Tilted Lemaître model and the dark flow

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Abstract In the last years, the peculiar velocities of many X-ray galaxies clusters with respect to the distance have been measured directly in the rest frame of the cosmic microwave background radiation (CBR), using the kinematic Sunyaev-Zeldovich (kSZ) effect. These measures prove that exists a highly coherent motion, extending out to at least to $1\,Gpc$, of the matter rest frame with respect to the CBR rest frame. This global motion was named “dark flow”. By using an inhomogeneous spherically symmetric “tilted” Lemaître model, we could explain the dark flow if we assume a linear increase with distance of the peculiar velocities, which is in principle allowed by these observations. This linear increase of the dark flow with the distance has the same behavior that the intrinsic dipole, due to the kinematic acceleration, which appears in the Hubble law of the Lemaître model. In the “tilted” Lemaître model considered, we consider that the radiation orthogonal congruence is a perfect fluid and the matter “tilted” congruence is an imperfect fluid with heat flux.

1 Tilted cosmological models

In cosmology it is essential to specify the set of observers, or rather, the congruences of world-lines from which observations are made.

Cosmological quantities depend on the choice of these congruences, specified by 4-velocities fields. For instance, with the same spacetime metric, one can have two different stress-energy-momentum tensors, corresponding to two different congru-
ences, both satisfying the Einstein equations. The interpretation of the universe of two observers, associated with two different congruences, can be radically different. These models are named “tilted” in the literature, which begins with \[1\].

If \( \Sigma \) is a global 3D spacelike space, which exists assuming zero vorticity, \( n^\alpha \) is the four velocity of the normal or orthogonal congruence to \( \Sigma \) and \( u^\alpha \) the four velocity of the tilted one.

The relationship between both velocities is, at low speed, a galilean transformation: \( u^\alpha = n^\alpha + v^\alpha \). The relative (“tilting”) peculiar velocity \( v^\alpha \) between the two congruences may be related to a physical phenomenon such as the observed motion of our galaxy relative to the cosmic microwave background radiation, the CBR dipole, which is usually interpreted as a Doppler effect.

Previous works on tilted models have been realized by using FLRW [2], Bianchi [1] and, recently, Lemaître-Tolman-Bondi (LTB) [3] and Szekeres metrics [4].

In this work and in the more detailed article [5], we will consider a different case: the Lemaître model. This is the generalization of the LTB model to the case of non-dust matter with a non-null pressure gradient, which gives rise to a kinematic acceleration. This can explain the acceleration of the expansion obtained by the SN1a supernovae distance measures, without considering dark energy, see [6].

2 The tilted Lemaître model

The metric of the Lemaître model in comoving coordinates is:

\[
ds^2 = -N(r,t)^2 dt^2 + B(r,t)^2 dr^2 + R(r,t)^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

Where \( N(r,t) \) is the lapse and \( R(r,t) \) is the areal radius (or warp factor). It is spherically symmetric, with three Killing vectors. Its symmetry group is a \( G_3/S^2 \). and it has a “local” preferred radial spatial direction at each point. Also, it belongs to Petrov type D and it has null magnetic Weyl tensor.

This metric is compatible with a general non-perfect (with heat flux and anisotropic pressures) comoving fluid, as Lemaître first pointed out in [7].

The comoving congruence is normal to a foliation of global 3D spacelike hypersurfaces, \( n^\alpha = N^{-1} \delta^\alpha_t \).

The kinematical quantities, expansion, shear tensor and acceleration four-vector of the Lemaître spacetime are all non-null and can be reduced to scalars. Moreover, as the vorticity \( \omega^\alpha_{\alpha\beta} = 0 \), it admits a global 1+3 splitting. Also, as a LRSIIb model, it admits a 1+1+2 threading.

Choosing the radially “tilted” non-comoving congruence \( u^\alpha \) as

\[
u^\alpha = \left( \frac{1}{N}, \frac{v_r}{B}, 0, 0 \right),
\]

where \( v_r \) means the radial peculiar velocity w.r.t. the normal \( n^\alpha \) frame. Due the spherical symmetry of the Lemaître model, \( v_r \) is a spherical average.
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All the kinematical quantities of the “tilted” Lemaître spacetime w.r.t. the “tilted” congruence $u^\alpha$ can be computed and related to the non-tilted ones.

