Flavour dependence of the pion and kaon form factors and parton distribution functions

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The separate quark flavour contributions to the pion and kaon valence quark distribution functions are studied, along with the corresponding electromagnetic form factors in the space-like region. The calculations are made using the solution of the Bethe-Salpeter equation for the model of Nambu and Jona-Lasinio with proper-time regularization. Both the pion and kaon form factors and the valence quark distribution functions reproduce many of the features of the available empirical data. The larger mass if the strange quark naturally explains the empirical fact that the ratio \( u^s \rightarrow (x) / u^q \rightarrow (x) \) drops below unity at large \( x \), with a value of approximately \( M^s_0 / M^q_0 \) as \( x \rightarrow 1 \). With regard to the elastic form factors we report a large flavour dependence, with the \( s \)-quark contribution to the kaon form factor being an order of magnitude smaller than that of the \( s \)-quark at large \( Q^2 \), which may be a sensitive measure of confinement effects in QCD. Surprisingly though, the total \( K^+ \) and \( \pi^0 \) form factors differ by only 10%.

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I. INTRODUCTION

In our quest to understand the structure of strongly interacting matter, parton distribution functions (PDFs) and electromagnetic form factors are of fundamental importance, and provide complementary information. In an infinite momentum frame picture the former describe the distribution of longitudinal momentum carried by each quark flavour, while the latter are related to their distribution transverse to the beam. There have been numerous studies of hadron PDFs and form factors within quark models of various degrees of sophistication and success, for example, see Refs. [1–18] and [19–32], respectively.

In this paper we focus on the structure of the pion and kaon, with a particular interest in the effects of the larger mass of the strange quark in the kaon. At present, a detailed understanding of pion and kaon structure is hampered by the rather small sample of experimental data [33, 34]. The pion PDF has been measured reasonably well in the valence region, and it is known that \( u^s \rightarrow (x) \) is somewhat softer than \( u^q \rightarrow (x) \) in the large-\( x \) region. While at the present time one does not know the separate flavour contributions to the kaon elastic form factor, it may prove possible to measure them in the future, for example, with a parity violating probe. Further, given the influence of the Drell-Yan-West relation [35, 36] and its phenomenological importance, it is of considerable interest to compare the flavour dependence of the large-\( Q^2 \) PDFs with the corresponding large-\( Q^2 \) behaviour of the separate flavour contributions to the elastic form factor.

We study the structure of the pion and kaon using the Nambu–Jona-Lasinio (NJL) model with proper-time regularization [37] to simulate the effect of quark confinement [38–40]. The separate contributions of each flavour to the pion and kaon elastic form factors are determined with and without the effect of vector-meson dressing at the quark-photon vertex. In comparison with existing experimental data the model shows excellent agreement. The PDFs are also calculated and the effect of the quark masses on the large-\( x \) behaviour is explored. We also investigate the effect of the spectator quark mass on the PDF for a given quark flavour, finding satisfactory agreement with the experimental ratio \( u^s \rightarrow (x) / u^q \rightarrow (x) \). We conclude with a discussion of the validity of the Drell-Yan-West relation within this framework.

II. NAMBU–JONA-LASINIO MODEL

The NJL model is a chiral effective theory that mimics many of the key features of quantum chromodynamics (QCD) and is therefore a useful tool to help understand non-perturbative phenomena in low energy QCD [41–45]. For example, the NJL model encapsulates dynamical chiral symmetry breaking, which gives rise to dynamically generated dressed quark masses. The NJL model has been successfully used to investi-gate a broad range of phenomena, including hadron properties [13, 46–52], heavy ion collisions [53], neutron stars [45, 54, 55], quark fragmentation functions [56, 57] and transverse momentum dependent phenomena [58].

The three-flavour NJL Lagrangian – containing only four-fermion interactions – takes the form

\[
\mathcal{L}_{NJL} = \bar{\psi} (i \slashed{D} - \hat{m}) \psi + G_\pi \left[ (\bar{\psi} \lambda_\alpha \gamma_5 \psi)^2 - G_\rho \left[ (\bar{\psi} \lambda_\alpha \gamma^a \gamma_5 \psi)^2 + (\bar{\psi} \lambda_\alpha \gamma^8 \gamma_5 \psi)^2 \right] \right],
\]

where the quark field has the flavour components \( \psi = (u, d, s) \), \( \hat{m} = \text{diag}(m_u, m_d, m_s) \) denotes the current quark mass matrix, and \( G_\pi, G_\rho \) are four-fermion coupling constants. A sum over \( a = 0, \ldots, 8 \) is implied in Eq. (1), where \( \lambda_1, \ldots, \lambda_8 \) are the Gell-Mann matrices in flavour space and \( \lambda_0 \equiv \sqrt{2} \mathbb{I} \). The elementary quark-antiquark interaction kernel derived from Eq. (1) takes the form

\[
\mathcal{K}_{ab,\gamma d} = \sum_{\Omega} \mathcal{K}_{\Omega} \Omega_{\gamma d} \bar{\Omega}_{ab}
\]

