Reviving non-Minimal Horndeski-like Theories after GW170817: Kinetic Coupling Corrected Einstein-Gauss-Bonnet Inflation

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After the recent GW170817 event of the two neutron stars merging, many string corrected cosmological theories confronted the non-viability peril. This was due to the fact that most of these theories produce massive gravitons primordially. Among these theories were the ones containing a non-minimal kinetic coupling correction term in the Lagrangian, which belong to a subclass of Horndeski theories. In this work we demonstrate how these theories may be revived and we show how these theories can produce primordial gravitational waves with speed $c^2_T = 1$ in natural units, thus complying with the GW170817 event. As we show, if the gravitational action of an Einstein-Gauss-Bonnet theory also contains a kinetic coupling of the form $\sim \xi(\phi) G^\mu_\nu \partial_\mu \phi \partial_\nu \phi$, the condition of having primordial massless gravitons, or equivalently primordial gravitational waves with speed $c^2_T = 1$ in natural units, results to certain conditions on the scalar field dependent coupling function of the Gauss-Bonnet term, which is also the non-minimal coupling of the kinetic coupling. We extensively study the phenomenological implications of such a theory focusing on the inflationary era, by only assuming slow-roll dynamics for the scalar field. Accordingly, we briefly study the case that the scalar field evolves in a constant-roll way. By using some illustrative examples, we demonstrate that the viability of the theoretical framework at hand may easily be achieved. Also, theories containing terms of the form $\sim \xi(\phi) \Box g^\mu_\nu \partial_\mu \phi \partial_\nu \phi$ and $\sim \xi(\phi) (g^\mu_\nu \partial_\mu \phi \partial_\nu \phi)^2$ also lead to the same gravitational wave speed as the theory we shall study in this paper, so this covers a larger class of Horndeski theories.

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I. INTRODUCTION

The primordial Universe is one of the mysteries that modern theoretical cosmologists and physicists are trying to understand. Our approach to the primordial Universe is through a regime of classical gravity towards to the unknown era of quantum gravity or the “theory of everything” which is believed to govern the small scale physics and unifies all forces in nature. In between the classical gravity regime and the quantum gravity regime, theoretical cosmologists believe that an era of abrupt acceleration occurred, known as inflationary era. This era is at the border of classical and quantum gravity, so many believe that it should have some imprints of the quantum gravity era, in the effective theory that controls the inflationary regime. The most appealing theoretical proposal for the “theory of everything” is nowadays string theory in all its different forms. If string theory controls the quantum gravity era, it is reasonable to expect that the effective theory that controls the inflationary era might have terms which originate in the complete M-theory Lagrangian. In some sense the classical inflationary theory should contain several string theory corrections. A particularly appealing class of theories which contains string theory motivated corrections is Einstein-Gauss-Bonnet theory of gravity [1-31]. These theories, along with several extensions containing kinetic coupling corrections [1] are known to provide a viable inflationary era which can in many cases be compatible with the observational data, see Refs. [1] and references therein for details.

However, Einstein-Gauss-Bonnet theories have a serious flaw, namely they predict a non-zero graviton mass during the primordial inflationary era. This was not a problem until recently, the GW170817 event utterly changed the scenery in modern theoretical cosmology, excluding many modified gravity theories from being viable descriptions of the Universe, see Ref. [32] for a detailed account on this topic. This is because the two neutron stars merging event GW170817 [33] indicated that the gravitational waves arrived almost simultaneously with the gamma rays emitted from the neutron stars merging. Correspondingly, this observation clearly shows that the gravitational wave speed $c_T$ is nearly equal to unity, that is $c^2_T = 1$ in natural units, which is equal to the speed of light. Thus theories that do not predict a massless graviton should no longer be considered as successful description of our Universe at large
or small scales. It is remarkable though that many modified gravity theories \[^{34,40}\], still remain compatible with the observations, like $f(R)$ gravity or Gauss-Bonnet theories.

Now the question is, why an event that is observed at small redshifts should affect so seriously theories that predict a primordial massive graviton? The answer to this question is simple, there is no reason for the graviton to change its mass during the evolution of the Universe. So the answer to the above question, can also be cast in a question, why should the graviton mass during the inflationary era be different from the gravitons emitted from low redshift astrophysical sources? No particle physics process can explain why the graviton should have different mass during inflation, the post-inflationary era and at present time, at least to our knowledge.

In view of this way of thinking, in a previous work we studied how Einstein-Gauss-Bonnet theories can be compatible to the GW170817 event, and produce primordial gravitational waves with speed equal to that of light \[^{29,31}\]. As we showed, the condition $c_f^2 = 1$ restricts significantly the functional forms of the Gauss-Bonnet scalar coupling function $\xi(\phi)$ and the scalar potential. We also demonstrated that the GW170817-compatible theories can produce a viable inflationary era, compatible with the latest Planck data \[^{41}\].

In this work, we extend the study performed in our previous work \[^{29,31}\], including in the effective action of Einstein-Gauss-Bonnet theories non-minimal kinetic coupling terms of the form $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. These type of theories belong to the more general class of Horndeski theories \[^{42,52}\], and are also known as kinetic coupling theories \[^{56–69}\] and after the GW170817 event, these theories were significantly looked down upon, due to the fact that Horndeski theories produce primordial gravitational waves with speed less than that of light. Actually, Horndeski theories in view of GW170817 were also studied and discussed in Refs. \[^{70–72}\]. In this paper, we shall revive in a concrete way the Horndeski theories containing kinetic couplings of the form $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$, by imposing the constraint that the gravitational wave speed is equal to that of light. As we show in detail, the constraint $c_f^2 = 1$ results to a differential equation that determines uniquely the dynamical evolution of the scalar field. In addition, the coupling function $\xi(\phi)$ and the scalar potential are not freely chosen, but must satisfy a specific differential equation. Then by assuming that the slow-roll conditions hold true, we show that the inflationary phenomenology of GW170817-compatible kinetic coupling corrected Einstein-Gauss-Bonnet theory can be compatible with the latest Planck data \[^{41}\]. We also perform the study in the case of a constant-roll evolution for the scalar field, and we show that the viability can be achieved in this case of dynamical evolution too. Finally, as we show, the same constraint that renders the kinetic coupling corrected Einstein-Gauss-Bonnet theories compatible with the GW170817 event, also renders theories containing terms $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and $\sim \xi(\phi) (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)^2$ also compatible with the GW170817 event. Thus a broader class of Horndeski theories can be revived, however we restrict ourselves to the simplest case of having only kinetic coupling terms, because the extended theories are quite more difficult to study with regard to their inflationary phenomenology.

