Magnetic field influence in deadweight force standard machines: a practical case

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Abstract. The intention of this study is the determination of the influence in uncertainty determination of the possible magnetic interactions in the CEM 500 kN deadweight force standard machine. For this, the magnetic susceptibility of the masses material has been determined as well as the magnetic flux density has been measured in several points. Based on these measurement results a model has been developed to determine the influence of the magnetic interactions with the help of the FEMM software.

1. Introduction
As it has already been established in EURAMET cg-4 guide [1] section 4.1 “The uncertainty budget for the machine also needs to consider possible force-generating mechanisms other than gravity and air buoyancy, including magnetic, electrostatic, and aerodynamic effects”. In this paper, the influence of the magnetic effects of the CEM 500 kN force standard machine is described. This machine is extensively described in [2]. It is a deadweight machine where the forces are generated by the effect of gravity acceleration on a set of different masses, which lie on a moving platform. The forces are generated as the platform is moving and the different masses are loaded. The set includes 36 masses that may generate forces of 5 kN, 10 kN, 15 kN, 20 kN, 25 kN and 50 kN. They are all made of low carbon steel with nickel coating ST52-3 as well as most parts of the machine, including the moving platform. The machine total height is more than 15 m and the part dedicated to the masses is around 6 m high. In general terms the smaller ones (5 kN and 10 kN, but the first 15 kN one) can be assumed as having a ring shape and the other ones as having a cylindrical shape, although their shape has been designed to be hanging upheld.

The intention of this work is to provide an estimation of the effect of the possible magnetic interaction among the masses and with the moving platform they lie on. These masses are quite big and can easily be magnetized during manufacturing and, although they were demagnetized before their installation in the machine, they may be re-magnetised again after more than 20 years since its installation, as the distance between them when the machine is loaded is 16 mm. By means of this study an estimation of this possible magnetic effect is determined to be included in the uncertainty determination of this machine.

2. Methodology
The methodology that has been followed to study this effect has two steps. The first step is to get as much information as possible from the experimental side. For that, two actions have been performed, the determination of magnetic susceptibility of the material and the determination of the magnetization of the masses from the measurement of the corresponding magnetic fields. The second step has been the system simulation in order to obtain the magnetic forces in the machine.

2.1. Determination of the magnetic susceptibility of the material
The magnetic susceptibility has been determined by means of the susceptometer method, which is described in section B.6.4 of the recommendation OIML R 111-1 [3]. For this, nine samples of the masses material were used. It was shown that the material was very inhomogeneous from the magnetic point of view. The obtained value was \( \chi = 0.1 \pm 0.1 \).
2.2. Measurement of the magnetic field

The flux density measurement of the magnetic field was performed by means of a gaussmeter (Brockhaus 455 DSP with transversal probe HMNT-4E04-VR-06). The objective was to measure the magnetic field on the surface of every mass. This is very difficult in practice as most of the masses surfaces are not accessible. Other important difficulty is the dispersion in the results of the magnetic field measurements for the same mass depending on the surface point, although this dispersion is similar to the one measured in one single point among the 36 masses for a certain distance between masses. As all the masses have been made with the same material and process, the measured magnetic field dispersion on one mass surface is similar to the one among masses and the only interest is to know what the possible influence of the magnetic effects is on the uncertainty, it is considered enough to take into account the magnetic field in one single point for every mass. Taking into consideration cylindrical coordinates the components in the r axis (tangential) and the z axis (normal) have been measured (the other component will be zero as a consequence of the system symmetry). As expected the components of the flux density in the r axis are very small and most times negligible if compared with the corresponding components in the z axis. Each measurement has been performed to a fixed distance from the surface (3.6 mm) and along the same vertical axis. The repeatability of each measurement was below 1 nT. There is one exception to this procedure, the moving platform, where its magnetic field was measured in four diametrically opposed points. These measurements have been performed for all the possible loads of the machine as the platform has been moved accordingly. In general terms an overall reduction of the magnetic flux of 15 % - 30 % has been observed when the machine is completely loaded if compared when it is unloaded.

