Research Article

3D TDOA/AOA Localization in MIMO Passive Radar with Transmitter and Receiver Position Errors

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This paper deals with the problem of determining the position of a single target from time difference of arrival (TDOA) and angle of arrival (AOA) measurements using multitransmitter multireceiver passive radar system with widely separated antennas. A practically motivating scenario where the transmitter and receiver positions are contaminated by errors is addressed. First, the reduction in localization accuracy due to the presence of transmitter and receiver position errors is derived through the Cramér-Rao lower bound (CRLB) analysis. Then, a novel algebraic localization algorithm based on weighted least squares minimization is proposed that takes the transmitter and receiver position errors into consideration to reduce the estimation error. The proposed solution is shown theoretically to reach the CRLB even when the transmitter and receiver positions have errors. Simulation results also verify the theoretical developments and the performance improvement of the proposed solution over existing algorithms.

1. Introduction

Passive radar detects and tracks potential targets by exploiting noncooperative transmitters as their sources of radar transmission [1–4]. Dispensing with the need for a dedicated transmitter makes passive radar inherently low cost and hence attractive for a broad range of applications. Recently, by taking the advantage of the spatial diversity from widely separated antenna configuration, passive radar with multiple separated receivers and noncooperative transmitters, also known as multiple-input multiple-output (MIMO) passive radar, has received growing attention due to its enormous potential in improving the detection and localization performances [5–8].

Target localization is one of the salient issues in the MIMO passive radar field. The time difference of arrival (TDOA) measurement, which usually comes from the crosscorrelation (CC) processing between the reference signal and the reflected target echo [9], is a common measurement used to determine the target position. And over the years, many localization methods have been developed based on TDOA measurements [10–14]. However, the TDOA-based localization with terrestrial transmitters and receivers suffers from poor accuracy in estimating the target height [11] and is overly dependent on the TDOA measurement accuracy. To overcome the fundamental flaw of TDOA-based localization, as suggested in [11], angle of arrival (AOA) measurement of the reflected target echo, which can be determined by subspace-based estimators [15], can be jointly utilized [16]. However, unlike TD-based localization which has been extensively studied [10–14], the hybrid TDOA/AOA localization is potentially more challenging due to the higher nonlinearity between the desired target position and TDOA/AOA measurements, and less effort has been devoted to hybrid TDOA/AOA localization.

Recently, borrowing the two-stage weighted least squares (TSWLS) idea originally proposed for radiation source localization [17], A. Noroozii et al. [18] developed a TSWLS algebraic solution for target localization with a MIMO passive radar using TDOA and AOA measurements. The performance analysis indicates that it can achieve the Cramer-Rao lower bound (CRLB) under mild noise conditions. By using a different way to linearize the TDOA and AOA measurement equations, R. Amiri et al. [19] explored a
different algebraic localization algorithm based on weighted least squares (WLS) minimization, which is also shown theoretically and numerically to achieve the CRLB. Unlike Noroozi1’s method which requires two WLS stages, Amiri’s method determines the target position in only one WLS stage. Nevertheless, the above studies are based on the ideal assumption that the transmitter and receiver positions are exactly known, which is certainly not practical. Actually, the transmitter and receiver positions need to be estimated before the localization of an unknown target can be achieved, and the transmitter and receiver positions cannot be precisely known, especially when the antennas are mounted on moving platforms [20–22]. GPS signal is sheltered, or the transmitters are extremely noncooperative (like the hostile radar radiation whose position could usually only roughly determined by electronic reconnaissance techniques [23]). The performance degradation created by transmitter and receiver position errors has been known for a while in the TDOA-based localization with MIMO passive radar [24]. And it was shown that the transmitter and receiver position errors can significantly degrade the localization performance of MIMO passive radars. Consequently, the errors in transmitter and receiver positions need to be taken into consideration in practical applications during the design of localization algorithms in MIMO passive radars.

In this paper, we address the target localization problem from TDOA and AOA measurements in the presence of transmitter and receiver position errors. We evaluate how much degradation the target localization accuracy is with respect to the amount of transmitter and receiver position errors by deriving the CRLB in the presence of transmitter and receiver position errors and examining the increase in CRLB due to the transmitter and receiver position errors. Then, a novel closed-form solution is proposed for the localization problem to reduce the performance degradation created by the transmitter and receiver position errors. The proposed solution is shown analytically, under some mild approximations, to reach the CRLB, even in the presence of transmitter and receiver position errors. Some numerical simulations will be conducted to support the theoretical development of the proposed solution.

1.1. Notations. This paper involves numerous symbols. By convention, uppercase and lowercase bold fonts denote matrices and vectors, respectively. The operations $(\cdot)^{T}$ and $(\cdot)^{-1}$ represent transpose and inverse, respectively. $I_k$ is the $k	imes k$ identity matrix, and $0_k$ is a $k\times 1$ vector with all elements equal to zero. diag[a] is a diagonal matrix with the elements of a on the main diagonal. $E{(\cdot)^{-1}}$, tr$(\cdot)^{-1}$, and $||\cdot||$ stand for the statistical expectation, trace operation, and 2-norm, respectively. The superscript $(\cdot)^{T}$ is the true value of the noisy vector. $[a]_{k}$ and $[a]_{j:k}$ are indexing expressions for a column vector $a$, with $[a]_{k}$ representing the $k$th element of $a$ and $[a]_{j:k}$ representing the $j$th to the $k$th elements of $a$. $[A]_{k}$, $[A]_{j:k}$, $[A]_{j,k}$, $[A]_{j,k:i}$, $[A]_{j:k,i}$, $[A]_{j:k,i:l}$ are indexing expressions for a matrix $A$, with $[A]_{k}$ representing the $k$th column of matrix $A$, $[A]_{j:k}$ representing the $k$th row of matrix $A$, $[A]_{j,k}$ representing a submatrix including the elements of $A$ indexed by $j:k$ in the first dimension and $i:l$ in the second dimension, $[A]_{j,k}$ and $[A]_{j:k}$ include all subscripts in one of the dimensions but use the vector $j:k$ to index in the other dimension.

