Reliability Evaluation and Optimization for Phased Mission Systems with Cascading Effects

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Abstract. The phased mission system (PMS) has been widely used in aerospace, data transmission, internet of things, which perform the entire mission with non-overlapping, consecutive and multiple phases of operation. The challenge for the PMS reliability evaluation is to consider both the dynamic of each component across phases and the functional dependence (FDEP) among components. In some cases, the component can fail individually or resulting from the cascading effect (CE) caused by internal FDEP among components. So, this paper presents the reliability evaluation method for PMSs with the CE based on recursive algorithm. Then, to find the best components’ allocation scheme with the maximized system reliability and the minimized manufacturing cost, the non-dominated sorting genetic algorithm (NSGA-II) is employed to solve the combinatorial optimization problem. Finally, the numerical example illustrates the effectiveness of proposed method and obtains solutions of the optimization model.

1. Introduction

As the developments of aerospace, nuclear power and communications networks, the phased mission system (PMS) becomes a more accurate and appropriate model to elaborate the statistic dependence and dynamics among different phases.

For the PMS with binary components, the common reliability analysis methods mainly include state space model, combination analysis and the recursive algorithm, among which the Markov-based model has lower efficiency than the latter two methods [1]. Combinatorial analysis is used to evaluate the system reliability grounded on decision diagrams and the total probability formula. Xing and Dugan [2] proposed a generalized PMS analysis method to calculate the reliability and the sensitivity of components’ performance variation. Tang et al [3] developed a new algorithm for reliability analysis of multi-mode PMS with dependencies by using binary decision diagrams. For further simplifying the model and computational complexity, Mo et al [4] applied the multi-value decision diagram to analyze the system reliability with multiple phases and fault mechanisms.

Compared with decision diagrams, there is a fully automated backward recursive algorithm to evaluate the system reliability exactly, which can take account of the time-varying and phase-dependent failure rates, and deal with different system structures and performance demand across missions. The method was firstly proposed to focus on k-out-of-n systems by Amari [5] in 2011 and was extended by Levitin [6] to deduce the system reliability of the PMS with arbitrary system structure and non-reparable binary components.
In recent years, the recursive algorithm was applied for structure optimization [7], as well as the reliability evaluation of PMSs with functional dependence (FDEP) failure mechanisms, especially for the common cause failure (CCF). Levitin and Amari [8] performed an extension of the recursive algorithm to consider the effect of CCFs caused by sudden changes in environment for the reliability analysis of PMSs. Yu et al [9] adopted this method in a phased-mission common bus system and employ a genetic algorithm (GA) to address the reliability optimization problem.

The cascading effect (CE) refers to the failure of a component in a system that causes a chain reaction or domino chain. It abounds in power grid systems, wireless sensor network systems and epidemic systems. Xing et al [10] analyzed the imperfect coverage system with the CE via the conditional failure probability of trigger elements. Zhao and Xing [11] established the reliability model of the internet of things system by decomposing the unreliability of the system, considering the CE and probabilistic competition failure.

In this paper, we evaluate the system reliability of the PMS with the CE using a recursive algorithm. To apply the recursive algorithm to the PMS with the CE, we improve it by deriving the transition probability considering the CE. Moreover, to maximize the system reliability and minimize the manufacturing cost, the non-dominated sorting genetic algorithm (NSGA-II) is utilized to search the optimal allocation strategy, which is a computationally fast and elitist multi-objective evolutionary algorithm based on a non-dominated sorting approach [12].

The remainder of the paper is organized as follows. Section 2 presents assumptions used in this paper. Section 3 describes the reliability modeling of a PMS with the CE. Section 4 presents the optimization model and lists procedures of optimization algorithm. Section 5 gives a numerical example and discusses the simulation results. Section 6 offers some conclusions.

2. Assumptions
The considered system in this paper is based on following assumptions:
1) The system mission consists of $M$ consecutive and non-overlapping phases and $N$ components.
2) Both the system and its components cannot repairable during the mission.
3) The overall mission fails if its performance cannot satisfy the requirement at any phase.
4) The environmental stress of components and the system structure can change with phases.
5) The components are phase-dependent across the entire mission and the duration of each phase is fixed.
6) The trigger component and its functional dependence components compose the cascading effect group (CEG).
7) In the case of the trigger component failed, the functional dependence components belonging to the CEG would fail.
8) A component can fail either due to an individual failure distribution or result from a CE. The individual failures of components and the CE are independent.