\section*{II. NON-MINIMAL KINETIC COUPLING CORRECTED EINSTEIN-GAUSS-BONNET INFLATIONARY PHENOMENOLOGY: ESSENTIAL FEATURES AND COMPATIBILITY WITH GW170817 CONSTRAINTS}

In principle, Einstein-Gauss-Bonnet gravity is simply a string corrected scalar field theory with minimal coupling, thus a natural extension of this string corrected theory can be obtained by simply adding a non-minimal kinetic coupling, or other higher order string corrections containing derivatives of the scalar field, see \[^{41}\] for more details on all the possible string corrections. In this work we shall assume that the gravitational action of a minimally coupled scalar field contains two string corrections, namely a Gauss-Bonnet term with a scalar field dependent coupling and all the possible string corrections. In this work we shall assume that the gravitational action of a minimally coupled scalar field, the one used previously which is $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$, and after the GW170817 event, these theories were significantly looked down upon, due to the fact that Horndeski theories produce primordial gravitational waves with speed less than that of light. Actually, Horndeski theories in view of GW170817 were also studied and discussed in Refs. \[^{70–72}\]. In this paper, we shall revive in a concrete way the Horndeski theories containing kinetic couplings of the form $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$, by imposing the constraint that the gravitational wave speed is equal to that of light. As we show in detail, the constraint $c_f^2 = 1$ results to a differential equation that determines uniquely the dynamical evolution of the scalar field. In addition, the coupling function $\xi(\phi)$ and the scalar potential are not freely chosen, but must satisfy a specific differential equation. Then by assuming that the slow-roll conditions hold true, we show that the inflationary phenomenology of GW170817-compatible kinetic coupling corrected Einstein-Gauss-Bonnet theory can be compatible with the latest Planck data \[^{41}\]. We also perform the study in the case of a constant-roll evolution for the scalar field, and we show that the viability can be achieved in this case of dynamical evolution too. Finally, as we show, the same constraint that renders the kinetic coupling corrected Einstein-Gauss-Bonnet theories compatible with the GW170817 event, also renders theories containing terms $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and $\sim \xi(\phi) (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)^2$ also compatible with the GW170817 event. Thus a broader class of Horndeski theories can be revived, however we restrict ourselves to the simplest case of having only kinetic coupling terms, because the extended theories are quite more difficult to study with regard to their inflationary phenomenology.

In the spirit of this way of thinking, in a previous work we studied how Einstein-Gauss-Bonnet theories can be compatible to the GW170817 event, and produce primordial gravitational waves with speed equal to that of light \[^{29,31}\]. As we showed, the condition $c_f^2 = 1$ restricts significantly the functional forms of the Gauss-Bonnet scalar coupling function $\xi(\phi)$ and the scalar potential. We also demonstrated that the GW170817-compatible theories can produce a viable inflationary era, compatible with the latest Planck data \[^{41}\]. We also perform the study in the case of a constant-roll evolution for the scalar field, and we show that the viability can be achieved in this case of dynamical evolution too. Finally, as we show, the same constraint that renders the kinetic coupling corrected Einstein-Gauss-Bonnet theories compatible with the GW170817 event, also renders theories containing terms $\sim \xi(\phi) G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and $\sim \xi(\phi) (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)^2$ also compatible with the GW170817 event. Thus a broader class of Horndeski theories can be revived, however we restrict ourselves to the simplest case of having only kinetic coupling terms, because the extended theories are quite more difficult to study with regard to their inflationary phenomenology.
For the purposes of studying inflationary dynamics in this paper, we shall assume that the gravitational background will be that of a flat Friedman-Robertson-Walker (FRW) with line element,

$$ ds^2 = -dt^2 + a^2 \sum_{i=1}^{3} (dx^i)^2, $$

(2)

where $a(t)$ denotes as usual the scale factor. This choice for the line element is also convenient in order to obtain simple functional forms for the curvature related terms, since now both the Ricci scalar and the Gauss-Bonnet topological invariant can be written in terms of Hubble’s parameter $H = \frac{\dot{a}}{a}$ as $R = 6H^2 + \dot{H}$ and $G = 24H^2(H + \dot{H})$, where the “dot” as usual implies differentiation with respect to cosmic time $t$. Also we shall assume that the scalar field is homogeneous, so it has a time dependence solely.

At this point let us investigate the implication of the requirement that the speed of the primordial gravitational waves is equal to unity, that is $c_T^2 = 1$ in natural units, for the theory of the action [1]. As it was shown in Ref. [1], the speed of the tensor mode of the primordial curvature perturbations for the action [1] is equal to,

$$ c_T^2 = 1 - \frac{Q_f}{2Q_t}, $$

(3)

where in the case at hand, the term $Q_f$ is equal to,

$$ Q_f = 16(\dddot{\xi} - 3H\ddot{\xi}) + 4c\dot{\phi}^2, $$

(4)

and in addition $Q_t = \frac{1}{c} - 8\dot{\xi}H + c\ddot{\phi}^2$. Due to the presence of the extra kinetic coupling string correction, the functional forms of the auxiliary parameters $Q_f$ and $Q_t$ are intrinsically different in the case at hand, in comparison to the usual Einstein-Gauss-Bonnet theory, as it can be seen by comparing the above with the ones we used in Ref. [29]. As it is obvious from Eq. [4], the compatibility with the GW170817 event can be once again restored by demanding that $Q_f = 0$ or at least $Q_f \approx 0$, which in view of the functional form of $Q_f$ given in Eq. [4] results to the following constraint,

$$ 4(\dddot{\xi} - H\ddot{\xi}) + c\dot{\phi}^2 = 0. $$

(5)