The paper is organized as follows. Section 2 presents the localization scenario and introduces the symbols involved. Section 3 evaluates the CRLB in the presence of transmitter and receiver position errors. Section 4 presents a novel proposed algebraic solution as well as a theoretical accuracy analysis. Section 5 contains the simulation results to verify the localization performance of the proposed solution, and Section 6 is the conclusion.

2. Problem Formulation

Consider a MIMO passive radar like the one illustrated in Figure 1. This MIMO passive radar system is comprised of $N$ geographically separated receivers with positions $s_{r,n} = [x_{r,n}^{o}, y_{r,n}^{o}, z_{r,n}^{o}]^{T}$, $n = 1, 2, \ldots, N$, and $M$ noncooperative transmitters with positions $s_{m}^{o} = [x_{m}^{o}, y_{m}^{o}, z_{m}^{o}]^{T}$, $m = 1, 2, \ldots, M$. Let $\mathbf{u}^{o} = [x_{o}^{o}, y_{o}^{o}, z_{o}^{o}]^{T}$ be unknown position coordinates of the target. The transmitters send out a set of waveforms, which are then reflected by potential targets. The receivers sense the direct path signals from the transmitters as well as the reflected signals from the target. After some processing, each receiver extracts TDOAs and one AOA pair (an elevation angle and an azimuth angle), which are then sent to the fusion center to determine the target position.

In a practical localization scenario, the actual positions of all transmitters and receivers are not known, and the available positions are as follows:

$$\begin{align*}
    s_{m} &= s_{o,m} + \Delta s_{m}, \quad m = 1, 2, \ldots, M, \\
    s_{r,n} &= s_{o,r,n} + \Delta s_{r,n}, \quad n = 1, 2, \ldots, N,
\end{align*}
$$

(1)

where $s_{o,m}$ and $s_{o,r,n}$ represent the available (noisy) positions of transmitter $m$ and receiver $n$, and $\Delta s_{m}$ and $\Delta s_{r,n}$ represent the corresponding position errors. For notation simplicity, we put the positions of the transmitters and receivers into a $3(M+N)$-by-1 column vector as

$$\beta = \beta^o + \Delta \beta,$$

(2)

where $\beta = [s_{1}^{T}, s_{2}^{T}]^{T}$ with $s_{i} = [s_{i,1}^{o}, \ldots, s_{i,M}^{o}]^{T}$ and $\Delta s_{i} = [\Delta s_{i,1}^{o}, \ldots, \Delta s_{i,N}^{o}]^{T}$ denotes the noisy transmitter and receiver position vector, $\beta^o = [\beta_{1}^{o}, \ldots, \beta_{M}^{o}]^{T}$ with $\beta_{m}^{o} = [s_{m}^{o,1}, \ldots, s_{m}^{o,M}]^{T}$ and $\Delta s_{m}^{o} = [\Delta s_{m,1}^{o}, \ldots, \Delta s_{m,N}^{o}]^{T}$ stands for the actual transmitter and receiver position vector, and $\Delta \beta = [\Delta s_{1}^{o}, \Delta s_{2}^{o}]^{T}$ with $\Delta s_{m} = [\Delta s_{m,1}^{o}, \ldots, \Delta s_{m,N}^{o}]^{T}$ and $\Delta s_{i} = [\Delta s_{i,1}^{o}, \ldots, \Delta s_{i,N}^{o}]^{T}$ stands for the transmitter and receiver position errors vector assumed to be zero-mean Gaussian with covariance matrix

$$E[\Delta \beta \Delta \beta^{T}] = Q_{\beta},$$

(3)

where the covariance matrix $Q_{\beta}$ can be determined from theoretical analysis and actual measurement [25].
Based on the above geometry, the range between transmitter m and the target is $R^0_{m} = \|u^o - s^o_{m}\|$, and the baseline distance between transmitter m and receiver n is $R^0_{m,n} = \|s^o_{m} - s^o_{n}\|$. Thus, the time difference between the direct path signal from transmitter m and the corresponding reflected signal arriving at receiver n can be given by

$$\tau_{m,n}^o = \frac{1}{c}(R^0_{m} + R^0_{n} - R^0_{m,n}).$$

(4)

The true AOA pair for receiver n, that is, the elevation angle denoted by $\theta^o_n$ and the azimuth angle denoted by $\phi^o_n$, is, respectively, given by

$$\theta^o_n = \arctan\left(\frac{y^o - y^o_{rn}}{x^o - x^o_{rn}}\right),$$

(5)

$$\phi^o_n = \arctan\left(\frac{z^o - z^o_{rn}}{\sqrt{(x^o - x^o_{rn})^2 + (y^o - y^o_{rn})^2}}\right).$$

Considering the unavoidable measurement noises in reality, the erroneous version of the TDOA and AOA measurements can be expressed as

$$\tau_{m,n} = \tau_{m,n}^o + \Delta \tau_{m,n},$$

$$\theta_n = \theta^o_n + \Delta \theta_n,$$

$$\phi_n = \phi^o_n + \Delta \phi_n,$$

(6)

where $\tau_{m,n}$, $\theta_n$, $\phi_n$ represent the noisy TDOA and AOA measurement, and $\Delta \tau_{m,n}$, $\Delta \theta_n$, $\Delta \phi_n$ are the corresponding measurement noises. For easier manipulation, we put these TDOA and AOA measurements into an $(MN+2N)$-by-1 column vector form as

$$\mathbf{a} = \mathbf{a}^o + \Delta \mathbf{a},$$

(7)

