Superfluid $^3$He, Particle Physics and Cosmology

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Superfluid $^3$He-A and high-temperature superconductors both have gapless fermionic quasiparticles with the "relativistic" spectrum close to the gap nodes. The interaction of the "relativistic" fermions with bosonic collective modes is described by the quantum field theory, which results in a close connection with particle physics. Many phenomena in high-energy physics and cosmology can thus be simulated in superfluid phases of $^3$He and in unconventional superconductors. This includes axial anomaly, vacuum polarization, zero-charge effect, fermionic charge of the vacuum, baryogenesis, event horizon, vacuum instability, Hawking radiation, etc. Analogs of some of these phenomena, which are related to the axial anomaly, have been experimentally simulated in superfluid $^3$He. This includes the baryogenesis by textures (Manchester), the baryogenesis by cosmic strings (Manchester) and the generation of the primordial magnetic field via the axial anomaly (Helsinki).

It is now well realized that the Universe and its broken symmetry ground state – the physical vacuum – may behave like a condensed matter system, in which the complicated degenerate ground state is developed due to the symmetry breaking. The condensed matter counterparts of Universe are represented by superconductors and by superfluid phases of $^3$He: both systems contain quantum Bose and Fermi fields, describing the interaction of the elementary quasiparticles with each other and with the degenerate vacuum. This analogy allows us to simulate in condensed matter many properties of the physical vacuum, while the direct experiments with the physical vacuum are still far from the realization.

The important property of the physical vacuum is its high anisotropy (Fig. 1).

The physical vacuum (superfluid ground state) of superfluid $^3$He-A resembles the electroweak vacuum. The fermionic excitations of the $^3$He-A in the vicinity of the gap nodes are "relativistic" chiral fermions (Fig. 2). They are the left-handed near the pole north, where they have a linear momentum $p$ close to $p_F$ (1 is the direction of spontaneous angular momentum in $^3$He-A) and they are right-handed near the south pole where $p = -p_F$. Another important similarity is that the symmetry breaking pattern $SU(2) \times U(1) \rightarrow U(1)$ in electro-weak interactions is the same as the symmetry breaking pattern $SO(3) \times U(1) \rightarrow U(1)$ in $^3$He-A (Fig. 3).

That is why the physical effects are actually the same in the vacuum of the high energy physics and in superfluid $^3$He-A. Thus the $^3$He-A is the right condensed matter for simulations of the vacuum properties (Fig. 4). The main difference is in the terminology, so we need the dictionary for the translation from the particles physics language to that of $^3$He-A (Fig. 5).

Here we discuss 3 experiments in superfluid $^3$He which simulate the processes in the early Universe. The first two are related to the cosmological problem why the Universe contains much more matter than antimatter. The

![Anisotropy of vacuum](image)

**FIG. 1.** (top): The spectrum of fermionic excitations of the physical vacuum contain the branch of the chiral particles: left-handed neutrino. The right-handed neutrino is absent: it is the remarkable manifestation of the violation of the left-right symmetry in the vacuum. Another symmetry, which is broken in the present Universe, is the $SU(2)$ symmetry of weak interactions. In the unbroken symmetry state of the early Universe, the left leptons (neutrino and left electron) formed the $SU(2)$ doublet, while the right electron was the $SU(2)$ singlet. (bottom): During the cool down of the Universe the phase transition (or the crossover) occurred, at which the $SU(2)$ symmetry was broken. As a result the left and right electrons were hybridized forming the present electronic spectrum with the gap $\Delta = m_e c^2$.
Close to north gap node quasiparticle is similar to left-handed positron

3He experiments allow to test different scenarios in which the nonconservation of the baryons is caused by the so-called axial anomaly, which governs the nucleation of the fermionic charges in the vacuum. The essence of the axial anomaly is presented in Fig. 1. The chiral anomaly equation is obtained using the picture of Landau levitation of fermions moving in the electromagnetic and gravity fields and obeying the relativistic equation \( g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0 \). Here \( e = \pm \) is the "electric charge" and simultaneously the chirality of the quasiparticles. The righthanded particles have positive charge \( e = +1 \) and are in the vicinity of the north pole, while the left particles with \( e = -1 \) are in the vicinity of the south pole. \( p_\mu \) is the 4-momentum; \( A_\mu = \mu_R \) plays the part of the chemical potential for the right-handed particles. The texture of \( \hat{l} \) plays the part of the chemical potential for the right-handed electron. Here \( \mu \) is the "electric charge" and simultaneously the metric tensor of the gravitational field: \( g^{ik} = c^2 (dx^i dx^k) + c^2 (dx^l dx^k) \), \( g^{00} = -1 \), the quantities \( c_l = p_F/m \) and \( c_\perp = \Delta_0/p_F \) correspond to velocities of "light" propagating along and transverse to \( \hat{l} \), here \( m \) is the mass of the \(^3\)He atom and \( \Delta_0 \) is the amplitude of the gap.

