Robotic Unicycle Intelligent Robust Control Pt I: Soft Computational Intelligence Toolkit

Ulyanov Sergey¹²³*, Ulyanov Viktor²³, Yamafuji Kazuo⁴

1. Dubna State University, Institute of system analysis and management, Dubna, Moscow, 141980, Russia
2. INESYS LLC (EFKO GROUP), Ovchinnikovskaya naberezhnaya 20, Bld 1, Business Centre “Central City Tower”, Moscow, 115035, Russia
3. NUST MISIS IYS Lab, Leninskiy prospekt 4, Moscow, 119049, Russia
4. Dept. of Mechanical and Intelligent Control Eng., University of Electro-Communications, 1-5-1 Chofu, Chofugaoka, 182 Tokyo, Japan

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ABSTRACT

The concept of an intelligent control system for a complex nonlinear biomechanical system of an extension cableless robotic unicycle discussed. A thermodynamic approach to study optimal control processes in complex nonlinear dynamic systems applied. The results of stochastic simulation of a fuzzy intelligent control system for various types of external/ internal excitations for a dynamic, globally unstable control object - extension cableless robotic unicycle based on Soft Computing (Computational Intelligence Toolkit - SCOptKB™) technology presented. A new approach to design an intelligent control system based on the principle of the minimum entropy production (minimum of useful resource losses) determination in the movement of the control object and the control system is developed. This determination as a fitness function in the genetic algorithm is used to achieve robust control of a robotic unicycle. An algorithm for entropy production computing and representation of their relationship with the Lyapunov function (a measure of stochastic robust stability) described.

1. Introduction: Intelligent Mechatronics as an Implementation Background of a New Types of Nonlinear Mechanical Systems Motion

The extraction of knowledge from a new movement types of real physical control objects is based on benchmarks mathematical models’ simulation. The robotic unicycle motion is one of such type of “benchmark movements” (benchmark model of nonlinear mechanics¹⁻⁵), described as nonlinear nonholonomic, global unstable dynamic system. Related research of such dynamic systems is interesting for nonlinear mechanics (to develop a new method of nonlinear effects research) and for modern control theory (to develop a new intelligent control algorithms).

Modern methods and algorithms of intelligent control development
The development of an algorithm and control system for robotic unicycle benchmark requires a new technology of unconventional computing - computational intelligence toolkit. The physical feature of robotic-unicycle is that the real unicycle bike's control is realized by skillful human being operator only. This leads to the studying of the robotic unicycle as a biomechanical system includes new approaches to the control system, such as intuition, instinct and emotions inherent to the human-operator (rider) and allowing to study the possibility of cognitive control by including the "human factor" in the control loop. The control of the robotic unicycle motion is based on the coordination of the complex movement components (pedaling and movement of the rider’s torso). Changing the components coordination type generates new types of movement (rectilinear movement, slalom, dance, jumping, etc.). From nonlinear mechanics view point it is 3D synergetic effects of energy transfer from one generalized coordinate to others apply nonlinear relationships between generalized coordinates described by system of nonlinear equations.

Related works. Previous studies conducted in the field of different unicycle robot mechanical models controlling (see, Table 1) considered the system only from the point of view of a mechanical model using classical control methods and / or a simplified, hybrid fuzzy proportional-differential controller (FPD) with empirical tables of fuzzy decision making (production) rules (look - up Tables) [1-4]. However, this become an algorithmically intractable problem for traditional control methods in the task solution of robust (stable) motion of the object and led to appearing of new approaches to solve this issue.

The concept of this research and development of robotic unicycle intelligent control system becomes the structure shown in Figure 1.

To solve the problem of this object controlling a cyber-physical model called as “Conceptual Logical Structure of the Distributed Knowledge Representation (Information Levels) in the Artificial Life of the Robotic Unicycle” (Figure 1 a, b) as a biomechanical model of movement and control was developed and proposed.

The main research objective is studying the control problem of the robotic unicycle nonlinear biomechanical model, as well as the creation and “training” of a control system by means of available soft computing methods and algorithms.

To assess the quality of control, a new physical principle: the minimum entropy production rate in the object’s movement and in the control system [2-5,7-13]. The physical measure of entropy production rate is applied as a fitness function in the genetic algorithm (GA).

### Table 1 Models of unicycle robot

| Model                  | Year  |
|------------------------|-------|
| Schoonwinkel model     | 1987  |
| D.W. Vos model         | 1992  |
| Yamafuji et al. model  | 1995  |

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Kim et al. model 2010 [18]

Murata Seiko Girl model 2011 [15]

Eric Wieser model 2017 [16]

J.F. De Vries model 2018 [17]

Shen J. model [27]

Figure 1(a). Conceptual Logical Structure of Distributed Knowledge Representation (on Information Levels) in Artificial Life of the Robotic Unicycle
This approach ensures the global stability of the dynamic control object and robustness it’s of the control system. Based on this approach the "Self-organizing structure of an artificial intelligence (AI), robust control system design with a new physical measure of control quality" (see below Figure 3) with a new type of intelligent feedback based on the principles of computational intelligence, as well as "Fuzzy Simulation structure of an intelligent control system design with soft computing algorithms "(see Figure 5 below) has been developed. In previous studies, the problem of external and internal excitations on the mechanical and control system was not considered, see [1-4]. As a result, the global dynamic stability in object’s control was not achieved.

In this article the modelling and optimization of intelligent control system with stochastic external / internal excitations simulation in the mechanical and control systems (floor roughness’s, mechanical vibrations, zero sensors drift etc.) using the structure of the forming filters [5] is represented. The results of the simulation and experiment confirming the efficiency of the model robotic unicycle control system.

