Right-Handed Sneutrinos as Curvatons

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Abstract

We consider the possibility that a right-handed sneutrino can serve as the source of energy density perturbations leading to structure formation in cosmology. The cosmological evolution of a coherently oscillating condensate of right-handed sneutrinos is studied for the case where reheating after inflation is due to perturbative inflaton decays. For the case of Dirac neutrinos, it is shown that some suppression of Planck scale-suppressed corrections to the right-handed neutrino superpotential is necessary in order to have sufficiently late decay of the right-handed sneutrinos. $cH^2$ corrections to the sneutrino mass squared term must also be suppressed during inflation ($|c| \lesssim 0.1$), in which case, depending on the magnitude of $|c|$ during inflation, a significantly blue (if $c > 0$) or red (if $c < 0$) perturbation spectrum is possible. R-parity must also be broken in order to ensure that the Universe is not overclosed by LSPs from the late decay (at temperatures 1 – 10 MeV) of the right-handed sneutrino condensate. The resulting expansion rate during inflation can be significantly smaller than in conventional supersymmetric inflation models (as low as $10^6$ GeV is possible). For the case of Majorana neutrinos, a more severe suppression of Planck-suppressed superpotential corrections is required. In addition, the Majorana sneutrino condensate is likely to be thermalised before it can dominate the energy density, which would exclude the Majorana right-handed sneutrino as a curvaton.

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1 Introduction

The observation of neutrino masses and mass splittings, via solar and atmospheric neutrinos [1], strongly suggests the existence of right-handed (r.h.) neutrinos. In extensions of the Minimal Supersymmetric Standard Model (MSSM) [2] which can accommodate neutrino masses we therefore expect to have right-handed sneutrinos. In the cosmology of the MSSM and its extensions, Bose condensates of scalar fields such as squarks and sleptons form naturally after inflation [4]. These may have important consequences for cosmology; for example, they allow for the possibility of baryogenesis and leptogenesis via the Affleck-Dine mechanism [4, 5]. Since right-handed sneutrinos may also form condensates, it is important to consider in some detail the cosmological evolution of a r.h. sneutrino condensate.

Due to the weak coupling of the r.h. sneutrinos to the MSSM fields, a condensate of r.h. sneutrinos will be long-lived and so may come to dominate the energy density of the Universe before it decays. The question of whether the r.h. sneutrino condensate can dominate the energy density of the Universe when it decays has recently acquired some importance. It has been noted [6, 7] that if a scalar dominates the energy density when it decays, and if that scalar is effectively massless during inflation, then quantum fluctuations of the scalar during inflation can in principle account for the primordial energy density perturbations leading to structure formation [6, 7, 8]. This has been labelled the curvaton scenario [9, 10]. Should the curvaton be able to account for the density perturbations, the parameters of the inflation model would be more weakly constrained than in the conventional case where density perturbations arise from quantum fluctuations of the inflaton.

Thus if there exists a natural curvaton candidate such as the r.h. sneutrino, it is important to confirm or exclude that candidate as a curvaton\(^1\). The main goal of this paper is to investigate the possibility of a r.h. sneutrino curvaton.

The masses of the r.h. sneutrinos and their coupling to the MSSM fields are determined by the model of neutrino masses, in particular whether they have Dirac

\(^1\)Models have recently been proposed where the curvaton corresponds to an MSSM flat direction scalar [11] and an MSSM Higgs scalar [12].
masses or Majorana masses via a see-saw mechanism [1]. The Yukawa coupling of the r.h. sneutrino to the MSSM fields plays a fundamental role in determining the evolution of the r.h. sneutrino condensate, in particular its decay temperature, its effective mass from interacting with the background of inflaton decay products [13] and its rate of thermalisation/scattering from the background.

Throughout this paper we will consider the simplest model for inflation and reheating, corresponding to a constant expansion rate during inflation followed by formation of a coherently oscillating inflaton condensate and reheating due to perturbative inflaton decays.

The paper is organised as follows. In Section 2 we discuss the cosmological environment due to perturbative inflaton decays. In Section 3 we consider the evolution of the r.h. sneutrino condensate in this environment. In Section 4 we present our conclusions.

2 Cosmological Background from Perturbative Inflaton Decays

After inflation we consider the inflaton $S$ to have a mass $m_S$ and to be coherently oscillating about the minimum of its potential. The inflaton is assumed to decay into pairs of relativistic MSSM particles with initial energy $m_S/2$. If the initial energy of the decay products is sufficiently large, the initial scattering rate of the inflaton decay products may be small enough that thermalisation only occurs once the Universe has expanded sufficiently for the decay product scattering rate $\Gamma_{sc}$ to exceed the expansion rate $H$. There are therefore two possibilities:

(a) **Instantaneous Thermalisation.** The inflaton decay products thermalise immediately after decay. In this case there will be two epochs: (i) Inflaton Matter Domination (IMD), where the energy density of the Universe is dominated by the coherently oscillating inflaton, and (ii) Radiation Domination (RD), defined to mean domination of the energy density of the Universe by relativistic inflaton decay products, not necessarily thermalised.
(b) Non-instantaneous Thermalisation. The relativistic inflaton decay products are unthermalised initially. In this case we will show that thermalisation cannot occur during IMD and so must occur during RD. Therefore in this case the background will have three distinct epochs: (i) IMD, (ii) RD pre-thermalisation and (iii) RD post-thermalisation.

2.1 Inflaton decay product thermalisation

Prior to thermalisation there is a spectrum of decay products, ranging from red-shifted products from the earliest decays (occurring at the end of inflation) to products from the most recent decays. Most of the energy density in decay products will come from the most recent decays. Following the discussion of [14], the spectrum of unthermalised decay products as a function of energy during IMD and RD epochs is given by

\[
\frac{dn}{dE} \approx \frac{3}{2} \left( \frac{H}{H_R} \right)^\gamma \frac{\rho_s (H_R) E^{1/2}}{m_s m_S^{3/2}},
\]

where \(\gamma = 1\) during IMD (3/4 during RD), \(n(E)\) is the number density of decay products with energy less than \(E\) and \(H_R\) is the expansion rate at the onset of radiation domination. Thus \(n(E) \propto E^{3/2}\) and so 80% of the decay products at a given time have energy between \(E_d\) and \(E_d/3\), where \(E_d\) is the energy of the most recent decay products. In addition, once thermalisation by scattering begins, the lower energy decay products tend first to increase their energy by scattering from the more numerous higher-energy decay products in the spectrum [15, 16], so that they may be regarded as higher-energy decay products as far as thermalisation is concerned. Thus we will consider the energy of the decay products at a given time to be approximately \(E_d\). During IMD, \(E_d \approx m_S/2\). Once the inflaton condensate has decayed away and the Universe enters the RD epoch, the energy of the dominant unthermalised decay products will be red-shifted to \(E_d \approx \left( \frac{a_R}{a} \right) \frac{m_S}{2}\), where \(a_R\) is the scale factor at the onset of radiation domination.

