Decision-making process in shipping finance: A stochastic approach

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Abstract: Shipping markets irregularity due to high-level volatility of freight rates and asset prices increases the risk of banks' invalid financial strategy. Risk is further increased due to the heterogeneous shipping market, despite the regulations set by the Basel Convention. Consistent with the above, the present work contributes to the existing methodological aspects of bank's financial strategy on shipping finance by enhancing the role of the credibility theory, which balances the individual bank policy with the market as a whole. This has been primarily forwarded on by the analysis of the operational environment's internal factors of an individual bank combined with the whole shipping banks' loans portfolio by estimating the credibility factor to the decision of the bank to either increase or decrease financing in the relevant market. The important factors extracted from the principal components analysis are linked with interest income on loan and operating profit accounts. The final model predicts that the optimal decision is positive driven by both the aforementioned dependent variables, while the interest income on loan variable has more influence compared with that of the operating profit variable. In the absence of the influence of the dependent variables, the bank's decision strategy matches the market's strategy by 77% that decreases as the dependent variables increase their influence.

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PUBLIC INTEREST STATEMENT

High-level volatility of freight rates and asset prices increases the risk banks undertake when getting involved in shipping finance. The relevant risk may influence the performance of corporate bank loans, especially during times of financial instability in the industry. Consistent with the above, the present work aims to contribute as a decision support tool that implicitly increases the validity of future financial strategy specifically on loan grants to shipping industry. This has been primarily forwarded by the analysis of the operational environments' internal factors of an individual bank combined with the whole shipping banks' loans portfolio. Finally, a specific methodological framework for shipping finance is developed that might be considered as an optimal decision of an individual bank to either increase or decrease a loan grant taking into account both its policy in the shipping market, as well as the most important variables arising from its internal operational environment.
1. Introduction

The presence of high risk in shipping markets due to high volatility in terms of freight rates and asset prices raises questions about banks’ decisions to continue financing such an irregular and heterogeneous market, despite the regulations set by the Basel Convention (Albertijn, Bessler, & Drobetz, 2011; Sambracos & Maniati, 2013). Bank finance is an important source of capital for the financing of the shipping industry, while the relevant default risk may also lead banks to bankruptcy (Chava & Purnanandam, 2011) and influence the performance of corporate bank loans during times of financial instability in the shipping industry (Mitroussi, Abouarghoub, Haider, Pettit, & Tigka, 2016).

Classification of factors that affect the amount of loans for the shipping industry the following year (based on previous years’ experience) might be critical for banks’ performance. In this paper, internal factors of the operational environment of an individual bank in relation with the whole shipping banks’ loans portfolio, are analysed by applying the credibility factor to the decision of a shipping bank to either increase or decrease financing in the relevant market.

Consistent with the variance of shipping loans granted, we present a stochastic model in order to estimate the premium for the next period in relation with past claims experience data (Zadeh & Stanford, 2016). This model is based on the credibility factor as a parameter used to quantify the individual bank’s outcome (decision) with respect to both the internal and external financial environment.

The earliest work on credibility theory was made by Mowbray (1914) and Whitney (1918) who referred to limited fluctuation, in order to incorporate in premiums as much as individual experience as possible. Bühlmann (1967) proceeded in formalising the principles for the credibility theory based on premiums, as well as in computing credibility factors $Z$ in the model with equal exposure units. Further analysis on credibility theory had been presented by Bühlmann and Straub (1970), Hachemeister (1975), De Vylder (1976), Goovaerts and Hoogstad (1987), and Frees (2003).

Considering that the risk is a random variant set out apart from the typical individual risks, Jewell (1974), Zehnwirth (1977), Klugman (1987), Makov, Smith, and Liu (1996) issued the Bayesian approach to credibility theory. McCullagh and Nelder (1989), Aitkin, Anderson, Francis, and Hinde (1989), Dobson (1990), Nelder and Verrall (1997), McCulloch and Searle (2001) and Demidenko (2004) linked general linear models (GLMs) with the credibility theory. Haberman and Renshaw (1996) reviewed the applications of GLMs to risk theory and Antonio and Beirlant (2006) applied mixed GLMs in actuarial science.

Recently, credibility models have been mainly applied in property and casualty insurance, in life insurance. Herbertsson, Meester and Sander, and Fackrell applied phase-type distributions to health care, finance and transportation infrastructure. There has been a large number of applications available concerning risk theory, where the claim sizes were frequently assumed to be phase-type distributed.

Principal component analysis (PCA), preceded in order to decrease the dimensionality of a bank’s internal environment system variables by defining those most significant that explained more than 95% of system variance. PCA was first introduced by Pearson (1901), and recently has been forwarded on with a large set of data by Hanschel and Monnin (2005), Illing and Liu (2003), Canbas, Cabuk, and Kiliç (2005), Ho and Wu (2009), Abu-Shanab and Pearson (2009), Baek, Balasubramanian, and Lee (2015).
Consistent with Ionită & Schiopu (2010), we follow PCA based on both balance sheet and profit and loss account data, as the dimension of data is reduced without much information loss. Moreover, our work follows Shih, Zhang, and Liu (2007) that derived four measures of a bank’s ability to perform the core task of financial intermediation based on PCA and, in order to understand the factors that drive Chinese bank performance.

This paper reveals the most important factors that arise from a bank’s internal environment based on PCA and implicitly contributes to the development of a specific methodological framework for shipping finance with respect to bank credibility. Essentially, it might be considered as a decision support tool, taking into account both the credibility factor in decision-making process, and the policy each bank wants to follow in the market, as well as the most important variables arising from its internal operational environment.

