Low-energy Electrons in Gamma-Ray Burst Afterglow Models

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Abstract

Observations of gamma-ray burst (GRB) afterglows have long provided the most detailed information about the origin of this spectacular phenomenon. The model that is most commonly used to extract physical properties of the event from the observations is the relativistic fireball model, where ejected material moving at relativistic speeds creates a shock wave when it interacts with the surrounding medium. Electrons are accelerated in the shock wave, generating the observed synchrotron emission through interactions with the magnetic field in the downstream medium. It is usually assumed that the accelerated electrons follow a simple power-law distribution in energy between specific energy boundaries, and that no electron exists outside these boundaries. This Letter explores the consequences of adding a low-energy power-law segment to the electron distribution with energy that contributes insignificantly to the total energy budget of the distribution. The low-energy electrons have a significant impact on the radio emission, providing synchrotron absorption and emission at these long wavelengths. Shorter wavelengths are affected through the normalization of the distribution. The new model is used to analyze the light curves of GRB 990510, and the resulting parameters are compared to a model without the extra electrons. The quality of the fit and the best-fit parameters are significantly affected by the additional model component. The new component is in one case found to strongly affect the X-ray light curves, showing how changes to the model at radio frequencies can affect light curves at other frequencies through changes in best-fit model parameters.

Key words: gamma-ray burst; general – methods: data analysis – radiation mechanisms: non-thermal

1. Introduction

Gamma-ray bursts (GRBs) are the most powerful explosions in the universe and can therefore be observed to very high redshift. They have been hypothesized to be tracers of star formation and thus to be a probe of the star formation history of the early universe (Chary et al. 2016). Using the bursts as effective tools in cosmological studies requires a solid understanding of the physics that drives the explosions and the observable consequences of the GRB events (Wang et al. 2015). Our understanding of GRBs comes mostly through observations of their afterglow emission at wavelengths ranging from radio to X-rays (e.g., Piran 2004; Gehrels et al. 2009). These observations are best interpreted with a model where the emission arises from shocks in a relativistically expanding jet, internal shocks for the prompt high-energy GRB emission, and external shocks for the afterglow. Electrons are accelerated to high energies in these shocks, giving rise to synchrotron emission as they interact with the magnetic field in the downstream medium.

The theory of particle acceleration in relativistic shocks is far from complete, but it is generally acknowledged that a population of high-energy particles form with distribution that can be approximated as a power law in momentum (Pelletier et al. 2017). These high-energy particles contain most of the energy of the distribution and are responsible for the non-thermal emission arising in the model. There are, however, a considerable number of lower energy electrons that can contribute to the long-wavelength emission and increase absorption. Ressler & Laskar (2017) studied the effect of adding a thermal population of electrons to the power-law distribution and found that it can provide significant effects even in the optical frequency range, depending on the chosen parameters. Their analysis showed calculated light curves for several models but they did not compare it to any observations to test the validity of the model.

This Letter also focuses on the effects of low-energy electrons by extending the electron distribution to low energies using a power-law segment. A power-law distribution is chosen over a thermal distribution for simplicity to demonstrate the effect on parameter determination from fitting afterglow observations. A thermal distribution requires at least two parameters and special care to get a continuous distribution, while the extra power-law segment requires only a single parameter and the distribution is automatically continuous. The extension is added to the GRB afterglow model of Jóhannesson et al. (2006) that has been used to analyze several GRB afterglows (de Ugarte Postigo et al. 2005, 2007; Resmi et al. 2012; Sánchez-Ramírez et al. 2017). To test the effect of the additional electrons, the model is used to analyze the afterglow observations of GRB 990510, which has been well studied (e.g., Panaitescu & Kumar 2001, 2002; Jóhannesson et al. 2006). The results show a statistical preference for models with additional low-energy electrons, indicating the need for further exploration of the electron distribution in GRB afterglows. This Letter is organized as follows: in Section 2 the model and the new extension is described and its effects explored, in Section 3 the results of the analysis of GRB 990510 are presented, and the Letter concludes with a discussion in Section 4.

