Model-independent analysis of soft masses
in heterotic string models
with anomalous $U(1)$ symmetry

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Abstract
We study the magnitudes of soft masses in heterotic string models
with anomalous $U(1)$ symmetry model-independently. In most cases,
$D$-term contribution to soft scalar masses is expected to be comparable
to or dominant over other contributions provided that supersymmetry
breaking is mediated by the gravitational interaction and/or an
anomalous $U(1)$ symmetry and the magnitude of vacuum energy is
not more than of order $m_{3/2}^2 M^2$.

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1 Introduction

Superstring theories are powerful candidates for the unification theory of all forces including gravity. The supergravity theory (SUGRA) is effectively constructed from 4-dimensional (4D) string model using several methods [1, 2, 3]. The structure of SUGRA is constrained by gauge symmetries including an anomalous $U(1)$ symmetry ($U(1)_A$) [4] and stringy symmetries such as duality [5].

4D string models have several open questions and two of them are pointed out here. The first one is what the origin of supersymmetry (SUSY) breaking is. Although interesting scenarios such as SUSY breaking mechanism due to gaugino condensation [3] and Scherk-Schwarz mechanism [4] have been proposed, realistic one has not been identified yet. The second one is how the vacuum expectation value (VEV) of dilaton field $S$ is stabilized. It is difficult to realize the stabilization with a realistic VEV of $S$ using a Kähler potential at the tree level alone without any conspiracy among several terms which appear in the superpotential [8]. A Kähler potential generally receives radiative corrections as well as non-perturbative ones. Such corrections may be sizable for the part related to $S$ [3, 10]. It is important to solve these enigmas in order not only to understand the structure of more fundamental theory at a high energy scale but also to know the complete SUSY particle spectrum at the weak scale, but it is not an easy task because of ignorance of the explicit forms of fully corrected total Kähler potential. At present, it would be meaningful to get any information on SUSY particle spectrum model-independently.

In this paper, we study the magnitudes of soft SUSY breaking parameters in heterotic string models with $U(1)_A$ and derive model-independent predictions for them without specifying SUSY breaking mechanism and the dilaton VEV fixing mechanism. The idea is based on that in the work by Ref.[12]. The soft SUSY breaking terms have been derived from “standard string model” and analyzed under the assumption that SUSY is broken by $F$-term condensations of the dilaton field and/or moduli fields $M^i$. We relax this assumption such that SUSY is broken by $F$-term condensation of $S$, $M^i$ and/or matter fields with non-vanishing $U(1)_A$ charge since the scenario

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1 The stability of $S$ and soft SUSY breaking parameters are discussed in the dilaton SUSY breaking scenario in Ref.[11].
based on $U(1)_A$ as a mediator of SUSY breaking is also possible \[13\]. In particular, we make a comparison of magnitudes between $D$-term contribution to scalar masses and $F$-term ones and a comparison of magnitudes among scalar masses, gaugino masses and $A$-parameters. The features of our analysis are as follows. The study is carried out in the framework of SUGRA model-independently, i.e., we do not specify SUSY breaking mechanism, extra matter contents, the structure of superpotential and the form of Kahler potential related to $S$. We treat all fields including $S$ and $M^i$ as dynamical fields.

The paper is organized as follows. In the next section, we explain the general structure of SUGRA briefly with some basic assumptions of SUSY breaking. We study the magnitudes of soft SUSY breaking parameters in heterotic string models with $U(1)_A$ model-independently in section 3. Section 4 is devoted to conclusions and some comments.

2 General structure of SUGRA

We begin by reviewing the scalar potential in SUGRA \[16, 17\]. It is specified by two functions, the total Kahler potential $G(\phi, \bar{\phi})$ and the gauge kinetic function $f_{\alpha\beta}(\phi)$ with $\alpha$, $\beta$ being indices of the adjoint representation of the gauge group. The former is a sum of the Kahler potential $K(\phi, \bar{\phi})$ and (the logarithm of) the superpotential $W(\phi)$

$$G(\phi, \bar{\phi}) = K(\phi, \bar{\phi}) + M^2 \ln |W(\phi)/M^3|^2$$

where $M = M_{Pl}/\sqrt{8\pi}$ with $M_{Pl}$ being the Planck mass, and is referred to as the gravitational scale. We have denoted scalar fields in the chiral multiplets by $\phi^I$ and their complex conjugate by $\bar{\phi}_J$. The scalar potential is given by

$$V = M^2 e^{G/M^2} (G_I(G^{-1})_J G^J - 3M^2) + \frac{1}{2} (Re f^{-1})_{\alpha\beta} \hat{D}^\alpha \hat{D}^\beta$$

where

$$\hat{D}^\alpha = G_I(T^\alpha \phi)^I = (\bar{\phi} T^\alpha)_J G^J.$$

Here $G_I = \partial G/\partial \phi^I$, $G^J = \partial G/\partial \bar{\phi}_J$ etc, and $T^\alpha$ are gauge transformation generators. Also in the above, $(Re f^{-1})_{\alpha\beta}$ and $(G^{-1})^I_J$ are the inverse matrices

\[2\]The model-dependent analyses are carried out in Ref.\[13, 14, 15\].
of $Re f_{\alpha\beta}$ and $G^J_I$, respectively, and a summation over $\alpha,...$ and $I,...$ is understood. The last equality in Eq.(3) comes from the gauge invariance of the total Kähler potential. The $F$-auxiliary fields of the chiral multiplets are given by

$$F^I = M e^{G/2M^2} (G^{-1})^I_J G^J.$$  

(4)

