Wavelet Interval Type-2 Takagi-Kang-Sugeno Hybrid Controller for Time-Series Prediction and Chaotic Synchronization

DUC-HUNG PHAM¹,², CHIH-MIN LIN² (Fellow, IEEE), VAN NAM GIAP³, TUAN-TU HUYNH⁴ (Member, IEEE), AND HSING-YUEH CHO²
¹Faculty of Electrical and Electronic Engineering, Hung Yen University of Technology and Education, Hai Duong 160000, Vietnam
²Department of Electrical Engineering, Yuan Ze University, Taoyuan 320, Taiwan
³School of Electrical & Electronic Engineering, Hanoi University of Science and Technology, Hai Ba Trung, Hanoi 100000, Vietnam
⁴Faculty of Mechatronics and Electronics, Lac Hong University, Bien Hoa 810000, Vietnam

Corresponding authors: Chih-Min Lin (cml@saturn.yzu.edu.tw) and Duc-Hung Pham (duchung.pham@utehy.edu.vn)

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ABSTRACT This paper presents a new hybrid neural network controller for time series prediction and chaotic synchronization. The proposed controller is called as a wavelet interval type-2 Takagi-Kang-Sugeno (TSK) fuzzy brain emotional learning cerebellar model articulation controller (WIT2TFBCC), and it consists of a wavelet interval type-2 TSK fuzzy brain emotional learning controller (WIT2TFBELC), and a wavelet interval type-2 TSK fuzzy cerebellar model articulation controller (WIT2TFCMAC). The proposed WIT2TFBCC can serve both as a control signal for chaotic master-slave synchronization and as a prediction output signal for the time series predictor. Moreover, a robust compensator is used to achieve robust ability of the system. A Lyapunov function was used to establish the adaptive laws and effectively adjust the system parameters online. Finally, two examples of the application are presented to illustrate the performance of proposed method.

INDEX TERMS TSK fuzzy system, interval type-2 wavelet function, brain emotional learning control, cerebellar model articulation controller, 5D chaotic system, Henon map time series.

I. INTRODUCTION
Prediction of chaotic time series is always a major problem in control engineering when dealing with unknown phenomena with multiple variables in nonlinear systems and time-varying systems [1]. Prediction has many applications in various fields of science, such as medicine [2], economics [3], COVID-19 pandemic [4], and especially in engineering [1], [5], [6], [7]. Chandra et al. [8] recommended a hybrid deep learning methods using convolutional neural networks, providing another comparison with simple neural networks that use stochastic gradient descent and adaptive moment estimation (Adam) for training. Li et al. [9] presented an echo state network with improved topology for accurate and efficient time series prediction.

Furthermore, many studies using the chaotic trajectory synchronization have been published in recent years, such as Giap et al. presented a 3D Lorentz chaos system for secure transmission [10]. The applications of synchronization of chaotic systems for the secure communication can be found in [11], [12], and [13]. Lin et al. recommended a 4D memristive chaos synchronization for image encryption [14]. Recently, a multilayer type-2 fuzzy control system was discussed for the synchronization of 5D nonlinear chaotic systems [15]. Hosny et al. introduced a 4D hyperchaotic Chen system for color image encryption [16]. Yan et al. presented a 5D fractional chaotic system synchronization, which could be a good choice for secure transmission [17]. A type-2 BELC was proposed for 3D Lorentz chaotic systems [18]. Huynh et al. introduced the wavelet type-2 fuzzy brain imitated neural network for the synchronization of 3D chaotic systems [19]. The synchronization of chaotic systems by using electronic components can be found in [20].
In fuzzy systems, there are generally two types, namely Takagi-Sugeno-Kang (TSK) fuzzy systems (FS) and Mamdani-Larsen fuzzy systems [21]. In the TSKFS, the introductory parts of the TSK rules are similar to the “IF” part of the other fuzzy inference (FI) rules. And the “THEN” part of the TSK rules is generally a polynomial function of the input variable. This means that the constant in a general inference process is replaced by the linear local equation in the previous parts of a TSK -based FI model. This makes it more expressive than the basic FI models. Therefore, the TSK fuzzy inference model can improve mapping and achieve accurate approximation performance, which makes it more powerful for various applications. Therefore, in this work, we combined TSK with CMAC and BELC channels to develop a new controller, the wavelet interval type-2 TSK fuzzy brain emotional learning cerebellar model articulation controller (WIT2TFBCC).

To date, type-1 and type-2 intelligent control systems based on fuzzy inference systems (FIS) have developed rapidly in many fields [22], [23]. Since the type-1 FIS (T1FIS) relies on fixed membership functions, it cannot efficiently deal with the uncertainties of the inputs and parameters of nonlinear systems. To overcome this problem, type-2 fuzzy inference systems (T2FIS) and interval type-2 FIS (IT2FIS) are generally used because they are more general than T1FIS, have greater design freedom to achieve better control performance, and improve the response to the input uncertainty of membership functions. Nevertheless, deciding the membership functions and network dimension for T2FIS and IT2FIS is certainly a difficult task. The trial-and-error approach is a common way to constrain the network structure dimension, but it usually takes a lot of time to train and learn and does not guarantee that the parameters and network structures can be precisely defined. For this reason, many studies have developed solutions that combine various effective techniques, additional networks and algorithms such as wavelet function [7], [22], function link network [24] and self-organising algorithms [22].

Due to the imprecision of system uncertainties and external disturbances, nonlinear chaotic synchronization systems always pose a challenge to researchers. To solve these problems, Lucas et al. [25] developed the Brain Emotional Learning Controller (BELC). BELC is comprised of sensory input, sensory learning, sensory output, input, emotional learning, emotional output, and output from the controller. BELC has been integrated with another system in order to boost its performance, such as CMAC [12], fuzzy system [13], TSK fuzzy system [14], type-2 fuzzy system [18], and wavelet type-2 fuzzy system [19]. BELC has been utilized for chaotic synchronization and secure communication [12], [13], [14].

The proposed method has combined the advantages of wavelet interval type-2 function with TSKFS to form a neural network (NN) structure for brain emotion learning. The simulation of Henon chaotic time series prediction and the simulation of 5-D hyperchaotic synchronization illustrate the performance of our proposed method. Subsequently, the proposed method can be extended and applied to other electromechanical underactuated systems such as [26], [27], and [28]. The main contributions of this work are listed below:

1. A new wavelet IT2FIS will be added to the BELC and CMAC channels to improve response to member function input uncertainty.

