Multi-choice Linear Programming in Fuzzy Random Hybrid Uncertainty Environment and Their Application in Multi-commodity Transportation Problem

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ABSTRACT

In this paper, due to increasing competition in the business world, which makes decision makers dealing with multiple options/information for optimal decisions on a single task, we will look at multi-choice programming in hybrid fuzzy random environment. Alternative choices multi-choice parameters are considered as fuzzy random variables. By using polynomials interpolation for each multi-choice parameter, the model is transformed into a fuzzy random programming problem. Then, to convert this model to its deterministic form, we use the concept of the mean value of fuzzy random variables. Finally, to validate the proposed mathematical operations, we solve a multi-commodity transportation problem with fuzzy random multi-choice parameters.

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1. Introduction

Starting Multi-Choice programming (MCP) refers to a study by Healy on a particular case of mixed-integer planning. When the parameters involved in mathematical programming are multi-choice, that is, one parameter may have a set of options, of which only one is selected to optimise the objective function, this formulation is known as MCP. In such circumstances, several decision-making problems fall into the multi-choice parameter scenario. Therefore, the increasing use of multi-choice parameters has caused researchers to pay attention to this problem. MCP is highly applicable in real-life decision making. For example, a logistics and distribution company such as FedEx may charge different shipping costs for the same product from different customers based on their responsiveness [1]. MCP has been widely studied in the literature. Most studies in this field have focused on the topic of Multi-Choice Goal Programming (MCGP) problem and Transportation Problem (TP). For example, Chang [2] proposed a new method for solving MCGP. By extending the idea introduced by Liao [3] presented a state of goal programming in which the coefficients of the decision variables are multi-choice. The reader can refer to [4–9] for further reading on MCGP problem. As an extending of the MCP problem, Pradhan and Biswal [10] considered the aspiration level of multi-choice parameters in a linear programming problem as random variables. Maiti
and Roy [11] also presented a bi-level Random programming problem with some multi-choice parameters. In their model, objective function cost coefficients were considered as multi-choice parameters and the constraint parameters are random variables with a normal distribution. In recent years, the multi-choice of supply or demand parameters in TP, in particular, investigating these problems in uncertain environments, such as fuzzy and random, has been of great interest to researchers. The first research on fuzzy multi-choice goal programming was presented by Tabrizi et al. [12] who considered the multiple aspiration levels of goals programming as triangular fuzzy numbers and solved using the Zimmerman [13] and by Chang [2]. Aggarwal et al. [14] have considered a fully fuzzy multi-objective programming problem where the parameters and decision variables are fuzzy variables. The resource parameters of the constraints are multi-choice with only two alternatives, represented by fuzzy numbers. They use ranking function to get the crisp value of the fuzzy number. Ramzannia-Keshteli et al. [15] presented a multi-parametric approach to solve flexible fuzzy multi-choice goal programming problem. They presented a new method to solve this model using linear multi-parametric programming while the minimum degree membership for constraints and goals are considered by decision-maker. Mahapatra [16,17] considered a multi-choice random transportation problem in which supply and demand parameters are constrained Weibull random variables. A new method for solving a multi-choice stochastic solid TP was proposed by Roy and Mahapatra [18]. Roy [19] also studied a TP with multi-choice cost and demand and random supply. In this paper, he analysed the multi-choice stochastic TP where the objective function cost coefficients and demand parameters for these constraints are multi-choice parameters. Al Qahtani et al. [20] studied a multi-choice multi-objective transportation problem in which at least one of the goals has multiple aspiration levels, and supply and demand parameters are random variables that have not been predetermined.

As we know, the probability distribution of a random variable must be obtained through statistical analysis and inference based on deterministic data with the appropriate measure. However, such data cannot be found in many situations and as a result, the opinions of the experts are replaced in imprecise form. In such cases where the probability distribution is faced with two fuzzy and random phenomena, the random variable alone cannot justify this combined phenomenon. Hence, the necessity of dealing with fuzzy random compound environments is created. There has been a good deal of research done in this regard [21]. In this paper, we consider an MCP problem in which the alternatives choices are fuzzy random variables. The purpose of using these variables is to deal with unknown factors in the problem. We can better interpret real-world problems when we use fuzzy and stochastic hybrid environments. Using the polynomial interpolation and then the concept of the mean value of the fuzzy random variable convert it into its deterministic form. Finally, we solve a Multi-Commodity Transportation Problem (MCTP) with fuzzy random multi-choice parameters. Our main motivation for using the multi-commodity problem is its widespread use in the transportation industry. This is the first time that the application of multi-choice linear programming in a stochastic and fuzzy hybrid environment has been used in a multi-commodity transportation problem. We believe that using multi-choice programming in uncertain environments in transportation problem can bring this much closer to reality.