The $D$-auxiliary fields of the vector multiplets are given by

$$D^\alpha = (Re f^{-1})_{\alpha\beta} \hat{D}^\beta.$$  

(5)

Using $F^I$ and $D^\alpha$, the scalar potential is rewritten down by

$$V = V_F + V_D,$$

$$V_F \equiv F_I K^I_J F^J - 3M^4 e^{G/M^2},$$

(6)

$$V_D \equiv \frac{1}{2} Re f_{\alpha\beta} D^\alpha D^\beta.$$  

(7)

Let us next summarize our assumptions on SUSY breaking. The gravitino mass $m_{3/2}$ is given by

$$m_{3/2} = \langle Me^{G/2M^2} \rangle$$  

(8)

where $\langle \cdots \rangle$ denotes the VEV. As a phase convention, it is taken to be real. We identify the gravitino mass with the weak scale in most cases. It is assumed that SUSY is spontaneously broken by some $F$-term condensations ($\langle F \rangle \neq 0$) for singlet fields under the standard model gauge group and/or some $D$-term condensations ($\langle D \rangle \neq 0$) for broken gauge symmetries. We require that the VEVs of $F^I$ and $D^\alpha$ should satisfy

$$\langle (F_I K^I_J F^J)^{1/2} \rangle \leq O(m_{3/2} M),$$

(9)

$$\langle D^\alpha \rangle \leq O(m_{3/2} M)$$  

(10)

for each pair $(I, J)$ in Eq.(3). Note that we allow the non-zero vacuum energy $\langle V \rangle$ of order $m_{3/2}^2 M^2$ at this level, which could be canceled by quantum corrections.

In order to discuss the magnitudes of several quantities, it is necessary to see consequences of the stationary condition $\langle \partial V / \partial \phi^I \rangle = 0$. From Eq.(3), we find

$$\partial V / \partial \phi^I = G_I \left( \frac{V_F}{M^2} + M^2 e^{G/M^2} \right) + Me^{G/2M^2} G_I^J F^J$$

$$- F_I G^I_J F^J - \frac{1}{2} (Re f_{\alpha\beta})_{IJ} D^\alpha D^\beta$$

$$+ D^\alpha (\hat{D} T^\alpha)_{IJ} G^J_I.$$  

(11)
Taking its VEV and using the stationary condition, we derive the formula

\[ m_{3/2} \langle G_{IJ} \rangle \langle F^J \rangle = -\langle G_I \rangle \left( \frac{V_F}{M^2} + m_{3/2}^2 \right) + \langle F^I \rangle \langle G^I \rangle \langle F^J \rangle + \frac{1}{2} \left( \langle Re f_{\alpha \beta} \rangle \langle D^\alpha \rangle \langle D^\beta \rangle - \langle D^\alpha \rangle \langle (\bar{\phi} T^\alpha) J \rangle \langle G^I \rangle \right). \]  

We can estimate the magnitude of SUSY mass parameter \( \mu_{IJ} \equiv m_{3/2} \langle G_{IJ} \rangle / \langle G_I \rangle / M^2 - \langle G^I \rangle / (G^{-1})^J_J \) using Eq. (12). By multiplying \( (T^\alpha \phi)^I \) to Eq. (11), a heavy-real direction is projected on. Using the identities derived from the gauge invariance of the total Kähler potential

\[ G_{IJ} (T^\alpha \phi)^I + G_J (T^\alpha) J - K_I^J (\bar{\phi} T^\alpha) J = 0, \]  
\[ K^J_J (T^\alpha \phi)^J + K_J^I (T^\alpha) J - [G^J (\bar{\phi} T^\alpha) J] J = 0, \]

we obtain

\[ \frac{\partial V}{\partial \phi^I} (T^\alpha \phi)^I = (\frac{V_F}{M^2} + 2M^2 e^{G/M^2}) \tilde{D}^\alpha - F^I F^J (\tilde{D}^\alpha)^J - \frac{1}{2} \left( \langle Re f_{\beta \gamma} \rangle \langle D^\alpha \rangle \right) (T^\alpha \phi)^I D^\beta + (\bar{\phi} T^\beta) J G^I_J (T^\alpha \phi)^I D^\beta. \]  

