Spontaneous Breaking of Lorentz-Invariance and Gravitons as Goldstone Particles

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Abstract

We consider some aspects of spontaneous breaking of Lorentz Invariance in field theories, discussing the possibility that the certain tensor operators may condensate in the ground state in which case the tensor Goldstone particles would appear. We analyze their dynamics and discuss to which extent such a theory could imitate the gravity. We are also interested if the universality of coupling of such ‘gravitons’ with other particles can be achieved in the infrared limit. Then we address the more complicated models when such tensor Goldstones coexist with the usual geometrical gravitons. At the end we examine the properties of possible cosmological scenarios in the case of goldstone gravity coexisting with geometrical gravity.

1 Introduction

The idea that the spontaneous breaking of Lorentz Invariance (LI) is accompanied by the non-scalar Goldstone particles was first discussed by Bjorken in the seminal paper [1], where these goldstones where associated with the photons. This idea was further extended to nonabelian case and also [2]-[3] to tensor condensates and corresponding goldstones were interpreted as graviton-like objects.

The possibility to represent some or all gauge fields and gravitons as the Goldstone particles connected with fluctuations of vector and tensor condensates can seem quite promising. Especially this concerns to gravitons, as far as the vector gauge fields interaction are anyway renormalizable while the bad behavior of gravity at high virtualities obliges to go to some different descriptions at small distances - just as in Loop Quantum Gravity or in String Theory approach.

But it is rather unclear how far can one advance in the direction of full (or partial) replacing of geometrical gravity by the composite goldstone tensor field. Firstly one must have answers to a number of “crucial” questions, in particular:

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• is it possible to make the interaction of such a goldstones universal, as is for a “usual” geometrical gravitons of the general relativity (GR) - this is necessary at least in the long-distance limit, where the universality of gravitation is experimentally tested with very high precision;

• is it possible to guarantee the Lorentz invariance of various measurable quantities at experimentally acceptable level in the presence of vector and tensor condensates.

These questions are not independent. If Lorentz invariance is restored at large distances, then probably the universality of interactions will be also restored, because many arguments are collected [4] (but see also [5]) that the only consistent Lorentz-invariant theory of interacting massless spin two particles is the general relativity.

Perhaps also the opposite statement is true - if at large distances the gravitational interactions become universal, then Lorentz non-invariant terms can die out or turn into gauge fixing terms, giving no contributions to the measurable quantities.

Unfortunately, there is no understanding if all this is really true, and there is no more or less reliable model calculations supporting these hypothesis.

But if these hypotheses are correct and one the non-universal and LI-violating contributions to the amplitudes are indeed suppressed to the “needed” level, then such an approach to the gravity could open up a number of interesting possibilities. In particular, one can try to incorporate (unify) the goldstone gravitons in the standard model picture, or in some of its generalization. It seems to us, that remain two main classes of possible models.

• In the models of the first type there is no geometrical gravity and the fundamental physics corresponds to some renormalizable theory in a flat four-dimensional space-time. This can be, for example, some supersymmetric grand unified generalization of the standard model like SO(10), E(6) or so, supplemented by additional gauge sector that becomes strongly coupled at some scale \( \Lambda \) (related to the Planck scale \( m_p \simeq 10^{19} \) GeV) at which the tensor condensates are formed and thus the Goldstone gravitons emerge. In this case no additional dimensions can be introduced without destroying the renormalizability. An important advantage (or disadvantage!? ) is that for the goldstone-gravity the cosmological constant is automatically zero [3], but perhaps it can be imitated by the non-LI corrections.

• In second type of models the goldstone-gravity emerging from the spontaneous breaking of LI coexist with the fundamental "geometrical” gravity whose scale \( M_P \gg \Lambda \). The latter could be originated e.g. by the “normal” gravity whose scale \( M_P \gg \Lambda \). The latter could be originated e.g. by the “normal” gravity whose scale \( M_P \gg \Lambda \). The latter could be originated e.g. by the “normal” string degrees of freedom at very small distances, related to the string scale \( \Lambda_s \sim M_P \), while at much larger distances, or at energy scale \( \Lambda \ll \Lambda_s \), where the field approximation already works, the condensation of the tensor operators takes place. In this bigravity case we can have two gravitons: primary graviton - coming (for example) from the massless modes of strings, and composite graviton, related to the Goldstone fluctuations of the tensor condensate.

In this paper we consider in details some of these questions and also discuss the main peculiarities of corresponding cosmological models. The content of the article is distributed in sections as follows:
Section 2: general structure of tensor condensates in renormalizable field theories with strong coupling.

Section 3: the Goldstone modes of such condensates and their effective action.

Section 4: tensor condensates and their fluctuations in the curved background.

Section 5: couplings of the goldstone gravitons to other particles and possibility of their universality in the infrared limit.

Section 6: the peculiarities of the tensor condensation in supersymmetric theories, where the Goldstone supergravity can appear.

Section 7: cosmological implications of the models with purely goldstone gravity, and the bigravity models when both - goldstone and the geometrical gravity are present. In the last case the geometrical gravity (string) scale can be shifted from $10^{19}$ GeV to much higher energies $10^{60}$ GeV.

Section 8: concluding remarks.

2 Tensor condensates

In the field theory the (composite) condensates can develop from the enough strong attraction between virtual particles, so that a tachyonic bound states with $m^2 < 0$ can be created. As a result the ground state is coherently filled with such tachyons which in addition can have non-trivial quantum numbers. In a more phenomenological approach one can simply forget about this dynamical stage and directly start from an effective Lagrangian applicable below some energy scale, where the effective composite fields get negative $m^2$. The last is a way usually followed when considering various applications of spontaneous symmetry breaking\footnote{In fact, the tachyon formation and condensation does not necessarily need the strong interaction of virtual particles and in some cases is possible also in a weak coupling regime. But the latter usually needs additional symmetry arguments or tuning and is rather a dynamical exclusion than a general rule. One possibility is when particles which bind in tachyons are almost massless, and forces between them are long range. The other known example is superconductivity, where only the electrons close to the Fermi surface are essential in pair condensation.}

The non-abelian gauge theories with adequately restricted multiplet content usually become strongly coupled below certain ‘confinement’ scale $\Lambda$, and various composite condensates can appear, like the quark $\langle \bar{q}q \rangle$ and gluonic $\langle BB \rangle$ condensates in the QCD. Unfortunately, about the formation of condensates we know mainly from the experiment (and lattice calculations) and not directly from theory. Only some special cases, e.g. in gauge theories in the large $N_c$ limit or in theories with large supersymmetry this phenomenon can be analyzed in more details. We also know (only from the experiment) that in the QCD case the condensates are Lorentz scalars, and yet we do not understand the dynamical reasons for this. But may be for other gauge groups with specific multiplet contents, different from QCD, the nonscalar condensates can also appear along with “usual” scalar condensates.

2.1 Model and general definitions

Let us consider a theory in a minskowskian space-time based on some non-abelian gauge symmetry $G_c$ and discuss a general composite symmetric tensor operator with the “vacuum”
must be Lorentz-invariant. Next, we suppose that for a slowly changing field \( t \)
freedom with the constraint, one can define the effective lagrangian for the effective tensor
connected with the operator
As far as we start with initially Lorentz-invariant theory, the effective Lagrangian /suppress L(\( t \))
approximately local and can be expanded in powers of \( t \)
\[ \delta \]
where we introduced the integration with the
no principal obstacles.
especially in a strong field region, is a rather complicated task, but in principle there are
no principal obstacles.
We illustrate this in a brief formal way. Let us introduce, as usual, the external current
\( J_{\mu \nu} \) linearly coupled with \( \tau_{\mu \nu} \). Then the response of the system to the excitation of
the degrees of freedom related to \( \tau_{\mu \nu} \) can be described by the generating functional
\[ Z[J_{\mu \nu}(x)] = \int D\psi \; \hat{\tau}_{\mu \nu} \; e^{i \int d^4x L(\psi)} \]
can be nonzero and spontaneously break LI, and let us study the behavior of the system
in a neighborhood of \( \langle \tau_{\mu \nu} \rangle \). To calculate explicitly the effective lagrangian /suppress L(\( \tau_{\mu \nu} \))
representing the response of the system to the excitation of corresponding degrees of freedom,
especially in a strong field region, is a rather complicated task, but in principle there are
no principal obstacles.
We illustrate this in a brief formal way. Let us introduce, as usual, the external current
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the degrees of freedom related to \( \tau_{\mu \nu}(x) \) can be described by the generating functional
\[ Z[J_{\mu \nu}(x)] = \int D\psi \; \hat{\tau}_{\mu \nu} \; e^{i \int d^4x [L(\psi) + \tau_{\mu \nu}(\psi) J_{\mu \nu}(x)]} = \int D\psi \; D t_{\mu \nu} \; \delta_x [t_{\mu \nu}(x) - \tau_{\mu \nu}(\psi)(x))] \; e^{i \int d^4x [L(\psi) + t_{\mu \nu}(x) J_{\mu \nu}(x)]} = \int D t_{\mu \nu} \; e^{i \int d^4x [L(t_{\mu \nu}) + t_{\mu \nu} J_{\mu \nu}]} \]
where we introduced the integration with the \( \delta_x \) functional over the auxiliary field \( t_{\mu \nu}(x) \)
connected with the operator \( \tau_{\mu \nu} \). In this way, by integrating out the fundamental degrees of
freedom with the constraint, one can define the effective lagrangian for the effective tensor
field \( t_{\mu \nu} \):
\[ i \int d^4x L(t_{\mu \nu}(x)) = \ln \left[ \int D\psi \; e^{i \int d^4x L(\psi)} \; \delta_x [t_{\mu \nu}(x) - \tau_{\mu \nu}(\psi)] \right] \]
As far as we start with initially Lorentz-invariant theory, the effective Lagrangian \( L(t_{\mu \nu}) \)
must be Lorentz-invariant. Next, we suppose that for a slowly changing \( t_{\mu \nu}(x) \) it is approxi-
mately local and can be expanded in powers of \( t_{\mu \nu} \) and its derivatives \( t_{\mu \nu,\alpha}(x) = \partial_\alpha t_{\mu \nu} \):
\[ L(t_{\mu \nu}) = - V(t_{\mu \nu}) + \Gamma^{\alpha \beta \gamma \delta \rho \sigma} t_{\alpha \beta \gamma} t_{\delta \rho \sigma} + W^{\alpha \beta \gamma \delta \rho \sigma \lambda \tau} t_{\alpha \beta \gamma} t_{\delta \rho \sigma} t_{\lambda \tau} + ... \]
\(^2\) As far as we are in a Minkowski space, the indices could be moved up and down by means of \( \eta_{\mu \nu} = \eta^{\mu \nu} = \text{diag}(1,1,1,-1) \), e.g. \( \hat{\tau}_{\mu \nu} = \eta^{\rho \sigma} \tau_{\rho \sigma} \).
\(^3\) For simplicity we do not discuss here a more general case when \( \tau_{\mu \nu} \) could have also some nontrivial internal quantum numbers, so that the condensation of \( \tau_{\mu \nu} \) would break additionally internal symmetries.
where generically also the coefficients $\Gamma, W, \ldots$ are functions of the field $t_{\mu \nu}(x)$, while in most general case the scalar potential $V(t_{\mu \nu})$ is a function of four independent Lorentz-invariant combinations:

$$s_1 = \text{Tr}(t) \quad s_2 = \text{Tr}(t^2), \quad s_3 = \text{Tr}(t^3), \quad s_4 = \text{Tr}(t^4) \quad (6)$$

Therefore, we are interested in a situation when near the confinement scale $\Lambda_c$ the nontrivial vacuum expectation value can be generated,

$$\langle \hat{t}_{\mu \nu} \rangle = \langle t_{\mu \nu} \rangle = n_{\mu \nu} \quad , \quad (7)$$

where $n_{\mu \nu}$ is a constant symmetric tensor with the elements of the order of 1.