2. TSKFS for dual-channel CMAC and BELC to update parameters more flexibly, improve mapping, and provide accurate approximation performance.

3. A proposed WIT2TFBCC controller is designed for chaotic system synchronization and prediction system.

4. Using Lyapunov stability theory to prove the stability of the proposed system.

The content of this paper consists of the following parts: Section I introduces the problem to be solved, Section II presents the proposed WIT2TFBCC for synchronization of chaotic systems and shows how the update of the controller parameters is determined by the Lyapunov method. Section III shows the proposed WIT2TFBCC for time series prediction and demonstrates the stability of the proposed method. Section IV presents the numerical simulation results for 5-D hyperchaotic in Matlab software. Section V shows the result of Henon map time series prediction. Finally, section VI presents the conclusion of the paper. The main differences of the proposed controller from others are listed in Table 1.

II. WAVELET INTERVAL TYPE-2 TAKAGI-SUGENO-KANG FUZZY BRAIN EMOTIONAL CEREBELLAR MODEL ARTICULATION CONTROLLER (WIT2TFBCC)

Consider a novel 5D hyperchaotic system expressed as [29]

\[
\begin{aligned}
\dot{x}_1 &= a_1 x_2 + a_2 x_4 \\
\dot{x}_2 &= -a_1 x_1 - a_2 x_1 x_4 \\
\dot{x}_3 &= a_3 x_4 \\
\dot{x}_4 &= -a_2 x_1 x_2 - a_4 x_3 \\
\dot{x}_5 &= -a_3 x_1
\end{aligned}
\]

where \(a_1, a_2, a_3\) and \(a_4\) can be exactly known, and the slave system can be provided as:

\[
\begin{aligned}
\dot{y}_1 &= a_1 y_2 + a_2 y_4 + \Delta Y(y_1) + n_d + u_1 \\
\dot{y}_2 &= -a_1 y_1 - a_2 y_1 y_4 + \Delta Y(y_2) + n_d + u_2 \\
\dot{y}_3 &= a_3 y_4 + \Delta Y(y_3) + n_d + u_3 \\
\dot{y}_4 &= -a_2 y_1 y_2 - a_4 y_3 + \Delta Y(y_4) + n_d + u_4 \\
\dot{y}_5 &= -a_3 x_1 + \Delta Y(y_5) + n_d + u_5
\end{aligned}
\]

where \(u = [u_1, u_2, u_3, u_4, u_5]^T\) is the control input vector, \(n_d = [n_{d1}, n_{d2}, n_{d3}, n_{d4}, n_{d5}]^T\) is the unknown external disturbance, \(\Delta Y(y) = [\Delta Y(y_1), \Delta Y(y_2), \Delta Y(y_3), \Delta Y(y_4), \Delta Y(y_5)]^T\) is the uncertainty of the system, and \(y_i(i = 1, 2, 3, 4, 5)\) is the slave state. The tracking error can be
TABLE 1. The main differences of the proposed controller from other methods.

| Method                  | The main structure and its application | Additional techniques | Membership function | System weights | Mathematical solution method to determine the adaptive learning laws and convergence analysis |
|-------------------------|----------------------------------------|-----------------------|---------------------|----------------|--------------------------------------------------------------------------------------------------|
| 1 Our method            | Consisting of one CMAC, one BEC, two wavelet interval type-2 TSK fuzzy inference systems (one can be used for BEC and one for CMAC). | Consisting of SMC to design the adaptive laws | Consisting of the wavelet type-2 function | Consisting of wavelet interval type-2 TSK fuzzy adaptive weights that can be updated online by adaptive laws. And consisting of relative coefficient weights of two subnetworks (CMAC and BEC) that can also be updated by adaptive laws. | Consisting of a Lyapunov function. |
| 2 WTSKFCMAC [7]         | Comprising TSK fuzzy rules, a CMAC     | Consisting of SMC to design the adaptive laws | Consisting of the Gaussian function | Consisting of adaptive weights | Consisting of a Lyapunov function |
| 3 RCFBC [12]            | Comprising a fuzzy inference system, a BEC, a RCMAC and it can be acted as a main controller | Consisting of SMC to design the adaptive laws | Consisting of the Gaussian function | Consisting of adaptive weights | Consisting of a Lyapunov function |
| 4 T2HC [30]             | Comprising a Type-2 fuzzy inference system, a Type-2 fuzzy CMAC, a BEC, and it can be acted as the main controller. | Consisting of sliding mode controller. | Consisting of the Gaussian function | Consisting of adaptive dynamic weights | Consisting of a Lyapunov function |
| 5 IT2RFRBF [31]         | Comprising an interval type-2 recurrent neural network. | None | Consisting of the ellipsoidal membership function | Consisting of adaptive weights | Consisting of a Lyapunov function |
| 6 AORFC [32]            | Comprising a CMAC, a TSK fuzzy membership function and it can be acted as a main controller | Consisting of backstepping control technique to design the adaptive laws | Consisting of the type-2 fuzzy elliptic membership function | Consisting of adaptive weights by TSK fuzzy membership function | Consisting of a Lyapunov function |
| 7 RWBEL [33]            | Comprising a BEC, a RNN, and it can be acted as a main controller. | Consisting of SMC to design the adaptive laws | Consisting of the Wavelet membership function | Consisting of adaptive weights | Consisting of a Lyapunov function |
| 8 TSKFCMAC [34]         | Comprising a TSK fuzzy rule, a CMAC and it can be acted as a main controller | Consisting of SMC to design the adaptive laws | Consisting of the Gaussian membership function | Consisting of adaptive weights | Consisting of a Lyapunov function |

calculated as

\[
\begin{align*}
    e_1 &= y_1 - x_1 \\
    e_2 &= y_2 - x_2 \\
    e_3 &= y_3 - x_3 \\
    e_4 &= y_4 - x_4 \\
    e_5 &= y_5 - x_5
\end{align*}
\] (3)