1 In principle the two flavour singlet pieces of the \( G_\rho \) term in Eq. (1) can appear in the NJL interaction Lagrangian with separate coupling constants, as they are individually chiral symmetric. Our choice of identical coupling avoids flavour mixing, giving the flavour content of the \( \omega \) meson as \( u\bar{d} + d\bar{u} \) and the \( \phi \) meson as \( s\bar{s} \).
where we assume that $m = m_d = m$, and with the Lagrangian of Eq. (1) the $\rho$ and $\omega$ mesons are therefore mass degenerate, differing only in their flavour structure.

The NJL model is non-renormalizable and hence a regularization scheme must be used to control divergences. Here the proper-time scheme is chosen, because it simulates aspects of quark confinement by eliminating on-shell quark propagation, while maintaining the symmetries of the theory, such as the Poincaré and chiral symmetries. As a result it has been widely used [38, 39, 51, 59–64]. Formally the proper-time regularization scheme is defined by

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau \tau^{n-1} e^{-\tau X}, \quad (3)$$

where $X^n$ is obtained by first introducing Feynman parametrization and then performing a Wick rotation of the loop momenta to Euclidean space. Only the ultraviolet cutoff, $\Lambda_{UV}$, is needed to render the theory finite. However, in bound states of quarks we also include the infrared cutoff, $\Lambda_R$, which eliminates unphysical thresholds for the decay of hadrons into quarks, therefore implementing quark confinement in the NJL model.

The standard NJL gap equation, illustrated in Fig. 1, provides the dressed quark propagator. The general solution of this gap equation has the form $S_q^{-1}(p) = \rho - M_q + i\epsilon$, where the dressed quark mass for each quark flavour $q = u, d$, satisfies

$$M_q = m_q - 4 G_\pi \langle \bar{q}q \rangle + m_q + 12 i G_\pi \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_D[S_q(k)]. \quad (4)$$

The quark condensate is denoted by $\langle \bar{q}q \rangle$ and $m_q$ is the current quark mass for each quark flavour. Introducing the proper-time regularization scheme gives

$$M_q = m_q + \frac{3 M_q G_\pi}{\pi^2} \int d\tau \frac{1}{\tau^2} e^{-\tau M_q^2}, \quad (5)$$

where here, and in the following, we drop the proper-time regularization parameters to aid readability. In the chiral limit ($\tilde{m} = 0$) the NJL Lagrangian respects the chiral $SU(3)_L \otimes SU(3)_R$ symmetry, however a non-trivial solution ($M_q \neq 0$) to Eq. (4) exists provided $G_\pi > G_{\text{critical}}$, which is a signature for dynamical chiral symmetry breaking (DCSB).

The mesons considered here are $\pi$, $K$, $\rho$, $\omega$ and $\phi$ are realized in the NJL model as quark-antiquark bound states whose properties are governed by the Bethe-Salpeter equation (BSE) illustrated in Fig. 2. This BSE takes the form

$$T(q) = \mathcal{K} + \int \frac{d^4 k}{(2\pi)^4} \mathcal{K} S(k + q) T(q) S(k), \quad (6)$$

where $q$ is the total 4-momentum of the two-body system and the Dirac, colour and flavour indices have been omitted.

The solution to the Bethe-Salpeter equation in the $\alpha = \pi, K$ and $\beta = \rho, \omega, \phi$ channels, respectively are

$$T_D(q)_{ab,cd} = [\gamma_5 \lambda_a]_{ab} \tau(q)[\gamma_5 \lambda^T_d]_{cd}, \quad (7)$$

$$T_P(q)_{ab,cd} = [\gamma_\rho \lambda_a]_{ab} \tau(\rho)(q)[\gamma_\omega \lambda^T_d]_{cd}, \quad (8)$$

where $\lambda_a$, $\lambda_b$ are the appropriate flavour matrices, for example, $\lambda_\rho = \lambda_3$, $\lambda_\omega = \sqrt{2} (\lambda_1 \pm i \lambda_2)$ and $\lambda_\phi = \frac{1}{\sqrt{2}} (\lambda_4 \pm i \lambda_5)$. The reduced $t$-matrices in these channels take the form