We found that: (i) the main factors are interest income on loan and operating profit accounts and the optimal decision is positively influenced by both variables. The first variable has more influence compared with the second one, (ii) in the absence of the influence of the dependent variables, the bank’s decision strategy approaches the market’s strategy by 77% and (iii) as the dependent variables increase their influence, the bank’s decision to either increase or decrease shipping loans limits to its own policy.

The paper is structured as follows: Section 2 briefly describes the data and the methodology used, while Section 3 introduces the empirical results of the present study. The proposed support tool is then briefly discussed in Section 4 and some concluding remarks are provided in Section 5.

2. Methodological issues
Data were derived from Bloomberg and Bankscope databases and all rules and restrictions were conditioned accordingly. Technical analysis was forwarded with MATLAB™ and all statistics were processed with SPSS (IBM COM, v.22).

2.1. The credibility coefficient estimate
Considering that the market consists of m-shipping banks that grant loans to the shipping sector the last n-observed years, the credibility $\{C_j\}_{j=1,..,m}$ equals $Z_j \cdot R_j + (1 - Z_j) \cdot H$, where $R_j = n^{-1} \sum_{i=1}^{n} R_{ij}$ is the observed mean of total loans $\{R_{ij}\}_{j=1,..,m}$ granted the last n-observed years by an individual j-shipping bank and $H = (n \cdot m)^{-1} \cdot \sum_{m=1}^{m} \sum_{j=1}^{n} R_j$ is the corresponding overall mean of a shipping banks portfolio. Credibility is the difference of the total loan that will be granted from the shipping bank the next year (n + 1) conditioned on a credibility factor $Z_j = (C_j - H) / (R_j - H)$. Consistent with Bühlmann’s theory, the factor $Z_j$ is in principle between values 0 and 1 and implicitly measures the amount of credence attached to the individual experience. In the current research, the lower and upper limits of the credibility factor are either extended below zero or above one. For instance, when either $C_j < H < R_j$ or $R_j < H < C_j$ then $Z_j < 0$. On the other hand, when either $H < R_j < C_j$ or $C_j < R_j < H$ then $Z_j > 1$.

2.2. The stochastic uniform hypothesis
Consistent to heterogeneous portfolios of loans granted by m-shipping banks during the last n-years, contracts are in principle conditioned to unknown risk factors $\{\theta_j\}_{j=1,..,m}$ that are different through years and among banks. The objective hypothesis is that all contracts are subject to a common risk factor, i.e. $\theta_j = \theta$, and the credibility factor $Z_j$ is commonly constrained as $Z = (1 + k/n)^{-1}$. The structural parameter $k = E[V(X|\theta)] / \sqrt{E[X|\theta]}$, where $E[V(X|\theta)]$ and $\sqrt{E[X|\theta]}$ are the mean variance and the variance of the mean, respectively, of the total loan grants X conditioned on the risk factor $\theta$. In terms of statistics, let a random variable (RV) $X_j^{n}$ conditioned on an unknown parameter $\theta \in \Theta$ that is the upper bound of the total loan grants of the j-shipping bank, $j = 1, ..., m$ at the year $i = 1, ..., n$. In the absence of any well-established, prior
information and in favour of totally randomness we might presume that the total loan grants are uniformly distributed at the range \(0-\theta\), i.e. \(X_i^{(j)}|\theta \sim U(0, \theta)\). According to that, the conditioned mean is \(E\left\{X_i^{(j)}|\theta \right\} = \theta/2\) and the mean of total loan grants of the \(j\)-bank at the \(i\)-year, irrelevant to the unknown parameter \(\theta\), is \(E\left\{X_i^{(j)}\right\} = E\left\{X_i^{(j)}|\theta \right\} = \theta/2\). Considering that the RVs \(X_i^{(j)}\) are identical and independently distributed (\(iid\)), the parametric mean \(\mu\) of total loan grants of all banks all years is unbiased estimated by the point statistic function (PSF) \(\hat{\mu} = (n \cdot m)^{-1} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} X_i^{(j)}\). Then, the point estimate of the mean of the unknown parameter \(\theta\), is \(\hat{\theta} = 2 \cdot \hat{\mu}\).

Similarly, the conditioned variance of the total loan grants of the \(j\)-bank at the \(i\)-year is \(V\left\{X_i^{(j)}|\theta \right\} = \sigma^2/12\) and the mean conditioned variance is \(E\left\{V\left\{X_i^{(j)}|\theta \right\}\right\} = \sigma^2/12\) as well as the variance of the conditioned mean is \(V\left\{E\left\{X_i^{(j)}|\theta \right\}\right\} = V\left\{\theta/2\right\} = \left(\frac{\sigma^2}{4} - 4 \cdot \mu^2\right)/4\).

