Waves in porous media saturated with bubbly liquid

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Abstract. Wave processes in a porous medium saturated with bubbly liquid, taking into account the nonlinearity of the oscillations of the bubbles, are numerically investigated. The nonlinearity is accounted in the equation of gas state and the equation of Rayleigh - Lamb. Numerical study in the framework of nonlinear model is performed for the first time. The dispersion relations for linear waves in a porous medium containing bubbly liquid, as well as for the main acoustic mode in a cylindrical cavity in such a medium, are obtained. Frequency dependences of velocity and attenuation of harmonic waves in a cylindrical waveguide are calculated. The influence properties of the bubbly liquid saturated porous medium on the wave velocity and attenuation and the penetration depth of perturbation into the surrounding porous medium are analyzed. The passage of the step wave from fluid into a porous medium saturated with bubbly liquid is investigated. The influence of porous medium parameters and intensity of the incident wave on the wave evolution in bubbly liquid saturated porous medium is studied. The calculation results show good qualitative agreement with the experimental data of other authors.

1. Introduction

The investigation of waves in porous medium saturated with bubbly liquid is of particular interest, because often the oil extraction process is accompanied by formation of gas bubbles in the reservoir. For example, when the pressure decreases in the reservoir, formation of bubbles from dissolved gas, or gas separation from gas condensate, or the decomposition of gas hydrates may occur. Gas is also formed during the acidizing of low-permeability zones.

The study of the properties of acoustic waves in a cylindrical waveguide is important in connection with numerous applications, for example, determination of the elastic properties of the surrounding space [1]. In particular, in geophysical research the acoustic waves in a cylindrical cavity are used to determine the characteristics of water-bearing and hydrocarbon reservoirs. The basic acoustic mode of such waves is usually called tube waves.

In traditional acoustic well studies, the rock porosity is determined by the first entry of the acoustic pulse. One of the next arrivals, the Stonely wave (also called the tube wave), is used to determine permeability, and the mechanical properties are determined by the whole wave train. For correct interpretation of acoustic logs, a deep understanding of the influence of rock properties (porosity, permeability, lithology, hydrocarbon saturation) on wave propagation in a cylindrical waveguide (borehole), is required.
In the present time in the scientific literature there is a number of experimental studies [2-5] for the wave propagation in porous media saturated with liquid with gas bubbles. The theoretical studies mainly focus on the dispersion relations for waves in such media and the propagation of perturbations in the framework of linear theory [2, 5, 6]. Insufficiently investigated there remains the problem of evolution of pressure perturbations in a porous medium saturated with bubbly liquid. In this work, we numerically investigated wave processes in a porous medium saturated with bubbly liquid, in the framework of nonlinear model. The nonlinearity is taken into account in the equation of gas state and the equation of Rayleigh - Lamb.

2. Basic Equations

To study wave propagation in a porous medium containing a bubbly liquid, two velocity, two stress tensors model of saturated porous medium is used [7, 8]. The pore space is filled two-phase mixture of liquid with gas bubbles, bubble size is calculated using the equation of Rayleigh-Lamb for a gas bubble in a porous medium.

The equations of mass and momentum balance are in the form:

\[
\frac{\partial \rho_{l+g}}{\partial t} + \nabla^i (\rho_{l+g} v_j^i) = 0, \quad \frac{\partial \rho_s}{\partial t} + \nabla^i (\rho_s v_j^i) = 0, \quad \frac{\partial \eta_h}{\partial t} + \nabla^i (\eta_h v_j^i) = 0
\]

(1)

\[
\rho_{l+g} \frac{d v_j^i}{dt} = -\alpha_{l+g} \nabla^i p_l - F_j^i, \quad \rho_s \frac{d v_j^i}{dt} = -\alpha_s \nabla^i p_l + \nabla^i \sigma_{s,ij}^i + F_j^i,
\]

(2)

where \(\rho, v, \alpha\) are the density, velocity, volumetric content of the \(j\) - th phase, the subscripts \(j = s, l, l+g\) relate to the skeleton of the porous medium, liquid or mixture of liquid and bubbles; \(\sigma_{s}, p_{l}\) are effective stress in the skeleton and the pressure in the liquid, respectively, \(n_h\) is the number of bubbles per unit volume.

