Oscillatory Flow and Variable Viscosity by the Heat for the Prandtl-Eyring Fluid through Porous Channel

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Abstract. This paper looked into the effect of heat transfer on the magnetohydrodynamics oscillatory flow of Prandtl-Eyring Fluid through a porous channel. The perturbation procedure is used to obtain accurate forms for velocity profiles. and discuss the solutions for two types of flow “Couette flow and Poiseuille flow” of this problem. The results are presented by graphs. The first section in your paper.

Keywords: Prandtl-Eyring fluid, Magneto-hydrodynamics (MHD), Oscillatory flow, Grashof number.

1. Introduction

In astronomical, geophysical, and engineering mechanics, the influence of the magnetic field on the electrical movement of the conductive fluid is crucial. In food engineering, petroleum processing, power engineering, and polymer solutions, as well as polymer melt in the plastic manufacturing sectors, heat transfer of non-newtonian fluids is a top priority. In different divisions of magnetohydodynamic power generation, nanotechnological production, and groundwater flow, heat transfer in the presence of intense magnetic fields is crucial. The magnetic and mechanical properties that follow, and this science is also looking at specifically producing electricity from hot gases evaporated ionizing generators that depend on this magnetic movement.

It's still looking at ways to simulate nuclear fusion in the lab by adding strong electromagnetic energy to a combination of deuterium and tritium, similar to what happens in the sun and in nuclear reactors using molten sodium molten metal. There is literature that looks at some of the various works related to this flow in a setting where magnetic fields are far from the container walls. Khudair and Al-Khafajy [5] explored the effect of heat transfer on magnetohydrodynamics oscillatory flow of Williamson fluid in a tube.

An unsteady MHD Couette flow with heat transfer of a viscoelastic fluid under an exponentially decaying pressure gradient was investigated by Attia and Ewis [3]. Lqaa and Al–Khafajy [6] investigated the effect of heat transfer on the oscillating flow of the hydrodynamics of a magnetizing Eyring-Power fluid of varying viscosity through a porous medium using a mathematical model see [7-13]. For many researchers, like [14-19], several experiments have been carried out in the study of Prandtl-Eyring fluid, a non-Newtonian fluid, in which the flow of this fluid has been observed in different conditions, channels, and cycles. Investigating the effect of heat transfer on the oscillatory magnetohydrodynamics in the flow
of Prandtl-Eyring Fluid through a Porous Channel media using a mathematical model. The "Poiseuille flow" and "Couette flow" modes of flow are solved using the perturbation technique series.

2. Mathematical Formulation:
Figure 1 depicts the transit of a Prandtl-Eyring fluid into an h-width channel under the control of an electrically applied magnetic field and nuclear heat transfer. The fluid is thought to have low electrical conductivity and produce too little electromagnetic force. We're working for a Cartesian coordinate system, so \((u(y,t),0,0)\) is a velocity vector with \(u\) as the \(x\)-component and \(y\) perpendicular to the \(x\)-axis.

![Figure 1 Channel format: (i) Poiseuille flow and (ii) Couette flow [6].](image)

Basic Equations:

[13] gives the fundamental equations governing the Prandtl-Eyring fluid.

The equation for continuity is:

\[
\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{u}}{\partial y} = 0
\]

(continuity equation) (1)

The equations for momentum are:

In the direction of \(x\):

\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \frac{\partial \mathbf{u}}{\partial x} + \mathbf{v} \frac{\partial \mathbf{u}}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \rho g \beta (T - T_0) - \sigma B_0^2 \mathbf{u} - \frac{\mu(T)}{k} \mathbf{u}
\]

(2)

In the direction of \(y\):

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \frac{\partial \mathbf{v}}{\partial x} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \rho g \beta (T - T_0) - \sigma B_0^2 \mathbf{v} - \frac{\mu(T)}{k} \mathbf{v}
\]

(3)

The following is the temperature equation:

\[
c_T \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \frac{\partial \mathbf{v}}{\partial x} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial y} \right) = K_T \left( \frac{\partial^2 \mathbf{v}}{\partial x^2} + \frac{\partial^2 \mathbf{v}}{\partial y^2} \right) - \frac{\partial q}{\partial y}
\]

(4)

Here they are \(V \equiv (u(y,t),0,0)\) denotes the velocity field. \(T(y,t)\) is the temperature of a fluid. The magnetic field frequency \(B_0\), \(\rho\) is a fluid density, \(\sigma\) is the fluid conductivity, \(k\) is a permeability, \(c_T\) is a specific heat at constant pressure, \(K_T\) is thermal conductivity, \(q\) is a nuclear heat flux, and \(\mu(T)\) fluid viscosity is temperature-dependent.

