How the merger of two white dwarfs depends on their mass ratio: orbital stability and detonations at contact

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Accepted 2012 February 21. Received 2012 February 21; in original form 2012 January 11

ABSTRACT
Despite their unique astrophysical relevance, the outcome of white dwarf binary mergers has so far only been studied for a very restricted number of systems. Here we present the results of a survey with more than 200 simulations systematically scanning the white dwarf binary parameter space. We consider white dwarf masses ranging from 0.2 to 1.2 M⊙ and account for their different chemical compositions. We find excellent agreement with the orbital evolution predicted by mass transfer stability analysis. Much of our effort in this paper is dedicated to determining which binary systems are prone to a thermonuclear explosion just prior to merger or at surface contact. We find that a large fraction of He-accreting binary systems explode: all dynamically unstable systems with accretor masses below 1.1 M⊙ and donor masses above ∼0.4 M⊙ are found to trigger a helium detonation at surface contact. A substantial fraction of these systems could explode at earlier times via detonations induced by instabilities in the accretion stream, as we have demonstrated in our previous work. We do not find definitive evidence for an explosion prior to merger or at surface contact in any of the studied double carbon–oxygen systems. Although we cannot exclude their occurrence if some helium is present, the available parameter space for a successful detonation in a white dwarf binary of pure carbon–oxygen composition is small. We demonstrate that a wide variety of dynamically unstable systems are viable Type Ia candidates. The next decade thus holds enormous promise for the study of these events, in particular with the advent of wide-field synoptic surveys allowing a detailed characterization of their explosive properties.

Key words: hydrodynamics – nuclear reactions, nucleosynthesis, abundances – supernovae: general – white dwarfs.

1 INTRODUCTION
Type Ia supernovae are lynchpins in measuring distances on cosmological scales (Perlmutter et al. 1998; Riess et al. 1998). These events are bright enough to be seen from a great distance such that many thousands have been catalogued by modern transient surveys such as Palomar Transient Factory (PTF) (Rau et al. 2009), Harvard-Smithsonian Center for Astrophysics (CfA) (Hicken et al. 2009a,b) and Lick Observatory Supernova Search (LOSS) (Leaman et al. 2011). Despite the large number of detected events, the possible progenitors of Type Ia supernovae are still not known (Howell et al. 2011).

A strong constraint comes from the Type Ia event recently discovered in M101, which placed an upper limit of 0.02 R⊙ on the size of the star that exploded (Nugent et al. 2011; Bloom et al. 2012). This all but eliminates the possibility that the companion was a red giant (Li et al. 2011) and restricts the pre-explosion mass transfer rate from any non-giant companion to a narrow range of values (Chomiuk et al. 2012; Horesh et al. 2012). Observations indicate that the event is a fairly standard Type Ia (Brown et al. 2011; Bloom et al. 2012), and thus the particular progenitor channel that led to this event is likely to be similar to a significant fraction of the Type Ia events observed. The strong restrictions and/or elimination of non-degenerate companions compel us to explore the possibility that the companion is also a degenerate star. This double degenerate (DD) scenario for Type Ia would also be consistent with the observed delay time distribution, which is approximately proportional to t−1 (Totani et al. 2008; Maoz, Sharon & Gal-Yam 2010; Ruiter et al. 2010). This progenitor channel would further explain why hydrogen has never been observed in a Type Ia (Howell 2011). Moreover, the recently observed super-Chandrasekhar mass events (Howell et al. 2006; Scalzo et al. 2010; Taubenberger et al. 2010).

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Monthly Notices of the Royal Astronomical Society: 2012 RAS
2 NUMERICAL APPROACH

The simulations of this investigation are performed with a three-dimensional smoothed particle hydrodynamics (SPH) code (Rosswog et al. 2008); for recent reviews of the SPH method, see e.g. Monaghan (2005) and Rosswog (2009). The orbital dynamics of a binary system is very sensitive to the redistribution of angular momentum during mass transfer and therefore accurate numerical conservation is key to a reliable simulation. SPH is a very powerful tool for this type of study because the equations conserve angular momentum by construction, see e.g. section 2.4 in Rosswog (2009). In practice, the quality of conservation is determined by the force evaluation (in our case by the opening criterion of our binary tree; Benz et al. 1990) and by the time-stepping accuracy, both of which are driven together via gravitational wave radiation, the signature of which is expected to be a dominant component of the ambient gravitational wave sky (e.g. Lipunov, Postnov & Prokhorov 1987; Hils, Bender & Webbink 1990; Nelemans, Yungelson & Portegies Zwart 2004; Liu et al. 2010).

A critical stage in the evolution of a compact WD binary is the period just after mass transfer has been initiated. For the binary to survive, the mass transfer must be stable, which depends sensitively on the internal structure of the donor star, the binary mass ratio and the angular momentum transport mechanisms (e.g. Marsh, Nelemans & Steeghs 2004; Golhale, Peng & Frank 2007). The torques applied by a disc that may form from accreted material, and the tides raised on the accretor, are crucial for the binary’s orbital stability (Iben, Tutukov & Fedorova 1998; Piro 2011). If these mechanisms are unable to widen the orbit, the system will merge quickly. The subsequent evolution can produce a R Corona Borealis star (Webbink 1984; Clayton et al. 2011), trigger an explosion as a Type Ia (Iben & Tutukov 1984; Webbink 1984; Piersanti et al. 2003a,b; Yoon, Podsiadlowski & Rosswog 2007), or undergo an accretion-induced collapse to a neutron star (Saio & Nomoto 1985; Nomoto & Kondo 1991; Saio & Nomoto 2004; Shen et al. 2012).

