Charged-current neutrino$^{208}$Pb reactions

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We present theoretical results on the non flux-averaged $^{208}$Pb($\nu_e, e^-)^{208}$Bi and $^{208}$Pb($\nu_\mu, \mu^-)^{208}$Bi reaction cross sections, obtained within the charge-exchange Random-Phase-Approximation. A detailed knowledge of these cross sections is important in different contexts. In particular, it is necessary to assess the possibility of using lead as a detector in future experiments on supernova neutrinos, such as OMNIS and LAND, and eventually detect neutrino oscillation signals by exploiting the spectroscopic properties of $^{208}$Bi. We discuss the present status on the theoretical predictions of the reaction cross sections.

The study of reactions induced by neutrinos on nuclei is at present an active field of research. A detailed knowledge of the reaction cross sections is interesting for different domains, going from high energy physics to astrophysics [1]. For example, they are necessary in the interpretation of current experiments on neutrinos as well as in the evaluation of possible new detectors for future experiments. The importance of neutrino-nuclei reactions in astrophysical processes, such as the r-process nucleosynthesis, is also being attentively studied [2-4]. In particular, $\nu - $Pb reactions have attracted much interest recently. Lead has been used as a shielding material in the recent experiments on neutrino oscillations performed by the LSND collaboration [4-5] so that estimates of the $\nu - $Pb reaction cross sections are necessary for the evaluation of backgrounds in these experiments; also projects on lead-based detectors [6], such as OMNIS [6] and LAND [7], are being studied for the purpose of detecting supernova neutrinos. These detectors might provide information on neutrino properties, such as oscillations in matter [10] or the mass [11] by measuring the time delay and/or spreading in the neutrino signal [12] as well as help in testing supernova models. From the practical point of view, lead-based detectors seem to present several of the characteristics required to be supernova observatories, namely high sensitivity to neutrinos of all flavors, simplicity, reliability with inexpensive materials [1]. Large cross sections for neutrinos in the supernova energy range are also an important condition since they determine the possible rates and therefore the maximum observable distance. Actually, $\nu$-nucleus reaction cross sections increase strongly with the charge of the nucleus. For example, if the neutrinos come from the Decay-At-Rest (DAR) of $\mu^+$, the cross sections of the flux-averaged charged-current (CC) reaction $\nu_e + Z \ X_N \rightarrow Z + X_{N-1} + e^- + \gamma$ goes from about $14 \times 10^{-42} \text{ cm}^2$ for $^{12}$C [12] to $2.56 \times 10^{-40} \text{ cm}^2$ in $^{56}$Fe [12] and is estimated to be $3.62 \times 10^{-39} \text{ in } ^{208}$Pb [12]. Besides these practical features which are essential in the choice of the nucleus to use to detect neutrinos, another important feature is the spectroscopic properties which may suggest attractive signals of supernova neutrino oscillations. In [17], for example, it has been shown that the measurement of events where two neutrons are emitted by $^{208}$Bi excited in the reaction $\nu_e + ^{208}$Pb $\rightarrow ^{208}$Bi $+ e^-$ is both flavor-specific and very sensitive to the mean energy of the $\nu_e$. In case when $\nu_\mu, \nu_\tau \rightarrow \nu_e$ oscillations take place, the hotter $\nu_e$ would increase the number of two neutron events by a factor of forty [10]. Another possible signal has been proposed in [15], that is, that the energy distribution of the neutrons emitted in the same CC reaction should have a peak at low energy more or less pronounced according to whether the oscillations occur or not. This peak would come from the excitation of a peak at around 8 MeV in the Gamow-Teller strength distribution. (One should however note that this peak has never been observed experimentally). Both the estimate of the CC $\nu - $Pb reaction cross section in [10] and the microscopic calculations of [12] show that a possible oscillation signal relies strongly on the knowledge of the spectral properties of $^{208}$Bi. In fact, the CC reaction cross section induced by $\nu_e$ scales almost as the square of the electron energy and is particularly sensitive to the detailed structure of the excitation spectrum as was already pointed out for the case of $^{12}$C [16]. It is then important either to get the cross sections directly from the experiment or/and to obtain different theoretical estimates in order to know the theoretical uncertainties and how they affect the reaction cross sections. This is crucial when the impinging neutrino energy increases because not only the allowed Gamow-Teller (GT) and Isobaric Analogue State (IAS) contribute significantly to these cross sections but also forbidden transitions, of first-, second-, third-order (which are not very well known experimentally).