The general form of potential $V$ for not very large $t_{\mu \nu}$ is

$$V(t_{\mu \nu}) = V_0 + g_1 t_{\mu}^\mu + g_2 t_{\mu}^\nu t_{\nu}^\mu + g_3 t_{\mu}^\nu t_{\nu}^\rho t_{\rho}^\mu + g_4 t_{\mu}^\nu t_{\nu}^\rho t_{\rho}^\sigma \ldots \ , \quad (8)$$

where generally the dimensional coefficients $g_i$ are expected to be related to the confinement scale $\Lambda$ as $g_i \sim \Lambda^4$. As far as $t_{\mu \nu}^2$ is not positively defined such potentials can easily lead to the nontrivial minima for very broad area of coefficients $g_i$. Therefore, one can expect that near to the confinement scale $\Lambda_c$ the nontrivial expectation value can be generated,

$$\langle \hat{t}_{\mu \nu} \rangle = \langle t_{\mu \nu} \rangle = n_{\mu \nu} \quad , \quad (9)$$

where $n_{\mu \nu}$ is some constant symmetrical tensor ($n_{\mu \nu} \neq \eta_{\mu \nu}$) with the elements of the order of 1.

In fact we will take later in mind a slightly more general effective lagrangian $L(t_{\mu \nu}, \phi_i)$, containing also the fields $\phi_i$ of the “standard model” (or its generalization), coupled to $\psi = (B, \chi, \varphi)$ fields by some mediators. We also suppose that these couplings are rather weak, so that on can neglect the contributions from $\phi_i$ fields to the $(B, \chi, \varphi)$ dynamics.

### 2.2 Configurations of the tensor condensate

The possible configurations of $n_{\mu \nu}$ can be classified in terms of four eigenvalues $\lambda^{(a)}$:

$$n_{\mu \nu} S^{(a)}_\mu = \lambda^{(a)} S^{(a)}_\mu \quad , \quad (10)$$

where $S^{(a)}_\mu$ are the orthonormal eigenvectors:

$$S^{(a)}_\mu S^{(b)}_\mu = \delta^{ab} \quad , \quad \sum_a S^{(a)}_\mu S^{(a)}_\nu = \eta_{\mu \nu} \quad , \quad \sum_a S^{(a)}_\mu S^{(a)}_\nu \lambda^{(a)} = n_{\mu \nu} \quad .$$

This means that by a rotations and busts on can chose such a coordinate system in with $n_{\mu \nu}$ is diagonal

$$n_{\mu \nu} = \begin{pmatrix}
\lambda_1 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 \\
0 & 0 & 0 & -\lambda_0
\end{pmatrix} \quad , \quad (11)$$

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4Here we only supposed that $V(t_{\mu \nu})$ is analytic at $t_{\mu \nu} = 0$. We also symbolically denoted as $t_{\mu \nu}^n$ invariants, containing n-th power of $t_{\mu \nu}$, for example $t_{\mu \nu}^2 = t_{\mu \nu} t_{\alpha \beta} \eta^{\mu \alpha} \eta^{\nu \beta}$, where $\eta_{\alpha \beta}$ is Minkovski metric.
or it has one of the possible ‘light-like’ forms

\[ n_{\mu\nu} = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_4 & \lambda_5 \\ 0 & 0 & \lambda_5 & -\lambda_4 \end{pmatrix}, \quad n_{\mu\nu} = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_4 - \lambda_5 & \lambda_5 \\ 0 & 0 & \lambda_5 & -\lambda_4 - \lambda_5 \end{pmatrix} \]  

(12)

In the light-like cases we will always have the preferential space direction. For the diagonal \( n_{\mu\nu} \) the eigenvectors in this frame are also diagonal \( S_{\mu}^a \sim \delta_{\mu}^a \lambda^a \), and for ‘light-like’ \( n_{\mu\nu} \) the eigenvectors \( S_{\mu}^a \) can have also two nondiagonal components - for example \( S_0^3 \) and \( S_3^0 \).

In the following, we consider only first case, with the diagonal \( n_{\mu\nu} \) in the form (11) with real eigenvalues \( \lambda^a \). The eigenvectors in this frame are also diagonal: \( S_{\mu}^a \sim \delta_{\mu}^a \lambda^a \).

Clearly, we are not interested in the case when LI is unbroken, i.e. \( n_{\mu\nu} \propto \eta_{\mu\nu} \). The LI breaking needs that the irreducible ”traceless” part of \( n_{\mu\nu}, \tilde{n}_{\mu\nu} \) is non-zero.

As far as we start with initially Lorentz-invariant theory, in most general case the scalar potential \( V(t_{\mu\nu}) \) is a function of four independent Lorentz-invariant combinations:

\[ V(t_{\mu\nu}) = V(s_1, s_2, s_3, s_4) \]  

(13)

of four independent invariants

\[ s_1 = \text{Tr}(t) = t_{\mu}^{\mu}, \quad s_2 = \text{Tr}(t^2) = t_{\mu\nu}^{\nu\mu}, \quad s_3 = \text{Tr}(t^3) = t_{\mu\nu}^{\rho}t_{\rho}^{\mu\nu}, \quad s_4 = \text{Tr}(t^4) = t_{\mu\nu}^{\rho}t_{\rho}^{\sigma\mu\nu\sigma} \]  

(14)

Therefore the the conditions for extremum \( \partial V/\partial t_{\mu\nu} = 0 \) are reduced to \( \partial V/\partial s_i = 0 \) and the stability conditions for condensate is \( \partial^2 V/\partial s_i \partial s_k > 0 \). The roots \( \tilde{s}_i \) of equation \( \partial V/\partial s_i = 0 \) are related to the eigenvalues of matrix \( n_{\mu\nu} \) :

\[ \tilde{s}_1 = \sum_{i=1}^{4} \lambda_i, \quad \tilde{s}_2 = \sum_{i=1}^{4} \lambda_i^2, \quad \tilde{s}_3 = \sum_{i=1}^{4} \lambda_i^3, \quad \tilde{s}_4 = \sum_{i=1}^{4} \lambda_i^4. \]  

(15)

The simplest model example of potential, such as

\[ V(s_i) = v_0 \sum_{i=1}^{4} \left( -a_i s_i + \frac{1}{2}s_i^2 \right), \]

already leads to general form of spontaneous LI breaking. In this case the roots, coming from conditions \( \partial V/\partial s_i = 0 \), are

\[ s_i = a_i, \quad V(a_i) = -\frac{1}{2}v_0 a_i, \quad \frac{\partial^2 V(a_i)}{\partial s_i^2} = v_0, \]

and, depending from values of \( a_i \), on can have configurations (11) or (12) with all different or equal \( \lambda_i \).

The degenerate case, when some of eigenvalues \( \lambda_i \) may coincide, can lead to number of simplifications. For example the case of maximal degeneracy \( \lambda_1 = \lambda_2 = \lambda_3 \neq \lambda_0 \). In this case on can go to a system where the space is isotropic and this will lead to additional restrictions on possible goldstone-gravity configurations (see next section). But there is no reason to expect such a degeneracy for the general potentials \( V(s_i) \), although on can meet the approximate degeneracy if the Lorentz-invariance is restored at long distances.

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When LI is spontaneously broken and $\langle t_{\mu\nu} \rangle \neq 0$, the mean value of energy-momentum tensor for the $\phi_i$ fields in vacuum can be also nontrivial and has the general form:

$$\langle T_{\mu\nu} \rangle = a_0 \eta_{\mu\nu} + a_1 n_{\mu\nu} + a_2 n_{\mu\gamma} n_{\gamma\nu} + \cdots$$  \hspace{1cm} (16)

In the Lorentz frames, where the $n_{\mu\nu}$ is diagonal, the $T_{\mu\nu}$ is also diagonal. In this case the $n_{\mu\nu}$ condensate can be interpreted as an anisotropic “solid media” filling the space with the energy density $T_{00}$ and anisotropic pressures $T_{kk}$.

The $a_0$ term in (16) is usually perturbatively divergent in renormalizable theories, and this leads to problems in GR, but here this Lorentz-invariant part $\sim \eta_{\mu\nu}$ can not be taken into account (it can be freely subtracted)\(^5\) because it does not interact with the goldstone gravitons. The non-LI contributions in $T_{\mu\nu}$ are probably nonuniversal and the coefficients $a_i$ are scale-dependent. Near the confinement scale $a_i \sim \Lambda^4$, but they, can become small on large distances as e.g. in $G_{\mu\nu}^{(i)}$ (21).

If usual general-relativistic geometrical gravity is also added to the renormalizable theory with $\psi = (B, \chi, \phi)$ fields, then the big mean invariant $T_{\mu\nu}$ can lead to unacceptable cosmological behavior. But at the same time here we have additional possibility for solving cosmological “constant” problem, because on can essentially restrict the strength of interaction of the geometrical gravitons with $\langle T_{\mu\nu} \rangle$ (up to the level of today’s value of cosmological constant) (See section 6.).

### 2.3 LI violations in observable

The constant tensor $n_{\mu\nu}$ can enter in various observable quantities, and we will interpret this as a violation of Lorentz invariance (LI). In general, all Green functions and scattering amplitudes may depend from $n_{\mu\nu}$. At the scales $|x_i - x_j| \gg \Lambda^{-1}$ the condensate $n_{\mu\nu}$ can be considered as a constant external tensor field. Then for the external momenta $k_i \ll \Lambda$ the amplitudes

$$A_m(k_1, k_2, \ldots, k_m, n_{\mu\nu})$$

(17)
can be calculated from some effective low-energy Lagrangian of the “physical” particle fields $\phi$ which can include the external “spurion” field $n_{\mu\nu}$ via various vertexes:

$$\phi \partial_\mu \partial_\nu \phi \ n_{\mu\nu} , \ \phi^k \partial_\mu \phi \partial_\nu \phi \ n_{\mu\nu} , \ \phi \partial_\mu \partial_\nu \phi \ n_{\mu\lambda} n_{\lambda\nu} , \cdots$$  \hspace{1cm} (18)

Then the $\phi$-particles propagators get tree level corrections depending on $n_{\mu\nu}$ which can be expressed as

$$G^{-1}(k) = k^2 - m^2 + k_\mu k_\nu N_{\mu\nu} + O(k^4) = G_{\mu\nu} k_\mu k_\nu - m^2 + O(k^4)$$

(19)

$$G_{\mu\nu} = n_{\mu\nu} + N_{\mu\nu} , \quad N_{\mu\nu} = c_1 n_{\mu\nu} + c_2 n_{\mu\lambda} n_{\lambda\nu} + \cdots ,$$

where coefficients $c_i$ are proportional to coupling constants, with which operators (18) enter the effective lagrangian. The term of type (18) come also from expansion of the vertices with loops in external momenta. There are also induced vertices of same general type (18), where some $n_{\mu\nu}$ are replaced by Minkowski metric $\eta_{\mu\nu}$.

So these vertexes can also be represented in combined form

$$\phi \partial_\mu \partial_\nu \phi G_{\mu\nu}^{(1)} , \ \phi^k \partial_\mu \phi \partial_\nu \phi G_{\mu\nu}^{(2)} , \ \phi \partial_\mu \partial_\nu \phi G_{\mu\lambda}^{(3)} G_{\lambda\nu}^{(4)} , \cdots$$  \hspace{1cm} (20)

\(^5\)As is usually done in field theories without gravity because it does not affect any measurable quantity.
where
\[ G_{\mu\nu}(i) = c_1^{(i)} \eta_{\mu\nu} + c_2^{(i)} n_{\mu\nu} + \cdots \]

Now, if coefficients \( G_{\mu\nu}(i) \) are expected to be universal, they all should be approximately the same
\[ G_{\mu\nu} = G_{\mu\nu}^{(1)} = G_{\mu\nu}^{(2)} = \cdots , \]
and hence \( G_{\mu\nu} \) can be interpreted as a new metric corresponding to flat space with non-orthogonal coordinates.