While theoretical advances detailing the possible outcomes of WD mergers have been made in the last decade (see review by Postnov & Yungelson 2006), simulations have only probed the parameter space of DD mergers for a very small number of systems, with the principal emphasis being on systems with a total mass greater than the Chandrasekhar mass. We have previously stressed the vital role of the numerical initial conditions in determining the outcome of a merger (Dan et al. 2011), simulations with approximate initial conditions start with too little angular momentum and predict a merger that occurs much more rapidly than what is realized in nature. Our previous results indicated that many DD systems with a donor composed largely of helium (He) are expected to be ignited by an interaction between the accretion stream and the torus of the material acquired from the donor (Guillochon et al. 2010). Here we greatly expand upon our initial study with 225 new simulations that fully explore the possible combinations of WD donors and accretors in the mass range between 0.2 and 1.2 M⊙. We demonstrate that our systematic study exhibits the orbital stability trends expected for merging DD systems, thus enabling us to evaluate which systems are expected to produce strong thermonuclear events in the lead up to or at the point of merger. We find that many systems with He donors may produce explosions, but that systems with carbon–oxygen (CO) donors are incapable of exploding without the inclusion of a significant He layer.

In Section 2, we recap our numerical approach and describe how our simulations cover the parameter space of DD systems. In Section 3, we give a brief overview of how dynamical orbital stability is determined and compare these expectations to our simulation results. Section 4 discusses detonations that are either triggered by the accretion stream or at the moment of merger for both He and CO mass-transferring donor stars. We summarize our findings and relate our models with Type Ia observations in Section 5. For reference, we provide fitting formulae bounding the region of parameter space where detonations are expected in Appendix A.
For all our simulations, the stars are initially synchronized, cold and isothermal ($T = 10^4$ K). In the first step, the stars are individually driven to an accurate numerical equilibrium by adding a velocity-dependent damping term to the momentum equation. Subsequently, we place the stars in the corotating frame at an initial separation that is large enough to avoid any immediate mass transfer. The orbital separation is then adiabatically reduced in a way that the time-scale over which the separation changes is much longer than the dynamical time-scale of the donor. During this process, spurious oscillations are minimized by applying a damping force. When the first SPH particle reaches the inner Lagrangian point $L_1$, we stop the relaxation process, transform the velocities to the fixed frame and set this moment as the time origin ($t = 0$) of the simulation. Most of the previous SPH WD merger calculations (Benz et al. 1990; Segretain, Chabrier & Mochkovitch 1997; Guerrero, García-Berro & Isern 2004; Yoon et al. 2007; Lören-Aguilar, Isern & García-Berro 2009; Pakmor et al. 2010, 2011) started from rather approximate ICs. In Dan et al. (2011), we have shown in detail that the initial conditions have a strong impact on all major aspects of the simulations. Here, we only briefly illustrate the differences between the approximate and our, accurate ICs in the remnant structures resulting from the merger of a $0.81 + 0.9 M_\odot$ system in Fig. 1. When using the accurate IC there is less mass in the disc and more mass in the trailing arm than when starting from the approximate IC. The approximate IC also overestimates the density and temperature in the region surrounding the accretor. As we explain in Section 5, the pollution of the ambient environment prior to any supernova-like event may have important observational consequences.

The compositions used in this study are shown in Fig. 2. Below $0.45 M_\odot$ the stars are made of pure He. In principle, WDs with masses below $0.45 M_\odot$ could harbour heavier elements in their cores (Iben & Tutukov 1985; Han, Tout & Eggleton 2000; Moroni & Straniero 2009), but Nelemans et al. (2001a) have shown that the probability for hybrid WDs in this range is 4–5 times smaller than that for He WDs. Based on Rappaport, Podsiadlowski & Horev (2009), who found that a $0.475 M_\odot$ WD has a He envelope of about 30 per cent of its total mass, between 0.45 and 0.6 $M_\odot$ we adopt a hybrid He–CO composition with a ~0.1 $M_\odot$ He mantle and a pure CO core. WDs between 0.6 and 1.05 $M_\odot$ are assumed to be made entirely of CO. According to Iben & Tutukov (1985), there is only a small amount of He at the surface of the CO core for this range of masses. It has been shown that tiny amounts of He can be important for CO detonations (e.g. Seitenzahl et al. 2009), but for the numerical resolution that can be afforded in such a large parameter study (20 000 SPH particles per star) the outer He layers cannot be properly resolved. For this range of masses there should be more oxygen than carbon (Iben & Tutukov 1985), and we have chosen mass fractions $X^{12}C = 0.4$ and $X^{16}O = 0.6$ uniformly distributed through the star, similar to Lören-Aguilar et al. (2009). For WDs with masses above 1.05 $M_\odot$ we use $X^{16}O = 0.60$, $X^{20}Ne = 0.35$ and $X^{24}Mg = 0.05$ (similar to the composition found by Gil-Pons & García-Berro 2001 for a 1.1-$M_\odot$ WD), distributed uniformly throughout the entire star.