Taking its VEV and using the stationary condition, we derive the formula

\[ \left\{ \left( \frac{M_{IJK}^2} {2g_{\alpha \beta}} \right)^\alpha \beta + \left( \frac{V_F}{M^2} + m_{3/2}^2 \right) \langle Re f_{\alpha \beta} \rangle \right\} \langle D^\beta \rangle = \langle F^I \rangle \langle F^J \rangle \langle (\tilde{D}^\alpha)^J \rangle + \frac{1}{2} \left( \langle Re f_{\beta \gamma} \rangle \langle (T^\alpha \phi)^I \rangle \langle D^\beta \rangle \langle D^\gamma \rangle \right) \]

where \( (M_{IJK}^2)_{\alpha \beta} = 2g_{\alpha \beta} \langle (\bar{\phi} T^\beta) J K^J_I (T^\alpha \phi)^I \rangle \) is the mass matrix of the gauge bosons and \( g_{\alpha} \) and \( g_{\beta} \) are the gauge coupling constants. Using Eq. (14), we can estimate the magnitude of \( D \)-term condensations \( \langle D^\beta \rangle \).

Using the scalar potential and gauge kinetic terms, we can obtain formulae of soft SUSY breaking scalar masses \( (m^2)^I \), soft SUSY breaking gaugino masses \( M_{\alpha} \) and \( A \)-parameters \( A_{IJK} \).

\[ (m^2)^I = (m^2_F)^I + (m^2_D)^I, \]
\[ (m^2_F)^I = m_{3/2}^2 + \langle V_F \rangle / M^2 \langle K_I \rangle. \]
\begin{equation}
(F'I)\langle(KJ')_I(K^{-1}I)_{J'}K_{J'I} - K_{J'I}J')\rangle(F_J') + \cdots,
\end{equation}
\begin{equation}
(m_D^2)_I^J \equiv \sum_\hat{\alpha} q^\hat{\alpha}_I \langle D^\hat{\alpha} \rangle \langle K_{J'I} \rangle,
\end{equation}
\begin{equation}
M_\alpha = \langle F'\rangle \langle(Ref_\alpha)^{-1}\rangle \langle f_{\alpha}J \rangle,
\end{equation}
\begin{equation}
A_{IJK} = \langle F'\rangle \langle(f_{IJK}J') \rangle + \frac{\langle K_{J'I} \rangle}{M^2} \langle f_{IJK} \rangle
- \langle K_{J'I} \rangle \langle(K^{-1})_J'\rangle \langle f_{J'IJK} \rangle
\end{equation}

where the index \(\hat{\alpha}\) runs over broken gauge generators, \(Ref_\alpha \equiv Ref_{\alpha \alpha}\) and \(f_{IJK}\)’s are Yukawa couplings some of which are moduli-dependent. The \((I \cdots JK)\) in Eq.\((21)\) stands for a cyclic permutation among \(I, J\) and \(K\). The ellipsis in \((m_D^2)_I^J\) stands for extra \(F\)-term contributions and so forth. The \((m_D^2)_I^J\) is a \(D\)-term contribution to scalar masses.

3 Heterotic string model with anomalous U(1)

Effective SUGRA is derived from 4D string models taking a field theory limit. In this section, we study soft SUSY breaking parameters in SUGRA from heterotic string model with \(U(1)_A\).\(^3\) Let us explain our starting point and assumptions first. The gauge group \(G = G_{SM} \times U(1)_A\) originates from the breakdown of \(E_8 \times E_8'\) gauge group. Here \(G_{SM}\) is a standard model gauge group \(SU(3)_C \times SU(2)_L \times U(1)_Y\) and \(U(1)_A\) is an anomalous \(U(1)\) symmetry. The anomaly is canceled by the Green-Schwarz mechanism.\(^2\) Chiral multiplets are classified into two categories. One is a set of \(G_{SM}\) singlet fields which the dilaton field \(S\), the moduli fields \(M^i\) and some of matter fields \(\phi^m\) belong to. The other one is a set of \(G_{SM}\) non-singlet fields \(\phi^k\). We denote two types of matter multiplet as \(\phi^\lambda = \{\phi^m, \phi^k\}\).

The dilaton field \(S\) transforms as \(S \rightarrow S - i\delta^A_{GS} M\theta(x)\) under \(U(1)_A\) with a space-time dependent parameter \(\theta(x)\). Here \(\delta^A_{GS}\) is so-called Green-Schwarz coefficient of \(U(1)_A\) and is given by

\begin{equation}
\delta^A_{GS} = \frac{1}{96\pi^2} Tr Q^A = \frac{1}{96\pi^2} \sum_\lambda q^A_\lambda,
\end{equation}

\(^3\) Based on the assumption that SUSY is broken by \(F\)-components of \(S\) and/or a moduli field, properties of soft SUSY breaking scalar masses have been studied in Ref.\([20, 21]\).
where \( Q^A \) is a \( U(1)_A \) charge operator, \( g^A_\lambda \) is a \( U(1)_A \) charge of \( \phi^\lambda \) and the Kac-Moody level of \( U(1)_A \) is rescaled as \( k_A = 1 \). We find \(|\delta^A_{GS}/q^A_m| = O(10^{-1}) \sim O(100)\) in explicit models \([23, 24]\).