2. Problem Statement and Research Purpose: Creation of the Robotic Unicycle Mathematical Model with Essentially Non-linear Intersection between Generalized Coordinates

As mentioned above, the objective of this research is development the intelligent control system for non-holonomic, essentially nonlinear, global spatially unstable with high amount of linking constraints model of the robotic unicycle. For this purpose, a new unicycle mathematical model was created for the “Real” unicycle’s coordinate system (see, Figure 2).

$$\begin{align*}
\frac{dx(t)}{dt} &= Rw \cdot \frac{d\theta w(t)}{dt} \cdot \cos(\alpha(t)) ; \\
\frac{dy(t)}{dt} &= Rw \cdot \frac{d\theta w(t)}{dt} \cdot \sin(\alpha(t)) \\
\end{align*}$$

(1.1)

where $Rw$ - wheel radius, \( \frac{d\theta w(t)}{dt} \) - velocity of wheel rotation, \( \alpha(t) \) - yaw angle. The coordinates $x_0, y_0$ are eliminated by substituting of Eq. (1.1) to kinematic Lagrangean part.

$$q_j(t) = [\alpha, \gamma, \beta, \theta w, \Psi, \theta_1, \theta_2, \theta_3, \theta_4, \eta], \quad j = 1,\ldots,9.$$
In Figure 2 notation: α - yaw angle; γ - roll angle; β - pitch angle; C.M. - center of mass; L 1,...,6 - links 1-6; θ1, ..., θ4 - links rotation angles; Ψ - initial position of pedals and the current angle of pedal rotation (link 5,6) is included in equations as summa - \( \Psi(t) = \theta_{w}(t) + \psi(\text{const}) \).

Thus, Lagrangian solving including represented above equations of nonholonomic/holonomic constraints and external forces of stochastic excitations, gives the following generalised stochastic equation of the robotic unicycle system motion with control:

\[
\begin{cases}
ME_i(q,q)(\ddot{q}_i(t) - \dot{c}_i(t)) + C_i(q,\dot{q})(\dot{q}_i(t) - \dot{c}_i(t)) - G_i(q,\dot{q}) + D_i(q,\dot{q}) \\
L_i(q,\dot{q})(\dot{q}_i(t) - \dot{c}_i(t)) - \dot{c}_i(t)
\end{cases}
\quad \text{(a)},
\]

\[
\begin{cases}
\lambda_i(q,\dot{q})(q_i(t) - \dot{c}_i(t)) + \Gamma_i(q,\dot{q}) + \Phi_i(q,\dot{q}) - \tau_i(t) - \gamma_i(t)\\
\end{cases}
\quad \text{(b)},
\]

where - \( i,j = 1..9 \); vector of generalized accelerations \( \ddot{q}_j(t) = [\ddot{u}, \dddot{v}, \dddot{w}, \dddot{\theta_1}, \dddot{\theta_2}, \dddot{\theta_3}, \dddot{\theta_4}, \dddot{\theta_5}] \); vector of generalised velocities \( \dot{q}_j(t) = [\dot{u}, \ddot{v}, \ddot{w}, \dot{\theta_1}, \dot{\theta_2}, \dot{\theta_3}, \dot{\theta_4}] \). In the system of equations Eq (1.2), equation (a) is the dynamical equation of motion for the whole unicycle model with stochastic excitations, and equation (b) is the description of Lagrangian multipliers \( \lambda_i \), where \( n = 1...4 \). Matrices, vectors and another terms of equations (1.2) described in detail in the following sections of this article.

Stochastic excitation appears in case of \( \xi_c(t) \) & \( \xi_i(t) \neq 0 \) and described via differential equation of Forming Filter as Gaussian (as in our case) random process with autocorrelation function \( R(\tau) = \sigma^2_c \exp(-\alpha_c \cdot |\tau|) \). This disturbance is included into equation of motion for some generalized coordinates, and it is modelling possible roughness of flow, jamming in closed-links mechanism, and inaccuracy of angular acceleration measuring (sensors zero drift).

Under these conditions obtained stochastic equation of motion with parametric excitations. All of this gives a possibility to simulate behavior of dynamic controlled system more realistically and to determine real parameters of intelligent controllers for error estimation and control robustness. Stochastic modelling via Forming Filters is described below in [3].

3. Stability Estimation of Robotic Unicycle System

For definition of (un)stability is used a Salvadory theorem about equilibrium of mechanical systems with dissipative forces of a Q(q,\dot{q}) type along with full energy of system E(q,\dot{q}) as Lyapunov function \( V(q,\dot{q}) = E(q,\dot{q}) = T(q,\dot{q}) + U(q) \); where \( T(q,\dot{q}) \) is kinetic energy of system, \( U(q) \) - potential energy of system. Under Lyapunov’s theorem conditions, if the function \( V(q,\dot{q}) \) is: 1) positively determined about any \( q,\dot{q} \) and have 0 at \( (q,\dot{q}) = 0 \), i.e. \( V(q,\dot{q}) \geq a(q,\dot{q}) \) & \( V(0) = 0 \), where \( a \) is a some continuous, strictly increasing function, satisfying to a condition \( a(0) = 0 \); 2) Derivative of function \( V \) by time \( t \) is negative, i.e. \( \dot{V}(q,\dot{q}) \leq 0 \); when origin is stable [6]:

Let’s considering conditions of Salvadori’s theorem that determine the dynamical systems stability.

Assume iff:

1) \( U(q) \) have minimum at \( q = 0 \);
2) \( U(q) \) don’t have minimum at \( q = 0 \);
3) equilibrium state \( q = 0 \) is stable; in the case 2), with condition 1a), equilibrium state \( q = 0 \) - unstable.

Basing on the both theorems lets define a stability condition of robotic unicycle system.

The equation for expression of robotic unicycle potential energy for \( q_i = (0,0,0,0,\theta_1,\theta_2,\theta_3,\theta_4, 0) \) has following form:

\[
\sum_{n} U_n(0) = g \cdot [Rw \cdot \sum_{n} M_n + M_1 \cdot (2 \cdot e1 - e2 \cdot \sin(\theta) + \sin(\theta_2)) + \Delta z \cdot M_3 \cdot (\cos(\theta_3) + \cos(\theta_4)) + M_4 \cdot e6]
\]

where: coordinates - \( \theta_1, \theta_2, \theta_3, \theta_4 \) cannot be equal to 0 at \( \gamma, \beta, \theta_w = 0 \) by mechanical constraints of closed-links mechanism; \( M_i \) - masses of robotic unicycle parts; \( Rw, e1,e2,e6, \Delta z \) - sizes in Figure 2. From Eq. (1.3) follows, that \( U(0) \) has maximum value in the equilibrium statement. The Eq. (1.3) satisfies to a condition 1a) of Salvadory theorem, that describes instability of robotic unicycle system at equilibrium statement. This enables to assert about global instability of the robotic unicycle autonomous dynamic system. However, as discussed in [6], in case the \( U(0) \) has maximum value it might happen that equilibrium will be stable owing to occurrence of external forces, such as gyroscopic or similar that in our case is controlled torques.

