On the quest for unification - simplicity and antisimplicity. 

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Abstract  

The road towards unification of elementary interactions is thought to start on the solid ground of a universal local gauge principle. I discuss the different types of bosonic gauge symmetries in gravitational and nongravitational (standard model) interactions and their extensions both fermionic, bosonic and with respect to space-time dimensions. The apparently paradoxical size and nature of the cosmological constant is sketched, which at first sight does not readily yield a clue as to the enveloping symmetry structure of a unified theory. Nevertheless a tentative outlook is given encouraging to proceed on this road.

1 Dedicated to Alberto Sirlin on the occasion of his birth day.
In collaboration with Sonja Kabana, Wolfgang Ochs and Luzi Bergamin.

Topics:

length scale

1) the ways of gravitation
gauging spin (- Lie group)
\[ e^a_\mu \leftrightarrow g_{\mu\nu} \]
(no) response to vacuum energy density

\[ \lambda > \frac{1}{2 eV} \sim 10^8 \text{ fm} \]

2) + QED gauging charge \( U1_{em} \)
\[ \lambda > \lambda_{\text{cl}}^e = \frac{\alpha}{m_e} \sim 3 \text{ fm} \]

what is the relation between charge and spin?

3) + QCD \( SU3_c \)
gauging color charges
\[ gl^A, (u, d; c, s; t, b)^c \]
and \( q \rightarrow \bar{q} \)

\[ \lambda > (\sqrt{2} G_F)^{1/2} \sim 10^{-3} \text{ fm} \]

4) + \( SU2_L \times U1_Y \) the electroweak completion

\[ SO(3, 1) \times SU3_c \times U1_{em} \rightarrow SU2_L \times U1_Y \]

\[ \begin{array}{cccccc}
  \text{gr} & \text{gl} & \gamma & W^{\pm} & Z & \varphi_H \\
  2 & 16 & 2 & 9 & 1 & \rightarrow 30 \\
\end{array} \]
topics continued

| length scale |
|--------------|
| $\lambda \sim 10^{-17} \text{ fm}$ |

5) unification of charges

$SU_3c \times SU_2L \times U_{1Y} \rightarrow SO_{10} \rightarrow E_6 \cdots$

6) gauging fermionic charges

susy $\leftrightarrow$ anti-simplicity

$\lambda \sim 10^{-4} \text{ fm}$

7) exact broken symmetries

+ gauge symmetries, CPT

leptonic numbers

$\lambda \sim 10^{-11} \text{ fm}$

8) primary breakdown

difficulties

$\lambda \sim 10^{-17} \text{ fm}$

9) 4 dimensions and cosmological term: a paradox

all

10) outlook ( quo vadis ? )

1 The ways of gravitation

The base quantities involve the structural forms with respect to tangent space of vierbein, spin connection and Riemann curvature

\begin{align*}
e^a &= (e_1)^a = dx^\mu e^a_\mu ; \quad \omega^a_b = (\omega_1)^a_b = dx^\mu (\omega_\mu)^a_b \\
R^a_b &= (R_2)^a_b = \frac{1}{2} dx^\mu \wedge dx^\nu (R_{\mu\nu})^a_b \\
g_{\mu\nu} &= e^a_\mu \eta_{ab} e^b_\nu ; \quad (\Gamma_\mu)^\nu_\varphi = \epsilon^a_\nu \left[ \partial_\mu e^a_\varphi + (\omega_\mu)^a_b e^b_\varphi \right] \\
R^a_\sigma_{\mu\nu} &= e^a_\mu e^b_\sigma (R_{\mu\nu})^a_b
\end{align*}

with the structural relations
\[ d \omega^a + \hat{\omega}^a b e^b = 2 T^a = 2 \left( T_2 \right)^a \]

\[ T^a = \frac{1}{2} d \omega^{a} \wedge d \omega^{e} T^{a}_{\mu e} \]

\[ T^\sigma_{[\mu e]} = e^\sigma_a T^a_{\mu e} = \frac{1}{2} \left[ (\Gamma^\sigma_\nu)^\mu - (\Gamma^\sigma_\mu)^\nu \right] \]

\[ R^a_{\phantom{abc}b} = - \left( d \omega + \omega^2 \right)^a_{\phantom{abc}b} \]

In eq. (2) \( g_{\mu\nu} \) and \( \Gamma^\nu_{\mu e} \) denote the metric and vector connection respectively. \( \eta_{ab} \) denotes the tangent space Lorentz invariant metric. Vector and spin connections are characterized by their minimal representatives

\[ \Gamma^\sigma_{\mu e} = (\Gamma^\sigma_\mu)^e \quad T^\sigma_{[\mu e]} = T^\sigma_{\mu e} = g^{\sigma\tau} T^\tau_{[\mu e] ; \tau} \]

\[ \gamma^\sigma_{\{\mu e\}} = g^{\sigma\tau} \gamma^\tau_{\{\mu e\}} ; \tau \quad \Gamma^\sigma_{\mu e} : \nu = \frac{1}{2} \left[ \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right] \]

\[ \hat{\omega} : d \omega^a + \hat{\omega}^a b e^b = 0 \]

\[ \hat{\omega}_{[ab]} = \eta^{bb'} (\hat{\omega}_{\mu}^a)^{a}_{b'} = \frac{1}{2} \begin{cases} e^{\nu b} \left( \partial_\nu e^a_\mu - \partial_\mu e^a_\nu \right) \\ - [a \leftrightarrow b] \\ + e^{\nu b} e^{aa'} e_{\mu c} \left( \partial_\nu e^c_\sigma - \partial_\sigma e^c_\nu \right) \end{cases} \]

In eqs. (2) and (3) and in the following tensor (or tangent space) indices in brackets denote antisymmetric [] and symmetric {} symmetric combinations respectively.

**Torsion and contorsion**

The difference of two connections, for a given metric, defines a covariant quantity, a three tensor in the case of vector or spin connections, with the respective relations

\[ \omega_{\mu ; \nu} = e_{\nu a} e^b_\mu (\omega_{\mu})^a_{b} ; \Gamma_{\mu e ; \nu} = g_{\nu\sigma} \Gamma^\sigma_{\mu e} \]

\[ \Gamma_{\mu e ; \nu} = e_{\nu a} \partial_\mu e^a_\nu + \omega_{\mu ; \nu} = \gamma^\nu_{\{\mu e\}} ; \nu + \Delta \Gamma_{\mu e ; \nu} \]

\[ \omega_{\mu ; [\nu e]} = \hat{\omega}_{\mu ; [\nu e]} + \Delta \omega_{\mu ; [\nu e]} \]
and

\[ \gamma_{\{\mu\nu\}}; \nu = e_{\nu a} \partial_{\mu} e^{a}_{\bar{\nu}} + \hat{\omega}_{\mu; [\nu\bar{\nu}]} \]

