The Irreducible Axion Background

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Why Axions?

- Strong CP
- Dark matter candidate
- Potential mediator to dark sector
- Prevalent in string theories.
- Goldstone bosons of global symmetries
Today’s Definition of Axions

\[ \mathcal{L} \supset \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2a^2 - \frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{aee}}{2m_e}(\partial_\mu a)\bar{e}\gamma^\mu\gamma_5e \]
Axion Parameter Space

If it solves strong CP (Canonically)

KSVZ
DFSZ

[C. O’Hare, GitHub]
Axion Parameter Space

If it exists

$|g_{a\gamma}|$ [GeV$^{-1}$] vs. $m_a$ [eV]

Astrophysical Production

KSVZ

DFSZ

[C. O’Hare, GitHub]
Axion Parameter Space

If it is all of DM
Axion Parameter Space

If it is all of DM

If it is all of DM

[KSVZ

DFSZ

Astrophysical

Production

DM Halos

DM Decays

[GeV

10^{-7}

10^{-8}

10^{-9}

10^{-10}

10^{-11}

10^{-12}

10^{-13}

10^{-14}

10^{-15}

10^{-16}

10^{-17}

10^{-18}

10^{-19}

m_a [eV]

[C. O’Hare, GitHub]
Axion Parameter Space

What if it is not ALL of DM?

[C. O’Hare, GitHub]
Axion Parameter Space

What if it is not ALL of DM?

Irreducible Axion Background (Freeze-in relics)

IRREVERSIBLE AXION BACKGROUND
(FREEZE-IN RELICS)
Irreducible Cosmic Abundance & Constraints
The General Picture

Dark matter may consist of **more than one species.**
Definition of $F_{\chi}$

\[ \rho_{\chi} \approx F_{\chi} \rho_{\text{DM}} e^{-t/\tau_{\chi}} \]  
(For non-relativistic $\chi$ after freeze-in)
Constraints on Sub-component DM

• Many constraints on $DM$ can immediately be modified to constraints on $\chi$.

For example (for $\tau_\chi \gg t_U$):

- $\sigma_{DM-N} \rightarrow F_\chi \times \sigma_{\chi-N}$
- $\Gamma_{DM\rightarrow\gamma\gamma} \rightarrow F_\chi \times \Gamma_{\chi\rightarrow\gamma\gamma}$
- $\langle \sigma_{DM+DM\rightarrow SM} \nu \rangle \rightarrow F_\chi^2 \times \langle \sigma_{\chi+\chi\rightarrow SM} \nu \rangle$

• Current indirect detection experiments for decay can probe down to $F_\chi \sim 10^{-12}$!
Constraints on Sub-component DM

Broadly speaking searches for a dark particle $\chi$ constrain the parameters $(m_\chi, g_\chi, F_\chi)$.

- **DM Approach**: $F_\chi = 1$ and $\tau_\chi \gg t_U$.
- **Agnostic Approach**: $F_\chi$ is an additional free parameter.
- **Calculational Approach**: $F_\chi = F_\chi(m_\chi, g_\chi, C)$, where $C$ is some cosmology.

Constraints depending on $C$ are not robust.
Irreducible Cosmic Abundance Constraints

Instead we calculate the irreducible abundance $F_{\chi,\text{irr}}(m_\chi, g_\chi)$.

This is determined by considering only production after the beginning of BBN ($T < 5$ MeV).

Constraints obtained using $F_{\chi,\text{irr}}(m_\chi, g_\chi)$ are robust under two mild assumptions:

1. $\chi$ does not decay/annihilate to a dark sector.
2. Standard cosmology holds from BBN on.
Summary

• Sub-components are well motivated and interesting in their own right.
• DM searches can also constrain subcomponents.
• There exists an irreducible abundance which can be used to obtain robust constraints.
Application to Axions
Irreducible Axion Background Constraints

1. Calculate $F_{a,\text{irr}}$

2. Apply astrophysical and cosmological constraints.
Production of Axions

• The irreducible axion background is obtained by freezing-in axions beginning at $T = 5 \text{ MeV}$.

• What does freezing-in mean?