This research proclaims that it is possible to create such intelligent control system, which can continuously stabilize dynamic motion of nonlinear robotic unicycle and the simulation results are shown below.

4. Methods for Task Solving - Conceptual Model of Biomechanical Robotic Unicycle Control System

To provide computational intelligence methods that can
coordinating the complex motion components, it is necessary to use qualitatively new control algorithms that can operate with linguistic variables \cite{8}. Soft computing methods fully satisfy to requirements, and that is determines their use. Based on the physical and sophisticated description of the biomechanical model, and using soft computing methods, the following structure of modeling the intelligent control system is represented.

**Biomechanical Model of Intelligent Control System.** The human riding control of the unicycle as logic-dynamic hierarchical process may be formed by:

1. mechanical dynamic system “human-rider - unicycle”;
2. decision-making process of unicycle intelligent control with different levels of “riding skills”;
3. logical behavior for human body motorists (legs, hands and torso coordination) based on intuition, instinct, and emotion mechanisms;
4. distributed information system for cooperative coordinating of sub-systems in biomechanical model \cite{9}.

In accordance with this representation of dynamic control process a hierarchical logic structure of distributed knowledge representation of the robotic unicycle artificial life is shown in Figure 1. For description of artificial life of robotic unicycle, the methods of qualitative physics for internal world representation based on mathematical model of unicycle motion used.

Logic structure of biomechanical control system for description a human riding of unicycle includes four levels:

1. distributed information levels with sub-levels;
2. logical system;
3. support decision-making system;
4. dynamic mechanical system.

Further the proposals of this structure are described in details. **Distributed information levels include four sub-levels:**

1. physical level and logic of virtual reality;
2. behavior and coordination level;
3. intelligent control levels with two sub-levels;
4. executive biomechanical level.

Intersections between the horizontal lines of distributed information levels and vertical lines of **logical system, support decision-making system, and dynamic system** (of unicycle motion and a human-rider behavior as biomechanical control model) realize the particular for human unicycle riding models with different skill levels of smart control tools using. Let’s consider here this approach with examples.

**Example 1:** Physical and logical level of virtual reality. The intersection of the first horizontal level (Physical and logical level of virtual reality) with the first vertical level (Logical system) gives the structure of the human learning process to ride (control) a unicycle. The intersection with the second vertical level (support decision making system) corresponds to the level of the central nervous system (CNS) as a biological control system. The intersection with the third level (Dynamic (mechanical) system) is introduced mechanical model of the of a unicycle movement as a dynamic system. The logical sum of these sublevels implements the physical level of the unicycle movement description and the physical interpretation of the experimental data (attempts). The mathematical background for describing the learning process is the quantum fuzzy logic. The functions of the CNS are realized as the knowledge base (KB) domain of possible stable states. But, to create a control system of such a high intelligent level is not currently possible.

**Example 2:** Behavior and coordination level. This structure includes the mechanisms of instinct, intuition and emotion. The mechanism of instinct is described in the logical structure as a local coordinator with fuzzy rules and corresponds to a control structure with active and passive adaptation based on a fuzzy neural network (FNN). The mechanism of intuition is represented as a global coordinator and realized in the control process as a decision-making process based on a genetic algorithm (GA). The mechanism of emotions is described basing on the information from motion sensors and represented in the form of lookup tables with different semantic expression of the linguistic description of the desired dynamic motion behavior (as examples, “smoothly”, “quickly” and so on). Thus, the intersection of two distributed information levels with logical systems is realizing the artificial brain unit for the process of unicycle control system self-organization.

**Example 3:** The Intelligent control level - an artificial intelligent control system with a distributed knowledge representation, includes “will” and “mind” (desires and opportunities) concepts, just like a human being \cite{9,10}. For the mechanisms of instinct and emotion, new lookup tables are determined using an FNN. The mechanism of intuition is realized on the GA basis and directs the two fuzzy controllers’ actions. Thus, the fuzzy simulation based on mathematical GA and FNN tools implements the soft computing algorithm in the robot’s intelligent control system.

From a qualitative physical description and movement simulation the domain of possible virtual stable states described by a strange attractor is obtained, as it was shown in \cite{14}. This suggests that the human postural control system is a highly organized complex system and the position...
of the human body changes (sways) stochastically. This fully corresponds to the results described in \[11\].

**Example 4:** Executive biomechanical level - the physical process of the robotic unicycle system realization. In this case, the vestibular system as a logical control system is realized by the balancing turntable (torso, shoulder girdle and arms), and the neuro-muscular system is formed by the closed-links mechanism of the legs.

Thus, the dynamic process of human unicycle riding control with different intelligent behavior levels should be described as the intersection of logical systems with distributed information levels.

**The intelligent robust control system structuring.** The development of complex systems robust motion control has two ways of research: 1) the study of stable motion processes; and 2) the study of unstable motion control with different intelligent behavior levels should be described as the intersection of logical systems with distributed information levels.

As mentioned above, our attention has been focused on the study of the robotic unicycle as a dynamic, global-spatially unstable controlled object. Such a global-spatially unstable dynamic object requires a new intelligent robust control algorithm based on knowledge description of an essentially nonlinear, unstable dynamic system movements \[3-9\]. The structure of the intelligent robust control algorithm in the conceptual form for the entire class of unstable dynamic control objects, was described in \[2-4,8\], here we apply it to the problem of controlling the robotic unicycle.

