Allosteric Voltage Gating of Potassium Channels I
mSlo Ionic Currents in the Absence of Ca\(^{2+}\)

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abstract Activation of large conductance Ca\(^{2+}\)-activated K\(^{+}\) channels is controlled by both cytoplasmic Ca\(^{2+}\) and membrane potential. To study the mechanism of voltage-dependent gating, we examined mSlo Ca\(^{2+}\)-activated K\(^{+}\) currents in excised macropatches from X. oocytes in the virtual absence of Ca\(^{2+}\) (<1 nM). In response to a voltage step, I\(_K\) activates with an exponential time course, following a brief delay. The delay suggests that rapid transitions precede channel opening. The later exponential time course suggests that activation also involves a slower rate-limiting step. However, the time constant of I\(_K\) relaxation (\(\tau(\text{I}_K)\)) exhibits a complex voltage dependence that is inconsistent with models that contain a single rate limiting step. \(\tau(\text{I}_K)\) increases weakly with voltage from -500 to -20 mV, with an equivalent charge (z) of only 0.14 e, and displays a stronger voltage dependence from +30 to +140 mV (z = 0.49 e), which then decreases from +180 to +240 mV (z = -0.29 e). Similarly, the steady state G\(_o\)-V relationship exhibits a maximum voltage dependence (z = 2 e) from 0 to +100 mV, and is weakly voltage dependent (z = 0.4 e) at more negative voltages, where P\(_o\) = 10\(^{-5}\) - 10\(^{-6}\). These results can be understood in terms of a gating scheme where a central transition between a closed and an open conformation is allosterically regulated by the state of four independent and identical voltage sensors. In the absence of Ca\(^{2+}\), this allosteric mechanism results in a gating scheme with five closed (C) and five open (O) states, where the majority of the channel's voltage dependence results from rapid C-C and O-O transitions, whereas the C-O transitions are rate limiting and weakly voltage dependent. These conclusions not only provide a framework for interpreting studies of large conductance Ca\(^{2+}\)-activated K\(^{+}\) channel voltage gating, but also have important implications for understanding the mechanism of Ca\(^{2+}\) sensitivity.

key words: calcium • K\(_{\text{Ca}}\) channel • large conductance Ca\(^{2+}\)-activated K\(^{+}\) channel • ion channel gating

introduction

Large conductance Ca\(^{2+}\)-activated K\(^{+}\) (BK)\(^1\) channels are sensitive to both membrane voltage and intracellular Ca\(^{2+}\) (Gorman and Thomas, 1980; Marty, 1981; Pallotta et al., 1981; Barrett et al., 1982; Latorre et al., 1982; Methfessel and Boheim, 1982; Moczydlowski and Latorre, 1983; D’Chiara and Reinhart, 1995; Cui et al., 1997). The response of BK channels to Ca\(^{2+}\) is key to their physiological role in a variety of cell types and has been studied extensively (Lewis and Hudspeth, 1983; Magleby and Pallotta, 1983a,b; Petersen and Maruyama, 1984; Storm, 1987; Lancaster et al., 1991; McManus and Magleby, 1991; Brayden and Nelson, 1992; Bielefeldt and Jackson, 1993; Crest and Gola, 1993; Robitaille et al., 1993; Nelson et al., 1995; Yazejian et al., 1997; Marrion and Tavalin, 1998; Safronov and Vogel, 1998). Attempts to identify Ca\(^{2+}\)-binding sites have also provided a focus for structure-function studies (Wei et al., 1994; Schreiber and Salkoff, 1997; Schreiber et al., 1999). In comparison, the voltage-dependent activation of BK channels has received less attention. Yet BK channels respond rapidly to voltage changes (Hudspeth and Lewis, 1988; Cui et al., 1997), suggesting that this process might contribute to the physiological function of BK channels. More importantly, interactions between Ca\(^{2+}\)- and voltage-dependent activation (Cox et al., 1997a; Cui et al., 1997) imply that understanding BK channel Ca\(^{2+}\) sensitivity depends on also understanding the mechanism of voltage-dependent gating.

Voltage-dependent activation of BK channels occurs on a millisecond time scale, similar to many purely voltage-gated K\(^{+}\) (K\(_v\)) channels. However, during an action potential, changes in voltage and voltage-dependent Ca\(^{2+}\) entry through Ca\(^{2+}\) channels contribute to BK channel activity such that the direct effect of voltage is difficult to assess. Using a spike-like voltage clamp, Crest and Gola (1993) examined the time course of Ca\(^{2+}\) and K currents during Ca\(^{2+}\) action potentials in molluscan neurons and concluded that BK channel voltage dependence is important for closing channels.
rapidly following action potentials and for terminating repetitive firing in response to sustained depolarization. The properties of mammalian BK channel activation studied under constant \([\text{Ca}^{2+}]\), suggest they could participate in similar processes. The voltage sensitivity of BK channel activation is weak in comparison with that of K, channels (Cui et al., 1997; Horrigan and Aldrich, 1999), but open probability \(P_o\) can increase from \(\sim 10\) to \(90\%\) of \(P_{\text{max}}\) in response to a 70-mV voltage change centered around the half-activation voltage \(V_{1/2}\) (Barrett et al., 1982; Cui et al., 1997). Calcium alters the kinetics of voltage-dependent activation and shifts \(V_h\) to more negative voltages (Barrett et al., 1982; Cui et al., 1997). Thus the change in \(P_o\) evoked by a given voltage stimulus can be "tuned" by \([\text{Ca}^{2+}]\).

Most studies of BK channel gating have been performed in the presence of \([\text{Ca}^{2+}]\) (Barrett et al., 1982; Latorre et al., 1982; Moczydlowski and Latorre, 1983; McManus and Magleby, 1991; Rothberg and Magleby, 1998), leaving open the possibility that the mechanism of voltage sensitivity reflects, to some extent, voltage-dependent changes in the affinity of \([\text{Ca}^{2+}]\) for its binding site rather than a direct effect of voltage on channel conformation (Moczydlowski and Latorre, 1983). Recent studies, however, involving several cloned homologues of the slo family of \([\text{Ca}^{2+}]-\)activated \(\text{K}^+\) channels demonstrate that BK channels can be activated by membrane depolarization in the absence of \([\text{Ca}^{2+}]\) binding (Meera et al., 1996; Cui et al., 1997), and that gating currents can be detected under these conditions (Stefani et al., 1997; Horrigan and Aldrich, 1999). These and other results indicate that BK channel voltage sensitivity reflects the action of an intrinsic voltage sensor (Cui et al., 1997; Stefani et al., 1997). Indeed, the amino acid sequence of slo BK channels contains a "core" domain that has many features in common with that of K, channels (Wei et al., 1994; Diaz et al., 1998). These include a p-region, homologous to the pore-forming region of K, channels, surrounded by six putative transmembrane segments including a charged S4 domain. The S4 domain forms part of the voltage sensor of Shaker and other voltage-gated channels (Yang and Horn, 1995; Aggarwal and Mackinnon, 1996; Larson et al., 1996; Mannuzzu et al., 1996; Seoh et al., 1996; Yang et al., 1996; Yusaf et al., 1996; Bao et al., 1999), and S4 mutations alter the voltage dependence of BK channels (Diaz et al., 1998; Cui, J., and R.W. Aldrich, manuscript in preparation). Thus, it is likely that structural and mechanistic similarities exist between BK and K, channels.

In the present study, we examine the response of mSlo \([\text{Ca}^{2+}]-\)activated \(\text{K}^+\) channels to voltage in the virtual absence of \([\text{Ca}^{2+}]\left(<1\ \text{nM}, \text{see met h o d s}\right)\) to help understand the mechanism of voltage-dependent gating. The behavior of mSlo in 0 \([\text{Ca}^{2+}]\) must reflect transitions between only a subset of the states that are available in the presence of \([\text{Ca}^{2+}]\). Thus, the 0 \([\text{Ca}^{2+}]\) condition should provide a limiting example of mSlo voltage-gating behavior that must be accounted for by any complete model of mSlo gating. The ability of BK channels to open in a voltage-dependent manner in the absence of \([\text{Ca}^{2+}]\) suggests that channel activation is fundamentally a voltage-dependent process that is modulated by \([\text{Ca}^{2+}]\) binding (Meera et al., 1996; Cox et al., 1997a). If so, the voltage-dependent activation pathway must be central to the \([\text{Ca}^{2+}]-\)dependent response, and delineation of this pathway in the absence of \([\text{Ca}^{2+}]\) will be a prerequisite to the establishment of a detailed \([\text{Ca}^{2+}]-\)dependent gating scheme or the interpretation of BK channel structure–function studies that seek to distinguish \([\text{Ca}^{2+}]\)- and voltage-dependent conformational changes.

Many of the effects of \([\text{Ca}^{2+}]\) and voltage on the kinetic and steady state properties of macroscopic mSlo \(I_K\) can be reproduced by a gating scheme (Cox et al., 1997a) based on the allosteric model of Monod et al. (1965). According to this voltage-dependent Monod-Wyman-Changeux (MWC) scheme (Scheme I), mSlo channels activate by undergoing a rate limiting, voltage-dependent transition between a closed (C) and an open (O) conformation and \([\text{Ca}^{2+}]\) binding alters the kinetic and equilibrium properties of this transition. Because mSlo channels are composed of four identical subunits (Shen et al., 1994), the model assumes that each channel contains four identical \([\text{Ca}^{2+}]\) binding sites. This results in a scheme with 10 states representing different \([\text{Ca}^{2+}]\)-bound versions of the closed and open conformation.

A key feature of this model is that the C to O conformational change is allosteric in that it not only opens the channel pore, but also alters the \([\text{Ca}^{2+}]-\)binding sites, causing their affinities for \([\text{Ca}^{2+}]\) to increase. This allosteric linkage between channel opening and \([\text{Ca}^{2+}]\) binding, represented by a factor, C, in the model, accounts for the ability of \([\text{Ca}^{2+}]\) to affect open probability. Another important feature of the model is that the transition from C to O is represented by a single step and is therefore assumed to be concerted in the sense that \([\text{Ca}^{2+}]\)-binding sites in all four subunits change simultaneously upon channel opening. Because the C–O transition is voltage

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**Scheme I**

- **C** ↔ **C\text{Ca}** ↔ **C\text{Ca}_2** ↔ **2C\text{Ca}** ↔ **2C\text{Ca}_3** ↔ **1/4C\text{Ca}_4**

- **O** ↔ **O\text{Ca}** ↔ **O\text{Ca}_2** ↔ **O\text{Ca}_3** ↔ **1/4O\text{Ca}_4**
dependent, Scheme I also implicitly assumes that voltage sensors, presumably present in each subunit, move in a concerted manner during channel activation.

Although Scheme I reproduces many features of mSlo activation, it is likely to be an oversimplification, particularly with regards to the mechanism of voltage-dependent gating. A basic prediction of this model is that activation can be described by a simple two-state process in the absence of Ca\(^{2+}\) binding, indicated by the highlighted C–O transition in the above diagram. Although a two-state mechanism can account for the basic features of activation, deviations from Scheme I–like behavior are observed in the presence and absence of Ca\(^{2+}\) that suggest BK channel voltage gating is more complicated (Cox et al., 1997a). These deviations include a brief delay in voltage-dependent activation (Cox et al., 1997a; Stefani et al., 1997), and a conductance–voltage relationship (G–V) that is best fit by a Boltzmann function raised to a power greater than one (Cui et al., 1997). The shape of the G–V also changes slightly with [Ca\(^{2+}\)], an effect that is not predicted by Scheme I (Cox et al., 1997a; Cui, J., and R.W. Aldrich, manuscript in preparation). Similarly, the voltage dependence of I\(_K\) relaxation kinetics deviates from the prediction of Scheme I at extreme voltages (Cox et al., 1997a). Finally, a rapid component of gating charge movement is observed that precedes channel opening (Stefani et al., 1997; Horrigan and Aldrich, 1999), whereas Scheme I requires that channel opening and voltage-sensor movement occur simultaneously.

In the present study, we examine in detail several aspects of mSlo behavior in the absence of Ca\(^{2+}\) that deviate from the predictions of Scheme I. The results can be explained by relaxing the assumption that channel opening involves a single concerted voltage-dependent transition. Instead, we suggest that mSlo voltage sensors can move independently and that channel opening and voltage-sensor movement represent distinct events that are allosterically coupled. The resulting model of voltage-dependent gating differs from many common schemes in that the channel can open while any number (or none) of the four voltage sensors are activated. This allosteric mechanism defines a 10-state voltage-gating scheme with multiple open and closed states arranged in parallel, analogous to Scheme I. Similar schemes have been proposed to describe the gating of other voltage-dependent channels (Marks and Jones, 1992; Rios et al., 1993; McCormack et al., 1994).

