Numerical Study of a Superconducting Glass Model

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An XY model with random phase shifts as a model for a superconducting glass is studied in two and three dimensions by a zero temperature domain wall renormalization group which allows one to follow the flows of both the coupling constant and the disorder strength with increasing length scale. Weak disorder is found to be marginal in two and probably irrelevant in three dimensions. For strong disorder the flow is towards a non-superconducting gauge glass fixed point in 2d and a superconducting glass in 3d. Our results are in agreement with recent analytic theory and are inconsistent with earlier predictions of a re-entrant transition to a disordered phase at very low temperature and with the loss of superconductivity for any finite amount of disorder.

64.40, 74.40, 75.10

The classical XY model with random phase shifts described by the Hamiltonian

\[ H = - \sum_{<ij>} J_{ij} \cos(\theta_i - \theta_j - A_{ij}) \]  (1)

has been the subject of much interest over the past decade. Here, \( \theta \) is the phase of the order parameter at the \( i \)th site of a square lattice in 2d or a simple cubic lattice in 3d and the sum is over all nearest neighbor bonds. The coupling constants \( J_{ij} \) are uniform \( J > 0 \) and the quenched random phase shifts \( A_{ij} \) are uncorrelated from bond to bond and uniformly distributed over the range \(-\alpha \pi \leq A_{ij} \leq \alpha \pi \) with \( 0 \leq \alpha \leq 1 \) so that \( < A_{ij} > = 0 \) and \( < |A_{ij}| > = \alpha \pi/2 \) where \( < > \) means an average over disorder. In \( d = 2 \), the model of eq. (1) is a description of an array of Josephson junctions in a magnetic field perpendicular to the plane of the array when \( \theta \) represents the phase of the superconducting order parameter of the \( i \)th grain and \( A_{ij} = (2\pi/\Phi_0) \int_{A}^{B} \vec{A} \cdot d\vec{r} \) with \( \vec{A} \) the vector potential of the external magnetic field and \( \Phi_0 = hc/2e \) is the quantum of flux. The \( A_{ij} \) become independent quenched random variables when the average flux through an elementary plaquette is an integer multiple of \( \Phi_0 \) but the superconducting grains are randomly displaced from their ideal lattice positions. It is also a model for an XY magnet with random Dzyaloshinskii-Moriya interactions.

Whatever the physical origin of the quenched random phase shifts, the system described by eq. (1) has been a theoretical challenge for a decade. Early work concluded that weak disorder (\( \alpha \ll 1 \)) does not destroy the superconducting phase at intermediate temperature \( T \) but at low \( T \) there is a re-entrant transition to a normal phase. However, this was not confirmed either numerically or experimentally. Some later theoretical work suggested that the bound vortex (KT) phase is destroyed at any finite \( T \) by arbitrarily small disorder and that the experimentally observed KT phase is a finite size effect. The more recent theoretical work based on the ideas of Cha and Fertig, on the other hand, argue for a more conventional phase diagram in which there is a superconducting phase for \( T < T_c(\alpha) \) where \( T_c(\alpha) \geq 0 \) for \( \alpha \leq \alpha_c \). The case of maximum disorder (\( \alpha = 1 \)) when the \( A_{ij} \) are uniformly distributed between \(-\pi \) and \(+\pi \) is called the gauge glass which has been studied numerically at \( T = 0 \) by a domain wall renormalization group (DWRG) in both \( d = 2 \) and \( d = 3 \) and the conclusions from these studies are that the stiffness to distortions of the phase vanishes in \( d = 2 \) so that the glass is not superconducting and that in \( d = 3 \) the gauge glass is probably superconducting, but the evidence is not conclusive. In view of the three conflicting scenarios in two dimensions, (i) re-entrant transition, (ii) destruction of superconductivity for any finite disorder and (iii) superconductivity for \( T < T_c(\alpha) \) with \( T_c(\alpha) > 0 \) for \( 0 \leq \alpha < \alpha_c \), this system is an ideal candidate for study by a numerical DWRG at \( T = 0 \) as this will distinguish scenario (iii) from the others as only scenario (iii) predicts a finite stiffness at \( T = 0 \) for \( 0 \leq \alpha < \alpha_c \).

The standard DWRG at \( T = 0 \) consists of computing the lowest energies of a set of systems of several linear sizes \( L \) with periodic and antiperiodic boundary conditions (BC) in one direction with some fixed BC in the other \( d - 1 \) directions. The difference \( \Delta E(L) = \langle |E_{sp}(L) - E_p(L)| \rangle \) is the domain wall energy and \( \Delta E(L)/2 \) is interpreted as an effective coupling constant \( J(L) \) at length scale \( L \) which one expects to scale as \( J(L) \sim L^\theta \) at large \( L \). The stiffness exponent \( \theta \) is a crucial quantity as its value will distinguish between an ordered superconducting phase at small but finite \( T \) (\( \theta \geq 0 \)) and a disordered phase (\( \theta < 0 \)). If \( \theta < 0 \), then the energy of an excitation of size \( L \) is \( \Delta E(L) \) which vanishes as \( L \to \infty \) and the probability of such a phase unwinding excitation \( P(\Delta E(L)) \sim \exp(-\Delta E(L)/kT) \) is large at any \( T > 0 \) so the stiffness to twists in the phase will vanish. If \( \theta > 0 \), the converse is true and the stiffness will be finite for \( T < T_c \) and the glass will be superconducting. This technique has been applied to random systems such as spin and gauge glasses at \( T = 0 \) but the the value of the exponent \( \theta \) obtained by this method is not very reliable because it is not clear if the asymptotic scaling regime
is reached for the small sizes $L$ it is usually possible to simulate.

