Twist-three effects in two-photon processes.

A.V. Belitsky\textsuperscript{a,b}, D. Müller\textsuperscript{b}

\textsuperscript{a}C.N. Yang Institute for Theoretical Physics
State University of New York at Stony Brook
NY 11794-3840, Stony Brook, USA

\textsuperscript{b}Institut für Theoretische Physik, Universität Regensburg
D-93040 Regensburg, Germany

Abstract

We give a general treatment of twist-three effects in two-photon reactions. We address the issue of the gauge invariance of the Compton amplitude in generalized Bjorken kinematics and relations of twist-three ‘transverse’ skewed parton distributions to twist-two ones and interaction dependent three-particle correlation functions. Finally, we discuss leading order evolution of twist-three functions and their impact on the deeply virtual Compton scattering.

Keywords: two-photon processes, deeply virtual Compton scattering, skewed parton distribution, twist-three contributions

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1 Introduction.

Exclusive two-photon reactions involving two hadrons (scattering or production), like deeply virtual Compton scattering (DVCS) \[ \gamma^* \gamma \rightarrow hh \] with small invariant mass of the hadron system \[ \gamma \gamma \rightarrow hh \] etc., are of special interest since they involve new nonperturbative characteristics which generalize conventional parton densities and/or distribution amplitudes. The leading twist factorization gives the amplitude of these processes in terms of a perturbatively calculable coefficient function and a skewed parton distribution (for DVCS) \[ \gamma \gamma \rightarrow hh \] which are responsible for soft physics. However, the twist-two analysis of the Compton scattering at non-zero \( t \)-channel momentum transfer, \( \Delta \neq 0 \), is inadequate since it violates explicitly the gauge invariance of the amplitude due to approximation involved, \( \Delta_\perp/\sqrt{-q^2} \ll 1 \). This calls for a consistent treatment of the effects suppressed in \( \Delta_\perp \), i.e. twist expansion\(^1\). As a first step one takes the contributions linear in \( \Delta_\perp \) which are of twist-three. Obviously, then the amplitude will be gauge invariant to the twist-four accuracy. Repeating the steps one improves the amplitude accordingly.

Let us demonstrate the peculiarities of the off-forward kinematics on a simple example of a free Dirac fermion theory. As we will see, even and odd parity structures ‘talk’ to each other in the case at hand, when the operators with total derivatives within the context of the operators product expansion are relevant. This will be the source of restoration of the electromagnetic gauge invariance of the two-photon amplitude defined by a chronological product \( T \{ j_\mu(x) j_\nu(y) \} \) of currents \( j_\mu(x) = \bar{\psi} \gamma_\mu \psi \). The leading light-cone singularity \( (x-y)^2 \rightarrow 0 \) arises from the hand-bag diagram and reads

\[
T \{ j_\mu(x) j_\nu(y) \} = i \bar{\psi}(x) \gamma_\mu S(x-y) \gamma_\nu \psi(y) + i \bar{\psi}(y) \gamma_\nu S(y-x) \gamma_\mu \psi(x), \tag{1}
\]

where \( S(x) = \frac{\not{x}}{2m} \) is the free quark propagator. Taking into account the equation of motion \( \not{\partial} \psi = 0 \) and \( \not{\partial} S(x) = -i \delta(x) \) it is a simple task to show that the hand-bag diagram respect current conservation. After performing the decomposition of the Dirac structure in Eq. (1) it reduces to

\[
T \{ j_\mu(x) j_\nu(y) \} = S_{\mu\nu,\rho\sigma} i S_\rho(x-y) \left\{ \bar{\psi}(x) \gamma_\sigma \gamma_5 \psi(y) - (x \leftrightarrow y) \right\} \tag{2}
- i \epsilon_{\mu\nu,\rho\sigma} i S_\rho(x-y) \left\{ \bar{\psi}(x) \gamma_\sigma \psi(y) + (x \leftrightarrow y) \right\},
\]

where \( S_{\mu\nu,\rho\sigma} = g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma} \). One finds that current conservation does not separately occur in both terms. Employing the density matrix \( |P_1\rangle \langle P_2| \) for free fields, we find, of course, that the amplitude \( \langle P_2| T \{ j_\mu(x) j_\nu(y) \} |P_1\rangle \) respects current conservation, however, there appears

\(^1\)Here we imply kinematical definition of twist.
\(^2\)We use the conventions for Dirac and Lorentz tensors from Itzykson and Zuber \[8\].
a cancellation between both matrix elements of different parity on the r.h.s. in Eq. (2). So we realize that the current conservation is not manifest in the decomposition (2). This arises due to operators with total derivatives in twist decomposition of \( \bar{\psi}(x)\gamma_\mu(1,\gamma_5)\psi(y) \), see later Eq. (28). However, once this decomposition is performed the gauge invariance is restored automatically. As compared to this simple example the only complication in QCD to leading order emerges due to the presence of the interaction dependent three-particle contributions. This does not present any difficulty and will be solved in the next section.

Our consequent presentation is organized as follows. The next section is devoted to a detailed description of the twist-three formalism for the generalized Compton amplitude and as a result the restoration of the gauge invariance sketched above. In section 3 we address the issue of a twist decomposition of light-ray (and local) operators in a situation when total derivatives are relevant. Then we turn in section 4 to the two cases of matrix elements of operators sandwiched between spin-0 and \( \frac{1}{2} \) hadrons and derive relations for twist-three two-particle ‘transverse’ functions in terms of twist-two (Wandzura-Wilczek contribution) and three-particle correlation functions. In section 5 we comment on the phenomenological consequences for different asymmetries measurable in the DVCS process, and then in section 6 point out that the evolution of twist-three skewed distributions is known in leading logarithmic approximation from analogous studies for forward kinematics. Finally, we give our conclusions.

## 2 Generalized two-photon amplitude.

The amplitude of scattering of two virtual photons with momenta \( q_1 \) and \( q_2 \) on a hadron target, with incoming (outgoing) momentum \( P_1 \) (\( P_2 \)), is given by a Fourier transform of a correlator of electromagnetic currents,

\[
T_{\mu\nu} = i \int d^4x \, e^{iq\cdot x} \langle P_2 | T \{ j_\mu(x/2) j_\nu(-x/2) \} | P_1 \rangle. \tag{3}
\]

One introduces the vectors \( q = \frac{1}{2}(q_1 + q_2) \) and \( P = P_1 + P_2, \Delta = P_2 - P_1 = q_1 - q_2 \) to describe the amplitude. They can be used to construct a pair of the light-cone vectors \( n_\mu \) and \( n^*_\mu \), such that \( n^2 = n^*2 = 0 \) and \( n \cdot n^* = 1 \), as follows

\[
n_\mu = -\frac{2\xi}{q^2\sqrt{1 - 4(\xi\delta)^2}} q_\mu - \frac{1 - \sqrt{1 - 4(\xi\delta)^2}}{2q^2\delta^2\sqrt{1 - 4(\xi\delta)^2}} P_\mu, \quad n^*_\mu = \frac{\xi\delta^2}{\sqrt{1 - 4(\xi\delta)^2}} q_\mu + \frac{1 + \sqrt{1 - 4(\xi\delta)^2}}{4\sqrt{1 - 4(\xi\delta)^2}} P_\mu, \tag{4}
\]

where \( \delta^2 \equiv (M^2 - \Delta^2/4)/q^2 \). The transverse metric and antisymmetric tensor are defined as,

\[
g_{\mu\nu}^\perp = g_{\mu\nu} - n_\mu n^*_\nu - n_\nu n^*_\mu, \quad \epsilon_{\mu\nu}^\perp \equiv \epsilon_{\mu\nu-} \epsilon_{\mu\nu+}. \]

Here and in the following we use the conventions for the generalized Bjorken variable \( \xi = -q^2/P \cdot q \) and skewedness \( \eta = \Delta \cdot q/P \cdot q \) which are scaling...
Figure 1: Two- and three-leg coefficient functions (plus crossed diagrams) for DVCS process which form a gauge invariant amplitude to twist-three accuracy.

variables of (3). Neglecting the corrections $O(\delta^2)$ we have then to twist-four accuracy

\[ n_\mu = -\frac{\xi}{q^2}(2q_\mu + \xi P_\mu), \quad n^*_\mu = \frac{1}{2} P_\mu, \quad \Delta_\mu = 2\eta n^*_\mu + \Delta^\perp_\mu. \]  

This approximation will be used throughout in our analysis.