\[ \hat{\omega}_{\mu; [\nu\bar{\nu}]} = \frac{1}{2} \left\{ e_{\nu a} \left( \partial_{\bar{\nu}} e^{a}_{\mu} - \partial_{\mu} e^{a}_{\bar{\nu}} \right) \right\} \]

contorsion: \( \Delta \omega_{\mu; [\nu\bar{\nu}]} \equiv K_{\mu; [\nu\bar{\nu}]} \); \( \Delta \Gamma_{\mu\bar{\nu} ; \nu} = K_{\mu ; [\nu\bar{\nu}]} \)

\[ T_{[\mu\nu]} ; \nu = \frac{1}{2} \left( \Delta \Gamma_{\mu\bar{\nu} ; \nu} - \Delta \Gamma_{\nu\bar{\mu} ; \nu} \right) = \frac{1}{2} \left( K_{\mu ; [\nu\bar{\nu}]} - K_{\bar{\nu} ; [\nu\mu]} \right) \]

\[ K_{\mu ; [\nu\bar{\nu}]} = T_{[\nu\bar{\nu}]} ; \mu - T_{[\nu\mu]} ; \bar{\nu} - T_{[\bar{\nu}\mu]} ; \nu \]

(5)

It follows that all covariant quantities derived from the difference of vector or spin connections and their minimal forms are equivalently determined from either the torsion or contorsion three tensor

\[ \Delta \Gamma_{\{\mu\nu\}} ; \nu = \frac{1}{2} \left( \Delta \Gamma_{\mu\bar{\nu} ; \nu} + \Delta \Gamma_{\nu\bar{\mu} ; \nu} \right) = \frac{1}{2} \left( K_{\mu ; [\nu\bar{\nu}]} + K_{\bar{\nu} ; [\nu\mu]} \right) \]

\[ = T_{[\nu\bar{\nu}]} ; \mu + T_{[\nu\mu]} ; \bar{\nu} \]

(6)

A special case arises if torsion and contorsion tensors are totally antisymmetric

\[ \Delta \Gamma_{\{\mu\nu\}} ; \nu = 0 \rightarrow T_{[\mu\bar{\nu}]} ; \nu = T_{[\mu\nu\bar{\nu}]} = K_{\mu ; [\nu\bar{\nu}]} = K_{[\mu\nu\bar{\nu}]} \]

(7)

Then autoparallel curves and geodesics, obeying inequivalent differential equations in general, coincide

\[ \ddot{Y}^{\nu} + \Gamma_{\mu e}^{\nu} \dot{Y}^{\mu} \dot{Y}^{e} = 0 \leftrightarrow \ddot{X}^{\nu} + \gamma_{\{\mu\nu\}}^{e} \dot{X}^{\mu} \dot{X}^{e} = 0 \]

(8)

**Action densities**

We will restrict the discussion to minimal connections in the following, because torsion (contorsion) fields become dynamically dependent on matter fields,
through their coupling to gravity. This is of course particularly so in supergravity theories where gravitinos give rise to torsion

\[ T_{[\mu\nu]} \propto \sum_i \overline{\psi}_\mu^i \gamma^a \psi^i_\nu \] (9)

In eq. (9) \( \psi^i_\mu = 1 \cdots N \) denote N irreducible spin 3/2 fields, obeying in \( d = 4 \) dimensions a Majorana condition \( C \gamma_0 (\psi^i_\mu)^\dagger = \psi^i_\mu \).

Let us consider a class of action densities of the Brans-Dicke type \([1]\)

\[ s = e Q R + e \mathcal{L}_{\text{matter}} \]

\[ Q (x) \rightarrow \begin{cases} 1 & 1 \\ 16\pi G_N & 16\pi l_P^2 \end{cases} \quad \text{for} \quad d \rightarrow 4 \]

for in the long range classical limit

\[ e = \det (e_{\mu}^\alpha) \quad l_P = 1.62 \times 10^{-20} \text{fm} \]

In eq. (10) \( Q \) is thought to be a quadratic function of a set of real scalar fields \( Q = \frac{1}{2} C^{\alpha\beta} \phi_\alpha (x) \phi_\beta (x) \) (11)

A nontrivial set of scalars \( \phi_\alpha \) is necessary to generate simultaneously and spontaneously curvature as well as the Newton constant.

Infinitesimal variation of \( s \) with respect to \( e \) or \( g \) then yields

\[ \delta s = e (\delta g^{\mu\nu}) s_{\mu\nu} \quad s_{\mu\nu} = \begin{cases} [D_\mu D_\nu - g_{\mu\nu} D^2] Q \\ + Q E_{\mu\nu} + \frac{1}{2} \partial^\text{matter}_{\mu\nu} \end{cases} \]

(12)

\[ E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \]

In eq. (12) covariant derivatives relate to the minimal (metric) connection. \( E_{\mu\nu}, R_{\mu\nu} \) denote Einstein and Ricci tensor respectively and \( R \) the curvature scalar. The Riemann curvature tensor takes its conventional form
\[ R^\mu_{\nu \sigma \tau} = -g^{\mu \nu'} \left( \gamma_{\{\mu'\tau\}}; \alpha g^{\alpha \beta} \gamma_{\{\sigma \nu\}}; \beta - \left[ \sigma \leftrightarrow \tau \right] \right) \]  
\[ - \partial_\tau \gamma_{\{\sigma \nu\}}; \mu' + \partial_\sigma \gamma_{\{\tau \nu\}}; \mu' \]  
\text{(13)}

It is instructive to retain the characteristic part \( \chi \) of the Riemann curvature tensor upon dropping terms containing (quadratically) single space time derivatives

\[ \chi^\mu_{\nu \sigma \tau} = \frac{1}{2} g^{\mu \nu'} \left( \partial_\tau \partial_\nu g_{\sigma \mu'} - \partial_\tau \partial_\mu' g_{\sigma \nu} \right) \]  
\[ - \partial_\sigma \partial_\nu g_{\tau \mu'} + \partial_\sigma \partial_\mu' g_{\tau \nu} \]  
\text{(14)}

For definiteness we exhibit the structure of the Ricci tensor and the curvature scalar explicitly

\[ R^\nu_{\nu \sigma \tau} = \left\{ - g^{\alpha' \beta'} g^{\alpha \beta} \left( \gamma_{\{\alpha' \tau\}}; \alpha \gamma_{\{\beta' \nu\}}; \beta - \gamma_{\{\alpha' \beta'\}}; \alpha \gamma_{\{\tau \nu\}}; \beta \right) \right\} \]  
\[ + g^{\alpha \beta} \left( \partial_\tau \gamma_{\{\alpha \nu\}}; \beta - \partial_\alpha \gamma_{\{\tau \nu\}}; \beta \right) \]  
\text{(15)}

The second order differential operator \( D^\nu_{\nu \tau} \) is understood to by symmetrized with respect to the index pair \( \nu' \tau' \).