This geometrical interpretation can be simply generalized to \( n_{\mu\nu} \rightarrow t_{\mu\nu}(x) \) slowly varying with \( x \) on the scale \( \Lambda^{-1} \).

But in general the coefficients \( G_{\mu\nu}(i) \) are not universal, at least near to the scale \( \Lambda \), and they depend on the particle \( \phi \) and vertex type. So it is probably impossible to interpret the additional terms in (19) and (18) as coming from motion of \( \phi \) particles in nonorthogonal coordinates, represented by pure gauge gravitational field \( G_{\mu\nu} \), if this motion is “measured” at distances \( \sim \Lambda^{-1} \). For such measurements vacuum will look like a highly anisotropic crystal.

But it is possible that at large distances \( r \gg \Lambda^{-1} \) the difference between various \( G_{\mu\nu}(i) \) can become very small. This can be represented as:
\[ G_{\mu\nu}(i) = G_{\mu\nu} + \frac{k^2}{\Lambda^2} Z_{\mu\nu}^{(i)} + \frac{k_\alpha k_\beta n_{\alpha\beta}}{\Lambda^2} Y_{\mu\nu}^{(i)} + \cdots , \]
where the nonuniversal contributions to \( G_{\mu\nu}(i) \) are strongly suppressed close to “our” scale.

We consider this possibility in more details in Section 5.

3 Tensor condensate oscillations

3.1 Goldstone gravitons

So let us suppose that the \((B, \chi, \varphi)\) field system is such that the condensate \( \langle \tau_{\mu\nu} \rangle \) is generated. As discussed in previous section this means that the minimum of potential \( V(t_{\mu\nu}) \), defined by the condition \( \partial V/\partial t_{\mu\nu} = 0 \) is in the nontrivial point \( \langle t_{\mu\nu} \rangle = n_{\mu\nu} = \eta_{\mu\nu} + \tilde{n}_{\mu\nu} \).

Further, because \( V(t_{\mu\nu}) \) depends only from invariants formed from \( t_{\mu\nu} \) and by definition does not depend on the derivatives of \( t_{\mu\nu} \) over \( x \), the field configurations \( n_{\mu\nu}(x) \) which can be obtained from \( n_{\mu\nu} \) by the arbitrary local Lorentz transformation
\[ t_{\mu\nu}(x) = \Omega_{\mu}^\lambda(x) n_{\lambda\sigma} \Omega_{\nu}^\sigma(x) = \eta_{\mu\nu} + \Omega_{\mu}^\lambda(x) \tilde{n}_{\lambda\sigma} \Omega_{\nu}^\sigma(x) , \]
give the same value of the potential as \( n_{\mu\nu} \): \( V(t_{\mu\nu}(x)) = V(n_{\mu\nu}) \). Thus, the variables \( \Omega_{\nu}^\sigma(x) \) are connected with the massless Goldstone-like degrees of freedom, whose Lagrangian

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6 Even for slightly nonuniversal \( G_{\mu\nu}^{(i)} \), we can have the nonstandard \((\varepsilon, \vec{k})\) dispersion relations like \( \varepsilon^2 = \gamma k^2 + m^2 \ldots \), where \( \gamma_\delta \) depends from particles type. This can lead to Cherenkov radiation in vacuum and to other “kinematically forbidden” process. All these topics is widely discussed in last years from various angles \[6, 7\], in particular in connection with GKZ cutoff.

7 Note that from experiment we have rather strong limitations on such a mean \( |\delta G_{\mu\nu}^{(i)}|_{\text{exper.}} < 10^{-15} \div 10^{-20} \), but which can be covered by the factor \( k^2/\Lambda^2 \).
are given only by the terms with derivatives \( \partial_\lambda t_{\mu\nu}(x) \) in (5). The local \( O(3,1) \) rotation matrices

\[
\Omega^\nu_\mu(x) = (\exp[\frac{1}{2} \omega_{ab}(x) \Sigma^{ab}])^\nu_\mu
\]  

(23)

depend on antisymmetric “angular” fields \( \omega_{ab}(x) \) which represent six independent flat directions. Here \( \Sigma^{ab} \) is the spin part of generators of Lorentz rotation in the vector representation:

\[
(\Sigma^{ab})^\nu_\mu = \eta^{\nu\sigma} \delta^b_\mu - \eta^{b\nu} \delta^a_\mu ,
\]

where the local \((a,b)\) and tensor \((\mu,\nu)\) indices are ‘mixed’. The full infinitesimal action of these operators on a tensor, \((\Sigma_{\rho\sigma})^{a\beta}_\mu t_{a\beta} \), is given by relation

\[
(\Sigma_{\rho\sigma})^{a\beta}_\mu = \eta_{\rho\mu} \delta^{a}_\sigma \delta^{\beta}_\nu - \eta_{\rho\nu} \delta^{a}_\sigma \delta^{\beta}_\mu + \eta_{\sigma\nu} \delta^{a}_\mu \delta^{\beta}_\rho - \eta_{\sigma\rho} \delta^{a}_\mu \delta^{\beta}_\nu .
\]

Therefore, taking all this into account, one can divide dynamical variables \( t_{\mu\nu}(x) \) in the massless Goldstone-like “rotational” modes connected with \( \Omega^\lambda_\mu(x) \) and the massive modes \( g^a(x) \) connected with the excitations of the eigenvalues \( \lambda^a \):

\[
t_{\mu\nu}(x) = \sum_{a=1}^4 w^a_\mu(x) (\lambda^{(a)} + g^a(x)) \ w^a_\nu(x) ,
\]

(24)

where \( w^a_\mu(x) = \Omega^\lambda_\mu(x) S^a_\lambda \) are the analogues of local tetrad vectors, often used for representing the gravitational field. Here these tetrads \( w^a_\mu(x) \) are restricted by a “gauge” condition that in every point of the space-time they can be reduced by the local Lorentz-transformation to the predefined constant vectors \( S^a_\lambda \).

One can also represent (23) in a slightly different form

\[
t_{\mu\nu}(x) = \Omega^\lambda_\mu(x) [\tilde{n}_{\mu\nu} + \tilde{t}_{\lambda\sigma}(x)] \ \Omega^\sigma_\nu(x) ,
\]

(25)

where \( \tilde{n}_{\mu\nu} \) is a traceless part of \( n_{\mu\nu} \) \((\eta^{\mu\nu} \tilde{n}_{\mu\nu} = 0) \) and

\[
\tilde{t}_{\mu\nu}(x) = S^a_\mu \ S^a_\nu \ g^a(x)
\]

(26)

is a part of \( t_{\mu\nu}(x) \) that contains only the massive modes while the massless modes are encoded in \( \Omega^\lambda_\mu \). (Here and below the summation over the “eigenvalue” index \( a \) is assumed as in (23).) For small \( \omega_{ab}(x) \) the massless fields \((\omega_{\mu\nu} = -\omega_{\nu\mu}) \) enter in linear form \( \Omega_{\mu\nu}(x) = \eta_{\mu\nu} + \omega_{\mu\nu} \), and hence the week field decomposition of \( t_{\mu\nu}(x) \) in massless fields is given by the series

\[
t_{\mu\nu}(x) \simeq n_{\mu\nu}(x) + \omega_{\mu\gamma}(x) \tilde{n}_{\gamma\nu}(x) + \tilde{n}_{\mu\gamma}(x) \omega_{\gamma\nu}(x) + \cdots
\]

(27)

8 As far as all loop corrections due to fundamental fields \( \psi \) are already included in the effective potential \( V(t_{\mu\nu}) \), we do not need to care about corrections to \( V(t_{\mu\nu}) \) from the Goldstone loops.

9 The masses of the fields \( g^a(x) \) are defined by the second derivatives \( \partial^2 V/\partial g^a_{\mu\nu} \) and in general are expected to be order \( \Lambda \). Note that one should not expect that they correspond directly to some new massive stable particles formed on the scale \( \Lambda \). They contain the operator combinations of the fundamental fields \( \psi = B, \chi, \varphi \) in approximately the same proportion as in the decomposition (4) for \( \tau_{\mu\nu} \), and can be highly unstable.
So the symmetric tensor $h_{\mu\nu}(x)$ entering in usual linearized description of weak gravitational field
\[ t_{\mu\nu}(x) \simeq n_{\mu\nu} + h_{\mu\nu}(x) , \quad |h_{\mu\nu}| \ll |n_{\mu\nu}| , \]
can be represented by antisymmetric goldstone fields $\omega_{\alpha\beta}$
\[ h_{\mu\nu} \simeq \omega_{\mu\beta}\tilde{n}_\nu^\beta - \omega_{\nu\beta}\tilde{n}_\mu^\beta , \quad \omega_{ab} \simeq h_{a\beta}\tilde{n}_{b\beta} - h_{b\beta}\tilde{n}_{a\beta} , \tag{28} \]
where $\tilde{n}_{\alpha\beta} = (n^{-1})_{\alpha\beta} \simeq n_{\alpha\beta}$ is function of $n_{\alpha\beta}$. In fact this representation of $h_{\mu\nu}$ in terms of $\omega_{\mu\nu}$ corresponds to a definite gauge choice in linear approximation \(^{10}\). The general “gauge fixing” corresponding to graviton-goldstones is given in \(^{10}\).

### 3.2 Lagrangian

Between the degrees of freedom contained in $t_{\mu\nu}(x)$ in the neighborhood of $n_{\mu\nu}$ should be no tachyons, as far we are already in the minimum of $V(t)$ with respect to all independent variations of components of $t_{\mu\nu}$. One should not expect any ghosts contributions in terms with derivatives $t_{\alpha\beta,\gamma}$, so that all signs of corresponding kinetic terms in \(^{5}\) should be correct – otherwise there will be a nocausal propagation of $t_{\mu\nu}$ fields, but it is impossible in terms of primary fields $(B, \chi, \varphi)$. These conditions restricts the form of coefficients $\Gamma_{\alpha\beta}\delta_{\rho\sigma}(t) , ... ,$ entering effective lagrangian \(^{5}\) near the $t_{\mu\nu} = n_{\mu\nu}$. And just this second term
\[ L_2 = \Gamma_{\alpha\beta,\gamma}\delta_{\rho\sigma} t_{\alpha\beta,\gamma} t_{\delta\rho,\sigma} , \tag{29} \]
in the lagrangian \(^{5}\) defines the minimal dynamics for the goldstone fields $\omega_{\mu\nu}$. The stability conditions for $n_{\mu\nu}$ can be also be formulated in terms of corresponding effective Hamiltonian
\[ \mathcal{H} = t_{\alpha\beta,0} \partial L_2 / \partial(t_{\alpha\beta,0}) - L_2 . \]
For this one must require that for all small variations $|\omega_{\alpha\beta}| \ll 1 , |\phi^a| \ll \Lambda$ of $t_{\mu\nu}$ fields
\[ \delta t_{\mu\nu}(x) \simeq \phi^a(x) S^a_{\alpha\mu} S^a_{\nu} + \omega_{\alpha\beta}(x) (g_{\mu\alpha}n_{\nu\beta} - n_{\mu\alpha}g_{\nu\beta} + g_{\nu\alpha}n_{\mu\beta} - n_{\nu\alpha}g_{\mu\beta}) \tag{30} \]
the Hamiltonian remains positively defined, and the corresponding variation $\delta \mathcal{H} \geq 0$ in the neighborhood of $t_{\mu\nu}(x) = n_{\mu\nu}$.