The control structure with a new intelligent feedback type is represented in Figure 3. It is based on the scheme of a conventional control system with linear feedback $P(I)D$, intelligent soft computing tools (fuzzy set theory, fuzzy neural networks (FNN), genetic algorithms (GA)); nonlinear model of the control object; entropy production rate calculating; stochastic simulation of random external/internal excitations.

In the structure Figure 3 the following designations are used: GA - Genetic Algorithm; f - Fitness Function of GA; S-$i$ - Entropy of System; $S_c$ - Entropy of Controller; $S_p$ - Entropy of Controlled Plant; e - Error; $u^*$ - Optimal Control Signal; $m(t)$ - Disturbances (random external/internal excitations); FC - Fuzzy Controller; FNN - Fuzzy Neural Network; FLCS - Fuzzy Logic Classifier System; SSCQ - Simulation System of Control Quality; K - Global Optimum Solution of Coefficient Gain Schedule (Teaching Signal); LPTR - Look-up Table of Fuzzy Rules; CGS - Coefficient Gain Schedule (in case of 2 PD controllers - $K = (k_1, k_2, k_3, k_4)$).

The control self-organization in this system, at the first stage, is provided by optimizing the control parameters of the $P(I)D$ controller by selecting the best solutions with a genetic algorithm, in which the selection criterion is the best fitted solution, calculated using the fines function. To determine solution fitness’s a new physical measure of control quality is used - a entropy production minimum rate. This measure is the difference between entropy production by the control object itself and the control system included in it. This allows you to adapt the parameters of the linear control system to a nonlinear control object \[7,12\].

The next adaptation stage is training the control system to ensure its robustness. This step is based on a fuzzy logic classifier that defines fuzzy rules for logical relationships of linear controller parameters. The classifier is the FNN for which the training signal is optimized control parameters obtained from the genetic algorithm output. This stage generates fuzzy lookup tables, adapted control parameters of the low-level controller $P(I)D$.

In this approach, solving the problem of a nonlinear object controlling, the criterion of the control quality (controllability) is the entropy production function, which is directly related to the Lyapunov function i.e. to a dynamic system stability, as it shown in \[7,12,13\]. The interrelation between these functions in an intelligent control system ensures its robustness, as shown in Figure 4. Further, the obtained lookup tables are used by a fuzzy controller (FC) to control the linear controller parameters.

Figure 3. Self-organizing structure of an artificial intelligent (AI), robust control system design with a new physical measure of control quality

**Description:**

1. GA - Genetic Algorithm;
2. f - Fitness Function of GA;
3. S-$i$ - Entropy of System;
4. $S_c$ - Entropy of Controller;
5. $S_p$ - Entropy of Controlled Plant;
6. e - Error;
7. $u^*$ - Optimal Control Signal;
8. $m(t)$ - Disturbance;
9. FC - Fuzzy Controller;
10. FNN - Fuzzy Neural Network;
11. FLCS - Fuzzy Logic Classifier System;
12. SSCQ - Simulation System of Control Quality;
13. $K$ - Global Optimum Solution of Coefficient Gain Schedule (Teaching Signal);
14. LPTR - Look-up Table of Fuzzy Rules;
15. CGS - Coefficient Gain Schedule $K = (k_1, k_2, k_3, k_4)$.
Based on the intelligent control structure and interrelation-ship in Figure 4, the Fuzzy Simulation structure of an intelligent control system design was developed in Figure 5.

Simulation is decomposed into two main stages: Off-Line and On-Line. At the first stage a controlled object mathematical model is creating and the thermodynamic equations of its states are founding to calculate the entropy. Further, the equations for entropy production forming the GA fitness function. GA in computer stochastic simulation mode optimizes the P(I)D controller parameters. The next step is the training of the control system based on the optimized controller parameters obtained from the GA and obtaining lookup tables (FC Knowledge Base) using the FNN.

In On-Line mode basing on the obtained lookup tables the P(I)D controller parameters of the robotic unicycle are changes by a fuzzy controller in real time. The structure of the robot control system is described below.

5. Mathematical System Modeling Using Soft Computing Methods

The simulation basis is a mathematical description of the control object motion represented in (1.2). Let us dwell on the equations (1.2) description.

For the mathematical simulation of a robotic unicycle movement the following parameters of the model (1.2) are adopted and graphically represented in Figure 2: where \( i,j = 1 \ldots 9 \); \( \dot{q}_i(t) \) - vector of generalized accelerations;

- vector of generalized velocities; In the system of equations Eq(1.2), equation (a) is the dynamic equation of motion for the whole unicycle model, and equation (b) is the description of Lagrangian multipliers \( \lambda_n \), where \( n = 1 \ldots 4 \).

In Eq.(1.2,a), \( ME_{i,j}(q) \) is a 9×9 block matrix that consists of inertial acceleration’s terms \( M(q) \), derived from Lagrange equations, and geometrical acceleration’s terms derived from equations of closed-links mechanism constraints \( E(q) \); \( BT_{i,j}(q,\dot{q}) \) is a 9×9 block matrix that consists of Coriolis and centrifugal \( B(q,\dot{q}) \) & \( T(q,\dot{q}) \) terms, derived from Lagrange equations, and equations of closed-links mechanism constraints, respectively; \( G(q) \) is a 9-dimensional vector of gravity terms \( G(q) = [0,G_x(q),G_y(q),G_z(q),0,0,0,0,0] \); \( D(q,\dot{q}) \) is a 9-dimensional vector of viscous friction forces terms \( D(q,\dot{q}) = [D_x(q,\dot{q}),D_y(q,\dot{q}),D_z(q,\dot{q}),0,0,0,0,0,0] \); \( \xi(t) \) is a 9-dimensional vector of given stochastic excitation.

