TensorFlow:
Large-Scale Machine Learning on Heterogeneous Distributed Systems
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Abstract

TensorFlow [1] is an interface for expressing machine learning algorithms, and an implementation for executing such algorithms. A computation expressed using TensorFlow can be executed with little or no change on a wide variety of heterogeneous systems, ranging from mobile devices such as phones and tablets up to large-scale distributed systems of hundreds of machines and thousands of computational devices such as GPU cards. The system is flexible and can be used to express a wide variety of algorithms, including training and inference algorithms for deep neural network models, and it has been used for conducting research and for deploying machine learning systems into production across more than a dozen areas of computer science and other fields, including speech recognition, computer vision, robotics, information retrieval, natural language processing, geographic information extraction, and computational drug discovery. This paper describes the TensorFlow interface and an implementation of that interface that we have built at Google. The TensorFlow API and a reference implementation were released as an open-source package under the Apache 2.0 license in November, 2015 and are available at www.tensorflow.org.

1 Introduction

The Google Brain project started in 2011 to explore the use of very-large-scale deep neural networks, both for research and for use in Google’s products. As part of the early work in this project, we built DistBelief, our first-generation scalable distributed training and inference system [14], and this system has served us well. We and others at Google have performed a wide variety of research using DistBelief including work on unsupervised learning [31], language representation [35, 52], models for image classification and object detection [16, 48], video classification [27], speech recognition [56, 21, 20], sequence prediction [47], move selection for Go [34], pedestrian detection [2], reinforcement learning [48], and other areas [17, 5]. In addition, often in close collaboration with the Google Brain team, more than 50 teams at Google and other Alphabet companies have deployed deep neural networks using DistBelief in a wide variety of products, including Google Search [11], our advertising products, our speech recognition systems [50, 6, 46], Google Photos [43], Google Maps and StreetView [19], Google Translate [18], YouTube, and many others.

Based on our experience with DistBelief and a more complete understanding of the desirable system properties and requirements for training and using neural networks, we have built TensorFlow, our second-generation system for the implementation and deployment of large-scale machine learning models. TensorFlow takes computations described using a dataflow-like model and maps them onto a wide variety of different hardware platforms, ranging from running inference on mobile device platforms such as Android and iOS to modest-sized training and inference systems using single machines containing one or many GPU cards to large-scale training systems running on hundreds of specialized machines with thousands of GPUs. Having a single system that can span such a broad range of platforms significantly simplifies the real-world use of machine learning system, as we have found that having separate systems for large-scale training and small-scale deployment leads to significant maintenance burdens and leaky abstractions. TensorFlow computations are expressed as stateful dataflow graphs (described in more detail in Section 2), and we have focused on making the system both flexible enough for quickly experimenting with new models for research purposes and sufficiently high performance and robust for production training and deployment of machine learning models. For scaling neural network training to larger deployments, TensorFlow allows clients to easily express various kinds of parallelism through replication and parallel execution of a core model dataflow
graph, with many different computational devices all collaborating to update a set of shared parameters or other state. Modest changes in the description of the computation allow a wide variety of different approaches to parallelism to be achieved and tried with low effort [14, 29, 42]. Some TensorFlow uses allow some flexibility in terms of the consistency of parameter updates, and we can easily express and take advantage of these relaxed synchronization requirements in some of our larger deployments. Compared to DistBelief, TensorFlow’s programming model is more flexible, its performance is significantly better, and it supports training and using a broader range of models on a wider variety of heterogeneous hardware platforms.

Dozens of our internal clients of DistBelief have already switched to TensorFlow. These clients rely on TensorFlow for research and production, with tasks as diverse as running inference for computer vision models on mobile phones to large-scale training of deep neural networks with hundreds of billions of parameters on hundreds of billions of example records using many hundreds of machines [11, 47, 48, 18, 53, 41]. Although these applications have concentrated on machine learning and deep neural networks in particular, we expect that TensorFlow’s abstractions will be useful in a variety of other domains, including other kinds of machine learning algorithms, and possibly other kinds of numerical computations. We have open-sourced the TensorFlow API and a reference implementation under the Apache 2.0 license in November, 2015, available at www.tensorflow.org.

The rest of this paper describes TensorFlow in more detail. Section 2 describes the programming model and basic concepts of the TensorFlow interface, and Section 3 describes both our single machine and distributed implementations. Section 4 describes several extensions to the basic programming model, and Section 5 describes several optimizations to the basic implementations. Section 6 describes some of our experiences in using TensorFlow. Section 7 describes several programming idioms we have found helpful when using TensorFlow, and Section 8 describes several auxiliary tools we have built around the core TensorFlow system. Sections 9 and 10 discuss future and related work, respectively, and Section 11 offers concluding thoughts.

2 Programming Model and Basic Concepts

A TensorFlow computation is described by a directed graph, which is composed of a set of nodes. The graph represents a dataflow computation, with extensions for allowing some kinds of nodes to maintain and update persistent state and for branching and looping control structures within the graph in a manner similar to Naiad [36]. Clients typically construct a computational graph using one of the supported frontend languages (C++ or Python). An example fragment to construct and then execute a TensorFlow graph using the Python front end is shown in Figure 1 and the resulting computation graph in Figure 2.

