THEORY, PHENOMENOLOGY, AND PROSPECTS FOR DETECTION OF SUPERSYMMETRIC DARK MATTER

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Abstract

One of the great attractions of minimal super-unified supersymmetric models is the prediction of a massive, stable, weakly interacting particle (the lightest supersymmetric partner, LSP) which can have the right relic abundance to be a cold dark matter candidate. In this paper we investigate the identity, mass, and properties of the LSP after requiring gauge coupling unification, proper electroweak symmetry breaking, and numerous phenomenological constraints. We then discuss the prospects for detecting the LSP. The experiments which we investigate are (1) space annihilations into positrons, anti-protons, and gamma rays, (2) large underground arrays to detect upward going muons arising from LSP capture and annihilation in the sun and earth, (3) elastic collisions on matter in a table top apparatus, and (4) production of LSPs or decays into LSPs at high energy colliders. Our conclusions are that space annihilation experiments and large underground detectors are of limited help in initially detecting the LSP although perhaps they could provide confirmation of a signal seen in other experiments, while table top detectors have considerable discover potential. Colliders such as LEP II, an upgraded Fermilab, and LHC, might be the best dark matter detectors of all. This paper improves on most previous analyses in the literature by (a) only considering parameters not already excluded by several physics constraints listed above, (b) presenting results that are independent of (usually untenable) parameter choices, (c) comparing opportunities to study the same cold dark matter, and (d) including minor technical improvements.

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1 Introduction

Recently there has been a great deal of effort investigating minimal supergravity models with
gauge coupling unification, and radiatively generated electroweak symmetry breaking (i.e., models
in which the Higgs mechanism is derived rather than being imposed) [1, 2, 3, 4, 5, 6, 7, 8]. It is
quite remarkable that these minimal super-unified models have survived the numerous fusillades by
theorists and experimentalists trying to constrain or even rule out supersymmetry. We are then left
with the inescapable phenomenological conclusion that minimal super-unified models are at least
as good as the Standard Model in describing the world around us. “Super-unified” means that the
three gauginos are given a common mass at the unification scale, and similarly the sfermions and
Higgses are all given a common mass there.

In addition to passing all current phenomenological tests, these supersymmetric models have
the potential to answer such vexing questions as, where does all the dark matter come from? It is
remarkable that when all known phenomenological constraints are applied to the minimal super-
unified solutions, those which remain often have an LSP (Lightest Supersymmetric Particle) which
does not overclose the universe and which generally has a significant enough relic density to be
cosmologically interesting – that is, provide enough mass density to the universe to be a viable cold
dark matter candidate.

In this paper we primarily investigate the LSP in the context of this constrained minimal
supersymmetric standard model [5] (CMSSM), and determine what predictions can be made for its
detection in many different experiments. The experiments we consider here are neutrino yield from
capture and annihilation of the LSP in the sun and earth, direct detection methods with various
nuclei, collider signatures, and space annihilation signatures such as anomalous positron fraction,
$\bar{p}$, and $\gamma$ ray production.

Although there is so far no clear phenomenological necessity to investigate supersymmetric
models which drastically differ from the minimal super-unified one, we nevertheless do consider
implications (such as non-universal soft masses) when the issue becomes interesting. But our main
goal is to make a definitive statement on the properties and detectability of the LSP within the
minimal constrained super-unified model.

2 The particle physics model

Before continuing further, we define what we mean by CMSSM (See Ref. 5 for a complete descrip-
tion). Briefly, CMSSM is the parameter space defined by minimal particle content (i.e. the Standard
Model spectrum with two Higgs doublets plus superpartners), the gauge group $SU(3) \times SU(2) \times U(1)$
below the scale of gauge coupling unification, common gaugino masses, common scalar masses, common
trilinear and bilinear soft masses at the unification scale, correct electroweak symmetry breaking
and conserved R–parity. Furthermore, a CMSSM solution must satisfy all known experimental
constraints including the requirement that relic particles not overclose the universe ($\Omega_{\text{TOT}}h^2 < 1$).
We do not impose additional constraints such as proton decay limits since that would require more
specific knowledge of the exact high scale theory than we believe is available today. In this regard,
our analysis is more general than previous ones 6, 7.
R–parity is extremely important to the analysis in this paper. R–parity is a discrete $Z_2$ symmetry which, when conserved, forbids baryon number and lepton number violating interactions in the superpotential, and dictates an absolutely stable LSP. But, the most general renormalizable and gauge invariant supersymmetric Lagrangian manifestly breaks R–parity and leads to the prediction of unacceptably rapid proton decay. Therefore, R–parity conservation was hypothesized \cite{10, 11, 12, 13} and tentatively accepted as the theoretical reason for a stable proton. From the perspective of supersymmetrizing the standard model Lagrangian one could argue that R–parity should not be viewed as an \textit{ad hoc} symmetry since the standard model Lagrangian has no relevant B or L violating interactions. Despite this, few were pleased with “old R–parity” for several reasons: (1) No one knew where it came from, (2) it was a global symmetry and therefore quantum gravity effects might possibly preclude its conservation, and (3) it was not a unique option – just forbidding B or L violating terms is sufficient to save the proton.

Progress has been made over the last several years and a “new R–parity” can now be viewed as more attractive. Krauss and Wilczek \cite{14} originally pointed out that discrete symmetries could result from spontaneous symmetry breaking of a continuous gauge symmetry. With a gauge symmetry producing the low energy discrete symmetry, all ruminations about the ills of global symmetries become irrelevant. Furthermore, with this possibility at our disposal we begin to understand that it might not be difficult to motivate the very existence of a discrete symmetry like R–parity from spontaneously broken gauge symmetries at the GUT scale. Indeed, several authors \cite{15, 16} have shown that R–parity can be derived from a continuous $B–L$ gauge symmetry, and that a large class of models which contain $U(1)_{B–L}$ at the high scale automatically conserves R–parity in the low energy phase.

The “new R–parity” which is much more highly motivated and understood than the “old R–parity” comes with a price: gauged $B–L$ at the high scale. There are several ways to envision a $B–L$ gauged symmetry. We could assume that it is an admixture of $U(1)$’s left over from compactification in a string theory along with the Standard Model gauge group. Or we can assume that $B–L$ is a subgroup of a simple grand unified group (which may or may not have come from the string), in which case we acknowledge $SO(10)$ as the leading candidate. Certainly there are other possibilities, but our viewpoint at the moment is that gauge coupling unification and gauged R–parity are mutually compatible and perhaps even imply each other. We expect R–parity conservation to eventually arise from the structure of the Higgs potential of the full theory, but we do not consider its origin further in this paper.

Another important aspect of the analysis is radiative electroweak symmetry breaking \cite{17, 18, 19, 20}, wherein the Higgs mechanism, and therefore symmetry breaking, can be \textit{derived} through the renormalization of the Higgs masses from the high scale, $m_{H}^2(Q^2 = M^2_{GUT}) > 0$, to the low scale, $m_{H}^2(Q^2 = M^2_{Z}) < 0$. With a large Yukawa coupling graciously provided by the top quark, such an aesthetic explanation of electroweak symmetry breaking is manifest in the minimal super-unified model. That the Higgs mechanism emerges from such a theory is one of its most attractive features. After GUT scale parameters are chosen and after all renormalization group equations are solved, we must carefully analyze the Higgs potential to see if it truly admits EWSB and we must make sure that the minimum of the potential will numerically reproduce the Z mass (i.e., $v_1^2 + v_2^2 = v^2 = 4M^2_Z/(g_1^2 + g_2^2)$). The relationship among the low energy Lagrangian parameters
Figure 1: Scatter plot of unphysical solutions in the $M_2-\mu(m_Z)$ plane with $m_0 = 300$ GeV, $\tan \beta = 3$, $A_0 = 0$, and $m_t = 170$ GeV held fixed. These solutions do not satisfy the constraints of electroweak symmetry breaking and yield an incorrect Z-boson mass. The solid line, however, is physical in that the Z-boson mass is fixed to its experimental value.

