How Functional Complexity affects the Scalability-Energy Efficiency Trade-Off of HCC WSN Clustering

Merim Dzaferagic, Nicholas Kaminski, Irene Macaluso, and Nicola Marchetti
CONNECT, Trinity College Dublin, Ireland, E-mail: {dzaferam, kaminskn, macalusi, marchetn}@tcd.ie

Abstract—Even though clustering algorithms in Wireless Sensor Networks (WSN) are a well investigate subject, the increasing interest in the Internet of Things (IoT) and 5G technologies has precipitated the need of new ways to comprehend and overcome a new set of challenges. While studies mainly propose new algorithms and compare these algorithms based on a set of properties (e.g. energy efficiency, scalability), none of them focuses on the underlying mechanisms and organizational patterns that lead to these properties. We address this lack of understanding by applying a complex systems science approach to investigate the properties of WSNs arising from the communication patterns of the network nodes. We represent different implementations of clustering in WSNs with a functional topology graph. Moreover, we employ a complexity metric - functional complexity ($C_F$) - to explain how local interactions give rise to the global behavior of the network. Our analysis shows that higher values of $C_F$ indicate higher scalability and lower energy efficiency.

Keywords—Complex systems science, functional complexity, wireless sensor networks, clustering, scalability, energy efficiency

I. INTRODUCTION

Clustering partitions a network of nodes into a number of smaller groups (clusters). Different approaches to clustering are available in the literature. The authors of [1]-[4] introduced different approaches, which involve adaptive clustering, random competition based clustering, Hierarchical Control Clustering (HCC), energy efficient hierarchical clustering, distributed clustering, Low Energy Adaptive Clustering Hierarchy (LEACH), and Hybrid Energy-Efficient Distributed (HEED) clustering. They also highlighted that the algorithms differ in properties like stability of the created clusters, objectives (e.g. scalability, fault-tolerance, connectivity, load balancing, redundancy elimination, rapid convergence, network lifetime), clustering criteria (e.g. identifier, position, cluster head frequency, residual energy), methodology (e.g. distributed, centralized, hybrid). Even though these algorithms are well studied, the analysis is missing an important aspect which involves the understanding of the mechanisms that lead to certain properties of these algorithms. Since the HCC algorithm proposed in [4] is the most popular multi-tier hierarchical clustering algorithm [1], in the course of this analysis we will focus on this algorithm.

Understanding the organizational and communication characteristics of different clustering algorithms allows us to comprehend which aspects of a specific implementation lead to certain characteristics, i.e. scalability and energy efficiency. In [5], we propose a framework which allows us to represent network functions with graphs called functional topologies. Therein, we also propose a metric to calculate the functional complexity of an implementation of a network function. Here, we employ our functional framework to model an implementation for clustering in Wireless Sensor Networks (WSNs). Our goal is to investigate the relationship between the functional complexity and certain properties of the implementation, such as scalability and energy efficiency. This allows us to understand the underlying mechanisms that are a product of complex interactions between functional entities.

II. NETWORK FUNCTIONAL FRAMEWORK

We employ the functional framework introduced in [5] to represent the implementation of the clustering algorithm with a functional topology. The functional topology is a graph that depicts the functional connectivity between system parts, where each node represents a functional entity related to the implementation, and each link indicates interactions between nodes.

The functional complexity metric, also proposed in [5], is based on the Shannon entropy ($H(x_n)$) as a key characteristic of any system. To describe the potential for a node to interact with other nodes we employ the Bernoulli random variable $x_n$. The probability of interactions $p(x_n = 1) = i_p / j$, where $i_p$ is the number...
of nodes that can reach node $n$ and $j$ is the number of nodes for the given subgraph.

Since we focus on one hop communication the complexity metric applied to our system turns out to be the following simplified single scale version of the general functional complexity expression introduced in [5], i.e.

$$C_F = \sum_{j=2}^{N} |\langle I(N^j) \rangle - \frac{j}{N} I(N)|,$$  \hspace{1cm} \text{(1)}$$

where $N$ is the total number of nodes in the functional topology and $N^j$ is a subgraph with $j$ nodes. The functional complexity compares the average uncertainty of interactions for a smaller subset ($\langle I(N^j) \rangle$) to the amount of information which is expected from the calculation performed on the whole system ($I(N)$).

