The excited bottom-charmed mesons in a nonrelativistic quark model

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Using the newly measured masses of \( B_c(1S) \) and \( B_c(2S) \) from the CMS Collaboration and the 1S hyperfine splitting determined from the lattice QCD as constrains, we calculate the \( B_c \) mass spectrum up to the 6S multiplet with a nonrelativistic linear potential model. Furthermore, using the wave functions from this model we calculate the radiative transitions between the \( B_c \) states within a constituent quark model. For the higher mass \( B_c \) states lying above \( DB \) threshold, we also evaluate the Okubo-Zweig-Iizuka (OZI) allowed two-body strong decays with the \( ^3P_0 \) model. Our study indicates that besides there are large potentials for the observations of the low-lying \( B_c \) states below the \( DB \) threshold via their radiative transitions, some higher mass \( B_c \) states, such as \( B_c(2^3P_2) \), \( B_c(2^3D_1) \), \( B_c(3^3D_1) \), \( B_c(4^3P_0) \), and the 1F-wave \( B_c \) states, might be first observed in their dominant strong decay channels \( DB, DB^* \) or \( D^*B \) at the LHC for their relatively narrow widths.

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I. INTRODUCTION

The \( B_c \) states composed of a bottom-charmed quark-antiquark pair, as an important family of hadron spectra was predicted in theory about 40 years ago [1], however, the experimental progress towards establishing the \( B_c \) spectrum is not obvious. Except for the ground state \( B_c \) meson observed in 1998 by the CDF Collaboration at Fermilab [2], until 2018, only the ATLAS Collaboration reported evidence of an excited \( B_c \) state with a mass of 6842 ± 9 MeV [3] consistent with the values predicted for \( B_c(2S) \), while it was not confirmed by the LHCb Collaboration by using their 8 TeV data sample [4]. The poor situation of the observations and measurements of \( B_c \) spectrum is due to that the production yields are significantly smaller than those of the charmonium and bottomonium (\( c\bar{c} \) and \( b\bar{b} \)) states. Fortunately, the LHC provides good opportunities for our search for the excited \( B_c \) states with its high collision energies and integrated luminosity. Very recently, two excited \( B_c^* \) states were observed in the \( B_c^+ \pi^+ \pi^- \) invariant mass spectrum by the CMS Collaboration [5]. Signals are consistent with the \( B_c(2S) \) and \( B_c^*(2S) \) states. These two states are well resolved from each other and are observed with a significance exceeding five standard deviations. The mass of \( B_c(2S) \) meson, 6871 ± 2.8 MeV, measured by the CMS Collaboration is inconsistent with the determination 6842 ± 9 MeV by the ATLAS Collaboration. The reason is that the peak observed by ATLAS could be the superposition of the \( B_c(2S) \) and \( B_c^*(2S) \) states, too closely spaced with respect to the resolution of the measurement [5].

The \( B_c \) states as the only conventional heavy mesons with different flavors have aroused great interests in theory. Compared with the \( c\bar{c} \) and \( b\bar{b} \) spectra, the \( B_c \) spectrum has several special features for the bottom-charmed quark-antiquark pair. (i) The \( B_c \) states cannot annihilate into gluons, thus, the low-lying excited \( B_c \) states below the \( DB \) threshold are more stable with a narrow width less than a few hundred keV, they mainly decay via the electromagnetic or hadronic transitions between two different \( B_c \) states. (ii) In the \( B_c \) meson spectrum there are configuration mixings between the states with different total spins but with the same total angular momentum, such as \( ^3P_1 \rightarrow ^1P_1, ^3D_2 \rightarrow ^1D_2, \) and \( ^3F_3 \rightarrow ^1F_3 \) mixings via the antisymmetric part of the spin-orbit potential. (iii) Additionally, the \( B_c \) states provide a unique window for studying the heavy-quark dynamics that is very different from those provided by the \( c\bar{c} \) and \( b\bar{b} \) states. In the past years, the \( B_c \) mass spectrum has been predicted with various models [6–34]. Furthermore, a few lattice calculations can be found in Refs. [35–39]. To estimate the production rates in experiments, the production of the excited \( B_c \) states were often discussed in the literature [40–53]. As the dominant decay modes, the electromagnetic transitions of the low-lying \( B_c \) states were also widely estimated in the literature [7, 16, 54–58]. However, the studies of the OZI-allowed strong decays for the high-lying \( B_c \) states are confined only to a few calculations [17, 18, 53, 60].

The successes of the observations of the radially excited \( B_c \) states \( B_c(2S) \) and \( B_c^*(2S) \) by the CMS Collaboration [5] have demonstrated that more excited \( B_c \) states are to be discovered in future LHC experiments. Stimulated by the great discovery potentials of the missing \( B_c \) states in future experiments, in present work we carry out a systematical study of the \( B_c \) spectrum. First, using the newly measured masses of \( B_c(1S) \) and \( B_c(2S) \) from the CMS Collaboration [5] and the 1S hyperfine splitting determined from the lattice QCD [36, 38] as constrains, we calculate the \( B_c \) mass spectrum up to the 6S multiplet with a nonrelativistic linear potential model. The slope parameter of the linear potential has been well determined in our previous study of the charmonium states [61]. To involve the spin-dependent corrections of the spatial wave functions, following the method adopted in Refs. [61, 62] we treat the spin-dependent interactions as nonperturbative terms in our calculations. With this nonperturbative treatment, we can reasonably include the effect of spin-dependent interactions on the spatial wave functions, which is essential for us to gain reliable predictions of the decay behaviors.

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Then, with the available wavefunctions from the potential model, we evaluate the electromagnetic transitions between the \( B_c \) states within a nonrelativistic constituent quark model developed in our previous works [61, 62]. With this approach the possible higher EM multipole contributions to a EM transition process can be included naturally. Considering the fact that the higher \( B_c \) states lying above the \( DB \) threshold may have enough possibilities to be produced at LHC, and they are easy to be established in the \( D^* B^* \) hadronic final states, thus to give useful references for the LHC observations, we further calculate the OZI-allowed strong decays of the higher \( B_c \) states within the widely used \( ^3P_0 \) model [63-65]. It is found that \( B_c(2^3D_2), B_c(2^3D_1), B_c(3^3D_1) \) together with the 1\( F \)-wave \( B_c \) states might be first observed in their dominant strong decay channels \( DB, DB' \) or \( D' B \) at LHC for their relatively narrow width.

This paper is organized as follows. In Sec. II, the \( B_c \) mass spectrum is calculated within a nonrelativistic linear potential model. Then, with the obtained \( B_c \) spectrum the radiative transitions between the \( B_c \) states are estimated in Sec. III within a nonrelativistic constituent quark model. In Sec. IV, the OZI-allowed two-body strong decays of the excited \( B_c \) state are also studied within the \( ^3P_0 \) model. In Sec. V we focus on the calculation results and discuss some strategies for looking for the \( B_c \) states in future experiments. Finally, a summary is given in Sec. VI.

II. MASS SPECTRUM

To describe a bottom-charmed meson system, we adopt a nonrelativistic linear potential model. In this model, the effective quark-antiquark potential is written as the sum of the spin-independent term \( H_0(r) \) and spin-dependent term \( H_{sd}(r) \), i.e.,

\[
V(r) = H_0(r) + H_{sd}(r),
\]

where

\[
H_0(r) = \frac{4}{5} \frac{\alpha_s}{r} + b r
\]

includes the standard color Coulomb interaction and the linear confinement. The spin-dependent part \( H_{sd}(r) \) can be expressed as [1, 9, 11]

\[
H_{sd}(r) = H_{SS} + H_T + H_{LS},
\]

where

\[
H_{SS} = \frac{32 \pi \alpha_s}{9 m_q m_{\bar{q}}} \delta_{\sigma}(r) S_q \cdot S_{\bar{q}}
\]

is the spin-spin contact hyperfine potential. Here, we take \( \delta_{\sigma}(r) = (\sigma / \sqrt{3}) e^{-\sigma r^2} \) as suggested in Ref. [60]. The tensor potential \( H_T \) is adopted as

\[
H_T = \frac{4}{3} \frac{\alpha_s}{m_q m_{\bar{q}}} \frac{1}{r^3} \left( S_q \cdot r S_{\bar{q}} \cdot r - S_q \cdot S_{\bar{q}} \right).
\]

For convenience in the calculations, the potential of the spin-orbit interaction \( H_{LS} \) is decomposed into symmetric part \( H_{sym} \) and antisymmetric part \( H_{anti} \).

\[
H_{LS} = H_{sym} + H_{anti},
\]

with

\[
H_{sym} = \frac{S_q \cdot L}{2} \left[ \left( \frac{1}{2m_q^2} + \frac{1}{2m_{\bar{q}}^2} \right) \left( 4 \frac{\alpha_s}{3} r^2 - b \right) + \frac{8 \alpha_s}{3m_q m_{\bar{q}} r^2} \right],
\]

\[
H_{anti} = \frac{S_q \cdot L}{2} \left( \frac{1}{2m_q^2} - \frac{1}{2m_{\bar{q}}^2} \right) \left( 4 \frac{\alpha_s}{3} r^2 - b \right).
\]

In these equations, \( L \) is the relative orbital angular momentum of the \( q \bar{q} \) system; \( S_q \) and \( S_{\bar{q}} \) are the spins of the quark \( q \) and antiquark \( \bar{q} \), respectively, and \( S_{\sigma} \equiv S_q \pm S_{\bar{q}} \); \( m_q \) and \( m_{\bar{q}} \) are the masses of quark \( q \) and antiquark \( \bar{q} \), respectively; \( \alpha_s \) is the running coupling constant of QCD; and \( r \) is the distance between the quark \( q \) and antiquark \( \bar{q} \). The five parameters in the above equations (\( \alpha_s, b, \sigma, m_r, m_c \)) are determined by fitting the spectrum.

We can get the masses and wave functions by solving the radial Schrödinger equation

\[
\frac{d^2 u(r)}{dr^2} + 2\mu_r \left[ E - V_{q\bar{q}}(r) - \frac{L(L+1)}{2\mu_r r^2} \right] u(r) = 0,
\]

with

\[
V_{q\bar{q}}(r) = V(r) + H_{SS} + H_{SL} + H_T,
\]

where \( \mu_r = m_q m_{\bar{q}} / (m_q + m_{\bar{q}}) \) is the reduced mass of the system, and \( E \) is the binding energy of the system. Then, the mass of a bottom-charmed state is obtained by

\[
M_{q\bar{q}} = m_q + m_{\bar{q}} + E.
\]

In this work, to reasonably include the corrections from these spin-dependent potentials to both the mass and wave function of a meson state, we deal with the spin-dependent interactions nonperturbatively. We solve the radial Schrödinger equation by using the three-point difference central method [67] from central \(( r = 0) \) towards outside \(( r \rightarrow \infty) \) point by point. This method was successfully to deal with the spectroscopies of \( cc \) and \( bb \) [61, 62]. To overcome the singular behavior of \( 1/r^2 \) in the spin-dependent potentials, following the method of our previous works [61, 62], we introduce a cutoff distance \( r_c \) in the calculation. Within a small range \( r \in (0, r_c) \), we let \( 1/r^2 \rightarrow 1/r_c^2 \).