In Eq.(2,b), \( MC_{ij}(q) \) is a 9×9 matrix of inertial acceleration’s terms \( M(q) \) derived from Lagrange equations; \( Bc_{ij}(q,\dot{q}) \) is a 9×9 matrix of Coriolis and centrifugal \( B(q,\dot{q}) \) terms, derived from Lagrange equations; \( GC(q) \) is a 9-dimensional vector \( GC(q) = [0,0,0,0,G_x(q),G_y(q),G_z(q),G_x(q),G_y(q),G_z(q)] \) of gravity terms; \( DC(q,\dot{q}) \) is a 9-dimensional vector of viscous friction forces terms; \( \xi_r \) is a 9-dimensional vector of torques \( \tau_r = [0,0,0,0,0,0,\tau_{r31},\tau_{r412},0] \); \( \xi_i(t) \) is given stochastic excitation; \( Ai_n(q) \) is a 9×4 matrix of...
geometrical terms derived from constraints equations of closed-links mechanism, correspond to motion equation by \( i \) index; \( \lambda \) - 4-dimensional vector of Lagrangian multipliers:

\[
M_{c_{ij}}(q) = \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
\end{bmatrix}; \\
B_{c_{ij}}(q, \dot{q}) = \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
\end{bmatrix}; \\
A_{c}(q) = \begin{bmatrix}
A_{1}(q) & A_{2}(q) & 0 & 0 \\
0 & A_{3}(q) & A_{4}(q) \\
A_{5}(q) & A_{6}(q) & 0 & 0 \\
0 & A_{7}(q) & A_{8}(q) \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The equations of the Robotic Unicycle system’s controlled torques. In the case of PD controller for the Links mechanism, the controlled torque is given as:

\[
\tau_{(\theta)12} = -k(\dot{\theta}) + kd(\ddot{\theta}) 
\]

and for the Rotor mechanism - as:

\[
\tau_{(\phi)12} = kp(\dot{\phi}) + kd(\ddot{\phi}) 
\]

GA generates P(1)D controller’s parameters \( kp(T), kd(T), kd(\dot{T}) \), selecting them basing on the results of the fitness function calculations, each sampling time \( T = 0.05 \)sec. This sampling time is defined from real controller scheme of Robotic Unicycle.

Information-thermodynamic criterion of the control qualities distribution. The thermodynamic ratio of the robust intelligent control quality to the optimization criterion, used in the quantum algorithm (QA) of a knowledge bases (KB) self-organization [8], are shown in Table 2. In Table 2 the following notations are introduced: \( V \) - Lyapunov function; \( S_{o}, S_{c} \) entropy production in the Control Object & Controller, respectively; \( V = \frac{1}{2} \sum_{i=1}^{n} q_{i}^{2} + \frac{1}{2} S^{2}; S = S_{o} - S_{c}; \dot{q} = \phi(q_{i},u,i) \) - the CO equation of motion; \( q_{i} \) - generalized coordinates; \( u \) - the desired control.

In the Table 2, the equation of a dynamic system control qualities distribution connects in an analytical form, on the basis of the phenomenological thermodynamics entropy concept, such a qualitative notions of control theory as - stability, controllability and robustness. As a result, the necessary distribution between the stability, controllability and robustness levels allows achieving the control goals in emergency situations with a minimum of useful resource consumption by using as a GA’s fitness function the criterion of minimum generalized entropy production. The thermodynamic definition of the S and information H entropies are interrelated by the von Neumann relation in the form: \( S = kH = -k \sum p_{i} \ln p_{i} \), where \( k \approx 1.38 \times 10^{-23} \) J/K is the Boltzmann constant. As a result (after substitution of this ratio), obtained an equation that also relates stability, controllability and robustness, but on the basis of the Shannon’s information entropy, which allows to determine the control to guaranteed achievement of the control goal in emergency situations with the requirement of a minimum information quantity about the external environment and CO states.

As mentioned above, to assess the quality of control, a new physical principle: the minimum entropy production rate in the object’s movement and in the control system [4-13]. The physical measure of entropy production rate is applied as a fitness function in the genetic algorithm (GA).

Table 2. Thermodynamic ratio of a robust intelligent control quality distribution

Therefore, these ratios are forming an equations system of control defining which guarantees the control goal achievement in emergency situations with a minimum useful resource consumption and the minimum required initial information [8-13].

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control quality rates. Assume that the control object is described in general form by the equation \( \dot{q}_i = \phi \left( q_i, t, S(t), u \right) \), where the generalized coordinate \( q_i \) describes the control object movement, \( u \) is the desired control and \( S(t) = S_o(t) - S_c(t) \) is the generalized entropy of the system represented as the difference between the control object’s entropy production \( S_o(t) \) and the controller’s entropy production \( S_c(t) \). Consider the following equation:

\[
\frac{dV}{dt} = \sum_{i=1}^{n} q_i \cdot \phi \left( q_i, t, S(t), u \right) + (S_o - S_c) \cdot (\dot{S}_o - \dot{S}_c) \leq 0
\]

(4.3)

Equation (4.3) in analytical form interrelates such qualitative notations of control theory as stability \( V \) (Lyapunov function), controllability and robustness basing on the concept of phenomenological thermodynamics entropy [8,9,19-22].

This approach allows, as noted earlier, to find the necessary distribution between the levels of stability, controllability and robustness, which permitting to achieve the control goal in emergency situations with a minimum useful resource consumption by using the minimum generalized entropy production as a fitness function in the genetic algorithm, which is included in the right part Eq. (4.3).

Consider now the (4.3) regarding to the interrelation of thermodynamic entropy to Shannon information entropy. The thermodynamic definition of \( S \) and \( H \) entropies are interrelated by the von Neumann relation in the:

\[
S = kH = -k \sum_i p_i \ln p_i
\]

Boltzmann constant. Substitution the Shannon information entropy \( -H \) instead of \( \dot{S}(t) \) in equation (4.3) give as a result:

\[
\frac{dV}{dt} = \sum_{i=1}^{n} q_i \cdot \phi \left( q_i, t, k(H_o - H_c), u \right) + k(H_o - H_c) \cdot (\dot{H}_o - \dot{H}_c) \leq 0
\]

(4.4)

Thus, equation (4.4) also interrelates stability, controllability and robustness, but already on the Shannon’s information entropy basis, which also allows defining controlling approaches for guaranteed achievement of the control goal in emergency situations with the requirements of a minimum information about the external environment and the control object state. Consequently the (4.3) and (4.4) forming an equations system, which solution are determining the controlling approaches that guarantees the achievement of the control goal in emergency situations with a minimum useful resource consumption and the minimum initial information requirements.