In this paper, we present new theoretical results for the CC $\nu_e + ^{208}$Pb $\rightarrow ^{208}$Bi $+ e^-$ reaction cross section. Our calculations, as opposed to [12], are performed in a self-consistent charge-exchange Random-Phase-Approximation (RPA) with effective Skyrme forces. Contrary to all the previously published calculations, we...
present non flux-averaged cross sections, obtained for both low-energy $\nu_e$ and high-energy $\nu_\mu$. These reaction cross sections given as a function of neutrino energy span a large energy range. They can be used to convolute with different neutrino fluxes in various contexts, for example for future experiments with astronomical neutrinos which are at present under study, for the very recent terrestrial experiments such as the LSND ones \cite{ref1} to estimate the background, or in the $r$-process nucleosynthesis.

We will emphasize the importance of the contribution of forbidden transitions and how it evolves as a function of neutrino energy. This is often not taken into account in many present $r$-process nucleosynthesis calculations and so the neutrino-nuclei cross sections are underestimated (in \cite{ref2} only the importance of first forbidden transitions in neutron-rich nuclei was emphasized).

We will compare our results with presently available calculations \cite{ref3,ref4}. With this aim, we will present two different flux-averaged cross sections, where the neutrino fluxes are given by either the DAR of $\mu^+$ and Decay-In-Flight (DIF) of $\pi^+$; or by a Fermi-Dirac spectrum for a supernovae explosion. Finally, we will discuss our results in relation to the suggested possible oscillation signals that would use the spectroscopic properties of $^{208}$Bi.

The general expression for the differential cross section as a function of the incident neutrino energy $E_\nu$ for the reaction $\nu_e +^{208}$Pb $\rightarrow l +^{208}$Bi ($l = e, \mu$) is \cite{ref5}

$$\sigma(E_\nu) = \frac{G^2}{2\pi} \cos^2 \theta_C \sum_f p_l E_l \int_{-1}^{1} d(\cos \theta) M_\beta,$$

where $G \cos \theta_C$ is the weak coupling constant, $\theta$ is the angle between the directions of the incident neutrino and the outgoing lepton, $E_l = E_\nu - E_f$ ($p_l$) is the outgoing lepton energy (momentum), $E_f$ being the energy transferred to the nucleus, $M_\beta$ are the nuclear Gamow-Teller and Fermi type transition probabilities [8].

In a nucleus as heavy as $^{208}$Pb the distortion of the outgoing lepton wavefunction due to the Coulomb field of the daughter nucleus becomes large and affects the integrated cross section considerably. In our treatment of this effect we follow the findings of ref. \cite{ref6}. In ref. \cite{ref7} it is found that the “Effective Momentum Approximation” (EMA) works well for high energy neutrinos. This approximation consists in using an effective momentum $p_{l}^{\text{eff}} = \sqrt{E_{l}^{2} - m_{l}^{2}}$ where $E_{l} = E - V_C(0)$ ($V_C(0)$ is the Coulomb potential at the origin) in calculating the angle integrated cross section and multiplying eq. (1) by $(p_{l}^{\text{eff}} / p_l)^2$. It is also shown that the Modified EMA (MEMA) works better than EMA for $p_l \mu$ of low and high energies. In this approximation eq. (1) is multiplied by $p_{l}^{\text{eff}} E_{l} / p_l E_l$. We use therefore this method in all our calculations of the ($\nu_\mu, e^-$) cross sections. In the case of the ($\nu_e, e^-$) process, the situation is somewhat more complicated. The Fermi function works only for very low energies, namely $E_\nu \leq 10$ MeV (where $p_l R \ll 1, R$ is the nuclear radius), whereas the EMA seems to be a good approximation for most energies of the outgoing electrons \cite{ref8}. As in ref. \cite{ref9}, for $\nu_e \mu$, we treat Coulomb corrections by interpolating between the Fermi function at low electron energies and the EMA approximation at high lepton energies.

To get flux-averaged cross sections it is necessary to convolute \cite{ref10} by the neutrino flux $f(E_\nu)$, that is

$$\langle \sigma \rangle_f = \int_{E_0}^{\infty} dE_\nu \sigma(E_\nu) f(E_\nu),$$

$E_0$ being the threshold energy. The choice of $f(E_\nu)$ depends on the neutrino source and can be taken for example equal to the supernova neutrino energy spectrum given by transport codes or the neutrino fluxes produced by a beam dump.