$$\mathbf{a} = [\theta^o, \phi^o, \tau^o]^T,$$

$$\tau = [\tau_1^o, \tau_2^o, \ldots, \tau_M^o]^T,$$

$$\tau_m = [\tau_{m,1}, \tau_{m,2}, \ldots, \tau_{m,N}]^T,$$

$$\theta = [\theta_1, \theta_2, \ldots, \theta_N]^T,$$

and $\phi = [\phi_1, \phi_2, \ldots, \phi_N]^T$ stands for the noisy TDOA and AOA measurement vector, $\mathbf{a}^o = [(\theta^o)^T, (\phi^o)^T, (\tau^o)^T]^T$ with $\tau^o = [(\tau_1^o)^T, (\tau_2^o)^T, \ldots, (\tau_M^o)^T]^T$, $\tau_m^o = [\tau_{m,1}^o, \tau_{m,2}^o, \ldots, \tau_{m,N}^o]^T$, $\theta^o = [\theta_1^o, \theta_2^o, \ldots, \theta_N^o]^T$, and $\phi^o = [\phi_1^o, \phi_2^o, \ldots, \phi_N^o]^T$, stands for the true TDOA and AOA vector, and $\Delta \mathbf{a} = [\Delta \theta^o, \Delta \phi^o, \Delta \tau^o]^T$ with $\Delta \tau = [\Delta \tau_1^o, \Delta \tau_2^o, \ldots, \Delta \tau_M^o]^T$, $\Delta \tau_m = [\Delta \tau_{m,1}, \Delta \tau_{m,2}, \ldots, \Delta \tau_{m,N}]^T$, $\Delta \theta = [\Delta \theta_1, \Delta \theta_2, \ldots, \Delta \theta_N]^T$, and $\Delta \phi = [\Delta \phi_1, \Delta \phi_2, \ldots, \Delta \phi_N]^T$ stands for the TDOA and AOA measurement noise vector assumed to be zero-mean Gaussian with covariance matrix.

$$E[\Delta \mathbf{a}\Delta \mathbf{a}^T] = \mathbf{Q}_\alpha,$$  

(8)

where the covariance matrix $\mathbf{Q}_\alpha$ can be determined from the specific signal conditions [26].

In this work, we are interested in identifying the unknown target position as accurately as possible, using the noisy transmitter/receiver positions and the TDOA/AOA measurements, as well as their error statistical characteristics. Nevertheless, this is a potentially challenging task since the desired target position is highly nonlinear with respect to the TDOA and AOA measurements.

3. CRLB Analysis

CRLB traces out the lowest possible variance of unbiased estimators and is often used as a benchmark for performance evaluation. In this section, we shall characterize the influence of transmitter and receiver position errors on the localization accuracy by deriving a novel CRLB for target position estimation with transmitter/receiver position errors and comparing it with the one without transmitter/receiver position error.

3.1. CRLB with Transmitter and Receiver Position Errors

In addition to TDOA/AOA measurement noises, the presence of transmitter and receiver position errors is included. Therefore, the unknown parameter vector for the CRLB evaluation is $\eta^o = [(\mathbf{a})^T, (\beta^o)^T]^T$, and the observation vector is $\mathbf{z} = [\alpha^T, \beta^T]^T$. As described in Section 2, the TDOA/AOA measurement noise vector $\Delta \mathbf{a}$ and transmitter/receiver position error vector $\Delta \mathbf{a}$ are independent of each other, and zero-mean Gaussian with covariance matrices $\mathbf{Q}_\alpha$ and $\mathbf{Q}_\beta$, respectively. Hence, the logarithm of the joint probability density function of $\mathbf{z}$ under $\eta^o$ is

$$\ln p(\mathbf{z} | \eta^o) = \ln p(\mathbf{z} | \mathbf{a}^o) + \ln p(\mathbf{z} | \beta^o)$$

$$= \kappa - \frac{1}{2} (\mathbf{a} - \mathbf{a}^o)^T \mathbf{Q}_\alpha^{-1} (\mathbf{a} - \mathbf{a}^o)$$

(9)

$$- \frac{1}{2} (\beta - \beta^o)^T \mathbf{Q}_\beta^{-1} (\beta - \beta^o),$$

where $\kappa$ is a constant independent of $\eta^o$. According to the structure of $\eta^o$, the Fisher information matrix (FIM) will be a $(3+3M+3N)$-by-$(3+3M+3N)$ matrix given by...
\[ FIM(\eta^o) = E \left[ \left( \frac{\partial \ln p(z|\eta^o)}{\partial \eta^o} \right)^T \left( \frac{\partial \ln p(z|\eta^o)}{\partial \eta^o} \right) \right], \]
\[ = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}, \]
and the blocks \( X, Y, \) and \( Z \) are given by
\[ X = \left( \frac{\partial \alpha^o}{\partial u^o} \right)^T Q_a^{-1} \left( \frac{\partial \alpha^o}{\partial u^o} \right), \]
\[ Y = \left( \frac{\partial \alpha^o}{\partial u^o} \right)^T Q_a^{-1} \left( \frac{\partial \alpha^o}{\partial \beta^o} \right), \]
\[ Z = \left( \frac{\partial \alpha^o}{\partial \beta^o} \right)^T Q_a^{-1} \left( \frac{\partial \alpha^o}{\partial \beta^o} \right) + Q_b^{-1}, \]
where \( \partial \alpha^o/\partial u^o \) and \( \partial \alpha^o/\partial \beta^o \) are the partial derivatives of the parametric form of \( \alpha^o \) with respect to \( u^o \) and \( \beta^o \), respectively, with their elements given by

