Dynamic circulation in curve with $R = 300$ m radius for C.F.R. 060 - EA 5100kW electric locomotive

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Abstract. In the paper we study the dynamic circulation in curve with 300 m radius of rail for C.F.R. 060-EA 5100kW electric locomotive. This study verifies simultaneous disposition of bogies in diagonal, intermediary and string positions through determination of dependences for lead forces, guide forces, centrifugal forces and of polar distance towards speed. We also verify safety against derailment, and we determine maximum circulation speed admitted in curve.

1. Simplifying hypothesis

On vehicle’s circulation in circular curves and constant speed, in between wheel and track contact points quasi - static forces of interaction appear. Their value depends of external forces of vehicle and they determine vehicle’s position at given speed.

Through dynamic enrollment of vehicles in curve we understand the determination of variation for quasi – static interaction forces between track and wheel in curve depending on movement speed. It is determined:

- Directive forces between rail track and wheel flange, $P_i$;
- Guiding forces, $Y_i$;
- Safety criteria against derailment $(Y/Q)_\text{lim}$.

Based on value of these parameters which has to have certain limits it is possible to establish the maximum speed for the vehicle to circulate through a curve with exact ray without the danger of derailment, rail track deripement or excessive usage of wheels flanges for guiding wheels and rail tracks.

To make easy the calculus of forces in directive points in all studies we have the following simplifying hypothesis [1-4]:

- The rolling surface of flanges is considered to be cylindrical and friction between wheel’s flange and rail track is neglected;
- It is considered that rail tracks and wheels are new, so the contact is in two points (A - support, B - guiding);
- $N_i$ forces that appear in guiding points B, directed after common normal of the two contact surfaces are replaced by their horizontal components;
- In equilibrium equation it is introduced $P_i$ forces named guiding forces;
- Advance to attack b is neglected being very small so it is considered that guiding forces act in vertical plan of the axle;
- Rolling speed, it is considered to be constant;
- Traction or braking forces are neglected (it is considered free movement that leads to maximum $P_i$);
- The loads on the wheels are considered the same and equal to $Q_0$ static load so the rotation pole is on bogie’s ax.
- The driveway is considered perfect circular with constant overwide and overhight;
- All the forces that act on vehicle are considered to be in the plan tangent to driveway surface;
- The driveway and the vehicle are considered undeformable in horizontal plan;
- The friction forces between the rolling surface of wheel and rail track which depend of load on wheel can be considered:
  - Coulombian forces with constant friction coefficient (Heumann);
  - With friction coefficient depending on pseudo slip of the wheel but isotropic (Lévi, Müller, ORE);
  - With anisotropic friction coefficient after longitudinal and transversal direction considering pseudo slip forces tangential $T_x$ và $T_y$ (Kalker).

2. The general scheme of bogie with acting forces and equilibrium equations
To assure the circulation through curve of engines with bogies they are frequently build with coupled bogies in between with a transversal or slanting couple, rigid or elastic.

The bonding couple between bogies has the role to decrease the guiding forces of wheels that roll the track in curves and it is called elastic couple.

Experimentally resulted that forces developed in these bonding couples reduces a lot the guiding forces especially at low speed and small rays of curves circulation. Also, the couples reduce screeching oscillations of bogies.

In certain case of locomotive $C_0C_0$, 060 – EA 5100 kW type, which has an elastic and slant couple and rolls in a curve with $R$ ray the forces and moments appeared are presented in Figure 1 and it is considered that $F_y \neq 0$, $F_x = 0$, $M_t \neq 0$, $\mu \neq$ constant and it is accepted friction coefficient after Müller (so it is considered the friction isotopic coefficient after the two directions).

![Figure 1. The forces and moments acting over bogies](image-url)

To systemize calculus the parameters are reported at a coordinate system xOy, the positive sense of forces and moments being the one indicated in the Figure 1.

At alignment circulation we have next certain parameters: rotation poles $P_1$ and $P_1$, distanced at $2l_p$; axles 1 ... 6 and distances between axles ($l_{13}$ and $l_{46}$), respective between rotation poles and external
axles noted with $l_p$; the length of axle in alignment $b_0$ and inclination angle in alignment $\gamma_0$; parameters $l$ and $t$ of couples braces.