The nuclear structure model used to evaluate the transition probabilities $M_\beta$ in \cite{ref10} is the charge-exchange Random-Phase-Approximation (RPA). The details of the approach can be found in \cite{ref11}. The calculations we present have been obtained in a self-consistent approach: the HF single-particles energies and wavefunctions as well as the residual particle-hole interaction are derived from the same effective forces, namely the SIII \cite{ref12} and SGII \cite{ref13} Skyrme forces. We have found that the model configuration space used is large enough for the Ikeda and Fermi sum rule to be satisfied as well as the non energy-weighted and energy-weighted sum rules for the forbidden transitions \cite{ref14}. The GT strength distribution we have obtained is peaked at 19.2 MeV, in agreement with the experimental value. This main peak exhausts about 60% of the Ikeda sum rule. The IAS results at 18.4 MeV and this value compares again well with the experimental finding (18.8 MeV). Apart from these two resonances and the spin-dipole, the experimental knowledge about states of higher multipolarity is rather poor. The recent experiment of Ref. \cite{ref15} shows that isovector monopole strength exists in $^{208}$Bi between 30 and 45 MeV and in the present calculation we find some strength in the same energy region.

In figs.1 and 2 we show the non flux-averaged $^{208}$Pb$\nu_e e^-$ $^{208}$Bi and $^{208}$Pb$\nu_\mu e^-$ $^{208}$Bi inclusive cross sections as a function of the neutrino energy, for a mesh of energies, namely $\Delta E = 2.5$ MeV for $E_\nu$, and $\Delta E = 5.0$ MeV for $E_{\nu_\mu}$. The dashed line in fig.1 shows the cross section obtained when only the Fermi function is used to include the Coulomb corrections. The results shown have been obtained with the SIII force, but we have found that with the SGII force we get quite similar results. All the multiplicities with $J \leq 6$ are included. We have checked that the contribution coming from $J = 7$ is small. (Note that, for higher multiplicities, a mean field description, neglecting the particle-hole residual interaction, can be used to evaluate the transition probabilities \cite{ref16}). In the calculations we present the
axial vector coupling constant has been taken equal to 1.26. Note that the use of an effective $g_a$ to take into account the problem of the “missing” GT strength will reduce the reaction cross section by $10−15\%$ as it was already discussed in [4].

Figure 3 shows the contribution of the different multipoles to the total cross section (fig.1), for the impinging neutrino energies $E_{\nu} = 15, 30, 50 \text{ MeV}$, which are characteristic average energies for supernova neutrinos. When $E_{\nu} = 15 \text{ MeV}$ (fig.3, up), $\sigma_{\nu}$ is dominated by the allowed Gamow-Teller $(J^z = 1^+)$ transition. As the neutrino energy increases (fig.3, middle), the allowed IAS and other forbidden transitions start to contribute significantly. Finally, when $E_{\nu} = 50 \text{ MeV}$ (fig.3, bottom), the GT and IAS transitions are not dominating at all, the cross section is being spread over many multipoles. These results suggest that $\tau$-process nucleosynthesis calculations such as [3], which include neutrino-nuclei reactions, should take into account forbidden transitions. This may be even more important if $\nu_\tau$, $\nu_\mu \rightarrow \nu_e$ oscillations occur, because in this case electron neutrino may have a higher average energy than it is usually expected from current supernovae models.

Let us now come to the comparison with other available calculations. Table 1 shows our flux-averaged cross sections, in comparison with those of [10]. The low-energy neutrino flux is given by a Fermi-Dirac spectrum [4,14].

$$f(E_{\nu}) = \frac{1}{c(\alpha)T^3} \exp \left[ \frac{E_{\nu}^2}{T} - \alpha \right] + 1$$

where $T, \alpha$ are fitted to numerical spectra and $c(\alpha)$ normalizes the spectrum to unit flux. The values of the parameters $T, \alpha$ have been chosen to be able to compare our results with those of [10]. As we can see from table 1, our predictions are in close agreement (the difference is at most $20−30\%$) with [10]. The results of [10] have been obtained in a CRPA approach. A variation of $20−30\%$ is actually to be expected for calculations based on the same approach but using different parametrization (for example for single particle wavefunctions and effective particle-hole interaction), because of the sensitivity of the flux-averaged cross sections to the detailed strength distributions [4,14], as we will discuss further. On the contrary, our results and those of ref. [14] present significant differences with those of [10], obtained using the allowed approximation and including the IAS, the GT and the first-forbidden contributions treated on the basis of the Goldhaber-Teller model.