3. Reliability evaluation
First, the conditional reliability and unreliability during the specific phase are calculated. Second, states combinations of all binary components are generated. Third, the transition probability considering the overlapping CE is derived. Finally, we substitute the third step into recursive function to obtain the overall system reliability.

3.1. Conditional reliability of components
Generally, the component reliability always degrades over the entire mission, we need to evaluate the component’s reliability during the specific state based on the premise that it works at previous state. Levitin [8] gave the calculation methods of the conditional reliability and unreliability as follows:
\[ q_n(m) = \frac{\Phi_n(m) - \Phi_n(m-1)}{1 - \Phi_n(m-1)} \]  
(1)

\[ p_n(m) = 1 - q_n(m) = \frac{1 - \Phi_n(m)}{1 - \Phi_n(m-1)} \]  
(2)

\[ \Phi_n(m) = F_n(\alpha_{n,1}t_1 + \alpha_{n,2}t_2 + \ldots + \alpha_{n,m}t_m) \]  
(3)

where \( q_n(m) \) is the conditional unreliability of component \( n \) and \( p_n(m) \) is the conditional reliability when the component remains working from beginning to phase \( m-1 \). \( \Phi_n(m) \) is the unreliability of component \( n \) at the end of phase \( m \). \( F_n \) is the baseline failure distribution of component \( n \). \( \alpha_{n,m} \) is the stress-dependent acceleration factor of component \( n \) during phase \( m \). \( t_m \) is the duration of phase \( m \).

### 3.2 Generating state combinations

The system reliability focuses on the probability that the system doesn’t fail at any phase. The failed component in the end of the mission can turn into failure at any phase rather than just the last phase. Consider a random state vector of \( N \) binary components \( x_n \) \((x_1(m), x_2(m), \ldots, x_N(m))\) at the end of phase \( m \), set \( y = x_n \). If \( y \) includes \( s \) zeros and \( 0 \leq s \leq N \), there are \( s \) out of \( N \) components, numbered as \( e(i) \) \((0 \leq i \leq s)\), are failed before the end of phase \( m \). Given the state vector at the end of \( m \) is \( y \), the combinations of component states during phase \( m \) are determined by \( \sigma \), noted as \( y_\sigma \). Let \( y_\sigma = y \) at the beginning. Levitin [8] presented a method to generate any combination of failed components during phase \( m \). The criterion of enumeration of \( y_\sigma \) with binary elements is complied with this equation:

\[ \Theta(\sigma,e(i)) = \text{mod}_2 \left[ \frac{\sigma}{2^{s-1}} \right] = \begin{cases} 1 & \text{component } e(i) \text{ works during phase } m \\ 0 & \text{component } e(i) \text{ fails during phase } m \end{cases} \]  
(4)

where \( \sigma \) is the integer index from 0 to \( 2^s - 1 \). \( \Theta(\sigma,e(i)) \) is a binary value and \( y_\sigma(e(i)) = \Theta(\sigma,e(i)) \).

Thus, we can obtain all possible cases of failed components in the duration of phase \( m \). For example, consider a system with three components and \( y=(1,0,0) \), then \( s=2 \) and \( e(1)=2, e(2)=3 \), which means components 2, 3 are failed at the end of the phase \( m \) and \( \sigma=0,1,2,3 \), there are four \( y_\sigma \) can be derived by equation (4), which are \( y_0=(1,0,0), y_1=(1,1,0), y_2=(1,0,1) \) and \( y_3=(1,1,1) \).

### 3.3 Transition probability with the CE

Follow the state vector \( y_\sigma \) of phase \( m \), we need to evaluate the \( \sigma \)-transition probability of each \( y_\sigma \) from the beginning of phase \( m \) to the end of phase \( m \), denoted as \( Q_m(y,\sigma) \). Suppose that there is no external disturbance or internal functional dependence, the transition probability is shown in equation (5).

\[ Q_m(y,\sigma) = \prod_{n=1}^{s} (q_n(m))^{y_n(n)} \]  
(5)

where \( y_n(n) \) represents the state of component \( n \) in \( y_\sigma \) and takes the value of 1 or 0.