\[ \left[ \frac{\partial \alpha^o}{\partial u^o} \right]_{n,1} = -\frac{(y^o - y_{r,n}^o)}{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial u^o} \right]_{n,2} = \frac{(y^o - y_{r,n}^o)}{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial u^o} \right]_{N+n,1} = \frac{-(x^o - x_{r,n}^o)(x^o - z_{r,n}^o)}{(R_{r,n}^o)^2 \sqrt{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial u^o} \right]_{N+n,2} = \frac{-(y^o - y_{r,n}^o)(x^o - z_{r,n}^o)}{(R_{r,n}^o)^2 \sqrt{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial u^o} \right]_{N+n,3} = \frac{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}{(R_{r,n}^o)^2}. \]
\[ \left[ \frac{\partial \alpha^o}{\partial u^o} \right]_{2N+(n-1)N+n+1:3} = \frac{(u^o - s_{r,m}^o)^T}{cR_{r,m}^o} + \frac{(u^o - s_{r,n}^o)^T}{cR_{r,n}^o}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial \beta^o} \right]_{n,3M+3n-2} = -\frac{(y^o - y_{r,n}^o)}{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial \beta^o} \right]_{n,3M+3n-1} = \frac{(x^o - x_{r,n}^o)}{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}, \]
\[ \left[ \frac{\partial \alpha^o}{\partial \beta^o} \right]_{N+n,3M+3n-2} = \frac{-(x^o - x_{r,n}^o)(z^o - z_{r,n}^o)}{(R_{r,n}^o)^2 \sqrt{(x^o - x_{r,n}^o)^2 + (y^o - y_{r,n}^o)^2}}. \]
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We shall characterize analytically the performance degradation from transmitter and receiver position errors by establishing the positive definiteness of the second term in (17), that is, $X^{-1}Y(Z - Y^TX^{-1}Y)^{-1}Y^TX^{-1}$. In order to prove $X^{-1}Y(Z - Y^TX^{-1}Y)^{-1}Y^TX^{-1}$ is positive definite, we only need to prove $Y(Z - Y^TX^{-1}Y)^{-1}Y^T$ is positive definite since $X^{-1}$ is full rank.

Since $X = (\partial a^o/\partial u^o)^T Q_u^{-1} (\partial a^o/\partial u^o)$ is invertible, we can deduce $Q_u^{-1} (\partial a^o/\partial u^o)$ has a full column rank of 3. Moreover, it can be inferred from the expression of $(\partial a^o/\partial u^o)^T$ that $(\partial a^o/\partial u^o)^T$ has a full column rank of $(MN+2N)$. Therefore, utilizing the expression of $Y$ in (11), $Y^T$ has a full column rank.

Next, we proceed to prove $Z - Y^TX^{-1}Y$ is positive definite. From the expressions of $Y$ and $Z$ in (11), we obtain (15) after some mathematical manipulations

$$Z - Y^TX^{-1}Y = \left(\frac{\partial a^o}{\partial \beta^o}\right)^T Q_u^{-1} Q_u \left(\frac{\partial a^o}{\partial u^o}\right) X^{-1} \left(\frac{\partial a^o}{\partial u^o}\right)^T Q_u^{-1} + Q_u^{-1},$$

(15)

where $Q_u^{-1}$ is an obvious positive definite matrix. By using the expression of $X$ in (11) and performing the Cholesky decomposition on $Q_u^{-1}$, that is, $Q_u^{-1} = L_u L_u^T$, we can rewrite the first term on the right side of (15) as

$$Q_u^{-1} - Q_u^{-1} \left(\frac{\partial a^o}{\partial u^o}\right) X^{-1} \left(\frac{\partial a^o}{\partial u^o}\right)^T Q_u^{-1} = L_u \left[I - \Phi(\Phi^T\Phi)^{-1}\Phi^T\right] L_u^T,$$

(16)

where $\Phi = L_u^T (\partial a^o/\partial u^o)$. It is readily to see from (16) that $Q_u^{-1} - Q_u^{-1} \left(\frac{\partial a^o}{\partial u^o}\right) X^{-1} \left(\frac{\partial a^o}{\partial u^o}\right)^T Q_u^{-1}$ is positive semi-definite since $I - \Phi(\Phi^T\Phi)^{-1}\Phi^T$ is a projection matrix. Therefore, $Z - Y^TX^{-1}Y$ in (15) is positive definite as long as $Q_u^{-1} \neq 0$.

Synthesizing the above results, that is, $Y^T$ has a full column rank, and $X^{-1}$ and $Z - Y^TX^{-1}Y$ are positive definite, we can infer that $X^{-1}Y(Z - Y^TX^{-1}Y)^{-1}Y^TX^{-1}$ is a positive definite matrix. That is to say, we can obtain (17) from (14) that

$$\text{CRLB}(u^o) > X^{-1},$$

(17)

where CRLB($u^o$)$>X^{-1}$ represents CRLB($u^o$) - $X^{-1}$ is positive definite. This condition implies $\text{tr}[\text{CRLB}(u^o)] > \text{tr}(X^{-1})$. $\text{tr}[\text{CRLB}(u^o)]$ and $\text{tr}(X^{-1})$ have the physical meaning, that is, the minimum possible mean-
square position errors for the target localization with and without transmitter/receiver position errors, respectively. Thus, it can be concluded that the presence of transmitter and receiver position errors indeed deteriorates the target localization accuracy, at least at the CRLB level.

4. Algorithm Development and Analysis

The degradation of localization accuracy in the presence of transmitter and receiver position errors has been shown in Section 3 through the derivation and analysis of the CRLB. In what follows, to minimize the influence of transmitter and receiver position errors on target localization accuracy, we will proceed to design a novel algebraic localization algorithm for the aforementioned practical localization scenario. After that, a theoretical analysis will be performed to show that the proposed solution achieves the CRLB when satisfying some mild conditions.

4.1. Localization Algorithm. The proposed solution is derived based on transforming the nonlinear TDOA and AOA equations into linear ones, where the target position can be estimated using a simple WLS minimization.