For the problem of interest with a variable disorder strength, this version of the DWRG, which considers only the scaling of the effective coupling $J(L)$ needs considerable modification as we also want to know how the disorder strength scales with $L$. For a single junction, the Hamiltonian is $H = -J \cos(\phi - A)$ where $\phi$ is the phase difference across the junction and the usual comparison of the energies with periodic and antiperiodic BC gives $\Delta E(1) = 2J \cos A$ which does not separate the disorder strength from the coupling constant. At large scale $L$, the interaction is $V_L(\phi - A(L))$ where $A(L)$ is the phase shift at scale $L$ and $V_L(\phi)$ is a $2\pi$-periodic function with a minimum when its argument is zero. Thus, if we impose BC with phase shifts $\Delta_\mu$ across the boundaries in the $d$ directions $\mu = 1, 2, \ldots, d$, minimizing the energy with respect to the phases $\theta_i$ will give the ground state energy of a system of linear size $L$ as a function of $\Delta_\mu$ which is $2\pi$ periodic in each of the $d$ directions

$$E_L(\Delta_\mu) = E_L(\Delta_\mu + 2\pi)$$

with a minimum at some $\Delta^0_\mu$ which depends on the precise realization of disorder. The key observation is that $\Delta^0_\mu$ is exactly the phase shift $A_\mu(L)$ which minimizes the energy at scale $L$. A measure of the strength of disorder at this scale is

$$|A(L)| \equiv < |\Delta^0_\mu| >$$

with $|A(1)| = \alpha \pi/2$. The coupling constant $J(L)$ at scale $L$ is found by first finding $E_L(\Delta^0_\mu)$, changing $\Delta^0_\mu$ by $\pi$ in one of the $d$ directions and then finding the energy minimum $E_L(\Delta^0_\mu + \pi)$ with these BC. As discussed above, the coupling constant $J(L)$ at scale $L$ is

$$J(L) \equiv < (E_L(\Delta^0_\mu + \pi) - E_L(\Delta^0_\mu)) >$$

and measuring $J(L)$ and $|A(L)|$ for several sizes $L$ gives renormalization group flows for both the coupling constant and disorder strength. Of interest are the stable fixed point values $J^* = J(L = \infty)$ and $A^* = |A(L = \infty)|$ as these determine the nature of the phases. There are several possibilities of which the simplest are $[J^* = \infty, A^* = 0], [J^* = \infty, A^* = \pi/2], [J^* = 0, A^* = \pi/2]$ corresponding respectively to a superconducting state with long range order, a superconducting glass and a non-superconducting glass. There are other possibilities such as a state with quasi long range order corresponding to a flow to a fixed line with finite $J^*$ and $A^*$ whose values depend on the initial values of coupling and disorder. This is the scenario in $d = 2$ predicted by recent analytic work [8].

Of course, since we do not know how to find the exact ground state of a disordered system of arbitrary linear size $L$, the best we can do is to numerically estimate $J(L)$ and $|A(L)|$ for a set of samples of different sizes $L$ up to some maximum and extrapolate to large $L$. We use simulated annealing [18] to estimate the ground state energies which is considerably more efficient than simple repeated quenches to $T = 0$ [19]. Also, we imposed periodic $\Delta_\mu = 0$ BC in $d - 1$ directions and twisted $\Delta \neq 0$ BC in the remaining direction and minimized the energy with respect to the the phases $\theta_i$ and to the twist $\Delta$ to find $\Delta^0$. To obtain the domain wall energy $\Delta E_L$ the twist is changed to $\Delta^0 + \pi$ and kept fixed while the energy is minimized with respect to $\theta_i$ only. According to our earlier discussion, the energy should be minimized with respect to global phase shifts in all $d$ directions and the domain wall energy $\Delta E_L$ obtained by increasing the phase shift by $\pi$ in one direction. To within the errors of our simulations, $\Delta E_L$ is independent of the choice of BC in the $d - 1$ transverse directions so, for simplicity, we imposed periodic or $\Delta_\mu = 0$ BC in these directions. As a consistency check [19], we simulated two identical copies of each system with different random number sequences to obtain two estimates $E_1, E_2$ of the ground state energy. In the event that the simulation finds the exact ground state, then $\delta E = E_1 - E_2 = 0$, which often occurs for our small $L$ values. If the simulation does not reach the exact minima, $< (\delta E)^2 >$ is a measure of the error. To minimize the errors caused by failure to reach the true energy minimum, we adjust the annealing schedule and the number of annealing attempts until $\delta E/E < N^{-1/2}$ where $N = 10^5$ in 2$d$ and $10^4$ in 3$d$ is the number of realizations of disorder. This consistency check makes the error due to not reaching the true ground state no worse than the statistical error in the averaging over disorder.