The error dynamics can be given by

\[
\begin{align*}
    \dot{e}_1 &= \dot{y}_1 - \dot{x}_1 = a_1 e_2 + a_2 e_4 + \Delta Y(y_1) + n_{d1} + u_1 \\
    \dot{e}_2 &= \dot{y}_2 - \dot{x}_2 = -a_1 e_1 - a_2 y_1 y_4 + a_2 x_1 x_4 + \Delta Y(y_2) + n_{d2} + u_2 \\
    \dot{e}_3 &= \dot{y}_3 - \dot{x}_3 = a_3 e_4 + \Delta Y(y_3) + n_{d3} + u_3 \\
    \dot{e}_4 &= \dot{y}_4 - \dot{x}_4 = -a_4 e_3 - a_2 y_1 y_2 + a_2 x_1 x_2 + \Delta Y(y_4) + n_{d4} + u_4 \\
    \dot{e}_5 &= \dot{y}_5 - \dot{x}_5 = -a_3 e_1 + \Delta Y(y_5) + n_{d5} + u_5
\end{align*}
\] (4)

Equation (4) can be presented by the vector form as

\[
\dot{e} = Te + \varphi + \Delta Y(y_5) + n_d + u
\] (5)

where

\[
e = [e_1, e_2, e_3, e_4, e_5]^T, \quad T = \begin{bmatrix}
    0 & a_1 & 0 & a_2 & 0 \\
    -a_1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & a_3 & 0 \\
    0 & 0 & -a_4 & 0 & 0 \\
    -a_3 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

and

\[
\varphi = \begin{bmatrix}
    0 \\
    -a_2 y_1 y_4 + a_2 x_1 x_4 \\
    0 \\
    -a_2 y_1 y_2 + a_2 x_1 x_2 \\
    0
\end{bmatrix}
\]
If $\Delta Y(y)$ and $n_d$ are known, an ideal control value can be defined as
\[
u^* = -Te - \varphi - \Delta Y(y) - n_d - \Psi e
\]
where $\Psi = \text{diag}(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5)$ is the feedback gain matrix. Inserting (6) into (5) attains:
\[
\dot{e} + \Psi e = 0
\]
(7)
If $\Psi$ can be regulated to fascinate the Hurwitz stability criterion, then $\lim_{t \to \infty} e \to 0$. In fact, the ideal controller in (7) is not obtained because the lumped uncertainty, $n_d(1)$, is not known for practical applications. Therefore, a wavelet-interval type-2 TSK fuzzy brain emotional learning cerebellar model articulation controller (WIT2TFBCC) is investigated to mimic the ideal controller. In this section, the proposed wavelet-interval type-2 TSK fuzzy brain emotional learning cerebellar model articulation controller (WIT2TFBCC) is presented. The proposed structure includes two channels: The Wavelet Interval Type-2 TSK Cerebellar Model Articulation Controller channel (WIT2FMCAC) and the Wavelet Interval Type-2 TSK Fuzzy Brain Emotional Learning Controller channel (WIT2TFBLC). The WIT2TFBLC channel contains four domains called input domain, association memory domain, output weight domain, and sub-output domain. The WIT2FMCAC channel has five domains, besides four domains as similar as the WIT2TFBLC, it has one more domain called receptive-field domain which is located between the associative memory domain and the weight memory domain. The structure of the proposed algorithm is described in Fig 1.

A. THE INPUT DOMAIN
In this domain, the input variables are denoted by a vector $x = [x_1, x_2, \ldots, x_i, \ldots, x_n]^T \in \mathbb{R}^n$, where $x_i$ and $n_i$ are the $i^{th}$ input variable and the input dimension, respectively.

B. WIT2TFBLC CHANNEL
1) ASSOCIATIVE MEMORY DOMAIN
This domain uses the wavelet interval type-2 membership function (WIT2MF).

\[
a_{iq} = \left( \frac{x_i - m_{iq}^B}{\bar{a}_{iq}^B} \right) \exp \left( -\frac{(x_i - m_{iq}^B)^2}{2\bar{a}_{iq}^B} \right)
\]
(8)
and
\[
a_{iq} = \left( \frac{x_i - m_{iq}^B}{\bar{a}_{iq}^B} \right) \exp \left( -\frac{(x_i - m_{iq}^B)^2}{2\bar{a}_{iq}^B} \right)
\]
(9)
where $m_{iq}^B$, $\bar{a}_{iq}^B$ and $\bar{a}_{iq}^B$ are respectively the center, the lower variance and the upper variance of the WIT2MF.

\[
\begin{align*}
a_q &= \sum_{i=1}^{n_i} a_{iq} \\
a_q^T &= \sum_{q=1}^{n_q} a_{q}^T
\end{align*}
\]
(10)
Set
\[
\begin{align*}
\bar{a} &= [a_1, \ldots, a_q, \ldots, a_{n_q}]^T \in \mathbb{R}^{n_q} \\
a &= [a_1, \ldots, a_q, \ldots, a_{n_q}]^T \in \mathbb{R}^{n_q}
\end{align*}
\]
(11)

2) WEIGHT MEMORY DOMAIN
In this research, the weight is formed by the TSK fuzzy inference system. The following reference rules are exploited:

\[
R^i: \text{If } x_1 \text{ is } a_{1q}, \ldots, a_{n_i} \text{ is } a_{n_{iq}}, \text{ then }
\]
\[
\begin{align*}
&w^i_q = x_1 t^i_q + x_2 t^i_{2q} + \ldots + x_n t^i_{n_q}
\end{align*}
\]
(12)
for $i = 1, 2, \ldots, n_i$, where $q = 1, 2, \ldots, n_q$ in which $n_q$ is the fuzzy rules dimension, and $a_{iq}$ is the fuzzy set for the $i^{th}$ input, $q^{th}$ output; and $w^i_q$ is the TSK-type output weight of WIT2TFBLC. Defining the weight matrix for WIT2TFBLC channel

\[
\bar{w}_i^B = \begin{bmatrix}
\bar{w}_1^B \\
\bar{w}_2^B \\
\vdots \\
\bar{w}_{n_i}^B \\
\end{bmatrix} = \begin{bmatrix}
\bar{t}_1^B \\
\bar{t}_2^B \\
\vdots \\
\bar{t}_{n_i}^B \\
\end{bmatrix}
\]
and

\[
\bar{w}_i^B = \begin{bmatrix}
\bar{w}_1^B \\
\bar{w}_2^B \\
\vdots \\
\bar{w}_{n_i}^B \\
\end{bmatrix} = \begin{bmatrix}
\bar{t}_1^B \\
\bar{t}_2^B \\
\vdots \\
\bar{t}_{n_i}^B \\
\end{bmatrix}
\]
(13)
where

\[
\bar{t}_i^B = \begin{bmatrix}
\bar{t}_{i1}^B \\
\bar{t}_{i2}^B \\
\vdots \\
\bar{t}_{i_{n_q}}^B \\
\end{bmatrix} \in \mathbb{R}^{n_i \times n_q}
\]
and

\[
\bar{t}_i^B = \begin{bmatrix}
\bar{t}_{i1}^B \\
\bar{t}_{i2}^B \\
\vdots \\
\bar{t}_{i_{n_q}}^B \\
\end{bmatrix} \in \mathbb{R}^{n_i \times n_q}
\]