$$\tau(q) = \frac{-2i G_\pi}{1 + 2 G_\pi \Pi_E(q^2)}, \quad (9)$$

$$\tau^{\mu\nu}(q) = \frac{-2i G_\rho}{1 + 2 G_\rho \Pi_P(q^2)} \left( g^{\mu\nu} + 2 G_\rho \Pi_P(q^2) \frac{q^{\mu}q^{\nu}}{q^2} \right), \quad (10)$$

where the bubble diagrams appearing read:

$$\Pi_\pi(q^2) = 6i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_D[\gamma_5 S_S(k)\gamma_5 S_S(k + q)], \quad (11)$$

$$\Pi_K(q^2) = 6i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_D[\gamma_5 S_S(k)\gamma_5 S_S(k + q)], \quad (12)$$

$$\Pi_\rho(q^2) P^\mu\nu_T(q^2) = 6i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_D[\gamma_\rho S_S(k)\gamma_\rho S_S(k + q)], \quad (13)$$

where $\Pi_\rho = \Pi_\omega = \Pi_\phi$, $\Pi_\pi = \Pi_\rho$ and $\ell \equiv u, d$. The trace is over Dirac indices only and $P^{\mu\nu}_T = g^{\mu\nu} - q^{\mu}q^{\nu}/q^2$.

The meson masses are defined by the pole in the corresponding $t$-matrix, and therefore the pion mass, for example, is determined by the pole condition:

$$1 + 2 G_\pi \Pi_\pi(k^2 = m_\pi^2) = 0, \quad (14)$$

where analogous results determine $m_K$, $m_\rho$, $m_\omega$ and $m_\phi$. In fact, because of DCSB the pion and kaon masses are given by the simple expressions:

$$m_\pi^2 = \frac{m}{M_\pi} \left[ \frac{2}{G_\pi I_{\pi\pi}(m_\pi^2)} \right], \quad (15)$$

$$m_K^2 = \frac{m}{M_K} + \frac{m}{M_\pi} \left[ \frac{1}{G_\pi I_{\pi\pi}(m_\pi^2)} + (M_K - M_\pi) \right], \quad (16)$$

where

$$I_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int_0^\infty d\tau \frac{d^2 k}{k^3} e^{-x(k^2 - 1)} \int e^{-[x(1-k^2)k^2 + x M_k^2(1-x)]} \text{Tr}_D[S_S(k)]. \quad (17)$$

Figure 1. (Colour online) The NJL gap equation in the Hartree-Fock approximation. The thin line is the bare quark propagator, $S_q^{-1}(k) = \tilde{p} - M_q + i\epsilon$, whereas the thick line is the dressed quark propagator $S(k)$. The $\bar{q}q$ interaction kernel is given by Eq. (2).

Figure 2. (Colour online) The Bethe-Salpeter equation illustrated here for quark and antiquark scattering.
This demonstrates the Goldstone boson nature of the pion and kaon in the chiral limit. The residue at a pole in the \( \hat{q}q \) \( t \)-matrix defines the effective meson-quark-quark coupling constant, and for the various mesons we obtain

\[
Z_a^{-1} = -\frac{\partial \Pi_a(q^2)}{\partial q^2} \bigg|_{q^2=m_a^2}, \quad \alpha = \pi, K, p, \omega, \phi. \tag{18}
\]

The parameters of our NJL model are therefore: the couplings in the NJL Lagrangian \( G_s \) and \( G_p \); the regularization parameters \( \Lambda_{IR} \) and \( \Lambda_{UV} \); and the \( u/d \) and \( s \) dressed quark masses (or alternatively their current quark masses). In QCD the confinement scale is set by \( \Lambda_{QCD} \) and therefore we fix \( \Lambda_{IR} = 240 \text{ MeV} \) and choose the dressed light quark mass as \( M = 400 \text{ MeV} \). The remaining parameters are then fit to the physical pion \( (m_\pi = 140 \text{ MeV}) \), kaon \( (m_K = 495 \text{ MeV}) \) and rho \( (m_\rho = 770 \text{ MeV}) \) masses, together with the pion decay constant \( (f_\pi = 93 \text{ MeV}) \). This gives \( \Lambda_{UV} = 19.04 \text{ GeV}^{-2} \), \( G_s = 11.04 \text{ GeV}^{-2} \), \( \Lambda_{UV} = 645 \text{ MeV} \) and \( M_s = 611 \text{ MeV} \).

