Effective Potentials and Symmetry Restoration in the Chiral Abelian Higgs Model

Jens O. Andersen
Institute of Physics
University of Oslo
P.O. BOX 1048, Blindern
N-0316 Oslo, Norway

Abstract

The chiral Abelian Higgs model is studied at finite temperature. By integrating out the heavy modes, we make a three-dimensional effective theory for the static modes. It is demonstrated that the plasma masses are correctly reproduced to leading order in $m^2/T^2$. The effective potential for the composite operator $\phi^\dagger \phi$ is calculated at one loop for the resulting three-dimensional theory and it is shown that the result is gauge parameter invariant. The numerical investigation of the potential reveals that the symmetry is restored via a first order phase transition. Comparison with the ordinary ring improved potential is made and it is found that the barrier height at $T_c$ is somewhat higher.

PACS numbers: 5.70.Fh, 11.15Ex, 12.15.Ji

1 Introduction

Quantum field theories at finite temperature has received considerable attention, since the work of Dolan and Jackiw twenty years ago [1]. It was first observed by Kirnitz and Linde [2] that symmetries which are spontaneously broken at zero temperature are normally restored at high temperatures. Recently, there has been much interest in the electroweak phase transition, mainly because of its role in the generation of the baryon asymmetry of the universe [3]. Important aspects are the order and strength of the phase transition as a function of the Higgs boson mass, and the calculation of nucleation rates.

Several approaches have been used in the study of the electroweak phase transition. This includes the use of three-dimensional effective theories [4,5], the $\epsilon$-expansion [6] and ring improved effective potentials [7]. Recently, Buchmüller et al. have proposed an effective potential for the operator $\sigma = \phi^\dagger \phi$ and have applied it to the Abelian Higgs model and SU(2) [8,9]. It has been claimed that this new effective potential is gauge invariant, which is a desirable property. The problems of gauge invariance and gauge
fixing dependence are important and must be taken seriously. Physical quantities such as the critical temperature must be gauge invariant. Normally, the effective potential is gauge fixing independent at the minimum, that is for the value of the field that is a solution of the effective field equations (“on shell”). However, the potential should also be gauge invariant away from the minimum (“off shell”) since the form of the potential (e.g. barrier height) determines the dynamics of the phase transition.

In the present work we consider the chiral Abelian Higgs model with a quartic self interaction in 3+1 dimensions. This model has previously been studied by Arnold and Espinosa [10], using resummation techniques. We shall use a different approach by making a three-dimensional effective theory for the static modes by integrating out the heavy modes. This method has been extensively used in finite temperature field theory in recent years. Applications to U(1) and SU(2) gauge theories are found in refs. [4,5] and we recommend them for further details.

The decoupling of the non-static (heavy) modes from the high temperature dynamics was proposed a long time ago and has been studied in detail in various theories, e.g. QCD and QED [11]. By doing this dimensional reduction [11-13] the infrared behaviour of the static modes improves due to the induced thermal masses. Secondly, this approach induces non-linear interaction between the static modes.

The dimensional reduction is carried out in section two and it is explicitly demonstrated that one obtains the same thermal masses to leading order in $m^2/T^2$ as those found by solving the Schwinger-Dyson equation in the full four-dimensional theory [10]. In section three we compute the effective potential for the composite operator $\sigma = \phi^\dagger \phi$ to one loop order. The calculations are carried out in the $R_\xi$ gauge and it is shown that the effective potential is independent of the gauge parameter at one loop. Although this does not imply gauge invariance of the effective action, this independence is of course a nice feature. The potential is investigated numerically at finite temperature and is compared with the ordinary ring improved effective potential. It is found that the symmetry is restored at high temperature via a first order phase transition. Finally, we summarize and make some comments on further developments in section four.

