Observation of Half-Quantum Vortices in an Antiferromagnetic Spinor Bose-Einstein Condensate

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We report the observation of half-quantum vortices (HQVs) in the easy-plane polar phase of an antiferromagnetic spinor Bose-Einstein condensate. Using in-situ magnetization-sensitive imaging, we observe that a singly charged quantum vortex dissociates into a pair of HQVs with opposite core magnetization. The dissociation dynamics is investigated by measuring the temporal evolution of the separation distance of the HQV pair. The separating behavior of the HQVs shows the short-range nature of their interactions. We observe that the dissociation process is activated faster with larger thermal spin fluctuations in the condensate, confirming the energetic instability of a singly charged vortex state.

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Quantum vortices are topological defects in a superfluid and the supercurrent circulation around them is quantized in units of $h/m$ because of $U(1)$ gauge symmetry [1], where $h$ is the Planck constant and $m$ is the particle mass. However, when a superfluid possesses additional internal degrees of freedom such as spin, there is an intriguing possibility for the superfluid to host quantum vortices of a fractional circulation of $h/m$. This originates from the interwinding of the superfluid phase with the spin orientation, imposing a new rule on the supercurrent circulation in connection with spin texture [2]. Quantum vortices having $h/2m$ circulation, so-called half-quantum vortices (HQVs) have been experimentally observed in a two-component spin mixture of atomic condensates [3], exciton-polariton condensates [4, 5], and triplet superconductors [6].

Fractional quantum vortices are of particular interest to study two-dimensional (2D) superfluidity. In two dimensions, long-range order is prohibited [7] and the superfluid phase transition is driven by unbinding of vortex-antivortex pairs as described by the Berezinskii-Kosterlitz-Thouless (BKT) theory [8, 9]. Therefore, introduction of fractional quantum vortices as point defects represents an opportunity to explore for new superfluid phases in 2D, possibly beyond the BKT physics. Recently, the polar phase of a spin-1 spinor Bose-Einstein condensate with antiferromagnetic interactions has been considered with great interest, where the order parameter manifold allows the existence of HQVs [10][12]. Theoretical studies predicted anomalous superfluid density jump at the phase transition [13][15] and a new superfluid state that has completely broken spin ordering [16][17].

In this Letter, we report the observation of HQVs in the easy-plane polar phase of an antiferromagnetic spin-1 Bose-Einstein condensate of $^{23}\text{Na}$ atoms. Using magnetization-sensitive dispersive imaging we observe that a singly charged quantum vortex dissociates into a pair of HQVs with opposite core magnetization. The separating behavior of the HQVs reveals their short-range repulsive interactions due to the oppositely magnetized cores. We also observe spin fluctuations in the spinor condensate, which mainly arise from thermal populations of the transverse magnon excitations, and we find that they accelerate the initiation of the dissociation process. Our result confirms the energetic instability of a singly charged vortex in the antiferromagnetic spinor condensate.

The spin-dependent part of the mean-field energy functional for a spin-1 condensate is given as

$$E_s = \frac{c_2 n}{2} \langle F \rangle^2 + p \langle F_z \rangle + q \langle F_z^2 \rangle,$$

(1)

where $n$ is the atomic number density and $\mathbf{F} = (F_x, F_y, F_z)$ is the single-particle spin operator [11][12]. The first term describes the spin-dependent interaction energy and the second and third terms are the linear and quadratic Zeeman energies, respectively, where the $z$ direction is defined by the external magnetic field. With antiferromagnetic interactions ($c_2 > 0$) and zero total magnetization ($p = 0$), the ground state of the system is a polar state with $\langle \mathbf{F} \rangle = 0$ [18]. This is a $|m_F = 0\rangle$ state along a certain direction which we denote by a unit vector $\hat{d}$. Depending on the sign of the quadratic Zeeman field $q$, the condensate has two distinctive phases: For $q > 0$, the easy-axis polar phase with fixed spin direction $\hat{d} \parallel \hat{z}$, giving $\langle F_z^2 \rangle = 0$, and for $q < 0$, the easy-plane polar phase with $\hat{d} \perp \hat{z}$ and $\langle F_z^2 \rangle = 1$. These two phases are referred to simply as polar (P) and antiferromagnetic (AF) phases, respectively.