The centre of mass (CM) cross-section of the relativistic inflaton decay products is

\[
\sigma_{sc} \approx \frac{\alpha_{sc}^2}{E_{CM}^2},
\]
where $E_{CM} \approx 2E_d$ and $\alpha_{sc} = g^2/4\pi$, where $g$ is a typical MSSM gauge or Yukawa coupling. Therefore the scattering rate of the inflaton decay products is $\Gamma_{sc} = n\sigma_{sc}$, where $n$ is the number density of inflaton decay products.

During IMD most of the decay products at a given scale factor are produced during the previous e-folding. The energy density in the inflaton condensate during IMD is

$$\rho_S = \left(\frac{a_e}{a}\right)^3 e^{-\Gamma_d t} \rho_I$$

(3)

where $\Gamma_d$ is the decay rate of the inflaton, $a_e$ is the scale factor at the end of inflation and $\rho_I$ is the energy density during inflation, assumed constant. Therefore the number of inflatons which decay during an e-folding ($\delta t \approx H^{-1}$) (and so the number of inflaton decay products produced) is

$$n \approx \frac{\Gamma_d \rho_S}{H m_S}$$

(4)

for $\Gamma_d/H < 1$. The condition for thermalisation to occur during IMD, $\Gamma_{sc} \gtrsim H$, is then

$$\frac{3\alpha_{sc}^2 M^2 \Gamma_d}{m_S^3} \gtrsim 1,$$

(5)

where $M = M_{Pl}/\sqrt{8\pi}$ and we have used $E_d = m_S/2$ and $\rho_S \approx 3M^2H^2$ during IMD. Eq. (5) is independent of the scale factor, so if it is not satisfied immediately at the end of inflation, it will not be satisfied during IMD. Therefore for the case of perturbative inflaton decay, thermalisation must either be instantaneous or must occur during RD.

After IMD, the number density and energy of the relativistic decay products is

$$n = \left(\frac{a_R}{a}\right)^3 n(a_R) \ ; \ E_d \approx \left(\frac{a_R}{a}\right) \frac{m_S}{2}$$

(6)

The inflaton decay rate, $\Gamma_d$, may be expressed in terms of the reheating temperature$^2$, $T_R$, as $\Gamma_d = k_{TR} T_R^2/M_{Pl}$, where $k_{TR} = (4\pi^3 g(T_R)/45)^{1/2}$ and $g(T_R)$ is the effective number of massless degrees of freedom in thermal equilibrium [20]. (In the following

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$^2$The reheating temperature is defined in the following to be the temperature of a thermalised Universe at the onset of radiation domination. It is therefore used generally to parameterise the energy density at the onset of radiation domination, even if the relativistic decay products at that time have not yet thermalised.
we will consider $k_{TR} \approx 20$, corresponding to the field content of the MSSM with $g(T_R) \approx 200$.

The thermalisation condition $n \sigma_{sc} > H$ (where $H = \left(\frac{m_{S}}{a}\right)^2 H(a_R)$ during RD) then implies that thermalisation occurs at scale factor $a_{th}$ given by

$$\frac{a_{th}}{a_e} \approx \frac{m_{S}^3}{3\alpha_{sc}^2 M^2 \Gamma_d \frac{(H_I)}{\Gamma_d}^{2/3}}$$

with the corresponding temperature given by

$$T_{th} \approx \frac{3}{\sqrt{8\pi}} \frac{k_{TR} \alpha_{sc}^2 M T_R^3}{m_{S}^3}.$$  

Here $H_I$ is the expansion rate during inflation.

2.2 Scalar field squared expectation value of the inflaton decay product background

When discussing the evolution of the r.h. sneutrino condensate, we will need the expectation value $<\Psi^2>$, where $\Psi$ represents a generic real MSSM scalar in the inflaton decay product background [13].

(i) Unthermalised Decay Products during IMD. If we consider the average momentum of the scalar modes in the inflaton decay product background to be $\sim k$, then the average energy density of a real massless scalar field $\Psi$ is

$$<\rho_{\Psi}> = <\frac{1}{2}\dot{\Psi}^2 + \frac{1}{2}(\nabla \Psi)^2 > \approx k^2 <\Psi^2>.$$  

Thus with $k \approx E_d \approx m_S/2$ and $<\rho_{\Psi}> = f_{\Psi} \rho_d$, where $\rho_d \approx (\Gamma_d/H) \rho_S$ is the energy density of the inflaton decay products during IMD and $f_{\Psi}$ is the fraction of the total inflaton decay product energy density in the real scalar field $\Psi$, we obtain

$$<\Psi^2> \approx \frac{f_{\Psi} \rho_d}{k^2} \approx \frac{12 f_{\Psi} \Gamma_d M^2 H}{m_{S}^3}.$$  

(ii) Unthermalised Decay Products during RD. In this case the energy of the dominant decay products red-shifts as $E_d \approx \left(\frac{a}{a_e}\right) \frac{m_{S}}{2}$ whilst $\rho_d \propto a^{-4}$ for relativistic
decay products, with \( \rho_d(a_R) \approx \rho_S(a_R) \). Thus \(<\Psi^2> \propto \rho_d/E_d^2 \propto a^{-2} \propto H \). Therefore the same relation, Eq. (10), between \(<\Psi^2>\) and \(H\) also holds during RD.

(iii) Thermalised Decay Products. In this case the energy density of the inflaton decay products is \( \rho_d = \frac{\pi^2 g(T) T^4}{30} \), where for a real scalar field \( g(T) = 1 \). Therefore, with \( k \approx T \) for effectively massless thermalised particles, we find

\[
<\Psi^2> \approx \gamma_T T^2 \; ; \; \gamma_T = \frac{\pi^2}{30} .
\]

We note that in the case where the inflaton decay products thermalise immediately at the end of inflation, the temperature during IMD is related to \( H \) by [20]

\[
T = k_r \left( M_{Pl} H T_R^2 \right)^{1/4} \; ; \; k_r = \left( \frac{9}{5 \pi^3 g(T)} \right)^{1/8} .
\]

3 Right-Handed Sneutrino Condensate Evolution

3.1 Neutrino masses and the r.h. sneutrino scalar potential

For simplicity we will consider a single neutrino generation. The superpotential of the r.h. neutrino superfield, \( N \), is given by

\[
W_\nu = \lambda_\nu N H_u L + \frac{M_N N^2}{2} ,
\]

where \( H_u \) and \( L \) are the MSSM Higgs and charged lepton superfield \( SU(2)_L \) doublets [2]. The corresponding scalar potential for the r.h. sneutrino is then \( V(N) = \frac{m_o^2}{2} N^2 \) (with \( N \) a conventionally normalised real scalar field), where \( m_{\tilde{N}}^2 = m_o^2 + M_N^2 + m_{\text{eff}}^2 \). Here \( m_o^2 \) is the conventional SUSY breaking mass squared term (\( m_o \sim 100 \text{ GeV} \)) whilst \( m_{\text{eff}} \equiv \lambda_\nu <\Psi^2> \) is the effective mass squared term due to the interaction of the r.h. sneutrinos with the inflaton decay product backround [13]. In addition, we expect terms due to Planck-scale suppressed interactions. For now we will consider the evolution of the sneutrino condensate in the absence of such terms.