Finally the curvature scalar becomes
\[ R = \left\{ \begin{array}{c}
- g^{\alpha\beta'} g^{\alpha''\beta''} g^{\alpha\beta} \\
\quad \left( \gamma^{\{\alpha'\alpha''\} ; \alpha} \gamma^{\{\beta'\beta''\} ; \beta} \\
- \gamma^{\{\alpha'\beta''\} ; \alpha} \gamma^{\{\alpha''\beta'\} ; \beta} \\
+ g^{\alpha\beta} g^{\alpha'\beta'} \\
\quad \left( \partial_{\alpha'} \gamma^{\{\alpha'\beta\} ; \beta} - \partial_{\alpha} \gamma^{\{\alpha'\beta\} ; \beta} \right) \end{array} \right\} \] (16)

\[ \chi = g^{\alpha\beta} g^{\alpha'\beta'} \left( \partial_{\alpha'} \partial_{\beta'} g^{\alpha\beta} - \partial_{\alpha'} \partial_{\alpha} g^{\beta\beta'} \right) = D^{\nu'\tau'} g^{\nu'\tau'} \]

\[ D^{\nu'\tau'} = \left( g^{\nu'\tau'} g^{\alpha\beta} - g^{\nu'\alpha} g^{\tau'\beta} \right) \partial_{\alpha} \partial_{\beta} \]

We give also the form of the action density \( s \) containing only first derivatives in quadratic form modulo a total divergence

\[ s = \left\{ \begin{array}{c}
e Q g^{\alpha\beta} \\
\quad \left( - \gamma^{\{\alpha\sigma\} \gamma^{\sigma\beta} ; \beta} + \gamma^{\{\alpha\beta\} \gamma^{\sigma\beta} ; \beta} \right) \\
+ e \left( \partial_{\alpha} Q \right) \left( g^{\alpha\sigma} \gamma^{\alpha\beta} ; \beta} - g^{\alpha\beta} \gamma^{\beta\beta} ; \beta} \right) \\
- \partial_{\alpha} \left[ e Q \left( g^{\alpha\sigma} \gamma^{\alpha\beta} ; \beta} - g^{\alpha\beta} \gamma^{\beta\beta} ; \beta} \right) \right] \end{array} \right\} \] (17)

We note the form of the second order differential operator corresponding to \( E_{\nu\tau} \)

\[ E_{\nu\tau}^{\nu'\tau'} \left( g ; \partial_{\nu} \right) = D_{\nu\tau}^{\nu'\tau'} - \frac{1}{2} g_{\nu\tau} D^{\nu'\tau'} \]

\[ E_{\nu\tau}^{\nu'\tau'} = \frac{1}{2} \left( \begin{array}{c}
\left( \delta_{\nu}^{\nu'} \delta_{\tau}^{\tau'} - g_{\nu\tau} g_{\nu'\tau'} \right) g^{\alpha\beta} \partial_{\alpha} \partial_{\beta} \\
\quad + g_{\nu'\tau'} \partial_{\nu} \partial_{\tau} \\
- \delta_{\nu}^{\nu'} g_{\tau'\alpha} \partial_{\tau} \partial_{\alpha} - \delta_{\tau}^{\tau'} g_{\nu'\alpha} \partial_{\nu} \partial_{\alpha} \\
\quad + g_{\nu\tau} g_{\nu'\beta} \partial_{\alpha} \partial_{\beta} \end{array} \right) \] (18)

The characteristic propagation cone is obtained replacing the partial derivatives in the expression for \( E_{\nu\tau}^{\nu'\tau'} \) in eq. (18) by the directional vector \( \partial_{\nu} \rightarrow \xi_{\nu} \):
\[ E_{\nu^\prime \tau^\prime}(g; \xi) = \frac{1}{2} \begin{pmatrix} (\delta^\nu_\nu \delta^\tau^\tau - g_{\nu\tau} g^{\nu^\prime \tau^\prime}) \xi^2 \\ + g^{\nu^\prime \tau^\prime} \xi_\nu \xi_\tau + g_{\nu\tau} \xi^\nu \xi^\tau' \\ - \delta^\nu_\nu \xi_\tau \xi^\tau' - \delta^\tau_\tau \xi_\nu \xi^\nu' \end{pmatrix} \]  

(19)

Upon symmetrization of \( E_{\nu^\prime \tau^\prime} \) with respect to \( \nu^\prime \tau^\prime \) we verify the transverse projection condition

\[ \xi^\nu E_{\nu^\prime \tau^\prime} = \frac{1}{2} \begin{pmatrix} \frac{1}{2} (\xi^\nu \delta^\tau^\tau + \xi^\tau' \delta^\nu_\nu - 2 \xi_\tau g^{\nu^\prime \tau^\prime}) \xi^2 \\ + g^{\nu^\prime \tau^\prime} \xi_\tau \xi^2 + \xi_\tau \xi^\nu \xi^\tau' \\ - \xi_\tau \xi^\nu \xi^\tau' - \frac{1}{2} (\delta^\tau^\tau \xi^\nu + \delta^\nu_\nu \xi^\tau') \xi^2 \end{pmatrix} \rightarrow \xi^\nu E_{\nu^\prime \tau^\prime} = 0 \]  

(20)

This allows to illustrate the quantized nature of configuration space variables, given the quantized nature of the gravitational field, as is evident from the equations of motion derived from eq. (12) (actio = reactio)

\[ [D_\mu D_\nu - g_{\mu\nu} D^2] Q + Q E_{\mu\nu} = - \frac{1}{2} \partial_{\mu\nu} \text{matter} \]  

(12)

The characteristic light cone variables \( \xi \) in eqs. (19, 20) appears as tangent vector to the light like geodesics and obey the geodesic equation (eq. 8)

\[ \dot{\xi}^\nu + \gamma_{\{\mu\nu\}}^\nu \xi^\mu \xi^\nu = 0 ; \ g_{\mu\nu} \xi^\mu \xi^\nu = 0 ; \ \dot{X}^\mu = \xi^\mu \]  

(22)

While the dot in eq. (22) denotes the derivative with respect to a classical path variable \( \tau_{cl} \) along the sought light like geodesic, the relations in eqs. (21, 22) reveal the configuration space variables \( \xi \) and \( X \) as quantized operators, depending together with \( \tau_{cl} \) on an exhaustive set of classical base space variables \( y_{cl} \).
Thus the configuration space variables $X$, necessarily the arguments of quantized fields in order to safeguard locality, describe a quantum deformation of the base space, spanned by $y_{\text{cl}}$, as is illustrated by target space and base space in string theory. The associated space geometry is sometimes referred to as noncommutative $[2]$

These considerations are simple and straightforward, yet the dimensionality of both target and base space is by no means clearly ten to eleven for the former and two for the latter. As a general warning let me state here, that the noncommutativity of target space variables $X^\mu \leftrightarrow y^{\nu}_{\text{cl}}$ is encoded in the dynamics, which influences causally dependent variables and the latter does not allow in general for any nontrivial algebraic reduction.