The proposed model has important implications for the interpretation and analysis of BK channel structure–function studies because complicated relationships will exist between elementary molecular events such as voltage-sensor movement or channel opening, which give rise to the apparently simple macroscopic features of I\(_K\). In addition, the complexity of the voltage-gating scheme greatly increases the minimum number of states that are required to describe BK channel gating in the presence of Ca\(^{2+}\). Finally, the demonstration that mSlo gating is a multistate process in the absence of Ca\(^{2+}\) raises fundamental questions concerning the identity of the step or steps in the activation pathway that are affected by Ca\(^{2+}\).

**Methods**

**Channel Expression**

Experiments were performed with the mbr5 clone of the mouse homologue of the Slo gene (mSlo), kindly provided by Dr. Larry Salkoff (Washington University School of Medicine, St. Louis, MO). The clone was modified to facilitate mutagenesis and was propagated and cRNA transcribed as previously described (Cox et al., 1997b). *Xenopus* oocytes were injected with ~0.5–5 ng of cRNA (50 nl, 0.01–0.1 ng/nl) 3–7 d before recording.

**Electrophysiology**

Currents were recorded using the patch clamp technique in the inside out configuration (Hamill et al., 1981). Upon excision, patches were transferred into a separate chamber and washed with at least 20 vol of internal solution. Internal solutions contained (mM): 104 KMeSO\(_3\), 6 KCl, and 20 HEPES, and 40 μM (+)18-crown-6-tetracarboxylic acid (18C6TA) to add chelate contaminant Ba\(^{2+}\) (Diaz et al., 1996; Neyton, 1996; Cox et al., 1997b). In addition "0 Ca\(^{2+}\)” solutions contained 2 mM EGTA, reducing free Ca\(^{2+}\) to an estimated 0.8 nM based on the presence of ~10 μM contaminant Ca\(^{2+}\) (Cox et al., 1997b). 4.5 μM Ca\(^{2+}\) solutions were buffered with 1 mM HEDTA and free Ca\(^{2+}\) was measured with a Ca\(^{2+}\) electrode (Orion Research, Inc.). The external (pipette) solution contained (mM): 108 KMeSO\(_3\), 2 KCl, 2 MgCl\(_2\), and 20 HEPES, pH was adjusted to 7.2. Experiments were carried out at 5° or 20°C (±1°C) as indicated.

Electrodes were made from thick-walled 1010 glass (World Precision Instruments, Inc.) or borosilicate glass (VWR Microptic). Their tips were coated with wax (KERR Sticky Wax) and fire polished before use. Pipette access resistance measured in the bath solution (0.5–1.5 MΩ) was used as an estimate of series resistance (R\(_s\)) to correct the pipette voltage (V\(_p\)) at which I\(_K\) was recorded. The corrected pipette voltage, V\(_m\), was used in determining membrane conductance (G\(_m\)) from tail current measurements and in plotting the voltage dependence of G\(_m\) or the time constant of I\(_K\) relaxation (1/τ\(_I\)(I\(_K\))). Series resistance error was <15 mV for all data presented and <10 mV for I\(_K\) measurements.

Data were acquired with an Axopatch 200-B amplifier that was modified to provide an increased voltage range (Axon Instruments) and set in patch mode. Currents were filtered at 100 kHz with the Axopatch’s internal four-pole bessel filter and subsequently by an eight-pole bessel filter (Frequency Devices, Inc.). Macroscopic currents were filtered at 30–50 kHz and sampled at 100 kHz with a 16 bit A/D converter (ITC-16; Instrutech Corp.). A P–4 protocol was used for leak subtraction (Armstrong and Bezanilla, 1974) from a holding potential of ~80 mV. To increase the signal to noise ratio, the response to four to eight pulses were typically averaged at each pulse voltage. A Macintosh-based computer system was used in combination with Pulse Control acquisition software (Herrington and Bookman, 1995) and Igor Pro for graphing and data analysis (Wavemetrics Inc.). A Levenberg-Marquardt algorithm was used to perform nonlinear least-squares fits.
Simulations

Simulations were performed using a fifth order Runge-Kutta algorithm with adaptive step size (Press et al., 1992) implemented in Igor Pro (Wavemetrics Inc.).

Filter and Instrumentation Delay

The effect of filtering on I\textsubscript{K} activation kinetics was tested by convolving simulated traces that closely match the data with the impulse response of an eight-pole bessel filter. The impulse response was determined as the derivative of the step response, measured for a 2 kHz filter and scaled along the time axis to correspond to a particular corner frequency. Simulations were calculated at 1 \mu s intervals and were filtered at 100 kHz, and then at 30 kHz to correspond to the experimental arrangement. This procedure introduced a delay of 22 \mu s, accounting for the majority of the instrumentation delay (25 \mu s), but had no detectable effect on the shape of the simulated traces. Therefore, simulations were left unfiltered and data were corrected for filtering by shifting I\textsubscript{K} traces along the time axis by 25 \mu s. Instrumentation delay was estimated by measuring the time between a voltage step command to the patch clamp and the peak of the capacitive transient (Sigworth and Zhou, 1992).

Single Channel Analysis

Single channel events were observed in patches containing hundreds of channels at voltages where open probability is low (<10\textsuperscript{-3}). Currents were typically filtered at 20 kHz, yielding a dead-time of ~10 \mu s, and were sampled at 50–100 kHz. At voltages where the closed level was clearly defined, total open probability (n\textsubscript{P\textsubscript{o}}) was determined from steady state recordings of 5–45 s duration. All-points amplitude histograms were compiled and the probability (P\textsubscript{k}) of occupying each open level (k) was evaluated using a 1/2 amplitude criterion. n\textsubscript{P\textsubscript{o}} was then determined as:

\[ nP_o \approx \sum_k kP_k. \]

n\textsubscript{P\textsubscript{o}} was also evaluated by fitting P\textsubscript{k} with a Poisson distribution:

\[ P_k = \frac{(nP_o)^k}{k!} e^{-nP_o}. \]

In all cases P\textsubscript{k} was well fit by a Poisson distribution and the values of n\textsubscript{P\textsubscript{o}} obtained by the two methods differed by <5%. This is consistent with the idea that I\textsubscript{K} represents the activity of a large population of channels with low P\textsubscript{o}, rather than a subpopulation with higher P\textsubscript{o}. Normalized open probability (P\textsubscript{k}/P\textsubscript{max} = nP\textsubscript{k}/nP\textsubscript{max}) was determined by combining nP\textsubscript{o} measurements with an estimate of P\textsubscript{max} obtained from the macroscopic G\textsubscript{K-V} relationship in the same patch (nP\textsubscript{max} = g-channel \cdot g\textsubscript{K}, where g\textsubscript{K} is the single channel conductance). Patches that were used to measure single channel activity at negative voltages often produced currents that were too large to measure (>20 nA) at voltages that activate mSlo channels maximally. In these cases, G\textsubscript{max} was estimated by fitting the macroscopic G\textsubscript{K-V} with a Boltzmann function (L + \exp[-z(V - V\textsubscript{h})/KT])\textsuperscript{-1} raised to the 3.2 power as in Eq. 6, B and C.

For voltages >60 mV from the reversal potential (0 mV), single channel amplitudes were large enough that false opening events due to noise were not detected using 20 kHz filtering. The prevalence of false events was assessed by evaluating the number of current transients from the closed level that exceed the 1/2 amplitude criterion in a direction opposite that of the channel opening. nP\textsubscript{o} was determined after digitally filtering current records until such false events were not observed at ±20 or ±20 mV. This procedure yielded a corner frequency of ~5 kHz. For V > 60 mV, no difference in nP\textsubscript{o} was observed with 5 or 20 kHz filtering.

However, for V < −60 mV, a decrease in nP\textsubscript{o} was observed at 5 kHz, reflecting the brevity of open times at these voltages. The largest decreases (~30%) were observed at the most negative voltages (approximately −120 mV). Thus, P\textsubscript{o} may be underestimated, but this effect is small when compared with patch-to-patch variation in P\textsubscript{o} observed at these voltages (see Fig. 6, E and F).

Shifts in Voltage-dependent Parameters

Patch-to-patch variations in half-activation voltage and other voltage-dependent parameters are observed for mSlo (V\textsubscript{h} = 190 ± 10 mV; SD, n = 20 in 0 Ca\textsuperscript{2+}) and hSlo (Stefani et al., 1997), possibly due to differences in redox state of the channel (DiChiara and Reinhart, 1997). Such shifts do not appreciably alter the shape of the G\textsubscript{K-V} or other voltage-dependent relationships, but make comparisons of data between different experiments difficult and alter the slope of averaged voltage-dependent relationships relative to those observed in individual experiments. To compensate for this effect, V\textsubscript{h} was determined for each patch and compared with the average for all experiments (V\textsubscript{h}) at the same [Ca\textsuperscript{2+}]. Data from individual experiments were then shifted along the voltage axis by ΔV\textsubscript{h} = (V\textsubscript{h}) − V\textsubscript{h}.

Results

Delay in I\textsubscript{K} Activation

In the absence of Ca\textsuperscript{2+}, mSlo Ca\textsuperscript{2+}-activated K\textsuperscript{+} channels open in response to membrane depolarization exhibiting a steady state half-activation voltage of approximately +190 mV (Cox et al., 1997a; Cui et al., 1997). Fig. 1 A1 shows mSlo I\textsubscript{K} evoked in response to a 20-ms pulse to +160 mV from a holding potential of −80 mV in 0 Ca\textsuperscript{2+} (20°C). The time course of activation and deactivation are well fit by exponential functions (Fig. 1 A1, dashed lines). Similar exponential kinetics are observed over a wide range of voltage and [Ca\textsuperscript{2+}], (Cui et al., 1997), suggesting a two-state model with a single voltage-dependent transition between a closed and an open state. However, the exponential activation of I\textsubscript{K} is preceded by a brief delay in the presence or absence of Ca\textsuperscript{2+} (Cox et al., 1997a; Stefani et al., 1997). Fig. 1 A2 shows the initial time course of I\textsubscript{K} activation on an expanded time scale. There is a delay of ~100 μs before the current begins to increase, and at least 300 μs is required to achieve an exponential time course (Fig. 1 A2, dashed line). Although this delay is brief compared with the subsequent relaxation of I\textsubscript{K}, it is inconsistent with a two-state gating scheme and suggests that mSlo channels undergo one or more transitions among closed states before opening. To better study these rapid transitions, we examined I\textsubscript{K} activation at a reduced temperature. A family of I\textsubscript{K} evoked by membrane depolarization at 5°C still exhibits activation kinetics that are well fit by single exponential functions (Fig. 1 B1). The activation is slowed relative to 20°C, and the delay is similarly prolonged (Fig. 1 B2). I\textsubscript{K} begins to increase after 250 μs and attains an exponential time course after 1 ms. A control trace evoked in response to a voltage pulse to −180 mV is also shown in Fig. 1 B2; the capaci-
tive transient decays within 30 μs, showing that voltage clamp speed and filter properties contribute little to the delay.

The delay in $I_K$ in the absence of Ca$^{2+}$ indicates that the voltage-dependent activation of mSlo cannot be described by a two-state model. This conclusion is inconsistent with the predictions of Scheme I, where channel opening involves a single concerted step. However, mSlo is a homotetramer and activation is likely to involve the participation of multiple subunits. Unless these subunits move in a strictly concerted manner, channel activation must be described by a multistate scheme that reflects conformational changes in individual subunits. The most extreme deviation from the behavior of a concerted model should occur when subunits act independently (i.e., noncooperatively). Shown below is an example of a completely noncooperative model (Scheme II). This scheme corresponds to that used by Hodgkin and Huxley (1952) to describe the activation of voltage-dependent K$^+$ channels in squid axon. It assumes that channel opening requires all four identical subunits to undergo independent transitions between a resting (R) and an activated (A) conformation.

$$\begin{align*}
R & \xrightarrow{\alpha} A \\
R & \xrightarrow{\beta} A \\
R & \xrightarrow{\gamma} A \\
R & \xrightarrow{\delta} A
\end{align*}$$

(Scheme II)

Scheme II can be reduced to a five-state kinetic scheme (Scheme III), where subscripts (0–4) indicate the number of activated subunits in each closed (C) or open (O) state.

$$\begin{align*}
C_0 & \xrightarrow{4\alpha} C_1 \xrightarrow{3\alpha} C_2 \xrightarrow{2\alpha} C_3 \xrightarrow{\alpha} O_4
\end{align*}$$

(Scheme III)

Scheme III predicts a delay, but it cannot reproduce the kinetics of mSlo activation. The Hodgkin-Huxley model produces an activation time course that is highly sigmoidal because the delay and subsequent activation of $I_K$ are both determined by a single process (subunit activation) and therefore occur on a similar time scale. When Scheme III is fit to the brief delay in mSlo $I_K$, it predicts an activation time course that is too rapid (Fig. 2, A1 and A2).

