The two Josephson junction flux qubit with large tunneling amplitude

V.I. Shnyrkov,1 A.A. Soroka,2 and S.I. Melnik3

1B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, av. Lenina 47, Kharkov 61103, Ukraine
2National Science Center "Kharkiv Institute of Physics and Technology", Akhiezer Institute for Theoretical Physics, st. Akademicheskaya 1, 61108 Kharkov, Ukraine
3M.K. Yankel Kharkov National University of Radio Electronics, Ministry of Education of Ukraine, av. Lenina 4, Kharkov 61161, Ukraine

In this paper we discuss solid-state nanoelectronic realizations of Josephson flux qubits with large tunneling amplitude between the two wells. The latter can be controlled via the height and form of the barrier, which is determined by quantum-state engineering of the flux qubit circuit. The simplest circuit of the flux qubit is a superconducting loop interrupted by a Josephson nanoscale tunnel junction. The tunneling amplitude between two macroscopically different states can be essentially increased, by engineering of the qubit circuit, if tunnel junction is replaced by a ScS contact. However, only Josephson tunnel junctions are particularly suitable for large-scale integration circuits and quantum detectors with preset-day technology. To overcome this difficulty we consider here novel flux qubit with high-level energy separation between "ground" and "excited" states, which consists of a superconducting loop with two low-capacitance Josephson tunnel junctions in series. We demonstrate that for real parameters of resonance superposition the splitting amplitude can reach values greater than 1 K. Analytical results obtained with instanton technique show good correlation with a numerical exact solution.

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Since successful demonstration of Rabi oscillations and Landau-Zener coherent effects,1,2,3,4,5 the superconducting qubits (quantum bits) based on mesoscopic Josephson junctions became the subject of consideration as possible candidates to be the basic elements of a quantum computer hardware.6,7 Including detectors to measure the state of an individual qubit.8,9,10,11 The Josephson junction (JJ) qubits have two energy scales which are the Josephson coupling energy EJ and the charging energy EC of the JJ, and they are subdivided into flux qubits, charge qubits, as well as charge-phase and persistent-current qubits. In principle, all circuits of a quantum computer can be fabricated by modern techniques using these superconducting qubits. However, it is but poor quality6,12 of the experimentally tested elements that is the limiting factor on the way of implementation of quantum registers. For example, an important but still unsolved problem in the physics of a qubit working in the charge regime with EC/EJ ≫ 1 is an essential decrease of high spectral density of the noise associated with the motion of charge in traps. In its turn, the phase qubit (EJ/EC ≫ 1), which utilizes the phase of the superconducting order parameter as a dynamic variable, is much less sensitive to the charge fluctuations but is subject to the influence of the noise in critical current of JJ, spin fluctuations and Nyquist noise currents generated by excess ambient temperature. The tunnel splitting of the energy levels arising from the coherent superposition of the macroscopic states is small usually, ΔE01 ≈ 150–250 mK. Taking into account the effective noise temperature, which can reach T eff ≈ 50–100 mK in experimental studies of the qubit dynamics, leads to13,14,15 a dramatic fall of the decoherence times τϕ and relaxation times τε. This means that, in order to enhance considerably the qubit quality,6 (the number of one-bit operations during the coherence timespan), the system with large (ΔE01 ≳ 10−23 – 10−22 J) tunnel splitting of the energy levels should be created.

Undoubtedly, the problem of creation of a quantum register based on Josephson qubits brings up many issues but presently the invention of a high-quality qubit is the most important one among them. It is easy to show that the rate of the energy exchange between two macroscopic states in a flux qubit is bounded by the "cosine" shape of the potential barrier and cannot be increased owing to decreasing the barrier height since the latter determines the characteristic rate of thermal decay of the current-flow states. A similar limitation associated with the lowering of the effective barrier height can appear also when highly increasing the pre-exponential factor. It is absolutely obvious that the ideal case for a flux qubit is when the tunnel barrier in the phase space looks like II-function having sufficiently large height and small action. It was this issue that motivated the authors of the Ref. 12 for analyzing the phase-slip qubit, whose creation required developing a new non-Josephson technology. In this paper we search for an improved barrier design for the JJ flux qubit.

