Research Article

Partial Component Consensus of Discrete-Time Multiagent Systems

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The multiagent system has the advantages of simple structure, strong function, and cost saving, which has received wide attention from different fields. Consensus is the most basic problem in multiagent systems. In this paper, firstly, the problem of partial component consensus in the first-order linear discrete-time multiagent systems with the directed network topology is discussed. Via designing an appropriate pinning control protocol, the corresponding error system is analyzed by using the matrix theory and the partial stability theory. Secondly, a sufficient condition is given to realize partial component consensus in multiagent systems. Finally, the numerical simulations are given to illustrate the theoretical results.

1. Introduction

In recent years, the theory of consistency, as the basis of coordinated control of multiagent system, has attracted extensive attention from many researchers [1–3]. The unified nature of multiagent systems is wildly applied to computing science [4], systems and control [5–7], and distributed sensor networks [8–10].

The consistency of the discrete multiagent systems is that the state of all agents in a discrete system model can achieve asymptotic convergence under certain conditions. Many researchers have discussed the consistency problem of multiagent systems [11–14] and have obtained a lot of research results. Xie Dongmei and Wang Shaokun considered the consensus of second-order discrete-time multiagent systems with fixed topology in [15]. In 2016, Gao Yulan et al. studied group consensus for second-order discrete-time multiagent systems with time-varying delays under switching topologies in [16]. At the same year, Cao Yanyan and Sun Yuanfeng discussed consensus of discrete-time third-order multiagent systems in directed networks in [17]. Furthermore, the consensus of leader-following multiagent has also received a lot of attention. Wang Yunpeng et al. proposed an algorithm to research the consensus of discrete-time linear multiagent systems with communication noises in [18]. Xu Xiaole et al. investigated the leader-following consensus problem of discrete-time multiagent systems through Lyapunov method in [19].

The consistency of discrete multiagent systems has more advantages than continuous multiagent systems. For example, it can reduce a lot of computation, and the speed of convergence is fast and so on. Therefore, the research of discrete consistency has some practical significance. Wu Binbin et al. have studied the partial component consensus of continuous multiagent systems in [20]. In this paper, we discuss the partial component consistency of discrete leader-following multiagent system. Based on the matrix theory and the partial stability theory, together with designing an appropriate pinning control protocol, a sufficient condition is proposed to realize partial component consensus in multiagent systems.

In detail, the remainder of this paper is organized as follows: Section 2 contains the problem statement and preliminaries; Section 3 presents the main result about the partial component consensus of discrete leader-following multiagent system; Section 4 provides a numerical example
to verify the effectiveness of the proposed results; Section 5 offers concluding remarks.

2. The Problem Statement and Preliminaries

In this section, we will give the basic concept of partial component consistency, basic matrix theory, and some definitions and lemmas. For details, refer to [20, 21].

We consider the following n-dimensional discrete system:

\[ x(k+1) = F(x(k)), \]

where \( F(x) \in \mathbb{C}[R^n, R^n] \), \( F(0) \equiv 0 \), \( x = (x_1, \ldots, x_m, x_{m+1}, \ldots, x_n) \in R \). Supposing \( y = (x_1, \ldots, x_m) \), \( z = (x_{m+1}, \ldots, x_n) \), \( (m + p = n) \), and \( \|x\| = \left(\sum_{i=1}^{m} x_i^2\right)^{1/2} \), \( \|y\| = \left(\sum_{i=m+1}^{n} x_i^2\right)^{1/2} \), \( \|z\| = \left(\sum_{i=m+1}^{n} x_i^2\right)^{1/2} \), \( k \in N \).

Similar to the definition of partial component stability for continuous system in [21], we give the following definition of partial component stability for discrete system.

**Definition 1.** The trivial solution of (1) is stable for vector \( y \), if \( \forall \varepsilon > 0, \forall k_0 \in N \), \( \exists \delta(k_0, \varepsilon) > 0 \), \( \forall k \geq k_0 \), \( \exists \hat{\delta}(k) \leq \delta(k_0, \varepsilon) \), and when \( k \geq k_0 \), there must be \( \|y(k, k_0, x_0)\| < \varepsilon \).

**Definition 2.** The trivial solution of (1) is attracted to vector \( y \), if \( \forall k \in N \), \( \exists \delta(k) > 0 \), \( \forall x \), \( \forall x_0 \in S_{\delta(k)} \), we get \( \|y(k, k_0, x_0)\| \leq \delta(k_0, \varepsilon) \), \( \exists \hat{\delta}(k) \leq \delta(k_0, \varepsilon) \), and when \( t \geq k_0 + T \), \( \exists \|y(k, k_0, x_0)\| < \varepsilon \), \( \exists \hat{\delta}(k) \leq \delta(k_0, \varepsilon) \). We can get the errorsystem from (2), (3), and (4) as follows:

\[ e_i(k+1) = Ae_i(k) - c(L + D) \Gamma e_i(k), \]

where \( c > 0 \) denotes coupling strength; \( \Gamma = \text{diag}(r_1, \ldots, r_n) \in R^{n \times n} \), \( r_k \geq 0, k = 1, \ldots, n \) is the intercoupling matrix; if the \( i \)th agent can receive the information from the \( j \)th agent, then \( a_{ij} > 0 \); otherwise, \( a_{ij} = 0 \). If the \( i \)th agent can receive the information from the \( j \)th agent, then \( d_j > 0 \); otherwise, \( d_j = 0 \).