The organisation of the paper is as follows: After presenting some of the concepts and definitions in Section 2, we present the mathematical model of the Multi-Choice Fuzzy Random Linear Programming (MCFRLP) problem in Section 3. In Section 4, the deterministic
equivalent form of the proposed model is given with the solution of the solution. To validate the proposed mathematical operation, we solve an MCTP with fuzzy random multi-choice parameters in section 5. Finally, in section 6, some future results and suggestions are presented.

2. Preliminaries

In this section, we first review a number of definitions and notations to explain the general concepts associated with the discussion.

In the following definitions, we assume that \((\Omega, F, P)\) is a probability space and \((\Theta, P(\Theta), Pos)\) is a possibility space where \(\Theta\) is universe, \(P(\Theta)\) is the power set of \(\Theta\) and \(Pos\) is a possibility measure defined on fuzzy sets. Furthermore, \(F_c(\mathbb{R})\) is a collection of all normalised fuzzy numbers whose \(\alpha\)-level sets are convex subsets of \(\mathbb{R}\) [22].

**Definition 2.1:** A fuzzy subset \(\tilde{A}\) of the real line with membership function \(\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]\) is called fuzzy number if

1. \(\tilde{A}\) is normal and convex fuzzy set.
2. Support of \(\tilde{A}\) must be bounded.

A fuzzy random variable is a random variable and a Borel measurable function whose actual value is a fuzzy number [23].

The following lemma, extracted from Reference [24], shows that the \(\alpha\)-cut of a fuzzy random variable is a random interval.

**Lemma 2.2:** If \(X\) is a fuzzy random, then an \(\alpha\)-cut

\[X_\alpha(\omega) = \{t \in \mathbb{R} | \mu_{X(\omega)}(t) \geq \alpha\} = [X^-_\alpha(\omega), X^+_\alpha(\omega)]\]

is a random interval for every \(\alpha \in (0, 1]\).

The expected value is one of the basic concepts of a fuzzy random variable. For this reason, several operators have been proposed to define it in the literature [25].

Here, a definition of scalar expected value and the scalar variance of a fuzzy random variable, shown as \(Er(X)\), is provided [26,27].

**Definition 2.3:** Let \(X\) be a fuzzy random variable then, the scalar expected value is defined as follows:

\[Er(X) = \frac{1}{2} \int_0^1 \{E(X^-_\alpha) + E(X^+_\alpha)\} d\alpha\]

where \(E(X^-_\alpha)\) and \(E(X^+_\alpha)\) are expected values of \(X^-_\alpha\) and \(X^+_\alpha\) respectively.

**Remark 1:** If \(X\) be a fuzzy random variable, then expected value \(X\) is a fuzzy number.

**Remark 2:** If \(X\) is a fuzzy random, then for any \(\omega \in \Omega\), \(X(\omega)\) is fuzzy number.
In the following definition [22], the mean value of a fuzzy number $X(\omega)$, denoted as $M(X(\omega))$, is presented.

**Definition 2.4:** If $X$ is a fuzzy random variable, the mean value of the fuzzy number $X(\omega)$ is defined as follows:

$$M(X(\omega)) = \frac{1}{2} \int_0^1 [X^-_\alpha(\omega) + X^+_\alpha(\omega)]d\alpha \quad \forall \omega \in \Omega$$

**Remark 3:** If $X$ is a fuzzy random variable, then $M(X)$ is random variable. Also, $E(M(X)) = Er(X)$ [22].

According to Remark 3, the definition of the scalar variance of a fuzzy random number, defined as $Vr(X)$, can be presented [22].

**Definition 2.5:** Let $X$ be a fuzzy random variable. The scalar variance of $X$ is defined as follows:

$$Vr(X) = Var(M(X)) = \int_\Omega (M(X(\omega)))^2 P(d\omega) - (Er(X))^2$$

**Corollary 2.1:** Let $X$ and $Y$ be fuzzy random variables and $\lambda \in \mathbb{R}$ then, $M(X + \lambda Y) = M(X) + \lambda M(Y)$.

*Proof:* see [22].

**Definition 2.6:** Let $X$ and $Y$ be fuzzy random variables. Then the relations ‘$\cong$’ and ‘$\preceq$’ are defined respectively as follows [21]:

(i) $X \cong Y$ iff $M(X) = M(Y)$,
(ii) $X \preceq Y$ iff $M(X) \leq M(Y)$.

