Transition to centrifuging granular flow in rotating tumblers: a modified Froude number

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Abstract. Centrifuging of granular material in a partially filled rotating circular tumbler occurs when particles are flung outward to form a ring of particles at the periphery of the tumbler rotating as a solid body. The critical rotation speed for centrifuging was studied experimentally in a quasi-two-dimensional tumbler as a function of particle diameter, tumbler fill fraction and interstitial fluid. A qualitative numerical study using the discrete element method was also conducted to obtain a better understanding of the impact of friction on the transition. Experimental results show that the critical rotational speed for dry systems is not affected by the particle diameter unless the fill fraction is above 75\%, where endwall friction begins to play a significant role. The critical speed is proportional to \((1 - \phi)^{-1/4}\), where \(\phi\) represents the tumbler fill fraction. The angle of repose, which represents inter-particle friction, also affects the transition to centrifuging. Finally, the interstitial fluid, or rather the density difference between the particles and the interstitial fluid, affects the measured critical speed. Correction terms for the critical rotational speed are proposed to more accurately characterize the transition to centrifuging for granular flow in rotating tumblers, resulting in a modified Froude number.

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1. Introduction

The nature of the flow of granular materials in a partially filled rotating circular tumbler, a canonical granular system, is often characterized by the Froude number, \( Fr = \omega^2 R/g \), where \( \omega \) is the angular speed of the tumbler, \( R \) its radius and \( g \) the acceleration due to gravity \([1]\). The Froude number is a measure of the ratio of the inertial forces to the gravitational forces. Although there are several viewpoints, the flow can be categorized into five regimes \([1–4]\) depending on the magnitude of the Froude number. Avalanching, in which wedges of particles intermittently avalanche down the slope when the angle of repose reaches a critical value, occurs at very low rotational speeds (low \( Fr \)). As the rotational speed increases, particles flow in a flat, continuously flowing layer called the rolling regime. Higher rotational speeds (or higher \( Fr \)) result in a curved flowing layer called the cascading regime. At a high enough rotational speed, a situation known as cataracting flow occurs, where the particles lose contact with one another as they are thrown off the upstream end of the flowing layer, landing back on the bed of particles further downstream after a short period of free flight. Finally, at still higher rotational speeds (higher \( Fr \)), the particles are flung outward centrifugally to form a ring of particles at the outer wall of the tumbler, rotating as a solid body. It is the transition to this centrifuging regime that we consider here.

The centrifuging regime for granular materials has been largely overlooked and understudied, partly because it is regarded as a trivial state, since there is no relative motion between particles. Newtonian mechanics predicts centrifuging to occur when the gravitational and centripetal forces balance each other (\( Fr = 1 \)). Thus, the critical rotational speed for centrifuging is

\[
\omega_c = \sqrt{\frac{g}{R}}.
\]  

While the Froude number provides an approximation of when centrifuging flow occurs in a rotating tumbler, it only takes into account \( \omega, R \) and \( g \). A handful of other system parameters could play a role in determining the critical rotation speed for centrifuging. For instance, to account for the size of the particles, the critical speed would be

\[
\omega_c = \sqrt{\frac{2g}{D-d}}.
\]
where $D$ is the tumbler diameter and $d$ is the particle diameter [5]. This equation is strictly valid only for a monolayer of particles at the wall of the tumbler and reduces to equation (1) in the limit of $d/D \rightarrow 0$. To account for the volume fill fraction $\phi$ of material within the tumbler exceeding a monolayer upon centrifugation to the tumbler wall, the critical rotation speed would be [6]

$$\omega_c = \sqrt{\frac{g}{R \sqrt{1 - \phi}^2}}, \quad (3)$$

which reduces to equation (1) in the limit of the fill fraction approaching zero. Finally, a modified Froude number has been proposed that would account for buoyancy in a liquid granular system with an interstitial fluid other than air. In this case, the critical speed for centrifuging is

$$\omega_c = \sqrt{\frac{g(\rho_p - \rho_t)}{R \rho_p}}, \quad (4)$$

where $\rho_{p, t}$ is the density of the particles or interstitial fluid, respectively [7]. This expression is valid for $\rho_p > \rho_t$ and again reduces to equation (1) when $\rho_t/\rho_p \rightarrow 0$, as is the case when the interstitial fluid is air.

