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Generation and stability of diversiform nonlinear localized modes in exciton–polariton condensates

Kun Zhang1, Wen Wen1,2,3, Ji Lin1 and Hui-jun Li1,*

1 Institute of Nonlinear Physics and Department of Physics, Zhejiang Normal University, Jinhua, 321004 Zhejiang, People’s Republic of China
2 Department of Mathematics and Physics, Hohai University, Changzhou, Jiangsu 213022, People’s Republic of China
3 College of Science, Hohai University, Nanjing, Jiangsu 210098, People’s Republic of China
* Author to whom any correspondence should be addressed.
E-mail: hjli@zjnu.cn

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Abstract
We propose a scheme to generate and stabilize one- and two-dimensional dark, bright, dark-like, bright-like solitons, and vortices with \( m = 1 \) and \( m = 2 \) in a nonresonantly incoherent pumped exciton–polariton condensate. A spatially modulating pumping is introduced, which can compensate (counteract) the loss (gain) originated from the nonlinear excitation of the stable homogeneous polariton. The numerical simulations show that the balance between the gain and loss in this scheme can support and stabilize various nonlinear modes, not just stable dark solitons which have been found in the previous studies. Our proposal may provide a way to generate, stabilize, and control nonlinear modes in the nonresonantly pumped exciton–polariton system.

1. Introduction
In the past years, exciton–polaritons in semiconductor microcavities, which can form Bose–Einstein condensates (BEC) at a few Kelvin or even at room temperature [1–7], have emerged as a suitable alternative to BEC in ultracold atomic gases. The strong light–matter interaction [8] observed in the exciton–polariton system allows it to be an ideal platform to study quantum and nonequilibrium physics and exotic properties of high-orbital condensates [9]. The complexity and the strong nonlinearity are also the key features for the formations of various nonlinear phenomena, such as bistability [10, 11], information processing [12–14], pattern formation [15–18], quantum vortices [19–25], and spatial solitons [22, 23, 26–38], and so on.

The exciton–polaritons can be made to form condensates out of equilibrium, which are best understood as a steady-state balance between pumping and decay, rather than true thermal equilibrium [39]. The exciton–polariton condensates are described by a high-dimensional saturated nonlinear Schrödinger equation (SNLSE) with the Kerr nonlinear term and the gain and loss terms, and the formations and stability of nonlinear modes in this system are of particular interest. For the case of resonant coherent pumping, the bright [27, 28] and dark solitons [26, 29, 34] were predicted and observed in experiments. However, for the nonresonant incoherent homogeneous pumping, dark solitons are unstable, and it can only evolve in a short time in one-dimensional [32] and two dimensional [33] cases. So the spatially periodic [23], ring-shaped [18, 24, 35] and Gaussian-shaped [22, 30, 31] pumping have been proposed to stabilize these localization states. However, these nonlinear localized states are still unstable, because the balance between the nonlinear gain and the constant loss is not realized.

In the previous studies on exciton–polariton condensations, the soliton is generated by the initial random noise, and formed by assuming a balance between gain and loss [30]. However, when we discuss the nonlinear excitations (such as solitons and vortices) on the basis of homogeneous condensates, it is difficult to realize the balance between the nonlinear gain and the invariable constant loss. The purpose of this paper is threefold. Firstly, based on the homogeneous steady state, we propose a spatially distribution pumping to obtain the balance between the nonlinear gain and the invariable loss. Secondly, we directly
solve the nonlinear steady state, rather than obtaining the solutions by evolving an initial noise. Finally, after introducing the spatial distribution pumping, we can obtain stable dark soliton, bright soliton, bright-like, dark-like soliton, and vortices with \( m = 1 \) and \( m = 2 \) by our numerical method.

In this paper, to achieve the balance between gain and loss, we construct an incoherent pumping which consists of a homogeneous pumping and a Gaussian pumping. Then, various nonlinear stable states are found directly. The stability of nonlinear modes is proved by the linear stability analysis and the evolution. Thanks to the spatial distribution pumping and the method of finding solution, we not only find the dark soliton, bright ground soliton, bright-like soliton, and dark-like soliton in one-dimensional system, the bright soliton, bright-like soliton, dark-like soliton, and the vortices with \( m = 1 \) and \( m = 2 \) in two-dimensional system, but also prove their stability by the evolution and the linear stability analysis.