The required to recover the Z mass is

$$\frac{1}{2} M_Z^2 + \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} + L(\tan \beta, m_t, m_{\tilde{t}}, \ldots)$$

(1)

where $m_{H_u}$ and $m_{H_d}$ are the Higgs soft masses and $L$ contains all effects of one loop corrections to the Higgs potential.

We want to emphasize that proper electroweak symmetry breaking (EWSB) is not a matter of taste or aesthetics; it is a requirement imposed by nature. Therefore, all analyses which do not require Eq. 1 be satisfied must necessarily suffer from unphysical model solutions and suspect conclusions; they cannot correctly describe our world.

To illustrate how important it is to require that the Higgs potential admits electroweak symmetry breaking, we have constructed a set of solutions which have EWSB imposed, and others which do not. To illustrate, we have chosen $m_0 = 300$ GeV, $\tan \beta = 3$, $A_0 = 0$, $m_t = 170$ GeV, and $\text{sgn}(\mu) = +$. (Recall that $m_0$ is the universal high scale mass for all scalars, $\tan \beta$ is the ratio of the vacuum expectation values of the up-Higgs to the down-Higgs, $A_0$ is the universal tri-linear soft supersymmetry breaking term at the high scale, and $\mu$ is the Higgs mixing coefficient in the superpotential) The actual numbers for these parameters are not important since the results that we present below are general for any set of input parameters. We then varied $M_2$ (or equivalently, $m_{1/2}$) and determined $\mu$ from the requirement of EWSB. The solid line in Fig. 1 shows where these solutions, which do have correct electroweak symmetry breaking, lie in the $M_2-\mu$ plane. We then made a second set of solutions by choosing $M_2$ and $\mu$ randomly with values up to 1 TeV. Unlike the first set of solutions with EWSB imposed, the values of $M_2$ and $\mu$ in the second set are not correlated with each other (no EWSB) and are represented by the dots in Fig. 1.

We should emphasize again that the dots in Fig. 1 represent unphysical supersymmetric models; only the line corresponds to a theory with EWSB and the correct value for $M_2$. Although the solid line in Fig. 1 will move around with different choices of $m_0$, $\tan \beta$, etc, the strong correlation between $M_2$ and $\mu$ will always exist, and needs to be calculated. Unfortunately, one often finds studies of supersymmetry phenomenology which effectively map the whole unphysical $M_2-\mu$ plane (or some optimistic part) onto some particular observable (e.g., neutrino flux from neutralino capture and annihilation in the sun). We illustrate with one example why such an approach can lead to orders of magnitude errors.

The example that we give is the spin dependent elastic cross ($\sigma_{SD}$) section with LSPs impinging on $^{73}$Ge. Fig. 2 is a complete mapping of all the solutions of Fig. 1 in the $M_2-\mu$ plane onto the $\sigma_{SD}-m_\chi$ plane. Notice that the line (physical solutions) is at least three orders of magnitude below the highest cross sections of the dots (unphysical solutions), and that well over half of the dots are

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1 For definitions of SUSY parameters and conventions used in this paper see Ref. [5]
Figure 2: A direct mapping of the solutions in Fig. 1, showing spin-dependent cross sections on $^{73}_{\text{Ge}}$ vs. LSP mass. The dots are unphysical solutions which do not have proper electroweak symmetry breaking, and the solid line is physical.

above the line. Since the counting rate for a nuclear detector scales linearly with $\sigma_{SD}$ one could be three orders of magnitude too optimistic about the prospects of detecting LSP cold dark matter if the unphysical solutions are used, as has happened.

As discussed above, requiring the Higgs potential to admit EWSB is an absolute requirement on any physically viable supersymmetric theory. Furthermore, experimental predictions using supersymmetric solutions which do not impose this requirement will often lead to grossly optimistic conclusions.

3 What is the LSP?

Besides such exotics as axions and axinos, minimal supergravity with conserved R–parity has several stable dark matter possibilities. If supersymmetry breaking is communicated to the visible sector through gravitational interactions then the gravitino mass ($m_{3/2}$) is of order the superpartner masses or greater. Since we want the superpartner masses to be of order the weak scale to solve the naturalness problem then the gravitino mass also must be that heavy. Furthermore, decays of the gravitino with a weak scale mass could greatly disrupt the successful description of big bang nucleosynthesis [21]. To escape this problem the gravitino is usually assumed to be heavy enough not to affect this analysis.

Ruling out all charged and colored objects [22], we are left with two possibilities for the LSP: the sneutrino and lightest neutralino. The sneutrino as LSP was first suggested by the authors of Ref. [23] and was recently considered again in finer detail by the authors of Ref. [24]. One of the conclusions of Ref. [24] was that sneutrinos with cosmologically interesting relic densities were already ruled out by experiment except for possibly a small region of parameter space near $m_{\tilde{\nu}} \simeq 600$ GeV.

We would like to point out, however, that within the minimal super-unified approach the sneutrino cannot be the LSP and have a mass above $M_W$. To demonstrate this we first write down the neutralino mass matrix [25] in the $\{\tilde{B}, \tilde{W}^3, \tilde{H}_d, \tilde{H}_u\}$ basis,

$$Y = \begin{pmatrix}
M_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\
0 & M_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\
-m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & \mu \\
m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & \mu & 0
\end{pmatrix}, \quad (2)$$

and note that an upper bound can be placed on the lightest neutralino from the diagonal elements of the neutralino mass matrix squared:

$$m_{\chi}^2 \leq \min\{(YY^\dagger)_{11}, (YY^\dagger)_{22}, (YY^\dagger)_{33}, (YY^\dagger)_{44}\}. \quad (3)$$
Choosing just one of these,
\[ m_\chi^2 \leq (Y Y^\dagger)_{11} = M_1^2 + M_2^2 \sin^2 \theta_W \]  
we obtain a working upper bound on the LSP mass:
\[ m_{\chi,\text{max}}^2 = M_1^2 + M_2^2 \sin^2 \theta_W = \eta m_{1/2}^2 + M_2^2 \sin^2 \theta_W \]  
where
\[ \eta = \left( \frac{5 \alpha_Y}{3 \alpha_{\text{GUT}}} \right)^2 \approx 0.18. \]  
The renormalized low scale sneutrino mass is
\[ m_{\tilde{\nu}}^2 = m_0^2 + b_\tilde{\nu} m_{1/2}^2 + \frac{1}{2} M_Z^2 \cos 2\beta \]  
where \( b_\tilde{\nu} \approx 0.5 \) \(^\text{[3]}\). Therefore,
\[ m_{\tilde{\nu},\text{min}}^2 \approx \frac{1}{2} (m_{1/2}^2 - m_Z^2). \]  
Since \( m_{\tilde{\nu}}^2 \) scales faster with \( m_{1/2}^2 \) than does \( m_\chi^2 \), there must be a maximum value of \( m_{\tilde{\nu}} \) for which it is still possible for the sneutrino to be the LSP: \( m_{\chi,\text{max}}^2 = m_{\tilde{\nu},\text{min}}^2 \). This value is a little less than \( m_W \). Therefore, a sneutrino with mass greater than \( m_W \) cannot be the LSP, a result which is independent of all parameters (\( \tan \beta, \mu, m_0, \ldots \)). Within the CMSSM we find no solutions with the sneutrino as LSP. That is, imposing all the physics constraints always leads to the LSP being the lightest neutralino.