It is obvious that for the case $c = 0$, one obtains again the case studied in [29], so the usual Einstein-Gauss-Bonnet theory. We can rewrite this expression in terms of the scalar field $\phi$ and by using $\dddot{\phi} = \dot{\phi} \dddot{\phi}$ then the constraint [4] can be rewritten as follows,

$$ 4(\dddot{\xi}\ddot{\phi}^2 + \dddot{\xi}\dot{\phi}^2 - H\dddot{\xi}\ddot{\phi}^2 + c\dddot{\phi}^2 = 0, $$

(6)

where the “prime” denotes differentiation with respect to the scalar field, and this notation will be kept for the rest of this paper. It is worth mentioning that the exactly same constraint would result in order for the gravitational wave speed to be equal to unity, even if the action contained additional string corrections of the form $c_1(\dddot{\xi}(\phi)D_\mu g^{\mu\nu} \partial_\nu \phi \partial_\mu \phi + c_2(\dddot{\xi}(\phi)(g^{\mu\nu} \partial_\nu \phi \partial_\mu \phi)^2$ [1], however this case would possibly lead to highly complicated and difficult to tackle analytically equations of motion, so we do not consider these in this paper.

Since we are aiming in studying the inflationary era, we shall assume that the slow-roll conditions hold true, that is $H \ll H^2$ and also that the slow-roll conditions hold true for the scalar field, that is $\dddot{\phi} \ll H\ddot{\phi}$. In effect, we also have $\dddot{\phi} \ll H\dddot{\phi}$. Thus, the differential equation [6] can be simplified and it reads,

$$ \dddot{\phi} = \frac{4H\dddot{\xi}}{4\dddot{\xi} + c\dddot{\phi}}. $$

(7)

It is important to make certain comments on this result. Firstly, it is reminiscing of the one previously acquired result in Ref. [29], which corresponds to the case of having $c = 0$, with the only difference being a factor 4, but this is a feature which was expected. Also, owning to the fact that the time derivative of the scalar field is written now proportionally to Hubble’s parameter and the Gauss-Bonnet coupling function, the same principles apply. In particular, all the functions can be written in terms of the scalar field instead of cosmic time $t$ and furthermore, the scalar functions of the model are interconnected. Particularly, the derivative of the scalar field $\phi$ must satisfy both Eq. [4] and the equation of motion of the scalar field. The existence of $c$ in the denominator makes it also a bit difficult to simplify the expression of Eq. [7], hence the strategy we followed in Ref. [29] in which case the choices of the coupling function $\dddot{\xi}(\phi)$ were done in such a way so that the ratio $\dddot{\xi}/\dddot{\phi}$ gets simplified, cannot be used in the present paper.
Now let us obtain the field equations for the gravitational action \( S \), by varying the action with respect to the metric and with respect to the scalar field. By doing so, one obtains the field equations for gravity, which in our case deviate from the Einstein field equations, and the equations of motion read,
\[
\frac{3H^2}{\kappa^2} = \frac{1}{2}\omega\dot{\phi}^2 + V + 24\xi H^3 - 9c\xi H^2\dot{\phi}^2 ,
\]
\[
-\frac{2\dot{H}}{\kappa^2} = \omega\dot{\phi}^2 - 8H^2(\ddot{\xi} - H\dot{\xi}) - 16\dot{\xi} H \dot{H} + c\left(2\xi(\ddot{H} - 3H^2)\dot{\phi}^2 + 4H\xi\ddot{\phi}\dot{\phi} + 2H\xi\dot{\phi}^2\right) ,
\]
\[
V' + \omega(\ddot{\phi} + 3H\dot{\phi}) + \xi G - 3c\left(H^2(\ddot{\xi} + 2\xi\ddot{\phi}) + 2H(2\ddot{H} + 3H^2)\ddot{\phi}\right) = 0 .
\]

The third equation stands for the differential equation of the scalar potential due to the fact that Eq. (8) was derived from the realization that the graviton must be massless during the inflationary era. It is the general expression with zero approximations assumed. As it is obvious, in the case at hand, the field equations corresponding to the action \( S \) have a lengthy functional form. Also the system of equations of motion is very complex and an analytical solution cannot be extracted easily. However, we shall try and simplify the equations above in order to obtain a viable phenomenology. In the following, we shall make two types of assumptions. Firstly, as it was mentioned previously, the slow-roll approximations for the Hubble rate and the scalar field will be implemented and in particular, we shall assume that,
\[
\dot{H} \ll H^2 \quad \frac{1}{2}\omega\dot{\phi}^2 \ll V \quad \ddot{\phi} \ll H\dot{\phi} .
\]
Furthermore we shall assume that the following two conditions hold true,
\[
24\xi H^3 \ll V, \quad -9c\xi H^2\dot{\phi}^2 \ll V ,
\]
and in the end we must explicitly check whether these conditions are satisfied for all the phenomenologically viable models we shall study. In view of the above assumptions, the equations of motion are greatly simplified and these read,
\[
H^2 = \frac{\kappa^2 V}{3} ,
\]
\[
\dot{H} = -\frac{\kappa^2\omega H^2}{2}\left(\frac{4\xi' + c\xi}{4\xi'' + c\xi}\right)^2 ,
\]
\[
V' + 3\omega H^2\left(\frac{4\xi'}{4\xi'' + c\xi}\right) = 0 ,
\]
where in the last equation, we made use of Eq. (7). The above equations of motion are simpler in comparison to those obtained without the slow-roll assumption, and thus in the case at hand analytic results can be obtained and the phenomenology of the non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet theory can be investigated in a simpler and direct way. The advantage of the slow-roll assumption in the case at hand is that, given the scalar coupling function \( \xi(\phi) \), the scalar potential can be determined directly by using the equations of motion, as we show in the illustrative examples we chose to present in the next section. It is important to note again however, something that we already mentioned earlier in this section, that is, in the end it is vital to check whether the approximations imposed hold true, for each model that yields a viable phenomenology. It is conceivable that a model that yields a viable phenomenology, but still fails to satisfy the initial assumptions, is intrinsically wrong.