In spite of these various proposals for the critical speed for centrifuging, there appears to be no systematic study in which a variety of parameters have been varied. Here, we use experiments and numerical computations to investigate the transition to the centrifuging regime for granular materials in rotating quasi-two-dimensional (2D) and 3D circular tumblers.

2. Methods

Dry granular experiments were conducted in a quasi-2D tumbler of diameter $D = 280$ mm and thickness $t = 9$ mm. Wet granular experiments were conducted in a quasi-2D tumbler of diameter $D = 175$ mm and thickness $t = 7$ mm. The tumblers were rotated about their axes by a dc stepper motor with a planetary gear drive controlled by a computer. The tumbler and endwalls were clear acrylic so that imaging was possible using a digital camera. Monodisperse clear spherical glass beads with diameters of 0.6, 1, 2, 3 and 4 mm and a density of $\rho = 2400$ kg m$^{-3}$ and monodisperse steel beads with diameters of 1 and 2 mm and a density of $\rho = 7500$ kg m$^{-3}$ were used. The tumbler fill fractions, measured using a graduated cylinder, were $\phi = 0.10$, 0.25, 0.40, 0.50, 0.60, 0.75 and 0.90. In a typical experiment, the rotational speed of the tumbler was increased from rest to the operational speed in about 5 s while observing the particles, the process being repeated several times to bracket the critical rotation speed to within $\pm 2\%$. The critical rotation speed was recorded when the particles were centrifuged to the periphery of the tumbler (figure 1(c)) with no particles falling off the ring of particles (as occurs in figures 1(a) and (b)). Identical results on the critical speed were achieved in the case of quasi-static acceleration. In many cases, the critical rotation speed was confirmed by slowly decreasing the rotational speed from just above the critical speed to zero in about 25 s until the particles fell off the ring of centrifuged particles.

To further investigate the effects of friction on the angle of repose and the critical speed for centrifuging, we simulated 2 mm monodisperse beads in a half-full, $\phi = 0.5$, quasi-2D circular tumbler with a diameter of $D = 70$ mm, using the discrete element method [8]. The method uses an explicit, constant time step to integrate Newton’s second law to describe the translational and rotational motion of individual ‘soft’ particles. As in our previous simulations [9–11],
Figure 1. A quasi-2D tumbler half-filled with dry monodisperse 2 mm glass beads (a, d) rotating clockwise at the critical speed of \( \omega_c = \sqrt{g/R} \) (equation (1)), (b, e) rotating at the critical speed given by equation (3) or (c, f) rotating above the critical speed such that the bulk granular material is centrifuging. ((a–c)—Experimental images with tumbler diameter \( D = 280 \) mm; (d–f)—numerical simulation images for \( D = 70 \) mm with red particles representing the tumbler wall.)

The centrifuging regime is defined as occurring when the bulk of the material is pushed against the tumbler wall with no relative motion between particles. The situation for a half-filled tumbler at \( Fr = 1 \), corresponding to equation (1) for the critical speed, is not centrifuging, but cataracting, in which particles lose contact with one another and are in free flight, as shown in figure 1(a), although much of the granular material is pushed against the tumbler wall. If the fill fraction is taken into account when calculating the critical speed, equation (3), more of the...
The measured critical rotation speed as a function of the fill fraction is shown for five different dry monodisperse glass samples. Error bars (typically of the order of the size of the symbols) are one standard deviation on either side of the mean. The theoretical critical values are shown by the curves plotted using equations (3) and (5) with an average angle of repose of $\beta_s \approx 27^\circ$. The $\star$ (red) at $\phi = 0.9$ represents the critical rotation speed for exiting centrifuging conditions for the five different bead sizes when decelerating from above the critical speed required for centrifuging flow.