The article is arranged as follows. In section 2, we give an introduction of the model under study. In section 3, various nonlinear stable states, their properties, and stability are studied for the one- and two-dimensional systems, respectively. In the last section, we summarize the main results.

2. Model

Using a mean-field theory, we describe the dynamics of two dimensional exciton–polariton condensates by a dissipative Gross–Pitaevskii equation for the polariton field \( \Psi \), coupled to the rate equation of the density of the excitonic reservoir \( n^e \)

\[
\frac{i\hbar}{\partial t} \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + g_C|\Psi|^2 + g_R n^R + \frac{i\hbar}{2} (R n^R - \gamma_C) \right] \Psi, \quad (1a)
\]

\[
\frac{\partial n^R}{\partial t} = P_u(r) - (\gamma_R + R|\Psi|^2) n^R, \quad (1b)
\]

where \( P_u(r) \) is the exciton creation rate determined by the incoherent pumping profile, \( m^* \) is the effective mass of lower polaritons, \( \gamma_C \) and \( \gamma_R \) are the polariton and exciton loss rates, \( R \) is the condensation rate, \( g_C \) represents the nonlinear interaction between polaritons, \( g_R \) is the interaction between polaritons and reservoir excitons. The incoherent pumping is constructed by \( P_u(r) = P_0 + P_1 \exp\left[-(x^2 + y^2)/w_0^2\right] \), which consists of a cw field \( P_0 \) and Gaussian field, respectively.

Equations (1a) and (1b) can be written into the dimensionless forms

\[
\frac{i}{\partial s} \frac{\partial u}{\partial s} = -\nabla_u^2 u - \sigma_1 |u|^2 u - \sigma_2 n u + i(\sigma_3 n - \sigma_4) u, \quad (2a)
\]

\[
\frac{\partial n}{\partial s} = \sigma_5 P(r) - \sigma_3 (1 + \sigma_5 |u|^2) n, \quad (2b)
\]

where \( s = t/\tau_0 \), \( (\xi, \eta) = (x, y)/R_u \), \( u = \Psi/\psi_0 \), \( n = n_R/n_R^0 \), \( w = w_0/R_u \), \( \nabla_u^2 = \partial^2/\partial \xi^2 + \partial^2/\partial \eta^2 \), and \( P(r) = \sigma_7 + \sigma_8 \exp(-\frac{r^2}{w_0^2}) \), with \( \tau_0 = (2m^* R_u^2)/\hbar \), \( R_u \), \( \psi_0^2 \) and \( n_R^0 \) being, respectively, characteristic time, beam radius, the condensate density, and the reservoir density. These coefficients in equations (2a) and (2b) are given by \( \sigma_1 = -g_C\psi_0^2 \gamma_0/h \), \( \sigma_2 = -g_R n_R^0 \tau_0/h \), \( \sigma_3 = R n_R^0 \tau_0/2 \), \( \sigma_4 = \gamma_C \tau_0/2 \), \( \sigma_5 = \gamma_R \tau_0 \), \( \sigma_6 = R \psi_0^2 / \gamma_R \), \( \sigma_7 = \frac{\tau_0}{w_0^2} \), and \( \sigma_8 = \sigma_5 P_1/P_0 \). Calculated by the parameters in reference [32], the characteristic time is \( \tau_0 = 5.45 \times 10^{-10} \) s.

3. Soliton solutions and properties

In order to discuss the soliton solutions of equations (2a) and (2b), their properties and stability, we first assume a plane-wave solution \( u = u_0 \exp(i\beta s) \) and the constant reservoir density \( n = n_0 \) of equations (2a) and (2b), and consider a homogeneous pumping \( P(r) = \sigma_7 \) (i.e. \( \sigma_8 = 0 \) or \( P_1 = 0 \)). When the pumping is weak, the reservoir density \( n_0 = \sigma_7 \), and the condensate \( u_0 = 0 \). With the increase of the pumping, the balance between loss and gain is obtained at the pumping threshold value \( P_{th} = \sigma_7 = \frac{\beta}{\sigma_3} \). Above the threshold value, the steady homogenous condensate density is given by

\[
u_0^2 = \frac{\sigma_3 \sigma_7 - \sigma_4}{\sigma_4 \sigma_6}, \quad (3)
\]

the condensate dimensionless energy is

\[
\beta = \sigma_3 u_0^2 + \frac{\sigma_2 \sigma_7}{1 + \sigma_6 u_0^2}, \quad (4)
\]