In a TensorFlow graph, each node has zero or more inputs and zero or more outputs, and represents the instantiation of an operation. Values that flow along normal edges in the graph (from outputs to inputs) are tensors, arbitrary dimensionality arrays where the underlying element type is specified or inferred at graph-construction time. Special edges, called control dependencies, can also exist in the graph: no data flows along such edges, but they indicate that the source node for the control dependence must finish executing before the destination node for the control dependence starts executing. Since our model includes mutable state, control dependencies can be used directly by clients to enforce happens before relationships. Our implementation also sometimes inserts control dependencies to enforce orderings between otherwise independent operations as a way of, for example, controlling the peak memory usage.

Operations and Kernels

An operation has a name and represents an abstract computation (e.g., “matrix multiply”, or “add”). An operation can have attributes, and all attributes must be provided or inferred at graph-construction time in order to instantiate a node to perform the operation. One common use of attributes is to make operations polymorphic over different tensor element types (e.g., add of two tensors of type float versus add of two tensors of type int32). A kernel is a particular implementation of an operation that can be run on a particular type of device (e.g., CPU or GPU). A TensorFlow binary defines the sets of operations and kernels available via a registration mechanism, and this set can be extended by linking in additional operation and/or kernel definitions/registrations. Table 1 shows some of the kinds of operations built into the core TensorFlow library.

Sessions

Clients programs interact with the TensorFlow system by creating a Session. To create a computation graph, the Session interface supports an Extend method to augment the current graph managed by the session with additional nodes and edges (the initial graph when a session is created is empty). The other primary operation supported
import tensorflow as tf

b = tf.Variable(tf.zeros([100]))  # 100-d vector, init to zeroes
W = tf.Variable(tf.random_uniform([784, 100], -1, 1))  # 784x100 matrix w/rnd vals
x = tf.placeholder(name="x")  # Placeholder for input
relu = tf.nn.relu(tf.matmul(W, x) + b)  # Relu(Wx+b)
C = [...]

s = tf.Session()
for step in xrange(0, 10):
    input = ...construct 100-D input array ...  # Create 100-d vector for input
    result = s.run(C, feed_dict={x: input})  # Fetch cost, feeding x=input
    print step, result

---

by the session interface is `Run`, which takes a set of output names that need to be computed, as well as an optional set of tensors to be fed into the graph in place of certain outputs of nodes. Using the arguments to Run, the TensorFlow implementation can compute the transitive closure of all nodes that must be executed in order to compute the outputs that were requested, and can then arrange to execute the appropriate nodes in an order that respects their dependencies (as described in more detail in[5,1]). Most of our uses of TensorFlow set up a Session with a graph once, and then execute the full graph or a few distinct subgraphs thousands or millions of times via Run calls.
Variables

In most computations a graph is executed multiple times. Most tensors do not survive past a single execution of the graph. However, a Variable is a special kind of operation that returns a handle to a persistent mutable tensor that survives across executions of a graph. Handles to these persistent mutable tensors can be passed to a handful of special operations, such as Assign and AssignAdd (equivalent to +=) that mutate the referenced tensor. For machine learning applications of TensorFlow, the parameters of the model are typically stored in tensors held in variables, and are updated as part of the Run of the training graph for the model.

3 Implementation

The main components in a TensorFlow system are the client, which uses the Session interface to communicate with the master, and one or more worker processes, with each worker process responsible for arbitrating access to one or more computational devices (such as CPU cores or GPU cards) and for executing graph nodes on those devices as instructed by the master. We have both local and distributed implementations of the TensorFlow interface. The local implementation is used when the client, the master, and the worker all run on a single machine in the context of a single operating system process (possibly with multiple devices, if for example, the machine has many GPU cards installed). The distributed implementation shares most of the code with the local implementation, but extends it with support for an environment where the client, the master, and the workers can all be in different processes on different machines. In our distributed environment, these different tasks are containers in jobs managed by a cluster scheduling system. These two different modes are illustrated in Figure 3. Most of the rest of this section discusses issues that are common to both implementations, while Section 3.3 discusses some issues that are particular to the distributed implementation.

Devices

Devices are the computational heart of TensorFlow. Each worker is responsible for one or more devices, and each device has a device type, and a name. Device names are composed of pieces that identify the device’s type, the device’s index within the worker, and, in our distributed setting, an identification of the job and task of the worker (or localhost for the case where the devices are local to the process). Example device names are "/job:localhost/device:cpu:0" or "/job:worker/task:17/device:gpu:3". We have implementations of our Device interface for CPUs and GPUs, and new device implementations for other device types can be provided via a registration mechanism. Each device object is responsible for managing allocation and deallocation of device memory, and for arranging for the execution of any kernels that are requested by higher levels in the TensorFlow implementation.

Tensors

A tensor in our implementation is a typed, multidimensional array. We support a variety of tensor element types, including signed and unsigned integers ranging in size from 8 bits to 64 bits, IEEE float and double types, a complex number type, and a string type (an arbitrary byte array). Backing store of the appropriate size is managed by an allocator that is specific to the device on which the tensor resides. Tensor backing store buffers are reference counted and are deallocated when no references remain.

3.1 Single-Device Execution

Let’s first consider the simplest execution scenario: a single worker process with a single device. The nodes of the graph are executed in an order that respects the dependencies between nodes. In particular, we keep track of a count per node of the number of dependencies of that node that have not yet been executed. Once this count drops to zero, the node is eligible for execution and is added to a ready queue. The ready queue is processed in some unspecified order, delegating execution of the kernel for a node to the device object. When a node has finished executing, the counts of all nodes that depend on the completed node are decremented.