• Definition: Freeze-in is the process where particles are created from the primordial plasma of the universe without ever being in a state of thermal equilibrium with it.

\[
\begin{align*}
\text{SM} & \not\rightarrow \chi \\
\text{SM} & \rightarrow \chi
\end{align*}
\]
Types of Freeze-In (Rough Idea)

There exist many types of freeze-in, but can generally be classified into two groups.

\[ Y = \frac{n}{s} \]

\( t_{\text{RH}} \)

\( t_{\text{cut}} \)

IR Freeze-In

UV Freeze-In
Logic of Freeze-In (Simplified)

\[ \frac{dn_a}{dt} + 3Hn_a = n_{\text{SM}}\Gamma_{\text{SM}\rightarrow a} - n_a\Gamma_{a\rightarrow \text{SM}} \approx 0 \]

Axion Production
Axion Destruction

Define: \( Y_a = \frac{n_a}{s} \sim n_aR^{-3} \)

\[ \frac{dY_a}{d\log T} \sim -\frac{\Gamma_{\text{SM}\rightarrow a}}{H} \implies Y_{a,\text{FI}} \sim \frac{\Gamma_{\text{SM}\rightarrow a}}{H} \bigg|_{T_*} \]
UV Freeze-In Example

\[ \Gamma \sim g_{a\gamma\gamma}^2 T^3 \implies \frac{\Gamma}{H} \sim g_{a\gamma\gamma}^2 M_{Pl} T \]
IR Freeze-In Example

\[ \Gamma \sim g_{a\gamma\gamma}^2 T^3 \text{ (Naively)} \]
IR Freeze-In Example

\[ \Gamma \sim \begin{cases} 0, & m_\gamma(T) > m_a/2 \\ g_{a\gamma\gamma}^2 m_a T^2, & m_\gamma(T) < m_a/2 \end{cases} \]

where \( m_\gamma(T) \approx eT/3 \)
General Production of Axions

| Process                     | $g_{a\gamma\gamma}$ | $g_{aee}$ |
|-----------------------------|-----------------------|-----------|
| **Photon Conversion**       | $\gamma e \rightarrow ae$ | $\gamma \rightarrow a$ |
| **Fermion Annihilation**    | $e^- e^+ \rightarrow a\gamma$ | $e^- e^+ \rightarrow a\gamma$ |
| **Inverse Decay**           | $\gamma\gamma \rightarrow a$ or $e^- e^+ \rightarrow a$ | $e^- e^+ \rightarrow a$ |
Irreducible Freeze-In Background

\[ m_a = T_{\text{RH}} \]
Irreducible Freeze-In Background

Photon Conversion

Exponentially suppressed for $m_a \gg T_{RH}$
Irreducible Freeze-In Background

Inverse Decay

Dominant at large $m_a$
Irreducible Freeze-In Background

Fermion Annihilation

\[ e^- \rightarrow e^+ \gamma \]

\[ e^- e^+ \rightarrow \gamma a \]

\[ \gamma \gamma \rightarrow a \]

\[ e\gamma \rightarrow e\alpha \]

Strictly sub-dominant
Irreducible Freeze-In Background

\begin{align*}
T_a &= 1 \\
T_a &= 10^{-5} \\
T_a &= 10^{-10}
\end{align*}

\begin{align*}
m_a &= \text{fixed} \\
mg &= \text{fixed}
\end{align*}

\begin{align*}
e\gamma &\to ea \\
\gamma\gamma &\to a \\
e^-e^+ &\to \gamma a
\end{align*}
Astrophysical and Cosmological Constraints
Intuition: X-rays

- Consider the benchmark constraint $\tau_{\text{DM}} > 10^{28}\text{s} \implies \tau_a > F_a \tau_{\text{DM}}$. 
Intuition: X-rays

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Intuition: X-rays

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- To observe decays today we require $\tau_a \gtrsim 0.1 \times t_U$. 
Intuition: X-rays

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Cosmological and Astrophysical Constraints

How we observe the decay of axions depends on when they decay.
Cosmological and Astrophysical Constraints

How we observe the decay of axions depends on when they decay.