$I(N)$ is the total amount of information of the whole functional topology. $I(N^j)$ is the uncertainty of interactions for a smaller subset, i.e. the total amount of information of the $k^{th}$ subgraph with $j$ nodes. It is calculated with equation \text{(2)}.

$$I(N^j) = \sum_{n \in N^j} H(x_n)$$  \hspace{1cm} \text{(2)}$$

$H(x_n)$ reaches its maximum if the probability of interaction with node $n$ is $p(x_n = 1) = 1/2$. As the distribution of links among nodes for a sparse graph is almost uniform, a sparse graph results in high values of $H(x_n)$. High values of $H(x_n)$ result in high values of $I(N^j)$. Therefore, the functional complexity is high for a sparse graph, with uniformly distributed links among nodes for subgraphs with the size $j < N$. The functional complexity is zero for a fully connected and for a disconnected graph. For more details about the functional framework and the complexity metric expressed by equation \text{(1)} the reader is referred to [5].

The physical topology of our Wireless Sensor Network (WSN) is created according to the Von Neumann neighborhood, which is basically an undirected lattice graph. Now we employ our functional framework to represent the implementation of the HCC algorithm as a functional topology. Our goal is to investigate the influence of the interactions among nodes after the clusters are established, on the objectives of clustering algorithms (i.e. scalability and energy efficiency). We focus on the maintenance phase of the algorithm. After the set-up phase the nodes establish connections to their neighboring nodes according to the Breadth-First Search (BFS) algorithm. Each node discovers its subtree, and exchanges information with its neighbors in the BFS tree. We imagine a virtual decision maker entity that is moving from one node to another. At each node the decision maker entity collects information from nodes that belong to its subtree and forwards this information to its parent. In other words, each node maintains its position in the BFS tree and therefore the functional topology of the HCC algorithm is equivalent to the BFS tree created upon the physical topology (Figure 1).

**III. Analysis**

Adding a new node to a cluster created according to the HCC algorithm simply means that the new node gets connected to one of the existing nodes which is going to be its parent node. The new node does not need to inform all nodes in the cluster about its arrival, which simplifies the process of adding nodes to the network and represents a highly scalable implementation. According to the authors of [1]–[3] and [6] hierarchical approaches proved themselves to be more scalable than their non-hierarchical counterpart. The authors of [6] showed that a trade-off exists between network scalability and energy consumption in clustering schemes. Our goal is to investigate the relationship between these objectives and the functional complexity, which would allow us to analyze them together.

Figure 2 depicts the relationship between the uncertainty of interactions for all subset sizes and the amount of information which is expected from the calculation performed on the whole system for the HCC algorithm, i.e. the functional complexity of HCC. As shown in Figure 1 the functional topology of the HCC algorithm has sparse intra and inter cluster connections. As the links in the functional topology represent functional dependencies between nodes, a sparse connectivity pattern indicates weak dependencies which result in high scalability. This follows from the fact that in order to add a node to the network, the new node only needs to establish a connection (send a message) to one of the nodes in the topology and to declare this node as its parent. The functional complexity of the
Fig. 2. Functional complexity of the HCC algorithm; the functional complexity is the area between the green and the blue curves.

HCC algorithm is 38.31. As the authors of [1]–[3] and [6] agree that the communication between the cluster-head and the base station consumes most energy, an approximation of the energy efficiency could be calculated as the ratio between the average number of intra-cluster connections and the total number of links between the base station and each cluster-head in the functional topology. The energy efficiency of the HCC algorithm is 0.61 which is very low.

IV. CONCLUSION

Our analysis confirms the observation made by the authors of [6], which highlights the trade-off between scalability and energy efficiency. With our complex systems science approach we showed that these aspects of different clustering algorithms can be analyzed together by analyzing the functional complexity of the specific implementation. Increasing values of $C_F$ lead to the increase of scalability and the decrease of energy efficiency. Our ongoing work also confirms this trend, by analyzing other WSN clustering algorithms.

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