### 3 ORBITAL STABILITY

After one or more common envelope phases, a WD binary can emerge close enough for gravitational radiation to drive the binary towards shorter orbital periods until mass transfer is initiated. This occurs at orbital periods between 2 and 15 min depending on the characteristics of the donor. For the binary to survive, the mass transfer must be stable, which depends sensitively on the binary mass ratio and the size of the donor star. Dynamical instability is guaranteed for systems in which the mass ratio of the donor star to the accretor star $q = M_{\text{d}}/M_1$ exceeds 2/3 (see e.g. Nelemans et al. 2001b). For $q < 2/3$, the system can still survive as an interacting binary if orbital angular momentum losses are balanced by other mechanisms. In many instances, the formation of a tidally truncated

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**Figure 1.** Remnant profiles both with accurate (solid red line) and approximate (dashed green line) ICs for the $0.81 + 0.9 M_\odot$ system. Shown are the spherically enclosed mass (left) and spherically averaged density (centre) and temperature (right) versus radius from the centre of the accretor.

**Figure 2.** The initial chemical composition of double WD systems with donor masses $M_2$ from 0.2 to 1.05 $M_\odot$ and accretor masses $M_1$ between 0.2 and 1.2 $M_\odot$. See text for additional information.
accretion disc can efficiently return the advected angular momentum to the orbit. The system can then survive and evolve to longer periods as a result of the mass transfer. On the other hand, mass transfer can proceed directly into the surface of the accretor. As a result, the system is destabilized by transforming orbital angular momentum into the accretor’s spin. A direct impact will occur if the radius of closest approach of the accretion stream \( r_{\text{min}} \) is less than the radius of the accretor \( R_{1} \). An estimate of \( r_{\text{min}} \) is given by (Lubow & Shu 1975; Nelemans et al. 2001b)

\[
\frac{r_{\text{min}}}{a} = 0.04948 - 0.03815 \log q + 0.0475 \log^2 q - 0.006973 \log^3 q,
\]

where \( a \) is the orbital separation. The two very short period binaries HM Cnc and V407 Vul belong to this so-called direct impact category as they both satisfy \( r_{\text{min}} < R_{1} \) (Marsh & Steeghs 2002; Ramsay, Hakala & Cropper 2002).

Tidal coupling can also return angular momentum to the orbit, heating both WD members in the process. This tidal heating can influence the donor’s mass transfer response and, as a result, affect the stability of the system. For the coupling to effectively stabilize the system, the synchronization torques must act on a time-scale shorter than the time-scale over which the rate of mass transfer exponentiates. Some WD binaries clearly avoid merger, as we see their survivors in the form of AM Canum Venaticorum (AM CVn) binaries (see reviews by Warner 1995; Nelemans 2005; Solheim 2010). Survival is however not guaranteed even when mass transfer is expected to be stable. Mass transfer can, for example, exceed the Eddington rate or spark a thermonuclear explosion that might destroy the binary.

In Fig. 3 we show the number of orbits each system survives prior to merger. The mass transfer rates we are able to numerically resolve are so far above the Eddington limit that photons are effectively trapped and, as a result, no significant mass loss is anticipated to occur. In our simulations, the degree to which the donor star overfills its Roche lobe is artificially enhanced by our finite resolution. This results in a mass transfer rate that increases too quickly and produces an undermassive accretion disc that cannot return sufficient angular momentum to the orbit. A number of systems in the disc accretion regime do not show any sign of a merger after as many as 90 orbital revolutions. As these systems are likely to be stable anyway, we do not continue the evolution of these systems beyond this limit.

Recent analytical estimates of Fuller & Lai (2012) show that the WD binaries should reach a high degree of synchronization before mass transfer sets in. For this reason and because constructing such initial conditions is challenging as desynchronicity is a strongly destabilizing effect, we start all our simulations with tidally locked stars. After a few orbital revolutions of mass transfer, however, the accreting star becomes desynchronized. In nature this would happen at separations that are larger than those at which we can numerically resolve mass transfer. In summary, our simulated systems show the transition from close to synchronized to desynchronized, but at a somewhat too small initial separation. Despite the quick build-up to this desynchronized state, our simulations are able to capture the basic angular momentum transport mechanisms correctly as illustrated by the dramatic increase in the number of orbits as we approach the disc accretion boundary. In comparison with previous work at higher resolution in which the WD binaries merge within only a few orbits, our results illustrate the importance of carefully constructed initial conditions.

### 4 DETONATIONS AT CONTACT

In this section we investigate which binary systems could produce immediate detonations at contact, first during the mass transfer, which in the following we refer to as ‘stream-induced detonations’, and, if detonations from the stream interaction are avoided, during the final coalescence that we call ‘contact detonations’. In Section 4.1 we briefly summarize the detonation criteria that we use in Sections 4.2 and 4.3 to discuss the detonation prospects for both He- and CO-transferring systems.

