Binding and Off-Mass-Shell Effects in Deep Inelastic Scattering on Deuteron.

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The role of relativistic off-mass-shell kinematics in the deep inelastic scattering on deuteron is analyzed. It is shown that the relativistic impulse approximation reproduces effects from the binding, Fermi motion, and two-nucleon contribution. The nonrelativistic limit of the deuteron structure function is in agreement with the nonrelativistic calculations.

The discovering of the EMC-effect initiated active studying of nuclear effects in the deep inelastic scattering (DIS) on the deuteron from the point of view of QCD and nuclear physics. These investigations allow one to rely that nuclear properties can be expressed in terms of the QCD parameters \[1\]. On the other hand, successful description of deep inelastic scattering on deuteron can give us information about the neutron structure function \[2\]. All these tasks demand consistent calculations of nuclear effects in DIS.

Due to recent success in the description of the deuteron low energy characteristics in the relativistic formalism \[4,5\] one hopes to construct in the framework of this formalism a consistent picture of different processes on the deuteron. In the present paper, we analyze the deep inelastic scattering on the deuteron in the framework of the approach based on the Bethe-Salpeter formalism.

I. BASIC APPROXIMATIONS

Most of the relativistic field theory approaches to DIS on nuclei are based on the following basic approximations \[3,7\]:

- Ladder approximation for the deuteron vertex function;
- Disregard of the interference terms in the square of the scattering amplitude;
- The same representation for the nucleon amplitude via scalar structure functions for both a bound and a free nucleon.

The first approximation makes it possible to solve the Bethe-Salpeter equation in the mesonic nucleon theory \[4,6\]. In this approximation, the interaction between nucleons is represented as an infinite ladder of one-mesonic exchanges. It has been shown that this approximation allows one to describe the basic properties of deuteron such as its mass, binding energy and magnetic and quadruple momenta simultaneously. \[4,5\] This approximation works well when the relative energy of constituents is small. Presupposing this in the deuteron we keep this approximation.

The second approximation permits one to consider the DIS amplitude as an incoherent sum of squared terms corresponding to the scattering on individual constituents. This approximation possibly works well in the Bjorken limit. Calculation at the finite \(Q^2\) and large and low \(x\) requires a detailed analysis of the interference terms. A simple analysis
of the Compton amplitude shows that most of the interference terms are suppressed as an additional power of $\frac{1}{q^2}$ [8]. Nevertheless, this approximation needs a more explicit investigation.

The third approximation was analyzed in several papers which considered additional structure functions in the representation for the hadron tensor. The analysis motivated by the Sullivan model [10–12] has shown that these additional structure functions can be neglected, and off-mass-shell nucleon amplitudes can be replaced by the on-mass-shell one.

$$W_{\mu\nu}(P,q) = \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right)F_1(x) + \frac{1}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)F_2(x). \quad (1)$$

In our opinion this approximation is not so clear and even if we could represent the hadron tensor through two structure functions, both of them are not identical to the on-mass-shell one. In the paper [8], we have shown that this approximation leads to the representation for the deuteron hadron tensor via convolution of the nucleon structure function and distribution function, satisfying the baryon sum rule:

$$\int \frac{d^4k}{(2\pi)^4} f(M_D, k) = 2$$

It follows from this relation that if we neglect the off-mass-shell effects and identify off-shell and on-shell nucleon structure functions, the ratio of the deuteron and isoscalar nucleon structure function becomes equal to 1 when $x \to 0$. However, application of this approximation gives no EMC-like behavior of this ratio, what means it has no nuclear effects but the Fermi motion. An attempt to introduce the dependence on the relative energy $k_0$ conserving on-mass-shell form of the hadron tensor and structure functions leads to changing of the ratio at $x = 0$ and thus to contradiction with the baryon sum rule.

We show in this paper how this contradiction can be removed by the expansion of the convolution formula near the nucleon mass-shell. Moreover inclusion of the off-mass-shell effects for the bound nucleon structure function reproduces the EMC-effect for the deuteron and the two-nucleon contributions.

II. COMPTON AMPLITUDE.

Following Bethe-Salpeter based approach [8] we can write the forward Compton amplitude on deuteron as

$$T_{\mu\nu}^D(P, q) = \int \frac{d^4k}{(2\pi)^4} \Gamma(P, k)\mathcal{G}_{6\mu\nu}(P, k, q)\Gamma(P, k). \quad (2)$$

Here $\Gamma(P, k)$ is the Bethe-Salpeter vertex function defined as

$$\Gamma(P, k) = S^{-1}(P, k) \int d^4X e^{ikx} e^{iPX} \langle 0|T(\psi(X - x/2)\psi(X + x/2))|D\rangle$$

and satisfying the homogeneous Bethe-Salpeter equation [3]:

$$\Gamma^S(P, k) = -\int \frac{d^4k'}{(2\pi)^4} \mathcal{G}_4(P, k, k')S^{(2)}(P, k')\Gamma^S(P, k'). \quad (3)$$
The normalization condition is fixed as the deuteron electromagnetic current at zero momentum transfer:

$$\langle D|J_\mu(0)|D\rangle = 2iP_\mu.$$ (4)