### 3. Multi-choice Fuzzy Random Linear Programming Problem

In this section, MCFRLP problem is introduced. The general form of this problem can be presented as:

**Problem 1:**

$$\begin{align*}
\text{Max } z &= \sum_{j=1}^n \{\tilde{c}_j^{(1)}, \tilde{c}_j^{(2)}, \ldots, \tilde{c}_j^{(k)}\} x_j \\
\text{s.t. } \sum_{j=1}^n \{\tilde{a}_j^{(1)}, \tilde{a}_j^{(2)}, \ldots, \tilde{a}_j^{(p)}\} x_j &\leq \{\tilde{b}_i^{(1)}, \tilde{b}_i^{(2)}, \ldots, \tilde{b}_i^{(r)}\}, \quad i = 1, 2, \ldots, m \\
x_j &\geq 0, \quad j = 1, 2, \ldots, n
\end{align*}$$

where $X = (x_1, x_2, \ldots, x_n)$ is a deterministic $n$ - dimensional decision vector. Each alternative value $\tilde{c}_j^{(r)}$, $(r = 1, 2, \ldots, k; j = 1, 2, \ldots, n)$, $\tilde{a}_j^{(s)}$, $(s = 1, 2, \ldots, p; j = 1, 2, \ldots, m)$, and $\tilde{b}_i^{(t)}$ $(t = 1, 2, \ldots, r; i = 1, 2, \ldots, m)$ of the multi-choice parameters $\tilde{c}_j$, $\tilde{a}_j$, and $\tilde{b}_i$ are considered as fuzzy random variables. Due to the fuzzy random alternate choices of the multi-choice parameters in **Problem 1**, it is not possible to solve this problem directly. Therefore, in order to solve this problem, we need to develop a suitable method.
Table 1. Node point for multi-choice parameter $\tilde{c}_j$.

| $u_j$ | 0  | 1  | 2  | ... | $k_j - 1$ |
|---|---|---|---|---|---|
| $f_{\tilde{c}_j}(u_j)$ | $\tilde{c}_j^{(1)}$ | $\tilde{c}_j^{(2)}$ | $\tilde{c}_j^{(3)}$ | ... | $\tilde{c}_j^{(k_j)}$ |

Table 2. Node point for multi-choice parameter $\tilde{a}_{ij}$.

| $w_j$ | 0  | 1  | 2  | ... | $p_{ij} - 1$ |
|---|---|---|---|---|---|
| $f_{\tilde{a}_{ij}}(w_j)$ | $\tilde{a}_{ij}^{(1)}$ | $\tilde{a}_{ij}^{(2)}$ | $\tilde{a}_{ij}^{(3)}$ | ... | $\tilde{a}_{ij}^{(p_{ij})}$ |

Table 3. Node point for multi-choice parameter $\tilde{b}_i$.

| $v_i$ | 0  | 1  | 2  | ... | $r_i - 1$ |
|---|---|---|---|---|---|
| $f_{\tilde{b}_i}(v_i)$ | $\tilde{b}_i^{(1)}$ | $\tilde{b}_i^{(2)}$ | $\tilde{b}_i^{(3)}$ | ... | $\tilde{b}_i^{(r_i)}$ |

4. Deterministic Model Formulation

Since the model presented in the previous section contains fuzzy random variables, it is necessary to obtain a deterministic form to solve it. We establish the equivalent deterministic model of the problem 1 by using the $E_r$-expected Value Model. On the other hand, problem 1 contains multi-choice parameters. Also, any alternative value of the multi-choice parameters is fuzzy random variables. We cannot apply any approach directly to the model because there are different options for each parameter. Therefore, we first convert these multiple-choice parameters using interpolated polynomials. Interpolating polynomials are formed by introducing an integer variable corresponding to each multi-choice parameter [28]. Each integer variable considers exactly the number of $k$ nodes if the relevant parameter has the number $k$. Each node corresponds exactly to a functional value of a multiple-choice parameter. Here the functional value of each node is a fuzzy random variable. Here, we replace the multi-choice parameter with the interpolation polynomial using the Lagrange’s formula.

4.1. Lagrange Interpolating Polynomials for the Multi-choice Parameters

Let us introduce the integer variable $u_j$ for the multi-choice parameter $\tilde{c}_j$ which takes $k_j$ number of values. Actually, let $u_j = 0, 1, 2, \ldots, k_j - 1$, be the node point where $f_{\tilde{c}_j}(u_j) = \tilde{c}_j^{(1)}, \tilde{c}_j^{(2)}, \ldots, \tilde{c}_j^{(k_j)}$ be respective functional values of the interpolating polynomial (Table 1).