#### 4.1 Detonation criteria

In its simplest form, a detonation is a shock wave that advances supersonically into an unburnt, reactive medium. Behind the shock wave, exothermic reactions take place that continuously drive the
shock to wear down possible dissipative effects. Whether a detonation forms or not depends delicately on the exact local conditions. The detonation conditions that have been applied in previous work (Fink, Hillebrandt & Röpke 2007; Röpke, Woosley & Hillebrandt 2007; Townsley et al. 2007; Jordan et al. 2008) all rather loosely decide whether detonation conditions are reached based solely on the peak temperature reached in the simulation above a certain density' (Seitenzahl et al. 2009). Apart from local density and temperature, the temperature profile and the size of the region being heated determines whether a detonation is triggered or not. Depending on these factors, the critical radii have been found to vary between centimetres and hundreds of kilometres (Seitenzahl et al. 2009). In general, these values are substantially below the resolution lengths in the relevant regions that we can afford in this study (\(10^7-10^8\) cm) with its more than 200 simulation runs.

Given the delicacy of the exact detonation conditions, we resort in the bulk of this work to a simple yet robust time-scale argument. We assume that dynamical burning sets in when the thermonuclear time-scale, \(\tau_{\text{dynamical}}\), drops below the local dynamical time-scale \(\tau_{\text{dyn}}\). When this happens, matter heats up more rapidly than it can expand, cool and quench the burning. To compute \(\tau_{\text{dynamical}}\), we use \(\tau_{\text{dynamical}} = c_s T / \epsilon_{\text{nuc}}\) (e.g. Taam 1980; Nomoto 1982), where the nuclear energy generation rate, \(\epsilon_{\text{nuc}}\), is taken directly from the reaction network and \(c_s\) is the specific heat at constant pressure, which we calculate from the Helmholtz equation of state. For the local dynamical time-scale, we use \(\tau_{\text{dyn}} = H / c_s\), where \(c_s\) is the sound speed and \(H\) is the local pressure scale height. We estimate \(H\) as \(P/\rho g\), where \(g\) is the local gravitational acceleration.

In our SPH formulation, the energy variable that is evolved forward in time is the internal energy. The temperature at each particle position is found at every time step by a Newton–Raphson iteration. This temperature \(T_{\text{a}, \text{i}}\) labelling the particle, is the best estimate for the local matter temperature, but small numerical fluctuations in the internal energy (say, due to finite numerical resolution, finite SPH neighbour number, finite time-stepping accuracy etc.) can lead to larger fluctuations in the SPH particle temperature estimate. This is especially so in regions where matter is degenerate and essentially independent of temperature. To some extent, this could be alleviated by designing a Fehlberg-type integration scheme that rejects and re-takes time steps if their temperature changes are too large. In addition to being computationally expensive, this approach may be ineffective where the fluctuations are not associated with the choice of time step. Therefore, one should be very cautious in interpreting temperatures of very few (substantially less than the typical neighbour number) hot particles, which may also be due to small internal energy fluctuations.

Although we stress that \(T_{\text{a}}\) is the best local temperature estimate, we also calculate the SPH-smoothed temperature value

\[
\langle T \rangle_a = \sum_b \frac{m_b}{\rho_b} T_b W(|r_a - r_b|, h_a),
\]

which is a robust lower limit. Here, \(\rho_b\) and \(m_b\) are the densities and masses of neighbour particles \(b\), \(W\) is the cubic spline kernel and \(h_a\) is the smoothing length of particle \(a\). In the limit of infinite resolution, both temperature estimates would coincide. Although it is plausible that a detonation would be triggered when \(\tau_{\text{nuc}}/\tau_{\text{dynamical}}(T) < 1\), we only consider a detonation as inescapable consequence beyond reasonable doubt if the more conservative ratio \(\langle \tau_{\text{nuc}}/\tau_{\text{dynamical}}(T) \rangle\) is also below unity.

As a matter of illustration, we show in Fig. 4 the evolution of temperature and density of a 0.6 + 0.9 M\(\odot\) system during the last stages of its merger. The left-hand panel shows the maximum particle temperature (green) together with the SPH-smoothed value (red) and the arithmetic mean value within the interaction radius (\(\approx 2h\)) of the hottest particle (white). The shaded regions indicate the 1\(\sigma\) and 2\(\sigma\) deviations. The corresponding quantities for the densities are plotted in the right-hand panel. To illustrate the geometry of the potential ignition region, we show in Fig. 5 (left and central panels) the particle temperatures within a slice around the orbital plane. For a fair comparison we chose the slice thickness in each case to be four times the smoothing length of the particle with the minimum ratio of \(\tau_{\text{nuc}}/\tau_{\text{dynamical}}(T)\). The interaction region of this particle is indicated in both panels by the red circles. The probability distribution functions (PDFs) of the particles in these interaction regions are shown in the right-hand panel. Despite the very low numerical resolution that can be afforded in this parameter study, the overall dynamics is captured accurately; see for example the excellent agreement of the numerical results with the analytical estimates for the orbital evolution shown in Fig. 3. Nevertheless, at the current low resolution the shown interaction regions can contain particles from different...
morphological parts of the flow. In the shown example, the most promising particle has neighbours in the cold ($T < 5 \times 10^7$ K) incoming accretion stream, in the hot interaction region ($\sim 2 \times 10^8$ K) and in the moderately hot inner disc regions ($\sim 10^8$ K). As a consequence, the PDFs can be multi-peaked. As resolution increases and the neighbour particles come from a smaller and smaller local volume, the PDFs will become narrower, single-peaked and $T$ and $\langle T \rangle$ will become increasingly similar.

