Transverse energy fluctuations and the pattern of \( J/\psi \) suppression in Pb–Pb collisions

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The NA50 collaboration has recently observed that the \( J/\psi \) production rate in Pb–Pb collisions decreases more rapidly as a function of the transverse energy for the most central collisions than for less central ones. We show that this phenomenon can be understood as an effect of transverse energy fluctuations in central collisions. A good fit of the data is obtained using a model which relates \( J/\psi \) suppression to the local energy density. Our results suggest that the \( J/\psi \) is completely suppressed at the highest densities achieved in Pb–Pb collisions.

Among the various particles which are produced in nucleus-nucleus collisions at high energy, the \( J/\psi \) meson plays a special role. Because of the large mass of the charm quark, \( \bar{c}c \) pairs are produced on a short time scale and their evolution probes the state of matter in the early stages of the collisions. In particular, their binding scale and their evolution probes the state of matter in the early stages of the collisions. In particular, their binding scale and their evolution probes the state of matter in the early stages of the collisions. In particular, their binding scale and their evolution probes the state of matter in the early stages of the collisions.

The model developed in Ref. [8], on which the present analysis relies, relates the anomalous \( J/\Psi \) suppression to the local energy density: if the energy density at the point where the \( J/\psi \) is produced exceeds a critical value \( \epsilon_c \), the \( J/\psi \) disappears. This model was motivated by the simple observation that the local energy density is higher in Pb–Pb collisions than in any other system which had been studied previously (e.g. S–U collisions), even though the average density in Pb–Pb collisions does not exceed that in other systems. This simple picture was shown to account quantitatively for the data with a single parameter, \( \epsilon_c \). Here, we extend this model taking into account transverse energy fluctuations, which were neglected in the numerical estimates presented in [8].

The transverse energy produced in a nucleus-nucleus collision can be used as a measure of the impact parameter, the largest values of \( E_T \) corresponding to the most violent collisions at small impact parameter. However the correlation between \( b \) and \( E_T \) is not one-to-one: for a given impact parameter, the produced transverse energy fluctuates. The effect of these fluctuations is particularly visible for the collisions producing the largest transverse energies: these correspond essentially to central collisions with nearly vanishing impact parameters. If one assumes that a fluctuation in \( E_T \) results from a fluctuation in the energy density over the entire collision zone, one sees that an \( E_T \) fluctuation is accompanied by an increase of the region where the energy density exceeds \( \epsilon_c \) and hence can be responsible for an amplification of the \( J/\psi \) suppression.

As a first estimate of this effect, let us consider central collisions of two identical nuclei described by sharp sphere densities, and ignore the nuclear absorption. At zero impact parameter, the energy density at point \( s \) in the transverse plane (see Fig. [8]) is proportional to the number of participants in a small tube centered at \( s \) (see below), that is \( \epsilon(s) \propto \sqrt{R^2-s^2} \). We assume that all the \( J/\psi \)'s emerging at points \( s < s_c \) where the energy density is \( \epsilon(s) > \epsilon_c \) are suppressed. The radius \( s_c \) of the total suppression zone is then related to \( \epsilon_c \) by:
where \( \epsilon_{\text{max}} = \epsilon(s = 0) \). The density of \( c\bar{c} \) pairs produced at point \( s \) is proportional to the number of nucleon-nucleon collisions in a small tube centered at \( s \), i.e. to \( R^2 - s^2 \). The survival probability follows from a simple geometrical counting:

\[
\frac{\int_{s}^{R} d^2s \ (R^2 - s^2)}{\int_0^{R} d^2s \ (R^2 - s^2)} = \left( \frac{R^2 - s^2}{R^2} \right) = \left( \frac{\epsilon_c}{\epsilon_{\text{max}}} \right)^4.
\]

This result provides a simple geometrical interpretation of the effect of a transverse energy fluctuation: assuming such a fluctuation to be evenly distributed over the whole transverse plane, an increase of \( E_T \) results in an increase of the volume in which the \( J/\psi \)'s are suppressed. At the same time, the number of \( c\bar{c} \) pairs, proportional to the number of nucleon-nucleon collisions, remains constant, independent of \( E_T \). This is the origin, in this picture, of the expected extra suppression at large \( E_T \). A fluctuation of \( E_T \) by about 10\% increases \( \epsilon_{\text{max}} \) by the same amount, and, according to Eq \( \mbox{(2)} \), decreases the \( J/\psi \) production by about 30\%; this is indeed the order of magnitude of the observed effect.