2. Model

The model in Jóhannesson et al. (2006) assumes that the afterglow emission arises from a relativistic shock wave traveling through the central engine’s surrounding medium. The shock wave is formed as a relativistic slab of matter with energy $E_0 = \Gamma_0 M_0 c^2$ is released into a cone with a half opening
The shock is assumed to accelerate electrons to relativistic speeds and a strong magnetic field is generated within the downstream medium, resulting in synchrotron emission. The dynamics of the system are determined from energy and momentum conservation, assuming that the downstream medium is a thin uniform shell that expands sideways at the local speed of sound and sweeps up everything in its way (Rhoads 1999). The density of the surrounding medium can have an arbitrary radial dependence, but in this Letter only two forms are considered: a constant density medium, \( \rho(r) = m_p n_0 \), and a wind-like medium, \( \rho(r) = C_w A_w r^{-2} \). Here \( r \) is the distance from the central engine, \( m_p \) is the mass of the proton, \( n_0 \) is the number density of the external medium, \( A_w \) is a normalization parameter, and \( C_w = 5.015 \cdot 10^{11} \text{g cm}^{-1} \) for a typical Wolf–Rayet star (Dai & Lu 1998; Chevalier & Li 1999).

The electron energy distribution in this model is based on the one described in Panaitescu & Kumar (2001), but extended to low energies with an additional power-law section. The electron distribution is

\[
\frac{dn}{d\gamma} = n_{e0}^{\alpha} \begin{cases} \frac{\gamma}{\gamma_1}^{-p_1}, & \gamma_0 < \gamma < \gamma_1, \\ \frac{\gamma}{\gamma_2}^{-p_2}, & \gamma_1 < \gamma < \gamma_2, \\ \frac{\gamma}{\gamma_M}^{-p_3}, & \gamma_2 < \gamma < \gamma_M, \end{cases}
\]

(1)

where \( \gamma_1 = \min \{ \gamma_0, \gamma_c \}, \gamma_2 = \max \{ \gamma_0, \gamma_c \}, p_c = p \) in the slow cooling phase where \( \gamma < \gamma_c \), and \( p_c = 2 \) in the fast cooling phase with \( \gamma_c < \gamma \). The lower limit of the distribution is fixed at \( \gamma_0 = 2 \) because our formalism for the emitting radiation is only valid for high-energy electrons. Here, \( \gamma_i \) is the injection break Lorentz factor, which is defined assuming the electrons at the injection break contain a fraction \( \epsilon_i \) of the total kinetic energy of the downstream medium

\[
\gamma_i = \epsilon_i \frac{m_p c}{m_e} (\Gamma - 1) + 1,
\]

(2)

where \( m_e \) is the mass of the electron. The cooling Lorentz factor is

\[
\gamma_c = \frac{4\pi m_e c}{\sigma_T B'^2 t'},
\]

(3)

which is found by equating the synchrotron energy loss at time \( t' \) with the energy of the electrons (Kardashev 1962). Primed quantities are evaluated in the co-moving rest-frame of the shock wave. Here \( \sigma_T \) is the Thompson’s cross section and \( B' = \sqrt{32\pi \epsilon_B \Gamma (\Gamma - 1) \rho c^2} \) is the magnetic field strength.

The energy of the distribution should be dominated by electrons with Lorentz factors above or around \( \gamma_1 \) so \( p_1 < 2 \). For large negative values of \( p_1 \), the distribution behaves effectively as a distribution without the low-energy extension. The normalization factor \( n_{e0} \) is determined from particle conservation. The maximum Lorentz factor \( \gamma_M \) is determined such that the acceleration timescale does not exceed the radiative loss timescale (Dai & Lu 1998), and the total energy of the electron distribution does not exceed a fraction \( \epsilon_c < 1 \) of the kinetic energy of the downstream medium. Depending on the exact values of the parameters, the latter condition can result in a sharp break in the emitted spectrum above the synchrotron frequency associated with \( \gamma_M \). This break can even extend down to the optical range or lower at late times for certain parameter values.