As it will be apparent shortly, the scalar coupling \( \xi(\phi) \) plays a crucial role in the theory at hand, and critically affects the slow-roll indices and the resulting observational indices, namely the spectral index of the primordial scalar perturbations, and the tensor-to-scalar ratio. Also as it will be shown shortly, an elegant choice of \( \xi(\phi) \) will results to quite elegant final expressions of the aforementioned phenomenology related parameters.

Now using the slow-roll assumptions we shall derive the analytic expressions for the slow-roll indices for the theory at hand. For the non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet theory the slow-roll indices have the general expressions \( \epsilon_i \),
\[
\epsilon_1 = \frac{\dot{H}}{H^2} \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}} \quad \epsilon_3 = 0 \quad \epsilon_4 = \frac{\dot{E}}{2H E} \quad \epsilon_5 = \frac{Q_\phi}{2HQ_t} \quad \epsilon_6 = \frac{\dot{Q}_t}{2HQ_t} .
\]
where \( Q_a = -8\dot{\xi}H^2 + 4c\xi H \dot{\phi}^2 \), \( E = \frac{1}{(\kappa \phi)^2} \left( \omega \dot{\phi}^2 + \frac{36\dot{\xi}^2}{2\xi^2} + Q_c \right) \) and \( Q_C = -6c\dot{\phi}^2 H^2 \). Thus, using the slow-roll equations of motion, namely Eqs. [13]-[15], we obtain,

\[
\epsilon_1 = \frac{\kappa^2 \omega}{2} \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2, 
\]

\[
\epsilon_2 = \frac{4\xi''}{4\xi'' + c\xi} - \epsilon_1 - \frac{4\xi' (4\xi'' + c\xi')}{(4\xi'' + c\xi)^2}, 
\]

\[
\epsilon_4 = \frac{2\xi'}{4\xi'' + c\xi} E', 
\]

\[
\epsilon_5 = \frac{-16\xi'^2 H^2 + 4c\xi H^2 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2}{1 - 32 \frac{\xi'^2 H^2}{4\xi'' + c\xi} + 32c\xi H^2 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2}, 
\]

\[
\epsilon_6 = -\frac{4H^2 \frac{4\xi'^2}{4\xi'' + c\xi} \left( \frac{4\xi'' + c\xi'}{4\xi'' + c\xi} + \epsilon_2 - \epsilon_1 \right) + cH^2 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2 \left( \frac{2\xi'^2}{4\xi'' + c\xi} + \xi \epsilon_2 \right)}{1 - 32 \frac{\xi'^2 H^2}{4\xi'' + c\xi} + 32c\xi H^2 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2},
\]

and the auxiliary parameters \( Q_a, Q_t, Q_d, Q_c \) in turn take the following form,

\[
Q_a = -32 \frac{\xi'^2}{4\xi'' + c\xi} H^3 + 4cH^3 \xi \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2, 
\]

\[
Q_t = \frac{1}{\kappa^2} - 32 \frac{\xi'^2}{4\xi'' + c\xi} H^2 + c\xi H^2 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2, 
\]

\[
Q_c = -6c\xi H^4 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2, 
\]

\[
Q_d = -4c\xi^2 H^2 \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2 \dot{H}, 
\]

\[
Q_e = -32 \frac{4\xi'^2 H}{4\xi'' + c\xi} \dot{H} + 16cH \frac{\xi'}{4\xi'' + c\xi} \left( \frac{4\xi'}{4\xi'' + c\xi} \right)^2 + 2 \xi \ddot{\phi} - 8\xi H^2 \frac{\xi'}{4\xi'' + c\xi}, 
\]

where in order not to make \( Q_e \) lengthy, the \( \ddot{\phi} \) is left as it is but essentially it is equal to \( \ddot{\phi} = \epsilon_2 H \ddot{\phi} = 4\epsilon_2 H^2 \frac{\xi'}{4\xi'' + c\xi} \). The parameter \( Q_e \) is not used here but is introduced now for convenience since it shall be utilized in the subsequent calculations. It is important to make two comments here. Firstly, the indices \( \epsilon_5 \) and \( \epsilon_6 \) have quite different forms thus it is not expected that their values will be similar. This is attributed to the non-minimal kinetic coupling term proportional to \( c \). Also, for \( c = 0 \), we obtain the same equations as in Ref. [29]. In the context of the present formalism, only the first two slow-roll indices have simple functional forms whereas the rest are quite complicated, especially the index \( \epsilon_4 \) which at best its form could be characterized as lengthy. Nonetheless, the slow-roll indices are of paramount importance since in this approach, the observational indices, namely the scalar spectral index of primordial curvature perturbations \( n_S \), the tensor spectral index \( n_T \) and the tensor-to-scalar ratio \( r \) are given in terms of the slow-roll indices as follows [1],

\[
n_S = 1 - 2\frac{\epsilon_1 + \epsilon_2 + \epsilon_4}{1 - \epsilon_1}, \quad n_T = -2\frac{\epsilon_1 + \epsilon_6}{1 - \epsilon_1}, \quad r = 16 \left( -\epsilon_1 - \frac{\kappa^2 (2Q_c + Q_d - H Q_e)}{4H^2} \right) \frac{c^2}{\kappa^2 Q_t},
\]
where, $c_A$ denotes the sound wave velocity and it is given by the formula,

$$c_A^2 = 1 + \frac{2Q_d Q_d + Q_a Q_e}{2\omega \phi^2 Q_t + 3Q_a^2 + 2Q_e Q_t}.$$  \hspace{1cm} (28)

Also it is important to note that the formulas quoted in Eq. (27) hold true only if the slow-roll assumptions hold true for the slow-roll indices, that is when $\epsilon_i \ll 1$, and a refined derivation of the formulas (27) was given in Ref. [73]. Lastly, a vital ingredient of a theory that is analytically tractable, is the functional form of the coupling function firstly.