The relationship between the delay and subsequent relaxation of $I_K$ can be defined precisely for models like Scheme II, which require n independent subunits to be activated before channels are open (Hodgkin and Huxley, 1952; Cole and Moore, 1960; Colquhoun and Hawkes, 1977). For such models, the time-dependent occupancy of the open state is:

$$O(t) = [A(t)]^n,$$

where $A(t)$ represents the probability that a subunit is activated.
A(t) = A_∞(1 - e^{-t/\tau}) \quad (2)

and \( \tau = 1/(\alpha + \beta) \) is the time constant of subunit activation, with \( A_∞ = \alpha / (\alpha + \beta) \) representing the steady state activation. The time course of \( I_K \) activation is determined by combining Eqs. 1 and 2 and expanding in a binomial series:

\[
I_K = I_∞(1 - e^{-t/\tau})^n = I_∞ \sum_{k=0}^{n} \frac{(-1)^k n!}{k!(n-k)!} e^{kt/\tau}, \quad (3)
\]

where \( I_∞ \) is proportional to \( A_∞ \) and represents the steady state amplitude of \( I_K \). The delay duration (\( \Delta t \)) is defined by fitting the slowest component of \( I_K \) relaxation after the delay (\( I_{slow} \)) with a single exponential function:

\[
I_{slow} = I_∞(1 - e^{-(\Delta t-t)/\tau}). \quad (4)
\]

This function intersects the time axis at \( t = \Delta t \). \( I_{slow} \) can be determined from the first two terms of the series in Eq. 3:

\[
I_{slow} = I_∞(1 - ne^{-t/\tau}). \quad (5)
\]

Combining Eqs. 4 and 5:

\[
\Delta t = ln(n) \tau. \quad (6)
\]

For a Hodgkin-Huxley model (\( n = 4 \)), \( \Delta t = 1.39 \tau(I_K) \) represents the slow time constant of \( I_K \) relaxation. In contrast, the data in Fig. 2 A indicate \( \Delta t = 0.12 \gamma(I_K) \).

The rapid attainment of an exponential time course in Fig. 2 A suggests that the transitions responsible for the delay equilibrate within 1 ms and that a much slower process limits activation during the subsequent 30 ms. Scheme II can reproduce such behavior only when subunits interact in a highly negatively cooperative manner such that one transition becomes rate limiting while others equilibrate rapidly. An alternative model (Scheme IV) can account for \( I_K \) kinetics with the assumption that subunits undergo rapid independent transitions, as in Scheme II, but that channel opening involves an additional conformational change that is slow and rate limiting (Ledwell and Aldrich, 1999).

![Scheme IV](image_url)
The equilibrium properties of Schemes V and VI are identical, but Scheme VI can fit \( I'_K(t) \) with \( X = 3 \) (Fig. 2 B2). Similar results are obtained if only the forward rate is effected by subunit activation, thereby altering the equilibrium constants (\( X = 3.5 \), data not shown). However, our most favored model, discussed later, can account for these results without abandoning the idea that subunits undergo independent transitions.

Several experimental factors might contribute to a deviation between \( I'_K \) kinetics and the prediction of Scheme V. The membrane charging time constant for an excised patch is expected to be very fast (\( \tau = C_m R_s \approx 1 \mu s \), see Methods) and should not affect \( I'_K \) kinetics. The relaxation of the capacitive transient in Fig. 1 B2 (control) is mainly limited by the filtering of the current signal. To test whether filtering affects \( I'_K \) kinetics, simulated traces in Fig. 2 were convolved with the impulse responses of 100- and 30-kHz eight-pole bessel filters to reproduce experimental conditions (see Methods). The filtered and unfiltered traces were indistinguishable after compensating for a 22-\( \mu s \) filter delay (data not shown). Thus, \( I'_K \) kinetics are not modified by filtering and appear to represent a genuine property of the channel.

It is conceivable that Scheme V could give rise to the observed delay kinetics if channels were distributed in states other than \( C_0 \) at the start of the voltage pulse. This possibility was ruled out by examining the effect of initial conditions on the delay. Fig. 3 A shows the time course of \( I_K \) evoked at +180 mV after a 1-ms prepulse to voltages between \(-80 \) and \(+120 \) mV. Prepulses to voltages \(-10 \) mV produced a progressive decrease in the delay, resulting in a shift of \( I_K \) along the time axis analogous to that reported by Cole and Moore (1960) for \( K^+ \) current in squid axon. A similar effect was reported for hSlo channels by Stefani et al. (1997). This Cole-Moore shift indicates that the initial distribution of channels among closed states is voltage dependent. However,
prepulses to −80 or 0 mV had no detectable effect on the delay, suggesting that the closed state distribution does not change at voltages <0 mV. This result supports the assumption that channels mainly occupy the ground state (C₀) at −80 mV.

Voltage Dependence of the Delay in Iₖ Activation

Although the delay kinetics deviate from the prediction of Scheme V, they are consistent with the idea that multiple closed-state transitions precede channel opening. In addition, the dependence of the delay on prepulse voltage implies that closed-state transitions are voltage dependent. To help characterize the voltage dependence of these early transitions, we measured the delay duration (Δt) during prepulses to different voltages. Δt was determined by fitting Iₖ with exponential functions (Eq. 4), as shown in Fig. 3 B, Fig. 3 C plots Δt on a log scale versus pulse voltage for two different experiments. The average Δt-V relationship is plotted in Fig. 3 D (mean ± SEM, n = 6). The delay is maximal at approximately +155 mV and exhibits a bell-shaped voltage dependence.

The two Δt-V relationships in Fig. 3 C are similar in shape but differ in magnitude by ~25%, possibly reflecting variations in temperature (T = 5 ± 1°C) (Δt has a Q₁₀ of 2.3 based on comparison of the delay at 20°C in Fig. 1 B (210 μs) to the mean delay measured at 5°C (725 μs) at +160 mV). To compensate for patch-to-patch variation in the magnitude of Δt, the average Δt-V relationship in Fig. 3 D was determined after first normalizing the component Δt-V’s to the mean Δt at +180–195 mV. The ability of the average and individual relationships to be fit by the same functions (Fig. 3, C and D, solid lines), discussed below, argues that the average accurately captures the shape of the Δt-V.

In general, the relationship between the Δt and the kinetics of closed-state transitions will not be as simple as described for Scheme II (Eq. 6). The time course of Iₖ, and therefore the delay duration, may be influenced by transitions between closed and open states and by O-O transitions if multiple open states exist. Previous
studies of Shaker K⁺ channels have taken the approach of measuring delay duration at high voltages to estimate the kinetics of closed-state transitions (Zagotta et al., 1994b; Schoppa and Sigworth, 1998a). At sufficiently positive voltages, the backward rate constant from open to closed is assumed to be small such that the occupancy of the open state reflects only the rate of leaving the closed state. In other words, the time course of I_K approximates the cumulative distribution of latencies to first opening, and Δt is highly dependent upon closed-state transition kinetics.

In this study, Δt was measured over a range of voltages where backward rate constants from open to closed may not be negligible. However, the initial time course of I_K(t) during the delay should be determined mainly by the rate of leaving the closed state when P_o is small (Eq. 10). Thus, the initial time course of I_K and the delay duration will be determined by closed-state transition kinetics provided I_K achieves an exponential time course while P_o is small. This condition appears to be satisfied at all voltages where Δt was measured for mSlo (see Figs. 1 B, 3 B). Therefore, the Δt–V relationship should provide information about the voltage dependence of closed-state transitions. Such an argument cannot be made in the case of Shaker K⁺ channels because the time course of activation has pronounced sigmoidicity such that P_o is not small when I_K achieves an exponential time course.

The precise relationship between closed-state transition kinetics and Δt is model dependent. For some models, including Scheme V, Δt will be proportional to the time constant of voltage sensor movement τ_a (provided I_K achieves an exponential time course when P_o is small and transitions among closed states are fast relative to channel opening); in which case, the Δt–V relationship can be used to determine the voltage dependence of the R–A transition (see appendix).

Consistent with the prediction that Δt reflects the voltage dependence of τ_a, the Δt–V relations can be fit by functions of the form Δt = 1/(a + b), where a = a_o e^{(z_o/2kT)}, b = b_o e^{(-z_o/2kT)} (z_o = z_a = 0.28 e) (Fig. 3, C and D, solid lines). The bell-shaped voltage dependence is consistent with a process governed by a single transition with voltage-dependent forward and backward rate constants. If Δt is proportional to τ_a(V) then z_a = z_o and z_b = z_o, and the fit to the average Δt–V relationship in Fig. 3 D implies that the transition from R to A involves a total charge movement (Δz = z_a + z_b) of 0.56 e with a half-activation voltage (V_ah) of +153 mV, corresponding to the peak of the Δt–V relationship (Fig. 3 D, dashed line). Although our final model is more complicated than Scheme V, we show later that it can reproduce the Δt–V relationship using similar parameters for the voltage-sensor transition (z = 0.55 e, V_ah = +145 mV). Moreover, gating current measurements in the companion article produce similar results (z = 0.55 e, V_ah = +155 mV) (Horrigan and Aldrich, 1999). Thus measurements of I_K delay appear to provide a reasonable method for characterizing mSlo voltage-sensor movement.

The Voltage Dependence of I_K Relaxation

The predominantly exponential time course of I_K suggests that mSlo activation is dominated by a single rate-limiting step. To study the properties of this transition, we examined the voltage dependence of I_K relaxation kinetics. The time constant of I_K relaxation, measured after the delay, changes with voltage, suggesting that the rate-limiting step may be voltage dependent (Cox et al., 1997a). But mSlo gating is a multistate process with rapid, voltage-dependent transitions among closed states. While the kinetics of closed-state transitions are too fast to limit the exponential relaxation of I_K, τ(I_K) may be influenced by the equilibrium distribution of closed states. Therefore, the voltage dependence of τ(I_K) may reflect a voltage dependence of the closed-state equilibria in addition to the rate-limiting step. For example, Scheme V predicts:

\[ \tau(I_K) = \frac{1}{\delta P_{C_L} + \gamma} \] (11)

where P_{C_L} represents the conditional probability that a closed channel occupies the last closed state [i.e., P(C_{4|C}|C) for Scheme V]. Eq. 11 is valid for any scheme with a single open state and a single closed–open transition, provided the C–O transition is rate limiting and the preceding C–C transitions are equilibrated. Although Eq. 11 contains a voltage-dependent contribution from closed-state equilibria (P_{C_L}), at extreme voltages, τ(I_K) depends only on the rate-limiting step. For example, at negative voltages where P_{C_L} is small, τ(I_K) = 1/Δt, at positive voltages where P_{C_L} = 1 and Δt >> γ, τ(I_K) = 1/δ. Even if P_{C_L} fails to achieve limiting values of 0 or 1, the voltage dependence of τ(I_K) at extreme voltages should reflect only the voltage dependence of δ or γ, provided P_{C_L} is relatively constant.

The voltage dependence of τ(I_K) was examined in an experiment illustrated in Fig. 4. I_K was activated by stepping from a holding potential of −80 mV to voltages between +100 and +240 mV (Fig. 4 A). I_K tail currents were recorded at more negative voltages, following a 50-ms depolarization to +120 mV (Fig. 4, B–D). In all cases, the time course of I_K was well fit by an exponential function after a brief delay (Fig. 4, solid lines). τ(I_K) is plotted from +30 to +240 mV in Fig. 5 A and exhibits a bell-shaped voltage dependence that can be fit by a two-state model (solid curve) (Cui et al., 1997). This behavior also appears consistent with the prediction of
Scheme V because \( \tau(\text{IK}) \) increases exponentially with voltage from +30 to +110 mV and decreases exponentially from +180 to +240 (Fig. 5 A, dashed lines) as if \( \tau(\text{IK}) \) is determined by single voltage-dependent rate constants at these voltages. However, our analysis of the delay in IK activation suggests that the voltage range in Fig. 5 A is insufficient to observe the limiting voltage dependence of \( \tau(\text{IK}) \). The Cole-Moore shift (Fig. 3 A) indicates that closed-state equilibria change from 140 to 1120 mV, and the weak voltage dependence of the delay (Fig. 3 C) suggests that these equilibria continue to change over a large voltage range.

To test the limiting behavior of \( \tau(\text{IK}) \), tail currents were measured at very negative voltages (Fig. 4, C and D). Fig. 5 B plots \( \tau(\text{IK}) \) for the data in Fig. 4 down to Figure 4. The voltage-dependence of IK relaxation. (A) Activation and (B–D) deactivation kinetics measured at 5°C are fit by exponential functions (solid lines). IK was activated in response to voltages from +100 to +240 mV. Deactivation was measured at the indicated voltages after a 50 ms depolarization to +120 mV. Figure 5. The voltage dependence of \( \tau(\text{IK}) \). (A) Time constants \([\tau(\text{IK})]\) from the fits in Fig. 4, A and B, are plotted versus voltage and fit (dashed line) by two exponential functions with the indicated equivalent charge (\( z \)). The prediction of a two-state model \([\tau(\text{IK})] = 1/(a + b), a = a \_1 \_e^{z \_1 e^{-\text{z \_1}}}, b = b \_1 \_e^{z \_1 e^{-\text{z \_1}}} \) is indicated by a solid line (\( z_1 = 0.37 \_e, z_2 = 0.67 \_e, z_2 = 4.86 \_s^{-1}, b_2 = 1.323 \_s^{-1} \)). (B) \( \tau(\text{IK}) \) is plotted on a log scale versus voltage for all the records in Fig. 4. Data were shifted along the voltage axis by \( \Delta V_h = +5.6 \) mV (see methods). Three regions of exponential voltage dependence are shown by dashed lines with the indicated equivalent charges (\( z \)). A solid curve indicates a fit to Scheme IX (Table I, Patch 1). (C) \( \tau(\text{IK}) \)–V plots obtained from multiple experiments at 5°C and 20°C (○) were normalized to mean \( \tau(\text{IK}) \) at −80 mV, and then averaged in 15-mV bins (●). Solid curves indicate fits of Scheme IX to the averaged data (Table I: average 5°C and 20°C). The dashed curve represents a fit of Scheme VIII to the average 5°C data for \( V < +100 \) mV (\( z_{\_1(0)} = -0.45 \_e, \_z_{\_1(0)} = 2.400 \_s^{-1}, z_{\_2(0)} = -0.14 \_e, \_z_{\_2(0)} = 700 \_s^{-1}, z_{\_2(0)} = -0.26 \_e, \_z_{\_2(0)} = 300 \_s^{-1} \)).