![FIG. 1. Geometry of a central collision: the shaded area](image)

We now turn to a more quantitative calculation. We assume that the transverse energy produced in a nucleus-nucleus (A–B) collision is proportional to the number of participants at impact parameter \( b \), \( N_p(b) \), as predicted in the wounded nucleon model \[13\]. The actual dependence may be somewhat stronger than a simple linear relation \[14\], and such deviations are important in co-moving scenarios \[8\], but we do not expect them to alter our results significantly. Furthermore, as in \[8\] we make this relation local, and take the energy density at point \( s \) and impact parameter \( b \) to be proportional to the density \( n_p(s, b) \) of participants in a plane orthogonal to the collision axis:

\[
n_p(s, b) = T_A(s) \left[ 1 - e^{-\sigma_N T_B(b-s)} \right] + T_B(b-s) \left[ 1 - e^{-\sigma_N T_A(s)} \right],
\]

where \( \sigma_N = 32 \text{ mb} \) is the nucleon-nucleon inelastic cross section, and \( T_A(s) = \int_{-\infty}^{\infty} dz \ \rho_A(s, z) \). The total number of participants at impact parameter \( b \) is \( N_p(b) = \int d^2s \ n_p(s, b) \). For our numerical estimates, we parametrize \( \rho_A(r) \) by:

\[
\rho_A(r) = \frac{\rho_0}{1 + \exp \left( \frac{-r}{\alpha} \right)}, \quad \int \rho_A(r) d^3r = A,
\]

with \( \alpha = 0.53 \text{ fm}, R_A = 1.1 \ A^{1/3} = 6.52 \text{ fm} \) and \( \rho_0 = 0.17 \text{ fm}^{-3} \) for \( ^{208}\text{Pb} \).

At fixed impact parameter, the number of participants may fluctuate. We shall ignore these fluctuations, focusing here only on the fluctuations of the transverse energy produced by a fixed number of participants or, equivalently, at a fixed impact parameter. The corresponding distribution is chosen to be the following gaussian distribution \[16\]:

\[
P(E_T|b) = \frac{1}{\sqrt{2\pi a q^2 N_p(b)}} \exp \left[ -\frac{(E_T - q N_p(b))^2}{2q^2 a N_p(b)} \right],
\]

The mean value of the transverse energy is \( \langle E_T \rangle(b) = q N_p(b) \), with \( q \) the average transverse energy per participant, and the dispersion is \( \sigma_{E_T}^2 = a q^2 N_p(b) \), with \( a \) a dimensionless parameter. The fit to the minimum bias \( E_T \)-distribution by NA50 yields \( q = 0.274 \text{ GeV} \) and \( a = 1.27 \) \[17\]. Then the “knee” of the \( E_T \)-distribution, defined as \( E_T^{\text{min}} = q N_p(0) \), sits at about 108 GeV.

We assume, as in the simple model discussed above, that the fluctuation in \( E_T \) at given \( b \) is distributed over the entire collision zone proportionally to \( n_p(s, b) \) so that the energy density can be written as:

\[
\epsilon \propto \frac{E_T}{\langle E_T \rangle(b)} n_p(s, b),
\]

where \( \langle E_T \rangle(b) \propto N_p(b) \) is the average transverse energy.

The production of \( c\bar{c} \) pairs in an A-B collision goes like the number of nucleon–nucleon collisions, and so does that of Drell–Yan pairs. Thus the probability \( P(\text{DY}|b) \) that a Drell–Yan pair be produced given that a nucleus–nucleus collision has taken place at impact parameter \( b \) is given by:

\[
P(MB|b) P(\text{DY}|b) = \sigma^\text{NN}_\text{DY} T_A(s) T_B(s-b) = \sigma^\text{NN}_\text{DY} T_{AB}(b),
\]

where \( P(MB|b) = 1 - P_0(b) = 1 - \left( 1 - \sigma_N T_{AB}(b) \right)^{AB} \simeq 1 - e^{-\sigma_N T_{AB}(b)} \) is the probability to have at least one inelastic collision (minimum bias) at impact parameter \( b \). For large impact parameters, \( P(\text{DY}|b) \) becomes independent of \( b \), \( P(\text{DY}|b) \rightarrow \sigma^\text{NN}_\text{DY} / \sigma_N \), which is the probability to produce a Drell–Yan pair in an inelastic nucleon–nucleon collision.