The temperatures at the channel's walls are expressed as:

\[T = T_0\text{ at } \bar{y} = 0,\text{ and } T = T_1\text{ at } \bar{y} = h\] (5)

The radioactive heat flux is calculated as follows:

\[
\frac{\partial q}{\partial y} = 4\eta^2(T_0 - T)
\] (6)

The absorption of radiation denoted by \(\eta\).

Prandtl-Eyring fluid has the following basic equation:

\[
S = -\bar{p} I + \tau
\]

(7)

\[
\tau = \frac{\mu}{\gamma} sinh^{-1} \left( \frac{\hat{g}}{C} \right) Y, \quad \gamma = \left[ \frac{1}{2} tr \left( Y \right)^2 \right], \quad Y = \nabla \mathbf{V} + (\nabla \mathbf{V})^T
\]

(8)
\[ \nabla \vec{V} = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}, \quad (\nabla \vec{V})^T = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix} \]

Then

\[ Y = \begin{bmatrix} 2u_x & u_y + v_x & u_z + w_x \\ u_x + v_y & 2v_y & v_z + w_y \\ w_x + u_y & w_y + v_z & 2w_x \end{bmatrix} \quad (9) \]

That means

\[ \tau_{xx} = 2 \begin{bmatrix} \frac{\sqrt{\tau}}{\left( \frac{\partial u}{\partial y} \right)} \sinh^{-1} \left( \frac{1}{\frac{\partial u}{\partial y}} \right) \end{bmatrix} u_x \quad (10) \]

\[ \tau_{xy} = 2 \begin{bmatrix} \frac{\sqrt{\tau}}{\left( \frac{\partial u}{\partial y} \right)} \sinh^{-1} \left( \frac{1}{\frac{\partial u}{\partial y}} \right) \end{bmatrix} (u_y + v_x) \quad (11) \]

\[ \tau_{yx} = 2 \begin{bmatrix} \frac{\sqrt{\tau}}{\left( \frac{\partial u}{\partial y} \right)} \sinh^{-1} \left( \frac{1}{\frac{\partial u}{\partial y}} \right) \end{bmatrix} (v_x + u_y) \quad (12) \]

\[ \tau_{yy} = 2 \begin{bmatrix} \frac{\sqrt{\tau}}{\left( \frac{\partial u}{\partial y} \right)} \sinh^{-1} \left( \frac{1}{\frac{\partial u}{\partial y}} \right) \end{bmatrix} v_y \quad (13) \]

By using the Maclaurin series, we have;

\[ \sinh^{-1} \left( \frac{1}{\frac{\partial u}{\partial y}} \right) = \frac{1}{\frac{\partial u}{\partial y}} - \frac{1}{6 \left( \frac{\partial u}{\partial y} \right)^3}, \quad \frac{\partial u}{\partial y} \ll 1 \quad (14) \]

So that

\[ \tau_{xx} = \tau_{yy} = 0 \quad \text{and} \quad \tau_{xy} = \tau_{yx} = \sqrt{\tau} B \left( \frac{1}{\frac{\partial u}{\partial y}} - \frac{1}{6 \left( \frac{\partial u}{\partial y} \right)^3} \right) \quad (15) \]

Substituting equation (6) into equation (4), so that the heat equation become;

\[ c_r \frac{\partial \theta}{\partial t} = K \frac{\partial^2 \theta}{\partial x^2} - 4\eta^2 (T_0 - T) \quad (16) \]

3. Method of Solution:

The motion equations that governing, The non-dimensional conditions that we can apply are as follows:

\[ \begin{align*}
    x & = \frac{x}{h}, \quad y = \frac{y}{h}, \quad u = \frac{u}{h}, \quad t = \frac{t}{h}, \quad \nu = \frac{\nu h}{\mu_0} \\
    \Delta T & = T_1 - T_0, \quad \theta = \frac{T - T_0}{\Delta T}, \quad \mu(\theta) = \frac{\mu(\theta)}{\mu_0}, \quad \tau_{xy} = \frac{h}{\mu_0 u} \tau_{xy},
\end{align*} \quad (17) \]

where \( U \) is the mean flow velocity.

