The challenge to reliably measure the general relativistic Lense-Thirring effect with a few percent accuracy

Lorenzo Iorio
Dipartimento Interateneo di Fisica dell' Università di Bari
Via Amendola 173, 70126
Bari, Italy
e-mail: Lorenzo.Iorio@libero.it

Abstract

In this paper we critically analyze the so far performed and proposed tests for measuring the general relativistic Lense-Thirring effect in the gravitational field of the Earth with some of the existing accurately tracked artificial satellites. The impact of the 2nd generation GRACE-only EIGEN-GRACE02S Earth gravity model and of the 1st CHAMP+GRACE+terrestrial gravity combined EIGEN-CG01C Earth gravity model is discussed. The role of the proposed LARES is discussed as well.

1 Introduction

The Einstein’s General Theory of Relativity (GTR), in its weak-field and slow-motion approximation (Soffel 1989) valid throughout the Solar System, predicts, among other things, that the rotation of a body of mass $M$ induces a so called ‘gravitomagnetic’ component of its gravitational field which acts on a test particle orbiting it with a non central, velocity-dependent force analogous to the Lorentz force of the Maxwellian electromagnetism. As a consequence, the longitude of the ascending node $\Omega$ and the argument of pericentre $\omega$ of its orbit undergo tiny secular precessions

$$\dot{\Omega}_{LT} = \frac{2GJ}{c^2a^3(1 - e^2)^{3/2}}, \dot{\omega}_{LT} = -\frac{6GJ \cos i}{c^2a^3(1 - e^2)^{3/2}},$$

(1)

where $G$ is the Newtonian constant of gravitation, $J$ is the proper angular momentum of the central body, $c$ is the speed of light, $a, e$ and $i$ are the semimajor axis, the eccentricity and the inclination, respectively, of the test particle’s orbit. It is the Lense-Thirring effect (Lense and Thirring 1918).

Recent years have seen increasing efforts devoted to the measurement of this kind of post-Newtonian gravitomagnetic effect in the gravitational field.
of the Earth by means of the analysis of the Satellite Laser Ranging (SLR) data to the existing LAGEOS \( (a = 12270 \text{ km}, i = 110 \text{ deg}, e = 0.0045) \) and LAGEOS II \( (a = 12163 \text{ km}, i = 52.65 \text{ deg}, e = 0.014) \) geodetic satellites (Ciufolini 2004; Ciufolini and Pavlis 2004; Iorio and Morea 2004; Iorio 2004) for which the Lense-Thirring effect amounts to a few tens of milliarcseconds per year (mas yr\(^{-1}\)). Alternative approaches including also the other existing SLR satellites, with particular emphasis on Ajisai, and the altimeter Jason-1 satellite have also been proposed (Iorio 2002; Iorio and Doornbos 2004; Vespe and Rutigliano 2004). The originally proposed LARES mission (Ciufolini 1986; 1998; Iorio et al. 2002; Iorio 2003a), which involves the launch of another SLR target whose data should be analyzed together with those from LAGEOS and LAGEOS II, has recently been investigated in the context of the relativity-dedicated OPTIS mission (Iorio et al. 2004; Lämmerzahl et al. 2004).

The major sources of systematic errors in such kind of measurements are due to the aliasing classical secular precessions (Iorio 2003b) induced by the even zonal harmonic coefficients \( J_\ell, \ell = 2, 4, 6, ... \) of the multipolar expansion of the terrestrial gravitational potential, called geopotential (Kaula 1966), and to the non-gravitational perturbations induced, e.g., by the direct solar radiation pressure, the Earth albedo, the Earth infrared radiation, the solar Yarkovsky-Schach effect, the terrestrial Yarkovsky-Rubincam effect, the asymmetric reflectivity (Lucchesi 2001; 2002; 2003; 2004; Lucchesi et al. 2004). The non-gravitational forces especially affect the perigees of the geodetic satellites of LAGEOS type, while the nodes are relatively insensitive to them. The observables used or proposed are suitable linear combinations of the orbital residuals of the rates of the nodes and the perigees of various existing and proposed satellites. Their main goal is to reduce the impact of the systematic error due to the mismodelling in the static and the time-varying parts of the geopotential’s coefficients by cancelling out as many even zonal harmonics as possible. On the other hand, a reasonable compromise with the fact that the impact of the non-gravitational forces has to be reduced as well must also be obtained. It is important to note that the perturbations of gravitational origin have the same linear temporal signature of the Lense-Thirring effect itself, or are even parabolic if the secular variations \( \dot{J}_\ell \) of the even zonal harmonics are accounted for, while many of the non-gravitational forces (and all the tidal perturbations (Iorio 2001)) are periodic. This means that, for a given observational time span \( T_{\text{obs}} \), the harmonic noises can be fitted and removed from the time series, provided that their periods \( P \) are shorter than \( T_{\text{obs}} \). This is not possible for the linear and parabolic biases without distorting the genuine relativistic
linear signal of interest. So, their impact on the performed measurement can and must only be assessed as more accurately and reliably as possible. Another important point to be noted is that the observables should also be chosen in order to reduce the effect of the a priori ‘memory imprint’ of GTR on the background reference Earth gravity models adopted in the analysis. Indeed it could drive the outcome of the tests just towards the expected result compromising their full reliability.