here, the reservoir density \( n_0 = \frac{\sigma_1}{\sigma_3} \).
Above the threshold pumping, we can consider the nonlinear excitation on the basis of homogeneous condensates. From the last term \(i(\sigma_n - \sigma_s) u\) of equation (2a), one can find that as \(u\) and \(n\) are the soliton profile being spatial dependence, the constant loss \(\sigma_s\) can not be balanced by the nonlinear saturated gain term \(\sigma_n\) directly, here we assume \(n = n(r) = \frac{\sigma_n}{\sigma_n - \sigma_s}\). Thus the spatial distribution pumping, i.e. Gaussian inhomogeneous pumping, is proposed to compensate the denominator of the nonlinear saturated gain term. Of course, the periodic-, Bessel- inhomogeneous pumping can also be used to realized the function.

To obtain the stationary soliton solutions, we substitute \(u(\xi, \eta, s) = \psi(\xi, \eta)e^{i\lambda s}\) and \(n(\xi, \eta, s) = n'(\xi, \eta)\) into equations (2a) and (2b), and obtain

\[
-\beta \psi + \nabla^2 \psi + \sigma_1 |\psi|^2 \psi + \sigma_2 n' \psi - i(\sigma_3 n' - \sigma_4) \psi = 0, \tag{5}
\]

here, \(n' = \frac{n \rho(\xi)}{1 + \sigma_s |\psi|^2}\). By using the Newton conjugate gradient method [40], the profiles and power

\[
P = \int_{-\infty}^{+\infty} |\psi|^2 d\xi \, d\eta \quad \text{or the renormalized power} \quad P = \int \int_{-\infty}^{+\infty} \|\psi\|^2 - |\psi_n|^2 d\xi \, d\eta \quad \text{(mainly for the solution with background)}
\]

of the soliton solutions are obtained in the following sections, here \(\psi_0\) is the amplitude of the background. Once the soliton solution \(\psi\) is obtained, one can analyze its stability by including a perturbation

\[
u(\xi, \eta, s) = \{\psi(\xi, \eta) + \epsilon [v_1(\xi, \eta)e^{i\lambda s} + v_2(\xi, \eta)e^{i\lambda s}]\}e^{i\lambda s}, \tag{6a}
\]

\[n(\xi, \eta, s) = n' + \epsilon [v_3(\xi, \eta)e^{i\lambda s} + v_4(\xi, \eta)e^{i\lambda s}]\]

where \(v_1, v_2\) and \(v_3\) are the normal modes, and \(\lambda\) is the corresponding eigenvalue of the perturbation. Substituting equations (6a) and (6b) into equations (2a) and (2b), one obtains the following linear eigenvalue problem

\[
\begin{pmatrix}
L_1 & L_2 & L_3 \\
-L_2 & L_1 & L_3 \\
L_4 & L_3 & L_5
\end{pmatrix}
\begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix}
= i\lambda
\begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix}, \tag{7}
\]

with \(L_1 = -\beta - \nabla^2 - 2\sigma_1 |\psi|^2 - \sigma_2 n' + i(\sigma_3 n' - \sigma_4)\), \(L_2 = -\sigma_1 |\psi|^2\), \(L_3 = -\sigma_2 n + i\sigma_3 n', L_1 = -i\sigma_3 n' |\psi|^2\), which can be solved numerically by using the Fourier collocation method [40]. The stable soliton solution \(\psi\) is characterized by the real parts of all the eigenvalues being negative or zero. The stability is then proved by the split-step Fourier evolution method. In generally, we use the parameters \(\sigma_1 = -1, \sigma_2 = 0.3, \sigma_3 = 0.15, \sigma_4 = 1, \sigma_5 = 4, \sigma_7 = 1\), which are obtained by substituting the parameters in [32] into these formulas below equation (2a), and \(w = 5\).

3.1. The soliton solutions for one-dimensional system

We now present the soliton solutions of equation (5) for one-dimensional system, and check their stability by using the numerical simulations. Under the homogeneous pumping, the condensate is steady and homogeneous, and the nonlinear modes are excited on a homogeneous background of the steady condensate wave function for the bright and dark solitons. Once the condensate wave function is the localization of the obtained steady states by solving equation (5). The designed pumping can suppress the imaginary part of the exciton–polariton system available, though an absolute balance of the gain and loss can not be realized.