It is written the equilibrium equations for the two bogies [1-4]:

$$
(\sum F_k)_y = 0 \quad \text{forces after Oy axis;}
$$

$$
(\sum M_k)_p = 0 \quad \text{the moment in relation to P pole of every bogie accordingly the signs conventions from Figure 1, and in diagonal $P_3 < 0$ and $P_6 < 0$.}
$$

The equilibrium equation system for bogie I is:

$$
\begin{align*}
(1) & \quad \begin{cases} 
P_1 + P_3 - C_l - F_v - F_{ly} - 2Q \sum_{i=1}^{3} (\mu_i \cos \xi_i) = 0, \\
|l_{13} - p_1|; (-p_1) \\

P_1p_1 - P_3(l_{13} - p_1) - C_i(p_1 - c_i) - F_v(p_1 - l_{13}) - M_{rl} + \\
+ F_{l_y}(l + l_{13} - p_1) + F_{lx}t - 2Q \sum_{i=1}^{3} (\mu_i d_i) = 0,
\end{cases}
\end{align*}
$$

and for bogie II is:

$$
\begin{align*}
(2) & \quad \begin{cases} 
P_4 + P_6 - C_{ll} - F_v - F_{l_{ly}} - 2Q \sum_{i=4}^{6} (\mu_i \cos \xi_i) = 0, \\
|l_{46} - p_4|; (-p_4) \\

P_4p_4 - P_6(l_{46} - p_4) - C_{ll}(p_4 + c_{ll} - l_{46}) - F_v(p_4 + l_{ll} - l_{46}) - M_{rll} + \\
+ F_{l_{ly}}(l + p_4) + F_{lx}t - 2Q \sum_{i=4}^{6} (\mu_i d_i) = 0,
\end{cases}
\end{align*}
$$

where $\mu$ is the friction coefficient, $\xi$ is the friction angle and $d$ is the brace of the friction forces.

Equations systems (1) and (2) contain unknown $P_1$, $P_3$, $P_4$, $P_6$, $F$, $p_1$, $p_4$ and $C$ (or $v$), so they are compatible undetermined systems. Therefor the systems can be solved only in relation to main unknown $P_1$, $P_3$, $P_4$ and $P_6$ which can be determined only if we know concrete simultaneous positions of the two bogies.

For this purpose, first relation from equations system (1) multiply with $(l_{13} - p_1)$, respective with $(-p_1)$ and the relation gather second from the same system obtaining for bogie I:

$$
P_1 = \frac{1}{l_{13}} \cdot \left[ C_l(l_{13} - c_l) + F_v(l_{13} - l_{13}) + M_{rl} - F_{l_y}l - F_{lx}t + \\
+ 2Q \left( \sum_{i=1}^{3} (\mu_i d_i) + (l_{13} - p_1) \cdot \sum_{i=1}^{3} (\mu_i \cos \xi_i) \right) \right].
$$

respective:

$$
P_3 = \frac{1}{l_{13}} \cdot \left[ C_i c_l + F_v l_{13} - M_{rl} + F_{l_y}(l + l_{13}) + F_{lx}t - \\
- 2Q \left( \sum_{i=1}^{3} (\mu_i d_i) - p_1 \sum_{i=1}^{3} (\mu_i \cos \xi_i) \right) \right],
$$

and first relation from equations system (2) multiply with $(l_{46} - p_4)$, respective with $(-p_4)$ gathering second relation from the same equations system resulting for bogie II:

$$
P_4 = \frac{1}{l_{46}} \cdot \left[ C_{ll}c_{ll} + F_v l_{46} + M_{rll} - F_{l_{ly}}(l + l_{46}) - F_{lx}t + \\
+ 2Q \left( \sum_{i=4}^{6} (\mu_i d_i) + (l_{46} - p_4) \cdot \sum_{i=4}^{6} (\mu_i \cos \xi_i) \right) \right]
$$