3.2 Multi-Device Execution

Once a system has multiple devices, there are two main complications: deciding which device to place the computation for each node in the graph, and then managing the required communication of data across device boundaries implied by these placement decisions. This subsection discusses these two issues.

3.2.1 Node Placement

Given a computation graph, one of the main responsibilities of the TensorFlow implementation is to map the computation onto the set of available devices. A simplified version of this algorithm is presented here. See Section 4 for extensions supported by this algorithm.

One input to the placement algorithm is a cost model, which contains estimates of the sizes (in bytes) of the
input and output tensors for each graph node, along with estimates of the computation time required for each node when presented with its input tensors. This cost model is either statically estimated based on heuristics associated with different operation types, or is measured based on an actual set of placement decisions for earlier executions of the graph.

The placement algorithm first runs a simulated execution of the graph. The simulation is described below and ends up picking a device for each node in the graph using greedy heuristics. The node to device placement generated by this simulation is also used as the placement for the real execution.

The placement algorithm starts with the sources of the computation graph, and simulates the activity on each device in the system as it progresses. For each node that is reached in this traversal, the set of feasible devices is considered (a device may not be feasible if the device does not provide a kernel that implements the particular operation). For nodes with multiple feasible devices, the placement algorithm uses a greedy heuristic that examines the effects on the completion time of the node of placing the node on each possible device. This heuristic takes into account the estimated or measured execution time of the operation on that kind of device from the cost model, and also includes the costs of any communication that would be introduced in order to transmit inputs to this node from other devices to the considered device. The device where the node’s operation would finish the soonest is selected as the device for that operation, and the placement process then continues onwards to make placement decisions for other nodes in the graph, including downstream nodes that are now ready for their own simulated execution. Section 4.3 describes some extensions that allow users to provide hints and partial constraints to guide the placement algorithm. The placement algorithm is an area of ongoing development within the system.

3.2.2 Cross-Device Communication

Once the node placement has been computed, the graph is partitioned into a set of subgraphs, one per device. Any cross-device edge from $x$ to $y$ is removed and replaced by an edge from $x$ to a new Send node in $x$'s subgraph and an edge from a corresponding Receive node to $y$ in $y$’s subgraph. See Figure 4 for an example of this graph transformation.

At runtime, the implementations of the Send and Receive nodes coordinate to transfer data across devices. This allows us to isolate all communication inside Send and Receive implementations, which simplifies the rest of the runtime.

When we insert Send and Receive nodes, we canonicalize all users of a particular tensor on a particular device to use a single Receive node, rather than one Receive node per downstream user on a particular device. This ensures that the data for the needed tensor is only transmitted once between a source device $\rightarrow$ destination device pair, and that memory for the tensor on the destination device is only allocated once, rather than multiple times (e.g., see nodes $b$ and $c$ in Figure 4).

By handling communication in this manner, we also allow the scheduling of individual nodes of the graph on different devices to be decentralized into the workers: the Send and Receive nodes impart the necessary
synchronization between different workers and devices, and the master only needs to issue a single Run request per graph execution to each worker that has any nodes for the graph, rather than being involved in the scheduling of every node or every cross-device communication. This makes the system much more scalable and allows much finer-granularity node executions than if the scheduling were forced to be done by the master.

### 3.3 Distributed Execution

Distributed execution of a graph is very similar to multi-device execution. After device placement, a subgraph is created per device. Send/Receive node pairs that communicate across worker processes use remote communication mechanisms such as TCP or RDMA to move data across machine boundaries.

#### Fault Tolerance

Failures in a distributed execution can be detected in a variety of places. The main ones we rely on are (a) an error in a communication between a Send and Receive node pair, and (b) periodic health-checks from the master process to every worker process.

When a failure is detected, the entire graph execution is aborted and restarted from scratch. Recall however that Variable nodes refer to tensors that persist across executions of the graph. We support consistent checkpointing and recovery of this state on a restart. In particular, each Variable node is connected to a Save node. These Save nodes are executed periodically, say once every N iterations, or once every N seconds. When they execute, the contents of the variables are written to persistent storage, e.g., a distributed file system. Similarly each Variable is connected to a Restore node that is only enabled in the first iteration after a restart. See Section 4.2 for details on how some nodes can only be enabled on some executions of the graph.

### 4 Extensions

In this section we describe several more advanced features of the basic programming model that was introduced in Section 2.

#### 4.1 Gradient Computation

Many optimization algorithms, including common machine learning training algorithms like stochastic gradient descent [45], compute the gradient of a cost function with respect to a set of inputs. Because this is such a common need, TensorFlow has built-in support for automatic gradient computation. If a tensor $C$ in a TensorFlow graph depends, perhaps through a complex subgraph of operations, on some set of tensors $\{X_k\}$, then there is a built-in function that will return the tensors $\{dC/dX_k\}$. Gradient tensors are computed, like other tensors, by extending the TensorFlow graph, using the following procedure.