The equation of motion for the Goldstone field $\delta \mathcal{L} / \delta \omega_{ab}(x) = 0$ follow from the terms with derivatives in \(^{5}\). On the other hand these equations must be also contained in the conservation lows $\partial_\mu J_{[\alpha\beta]\mu} = 0$ for the Noetter currents, connected with the symmetry which is spontaneously broken. In our case these currents are the full angular momentum densities $J_{[\alpha\beta]\mu}$ for field system $\psi = (B, \chi, \varphi)$, or for lagrangian \(^{5}\) for the effective fields $t_{\mu\nu}$. Since the Lagrangians are translation invariant these equations reduce to
\[ T_{\alpha\beta} - T_{\beta\alpha} - \partial_\mu S_{[\alpha\beta]\mu} = 0 , \]
where $T_{\alpha\beta}$ is canonical energy-momentum tensor, and $S_{\alpha\beta\mu}$ - is the spin part of $J_{[\alpha\beta]\mu}$.

\(^{10}\) Note, that this gauge in which the goldstone gravitons appear is similar to the axial gauge $n_{\mu}A^\mu = 0$ for vector goldstone particles, where $n_\mu$ is proportional to the value of vector condensate
To find the form of the Lagrangian $L_2$ for weak fields, including also their minimal interactions, we need the derivatives of $t_{\alpha \beta}$ up to the second order in physical fields

$$
\partial_\lambda t_{\mu \nu} \simeq \partial_\lambda \partial_\nu \left( S^a_{\mu} S^a_{\nu} + \omega_{\mu \alpha} S^a_{\alpha} S^a_{\nu} + \omega_{\nu \alpha} S^a_{\alpha} S^a_{\mu} \right) +
$$

$$
+ \partial_\lambda \omega_{\alpha \beta} \left( g_{\mu \alpha} \tilde{t}_{\nu \beta} + g_{\nu \alpha} \tilde{t}_{\mu \beta} + n_{\mu \alpha} \omega_{\beta \nu} + n_{\nu \alpha} \omega_{\beta \mu} \right) + \cdots
$$

(31)

In the weak field approximation tensor $\Gamma$ can be taken as a constant, depending only from $\eta_{ik}$ and $n_{ik}$. Then the general form of the Lagrangian $L_2$ reads as

$$
L_2 \simeq (P^{\gamma \alpha \beta \delta \rho} + \omega_{ab} \eta^{ab}) \partial_\eta t_{\alpha \beta} \partial_\eta \tilde{t}_{\delta \rho} +
$$

$$
(\partial_\gamma \omega_{ab} \partial_\sigma \omega_{mn}) (\tilde{t}_{\alpha \beta} \partial_\eta \tilde{t}_{\delta \rho}) (H^{1 \rho}_{ab} \cdots + \omega_{\gamma} H_{2 \cdots}) +
$$

$$
\partial_\gamma \omega_{ab} \cdot (\tilde{t}_{\alpha \beta} \partial_\gamma \tilde{t}_{\delta \rho}) U^{\gamma a \cdots \rho} + \cdots,
$$

(32)

where $P^\cdots, Q^\cdots, H^\cdots$ are constant tensors, constructed from $\eta^{ik}$ and $n^{ik}$, so that to fulfill the $H$ stability conditions. In (32) the first and second lines correspond to kinetic terms for the massive and goldstone modes and the third line describes their interaction in which, as is usual for goldstone particles, fields $\omega_{\alpha \beta}$ enter only through the derivative terms.

It is very essential and specific for the non-scalar goldstone particles that the massless fields $\omega_{\alpha \beta}$ can enter also in the coefficient of the kinetic term for massive mode without derivatives (first line in (32)). For the scalar goldstones such terms are cancelled in all orders and the goldstone interaction with other fields enter only through the derivative terms - like in the third line in (32).

From all possible field combinations, entering (32) and linear in $\omega_{\alpha \beta}$, only the term

$$
\omega_{\alpha \beta} J_{\alpha \beta} \quad \text{where} \quad J_{\alpha \beta} = (S^a_{\alpha} \partial_\beta \psi^a - S^a_{\beta} \partial_\alpha \psi^a) (S^a_{\alpha} \partial_\mu \psi^a) + \partial_\mu \omega_{n} \partial_\nu \omega_{\beta]}
$$

(33)

gives the finite contribution in the soft limit. Expression (33), representing the soft interaction of goldstone-gravitons with the rest of the fields (matter), can be written in a more “standard” way using (28):

$$
h_{\mu \nu} T^{\mu \nu}, \quad \text{where} \quad T^{\mu \nu} = 2 n^{\mu}_{\lambda} J^{\lambda \nu}, \quad h_{\mu \nu} \simeq \omega_{\mu \beta} n_{\beta \nu} - \omega_{\nu \beta} n_{\beta \mu}
$$

The kinetic terms for the goldstone fields $\omega_{\alpha \beta}$ (the second line in (32)) can be expressed also by means of $h_{\alpha \beta}$:

$$
(\partial_\gamma \omega_{ab} \partial_\eta \omega_{mn}) (n_{\alpha \beta} n_{\gamma \delta}) H^{\gamma a \cdots \rho} \quad \Rightarrow \quad (\partial_\gamma h_{ab} \partial_\eta h_{mn}) \tilde{H}^{\gamma a \cdots \rho},
$$

(34)

11 Notice that the states described by $\omega_{ab}$ and $\tilde{t}_{\alpha \beta}$ are orthogonal and so should be no direct mixing between them, i.e. no terms of the type $\partial_\omega \omega_{ab} \partial_\rho t_{ab}$.

12 For rather general $L = \partial_\alpha \phi \partial_\beta \phi - V(\phi)$ where $\phi$ - arbitrary column vector we can localize $\Omega$ - the global symmetry of $V(\phi)$ separating the massive $\phi$ and the “massless” modes $\partial_\alpha (\Omega_\phi)^{\dagger} \partial_\alpha (\Omega_\phi)$ where $\Omega(x) = \exp(i \tilde{\omega}(x))$). For $\rho > 0$ the goldstone fields $\tilde{\omega}(x)$ fully decouple from massive kinetic term and enter in only through derivative $L = \partial_\alpha \rho \partial_\beta \rho + (\rho \rho \rho) \partial_\alpha \tilde{\omega} \partial_\beta \tilde{\omega} + \partial_\gamma \tilde{\omega} \partial_\gamma \tilde{\omega} - V(\rho)$ where the current $J_{\mu} = i \partial_\mu \rho \partial_\rho \rho - \rho \rho \rho \partial_\rho \rho$. So the interaction of such a $\tilde{\omega}$-goldstones is switched off when their momenta is going to zero. The ‘diagonal’ terms in $L_2$ of type $\partial \tilde{t}_{\alpha \beta} \partial \tilde{t}_{\alpha \beta}$ analogously do not give the contribution to nonderivative goldstone coupling, but the asymmetrical terms like $\partial \tilde{t}_{\alpha \beta} \partial \tilde{t}_{\lambda \beta}$ - already can give. In fact these are the same terms in the kinetic part of $L_2$ that contribute to the spin operators.

13 Note that terms (33) in $L_2$, although linear in $\omega_{\alpha \beta}$, does not contribute to the potential $V(t_{\mu \nu})$ even after taking into account loop corrections.
where the symmetric “metric” fluctuations $h_{\alpha\beta}$ are given by (28) and $\tilde{H}^{\gamma\alpha\beta\mu\nu}$ in low energy limit is a constant tensor constructed from $n_{\alpha\beta}$ and $\eta_{\alpha\beta}$.

If LI is restored at long distances then expression (34) should reduce to sum of the Pauli-Fierz Lagrangian

$$L_{pf} = \partial_\alpha h_{\mu\nu} \partial^{\alpha} h^{\mu\nu} - \frac{1}{2} \partial_\alpha h_{\mu\nu} \partial^{\alpha} h^{\mu\nu} - f_\alpha \partial_\alpha h + \frac{1}{2} \partial_\mu h \partial_\mu h,$$

(35)

where $h = \eta_{\mu\nu} h^{\mu\nu}$, $f_\mu = \partial_\nu h^{\mu\nu}$ and terms $\tilde{L}$, depending from $n_{\mu\nu}$ in such a form that they can be considered as a gauge fixing terms (depending only on $s_i$ and their derivatives). This non LI gauge can be defined by a condition which implies that $h_{\mu\nu}$ can be represented in the form $h_{\mu\nu} = \omega_{\mu\beta} n^{\beta\nu} - \omega_{\beta\nu} n^{\beta\mu}$. Hence now $\omega_{\nu\beta}$ are new variables in (35), $h = 0$, and only first two terms in right hand side of (35) remain.

The antisymmetric fields $\omega_{\mu\nu}$ contain 6 local quantities, connected with the massless spin two fields. Two of them represent the transverse spin-two massless particles (gravitons). The remaining four degrees of freedom are not propagating. In the GR the corresponding quantities correspond to a Coulomb (Newton) field coupled to the energy of sources, and to the 3-vector $g$ with components $\sim g_{0i}/g_{00}$ connected with the angular velocity ($\sim \text{rot} \ g$) of the local system.

Note that after spontaneous breaking of LI we have in fact a theory with two metrics – one is the flat background Minkowski metric $\eta_{\mu\nu}$ and the other is the dynamical ‘curved metric’ $t_{\mu\nu}(x)$.

It is remarkable, that the goldstone gravitons do not interact with the Lorentz invariant contribution to energy momentum tensor $T_{\mu\nu} \sim \eta_{\mu\nu}$ because local Lorentz rotations do not change it (See [3]). The conditions (40) reflect this fact. Therefore in such a systems on can not have the de-Sitter type behavior and thus the cosmological inflation stage driven by almost constant scalar fields becomes problematic.

### 3.3 Degenerate case

It is instructive to consider briefly the simplified case when the condensate $n_{\mu\nu}$ is maximally degenerate: $\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_0$. Then in a coordinate system where $n_{\mu\nu}$ is diagonal one has $n_{11} = n_{11} = n_{11} = \lambda_1$, $n_{00} = \lambda_0$, and there emerge only tree Goldstone modes. In the weak field limit these are the components $h_{i0} = \omega_{i0}(\lambda_0 - \lambda_1)$.

This means that not all gravitational configurations can be reproduced in this case, but only the ones that correspond to the ansatz

$$ds^2 = dt^2 - 2v_i \, dx^0 \, dx^i - (\delta_{ik} + v_i v_k) \, dx^i \, dx^k,$$

(36)

where $v_k(x^i, x^0)$ are some functions of $x^i$ and $x^0$. Let us consider briefly the case of weak field, with $v_i \approx h_{i0} \ll 1$. Then the “electric” components of the curvature tensor read as $R^{0i0k} \sim \partial^0 (\partial^i h^{0k} + \partial^k h^{0i})$ while the other components are small in a weak field limit.

---

14 For the similar case of vector condensate $n_{\mu} = \langle t_{\mu} \rangle$ we have 3 goldstone objects, formed from $n_{\mu}$ by local transformations $\omega_{\mu\nu}(x)$ which rotate $n_{\mu}$. Two of them represent the transverse vector particles (photons), and third is the classical Coulomb field.
The gravitational field of static mass \( m \) follows in this case from equations \( R_{00} \sim m_p^{-2} T_{00} \Rightarrow \partial^0 \partial^i h^{0i} \sim m_p^{-2} m \delta^3(x) \), with a simple solution

\[
\omega^{0i} \simeq h^{0i} \simeq \frac{m}{m_p^2} \frac{x^0 x^i}{|x|^3}, \quad R_{0i0k} \sim (m/m_p^2) \frac{1}{|x|^3} \left( \delta^{ik} - 3 \frac{x^i x^k}{|x|^2} \right)
\]

which leads to the Newton law \([13]\). The force acting on a test particle is \( f^k \sim \Gamma^k_{00} \sim \partial^0 h^{k0} \sim x^k/|x|^3 \).

