Dark Scalar Doublets and Neutrino Tribimaximal Mixing from $A_4$ Symmetry

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Abstract

In the context of $A_4$ symmetry, neutrino tribimaximal mixing is achieved through the breaking of $A_4$ to $Z_3$ ($Z_2$) in the charged-lepton (neutrino) sector respectively. The implied vacuum misalignment of the $(1,1,1)$ and $(1,0,0)$ directions in $A_4$ space is a difficult technical problem, and cannot be treated without many auxiliary fields and symmetries (and perhaps extra dimensions). It is pointed out here that an alternative scenario exists with $A_4$ alone and no redundant fields, if neutrino masses are “scotogenic”, i.e. radiatively induced by dark scalar doublets as recently proposed.
The neutrino mixing angles are now known to some accuracy. Based on a recent global analysis [1],
\[
\begin{align*}
\theta_{23} &= 42.3 \pm 5.1 \pm 3.3, \\
\theta_{12} &= 34.5 \pm 1.4, \\
\theta_{13} &= 0.0 \pm 7.9 \pm 0.0,
\end{align*}
\]
at the 1σ level. Thus the central values of \(\sin^2 2\theta_{23}\), \(\tan^2 \theta_{12}\), and \(\theta_{13}\) are 0.99, 0.47, and 0 respectively. These numbers agree well with the hypothesis of tribimaximal mixing [2], i.e.
\[
\begin{align*}
\sin^2 2\theta_{23} &= 1, \\
\tan^2 \theta_{12} &= 0.5, \\
\theta_{13} &= 0.
\end{align*}
\]
Such a pattern is best understood as the result of a family symmetry and the non-Abelian finite group \(A_4\) has proved to be useful in this regard [3, 4, 5]. Specifically, it was shown [6, 7, 8] how this may be achieved by the breaking of \(A_4\) in a prescribed manner [9], i.e. \(A_4 \to Z_3\) in the charged-lepton sector and \(A_4 \to Z_2\) in the neutrino sector. The group-theoretical framework of how this works in general has also been discussed [10, 11]. For a brief history, see Ref. [12].

In another development, it has been proposed recently [13] that neutrino mass is radiative in origin such that the particles in the loop are odd under a new discrete \(Z_2'\) symmetry, thereby accommodating a dark-matter candidate. The simplest realization of this “scotogenic” neutrino mass is depicted in Fig. 1. Here \(N_k\) are heavy Majorana fermion singlets

![Figure 1: One-loop generation of seesaw neutrino mass.](image-url)
odd under $Z'_2$ and $(\eta^+, \eta^0)$ is a scalar doublet also odd under $Z'_2$ [14], whereas the standard-
model $(\phi^+, \phi^0)$ is even. Exact conservation of $Z'_2$ means of course that $\eta^0$ has no vacuum
expectation value, so that $N$ is not the Dirac mass partner of $\nu$ as usually assumed. The
allowed quartic coupling $(\lambda_5/2)(\Phi^\dagger \eta^2 + H.c.)$ splits $\text{Re}(\eta^0)$ and $\text{Im}(\eta^0)$ so that whichever is
lighter is a good dark-matter candidate [13, 15, 16, 17]. The collider signatures of $\text{Re}(\eta^0)$
and $\text{Im}(\eta^0)$ have also been discussed [18]. For a brief review of the further developments of
this idea, see Ref. [19].

Going back to $A_4$, let $(\nu_i, l_i) \sim 3$ and either (I) $l_i^c \sim 1, 1', 1''$, or (II) $l_i^c \sim 3$, then with the
Higgs fields (I) $(\phi_i^+, \phi_i^0) \sim 3$, or (II) $(\phi_i^+, \phi_i^0) \sim 3$ and $(\zeta^+, \zeta^0) \sim 1$, the mass matrix linking
$l_i$ to $l_j^c$ is diagonalized on the left by [9]

$$U_{\nu} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

(3)