When TensorFlow needs to compute the gradient of a tensor $C$ with respect to some tensor $I$ on which $C$ depends, it first finds the path in the computation graph from $I$ to $C$. Then it backtracks from $C$ to $I$, and for each operation on the backward path it adds a node to the TensorFlow graph, composing the partial gradients along the backwards path using the chain rule. The newly added node computes the “gradient function” for the corresponding operation in the forward path. A gradient function may be registered by any operation. This function takes as input not only the partial gradients computed already along the backward path, but also, optionally, the inputs and outputs of the forward operation. Figure 5 shows gradients for a cost computed from the example of Figure 2. Grey arrows show potential inputs to gradient functions that are not used for the particular operations shown. The addition needed to Figure 1 to compute these gradients is:

$$[db, dW, dx] = tf.gradients(C, [b, W, x])$$

In general an operation may have multiple outputs, and $C$ may only depend on some of them. If, for example, operation $O$ has two outputs $y_1$ and $y_2$, and $C$ only depends on $y_2$, then the first input to $O$’s gradient function is set to 0 since $dC/dy_1 = 0$.

Automatic gradient computation complicates optimization, particularly of memory usage. When executing “forward” computation subgraphs, i.e., those that are explicitly constructed by the user, a sensible heuristic breaks ties when deciding which node to execute next by observing the order in which the graph was constructed.

![Figure 5: Gradients computed for graph in Figure 2](image-url)
This generally means that temporary outputs are consumed soon after being constructed, so their memory can be reused quickly. When the heuristic is ineffective, the user can change the order of graph construction, or add control dependencies as described in Section 5. When gradient nodes are automatically added to the graph, the user has less control, and the heuristics may break down. In particular, because gradients reverse the forward computation order, tensors that are used early in a graph’s execution are frequently needed again near the end of a gradient computation. Such tensors can hold on to a lot of scarce GPU memory and unnecessarily limit the size of computations. We are actively working on improvements to memory management to deal better with such cases. Options include using more sophisticated heuristics to determine the order of graph execution, recomputing tensors instead of retaining them in memory, and swapping out long-lived tensors from GPU memory to more plentiful host CPU memory.

4.2 Partial Execution

Often a client wants to execute just a subgraph of the entire execution graph. To support this, once the client has set up a computation graph in a Session, our Run method allows them to execute an arbitrary subgraph of the whole graph, and to inject arbitrary data along any edge in the graph, and to retrieve data flowing along any edge in the graph.

Each node in the graph has a name, and each output of a node is identified by the source node name and the output port from the node, numbered from 0 (e.g., “bar:0” refers to the 1st output of the “bar” node, while “bar:1” refers to the 2nd output).

Two arguments to the Run call help define the exact subgraph of the computation graph that will be executed. First, the Run call accepts inputs, an optional mapping of name:port names to “fed” tensors values. Second, the Run call accepts output_names, a list of output name[:port] specifications indicating which nodes should be executed, and, if the port portion is present in a name, that that particular output tensor value for the node should be returned to the client if the Run call completes successfully.

The graph is transformed based on the values of inputs and outputs. Each node:port specified in inputs and outputs. Each node:port specified in inputs is replaced with a feed node, which will pick up the provided input tensor from specially-initialized entries in a Rendezvous object used for the Run call. Similarly, each output name with a port is connected to a special fetch node that arranges to save the output tensor and return it to the client when the Run call is complete. Finally, once the graph has been rewritten with the insertion of these

special feed and fetch nodes, the set of nodes to execute can be determined by starting at each of the nodes named by any output and working backwards in the graph using the graph dependencies to determine the full set of nodes that must be executed in the rewritten graph in order to compute the outputs. Figure 6 shows an original graph on the left, and the transformed graph that results when Run is invoked with inputs=={b} and outputs=={f:0}. Since we only need to compute the output of node f, we will not execute nodes d and e, since they have no contribution to the output of f.

4.3 Device Constraints

TensorFlow clients can control the placement of nodes on devices by providing partial constraints for a node about which devices it can execute on. For example, “only place this node on a device of type GPU”, or “this node can be placed on any device in /job:worker/task:17”, or “Colocate this node with the node named variable13”. Within the confines of these constraints, the placement algorithm is responsible for choosing an assignment of nodes to devices that provides fast execution of the computation and also satisfies various constraints imposed by the devices themselves, such as limiting the total amount of memory needed on a device in order to execute its subset of graph nodes.

Supporting such constraints requires changes to the placement algorithm described in Section 3.2.1 We first compute the feasible set of devices for each node, and then use union-find on the graph of colocation constraints to compute the graph components that must be placed together. For each such component, we compute the intersection of the feasible device sets. The computed feasible device set per node fits easily into the placement algorithm’s simulator.


4.4 Control Flow

Although dataflow graphs without any explicit control flow are quite expressive, we have observed a number of cases where supporting conditionals and loops can lead to more concise and efficient representations of machine learning algorithms.

Much as in the dataflow-machine approach described by Arvind [3], we introduce a small set of primitive control flow operators into TensorFlow and generalize TensorFlow to handle cyclic dataflow graphs. The Switch and Merge operators allow us to skip the execution of an entire subgraph based on the value of a boolean tensor. The Enter, Leave, and NextIteration operators allow us to express iteration. High-level programming constructs such as if-conditionals and while-loops can be easily compiled into dataflow graphs with these control flow operators.

The TensorFlow runtime implements a notion of tags and frames conceptually similar to the MIT Tagged-Token machine [4]. Each iteration of a loop is uniquely identified by a tag, and its execution state is represented by a frame. An input can enter an iteration whenever it becomes available; thus, multiple iterations can be executed concurrently.