The synchrotron radiation is calculated using the standard assumption that the pitch angle between the electrons and the magnetic field is isotropic. The radiation is calculated numerically in the model by integrating the synchrotron power per electron over the electron distribution. Using standard assumptions (e.g., Sari et al. 1998) one can easily derive an approximate power-law behavior for the resulting co-moving frame radiation power

\[
P'_\nu(\nu') \propto \begin{cases} \nu'^{1/3}, & \text{if } \nu' < \nu'_i, \\ \nu'^{-(\gamma_i - 1)/2}, & \text{if } \nu'_i \leq \nu' < \nu'_j, \\ \nu'^{-(\gamma_c - 1)/2}, & \text{if } \nu'_j \leq \nu' < \nu'_k, \\ \nu'^{-p_2/2}, & \text{if } \nu'_k \leq \nu' < \nu'_M. \end{cases}
\]

(4)

Here \( \nu'_i \) is the synchrotron frequency corresponding to the Lorentz factor \( \gamma_i \). It is assumed that \( p_1 > 1/3 \). The additional low-energy electrons therefore contribute to the emitted spectrum for \( \nu' < \nu'_i \). The contribution to the synchrotron absorption coefficient can also be evaluated and if \( p_1 > -2/3 \), then

\[
\alpha'_\nu(\nu') \propto \begin{cases} \nu'^{-5/3}, & \text{if } \nu' < \nu'_i, \\ \nu'^{-(\gamma_i + 4)/2}, & \text{if } \nu'_i \leq \nu' < \nu'_j, \\ \nu'^{-(\gamma_c + 4)/2}, & \text{if } \nu'_j \leq \nu' < \nu'_k, \\ \nu'^{-(\gamma_c + 5)/2}, & \text{if } \nu'_k \leq \nu' < \nu'_M. \end{cases}
\]

(5)

The low-energy electrons contribute to the absorption of the synchrotron spectrum even in the case where their contribution to the emission is insignificant. In addition to the change in spectrum, the low-energy electrons affect the estimation of the normalization constant \( n_{e0} \), and therefore the normalization of the emission at all frequencies. Numerical calculations show that \( p_1 \lesssim -10 \) is required before the effects of the extra component can be completely neglected, although the effect is small up to \( p_1 \lesssim -2 \).

The effect of the low-energy extension is illustrated in Figure 1, which shows results of model calculations for several values of \( p_1 \). The other parameters are fixed at typical values for GRB afterglows: \( E_0 = 10^{51} \text{erg}, \Gamma_0 = 1000, \theta_0 = 3^\circ, n_0 = 1 \text{cm}^{-3}, p = 2.2, \epsilon_e = 0.3, \epsilon_i = 0.01, \) and \( \epsilon_B = 0.001 \). As expected from the analytical approximations in Equation (4), the effect is largest for the radio and mm light curves with frequencies that are below \( \nu'_i \) before 10 and 1 days, respectively. The radio light curve is particularly affected because of increased synchrotron self-absorption for \( p_1 > 0.33 \) that shows up in the fast rise of the early radio light curve. The effect on the near-IR to X-rays is in this case only through the normalization of the light curve. The increased number of electrons at low energies results in lower emission, even though the total energy of the electron distribution is dominated by electrons around \( \gamma_i \).

3. Application to GRB 990510

The afterglow of GRB 990510 has been referred to as the canonical afterglow light curve due to its smooth decline that
The data used in this analysis is from Harrison et al. 1999; Stanek et al. 1999; Holland et al. 2000; Kuulkers et al. 2000; Panaitescu & Kumar 2001; Jóhannesson et al. 2006. Its afterglow is well sampled with data at many wavelengths from radio to X-rays. The data used in this analysis is from Harrison et al. (1999; optical and radio), Stanek et al. (1999; optical), and Kuulkers et al. (2000; X-rays). The optical data is corrected for Milky Way dust extinction of $E(B-V) = 0.2$ (Schlegel et al. 1998).