Using the Newton conjugate gradient method [40] and the trial solution [41], we first find the nonlinear steady state of equation (5), then make the linear stability analysis by equation (7) and study the evolutions of the obtained steady states by numerically solving equations (2a) and (2b). The dark solitons and their stability with \(\beta = 0.1\) are shown in figure 1. From the profile of dark soliton, since a hump contributes to the nonlinear saturated gain term in the denominator, the coefficients of inhomogeneous pumping \(\sigma_s < 0\) are taken to reduce the gain. Figures 1(a) and (b) illustrate the power and stability curves as a function of the intensity of inhomogeneous pumping \(\sigma_s\). From the stability curves, one can find the negative inhomogeneous pumping stabilizes the dark solitons. In figures 1(c1)–(f1), the red solid line (blue dashed–dotted line) denotes the profile \(|\psi|\) (the phase \(\phi\) of the steady state by solving equation (5)).

The stability is proved further by a numerical evolution of equations (2a) and (2b), and adding a random perturbations into the initial value of evolution, i.e., the initial value is taken as \(u(s = 0, \xi, \eta) = \psi(\xi, \eta)\) and \(n(s = 0, \xi, \eta) = n'(\xi, \eta)\) where \(\epsilon = 0.1\) and \(f_{1,2}\) are the random variables uniformly distributed in the interval \([0, 1]\), and \(s = 100\) denotes 54.5 ns. Comparing with the stability of the dark solitons obtained in [32], the evolving stability is improved, which results from the introduced inhomogeneous pumping and the calculation method. Here, the initial conditions for the evolutions are actually closer to the physical reality than ones in [32]. In figures 1(c2)–(f2), the projections
of the evolution are shown in the left panels, the red solid and blue dashed-dotted lines denoted the profile and phase after propagating the time $s$ are shown in the right panels.

The phase jump is the typical characteristics of dark soliton. From figures 1(e2)–(f2), it is seen that there are some oscillations of phase near $\xi = 0$ at certain time, but they disappear and a reverse of phase happen for a long evolution time, and the phase jump is clear. It should be noted that the dark soliton without the inhomogeneous pumping [32] or with the positive inhomogeneous pumping decays very quickly in a short time. For the negative inhomogeneous pumping, the dark solitons are stable when reaching the threshold value $\sigma_8 = -0.2$ as shown in figures 1(b) and (e1)–(f1).

When we find the stable state solution, we use the periodic boundary condition for bright soliton and the nonlinear modes with uniform background. For the dark soliton, the same boundary condition is also taken by taking $u(\xi) = u(\xi) - u(\xi - \xi_0)$ as the solution, where $u(\xi)$ and $u(\xi - \xi_0)$ are both dark soliton solutions of equation (5) due to the translational invariance of equations (2a) and (2b). If $\xi_0$ is enough large, $u(\xi) - u(\xi - \xi_0)$ is also the solution, thus, the periodic boundary condition is reasonable. And the finite boundary condition

$$u(s, \xi) = \begin{cases} u_0' - u_0' \exp \left[ -\frac{(\xi - \xi_0)^2}{w^2} \right] - u_0' \exp \left[ -\frac{(\xi + \xi_0)^2}{w^2} \right] & -\xi_0 \leq \xi \leq \xi_0 \text{ and in the neighbourhood of } \pm \xi_0 \\ 0 & \xi < -\xi_0 \text{ or } \xi > \xi_0, \end{cases}$$

is used to obtain the evolution results. Where we take $\xi_0 = 80$ and $u_0'$ is the value of the stable state in the neighborhood of $\pm \xi_0$. We also adopt the similar boundary condition for the two-dimensional system.

Generally, it is difficult to obtain the dark and bright solitons in the same system, since the dark soliton results from the defocusing nonlinearity and the bright soliton results from the focusing nonlinearity. However, in the exciton–polariton condensate system, there exists the Kerr nonlinearity, saturated nonlinearity and the nonresonant pumping simultaneously, which can support bright solitons.