The recent Ref. 10 demonstrated how the level splitting can be increased at low temperatures (T → 0) by an order of magnitude with the potential barrier height kept unchanged by modifying the qubit’s potential barrier shape due to using the clean-limit ScS junction in the superconducting ring. However, the fabrication difficulties of obtaining pure and reproducible ScS junctions are the serious hindrance in the way of designing large-scale integrated qubit circuits.

To solve this problem, the analysis is carried out in the...
of $\Phi$ current qubit have to satisfy the usual condition for flux qubits in the phase space being modified. To retain this classi-
SQUIDs with magnetic flux $\Phi$ as compared to the "junction 2" Josephson energy, the critical current and the capaci-
tance as compared to the "junction 2" would have greater or equal values of the
"junction 1". The external magnetic flux $\Phi_e$ can be coupled to the qubit by a se-
parate coil located in close proximity to the qubit’s loop. It is well known that in classical limit the circulating current $I_s$ as a function of external magnetic flux for dc SQUIDs with $I_{c1} = I_{c2}$ has the singularity in the points of $\Phi_e = \Phi_0(n + 1/2)$ ($\Phi_0$ is the flux quantum) so that the two-junction interferometer can be considered as a "single-junction" one, with the potential energy shape in the phase space being modified. To retain this classical effect in the quantum regime the proposed 2JJ flux qubit have to satisfy the usual condition for flux qubits $g = E_{j1}/E_{c1} \gg 1$, at which phase is a good quantum variable and the charging effect on the island between JJ contacts is negligible.

The problem lies in determining and analyzing the tunnel splitting $\Delta E_{01} = E_1 - E_0$ of the degenerated zero energy level in the double-well symmetric potential of a 2JJ flux qubit (at corresponding external conditions) resulted from the coherent quantum tunneling of the magnetic flux between the wells. In the proposed mesoscopic system in quantum regime, the two lower energy levels $E_0$ and $E_1$ arising from coherent superposition of the macro-
scopically distinct flux or persistent-current states form a quantum bit (qubit). It turns out that, because of the
change in the form of the potential energy of the 2JJ flux qubit as compared to the 1JJ qubit, the tunnel splitting $\Delta E_{01}$ can multiply rise reaching the values $\geq 1$ (in temperature units) and substantially enhance the properties of the qubit as a basic element for quantum computations. However, even slightly away from the symmetry condition $I_{c1} = I_{c2}$, the splitting energy is reduced. The sensitivity of the $\Delta E_{01}$ magnitude to $\lambda$ as well as to the junction parameters can limit applications based on the 2JJ flux qubit both for quantum computation and quantum detectors.

With this sake, we will discuss the 2JJ flux qubit in the approximation of the Hamiltonian of an isolated system in the zero temperature limit. All the dissipative processes associated with the own and the external, regarding the system, degrees of freedom (the quasiparticles, the magnetic flux fluctuations in the qubit and in the outer measuring circuit, etc.) are neglected in this approximation. In the framework of this approximation, only the supercurrent component flows in the qubit ring which in classical regime, according to the Josephson relation, is equal to

$$I_s = I_{c1} \sin \varphi_1 = I_{c2} \sin \varphi_2,$$

where $\varphi_1, \varphi_2$ are the order parameter phase differences at corresponding tunnel junctions. It is convenient to count the values of the supercurrent $I_s$ and the phase differences at the junctions clockwise, the applied magnetic flux $\Phi_e$, the total magnetic flux in the ring $\Phi$ and the supercurrent $I_s$ being tied by the relation $\Phi = \Phi_e - LI_s(\Phi)$. The classic Hamiltonian of the 2JJ flux qubit in the approximation of the isolated system contains the contributions of the electrostatic energy of the charges in the junction capaci-
tances, the junction Josephson energies and the magnetic energy of the supercurrent in the ring, and has the form:

$$H = \frac{(2eN_0)^2}{2} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) - (E_{j1} \cos \varphi_1 + E_{j2} \cos \varphi_2) + \frac{(\Phi - \Phi_e)^2}{2L} + E_0,$$

where $N_0$ is the number of the excess (deficient) Cooper pairs in the banks of the SIS Josephson junctions, $E_0$ is the constant fixing the reference level for the poten-
tial energy. Using relation (1), we will reduce expression for the Josephson energy in Hamiltonian (2) to the form $E_j(\phi) = -(E_{j1} \cos \varphi_1 + E_{j2} \cos \varphi_2) = -E_{j1} \sqrt{(1 - \lambda)^2 + 4\lambda \cos^2(\phi/2)}$, where a new variable of the overall phase $\phi = \varphi_1 + \varphi_2$ is introduced. As follows from the theory of charge-phase qubits [6], this classical expression $E_j(\phi)$ for the Josephson energy of the two-
junction interferometer remains unchanged in the quantum regime of system dynamics in the limit $g \to \infty$. Mathematically it means a fulfillment of the relationship (1) between semiclassical variables $\varphi_1, \varphi_2$ in the quantum regime at $g \gg 1$, and physically disappearance of phase fluctuations induced by the charging effect on the island between JJ contacts [6].
Due to the unambiguity of the superconductor’s order parameter the variable $\phi$ satisfies the condition

$$\phi = \phi_1 + \phi_2 = 2\pi \frac{\Phi}{\Phi_0} + 2\pi n, \quad \Phi = n\Phi_0 + \frac{\phi}{2\pi}\Phi_0, \quad \Phi_0 = \pi\hbar/e,$$

where $n$ is the integer number of the flux quanta $\Phi_0$ in the total magnetic flux $\Phi$ (below, we will consider the qubit to work in the $n = 0$ mode). Owing to the relationships (1), (3) there is the only independent phase variable from $\phi_1, \phi_2, \phi$, and in the quantum regime the physical fluctuating variable is the total phase difference $\phi$ at the both junctions which equals, to within $2\pi$ factor, to the total magnetic flux in the ring in the units of flux quantum $(\phi/2\pi = \Phi/\Phi_0)$. The other two variables, $\phi_1(\phi)$ and $\phi_2(\phi)$, are unambiguously dependent on $\phi$, i.e., the total phase difference $\phi$ uniquely expands into the components $\phi_1, \phi_2$ (see below).

The transition to the quantum description of the flux qubit consists in associating the value $N_0$ of the Cooper pairs tunneling through the junctions with the operator $\hat{N}_0 = -i\frac{\partial}{\partial \phi}$, conjugated to the phase operator $\hat{\phi}$ ([$\hat{N}_0, \hat{\phi}$] = $-i$), and solving the Schrodinger equation with the obtained quantum Hamiltonian in the particle with the mass $M$ moving in the potential $U(\phi)$. Here $\hat{\phi} = \hbar\hat{\phi}$ and $\hat{\phi}$ corresponds to the Schrodinger equation for masses $M$ in curves $U(\phi)$.

$$\hat{H} = \frac{\hat{p}^2}{2M} + U(\phi) = -\hbar^2 \frac{\partial^2}{2M \partial \phi^2} + E_J(\phi)\left(\varepsilon_0 - \sqrt{(1 - \lambda)^2 + 4\lambda \cos^2 \phi/2 + \left(\frac{\phi - \phi_e}{2\beta_L}\right)^2}\right),$$

which can be considered as the Hamiltonian of a quantum particle with the mass $M$ moving in the potential $U(\phi)$. Here $\hat{p} = \hbar\hat{N}_0 = -i\hbar\frac{\partial}{\partial \phi}$ and $\hat{\phi}$ corresponds to the Schrodinger equation for masses $M$ in curves $U(\phi)$.