Similarly, we formulate a Lagrange interpolating polynomials $f_{\tilde{a}_{ij}}(w_j)$ and $f_{\tilde{b}_i}(v_i)$ which passes through all the $p_{ij}$ and $r_i$ numbers of points given by Tables 2 and 3, respectively.

Hence, the interpolating polynomial for multi-choice parameters $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ can be derived as:

$$f_{\tilde{c}_j}(u_j; \tilde{c}_j^{(1)}, \tilde{c}_j^{(2)}, \ldots, \tilde{c}_j^{(k_j)}) = \frac{(u_j - 1)(u_j - 1) \cdots (u_j - k_j + 1)\tilde{c}_j^{(1)}}{(-1)^{(k_j-1)}(k_j - 1)!} c_j^{(1)} + \frac{u_j(u_j - 2)(u_j - 3) \cdots (u_j - k_j + 1)\tilde{c}_j^{(2)}}{(-1)^{(k_j-2)}1!(k_j - 2)!} c_j^{(2)} + \cdots$$
The value of fuzzy random variables. By adopting this case due to fuzzy randomness of parameters. Therefore, we used the concept of mean suitable to real-world problems. However, the model is not well defined theoretically in

\[ \text{Problem 3:} \]

\[ \begin{align*}
\text{Max } z &= \sum_{j=1}^{n} \frac{u_j(u_j-1)(u_j-2) \cdots (u_j-k_j+2) \cdot \bar{z}^{(k_j)}_{j}}{(k_j-1)!} \\
\end{align*} \]

\[ j = 1, 2, \ldots, n. \]  

\[ \begin{align*}
f_{\tilde{a}_{ij}}(w_{ij}; \tilde{a}_{ij}^{(1)}, \tilde{a}_{ij}^{(2)}, \ldots, \tilde{a}_{ij}^{(p_{ij})}) &= \frac{(w_{ij} - 1)(w_{ij} - 2) \cdots (w_{ij} - p_{ij} + 1) \cdot \tilde{a}_{ij}^{(1)}}{(-1)^{(p_{ij} - 1)}(p_{ij} - 1)!} \\
&+ \frac{w_{ij}(w_{ij} - 2)(w_{ij} - 3) \cdots (w_{ij} - p_{ij} + 1) \cdot \tilde{a}_{ij}^{(2)}}{(-1)^{(p_{ij} - 2)}1!(p_{ij} - 2)!} + \cdots \\
i &= 1, 2, \ldots, m, j = 1, 2, \ldots, n. \]

\[ \begin{align*}
f_{\tilde{b}_{ij}}(v_{ij}; \tilde{b}_{ij}^{(1)}, \tilde{b}_{ij}^{(2)}, \ldots, \tilde{b}_{ij}^{(r_{ij})}) &= \frac{(v_{ij} - 1)(v_{ij} - 2) \cdots (v_{ij} - r_{ij} + 1) \cdot \tilde{b}_{ij}^{(1)}}{(-1)^{(r_{ij} - 1)}(r_{ij} - 1)!} \\
&+ \frac{v_{ij}(v_{ij} - 2)(v_{ij} - 3) \cdots (v_{ij} - r_{ij} + 1) \cdot \tilde{b}_{ij}^{(2)}}{(-1)^{(r_{ij} - 2)}1!(r_{ij} - 2)!} + \cdots \\
i &= 1, 2, \ldots, m. \]

So, by replacing (1), (2) and (3) in problem 1, we have:

**Problem 2:**

\[ \text{Max } z = \sum_{j=1}^{n} f_{\tilde{c}_{ij}}^{r}(u_{ij}; \tilde{c}_{ij}^{1}, \tilde{c}_{ij}^{2}, \ldots, \tilde{c}_{ij}^{k_j})x_j \]

\[ \begin{align*}
s.t. \quad & \sum_{j=1}^{n} f_{\tilde{a}_{ij}}(w_{ij}; \tilde{a}_{ij}^{(1)}, \tilde{a}_{ij}^{(2)}, \ldots, \tilde{a}_{ij}^{(p_{ij})})x_j \leq f_{\tilde{b}_{ij}}(v_{ij}; \tilde{b}_{ij}^{(1)}, \tilde{b}_{ij}^{(2)}, \ldots, \tilde{b}_{ij}^{(r_{ij})}), \quad i = 1, 2, \ldots, m \\
x_j & \geq 0, \quad j = 1, 2, \ldots, n \\
0 & \leq u_{ij} \leq k_j - 1 \quad \text{and integers} \\
0 & \leq w_{ij} \leq p_{ij} - 1, \quad \text{and integers} \\
0 & \leq v_{ij} \leq r_{ij} - 1, \quad \text{and integers} \]

