The Numerical Calculation and Experimental Measurement of the Inductance Parameters for Permanent Magnet Synchronous Motor in Electric Vehicle

Chao Jiang, Mingzhong Qiao, Peng Zhu

College of Electrical Engineering, Naval University of Engineering, Wuhan 430033, China

Email address: 653462459@qq.com (Chao Jiang)

Abstract. A permanent magnet synchronous motor with radial magnetic circuit and built-in permanent magnet is designed for the electric vehicle. Finite element numerical calculation and experimental measurement are adopted to obtain the direct axis and quadrature axis inductance parameters of the motor which are vital important for the motor control. The calculation method is simple, the measuring principle is clear, the results of numerical calculation and experimental measurement are mutual confirmation. A quick and effective method is provided to obtain the direct axis and quadrature axis inductance parameters of the motor, and then improve the design of motor or adjust the control parameters of the motor controller.

1. Introduction

Since the 21st century, environmental issues have increasingly become a major global problem. To achieve zero emissions, the electric vehicles have been widely concerned [1]. Permanent magnet synchronous motor has the advantages of high power density, high efficiency and wide speed range. It is especially suitable for electric vehicles, which require long mileage, frequent start and stop, frequent acceleration and deceleration. The inductance parameters of the motor have a significant effect on its steady and transient performance. However, the permanent magnet synchronous motor adopts permanent magnet excitation, its magnetic circuit characteristics and the method of obtaining the direct axis and quadrature axis inductance parameters are much different from those of the ordinary electrically excited motor. Electric vehicle permanent magnet synchronous motor is usually powered by the inverter, no matter which control method is used, vector control, direct torque control or other control methods, the direct axis and quadrature axis inductance parameters are important control parameters [2-5].

At present, there are many ways to obtain the inductance parameters of permanent magnet motor, which can be divided into analytic method, numerical calculation method and experimental measurement method. Numerical calculation can be achieved by using static magnetic field or transient magnetic field. Experimental measurement can be done by using off-line measurement, such as direct load method, voltage integration method and current decay method, as well as on-line identification [6-7].

In this paper, the finite element method based on static magnetic field is used to calculate the of the direct axis and quadrature axis inductance parameters of the motor designed. Then, experimental measurement is adopted to obtain the direct axis and quadrature axis inductance parameters of the
motor. The methods are simple and practical, while the results of calculation and measurement are confirmed by each other.

2. Design of permanent magnet synchronous motor for electric vehicle

The design requirements of an electric vehicle motor are shown in Table 1.

| Item               | Technical Specification | Item               | Technical Specification |
|--------------------|-------------------------|--------------------|-------------------------|
| Bus voltage        | 72VDC                   | Number of phases   | 3                       |
| Rated speed        | 2500rpm                 | Peak torque        | 120N\cdot m             |
| Rated power        | 12kW                    | Peak power         | 24kW                    |
| Rated torque       | 38.2N\cdot m            | Maximum speed      | 6000rpm                 |
| Maximum current    | 396A                    | Efficiency         | 90%                     |

According to the design theory of permanent magnet motor and the running characteristics of electric vehicle, the structure of built-in permanent magnet and radial magnetic circuit is selected, for its strong field weakening ability and high structural strength. The winding is single-layer cross winding, the core material is DW540-50 and the permanent magnet material is NdFe30. The entire motor is a fully enclosed structure, eliminating the need for ventilation and cooling system to increase the efficiency. The specific design process is shown in Figure 1.

- **Figure 1.** The design process of permanent magnet synchronous motor

The motor design result is shown in Table 2, the trial prototype is shown in Figure 2.

| Item                  | Design Value | Item                      | Design Value |
|-----------------------|--------------|---------------------------|--------------|
| Stator outer diameter | 175mm        | Number of parallel branches | 4            |
| Stator inner diameter | 105mm        | Rotor outer diameter      | 103.5mm      |
| Effective core length | 141.5mm      | Rotor inner diameter      | 70mm         |
| Number of pole pairs  | 4            | Magnetic bridge width     | 1.5mm        |
| Number of stator slots| 48           | Permanent magnet spacing  | 6mm          |
| Stator slot type      | Pear-shaped  | Permanent magnet width    | 26mm         |
The finite element method is used to simulate the electromagnetic field of the motor at the rated load. Figure 3 shows the magnetic flux distribution of the motor at rated load.