The potential $U(\phi)$ shape depends on the parameters $\lambda, \beta_L, \phi_e$. We are interested in the case of symmetric potential which, according to (3), is realized at $\phi_e = \pi$ ($\Phi_e = \Phi_0/2$). It should be noticed that in the extreme case of identical junctions, at $\lambda = 1$, the potential $U(\phi)$ coincides with the potential of the flux qubit based on the clean ScS contact studied in Ref. (Equation (3)):

$$U_{ScS}(\phi) = E_J(-2\cos(\phi/2) + \frac{(\phi - \phi_e)^2}{2\beta_L^2}).$$

At the same time, owing to renormalization of the mass $M$ for the 2JJ flux qubit by the factor $\lambda/(\lambda + 1)$ in respect to the corresponding mass for the ScS flux qubit (provided that the capacitances of SIS and ScS junctions are equal, $C_1 = C$), for $\lambda = 1$, the relation for masses is $M_{2JJ} = M_{ScS}/2$. Hence, at $\lambda \approx 1$ the splitting $\Delta E_{01}$ in 2JJ qubit is expected yet more than in the ScS qubit. The parameter $\beta_L$ determines the height of the potential barrier of the double-well potential, so that the barrier height goes down while reducing $\beta_L$. Like in the case of ScS qubit, the 2JJ qubit potential has two local minima even at $\beta < 1$ (unlike the SIS qubit where the double-well potential exists at $\beta > 1$ only), which gives a possibility of considerable scaling down the geometric dimension (inductance) of the system with the mesoscopic junctions.

Fig. 2 shows the potential $U(\Phi/\Phi_0)/k_B$ of 2JJ flux qubit for several parameters couples ($\lambda, \beta_L$) and also, for the comparison sake, the well-known potential $U_{ScS}(\Phi/\Phi_0)/k_B$ of 1JJ flux qubit at external magnetic flux $\Phi_e = \Phi_0/2$. The inductances $L$ for both types of the qubits can be supposed equal (to specify the magnetic flux fluctuation level) while the parameter $\beta_L$ (i.e., the critical currents $I_1, L$ of the corresponding SIS junctions) in all the dependences are chosen in such a way so that the potential barriers in all the potentials were of the same height $U_0$. The latter requirement implies roughly equal decay rates for the metastable states due to thermal fluctuations, taking them into account being beyond our consideration. Apparently, to realize the quantum regime in a physical experiment, the value $U_0/k_B$ must highly
exceed the system temperature. As seen from Fig. 2 the potentials \( U(\Phi/\Phi_0)/k_B \) for a 2JJ qubit have lesser width (between the potential minima points) as compared to the corresponding potentials for a 1JJ qubit while the area under the potential curve between the points of its minima for the 2JJ qubit shrinks greatly against the corresponding area for the 1JJ qubit. Additionally, if the corresponding capacitances of the SIS junctions in both 2JJ and 1JJ qubits are equal \( (C_1 = C) \) then the ratio of the effective masses for these qubits is \( \lambda/(\lambda + 1) \). As it will be shown below, it is the change in the potential shape and the decrease of the effective mass in 2JJ qubit that lead to multiple rise in the amplitude of its tunnel splitting.

**FIG. 3:** a) Integral phase-current relation \( I_s(\phi/2\pi)/I_{c1} \) for 2JJ qubit at various \( \lambda \): 0.99 - 1, 0.9 - 2, 0.8 - 3, 0.5 - 4. b) Functions \( (\varphi_1/2\pi)[\phi/2\pi] - I_1 \) and \( (\varphi_2/2\pi)[\phi/2\pi] - 2 \) for 2JJ qubit at \( \lambda = 0.9 \). The straight line \( \varphi_1(\phi) + \varphi_2(\phi) = \phi - 3 \) corresponds to the \( \phi \) definition. The values of \( \varphi_1/2\pi = \arcsin(\lambda)/2\pi \approx 0.18 \) and \( \varphi_2/2\pi = 0.25 \) (the latter being \( \lambda \)-independent) correspond to \( \phi_m/2\pi = \arccos(-0.9)/2\pi \approx 0.43 \).