The requirement of \( U(1)_A \) gauge invariance yields the form of Kähler potential \( K \) as,

\[
K = K(S + \bar{S} + \delta^A_{GS}V_A, M^i, \bar{M}^i, \bar{\phi}_\mu e^{\eta_\mu V_A}, \phi^\lambda)
\]

up to the dependence on \( G_{SM} \) vector multiplets. We assume that derivatives of the Kähler potential \( K \) with respect to fields including moduli fields or matter fields are at most of order unity in the units where \( M \) is taken to be unity. However we do not specify the magnitude of derivatives of \( K \) by \( S \) alone. The VEVs of \( S \) and \( M^i \) are supposed to be fixed non-vanishing values by some non-perturbative effects. It is expected that the stabilization of \( S \) is due to the physics at the gravitational scale \( M \) or at the lower scale than \( M \). Moreover we assume that the VEV is much bigger than the weak scale, i.e., \( O(m_{3/2}) \ll \langle K_S \rangle \). The non-trivial transformation property of \( S \) under \( U(1)_A \) implies that \( U(1)_A \) is broken down at some high energy scale \( M_I \).

Hereafter we consider only the case with overall modulus field \( T \) for simplicity. It is straightforward to apply our method to more complicated situations with multi-moduli fields. The Kähler potential is, in general, written by

\[
K = K(S + \bar{S} + \delta^A_{GS}V_A) + K(T + \bar{T}) + K^{(S,T)} + \sum_{\lambda, \mu} (s^\mu_\lambda (S + \bar{S} + \delta^A_{GS}V_A) + t^\mu_\lambda (T + \bar{T}) + u^{(S,T)}_\lambda \phi^\lambda \bar{\phi}_\mu + \cdots
\]

where \( K^{(S,T)} \) and \( u^{(S,T)}_\lambda \) are mixing terms between \( S \) and \( T \). The magnitudes of \( \langle K^{(S,T)} \rangle \), \( \langle s^\mu_\lambda \rangle \) and \( \langle u^{(S,T)}_\lambda \rangle \) are assumed to be \( O(\epsilon_1 M^2) \), \( O(\epsilon_2) \) and \( \epsilon_3 \) where \( \epsilon_n \)’s \((n = 1, 2, 3)\) are model-dependent parameters whose orders are expected not to be more than one.\footnote{The existence of \( s^\mu_\lambda \phi^\lambda \bar{\phi}_\mu \) term in \( K \) and its contribution to soft scalar masses are discussed in 4D effective theory derived through the standard embedding from heterotic M-theory \([25]\).}

We estimate the VEV of derivatives of \( K \) in the form including \( \epsilon_n \). For example, \( \langle K^{(S,T)}_{\lambda \bar{S}} \rangle \leq O(\epsilon_p M) \) \((p = 2, 3)\). Our consideration is applicable to models in which some of \( \phi^\lambda \) are composite fields made of original matter multiplets in string models if the Kähler potential
meets the above requirements. Using the Kähler potential \( \hat{D}^A \), \( \hat{D}^A \) is given by
\[
\hat{D}^A = -K_S \delta_{GS}^A M + \sum_{\lambda, \mu} K^\mu_\lambda \phi_\mu(q^A \phi)^\lambda + \cdots. \tag{25}
\]
The breaking scale of \( U(1)_A \) defined by \( M_I \equiv |\langle \phi^m \rangle| \) is estimated as \( M_I = O((\langle K_S \rangle \delta_{GS}^A M/q_m^A)^{1/2}) \) from the requirement \( \langle D^A \rangle \leq O(m_{3/2}M) \). We require that \( M_I \) should be equal to or be less than \( M \), and then we find that the VEV of \( K_S \) has an upper bound such as \( \langle K_S \rangle \leq O(q_m^A M/\delta_{GS}^A) \).

The \( U(1)_A \) gauge boson mass squared \( (M_V^2)^A \) is given by
\[
(M_V^2)^A = 2g_A^2 (\langle K_S^A \rangle)^2 + \sum_{m,n} q_m^A q_n^A (\langle \phi^m \rangle \langle \bar{\phi}^n \rangle) \tag{26}
\]
where \( g_A \) is a \( U(1)_A \) gauge coupling constant. The magnitude of \( (M_V^2)^A/g_A^2 \) is estimated as \( Max(O(\langle K_S^A \rangle^2, O(q_m^A M_I)^2)) \). We assume that the magnitude of \( (M_V^2)^A/g_A^2 \) is \( O(q_m^A M_I^2) \). It leads to the inequality \( \langle K_S^A \rangle \leq O((q_m^A M_I/\delta_{GS}^A M_I)^2) \).