In figure 2, we show the bright soliton with the uniform background (the bright-like soliton) and the dip-type soliton (the dark-like soliton sharing a similar intensity profile of the dark soliton without the phase jump). Figures 2(a) and (b) show the power and stability curves as a function of the intensity of inhomogeneous pumping $\sigma_8$. Figures 2(c1)–(e1) show the initial profiles by solving the steady state equation (5), and figures 2(c2)–(e2) corresponds to the evolutions. It is seen from the power curve in figure 2(a) that the bright-like soliton exists for $\sigma_8 > 0$, the dark-like soliton for $\sigma_8 < 0$, but no bright-like
Figure 2. (a, b) Power and stability curves of nonlinear modes as a function of $\sigma_8$. (c1–e1) Profiles of the bright-like soliton and the dark-like soliton with the different $\sigma_8$ marked by the red dots and the letters c–e in panels (a) and (b). The red solid line (blue dashed-dotted line) denote the profile $|\psi|$ (the phase $\phi$) of nonlinear steady state obtained by solving equation (5). The real part and imaginary part of amplitude $\psi$ are shown in the inset. (c2–e2) The projections and profiles of the evolution results. In the left panels, the projections of the evolution are shown. The profile and phase of evolution at the special time $s$ marked by the green line of the left panels are shown in the right panels.

3.2. The soliton solutions for two-dimensional system

It is well known that the solitons of the high-dimensional SNLSE are generally unstable except for a bright ground soliton. However, it is interesting to find stable high-dimensional solitons in the exciton–polariton model (2a) and (2b), because the SNLSE has the imaginary part and the Kerr nonlinearity. Except for the vortex, there has been no report on the high-dimensional stable spatial solitons. In this subsection, we study the interaction between the imaginary part and the Kerr- and saturated nonlinearities and find other stable high-dimensional spatial solitons.

The two dimensional bright-like soliton and dark-like soliton are shown in figure 3. Figures 3(a) and (b) show the power and stability curves as a function of the intensity of inhomogeneous pumping $\sigma_8$, and figures 3(c1)–(e1) and (c2)–(e2) are the profiles and evolutions, respectively. The power curve plotted in figure 3(a) is different from the case in figure 2(a), but sharing a similar tendency. Once use the homogeneous incoherent pumping ($\sigma_8 = 0$), the bright-like soliton and the dark-like soliton can not be obtained. It is seen from the stability curve in figure 3(b) that the stability is strengthened with the increasing of $|\sigma_8|$ for $\sigma_8 < 0$, and all above these two-dimensional nonlinear modes are stable. Although the
stability curve in figure 3(b) shows that the bright-like soliton are unstable, the numerical evolution in figure 3(c2) shows that the profile of the soliton is still conserved at evolution time 436 ns.

If a phase factor $\exp(i m \theta)$ ($\theta = \arctan \frac{\eta}{\xi}$ is the azimuthal coordinate) is imprinted onto the initial state, the vortex steady state can be found. Though there exists the vortex in the exciton–polariton system, it is still interesting to study how to enhance stability of vortex and find a stable high-charged vortex. For balancing the nonlinear gain and constant loss, we still search the vortex with the uniform background, rather than ones in [22].

In figure 4, we show the vortex with $m = 1$ and its stability. The power and stability curves as a function of the intensity of inhomogeneous pumping $\sigma_8$ are shown in figures 4(a) and (b), and figures 4(c1)–(f1) and (c2)–(f2) are the profiles and the evolution, respectively. From figure 4(b), we find the vortices with $m = 1$ become more stable with increasing $|\sigma_8|$ for $\sigma_8 < 0$, and unstable for $\sigma_8 > 0$. Compared figure 4(c1) with its evolution result shown in figure 4(c2), the vortex will disappear for a long evolution time when $\sigma_8 = 0$. Through the linear stability analysis in figure 4(b), one can find the vortex in figure 4(d1) is unstable for $\sigma_8 = 0$. Because the value $\text{Re}(\lambda)$ is so small, the profile and phase are still remained after evolving 436 ns, which means that the vortex of $m = 1$ is stable even in the absence of the inhomogeneous pumping. By examining the initial phase with the evolution results, one can find the vortex rotates. For $\sigma_8 = -3.0$, the vortex has a core at the center, characterized the phase shown in the inset. The core disappears after a long evolution time, but the vortex can be considered to be stable as its profile and phase are still remained. Thus, for the case of $m = 1$, all the vortices with uniform background are stable so long as $\sigma_8 \leq 0$.