Consider a system with the CE, the CEG is usually composed of a trigger component and its dependent components. The cascading failure is usually modeled by FDEP gates, which can form a domino chain in order. Some components in the chain are dual components, which can perform as the trigger event and dependent event at the same time. The failure of the dual component only affects the following dependent components rather than former components in the cascading chain.

Thus, a CEG can be composed of the trigger component or dual components and its individual dependent components. For example, the cascading relationship is shown in figure 1.
From figure 1, component 4 is a dual component, so the CEGs in this system are (1,2,4,5), (4,5) and (3,5).

Let \( \sigma \) be the set of CEGs, \( \sigma \) and \( \sigma' \) are the sets of failed components in \( \gamma \) and \( \gamma' \) respectively, \( \sigma_i = \{tr_k, dep_{i1}, dep_{i2}\ldots\} \) contains a trigger component \( tr_k \) and its corresponding functional dependent components. The CEG \( \sigma_i \) belongs to \( \sigma \) if \( \sigma_i \in \sigma \) and the set of \( \sigma_i \cup \gamma' \) includes the first element \( tr_k \). In other words, the \( k \)th CE only occurs during phase \( m \) if all components from CEG \( \sigma_i \) have been failed by the end of phase \( m \) and the \( tr_k \) fails during phase \( m \).

The \( \omega(\gamma') \) covers all the CEGs that can fail resulting from CEs during phase \( m \). With respect to the characteristic of CE, if both CEG \( \sigma \) and CEG \( \sigma_i \) exist in \( \omega(\gamma') \), and they include the same element \( tr_k \), the CEG \( \sigma \) can be ignored in \( \sigma \)-transition with the reason that \( tr_k \) plays a role as the dependent component on the \( tr_j \) of the CEG. Thus, the simplified set of CEGs can be determined as \( \omega(\gamma') \) by this rule. If \( \omega(\gamma') \) includes \( K \) elements, the \( 2^K \) subsets \( \omega(\gamma') \) are all cases of occurrence of CEs except for \( \emptyset \) during phase \( m \), which are enumerated by the equation (4). When CEG \( \sigma_i \) belongs to \( \omega(\gamma') \), \( \Theta(sub,k) = 1 \), otherwise, \( \Theta(sub,k) = 0 \), where \( sub \) is the integer index from 1 to \( 2^K \). Given the \( \gamma \), by the end of phase \( m \), the total probability of \( \sigma \)-transition during phase \( m \) is shown as equation (6).

\[
Q_m(\gamma, \sigma) = \sum_{w_{sub}=1}^{2^K-1} \left( \prod_{l=1}^{K} (q_{w_l}(m))^{\Theta(sub,k)} \times \prod_{c_{w_{sub}}}=1}^{K} q_{c_{w_{sub}}}(m) \right)
\]

(6)

where \( w_{0} = \gamma' - w_{sub} \) is the set of components that fail in phase \( m \) with individual failure instead of the CE. Note that when \( \sigma = 0 \), \( \gamma = (0,0,\ldots,0) \), define \( Q_m(\gamma,0) = 1 \) whatever \( \gamma \) is.

### 3.4. Recursive algorithm of system reliability

According to backward recursion algorithm, the probability of \( x_m = y \) satisfies the Markov Property, which means it only depends on \( x_{m-1} \), so the probability of \( x_m = y \) is obtained as equation (7).

\[
Z_{m,y} = Pr\{x_m = y; \psi_{m-1}(x_{m-1}) = 1, \ldots, \psi_1(x_1) = 1\} = \prod_{n=1}^{N} (p_n(m))^{y(n)} \times \sum_{\sigma=0}^{y} (\psi_{m-1}(y - y_\sigma) \times Z_{m-1,y-y_\sigma} \times Q_m(y,\sigma))
\]

(7)

where \( \psi_m \) is the system structure function of phase \( m \). \( Z_{m,y} \) represents the probability that the state vector of components is \( y \) at the end of the phase \( m \).