Looking back at eqs. (10) and (11) we show the effect of a Weyl transformation with a constant scale function $f$:

$$
e^a_\mu \rightarrow f e^a_\mu ; \quad R \rightarrow f^{-2} R = \overline{R}$$

$$e Q R \rightarrow \overline{e} \overline{Q} \overline{R} ; \quad \overline{Q} = f^{d-2} Q$$

(23)

When the rescaling function $f$ is chosen $x$-dependent in order to eliminate the factor $Q (\varphi)$ in the expression for the action density $s$ in eq. (10) the quadratic dependence of $s$ including the kinetic terms in $\mathcal{L}_{\text{matter}}$ is lost.

In the flat space limit and $d \rightarrow 4$ as indicated in eq. (10) and neglecting further gravitational effects in the remaining matter interactions we reduce eq. (12) to the (approximate) Einstein equations focusing on QCD in flat space-time:

$$E_{\mu \nu} = -8\pi G_N \theta_{\mu \nu} ; \quad \text{QCD} : \quad \langle \Omega | \theta_{\mu \nu} | \Omega \rangle \varepsilon g_{\mu \nu} \approx \varepsilon \eta_{\mu \nu}$$

$$g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu} \approx \eta_{\mu \nu} ; \quad \varepsilon \rightarrow \varepsilon_{\text{QCD}}$$

$$-\varepsilon_{\text{QCD}} = (0.23 \text{ GeV})^4 \div (0.28 \text{ GeV})^4$$

(24)

In eq. (23) $\eta_{\mu \nu}$ denotes the conventional Lorentz metric, normalized to $\eta_{00} = 1$

The negative vacuum energy density (or positive vacuum pressure) was deduced first by Shifman, Vainshtain and Zakharov $[3]$ from charmonium sum rules $[4], [5]$

The dog that did not bark
The content of eq. (24) brings about a paradox, since restricting our view to $d = 1 + 3$ dimensions space time becomes a homogeneous space or as the mathematical language says 'inherits a cosmological term':

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = - 8 \pi G_N \varepsilon g_{\mu \nu} \rightarrow$$

$$R_{\mu \nu} = \left( 8 \pi G_N \varepsilon / \left( \frac{1}{2} d - 1 \right) \right) g_{\mu \nu} \rightarrow$$

$$R_{[\alpha \mu]} ; [\beta \nu] = \sigma a^{-2} \left( g_{\alpha \beta} g_{\mu \nu} - g_{\alpha \nu} g_{\beta \mu} \right) ; \; \sigma = \pm 1$$

$$R_{\mu \nu} = (d - 1) \sigma a^{-2} g_{\mu \nu} \rightarrow$$

$$\sigma a^{-2} = \frac{1}{8 \pi G_N \varepsilon} \rightarrow$$

$$\left( d - 1 \right) \left( \frac{1}{2} d - 1 \right)$$

$$\sigma = \varepsilon / |\varepsilon| ; \; \sigma_{QCD} = - 1$$

$$d \rightarrow 4 : \; a_{QCD} = \frac{\sqrt{\frac{3}{8 \pi}} \frac{m_{pl}}{\varepsilon_{QCD}}}{|\varepsilon_{QCD}|^{1/4}}$$

The negative sign $\sigma_{QCD} = - 1$ selects Anti de Sitter space

$$AdS\;4 = SO\left(2, 3\right) / SO\left(1, 3\right)$$

as homogeneous four dimensional space time.

The positive sign would select de Sitter space

$$dS\;4 = SO\left(1, 4\right) / SO\left(4\right)$$

The curvature radius becomes

$$a_{QCD} = 13.5\;km \left( \frac{0.25\;GeV}{|\varepsilon_{QCD}|^{1/4}} \right)^2$$ (26)
New York would not be the same.
There is no doubt that the above curvature is not present in the (almost) flat four dimensions, and an interesting paradox arises. Yet if the above deduced curvature in the form

\[ (\sigma a^{-2})_{QCD} \]  \hspace{1cm} (27)

does not curve the four known dimensions it should then curve some other space.

**cosmological acceleration and negative pressure**
If we interpret the large scale cosmological *acceleration* found by the large z supernova Ia redshift surveys \(^{[6]}\), \(^{[7]}\) as due to a cosmological constant (constant in time) then it follows

\[ \sigma_{cos} = +1 \leftrightarrow \sigma_{QCD} = -1 \]

\[ a_{cos} = \frac{c}{\sqrt{\Omega_{cos} H_0}} = 2.2 \times 10^{23} \text{ km} \left( \frac{0.7}{\Omega_{cos}} \right)^{1/2} / h_{50} \]  \hspace{1cm} (28)

\[ h_{50} = \frac{H_0}{50 \text{ km/sec/Mpsec}} \]

In eq. (28) \( H_0 \) denotes the Hubble expansion parameter at present and \( \Omega_{cos} = \varepsilon / \varepsilon_{cr} \) where the critical energy density is

\[ \varepsilon_{cr} = 3 \left( H_0 \right)^2 / (8 \pi G N) \]  \hspace{1cm} (29)

The large curvature radius \( a_{cos} \) as deduced in eq. (28) while acceptable phenomenologically together with the notable change of sign \( \sigma_{cos} = +1 \) does not seem to resolve the paradoxical situation with respect to the cosmological term, which we summarize in table 1:
origin $\sigma \sim |\text{energy density}|^{1/4} \sim \text{curvature radius (km)}$

|        | $\sim$ | $|\text{energy density}|^{1/4}$ | $\sim$ | curvature radius (km) |
|--------|-------|-------------------------------|-------|------------------------|
| QCD    | -1    | 0.25 GeV                      |       | 13.5                   |
| cosmos | +1    | $2 \times 10^{-3}$ eV        |       | $2.2 \times 10^{23}$   |

Table 1: Survey of candidate cosmological curvatures.