In the remainder of the paper, we explore the $M_1-M_2$ plane to determine the regions in which the detonation conditions we have described are fulfilled.

4.2 Helium mass transferring systems

As discussed in Section 2.1, some of the donors we consider are composed of He in their outermost layers. For those systems with either He cores or thick He shells, the mass transferred to the accretor will be pure He until either the donor is disrupted, or until the He layer on the donor is exhausted. As donors with a significant fraction of He may exist for $0.2 \lesssim M_1 \lesssim 0.6 M_\odot$, the mass ratio of systems with He donors can vary quite widely, and thus both dynamically stable and unstable systems with He donors are expected to exist. Those dynamically stable systems that are close enough to one another to initiate mass transfer will appear as AM CVn systems, whereas the dynamically unstable systems, which experience an exponential growth in the accretion rate, are only likely to be observed prior to contact. As WDs are supported by degeneracy pressure, the mass–radius relationship of WDs only weakly depends on the WDs’ composition, and thus the ratio of He to CO in WDs is largely unknown, aside from those WDs that are too massive to contain any significant He.

4.2.1 He detonations in stable systems

Observed AM CVn systems are primarily expected to accrete through a disc (Nelemans 2005), although a few systems are likely undergoing direct impact accretion (Marsh & Steeghs 2002; Ramsay et al. 2002). While the temperature necessary for dynamical He burning is somewhat higher than the virial temperature of a $\sim 0.6 M_\odot$ WD, slow He burning can proceed in the accretor’s acquired He shell over a time-scale shorter than the thermal time-scale, raising the entropy of the accreted material before the excess heat can be radiated away into space. This is in fact the reason why no single WDs with a significant He layer are expected for $M \gtrsim 0.6 M_\odot$, as even if the WD begins its life with a layer of He, this layer will be converted into CO on a time-scale that is significantly shorter than the Hubble time. If the density at the base of the He layer is large enough (i.e. if the He shell is massive enough), the temperature eventually rises so much that the burning time-scale becomes shorter than the convective time-scale in the envelope, leading to a runaway process that produces a detonation (Bildsten et al. 2007).

However, a detonation can only occur in these kinds of systems if the donor is massive enough such that it can provide the minimum required mass. It turns out that the minimum mass necessary is larger than $\sim 0.2 M_\odot$ for accretors with mass $M_1 < 0.6 M_\odot$ (Shen et al. 2009), systems that may or may not be dynamically stable depending on the synchronization time-scale.

4.2.2 Stream-induced detonations prior to contact

As we described in Guillochon et al. (2010), detonations of the He torus can be triggered by the accretion stream several orbits prior to the merger event. Using our significantly larger set of accreting systems, we have provided an updated version of the figure originally presented in Dan et al. (2011) showing the systems that are expected to have surface detonations triggered by the accretion stream, see Fig. 6. A thermonuclear runaway will occur prior to the merging event if $\tau_{nuc}/\tau_{dyn} \sim 1$, a condition that depends on the thermodynamic state of the He torus acquired through the accretion process. Due to our limited numerical resolution, the region of hydrodynamical instability between the stream and the torus is not well resolved. As in Dan et al. (2011), we measure the increase in the density and temperature as the accretion stream compresses the torus in order to determine whether dynamical burning occurs prior to merger. The conditions for burning are more easily met for systems in which the temperature and density of the material acquired pre-merger are large to begin with, leading to conditions favourable for uncontrolled burning once the material is compressed by the dynamical interaction, whether by the stream or by the donor itself.

The pre-interaction conditions in the He torus depend on three primary factors: the mass of the accretor, the component of the stream velocity normal to the accretor’s surface at impact and the evolution of $\dot{M}$ with time. For an increasing accretor mass, the larger gravitational pressure at the accretor’s surface leads to larger hydrostatic pressures, and thus greater equilibrium densities and temperatures. In contrast, an increasing accretor mass shrinks the accretor’s radius relative to the circularization radius, resulting in more ‘disc-like’ accretion and thus a less concentrated torus that is

**Figure 5.** Particle temperatures within a cross-section slice with a thickness equal to four smoothing lengths for a $0.3 + 0.6 M_\odot$ system using $4 \times 10^4$ (left) and $2 \times 10^5$ (centre) SPH particles. The right-hand panel shows the PDFs of the temperature within the red circle (interaction radius of the particle with minimum $(\tau_{nuc}/\tau_{dy})(T)$) shown in the left and central panels.
Figure 6. Stream-induced detonations prior to contact: torus masses and the ratio between the thermonuclear time-scale \( \tau_{\text{nuc}} \) and the local dynamical time-scale \( \tau_{\text{dyn}} \) for the He donor systems. If \( \tau_{\text{nuc}} \leq \tau_{\text{dyn}} \), we measure the masses when the time-scales for the first time become equal, otherwise we take the mass when the ratio \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) is minimal. The white solid lines show where \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) is equal to 0.1, 1 and 10. The black dashed line is our fit to the contour of ratio unity, see equation (A3) in Appendix A. A comparison with Fig. 8 shows that a large fraction of WDs with He donors undergo accretion stream triggered He detonations prior to contact. The torus masses at detonation can be as large as \( \sim 0.1 \text{ M}_\odot \), but also significantly smaller for larger \( M_{\text{tot}} \). Systems with a massive He donor and small disc masses at detonation may be capable of producing multiple sub-luminous events prior to the final coalescence.

more supported by rotation than fluid pressure. A shrinking accretor also reduces the normal component of the stream velocity relative to the accretor’s surface, and as a result the degree of compression achieved as a result of the stream’s impact on to the accretor’s surface is more limited. These effects are manifest as the larger \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) ratios towards the highest accretor masses in Fig. 6.

