Orbital and Spin Magnetic Dipole Strength
in a shell model calculation with $\Delta N=2$ excitations: $^8$Be

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Abstract

The magnetic dipole strength and energy-weighted strength distribution is calculated in $^8$Be, as well as the separate orbit and spin parts. All $\Delta N=2$ excitations over and above (and including) the configuration $0s^40p^4$ are included. The interaction has a central, two-body spin-orbit and a tensor part. The energy-independent and energy-weighted orbital strength distribution is remarkably insensitive to the presence or absence of the spin-orbit or tensor interaction - not so the spin strength. The energy-weighted strength distribution can be divided into a low energy and a high energy part. The high energy orbital part is somewhat less but close to the low energy part, in fair agreement with a prediction that they be equal by de Guerra and Zamick and by Nojarov. There is a wide plateau separating the low energy part from the high energy part.
1 Introduction

In this work we wish to study the orbit and spin magnetic dipole strength distribution, both energy-independent and energy-weighted in a deformed nucleus. Our interests are slanted more towards the orbital distribution because of the intense work in recent years on scissors mode excitations [1], [2], [3]. The best milieux for scissors mode excitations are strongly deformed nuclei. We pick for our study the nucleus $^8$Be. The reasons for this choice are two-fold: it is known to be strongly deformed and we can perform a large shell model calculation which includes not only the basic configuration $0s^40p^4$ but also all $\Delta N=2$ excitations. Thus we can get a low energy and a high energy strength distribution. There are of course some atypical properties of $^8$Be. This nucleus is not stable and therefore is not amenable to the most direct way of reaching scissors modes -inelastic electron or photon scattering. Secondly, being an $N=Z$ nucleus, the scissors modes will have isospin $T=1$, whereas the ground state has isospin zero. The scissors modes will be at a much higher energy than in a typical heavy deformed nucleus where, despite the fact that the scissors modes are isovector excitations, the $J=1^+$ states that have been observed have the same isospin as the ground state.

Theoretical studies of 'scissors mode' excitations of $N=Z$ have been carried out before, e.g. by Chaves and Poves [4] for $^{20}$Ne. They show that the $1^+ T=1$ states at rather high excitation energies have all the right properties to be called scissors mode excitations.

We emphasize again that if we were to carry out the shell model calculation in the small model space $0s^40p^4$ we would not be adding much new to the subject. But by performing a calculation in a larger space we are able to settle some questions about the strength distribution. Another point we wish to pursue is how sensitive are the energies and strength distributions to various parts of the nucleon-nucleon interaction, in particular the spin-orbit
and tensor interactions. To this end, we use an interaction which we have used before for other purposes - the schematic interaction of the form

\[ V_{\text{sche}} = V_c + xV_{so} + yV_t \]  

where \( c \), \( s.o. \) and \( t \) stand for central, spin-orbit and tensor respectively. The parameters of \( V \), for \( x=1 \) and \( y=1 \), were loosely fitted to the matrix elements of the Bonn A interaction. We can vary the strength of the spin-orbit and tensor interactions by varying \( x \) and \( y \).

2 Results:

We perform OXBASH calculations for the \( J=0^+ T=0 \) ground state and \( J=1^+ T=1 \) states of \({}^8\text{Be}\). We calculate the B(M1)'s from the ground state to the \( 1^+ \) states for three cases:

1. Spin + Orbit: \( g_{l\pi}=1 \quad g_{l\nu}=0 \quad g_{s\pi}=5.586 \quad g_{s\nu}=-3.826 \)
2. Spin Only: \( g_{l\pi}=0 \quad g_{l\nu}=0 \quad g_{s\pi}=5.586 \quad g_{s\nu}=-3.826 \)
3. Orbit Only: \( g_{l\pi}=1 \quad g_{l\nu}=0 \quad g_{s\pi}=0 \quad g_{s\nu}=0 \)

2 (a) The first 7 states

In Table I we present results for energies and B(M1) for the first 7 states. We consider the three different sets of M1 operators as above and various choices of \( x \) and \( y \), the spin-orbit and tensor strength.