3) SUB-OUTPUT DOMAIN
\[
\begin{align*}
a_{i}^T &= \sum_{q=1}^{n_q} a_{iq} t^B_{aq} x \\
&= \sum_{q=1}^{n_q} a_{iq} t^B_{aq} x
\end{align*}
\]
(14)
\[ a'_q = \sum_{q=1}^{n_q} a'_q w'_q = \frac{\sum_{q=1}^{n_q} a'_q f_q^T x}{\sum_{q=1}^{n_q} a'_q} \]  
where \( a'_q \) and \( a'_{q,k} \) are determined by Karnik-Mendel algorithm [28]

\[ a'_q = \begin{cases} 
  a_q, & q \leq L_s \\
  a_q, & q > L_s 
\end{cases} \]  
and

\[ a'_q = \begin{cases} 
  a_q, & q \leq R_s \\
  a_q, & q > R_s 
\end{cases} \]  
where and \( L_s \) and \( R_s \) are the left and right switch points, then

\[ a_q = \frac{a'_q + a'_{q,k}}{2} \]  

C. WIT2FMC MAC CHANNEL

1) ASSOCIATIVE MEMORY DOMAIN

\[ o_{iq} = \frac{x_i - m_{iq}^C}{d_{iq}} \exp \left( -\frac{(x_i - m_{iq}^C)^2}{2d_{iq}^2} \right) \]  
and

\[ \bar{o}_{iq} = \frac{x_i - m_{iq}^C}{d_{iq}} \exp \left( -\frac{(x_i - m_{iq}^C)^2}{2d_{iq}^2} \right) \]  
where \( m_{iq}^C \), \( d_{iq}^C \) and \( \bar{d}_{iq}^C \) are the center, the lower variance and the upper variance of the WIT2MF, respectively.

2) RECEPTIVE-FIELD DOMAIN

Each block value is the product of the equivalent block of the associative memory domain, which is determined by:

\[ o_q = \prod_{i=1}^{n_q} o_{iq} \]  
and

\[ \bar{o}_q = \prod_{i=1}^{n_q} \bar{o}_{iq} \]

Set

\[ \bar{o} = [\bar{o}_1, \ldots, \bar{o}_q, \ldots, \bar{o}_{n_q}]^T \in \mathbb{R}^{n_q} \]  

and

\[ o = [o_1, \ldots, o_q, \ldots, o_{n_q}]^T \in \mathbb{R}^{n_q} \]  

3) WEIGHT MEMORY DOMAIN

Define the weight matrix for the CMAC channel with TSK fuzzy reasoning as follows

\[ R^f : \text{If } x_1 \text{ is } o_{1q}, x_2 \text{ is } o_{2q}, \ldots, \text{ and } x_n \text{ is } o_{nq}, \text{ then } w^C_q = x_1 t_{1q}^C + x_2 t_{2q}^C + \ldots + x_n t_{nq}^C \]
for \( i = 1, 2, \ldots, n_i \)  

where \( w^C_q \) is the TSK-type output weight of TSKCMAC. For detail, using the wavelet interval type-2 function, the weight matrix is defined as follows:

\[ \bar{w}^C = \begin{bmatrix} \bar{w}^C_1 \\ \vdots \\ \bar{w}^C_{n_q} \end{bmatrix} = \begin{bmatrix} \bar{t}^C_{11} & \ldots & \bar{t}^C_{1l} & \ldots & \bar{t}^C_{1n_q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{t}^C_{l1} & \ldots & \bar{t}^C_{l1} & \ldots & \bar{t}^C_{ln_q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{t}^C_{1n_q} & \ldots & \bar{t}^C_{1n_q} & \ldots & \bar{t}^C_{n_q n_q} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_{n_q} \end{bmatrix} \]  

\[ w^C = \begin{bmatrix} w^C_1 \\ \vdots \\ w^C_{n_q} \end{bmatrix} = \begin{bmatrix} t^C_{11} & \ldots & t^C_{1l} & \ldots & t^C_{1n_q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ t^C_{l1} & \ldots & t^C_{l1} & \ldots & t^C_{ln_q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ t^C_{1n_q} & \ldots & t^C_{1n_q} & \ldots & t^C_{n_q n_q} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_{n_q} \end{bmatrix} \]

where

\[ t^C = \begin{bmatrix} t^C_1 \\ \vdots \\ t^C_{n_q} \end{bmatrix} \quad \in \mathbb{R}^{n_q} \quad x \quad \in \mathbb{R}^{n_q} \]

and

\[ t^C = \begin{bmatrix} t^C_1 \\ \vdots \\ t^C_{n_q} \end{bmatrix} \quad \in \mathbb{R}^{n_q} \quad x \quad \in \mathbb{R}^{n_q} \]

4) SUB-OUTPUT DOMAIN

The \( q \)-th output of the network is determined by the weight memory domain and the Associative memory domain.

\[ o'_q = \frac{\sum_{q=1}^{n_q} o'_q w^C_q x}{\sum_{q=1}^{n_q} o'_q} = \frac{\sum_{q=1}^{n_q} o'_q w^C_q x}{\sum_{q=1}^{n_q} o'_q} \]

\[ o^f_q = \frac{\sum_{q=1}^{n_q} o^f_q w^C_q x}{\sum_{q=1}^{n_q} o^f_q} = \frac{\sum_{q=1}^{n_q} o^f_q w^C_q x}{\sum_{q=1}^{n_q} o^f_q} \]

where \( o'_q \) and \( o^f_q \) are determined by Karnik-Mendel algorithm [28], and \( L_s \) and \( R_s \) are the left and right switch points:

\[ o'_q = \begin{cases} 
  a_q, & q \leq L_s \\
  a_q, & q > L_s 
\end{cases} \]

\[ o^f_q = \begin{cases} 
  a_q, & q \leq R_s \\
  a_q, & q > R_s 
\end{cases} \]

then

\[ o_q = \frac{o'_q + o^f_q}{2} \]
where \( u_{tb} \) is a robust controller which rejects the approximate error \( \varepsilon \). The estimation error, \( \hat{u} \), is calculated by subtracting (33) from (32) as follows:

\[
\hat{u} = u^* - u = A^*H^a - O^*H^o + (A\hat{H}^a - O\hat{H}^o) + \varepsilon - u_{tb}
\]

\[
\hat{u} = A\hat{H}^a + \hat{A}\hat{H}^a + (\hat{O}\hat{H}^o - \hat{O}\hat{H}^o) + \varepsilon - u_{tb}
\]

\[
\hat{u} = A\hat{H}^a + \hat{A}\hat{H}^a - \hat{O}\hat{H}^o - \hat{O}\hat{H}^o + \varepsilon - u_{tb}
\]