In the generalized Bjorken kinematics $-q^2 \to \infty$, $P \cdot q \to \infty$, $\Delta \cdot q \to \infty$, $\Delta^2 = \text{finite}$, $\xi$ and $\eta$ being fixed the contribution to (3) comes from the diagrams in Fig. 1

\[ T_{\mu\nu} = T^2_{\mu\nu} + T^3_{\mu\nu}, \]  

where $T^2_{\mu\nu}$ comes from hand-bag diagram (left) while $T^3_{\mu\nu}$ from antiquark-gluon-quark one (right). Explicit calculation, done in the light cone gauge $B^+ = 0$, gives

\[ T^2_{\mu\nu} = -\frac{1}{2q^2} \int dx \left\{ C^+(x, \xi) \left[ S_{\mu\nu;\rho-} V K_\rho(x, \eta) + 2i\epsilon_{\mu\nu\rho\sigma} q_\sigma A O_\rho(x, \eta) \right] \right. \]
\[ - C^-(x, \xi) \left. \left[ q^2 \xi S_{\mu\nu;\rho+} V O_\rho(x, \eta) - i\epsilon_{\mu\nu\rho-} \left( 2x A O_\rho(x, \eta) - A K_\rho(x, \eta) \right) \right] \right\}, \]

for the first one, and

\[ T^3_{\mu\nu} = -\frac{1}{2q^2} \int dx \int \frac{d\tau}{\tau - i0} \left\{ C^+(x, \xi) S_{\mu\nu;\rho-} \left[ V S^-_\rho(x, x - \tau, \eta) + V S^+_\rho(x + \tau, x, \eta) \right] \right. \]
\[ + C^-(x, \xi) S_{\mu\nu;\rho+} \left. \left[ V S^-_\rho(x, x - \tau, \eta) - V S^+_\rho(x + \tau, x, \eta) \right] \right\}, \]

for the second. Here $S_{\mu\nu;\rho\sigma}$ tensor defined in the introduction and tree coefficient functions are

\[ C^\pm(x, \xi) = (1 - x/\xi - i0)^{-1} \pm (1 + x/\xi - i0)^{-1}. \]

We use here the following conventions for generalized functions

\[ ^1O_\rho(x, \eta) = \int \frac{dk}{2\pi} e^{i\kappa x} \langle P_2 | ^1O_\rho \left( \frac{\kappa}{2}, -\frac{\kappa}{2} \right) | P_1 \rangle, \quad ^1K_\rho(x, \eta) = \int \frac{dk}{2\pi} e^{i\kappa x} \langle P_2 | ^1K_\rho \left( \frac{\kappa}{2}, -\frac{\kappa}{2} \right) | P_1 \rangle, \]
\[ ^1S^\pm_\rho(x_1, x_2, \eta) = \int \frac{d\kappa_1 d\kappa_2}{2\pi 2\pi} e^{i\kappa_1 (x_1 + x_2) + i\kappa_2 (x_1 - x_2)} \langle P_2 | ^1S^\pm_\rho \left( \frac{\kappa_1}{2}, \kappa_2, -\frac{\kappa_1}{2} \right) | P_1 \rangle, \]
where the two-quark operators are defined by

\[ IO_\rho(\kappa, -\kappa) = \bar{\psi}(-\kappa) i \Gamma_\rho \psi(\kappa), \quad IK_\rho(\kappa, -\kappa) = \bar{\psi}(-\kappa) i \gamma^\rho \partial^\rho \psi(\kappa), \] (11)

with \( \{ \gamma_\rho, \gamma_\rho \gamma_5 \} \) for \( I = V, A \) and antiquark-gluon-quark ones read

\[ IS_\rho^\pm(\kappa_1, \kappa_2, \kappa_3) = i g \bar{\psi}(\kappa_3 n) \left[ \gamma_+ G_{+\rho}(\kappa_2 n) \pm i \gamma_+ \gamma_5 G_{+\rho}(\kappa_2 n) \right] \psi(\kappa_1 n), \]
\[ AS_\rho^\pm(\kappa_1, \kappa_2, \kappa_3) = i g \bar{\psi}(\kappa_3 n) \left[ \gamma_+ \gamma_5 G_{+\rho}(\kappa_2 n) \pm i \gamma_+ \bar{G}_{+\rho}(\kappa_2 n) \right] \psi(\kappa_1 n). \] (12)

The latter two are related to each other by the ‘duality’ equation

\[ i \epsilon_\rho \Delta^\pm \rho \pm = \pm \beta^\pm. \] (13)

By means of the quark equation of motion, \( \mathcal{D} \psi = 0 \), we can obtain the following relation between the correlation functions introduced so far

\[ \frac{\partial}{\partial \kappa} IO_\rho^\pm(\kappa, -\kappa) - i \epsilon_\rho \partial \pm A O_\sigma^\pm(\kappa, -\kappa) + \frac{1}{2} \partial_\kappa V_\kappa(\kappa, -\kappa) + i \epsilon_\rho \partial \pm A O_\sigma^\pm(\kappa, -\kappa) \]
\[ + \int d\lambda \left\{ w(\lambda - \kappa) V S^\pm_\rho(\kappa, \lambda, -\kappa) + w(\lambda + \kappa) V S^\pm_\rho(\kappa, \lambda, -\kappa) \right\} = 0, \]
\[ \partial \pm O_\rho^\pm(\kappa, -\kappa) - i \epsilon_\rho \partial \pm A O_\sigma^\pm(\kappa, -\kappa) + \epsilon_\rho \partial \pm A K_\sigma(\kappa, -\kappa) - \partial \pm O_\rho^\pm(\kappa, -\kappa) \]
\[ + \int d\lambda \left\{ w(\lambda - \kappa) V S^\pm_\rho(\kappa, \lambda, -\kappa) - w(\lambda + \kappa) V S^\pm_\rho(\kappa, \lambda, -\kappa) \right\} = 0, \] (14)

where \( w(\kappa) = -\theta(\kappa) \) for the ML prescription on the infrared pole in the gluon propagator. Here \( \partial \) stands for the total derivative. In terms of the momentum fraction space functions introduced in Eqs. (10) these relations read

\[ 2x V O_\rho^\pm(x, \eta) + 2 \eta i \epsilon_\rho A O_\sigma^\pm(x, \eta) - i \epsilon_\rho A \Delta^\pm \rho \pm A O_\sigma^\pm(x, \eta) - \frac{1}{2} \partial \pm A O_\rho^\pm(x, \eta) \]
\[ - \int \frac{d\tau}{\tau - i \partial} \left\{ V S^\pm_\rho(x, -x + \tau, \eta) + V S^\pm_\rho(x, x + \tau, \eta) \right\} = 0, \]
\[ 2x i \epsilon_\rho A O_\sigma^\pm(x, \eta) + 2 \eta V O_\rho^\pm(x, \eta) - \Delta^\pm \rho \pm V O_\rho^\pm(x, \eta) - i \epsilon_\rho A K_\sigma^\pm(x, \eta) \]
\[ + \int \frac{d\tau}{\tau - i \partial} \left\{ V S^\pm_\rho(x, -x + \tau, \eta) - V S^\pm_\rho(x, x + \tau, \eta) \right\} = 0. \] (15)

So that expressing \( K \) in terms of other correlation functions in Eq. (10) we get the result

\[ T_{\mu \nu} = -\frac{1}{q^2} \int dx \left\{ T_{\mu \nu}^{(1)} C^{(-)}(x, \xi) V O_\rho^\pm(x, \eta) + T_{\mu \nu}^{(2)} C^{(+)}(x, \xi) A O_\rho^\pm(x, \eta) \right\} \]
\[ + T_{\mu \nu}^{(3)} C^{(-)}(x, \xi) V O_\rho^\pm(x, \eta) + T_{\mu \nu}^{(4)} C^{(+)}(x, \xi) A O_\rho^\pm(x, \eta) \}, \] (16)

where the support properties of distributions restrict the integration range of the variable \( x \) within the interval \([-1, 1]\). Here the Lorentz tensors are

\[ T_{\mu \nu}^{(1)} = n_\mu (q_\nu + \xi n_\nu - \frac{1}{2} \Delta_\nu^\pm) + n_\nu (q_\mu + \xi n_\mu + \frac{1}{2} \Delta_\mu^\pm) - q \cdot n^* g_{\mu \nu}, \]

and

\[ T_{\mu \nu}^{(2)} = n_\mu (q_\nu + \xi n_\nu - \frac{1}{2} \Delta_\nu^\pm) + n_\nu (q_\mu + \xi n_\mu + \frac{1}{2} \Delta_\mu^\pm) - q \cdot n^* g_{\mu \nu}. \]
\[ \mathcal{T}^{(2)}_{\mu\nu} = i\varepsilon_{\mu\nu\sigma\rho} q_\sigma - \frac{i}{2}\varepsilon_{\rho\sigma} \Delta_\sigma \left( n^*_\rho g_{\mu\nu} + n^*_\nu g_{\mu\rho} \right), \]
\[ \mathcal{T}^{(3)}_{\mu\nu\rho} = (q_\nu + (2\xi - \eta)n^*_\nu) g_{\mu\rho} + (q_\mu + (2\xi + \eta)n^*_\mu) g_{\nu\rho}, \]
\[ \mathcal{T}^{(4)}_{\mu\nu\rho} = i\varepsilon_{\mu\nu\rho\sigma} q_\sigma - i\eta\varepsilon_{\rho\sigma} \left( n^*_\rho g_{\sigma\nu} + n^*_\nu g_{\sigma\rho} \right). \]

These expressions are obviously target independent. In the case of scalar target our result reduces to the one obtained in Ref. [9].

