Late-time effects of Planck-scale cosmology: dilatonic interpretation of the dark energy field

M. Gasperini

Dipartimento di Fisica, Università di Bari,
Via Amendola 173, 70126 Bari, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

Abstract

We present a model of dark energy based on the string effective action, and on the assumption that the dilaton is strongly coupled to dark matter. We discuss the main differences between this class of models and more conventional models of quintessence, uncoupled to dark matter. This paper is based on talks presented at the “VII Congresso Nazionale di Cosmologia” (Osservatorio Astronomico di Roma, Monte Porzio Catone, November 2002), and at the Meeting “Dark Energy Day” (University of Milano-Bicocca, November 2002). To appear in the Proc. of the International Conference on “Thinking, Observing and Mining the Universe” (Sorrento, September 2003), eds. G. Longo and G. Miele (World Scientific, Singapore).
LATE-TIME EFFECTS OF PLANCK-SCALE COSMOLOGY:
DILATONIC INTERPRETATION OF THE DARK ENERGY FIELD

MAURIZIO GASPERINI
Dipartimento di Fisica, Università di Bari,
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari,
Via Amendola 173, 70126 Bari, Italy

We present a model of dark energy based on the string effective action, and on the assumption that the dilaton is strongly coupled to dark matter. We discuss the main differences between this class of models and more conventional models of quintessence, uncoupled to dark matter.

1. Introduction

Understanding Planck-scale physics is one of the main objects of modern theoretical physics, as an (almost) compulsory ingredient for a successful unification of all interactions. Present theoretical attempts, mainly based on supersymmetric models of strings and membranes, provide (at least in principle) consistent unifications schemes including quantized gravitational interactions, at all energy scales. Their direct verification, however, seems to be out of reach of the conventional experimental approach to high energy physics, unless we believe in the (probably unlikely) possibility of living in a “brane-world” Universe characterized by a very low scale of (higher-dimensional) bulk gravity.

Fortunately enough, as discussed for instance by Starobinski and Amelino-Camelia also at this Conference, direct and important experimental information on quantum gravity and Planck scale physics is presently coming (or is expected to come soon) from many astrophysical and cosmological observations. Here, in particular, I will discuss the possibility of interpreting the large-scale acceleration of our present Universe as a direct “late-time” effect of dilatonic interactions, correctly described at the Planck scale by string and M-theory models.

Let me start by recalling that the string effective action contains, even to lowest order, at least two fundamental fields, the metric and the dilaton:

\[ S = -\frac{1}{2\lambda^2} \int d^4 x \sqrt{-g} e^{-\phi} \left[ R + (\nabla \phi)^2 + V(\phi) + \ldots \right]. \quad (1.1) \]

The dilaton \( \phi \) is scalar field which, from a physical point of view, controls the strength of all interactions, in the context of unified and grand-unified models. From a geometric point of view it may represent the radius of the 11-th dimensions, in the context of M-theory models. Aside from its possible interpretation, the dilaton is a scalar field necessarily present in the action, it is non-minimally coupled to the metric and to the other fields, and a question which may arise naturally is whether or not such a field can automatically provide a model of “quintessence”, to explain the cosmic acceleration presently observed on large scales.

To answer this question we should ask, first of all, what happens to the dilaton in string cosmology models. Here we shall analyze, in particular, pre-big bang models, where the dilaton tends to grow starting from an initial state called “perturbative vacuum”, and corresponding to the asymptotic limit \( \phi \to -\infty \). Such
a growth is not damped, at least initially, by the potential, which is very flat in
the perturbative region (where \( V \sim e^{-\exp(-\phi)} \)), so that the dilaton, sooner or later,
necessarily enters the strong coupling regime. In such a context, large times are
synonym of large values of \( \phi \) and large couplings, and what happens today thus
depends on the form of the dilaton potential in the strong coupling regime. We
have, in principle, two possibilities.