(34)

D. THE OUTPUT DOMAIN

Set

\[
A = [a_1, \ldots, a_q, \ldots, a_{nq}], \quad O = [a_1, \ldots, a_q, \ldots, a_{nq}],
\]

\[
H^a = [h^a_1, \ldots, h^a_q, \ldots, h^a_{nq}]^T,
\]

\[
H^o = [h^o_1, \ldots, h^o_q, \ldots, h^o_{nq}]^T.
\]

Fig. 1 illustrates the proposed WIT2TBCC. The controller output is defined as:

\[
u_{WIT2TBCC} = AH^a - O\hat{H}^o
\]

(31)

E. CONVERGENCE ANALYSIS

In practice there is always an error between the ideal controller and the WIT2TBCC controller

\[
u = \hat{u}_{WIT2TBCC} + \varepsilon = \hat{A}\hat{H}^a - \hat{O}\hat{H}^o + \varepsilon
\]

(32)

where \( \hat{A} \), \( \hat{H}^a \), \( \hat{O} \), and \( \hat{H}^o \) are respectively the estimated values of \( A \), \( H^a \), \( O \), and \( H^o \).

\( \varepsilon \) is the minimum error between \( u^* \) and \( u_{WIT2TBCC} \) (for \( 0 \leq \varepsilon \leq \kappa \), where \( \kappa \) is constant vector), \( A^* \) and \( O^* \) are optimal parameters for \( A \) and \( O \), respectively. But the \( u_{WIT2TBCC} \) can be unattained, therefore an online estimation of the WIT2TBCC, \( u_{WIT2TBCC} \), is applied to estimate \( u_{WIT2TBCC} \). Using (25), equation (26) becomes

\[
u = u_{WIT2TBCC} + u_{tb} = \hat{A}\hat{H}^a - \hat{O}\hat{H}^o + u_{tb}
\]

(33)
\[
\begin{bmatrix}
\frac{\partial \alpha}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
0, \ldots, 0, \frac{\partial \alpha}{\partial \sigma_{l_1 q}}, \ldots, 0
\end{bmatrix}_{(q-1) \times n_i}
\] (44)
\[
\begin{bmatrix}
\frac{\partial \beta}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
0, \ldots, 0, \frac{\partial \beta}{\partial \sigma_{l_1 q}}, \ldots, 0
\end{bmatrix}_{(q-1) \times n_i}
\] (45)

Inserting (35) into (34) obtains
\[
\ddot{u} = \ddot{A}H^a + (A_m^a \dot{m}^a + A_{d_1}^a \ddot{d}^B + A_{d_2}^a \dot{d}^B + A_{d_3}^a \ddot{d}^B + A_{d_4}^a \dot{d}^B + T_\alpha)
\times \dot{H}^a - \ddot{H}^o - (O_m c \dot{m}^a + O_{d_1}^a \dot{d}^C + O_{d_2}^a \ddot{d}^C + O_{d_3}^a \dot{d}^C)
\times \dot{H}^a + \Delta \dot{H}^a - \Delta \ddot{H}^o - \Delta \dot{H}^o - \Delta v\dot{u}_b
\]
\[
= \ddot{A}H^a - \ddot{H}^o + A_m^a \dot{m}^a \dot{H}^a + A_{d_1}^a \ddot{d}^B \dot{H}^a + A_{d_2}^a \dot{d}^B \dot{H}^a
\times (O_m c \dot{m}^a + O_{d_1}^a \dot{d}^C + O_{d_2}^a \ddot{d}^C + O_{d_3}^a \dot{d}^C)
\times \dot{H}^a + \Delta \dot{H}^a - \Delta \ddot{H}^o - \Delta \dot{H}^o + \theta - \Delta \dot{u}_b
\]
\[
= \ddot{A}H^a - \ddot{H}^o + A_m^a \dot{m}^a \dot{H}^a + A_{d_1}^a \ddot{d}^B \dot{H}^a + A_{d_2}^a \dot{d}^B \dot{H}^a + A_{d_3}^a \ddot{d}^B \dot{H}^a
\times (O_m c \dot{m}^a + O_{d_1}^a \dot{d}^C + O_{d_2}^a \ddot{d}^C + O_{d_3}^a \dot{d}^C)
\times \dot{H}^a + \Delta \dot{H}^a - \Delta \ddot{H}^o - \Delta \dot{H}^o + \theta - \Delta \dot{u}_b
\] (46)

where the cumulative error, \(\theta = T_\alpha \dot{H}^a - T_\alpha \dot{H}^o + \Delta \dot{H}^a - \Delta \ddot{H}^o + \Delta v\dot{u}_b\) is assumed to be bounded by \(\|\theta\| < \kappa\). Defining the sliding surface as \(s = \dot{e} + \dot{\Psi} \int e(\tau) d\tau\). The derivative of \(s\) can be calculated by (6), (7), and (34) as follows:
\[
\dot{s} = \dot{e} + \dot{\Psi} e = u^* - u
\]
\[
= \ddot{A}H^a - \ddot{H}^o + \dot{H}^o T_{\alpha} A_m^a \dot{m}^a + \dot{H}^o T_{\alpha} A_{d_1}^a \ddot{d}^B + \dot{H}^o T_{\alpha} A_{d_2}^a \dot{d}^B + \dot{H}^o T_{\alpha} A_{d_3}^a \ddot{d}^B + \dot{H}^o T_{\alpha} A_{d_4}^a \dot{d}^B
\times (O_m c \dot{m}^a + O_{d_1}^a \dot{d}^C + O_{d_2}^a \ddot{d}^C + O_{d_3}^a \dot{d}^C)
\times \dot{H}^a + \Delta \dot{H}^a - \Delta \ddot{H}^o - \Delta \dot{H}^o + \theta - \Delta \dot{u}_b
\]
\[
= \ddot{A}H^a - \ddot{H}^o + A_m^a \dot{m}^a \dot{H}^a + A_{d_1}^a \ddot{d}^B \dot{H}^a + A_{d_2}^a \dot{d}^B \dot{H}^a + A_{d_3}^a \ddot{d}^B \dot{H}^a
\times (O_m c \dot{m}^a + O_{d_1}^a \dot{d}^C + O_{d_2}^a \ddot{d}^C + O_{d_3}^a \dot{d}^C)
\times \dot{H}^a + \Delta \dot{H}^a - \Delta \ddot{H}^o - \Delta \dot{H}^o + \theta - \Delta \dot{u}_b
\] (47)

