Generation of IDF Equation from Catchment Delineation Using GIS

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Abstract

The study was intended to find out the catchment characteristics of an area and the IDF (Intensity-Duration-Frequency) analysis of rainfall of that area using Gumbel method and Log Pearson type III method. In different studies, the IDF equations are generated for an area without acknowledging the catchment, stream length of the catchment and the time of concentration. In our study the Digital Elevation Map (DEM) of Dhaka was used. The DEM was analyzed using QGIS to find out different catchments, catchment area, catchment slope, stream length etc. The rainfall data of 18 years was collected from BMD (Bangladesh Meteorological Department) at 24 hours’ interval. This rainfall data was analyzed using Gumbel method and Log Pearson Type III method because these methods give accurate prediction for return period more than the range of the acquired data. The return period was taken as 2 years, 5 years, 10 years, 25 years, 50 years and 100 years. Finally, the intensity for different return period was plotted against the duration of the rainfall to find out the IDF curve. For any kind of rainfall analysis to find the peak discharge for designing storm sewage lines, small hydraulic structures or calculating surface runoff, infiltration, sub-surface runoff, discharge at rivers or water bodies, the derived IDF equations can be used. Even to find out peak discharge with a return period more than 18 years these set of equation can be used. This method can also be used in other sub urban or urban areas to find out the time of concentration of that catchment and IDF relationships for short duration rainfalls.

Keywords: IDF Equation; Catchment Delineation; Rainfall; Time of Concentration; QGIS.

1. Introduction

A drainage basin or catchment basin is an area of land where precipitation is collected and this collected volume is drained off into a common outlet such as into a river, bay or other body of water. In our case, the catchment basin of Dhaka city was determined by QGIS. GIS is proved to be a powerful tool for any hydrological assessments. Many researchers used to analyze DEM to model flooding [1]. From the drainage basin the highest concentration of runoff was calculated using two equations (Kirpich equation and Bransby-Williams formula). The Kirpich equation (Maidment 1993) [2] was developed using data from seven rural watersheds on a farm in Tennessee with well-defined channels and steep slopes [3]. Time of concentration is the time required for precipitation of the farthest corner to reach the watershed outlet. Time of concentration is used to find the peak discharge of runoff for the catchment. For drainage design of a small catchment, the peak storm water runoff volume is estimated based on rainfall intensity whose duration of rainfall equals to time of concentration (t_c) of the catchment (Chen and Wong 1993) [4].

Bell F. C. suggests that Rainfall-duration-frequency estimations for engineering purposes are usually subject to large sampling errors and extensive generalizations of the basic data are therefore advantageous from the point of view of estimation efficiency. Relationships between high-intensity rainfall values for durations up to 2 hr and return periods up to 100 years are approximately expressed by means of 'rainfall-duration' and 'rainfall-frequency' ratios to
generate IDF equations [5]. In this study the set of IDF equation was generated by using the value generated from Gumbel method and LPT-III method. Both the methods give short duration rainfall amount for different duration. The duration of the rainfall was taken from the time of concentration. In this study the catchment profile including stream length, area, and time of concentration are used to get a better understanding of the IDF curves. This study also compares findings of other studies that followed different approach to generate IDF equations. The findings of the study prove to be vital for designing storm sewage lines, small hydraulic structures or calculating surface runoff, infiltration, discharge at rivers or water bodies, urban flood extent delineation etc.

1.1. Structure of the Paper

The study area chosen for this research is Dhaka, the capital of Bangladesh which has one of the highest population density among the big cities of the world. The study area extends to 23°40’ to 23°N latitude and 90°20’ to 90°30’E longitude. It is surrounded by Buriganga River to the south, Turag River to the west, Tongi khal to the north and Balu River to the East. Dhaka city is located in Modhupur terrace which is an alluvial terrace. Topographically, Dhaka city is located in flat land with elevation ranges from 1 to 14 meters. Dhaka city is built up in a flood plain with numerous water bodies are present here to drain the excess water in the monsoon period. Dholai Khal and Begunbari Khal are the main water bodies that drains the massive runoff from Dhaka city. But as the population boom in Dhaka, large number of water bodies are encroached and lost their ability to drain and store water. According to BMD annual average rainfall for Dhaka is 2076 mm. A growing concern is that, in a changing climate, characterized by heavier and more erratic rainfall in the Ganges-Brahmaputra-Meghna (GBM) basin during the monsoon season, the situation may worsen. To measure the excess volume of water during urban flooding or to design any hydraulic structure the rainfall amount and period is required to measure. For this reason, this study is concerned to determine the catchment basin of Dhaka and IDF equation for Dhaka.