2 + 3 QED and QCD

We restrict this section to the structure of the QCD Lagrangean and the ensuing two central anomalies, insofar as these are relevant for the study of vacuum energy density and potential induced cosmological terms. For a review see e.g. refs. [8], [9], [10]. The Lagrangean (flat space) density with Fermi gauge parameter $\eta$ is of the form

$$L_{QCD} = \left\{ \begin{array}{c}
\frac{1}{2} C^A C^A + 2 \text{tr } \partial \mu \bar{\sigma} D_{\mu} c \\
2 \eta g^2 \\
\frac{1}{4} F_{\mu\nu}^A F^{\mu\nu A} + \frac{e}{g^2} i \gamma^\mu D_{\mu} q - m_q \bar{q} q \\
4 g^2
\end{array} \right\}$$ \hspace{1cm} (30)

$D_{\mu} = \partial_{\mu} + W_{\mu}; \ W_{\mu} = i D^A V^A_{\mu}$

$C^A = \partial_{\mu} V^A_{\mu}; \ c = i D^A c^A; \ \bar{c} = -i D^{A*} c$

$F_{\mu\nu}^A = \partial_{\nu} V^A_{\mu} - \partial_{\mu} V^A_{\nu} - f_{ABC} V^B_{\nu} V^C_{\mu}$

In eq. (30) $D_A$ denotes the fundamental representation of $S U(N_c)$ with $N_c \to 3$ and the conventional normalization.
The central anomalies become 'trace' or conformal and singlet axial current anomalies in flat $d = 4$ space time

\[ \theta^\mu = \frac{b_1}{8 \pi^2} \left( -\frac{1}{4} F_{\mu \nu}^A F^{\mu \nu} A \right)_{\text{ren.gr.inv.}} + m_q \overline{q} q \]

\[ j_\mu^5 = \sum_q \overline{q} \gamma_\mu \gamma_5^R q ; \quad \gamma_5^R = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3 \rightarrow \gamma_5 \]

\[ \partial^\mu j_\mu^5 = \frac{1}{2 n_{fl}} \left( \frac{1}{4} F_{\mu \nu}^A \tilde{F}^{\mu \nu} A \right)_{\text{ren.gr.inv.}} + 2 m_q \overline{q} i \gamma_5 q \]

\[ \left( \frac{1}{4} F_{\mu \nu}^A \tilde{F}^{\mu \nu} A \right)_{\text{ren.gr.inv.}} \equiv \text{ch}_2 ( F ) \frac{1}{8 \pi^2} \]

In eq. (32) $n_{fl}$ denotes the number of quark flavors and $b_1$ refers to the (perturbative) rescaling function

\[ b ( \kappa ) = -\beta ( g ) / g = \sum_1^\infty b_n \kappa^n ; \quad \kappa = \frac{1}{16 \pi^2} g^2 \]

\[ b_1 = \frac{11}{3} C_2 ( G ) - \frac{2}{3} n_{fl} \]

\[ b_2 = \frac{44}{3} ( C_2 ( G ) )^2 - 2 C_2 ( q ) n_{fl} - \frac{10}{3} C_2 ( G ) n_{fl} \]

\[ \ldots \]

In eq. (33) $C_2 ( G )$, $C_2 ( q )$ denote the second Casimir invariant for the adjoint $( G )$ and the defining $( q )$ representations of the gauge group respectively.
For $G = SUN_c$ and $N_c \to 3$ these quantities become

$$C_2 ( G ) \to N_c \to 3 ; \quad C_2 ( q ) \to \frac{N_c^2 - 1}{2 N_c} \to \frac{4}{3}$$

$$b_1 \to 11 - \frac{2}{3} n_{fl} , \quad b_2 \to 102 - \frac{38}{3} n_{fl}$$

The 'trace' $\eta_{\mu\nu}$ in eq. (32) relates to the 'base' condensates and to the vacuum energy density $\varepsilon_{QCD} = - p_0$ defined in eq. (24)

$$\langle \Omega | \frac{1}{4} F_{\mu\nu}^A F^{\mu\nu A} | \Omega \rangle = B^2 = \left\{ \begin{array}{c} 0.250 \text{ GeV}^4 \quad [1] \\ 0.125 \text{ GeV}^4 \quad [2] \end{array} \right.$$ (35)

$$\langle \Omega | ( - \pi u ) | \Omega \rangle = f^2 \pi M_0 \simeq (0.24 \text{ GeV})^3$$

$$M_0 = \frac{m^2}{m_u + m_d} \simeq 1.3 \text{ GeV}$$

The quantity $M_0$ in eq. (35) is 'scale' dependent and corresponds to that scale here where $m_u + m_d = 14 \text{ MeV}$.

Then the spontaneous QCD parameters $\varepsilon_{QCD} = - p_0$ (eqs. (24), (35)) become restricting QCD to the three light flavors u, d, s

$$\langle \Omega | \vartheta_{\mu\nu}^{QCD} | \Omega \rangle = \varepsilon_{QCD} \eta_{\mu\nu} ; \quad - \varepsilon_{QCD} = p_0$$

$$p_0 = \frac{9}{32 \pi^2} B^2 + \sim f^2 \pi \left( \frac{1}{2} m^2_{\pi} + m^2_K \right) = (0.23 \div 0.28 \text{ GeV})^4$$

(36)

The range of values for the vacuum pressure $p_0$ in eq. (36) corresponds to the lower and higher value for the spontaneous parameter $B^2$ given in eq. (35).
This led to the attempt to map out the thermodynamically the phase structure of QCD in nucleus-nucleus, $p - \overline{p}$ and $e^+ - e^-$ collisions in ref. [5], extrapolating all systems to zero chemical potentials. The Gibbs potentials $g_T = p_T / T$ for the hadronic ($g^H$) and the plasma ($g^{QGP}$) phase, where hadronic interactions in the hadron phase are replaced by a representative set of noninteracting resonances, while in the plasma phase $u, d, s$ flavored quarks are endowed with masses and further interactions are neglected are shown in figure 1.

Figure 1: Gibbs potentials as a function of the temperature for three values of the gluon condensate in the ground state, and for free quark flavors $u, d, s$ and gluons.

The determination of the temperature characterizing various collision systems are shown in figure 2.

Equating the pressure for the two phases yields for the critical temperature at zero chemical potentials the estimate

$$T_{cr} = 194 \pm 18 MeV$$  \hspace{1cm} (37)

In the error for $T_{cr}$ in eq. (37) the systematic errors both in theoretical approximations and in the thermal description of the scattering dynamics are included.
Figure 2: Temperature at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus+nucleus, hadron+hadron and lepton+lepton collisions. We demand for the fits confidence level > 10%.

4 Completing the standard model

In this section I concentrate on spontaneous gauge symmetry breaking [11], [12], [13]. The new feature is the appearance of a multiplet of scalar elementary fields. The minimal single doublet represents a quaternion, two doublets an octonion:

\[
\begin{pmatrix}
\nu_e \\
 e^-
\end{pmatrix} \rightarrow \begin{pmatrix}
\varphi^0 \\
\varphi^-
\end{pmatrix} \rightarrow \Phi = \begin{pmatrix}
\varphi^0 & - (\varphi^-)^* \\
\varphi^- & (\varphi^0)^*
\end{pmatrix}
\]

\[
\sqrt{2} \Phi = \begin{pmatrix}
\Phi_0 - i \Phi_3 - \Phi_2 - i \Phi_1 \\
\Phi_2 - i \Phi_1 & \Phi_0 + i \Phi_3
\end{pmatrix} = \Phi_0 + \frac{1}{i} \vec{\sigma} \vec{\Phi} = \xi
\]
The invariant of the electroweak gauge group $SU_2 \times U_1$ exhibits the enlarged $SU_2 \times SU'_2$ invariance.