Elementary results in this NJL model are presented in Tab. I. A focus herein is the effect of explicit chiral symmetry and flavour symmetry violation. As a starting point we can consider the Goldberger–Treiman relation at the quark level, and the Gell-Mann–Oakes–Renner relation. For the pion these read

\[
f_\pi \sqrt{Z_\pi} = \frac{1}{2} (m_u + m_d), \tag{19}
\]

\[
f_\pi^2 m_\pi^2 = -\frac{1}{2} (m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle, \tag{20}
\]

and in the chiral limit these relations are satisfied exactly. With the parameters above we find violation at the 1% level for the pion. However, for the analogous relations for the kaon we find violations at the 20-25% level, which is sizeable, but much less than what may be expected from the current quark mass ratio \( 2m_s/(m_u + m_d) = 27.5 \pm 1.0 \) [65, 66].

III. ELASTIC FORM FACTORS

To determine the electromagnetic current of the pion or kaon we couple the electromagnetic field to the quark fields via minimal substitution: \( i \partial^a \to i \partial^a - \hat{Q} A^a \gamma^\mu \), where \( A^a \) is electromagnetic potential, \( e \) is the positron charge and \( \hat{Q} = \text{diag} \{ e_u, e_d, e_s \} = \frac{1}{2} (\Lambda^1 + \frac{1}{\sqrt{3}} \Lambda^3) \) is the quark charge operator, where \( e_q \) are the quark charges. The matrix element of the electromagnetic current for a pseudoscalar meson reads

\[
f_\pi^\mu(p', p) = (p'^\mu + p^\mu) F_\pi^{( \mu)} (Q^2), \quad \alpha = \pi, K, \tag{21}
\]

where \( p \) and \( p' \) denote the initial and final four momenta of the pseudoscalar meson, \( q^2 = (p' - p)^2 = -Q^2 \) and \( F_\pi(Q^2) \) is the pion or kaon form factor.

These results are denoted as “bare” because the quark-photon vertex is the elementary result, that is, \( \mathcal{N}_{\alpha \rho}^{(bare)} = \hat{Q} \gamma^\rho \). Importantly, these expressions satisfy charge conservation exactly.

The quark-sector form factors for a hadron \( \alpha \) are defined by

\[
F_\alpha(Q^2) = e_\alpha F_\alpha^{(\pi)} (Q^2) + e_d F_\alpha^{(K)} (Q^2) + e_s F_\alpha^{(\bar{K})} (Q^2) + \ldots \tag{28}
\]

Therefore the “bare” pseudoscalar meson quark-sector form factors are easily read off from Eqs. (24)-(26).

In general the quark-photon vertex is not elementary \( \hat{Q} \gamma^\rho \) but is instead dressed, with this dressing given by the inhomogeneous Bethe-Salpeter equation, which is illustrated in Fig. 4. With the NJL kernel of Eq. (2), the general solution for
the dressed quark-photon vertex for a quark of flavour $q$, has the form

$$\mathcal{N}_{q,Q}^{\mu}(p', p) = e_q \gamma^\mu + \left( \frac{q^\mu q}{q^2} \right) F_0(Q^2) \to \gamma^\mu F_{1q}(Q^2),$$

(29)

where the final result is used because the $q^\mu q/q^2$ term cannot contribute to a hadron electromagnetic current because of current conservation. Note, the result after the equality in Eq. (29) clearly satisfies the Ward-Takahashi identity:

$$q_{\mu} \mathcal{N}_{q,Q}^{\mu}(p', p) = e_q \left[ S_{q}^{-1}(p') - S_{q}^{-1}(p) \right].$$

(30)

For the dressed $u$ and $d$ quarks we find

$$F_{1U/D}(Q^2) = e_{u/d} \frac{1}{1 + 2 G_\rho \Pi_{\nu}^{ul}(Q^2)},$$

(31)

$$F_{1S}(Q^2) = e_s \frac{1}{1 + 2 G_\rho \Pi_{\nu}^{us}(Q^2)},$$

(32)

where the explicit form of the bubble diagram is

$$\Pi_{\nu}^{qq}(Q^2) = \frac{3 Q^2}{8 \pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} x(1-x) e^{-\frac{1}{2} [M^2_{\rho + x(1-x)Q^2}]}.$$  

(33)