Somewhat higher mass sneutrinos as LSP could be obtained by relaxing universal gaugino mass conditions, but one would be hard pressed to get \( m_{\tilde{\nu}} \approx 600 \) GeV. It is conceivable that such massive sneutrino LSPs are possible in unusual extended gauge group theories, but we know of no examples. We merely state that within the present framework we must reject the sneutrino as a cosmologically interesting LSP. If it so happens that the sneutrino is the LSP with mass less than \( m_W \) then it will be detected at LEP II. Sneutrinos in this mass range do not give substantial \( \Omega h^2 \), so they are not interesting cold dark matter candidates.

We are now left with the neutralino as the sole surviving dark matter candidate. Whereas a sneutrino with mass of \( \mathcal{O}(m_W) \) yields negligible and uninteresting relic abundance, the neutralino can yield just the right amount of dark matter depending on its composition \(^\text{[26, 27, 22]}\). We express the LSP composition as a superposition of the four supersymmetric weak eigenstates,
\[ \chi = Z_{11} \tilde{B} + Z_{12} \tilde{W}^3 + Z_{13} \tilde{H}_d + Z_{14} \tilde{H}_u \]  
A compelling argument for the minimal super-unified approach to supersymmetry model building is that it automatically outputs a neutralino as the LSP with the correct composition to be a cold dark matter candidate. Specifically, gaugino mass renormalization from the high scale and the requirement of radiative electroweak symmetry breaking invariably produce a lightest neutralino which is Bino-like (see Fig. \(^\text{[8]}\) and caption). This preferred requirement on the neutralino wave function was first noticed outside the context of supergravity model building \(^\text{[28]}\).
Figure 3: We show how the renormalization group running of the gaugino mass terms and $\mu$ automatically generate a bino-like LSP. The lower solid lines are $M_2$ (upper line) and $M_1$ (lower line). Their initial values are input at the high scale with the universal gaugino mass assumption. The dotted lines are the values of $m_{H_u}^2$ (lower line) and $m_{H_d}^2$ (upper line). (Note that $m_{H_u}^2$ runs negative at low scale as required by radiative electroweak symmetry breaking.) Their values are input at the high scale with the common scalar mass assumption. The value of $\mu$, here denoted by the dot-dashed line, is determined at the low scale by the radiative electroweak symmetry breaking condition of Eq. 1, and the values at other scales are calculated from the renormalization group equations. The Bino ($\tilde{B}$) mass $M_1$ is the lightest mass parameter in the neutralino mass matrix and so the LSP is a Bino-like neutralino.

The existence of this stable weakly interacting massive particle (WIMP) in supersymmetry does not depend on astrophysics. It is a property of a class of supersymmetry theories which were motivated by reasons unrelated to what one might see in a telescope. Nevertheless, we do not believe that it is an accident that supersymmetry predicts a WIMP with generically large relic abundance, AND that the astrophysics data probably does imply the existence of a WIMP with large relic abundance $^{29, 30, 31, 32, 33, 34}$. This position is further strengthened by the paucity of visible matter compared to inflation’s preferred value of $\Omega_{TOT} = 1$. We do not wish to review further the arguments for the existence of dark matter since our purpose is to study what SUSY predicts regardless of the astrophysics and cosmology. When needed, however, we do assume that the local density of the dark matter in the galactic halo is $0.2 \text{ GeV/cm}^3 < \rho_{loc} < 0.4 \text{ GeV/cm}^3$ $^{35}$. Unless otherwise stated, we always use $\rho_{loc} = 0.4 \text{ GeV/cm}^3$ in our numerical work. Astrophysics and cosmology only enter our analysis in the calculation of $\Omega_\chi h^2$ and in considering the detection of LSPs, not in determining the LSP properties.

To obtain the relic density of the LSP, we calculate using well known analytic techniques $^{36, 37, 38, 39, 40, 41, 42}$ and compare with or use many formulas in the literature $^{43, 44, 1, 15, 49, 47}$. For each solution we calculate the thermal average by considering all possible final states that the neutralino can annihilate into, $f \bar{f}$, $W^+W^-$, $W^\pm H^\mp$, $ZZ$, $Zh$, $ZH$, $ZA$, $hA$, $HA$, $hH$, $hh$, $HH$, $AA$, $H^\pm H^\mp$. Furthermore, for each solution we calculate the freeze-out temperature, photon reheating factor, and the number of degrees of freedom at freeze-out in our attempt to perform a precise relic density calculation. We realize that the analytic techniques used to solve the Boltzmann equation, along with cosmological uncertainties, can lead to a predicted relic abundance which is off by 20% or more $^{48, 49, 50}$. One such difficulty, first pointed out in Ref. $^{51}$ and quantitatively studied in Ref. $^{50, 10, 52}$, is the possibility of neutralinos annihilating through a resonance. The Taylor series expansion of the annihilation cross-section becomes untrustworthy and special techniques must be used to obtain the correct relic density. It is for these reasons that we use the relic density calculation only as a rejection criteria and not as a measure of the local density as some authors have done. To be precise, we reject all solutions which are not in the range expressed by $0.05 < \Omega_\chi h^2 < 1.0$, and then assume that $\rho_{loc} \approx 0.4 \text{ GeV/cm}^3$ independent of the calculated $\Omega_\chi h^2$. It turns out that our results do not depend sensitively on the above-quoted lower and upper values of $\Omega h^2$ in our definition for cosmologically interesting solutions as long as these values are
Figure 4: Histogram of $\Omega_\chi h^2$ for all solutions with $m_t = 170$ GeV. Note the peak centered at $\Omega_\chi h^2 \approx 0.1$. Given the measure on our input parameter space and $h = 0.35$, the constrained solutions seem to prefer $\Omega_{CDM} \approx 0.8$ in the “cosmologically interesting” region of $0.05 < \Omega_\chi h^2 < 1$.

Figure 5: A spectrum scatter plot of the constrained minimal supersymmetry parameter space. Each dot represents a mass of a particle particle labelled directly below and belongs to a supersymmetric solution which satisfies all theoretical and experimental requirements described in the text. The horizontal banding is due to numerical sampling and is not of significance. The LSP is the first of the four vertical bands collectively labeled $\chi^0$.

reasonable. We have compared our relic density calculation with others and have found good agreement. Figure 4 is a histogram of $\Omega_\chi h^2$ for all solutions with $m_t = 170$ GeV. We keep the solutions with $0.05 < \Omega_\chi h^2 < 1$, leaving us with many solutions to analyze.

Before describing different techniques to detect the LSP we want to emphasize one more time that each solution which we consider in the subsequent analysis is “fully constrained”. That is, each solution is consistent with gauge coupling unification, electroweak symmetry breaking, invisible width of the Z, $b \rightarrow s\gamma$, all direct production collider constraints, etc. Many past analysis have ignored important phenomenological constraints in their models. For example, Bottino et al. [53, 54] consider a large region of parameter space which is incompatible with the Z width constraint and the $b \rightarrow s\gamma$ constraint. (Note that ignoring the Z-width and $b \rightarrow s\gamma$ constraints and allowing such low generic Higgs and sfermion masses lead to substantially more optimistic conclusions for LSP detectability [55].) Furthermore, they choose to drastically simplify their SUSY parameter space by allowing many particles to obtain masses which violate present experimental limits: $m_h = 50$ GeV, $m_{sfermions} = 1.2 m_\chi$ for all $m_\chi > 45$ GeV, etc. We do not reduce the effective dimensionality of SUSY parameter space in such an arbitrary manner, but rather choose a well motivated theory in which to work. We attempt to cover all of the physically realizable supersymmetric parameter space, and allow only those parameters which are consistent with the above-described theoretical and experimental requirements. In Fig. 5 we show a scatter plot of all masses in the CMSSM. Each point represents a mass of one particle in a supersymmetric solution which satisfies all the above theoretical and experimental constraints. In Fig. 6 we show just one solution out of the thousands contained in Fig. 5 to give the reader a feel for the typical mass correlations. We now consider several different LSP detection schemes using the CMSSM parameter space as shown in Fig. 5 to study LSP detection.