What emerges from Table I is that there is a well defined scissors mode in \({}^8\text{Be}\). If we focus on the orbital column, we see that for a pure central interaction \( (x=0, y=0) \), the dominant state is the second one at 19.92 MeV with a strength \( B(M1) = 0.61\mu_N^2 \). It can be seen from Table II that the total orbital strength is \( 0.87\mu_N^2 \) so that one gets 71% of the strength.
concentrated in one state when a central interaction is used. The orbital \( B(M1) \) of the 19.92 MeV state has nearly the same value as \( B(M1) \) with the full operator i.e. spin + orbit. This shows that the mode is dominantly orbital. When the spin-orbit and tensor interactions are turned on \((x=1, y=1)\), there is some fragmentation of the orbital strength. The lowest 1\(^+\) state, at 17.96 MeV, gets a strength of 0.398\(\mu_N^2\) and the next one, at 20.8 MeV gets 0.198\(\mu_N^2\). The summed strengths of these two states is about the same as in the \( x=0, y=0 \) (central only) case. Further examination shows that this fragmentation is mainly due to the spin-orbit interaction -the tensor interaction is much less important. Note that the other states listed carry negligible strength.

2 (b) The Total Strength and Energy-Weighted Strength

There has been considerable interest in magnetic dipole strength \( S \) and energy weighted strength \( E.W.S \) distributions. First the interest was more on the spin strength related to quenching of both isovector magnetic dipole strength and the closely related Gamow-Teller strength. More recently, there has been a focus on orbital strength and energy weighted strength. This is related to the experimental observation by W. Ziegker \[3\] et. al. and C. Rangacharyulu et. al. \[4\] that there is a close relation between summed magnetic dipole orbital 'scissors mode' strength and electric quadrupole strength \( B(E2) \) from the \( J=0^+ \) ground state to the first \( 2^+ \) state in even-even deformed nuclei.

This observation has lead to much theoretical work on energy weighted sum rules, including works of Heyde and de Coster \[3\], Zamick and Zheng \[4\], de Guerra and Zamick \[10\], Nojarov \[11\] and Hamamoto and Nazarewicz \[12\]. The above works relate to heavy deformed nuclei where complete \( \Delta N=2 \)
shell model calculations are not possible. By here considering a light strongly
deformed system $^8$Be, we hope to cast some light on the theoretical works in
heavier systems.

In Tables II, III and IV we present the total strength and energy-
weighted strength. We do this for the total $M1$ operator, the spin part,
and the orbital part and for various $x$ and $y$. There are many interesting
comments to be made about this table.

First of all, both the summed and energy-weighted summed orbital strength
is remarkably insensitive to $x$ and $y$—that is whether the spin-orbit interaction
and/or tensor interaction are present or not present. The values of the
summed strengths for $(x, y) = (0, 0), (0, 1), (1, 0), (1, 1)$ are respectively $0.75,$
$0.74, 0.72$ and $0.73 \mu N^2$ while the corresponding energy weighted numbers
are $20.48, 21.03, 20.05$ and $20.56 \mu N^2 MeV$. It is especially surprising that
when a spin-orbit splitting is introduced within the $0p$ shell, it has very little
effect. This is undoubtedly due to the fact that $^8$Be is strongly deformed, so
the asymptotic wave functions are approximately valid in all cases.

For the spin strength, there is much more sensitivity to the interaction.
It was noted by Kurath [13] that the spin-orbit interaction is very important
for magnetic dipole spin interactions—his energy weighted sum rule uses the
spin-orbit interaction. It was noted by Zamick, Abbas and Halemane [14]
that the tensor interaction can also have a large effect provided one allows
for ground state correlations in the nucleus.

The results in table III support the claims of these authors. When the
tensor interaction is turned off ($y = 0$) then the summed spin strength $S$
without a spin-orbit interaction is $0.12 \mu N^2$. When the spin-orbit interaction
is turned on $S$ more than triples to $0.38 \mu N^2$. The corresponding energy-
ighted numbers $E.W.S$ are $5.3 \mu N^2 MeV$ and $11.6 \mu N^2 MeV$. 
On the other hand, with the spin-orbit interaction turned off, the value of $S$ changes from $0.12 \mu N^2$ to $0.40 \mu N^2$ when the tensor interaction is turned on and $E.W.S$ increases by about a factor of four, from $5.3 \mu N^2 MeV$ to $21.3 \mu N^2 MeV$.