## 2 The Three-dimensional Effective Theory

### The Abelian Higgs model. Let us first consider the Abelian Higgs model without fermions. The Euclidean Lagrangian reads:

\[
\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} (D_\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{2} \epsilon^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2. \tag{1}
\]

plus gauge fixing terms. Here $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative. The corre-
The corresponding action is then

\[ S = \int_0^\beta d\tau \int \mathcal{L} \, d^3x. \quad (2) \]

At finite temperature we expand the fields as

\[ A_i(x, \tau) = \beta^{-\frac{1}{2}} \left[ A_i(x) + \sum_{n \neq 0} a_{i,n}(x)e^{2\pi in\tau/\beta} \right], \quad i = 1, 2, 3 \quad (3) \]

\[ A_\tau(x, \tau) = \beta^{-\frac{1}{2}} \left[ \rho(x) + \sum_{n \neq 0} a_{\tau,n}(x)e^{2\pi in\tau/\beta} \right] \quad (4) \]

\[ \Phi(x, \tau) = \beta^{-\frac{1}{2}} \left[ \phi_0(x) + \sum_{n \neq 0} \phi_n(x)e^{2\pi in\tau/\beta} \right]. \quad (5) \]

The calculations will be carried out in the thermal static gauge \[14\]:

\[ a_{\tau,n}(x) = 0, \quad \forall \ n. \quad (6) \]

We integrate over \( \tau \) and exploit the orthonormality of the modes. There will now be terms in the action which involve only the static modes

\[ S^{(0)} = \int \left[ \frac{1}{4} F_{ij}F_{ij} + \frac{1}{2}(\partial_i \rho)^2 + \frac{1}{2}(\partial_i \phi_0)^\dagger(\partial_i \phi_0) - \frac{1}{2} e^2 \phi_0^\dagger \phi_0 + \frac{\lambda T}{4} (\phi_0^\dagger \phi_0)^2 
+ \frac{e^2 T}{2} (A_i^2 + \rho^2)\phi_0^\dagger \phi_0 + \frac{ie A_i T^\frac{1}{2}}{2} (\phi_0 \partial_i \phi_0^\dagger - \phi_0^\dagger \partial_i \phi_0) \right] d^3x. \quad (7) \]

The terms that are quadratic in non-static modes are

\[ S^{(2)} = \sum_{n \neq 0} \int \left[ \frac{1}{2} (\partial_i a_{j,n})^\dagger(\partial_i a_{j,n}) - \frac{1}{2} (\partial_i a_{i,n})^\dagger(\partial_i a_{i,n}) + \frac{1}{2}(2\pi n T)^2 a_{i,n}^\dagger a_{i,n} 
+ \frac{1}{2}(2\pi n T)^2 \phi_n^\dagger \phi_n + \frac{1}{2} (\partial_i \phi_n)^\dagger(\partial_i \phi_n) - \frac{1}{2} e^2 \phi_n^\dagger \phi_n \right] d^3x, \quad (8) \]

and finally there are terms representing the interactions between the static and the non-static modes. These terms generate the effective thermal masses of the zero modes \( \phi_0 \) and \( \rho \)

\[ S^{(2)}_{\text{int}} = \sum_{n \neq 0} \int \left[ \frac{1}{2} e^2 T a_{i,n}^\dagger a_{i,n}(\phi_0^\dagger \phi_0) + e^2 \pi n T^2 \phi_n^\dagger \phi_n + \frac{\lambda T}{4} 4\phi_0^\dagger \phi_0 \phi_n^\dagger \phi_n 
+ \frac{ie T^\frac{1}{2}}{2} a_{i,n}(\phi_0 \partial_i \phi_n^\dagger + \phi_n \partial_i \phi_0^\dagger - \phi_0^\dagger \partial_i \phi_n - \phi_n^\dagger \partial_i \phi_0) + \frac{1}{2} e^2 \rho^2 T(\phi_n^\dagger \phi_n) \right] d^3x. \quad (9) \]

We should make the remark that we in eq. \[9\] have set \( A_i(x) = 0 \) since these terms only affect the kinetic part of the effective theory (see refs. \[15,16\]). The omission of these terms then correspond to the neglect of wave function renormalization and also some finite higher order corrections to the interaction between \( \rho \) and \( \phi_0 \) and to the scalar potential of the \( \rho \)-field. These corrections are of order \( e^4 \) and should hence not be included since
we calculate consistently to order $e^2$. The fact that the spatial part of $A_\mu(x)$ remains massless and therefore acts as the gauge field in the dimensionally reduced theory could also have been predicted on general grounds by considering Ward identities in the high temperature limit [13,18].