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2We would like to thank M. Drees and M. Nojiri, and also L. Roszkowski, for allowing us to compare results of our relic density calculations with results from their programs.

Figure 6: Mass spectra of a typical constrained minimal supersymmetry solution. This model satisfies all the theoretical and experimental constrains discussed in the text. The LSP is the lightest of the four $\chi^0$ states.
4 LSP Detection through Space Annihilation

While gently floating around in the galactic halo, the LSPs have some finite probability of annihilating with other LSPs into quarks, leptons, gauge bosons or Higgs bosons, which in turn fragment into photons, positrons, and anti-protons in the process. These annihilation products then can find their way into an earth-based or space-based detector, and the measurement of anomalously large fluxes of photons, positrons, or anti-protons could indicate a high density of LSPs in the halo. In this section, we present a quantitative measure for the prospects of finding these space annihilation signals. Note that all cases depend on rather uncertain astrophysics assumptions and background estimates. Before launching into the specifics of each method, we first construct a reference formalism which is applicable for all the methods. Then we will apply the formalism to each detection scheme independently.

First, for annihilations into general two body final states, $\chi\chi \rightarrow AB$, we define the “annihilation strength function” as

$$S_{AB}(\vec{r}) = \frac{\rho^2(\vec{r})}{m^2_{\chi}} \sigma v_{AB},$$

(10)

($\rho(\vec{r})$ is the mass density at position $\vec{r}$ and $v$ is relative velocity) which is interpreted as

$$[S_{AB}(\vec{r})] = \text{# of AB pairs cm}^3\text{s} \text{ at position } \vec{r}.$$ (11)

Since the LSPs are highly non-relativistic in the halo, the center of mass energy is $2m_{\chi}$.

But we are most often interested in a particular annihilation product (say, particle $y$) of the $AB$ pair, and so we must introduce a general “fragmentation yield function” $F_{y/AB}(E, m_{\chi})$ which describes the number and energy distribution of the particle $y$ originating from the decays of the $AB$ annihilation products. The “source strength function” of the $y$ particles is then

$$\frac{dS_y(E, \vec{r})}{dE} = S_{AB}(\vec{r})F_{y/AB}(E, m_{\chi}).$$ (12)

By integrating Eq. (12) over energy one can easily see that

$$\int dEF_{y/AB}(E, m_{\chi}) = \text{# of } y’s \text{ per } 1 \ AB \text{ pair produced at } E_{cm}/2 = m_{\chi}.$$ (13)

To obtain the area flux at a position $\vec{r}_{det}$ (a detector location) we must propagate the particles from the position $\vec{r}$ to $\vec{r}_{det}$. Furthermore, the energies of the produced particles could be dramatically attenuated during its long journey to earth. The effects of this modulation will be described by a general “transport modulation function” $M_y(E, E’).$ In general $M_y$ does depend on position, but we employ the standard approximation and ignore this dependence. Therefore, placing the detector at $\vec{r} = 0$, defining $\theta = 0$ as the direction to the galactic center, and summing over all possible direct annihilation products, we obtain a general formula for the differential flux at the detector:

$$\frac{dA_y(E, \theta)}{dEd\Omega} = \sum_{AB} \frac{1}{4\pi} \int dE' \int d\vec{r} M_y(E, E') F_{y/AB}(E', m_{\chi}) S_{AB}(r, \theta)$$ (14)

8
or
\[
\frac{dA_y(E, \theta)}{dE d\Omega} = \sum_{AB} \frac{1}{4\pi} \int dE' M_y(E, E') F_{y/AB}(E', m_\chi) \frac{(\sigma v)_{AB}}{m_\chi^2} \int_{\text{halo}} dr \rho^2(r, \theta).
\]

The integral over \( r \) is required to sum the volume density production rate at all points in the halo. The \( \theta \) dependence comes about from the coordinate transformation on \( \rho(\vec{r}) \) (the origin is now shifted from the galactic center to the detector and therefore angular invariance of the dark matter distribution is no longer preserved). The units of \( dA_y/dE d\Omega \) are \( \text{cm}^{-2} \text{ sec}^{-1} \text{ GeV}^{-1} \text{ sr}^{-1} \).

4.1 Gamma Rays

Since photons with energies in the GeV range experience very little modulation in the galactic halo, we can approximate \( M_\gamma \) by \( M_\gamma(E, E') = \delta(E - E') \). Putting this into Eq. (15) gives us a working equation to analyze the photon flux from LSP annihilation:

\[
\frac{dA_\gamma(E, \theta)}{dE d\Omega} = \sum_{AB} \frac{1}{4\pi} F_{\gamma/AB}(E, m_\chi) \frac{(\sigma v)_{AB}}{m_\chi^2} \int_{\text{halo}} dr \rho^2(r, \theta).
\]

The integral over \( \rho^2 \) can be represented by \( r_{gc}^2 \rho_{\text{loc}}^2 I(\theta) \) where \( r_{gc} \) is distance to the galactic center, and \( I(\theta) \) is an angular factor \([56, 57, 58]\) which depends on the specifics of the halo distribution and is a maximum at \( \theta = 0 \) (toward the galactic center). Since we are most interested in diffuse gammas from high galactic latitudes, we set \( I(\theta) = 1 \) \([56]\). When the LSPs directly annihilate into photons then \( F_{\gamma/AB} = 2\delta(E - m_\chi) \) and the differential energy flux is simply

\[
\frac{dA_\gamma}{dE d\Omega} = \frac{1}{2\pi} \delta(E - m_\chi) \frac{\rho_{\text{loc}}^2}{m_\chi^2} r_{gc}(\sigma v)_{\gamma\gamma}.
\]

The diffuse photon background is not known for \( E \gtrsim 1 \text{ GeV} \). Using data from the MeV region and extrapolating to higher energies, the photon spectrum is inferred to be \([59]\)

\[
\frac{d\bar{A}_\gamma}{dE d\Omega} = 8 \times 10^{-7} \left( \frac{1 \text{ GeV}}{E} \right)^{2.7} \text{ cm}^{-2} \text{ sec}^{-1} \text{ GeV}^{-1} \text{ sr}^{-1}.
\]

The bar on top of the \( \bar{A}_\gamma \) indicates expected background flux.

In this section we investigate the possibility that neutralinos annihilating in the galactic halo produce copious amounts of photons directly (\( \chi\chi \to \gamma\gamma \)) or indirectly (\( \chi\chi \to AB \to \gamma's + X \)) which clearly stand out from the expected background. First, we compare the expected background flux to monochromatic photon flux from direct neutralino annihilation. A useful observable for this comparison is the ratio of the integrated signal flux to the integrated background flux:

\[
R_\gamma = \frac{\int_{m_\chi - \epsilon}^{\infty} \frac{dA_\gamma}{dE d\Omega}}{\int_{m_\chi - \epsilon}^{\infty} \frac{d\bar{A}_\gamma}{dE d\Omega}} \approx (1.6 \times 10^{10}) \left( \frac{\langle \sigma v \rangle_{\gamma\gamma}}{\text{GeV}^{-2}} \right) \left( \frac{\text{GeV}}{m_\chi} \right)^{0.3}.
\]
Figure 7: Scatter plot of $R_\gamma$ vs. $m_\chi$ for all allowed solutions. For a solution to be detectable $R_\gamma \gtrsim 1$ is required. Near the top mass some solutions could be detected with a high resolution detector, capable of measuring the integrated gamma ray flux at energies very close to and above the LSP mass.

