Kaon to Pion ratio in Heavy Ion Collisions

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The momentum integrated Boltzmann equation has been used to study the evolution of strangeness of the strongly interacting system formed after the heavy ion collisions at relativistic energies. We argue that the experimentally observed non-monotonic, horn-like structure in the variation of the $K^+/\pi^+$ with colliding energy appears due to the release of large number of colour degrees of freedom.

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Calculations based on lattice QCD [1] indicate a phase transition from hadronic matter to a deconfined state of quarks and gluons, called quark gluon plasma (QGP), at a temperature, $T_c \sim 175$ MeV. The quest for creating QGP in laboratory has led to heavy ion collision experiments being carried out at centre of mass energy ($\sqrt{s_{NN}}$) ranging from a few GeV to 200 GeV per nucleon. In the experimental side judicious choices of beam energy, colliding ions, impact parameter and other suitable variables have been used to confirm the existence of QGP [2].

The non-monotonic behaviour of $K^+/\pi^+$ with colliding energy of the nuclei [3] has led to intense theoretical activities [4–7]. In the present work, we study the evolution of the strangeness of the system formed in heavy ion collision for various initial conditions within the framework of momentum integrated Boltzmann equation.

In the QGP phase the evolution equation of strangeness is given by:

$$\frac{dr_s}{d\tau} = \frac{R_s(T)}{n_s^{eq}}[1 - r_s r_s]$$

(1)

where, $r_i = n_i/n_i^{eq}$, $n_i$ and $n_i^{eq}$ are the non-equilibrium and equilibrium densities of the species $i$ in the QGP phase. $R_i$ is the rate of production of particle $i$ at temperature $T$ and $\tau$ is the proper time. The reactions considered for the production of $s$ are: $q\bar{q} \rightarrow s\bar{s}$, $g\bar{g} \rightarrow s\bar{s}$. The cooling of the heat bath is governed by the following equation [8]:

$$\frac{dT}{d\tau} = -c_s \frac{T}{\tau} - \frac{b(r_s + r_s)}{\alpha(a + b(r_s + r_s))}$$

(2)

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Fig. 1. The time evolution of the ratio of non-equilibrium to equilibrium density of quarks/various hadrons when initial phase is assumed to be QGP ($T_i > T_c$). Here $\bar{s}$, $K^+$ etc. stand for their corresponding number densities.

where $\alpha = (1 + c_s^2)/c_s^2$, $a = 8\pi^2/(45c_s^2)$ and $b = 7\pi^2n_F/(120c_s^2)$. $c_s^2$ is the velocity of sound, $n_F$ is the number of flavours and $\dot{r}_s = dr_s/d\tau$. In case of (equilibrium) Bjorken’s scaling solution [9] the last term in Eq. 2 vanishes.

When the temperature of the QGP phase approaches the transition temperature $T_c$ due to expansion, then the $s$ and $\bar{s}$ quarks hadronise to strange hadrons like $K^+$, $K^-$, $\Lambda$ etc. An exhaustive set of hadronic reactions has been considered for the production of $K^+$ in the hadronic phase [10] (see also [11]).

The evolution of $K^+$ ($u\bar{s}$) in the mixed phase is governed by the following equation:

$$\frac{d\dot{r}_{K^+}}{d\tau} = \frac{R_{K^+}(T_c)}{n_{eq}^{K^+}} \left( 1 - r_{K^+}r_{K^+} \right) + \frac{R_{\Lambda}(T_c)}{n_{eq}^{\Lambda}} \left( 1 - r_{K^+}r_{\Lambda} \right) + \frac{R_{\Sigma}(T_c)}{n_{eq}^{\Sigma}} \left( 1 - r_{K^+}r_{\Sigma} \right) + \frac{1}{f} \frac{df}{d\tau} \left( \delta r_{\bar{s}^eq}s_{\bar{s}^eq} - r_{K^+} \right),$$

Similar coupled equations can be written for $\Lambda$ and $\Sigma$. In the above equation $f$ represents the fraction of hadrons in the mixed phase at time $\tau$. The last term stands for the hadronization of $\bar{s}$ quarks to $K^+$ [12, 13]. Here $\delta$ is a parameter which indicates the fraction of $\bar{s}$ quarks hadronizing to $K^+$. The value of $\delta = 0.5$ if we consider only $K^+$ and $K^0$ formation in the mixed phase. The initial values of $\bar{s}$ quarks are taken close to their equilibrium values. However, a small change in the initial value of $r_{\bar{s}}$ does not change the final results significantly. Even with lower initial values of $r_{\bar{s}}$ the system reaches equilibrium very fast due to their production in the high temperature heat bath. To obtain the particle density we solve the above-mentioned coupled set of differential equations numerically.

