Microwave toroidal dipolar response in an asymmetric planar metamaterial

Linyan Guo 1, , Xiaojun Huang 2,3, , Qisheng Zhang 4, , and Minjie Guo 5

1 School of Geophysics and Information Technology, China University of Geosciences, Beijing 100083, People’s Republic of China
2 College of Communication & Information Engineering, Xi’an University of Science and Technology, Xi’an, Shaanxi 710054, People’s Republic of China
3 State Key Laboratory of Millimeter Waves, Southeast University, Nanjing 210096, People’s Republic of China
4 Author to whom any correspondence should be addressed.

E-mail: zqs@cugb.edu.cn

Keywords: toroidal resonance, metamaterial, high quality

Abstract

Toroidal multipoles are the third type of multipoles which are fundamentally different from conventional electric multipoles and magnetic multipoles. The implementation of toroidal multipoles is hard since its energy is too low to be discovered. This paper proposes a simple planar metamaterial to achieve an often ignored toroidal dipolar response in the microwave band. The metamaterial is designed with a special asymmetric arrangement of asymmetric U-ring resonators to attain a toroidal resonance with high quality factor at 13.5 GHz. Its toroidal nature can be verified by calculated scattered power by multipoles, magnetic field and the surface current. The presented planar metamaterial owns toroidal resonance with high quality factor and it can be applied in the field of antennas, photonics, sensing and energy.

1. Introduction

Toroidal dipole is the most primitive member of toroidal moments, which was firstly considered by Zel’dovich in 1957 [1]. This kind of novel moments are fundamentally different from classical electric moments and magnetic moments. As we all know, an electric dipole is created by the interaction of positive and negative charges, while a magnetic dipole is generated by a closed loop of current (see figures 1(a) and (b)). Unlike electric dipole and magnetic dipole, a toroidal dipole is produced by currents flowing on a surface of a torus along its meridians, which is characterized by a vortex distribution of magnetic dipoles (see figure 1(c)). In classical electrodynamics, the multipole expansion of electromagnetic potentials includes electric/magnetic dipoles and their multipoles, without including toroidal multipoles. Currently, it is proposed that a complete multipole expansion should include the toroidal moments along with electric moments and magnetic moments [2].

In natural matter, the resonance of electric multipole moments and magnetic multipole moments is much stronger than that of toroidal moments. This makes the electromagnetic manifestations of toroidal dipole and multipoles are masked by much stronger effects of electric and magnetic moments, resulting in few experimental observations of toroidal moments [3, 4]. Hence, suppressing traditional electric and magnetic multipoles and enhancing toroidal multipoles response are essential towards providing interesting phenomena associated with toroidal moments. In 2010, Zheludev et al firstly declared their numerical and experimental observations of a toroidal dipolar response by the means of metamaterials in the microwave region [5]. This novel discovery causes great concern on the often ignored toroidal multipoles and inspires many related researches on toroidal responses in microwave [6–9], terahertz [10, 11], mid-infrared [12] and optical regions [13, 14]. At the same time, it also provides a basis for related researches in resonant transparency [15], permittivity sensor [16], polarization twist [17], toroidal lasing spacer [18], electromagnetically induced transparency [19], and so on. Based on our best knowledge, most of the previous papers have used similar models to achieve toroidal dipole response in different frequency bands, such as microwave band, THz band, infrared band, and optical band [7, 20, 21]. These researches and applications of toroidal dipoles in different frequency bands are equally important. Studies have shown that the toroidal metamaterials in the microwave...
band have broad application prospects in high-performance emitters, polarization twisters and sensors. Therefore, it is necessary to design metamaterials to realize toroidal dipole resonance in the microwave region. At present, many metamaterials with toroidal dipolar response are three-dimensional structures, which is hard to implement and experiment. This paper proposes a planar structure that is conducive to the related study of toroidal response with high quality factor.

