A measure of the importance of roads based on topography and traffic intensity

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Abstract. Mathematical models of street traffic allowing assessment of the importance of their individual segments for the functionality of the street system is considering. Based on methods of cooperative games and the reliability theory the suitable measure is constructed. The main goal is to analyze methods for assessing the importance (rank) of road fragments, including their functions. A relevance of these elements for effective accessibility for the entire system will be considered.

Keywords: component importance · coherent system · road classification · graph models · traffic modelling.

Subject Classifications:MSC 68Q80 · (90B20; 90D80)

1 Introduction.

1.1 Historical remarks and motivations.

The function of a road network is to facilitate movement from one area to another. As such, it has an important role to play in the urban environment to facilitate mobility. It furthermore determines the accessibility of an (urban) area (together with public transport options). In many studies on the design and maintenance of roads, the authors raise the problem of alternative connections needed to ensure efficient transport between strategic places (cf. Lin (2010), Tacnet et al. (2012)). It is known that individual segments of the road structure are exposed to various types of threats, resulting in temporary disconnection of such couplings. As a result, the road network determines the quality of life in the analyzed area. Therefore, it is worth trying to define measurable parameters, the quality of road connections, the road system constituting the infrastructure used for transport. Further considerations focus on road systems for road transport. However, the proposed approach can be successfully applied to other similar structures.

When designing, it is worth conducting an analysis of the effects of excluding individual segments and determining the measures that allow for the identification of critical ones. However, the difficulty with this kind of economic appraisal is first of all that it is not easy to measure the valuation of travel time. Different people and organizations value travel time in different ways, depending on many factors such as income, goal of the trip, social background, etc (cf. Cherlow (1981)). It is relatively easier to measure the value travel time than the highway security measure (v. Sharpe (2012)). The purpose

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of the work is to determine the importance of roads segment in road traffic. Consideration will be commonly known measures of significance used to evaluate components of binary systems. Road topography is a long-term process that cannot be changed in a short time. Therefore, it is important to ensure safe road traffic when planning communication infrastructure. To this end, it is important to introduce objective methods for assessing weak links in the road system. Using methods of stochastic processes and game theory, a quantitative approach to the importance of various elements of infrastructure will be proposed. The introduced connection assessment proposals will be illustrated using information about the actual local road network in the selected city (see Example 1.2).

1.2 A motivating example.

In the presented work, the network of streets ensuring access from point $A$ to point $B$ in Zduńska Wola will be treated as a system. The diagram of the streets analyzed can be seen in Figure 1a. Let us emphasize that the purpose of modeling is not to reflect the current traffic on the network, as shown in Figure 1b, but to establish the importance of network elements due to their objective importance for the functioning of the road system.

![Fig.1: Analysed traffic network.](image)

In research, there are many measures that allow assessing the importance of individual components, based on the system structures, lifetimes and reliability of individual components or methods of estimating significance based on the methods of turning on and off. The most classic methods based on reliability theory will be used in the paper. To this end, the street network will be presented in the form of a system, where each road is presented as a separate component. Then, based on the construction of the system, the structure function will be determined, thanks to which it will be possible to calculate the meaning of individual components and the corresponding streets. The next stage will be defining the theory of traffic in the context of significance measures. This area will be examined in relation to the satisfaction and comfort of drivers. Drivers satisfaction means that the system works properly, if not the system is failed. So as reliability of particular road we consider probability of driver’s satisfaction from the journey. A road connection system in a given area should allow transport in a predictable time

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3 Source: Google Maps.
A measure of the importance of roads

between different points. Extending this time has a negative effect on drivers. As a result, their right ride quality is compromised and they are more likely to fail to comply with the rules. Therefore, providing drivers with driving comfort and satisfaction is also important for general road safety. The use of this approach can, therefore, be a guide for both drivers and road builders planning road infrastructure.

1.3 The paper organization.

The purpose of the research presented here is to implement of various importance measures introduced in the reliability theory to analyze the impact of elements of road networks. The theory related to significance measures and their use in traffic theory is described in the section 2. There is a close relationship between road delays and the construction and function of both the road and the intersection that forms part of it. For this purpose, simulations of vehicle traffic on the analyzed roads were performed. The theory related to the method of modeling vehicle traffic and their behavior at intersections is described in section 3. Section 4 describes the real traffic network, its transfer to the simulation model, and the results obtained in this way. Then, on this basis, the importance of individual fragments was calculated depending on the intensity of traffic on these roads.

Considering this work is a look at the impact on the comfort of communication of the road structure in connection with traffic without directly referring to the behavior of drivers, which was devoted the paper Szajowski and Włodarczyk (2020). In these previous works, significant dependence on traffic quality on drivers’ compliance with applicable rules was shown. Here, a similar approach was applied to the condition of changing behavior to incorrect, which may further result in a deterioration in traffic quality. Therefore, the results obtained show important elements of the road network that have an impact on road safety and properly functioning.

2 Importance measure.

The operation of most systems depends on the functioning of its individual components. It is important to ensure the proper running of the entire system. To this end, it is important to assess the contribution of individual components. In road networks, network curves model road segments, intersections, and special places on the road that have a significant impact on the flow of traffic, such as railway crossings, tunnels, bridges, viaducts or road narrowing. In order to estimate the importance of particular elements, the concept of importance measures was introduced (for detailed description of the concept and its extension to multilevel elements and systems see review paper by Amrutkar and Kamalja (2017)). Since 1969 researchers offer various numerical representations to determine which components are the most significant for system reliability. It is obvious that the greater are these values, the more this element have on the functioning of the entire system. The significance of individual elements depends on the system structure as well as the specificity and failure rate of individual elements. There are three basic classes of measures of importance (v. Amrutkar and Kamalja (2017), Birnbaum (1969), Średnicka (2020)):

i Reliability importance measures subordinate changes in the reliability of the system depending on the change in the reliability of individual elements over a given period of time and in depend on the structure of the system.

ii Structural importance measures are using when just the structure of the system is known. Depending on the position of the components in the system, their relative importance is measured.
iii **Lifetime importance measures** focus on both, components position in the system and lifetime distribution of each element. According to Kuo and Zhu if it is a function of the time it can be classified as Time-Depend Lifetime (TDL) importance and if it is not a function of time we have Time Independent Lifetime (TIL) importance.

Moreover, depending on the number of states, systems can be divided into two types:

i **Binary systems** — comprised of \( n \) components, where each of them can have precisely one of two states. State 0 when the component is damaged and state 1 when is working.

ii **Multistate systems (MSS)** — comprised of \( n \) components, which can undergo a partial failure, but they do not cease to perform their functions and do not cause damage to the entire system.

### 2.1 Concepts of importance measures.

Establishing the hierarchy of components of a complex system has been reduced to measuring the influence of the element state on the status of the entire system. The concept of an element (system) state depends on the context. For the needs of the road network analysis, we assume, similarly to the reliability theory, a binary description of both elements and the system (v. also Ramamurthy (1990)). For this purpose, we will use the known results on significance measures obtained in research on this subject developed in recent years. Importance measures have been developed in many directions and under many definitions. However, one of the most popular areas of development and application are:

- theory of cooperative games (in simple game)
- reliability theory (in coherent and semi-coherent structure)

Many methods have been developed to combine and standardize the terminology associated with both applications. Therefore, to begin with, we must briefly mention the relationship between importance measure theory in the context of both concepts. The first attempt to define it was made by Ramamurthy (1990), so the following notation was proposed:

\begin{align*}
(1) \quad & \emptyset \in P, \text{ where } P \text{ is set of subset of } N; \\
(2) \quad & N \in P, \text{ where } N \text{ is finite, nonempty set}; \\
(3) \quad & S \subseteq T \subseteq N \text{ and } S \in P \text{ imply } T \in P.
\end{align*}

The concepts related to game theory and reliability theory were compared with each other and on this basis, it was possible to define the relationship between these concepts. To begin with, it is easy to see the relationship between players and components. According to game theory, we have a set of players \( N = \{1, 2, 3, \ldots, n\} \) and a family of coalitions \( 2^N \). In the theory of reliability, we have a set of components \( N = \{1, 2, 3, \ldots, n\} \), where the components and the entire system can be in two states, state 1 for functioning and state 0 for failed. Similarly is in game theory, where \( \lambda : 2^N \to \{0, 1\} \), which is applied in simple game if on set \( N \) form of characteristic function fulfils

\begin{align*}
(1) \quad & \lambda(\emptyset) = 0; \\
(2) \quad & \lambda(N) = 1; \\
(3) \quad & S \subseteq T \subseteq N \text{ implies } \lambda(S) \leq \lambda(T).
\end{align*}

Here this characteristic function has its counterpart as a structure function, and simple games as semi-coherent structures. In addition, also winning and blocking coalitions are comparable to path and cut sets.
2.2 Important measures on binary reliability systems.

In the classical approach the system and their elements are binary (v. Birnbaum(1969), Birnbaum et al.(1961)). Let the system comprised of \( n \) components can be denoted by \( c = (c_1, c_2, ..., c_n) \). The description of the vector of component states (in the short state vector) \( x = (x_1, x_2, ..., x_n) \), where each \( x_i = \chi_W(c_i) \), \( c_i \in \{ W, F \} \) (\( W \) - means the element is functioning; \( F \) - means the element is failed). For state vector, we can use below notations [5]

\[
\begin{align*}
\bar{x} &\leq \bar{y} \quad \text{if} \quad x_i \leq y_i \quad \text{for} \quad i \in \{1, \ldots, n\} \\
\bar{x} &= \bar{y} \quad \text{if} \quad x_i = y_i \quad \text{for} \quad \forall i \in \{1, \ldots, n\} \\
\bar{x} &< \bar{y} \quad \text{if} \quad \bar{x} \leq \bar{y} \quad \text{and} \quad x \neq y \\
(1, x) &= (x_1, x_2, x_3, \ldots, x_{i-1}, 1, x_{i+1}, \ldots, x_n) = (1, x_i) \\
(0, x) &= (x_1, x_2, x_3, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n) = (0, x_i) \\
\bar{0} &= (0, 0, \ldots, 0) \quad \bar{1} = (1, 1, \ldots, 1).
\end{align*}
\]

If the structure of the system is known, we can define the state of the system \( \phi(\bar{x}) \) as Boolean function (structure function ) of the state vector.

If from \( x_i \leq y_i \) for \( i \in \{1, \ldots, n\} \) results \( \phi(\bar{x}) \leq \phi(\bar{y}) \), and \( \phi(\bar{1}) = 1 \), \( \phi(\bar{0}) = 0 \), then we call the system coherent. It is known (v. Birnbaum (1969)) that for every \( i = 1, 2, \ldots, n \) structure function can be decomposed as follows:

\[
\phi(\bar{x}) = x_i \cdot \delta_i(\bar{x}) + \mu_i(\bar{x}),
\]

where \( \delta_i(\bar{x}) = \phi(1_i, \bar{x}) - \phi(0_i, \bar{x}) \), \( \mu_i(\bar{x}) = \phi(0_i, \bar{x}) \) are independent of the state \( x_i \) of the component \( c_i \).

In addition, we can observe situations where the system can be functioning even if some components are failed. The smallest set of functioning elements that ensures the operation of the entire system is called minimal path. The opposite situation is observed in the case of minimal cut set, which is the minimum set of components whose failure cause the whole system to fail. We can define the structure function as a parallel structure of minimal paths. According to the definition, this structure is damaged if, and only if all of the components are failed. So the system consists of \( n \) minimal paths series, denoted by \( \rho_i(\cdot) \), for \( i = 1, 2, \ldots, n \), can be presented as:

\[
\phi(\bar{x}) = \prod_{i=1}^{n} \rho_i(\bar{x}) = 1 - \prod_{i=1}^{n} \left[1 - \rho_i(\bar{x}) \right].
\]

Similarly, the structure function can be presented as series of minimal cut sets. So for \( n \) minimal cut parallel structures, marked by \( \kappa_i(\cdot) \), for \( i = 1, 2, \ldots, n \), structure function looks as follows:

\[
\phi(\bar{x}) = \prod_{i=1}^{n} \kappa_i(\bar{x}).
\]

If we simply replace the minimum paths and minimum cut sets with components, the formulas (2) and (3) apply for serial and parallel components.