2. Research Methodology

In this study, the catchments of Dhaka city are analyzed using GIS. The catchment map (Figure 2) was produced from the DEM (Digital Elevation Model) using GIS. The catchment map also includes stream lines (rivers, canals etc.) which are overlaid on the DEM map. This map produced catchment area, stream length, and slope, for the selected Dhaka city catchment. These data were used in Equations 1 and 2 to find out the time of concentration ($t_c$). The time of concentration is required to determine the maximum discharge. The calculated time of concentration was used along with the rainfall data to produce IDF equations. The Rainfall Data for this study was collected from Bangladesh Meteorological Department (BMD). The rainfall data was collected at 24 hours’ interval from BMD’s Dhaka rain gauge station. The collected data was taken in between the range of 2000 to 2018. The average rainfall per day was used along with different return periods (2 years, 5 years, 10 years, 20 years, 50 years and 100 years) to generate Intensity for different duration of rainfall. The range of the duration of rainfall was determined from the maximum and minimum time of concentration of Table 1. Two different approach (Gumbel Method and Log Pearson Type-III Method) was used to determine the intensity described previously. IDF equations and IDF curves produced by the above analysis were shown in Table 2, Table 3 and Figure 3. The flow chart of this research is given in Figure.

2.1. Catchment Delineate

Initially Advanced Space borne Thermal Emission and Reflection Radiometer (ASTER) Digital Elevation Model (DEM) data is collected from United State Geological Survey website. ASTER GDEM (Global Digital Elevation Model) boasted a global resolution of 1 arc-second (approximately 30 m) resolution. The elevations in this Digital Elevation Model (DEM) represent the topographic bare-earth surface. Contour of the study area was prepared using this DEM data. Slope map (Figure 2) was prepared from DEM data using QGIS Spatial Analyst tool in this case Fill tool. After that flow generation has been developed to observe the flow direction of the study area. The map was finalized using the basin tool and catchment basin layout was prepared (Figure 2).
Figure 1. Flowchart of the research methodology
2.2. Time of Concentration

Kirpich’s Equation (1940) was developed for small, agricultural watersheds. It was derived by examining the required time for the stream to rise from low to maximum stage during a storm. The time of concentration was then assumed equal to that time.

Time of concentration, \( t_c = 0.01947 \times L^{0.77} \times S^{-0.385} \) (Kirpich Formula)  \( (1) \)

Bransby Williams formula is described as an “arbitrary but reasonable approach” where there is no specific formula available for a particular region. This recommendation was based as per Beran (1980) [6] who tested eight formulae for estimating the time of concentration and showed the Bransby Williams formula was least biased and straightforward to calculate.

Time of concentration \( t_c = \frac{FL}{(A^{0.3})^{(S^{0.2})}} \) (Bransby-Williams formula)  \( (2) \)

Here, \( L \) = Maximum length of travel of water (m); \( S \) = Average slope of the catchment \( F \) = Factor of proportionality which is 58.5 when \( A \) in \( \text{km}^2 \); \( L \) = Maximum length of travel for water (km); \( S \) = Slope of Catchment (m/km); \( A \) = Area of the Catchment (\( \text{km}^2 \)).

| Length of Stream (m) | Average slope of Catchment | Catchment Area (sqm) | Time of concentration, \( t_c \) (min) (Kirpich Equation) | Time of concentration, \( t_c \) (min) (Bransby-Williams formula) |
|----------------------|-----------------------------|----------------------|----------------------------------------------------------|----------------------------------------------------------|
| 51378.298            | 0.035                       | 4207000000           | 298.127                                                  | 194.664                                                  |
| 2015.386             | 0.035                       | 4207000000           | 24.629                                                   | 7.635                                                    |

The stream length, slope of the catchment and area of the catchment was measured by QGIS. For small catchments the peak discharge can be found using time of concentration. The duration of rainfall was taken in between the maximum and minimum time of concentration range at random. It is suggested that duration of 2 hours is adequate for peak discharge at a sub catchment of Dhaka [7] which is in between the range of Table 1.
2.3. Source of Rainfall Data

The Rainfall Data for this study was collected from Bangladesh Meteorological Department (BMD). The rainfall data was collected at 24 hours’ interval from BMD’s Dhaka rain gauge station. The collected data was taken in between the range of 2000 to 2018. This data was used to find out the IDF curve for the catchment of Dhaka. Only the significant rainfall event was taken into account. Total 1386 significant rainfall data was found in the 18 years’ period. Maximum 341 mm rainfall was recorded. The average of total rainfall was 5.39 mm.