Irrelevant to the unknown parameter \(\theta\), the variance of total loan grants of the \(j\)-bank at the \(i\)-year is \(V\left\{X_i^{(j)}\right\} = V\left\{E\left\{X_i^{(j)}|\theta \right\}\right\} + E\left\{V\left\{X_i^{(j)}|\theta \right\}\right\}\) and following \(iid\), the parametric variance \(\sigma^2\) of total loan grants of all banks all years is unbiased estimated by the PSF \(\hat{\sigma}^2 = (n \cdot m - 1)^{-1} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} (X_i^{(j)} - \hat{\mu})^2\). Therefore, the PSF of the second moment of the unknown parameter \(\theta\), is \(\hat{\theta} = 3 \cdot (\hat{\sigma}^2 - \hat{\mu}^2)\) that subsequently leads to the point estimates of

(i) the mean conditioned variance \(\hat{E}\left\{V\left\{X_i^{(j)}|\theta \right\}\right\} = (\hat{\sigma}^2 - \hat{\mu}^2)/4\), (ii) the variance of the conditioned mean \(\hat{V}\left\{E\left\{X_i^{(j)}|\theta \right\}\right\} = 3 \cdot (\hat{\sigma}^2 - 7 \cdot \hat{\mu}^2)/4\), (iii) the structural parameter \(\hat{k} = \frac{\hat{E}\left\{X_i^{(j)}|\theta \right\}}{\hat{V}\left\{X_i^{(j)}|\theta \right\}} = \frac{3 \cdot \hat{\sigma}^2 - 7 \cdot \hat{\mu}^2}{3 \cdot \hat{\sigma}^2 - \hat{\mu}^2}\) restricted to \(\hat{\sigma}^2 > 7/3 \cdot \hat{\mu}^2\) and (iv) the credibility coefficient \(\hat{Z} = (1 + \hat{k}/n)^{-1}\).

### Table 1. Summary of data of credibility and correlation coefficients

| \(s\) | Internal variable | Group | \(m\) | \(Z_{as}\) | \(Z_{s}\) | \(r_s\) |
|------|------------------|------|------|---------|---------|------|
| 1    | Loans            | 91   | >1   | 0.59    | −       |      |
| 2    | Net income       | I    | 91   | >1     | 0.71    | 0.52**|
| 3    | Total assets     | II   | 92   | >1     | 0.61    | 0.65**|
| 4    | Deposit & short funding | II  | 92   | >1     | 0.62    | 0.49**|
| 5    | Net interest     | I    | 91   | >1     | 0.64    | 0.71**|
| 6    | Profit before tax| I    | 91   | 0.71   | 0.69    | 0.29 |
| 7    | Interest income on loan | I   | 88   | 0.33   | 0.62    | 0.73**|
| 8    | Interest expenses on customer deposits | II | 83   | 0.81   | 0.64    | 0.78**|
| 9    | Total interest expenses | I   | 91   | >1     | 0.66    | 0.53**|
| 10   | Net interest income | I   | 89   | >1     | 0.64    | 0.47**|
| 11   | Operating profit | I    | 91   | 0.73   | 0.7     | 0.66**|
| 12   | Personnel expenses| I    | 81   | >1     | 0.65    | 0.62**|
| 13   | Corporate commercial loans | II | 55   | 0.82   | 0.63    | 0.57**|
| 14   | Customer deposits | II   | 81   | >1     | 0.66    | 0.46**|
| 15   | Total customer deposits | II | 89   | <1     | 0.63    | 0.52**|

Notes: \(m\): sample size (banks); \(Z_{as}\): sample mean of the \(s\)-internal variable; \(Z_{s}\): credibility coefficient estimate conditioned on the uniform hypothesis assigned to the \(s\)-internal variable; \(r_s\): Spearman correlation coefficient between internal variables and loans.

*Significance level at \(p < 5\%\).
**Significance level at \(p < 1\%).
We applied both methods described above to extract the credibility coefficient for all m-banks and p-internal data variants (see Table 1). The array $Z = \begin{bmatrix} Z_{1} \ldots Z_{m} \end{bmatrix}_{mp}$ contains the credibility coefficients estimates $Z_{is}$ of j-bank ($i = 1, \ldots, m$) for the s-internal variable ($s = 1, \ldots, p$) and $\bar{Z}_{is} = m^{-1} \sum_{j=1}^{m} Z_{js}$ is the sample mean of the s-internal variable. Similarly, the credibility coefficient constrained on the uniform hypothesis $\bar{Z}_{is}$ is for the s-internal variable. Finally, the non-parametric Wald–Wolfowitz test was applied to test the validity of the objective hypothesis that the parametric mean of credibility coefficients of an internal variable equals the corresponding estimate value conditioned on the uniform hypothesis.

2.3. Principal component analysis

Principal component analysis (PCA) was applied separately on the internal data of two sets of banks derived from: group (I) profit and loss accounts (net income, net interest income, interest income on loans, total interest expense, personnel expenses, operating profit, profit before tax, net interest) and group (II) balance sheet (total assets, corporate and commercial loans, customer deposits, total customer deposits, deposits and short-term funding, interest expense on customer deposits).

Both the Spearman correlation coefficient ($r_s$) and the Kaiser–Meyer–Olkin (KMO) statistics were extracted. For each set of data, the rationale behind applying PCA was to examine the relationships among covarying variables, by transforming the original data to a new set of equal number of orthogonal variables, i.e. the PCs.