The formula of soft SUSY breaking scalar masses on \( G_{SM} \) non-singlet fields is given by \[21\]
\[
(m^{2})_{l}^{k} = (m_{3/2}^{2} + \langle V_{F} \rangle M^{2}) \langle K_{I}^{k} \rangle + \langle F^{I} \rangle \langle F_{J} \rangle (\langle R_{HI}^{jk} \rangle + \langle X_{HI}^{jk} \rangle), \tag{27}
\]
\[
\langle R_{HI}^{jk} \rangle \equiv \langle (K_{HI}^{I} K^{-1}_{I} K_{HI}^{J} - K_{HI}^{J}) \rangle, \tag{28}
\]
\[
\langle X_{HI}^{jk} \rangle \equiv q_{l}^{A} ((M_{V}^{2})^{A})^{-1} \langle (D_{I}^{A})^{J} \rangle \langle K_{I}^{k} \rangle. \tag{29}
\]
Here we neglect extra \( F \)-term contributions and so forth since they are model-dependent. The neglect of extra \( F \)-term contributions is justified if Yukawa couplings between heavy and light fields are small enough and the \( R \)-parity violation is also tiny enough. We have used Eq.\([16]\) to derive the part related to \( D \)-term contribution. Note that the last term in r.h.s. of Eq.\([16]\) is negligible when \( (M_{V}^{2})^{A}/g_{A}^{2} \) is much bigger than \( m_{3/2}^{2} \). Using the above mass formula, the magnitudes of \( \langle R_{HI}^{jk} \rangle \) and \( \langle X_{HI}^{jk} \rangle \) are estimated and given in Table 1. Here we assume \( q_{l}^{A}/q_{m}^{A} = O(1) \).

Now we obtain the following generic features on \( (m^{2})_{l}^{k} \).

1. The order of magnitude of \( \langle X_{HI}^{jk} \rangle \) is equal to or bigger than that of \( \langle R_{HI}^{jk} \rangle \) except for an off-diagonal part \( (I, J) = (S, T) \). Hence the magnitude of \( D \)-term contribution is comparable to or bigger than that of \( F \)-term contribution.
Table 1: The magnitudes of $\langle R_{ik}^j \rangle$ and $\langle X_{ik}^j \rangle$

| $(I,J)$ | $\langle R_{ik}^j \rangle$ | $\langle X_{ik}^j \rangle$ |
|---------|-----------------|-----------------|
| $(S,S)$ | $O(\epsilon_p/M^2)$ | $Max(O((K_S^2)/\langle K_S^2 \rangle), O(\epsilon_p/M^2))$ |
| $(T,T)$ | $O(1/M^2)$ | $Max(O(\epsilon_1/\langle K_T^2 \rangle), O(1/M^2))$ |
| $(m,m)$ | $O(1/M^2)$ | $O(1/M^2)$ |
| $(S,T)$ | $O(\epsilon_p/M^2)$ | $Max(O(\epsilon_1/\langle K_S^2 \rangle M), O(\epsilon_3/M^2))$ |
| $(S,m)$ | $O(\epsilon_p M_I/M^3)$ | $Max(O(\epsilon_p/\langle K_S^2 \rangle M), O(\epsilon_p/(MM_I))$ |
| $(T,m)$ | $O(M_I/M^3)$ | $Max(O(\epsilon_3/\langle K_S^2 \rangle M), O(1/(MM_I))$ |

except for the universal part $(m^2_3/2 + \langle V_F \rangle/M^2)\langle K_I^k \rangle$.

(2) In case where the magnitude of $\langle F_m \rangle$ is bigger than $O(m^3_3/M_I)$ and $M > M_I$, we get the inequality $(m^2_D)_k > O(m^2_3/2)$ since the magnitude of $\langle \hat{D}^A \rangle$ is bigger than $O(m^2_3/2)$.

(3) In order to get the inequality $O((m^2_T)_k) > O((m^2_D)_k)$, the following conditions must be satisfied simultaneously,

$$\langle F_T \rangle, \langle F_m \rangle \ll O(m^3_3/M_I), \quad \langle F_S \rangle = O\left(\frac{m^3_3 M}{\langle K_S^2 \rangle^{1/2}}\right)$$

$$\frac{M^2 \langle K_S^2 \rangle^{1/2}}{\langle K_S^2 \rangle^{1/2}} < O(1), \quad \frac{\epsilon_p}{\langle K_S^2 \rangle} < O(1), \quad (p = 2, 3) \quad (30)$$

unless an accidental cancellation among terms in $\langle \hat{D}^A \rangle$ happens. To fulfill the condition $\langle F_{T,m} \rangle \ll O(m^3_3/M_I)$, a cancellation among various terms including $\langle K_I \rangle$ and $\langle M^2 W_I / W \rangle$ is required. Note that the magnitudes of $\langle K_T \rangle$ and $\langle K_m \rangle$ are estimated as $O(M)$ and $O(M_I)$, respectively.