Next we consider the problem of partial component conformance of discrete leader-following multiagent system under directed network topology. We design the controller as follows:

\[ u_i(k) = c \sum_{j=1}^{N} a_{ij} \Gamma (x_j(k) - x_i(k)) \]

\[ -cd_i \Gamma (x_j(k) - x_i(k)), \]

where \( c > 0 \) denotes coupling strength; \( \Gamma = \text{diag}(r_1, \ldots, r_n) \in R^{n \times n} \), \( r_k \geq 0, k = 1, \ldots, n \) is the intercoupling matrix; if the \( i \)th agent can receive the information from the \( j \)th agent, then \( a_{ij} > 0 \); otherwise, \( a_{ij} = 0 \). If the \( i \)th agent can receive the information from the \( j \)th agent, then \( d_j > 0 \); otherwise, \( d_j = 0 \).

Next we consider the problem of partial component conformance of discrete leader-following multiagent system.
Proof. Define the following Lyapunov function candidate:

\[ V(k) = \sum_{j=1}^{N} \sum_{i=1}^{N} \epsilon_j^T(i,k) \epsilon_i(k) = e^T(k) \wedge \tilde{e}(k), \]

where \(\wedge = \text{diag}(1, \ldots, 1, 0, \ldots, 0)\), \(\tilde{e}(k) = (\epsilon_1^T(k), \ldots, \epsilon_n^T(k))^T\).

Similar to literature [19], we can obtain the following:

\[ \Delta V(k) = V(k+1) - V(k) = \tilde{e}^T(k) \left( [A \otimes I_N - c \Gamma \otimes (L + D)] - I_N \right) \tilde{e}(k) \]
\[ = \sum_{j=1}^{l} \epsilon_j^T(k) (M_j^T M_j - I_N) \epsilon_j(k) \]
\[ \leq -\sum_{j=1}^{l} \beta \epsilon_j^T(k) \epsilon_j(k) = -\beta V(k). \]

In this case, if (9) can be held, then there exist \(\beta \in (0, 1)\), such that

\[ \Delta V(k) = V(k+1) - V(k) \leq -\beta V(k). \]

Therefore,

\[ V(k+1) \leq V(k) - \beta V(k) = (1 - \beta)^2 V(k - 1) \leq \cdots \leq (1 - \beta)^{k+1} V(0), \]

when \(k \rightarrow \infty\), \(V(k+1) \leq (1 - \beta)^{k+1} V(0)\), i.e., \(\lim_{k \rightarrow \infty} \epsilon_j(k) = 0\). Therefore, \(\lim_{k \rightarrow \infty} \sum_{j=1}^{l} \|\epsilon_j(k)\| = 0\); then the solution of (8) is asymptotically stable for partial vector element; i.e., system (2) and (3) can achieve consensus for the first \(l\) components. \(\square\)

4. Numerical Examples

In this section, a numerical example has been given to show that our theoretical result obtained above is effective.

Example 1. Given the parameter \(n\) and \(N\) in system (2) as \(n = 3\) and \(N = 4\), we consider the consensus of the discrete leader-following multiagent system for the first two components (i.e., \(l = 2\)). Supposing the state of the \(i\)th agent (the subscript of the state of the leader denotes 0) is \(x_i(k+1) = Ax_i(k)\), \(A = \text{diag}(1.01, 1.01, 1.03)\). Designing \(L\) and \(D\) as follows:

\[ L = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}, \]
\[ D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \]

then bring \(A, L,\) and \(D\) into (4). Through simple calculations, at the same time, we set \(c = 0.3\) and \(\Gamma = \text{diag}(1, 1, 0)\); then, (9) will be held. In this case, it is straightforward to check that all the conditions in Theorem 7 hold. Next we will give the topology diagram of agent connection and the error trajectories of system (2) and (3) through Matlab software.

Figure 1 expresses the topology connection of the given agent. Next we give the evolution diagram of the state error of the leader-following agent (Figure 2); Figures 2(a) and 2(b) represent the consensus of system (2) and (3) for the first two components and Figure 2(c) represents that system (2) and (3) cannot achieve consensus for the third component.

5. Conclusion

In this paper, the partial component consensus has been investigated for the first-order discrete leader-following multiagent system. By establishing the suitable control term and using the matrix theory together with the stability theory, the sufficient conditions for the partial component conformance of the discrete system are derived. Furthermore, a numerical example has been given to illustrate the effectiveness of the present results. As an extension to this work, we plan to discuss the partial component consensus for the high-order discrete leader-following multiagent system.

Data Availability

No data were used to support this study.
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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