In most of the considerations about the functioning of systems, it is assumed that elements work independently. Then the state of \( i \)-th element is a binary random variable \( X_i \) and the reliability that the element \( i \) is unimpaired will be denoted by \( p_i \), where

\[
p_i = P(X_i = 1) = 1 - P(X_i = 0).
\]
We also define the vector of reliabilities for \( n \) elements by
\[
\vec{p} = (p_1, p_2, \ldots, p_n).
\] (5)

Based on reliabilities vector and structure function we can define the probability of the system functioning
\[
P(\phi(x) = 1|\vec{p}) = E[\phi(x)|\vec{p}] = h_\phi(\vec{p}).
\] (6)

For the structure \( \phi(\vec{x}) \) function \( h_\phi(\vec{x}) \) is called reliability function.

### 2.3 Reliability importance measure.

As was introduce, reliability importance measures are based on changes in reliabilities of components and on the system structure. This measure first time was introduced by Birnbaum (1969). At the beginning, from formulas (4), (5) and (1) he express the reliability function by \( h_\phi(\vec{p}) = p_i \cdot E[\delta_i(X)] + E[\mu_i(X)] \), where, for every \( i = 1, 2, \ldots, n \) and according to equation (1), we have
\[
\frac{\partial h_\phi(\vec{p})}{\partial p_i} = E[\delta_i(X)] = E \left[ \frac{\partial \phi(X)}{\partial X_i} \right].
\]

According to Birnbaum (1969) the reliability importance of the component \( c_i \) for structure \( \phi(\cdot) \) is defined as \( I_i(\phi; p) = I_i(\phi; 1; p) + I_i(\phi; 0; p) \), where \( I_i(\phi; 1; p) = P\{\phi(X) = 1|X_i = 1; \vec{p}\} - P\{\phi(\vec{X}) = 1; \vec{p}\} \), and \( I_i(\phi; 0; p) = P\{\phi(X) = 0|X_i = 0; p\} - P\{\phi(X) = 0; \vec{p}\} \)\(^4\). We have the following useful identities
\[
I_i(\phi; 1; \vec{p}) = (1 - p_i) \cdot \frac{\partial h_\phi(\vec{p})}{\partial p_i} = E[(1 - X_i)\delta_i(X)]
\]
\[
I_i(\phi; 0; \vec{p}) = p_i \cdot \frac{\partial h_\phi(\vec{p})}{\partial p_i} = E[X_i\delta_i(X)]
\]
\[
I_i(\phi; \vec{p}) = \frac{\partial h_\phi(\vec{p})}{\partial p_i} = E[\delta_i(X)].
\]

The Birnbaum importance measures for \( i = 1, 2, \ldots, n \) have forms (the symbol \( \phi \) is dropped for short)
\[
B(i|\vec{p}) = \frac{\partial h_\phi(\vec{p})}{\partial p_i} = \frac{\partial [1 - h_\phi(\vec{p})]}{\partial [1 - p_i]},
\] (7)

here \( B(i|p) \) is \( p \) dependent. In case when reliabilities vector \( \vec{p} \) is unknown, we have to consider structural importance defined for \( i = 1, 2, \ldots, n \) in the following way
\[
B(i) = I_i(\phi) = \left. \frac{\partial h_\phi(\vec{p})}{\partial p_i} \right|_{p_1 = \ldots = p_n = \frac{1}{2}},
\] (8)

this information will be useful in the next section.

### 2.4 Structural importance measures.

When we looking for relevant component \( c_i \) for the structure \( \phi(\cdot) \) and the state vector \( \vec{x} \) is known, we are going as follow definition \( \delta_i(\vec{x}) = \phi(1_i, \vec{x}) - \phi(0_i, \vec{x}) = 1 \). We can also highlight definitions if the component \( c_i \) is relevant for the functioning of structure\(^4\) \( I_i(\phi; 1; \vec{p}) \) and \( I_i(\phi; 0; \vec{p}) \) are the reliability importance of the \( c_i \) component for functioning and failure of the structure, respectively.
φ(·) at the state vector \( \vec{x} \) if \((1 - x_i) \cdot \delta_i(\vec{x}) = 1\), and, if the component \( c_i \) is relevant for the failure of structure \( \phi(\cdot) \) at the state vector \( \vec{x} \) gives \( x_i \cdot \delta_i(x) = 1 \). Distinctly, depends on if the coordinate \( x_i \) of the vertex \( \vec{x} \) is equal to 0 or 1, then \( c_i \) is relevant for functioning or failure of the system.

Birnbaum (1969) defined structural importance measure of component \( c_i \) for the functioning of the structure \( \phi(\cdot) \) as \( I_i(\phi, 1) = 2^{-n} \sum (1 - x_i) \cdot \delta_i(x) \), where sum extends on all combinations \( 2^n \) of vertices of the state vectors. In the similar way is defined structural importance measure of the component \( c_i \) for the failure of structure \( \phi(\cdot) \) by \( I_i(\phi, 0) = 2^{-n} \sum x_i \cdot \delta_i(x) \). Finally, by summarizing, the structural importance measure of the component \( c_i \) for the structure \( \phi(\cdot) \) is \( I_i(\phi) = I_i(\phi, 1) + I_i(\phi, 0) = 2^{-n} \sum \delta_i(x) \).

Barlow and Proschan (1975) used a more extended approach to structural measures. Their point of view assumes that all components have a continuous lifetime distribution, denoted by \( F_i \), for \( i = 1, 2, \ldots, n \). It is possible to calculate the probability of a system failure caused by the \( c_i \) component. For the \( c_i \) component, which is described by the distribution \( F_i \) and the density function \( f_i \), the probability that a system failure at time \( t \) was caused by the \( c_i \) component can be described as follows

\[
\frac{\int_0^{t} [h(1, \bar{F}(u)) - h(0, \bar{F}(u))]dF_i(u)}{\int_0^{t} \sum_{k=1}^n [h(1_k, \bar{F}(u)) - h(0_k, \bar{F}(u))]dF_k(u)}.
\]

In the consequence of (9), it obvious to define the probability that failure of the system in \([0, t]\) was caused by the \( c_i \) component is

\[
I_i^{BS}(\phi) = \int_0^1 [h(1, p) - h(0, p)]dp.
\]

where \((1, p)\) and \((0, p)\) is a probability vector where \( i \)-th component has probability equal 1 or 0, relatively.

For further calculations, let us remind quick note from Section 2.2, that minimal path is the minimal set of elements, which ensures the proper functioning of the system. Based on this we can define critical path set for component \( c_i \) as \( \{i\} \cup \{j|x_j = 1, i \neq j\} \). In this way, information about the system is functioning or failed is determined by the \( c_i \) component functions or fails. A critical path vector (or set) for the component \( c_i \), and its size \( r \), we have \( 1 + \sum_{i \neq j} x_j = r \), for \( r = 1, 2, \ldots, n \). The formula for counting the number of vectors of critical paths for the component \( c_i \) with size \( r \) is the following

\[
n_r(i) = \sum_{\sum_{j \neq x_i} x_j = r-1} [\phi(1_j, x) - \phi(0_j, x)].
\]

Finally, we can define the structural importance of the component \( c_i \) using the number of vectors of critical paths \( n_r(i) \) as follows

\[
I_i^{BP}(\phi) = \sum_{r=1}^n n_r(i) \cdot \frac{(r-1)!(n-r)!}{n!}.
\]
The equation (11) can be also presented in two more interesting expressions. The first expression is the following

\[ I_{BP}^{i}(\phi) = \frac{1}{n} \sum_{r=1}^{n} n_r(i) \binom{n-1}{r-1}^{-1}, \]

where \( n_r(i) \) describes the number of vectors of critical paths with size \( r \). The denominator in the above equation represents the amount of results in which precisely \( r - 1 \) components are in operation among the \( n - 1 \) components without \( c_i \) component. Second additional representation of equation (11) can be written as follows

\[ I_{BP}^{i}(\phi) = \int_{0}^{1} \left[ \sum_{r=1}^{n} n_r(i) \cdot \binom{n-1}{r-1}^{-1} \cdot \binom{n-1}{r-1} \cdot (1-p)^{n-r} \cdot p^{r-1} \right] dp, \]

here \( \binom{n-1}{r-1}(1-p)^{n-r}p^{r-1} \) means the probability that from the \( n - 1 \) components without \( c_i \) component, \( r - 1 \) elements are functioning. What’s more, \( n_r(i) \binom{n-1}{r-1}^{-1} \) means the probability that \( r - 1 \) functioning elements including \( c_i \) component determine the critical path set to the \( c_i \) component. So multiplication of them means the probability that components \( c_i \) is responsible for system failure and integral of it over \( p \) is that reliability for the component \( c_i \) is a uniform distribution \( p \sim U(0, 1) \).

As it was written at the beginning in Section 2.1 there is a big connection between the concepts related to game theory and the theory of reliability. The measure introduced by Barlow and Proschan is an example of this. This definition is reflected in cooperative games as Shapley’s value, which informs what profit a given coalition player can expect, taking into account his contribution to any coalition.

2.5 Importance measures of road segments based on traffic flow in example 1.2.

As was said in section 1, binary systems are considered. The analyzed system is a street network allowing access from \( A \) to \( B \), it is possible in several ways. We assume that drivers drive only from \( A \) to \( B \), straight, without unnecessary U-turns on the route. Streets were presented at the beginning in Fig. 1a and can be transform to the form of the system (a scheme) as on Fig. 2.

Fig. 2: Analysed traffic network presented in system form.
Based on the system representation of the streets network we can determine the structure function. As we know, the structure function can be defined using either minimal path set or minimal cut set. So for the given structure both sets are presented in the tables 1 and 2.

Table 1: Minimal path set.

| Path | Elements           |
|------|-------------------|
| 1    | 1 2 3 8 12        |
| 2    | 1 2 5 9 11 12     |
| 3    | 4 6 9 11 12       |
| 4    | 4 7 10 11 12      |

Based on tables 1 and 2, it is possible to define minimal path series structures represented by the following equations

\[
\rho_1(x) = \prod_{\{1,2,3,8,12\}} x_i = x_1 \cdot x_2 \cdot x_3 \cdot x_8 \cdot x_{12} \quad \rho_2(x) = \prod_{\{1,2,5,9,11,12\}} x_i
\]

\[
\rho_3(x) = \prod_{\{4,6,9,11,12\}} x_i \quad \rho_4(x) = \prod_{\{4,7,10,11,12\}} x_i
\]

and minimal cut parallel structures described as follows

\[
\kappa_1(x) = \prod_{\{1,4\}} x_i = x_1 \parallel x_4 \quad \kappa_2(x) = \prod_{\{2,4\}} x_i \quad \kappa_3(x) = \prod_{\{1,6,7\}} x_i
\]

\[
\kappa_4(x) = \prod_{\{2,6,7\}} x_i = x_2 \parallel x_6 \parallel x_7 \quad \kappa_5(x) = \prod_{\{4,5,3\}} x_i \quad \kappa_6(x) = \prod_{\{4,5,8\}} x_i
\]

\[
\kappa_7(x) = \prod_{\{1,6,10\}} x_i = x_1 \parallel x_6 \parallel x_{10} \quad \kappa_8(x) = \prod_{\{2,6,10\}} x_i \quad \kappa_9(x) = \prod_{\{3,5,6,7\}} x_i
\]

\[
\kappa_{10}(x) = \prod_{\{3,9,7\}} x_i = x_3 \parallel x_9 \parallel x_7 \quad \kappa_{11}(x) = \prod_{\{3,9,10\}} x_i \quad \kappa_{12}(x) = \prod_{\{8,9,7\}} x_i
\]

\[
\kappa_{13}(x) = \prod_{\{8,9,10\}} x_i = x_8 \parallel x_9 \parallel x_{10} \quad \kappa_{14}(x) = \prod_{\{3,4,9\}} x_i \quad \kappa_{15}(x) = \prod_{\{4,8,9\}} x_i
\]

\[
\kappa_{16}(x) = \prod_{\{3,11\}} x_i = x_3 \parallel x_{11} \quad \kappa_{17}(x) = \prod_{\{8,11\}} x_i \quad \kappa_{18}(x) = \prod_{\{1,11\}} x_i
\]

\[
\kappa_{19}(x) = \prod_{\{2,11\}} x_i = x_2 \parallel x_{11} \quad \kappa_{20}(x) = \prod_{\{12\}} x_i = x_{12}
\]

From the definition in equation (2) and based on the above equations, we can write the structure function of the presented system as follows

\[
\phi(x) = \rho_1(x) \parallel \rho_2(x) \parallel \rho_3(x) \parallel \rho_4(x) = 1 - (1 - \rho_1(x))(1 - \rho_2(x))(1 - \rho_3(x))(1 - \rho_4(x))
\]

Table 2: Minimal cut set.

| Cut Elements | Cut Elements |
|--------------|--------------|
| 1            | 1 4          |
| 2            | 2 4          |
| 3            | 1 6 7        |
| 4            | 2 6 7        |
| 5            | 4 5 3        |
| 6            | 4 5 8        |
| 7            | 1 6 10       |
| 8            | 2 6 10       |
| 9            | 3 5 6 7      |
| 10           | 3 9 7        |
|              | 11 3 9 10    |
|              | 12           |
|              | 13           |
|              | 14           |
|              | 15           |
|              | 16           |
|              | 17           |
|              | 18           |
|              | 19           |
|              | 20           |
|              | 12           |
In addition, our structure function can be also expressed by the series of minimal cut structures

\[ \phi(x) = \prod_{i=1}^{20} \kappa_i(x). \]

And finally, thanks to equation (6), we can write the reliability function of the analyzed system

\[
h_{\phi}(p) = 1 - (1 - \prod_{\{1,2,3,8,12\}} p_i)(1 - \prod_{\{1,2,5,9,11,12\}} p_i)(1 - \prod_{\{4,6,9,11,12\}} p_i) \\
\times (1 - \prod_{\{4,7,10,11,12\}} p_i),
\]

where \( p_i \), for \( i = 1, 2, \ldots, 12 \), are some probabilities, which definition will be introduced in the next section.