The gauge kinetic function is given by

$$f_{\alpha \beta} = k_{\alpha} \frac{S_M}{\langle K_S^2 \rangle} \delta_{\alpha \beta} + \epsilon_{\alpha} \frac{T}{M} \delta_{\alpha \beta} + f_{\alpha \beta}^{(m)}(\phi^\lambda) \quad (31)$$

where $k_{\alpha}$’s are Kac-Moody levels and $\epsilon_{\alpha}$ is a model-dependent parameter [29].

The gauge coupling constants $g_{\alpha}$’s are related to the real part of gauge kinetic functions such that $g_{\alpha}^2 = \langle Re f_{\alpha \alpha} \rangle$. The magnitudes of gaugino masses and $A$-parameters in MSSM particles are estimated using the formulae

$$M_a = \langle F_I \rangle \langle h_{aI} \rangle, \quad (32)$$
Table 2: The magnitudes of $\langle h_{aI} \rangle$ and $\langle a_{kl'I} \rangle$

| I | $\langle h_{aI} \rangle$ | $\langle a_{kl'I} \rangle$ |
|---|----------------|----------------|
| $S$ | $O(1/M)$ | $Max(O(\langle K_S \rangle/M^2), O(\epsilon_p/M))$ |
| $T$ | $O(\epsilon_\alpha/M)$ | $O(1/M)$ |
| $m$ | $O(M_1/M^2)$ | $O(M_1/M^2)$ |

$\langle h_{aI} \rangle \equiv \langle Ref_a \rangle^{-1} \langle f_{a,I} \rangle$ \hspace{1cm} (33)

$A_{kl'} = (F^I)\langle a_{kl'I} \rangle$, \hspace{1cm} (34)

$\langle a_{kl'I} \rangle \equiv \langle f_{kl',I} \rangle + \frac{\langle K_I \rangle}{M^2} \langle f_{kl'} \rangle - \langle K_{kl}'' \rangle \langle (K^{-1})_I' \rangle \langle f_{kl'} \rangle$. \hspace{1cm} (35)

The result is given in Table 2. Here we assume that $g^{-2} = O(1)$.

In case that SUSY is broken by the mixture of $S$, $T$ and matter $F$-components such that $\langle (K_S^{1/2} F_S) \rangle$, $\langle F_T \rangle$, $\langle F_m \rangle = O(m_{3/2}/M)$, we get the following relations among soft SUSY breaking parameters

$$(m^2)_k \geq (m^{2}_{D})_k = O(m^{2}_{3/2} M^2) \geq (A_{kl'})^2 = O(m^2_{3/2}),$$ \hspace{1cm} (36)

$$(M_a)^2 = O(m^2_{3/2}) \cdot Max(O(\langle K_S \rangle^{-1}), O(\epsilon_\alpha^2), O(M^2/M^2)).$$ \hspace{1cm} (37)

Finally we discuss the three special cases of SUSY breaking scenario.

1. In the dilaton dominant SUSY breaking scenario

$$\langle (K_S^{1/2} F_S) \rangle = O(m_{3/2}/M) \gg \langle F_T \rangle, \langle F_m \rangle,$$ \hspace{1cm} (38)

the magnitudes of soft SUSY breaking parameters are estimated as

$$(m^2)_k = O(m^{2}_{3/2}) \cdot Max(O(1), O(M^2/\langle K_S \rangle), O(\epsilon_p/\langle K_S \rangle)),$$

$$M_a = O\left(\frac{m_{3/2}}{\langle K_S \rangle^{1/2}}\right), \quad A_{kl'} = O(m_{3/2}) \cdot Max(O(\langle K_S \rangle/M), O(\epsilon_p)).$$

Hence we have a relation such that $O((m^2)_k) \geq O((A_{kl'})^2)$. 

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As discussed in Ref. [14], gauginos can be heavier than scalar fields if \( \langle K_S \rangle \) is small enough and \( O(M^2 \langle K_S^2 \rangle) < O(\langle K_S \rangle) \). In this case, dangerous flavor changing neutral current (FCNC) effects from squark mass non-degeneracy are avoided because the radiative correction due to gauginos dominates in scalar masses at the weak scale. On the other hand, in Ref. [15], it is shown that gauginos are much lighter than scalar fields from the requirement of the condition of vanishing vacuum energy in the SUGRA version of model proposed in Ref. [27]. In appendix, we discuss the relations among the magnitudes of \( \langle K_S \rangle \), \( \langle K_S^2 \rangle \) and \( \langle K_S^2 \rangle \) under some assumptions.