It is an easy exercise to check that, after accounting for twist-three corrections, gauge invariance is fulfilled up to \( \mathcal{O}(\Delta^2) \), i.e. twist-four effects: 
\( q_1\mathcal{T}^{(i)}_{\mu\nu} = q_2\mathcal{T}^{(i)}_{\mu\nu} = \mathcal{O}(\Delta^2) \), where we have used Sudakov decomposition for the photon momenta 
\( q_1 = \frac{q_2^2}{2\xi} n_\mu - (\xi - \eta)n^*_\mu + \frac{1}{2}\Delta^\perp_\mu \) and 
\( q_2 = -\frac{q_2^2}{2\xi} n_\mu - (\xi + \eta)n^*_\mu - \frac{1}{2}\Delta^\perp_\mu \). Interesting to note that the last two structures \( \mathcal{T}^{(i)}_{\mu\nu\rho} \) are gauge invariant provided we simultaneously contract them both with incoming and outgoing photons 
\( q_1 q_2 \mathcal{T}^{(i)}_{\mu\nu\rho} = 0 \). Namely, e.g. for the last tensor we get 
\( q_1\mathcal{T}^{(4)}_{\mu\nu\rho} = -\frac{1}{2}\xi P_{\mu\nu} P_\rho \tilde{\Delta}^\perp_\mu \) and 
\( q_2\mathcal{T}^{(4)}_{\mu\nu\rho} = -\frac{1}{2}\xi P_{\mu\nu} P_\rho \tilde{\Delta}^\perp_\mu \),
where we use throughout the convention 
\( \tilde{\Delta}^\perp_\mu \equiv i\varepsilon^\perp_\mu \Delta_\rho \) and introduced a projector \( P_{\mu\nu} = g_{\mu\nu} - q_1 q_2 / q_1 \cdot q_2 \) which gives 
\( \xi P_{\mu} P_{\nu} = q_\nu + \frac{1}{2}(2\xi - \eta)P_\nu \) and 
\( \xi P_{\mu} P_\nu = q_\mu + \frac{1}{2}(2\xi + \eta)P_\mu \)
up to terms of order \( \mathcal{O}(\Delta) \) which we drop since they would exceed the accuracy we work to. A similar result we get for \( \mathcal{T}^{(3)}_{\mu\nu\rho} \) with \( \tilde{\Delta} \) being replaced by \( \Delta \). Using the definitions of the photon and hadron momenta in terms of light-cone vectors, consequent simple algebra leads to the following structures in photon and hadron momenta

\[ \mathcal{T}^{(1)}_{\mu\nu} = \frac{q_1 q_2}{2\xi} \left( g_{\mu\nu} - \frac{q_1 q_2}{q_1 \cdot q_2} \right) + \frac{\xi}{2} \left( P_{\mu} - \frac{P \cdot q_2}{q_1 \cdot q_2} q_1 \right) \left( P_\nu - \frac{P \cdot q_1}{q_1 \cdot q_2} q_2 \right), \]
\[ \mathcal{T}^{(2)}_{\mu\nu} = \frac{i}{2}\varepsilon_{\mu\nu\rho\sigma} P_\rho q_\sigma \left( g_{\mu\theta} - \frac{P_{\mu} q_\theta}{P \cdot q_2} \right) \left( g_{\nu\lambda} - \frac{P_{\nu} q_\lambda}{P \cdot q_1} \right), \]
\[ \mathcal{T}^{(3)}_{\mu\nu\rho} = \xi \left( g_{\mu\rho} - \frac{q_1 q_2}{q_1 \cdot q_2} \right) \left( P_{\nu} - \frac{P \cdot q_1}{q_1 \cdot q_2} q_2 \right) + \xi \left( g_{\nu\rho} - \frac{q_1 q_2}{q_1 \cdot q_2} \right) \left( P_{\mu} - \frac{P \cdot q_2}{q_1 \cdot q_2} q_1 \right), \]
\[ \mathcal{T}^{(4)}_{\mu\nu\rho} = i\varepsilon_{\mu\nu\rho\sigma} q_\sigma \left( g_{\mu\theta} - \frac{P_{\mu} q_\theta}{P \cdot q_2} \right) \left( g_{\nu\lambda} - \frac{P_{\nu} q_\lambda}{P \cdot q_1} \right), \]

where a missing twist-four part is restored minimally according to previous discussion. All structures have a well defined forward limit. It will be shown in section [3] that these terms are equivalent to those used in our previous studies [10, 11]. In subsequent sections we derive relations between the ‘transverse’ generalized distributions introduced here and conventional twist-two ones as well as three-particle correlation functions.

### 3 Twist decomposition.

In this section we present a decomposition of the two-quark operators into separate twist components to twist-three accuracy. Similar analyses have been done in Refs. [12, 13, 14]. However, an
essential new ingredient of our study is the treatment of operators with total derivatives. Since the group-theoretical notion of twist as the dimension minus spin of an operator is well defined for local operators, our strategy will be thus: first, an expansion of a light cone operator in infinite Taylor series in local ones, second, an extraction of definite twist components and as a final step the resummation of the result back into nonlocal form. An alternative approach directly based in terms of light-ray operators is described in Ref. [13]. In the following we will not take care of trace terms, proportional to \( n_\rho \), since they only contribute at twist-four level.

The Taylor expansion of Eq. (11) in terms of local operators simply reads

\[
\mathcal{O}_\rho(\kappa, -\kappa) = \sum_{j=0}^{\infty} \frac{(-i\kappa)^j}{j!} n_{\mu_1} \ldots n_{\mu_j} \mathcal{O}_{\rho;\mu_1 \ldots \mu_j}, \quad \text{with} \quad \mathcal{O}_{\rho;\mu_1 \ldots \mu_j} = \bar{\psi} \Gamma_\rho i \overset{\leftrightarrow}{\mathcal{D}}_{\mu_1} \ldots i \overset{\leftrightarrow}{\mathcal{D}}_{\mu_j} \psi,
\]

where \( \overset{\leftrightarrow}{\mathcal{D}}_\mu = \mathcal{D}_\mu - \mathcal{D}_\mu \) and \( \mathcal{D}_\mu = \partial_\mu - igB_\mu \). To have a well-defined decomposition into twist-two and three contributions we extract tensors \( \mathcal{R}_{\rho;\mu_1 \ldots \mu_j} \) with \( (j+1) \) indices corresponding to the two Young tables \( \begin{array}{|c|c|c|c|c|} \hline \mu & \mu & \cdots & \mu \hline \end{array} \) and \( \begin{array}{|c|c|c|c|} \hline \mu & \mu & \cdots & \mu \hline \end{array} \). This can easily be done with the result

\[
\mathcal{O}_\rho(\kappa, -\kappa) = \mathcal{R}^2_{\rho}(\kappa, -\kappa) + \mathcal{R}^3_{\rho}(\kappa, -\kappa)
\]

\[
= \sum_{j=0}^{\infty} \frac{(-i\kappa)^j}{j!} n_{\mu_1} \ldots n_{\mu_j} \left\{ \mathcal{R}^2_{\rho;\mu_1 \ldots \mu_j} + \frac{2j}{j+1} \mathcal{R}^3_{\rho;\mu_1 \ldots \mu_j} \right\},
\]

with

\[
\mathcal{R}^2_{\rho;\mu_1 \ldots \mu_j} = \mathcal{S}_{\mu_1 \ldots \mu_j} \bar{\psi} \Gamma_\rho i \overset{\leftrightarrow}{\mathcal{D}}_{\mu_1} \ldots i \overset{\leftrightarrow}{\mathcal{D}}_{\mu_j} \psi, \quad \mathcal{R}^3_{\rho;\mu_1 \ldots \mu_j} = \mathcal{S} \mathcal{A} \mathcal{S}_{\mu_1 \ldots \mu_j} \mu_1 \ldots \mu_j \bar{\psi} \Gamma_\rho i \overset{\leftrightarrow}{\mathcal{D}}_{\mu_1} \ldots i \overset{\leftrightarrow}{\mathcal{D}}_{\mu_j} \psi,
\]

where \( \mathcal{S}_{\mu_1 \ldots \mu_j} \) is a symmetrization (and trace subtraction) operation of \( j \) indices with weight \( \frac{1}{j!} \) and antisymmetrization being defined by \( \mathcal{A} \mu_1 \mu_2 = \frac{1}{2} (t_{\mu_1 \mu_2} - t_{\mu_2 \mu_1}) \). In terms of light-ray operators the twist-two part reads

\[
\mathcal{R}^2_{\rho}(\kappa, -\kappa) = \int_0^1 du \left\{ \bar{\psi}(-u\kappa) \Gamma_\rho \psi(u\kappa) \right\} + \frac{\kappa}{2} \int_0^u dt \bar{\psi}(-u\kappa) \Gamma_+ \left[ \overset{\to}{\partial}_\rho (-u\kappa) - \overset{\to}{\partial}_\rho (-u\kappa) - 2i\kappa gB_\rho(\tau\kappa) \right] \psi(u\kappa)
\]

in the light-cone gauge \( B_+ = 0 \). If we would work in a covariant gauge this would result, apart from restoration of the gauge-link factors, to the substitution in the square brackets

\[
\overset{\to}{\partial}_\rho (-u\kappa) - \overset{\to}{\partial}_\rho (-u\kappa) - 2i\kappa gB_\rho(\tau\kappa) \rightarrow \overset{\to}{\mathcal{D}}_\rho (-u\kappa) - \overset{\to}{\mathcal{D}}_\rho (-u\kappa) + 2i\kappa gG_+(\tau\kappa).
\]