3.1. Dry granular systems

The measured critical rotation speeds during acceleration for glass beads of diameter $d = 0.6, 1, 2, 3$ and $4$ mm at tumbler fill fractions $0.1 \leq \phi \leq 0.9$ are shown in figure 2. For low fill fractions ($\phi < 0.75$) the particle diameter does not affect the critical rotation speed for centrifuging. For example, at a fill fraction of $\phi = 0.25$, the critical speed for all particle diameters was 120 rpm, whereas at a fill fraction of 0.50, the critical speed was 135 rpm. However, at the highest fill fractions ($\phi \geq 0.75$), the critical speed for centrifuging depends on the particle diameter. For example, the average critical speed for $\phi = 0.75$ was $165 \pm 10$ rpm, with the critical speed being lower for smaller particles and higher for large particles.

The dependence of the critical speed on particle diameter at high fill fractions is a consequence of the frictional forces at the tumbler endwalls. As the rotational speed of the tumbler increases, the bulk material has to overcome not only gravitational forces but also frictional forces in order to reach the centrifuging regime, thus requiring higher rotational speeds. When the tumbler was slowly decelerated at a constant rate after the material had
Table 1. The angle of repose for spherical glass and steel beads measured in a quasi-2D heap flow device.

| Material | Diameter (mm) | Angle (deg) |
|----------|---------------|-------------|
| Glass    | 0.6           | 24.0 ± 0.3  |
|          | 1.0           | 25.0 ± 0.5  |
|          | 2.0           | 26.9 ± 1.6  |
|          | 3.0           | 28.0 ± 0.6  |
|          | 4.0           | 30.0 ± 0.4  |
| Steel    | 1.0           | 29.2 ± 1.3  |
|          | 2.0           | 21.5 ± 1.0  |

reached the centrifuging regime, the material remained in the centrifuging regime at speeds slightly below those for acceleration for larger particles. This is shown by the star data point for $\phi = 0.9$ in figure 2. For other fill fractions the critical speed for deceleration is similar to that for acceleration. Thus, for the largest particles and the highest fill fraction, endwall friction restricted the bulk of particles from entering the centrifuging regime until higher speeds were reached.

It is evident that the critical speed indeed depends on the fill fraction of material in the tumbler, but the measured values of the critical speed for $\phi < 0.75$ are 40–50 rpm higher than the values predicted from equation (3), which is the dashed curve in figure 2. This suggests that there is another system parameter that needs to be taken into account, as described in the next section.

3.2. Friction and material angle of repose

Of the parameters noted earlier, the friction between particles has not been considered to this point. Walton and Braun [15] studied this problem via numerical simulations for a half-filled tumbler. On the basis of the Mohr–Coulomb failure criterion, they proposed the expression

$$\omega_c = \left(\frac{g}{R \sin \beta_s \sqrt{1 - \phi}}\right)^{1/2},$$

where $\beta_s$ is the material angle of repose [15]. The modification to include the fill fraction, $\phi$, was added at a later date [12].

The angle of repose of a granular mixture depends on the density, shape, coefficient of friction and dispersity of the material. Numerous studies have been dedicated to understanding how the angle of repose is related to the material properties [16–25]. The angle of repose was measured for each particle size in a quasi-2D heap flow device, and the values are shown in table 1. The experimental centrifuging data can be compared with equation (5) using the average angle of repose for the five sizes of glass beads, $\beta_s \approx 27^\circ$. It is evident from figure 2 that when the angle of repose is included, the calculated (solid curve, equation (5)) and measured critical speeds for the transition to centrifuging agree very well.

The connection between centrifuging flow and the angle of repose is not immediately obvious. However, two simple thought experiments can explain this result. First, consider the case of frictionless particles in which $\beta_s = 0$. In this case, we would expect the particles to remain at the bottom of the tumbler due to gravity no matter how fast the tumbler rotates, simply
because the particles are frictionless, slipping freely against each other and the tumbler wall. Thus, the critical rotational speed cannot be achieved (it is infinite), as predicted by equation (5) for $\beta_s = 0$. Second, consider frictional particles in the tumbler in the rolling or cascading regimes of flow, prior to centrifuging. As the angle of repose of the particles increases due to particle properties such as friction, the upper end of the free surface at the circumference of the tumbler extends higher, thus making it easier for centrifugation to be initiated. In this way, it is evident that the larger the value of $\beta_s$, the lower the value of $\omega_c$, just as predicted by equation (5).