Consider the Lemaître metric $g_{\alpha\beta}$ as the solution of the Einstein equations for the two different fluid congruences. Where the two stress-energy-momentum tensors $T^*_\alpha\beta$ and $T_{\alpha\beta}$ are:

$$T^*_\alpha\beta = \frac{4}{3} \mu^* n^\alpha n^\beta + \frac{1}{3} \mu^* g_{\alpha\beta},$$

which corresponds to an inhomogeneous radiation fluid with energy density $\mu^*(r,t)$ and

$$T_{\alpha\beta} = (\mu + p) u^\alpha u^\beta + p g_{\alpha\beta} + q_a u^\beta + u^a q_\beta,$$

which corresponds to an inhomogeneous imperfect matter fluid, with heat flux, $q_\alpha = q s_\alpha(r,t)$, in the radial direction of the observer.

Since the Einstein tensor is the same for the tilted (matter) and orthogonal (radiation) congruences, imposing $T_{\alpha\beta} = T^*_\alpha\beta$, we should have the following relations between dynamical quantities:

$$\mu = \mu^* (1 + \frac{4}{3} v_r^2); \quad p = \mu^* (\frac{1}{3} + \frac{4}{3} v_r^2); \quad q = \frac{4}{3} v_r.$$

3 The CBR dipole and the dark flow

In the $\Lambda$CDM model the Hubble expansion is assumed to be uniform, so that the differences between peculiar velocities of galaxies (or clusters) $v_r$ and the observer velocity, are deviations from the isotropy of the usual Hubble law of the FLRW models. In the concordance $\Lambda$CDM model, the spherically averaged peculiar bulk velocity has a hyperbolic $1/r$ dependence with distance.

On the other hand, the kinematic Sunyaev-Zeldovich (kSZ) effect, which measures the dipole anisotropy of the CBR through a tiny temperature shift (of the order of $\mu K$) in the spectrum of CMB photons scattered from hot gas in clusters of galaxies, gives us the peculiar velocity of any cluster directly in the rest frame of the CMB. By using the kSZ effect, the authors in the review [8] claim to have detected a highly significant CBR dipole for $\sim 1200$ clusters with redshift $z \leq 0.12$ up to $z = 0.6$. This detection proves that exists a highly coherent motion, extending out to at least $1 Gpc$, of the matter rest frame with respect to the CBR rest frame congruence. This global motion was named “dark flow” by the authors of [8] and supposed by them to be approximately constant, but see Fig. [1] This dark flow do not has the $1/r$ dependence with distance, so this is in contradiction with the result of the $\Lambda$CDM model. Also, it appears that the dark flow may extend across the entire observable Universe horizon. This can be considered as an evidence for a “tilted” Universe. In [9] it was shown that, in the Lemaître model, the following generalized Hubble law is verified:
Fig. 1 In colored rectangles, the kSZ measured peculiar velocities as a function of the distance. Shaded grey region shows the peculiar velocity in the concordance $ΛCDM$ model. The red line is our hypothesis of linear increasing with the distance of the kinematic dipole of the dark flow. Figure taken, without the red line, from A. Kashlinsky et al., [8].

\[
c z = \left( \frac{\dot{R}}{R} - A \cos \Psi + \sqrt{3} \sigma \cos^2 \Psi \right) \frac{d_\theta}{0}.
\] (6)

This law shows an intrinsic dipole, due to the acceleration, increasing linearly with the distance as in our hypothesis. Where, $Ψ$ is the angle between the direction of observation of a light ray and the preferred radial vector $e_r$ of the Lemaître model, $σ$ is the scalar shear, $A$ is the kinematic acceleration and $d_\theta$ is the angular diameter distance. However, up to now, there is not the necessary accuracy to obtain the possible shear using the kSZ effect. Note that in the pure dust Lemaître-Tolman-Bondi model the intrinsic dipole, due to the acceleration, is absent.

In conclusion, accepting that the dark flow is real and that it increases linearly with the distance, the tilted Lemaître model could be considered as a candidate to explain it. Then there exists a preferred radial spatial direction in the Universe, given by the matter dark flow. Is this “the axis of evil” or better “the rebel or guerrillero axis”, because it reappears when the dark flow is considered?.

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