A necessary condition for successful stream detonations is that the stream’s ram pressure is considerably larger than the hydrostatic pressure of the torus, which is only possible if the mass of the stream is comparable in mass to the torus that has already accumulated. This implies that the second derivative of the amount of mass transferred, \( \dot{M} \), must be comparable to \( \dot{M} \) itself. Therefore, stream detonations are only possible when an exponentially growing \( \dot{M} \) can be sustained for several orbits, a condition that is typical of systems with abrupt orbital evolution. Because of the stochastic nature of the triggering event, a detonation can be realized at any point in time where the He torus mass is larger than the minimum shell mass required for a successful detonation (Bildsten et al. 2007; Shen & Bildsten 2009; Woosley & Kasen 2011), which we find can precede the final merger event by many orbits.

4.2.3 Contact detonations

If detonations from the stream interaction are avoided, the system has another chance to initiate a He detonation during the final coalescence of the two systems. Except for WDs that are almost equal in mass, the dynamical interaction during the final coalescence involves the donor being ripped apart by tides, while the accretor remains relatively unaffected. Approximately half of the donor’s remaining mass falls on top of the previously accreted He, but at a speed that is only a fraction of the escape speed from the accretor as the shredded donor is partly held aloft by its angular momentum (Fig. 7). Thus, the final coalescence can be viewed as a low-speed collision (Rosswog et al. 2009). In simulations of WD–WD head-on collisions, collisions between pairs of WDs with mass as low as 0.4 M\(_\odot\), He WDs were seen to produce detonations. The mutual escape speed of two 0.4 M\(_\odot\) at the point of collision is \( \sqrt{2 G (M_1 + M_2) / (R_1 + R_2) } = 3 \times 10^8 \text{ cm s}^{-1} \), which is approximately one-third of the escape speed from a 0.8-M\(_\odot\) accretor.

At the point of disruption, half of the donor star falls on to the accretor as the donor’s Roche lobe rapidly shrinks. Once this has occurred, the donor star can be viewed as material whose orbital dynamics are primarily determined by the surviving accretor. This means that the common centre of motion, the barycentre, shifts from being located at a point lying between the two WDs to lying within the centre of the accretor. From the plunging donor material’s point of view, which originally moved on a circular orbit, the point around which it orbits moves, placing the material on a highly eccentric orbit with a large \( r \). This results in a collision that is more violent, with a plunging speed that can be almost as large as the accretor’s escape speed. In systems with a large \( q \), the barycentre is usually already close to the accretor’s centre of mass, and thus there is no rapid acceleration within the corotating frame, and thus the final coalescence is much more gentle. This behaviour is clearly visible in the right-hand panel of Fig. 9, which shows that the temperature at minimum \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) tends to increase with increasing \( q \) for a fixed total mass.

The optimal conditions for ignition are reached when the temperature is at its peak, close to the time of the donor’s disruption, provided that the accretor is below 1.1 M\(_\odot\) and the donor is above 0.4 M\(_\odot\), see the upper-left panel of Fig. 8. For all systems with \( M_{\text{tot}} > 0.9 \text{ M}_\odot\), individual particle temperatures \( T \) of 10\(^6\) K and greater are reached in the torus formed via the mass transfer, but the density is too low to lead to a dynamical burning event. If we consider the more conservative smoothed temperature criteria, only systems with \( M_{\text{tot}} > 1.3 \text{ M}_\odot\) are capable of detonating at contact. Under both criteria, systems with \( q \sim 1 \) are more prone to produce detonations. Systems that do not satisfy the criteria for detonating either by the stream mechanism or at the point of merger may be progenitors of He novae instead (e.g. Yoon & Langer 2004).

The ratio \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) is very large in the right-lower corner of Fig. 8 (left-hand panels), close to the region of disc accretion. As described in Section 3, we expect that the systems should be dynamically stable, as they do not show any sign of a merger after 70–90 orbits of mass transfer.

4.3 Carbon–oxygen mass transfer

While we find that He-accreting WDs yield detonations, systems accreting CO are unlikely to explode at or prior to merger; see the right-hand panels of Fig. 8. Fig. 9 shows the region of the \( \rho–T \) plane where a CO detonation can be successfully ignited (Seitenzahl et al. 2009), together with the temperature, both \( T \) and \( T' \), and the density of all CO mass-transferring systems at the moment when \( \tau_{\text{nuc}}/\tau_{\text{dyn}}(T) \) is minimal. Overall, there is a clear tendency to increase the temperature at minimum \( \tau_{\text{nuc}}/\tau_{\text{dyn}}(T) \) with the mass ratio for a fixed total mass; see the right-hand panel of Fig. 9.