3.3. Discrete element simulations

Discrete element simulations ($D = 70$ mm, $d = 2$ mm glass beads) allow further investigation of the effect of friction on the critical speed for a centrifuging flow. Of course, because of the computational intensity of such simulations it is only possible to consider a limited number of specific values for the parameters. Thus, it is difficult to ramp up the rotational speed until the transition to centrifuging flow is achieved as was done experimentally. Furthermore, constraints on the number of particles that can be simulated restrict the size of the tumbler (nearly 18,000 particles are included in the simulation). As a result, it is difficult to obtain an angle of repose that is identical to that in the experiments, and the centrifuged layer of particles at the periphery is only a few particles thick. This makes both quantitative comparisons to the experimental results and predictions of the critical rotational speed based on equation (5) quite difficult. Nevertheless, the effect of friction on the transition to centrifuging can be demonstrated qualitatively for a 3D tumbler of axial length $D$ with periodic endwall conditions. For the half-full tumbler simulations shown in figure 3, the critical rotational speed based on equation (1) is 160 rpm and that based on equation (3), which includes the influence of the fill fraction, is 190 rpm.

As shown in figure 3(a), the contours of the number density of particles, $N$, are uniformly distributed about the circumference of the tumbler, indicating that the material is centrifuging as a tightly packed solid body for a high coefficient of friction ($f = 0.9$) at 240 rpm. (Slight variations in the contours are a consequence of the resolution of the contour algorithm, the finite size of the particles and the local packing fraction.) When the particle coefficient of friction is reduced to $f = 0.1$, shown in figure 3(b), the material is unevenly distributed about the circumference of the tumbler with the thinnest layer of particles from one to three o’clock and a much thicker layer from five to ten o’clock. This is a consequence of looser packing throughout the entire ring of particles, like that in figure 1(a), because the particles are moving relative to one another and even falling out of the ring of centrifuged particles. However, at a higher speed, 300 rpm, for the same low coefficient of friction, shown in figure 3(c), the material becomes uniformly distributed, because it is rotating as a tightly packed solid body. Computational results for frictional endwalls and for the quasi-2D case are nearly identical. Hence, it is clear that the coefficient of friction between particles affects the transition to centrifuging flow, and apparently the angle of repose, which depends on the particle coefficient of friction, accounts for this effect.

3.4. Liquid granular systems

Finally, experiments in liquid granular systems were considered in order to study the effect of buoyancy due to the interstitial fluid on the critical speed for centrifuging. Assuming that

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Figure 3. Comparison of simulations for a half-full tumbler of axial length and diameter equal to 70 mm rotating clockwise with $d = 2.0$ mm glass particles using periodic endwall conditions. $N$ is the particle number density.

The following expression is proposed to account for the densities of the particles and interstitial medium, $\rho_p, \rho_f$, respectively. A total of five sizes of monodisperse glass or steel beads were used with water ($\rho_f = 1000$ kg m$^{-3}$) as the interstitial fluid. However, only two examples will be described (one glass and one steel).

Consider first the $d = 1$ mm spherical glass beads in water. The angle of repose for these beads from table 1 is $\beta_s = 25.0^\circ$, and the density of the glass beads is $\rho_p = 2400$ kg m$^{-3}$. The experimental data for acceleration along with equation (6) are plotted as a function of fill fraction in figure 4. The calculated critical speed for centrifuging agrees reasonably well with the data for fill fractions 0.1 $\leq \phi \leq 0.75$.

For $d = 1$ mm spherical steel beads, the angle of repose is $\beta_s = 29.2^\circ$, and the density of the steel beads is $\rho_p = 7500$ kg m$^{-3}$. As shown in figure 5, the calculated critical speed for centrifuging agrees well for the measured fill fractions for 0.25 $\leq \phi \leq 0.75$, but not...
Figure 4. The critical rotation speed for acceleration as a function of the fill fraction is shown for monodisperse $d = 1$ mm glass beads in water. The theoretical critical values based on equation (6) are shown by the solid curve.