Finally, it should be mentioned that the antisymmetric part of the spin-orbit potential, \( H_{anti} \), can let the states with different total spins but with the same total angular momentum, such as \( B_c(n^3L_J) \) and \( B_c(n^1L_J) \), mix with each other. Thus, as mixing states between \( B_c(n^3L_J) \) and \( B_c(n^1L_J) \), the physical \( B_c \) states \( B_c(nL) \) and \( B_c(nL') \) are expressed as

\[
\begin{pmatrix}
B_c(n^3L_J) \\
B_c(n^1L_J)
\end{pmatrix} = \begin{pmatrix}
\cos \theta_{st} & \sin \theta_{st} \\
-\sin \theta_{st} & \cos \theta_{st}
\end{pmatrix} \begin{pmatrix}
B_c(n^3L_J) \\
B_c(n^1L_J)
\end{pmatrix},
\]
where $J = L = 1, 2, 3 \cdots$, and the $\theta_{nl}$ is the mixing angle. In this work $B_s(nL)$ corresponds to the higher mass mixed state as often adopted in the literature.

In this work the parameter set is taken as $\alpha_s = 0.5021$, $b = 0.1425\text{GeV}^2$, $m_b = 4.852\text{ GeV}$, $m_c = 1.483\text{ GeV}$, $\sigma = 1.3$ GeV and $r_c = 0.16$ fm. To consistent with our previous study [6]], the charmed quark mass $m_c$ and the slope for the linear confining potential are taken the determinations, i.e.,
$m_c = 1.483$ GeV and $b = 0.1425$ GeV$^2$. The other three parameters ($m_b$, $\alpha_s$, $\alpha$) are determined by fitting the masses of the $B_c$, $B_c^*$ and $B_c(2S)$ mesons. The masses of $B_c$ and $B_c(2S)$ are taken from the recent measurements of the CMS Collaboration [5]. Although the $B_c^*$ meson is still not measured in experiments, the mass difference between the $B_c^*$ and $B_c$ is predicted to be around 55 MeV from lattice QCD [36–38].

Thus, combining it with the measured mass 6271 MeV for $B_c$, in present work we estimate the mass of $B_c^*$ as $\sim 6326$ MeV. The cutoff distance $r_c$ is determined by the mass of $B_c(1^3P_0)$. To determined the mass of $B_c(1^3P_0)$, we adopt a method of perturbation, i.e., we let $H = H_0 + H'$, where $H'$ is a part which contained the term of $1/r^3$. By solving the equation of $H_0|\psi_n^{(0)}\rangle = E_0|\psi_n^{(0)}\rangle$, we can get the energy $E_0$ and wave function $|\psi_n^{(0)}\rangle$, then, we obtain the mass of $B_c(1^3P_0)$, $M = m_b + m_c + E_0 + \langle \psi_n^{(0)}|H'|\psi_n^{(0)}\rangle$.

By solving the radial Schrödinger equation and with the determined parameter set, we obtain the masses of the bottom-charmed states, which have been listed in Tab. 1 and shown in Fig. 1. For comparison, the other model predictions in Refs. [7,11,15,16] are listed in the same table as well.

It is found that the masses of the low-lying $1S^-$, $2S^-$, $3S^-$, $1P^-$, $2P^-$, $1D$-wave $B_c$ states predicted in this work are compatible with the other potential model predictions. For the higher mass states, such as $4S^-$, $5S^-$, $6S^-$, $3P^-$, $4P^-$, $2D^-$, $2F^-$, $3F$-wave states, the masses predicted by us are very close to those predicted with a relativistic model in Ref. [8], while are about 100 – 200 MeV smaller than those predicted in Refs. [13,16]. Furthermore, the hyperfine splitting between $B_c^*(2S)$ and $B_c(2S)$ is predicted to be 19 MeV, which

\begin{table}[h]
\centering
\caption{Mixing angles.}
\begin{tabular}{|c|c|c|c|c|}
\hline
Mixing angle & ours [11] [14] [10] \\
\hline
$\theta_{1F}$ & 35.5$^\circ$ & 22.4$^\circ$ & 20.57$^\circ$ & 20.4$^\circ$ \\
$\theta_{2F}$ & 38.0$^\circ$ & 18.9$^\circ$ & 19.94$^\circ$ & 23.2$^\circ$ \\
$\theta_{3F}$ & 39.7$^\circ$ & \ldots & 17.68$^\circ$ & \ldots \\
$\theta_{4F}$ & 39.7$^\circ$ & \ldots & \ldots & \ldots \\
$\theta_{1D}$ & 45.0$^\circ$ & 44.5$^\circ$ & -2.49$^\circ$ & -35.9$^\circ$ \\
$\theta_{2D}$ & 45.0$^\circ$ & \ldots & -2.8$^\circ$ & \ldots \\
$\theta_{3D}$ & 45.0$^\circ$ & \ldots & \ldots & \ldots \\
$\theta_{4D}$ & 41.4$^\circ$ & 41.4$^\circ$ & \ldots & \ldots \\
$\theta_{1F}$ & 43.4$^\circ$ & \ldots & \ldots & \ldots \\
$\theta_{2F}$ & 42.4$^\circ$ & \ldots & \ldots & \ldots \\
\hline
\end{tabular}
\end{table}
is slightly smaller than 30 – 45 MeV predicted in previous works [7,11,15,16,36,38]. Finally, it should be pointed out the mixing angles for 3P_1 - 1P_1, 3D_2 - 1D_2, and 3F_3 - 1F_3 have obvious model dependencies (see Tab. III).

### III. RADIATIVE TRANSITIONS

We use the nonrelativistic constituent quark model as adopted in Refs. [61,62,63,71,72] to calculate the radiative transitions between the B_c states. In this model, the quark-photon EM coupling at the tree level is taken as

\[ H_c = -\sum_j e \bar{\psi}_j \gamma^\mu A^\mu(k, r) \psi_j, \]

where \( A^\mu \) represents the photon field with three momenta \( k \); while \( e_j \) and \( r_j \) stand for the charge and coordinate of the constituent quark \( \psi_j \), respectively. In order to match the nonrelativistic wave functions of the \( B_c \) states, we adopt the nonrelativistic form of Eq. (13), which is given by [73,78],

\[ H_c'' = \sum_j \left[ e_j r_j \cdot e - \frac{e_j}{2m_j} \sigma_j \cdot (e \times k) \right] e^{-ik r_j}, \]

where \( e \) is the polarization vector of the initial photon, \( m_j \) and \( \sigma_j \) stand for the constituent mass and Pauli spin vector for the \( j \)th quark. The helicity amplitude \( \mathcal{A} \) can be expressed as

\[ \mathcal{A} = -i \sqrt{\frac{2}{2J_c + 1}} |H_c''| \). \]

Finally, we obtain the partial decay width of a radiative transition by

\[ \Gamma = \frac{|k|^2}{\pi} \frac{2}{2J_c + 1} M_f \sum_{J_f, J_c} |\mathcal{A}_{J_f, J_c}|^2, \]

where \( J_c \) is the total angular momentum of an initial meson, and \( J_{f} \) and \( J_{c} \) are the components of the total angular momenta along the z axis of initial and final mesons, respectively. \( M_i \) and \( M_f \) correspond to the masses of the initial and final \( B_c \) states, respectively.

The radiative decays properties for the \( B_c \) states have been listed in Tables [II][VIII]. For a comparison, some other predictions of the low-lying \( B_c \) states from Refs. [7,8,11] are also given in the tables.

### IV. STRONG DECAYS

In this work, we use the \( 3P_0 \) model [63,65] to calculate the OZI-allowed strong decays of the bottom-charmed mesons. In this model, it assumes that the vacuum produces a quark-antiquark pair with the quantum number \( 0^{++} \) and the heavy meson decay takes place via the rearrangement of the four quarks. The transition operator \( \hat{T} \) in this model can be written as

\[ \hat{T} = -3\gamma \sqrt{96\pi} \sum_m \langle m_1 - m | 00 \rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \]

\[ \times Y_{2}^{m_3} \left( \frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{34} \omega_0^{34} (\mathbf{p}_3 \cdot \mathbf{p}_4) d_{ij}, \]

where \( \gamma \) is a dimensionless constant that denotes the strength of the quark-antiquark pair creation with momentum \( \mathbf{p}_3 \) and \( \mathbf{p}_4 \) from vacuum; \( d_{ij} \) are the creation operators for the quark and antiquark, respectively; the subscripts, \( i \) and \( j \), are the SU(3)-color indices of the created quark and anti-quark; \( \chi_{34} \) corresponds to flavor and color singlets, respectively.

For an OZI allowed two-body strong decay process \( A \to B + C \), the helicity amplitudes \( M_{M_A, M_B, M_C}(\mathbf{P}) \) can be derived as follow

\[ \langle BC|TA \rangle = \delta(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C) M_{M_A, M_B, M_C}(\mathbf{P}). \]

Using the Jacob-Wick formula [79], one can convert the helicity amplitudes \( M_{M_A, M_B, M_C}(\mathbf{P}) \) to the partial wave amplitudes \( M^{HL} \) via

\[ M^{HL}(A \to BC) = \frac{\sqrt{4\pi(2J_c + 1)}}{2J_c + 1} \sum_{M_B, M_C} \langle J_B M_B J_C M_C | J_A M_A \rangle M^{HL}(A \to BC) \]

\[ \times \langle J_B M_B J_C M_C | J_A M_A \rangle \mathcal{M}_{M_A, M_B, M_C}(\mathbf{P}). \]

In the above equations, \( (J_A, J_B, J_C) \), \( (L_A, L_B) \) and \( (S_A, S_B, S_C) \) are the quantum numbers of the total angular momenta, orbital angular momenta and total spin for hadrons \( A, B, C \), respectively; \( M_{J_c} = M_{J_c} + M_{J_c} \), \( J = J_B + J_C \) and \( J_A = J_B + J_C + L \). In the c.m. frame of hadron \( A \), the momenta \( \mathbf{P}_B \) and \( \mathbf{P}_C \) of mesons \( B \) and \( C \) satisfy \( \mathbf{P}_B = -\mathbf{P}_C \equiv \mathbf{P} \).

Then the strong decay partial width for a given decay mode of \( A \to B + C \) is given by

\[ \Gamma = 2\pi |\mathbf{P}| E_B E_C M_A \sum_{J_L} |M^{HL}|^2, \]

where \( M_A \) is the mass of the initial hadron \( A \), while \( E_B \) and \( E_C \) stand for the energies of final hadrons \( B \) and \( C \), respectively. The details of the \( 3P_0 \) model can be found in our recent paper [80].