TensorFlow uses a distributed coordination mechanism to execute graphs with control flow. In general, a loop can contain nodes that are assigned to many different devices. Therefore, managing the state of a loop becomes a problem of distributed termination detection. TensorFlow’s solution is based on graph rewriting. During the graph partitioning, we automatically add control nodes to each partition. These nodes implement a small state machine that orchestrates the start and termination of each iteration, and decides the termination of the loop. For each iteration, the device that owns the loop termination predicate sends a tiny control message to every participating device.

As explained above, we often train machine learning models by gradient descent, and represent gradient computations as part of dataflow graphs. When a model includes control-flow operations, we must account for them in the corresponding gradient computation. For example, the gradient computation for a model with an if-conditional will need to know which branch of the conditional was taken, then apply the gradient logic to this branch. Similarly, the gradient computation for a model with a while-loop will need to know how many iterations were taken, and will also rely on the intermediate values computed during those iterations. The basic technique is to rewrite the graph so to memorize the values needed for the gradient computation. We omit the somewhat intricate details of this encoding.

4.5 Input Operations

Although input data can be provided to a computation via feed nodes, another common mechanism used for training large-scale machine learning models is to have special input operation nodes in the graph, which are typically configured with a set of filenames and which yield a tensor containing one or more examples from the data stored in that set of files each time they are executed. This allows data to be read directly from the underlying storage system into the memory of the machine that will perform subsequent processing on the data. In configurations where the client process is separate from the worker process, if the data were fed, it typically would require an extra network hop (from the storage system to the client and then from the client to the worker vs. directly from the storage system to their worker when using an input node).

4.6 Queues

Queues are a useful feature that we have added to TensorFlow. They allow different portions of the graph to execute asynchronously, possibly at different cadences, and to hand off data through Enqueue and Dequeue operations. Enqueue operations can block until space becomes available in the queue, and Dequeue operations can block until a desired minimum number of elements are available in the queue. One use of queues is to allow input data to be prefetched from disk files while a previous batch of data is still being processed by the computational portion of a machine learning model. They can also be used for other kinds of grouping, including accumulating many gradients in order to compute some more complex combination of gradients over a larger batch, or to group different input sentences for recurrent language models into bins of sentences that are approximately the same length, which can then be processed more efficiently.

In addition to normal FIFO queues, we have also implemented a shuffling queue, which randomly shuffles its elements within a large in-memory buffer. This shuffling functionality is useful for machine learning algorithms that want to randomize the order in which they process examples, for example.

4.7 Containers

A Container is the mechanism within TensorFlow for managing longer-lived mutable state. The backing store for a Variable lives in a container. The default container is one that persists until the process terminates, but we also allow other named containers. A container
can be reset by clearing it of its contents entirely. Using containers, it is possible to share state even across completely disjoint computation graphs associated with different Sessions.

5 Optimizations

In this section, we describe some of the optimizations in the TensorFlow implementation that improve performance or resource usage of the system.

5.1 Common Subexpression Elimination

Since the construction of computation graphs is often done by many different layers of abstractions in the client code, computation graphs can easily end up with redundant copies of the same computation. To handle this, we have implemented a common subexpression pass similar to the algorithm described by Click [12] that runs over the computation graph and canonicalizes multiple copies of operations with identical inputs and operation types to just a single one of these nodes, and redirects graph edges appropriately to reflect this canonicalization.

5.2 Controlling Data Communication and Memory Usage

Careful scheduling of TensorFlow operations can result in better performance of the system, in particular with respect to data transfers and memory usage. Specifically, scheduling can reduce the time window during which intermediate results need to be kept in memory in between operations and hence the peak memory consumption. This reduction is particularly important for GPU devices where memory is scarce. Furthermore, orchestrating the communication of data across devices can reduce contention for network resources.

While there are many opportunities for scheduling optimizations, here we focus on one that we found particularly necessary and effective. It concerns the scheduling of Receive nodes for reading remote values. If no precautions are taken, these nodes may start much earlier than necessary, possibly all at once when execution starts. By performing an as-soon-as-possible/as-late-as-possible (ASAP/ALAP) calculation, of the kind common in operations research, we analyze the critical paths of graphs, in order to estimate when to start the Receive nodes. We then insert control edges with the aim of delaying the start of these nodes until just before their results are needed.

5.3 Asynchronous Kernels

In addition to normal synchronous kernels that complete their execution at the end of the Compute method, our framework also supports non-blocking kernels. Such non-blocking kernels use a slightly different interface whereby the Compute method is passed a continuation that should be invoked when the kernel’s execution is complete. This is an optimization for environments where having many active threads is relatively expensive in terms of memory usage or other resources, and allows us to avoid tying up an execution thread for unbounded periods of time while waiting for I/O or other events to occur. Examples of asynchronous kernels include the Receive kernel, and the Enqueue and Dequeue kernels (which might need to block if queue space is not available or if no data is available to be read, respectively).

5.4 Optimized Libraries for Kernel Implementations

We often make use of pre-existing highly-optimized numerical libraries to implement kernels for some operations. For example, there are a number of optimized libraries for performing matrix multiplies on different devices, including BLAS [15] and cuBLAS [39], or GPU libraries for convolutional kernels for deep neural nets such as cuda-convnet [28] and cuDNN [9]. Many of our kernel implementations are relatively thin wrappers around such optimized libraries.