Introducing the two real fields by $\phi_n = \phi_{1,n} + i\phi_{2,n}$, we define the propagators for the fields $a_{i,n}, \phi_{1,n}$ and $\phi_{2,n}$ by:

$$\begin{aligned}
\left[ -\nabla^2 + (2\pi nT)^2 \right] \delta_{ij} + \partial_i \partial_j \right] D_{jk,n}(x,x') &= \delta_{jk}(x,x') \\
\left[ -\nabla^2 - c^2 + (2\pi nT)^2 \right] \Delta_{i,n}(x,x') &= \delta(x,x'), \quad i = 1, 2.
\end{aligned} \quad (10) \quad (11)$$

We may write the effective action for the zero modes as

$$S_{eff} = S^{(0)} + S^{(2)} + \Delta S \quad (12)$$

where

$$\Delta S = \langle S^{(2)}_{\text{int}} \rangle - \frac{1}{2} \langle (S^{(2)}_{\text{int}})^2 \rangle + \ldots \quad (13)$$

and $S^{(2)}$ is the quadratic contribution from eq. (8). The second term in eq. (13) is necessary in order to calculate consistently to order $\lambda$ and $e^2$. Using the propagators we find the different contributions to the effective theory. The contribution from the heavy scalar modes to the static scalar mode is easily found:

$$\Delta S_{\text{scalar}} = \sum_{n \neq 0} \int \frac{\lambda T \phi_0^+ \phi_0 \Delta_{i,n}(x,x)}{d^3x}$$

$$= \sum_{n \neq 0} \int \left[ 2\lambda T \phi_0^+ \phi_0 \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 - c^2 + (2\pi nT)^2} \right] d^3x$$

$$= \int \left[ \frac{2\lambda T \phi_0^+ \phi_0}{2\pi^2} \int_0^\infty \frac{pdp}{(\exp \beta p - 1)} + O\left(\frac{c^2}{T^2}\right) \right] d^3x$$

$$= \int \left[ \lambda \phi_0^+ \phi_0 \frac{T^2}{6} + O\left(\frac{c^2}{T^2}\right) \right] d^3x. \quad (14)$$

In the above equation we have dropped a divergence, which corresponds to a mass renormalization (see also ref. [24]). The corresponding Feynman diagrams are shown in fig. 1a. We would also like to make the remark that our renormalization procedure differs from that of Jakovac and Patkos [14]. This is partly due to their introduction of the auxiliary fields $\chi$ by a Hubbard-Stratonovich transformation [24].

The contributions from the vector particles are calculated in a similar way, and after some lengthy algebra we find

$$\Delta S = \int \left[ \frac{1}{2} \phi_0^+ \phi_0 (4\lambda + 3e^2) \frac{T^2}{12} + \frac{1}{6} \rho^2 e^2 T^2 \right] d^3x. \quad (15)$$
The Feynman diagrams are displayed in figs. 1b - 1c and 2a - 2b. Our result is to leading order in $c^2/T^2$ in accordance with that of Arnold and Espinosa [10], who used the Schwinger-Dyson equations for the propagators. The result is also in agreement with that obtained by Jakov´ac and Patk´os [14] to order $\lambda$ and $e^2$.