Substituting equation (17) into equations (1), (2), (3), (5), (15) and (16), we have the following of non-d

\[ \frac{\partial u}{\partial x} + \frac{\partial \nu}{\partial y} = 0 \quad (18) \]

\[ Re \frac{\partial \nu}{\partial x} = -\frac{\partial p}{\partial x} + \sqrt{2\tau} \frac{\partial^2 u}{\partial y^2} - 3\sqrt{2S} \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + Gr \theta - \left( M^2 + \frac{\mu(\theta)}{\mu_0} \right) u \quad (19) \]

\[ Pe \frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \quad (20) \]

with \( \theta = 0 \) at \( y = 0 \) and \( \theta = 1 \) at \( y = 1 \)
Where \( Re = \frac{\rho h U}{\mu_0} \) is a Reynolds number, \( Da = \frac{k}{h^2} \) is a Darcy number, \( M = \frac{\sqrt{\varepsilon k_0 h^2}}{\mu_0} \) is a magnetic parameter, \( Pe = \frac{\rho h U c_T}{\kappa_T} \) is a Peclet number, \( N = \frac{4\eta^2 h^2}{K_T} \) is the radiation parameter, \( Gr = \frac{\rho g h^2 \Delta T}{\mu_0 U} \) is a Grashof number, \( Pr = \frac{\mu_0 c_T}{K_T} \) is a Prandtl number, \( Ec = \frac{U^2}{c_T \Delta T} \) is an Eckert number, \( Br = Pr Ec \) Brinkman number, and \( (\mathcal{F} = \frac{B}{\mu_0 A}, S = \frac{F u^2}{6h^2 A^2}) \) are the dimensionless Prandtl-Eyring fluid parameters.

The solution of equations (19) and (20) has been computed in the next section for two types of flows (Poiseuille flow and Couette flow).

with the boundary conditions:

\[ \theta(0) = 0, \quad \theta(1) = 1 \]  

The heat equation was solved in the second semester

4. Reynold’s Model of Viscosity[1]:

The following are the definitions of the Reynold's model and viscosity difference with temperature:

\[ \mu(\theta) = e^{-\alpha \theta} \]  

We get the following results when we use the Maclaurin series:

\[ \mu(\theta) = 1 - \alpha \theta, \quad \alpha << 1 \]  

When the viscosity is set to \( \alpha = 0 \) according to this condition, we can get the following result by replacing equation (24) with equation (19):

\[ Re \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} + \sqrt{\mathcal{F} T} \frac{\partial u}{\partial y} - 3 \sqrt{S} \frac{\partial u}{\partial y} + Gr \theta - \left( M^2 + \frac{(1-\alpha \theta)}{\partial x} \right) u \]  

5. Solution of the problem:

5.1. The temperature equation’s solution

The temperature equation is solved (20) with boundary conditions \( \theta(0) = 0, \theta(1) = 1. \) Let \( \theta(y, t) = \theta_0(y) e^{i\omega t} \) and by substituting into equation (14), after simplify the result, we obtain

\[ \frac{\partial^2}{\partial y^2} \theta_0(y) + (N^2 - i\omega Pe) \theta_0(y) = 0 \]  

\[ \frac{\partial^2 \theta_0}{\partial y^2} + (N^2 - i\omega Pe) \theta_0 = 0 \]  

The solution of equation (27), is:

\[ \theta_0(y) = \csc(\varphi) \sin(\varphi y) \]

where \( \varphi = \sqrt{N^2 - i\omega Pe} \).