In the calculations, the wavefunctions of the initial \( B_c \) states are adopted our quark model predictions. Furthermore, we need the wavefunctions of the final hadrons, i.e., the \( B^{*0}, B^{*+}, D^{*0}, D^{*+}, D_c^{*0} \) mesons and some of their excitations, which are adopted from the quark model predictions of Refs. [81,82].

In this work, for the masses of the light constituent \( u, d \) and \( s \) quarks, we set \( m_u = m_d = 0.33 \text{ GeV}, m_s = 0.45 \text{ GeV} \); while for the heavy \( b \) and \( c \) quarks, their masses are taken to be \( m_b = 4.852 \text{ GeV} \) and \( m_c = 1.483 \text{ GeV} \) as the determinations in the calculations of the \( B_c \) mass spectrum. The masses of the final hadron states in the decay processes are adopted from the
TABLE III: Partial widths of the $M1$ transitions for the low-lying 1S-, 2S-, and 3S-wave $B_c$ states compared with the other model predictions.

| Initial state | Final state | $E_γ$ (MeV) | $Γ_{M1}$ (eV) | $Γ_{M1}$ (eV) |
|---------------|-------------|-------------|---------------|---------------|
| $1^3S_1$      | $1^3S_0$    | 72          | 62            | 67            | 64            | 55            | 134.5          | 73             | 80             | 60             | 57             |
| $2^3S_1$      | $2^3S_0$    | 43          | 46            | 32            | 35            | 19            | 28.9           | 30             | 10             | 10             | 2.4            |
| $1^3S_0$      |              | 606         | 584           | 588           | 649           | 591           | 123.4          | 141            | 600            | 98             | 1205           |
| $2^3S_0$      | $1^3S_1$    | 499         | 484           | 498           | 550           | 523           | 93.3           | 160            | 300            | 96             | 99             |
| $3^3S_1$      | $3^3S_0$    | 22          | 13            | 3             | 0.8           |               |                |                |                |                |                |
| $2^3S_0$      |              | 405         | 371           | 200           | 356           |               |                |                |                |                |                |
| $1^3S_0$      |              | 932         | 915           | 600           | 1885          |               |                |                |                |                |                |
| $3^3S_0$      | $2^3S_1$    | 354         | 341           | 60            | 152           |               |                |                |                |                |                |
| $1^3S_1$      |              | 855         | 855           | 4200          | 510           |               |                |                |                |                |                |

TABLE IV: Partial widths of the $M1$ transitions for the higher $nS$-wave ($n = 4, 5, 6$) $B_c$ states.

| Initial state | Final state | $E_γ$ (MeV) | $Γ_{M1}(eV)$ | $Γ_{M1}(eV)$ |
|---------------|-------------|-------------|--------------|--------------|
| $4^3S_0$      | $3^3S_1$    | 283         | 186          | 4             |
| $2^3S_1$      |              | 622         | 579          | 3             |
| $1^3S_1$      |              | 1116        | 1122         | 2             |
| $5^3S_0$      | $4^3S_1$    | 251         | 209          | 5             |
| $3^3S_1$      |              | 533         | 720          | 4             |
| $2^3S_1$      |              | 861         | 1260         | 3             |
| $1^3S_1$      |              | 1339        | 1893         | 2             |
| $6^3S_0$      | $5^3S_1$    | 230         | 225          | 6             |
| $4^3S_1$      |              | 481         | 849          | 5             |
| $3^3S_1$      |              | 755         | 1613         | 4             |
| $2^3S_1$      |              | 1073        | 2203         | 3             |
| $1^3S_1$      |              | 1536        | 2822         | 2             |

TABLE V: The masses (MeV) of the final hadrons appearing in the strong decay processes of the $B_c$ states. The masses are taken from the Particle Data Group [83] if there are experimental data, otherwise we take the quark model predictions in Refs. [81, 82].

| State | $1^1S_0$ | $1^3S_1$ | $1^3P_0$ | $1P_1$ | $1^3P_2$ |
|-------|----------|----------|----------|--------|----------|
| $B_c$ | 5279     | 5325     | 5683     | 5729   | 5754     | 5768     |
| $B_c^*$| 5367     | 5415     | 5756     | 5801   | 5836     | 5851     |
| $D$   | 1870     | 2010     | 2252     | 2402   | 2417     | 2466     |
| $D_c$ | 1968     | 2112     | 2344     | 2488   | 2510     | 2559     |

Recently, signals of two excited $\bar{b}c$ states $B_c(2S)$ and $B_c^*(2S)$ were observed in the $B_c^0\pi^+\pi^-$ invariant mass spectrum by the CMS Collaboration at LHC. These two states are well resolved from each other and are observed with a significance exceeding five standard deviations. The mass of $B_c(2S)$ meson is measured to be $6871 \pm 2.8$ MeV. Furthermore, a more precise mass of $B_c(2S)$, $M(B_c^*) = 6871.1 \pm 0.5$ MeV, is measured by the CMS Collaboration as well. Combining these newest measurements, we predict that the mass of $B_c(2S)$ might be $\sim 6890$ MeV, and the mass hyperfine splitting be-

V. DISCUSSION

1. $S$-wave states

Particle Data Group [83] if there are measured values, otherwise we take the quark model predictions of Refs. [81, 82] (see Table V). There is no experimental data which can be used to determine the quark pair creation strength, thus, in this work we adopt a typical value $\gamma = 0.4$ that gives a reasonably accurate description of the overall scale of decay widths of both light and heavy mesons [66, 84, 85]. The strong decays properties for the bottom-charmed states are presented in Tab. X to XV.
between $B_c^*(2S)$ and $B_c(2S)$,
\[ \Delta m(2S) \approx 20 \text{ MeV}, \]  
(21)
is slightly smaller than $30 - 45 \text{ MeV}$ predicted in previous works (see Table I). The predicted masses for the other higher $S$-wave states compared with other works are also given in Table I. Obvious differences can be found in various theoretical predictions.

The $M1$ transitions of the low-lying $S$-wave states $B_c^*(2S)$ and $B_c^*(1S)$ were often discussed in the literature for these transitions which might be used to establish them in experiments. In this work we also calculate their $M1$ transitions. Our results compared with the some other predictions are listed Table III. Obvious model dependence can be seen in various calculations. Our predicted partial width
\[ \Gamma(B_c^*(2S) \rightarrow B_c \gamma) \approx 1.2 \text{ keV}, \]  
(22)
for the $M1$ transition $B_c^*(2S) \rightarrow B_c \gamma$ is about an order of magnitude larger than that predicted in Refs. [7, 9, 11], and about a factor 2 larger than the value predicted within the GI model [11]. Combining our calculations of the EM transitions $B_c^*(2S) \rightarrow 1P_\gamma$ and the strong transitions $B_c^*(2S) \rightarrow B_c \pi \pi$ predicted in [11], the total decay width of $B_c^*(2S)$ meson is estimated to be $I_{\text{total}} \sim 75 \text{ keV}$, then the branching fraction for $M1$ transition $B_c^*(2S) \rightarrow B_c \gamma$ is predicted to be
\[ Br[B_c^*(2S) \rightarrow B_c \gamma] \sim 2\%. \]  
(23)
The fairly large branching fraction may give a good opportunity for us to observe the $B_c^*(2S)$ via the $M1$ transition $B_c^*(2S) \rightarrow B_c \gamma$. This process may be used to determined the mass of $B_c^*(2S)$ in future experiments.

The masses of $3S$-wave states $B_c(3^1 S_0)$ and $B_c(3^3 S_1)$ are predicted to be $\sim 7.24 \text{ GeV}$ and $\sim 7.25 \text{ GeV}$, respectively, which are just above the $DB^*$ threshold. Their radiative and strong decay properties are estimated in this work. The results for the $M1$ transitions, $E1$ dominant transitions and strong decays of the $3S$-wave states are given in Tables III, VII and XI, respectively. There are only a few works about the radiative and strong decay properties of the $3S$-wave states [11, 18, 55, 60]. The $M1$ transitions of the $3S$-wave states roughly agree with the predictions in Ref. [11], except that our predicted partial width $\Gamma(3^3 S_1 \rightarrow 1^1 S_0 + \gamma) \approx 510 \text{ eV}$ for the $M1$ transition $3^3 S_1 \rightarrow 1^1 S_0 + \gamma$ is about an order of magnitude smaller than that in Ref. [11]. The strong decay widths of $B_c(3^1 S_0)$ and $B_c(3^3 S_1)$ predicted by us are comparable to those predicted in recent works [18, 55]. Both $B_c(3^1 S_0)$ and $B_c(3^3 S_1)$ might be broad states with a width of $\sim 100 \text{ MeV}$. The $B_c(3^1 S_0)$ dominantly decay into $DB^*$ channel, while $B_c(3^3 S_1)$ dominantly decay into both $DB$ and $DB^*$ channels. The production rates of the $3S$-wave $B_c$ states in $pp$ collisions at the LHC may be comparable with those of the $2S$-wave $B_c$ states [18]; thus, the $3S$-wave $B_c$ states may have large potentials to be established in the $DB^*$ final states.

The higher $S$-wave states $B_c(n^1 S_0)$ and $B_c(n^3 S_1)$ ($n \geq 4$) are far from the $DB$ threshold, thus many $OZI$-allowed two-body strong decay channels are open. There are few discussions of the decay properties of the higher mass $S$-wave states in the literature. To know some decay properties of these higher $S$-wave states, in this work we give our predictions of the $M1$ transitions and strong decays of $B_c(nS)$ ($n = 4, 5, 6$), which are listed in Table IV and XII, respectively. It is found these higher mass $S$-wave states are broad states with a width of $\sim 100 - 400 \text{ MeV}$. Combining $M1$ transitions of higher $S$-wave states with their strong decays, we found that the branching fractions of the $M1$ transitions $B_c(nS) \rightarrow B_c(1S) + \gamma$ may reach up to a sizeable value $O(10^{-3})$.