We make fairly extensive use of the open-source Eigen linear algebra library [25] for many of the kernel implementations in the system. As one part of the development of TensorFlow, our team (primarily Benoit Steiner) has extended the open source Eigen library with support for arbitrary dimensionality tensor operations.

5.5 Lossy Compression

Some machine learning algorithms, including those typically used for training neural networks, are tolerant of noise and reduced precision arithmetic. In a manner similar to the DistBelief system [14], we often use lossy compression of higher precision internal representations when sending data between devices (sometimes within the same machine but especially across machine boundaries). For example, we often insert special conversion nodes that convert 32-bit floating point representations into a 16-bit floating point representation (not the proposed IEEE 16-bit floating point standard, but rather just a 32-bit IEEE 794 float format, but with 16 bits less precision in the mantissa), and then convert back to a 32-bit representation on the other side of the communication channel (by just filling in zeroes for the lost portion.
of the mantissa, since that’s less computationally expensive than doing the mathematically correct probabilistic rounding when doing this $32 \rightarrow 16 \rightarrow 32$-bit conversion).

6 Status and Experience

The TensorFlow interface and a reference implementation have been open sourced under an Apache 2.0 license, and the system is available for download at www.tensorflow.org. The system includes detailed documentation, a number of tutorials, and a number of examples demonstrating how to use the system for a variety of different machine learning tasks. The examples include models for classifying hand-written digits from the MNIST dataset (the “hello world” of machine learning algorithms 32, classifying images from the CIFAR-10 dataset 30, doing language modeling using a recurrent LSTM 22 network, training word embedding vectors 35 and more.

The system includes front-ends for specifying TensorFlow computations in Python and C++, and we expect other front-ends to be added over time in response to the desires of both internal Google users and the broader open-source community.

We have quite a few machine learning models in our previous DistBelief system 14 that we have migrated over to TensorFlow. The rest of this section discusses some lessons we have learned that are generalizable for any such migration of machine learning models from one system to another, and therefore may be valuable to others.

In particular, we focus on our lessons from porting a state-of-the-art convolutional neural network for image recognition termed Inception 23. This image recognition system classifies $224 \times 224$ pixel images into one of 1000 labels (e.g., “cheetah”, “garbage truck”, etc.). Such a model comprises 13.6 million learnable parameters and 36,000 operations when expressed as a TensorFlow graph. Running inference on a single image requires 2 billion multiply-add operations.

After building all necessary mathematical operations in TensorFlow, assembling and debugging all 36,000 operations into the correct graph structure proved challenging. Validating correctness is a difficult enterprise because the system is inherently stochastic and only intended to behave in a certain way in expectation — potentially after hours of computation. Given these circumstances, we found the following strategies critical for porting the Inception model to TensorFlow:

1. Build tools to gain insight into the exact number of parameters in a given model. Such tools demonstrated subtle flaws in a complex network architecture specification. In particular we were able to identify operations and variables instantiated incorrectly due to automatic broadcasting in a mathematical operation across a dimension.

2. Start small and scale up. The first convolutional neural network that we ported from our previous system was a small network employed on the CIFAR-10 data set 30. Debugging such a network elucidated subtle edge cases in individual operations (e.g., max-pooling) within the machine learning system that would have been practically indecipherable in more complex models.

3. Always ensure that the objective (loss function) matches between machine learning systems when learning is turned off. Setting the learning rate to be zero helped us identify unexpected behavior in how we had randomly initialized variables in a model. Such an error would have been difficult to identify in a dynamic, training network.

4. Make a single machine implementation match before debugging a distributed implementation. This strategy helped us delineate and debug discrepancies in training performance between machine learning systems. In particular, we identified bugs due to race conditions and non-atomic operations incorrectly assumed to be atomic.

5. Guard against numerical errors. Numerical libraries are inconsistent in how they handle non-finite floating point values. Convolutional neural networks are particularly susceptible to numerical instability and will tend to diverge quite regularly during experimentation and debugging phases. Guarding against this behavior by checking for non-finite floating point values allows one to detect errors in real time as opposed to identifying divergent behavior post-hoc.

6. Analyze pieces of a network and understand the magnitude of numerical error. Running subsections of a neural network in parallel on two machine learning systems provides a precise method to ensure that a numerical algorithm is identical across two systems. Given that such algorithms run with floating point precision, it is important to predict and understand the magnitude of expected numerical error in order to judge whether a given component is correctly implemented (e.g., distinguishing between “within 1e-2, great!” and “within 1e-2: why is it so incorrect??”).
Validating complex mathematical operations in the presence of an inherently stochastic system is quite challenging. The strategies outlined above proved invaluable in gaining confidence in the system and ultimately in instantiating the Inception model in TensorFlow. The end result of these efforts resulted in a 6-fold speed improvement in training time versus our existing DistBelief implementation of the model and such speed gains proved indispensable in training a new class of larger-scale image recognition models.

7 Common Programming Idioms

TensorFlow’s basic dataflow graph model can be used in a variety of ways for machine learning applications. One domain we care about is speeding up training of computationally intensive neural network models on large datasets. This section describes several techniques that we and others have developed in order to accomplish this, and illustrates how to use TensorFlow to realize these various approaches.

The approaches in this subsection assume that the model is being trained using stochastic gradient descent (SGD) with relatively modest-sized mini-batches of 100 to 1000 examples.

