A sparse least squares support vector machine used for SOC estimation of Li-ion Batteries

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Abstract: Li-ion batteries have been widely used in electric vehicles, power systems and home electronics products. Accurate real-time state-of-charge (SOC) estimation is a key function in the battery management systems to improve the operation safety, prolong the life span and increase the performance of Li-ion batteries. Kalman Filter has shown to be a very efficient method to estimate the battery SOC. However, the battery models are often built off-line in the literature. In this paper, a least squares support vector machine (LS-SVM) model trained with a small set of samples is applied to capture the dynamic characteristics of Li-ion batteries, enabling the online application of the modelling approach. In order to improve the model performance of battery model, a sparse LS-SVM model is first built by a fast recursive algorithm. Then, the batteries SOC is estimated using an unscented Kalman filter (UKF) based on the sparse LS-SVM battery dynamic model. Simulation results on the Hybrid Pulse Power Characteristic (HPPC) test data and the Federal Urban Drive Schedule (FUDS) test data confirm that the proposed approach can produce simplified yet more accurate model.

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1. INTRODUCTION

To tackle the global challenges such as climate change and environment pollutions due to the extensive consumption of fossil fuels, batteries energy storage systems have become increasingly popular in various applications to improve the acceptance and usage of electricity generated from renewables, such as power systems and electric vehicles (EVs) (Zhang et al. [2015]). Due to the merits of high energy density and long cycle life (Meng et al. [2016]), Li-ion batteries have been widely adopted as the energy sources for EVs and power systems. However, limited by its chemistry of a single cell, the storage should use a battery pack consisting of hundreds or more battery cells which is managed by a battery management system (BMS) for the safety. The state of charge (SOC), like a fuel gauge in an ICE vehicle, is an important performance indicator for managing the operation and control of batteries.

Over the years, a number researchers have paid considerable attention to the SOC estimations using various methods. These methods can be divided to two groups. The first one is model-free methods where the Ampere-hour method (Jeong et al. [2014]) and the open circuit voltage (OCV) based method (Petzl and Danzer [2013]) are among the most popular approaches (Zhang et al. [2015]). However, the measurement errors will be accumulated, leading to estimation deviations. The second one is the model-based methods, for example the equivalent electric circuit models (ECCMs) (Zhang et al. [2015]), artificial neural networks (Zhang et al. [2017]) and least square support vector machine (LS-SVM) (Meng et al. [2016], Shi et al. [2008]), etc.. Most of these methods are used to build model offline, and experiments need to be conducted to acquire sufficient data samples to train the model.

The Kalman filter is often used to improve the SOC estimation accuracy. Considering the nonlinear characteristics of the battery electrical dynamics, the extended Kalman filter (EKF) (Chen et al. [2013]) or the unscented Kalman filter (UKF) (Xiong et al. [2013]) and adaptive Kalman...
filter variants (Meng et al. [2016]) have been used in the battery SOC estimation. While the existing model based methods for the battery SOC estimation lead to more accurate results, the generalization performance of these method can be degraded if a number of unseen data samples, which are not used in the offline modelling, emerge in the applications. To overcome the drawback, online battery model identification have been used in the last few years, such as the RTLS-based observer method (Wei et al. [2018]) and the Gaussian process regression method (Shahinghui et al. [2018]). This paper follows this technical route, and an online battery state-space model is built. In order to reduce the calculation time, a least square support vector machine (LS-SVM) model using a small set of samples is built for SOC estimation.

The general LS-SVM method has a well-known drawback in terms of the sparsity. In this paper, a sparse least square support vector machine proposed in Zhang et al. [2012] is first used for the battery state-space model. From the reference Zhang et al. [2012], Li presented a sparsity solution to build a the least square support vector machine is obtained by selecting the features in high dimensions using the fast recursive algorithm (FRA) (Li et al. [2005]). The sparse least square support vector machine method uses a mapping function instead of the kener function which remains the information of the all samples.

The remainder of this paper is organized as follows. The battery state-space model is formulated in Section 2 for the batteries SOC estimation. Section 3 is introduces the sparse LS-SVM solution. In Section 4, the SOC estimation and the test data are presented in detail. The simulation results are analysed in Section 5. Finally, Section 6 concludes the paper.