In current model, parameters are assumed to be fuzzy random variables which are more suitable to real-world problems. However, the model is not well defined theoretically in this case due to fuzzy randomness of parameters. Therefore, we used the concept of mean value of fuzzy random variables. By adopting the Er-expected value model and considering **Corollary 1,** problem 2 can be rewritten as:

**Problem 3:**

\[ \begin{align*}
\text{Max } z &= \sum_{j=1}^{n} \text{Er}(f_{\tilde{c}_{ij}}^{r}(u_{ij}; \tilde{c}_{ij}^{1}, \tilde{c}_{ij}^{2}, \ldots, \tilde{c}_{ij}^{k_j})x_j) \\
\end{align*} \]

\[ \begin{align*}
s.t. \quad & \sum_{j=1}^{n} \text{Er}(f_{\tilde{a}_{ij}}(w_{ij}; \tilde{a}_{ij}^{(1)}, \tilde{a}_{ij}^{(2)}, \ldots, \tilde{a}_{ij}^{(p_{ij})})x_j) \]

...
\begin{align*}
&\leq Er\left(f_{b_i}(v_i; b_i^{(1)}, b_i^{(2)}, \ldots, b_i^{(r_i)}), i = 1, 2, \ldots might) \\
x_j &\geq 0, \quad j = 1, 2, \ldots, n \\
0 &\leq u_j \leq k_j - 1 \quad \text{and integers} \\
0 &\leq w_{ij} \leq p_{ij} - 1, \quad \text{and integers} \\
0 &\leq v_i \leq r_i - 1, \quad \text{and integers}
\end{align*}

Using this model with all its real parameters, solutions with optimal \(Er\)-expected return subject to \(Er\)-expected constraints will be obtained. Now, we present an algorithm for solving MCFRLP problem.

**Algorithm 1 (MCFRLP problem Algorithm)**

**Step 1.** Define multi-choice fuzzy random parameters of MCFRLP problem by using information of experts or decision makers.

**Step 2.** Obtain interpolated polynomials for the multi-choice parameter using the Lagrange’s formula.

**Step 3.** Convert Problem 1 to Problem 2 by replacing multi-choice parameters with interpolating polynomial obtained from Lagrange’s formula in Step 2.

**Step 4.** Convert Problem 2 to Problem 3 by using \(Er\)-expected model and determine their \(Er\)-expected values.

**Step 5.** Solve \(Er\)-MCFRLP problem which is a non-linear programming. The obtained optimal solution is called \(Er\)-optimal solution of the original problem.

### 5. Multi-commodity Multi-choice Fuzzy Stochastic Transportation Problem

Transportation problem or moving goods from one place to another is one of the most important problems in the economy of a country. The displacement of millions of goods per day poses the role of transportation programming as a science. In the meantime, Multi-Commodity Transportation Problem (MCTP) is particularly important. In a multi-commodity transportation network, there are usually 3 parameters. Number of source nodes (warehouses), number of destination nodes (market) and number of goods which we show them with \(m\), \(n\) and \(k\), respectively. In fact, a directed network with \(k\) commodities is a multi-commodity transportation network, whenever \(|V| = m + n, E = \{(i,j)|i \text{ source node, } j \text{ destination node}\}\) and \(|E| = mn\). The following model represents a general MCTP):

**Problem 4:**

\[
\begin{align*}
\text{Min} & \quad \sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}^k \\
\text{s.t.} & \quad \sum_{i=1}^{m} x_{ij}^k = b_j^k, \quad \forall j, k
\end{align*}
\]
\[
\sum_{i=1}^{n} x_{ij}^k = a_i^k, \quad \forall i, k
\]
\[
\sum_{k=1}^{K} x_{ij}^k \leq u_{ij}, \quad \forall i, j
\]
\[
x_{ij}^k \geq 0, \quad \forall i, j, k
\]