Figure 8: Typical continuous photon spectrum from LSP annihilations. The dotted line is the expected background in the multi-GeV region, and the solid line is the calculated photon spectrum from the decays of all final state annihilation channels. If this background estimate is correct, it is difficult to detect the signal.

A continuous spectrum of photons is also expected from LSP annihilations and we have decayed all the final states of neutralino annihilations, counted the photons, and analyzed their energy spectra to determine whether or not a detectable signal is possible. We use JETSET to determine the photon yield from final state annihilation products. It appears that generally the continuous photon spectrum from LSP annihilation is not enough to clearly stand out from background. Fig. 8 represents a typical case where only a small portion of the signal spectrum smoothly rises above the expected background, not enough to be easily discernible. Not only is the signal low, but the measurement is extremely difficult even with a large signal because electron contamination makes it difficult to identify legitimate diffuse gamma rays. However, we note that a signal does exist, so a clever way to separate signal from background could be fruitful. Many experiments, such as those conducted at the Whipple observatory, only look at high energy gammas from a point source and so partially can solve the problem of electron contamination by employing a highly restrictive solid angle cut. Such techniques to enhance the electron/photon rejection ratio are not permissible when trying to detect high energy diffuse gamma radiation. Therefore, not only is the experiment a very difficult one, but the signal is low for LSP annihilations into photons.

4.2 Anomalous Positron Fraction

Just as photons can be created in LSP annihilations, making their way to earth, so too can positrons. The working equation for the production and propagation of positrons is not as simple as that for photons. Additional complications arise since positrons get “trapped” in the galactic magnetic field and since their energy distributions broaden and soften over time from Thomson scattering and synchrotron radiation.

A containment time $\tau_{e^+}$ is usually introduced to parametrize the amount of time that a positron is trapped in the galactic halo. Since $c\tau_{e^+}$ is estimated to be several orders of magnitude beyond the
Figure 9: Scatter plot of $R_{e^+}^{VV}$ vs. $m_\chi$ for all solutions with $m_t = 170\text{ GeV}$. Only for values of $R_{e^+}^{VV} \gtrsim 1$ would we expect a significant enough bump in the $e^+/(e^- + e^+)$ spectrum to indicate possible LSP annihilations through vector boson pairs. Our LSPs, therefore, are not detectable through this mode.

characteristic length scale of the galactic halo, the positrons in some sense sample the entire halo before finally escaping. This effect will raise the expected flux at the earth and can be approximated by the following substitution

$$\int_{\text{halo}} d\rho (r, \theta) \rightarrow \left( c\tau_{e^+} \right) \bar{\rho}^2$$

where $\bar{\rho}^2$ is the average density over the galactic halo (or more precisely, the containment volume). As an approximation to this we will identify $\bar{\rho}^2$ as the local density $\rho_{\text{loc}}^2$. We have essentially replaced the length scale of the galactic halo with the higher length scale of the containment time. Then using the model of Ref. [70] we can extract $M_{e^+}(E, E')$: \begin{equation}
M_{e^+}(E, E') = \frac{1}{E'^2 \tau_{e^+}} \exp \left[ \frac{90}{\tau_{e^+}} \left( \frac{1}{E'} - \frac{1}{E} \right) \right].
\end{equation}

The differential energy flux is

$$\frac{dA_{e^+}}{dE d\Omega} = \sum_{AB} \frac{1}{4\pi} \int dE' M_{e^+}(E, E') F_{e^+/AB}(E', m_\chi)(\sigma v)_{AB} \frac{\rho_{\text{loc}}^2}{m_\chi^2} (c\tau_{e^+}).$$

Kamionkowski and Turner [71] have suggested that neutralino annihilation into W’s and Z’s could produce an almost monochromatic line spectrum of positrons from the decays of these vector bosons. In order for the signal to be detectable in the $e^+/(e^- + e^+)$ spectrum, the neutralino must be almost completely Higgsino-like in order to give a sufficiently large annihilation cross section. And even then, the effect is quite small and required the authors to multiply their signal by a factor of 10 just for it to show up on their graphs. Unfortunately, such Higgsino-like LSPs do not give cosmologically interesting $\Omega_\chi h^2$ nor do they occur in the CMSSM spectrum, and the signal which we obtain from positrons via vector boson production and decays is even smaller than that obtained by Kamionkowski and Turner. In Fig. 9 we plot $R_{e^+}^{VV}$ vs. $m_\chi$ where $R_{e^+}^{VV}$ is defined to be

$$R_{e^+}^{VV} = \frac{e^+ \ \text{from} \ \chi \chi \rightarrow W^+W^-, ZZ \ \text{at} \ E = E_{\text{peak}}}{e^+ \ \text{total}}.$$

Clearly, positrons from vector boson annihilation final states cannot provide a clear signature of LSP annihilations in the galactic halo.

We can extend the analysis and sum over all final states of the LSP annihilations and then determine the number and energy of final state positrons and electrons. The relevant $F_{e^+/AB}$ are determined by the JETSET Monte Carlo [63]. One final state of particular interest is the top quark which decays approximately 100 percent of the time to $bW^+$ and so produces a positron from the subsequent $W^+$ decay. Unfortunately, even this mode ($\chi \chi \rightarrow t\bar{t}$) will not produce a significant enough positron source to make a large bump in the $e^+/(e^- + e^+)$ spectrum. In Fig. 10 we plot
Figure 10: Our best case scenario is plotted for the case of $m_\chi = 275$ GeV using all positrons (and electrons) from the decays of all annihilation final states (not just vector bosons). Here the result is somewhat encouraging, although the signal to background is still small. However, a signal does exist, so it is worth looking for a better way to reduce background.

For the best case scenario which we find among our solutions. For this solution $m_\chi = 275$ GeV and $\Omega_\chi h^2 = 0.05$. The two humps come from top quarks decaying into positrons, and bottom quarks and tau’s decaying into positrons. Note that if we were to scale the LSP density with the calculated $\Omega_\chi$ we would have obtained a substantially smaller signal. We therefore conclude that positrons from LSP annihilations will be a difficult signal to extract above the background but perhaps not impossible.

4.3 Anti-protons

At low energies the ratio of anti-protons to protons in the cosmic ray spectrum is expected [72] to be extremely low (less than about $10^{-5}$ for $E < 1$ GeV). When the LSP annihilates into quarks and gluons, the stable particles in the final state after hadronization will in general contain protons and anti-protons. If enough anti-protons are created and they hit the detector at low energies then this would be a possible signal of LSP annihilations [73, 74].

Since the proton is trapped in the galactic magnetic field and has a rather large containment time compared to the size of the galaxy, we can use a very similar formalism to that which was used for positrons. Namely, we rewrite the integral over $r$ in Eq. [15] as

$$\int_{\text{halo}} d\tau \rho^2(\vec{r}) \rightarrow (v_\beta \tau_\beta) \rho^2_{\text{loc}}$$

and then the differential energy flux becomes

$$\frac{dA_{\bar{p}}}{dE d\Omega} = \sum_{AB} \frac{1}{4\pi} \int dE' M_{\bar{p}}(E, E') F_{\bar{p}/AB}(E', m_\chi) \rho^2_{\text{loc}} \frac{\sigma v}{m_\chi} (\bar{v}_\beta) \rho_{\text{loc}} (\bar{v}_\beta)$$

We use $\tau_\beta = 5 \times 10^7$ yrs in our numerical calculations, but the reader should be aware that there is at least a factor of ten uncertainty in this number [75].