The Bayesian method is used to test the effect of the additional low-energy electrons by comparing the Bayesian factor of models with and without the low-energy component. Models with both a constant density medium (CM, CMo) and a wind-like medium (WM, WMo) are used. The “o” in the model names stands for “without the low-energy electrons”. The Bayesian evidence is calculated using MultiNest, which also provides posterior distributions for the free parameters of the model (Feroz & Hobson 2008; Feroz et al. 2009, 2013). The likelihood is calculated assuming that the afterglow flux data is sampled from a log-normal distribution that is equivalent to the apparent magnitude being distributed normally. The prior distributions of the parameters are mostly non-informative uniform or log-uniform distributions bounded only by physical constraints of the model. $\theta_0$ is bound from above to be no larger than $90^\circ$, $\epsilon_e$ and $\epsilon_B$ are constrained to be less than 0.5, and $\epsilon_i$ is then constrained to be less than $\epsilon_e$. The initial energy release, $E_0$, is constrained to be less than $10^{52}$ erg, which is about 10% of the energy expected to be released in the gravitational collapse of a massive star. The value of $p_i$ is also constrained to be less than 1, so both the energy and number of electrons in the distribution peaks at around $\gamma_i$. Other boundaries are set such that they do not affect the results.

The best-fit model is the WM model, where the logarithm of the Bayesian evidence is $\log(Z) = -266$. This is considerably higher than the CM model, which has $\log(Z) = -280$ giving a value of 10 for the log of the Bayes factor indicating strong evidence. The WM model gives a better fit to the R-band data, while the radio data is better fit with the CM model. Other bands are similar for the models, and the WM model thus provides a larger Bayesian evidence because the number of points in the R-band is much larger than that in radio. The WMo and CMo models result in significantly worse Bayesian evidence than the corresponding WM and CM models. The Bayesian factor between the CM and CMo models is 37 and for WM and WMo it is 189, providing very strong evidence for the addition of the low-energy electrons in this analysis.

The best-fit model curves are shown overlaid on the data in Figure 2 for all models. Even though the WM model is statistically better, the CM model looks better because it follows the trend of the radio points and the difference in the R-band is barely visible. It is also clear that the WMo model provides a poor fit to the X-ray data. The low-energy electrons thus significantly affect the quality of the fit at X-ray wavelengths through changes in the model parameters, even though they do not contribute to the emission at those wavelengths. It also demonstrates how important multi-wavelength data is for model selection.

The 1D marginal posterior distributions for the model parameters are shown in Figure 3 for the models considered in the analysis. Table 1 shows the parameters posterior mean values, the 68% confidence regions, and the maximum likelihood values. The WM model clearly stands out and the posterior distributions are often cut off abruptly by the prior range. The WM model requires large values for the initial energy release, $E_0$, which is only constrained by the selection of the prior. Increasing the size of the prior results in the best-fit model having even higher values of $E_0$, which are beyond reasonable estimates of the available energy from the central engine. The posterior distribution for $p_i$ is also at the boundary set by the prior. Because of these extreme parameter values, and the fact that the CM model better reproduces the radio data, the CM model is considered a better model of this event even though the WM model is statistically favored.

There is considerable difference between the posterior distributions of the models with and without the low-energy...
electrons, in particular for the WM and WMo models. The posterior distributions for the WM and CM models are broader compared to the WMo and CMo counterparts, and the means of the distributions are also shifted. The effect differs somewhat between the CM and WM models in detail, but the shift is in all cases in the same direction except for the external density. There is a decrease in the posterior mean for $A_*$ in the WM model compared to the WMo model, while $n_0$ is increased in the CM model compared to the CMo model. The 2D marginal posterior distributions (not shown) indicate that $p_1$ is in both models well constrained, and there is very little correlation between $p_1$ and the other model parameters.

