Soft Pomeron in QCD

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This talk is a brief presentation of our view on the Pomeron, as a non-perturbative QCD phenomenon but from sufficiently short distances. Our approach is based on the scale anomaly of QCD and emphasizes the role of semi-classical QCD vacuum fields. We show that both the intercept and the slope of Pomeron trajectory appear to be determined by the energy density of non-perturbative QCD vacuum. The particular example of semi-classical QCD vacuum fields is discussed based on a new type of instanton-induced interactions ("instanton ladder") that leads to the rising with energy cross section $\sigma \sim s^{\Delta P}$ of Regge type (the Pomeron).

I. MAIN IDEA

It is well known that the Pomeron structure is one of the most challenging problem of QCD. We need soft Pomeron to describe the experimental data on soft processes [1] but we have no idea why a Regge pole with the intercept close to unity, which we call soft Pomeron, could appear in our microscopic theory-QCD. Our main idea [2] is that soft Pomeron $\rightarrow$ non-perturbative QCD but at sufficiently short distances.

Indeed, pQCD calculations [3] lead to so called BFKL Pomeron which is not a Regge pole. On the other hand, perturbative approach for 1+2 dimensional QCD [4] gives a reggeon with the intercept larger than unity if we introduce a scale in our theory ( gluon mass ). For massless 1+2 QCD we still do not have a Pomeron. The lesson which we learned from this approach is that we have to find the origin of a violation of the scale invariance in QCD which leads to an appearance of a typical scale in massless QCD. This momentum scale should be large enough since our phenomenological Pomeron has sufficiently large typical momentum. Indeed, (i) the slope of the Pomeron trajectory $\alpha_P(t) = 1 + \Delta_P + \alpha_P'(0) t$ is equal to $\alpha_P'(0) = 0.25 GeV^{-2} \ll \alpha_R'(0) = 1 GeV^{-2}$, where $\alpha_R'$ is the slope of secondary trajectories; and (ii) the $t$ - slope of the triple Pomeron vertex $G_{3P}(t)$ is very small.

Therefore, we have three principle questions to answer:(i) why we have a Pomeron in QCD; (ii) why the Pomeron intercept is so small ($\Delta_P \approx 0.08 \div 0.1$ ) for non-perturbative QCD and (iii) why a typical momentum scale is so high ($M_0 \propto 1/\alpha_P' \approx 2 GeV$ ) for “soft” Pomeron.

II. HIGH MOMENTUM SCALE FOR POMERON

The first question which we would like to answer is does sufficiently large mass scale $M_0 \approx 2 GeV$ appear in non-perturbative QCD?. Our answer is yes.
As have been mentioned, the key ingredient of our approach is the breakdown of the scale invariance in QCD, reflected in scale anomaly \cite{5}. QCD is scale invariant on the classical tree level in the chiral limit of the massless quarks. However, this invariance is broken due to scale dependence of the QCD coupling constant, which introduces a dimensionful scale (\Lambda). On the formal level, the breakdown of scale invariance in the theory is reflected by non-conservation of scale current, and thus in the non-zero trace of the the energy-momentum tensor \(\Theta^\mu_{\mu}\) \cite{6}. Scale anomaly leads to a set of powerful low energy theorems \cite{6}, from which we are going to use

\[ i \int dx \langle T \{ \Theta(x) \Theta(0) \} \rangle = \int \frac{dM^2}{M^2} \left[ \rho^{\text{phys}}_\Theta(M^2) - \rho^{\text{gt}}_\Theta(M^2) \right] = -4 \langle 0 | \Theta | 0 \rangle = -16 \epsilon_{\text{vac}} \neq 0 \] \hspace{1cm} (1)

Let us make a simple estimate counting the number of coupling constant in Eq. (1). On the l.h.s. we have each term of the order of \(\alpha^2\) while on the r.h.s. \(\epsilon_{\text{vac}} \propto \alpha_S\). Therefore, Eq. (1) holds only because the range of integration over \(M^2\) is of the order of \(1/\alpha_S\) or, in other words, typical \(M^2 \propto 1/\alpha_S\). This is a reason for large momentum scale in QCD. The real estimates for the value of \(M^2_0\) has been performed using the chiral limit of QCD in which 

\[ \Theta^\mu_{\mu} = -\partial_\mu \pi^a \partial^\mu \pi^a + 2m^2\pi^a \pi^a + \cdots \quad \text{and} \quad \langle \pi^+ \pi^- | \Theta^\mu_{\mu} | 0 \rangle = M^2 = (\alpha_S)^0. \] \hspace{1cm} (2)

Substituting Eq. (2) in Eq. (1) one can get the estimates for the value of typical mass, namely

\[ M^2_0 \simeq 32\pi \left\{ \frac{\left| \epsilon_{\text{vac}} \right|}{N_f - 1} \right\}^{1/2} = 4 \div 6 \text{ GeV}^2 \approx 1/\alpha'_p \ll \Lambda, \] \hspace{1cm} (3)

for \(\epsilon_{\text{vac}} \approx -(0.24 \text{ GeV})^4\). The physical way of understanding the disappearance of the coupling constant in \(\Theta^\mu_{\mu} \propto g^2 F^2\) is to assume that the non-perturbative QCD vacuum is dominated by the semi-classical fluctuations of the gluonic fields with \(F \propto 1/g\).

**III. SCALE ANOMALY OF QCD \(\rightarrow\) SOFT POMERON**

Armed with this knowledge on QCD vacuum we can formulate our strategy for searching of non-perturbative Pomeron contribution.
It consists of two steps: (i) first is to find the $\Theta^2_2$ contribution to the next-to-leading order BFKL kernel which is formally $\sim O(\alpha_s^2)$; (ii) second to replace this contribution by the non-perturbative one which is of the order of $\alpha_s^0$ due to scale anomaly. These steps are shown in Fig.1.