In this section we shall investigate several models that may provide a viable phenomenology, by appropriately choosing the scalar coupling function $\xi(\phi)$. The procedure we shall follow is to choose and fix the Gauss-Bonnet scalar coupling function firstly $\xi(\phi)$, accordingly we shall find the corresponding scalar potential, and afterwards we derive the corresponding expression of the scalar field from Eq. (15). Up on finding its expression, we shall demonstrate how the slow-roll indices are influenced by such selection and finally, by utilizing the form of slow-roll index $n$ the corresponding expression of the scalar field from Eq. (15), one obtains the following result,

$$N = \int_{\phi_i}^{\phi_f} \frac{4\xi'' + c\xi'}{4\xi'} d\phi,$$  \hspace{1cm} (29)

It is clear that the overall phenomenology acquired for the non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet theory is not fundamentally different from the one obtained from the simple Einstein-Gauss-Bonnet theory. In fact, many, if not every parameter here is similar to the corresponding ones in Ref. [29]. Hence, it is expected that the same steps will yield appealing characteristics, if not viability. Specifically, since each object is written in terms of the scalar field $\phi$, we shall evaluate its value during the initial and final stage of the inflationary era. The latter can be extracted by letting the first slow-roll index $\epsilon_1$ become of order $O(1)$. Afterwards, the initial value of the scalar field $\phi_i$ at the first horizon crossing can be obtained easily from the $e$-foldings number, hence this is the reason why we rewriten it in terms of the scalar field. In the following section, we shall extensively study the phenomenological viability of several appropriately chosen models and when possible, we shall directly compare the non-minimal kinetic coupling Einstein-Gauss-Bonnet theory with the simple Einstein-Gauss-Bonnet theory.

III. CONFRONTING THE GW170817-COMPATIBLE NON-MINIMAL KINETIC COUPLING CORRECTED EINSTEIN-GAUSS-BONNET WITH THE PLANCK DATA

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A. The Choice Of A Trigonometric Scalar Coupling $\xi(\phi)$

We commence our study by assuming the Gauss-Bonnet has the following form,

$$\xi(\phi) = \lambda_1 \sin(\gamma_1 \kappa \phi),$$  \hspace{1cm} (30)

where $\lambda_1$ and $\gamma_1$ are the free parameters of the model. It can easily be inferred that this choice is somewhat beneficial since $\xi'' = -\gamma_1^2 \kappa^2 \xi$, therefore the ratio $\frac{4\xi'}{4\xi'}$ can be simplified. Consequently, the corresponding scalar potential reads,

$$V(\phi) = V_1 \sin(\gamma_1 \kappa \phi) \frac{4\xi'\xi'' - \xi'^2}{4\xi'^2 + \xi''^2},$$  \hspace{1cm} (31)

where $c$ is the integration constant with mass dimensions $[m]$, for consistency. In this case, we can see that the scalar potential is also trigonometric as the Gauss-Bonnet Coupling is. Concerning the first slow-roll index, it takes the
The spectral index of primordial curvature perturbations obtains the value $n_s = 0.965546$, the tensor spectral index is equal to $n_T = -0.0019894$ and finally the tensor-to-scalar ratio becomes $r = 0.015899$ which are obviously accepted values.

Finally the numerical values of the slow-roll indices are $\epsilon_1 = 0.0009936$, $\epsilon_2 = 0.015228$, $\epsilon_4 = -4.05 \cdot 10^{-7}$, $\epsilon_5 = -3.78 \cdot 10^{-8}$ and finally $\epsilon_6 = 5 \cdot 10^{-16}$ which essentially is zero. These values are indicative of the fact that the slow-roll conditions implemented previously in the equations of motion are indeed valid. The previous designation might seem bizarre but was chosen since it led to a viable phenomenology but as we also show, the assumptions and approximations made in the previous sections are also satisfied. Also in Fig. 1 we can see that the model is phenomenologically viable for a wide range of the free parameters. The difference between the $\xi G$ and $cG_{\mu \nu} \partial_\mu \phi \partial_\nu \phi$ seems to be the constants and in particular the constant $\lambda_1$ of the Gauss-Bonnet coupling and $c$. Our analysis showed that the parameter $\lambda_1$ must be orders of magnitude lesser than unity in order to achieve a phenomenologically acceptable inflationary era. However, this can be attributed to the value of $c$ which is trivial in this case. In addition, the parameters $\lambda_1$ and $V_1$ seem to affect drastically the scalar spectral index. This was also the case for $\gamma_1$ being $-0.15$ since it was the only value capable of doing so when $\lambda_1$ and $V_1$ were fixed in these values, along with $c$, so as
not to violate the approximations made. Specifically, $c$ must be negative otherwise the scalar field obtains a complex value during the first horizon crossing.