If \( M_N = 0 \) we will have Dirac neutrino masses, with \( m_\nu = \lambda_\nu v_u \). The r.h. sneutrino mass is then given by \( m_{\tilde{N}}^2 = m_o^2 + m_{\text{eff}}^2 \). If \( M_N \gg \lambda_\nu v_u \) (where \( v_u = <H_u> \)) we will
have Majorana neutrino masses from the see-saw mechanism, $m_\nu = \frac{\lambda_\nu^2 M_N}{v_u^2}$, such that the Yukawa coupling $\lambda_\nu$ can be expressed as a function of $m_\nu$,

$$\lambda_\nu = \left(\frac{m_\nu M_N}{v_u^2}\right)^{1/2}. \quad (14)$$

The usual idea of the see-saw mechanism [1] is to consider the magnitude of $\lambda_\nu$ to be similar to the charged lepton Yukawa couplings, which requires e.g. $M_N \approx 10^9$ GeV for $\lambda_\nu \approx \lambda_\tau \approx 10^{-2}$ and $m_\nu \approx 0.1$eV. However, other natural mass scales could also be of interest, for example $M_N \sim 100$ GeV $-$ 1 TeV, as suggested by the electroweak scale and by the scale of the SUSY mass term $\mu H_u H_d$ of the MSSM superpotential [2].

### 3.2 Conditions for r.h. sneutrino to act as a curvaton

R.h. sneutrino oscillations begin once the r.h. sneutrino mass satisfies $m_{\tilde{N}} \gtrsim H$. We denote the scale factor at this time by $a_{\text{osc}}$ and the homogeneous sneutrino expectation value by $N_{\text{osc}}$. During inflation, in order to serve as a curvaton, the r.h. sneutrino must be effectively massless, $m_{\tilde{N}} \ll H_I$. The quantum fluctuation of an effectively massless r.h. sneutrino mode at horizon crossing is given by $\delta N \approx H_I/2\pi$ [20]. In the case of a scalar potential consisting purely of a mass term ($\propto N^2$), once the perturbation mode is stretched outside the horizon by the expansion of the Universe, its amplitude on sub-horizon scales will evolve in the same way as the homogeneous field. This can be seen by considering $N = N_o + \delta N$, where $N_o$ is the homogeneous field and $\delta N$ is a perturbation of wavenumber $k$. The equations of motion for these are

$$\ddot{N}_o + 3H \dot{N}_o = -V'(N_o) \quad (15)$$

and

$$\delta \ddot{N} + 3H \delta \dot{N} - \frac{k^2}{a^2} \delta N = -V''(N_o) \delta N . \quad (16)$$

For a mode outside the horizon, the $k^2/a^2$ term ($\ll H^2$) will effectively play no role in the evolution of the scalar field. Therefore, for $V(N) \propto N^2$, Eq. (15) $\leftrightarrow$ Eq. (16) under $N_o \leftrightarrow \delta N$. Thus $N_o$ and $\delta N$ will evolve in the same way. Therefore $\delta N/N$ for a superhorizon perturbation will be fixed by its value at the onset of oscillations,
(\delta N/N)_{osc}. Once coherent oscillations of the r.h. sneutrino begin, the energy density in
the r.h. sneutrino field will be proportional to its amplitude squared. Therefore if the
energy density of the Universe becomes dominated by the r.h. sneutrino oscillations
before the sneutrinos decay, the energy density perturbation when a given mode re-
enters the horizon will be given by

$$\delta \rho \equiv \frac{\delta \rho}{\rho} \approx \left(\frac{2\delta N}{N}\right)_{osc} = \frac{H_I}{\pi N_I}.$$  (17)

(A more precise calculation gives the same result up to a factor of the order of 1 [9].)
For \(H_I\) and \(N_I\) constant this corresponds to a scale-invariant perturbation spectrum.
In order to account for the observed CMB temperature fluctuations, we then require
that \(\delta \rho \approx 10^{-5}\) [22].

### 3.3 Condensate evolution without Planck-suppressed terms

The evolution of the r.h. sneutrino expectation value depends on the inflaton decay
product background and reheating temperature. We will consider the case where
the r.h. sneutrino begins coherent oscillations during the IMD epoch, \(a_{osc} < a_R\).
Oscillations begin at \(H_{osc} \approx m_{\tilde{N}} > 100\) GeV. The condition \(H_R < H_{osc} \approx m_{\tilde{N}}\) then
implies an upper bound on \(T_R\),

$$T_R \lesssim \left(\frac{m_{\tilde{N}} M_{Pl}}{k_{TR}}\right)^{1/2} \approx 7.8 \times 10^9 \left(\frac{m_{\tilde{N}}}{100 \text{ GeV}}\right)^{1/2} \text{ GeV}.$$  (18)

In particular, if \(T_R \lesssim 10^8\) GeV, as would be required by the thermal gravitino upper
bound [21, 24] if the inflaton decay products are thermalised at the onset of radiation
domination, then r.h. sneutrino oscillations would generally begin during IMD. However,
it is possible that thermalisation of the relativistic decay products could occur at
\(T_{th} \lesssim 10^8\) GeV even though \(T_R > 10^8\) GeV, in which case the thermal gravitino bound
on \(T_R\) could be evaded.

We will also assume that thermalisation of the inflaton decay products occurs after
the Universe becomes radiation dominated. This is true if Eq. (5) is not satisfied,
which in turn requires that the inflaton mass satisfies

$$m_S \gtrsim 6 \times 10^{11} \alpha_{sc}^{2/3} \left(\frac{T_R}{10^8 \text{ GeV}}\right)^{2/3} \text{ GeV},$$  (19)
where $\alpha_{sc}$, being due to typical MSSM couplings, is not expected to be much smaller than 1. We finally assume that $m_{\tilde{N}}^2$ is dominated by the time-independent terms when the sneutrino oscillations begin, $m_{\tilde{N}}^2 \approx m_{\tilde{N}}^2 \equiv m_{\nu}^2 + M_{\tilde{N}}^2$. This requires that $m_{\text{eff}}^2 < m_{\tilde{N}}^2$ at $H_{\text{osc}} \approx m_{\tilde{N}}$. Using $\langle \Psi^2 \rangle$ for an unthermalised background during IMD, Eq. (10), this requires that

$$m_{\tilde{N}}^2 \approx \frac{12 k T_R f_{\Psi} \lambda^2 M T_R^2}{\sqrt{8\pi m_{N_c}}}.$$  \hspace{1cm} (20)