The shape of the potential \( U(\phi) \) for 2JJ qubit is directly related to the form of the current-phase relation derived from \( \mathbf{1}, \mathbf{3} \):

\[
\frac{I_s(\phi)}{I_{c1}} = \sin \varphi_1 = \lambda \sin \varphi_2 = \frac{\lambda \sin \phi}{\sqrt{(1 - \lambda)^2 + 4\lambda \cos^2(\phi/2)}}. \tag{5}
\]

The current-phase relation \( I_s(\phi) \) extrema (which are equal by their absolute values) are located in the points \( \phi_m = \arccos(-\lambda) \) (maximum; \( \pi/2 \leq \phi_m \leq \pi \)) and \( \phi_{m1} = 2\pi - \arccos(-\lambda) \) (minimum) symmetrically around the point \( \phi = \pi \), where the supercurrent vanishes to zero \( (I_s = 0) \) alternating its direction. Thus, at \( \lambda \) being near the unity, in the interval \( (\phi_m, \phi_{m1}) \), the supercurrent \( I_s \) changes dramatically from its maximum to minimum value with alternating the current direction in the point \( \phi = \pi \). Fig. **3**(a) displays the integral current-phase dependence \( I_s(\phi/2\pi)/I_{c1} \) for 2JJ qubit for several parameters \( \lambda \). The interval \( (\phi_m, \phi_{m1}) \) shrinks as the parameter \( \lambda \) increases and the maximum-to-minimum by-current transition becomes more sharp. The extreme case \( \lambda = 1 \) corresponds to \( \phi_m = \phi_{m1} = \pi \) with the infinite derivative of the current-phase relation in the point \( \pi \). Let us also consider the order parameter phase differences \( \varphi_1(\phi), \varphi_2(\phi) \), derived directly from \( \mathbf{3} \). The analysis of formula \( \mathbf{5} \) shows that the function \( \varphi_1(\phi) \) for a junction with high critical current has extrema in the points \( \phi_m, \phi_{m1} \). The transition from the maximum positive value \( \varphi_1(\phi_m) = \arcsin \lambda \) \( (0 \leq \varphi_1(\phi_m) \leq \pi) \) to the minimum negative value \( \varphi_1(\phi_{m1}) = -\arcsin \lambda \) with alternating the phase difference sign in the point \( \pi \) \( (\varphi_1(\pi) = 0) \) takes place in the interval \( (\phi_m, \phi_{m1}) \), and \( \varphi_1(0) = \varphi_1(2\pi) = 0 \). The function \( \varphi_2(\phi) \) for a junction with lower critical current is a monotonically increasing one from \( \varphi_2(0) = 0 \) to \( \varphi_2(2\pi) = 2\pi \), which is symmetrical with respect to the line \( y = \phi/2 \). The function \( \varphi_2(\phi) \) behaves as follows: \( \varphi_1 = \varphi_2 = \phi/2 \) at \( 0 \leq \phi < \pi \); at the point \( \pi \) a jump appears in the function \( \varphi_1(\phi) \) between the values \( \pi/2, -\pi/2 \) with
further linear rise up to \( \varphi_1(2\pi) = 0 \), while the function \( \varphi_2(\phi) \) demonstrates a jump between the values \( \pi/2, 3\pi/2 \) with further linear increase up to \( \varphi_3(2\pi) = 2\pi \). Fig. 1(b) exhibits dependences \( (\varphi_1/2\pi)|\phi/2\pi|, (\varphi_2/2\pi)|\phi/2\pi| \) for a certain \( \lambda \), with their distinctive appearance. The straight line \( \varphi_1(\phi) + \varphi_2(\phi) = \phi \) corresponds to the \( \phi \) definition showing the expansion of the total phase difference over the both junctions into the component phase differences of the order parameter over each of them.