Therefore, with the NJL Lagrangian of Eq. (1) there is no flavour mixing in the dressed quark form factors, in analogy with the dressed quark masses. The dressed quark form factors are illustrated in Fig. 5. In the limit $Q^2 \to \infty$ these form factors reduce to the elementary quark charges, as expected because of asymptotic freedom in QCD. For small $Q^2$ these results are similar to expectations form vector meson dominance, where the dressed $u$ and $d$ quarks are dressed by $\rho$ and $\omega$ mesons and the dressed $s$ quark by the $\phi$ meson. Note, the denominators in Eqs. (31) and (32) are the same as the pole condition obtained by solving the Bethe-Salpeter equation in the $\rho$, $\omega$ or $\phi$ channels. Therefore, the dressed $u$ and $d$ quark form factors have poles at $Q^2 = -m_{\rho}^2 = -m_{\omega}^2$, and the dressed $s$ quark form factor has a pole at $Q^2 = -m_{\phi}^2$.

The complete results for the pseudoscalar meson form factors — with a dressed quark-photon vertex — read

$$F_{\pi^0}(Q^2) = \left| F_{1U}(Q^2) \right| \frac{F_{\pi}(Q^2)}{F_{\pi}(Q^2)},$$

(34)

$$F_{K^0}(Q^2) = F_{1D}(Q^2) f_{K^0}^d(Q^2) - F_{1S}(Q^2) f_{K^0}^s(Q^2),$$

(35)

$$F_{K^+}(Q^2) = F_{1D}(Q^2) f_{K^+}^d(Q^2) - F_{1S}(Q^2) f_{K^+}^s(Q^2).$$

(36)

IV. VALENCE QUARK DISTRIBUTIONS OF THE KAON

The twist-2 quark distributions in a hadron $\alpha$ are defined by

$$q_{\alpha}(x) = p^+ \int \frac{d\xi}{2\pi} \ e^{ix p^+ \xi} \left< \alpha | \tilde{\psi}_q(0) \gamma^\mu \psi_q(\xi) | \alpha \right>,$$

(37)

where $q$ is the quark flavour, $c$ denotes a connected matrix element and $x = \frac{k^+}{p^+}$ is the Bjorken scaling variable, where $p^+$ is the plus-component of the hadron momentum and $k^+$ is the plus-component of the struck quark momentum. Note, in the NJL model the gluons are “integrated out” and therefore the gauge-link which should appear in Eq. (37) is unity.

From Eq. (37) one may readily show that the valence quark distribution functions of the pion or kaon are given by the two Feynman diagrams in Fig. 6, where the operator insertion is given by $\gamma^\mu \delta(p^+ x - k^+) \tilde{P}_q$ and $\tilde{P}_q$ is the projection operator for quarks of flavour $q$:

$$\tilde{P}_{u/d} = \frac{1}{2} \left( \frac{2}{3} \mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right), \quad \tilde{P}_s = \frac{1}{3} \mathbb{1} - \frac{1}{\sqrt{3}} \lambda_8.$$

(38)

Using the relation $\tilde{q}(x) = -q(-x)$ the valence quark and anti-quark distributions can be expressed as

$$\begin{align*}
\tilde{q}_{u/d}(x) &= \frac{1}{2} \left( \frac{2}{3} \mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right) \tilde{P}_q, \\
\tilde{q}_s(x) &= \frac{1}{3} \mathbb{1} - \frac{1}{\sqrt{3}} \lambda_8 \tilde{P}_q.
\end{align*}$$

(39)

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\tilde{q}_s(x) &= \frac{1}{3} \mathbb{1} - \frac{1}{\sqrt{3}} \lambda_8 \tilde{P}_q.
\end{align*}$$

(39)
quark distributions in the pion or kaon are given by
\[ q_\alpha (x) = i Z_\alpha \int \frac{d^4 k}{(2\pi)^4} \delta (p^+ x - k^+) \times \text{Tr} \left[ \gamma_5 \lambda_\alpha S(k) \gamma^+ \not{P} q S(k) \gamma_5 \lambda_\alpha S(k - p) \right] \] \[ \bar{q}_\alpha (x) = -i Z_\alpha \int \frac{d^4 k}{(2\pi)^4} \delta (p^+ x + k^+) \times \text{Tr} \left[ \gamma_5 \lambda_\alpha S(k) \gamma^+ \not{P} q S(k) \gamma_5 \lambda_\alpha S(k + p) \right] \]