2. In the moduli dominant SUSY breaking scenario

\[
\langle F_T \rangle = O(m_{3/2}M) \gg \langle (K_S^2)^{1/2} F_S \rangle, \langle F_m \rangle,
\]

the magnitudes of soft SUSY breaking parameters are estimated as

\[
(m^2)_k = O(m_{3/2}^2) \cdot \text{Max}(O(1), O(\frac{\epsilon_1 M}{\langle K_S \rangle})),
M_a = O(\epsilon_a m_{3/2}), \quad A_{klm} = O(m_{3/2}).
\]

Hence we have a relation such that \( O((m^2)_k) \geq O((A_{klm})^2) \geq O((M_a)^2). \) The magnitude of \( \mu_{TT} \) is estimated as \( \mu_{TT} = O(m_{3/2}) \).

3. In the matter dominant SUSY breaking scenario

\[
\langle F_m \rangle = O(m_{3/2}M) \gg \langle (K_S^2)^{1/2} F_S \rangle, \langle F_T \rangle,
\]

the magnitudes of soft SUSY breaking parameters are estimated as

\[
(m^2)_k = O(m_{3/2}^2 \frac{M^2}{M^2_f}), \quad M_a, A_{klm} = O(m_{3/2} \frac{M_f}{M}).
\]

The relation \( (m^2)_k \gg O((M_a)^2) = O((A_{klm})^2) \) is derived when \( M \gg M_f \). The magnitude of \( \mu_{mn} \) is estimated as \( \mu_{mn} = O(m_{3/2}M/M_f) \). This value is consistent with that in Ref. [13].
4 Conclusions

We have studied the magnitudes of soft SUSY breaking parameters in het-
erotic string models with $G_{SM} \times U(1)_A$, which originates from the breakdown
of $E_8 \times E'_8$, and derive model-independent predictions for them without spec-
ifying SUSY breaking mechanism and the dilaton VEV fixing mechanism.
In particular, we have made a comparison of magnitudes between $D$-term
contribution to scalar masses and $F$-term ones and a comparison of mag-
nitudes among scalar masses, gaugino masses and $A$-parameters under the
condition that $O(m_{3/2}^3) \ll \langle K_S \rangle \leq O(q_m^3 M/\delta_{GS}^4),\,(M_F^2)^A/g_A^2 = O(q_m^{12} M_I^2)$
and $\langle V \rangle \leq O(m_{3/2}^2 M^2)$. The order of magnitude of $D$-term contribution of
$U(1)_A$ to scalar masses is comparable to or bigger than that of $F$-term con-
tribution $\langle F_I \rangle \langle F_J \rangle \langle R_{IkJ} \rangle$ except for the universal part $(m_{3/2}^2 + \langle V_F \rangle/M^2)(K_I^I)$. If the magnitude of $F$-term condensation of matter fields $\langle F_m \rangle$ is bigger than
$O(m_{3/2} M_I)$, the magnitude of $D$-term contribution $(m_D^2)^I_k$ is bigger than
$O(m_{3/2}^2)$. In general, it is difficult to realize the inequality $O((m_D^2)^I_k) < O((m_F^2)^I_k)$ unless conditions such as Eq. (30) are fulfilled. We have also dis-
cussed relations among soft SUSY breaking parameters in three special sce-
narios on SUSY breaking, i.e., dilaton dominant SUSY breaking scenario,
moduli dominant SUSY breaking scenario and matter dominant SUSY breaking
scenario.

The $D$-term contribution to scalar masses with different broken charges as
well as the $F$-term contribution from the difference among modular weights
can destroy universality among scalar masses. The non-degeneracy among
squark masses of first and second families endangers the discussion of the
suppression of FCNC process. On the other hand, the difference among bro-
ken charges is crucial for the scenario of fermion mass hierarchy generation
\cite{28}. It seems to be difficult to make two discussions compatible. There are
several way outs. The first one is to construct a model that the fermion
mass hierarchy is generated due to non-anomalous $U(1)$ symmetries. In the
model, $D$-term contributions of non-anomalous $U(1)$ symmetries vanish in
the dilaton dominant SUSY breaking case and it is supposed that anoma-
lies from contributions of the MSSM matter fields are canceled out by an
addition of extra matter fields. The second one is to use “stringy” symme-
tries for fermion mass generation in the situation with degenerate soft scalar
masses \cite{29}. The third one is to use a parameter region that the radiative
correction due to gauginos, which is flavor independent, dominates in scalar
masses at the weak scale. It can be realized when $\langle K_S^S \rangle$ is small enough and $O(M^2\langle K_S^S \rangle) < O(\langle K_S \rangle)$.