PCs are derived in a decreasing order of importance, so that the first and the last PC, respectively, account for the maximal and minimal amount of variance in the original data. In addition, they linearly combine the original variables. Weight coefficients are the contributions of each original variable to each one of the PCs and in absolute terms, the original variable with the maximum contribution to a significant principal component is selected for further process. Percentages of the variation in the original set of p-variables where the PCs account for are equal to the normalised eigenvalues $\hat{\lambda}_{s} = \lambda_{s} / \sum_{s=1}^{p} \lambda_{s}$, $1 \leq s < s_{k} \leq p$ of the covariance matrix $\Sigma = Z' \cdot Z$ ($Z'$ is the transpose of $Z$) where $Z = \begin{bmatrix} Z_{1} \ldots Z_{m} \end{bmatrix}_{mp}$ and $Z_{is}$ refers to the data of the s-th-internal variable of the j-th-bank. By multiplying the array of original variables $Z_{mp}$ with the array of eigen vectors $v_{s}$ assigned to the covariance matrix $\Sigma$ we get $L = Z \cdot v$ where $L_{mp}$ is the array of the PCs. These are organised in columns of an increasing order of importance, such that the p-th column contains the 1st principal component accounting for the percentage $\hat{\lambda}_{1}$ of the variance of the original variables. In general, the $s$th column of array $L_{mp}$ contains the $s$th principal component $L_{s} = \begin{bmatrix} L_{s,1} \ldots L_{s,m} \end{bmatrix}_{mp}$ accounting for the percentage $\hat{\lambda}_{s}$ of the variance of the original variables. Then, the contribution of the original variable $Z_{s} = \begin{bmatrix} Z_{s,1} \ldots Z_{s,m} \end{bmatrix}_{mp}$ to the PC $L_{s}$ is $\alpha_{s,s} = \left| v_{s,s} \right| / \sum_{j=1}^{p} \left| v_{s,j} \right|$ Eigenvectors $(\lambda_{s})_{s=1,\ldots,p}$ and their corresponding eigenvalues $(\lambda_{s})_{s=1,\ldots,p}$ were computed using the MATLAB function eig.

2.4. PCs’ statistics

Bartlett’s test of sphericity was used to test the null hypothesis of equal eigenvalues of PCs. For the $s_{p}$-PCs of lower importance $L_{p} \ldots L_{s}$, with corresponding eigenvalues $\lambda_{s} < \ldots < \lambda_{1}$, this had to be accepted if the test statistic function $\chi = -(m-1) \cdot \sum_{s=1}^{s_{p}} \log (\lambda_{s}) + (m-1) \cdot s_{p} \cdot \log \left( \frac{s_{p} - 1}{s_{p} + 2} \right)$ is not greater than the critical value of chi-square distribution with $(s_{p} - 1)(s_{p} + 2)/2$ degrees of freedom. As Bartlett’s test is insufficient for testing the significance of the last two PCs, i.e. those of the lowest importance ($s = 1, 2$), a port hoc test for proportion of total variance was applied. A desired proportion of the minimum of the total variance at 95.0% was selected and then excluded all PCs which accounted for less than 1% of the total variance.
2.5. Multilinear regression

Let $w$, $1 \leq w \leq p$ be the number of significant PCs and $Z_w = \left[ Z_{sw} \right]_{m \times w}$ is the array of all credibility coefficient vectors $Z_{sw}$, $s = 1, ..., w$ assigned to $w$-significant PCs (see PCs' statistics). The linear regression model $Z = \left[ 1 \ Z_w \right]_{m \times (w+1)} \cdot B + \varepsilon$, expresses the linear combination of credibility coefficient vectors $Z_{sw}$ to the dependent vector variable $Z = \left[ Z_j \right]_{m \times 1}$ of credibility coefficient estimates for loan granted data for all $m$-banks in the presence of random errors $\varepsilon = \left[ \varepsilon_j \right]_{m \times 1}$. The vector $B = \left[ \beta_{sw} \right]_{(w+1) \times 1}$ consists of a constant term $\beta_0$ and $w$ sloping coefficients such that the coefficient $\beta_s$ explains the influence of the credibility coefficient vector $Z_s$ to the dependent vector variable $Z$.

Method of ordinary least squares (ols) solves the linear regression model and estimate of the vector $B$ is $\hat{B} = \left( Z_w' \cdot Z_w \right)^{-1} \cdot Z_w' \cdot Z$. All diagnostic tests for residual errors were passed in order to confirm the linear, unbiased and minimum variance estimate $\hat{B}$.

3. Data and empirical results

Data for eighty eight (88) banks worldwide involved in shipping finance were derived from Bloomberg and Bankscope databases for the time period 2005–2010 that the relevant industry displaced both its peak and least value.

For nine internal variables over 15 (see Table 1), the sample mean $\bar{Z}_s$ of the credibility coefficient was found to exceed the value 1 and for a single case (Customer Deposits) was found to be negative. In all other cases (5/15), the sample mean ranged from 0 to 1 and the parametric mean of the credibility coefficient for each one of the internal variables except Loan ($s = 1$) might be equal the corresponding estimate $\hat{Z}_s$ ($s = 2–15$) conditioned on the uniform hypothesis (Wald–Wolfowitz; $s = 1$, $p = 0.02$, $s = 2–15$, $p > 0.05$). For all the internal variables except Loan where $\hat{Z}_1 = 0.59$, this estimate lies in a narrow range 0.61–0.71.