### 2.6 Reliability importance applied to road networks.

To consider reliability importance we need to define what exactly means that system is functioning or failed. We assume that the condition of the system’s functioning is the comfort and satisfaction of drivers. The state 1 will mean the driver’s satisfaction with a given road section or route, the state 0 — dissatisfaction. For drivers, the measure of satisfaction is the travel time on a given section of the road, and more precisely, the realization of the road according to the planned travel time. Drivers want to finish the journey in the shortest possible time. The excess of this time, i.e. the delay on a given section of the road after exceeding a certain critical level causes dissatisfaction of drivers with the journey. This critical level that causes dissatisfaction may be different for each driver and is close to the lifetime. Weibull distribution is often used to represent the lifetime of objects. A similar approach was used in a paper published by Fan et al. in 2014. The cited article considered a situation when, while waiting before entering the intersection, the waiting time for a given driver exceeded a certain critical value, the driver stopped complying with traffic rules. Like here, this critical value was determined by Weibull distribution. The variable from the Weibull distribution can be represented as the cumulative distribution function given by the following formula:

\[
F(t) = \begin{cases} 
1 - \exp \left( - \left( \frac{t}{\lambda} \right)^k \right), & \text{for } t > 0, \\
0, & \text{otherwise.} 
\end{cases}
\]

Based on the cumulative distribution function, it is possible to calculate the reliability function, i.e. the function that tells the probability of correct functioning of an object. We parametrize the segments by the acceptable delay time \( t \) by the driver. The population of the diver is not homogeneous. The acceptable delay is the random variable with some distribution \( \Pi \). The delay of travel \( \tau \) is a consequence of various factors. Let us assume that its cumulative distribution is \( F(t) \). We will say that the segment is reliable or works for given driver with accepted delay \( t \) if \( \tau(\omega) \geq t \). The probability \( Q(t) \) of the event is the subjective driver reliability of the segment. Its expected probability with respect to \( \Pi \), \( p = \int_0^\infty Q(u)d\Pi(u) \) is mean reliability of the segment. For the homogeneous class of drivers the delay time \( \xi \) on given road section is common value for all drivers, so (mean) reliability will means probability that for assumed delay time driver is satisfied from the journey. Therefore, according to the theory of importance measures, \( p_i \), which indicates the reliability of the segment will be determined as probability of driver’s satisfaction and \( 1 - p_i \) will means probability that driver is dissatisfied of journey for assumed delay time. It can be determine as following formula:

\[
p_i = P(X_i = 1|t = \xi) = 1 - P(X_i = 0|t = \xi) = Q(\xi),
\]
where \( \xi \) is the delay time, \( X = 1 \) means the driver is satisfied with the road, \( X = 0 \) he is dissatisfied. The paper adopts the same Weibull distribution parameters as in paper by Fan et al. (2014), i.e. \( \lambda = 30, k = 2.92 \). When we know the relationship between street reliability from the time of delays, we can define the reliability of these route fragments for a given traffic intensity. Different road sections react differently to increasing traffic intensity, which is why reliabilities will be different. Using simulations we determine the dependence of traffic intensity and delay times.

2.7 Continuing example of section 1.2

We will now proceed to briefly introduce these definitions on a simple example. Let us assume that we have a shortened road network scheme limited to Piwna 1, Zlotnickiego, Laska, Sieradzka 1 streets. This scheme is presented in the way shown in Figure 3. Here we have the components \( c_4, c_{11} \) in series, and the components \( c_6, c_{10} \) in parallel. So we can define this system as "\( k \)-out-of-\( n \)" structure, where \( n \) is number of all components, \( k \) means number of components in series, and \( n - k \) is the number of components in parallel. On this basis, we can define the structure function as

\[
\phi(\vec{x}) = x_4 \cdot (1 - (1 - x_6) \cdot (1 - x_{10})) \cdot x_{11},
\]

and the system reliability function corresponding to the above

\[
h(\vec{p}) = p_4 \cdot (1 - (1 - p_6) \cdot (1 - p_{10})) \cdot p_{11}.
\]

To begin with, we assume that the reliability of individual components is unknown, so only structural measures of significance can be calculated. They will be calculated based on the definitions introduced in Section 2.4. Two proposals for structural measures have been introduced; first proposed by Birnbaum, which assume that each reliability \( p_i \) of components \( c_i \), for \( i = 1, 2, \ldots, n \) are the same and equal to \( \frac{1}{2} \) and second the Barlow and Proschan Importance Measures, which is define for \( p \in [0, 1] \). So using this theory and definitions in equation (8) for Birnbaum Importance and in (10) for Barlow and Proschan Importance, we count the importance of analyzed components. Obtained results are presented in Table 3. We see that roads connected in series are more important than roads connected in a parallel way. This is consistent with the logic, if one
of the parallel roads is blocked, you can always choose a different route that will allow you to reach your destination. For streets in a serial connection, this is not possible. We also note the differences in the values of the importance measures calculated using the Birnbaum and Barlow-Proshcan definitions, this is because the first measure is calculated for the constant reliability of the elements equal to $\frac{1}{2}$, so it only examines the relationship between element positions. The second measure takes into account, apart from the structure itself, also the variability of reliability of individual elements.

Now we will examine the reliability importance measures for the simplified system shown in Figure 3. Let us assume that for a given traffic intensity, we have certain delay times, on this basis, we will calculate the probability that drivers are still satisfied with the travel along the road, i.e. the reliability of the road. Next, for these values, using the formula (7), we calculate the value of the measures of significance defined by Birnbaum. The assumed delay times, as well as the corresponding reliability and importance, will be presented in Table 4. As we can see, with a road delay of about 25 seconds, the likelihood of driver satisfaction is close to $\frac{1}{2}$, and in the case of delays of about 5 seconds, drivers do not experience almost the negative effects of a slowdown in traffic. With such reliability of streets and with such a scheme, it is easy to notice some issues: streets in a parallel position have less contribution to potential nervousness or driver satisfaction than in a serial connection, in addition, in the case of streets in a series, those with less reliability are more important, so these should be paid greater attention to maintain proper traffic quality. In the case of streets in parallel connection, streets with greater reliability are more important. It is logical that drivers knowing that the road is a better way will choose it, so it is important to constantly maintain it in good condition because when it fails the whole connection will lose much reliability.

### Table 4: Hypothetical calculations of importance measures in the example system.

| Id | Street name | Delay $\xi$ | Probability of satisfaction $Q(\xi)$ | Importance $B(i|p)$ |
|----|-------------|-------------|--------------------------------------|--------------------|
| 4  | Piwna 1     | 25 s        | 0.5559                               | 0.8513             |
| 6  | Zlotnickiego| 20 s        | 0.7363                               | 0.0025             |
| 10 | Laska       | 5 s         | 0.9947                               | 0.1249             |
| 11 | Sieradzka 1 | 16 s        | 0.8526                               | 0.5551             |

2.8 Structural importance of real traffic network.

As was presented in the previous sections, if the reliability of individual components is unknown, it is possible to use structural measures of significance. Therefore, we will begin our considerations about the analyzed system by calculating the structural significance of individual roads. In the same way as in the previous section, the definition of significance measures introduced by Birnbaum, and Barlow and Proschan presented in section 2.3 by the equations (8) and (10), respectively, were used. The results obtained are presented in Table 5. We see that the results of both measures are similar. As was expected, the most important for the entire route is street Sieradzka 2, because each route finally leads along this street, for B-P importance for these streets is bigger than for B-importance. Next, the most important part of the route is Sieradzka 1, we see that 3 of 4 ways to obtain point $B$ are going by this street. For this street, Birnbaum's value is smaller than the Barlow-Proshcan's value. The importance of Piwna 1 is the last value of importance bigger than 0.1, anyway similar to this value are importances of Dolna, Zlota, and Nyska 2. Surmise that the significance of the Zlota and Dolna will be close
Table 5: Structural importance of roads in the analyzed system.

| Id | Street name | Birnbaum Importance $B(i;\phi)$ | Barlow-Proshan Importance $I_{BP}^{R}(\phi)$ |
|----|-------------|-------------------------------|----------------------------------|
| 1  | Dolna       | 0.0861                        | 0.0973                           |
| 2  | Zlota       | 0.0861                        | 0.0973                           |
| 3  | Mickiewicza | 0.0577                        | 0.0531                           |
| 4  | Piwna 1     | 0.1155                        | 0.1202                           |
| 5  | Nyska 1     | 0.0284                        | 0.0338                           |
| 6  | Zlotnickiego| 0.0577                        | 0.0531                           |
| 7  | Piwna 2     | 0.0577                        | 0.0531                           |
| 8  | Jasna       | 0.0577                        | 0.0531                           |
| 9  | Nyska 2     | 0.0861                        | 0.0973                           |
| 10 | Laska       | 0.0577                        | 0.0531                           |
| 11 | Sieradzka 1 | 0.1439                        | 0.1882                           |
| 12 | Sieradzka 2 | 0.2016                        | 0.3690                           |

to the value calculated for Piwna 1 was not difficult. However, it is not so easy to guess the similarity of the importance of Nyska 2 street to Dolna and Zlota. Mickiewicza, Zlotnickiego, Piwna 2, Jasna, Laska and Nyska 1 streets have the smallest contribution to the proper functioning of the entire connections between $A$ and $B$.

3 Traffic modelling

3.1 Review and history.

Traffic modeling is a particularly complex issue. There are both modeling of individual phenomena occurring on roads and entire road networks. The first research into vehicle movement and traffic modeling theory began with the work of Bruce D. Greenshields (1935). On the basis of photographic measurement methods, he proposed basic and empirical relationships between flow, density, and speed occurring in vehicle traffic. Next, Lighthill and Whitham (1955) and Richards (1956) introduced the first theory of movement flow. They presented a model based on the analogy of vehicles in traffic and fluid particles. Interest in this field has increased significantly since the nineties, mainly due to the high development of road traffic. As a result, many models were created describing various aspects of road traffic. As a result, many models were created describing various aspects of road traffic and focusing on different detail models, we can distinguish:

– microscopic models
– mesoscopic models
– macroscopic models

The differences in the models are at the level of aggregation of modeled elements. Mesoscopic models based mainly on gas kinetic models. Macroscopic models based on first and second-order differential equations, derived from Lighthill-Whitham-Richards (LWR) theory. Microscopic models focus on the simulation of individual vehicles and their interactions. One of the most popular are car-following models and cellular automata models, the last is used in this paper. The most popular cellular automata traffic model is the Nagel-Schereckenberg (1992) model, but also very interesting model is LAI model (cf. Lárraga and Alvarez-Icaza (2010)), which is more advance than NaSch model. LAI model is used in this paper, therefore, in the next section theory about cellular automata will be introduced and later will be a more detailed description of LAI model.
3.2 Cellular automaton

Janos von Neumann, a Hungarian scientist working at Princeton, is the creator of cellular automata theory. In addition, the development of this area was significantly influenced by the Lviv mathematician Stanislaw Ulam, who is responsible for discrediting the time and space of automats and is considered to be the creator of the definition of cellular automats as "imaginary physics"[19]. According to a book written by Ilachinski, cellular automata can reliably reflect many complex phenomena with simple rules and local interactions. Cellular automata are a network of identical cells, each of which can take one specific state, with the number of states being arbitrarily large and finite. The processes of changing the state of the cells run parallel and according to the rules. These rules usually depend on the current state of the cell or the state of neighboring cells. From the mathematical point of view, cellular automatas are defined by the following parameters [22] [35]:

- **State space** — a finite, $k$-element set of values defined for each individual cell.
- **Cell grid** — discrete, $D$-dimensional space divided into identical cells, each of which at a given time $t_h$ has one, strictly defined state of all possible $k$ states. In the case of the 2D network, the cell status at $i, j$ is indicated by the symbol $\sigma_{ij}$.
- **Neighborhood** — parameter determining the states of the nearest neighbors of a given cell $i,j$, marked with the symbol $N_{ij}$.
- **Transition rules** — rules determining the cell state in a discrete time $t_{h+1}$ depending on the current state of this cell and the states of neighboring cells. The state of the cell in the next step is presented in the following relationship:

$$\sigma_{ij}(t_{h+1}) = F(\sigma_{ij}(t_h), N_{ij}(t_h)),$$

where:

- $\sigma_{ij}(t_{h+1})$ — cell state in position $i,j$ in step $t_{h+1}$,
- $\sigma_{ij}(t_h)$ — cell state in position $i,j$ in step $t_h$,
- $N_{ij}(t_h)$ — cells in the neighborhood of a cell in position $i,j$ in step $t_h$.