To evaluate these expressions we first take the moments:
\[ \mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \]

where \( n = 1, 2, \ldots \) is an integer. Using the Ward-like identity \( S(k) \gamma^+ S(k) = -\partial S(k)/\partial k_+ \) and introducing the Feynman parametrization, the quark and anti-quark distributions can then be straightforwardly obtained. For the valence quark and anti-quark distributions of the \( K^+ \) we find:
\[ q_{K^+} (x) = \frac{3 Z_K}{4 \pi^2} \int d\tau e^{-\tau |x(1-x)M^2 + (1-x)M^2_\tau|} \times \left[ \frac{1}{\tau} + x(1-x) \left( m_K^2 - (M_\tau - M_\tau)^2 \right) \right] \]
\[ \bar{q}_{K^+} (x) = \frac{3 Z_K}{4 \pi^2} \int d\tau e^{-\tau |x(1-x)M^2 + (1-x)M^2_\tau|} \times \left[ \frac{1}{\tau} + x(1-x) \left( m_K^2 - (M_\tau - M_\tau)^2 \right) \right] \]

Results for the \( \pi^+ \) are obtained by \( M_\pi \rightarrow M_\pi \) and \( Z_K \rightarrow Z_\pi \), giving the result \( u_{\pi^+} (x) = \bar{d}_{\pi^+} (x) \). The quark distributions for the other pseudoscalar mesons can be obtained using flavour symmetries.

The quark distributions satisfy the baryon number and momentum sum rules, which for the \( K^+ \) read:
\[ \int_0^1 dx [u_{K^+} (x) - \bar{u}_{K^+} (x)] = \int_0^1 dx [\bar{s}_{K^+} (x) - s_{K^+} (x)] = 1 \]
for the number sum rule and at the model scale the momentum sum rule is given by
\[ \int_0^1 dx \ x [u_{K^+} (x) + \bar{u}_{K^+} (x) + s_{K^+} (x) + \bar{s}_{K^+} (x)] = 1 \]

Analogous results hold for the remaining kaons and the pions.

V. ELASTIC FORM FACTORS RESULTS

Results for the pion form factor – including effects from the dressed quark-photon vertex – are presented in Figs. 7 and 8, where comparisons to data [67–72] and an empirical parametrization [67] and the Dyson-Schwinger equation (DSE) result of Ref. [31] have been made. We find excellent agreement with existing data and the modest differences with the DSE result for \( Q^2 \lesssim 6 \text{ GeV}^2 \) are easily understood. The DSE result drops more rapidly that our NJL result primarily because the Bethe-Salpeter vertices in the DSE approach are non-pointlike and thereby suppress large relative moment between the dressed-quark and dressed-antiquark in the bound state. Our result for \( Q^2 F_\pi (Q^2) \) is very similar to the empirical monopole result and begins to plateau for \( Q^2 \gtrsim 6 \text{ GeV}^2 \) where \( Q^2 F_\pi (Q^2) \approx 0.49 \). This maximum is almost identical to that obtained in the DSE, which is not surprising because in both formalisms it is driven by dynamical chiral symmetry breaking [41, 42, 73]. For \( Q^2 \gtrsim 6 \text{ GeV}^2 \) the DSE result for \( Q^2 F_\pi (Q^2) \) begins to decrease, which is a consequence of QCD’s running coupling and a feature which is absent in our NJL calculations.

Results for the \( K^+ \) form factor and the quark-sector components – each including effects from the dressed quark-photon vertex – are given in Figs. 9 and 10. We find excellent agreement with the data from Ref. [74] and the empirical monopole \( F_K (Q^2) = [1 + Q^2/\Lambda^2_K]^{-1} \) determined by reproducing the charge radius of Ref. [74]. In contrast to the pion, all existing data for the kaon form factor lies in the domain \( 0 < Q^2 < 0.1 \text{ GeV}^2 \), and therefore we eagerly await any new data at \( Q^2 \) similar to the pion [75]. For the quark-sector form factors we observe a very
Figure 9. (Colour online) The $K^+$ form factor (solid line) together with the up (dashed-dotted line) and strange (dashed line) quark sector contributions. The dotted-line is the fit to data using the form $F_K(Q^2) = [1 + Q^2/\Lambda_K^2]^{-1}$, giving $\Lambda_K = 0.687$ GeV$^2$, and the insert compares our results with existing data taken from Ref. [74].

Figure 10. (Colour online) Results for $Q^2 F_{K^+}(Q^2)$ together with the charge-weighted quark-sector contributions and the empirical result obtained from Ref. [74]. This result clearly illustrates that the $s$ quark dominates the form factor at large $Q^2$.

Figure 11. (Colour online) We illustrate various pion and kaon form factor ratios, including for the quark sector form factors, to ascertain a measure of flavour breaking and environment sensitivity as a function of $Q^2$. Note, all ratios would be unity for all $Q^2$ in the $SU(3)$ flavour limit.