The $G_4(P,k,k')$ and $G_{6\mu
u}(P,k,q)$ denote the irreducible two-nucleon Green functions. The second is two-nucleon Green function with the insertion of a T-product of the nucleon electromagnetic current operators. Formally, the irreducible kernel for the Bethe-Salpeter equation can be defined via the exact two-nucleon Green function $G_4(P,k,k')$:

$$G_4(P;k,k') = S^{(2)}(P,k) \left( (2\pi)^4 \delta(k-k') + \sum_{n\geq 1} \frac{1}{n!} \int \frac{d^4k_1}{(2\pi)^4} \cdots \frac{d^4k_n}{(2\pi)^4} G_4(P;k,k_1) \cdots G_4(P;k_n,k') S^{(2)}(P,k') \right)$$ (5)

where $S^{(2)}(P,k)$ is a direct product of the two nucleon propagators. The irreducible two-nucleon Green function with the insertion of the T-product of the nucleon electromagnetic current operators can be defined as:

$$G_{6\mu
u}(q,P,k',k) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} G_4^{-1}(P;k,k_1) G_6^{\mu\nu}(q,P,k_1,k_2) G_4^{-1}(P;k_2,k')$$ (6)

where

$$G_6^{\mu\nu}(q,P,k',k) = i \int d^4xd^4y'd^4y d^4Yd^4Y' e^{-iky+ik'y'} e^{-iP(Y-Y')} e^{-iP(Y+Y')} \langle 0|T(\bar{\psi}(Y+\frac{y}{2})\psi(Y-\frac{y}{2})J_\mu(x)J_\nu(0)\bar{\psi}(Y'+\frac{y'}{2})\psi(Y'-\frac{y'}{2}))|0 \rangle.$$ (7)

The diagrams on fig.1. schematically present different contributions to the forward Compton scattering amplitude on the deuteron. The term a) corresponds to the relativistic impulse approximation. In the ladder approximation, the term f) contains only mesonic exchange current contribution. Both of them define the convolution approximation. The term f) affects the behavior of the structure function only at small $x$. Thus, in the relativistic convolution approximation there are no mesonic corrections ensuring the EMC-behavior.

Analyzing terms behind the convolution approximation, we can note that their imaginary part contains a nucleon propagator with high momentum. So it has an additional power of $\frac{1}{Q^2}$. We cannot say that about the terms b) and e). But we can assume that these terms can be suppressed because of high nucleon relative energy. In any case, these terms correspond to the scattering on the deuteron as on a whole and we presuppose that they will be visible at high $x$ or finite $Q^2$. Thus if we neglect the terms e) and b), we get only the relativistic impulse approximation to describe the EMC-behavior of the deuteron and nucleon structure function ratio.

![Diagrams](image)
Taking only the term in (2) corresponding to relativistic impulse approximation we get the deuteron hadron tensor in the form:

\[
W_{\mu\nu}^{D}(P, q) = \int \frac{d^4k}{(2\pi)^4} W_{\mu\nu}^{N} \left( \frac{P}{2} + k, q \right) f^{N}(P, k) + \int \frac{d^4k}{(2\pi)^4} W_{\mu\nu}^{\overline{N}} \left( \frac{P}{2} + k, q \right) f^{\overline{N}}(P, k)
\]

where \( k \) is the relative momentum of nucleons inside the deuteron. The distribution functions \( f^{N}(P, k) \) have the following form:

\[
f^{N}(P, k) = \frac{im^2}{2\varepsilon} \left( \frac{M_D}{2} + k_0 - E \right)^2 \left[ \frac{\Phi_{++}^2(P, k)}{-k_0 - (E - M_D/2)} + \frac{\Phi_{--}^2(P, k)}{-k_0 + (E + M_D/2)} \right]
\]

\[
f^{\overline{N}}(P, k) = \frac{im^2}{2\varepsilon} \left( \frac{M_D}{2} + k_0 + E \right)^2 \left[ \frac{\Phi_{+-}^2(P, k)}{-k_0 + (E + M_D/2)} + \frac{\Phi_{-+}^2(P, k)}{-k_0 - (E - M_D/2)} \right].
\]