The way the cell neighborhood is defined has a significant impact on the calculation results. The most common are two types:

- **Von Neumann neighborhood** Each cell is surrounded by four neighbors, immediately adjacent to each side of the cell being analyzed. The neighborhood for $i,j$ constructed in this way is as follows:

$$N_{i,j}(t_h) = \begin{pmatrix}
\sigma_{i-1,j}(t_h) \\
\sigma_{i,j-1}(t_h) \\
\sigma_{i,j}(t_h) \\
\sigma_{i,j+1}(t_h) \\
\sigma_{i+1,j}(t_h)
\end{pmatrix},$$

- **Moore neighborhood** Each cell is surrounded by eight neighbors, four directly adjacent to the sides of the analyzed cell, and four on the corners of the analyzed cell. The neighbor cell matrix for $i, j$ looks like this:

$$N_{i,j}(t_h) = \begin{pmatrix}
\sigma_{i-1,j-1}(t_h) & \sigma_{i-1,j}(t_h) & \sigma_{i-1,j+1}(t_h) \\
\sigma_{i,j-1}(t_h) & \sigma_{i,j}(t_h) & \sigma_{i,j+1}(t_h) \\
\sigma_{i+1,j-1}(t_h) & \sigma_{i+1,j}(t_h) & \sigma_{i+1,j+1}(t_h)
\end{pmatrix}.$$

There are also modifications to the above types, such as the combined neighborhood of Moore and von Neumann, as well as numerous modifications to the Moore neighborhood itself, and a different way defined by Margolus to simulate falling sand.
In addition, boundary conditions are an important aspect of cellular automata theory. Since it is impossible to produce an infinite cellular automaton, some of the simulations would be impossible because with the end of the automaton’s grid the history of a given object or group of objects would end. For this purpose, boundary conditions at the ends of the grid were introduced. There are the following types of boundary conditions:

- periodic boundaries — cells at the edge of the grid behind neighbors have cells on the opposite side. In this way, the continuity of traffic and ongoing processes is ensured.
- open boundaries — elements extending beyond the boundaries of the grid cease to exist. This is used when new objects are constantly generated, which prevents too high density of objects on the grid.
- reflective boundaries — on the edge of the automaton a border is created, from which the simulated objects are reflected, most often it serves to imitate the movement of particles in closed rooms.

In the next section the model using cellular automata used in the simulation will be presented. Open boundary conditions are used in our simulations. After leaving the street, vehicles disappear. This is in line with logic, new vehicles are constantly appearing and disappearing on the roads. The applied neighborhood is a modified version of the presented neighborhoods, because vehicles as their neighbors take those vehicles that are nearby, and more specifically the nearest vehicle on the road, even if it is not directly adjacent to the analyzed vehicle, and also cars move by more cell. We can assume that it is a more extended version of the Von Neumann neighborhood.

### 3.3 The vehicles movement.

In order to define vehicle traffic rules and simulate their movement, the model proposed by Lárraga and Alvarez-Icaza (2010) was used. The proposed model meets the general behavior of vehicles on the road. Drivers with free space ahead are traveling at maximum speed. Approaching the second vehicle, drivers react to changes in its speed, providing themselves with a constant space for collision-free braking. This model is often called LAI model, from the authors’ names. This part of the work will include a description of this model and also comments on possible assumptions.

The model presents traffic flow at a single-lane road, where vehicles move from left to right. The road is divided into 2.5-meter sections, and each is presented as a separate cell. The length of the car is taken as 5 meters what is represented as two cells. Each cell can be empty or occupied only by part of one vehicle. The position of the vehicle is determined by the position of its front bumper. Vehicles run at speeds from 0 to $v_{\text{max}}$, which symbolize the number of cells a vehicle can move in one-time step $t$. The time step corresponds to one second. The speed conversion from simulation to real is presented in the Table 6.

Here in the first column, we have the velocity used in the model, next column presents how distance is done in a one-time step (1 second), the next columns present real velocity in m/s and km/h for better imagine how the model works. In simulations we decide to used maximum speed equals to 45 km/h, because traffic flow in the city is considered, so drivers have not too much space to fast driving.

The model takes into account the limited acceleration and braking capabilities of vehicles and also ensures appropriate distances between vehicles to guarantee safe driving. Three distances calculated for the car following to its predecessor are included. These values calculate the distance needed for safe driving in the event that the driver wants to slow down ($d_{\text{dec}}$), accelerate ($d_{\text{acc}}$) or maintain the current speed ($d_{\text{keep}}$), assuming
Table 6: The relationship between real and simulation speeds in the model used.

| Distance | Real speed | Real speed |
|----------|------------|------------|
| 2.5 m    | 2.5 m/s    | 9 km/h     |
| 5 m      | 5 m/s      | 18 km/h    |
| 7.5 m    | 7.5 m/s    | 27 km/h    |
| 10 m     | 10 m/s     | 36 km/h    |
| 12.5 m   | 12.5 m/s   | 45 km/h    |
| 15 m     | 15 m/s     | 54 km/h    |
| 17.5 m   | 17.5 m/s   | 63 km/h    |

that the predecessor will want to suddenly start slowing down with the maximum force $M$ until to stop. They are calculated as follows:

$$d_{acc} = \max \left(0, \sum_{i=0}^{\left\lfloor \frac{v_n(t) + \Delta v}{M} \right\rfloor} \left[ \left( v_n(t) + \Delta v \right) - i \cdot M \right] - \sum_{i=0}^{\left\lfloor \frac{v_{n+1}(t) - M}{M} \right\rfloor} \left[ \left( v_{n+1}(t) - M \right) - i \cdot M \right] \right) \quad (13a)$$

$$d_{keep} = \max \left(0, \sum_{i=0}^{\left\lfloor \frac{v_n(t)}{M} \right\rfloor} \left[ v_n(t) - i \cdot M \right] - \sum_{i=0}^{\left\lfloor \frac{v_{n+1}(t) - M}{M} \right\rfloor} \left[ \left( v_{n+1}(t) - M \right) - i \cdot M \right] \right) \quad (13b)$$

$$d_{dec} = \max \left(0, \sum_{i=0}^{\left\lfloor \frac{v_n(t) - \Delta v}{M} \right\rfloor} \left[ \left( v_n(t) - \Delta v \right) - i \cdot M \right] - \sum_{i=0}^{\left\lfloor \frac{v_{n+1}(t) - M}{M} \right\rfloor} \left[ \left( v_{n+1}(t) - M \right) - i \cdot M \right] \right) \quad (13c)$$

Here, vehicle $n$ is the follower, and $n + 1$ is the preceding car. $v_n(t)$ means the value of the velocity of vehicle $n$ in time $t$, $\Delta v$ is the ability to accelerate in one-time step and $M$ is ability to emergency braking.

Updating vehicle traffic takes place in four steps, which are done parallel for each of the vehicles.

I. Calculation of safe distances $d_{dec_n}$, $d_{acc_n}$, $d_{keep_n}$.

II. Calculation of the probability of slow acceleration.

III. Speed update.

IV. Updating position.

**Safe distances.** According to formulas 13 safe distances are counted for each vehicles. The calculation of these values is based on the assumption that if the vehicle in the next time step $t + 1$ increases its speed (or maintains it or slows it down respectively) and the driver preceding from the moment $t$ will constantly slow down to speed 0 (with maximum ability to emergency braking), there will be no collision. The different between these equation is just in first part, which define traveled distance by vehicle $n$ if it decelerate ($v_n(t + 1) = v_n(t) - \Delta v$), keep velocity ($v_n(t + 1) = v_n(t)$) or accelerate ($v_n(t + 1) = v_n(t) + \Delta v$), in next time step, and next begins to brake rapidly. The second part of equation determines the distance traveled by the preceding vehicle if it starts to braking with maximum force $M$.

**Calculation of the probability of slow acceleration.** The second step in the vehicle movement procedure focuses on calculating the stochastic parameter $R_a$ responsible for slowing down vehicle acceleration. It is assumed that low-speed vehicles have more troubles to accelerate. According to human nature and the mechanism of the car, it is true that is that the faster we go, the easier we manage to accelerate, and standing or driving very slowly cause slower acceleration. The limiting speed at which acceleration comes easier is assumed to be 3, which corresponds to 27 km/h. The value of $R_a$
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Parameter is calculated based on the formula

\[ R_a = \min(R_d, R_0 + v_n(t) \cdot (R_d - R_0)/v_s), \]  
(14)

where \( R_0 \) and \( R_d \) are fixed stochastic parameters, mean respectively probability to accelerate when the speed is equal to 0, and probability to accelerate when the speed is equal or more than \( v_s \), and \( v_s \) limit speed below which acceleration is harder.

Easily can be seen, that the relationship between the probability of acceleration at speed 0 and at a speed greater than the limit is interpolated linearly, which is taken from the idea presented also by Lee et al. [16]. In the simulations, 0.8 and 1 were adopted as \( R_0 \) and \( R_d \) parameters, respectively, which will not cause frequent difficulties in accelerating vehicles, however, the stochastic nature of this process will be taken into account. The graph of the \( R_a \) parameter change for the other parameters adopted in this way is presented in Figure 4.

![Fig. 4: Values of \( R_a \) parameters for fixed \( R_0, R_d \) and \( v_s \) [15].](image)

**Speed update.** In the beginning, as mentioned before \( \Delta v \) means speed increase in one time step, fixed for all vehicles. \( v_n(t) \) and \( x_n(t) \) determine the velocity and the position of vehicle \( n \) in time \( t \). Distance from vehicle \( n \) to vehicle \( n+1 \) is counted by the following formula

\[ d_n(t) = x_{n+1}(t) - x_n(t) - l_s, \]

which exactly means the distance from front bumper of vehicle \( n \) pointed by \( x_n(t) \) to rear bumper of the vehicle in the front, presented by the difference between the position of the front bumper \( x_{n+1}(t) \) and the length of the vehicle \( l_s \) (in cells). The speed update is done in four steps, the order of which does not matter.

1. **Acceleration.** If the distance to the preceding vehicle is greater than \( d_{aac_n} \) then the vehicle \( n \) increase velocity by \( \Delta v \) with probability \( R_a \), what is presented as follows

\[ v_n(t + 1) = \begin{cases} \min(v_n(t) + \Delta v, v_{max}), & \text{with prob. } R_a \\ v_n(t), & \text{otherwise} \end{cases} \]

In this rule is assumed that all drivers strive to achieve the maximum velocity if it is possible. Here is include irregular ability to accelerate depends on the distance to preceding vehicles, relevant velocities of both, and stochastic parameter responsible for slower acceleration defined in Step II.