Finally we give a comment on moduli problem [30]. If the masses of dilaton or moduli fields are of order of the weak scale, the standard nucleosynthesis should be modified because of a huge amount of entropy production. The dilaton field does not cause dangerous contributions in the case with $\langle (K_S^S)^{1/2} F_S \rangle = O(m_{3/2}M)$ if the magnitude of $\langle K_S^S \rangle$ is small enough.\footnote{This possibility has been pointed out in the last reference in [10].}

Because the magnitudes of $(m_F^2)_S$ is given by $O(m_{3/2}^2 / \langle K_S^S \rangle^2)$.

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**A On derivatives of Kähler potential related to dilaton**

We discuss the relations among $\langle K_S \rangle$, $\langle K_S^S \rangle$ and $\langle K_S^S \rangle$ using SUSY breaking conditions and the stationary conditions of scalar potential. We list our assumptions first.

1. SUSY is broken by the mixture of $S$, $T$ and matter $F$-components such that $\langle (K_S^S)^{1/2} F_S \rangle$, $\langle F_T \rangle$, $\langle F_m \rangle = O(m_{3/2}M)$.

2. The magnitude of $\langle K_S \rangle$ is much bigger than $O(m_{3/2})$ and it is comparable to or smaller than $O(q_A^A M / \delta_{GS}^A)$. The latter is equivalent to the condition that the magnitude of $M_I \equiv |\langle \phi^m \rangle|$ is at most $O(M)$.

3. The magnitude of $\langle K_S^S \rangle^{1/2}$ is much bigger than those of $\langle K_S^T \rangle$ and $\langle K_S^m \rangle$, and it is comparable to or smaller than $O(q_m^A M_I / \delta_{GS}^A M)$. The latter is equivalent to the condition that the magnitude of $(M_V^2)^A / g_A^2$ is $O(q_m^A M_I^2)$.  

4. The magnitude of $\delta_{GS}^A / q_m^A$ is $O(1/10) \sim O(1/100)$. 

5. No cancellation happens among terms in $\langle K_S \rangle$ and $M^2\langle W_S \rangle /\langle W \rangle$. On a later discussion, we relax this assumption.

Under these assumptions, the following relation is derived

$$\langle K_S^S \rangle^{1/2} = O(\frac{\langle G_S \rangle}{M}) = O(\frac{\langle K_S \rangle}{M} + M \frac{\langle W_S \rangle}{\langle W \rangle})$$

(41)

by the use of the definition (1). If $\langle K_S \rangle$ is bigger than $M^2\langle W_S \rangle /\langle W \rangle$, we find that $\langle K_S^S \rangle^{1/2} = O(\langle K_S \rangle/M) \leq O(q_m^A/\delta_{GS}^A)$.

Further we can get the following relation among $\langle K_S \rangle$, $\langle K_S^S \rangle$ and $\langle K_{SS}^S \rangle$ from the stationary conditions (12) and (16),

$$\frac{\langle K_{SS}^S \rangle}{\langle K_S^S \rangle} = \text{Max}(O(\frac{\langle K_S^S \rangle}{\langle K_S \rangle}), O(\frac{\langle K_S^S \rangle^{1/2}}{M})).$$

(42)

Let us consider a typical case with non-perturbative superpotential derived from SUSY breaking scenario by gaugino condensations. The non-perturbative superpotential $W_{non}$ is, generally, given by

$$W_{non} = \sum_i a_i(\phi^A, T)exp(-\frac{b_iS}{\delta_{GS}^AM})$$

(43)

where $a_i$'s are some functions of $\phi^A$ and $T$, and $b_i$'s are model-dependent parameters of $O(q_m^A)$. Using the second assumption, Eqs.(11) and (13), we get the relation $\langle K_{SS}^S \rangle^{1/2} = M \langle W_S \rangle /\langle W \rangle = O(q_m^A/\delta_{GS}^A)$ if $O(\langle W_{non} \rangle) = O(\langle W \rangle)$. This relation means $M_I = M$ from the third assumption, and it leads to the relation such that $M\langle K_{SS}^S \rangle /\langle K_S^S \rangle = O(q_m^A/\delta_{GS}^A)$. We obtain the relation $M\langle K_{SS}^S \rangle /\langle K_S^S \rangle = O(q_m^A/\delta_{GS}^A)$ using Eq.(12). Finally we discuss the case where the cancellation happens among terms in $\langle K_S \rangle$ and $M^2\langle W_S \rangle /\langle W \rangle$. Then the magnitude of $\langle K_{SS}^S \rangle^{1/2}$ and $M\langle K_{SS}^S \rangle /\langle K_S^S \rangle$ can be smaller than $O(q_m^A/\delta_{GS}^A)$. Hence the magnitude of gaugino masses can be bigger than those of scalar masses in case where $\langle K_S^S \rangle$ is small enough and $M^2\langle K_{SS}^S \rangle /\langle K_S^S \rangle < O(1)$.

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