Assume that \( r(m) \) represents the system performance requirement of phase \( m \), the system performance based on the system structure function is the cumulative performance of all component. The performance of each component in phase \( m \) depends on its state in phase \( m \) and the nominal performance. For example, the cumulative performance of the parallel system is the sum of the component performance, the state of the parallel system in phase \( m \) depends on whether the cumulative performance of the system achieves \( r(m) \), as shown in equation (8).
The objective function of such fields as network design and system maintenance [1] is elaborated as follows.

\[ \psi_{m}(g_{1}(m)\chi_{1}(m), g_{2}(m)\chi_{2}(m), \ldots, g_{n}(m)\chi_{n}(m)) = \begin{cases} 1 & \sum_{N} g_{n}(m)\chi_{n}(m) \geq r(m) \\ 0 & \sum_{N} g_{n}(m)\chi_{n}(m) < r(m) \end{cases} \]  

where \( \psi_{m}(\bullet) \) is the state of binary system. \( g_{n}(m) \) is the nominal performance of component \( n \) at phase \( m \). \( \sum_{N} g_{n}(m)\chi_{n}(m) \) is the cumulative performance of the system achieves \( r(m) \).

Therefore, the backward recursive calculation method of the phased mission system reliability at the end of phase \( M \) can be expressed in equation (9).

\[ R = \sum_{\sigma=0}^{2^{N}-1} \psi_{M}(\Delta y) \times Z_{M,\Delta y - y_{\sigma}} \]  

where \( \Delta y \) means all possible combinations of components’ states at the end of mission given \( \sigma=0,1,\ldots,2^{N}-1 \).

4. Reliability optimization

4.1. Optimization model

For improving the reliability of the non-repairable PMS, it is necessary to select each component among several distinct quality levels, which have different performance levels and individual manufacturing cost. In order to obtain the cost-efficient allocation scheme of components, the decision variable is a set of numbers of each component \( \sigma \) and the cost on the condition that it cannot exceed the acceptable manufacturing cost.

Therefore, the decision variable is a set of numbers of each component \( \sigma \) and the conditional unreliability \( q_{n}(m) \) depend on \( \sigma \), noted as \( \Phi_{n}(\sigma(n); m), p_{n}(\sigma(n); m), q_{n}(\sigma(n); m) \), respectively.

The optimization model includes two objectives, which are maximizing system reliability with the constraint of the lowest system reliability requirement and minimizing system manufacturing cost on the condition that it cannot exceed the acceptable manufacturing cost.

Considering the constraints of system reliability, manufacturing cost and the alternative quality levels of components, the optimization model of the PMS can be formulated as follows:

\[ \begin{align*}
\text{max } R(\sigma) &= \sum_{\sigma=0}^{2^{N}-1} \psi_{M}(\Delta y) \times Z_{M,\Delta y - y_{\sigma}} (\sigma) \\
\text{min } C(\sigma) &= \sum_{n=1}^{N} c_{v(n)} \\
\text{s.t. } R(\sigma) &\geq R_{0} \\
\sum_{n=1}^{N} c_{v(n)} &\leq C_{0} \\
1 &\leq \sigma(n) \leq \sigma(n,\text{max})
\end{align*} \]  

where \( c_{v(n)} \) is the manufacturing cost of \( \sigma(n) \)th quality level of component \( n \). \( R_{0} \) is the basic demand of system reliability, while \( C_{0} \) is the highest acceptable manufacturing cost of the PMS. The quality level of component \( n \) should not be larger than total number \( \sigma(n,\text{max}) \).

4.2. Recursive algorithm of system reliability

To address this multi-objective optimization problem, it is unrealistic to exhaustively enumerate all allocation solutions, there is even no solution for this Non-deterministic Polynomial problem within the reasonable time [13]. The NSGA-II is widely applied to find the non-inferior solution for multi-objective optimization of such fields as network design and system maintenance [14]. The detailed procedure is elaborated as follows.
(1) Determine the penalty function of two objectives. As shown in equation (11), the penalty function is introduced to remove the non-compliant solutions with respect to the constraints of two objectives, that is, if the allocation scheme lower than system reliability requirement, the system reliability will decrease sharply; in contrast, once the system manufacturing cost of a solution is higher than cost limitation, the value of second function surge to infinity.

$$
\begin{align*}
R(v) &= R(v) - \text{Inf} \cdot \max \left\{ 0, 1 - \frac{R(v)}{R_0} \right\} \\
C(v) &= C(v) + \text{Inf} \cdot \max \left\{ 0, 1 - \frac{C(v)}{C_0} \right\}
\end{align*}
$$

where \(\text{Inf}\) is an infinite number.

(2) Determine the coding space. In the problem, each solution is identified a chromosome with \(N\) integer numbers which range from 1 to their individual alternative amount of quality levels. Consider a permutation of \((4,2,5,6)\), it means quality level 4 selected for component 1, quality level 2 selected for component 2, quality level 5 selected for component 3 and quality level 6 selected for component 4.