We will find the tunnel splitting \( \Delta E_{01} \) of the degenerated zero level in the symmetrical (at \( \phi_e = \pi \)) double-well potential \( U(\phi) \) in the 2JJ flux qubit by numerical solution of the Schrodinger equation and analytically by using instanton technique in the semiclassical approximation. To find a numeric solution of the Schrodinger stationary equation

\[
\hat{H}\Psi(\phi) = E\Psi(\phi)
\]

with Hamiltonian (4), a kind of the finite elements method is used with approximation of the potential \( U(\phi) \) by a piecewise constant function. Zero boundary conditions are used for the wave function \( \Psi(\phi) \), the domain width and the element quantity being set so that provide good accuracy of the calculation.

In the semiclassical approximation the problem of a tunneling quantum particle can be solved using the instanton technique. For a particle of the mass \( M \), moving at zero temperature in symmetric double-well potential \( V(x) \), referenced from its minimum level \( V(\pm a) = 0 \), \( \pm a \) are the minimum points), the expressions for the energy levels \( E_{1,0} \) and the tunnel splitting \( \Delta E_{01} \) read like

\[
E_{1,0} = E_0 \pm \frac{\Delta E_{01}}{2} = \frac{\hbar \omega_0}{2} \pm \hbar K \exp \left( -\frac{S_0}{\hbar} \right),
\]

\[
S_0 = \int_{-a}^{a} dx \sqrt{2MV(x)},
\]

\[
K = A\omega_0 \sqrt{\frac{M\omega_0 a^2}{\pi \hbar}}, \quad \omega_0^2 = \frac{V''(\pm a)}{M},
\]

\[
\Delta E_{01} = 2A\hbar\omega_0 \sqrt{\frac{M\omega_0 a^2}{\pi \hbar}} \exp \left( -\frac{S_0}{\hbar} \right).
\]

Here \( \omega_0 \) is the frequency of the particle zero oscillations in each of the wells, \( S_0 \) is the particle action on the instanton trajectory, the dimensionless constant \( A \) is found from the equation for the instanton’s function \( t(x) \) in the asymptotic limit:

\[
t(x)|_{x \to a} = \int_{0}^{x-a} \frac{dx}{\sqrt{2V(x)/M}} = \frac{-1}{\omega_0} \ln \frac{a-x}{Aa},
\]

\[
A = \lim_{x \to a} \frac{(a-x)}{a} \exp \left( \int_{0}^{x} dx \sqrt{\frac{M\omega_0^2}{2V(x)}} \right).
\]

Starting from the Hamiltonian (4) and using formulae (7), (8), we obtain the tunnel splitting \( \Delta E_{01} \) for the 2JJ flux qubit in the case of symmetric double-well potential:

\[
\Delta E_{01} = 4A\hbar\omega_0 \sqrt{\frac{M\omega_0 a^2}{\pi \hbar}} \exp \left( -\frac{S_0}{\hbar} \right),
\]

\[
S_0 = \frac{2\hbar}{a} \sqrt{\frac{\lambda C_1}{(\lambda+1)L}} \int_{0}^{a} d\alpha \sqrt{2\beta L \left( \sqrt{(1-\lambda)^2/4 + \lambda \sin^2 \alpha} - \sqrt{(1-\lambda)^2/4 + \lambda \sin^2 \alpha} + \alpha^2 - \alpha_0^2 \right)},
\]

\[
\omega_0 = \frac{(\lambda+1)}{\lambda C_1 L} \left( 1 - \frac{1}{2} \frac{d^2}{d \alpha^2} \left( \sqrt{(1-\lambda)^2/4 + \lambda \sin^2 \alpha} \right) \right), \quad M = \left( \frac{2\pi}{\Phi_0} \right)^2 \frac{\lambda C_1}{(\lambda+1)},
\]

\[
A = \lim_{\alpha \to \alpha_0} \frac{(\alpha_0 - \alpha)}{\alpha_0} \exp \left( \frac{\sqrt{M\omega_0^2}}{E_{11}} \int_{0}^{a} \frac{d\alpha}{\sqrt{(1-\lambda)^2/4 + \lambda \sin^2 \alpha} - \sqrt{(1-\lambda)^2/4 + \lambda \sin^2 \alpha} + (\alpha^2 - \alpha_0^2)/\beta L} \right),
\]

\[
2a_0 \frac{\beta L}{\lambda \sin \alpha_0 \cos \alpha_0} \sqrt{(1-\lambda)^2/4 + \lambda \sin^2 \alpha_0} = 0.
\]

