The stellar cusp around the Milky Way’s central black hole

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Abstract. The existence of stellar cusps in dense clusters around massive black holes is a fundamental, decades-old prediction of theoretical stellar dynamics. Yet, observational evidence has been difficult to obtain. With a new, improved analysis of high-angular resolution images of the central parsecs of the Galactic Center, we are finally able to provide the first solid evidence for the existence of a stellar cusp around the Milky Way’s massive black hole. The existence of stellar cusps has a significant impact on predicted event rates of phenomena like tidal disruptions of stars and extreme mass ratio inspirals.

1. Introduction
It is a standard paradigm of modern astrophysics that the majority of galaxies – possibly with the exception of very low mass galaxies and/or dwarf irregulars – contains massive black holes (MBHs) at their centres (see, e.g., [1] and references therein). Over the past two decades, mainly thanks to studies with the Hubble Space Telescope and high-angular resolution observations with Adaptive Optics at major ground-based telescopes, we have also learned that the majority of galaxies contains so-called nuclear star clusters (NSCs) at their photometric and dynamical centres. These clusters are the densest and most massive clusters that can be found in the present-day Universe. They have masses between a few $10^5$ to $10^8 \, \text{M}_\odot$ and half light radii of a few to a few tens of parsecs. NSCs do not consist of single-age stellar populations, but rather show signs of repeated star formation along their host galaxies’ lifetime, i.e. they are in their properties significantly different from globular clusters. Most intriguingly, NSCs have been found to coexist with massive black holes (for the properties of NSCs, see [2, 3, 4] and references therein).

The evolution of a star cluster containing an MBH is a decades-old problem of stellar dynamics that has been studied by a large number of authors with a wide range of techniques (analytical, Fokker-Planck, Nbody). Peebles (1972) [5] proved that, while the statistical thermal equilibrium must be violated close to a MBH in a galactic nucleus due to tidal disruptions, stellar collisions
and gravitational captures, there exists a steady state with net inward flux of stars. This is a quasi-steady state solution, where the stellar density takes a power-law form $\rho \propto r^{-\gamma}$. This finding was corroborated for a single-mass population by Bahcall and Wolf [6] and subsequently for multi-mass stellar populations with realistic number fractions (e.g., [7, 8, 9, 10, 11, 12, 13, 14] and references therein). This so-called stellar density cusp will be fully developed after a so-called relaxation time, the time necessary for the randomisation of the cluster phase space via close encounters between stars (so-called two-body relaxation), which is typically below a Hubble time for a NSC in a Milky Way-like galaxy. For a cluster composed only of stars of a single mass the predicted value is $\gamma = 1.75$. For realistic, multi-mass clusters, the lower mass stars have $\gamma = 1.5$, while the heavier stars (and in particular stellar mass black holes) will follow a steeper distribution ($\gamma_{BH} \approx 2$; see [13, 14]). The cusp will be well developed inside the radius of influence of the MBH, which is roughly the radius of a sphere that contains once to twice the black hole mass in form of stars (or stellar remnants) [11].

Although stellar cusps around MBHs are a robust theoretical prediction, they have been rather elusive observationally. That is mainly because of the great distances of MBHs: Their radii of influence are often resolved only by a few pixels in imaging or spectroscopy, which means that a handful of rare bright stars can bias the measured cluster structure significantly, as can be seen very well in the case of our own Milky Way (see discussion in [15]). When comparing theoretical predictions with observations in nature, it is also important to be aware that theoretical models are often highly simplified, e.g. they typically are setup with clusters of a single age stellar population, and do not suffer from observational effects such as strong and spatially variable interstellar extinction.

Figure 1. A progressive zoom into the Galactic Center. Left: Spitzer 4.5 $\mu$m image [16] of the central 100 pc $\times$ 100 pc of the Galaxy. Coordinates are given as offsets from the MBH Sgr A*. The NSC is the bright, compact structure in the centre of the image. Middle: NACO/VLT 2.2 $\mu$m image of the central 3 pc $\times$ 3 pc around Sgr A*. Right: NACO/VLT 2.2 $\mu$m image of the central 1.6" $\times$ 1.6" (0.06 pc $\times$ 0.06 pc) around Sgr A*. Stellar orbits determined by [17] are over-plotted onto the image. Sgr A* is visible as a near-infrared source at the image centre.