Table II. Charge radius results for the pion and kaon, together with the various quark-sector contributions. All radii are in units of fm and the empirical results are from Refs. [65, 76].

|          | $r_{exp}$ | $r_u$ | $r_s$ | $r_d$ | $r_s$ |
|----------|-----------|-------|-------|-------|-------|
| $\pi^+$  | 0.672 ± 0.008 | 0.629 | 0.629 | -0.629 | 0     |
| $K^+$    | 0.560 ± 0.031 | 0.586 | 0.646 | 0     | -0.441|
| $K^0$    | 0.277 ± 0.018 | -0.272 | 0     | 0.646 | -0.441|

Large difference in their $Q^2$ evolution, with the $s$ quark component much harder than the $u$ quark form factor. When weighted by the charges, as in Fig. 10, we find that the $s$ quark component begins to dominate the $K^+$ form factor for $Q^2 > 1.6$ GeV$^2$, becoming completely dominant at very large $Q^2$.

Results for the pion and kaon radii are listed in Tab. II. For the pion we find a radius 6% smaller than the Particle Data Group value [76] and agree within errors for both the $K^+$ and $K^0$ radii. We find that $r_{K^+}$ is about 7% smaller than $r_{\pi^+}$, which is driven by the quark-sector result $|r_{K^+}^u| < |r_{K^+}^s|$, with $r_{K^+}^u/r_{K^+}^d = 0.70$. We find the perhaps surprising result that $r_{K^+}^s > r_{\pi^+}^s$, with $r_{K^+}^s/r_{\pi^+}^s = 1.027$ a measure of environment sensitivity for the $u$ quark. These quark-sector radii are listed in Tab. II. As a measure of flavour breaking we have $|r_{\pi^+} - r_{K^+}| / |r_{\pi^+} + r_{K^+}| = 0.035$ and $|r_{K^+}^s + r_{K^+}^u| / |r_{K^+}^s - r_{K^+}^u| = 0.19$, which would vanish in the $SU(3)$ flavour limit. We therefore find that in some observables flavour breaking effects may be as large as 20%.

In Fig. 11 we illustrate the ratio $F_{K^+}(Q^2)/F_{\pi^+}(Q^2)$ which is always greater than unity and becomes almost constant for $Q^2 > 3$ GeV$^2$. For very large $Q^2$ this ratio plateaus to the value $f_K^2/f_{\pi^+}^2 = 1.10$, in agreement with the QCD result in the conformal limit [77]:

$$F_{K^+}(Q^2)/F_{\pi^+}(Q^2) \underset{Q^2 \gg \Lambda_{QCD}^2}{\rightarrow} f_K^2/f_{\pi^+}^2,$$  \hspace{1cm} (46)

however we find $f_K = 97.3$ MeV whereas the empirical value is $f_K = 110.4 \pm 0.8$ [76]. When expressed in terms of the quark sector form factors, and in the $m_u = m_d$ limit, we have

$$F_{K^+}(Q^2)/F_{\pi^+}(Q^2) = e_u F_{K^+}^u(Q^2) - e_s F_{K^+}^s(Q^2) - e_d F_{K^+}^d(Q^2),$$  \hspace{1cm} (47)

where the various quark-sector ratios are also given in Fig. 11. It is clear therefore, that the large constant ratio $F_{K^+}(Q^2)/F_{\pi^+}(Q^2)$ conceals dramatic flavour breaking effects in the quark-sector form factors that grow with increasing $Q^2$. In the $SU(3)$ flavour limit all ratios in Fig. 11 would be unity for all $Q^2$. However, at $Q^2 = 10$ GeV$^2$ we find $F_{K^+}^u/F_{\pi^+}^u \approx 0.36$ and $F_{K^+}^d/F_{\pi^+}^d \approx 2.74$. Therefore, at large $Q^2$ we find very large flavour breaking and environment sensitivity effects. The final ratio illustrated in Fig. 11 is $F_{K^+}^s(Q^2)/F_{\pi^+}^s(Q^2)$, which rapidly drops to zero with increasing $Q^2$. This behaviour can be understood by noting that a form factor is a measure of the ability of a hadron to absorb an electromagnetic current and remain a hadron. In the case of the $K^+$, if the $u$-quark interacts with the electromagnetic current it must drag along the heavier $s$-quark for the $K^+$ to remain intact, which becomes increasingly more
difficult at larger $Q^2$ than if the struck quark is an $s$-quark. Therefore this ratio may well be a very sensitive measure of confinement effects in QCD.