One can easily project onto the twist-two part from the operator \( \mathcal{O}_\rho \) by contraction with the vector \( n_\rho \), namely, \( \mathcal{R}^2_+ = \mathcal{O}_+ \).
The conversion of the twist-three part in terms of three-particle operators is more tricky as compared to the forward case due to relevance of operators involving total derivatives. From the Eqs. (20) and (22) one can easily derive, however, the following equation, cf. [12],

$$V\mathcal{O}_\rho(\kappa, -\kappa) = V\mathcal{R}^2_\rho(\kappa, -\kappa) + \kappa \int_0^1 du \left\{ i u \epsilon_{+\rho\mu\nu} \partial_\mu A \mathcal{O}_\rho(u\kappa, -u\kappa) + \frac{\kappa}{2} \int_{-u}^u d\tau \left[ (u - \tau) V\mathcal{S}^+_\rho(u\kappa, \tau\kappa, -u\kappa) - (u + \tau) V\mathcal{S}^-_\rho(u\kappa, \tau\kappa, -u\kappa) \right] \right\},$$

(24)

with $\partial_\mu = \partial_\mu^+ + \partial_\mu^-$ being the total derivative, and similar expression for $A\mathcal{O}_\rho$ which is obtained upon substitution $\Gamma_\rho \to \Gamma_\rho \gamma_5$. These two equations set an infinite iteration in total derivatives. We can represent these relations in a schematic form

$$V\mathcal{O} = V\mathcal{R}^2 + \mathcal{J} \ast (\epsilon \partial A\mathcal{O} + V\mathcal{S}), \quad A\mathcal{O} = A\mathcal{R}^2 + \mathcal{J} \ast (\epsilon \partial V\mathcal{O} + A\mathcal{S}),$$

with $\mathcal{J}$ being an integral operator. A formal solution to this system of equations reads

$$V\mathcal{O} = \left(1 - \mathcal{J}^2 \partial^2\right)^{-1} \ast \left\{ V\mathcal{R}^2 + \epsilon \partial A\mathcal{O} + V\mathcal{S} \right\},$$

and similar for $A\mathcal{O}$. So that $V\mathcal{O} - A\mathcal{R}^2$ is a twist-three part of $V\mathcal{O}$. The series can be summed up into total translations in the light cone formalism. Let us demonstrate first the effect of total derivatives in the basis of local operators. The moments of Eq. (24) look like

$$V\mathcal{O}_{\rho;j} = V\mathcal{R}^2_{\rho;j} + \frac{j}{j+1} i \epsilon_{+\rho\mu\nu} i \partial_\mu A \mathcal{O}_{\nu;j-1} - \frac{2}{j+1} \sum_{k=1}^{j-1} \left\{ (j-k) V\mathcal{S}^+_{\rho;j,k} - k V\mathcal{S}^-_{\rho;j,k} \right\},$$

(25)

where obviously $\mathcal{O}_{\rho;j} = n_{\mu_1} \ldots n_{\mu_j} \mathcal{O}_{\mu_1 \ldots \mu_j}$ and we have introduced three-particle operators

$$V\mathcal{S}^\pm_{\rho;j,k} = i g v^{-2} \psi^+ \partial_+^{k-1} \left[ \gamma_+ G_{-\rho} \pm i \gamma_+ \gamma_5 \tilde{G}_{-\rho} \right] \partial_+^{j-k-1} \psi,$$

(26)

and the analogous one for odd parity, i.e. $V \to A$ and $\gamma_+ \to \gamma_+ \gamma_5$. The solution for $\mathcal{R}^3$ reads

$$V\mathcal{R}^3_{\rho;j} = \frac{1}{2j} \sum_{l=0}^{j-1} (j-l) (i \partial_+)^l \left\{ \sigma_{l+1} i \epsilon_{+\rho\mu\nu} i \partial_\mu A \mathcal{R}^2_{\nu;j-l-1} - \sigma_l (i \partial_\rho n_{\sigma} - g_{\rho\sigma} i \partial_\sigma) V\mathcal{R}^2_{\sigma;j-l-1} \right\}

- \frac{1}{j} \sum_{l=0}^{j-1} \sum_{k=1}^{j-l-1} (i \partial_+)^l \left\{ (j-k-l) V\mathcal{S}^+_{\rho;j-l,k} - (1)^l k V\mathcal{S}^-_{\rho;j-l,k} \right\},$$

(27)

with $\sigma_l = \frac{1}{2} [1 - (-1)^l]$. Now we can resum this equation back into light-ray operators, or this can directly be obtained from Eq. (25), with the result

$$V\mathcal{R}^3_\rho(\kappa, -\kappa) = \frac{\kappa}{2} \int_0^1 du \left\{ u i \epsilon_{+\rho\mu\nu} \partial_\mu A \mathcal{R}^2_\nu((\bar{u} - u) \kappa, (\bar{u} - u) \kappa) + A \mathcal{R}^2_\rho((u - \bar{u}) \kappa, -\kappa) \right\}

- u \left( \partial_\rho n_\sigma - g_{\rho\sigma} \partial_\sigma \right) \left[ V\mathcal{R}^2_\rho((\bar{u} - u) \kappa, (\bar{u} - u) \kappa) - V\mathcal{R}^2_\rho((u - \bar{u}) \kappa, -\kappa) \right]

+ \kappa \int_{-u}^u d\tau \left[ (u - \tau) V\mathcal{S}^+_\rho((\tau + \bar{u}) \kappa, (\tau + \bar{u}) \kappa) - (u + \tau) V\mathcal{S}^-_\rho((u - \bar{u}) \kappa, (\tau - \bar{u}) \kappa, -\kappa) \right].$$

(28)
and same for $V \leftrightarrow A$. Contrary to the forward scattering there appeared contributions from different parity operators to a given twist-three one and the ‘center-of-mass’ of two- and three-particle operators gets shifted by a total translation $\exp(\pm i\vec{u}\kappa\partial)$. This equation gives a relation between skewed parton distributions of different ‘twists’ when sandwiched between hadronic states, which we discuss in the next section.

4 Twist-three skewed parton distributions.

In this section we define twist-three skewed parton distributions and their relation to twist-two ones. Before we deal with spin-$\frac{1}{2}$ functions, we consider first a more simple case of spinless target. For generality, we deal with the incoming and outgoing hadrons of different masses $P_1^2 = M_1^2 \neq P_2^2 = M_2^2$.

4.1 Spin-0 target.

It is instructive to start the analysis with the expectation values of local operators. Since for spin-0 hadron we can not form a ‘twist-two’ axial-vector, only the vector twist-two operator develops non-zero reduced matrix elements, which are given by

$$\langle P_2|\mathcal{R}_{\rho\mu_1\ldots\mu_j}^2|P_1\rangle = S_{\rho\mu_1\ldots\mu_j} \left\{ P_\rho \ldots P_{\mu_j} B_{j+1,j+1} + \Delta_\rho P_{\mu_1} \ldots P_{\mu_j} B_{j+1,j} + \cdots + \Delta_\rho \ldots \Delta_{\mu_j} B_{j+1,0} \right\},$$

$$\langle P_2|A_{\rho\mu_1\ldots\mu_j}^2|P_1\rangle = 0.$$  \hfill (29)

The reduced matrix elements $B_{jk}$ are defined as moments of a skewed parton distribution $B(x, \eta)$:

$$B_{j,-k} = \frac{1}{k!} \frac{d^k}{d\eta^k} \int_{-1}^{1} dx \ x^{j-1} B(x, \eta)|_{\eta=0}, \quad \text{where} \quad 0 \leq k \leq j, \quad 1 \leq j.$$  \hfill (30)