On the other hand, the vertical component of the magnetic field of the Earth was determined in the machine platform (30 nT). The vertical component of the magnetic field of the Earth was also determined outside the building (38 nT). This value agrees quite well the value calculated by the NOAA calculator [4] (37 nT). This fact provides confidence in the measurement results.

2.3. Determination of the magnetization of the masses

The magnetization of each mass is determined from the magnetic field measurements considering that each mass is a magnet. According to the recommendation OIML R 111, equation B.6.2-1, [3] the permanent magnetization $M$ can be determined as

$$M = \frac{2B/\mu_0}{\sqrt{R^2 + (D + z)^2} - \sqrt{R^2 + z^2}} - f(B_E)$$

This equation corresponds to the case of a cylindrical magnet with radius $R$ and height $D$ as any magnet can be modelled as a solenoid, where $B$ is the flux density measured in the z axis at a distance $z$. It has been corrected by a factor that depends on the magnetic field of the Earth $B_E$ but, in our case this correction is negligible. The previous equation can be used when the mass is a cylinder. When the mass is a ring the associated magnet can be modelled as two concentric solenoids with radius $R$ (external) and $S$ (internal) with the same density current which circulates in opposite directions. Then the permanent magnetization $M$ can be determined as follows,

$$M = \frac{2B/\mu_0}{\sqrt{R^2 + (D + z)^2} - \sqrt{R^2 + z^2}} - \frac{2B/\mu_0}{\sqrt{S^2 + (D + z)^2} - \sqrt{S^2 + z^2}} - f(B_E)$$

The case for the moving platform is also different as it is a parallelepiped with width and length $W$ and height $D$. In this case the permanent magnetization $M$ can be determined as follows,

$$M = \frac{\pi B/\mu_0}{atan\left(\frac{W^2}{2z\sqrt{4z^2 + 2W^2}}\right) - atan\left(\frac{W^2}{2(D + z)\sqrt{4(D + z)^2 + 2W^2}}\right)}$$

In order to use the previous equation $B$ has to be measured in the z axis (symmetry axis), but this is not our case, as it has been measured at 70.2 cm from the axis ($R = 75$ cm, $S = 55$ cm and $W = 177$ cm). As a consequence, some correction has to be performed. When the flux density of the magnetic field produced by a turn with radius $a$ is considered, the ratio of the flux densities for $r = 0$ and $r = y'$ is given by the following expression, where $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kinds,

$$K(m) = \frac{1}{\sqrt{1 - m^2}}$$

$$E(m) = \int_0^\infty \frac{1}{\sqrt{1 - m^2 \sin^2 \theta}} d\theta$$
\[ B(r = y) = \frac{(z^2 + a^2)^{3/2} \sqrt{2m}}{\pi r (2ay)} \left( E(m) (am - y(2 - m)) \right) + yK(m) \]

and \( m \) is given by the following equation,

\[ m = \frac{4ay}{a^2 + y^2 + z^2 + 2ay} \]

Thanks to the previous expressions, the equations for the magnetization can be corrected as a solenoid is a set of turns. For the case of the platform this is not necessary because the magnetization is determined from the mean of flux densities measured in four diametrically opposite points.

2.4. Simulation

The simulation has been performed by the open source software Finite Element Method Magnetics (FEMM) [5]. FEMM is a suite of programs for solving low frequency electromagnetic problems on two-dimensional planar and axisymmetric domains. In our case, given the symmetry in the shape of the machine masses, the axisymmetric case can be applied.