As compared with the isovector orbital transition, the isovector spin transition has a factor $(9.413)^2$, which in general makes spin transitions much larger than orbital ones. However, we see here that the summed orbital strength is comparable -indeed somewhat larger than the summed spin strength. This is a manifestation of the strong deformation in $^8$Be. In the $SU(4)$ limit, the spin transition rates will be zero. The asymptotic wave functions in the $0p$ shell become $zP$, $yP$ and $xP$ where

$$P = N \exp\left(-\frac{x^2}{2b_x^2} - \frac{y^2}{2b_y^2} - \frac{z^2}{2b_z^2}\right) \quad (2)$$

The occupied orbits have the quanta in the $z$ direction. A transition from $'z'$ to $'x'$ or $'y'$ cannot be induced by the spin operator $\sigma$.

2 (c) The Energy-Weighted Distribution

We now switch from tables to figures. We show the cumulative sum of $(E_n - E_0)B(M1)$ for the total, the spin and the orbital magnetic dipole operators in Figures 1, 2 and 3, where we consider only the full interaction $x=1, y=1$. The curves for other values of $(x, y)$ are qualitatively similar. Let us first focus on the orbital excitations. The energy weighted sum shoots up to about $12 \mu N^2 MeV$, then there is a wide plateau from about $20 MeV$ to $60 MeV$ and then a rapid rise to $20.6 \mu N^2 MeV$ and another plateau. Indeed there is no further change.

We can clearly identify the low-lying strength and the high-lying strength. The low-lying energy weighted value is about $12 \mu N^2 MeV$ and the total
strength is 20.56 $\mu N^2 MeV$. There had been a prediction using a simple Nilsson model by de Guerra and Zamick [10, 15] that the high-lying energy weighted strength should equal the low-lying energy weighted strength. A similar result was obtained by Nojarov [11] with a somewhat different approach.

Our shell model calculation gives the high-lying energy weighted strength to be 71% of the low-lying strength, in fair agreement with the previous predictions. One possible reason for a deviation is that our single-particle splittings are not $n\hbar\omega$ but rather are implicitly calculated in OXBASH with the schematic interaction described in the introduction. Our single-particle splittings are larger than $n\hbar\omega$.

Our shell model results with a $\Delta N=2$ truncation do not support the claim of Hamamoto and Nazarewicz [12] that the high lying energy weighted orbital strength should be much larger than the corresponding low lying strength. Our results go somewhat in the other direction.

We end by saying what we feel are the main points of interest in this work. Firstly, from Table IV we see the surprising insensitivity of the orbital scissors mode summed strength $S$ or $E.W.S$ to the presence or absence of the tensor and spin-orbit interactions. Secondly, there are the shapes of the figures of cumulative $E.W.S$ strength versus excitation energy, with a wide plateau, especially in the orbital case, separating the low lying from the high lying strength.