To our knowledge, there is no analogue for this disordered $XY$ system of the "branch and cut" algorithms to find exact ground states of Ising spin glasses [20] in fairly large systems in a reasonable amount of CPU time. For repeated simulated annealings of $N$ different samples the CPU time becomes prohibitive for $L > 8$ in 2$d$ and $L > 4$ in 3$d$. We therefore chose sizes $L = 2, 4, 8$ in 2$d$ and $L = 2, 3, 4$ in 3$d$ and the results are summarized in Fig.(1) for 2$d$ and in Fig.(2) for 3$d$.

In 2$d$, for small disorder $\alpha < \alpha_c \approx 0.37$, $J(L)$ increases more slowly than a power of $L$ and seems to flow to a finite disorder dependent value $J^*(\alpha)$ and the disorder strength $|A(L)|$ does not change with $L$, at least for our small sizes, both of which are completely consistent with analytic RG calculations [14]. At larger disorder strength $\alpha > \alpha_c$, $|A(L)|$ increases and $J(L)$ decreases as $L$ increases, in agreement with the analytic theory [2,14]. In this range of disorder strengths, the system is probably flowing to the non-superconducting glass fixed point at $J^* = 0, A^* = \pi/2$. When the disorder is maximal ($\alpha = 1$), $|A(L)|$ remains fixed at $\pi/2$ and $J(L) \sim L^0$ with the stiffness exponent $\theta \approx -1/2$, in agreement with other simulations of the gauge glass in 2$d$ [14]. The flows shown in Fig.(1) may be regarded as RG flows in a higher dimensional parameter space projected on to the $(J(L), |A(L)|)$ plane, so the crossing of the two trajectories for $\alpha = 0.45$ and $\alpha = 0.5$ does not violate the non-crossing rule. For the 2$d$ system, the RG flows are
in the three parameter space of $J, |A|$ and the vortex fugacity $y$ \cite{4}. From the data of Fig.(1) and assuming that the trends for small $L$ continue when $L$ is large, one would conclude that the most likely scenario for the disordered system in $2d$ is, for weak disorder $\alpha < \alpha_c$, $J(L) \to J^*(\alpha)$ and $|A(L)| = |A(1)|$, so that the coupling constant scales to a finite but disorder dependent value while the disorder is marginal. For larger disorder $\alpha > \alpha_c$, $J(L) \to J^* = 0$ and $|A(L)| \to \pi/2$ which is a non-superconducting disordered state. Our results are consistent in all respects with recent analytical theory \cite{10} and inconsistent with the re-entrant \cite{3,2} and superconducting \cite{9,10} scenarios. However, they are consistent with a re-entrant scenario in which the ordered phase extends to $T = 0$ for a range of $\alpha$ \cite{21}. The flows for $2d$ of Fig.(1) are consistent with a discontinuous jump in $J^*$ at $\alpha_c$ as predicted analytically \cite{9}.

The results of the simulations of the random gauge model in $3d$ are shown in Fig.(2). The system sizes are very small ($L = 2, 3, 4$) because of the CPU time needed to get close to the minimum energies so that irrelevant variables are giving large corrections to scaling. Nevertheless, some qualitative features are apparent, assuming that the small $L$ trends continue. For small disorder $\alpha < \alpha_c \approx 0.55$, $J(L) \sim L^{d-2}$ as expected and the disorder strength seems to decrease. It is impossible to say if $|A(L)| \to 0$ as one expects but the data is consistent with this. One is tempted to conclude that, in this regime of weak disorder, the DWRG flows are to a stable fixed point at $J^* = \infty$, $A^* = 0$ corresponding to a true superconducting phase. For larger disorder $\alpha_c < \alpha \leq \pi/2$, the disorder increases with $L$ and seems to flow to its maximum value of $\pi/2$. The coupling $J(L)$ seems to flow to a finite value which corresponds to a stiffness ex-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{DWRG flows for the $2d$ superconducting glass model in the coupling constant-disorder strength ($J, |A|$) plane. The initial value of $J$ is $J(1) = 1$ and the initial disorder strength $\alpha$ is indicated near corresponding symbols.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{DWRG flows for the model in $3d$.}
\end{figure}

Our conclusions from our new $T = 0$ DWRG which follows the flows in two parameter space are in $2d$, the recent analytic theory which predicts a quasi long range ordered state for $T < T_c(\alpha)$ is the correct scenario and earlier suggestions of a re-entrant transition to a disordered phase at low $T$ or no superconductivity at any finite disorder are ruled out. In $3d$, weak disorder has little or no effect on the superconducting phase and there is a critical disorder strength parametrized by $\alpha = \alpha_c$, above which the system is a superconducting glass at low $T$.

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