The functions \( \Phi \) are expressed via the BS-vertex function \( \Gamma(P, k) \):

\[
\begin{align*}
\Phi_{++}^2(M_D, k) &= \sum_{s} \Gamma_{\alpha\beta}^S(M_D, k) \sum_{s} u_{\alpha}^s(k) \bar{\tau}_{\alpha}^s(k) \sum_{s} u_{\beta}^s(-k) \bar{\tau}_{\beta}^s(-k) \Gamma_{\delta\gamma}^S(M_D, k) \\
\Phi_{+-}^2(M_D, k) &= -\sum_{s} \Gamma_{\alpha\beta}^S(M_D, k) \sum_{s} v_{\alpha}^s(k) \bar{\tau}_{\alpha}^s(k) \sum_{s} v_{\beta}^s(-k) \bar{\tau}_{\beta}^s(-k) \Gamma_{\delta\gamma}^S(M_D, k) \\
\Phi_{-+}^2(M_D, k) &= -\sum_{s} \Gamma_{\alpha\beta}^S(M_D, k) \sum_{s} v_{\alpha}^s(-k) \bar{\tau}_{\alpha}^s(-k) \sum_{s} v_{\beta}^s(k) \bar{\tau}_{\beta}^s(k) \Gamma_{\delta\gamma}^S(M_D, k) \\
\Phi_{--}^2(M_D, k) &= \sum_{s} \Gamma_{\alpha\beta}^S(M_D, k) \sum_{s} v_{\alpha}^s(-k) \bar{\tau}_{\alpha}^s(-k) \sum_{s} v_{\beta}^s(-k) \bar{\tau}_{\beta}^s(-k) \Gamma_{\delta\gamma}^S(M_D, k)
\end{align*}
\]

The elementary nucleon (antinucleon) amplitude \( W_{\mu\nu}^{N(\overline{N})} \left( \frac{P}{2} + k, q \right) \) differs from the amplitude of a physical nucleon. It appears in the dependence on the time component of the nucleon relative momentum, and in the difference between the total momentum the square of this nucleon and square of its mass \( m \). Also, it cannot be represented via the two structure functions only. To expand expression (8) in terms of the on-mass-shell nucleon hadron tensor, we presuppose the leading contribution of the amplitude near the mass-shell of the nucleon interacting with the photon and approximate expression (8) by the residue at \( \frac{M_D}{2} + k_0 = E \) for the nucleon contribution part and at \( \frac{M_D}{2} + k_0 = -E \) for the antinucleon:

\[
W_{\mu\nu}^{D}(M_D, q) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{E_3(M_D - 2E)} \left\{ \Phi_{++}^2(M_D, k)W_{\mu\nu}^{N}(k, q) + \frac{(M_D - 2E)\partial}{\partial k_0} \left( W_{\mu\nu}^{N}(k, q)\Phi_{++}^2(M_D, k) \right)_{k_0 = k_0^N} + \frac{(M_D - 2E)^2}{M_D^2} \left[ \Phi_{+-}^2(M_D, k)W_{\mu\nu}^{N}(k, q) + \Phi_{-+}^2(M_D, k)W_{\mu\nu}^{\overline{N}}(k, q) + M_D\frac{\partial}{\partial k_0} \left( W_{\mu\nu}^{N}(k, q)\Phi_{++}^2(M_D, k) \right)_{k_0 = k_0^N} + M_D^2 \frac{\partial}{\partial k_0} \left( W_{\mu\nu}^{\overline{N}}(k, q)\Phi_{++}^2(M_D, k) \right)_{k_0 = k_0^N} + \frac{M_D^2}{(M_D + 2E)} \frac{\partial}{\partial k_0} \left( W_{\mu\nu}^{\overline{N}}(k, q)\Phi_{+-}^2(M_D, k) \right)_{k_0 = k_0^N} + \frac{M_D^2}{(M_D + 2E)^2} \Phi_{-+}^2(M_D, k)W_{\mu\nu}^{\overline{N}}(k, q) \right] \right\}
\]
As it follows from the energy conservation law, the construction \( M_D - 2E \) is the kinematic expression of a binding potential. Thus, we can conclude that the contribution of the antinucleon states is suppressed as an additional power of the potential. The contribution of the BS-vertex function derivative over the relative energy in the relativistic impulse approximation obviously is connected with the two-nucleon contribution.

We can observe here that expression (10) contains only a free nucleon hadron tensor and its derivatives at the nucleon mass-shell. Thus, neglecting the derivatives of additional structure functions in the representation of the hadron tensor, we can get the structure function \( F_2^D \) in the Bjorken limit:

\[
\lim_{Q^2 \to \infty} g_{\mu\nu} W_{\mu\nu}^D(P, q) = -\frac{1}{x} F_2(x).
\]

\[
\frac{\partial}{\partial k_0} \lim_{Q^2 \to \infty} g_{\mu\nu} W_{\mu\nu}^D(P, q) = \left[ \frac{1}{x^2} F_2(x) - \frac{1}{x} \frac{d}{dx} F_2(x) \right] \left( \frac{\partial x}{\partial k_0} \right)_{k_0 = k_0^N}.
\]