2. Planar toroidal metamaterial design

A typical example of implementing a toroidal dipole is a solenoid that is bend into a ring [4], equivalently, a closed loop of azimuthally oriented magnetic dipoles. Usually, a split ring resonator or U-ring resonator can be regarded as an artificial atom with a magnetic dipolar moment at microwave frequencies. In this paper, a series of asymmetric U-rings are arranged to obtain the closed loop of magnetic dipolar moments.

In addition, toroidal moments swap their signs under either spatial inversion or time reversal [22]. This means that we need to break the symmetry of a metamolecule. In figure 2(a), the proposed toroidal dipole model is constructed by asymmetric planar metamaterial, which is composed of 8-fold asymmetric U-ring metamolecules. Here, ‘asymmetric’ U-ring means different lengths of the ring’s bars (l₁ ≠ l₂), while the ‘asymmetric’ planar metamaterial means the rotation symmetry is broken with respect to the z-axis.
For the implementation, the dielectric substrate, Rogers RO4003C, is chosen, whose relative permittivity and tangent loss are \( \varepsilon_r = 3.38 \) and \( \tan \delta = 0.0027 \). Its thickness is \( h = 0.813 \) mm. The metal is copper with a thickness of 0.035 mm and conductivity of \( \sigma = 5.9 \times 10^7 \) S m\(^{-1}\). The toroidal metamolecule proposed herein has 8-fold asymmetric U-ring resonators. The asymmetric U-rings are flipped between the up and down halves to break the spatial rotation symmetry. This spatial asymmetry plays a crucial role in the generation of the toroidal dipolar response. Eight U-rings are arranged to form a annulus with the radius, \( r \). Each ring consists of metal vias and bars with the different length, \( l_1 \) and \( l_2 \), and the same width, \( w \). These metal vias can strengthen the coupling between the short bars and long bars of each U-ring. In figure 2(a), the short metal bars are shown in red and long metal bars are in blue, while the metal vias are in yellow. The toroidal metamolecule has an overall size of \( d \times d \times s \). The following values are used for the metamolecules: \( r = 1 \) mm, \( l_1 = 3.5 \) mm, \( l_2 = 3 \) mm, \( w = 0.3 \) mm, \( d = 10 \) mm and \( s = 2.883 \) mm. For numerical simulations, the commercial solver CST Microwave Studio is used. In the simulation, the metamolecules of the metamaterial array is placed in a rectangular unit cell with periodic boundary conditions imposed along \( x \)- and \( y \)-axes. This toroidal model works with \( x \)-polarized incident wave impinging normally to the planar metamaterial.

For experimental measurements, due to the limitation of printed circuit board technology, the vertical metal vias are cylinders with diameter \( w \) electroplated by copper on the inner wall. As shown in figure 2(d), the metamolecule is periodically arranged into an array along \( x \)- and \( y \)-axes to form a planar metamaterial slab. The metamaterial slab is formed by \( 20 \times 20 \) array of toroidal metamolecules, which has an overall size of \( 200 \times 200 \times 0.813 \) mm\(^3\). The axis of the toroidal structure points normal to the dielectric substrate (parallel to \( z \)-axis). That means the normally incident electromagnetic wave propagates along its axis, i.e., \( z \)-axis. The transmission and reflection spectra are measured by a vector network analyser Agilent PNA E8362B and two linearly polarized double ridge horn antennas. In the experimental measurements, the polarization direction of the horn antennas is paralleled to the \( x \)-axis.