For $\tau_a \gtrsim t_U$, we can look for decays in local sources of axions:

- Galactic Center
- Dwarf Spheroidal Galaxies

Very schematic!
Cosmological and Astrophysical Constraints

How we observe the decay of axions depends on when they decay.

For $t_{\text{CMB}} \ll \tau_a \lesssim t_U$, we can look for decays in the diffuse axion background. [Zurek et al, 2013]

$$\frac{d\Phi}{dE} = \frac{2\rho_a}{m_a E H_0} \frac{e^{-t(E)/\tau_a}}{\tau_{a\rightarrow\gamma\gamma}} \frac{e^{-\kappa(z,E)}}{\sqrt{\Omega_m(m_a/2E)^3 + \Omega_\Lambda}} \Theta(m_a - 2E)$$
Cosmological and Astrophysical Constraints

How we observe the decay of axions depends on when they decay.

For $t_{\text{CMB}} \lesssim \tau_a$, we can look for the effect of decays on CMB anisotropies ($z \lesssim 1100$)

[Slater and Wu, 2016]
Cosmological and Astrophysical Constraints

How we observe the decay of axions depends on when they decay.

For $\tau_a \lesssim t_{\text{CMB}}$, we can look for the effect of decays on CMB spectral distortions 

$(1100 \lesssim z \lesssim 2 \times 10^6)$  

[Balzas et al, 2022]
Cosmological and Astrophysical Constraints

How we observe the decay of axions depends on when they decay.

For $t_{BBN} \lesssim \tau_a$, we can look for the effect of decays light element abundance
How we observe the decay of axions depends on when they decay.

$$t_{BBN} \lesssim \tau_a \lesssim t_{CMB}$$, we can look for the effect of decays in $\Delta N_{\text{eff}}$.

[Depta, Hufnagel, and Schmidt-Hoberg, 2020]
Photophilic Axion Bounds

$g_{\gamma\gamma} \left[ \text{GeV}^{-1} \right]$

$T_{RH} = 5 \text{ MeV}$

$m_a \left[ \text{keV} \right]$

Photophilic ALP

HB stars

CRB

Basin

SD

SN1987A Decays

BBN

SN1987A $\nu$ burst

$T_{\text{hot}} = 10 \text{ MeV}$

CMB

X-ray

CRB

$F_\gamma = 10^{-10}$

$F_\gamma = 10^{-16}$
Photophilic Axion Bounds

- HB stars
- SN1987A Decays
- SN1987A ν burst
- BBN
  - $T_{\text{RH}} = 10$ MeV
- CMB
  - $T_{\text{RH}} = 5$ MeV
- X-ray CRB
- $\tau = 10^{15}$ s
- $\tau = 10^{13}$ s
- $\tau = 10^3$ s

$g_{\gamma\gamma} \gamma^{-1}$ vs. $m_a$ [keV]

$T_{\text{RH}} = 5$ MeV
Generalizations
### Photophobic Axion Constraints

| Process                                      | Diagrams |
|----------------------------------------------|----------|
| **Photon Conversion**                        | $g_{\gamma\gamma}$ |
| $\gamma e \rightarrow a e$                   | ![Photon Conversion Diagram] |

| **Fermion Annihilation**                     | $g_{\alpha ee}$ |
| $e^-e^+ \rightarrow a\gamma$                 | ![Fermion Annihilation Diagram] |

| **Inverse Decay**                            |           |
| $\gamma\gamma \rightarrow a$                 | ![Inverse Decay Diagram] |
| or $e^-e^+ \rightarrow a$                     |           |
Photophobic Axion Constraints
Photophilic Axions with Misalignment

- $T_{RH} = 5 \text{ MeV}$
- $\Omega_a h^2 > 0.12$
- $\theta_0 = 1$
- $\theta_0 = 0.1$
- $\theta_0 = 0.01$
- $\mu_0 = 0$
- $\mu_0 = 0$
- $\Delta = 1$
- $\Delta = 12$

$g_{\gamma\gamma}$ vs. $m_a$ in [keV]
Production of Sterile Neutrinos

• For simplicity, assume sterile neutrino mixes only with $\nu_e$.

\[
\begin{pmatrix}
\nu_e \\
\nu_s
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]