A first possibility is that the potential develops some structures when approach-
ing the strong coupling regime, and that the dilaton today is frozen, trapped inside
a minimum of the potential, in a range of values which are perturbative enough so
as to keep small enough the effective coupling \( \alpha_{\text{GUT}} \) of grand-unified models (see
Fig. 1a). For instance\(^{11} \), \( \alpha_{\text{GUT}} \simeq \exp(\phi_0) \), with \( \phi_0 \sim -3 \). A typical example of
potential corresponding to such a scenario is the following\(^{12} \):

\[
V = m^2 \left[ e^{k_1(\phi - \phi_1)} + \beta e^{-k_2(\phi - \phi_1)} \right] e^{-e \exp[-\gamma(\phi - \phi_1)]},
\] (1.2)

where \( k_1, k_2, \phi_1, \epsilon, \beta, \gamma \) are (model-dependent) dimensionless numbers of order one.
In such a context it is possible to obtain realistic cosmological solution\(^{13} \), describing
a present Universe dominated by the dilaton potential and consistent with low-
energy gravitational phenomenology. However, it turns out difficult (if not impos-
sible) to solve the usual “fine tuning” and “coincidence” problems (i.e. to explain
why the energy density of the vacuum is so small, and why it is just of the same
order as the present dark-matter energy density).

The second possibility is that the dilaton has not been stopped by the potential,
and that today is free to run to plus infinity, rolling down a (probably exponentially)
suppressed potential (see Fig. 1b). In that case the strong coupling corrections to
the effective action become more and more important as time goes on, and one
needs an appropriate “saturation” mechanism to keep the effective couplings small
enough, to be compatible with present phenomenology. For instance\(^{14} \):

\[
\alpha_{\text{GUT}} \simeq \exp(\phi_0) \left( 1 + N e^{\phi_0} \right)^{-1}, \quad N \sim 10^2, \quad \phi \rightarrow +\infty,
\] (1.3)

as will be illustrated in the next section. A typical example of “bell-like” potential,
corresponding to this scenario, is the following\(^{15} \):

\[
V = m^2 \left[ \exp \left( -e^{-\phi}/\alpha_1 \right) - \exp \left( -e^{-\phi}/\alpha_2 \right) \right],
\] (1.4)

where \( \alpha_1 > \alpha_2 > 0 \), and \( c_1 \) is a number of order 100. In such a context it is
possible to obtain realistic cosmological solutions\(^{15} \) in which the present Universe
is dominated by a mixture of kinetic and potential dilaton energy density, and it
becomes possible to solve –or at least to relax– the coincidence problem. The rest
of this paper will be devoted to the discussion on this second possibility.
2. Late-time saturation of the dilaton couplings

Let us thus consider the large “bare-coupling” limit, in which \( \phi \to +\infty \). The string effective action, to lowest order in the higher-derivative \( \alpha' \) expansion, but to all orders in the dilaton loop corrections, can be written in this form,

\[
S = -\frac{1}{2\lambda_2^2} \int d^4 x \sqrt{-g} \left[ e^{-\Psi} R + Z(\phi) \partial_\mu \phi \partial^\mu \phi + 2\lambda_s^2 V(\phi) \right] + S_m(\phi, g, \text{matt}),
\]

where \( \Psi \) and \( Z \) are the “form factors” due to the loop corrections, and other corrections are included into the potential and the matter action \( S_m \).

In order to obtain the saturation of such corrections we shall follow the spirit of the old “induced-gravity” models, assuming the validity of an asymptotic expansion in inverse powers of the “bare” coupling constant \( g_s^2 = e^\phi \), both for the loop form factors, for the potential, and for the dilatonic charge \( q \) of the matter fields, defined by \( q = -(2/\rho_m \sqrt{-g})(\delta S_m/\delta \phi) \). We will set, in particular,

\[
e^{-\Psi} = c_1^2 + e^{-\phi} + \mathcal{O}(e^{-2\phi}), \quad Z = -c_2^2 + e^{-\phi} + \mathcal{O}(e^{-2\phi}), \quad V = V_0 e^{-\phi} + \mathcal{O}(e^{-2\phi}), \quad q = q_0 + \mathcal{O}(e^{-2\phi}).
\]

Here \( c_1^2, c_2^2 \) are dimensionless parameters of order one hundred, as they are controlled by the number of fundamental fields contributing to the dilatonic loops (this number is large in the context of unified models).

In the case of long-range dilaton interactions, that we are assuming here, we know that the present value of the effective dilatonic charge \( q(\phi) \) has to be extremely suppressed for ordinary macroscopic matter \( (q \ll 1) \), to avoid contradiction with the observed gravitational phenomenology at low energy. For the (possibly) more exotic components of dark matter, on the other hand, there is no need of such suppression, and the asymptotic charge \( q_0 \) could be non-zero, and even large, in principle. If this is the case we are lead to an interesting and non-standard cosmological scenario\(^{15} \).