6. The Cognitive Intelligent Control System Information-Thermodynamic Analysis

Result of equations (4.3) and (4.4) generalization is the following equations system:

\[
\frac{dV}{dt} = \sum_{i=1}^{n} q_i \cdot \phi \left( q_i, t, k(S_o - (S_{tc} + S_{cc})), u \right)
\]

\[
+ (S_o - (S_{tc} + S_{cc})) \cdot (\dot{S}_o - (\dot{S}_{tc} + \dot{S}_{cc})) \leq 0
\]

(5.1)

\[
\frac{dV}{dt} = \sum_{i=1}^{n} q_i \cdot \phi \left( q_i, t, k(H_o -(H_{tc} + H_{cc})), u \right)
\]

\[
+ (H_o - (H_{tc} + H_{cc})) \cdot (\dot{H}_o - (\dot{H}_{tc} + \dot{H}_{cc})) \leq 0
\]

(5.2)

where \( (S_{tc} + S_{cc}) \) and \( (H_{tc} + H_{cc}) \) means the total thermodynamic and information entropies of the Technical intelligent (Tc) and Cognitive (Cc) controllers, respectively.

From equation (5.1) follows that the robustness of the intelligent control system can be increased by the cognitive regulator entropy producing, which reduces the useful resource consumption, and equation (5.2) shows that the negentropy of the cognitive controller reduces the minimum requirements for the initial information to achieve robustness. Moreover, information based on knowledge in the cognitive controller’s KB allows to obtain an additional resource for effective capacity, which is equivalent to the appearance of a targeted action on the control object to ensure the achievement of the control goal.

One of the key tasks of modern robotics is the development of technologies for the cognitive interaction of robotic systems, which allow solving the tasks of intelligent hierarchical control by redistributing knowledge and control functions, for example, traditional “master - slave” system. Modern approaches to solving this issue are based on the theory of multi-agent systems, theory of a swarm.
artificial intelligence, and many others [8,9,23-25].

According to [24-26], in a multi-agent system there is a new synergetic information effect of knowledge bases self-organization and formation of an additional information resource arising from the information and knowledge exchanges between active agents (swarm synergetic information effect). Extraction of an additional resource in the form of quantum information, which hidden in classical states, is realized on the basis of quantum fuzzy inference, which in turn, is a new quantum search algorithm and a special case of a quantum self-organization algorithm.

Due to the synergistic effect, an additional information resource is created and the multi-agent system is able to solve complex dynamic tasks for the joint work implementation. The assigned task may not be performed by each element (agent) of the system individually in environments variety without external control, monitoring or coordination, but the exchange of knowledge and information allows to perform joint effective work to achieve the control goal in the conditions of initial information uncertainty and limitations on the useful resource consumption [14]. In particular, it is well known that for feedback control systems, the amount of recoverable useful work \( W \) satisfies the inequality \( W_{\text{max}}(t) = k T_{\text{min}} \int_0^t I \, dt \leq k T I_t \), where \( k \) is the Boltzmann constant, \( T_{\text{min}}(t) \) is interpreted as the lowest achievable by the system temperature in time \( t \) under feedback control, assuming \( T_{\text{min}}(0) = T \), and \( I \) determines the Shannon information quantity (entropy transfer) extracted by the system from the measurement process [23].

The synergetic effect physically means the self-organization of knowledge and creation of an additional information quantity that allows to the multi-agent system making the most useful work with a minimum useful resource consumption and with a minimum initial information requirement, nondestructive the lower executive control system level [22-25]. Together with the information-thermodynamic intelligent control law (optimal distribution of control qualities “stability -controllability - robustness”), an intelligent control system (ICS) for multi-agent systems is designed, which guarantees the achievement of the control goal in the conditions of initial information uncertainty and limited useful resource [14,19,25].

Let’s consider these statements in more detail on the basis of the interrelationship analysis between the information quantity and extracted on its basis useful work and free energy.

As noted above, if microscopic degrees of freedom are available to the Maxwell demon observer form, then the second thermodynamics law can be violated. Szilárd showed from the Maxwell demon model analysis that the work in the form \( k T \ln 2 \) is extracted from the thermodynamic cycle. Moreover, it was shown that the extracted work \( W_{\text{ext}} \) from the system is determined by the information quantity (or quantum-classical mutual information) \( I \) that determine the knowledge about the system during measurement. At the same time, a similar ratio in the form of a lower bound exists for the total measurement cost \( W_{\text{ext}}^s \) and information erasing \( W_{\text{ext}}^s \leq -\Delta S + k T I \) and \( W_{\text{ext}}^s \geq k T I \), where \( \Delta S \) determines the free energy of the system. Then it is easy to notice that the speed of the extracted work \( W_{\text{ext}} \) is limited by the value \( W_{\text{ext}} \leq k T I \), i.e. it is limited by the speed of the extracted information.

The proposed quantum algorithm model of ICS self-organization, based on the minimum information entropy principles (in the “intelligent” state of control signals) and on a generalized thermodynamic measure of entropy production (in the system “control object + controller”). The main result of the self-organization process application is the acquisition of the robustness necessary level and the reproducible structure flexibility (adaptability). It is noted that the robustness property (by its physical nature) is an integral part of self-organization, and the required level of ICS robustness is achieved by fulfilling the minimum generalization entropy production principle, described above. The minimum entropy production principle in the CO and control system [14] acting as the physical principle of optimal functioning with minimal useful work consumption and underlies the development of robust ICS. This statement is based on the fact that, for the general case of dynamic objects controlling, the optimal solution to the finite variational problem of the maximum useful work \( W \) determining is equivalent, according to [14], to the solution of the finite variational problem of finding the minimum entropy production \( S \). Thus, the study of the conditions of the maximum functionality \( \max_{q,u} W ) \) (where \( q,u \) a CO’s generalized coordinates and the control signal respectively) are equivalent, accordingly to [25], to study the associated problem of the minimum entropy production, i.e. \( \min_{q,u} S \).