The best case for testing the stellar cusp hypothesis is the centre of the Milky Way. In the Galactic Centre (GC), an MBH of about $4 \times 10^6 M_\odot$, called Sagittarius A* (Sgr A*), is surrounded by an NSC of about $2.5 \times 10^7 M_\odot$, at a distance of $\sim$8 kpc ([18, 19, 17, 20] and references therein). With high angular resolution observations we can therefore probe the cluster structure and dynamics on scales of milli-parsecs, which means that the GC is a prime laboratory for studying the interaction of a stellar cluster with an MBH [21]. Figure 1 provides an overview of the GC and a progressive zoom toward the MBH Sagittarius A* (Sgr A*).
It is important to keep in mind the limitations on observational studies of the GC: (1) Due to the extremely high interstellar extinction, stars can only be detected with reasonable sensitivity in the near-infrared. In this region, however, intrinsic stellar colours are small and in combination with the strong differential reddening caused by the small-scale variability of extinction it is therefore very hard to classify the stars. For example, it is not trivial to distinguish between an old, cold giant star and a young, hot massive star by means of imaging and stellar colours. (2) Due to the extreme density of the NSC, one needs to work with high angular resolution, typically at the diffraction limit of ground-based 8-10m-class telescopes, which can only be achieved with special techniques (Adaptive Optics and speckle imaging). Even then, most faint stars (main sequence stars of less than two solar masses) cannot be detected because of the source crowding. Hence, we can only observe the tip of the iceberg, on the order a few to 10% of all the stars suspected to exist at the GC.

Because of the observational limitations, evidence on the existence of a stellar cusp at the GC was elusive. Although the detection of a cusp was claimed by first high angular resolution observations [15, 22], the actual situation turned out to be more complex. Only stars older than the cluster relaxation time can serve as suitable tracers of a cusp, but there are many massive young stars present in the central parsec around Sgr A*. These are too young to be dynamically relaxed. Once the pollution from young stars was taken into account, it appeared that the stellar surface number density within a few 0.1 pc of the MBH at the GC was close to flat, or even decreasing [23, 24, 25]. The lack of late-type giants close to the MBH had been reported before (see, e.g., discussion fo this topic in the review by [18]), but these more detailed studies were now considered to be incompatible with the existence of a stellar cusp. This is the origin of the missing cusp problem.

The existence or not of a stellar cusp around the MBH at the GC has implications for other galaxies as well, assuming that the Milky Way’s centre is representative for the nuclei of normal galaxies. Observational confirmation or rejection of a stellar cusp would not only have fundamental importance for our understanding of stellar dynamics. It is also of particular importance for gravitational wave astronomy. So called Extreme Mass Ratio Inspirals (EMRIs) are considered the most exquisite probes of General Relativity and of the related astrophysics of stellar remnant plus MBH systems [26]. The possibility of observing EMRIs with future gravitational wave observatories in space, such as LISA (Laser Interferometer Space Antenna ) [27, 28] or Taiji [29] is directly linked to the question whether stellar cusps exist around MBHs or not [30, 14].

Motivated by the fundamental importance of this topic, we have recently undertaken new observational and theoretical studies. Observationally, we pushed the detection limit to fainter stars. The observations were then compared to new, more realistic Nbody simulations. We have found that a stellar cusp does indeed exist at the GC and that its properties agree with our expectations. The details of our work can be consulted in these three papers: 1. Gallego-Cano et al. (submitted to A&A, arXiv:1701.03816); 2. Schödel et al. (submitted to A&A, arXiv:1701.03817); 3. Baumgardt et al. (submitted to A&A, arXiv:1701.03818). We will briefly describe our findings in the following sections.

2. Surface density of stars around the MBH Sgr A*
Before interpreting the stellar surface and light densities, it is important to have an approximate understanding of the stellar population we expect to observe at the GC. While we are still far from having a detailed understanding of the stellar population and its formation history in the inner few parsecs of the Milky Way, the following assumptions appear to be robust, according to our current best knowledge: The NSC has undergone repeated star formation episodes throughout its lifetime, with the most recent events occurring a few 10⁶ and a few 10 Myr ago. The majority of the stars are old, with an estimated 80% having formed more than
Figure 2. Properties of the observable stellar population at the GC. Left: Simple population synthesis model of the GC stellar population, assuming continuous star formation, modified from Fig. 16 in [15] (Reproduced with permission from Astronomy & Astrophysics, ©ESO). The distance modulus and extinction were added to the stellar magnitudes, which means that the magnitude values on the x-axis correspond to observed values. The upper panel shows the mean mass of all stars and of old stars (lifetime > 1 Gyr) only. The middle panel shows a luminosity function. The lower panel displays the fraction of old stars in each magnitude interval. The completeness limit due to source crowding of previous studies was limited to $K_s \lesssim 17.5$, while the sensitivity limit of spectroscopic studies is $K_s \approx 16$. At $K_s < 17.5$, number counts of old stars in the GC will therefore be dominated by horizontal branch/red clump (HB/RC) stars, core-helium burning giants. Right: $K_s$ luminosity function of stars within a projected distance of $R \lesssim 10''$ of Sgr A* extracted from a deep NACO/VLT S13 image. The bump centred at $K_s \approx 15.75$ corresponds to the RC. The sharp drop in detected stars at $K_s > 18$ is due to extreme source crowding, which impedes the detection of fainter stars. While RC stars and brighter giants have dominated the stellar density measurements of all previous work, we use measurements of resolved and unresolved stars at $K_s \gtrsim 17.5$ in our new work.