For commodity \(k\), \(a_i^k\) is the availability of the commodity at the \(i\)-th source, \(b_j^k\) is the demand of the commodity at the \(j\)-th destination, \(c_{ij}^k\) is the cost for transporting one unit of the commodity from the \(i\)-th source to the \(j\)-th destination, \(u_{ij}\) is the capacity of arc \((i, j)\) and \(x_{ij}^k\) is the number of units of the product that should be transported from the \(i\)-th source to \(j\)-th destination. Assume that for each commodity \(k\), the amount of inventory at all source nodes is equal to the demand for that commodity at all destination nodes. In other words:
\[
\sum_{i=1}^{m} a_i^k = \sum_{j=1}^{n} b_j^k, \quad k = 1, 2, \ldots, K
\]

Obviously, if this relationship is not established, we can design a new network using artificial nodes where the above relation is established. In this paper, we consider a mathematical model for a multi-commodity multi-choice fuzzy stochastic TP as follows:

**Problem 5:**

\[
\text{Min} \sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} \{ \tilde{c}_{ij}^{(1)}(r), \tilde{c}_{ij}^{(2)}(r), \tilde{c}_{ij}^{(3)}(r), \ldots, \tilde{c}_{ij}^{(k)}(r) \} x_{ij}^k
\]
\[
\text{s.t.} \quad \sum_{i=1}^{m} x_{ij}^k = b_j^k, \quad \forall j, k
\]
\[
\sum_{i=1}^{n} x_{ij}^k = a_i^k, \quad \forall i, k
\]
\[
\sum_{k=1}^{K} x_{ij}^k \leq \tilde{u}_{ij}, \quad \forall i, j
\]
\[
x_{ij}^k \geq 0, \quad \forall i, j, k
\]

Where each alternative value \(\tilde{c}_{ij}^{(r)}\), \(r = 1, 2, \ldots, k_{ij}\) of the multi-choice parameters and \(\tilde{u}_{ij}\) are considered as fuzzy random variables. Using the interpolation polynomial and the \(Er\)-expected value model, **Problem 1** can be converted to its deterministic form.

The present method serves as a useful decision-making tool for a decision-maker to find the optimal solution with the best alternative for a multi-choice parameter. Since this paper is a first step to study the application of multi-choice fuzzy random linear programming problems in multi-commodity transportation problem and so far no serious research has been investigated, so we need further research to see the effectiveness of our method in more practical cases.
6. Numerical Example

To illustrate the method discussed in the previous section, two numerical examples will be presented in the following:

Example 1: Consider the following MCFRLP problem:

Problem 6:

\[
\begin{align*}
\text{Max } z &= \{\tilde{c}_1, \tilde{c}_2, \tilde{c}_3\}x_1 + \{\tilde{c}_4, \tilde{c}_5, \tilde{c}_6\}x_2 \\
\text{s.t. } &\{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3\}x_1 + \{\tilde{a}_4, \tilde{a}_5, \tilde{a}_6\}x_2 \leq \{\tilde{b}_1, \tilde{b}_2, \tilde{b}_3\} \\
&\{\tilde{a}_7, \tilde{a}_8, \tilde{a}_9\}x_1 + \{\tilde{a}_{10}, \tilde{a}_{11}, \tilde{a}_{12}\}x_2 \leq \{\tilde{b}_4, \tilde{b}_5, \tilde{b}_6\} \\
x_1, x_2 &\geq 0
\end{align*}
\]

where each alternative values of the multi-choice parameters are fuzzy random variables which are given as follows:

\[
\begin{align*}
\tilde{c}_1 &= (N\sim(17, 5^2), 2.3, 4.3) & \tilde{c}_2 &= (N\sim(18, 4^2), 3.4, 4.4) & \tilde{c}_3 &= (N\sim(19, 5^2), 3.7, 5.7) \\
\tilde{c}_4 &= (N\sim(27, 4^2), 2.7, 3.7) & \tilde{c}_5 &= (N\sim(28, 3^2), 2.5, 0.5) & \tilde{c}_6 &= (N\sim(29, 2^2), 1.8, 0.8) \\
\tilde{a}_1 &= (N\sim(3, 1^2), 1.5, 2.5) & \tilde{a}_2 &= (N\sim(4, 0.5^2), 2.3, 4.3) & \tilde{a}_3 &= (N\sim(5, 2^2), 3.8, 2.8) \\
\tilde{a}_4 &= (N\sim(1, 0.5^2), 2.5, 1.5) & \tilde{a}_5 &= (N\sim(2, 0.5^2), 1.8, 1) & \tilde{a}_6 &= (N\sim(3, 1^2), 2.2, 4.2) \\
\tilde{a}_7 &= (N\sim(2, 1.5^2), 1.7, 3.7) & \tilde{a}_8 &= (N\sim(3, 2^2), 3.7, 2.7) & \tilde{a}_9 &= (N\sim(4, 0.25^2), 2.8, 1.5) \\
\tilde{a}_{10} &= (N\sim(5, 3^2), 2.8, 1.8) & \tilde{a}_{11} &= (N\sim(6, 2.5^2), 3.75, 1.75) & \tilde{a}_{12} &= (N\sim(7, 4^2), 2.3, 4.3) \\
\tilde{b}_1 &= (N\sim(33, 4^2), 1.8, 3.8) & \tilde{b}_2 &= (N\sim(34, 1.5^2), 1.5, 2.5) & \tilde{b}_3 &= (N\sim(35, 0.5^2), 3.7, 2.7) \\
\tilde{b}_4 &= (N\sim(37, 5^2), 4.2, 2.2) & \tilde{b}_5 &= (N\sim(38, 3^2), 3.7, 1.7) & \tilde{b}_6 &= (N\sim(39, 2^2), 1.8, 0.8)
\end{align*}
\]

