Uncertainty evaluation for the dynamic measurement of deflections with a falling-weight-type impulse load device

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Abstract. The paper describes the uncertainty evaluation related to the dynamic measurement of deflections using a falling-weight-type impulse load device. This equipment is used in road and airfield pavements in-situ testing, contributing for the assessment of their performance analysis and characterization. This study aims at the determination of the deflection measurement uncertainty, following the GUM approach, where quantities are considered constant in time, and its comparison with the results obtained from recently developed approaches dedicated to time-varying measurement quantities.

1. Introduction
Testing activities related to road and airfield pavements have an important role in assuring the safety of persons and vehicles in transportation networks, which is a major concern to all the entities related to their design, construction, maintenance and management. In this context, several standard test methods are available, contributing for knowledge improvement of the tested pavements, namely, their structural performance and characterization.

Deflection measurement with a falling-weight-type impulse load device [1-2], often referred as Falling Weight Deflectometer (FWD), is one of these standard test methods, in which an impulse load is applied to the pavement surface and its vertical deflection response is measured at points spaced radially outward from the load axis. The impulse load is generated by a weight dropped on a buffer system and is transmitted through a plate resting on the pavement surface. In general, the test apparatus is mounted in a vehicle or on a trailer towed by a vehicle, as shown in Figure 1.

Figure 1. Airfield pavement testing using the FWD.
In addition to the measurement of influence quantities such as air and pavement temperature, the two main quantities measured in the FWD test – deflection and force – constitute time-varying quantities with standard requirements, namely, for the measurement accuracy. In order to comply with this requirement, it is crucial to establish SI traceability through the calibration of the individual measurement chains and to perform a rigorous measurement uncertainty evaluation, which is particularly relevant when flexible pavements are evaluated by FWD, considering that temperature has a relevant influence on pavement behavior and on its dynamic response under the applied load.

The main focus of GUM framework [3,4] is on quantities that are constant in time, considered suitable for a wide range of measurement applications. Measurement accuracy requirements are often more demanding and often time-dependent, thus creating challenges to metrology research, to traceability and to measurement uncertainty evaluation for dynamic quantities [5-9].

The main objective of this paper is, at a first stage, to perform a measurement uncertainty analysis based on the current GUM method and, at a second stage, to compare the results with those produced by recent developed approaches dedicated to the evaluation of dynamic measurement uncertainty, in this case, applied to deflection measurement with a FWD. The measurement process will be briefly described in a dedicated section, as well as the theoretical background on measurement uncertainty evaluation for both time-constant and time-variable quantities. Results obtained from both approaches will be present and compared in order to draw conclusions about the most suitable measurement uncertainty evaluation, taking into account the standard accuracy requirement related to the studied pavement test.

2. Deflection measurements with a falling-weight-type impulse load device
The FWD is the most widely used equipment in Europe and USA for pavement structural evaluation, as it performs non-destructive tests and allows the evaluation of distinct layers that constitute the pavement structure [10]. The use of FWD is even more important nowadays for rehabilitation purposes of the existing pavements, in order to adopt proper reinforcement solutions according to the real condition of the in service pavement.

In the studied standard test method, the FWD is placed over the desired test location, followed by the lowering of a loading plate (with a 300 mm or a 450 mm diameter) and of the deflection sensors (typically nine geophones are used, radially distributed outward from the load axis) to the pavement surface, which must be as clean as possible of loose rock particles and debris. Both the loading plate and the deflection sensors must rest on a firm and stable surface in order to assure suitable test repeatability.

Afterwards, the FWD weight is raised to the height that, when dropped, will apply the required pulse force to the pavement, with an approximate shape of a half-sine wave and a peak force between 45 kN and 250 kN, depending on the characteristics of pavement to be tested (typically, lower dynamic loads for road pavements and higher loads for airfields). When the weight is dropped on the pavement surface, the deflection sensors in the FWD measure the resulting vertical movement whereas a load cell mounted over the load plate measures the corresponding applied pulse force. At least three loading sequences are performed, one to ensure a proper contact of the plate on the pavement surface and the other two in order to evaluate the test repeatability. The main metrological requirements for the mentioned measuring chains are: (i) 1 µm resolution, repeatability of 2 µm and 2% accuracy for the deflection measurement; (ii) 0,2 kN resolution and 2% accuracy for the force measurement.

3. Experimental data
The experimental data used in the measurement uncertainty evaluation presented in this paper was obtained by LNEC from real pavement testing with a FWD (manufactured by Carl Bro, model PRI 2100). SI traceability of the measuring chains is provided by an accredited calibration laboratory.