and

$$
P_6 = \frac{1}{l_{46}} \cdot \left[ C_{ll}(l_{46} - c_{ll}) + F_v(l_{46} - l_{ll}) - M_{rll} + F_{lx}l + F_{lx}t - \\
- 2Q \sum_{i=4}^{6} (\mu_i d_i) - p_4 \sum_{i=4}^{6} (\mu_i \cos \xi_i) \right].
$$
In these relations we have locomotive weight with \( G_l = 12Q \):
\[
G_l = C_{l1} = C = \frac{G_l}{2g} \left( \frac{x^2}{R} - \frac{gh}{2s} \right) = \frac{6Q}{g} \left( \frac{x^2}{R} - \frac{gh}{2s} \right).
\] (7)
\[
d_i = \sqrt{p_i^2 + s^2}; \quad \cos \xi_i = \frac{p_i}{d_i}; \quad p_i = p - l_{1i}.
\] (8)

The systems of equations are solved imposing the position of bogies and giving values to polar distance \( p_1 \) and \( p_4 \) or to speed \( v \) in m/s (only when both bogies are in diagonal or string position).

3. Determination of rappel moment (\( M_r \))

The rappel moment is expressed through relation [1-4]:
\[
M_r = k_r \cdot \beta,
\] (9)

where \( k_r \) is angular rigidity of dispositive of rappel and it is determined with a calculus relation specific to every railway vehicle to which is calculated the dynamic circulation in curve for a given \( R \) ray and \( \beta \) is the rotation angle of bogies towards the box.

For electric locomotive C.F.R. 060 – EA of 5100 kW in [5] is determined the variation \( M_r(\beta) \) which is represented in Figure 2.

![Figure 2. The variation of rappel moment](image)

When during rolling on rail track the axles 1 and 4 are in contact with external part of track the rotation angles \( \beta_1 \) and \( \beta_2 \) of bogies towards the box, respective the movement of the center of the box towards the normal through the center of curve on box’s ax x (see Figure 3) is determined with relations [1-4]:
\[
x = \frac{1}{4l_p} \left( 2p_4 - l_{13} - 2l_p (p_1 + p_4 - l_{13}) + 2R \right),
\] (10)
\[
\beta_1 = \arctg \left[ \frac{-(p_1 - l_{1p}) \sqrt{R^2 - p_1^2 + (l_p + x)^2} \sqrt{R^2 - p_1^2 - (l_p - x)^2 + (p_1 - l_{1p})^2}}{R^2 - p_1^2 - (l_p + x)^2} \right],
\] (11)
\[
\beta_2 = -\arctg \left[ \frac{(p_4 + l_{1p} - l_{14}) \sqrt{(R - l_4)^2 - p_4^2 + (l_p - x)^2} \sqrt{(R - l_4)^2 - p_4^2 - (l_p - x)^2 + (p_4 + l_{1p} - l_{14})^2}}{(R - l_4)^2 - p_4^2 - (l_p - x)^2} \right].
\] (12)
4. The calculus of couple’s parameters and of force from the couple

If the vectorial contour is projected A-3-I-II-4-B-A from Figure 4 on horizontal and vertical the transversal inclination angle of transversal inclined couple is given by relation [1-4]:

\[ \gamma = \arctg \left( \frac{2 \xi - l_{x} + d \left( \cos \beta_{1} + \cos \beta_{2} \right) + t \left( \sin \beta_{1} + \sin \beta_{2} \right)}{l_{x} + d \left( \sin \beta_{1} + \sin \beta_{2} \right) + t \left( \cos \beta_{1} + \cos \beta_{2} \right)} \right) \]  \tag{13}

and the length of the couple in curve is:

\[ b = \frac{f \left( \cos \beta_{1} + \cos \beta_{2} \right) + l_{x} + d \left( \sin \beta_{1} + \sin \beta_{2} \right)}{\cos \gamma} \] \tag{14}

During the circulation in curve with R ray the bogies are spinning towards the box of the engine with angles \( \beta_{1} \) and \( \beta_{2} \), as it is presented in figure 4 and length b as much as inclination \( \gamma \) of couple is modified.

The length’s modifying in curve towards alignment (couple’s arrow) is [1-4]:

\[ f = |b_{0} - b| \] \tag{15}

and leads to deformation of arc only if exceeds free movement \( j_{c} \) from the couple, that would be \( f > j_{c} \).