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Table I. The excitation energies of the seven lowest $J=1^+ \ T=1$ states in $^8\text{Be}$ and the corresponding $B(M1)$ strengths for the three cases of interest.
| Interaction | $E_x(1^+)$ | Total $B(M1)$ | Spin $B(M1)$ | Orbital $B(M1)$ |
|-------------|-----------|---------------|--------------|-----------------|
| $x = 0, y = 0$ | 18.84   | 0.6773 $10^{-4}$ | 0.7337 $10^{-4}$ | 0.1131 $10^{-6}$ |
|             | 19.92   | 0.6079         | 0.3873 $10^{-8}$ | 0.6080          |
|             | 22.38   | 0.6403 $10^{-6}$ | 0.3327 $10^{-8}$ | 0.5512 $10^{-6}$ |
|             | 27.06   | 0.0687         | 0.0688        | 0.2178 $10^{-7}$ |
|             | 31.28   | 0.1490 $10^{-5}$ | 0.1918 $10^{-5}$ | 0.2706 $10^{-7}$ |
|             | 32.47   | 0.1957 $10^{-5}$ | 0.2399 $10^{-5}$ | 0.2245 $10^{-7}$ |
|             | 35.93   | 0.2732 $10^{-5}$ | 0.2751 $10^{-6}$ | 0.1273 $10^{-5}$ |
| $x = 0, y = 1$ | 18.72   | 0.5031 $10^{-2}$ | 0.4688 $10^{-3}$ | 0.7457 $10^{-2}$ |
|             | 19.90   | 0.5918         | 0.1015 $10^{-4}$ | 0.5873          |
|             | 21.65   | 0.0201         | 0.9593 $10^{-2}$ | 0.1796 $10^{-2}$ |
|             | 27.57   | 0.0677         | 0.0643        | 0.1694 $10^{-4}$ |
|             | 30.97   | 0.8629 $10^{-2}$ | 0.0177        | 0.1638 $10^{-2}$ |
|             | 32.42   | 0.0508         | 0.0455        | 0.1703 $10^{-3}$ |
|             | 36.00   | 0.5522 $10^{-3}$ | 0.6005 $10^{-3}$ | 0.1476 $10^{-5}$ |
| $x = 1, y = 0$ | 18.28   | 0.6816         | 0.1103        | 0.2430          |
|             | 21.06   | 0.0577         | 0.0955        | 0.3022          |
|             | 23.74   | 0.0630         | 0.2918 $10^{-2}$ | 0.0386         |
|             | 26.75   | 0.0799         | 0.0888        | 0.3485 $10^{-3}$ |
|             | 33.58   | 0.4531 $10^{-3}$ | 0.9509 $10^{-4}$ | 0.1524 $10^{-3}$ |
|             | 34.74   | 0.2743 $10^{-2}$ | 0.5405 $10^{-2}$ | 0.4449 $10^{-3}$ |
|             | 36.75   | 0.1004 $10^{-5}$ | 0.1080 $10^{-3}$ | 0.1541 $10^{-3}$ |
| $x = 1, y = 1$ | 17.96   | 0.6216         | 0.1015        | 0.2208          |
|             | 20.80   | 0.1196         | 0.0397        | 0.2972          |
|             | 22.89   | 0.0211         | 0.0128        | 0.0667          |
|             | 27.04   | 0.0943         | 0.0915        | 0.2214 $10^{-4}$ |
|             | 33.51   | 0.8396 $10^{-2}$ | 0.0137       | 0.6453 $10^{-3}$ |
|             | 34.57   | 0.0253         | 0.0249        | 0.1434 $10^{-5}$ |
|             | 35.73   | 0.7579 $10^{-5}$ | 0.8102 $10^{-3}$ | 0.6613 $10^{-3}$ |
Table II. The total strength $S$ of $B(M1)$ and the total energy weighted strength $E.W.S$ in $^8$Be for the four types of interaction.

| Interaction | $S$     | $E.W.S$ |
|-------------|---------|---------|
| $x = 0, y = 0$ | 0.86713 | 25.810  |
| $x = 0, y = 1$ | 1.1426  | 42.186  |
| $x = 1, y = 0$ | 1.0814  | 29.707  |
| $x = 1, y = 1$ | 1.2650  | 42.172  |

Table III. The Spin strength $S$ of $B(M1)$ and the Spin energy weighted strength $E.W.S$ in $^8$Be for the four types of interaction.

| Interaction | $S$     | $E.W.S$ |
|-------------|---------|---------|
| $x = 0, y = 0$ | 0.12224 | 5.3329  |
| $x = 0, y = 1$ | 0.39662 | 21.267  |
| $x = 1, y = 0$ | 0.38415 | 11.604  |
| $x = 1, y = 1$ | 0.53413 | 23.126  |

Table IV. The Orbital strength $S$ of $B(M1)$ and the Orbital energy weighted strength $E.W.S$ in $^8$Be for the four types of interaction.

| Interaction | $S$     | $E.W.S$ |
|-------------|---------|---------|
| $x = 0, y = 0$ | 0.7451  | 20.479  |
| $x = 0, y = 1$ | 0.7426  | 21.032  |
| $x = 1, y = 0$ | 0.7196  | 20.005  |
| $x = 1, y = 1$ | 0.7261  | 20.565  |
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