Finally, we address the validity of the approximations made in the previous section. Concerning the slow-roll indices, it was mentioned some paragraphs above, that these respect the slow-roll conditions. Also using for example the values $(\omega, \lambda_1, V_1, N, \gamma_1, c) = (1, 10^{-8}, 10^{3}, 60, -0.15, -7)$, we have $\dot{H} \sim O(10^{-1})$ whereas $H^2 \sim O(10^2)$, so the condition $\dot{H} \ll H^2$ holds true. In addition, $\frac{1}{3} \omega \dot{\phi}^2 \sim O(10^{-1})$ and similarly $V \sim O(10^2)$, so the assumption $\frac{1}{3} \omega \dot{\phi}^2 \ll V$ holds true. Finally, $\ddot{\phi} \sim O(10^{-1})$ compared to $H \dot{\phi} \sim O(10)$, so the condition $\dot{\phi} \ll H \dot{\phi}$ holds true. Now let us see whether the rest of the assumptions made in the previous sections indeed hold true. In particular, for the first condition $\dot{\phi}$ in Eq. (12), we have $24 \dot{\phi}$ with regard to (10), we have $24 \dot{\phi}$ holds true. In addition, $\frac{1}{3} \omega \dot{\phi}^2 \sim O(10^{-1})$ and similarly $V \sim O(10^2)$, so the assumption $\frac{1}{3} \omega \dot{\phi}^2 \ll V$ holds true. Finally, $\ddot{\phi} \sim O(10^{-1})$ compared to $H \dot{\phi} \sim O(10)$, so the condition $\dot{\phi} \ll H \dot{\phi}$ holds true. Now let us see whether the rest of the assumptions made in the previous sections indeed hold true. In particular, for the first equation of motion Eq. (8), we have $24 \dot{\phi}$ holds true. Also, we have $16 \dot{H} \dot{H} \sim O(10^{-8})$, $8(\dot{\xi} - \dot{H} \xi) \sim O(10^{-5})$ and $c \left(2 \dot{\xi} \dot{\phi}^2 \ddot{H} - 6 \dot{\xi} \dot{\phi}^2 H^2 + 4H \dot{\xi} \dot{\phi} \ddot{\phi} + 2H \dot{\xi} \dddot{\phi} \right) \sim O(10^{-5})$ while $\omega \dot{\phi}^2 \sim O(10^{-1})$ hence Eq. (11) is indeed valid. Lastly, with regard to (10), we have $\xi \dot{\phi}^2 \sim O(10^{-3})$, $3c \left(H^2 (\dddot{\xi} \dot{H} + 2 \dddot{\phi} \dot{\phi}) + 2H (2 \dot{H} + 3H^2) \dddot{\phi} \right) \sim O(10^{-3})$ while $V' \sim O(10)$. As a result, all the approximations made are valid.

As a last comment, it is worth mentioning that the choice for a cosine as a Gauss-Bonnet coupling function is also capable of producing a viable phenomenology. In particular, assuming that $\xi(\phi) = \lambda_1 \cos(\gamma_1 \phi)$ then by altering only $\gamma_1$ and $c$ to $c = -3$ and $\gamma_1 = 0.1$ manages to produce viable observed indices as $n_S = 0.96239$, $r = 0.0355884$ and $n_T = -0.00445847$. In Fig. 2 we present the dependence of the observational indices on the same free parameters, namely $c$ and $\gamma_1$. Moreover, all the approximations made in the previous section, are satisfied once again. It becomes apparent that the same phenomenology is gained in the case of a cosine as well. Thus, one could argue that the general Gauss-Bonnet scalar coupling function reads $\xi(\phi) = \lambda_1 \sin(\gamma_1 \phi + \theta)$ where $\theta$ is an arbitrary phase, or in other words a connection between the sine and cosine description. This however does not mean that each and every trigonometric function is capable of producing a viable description. Indeed, the case of a tangent Gauss-Bonnet coupling or in general a hyperbolic trigonometric case results in complex values for the scalar field. The most promising choice would be the hyperbolic sine which produced a real scalar field, however the model, dependent on $c$ and $\gamma_1$, cannot produce compatible values for both the scalar spectral index of primordial curvature perturbations and the tensor-to-scalar ratio. In general, the same argument could be used for the exponential choice or an exponential like choice for the Gauss-Bonnet coupling. The choice of $\xi(\phi) = \lambda_1 (1 - e^{\gamma_1 \phi})$ is described only by a complex scalar field and furthermore, the choice of a pure exponential, meaning $\xi(\phi) = \lambda_1 e^{\gamma_1 \phi}$ results in eternal or no inflation since slow-roll index $\epsilon_1$ turns out to be $\phi$-independent. In fact, $\epsilon_1 = \frac{d}{2} \left(\frac{4 \gamma_1^2 \gamma_1}{4 \gamma_1^2 \phi^2 - c} \right)$ which according to the choice of $\gamma_1$ and $c$ leads to either eternal inflation for $\epsilon_1 < 1$ or no inflation for $\epsilon_1 > 1$. It seems that this description is suitable for trigonometric functions. In principle there are many other models that may lead to a viable phenomenology, however we refrain from discussing other examples for brevity. The procedure is more or less the same, and nothing new is added to our argument. In the next section, we shall discuss another interesting case, the case that the scalar field evolves dynamically in a constant-roll way.
In this section, we shall study a different scenario compared to the slow-roll assumption for the scalar field. Particularly, we shall assume that the scalar field evolves dynamically in a constant-roll way \([74–108]\), and as we shall see most of the equations we used previously, shall remain the same, hence for simplicity we shall present only the equations that differ from the previous case. Essentially, the constant-roll assumption states that,

\[
\ddot{\phi} = \beta H \dot{\phi},
\]

where \(\beta\) denotes the constant-roll parameter. As a result, the overall phenomenology experiences certain changes. Firstly, the equation for the scalar field shall be rewritten. Recalling Eq. (6), in view of the constant-roll condition (35) we have,

\[
\dot{\phi} = 4H(1 - \beta) \frac{\xi'}{4\xi'' + c\xi},
\]

Here, it is worth making some comments. Firstly, this formula is exact as no approximations were made. Secondly, for \(\beta \ll 1\), one obtains the same equation as in Eq. (7). Finally, as expected, for \(c \to 0\), we obtain the same formula as in the case of Einstein-Gauss-Bonnet theory, either for the slow-roll or constant-roll assumption respectively. Concerning the equations of motion, they remain exactly the same, namely Eqs. (8)-(10). Furthermore, we shall assume the exact same approximations, meaning the slow-roll conditions and also the ones made in Eqs. (11) (only the first two equations of course of Eq. (11)) and (12). Thus, the simplified equations of motion read,

\[
H^2 = \frac{\kappa^2 V}{3},
\]

\[
\dot{H} = -\frac{\kappa^2 \omega H^2}{2}(1 - \beta)^2 \left(\frac{4\epsilon'}{4\xi'' + c\xi}\right)^2,
\]

\[
V' + 3(1 - \beta)(1 + \frac{\beta}{3} H^2 \frac{4\epsilon'}{4\xi'' + c\xi}) = 0.
\]