For the analysis of the moments of skewed parton distributions we have to project both sides of (29) with the light-cone vectors $n_{\mu_1} \ldots n_{\mu_j}$. Since

$$n_{\mu_1} \ldots n_{\mu_j} S_{\rho\mu_1\ldots\mu_j} \Delta_\rho \ldots \Delta_{\mu_j} P_{\mu_{k+1}} \ldots P_{\mu_j} = \frac{1}{j+1} \left\{ \Delta_\rho \Delta_\rho^{k+1} \frac{d^{j+1}}{d\eta^{j+1}} + (j+1-k)P_\rho \Delta_\rho^{k+1} \right\} P_{\rho}$$

the coefficients in front of the two monomials are generated by derivatives of $B_{j+1}(\eta)$ w.r.t. the skewedness parameter, namely,

$$\langle P_2|\mathcal{R}_{\rho\mu_j}^2|P_1\rangle = P_\rho P_{\rho}^{j+1} \left( 1 - \frac{\eta}{j+1} \frac{d}{d\eta} \right) B_{j+1}(\eta) + \Delta_\rho P_{\rho}^{j+1} \frac{d}{d\eta} B_{j+1}(\eta),$$

$$\langle P_2|\mathcal{R}_{\rho\mu_j}^2|P_1\rangle n_\rho = P_{\rho}^{j+1} B_{j+1}(\eta),$$  \hfill (32)
with

\[ B_{j+1}(\eta) = \sum_{k=0}^{j+1} \eta^k B_{j+1,j+1-k} = \int_{-1}^{1} dx \; x^j B(x, \eta), \]

The moments \( B_{j+1}(\eta) \) can be taken from the + component of the operator \( \mathcal{R}_+ \). As a consequence of symmetrization the first equation contains a Wandzura-Wilczek term proportional to \( \Delta^\perp_\rho = \Delta_\rho - \eta \rho_\rho \), which effectively enters as a twist-three contribution to the scattering amplitude. The matrix element of the light-ray operator can be obtained in a straightforward manner by resummation:

\[
\langle P_2 | \mathcal{R}_\rho^2(\kappa, -\kappa) | P_1 \rangle = \int_{-1}^{1} dx e^{-i k x} \left( P_\rho B(x, \eta) + \Delta^\perp_\rho \int_{-1}^{1} dy \; W_2(x, y) \frac{d}{d\eta} B(x, \eta) \right), \tag{33}\]

where the kernel reads \( W_2(x, y) = \theta(x)\theta(y-x)/y + \theta(-x)\theta(x-y)/(-x) \).

Next we define the reduced matrix elements of the antiquark-gluon-quark operators. Since these operators are partially antisymmetrized, we have obviously two vectors \( \Delta^\perp_\rho \) and the dual ones \( \tilde{\Delta}^\perp_\rho = i \epsilon^\perp_\rho \Delta^\perp_\rho \), introduced above, in our disposal. Thus, the general decomposition of reduced matrix elements reads

\[
\langle P_2 | \mathcal{S}_{\rho,j,k}^\pm | P_1 \rangle = \Delta^\perp_\rho S_{j,k}^\pm P_+^j, \quad \langle P_2 | \mathcal{A}_{\rho,j,k}^\pm | P_1 \rangle = \tilde{\Delta}^\perp_\rho \tilde{R}_{j,k}^\pm P_+^j. \tag{34}\]

The duality relation (13) reduces immediately the number of independent contributions to two instead of four, namely, \( \tilde{R}_{j,k}^\pm = S_{j,k}^\pm \). It turns out convenient to work in a mixed representation for the skewed parton distributions. We introduce a representation that depends on the position of the gluon field and a Fourier conjugate variable with respect to a distance between both quark fields:

\[
\langle P_2 | \{ \mathcal{V}_\rho^\pm(\kappa, \eta, -\kappa), \mathcal{A}_\rho^\pm(\kappa, \eta, -\kappa) \} | P_1 \rangle = P_+^2 \left\{ \begin{array}{c} \Delta^\perp_\rho \\ \tilde{\Delta}^\perp_\rho \end{array} \right\} \int dx e^{-i \kappa x} S^\pm(x, u, \eta). \tag{35}\]

The moments with respect to the momentum fraction \( x \) are given by a polynomial of order \( j - 2 \) in the variable \( u \):

\[
S^\pm_j(u, \eta) \equiv \int_{-1}^{1} dx \; x^{j-2} S^\pm_j(x, u, \eta) = \sum_{k=1}^{j-1} \left( \begin{array}{c} j - 2 \\ k - 1 \end{array} \right) \left( \frac{1+u}{2} \right)^{k-1} \left( \frac{1-u}{2} \right)^{j-k-1} S^\pm_{j,k}(\eta), \tag{36}\]

where \( S_{j,k}(\eta) \) are polynomials in \( \eta \) of order \( j \) defined in Eq. (34). As a simple consequence of our definition we have

\[
\int_{-1}^{1} du \left[ \frac{1+u}{2} S^\pm_j(u, \eta) \right] = 2 \sum_{k=1}^{j-1} \frac{k}{j(j-1)} S^\pm_{j,k}(\eta), \quad \int_{-1}^{1} du \left[ \frac{1-u}{2} S^\pm_j(u, \eta) \right] = 2 \sum_{k=1}^{j-1} \frac{j-k}{j(j-1)} S^\pm_{j,k}(\eta). \tag{37}\]
Finally, to find the matrix element of the operator $O_{\rho,j} = R^2_{\rho,j} + \frac{2j}{j+1} R^3_{\rho,j}$, it remains to insert our findings (32) and (37) into the solution (27):

$$P^{-j}_+ \langle P_2 | \mathcal{O}_{\rho,j} | P_1 \rangle = \left( P_\rho + \frac{\Delta^\perp_\rho}{\eta} \right) B_{j+1}(\eta) + \Delta^\perp_\rho \left\{ \frac{1}{\eta} \sum_{j=0}^{j} \frac{\sigma_{j+1-k}(-\eta)^{j-k}}{j+1} \int_{-1}^{1} du \left\{ \frac{1-u}{2} S^+_k(u, \eta) - (-1)^{j-k} \frac{1+u}{2} S^-_k(u, \eta) \right\} \right\}$$

(38)

Now we consider the axial-vector case, which is handled in the same way. The main difference is that there is no twist-two part for spinless target. However, a non-vanishing twist-two part of the vector operator induces now a Wandzura-Wilczek type relation due to the epsilon tensor in Eq. (27) and provides

$$P^{-j}_+ \langle P_2 | A \mathcal{O}_{\rho,j} | P_1 \rangle = \Delta^\perp_\rho \sum_{k=0}^{j} \frac{\sigma_{j-k}(-\eta)^{j-k}}{j+1} \int_{-1}^{1} du \left\{ \frac{1-u}{2} R^+_k(u, \eta) - (-1)^{j-k} \frac{1+u}{2} R^-_k(u, \eta) \right\}$$

(39)

The final step is a summation of local operators, see Eq. (24), which leads to the expectation values of the light-ray operators in terms of Fourier transform of skewed parton distributions in parity even and odd cases

$$\langle P_2 | \mathcal{O}_\rho(\kappa, -\kappa) | P_1 \rangle = \int_{-1}^{1} dx e^{-i k x} \left\{ \Delta^\perp_\rho \int_{-1}^{1} \frac{dy}{|\eta|} W_+ \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial y} \frac{\partial}{\partial y} \right) B(y, \eta) \right\}$$

$$+ \left( P_\rho + \frac{1}{\eta} \Delta^\perp_\rho \right) B(x, \eta)$$

$$- \Delta^\perp_\rho \int_{-1}^{1} \frac{dy}{|\eta|} \int_{-1}^{1} du \left\{ \int_{-1}^{1} \frac{dy}{|\eta|} W_n \left( \frac{x}{\eta}, \frac{y}{\eta} \right) S^+(y, u, \eta) + \frac{1+u}{2} W_n \left( \frac{x}{\eta}, \frac{y}{\eta} \right) S^-(y, u, \eta) \right\}$$

$$\langle P_2 | A \mathcal{O}_\rho(\kappa, -\kappa) | P_1 \rangle = \int_{-1}^{1} dx e^{-i k x} \left\{ \Delta^\perp_\rho \int_{-1}^{1} \frac{dy}{|\eta|} W_- \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial y} \frac{\partial}{\partial y} \right) B(y, \eta) \right\}$$

$$- \Delta^\perp_\rho \int_{-1}^{1} \frac{dy}{|\eta|} \int_{-1}^{1} du \left\{ \int_{-1}^{1} \frac{dy}{|\eta|} W_n \left( \frac{x}{\eta}, \frac{y}{\eta} \right) R^+(y, u, \eta) + \frac{1+u}{2} W_n \left( \frac{x}{\eta}, \frac{y}{\eta} \right) R^-(y, u, \eta) \right\}$$

(40)

Here $W_n \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \equiv \frac{d^2}{dy^2} W \left( \frac{x}{\eta}, \frac{y}{\eta} \right)$ and the $W$ kernels read

$$W(x, y) = \frac{\Theta(x, y)}{1+y}, \quad \Theta(x, y) = \text{sign}(1+y) \theta \left( \frac{1+x}{1+y} \right) \theta \left( \frac{y-x}{1+y} \right),$$

$$W_{\pm}(x, y) = \frac{1}{2} \left\{ W(x, y) \pm W(-x, -y) \right\}.$$
In restoration of non-local form one uses the following result for Mellin moments of $W$-kernels
\[\int_{-1}^{1} \frac{dx}{|\eta|} x^j W\left(\frac{x}{\eta}, \frac{y}{\eta}\right) = \sum_{k=0}^{j} \frac{(-1)^{j-k} \eta^{j-k} y^k}{j+1}, \quad \int_{-1}^{1} \frac{dx}{|\eta|} x^j W_{\pm}\left(\frac{x}{\eta}, \frac{y}{\eta}\right) = \sum_{k=0}^{j} \frac{(-1)^{j-k} \pm 1}{2(j+1)} \eta^{j-k} y^k.\] (43)

Note that the piece $\frac{1}{\eta} \Delta^\perp \gamma_{\rho} B(x, \eta)$ in Eq. (40) is cancelled by a term arising from the convolution with the $W_+\,$ kernel.