(3) Perform crossover on selected chromosomes. Select individuals randomly from the initialized population with crossover rate, the two-point crossover is conducted on the chosen chromosomes, exchange a set of corresponding genes between two randomly defined sites.

(4) Perform mutation on selected chromosomes. The single-point mutation is used in the permutation part of the chromosomes, determine the mutation site randomly among \(N\) positions of each selected individuals. One gene in each selected chromosome is replaced with a random value ranging from 1 to the number of each corresponding component’s quality level.

As for generating initial population, fast non-dominated sorting, calculating the crowding distance, ranking all individuals based on the crowding distance and the Pareto level, implementing the elitist strategy and terminal condition setting, all of those steps are detailed by Ma et al [14].

5. Numerical example

Consider a spatial data collection system designed for various computing phases used in collection and analysis of data. The facility mainly consists of peripheral devices, I/O controllers and I/O modules. Peripheral devices will be inaccessible once the I/O controllers or modules fail.

Assumed that the data collection system contains 10 components involving one computer, two I/O controllers, two peripheral devices and other five independent components, the cascading effect among FDEP components are roughly modeled with FDEP gates in figure 2.

**Figure 2.** The CE of the system.

**Figure 3.** System structure during phase 1.

5.1. Structure function

Figure 3-5 show the system structures for the three consecutive phases. The accumulative performance of series structure equals to the minimum performance of these components. The parallel connection of components perform the same task simultaneously, it accumulates the performance by summing the components’ performance. When it comes to \(k\)-out-of-\(n\) configuration, the system works if and only if at least \(k\) components do not fail during phase 3, the alternatives containing \(k\) components are classified into \(n-k+1\) groups by the set with a same components in
working, each alternative is series, calculate the maximum performance among all alternatives in each group, and the accumulative performance is the summary of the \( n-k+1 \) maximum values.

As shown in figure 3, consider that the system requirement is 7, the nominal components’ performances are \( g(1)=(g_1(1), \ldots, g_{10}(1)) = (8, 8, 9, 9, 9, 10, 10, u, u, u) \), where \( u \) represents the unknown component’s performance have no contribution at phase 1, so the constraints of structure function is:

\[
\psi(x) = \{ \min(8x_1 + 8x_2, 9x_3 + 9x_4 + 9x_5 + 10x_6 + 10x_7) \geq 7 \} \quad (12)
\]

At phase 2, system function is presented in figure 4, the system requirement is 10, the nominal components’ performances are expressed as \( g(2)=(g_1(2), \ldots, g_{10}(2)) = (10, 10, 10, 10, u, 10, 10, u, u, u) \), the corresponding structure function is shown as equation (13).

\[
\psi_2(x) = \{ \min(10x_1 + 10x_2, 10x_3) + \min(10x_4 + 10x_5, 10x_7) \geq 10 \} \quad (13)
\]

![Figure 4. System structure during phase 2.](image)

![Figure 5. System structure during phase 3.](image)

At phase 3, with the performance demand of Figure 5, the nominal components’ performances are \( g(3)=(g_1(3), \ldots, g_{10}(3)) = (u, u, u, u, u, 5, 5, 5, 6, 6) \). Thus, we have:

\[
\psi_3(x) = \{ \max[\min(5x_1,5x_2), \min(5x_3,5x_4), \min(5x_5,6x_6), \min(5x_7,6x_8), \min(5x_9,6x_{10})] + \max[5x_1,5x_2], \min(5x_3,6x_4), \min(5x_5,6x_6) \} \quad (14)
\]

5.2. Parameter setting

There are several alternative quality levels of each component, and the failure probabilities of all components in the data collection system have the Weibull distribution. The baseline time distribution is shown in equation (15).

\[
F_n(t) = 1 - \exp(-\frac{t}{\eta_n(v_n)})^{\beta_n(v_n)} \quad (15)
\]

where \( \eta_n(v_n) \) is the scale parameter and \( \beta_n(v_n) \) is the shape parameter set of quality levels of component \( n \), the quality levels’ characteristic of each component vary from the \( \eta_n(v_n) \) and \( \beta_n(v_n) \). The two parameters are estimated by maximum-likelihood estimation Method referring to [8], which are presented in table 1.

