mach numbers; b – pressure \((p/p_\infty)\); c – density \((\rho/\rho_\infty)\); d – temperature (in K); e – longitudinal velocity \((u/V_\infty)\), f – transversal velocity \((v/V_\infty)\).
Figure 5. Gas-dynamic parameters with gas discharge of the first configuration and with magnetic field of $B_z=0.1 \, \text{T}$: a – Mach numbers; b – pressure ($p/p_\infty$); c – density ($\rho/\rho_\infty$); d – temperature (in K); e – longitudinal velocity ($u/V_\infty$); f – transversal velocity ($v/V_\infty$).

Distributions of the Stanton numbers, of the pressure coefficient, as well as the friction coefficient of surface tension (figure 11) along the surface, are shown in figures 9–11.

The Stanton number was calculated by the formula

$$St = \frac{q_{w,c}}{\frac{1}{2} \rho_\infty V_\infty^3},$$

where $q_{w,c}$ is the convective heat flux.

The coefficients of the pressure and friction are calculated as following.
\[ c_p = \frac{p - p_w}{\frac{1}{2} \rho_w V_w^2}, \quad c_f = \frac{\tau_w}{\frac{1}{2} \rho_w V_w^2} = \frac{\mu}{\rho_w V_w^2} \left( \frac{\partial u}{\partial y} \right). \]

Figure 6. Gas-dynamic parameters with gas discharge of the second configuration and with magnetic field of \(B_z = 0.1\) T: a – Mach numbers; b – pressure (\(p/p_w\)); c – density (\(\rho/\rho_w\)); d – temperature (in K); e – longitudinal velocity (\(u/V_w\)); f – transversal velocity (\(v/V_w\)).
Figure 7. Electrodynamic parameters with gas discharge of the first configuration and with magnetic field of $B_z=0.1$ T: a – electric potential ($\phi/\varepsilon$), b – $x$-component of the magnetic force (dyn), c – $y$-component of the magnetic force (dyn), d – current density (mA/cm$^2$), e – concentration of electron in $10^{10}$ cm$^{-3}$.

The Stanton coefficient very strongly increases in the domain between cathode and anode without discharge, especially with the magnetic field.

An important effect of the electric discharge and magnetic field is a shift of the heating domain by the amount of ~3 cm along the inclined surface.

A similar effect is observed in the distribution of the pressure coefficient $c_p$ with the difference that it increases at displacement of the maximal pressure along the surface.

The distribution of the surface friction coefficient $c_f$ points to increasing the vortex in gas discharge with magnetic field (figures 11 a, b).
Conclusion
The problem of hypersonic flow past the surface with sharp edge has been considered. This surface forms an obtuse angle therefore the shock wave generated by the leading edge interacts with the surface.

The effect of influence of the surface direct current discharge and a transverse magnetic field on the gas dynamic characteristics is investigated. To solve this problem a numerical simulation model has been used. This model is based on set of the Navier – Stokes and energy conservation equations, as well as on equations of electrodynamics in the ambipolar approximation, and the Poisson equation.

Figure 8. Electrodynamic parameters with gas discharge for the second configuration and with magnetic field of $B_z=0.1$ T: a – electric potential ($\phi/\varepsilon$), b – x-component of the magnetic force (dyn), c – y-component of the magnetic force (dyn), d – current density (mA/cm$^2$), e – concentration of electron in $10^{10}$ cm$^{-3}$.
Figure 9. Distribution of the St-number along surface.

Figure 10. Distribution of the $C_p$-coefficient along surface.

Figure 11. Distribution of $C_f$-coefficient along surface.
The results of numerical modeling the gas dynamics and electrodynamics of gas discharge with magnetic field point out to a change in the structure of the shock-wave interaction with surface far from location of the gas discharge.

So, by the calculations it is shown that the low-current glow discharge can be used as an electromagnetic actuator in hypersonic flows.

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