Therefore, in the developed quantum algorithm model, the applied principle of minimum informational entropy guarantees the necessary condition for self-organization - the minimum of the required initial information in the learning signals; the thermodynamic criterion of a new measure the generalized entropy production minimum provides a sufficient condition for self-organization - the control processes robustness with a minimum consumption of a useful resource.

More significant is the fact that the averaged val-
ue of the produced work by the dissipation forces -
\[
\frac{W_{\text{diss}}}{kT} = S_{KL}(P_P P_B),
\]
i.e. the dissipation forces work is determined by the Kulbak-Leibler divergence for the probability distributions \(P_P\), \(P_B\). Note that the left part of this ratio represents the physically a thermal energy, and the right part determines a purely system’s information.

Information entropy is a measure of the information quantity about a system and the Kulbak-Leibler divergence, as well as the Fisher’s information quantity determination. The similar interrelation exists between the work produced by the dissipation forces and the Renyi divergence.

Thus, substituting into (5.1) and (5.2) represented relationship between information - the extracted free energy and the work obtaining the conclusion noted above - the intelligent control systems robustness may increase by the cognitive controller entropy production, which reduces the useful resource consumption of the control object, and same time, the negentropy of cognitive control reduces the minimum initial information requirements to achieve robustness. Therefore, the extracted information, based on knowledge in the cognitive controller’s KB allows obtaining an additional resource for useful work, which is equivalent to appearance of a targeted action on the control object to guarantee the achievement of the control goal.

**General mathematical simulation structure.** The structure of Fuzzy Simulation structure of an intelligent control system design represented in Figure 5, and decomposed into two main stages: Off-Line and On-Line as mentioned earlier.

In the Off-Line a controlled object mathematical model is creating and the thermodynamic equations of its states are founding to calculate the entropy which forming the GA’s fitness function and computer stochastic simulation with GA optimizes the P(I)D controller parameters.

In the next step GA randomly selects an optimized PD’s controller parameters in the all possible solutions domain, using the minimum entropy production in the intelligent control system and in the complex nonlinear model dynamic behavior as a criterion for the solution suitability (fitness function). The fitness function of the GA is represented as:

\[
\text{Eval} = \min((S^\alpha - S^\beta)(\frac{dS^\alpha}{dt} - \frac{dS^\beta}{dt}))
\]

where \(\frac{dS^C}{dt}\) - is the control system’s entropy production rate; \(\frac{dS^O}{dt}\) - is entropy production rate in the motion of the robotic unicycle (Object) with following condition:

\[
\frac{dS^C}{dt} > \frac{dS^O}{dt}.
\]

Description of entropy production rate calculation is presented in \([8,9]\).

**Thermodynamic equation of motion.** The equations for calculation of the entropy production rate for intelligent control system and dynamic motion of robotic Unicycle are derived from approach as described in \([3]\). These equations are described in the following form:

\[
\begin{bmatrix}
\frac{dS^P}{dt} \\
\frac{dS^C}{dt}
\end{bmatrix} =
\begin{bmatrix}
M_{ij}(q) & 0 \\
0 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
B_{ij}(q,\dot{q}) \cdot \dot{q}^T(t) + D^T(\dot{q})
\end{bmatrix}
\cdot \dot{q}(t)
\]

(5.4)

where: \(i,j=1..9; M_{ij}(q)\) is a 9×9 diagonal matrix of inertial acceleration’s terms \(M(q)\) derived from Lagrange equations; \(B_{ij}(q,\dot{q})\) is a 9×9 diagonal matrix of Coriolis and centrifugal terms, derived from Lagrange equations; \(D(\dot{q})\) is a 9-dimensional vector of viscous friction forces terms \(D_i(\dot{q}) = [D_1(q), D_2(q), D_3(q), D_4(q), D_5(q), D_6(q), D_7(q), D_8(q), D_9(q)]\); \(\tau_T\) is a 9-dimensional vector of torque’s dissipative parts \(k_d \cdot [\tau_a \gamma] \cdot \tau_a = [0, \tau_1, \tau_2, 0, 0, 0, 0, 0, 0] \).

Following the Figure 5, the next step is the training of the control system based on the optimized controller parameters obtained from the GA and obtaining lookup tables (FC Knowledge Base) using the FNN.

In On-Line mode basing on the obtained lookup tables the P(I)D controller parameters of the robotic unicycle are changes by a fuzzy controller in real time.

7. **Mathematical Simulation and Experimental Results**

Soft Computing Simulation structure in MathLab Simulink® system is shown in Scheme 1. It consists from following parts:

(1) Block of main equations;
(2) Block of random excitation;
(3) Blocks of equation’s coefficients;
(4) Blocks of Lagrangian multipliers calculation;
(5) Block of Intelligent control system based on Soft Computing - GA or FNN.

In all simulation cases, the real parameters of the robotic unicycle model were used see the Figure 6, and the corresponding stochastic effects: disturbing from the floor to the yaw rotation angle and jamming in the closed-links.
mechanism. (see Figure 7).

The method of algebraic loops expulsion, described in detail in the patent [19], was applied to accelerate the simulation processes. This method allowed to accelerate the computer simulation process - integration is about 190 times down with the difference in integration result less than 1%, as shown in Figure 8 and Figure 9.

**Simulation results discussion.** In Figure 4 shown the comparison of three types of control approaches:

1. Conventional PD controller with fixed gain coefficients - temporal mechanical and thermodynamic controlled system behavior;
2. The GA with fitness function as minimum of entropy production rate for conventional PD controller.
3. The Fuzzy PD controller with lookup tables obtained after learning process by FNN with pattern from GA.

Such structure is the most applicable because of its flexibility, it has an opportunity to change only necessary separated blocks, such as control, main equations, excitation etc., without changing the whole structure.

From presented results it is visible, that: (1) usage of the approach described above, with application of a minimum entropy production rate as fitness function in GA and learning process by FNN, is completely justified; (2) dynamic motion occurs more smoothly even the control signal’s discretization time is use in PD-GA and Fuzzy PD controllers with sampling time = 0.05 sec.