**4.2 Spin-$\frac{1}{2}$ target.**

Now we are in a position to discuss a spin-$\frac{1}{2}$ target. It is convenient to express the expectation value of local operators in terms of spinor bilinears
\[
\begin{align*}
(b, \tilde{b}) &= \bar{U}(P_2, S_2) (1, \gamma_5) U(P_1, S_1), \\
(h_\rho, \tilde{h}_\rho) &= \bar{U}(P_2, S_2) \gamma_\rho (1, \gamma_5) U(P_1, S_1), \\
(t_{\rho\sigma}, \tilde{t}_{\rho\sigma}) &= \bar{U}(P_2, S_2) i\sigma_{\rho\sigma} (1, \gamma_5) U(P_1, S_1),
\end{align*}
\] (44)

Obviously, the dual tensor form factor $\tilde{t}_{\rho\sigma}$ is obtained from $t_{\rho\sigma}$ by contraction with the $\epsilon$-tensor and can, therefore, be eliminated. Furthermore, equation of motion shows that in each parity sector we have the relation between the structures (44)
\[
\begin{align*}
P_\rho b &= M_+ h_\rho - t_{\rho\sigma} \Delta^\sigma, \quad \Delta_\rho b &= M_+ h_\rho - t_{\rho\sigma} P^\sigma, \\
\Delta_\rho \tilde{b} &= M_+ \tilde{h}_\rho - \tilde{t}_{\rho\sigma} P^\sigma, \quad P_\rho \tilde{b} = M_+ \tilde{h}_\rho - \tilde{t}_{\rho\sigma} \Delta^\sigma,
\end{align*}
\] (45)

where $M_\pm = M_2 \pm M_1$. As demonstrated above for a scalar target, the symmetrization provides us a Wandzura–Wilczek term proportional to $\Delta^\perp$. To take advantage of the analysis already performed in the preceding section and for a symmetrical handling of even and odd parity sectors, our basis is spanned by the (pseudo) scalar and the (axial) vector bilinears. At the end, we express the result in terms of conventional ones, introduced by Ji [2],
\[
\begin{align*}
h_\rho, \quad e_\rho &= t_{\rho\sigma} \frac{\Delta^\sigma}{M_+}, \quad \tilde{h}_\rho, \quad \tilde{e}_\rho &= \frac{\Delta_\rho \tilde{b}}{M_+}.
\end{align*}
\] (46)

Obviously, for the (pseudo) scalar bilinears we can take the results deduced for scalar target. The only new structure for the spin-$\frac{1}{2}$ target is proportional to the $h_\rho, \tilde{h}_\rho$.

In parallel to section 4.1, the local matrix elements read
\[
\begin{align*}
\langle P_2 | V R^2_{\rho, \mu_1 \ldots \mu_j} | P_1 \rangle &= \sum_{\mu_1 \ldots \mu_j} h_\rho \left\{ P_{\mu_1} \ldots P_{\mu_j} A_{j+1, j+1} + \cdots + \Delta_{\mu_1} \ldots \Delta_{\mu_j} A_{j+1, 0} \right\} \\
&\quad + \frac{b}{M_+} \cdots,
\end{align*}
\] (47)

where the ellipses (here and later on) stand for the r.h.s. of Eq. (28) (and corresponding equations from the preceding subsection). Analogous relation holds for the parity odd case. The moments
of the skewed parton distribution are defined analogously to Eq. (30), \( A_{j+1}(\eta) = \int_{-1}^{1} dx^j A(x, \eta) \). The difference arises only from the symmetrization procedure

\[
n_{\mu_1} \cdots n_{\mu_j} \mathbf{S}_{\mu_1 \cdots \mu_j} h_{\rho} \Delta_{\mu_1} \cdots \Delta_{\mu_k} P_{\mu_{k+1}} \cdots P_{\mu_j} = \frac{P_{\mu_j}^{j-1}}{j+1} \left\{ h_{\rho} \eta^k P_+ + (j-k) P_{\rho} h_{\eta^k} + k \Delta_{\rho} h_{\eta^{k-1}} \right\}. \tag{48}
\]

and provides now a new structure

\[
\langle P_2 | \mathcal{R}_{\rho_\beta}^2 | P_1 \rangle = \frac{P_{\mu_j}^{j-1}}{j+1} \left( (j+1) P_{\rho} h_{\eta^k} + h_{\rho} - P_{\rho} h_{\eta^k} + \Delta_{\rho} h_{\eta^{k-1}} \frac{d}{d\eta} \right) A_{j+1}(\eta) + \frac{b}{M_+} \cdots,
\]

\[
\langle P_2 | \mathcal{R}_{\rho_\beta}^2 | P_1 \rangle n_\rho = h_{\rho} P_{\mu_j}^{j-1} A_{j+1}(\eta) + \frac{b}{M_+} P_{\mu_j}^{j+1} B_{j+1}(\eta). \tag{49}
\]

Next we resum the local result and get

\[
\langle P_2 | \mathcal{R}_{\rho_\beta}^2(\kappa, -\kappa) | P_1 \rangle = \int_{-1}^{1} dx e^{-iP_+ x} \left\{ P_{\rho} \left( \frac{h_{\rho}}{P_+} A(x, \eta) + \frac{b}{M_+} B(x, \eta) \right) \right. \tag{50}
\]

\[+ \int_{-1}^{1} dy W_2(x, y) \left[ \Delta_{\rho} \left( \frac{h_{\rho}}{P_+} \frac{d}{d\eta} A(y, \eta) + \frac{b}{M_+} \frac{d}{d\eta} B(y, \eta) \right) + \left( h_{\rho} - P_{\rho} \frac{h_{\rho}}{P_+} \right) A(y, \eta) \right]. \]

Projecting this expression with vector \( n_\rho \) we obtain conventional definitions for twist-two skewed parton distributions. The basis used presently can be easily expressed in terms of Ji’s parametrization as follows, for Dirac structures

\[
\tilde{b} = \frac{M_+}{\Delta_+} \tilde{e}_+, \quad b = \frac{M_+}{P_+} (h_+ - e_+), \tag{51}
\]

and skewed parton distributions

\[
\tilde{A} = \tilde{H}, \quad \tilde{B} = \eta \tilde{E}, \quad A = H + E, \quad B = -E. \tag{52}
\]

For the following, it is useful to note as well that \( M_+ \left( h_{\rho} - \frac{P_{\rho}}{P_+} h_+ \right) = \left( t_{\rho\sigma} - \frac{P_{\rho}}{P_+} t_{\sigma} \right) \Delta_{\sigma} \).

For the time being we introduce three-particle skewed parton distributions without an explicit spinor bilinear decomposition,

\[
\langle P_2 \left\{ \mathcal{S}_{\rho_\beta}^\pm(\kappa, \umu \kappa, -\kappa) \right\} | P_1 \rangle = P_+^2 \int dx e^{-iP_+ x} \left\{ \mathcal{S}_{\rho_\beta}^\pm(x, \umu \eta) \right\}. \tag{53}
\]

A discussion of the parametrization will be given below.