Figure 5. The critical rotation speed as a function of the fill fraction shown for monodisperse $d = 1$ mm steel beads in water. The theoretical critical values based on equation (6) are shown by the solid curve. The ⋆ (red) at $\phi = 0.1$ represents the critical rotation speed for exiting centrifuging conditions while decelerating from above the critical speed required for centrifuging flow.

for $\phi = 0.10$. The poor agreement in this case at low fill fractions is the opposite of what was observed for dry granular systems. This arises from fluid inertia, which is amplified at low fill fractions, because a majority of the system is comprised of fluid. At these low fill fractions, particles are carried along with the tumbler upward as the centrifuging is imminent, but then fall
Figure 6. (a) Correlation between the experimentally measured critical rotational speeds to enter centrifuging flow and the theoretical rotational speeds given by equation (3). (b) Correlation between the experimentally measured critical rotational speeds and the theoretical rotational speeds given by equation (6). All glass and steel (s) beads in both air (black) and water (blue), as well as the experimentally measured speeds to exit centrifuging flow after deceleration from above the critical speed ⋆ (red), are shown.

off the tumbler wall because the rotational speed is not quite adequate for centrifugation. This leads to strong chaotic interactions between the particles and the fluid with the particle motion affecting the fluid motion and vice versa, delaying the establishment of the centrifuging ring of particles. The fluid flows and particle motions interact so strongly that centrifuging is difficult to achieve. At higher fill fractions, where the ratio of fluid to particle volume decreases, the particle inertia dominates, and the measured critical speeds agree well with calculated speeds. To verify this effect, the tumbler speed was reduced slowly for $\phi = 0.1$ with steel beads after achieving centrifuging until particles were observed to start falling out of the ring of centrifuged particles. The rotational speed when particles exited the centrifuging regime agrees well with the theoretical curve from equation (6), as shown in figure 5. Of course, in this case the interactions between the particles and the fluid are minimal, because the ring of centrifuged particles has already been established. Having several particles fall out of the centrifuging mass of particles signals transition, but their effect on the fluid is quite small.

4. Conclusions

We have shown that by including other system parameters in the expression for the critical speed for centrifuging to occur, the expression becomes much more accurate. This is most evident in the correlation between the measured and calculated speeds for centrifuging in figure 6 in the case of all of the particle sizes, tumbler diameters, fill fractions, particle densities and interstitial fluids that were measured. We compare the measured critical speed to equation (3) in figure 6(a) and to equation (6) in figure 6(b). It is quite clear that the estimate of the critical speed must include frictional and fluid effects as well as the fill fraction. For nearly all cases in figure 6(b),
the measured critical speed matches the critical speed calculated from equation (6). The only exceptions correspond to high fill fractions in air and 1 mm steel particles at the lowest fill fraction in water, although these deviations disappear for slow deceleration.

Even though equation (5) (without the fluid density correction) was proposed nearly 20 years ago, these are the first experimental data that verify the relation. Furthermore, the previous numerical studies were only carried out for a tumbler fill fraction of $\phi = 0.50$ [26], whereas the experimental data agree with this expression over a wide range of fill fractions, from $\phi = 0.10$ to 0.90. Based on these results, we propose the following modified Froude number,

$$Fr = \frac{\omega^2 R}{g} \left( \frac{\sin \beta_s \sqrt{1 - \phi}}{(1 - \rho_f / \rho_p)} \right),$$

(7)

to accurately describe the nature of granular flow in rotating tumblers. However, some questions remain. For example, what is the physical mechanism by which the angle of repose affects the transition for centrifugation, particularly since the angle of repose is a function of particle size, shape, density and surface properties? Another key question is whether the modified Froude number (equation (7)) can be used to characterize transitions for other regimes of granular flow (avalanching, rolling or cataracting) more accurately. This may be important in that it may permit better prediction of transitions, since it includes significant parameters related to the particles and the interstitial fluid in the system.

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