For a uniformly distributed energy sources \( T^{00}(x) = \varrho_0 \) the equation \( R_{00} \sim \partial^0 \partial^j h^{0i} \sim \varrho_0 \) gives regular solution \( h^{0i} \sim \varrho_0 x^i(x^0 \pm c) \), with nonzero components of curvature tensor \( R_{0i0k} \sim \varrho_0/m_p^2 \). This corresponds to a 3-space metric at small \( x^i \)

\[
g_{ik} = \eta_{ik} + h_{0i} h_{0k} = \eta_{ik} + \frac{x^i x^k}{a^2(x^0)}, \quad a(x^0) \sim \frac{m_p^2}{(c \pm x^0) \varrho_0} \sim a_0(1 \pm \frac{\varrho_0}{m_p^2} a_0 x^0 + ...)
\]

with the time varying “cosmological” scale factor \( a(x^0) \) \([16]\). The other way to “measure” this scale variation can be found from the equation for geodesic deviation, which in the case of two standing test particles, separated by the distance \( \delta x^i \), reduces to \( \partial^0 \partial^i \delta x^i \sim R_{0i0k} \delta x^k \). Because \( (\partial^0 \partial^i \delta x^i)/\partial x^i = (\partial^0 \partial^i a)/a \), this corresponds to the standard evolution equation \( (\partial^0 \partial^i a)/a \sim \varrho_0/m_p^2 \) for homogenous cosmology.

The goldstone fluctuations of the degenerate light-like condensates of the type \([12]\) can be used in the same way to describe the transverse gravitational waves.

### 3.4 Einstein equations

Going to the general case of strong goldstone fields it is interesting to find under what conditions the first derivative term \([29]\), without massive fields \( \partial^2 \), can be reduced to the Einstein-Hilbert Lagrangian, with \( t_{\mu
u}(x) \) playing the role of metric. The answer to this question, without entering into the deep details, can be formulated in a rather simple and natural form.

The **first condition** is that tensor \( \Gamma^{\alpha\beta\gamma\delta\rho\sigma} \) must be expressed only through the “contravariant” \( t^{\mu\nu} \), which should be defined as inverse to \( t_{\mu\nu} \), i.e. through \( t_{\gamma\nu} t^{\gamma\mu} = \delta^\mu_\nu \). Solving this equation, we have

\[
\ell^{\alpha\beta} = \frac{1}{4!} \epsilon^{\alpha\beta\gamma\delta\rho\sigma} t_{\lambda \sigma} t_{\rho \mu} t_{\gamma \nu}, \quad (37)
\]

where \( t = \det(t_{\mu\nu}(x)) \). After that we get the general expression for \([29]\):

\[
L_2 = L_2^{eh} + \tilde{L}_2, \quad L_2^{eh} = t_{\alpha\beta\gamma} t_{\delta\rho\sigma} \left( c_1 t^{\alpha\delta} t^{\beta\sigma} t^{\gamma\rho} + c_2 t^{\alpha\delta} t^{\beta\rho} t^{\gamma\sigma} + c_3 t^{\alpha\gamma} t^{\beta\rho} t^{\delta\sigma} + c_4 t^{\alpha\gamma} t^{\beta\sigma} t^{\delta\rho} + c_5 t^{\alpha\beta} t^{\delta\rho} t^{\gamma\sigma} \right), \quad (38)
\]

The generalization to strong fields can also be found in this gauge. It corresponds to Schwarzschild solution in some special synchronous system connected with form \([35]\), and i.e. applicable up to distances \( \sim |x^0| \sim |x| \sim \Lambda^{-1} \) where the goldstones can disappear due to the melting of the tensor condensate \( n_{\mu\nu} \).

Comparing this behavior of \( a(x^i) \) with the case of the cosmological scale factor in the GR \( (\partial^0 a/a)^2 \sim \varrho_0 m_p^{-2} \), one can try to fix the Hubble parameter as \( H \sim \sqrt{\varrho_0}/m_p \). But this is the result of linearized approximation. Evidently, all these solutions are applicable only in limited intervals of \( x^0 \) when \( h_{0i} \) are small.

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where \( c_i \) are some scalar functions from invariants formed out of \( t_{\mu\nu} \). All such invariants can be expressed as functions from \( s_i \), and they should be taken at extremal values as far as we have excluded heavy fields \( \rho_i \). In this way, all \( c_i \) in \((38)\) are constants. The part \( \tilde{L}_2 \) contains terms with the same structure as in \((38)\) but in which some “contravariant” \( t^{\mu\nu} \) used for index contraction are changed by \( t_{\mu\nu} \) or \( \eta_{\mu\nu} \).

Then the first condition is that \( \tilde{L}_2 \) = 0, or it can be represented as a function from 4 scalar combinations \( s_i \) defined in \((14)\) and of their derivatives in \( x^\mu \). Then \( \tilde{L}_2 \) can be considered as gauge fixing term which does not change the dynamics of \( t_{\mu\nu} \) fields. Note that this condition, although simply formulated, is dynamically very restrictive and it would be strange if it is fulfilled “by itself” without fine tuning for some parameters in the \( \psi = (B, \chi, \varphi) \) field system.

The second condition is that in the “weak field limit”

\[
t_{\mu\nu}(x) = n_{\mu\nu} + h_{\mu\nu}(x), \quad h_{\mu\nu} \to 0
\]

the expression \((38)\) transforms into the Pauli-Fierz Lagrangian \((35)\) for massless spin-two particles in a nonorthogonal coordinates with metric \( n_{\mu\nu} \). In fact such a condition removes the spin zero massless modes, which are present in general in \((38)\). As it can be simply shown it leads to relations for coefficients in \((38)\):

\[
c_2 = -2c_1, \quad c_3 = c_4 = c_5 = 0.
\]

These two conditions fix the form of \( L_2 \) up to overall constant \((c_1 \sim \Lambda^2 \to m_B^2)\) and after that we obtain finally that

\[
L_2^{(eh)} = c_1 t_{\alpha\beta,\gamma} t_{\delta\sigma,\rho} (2t^{\alpha\delta}t^{\beta\rho}t^{\gamma\sigma} - t^{\alpha\delta}t^{\beta\sigma}t^{\gamma\rho}), \quad (39)
\]

where \( t^{\alpha\beta} \) is given by \((37)\). Possibly this condition can be relaxed, because in fact what we need here is to truncate the contributions from spin 0 and 1 under specific “gauge chose”.

This expression \((39)\) precisely coincides with the Hilbert-Einstein Lagrangian in gauges which fixed value of \( t(x) = \det[t_{\mu\nu}(x)] = \text{const} \). On the other hand, here we have just this case - the “gauge” of goldstones in \( t_{\mu\nu}(x) \) is fixed by the conditions

\[
Tr(t_{\mu\nu}) = a_1, \quad Tr(t^2) = a_2, \quad Tr(t^3) = a_3, \quad Tr(t^4) = a_4, \quad (40)
\]

where constants \( a_k \) coincide with roots of equations \( \partial V/\partial s_k = 0 \) (See \((15)\)), connected to eigenvalues of \( t_{\mu\nu} \) and as a result the \( \det[t_{\mu\nu}(x)] = \lambda^{(1)}\lambda^{(2)}\lambda^{(3)}\lambda^{(4)} = \text{const} \). This is by itself very interesting gauge. In this case \( \det[t_{\mu\nu}] \) is also fixed by the fact that the \( t_{\mu\nu} \) - goldstones correspond to the 4-volume preserving fluctuations of metric. In other way, this explains why the goldstone gravitons does not interact with the terms corresponding to cosmological constant.

The Hilbert-Einstein Lagrangian \( L_2 \) in \((39)\) is polynomial in \( t_{\mu\nu}(x) \), and in fact such a theory looks like some special \( \sigma \)-model \(^{17}\). It takes even more simple form, when we substitute in \((39)\) and \((37)\) \( t_{\mu\nu} \) in form \((22)\) and use \( \Omega^\mu_\nu \) as a main variables, describing the gravitational field.

\(^{17}\) The accidental scale-invariance of \( \hat{L} \) in \((39)\) of type \( x \to \alpha x, \quad t_{\mu\nu} \to \alpha^{2/11}t_{\mu\nu} \) is probably fictitious because gauge fixing conditions \((10)\) are not invariant under such a transformation.
The terms with higher then two powers of derivatives in (5) can lead to higher order in “curvature” terms in L, and near to the “Planck” scale where $\partial_\mu \sim \Lambda$ all these terms can be of same order as $L_2$.

Here again as for $L_2$ we come two type of terms. Terms that we construct by using only the covariant $t^{\mu\nu}$ for contraction of indices and other terms - in which $\eta^{\mu\nu}$ and $t^{\mu\nu}$ are also used. Then from the first we come to expressions containing invariants constructed only from higher powers of the curvature tensor and taken in gauge (10). The other terms are different, and perhaps they can be interpreted as the gauge fixing terms for the “invariant” part of the Lagrangian. In this case we have the Lorentz invariant description at all scales.

The last possibility is attractive but unfortunately we do not see clear reasons for its realization. Less restrictive is the possibility that the coefficients entering in such an expansion can be scale dependent, and it is not excluded that at distances $|x_i - x_j| \gg \Lambda^{-1}$ only the invariant part survives and the “additional” terms becomes comparatively small, for example like $\sim 1/(|x_i - x_j| \Lambda)^2$ or turn into “gauge fixing” expressions.

4 Tensor condensates in curved space and bigravity theory

Here we briefly consider what modifications can be expected if we repeat the constructions of previous sections with slowly varying condensate $\langle \tau^{\mu\nu}(x) \rangle$ and its fluctuations in the external curved background $g_{\mu\nu}(x)$, instead of flat Minkowski metric $\eta_{\mu\nu}$ without back reaction on metric $g_{\mu\nu}(x)$ from the system of $\psi = (B, \chi, \varphi)$ fields.

For small curvatures, $|R^{\mu\nu\lambda\sigma}(x)| \ll \Lambda^2$, the local dynamics which leads to the condensation of some composite operators in field system $\psi = (B, \chi, \varphi)$ remains approximately the same as in a flat case. Therefore, the mean value

$$\langle \tau^{\mu\nu}(x) \rangle = n^{\mu\nu}(x),$$

should be a covariantly constant tensor which in the local Lorentz frame for every point $x$ reduces to approximately the same $n^{\mu\nu}$ as before, with a possible corrections of order $|R^{\mu\nu\lambda\sigma}(x)|/\Lambda^2$.

If we now consider the geometrical gravity as a dynamical field also, then on should expect that in a weak field limit the propagation of $t^{\mu\nu} = n^{\mu\nu} + h^{\mu\nu}$ and $g^{\mu\nu} = \eta^{\mu\nu} + \beta^{\mu\nu}$ will be independent. And the form of effective $L(h^{\mu\nu}, \beta^{\mu\nu})$ - in first approximation is given by a sum of two Pauli-Fierz Lagrangians for $h^{\mu\nu}$ and $\beta^{\mu\nu}$ (if propagation of $\beta^{\alpha\beta}$ is Lorentz-covariant). The interaction of these fields with other massive particles will be also independent in this approximation. If in addition the goldstone gravitons interact in the long range limit universally (i.e. with the energy momentum tensor), then their actions will be indistinguishable.

It seems that the dynamical mixing between $t^{\mu\nu}$ and $g^{\mu\nu}$ is impossible (as between massless particles), so at long distances there remain two massless particles, which act in a rather similar way. The presence of the second (true) ‘gravitational field’ can be manifested only in possible small nonuniversality of $t^{\mu\nu}$ interactions.

18 In fact the same situation takes place for the usual geometrical gravity, where the “natural” general form of the Lagrangian is $\sum_n c_n R^n$ where $c_n \sim m^2^n$ and the invariants $R_n \sim (R_{abcd})^n$ are constructed from n-th powers of the curvature tensor.
When both fields $t_{\mu\nu}$ and $g_{\mu\nu}$ are strong (it is when scales $\Lambda_s$ and $\Lambda$ are also close) then their interaction can be highly asymmetrical and complicated.