Furthermore, we investigate whether the above conclusions are suitable for the higher-charged vortices. In figure 5, the vortices of $m = 2$ and their properties are shown. Figures 5(a) and (b) are the power and stability curves as a function of the intensity of inhomogeneous pumping $\sigma_8$. Figures 5(c1)–(f1) and (c2)–(f2) are the profiles and the results of evolution, respectively. Comparing figures 5(a) and (b) with figures 4(a) and (b), one can find the tendency of power curves is same, whereas the stabilities are different. Through the stability analysis and evolving results, one can find the vortex with $\sigma_8 > 0$ and one with
Figure 4. (a, b) Power and stability curves of the vortices with $m = 1$ as a function of $\sigma_8$. (c1–f1, c2–f2) Profiles and evolution results of the vortices with the different $\sigma_8$ marked by the red dots and the letters c–f in panels (a) and (b). These insets show the phases of vortices.

Figure 5. (a, b) Power and stability curves of the vortices with $m = 2$ as a function of $\sigma_8$. (c1–f1, c2–f2) Profiles and evolution results of the vortices with the different $\sigma_8$ marked by the red dots and the letters c–f in panels (a) and (b). These insets show the phases of vortices.
Figure 6. (a, b) The power and stability curves of the one- (the red solid line) and two-dimensional (the blue dashed line) bright ground soliton as a function of $\sigma_8$. (c1, c2) The profile and its evolution results of the one-dimensional bright ground soliton marked by the red dot (and the letter c) in panel (a) and (b). (d1, d2) The profile and its evolution results of the two-dimensional bright ground soliton marked by the blue dot (and the letter d) in panel (a) and (b).

$\sigma_8 = 0$ are unstable, which break down into two vortices, and are stable for $\sigma_8 < -0.1$. So, we see that the inhomogeneous pumping is very important for stabilizing the higher-charged vortices.

3.3. Some special soliton solutions

By taking another sets of parameters, $\sigma_3 = 0.2$ achieved by setting $P_0, \beta = 0.35$, and the other parameters being same, we obtain one- and two-dimensional bright ground solitons shown in figure 6. The dipole soliton and the vortex with the zero-background can also be found, but they are unstable. Though there is a wide existence interval for the bright ground soliton from the power curves in figure 6(a), very narrow interval of $\sigma_8$ can be found to stabilize these solitons, only in the neighborhood of the intersection point shown in figure 6(b). From these results, we know the Gaussian potential possesses the trapping function for these nonlinear modes, but its balance function is more important for the stability of soliton. From these evolution results, we know that it is easier to generate and stabilize the nonlinear modes with the uniform background than ones with the zero-background.

After some numerical simulations, we find all above results are independent on the width of Gaussian pumping and the function form, the similar nonlinear solution for equations (1a) and (1b) can be obtained by replacing the Gaussian pumping with other pumping, such as Bessel pumping, periodic pumping, and so on. The uniform part in the pumping is also very important to stabilize the nonlinear modes, because it can balance the constant loss.

4. Summary

In conclusion, we have proposed a scheme to generate and stabilize the nonlinear modes by introducing the incoherent pumping in the exciton–polariton condensate. The introduced pumping contains a homogeneous part that balances the constant loss, in addition to an inhomogeneous part that compensates the gain or loss caused by the denominator of nonlinear saturated gain terms. The bright, dark, bright-like, and dark-like solitons in one dimensional system, and the bright, bright-like, dark-like solitons and vortices with $m = 1$ and $m = 2$ in two-dimensional system are found. It is demonstrated that the stabilities of these nonlinear modes can be realized by engineering the inhomogeneous pumping. The realizations of the diversiform and stable nonlinear modes are resulted from the balance between the nonlinear saturated gain and the constant loss available in the exciton–polariton system. The results presented here may be useful for understanding the physical properties of condensates out of equilibrium, and guiding experimental studying of condensate soliton, which may have potential applications in polariton condensates for information storage and processing or quantum simulators.
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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

Wen Wen https://orcid.org/0000-0002-0697-4522
Hui-jun Li https://orcid.org/0000-0002-8280-3450