Noting with \( F_{oc} \) the pretension force and \( k_{c} \) the rigidity of the arc, the force from the couple can be defined through relations [1-4]:

\[
F = 0, \text{ if } f < j_{c}; \\
F \in [0; F_{oc}], \text{ if } f = j_{c}; \\
F = F_{oc} + k_{c} \left( f - j_{c} \right), \text{ if } f > j_{c}. \tag{16}
\]

Into a curve orientated through right towards the given sense and to brace position as in Figure 4, the couple condense (\( b < b_{0} \)) and F forces are orientated as in figure forming angles \( \gamma - \beta_{2} \) towards the normals of bogies axes having the components:
\[
F_{1x} = F \cdot \cos(\gamma - \beta_1); \quad F_{2x} = F \cdot \cos(\gamma - \beta_2); \\
F_{1y} = F \cdot \sin(\gamma - \beta_1); \quad F_{2y} = F \cdot \sin(\gamma - \beta_2).
\]

(17)

5. Circulation with both bogies in diagonal

It is considered the situation when bogies at low speed circulate in diagonal, meaning \(p_1 = p_4 = p_{\text{max}}\).

At \(P_3 = 0\) and \(P_6 = 0\), at speeds of detachment from diagonal of the two bogies (\(v_{\text{dI}}\) and \(v_{\text{dII}}\)) we have for bogie I [1-4]:

\[
C_i = \frac{1}{e_i} \cdot \left\{ -F_{\nu}l_{\nu 1} + M_{rff} - F_{\nu}y(l + l_{13}) - F_{\nu}x + 2Q \left[ \sum_{i=1}^{3} \left( \mu_i d_i \right) - p_1 \sum_{i=1}^{3} \left( \mu_i \cos \xi_i \right) \right] \right\}
\]

and for bogie II:

\[
C_{II} = \frac{1}{e_{II}} \cdot \left\{ -F_{\nu}(l_{46} - l_{\nu 11}) + M_{rff} - F_{\nu}y - F_{\nu}x + 2Q \left[ \sum_{i=1}^{6} \left( \mu_i d_i \right) - p_4 \sum_{i=1}^{6} \left( \mu_i \cos \xi_i \right) \right] \right\}.
\]

(18)

Having:

\[
C_i = C_{II} = C,
\]

result that detaching speed from diagonal of first bogie is:

\[
v_{\text{dI}}^2 = Rg \cdot \left\{ \frac{h}{2s} - \frac{F_{\nu}(l_{46} - l_{\nu 11}) - M_{rff} + F_{\nu}y + F_{\nu}x \tau}{6Q_{\nu}} + \frac{1}{3e_i} \cdot \left[ \sum_{i=1}^{3} \left( \mu_i d_i \right) - p_1 \sum_{i=1}^{3} \left( \mu_i \cos \xi_i \right) \right] \right\}
\]

(19)

and of the second bogie is:

\[
v_{\text{dII}}^2 = Rg \cdot \left\{ \frac{h}{2s} - \frac{F_{\nu}(l_{46} - l_{\nu 11}) - M_{rff} + F_{\nu}y + F_{\nu}x \tau}{6Q_{\nu}} + \frac{1}{3e_{II}} \cdot \left[ \sum_{i=1}^{6} \left( \mu_i d_i \right) - p_4 \sum_{i=1}^{6} \left( \mu_i \cos \xi_i \right) \right] \right\}
\]

(20)

If \(v_{\text{dI}}^2 > 0\) and \(v_{\text{dII}}^2 > 0\), which is less probable for the bogies tied through elastic couple result that the two bogies circulate simultaneously in diagonal and with relation (20) we determine the detaching speed \(v_{\text{d}}\).

For different values of speed \(v \in [0, v_{\text{dI}}]\) it is determined the centrifugal uncompensated force \(C\) with equation (7), sizes \(x, \beta_1, \beta_2, \gamma, \beta, f, F, F_{1x}, F_{1y}, F_{2x}, F_{2y}, M_{r1}, M_{r2}, p_i, d_i, \xi_i, F_6\) with afferent relations and from relations (3) … (6) results the guiding forces \(P_1, P_2, P_3\) and \(P_4\). If results that \(v_{\text{dI}}^2 < 0\) and \(v_{\text{dII}}^2 < 0\), that means that at \(v = 0\) first bogie is no longer in diagonal position but only the second.
is. Actually, the force moment F in relation to axle I being bigger than in relation with axle 4, it is possible that at \( \nu = 0 \) first bogie to circulate in free position. To determine the position of first bogie at \( \nu = 0 \) when \( \nu_1^2 < 0 \) and \( \nu_2^2 < 0 \), we determine:

\[
C_0 = -\frac{g_1h}{2s},
\]

and at \( p_2 = p_{max} \), from equalizing centrifugal forces \( C_0 \) and \( C_1 \) calculated with relation (18) results the polar distance \( p_{max} < p_{max} \).