Data Parallel Training

One simple technique for speeding up SGD is to parallelize the computation of the gradient for a mini-batch across mini-batch elements. For example, if we are using a mini-batch size of 1000 elements, we can use 10 replicas of the model to each compute the gradient for 100 elements, and then combine the gradients and apply updates to the parameters synchronously, in order to behave exactly as if we were running the sequential SGD algorithm with a batch size of 1000 elements. In this case, the TensorFlow graph simply has many replicas of the portion of the graph that does the bulk of the model computation, and a single client thread drives the entire training loop for this large graph. This is illustrated in the top portion of Figure 7.

This approach can also be made asynchronous, where the TensorFlow graph has many replicas of the portion of the graph that does the bulk of the model computation, and each one of these replicas also applies the parameter updates to the model parameters asynchronously. In this configuration, there is one client thread for each of the graph replicas. This is illustrated in the bottom portion of Figure 7. This synchronous approach was also described in [14].
8 Performance

A future version of this white paper will have a comprehensive performance evaluation section of both the single machine and distributed implementations.

9 Tools

This section describes some tools we have developed that sit alongside the core TensorFlow graph execution engine.

9.1 TensorBoard: Visualization of graph structures and summary statistics

In order to help users understand the structure of their computation graphs and also to understand the overall behavior of machine learning models, we have built TensorBoard, a companion visualization tool for TensorFlow that is included in the open source release.

Visualization of Computation Graphs

Many of the computation graphs for deep neural networks can be quite complex. For example, the computation graph for training a model similar to Google’s Inception model [48], a deep convolutional neural net that had the best classification performance in the ImageNet 2014 contest, has over 36,000 nodes in its TensorFlow computation graph, and some deep recurrent LSTM models for language modeling have more than 15,000 nodes.

Due to the size and topology of these graphs, naive visualization techniques often produce cluttered and overwhelming diagrams. To help users see the underlying organization of the graphs, the algorithms in TensorBoard collapse nodes into high-level blocks, highlighting groups with identical structures. The system also separates out high-degree nodes, which often serve bookkeeping functions, into a separate area of the screen. Doing so reduces visual clutter and focuses attention on the core sections of the computation graph.

The entire visualization is interactive: users can pan, zoom, and expand grouped nodes to drill down for details. An example of the visualization for the graph of a deep convolutional image model is shown in Figure 10.

Visualization of Summary Data

When training machine learning models, users often want to be able to examine the state of various aspects of the model, and how this state changes over time. To this end, TensorFlow supports a collection of different Summary operations that can be inserted into the graph,
including scalar summaries (e.g., for examining overall properties of the model, such as the value of the loss function averaged across a collection of examples, or the time taken to execute the computation graph), histogram-based summaries (e.g., the distribution of weight values in a neural network layer), or image-based summaries (e.g., a visualization of the filter weights learned in a convolutional neural network). Typically computation graphs are set up so that Summary nodes are included to monitor various interesting values, and every so often during execution of the training graph, the set of summary nodes are also executed, in addition to the normal set of nodes that are executed, and the client driver program writes the summary data to a log file associated with the model training. The TensorBoard program is then configured to watch this log file for new summary records, and can display this summary information and how it changes over time (with the ability to select the measurement of “time” to be relative wall time since the beginning of the execution of the TensorFlow program, absolute time, or “steps”, a numeric measure of the number of graph executions that have occurred since the beginning of execution of the TensorFlow program). A screen shot of the visualization of summary values in TensorBoard is shown in Figure 11.

9.2 Performance Tracing

We also have an internal tool called EEG (not included in the initial open source release in November, 2015) that we use to collect and visualize very fine-grained information about the exact ordering and performance character-
istics of the execution of TensorFlow graphs. This tool works in both our single machine and distributed implementations, and is very useful for understanding the bottlenecks in the computation and communication patterns of a TensorFlow program.

Traces are collected simultaneously on each machine in the system from a variety of sources including Linux kernel ftrace, our own lightweight thread tracing tools and the CUDA Profiling Tools Interface (CUPTI). With these logs we can reconstruct the execution of a distributed training step with microsecond-level details of every thread-switch, CUDA kernel launch and DMA operation.

Traces are combined in a visualization server which is designed to rapidly extract events in a specified timerange and summarize at appropriate detail level for the user-interface resolution. Any significant delays due to communication, synchronization or DMA-related stalls are identified and highlighted using arrows in the visualization. Initially the UI provides an overview of the entire trace, with only the most significant performance artifacts highlighted. As the user progressively zooms in, increasingly fine resolution details are rendered.

Figure 12 shows an example EEG visualization of a model being trained on a multi-core CPU platform. The top third of the screenshot shows TensorFlow operations being dispatched in parallel, according to the dataflow constraints. The bottom section of the trace shows how most operations are decomposed into multiple workitems which are executed concurrently in a thread pool. The diagonal arrows on the right hand size show where queueing delay is building up in the thread pool. Figure 13 shows another EEG visualization with computation mainly happening on the GPU. Host threads can be seen enqueuing TensorFlow GPU operations as they become runnable (the light blue thread pool), and background housekeeping threads can be seen in other colors being migrated across processor cores. Once again, arrows show where threads are stalled on GPU to CPU transfers, or where ops experience significant queueing delay.

Finally, Figure 14 shows a more detailed view which allows us to examine how TensorFlow GPU operators are assigned to multiple GPU streams. Whenever the dataflow graph allows parallel execution or data transfer we endeavour to expose the ordering constraints to the GPU device using streams and stream dependency primitives.