In doing so, we obtained that the total cross section is proportional to $s^{\Delta P}$ with

$$\Delta P = \frac{\pi^2}{2} \left( \frac{8\pi}{b} \right)^2 \frac{18}{32\pi^2} \int \frac{dM^2}{M^6} \left( \rho^{\text{phys}}_\theta(M^2) - \rho^{\text{QCD}}_\theta(M^2) \right) = \frac{1}{48} \ln \frac{M_0^2}{4m^2_e};$$

where we use Eq. (3) for the non-perturbative contribution to $\Theta^2$. Numerically, Eq. (4) gives $\Delta P = 0.08 \div 0.1$ for $M_0^2 = 4 \div 6 \text{GeV}^2$ in a good agreement with the experimental data [1].

Therefore, we obtained a soft Pomeron that we needed. All corrections to our approach are small since they are proportional to $\alpha_S(M_0^2) \ll 1$. However, we would like to stress that we cannot control all other contributions which stem from long distances $r \gg 1/M_0$. We believe that they are irrelevant to the Pomeron structure.

The physical interpretation is very simple: $t$-channel gluon scatters off semi-classical vacuum gluon fields (see Fig.2-a). This scattering leads to an excitation of the quark zero modes $\rightarrow$ pion production. In general, This picture is very close to one that J. Bjorken has guessed [10].

### IV. QCD INSTANTONS AND THE SOFT POMERON

The picture, that has been discussed, suggests two key observation: (i) a soft Pomeron is a coherent state of strong semi-classical gluon field $F_{\mu\nu} \propto 1/g(M_0) \gg 1$; and (ii) the typical momentum scale for these fields is rather large $M_0 \propto 1/g(M_0)$. A natural candidate for such fields is an instanton contribution. It is well known that instantons are the only classical solutions of QCD which describe the transition between different QCD vacua [11]. We found instructive to view a Pomeron [12,13], as a process shown in Fig.2-b, in which $t$-channel gluon propagates through space inducing a chain of instanton transition between different vacua and producing more gluons in each transition. Practically, it means that we calculate a ladder diagram of Fig.3-a with the vertices defined in Fig.3-b. Our main assumption is that interactions between instantons result in diluted gas of instantons with a definite size $\rho = \rho_0 \approx 0.3 \text{ fm}$. We see a strong support of this hypothesis in lattice calculations (see Ref. [14] and references therein). We can reduce the problem of high energy asymptotic to the ladder diagram of Fig.3-a using the following parameters:

$$\alpha_S(\rho_0) \ll 1 \quad e^{-\frac{2\pi^2}{\alpha_S(\rho_0)}} \ll 1;$$

$$e^{-\frac{2\pi^2}{\alpha_S(\rho_0)}} \ln s \geq 1 \quad \frac{\rho_0 M_0}{\alpha_S} \geq 1.$$

The ladder diagram sums $(e^{-\frac{2\pi^2}{\alpha_S(\rho_0)}} \ln s)^n \left( \frac{\rho_0 M_0}{\alpha_S} \right)^m$ contributions.
Our hope is that $\Delta P \propto \rho_0^4 \epsilon_{vac} \approx 0.016$ (see Fig.4).

Actually, the value of intercept $\Delta P \propto \int_{M_0}^{M_2} dM^2 \sigma(G + G \rightarrow I \rightarrow nG)$. We believe that $M_0 \approx E_{sph}$ and $\sigma(G + G \rightarrow I \rightarrow nG) \propto \exp \left( \frac{1}{t_{sph}} F(M_{E_{sph}}) \right)$. The form of the “holy grail” function $F(M_{E_{sph}})$ is not known. In our calculation we just cut integral at $M_0 \approx E_{sph}$. The result strongly depends on the value of cutoff. However, our main idea is to find a relation between the value of the Pomeron intercept and the vacuum energy (see Fig.4-a and Fig.4-b). We have not found such a relation yet and we just calculate the intercept.
using \( M_0 = 2.4 \text{GeV} \). Fig.5-a gives an answer to the question why the non-perturbative intercept is so small. One can see that the value of the intercept has a maximum as function of \( \alpha_S \). As it has been mentioned we cannot guarantee the value of \( \Delta_P \) but our approach provides an explanation how this value could be small even for large \( \alpha_S \). Fig.5-b shows the calculated Pomeron trajectory which turns out to be nonlinear. It is interesting to notice that the slope of the Pomeron trajectory \( \alpha'_P = \Delta_P \times 2.0 \text{GeV}^{-2} \) in a good agreement with the experimental data.

\[ \Delta_P \]

\[ \alpha'_P = \Delta_{\text{soft}} \times 2 \text{GeV}^{-2} \]

V. RESUME

Concluding this talk I would like to repeat our point of view: the soft Pomeron has a close relation to the structure of QCD vacuum with typical semi-classical strong fields \( A_\mu \propto 1/g \) and typical large momentum scale \( M_0 = E_{\text{sph}} \propto 1/\rho_0 \alpha_S \). We demonstrate that such strong field give rise to Reggeon with reasonable intercept. We believe that all other contributions are still small at scale \( M_0 \) since \( \alpha_S(M_0) \ll 1 \).

The difference of our approach from others on the market \[1\] is that there is no clear relation to property of QCD vacua in them, while we have such a relation in our approach.

We want to stress that our approach as well as other ones is based on the assumption that long distance physics which provides confinement of quark and gluons cannot lead to a Pomeron contribution. The reason for this assumption is the only one: our phenomenological Pomeron has a sufficiently large momentum scale.

We hope that our approach will trigger a return to theoretical discussion of the Pomeron structure with direct addressing to an achieved knowledge of non-perturbative QCD.

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