Therefore, the temperature function is:

\[ \theta(y, t) = \csc(\varphi) \sin(\varphi y) e^{i\omega t} \]  

5.2. Solution of the momentum equation

5.2.1. Poiseuille flow

By above assumption (no-slip) condition the boundary conditions are \( \bar{u} = 0 \) at \( \bar{y} = 0, h. \) From equations (25) the non-dimensional boundary conditions are \( u(0) = u(1) = 0. \)
From equation (3), we have \( \frac{\partial p}{\partial y} = 0 \), so the pressure \( p \) depends on \( x \) only. Set \( \omega \) be an oscillation of frequency, and let
\[
\frac{\partial p}{\partial x} = \lambda e^{i\omega t}, \quad u(y, t) = u_0(y)e^{i\omega t}
\]
where \( \lambda \) is a real constant.

By Substituting equations (29) by equation (25), we have
\[
-\lambda = \sqrt{2\pi} \frac{\partial^2 u_0(y)}{\partial y^2} - 3\sqrt{2}e^{2i\omega t}S \left( \frac{\partial^2 u_0(y)}{\partial y^2} \right) \left( \frac{\partial u_0(y)}{\partial y} \right)^2 + Gr\theta_0 - \left( i\omega Re + M^2 + \frac{(1-\alpha\theta)}{\partial a} \right) u_0(y)
\]
(30)

Since the equation (30) is a nonlinear differential equation with a difficult exact solution, the perturbation technique would be used to find the solution:
\[
u_0 = u_0_0 + S u_0_1 + O(S^2)
\]
(31)

We obtain the following results by replacing equation (30) for equation (31), with maximum displacement \( u_0(0) = u_0(1) = 0 \), and then equating the like powers of \( S \), as seen in the following subsections:

1. **Zeros-order system (\( S^0 \))**
\[
\frac{\partial^2 u_{00}}{\partial y^2} - \frac{\left( i\omega Re + M^2 + \frac{(1-\alpha\theta)}{\partial a} \right) u_{00}}{\sqrt{2\pi}} = -\frac{(\lambda + Gr\theta_0)}{\sqrt{2\pi}}
\]
(32)
The boundary conditions are as follows: \( u_{00}(0) = u_{00}(1) = 0 \)

2. **First-order system (\( S^1 \))**
\[
\frac{\partial^2 u_{01}}{\partial y^2} - \frac{\left( i\omega Re + M^2 + \frac{(1-\alpha\theta)}{\partial a} \right) u_{01}}{\sqrt{2\pi}} = \frac{3}{\sqrt{2\pi}} e^{2i\omega t} \left( \frac{\partial u_{00}}{\partial y} \right)^2 \left( \frac{\partial^2 u_{00}}{\partial y^2} \right)
\]
(33)
The boundary conditions are as follows: \( u_{01}(0) = u_{01}(1) = 0 \)

Equations (32) and (33) are solved and the resulting substitution in equation (29) after substitution in equation (31), we obtain the solution of the equation of motion in the case of Poiseuille flow
\[
u(y, t) = \begin{pmatrix}
\frac{A}{B} + e^{i\sqrt{B}} \left( \frac{A}{B(1+e^{i\sqrt{B}})} \right) + e^{-i\sqrt{B}} \left( \frac{Ae^{-i\sqrt{B}}}{B(1+e^{-i\sqrt{B}})} \right) \\
-\frac{1}{HH^2(1+e^{i\sqrt{B}})^2} \left( 3A^3 e^{2i\omega t-3i\sqrt{B}y} + e^{i\sqrt{B}} \left( 4e^{i\sqrt{B}} + e^{-i\sqrt{B}} - 4e^{i\sqrt{B}}e^{(1+y)} - 4e^{-i\sqrt{B}}e^{(1+y)} \right) + 4\sqrt{BH}e^{i(1+y)}e^{(1+y)} - 4\sqrt{BH}e^{i(1+y)}e^{(1+y)} \right) \\
+ \frac{3A^2 e^{2i\omega t}(1-4e^{i\sqrt{B}}e^{1+y} - 4e^{-i\sqrt{B}}e^{1+y})}{HH^2(1+e^{i\sqrt{B}})^2} e^{i\sqrt{B}y} + \frac{3A^2 e^{2i\omega t}(1-4e^{-i\sqrt{B}}e^{1+y} - 4e^{i\sqrt{B}}e^{1+y})}{HH^2(1+e^{-i\sqrt{B}})^2} e^{-i\sqrt{B}y}
\end{pmatrix} e^{i\omega t}
\]
(34)

5.2.2. **Couette flow**

**Solution of the momentum equation**
In non-dimensional form, the boundary conditions are \( u(0) = 0, u(1) = U_0 \). The solution was determined using the perturbation method, and the results were shown in diagrams.