The skeleton of the porous medium is assumed to be elastic with effective moduli of elasticity \(\lambda_{s}^{*}, \mu_{s}^{*}\):

\[
\sigma_{s,ij}^{*} = \alpha_{s} \left( \lambda_{s}^{*} \delta_{ij} e_{s,mm}^{*} + 2 \mu_{s}^{*} \varepsilon_{s,j}^{ij} + \gamma_{s}^{*} \delta_{ij} p_l \right), \quad \gamma_{s}^{*} = \beta_{s} \left( \lambda_{s}^{*} + 2/3 \mu_{s}^{*} \right),
\]

(3)

\[
\frac{d \varepsilon_{s,j}^{ij}}{dt} = \frac{1}{2} \left( \nabla^i v_{j}^{i} + \nabla^j v_{i}^{i} \right),
\]

where \(\varepsilon_{s}\) is strain of the solid phase, \(\gamma_{s}\) is the ratio of bulk elastic moduli of the skeleton and the solid.

The state equations for solid and liquid phases are given in the acoustic approximation:

\[
\rho_{l} - \rho_{l0} = C_{s}^{2} \left( \rho_{s} - \rho_{s0} \right) \quad p_{l} - p_{l0} = C_{l}^{2} \left( \rho_{l} - \rho_{l0} \right),
\]

here subscript 0 indicates the unperturbed value, \(\rho_{s,j}\) is the true density, \(C_{s,j}\) is the sound velocity in the material of the \(j\) - th phase, \(p_{l}\) is true pressure of the solid phase.

Interphase interaction is a sum of viscous friction force \(F_{\mu}\) and added masses force \(F_{m}\):

\[
F = F_{m} + F_{\mu}, \quad F_{m} = \frac{1}{2} \eta_{m} \alpha_{s} \rho_{l+g} \left( \frac{d v_{l}^{i}}{dt} - \frac{d v_{s}^{i}}{dt} \right), \quad F_{\mu} = \eta_{\mu} \alpha_{s} \alpha_{l+g} \mu_{l} \mu_{s}^{2} \left( v_{l}^{i} - v_{s}^{i} \right),
\]

(4)

where \(\alpha_{s}\) is the characteristic size of the grains of the skeleton, \(\mu_{l}\) is viscosity of the fluid, \(\eta_{m}, \eta_{\mu}\) are the coefficients of interaction between phases dependent on the structure of the pore space.

The gas in the bubbles is adiabatically compressed (\(\gamma\) is the ratio of specific heats):
bubble radius $a_b$ changes according to the equation of Rayleigh-Lamb for the bubble in a porous medium [2]:

$$\frac{d a_b}{d t} = w_R + w_A, \quad w_A = \frac{p_g - p_l - \frac{2 \Sigma}{a_b}}{\rho_0 C_l x_g^{\frac{1}{3}}}, \quad (6)$$

$$\rho_0 w_R a_b + \frac{3}{2} w_R^2 = p_g - p_l - \frac{2 \Sigma}{a_b} - 4 \mu_l \frac{w_R}{a_b} \left( 1 + \frac{1}{4} \alpha_s \eta \mu \left( \frac{a_b}{a_s} \right)^2 \right)$$

here $x_g$ is the gas volume fraction in the bubbly liquid, $\Sigma$ is the coefficient of surface tension at the liquid and gas boundary, $w_s$ takes into account the compressibility of the liquid [9].

To close the system of equations the relations between the true pressures $p_b, p_l$ in the phases and the effective pressure $p_s$, in the skeleton are used

$$\rho_s = \alpha_s \rho_s^*, \quad \rho_l = \alpha_l \rho_l^* + \alpha_s \rho_s^*, \quad \alpha_s + \alpha_l + \alpha_g = 1, \quad \alpha_g = \frac{4}{3} \pi a_b^3 n_b, \quad (7)$$

$$p_s = \alpha_s \left( p_s - p_l \right), \quad p_s^* = -\frac{1}{3} \sigma_{s*}^{*m}.$$

3. Dispersion Relations

After linearization the above system of equations the dispersion relation was obtained, and the velocity and the damping of linear waves in porous media saturated with bubbly liquid were calculated.