Due to length of the consequent formulas we give below only results for the vector case since the axial one follows from the former by substitutions. Combining the Eq. (19) with the two-particle part of Eq. (27) we get for the \( h \)-part of the local operators \( \mathcal{O}_{\rho} \)

\[
\langle P_2 | \mathcal{O}_{\rho_\beta}^{1-j} | P_1 \rangle = \left( P_{\rho} + \frac{\Delta_{\rho_\beta}}{\eta} \right) h_{\eta} A_{j+1} + \sum_{k=0}^{j} \frac{\sigma_{j+1-k}}{j+1} (-\eta)^{j-k} \left\{ \Delta_{\rho} h_{\eta} \left( \frac{d}{d\eta} - \frac{k+1}{\eta} \right) \right. \tag{54}
\]

\[+ \left( h_{\rho} P_+ - P_{\rho} h_+ \right) \right\} A_{k+1} + \sum_{k=0}^{j} \frac{\sigma_{j-k}}{j+1} (-\eta)^{j-k} \left\{ \Delta_{\rho} \tilde{h}_{\eta} \left( \frac{d}{d\eta} - \frac{k+1}{\eta} \right) \right.
\]

\[+ \left. i \epsilon_{\rho\sigma} \left( h_{\sigma} P_+ - P_{\sigma} \tilde{h}_+ \right) \right\} \tilde{A}_{k+1} + \cdots.
\]
Transforming it finally to the non-local form, and adding the \( b \)-term and the missing three-particle piece, one finds

\[
\langle P_2 | \mathcal{O}_\rho ( -\kappa, \kappa ) | P_1 \rangle = \int_{-1}^{1} dx e^{-i\kappa P_+ x} \left\{ \left( P_\rho + \frac{1}{\eta} \Delta_\rho \right) D(x, \eta) \right\} (55)
\]

\[
+ \int_{-1}^{1} dy \left[ W_+ \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \Delta_\rho \left( \frac{d}{dy} \frac{d}{dy} - \frac{y}{\eta} \frac{d}{dy} \right) D(y, \eta) + \left( h_\rho - P_\rho \frac{h_+}{P_+} \right) A(y, \eta) \right] + W_- \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \left( \bar{\Delta}_\rho \left( \frac{d}{dy} \frac{d}{dy} - \frac{y}{\eta} \frac{d}{dy} \right) \bar{D}(y, \eta) + i \epsilon_{\rho \sigma} \left( \bar{h}_\sigma - P_\sigma \frac{\bar{h}_+}{P_+} \right) \bar{A}(y, \eta) \right)
\]

\[- \int_{-1}^{1} du \left( \frac{1 - u}{2} W'' \left( \frac{x}{\eta}, \frac{y}{\eta} \right) S_\rho^+ (y, u, \eta) + \frac{1 + u}{2} W'' \left( -\frac{x}{\eta}, -\frac{y}{\eta} \right) S_\rho^- (y, u, \eta) \right) \right\},
\]

where we have introduced a shorthand notation for the combinations

\[ D(x, \eta) = \frac{h_+}{P_+} A(x, \eta) + \frac{b}{M_+} B(x, \eta), \quad \bar{D}(y, \eta) = \frac{\bar{h}_+}{P_+} \bar{A}(y, \eta) + \frac{\bar{b}}{M_+} \bar{B}(y, \eta). \] (56)

The derivatives w.r.t. \( \eta \) in Eq. (53) acts only on skewed parton distributions but not on spinor bilinears. The axial-vector case is deduced by the following trivial substitutions: \( b \leftrightarrow \bar{b}, h_\rho \leftrightarrow \bar{h}_\rho, B \leftrightarrow \bar{B}, A \leftrightarrow \bar{A} \) and \( S_\rho^+ \rightarrow R_\rho^+ \). Conventional form is obtained by means of substitutions (54,52). Equation (53) is our final result. Its transverse part gives an expression for the ‘transverse’ twist-three skewed functions in terms of ‘known’ leading twist and three particle functions.

Let us address briefly the parametrization of genuine twist-three part of the correlation functions. An analysis shows that there exist four independent twist-three spinor bilinears, which we can choose to be

\[ \Delta_\rho \frac{b}{M_+}, \quad \Delta_\rho \frac{h_+}{P_+}, \quad \bar{\Delta}_\rho \frac{\bar{b}}{M_+}, \quad \bar{\Delta}_\rho \frac{\bar{h}_+}{P_+}; \] (57)

while the dual ones read in the same sequence

\[ \bar{\Delta}_\rho \frac{b}{M_+}, \quad \bar{\Delta}_\rho \frac{h_+}{P_+}, \quad \bar{\Delta}_\rho \frac{\bar{b}}{M_+}, \quad \bar{\Delta}_\rho \frac{\bar{h}_+}{P_+}. \] (58)

Note that a fifth candidate, \( M_+ \frac{t_{\rho +}}{P_+} \), and trace terms proportional to \( M_+^2 n_\rho \) enter at twist-four level. One can immediately see that bilinears used in Eq. (55) can be spanned by this basis via relations \( h_\rho - P_\rho \frac{h_+}{P_+} = -\frac{1}{1-\eta^2} \Delta_\rho \frac{b}{P_+} + \frac{1}{1-\eta^2} \Delta_\rho \frac{\bar{b}}{P_+} \) and \( \bar{h}_\rho - P_\rho \frac{\bar{h}_+}{P_+} = -\frac{1}{1-\eta^2} \bar{\Delta}_\rho \frac{b}{P_+} + \frac{1}{1-\eta^2} \bar{\Delta}_\rho \frac{\bar{b}}{P_+} \). The parametrization of correlation functions \( S \) and \( R \) then reads

\[ S_\rho^\pm (x, u, \eta) = \Delta_\rho \frac{b}{M_+} S_1^\pm + \Delta_\rho \frac{h_+}{P_+} S_2^\pm + \bar{\Delta}_\rho \frac{\bar{b}}{M_+} \bar{S}_1^\pm + \bar{\Delta}_\rho \frac{\bar{h}_+}{P_+} \bar{S}_2^\pm, \]

\[ R_\rho^\pm (x, u, \eta) = \Delta_\rho \frac{b}{M_+} R_1^\pm + \Delta_\rho \frac{h_+}{P_+} R_2^\pm + \bar{\Delta}_\rho \frac{\bar{b}}{M_+} \bar{R}_1^\pm + \bar{\Delta}_\rho \frac{\bar{h}_+}{P_+} \bar{R}_2^\pm, \] (59)
where duality (13) requires \( \tilde{S}_i^\pm = R_i^\pm, \tilde{R}_i^\pm = S_i^\pm \). To reexpress the parity even and odd sector only in terms of vector and axial-vector form vectors, respectively, one can equally use an alternative basis, where the independent elements are

\[
\Delta_{\rho}^+ \frac{b}{M_+}, \quad h_{\rho} - P_{\rho} \tilde{h}_{\rho}, \quad \Delta_{\rho}^+ \frac{\tilde{b}}{M_+}, \quad \tilde{h}_{\rho} - P_{\rho} \tilde{h}_{\rho},
\]

while the dual ones read in the same sequence

\[
\tilde{\Delta}_{\rho}^+ \frac{b}{M_+}, \quad \tilde{\Delta}_{\rho}^+ \frac{\tilde{b}}{P_+}, \quad \Delta_{\rho} h_{\rho} - P_{\rho} \Delta_{\rho}, \quad \tilde{\Delta}_{\rho} \tilde{h}_{\rho} - \tilde{h}_{\rho} \Delta_{\rho},
\]

(60)

while the dual ones read in the same sequence

\[
\Delta_{\rho}^+ \frac{b}{M_+}, \quad h_{\rho} - P_{\rho} P_{\rho}, \quad \Delta_{\rho}^+ \frac{\tilde{b}}{P_+}, \quad \tilde{h}_{\rho} - P_{\rho} \tilde{h}_{\rho},
\]

(61)

## 5 Power suppression of twist-three effects in DVCS.

Now we point out the phenomenological consequences of our analysis for the DVCS process, \( eN \rightarrow e'N'\gamma \), with \( \eta = -\xi \). Since DVCS interferes with the Bethe-Heitler process, the possibility exists to get a direct access to the skewed parton distributions via the interference term. In leading twist-two approximation of the DVCS hadronic tensor the amplitude squares for unpolarized or longitudinal polarized nucleon behave as:

\[
|T_{\text{DVCS}}|^2 \propto \frac{1}{Q^2}, \quad T_{\text{BH}} T_{\text{DVCS}}^* \propto \frac{1}{Q^2} \left( 1 - \frac{\Delta_{\text{min}}^2}{\Delta^2} \right)^{1/2}, \quad |T_{\text{BH}}|^2 \propto \frac{1}{\Delta^2},
\]

(62)

where \( \Delta_{\text{min}}^2 = -4M^2\xi^2/(1 - \xi^2) \) follows from the kinematical boundary on which \( \Delta^\perp \) vanish. Consequently, the interference terms and so also the charge and spin asymmetries vanish (this is not the case for transversely polarized target). Since the leading \( \Delta^\perp \) dependence could also arise from the twist-three contributions to the DVCS amplitude, one may argue that those enter in the interference term without power suppression. Consequently, the whole twist-two analyses would be spoiled. This possible complication has been already studied in the past [11], unfortunately, it was not clearly emphasized that twist-three contributions are power suppressed.