A variable \( \alpha = (\phi - \pi)/2 \) is introduced in formulae (9) (due to the potential symmetry condition \( \phi_e = \pi \)), the minima point of \( \alpha_0 > 0 \) of the potential \( U(\alpha) \) satisfying equation (9). The accuracy of the semiclassical approximation is high provided that \( S_0/\hbar \gg 1 \), the method accuracy degrades as the dimensionless variable \( S_0/\hbar \) diminishes approaching the unity. The results of a numerical analysis is of great importance in this region.
FIG. 4: (a) Function $\Delta E_{01}(\beta_L)/k_B$ for 2JJ qubit at $\lambda = 0.99$ – 1, ScS qubit – 2, 1JJ qubit – 3, the points in the numerical curves corresponding to equal height (9.64 K) of the potential barrier for all the qubits are indicated by arrows. (b) Function $\Delta E_{01}(\beta_L)/k_B$ for 2JJ qubit and various $\lambda$: 0.99 – 1, 0.9 – 2, 0.8 – 3 and ”level line” of equal heights (9.64 K) of the potential barriers at varying $\lambda - 4$. The numerically obtained results are pointed by hollow circles, the analytically obtained ones are plotted by solid lines in the graphs (a) and (b). Dashed lines show the lowest boundary $\beta_L$, at which the level $E_1$ height becomes equal to the potential barrier height $U_0$. For 1JJ and ScS qubits, the capacitance of corresponding (SIS and ScS) junctions is $C = 2.7 \times 10^{-15}$ F while for 2JJ qubit the capacitance of the larger SIS junction is $C_1 = 2.7 \times 10^{-15}$ F. The geometric ring inductance is $L = 3.0 \cdot 10^{-10}$ H, parameter $g \approx 76\beta_L$ for all the qubits.

Fig. 4(a,b) presents the $\beta_L$-dependences of the tunnel splitting $\Delta E_{01}(\beta_L)/k_B$ for 2JJ, ScS and SIS flux qubits at the equal capacitances of the corresponding junctions $C_1 = C = 2.7$ pF and at the inductance $L = 0.3$ nH of the qubits loop. In both plots the curves calculated numerically are pointed by hollow circles while the ones obtained analytically using the instanton technique are plotted by solid lines. The formulae (9) were used for 2JJ qubit while similar formulae were taken for ScS and SIS qubits based on the forms of their potentials. The change in the parameter $\beta_L$ means the variation of the critical currents $I_{L,1}, I_c$ of the corresponding junctions at a fixed inductance $L$. The double-well potential height decreases with lowering the parameter $\beta_L$, the energy level $E_1$ being equalized to the potential barrier height $U_0$ at a certain $\beta_{L0}$ ($E_1 = U_0$) and exceeding it with further $\beta_L$ lowering. Then, the wave function corresponding to the level $E_1$ is no further a superposition of the states localized in the left and right wells. The boundary values $\beta_{L0}$ for the curves in the figure are indicated by dash lines. In the vicinity of $\beta_{L0}$, at ($U_0 - E_1) \sim k_BT$, the quantum coherence will be destroyed due to thermal fluctuation causing the over-barrier transitions. One can see from Fig. 4 that the numerically and the analytically obtained curves almost coincide at large $\beta_L$ and begin to diverge at lower $\beta_L$. This is because of the condition of semiclassicity $S_0/h \gg 1$ starts to fail when diminishing $\beta_L$. This, in its turn, is caused by decreasing of the barrier height $U_0$ and therefore the action $S_0$. The analysis of the dependences $S_0(\beta_L)/h$ reveals that $S_0/h \sim 1$ at $\beta \sim \beta_{L0}$ and the relative divergence between the numerical and the analytical results for 2JJ and ScS qubits is within 2 to 10 per cent. For a SIS qubit a fit of the numerical and analytical results requires the more accurate fulfillment of the semiclassicity condition. However, it is follows even from this analysis that obtaining the tunnel splitting $\Delta E_{01} > 1$ K in the flux qubit based on a single SIS junction is impossible under condition of weak $(U_0 - E_1 \gg k_BT)$ influence of thermal fluctuations on the metastable states decay. The dependences $S_0(\beta_L)/h$ for 2JJ, ScS and SIS qubits are close to linear ones, whose slope (the action $S_0$ from $\beta_L$ rate of increase) being higher in the indicated order. The value of tunnel splitting in the region of its exponential smallness $S_0(\beta_L)/h \gg 1$ diminishes in the same sequence.