**6. Results and Discussion**
The impact of fluid parameters on velocity and temperature in the MHD oscillatory flow of Prandtl-Eyring fluid through a porous medium for Poiseuille flow and Couette flow in the basis of some observations during the graphics illustrations to have a clearer description of the physiological matter. The statistical solutions and diagrams were collected using the (MATHEMATICA-12) software. The
perturbation procedure is used to solve the momentum equation, and all of the results are graphed in area $0 \leq y \leq 1$, which is the diameter of the flow channel.

The velocity profile is seen in figures (2-13); each figure is divided into two parts, with (a) representing Poiseuille flow and (b) representing Couette flow, respectively. Both of these forms will be explained in terms of the two geometric categories of flow states. Figures (2-13) demonstrate the velocity profile of the Poiseuille flow (a). The velocity profile $u$ increases as $N$ increases in Figure 2. Figure 3 indicates that as the parameters $Pe$ are increased, the velocity profiles decrease. The velocity profile $u$ increases as $Gr$ increases in Figure 4. Figure 5 illustrates how the effect of $t$ on velocity profiles $u$ decreases. The velocity profile $u$ increases as $S$ increases in Figure 6. The velocity profile $u$ increases as $Da$ increases, as seen in Figure 7. Figure 8 illustrates that as the parameters $M$ are increased, the velocity profiles $u$ decrease. Figure 9 illustrates how the effect of $F$ on velocity profiles $u$ decreases. Figure 10 reveals that as the parameters $Re$ are increased, the velocity profiles $u$ rise. The effect of $\lambda$ on the velocity profiles feature $u$ vs. $y$ is depicted in Figure 11. Figure 12 illustrates how the velocity profile $u$ rises as the number $\alpha$ rises. Figure 13 indicates that as the parameters $\omega$ are increased, the velocity profiles decrease.

Figures (2-13) (b) depict the velocity profile of a (b) Couette flow. The velocity profile $u$ increases as each parameter $N$, $Gr$, $Da$, $\lambda$, $Re$, $S$ and $\alpha$ is increased, whereas $u$ decreases as $\omega$, $F$, $Pe$, $M$, and $t$ are increased. Since the upper wall of the channel moves at a steady rate($U_0 = 0.3$), the fluid flow velocity in the case of Couette flow is higher than in the case of Poiseuille flow. As a result, in Poiseuille flow, the maximum fluid velocity is in the center of the flow path, while in Couette flow, the highest speed is at the upper wall. Tables comparing the two modes of flow: Poiseuille and Couette. We can see in Figure 14 that as $Pe$ rises, the temperature $\theta$ rises as well. Figure 15 shows that as $N$ increases, the temperature $\theta$ rises. We can see in Figure 16 that the temperature $\theta$ drops as the temperature $\theta$ rises. We can see in Figure 17 that as $t$ increases, the temperature $\theta$ decreases.

![Fig (2) Velocity profile with different values $N = \{1.25,1.5,1.75,2\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \lambda = 0.9, \alpha = 0.5, Re = 1, F = 0.5, M = 1, Da = 1, Gr = 1, Pe = 2, t = 0.5, \xi = 0.05, U_0 = 0.3.$](image-url)
Fig (3) Velocity profile with different values $Pe = \{1,2,3,4\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \lambda = 0.9, Re = 1, F = 0.5, M = 1, Da = 1, Gr = 1, N = 1.25, t = 0.5, \alpha = 0.5, Pe = 2, \xi = 0.05, U_0 = 0.3$.

Fig (4) Velocity profile with different values $Gr = \{0.8,1,1.2,1.4\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \lambda = 0.9, Re = 1, F = 0.5, M = 1, Da = 1, t = 0.5, \xi = 0.05, N = 1.25, \alpha = 0.5, Gr = 1, Pe = 2, U_0 = 0.3$.