According to the action (2.1) we must include in fact the dilaton energy density \( \rho_\phi \) among the matter sources, in addition to radiation, baryons and cold dark matter. The Einstein equations (in units \( 16\pi G = 1 \)) can be written in the usual form,

\[
6H^2 = \rho_{\text{rad}} + \rho_{\text{bar}} + \rho_{\text{cdm}} + \rho_\phi,
\]

but the various components of the cosmic fluid are now differently coupled to the dilaton. Consider, in particular, a model in which the coupling to ordinary matter decays exponentially with time (i.e. \( q_0 = 0 \) for radiation and baryons), while the coupling to dark matter grows with time, and tends to saturate to a constant value \( q_0 \) as \( t \to \infty \). At late enough times we then recover the usual conservation equation for radiation and baryons,

\[
\dot{\rho}_{\text{rad}} + 4H \rho_{\text{rad}} = 0, \quad \dot{\rho}_{\text{bar}} + 3H \rho_{\text{bar}} = 0,
\]

while the evolution of dark matter is strongly coupled to the dilaton evolution\(^{16} \), through a model-dependent coupling function \( \epsilon(\phi) \),

\[
\dot{\rho}_{\text{cdm}} + 3H \rho_{\text{cdm}} - \frac{1}{2} \epsilon(\phi) \dot{\phi} \rho_{\text{cdm}} = 0, \quad \dot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} + \frac{1}{2} \epsilon(\phi) \rho_{\text{cdm}} = 0.
\]

Here \( \dot{\phi} \) is the canonically rescaled dilaton field in the Einstein frame, and \( \epsilon(\phi) \) is determined\(^{15,17} \) by the explicit form of the asymptotic expansion (2.3).

Because of this coupling, there are possible drastic modifications of the standard scenario after equilibrium. When the Universe become matter-dominated,
the dilaton (still subdominant) is “dragged” by dark matter, and we may have a first (very slight) modification of the standard decelerated evolution, since $\rho_{\text{cdm}} \sim \rho_0 \sim a^{-(3+\epsilon^2)}$, where $a$ is the scale factor (of course, $\epsilon < 1$ to avoid contradictions with standard gravitational phenomenology). When the dilaton potential comes into play, eventually, the Universe enters a final phase in which $\rho_{\text{cdm}} \sim \rho_0 \sim V \sim a^{-6/(2+q_0)} \sim H^2$, which is accelerated for $q_0 > 1$, and in which the ratio of the dilaton to dark-matter energy density is frozen, asymptotically, to a constant number of order one (depending on $q_0, c_1, c_2$). The fraction of kinetic to potential energy density of the dilaton (i.e., the equation of state of the dark energy component) is also fixed by the parameters of the asymptotic expansion (2.3).

To give a qualitative illustration of such scenario we present in Fig. 2 a plot of the various energy densities, as a function of $a$ and of the red-shift parameter $z = a_0/a - 1$. The curves have been obtained through a numerical integration of the string cosmology equations\(^{15}\) obtained from the action (2.1), with the potential (1.4) and with $\Psi, \dot{Z}$ defined by the expansion (2.2) truncated to first order. We have set $q(\phi) = q_0 e^{q_0 \phi}/(c_1^2 + e^{q_0 \phi}), q_0 = 2.5, c_1 = 150, c_2 = 100, c_2^1 = 30, c_2 = 30, \alpha_1 = 10 = 2\alpha_2$. Finally, we have appropriately tuned the potential by choosing $m = 10^{-3}H_{\text{eq}}$, so that today (i.e. for $z = 0$) we are already inside the dilaton-dominated evolution. In the radiation phase we can see the existence of a “focusing effect”, by which the (subdominant) dilaton energy tends to approach the other components. In the dragging phase the dilaton kinetic energy closely follows the dark-matter evolution. In the final freezing phase the evolution of the dilaton and dark-matter energy densities are closely tied together, and their ratio is fixed for ever to a number of order one.

![Figure 2](image-url)

**Figure 2:** Time evolution of the various energy densities. Radiation and baryons are uncoupled to the dilaton, and thus obey the standard scaling behaviour ($\rho \sim a^{-4}$ and $\rho \sim a^{-3}$, respectively).

### 3. Phenomenological consequences of a “running-dilaton” cosmology

There are three main differences between the dilatonic models of dark energy (coupled to dark matter) illustrated before, and the more conventional (uncoupled) models of dark energy.