![Figure 6. Robotic Unicycle model](image)

![Figure 7. Simulated stochastic excitations - from a floor roughness’s and jamming in closed-links mechanisms](image)

![Figure 8. The accelerated and standard model’s an integrating accuracy example (with & without algebraic loops)](image)
Entropy production rate for the Pitch angle after the learning by FNN is decreased to 10 times. For the Yaw and Roll angles Entropy production rate is 10 times less for PD-GA and 1000 times less for the Fuzzy PD than for conventional PD controller.

However, such energy transmitting increases amplitude of Pitch angle in case of PD-GA controller that conducts to increase torque in Links control system. But, after learning by FNN the motion in the pitch direction becomes smooth with small amplitude. It confirms about learnability and intellectualization of the Robotic Unicycle control system.

As can be seen from Figure 11 a, the movement of the model is smoother, which leads to a saving of the resource of the system as a whole. Also shown are the changes in the gain $k_p$, $k_d$, $k_p$, $k_d$ of the control equations (4.1) and (4.2). From the Figure 11a its visible that the model movement is going...
smoother, which leads to the system resource saving at whole. There are also shown the gain coefficients $k_p, k_d, k_p, k_d$ changes in the control equation (4.1) and (4.2).
Figure 10. Stochastic simulation results of Robotic Unicycle Model: Temporal mechanical and thermodynamic behavior of Robotic Unicycle with: (a) conventional PD controller; (b) PD-GA controllers; (c) Fuzzy PD controllers; Temporal mechanical and thermodynamic behavior; (d) conventional PD control system torques; (e) PD-GA control system torques; (f) Fuzzy PD control system torques.
The introduction of the GA fitness function in the quadratic form of a generalized function excludes the controls leading to local instability (negative value of the generalized entropy). As a consequence, it gives improved characteristics of control quality (minimum complexity in the implementation of the gain coefficients change laws time, minimum driving mechanisms efforts and useful resource consumption). In this case, the information-thermodynamic law of the compromise distribution of the conflicting control qualities (stability, controllability and robustness) is fully satisfied.

**Experimental Results Discussion.** Created in 1997-2000, the robotic unicycle is shown in Figure 6. The experimental results are presented in Figure 12-13. The time of the full-scale experiment was limited to 8 seconds due to the adverse effect of the gyroscopic sensors drift signal.

Though, it should be noted that sampling (more than 0.001 sec) of control signal from conventional PD, as it is present in real model, offers the Unicycle simulation system to “falling” after 8-10 sec.

In Figure 12 shows the experimental results for the cableless unicycle model. As it shown, the robot’s lateral stability - in the roll direction $\gamma$, and posture in the pitch direction $\beta$ is obtained.

In Figure 13 shown the temporal behavior of the fuzzy gains $k_p1$, $k_d2$, $k_p2$, $k_d2$ for 2 PD controllers (Eq. 4.1, 4.2). From the result in Figure 11c absorbed that the robot’s posture in the yaw direction $\alpha$ is changed rapidly during the experiment, which indicates a satisfactory redistribution of control energy that provides lateral stability of

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**Figure 11.** Stochastic simulation results of Robotic Unicycle Model with different GA’s fitness functions - Eval 1, Eval2

**Figure 12.** Experimental results, angles - pitch $\beta$, roll $\gamma$ and yaw $\alpha$ angles

**Figure 13.** Experimental results of the fuzzy gains temporal behavior $k_p1$, $k_d2$, $k_p2$, $k_d2$ for 2 PD controllers
the robot (roll $\gamma$) and tracking stochastic excitations on the model (floor roughness’s and jamming in closed-links mechanisms) by controlling the yaw angle $\alpha$ and pitch angle $b$.

Remark 1. The obtained experimental results were achieved using empirically generated fuzzification and defuzzification functions for the gain coefficients $kp1$, $kd2$, $kp2$, $kd2$ of two fuzzy PD controllers, which were generated on the basis of preliminary results of GA simulation with the fitness function - reduction only the entropy production rate of the control system, i.e. incomplete simulation process of soft computing technology. The simulation results presented above were obtained later, upon completion of the of a robotic unicycle mathematical model development (Figure 2, Eq. (1.2)), formation the soft computing process technology (Figure 3), and most importantly, the occurrence of this approach calculating possibility - appropriate computational capabilities, without which this process was extremely difficult.

Remark 2. Despite of this, the obtained result at this time and with those computational capabilities, leads to confirmation of quite satisfactory operation of the represented structure of the intelligent control system. The represented structure of the process, as well as the new developments in this direction, is planned to be fully applied in the new prototype - an Autonomous Flexible Robotic Unicycle.

8. Conclusions

(1) In this work represents the basic idea of intelligent control of dynamical, globally unstable, nonlinear objects on the robotic unicycle example. The basis of this approach is a qualitative physical analysis of the robot dynamic movement with the introduction of intelligent feedback in the control system and the implementation of instinct and intuition mechanisms based on the FNN and GA.

(2) The main components of an intelligent control system based on soft computing and robustness determination are also presented. Thus, there is an adaptation of the two fuzzy PD controllers’ parameters to achieve a stable motion of the robotic unicycle over a long(finite) time interval, without changing the structure of the control system executive level, is achieved.

(3) The introduction of these two new mechanisms to an intelligent control system is based on the principle of minimum entropy production in the robot unicycle’s motion and the control system itself. The fuzzy stochastic simulation of thermodynamic equations of motion and the intelligent control system confirm the effectiveness of the robot’s postural stability control to handle the system’s nonlinearity.

(4) In this case the unicycle robot model is a new benchmark for intelligent fuzzy controlled motion of a nonlinear dynamic system with two (local and global) 3D unstable states.

(5) The use of a fuzzy gain schedule PD controller with look-up tables calculated by FNN, offers the ability to use instinct and intuition mechanisms in on line to intellectualize the intelligent control system levels.

(6) Quantum soft computational intelligence toolkit [23–26] applied to design of self-organized conventional PD controllers can increase the robustness of robotic unicycle.

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