The fragmentation function $F_{\bar{p}/AB}(E', m_\chi)$ we extract from JETSET. The transport modulation function $M_{\bar{p}}(E, E')$ (here due to solar wind effects) can be extracted from Ref. [76] and is

$$M_{\bar{p}}(E, E') = \delta(E' - f(E)) \theta(p_c - p) + \delta(E' - E - \Delta E) \theta(p - p_c)$$

where

$$f(E) = p_c \ln \frac{p + E}{p_c + E} + E_c + \Delta E.$$  

Furthermore, we have chosen $p_c = 1.015$ GeV and $\Delta E = 0.5$ GeV which corresponds to the minimum solar activity.
Figure 11: Scatter plot of $R_\bar{p}$ vs. $m_\chi$ for all solutions. $R_\bar{p}$ is the ratio of low-energy antiprotons expected from LSP annihilation to those expected from spallation in the energy range of 100 MeV to 200 MeV. Detecting an effect would probably require $R_\bar{p}$ to approach unity. Detecting LSP annihilations through an anomalously large anti-proton flux appears to be quite difficult since the signal $\bar{p}$'s are never more numerous than the expected background $\bar{p}$'s in this energy window. Note that the region below $m_\chi \approx 30$ GeV will be covered at FNAL and LEP (see section 7).

We present the results in Fig. 11 of our anti-proton signal analysis as a scatter plot of $R_\bar{p}$ vs. $m_\chi$ where $R_\bar{p}$ is defined as

$$R_\bar{p} = \frac{\# \text{ signal } \bar{p}'s \text{ with } 100 \text{ MeV} < E < 200 \text{ MeV}}{\# \text{ background } \bar{p}'s \text{ with } 100 \text{ MeV} < E < 200 \text{ MeV}}$$  (28)

Our results are considerably less optimistic than those found in Ref. [77]. However, the reasons are clear. When radiative symmetry breaking is included in the analysis there are correlations between the LSP and squark masses, and the squarks are typically heavier than what is usually optimistically assumed otherwise. Therefore the production of gluons through squark-quark loops, which Ref. [77] found to be so important, become quite small and the number of anti-protons is reduced. We note again that there is a signal, and so as sensitivity to $\bar{p}$'s is increased at low energies, a shelf in the energy spectrum could become visible, indicating halo annihilations.

5 LSP Detection through Earth and Sun Capture

Given enough LSPs and a sufficient interaction strength between them and ordinary matter, LSPs would be captured by the Sun or Earth and would annihilate into neutrinos which could then be detected as upward going muons in underground detectors. Data on upward muons from Earth and Sun from several underground experiments, notably Kamiokande [78] and IMB [79], has been used to establish limits on WIMPs [80]. This technique will be exploited in the future by several experiments such a MACRO in Italy, the largest neutrino telescope currently in operation, and several potentially much larger underwater or under ice neutrino telescopes such as Dumand off Hawaii, Amanda at the South pole, Baikal in Lake Baikal, and Nestor off the Greek island of Pylos. The hope is that a 1 km$^2$ neutrino telescope may eventually be constructed which would greatly expand the potential of this technique. In this section we examine LSP detection by this method.

5.1 Expected flux of upward muons from LSP annihilation

The basic scenario for LSP detection via the observation of upward muons in an underground detector is that halo LSPs lose energy and are captured by scattering from nuclei in either the Earth or Sun. Subsequent scatterings result in the LSPs settling to the center whereupon LSP and anti-LSP annihilate into, among other things, neutrinos. The neutrinos travel relatively freely to the Earth, where they may undergo a charged current interaction and produce an upward going muon in an underground detector. Only high energy ($> 1$ GeV) neutrinos may produce upward going
muons, and the only known high energy neutrinos are those produced in atmospheric cosmic ray showers. A sharp reduction in this background may be made by considering angular windows around the Earth and Sun.

We calculate the expected rate of upward muons using the following steps (for more detail see [81]):

1. Calculate the capture rate of LSPs by the Earth and Sun due to scalar and spin-dependent scattering. We use the formula of Ref. [82, 80] for the capture rate calculations. The fraction of full capture rate [83] has been considered taking into account the competition between capture and annihilation rates.

2. Calculate the LSP annihilation branching ratios into all relevant final states.

3. Determine the neutrino yield from all possible annihilation products of the LSP. We take into consideration energy loss of heavy quark annihilation products [84]. We use JETSET [68] to determine neutrino yield from the subsequent decays of the annihilation products (e.g., \(b \to \nu_e + X\)). We keep track of \(\nu_\tau\) yield since it can undergo a charge-current conversion to a \(\tau\) which subsequently decays into a \(\mu\) with a branching fraction of 0.187.

4. Determine energy loss and flux loss of neutrinos undergoing neutral current or charged current interactions as they travel through the sun [84].

5. Calculate the upward going muon rate given neutrino flux. This is done by first determining the charged current conversion rate in rock [85], and then by simulating the muon energy loss as it propagates through the rock on its way into the detector [86].

6. Finally, estimate the atmospheric neutrino background [87, 88], and scale it to recent experimental data [81, 78] for higher accuracy.

It is a complicated question to evaluate whether the Earth or Sun will yield a better upward muon signal. The answer depends on LSP mass, as well as the LSP interaction cross sections and annihilation branching ratios. [We have compared our spin-dependent and spin-independent cross section calculations with those of Drees and Nojiri and find excellent agreement for solutions with higher mass LSPs (\(m_\chi \gtrsim 100\) GeV). For solutions with lower mass LSPs we invariably have somewhat higher cross sections, which leads to a larger signal of upward going muons (and LSP-nucleon scattering which we discuss in the next section).] Crudely speaking, for LSPs less than about 100 GeV and having substantial spin-independent interactions, the Earth signal could be larger. For heavy LSPs and LSPs with primarily spin-dependent interactions, the signal from the Sun will be larger. In the case of CMSSM neutralinos considered here, the signal from the Sun is almost always far larger than that of the Earth since gaugino-like neutralinos have primarily spin-dependent interactions for which the Earth provides little signal. Hence, we show only the detection possibilities from upward muons from LSP annihilations in the Sun.

We present our results in Fig. [12] as the area required to have an upward-going muon signal above background with a 4\(\sigma\) significance. In order to make this plot it was assumed that the idealized detector is located at the equator (which has the largest exposure to the sun), and is
Figure 12: The detector area required for detection of upward going muons with a 4σ statistical significance above background. Such a signal would be possible evidence for LSP capture and annihilation in the sun. Only a few solutions with $m_\chi \leq 40$ GeV would be ruled out if a 1 km$^2$ detector did not observe an effect, and those solutions are detectable at LEP II and FNAL.

| Latitude       | Factor |
|----------------|--------|
| 0°             | 1.00   |
| 20° (DUMAND)   | 1.09   |
| 42.5° (MACRO)  | 1.46   |
| 90° (AMANDA)   | 3.43   |

Table 1: Exposure factors for various detector locations. The areas in Fig. 12 must be multiplied by the appropriate scale factor.

100% efficient in detecting upward muons (i.e. any muon below the horizontal) of energy greater than 2 GeV. The background of atmospheric upward muons was considered in the angular bins required to observe 90% of the LSP upward muon flux. These bin sizes were determined by a Monte Carlo performed by the Kamionkande collaboration [89]. The detector area required depends on the exposure time to the sun. (Higher latitudes mean lower exposure time). The areas in Fig. 12 must be multiplied by the factors listed in Table 1 to account for this effect in actual experiments. Also, we have not corrected for different energy thresholds in different detectors, which would decrease the effectiveness of Amanda and Dumand relative to the MACRO thresholds we use.