It can be seen that the magnetic flux density distribution in the air gap, rotor yoke, stator tooth is reasonable. The magnetic flux density is too high only in the small magnetic bridge, which needs to be optimized in the further.

3. Numerical calculation of direct axis and quadrature axis inductance

3.1. Direct axis and quadrature axis current convert to three-phase current

Based on the finite element method, when the inductance parameter is calculated by the static magnetic field, the applied excitation is three-phase current, so the direct current is converted into three-phase current.

Firstly, the space current vector is transformed from three-phase stationary A-B-C coordinate system to two-phase stationary D-Q coordinate system. Under the condition of constant power constraint, the vector transformation is shown as formula (1):

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} = \sqrt{2/3} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
  2 & -\sqrt{3} & \sqrt{3}
\end{bmatrix} \begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]

(1)

And then transform it from the two-phase stationary D-Q coordinate system to two-phase synchronous rotation d-q coordinate system, the vector transformation is shown as formula (2):

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} = \begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
\]

(2)

Where \( \theta \) is the angle between the rotor direct axis and the stator A-phase winding axis.

So, the vector transformation from the three-phase stationary A-B-C coordinate system to the two-phase synchronous rotation d-q coordinate system is:

\[
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix} = \sqrt{2/3} \begin{bmatrix}
  \cos \theta & \cos(\theta - 120^\circ) & \cos(\theta - 240^\circ) \\
  \sin \theta & \sin(\theta - 120^\circ) & \sin(\theta - 240^\circ)
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q \\
  i_r
\end{bmatrix}
\]

(3)

By solving the above equation, the three-phase current can be expressed as:

\[
\begin{cases}
  i_a = \sqrt{2/3} (i_d \cos \theta - i_q \sin \theta) \\
  i_b = \sqrt{2/3} (i_d \cos(\theta - 120^\circ) - i_q \sin(\theta - 120^\circ)) \\
  i_c = \sqrt{2/3} (i_d \cos(\theta - 240^\circ) - i_q \sin(\theta - 240^\circ))
\end{cases}
\]

(4)

The direct axis and quadrature axis current can be expressed by its stator current as:

\[
\begin{cases}
  i_d = \sqrt{3} i_s \sin \beta \\
  i_q = \sqrt{3} i_s \cos \beta
\end{cases}
\]

(5)

Where \( i_s \) is the stator current and \( \beta \) is the angle between \( i_q \) and \( i_r \).
Take (4) & (5), the three-phase current excitation can be expressed as:
\[
\begin{align*}
    i_a &= \sqrt{2}i_k \sin(\beta - \theta) \\
    i_b &= \sqrt{2}i_k \sin(\beta - \theta + 120^\circ) \\
    i_c &= \sqrt{2}i_k \sin(\beta - \theta + 240^\circ)
\end{align*}
\]
(6)

It can be seen from the above formula that when the stator current is constant, by changing $\theta$ and $\beta$, it is possible to obtain different phase current, that is, different static magnetic field.

### 3.2. three-phase inductance convert to direct axis and quadrature axis inductance

As the result of static magnetic field calculation, the inductance matrix is three-phase inductance matrix $L_{abc}$. Only by converting $L_{abc}$ to $L_{dq}$, can the direct axis and quadrature axis inductance be obtained, the specific transformation relationship is:
\[
L_{dq} = C^T \cdot L_{abc} \cdot C
\]
(7)

Where,
\[
C = \frac{1}{\sqrt{3}} \begin{bmatrix} 
\cos \theta & -\sin \theta \\
\cos(\theta - 120^\circ) & -\sin(\theta - 120^\circ) \\
\cos(\theta - 240^\circ) & -\sin(\theta - 240^\circ)
\end{bmatrix}
\]

Since the default effective core length is 1m in the two-dimensional field calculation, the default number of conductors per slot is 1, and the default number of parallel branches per phase is 1, to obtain the actual direct axis and quadrature axis inductance, it is also necessary to consider the effective length of the motor core, the number of conductors per slot and the number of parallel branches per phase. So the formula (7) is amended as follows:
\[
L_{dq} = C^T \cdot L_{abc} \cdot C \left( \frac{N_s}{P_B} \right)^2 \cdot l_{ef}
\]
(8)

Where, $N_s$ is the number of conductors per slot, $P_B$ is the number of parallel branches per phase per phase, $l_{ef}$ is the effective length of the core.