3. Toroidal dipolar response

Figure 3 shows the simulated and measured spectra of the planar metamaterial based on 8-fold U-ring toroidal metamolecules. The simulated and measured results are in a good agreement except a few mismatches. This is due to the influence of the printed circuit board manufacturing process and the limited size of the metamaterial. In addition, figure 3(a) illustrates the simulated S parameters of the infinite size of the toroidal metamaterial in the \( O-xy \) plane. However, a small-sized metamaterial composed of a limited number of metamolecules is generally used in experiments and practical applications. The boundary condition of the infinite metamaterial is different from that of the small-sized metamaterial. It means the former uses a periodic boundary, while the latter is surrounded by free space. As can be seen from figure 3, there are two resonances which are expressed as reflection peaks and sharp transmission dips. The simulated transmission dips are located at 13.5 GHz and 13.9 GHz, while the measured results are at 13.8 GHz and 14.5 GHz. Moreover, the quality factors (Q factors), which is the ratio of resonance frequency to the full width at half maximum, are 268 and 271 for each resonances,
respectively. These two Q factors are calculated from the simulated transmission coefficients. Compared with other 2D and 3D toroidal structures, this toroidal metamaterial proposed in this paper has a high Q factor.

To analyse these resonances quantitatively, the numerical calculation method is applied to derive the characteristic of scattered power. The scattered power of different moments is obtained from the surface current density in one toroidal metamolecule. The calculation method of the scattered power can be found in [5]. Thus, the intensity of different multipole moments can be compared by their scattered power. Figure 4 shows the scattered power of five major multipoles: $x$-component of electric dipole $P_x$, $y$-component of magnetic dipole $M_y$, $z$-component of toroidal dipole $T_z$, electric quadrupole $Q_e$, and magnetic quadrupole $Q_m$. Since the components in other directions are small, these components of electric, magnetic, and toroidal moments are not given here.

Figure 4 shows that the 8-fold metamolecule ensures some distinct electromagnetic properties: apart from supporting the traditional electric and magnetic dipole moments, it also supports an obvious toroidal dipole at 13.5 GHz. Furthermore, the scattered power of the electric dipole is very strong throughout the whole frequency band. This is mainly due to the presence of the splits between the short bars and long bars of each U-ring, and the $x$-polarized electric field of the incident wave. At 13.5 GHz, traditional electric and magnetic multipole excitations of the metamolecule are not resonant, which suppresses their scattered power. Therefore the conventional multipole excitations are not responsible for the resonance at this frequency. This illustrates that this resonance is probably due to the toroidal dipole, which has a strongest contribution provided by $z$-component of the toroidal dipole, $T_z$. The scattered power of the toroidal dipole is about 5 times stronger than the second one, $P_x$. Besides, while the scattered power of the toroidal dipole reaches a maximum, the power of the electric dipole is drastically reduced. The reason is that the electric dipoles produced by the U-rings in the upper half and those in the lower half of figure 2(a) are opposite. In addition, the toroidal dipole resonance at 13.5 GHz has a very narrow band with a high Q factor $\sim 268$. Comparing with conventional electric / magnetic multipole resonance in other papers, the high Q factor of the toroidal mode is a direct result of its strong confinement and weak coupling to free space. At 13.9 GHz, the three strongest contributions to this resonance are mainly provided by the electric quadrupole, the $x$-component of the electric dipole, and the $y$-component of the magnetic dipole. Two resonances in figure 3 are predominantly caused by the toroidal dipole and the electric quadrupole, respectively. Since the magnetic field of the toroidal dipole is a head-to-tail configuration of closed loops, this paper focuses on its magnetic dipole response at 13.9 GHz to directly compare their magnetic field.

The nature of the resonant excitations at 13.5 GHz and 13.9 GHz can be explained by the simulated local magnetic fields and surface currents of one toroidal metamolecule. The simulated results are shown in figure 5. The first column and the second column show the magnetic field and surface current at 13.5 GHz and 13.9 GHz, respectively. The local magnetic field of toroidal resonance in the $O$-$xy$ plane is shown in figure 5(a). It shows that the calculated arrows of the local magnetic field construct a head-to-tail configuration of closed loops, which are constrained within the toroidal metamolecule. The head-to-tail configuration leads to a non-zero $z$-component of the toroidal dipolar moment orienting along the metamolecule’s axis. In figure 5(b), the calculated magnetic arrows at 13.9 GHz are split in two bundles. These arrows first travel through the U-rings in the upper and lower half of the figure and then join immediately. The local magnetic arrows travel along the direction of external