Our results for detectability of upward going muons are considerably less optimistic than previous analyses [90]. The highly correlated mass spectra in minimal supergravity (see Fig. 6) forbid one from randomly choosing squark masses, Higgs masses, gaugino masses, and LSP composition independent from one another. The resulting parameter space then contains gaugino-like LSPs with typically small spin-dependent cross-sections and even smaller spin-independent cross-sections. Furthermore, our refinements in each step of the calculation (such as including thresholds) turned out to reduce the signal even further. Note that the largest working neutrino telescope, MACRO, has an area of only about $10^{-3}$ km$^2$ and so cannot constrain any neutralino models with one year’s observation. Unfortunately, only a few solutions with $m_\chi \lesssim 40$ GeV are detectable even with a 1 km$^2$ neutrino telescope, and those will be covered by LEP II and FNAL.

6 LSP Detection through elastic collisions with nuclei

LSPs can also be detected by directly searching for their elastic scattering off nuclei in a tabletop detector. There have been many calculations of the LSP elastic scattering cross section off matter [91, 51, 92, 93, 94, 95, 96] which we use. As the illustration of this technique, we have
Figure 13: Scatter plot of the counting rate for the $^{73}$Ge nuclei vs. $m_\chi$. The upper line is the limit that would be placed on the rate if 1 kg of Ge were collecting data for 1 year and could suppress background enough to be sensitive to 0.1 event/kg/day. Likewise, the lower line is for 10 kg of Ge with sensitivity up to 0.01 events/kg/day in which case many solutions would be either detected or ruled out for $m_\chi \lesssim 200$ GeV.

Figure 14: Scatter plot of the counting rate for the $^{93}$Nb nuclei vs. $m_\chi$. See Fig. 13 caption for further explanation.

chosen to investigate two separate elements which have different nuclear properties: $^{73}$Ge, which has (essentially) just one unpaired neutron, and $^{93}$Nb, which has (essentially) just one unpaired proton. Both these elements have large nuclear masses, and therefore the coherent spin-independent scattering amplitude, which couples to the mass of the nucleus rather than the nucleon, will be enhanced.

Germanium is an excellent case study because the nuclear properties have been studied quite thoroughly making the calculation of expected rates more reliable than with other elements where one must rely on simplistic models of nuclear properties. Furthermore, a strong experimental program is already underway. Although there is still some uncertainty coming from the quark spin content of nucleons, these errors have diminished substantially with more precise experimental numbers and improved theoretical understanding of the measurements which have been conducted over the years. Therefore, quark spin content of the nucleons is no longer a major source of uncertainty in the spin dependent cross section calculation. We utilize the values for $\Delta q$ ($\Delta u = 0.83 \pm 0.03$, $\Delta d = -0.43 \pm 0.03$, and $\Delta s = -0.10 \pm 0.03$ for the proton) as suggested by Ref. [100].

The current sensitivity of LSP-nucleon collisions is generally quoted as 0.1 events per day, with future goals focusing on about 0.01 events per day sensitivity. With this in mind we plot in Figs. 13 and 14 the counting rate per kilogram per year for a $^{73}$Ge and a $^{93}$Nb detector. We note that LSPs up to approximately 100 GeV are detectable with a 1 kg $^{73}$Ge detector. Furthermore, if the sensitivity is increased by a factor of 10 (0.01 events per day) as some suggest is possible then solutions with $m_\chi$ up to about 200 GeV are detectable with 10 kg.

In Fig. 13 we compare the prospects of detecting the LSP using a 1 km$^2$ neutrino telescope and a 10 kg $^{73}$Ge detector. The lower left quadrant corresponds to the region where the LSP will be detected by at least one of the methods. The 10 kg Germanium detector covers a substantially larger fraction of the parameter space than the 1 km$^2$ neutrino telescope. This leads us to conclude that a 10 kg Germanium detector has more discovery potential than a 1 km$^2$ neutrino telescope.

Figure 15: Scatter plot comparing the effectiveness of a neutrino telescope to a Ge table top experiment. All solutions in the lower left quadrant would either be detected by the neutrino telescope and/or the table top experiment. Note that 10 kg of Ge covers many more solutions than a 1 km$^2$ neutrino telescope.
Figure 16: Constraints on the LSP mass and mixing from the Z invisible width contributions of the LSP and from the direct search limits on the lightest chargino. The region above the curved dotted line is excluded by $\Delta \Gamma_{\text{inv}}$ width constraint. The region to the left of the vertical dotted line is excluded by the non-observation of $\chi^+\chi^-$ production at LEP. Both of these constraints must be applied on solutions with $m_\chi < m_Z/2$. The dots are solutions which pass these cuts.

But we should keep in mind that if table top experiments are to have a significant constraining impact on super-unified models the sensitivity must increase to 0.01 events per day and 10 kg or more of nuclear detector material will be needed.

7 LSP Detection through Collider Experiments

If the LSP mass is less than $m_Z/2$ then experiments at the Z-peak could possibly witness $Z \rightarrow \chi\chi$ decays. Since the neutralinos are invisible to the detectors of LEP and SLD, we must ensure that such decays do not violate the invisible width constraints. The maximum contribution possible to the invisible Z width from non-standard physics is $\Delta \Gamma \lesssim 12 \text{ MeV}$ [102].

Fig. 16 shows the constraints on the LSP from the invisible Z width in the $\eta_Z - m_\chi$ plane where $\eta_Z$ is the neutralino mixing factor in the coupling of the Z with the LSP ($\eta_Z = |Z_{13}^2 - Z_{14}^2|$). An invisible width contour is shown for $\Gamma_{\text{inv}} = 12 \text{ MeV}$. All neutralinos which fall to the left or above this curve are excluded from the invisible width constraint. There is also a constraint on the neutralino from non-observation of $Z \rightarrow \chi^+\chi^-$ decays. This constraint is possible because of correlations between the chargino mass matrix and the neutralino mass matrix through $\mu$, $M_2$ and (in the case of common $m_{1/2}$) $M_1$. We have used $\tan \beta > 1.3$ to plot this chargino correlation bound on the LSP, because $\tan \beta < 1.3$ would cause the top quark Yukawa coupling to go strong below the high scale for $m_t \gtrsim 150 \text{ GeV}$ (which is implied by top searches at Fermilab [103, 104] and LEP precision data [105, 106]), so the theory would no longer be perturbative (as it is apparently observed to be by the gauge coupling unification). Since the minimal super-unified model almost invariably gives large gaugino fraction, we expect that there will be many viable solutions with small $\eta_Z$ and $m_\chi < m_Z/2$. In Fig. 16 we have superimposed a scatter plot of super-unified solutions to show that low mass $m_\chi$ are copious and do not violate the $\Gamma_{\text{inv}}$ bound.

Measurements of $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{had})$ at LEP might be indicating that the lightest stop and chargino are less massive than about $m_W$ [107]. However, if universal gaugino masses at the GUT scale are the correct description of nature, then supersymmetric solutions which predict $R_b$ within one standard deviation of the experimental measurement cannot provide sufficient dark matter to be cosmologically interesting. This is because scenarios with good $R_b$ require the LSP to be almost pure Higgsino-like, and annihilations through the Z boson become much too efficient and the final relic density is negligible. By employing non-minimal GUT scale boundary conditions one could “decouple” the neutralino from the Z-boson (e.g., choose $M_1 << M_2$) and allow for a large relic abundance of LSPs. One has to worry about squark and slepton t-channel exchanges yielding large cross sections as well, but that is a problem uncommon scalar masses can solve. In any event, if supersymmetry does enhance $R_b$ to within its 1σ experimental limit, then the lightest
stop and/or the lightest chargino will be discovered at LEP II and the mass of the LSP would be quickly measured. Determining whether or not the LSP is in fact the cold dark matter would require other experiments which could directly measure its existence in the halo, or experiments which would pin down all other relevant parameters in the supersymmetric Lagrangian for a reliable calculation of the expected relic abundance.