2. $P$-wave states

The masses of $1P$-wave states $B_c(1P)$ might lie in the range of $(6710, 6790) \text{ MeV}$, which are consistent with the other predictions with potential models [7, 11], and the recent lattice calculations [16]. The $1P$-wave $B_c(1P)$ states mainly decays via the E1 dominate transitions $1P \rightarrow 1S$. We have calculated the partial decay widths for the $EM$ transitions $1P \rightarrow 1S$, our results compared with some other predictions are listed in Table VII. Most of our results are compatible with the predictions in [9, 11], except our predicted partial decay widths of $\Gamma(B_c(1P_1) \rightarrow B_c \gamma) \approx 35 \text{ keV}$ and $\Gamma(B_c(1P_1') \rightarrow B_c \gamma) \approx 40 \text{ keV}$ are about a factor of $3 \sim 5$ larger than the predictions in Refs. [9, 11]. The $B_c(1P_1)$ and $B_c(1P_1')$ states might be first found in the $B_c \gamma$ final state via their radiative transitions. The branching fractions for $B_c(1P_1)$ and $B_c(1P_1')$ decay into $B_c \gamma$ are predicted to be
\[ Br[B_c(1P_1) \rightarrow B_c \gamma] \sim 33\%, \]  
(24)
\[ Br[B_c(1P_1') \rightarrow B_c \gamma] \sim 65\%. \]  
(25)
While the $B_c(1^3 P_0)$ and $B_c(1^3 P_2)$ states dominantly decay into $B_c^* \gamma$ final state with a decay rate $\sim 100\%$, thus, they have good potentials to be found via the radiative decay chains $B_c(1^3 P_0) \rightarrow B_c(1^3 S_1) \gamma \rightarrow B_c(1^3 S_1) \gamma \gamma$ and $B_c(1^3 P_2) \rightarrow B_c(1^3 S_1) \gamma \gamma$. The $B_c(1^3 S_1) \gamma \gamma$ states might be first found in the $B_c \gamma$ final state via their radiative decays. The radiative decay rates into the $B_c(1^3 S_1) \gamma$ $(n = 1, 2)$ are also sizeable. Their partial widths are predicted to be
\[ \Gamma[B_c(2^3 P_2) \rightarrow DB] \approx 760 \text{ keV}, \]  
(26)
\[ \Gamma[B_c(2^3 P_2) \rightarrow B_c^* \gamma] \approx 52 \text{ keV}, \]  
(27)
\[ \Gamma[B_c(2^3 P_2) \rightarrow B_c^*(2S) \gamma ] \approx 50 \text{ keV}, \]  
(28)
Thus, the total width of $B_c(2^3 P_2)$ is $\Gamma_{\text{total}}[B_c(2^3 P_2)] \approx 880 \text{ keV}$. The $B_c(2^3 P_2)$ state may have potentials to be observed in the $DB$ and $B_c \gamma$ final states. While for $B_c(2^1 P_0)$, $B_c(2P_1)$ and $B_c(2P_1')$ states, their decays are governed by the $EM$ transitions. The radiative decay properties of these states have been given in Table VII. With these predictions, the total widths for $B_c(2^3 P_0)$, $B_c(2P_1)$ and $B_c(2P_1')$ are estimated to be...
\( \Gamma_{\text{total}}[B_c(2^3P_0)] \approx 100 \text{ keV}, \Gamma_{\text{total}}[B_c(2^1P_1)] \approx 120 \text{ keV}, \) and \( \Gamma_{\text{total}}[B_c(2^3P_1)] \approx 133 \text{ keV}, \) respectively. The branching fractions for \( B_c(2P_1) \to B_c^{\gamma}, B_c(2P_1^\prime) \to B_c^{\gamma} \) and \( B_c(2^3P_0) \to B_c^{\gamma} \) are predicted to be

\[
\begin{align*}
Br[B_c(2P_1) \to B_c^{\gamma}] &\approx 20\%, \\
Br[B_c(2P_1^\prime) \to B_c^{\gamma}] &\approx 33\%, \\
Br[B_c(2^3P_0) \to B_c^{\gamma}] &\approx 41\%.
\end{align*}
\]

The large branching fractions indicate that these states usually are broad states with a width of about 150 MeV smaller than those predicted in Refs. [15, 16]. The 1D-wave states mainly decay via the \( \pi \pi \) channel, while the \( B_c(2^3P_0) \) may be observed via the radiative decay chain.

\[
B_c(2^3P_0) \to B_c^{\gamma} \to B_c(2^3P_0^*) \to B_c^{\gamma} \gamma. \]

It should be pointed out that the \( B_c(2P_1) \) and \( B_c(2P_1^\prime) \) states may lie above the \( B^D \) threshold, so they may have fairly large strong decay widths \( O(100 - 1000) \text{ MeV} \) into \( B^D \) and/or \( BD \) channels as predicted in Ref. [17].

For the higher \( P \)-wave states \( B_c(nP) \) and \( B_c(nP^\prime) \), the weak decay widths are in reasonable agreement with the other predictions. Our study indicates that the \( B_c(1^3D_1) \) state may have a relatively large potential to be observed via the radiative decay chain

\[
B_c(1^3D_1) \to B_c(1^1P_1) \to B_c(1^3S_1) \to B_c(1^1S_0) \gamma, \]

and the branching fraction for this chain is estimated to be \( \sim 100\% \). The optimal decay chain for the observations of \( B_c(1^3D_1) \) is

\[
B_c(1^3D_1) \to B_c(1^1P_1) \to B_c(1^3S_1) \gamma, \]

and the branching fraction for this chain is estimated to be \( \sim 60\% \). The optimal decay chains for the observations of \( B_c(1D_2) \) are

\[
B_c(1D_2) \to B_c(1P_1) \gamma, \]

and the branching fraction for these chains are estimated to be \( \sim 50\% \) and \( \sim 30\% \), respectively. While for the observations of \( B_c(1D_2) \), the optimal decay chains are

\[
B_c(1D_2) \to B_c(1P_1) \gamma, \]

and the branching fraction for these chains are estimated to be \( \sim 35\% \) and \( \sim 47\% \), respectively.

The masses of the 2D states are predicted to be \( \sim 7.34 \text{ GeV} \), which is very close to the \( D_B \) threshold. Their decays are governed by the strong decay modes, such as \( DB, DB^*, BD^* \) or \( B^D^* \). Their strong decay properties predicted by us have been listed in Table XIII. There are few discussions about the radiative decays of the 2D-wave \( B_c \) states in the literature. In this work, we also calculate their radiative decay properties, our results are given in Table XIV. It is found that the \( B_c(2^3D_1) \) state has a relatively narrow width of \( \Gamma \sim 58 \text{ MeV} \). The decays of \( B_c(2^3D_1) \) are governed by the \( BD^* \) mode with a branching fraction

\[
Br[B_c(2^3D_1) \to BD^*] \approx 87\%. \]

The other three 2D states \( B_c(2^3D_2), B_c(2^3D_2), \) and \( B_c(2^3D_2') \) are broad states with a width of \( \sim 100 - 200 \text{ MeV} \). The \( B_c(2^3D_2) \) mainly decays into \( DB, DB^*, \) and \( B^D^* \) channels. While the \( B_c(2^3D_2) \) and \( B_c(2^3D_2') \) states dominantly decay into \( DB^*, BD^* \) or \( B^D^* \) channels. Combing the strong and radiative decay properties with each other, it is found that the branching fractions of the dominant \( \ell M \) decay processes \( B_c(2D) \to B_c(nP) \) \( (n = 1, 2) \) are \( O(10^{-4}) \). The observations of the \( DB, DB^*, BD^* \) or \( B^D^* \) final states might be useful to search for these missing 2D states in future experiments.

The higher 3D-wave states \( B_c(3D) \) are also studied in present work. The masses predicted by us are about 7.62 GeV, which are comparable with those predicted in Ref. [15], while about 150 MeV smaller than those predicted in Refs. [16, 17]. The strong decay properties are shown in Table XIII. It is found that these higher 3D-wave states have a width of \( \sim 100 \text{ MeV} \). These higher states might be observed in their dominant strong decay channels.

\[\text{VI. SUMMARY}\]

In this paper, we have calculated the \( B_c \) meson spectrum up to the 6S states with a nonrelativistic linear potential model by further constraining the model parameters with the mass
of $B_c(2S)$ newly measured by the CMS Collaboration. As important tasks of this work, the radiative transitions between the $B_c$ states and the OZI allowed two body strong decays for the higher mass excited $B_c$ states are evaluated with the wavefunctions obtained from the linear potential model. Our calculations may provide useful references to search for the excited $B_c$ states. The main results are emphasized as follows.

For the $S$-wave states, the $2S$ hyperfine splitting is predicted to be $m[B_c^*(2S)] - m[B_c(2S)] \approx 19$ MeV. The mass of the newly observed $B_c^*(2S)$ state might be determined via the $M1$ transition $B_c^*(2S) \rightarrow B_c \gamma$ in future experiments. The $3S$-wave states $B_c(3^1S_0)$ and $B_c(3^3S_1)$ are about 50 MeV above the $BB^*$ thresholds, their widths are estimated to be $\sim 100$ MeV. Since production rates of the $3S$-wave $B_c$ states in $p\bar{p}$ collisions at the LHC are comparable with those of the $2S$-wave $B_c$ states \cite{13}, both $B_c(3^1S_0)$ and $B_c(3^3S_1)$ states may have large possibilities to be established in the $BB^*$ final state, while $B_c(3^3S_1)$ might be observed in the $DB$ final state as well.

For the $P$-wave states, it is found that the decays of the $2P$-wave states, $B_c(2^1P_0)$, $B_c(2^3P_1)$ and $B_c(2^3P_1')$ together with all of the $1P$-wave states are governed by the $E1$ transitions, their typical decay widths are $\sim 100$ keV. It should be possible to observe these $P$-wave states via their dominant radiative decay processes with the higher statistics of the LHC. The $B_c(2^3P_1)$ state is just $\sim 20$ MeV above the $DB$ threshold. It mainly decays into the $DB$ channel with a very narrow width of $\Gamma \approx 1$ MeV, so it has a large potential to be first observed in the $DB$ final state. The predicted masses of $3P$-wave states are in the range of (7420,7470) MeV. They are broad states with widths of $\sim 200$ MeV, and strongly couple to the $B^*D^*$ final state. It is interesting found that the $4P$-wave states $B_c(4^3P_0)$, $B_c(4^3P_1)$ and $B_c(4^3P_1')$ with a mass around 7.7 GeV may have relatively narrow widths $\sim O(100)$ MeV, these higher $P$-wave states might be first observed in their dominant channel $DB$ or $D^*B$.

The $1D$-wave states mainly decay via the $EM$ transitions. Our study indicates that these $1D$-wave states may have a relatively large potential to be observed via the radiative decay chains. For example, to look for the $B_c(1^1D_1)$ state, the $B_c(1^1D_1) \rightarrow B_c(1^3P_3)\gamma \rightarrow B_c(1^3S_1)\gamma \rightarrow B_c(1^3S_1)\gamma \rightarrow B_c(1^3S_0)\gamma \rightarrow B_c(1^3S_0)\gamma \gamma \gamma$ is worthy to be searched, for the branching fraction of this chain is estimated to be $\sim 100\%$. The masses of the $2D$ and $3D$ states are predicted to be $\sim 7.34$ and 7.62 GeV, respectively. Their decays are governed by the strong decay modes, such as $DB$, $DB^*$, $BD^*$ or $B^*D^*$. These higher $D$-wave states usually have a width of $O(100)$ MeV. The observations of the $DB$, $DB^*$, $BD^*$ or $B^*D^*$ final states might be useful to search for these missing $2D$ and $3D$ states in future experiments.