2. **Random slowing down.** This rule allows drivers to maintain the current speed, if it allows safe driving, it also takes into account traffic disturbances, which are an indispensable element of traffic flow. The probability of random events is determined
by the \( R_s \) parameter. If \( d_{acc,n} > d_n(t) \geq d_{keep,n} \), then the updated speed is determined according to the formula

\[
v_n(t + 1) = \begin{cases} 
\max(v_n(t) - \Delta v, 0), & \text{with prob. } R_s \\
v_n(t), & \text{otherwise}
\end{cases}
\]

3. Braking. This rule ensures that the drivers keep an adequate distance from front vehicles. Rapid braking is not desirable, so in order to ensure a moderate braking process for the driver, when the free space in front of the car is too small, the vehicle speed is reduced by \( \Delta v \), which reflects optimal braking.

\[v_n(t + 1) = \max(v_n(t) - \Delta v, 0) \quad \text{if} \quad d_{keep,n} > d_n(t) \geq d_{dec,n}\]

4. Emergency braking. As can be seen in real life, it is not always possible to brake calmly. What is more, road situations often force more aggressive braking. Such situations are included in this rule. When the driver gets too close to the other car, or when the other car brakes too much, it forces emergency braking. If the distance is at least \( d_{dec} \), this rule is not applied. According to the commonly accepted standard proposed in the literature (v. Alvarez and Horowitz Larraga and Alvarez-Icaza, the emergency braking force is set to \(-5\) m/s². With respect to the assumed model parameters the value of \( M \) is 2. This step is described by

\[v_n(t + 1) = \max(v_n(t) - M, 0) \quad \text{if} \quad d_n(t) < d_{dec,n}\]

**Updating position** Finally, with updated vehicle speed, it is possible to actualize vehicle positions. The vehicles are moved by the number of cells according to their speed. This is described by means of

\[x_n(t + 1) = x_n(t) + v_n(t + 1),\]

where \( x_n(t + 1) \) is actualized position, \( v_n(t + 1) \) is the previously determined vehicle speed, and \( x_n(t) \) is last position of vehicle.

### 3.4 Intersections

Intersections are an inseparable element of road traffic, they are an intersection with a road at one level. All connections and crossroads also count in intersections. There are the following types of intersections:

- uncontrolled intersections
- intersections with traffic signs
- crossings with controlled traffic (traffic lights or authorized person)

Modeling of traffic at intersections is an important element of road traffic modeling, many models have been created on this subject, such as models simulating the movement of vehicles at intersections of type T [34], describing the movement at un-signalized intersections as in the case of [28], [10] and those considering traffic at intersections with traffic lights [4]. Typically, these models consist of two aspects, modeling vehicle traffic and modeling interactions at intersections. General rules are set for intersections, however, the behavior of drivers who may or may not comply with these rules is also taken into account. Modeling of such behavior is also different, which usually distinguishes these models. This aspect was consider in my engineering thesis [31]. Helpful in modeling interactions at intersections is game theory, which facilitates the decision
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about the right of way, where players are drivers in conflict at the intersection, examples of such use can be seen in [21]. Additionally, signalized intersection models using Markov chain are often used, as in the case of [33].

However, the purpose of the work is simple modeling of road traffic, therefore advanced intersection modeling methods will not be considered. It is assumed that all drivers comply with traffic regulations and follow road safety. The consequences of changing behavior to incorrect and inconsistent with traffic rules are not investigated. The purpose of the work is to find elements that affect the potential threat affecting the reluctance of drivers to comply with traffic rules. Depending on the maneuver performed by the drivers and the type of intersection, the following situations need to be modeled:

- turn right from the road without right of way
- turn right from the road with right of way
- turn left from the road with right of way
- turn left from the road without right of way
- go straight ahead at traffic lights
- turn left at traffic lights

When modeling the above situations, two basic rules were used:

Rule 1 a driver who wants to join the traffic on the main road can do, if and only if, during the whole process, until the maximum speed is reached, he does not disturb the driving of other vehicles on the main road. This maneuver may be described by the following formula:

\[
l_x - v_x - \sum_{\Delta v=2}^{v_{max}} \min(v_{max}, v_x + \Delta v - 1) + \sum_{\Delta v=2}^{v_{max}} \Delta v > d_{keep_x},\]

where \(l_x\) is the distance of the vehicle on the main road to the intersection, \(v_x\) is his current vehicle speed. The first sum symbolizes the distance traveled by the vehicle on the main road until the passing vehicle reaches maximum speed. The second sum represents the distance traveled by the vehicle joining the traffic until it reaches maximum speed, assuming that in the first second the vehicle will be at an intersection with a speed equal 1. Both vehicles increase their speed by 1 in each second and they do not exceed the maximum speed. The value of the left side of the inequality must be greater than the distance needed by the driver on the main road to maintain his speed. Otherwise, the driver would be forced to brake which would disturb his movement.

Rule 2 The driver wanting to cross the opposite direction road can do it if there is no collision with the opposite direction during the time needed to complete it and the opposite driver will not be forced to brake. The time it takes to complete the maneuver depends on the initial speed at the start of the maneuver. This relationship is described in the Table 7.

| Velocity \(v_n\) | Need time \(\tau_n\) |
|-----------------|---------------------|
| 2               | 1s                  |
| 1               | 2s                  |
| 0               | 3s                  |

Table 7: Relationship between the time of crossing of the opposite road and the initial speed.
The condition ensuring the correct execution of the maneuver can be described by the following inequality:

\[ l_x - \sum_{\Delta v=0}^{\tau_x} \min(v_{\text{max}}, v_x + \Delta v - 1) > d_{\text{dec}} \]

where \( l_x \) is the distance of the opposite car opposite to the intersection and the sum is responsible for calculating the distance traveled by this car in the time needed to complete the turn. The value of the left side of the inequality must be greater than the speed needed for safe braking by the vehicle. Otherwise, it would force the driver to emergency braking, which is not desirable, and in the event of a possible delayed reaction of the driver could lead to an accident.

The above rules are the basis used to define behavior at intersections, more information on where the rules were applied, and why, it will be described in Section 4.1.

In addition, modeling of traffic lights was needed. The traffic light scheme was used in accordance with the Polish regulations. The traffic light cycle follows the diagram 5. The duration and meaning of individual signals are as follows:

1. **Red light** — no entry behind the signal light. The duration is 60 seconds.
2. **Red and yellow light** — means that in a moment will be a green signal. According to the regulations, it lasts 1 s.
3. **Green light** — allows entry after the signal light if it is possible to continue driving and this will not cause a road safety hazard. The duration is the same as for the red signal, equal to 60 s.
4. **Yellow light** — does not allow entry behind the signal light, unless stopping the vehicle would cause an emergency brake. According to the regulations, it should last at least 3 seconds.

Such a traffic light cycle and the duration of each signal were adopted in the simulation. Of course, there is also a relationship between the capacity of intersections and the time of the traffic light cycle. However, the most standard signaling scheme was adopted to ensure optimal intersection capacity. In addition, it was assumed that both directions of travel are equivalent, which is why this cycle is the same on both roads.

In accordance with the theory described for traffic modeling, as well as with the proposed method of traffic conditioning at intersections. For each street from the diagram in the drawing 1, traffic simulations were performed in the MATLAB package. Real and simulated street sizes are presented in Section 4.1 in the next chapter. For each simulation, the time it took me from the beginning of the road to leaving the intersection at its end was calculated for each vehicle. Simulations have been carried out many times for different probabilities of a new driver appearing on the road, which in the further understanding will be taken as traffic intensity.
3.5 Model calibration

An important aspect in the case of traffic modeling, which we could not fail to mention in this chapter, is the calibration of models. In general, this topic is part of a larger problem, which is simulation optimization. The area of development of simulation optimization in recent years has enjoyed great interest among researchers and practitioners. Simulation optimization is the pursuit of the maximum performance of a simulated real system. The system performance is assessed based on the simulation results, and the model parameters are the decision variables. The assessed performance in this case is the model’s ability to recreate reality. Therefore, it is a very important topic in modeling traffic, which aims to enable the reconstruction of real vehicle traffic, so correctly choose the model parameters so that the model used is a reliable model and correctly shows the modeled behavior characteristics. Optimization in the context of motion simulation models has evolved in many areas and the only ones were optimization and calibration of motion, but they were not often combined with optimization theory, where some of the problems in motion modeling are well known. One of the most important conclusions is that there is no algorithm that is suitable for all problems and needs and that the choice of the right algorithm depends on the example being examined (v. Spall et al.(2006)). Most studies focused on testing the performance of the optimization algorithm, where models are evaluated against actual traffic data, e.g. Hollander and Liu in 2008. However, based on real traffic data, it is not possible to evaluate the effectiveness of the algorithm and the entire calibration process. Another approach proposed in the literature is the use of synthetic measurements, i.e. data obtained from the model itself. This approach was proposed e.g. by Ossen and Hoogendoorn(2008), and tested changes in model calibration due to the use of errors in synthetic motion trajectories by Ciuffo et al.(2007), which used tests with synthetic data to configure the process of calibration of microscopic motion models, based on trial and error.

4 Simulation

4.1 Description of real traffic network

Using the models proposed in section 3, a simulation of vehicle movement was performed on each street presented in the Figure 1. The model of vehicle traffic along a straight road is presented in Section 3.3. The modeling movement between streets was more complicated. Section 3.4 describes the general rules needed to define traffic at intersections. There are, various maneuvers required simulation. In addition, the actual road lengths have been converted into simulation values to best reflect the road traffic. Table 8 describes real and simulation road lengths and maneuvers that should be performed on a given road section. In the case of Nyska 1, Sieradzka 1, and Sieradzka 2 streets, drivers go through given section with priority, driving straight ahead.

For Dolna and Zlotnickiego roads, drivers join the traffic on the main road being on a road without the right of way. Rule 1 was applied, assuming that when approaching an intersection, drivers must slow down to a speed of 0 or 1, and then decide according to the condition described.

At Mickiewicza street, at the end of the road, the driver is forced to slow down to 2, which corresponds to the real speed of 18 km/h, we can assume that this is a reasonable speed to make a turn. After decelerating, drivers can leave the intersection.

For Zlota and Piwna 1 streets, drivers with probability $\frac{1}{3}$ turn left, otherwise they go straight. Before turning, the drivers slow down to at least speed 2, if they can cross the opposite direction lane they continue driving if they do not slow down more. Therefore, drivers must give way to oncoming vehicles, Rule 2 applies.
Table 8: Description and base information on analysed roads.

| Id | Street name | Length In meters | Intersections and turning |
|----|-------------|------------------|--------------------------|
| 1  | Dolna       | 300 m            | Give way on Zlota and turn right |
| 2  | Zlota       | 350 m            | Give way oncoming vehicles and turn left or go straight |
| 3  | Mickiewicza | 500 m            | Turn right with right of way |
| 4  | Piwna 1     | 450 m            | Give way oncoming vehicles and turn left or go straight |
| 5  | Nyska 1     | 160 m            | Go ahead with right of way |
| 6  | Zlotnickiego| 500 m            | Give way on Nyska 1 and turn right |
| 7  | Piwna 2     | 180 m            | Give way vehicles on the main road and turn left |
| 8  | Jasna       | 400 m            | and Give way vehicles on the main road and turn left |
| 9  | Nyska 2     | 200 m            | Give way oncoming vehicles and turn left on intersection |
| 10 | Laska       | 500 m            | Go ahead, but wait on traffic lights |
| 11 | Sieradzka 1 | 500 m            | Go ahead with right of way |
| 12 | Sieradzka 2 | 500 m            | Go ahead to the end of road |

In the case of Piwna 2 and Jasna streets, we assume that the drivers slow down before the intersection to 0 or 1 and with probability \( \frac{1}{2} \) turn right or left. In both situations, it is necessary to apply Rule 1, because drivers must give way to vehicles that are on the road they are turning, in addition in the case of a left turn, Rule 2 should be applied too because the vehicle will cross the opposite direction.

The last two traffic situations relate to traffic at intersections with traffic lights. When driving along Laska Street, in the event of red light, drivers wait at the intersection, then they can leave it. The case where drivers would like to turn left is not being considered because in real life a left lane is intended for a left turn. When leaving Nyska 2 Street, drivers may ride to the right, left, or straight. In the case of a right turn or straight ahead, the process goes without any problems, so we allow drivers to leave the intersection. When turning left, you must pass vehicles driving in the opposite direction, so Rule 2 applies.

According to the above assumptions, simulations were made, and repeated 1000 times for each traffic intensity to obtain the average values of delays depending on the intensity. The sample code and description of the program are at the end of the work in Appendix A.