10 Future Work

We have several different directions for future work. We will continue to use TensorFlow to develop new and interesting machine learning models for artificial intelligence, and in the course of doing this, we may discover ways in which we will need to extend the basic TensorFlow system. The open source community may also come up with new and interesting directions for the TensorFlow implementation.

One extension to the basic programming model that we are considering is a function mechanism, whereby a user can specify an entire subgraph of a TensorFlow computation to be a reusable component. In the implementation we have designed, these functions can become reusable components even across different front-end languages for TensorFlow, so that a user could define a function using the Python front end, but then use that function as a basic building block from within the C++ frontend. We are hopeful that this cross-language reusability will bootstrap a vibrant community of machine learning researchers publishing not just whole examples of their research, but also small reusable components from their work that can be reused in other contexts.

We also have a number of concrete directions to improve the performance of TensorFlow. One such direction is our initial work on a just-in-time compiler that can take a subgraph of a TensorFlow execution, perhaps with some runtime profiling information about the typical sizes and shapes of tensors, and can generate an optimized routine for this subgraph. This compiler will understand the semantics of perform a number of optimizations such as loop fusion, blocking and tiling for locality, specialization for particular shapes and sizes, etc.

We also imagine that a significant area for future work will be in improving the placement and node scheduling algorithms used to decide where different nodes will execute, and when they should start executing. We have currently implemented a number of heuristics in these subsystems, and we’d like to have the system instead learn to make good placement decisions (perhaps using a deep neural network, combined with a reinforcement learning objective function).

11 Related Work

There are many other systems that are comparable in various ways with TensorFlow. Theano [7], Torch [13], Caffe [26], Chainer [49] and the Computational Network Toolkit [54] are a few systems designed primarily for the training of neural networks. Each of these systems maps the computation onto a single machine, unlike the distributed TensorFlow implementation. Like Theano and Chainer, TensorFlow supports symbolic differentiation, thus making it easier to define and work with gradient-based optimization algorithms. Like Caffe, TensorFlow has a core written in C++, simplifying the deployment
The TensorFlow system shares some design characteristics with its predecessor system, DistBelief [14], and with later systems with similar designs like Project Adam [10] and the Parameter Server project [33]. Like DistBelief and Project Adam, TensorFlow allows computations to be spread out across many computational devices across many machines, and allows users to specify machine learning models using relatively high-level descriptions. Unlike DistBelief and Project Adam, though, the general-purpose dataflow graph model in TensorFlow is more flexible and more amenable to expressing a wider variety of machine learning models and optimization algorithms. It also permits a significant simplification by allowing the expression of stateful parameter nodes as variables, and variable update operations that are just additional nodes in the graph; in contrast, DistBelief, Project Adam and the Parameter Server systems all have
whole separate parameter server subsystems devoted to communicating and updating parameter values.

The Halide system \cite{40} for expressing image processing pipelines uses a similar intermediate representation to the TensorFlow dataflow graph. Unlike TensorFlow, though, the Halide system actually has high-level knowledge of the semantics of its operations and uses this knowledge to generate highly optimized pieces of code that combine multiple operations, taking into account parallelism and locality. Halide runs the resulting computations only on a single machine, and not in a distributed setting. In future work we are hoping to extend TensorFlow with a similar cross-operation dynamic compilation framework.

Like TensorFlow, several other distributed systems have been developed for executing dataflow graphs across a cluster. Dryad\cite{24} and Flume\cite{8} demonstrate how a complex workflow can be represented as a dataflow graph. CIEL\cite{37} and Naiad\cite{36} introduce generic support for data-dependent control flow: CIEL represents iteration as a DAG that dynamically unfolds, whereas Naiad uses a static graph with cycles to support lower-latency iteration. Spark\cite{55} is optimized for computations that access the same data repeatedly, using “reliable distributed datasets” (RDDs), which are soft-state cached outputs of earlier computations. Dandelion\cite{44} executes dataflow graphs across a cluster of heterogeneous devices, including GPUs. TensorFlow uses a hybrid dataflow model that borrows elements from each of these systems. Its dataflow scheduler, which is the component that chooses the next node to execute, uses the same basic algorithm as Dryad, Flume, CIEL, and Spark. Its distributed architecture is closest to Naiad, in that the system uses a single, optimized dataflow graph to represent the entire computation, and caches information about that graph on each device to minimize coordination overhead. Like Spark and Naiad, TensorFlow works best when there is sufficient RAM in the cluster to hold the working set of the computation. Iteration in TensorFlow uses a hybrid approach: multiple replicas of the same dataflow graph may be executing at once, while sharing the same set of variables. Replicas can share data asynchronously through the variables, or use synchronization mechanisms in the graph, such as queues, to operate synchronously. TensorFlow also supports iteration within a graph, which is a hybrid of CIEL and Naiad: for simplicity, each node fires only when all of its inputs are ready (like CIEL); but for efficiency the graph is represented as a static, cyclic dataflow (like Naiad).

12 Conclusions

We have described TensorFlow, a flexible data flow-based programming model, as well as single machine and distributed implementations of this programming model. The system is borne from real-world experience in conducting research and deploying more than one hundred machine learning projects throughout a wide range of Google products and services. We have open sourced a version of TensorFlow, and hope that a vibrant shared community develops around the use of TensorFlow. We are excited to see how others outside of Google make use of TensorFlow in their own work.
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