Scheme VIII
TABLE I

Scheme IX Kinetic Parameters

| Parameters | Patch 1 | Patch 2 | Average 5°C | Average 20°C |
|------------|---------|---------|-------------|-------------|
| α(0)*     | 276     | 257     | 238         | 1276        |
| β(0)      | 7650    | 7129    | 6596        | 35370       |
| δ1(0)*    | 0.00128 | 0.00384 | 0.00138     | 0.0074      |
| δ2(0)*    | 0.0218  | 0.0653  | 0.0235      | 0.126       |
| δ3(0)     | 0.370   | 1.11    | 0.399       | 2.14        |
| δ4(0)*    | 2.27    | 6.99    | 2.51        | 25.7        |
| γ1(0)     | 640     | 1923    | 690         | 3700        |
| γ2(0)     | 640     | 1923    | 690         | 3700        |
| γ3(0)     | 640     | 1923    | 690         | 3700        |
| γ4(0)     | 269     | 712     | 256         | 2612        |
| γ5(0)     | 98.3    | 294     | 94.8        | 295         |

*T these rate constants in combination with the following parameters are sufficient to define the kinetic behavior of the model (12)

\[ \alpha \beta = 1 \text{ at } +145 \text{ mV} \]



-360 mV and, in other experiments, \( \tau(I_K) \) was measured at voltages as low as -500 mV (Fig. 5 C). Tail currents were always well fit by exponential functions, but the \( \tau(I_K) \)-V relationship at negative voltages departs from that observed in Fig. 5 A. For \( V < +30 \text{ mV} \), the slope of the \( \tau(I_K) \)-V relationship decreases and achieves an exponential voltage dependence of only e-fold per 170 mV (0.14 e equivalent charge) from -360 to -40 mV. Fits to \( \tau(I_K) \)-V in Fig. 5 B define three regions of exponential voltage dependence, characterized by mean equivalent charges of +0.143 e ± 0.003, +0.49 e ± 0.02, and -0.29 e ± 0.02 (mean ± SEM, n = 5) over voltage ranges of -500 to -20 mV, +30 to +140 mV, and +180 to +280 mV, respectively.

A similar voltage dependence is observed at 20°C and 5°C (Fig. 5 C). The individual plots were normalized to the average time constants measured at -80 mV for 5°C (0.95 ± 0.04 ms, n = 6) or 20°C (0.172 ± 0.015 ms, n = 6). The increase in temperature speeds \( I_K \) relaxation 5.5-fold at all voltages (Q10 = 3.1) such that the shape of the \( \tau(I_K) \)-V is essentially unchanged. This also demonstrates that measurements of the \( \tau(I_K) \)-V relationship at negative voltages at 5°C are not limited by our ability to resolve fast tail currents. Nor is the limiting voltage dependence affected by series resistance error because tail current amplitudes saturate at voltages less than -150 mV (Fig. 4 D). In this experiment, the tail current amplitude was <5 nA at even the most negative voltages, the electrode resistance was 1 MΩ, and the series resistance error was ~5 mV or less and constant from -150 to -360 mV.

The \( \tau(I_K) \)-V relationship at negative voltages is likely to represent the limiting behavior of \( \tau(I_K) \) since no deviation from exponential voltage dependence was observed down to ~500 mV (Fig. 5 C). Unfortunately, the voltage dependence of \( \tau(I_K) \) could not be tested at very positive voltages. Recordings of \( I_K \) at extreme negative voltages were possible because tail currents decayed rapidly and the time spent at these voltages could therefore be minimized (e.g., 0.5 ms at -500 mV). Measuring the slower time course of \( I_K \) activation required longer voltage pulses (10 ms, 5°C), which tends to compromise patch stability above +300 mV. Shorter pulses can be used at 20°C because activation is faster, but higher temperatures also tend to reduce patch stability and thereby offset the benefit of reduced pulse duration.

Scheme V predicts that \( \tau(I_K) \) should achieve a limiting exponential voltage dependence, but the \( \tau(I_K) \)-V relationship is inconsistent with this model. According to Scheme V, \( \tau(I_K) \) = 1/ \( \gamma \) at negative voltages, and the exponential relationship defined by \( \tau(I_K) \)-V between -360 and -40 mV in Fig. 5 B (\( \tau_lim(\alpha) \)) should represent 1/ \( \gamma \). However, Scheme V also requires that \( \tau(I_K) \) satisfy the inequality \( \tau(I_K) \leq 1/ \gamma \) at all voltages because \( \delta PC_l + \gamma \geq \gamma \) (Eq. 11). The data clearly do not meet this condition because \( \tau(I_K) \) measured from +10 to +230 mV is up to fivefold greater than \( \tau_{lim(\alpha)} \) (Fig. 5 B).

The complex voltage dependence of \( \tau(I_K) \) could be accounted for by a sequential gating scheme that, unlike Scheme V, contains multiple rate-limiting transitions. In general, the relaxation of any system of n states will be multieponential with n - 1 characteristic time constants. In Scheme V, all but one of these time constants is fast and relaxes during the delay in \( I_K \) activation. The remaining slow time constant, reflecting the C-O transition, dominates the time course of \( I_K \) relaxation, and the voltage dependence of \( \tau(I_K) \) exhibits two regions of exponential voltage dependence associated with the forward and backward rate constants for this transition. If additional transitions in the activation pathway were slow, then they could also limit the time course of \( I_K \) relaxation over some voltage range, possibly contributing additional regions of exponential voltage dependence to the \( \tau(I_K) \)-V relationship.

A sequential scheme containing multiple rate-limiting transitions cannot be ruled out based on the data presented thus far. However such a model is difficult to reconcile with the observation that \( I_K \) relaxes with a predominantly single exponential time course at all voltages. The problem can be illustrated by attempting
to fit the $\tau(I_K) - V$ with a general sequential scheme containing a single open state (Scheme VII).

\[
C_0 \leftarrow \cdots \leftarrow C_{n-1} \xleftarrow{\alpha_{n-1}} C_n \xrightarrow{\beta_{n-1}} \overset{\alpha_n}{\underset{\beta_n}{O}}
\]

(SCHEME VII)

We will restrict our analysis to voltages less than +100 mV where steady state open probability is small (<10^{-2}), and consider the case where channels begin in the open state (i.e., tail currents). For Scheme VII, as for Scheme V, $\tau(I_K)$ is determined at negative voltages by the rate of leaving O [\(\tau(I_K) = \tau_{lim(-)} = 1/\beta_n\)]. Under what conditions will $I_K$ relax with $\tau(I_K) > 1/\beta_n$, as observed at more positive voltages? First, $\tau(I_K)$ can differ from 1/\(\beta_n\) only when $\alpha_n > 0$. Moreover, $\alpha_n$ must be large compared with $\beta_n$ for $I_K$ to relax with an exponential time course. To see this, consider the relaxation of $I_K$ at +100 mV where $\tau(I_K)$ is approximately fourfold greater than $\tau_{lim(-)}$ (Fig. 5 B). If $I_K$ is described by a single exponential function with $\tau(I_K) = 4\tau_{lim(-)} = 4/\beta_n$, then:

\[
O'(t) = \left(\frac{-\beta_n}{4}\right)O(t).
\]

When $O(0) = 1$, the initial rate of $I_K$ decay is given by $O'(0) = -\beta_n/4$, but the general scheme (Scheme VII) predicts a much faster initial decay $O'(0) = -\beta_n$, as required by the expression:

\[
O'(t) = \alpha_nC_n(t) - \beta_nO(t).
\]

Hence there must exist a fast component of $I_K$ relaxation that is not evident in the data.

If the amplitude of the fast component is small, then, after it decays, Eq. 13 should be approximated by Eq. 12. Equating Eqs. 12 and 13 gives:

\[
\alpha_n = 0.75 \beta_n \left[O(t) \right. \left. \frac{C_n(t)}{C_0(t)} \right]
\]

(SCHEME VIII)

If $O(t)$ is still large after the fast component has decayed, then $[O(t)/C_n(t)]$ will be large, implying $\alpha_n > \beta_n$, and the final equilibrium constant for the C-O transition will be large ($\alpha_n/\beta_n >> 1$) even when $P_o$ is small (<10^{-2}), which suggests that some other forward rate constant in the activation scheme is small at +100 mV. If $\alpha_{n-1}$ is very small when $V < +100$ mV, then Scheme VII can be approximated by a three-state model (Scheme VIII), which can be solved exactly and is characterized by two time constants: $\tau_{slow}$ and $\tau_{fast}$.

The dashed line in Fig. 5 C shows that this scheme can fit the $\tau(I_K) - V$ relationship at 5°C (for $V \approx +100$ mV). Using the parameters in the figure legend, $\tau(I_K)$ is equal to $\tau_{slow}$ and $\tau_{fast}$ will be at least 4.5x smaller than $\tau_{slow}$. Such a fast component would be observable if its amplitude were significant, but the amplitude of the fast component was adjusted to <20% of the total by requiring $\alpha_n/\beta_n$ to be large ($\alpha_n/\beta_n = 2.1$ at +100 mV). Thus the fast component can be made small, giving the appearance of a monoexponential decay.

An Allosteric Model of Voltage-dependent Gating

Although the $\tau(I_K) - V$ relationship is inconsistent with Scheme V, the data can be explained in terms of the conformational events outlined in Scheme IV. If mSlo channels can open even when one or more subunits are in the R conformation, Scheme IV can be represented by a 10-state gating scheme with 5 open and 5 closed states (Scheme IX), where each horizontal transition (C-C or O-O) represents a subunit conformational change, and vertical transitions represent channel opening.

(SCHEME IX)
As before, subscripts (0–4) denote the number of activated subunits in each open and closed state. Scheme IX predicts that channels can open even if no subunits are activated (C0–O0). However, such an event should be rare because subunit activation is assumed to increase the probability of channel opening. This interaction is represented by a factor D. The equilibrium constants for the C–O transitions increase D-fold for each subunit that is activated. As will be shown later, this requirement favors the possibility that channels will pass through several closed states before opening in response to a voltage step, consistent with the presence of a delay in Iκ activation. In addition, if the C–O transitions are slow and rate limiting, while C–C and O–O transitions equilibrate rapidly, this model can account for Iκ kinetics that are essentially exponential after the delay at all voltages. At the same time, Scheme IX can reproduce the complex voltage dependence of t(ικ) because it contains multiple rate-limiting C–O transitions that dominate Iκ relaxation over different voltage ranges. Both the individual and average t(ικ)–V relationships in Fig. 5, B and C, are well (solid curves) fit by Scheme IX. The observation that a change in temperature has little effect on the shape of the t(ικ)–V relationship (Fig. 5 C) is also consistent with the idea that a single type of conformational change (C–O transition) limits Iκ relaxation at all voltages.

Scheme IX describes only the response of mSlo channels to voltage and contains no Ca2+-bound states; however, it clearly resembles Scheme I, used earlier to describe the interaction of Ca2+ with mSlo channels. Indeed these two models are strictly analogous. Like Scheme I (Cox et al., 1997a), Scheme IX assumes that channels undergo a central rate-limiting conformational change from closed to open, and this transition is allosterically coupled. In Scheme I, interaction of Ca2+ with binding sites on each of the four subunits enhances channel opening. In Scheme IX, voltage-dependent conformational changes in each subunit influence channel opening.

An important prediction of Scheme IX is that the limiting behavior of t(ικ) should reflect the voltage dependence of C–O transitions. According to the model, horizontal transitions equilibrate rapidly and open channels tend to occupy the left-most state (O0) at negative voltages such that the time constant of Iκ deactivation is determined by the rate constant (γ0) associated with the O0 to C0 transition [t(ικ) = 1/ γ0]. The t(ικ)–V relationship measured at limiting negative voltages therefore determines the value of γ0 used in the model and implies that channel closing is weakly voltage dependent (z0 = 0.138 ± 0.003 e, n = 11). For simplicity, we also assume for Scheme IX that all O to C transitions have the same voltage dependence.