Neglecting terms of \( (M_D - 2E)^2 \) order we obtain following expression for the deuteron structure function:

\[
F_2^D(x_D) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^3(M_D - 2E)^2} \left\{ F_2^N(x_N) \left( 1 - \frac{E + k_3}{M_D} \right) \Phi^2(M_D, k) - \frac{M_D - 2E}{M_D} \frac{dF_2^N(x_N)}{dx_N} \Phi^2(M_D, k) + F_2^N(x_N) \frac{E - k_3}{M_D} (M_D - 2E) \frac{\partial}{\partial k_0} \Phi^2(M_D, k) \right\} k_0 = E - \frac{M_D}{2}.
\]

**IV. NONRELATIVISTIC LIMIT**

To compare our result with the nonrelativistic calculations, we expand \( E \) in powers of \( \frac{p^2}{m^2} \) in (11) and discard the pure relativistic contributions connected with the two-nucleon effects. This gives us the following expression for the structure function \( F_2^D \):

\[
F_2^D(x_D) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^3(M_D - 2E)^2} \left\{ F_2^N(x_N) \left( 1 - \frac{k_3}{m} \right) \Psi^2(k) - \frac{-T + \epsilon}{m} \frac{dF_2^N(x_N)}{dx_N} \Psi^2(k) \right\}
= \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^2M_D(M_D - 2E)^2} \left\{ \Phi^2(M_D, k) \right\} k_0 = E - \frac{M_D}{2}
\]

\( T = 2E - 2m \) is the kinetic energy of nucleons, and \( \epsilon = M - 2m \) is the binding energy.

Here, we introduce an analog of the nonrelativistic wave function \( \Psi^2(k) \):

\[
\Psi^2(k) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^2M_D(M_D - 2E)^2} \left\{ \Phi^2(M_D, k) \right\} k_0 = E - \frac{M_D}{2}
\]

with the usual normalization condition

\[
\int \frac{d^3k}{(2\pi)^3} \Psi^2(k) = 1.
\]

Comparing (12) with the nonrelativistic calculations [9], we can conclude that we have got an analog of the nonrelativistic impulse approximation with interaction corrections. Distinction from calculation [9] consists only in convolution with the distribution function in the second term containing the derivative of the nucleon structure function.


V. NUMERICAL RESULTS

Numerical calculations of (12) are presented on fig.2. by the continuous curve. The calculation was performed with the Bethe-Salpeter vertex function calculated in the approach with the separable form of interaction [13]. The parametrization for the isoscalar nucleon structure function is taken from [9].

At small $x \approx 0$ the ratio of the deuteron and nucleon structure functions equals 1. It is connected with the neglect of the two-nucleon contribution expressed by the derivative with respect to relative energy of the Bethe-Salpeter vertex function, and the relativistic mesonic exchange currents which affect the ratio at small $x$.

At intermediate $x$ ($0.1 \leq x \leq 0.6$) the deuteron structure function is suppressed in comparison with the nucleon one. This suppression is a corollary of the off-mass-shell behavior of the nucleon structure function, which is ensured by the derivative of the nucleon structure function in expression (11). This result is in qualitative agreement with calculation including the binding by the mesonic corrections [14], which is presented by the dashed curve on figure 2.

The relativistic Fermi motion provides a rapid growth of the ratio at large $x \geq 0.6$. The ratio obtained from (11) has a softer growth in this region of $x$ and smaller EMC-effect at intermediate $x$. It is connected with the distribution over the whole region of $x$ of the contribution of the structure function derivative.

VI. CONCLUSION.

In this paper we have analyzed the connexion between the off-mass-shell kinematics of nucleons inside the deuteron and the binding effects in the deep inelastic scattering on
the deuteron. It is shown that assuming nucleons near the mass-shell we can express
the deuteron structure function $F_2^D$ in terms of a free nucleon structure function and its
derivatives. The contribution of $P$, $S^-$, $D^-$ waves proves to be suppressed as a second
power of nucleon-nucleon potential in comparison with the $S^{++}$, $D^{++}$ wave contribution.

Comparison of the obtained results with the previous nonrelativistic calculations shows
that the influence of mesonic degrees of freedom in the nonrelativistic models, leading
to binding effects, can be reproduced in the relativistic impulse approximation in the
framework of the Bethe-Salpeter formalism. So we can conclude that these corrections
are to some extent nonrelativistic parametrization of the off-mass-shell effects.

As it has been assumed in this paper, relativistic mesonic corrections do not influence
the EMC behavior of the ratio of the deuteron and nucleon structure functions ratio.
Possibly, they can significantly influence at large and low $x$. Further investigations of
terms beyond the convolution approximation can help one answer this question and give
a description of the deuteron structure at finite $Q^2$ and at $x > 1$.

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