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, if $\langle \phi_i^0 \rangle = \langle \phi_j^0 \rangle = \langle \phi_k^0 \rangle = v$. This is a natural
minimum of the Higgs potential [3] because it corresponds to a $Z_3$ residual symmetry with
e $\sim 1, \mu \sim \omega^2, \tau \sim \omega$, whereas $\Phi \equiv (\Phi_1 + \Phi_2 + \Phi_3)/\sqrt{3} \sim 1$, $\Phi' \equiv (\Phi_1 + \omega \Phi_2 + \omega^2 \Phi_3)/\sqrt{3} \sim \omega^2$, and $\Phi'' \equiv (\Phi_1 + \omega^2 \Phi_2 + \omega \Phi_3)/\sqrt{3} \sim \omega$.

To obtain tribimaximal mixing, what is required for the Majorana neutrino mass matrix
$M_{\nu}$ is [6] $2 - 3$ symmetry and zero $1 - 2$ and $1 - 3$ entries. Since $123 + 231 + 312$ and
$132 + 321 + 213$ are $A_4$ invariants and $M_{\nu}$ must be symmetric, the simplest implementation
is to have [7]

$$M_{\nu} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix},$$

(4)

which requires effective scalar triplet fields $(\xi_i^{++}, \xi_i^+, \xi_i^0)$ transforming as $3$ with $\langle \xi_i^0 \rangle \neq 0$ and
$\langle \xi_{2,3}^0 \rangle = 0$, thereby breaking $A_4 \rightarrow Z_2$. Let the eigenvalues of $M_{\nu}$ be denoted by

$$m_1 = a + d, \quad m_2 = a, \quad m_3 = -a + d,$$

(5)

3
then the mixing matrix linking $\nu_{e,\mu,\tau}$ to $\nu_{1,2,3}$ is given by \[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \] i.e. tribimaximal mixing.

Because the scalar fields $\Phi_i$ and $\xi_i$ are both $\mathbb{3}$ under $A_4$, the requirement that they break the vacuum in different directions is incompatible with the most general Higgs potential allowed by $A_4$ alone. Complicated sets of auxiliary fields and symmetries (and/or possible extra dimensions) are then needed \[7, 8, 20, 21, 22\] for it to happen. This is perhaps the one stumbling block of the application of $A_4$ to tribimaximal mixing.

The reason that the two breaking directions are incompatible is because $A_4$ allows $\mathbb{3} \times \mathbb{3}$ to be invariant, so if one $\mathbb{3}$ has a vacuum expectation value along a certain direction, the other is forced to as well. This is of course not a problem if only one $\mathbb{3}$ is required to have vacuum expectation values and not the other, because that corresponds to having an exactly conserved $Z_2'$ under which the second $\mathbb{3}$ is odd. Specifically, let the charged leptons acquire mass from $\Phi_i$, but the neutrino masses are obtained radiatively as discussed earlier, without any vacuum expectation value for $\eta^0$.

![Figure 2: One-loop generation of seesaw neutrino mass.](image)

Instead of having three $N$'s (which would have been necessary in the canonical seesaw mechanism), assume just one $N$ but three scalar $\eta$ doublets, as shown in Fig. 2. Let $(\eta^+_i, \eta^0_i)$
transform as 3 under $A_4$, then $\mathcal{M}_\nu$ is proportional to the unit matrix. Suppose $A_4$ is now softly broken by the quadratic terms $\eta_2^\dagger \eta_3 + \eta_3^\dagger \eta_2$ and $2\eta_1^\dagger \eta_1 - \eta_2^\dagger \eta_2 - \eta_3^\dagger \eta_3$. Then $\mathcal{M}_\nu$ is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} a + 2b & 0 & 0 \\ 0 & a - b & d \\ 0 & d & a - b \end{pmatrix},$$

which will lead to tribimaximal mixing [6], with

$$m_1 = a - b + d, \quad m_2 = a + 2b, \quad m_3 = -a + b + d.$$ (8)

Since the origin of $\mathcal{M}_\nu$ is the mass-squared matrix of $\eta_{1,2,3}$, this model may be tested at least in principle. Note that $b = 0$ cannot be a solution here as in Ref. [7] because that would require a negative mass-squared eigenvalue for $\eta_i^0$. As it is, $\Delta m_{sol}^2 << \Delta m_{atm}^2$ implies $d \simeq 3b$ or $-2a - b$ in this scenario.

