Energy flows in tight focus of optical vortices

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Abstract. In this paper we investigated tight focusing of optical vortex with topological charge \( m = 2 \) and left circular polarization. The simulation was based on Richards-Wolf equation. Light with wavelength 532 nm was focused by aplanatic lens with numerical aperture NA=0.95. It was shown that the longitudinal component of Poynting vector has negative values on the optical axis. The reason of the energy backflow is due to the fact that the projection of the spin flow onto the optical axis is negative and exceeds in absolute value the projection of the orbital energy flow, which is always positive.

1. Introduction

The effect of the reverse energy flux has been known in optics for a long time [1] - [5]. When focusing beams with a phase or polarization singularity at the focus there are regions with the direction of the Poynting vector opposite to the direction of beam propagation [6].

In this paper we investigated tight focusing of optical vortex with topological charge \( m = 2 \) and left circular polarization. The simulation was based on Richards-Wolf equation. Light with wavelength 532 nm was focused by aplanatic lens with numerical aperture NA=0.95. It was shown that the longitudinal component of Poynting vector has negative values on the optical axis. Previously in [7] A. Bekshaev shown that Poynting vector (energy flow) is equal to the sum of orbital energy flow and spin flow. It was shown that the reason of the energy backflow is due to the fact that the projection of the spin flow onto the optical axis is negative and exceeds in absolute value the projection of the orbital energy flow, which is always positive. It was also shown that the transverse energy flow in the focal region and spin angular momentum rotate in different directions.

2. Spin and orbital energy flows

In 2007 A. Bekshaev shown [7] that Poynting vector \( \mathbf{P} \) (energy flow) is equal to the sum of orbital energy flow \( \mathbf{P}_o \) and spin flow \( \mathbf{P}_s \):

\[
\mathbf{P} = \frac{\text{Re}}{2} (\mathbf{E}^* \times \mathbf{H}) = \mathbf{P}_o + \mathbf{P}_s,
\]

\[
\mathbf{P}_o = \frac{\text{Im}}{2k} (\mathbf{E}^* (\nabla \mathbf{E})), \quad \mathbf{P}_s = \frac{1}{4k} (\nabla \times \text{Im} (\mathbf{E}^* \times \mathbf{E})),
\]

(1)
where \( \mathbf{E} \) and \( \mathbf{H} \) is the electrical and magnetic field, \( \text{Re}(\cdot) \) and \( \text{Im}(\cdot) \) is real and image part of the number, \( k \) is the wavelength. From (2) it follows that the spin flow \( \mathbf{P}_s \) is the curl from spin angular momentum \( \mathbf{S} \) [8]:

\[
\mathbf{S} = \frac{\text{Im}}{2} (\mathbf{E}^* \times \mathbf{E}).
\]

In the Cartesian coordinate system, the components of the orbital energy flow vector are equal to:

\[
P_{x,x} = \frac{\text{Im}}{2k} \left( E_x^* \frac{\partial}{\partial x} E_x + E_y^* \frac{\partial}{\partial y} E_x + E_z^* \frac{\partial}{\partial z} E_x \right),
\]

\[
P_{x,y} = \frac{\text{Im}}{2k} \left( E_x^* \frac{\partial}{\partial y} E_x + E_y^* \frac{\partial}{\partial y} E_y + E_z^* \frac{\partial}{\partial z} E_y \right),
\]

\[
P_{x,z} = \frac{\text{Im}}{2k} \left( E_x^* \frac{\partial}{\partial z} E_x + E_y^* \frac{\partial}{\partial y} E_y + E_z^* \frac{\partial}{\partial z} E_z \right).
\]

Similarly, the spin flow components could be written as:

\[
P_{s,x} = \frac{1}{2k} \left( \frac{\partial}{\partial y} \text{Im}(E_x^* E_y) + \frac{\partial}{\partial z} \text{Im}(E_x^* E_z) \right),
\]

\[
P_{s,y} = \frac{1}{2k} \left( \frac{\partial}{\partial z} \text{Im}(E_x^* E_z) + \frac{\partial}{\partial x} \text{Im}(E_x^* E_x) \right),
\]

\[
P_{s,z} = \frac{1}{2k} \left( \frac{\partial}{\partial x} \text{Im}(E_x^* E_x) + \frac{\partial}{\partial y} \text{Im}(E_x^* E_y) \right).
\]