The effective theory for the static modes is now obtained:

$$S_{\text{eff}} = \int \left[ \frac{1}{4} F_{ij} F_{ij} + \frac{1}{2} (\partial_i \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} (D_\mu \phi_0)^\dagger (D_\mu \phi_0) + \frac{1}{2} m^2 \phi_0^\dagger \phi_0 + \frac{\lambda T}{4} (\phi_0^\dagger \phi_0)^2 \\ + \frac{e^2 T}{2} \rho^2 \phi_0^\dagger \phi_0 \right] d^3x,$$

(16)

where the thermal masses are given by:

$$m^2 = -c^2 + (4 \lambda + 3 e^2) T^2 \frac{T^2}{12},$$

(17)

$$m_\rho^2 = \frac{e^2 T^2}{3}.$$  

(18)

Note that we discard $S^{(2)}$ from the effective theory since it is independent of the static mode. It only gives a temperature dependent contribution to the effective action and will not affect the critical temperature. From eq. (16) one observes that the zeroth component of the vector potential $A_\mu(x, \tau)$ plays the role of an extra scalar field in this effective theory and that it is coupled non-linearly to the static mode of the scalar field $\Phi(x, \tau)$.

The chiral Abelian Higgs model. Let us next couple Dirac fermions chirally to the Abelian Higgs model:

$$L' = L + \bar{\Psi} \left[ \Phi^\dagger \frac{1 - \gamma_5}{2} + \Phi \frac{1 + \gamma_5}{2} \right] \Psi + \bar{\Psi} \left[ \gamma^\mu \partial_\mu - ie \gamma^\mu A_\mu \frac{1 - \gamma_5}{2} \right] \Psi.$$  

(19)

At finite temperature we expand the fermionic field as

$$\Psi(x, \tau) = \beta^{-\frac{1}{2}} \sum_n \psi_n(x) e^{i\pi(2n+1)/\beta}.$$  

(20)

The fermions are antiperiodic in time, which implies that they do not contribute to $S^{(0)}$ in eq. (10). We then write

$$S_{(2)'}^{(2')} = S^{(2)} + \sum_n \int \bar{\psi}_n \gamma^\mu \partial_\mu \psi_n \, d^3x,$$

(21)

$$S_{\text{int}}^{(2)'} = S^{(2)}_{\text{int}} + \sum_n \int \left[ gT^\frac{1}{2} \bar{\psi}_n \phi_0^\dagger \frac{1 - \gamma_5}{2} \psi_n + gT^\frac{1}{2} \bar{\psi}_n \phi_0 \frac{1 + \gamma_5}{2} \psi_n \\ -ie\rho T^\frac{1}{2} \bar{\psi}_n \gamma^0 \frac{1 - \gamma_5}{2} \psi_n \right] d^3x.$$  

(22)

Note that the contribution from fig. 1b depends on the external momentum $k$. We have made a high temperature expansion and included the dominant $T^2$ term. The $k^2$ pieces contribute to wave function renormalization.
where \( \gamma^\mu \partial_\mu \) now means \( \gamma^i \partial_i + \gamma^0 (2n+1)\pi T \). The fermion propagators are defined by

\[
(\gamma^\mu \partial_\mu) S_{F,n}(x, x') = \delta(x, x'),
\]

(23)

We can now compute the fermion contribution to the effective action \( S_{eff} \) for the static modes. The calculations are carried out consistently to order \( g^2 \) applying the same techniques as previously. After some manipulations one finds that the fermion contribution to \( \langle S^{(2)_{int}} \rangle \) vanishes identically due to the properties of the gamma matrices. Thus one is left with the correction:

\[
\Delta S_{\text{fermion}} = \sum_n \text{Tr} \int \left[ \frac{1}{2} g^2 T \phi_0^\dagger \phi_0 \Delta F_n(x, x') \left( \frac{1 - \gamma^5}{2} \right) \Delta F_n(x, x') \left( \frac{1 + \gamma^5}{2} \right) \right] d^3x d^3x'
\]

(24)

The diagrams are shown in figs. 1d and 2c. Again, we have only kept the dominant temperature contribution. Using eqs. (15) and (24) one obtains the following effective masses

\[
m^2 = -c^2 + (4\lambda + 3e^2 + g^2) \frac{T^2}{12}
\]

\[
m^2_\rho = \frac{e^2 T^2}{2}.
\]

(25)

Our result is again in agreement with that of Arnold and Espinosa [10].