In this paper we wish to critically discuss the performed and proposed attempts to detect the Lense-Thirring effect in view of the progress in our knowledge of the classical part of the terrestrial gravitational field due to the dedicated CHAMP (Pavlis 2000) and, especially, GRACE (Ries et al. 2003a) missions.

2 The node-node-perigee tests

The first attempts to measure the Lense-Thirring effect were made with the following combination of the orbital residuals of the rates of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II (Ciufolini 1996)

\[ \delta \dot{\Omega}^{\text{LAGEOS}} + c_1 \delta \dot{\Omega}^{\text{LAGEOS II}} + c_2 \delta \omega^{\text{LAGEOS II}} \sim \mu 60.2, \]

where \( c_1 = 0.304 \), \( c_2 = -0.350 \) and \( \mu \) is the solved-for least squares parameter which is 0 in Newtonian mechanics and 1 in GTR. The predicted relativistic signal is a linear trend with a slope of 60.2 mas yr\(^{-1}\). The combination of Eq. (2) cancels out \( J_2 \) and \( J_4 \) along with their temporal variations.

In (Ciufolini 2004) the results of the tests performed with Eq. (2) and the pre-CHAMP/GRACE EGM96 Earth gravity model (Lemoine et al. 1998) are reported with a claimed total error of 20-25%. In reality, this estimate is widely optimistic, as pointed out by a number of authors (Ries et al. 2003b; Iorio 2003b; Iorio and Morea 2004; Vespe and Rutigliano 2004). Indeed, in the EGM96 solution the retrieved even zonal harmonics are reciprocally strongly correlated, so that a realistic evaluation of the systematic error induced by them should be performed by linearly adding the absolute values of the individual errors. In this case a 1-\( \sigma \) 83\% error is obtained. Instead, Ciufolini has used the full covariance matrix of EGM96 obtaining a 1-\( \sigma \) 13\% error which comes from a luckily correlation between the uncancelled \( J_6 \) and \( J_8 \). The point is that such covariance matrix has been obtained from a multidecadal analysis of the data from a large number of SLR satellites among which LAGEOS and LAGEOS II played an important role. The Lense-Thirring tests, instead, have been conducted over time spans few years long,
so that nothing assures that the EGM96 covariance matrix realistically reflects the correlations among the even zonal harmonics during any particular relatively short time spans.

Also the impact of the non-gravitational perturbations on the perigee of LAGEOS II has been underestimated. According to Lucchesi (2002), their systematic error would amount to almost 28% over 7 years. Moreover, the effect of the Earth’s penumbra on it (Vespe 1999) has not been considered at all: it would amount to 10% over 4 years. Finally, although mainly concentrated in $J_2$ and $J_4$, the Lense-Thirring ‘imprint’ is also presented in EGM96 which is largely based just on the LAGEOS satellites which have been used for measuring the gravitomagnetic effect.

3 The node-node tests

The notable improvements in our knowledge of the Earth’s gravitational field thanks to the CHAMP and GRACE missions have allowed to look for alternative combinations capable of reducing the total error. In (Iorio and Morea 2004; Iorio 2005a) the following combination has been explicitly proposed

$$\delta \Omega_\text{LAGEOS}^{\text{obs}} + k_1 \delta \Omega_\text{LAGEOS II}^{\text{obs}} \sim \mu_{LT}48.2,$$

where $k_1 = 0.546$ and 48.2 is the slope, in mas yr$^{-1}$, of the expected gravitomagnetic linear trend. Eq. (3) dramatically reduces the systematic error due to the non-gravitational perturbations to $\sim 1\%$ because it does not include the perigee of LAGEOS II.