2.4. Gumbel Theory of Distribution

In the probability theory and statistics, the Gumbel Distribution is used to model the distribution of the maximum of a number of samples of various distributions. In our case, Gumbel distribution theory was used to find out the maximum intensity of rainfall from 2000 to 2018. Gumbel theory was selected for IDF analysis because it’s relatively simple and it counts only extreme rainfall events [8]. In this method, the different return period $T$ (2 years, 5 years, 10 years, 25 years, 50 years, and 100 years) was taken into consideration. For this different return period the highest maximum precipitation $P_T$ was derived for each case. From this precipitation the rainfall intensity was calculated by taking duration of the rainfall $T_d$ (min) into consideration. Rainfall intensity $I_T$ (mm/hr) was found by dividing the frequency precipitation value to the time of concentration, $T_d$ (min). In our case the duration of the rainfall was taken in between the range of maximum and minimum time of concentration for different stream length. For the calculation of $P_T$ (mm/hr) the average rainfall (mm/hr) of 18 years $P_{ave}$ was calculated and two constant $K$ and $S$ was also calculated. Here $S$ is the standard deviation of the total rainfall data. The equations are given below:

$$P_T = P_{ave} + K$$

$$K = \sqrt[6]{\frac{2}{\pi}} \left[ 0.5772 + \ln \left( \frac{T}{T-1} \right) \right]$$

$$P_{ave} = \frac{1}{n} \sum_{i=1}^{n} P_i$$

$$S = \frac{1}{n-1} \sum_{i=1}^{n} ((P_i - P_{ave})^2)^{1/2}$$

The rainfall intensity, $I_T$ (mm/hr) for any return period $T$ is obtained from the following equation:

$$I_T = \frac{P_T}{T_d}$$

2.5. Log Pearson Type III

The log Pearson type III distribution is a statistical technique for fitting frequency distribution data to predict the design flood or rainfall. The advantage of this particular technique is that extrapolation can be made of the values for events with return periods well beyond the observed flood events or rainfall events. The equations of Log Pearson Type III are given below:

$$P^* = \log(P_i)$$

$$P_T^* = P_{ave}^* + K_T S^*$$

$$P_{ave}^* = \frac{1}{n} \sum_{i=1}^{n} P^*$$

$$S^* = \left( \frac{1}{n-1} \sum_{i=1}^{n} (P^* - P_{ave}^*)^2 \right)^{1/2}$$

This method involves logarithmic values of the previously described data. The average precipitation $P_T^*$ (mm/hr) and the standard deviation was calculated using the logarithmic transformed data.

In Equation 9, $K_T$ is the Pearson frequency factor which depends on return period $T$ (years) and skewness coefficient (Cs). The skewness coefficient is computed by following Equation.

$$C_s = \frac{n \sum_{i=1}^{n} (P_i - P_{ave})^3}{(n-1)(n-2)(s^3)}$$

$K_T$ values can be obtained from tables in many hydrology references. By knowing the skewness coefficient and the recurrence interval, the frequency factor, $K_T$ for the LPT III distribution can be extracted. The antilog $P_T$ of will provide the estimated extreme value for the given return period.
2.6. IDF Empirical Equation

IDF empirical equations are the equation that estimates the maximum rainfall intensity for different duration and return period. Equation 13 is the form of IDF empirical equation which is used in this study:

\[ i = x^a (td)^y \]  

(13)

Where, \( i \) is the rainfall intensity in mm/hr, \( td \) is the rainfall duration in min. \( x \) and \( y \) are the fitting parameter. Least-square method was applied to find the parameter \( x \) and \( y \) for the rainfall IDF empirical formula. Correlation coefficient (R) was estimated to find the best fit IDF empirical equation.