Based on the detonation criteria of Seitenzahl et al. (2009), Pakmor et al. (2011) have found detonations in their merger study of (nearly) equal-mass WDs. Because Pakmor et al. (2011) used approximate ICs that result in higher temperatures and densities during the evolution of mass transfer (Dan et al. 2011), we run a direct comparison with their 0.81 + 0.9 M\(_\odot\) system. We compared the minimum obtained value of \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) with accurate (Dan et al.
Figure 7. Column density snapshots corresponding to the moment when the system first crosses \( (\tau_{\text{nuc}}/\tau_{\text{dyn}})(T) < 1 \), or, if that does not occur, when \( (\tau_{\text{nuc}}/\tau_{\text{dyn}})(T) \) is minimum. The 10 snapshots shown here are representative of all DD chemical composition and mass combinations shown in Fig. 2. In this parameter space, we fully explore the possible combinations of WD donors and accretors and investigate their orbital stability and whether detonations prior or at the merger moment are possible. Systems with a mass ratio close to one (diagonal dashed line) show a quick disruption, within about 20 orbits, while those away from this line show a slow depletion of the donor (i.e. low \( \dot{M} \)) over dozens of orbits of mass transfer and the formation of an extended atmosphere surrounding the accretor. We find that a large fraction of the systems with He donors (below the horizontal dashed line) are expected to explode prior to or at the point of the merger. In contrast, the CO-accreting systems (above the horizontal dashed line) are unlikely to explode at or prior to the merger.

2011) and approximate (same as in Pakmor et al. 2011) ICs for non-rotating stars and found a difference of only 5 per cent, with \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \approx 7 \). The difference is much larger, however, for systems with lower mass ratios. For the corotating system with 0.6 + 0.9 \( M_{\odot} \) components, \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) for the approximate ICs is four orders of magnitude smaller than the value obtained for using the accurate ICs. The two ICs also lead to important differences in the merger remnant profiles at the moment when \( \tau_{\text{nuc}}/\tau_{\text{dyn}} \) is minimum: the temperature is overestimated when using the approximate ICs and the density is larger in the hot envelope surrounding the accretor, where the dynamical burning takes place (Fig. 1).

5 DISCUSSION

Although small patches of the WD merger parameter space have been explored previously in a moderate number of simulations (Benz et al. 1990; Rasio & Shapiro 1995; Segretain et al. 1997; Guerrero et al. 2004; D’Souza et al. 2006; Motl et al. 2007; Yoon et al. 2007; Lorent-Aguilar et al. 2009; Fryer et al. 2010; Pakmor et al. 2010; Dan et al. 2011; Pakmor et al. 2011; Raskin et al. 2012), the basic question: Which white dwarf systems produce explosions prior to or directly at merger? has so far remained unanswered. This serious gap in our understanding has in particular hampered our ability to
DWD orbital stability and detonations

Figure 8. Contact-induced detonations: colour coded is the minimum ratio between the thermonuclear time-scale $\tau_{\text{nuc}}$ and the local dynamical time-scale $\tau_{\text{dyn}}$ over a grid spanning the entire parameter space of He (left) and CO donor (right) masses. The ratio reaches a minimum value at the moment of the merger when the temperature reaches its peak. The time-scales were computed using both the individual particle temperature $T$ (upper panels) and the SPH-smoothed temperature $\langle T \rangle$ (lower panels, see equation 2). The dashed lines represent fits (see Appendix A) to the contours with $\tau_{\text{nuc}}/\tau_{\text{dyn}} = 1$, above which thermonuclear runaway is expected.

Figure 9. Left-hand panel: the filled diamonds show $(\rho, T)$ at minimum $(\tau_{\text{nuc}}/\tau_{\text{dyn}})(T)$ and are connected by black lines to $(\rho, \langle T \rangle)$, represented by filled squares. Colour coded is the total mass of the system in solar units. $\langle T \rangle$ is computed at the location of the particle with temperature $T$. None of the CO mass-transferring systems reaches the CO detonation criteria of Seitenzahl et al. (2009) (shaded region) or the $\tau_{\text{nuc}}/\tau_{\text{dyn}} < 1$ condition (see the right-hand panels of Fig. 8). Right-hand panel: temperature at minimum $(\tau_{\text{nuc}}/\tau_{\text{dyn}})(T)$ in the $q-M_{\text{tot}}$ plane. The temperature shows a clear trend towards higher values with increasing mass ratio and/or fixed total mass.

confidently judge the potential of DD mergers as Type Ia progenitor systems. In this paper, we close this gap by methodically covering the whole parameter space of WD mergers using more than 200 simulation runs. We have systematically scanned the WD mass range from 0.2 to 1.2 $M_\odot$ and accounted for their different initial chemical compositions.