For the case of Dirac neutrinos this implies that

$$m_S \sim 1.1 \times 10^5 f_{\Psi}^{1/2} \left( \frac{100 \text{ GeV}}{m_{N_c}} \right)^{1/2} \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{m_{\nu}}{0.1 \text{ eV}} \right) \text{ GeV},$$  \hspace{1cm} (21)

where we have assumed $v_u \approx 100 \text{ GeV}$, whilst for the case of Majorana neutrino masses

$$m_S \sim 1 \times 10^{11} f_{\Psi}^{1/2} \left( \frac{M_{\nu}}{m_{N_c}} \right)^{1/2} \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{m_{\nu}}{0.1 \text{ eV}} \right)^{1/2} \text{ GeV}.$$  \hspace{1cm} (22)

This assumption will give the largest r.h. sneutrino energy density at late times for a given $T_R$ and $N_{\text{osc}}$, since the effect of having $m_{\text{eff}} > m_{\tilde{N}}$ at $H \approx m_{\tilde{N}}$ would be to cause r.h. sneutrino oscillations to begin earlier and so to experience a greater dilution of the r.h. sneutrino energy density due to expansion. From now on we will consider $m_{N} \approx m_{\tilde{N}}$.

A fundamental condition for the r.h. sneutrino to play the role of a curvaton is that the r.h. sneutrinos decay after the Universe becomes dominated by the energy density in their coherent oscillations. The r.h. sneutrino decay rate is given by

$$\Gamma_{\tilde{N}_d} \approx \frac{\lambda^2 m_{\tilde{N}}}{4\pi}.$$  \hspace{1cm} (23)

The r.h. sneutrinos decay once $\Gamma_{\tilde{N}_d} \approx H$. At the onset of r.h. sneutrino oscillations, $\rho_{\tilde{N}} \approx m_{\tilde{N}}^2 N_{\text{osc}}^2/2$ whilst $\rho_S \approx 3 H_{\text{osc}}^2 M^2 \approx 3 m_{\tilde{N}}^2 M^2$. During IMD, the energy density in the inflaton oscillations and the r.h. sneutrino oscillations are both evolve as $a^{-3}$. Thus $\rho_{\tilde{N}}/\rho_S$ is constant. Once the Universe is radiation dominated, the energy density in the dominant relativistic background, $\rho$, evolves as $a^{-4}$ whilst the energy density in the r.h. sneutrino evolves as $a^{-3}$. Therefore once $a > a_R$,

$$\frac{\rho_{\tilde{N}}}{\rho} = \left( \frac{a}{a_R} \right) \frac{\rho_{\tilde{N}}}{\rho_S} \approx \left( \frac{a}{a_R} \right) \frac{N_{\text{osc}}^2}{6 M^2}.$$  \hspace{1cm} (24)

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Thus the Universe becomes r.h. sneutrino dominated once $H < H_{\text{dom}}$, where

$$H_{\text{dom}} \approx \left( \frac{N_{\text{osc}}^2}{6 M^2} \right)^2 H_R .$$

(25)

The condition that r.h. sneutrino decay occurs after r.h. sneutrino domination, $\Gamma_{\tilde{N}_d} < H_{\text{dom}}$, then implies that

$$m_{\tilde{N}} < \frac{4 \pi k_{T_R} T_R^2}{\lambda_{\nu}^2 M_{\text{Pl}}} \frac{N_{\text{osc}}^2}{6 M^2} .$$

(26)

We will refer to Eq. (26) as the late decay condition in the following.

For the case of Dirac neutrino masses the late decay condition becomes

$$m_{\tilde{N}} < \frac{4 \pi v_u^2}{m_{\nu}^2} \left( \frac{k_{T_R} T_R^2}{M_{\text{Pl}}} \right) \frac{N_{\text{osc}}^2}{6 M^2} ,$$

(27)

which implies that

$$m_{\tilde{N}} < 5.8 \times 10^{21} \left( \frac{0.1 \text{ eV}}{m_{\nu}} \right)^2 \left( \frac{T_R}{10^8 \text{ GeV}} \right)^2 \left( \frac{N_{\text{osc}}}{M} \right)^4 \text{ GeV} .$$

(28)

Thus in order to have $m_{\tilde{N}} > m_{\nu} \approx 100 \text{ GeV}$ we require that $N_{\text{osc}}/M \gtrsim 1 \times 10^{-5}$. We will see that values of $N_{\text{osc}}/M$ in this range require some suppression of Planck-scale suppressed non-renormalisable corrections to the r.h. neutrino superpotential.

For the case of Majorana neutrino masses the late decay condition becomes

$$m_{\tilde{N}} < \left( \frac{m_{\tilde{N}}}{M_N} \right)^{1/2} \left( \frac{4 \pi v_u^2}{m_{\nu}} \right)^{1/2} \left( \frac{k_{T_R} T_R^2}{M_{\text{Pl}}} \right)^{1/2} \frac{N_{\text{osc}}^2}{6 M^2} ,$$

(29)

such that

$$m_{\tilde{N}} < 7.6 \times 10^5 \left( \frac{m_{\tilde{N}}}{M_N} \right)^{1/2} \left( \frac{0.1 \text{ eV}}{m_{\nu}} \right)^{1/2} \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{N_{\text{osc}}}{M} \right)^2 \text{ GeV} .$$

(30)

We will restrict attention to r.h. neutrino masses $M_N \gtrsim m_{\nu} \approx 100 \text{ GeV}$, since masses smaller than this would have little motivation from neutrino mass models or natural particle physics scales. In this case $m_{\tilde{N}} \approx M_N$. Eq. (30) then shows that a right-handed Majorana sneutrino curvaton with $M_N \gtrsim 100 \text{ GeV}$ is possible only if $N_{\text{osc}}$ is not very small compared with the reduced Planck scale $M$. ($N_{\text{osc}}/M \gtrsim 0.01$ if $T_R \lesssim 10^8 \text{ GeV}$). It also shows that $T_R$ cannot be very small compared with the thermal
gravitino upper bound of $10^8$ GeV if $N_{osc} \lesssim M$. The requirement that $N_{osc}$ is not very small compared with $M$ is a strong constraint on a Majorana r.h. sneutrino curvaton, since, as we discuss below, it requires a high degree of suppression of Planck-scale suppressed contributions to the r.h. sneutrino superpotential.

From Eq. (17) and the density perturbation constraint, $\delta_\rho \approx 10^{-5}$, the expansion rate during inflation is $H_I = \pi \delta_\rho N_I \gtrsim 10^9$ GeV if $N_I \approx N_{osc} \gtrsim 1 \times 10^{-5} M$, corresponding to the case of Dirac neutrino masses. This can be substantially lower than the typical value of the expansion rate during inflation found in conventional SUSY inflation models, $H_I \approx 10^{13}$ GeV [25, 26, 27]. For the case of Majorana neutrino masses with $N_I \approx N_{osc} \gtrsim 0.01 M$, the corresponding bound on $H_I$ is $H_I \gtrsim 7 \times 10^{11}$ GeV. These bounds assume that $N$ does not evolve significantly from the end of inflation to the onset of sneutrino oscillations, so that $N_{osc} \approx N_I$. We will see that the range of allowed $H_I$ can be increased if this assumption is altered.