- The first difference, also evident from Fig. 2, is that in the final freezing phase the energy density of dark matter is diluted in time at a slower ratio than the energy density of baryons, which are uncoupled to the dilaton. As a consequence, the ratio $\rho_{\text{bar}}/\rho_{\text{cdm}} \sim a^{-3q_0/(2+q_0)}$ decreases in time during the accelerated phase, and this
could probably explain why today the fraction of baryons is so small ($\sim 10^{-2}$) in critical units. Experimental information on the past value of the ratio $\frac{\rho_{\text{baryons}}}{\rho_{\text{CDM}}}$, if available, and if compared with the present value of this ratio, would immediately provide a direct test of this class of dilatonic models.

- The second difference is that the coincidence problem, if not solved, is at least relaxed, because the dilatonic (dark-energy) density and the dark-matter density are of the same order not only today, but also in the future (for ever), and possibly in the past, depending on the beginning of the freezing epoch (i.e., on the specific amplitude $V_0$ of the potential).
- The possibility of an early beginning of the accelerated epoch, even at redshift values $z_{\text{acc}}$ much larger than one, is indeed the third important difference characterizing our class of dilatonic models. In models of uncoupled dark energy, indeed, $z_{\text{acc}}$ is always smaller than one, or at most one.

For a more detailed illustration of this important point consider in fact the Einstein equations, for a model of uncoupled quintessence with energy density $\rho_Q$,

$$6H^2 = \rho_m + \rho_Q, \quad 4\dot{H} + 6H^2 = -p_Q = -w\rho_Q, \quad (3.1)$$

and with fixed equation of state,

$$-1 \lesssim p_Q/\rho_Q \equiv w \lesssim -1/3 \quad (3.2)$$

(Here $\rho_m$ includes both baryons and dark matter). In such a context, $\rho_m \sim a^{-3}$ is diluted faster than $\rho_Q \sim a^{-3(1+w)}$. So, even if today $\rho_Q$ dominates, and the expansion is accelerated ($\ddot{a}/a = H^2 + 2 > 0$), at some time in the past $\rho_m$ necessarily has to become dominant, in a decelerated Universe. The acceleration, in particular, switches off ($\ddot{a} = 0$) at

$$z_{\text{acc}} = \frac{a_0}{a_{\text{acc}}} - 1 = \left[1 + 3w \left(\frac{\Omega_m - 1}{\Omega_m}\right)\right]^{-1/3w} - 1, \quad (3.3)$$

where $\Omega_m = \rho_m/6H^2$. If we plot this function $z_{\text{acc}}(w)$, at (fixed) realistic values of $\Omega_m$, we can easily check that $z_{\text{acc}} \lesssim 1$ for realistic (i.e., observationally compatible) values of the dark-energy equation of state (in the range of eq. (3.2)), as illustrated in Fig. 3.

![Figure 3: Possible beginning of the accelerated epoch for models of uncoupled dark energy with fixed equation of state.](image)

The asymptotic freezing phase of dilatonic models, on the contrary, is characterized by a constant positive acceleration $\ddot{a}/(aH^2) = (q_0 - 1)/(q_0 + 2) > 0$, and its extension towards the past is in principle constrained by the present fraction...
of baryon energy density, which grows as we go back in time, in such a way that baryons tend to become over-dominant. By imposing that this is not the case, we can obtain more significant constraints on $z_{\text{acc}}$ from the measured value of the so-called density contrast $\sigma_8$, characterizing the level of dark-matter fluctuations over a distance scale $R_8 \simeq 8\,\text{Mpc}$.

We have used the current SNIa observations\textsuperscript{18} to extract information on the parameters $c_1$, $c_2$, $q_0$ of the freezing phase (determining the asymptotic value of $w$), and we have fixed the present values of $\rho_{\text{cdm}}$, $\rho_{\text{bar}}$ to the standard values $\Omega_{\text{cdm}} = 0.3$, $\Omega_{\text{bar}} h^2 = 0.02$, $h = 0.65$. By imposing the $\sigma_8$ constraint on the density fluctuations predicted in models of coupled dark energy\textsuperscript{19}, one then finds\textsuperscript{17} that the beginning of the dilaton-dominated epoch is allowed up to $z_{\text{acc}} \simeq 3.5$, according to the best fit value of $w$ from present SNIa data; taking into account one sigma and two sigma deviations from the best fit one can also obtain, respectively, $z_{\text{acc}} \lesssim 5$ and $z_{\text{acc}} \lesssim 8$. In any case, the possible past extension of the accelerated phase is considerably increased with respect to the standard predictions of Fig. 3, thus providing a further relaxation of the coincidence problem.