4. Discussion and Summary

The best-fit model parameters presented here are in reasonable agreement with previous analyses. Panaitescu & Kumar (2001) used a CMo model very similar to the one used in this analysis. Their best-fit values are all within the 99% confidence intervals of our posterior distributions, apart from the value of $\theta_0$. Their value for $\theta_0$ is 26$^\circ$, which is considerably off the posterior distribution determined here. A follow-up study was performed in Panaitescu & Kumar (2002) where the parameters changed significantly. In particular, their values of $\theta_0$ and $n_0$ are larger, and both are outside the posterior distributions presented in Figure 3. Their value of $p$ is also considerably smaller but with large uncertainties, and it agrees with the posterior at the 2$\sigma$ level. Like the present analysis, Panaitescu & Kumar (2002) found that a constant density external medium better fits the data. This afterglow data was also analyzed by Jóhannesson et al. (2006) using an older version of the code used here and a different fitting technique. As expected, the parameter estimates all fall within the 68% confidence intervals of the current posterior distributions.

The results of the analysis of GRB 990510 and the statistical preference for the additional electron component lend support to the need for more detailed treatment of the electron distribution in GRB afterglow modeling. In particular, this can affect the determination of the energetics of the outflow and the density structure of the external medium. The power-law segment added in this Letter is just a simple modification of the electron energy distribution to explore its effects, and further work is needed to get a more accurate physical picture. One such method is to simultaneously solve for the dynamics of the afterglow and the distribution of electrons. This was done in the work of Geng et al. (2018), but their approach is limited to electron cooling and lacks the thermalization effect of the

![Figure 3. Marginalized posterior distributions for the parameters of the afterglow tuned to the data of GRB 990510 shown in Figure 2. The distributions are normalized to 1 at the peak. Shown are both the actual sampled distributions (as half-transparent colors) and the Gaussian kernel density estimate (as smooth curves). CM is green, CMo is blue, WM is black, and WMo is red. The best-fit values used for the model curves in Figure 2 are shown as symbols at the top of the panels. Note that the best fit values are not always near the peak of the distribution.](image-url)
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### Table 1

| Parameter                  | CMo               | WMo               | CM               | WM               |
|----------------------------|-------------------|-------------------|------------------|------------------|
| Bayesian Evidence          | $-317.3 \pm 0.4$  | $-454.81 \pm 0.03$| $-280.01 \pm 0.03$| $-265.6 \pm 0.2$ |
| Minimum $\chi^2$           | 549.8             | 814.7             | 470.1            | 444.1            |
| $E_0$ [10$^{48}$ erg]      | $2.65^{+1.86}_{-1.56}$ (1.44) | $4.54^{+0.31}_{-0.33}$ (4.25) | $5.3^{+1.6}_{-1.6}$ (5.5) | $89.5^{+6.0}_{-11}$ (99.6) |
| $A_4$ [10$^{-3}$]          | ...               | ...               | ...              | ...              |
| $n_0$ [cm$^{-3}$]          | $0.0134^{+0.005}_{-0.0025}$ (0.0026) | ...               | $0.025^{+0.029}_{-0.018}$ (0.032) | ...              |
| $\theta_0$ [deg]           | $1.79^{+0.15}_{-0.15}$ (1.56) | $1.80^{+0.15}_{-0.28}$ (1.69) | $1.75^{+0.11}_{-0.28}$ (1.89) | $3.15^{+0.70}_{-0.60}$ (2.07) |
| $p$                        | $2.11^{+0.03}_{-0.03}$ (2.101) | $1.471^{+0.011}_{-0.012}$ (1.474) | $2.14^{+0.034}_{-0.034}$ (2.165) | $2.64^{+0.042}_{-0.044}$ (2.678) |
| $\epsilon_e$              | $0.30^{+0.11}_{-0.23}$ (0.23) | $0.0181^{+0.0016}_{-0.0016}$ (0.0181) | $0.29^{+0.13}_{-0.11}$ (0.32) | $0.075^{+0.109}_{-0.094}$ (0.376) |
| $\epsilon_i$              | $0.039^{+0.015}_{-0.011}$ (0.025) | $0.0013^{+0.00024}_{-0.0002} (0.00137$ | $0.073^{+0.031}_{-0.024}$ (0.082) | $0.047^{+0.015}_{-0.0055}$ (0.0342) |
| $\epsilon_B$              | $0.0232^{+0.0096}_{-0.0036}$ (0.0467) | $0.481^{+0.0096}_{-0.0021}$ (0.4971) | $0.0103^{+0.015}_{-0.0026}$ (0.012) | $0.28^{+0.14}_{-0.17}$ (0.45) |