### 3 The One-Loop Effective Potential

**The one-loop effective potential.** With the effective three-dimensional theory at hand, we now calculate the effective potential for the composite field \( \sigma = \phi^\dagger \phi \) [8,9] in the one-loop approximation. The calculations for the Abelian Higgs model and the chiral Abelian Higgs model are identical; the only difference is that different thermal masses enter into the final result. (In the following we drop the subscript on the scalar field and hence write \( \phi \) instead of \( \phi_0 \)). In order to do so we compute the free energy in the presence of a constant external source \( J \):

\[
e^{-\Omega W(J)} = \int \mathcal{D}A_i \mathcal{D}\phi^\dagger \mathcal{D}\phi \mathcal{D}\rho e^{-S_{eff}(A_i, \phi^\dagger, \phi, \rho) - \int \phi^\dagger \phi J d^3x},
\]

(26)

where \( \Omega \) is the three-dimensional volume. The composite field \( \sigma \) is defined through the relation:

\[
\frac{\delta W(J)}{\delta J} = \sigma.
\]

(27)
The effective potential is then obtained as a Legendre transform in the usual way:

\[ V(\sigma) = W(J) - \sigma J. \]  

(28)

The classical potential is

\[ V_0(\phi^\dagger \phi) = \frac{1}{2} (m^2 + 2J) \phi^\dagger \phi + \frac{1}{4} \lambda T (\phi^\dagger \phi)^2 + \frac{1}{2} m^2_\rho \rho^2 + \frac{1}{2} e^2 T \rho^2 \phi^\dagger \phi. \]  

(29)

The classical equations of motion then read

\[ e^2 T \rho^2 \phi + (m^2 + 2J + \lambda T \phi^\dagger \phi) \phi = 0, \quad e^2 T \rho \phi^\dagger \phi + m^2_\rho \rho = 0, \]  

(30)

and have two solutions:

\[ \overline{\rho} = 0, \quad \overline{\phi} = \phi_s = 0 \]  

(31)

\[ \overline{\rho} = 0, \quad \overline{\phi} = \phi_b = \left[ -\frac{1}{\lambda T} (m^2 + 2J) \right]^{\frac{1}{2}} e^{i\alpha}. \]  

(32)

Here \( \alpha \) is a phase. The solutions (31) and (32) correspond to the global minimum of the classical action in the presence of the source \( J \) for \( m^2 + 2J > 0 \) and \( m^2 + 2J < 0 \), respectively.

The masses of the particles are given by the following expressions

\[ m_A^2 = e^2 T |\phi|^2, \quad m_\phi^2 = m^2 + 2J + 3\lambda T |\phi|^2, \]  

(33)

\[ m_\chi^2 = m^2 + 2J + \lambda T |\phi|^2, \quad m_\rho^2 = \frac{e^2 T^2}{2} + e^2 T |\phi|^2. \]  

(34)

In the symmetric phase the masses are

\[ m_A^2 = 0, \quad m_\phi^2 = m^2 + 2J, \quad m_\chi^2 = m^2 + 2J, \quad m_\rho^2 = \frac{e^2 T^2}{2}, \]  

(36)

while in the broken phase one obtains

\[ m_A^2 = -\frac{e^2}{\lambda} (m^2 + 2J), \quad m_\phi^2 = -2(m^2 + 2J), \quad m_\chi^2 = 0, \quad m_\rho^2 = \frac{e^2 T^2}{2} - \frac{e^2}{\lambda} (m^2 + 2J). \]  

(37)

We shall work in the \( R_\xi \) gauge, where the gauge fixing term is

\[ \mathcal{L}_{GF} = \frac{1}{2} \xi (\partial_i A_i + \xi e^2 T \phi \phi_2)^2. \]  