• The Boltzmann equation will have the following form:

\[
\left( \frac{\partial}{\partial t} - H_p \frac{\partial}{\partial p} \right) f_s(T, p) = (f_s^{eq} - f_s) \Gamma_s(T, p)
\]

\[
\Gamma_s(T, p) \approx \begin{cases} 
\frac{1}{4} \sin^2(2\theta) d_e G_F^2 E T^4, \\
\frac{1}{\tau_s} \left[ \frac{m_s}{E} + \frac{288 \zeta(3) T^3}{m_s^3} + \frac{112 \pi^4 T^3}{3 m_s^5} \left( E T + \frac{p^2 T}{3 E} \right) \right], & m_s \ll T_{RH} \\
\end{cases}
\]

\[
\end{cases}
\]

\[
\begin{cases} 
\frac{1}{\tau_s} \left[ \frac{m_s}{E} + \frac{288 \zeta(3) T^3}{m_s^3} + \frac{112 \pi^4 T^3}{3 m_s^5} \left( E T + \frac{p^2 T}{3 E} \right) \right], & m_s \gg T_{RH}
\end{cases}
\]

[G. Gelmini, E. Osoba, S. Palomares-Ruiz, S. Pascoli, 2008]
Cosmological and Astrophysical Constraints

The easiest way to observe sterile neutrinos is through their radiative decay.

However, their lifetime is determined by a different process

(Also to $e^+e^-$ when $m_s > 2m_e$)

[Diagrams from Kopp and Dasgupta, 2021]
Sterile Neutrino Constraints

\[ \sin^2(2\theta) = 10^\circ \]

\[ T_{RH} = 5 \text{ MeV} \]

[Gray Constraints taken from Patrick Bolton]
Thank You

Upcoming work:
• Axiverse with SU(N) SYM Domain Walls
• PBH Production with SUSY Axions
• Observing Light BSM Particles with Muon Decay
• Axion Dark Matter in the Mirror World
Photophilic Axions with $T_{RH} = 100$ MeV
Axions with “Universal” Couplings

\[ T_{\text{RH}} = 5 \text{ MeV} \]
Production of Axions

- Abundance obtained from solving the Boltzmann equation.

\[ \dot{n}_a + 3Hn_a = R(t) \]

\[ R(t) = \sum_{\text{process}} \int \left( \prod_i d\Pi_i \right) \left( \prod_f d\Pi_f \right) (2\pi)^4 \delta^4 \left( \sum_i p_i - \sum_f p_f \right) |M_{i\rightarrow f}|^2 \times \Phi \]

\[ \Phi = (f_a^{eq} - f_a) \times \left[ \prod_i \left( 1 \pm f_i^{eq} \right) \prod_{f \neq a} f_f^{eq} - \prod_{f \neq a} \left( 1 \pm f_f^{eq} \right) \prod_i f_i^{eq} \right] \]

Note dropping \( f_a \) decouples the Boltzmann equation!
(Production by different processes are independent)
Production of Axions

Make the following definitions:

1. \( x = \frac{m_a}{T} \)
2. \( Y_a = \frac{n_a}{s} \)
3. \( \tilde{g}(x) = 1 - \frac{1}{3} \frac{d \log g_{\ast,s}}{d \log x} \)

The Boltzmann equation simplifies to

\[
\frac{dY_a}{dx} = \frac{\tilde{g}(x)}{xH(x)s(x)} R(x) \implies \mathcal{F}_a \simeq \frac{m_\alpha s_0}{\rho_{DM,0}} Y_a(\infty) \quad \text{(Ignoring Axion Decay)}
\]
Production of Axions (Inverse Decay)

\[
\sum |M_{\gamma\gamma \rightarrow a}|^2 = \frac{1}{2} g_{a\gamma\gamma} m_a^2 \left( m_a^2 - 4m_\gamma^2 \right)
\]

\[
R_{ID}(T) = \sum |M_{\gamma\gamma \rightarrow a}|^2 \int d\Pi_1 d\Pi_2 d\Pi_a (2\pi)^4 \delta^4 (p_1 + p_2 - p_a) \times f_a^{eq}[1 + (f_1^{eq} + f_2^{eq})]
\]

\[
= \sum \frac{|M_{\gamma\gamma \rightarrow a}|^2}{32\pi^3} \int_{m_a}^{\infty} dE_a f_a^{eq} \left( \beta p_a + 2T \ln \left[ \frac{1 - e^{-E_+/T}}{1 - e^{-E_-/T}} \right] \right)
\]

\[
\beta = \sqrt{1 - 4m_\gamma(T)^2/m_a^2} \quad E_{\pm} = (E_a \pm \beta p_a)/2
\]