The forward transitions from C to O also appear to be weakly voltage dependent. The rate constant from C4 to O4 can be determined by measuring the τ(ικ)–V relationship at limiting positive voltages, where τ(ικ) = 1/ δυ. Simulations of Scheme IX suggest that this limiting behavior will be observed at voltages greater than +300 mV, which cannot be attained under our experimental conditions. Nonetheless, the voltage dependence of τ(ικ)–V measured from +180 to +280 mV (z = 0.29 ± 0.02 e) places an upper limit on the charge associated with channel opening (see discussion). We assign a charge z0 = 0.26 e to the forward C–O transitions in the model, giving a total charge of zL = 0.40 e for the C–O equilibrium (zL = z0 + zL). This value of z0 provided reasonable fits to the τ(ικ)–V relationships (Fig. 5), and the resultant value of zL is also consistent with the voltage dependence of steady state open probability discussed later.

If the C–O transitions in Scheme IX are weakly voltage dependent, then the horizontal transitions involving subunit activation must account for the bulk of the channel’s voltage sensitivity. To account for the Δτ–V relationship, the Cole–Moore effect, and the voltage dependence of P0, we have assigned a charge zL = 0.55 e and half activation voltage Vh(J) = +145 mV to the equilibrium constant for subunit activation:

$$J = e^{(V - Vh_J)/z_L}.$$

Thus, a total charge of 4zL = 2.2 e should be associated with the horizontal transitions, and zL = 0.4 e, or 15% of the estimated total charge (zT = zL + 4zL = 2.6 e), is associated with channel opening.

From a mechanistic standpoint, the assignment of most of the charge to the horizontal transitions in Scheme IX implies that subunit activation involves movement of the channel’s intrinsic voltage sensor. Thus Scheme IX not only divides mSlo activation into fast and slow transitions, but also separates voltage-sensor movement from channel opening. In other words, the model suggests that voltage-sensor activation and channel opening represent distinct conformational events that are allosterically coupled.

The Voltage Dependence of Steady State Activation

The conductance–voltage (Gκ–V) relationship was measured in 0 Ca2+ (at 20°C) using both macroscopic and single channel currents to examine steady state activation over a wide range of voltage and open probability (Fig. 6). Fig. 6 A shows a family of macroscopic Iκ evoked in response to 20-ms pulses to different voltages. Steady state conductance (Gκ) was determined from tail current amplitudes, normalized to maximal conductance (Gmax), and plotted against voltage (Fig. 6, B and C). Data from many individual experiments
are plotted (○). To compensate for patch-to-patch variation in half-activation voltage, individual plots were shifted along the voltage axis to align them with the mean $V_h$ (190 ± 2.3 mV; SEM, $n = 20$) (see methods). These shifted data were then combined in 15-mV bins to determine the average G–V (Fig. 6, B and C, ●).

The predictions of sequential Scheme V and the allosteric Scheme IX are superimposed on the data in Fig. 6 B (dashed lines) and are essentially indistinguishable from each other over this voltage range. The G–V can also be well fit by a Boltzmann function that is raised to a power of 3.2 (Fig. 6, B and C, solid line) (Cui et al., 1997). Many sequential models like Scheme V predict a G–V relationship that can be approximated by a Boltzmann function raised to power $>1$ (Wyman and Gill, 1990; Zagotta et al., 1994b). Such Boltzmann-like functions achieve a maximal limiting voltage dependence at negative voltages. The semi-log plot in Fig. 6 C demonstrates that the data are consistent with a such a relationship over a large range of open probability, and appear to achieve a limiting logarithmic slope of e-fold per 12.6 mV ($z = 2\pi$) (Fig. 6 C, dashed line).
However, while this slope does indicate the maximum voltage dependence of the mSlo G–V, it does not represent the limiting voltage dependence. Measurements of channel activity at more negative voltages (Fig. 6 D) reveal a marked decrease in the voltage dependence of steady state activation (Fig. 6 E), which deviates from a Boltzmann-like G–V relationship (Fig. 6 F) and provides evidence for the presence of multiple open states.

The data in Fig. 6 D were recorded at +80 and −80 mV from a macropatch containing several hundred mSlo channels. At these voltages, where \( P_o \) is small (<10\(^{-3}\)), single channel openings are observed. All-point amplitude histograms like those in Fig. 6 D were constructed by recording such events for 5–45 s and were used to evaluate total open probability at each voltage (V ≤ +80 mV; see methods). These data were then normalized based on macroscopic currents recorded in the same patch to determine normalized open probability (\( P_{o/\text{omax}} \), which is plotted against voltage in Fig. 6 E.

The normalized \( P_{o/V} \) relationships in Fig. 6 E were obtained from several experiments in 0 Ca\(^{2+}\) or 4.5 \( \mu \)M Ca\(^{2+}\). \( P_{o/\text{omax}} \) at +80 mV in 0 Ca\(^{2+}\) are comparable with those measured from macroscopic currents at the same voltage (Fig. 6 C). Similarly, the voltage dependence of \( P_o \) from +20 to +80 mV is comparable with the maximal slope of the macroscopic G–V, indicated by a dashed line in Fig. 6 E. However, a decrease in the slope of the \( P_{o/V} \) relationship is observed at more negative voltages both in the presence and absence of Ca\(^{2+}\). That is, \( P_o \) at negative voltages is greater than predicted from the Boltzmann-like fit to the macroscopic G–V. This deviation cannot be due to a failure to detect brief openings, since missed events should be more prominent at negative voltages and lead to an underestimate of \( P_o \). Furthermore, the shape of the normalized \( P_{o/V} \) relationship is similar in the presence or absence of Ca\(^{2+}\) despite the fact that Ca\(^{2+}\) increases the mean open time (Magleby and Pallotta, 1983b).

The normalized \( P_{o/V} \) relationships obtained from many experiments in 0 Ca\(^{2+}\) are plotted on a semi-log scale in Fig. 6 F for voltages from −120 to +300 mV. Filled symbols indicate averages (mean ± SEM, 15-mV bin width) while open symbols represent data from individual experiments. The data from single channel activity (\( P_{o/\text{omax}} < 10^{-3} \)) are continuous with the macroscopic data and are weakly voltage dependent at negative voltages where \( P_{o/\text{omax}} = 10^{-5} - 10^{-6} \). The prediction of sequential Scheme \( V \) is superimposed on the plot (dashed line), the data deviates from this Boltzmann-like relationship for \( P_{o/\text{omax}} < 10^{-4} \). The allosteric model, Scheme IX provides an excellent fit (Fig. 6 F, solid line) over the entire voltage range using the same parameters that describe the \( \tau(I_K)–V \) relationship in Fig. 5.

The decrease in voltage dependence of steady state activation observed at low \( P_o \) could be caused by a small subpopulation of malformed mSlo channels or endogenous channels that fail to close in a normal voltage-dependent manner. However, several lines of evidence argue that the weak voltage dependence of \( P_o \) represents the behavior of normal mSlo channels. First, the normalized \( P_{o/V} \) relationships recorded from different experiments are similar (Fig. 6 E) and are therefore unlikely to represent a variable mixture of channel types. Second, the large amplitude of single channel events measured at negative voltages (Fig. 6 D) clearly identify them as mSlo channel currents. Third, \( P_o \) is Ca\(^{2+}\) sensitive even at potentials where the voltage dependence of \( P_o \) is weak (Fig. 6 E). Fourth, the relative probability of observing multichannel openings was well described by a Poisson distribution (data not shown), consistent with the presence of a large uniform population of channels with low \( P_o \), rather than a small subpopulation of malformed channels with high \( P_o \). Finally, although the number of events collected were insufficient to analyze the single channel kinetics in detail, open times observed at −80 mV in 0 Ca\(^{2+}\) were very brief (95% were <160 ms, measured with a 50% amplitude criterion), consistent with the observation that macroscopic \( I_K \) deactivation is very fast at the same voltage (mean \( \tau(I_K) = 172 \mu s \) at 20°C).

The complex voltage dependence of steady state activation provides support for the conclusions that mSlo voltage gating involves multiple open states and that the C–O transitions are weakly voltage dependent. The limiting voltage dependence of mSlo indicates that a weakly voltage-dependent pathway exists between the resting closed state and an open state even though channel opening at more positive voltages proceeds through one or more voltage-dependent routes. Thus the “limiting slope” of the G–V relationship cannot be used as an estimate of total gating charge for BK channels. This behavior can be explained by models like Scheme IX, where multiple voltage-dependent pathways exist between closed and open states. According to Scheme IX:

\[
P_o = \frac{1}{1 + \left(\frac{1 + j}{L + 1 + D}\right)^4}.\]  

(15)

At negative voltages, where \( j \) is small (\( j << 1/D \)), this reduces to

\[
P_o = \frac{L}{1 + L}.
\]

(16)

if \( P_o \) is also small (\( L << 1 \):

\[
P_o = L.
\]

(17)
Thus, at negative voltages, \( P_0 \) is determined by the \( C_0-O_0 \) equilibrium constant \( (L) \). That \( P_0 \) is weakly voltage dependent is consistent with the notion that C–O transitions are weakly voltage dependent. The fit of Scheme IX to the average data in Fig. 6 F suggests that this limiting behavior of \( P_0 \) was obtained, and the value of \( L \) used in the model \( (2 \times 10^{-6}) \) is constrained by \( P_0 \) at negative voltages. However, the limiting voltage dependence of \( P_0 \) was not measured over a large enough voltage range to directly determine the voltage dependence of \( L \). Therefore \( z \) was adjusted to provide a reasonable fit to the overall shape of the \( P_0 \–V \) and \( \tau(I_K)–V \) relationships, as well as the limiting behavior of \( P_0 \).

Reproducing "Sequential" Behavior with the Allosteric Gating Scheme

At negative voltages, both the \( \tau(I_K)–V \) and \( P_0–V \) relations are weakly voltage dependent, consistent with the hypothesis that mSlo channels can open and close in a manner that does not involve voltage-sensor movement. We also have described features of mSlo behavior that can be described by a more conventional sequential gating scheme (Scheme V) \((\text{e.g., the kinetics and voltage dependence of the delay in } I_K \text{ activation})\) and the ability of the macroscopic G–V to be approximated by a Boltzmann function raised to a power greater than one over a large range of \( P_0 \). The allosteric model (Scheme IX) also can account for these latter results.

The \( P_0–V \) relationship predicted by Scheme IX (Fig. 6 F, Scheme IIb) is almost indistinguishable from that predicted by Scheme V for \( P_0 > 10^{-3} \). To reproduce this behavior, the allosteric factor \( (D) \) in Scheme IX must be large. If \( D \) is large, channel opening at positive voltages most likely will occur only after all four voltage sensors have been activated, and the allosteric model then behaves much like a sequential scheme. The value of \( D \) \((17) \) used in the model was also constrained by the overall shape of the \( P_0–V \) relationship. At very negative voltages, \( P_0 \) is determined by the \( C_0–O_0 \) equilibrium constant \( L \); at positive voltages, the \( C_4–O_4 \) equilibrium constant \( (LD^4) \) becomes more important. Thus, the value of \( D \) is critical in determining the relative magnitude of \( P_0 \) at negative and positive voltages. The half-activation voltage of the G–V \( [V_h(P_0)] \) also depends upon \( D \), as well as the equilibrium constants \( L \) and \( J \). \( L \) is fixed by the limiting value of \( P_0 \) at negative voltages, \( J \) is constrained by the voltage dependence of the delay in \( I_K \) (see below), and there is little freedom to adjust \( D \) without producing unacceptable changes in \( V_h(P_0) \).

Figs. 7 and 8 show that Scheme IX can fit the kinetics of \( I_K \) activation, using the same parameters that reproduce the \( \tau(I_K)–V \) and \( P_0–V \) relationships. These fits were critical in constraining the model parameters associated with the horizontal transitions, corresponding to voltage-sensor activation.

The results presented in Figs. 1–3 show that mSlo \( I_K \) activates with a brief delay and that these kinetics can be approximated by a sequential model (Scheme V) that contains fast voltage-dependent transitions followed by a rate-limiting opening step. However the time course of \( I_K \) was not precisely reproduced by the
sequential scheme unless voltage sensors were assumed to interact in a cooperative manner (Scheme VI). The allosteric model can account for these kinetics while assuming voltage sensors act independently. Fig. 7 shows that Scheme IX reproduces the time course and delay in I_K activation (Fig. 7, A, B, and C) as well as I'_K (Fig. 7 B) at +180 mV, and the Cole-Moore shift (Fig. 7 C).