The points in the numerical curves corresponding to
the equal heights of the potential barriers ($U_0 = 9.64\, \text{K}$) are indicated by arrows in Fig. 4(a). The corresponding values of the parameter couples ($\beta_L, E_0(\beta_L)/k_B$) for 2JJ ($\lambda = 0.99$), ScS and SIS flux qubits are: (0.9, 3.83K), (0.88, 1.79K), (1.60, 0.16K). It is seen that, under this condition, the tunnel splitting in a 2JJ qubit is more than twice the splitting in a ScS qubit and more than 20 times higher than that of a SIS qubit. The curve for the tunnel splitting in 2JJ lies completely above the curves for ScS and SIS qubits, and the tunnel splitting for a 2JJ qubit is more than twice the splitting in a ScS qubit and more than 20 times higher than that of a SIS qubit. The curve for the tunnel splitting of a 2JJ qubit is less by a factor of about two. Fig. 4(b) shows the dependence $\Delta E_{01}(\beta_L)/k_B$ corresponding to equal height (9.64 K) of the potential barriers in 2JJ qubit with varying $\lambda$. The curve $\Delta E_{01}(\beta_L)/k_B$ shifts right when decreasing $\lambda$, and the smaller being the value of $\lambda$, the higher the tunnel splitting at a fixed $\beta_L$. This, however, is due to the lowering of the barrier $U_0$ height when decreasing $\lambda$ that leads to the exponential rise of the thermal decay rate. Note that when desymmetrizing the junctions a fit between the numerical and the analytical curves gets worse because $S_0(\beta_L)/h$ decreases. As seen from the plot, the value of the tunnel splitting gradually diminishes while moving along the level line with the equal height of the potential barriers towards the lower values of the junction symmetry parameter $\lambda$ (and the higher $\beta_L$).

It should be emphasized that the principal requirements to 2JJ flux qubits, namely: $\lambda \lesssim 0.95$; $C \lesssim 50\, \text{fF}/\mu\text{m}^2$; $j_c \sim 10^3\, \text{A/cm}^2$; $I_c \sim 1\, \mu\text{A}$ at the JJ area $S_J \sim 0.1\, \mu\text{m}^2$; $L \sim 0.3\, \text{nH}$, $\beta_L \sim 1$ can be met with the present-day technology based on Nb, NbN, MoRe materials with superconductivity gap $\Delta(0) \sim 10\, \text{K}$ (see, e.g., Ref. 20). One can notice in conclusion that 2JJ flux qubit with large amplitude of tunnel splitting potentially has some strong advantages: (i) weak sensitivity to the motion of charge in traps; (ii) extremely fast excitation (pumping frequency) in qubit-based readout as well as in computer circuits due to considerable increasing of the quantum tunneling rate $\nu \sim \Delta E_{01}$; (iii) macroscopically large energy relaxation times $\tau_\nu$ (see, e.g., Ref. 3 and Refs. therein); (iv) further improvement of qubit coherence characteristics.

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* Electronic address: Shnyrkov@ilt.kharkov.ua

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