2. THE BATTERY MODEL

Using the UKF method for the SOC estimation of Li-ion batteries, an accurate model needs to be built in the first. The model is described in state space equation including the last few years, such as the RTLS-based observer method (Meng et al. [2016]) have been used in the battery state-space model is formulated using LS-SVM:

\[
\begin{align*}
\dot{x}_k &= f(x_{k-1}, i_k) + s_k \\
V_k &= h(x_{k}, i_k) + v_k
\end{align*}
\]

where \(i_k, X_k\) and \(V_k\) are the terminal current, SOC and terminal voltage at the time instant \(k\) respectively, \(s_{k-1}\) and \(v_k\) represent the process and the measurement noise respectively which are determined by the estimation error \(e\).

2.1 State Equation

Battery SOC indicates the residual capacity and can be expressed in a recursive form

\[
X_k = X_{k-1} - \eta \cdot \Delta t \cdot i_k/Q_n
\]

where \(Q_n\) is the nominal capacity and the constant \(\eta = 1/3600\). \(X_k\) and \(i_k\) are the SOC and terminal current at the time instant \(k\) respectively, and \(\Delta t\) is the sampling time.

2.2 Measurement Equation

Suppose the discharging current, SOC and terminal voltage at the time instant \(k\) are \(i_k, SOC_k\), and \(V_k\) respectively, the measurement equation using the LS-SVM method referring for (Zhang et al. [2012] is expressed as follows

\[
V_k = h(u_k) = \sum_{i=1}^{m} w_{m,j} \exp\left\{-\frac{1}{2} (u_k - uc_j)^T \Gamma_j^{-1} (u_k - uc_j)\right\}
\]

where \(u_k = \{i_k, SOC_k\}\) is the control variable in the UKF method. \(w_{m,j}\) are linear coefficients to the Gaussian function \(\exp\{-\frac{1}{2} (u_k - uc_j)^T \Gamma_j^{-1} (u_k - uc_j)\}\) which is used to produce the \(j\)th dimension space, and \(m\) is the final dimension of the support vector space. \(uc_j\) and \(\Gamma_j\) are some training vectors and width to produce the support vectors.

3. THE SPARSE LS-SVM ALGORITHM

It is well-known the traditional LS-SVM method lacks of sparseness which affects the models generalization performance. Accordingly, a fast recursive algorithm is applied to select the features which is produced by the mapping function instead of the kernel function.

3.1 The conventional LS-SVM method

For \(N\) pairs of training samples \(\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}\), \(x \in \mathbb{R}^{N \times k}\) and \(y \in \mathbb{R}^{N \times 1}\), the system is formulated using LS-SVM:

\[
y = \Phi(x) \cdot w
\]

where \(w \in \mathbb{R}^{m \times 1}\) is the linear parameters, \(\Phi = \{\Phi_1(x), \Phi_2(x), \ldots, \Phi_N(x)\} \in \mathbb{R}^{N \times N}\) are basic mapping functions which are used for the space projection.

The least squares algorithm (Lapin et al. [2014]) is applied as follows:

\[
\min_{w, e} J = \frac{1}{2} ||w||^2 + \frac{1}{2\epsilon} \sum_{i=1}^{N} e_i^2
\]

\[
st \quad e_i = y_i - \Phi(x_i) \cdot w
\]
where $c$ is the regularization term. $\Phi(x_i) = [\Phi_1(x_i), \Phi_2(x_i), \ldots, \Phi_N(x_i)]$ is $N$-dimensional feature space. $e_i$ is the estimate error.

Using the Lagrangian method, the problem is reformulated as:
\[
\mathcal{L} = \frac{1}{2} ||w||^2 + \frac{1}{2c} \sum_{i=1}^{N} e_i^2 - \sum_{i=1}^{N} \alpha_i \{ \Phi(x_i) \cdot w + e_i - y_i \} \tag{6}
\]
where $\alpha = [\alpha_1, \ldots, \alpha_N]^T$ a vector of Lagrange multipliers.