The assessment of the systematic error of gravitational origin is rather subtle (Iorio 2005b) because of the fact that the combination of Eq. (3) only cancels out $J_2$ along with its temporal variations. Instead, $J_4, J_6, J_8, \ldots$ do affect it. In particular, the impact of $\dot{J}_4$ and $\dot{J}_6$, for which large uncertainties still exist (Cox et al. 2003), may be a limiting factor, especially over time spans many years long. Indeed, as already pointed out, they would induce a parabolic noise signal which could not safely be fitted and removed from the time series without distorting the trend of interest as well.

Moreover, also the problem of the a priori GTR ‘memory’ effect would still be present even with the GRACE-based model. In fact, GTR has not been modelled in the currently released GeoForschungZentrum (GFZ, 1}

1The possibility of using only the nodes of LAGEOS and LAGEOS II in view of the future benefits from the GRACE mission was put forth in (Ries et al. 2003b) for the first time, although without quantitative details.
Potsdam) GRACE-based models (F. Flechtner, GFZ team, private communication, 2004) like EIGEN-GRACE02S (Reigber et al. 2005) and EIGENCG01C (Reigber et al. 2004). GRACE recovers the low degree even zonal harmonics from the tracking of both satellites by GPS and the medium-high degree geopotential coefficients from the observed intersatellite distance variations. From (Cheng 2002) it can be noted that the variation equations for the Satellite-to-Satellite Tracking (SST) range $\Delta \rho$ and range rate $\Delta \dot{\rho}$ of GRACE can be written in terms of the in-plane radial and, especially, along-track components $R, T, V_R, V_T$ of the position and velocity vectors, respectively. In turns, they can be expressed as functions of the perturbations in all the six Keplerian orbital elements (see (10)-(11), (A4)-(A6), (A14)-(A16) and (A28)-(A30) of (Cheng 2002)). Now, the gravitomagnetic off-diagonal components of the spacetime metric also induce short-periodic 1 cycle per revolution (1 cpr) effects (Lense and Thirring 1918; Soffel 1989) on all the Keplerian orbital elements, apart from the secular trends on the node and the pericentre. This means that there is also a Lense-Thirring signature in all the other typical satellite and intersatellite observables like ranges and range-rates. It is likely that it mainly affects just the low-degree even zonal harmonics to which the $J_2$-free combination of Eq. (3) is sensitive.

In regard to the static part of the geopotential, the systematic error induced by it amounts to 4% (1-$\sigma$ upper bound) according to the GRACE-only EIGEN-GRACE02S model and to 6% (1-$\sigma$ upper bound) according to EIGEN-CG01C which combines data from CHAMP, GRACE and terrestrial gravimetry. According to the evaluations of Cox et al. (2003), the impact of $\dot{J}_4$ and $\dot{J}_6$, which grows linearly in time by assuming that no inversions in their rates of change occur, is of the order of 1% yr$^{-1}$ (1-$\sigma$).

Another point to be noted is that in the aforementioned GFZ models $\dot{J}_2$ and $\dot{J}_4$ have not been solved for: instead, they have been held fixed to given values obtained from long time series of SLR data to the geodetic satellites among which LAGEOS and LAGEOS II, again, played a relevant role. $\dot{J}_6$ is not present at all. This means that in the recovered even zonal harmonics an ‘imprint’ from such secular variations is probably present, so that in evaluating the total error of gravitational origin it would be more conservative and realistic to linearly add the effects due to $J_\ell$ and those due to $\dot{J}_\ell$.