### 3.4 Effect of Planck-suppressed terms

In general, in addition to the globally SUSY scalar potential we expect (in the absence of specific symmetries) terms suppressed by powers of the reduced Planck mass, $M = M_{Pl}/\sqrt{8\pi}$, corresponding to the natural scale of supergravity (SUGRA) corrections [3, 19]. Thus we expect Planck-scale suppressed non-renormalisable terms to appear in the r.h. neutrino superpotential. We also expect contributions to the mass squared term of the form $cH^2$, where $|c|$ is model-dependent but expected to be of the order of 1 in the simplest models. These arise from terms in the full Lagrangian of the form $\frac{1}{M^2} \int d^4\theta S^\dagger S N^\dagger N = |F_S|^2/M^2$, where $S$ is the inflaton field (or any other field with a non-zero F-term contributing to the energy density of the Universe).

(i) $cH^2$ corrections

During inflation the value of $|c|$ is constrained by the deviation of the curvaton perturbation from scale-invariance. We consider the r.h. sneutrino potential during inflation to be

$$V(N) \approx \frac{1}{2} cH^2 N^2 .$$

(31)
The expansion rate during inflation is then
\[ H_I^2 = \frac{1}{3M^2} \left( \rho_S + \frac{cH_I^2N^2}{2} \right) \]  
(32)
where \( \rho_S \) is the energy density of the inflaton field, which we assume to be constant.

The index \( n \) of the perturbation spectrum as a function of present wavenumber \( k \) is given by
\[ n = 1 + \frac{2k}{\delta \rho} \frac{d\delta \rho}{dk} , \]  
(33)
such that \( \delta \rho/\rho \propto k^{n-1} \) and \( n = 1 \) corresponds to scale-invariance. \( \delta \rho \) will remain constant once the perturbation is outside the horizon, since both \( N \) and \( \delta N \) evolve in the same way for \( V(N) \propto N^2 \). Therefore we have
\[ \delta \rho \equiv \frac{2\delta N}{N} = \left( \frac{H_I}{\pi N} \right)_{a_\lambda} , \]  
(34)
where \( a_\lambda \) is the scale-factor at which a perturbation of wavelength \( \lambda \) exits the horizon.

The value of \( N \) as a function of the scale factor during inflation is given by the solution of
\[ \ddot{N} + 3H \dot{N} = -cH^2N \]  
(35)
With \( H \propto a^{-m} \), this has a solution of the form \( N \propto a^\gamma \), where
\[ \gamma = \frac{1}{2} \left[ -(3-m) + \sqrt{(3-m)^2 - 4c} \right] . \]  
(36)
Thus during inflation \( (m = 0) \) the solution corresponds to
\[ \gamma = \frac{1}{2} \left[ -3 + \sqrt{9 - 4c} \right] . \]  
(37)
If \( |4c| \ll 9 \) then during inflation \( \gamma \approx -c/3 \) and so \( N/N_\lambda = (a/a_\lambda)^{-c/3} \), where \( N_\lambda \) is the value at \( a_\lambda \). Thus for a given \( N \) and \( a \) (for example, their values at the end of inflation), \( N_\lambda \propto a_\lambda^{-c/3} \). The wavenumber at present is related to the scale factor at horizon exit \( (\lambda \approx H_I^{-1}) \) by \( k = 2\pi H_Ia_\lambda/a_p \), where \( a_p \) is the scale factor at present. Thus
\[ \frac{dN_\lambda}{dk} = \frac{dN_\lambda}{da_\lambda} \frac{da_\lambda}{dk} = -\frac{cN_\lambda}{3k} . \]  
(38)
For the case where the energy density is dominated by $\rho_S$, we have

$$\frac{d\delta\rho}{dN_\lambda} \approx - \frac{H}{\pi N_\lambda^2}. \tag{39}$$

Therefore

$$n = 1 + 2k \frac{d\delta\rho}{\delta\rho} \frac{dN_\lambda}{dk} \approx 1 + \frac{2c}{3}. \tag{40}$$

Note that $c > 0 ( < 0)$ results in a blue (red) spectrum of perturbations. Observation requires that $|\Delta n| < 0.1 \ [22]$. Therefore during inflation we must have

$$|c| < 0.15 \left| \frac{\Delta n}{0.1} \right|. \tag{41}$$

If $|c|$ is close to this upper bound then a significant blue or red perturbation spectrum is expected. We note that this would allow the r.h. sneutrino curvaton scenario to be consistent with the recent observation by WMAP of a blue perturbation spectrum on comoving scales of the order of 500 Mpc, with $n = 1.10^{+0.07}_{-0.06} \ [23]$.

Thus $|c|$ is constrained to be not much larger than 0.1 during inflation. This is significantly larger than the value $|c| \sim 1$ expected on dimensional grounds in SUGRA models [18]. This problem is similar to the conventional `$\eta$-problem' encountered in SUSY inflation models [25, 26, 27], where order $H^2$ corrections lead both to a large deviation from scale-invariance and to insufficient slow-roll inflation [18]. It has a natural solution in the case of SUSY D-term hybrid inflation (driven by the energy density of a Fayet-Illiopoulos D-term) [25], in which case $|F| = 0$ and so $|c| = 0$ during inflation (although a non-zero value is expected once inflation ends and coherent inflaton oscillations begin, since $f \, d^4 \theta S^\dagger S \Phi^\dagger \Phi = (|\partial_\mu S|^2 + |F_S|^2 + ...)|\Phi|^2$, with $\Phi$ a general scalar superfield). Since in the r.h. sneutrino curvaton scenario inflation must still be driven by an inflaton, it is possible that the mechanism which suppresses $|c|$ for the inflaton during inflation also suppresses $|c|$ for the curvaton.

After inflation we generally expect a non-zero F-term from the energy density of the coherently oscillating inflaton field. The solution of Eq. (35) during IMD corresponds to Eq. (36) with $m = 3/2$. Thus assuming that $N$ has a constant value during inflation, $N_I$, we find

$$\frac{N_{osc}}{N_I} = \left( \frac{a_{osc}}{a_e} \right)^\gamma = \left( \frac{H_I}{H_{osc}} \right)^{2\gamma/3}, \tag{42}$$
where $H_{\text{osc}} \approx m_{\tilde{N}}$.