In Scheme IX, the delay duration (\(\Delta t\)) is influenced by the time constant associated with voltage-sensor activation while the channel is closed \([\tau_i = 1/(\alpha + \beta)]\) and therefore constrains the rate constants associated with C-C transitions. The equilibrium constant \(J\) and charge \(z_j\) associated with voltage-sensor activation are mainly constrained by the \(\Delta t\)-V and \(P_o\)-V relationships. Fig. 8 A shows that the allosteric scheme reproduces the initial time course of I_K activation, and therefore the delay at different voltages. Fig. 8 B compares the average \(\Delta t\)-V data to the predicted \(\Delta t\) (solid line) and \(\tau_i\) (dashed line). Scheme IX predicts that the \(\Delta t\)-V and \(\tau_i\)-V relationships will be similar in shape but, in contrast to Scheme V, their maxima will not be at the same voltage \([V_{\text{max}}(\Delta t) = 153 \, \text{mV}, V_{\text{max}}(\tau_i) = 145 \, \text{mV}]\). This small difference, representing a voltage-dependent change in the relationship between \(\Delta t\) and \(\tau_i\), reflects the ability of channels to open before all four voltage sensors are activated. The \(\Delta t\)-V relationship was measured mainly at voltages more positive than \(V_{\text{max}}(\Delta t)\), and therefore constrains the charge associated with voltage-sensor activation \((z_j)\) more tightly than that associated with deactivation \((z_d)\). For simplicity, \(\Delta t\)-V was fit with the assumption that \(\tau_i\) is symmetrically voltage dependent, yielding \(z_a = z_b = 0.275\). Although the \(\Delta t\)-V data do not require symmetry, values of \(z_a\) and \(z_b\) close to these estimates are necessary to assign a reasonable total charge \([z_j = (z_a + z_b) = 0.55]\) to voltage-sensor activation. \(z_j\) is constrained by the fit to the \(P_o\)-V relationship (Fig. 6 F) and is consistent with the \(\tau_i\)-V relationship (Fig. 5, B and C) and Cole-Moore shift (Fig. 7 C). The accompanying paper shows that this estimate of \(z_j\) is also consistent with gating current measurements (Horrigan and Aldrich, 1999).

Delay in I_K Deactivation

One feature of Scheme IX that distinguishes it from models with a single open state is that it predicts a delay in I_K deactivation because channels can pass through several open states before closing. Tail currents in Fig. 4 C exhibit a slight delay, which is evident as a deviation from an exponential fit during the first 200 ms after a voltage pulse. Even at 5°C, this delay is brief and could be influenced by filter properties or series-resistance error. However, tail currents at more negative voltages in the same patch (Fig. 4 D) did not show such a deviation from exponential decay, suggesting that the tail current delay is not an artifact. The allosteric model reproduces these tail current kinetics, as shown in Fig. 8 C. Fig. 8 D compares tail currents and simulations at −40 and −360 mV. The data and prediction of Scheme IX (solid lines) deviate from an exponential time course (dashed lines) at −40 but not −360 mV. The delay disappears at very negative voltages because the time con-
stant associated with open-state transitions, representing voltage-sensor deactivation, decreases at negative voltages. It should be noted that, according to the allosteric model, the equilibrium constants for open-state transitions differ from those for closed state transitions by a factor \(D\). Thus, open- and closed-state transitions are expected to be characterized by different time constants. We expressed this difference as an increase in the forward \(O-O\) rate constants by \(\sqrt{D}\) and a decrease in the backward rates by the same factor relative to the corresponding \(C-C\) transitions.

**discussion**

We have studied mSlo channel currents at very low \([\text{Ca}^{2+}]\), to elucidate the mechanism of voltage-dependent gating. This procedure limits the number of accessible conformations to those without \(\text{Ca}^{2+}\) bound. To a first approximation, the voltage response of mSlo channels appears simpler than that of many voltage-sensitive channels and can reasonably be described by a two-state gating scheme (Cui et al., 1997). For example, the time courses of macroscopic \(I_K\) activation is well fit by an exponential function after a brief delay, and the time constant of \(I_K\) relaxation is much slower than the delay. At voltages where steady state \(P_o\) is significant (>10\(^{-3}\)), the kinetics of both activation and deactivation exhibit an exponential voltage dependence (Fig. 5 A) consistent with the presence of voltage-dependent forward and backward transitions between a closed and an open state. Finally, when estimates of equivalent charge associated with these transitions are summed, they are similar to that obtained by fitting the steady state \(G_{\text{K-V}}\) relationship with a Boltzmann function (Cox et al., 1997a).

These characteristics of mSlo gating appear simple and self-consistent, but closer examination of \(I_K\) kinetics and voltage dependence over a wider range of conditions reveals deviations from two-state behavior that imply a surprisingly complex underlying gating mechanism. An important conclusion of this study is that mSlo channel opening and voltage-sensor activation reflect distinct conformational events that occur on different time scales but are allosterically coupled. Based on the assumption that the channel has a voltage sensor in each of four identical subunits, this mechanism results in a 10-state gating scheme with five open and five closed states arranged in parallel. In this allosteric model, transitions among closed \((C-C)\) or open \((O-O)\) states are governed by rapid voltage sensor movements, while closed-open \((C-O)\) transitions are weakly voltage dependent and rate limiting. Because voltage-sensor activation is assumed to be much faster than channel opening, this scheme predicts exponential relaxation kinetics and other “simple” behaviors that can be approximated by a two-state model or a sequential gating scheme. However, these properties change with voltage in a manner that reflects the complexity of the underlying mechanism. In particular, the kinetic and steady state properties of \(I_K\) activation become weakly voltage dependent at negative voltages, suggesting that channel opening can occur in the absence of voltage-sensor activation.

The allosteric model of mSlo voltage gating has implications for understanding BK channel activation and voltage-dependent channel gating in general. First, the model establishes a framework for evaluating the effects of mutation on voltage-dependent BK channel gating. Second, the scheme forms a basis for understanding the effects of \(\text{Ca}^{2+}\) on BK channel gating. The demonstrated complexity of mSlo voltage gating in the absence of \(\text{Ca}^{2+}\) greatly increases the minimum complexity of models that include \(\text{Ca}^{2+}\)-bound states. The voltage-gating mechanism also raises the fundamental question whether \(\text{Ca}^{2+}\) acts by modulating voltage-sensor movement, channel opening, or some combination of both of these processes. Finally, the allosteric scheme may apply to other voltage-dependent channels. The following discussion considers these implications of the model in detail.

**Allosteric Voltage Gating and the Effect of S4 Mutation**

The allosteric gating scheme has implications for interpreting BK channel structure-function studies because some apparently simple features of macroscopic \(I_K\) kinetics and voltage dependence may be related in a complicated manner to elementary molecular events such as voltage-sensor movement and channel opening. This is illustrated by the example in Fig. 9.

Neutralization of a charged residue in the S4 segment of mSlo (R207Q) produces a marked decrease in the steepness of the G-V relationship and a shift of almost −100 mV in the half-activation voltage (Fig. 9 A) (Diaz et al., 1998; Cui, J., and R.W. Aldrich, manuscript in preparation). The S4 segment is thought to form part of the voltage sensor in voltage-gated channels and a reasonable hypothesis is that the mutation reduces the charge associated with voltage-sensor movement, thereby reducing the voltage dependence of channel activation. If mSlo channel voltage gating could be described by a two-state model, a decrease in gating charge would be required to account for a change in the steepness of the G-V. However, the allosteric model suggests a different explanation.

The wild-type (WT) G-V in Fig. 9 A was fitted by allosteric Scheme IX (solid line) using the same parameters as in Fig. 6 F. The R207Q G-V was then fitted (solid line) by changing the half-activation voltage for the voltage sensor \([\text{V}_{\text{h}}(j)]\) from +145 to −100 mV and leaving all other parameters the same as for the WT. That is, the effect of the S4 mutation can be accounted for
by increasing the equilibrium constant J 207-fold (ΔΔG = 5.33 kT) without changing the voltage sensor charge (z), the allosteric factor (D), the C–O equilibrium constant (L), or the charge associated with channel opening (z). According to the allosteric model, the shape of the G–V for R207Q mainly reflects the charge associated with the C4–O4 transition (z) because the voltage sensors are largely activated at voltages where P₀ is small. Thus the WT G–V is steeper because voltage-sensor activation and channel opening occur over the same voltage range. To test this conclusion, R207Q activation was examined at low P₀ (Fig. 9 B). The voltage dependence of P₀ for the mutant increases at negative voltages to a maximum slope like that exhibited by the WT (Diaz et al., 1998). This behavior is reproduced by Scheme IX (Fig. 9, solid lines), reflecting the ability of mutant voltage sensors to become deactivated at negative voltages, and consistent with the idea that the total gating charge for the two channels are similar. Gating charge measurements for WT and mutant channels in the companion article (Horrigan and Aldrich, 1999) support this conclusion.

Allosteric Voltage Gating and the Voltage Dependence of τ(Ik)

The voltage dependence of Iₖ relaxation kinetics is another feature of mSlo gating that illustrates a complex relationship between molecular events and macroscopic behavior. As shown in Fig. 4 C and previously reported by Cox et al. (1997a), the time constant of Iₖ relaxation exhibits a bell-shaped voltage dependence over a 250-mV voltage range centered near the half-activation voltage for mSlo. The two regions of exponential voltage dependence might reasonably be interpreted as representing the voltage dependencies of rate limiting forward and backward transitions between single closed and open states. However, the allosteric scheme suggests that this apparently simple voltage dependence is an emergent feature of all transitions in the model and cannot be attributed to properties of individual rate constants.

According to Scheme IX, the horizontal C–C and O–O transitions will be so fast that they do not affect τ(Iₖ). This point was confirmed in Fig. 10 A by showing that the values of τ(Iₖ) measured from Iₖ simulations (symbols) can be reproduced by an analytical approximation of the τ(Iₖ)–V relationship (solid line) that assumes horizontal transitions are equilibrated:

$$\tau(I_k) = \left[ \sum_i (\delta_i p_{C_i} + \gamma_i p_{O_i}) \right]^{-1}, \quad (18)$$

where δ and γ are rate constants for the C–O transitions and pC and pO are conditional occupancies of the open and closed states [pC = p(C) and pO = p(O)].

The shape of the τ(Iₖ)–V relationship can to some extent be explained by comparing it to the time constants of the individual C–O transitions in the model [τ_i = (δ_i + γ_i)^{-1}] (Fig. 10 A, τ₀–τₙ, dashed lines). At limiting negative voltages, τ(Iₖ) is determined by the time constant of the Cₐ–Oₐ transition (τ₀) and at positive voltages by the Cₙ–Oₙ transition (τₙ). At intermediate voltages, τ(Iₖ) represents a weighted sum of the rate constants for all C–O transitions (Eq. 18), where the relative weighting depends on the equilibrium distributions of different closed and open states [pC(V) and pO(V)]. The time constants of individual C–O transitions (τ_i) provide a rough indication of the range spanned by τ(Iₖ) at intermediate voltages. However, τ(Iₖ) measured from +30 to +240 mV in Fig. 4 C reflects the voltage dependence of the horizontal equilibrium in the model as well as the kinetics of the different C–O transitions, and cannot be attributed to a particular rate-limiting step. Thus, the exponential voltage dependence of τ(Iₖ) at intermediate voltages is coincidental and does not represent the voltage dependence of any one rate constant [although τ(Iₖ) can not be attributed to single rate constants, the rate-limiting transition always represent the C–O conformational change].

The simulation in Fig. 10 A predicts that τ(Iₖ) will achieve a limiting voltage dependence, representing the
Allosteric Voltage Gating of K Channels I

charge associated with the C4–O4 transition (z4), at voltages (>300 mV) that exceed our experimental range. Therefore, we used the exponential voltage dependence of \( \tau_{1v} \) between +180 and +280 mV (\( z_{1v} = 0.29 \pm 0.05 \text{ e} \)) as an upper limit for \( z_v \). For parameters that describe our data, Scheme IX always predicted \( z_{1v} = z_v \). This is not a general property of the model, however.

The allosteric model requires that the equilibrium constant for C–O transitions increase D-fold for each voltage sensor that is activated. The forward and backward rate constants are not otherwise constrained. \( \tau_{1v} \)-V relationships in Fig. 5 were fit with the additional assumption that forward rates increase and backward rates decrease monotonically with each voltage sensor activated. A conclusion of this analysis is that voltage-sensor activation affects mainly the forward rate constants. Fits to the 5° or 20°C data in Fig. 5 C required that the forward rates increase 11,500- or 6,700-fold when all four voltage sensors are activated, whereas the backward rates decrease only 7.3- or 12.5-fold, respectively. Indeed, the backward rates for the first three C–O transitions were assumed to be identical such that \( \tau_0, \tau_1, \tau_2 \) are identical at negative voltages as shown in Fig. 10 A. This result suggests that voltage-sensor activation may destabilize the closed conformation with little effect on the free energy of the open conformation or the transition barrier between O and C.

Allosteric Versus Sequential Gating Schemes

An important feature of the mSlo data that is reproduced by the allosteric model is that many kinetic and steady state properties of \( I_k \) can be approximated by a sequential gating scheme (Scheme V), except at extreme voltages. This is significant because many voltage-gated channels have been described by sequential models analogous to Scheme V; it is possible that such channels also operate through an allosteric mechanism, but have not been studied under conditions that reveal such a mechanism.