### 3.3. Calculation and analysis of direct axis and quadrature axis inductance

The finite element analysis model is established for the designed permanent magnet synchronous motor. For the convenience of calculating, the angle $\theta$ is taken as 0, that is, the rotor pole is aligned with the axis of A-phase winding, as shown in Figure 4.

![Figure 4. The finite element model of permanent magnet synchronous motor](image)

According to formula (6), the current excitation in the three-phase winding is set. Solving finite element static magnetic field can result in $L_{abc}$, and then the direct axis and quadrature axis inductance can be obtained by formula (8). Direct axis and quadrature axis current ranges from 10A to 280A, the sweeping step size is 10A. The angle $\beta$ is set to 90 $^\circ$, 0 $^\circ$, and 45 $^\circ$ respectively, to simulate the case where only the direct axis current is loaded, only the quadrature current is loaded, the current of both axes are loaded.

When only the direct axis current is loaded, $i_b = i_c = -i_a/2$, $i_q = 0$, how the direct axis and quadrature axis inductance changes with the current is shown in Figure 5(a). It can be seen that the
curves of both direct axis and quadrature axis inductance are relatively flat. The direct axis inductance decreased by 16.3%, the quadrature axis inductance decreased by 5.71%.

When only the quadrature axis current is loaded, \( i_a = 0 \), \( i_b = -i_c \), \( i_d = 0 \), how the direct axis and quadrature axis inductance changes with the current is shown in Figure 5(b). It can be seen that the curve of direct axis is still relatively gentle, but the quadrature axis inductance decreased by 33.1%.

When the direct axis and quadrature axis currents are loaded at the same time, the direct axis magnetic path and the quadrature axis magnetic path are coupled together, how the direct axis and quadrature axis inductance changes with the current is shown in Figure 5(c). Direct axis inductance declined by 13.6%, it is not much different compared with the previous two situations. The quadrature axis inductance decreased by 16.9%, between the previous two cases.

\[
\begin{align*}
\text{(a)} & \quad u_d = R i_d - \omega_L i_q + L_d \frac{di_d}{dt} \\
\text{(b)} & \quad u_q = R i_q + \omega_L i_d + L_q \frac{di_q}{dt} + \omega \psi_f \\
\text{(c)} & \quad u_d = R i_d + L_d \frac{di_d}{dt} \\
& \quad u_q = R i_q + L_q \frac{di_q}{dt}
\end{align*}
\]

4. Experimental measurement of direct axis and quadrature axis inductance

The principle of experimental measurement is from the voltage equation of permanent magnet synchronous motor. By measuring the current response curve with the DC voltage act on the motor, inductance parameters are obtained.

In the d-q coordinate system, the voltage equation of the permanent magnet synchronous motor is:

\[
\begin{align*}
\text{(9)} & \quad u_d &= R i_d - \omega_L i_q + L_d \frac{di_d}{dt} \\
\text{(10)} & \quad u_q &= R i_q + \omega_L i_d + L_q \frac{di_q}{dt} + \omega \psi_f
\end{align*}
\]

Where \( R \) is the resistance, \( \omega_L \) is the electrical angular velocity, \( \psi_f \) is the rotor magnetic flux linkage.

When the motor rotor is stationary, i.e. \( \omega_L = 0 \), the voltage equation can be simplified as:

\[
\begin{align*}
\text{(11)} & \quad u_d &= R i_d + L_d \frac{di_d}{dt} \\
\text{(12)} & \quad u_q &= R i_q + L_q \frac{di_q}{dt}
\end{align*}
\]
When applying a constant direct axis voltage, the corresponding current response is:

\[ i_d = \frac{U_d}{R} \left(1 - e^{-\frac{R}{L_d}t} \right) \]  

(13)

This is a first-order inertia, the current response increase exponentially, \( L_d / R_s \) is the time constant, when \( t = L_d / R_s \), there is:

\[ i_d = \frac{U_d}{R} \left(1 - e^{-\frac{R}{L_d} \frac{R_s}{R}} \right) \approx 0.632 \frac{U_d}{R} \]  

(14)

So:

\[ L_d = t_{d0.632} R \]  

(15)

\( t_{d0.632} \) is the time needed for the current to rise to a value about 0.632 times of stable value \( U_d / R \).

Similarly:

\[ L_q = t_{q0.632} R \]  

(16)

\( t_{q0.632} \) is the time needed for the current to rise to a value about 0.632 times of stable value \( U_q / R \).