The time evolutions of $r_i$ are shown in Fig. 1. We observe a clear over saturation in number density of kaons at the end of the mixed phase. For $T_i = T_c$ or $T_i > T_c$, we get $r_{K^+} \sim 1.2 - 1.4$ at the end of the mixed phase, before it reaches the equilibrium value of 1 in the hadronic phase. For $T_i < T_c$ i.e. if the system is formed in hadronic phase, we observe that $r_{K^+} < 1$, for
\[ \delta = 0.5. \] For smaller values of \( \delta \), \( r_{K^+} \) will be even smaller. This indicates that the strangeness remains out of chemical equilibrium if the system is formed in the hadronic phase. For a given centrality and \( \sqrt{s_{NN}} \) (c.m.energy) we take particle multiplicity (\( dN/d\eta \)) and evaluate the initial temperature using

\[ T_i^3 = \frac{2\pi^4}{45\zeta(3)} \frac{1}{\pi R^2 \tau_i} \frac{90}{4\pi^2 g_{eff}} \frac{dN}{d\eta}, \]

where \( \zeta(3) \) denotes the Riemann zeta function, \( R \) is the transverse radius \( \sim 1.1(N_{\text{part}}/2)^{1/3}, \) \( N_{\text{part}} \) is the number of participant nucleons of the colliding system, \( \tau_i \) is the initial time and \( g_{eff} \) is the statistical degeneracy. For initial temperatures corresponding to each \( dN/d\eta \) (hence \( \sqrt{s_{NN}} \)) we solve for \( S_{\text{theory}} (\equiv \text{anti-strangeness/entropy}) \) which is proportional to \( K^+/\pi^+ \).

In Fig. 2 (left) we depict the variation of \( S_{\text{theory}} \) with \( \sqrt{s_{NN}} \) which shows a non-monotonic (horn-like) behaviour.

Taking \( dN_{K^+}/d\eta \), \( dN_{K^-}/d\eta \) and \( dN_{\text{tot}}/d\eta \) from experiments [3, 14] we calculate the fractional entropy carried by the strange sector, which will be proportional to \( S_{\text{expt}} (\equiv g_{s_{eff}}/g_{tot_{eff}}) \), where \( g_{s_{eff}} \) (\( g_{tot_{eff}} \)) is the degrees of freedom associated with the strange (all) hadrons. In Fig. 2 (right) we show the variation of \( S_{\text{expt}} \) with \( \sqrt{s_{NN}} \). Does lattice QCD indicate any such behaviour? To understand this we plot the quantity, \( R_{\text{lattice}} \equiv \frac{g^{(2+1)F} - g^{2F}}{g^{(2+1)F}} \) derived from lattice QCD as a function of temperature \( (T) \) in Fig. 3. Here \( g^{(2+1)F} \) and \( g^{2F} \) are the effective statistical degeneracies for \( (2+1) \) flavours and 2 flavours respectively. The quantity, \( R_{\text{lattice}} \) reflects the effective strange degrees of freedom relative to the total i.e. it is the fractional entropy carried by the strange and anti-strange particles. One clearly observes a peak near \( T_c \) (\( \sim 175 \) MeV) followed by a reduction in the values at higher temperatures. This non-monotonic behaviour indicates the deconfinement of the quarks and gluons.

In summary, we have used the Boltzmann equation to study the evolution of \( \bar{s} \) and \( K^+ \) in heavy ion collision. The horn-like structure observed in the \( K^+/\pi^+ \) arises due to the sudden increase of entropy resulting from the release of large number of (colour) degrees of freedom at the deconfinement transition point.

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**References**

![Fig. 2. The variation of anti-strangeness to entropy ratio, \( S_{\text{theory}} \) (left) and \( S_{\text{expt}} \) (right) with \( \sqrt{s_{NN}} \) (see text).](image-url)
Fig. 3. Variation of $R_{lattice}$ with temperature. Lattice QCD results taken from Ref. [1].

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