Our aim is to show that this is indeed the case, while the complete interference term will be published elsewhere [13]. In our analysis [11] we used the following parametrization of the hadronic scattering amplitude that arises from the operator product expansion in the free fermion theory, where current conservation was restored in calculations by means of projection operators:

\[
T_{\mu\nu}(q, P, \Delta) = -P_{\mu\sigma} g_{\sigma\tau} P_{\tau\nu} \frac{q \cdot V_1}{P \cdot q} + (P_{\mu\rho} P_{\sigma\nu} + P_{\mu\nu} P_{\sigma\rho}) \frac{V_{2\rho}}{P \cdot q^2}
\]

(63)

\[-P_{\mu\sigma} i\epsilon_{\sigma\tau\rho\sigma} P_{\tau\nu} \frac{A_{1\rho}}{P \cdot q} - P_{\mu\sigma} i\epsilon_{\sigma\tau\rho\sigma} P_{\tau\nu} \frac{A_{2\rho}}{P \cdot q},
\]

where \( P_{\mu\nu} \) has been introduced in section 2. Comparing with the results given in Eqs. (16,18), we find that the form factors are related to each other in the following manner

\[
V_{1\mu} = \int \frac{dx}{\xi} C^{(-)}(x, \xi) O_\mu(x, \eta), \quad A_{1\mu} = \int \frac{dx}{\xi} C^{(+)}(x, \xi) A_\mu(x, \eta), \quad A_{2\mu} = 0,
\]

(64)
\[ V_{2\mu} = \int \frac{dx}{\xi} \left\{ \xi C^{(-)}(x, \xi) \left( \bar{O}_\mu(x, \eta) - \frac{1}{2P \cdot q} q \cdot \bar{V}O(x, \eta) \right) + C^{(+)}(x, \xi) \frac{i}{2} \frac{\epsilon_{\mu\nu\Delta q}}{P \cdot q} A_{\nu}(x, \eta) \right\}. \]

A straightforward calculation and power counting show that in the case of DVCS kinematics, where \( \eta = -\xi \), the contributions of \( V_{2} \) and \( A_{2} \) are power suppressed in comparison to the known twist-two result:

\[
T_{BH}T_{DVCS} = \,^{2-2y+y^2} \frac{\xi}{1-y} \frac{1}{\Delta^2 q^4} \left\{ \left( k_\sigma - \frac{1}{y} q_\sigma \right) \left( J_\sigma - \frac{q \cdot J}{q^2} \right) q \cdot V_1^\dagger + i\epsilon_{kq} A_1^\dagger \right\} + \lambda \frac{(2-y)y}{1-y} \frac{\xi}{\Delta^2 q^4} \left\{ \left( k^\sigma - \frac{1}{y} q^\sigma \right) \left( J_\sigma - \frac{q \cdot J}{q^2} \right) q \cdot A_1^\dagger + i\epsilon_{kq} A_1 \right\}. \tag{65}
\]

Here \( J_\mu \) is the electromagnetic current, \( \lambda \) and \( k_\mu \) denote the polarization and momentum of the incoming electron. We employed a simple rule that \( q_\mu, k_\mu - \frac{1}{y} q_\mu, \) and \( P_\mu, \Delta_\mu, \) when contracted with an electromagnetic current or spinor bilinears, give terms of order \( Q^2, Q^1, \) and \( Q^0, \) respectively. Note that in agreement with this counting our results for unpolarized nucleon target coincide with Ref. [16], where the contribution of \( V_{2} \) has been taken into account. Let us remark, that also in the case of a scalar target the same situation holds true, as it already has been shown in [9].

Since, the twist-three contributions enter as \( 1/Q \) power suppressed term to all single spin and charge asymmetries, the comparison of models for twist-2 skewed parton distributions with experimental data will be contaminated in an expected manner. Moreover, we emphasize again that the twist-3 functions are completely known in terms of twist-two ones in absence of the gluonic contributions as it is the case in the naive parton model. It is an interesting non-perturbative problem to estimate the size of the gluonic contributions in comparison to the Wandzura-Wilczek term. Assuming that the dynamical twist-three effects encoded in the three-particle operators are small, which presumably happens in view of recent experimental data [17] and new lattice results [18] for the forward kinematics, one can use the Wandzura-Wilczek part of the relation (63) as a model for the ‘transverse’ twist-three skewed parton distributions, \( \langle P_2 | \bar{O}_{\mu}^\dagger | P_1 \rangle \). This will allow to give a numerical estimate of the power suppressed contributions to asymmetries for the kinematics of present experiments [18].

6 Logarithmic scaling violation.

Let us add a final remark on the logarithmic scaling violation of twist-three functions. Obviously the Wandzura-Wilczek part evolves via the familiar twist-two generalized exclusive evolution equation known to two-loop accuracy nowadays [19]. The three-particle piece does not require a new study as well, since it was elaborated in great detail recently in the context of twist-three functions measured in inclusive reactions (forward limit restrictions was not made in those studies). Namely, the operators \( S \) alluded to above fall into a class of the so-called quasi-partonic
ones \( \text{[21]} \) and thus at leading order in the coupling constant their evolution kernel is given by a sum of conventional skewed twist-two ones with appropriate quantum numbers in subchannels: 
\[ K_{\bar{q}gq} = K_{\bar{q}g} + K_{gq} + K_{\bar{q}q}. \]
The issue of its diagonalization (see a recent review \[ \text{[21]} \]) has been addressed in detail recently \[ \text{[22, 23, 24]} \] within the context of integrable open spin chain models which arise in multicolour limit\(^3\). Namely, for \( N_c \to \infty \) the kernel \( K_{\bar{q}q} \) can be neglected and, e.g. 
\[ \frac{1}{N_c} K_{\bar{q}g} = \psi(\hat{J}_{\bar{q}g} + \frac{3}{2}) + \psi(\hat{J}_{\bar{q}g} - \frac{3}{2}) - 2\psi(1) \]
and 
\[ \frac{1}{N_c} K_{gq} = \psi(\hat{J}_{gq} + \frac{1}{2}) + \psi(\hat{J}_{gq} - \frac{1}{2}) - 2\psi(1) \]
for \( S^+ \), and with \( \frac{3}{2} \) and \( \frac{1}{2} \) being interchanged for \( S^- \). Due to conformal invariance at tree level, the kernels depend on the quadratic Casimir operator of the collinear conformal group \( SO(2, 1) \) or, in other words, on the conformal spin in the subchannel only, \( \hat{J}^2 = \hat{J}(\hat{J} - 1) \). It turns out that antiquark-gluon-quark system admits an extra integral of motion \( Q_S \) and is thus completely integrable. The result of the analysis allows to find the eigenfunctions \( \Psi \) and eigenvalues \( E \) of the system and thus solve the evolution equation,
\[
S(x_1, x_2, x_3| Q^2) = \sum_{\{\alpha\}} \Psi_{\{\alpha\}}(x_1, x_2, x_3) \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{N_c E_{\{\alpha\}}/\beta_0} \langle \langle S_{\{\alpha\}}(Q_0^2) \rangle \rangle, \quad \sum_{i=1}^{3} x_i = \eta.
\]
Here \( \beta_0 = \frac{4}{3} T_F N_f - \frac{11}{3} C_A \) is the QCD beta function and a set of quantum numbers \( \{\alpha\} \) parametrizes solutions and can be chosen as eigenvalues of the conformal spin \( J \) and the charge \( Q_S \). Namely, the lowest anomalous dimension, \( E_0(J) \), is known exactly \[ \text{[23]}, \] while the rest of the spectrum, \( E_q(J) \), can be described with a high accuracy using WKB approximation \[ \text{[22, 23, 24]} \] with the results
\[
E_0(J) = \psi(J + 3) + \psi(J + 4) - 2\psi(1) - \frac{1}{2}, \quad E_q(J) = 2 \ln J - 4\psi(1) + 2 \Re \psi \left( \frac{3}{2} + i\eta_S \right) - \frac{3}{2},
\]
where \( \eta_S \) is related to the conserved charge by the relation \( \eta_S \equiv \frac{1}{2} \sqrt{2 Q_S/J^2 - 3} \) and obeys a WKB quantization condition which, once solved, gives quantized values of \( E_Q \).

For practical purposes, one may also generalize the polynomial reconstruction method as it was applied for the forward kinematics in Ref. \[ \text{[26]} \] or use a brute force numerical integration to solve the evolution equations.

### 7 Conclusions.

In this paper we have studied twist-three effects for the two-photon amplitudes in the generalized Bjorken kinematics. We have demonstrated explicitly the restoration of the electromagnetic gauge invariance for a Compton-type amplitude to twist-four accuracy. We have given as well the exact
Lorentz structure of the amplitude motivated by our results at leading order in the coupling constant. Obviously, beyond leading order, e.g. different Lorentz structures in $\mathcal{T}_{\mu\nu}^{(1)}$ will be multiplied by independent functions (see Ref. [1]), analogues of $F_1$ and $F_2$ in the forward scattering. Our analysis demonstrates a necessity of introduction of twist-three two-particle generalized distributions. The latter can be related by means of QCD equation of motion and Lorentz invariance to the familiar twist-two ones and interaction dependent antiquark-gluon-quark correlation functions. The former ones represent a generalization of the Wandzura-Wilczek type relation, while the last ones give dynamical twist-three contributions. With the assumption of smallness of dynamical contributions the Wandzura-Wilczek part with parametrization of leading twist skewed parton distributions can serve as a model for ‘transverse’ functions. Contributions of these functions to diverse asymmetries [11] will be considered elsewhere [15].

Note added: Recently there appeared a note [27] where the quark hand-bag diagram is calculated in parton model with transverse momentum being kept. This result overlaps with a part of our analysis in section 2.

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