### 4.2 Simulations results

In accordance with the characteristics described in the previous section, simulations of motion were made. The results of road delays are shown in graph 6. We see that the delay increase characteristics for different roads are different. It is easy to see that one of the most difficult streets to travel are Jasna and Piwna 2, we see here a high sensitivity to traffic intensity. Another group of streets in terms of delays are Zlotnickiego and Dolna, and also Laska Street is similar to them, although the growth characteristics are different. Nyska 2 has a completely different behavior from the rest, but it is the only street with such a complex intersection, including traffic lights. In this case, the delay increases very quickly, reaching a critical level for this street, related to the capacity of the road. Therefore, despite the fact that the final result of the delay is not the largest, it can be considered that the efficiency of this intersection is the worst. The next, but
\textbf{A measure of the importance of roads} \hfill 23

Delays on streets depend on traffic intensity

Fig. 6: Time of delays depend on traffic intensity for different roads.

definitely more efficient streets are Piwna 1 and Zlota, and the most fluid traffic can be seen on the last 4 streets, where there are no intersections and traffic disturbances. In addition, both Sieradzka streets have the same delay times, because they both are without intersections streets and they have the same length.

Next, using the approach introduced in Section 2.6 and based on the calculated delay times, it is possible to determine the probability of driver satisfaction with a given section of the route. An undesirable phenomenon is exceeding a certain critical level of delay time, which will cause dissatisfaction to the driver. The probability that the critical value for a given delay is not exceeded is described by the reliability function of the Weibull distribution. Using this, the probability of driver satisfaction for a given delay on each road will be calculated depending on traffic intensity. These probabilities are presented in Figure 7. We see that the reliability of individual streets is different. Ones of the streets already at a low traffic intensity reach a critical state, which will cause drivers’ dissatisfaction for sure (these are e.g. ulice Nyska 2, Piwna 2, Jasna, Laska, Dolna, Zlotnickiego). We can also observe streets such as Nyska 1, where traffic is constantly flowing and does not irritate drivers. This drawing shows us that it is true that individual streets react differently to increasing traffic. Therefore, it is worth examining how these changes affect the overall functioning of the traffic network and the importance of individual fragments.

Fig. 7: Probabilities of drivers’ satisfaction vs. traffic intensity for different roads.
4.3 Values of Importance Measures

To begin with, we calculate the reliability of the entire system depending on traffic and
the reliability of each route. In this way, we obtain the probability of finish the journey
with the satisfaction of all roads. In order to calculate the reliability values of individual
roads \( p_i \), for \( i = 1, 2, \ldots, 12 \) are substituted the appropriate values in the formula
(12). In addition, we will calculate the reliability of individual routes that correspond
to the minimum paths. The relationship between the elements of each route is in series.
Individual routes include the following streets:

**Route 1:** Dolna, Zlota, Mickiewicza, Jasna, Sieradzka 2

**Route 2:** Dolna, Zlota, Nyska 1, Nyska 2, Sieradzka 1, Sieradzka 2

**Route 3:** Piwna 1, Zlotnickiego, Nyska 2, Sieradzka 1, Sieradzka 2

**Route 4:** Piwna 1, Piwna 2, Laska, Sieradzka 1, Sieradzka 2

| Traffic intensity | Probability of satisfaction from All routes Route 1 Route 2 Route 3 Route 4 |
|-------------------|-----------------------------------------------|-------------------------------|----------------|----------------|----------------|
| 0.050             | 1.0000                                       | 0.9974                        | 0.9973         | 0.9948         | 0.9948         |
| 0.075             | 1.0000                                       | 0.9989                        | 0.9970         | 0.9961         | 0.9969         |
| 0.100             | 1.0000                                       | 0.9855                        | 0.9598         | 0.9574         | 0.9807         |
| 0.125             | 0.9963                                       | 0.8737                        | 0.5440         | 0.5422         | 0.8596         |
| 0.150             | 0.1396                                       | 0.0841                        | 0.0006         | 0.0006         | 0.0595         |
| 0.175             | 0                                            | 0                             | 0              | 0              | 0              |
| 0.200             | 0                                            | 0                             | 0              | 0              | 0              |
| 0.600             | 0                                            | 0                             | 0              | 0              | 0              |

The calculated reliability values are presented in Table 9. We can see that the system
is no longer efficient at a traffic intensity of 0.175. In addition, we see that the capacity
of the system is always greater than the efficiency of individual roads. This is important
information regarding the critical value of traffic intensity that causes failure of the en-
tire network. In addition, we can see that the capacity of Routes 1 and 4 is greater,
which may suggest that with heavy traffic it is better to choose one of these two routes
to ensure a better chance of a quiet ride.

We will now proceed to calculate the importance of individual roads in the function-
ing of the entire system. The calculated values are shown in Table 10. For each street,
received values of measure of significance at a given traffic intensity were presented. As
mentioned before, these values are calculated on the basis of the structure function (12)
and importance measures theory introduced by Birnbaum (7) using the received reli-
bility for individual traffic intensities. As we can see, the most interesting results were
obtained for the traffic intensity of 0.125 and 0.150. At low traffic intensities, the re-
liability of individual elements does not affect the functioning of the system, because
the whole system works properly and the reliability of the roads are close to 1. For
the intensity of 0.125, the contribution of individual streets begins to be noticeable. We
see that, according to structural measures, the largest contribution to the functioning
of the network has Sieradzka 2 street, the next streets have the value of importance
close to 0.03, with the exception of Nyska 1, Zlotnickiego, and Nyska 2, which are
smaller. For the intensity of 0.150, we can see that there are difficulties in movement. The first thing that draws our attention is the importance of Nyska 2 Street, which was one of the smallest before, now it has become the most significant element. Another important component of the system is again Sieradzka 2, which is obvious. However, the streets that are worth paying attention to are Piwna 2 and Jasna, whose significance has also risen dramatically. With subsequent increases in intensity, we see that only these 3 streets really affect the quality of traffic, and of them the most street Nyska 2.

### 4.4 Comparison with real life data.

Based on the analysis made in the previous section, the streets Nyska 2, Piwna 2, and Jasna have the greatest importance for the appropriate functioning of the entire system at high traffic intensity. The analyzed traffic system is a real traffic network, which is why we know what traffic really looks like on individual roads. The presented scheme of travel from A to B shows the travel from two strategic positions in the city. The main streets in the city are Laska and Sieradzka, they pass through the center of the city. The traffic "on top" of Laska Street is greater than on Sieradzka Street because here we are already approaching the exit from the city. The results obtained are in line with expectations. One of the most important points in the city is the Nyska–Laska–Sieradzka intersection. In fact, this intersection is more extensive and we can see that a lot of work has been put into its proper functioning. Many simplifications are used there, which would also slightly change the results obtained from the simulation. For example, vehicles turning left into Sieradzka Street have more space so, when they are waiting for a turn, they do not obstruct the traffic of other vehicles going straight or turning right. In addition, time counters are used on the traffic lights that increase drivers’ watchfulness and their start when the green light comes on.

The intersection of Piwna and Laska streets was critical enough that it was impossible to turn left there, the sign ‘right to turn right’ was in force. This was a major impediment to general traffic as well as to the routes presented in the paper. That is why a roundabout intersection has recently been built here. This decision certainly required a lot of consideration by the city authorities, because there is not enough space for a full-size roundabout here, so it has a slightly flattened one side. However, as can be seen from the results obtained, it was one of the critical parts of traffic in the city, so this decision seems sensible.

The last problematic street is Jasna, but here in real traffic, there is no such intensity of vehicles, both on Jasna Street and the "bottom" part of Sieradzka Street. Assuming
that traffic in this part of the city is smaller and that turning into Mickiewicza street is not very problematic gives important information to drivers who considered which of these two roads is better.

5 Summarizing and conclusions.

A quantitative approach to road quality assessment is proposed. The measures of significance defined for the reliability systems were used as a tool to calculate the importance of individual road fragments. An actual traffic network diagram was analyzed, ensuring access from point A to point B. To begin with, assuming that only the structure of the analyzed road network is known, the structural significance of individual road fragments was calculated. For this purpose, two measures were used: proposed by Birnbaum, assuming constant reliability of individual road fragments, and Barlow and Proschan measure, which takes into account the variability of individual element reliability. There were noticeable differences between the received values, but the final result was similar in both cases. The most important for maintaining the efficiency of the analyzed road network are the streets that occur in the largest number of possible routes to the point B as a serial connection. This confirmed our expectations, but also helped to locate some of the roads that at the first consideration were not potentially important routes. When comparing the differences between Birnbaum and Barlow-Proschan measures, the second one was considered more appropriate for use in the context of road traffic, because the reliability of individual roads are not the same, many factors affect on them.

Then a method of assessing the reliability of street elements was proposed. For this purpose, it was assumed that the quality of roads is the satisfaction of drivers with the route traveled, and the delay time on individual roads was used as a measure of this. It was assumed that drivers have limited patience, which is close to a lifetime and was presented as a variable from the Weibull distribution. Having calculated the delay times on individual roads, it was possible to determine the probability of upset the driver at such a delay. However, in order for the obtained value to be able to be used in the theory of measures of importance, it had to be transformed so that it was responsible for the reliability of a given element. Therefore, the Weibull distribution reliability function was used, which in our example reflected the probability that with a given road delay, the driver would still be satisfied. Alternate method of reliability assessing to the net of roads has been used by Pilch and Szybka (2009).

In the paper by Szajowski and Włodarczyk (2020), it was shown that if drivers are dissatisfied with driving then they can stop complying with traffic rules. And one of the factors influencing their change and negative behavior on the road is delays. Therefore, the measures defined in this way are a guide for both drivers and traffic managers. For drivers, it shows which road is better to avoid because there is a chance of potential nervousness, and gives road drivers information about dangerous points in the city and points that have a negative impact on drivers. In addition, the large delay time on individual roads indicates the failure of the fragments concerned. Based on the simulations performed, the delay times on each road were calculated. Then it was shown which road fragments are the most important. The obtained results were confronted with the actual feelings regarding the given fragments. And they were considered likely because with the network as defined it was used as the most significant elements that were improved in real traffic. Which proves the real importance of these elements.

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Abbreviations

The following abbreviations are used in this article:

- **TDL**: Time-Dependent Lifetime (p. 4)
- **TIL**: Time Independent Lifetime (p. 4)
- **LAI model**: Model of general behavior of vehicles on the road (p. 15; v. [15])
Bibliography

[1] L. Alvarez and R. Horowitz. Safe platooning in automated highway systems part i: Safety regions design. *Vehicle System Dynamics*, 32(1):23–55, 1999. https://doi.org/10.1076/vesd.32.1.23.4228. Cited on p. 18.

[2] K. P. Amrutkar and K. K. Kamalja. An overview of various importance measures of reliability system. *International Journal of Mathematical, Engineering and Management Sciences*, 2(3):150–171, 2017. Cited on p. 3.

[3] R. E. Barlow and F. Proschan. Importance of system components and fault tree events. *Stochastic Processes and their Applications*, 3(2):153–173, 1975. ISSN 0304-4149. https://doi.org/10.1016/0304-4149(75)90013-7. Cited on p. 7.

[4] S. Belbasi and M. E. Foulaadvand. Simulation of traffic flow at a signalized intersection. *Journal of Statistical Mechanics: Theory and Experiment*, 2008(07):P07021, jul 2008. https://doi.org/10.1088/1742-5468/2008/07/p07021. Cited on p. 18.

[5] Z. W. Birnbaum. On the importance of different components in a multicomponent system. In P. Krishnaiah, editor, *Multivariate Analysis, II (Proc. Second Internat. Sympos., Dayton, Ohio, 1968)*, pages 581–592. Academic Press, New York, 1969. Cited on pp. 3, 5, 6, and 7.

[6] Z. W. Birnbaum, J. D. Esary, and S. C. Saunders. Multi-component systems and structures and their reliability. *Technometrics*, 3(1):55–77, 1961. ISSN 0040-1706. Cited on p. 5.

[7] J. R. Cherlow. Measuring values of travel time savings. *Journal of Consumer Research*, 7(4):360–371, 1981. ISSN 0093–5301, 1537–5277. https://doi.org/10.2307/2488690. Cited on p. 1.

[8] B. F. Ciuffo, V. Punzo, and V. Torrieri. A framework for the calibration of microscopic traffic flow models. *Transportation Research Board 86th Annual Meeting of the Transportation Research Board, Washington, D.C.*, 2007. Cited on p. 21.

[9] H. Fan, B. Jia, J. Tian, and L. Yun. Characteristics of traffic flow at a non-signalized intersection in the framework of game theory. *Physica A: Statistical Mechanics and its Applications*, 415(C):172–180, 2014. https://doi.org/10.1016/j.physa.2014.07.0. URL https://ideas.repec.org/a/eee/phsmap/v415y2014icp172-180.html. Cited on pp. 10 and 11.

[10] M. E. Foulaadvand and S. Belbasi. Vehicular traffic flow at a non-signalized intersection. *Journal of Physics A: Mathematical and Theoretical*, 40(29):8289–8297, jul 2007. https://doi.org/10.1088/1751-8113/40/29/006. Cited on p. 18.