VI. PARTON DISTRIBUTION FUNCTION RESULTS

Results for the pion and kaon valence PDFs at $Q^2 = 16 \text{ GeV}^2$ are presented in Fig. 12 and compared to empirical data for the pion valence PDF from Ref. [34].\textsuperscript{2} We find reasonable agreement over the entire $x$ domain where data is available. Our results have been evolved using the next-to-leading order (NLO) DGLAP evolution equations \cite{78-81} from a model scale of $Q_0^2 = 0.16 \text{ GeV}^2$, which was independently determined in Ref. [13] in the study of nucleon PDFs. At the model scale we find that the momentum fraction carried by the $u$ and $s$ quarks in the $K^+$ equal $(x_u) = 0.42$ and $(x_s) = 0.58$ (at this scale gluons carry no momentum so these results saturate the momentum sum rule). We therefore find flavour breaking effects of $(x_s - x_u)/[(x_s + x_u)] \approx 16\%$ which is similar to that seen in the masses: $[M_{K^+} - M_{K^0}]/[M_{K^+} + M_{K^0}] \approx 21\%$ and quark-sector radii. As another measure of $SU(3)$ flavour breaking we note that at the model scale $u_K(x)$ peaks at $x_u = 0.237$ and $s_K(x)$ peaks at $x_s = 1 - x_u = 0.763$, which implies flavour breaking effects of around $[x_s - x_u]/[x_s + x_u] \approx 53\%$. Note that in the $SU(3)$ flavour limit these distributions would peak at $x = 0.5$, which is the case for the pion when $m_u = m_s$.

The ratio $u_K(x)/u_{\pi}(x)$ is illustrated in Fig. 13 at $Q^2 = 16 \text{ GeV}^2$, however this ratio has only a slight $Q^2$ dependence and in the limit $x \rightarrow 1$ is a fixed point in $Q^2$. We find $u_K^u / u_{\pi}^u \rightarrow 0.434 \approx M_{\pi}^2 / M_{K}^2$ as $x \rightarrow 1$, in good agreement with existing data from Ref. [83]. However, the $x$ dependence differs from much of the data in the valence region, the reason for this discrepancy is not clear, however it may lie with the absence of momentum dependence in standard NJL Bethe-

\textsuperscript{2} This data has been reanalyzed in Ref. [82], where the new empirical parametrization would imply that the data shown in Fig. 12 should be shifted down for large-$x$ and shifted up for moderate-$x$.
VII. SUMMARY

We have used the NJL model – with proper-time regularization to simulate the effect of confinement – to calculate the electromagnetic form factors and PDFs of the pion and kaon. For the former we included the effect of vertex dressing through vector meson like correlations in the $t$-channel, which do not contribute to the PDFs. Particular attention was paid to the individual quark flavour contributions and the associate flavour breaking and environment sensitivity effects.

This work produced several remarkable results. Firstly, as illustrated in Figs. 9–11, the effect of the larger mass of the strange quark on the electromagnetic form factors is dramatic. Indeed, even though $|e_s| < |e_u|$ the $s$-quark dominates the total elastic form factor of the $K^+$ for large $Q^2$. Surprisingly, as shown in Fig. 11, even though there are very significant changes in the individual flavour contributions in the kaon, the total pion and kaon form factors lie within about 10–15% for all $Q^2$, with the environmental suppression of the $u$-quark form factor in the $K^+$ more or less compensated by the increase in the strange quark form factor over that of the $d$ quark. In terms of the overall agreement with experiment, the total kaon form factor agrees very well with the limited existing data. In the case of the pion, the data extends to much larger $Q^2$, where again we find excellent agreement.

The effects of the strange quark mass on the PDFs is less spectacular. In Fig. 12 we saw that the strange quark PDF in the $K^+$ is considerably enhanced over that of the $u$-quark in the valence region. Most importantly, as we see in Fig. 13, the empirical suppression of $u_{K^+}$ compared with $u_\pi$ is rather well described.

The comparison of the asymptotic behaviour of the individual flavour form factors and parton distributions is fascinating. While all elastic form factors in this model behave as $1/Q^2$ at large $Q^2$, $F^u_k(Q^2)/F^u_\pi(Q^2) \sim 10$ at $Q^2 = 10$ GeV$^2$. Nevertheless, as already noted, the total $K^+$ and $\pi^+$ form factors only differ by 10–15%. Numerous other effects of flavour breaking have also been determined, for example, the pion and kaon charge radii, where effects of around 20% were typically observed.

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