For the $F$-wave states, one should pay more attention to $1F$-wave $B_c$ states in future observations. They have a mass of $\sim 7.23$ GeV, lie between the $DB$ and $B^*D$ mass thresholds. They are narrow states with a width of several MeV to several ten MeV, and dominantly decay into $DB$ or $B^*D$ channels. For example, the $B_c(1^3F_4)$ state might be very narrow state with a width of $\sim 1$ MeV, its decays are governed by the $DB$ mode. To look for the missing $1F$-wave $B_c$ states, the $DB$ and $B^*D$ final states are worth observing.

Finally, it should be pointed out the strong decay widths of the excited $B_c$ states predicted in this work may have large uncertainties, for the parameter $\gamma$ cannot be directly determined by the strong decay processes of $B_c$ states. Fortunately, the uncertainties of the total strong decay widths of the excited $B_c$ states do not affect the important information, such as the dominant decay modes and corresponding decay rates, for our searching for the excited $B_c$ states in future experiments. Furthermore, the mixing angles for $3P_1 \rightarrow 1P_1$, $3D_2 \rightarrow 1D_2$, and $3F_1 \rightarrow F_1$ have obvious model dependencies. The uncertainties of the mixing angles also affect our predictions of the decay properties of the mixed states.

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Initial state & Final state & $E_\gamma$ (MeV) & $\Gamma_{E1}$ (keV) & $\Gamma_{SM}$ (keV) \\
--- & --- & --- & --- & --- \\
$1^3P_2$ & $1^3S_1$ & 397 & 416 & 416 & 426 & 445 & 1126 & 122 & 83 & 102.9 & 87 \\
$1P_1$ & & 387 & 405 & 399 & 412 & 433 & 0.1 & 13.7 & 11 & 8.1 & 40 \\
$1P_1$ & & 382 & 389 & 391 & 400 & 416 & 99.5 & 87.1 & 60 & 77.8 & 70 \\
$1^3P_0$ & & 353 & 355 & 358 & 366 & 377 & 79.2 & 75.5 & 55 & 65.3 & 96 \\
$1P'_1$ & $1^3S_0$ & 455 & 463 & 462 & 476 & 484 & 56.4 & 147 & 80 & 131.1 & 74 \\
$1P_1$ & & 450 & 447 & 454 & 464 & 468 & 0 & 18.4 & 13 & 11.6 & 35 \\
$1^3D_3$ & $1^3P_2$ & 258 & 312 & 272 & 264 & 239 & 98.7 & 149 & 78 & 76.9 & 67 \\
$1^3D_2$ & $1^3P_2$ & 310 & 263 & 273 & 241 & 12.6 & 8.8 & 6.8 & 8.3 \\
$1P'_1$ & & 321 & 280 & 287 & 253 & 143 & 63 & 46.0 & 41 \\
$1P_1$ & & 338 & 289 & 301 & 271 & 7.1 & 7 & 25.0 & 0.39 \\
$1^3D_2$ & $1^3P_2$ & 308 & 268 & 258 & 233 & 23.6 & 9.6 & 12.2 & 8.7 \\
$1P'_1$ & & 319 & 285 & 272 & 246 & 14.9 & 15 & 18.4 & 1.09 \\
$1P_1$ & & 335 & 294 & 284 & 263 & 139 & 64 & 44.6 & 44 \\
$1^3D_1$ & $1^3P_2$ & 258 & 303 & 255 & 265 & 229 & 2.7 & 3.82 & 1.8 & 2.2 & 0.7 \\
$1P'_1$ & & 268 & 315 & 273 & 279 & 242 & 0 & 7.81 & 4.4 & 3.3 & 12 \\
$1P_1$ & & 331 & 315 & 281 & 291 & 259 & 49.3 & 65.3 & 28 & 39.2 & 29 \\
$1^3D_0$ & & 302 & 365 & 315 & 325 & 299 & 88.6 & 133 & 55 & 79.9 & 65 \\
$1^3F_4$ & $1^3D_3$ & 222 & 194 & 81 & 69 \\
$1F_3$ & $1^3D_3$ & 227 & 207 & 5.4 & 4.76 \\
$1D_2$ & & 231 & 205 & 82 & 32 \\
$1D_2$ & & 236 & 212 & 0.04 & 0.04 \\
$1F_3$ & $1^3D_3$ & 218 & 191 & 3.7 & 4.91 \\
$1D_2$ & & 222 & 189 & 0.5 & 0.22 \\
$1D_2$ & & 226 & 197 & 78 & 29 \\
$1^3F_2$ & $1^3D_3$ & 221 & 202 & 0.4 & 0.12 \\
$1D'_2$ & & 224 & 200 & 6.3 & 5.72 \\
$1D_2$ & & 229 & 208 & 6.5 & 6.36 \\
$1^3D_1$ & & 237 & 212 & 75 & 78

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TABLE VII: Partial widths of the $E1$ dominant radiative transitions for the $2D$-wave $B_c$ states.

| Initial state | Final state | $E_r$ (MeV) | $\Gamma_{EM}$ (keV) | Initial state | Final state | $E_r$ (MeV) | $\Gamma_{EM}$ (keV) |
|---------------|-------------|-------------|---------------------|---------------|-------------|-------------|---------------------|
| $2^3D_1$      | $1^3P_2$    | 528         | 8.13                | $2^3D_1$      | $1^3P_2$    | 540         | 32                  |
| $1P^*$        |             | 540         | 7.6                 | $1P^*$        |             | 552         | 0.54               |
| $1P$          |             | 557         | 12.5                | $1P$          |             | 568         | 1.23               |
| $1^3P_0$      |             | 596         | 41.8                | $1^3P_0$      |             | 607         | 2.04               |
| $2^3P_2$      |             | 174         | 0.58                | $2^3P_2$      |             | 186         | 54                  |
| $2P^*$        |             | 184         | 10.15               | $2P^*$        |             | 195         | 0.09               |
| $2P$          |             | 199         | 20.88               | $2P$          |             | 211         | 0.23               |
| $2^3P_0$      |             | 225         | 46                  | $2^3P_0$      |             | 237         | 0.05               |
| $2D_2$        | $1^3P_2$    | 535         | 7.04                | $2D_2$        | $1^3P_2$    | 539         | 7.28               |
| $1P^*$        |             | 547         | 0.12                | $1P^*$        |             | 551         | 19                  |
| $1P$          |             | 564         | 22.6                | $1P$          |             | 567         | 1.48               |
| $1^3P_0$      |             | 602         | 0.29                | $1^3P_0$      |             | 606         | 0.3                |
| $2^3P_2$      |             | 181         | 6.33                | $2^3P_2$      |             | 185         | 6.71               |
| $2P^*$        |             | 190         | 0.74                | $2P^*$        |             | 194         | 29                  |
| $2P$          |             | 206         | 34                  | $2P$          |             | 210         | 0.24               |
| $2^3P_0$      |             | 232         | 0.04                | $2^3P_0$      |             | 236         | 0.05               |

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TABLE VIII: Partial widths of the $E_1$ dominant radiative transitions for the $2S^{-}$, $2P^{-}$-wave $B_c$ states. For comparison, the predictions from the relativistic quark model $[10]$, relativized quark model $[11]$, nonrelativistic constituent quark models $[7, 9]$ are listed in the table as well.