Consider now the scalar sector in more detail. Since $\eta_i$ are odd under the new exactly conserved $Z'_2$ for dark matter, and have no vacuum expectation value. The bilinear terms $\Phi_i^\dagger \eta_j$ are forbidden, and the quartic terms must contain an even number of $\Phi_i$ and $\eta_j$. The scalar potential consisting of only $\Phi_i$ is given by [3]

$$V_\Phi = m^2 \sum_i \Phi_i^\dagger \Phi_i + \frac{1}{2} \lambda_1 \left( \sum_i \Phi_i^\dagger \Phi_i \right)^2 + \lambda_2 (\Phi_1^\dagger \Phi_1 + \omega^2 \Phi_2^\dagger \Phi_2 + \omega \Phi_3^\dagger \Phi_3)(\Phi_1^\dagger \Phi_1 + \omega \Phi_2^\dagger \Phi_2 + \omega^2 \Phi_3^\dagger \Phi_3) + \lambda_3 (\Phi_2^\dagger \Phi_3)(\Phi_3^\dagger \Phi_2) + (\Phi_1^\dagger \Phi_3)(\Phi_3^\dagger \Phi_1) + (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \frac{1}{2} \lambda_4 [ (\Phi_2^\dagger \Phi_3)^2 + (\Phi_3^\dagger \Phi_2)^2 ] + H.c. \right\}.$$ (9)

The parameters $m^2$ and $\lambda_{1,2,3}$ are automatically real, and $\lambda_4$ may be chosen real by rotating the overall phase of $\Phi_i$. The vacuum solution

$$v_1 = v_2 = v_3 = v = [-m^2/(3\lambda_1 + 2\lambda_3 + 2\lambda_4)]^{1/2}$$

is protected by the residual symmetry $Z_3$, under which

$$\Phi \equiv \frac{1}{\sqrt{3}}(\Phi_1 + \Phi_2 + \Phi_3) \sim 1$$

(11)
\[ \Phi' \equiv \frac{1}{\sqrt{3}}(\Phi_1 + \omega \Phi_2 + \omega^2 \Phi_3) \sim \omega^2, \]  
\[ \Phi'' \equiv \frac{1}{\sqrt{3}}(\Phi_1 + \omega^2 \Phi_2 + \omega \Phi_3) \sim \omega, \]  
(12)  
(13)
as already mentioned. The scalar doublet \( \Phi \) has the properties of the standard-model Higgs doublet with mass-squared eigenvalues \( 2(3\lambda_1 + 2\lambda_3 + \lambda_4)v^2, 0, \) and \( 0 \) for \( \sqrt{2}\text{Re}\phi^0, \sqrt{2}\text{Im}\phi^0, \) and \( \phi^0 \) respectively. The charged scalars \( \phi^{\prime \pm} \) and \( \phi''^{\prime \pm} \) have \( m^2_{\pm} = -3(\lambda_3 + \lambda_4)v^2, \) whereas \( \phi'^0 \) and \( \phi''^0 \) are not mass eigenstates, but rather \( \phi'^0 = (\psi_1 + \psi_2)/\sqrt{2} \) and \( \phi'^0 = (\psi_1^* - \psi_2^*)/\sqrt{2}, \) \( \psi_1 = \frac{1}{\sqrt{2}}\text{Re}(\phi'^0 + \phi''^0) + \frac{i}{\sqrt{2}}\text{Im}(\phi'^0 - \phi''^0) \sim \omega^2, \)  
(14)  
(15)
with \( m^2_1 = 2(3\lambda_2 - \lambda_3 - \lambda_4)v^2 \) and \( m^2_2 = -6\lambda_4v^2. \) This subtlety in the mass spectrum of \( \phi'^0 \) and \( \phi''^0 \) was not recognized in Ref. [3], where \( \tau^- \rightarrow \mu^- \mu^+e^- \) and \( \mu \rightarrow e\gamma \) were thought to be nonzero. In fact, they are forbidden by the residual \( Z_3 \) symmetry.