When considering the wave propagation along a cylindrical cavity in a porous medium saturated with bubbly liquid it was assumed that the cavity is also filled with bubbly liquid. The boundary conditions at the interface between the cavity and a porous medium are the continuity of the liquid flow, pressure in a liquid and total stress. The cases of open pores (when fluid flow between the cavity and the porous medium is free) or closed pores (there is no flow at the interface) at the boundary are considered. The equations of motion are written in cylindrical polar coordinates, the system of equations is linearized, and we seek a solution in the form of a harmonic wave corresponding to the propagation of perturbations along the cylindrical cavity. In this case the perturbation in the porous medium is a combination of heterogeneous bulk longitudinal and transverse waves. The dispersion relation for tube waves is derived from the conditions at the boundary of the cavity and the porous medium.

In figure 1 the phase velocity and the linear damping decrement of the deformational (1) and filtrational (2) waves in bubbly liquid saturated porous medium (black lines) are presented. For comparison, the velocity and attenuation of waves in bubbly liquid (red lines), and liquid saturated porous medium (blue lines) are also shown.

The equilibrium parameters of the medium used in calculations and presented in figure 1, are the following: bubbly liquid is water + methane, the equilibrium pressure is 50 bar ($\rho_g = 34.36$ kg/m3), the volume fraction of gas in the bubbly liquid is 0.01, the radius of the bubbles is 1 mm, the skeleton of the porous medium is quartz, with porosity of 0.4 and a typical pore radius of 0.1 mm; effective elastic moduli of skeleton are $\lambda_s = \mu_s = 2 \cdot 10^9$ Pa.

$$p_g = \left( \frac{\rho_g^*}{\rho_g} \right)^{\gamma} \left( \frac{a_b}{a_b} \right)^{3 \gamma}, \quad (5)$$
Figure 1. The phase velocity and the linear damping decrement of the deformational (1) and filtrational (2) waves in bubbly liquid saturated porous medium (blue lines), in a bubbly liquid (red lines), and liquid saturated porous medium (black lines).

Minimum velocity of wave propagation in bubbly fluid (red line) corresponds to the natural frequency of the bubbles which is equal to 22.4 kHz. At these and lower frequencies the velocity of the fast and slow waves in porous medium saturated with bubbly liquid is lower and the attenuation is higher than that in a porous medium saturated with liquid without gas bubbles. With increasing frequency the velocities reach the same values as in a porous medium with a liquid without gas bubbles.

Analysis of the obtained numerical results showed that the increase in the equilibrium pressure leads to increase of the velocity of deformational and filtrational waves at frequencies below the natural frequency of the bubbles. Damping of waves in this frequency range is somewhat reduced.

4. Propagation of stepwise wave
Methodology for calculating the motion of a porous medium containing liquid with gas bubbles based on the method of Lax – Wendroff was developed, and the numerical study of pressure wave propagation in such medium was made. Perturbation in the porous medium (x>0) is generated by a wave incident from the pure liquid (x<0).

We have investigated the influence of medium parameters and of the initial pulse on the wave evolution in a porous medium containing the bubbly liquid.
In figure 2 shows the variation of the pressure in the liquid $p$ and the total stress $\sigma = \sigma_s - p$ in the step wave of the amplitude equal to $0.1 \cdot p_0$ passing from the liquid to the bubbly liquid saturated porous medium. Equilibrium pressure is $p_0 = 10$ bar (left) and $p_0 = 50$ bar (right). The other parameters of the porous medium and bubbly liquid are the same as in figure 1.

When entering a porous medium initial pulse splits into fast (deformational) and slow (filtrational) waves. At the fast wave passage the abrupt increase in full stress $\sigma$ is observed (black line), and the arrival of the slow wave is characterized by a gradual increase in pore pressure and subsequent damped oscillations associated with pulsations of the bubbles. The observed period of oscillations coincides with the period of natural oscillations of the bubbles (in this case about $0.10$ ms (left) and $0.045$ ms (right)). If you increase the amplitude of the original incident wave, pressure variations become less regular. In a medium with a higher equilibrium pressure, there is a high propagation velocity of both fast (deformational) and slow (filtrational) waves.

With decreasing bubble size the amplitude and period of oscillations decrease. With increasing gas content in the bubbly liquid decreases the velocity of waves in a porous medium containing bubbly liquid, and its acoustic stiffness. As a result, with increase in gas content the pressure increasing in the wave passed into the porous medium is slower.

The calculation results show good qualitative agreement with the experimental data on the passage of step waves from water to porous medium saturated with water with air bubbles [2, 3]. A quantitative comparison is not possible, as in these works are not all the medium parameters are specified.

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