The distinction between allosteric and sequential schemes is important because the models make different predictions concerning the possible molecular events that link voltage-sensor movement to channel opening (discussed below). In addition, the allosteric scheme provides a simple explanation for cooperative interaction of voltage-sensors in voltage-dependent gating. Many channels exhibit behaviors that deviate from the predictions of completely independent schemes such as the Hodgkin-Huxley model (Hodgkin and Huxley, 1952). Such results can be described by gating schemes in which the voltage sensors interact in a cooperative manner such that the activation of one affects the movement of the others (Vandenberg and Bezania, 1991; Perozo et al., 1992; Tytgat and Hess, 1992; Zagotta et al., 1994a). We have used such a model (Scheme VI) to fit the delay in mSlo \( I_k \) activation. Scheme VI implies that there is direct communication between voltage sensors in different subunits. The allosteric model (Scheme IX) can account for the delay kinetics, as well as other features that are not reproduced by Scheme VI, using the simpler assumption that voltage sensors act independently while the channel is either closed or open. Cooperativity is embodied in the allosteric transition between these two conformations, a mechanism of subunit-subunit communication that is known to exist in many proteins (Perutz, 1989).

Given the ability of allosteric and sequential models to act similarly, it is important to define the conditions that allow them to be distinguished. The ability of the allosteric model to act like a sequential scheme was il-
illustrated by the comparison of $P_o$-$V$ relationships for Schemes V and IX (Fig. 6 F). The two predictions deviate significantly from each other only when $P_o$ is very small ($P_o < 10^{-4}$); the two models cannot be distinguished based on conventional G-V measurements. The allosteric model for mSlo behaves like a sequential scheme because the allosteric factor is large ($D = 17$), such that the equilibrium constant for the C-O transition increases by a factor of 83,000 ($D^4$) when all voltage sensors are in the activated state. Channel opening is most likely to occur after all voltage sensors have been activated, as in a sequential scheme.

The conditions under which Schemes V and IX converge can be defined by comparing the expression for $P_o$ from Scheme IX (Eq. 15) with that for a version of Scheme V (see Eq. 19), where the C$_4$-O$_4$ transition is assigned an equilibrium constant of $LD^4$, equivalent to the C$_4$-O$_4$ transition in Scheme IX.

$$P_o = \frac{1}{1 + \frac{(1 + J)^4}{LD^4}}$$  \hspace{1cm} (19)

Eqs. 15 and 19 are equivalent when $DJ$ is large ($>1$), so we can define a value of $J (J_{eq})$ such that Schemes IX and V behave equivalently when $J \approx J_{eq}$.

$$J_{eq} = 1/D.$$  \hspace{1cm} (20)

The value of $P_o$ where these two models converge ($P_{eq}$) is obtained by substituting $J_{eq}$ into Eq. 19:

$$P_{eq} = \frac{1}{1 + \frac{(1 + J_{eq})^4}{LD^4(J_{eq})^4}}.$$  \hspace{1cm} (21)

For constant $LD^4$, $P_{eq}$ will decrease when $J_{eq}$ decreases (when $D$ increases). For large $D$s, it is necessary to measure small $P_o$ to observe a divergence between the predictions of the allosteric and sequential models. This is illustrated in Fig. 10 B, which compares $P_o$-$V$ relationships predicted by Scheme IX for different values of $D$ (5-160). As $D$ is increased, the predictions of Scheme IX (solid lines) become increasingly difficult to distinguish from that of Scheme V (Fig. 10 B, dashed line).

Detection of Allosteric Voltage-gating Behavior

It is quite possible that a channel that activates through an allosteric voltage-gating mechanism mistakenly could be described by a sequential model if the allosteric factor ($D$) is large. The mSlo data was fit with $D = 17$, indicating that each activated voltage sensor changes the free energy difference between closed and open conformations by $\Delta G_{co} = \ln(D) = 2.83$ kT. If this interaction energy were increased by only 55% such that $D = 80$, measurements at $P_o < 10^{-4}$ would be required before deviations from a sequential scheme could be detected (Fig. 10 B).

Additional factors could serve to prevent identification of an allosteric mechanism. Deviations from sequential gating behavior were only clearly demonstrated for mSlo by studying ionic currents at extreme negative voltages and very low $P_o$, where the voltage sensors are presumed to be in a resting state such that the kinetic and steady state properties of $I_K$ are determined by the C$_0$-O$_0$ transition. In the case of mSlo, the equilibrium constant for this transition ($L$) is large enough that $P_o$ can be measured when voltage sensors are not activated. Similarly, the kinetics and weak voltage dependence of the O$_0$-C$_0$ transition allow us to resolve tail current kinetics at voltages as low as $-500$ mV. In channels where the O$_0$-C$_0$ transition is faster or its equilibrium constant smaller than that of mSlo, measurement of ionic current properties may be impractical under conditions that distinguish allosteric and sequential schemes.

The voltages at which $P_o$ can be measured may also limit the identification of an allosteric mechanism. This point is illustrated by the behavior of the mSlo R207Q mutant (Fig. 9). In contrast to the wild type, the slope of the $P_o$-$V$ relationship for R207Q does not appear to decrease at negative voltages (Fig. 9 B), even though measurements were made down to $-180$ mV where $P_o$ is very small ($10^{-3}$). However, both WT and R207Q $P_o$-$V$ relationships are well fit by the allosteric model using identical values of $L$, $D$, $z_1$, and $z_2$, and different values of $V_h(\langle j \rangle)$ (Fig. 9 B, solid lines). These data imply that $P_o$ is more steeply voltage dependent for the mutant because the voltage sensors are significantly activated even at $-180$ mV [$V_h(\langle j \rangle) = -100$ mV]. The model predicts that R207Q will achieve the same limiting slope as the WT, but only at voltages more negative than $-200$ mV (Fig. 9 B). Thus, the identification of a weakly voltage-dependent limiting slope may be impossible if $P_o$ cannot be measured at voltages sufficiently negative to force the voltage sensors into the resting state.

Previous Evidence for Allosteric Voltage Gating

Allosteric models analogous to Scheme IX have been proposed to describe the voltage-dependent activation of several channels, including L-type Ca$^{2+}$ channels (Marks and Jones, 1992), Shaker K$^+$ channels (McCormack et al., 1994), and SR Ca$^{2+}$-release channels (Ríos et al., 1993). These examples are relevant to the above discussion, present some interesting contrasts to mSlo, and illustrate an alternative approach to testing the allosteric model. In all three studies, it was the action of a ligand rather than the properties of gating under control (ligand-free) conditions that provided the most compelling evidence for an allosteric voltage-gating mechanism. In other words, gating properties in the
absence of ligand were consistent with an allosteric model, but were also adequately described by sequential schemes analogous to Scheme V or VI. Measurements were not performed at extreme voltages or low $P_o$ to distinguish these possibilities. And the allosteric model parameters $L$ and $D$ that were used to describe these channels suggest that such measurements would be at least as difficult as they are for mSlo: $L = 7.8 \times 10^{-10}$, $D = 225$ (Marks and Jones, 1992); $L = 1.7 \times 10^{-5}$, $D = 49$ (McCormack et al., 1994); and $L = 1.3 \times 10^{-6}$, $D = 25$ (Rios et al., 1993). However, upon ligand application, channel gating was altered in complex ways that were most simply explained in the context of an allosteric voltage-gating mechanism.

Marks and Jones (1992) found that application of dihydropyridine agonists to Ca$^{2+}$ channels caused a decreased latency to first opening, an increase in the maximum $P_o$ and a slowing of $I_C$, tail currents. In addition, the $P_o$–V relationship was shifted to more negative voltages and became steeper. These diverse effects cannot be accounted for by a sequential scheme without assuming that ligand binding has complex effects on multiple transitions. In contrast, an allosteric scheme reproduces all the results with the simple assumption that agonist-binding increases the equilibrium constant ($L$) for the allosteric C–O transition. The decrease in first latency reflects that an increase in $L$ will allow channels to open when fewer than four voltage sensors are activated. Consistent with this prediction, two open dwell-time components were detected in the presence of ligand, providing evidence for multiple open states. And the predictions of the allosteric model concerning first latency are analogous to those concerning the kinetics of the macroscopic delay in the present study.

Rios et al. (1993) also concluded that multiple effects of an agonist (perchlorate) on SR Ca$^{2+}$-release channel conductance and gating currents could be explained simply in terms of an allosteric model where agonist binding enhances the allosteric transition (increasing $L$). Conversely, McCormack et al. (1994) proposed that inhibitory effects of 4AP on ionic and gating currents in Shaker channels can be explained in terms of a decrease in $L$ that favors the closed conformation.

In line with our results for mSlo, the three studies discussed above concluded that the allosteric transition between C and O was weakly voltage dependent or voltage independent. An important difference between these channels and mSlo is that their activation kinetics appear to be limited by voltage-sensor movement rather than channel opening. In the case of the Ca$^{2+}$ channel (Marks and Jones, 1992), C–O transitions are $\sim$100-fold faster than voltage-sensor activation, essentially opposite the relationship observed for mSlo. This difference results in activation kinetics for Ca$^{2+}$ channels and Shaker K$^+$ channels that are more sigmoidal than observed for mSlo (Zagotta et al., 1994b). Related differences in channel behavior are observed at the level of gating current kinetics (Horrigan and Aldrich, 1999). The relative speed of voltage-sensor movement and channel opening for mSlo turn out to be advantageous for dissecting transitions in the allosteric scheme. For example, we are able to assume that the delay in $I_k$ activation mainly reflects voltage-sensor properties because channel opening is slow. Similarly, the $I_k$ relaxation time constant $\tau(I_k)$ was analyzed with the assumption that voltage sensors are equilibrated.

Limiting Slope of the $P_o$–V Relationship and Multiple Open States

Measurements of the limiting voltage dependence or "limiting slope" of the G–V relationship have been used extensively to estimate the total gating charge associated with activation of many voltage-gated channels (Almers, 1978; Zagotta et al., 1994b; Hirshberg et al., 1995; Noceti et al., 1996; Seoh et al., 1996) including BK channels (Diaz et al., 1998). Almers (1978) showed that if channel gating can be described by a linear sequence of closed states followed by a single open state, then the voltage dependence of $P_o$ will be maximal at limiting negative voltages. Sigg and Bezanilla (1997) generalized this conclusion to any model containing a single open state. For such schemes, the limiting voltage dependence of $P_o$ denotes the total gating charge moved during a transition from the resting closed state occupied at negative voltages (e.g., C0 in Scheme V) to the open state. In the few previous instances where single channel currents have been used to measure very low values of $P_o$, the voltage dependence of $P_o$ appeared to achieve a maximum at negative voltages consistent with the presence of a single open state (Hirshberg et al., 1995; Islas and Sigworth, 1996). However our results show that BK channel gating is not consistent with such a model since the voltage dependence of $P_o$ for mSlo decreases at negative voltages.

When the limiting and maximum voltage dependence of $P_o$ are different, as for mSlo, then the channel must have multiple open states and there must be voltage-dependent pathways between the open states (Sigg and Bezanilla, 1997). The allosteric voltage-gating scheme encompasses these general conclusions. A particular feature of this scheme is that the maximum slope of the $P_o$–V relationship reflects the charge associated with a subset of transitions and therefore underestimates the total gating charge. For example, the maximum slope of the fit in Fig. 6 F represents an equivalent charge of 1.74 e, underestimating the total charge (2.6 e) by 33%. In addition, the relationship between maximum slope and total charge may change when model parameters $L$ or $D$ are altered, and changes
Molecular Mechanism of Allosteric Voltage Gating

Experiments in many voltage-gated channels suggest that the S4 transmembrane segment forms at least part of the voltage sensor (Yang and Horn, 1995; Aggarwal and MacKinnon, 1996; Bao et al., 1999; Larsson et al., 1996; Mannuzzu et al., 1996; Seoh et al., 1996; Yang et al., 1996; Yusaf et al., 1996). Residues have also been identified that may form part of an activation gate that controls the flow of ions through the pore (Liu et al., 1997; Holmgren et al., 1998; Perozo et al., 1998). However, little is known about the molecular nature of the interaction between voltage sensors and the activation gates. Sequential models of voltage gating (Schemes III or V) suggest that voltage sensors form part of the activation gate or are directly linked to the gate in such a way that channel opening can only occur when all voltage sensors are in an activated conformation (Fig. 11 A). In contrast, the allosteric voltage-gating model for mSlo implies a less direct interaction between voltage sensor and channel pore, which is more difficult to envisage in terms of a physical model.