References

[1] Deng H, Weihs G, Santori C, Bloch J and Yamamoto Y 2002 Science 298 199
[2] Kasprzak J et al 2006 Nature 443 409
[3] Ballili R, Hartwell V, Snoke D, Pfeiffer L and West K 2007 Science 316 1007
[4] Deng H, Haug H and Yamamoto Y 2010 Rev. Mod. Phys. 82 1489
[5] Christopoulos S et al 2007 Phys. Rev. Lett. 98 126405
[6] Christmann G, Buttì R, Fetlin E, Carlin J-F and Grandjean N 2008 Appl. Phys. Lett. 93 051102
[7] Baumberg J et al 2006 Nature 443 409
[8] Balili R, Hartwell V, Snoke D, Pfeiffer L and West K 2007 Science 316 1007
[9] Deng H, Haug H and Yamamoto Y 2010 Rev. Mod. Phys. 82 1489
[10] Wang Y et al 2011 Nat. Phys. 7 681
[11] Balili R, Hartwell V, Snoke D, Pfeiffer L and West K 2007 Science 316 1007
[12] Deng H, Haug H and Yamamoto Y 2010 Rev. Mod. Phys. 82 1489
[13] Christmann G, Buttì R, Fetlin E, Carlin J-F and Grandjean N 2008 Appl. Phys. Lett. 93 051102
[14] Christmann G, Buttì R, Fetlin E, Carlin J-F and Grandjean N 2008 Appl. Phys. Lett. 93 051102
[15] Christmann G, Buttì R, Fetlin E, Carlin J-F and Grandjean N 2008 Appl. Phys. Lett. 93 051102
[16] Bajoni D, Semenova E, Lemaitre A, Bouchoule S, Wertz E, Senellart P, Barbay S, Kuszelewicz R and Bloch J 2008 Phys. Rev. Lett. 101 266402
[17] Goblot V, Nguyen H S, Carusotto I, Galopin E, Lemaître A, Sagnes I, Amo A and Bloch J 2008 Phys. Rev. Lett. 101 266402
[18] Liew T C H, Kavokin A V and Shelykh I A 2008 Phys. Rev. B 75 241301(R)
[19] Kavokin A V and Shelykh I A 2008 Phys. Rev. B 75 241301(R)
[20] Rubo Y 2007 Phys. Rev. Lett. 99 106401
[21] Voronova S N and Lozovik Y E 2012 Phys. Rev. B 86 195305
[22] Ostrovskaya E A, Abdullaev J, Desyatnikov A S, Fraser M D and Kivshar Y S 2012 Phys. Rev. A 86 013636
[23] Ma X, Egorov O A and Schumacher S 2017 Phys. Rev. Lett. 118 157401
[24] Ma X and Chang S 2017 Phys. Rev. B 95 235301
[25] Dominici L et al 2018 Nat. Commun. 9 1467
[26] Yulin A V, Egorov O A, Lederer F and Skryabin D V 2008 Phys. Rev. B 78 061801(R)
[27] Egorov O A, Skryabin D V, Yulin A V and Lederer F 2009 Phys. Rev. Lett. 102 155304
[28] Sich M et al 2012 Nat. Photon. 6 50
[29] Grosso G, Nardini G, Morier-Genoud F, Léger Y and Deveaud-Plédran B 2012 Phys. Rev. B 86 020509(R)
[30] Ostrovskaya E A, Abdullaev J, Fraser M D, Desyatnikov A S and Kivshar Y S 2013 Phys. Rev. Lett. 110 170407
[31] Tanese D et al 2013 Nat. Commun. 4 1749
[32] Xue Y and Matuszewski M 2014 Phys. Rev. Lett. 112 216401
[33] Smirnov L A, Smirnov N A, Ostrovskaya E A and Kivshar Y S 2014 Phys. Rev. B 89 235310
[34] Cilibriani P, Ohashi H, Noguchi K, Askitopoulos A, Langbein W and Lagoudakis P 2014 Phys. Rev. Lett. 113 103901
[35] Kulczykowski M, Bobrovska N and Matuszewski M 2015 Phys. Rev. B 91 245310
[36] Koll G R 2017 Eur. Phys. J. Plus 132 530
[37] Opala A, Pieczarka M, Bobrovska N and Matuszewski M 2018 Phys. Rev. B 97 155304
[38] Hivet R et al 2012 Nat. Phys. 8 724
[39] Keeling J and Berloff N G 2008 Phys. Rev. Lett. 100 250401
[40] Yang J 2011 Nonlinear Waves in Integrable and Nonintegrable Systems (Philadelphia, PA: SIAM)
[41] Zhang K, Liang Y Z, Lin J and H-j L 2018 Phys. Rev. A 97 023844