In the magnetostatics case, the field intensity \( H \) and flux density \( B \) must obey

\[ \nabla \times H = J \]
\[ \nabla \cdot B = 0 \]

where \( J \) is the current density and subject to a constitutive relationship between \( B \) and \( H \) for each material. FEMM goes about finding a field that satisfies the previous system of equations via a magnetic vector potential approach. Flux density is written in terms of the vector potential, \( A \), as:

\[ B = \nabla \times A \]

As the mass material is considered to have a fixed magnetic susceptibility (linear isotropic material) and assuming the Coulomb gauge, \( \nabla \cdot A = 0 \), the problem can be reduce to solve this equation,

\[ \nabla^2 A = -\mu J \]

In the general 3-D case, \( A \) is a vector with three components. However, in the 2-D planar and axisymmetric cases, two of these three components are zero, leaving just the component in the “out of the page” direction. The advantage of using the vector potential formulation is that all the conditions to be satisfied have been combined into a single equation. If \( A \) is found, \( B \) and \( H \) can then be deduced by differentiating \( A \).

On the other hand, the force is determined by means of the Maxwell’s stress tensor, which prescribes a force per unit area produced by the magnetic field on a surface. The differential force produced is

\[ dF = \frac{1}{2} (H \cdot B) n + B \cdot H n - (H \cdot B) n \]

where \( n \) denotes the direction normal to the surface at the point of interest. The net force on an object is obtained by creating a surface totally enclosing the object of interest and integrating the magnetic stress over that surface.

Additionally, the Lua extension language has been used to add scripting/batch processing facilities to FEMM. Lua is a complete, open-source scripting language [5], which has facilitated the FEMM performance so that the tangential and normal components for the flux density of the magnetic field in a certain axis parallel to \( z \) axis and the force can be obtained in one single run for each possible force that can be generated by the machine. This is performed because, for every step of the script, the moving platform and different masses can be displaced so that different loads are applied. As an example, Figure 1 shows a direct result of the software: the density plot of the magnetic flux density modulus when the machine is completely unloaded.

3. Results

The first check has to be if the simulated magnetic field is similar to the measured one. In Figure 2 the normal component (\( z \) axis) of the magnetic flux density calculated by the software in the same vertical line where the experimental data were obtained is presented as a function of the distance as well as the corresponding experimental data. They are in quite good agreement taking into consideration all the approximations that have been performed. This component of the flux density is the important one that has influence in the force determination in the \( z \) axis. The experimentally observed decrease in the magnetic flux density when the machine is completely unloaded if compared when it is loaded has been observed in the simulation results.

Figure 3 shows the final results of the simulation. For each possible load it presents the force caused by the magnetic interaction relative to the generated force versus the generated force.
In absolute terms the magnetic interaction forces relative to the forces generated by the machine are more important for the biggest loads (the maximum force is 1.09 N, which affects to 550 kN (preload), the other forces are at least 5 times smaller, and they can be as small as 1 mN) but, as it can be seen, the magnetic force has a more important influence for the smallest loads.

As possible influences on the uncertainty determination there are the influence in the magnetic flux density and the influence of the magnetic susceptibility. In the first case the influence in the force is clearly proportional to the squared of the magnetic flux density \( F \approx B^2 \) and the second case is presented in Figure 4. In this figure each series of data is the relative force difference for the cases \( \chi = 0 \) and \( \chi = 0.2 \) and the case \( \chi = 0.1 \) relative to the generated force versus the generated force for each load. It can be seen that this influence is around 10 % of the magnetic interaction effect.

On the other hand, the influence in the distance between masses has also been studied. It has been obtained that the difference in the magnetic interaction force for the whole machine depending on the masses position (loaded or unloaded) provides a maximum relative difference with the generated force of \( 4.4 \times 10^{-7} \).

In conclusion, as the expected expanded relative uncertainty for this deadweight force standard machine is \( 2 \times 10^{-5} \), the magnetic interaction is an effect to be cared about so that it has to be considered in the uncertainty determination, especially for the smallest loads. Further studies are planned for the future that are expected to be able to describe and quantify this effect with more detail.

References

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