The combination of Eq. (3) has been used for tests with real data by Ciufolini and Pavlis (2004) over an observational time span of 11 years with EIGEN-GRACE02S. They incorrectly attribute the $J_2$–free node-only combination to themselves by means of (Ciufolini 1986) which, instead, has nothing to do with it. Moreover, they claim a total error ranging from 5%
(1-σ) to 10% (3-σ), but, again, it seems to be too optimistic and incorrectly evaluated (Iorio 2005b). E.g., the issues of the time-varying part of the geopotential and of the Lense-Thirring ‘imprint’ have been completely neglected. Moreover, root-sum-square calculations have been often ad-hoc used, when caution would have advised to linearly sum, e.g., the various errors of gravitational origin which can hardly be considered as independent. More realistic evaluations including also the effects of $J_4$ and $J_6$ points toward a 15%(1 − σ) − 45%(3 − σ) range error. Even by assuming the unsupported 2% claimed by Ciufolini and Pavlis for the time-dependent gravitational part of the systematic error, a more realistic evaluation of the total uncertainty yields a 6%(1 − σ) − 19%(3 − σ) interval. Finally, a scatter plot obtained by using different Earth gravity models and different observational time spans should have been produced.

4 An alternative combination

A way to reduce to systematic error due to the geopotential is, in principle, to suitably combine the nodes of $N$ satellites so to cancel out the first $N−1$ even zonal harmonics (Iorio 2002; Iorio and Doornbos 2004; Vespe and Rutigliano 2004). This possibility is very appealing because for, say, $N = 4$ $J_2$, $J_4$ and $J_6$, along with their temporal variations and a large part of the a priori GTR ‘memory’ effects would be canceled out. The main practical problem is that the other existing SLR satellites are too low (Starlette, Stella, Larets) or too high (ETALON1, ETALON2) with respect to LAGEOS and LAGEOS II.

The use of the ETALON satellites ($a = 25498$ km) would imply huge coefficients of their nodes which would enhance the effect of the uncanceled $\ell = 2, m = 1$ tesseral $K_1$ tide on the obtainable combinations. Indeed, the periods of such orbital perturbations are equal to the periods of the nodes which, for the ETALON satellites, amount to tens of years. Moreover, the nominal amplitudes of such semisecular signals are of the order of thousands of milliarcseconds.

On the other hand, the lower satellites with $a \sim 7000$ km would not pose problems in regard to the harmonic perturbations because their periods amount to months or a few years; the major drawback is represented by the uncanceled even zonal harmonics of higher degree to which such satellites are much more sensitive than LAGEOS and LAGEOS II.

The SLR Ajisai satellite ($a = 7870$ km) and the altimeter Jason-1 satellite ($a = 7713$ km) lie in an intermediate position. Indeed, it turns out that their nodes could be usefully combined, in principle, with those of LAGEOS
and LAGEOS II in order to reduce the impact of the geopotential to the 1% level without the uncertainties related to the $J_\ell$ and the a priori imprints of the background Earth gravity models to be used. In (Iorio and Doornbos 2004) the following $J_2 - J_4 - J_6$-free combination has been proposed

$$\delta \dot{\Omega}^L + h_1 \delta \dot{\Omega}^{L\,II} + h_2 \delta \dot{\Omega}^{Aji} + h_3 \delta \dot{\Omega}^{Jason} \sim \mu 49.5, \quad (4)$$

with

$$h_1 = 0.347, \quad h_2 = -0.005, \quad h_3 = 0.068. \quad (5)$$

According to EIGEN-GRACE02S, the 1-σ upper bound for the systematic error due to the geopotential amounts to 2%, while it is 1.6% according to EIGEN-CG01C. Note that, in regard to the Lense-Thirring effect, Eq. (4) is sensitive to the even zonal harmonics up to $\ell = 20$: this allows for accurate and reliable evaluations of their systematic error also in an analytical way (Iorio 2003b). It is likely that the forthcoming models will push the systematic error of gravitational origin below the 1% level. In regard to the most insidious uncanceled tidal perturbations like $K_1$ acting on the nodes of Ajisai and Jason-1, their periods amount to almost half a year. The major drawback of the combination of Eq. (4) is represented by the impact of the non-gravitational perturbations on Jason-1 which should be modelled in a truly accurate dynamical way: indeed its area-to-mass ratio, to which the non-conservative forces are proportional, amounts to $\sim 2.7 \times 10^{-2} \text{ m}^2 \text{ kg}^{-1}$, contrary to $7 \times 10^{-4} \text{ m}^2 \text{ kg}^{-1}$ of LAGEOS. However, according to the evaluations in (Iorio and Doornbos 2004) they should mainly have harmonic signatures with periods of the order of 1 year. This is a very important feature because they could, then, be fitted and removed from the time series over not too long observational time intervals. Moreover, the small magnitude of the coefficient $h_3$ which weighs the Jason’s node would be helpful in keeping the non-conservative forces within the few percent level and in reducing the measurement errors. It should be pointed out that, up to now, there are no long time series of the out-of-plane cross track Keplerian orbital elements of Jason available, contrary to the in-plane radial and along-track components of its orbit due to its oceanographic and altimetric use. Moreover, the current (radial) 1-cm accuracy in reconstructing its orbit is obtained in a reduced-dynamic fashion which would be unsuitable for Lense-Thirring tests. Also the orbital maneuvers may affect the possibility of getting smooth time series some years long, although they are mainly performed in its orbital plane.
5 The LARES mission