**Theorem 1**: The nonlinear 5-D hyper chaotic system given in using (1). The proposed WIT2TFBCC is shown in (31) using the update laws from (48) to (60) combining with the robust controller in (60). Then the robustness of the system can be definite.

\[
\ddot{H}^a = l_{H^a} s \dot{H}^a
\] (48)
\[
\dot{H}^o = l_{H^o} s \dot{H}^o
\] (49)
\[
\ddot{d}^B = l_{d^B} s \dot{d}^B
\] (50)
\[
\dot{d}^B = l_{d^B} s \dot{d}^B
\] (51)
\[
\ddot{d}^C = l_{d^C} s \dot{d}^C
\] (52)
\[
\dot{d}^C = l_{d^C} s \dot{d}^C
\] (53)
\[
\dot{m}^B = l_{m^B} s A_m^B \dot{H}^a
\] (54)
Remark 1: $H^a, H^o$, $\theta^o$, $\theta^t$, $t^C$, $t^B$, $t^B_m$, $m^C$, $d^B$, $d^C$, $d^B_m$, $d^C_m$ are constants, therefore, $\dot{H}^a = -\dot{H}^o$, $\dot{H}^o = -\dot{H}^a$, $\dot{t}^B = \dot{t}^C = \dot{t}^B_m = \dot{t}^C_m = \dot{t}^C = \dot{m}^B = -\dot{m}^B$, $\dot{m}^C = -\dot{m}^C, \dot{d}^B = \dot{d}^B, \dot{d}^C = \dot{d}^C, \dot{d}^B = \dot{d}^B, \dot{d}^C = \dot{d}^C$.

Remark 2:

\[
\begin{align*}
\dot{H}^a A_m m^B &= \dot{m}^B A_m \dot{H}^a \\
\dot{H}^o A_d \dot{d}^B &= \dot{d}^B A_d \dot{H}^o \\
\dot{H}^o A_d \dot{d}^C &= \dot{d}^C A_d \dot{H}^o \\
\dot{H}^o O_m C^C &= \dot{m}^C O_m \dot{H}^a \\
\dot{H}^o O_d \dot{d}^C &= \dot{C} O_d \dot{H}^a \\
\dot{H}^o O_d \dot{d}^C &= \dot{C} O_d \dot{H}^a
\end{align*}
\]

and

\[
\dot{V} = \sum_{q=1}^{n_q} \dot{H}^o T^a \left( s^T \dot{A} - \frac{\dot{H}^a}{l^H} \right)
\]

FIGURE 2. The structure of the proposed synchronization method.

FIGURE 3. The structure of nonlinear system prediction using proposed WIT2TFBCC.

\[
-\dot{m}^C T^a \left( s O_m^T \dot{H}^o + \frac{\dot{m}^C}{m^C} \right) - \dot{d}^C T^a \left( s O_d^T \dot{H}^o + \frac{\dot{d}^C}{d^C} \right)
\]

Via the compensator in (60) and the update parameters in (43)-(54), (64) becomes:

\[
\dot{V} = s^T (\theta - u_{th}) + \frac{\dot{\theta}}{l_k} - \frac{\dot{\theta}}{l_k} \| s \| + \frac{\dot{\theta}}{l_k}
\]

If the adaptive law of the error bound is adjusted by as below: $\dot{\theta} = -\dot{\theta} = -l_k \| s \|$, then (65) becomes:

\[
\dot{V} = s^T \theta - \dot{\theta} \| s \| - (\kappa - \dot{\theta}) \| s \| = s^T \theta - \kappa \| s \| \leq \| s \| - \kappa \| s \| \leq 0
\]

Since $\dot{V}(k)$ is negative semi-definite, it shows that $\dot{k}$ and $s(t)$ are bounded. Define $\Gamma = (\kappa - \| \theta \|) \cdot \| s \| \leq (\kappa - \| \theta \|) \cdot \| s \| \leq -\dot{V}$. Integrating $\Gamma$ with admiration to
time, obtains:

\[ \int_0^t \Gamma(\xi)d\xi \leq V(0) - V(t) \quad (67) \]

Because \( V(0) \) is bounded, \( \dot{V} \) does not increase and bounded, so \( \lim_{t \to \infty} \int_0^t \Gamma(\xi)d\xi < \infty \). This indicates that when \( t \to \infty \) then \( s \to 0 \). Therefore, the stability for the proposed WIT2TFBCC algorithm is proved.

### III. THE WIT2FBCC PARAMETER LEARNING ALGORITHM FOR TIME SERIES PREDICTION

Consider a nonlinear system described as [1]

\[ y(t + 1) = f(y(t), y(t - 1), \ldots, y(t - n)), r(t), \quad r(t - 1), \ldots, r(t - n) \quad (68) \]

where \( f(.) \) is the unknown function \( f : \mathbb{R}^n \to \mathbb{R} \), \( r \) and \( y \) are the input and output of the system, respectively. The input data \( r(t) \) is used to train the network and the final output from the neural network is also the output of the system.

Considering the single input-single output system for simplicity, the tracking error for the system is defined as

\[ e(t) = y(t) - y_N(t) \quad (69) \]

where \( y_N(t) \) is the desired output for the system and \( y_N(t) = u_{\text{WIT2TFBCC}}(t) \) is the output of the neural network.