The slow-roll indices do not experience any dramatic changes, except of course for the slow-roll index \(\epsilon_2\). In fact, the same forms as before are still applicable with the only difference being a factor of \(1 - \beta\). The same applies to the auxiliary parameters \(Q_i\) however we shall only present the slow-roll indices since they are more important. In fact, concerning the slow-roll indices, the only significant difference is found as expected in the slow-roll index \(\epsilon_2\), which in the case at hand is \(\epsilon_2 = \beta\). The slow-roll indices under the constant-roll assumption are shown below,

\[
\epsilon_1 = \frac{\omega}{2}(1 - \beta)^2 \left(\frac{4\kappa\epsilon'}{4\xi'' + c\xi}\right)^2,
\]

\[
\epsilon_2 = \beta,
\]

\[
\epsilon_4 = (1 - \beta) \frac{2\epsilon'}{4\xi'' + c\xi} \frac{E'}{E},
\]

\[
\epsilon_5 = \frac{-16(1 - \beta) \xi'^2 H^2}{4\xi'' + c\xi} + 4c\xi H^2 \left(\frac{4(1 - \beta)\epsilon'}{4\xi'' + c\xi}\right)^2
\]

\[
-\frac{1}{\kappa^2} - 32(1 - \beta) \frac{\xi'^2 H^2}{4\xi'' + c\xi} + 32c\xi H^2 \left(\frac{(1 - \beta)\epsilon'}{4\xi'' + c\xi}\right)^2,
\]

\[
\epsilon_6 = -\frac{4H^2 \frac{4\xi'^2(1 - \beta)}{4\xi'' + c\xi} + (4\xi''(1 - \beta) + \beta - \epsilon_1) + cH^2 \left(\frac{4\xi'(1 - \beta)}{4\xi'' + c\xi}\right)^2 \left(\frac{2\xi'^2(1 - \beta)}{4\xi'' + c\xi} + \xi\beta\right)}{\frac{1}{\kappa^2} - 32\frac{\xi'^2(1 - \beta)}{4\xi'' + c\xi} H^2 + c\xi H^2 \left(\frac{4\xi'(1 - \beta)}{4\xi'' + c\xi}\right)^2}.
\]
In this case as well, it is clear that the first two slow-roll indices are simple while the rest are perplexed. Also, only a factor of $1 - \beta$ is now emergent, in contrast to the slow-roll case, as mentioned before. Lastly, the $\epsilon$-foldings number is also slightly changed in the constant-roll case, as is shown below,

$$N = \frac{1}{1 - \beta} \int_{\phi_i}^{\phi_f} \frac{4 \xi'' + c \xi'}{4 \xi'} d\phi.$$

(45)

It can easily be inferred that the overall phenomenology is similar to the one acquired previously. In particular, for $\beta \ll 1$, the results are quite similar. In the following we shall study a specific model and examine whether the constant-roll assumption in general can produce compatible results. We shall present an interesting example with the Gauss-Bonnet scalar coupling being chosen to be a linear function of the scalar field.

A. The Choice Of A Linear Gauss-Bonnet Coupling

Let us assume a simple form for the Gauss-Bonnet scalar coupling function, and particularly, let $\xi(\phi)$ be equal to,

$$\xi(\phi) = \lambda \kappa \phi,$$

(46)

so $\xi(\phi)$ is a linear function of the scalar field. In this case, the second derivative with respect to the scalar field is zero, meaning $\xi'' = 0$. As we shall show shortly, the condition $\xi''$ being equal to zero does not affect dramatically the phenomenology, on the contrary it makes our study easier, since $\dot{\phi}$ is simplified. The corresponding scalar potential also has a power-law form, and it can easily be found for the linear scalar coupling, and it reads,

$$V(\phi) = V_2(\kappa \phi)^{-4(1 - \beta) \kappa^2 c^2}.$$

(47)

It is clear that the exponent is once again general and is not necessarily integer. Let us now proceed with the slow-roll indices. Due to the linear choice of the scalar coupling function, we have,

$$\epsilon_1 = 2\omega \left(\frac{2(1 - \beta)\kappa}{c\phi}\right)^2.$$

(48)

Due to the simple form of the slow-roll index $\epsilon_1$, the initial and final value of the scalar field are easily obtained and in this case are written as,

$$\phi_i = \pm \sqrt{c \phi_f^2 + 8N(1 - \beta)^2 - c},$$

(49)

$$\phi_f = \pm \frac{2\kappa(1 - \beta)\sqrt{2\omega}}{c}.$$

(50)

The resulting theory can be compatible with the observational data for a wide range of values of the free parameters. For example by choosing $(\omega, \lambda_2, V_2, N, c \beta) = (1, 10^{-8}, 10, 60, -5, 0.009)$ in reduced Planck units ($\kappa^2 = 1$), then the observational indices read $n_S = 0.968604$, $n_T = -0.0066667$ and $r = 0.0531572$ which are obviously compatible with the data coming from the Planck 2018 collaboration. Moreover, the scalar field decreases with time as $\phi_i = 9.72595$ whereas $\phi_f = 0.560594$ in Planck units and in addition, the slow-roll conditions seem to be valid since the numerical values of the slow-roll indices are of order $O(10^{-3})$ and lesser. In particular, $\epsilon_1 = 0.0033223$, $\epsilon_4 = 1.2 \cdot 10^{-6}$, $\epsilon_5 = -6.4 \cdot 10^{-8}$ and $\epsilon_6 = 5 \cdot 10^{-11}$. In Fig. 3 we present the dependence of the observational indices on the constant-roll parameter $\beta$ and the parameter $c$. In general, these are not the only parameters which affect the results. Indeed, $\lambda_2$ also affects the overall phenomenology as $\lambda_2 = 0.0001$ for instance produces incompatible results, namely $n_S = 0.944381$ and $r = 0.0751259$. The coefficient of the scalar potential also affects the results. Here, increasing $V_2$ affects mildly the scalar spectral index of the primordial curvature perturbations. On the other hand, the tensor-to-scalar ratio varies since for $V_2 = 10^3$, we have $r = 0.0533064$ but for $V_2 = 10^5$, while $n_S$ is altered slightly, the tensor-to-scalar ratio is increased by an order, in particular $r = 0.115128$, hence it is necessary to choose this value wisely.