The addition of \( \eta_i \) to the scalar potential does not change the above because \( \langle \eta^0_i \rangle = 0 \) and \( Z'_2 \) remains exactly conserved. However, the breaking of \( A_4 \rightarrow Z_3 \) by \( \langle \phi^0_i \rangle \) generates additional contributions to the \( \eta^0_i \) mass-squared matrix of the form
\[
\Delta^2 \{ \eta^*_1 \eta_1 + \eta^*_2 \eta_2 + \eta^*_3 \eta_3 \} + \Delta^2 \{ \eta^*_1 \eta_2 + \eta^*_2 \eta_3 + \eta^*_3 \eta_1 \} + \text{c.c.} \}
+ \{ \frac{1}{2} \Delta^2 \{ \eta^2_1 + \eta^2_2 + \eta^2_3 \} + \text{c.c.} \} + \Delta^2 \{ \eta_1 \eta_2 + \eta_2 \eta_3 + \eta_3 \eta_1 \} + \text{c.c.}. \]  
(16)
In other words, except for soft terms, the complete Higgs potential remains invariant under \( Z_3 \) after spontaneous symmetry breaking. The induced neutrino mass matrix of Eq. (7) is then modified:
\[
\mathcal{M}_\nu = \begin{pmatrix}
a + 2b & e & e \\
e & a - b & d \\
e & d & a - b
\end{pmatrix}.
\]  
(17)
Since the one-loop neutrino mass of Fig. 1 is proportional to \( \Delta^2_3 \) and \( \Delta^2_1 \) which split \( \text{Re}(\eta^0_i) \) and \( \text{Im}(\eta^0_i) \), these parameters should be relatively small. Assuming that \( \Delta^2_2 \) is also small,
then $e$ should be small compared to $a, b, d$ in Eq. (17). This means that \[ \sin^2 2\theta_{23} = 1 \]
and $\theta_{13} = 0$ as before, but the solar mixing angle is now given by

$$\tan^2 \theta_{12} = \frac{1}{2}(1 - 6\epsilon + 15\epsilon^2),$$

where $\epsilon = e/(d - 3b)$. Thus $\tan^2 \theta_{12} = 0.47$ is obtained for $\epsilon = 0.01$.

One possible explanation of the smallness of the terms in Eq. (16) is that $\Phi$ and $\eta$ are separated in an extra dimension so that they communicate only through a singlet in the bulk. In the limit this effect vanishes, there would be no mass splitting between $\text{Re}(\eta^0)$ and $\text{Im}(\eta^0)$, resulting in zero neutrino mass and no viable dark-matter candidate. With it, neutrinos acquire small radiative Majorana seesaw masses, $\text{Re}(\eta^0)$ is a good dark-matter candidate, and near tribimaximal mixing is possible.

In conclusion, it has been shown how $A_4$ symmetry may be implemented in a model of “scotogenic” neutrino mass with dark scalar doublets. The neutrino mass matrix is induced by the neutral scalar mass-squared matrix spanning $\text{Re}(\eta_{1,2,3}^0)$ and $\text{Im}(\eta_{1,2,3}^0)$. This scheme allows the neutrino mixing angles $\theta_{23}$ and $\theta_{13}$ to be exactly $\pi/4$ and 0, whereas $\tan^2 \theta_{12}$ should not be exactly 1/2. Suppose the lightest $\text{Re}(\eta^0)$ is dark matter, then its possible discovery \[18\] at the LHC together with the other $\eta$ particles in accordance with Fig. 2 would be a verifiable test of this proposal.

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