Using the Karush-Kuhn-Tucker (KKT) optimality condition:
\[
\frac{\partial \mathcal{L}}{\partial w} = 0 \Rightarrow w = \sum_{i=1}^{N} \alpha_i \Phi(x_i),
\]

\[
\frac{\partial \mathcal{L}}{\partial e_i} = 0 \Rightarrow \alpha_i = \frac{e_i}{c}, \forall i \in \{1, \ldots, N\} \tag{7}
\]

\[
\frac{\partial \mathcal{L}}{\partial \alpha_i} = 0 \Rightarrow \alpha_i \Phi(x_i) + e_i - y_i = 0, \forall i \in \{1, \ldots, N\}
\]

the optimality problem is rewritten as
\[
M \alpha = y \tag{8}
\]
where $M = K + c \cdot I$ is the definite symmetric matrix. $I$ and $K$ is the unit matrix and the kernel function matrix respectively.

### 3.2 A sparse solution

A sparse solution proposed in Zhang et al. [2012] is proved to have good generalization performance in the classifier. The sparse model is built using FRA which avoids to solve a set of linear equations and retains the samples information by the mapping function. The mapping function using the Gaussian Basis function generates a $m$ dimension support vector space as
\[
\Phi_m(x_i) = [\Phi_1(x_i), \ldots, \Phi_m(x_i)] \tag{9}
\]

\[
\Phi_j(x_i) = \exp\left(\frac{1}{2}(x_i - s_j)^T \Gamma_{j+1}^{-1}(x_i - s_j)\right)
\]

where $j = 1, \ldots, m$ and $m$ is the maximal dimension of the mapped high dimensional space.

According to ([Zhang et al. 2012], define a recursive matrix $R$ of $k + 1$ regression terms as
\[
R_k = \mathbf{I} - \Phi_k [\Phi_k^T \Phi_k + cI]^{-1} \Phi_k^T \tag{10}
\]

and two intermediate matrices both $A \in \mathbb{R}^{m \times N} = \{a_{k+1,i}\}$ and $Ay \in \mathbb{R}^{m} = \{b_{k+1}\}$ is calculated
\[
a_{k+1,i} = \Phi_k^T R_k^{-1} \Phi_i \tag{11}
\]

\[
b_{k+1} = y^T R_k^{-1} \Phi_k + 1 \]

where $R_0 = \mathbf{I}$, $k = 0, \ldots, m - 1$ and $i = 1, \ldots, N$.

$a_{k+1,i}$ can be iteratively calculated as follows:
\[
a_{i,k+1} = \begin{cases} 
\Phi_k^T \Phi_{k+1} - \sum_{j=1}^{k} a_{j,i} a_{j,k+1} + 1, & i \leq k + 1 \\
\frac{c \cdot a_{k+1,i}}{c + a_{k+1,k+1}} - \sum_{j=k+2}^{i} a_{j,k+1} + 1, & i > k + 1 
\end{cases} \tag{12}
\]

\[
b_{k+1} = y^T \Phi_{k+1} - \sum_{j=1}^{k} a_{j,k+1} b_{j} + 1 \tag{13}
\]

where $i$ and $k$ is the position of candidate regressors in the matrix $\Phi$ given in (9).

Thus the net contribution of a candidate regressor is calculated as
\[
\Delta R_{k+1} = \frac{1}{2c} \frac{(b_{k+1})^2}{c + a_{k+1,k+1}} \tag{14}
\]

Finally, $m$ support vectors are selected, the linear coefficient vector $w$ is computed by
\[
\hat{w}_{m,i} = \frac{b_i}{c + a_{i,i}} - \frac{1}{c} \sum_{j=i+1}^{m} a_{j,i} b_j \tag{15}
\]

Accordingly, the procedure of the LS-SVM model building is detailed as follows.

#### Step 1. Initialization:

(a) Initialize the hyperparameters $c$, $s_j$ and $\Gamma_j^{-1}$, the max dimension $n$ of the high-dimension feature space and set the initial dimension $k = 0$.