(38)

where we have used the global O(2) symmetry to make \( \phi \) purely real and \( \phi_2 \) is the imaginary part of the quantum field \( \phi \). This gauge is particularly simple since the cross
terms between the scalar field and the gauge field in the effective theory disappear. This makes it rather easy to calculate the one loop correction to the classical potential:

\[
W(J) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[ 2 \log(k^2 + m_A^2) + \log(k^2 + m_\phi^2) + \log(k^2 + m_\chi^2 + \xi m_A^2) + \log(k^2 + \xi e^2 T^2 \phi^2) + (k^2 + m_\rho^2) \right].
\]

(39)

The corresponding ghost contribution is found to be

\[
S_{\text{ghost}} = -\int \frac{d^3k}{(2\pi)^3} \log(k^2 + \xi e^2 T^2 \phi^2).
\]

(40)

Using dimensional regularization (see e.g. ref. [19]) the above integrals are easily computed:

\[
\int \frac{d^3k}{(2\pi)^3} \log(k^2 + M^2) = -\frac{1}{6\pi} M^3.
\]

(41)

The result is perfectly finite after regularization and is independent of the renormalization scale \(\mu\).

We see that the terms involving \(\xi\) cancel since one of the masses, either \(m_A^2\) or \(m_\chi^2\), vanishes. The effective potential is thus gauge parameter independent (to one loop order), in contrast to the ordinary effective potential [1]. Moreover, it can also be shown that eqs. (42) and (43) below can be obtained in Lorentz gauge.

There is some confusion in the literature about gauge invariance and gauge parameter independence. We emphasize that gauge parameter independence and gauge invariance are related issues, but not equivalent. One may have gauge invariance with respect to gauge transformations of the background fields, but still have dependence on how one fixes the gauge of the quantum fields. This is the case when applying the method of mean field gauges. See ref. [17] for details.

In the symmetric phase one finds the free energy

\[
W_s(J) = -\frac{1}{6\pi} (m^2 + 2J)^{\frac{3}{2}}.
\]

(42)

In the broken phase we get

\[
W_b(J) = -\frac{1}{4\lambda T} (m^2 + 2J)^2 - \frac{1}{6\pi} \left[ -\frac{e^2}{\lambda} (m^2 + 2J)^{\frac{3}{2}} \right] - \frac{1}{12\pi} \left[ -2(m^2 + 2J)^{\frac{3}{2}} \right] - \frac{1}{12\pi} \left[ \frac{e^2 T^2}{2} - \frac{e^2}{\lambda} (m^2 + 2J)^{\frac{3}{2}} \right].
\]

(43)

\(^2\)Working in Lorentz gauge, where \(L_{GF} = \frac{1}{4\pi} (\partial_i A_i)^2\), one finds that the masses and hence the effective potential are explicitly dependent on the gauge parameter. Furthermore, this dependence disappears at the minimum of the potential, as explained in the introduction.
Notice that the contribution from the $\rho$ particles is independent of $J$ in the symmetric phase. The contribution to the effective potential is therefore independent of the background field and is discarded. A similar remark applies to the vector meson in the symmetric phase, since the mass vanishes.

In the broken phase it is sufficient to use the tree level expression for the free energy in order to calculate $\sigma$ from eq. (27) [8]. It is now straightforward to derive the results

$$V_s(\sigma) = \frac{1}{2} m^2 \sigma - \frac{10\pi^2}{3} \sigma^3 \quad \text{(44)}$$

and

$$V_b(\sigma) = \frac{1}{2} m^2 \sigma + \frac{1}{4} \lambda T \sigma^2 - \frac{1}{6\pi} e^3(\sigma T)^3 - \frac{1}{12\pi} (2\lambda \sigma T)^\frac{3}{2} - \frac{1}{12\pi} \left(\frac{e^2 T^2}{2} + e^2 \sigma T\right)^\frac{3}{2}. \quad \text{(45)}$$