The scalar self interaction generates after $\varepsilon_{QCD}$ the next higher spontaneous effect in mass scale. The former is responsible for most of the nucleon mass ($\sim 900$ out of $938$ MeV), while the latter generates mass for all quark and charged lepton flavors, the W and Z gauge bosons and minimally the one remaining scalar Higgs boson:

$$V(\Phi) = -\frac{1}{2} \mu^2 \Phi^2 + \frac{1}{8} \lambda (\Phi^2)^2 \ (\text{constant})$$

$$\Phi_0 = v + H(x)$$

$$v = \sqrt{2} \mu / \sqrt{\lambda} = (\sqrt{2} G_F)^{-1/2} \approx 246.2 \text{ GeV}$$

$$V(v + H) = -\frac{1}{2} \mu^2 v^2 + \frac{1}{8} \lambda v^4 + \frac{1}{2} \left(-\mu^2 + \frac{3}{2} \lambda v^2\right) H^2$$

$$\to m_H^2 = 2 \mu^2 = \lambda v^2$$

$$V(v) = -\frac{1}{8} m_H^2 v^2 \ (\text{constant})$$

$$\to -\varepsilon_{\Phi_{\text{vac}}} = (104.3 \text{ GeV})^4 (m_H / 125 \text{ GeV})^2$$

Upon neglecting the free constant in the definition of the $\Phi$ potential we obtain a negative induced curvature as in the case of QCD with a correspondingly smaller curvature radius

$$\left(\frac{\varepsilon_{\Phi_{\text{vac}}}}{\varepsilon_{QCD}}\right)^{1/2} \sim (417)^2 (m_H / 125 \text{ GeV})^2$$

$$a_{\Phi} 4-d = 417^{-2} a_{QCD} \sim \frac{1}{2} cm \text{ for } m_H = 125 \text{ GeV}$$

"... they prove the impossible and disprove the obvious ... ".
### Table 2: Survey of candidate cosmological curvatures.

| Origin  | \( \sigma \) | Energy Density \( |\sigma|^{1/4} \) | Curvature Radius (km) |
|---------|---------------|-------------------------------|-----------------------|
| QCD     | -1            | 0.25 GeV                      | 13.5                  |
| Cosmos  | +1            | \( 2 \times 10^{-3} \) eV     | 2.2 \( 10^{23} \)     |
| El. Weak| -1            | 105 GeV                       | 0.5 \( 10^{-5} \)     |

5 + 6 Unification of charges and susy

Susy and vacuum energy density

As in the preceding section we focus on the aspect of spontaneous vacuum energy density in the context of initially flat 4-d rigid supersymmetry. For a recent overview see e.g. [14].

We consider the ’once local’ form of the algebra of \( N \) susy charges [15]

\[
\left\{ Q^i_\alpha, Q^k_\beta \right\} = \delta^{ik} \sigma^\nu_{\alpha\beta} P_\nu \ ; \ i, k = 1 \cdots N \rightarrow \\
\left\{ j^i_{\mu\alpha}(x), Q^k_\beta \right\} = \delta^{ik} \sigma^\nu_{\alpha\beta} \vartheta_{\mu\nu}(x) \tag{42}
\]

In eq. (42) \( j^i_{\mu\alpha}(x) \) denote the \( N \) local and conserved spinorial susy currents, whereas \( \vartheta_{\mu\nu}(x) \) is the (conserved) energy momentum density operator, in \( d = 1 + 3 \) flat dimensions.

An eventual spontaneous vacuum energy density of the form defined in eq. (24)

\[
susy : \langle \Omega | \vartheta_{\mu\nu} | \Omega \rangle = \varepsilon_{susy} g_{\mu\nu} \approx \varepsilon_{susy} \eta_{\mu\nu} \tag{43}
\]

in the susy environment leads not only to the spontaneous breakdown of all \( N \) supersymmetries but to a most notable change of sign, relative to the case encountered in QCD and in the Higgs effect.
\[ \varepsilon_{\text{susy}} \geq 0 \iff \langle \Omega \mid \{ j^{i \alpha}(x), j^{*k \beta}(y) \} \mid \Omega \rangle = \delta^{ik} \Gamma_{\mu \nu \theta}(z) \sigma^\theta_{\alpha \beta} ; z = x - y \]

\[ \Gamma_{\mu \nu \theta}(z) = (2\pi)^{-3} \int d^4 q \varepsilon(q^0) \tilde{\Gamma}_{\mu \nu \theta}(q, \varepsilon_{\text{susy}}) \]

\[ \tilde{\Gamma}_{\mu \nu \theta}(q, \varepsilon_{\text{susy}}) = \varepsilon_{\text{susy}} \delta(q^2) \gamma_{\mu \nu \theta}(q) + \Delta \tilde{\Gamma}_{\mu \nu \theta}(q, \varepsilon_{\text{susy}}) \]

\[ \gamma_{\mu \nu \theta}(q) = \eta_{\mu \theta} q_{\nu} + \eta_{\nu \theta} q_{\mu} - \eta_{\mu \nu} q_{\theta} \]

In the second last relation in eq. (44) the leading spectral singularity \((\propto \delta(q^2))\) arises for \(\varepsilon_{\text{susy}} > 0\) from \(N\) goldstino modes, as a consequence of the (universal) spontaneous breaking of all \(N\) supersymmetries. \(\Delta \tilde{\Gamma}\) denotes all residual contribution to the susy current current correlation function, less singular for \(q^2 \to 0\).

The condition \(\varepsilon_{\text{susy}} \geq 0\) follows from the Källen-Lehmann representation for a local anticommutator and the positivity of \(j j^*\) products.

We note the Christoffel-symbol like structure of \(\gamma_{\mu \nu \theta}\) as displayed in eq. (44), which is not accidental.

With respect to primordial cosmological inflation and the apparent present day remnant as shown in table 2 the sign of \(\varepsilon_{\text{susy}}\) is the same.