6. Circulation with bogie I in free position and bogie II in diagonal position

In this case \( p_1 < p_{max} \), respective \( f > j_c \) and \( P_3 = 0 \). At speed increase also second bogie detach from diagonal at \( \nu_2^2 > \nu_1^2 \) resulting \( P_6 = 0 \), when we still have \( p_4 = p_{max} \) (1-4).

From equality of centrifugal forces \( C_i \) given by relation (18) and \( C_{II} \) given from relation (7) at \( p_4 = p_{max} \) we obtain value \( p_1 \), which is actually the polar distance of bogie I when bogie II detaches from diagonal.

At circulation in diagonal of bogie II with \( \nu \in (\nu_{dI}, \nu_{dII}) \), respective \( \nu \in (0, \nu_{dII}) \) if bogie I haven’t circulated in diagonal, choosing \( p_1 \in (p_{max}, p_1') \), respective \( p_1 \in (p_{max}, p_1'') \) and \( p_4 = p_{max} \), calculating all necessary and dynamic geometrical parameters, from (18) and (7) result \( C_i \), respective \( C_{II} \), from (3) result \( P_1 \), from (5) result \( P_4 \) and from (6) result \( P_6 \), respective from (16) result \( F \).

If also \( \nu_{dII}^2 < 0 \), so at \( \nu = 0 \) none of the two bogies does not circulate in diagonal then from equaling centrifugal forces \( C_0 \) with \( C_1 \) given by relation (18), respective \( C_0 \) with \( C_{II} \) given by relation (19) result the polar distances \( p_{max}'' < p_{max} \) and \( p_{max}' < p_{max} \).

Due to interdependency between every centrifugal force and both polar distances through force from couple respective of all coupled’s parameters and dynamics mentioned this calculus is very complex and ask for a long time to be solved (iteration with two variable parameters \( p_1 \) and \( p_4 \)). That is why it is recommended that in this situation at determination of dependency between guiding forces and speed to start from a high speed, close to maximum speed where in general bogies circulate in string position so a situation well determined \( (p_1 = p_4 = p_{min}) \).

7. Circulation with both bogies in free position

After the second bogie detached from diagonal meaning also \( P_6 = 0 \), values are given to \( p_1 \in (p_1', p_{max}) \) or \( p_1 \in (p_1'', p_{min}) \) and from equality of centrifugal forces \( C_i = C_{II} \) given by relations (18) and (19) results \( p_4' \) which is polar distance of bogie II [1-4].

At \( p_1 = p_{min} \), result \( p_4' \) which represent polar distance of bogie II when bogie I enter in string having still \( P_3 = 0 \).

For \( p_1 = p_{min} \), \( p_4' \) and \( P_3 = 0 \) with a relation similar to (20) it is determined entrance in string speed \( v_{cl} \) of bogie I.

At circulation with both bogies in intermediary position with \( f > j_c \) and \( \nu \in (\nu_{dII}, \nu_{cl}) \), respective \( \nu \in (0, \nu_{cl}) \), values are given to polar distances of both bogies like: \( p_1 \in (p_1', p_{min}) \) and \( p_4 \in (p_{max}, p_4'') \), respective \( p_1 \in (p_{max}', p_{min}) \) and \( p_4 \in (p_{max}', p_4'') \), from (18) and (19) results \( C_i \), respective \( C_{II} \), from (3) results \( P_1 \) and from (5) results \( P_4 \), respective from (16) results \( F \).