| Initial state | Final state | $E_r$ (MeV) | $\Gamma_{E1}$ (keV) | $\Gamma_{EM}$ (keV) |
|---------------|-------------|-------------|---------------------|---------------------|
| $2^3S_1$      | $1^3P_2$    | 151         | 118                 | 118                 | 159                 | 102                 | 17.7               | 7.59               | 5.7               | 14.8               | 9.68               |
| $1^3P_1'$     | 161         | 130         | 136                 | 173                 | 115                 | 14.5               | 7.65               | 4.7               | 12.8               | 4.62               |
| $1^3P_1$      | 167         | 146         | 144                 | 185                 | 133                 | 7.8                | 5.53               | 2.9               | 7.7               | 3.48               |
| $1^3P_0$      | 196         | 181         | 179                 | 219                 | 174                 |                      |                    |                   |                   |                    |
| $2^1S_0$      | $1^1P_1$    | 119         | 84                  | 104                 | 138                 | 96                 | 5.2                | 4.40               | 6.1               | 15.9               | 6.38               |
|               | $1^1P_1$    | 125         | 101                 | 113                 | 150                 | 114                 | 0                  | 1.05               | 1.3               | 1.9               | 5.33               |
| $2^3P_2$      | $1^3D_3$    | 142         | 75                  | 118                 | 127                 | 129                 | 17.8               | 2.08               | 6.8               | 10.9               | 14                 |
|               | $1^3D_2'$   | 77          | 122                 | 118                 | 127                 |                      |                    |                   |                   |                   |                    |
|               | $1^3D_2$    | 79          | 127                 | 133                 | 135                 |                      |                    |                   |                   |                   |                    |
|               | $1^3D_1$    | 142         | 84                  | 135                 | 126                 | 139                 | 0.2                | 0.035              | 0.1               | 0.1                | 0.13               |
| $2^1S_1$      | 249         | 270         | 272                 | 232                 | 265                 | 73.8               | 75.3               | 55                | 49.4               | 50                 |
|               | $1^1S_1$    | 770         |                     | 778                 | 817                 | 785                 | 25.8               | 14                 | 25.8               | 52                 |
| $2^1S_1$      | 66          | 113         | 108                 | 117                 |                      | 1.49               | 5.5                | 3.5               | 1.05               |                    |
| $1^3S_0$      | 303         | 289         | 257                 | 274                 |                      | 90.5               | 52                 | 58.0               | 36                 |
|               | 825         | 871         | 825                 | 19                  | 131.1               | 44                 |                    |                   |                   |                    |
| $2^3P_1$      | $1^3D_2'$   | 47          | 108                 | 97                  | 101                 |                      | 0.023              | 0.8                | 1.2               | 0.006              |
|               | $1^3D_2$    | 49          | 103                 | 112                 | 109                 |                      | 0.517              | 3.6                | 3.9               | 0.84               |
|               | $1^3D_1$    | 125         | 54                  | 116                 | 105                 | 113                 | 0.3                | 0.204              | 1.6               | 1.6                | 1.45               |
| $2^3S_1$      | 232         | 241         | 253                 | 211                 | 240                 | 54.3               | 45.3               | 45                | 32.1               | 34                 |
|               | $1^3S_1$    | 754         | 761                 | 796                 | 762                 | 22.1               | 5.4                | 15.3               | 40                 |
| $2^1S_1$      | 285         | 284         | 246                 | 258                 |                      | 13.8               | 5.7                | 8.1               | 19                 |
|               | 820         | 860         | 811                 |                      |                      | 1.2               | 3.1                | 25                 |
| $2^3P_0$      | $1^3D_1$    | 98          | 19                  | 93                  | 86                  | 6.9                | 0.041              | 4.2               | 3.2               | 5.6                |
|               | $2^3S_1$    | 205         | 207                 | 231                 | 186                 | 214                 | 41.2               | 34                 | 42                 | 25.5               | 53                 |
|               | $1^3S_1$    | 729         | 741                 | 771                 | 738                 | 21.9               | 1                  | 16.1               | 41                 |                    |
| Initial state | Final state | $E_\gamma$ (MeV) | $\Gamma_{EM}$ (keV) | Initial state | Final state | $E_\gamma$ (MeV) | $\Gamma_{EM}$ (keV) |
|--------------|-------------|-----------------|-------------------|--------------|-------------|-----------------|-------------------|
| $3^1S_0$     | $2P'$       | 88              | 11.13             | $3^3S_1$     | $2^3P_2$    | 91              | 11.89             |
| $2P$         |             | 104             | 10.93             | $2P'$        |             | 101             | 2.92              |
| $1P'$        |             | 450             | 1.74              | $2P$         |             | 117             | 7.2               |
| $1P$         |             | 467             | 1.25              | $2^3P_0$     |             | 144             | 5                 |
|              |             |                 |                   | $1^3P_2$     |             | 450             | 1.58              |
|              |             |                 |                   | $1P'$        |             | 462             | 0.7               |
|              |             |                 |                   | $1P$         |             | 479             | 1.72              |
|              |             |                 |                   | $1^3P_0$     |             | 518             | 1.73              |
| $4^1S_0$     | $1P'$       | 727             | 1.93              | $4^3S_1$     | $1^3P_2$    | 724             | 1.88              |
| $1P$         |             | 743             | 1.7               | $1P'$        |             | 736             | 0.82              |
| $2P'$        |             | 380             | 6.31              | $1P$         |             | 752             | 1.37              |
| $2P$         |             | 395             | 5.14              | $1^3P_0$     |             | 790             | 1.3               |
| $3P'$        |             | 82              | 13                | $2^3P_2$     |             | 380             | 5.78              |
| $3P$         |             | 98              | 17                | $2P'$        |             | 389             | 1.96              |
|              |             |                 |                   | $2P$         |             | 405             | 4.04              |
|              |             |                 |                   | $2^3P_0$     |             | 430             | 3.28              |
|              |             |                 |                   | $3^3P_2$     |             | 86              | 16                |
|              |             |                 |                   | $3P'$        |             | 91              | 4.06              |
|              |             |                 |                   | $3P$         |             | 108             | 8.71              |
|              |             |                 |                   | $3^3P_0$     |             | 129             | 6.12              |
| $3^3P_0$     | $2^3D_1$    | 84              | 10.93             | $3^3P_2$     | $2^3D_3$    | 115             | 22                |
| $1^3D_1$     |             | 389             | 1.84              | $2^3D'$      |             | 116             | 1.57              |
| $3^3S_1$     |             | 166             | 45                | $2D$         |             | 120             | 1.72              |
| $2^3S_1$     |             | 511             | 36                | $2^3D_1$     |             | 127             | 0.23              |
| $1^3S_1$     |             | 1013            | 30                | $1^3D_2$     |             | 421             | 9.07              |
|              |             |                 |                   | $1D'$        |             | 419             | 1.06              |
|              |             |                 |                   | $1D$         |             | 427             | 1.16              |
|              |             |                 |                   | $1^3D_1$     |             | 431             | 0.94              |
|              |             |                 |                   | $3^3S_1$     |             | 209             | 43                |
|              |             |                 |                   | $2^3S_1$     |             | 552             | 39                |
|              |             |                 |                   | $1^3S_1$     |             | 1051            | 42                |
| $3P_1$       | $2D'$       | 93              | 0.003             | $3^3P'$      | $2D'$       | 110             | 1.9               |
| $2D$         |             | 97              | 1.3               | $2D$         |             | 114             | 0.05              |
| $2^3D_1$     |             | 104             | 2.39              | $2^3D_1$     |             | 121             | 2.47              |
| $1D'$        |             | 398             | 0.74              | $1D'$        |             | 414             | 0.19              |
| $1D$         |             | 405             | 0.31              | $1D$         |             | 421             | 0.93              |
| $1^3D_1$     |             | 409             | 0.67              | $1^3D_1$     |             | 425             | 0.61              |
| $3^3S_1$     |             | 187             | 26                | $3^3S_1$     |             | 203             | 25                |
| $2^3S_1$     |             | 531             | 27                | $2^3S_1$     |             | 546             | 22                |
| $1^3S_1$     |             | 1031            | 30                | $1^3S_1$     |             | 1046            | 24                |
| $3^3S_0$     |             | 199             | 18                | $3^3S_0$     |             | 216             | 30                |
| $2^3S_0$     |             | 548             | 19                | $2^3S_0$     |             | 564             | 28                |
| $1^3S_0$     |             | 1078            | 23                | $1^3S_0$     |             | 1093            | 32                |

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| State       | Decay mode | $\Gamma_{th}$ (MeV) | $B_r$(%) | State       | Decay mode | $\Gamma_{th}$ (MeV) | $B_r$(%) |
|------------|------------|---------------------|----------|------------|------------|---------------------|----------|
| $3^1S_0(7239)$ | $B'D$       | 161                 | 100      | $3^1S_1(7252)$ | $BD$       | 28                  | 21       |
| Total      |             | 161                 | 100      | Total      |             | 133                 | 100      |
| $4^1S_0(7540)$ | $B'D$       | 0.14                | 0.1      | $4^1S_1(7550)$ | $BD$       | 4.53                | 2.7      |
| $BD'$      | 34.9       | 18.3                |          | $B'D$      | 0.41       | 0.2                 |          |
| $B'D'$     | 104        | 54                  |          | $BD'$      | 17.0       | 10                  |          |
| $B_0^0D_s^*$ | 6.7        | 3.5                 |          | $B_s^*D_s^*$ | 112        | 66                  |          |
| $B_1^0D_s^*$ | 5.8        | 3.1                 |          | $B_1^0D_s^*$ | 2.81       | 1.6                 |          |
| $B_2^0D_s^*$ | 15.5       | 8.1                 |          | $B_2^0D_s^*$ | 5.29       | 3.1                 |          |
| $BD(1^3P_0)$ | 24         | 12.6                |          | $B_0^0D_s^*$ | 1.83       | 1.1                 |          |
| Total      |             | 191                 | 100      | Total      |             | 171                 | 100      |
| $5^1S_0(7805)$ | $B'D$       | 24.5                | 5.9      | $5^1S_1(7813)$ | $BD$       | 15.81               | 3.9      |
| $BD'$      | 1.5        | 0.4                 |          | $B'D$      | 20.18      | 5                   |          |
| $B'D'$     | 2.28       | 0.6                 |          | $BD'$      | 2.65       | 0.7                 |          |
| $B_0^0D_s^*$ | 1.62       | 0.4                 |          | $B_0^0D_s^*$ | 0.19       | 0.05                |          |
| $B_1^0D_s^*$ | 4.65       | 1.1                 |          | $B_1^0D_s^*$ | 0.02       | 0.005               |          |
| $B_2^0D_s^*$ | 5.75       | 1.4                 |          | $B_2^0D_s^*$ | 0.62       | 0.2                 |          |
| $B(1^3P_0)D$ | 18.6       | 4.5                 |          | $B_0^0D_s^*$ | 3.02       | 0.8                 |          |
| $B(1^3P_2)D$ | 27.6       | 6.7                 |          | $B_0^0D_s^*$ | 8.09       | 2                   |          |
| $B(1^1P_0)D$ | 72         | 19.9                |          | $B(1^3P_0)D$ | 18.96      | 4.7                 |          |
| $B(1^1P_2)D$ | 6.2        | 1.5                 |          | $B(1^3P_2)D$ | 13.34      | 3.3                 |          |
| $B(1^3P_0)D$ | 56.5       | 13.7                |          | $B(1^3P_2)D$ | 16.1       | 4                   |          |
| $BD(1^3P_0)$ | 23.5       | 5.7                 |          | $B(1^3P_0)D^*$ | 0.04      | 0.01                |          |
| $BD(1^3P_2)$ | 48.2       | 11.7                |          | $B(1^3P_0)D^*$ | 53.93      | 13.4                |          |
| $B'D(1^1P_0)$ | 70.9       | 17.2                |          | $B(1^3P_0)D^*$ | 5.19       | 1.3                 |          |
| $B'D(1^3P_0)$ | 12.3       | 3.0                 |          | $B(1^3P_2)D^*$ | 96        | 24                  |          |
| $B'D(1^3P_2)$ | 25.7       | 6.2                 |          | $BD(1^3P_2)$ | 0.89       | 0.2                 |          |
| $B_s(1^3P_0)D_S$ | 0.17      | 0.04                |          | $BD(1^3P_2)$ | 0.63       | 0.2                 |          |
| $B_sD_s(1^3P_0)$ | 0.56     | 0.14                |          | $BD(1^3P_0)D_S$ | 17.34     | 4.3                 |          |
| Total      |             | 413                 | 100      | Total      |             | 401                 | 100      |

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### TABLE XI: Strong decay properties for the 6s-wave $B_s$ states.