In this case, deflection measurements with LNEC’s FWD are supported by the use of nine geophones, which typically have a non-linear frequency response in the low frequency range (approximately comprised between 0,2 Hz and 7 Hz). This fact justifies the addition of a dedicated
filter to each of the nine geophones, with a natural frequency equal to the geophone’s cut-off frequency, in order obtain a linear frequency response in a frequency interval between 0.2 Hz and 300 Hz. The knowledge of the geophones cut-off frequencies (Table 1) is critical for filter compensation (symmetrical to the uncompensated geophone frequency response) and consequently for the deflection measurement accuracy (for example, a 0.5 Hz deviation in the geophone cut-off frequency of the compensation filter can result in a 5% deviation in the deflection peak value).

Table 1. Cut-off frequencies of LNEC’s FWD geophones.

| Geophone identification | Cut-off frequency / Hz | Geophone identification | Cut-off frequency / Hz |
|-------------------------|------------------------|-------------------------|------------------------|
| 1                       | 4,435                  | 6                       | 4,423                  |
| 2                       | 4,546                  | 7                       | 4,833                  |
| 3                       | 4,695                  | 8                       | 4,473                  |
| 4                       | 4,742                  | 9                       | 4,882                  |
| 5                       | 4,594                  | ---                     | ---                    |

The calibration of LNEC’s FWD geophones is carried out on a controlled dynamic shaker, which includes a SI traceable displacement measurement standard. The adopted metrological procedure includes three sequential tests: (i) the quantification of the linear relation (defined by the slope and offset parameters) between the geophone output digital signal and the displacement reference value, based on the peak values synchronization. This assumes a maximum permissible deviation of 0.1% between the geophone and the standard displacement measurements (Table 2 indicates the parameters estimates for the studied FWD, based on its last calibration certificate); (ii) the analysis of the geophone frequency response, in order to assess the linearization effect resulting from the applied compensation filter; (iii) the linearity test, in which calibration deviations are evaluated for an extended range of dynamic pulses in terms of rise time and amplitude.

Table 2. LNEC’s FWD geophone parameterization.

| Geophone identification | Slope / µm | Offset | Geophone identification | Slope / µm | Offset |
|-------------------------|------------|--------|-------------------------|------------|--------|
| 1                       | 11,348     | 13,666 | 6                       | 11,326     | 11,327 |
| 2                       | 11,280     | -2,424 | 7                       | 10,994     | -0,681 |
| 3                       | 10,864     | 2,7945 | 8                       | 11,335     | -15,6025 |
| 4                       | 10,896     | -1,121 | 9                       | 10,663     | 16,8795 |
| 5                       | 11,216     | 5,916  | ---                     | ---        | ---    |

The evaluation of the dynamic measurement uncertainty, described in the following section, was supported by experimental data obtained in a road test performed in November 2016 by LNEC’s Transport Infrastructures Division (Transportation Department), using a load circular plate with a 150 mm diameter and a low-pass filter with a cut-off frequency equal to 150 Hz. Table 3 presents an example of the experimental deflection peak values in one drop, while Figure 2 presents the graphical representation of the corresponding deflection dynamic record.

Table 3. Experimental deflection peak estimates values - drop no. 1, 2016/11.

| Geophone identification | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------------------------|---|---|---|---|---|---|---|---|---|
| Radial offset, /cm      | 0 | 30| 45| 60| 90| 120| 150| 180| 210|
| Deflection peak value, /µm | 60 | 33| 20| 12| 6 | 5  | 4  | 3  | 2 |
4. Evaluation of measurement uncertainty

4.1 Time-constant approach

This measurement uncertainty evaluation approach follows the conventional guidelines of the GUM [3], applied to the deflection measurement in the FWD test. In this case, the following uncertainty components were identified (quantification details are presented in Table 4):

- **calibration of the FWD’s geophones** – this uncertainty component corresponds to the calibration uncertainty of the reference displacement transducer (LVDT) related to the shaker used in the geophone calibration experimental apparatus;
- **instrumental drift** – based on the historical record of calibration certificates related to LNEC’s FWD, the geophones instrumental drift was calculated and the obtained maximum value (for a time period of 36 months) was used in the quantification of this measurement uncertainty;
- **short-term repeatability of the deflection measurement** – this uncertainty component was quantified based on available information regarding short-term repeatability tests performed after the FWD calibration; an average experimental standard deviation was determined taking into account the nine geophones of LNEC’s FWD;
- **relative deviations between geophones** – following the FWD calibration and in addition to the short-term repeatability test, another test is performed – the stacking tower test – where all the FWD’s geophones are installed in a dedicated structure in order to simultaneously measure the same deflection, which allows determining relative deviations between geophones; this uncertainty component was quantified based on the maximum experimental standard deviation of the geophones relative deviations, observed in LNEC’s FWD calibration records;
- **linearity** – this uncertainty component was quantified based on available information of LNEC’s FWD manufacturer, regarding the maximum linearity deviation obtained from experimental tests with similar equipment;
- **systematic deviation of the deflection peak value** – corresponds to the typical deviation (according to the LNEC’s FWD manufacturer) between deflection peak values measured by the geophone and the reference measurement standard in the FWD calibration process, considering the use of a compensation filter in agreement with each geophone cut-off frequency;
- **rounding-off of deflection peak value** – the output file of LNEC’s FWD provides the geophones deflection peak values estimates rounded to µm, as mentioned in the test standard [1].