To achieve this, we first rearrange the TDOA equation in (5) as

\[ cT_{m,n}^o + R_{t,m,n}^o - R_{t,n}^o = R_{t,m}^o. \]  

Squaring both sides of (18), and then rearranging it as

\[
2(x_{t,n} - x_{t,m})x^o + 2(y_{t,n} - y_{t,m})y^o + 2(z_{t,n} - z_{t,m})z^o + \left[ \frac{2(cT_{m,n} + R_{t,m,n})}{\sin(\phi_{n})} \right] z^o
\]

\[
= 2(cT_{m,n} + R_{t,m,n}) \frac{z_{t,n}}{\sin(\phi_{n})} + s_{t,n}^T s_{t,n} - s_{t,m}^T s_{t,m} + (cT_{m,n} + R_{t,m,n})^2
\]

\[
+ \left[ -2u^o + \left( \frac{z^o - z_{t,n}}{\sin(\phi_{n})} \frac{s_{t,m} - s_{t,n}}{R_{t,m,n}} \right) \frac{2(cT_{m,n} + R_{t,m,n})}{R_{t,m,n}} \frac{s_{t,m} - s_{t,n}}{R_{t,m,n}} + 2s_{t,n} \right] \Delta s_{t,m}
\]

\[
+ \left[ 2u^o \sin(\phi_{n}) - \left( \frac{z^o - z_{t,n}}{\sin(\phi_{n})} \frac{s_{t,m} - s_{t,n}}{R_{t,m,n}} \right) \frac{2(cT_{m,n} + R_{t,m,n})}{R_{t,m,n}} \frac{s_{t,m} - s_{t,n}}{R_{t,m,n}} + 2s_{t,n} \right] ^T \Delta s_{t,n}
\]

\[
- \frac{2(cT_{m,n} + R_{t,m,n})}{\sin(\phi_{n})} \Delta z_{t,n} - 2\frac{(x_{t,n} - x_{t,m})}{\sin(\phi_{n})} \Delta \phi_{n} - 2R_{t,m,n} \Delta r_{m,n}.
\]

To linearize the AOA equations, rewrite (6) and (7) as

\[
x^o \tan(\theta_{n}^o) - y^o = x_{t,n}^o \tan(\theta_{n}^o) - y_{t,n}^o,
\]

\[
x^o \tan(\theta_{n}^o) \sqrt{1 + \tan^2(\theta_{n}^o)} - z^o = x_{t,n}^o \tan(\theta_{n}^o) \sqrt{1 + \tan^2(\theta_{n}^o)} - z_{t,n}^o.
\]
Substitute $\theta_n^0 = \theta_n - \Delta \theta_n$, $\varphi_n^0 = \varphi_n - \Delta \varphi_n$, and $s_{r,n}^0 = s_{r,n} - \Delta s_{r,n}$ into (25) and (23), expand them in Taylor’s series keeping only terms below second order:

$$\tan(\theta_n)x^0 - y^0 = \tan(\theta_n)x_{r,n} - y_{r,n} + \frac{(x^0 - x_{r,n})}{\cos(\theta_n)}\Delta \theta_n - \tan(\theta_n)\Delta x_{r,n} + \Delta y_{r,n},$$

$$x^0 \tan(\varphi_n)\sqrt{1 + \tan(\varphi_n)^2} - z^0 = \tan(\varphi_n)\sqrt{1 + \tan(\varphi_n)^2}x_{r,n} - z_{r,n} - \tan(\varphi_n)\sqrt{1 + \tan(\varphi_n)^2}\Delta x_{r,n}$$

$$+ \Delta z_{r,n} + \left(\tan(\varphi_n)\frac{\tan(\varphi_n)\tan(\theta_n)}{\cos(\theta_n)^2}\Delta \varphi_n + \left(x^0 - x_{r,n}\right)\frac{\tan(\varphi_n)\tan(\theta_n)}{\cos(\theta_n)^2}\Delta \theta_n + \left(x^0 - x_{r,n}\right)\frac{\sqrt{1 + \tan(\theta_n)^2}}{\cos(\theta_n)^2}\Delta \varphi_n\right)$$

$$= \sqrt{1 + \tan(\theta_n)^2} + \frac{\tan(\varphi_n)\tan(\theta_n)}{\cos(\theta_n)^2}\Delta \theta_n + \frac{\tan(\varphi_n)\tan(\theta_n)}{\cos(\theta_n)^2}\Delta \varphi_n,$$

(24)

Now, collecting (25), (28), and (29) with respect to the $M$ transmitters and $N$ receivers, we can rearrange them in matrix form as

$$[G]_{n_r} = [\tan(\theta_n), -1, 0],$$

$$[G]_{N+n_r} = \left[\tan(\varphi_n)\sqrt{1 + \tan(\theta_n)^2}, 0, -1\right],$$

$$[G]_{2N+(m-1)N+n_r,1} = \left[2(x_{r,n} - x_{r,m}), 2(y_{r,n} - y_{r,m})\right],$$

$$[G]_{2N+(m-1)N+n_r,2} = \frac{2(\tau_{r,m} + R_{r,m,t,n})}{\sin(\varphi_n)},$$

$$[h]_{n_r} = \tan(\theta_n)x_{r,n} - y_{r,n}$$

$$[h]_{N+n_r} = x_{r,n}\tan(\varphi_n)\sqrt{1 + \tan(\theta_n)^2} - z_{r,n}$$

$$[h]_{2N+(m-1)N+n_r} = \frac{2(\tau_{r,m} + R_{r,m,t,n})\frac{\tau_{r,m}}{\sin(\varphi_n)} + (\tau_{r,m} + R_{r,m,t,n})^2}{\sin(\varphi_n)}$$

$$+ s_{r,n}^T s_{r,m} - s_{t,m}^T s_{t,n},$$

for $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$. The error vector $\Delta h$ can be expressed as

$$\Delta h = B\Delta a + D\Delta \beta,$$

(27)

where $B$ and $D$ are $(MN+2N)$-by-$(MN+2N)$ matrix and $(MN+2N)\times(3M+3N)$ matrix, respectively, with their elements given by