To a first approximation the mass reach at LEP for charginos and stops will be somewhere close to half the beam energy. Exceptions to this first approximation include stops mixing such that the lightest stop does not couple to the $Z$, or that the light stop and/or chargino is close enough to the LSP mass that they decay into very soft leptons or jets and the events are never detected. Not much can be done about the first problem, which reduces the sensitivity to stops by a few GeV (since there is still a photon diagram), but the latter problem can be overcome by making direct measurements of the “invisible rate” from initial state photon radiation. So if superpartners are being created in large numbers but then are decaying to untriggerable final states, the invisible rate measurement should register it.

We have studied our solutions in an effort to determine the likelihood of the above-mentioned detection scenarios. In the case of the chargino there is always sufficient mass difference between it and the LSP to provide the leptons or jets with plenty of energy to make chargino production a clear signal at LEP II (provided the chargino is light enough to be produced). Then mass reconstruction algorithms could conceivably determine the mass of the LSP from these events. The same is true for many of the low mass stop solutions, but there are some solutions which have a light stop and LSP which are nearly degenerate. For such solutions the stop events could not be directly tagged and the LSP mass could not be reconstructed. It is also possible, though not too likely, for $\tilde{\chi}^+ \rightarrow \tilde{t}_1 + b$, followed by $\tilde{t}_1 \rightarrow c + \text{LSP}$, which would make reconstruction of the LSP mass rather difficult. It is interesting to note that all the solutions which yield direct tagged chargino or stop events at LEP II have an LSP mass below about 50 GeV.

A signal of supersymmetry also can be deduced cleanly at hadron colliders from the production of charginos and neutralinos [108, 109, 110, 111]. For a large region of parameter space these particles will decay with a large branching fraction into the LSP plus three isolated, high $p_T$ leptons plus missing $E_T$. One such process is represented by the Feynman diagram in Fig. 17. We simulate these trilepton signal events using a full Monte Carlo event generator with a toy detector simulator. To reduce background we make the following cuts: one “trigger” lepton with $|\eta| < 1.0$ and $p_T > 10$ GeV; the two remaining leptons must have $|\eta| < 2.5$ and $p_T > 4$ GeV; $\Delta R_{ll} > 0.4$; and $m_{l+1} \neq M_Z \pm 10$ GeV. The standard model background rate is less than about 1 fb [111] given similar background cuts and a sufficiently well-designed detector which reduces lepton fakes.

Fig. 18 summarizes the results of our preliminary simulation [112]. We make a scatter plot of $m_\chi$ vs. $\epsilon_{tot} \sigma(3l)$ for our entire parameter space with $m_\chi$ below 100 GeV. (The variable $\sigma(3l)$ is the total cross section for trilepton production through chargino plus neutralino, and $\epsilon_{tot}$ is the total efficiency from the previously mentioned detector cuts.) Each point on the graph represents a complete minimal super-unified model solution. The top horizontal line is the 10 event contour with
Figure 18: Scatter plot of expected trilepton event rates at Fermilab for the $\chi^+_1 \chi^-_2 \rightarrow 3l$ signal with the cuts described in the text. The upper line is the 10 event contour with 100 pb$^{-1}$ of data. The lower dotted line is the 10 event contour with 1 fb$^{-1}$ of data. The mass reach for the latter is $m_\chi \gtrsim m_Z$ independent of all parameters such as tan $\beta$, etc.

100 pb$^{-1}$, and the lower horizontal line is the 10 event contour with 1 fb$^{-1}$ of data. From Fig. 18 we see that with 1 fb$^{-1}$ of data, all neutralinos below about 90 GeV will be detected (indirectly) through the trilepton signal. This result is completely independent of parameters in the model, such as $m_0$ and tan $\beta$, etc. Since the LSP occurs in these decays, its presence can be deduced in principle from the energy it carries away, and its mass and couplings inferred from an analysis of the events. Note that it is important to include the solution-by-solution detectability since that can vary considerably even for a given $m_\chi$.

At the LHC gluino production and decay through like-sign leptons or jets with large missing $E_T$ might be the best way to detect the neutralino. Using different modes one can obtain viable signals of gluino pair production for gluinos above 1 TeV [113]. Since the gluino mass is strongly correlated with the lightest neutralino mass ($m_\chi \sim 0.15 m_{\tilde{g}}$), the determination of the gluino mass essentially determines the neutralino mass. If a 1 TeV gluino were discovered, then we would know that the LSP mass is approximately 150 GeV, assuming the data were consistent with the CMSSM.

The above is a summary of some of the more promising ways to get at the LSP mass at a collider. But there are more ways. Indeed, if R-parity is conserved then a very model independent statement is that all particles which reach the detector are standard model states plus an even number of LSPs:

$$pp \rightarrow 2n\chi + X_{\text{sm}}.$$  \hspace{1cm} (29)

If a supersymmetry signal is found through any channel (like-sign dileptons, tri-leptons, ...) then one always has a fighting chance at reconstructing the LSP mass given enough events and sufficient understanding of the theory. Extracting results such as the LSP mass from LHC signals may be very difficult. To our knowledge the most likely technique may be one using the structure of the theory, as described in section 9.7 of Ref. [3]. We suggest, therefore, that colliders be viewed as an important player in dark matter detection. The mass reach of an upgraded Tevatron and certainly the LHC rival and generally exceed other experiments which have dark matter detection as one of their main goals. Of course, what is explicitly demonstrated once superpartners are observed at a collider is the existence of an LSP that lives longer than about $10^{-8}$ seconds. Direct confirmation of its role as dark matter will require one or more of the previously discussed experiments.

8 Conclusions and Comments

We have carried out a comparative analysis of supersymmetric cold dark matter and its detectability. We use the constrained minimal supersymmetric standard model (CMSSM) so all solutions examined are known to describe nature, including gauge unification, electroweak symmetry breaking, and experimental data. As the experimental data increases and our knowledge of the theory increases, the parameter space of CMSSM continues to shrink. For each set of CMSSM parameters
\((m_0, m_{1/2}, \mu, A_0, \tan \beta)\) a unique SUSY spectrum of masses and couplings is calculated. All LSP annihilation and scattering diagrams and the resulting rates are calculated for those parameters. Since all solutions satisfy all known constraints, we show scatter plots of results for the allowed parameter sets rather than selecting arbitrarily among them. Our conclusions about detectability do not confirm those of many previous analyses, because these earlier analyses (a) did not satisfy conditions such as gauge coupling unification or electroweak symmetry breaking, (b) did not use constrained parameters from all experimental data simultaneously, (c) chose optimistic and inconsistent sets of parameters, or (d) made other optimistic assumptions. We emphasize that the scatter plots we have shown, while not eliminating some dependence on the basic SUSY assumptions, do not depend at all on the usual parameters (\(\tan \beta, m_0, m_{1/2}, \text{etc.}\)). They are the best that one can do without new (not generally available) sources of knowledge.

After constructing many supersymmetric solutions and then subsequently using this same set of solutions to calculate the LSP detection signal of various currently running or proposed experiments, we find lower detection prospects than were previously thought (except in the case of dark matter detection by high energy colliders). We note that in spite of large uncertainties in astrophysical assumptions, which affect particularly the halo annihilation and solar annihilation cases, the range of possible rates coming from lack of knowledge of SUSY parameters is probably the dominant uncertainty in calculating detectability. That means that doing the SUSY physics right is the most important thing, and that is what we have emphasized.

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