### A. CONVERGENCE ANALYSIS FOR CHAOS TIME-SERIES PREDICTION

A Lyapunov function is given as follows:

\[ V_p(t) = \frac{1}{2} e^2(t) \quad (70) \]
The derivate of $V$:

$$\Delta V_p(t) = V_p(t + 1) - V_p(t) = \frac{1}{2}(e^2(t + 1) - e^2(t)) \quad (71)$$

According to the literature [19], the error can be represented by

$$e(t + 1) = e(t) - \Delta e(t) \equiv e(t) + \left(\frac{\partial e(t)}{\partial \Upsilon}\right)^T \Delta \Upsilon \quad (72)$$

where $\Delta \Upsilon$ is the change of $\Upsilon$. From (12) to (13), obtains

$$\Delta \Upsilon = l_{\Upsilon} \left( -\frac{\partial E_\Upsilon(t)}{\partial \Upsilon} \right) = l_{\Upsilon} e_\Upsilon(t) \frac{\partial \hat{y}_\Upsilon(t)}{\partial \Upsilon} \quad (73)$$

where $l_2$ can be one of these learning rates $l_{H^a}, l_{H^b}, l_{H^c}, l_{m^a}, l_{m^b}, l_{d^a}, l_{d^b}, l_{m^c}, l_{d^c}$. Thus,

$$\Delta V_p(t) = \frac{1}{2} \left( e^2(t + 1) - e^2(t) \right)$$

$$= \frac{1}{2} (e(t + 1) - e(t))(e(t + 1) + e(t))$$

$$= \frac{1}{2} \Delta e(t) \left[ e(t) + \frac{1}{2} \Delta e(t) \right]$$

$$= \left[ \frac{\partial e(t)}{\partial \Upsilon} \right]^T l_{\Upsilon} e(t) \frac{\partial \hat{y}_\Upsilon(t)}{\partial \Upsilon}$$

$$\times \left[ e(t) + \frac{1}{2} \left( \frac{\partial e(t)}{\partial \Upsilon} \right)^T l_{\Upsilon} e(t) \frac{\partial \hat{y}_\Upsilon(t)}{\partial \Upsilon} \right]$$


TABLE 2. The parameters of 5D hyper chaotic system.

| Parameter systems       | Values            |
|-------------------------|-------------------|
| Parameter systems       | \( a_i = 30, a_i = 30, a_i = 10, a_i = 30 \) |
| Initial conditions      | \( x(0) = [0.8, 4.9, 7.6, 3.7, 6.5]^T \), \( y(0) = [1.8, 5.9, 5.6, 2.65, 6]^T \) |
| External perturbations  | \( n(t) = \text{rand}(.) \times [0.02 \times t, 0.02 \times \text{square}(5 \times t), 0.03 \times \text{sin}(2 \times t), 0.02 \times \text{sawtooth}(10 \times t), 0.02 \times (\text{square}(5 \times t) + \cos(t^2))] \) |

TABLE 3. Initial parameters of the proposed WIT2TFBCC.

| Parameter                              | WIT2TFBELC | WIT2TFCMAC |
|----------------------------------------|------------|------------|
| Total of blocks for associative domain | 5          | 5          |
| The block number of receptive field domain | 5          | 5          |
| The block number of memory domain      | 5          | 5          |
| Number of output domain                | 2          | 2          |
| The initialization range for centers \( m^B_{iw} \) and \( m^C_{iw} \) | \([-0.2, 0.2]\) | \([-0.2, 0.2]\) |
| Initial upper variances \( \hat{d}^B_{iw} \) and \( \hat{d}^C_{iw} \) | 0.005      | 0.005      |
| Initial lower variances \( \hat{d}^B_{iw} \) and \( \hat{d}^C_{iw} \) | 0.005      | 0.0005     |
| The initialization range for \( \bar{l}^B_{iw} \) and \( \bar{l}^C_{iw} \) | \([-0.5, 0.5]\) | \([-0.5, 0.5]\) |
| The initialization range for \( \bar{l}^B_{iw} \) and \( \bar{l}^C_{iw} \) | \([-0.5, 0.5]\) | \([-0.5, 0.5]\) |
| Learning rates for \( l^B_{iw} \) and \( l^C_{iw} \) | 0.005      | 0.005      |
| Learning rates for TSK \( l^B_{iw}, l^C_{iw} \), and \( l^B_{iw}, l^C_{iw} \) | 0.002, 0.002 | 0.002, 0.002 |
| Learning-rate for \( l^B_{iw}, l^C_{iw} \), and \( l^B_{iw}, l^C_{iw} \) | 0.0001, 0.0001, 0.0001 | 0.001, 0.001, 0.001 |
| Learning-rate of robust compensator \( l_c \) | 0.01       | 0.01       |

TABLE 4. Comparison in root mean square error (RMSE) of WIT2TFBCC and other methods.

| Method                  | Computation Time (ms) | RMSE1 | RMSE2 | RMSE3 | RMSE4 | RMSE5 | Average RMSE |
|-------------------------|-----------------------|-------|-------|-------|-------|-------|--------------|
| CMAC [36]               | 0.15                  | 0.0111| 0.0111| 0.2648| 0.0105| 0.2251| 0.1047       |
| TSKCMAC [35]            | 0.17                  | 0.0090| 0.0061| 0.1916| 0.0057| 0.2703| 0.0965       |
| BELC [18]               | 0.18                  | 0.0066| 0.0027| 0.1227| 0.0026| 0.3329| 0.0935       |
| RCFCBC [15]             | 0.20                  | 0.0052| 0.0016| 0.0894| 0.0015| 0.3751| 0.0946       |
| T2FBEBC [18]            | 0.205                 | 0.0047| 0.0013| 0.0790| 0.0016| 0.3815| 0.0936       |
| DFLFBCE [15]            | 0.21                  | 0.0061| 0.0020| 0.1211| 0.0020| 0.3264| 0.0915       |
| Our method              | 0.22                  | 0.0040| 0.0010| 0.0532| 0.0010| 0.3977| 0.0914       |

\[ \Delta V_{\nu}(t) = -\beta_{\nu} e^2(t) |Z_T(t)|^2 \left[ 1 - \frac{1}{2} l_T (Z_T(t))^2 \right] \]  

(75)

If \( l_c \) is chosen as

\[ 0 < l_c < \frac{2}{(\max(Z_T))^2} \]  

(76)
Therefore, \( \frac{1}{2} \hat{h}(Z_T(t))^2 \leq 1 \), so \( \Delta V_p(t) < 0 \). Thus, the convergence of the online learning algorithm is guaranteed by the Lyapunov stability theorem in Section II. The error between the desired output and the output of the WIT2TFBCC converges to zero if \( t \to \infty \).