In its originally proposed version the LAGEOS III/LARES satellite is a SLR twin of LAGEOS which has to be launched in the same orbit of LAGEOS, apart from the inclination whose nominal value is $i = 70$ deg (Ciufolini 1986) and the eccentricity whose nominal value is $e = 0.04$ (Ciufolini 1998). The observable is the simple sum of the nodes of LAGEOS and LARES which would cancel, to a certain degree of accuracy depending on the precision of the launch and, consequently, on the quality and the cost of the launcher, all the classical precessions of the geopotential, which are proportional to $\cos i$ (Iorio 2003b), and would enforce the Lense-Thirring total signature, which, instead, is independent of $i$. However, according to the EGM96 Earth gravity model, departures from the nominal inclination up to 1 deg would induce a gravitational error of 10% (1-$\sigma$ upper bound). The more recent EIGEN-CG01C model reduces this limit to 2%. Moreover, also the $\dot{J}_2$, $\dot{J}_4$, $\dot{J}_6$, to which the sum of the nodes is sensitive, would further corrupt the obtainable accuracy over a time span of some years.

Later, in (Iorio et al. 2002), it was proposed to suitably combine the node and the perigee of LARES with the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II in order to greatly reduce the dependence on the unavoidable orbital injection errors, so to somewhat relax the original very stringent requirements on the required LARES orbital configuration. The systematic error due to the remaining uncanceled even zonal harmonics, calculated with the EGM96 Earth gravity model, was well below the 1% without the uncertainties related to the $\dot{J}_2$.

It is important to note that, since LARES is totally passive as LAGEOS and LAGEOS II and since its data must be combined together with those of its already orbiting twins, the level of the obtainable accuracy in measuring the Lense–Thirring effect with it would be set by the non-gravitational perturbations, i.e. $\sim 1\%$. Then, it would be unnecessary to push the systematic error of gravitational origin much below the 1% level. This consideration, together with the precision reached by the present-day (and future) Earth gravity models, allow for a much greater freedom in choosing the orbital configuration of LARES with respect to its original configuration. In particular, it would be possible to greatly reduce the costs of the launch by inserting LARES in a much lower orbit with respect to that of LAGEOS. E.g., an orbital configuration like that of Jason-1 would be well suited: the combination of Eq. (4), with Jason’s node replaced by the node of a low-altitude LARES with its orbital parameters, would easily reach a $\sim 1\%$ level of accuracy. It is also possible to show that with a three-node combination of
the nodes of the LAGEOS satellites and of a relatively low-altitude LARES ($a \sim 8000$ km) a satisfactory gravitational error of less than 1% ($1 - \sigma$ upper bound obtained with EIGEN-CG01C) could be achieved.

Of course, if the implementation of the combination of Eq. (4) with Jason-1 will be really feasible and/or the uncertainties related to the use of Eq. (3) will be reduced in some ways, the cost of an entirely new dedicated mission which would allow to only reach a $\sim 1\%$ measurement of the Lense-Thirring effect by means of a passive SLR satellite could be judged unjustified. The launch of at least two drag-free satellites could, in principle, be better justified because it would allow a really notable improvement in the error budget thanks to the active reduction of the non-gravitational perturbations. Note also that an active compensation of the non-conservative forces would also make feasible the use of their perigees along with their nodes without resorting to the passive LAGEOS and LAGEOS II; with the supplementary plane option it would also be possible to measure the difference of the perigees (Iorio and Lucchesi 2003). The forthcoming CHAMP/GRACE Earth gravity models would do the remaining job.

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\footnote{The situation with the OPTIS mission is different because the measurement of the Lense-Thirring effect would be one of its many other relativistic tasks which could be implemented with comparatively small modifications of the original concept.}
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