Let \( \tilde{z} = (r, \beta, \gamma) \) be a fuzzy random variable where \( r \) is a normal random variable with \( E(r) = \mu, \text{Var}(r) = \sigma^2 \). According to definition 2.4, the mean value of \( \tilde{z} \) is calculated as follows which is random variable [18]:

\[
\begin{align*}
\tilde{z}_\alpha^- &= r + \beta(\alpha - 1), \tilde{z}_\alpha^+ = r + \gamma(1 - \alpha) & \forall \alpha \in (0, 1] \\
M(\tilde{z}) &= r + \frac{1}{4}(\gamma - \beta) = \tilde{z}
\end{align*}
\]

Now, by adopting Algorithm 1 for the above problem, the optimal solution to the problem is obtained as follows:

\[
(z^*, X^*, u, w, v) = (279.3801, (10.30414, 2.682027), (2, 2), (0, 0, 2, 0), (0, 2))
\]

The next example, illustrates the application of the proposed model in multi-commodity transportation problem.

Example 2: Consider the following two-commodity transportation network:
Now, let us consider the multi-commodity multi-choice fuzzy stochastic TP as follows:

Problem 7:

\[
\begin{align*}
\text{Min } z &= \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{11} + \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{12} \\
&+ \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{13} + \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{21} \\
&+ \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{22} + \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{23} \\
&+ \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{31} + \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{32} \\
&+ \{ \tilde{c}_{ij}^{(1)}, \tilde{c}_{ij}^{(2)}, \tilde{c}_{ij}^{(3)} \} x_{33}
\end{align*}
\]

\[
\text{s.t. } \sum_{i=1}^{m} x_{ij}^k = b_j^k, \quad \forall j, k
\]

\[
\sum_{i=1}^{n} x_{ij}^k = a_i^k, \quad \forall i, k
\]

\[
\sum_{k=1}^{K} x_{ij}^k \leq \tilde{u}_{ij}, \quad \forall i, j
\]

\[
\sum_{i=1}^{n} x_{ij}^k \geq 0, \quad \forall i, j, k
\]

where each alternative value \(\tilde{c}_{ij}^{(r)}\), \(r = 1, 2, \ldots, k_{ij}\) of the multi-choice parameters and \(\tilde{u}_{ij}\) are considered as fuzzy random variables. Also, for commodity \(k\), \(a_i^k\) is the availability of the commodity at the \(i\)-th source and \(b_j^k\) is the demand of the commodity at the \(j\)-th destination. For simplicity we have assumed that all alternative choices of multi-choice parameters and the capacity of arc \((i, j)\) are fuzzy random variable and their values are given in Table 4.
The importance of the examples presented becomes clear when we do not have the appropriate amount of data to determine the distribution and need to use the imprecise opinion of an expert to estimate. Especially for the second example, when airlines, railways

In this example, it is assumed that $r$ is a random variable with a normal distribution. Applying the interpolation polynomial and the $E_r$-expected value model to problem 7 and solve the obtained deterministic problem. The obtained mathematical programming model is treated as a non-linear programming problem which solved by Lingo14 package. The optimal solution for the mathematical model is presented in Table 5. The minimum cost of the objective function is 26.0.