[11] B. Greenshields. A study of traffic capacity. *Proc. of the Highway Research Board*, 14:448–477, 1935. Cited on p. 13.

[12] Y. Hollander and R. Liu. Estimation of the distribution of travel times by repeated simulation. *Transportation Research Part C*, 16(2):212–231, 2008. https://doi.org/10.1016/j.trc.2007.07.005. Cited on p. 21.

[13] A. Ilachinski. *Cellular Automata: a Discrete Universe*. World Scientific, 2001. ISBN 9789813102569. URL https://books.google.pl/books?id=8PY7DQAQBAJ. Cited on p. 14.

[14] W. Kuo and X. Zhu. Relations and generalizations of importance measures in reliability. *IEEE Transactions on Reliability*, 61(3):659–674, 2012. Cited on p. 4.

[15] M. Lárraga and L. Alvarez-Icaza. Cellular automaton model for traffic flow based on safe driving policies and human reactions. *Physica A: Statistical Mechanics and its Applications*, 389(23):5425–5438, 2010. https://doi.org/10.1016/j.physa.2010.08.0. Cited on pp. 13, 15, 17, 18, and 27.

[16] H. K. Lee, R. Barlovic, M. Schreckenberg, and D. Kim. Mechanical restriction versus human overreaction triggering congested traffic states. *Phys. Rev. Lett.*, 92:
298702, Jun 2004. https://doi.org/10.1103/PhysRevLett.92.238702. Cited on p. 17.

[17] M. J. Lighthill and G. B. Whitham. On kinematic waves I. Flood movement in long rivers. On kinematic waves II. A theory of traffic flow on long crowded roads. Proc. of the Royal Society of London. Series A. Math. and Physical Sci., 229(1178):281–345, 1955. Cited on p. 13.

[18] Y.-K. Lin. System reliability for quickest path problems under time threshold and budget. Computers & Mathematics with Applications, 60(8):2326 – 2332, 2010. ISSN 0898-1221. https://doi.org/10.1016/j.camwa.2010.08.026. Cited on p. 1.

[19] K. Malecki and K. Szmajdziński. Symulator do mikroskopowej analizy ruchu drogowego. Logistyka, 3:8, 2013. URL https://www.czasopismologistyka.pl/artykuly-naukowe/send/239-artykuly-na-plycie-cd/2670-artykul. Bibliogr. 13 poz., rys., wykr., pelen tekst na CD. Cited on p. 14.

[20] K. Nagel and M. Schreckenberg. A cellular automaton model for freeway traffic. Journal de Physique I France, 2(12):2221–2229, 1992. https://doi.org/10.1051/jp1:1992277. Cited on p. 13.

[21] M. Nakata, A. Yamauchi, J. Tanimoto, and A. Hagishima. Dilemma game structure hidden in traffic flow at a bottleneck due to a 2 into 1 lane junction. Physica A: Statistical Mechanics and its Applications, 389:5353–5361, 12 2010. https://doi.org/10.1016/j.physa.2010.08.005. Cited on p. 19.

[22] J. Opara. Metoda automatów komórkowych - zastosowanie w modelowaniu procesów przemian fazowych. Prace Instytutu Metalurgii Żelaza, T. 62, nr 4:21–34, 2010. Cited on p. 14.

[23] S. Ossen and S. P. Hoogendoorn. Validity of trajectory-based calibration approach of car-following models in presence of measurement errors. Transportation Research Record, 2088(1):117–125, 2008. https://doi.org/10.3141/2088-13. Cited on p. 21.

[24] R. Pilch and J. Szybka. Estimation reliability of road net. Journal of Machine Construction and Maintenance–Problemy Eksploatacji, 1:157–165, 2009. ISSN 1232-9312. oryg.title in Polish "Ocena niezawodności sieci komunikacyjnych". Cited on p. 26.

[25] K. G. Ramamurthy. Coherent structures and simple games, volume 6 of Theory and Decision Library. Series C: Game Theory, Mathematical Programming and Operations Research. Kluwer Academic Publishers Group, Dordrecht, 1990. ISBN 0-7923-0869-7. https://doi.org/10.1007/978-94-009-2099-6. Cited on p. 4.

[26] M. Średnicka. Importance measures in multistate systems reliability. Technical report, Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology, Wrocław, 2020. 38p. Master’s Thesis. Cited on p. 3.

[27] P. I. Richards. Shock waves on the highway. Operations Research, 4(1):42–51, 1956. https://doi.org/10.1287/opre.4.1.42. Cited on p. 13.

[28] H. J. Ruskin and R. Wang. Modeling traffic flow at an urban unsignalized intersection. In P. M. A. Sloot, A. G. Hoekstra, C. J. K. Tan, and J. J. Dongarra, editors, Computational Science — ICCS 2002, pages 381–390, Berlin, Heidelberg, 2002. Springer Berlin Heidelberg. ISBN 978-3-540-46043-5. Cited on p. 18.

[29] L. Sharpe. Highway security measures 'are hardly ever cost-effective'. Engineering & Technology, 7(10):13–14, 2012. ISSN 1750-9637. Cited on p. 1.

[30] J. Spall, S. Hill, and D. Stark. Theoretical framework for comparing several stochastic optimization approaches. In G. Calafiore and F. Dabbene, editors, Probabilistic and Randomized Methods for Design under Uncertainty, pages 99–110. Springer London, 2006. https://doi.org/10.1007/1-84628-095-8_3. Cited on p. 21.

[31] K. J. Szajowski and K. Włodarczyk. Drivers’ skills and behavior vs. traffic at intersections. Mathematics, 8(3):paper:433, pages:20, mar 2020. https://doi.org/10.3390/math8030433. Cited on pp. 3, 18, and 26.
A Code of modelling and simulation

This work uses the LAI model presented in Section 3.3, which was then modified to add restrictions at intersections. Each of the streets included in the intersection was simulated individually and the interactions at the intersection were examined. As it was mentioned before, streets and intersections can be divided into several types. In this appendix, we will present the most advanced in terms of regulations used, i.e. the intersection on Piwna 2 and Jasna streets. Cars there may go to the right joining the traffic in this direction or turn left crossing the second direction of travel in addition. Both rules apply at intersections. Code presentations will start with the most basic ones and then we will go to the main code. To calculate the distance to the vehicles preceding, which will provide the possibility of acceleration, maintain speed or deceleration was calculated using three functions `d_acc.m`, `d_keep.m`, `d_dec.m`, listed below.

```matlab
function result = d_acc(n, speeds, M, dv)
    % n — id of car
    % speeds — vector of speeds
    % M — max ability to decelerate
    % dv — ability to accelerate
    result = max(0, sum((speeds(n) + dv) - (0:(floor((speeds(n)+dv)/M))*M) - ...
            sum((speeds(n+1) - M) - (0:(floor((speeds(n+1)-M)/M))*M));
end

function result = d_keep(n, speeds, M, dv)
    result = max(0, sum(speeds(n) - (0:(floor(speeds(n)/M))*M) - ...
            sum((speeds(n+1) - M) - (0:(floor((speeds(n+1)-M)/M))*M));
end

function result = d_dec(n, speeds, M, dv)
    result = max(0, sum(speeds(n) - dv) - (0:(floor((speeds(n)-dv)/M))*M) - ...
            sum((speeds(n+1) - M) - (0:(floor((speeds(n+1)-M)/M))*M));
end
```

Another important element is adding a new vehicle on the road, each vehicle has its own index, speed, position, and information where it goes. Depending on the simulation being performed, the probability of route selection can be set to others, if it is not needed, the `where` parameter is not given.
function [pos, speeds, ids, where] =
    new_car(pos, speeds, ids, v_min, prob_new_car, M, dv, ls, L, where)

% where = where the vehicle are going -1 right , 0 - left
if nargin < 10
    where = [ ];
end

% Generating a new car, with some probability. New cars are added with
% v_min velocity
if isempty(pos)
    if rand() <= prob_new_car
        pos = [2, pos];
        speeds = [v_min, speeds];
        where = [rand() > 1/2, where];
        ids = 1;
    end
else
    temp = 1:(L/2);
    if pos(1) ~= 2
        speeds_temp = [v_min, speeds];
        d_ac = d_acc(1, speeds_temp, M, dv);
        d_ke = d_keep(1, speeds_temp, M, dv);
        d_de = d_dec(1, speeds_temp, M, dv);
        % Acceleration
        if (pos(n+1) - pos(n) - ls) >= d_ac
            new_speeds = speeds(n);
            if rand() <= min(Rd, R0 + speeds(n) * (Rd - R0) / vs)
                new_speeds = max(speeds(n) - dv, 0);
            end
            speeds(n) = new_speeds;
        end
        % Random slowing down
        if d_ac > (pos(n+1) - pos(n) - ls) && (pos(n+1) - pos(n) - ls) >= d_ke
            new_speeds = speeds(n);
            if rand() <= Rs
                new_speeds = max(speeds(n) - dv, 0);
            end
            speeds(n) = new_speeds;
        end
        % Braking
        if d_ke > (pos(n+1) - pos(n) - ls) && (pos(n+1) - pos(n) - ls) >= d_de
            new_speeds = max(speeds(n) - dv, 0);
            speeds(n) = new_speeds;
        end
        % Emergency braking
        if (pos(n+1) - pos(n) - ls) < d_de
            new_speeds = max(speeds(n) - M, 0);
            speeds(n) = new_speeds;
        end
    end
end
end
end
end

Then, a speed update is performed for each vehicle according to the diagram described
in chapter 3.3. In addition, parameters are used to say whether the vehicle can leave
the intersection or not, and to what speed it should slow down before the intersection.

function [speeds, can_go] = velocity_update(n, speeds, pos, ls, dv, Rd, R0, Rs,
    vs, M, vmax, L, can_go, dec_to)

% Velocity update for vehicle 'n'
% can_go ---
% 0 — the car cannot leave the intersection,
% 1 — the car can leave the intersection
% dec_to — velocity do decelerate before intersection
if n == numel(pos) % for last car no dependence of previous car
    d_ac = d_acc(n, speeds, M, dv);
    d_ke = d_keep(n, speeds, M, dv);
    d_de = d_dec(n, speeds, M, dv);
    % Acceleration
    if (pos(n+1) - pos(n) - ls) >= d_ac
        new_speeds = speeds(n);
        if rand() <= min(Rd, R0 + speeds(n) * (Rd - R0) / vs)
            new_speeds = max(speeds(n) - dv, 0);
        end
        speeds(n) = new_speeds;
    end
    % Random slowing down
    if d_ac > (pos(n+1) - pos(n) - ls) && (pos(n+1) - pos(n) - ls) >= d_ke
        new_speeds = speeds(n);
        if rand() <= Rs
            new_speeds = max(speeds(n) - dv, 0);
        end
        speeds(n) = new_speeds;
    end
    % Braking
    if d_ke > (pos(n+1) - pos(n) - ls) && (pos(n+1) - pos(n) - ls) >= d_de
        new_speeds = max(speeds(n) - dv, 0);
        speeds(n) = new_speeds;
    end
    % Emergency braking
    if (pos(n+1) - pos(n) - ls) < d_de
        new_speeds = max(speeds(n) - M, 0);
        speeds(n) = new_speeds;
    end
else
    temp = 1:(L/2);
    if pos(1) == 2
        speeds_temp = [v_min, speeds];
        d_ac = d_acc(1, speeds_temp, M, dv);
        d_ke = d_keep(1, speeds_temp, M, dv);
        % % Velocity update for vehicle 'n'
        % can_go ---
        % 0 — the car cannot leave the intersection,
        % 1 — the car can leave the intersection
        % dec_to — velocity do decelerate before intersection
        if n == numel(pos) % for last car no dependence of previous car
            d_ac = d_acc(n, speeds, M, dv);
            d_ke = d_keep(n, speeds, M, dv);
            d_de = d_dec(n, speeds, M, dv);
            % Acceleration
            if (pos(n+1) - pos(n) - ls) >= d_ac
                new_speeds = speeds(n);
                if rand() <= min(Rd, R0 + speeds(n) * (Rd - R0) / vs)
                    new_speeds = max(speeds(n) - dv, 0);
                end
                speeds(n) = new_speeds;
            end
            % Random slowing down
            if d_ac > (pos(n+1) - pos(n) - ls) && (pos(n+1) - pos(n) - ls) >= d_ke
                new_speeds = speeds(n);
                if rand() <= Rs
                    new_speeds = max(speeds(n) - dv, 0);
                end
                speeds(n) = new_speeds;
            end
            % Braking
            if d_ke > (pos(n+1) - pos(n) - ls) && (pos(n+1) - pos(n) - ls) >= d_de
                new_speeds = max(speeds(n) - dv, 0);
                speeds(n) = new_speeds;
            end
            % Emergency braking
            if (pos(n+1) - pos(n) - ls) < d_de
                new_speeds = max(speeds(n) - M, 0);
                speeds(n) = new_speeds;
            end
else