- \( R_{ID}(T) = 0 \) for \( 2m_\gamma(T) > m_a \) where \( m_\gamma(T) \approx eT/3 \sim T/10 \)

- Similar calculation for electrons.
Production of Axions (2→2)

\[ R_{2\to 2}(T) \approx \frac{g_1 g_2 T}{32\pi^4} \int_{s_{\text{min}}}^{\infty} ds \, \lambda(s, m_1^2, m_2^2) \frac{K_1(\sqrt{s}/T)}{\sqrt{s}} \sigma_{12\to 3a}(s) \]  
[D’Eramo et al. 2017]

\[ \sigma_{FA}(s) = \frac{\alpha g_{a\gamma\gamma}^2}{24\beta} \left( 1 - \frac{m_a}{s} \right)^3 \left( 1 + \frac{2m_c^2}{s} \right) + \frac{\alpha g_{aee}^2}{2s^2(s - m_a^2)\beta^2} \left[ (s^2 - 4m_c^2m_a^2 + m_a^4) \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2\beta m_a^2s \right] \]

\[ \sigma_{PC}(s) = \frac{\alpha g_{a\gamma\gamma}^2}{32s^2} \left[ 2(2s^2 - 2m_a^2s + m_a^4) \ln \left( \frac{s - m_a^2}{m_\gamma^2} \right) - 7s^2 + 10m_a^2s - 5m_a^4 \right] \]

\[ + \frac{\alpha g_{aee}^2}{8s^3} \left[ 2 \left( 2s^2 - 2m_a^2s + m_a^4 \right) \ln \left( \frac{s}{m_e^2} \right) - 3s^2 + 10m_a^2s - 7m_a^4 \right] \]

\[ - \frac{\alpha g_{a\gamma\gamma}g_{aee}m_e}{8s^3(s - m_a^2 + m_e^2)} \left[ 2(s^3 + m_a^6) \ln \left( \frac{(s - m_a^2)^2}{(s + m_a^2)m_e^2} \right) - 3(s + m_a^2)(s - m_a^2)^2 \right] \]
Irreducible Cosmic Abundance Constraints

How do you calculate $F_{\chi, \text{irr}}(m_\chi, g_\chi)$?

- Generally found by setting $n_\chi(T = 5 \text{ MeV}) = 0$ and having $\chi$ freeze-in.
- Roughly equivalent to having reheating occur at $T_{\text{RH}} = 5 \text{ MeV}$.
- Generally $F_{\chi, \text{irr}}(m_\chi, g_\chi) \approx 0$ for $m_\chi \gg 100 \text{ MeV}$ because of Boltzmann suppression.
Evidence of dark matter

CMB

Bullet Cluster

Rotation Curves

Structure Formation
Model independent facts about dark matter

• It exists.
• It is not hot.
• Rough idea of DM halo density.
• Mass dependent constraints on self interactions.
• No known consistent explanation within SM.
• Thats about it...
Dark Matter Mass Range

- $10^{-2}$ eV
- $10^{-1}$ eV
- $10^{+}$ eV

"Classical" QCD Axion

- $10^{-4}$ eV
- $10^{-6}$ eV
- $10^{-22}$ eV

Constrained by BBN and stellar cooling

- $10^{-1}$ keV
- $1$ MeV
- $10$ GeV

Fermionic DM

- $100$ GeV
- $100$ TeV
- $10^{57}$ GeV

Thermal DM

WIMPS

SIMPS

ADM

$\nu_s$

Q-balls

Dark Quark Nuggets

Primordial Black Holes

etc…

Constrained by BBN and stellar cooling
My talk will focus on sub-components of DM, not DM itself. More flexible mass ranges.