#### Step 2. Forward regression vectors selection:

(a) Set $k = k + 1$, calculate $a_{i,j}$ $(i = 1, \ldots, k)$ and $b_j$ $(j = 1, \ldots, N)$ using (12) and (13).

(b) Calculate the net contributions of all candidate regressors using (14).

(c) Select the regressor with the maximum net contribution save as $k^{th}$ feature.

(d) If $k = m$ step, move to Step 3. Otherwise, return to 2(a).

#### Step 3. calculate the linear parameters:

(a) Calculate the weights using (15).

### 4. BATTERY SOC ESTIMATION USING UKF METHOD

The UKF method is proved more accurate for SOC estimation than the EKF method. The UKF estimate SOC using the sample distribution instead of the Jacobian matrix in the EKF. As discussed above, the state-space model of the Li-ion battery is formulated as follows
\[
\begin{align*}
X_k &= X_{k-1} - \eta \cdot \Delta t \cdot i_k / Q_n + s_k \\
Z_k &= \sum_{i=1}^{m} w_{m,j} \exp\left(\frac{1}{2}(u_k - u_{c,j})^T \Gamma_j^{-1}(u_k - u_{c,j})\right) + v_k
\end{align*} \tag{16}
\]

where $X_k$ and $Z_k$ is the SOC and the terminal voltage at the time instant $k$, $u_k = [i_k, X_k]$.

According to the model, the SOC estimation of Li-ion batteries using the UKF method is implement as

#### Step 1 Initialization.

Initialize the initial states $\hat{X}_0$ and the states covariance matrix $P_0$.

#### Step 2 Unscented Transformation.

1) Calculate the sigma points according the prior distribution.
\[
\begin{align*}
\chi_{k-1,i} &= \begin{cases} 
\hat{X}_{k-1}^i, & i = 0 \\
\hat{X}_{k-1}^i + (\sqrt{(L+\lambda)P_{k-1}^a}), & i = 1, \ldots, L \\
\hat{X}_{k-1}^i - (\sqrt{(L+\lambda)P_{k-1}^a}), & i = L+1, \ldots, 2L
\end{cases} \tag{17}
\end{align*}
\]
2) Calculate the weights to estimate the mean and covariance of the posteriori distribution

\[
\begin{align*}
\alpha^{(m)}_0 &= \frac{1}{(L + \lambda)} \\
\beta^{(c)}_0 &= \frac{1}{(L + \lambda)} + (1 - \alpha^2 + \beta) \\
\beta^{(m)}_i &= \frac{\lambda}{2(L + \lambda)} & i = 1, \ldots, 2L
\end{align*}
\] (18)

Step 3 Prediction. Update the predicted state and the corresponding covariance based on Equ. (1).

\[
\begin{align*}
\hat{X}_{k|k-1} &= \sum_{i=0}^{2L} c^{(m)}_i f(\chi_{k-1,i}, i_k) \\
P_{k|k-1} &= \sum_{i=0}^{2L} c^{(c)}_i (f(\chi_{k-1,i}, i_k) - \hat{X}_{k|k-1}) \cdot (f(\chi_{k-1,i}, i_k) - \hat{X}_{k|k-1})^T
\end{align*}
\] (19) (20)

Step 4 Correction (Estimation). Using the prediction error to compensate the SOC based on Equ. (1).

\[
\begin{align*}
\tilde{Z}_{k|k-1} &= \sum_{i=0}^{2L} c^{(m)}_i h(\hat{X}_{k|k-1}, i_k) \\
P_{z,k} &= \sum_{i=0}^{2L} (h(\hat{X}_{k|k-1}, i_k) - \tilde{Z}_{k|k-1})(h(\hat{X}_{k|k-1}, i_k) - \tilde{Z}_{k|k-1})^T \\
P_{zz,k} &= \sum_{i=0}^{2L} c^{(c)}_i (f(\chi_{k-1,i}, i_k) - \hat{X}_{k|k-1}) (h(\hat{X}_{k|k-1}, i_k) - \tilde{Z}_{k|k-1})^T \\
K &= P_{zz,k} P^{-1}_{z,z,k} \\
\hat{X}_k &= \hat{X}_{k|k-1} + K_k (Z_k - \tilde{Z}_{k|k-1})
\end{align*}
\] (21) (22) (23) (24) (25)

where \( L = 3 \), \( \lambda = \alpha^2(L + t) - L \) indicates the distribution of the sigma samples where \( \alpha \in [0.02, 1] \).