Thirteen internal variables were found significantly correlated with Loan ($s = 1$) except the internal variable Profit Before Tax (Spearman; $s = 2–5$, 7–15, $r_s \geq 0.46$, $p < 0.01$; $s = 6$, $r_s = 0.29$, $p = 0.206$). Within each group I and II internal variables there were found significant correlated and eligible for factor analysis (group I, KMO = 0.98, $p < 0.001$; group II, KMO = 0.99, $p < 0.001$).

| Table 2. Spearman correlation coefficients between internal variables |
|--------------------------|----------------|----------------|----------------|----------------|----------------|
| **Group I**              | **Net interest income** | **Interest income on loans** | **Total interest expenses** | **Personnel expenses** | **Operating profit** |
| Net income               | 0.46            | 0.24            | 0.06            | 0.14            | 0.99            | 0.82            |
| Net interest income      | 0.97            | 0.91            | 0.94            | 0.57            | 0.57            | 0.88            |
| Interest income on loans |                 | 0.98            | 0.99            | 0.36            | 0.36            | 0.75            |
| Total interest expenses  |                 |                 | 0.99            | 0.18            | 0.62            |
| Personnel expenses       |                 |                 |                 | 0.26            | 0.68            |
| Operating profit         |                 |                 |                 |                 | 0.89            |
| **Group II**             | **Corporate commercial loans** | **Customer deposits** | **Total customer deposits** | **Deposits & short term funding** | **Interest expenses on customer deposits** |
| Total assets             | 0.98            | 0.86            | 0.96            | 0.99            | 0.83            |
| Corporate commercial loans | 0.90            | 0.98            | 0.99            | 0.99            | 0.83            |
| Customer deposits        |                 | 0.90            | 0.86            | 0.83            |
| Total customer deposits  |                 |                 | 0.99            | 0.93            |
| Deposits & short-term funding |                 |                 |                 | 0.88            |
By applying PCA, it found that there two significant principal components that together accounted for 99.9% of the total variance of all internal variables of group I; these were modest correlated (Table 2, Spearman’s $r_s = 0.36$, $p < 0.001$) variables Interest Income on Loan ($s = 7$, $\lambda_s = 5.03$, 71.8%, $\alpha = 0.98$) and Operating Profit ($s = 11$, $\lambda_s = 1.97$, 28.1%, $\alpha = 0.99$). Similarly in group II, only a single component was found that accounted for the 92.9% of the total variance; this was the Total Assets ($s = 3$, $\lambda_s = 5.57$, $\alpha = 0.83$). Neither Interest Income on Loan nor Operating Profit was highly correlated to Total Assets (Spearman’s $r_s = 0.58$ and $0.63 < 0.7$, respectively).

The univariate linear regression model of the credibility coefficient of Loans was estimated:

$$Z_{\text{loan}} = 0.15 + 0.18 \cdot \text{Total Assets} + 0.45 \cdot \text{Interest Income on Loan} + 0.25 \cdot \text{Operating profit}$$

$$R^2 = 0.89, R^2_{\text{adj}} = 0.855, s = 0.074, F = 22.56^{**}, \quad ^{**}p < 0.01$$

and the step-wise method specified that the optimal model is:

$$Z_{\text{loan}} = 0.23 + 0.49 \cdot \text{Interest Income on Loan} + 0.29 \cdot \text{Operating array}$$

$$R^2 = 0.88, R^2_{\text{adj}} = 0.86, s = 0.072, F = 33.53^{**}, \quad ^{**}p < 0.01$$

Table 3 contains paired values of credibility coefficients for both the independent variables Interest Income on Loan (second column) and Operating Profit (third column), that were used as inputs to the above model to estimate the credibility coefficient of dependent variable Loans (fourth column).

Finally, we preferred to show the estimated values if they were ranged from 0 to 1, while all others were qualitatively represented either as negative signed (<0) or greater than one (>1). Regarding the aforementioned model, the estimated values $\hat{Z}_{\text{loan}}$ are positioned in a plane surface (x-axis: Interest Income on Loan, y-axis: Operating Profit), that regresses through the data points (z-axis: $Z_{\text{loan}}$) with a goodness of fit $R^2 = 88\%$ (Figure 1).

Comparing $Z_{\text{loan}}$ data with $\hat{Z}_{\text{loan}}$ estimates, 14/88 matching cases were found where in 3 cases ($pa = 3/14$) both $Z_{\text{loan}}$ and $\hat{Z}_{\text{loan}} < 0$ and in 11 cases ($pd = 11/14$) both $Z_{\text{loan}}$ and $\hat{Z}_{\text{loan}} > 1$. On the other hand, 5 over 88 unmatched cases were found, where in 2 cases ($pb = 2/5$) $Z_{\text{loan}} < 0$ and $\hat{Z}_{\text{loan}} > 1$ and in 3 cases ($pc = 3/5$) $Z_{\text{loan}} > 1$ and $\hat{Z}_{\text{loan}} < 0$. The McNemar’s test did not reject the null hypothesis that the two marginal probabilities for each outcome were the same, i.e. $pa + pb = pa + pc$ and $pc + pd = pb + pd$ ($X^2 = 0.2, df = 1, p = 0.655$) which implicitly states the coherency between raw data ($Z_{\text{loan}}$) end estimates ($\hat{Z}_{\text{loan}}$) for the values and took into account out of the interval 0–1.

In one case $Z_{\text{loan}} < 0$, $\hat{Z}_{\text{loan}}$ was equal to 0.18. In 13 over 88 cases where $Z_{\text{loan}} > 1$, the estimates that were found left skewed from 0 to 1 with a mean 0.81 (standard deviation 0.16). In 5 over 88 cases where values of $Z_{\text{loan}}$ were at the interval 0, 1 (mean and standard deviation were 0.55 and 0.32, respectively), their estimates $\hat{Z}_{\text{loan}}$ were found as negative. In 24 over 88 cases where values of $Z_{\text{loan}}$ ranged from 0 to 1, the estimates $\hat{Z}_{\text{loan}}$ were found greater than 1 with a mean 1.13 (standard deviation 0.23), respectively. Finally, in 26 over 88 cases both $Z_{\text{loan}}$ data and corresponding estimates $\hat{Z}_{\text{loan}}$ ranged from 0 to 1 with mean residual error equalled to ~0.08 (standard deviation 0.22). Despite the negative sign of the mean difference $Z_{\text{loan}} - \hat{Z}_{\text{loan}}$ we presume that the estimates are unbiased (Student’s test value = 0, $t = -1.95$, $p = 0.063$).