Note that for a fixed impact parameter, the number of nucleon–nucleon collisions may also fluctuate. We shall ignore such fluctuations, recognizing that this is a source of uncertainty on the \( E_T \)-distribution of the Drell–Yan pair production. With this simplification, the number of Drell–Yan pairs is entirely determined by the nuclear geometry, i.e., by the impact parameter of the collision. Besides, the production of a Drell–Yan pair is not affected by final state interactions, thus it is not directly
sensitive to the transverse energy produced in the collision (it will become indirectly related to $E_T$ through the relation between $b$ and $E_T$).

Final state interactions, however, strongly modify the $J/\psi$ production cross section. In addition to the standard nuclear absorption, we model the anomalous suppression by simply assuming \( \frac{n_e}{\sigma_{NN}} \) that any $J/\psi$ emerging at point $s$ is suppressed if the energy density $\epsilon(s)$ is higher than $\epsilon_c$. Then, the probability to produce a $J/\psi$ given that a collision has taken place at impact parameter $b$ and has produced a transverse energy $E_T$ is written as follows:

$$
P(\text{MB}|b) P(\psi|E_T, b) = \sigma_{NN} \int d^2 s \frac{1}{\sigma_a} \left(1 - e^{-\sigma_s T_a(s)}\right)$$

$$\times \left(1 - e^{-\sigma_c T_b(s-b)}\right) \Theta \left(n_c - \frac{E_T}{(E_T)^2} n_p(s, b)\right),$$

(8)

where $n_c$ is a parameter proportional to the critical energy density $\epsilon_c$ defined above. In this formula, $\sigma_a \approx 6.4$ nb is the absorption cross section \( \frac{\sigma_{NN}}{\sigma_{NN}} \), and $\sigma_{NN}^a$ is the $J/\psi$ production cross section in a nucleon–nucleon collision.

Putting everything together, we can write the probability to produce a $J/\psi$ in a collision at a given $E_T$ as:

$$
\frac{d^2 p}{d^2 b} P(\text{MB}|b) P(\psi|E_T, b) P(E_T|b) = \frac{d\sigma_{\psi}/dE_T}{d\sigma_{MB}/dE_T}.
$$

(9)

By replacing in this formula $P(\psi|E_T, b)$ by $P(DY|b)$, we obtain the probability to produce a Drell–Yan pair at a given $E_T$. We can then construct the ratio:

$$
\mathcal{N}(E_T) = \left(\frac{d\sigma_{\psi}/dE_T}{d\sigma_{MB}/dE_T}\right) / \left(\frac{d\sigma_{DY}/dE_T}{d\sigma_{MB}/dE_T}\right),
$$

(10)

which we shall use in order to compare the model with experimental data.

Indeed, the NA50 collaboration \( \frac{13}{\text{NA50}} \) presents its results essentially as the same ratio, where the numerator is an experimentally measured quantity, while the denominator, which is model dependent, is deduced from a theoretical analysis that is identical to the one presented above. In order to complete our theoretical estimate, we need an extra input, the normalisation factor $\sigma_{NN}/\sigma_{DY}$ (multiplied by the branching ratio to the dimuon decay channel) which is obtained from experiments with lighter projectiles and targets \( \frac{14}{\text{NA50}} \). The value of this normalisation factor given by NA50 is $\sigma_{NN}/\sigma_{DY} = 53.5 \pm 3$ \( \frac{15}{\text{NA50}} \). The fits in Figs\( \frac{2}{\text{EVT}} \) and \( \frac{3}{\text{EVT}} \) have been obtained with the values 52 and 53.5 respectively.

A plot of the calculated $\mathcal{N}(E_T)$ is shown in Fig\( \frac{2}{\text{EVT}} \) together with data from NA50. A reasonable fit of the 1998 data is obtained with a single parameter $n_c = 3.7$ fm$^{-2}$. Above 40 GeV, the curve deviates from nuclear absorption because the anomalous suppression mechanism sets in. Note that the points at low $E_T$ from 1996 data \( \frac{16}{\text{NA50}} \), on the other hand, are above the curve, i.e. they show less $J/\psi$ absorption than expected from an extrapolation of proton–nucleus and nucleus–nucleus data, a fact still unexplained. Above 100 GeV, i.e. approximately at the position of the knee, a second drop occurs which, as explained above, is associated with the increase of $E_T$ due to fluctuations. The fact that we reproduce quantitatively the relative magnitude of this drop suggests, according to the discussion following Eq.(2), that all the $J/\psi$ mesons disappear at the highest densities achieved in the system.