We previously derived lower bounds on the value of $H_I$ compatible with the observed density perturbations based on the assumption that $N_{\text{osc}} \approx N_I$. However, if $c < 0$ during IMD then the growth of $N$ after inflation will allow a wider range of $H_I$ to be compatible with a given value of $N_{\text{osc}},$

$$\frac{2\gamma}{3} = \frac{\ln (\frac{\pi \delta \rho_{\text{osc}}}{H_I})}{\ln (\frac{H_I}{m_{\tilde{N}}})},$$

(43)

where we have used $N_I \approx H_I/\pi \delta \rho$ in Eq. (42). For example, for the case of Dirac neutrino masses, with $\delta \rho \approx 10^{-5}$, $m_{\tilde{N}} \approx 100$ GeV and $N_{\text{osc}} \approx 10^{-4}M$, it is possible to have $H_I \approx 10^6$ GeV if $\gamma = 1.45$, corresponding to $c = -4.3$. This also shows that $|c|$ need not be small compared with 1 after inflation.

We conclude that the $cH^2$ correction to the r.h. sneutrino mass squared term is no more problematical for the curvaton than for the conventional inflaton: we require that $|c| \lesssim 0.1$ during inflation, in which case a significantly blue or red perturbation spectrum can arise depending on the sign and magnitude of $c$. After inflation $|c|$ need not be small compared with 1 (in the case where $c < 0$) and the $cH^2$ term can even widen the range of expansion rate and energy density during inflation which is compatible with the observed density perturbations.

(ii) **Non-renormalisable superpotential corrections**

We next consider adding Planck scale-suppressed non-renormalisable terms (NRTs) to the r.h. sneutrino superpotential,

$$W_N = \frac{1}{2} M_N N^2 + \frac{\lambda_N N^n}{n! M^{n-3}},$$

(44)

where $n!$ is a symmetry factor and $\lambda_n \approx 1$. For large enough $|N|$ the scalar potential becomes dominated by the NRT contribution,

$$V(N) \approx \frac{\lambda_n^2 N^{2(n-1)}}{2^{n-1} (n-1)!^2 M^{2(n-3)}}.$$  

(45)

The effect of the NRTs is to place an upper limit on the value of $N_{\text{osc}}$. During inflation, the effective mass $V''(N)$ must be small compared with $H^2$. After inflation and during
IMD, $H^2 \propto a^{-3}$. Therefore $V'(N)$ may become larger than $H^2$, at which point the $N$ field will begin to evolve. $N$ will then track the value at which $V''(N) \approx H^2$, since as $H^2$ decreases below $V''(N)$, the rate of roll of $N$ will increase until the rate of decrease of $V''(N)$ matches rate of the decrease of $H^2$. In particular, when $N$ will first begins to slow-roll we have $3H\dot{N}\approx -V'(N)$. For $V(N) \propto N^{2(n-1)}$, this has a solution $N \propto a^{3/(4-2n)}$, such that $V''(N) \propto a^{-3} \propto H^2$. So the slow-rolling solution will be such that $V''(N)$ tracks $H^2$ and is of the same order of magnitude as $H^2$. Thus the value of $N$ at which $V''(N) \approx H^2$ places an upper limit on $N$ for a given value of $H$, $N_{\text{lim}}$, given by

$$\frac{N_{\text{lim}}}{M} \approx \frac{\alpha_n}{\lambda_n^{1/2}} \left( \frac{H}{M} \right)^{n-2},$$

(46)

where $\alpha_n$ is a constant of order 1. Therefore for $N_{\text{lim}}$ to be greater than $N_{\text{osc}}$ at $H_{\text{osc}}$ we require that $n$ is greater than $n_{\text{lim}}$, where

$$n_{\text{lim}} = 2 + \frac{\ln \left( \frac{H_{\text{osc}}}{M} \right)}{\ln \left( \frac{\alpha_n}{\lambda_n^{1/2} N_{\text{osc}} / \alpha_n M} \right)}.$$  

(47)

For the case of Dirac neutrino masses, we require from the late decay condition that $N_{\text{osc}}/M \gtrsim 1 \times 10^{-5}$. Thus with $H_{\text{osc}} \approx m_{\tilde{N}} = 100 \text{ GeV}$, $N_{\text{osc}}/M \approx 1 \times 10^{-5}$ and $\lambda_n \approx \alpha_n \approx 1$, we require that $n > n_{\text{lim}} = 5.3$. Thus a suppression of Planck-scale suppressed NRTs in the r.h. neutrino superpotential of dimension less than 6 is required in this case. This might be achieved by a modest discrete symmetry.

For the case of Majorana neutrino masses, with $H_{\text{osc}} \approx m_{\tilde{N}} = 100 \text{ GeV}$, $N_{\text{osc}}/M \approx 0.01$ and $\lambda_n \approx \alpha_n \approx 1$, we require that $n > n_{\text{lim}} = 10.2$. This is a significant problem for a Majorana r.h. sneutrino curvaton. It requires a high degree of suppression of Planck-suppressed non-renormalisable terms, eliminating all NRTs in the r.h. neutrino superpotential up to $n < 11$. However, as we will discuss, there is likely to be a more severe problem for the Majorana r.h. sneutrino curvaton, namely the survival of the r.h. sneutrino condensate in the inflaton decay product background.

Although Planck scale-suppressed NRTs appear to disfavour a r.h. sneutrino curvaton, it should be noted that conventional SUSY inflation models also have problems
with Planck scale-suppressed NRTs. Chaotic inflation models require that the inflaton expectation value is greater than $M$ [27], whilst SUSY hybrid inflation models require that $N$ is close to $M$ when scales corresponding to observed cosmic microwave background (CMB) perturbations exit the horizon (assuming natural values of the renormalisable gauge and Yukawa couplings) [25, 26]. In this sense the r.h. sneutrino curvaton may be no more problematical that conventional SUSY inflation models with respect to Planck scale-suppressed superpotential terms.

3.5 Decay temperature of the Dirac and Majorana r.h. sneutrino condensate

The neutrino mass is related to the Yukawa coupling by $m_\nu = \lambda_\nu v_u$. The r.h. sneutrino mass is simply given by the SUSY breaking mass term, $m_\tilde{N} = m_o \approx 100$ GeV. Thus the temperature of the Universe when the condensate decays, $T_{\tilde{N}d}$, is given by $\Gamma_{\tilde{N}d} \approx H(T_{\tilde{N}d})$, where $H(T) = k_T T^2 / M_{Pl}$ and $k_T = (4\pi^3 g(T)/45)^{1/2}$. Thus with the r.h. sneutrino decay rate given by Eq. (23), the temperature of r.h. sneutrino decay is, in general,

$$T_{\tilde{N}d} = \left( \frac{\lambda_\nu^2 m_\tilde{N} M_{Pl}}{4\pi k_{T_{\tilde{N}d}}} \right)^{1/2}. \quad (48)$$