Figure 2 presents the cases where either $Z_{\text{loan}}$ data (A, B and C) or the estimates $\hat{Z}_{\text{loan}}$ (B, C and D) range from 0 to 1. Top left and bottom panels present the histogram of values (mean ± standard deviation) fitted by the normal curve. The top right panel is a scattered plot of estimates $\hat{Z}_{\text{loan}}$ versus $Z_{\text{loan}}$ data against the orthogonal axis bisector line.
Table 3. Results of $Z_{\text{loan}}$ and estimate of $\hat{Z}_{\text{loan}}$ based on the optimal model

| $Z_{\text{loan}}$ | Int in loan | Oper prof | $\hat{Z}_{\text{loan}}$ | $Z_{\text{loan}}$ | Int in loan | Oper prof | $\hat{Z}_{\text{loan}}$ |
|-------------------|-------------|-----------|------------------------|-------------------|-------------|-----------|------------------------|
| 0.028             | 0.677       | 2.366     | >1                     | 0.989             | 0.992       | 1.025     | >1                     |
| 0.106             | 0.249       | -5.341    | >1                     | 0.989             | 0.958       | 1.011     | 0.992                  |
| 0.157             | -3.661      | 0.571     | <0                     | 0.994             | 0.996       | 1.089     | >1                     |
| 0.316             | 0.260       | 2.092     | 0.964                  | 0.996             | 0.993       | 1.007     | >1                     |
| 0.318             | 0.300       | 0.091     | 0.403                  | 0.997             | 0.938       | 1.010     | 0.983                  |
| 0.406             | 2.574       | 3.882     | >1                     | 0.997             | 0.998       | 0.997     | >1                     |
| 0.483             | 0.849       | 1.086     | 0.961                  | 0.997             | 1.002       | 1.000     | >1                     |
| 0.523             | 0.864       | -9.631    | <0                     | 0.998             | 0.999       | 0.917     | 0.985                  |
| 0.571             | 1.025       | 0.682     | 0.930                  | 0.998             | 0.964       | 0.921     | 0.970                  |
| 0.578             | 0.152       | 1.941     | 0.867                  | 0.999             | 0.997       | 1.045     | >1                     |
| 0.585             | 0.757       | 0.823     | 0.840                  | 1.00              | 1.000       | 1.001     | >1                     |
| 0.626             | -21.711     | 4.090     | <0                     | -11.452           | 12.852      | <0        | >1                     |
| 0.652             | 0.617       | 1.758     | >1                     | <0                | -4.049      | 1.103     | <0                     |
| 0.683             | 0.302       | 0.855     | 0.626                  | <0                | -3.489      | 1.532     | <0                     |
| 0.764             | 0.681       | 1.270     | 0.932                  | <0                | -3.206      | 5.255     | 0.183                  |
| 0.768             | 1.041       | 1.076     | >1                     | <0                | 0.340       | 2.526     | >1                     |
| 0.768             | 0.341       | 1.356     | 0.790                  | <0                | 1.285       | 7.739     | >1                     |
| 0.802             | 0.867       | 1.161     | 0.992                  | >1                | 1.130       | -37.359   | <0                     |
| 0.807             | 1.212       | 0.946     | >1                     | >1                | 0.428       | -2.678    | <0                     |
| 0.847             | 1.401       | 0.582     | >1                     | >1                | 1.506       | -4.394    | <0                     |
| 0.850             | 0.852       | 1.058     | 0.955                  | >1                | 0.639       | -0.343    | 0.444                  |
| 0.853             | -1.280      | 1.028     | <0                     | >1                | 0.868       | -0.239    | 0.586                  |
| 0.886             | 0.964       | 0.505     | 0.849                  | >1                | 1.114       | -0.228    | 0.710                  |
| 0.894             | 0.963       | 0.974     | 0.985                  | >1                | 0.652       | 0.673     | 0.745                  |
| 0.905             | 0.893       | 0.975     | 0.950                  | >1                | 0.743       | 0.704     | 0.798                  |
| 0.907             | 0.977       | 1.129     | >1                     | >1                | 0.779       | 0.689     | 0.812                  |
| 0.913             | 0.994       | 0.892     | 0.976                  | >1                | 0.817       | 0.688     | 0.830                  |
| 0.918             | 0.835       | 1.092     | 0.956                  | >1                | 0.811       | 0.809     | 0.862                  |
| 0.931             | 0.931       | 1.284     | >1                     | >1                | 0.787       | 0.875     | 0.869                  |
| 0.932             | 1.183       | 1.470     | >1                     | >1                | 0.384       | 1.777     | 0.934                  |
| 0.939             | 0.982       | 1.031     | >1                     | >1                | 0.238       | 2.114     | 0.960                  |
| 0.951             | 1.009       | 1.252     | >1                     | >1                | 1.016       | 0.887     | 0.985                  |
| 0.956             | 1.227       | 0.968     | >1                     | >1                | 1.160       | 0.681     | 0.996                  |
| 0.957             | 0.849       | 0.607     | 0.822                  | >1                | 1.015       | 1.069     | >1                     |
| 0.959             | 0.836       | -0.346    | 0.539                  | >1                | 1.115       | 1.097     | >1                     |
| 0.959             | 1.052       | 1.002     | >1                     | >1                | 0.990       | 1.367     | >1                     |
| 0.970             | 0.876       | 0.905     | 0.922                  | >1                | 1.234       | 1.388     | >1                     |
| 0.970             | 0.903       | 0.870     | 0.925                  | >1                | 1.183       | 1.614     | >1                     |
| 0.972             | 0.992       | 1.073     | >1                     | >1                | 1.451       | 1.413     | >1                     |
| 0.973             | 1.106       | 1.136     | >1                     | >1                | 1.813       | 1.068     | >1                     |
| 0.975             | 1.024       | 1.007     | >1                     | >1                | 1.629       | 2.179     | >1                     |
| 0.981             | 0.984       | 0.963     | 0.992                  | >1                | 1.910       | 2.128     | >1                     |
| 0.982             | 0.952       | 0.988     | 0.983                  | >1                | 1.946       | 2.119     | >1                     |
| 0.987             | 1.092       | 0.851     | >1                     | >1                | 1.600       | 7.840     | >1                     |