But if a coupling constants in the ‘fundamental lagrangian’ $L(\chi, B, \varphi)$ are such that the difference between $\Lambda$ and the geometrical Planck(string) scale $\Lambda_s$ is very big then the quantum fluctuations of geometry near the scale $\Lambda$ would be small, and then the geometrical and goldstone gravity will in fact decouple. In this case only the composite gravity ($t_{\mu\nu}$) is essential at the macroscopic distances.

The geometrical gravity can again start to contribute only at maximal cosmological scales (See Section 6) due to the interaction with various contribution to $\langle T_{\mu\nu} \rangle \sim g_{\mu\nu}$ imitating the cosmological constant $\Omega$.

5 The Goldstone graviton coupling to other particles: universality problem

In the geometrical (string) gravity the gravitons are universally coupled to all particles and this is probably one of the main experimentally established property of the gravitation. The general relativity ‘explains’ this fact in a simple and natural way. For the $t_{\mu\nu}$ gravitons the situation is more complicated.

Note that in fact we “do not need” the universality of $t_{\mu\nu}$ (gravitons) interactions at all scales, but only in the long-distance limit, where we know (from experiment) that it is fulfilled with an extreme precision.

Since goldstones $g_{\mu\nu}$ are not geometrical objects, the universality of their coupling with other fields are not automatic at all scales and must be probably tuned to reach the agreement with the experiment. At the same time, although the local Lorentz-invariance is spontaneously broken, the translation invariance remains. Usually just this invariance leads, after localization, to diffeomorphism-invariance and universality of graviton interactions. Therefore, one can hope that at the distances much larger than $\sim \Lambda^{-1}$, at which the local Lorentz-invariance is broken, the approximate diffeomorphism invariance remains, and as a result we come to the universality of the long-range goldstone graviton mediated interactions.

But evidently at the scales close to $\sim \Lambda^{-1}$ one must expect big nonuniversalities. This can be directly seen in a models of the following type.

Suppose that in addition to the sector of $\psi$-particles that we considered before, $\psi = (B \chi, \varphi)$, and from which the Goldstone modes are formed, there exist also a sector of $s$ particles, only weekly coupled to particles of $\psi$ sector by some mediator (gauge) particles $\gamma$ with small coupling $e$. This can be, for example, the sector of standard model or some of its generalization, and $\gamma$ - some gauge fields coupled to both sectors.

Then we have tree classes of values of couplings ($\sim 1$, $\sim e^2$, $\sim e^4$) for goldstone-graviton vertexes ($t_{\psi\psi}$, $t_{\gamma\gamma}$, $t_{ss}$), corresponding to diagrams shown in Fig. 1. Because $e^2$ can be “chosen” arbitrary small on the scale $\Lambda$, on can have very big nonuniversality of $t_{\mu\nu}$

\[^{19}\text{It is not excluded that is possible to remove one of the massless particles in \(t_{\mu\nu}, g_{\mu\nu}\) system by some form of Higgs effect for tensor particles. For this we additionally need massless vector and scalar Goldstone particles interacting in a special way with \(t_{\mu\nu}\). May be the spontaneous breaking of LI is in some “virtual” form realized also in QCD like theories, so that the corresponding Higgs effect makes the tensor and vector goldstones massive - and the remnants of it are the massive vector and tensor resonances, whose interactions reminds the gauge coupling}\]
In the long range limit only the \( \partial L_2 / \partial \omega_{ab} \) term in (41) contributes to the interaction-current, connecting \( \omega_{ab} \) with other fields:

\[
\frac{\partial L_2}{\partial \omega_{ab}} \sim (\Gamma \cdots Q_{a}^{\beta} \gamma^\sigma)(\partial_{\nu} \tilde{t}_{\alpha\beta} \partial_{\sigma} \tilde{t}_{\delta\rho}) = \Upsilon_{ab}^{\mu\nu}(\partial_{\mu} \delta^n \partial_{\nu} \delta^n),
\]

where \( \Upsilon_{ab}^{\mu\nu} \) some constant tensors. The structure of \( \omega_{ab} \) interactions, coming from the term \( \partial L_2 / \partial \omega_{ab} \) in (41), evidently remains the interaction of gravitons in general relativity, coming from analogous terms \( T_{\mu\nu} = \partial L / \partial g_{\mu\nu} \). Note that in the weak field limit \( \omega_{ab} \) and \( h_{\mu\nu} \) are linearly related as in (28).

The fundamental fields and physical particles do not enter in \( g^{\mu\nu} \) symmetrically on the scale \( \Lambda \). Therefore, as follows from (42), there is no universality in the goldstone-graviton coupling at such a momenta. On much larger distances \( x_{ij} \gg \Lambda^{-1} \) operators \( \partial L_2 / \partial \omega_{ab} \) in (42) are renormalized, and so is not excluded that for \( t \)-gravitons with momenta \( k \to 0 \) the universality is reached.

There exist rather simple general argument [8] that in Lorentz-invariant case the massless spin two particles must be coupled universally to all particles in the limit \( k \to 0 \). This construction can be generalized to the case with spontaneous breaking of LI as follows.

Consider some general \( N \)-particle amplitude with soft emission of \( t_{\mu\nu} \)-graviton with momenta \( k \to 0 \). As usual, in this limit the main contribution comes from diagrams with the graviton emission from all external lines (See Fig.2), and has the form:

\[
A_{\mu\nu}^{(N)}(p_1, \ldots, p_N, k) \sim B^{(N)}(p_1, \ldots, p_N) \sum_{i=1}^{N} \frac{\Gamma^i_{\mu\nu}(p_i)}{(2p_{i\alpha} k_{\beta} G^{\alpha\beta})},
\]

Let us remark, however, that we know very little (from a direct experiment) whether the gravitational interaction of some weakly interacting particles (for example dark matter) is exactly universal.
where we supposed that propagators of $p_i$-particles near mass shell have the form $2(p_{i\alpha} k_\beta G_{\alpha\beta} - m_i^2)^{-1}$, and the tensors $G_{i\alpha\beta}$ can depend from $n_{i\mu\nu}$. The vertexes $\Gamma_{\mu\nu}^i$ for a on mass shell graviton emission by particle of type (i) can also contain all possible combinations of $p_\mu$ and $n_{i\mu\nu}$

$$\Gamma_{\mu\nu}^i(p) = C_i^\nu p_\mu + (H_1^i)_{\mu\nu} + p_\mu (H_2^i)_{\nu\lambda} p_\lambda +$$

$$+ p_\lambda (H_3^i)_{\lambda\mu} p_\nu + (H_3^i)_{\mu\lambda} p_\lambda (H_3^i)_{\nu\beta} p_\beta ,$$

(43)

where the tensors

$$(H_k^i)_{\mu\nu} = (C_k^i n + \hat{C}_k^i nnn + \hat{C}_k^i nnnn)_{\mu\nu} , \quad i, k = 1, 2, 3$$

(44)

and $C_k^i$, $\hat{C}_k^i$, ... - some invariant functions depending at $k = 0$ only from particles type. In the usual case without LI breaking we have in (43) only the first term $p_\mu p_\nu$ in graviton particle coupling. Due to masslessness of $t_{\mu\nu}$-gravitons the amplitudes $A_{\mu\nu}^{(N)}$ should be transverse at all $N$ and particles momenta $p_i$:

$$2k_\mu A_{\mu\nu}^{(N)} = B(p_1, ..., p_N) \sum_{i=1}^{N} \left( C_i^\nu p_\mu + (H_2^i)_{\nu\lambda} p_\lambda +\right.$$  

$$+ \frac{1}{(G_{\alpha\beta}^i k_\alpha p_i^\beta)} \left( (H_1^i)_{\nu\lambda} + (k H_2^i)_{\nu\lambda} + (p^j H_3^i)_{\nu\lambda} (k p^j) \right) \bigg) = 0 .$$

(45)

The only possibility that this conditions can be fulfilled identically at all $N$ and $p_i$ is when equation (45) is reduced to energy-momentum conservation $\sum_i p_i = 0$. And the momentum conservation take place because the translation invariance is unbroken by tensor condensate. This impose conditions on coefficients entering equation (45) which has the general solution of the form:

$$(H_1^i)_{\mu\nu} = 0 ; \quad (H_2^i)_{\mu\nu} = a_i G_{\mu\nu}^i ; \quad (H_3^i)_{\mu\nu} = b_i G_{\mu\nu}^i = b_i \eta_{\mu\nu} ;$$

$$C_i + 2a_i + (b_i)^2 = C.$$

21This point can look not a certain because LI is already broken. It corresponds to removing of scalar part of $t_{\mu\nu}$ with respect to $\eta_{\mu\nu}$. 

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And so we end with the universal (for $k \to 0$) vertexes for the soft $t_{\mu\nu}$-graviton interactions with particles

$$\Gamma^i_{\mu\nu}(p,k) = C p_{\mu} p_{\nu} + \frac{k^2}{\Lambda^2} \hat{\Gamma}^i_{\mu\nu}(p) + \ldots ,$$

(46)

where $\hat{\Gamma}^i_{\mu\nu}(p)$ are nonuniversal and can depend on $n_{\mu\nu}$.

The details of mechanism, which can lead to the behavior of the type (46) are not quit clear, but it can be somehow connected to a different scale behavior of operators $\sim p_{\mu} p_{\nu}$ and the dependent on $n_{\mu\nu}$ contributions to vertices $\Gamma_{\mu\nu}$.

If we suppose that (46) take place, the corrections to ‘universal’ vertexes (which $\sim T_{\mu\nu}$), from various nonuniversal operators in $\tau_{\mu\nu}$ can decrease with scale $\omega$ as $(\omega/\Lambda)^n$, where $n \geq 1$. For $\omega < 10^{-3} \div 10^{-5} \text{eV}$, where we already have experimental, data confirming the universality of gravity, the factor $(\omega/\Lambda)$ is $< 10^{-31}$. This can be compared with the best experimental limits on the universality of $|1 - m_{\text{grav}}/m_{\text{inert}}| < 10^{-15}$, coming from last Etvesh like measurements\(^{22}\).

From a more general point of view, when on looks on the possibility of restoration of maximal symmetry on long distances, many example of this known. But it is complicated to formulate the universal criteria for this. The simplest examples come from various lattice calculations (and experiments) - here not only isotropy but even translation invariance is broken on the scale of lattice spacing $a$. But at distances $x \gg a$ the fast isotropization take place, and the direction dependent contributions decrease as powers of $(a/x)$. On should also mention the calculations in [9], where the isotropization in renormalization flow to large distances is observed. But at the same time we know that there exist also such a media, whose macroscopic anisotropy reflects its the microscopic asymmetry.

From various of examples, we know, that the main reason for such an isotropization is connected with the fact that the quantities responsible for the spacial symmetry breaking are represented by a dimensional quantities (like lattice spacing or tensor $\Lambda^4 n_{\mu\nu}$). Therefor the operators, in which their enter, have different scale dependence, in comparison with more symmetric operator contribution, representing the same physical quantity.

Note, that the different approach to the universality of interactions of nonscalar goldstone particles was proposed [11]. In [11] the primary condition is that the non-Lorentz invariant terms in the effective Lagrangian can affect only the “gauge-dependent” components of Green functions, so that they should not enter (at all frequencies) into measurable quantities. Is unclear for us if is possible in dynamical theory of the type we consider, i.e. in the situation when there exist a real “oriented” condensate, which in principle can enter in various amplitudes.