![Figure 2](image-url)

**FIG. 2.** The $J/\psi$ survival probability in a Pb–Pb collision as a function of the transverse energy in GeV, measured by NA50 in 1996 (open circles) and in 1998 (close circles); after absorption in nuclear matter alone (dot-dashes); and after dissolution in a quark-gluon plasma with a critical density $n_c = 3.7$ fm$^{-2}$, without (dashes) and with fluctuations (full curve).

We have to point out that the rapidity window used to measure the $J/\psi$ and the one used to determine the $E_T$–distribution are different. Indeed, muons pairs are selected for $2.8 < y < 3.92$, whereas the neutral transverse energy is measured in the $1.1 – 2.3$ pseudo-rapidity window \( \frac{18}{\text{NA50}} \). In the calculation presented above, we have implicitly assumed that the fluctuations in these two rapidity windows are strongly correlated.

If, on the contrary, there were no correlation between $E_T$ fluctuations and the energy density at the point where the $c\bar{c}$ pair is produced, then the suppression criterion should involve only the average transverse energy at a given impact parameter, and \( \frac{19}{\text{NA50}} \) should be replaced by $n_p(s, b) > n_c$. The resulting $\mathcal{N}(E_T)$ is shown as the dashed curve in Fig\( \frac{2}{\text{EVT}} \). In this scenario, the suppression is a function of the impact parameter $b$ only: as a consequence, the suppression saturates at large $E_T$, to its value at $b = 0$. Such a model for $J/\Psi$ suppression is pro-
posed in [20], where, indeed, the criterion for suppression depends only on b. Although it refers to the density of strings rather than to that of participants, this bears little influence on the results.

Clearly, our theoretical curve in Fig. 2 does not provide a perfect fit to NA50 data, which have small error bars. In order to improve the quality of the fit, we did another calculation with a more gradual suppression mechanism: namely, we replaced the Θ-function in Eq. (8) by 
\[ 1 - \tanh(\lambda(n - n_c))/2 \]

The results for \( n_c = 3.75 \text{ fm}^2 \) and \( \lambda = 2 \text{ fm}^2 \) are displayed in Fig. 3. A perfect fit of the 1998 data is obtained over the entire \( E_T \) range from 40 GeV to 120 GeV. Note that above 80 GeV, the suppression becomes total at the hottest point of the system, so that the behaviour at large \( E_T \) is essentially the same as in Fig. 2. We obtain a fit indistinguishable from that of Fig. 3 with a two-threshold scenario [10], by assuming that 40% of the \( J/\psi \) are suppressed if the density lies between \( n_{c1} = 3.3 \text{ fm}^2 \) and \( n_{c2} = 4.0 \text{ fm}^2 \) and 100% above \( n_{c2} \). Note that the second threshold does not produce any visible structure in the \( E_T \) dependence of \( \sigma_\psi/\sigma_{DY} \).

In summary, we have shown that the second drop in the \( J/\psi \) yield around the knee of the \( E_T \)-distribution can be interpreted as an effect of \( E_T \) fluctuations, if one assumes that \( J/\psi \) suppression is related to the local energy density in the system. We have achieved our best fit by assuming that the suppression increases gradually with the energy density; in particular, there is no indication in the data that the suppression occurs in two steps. However, in order to reproduce the magnitude of the observed “second drop”, we need to assume that all the \( J/\psi \)'s disappear at the highest densities achieved in a Pb–Pb collision.

\[ \sigma_\psi = \frac{\sigma_{DY}}{2} \]

FIG. 3. Same as previous figure, with a gradual onset of anomalous \( J/\psi \) suppression (see text).

Finally, we would like to compare our model with that recently proposed in [20], where the \( J/\Psi \) suppression is explained by interactions with comoving particles. The density of comovers is related to the transverse energy in the same way as the energy density in our model. Thus these authors also obtain increased \( J/\Psi \) suppression, but much less pronounced than in NA50 data. This is because, as explained in detail in [20], absorption by comovers depends more weakly on transverse energy fluctuations than in the present model.

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