![Figure 4. Scattered power of different multipoles induced in one metamolecule in 12.5–14.5 GHz. The two main resonances are located at 13.5 GHz and 13.9 GHz.](image-url)
magnetic field. This configuration of closed loops illustrates that the magnetic dipole has a strong coupling to free space. Figures 5(c) and (d) show the $y$-component of the local magnetic field in the $O-xz$ plane. At 13.5 GHz, the magnetic arrows point to different directions of the upper and lower half, and the $y$-component of the magnetic field appear in the opposite phase. Contrarily, at 13.9 GHz, the magnetic arrows point to the same direction and the $y$-component of the magnetic field appear in the same phase.

Figures 5(e) and (f) show the calculated current distributions induced in one toroidal metamolecule. Through the analysis of the current distributions, the generation mechanisms of toroidal dipolar response can be revealed. The long and short bars of the U-rings form splits which interact with the $x$-polarized electric field and then generate circular currents flowing on the metal surface. Due to the different locations of the splits, the external electric field reaches the splits with a time difference, i.e., a phase delay. At 13.5 GHz, the U-rings in the upper and lower half are induced to generate currents in different directions. That is to say, the $x$-components of electric dipole moments cancel each other, resulting in a suppressed $P_x$. Therefore, the phase delay of 180° gives rise to the magnetic dipoles along opposite directions. This forms a head-to-tail shape of magnetic dipoles and then induces a non-zero toroidal dipolar moment along $z$-axis. In this case, the moments of the magnetic dipole and the magnetic quadrupole are negligible. Due to the closed shape of magnetic dipoles, the produced dipolar toroidization is an electric-induced toroidal dipolar response. As also shown in figure 4, it is represented by an attenuated power of $P_x$. Thus, a toroidal ordering is generated by purposely asymmetric structure and arrangement of U-rings. But in symmetric U-rings cases this phenomenon would not happen. In figure 5(e), all the current arrows whirl in the same directions and the induced magnetic dipoles also point to the same directions. Therefore, a non-zero component of net magnetic dipole moment paralleling to external magnetic field is generated.

4. Discussions

This section focuses on the impact of different folds in one metamolecule. The toroidal response is obtained by a combination of multiple magnetic dipoles, and it is largely dependent on the U-rings’ structure. The structure
dependence of the toroidal dipole resonance is presented in figure 6. Figure 6(a) shows the transmittance $|S_{21}|$ of a toroidal metamolecule with different folds. Each metamolecule has two resonances: the toroidal response at lower frequency and the magnetic response at higher frequency. For an easy comparison, figure 6(b) shows the resonant frequencies and Q factors of the two resonances. It can be seen from the figure that as the number of U-rings increases, the resonant frequencies move to the low frequency band. But both resonances retain high Q factors.

Actually, the more folds of U-rings the metamolecule have, the more currents are generated along the U-rings. This reduces the energy required to build a toroidal system, which makes the toroidal dipole resonate at a lower frequency. We demonstrate this phenomenon by modelling the metamolecules with different folds of U-rings. Figure 7 shows the scattered power and simulated magnetic field of toroidal dipolar moments of different metamolecules. Consistent with the results of figure 6, as the number of U-rings increases, the resonant frequency of toroidal moves to the low frequency band. But the intensity of the scattered power of toroidal moments does not change much. As shown in figures 7(b)–(e), local magnetic field are clearly observed to form head-to-tail configurations of closed loops, which are largely confined within the metamolecules. All of the head-to-tail configurations of magnetic arrows give rise to a non-zero $z$-component of the induced toroidal dipolar moment oriented along the axis of the metamolecules.