Connect the winding circuit as shown in Figure 6, \( R_s \) is a current limiting resistor, \( R_p \) is phase winding resistance, \( R = R_p + R_s \). Measured by the milliohm meter, \( R_s = 100 \text{m}\Omega \), \( R_p = 4.5 \text{m}\Omega \).

When the constant terminal voltage \( U \) is applied, the stator current \( I \) will produce a constant magnetic field coinciding with the axis of the A-phase winding when it is stabilized. The rotor magnetic pole will be aligned with the A-phase winding axis under the magnetic force of the stator magnetic field. After disconnecting the DC power supply, the direct axis of rotor will continue to align with the axis of A-phase winding.

According to the method in Figure 6, the power is turned on again, there is \( U = U_d \). There is a current response in the main circuit, \( i = i_d \). \( t_{d0.632} \) is the time required to rise to 0.632 times of the steady-state value, which can be obtained by the current response curve shown in Figure 7(a), \( t_{d0.632} = 0.819 \text{ms} \).
Disconnect the DC power supply and rotate the rotor clockwise or counter-clockwise by 90° under the guidance of the angular position indicator, so that the quadrature axis of rotor is aligned with the axis of A-phase winding, and then fix the rotor. In the same way, turned on the DC power again, and there is $U = U_q$. There is a current response in the main circuit, $i = i_q$. $t_{q,0.632}$ is the time required to rise to 0.632 times of the steady-state value, which can be obtained by the current response curve shown in Figure 7(b), $t_{q,0.632} = 1.633 ms$.

Take formula (15) and (16):

$$L_d = 0.08743 mH, \quad L_q = 0.1743 mH$$

Compared with the numerical calculation, the error of the direct axis and quadrature inductance is 3.3% and 3.4% respectively. It may be due to the ignorance of end leakage inductance in two-dimensional finite element calculation. From the perspective of engineering, the error is within the acceptable range.

5. Conclusion
In this paper, the built-in permanent magnet and radial magnetic circuit structure are used to design the permanent magnet synchronous motor of electric vehicle. On this basis, the finite element numerical calculation and experimental measurement are used to obtain the direct axis and quadrature inductance parameters which are important in the motor control process. From the calculated electromagnetic field distribution, it can be seen that the design of the motor is reasonable. From the change of the inductance with the current, it can be seen that the quadrature axis is more affected by the current, while the direct axis inductance is less affected by the current. The calculation method is simple, the principle of measurement is clear, the operation is easy, the error of calculation and measurement is small, the results of calculation and measurement are mutual confirmation. This article provides a quick and effective method for motor designers and controller designers of permanent magnet synchronous motor used in electric vehicles to quickly obtain the inductance parameters of the direct axis and quadrature axis, so that they can improve the design or adjust the control parameters.

References
[1] Z. Q. Zhu, D. Howe. Induction and switched-reluctance machines can provide the needed characteristics, but permanent magnet brushless machines offer a higher efficiency and torque density [P]. Electrical Machines and Drives for Electric, Hybrid, and Fuel Cell Vehicles. 2007: 746-765.
[2] R. Duta, M. F. Rahman. A comparative Analysis of Two Test Methods of Measuring d- and q Axes Inductances of Interior Permanent-Magnet Machine[J]. IEEE Transactions on Magnetics. 2006: 3712-3718.

[3] K. J. Meessen, P. Helin, et al. Inductance Calculations of Permanent-Magnet Synchronous Machines Including Flux Change and Self-and Cross-Saturations [J]. IEEE Transactions on Magnetics. 2008: 2324-2331.

[4] L. Chen Chang. An Improved FE Inductance Calculation for Electrical Machines [J]. IEEE Transactions on Magnetics. 1996: 3237-3245.

[5] Tao Sun, Soon-O Kwon, Suk-Hee, et al. Investigation and comparison of inductance calculation methods in interior permanent magnet synchronous motors[J]. Proceedings of the 11th International Conference on Electrical Machines and Systems. 2008:3131-3136.

[6] Y. S. Chen, Z. Q. Zhu. Calculation of d- and q-Axis Inductances of PM Brushless ac Machines Accounting for Skew [J]. IEEE Transactions on Magnetics. 2005: 3940-3942.

[7] T. J. E. Miller, Performance Estimation of Interior Permanent-Magnet Brushless Motors Using the Voltage-Driven Flux-MMF Diagram[J]. IEEE Transactions on Magnetics. 2006: 1867-1872.