Many allosteric proteins have been studied whose molecular structure is known in detail, such as hemoglobin, glycogen phosphorylase, phosphofructokinase, and aspartate transcarbamoylase (reviewed by Perutz, 1989). These studies provide guidelines concerning the molecular nature of allosteric interactions that could prove useful in understanding how voltage-sensor movement and channel opening might interact in mSlo. These examples are multimeric proteins with multiple ligand-binding sites. Cooperative ligand interactions with these proteins are well described by models analogous to the MWC scheme, where the protein can undergo an allosteric conformational change that in turn affects the affinity of ligand binding sites. In all cases, the high and low affinity conformations of these proteins have been determined by x-ray crystallography. Two important conclusions from these studies are that allosteric transitions generally involve a concerted quaternary conformational change, and that the interaction of ligand-binding sites with the allosteric transition is mediated through subunit interfaces. One explanation for these results is that molecular interactions between subunits are weaker than those within subunits. Thus, a change in the tertiary conformation of a subunit due to ligand binding is more likely to alter the relative position and bonds between subunits than it is to perturb the tertiary conformation of an adjacent subunit. Ligand-binding sites in allosteric proteins are linked to subunit interfaces in such a way that a change in quaternary conformation is translated into a change in the structure of the binding site. Quaternary conformational changes often preserve a symmetric arrangement of subunits and are therefore concerted.

By analogy with other allosteric proteins, it is reasonable to propose that mSlo channel opening involves a quaternary conformational change and that voltage sensors affect the interaction between subunits. Structural studies of gap-junction channels and nicotinic Ach-receptor channels by Unwin and co-workers (Unwin and Ennis, 1984; Unwin et al., 1988; Unwin, 1998) support the idea that channel opening involves a quaternary rearrangement of subunits. Experiments examining the accessibility of Shaker channels to cysteine-modifying reagents are also suggestive of such conformational changes in voltage-gated channels (Holmgren et al., 1998). Voltage sensors might then affect channel opening if their activation produces a change in the steric or binding interactions between subunits. Fig. 11 B illustrates how such a mechanism might work in the context of the allosteric voltage-gating scheme. In this cartoon, channel opening is depicted as a concerted rotation of four subunits and voltage-sensor activation is shown as a tertiary conformational change within each subunit. For simplicity, only the states with all four voltage sensors in the resting (−) or activated
(±) states are shown, and transitions involving independent voltage-sensor movement are abbreviated by dashed arrows. Shaded areas between subunits represent hypothetical regions of subunit-subunit interaction. Voltage sensors are shown as interacting strongly with adjacent subunits only when they are in the resting conformation and when the channel is closed. Thus, voltage-sensor activation promotes channel opening by selectively destabilizing the closed conformation, as suggested by the kinetic data. Conversely, the equilibrium constant for voltage-sensor activation increases when the channel is open because the resting (±) state is no longer stabilized via interaction with adjacent subunits. Although the details of Fig. 11B are speculative, they are consistent with our data and suggest a possible molecular mechanism for interaction between voltage sensors and channel opening that may guide future investigation.

The Voltage Dependence of Channel Opening

The idea that mSlo channel opening involves a quaternary conformational change is attractive not only because it is consistent with an allosteric mechanism, but also because it provides a possible explanation for the voltage dependence of channel opening. The allosteric model assumes that channel opening and voltage-sensor activation are distinct events. Yet the data suggest that C–O transitions are voltage dependent and account for ~15% of the total gating charge movement. This result may appear to contradict the assumption that the C–O conformational change does not involve voltage-sensor activation. However, it is likely that channel opening could involve the movement of charged groups in the voltage sensor without requiring that voltage sensors “activate.” Quaternary conformational changes in proteins often involve rotations of subunits about an axis perpendicular to the axis of symmetry (Perutz, 1989). In the case of ion channels, such a motion might allow voltage sensors to rotate within the membrane electric field. Thus, channel opening could involve a displacement of voltage-sensor charge without requiring that voltage sensors undergo a tertiary conformational change (from R to A). This mechanism suggests that an alteration in voltage-sensor charge could affect the charge movement associated with both voltage-sensor activation and channel opening. We did not test this prediction, but several investigations of Shaker K⁺ channels have concluded that 10–17% of the total gating charge (z₁) is associated with channel opening (McCormack et al., 1994; Zagotta et al., 1994a; Smith-Maxwell et al., 1998; Ledwell and Aldrich, 1999) or final cooperative transitions (Schoppa et al., 1992; Sigworth, 1994; Schoppa and Sigworth, 1998b). Thus, the fraction of z₁ associated with channel opening is similar for Shaker and mSlo even though z₁ is approximately fivefold greater for Shaker (Horrigan and Aldrich, 1999).

This is consistent with mSlo and Shaker undergoing similar conformational changes, with the difference that the Shaker voltage sensor is more highly charged. In addition, mutations to the Shaker S4 segment can almost eliminate the voltage dependence of channel gating (Bao et al., 1999), consistent with a reduction of voltage-sensor charge contributing to a decrease in the voltage dependence of the channel opening transition.

Implications for Calcium-dependent Activation

The allosteric voltage-gating scheme has implications for understanding the effects of Ca²⁺ on BK channel function. Many of the effects of Ca²⁺ on mSlo channel activation can be described by a voltage-dependent MWC scheme, which operates under the assumption that Ca²⁺ allosterically modulates voltage-dependent channel activation (Cox et al., 1997a; Cui et al., 1997). This scheme further assumes that channel opening can be described by a single concerted voltage-dependent transition between a closed and open conformation. However, we have shown here that the voltage-dependent activation pathway is more complicated and can be separated into voltage-sensor movement and channel-opening steps. This description of voltage gating raises a question concerning the mechanism of Ca²⁺-dependent action: Does Ca²⁺ alter voltage-sensor movement or channel opening, or some combination of both? This question will be addressed in a subsequent article (Horrigan, F.T., and R.W. Aldrich, manuscript in preparation).

Regardless of the mechanism of Ca²⁺ sensitivity, the complexity of the allosteric voltage-gating scheme greatly increases the minimal complexity of any model of Ca²⁺-dependent activation. Previous work has demonstrated that mSlo exhibits a dose–response relationship for Ca²⁺ that is characterized by a Hill coefficient of ~1–3 (Cui et al., 1997). Thus, the channel must have at least three Ca²⁺ binding sites and it is reasonable to assume that there might four, one for each subunit. Given a 10-state model of voltage gating and four Ca²⁺ binding sites, a complete model of mSlo gating must contain a minimum of 50 states (for each state in the voltage-gating scheme there will exist at least one state with 0–4 Ca²⁺ ions bound). If Ca²⁺ binding affects voltage-sensor movement, a 70-state model is necessary. Even more states may be required if the relative position of Ca²⁺-bound subunits and activated voltage sensors within the channel homotetramer affect their interaction (Cox et al., 1997a; Horrigan, F.T., and R.W. Aldrich, manuscript in preparation).

Limitations of the Data

According to Scheme IX, measurements of τ(IK) and P₀ at negative voltages provide direct information about the kinetics and voltage dependence of the C₀–O₀ transition. However, not all parameters in the allosteric
model are as tightly constrained, and several simplifying assumptions have been made that require confirmation. For example, the properties of closed-state transitions (C–C) were determined based upon analysis of the delay in \( I_K \) activation as well as the voltage dependence of \( \tau(I_K) \) and \( P_o \). These data are consistent with the assumption that horizontal transitions represent the activation of four independent and identical voltage sensors and that the forward and backward rate constants for these transitions are symmetrically voltage dependent. Similarly, the assumption that forward and backward rate constants for voltage-sensor activation are symmetrically affected by channel opening appears consistent with the presence of a brief delay in the decay of potassium tail currents. However, ionic current data provides only an indirect measurement of voltage-sensor movement, and deviations from these simple assumptions are possible.

It thus becomes important to characterize the properties of voltage-sensor movement. In the companion article (Horrigan and Aldrich, 1999), gating current measurements were used to directly examine voltage-sensor movement. The results generally confirm the above assumptions and provide further support for the allosteric voltage-gating scheme.

**appendix**

To derive an expression for the delay duration \( (\Delta t) \), Eq. 4 is rewritten in terms of open-state occupancy:

\[
O(t) = O_o[1 - e^{(\Delta t - t)/\tau}], \quad t \geq T, \tag{22}
\]

where \( T \) is the time where \( O(t) \) achieves an exponential time course, and \( \tau \) is the time constant of \( I_K \) relaxation [\( \tau(I_K) \)]. \( dO/dt \) can then be defined after the delay:

\[
O'(t) = \frac{O_o}{\tau}e^{(\Delta t - t)/\tau}, \quad t \geq T. \tag{23}
\]

Combining Eqs. 22 and 23 gives:

\[
\left[ \frac{\tau O'(T)}{O(T) + \tau O'(T)} \right] = e^{(\Delta t - T)/\tau}. \tag{24}
\]

Eq. 24 can be linearized when \( ((\Delta t - t)/\tau) \) is small (when \( O \ll \tau O' \)). Such is the case for mSlo at \( t = T \), when \( I_K \) achieves an exponential time course:

\[
\left[ \frac{\tau O'(T)}{O(T) + \tau O'(T)} \right] = 1 + \frac{(\Delta t - T)}{\tau}; \tag{25}
\]

hence

\[
\Delta t = T - \frac{O(T)}{O'(T)}. \tag{26}
\]

Because \( O(T) \) is small during the delay \( O'(t) \) can be approximated by Eq. 10:

\[
O'(t) = \delta C_L(t) \quad t \leq T, \tag{27}
\]

and

\[
O(T) = \int_0^T O'(t)dt = \delta \int_0^T C_L(t)dt. \tag{28}
\]

Combining Eqs. 26–28 gives:

\[
\Delta t = T - \int_0^T \frac{C_L(t)dt}{C_L(T)}. \tag{29}
\]

Thus \( \Delta t \) is determined only by \( C_L(t) \). If transitions among closed states are assumed to be fast relative to channel opening, as in Scheme V, then the amplitude and time course of \( C_L \) should be functions mainly of voltage-dependent rate constants that characterize closed-state transitions.

In Scheme V, \( C_L \) can be expressed as the product of a voltage-dependent amplitude function \( F[v] = A[v]^4 \) and a time-dependent function of the voltage-sensitive activation time constant \( \tau_j \): \( G(t, v) = [1 - e^{-t/\tau_j}]^4 \) (see Eqs. 2 and 8):

\[
C_L(v, t) = F(v)G(t, v) \tag{30}
\]

For any model where \( C_L \) can be expressed in terms of \( G(t, v) \), Eq. 29 can be expressed in terms of only \( G(t, v) \):

\[
\Delta t = T - \int_0^T \frac{G(t, v)dt}{G(t, v)T}. \tag{31}
\]

Thus \( \Delta t \) is a function of \( \tau_j \). Now we determine the conditions under which \( \Delta t \) will be proportional to \( \tau_j \) or, equivalently,

\[
\frac{\Delta t(\tau_j[v_2])}{\Delta t(\tau_j[v_1])} = \frac{\tau_j[v_2]}{\tau_j[v_1]} = s \tag{32}
\]

or

\[
\Delta t[s\tau] = s\Delta t[\tau], \tag{33}
\]

where \( \tau = \tau_j[v_1] \) and \( s\tau = \tau_j[v_2] \). If Eq. 33 is true, then, by combining Eqs. 31 and 33,

\[
T - \int_0^T \frac{G(s\tau, t)dt}{G(s\tau, T)} = \left(T - \int_0^T \frac{G(\tau, t)dt}{G(\tau, T)}\right), \tag{34}
\]

where

\[
G(s\tau, t) = G_\infty \tag{35}
\]
for \( t \geq T \), Eq. 34 is true if
\[
G[\tau,t] = G[st, t] \quad (36)
\]
(a condition that holds for Scheme V). Eq. 36 implies:
\[
\int_0^T \frac{G[\tau,t]}{G[\tau,T]} \, dt = \int_0^T \frac{G[st, st]}{G[st, st]} \, dt. \quad (37)
\]
By changing the variable of integration \((r = st)\) this becomes:
\[
\int_0^T \frac{G[st, st]}{G[st, st]} \, dt = \frac{1}{s} \left[ \int_0^T \frac{G[st, r]}{G[st, T]} \, dr + \int_T^T \frac{G[st, r]}{G[st, T]} \, dr \right]. \quad (38)
\]
Given Eq. 35 then:
\[
\frac{G[st,r]}{G[st,T]} \, dr = 1 \quad (39)
\]
for \( r \geq T \). Combining Eqs. 37–39 and replacing \( r \) with \( t \) yields:
\[
\int_0^T \frac{G[\tau,t]}{G[\tau,T]} \, dt = \frac{1}{s} \left[ \int_0^T \frac{G[st, t]}{G[st, T]} \, dt + T (s - 1) \right], \quad (40)
\]
which can be rearranged to yield Eq. 34. Thus \( \Delta t \) is proportional to \( \gamma[v] \) for Scheme V under the conditions that, first, \( I_K \) achieves an exponential time course when \( P_0 \) is small and, second, transitions among closed states are assumed to be fast relative to channel opening.

We thank Dan Cox and Gargi Talukder for helpful comments on the manuscript.
This work was supported by a grant from the National Institutes of Health (NS23294) and by a National Institute of Mental Health Silvio Conte Center for Neuroscience Research grant (MH48108). R.W. Aldrich is an investigator with the Howard Hughes Medical Institute.

Submitted: 16 March 1999 Revised: 1 June 1999 Accepted: 7 June 1999

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