We have checked that the differences do not come from the higher order forbidden transitions which are not included in the calculations of [10]. The three calculations satisfy the same constraints, namely they reproduce the centroid of the resonances and satisfy the sum-rules.

We believe that the significant differences (by a factor of 2) with [10] may have two origins. The first possible origin might be the way the Coulomb corrections are treated. In [10], the Coulomb distortion of the outgoing electron wave function was taken into account by multiplying the cross section by a Fermi function. In order to see the effect of using only the Fermi function instead of making an interpolation between the Fermi function and the EMA approximation, we have calculated the reaction cross sections using these two possible corrections. As figure 4 shows, the two cross sections have a quite different behaviour as a function of the neutrino energy so that this difference on the flux-averaged cross section may vary according to the particular neutrino flux considered. To get a quantitative idea of the variation, we have calculated the flux-averaged cross sections by convoluting the two curves of fig.4 with [4]. If we use the Fermi function only, the reaction cross sections increase, on average, by $50\%$.

The second possible origin of the discrepancies between our work, [14] and [10] might be the sensitivity of the flux-averaged cross sections to the detailed strength distributions in $^{208}\text{Bi}$. In fact, it has already been discussed in [4], that for low-energy neutrinos, the flux-averaged cross sections are very sensitive to the energy of the excited states in the final nucleus. The reason is twofold. First, due to the small electron mass, the non flux-averaged cross section scales as the square of energy of the states. Second, the energy dependence of the neutrino flux may emphasize differences in the non flux-averaged cross sections due to variations in the energy of the states. As it was discussed in [4], these two effects may modify the flux-averaged reaction cross sections by $20−30\%$.

To complete our comparison with the calculations of [10], we have calculated two more flux-averaged cross sections, using the neutrino fluxes of both $\nu_\mu$ coming from the DIF of $\pi^+$ and $\nu_\tau$ coming from the DAR of $\mu^+$. The neutrino fluxes $f(E_{\nu})$ were taken from [20]. These neutrino fluxes have been used in the recent experiments $\nu_\mu \rightarrow \nu_e$ [4,17], $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ [28] or $\nu_\mu \rightarrow \nu_\tau$ [29] performed by the LSND and KARMEN collaborations. The DAR($\nu_\tau, e^-$) cross section calculated is $\sigma_{\text{DAR}} = 44.39 \cdot 10^{-40} \text{ cm}^2$ which is very close to $36.2 \cdot 10^{-40} \text{ cm}^2$ obtained in [14]. On the contrary, our $\text{DIF}(\nu_\mu, \mu^-)$ is $\sigma_{\text{DIF}} = 399.2 \cdot 10^{-40} \text{ cm}^2$; whereas the one of [10] is $115 \cdot 10^{-40} \text{ cm}^2$. We believe that some of the disagreement may come from differences in the strength distributions of the high order (higher than 2) forbidden transitions. In fact, contrary to the reactions of neutrinos on light nuclei such as carbon, where these states contribute only by $20\%$ to the total DIF cross section, their contribution represents $65\%$ of the total cross section when the nucleus is as heavy as lead.

Let us finally discuss the two possible neutrino oscillation $\nu_\mu, \nu_\tau \rightarrow \nu_e$ signals based on the spectroscopic properties of $^{208}\text{Bi}$ excited in the CC reaction that have been proposed recently. In [10], it was shown that the 2-
neutron events associated with the deexcitation of $^{208}Bi$ are very sensitive to the mean electron neutrino energy. This signal relies on the fact that most of the IAS, GT and first-forbidden strength distributions are above the $2n$ emission threshold (14.98 MeV) in $^{208}Bi$. Our results show that not only the allowed and spin-dipole strengths are above this threshold, but also a fraction of the strength distributions associated with other forbidden transitions (fig.3) will contribute to the $2n$ decay. All the arguments given in [10] are based on the statistical calculations of $1n$ and $2n$ decays. The direct $1n$ emission represents about 50% of the total width in the case of the IAS, and $5 - 10\%$ in the case of the GT [2].