Notes: Paired values of credibility coefficients for both the independent variables interest income on loan (second column) and operating profit (third column).
Figure 1. Model’s surface regression estimate \( z = 0.23 + 0.49 x + 0.29 y \). \( R^2 = 0.88 \) versus data points of credibility coefficient on loan (z-axis) with respect to interest income on loan (x-axis) and operating profit (y-axis).

Notes: All values above 1 and below 0 are upper and lower limited to 1.01 and -0.01, respectively, in this plot.

Figure 2. Histograms of values of \( Z_{loan} \) when \( \hat{Z}_{loan} < 0 \) (top left a) and \( \hat{Z}_{loan} > 1 \) (bottom left c) and histogram of estimates \( \hat{Z}_{loan} \) when \( Z_{loan} > 1 \) (bottom right d). In all histograms the solid line is the normal fit curve adjusted to the data. At the top right (b) is the scatter plot (plus symbols) of \( \hat{Z}_{loan} \) (vertical axis) vs \( Z_{loan} \) (horizontal axis) together with the bisector (dashed) line.
4. Discussion

Focusing on the most essential operational environments’ internal factors of an individual bank combined with the whole shipping banks’ loans portfolio that may influence its decision to either increase or decrease financing in the relevant market, we developed a specific methodological framework for shipping finance with respect to bank credibility. More specifically, we found that the most important variables from shipping bank’s internal environment are linked to interest income on loan and operating profit accounts, while its decision for loan grants is related to its policy in comparison with the whole market (total of shipping banks).

According to Buhlmann’s theory a decision (C) making process contains two components: the individual unit (R) and the whole population (H), that in our case is the bank and the mass of banks, respectively. They are both combined with weight coefficients $Z$ and $1-Z$ and form a linear system $C = Z \cdot R + (1 - Z) \cdot H$.

If the $Z$ coefficient is from 0 to 1, then the linear system illustrates how credible the individual unit is; $Z = 0$ means total non-credible and the decision is made based on, if the unit depends exclusively on the population ($C = H$); $0 < Z < 1$ means partial credible; for instance, if $Z = 0.5$, then the decision is made based upon the average between the unit and the population units ($C = (R + H)/2$); and finally $Z = 1$ means fully credible and the decision ignores the population and is made exclusively by the unit ($C = R$).

For each internal factor the corresponding credibility coefficient was calculated. Consistent with the absence of any information for risk factors that influence the bank’s policy, we may consider an unknown upper limit for loan grants and all grants could be randomly distributed. Thus, we selected the uniform distribution as the most appropriate probabilistic conceptualisation for the assumption of the size of the internal factor. In Table 1, for each internal factor both the classic mean ($\overline{Z} \cdot s$) and stochastic ($\hat{Z}_s$) credibility coefficient were presented. Despite the wide range of means found from below 0 to above 1, the stochastic was ranged narrowly from 0.59 through to 0.71. Rejection of the credibility coefficients data normality allowed the non-parametric runs test rather than the student parametric test, and forwarded that the credibility coefficient might be insignificantly different to the stochastic value extracted under the uniform hypothesis for all 14 internal factors except the internal factor for loan grants.

Finally, we had two contradictory estimates: the mean estimate was greater than 1 and implied either a sharp aggressive or passive defensive strategy (see below). On the other hand, the stochastic value 0.59 implied balanced weights between the individual (59%) and the population market’s (41%) strategy.

Since no prior work has been done regarding the issue of decision-making process strategy based on the credibility coefficients, we made two models: the “detailed” model that is fully presented here and the “lumped” model that will be explicitly analysed in forthcoming research.

The “detailed” model took into account all the single values of the credibility coefficients and all of the results in this paper are based on it. On the other hand, the “lumped” model evaluated only the stochastic values of credibility coefficients. In both models, the dependent variable is the credibility coefficient of loan grants internal factor and the independents are the most essential internal factors found from the PCA.

PCA was extracted from those internal factors which account for most of the variance of the system of all internal factors. Since large values of correlation coefficients were found between internal factors, the idea was that many of them were explained from others and that they were redundant. The aim of this analysis was to minimise the redundancy without seriously affecting the system’s variance; in fact, the last was secured by setting a lower limit of 95% of the system’s variance. Consistent with the method’s limitations, we created two groups, each with homogeneous internal
factors. To this end, PCA extracted three internal factors as the most essential: Total Assets, Interest Income on Loans and Operation Profit.