![Figure 2. Studied deflection dynamic record – drop no. 1, 2016/11.](image-url)
### Table 4. Deflection measurement uncertainty budget.

| Standard uncertainty component $u(x_i)$ | Uncertainty source | Probability distribution | Relative standard uncertainty $u_r(x_i)$ | Degrees of freedom $\nu$ |
|-----------------------------------------|--------------------|--------------------------|------------------------------------------|------------------------|
| $u_{cal}(d)$                            | Calibration       | Gaussian                 | 0,007 5%                                 | 50                     |
| $u_{drf}(d)$                            | Instrumental drift| Uniform                  | 0,003 6% $/ \sqrt{3} = 0,002 1%$         | 50                     |
| $u_{rep}(d)$                            | Short-term repeatability | Gaussian            | 0,50 %                                  | 17                     |
| $u_{dev}(d)$                            | Relative deviations| Gaussian                 | 0,32 %                                  | 8                      |
| $u_{lin}(d)$                            | Linearity          | Uniform                  | 0,87% $/ \sqrt{3} = 0,50%$               | 50                     |
| $u_{sys}(d)$                            | Systematic deviation| Uniform                | 0,18% $/ \sqrt{3} = 0,10%$               | 50                     |
| $u_{rnd}(d)$                            | Rounding-off       | Uniform                  | 0,023% $/ \sqrt{3} = 0,013%$             | 50                     |

Combined relative standard uncertainty, $u_r(x) = 0,78%$

Effective degrees of freedom, $\nu_{eff} = 60$

Expansion factor, $k = 2,00$

95% expanded relative uncertainty, $U_{95\%}(d) = 1,6%$

The obtained 1,6% expanded measurement uncertainty value complies with the deflection measurement accuracy requirement (2%) mentioned in the FWD test standard [1]. The main uncertainty contributions are related to the short-term repeatability and the linearity components.

### 4.2 Time-varying approach

In recent years, the theoretical background regarding the evaluation of dynamic measurement uncertainty was established by several authors [5-9] and is available for use in both calibration and testing applications, as it is the case of the deflection dynamic measurement with a FWD studied in this paper.

In the analysis of a dynamic measurement (shown in Figure 3) [9], the measurement system is assumed to be linear and time-invariant and with a known dynamic behavior, where the measurand is a continuous function $x(t)$. The performed $n$ discrete-time observations $y[n]$ are considered as the result of an analogue-to-digital conversion (ADC) of the system output signal, affected by noise disturbance, $\varepsilon[n]$. In this approach, a linear estimation in the discrete-time domain, $x[n]$, is performed by digital filtering with an appropriate deconvolution filter, and is used later to make an inference of the continuous-time measurand $x(t)$. This approach and the corresponding measurement uncertainty evaluation were implemented in the PyDynamic software package for Python 3.x, developed and validated by PTB and NPL [11].

![Figure 3. Analysis of a dynamic measurement.](image)

The following input information was used for the study of the dynamic measurement uncertainty:
- the time record of the deflection estimates obtained from geophone no. 1 (shown in Figure 2), where a deflection peak value of 60 $\mu$m was obtained in a measuring interval of 120 ms with a sampling frequency of 40 kHz;
- the measurement uncertainty contribution of 0,78% (see Table 4, in the previous section 4.1) for the deflection output signal;
• a low-pass filter for deconvolution with a cut-off frequency equal to 150 Hz, since the deconvolution process is an ill-posed inverse problem, requiring regularization in order to provide reasonable estimates;
• the geophone’s nominal frequency response (amplitude and phase estimates and uncertainties), according to the manufacturer’s specifications [12]; it should be noticed that correlation between different frequencies as well as between amplitude and phase values is assumed to be negligible.

The sequential calculation procedure included several steps of uncertainty propagation [5]: (i) through the frequency domain by the Direct Fourier Transform; (ii) to the real and imaginary parts of the geophone frequency response; (iii) through the inverse system; (iv) through the low-pass filter; (v) and back to the time domain. A 95% expanded relative uncertainty of 0.25% was obtained from the time-variant approach for the deflection peak value estimate of 60 µm, which is quite lower than the 1.6% dispersion value obtained by the time-constant value.

5. Conclusions

The performed uncertainty evaluation showed significant differences between the dispersion values obtained from the time-constant and time-varying approaches. However, both complying with the accuracy requirement mentioned in the FWD test standard (lower than 2%).

Moreover, further improvements are still required in the implemented time-varying approach in order to validate the measurement uncertainty obtained from experimental data, namely, the use of the real geophone frequency response (instead of the nominal response given by the geophone manufacturer) and deformation reference data, in order to study the occurrence of systematic deviations induced by the regularization process in the time-varying approach.

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