For the case of the Dirac r.h. sneutrinos, Eq. (48) implies that

$$T_{\tilde{N}d} = \left( \frac{m_\nu^2 m_\tilde{N} M_{Pl}}{4\pi v_u^2 k_{T_{\tilde{N}d}}} \right)^{1/2} = 4.4 \left( \frac{m_\nu}{0.1 \text{ eV}} \right) \left( \frac{m_\tilde{N}}{100 \text{ GeV}} \right)^{1/2} \text{ MeV}, \quad (49)$$

where we have used $v_u \approx 100$ GeV and $k_{T_{\tilde{N}d}} \approx 5$, corresponding to $\gamma$, $e^\pm$ and $\nu_i$ ($i = 1, 2, 3$) as light degrees of freedom [20]. Thus the Dirac r.h. sneutrino condensate typically decays in the temperature range $1 - 10$ MeV. (Note that if the energy density of the r.h. sneutrino condensate dominates the energy density of the Universe when it decays, $T_{\tilde{N}d}$ should then be interpreted as the temperature to which the Universe reheats after the condensate decays.) Since the Dirac r.h. sneutrino condensate decays well below the temperature at which weakly interacting particles of mass of the order of $m_W$ freeze out of chemical equilibrium ($T_{\text{freeze}} \approx 1 - 10$ GeV), an important constraint on the Dirac r.h. sneutrino curvaton is the requirement that the lightest
supersymmetric particles (LSPs) produced in decay of the r.h. sneutrino condensate do not overclose the Universe. This requires that the LSPs decay before nucleosynthesis at $T \approx 1 \text{ MeV}$, in order that the light element abundances are not disrupted by photo-dissociation due to LSP decay cascades [24]. Therefore the LSP lifetime must satisfy

$$\tau_{\text{LSP}} \lesssim H^{-1}(T \approx 1 \text{ MeV}) \approx 1 \text{ s}.$$  \hspace{1cm} (50)

Thus although the LSP must be unstable its lifetime can be much longer than the time required to escape particle detectors ($\sim 10^{-8} \text{s}$), in which case experimental searches for SUSY particles would be unaffected. A wider range of LSP candidates would be allowed than in the R-parity conserving case where the LSP properties are constrained by the thermal relic cold dark matter density. This might be testable at future colliders such as the CERN Large Hadron Collider.

For the case of Majorana r.h. sneutrinos, Eq. (48) implies that

$$T_{\tilde{N}d} = \left( \frac{m_\nu m_\tilde{N}^2 M_{Pl}}{4\pi v^2 k T_{\tilde{N}d}} \right)^{1/2} = 2.2 \times 10^3 \left( \frac{m_\nu}{0.1 \text{ eV}} \right)^{1/2} \left( \frac{m_\tilde{N}}{100 \text{ GeV}} \right) \text{ GeV}.$$  \hspace{1cm} (51)

Thus since $m_\tilde{N}$ typically will not be much larger than 1 TeV as a result of the late decay condition, Eq. (30), we expect that $T_{\tilde{N}d} \sim 10^4 \text{ GeV}$ for the Majorana r.h. sneutrino condensate.

### 3.6 Thermalisation of the right-handed sneutrino condensate

We have been assuming throughout the preceding discussion that the r.h. sneutrino condensate survives in the environment of the inflaton decay products. However, it is possible that r.h. sneutrinos in the condensate could be inelastically scattered by collisions with the MSSM particles in the inflaton decay product background. If the relativistic particles in the background have energy $E_d$, then the scattering cross-section of a r.h. sneutrino, at rest in the condensate, from a relativistic particle in the inflaton decay product background via Higgs(ino) or (s)lepton exchange is expected to be

$$\sigma_{\tilde{N}sc} \approx \frac{\alpha_\nu \alpha_g}{E_{CM}^2},$$  \hspace{1cm} (52)
where $\alpha = \lambda^2 / 4\pi$, $\alpha_g = g^2 / 4\pi$, where $g$ is a typical MSSM gauge/Yukawa coupling and $E_{CM} \approx \sqrt{E_d m_{\tilde{N}}}$ is the centre of mass energy of the process. The scattering rate from the decay product background is then $\Gamma_{\tilde{N}_{sc}} = n \sigma_{\tilde{N}_{sc}}$, where $n$ is the number density of particles in the inflaton decay product background. The largest scattering rate will occur for the largest $n$ and smallest $E_d$, which corresponds to thermalised decay products ($T \lesssim T_{th}$). In this case $n = g(T) T^3 / \pi^2$ and $E_d \approx T$ [20]. Therefore

$$\Gamma_{\tilde{N}_{sc}} \approx \frac{g(T) \alpha \alpha_g}{\pi^2} \frac{T^2}{m_{\tilde{N}}}. \quad (53)$$

Then assuming that thermalisation of the inflaton decay products occurs during RD, the condition that the condensate is unthermalised is $\Gamma_{\tilde{N}_{sc}} \lesssim H(T) \equiv k_T T^2 / M_{Pl}$, which implies

$$\lambda_{\nu} \lesssim \left( \frac{4 \pi^3 k_T}{\alpha_g g(T)} \right)^{1/2} \left( \frac{m_{\tilde{N}}}{M_{Pl}} \right)^{1/2}. \quad (54)$$

For the case of Dirac neutrino masses, this condition becomes

$$m_{\nu} \lesssim \left( \frac{4 \pi^3 k_T v_u^2}{\alpha_g g(T)} \right)^{1/2} \left( \frac{m_{\tilde{N}}}{M_{Pl}} \right)^{1/2} \approx \frac{10^{-5}}{\alpha_g} \left( \frac{m_{\tilde{N}}}{100 \text{ GeV}} \right)^{1/2} \left( \frac{v_u}{100 \text{ GeV}} \right) \text{keV}, \quad (55)$$

where we have used $g(T) \approx 200$ and $k_T \approx 20$. Therefore, since $m_{\nu} \approx 0.1 \text{eV}$, the Dirac r.h. sneutrino condensate will not be thermalised.

For the case of Majorana neutrino masses, the non-thermalisation condition becomes

$$m_{\nu} \lesssim \left( \frac{4 \pi^3 k_T v_u^2}{\alpha_g g(T)} \right) \frac{v_u}{M_{Pl}} \approx \frac{10^{-5}}{\alpha_g} \left( \frac{v_u}{100 \text{ GeV}} \right)^2 \text{eV}. \quad (56)$$

Thus typically we expect that this will not be satisfied. For example, if we consider $\alpha_g$ to correspond to the top quark Yukawa (r.h. sneutrinos scattering from thermal top quarks via $H_u$ exchange) then $\alpha_g \approx 0.1$ and so $m_{\nu} \lesssim 10^{-4} \text{eV}$ would be required to evade thermalisation. Therefore with $m_{\nu} \approx 0.1 \text{eV}$ the Majorana r.h. sneutrino condensate will be thermalised at $T_{th}$, when the inflaton decay product background thermalises.

One possible escape from this conclusion is that the inflaton decay product background could remain unthermalised until the energy density of the sneutrino condensate comes to dominate the energy density of the Universe. Since the Majorana r.h. sneutrino condensate decays at a temperature typically around $10^4 \text{ GeV}$,
if $T_{th} \lesssim 10^4$ GeV then the Majorana r.h. sneutrino curvaton may remain a possibility. For the case of perturbative inflaton decays, the thermalisation temperature of the inflaton decay product background $T_{th}$, Eq. (8), is proportional to $(T_R/m_S)^3$. Therefore a very low background thermalisation temperature is a possibility if $T_R \ll m_S$.