$$Gu^0 = h + \Delta h,$$

(25)
\[
[B]_{n,n} = \frac{(x^n - x_{r,n})}{\cos(\theta_n)^2}
\]
\[
[B]_{N+n,n} = \frac{x^n - x_{r,n}}{1 + \tan(\theta_n)^2 \cos(\theta_n)^2}
\]
\[
[B]_{N+n,N+n} = \frac{\sqrt{1 + \tan(\theta_n)^2}}{\cos(\varphi_n)^2}
\]
\[
[B]_{2N+m-1,N+n} = -2(\tau_m + R_{tm,n}) \left( \frac{z^n - z_{r,n}}{\sin(\varphi_n)} \right)
\]
\[
[B]_{2N+m-1,N+n} = -2c R^0_{km}
\]
\[
[D]_{M+3n-2} = -\tan(\theta_n)
\]
\[
[D]_{M+3n-1} = 1
\]
\[
[D]_{M+3n} = -\tan(\varphi_n) \sqrt{1 + \tan(\theta_n)^2}
\]
\[
[D]_{M+3n+1} = 1
\]
\[
[D]_{2N+m-1,N+n} = \left[ -2u^o + \frac{(z^n - z_{r,n})}{\sin(\varphi_n)} \right] \left[ \frac{\left( s_{km} - s_{r,n} \right)}{R_{km,n}} - 2(\tau_m + R_{tm,n}) \left( \frac{s_{km} - s_{r,n}}{R_{km,n}} \right) + 2s_{km} \right]^T
\]
\[
[D]_{2N+m-1,N+n} = 2x^n + \frac{(z^n - z_{r,n})}{\sin(\varphi_n)} \left( x_{r,n} - x_{r,m} \right) \left( \frac{x_{r,n} - x_{r,m}}{R_{km,n}} \right) - 2(\tau_m + R_{tm,n}) \left( \frac{x_{r,n} - x_{r,m}}{R_{km,n}} \right) + 2x_{r,n}
\]
\[
[D]_{2N+m-1,N+n} = 2y^n + \frac{(z^n - z_{r,n})}{\sin(\varphi_n)} \left( y_{r,n} - y_{r,m} \right) \left( \frac{y_{r,n} - y_{r,m}}{R_{km,n}} \right) - 2(\tau_m + R_{tm,n}) \left( \frac{y_{r,n} - y_{r,m}}{R_{km,n}} \right) + 2y_{r,n}
\]
\[
[D]_{2N+m-1,N+n} = 2z^n + \frac{(z^n - z_{r,n})}{\sin(\varphi_n)} \left( z_{r,n} - z_{r,m} \right) \left( \frac{z_{r,n} - z_{r,m}}{R_{km,n}} \right) - 2(\tau_m + R_{tm,n}) \left( \frac{z_{r,n} - z_{r,m}}{R_{km,n}} \right) + 2z_{r,n} - \frac{2(\tau_m + R_{tm,n})}{\sin(\varphi_n)}
\]

for \(m = 1, 2, \ldots, M, n = 1, 2, \ldots, N\), and zeros elsewhere.

Note that (25) is a linear set of equations with respect to the target position \(u^o\), from which the target position \(u^o\) can be estimated using the WLS minimization. The WLS solution of (25), which minimizes the cost function \(E(\Delta h^T W \Delta h)\) with respect to the target position \(u^o\), is given by

\[
u = (G^T W G)^{-1} G^T W h.
\]

where \(W\) is the weighting matrix, and its optimal choice that gives the minimum parameter variance is given by

\[
W = \left[ E(\Delta h \Delta h^T) \right]^{-1}
\]

\[
= \left[ BQ_n B^T + DQ_q D^T \right]^{-1}.
\]

However, as suggested in (30), the computation of \(W\) relies on the unknown target position \(u^o\) via \(B\) and \(D\). Thus, it is unrealistic to evaluate \(W\) directly. To resolve this difficulty, a realistic and workable way is to form a least squares (LS) solution of target position \(u^o\) using (29) with \(W = I_{MN+2N}\). Using this LS solution, we update weighting matrix \(W\) via (33) and employ (32) once again to form a more accurate estimate of the target position \(u^o\).

Rewriting \(u^o\) as the identical equation \(u^o = (G^T W G)^{-1} G^T W h\) and subtracting it from sides of (29) yield

\[
\Delta u = (G^T W G)^{-1} G^T W \Delta h.
\]

Ignoring the second- and higher-order error terms in (31) and taking expectation result in that \(E(\Delta u) = 0\), that is, the target position estimate \(u\) in (29) is asymptotically unbiased. By multiplying (34) by its transpose and taking expectation, we have the covariance matrix of \(u\) as

\[
\text{cov}(u) = (G^T W G)^{-1}.
\]
4.2. Performance Analysis. We shall evaluate the efficiency of the proposed solution by comparing its covariance matrix with the CRLB in (14). By inserting (33) into (35), we have after some mathematical manipulations

\[
\text{cov}(\mathbf{u}) = \left( \tilde{X} - \tilde{Y} \tilde{Z}^{-1} \tilde{Y} \right)^{-1},
\]

where

\[
\tilde{X} = G_a^T Q_a^{-1} G_a, \\
\tilde{Y} = G_a^T Q_a^{-1} G_b, \\
\tilde{Z} = G_b^T Q_a^{-1} G_b + Q_b^{-1},
\]

with

\[
G_a = B^{-1} G, \\
G_b = B^{-1} D.
\]

Comparing (36) with the CRLB in (17), we observe that CRLB(\(\mathbf{u}^o\)) and \(\text{cov}(\mathbf{u})\) have the same structural form. Assuming the transmitter/receiver position errors and the TDOA/AOA measurement noises are sufficiently small, we can derive after tedious algebraic manipulations that

\[
G_a \approx \frac{\partial \mathbf{u}^o}{\partial \mathbf{u}^o}, \\
G_b \approx \frac{\partial \mathbf{u}^o}{\partial \mathbf{p}^o}.
\]

Based on this, it can be inferred that

\[
\text{cov}(\mathbf{u}) \approx \text{CRLB}(\mathbf{u}^o).
\]

In other words, the covariance matrix of the proposed solution accomplishes the CRLB given sufficiently small transmitter/receiver position errors and TDOA/AOA measurement noises.