8. Circulation with bogie I in string and bogie II in free position

In this situation first bogie circulate in string meaning \( f > j_c \), \( p_1 = p_{min} \) and \( P_3 \neq 0 \), respective second bogie circulate in intermediary position so \( p_4 \in (p_4', p_{min}) \) and \( P_6 = 0 \) [1-4].

When it arrives to \( p_1 = p_4 = p_{min} \), then \( F = 0 \) (see relation (15)) and results that force from the couple \( F = 0 \), than considering that still \( P_4 = 0 \) from relations (7) and (19) results \( C_i \) and \( C_{II} \), which are equal as value respective with a similar relation (21) results \( v_{cl} > v_{cl} \) which is entrance speed in string of the second bogie, both bogies circulating as free bogies.

Given values to \( p_4 \in (p_4', p_{min}) \) results \( C_{II} \) and circulation speed \( \nu \in (V_{cl}, v_{cl}) \). Knowing these parameters, we determine \( P_4 \) with relation (5), \( P_1 \) with relation (3) and \( P_3 \) with relation (4) and \( F \) with relation (16), where \( p_1 = p_{min} \).
Remark: On speed interval $v \in (v_{cI}, v_{cII})$, or even at speed $v < v_{cI}$ there’s the possibility that the couple’s arrow to be equal with the movement of couple that is $f = j_c$, when for a $p_1 = \text{constant}$ and a $p_4 = \text{constant}$ the force from the couple became equal with pretension force of couple that is $F = F_{oc}$. We keep $f = j_c$ and force from the couple decrease from the pretension force to zero that is $F \in [F_{oc}, 0]$, keeping constant polar distances found for the two bogies. After we get to $F = 0$ couple’s arrow became $f < j_c$, the two bogies behaving as two independent bogies, the context being as in case of free bogies.

9. Circulation with both bogies in string

When engine circulates with both bogies in string we will have: $p_1 = p_4 = p_{min}$, respective according to relation (15) $f = 0$, respective to relation (16) $F = 0$, so articulated bogies behave like free bogies [1-4].

On speed interval $v \in (v_{cII}, v_{max})$ we give values to speed, from relation (7) results $C$ for both bogies respective $P_1$ with relation (3), $P_3$ with relation (4), $P_4$ with relation (5) $P_6$ with relation (6).

knowing the values of guiding forces in all phases of circulation on the railway we can determine also the guiding forces similar to situation for free bogies.

10. Dynamic horizontal passport

For all circulation regime is determined the guiding force [1-4]:

$$Y_1 = P_1 - F_{fly} = P_1 - Q \cdot \mu_1 \cdot \cos \xi_1.$$  \hspace{1cm} (23)

With relations we already have determined and locomotive’s parameters [6] we can determine and represent the dynamic horizontal passport of electrical C.F.R. 060-EA locomotive of 5100 kW formed from curves $P_1(v)$, $P_3(v)$, $Y_1(v)$, $Y_3(v)$ and $p_1(v)$ for first bogie, curves $P_4(v)$, $P_6(v)$, $Y_4(v)$ $Y_6(v)$ and $p_4(v)$ for second bogie (Figures 5 and 6), respective $F_{cI}(V)$ (Figure 7) [7].

![Figure 5. Guiding and directive forces](image)

All the calculus has been done with Mathcad and the figures were designed in Microsoft Excel.
11. The guiding capacity of vehicles and safety against derailment

The derailment phenomena were studied by different researchers, admitting different hypothesis of calculus. What all this hypothesis has in common is that horizontal forces favor the derailment and the vertical down orientated are stopping it [1-4].

It is supposed that guiding wheel attack the rail track on a whatever angle $\alpha$, small not to influence the derailment process and the wheel track contact is only in driving point B, when the wheel has the tendency to derail. In this case on the wheel act forces $Q$ and $Y$ from vehicle and normal force $N$ from the track as in Figure 8.

To avoid derailment the wheel must get down from this position (back to two points contact).