Finally, we go to the main program codes. Depending on the exact type of intersection, the program looks slightly different, but the overall characteristics and construction are preserved. At the beginning we define the variables used in the model, then we have loops after repetitions for different probabilities of a new vehicle. We define new empty roads in each loop. Then we add the first vehicle on each road and go on to further processes. In the original program, before starting the loop after repeating the update on the road, the road was filled with vehicles. At each step, we update speeds and add a new vehicle on the road, according to the probability. Then update the speeds and remove those vehicles whose position has exceeded the length of the road. Finally, we analyze the interactions of drivers at intersections, resulting in a change in the parameter saying whether the vehicle can leave the intersection or not. This is done in accordance with the previously described assumptions. On the posted program we have an example for Piwna Street 2, where the driver turning right gives way to other vehicles on this road and turning left gives way to vehicles driving in the opposite direction than he plans because he crosses their lane. Below is the code.

```matlab
M_C = 1000; % Monte Carlo
dx = 2.5; % cell size
ls = 2; % car length, 5 m
vmax = 5; % 5 -> 12.5 m/s -> 45 km/h lub 6 -> 15 m/s -> 54 km/h
Rs = 0.01; % prob. of emergency
Rd = 1; % max. prob of acceleration
R0 = 0.8; % min. prob. of acceleration
vs = 5/dx + 1; % min velocity for faster driving
M = 2; % ability to emergency driving 5m/s2
dv = 2.5/dx; % ability to accelerate
dec_to = 0; % velocity to slow down before intersection
same_probs = 0.05:0.025:0.6; % different traffic intensity
it = 1;
for same_prob = same_probs
disp(same_prob)
SAVE = zeros(1,M_C);
for M = 1:M_C
% Intersection Piwna2 i Laska, z prawd. 1/2 turn right or left
% Inputs for Piwna2
L_Piwna2 = 180/dx; % road length
prob_new_car_Piwna2 = same_prob;
pos_Piwna2 = [];
speeds_Piwna2 = [];
```
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v_min_Piwna2 = 4;
can_go_Piwna2 = 0;

% Inputs for Laska1, from right
L_Laska1 = 100/dx;
prob_new_car_Laska1 = same_prob;
pos_Laska1 = [];
speeds_Laska1 = [];
v_min_Laska1 = 4;
can_go_Laska1 = 1;

% Inputs for Laska2, from left
L_Laska2 = 100/dx;
prob_new_car_Laska2 = same_prob;
pos_Laska2 = [];
speeds_Laska2 = [];
v_min_Laska2 = 4;
can_go_Laska2 = 1;

% First car on Piwna2, Laska1 and Laska2
if isempty (pos_Piwna2) % if on road there is no car
    pos_Piwna2(1) = 2;
speeds_Piwna2(1) = v_min_Piwna2;
ids_Piwna2 = 1;
where_Piwna2 = [1]; % 1 - right, 0 - left
end

if isempty (pos_Laska1) % if on road there is no car
    pos_Laska1(1) = 2;
speeds_Laska1(1) = v_min_Laska1;
ids_Laska1 = 1;
end

if isempty (pos_Laska2) % if on road there is no car
    pos_Laska2(1) = 2;
speeds_Laska2(1) = v_min_Laska2;
ids_Laska2 = 1;
end

point = L_Laska1;
% intersection point on Laska1 i Laska 2, at the end od roads
stop = zeros(1,3);
lizc_zas_Piwna2 = zeros(size(1:(L_Piwna2/2)));
lizc = 1;
saved_zas_Piwna2 = [];
can_go_Piwna2 = 0;
% must always stop before the intersection
for i = 1:1000
    % Adding new car with v_min velocity on Piwna2
    [pos_Piwna2, speeds_Piwna2, ids_Piwna2, where_Piwna2] = new_car (pos_Piwna2 , speeds_Piwna2, ids_Piwna2, v_min_Piwna2, prob_new_car_Piwna2, M, dv, ls, L_Piwna2, where_Piwna2);
    % Velocity updating for each vehicle on road Piwna2
    for n = 1:numel(pos_Piwna2)
        speeds_Piwna2(n) = velocity_update(n, speeds_Piwna2, pos_Piwna2, ls , dv, Rd, R0, Rs, vs, M, vmax, L_Piwna2, can_go_Piwna2, dec_to);
    end
end

% Adding new car with v_min velocity on Laska1
[pos_Laska1, speeds_Laska1, ids_Laska1] = new_car (pos_Laska1 , speeds_Laska1, ids_Laska1, v_min_Laska1, prob_new_car_Laska1, M, dv, ls, L_Laska1);
% Velocity updating for each vehicle on road Laska1
for n = 1:numel(pos_Laska1)
    speeds_Laska1(n) = velocity_update(n, speeds_Laska1, pos_Laska1, ls , dv, Rd, R0, Rs, vs, M, vmax, L_Laska1, can_go_Laska1, dec_to);
end

% Adding new car with v_min velocity on Laska2
[pos_Laska2, speeds_Laska2, ids_Laska2] = new_car (pos_Laska2, speeds_Laska2, ids_Laska2, v_min_Laska2, prob_new_car_Laska2, M, dv, ls, L_Laska2);
% Velocity updating for each vehicle on road Laska2
for n = 1:numel(pos_Laska2)
    speeds_Laska2(n) = velocity_update(n, speeds_Laska2, pos_Laska2 , ls , dv, Rd, R0, Rs, vs, M, vmax, L_Laska2, can_go_Laska2, dec_to);
end
% Position update
pos_Piwna2 = pos_Piwna2 + speeds_Piwna2;
pos_Laska1 = pos_Laska1 + speeds_Laska1;
pos_Laska2 = pos_Laska2 + speeds_Laska2;

% Removing cars
if sum(pos_Piwna2 > L_Piwna2)
speeds_Piwna2(pos_Piwna2 > L_Piwna2) = [];
where_Piwna2(pos_Piwna2 > L_Piwna2) = [];
ids_Piwna2(pos_Piwna2 > L_Piwna2) = [];
saved_czas_Piwna2(licz) = max(licz_czas_Piwna2);
licz = licz + 1;
saved_czas_Piwna2(licz_czas_Piwna2 == ...) = 0;
pos_Piwna2(pos_Piwna2 > L_Piwna2) = [];
end

if sum(pos_Laska1 > L_Laska1)
speeds_Laska1(pos_Laska1 > L_Laska1) = [];
pos_Laska1(pos_Laska1 > L_Laska1) = [];
end

if sum(pos_Laska2 > L_Laska2)
speeds_Laska2(pos_Laska2 > L_Laska2) = [];
pos_Laska2(pos_Laska2 > L_Laska2) = [];
end
licz_czas_Piwna2(ids_Piwna2) = licz_czas_Piwna2(ids_Piwna2) + 1;

% Intersection interaction
if ~isempty(speeds_Piwna2)
if sum(pos_Piwna2 == L_Piwna2)
  % some vehicle is at the end of the road
  if where_Piwna2(end) == 1
    to_point_on_left = pos_Laska2 - point;
    finds_on_left = find(to_point_on_left < 0);
    if ~isempty(finds_on_left)
      found_on_left = find(to_point_on_left(finds_on_left) == ...
      max(to_point_on_left(finds_on_left)));
      % founded car on Zlota is the closest to intersections
      dist_to_point_on_left = abs(pos_Laska2(found_on_left) - point);
      d_keep_on_left = max(0, sum(vmax - (0:(floor((vmax-M)/M)))*M) - ...
      sum((vmax-M) - (0:(floor((vmax-M)/M)))*M));
    end
    % distance which will not force the driver
    % to be released on the main road at the maximum speeds of both vehicles
    if dist_to_point_on_left <= d_keep_on_left
      can_go_Piwna2 = 1;
    end
  end
else % where=0, in left
  cond = 0;
  % Checks if it is free on the right
  to_point_on_right = pos_Laska1 - point;
  finds_on_right = find(to_point_on_right < 0);
  if ~isempty(finds_on_right)
    found_on_right = find(to_point_on_right(finds_on_right) == ...
    max(to_point_on_right(finds_on_right)));
    % founded car on Zlota is the closest to intersections
    dist_to_point_on_right = abs(pos_Laska1(found_on_right) - point);
    d_keep_on_right = max(0, sum(vmax - (0:(floor((vmax-M)/M)))*M) - ...
    sum((vmax-M) - (0:(floor((vmax-M)/M)))*M));
  end
  % distance which will not force the driver
  % to be released on the main road at the maximum speeds of both vehicles
  if dist_to_point_on_right <= ...
    sum(min(vmax, speeds_Laska1(found_on_right) + (2:vmax) - 1)) + ...
    sum(2:vmax) >= d_keep_on_right
  end
  can_go_Piwna2 = 1;
end
else % when = 0, in left
  cond = 0;
end
end
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182 \[ \text{sum}(2:vmax) >= d\text{\_keep\_on\_right} \]
183 \% if this condition is met on the right we have
184 \% the opportunity to enter the intersection
185 cond = cond + 1;
186 end
187 else
188 end
189 cond = cond + 1;
190 end
191
192 \% Checking if it is free on the left
193 to\_point\_on\_left = pos\_Laska2 - point;
194 finds\_on\_left = find(to\_point\_on\_left <0);
195 if ~isempty(finds\_on\_left)
196 found\_on\_left = find(to\_point\_on\_left(finds\_on\_left) == ...,
197 max(to\_point\_on\_left(finds\_on\_left)));
198 dist\_to\_point\_on\_left = abs(pos\_Laska2(found\_on\_left) - point);
199 speeds\_temp = [speeds\_Laska2(found\_on\_left) ,0];
200 d\_ke\_on\_left = d\_keep(1,speeds\_temp,M,dv);
201 d\_de\_on\_left = d\_dec(1,speeds\_temp,M,dv);
202 needed\_time = 3;
203 if dist\_to\_point\_on\_left - speeds\_Laska2(found\_on\_left) - ...,
204 min(speeds\_Laska2(found\_on\_left)+1,vmax) <= d\_de\_on\_left
205 cond = cond + 1; \% second condition is met
206 end
207 else
208 cond = cond + 1; \% second condition is met
209 end
210
211 if cond == 2 \% if both conditions were met vehicle can go
212 can\_go\_Piwna2 = 1;
213 else
214 can\_go\_Piwna2 = 0;
215 end
216 end
217
218 end
219
220 \% Checking if it is free on the left
221 to\_point\_on\_left = pos\_Laska2 - point;
222 finds\_on\_left = find(to\_point\_on\_left <0);
223 if ~isempty(finds\_on\_left)
224 found\_on\_left = find(to\_point\_on\_left(finds\_on\_left) == ...,
225 max(to\_point\_on\_left(finds\_on\_left)));
226 dist\_to\_point\_on\_left = abs(pos\_Laska2(found\_on\_left) - point);
227 speeds\_temp = [speeds\_Laska2(found\_on\_left) ,0];
228 d\_ke\_on\_left = d\_keep(1,speeds\_temp,M,dv);
229 d\_de\_on\_left = d\_dec(1,speeds\_temp,M,dv);
230 needed\_time = 3;
231 if dist\_to\_point\_on\_left - speeds\_Laska2(found\_on\_left) - ...,
232 min(speeds\_Laska2(found\_on\_left)+1,vmax) <= d\_de\_on\_left
233 cond = cond + 1; \% second condition is met
234 end
235 else
236 cond = cond + 1; \% second condition is met
237 end
238
239 if cond == 2 \% if both conditions were met vehicle can go
240 can\_go\_Piwna2 = 1;
241 else
242 can\_go\_Piwna2 = 0;
243 end
244
245 end
246
247 end
248
249 SAVE(M) = nanmean(saved\_czas\_Piwna2(5:end));
250 end
251 SAVE\_MC(it) = nanmean(SAVE(SAVE>0));
252 it = it +1;
253 end

Similarly, all simulations carried out at work were carried out, thus obtaining the travel
time for each section. On this basis, the road delay was calculated as the difference
between each reading, the smallest value.