5 Gyr ago [18, 31].

The left panel in Fig. 2 shows a simple model of the stellar population of the NSC assuming a continuous star formation history (taken from Fig. 16 in [15]). As mentioned above, it is very hard to classify stars at the GC because photometric studies do usually not cover a sufficient wavelength range and are usually not accurate and precise enough to break the degeneracy between intrinsic stellar colours and interstellar reddening and because spectroscopic studies are extremely time-consuming and limited to the brightest stars. Therefore, the diagram in the left panel in Fig. 2 is valuable because it can give us an approximate idea of the ages and masses of stars of a given observed brightness at the GC. If we are interested in identifying a stellar cusp, then it is important to focus on stars that are at least a few Gyr old. We can do this by selecting spectroscopically classified stars and/or by limiting the brightness range of the tracer population. Previous studies of the NSC structure ([23, 24, 25]) were dominated by stars of K-band magnitudes brighter than $\sim 17.5$, in particular by Red Clump (RC) stars, helium core burning giants with observed K-band magnitudes 15 – 16 at the distance and extinction of the GC.

RC stars are a convenient tracer population because they are, on average, a few Gyr old. We
can also see in the left panel in Fig. 2 that stars of K-magnitudes 18 or fainter would also be suitable tracer populations because they have low mean masses and are therefore probably, on average, at least a few Gyr old.

In our new work, we focused explicitly on this faint, old, low mass stellar population. We used K-band high angular resolution observations from the camera NACO at the ESO VLT 8m telescope. Several epochs of high quality images were stacked in order to increase the sensitivity. Subsequently, correction factors for stellar crowding and interstellar extinction were determined and applied to the data. Systematic effects arising from different choices of key parameters in the software packages that identify the stars in images were explored. Finally, the stellar surface densities were determined as a function of distance from the central MBH. Spherical symmetry of the cluster was assumed, which is not strictly correct, but accurate on a 10-20% level. In parallel, we also explored the diffuse light density from unresolved stars that could be measured in the images after subtracting all identified, resolved stars and the emission from diffuse, ionised gas.

In order to constrain the structure of the entire NSC and to be able to deproject the observed quantities, these data were complemented, at projected distances \( R \gtrsim 1 \text{pc} \) from Sgr A*, by star density/surface brightness measurements from other sources, with lower angular resolution and/or sensitivity, but taken over a larger field ([20, 32]). The latter were scaled to our data in the overlapping regions. In Fig. 3 we show the surface brightness from unresolved stars from 0.01 pc to 20 pc as well as the star counts for faint resolved stars (K-magnitudes \( \sim 18 \)) and RC stars (K-magnitudes \( \sim 15-16 \)). The diffuse flux from the unresolved stars traces sub-giants and main sequence stars of K-band magnitude 19 – 20, which have masses of about 0.8 – 1.8 \( M_\odot \). All these three tracer populations have similar masses and can be old enough to be dynamically relaxed.

### 3. Properties of the stellar cusp at the GC

As Fig. 3 shows, the three studied tracer populations show a very similar distribution. The projected densities can all be approximated well by single power-laws at projected distances \( R < 1 \text{pc} \). Only the brightest population, the red clump giants, show some systematic deviations at around \( R = 0.2 \text{pc} \) and a possible decline at \( R < 0.05 \text{pc} \). The red line in Fig. 3 indicates a fit with a Nuker law [33]:

\[
\rho(r) = \rho(r_b)2^{(\beta-\gamma)/\alpha} \left( \frac{r}{r_b} \right)^{-\gamma} \left[ 1 + \left( \frac{r}{r_b} \right)^{\alpha} \right]^{(\gamma-\beta)/\alpha},
\]

(1)

where \( r \) is the 3D distance from the central MBH, \( r_b \) is the break radius, \( \rho \) is the 3D density, \( \gamma \) is the exponent of the inner and \( \beta \) the one of the outer power-law, and \( \alpha \) defines the sharpness of the transition. We carried out fits with different parameters (e.g., for the value of \( \alpha \) that was kept fixed at a value of 10 during the fits) and data (data at large \( R \) from [20] or [32]) and determined mean parameters and uncertainties from the resulting best-fit parameters of the different tries (the formal uncertainties were always very small).