Finally, let us discuss here and validate whether the approximations assumed in the section II hold true. Firstly, by choosing $(\omega, \lambda_2, V_2, N, c \beta) = (1, 10^{-8}, 10, 60, -5, 0.009)$, we have $\dot{H} \sim O(10^{-2})$ compared to $H^2 \sim O(10)$, so the condition $\dot{H} \ll H^2$ holds true. Similarly, $\frac{1}{2} \omega \dot{\phi}^2 \sim O(10^{-2})$ while $V \sim O(10)$, so the assumption $\frac{1}{2} \omega \dot{\phi}^2 \ll V$
FIG. 3: The scalar spectral index $n_S$ (left) and tensor-to-scalar ratio $r$ (right) with respect to constant-roll parameter $\beta$ and the parameter $c$. Their values range from [0.005, 0.01] and [-8, -2] respectively. It becomes apparent that spectral index $n_S$ depends on both the free parameters, whereas $r$ is only affected by $c$. In particular, the observational indices are very sensitive since in such small area of designation for the free parameters, the possible values for $n_s$ and $r$ have a wide range.

also holds true. In addition, we have $24\xi H^3 \sim O(10^{-3})$ and $9\xi \ddot{\phi}^2 H^2 \sim O(10^{-5})$ which are both inferior to $V \sim O(10)$, so the conditions in Eq. (12), hold true. Also, $16\xi H^2 H \sim O(10^{-8})$, $8(\dot{\xi} - H\xi) \sim O(10^{-7})$ and $c(2\xi \ddot{\phi}^2 - 4H\xi \dot{\phi}^2 - 6\xi \dddot{\phi} H^2 + 4H\xi \dddot{\phi}^2) \sim O(10^{-5})$ while $\omega \phi \dddot{\phi} \sim O(10^{-1})$. Lastly, we note that $\xi \mathcal{G} \sim O(10^{-4})$, $3c(H^2(\ddot{\xi} H + 2\ddot{\phi}) + 8H^2) \sim O(10^{-4})$ and $9(V') \sim O(10)$.

As a result, the kinetic coupling-corrected Einstein-Gauss-Bonnet gravity with a linear coupling function, can lead to a viable phenomenology assuming that graviton is massless. This is in contrast to the simple GW170817-compatible Einstein-Gauss-Bonnet gravity, which as we show in Ref. [109], a linear coupling leads to the constant-roll condition inevitably, and the resulting theory is phenomenologically questionable, with the regard to the inflationary phenomenology. Actually, as we show in [109], the resulting scalar spectral index is always quite close to the value $n_S \sim -1$.

V. CONCLUSIONS

In this work we studied non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet theories in view of the GW170817 event, which restricted the gravitational wave speed to be equal to that of light’s. The constraint $c^2_T = 1$ as we showed, constrained the functional form of the scalar coupling and of the scalar potential, and by using the slow-roll assumption, we showed how a viable phenomenology can be obtained by the present theoretical framework. We derived analytic formulas for the slow-roll indices and for the observational indices characterizing the inflationary era, and by using several illustrative examples, we demonstrated how the compatibility of the inflationary phenomenology corresponding to the present theory can be achieved. In principle several choices of the scalar coupling function can also yield viable results, but we refrained from presenting more models than a small class of models, for brevity.

Our motivation of imposing the constraint $c^2_T = 1$ in the primordial gravitational waves, is mainly the fact that there is no particle physics process related to the inflationary and post-inflationary era that alters the graviton mass. The graviton has a specific mass which can be determined by the string theory governing the physics of the pre-inflationary epoch, so if the graviton mass is specific during the inflationary era, there is no fundamental reason that may allow it to change, at least to our knowledge. As we showed, the constraint $c^2_T = 1$ actually leads to a phenomenologically viable non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet inflationary theory. Also, it is worth mentioning that the constraint $c^2_T = 1$ for the non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet also covers theories with string correction terms of the form $\sim \xi(\phi) \Box g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and $\sim \xi(\phi) (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)^2$, in which case the full effective
Lagrangian is of the form,
\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa^2} - \frac{\omega}{2} g^\mu\nu \partial_\mu \phi \partial_\nu \phi - V(\phi) - \xi(\phi)G^\mu\nu \partial_\mu \phi \partial_\nu \phi - c_1 \Box \phi g^\mu\nu \partial_\mu \phi \partial_\nu \phi - c_2 (g^\mu\nu \partial_\mu \phi \partial_\nu \phi)^2 \right),
\]
(51)
results to the same differential equation. Theories of the form (51) belong to a wider class of Horndeski theories, so in this paper we demonstrated how these theories may be revived formally, in view of the GW170817 event. We did not study in detail however theories of the form (51) since with this paper we just wanted to demonstrate how the simplest class of these theories can be revived. In a future work we might address in detail the analysis of the full theory.

Finally, we discussed also the constant-roll scenario in the context of the present theory. It is interesting to note that the constant-roll condition is related to non-Gaussianities \[79, 108\], so in a future work it would be interesting to address this issue in detail in the context of the non-minimal kinetic coupling corrected Einstein-Gauss-Bonnet theories.

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