6 Supersymmetric generalization

It is interesting to generalize the above construction of spontaneous breaking of LI and of corresponding graviton-goldstones to the supersymmetric case. This, in particular, can also be essential for the cosmological mechanisms and various unification models. In bigravity models by changing the relation between the geometrical an goldstone scales on can try to regulate the value of the long-scale cosmological constant. Besides, this can be natural for

\(^{22}\) This comparison also shows that the minimal value on which on can now lower the value of $\Lambda$ by various brane-world mechanisms is $\geq 10^3 \text{GeV}$.\]
bigravity models, when on a small distances we have the string description, which usually leads to a supersymmetric field theory on a larger distances. If the “basic” field theory $(B, \chi, \varphi)$ is supersymmetric then on the larger scale, where the $n_{\mu\nu}$ condensate, we have two main possibilities:

- the first case - when the supersymmetry is also broken at the scale $\Lambda$;
- the other case - the supersymmetry is not broken by the mechanisms which generate the $<\tau_{\mu\nu}>$ condensate and is broken by the other mechanism at much larger distances.

The difference between these versions is evidently reflected in the behavior of $<T_{\mu\nu}>$. In a first case we can have nonzero mean $T_{\mu\nu} \sim \Lambda^4$ in (16), and in the case of non broken supersymmetry we have $<T_{\mu\nu}> = 0$ at $\Lambda$ scale and at some larger distances. In (16) this looks as an additional relation between coefficients. The first possibility is more general.

And in this Section we briefly consider the second case, to define conditions when it can take place.

So, as above, we suppose that there exist special sector, now supersymmetric, with becomes strongly coupled on some scale $\Lambda$. We assume that the dynamics of this sector is described by the effective renormalizable theory - otherwise it is complicated to have the grow of couplings with distance. Its lagrangian $L(V^i, \phi^i)$ can contain only vector gauge superfields $V^i$ in adjoint representation of some nonabelian group (say $SU(N)$) , and also a number of chiral superfields $\phi^i$ in multiplets of the same group. These fields also interact with a sector of “our” supersymmetric matter (using some mediators). Such Lagrangian is fixed up to values of constants, and something is known about the strong coupling behavior of such a theories. And we want to know if it is possible such a situation, when some vector and (or) tensor quantities have nonzero vacuum expectation values, but at the same time the supersymmetry is not spontaneously broken, and how natural is such a behavior. Here we use the simplest approach to this question.

Begin from the supersymmetric generalization of tensor operator $\tau_{\mu\nu}$ decomposition in terms of the fundamental renormalizable superfields $V^i, \phi^i$. For this we firstly include $\tau_{\mu\nu}$ in some adequate superfield as its lowest component. If we simply act on $\tau_{\mu\nu}$ by the global supersymmetry transformation

$$H_{\mu\nu}(x, \theta, \bar{\theta}) = \exp(\xi \bar{Q} + Q \xi) \tau_{\mu\nu}(x)$$ (47)

with parameters $\xi$ and generators $Q$, we also generate the other components of this superfield. Such a superfield is in general reducible - sum of real plus chiral and antichiral. We suppose that only chiral (plus antichiral) component is present

$$H_{\mu\nu} = \tau_{\mu\nu} + \theta \chi_{\mu\nu} + \theta \theta f_{\mu\nu}$$ (48)

Such a chose of multiplet, in which we inserted $\tau_{\mu\nu}$, essentially restricts the model and allows not to break LI on condensate scale. We can decompose

$$H_{\mu\nu} \equiv U_{\mu}^a U_{\nu}^a = s_{\mu}^a s_{\nu}^a + \theta \left( s_{\mu}^a \chi_{\nu}^a + s_{\mu}^a \chi_{\nu}^a \right) +$$
$$\theta \theta \left( s_{\mu}^a \sigma^a_{\nu} + \sigma^a_{\mu} s_{\nu}^a + \chi_{\mu} \chi_{\nu}^a \right),$$ (49)

Note that if $\tau_{\mu\nu}$ is chosen as component of a real superfield, than probably we will always break supersymmetry together with LI by condensing $\tau_{\mu\nu}$.
so that the lowest component of chiral superfield

$$U^a_\mu = s^a_\mu + \theta \chi^a_\mu + \theta \theta \sigma^a_\mu$$

coincides with vierbein eigenvectors $s^a_\mu$ (see (10)), in with we also included factors $\sqrt{\lambda^a}$ from eigenvalues.

In terms of ‘fundamental’ superfields $V^i$ and $\phi^i$ the operator decomposition of $\hat{H}_{\mu\nu}$ (the analog of (1)) can look as

$$\hat{H}_{\mu\nu} = P_1(\phi) \partial_\mu \phi^i \partial_\nu \phi^i + P_2(\phi) W^i_\alpha \sigma^{\alpha\beta} \partial_\nu W^i_\beta + \ldots ,$$

(50)

where $W^i_\alpha = \bar{D} \bar{D} \bar{D} V^i$ - is supergauge strengths, $P_i$ some real functions from chiral fields $\phi^i$ and summation over indexes of gauge group G is assumed.

After we integrate over fields $V^i$ and $\phi^i$ (at fixed $H_{\mu\nu}(x, \theta)$) we come to (as in (3)) the supersymmetric effective lagrangian $L(H_{\mu\nu})$ depending on $H_{\mu\nu}(x, \theta)$. We write it in a form of symbolic decomposition in powers of superfield $H_{\mu\nu}$ and its derivatives in coordinates:

$$L(H_{\mu\nu}) = F^{(0)}(H)_{\theta\theta} + F^{(1)}(H \cdot H^+)_{\theta\theta\theta\theta} + \partial_\alpha \partial_\beta F^{(2)}_{\alpha\beta}(H \cdot H^+)_{\theta\theta\theta\theta} + \ldots$$

(51)

with some functions $F^{(i)}$, defined by the dynamics on scale $\Lambda$. The first terms contribute to potential and higher describe the propagation of degrees of freedom contained in $H_{\mu\nu}$.

The simple example of terms in (51) is given by

$$L(H_{\mu\nu}) = F^{(0)}(H)_{\theta\theta} + (H \cdot H^+)_{\theta\theta\theta\theta} + \ldots$$

The corresponding potential energy will have the form

$$V(t_{\alpha\beta}) = f_{\alpha\beta}(t)f^{+}_{\alpha\beta}(t) ,$$

(52)

where

$$f^{+}_{\alpha\beta}(t) = a_1 t_{\alpha\beta} + a_2 t_{\alpha\gamma} t_{\gamma\beta} + a_3 t_{\alpha\gamma} t_{\gamma\delta} t_{\delta\beta} + \ldots$$

(53)

is the highest component of $H^{+}_{\alpha\beta}$, and the coefficients $a_i$ correspond to expansion of $F^{(0)}$ in powers of $H$. Expression (52) is the special case of potential (13) which now it is positive definite. This follows because we supposed that in every point $t_{\alpha\beta}$ can be reduced to diagonal form by local Lorentz-transformation. This leads to diagonal form of $f_{\alpha\beta}$ and to positivity of right hand side of (52). Therefore $t_{\alpha\beta} = 0$ is always the solution, corresponding to a minimum of potential with $V(0) = 0$. In this case LI is not broken spontaneously.

There can also exist other minima in (52) with $t_{\alpha\beta} = n_{\alpha\beta} \neq 0$, and $V(n_{\alpha\beta}) = 0$ - at least for some regions of parameters $a_i$. In diagonal basis for $f_{\alpha\beta}$ we become for potential

$$V = \sum_{i=0}^{3} (f(t_{ii}))^2 , \quad f(t) = \sum_{m=1}^{\infty} a^m t^m .$$

And because the equation $f(t) = 0$ can have many nontrivial zeros without fine tuning of $a_m$, such zeros will generate minima of $V(t_{\alpha\beta})$ with the same $V(n_{\alpha\beta}) = 0$. And therefore
supersymmetry is not yet broken in these minima, but the LI is broken\textsuperscript{24}, and these minima are stable\textsuperscript{25}.

At scales where the supersymmetry is broken the energy of these vacuums can become different - of the order of supersymmetry breaking scale $\Lambda_{\text{ss}}^4$. Which of them will be lower, depend from details of dynamics - this was already discussed in previous sections.

The oscillations of $t_{\alpha\gamma}$ corresponding to the local Lorentz rotations give the goldstone gravitons - for long wavelength they almost does not change the potential $V(t_{\mu\nu})$. But here the the local supersymmetry transformations\textsuperscript{24} with Majorana spinor parameters $\xi(x)$ also do not change the energy in the long wavelength limit and correspond to massless goldstone fermions - gravitinos. These $\xi(x)$ give the transverse components of such goldstone-gravitino in a specific gauge, defined by the condition that in every point gravitino field $\chi_{\mu}(x)$ can be transformed into the same constant eigenvectors $S^a_{\mu}$ by the local supersymmetry transformation with parameters $\xi(x)$. That is goldstone-gravitino can be represented in the form

$$\chi_{\mu}(x) \sim \left( \exp(iQ\xi(x)) - 1 \right) \sigma^a S^a_{\mu}.$$  

in such gauge.

After substitution of (48) in (51) we come to some effective constrained supergravity lagrangian, where gravitinos are described by spinor $\xi(x)$ and gravitons - by the antisymmetric matrixes $\omega_{\mu\nu}(x)$ corresponding to local Lorentz transformations.

Below the supersymmetry breaking scale these gravitinos are already not a purely goldstone particles and become a mass, depending from mechanism by which the supersymmetry breaking takes place.

### 7 Cosmology

If goldstone-gravity lagrangian approximately coincides with Einstein lagrangian as in (39), then probably all main stages of standard Friedman-Robertson-Walker (FRW) cosmological model can ran here without additional tuning.

There is even some advantage - at temperatures $> \Lambda$ the composite gravity disappears (because the $< t_{\mu\nu} > \sim \eta_{\mu\nu}$), and it gives additional possibilities to avoid the cosmological singularity.

The important property of goldstone gravitons is that they do not interact with contributions to $< T_{\mu\nu} > \sim \eta_{\mu\nu}$. This is so because this expectation value is not changed by the local Lorentz transformations $\Omega^\lambda_{\mu}(x)$ which “contain” the graviton-goldstone degrees of freedom. In particular, this means that the inflation stages can not appear in such a purely goldstone gravity from large coherent fluctuations of some effective scalar field.

In the more interesting (bigravity) case\textsuperscript{26} when the geometrical gravity is also present,\textsuperscript{26} in a more general case\textsuperscript{25} the potential will have similar to (52) form $V(t_{\mu\nu}) = f_{\alpha\beta}(t)f^+_{\alpha\beta}(t)/F(tt^*)$, where $F(z) \sim \partial F^{(1)}(z)/\partial z$. Here we come to similar conclusions, although the region of allowed parameters, for not to brake supersymmetry, can be more narrow.

\textsuperscript{24} For this case we can have the domain structure of vacuum in with regions with different Lorentz structures $n_{\mu\nu}$ are separated with massive supersymmetric walls. At large distances the supersymmetry expected to be broken, and domain structure becomes unstable.

\textsuperscript{25} Number of models was proposed which include many “metric” and which look geometrically unusual but do not contradicting to experiment. See for example [12], [13].
only the geometrical component of metric can “feel” the \( T_{\mu\nu} \sim \eta_{\mu\nu} \) component. Then the fundamental geometrical planck scale \( M_P \) can be chosen so that \( M_P \gg \Lambda \). And this way on can “naturally” make the observable cosmological constant small.

In the next sub-sections we briefly consider main features of the two type scenarios.

- In first there is no fundamental geometrical gravity - only of the goldstone type.
- In the second case we have special bigravity: there are both - geometrical and goldstone tensor fields with scales adjusted such that \( M_P \gg \Lambda^{27} \).

Below we also suppose that all possible macroscopic effects coming from a possible breaking of LI on scale \( \Lambda \), due to existence of \( n_{\mu\nu} \), can be made small - less than seen in current experiments.