Fig (5) Velocity profile with different values $t = \{0.3,0.5,0.7,0.9\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \lambda = 0.9, Re = 1, \alpha = 0.5, F = 0.5, M = 1, Da = 1, \xi = 0.05, N = 1.25, Gr = 1, Pe = 2, U_0 = 0.3$. 
Fig (6) Velocity profile with different values $\xi = \{0.05, 0.1, 0.15, 0.2\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \lambda = 0.9, \text{Re} = 1, F = 0.5, M = 1, \text{Da} = 1, Gr = 1, Pe = 2, N = 1.25, \alpha = 0.5, t = 0.5, U_0 = 0.3$

Fig (7) Velocity profile with different values $\text{Da} = \{0.8, 1, 1.2, 1.4\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \lambda = 0.9, \text{Re} = 1, F = 0.5, M = 1, t = 0.5, \xi = 0.05, Gr = 1, Pe = 2, \alpha = 0.5, N = 1.25, U_0 = 0.3$

Fig (8) Velocity profile with different values $M = \{0.8, 1, 1.2, 1.4\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, t = 0.5, F = 0.5, \lambda = 0.9, \text{Re} = 1, \text{Da} = 1, \xi = 0.05, Gr = 1, Pe = 2, \alpha = 0.5, N = 1.25, U_0 = 0.3$
Fig (9) Velocity profile with different values $F = \{0.4, 0.5, 0.6, 0.7\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \tau = 0.5, \text{Re} = 1, \lambda = 0.9, \text{M} = 1, \text{Da} = 1, \xi = 0.05, \text{Gr} = 1, \text{Pe} = 2, \alpha = 0.5, N = 1.25, U_0 = 0.3$

Fig (10) Velocity profile with different values $\text{Re} = \{0.5, 1, 1.5, 2\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \tau = 0.5, F = 0.5, N = 1.25, \text{Gr} = 1, \text{Pe} = 2, \lambda = 0.9, \text{M} = 1, \text{Da} = 1, \xi = 0.05, \alpha = 0.5, U_0 = 0.3$

Fig (11) Velocity profile with different values $\lambda = \{0.7, 0.9, 1.1, 1.3\}$ for (a) Poiseuille flow and (b) Couette flow, with $\omega = 1, \tau = 0.5, \text{Re} = 1, F = 0.5, \text{M} = 1, \text{Da} = 1, N = 1.25, \text{Gr} = 1, \text{Pe} = 2, \xi = 0.05, \alpha = 0.5, U_0 = 0.3$. 
Fig (12) Velocity profile with different values $\alpha = \{0.3, 0.5, 0.7, 0.9\}$ for (a) Poiseuille flow and (b) Couette flow, with $t = 0.5, \lambda = 0.9, Re = 1, F = 0.5, M = 1, \omega = 1, Da = 1, N = 1.25, Gr = 1, Pe = 2, \xi = 0.05, U_0 = 0.3$.

Fig (13) Velocity profile with different values $\omega = \{0.8, 1, 1.2, 1.4\}$ for (a) Poiseuille flow and (b) Couette flow, with $t = 0.5, \lambda = 0.9, Re = 1, F = 0.5, \alpha = 0.5, M = 1, Da = 1, N = 1.25, Gr = 1, Pe = 2, \xi = 0.05, U_0 = 0.3$.

Fig (14) Influence of $Pe$ on Temperature for $\omega = 1, N = 1.25, t = 0.5$. 

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Fig (15) Influence of $N$ on Temperature for $\omega = 1, Pe = 2, t = 0.5$

Fig (16) Influence of $\omega$ on Temperature for $N = 1.25, Pe = 2, t = 0.5$

Fig (17) Influence of $t$ on Temperature for $N = 1.25, Pe = 2, \omega = 1$
7. Conclusion
The transient incompressible MHD oscillatory flow of Prandtl-Eyring non-Newtonian fluid in a porous medium under the influence of thermal radiation and magnetic field was investigated in this research. The numerical effects programming kit (MATHEMATICA-12) was used to find the solution, which was achieved using the perturbation method.

- The results show that the velocity of both types of flow, Poiseuille and Couette, increases as the parameters Reynolds, Darcy, Grashof number, radiation parameter, and static pressure rise.
- The velocity components shrink as the frequency of the oscillation, Peclet number, magnetic parameter, and time increased.
- We will illustrate this by increasing the radiation parameter and the Peclet number. The temperature decreased as the time and frequency of the oscillations increased.

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