**Note.** Number in parentheses is the value associated with the maximum likelihood used to create the model curves in Figure 2.

Electrons that may be important for the lowest energy electrons. They also excluded several important effects, such as the equal arrival time surface. Clearly, there is room for considerable improvements in this area of GRB afterglow modeling.

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**References**

Chary, R., Petitjean, P., Robertson, B., Trenti, M., & Vangioni, E. 2016, SSRv, 202, 181
Chevalier, R. A., & Li, Z.-Y. 1999, ApJL, 520, L29
Dai, Z. G., & Lu, T. 1998, MNRAS, 298, 87
de Ugarte Postigo, A., Castro-Tirado, A. J., Gorosabel, J., et al. 2005, A&A, 443, 841
de Ugarte Postigo, A., Fatkhullin, T. A., Jóhannesson, G., et al. 2007, A&A, 462, L57
Feroz, F., & Hobson, M. P. 2008, MNRAS, 384, 449
Feroz, F., Hobson, M. P., & Bridges, M. 2009, MNRAS, 398, 1601
Feroz, F., Hobson, M. P., Cameron, E., & Pettitt, A. N. 2013, arXiv:1306.2144
Gehrels, N., Ramirez-Ruiz, E., & Fox, D. B. 2009, ARA&A, 47, 567
Geng, J.-J., Huang, Y.-F., Wu, X.-F., Zhang, B., & Zong, H.-S. 2018, ApJS, 234, 3
Harrison, F. A., Bloom, J. S., Frail, D. A., et al. 1999, ApJL, 523, L121
Holland, S., Björnsson, G., Hjorth, J., & Thomesen, B. 2000, A&A, 364, 467
Jóhannesson, G., Björnsson, G., & Gudmundsson, E. H. 2006, ApJ, 647, 1238
Kardashev, N. S. 1962, SvA, 6, 317
Kuulkers, E., Antonelli, L. A., Kuiper, L., et al. 2000, ApJ, 538, 638
Panaítescu, A., & Kumar, P. 2001, ApJ, 554, 667
Panaítescu, A., & Kumar, P. 2002, ApJ, 571, 779
Pelletier, G., Bykov, A., Ellison, D., & Lemoine, M. 2017, SSRv, 207, 319
Piran, T. 2004, RevMP, 76, 1143
Resmi, L., Misra, K., Jóhannesson, G., et al. 2012, MNRAS, 427, 288
Ressler, S. M., & Laskar, T. 2017, ApJ, 845, 150
Rhoads, J. E. 1999, ApJL, 525, 737
Sánchez-Ramírez, R., Hancock, P. J., Jóhannesson, G., et al. 2017, MNRAS, 464, 4624
Sari, R., Piran, T., & Narayan, R. 1998, ApJL, 497, L17
Schlegel, D. J., Finkbeiner, D. P., & Davis, M. 1998, ApJ, 500, 525
Stanek, K. Z., Garnavich, P. M., Kaluzny, J., Pych, W., & Thompson, I. 1999, ApJL, 522, L39
Wang, F. Y., Dai, Z. G., & Liang, E. W. 2015, NewAR, 67, 1