| State                  | Decay mode | $\Gamma_{ik}$ (MeV) | $B_i(\%)$ | State                  | Decay mode | $\Gamma_{ik}$ (MeV) | $B_i(\%)$ |
|------------------------|------------|----------------------|-----------|------------------------|------------|----------------------|-----------|
| $6^1S_0(8046)$         | $B^*D$     | 44.4                 | 12        | $6^1S_1(8054)$         | $BD$       | 17.6                 | 4.7       |
| $BD^*$                 |            | 24.3                 | 6.7       | $BD^*$                 |            | 31                   | 8.3       |
| $B^*D^*$               |            | 24.3                 | 6.7       | $B^*D^*$               |            | 19.1                 | 5.1       |
| $B^0D^0_s$             |            | 1.11                 | 0.3       | $B^0D^0_s$             |            | 0.3                  | 0.3       |
| $B^0D^0_s^*$           |            | 3.33                 | 0.9       | $B^0D^0_s^*$           |            | 1.22                 | 0.3       |
| $B(1P_0)D$             |            | 11.3                 | 3.1       | $B(1P_0)D$             |            | 0.09                 | 0.02      |
| $B(1P_2)D$             |            | 4.85                 | 1.3       | $B(1P_2)D$             |            | 2.96                 | 0.8       |
| $B(1P_0)D$             |            | 28.3                 | 7.8       | $B(1P_0)D$             |            | 0.25                 | 0.07      |
| $B(1P_1)D$             |            | 24.7                 | 6.8       | $B(1P_1)D$             |            | 11.1                 | 3         |
| $B(1P_2)D$             |            | 20.6                 | 5.7       | $B(1P_2)D$             |            | 1.09                 | 0.3       |
| $BD(1P_0)$             |            | 13.2                 | 3.6       | $BD(1P_0)$             |            | 10.9                 | 3         |
| $BD(1P_2)$             |            | 28.9                 | 8         | $BD(1P_2)$             |            | 21.2                 | 5.7       |
| $B(1P_0)D^*$           |            | 46.8                 | 13        | $B(1P_0)D^*$           |            | 17                   | 4.6       |
| $B(1P_0)D^*$           |            | 41.4                 | 11.4      | $B(1P_0)D^*$           |            | 9.1                  |           |
| $B(1P_1)D^*$           |            | 23.5                 | 6.5       | $B(1P_1)D^*$           |            | 34                   |           |
| $B(1P_2)D^*$           |            | 5.5                  | 1.5       | $B(1P_2)D^*$           |            | 16.6                 | 4.4       |
| $B(1P_0)D^*_s$         |            | 0.17                 | 0.05      | $B(1P_0)D^*_s$         |            | 14.1                 | 3.8       |
| $B(1P_1)D^*_s$         |            | 0.88                 | 0.24      | $B(1P_1)D^*_s$         |            | 12.9                 | 3.5       |
| $B(1P_2)D^*_s$         |            | 0.03                 | 0.01      | $B(1P_2)D^*_s$         |            | 30.5                 | 8.2       |
| $B(1P_0)D^*_s$         |            | 0.02                 | 0.01      | $B(1P_0)D^*_s$         |            | 27.9                 | 7.5       |
| $B(1P_1)D^*_s$         |            | 6.62                 | 1.8       | $B(1P_1)D^*_s$         |            | 39.9                 | 10.7      |
| $B(1P_2)D^*_s$         |            | 2.47                 | 0.68      | $B(1P_2)D^*_s$         |            | 0.61                 | 0.2       |
| $B(1P_0)D^*_s$         |            | 4.14                 | 1.1       | $B(1P_0)D^*_s$         |            | 3.06                 | 0.8       |
| $B(1P_1)D^*_s$         |            | 0.23                 | 0.06      | $B(1P_1)D^*_s$         |            | 0.24                 | 0.1       |
| $B(1P_2)D^*_s$         |            | 0.18                 | 0.05      | $B(1P_2)D^*_s$         |            | 0.001                | 0.0003    |
| Total                  |            | 361                  | 100       | Total                  |            | 372                  | 100       |

[88] F. E. Close and E. S. Swanson, Dynamics and decay of heavy-light hadrons, Phys. Rev. D 72, 094004 (2005).
| State          | Decay mode | $\Gamma_{th}$ (MeV) | $B_s(\%)$ | State          | Decay mode | $\Gamma_{th}$ (MeV) | $B_s(\%)$ |
|---------------|------------|---------------------|-----------|---------------|------------|---------------------|-----------|
| $3^3P_0(7420)$| $BD$       | 9.6                 | 3.5       | $3^3P_2(7464)$| $BD$       | 22                  | 11.1      |
|               | $B'D'$     | 255                 | 93        |               | $B'D'$     | 16                  | 8.1       |
|               | $B_0^0D_s^*$| 9.7                 | 3.5       |               | $BD'$      | 3.4                 | 1.7       |
|               |            |                     |           |               | $B'D'$     | 146                 | 74        |
|               |            |                     |           |               | $B_0^0D_s^{*}$| 2.7               | 1.4       |
|               |            |                     |           |               | $B_0^2D_s^{*}$| 7.8               | 4         |
| Total         |            | 274                 | 100       | $3P_{1/2}(7458)$| $B'D'$     | 9.3                 | 4.3       |
|               |            |                     |           |               | $BD'$      | 62                  | 28.1      |
|               |            |                     |           |               | $B'D'$     | 145                 | 65.8      |
|               |            |                     |           |               | $B_0^0D_s^*$| 4.0                | 1.8       |
|               |            |                     |           |               | Total      | 198                 | 100       |
|               |            |                     |           | $3P_{1/2}(7441)$| $B'D'$     | 9.3                 | 4.3       |
|               |            |                     |           |               | $BD'$      | 62                  | 28.1      |
|               |            |                     |           |               | $B'D'$     | 145                 | 65.8      |
|               |            |                     |           |               | $B_0^0D_s^*$| 4.0                | 1.8       |
|               |            |                     |           |               | Total      | 220                 | 100       |
| $4^3P_0(7693)$| $BD$       | 13.6                | 25.6      | $4^3P_2(7732)$| $BD$       | 21.76               | 11.4      |
|               | $B'D'$     | 14                  | 26.4      |               | $BD'$      | 30.1                | 15.8      |
|               | $B_0^0D_s^*$| 7.16                | 13.5      |               | $BD'$      | 13.9                | 7.3       |
|               | $B_0^2D_s^{*}$| 4.6              | 8.7       |               | $B'D'$     | 7.82                | 4.1       |
|               | $B(1P')D$  | 7.66                | 14.4      |               | $B_0^0D_s^*$| 0.84               | 0.4       |
|               | $B(1P)D$   | 0.44                | 0.83      |               | $B_0^0D_s^{*}$| 0.01             | 0.005     |
|               | $BD(1P)$   | 0.07                | 0.13      |               | $B_0^0D_s^{*}$| 2.34              | 1.2       |
|               | $B'D(1^3P_0)$| 5.5              | 10.4      |               | $B_0^0D_s^{*}$| 11.1              | 5.8       |
|               |            |                     |           |               | $B(1P')D$  | 27.7                | 14.5      |
|               |            |                     |           |               | $B(1P)D$   | 6.95                | 3.6       |
|               |            |                     |           |               | $B(1^3P_2)D$| 20.2               | 10.6      |
|               |            |                     |           |               | $B(1^3P_0)D'$| 8.8               | 4.6       |
|               |            |                     |           |               | $BD(1P')$  | 13.1                | 6.9       |
|               |            |                     |           |               | $BD(1P)$   | 6.61                | 3.5       |
|               |            |                     |           |               | $B'D(1^3P_0)$| 10.1              | 5.3       |
|               |            |                     |           |               | $B'D(1P)$  | 9.22                | 4.8       |
| Total         |            | 53                  | 100       | $4P_{1/2}(7727)$| $B'D$     | 24.5                | 19.4      |
|               |            |                     |           |               | $BD'$      | 3.7                 | 2.9       |
|               |            |                     |           |               | $B'D'$     | 0.86                | 0.7       |
|               |            |                     |           |               | $B_0^0D_s^*$| 4.4                | 3.5       |
|               |            |                     |           |               | $B_0^2D_s^{*}$| 6.78              | 5.4       |
|               |            |                     |           |               | $B_0^0D_s^{*}$| 6.66              | 5.3       |
|               | $B(1^3P_0)D$| 10.4              | 7.3       |               | $B(1^3P_0)D$| 0.002              | 0.002     |
|               | $B(1P')D$  | 0.03                | 0.002     |               | $B(1P')D$  | 0.4                 | 0.3       |
|               | $B(1P)D$   | 0.01                | 0.01      |               | $B(1P)D$   | 3.32                | 2.6       |
|               | $B(1^3P_2)D$| 36.6              | 25.6      |               | $B(1^3P_2)D$| 15                | 11.9      |
|               | $B(1^3P_0)D'$| 0.02             | 0.01      |               | $B(1^3P_0)D'$| 11.8              | 9.4       |
|               | $BD(1^3P_0)$| 13.6               | 9.5       |               | $BD(1^3P_0)$| 0.1                | 0.08      |
|               | $BD(1P)$   | 0.009               | 0.006     |               | $BD(1P')$  | 15.32               | 12.2      |
|               | $BD(1P)$   | 0.05                | 0.03      |               | $BD(1P)$   | 23.03               | 18.3      |
|               | $B'D(1^3P_0)$| 0.1              | 0.07      |               | $B'D(1^3P_0)$| 10.02             | 8.0       |
|               | $B'D(1P)$  | 0.31                | 0.22      |               |            |                     |           |
|               | $B_0D_s(1^3P_0)$| 4.75           | 3.3       |               |            |                     |           |
|               | $B_0(1^3P_0)D_s$| 0.41          | 0.3       |               |            |                     |           |
| Total         |            | 143                 | 100       |               |            | 126                 | 100       |
TABLE XIII: Strong decay properties for the $2D$-, $3D$-wave $B_c$ states.