Safety criteria against derailment is given by relation [1-4]:

$$\frac{Y}{Q} \leq \frac{\tan \beta - \mu_b}{1 + \mu_b \tan \beta}$$  \hspace{1cm} (24)

According to safety criteria against derailment $(Y/Q)_{\text{lim}}$, with $Y_{\text{lim}} = Y_1$ for bogie 1, respective $Y_{\text{lim}} = Y_4$ for bogie 2, the inclination angle of lip of the bandage $\beta = 60^\circ$ and friction coefficient of lip of the bandage $\mu_b = 0.36$ (according to ERRI B 55 and B 136 of UIC Committee) at circulation through curve with ray $R = 300$ m, the maximum circulation speed for both bogies results through interpolation in figure 5 like [7]:

- for bogie 1: at $Y_{\text{lim}} = 84509.91$ N results $V_{\text{lim}} = 94.3$ km/h;
for bogie 2: at $Y_{\text{dilim}} = 84509,91$ N results $V_{\text{dinlim}} = 97,9$ km/h.

12. **Maximum speed admitted in curves**

On C.F.R. network the admitted speeds are given for normal gauge and effective overheight considering the overheight admitted insufficiencies \[1-4\].

Considering effective overheight $h = 120$ mm and overheight insufficiency $I = 90$ mm results:

$$V_{\text{max}} = 4,25 \cdot \sqrt{R} = 73,612 \text{ [km/h]}.$$  

From dynamic horizontal passport results that $V_{\text{lim}} > V_{\text{max}}$, then the vehicle satisfy safety the circulation conditions on the rail track and the danger of derailment does not exist \[7\].

13. **Conclusions**

Based on the calculus we can conclude:

- Starting from maximum speed $V_{\text{max}} = 120$ km/h, ten variation of guiding and conductive forces for both situations, free and articulated bogies, is similar until the moment the couple between bogies starts to work;
- Once the elastic inclined couple starts to work it is observed a drastic decrease at ax 1 and 4 of these guiding forces respective it’s cancellation at ax 3 and 6 because the two bogies circulate at low speed in intermediary position.
- At speed $V = 0$ km/h it is observed the decrease of directive force $P_1$ to 38000 N, respective of directive force $P_4$ which decrease until almost 0 N;
- The benefic influence of elastic couple is evident through the decrease of guiding and directive forces which leads to reducing the wear in wheel – track area.
- There’s a disadvantage due to high pretension force of arc from the couple $F_{oc}$, which leads to circulation with forced string position of bogie I. Bogie I circulates in string position at speed $V \in [40,8; 101,5]$ km/h, speed interval where in case of free bogies circulation this circulates in intermediary position. Therefor appears the contact between ax 3 and exterior wire of the curve, frictions in contact area appears resulting an increase of wear of both wheel and rail track.

**References**

[1] Ghita E and Turos Gh 2006 *Dinamica vehiculelor feroviare (Dynamics of rail vehicles)*, Editura Eurostampa, Timișoara, Romania
[2] Sebeșan I 1995 *Dinamica vehiculelor feroviare (Dynamics of rail vehicles)*, Editura Eurostampa, Timișoara, Romania
[3] Ursu C 1969 *Dinamica materialului rulant de cale ferată (Dynamics of rail vehicles)*, Lito I.P.T., Facultatea de Mecanică, Timișoara, Romania
[4] Ursu C 1981 *Dinamica materialului rulant de cale ferată, Vol. I și II (Dynamics of rail vehicles)*, Lito I.P.T., Facultatea de Mecanică, Timișoara, Romania
[5] Dungan L I 2008 *Contribuții la studiul și cercetarea comportării arcurilor de tip flexicoil de la locomotiva electrică C.F.R. 060-EA de 5100 kW. Teză de doctorat (Contributions at study and behavior of flexicoil springs at C.F.R. 060 EA 5100 kW electric locomotive) PhD thesis*, Editura Politehnica, Timişoara, Romania
[6] Drăghici A, Câlceanu I 1980 *Cartea mecanicului de locomotive electrice (Book of electric locomotive driver)*, Ministerul Transporturilor și Telecomunicațiilor, Departamentul Căilor ferate, Direcția Tracțiune și Vagoane, Romania
[7] Ursu-Neamț G V 2008 *Contribuții la optimizarea parametrilor cuplui elastice și a influenței acesteia asupra circulației în curbă a locomotivelor cu boghiuri articulate, (Contributions at optimization of elastic couple parameters and its influence on circulation in curve of locomotives with articulated bogies)* PhD thesis, Editura Politehnica, Timișoara, Romania