5. EXPERIMENTAL RESULTS

A 5-Ah LiFePo4 battery was tested in a 25°C temperature chamber. The terminal voltage and current are recorded every second with the 0.02% measurement error of the full scale range (FSR) at low power applications and 0.05% measurement error of the FSR at high power applications. Both the Hybrid Pulse Power Characterization (HPPC) and the Federal Urban Drive Schedule (FUDS) test procedures were implemented. The test data are discussed in detail in Zhang et al. [2015].

Based on a number of simulation experiments, the regulation parameter in (5) is chosen as \( c = 8.8 \), the width \( \Gamma_j \) is rounded at 0.4 and \( s_j \) is initialized considering all training samples. In order to reduce the computationa complexity, the measurement equation is retrained using a smaller data set which has only 10 samples. Firstly, the S-LSSVM model is evaluated by comparing with the general LS-SVM model (Meng et al. [2016]) and the SVM model (Suykens and Vandewalle [1999]). The LS-SVM method is built using the RBF kernel function which is tuned by k-fold cross validation. A SVM model is also optimized using the cross-validation. And the regulation parameter in the two method is set as 0.1.

The average time to implement the S-LSSVM method which is 0.0203 s is evidently less than the other two methods in the same computer. This is because the selection operation reduces the algorithm complexity. The detailed simulation results are illustrated in Fig 2 to 7. The absolute errors represent the differences between the measured values and model outputs.
LS-SVM model. The absolute errors of the model produced by the proposed method on the validation data are within the same level as on the training data, while the absolute errors of the LS-SVM model are even greater than 1. It is further shown that the LS-SVM model is poor in sparseness. The generalization capability of the proposed model is better than the others as it is shown that the validation accuracy of the S-LSSVM model is much better than the other methods. Accordingly, the S-LSSVM model is most accurate one compared to the other models.

Then, the batteries SOC is estimated using UKF algorithm where the measurement noise covariance is \( \sigma^2 = 0.06 \), \( L = 3 \), \( t = 1 \), \( \alpha = 1 \), \( \beta = 0 \). The simulation results are illustrated in Figures 8 -10. The estimated SOC matches the actual measurements very well. For the HPPC test, the absolute error ranges from \(-0.06\%\) to \(+0.08\%\) as shown in Fig 8 and the absolute error ranges from \(-0.02\%\) to \(+1.2\%\) as shown in Fig 9. Particularly, the absolute error ranges from \(-0.02\%\) to \(+0.06\%\) when the SOC is between 10\% and 90\% as shown in Fig 9 which is better than \([-2\%, +2\%]\) in the literature (Meng et al. [2016]). Similarly, the absolute error is range from \(-0.01\%\) to \(+0.02\%\) for the FUDS test data, while the errors are between \(-0.79\%\) and \(0.94\%\) as reported in (Meng et al. [2016]). When compared to the results reported in (Antón et al. [2013]), the absolute error using the Ampere hour counting method and the EKF method are within \(+15\%\) and \(5\%\) respectively on average. Thus, the SOC estimation results of the proposed method in this paper is more accurate.

6. CONCLUSION AND FUTURE WORKS

A sparse LS-SVM model has been built for online estimation of battery SOC the Li-ion. The developed model is applied to both the HPPC and the FUDS test data sets. The experimental results confirm that the method proposed by the authors not only produces accurate estimations for classification problems but also for modelling problems. The battery model produced by the sparse LS-SVM method is shown to outperform two other methods. Furthermore, the computing time is much less than the other method which leads to real-time estimation poten-
Once the battery model is built, the UKF algorithm is applied to battery SOC estimation. It is shown that for both the HPPC and the FUDS test data sets, the estimated SOC values match well with the actual SOC with a $10^{-7}$ mean square error for HPPC data set and $10^{-9}$ mean square error of the FUDS test data.

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