### 7.1 Cosmology with only goldstone gravity

Here the most natural cosmological scenario corresponds to FRW geometry with spatially flat infinite metric with the “standard” stages with scale factor \( a(t) \sim |t|^\alpha \quad \alpha = 1/3, \ 2/3 \) . Now this is applicable in time intervals \(-\infty < t < -t_0, \ t_0 < t < \infty \). The value of \( t_0 \) is defined such that at \(-t_0 < t < t_0 \) the temperature of media is greater then \( \Lambda \), so that the \( < \tau_{\mu\nu} > \) condensate evaporates and there is no gravitational interaction in this time interval. Therefore the compressed media at \(-t_0 = t \) enters in the inertial stage of compression, and as a result the compression stops at some maximal density of order \( \Lambda^{4} \) (we can chose time \( t=0 \) for this moment). After that the media begins to expand, initially without deceleration, but the temperature is falling. When temperature reaches some critical value of order \( \Lambda^{27} \)

![Figure 3: Scale factor variation with time. In the interval from \(-t_0 \) to \( t_0 \) the Goldstone-gravity is switched off.](image)

the \( < \tau_{\mu\nu} > \) condensate appear and the goldstone gravity is switched on again and the system enters at the \( t_0 < t < \infty \) an FRW stage\(^{28}\).

\(^{27}\) One can also mention the even more sophisticated possibility, when the field theory model with only the goldstone-composite gravity (for some specially adjusted \( B_{\chi, \varphi} \) fields) is dual to string systems (in a special background), with contain long-range “geometrical” gravity. Then on can even expect that the cosmology with a pure goldstone-gravity should give approximately the same picture as a some solution of standard geometrical cosmology.

\(^{28}\) In this model the initial conditions for \( < \tau_{\mu\nu} > \) and the matter fields at \( t = -\infty \) can be prepared in some complicated nonisotropic and nonhomogeneous form. But the configuration of newly created \( < \tau_{\mu\nu} > \)
In this scenario some contemporary experimental data about the character of universe expansion, which are most often interpreted using small cosmological constant, can only be explained by some different mechanisms - possibly coming from some universal corrections.

Note that in such a model we do not meet the naturalness-type problems with flatness, isotropy etc., which are usually solved by the inflation stage. But the problem with “washing out” of “unneeded objects”, like monopoles, remains. The initial and boundary conditions in such a model are set at \( t = -\infty \), and so this can naturally lead to almost arbitrary density perturbations, which can penetrate through the \((-t_0) \div (t_0)\) interval and initiate various structures at the FRW stage \((t_0) \div \infty\). Here on can meet some troubles, because the gravitational instabilities can grow on the implosion stage \(-\infty \div (-t_0)\). As a result the perturbations at \( t \sim -t_0 \) can become too big, so that \( \delta \rho/\rho \sim 1 \) on all scales for rather general initial conditions at \( t = -\infty \). But, in the high-temperature time interval \((-t_0) \div (t_0)\) the density irregularities can essentially dump, possibly up to needed level. This question needs additional consideration to make any quantitative estimates.

### 7.2 Cosmology with two gravities

Here we have much more possibilities. The system becomes interesting when we put a limitation on the geometrical planck scale \( M_P \) such that:

\[
M_P \gg \Lambda \sim m_p \sim 10^{19} \text{GeV}
\]

then the \( < \tau_{\mu\nu} > \) - condensate scale \( \Lambda \) is located in “field theory region”, far from \( \Lambda_s \). Therefore the geometrical fluctuation of metric near \( \Lambda \) are small \(^{29}\).

It is also favorable to chose the \( L(B, \chi, \varphi) \) so that the physics is supersymmetric from planck(string) scale \( \Lambda_s \) up to \( \Lambda \). This by itself is a natural long range situation in string theory. Then there will be no contribution to the geometrical cosmological constant from distances \( (M_P^{-1} \div \Lambda^{-1}) \).

From the other hand, all contributions to the mean \( < T_{\mu\nu} > \), coming from scale \( \Lambda^{-1} \) and all larger distances, don’t interacts with the goldstone gravity. But in the same time they look like a cosmological constant for the geometrical gravity. So we can “adjust” the \( M_P \)-scale such \(^{30}\) that the mean \( < T_{\mu\nu} > \) gives no more than the now “observable” cosmological constant \( \rho_{\text{exp}} \sim (10^{-3} eV) \). This corresponds to condition on \( \Lambda \) and \( M_P \): \( \rho_{\text{exp}} \sim \Lambda^4 \left( \frac{\Lambda}{M_P} \right)^2 \),

which gives

\[
M_P = m_p \xi \sim 10^{81} \text{GeV} \quad , \quad \xi = M_P/m_p \sim \left( \frac{m_p}{\rho_{\text{exp}}^{1/4}} \right)^2 \sim (10^{31})^2 \text{GeV}
\]  

\(^{29}\) This scale ordering looks even natural in such a type models. The logarithmic grow of gauge constants in \( (B, \chi, \varphi) \) field sector is slow and on can need many orders in scale before on reaches the distances \( \Lambda^{-1} \) when interactions become strong.

\(^{30}\) This corresponds to the choice of \( L(B, \chi, \varphi) \) parameters on the \( M_P \) scale. And because this choice of \( L \) is not “free” and is in fact defined by the dynamics of strings probably all such an adjusting can only be considered as corresponding to the ‘anthropological’ selection of successful string vacuum.
for the geometrical Planck (string) scale $M_P$.

Note that this simplest construction (although it can “explain” the value of cosmological constant) is not fully perfect, because in this case the supersymmetry is supposed to be broken already at $\sim \Lambda$ i.e. at $10^{19}GeV$. We will have more preferable picture if supersymmetry is broken at much lower energy scale $\mu \ll m_p$. In this case to transform this $\mu$ to the cosmological constant $\rho_{exp}$ we need string planck mass $M_s$ smaller then in (54):

$$\begin{align*}
M_P &= m_p \xi \sim 10^{63}GeV, \quad \xi \sim \left(\frac{\mu}{\rho_{exp}}\right)^{2} \sim 10^{44},
\end{align*}$$

where the numbers are shown for $\mu \sim 10^{10}GeV$.

In brane-like models on can try to make the geometrical planck mass $M_P$ even less. The actual value depends from a connection between $\Lambda$, the brane scale $\kappa$ and the “size” of large dimensions $\varepsilon^{-1}$. We have two possibilities.

If the goldstone gravity is localized on the 3-brane, and the geometrical gravity can propagate also in bulk, then $\kappa > \Lambda$. The mass $M_P$ can be reduced by the factor $\eta = (\kappa/\varepsilon)^{d\perp}$, where $d\perp$ is the number of “large” dimensions; so here

$$\begin{align*}
M_P &\sim m_p \frac{\xi}{\eta} \sim 10^{63}/\eta GeV \quad \text{for} \quad \mu \sim 10^{10} \text{GeV}
\end{align*}$$

where $\eta$ can vary in wide interval but with conditions that $M_P > \Lambda$ - this corresponds to $\eta < 10^{44}$. For $\kappa \sim \Lambda \sim m_p$ this gives the estimates of size of large dimensions

$$\begin{align*}
\varepsilon^{-1} &\sim \kappa^{-1} \eta^{1/d\perp} < \kappa^{-1} \xi^{1/d\perp} \sim 10^{-28} \xi^{1/d\perp} eV^{-1} \Rightarrow 10^{-28+43/d\perp}
\end{align*}$$

where the last estimate is for $\mu \sim 10^{10}GeV$.

### 7.3 Inflation in bigravity models

Now briefly consider the specifics of inflation in such bigravity models. Here as in “usual” case an inflation can start from the large and slowly varying coherent fluctuations of some appropriate “inflaton” scalar $\phi_{in}$ field. The goldstone gravity do not interact with such coherent fields - their energy-momentum tensor is almost $\sim \eta_{\mu\nu}$ inside such fluctuation - and so does not leads to inflative expansion $^{31}$.

But inflation can be caused by the coupling of the geometrical gravity field to such a big fluctuation of this inflaton field $\phi_{in}$ arising on the energy scale $\Lambda_{in}$. If the energy density inside such a fluctuation is $\Lambda_{in}^4$, then the corresponding de-Sitter expansion stage is characterized by the scale factor $a(t) \sim \exp(H\delta t)$, where the mean

$$H \sim \Lambda_{in}^{2}/M_P = m_p (\Lambda_{in}/m_p)^2/\xi,$$

and the relaxation time $\delta t$ of $H$ depends from a model for $\phi_{in}$. Thus to have big value of $H \sim 10^{12}GeV$, as supposed in many approaches, we need $\Lambda_{in} \sim m_p 10^{8\pm 9}$. Such $\Lambda_{in}$ is still $\ll M_P$ given by $^{55}$ and where the locale physics corresponds to renormalized field theory. But from the other side, such a fluctuations are very big for the $\Lambda$ scale physics,

$^{31}$But it can interact with the surface of such a bubble, where the energy density changes, and can lead to a fast collapse of this fluctuation.
and therefore very improbable. Possibly some more complicated mechanisms (hybrid, etc) can cure this.\footnote{Note that to have such an inflation in the case of brane world with large dimensions we have limitation on the parameter $\eta < \xi(m_p/\Lambda_{in})$.}

What concerns the spectrum of initial density perturbations generated during such inflation stage, with $\Lambda_{in} > \Lambda$, they are prepared in pre-FRW stage. And so they will look in the goldstone gravity mediated FRW epoch as coming directly from the big-bang.\footnote{It is possible that the fluctuations corresponding to the quadrupole harmonics in CMB can be additionally affected, due to the interference with the vacuum tensor components $n_{\mu \nu}$. We thank to F. Viviani for this remark.}

The general picture of cosmological history in such a bigravity models can look as the following. We have the slow permanent de-Sitter expansion caused by the geometrical component of gravity - their rate is defined by the value of $\Lambda^6/\Lambda_{in}^2$ - and corresponds to the $H = \dot{a}/a$ of order of “modern time” experimental value.

This de-Sitter background is very rarely populated by the bubbles (Universes like ours) of normal $L(B, \chi, \varphi)$ matter. These bubbles are created by the extremely rare inflation-fluctuation on scales $\Lambda_{in} \gg \Lambda$ as discussed above.\footnote{Because the probability of inflation initializing fluctuation can be very small, there will be no more than one such island inside the primary de-Sitter horizon.}

Inflation with high $H$ is the first fast stage of grow of these matter-islands and it is driven by the geometrical gravity. Then follows the thermalization and the creation of matter with density and temperature possibly exiting $\Lambda$. After that this bubble enters the “weak” FRW stage of almost free expansion until their temperature $T$ is greater than $\Lambda$.

Further, when temperature becomes less then $\Lambda$, goldstone gravity with strength $G = m_p^2 \sim \Lambda^2$ switches on, and the expansion enters the “normal” FRW phase, with the rate defined entirely by the goldstone gravity. This will take place up to times when the mean falling matter density becomes of the order of primary effective cosmological constant.

After that the de-Sitter expansion of bubbles, driven by the geometrical gravity, becomes more essential (fast)\footnote{In such a model this epoch corresponds to the “today” of our Universe.} and they gradually dissolve and disappear.

In fact such type of cosmological scenario can be “embedded” in almost every bigravity theory with strongly separated values of gravitational constants and if for the more long-range gravity component the cosmological constant is cancelled by some effective mechanism. One can make future speculations and include this scenario in various multiverse models in which only some branches are equipped with the goldstone-gravitational physics.

\section{Concluding remarks}

The composite goldstone gravity is an interesting possibility which could open new ways, especially in cosmological aspects. It gives rise to a number of interesting questions with can be discussed for finding a different insight to the existing problems. These include the structure of horizons and various solitonic objects like black holes.

Trying to explore the goldstone gravity we encounter serious problems, and it is yet unclear if they can be solved in a appropriate way. These questions are purely technical, in spite of their technical complexity, and we hope that they can be answered somehow.
Evidently, the main question is if it is possible to organize the dynamics of tensor condensates in such a way to make the breaking of LI “invisible” with a sufficient precision. Moreover, one also needs a better understanding when and how the tensor condensates can form in the strong coupled gauge theories. Probably, this can be established by the lattice stimulation methods.

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