From eq. (27) one finds that the symmetric phase is represented by $\sigma < 0$ while the broken phase is represented by $\sigma > 0$. Thus, we can write our result as

$$V(\sigma) = V_b(\sigma) \Theta(\sigma) + V_s(\sigma) \Theta(-\sigma). \quad \text{(46)}$$

Some comments are in order. Firstly, we note that we have a contribution to the potential from the zeroth component of the gauge field. This is in turn a consequence of the non-linear interaction between the fields $\phi$ and $\rho$ that was a result of the dimensional reduction. Secondly, the non-analytic terms in the effective potential are independent of $m^2$. This implies that $V(\sigma)$ is valid also below the barrier temperature (in the sense that it is purely real), contrary to the ring improved effective potential (see eq. (50) below).

To lowest order in the couplings [5,8] one has

$$V_4(\sigma) = TV_3(\frac{\sigma}{T}), \quad \text{(47)}$$

where $V_4$ is the effective potential in the full four-dimensional theory and $V_3$ is the effective potential in the three-dimensional theory. This implies that

$$V_s(\sigma) = \frac{1}{2} m^2 \sigma - \frac{10\pi^2}{3} \frac{\sigma^3}{T^2} \quad \text{(48)}$$

and

$$V_b(\sigma) = \frac{1}{2} m^2 \sigma + \frac{\lambda}{4} \sigma^2 - \frac{T}{6\pi} (e\sigma)^\frac{3}{2} - \frac{T}{12\pi} (2\lambda \sigma)^\frac{3}{2} - \frac{T}{12\pi} \left(\frac{e^2 T^2}{2} + e^2 \sigma T\right)^\frac{3}{2}. \quad \text{(49)}$$

The ring improved effective potential. The ordinary ring improved effective potential for the chiral Abelian Higgs model computed in Landau gauge is [10]

$$V_{\text{ring}}(\phi_0) = \frac{1}{24} (4\lambda + 3e^2 + g^2)(T^2 - T_b^2)\phi_0^2 - \frac{T}{12\pi} (2M_T^3 + M_L^3 + m_1^3 + m_2^3) + \frac{1}{4} \lambda \phi_0^4, \quad \text{(50)}$$
where \( T_b \) is the barrier temperature:

\[
T_b^2 = \frac{12c^2}{4\lambda + 3e^2 + g^2}
\]

and the thermal masses are

\[
M_T^2 = e^2\phi_0^2
\]

\[
M_L^2 = e^2\phi_0^2 + \frac{e^2T^2}{2}
\]

\[
m_1^2 = -c^2 + \lambda\phi_0^2 + (4\lambda + 3e^2 + g^2)\frac{T^2}{12}
\]

\[
m_2^2 = -c^2 + 3\lambda\phi_0^2 + (4\lambda + 3e^2 + g^2)\frac{T^2}{12}
\]

Here \( \phi_0 \) is the background field, which is always larger than or equal to zero. The symmetric phase is represented by \( \phi_0 = 0 \), while the broken phase is represented by \( \phi_0 > 0 \). Notice also that the masses \( m_1^2 \) and \( m_2^2 \) becomes negative below the barrier temperature, implying that the effective potential becomes complex.

In fig. 3 we have shown \( V(\sigma) \) at the critical temperature for a Higgs mass of 55 GeV, and gauge and Yukawa couplings of 0.45 and 0.6, respectively. In fig. 4 we have shown the corresponding ring improved effective potential. Both potentials show that the symmetry is restored via a first order phase transition. This is to be expected from a renormalization group argument, namely that the renormalization group equations do not have a non-trivial fixed point \[18\]. We also see that the form of the potentials in the symmetric phase is qualitatively the same, although the barrier height is approximately 30% higher for \( V(\sigma) \). In turn, this will affect the phase transition. Moreover, the potential in the symmetric phase increases rapidly for small values of \( \sigma \).