All three internal factors were used as independent variables in a linear univariate regression model for prediction of the credibility coefficients of loan grants that were used as the dependent variable \(Z_{\text{loan}}\). Stepwise regression analysis excluded the total assets variable by increasing the adjusted goodness of fit from 0.855 to 0.86. The final model predicts that (i) the values of \(Z_{\text{loan}}\) are positively driven by each dependent variable, (ii) the interest income on loan variable has more influence compared to the operating profit variable (0.49 versus 0.29), (iii) if both variables together equal 0 or 1, i.e. reach their minimum or maximum value from zero through to one, then the \(Z_{\text{loan}}\) equals 0.23 (the bank’s decision strategy follows the market’s strategy by 77%) or reaches its maximum value (the bank strictly follows its own strategy) respectively.

By applying the “detailed” model in our study, we counted 55 over 88 banks (62.5%) of \(Z_{\text{loan}}\) values from 0 to 1 and mean value of 0.8 that is actually quite close to 1. This is presumably an indication that the decision-making process on loan grants is more so based on the individual strategy rather than on the whole market statement. For those banks, about half (47.3%) were unbiased estimated by the model developed in this research, 9% were false estimated, given that estimates were found below zero and 43.7% were overestimated, given that estimates were found above 1. For the last case, the mean error estimate is about 0.12 and might be attributed to the other bank’s internal factors with no significant influence to the values of \(Z_{\text{loan}}\).

In contradiction with the most frequent values of \(Z_{\text{loan}}\) that were found from 0 to 1, there were also only 6 banks over 88 (6.8%) of negative signed \(Z_{\text{loan}}\) values. In half of the cases, the \(Z_{\text{loan}}\) values were adequately estimated from the model, in one case it was overestimated with error 0.18 and in two cases we had false estimates above 1. Negative values of \(Z_{\text{loan}}\) two strategies regarding the next year \((n + 1)\). One case, is the optimistic strategy that leads the bank to overcome the market’s average amount of loan grants \((C > H)\), while the previous year’s had followed a pure conservative strategy by holding the average amount of loan grants below the market’s corresponding \((R < H)\). The other case, is the defensive strategy that leads the bank to grant loans below the market’s average amount \((C < H)\), while the previous year’s had followed a regressive strategy by granting loans above the market’s corresponding \((R > H)\).

Finally, in 27 banks over 88 (30.6%) the \(Z_{\text{loan}}\) values were found above 1. In 11 of those banks, \(Z_{\text{loan}}\) values were matched from the model, in 13 banks the \(Z_{\text{loan}}\) values were underestimated with mean error 0.18 and in only 3 banks there were false estimates, i.e. below zero. Values of \(Z_{\text{loan}}\) above 1 implicate either an extreme aggressive or passive defensive strategy; the bank either increases (aggressive) or decreases (defensive) the loan grants the next period \((n + 1)\) when the previous periods granted more \((H < R < C)\) or less \((C < R < H)\), respectively, than the population average. Both cases might be characterised as risky, since the individual policy for loan grants is further distant from the population average.

In summary, the importance of this paper concerns the methodological steps, (i) PCA as a powerful technique to find the most important internal factors that explain most of a bank’s system variance and (ii) the credibility coefficient that balances the individual bank’s policy with the market as a whole to either increase or decrease its loan grants to a shipping market. The main advantage of the proposed methodological framework is that it takes into account the current market trends and balances its own optimal policy concerning the loan grants.

5. Conclusions
The purpose of this study was threefold: (i) to analyse the operational environments’ internal factors of an individual bank combined with the whole shipping banks’ loans portfolio by estimating the credibility factor to the decision of the bank to either increase or decrease financing in the relevant market, (ii) to reveal the essential factors arising from a bank’s internal environment and, finally,
(iii) to develop a specific methodological framework for shipping finance with respect to bank credibility.

We found that the most important variables from shipping banks’ internal environment that affect a bank’s decision to either increase or decrease the shipping portfolio are linked to interest income on loan and operating profit accounts. In addition, an individual bank’s decision for loan grants is related to its policy in comparison with the whole market (total of shipping banks); prices of \(Z_{\text{loan}}\) close to or below zero mean that the bank’s top management has little confidence in its own estimations and follows what its competitors do, while prices close to or above 1 show that despite, what the majority of the shipping banks decide for loan grants the following year \(n + 1\), the individual bank will decide its own policy based on its own estimations.

Considering the aforementioned, key implications that arise from these findings concern (i) methodology, (ii) the natural basis of raw data, (iii) the significance of the improved financial strategy concerning the loan grant into the irregular and heterogeneous shipping market.

The behaviour of the model derived from the estimates supported the idea that values of \(Z_{\text{loan}}\) implicate either an extreme aggressive or passive defensive strategy: the bank either increases (aggressive) or decreases (defensive) the loan grants the following period \((n + 1)\) depending on its previous years’ policy than the population average. The relevant formulas accumulate the internal forces within the bank, as well as the general environment of all banks. Verification is an indication of credibility of the proposed model based on: (a) the principal component analysis, (b) the regression analysis and (c) the Bühlmann credibility model applied at stochastic and empirical level. To our knowledge, this is the first model to do so.

Further understanding of the proposed decision support tool requires quantitative assessment of the most important factors deriving from the external environment of the shipping banks, e.g. shipping market analysis per sector. The definition of the most important factors related to each shipping sub-sector is expected to integrate the proposed model as it will co-estimate both internal and external factors that may influence shipping finance taking into account each individual bank’s policy with respect to decision’s credibility factor.

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