**IV. WIT2TFBCC FOR 5-D HYPERCHAOTIC SYNCHRONIZATION**

The parameters of the 5D hyper chaotic system are given in Table 5. where \( \text{rand}(\cdot) \) builds a random number in range \([0, 1]\). The parameters for the proposed WIT2TFBCC control system are selected for WIT2TFBCC, the parameters of the proposed WIT2TFBCC are shown in Table 2.
The external perturbations and uncertainties of the hyperchaotic system are shown in Figs. 4(a) and (b), respectively. The 3D projections of the synchronization for the 5D hyperchaotic system using the WIT2TFBCC are shown in Fig. 5, and the synchronization of each axis is shown in Fig. 6. The tracking errors are shown in Fig. 7, and the control effort is shown in Fig. 8. From the simulation results, the proposed method synchronizes well with small tracking errors. In particular, the average RMSE of WIT2TFBCC is reduced and reaches the smallest value among all (see Table 4). However, WIT2TFBCC requires longer computation time due to its complex structure. This is reasonable because the higher the performance, the larger the computational cost.

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V. SIMULATION RESULTS OF WIT2TFBCC FOR HENON MAP TIME SERIES PREDICTION

Our method can be applied to the prediction of time series systems such as Henon map time series to illustrate the performance of our method. The Henon map is chosen as the prediction plant, which is referred to as [1] and [7]:

\[ y(k + 1) = -1.4y^2(k) + 0.3y(k - 1) + 1 \]  

(77)

This system has the past input-output values \( y(k) \), \( y(k - 1) \) therefore a parallel model is needed for the time series prediction. 1000 data points are chosen from the system over the interval \([-1.51.5]\). The initial conditions are defined as \( y(0) = y(1) = 0.4 \). Next, we use the training architecture for chaos time series prediction in Fig. 9, which shows the reference output and the predicted output of training and test data for the chaotic Henon map system. Fig. 10 shows the training process with 1000 samples between the blue solid line (the reference output) and the red dotted line (the output prediction with WIT2TFBCC). Fig. 11 (a) shows the testing process with 100 samples and Fig. 11(b) shows the result of the error signal between the reference output and the prediction output with our method. Fig. 12 shows the RMSE between our method and the others FBELC, RCFBC and T2BELC, the RMSE of our method is the best among them all because it converges to zero faster and more elastic. Table 5 shows the comparison of our method with the others, the test RMSE of our method is the smallest among all, but the calculation time is the longest because of the complexity of our method.

VI. CONCLUSION

In this study, the method for designing the WIT2TFBCC controller for synchronizing chaotic systems and predicting time series is proposed. The proposed controller has been successfully tested for synchronization of 5-D hyperchaotic systems and prediction of Henon Map time series. For the convergence of the chaotic synchronization system, an additional compensator with the sign function is used due to the complexity of the chaotic system. However, for the prediction of time series, no compensator is used, only optimal convergence according to the learning rate must be used, since the time series change continuously with time. Future studies can be 1) using optimal algorithms, such as modified grey wolf optimization (MGWO), to find the optimal learning rates faster, 2) applying our method for hardware applications.

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**TABLE 5. Comparison of proposed method with others.**

| Methods       | Computation time (ms) | Testing RMSE |
|---------------|-----------------------|--------------|
| WIT2TFBCC     | 18.45                 | 0.0128       |
| RCFBC         | 11.85                 | 0.0234       |
| T2BELC        | 17.23                 | 0.0217       |
| FBELC         | 10.75                 | 0.0245       |
| WITFCMNN [7]  | --                    | 0.0186       |
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CHIH-MIN LIN (Fellow, IEEE) was born in Changhua, Taiwan, in 1959. He received the B.S. and M.S. degrees from the Department of Control Engineering, National Chiao Tung University, Hsinchu, Taiwan, in 1981 and 1983, respectively, and the Ph.D. degree from the Institute of Electronics Engineering, National Chiao Tung University, in 1986. He is currently a Chair Professor of Yuan Ze University, Taoyuan, Taiwan. He also serves as an Associate Editor for IEEE Transactions on Cybernetics and IEEE Transactions on Fuzzy Systems. From 1997 to 1998, he was the Honor Research Fellow at the University of Auckland, New Zealand. He has published more than 200 journal articles and 160 conference papers. His research interests include fuzzy neural networks, cerebellar model articulation controller, intelligent control systems, adaptive signal processing, and classification problem.

VAN NAM GIAP received the B.S. degree in control engineering and automation from the Hanoi University of Science and Technology, Hanoi, Vietnam, in 2015, the master’s degree in electronic engineering from the National Kaohsiung University of Applied and Sciences, Kaohsiung, Taiwan, in 2017, and the Ph.D. degree in mechanical engineering from the National Kaohsiung University of Science and Technology, Taiwan, in June 2021. He is currently with the Hanoi University of Science and Technology, Vietnam. His research interests include sliding mode control, disturbance and uncertainty estimation, fuzzy logic control, secure communication, the magnetic bearing system and its applications, and self-bearing motors.

TUAN-TU HUYNH (Member, IEEE) was born in Ho Chi Minh City, Vietnam, in 1982. He received the B.S. degree in electrical & electronics from the Department of Electrical & Electronics Engineering, Ho Chi Minh University of Technology and Education, Vietnam, in 2005, the M.S. degree in automation from the Ho Chi Minh City University of Transport, Vietnam, in 2010, and the Ph.D. degree in electrical engineering from Yuan Ze University, Chung-Li, Taoyuan, Taiwan, in 2018. He is currently a Research Fellow with the Department of Electrical Engineering, Yuan Ze University. He is also a Lecturer with the Faculty of Mechatronics and Electronics, Lac Hong University, Vietnam. His research interests include MCDM, fuzzy logic control, neural networks, cerebellar model articulation controller, brain emotional learning-based intelligent controller, deep learning, and intelligent control systems.

HSING-YUEH CHO was born in Taiwan, in 1989. He received the master’s degree in electrical engineering from the Chien Hsin University of Science and Technology, Zhongli, Taiwan, in 2013. He is currently pursuing the Ph.D. degree in electrical engineering with Yuan Ze University, Taoyuan. His research interests include fuzzy logic control, adaptive control, cerebellar model articulation controllers, and intelligent control systems.

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