In [10], it was pointed out that the energy distribution of the neutrons in the $1n$ events should form a peak at low energy, more or less pronounced according to the occurrence or absence of oscillations. This peak comes from the GT strength distribution at around 7.6 MeV which is located above the $1n$ threshold emission at 6.9 MeV. Our GT distribution also shows a peak at around 7.5 MeV. We have checked that its location is not sensitive to the choice of the effective forces used. Still one should be careful about conclusions, because predictions of different models about the energy location and strength of that peak are at variance.

In summary, we have presented the non flux-averaged $^{208}Pb(\nu_e, e^-)^{208}Bi$ and $^{208}Pb(\nu_\mu, \mu^-)^{208}Bi$ reaction cross sections, calculated in a self-consistent charge-exchange Random-Phase-Approximation with Skyrme effective forces. These predictions can be employed for very different purposes, such as for the interpretation of the recent experiments on neutrino oscillations performed by the LSND collaboration (where reactions induced by neutrinos on lead contribute significantly to the background) and to evaluate the feasibility of future projects in which lead should be used as detector for supernova neutrinos. We have emphasized that forbidden transitions contribute significantly to the neutrino-nuclei reaction cross sections even at the “astrophysical neutrino energies” and they should be included in present r-process nucleosynthesis calculations. We have discussed the present status on the theoretical predictions on the reaction cross sections for the $\nu_e$ having typical energies from present models on supernovae. If on one hand our calculations agree with those of ref. [13], which are also based on RPA; on the other hand, they both significantly disagree with those of ref. [10]. We point out that the origin of the discrepancy might be mainly the different treatment of Coulomb corrections, but also the sensitivity of the reaction cross sections to the detailed energy spectrum of the final nucleus. We have also compared our flux-averaged reaction cross sections with $\nu_\mu$ coming from the DIF of $\pi^+$ and with $\nu_e$ coming from the DAR of $\mu^+$, with the ones of [15]. As expected, the DAR cross sections are very close. On the contrary our DIF cross section differs significantly from the one of [15]. We have pointed out that the two predictions may differ because of differences in the strength distributions of forbidden transitions of high multipolarity which represent the main contribution in reactions of neutrinos on nuclei as heavy as lead. Finally, we have discussed our results in relation with recently proposed signals to measure supernova neutrino oscillations.

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![Image](image1.png)

**FIG. 1.** Differential $^{208}$Pb$(\nu_e, e^-)^{208}$Bi cross section as a function of electron neutrino energy for a mesh of energies ($\Delta E_{\nu_e} = 2.5 \text{ MeV}$). As far as the treatment of the Coulomb distortion is concerned, the results are obtained by interpolating between the Fermi function, good at low electron energies and the Modified Effective Momentum Approximation, good at high outgoing electron energies. The dashed line shows the result obtained if only a Fermi function is used.

![Image](image2.png)

**TABLE I.** Flux-averaged cross sections ($10^{-40}$ cm$^2$) obtained by convoluting the inclusive cross sections of fig.1 by a Fermi-Dirac spectrum (3) for neutrinos emitted in a supernova explosion. Different temperatures $T$ and $\alpha$ values are considered. The results of recent calculations are shown for comparison.

| $(T, \alpha)$ | this work | ref. [13] | ref. [10] |
|---------------|-----------|-----------|-----------|
| (6, 0)        | 14.06     | 11.       | 27.84     |
| (8, 0)        | 23.3      | 25.       | 57.99     |
| (10, 0)       | 34.91     | 45.       | 96.14     |
| (6.26, 3)     | 25.21     | 21.       | 47.50     |

![Image](image3.png)

**FIG. 2.** Differential $^{208}$Pb$(\nu_\mu, \mu^-)^{208}$Bi cross section, obtained with the MEMA approximation, as a function of muon neutrino energy for a mesh of energies ($\Delta E_{\nu_\mu} = 5.0 \text{ MeV}$).

![Image](image4.png)

**FIG. 3.** Contribution of the different multipolarities to the differential $^{208}$Pb$(\nu_e, e^-)^{208}$Bi cross section ($10^{-40}$ cm$^2$) of fig.1 for $E_{\nu_e} = 15 \text{ MeV}$ (up), 30 MeV (middle), 50 MeV (bottom).