| Component Parameter | \( v(n)=1 \) | \( v(n)=2 \) | \( v(n)=3 \) | \( v(n)=4 \) | \( v(n)=5 \) | \( v(n)=6 \) | \( v(n)=7 \) |
|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( n=1, 2 \)        | \( \beta_n(v_n) \) | 1.9         | 1.2         | 1.5         | 2.1         | 2           | 2.1         | 2.2         |
|                     | \( \eta_n(v_n) \) | 3700        | 4000        | 3560        | 3560        | 3600        | 3700        | 3750        |
|                     | \( c_n(v_n) \)   | 18          | 12          | 15          | 22          | 20          | 28          | 30          |
|                     | \( \beta_n(v_n) \) | 1.8         | 1.8         | 2.3         | 2.5         | -           | -           | -           |
| \( n=3, 4 \)        | \( \eta_n(v_n) \) | 2000        | 2200        | 2080        | 1960        | -           | -           | -           |
|                     | \( c_n(v_n) \)   | 3           | 5           | 12          | 8           | -           | -           | -           |
|                     | \( \beta_n(v_n) \) | 1           | 1           | 1           | 1           | 1           | -           | -           |
| \( n=5, 6 \)        | \( \eta_n(v_n) \) | 16900       | 9000        | 10700       | 12200       | 17500       | -           | -           |
The acceleration factors of components in each phase are listed in Table 2, the unreliability of the \( v(n) \)th level of component \( n \) at the end of phase \( m \) can be calculated by equation (3).

In addition, the duration in three phases are \( t_1=240, t_2=280, \) and \( t_3=320 \) and they are fixed during the entire mission. For NSGA-II, the iteration is 100, as well as population size. The crossover rate is 0.5 and the mutation rate is 0.2. \( R_0 \) equals to 0.90 and \( C_0 \) is 200.

**Table 2.** Acceleration factors of components.

| Component | \( m=1 \) | \( m=2 \) | \( m=3 \) |
|-----------|----------|----------|----------|
| \( n=1,2 \) | 0.0 | 1.8 | 1.5 |
| \( n=3,4 \) | 0.0 | 3.0 | 2.0 |
| \( n=5,6 \) | 2.5 | 2.5 | 0.8 |
| \( n=7,8 \) | 1.0 | 1.7 | 1.0 |
| \( n=9,10 \) | 1.5 | 1.2 | 2.0 |

5.3. **Result analysis**

According to system structure function and parameter settings, the NSGA-II is implemented to address the multi-objective optimization problem. For further analysis, the comparison group, a general non-cascading PMS with same parameters of our experiment, is introduced to highlight the impact of functional dependence among components caused by cascading effect on system performance and manufacturing cost. The optimization results obtained by NSGA-II are shown in figure 6.

![Figure 6](image.png)

**Figure 6.** The optimization results obtained by NSGA-II.

Figure 6 shows the Pareto fronts obtained by NSGA-II. All of points presented in this figure conform to constraints in the optimization model, among which the red point set represents the allocation strategies of the data collection system with the CE, while the blue are strategies of the
general PMS previously mentioned. Actually, the closer the point to the (1,0) of coordinates, the allocation strategy can obtained with higher system reliability and lower manufacturing cost. Therefore, from the red point set, we can find that the satisfactory allocation of components’ quality levels in our example is [1,2,1,2,2,2,3,6,6], the corresponding system reliability is 0.903, and the system manufacturing cost is 105.6.

Moreover, the Pareto fronts with the CE tends to top-left compared to the comparison group, because the functional dependent component can fail caused by either independent failure or cascading failure. In other words, the CE can amplify the unreliability of functional dependent components in the system. As a result, the PMS with the CE is more likely to fail than that without the CE with the same manufacturing cost, the PMS with the CE demands the more expensive components with higher quality level than that without the CE to achieve the same system reliability.

6. Conclusions
This paper mainly discusses the reliability evaluation and optimization method for PMS with cascading effect. Firstly, based on the existing reliability analytical method of recursive algorithm, we extend the phase transition probability with respect to the CE. Secondly, considering the reliability and manufacturing cost of the system, we present the optimization model and employ NSGA-II to solve it. Thirdly, we apply the reliability analysis method and optimization technique by a numerical study to get the satisfactory allocation strategies and identify the effect of cascading failure on the PMS.

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