The order parameter of the condensate in the AF phase can be expressed with a three-component spinor as

$$\Psi_{\text{AF}} = \begin{pmatrix} \psi_+ \\ \psi_0 \\ \psi_- \end{pmatrix} = \sqrt{n} e^{i\phi} \begin{pmatrix} e^{-i\theta} \\ 0 \\ e^{i\theta} \end{pmatrix},$$

(2)

where $\psi_l$ is the $m_z = l$ spin component along the $z$ direction ($l = 0, \pm 1$), $\theta$ is the superfluid phase and $\phi$ is the azimuthal angle of the spin orientation, i.e.
spin winding number, \( q_s \)

| supercurrent winding number, \( q_n \) |
|------------------|---|
| 1/2              | -1/2 |

-1/2          1/2

FIG. 1: (color online). Schematic illustration of the half-quantum vortices (HQVs) in the antiferromagnetic (AF) spinor condensate. The superfluid phase \( \theta \) and the spin orientation \( \vec{d} \) rotate by \( \pi \) around vortex cores having non-zero magnetization \( M_z \). The order parameter of the condensate is continuous over the disclinations indicated by dashed lines, because it is invariant under the operation of \( \theta \rightarrow \theta + \pi \) and \( \vec{d} \rightarrow -\vec{d} \).

\( \vec{d} = (\cos \phi, \sin \phi, 0) \). The order parameter manifold is \( M_\text{AF} = |U(1) \times S^1|/\mathbb{Z}_2 \) [10, 11], where \( U(1) \) is the gauge symmetry, \( S^1 \) comes from the rotational symmetry of the spin, and \( \mathbb{Z}_2 \) arises from the invariance under the operation of \( \theta \rightarrow \theta + \pi \) and \( \phi \rightarrow \phi + \pi \). When the winding numbers of \( \theta \) and \( \phi \) around a quantum vortex are \( q_n \) and \( q_s \) in units of \( 2\pi \), respectively, the single-valuedness of the order parameter requires \( q_n \pm q_s \) to be integer. Therefore, quantum vortices with half-integer supercurrent winding number can exist in the AF phase with the aid of spin winding.

The spatial structures of the four fundamental HQVs with \( q_n = q_s = \frac{1}{2} \) are described in Fig. 1. When \( q_n + q_s = 0 \) (\( q_n - q_s = 0 \)), the \( m_z = -1 \) (\( m_z = 1 \)) component has no vorticity and it can fill up the core region of the HQV. Although having a ferromagnetic core is costly for the AF spin interactions, the core filling is energetically favored because it would reduce the kinetic energy by suppressing the density of the circulating spin component in the core region.

In the AF phase, a singly charged vortex with \( (q_n, q_s) = (\pm 1, 0) \) can be regarded as a sum of two HQVs with \( (q_n, q_s) = (\pm \frac{1}{2}, \frac{1}{2}) \) and \( (\pm \frac{1}{2}, -\frac{1}{2}) \). Note that the two HQVs have opposite core magnetization. Mean-field calculations showed that a singly charged vortex state is energetically unstable to decay into the two HQVs [19, 21]. In this work, to observe HQVs we investigate dynamics of a singly charged quantum vortex in an AF spinor condensate by measuring the magnetization distribution of the condensate.

Our experiment starts with generating a Bose-Einstein condensate of \( ^{23}\text{Na} \) atoms in the \( |F = 1, m_F = 0 \rangle \) hyperfine spin state in an optical dipole trap [22]. The trapping frequencies are \( (\omega_x, \omega_y, \omega_z)/2\pi = (4.2, 5.3, 480) \) Hz. A typical condensate containing about \( 3.5 \times 10^6 \) atoms has the Thomas-Fermi radii \( (R_x, R_y, R_z) \approx (185, 150, 1.6) \mu\text{m} \). For the peak atom density, the spin healing length is \( \xi_s = \hbar/2\sqrt{2m_2\omega_2} \approx 4.5 \mu\text{m} \) [23], which is larger than the sample thickness, and the spin dynamics in the highly oblate condensate is of 2D character. The external magnetic field \( B_z = 30 \mu\text{G} \) is applied, giving \( q/h = 0.24 \) Hz. The residual field gradient is compensated to be less than \( 40 \mu\text{G/cm} \).