**Unification of charges** \(SU3_c \times SU2_L \times U1_Y\)

The three running coupling constants \(g_k(\mu^2) ; k = 1,2,3\)

\[ \bar{g}_3 \leftrightarrow SU3_c ; \bar{g}_2 \leftrightarrow SU2_L ; \bar{g}_1 \leftrightarrow U1_Y \]

defined in the \(\overline{\text{MS}}\) scheme and normalized to a common Casimir operator of one chiral fermion family are defined as

\[ \alpha_1 = \frac{5}{3} \tan^2 \bar{\theta}_W \alpha_2 ; \alpha_{2(3)} = \bar{g}^2_{2(3)} / (4\pi) \]

\[ \alpha_3 = \frac{5}{3} \left( 1 / \cos^2 \bar{\theta}_W \right) \alpha_{em} ; \bar{\alpha}_3 = \left( 1 / \sin^2 \bar{\theta}_W \right) \alpha_{em} \]

We thus pick up the three coupling constants at the scale \(\mu = m_Z\)
at \( m_Z : \sin^2 \vartheta_W = 0.231078 \); \( (\alpha_{em})^{-1} = 127.934(27) \)

\( (\alpha_1)^{-1} = 59.02 (2) \); \( (\alpha_2)^{-1} = 29.56 (1) \); \( (\alpha_3)^{-1} = 8.47 (14) \) \( (46) \)

The rescaling equations are

\[
\frac{d}{d \tau} \kappa_k = \frac{1}{4\pi} \kappa_k ; \quad \kappa_k = -\frac{b_k}{\tau} \left( \kappa_{1,2,3} \right) \kappa_k
\]

\( (47) \)

\[
t = \log \left( \frac{\mu^2}{m_Z^2} \right) ; \quad b_k = -\beta_k / \mu
\]

\[
b_k = B_k \kappa_k + \cdots
\]

We only illustrate the coupling constant evolution to one loop and ignore the threshold effects above \( m_Z \) caused by the interestingly large value of the top quark mass (\( \simeq 174 \text{ GeV} \)):

\[
(\alpha_k)^{-1} \simeq (\alpha_{k(0)})^{-1} + \frac{1}{4\pi} B_k t
\]

\( (48) \)

\[
B_1 = -\frac{3}{5} \sum \mathcal{Y}^2 \left\{ \frac{2}{3} \text{ chiral fermions} \right\} \left\{ \frac{1}{3} \text{ complex scalars} \right\} \]

\[
B_2 = 7 \frac{1}{3} (6)_{\text{susy}} - \sum_{\text{doublets}} \left\{ \frac{1}{3} \text{ chiral fermions} \right\} \left\{ \frac{1}{6} \text{ complex scalars} \right\} \]

\[
B_3 = 11 (9)_{\text{susy}} - \sum_{3 \& \bar{3}} \left\{ \frac{1}{3} \text{ chiral fermions} \right\} \left\{ \frac{1}{6} \text{ complex scalars} \right\}
\]

In eq. (48) \( \alpha_{k(0)} \) denote the values of the coupling constants at scale \( \mu = m_Z \) as given in eq. (46).

We list the three constants \( B_k \) for the standard model (SM) and the minimal supersymmetric standard model (MSSM) in table 3 below (abbreviating doublets by dblts).
Table 3: Matter fields in SM and MSSM.

The three pairs of coupling constants $k \leftrightarrow l$ with $k \neq l$ cross at three respective crossing points

$$t_{kl} = \frac{4\pi}{B_l - B_k} \left( \left( \alpha_{k(0)} \right)^{-1} - \left( \alpha_{l(0)} \right)^{-1} \right)$$

The evolution of the (inverse) coupling constants according to the SM and the MSSM [17], [18] is shown in figures 3 (a) and (b) respectively as a function of $t = \log \left( \frac{\mu^2}{m_Z^2} \right)$

Figure 3: Coupling constant evolution according to SM (a) and MSSM (b) field contents.
The scales $M_{kl}$ corresponding to the crossing points $t_{kl}$, $k,l = 12, 13, 23$ for the SM and the almost perfect unification scale $M_{susy}$ for MSSM are shown in table 4.

| $M_{12}$ (GeV) | $M_{13}$ (GeV) | $M_{23}$ (GeV) | $M_{susy}$ (GeV) |
|----------------|----------------|----------------|------------------|
| $1.05 \times 10^{13}$ | $2.4 \times 10^{14}$ | $9 \times 10^{17}$ | $2.1 \times 10^{16}$ |

Table 4: SM crossing scales and MSSM unification scale.

7 + 8 SO10 and ($\nu$, $N$) neutrino sector

We focus here on the completion of the fermion families to three 16 (spinor) representations of SO10 (spin10).

The two decay chains of SO10 \cite{19}, \cite{20} proceed along SU5 and $SU_4 \times SU_2 \times SU_2$\cite{21} as largest simple subgroups

$$SU_4 \times SU_2 \times SU_2$$

$$SU_3 \times U_1 \times SU_2$$

The group $SU_4$ in eq. \cite{50} is the largest subgroup of SO10 realized in a vectorlike fashion in the 16 spin10 representation attributed to one family of (spin 1/2) fermions in the left chiral 4-d spinor basis:
\[ \frac{1}{2} \left( 1 + \gamma \frac{L}{5} \right) f \rightarrow f \hat{\gamma} \]

\[ f \hat{\gamma} : 16 = \left( \begin{array}{cccc} 4 & 2 & 1 & + \\ SU4_c & SU2_L & SU2_R & SU4_c \end{array} \right) \left( \begin{array}{cccc} \mathbb{T} & 1 & 2 \\ SU2_L & SU2_R & SU2_L & SU2_R \end{array} \right) \]

\[ \downarrow SU2_L \left( \begin{array}{ccc} u^1 & u^2 & u^3 & \nu_e \\ \hat{u}^1 & \hat{u}^2 & \hat{u}^3 \end{array} \right) \begin{array}{cc} N_e & e^+ \\ \hat{e}^+ & e^+ \end{array} \uparrow SU2_L \]

\[ \begin{array}{ccc} \nu_e & \hat{u}^1 & \hat{u}^2 \\ e^- & \hat{d}^1 & \hat{d}^2 \end{array} \]

\[ \leftrightarrow \quad \leftrightarrow \]

\[ SU4_c \quad SU4_c \]

(51)

In the \( SU5 \times U1_N \) basis the 16 representation decomposes into

\[ (1, 5) + (10, 1) + (\overline{5}, -3) \]

\[ U1_N \quad SU5 \]

\[ \begin{array}{c|c|c} 5 & N_e & 1 \\ \hline 1 & \hat{u}^1 \hat{u}^2 \hat{u}^3 & 10 \\ \hline -3 & e^- & \overline{5} \end{array} \]

\[ \begin{array}{c|c} \nu_e & \hat{d}^1 \hat{d}^2 \hat{d}^3 \\ e^+ & \hat{e}^+ \end{array} \]

(52)

and:

\[ \begin{array}{c|c|c|c} u \nu_e & c \nu_\mu & t \nu_\tau \\ d \ e^- & s \mu^- & b \tau^- \end{array} \]
The integer eigenvalues of the $U_1$ generator in eq. (52) are normalized to

$$\sum_{\{16\}} (U_1) = 50$$

(53)

In order to achieve the same normalization as for $I_{3w}$ we would substitute

$$U_1 \rightarrow J_N = \sqrt{\frac{2}{50}} U_1 = \frac{1}{5} U_1$$

(54)

$(\nu, N)$ mass matrix

The fields $N_{e,\mu,\tau}$ are singlets under SU5 and a fortiori under the SM gauge group. They can participate however in the so extended SM through Yukawa interactions with the (or several) doublet scalars

$$-L_Y (N \ell \varphi^*) = g_{IJN}^{(\nu,N)} N_{iJ} \left[ \nu \gamma^J (\varphi^0)^* + \ell \gamma^J (\varphi^-)^* \right] + h.c.$$

(55)

$I, J = 1, 2, 3$

The $u, c, t$-like Yukawa couplings defined in eq. (55) induce through the Higgs scalar vacuum expected value (eq. 40) a Dirac mass matrix, pairing $\nu \leftrightarrow N$, of the form

$$(\mu = \frac{1}{\sqrt{2}} v g^{(\nu,N)})_{IJ}$$

(56)

The above notation of $\mu$ as the part of the neutrino mass matrix induced by Yukawa couplings of the form given in eq. (56) shall not be confused with the scale parameter $\mu$ used in the previous section.