The above applies to condensate thermalisation during RD. It is straightforward to see that if condensate thermalisation does not occur during RD then it will not occur earlier during IMD. If we assume that the background is thermalised during IMD (which should give the largest rate of condensate thermalisation), then since during IMD $\Gamma_{\tilde{N}_{sc}} \propto T^2$ and $H \propto T^4$, it follows that once $T > T_R$ the thermalisation condition $\Gamma_{\tilde{N}_{sc}} \gtrsim H$ will become more difficult to satisfy. Thus if the r.h. sneutrino condensate does not thermalise during RD it will not thermalise at all.

In discussing the thermalisation rate, we have implicitly assumed that the mass of the $H_u$ and $L$ fields coupling directly to the r.h. sneutrino condensate is small compared with $T$ and $E_{CM} \approx (T m_{\tilde{N}})^{1/2}$. The effective mass is given by $\lambda_\nu < N >$, where $< N >$ is the amplitude of the coherent oscillations. In general $< N >$ is given by

$$< N > = \left( \frac{a_{osc}}{a} \right)^{3/2} N_{osc} \approx \left( \frac{T}{T_R} \right)^{3/2} \left( \frac{H_R}{H_{osc}} \right) N_{osc} \cdot$$

Thus for the case of Majorana r.h. sneutrinos the effective mass is given by

$$\lambda_\nu < N > = \frac{k_T T^{3/2} T_R^{1/2}}{\sqrt{8\pi}} \left( \frac{m_\nu}{m_{\tilde{N}} v_u^2} \right)^{1/2} \left( \frac{N_{osc}}{M} \right),$$

which implies that

$$\lambda_\nu < N > = 4 \times 10^2 \left( \frac{m_\nu}{0.1 \text{ eV}} \right)^{1/2} \left( \frac{T}{10^4 \text{ GeV}} \right)^{3/2} \times \left( \frac{T_R}{10^8 \text{ GeV}} \right)^{1/2} \left( \frac{100 \text{ GeV}}{m_{\tilde{N}}} \right)^{1/2} \left( \frac{N_{osc}}{M} \right) \text{ GeV},$$

where we have used $k_T \approx 20$ and $v_u = 100$ GeV. Thus for $N_{osc}/M < 1$, $m_{\tilde{N}} \gtrsim 100$ GeV and for a decay temperature of $10^4$ GeV for the Majorana r.h. sneutrino, we find that $\lambda_\nu < N >$ is smaller than $T$ and $E_{CM}$ and so may be neglected when considering condensate thermalisation.
For the case of Dirac r.h. sneutrinos the effective mass is given by

$$\lambda_\nu < N > = \frac{k_T T^{3/2}}{\sqrt{8\pi v u m_{\tilde{N}}}} \left( \frac{N_{osc}}{M} \right),$$

which implies that

$$\lambda_\nu < N > = 3 \times 10^{-15} \left( \frac{m_\nu}{0.1 \text{ eV}} \right) \left( \frac{T}{10 \text{ MeV}} \right)^{3/2} \times \left( \frac{T_R}{10^8 \text{ GeV}} \right)^{1/2} \left( \frac{100 \text{ GeV}}{m_{\tilde{N}}} \right) \left( \frac{N_{osc}}{M} \right) \text{ GeV}. \quad (61)$$

Thus the effective mass is generally negligible in the case of Dirac r.h. sneutrinos.

4 Conclusions

We have considered the possibility that a r.h. sneutrino could play the role of a curvaton in the cosmology of the MSSM extended to accommodate neutrino masses.

In the case of a Dirac r.h. sneutrino, the expectation value of the r.h. sneutrino at the onset of its coherent oscillations must satisfy $N_{osc}/M \gtrsim 10^{-5}$, in order that the energy density of the r.h. sneutrino condensate dominates the Universe when it decays. As a result, Planck-scale corrections to the r.h. neutrino superpotential must be suppressed, eliminating all superpotential terms $\propto N^n$ with $n < 6$. $cH^2$ corrections to the r.h. sneutrino mass squared must also be suppressed during inflation ($|c| \lesssim 0.1$).

The inflaton sector of the Dirac r.h. sneutrino curvaton scenario can have a much smaller expansion rate during inflation than conventional SUSY inflation models, with $H_I \approx 10^9 \text{ GeV}$ possible. (Including the effect of a negative $cH^2$ term after inflation, values of $H_I$ as small as $10^6 \text{ GeV}$ or less are possible.)

In addition, depending on the sign and magnitude of the $cH^2$ correction, it is possible to have a significantly blue ($c > 0$) or red ($c < 0$) perturbation spectrum. The recent suggestion from WMAP observations [23] of a blue perturbation spectrum on comoving scales of the order of 500 Mpc could therefore be accommodated within the curvaton scenario.

The late decay of the Dirac r.h. sneutrino condensate (at $T \approx 1 - 10 \text{ MeV}$) requires that R-parity be broken and that the LSP decays before nucleosynthesis, corresponding
to a lifetime shorter than 1s. (Since this would result in the loss of LSP cold dark matter, a new dark matter candidate would also be needed.) Thus the Dirac curvaton scenario predicts that LSP properties will typically be inconsistent with thermal relic cold dark matter.

In the case of a Majorana r.h. sneutrino curvaton, we find that the requirement that the r.h. sneutrino condensate dominates the energy density of the Universe before it decays implies that $N_{\text{osc}}/M \gtrsim 0.01$. This imposes a strong constraint on Planck-scale suppressed contributions to the r.h. neutrino superpotential, requiring elimination of all terms $\propto N^n$ with $n < 11$. However, a more severe problem may arise from scattering of the condensate sneutrinos by particles in the thermal background, since it is likely that the Majorana r.h. sneutrino condensate will be thermalised as soon as the inflaton decay product background thermalises. Thus if the inflaton decay products thermalise earlier than the time of r.h. sneutrino condensate domination of the energy density then the Majorana curvaton scenario will be ruled out. Only a sufficiently low background thermalisation temperature could evade this conclusion. Therefore, although not absolutely excluded, the Majorana r.h. sneutrino curvaton scenario appears to be more difficult to implement than the Dirac curvaton scenario.

In conclusion, we find that a r.h. sneutrino can serve as the source of the density perturbations leading to structure formation. For the favoured case of a Dirac r.h. sneutrino, this requires some suppression of its Planck-scale suppressed superpotential self-interactions together with sufficiently rapid R-parity violation. If these conditions are met, the Dirac r.h. sneutrino would provide us with the only curvaton candidate which is strongly motivated by particle physics.

Note Added: After completing this work we became aware of [28] and [29], which also discuss the case of a Majorana r.h. sneutrino curvaton.

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