The condensate is prepared in the AF phase by tuning the quadratic Zeeman field \( q \) with the microwave dressing technique [24, 25]. We first apply a radio-frequency pulse to rotate the spin direction \( \vec{d} \) from \( +\hat{z} \) to \( +\hat{x} \), forming a superposition state of the \( m_z = \pm 1 \) components. Then, we immediately turn on a microwave field whose frequency is detuned by \( -300 \) MHz with respect to the \( |F = 1, m_F = 0 \rangle \rightarrow |F = 2, m_F = 0 \rangle \) transition, resulting in \( q/h = -10 \) Hz. With Stern-Gerlach spin separation measurements, we confirm that the \( m_z = 0 \) component keeps being absent in the microwave dressing.
The spatial distribution of the magnetization of the condensate is measured with a spin-dependent phase-contrast imaging method [30–32]. The probe light is circularly polarized and its frequency is detuned by $-20$ MHz from the $3S_{1/2}[F = 1] \rightarrow 3P_{1/2}[F' = 2]$ transition [33]. Because the signs of the phase shift of the probe beam for the $m_z = \pm 1$ components are opposite, the optical signal in the phase-contrast imaging is proportional to the density difference between the $m_z = \pm 1$ components, i.e., the magnetization $M_z = n\langle F_z \rangle$ of the sample.

We generate vortices in the condensate by stirring it with a repulsive laser beam [34]. The stirring is applied before the spin rotation when the condensate is in the $P$ phase with $U(1)$ symmetry and we ensure that the generated vortices are singly charged because of the instability of multiply charged vortices [35] [Fig. 2(a)]. When the condensate containing vortices is transferred to the AF phase, we observe that pairs of point defects with opposite magnetization emerge in the condensate [Fig. 2(b)]. These are the HQV pairs that develop from the singly charged vortices. The separation directions of the HQV pairs appear random over the sample, which clearly indicates that the dissociation dynamics is driven not by the residual magnetic field gradient but by the intrinsic instability of a singly charged vortex state.

The dissociation process is investigated with in-situ measurements of the magnetization distribution of the condensate. HQV pairs appear to be discernible after a hold time $t_h \sim 0.7$ s in the microwave dressing [Fig. 3(b)], and the separation distance of the pair and the magnitude of the core magnetization gradually increase [Fig. 3(e) and (f)]. We see that the growing behavior almost ceases after $t_h \sim 1.3$ s when the separation distance reaches about $s_0 = 17.6 \mu m$. This seems to be consistent with the short-range nature of the interactions between HQVs with opposite core magnetization [19, 20].

Taking into account the imaging resolution of $\approx 4 \mu m$, we estimate the FWHM of the magnetized HQV core to be $\approx 8.4 \mu m$ that corresponds to $\sim 1.7\xi_z$, where $\xi_z$ is the average value of the spin healing lengths at the positions of the HQV pairs. This is in good quantitative agreement with mean-field predictions [19, 21].

We observe that spatial fluctuations of magnetization develop in the condensate when it is transferred to the AF phase (Fig. 4). The AF condensate has two gapless excitation modes: phonons and transverse magnons, and spin fluctuations arise mainly from thermal population of the magnon mode [11, 12, 38, 39]. In our experiment, the initial AF condensate is prepared by the spin rotation such that all the spins point to $+x$ direction. This spin texture corresponds to zero temperature for magnons and thus, thermal relaxation would subsequently occur, leading to the development of spin fluctuations as observed. Because of the 2D character for $\xi_z > R_z$, further enhancement of spin fluctuations is expected in our sys-
In conclusion, we have observed the dissociation of a singly charged quantum vortex into a pair of HQVs with opposite core magnetization in the AF spinor condensate and confirmed the energetic instability of a singly charged vortex state. It would be highly interesting to extend this work to a 2D regime [10, 11], where the pair superfluid state without spin ordering was predicted to exist at finite temperature [16, 17]. Magnetic order of the superfluid phase might be probed with spin-sensitive Bragg spectroscopy [42, 43] or matter wave interference methods [44].

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Because the probe light is not far from the resonance, the phase-contrast imaging involves non-negligible absorption effects and we empirically tune the probe light frequency to obtain a most flat signal for the condensate in the AF phase.

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