| State       | Decay mode | $Γ_{th}$ (MeV) | $B_r$ (%) | State       | Decay mode | $Γ_{th}$ (MeV) | $B_r$ (%) |
|-------------|------------|----------------|-----------|-------------|------------|----------------|-----------|
| $2^1D_1(7336)$ | $BD$       | 0.55           | 1.0       | $2^1D_1(7348)$ | $BD$       | 41.6          | 22.1      |
| $B'D$       | 6.24       | 10.9           |           | $B'D$       | 50.8       | 26.9           |           |
| $BD'$       | 50.1       | 87             |           | $BD'$       | 9.29       | 4.9            |           |
| $B'D'$      | 0.48       | 0.8            |           | $B'D'$      | 87         | 46.1           |           |
| $B'^0D'^*_1$ | 0.18       | 0.3            |           | $B'^0D'^*_1$ | 0.013      | 0.01           |           |
| Total       | 57         | 100            |           | Total       | 189        | 100            |           |
| $2^3D_3(7347)$ | $BD$       | 57.1           | 34.7      | $2^3D_3(7343)$ | $BD$       | 38.2          | 27        |
| $BD'$       | 66.8       | 40.7           |           | $BD'$       | 89         | 64             |           |
| $B'D'$      | 40.4       | 24.6           |           | $B'D'$      | 12.3       | 9              |           |
| Total       | 164        | 100            |           | Total       | 139        | 100            |           |
| $3^1D_1(7611)$ | $BD$       | 25.2           | 28.2      | $3^1D_3(7625)$ | $BD$       | 19.3          | 17        |
| $B'D$       | 5.65       | 6.3            |           | $B'D$       | 29.7       | 26.5           |           |
| $BD'$       | 0.48       | 0.5            |           | $BD'$       | 20.8       | 18.6           |           |
| $B'^0D'^*_1$ | 19.5       | 21.9           |           | $B'^0D'^*_1$ | 18.4       | 16.4           |           |
| $B'^0D'^*_2$ | 2.27       | 2.5            |           | $B'^0D'^*_2$ | 1.45       | 1.3            |           |
| $B'^0D'^*_3$ | 3.16       | 3.5            |           | $B'^0D'^*_3$ | 0.12       | 0.1            |           |
| $B'^0D'^*_4$ | 1.82       | 2.0            |           | $B'^0D'^*_4$ | 2.94       | 2.6            |           |
| $B'^0D'^*_5$ | 16.5       | 18.5           |           | $B'^0D'^*_5$ | 6.6        | 5.9            |           |
| $B(1P)D$    | 0.76       | 0.9            |           | $B(1P')D$   | 0.001      | 0.001          |           |
| $B'D(1^3P_0)$ | 13.9      | 15.6           |           | $B'D(1^3P_0)$ | 4.62       | 4.1            |           |
| Total       | 89         | 100            |           | Total       | 112        | 100            |           |
| $3^3D_1(7623)$ | $BD$       | 45.8           | 34.6      | $3^3D_3(7620)$ | $BD$       | 38.9          | 34.2      |
| $BD'$       | 20.6       | 15.6           |           | $BD'$       | 13.8       | 12             |           |
| $B'D'$      | 21.1       | 16             |           | $B'D'$      | 22.1       | 19             |           |
| $B'^0D'^*_1$ | 2.25       | 1.7            |           | $B'^0D'^*_1$ | 3.89       | 3.4            |           |
| $B'^0D'^*_2$ | 6.33       | 4.8            |           | $B'^0D'^*_2$ | 6.46       | 5.7            |           |
| $B'^0D'^*_3$ | 9.07       | 6.8            |           | $B'^0D'^*_3$ | 11.6       | 10             |           |
| $B(1P)D$    | 12.1       | 9.1            |           | $B(1P')D$   | 0.03       | 0.03           |           |
| $B(1P)D$    | 0.02       | 0.02           |           | $B(1P')D$   | 2.82       | 2.5            |           |
| $BD(1^3P_0)$ | 14.4       | 10.9           |           | $BD(1^3P_0)$ | 0.65       | 0.6            |           |
| $B'D(1^3P_0)$ | 0.65       | 0.5            |           | $B'D(1^3P_0)$ | 13.6       | 12             |           |
| Total       | 132        | 100            |           | Total       | 114        | 100            |           |
| State        | Decay mode | $\Gamma_{th}$ (MeV) | $B_r$ (%) | State        | Decay mode | $\Gamma_{th}$ (MeV) | $B_r$ (%) |
|--------------|------------|---------------------|-----------|--------------|------------|---------------------|-----------|
| $^1F_2(7235)$ | $BD$       | 61.9                | 85        | $^3F_4(7227)$| $BD$       | 0.85                | 97        |
|              | $B'D$      | 11.1                | 15        | $B'D$       |            | 0.03                | 3         |
| Total        |            | 73                  | 100       | Total        |            | 0.88                | 100       |
| $^1F_3(7240)$| $B'D$      | 15.1                | 100       | $^3F_4(7224)$| $B'D$      | 8.53                | 100       |
| Total        |            | 15.1                | 100       | Total        |            | 8.53                | 100       |
| $^2F_2(7518)$| $BD$       | 45.1                | 20.2      | $^3F_4(7514)$| $BD$       | 8                   | 6         |
|              | $B'D$      | 19.2                | 8.6       | $B'D$       |            | 20.9                | 16        |
|              | $BD^*$     | 0.39                | 0.2       | $BD^*$      |            | 37.7                | 29        |
|              | $B'D^*$    | 151                 | 68        | $B'D^*$     |            | 57                  | 43        |
|              | $^{0}D_{1}^*$ | 0.68             | 0.3       | $^{0}D_{1}^*$ |            | 4.48                | 3.4       |
|              | $^{0}D_{1}^*$ | 3.63             | 1.6       | $^{0}D_{1}^*$ |            | 3.26                | 2.5       |
|              | $^{2}D_{1}^*$ | 3.17             | 1.4       | $^{2}D_{1}^*$ |            | 0.05                | 0.04      |
| Total        |            | 223                 | 100       | Total        |            | 131                 | 100       |
| $^2F_3(7525)$| $B'D$      | 45.2                | 25        | $^3F_4(7508)$| $B'D$      | 43.9                | 25        |
|              | $BD^*$     | 41.0                | 23        | $BD^*$      |            | 30.2                | 17        |
|              | $B'D^*$    | 80.3                | 45        | $B'D^*$     |            | 90.2                | 52        |
|              | $^{0}D_{1}^*$ | 7.19             | 4         | $^{0}D_{1}^*$ |            | 7.78                | 4.5       |
|              | $^{2}D_{1}^*$ | 4.53             | 3         | $^{2}D_{1}^*$ |            | 2.57                | 1.5       |
| Total        |            | 178                 | 100       | Total        |            | 175                 | 100       |
### TABLE XV: Strong decay properties for the $3F$-wave $B_s$ states.

| State          | Decay mode $| \Gamma_{ik} (\text{MeV}) \mid R_i (%) \mid \text{State} | Decay mode $| \Gamma_{ik} (\text{MeV}) \mid R_i (%) |
|----------------|-------------|---------------|-------------|---------------|-------------|---------------|-------------|---------------|
| $3^3F_2(7730)$ | $BD$        | 32.1          | 14          | $3^3F_2(7771)$| $BD$        | 2.82         | 1.6         |
|                | $B'D$       | 16.1          | 7           | $B'D$        | 8.9         | 5.0          |
|                | $BD'$       | 3.89          | 1.7         | $BD'$        | 20.2        | 11.4         |
|                | $B'D'$      | 72            | 31.5        | $B'D'$       | 50.9        | 28.7         |
|                | $B_0^*D_s^*$| 0.38          | 0.2         | $B_0^*D_s^*$ | 3.2         | 1.8          |
|                | $B_0^*D_s^*$| 0.11          | 0.05        | $B_0^*D_s^*$ | 3.2         | 1.8          |
|                | $B_0^*D_s^{**}$ | 2.09   | 0.9         | $B_0^*D_s^{**}$ | 0.4   | 0.2          |
|                | $B_0^*D_s^{**}$ | 5.25   | 2.3         | $B_0^*D_s^{**}$ | 9.19   | 5.2          |
|                | $B(1P')D$   | 19.5          | 8.5         | $B(1P')D$   | 19.4        | 10.9         |
|                | $B(1P')D$   | 5.03          | 2.2         | $B(1P')D$   | 1.85        | 1.04         |
|                | $B(1^3P_2)D$| 12.1          | 5.3         | $B(1^3P_2)D$| 12.3        | 6.9          |
|                | $B(1^3P_0)D'$| 2.56         | 1.1         | $B(1^3P_0)D'$| 6.82        | 3.8          |
|                | $BD(1P')$   | 45.8          | 20          | $BD(1P')$   | 0.02        | 0.01         |
|                | $BD(1P)$    | 2.3           | 1           | $BD(1P)$    | 3.03        | 1.7          |
|                | $B'D(1^3P_0)$| 9.14         | 4           | $B'D(1^3P_0)$| 9.19        | 5.2          |
|                | $B'D(1P)$   | 0.48          | 0.2         | $B'D(1P)$   | 11.2        | 6.3          |
|                | $BD(1^3P_0)$| 0.008         | 0.003       | $BD(1^3P_0)$| 0.01        | 0.003        |
|                | $B(1P)D$    | 0.01          | 0.003       | $B(1P)D$    | 6.25        | 1.9          |
|                | $B(1P)D$    | < 0.001       | ≃ 0         | $B(1P)D$    | 2.26        | 0.7          |
|                | $B(1^3P_2)D$| 36.7          | 12          | $B(1^3P_2)D$| 27.6        | 8.4          |
|                | $B(1^3P_0)D'$| 0.08         | 0.03        | $B(1^3P_0)D'$| 8.06        | 2.5          |
|                | $B(1^3P_0)D'$| 30.4         | 10          | $B(1^3P_0)D'$| 8.69        | 2.6          |
|                | $B(1P)D'$   | 7.75          | 2.5         | $B(1P)D'$   | 2.3         | 0.7          |
|                | $B(1^3P_2)D'$| 0.68         | 0.2         | $B(1^3P_2)D'$| 0.6         | 0.2          |
|                | $BD(1^3P_0)$| 0.11          | 0.04        | $BD(1^3P_0)$| 11.6        | 3.5          |
|                | $BD(1P)$    | 0.07          | 0.02        | $BD(1P)$    | 16.6        | 5.1          |
|                | $BD(1P)$    | 0.56          | 0.2         | $BD(1^3P_2)$| 34.2        | 10           |
|                | $BD(1^3P_2)$| 27.1          | 8.9         | $BD(1^3P_2)$| 1.73        | 0.53         |
|                | $B'D(1^3P_0)$| 0.13         | 0.04        | $B'D(1^3P_0)$| 57.4        | 17.5         |
|                | $B'D(1P)$   | 38.9          | 12.8        | $B'D(1P)$   | 8.23        | 2.5          |
|                | $B'D(1P)$   | 19.2          | 6.3         | $B'D(1^3P_0)$| 0.003       | ≃ 0          |
|                | $B(1^3P_0)D_s$| 1.38         | 0.45        | $B(1^3P_0)D_s$| 0.14        | 0.04         |
|                | $B(1P)D_s$  | < 0.0001      | ≃ 0         | $B(1P)D_s$  | 0.01        | 0.003        |
|                | $B_s D_s (1^3P_0)$| 3.03        | 1.0         | $B_s D_s (1^3P_0)$| 3.29        | 10           |
|                | $B_s D_s (1^3P_0)$| 0.01        | 0.003       | $B_s D_s (1^3P_0)$| 3.29        | 10           |

Total: 228 100

| Total 228 | 100 | Total 177 | 100 |
| 3F_2(7779) | $B'D$ | 33.6 | 11 |
|           | $BD'$ | 34.4 | 11.3 |
|           | $B'D'$ | 59.9 | 19.6 |
|           | $B_0^*D_s^*$ | 4.2 | 1.4 |
|           | $B_0^*D_s^{**}$ | 1.69 | 0.6 |
|           | $B_0^*D_s^{**}$ | 4.85 | 1.6 |
|           | $B(1P)D$ | 0.008 | 0.003 |
|           | $B(1P)D$ | 0.01 | 0.003 |
|           | $B(1P)D$ | < 0.001 | ≃ 0 |
|           | $B(1P)D$ | 36.7 | 12 |
|           | $B(1P)D$ | 36.7 | 12 |
|           | $B(1P)D$ | 0.08 | 0.03 |
|           | $B(1P)D$ | 30.4 | 10 |
|           | $B(1P)D$ | 7.75 | 2.5 |
|           | $B(1P)D$ | 0.68 | 0.2 |
|           | $BD(1P)$ | 0.11 | 0.04 |
|           | $BD(1P)$ | 0.07 | 0.02 |
|           | $BD(1P)$ | 0.56 | 0.2 |
|           | $BD(1P)$ | 27.1 | 8.9 |
|           | $BD(1P)$ | 0.13 | 0.04 |
|           | $BD(1P)$ | 38.9 | 12.8 |
|           | $BD(1P)$ | 19.2 | 6.3 |
|           | $B(1P)D_s$ | 1.38 | 0.45 |
|           | $B(1P)D_s$ | < 0.0001 | ≃ 0 |
|           | $B_s D_s (1^3P_0)$ | 3.03 | 1.0 |
|           | $B_s D_s (1^3P_0)$ | 0.01 | 0.003 |

Total: 305 100

| Total 305 | 100 | Total 329 | 100 |