It is finally worth stressing that such an early beginning of the acceleration is compatible not only with $\sigma_8$ measurements, but also with the recent observations of the farthest Supernova SN1997ff at $z = 1.7$. This point is illustrated in Fig. 4, where we have reported the distance-modulus versus the redshift, for all the high-redshift Supernovae known so far. The dashed curve corresponds to the luminosity-distance relation for a standard $\Lambda\text{CDM}$ model, which is in good agreement with all data, but which is accelerated only up to $z \simeq 0.5$. The full curves correspond instead to dilatonic dark energy models. The bold ones also include baryons, which become more and more important as we go back in time. We have plotted, particular, two curves, corresponding to the best fit ($\beta_2 = 4.02$) and to the one-sigma deviation ($\beta_2 = 2.35$) of the low-redshift data. Both curves are also in agreement with the Supernova at $z = 1.7$, in spite of the fact that the acceleration, for the associated cosmological model, starts well above $z = 1.7$.

Figure 4: The farthest Supernova at $z = 1.7$ is compatible both with an early decelerated Universe (in the context of uncoupled models of dark energy), and with an early accelerated Universe (in the context of the dilatonic models discussed in this paper).
4. Concluding remarks

In the context of string cosmology it is possible to formulate consistent models in which our present accelerated Universe is dominated by a mixture of kinetic plus potential dilaton energy-density. This requires that the dilaton loop corrections are asymptotically saturated (to keep small enough the effective couplings), and non-universal (a strong coupling to dark matter is required, in particular). When the above assumptions are satisfied it can be shown, in addition, that the approximate equality of dark-matter and dark-energy density is no longer a coincidence typical of the present epoch.

There are two important phenomenological signatures of such a class of dilatonic dark-energy models: 1) the time variation of the ratio $\rho_{\text{bar}}/\rho_{\text{cdm}}$ during the accelerated phase; 2) the possibility of an early beginning of the acceleration, well above $z = 1$. A direct test of this second prediction is possibly expected from work in progress on the luminosity-redshift distributions of gamma-ray bursts\(^{21}\), using their sources as standard candles covering a range of redshift-values much larger than in the case of Supernovae observations.

Acknowledgments: I am very grateful to Luca Amendola, Federico Piazza, Domenico Tocchini-Valentini, Carlo Ungarelli and Gabriele Veneziano for the pleasant and fruitful collaboration leading to the results presented in this paper.

1. J. Polchinski, *String theory*, (University Press, Cambridge, 1998).
2. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B\textbf{436}, 257 (1998).
3. A. Starobinski, “Trans-Planckian effects in cosmology”, these proceedings.
4. G. Amelino Camelia, “Planck scale phenomenology”, these proceedings.
5. E. Witten Phys. Lett. B \textbf{149}, 351 (1984).
6. E. Witten, Nucl. Phys. B\textbf{443}, 85 (1995).
7. R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. \textbf{80}, 582 (1998).
8. A. G. Riess et al., Astron. J. \textbf{116}, 1009 (1998).
9. M. Gasperini and G. Veneziano, Phys. Rep. \textbf{373}, 1 (2003).
10. G. Veneziano, Phys. Lett. B \textbf{265}, 287 (1991); M. Gasperini and G. Veneziano, Astropart. Phys. 1, 317 (1993); .
11. V. Kaplunovsky, Phys. Rev. Lett. \textbf{55}, 1036 (1985).
12. N. Kaloper and K. A. Olive, Astropart. Phys. 1, 185 (1993).
13. M. Gasperini, Phys. Rev. D\textbf{64}, 043510 (2001).
14. G. Veneziano, JHEP \textbf{0206}, 051 (2002).
15. M. Gasperini, F. Piazza and G. Veneziano, Phys. Rev. D \textbf{65}, 023508 (2001).
16. L. Amendola, Phys. Rev. D \textbf{62}, 043511 (2000); L. Amendola and D. Tocchini-Valentini, Phys. Rev. D \textbf{64}, 043509 (2001).
17. L. Amendola, M. Gasperini, D. Tocchini-Valentini and C. Ungarelli, Phys. Rev. D \textbf{67}, 043512 (2003).
18. N. Dalal et al., Phys. Rev. Lett. \textbf{87}, 141302 (2001).
19. L. Amendola and D. Tocchini-Valentini, Phys. Rev. D \textbf{66}, 043528 (2002).
20. A. Riess et al., Ap. J. \textbf{560}, 49 (2001); N. Benitez et al., Ap. J. \textbf{577}, L1 (2002).
21. T. Di Girolamo, M. Vietri and G. Di Sciascio, work in progress (preprint SNS-PISA, 05/2003).