### 3. Numerical simulation

Our analysis relies on the Richard-Wolf integral [2]:

\[
U(\rho, \psi, z) = \frac{\text{Im}}{\pi} \int_0^{\alpha_{\text{max}}} \int_0^{2\pi} B(\theta, \phi) T(\theta) \mathbf{P}(\theta, \phi) \exp \left\{ ik \left[ \rho \sin \theta \cos(\phi - \psi) + z \cos \theta \right] \right\} \sin \theta d\theta d\phi
\]

where \( U(\rho, \psi, z) \) is the electrical or magnetic field in the focal spot, \( B(\theta, \phi) \) is the incident electrical or magnetic field (\( \theta \) is the polar angle and \( \phi \) is the azimuthal angle), \( T(\theta) \) is apodization function (for an aplanatic lens the apodization function is equal to \( T(\theta) = \cos^{\frac{\theta}{2}} \), and for the flat diffractive lens it is equal to \( T(\theta) = \cos^{\frac{3\theta}{2}} \), \( k = 2\pi/\lambda \) is the wavenumber, \( \lambda \) is the wavelength, \( \alpha_{\text{max}} \) is the maximal polar angle determined by the numerical aperture of the lens (\( \text{NA} = \sin \alpha_{\text{max}} \)), and \( \mathbf{P}(\theta, \phi) \) is the polarization matrix for the electric and magnetic fields:

\[
\mathbf{P}(\theta, \phi) = \begin{bmatrix}
1 + \cos^2 \phi (\cos \theta - 1) \\
\sin \phi \cos \phi (\cos \theta - 1) \\
-\sin \theta \cos \phi
\end{bmatrix} a(\theta, \phi) + \begin{bmatrix}
\sin \phi \cos \phi (\cos \theta - 1) \\
1 + \sin^2 \phi (\cos \theta - 1) \\
-\sin \theta \sin \phi
\end{bmatrix} b(\theta, \phi)
\]

where \( a(\theta, \phi) \) and \( b(\theta, \phi) \) are polarization functions for the x- and y- components of the incident beam.

For example, for light with circular polarization the polarization functions have the form:

\[
U_e(\theta, \phi) = \begin{pmatrix}
a(\theta, \phi) \\
b(\theta, \phi)
\end{pmatrix} = \begin{pmatrix} 1 \\ \sigma i \end{pmatrix}
\]

for the electric field, and
\[
U_H(\theta, \phi) = \begin{pmatrix}
  a(\theta, \phi) \\
  b(\theta, \phi)
\end{pmatrix} = \begin{pmatrix}
  -\sigma i \\
  1
\end{pmatrix}
\]

(9)

for the magnetic field, where \(\sigma = 1\) for right circular, and \(\sigma = -1\) for left circular polarization. We simulated the focusing of left-circularly polarized optical vortex with topological charge \(m=2\) and wavelength 532 nm. Figures 1-5 show the results of the simulation.

**Figure 1.** Distribution of different components of intensity \(I_x\) (a), \(I_y\) (b), \(I_z\) (c), and intensity \(I = I_x + I_y + I_z\) (d) in the focal plane.

**Figure 2.** The longitudinal component of Poynting vector \(P_z\).

**Figure 3.** Spin flow components \(P_{s,x}\) (a), \(P_{s,y}\) (b) and \(P_{s,z}\) (c) in the focal plane.

**Figure 4.** Orbital energy flow components \(P_{o,x}\) (a), \(P_{o,y}\) (b) and \(P_{o,z}\) (c) in the focal plane.
Figure 5. Sum of spin flow and orbital energy flow $P_{o,x} + P_{s,x}$ (a) and $P_{o,z} + P_{s,z}$ (b)

4. Conclusion
In this paper we investigated tight focusing of optical vortex with topological charge $m = 2$ and left circular polarization. The simulation was based on Richards-Wolf equation. Light with wavelength 532 nm was focused by aplanatic lens with numerical aperture NA=0.95. It was shown that the reason of the energy backflow is due to the fact that the projection of the spin flow onto the optical axis is negative and exceeds in absolute value the projection of the orbital energy flow, which is always positive. It was also shown that the transverse energy flow in the focal region and spin angular momentum rotate in different directions.

Acknowledgments
This work was supported by the Ministry of Science and Higher Education within the State assignment FSRC «Crystallography and Photonics» RAS in part of «Introduction» and «Conclusion», Russian Science Foundation (Project No. 18-19-00595) in part of «Numerical simulation», Russian Foundation for Basic Research (Project No. 18-29-2003) in part of «Spin and orbital energy flows».

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