### Summary and Final Remarks

We have calculated a three-dimensional effective theory for the static modes in the chiral Abelian Higgs model by integrating out the heavy modes. The thermal masses are seen to be correctly reproduced. Using this three-dimensional model, a one-loop calculation for the composite operator \( \phi^\dagger \phi \) has been performed. We have then used the obtained potential to investigate the phase transition. The potential is similar to the ring improved effective potential at the critical temperature and the symmetry is restored via a first order phase transition.

We have also noted that the effective potential is gauge parameter independent in the one-loop approximation. The questions of gauge invariance and gauge fixing dependence are important and will be subject of further investigation, particularly in connection with
the gauge invariant Vilkovisky-DeWitt effective action \[22\]. This work is in progress. Finally, it would be of interest to extend the present work by doing a Hubbard-Stratonovich transformation \[21\]. One could then carry out a two-variable saddle point approximation for the auxiliary fields. This method has previously been applied to \(\lambda \phi^4\) and has correctly reproduced the second order phase transition this model undergoes \[23\].

The author would like to thank Finn Ravndal for useful comments and suggestions.

References

[1] L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320, 1974.
[2] D. A. Kirzhnits and A. D. Linde, Phys. Lett. B 42, 471, 1972.
[3] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Annu. Rev. Nucl. Part. Sci. 43, 27, 1993.
[4] A. Jakováč, K. Kajantie and A. Patkós, Phys. Rev. D 49, 6810, 1994.
[5] K. Farakos, K. Kajantie, K. Rummukainen, and M. Shaposhnikov, Nucl. Phys. B 425, 67, 1994.
[6] P. Arnold and L. G. Jaffe, Phys. Rev. D 49, 3003, 1994.
[7] M. Carrington, Phys. Rev. D 45, 2933, 1992.
[8] W. Buchmüller, Z. Fodor and A. Hebecker, Phys. Lett B 331, 131, 1994.
[9] W. Buchmüller and Z. Fodor, Phys. Lett B 331, 124, 1994.
[10] P. Arnold and O. Espinosa, Phys. Rev. D 47, 3546, 1993.
[11] A. N. Jourjine, Ann. Phys 155, 305, 1984.
[12] T. Appelquist and R. Pisarski, Phys. Rev. D 23, 2305, 1981.
[13] N. P. Landsman. Nucl. Phys. B 322, 498, 1989.
[14] A. Jakováč and A. Patkós, Z. Phys. C 60, 361, 1993.
[15] I. Moss, D. Toms and A. Wright, Phys. Rev D 46, 1671, 218z1992.
[16] A. Jakováč and A. Patkós, Phys. Lett. B 334, 391, 1994.
[17] L. F. Abbott, Nucl. Phys. B185, 189, 1981.
[18] P. Ginsparg, Nucl. Phys B 170, 388, 1980.
[19] L. H. Ryder, *Quantum Field Theory*, Cambridge University Press, Cambridge, 1985.

[20] P. Fendley, Phys. Lett. B 196, 175, 1987.

[21] J. Hubbard, Phys. Rev. Lett. 3, 77, 1959; R. L. Stratonovich, Dokl. Akad. Nauk SSSR 115, 1957.

[22] G. A. Vilkovisky, Nucl. Phys B 234, 125, 1984.

[23] H. Meyer-Ortmanns and A. Patkós, Phys. Lett. B 297, 331, 1993.
FIGURE CAPTIONS:

Figure 1: Leading contributions to the scalar mass in the high temperature limit.

Figure 2: Dominant contribution to the vector mass in the high temperature limit.

Figure 3: Effective potential for the composite operator $\sigma = \phi^\dagger \phi$ at the critical temperature. The Higgs mass is 55 GeV, $g = 0.6$ and $e = 0.45$.

Figure 4: Ring improved effective potential at the critical temperature. The Higgs mass is 55 GeV, $g = 0.6$ and $e = 0.45$. 
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9501413v2
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9501413v2
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9501413v2
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9501413v2