In addition an unrestricted Majorana mass term is compatible with the SM gauge group, of the form

$$-L_M = \frac{1}{2} M_{IJ} N_{iJ} N_{iJ} + h.c.; \quad M_{IJ} = M_{JI}$$

(57)

Apart from the symmetry condition for $M$ in eq. (57) the three by three matrices $\mu$ and $M$ are (a priori) complex arbitrary and thus represent $18 + 12$ real parameters. They form the symmetric restricted six by six $(\nu, N)$ mass matrix.
\[ M = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \]

In eq. (58) \( \mu^T \) denotes the transpose matrix of \( \mu \).

B - L invariance, with \( L(\mathcal{N}) = -1 \), is violated by the Majorana mass term at some large scale.

For details of mass and mixing structure see e.g. ref. [22].

The 0 entry in \( M \) gives rise to one exact mass relation, valid through renormalization. This involves the 3 nonnegative eigenvalues of the hermitian three by three matrix \( \mu \mu^\dagger \) and the six physical neutrino masses of the full (hermitian) mass matrix \( \mathcal{M} \mu^\dagger \). The latter are assumed to contain three light and three heavy masses:

\[
\mu \mu^\dagger \rightarrow \mu_{1,2,3} \geq 0
\]

\[
\mathcal{M} \mu^\dagger \rightarrow m_{1,2,3} , M_{1,2,3} \geq 0 ; \quad M_J \gg m_K
\]

The relation which is known by the name sea-saw \cite{23}, \cite{24} becomes

\[
\text{Det}_{6x6} \mathcal{M} = (\text{Det}_{3x3} \mu)^2 \rightarrow m_{1/3} M_{1/3} = (\mu_{1/3})^2
\]

\[
m_{1/3} = (m_1 m_2 m_3)^{1/3} ; \quad M_{1/3} = (M_1 M_2 M_3)^{1/3}
\]

A generic estimate can be obtained assuming at unification scale the SO10 inspired relation of equal mass matrices for the \( (\nu, N) \) Yukawa couplings and for u,c,t quarks and values of the light neutrino masses characteristic of the mass square differences compatible with the solar neutrino deficit and the atmospheric neutrino anomaly \cite{25}, \cite{26}, \cite{27}

\[
\overline{\mu}_{\text{unif}} = \overline{\mu}_{u,c,t} \rightarrow \mu_{1/3} \simeq \frac{1}{3} (m_u m_c m_t)^{1/3} \simeq 0.4 \text{ GeV}
\]

\[
m_1 \simeq m_2 \simeq 10^{-3} \text{ eV} ; \quad m_3 \simeq 5 \times 10^{-2} \text{ eV}
\]

\[
\rightarrow m_{1/3} \simeq 3.6 \times 10^{-3} \text{ eV} \quad \Rightarrow \quad M_{1/3} \simeq 4.4 \times 10^{11} \text{ GeV}
\]
The order of magnitude estimate in eq. (61) \( M^{1/3} \approx 4.4 \times 10^{11} \text{GeV} \) reveals a high mass scale which serves through inverse powers as a protection of lepton number(s) as well as B-L approximate conservation at low energy. Nevertheless this mass scale is several orders of magnitude smaller than the unification scale obtained from the MSSM in table II. Thus the successful neutrino oscillation interpretation of the associated solar and atmospheric anomalies \[25, 26, 27\] is, since this workshop took place, augmented by the hint of the observation of a light Higgs boson with a mass \( \approx 115 \text{GeV} \) at LEP \[28, 29\] and by the measurement of the anomalous magnetic moment of the muon at BNL \[30, 31\] as yet uncertain but characteristic signatures of small deviations from SM expectations. They all point towards intermediary mass scales between the electroweak scale as characterized by \( v \) or \( m_Z \) and the unification scale for charges, represented within the MSSM by \( M_{\text{susy}} \approx 2 \times 10^{16} \text{GeV} \), bearing on physics beyond the standard model.

One aspect related to spontaneous generation of mass in QCD for (the main mass of) nucleons and through the Higgs effect in the SM or its supersymmetric extensions for all elementary quanta except neutrinos, was the central topic in our discussion of the cosmological constant in sections 1-6 as summarized in table II. This topic has an apparently paradoxical yet important bearing on the unification of charge like and spin like gauge interactions, including gravity among the latter.

I would like to appreciate the outstanding contributions of Alberto Sirlin to the backbones of the perturbative structure of gauge field theory. They were for me guidelines since student years and they serve to substantify that the quest for unification is not a dream but based on physical and logical reality - - for all I know.

9 + 10 Outlook ( quo vadis ? )

1. Spontaneous susy breaking in flat 4-d space time generates vacuum energy density

\[
\langle \Omega | \partial_{\mu \nu} | \Omega \rangle = \varepsilon_{\text{susy}} g_{\mu \nu} \quad \text{with} \quad \varepsilon_{\text{susy}} > 0
\]

This sign is opposite to the corresponding vacuum energy densities in QCD and with respect to the Higgs effect.

2. The metric \( g_{\mu \nu} \) does not react through d=4 curvature to any of the vacuum energy densities \( \varepsilon_{\text{susy}} > 0 \), \( \varepsilon_{\text{QCD,Higgs effect}} < 0 \). Through
the absorption of goldstino modes into gravitinos the mass scale of susy braking is linked to gravitino masses, which at present remain unknown.

3. This absence of 4-d curvature \((AdS_4 \leftrightarrow \varepsilon < 0, \ dS_4 \leftrightarrow \varepsilon > 0)\) indicates indirectly the presence of extra dimensions with unknown curvature scale.

4. All ungauged U1-like symmetries : B, \(L_{e,\mu,\tau}\), B-L \(\cdots\) are broken and protected, except for B, at low energy by the mass of heavy neutrino flavors \(N : M_{1/3} (N) \sim 10^{11} \text{GeV}\).

5. While signatures of the path towards unification (of all gauge and associated symmetries) appear – to me – to be elusive as to the scale of \(\sim 1\) TeV, they hopefully will manifest themselves first in small effects or the absence of expected large ones and this rightly so will demand a definite alertness in spirit and a wide open mind towards consistent speculations.
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