We obtain, for the unresolved stellar light \( r_{b,diffuse} = 3.2 \pm 0.2 \text{pc} \), \( \gamma_{diffuse} = 1.16 \pm 0.02 \), \( \beta_{diffuse} = 3.2 \pm 0.3 \), and \( \rho_{diffuse}(r_b) = 0.67 \pm 0.05 \text{mJy arcsec}^{-3} \), for the star counts of faint, resolved stars \( r_{b,faint} = 3.0 \pm 0.4 \text{pc} \), \( \gamma_{faint} = 1.29 \pm 0.02 \), \( \beta_{faint} = 2.1 \pm 0.1 \), and a density at the break radius of \( \rho_{faint}(r_b) = 3900 \pm 900 \text{pc}^{-3} \), and for the RC stars \( r_{b,RC} = 3.4 \pm 0.5 \text{pc} \), \( \gamma_{RC} = 1.28 \pm 0.07 \), \( \beta_{RC} = 2.26 \pm 0.04 \), and \( \rho_{RC}(r_b) = 1700 \pm 500 \text{pc}^{-3} \).

We cannot directly compare the values of the normalisation parameters \( \rho \). As concerns \( \beta \), it is not very well constrained. On the other hand, the break radius \( r_b \) is well constrained around 3 pc and the inner power-law index \( \gamma \) as well. The break radius corresponds roughly to the radius of influence of the Sgr A*, i.e. the radius inside which we expect the stellar cusp to appear [11]. Both the stellar number and light densities may suffer from not well constrained...
Figure 3. Surface light/stellar number density at the GC plotted against the projected distance from Sgr A*. The black line is the surface light density from the unresolved stars (Schödel et al., submitted to A&A, arXiv:1701.03817). In order to constrain the shape of the NSC at distances > 2 pc, we used the data from [32] and scaled them in the overlap region to our data. A constant flux of 6.4 mJy arcsec$^{-2}$ was subtracted to remove the contamination by the surrounding nuclear bulge and Galactic bar, which can be approximated by a constant on these scales. The red line is a best-fit Nuker model. The blue and green lines are scaled stellar number densities from Gallego-Cano et al. (submitted to A&A, arXiv:1701.03816) for RC giants and faint subgiants/main sequence stars.

systematic biases (uncertainties in sky background subtraction or completeness corrections etc.). If we adopt conservative values from averaging the three values, we obtain $r_b = 3.2 \pm 0.2$ and $\gamma = 1.24 \pm 0.07$, taking the standard deviations as uncertainties. Obviously, the stellar density displays a cusp and a core-like (flat) density law can be excluded with very high confidence.

4. Discussion and conclusions
We find that the stars at the GC follow a cusp-like density distribution inside the radius of influence of the central MBH. Our findings resolve the ambiguity from previous work. The cusp is less steep than what is expected for a single age population that is older than the relaxation time. In the latter case one would expect to observe a so-called Bahcall-Wolf cusp with $\gamma \geq 1.5$. Numerical experiments were performed by Baumgardt et al. (arXiv:1701.03818) to interpret our data. They performed Nbody simulations of a more realistic NSC, assuming repeated star
formation episodes every 1 Gyr. This means that most stars will not have had the time to fully relax dynamically, which will flatten the cusp. The results of the simulations agree very well with the data.

There appears to be a lack of giant stars at small projected distances of $R \lesssim 0.1$ pc from Sagittarius A*. The missing few dozens of giants could be explained if their envelopes were removed in the past, thus rendering them invisible to observations. While collisions between stars or between stellar remnants and stars are probably not effective enough, collisions with high-density clumps in a formerly existing star-forming gas disc around Sagittarius A* provide a viable mechanism [34]. We know that such a disc must have existed in the past because of the presence of many young, massive stars in this region that move in a coherent, disc-like pattern (e.g., [35, 36, 18]).

We conclude that a stellar cusp exists around Sagittarius A* and that its properties agree with what we expect theoretically. Densities in excess of a few $10^7 M_\odot$ pc$^{-3}$ are reached at distances $r < 0.01$ pc from the central black hole (Schödel et al., submitted to A&A, arXiv:1701.03817). This has significant implications for gravitational wave astronomy. The existence of a stellar cusp in the Milky Way implies the existence of such structures in other galaxies with similar or smaller MBHs. Therefore we can expect to observe significant numbers of EMRIs with future space-based gravitational wave observatories [30, 37, 27, 28].

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