5. Numerical Examples

This section contains some numerical simulations to evaluate the performance of the proposed solution. The localization scenario is set as shown in Figure 2, where a MIMO passive radar system comprised of \(N = 6\) geographically separated receivers and \(M = 4\) transmitters is deployed to locate a target at position \(\mathbf{u}^o = [100, 200, 100]^T\ m\). The positions of the transmitters and receivers are listed in Table 1. \(MN = 24\) TDOAs and \(N = 6\) AOA pairs are extracted to determine the target position.

In order to simulate a practical localization scenario, zero-mean Gaussian noises with known covariance matrices \(Q_a = d_i a_i^3 T_{MN,w} \delta_1^2 I_{MN}\) and \(Q_b = a_3^2 T_{(MN,N)}\) are added to actual true TDOAs/ AOAs and transmitter/receiver positions, respectively. \(\sigma_t\) represents TDOA measurement noise level, \(\sigma_{a}\) represents AOA measurement noise level, and \(\sigma_i\) represents the transmitter/receiver position error level. The localization accuracy is assessed using root mean squares error (RMSE) defined as

\[
\text{RMSE}(\mathbf{u}) = \sqrt{\frac{1}{L} \sum_{l=1}^{L} \| \mathbf{u}_l - \mathbf{u}^o \|^2},
\]

where \(\mathbf{u}_l\) is the estimation of \(\mathbf{u}^o\) at the \(l\)th Monte Carlo trial, and the number of Monte Carlo trials for each simulation is set as \(L = 5000\).

5.1. CRLB Comparison. In this section, in order to evaluate how sensitive the target localization accuracy is with respect to the transmitter and receiver position errors, we compare the CRLBs with and without transmitter/receiver position errors, under different TDOA/ AOA measurement noise and transmitter/receiver position error levels. The comparison results are exhibited in Figure 3.

Figure 3(a) presents the CRLBs with and without transmitter/receiver position error as TDOA measurement noise level \(\sigma_t\) changes from 0.01us to 10us while \(\sigma_t = 0.1^\circ\) and \(\sigma_1 = 50\) m. It is readily seen that the CRLB with transmitter and receiver position errors is generally above the one without and with the former larger than the latter about one order of magnitude. Figure 3(b) compares the traces of the two CRLBs as AOA measurement noise level \(\sigma_t\) changes from 0.01us to 10us while \(\sigma_i = 0.1^\circ\) and \(\sigma_i = 50\) m. When the AOA measurement noise is small, the localization accuracy with transmitter and receiver position errors level is apparently lower than that without. As the AOA measurement noise level increases, the gap between the two CRLBs is becoming smaller and smaller. When the AOA measurement noise level is sufficiently large, the two CRLBs are nearly the same. The CRLB comparison versus the transmitter/receiver position error level \(\sigma_i\) is plotted in Figure 3(c) where the AOA measurement noise level is fixed at \(\sigma_i = 0.1^\circ\) and \(\sigma_i = 0.1\) m. It can be seen that the difference between the two CRLBs becomes greater and greater with the increase of the transmitter/receiver position error level \(\sigma_i\). At a transmitter/receiver location error level of \(\sigma_i = 100\) m which is not rare in practical applications, the localization CRLB with transmitter and receiver position errors is ten times as the localization CRLB without. This reconfirms the significance of including the transmitter and receiver position errors in the design of the localization algorithm for MIMO passive radar.

5.2. Performance Comparison. Now, we proceed to evaluate the localization RMSE of the proposed solution by comparing with two typical algorithms, that is, Noroozi's method in [18] and Amiri’s method in [19], under different measurement noise and transmitter/receiver position error levels. In order to achieve a more comprehensive insight on the performance of the proposed solution, we consider two cases, that is, an ideal case where the transmitter and receiver position errors are negligibly small and a nonideal case where the transmitter and receiver position errors are significant. We first address the ideal case. The comparison results are presented in Figure 4.
Figure 2: Localization geometry for simulations.

Table 1: Positions of the transmitters and receivers.

| Transmitter | TX1 | TX2 | TX3 | TX4 | Receiver | RX1 | RX2 | RX3 | RX4 | RX5 | RX6 |
|-------------|-----|-----|-----|-----|----------|-----|-----|-----|-----|-----|-----|
| x₀_TX1 (m)  | -200| -200| 200 | 200 | x₀_RX1   | -450| 450 | 0   | 600 | -600| 0   |
| y₀_TX1 (m)  | -300| 300 | 300 | -300| y₀_RX1   | -450| 450 | 600 | 0   | 0   | -600|
| z₀_TX1 (m)  | 250 | 100 | 80  | 120 | z₀_RX1   | 200 | 100 | 200 | 150 | 150 | 100 |

Figure 3: Continued.
Figure 4 depicts the localization RMSE curves of the algorithms for different TDOA/AOA measurement noise levels in the ideal case. As expected, when there are no transmitter and receiver position errors, three algorithms generally perform comparably on the whole. At small TDOA/AOA measurement noise levels, the RMSE curves of the three algorithms match the CRLB very well. With the increase of TDOA/AOA measurement noise levels, the RMSE curves rise correspondingly and deviate gradually from the CRLB. And the deviation from the CRLB, known as the thresholding phenomenon, is owing to the discarded second and higher-order error terms in the derivation of the algorithms, which is invalid for large measurement noise levels. However, it is worth noting that, after deviating from CRLB, the localization RMSE of Noroozi’s method is slightly higher than that of Amiri’s method and the proposed solution. The reason may be that Noroozi’s method requires two WLS stages while Amiri’s method and the proposed solution require only one WLS stage, which means Noroozi’s method has to discard more second and higher-order error terms.

Next, we consider a more practical case, where the transmitter and receiver position errors cannot be ignored. The results are given in Figure 5.