| Source | Customer | Fuzzy Random Capacity | Fuzzy Random Multi-choice cost (commodity 1) | Fuzzy Random Multi-choice cost (commodity 1) |
|--------|----------|-----------------------|--------------------------------------------|--------------------------------------------|
| 1 1    | $(N \sim (1.5, 0.1), 0.5, 2.5)$ | $\tilde{c}_{11}^{(1)} = (N \sim (2, 0.5), 5.3, 1.3)$ | $\tilde{c}_{11}^{(1)} = (N \sim (3.5, 0.2), 2.3, 0.3)$ |
| 1 2    | $(N \sim (3.3, 0.2), 2.8, 5.6)$ | $\tilde{c}_{12}^{(1)} = (N \sim (6.5, 0.5), 2.5, 4.5)$ | $\tilde{c}_{12}^{(1)} = (N \sim (1.5, 0.2), 2.6, 4.6)$ |
| 1 3    | $(N \sim (2.7, 0.5), 0.2, 1.4)$ | $\tilde{c}_{13}^{(1)} = (N \sim (7.2, 0.3), 2.7, 5.9)$ | $\tilde{c}_{13}^{(1)} = (N \sim (7.5, 0.3), 3.1, 1.1)$ |
| 2 1    | $(N \sim (3.5, 0.2), 3.7, 1.7)$ | $\tilde{c}_{21}^{(1)} = (N \sim (8.5, 0.6), 2.1, 4.1)$ | $\tilde{c}_{21}^{(1)} = (N \sim (7.6, 0.1), 0.5, 2.1)$ |
| 2 2    | $(N \sim (2.7, 0.7), 0.7, 4.7)$ | $\tilde{c}_{22}^{(1)} = (N \sim (10.6, 0.7), 4.5, 2.1)$ | $\tilde{c}_{22}^{(1)} = (N \sim (8.5, 0.7), 4.2, 6.2)$ |
| 2 3    | $(N \sim (4.0, 0.25), 5.7, 1.7)$ | $\tilde{c}_{23}^{(1)} = (N \sim (10.1, 0.5), 2.7, 6.3)$ | $\tilde{c}_{23}^{(1)} = (N \sim (7.6, 0.3), 3.0, 1.3)$ |
| 3 1    | $(N \sim (3.4, 1.0), 0.4, 2.8)$ | $\tilde{c}_{31}^{(1)} = (N \sim (2, 0.65), 0.4, 4.4)$ | $\tilde{c}_{31}^{(1)} = (N \sim (16.4, 0.6), 2.4, 4.8)$ |
| 3 2    | $(N \sim (2.2, 0.2), 3.2, 6.4)$ | $\tilde{c}_{32}^{(1)} = (N \sim (4.5, 0.2), 3.3, 1.3)$ | $\tilde{c}_{32}^{(1)} = (N \sim (7.5, 0.7), 3.5, 5.5)$ |
| 3 3    | $(N \sim (2.4, 0.6), 1.3, 3.7)$ | $\tilde{c}_{33}^{(1)} = (N \sim (4.7, 0.1), 0.5, 1.7)$ | $\tilde{c}_{33}^{(1)} = (N \sim (18.5, 0.8), 2.8, 4.8)$ |
Table 5. The optimal solution for the mathematical model.

| $x_{ij}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Commodity 1 | 1.5      | 0.0      | 0.5      | 0.0      | 2.0      | 0.0      | 0.5      | 0.0      | 1.5      |
| Commodity 2 | 0.5      | 1.5      | 0.0      | 1.5      | 0.0      | 0.5      | 0.0      | 0.5      | 1.5      |

or road lines are newly established and no historical data is available. Also, there are several decision-making situations where we have to choose one value from a set of values of a parameter. In this case, we encounter a fuzzy phenomenon between random values. In such a situation, we conclude that the consideration of the multi-choice fuzzy random parameters in the parametric space is very logical and helpful for the decision-makers to take the proper decision.

7. Conclusions

In this paper, an appropriate method for solving the MCP problem is presented. In this model, the alternatives choices are fuzzy random variables. To transform the problem into its deterministic form, we used the Lagrange polynomial interpolation and the concept of the mean value of the fuzzy random variable. In the following, we discuss MCTP with fuzzy random multi-choice parameters. It should be noted that the problem of transportation or moving commodities from one place to another is one of the most important problems in the economy of a country. The movement of millions of commodities a day brings up the role of transportation problems as a science. In the meantime, multi-commodity transportation has a special place. In this paper, we have tried to focus more on the theoretical aspects of the problem and provide a simple model of it. This model can also be applied to other applications of transportation problems. It is noteworthy that considering the probability distribution function and the effect of variance on optimal solutions has a direct effect and the optimal solution is more reliable than other optimal solutions. Unfortunately, obtaining the optimal solution is not an easy task due to the complexity of the final nonlinear programming model if some other parameters of the fuzzy random variables are taken into consideration and the generalisation of the model, in this case, can be an interesting idea for future researches.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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