5. Conclusions

In summary, elusive toroidal response can be achieved in an asymmetric planar toroidal metamaterial constructed of asymmetric U-ring resonators in the microwave band. It exhibits a high Q resonance at 13.5 GHz. The scattered power of multipoles, magnetic field distributions and surface currents clearly reveal the existence of the elusive toroidal dipole at this resonance. The discussions on the toroidal metamaterial show that the asymmetric U-rings have a great influence on the toroidal response. Due to the simple structure, the toroidal...
model proposed in this paper is also suitable for achieving toroidal excitations in terahertz and optical frequency bands and studying the related properties. Additionally, the feasible planar toroidal metamaterial is useful to verify the theoretical results derived in previous literatures by experiments, such as toroidal metamaterial based antennas and other microwave devices.

Acknowledgments

This work was supported by the National Natural Science Foundation of China [Grant no. 41704176, 61701206 and 41574131], the National Key Research and Development Program of China [Grant no. 2017YFF0105704], the Open Research Funds of State Key Laboratory of Millimeter Waves (Grant No. K201803) and the Fundamental Research Funds for the Central Universities from China.

ORCID iDs

Linyan Guo ⊗ https://orcid.org/0000-0001-7274-2976
Xiaojun Huang ⊗ https://orcid.org/0000-0002-7685-2678
Qisheng Zhang ⊗ https://orcid.org/0000-0002-6012-7308

Figure 7. (a) Scattered power of toroidal dipoles of different metamolecules. Simulated magnetic field of (b) 4-fold at 14.07 GHz, (c) 6-fold at 13.84 GHz, (d) 10-fold at 13.11 GHz and (e) 12-fold at 12.72 GHz.
References

[1] Zel’Dovich I B 1958 Soviet Physics - JETP 6 1184
[2] Dubovik V and Tugushev V 1990 Physics Reports 187 145
[3] Sawada K and Nagaosa N 2005 Physical Review Letters 95 237402
[4] Marinov K, Boardman A, Fedotov V and Zheludev N 2007 New Journal of Physics 9 324
[5] Kaelberer T, Fedotov V, Papastmakis N, Tsai D and Zheludev N 2010 Science 330 1510
[6] Guo L, Li M, Yang H, Huang X and Wu S 2014 Journal of Physics D: Applied Physics 47 415501
[7] Fan Y, Wei L H, Chen H and Soukoulis C 2013 Physical Review B 87 115417
[8] Xiang T, Lei T, Hu S, Chen J, Huang X and Yang H 2018 Journal of Applied Physics 123 095104
[9] Gao J, Zhang K, Yang G and Wu Q 2014 IEEE Transactions on Magnetics 50 4002104
[10] Gerislioglu B, Ahmadiavand A and Pala N 2018 Physical Review B 97 161405
[11] Wang S, Zhao X, Wang S, Zhu J, Li Q and Wang Y 2019 Optical Materials Express 9 3657
[12] Liu Z, Du S, Cui A, Li Z, Fan Y, Chen S, Li W, Li J and Gu C 2017 Advanced Materials 29 1606298
[13] Liu G, Zhai X, Xia S, Liu Q, Zhao C and Wang J 2017 Optics Express 25 26045
[14] Dong Z, Zhu J, Rho J, Li J, Lu C, Yin X and Zhang X 2012 Applied Physics Letters 101 144105
[15] Fedotov V, Rogacheva A, Savinov V, Tsai D and Zheludev N 2013 Scientific Reports 3 2967
[16] Ye Q, Guo L, Li M, Liu Y, Xiao B and Yang H 2013 Physica Scripta 88 055002
[17] Guo L, Li M, Huang X and Yang H 2014 Applied Physics Letters 105 033507
[18] Huang Y, Chen W, Wu P, Fedotov V, Zheludev N and Tsai D 2013 Scientific Reports 3 1237
[19] Zhu L, Dong L, Guo J, Meng F, He X, Zhao C and Wu Q 2017 RSC Advances 7 55897
[20] Gupta M, Srivastava Y K and Singh R 2017 Advanced Materials 30 1704845
[21] Gupta M, Srivastava Y K, Manjappa M and Singh R 2017 Applied Physics Letters 110 121108
[22] Ogist R, Talebi N, Vogelgesang R, Sigle W and Aken P 2012 Nano Letters 12 5239