While such a broad study is necessarily limited in the numerical resolution that can be afforded in each simulation, we have...
found excellent qualitative agreement with previous numerical studies (Dan et al. 2011) and semi-analytical work (Marsh et al. 2004) on the orbital stability of WD binary systems. One of the main conclusions of our previous study (Dan et al. 2011) was that the accuracy of the numerical initial conditions has a major impact on all subsequent evolutionary stages and can be more important than numerical resolution. All our simulations were started from carefully constructed, initially tidally locked binary configurations. However, in all of our simulations the accretor star becomes desynchronized with the orbital period after about 5–10 orbital revolutions. In nature, WD binaries should be synchronous when mass transfer occurs (Fuller & Lai 2012), but at initial separations that are larger than those where numerically resolvable mass transfer sets in, so that the stars could already be desynchronized at this stage. We find that systems that are near the dividing line between disc and direct impact accretion can still persist for many dozens of orbits prior to merging, despite the fact that the accretor is already desynchronized. We also find that as the binary approaches merger, the orbital period changes so quickly that corotation cannot be maintained. In other words, while our initial conditions may not represent the physical state at the onset of resolvable mass transfer, the systems quickly evolve to the state that is likely realized in nature.

As expected, systems of a given mass ratio lead to higher peak temperatures as the total system mass increases. We additionally find that there is a clear trend to produce higher peak temperatures as the mass ratio approaches unity (Fig. 9). It comes as an interesting and maybe somewhat surprising result that a large fraction of the He-accreting systems are expected to explode prior to merger or at surface contact: we find that all systems with accretor masses below 1.1 M\(^\odot\) undergo such explosions, provided that the He-donating WD exceeds ~0.4 M\(^\odot\) (Fig. 8). This opens up a large fraction of unexplored parameter space for thermonuclear explosions. A substantial fraction of these systems could even explode earlier via stream-induced detonations, as we have demonstrated in our earlier work (Guillochon et al. 2010). DD binaries made entirely of carbon and oxygen, in contrast, are unlikely to explode prior to or during the merger. This conclusion may need to be altered for cases where the CO WDs possess more than a critical amount of He in their outer layers (Shen & Bildsten 2009; Raskin et al. 2012). Systems that do not undergo an early thermonuclear event may still be prone to an explosion long after the merger (Yoon et al. 2007).

Based on a previous calculation (Fryer et al. 2010), it has been argued that the environment surrounding exploding DD systems is polluted with tens of per cent of a solar mass and would have produced a detectable optical excess during the early evolution of 2011fe (Nugent et al. 2011). However, in none of our exploding systems do we find more than 1 per cent of a solar mass exterior to 10\(^{10}\) cm, and in fact many of our systems have no measurable mass beyond that distance (Fig. 10). Because we are starting with initially synchronized WD binaries, the total amount of angular momentum in our simulations is a robust upper limit. Therefore, we can conclude that the ambient environment will contain even less mass at these distances than what our current simulations predict.

We have shown that DD mergers provide a broad channel for producing thermonuclear explosions. While our simulations indicate that systems undergo detonations within the He layer, it remains an open question whether this primary detonation can actually trigger the explosion of the CO core. In the event that the CO core fails to detonate, these events may still resemble sub-luminous Type Ia (Shen et al. 2010). Future research has to investigate which of these thermonuclear explosions are consistent with current Type Ia supernova observations.

**ACKNOWLEDGMENTS**

We thank Marcus Brüggen, Philipp Podsiadlowski and Ken Shen for insightful comments and stimulating discussions. We acknowledge support from DFG grant RO 3399 (MD and SR), the David and Lucile Packard Foundation (JG, SR, ER-R), NSF grant AST-0847563 (ER-R) and the NASA Earth and Space Science Fellowship (JG).
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APPENDIX A: POLYNOMIAL FITTING FUNCTIONS FOR HE DETONATIONS

Approximate formulae obtained by polynomial fitting of the contours lines with $\tau_{\text{acc}}/\tau_{\text{dyn}} = 1$ are presented for the He mass-transferring systems (see Section 4.2). The contour $(\tau_{\text{acc}}/\tau_{\text{dyn}})(T) = 1$ shown in the upper-left panel of Fig. 8 is approximated to better
than 4 per cent by (in solar units)

\[ M_2 = 21.065M_1^4 - 60.751M_1^3 + 63.515M_1^2 
- 28.569M_1 + 5.1209, \]  
(A1)

with \( M_1 \) between 0.46 and 1.13 \( M_\odot \).

The conservative estimate for contact He detonations
\((\tau_{\text{nuc}}/\tau_{\text{dyn}})((T)) = 1 \) (lower-left panel of Fig. 8) is approximated
to within 3 per cent with

\[ M_2 = 39.431M_1^4 - 127.35M_1^3 + 152.17M_1^2 
- 80.163M_1 + 16.329, \]  
(A2)

with \( M_1 \) between 0.65 and 1.13 \( M_\odot \).

For stream detonations (see Fig. 6), a rational function was used
to approximate the data at \( 0.877 \leq M_1 \leq 1.2 \, M_\odot \):

\[ M_2 = \frac{0.39427M_1^4 - 1.18432M_1^3 + 1.18856M_1 - 0.39735}{M_1^4 - 3.05295M_1^3 + 3.11186M_1 - 1.05541}, \]  
(A3)

With this function we are slightly overestimating the number of
systems that may lead to a detonation as the curve is not single-valued for \( M_1 \leq 0.93 \, M_\odot \).