3. Result and Discussion

In the study, the maximum and minimum length of stream was found 51378.298 meters and 2015.386 meters respectively using Digital Elevation Model. The catchment area and average slope of the catchment was found to be 4207 km\(^2\) and 0.035. These catchment data was used to determine the time of concentration. Kirpich equation and Bransby-Williams formula was used to determine the time of concentration. The maximum time of concentration for our catchment was 298.127 minutes and the minimum time of concentration was 24.629 minutes. Gumbel method and Log-Pearson type-III method was used to determine the frequency precipitation PT(mm) for each duration with the return period of 2 years, 5 years, 10 years, 25 years, 50 years and 100 years.

The IDF equation was generated using the least square regression method and the correlation coefficient for each analysis was found 1. That means every IDF curve drawn for both the methods are best fit. The frequency precipitation value was found higher in Gumbel method than Log-Pearson type-III method. But the difference between corresponding parameters in Gumbel method and LPT-III method were significantly higher in our study than one of the previous research [9]. The statistical parameters \( x \) were significantly higher in our study averaging 4608 in Gumbel Method and 1680 for LPT-III method comparing to the Rasel and Tashfique Uddin (2015) studies [9] which are 1181 for Gumbel Method and 1151 for LPT-III method. In this study 18 years (2000-2018) precipitation data was used in camparison to the previous study where 40 years (1974-2014) data was used. In Gumbel method the frequency precipitation, PT values are 83.55 mm, 135.39 mm, 169.54 mm, 212.82 mm, 244.94 mm and 276.81 mm whereas for Log-Pearson type-III method the frequency precipitation, PT values are 25.43 mm, 53.96 mm, 68.88 mm, 81.62 mm, 36 mm and 91.77 mm for the return period of 2 years, 5 years, 10 years, 25 years, 50 years and 100 years at a minimum duration of 24.63 min. For this the rainfall intensity values IT are also higher for Gumbel method than for Log-Pearson type-III method in every analysis. This study is in agreement with the findings of Matin and Ahmed (1984) [10] that suggested Gumbel method estimating relatively higher rainfall intensity compared to those obtained by LPT III distribution. The IDF curves for both the methods are reverse exponential.

The set of equations in Tables 2 and 3 can be used to determine the intensity with a predefined return period. Also from the curves of Figure 3 the intensity of rainfall can be determined with a given duration and recurrence period. Moreover, the IDF equation of Indian Method for Dhaka described by Rimi and Matin et al. (2016) [11] gives similar value to the equation of Gumbel Method Table 2 at smaller return periods.
Table 2. Rainfall IDF empirical equation using Gumbel method and their correlation coefficient, R for Dhaka

| Return Period (yr) | Equation         | Correlation Coefficient, R |
|-------------------|------------------|----------------------------|
| 2                 | \[ Y = 2052.311X^{-1} \] | 1                          |
| 5                 | \[ Y = 3332.173X^{-1} \] | 1                          |
| 10                | \[ Y = 4175.88X^{-1} \]  | 1                          |
| 25                | \[ Y = 5241.896X^{-1} \] | 1                          |
| 50                | \[ Y = 6032.73X^{-1} \]  | 1                          |
| 100               | \[ Y = 6817.72X^{-1} \]  | 1                          |

Table 3. Rainfall IDF empirical equation using LPTIII method and their correlation coefficient, R for Dhaka

| Return Period (yr) | Equation         | Correlation Coefficient, R |
|-------------------|------------------|----------------------------|
| 2                 | \[ Y = 626.466X^{-1} \] | 1                          |
| 5                 | \[ Y = 1329.174X^{-1} \] | 1                          |
| 10                | \[ Y = 1696.524X^{-1} \] | 1                          |
| 25                | \[ Y = 2010.307X^{-1} \] | 1                          |
| 50                | \[ Y = 2161.195X^{-1} \] | 1                          |
| 100               | \[ Y = 2260.428X^{-1} \] | 1                          |

4. Conclusion

The collected rainfall data shows an erratic nature of rainfall in Dhaka. The catchment analysis suggests that the kirpich formula gives higher values of time of concentration than Bransby-Williams formula. The time of concentration also increases with stream length. The IDF curve on the other hand shows that for same catchment and duration Gumbel Method gives a higher intensity of rainfall than LPT III. This suggests that for any hydraulic design Gumbel Method is safer having larger value of intensity. The Correlation factor of the IDF curves is found to be 1 which suggests the curves are in good shape. So the IDF curves can be used for any given duration and return period. It also may be suggested that if the rainfall values collected by BMD were recorded at 1hour or 2 hours’ interval the generated IDF equation will be more accurate.

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6. Conflicts of Interest

The authors declare no conflict of interest.

7. References

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