Figures 5(a) and 5(b) give the RMSE curves of the algorithms under different TDOA measurement noise levels and AOA measurement noises, respectively. The transmitter and receiver position errors level is set to $\sigma_s = 50m$. As can be seen from Figures 5(a) and 5(b), when there are non-negligible transmitter and receiver position errors, the localization RMSEs of Noroozi’s method and Amiri’s method cannot reach the CRLB, even at small TDOA/AOA measurement noise regions. This is because both Noroozi’s method and Amiri’s method assume transmitter and receiver positions are accurate but in fact have errors. By contrast, the proposed solution provides better localization performance, with its RMSE reaching the CRLB at small TDOA/AOA measurement noise levels. Although the RMSE of the proposed solution starts to deviate from the CRLB when the TDOA/DOA measurement noises are large, it is still much smaller than that of Noroozi’s method and Amiri’s method. Figure 5(c) presents the RMSE comparison of the algorithms as transmitter and receiver position errors level $\sigma_s$ varies from 1m to 1000m, and the TDOA/AOA measurement noise level is fixed at $\sigma_t = 0.1us$ and $\sigma_a = 0.1\degree$. Within expectation, when the transmitter and receiver position errors are sufficiently small, the RMSE curves of the three algorithms are very close. However, with the increase
Figure 4: RMSE comparison of different algorithms when there are no transmitter and receiver position errors. (a) RMSE comparison as TDOA measurement noise level $\sigma_t$ increases and $\sigma_a = 0.1^\circ$. (b) RMSE comparison as AOA measurement noise level $\sigma_a$ increases and $\sigma_t = 0.1\mu s$.

Figure 5: Continued.
of transmitter and receiver position errors level, the performance deterioration brought by the presence of transmitter and receiver position errors becomes more and more obvious for Noroozi’s method and Amiri’s method. Thus, the proposed solution performs much better than Noroozi’s method and Amiri’s method over large transmitter and receiver position errors regions. This verifies again that the presence of transmitter and receiver location errors can deteriorate the localization accuracy and the necessity of taking the transmitter and receiver position errors into consideration in the design of localization algorithm for MIMO passive radar systems.

5.3. GDOP Analysis. In order to assess the effect of varying the target position on the proposed solution, we use the contour plots of the geometrical dilution of precision (GDOP) values defined as GDOP(\(\mathbf{u}\)) = \(\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}\) where \(\sigma_x\), \(\sigma_y\), and \(\sigma_z\) are the variances of localization on the \(x\), \(y\), and \(z\) axes, respectively. In all GDOP simulations, the TDOA/AOA measurement noise level and transmitter/receiver position error level are set to be \(\sigma_t = 0.1\)us, \(\sigma_s = 0.1^\circ\), \(\sigma_z = 50\)m, respectively. It should be pointed out that the setting of \(\sigma_t = 0.1\)us, \(\sigma_s = 0.1^\circ\), and \(\sigma_z = 50\)m is of practical value under the circumstance of the target echoes being contaminated by moderate noises and GPS signal being sheltered or jammed. The contour plots of the GDOP values in an \(4\)km \(\times\) \(4\)km area around the MIMO passive radar are provided in Figure 6.

Figure 6(a) presents the GDOP contours at \(z^o = 50\)m, from which we observe that the GDOP plots have a relatively symmetric form corresponding to the transmitter/receiver layout. As the target position moves towards the receivers, the GDOP values decrease, so that the minimum level occurs near the receiver positions. Figure 6(b) presents the GDOP contours at \(z^o = 500\)m. Comparing Figure 6(a) with Figure 6(b), we see that by increasing the \(z\)-coordinate of the target, the GDOP contours become more symmetric. Furthermore, when the target \(x\)-coordinate and \(y\)-coordinate are the same, the larger the target \(z\)-coordinate, the larger the GDOP values, and the smaller the influence of the transmitter/receiver position on target localization accuracy. This simulation shows that when the target \(z\)-coordinate becomes larger, the GDOP also becomes larger and the influence of transmitter/receiver position errors decreases. This can be useful in our application.

5.4. Computation Complexity Comparison. The computational complexity is also an important index for performance evaluation. In what follows, to evaluate the proposed solution in terms of computational complexity, we count the average running time of the algorithms from 5000
independent Monte Carlo runs. The main configuration of
the computer is shown as follows: Intel(R) Core(TM) CPU
i5-7200U@2.50GHz; 8.00G RAM; Windows 10 64bit Op-
erating System; Matlab 2018a Software. The comparison
results are given in Table 2.

As presented in Table 2, the time cost of Noroozi1’s
method is almost twice higher than that of Amiri’s method.
This is because Noroozi1’s method needs two WLS stages
while Amiri’s method identifies the target position in only
one WLS stage. The proposed solution incurs the highest
time cost among the algorithms, approximately 3 times
higher than Amiri’s method. This is not surprising since the
proposed solution took the transmitter and receiver position
errors into account while Noroozi1 and Amiri’s methods do
not. That is to say, it is at the expense of the higher com-
putation cost that the proposed solution achieves higher
localization accuracy. However, in view of the significant
performance enhancement, the increased computation cost
is worthy and acceptable.

6. Conclusions

Target localization from time difference of arrival (TDOA)
and angle of arrival (AOA) measurements using multi-
transmitter multireceiver passive radar system requires very
precise knowledge of the transmitter and receiver positions.
A small error in the transmitter and receiver positions may
result in a significant degradation in target localization
accuracy. Hence, this paper addresses a practically motivated
scenario, where the transmitter and receiver positions are
not known perfectly and only the nominal values are
available for processing. To minimize the influence of
transmitter and receiver position errors on target localiza-
tion accuracy, we proposed a novel algebraic solution to
improve the target position estimate. By taking the trans-
mitter and receiver position errors into consideration in the
measurement model, the proposed solution can achieve the
CRLB, no matter whether there exist transmitter and re-
ceiver position errors or not. Both theoretical performance
analysis and numerical simulations are performed to
demonstrate the superiority of the proposed solution over
existing algorithms.

Data Availability

The data used to support the findings of this study are in-
cluded within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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