Why \(^3\)He can help

- We need broken symmetry condensed matter to simulate quantum vacuum
- liquid crystals but they are not quantum
- superfluid 4He but it does not contain Fermi quasiparticles
- superconductors: Bogoliubov fermionic quasiparticles vs quarks
- \(^3\)He - Jona-Lasinio model of quark condensate, but the limited number of degrees of freedom
- \(^3\)He phases of \(^3\)He are right systems
- Superfluid \(^3\)He represent interacting Fermi and Bose quantum fields analogs of leptons, gauge fields, Higgs fields, gravitons, W-bosons, ...
- Symmetry breaking is similar to that in physical vacuum
- \(^3\)He-A = electroweak vacuum
- Various topological objects
- 3 vortices + vortex sheet in \(^3\)He-A and 4 vortices in \(^3\)He-B (observed in Helsinki)
- \(^3\)He-A vortices = Z- and W- cosmic strings
- \(^3\)He-A vortices = Alice string
- Superfluid motion of texture = event horizon in black hole
- Nucleation of vortices by neutrons = strings formation in Big-Bang
- Probing of quantum vacuum
- Flow instability = vacuum instability in strong fields
- Superfluid motion of texture = event horizon in black hole
- Anomalies in quantum field theory
- Anomalous forces on vortex (Manchester experiment) = baryogenesis
- Modelling of processes in hot Universe (Helsinki)

Anomalies in quantum field theory

Anomalous forces on vortex (Manchester experiments) = baryogenesis

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Anomalous production of the linear momentum leads to the additional force acting on the continuous vortex in \(^3\)He-A (Fig. 3), which was measured in the Manchester experiment. The axial anomaly equation has thus been verified in the \(^3\)He-A experiments with the moving vortices. The axial anomaly is also responsible for the dynamics of singular vortices and for the baryogenesis by the singular cosmic string. In this case the spectral flow from the vacuum is realized by...
the bound states of quasiparticles in the core of vortices and strings (Figs. 11,12). The baryoproduction in the core of the cosmic string was simulated in the Manchester experiment on dynamics of singular vortices in $^3$He-B [5]. There is also an opposite effect: the anomalous transformation of the chiral particles into the magnetic field. In the Universe this mechanism can be responsible for the formation of the primordial magnetic field [1], while in the $^3$He-A the similar anomaly equations describes the collapse of the excitation momenta towards the formation of the textures – counterpart of the magnetic field in cosmology (Figs. 11,12). Such collapse was recently observed in Helsinki rotating cryostat [7].

We discussed only 3 experiments in superfluid $^3$He related to the properties of the electroweak vacuum. In all of them the chiral anomaly is an important mechanism. It regulates the nucleation of the fermionic charge from the vacuum as was observed in [9] and the inverse process of the nucleation of the effective magnetic field from the fermion current as was observed in [8]. There are many other connections between the superfluid $^3$He and other branches of physics which should be exploited (Fig. 13).

FIG. 5. Dictionary

FIG. 6. Chiral anomaly: the spectral flow from the occupied energy levels caused by the applied electric field leads to the creation of the fermionic charge from the vacuum if the left-right symmetry is violated.

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**Anomalous nonconservation of baryonic charge and 3He momentum**

Baryon nonconservation in Electroweak Theory

\[
\mathbf{\dot{B}} = \left(\frac{1}{4\pi^2}\right) B_Y \dot{E} \mathbf{Y} + \left(\frac{1}{4\pi^2}\right) B_W \dot{E} \mathbf{W}
\]

where

- \( B_Y \) and \( E \) = weak magnetic and electric fields
- \( B_W \) and \( W \) = hypercharge of right and left quark

\[
\mathbf{\dot{B}} = \left[ B_Y \mathbf{E}_Y - B_W \mathbf{E}_W \right]
\]

**Left-Right asymmetry gives net product of baryonic charge**

\[
\mathbf{\dot{B}} = \left(\frac{3}{8}\right) \mathbf{P}_R - \left(\frac{1}{4}\right) \mathbf{P}_L
\]

**Momentum transfer in Superfluid 3He-A**

\[
\mathbf{\dot{P}} = \left(\frac{1}{4\pi^2}\right) \mathbf{B} \mathbf{E} - \mathbf{F}
\]

where

- \( \mathbf{P} \) = total momentum of quasiparticles
- \( \mathbf{B} \) = magnetic and electric fields of hypercharge
- \( \mathbf{E} \) = weak magnetic and electric fields

**Left-Right asymmetry**

\[
\mathbf{\dot{P}} = \left(\frac{1}{2\pi^3}\right) \mathbf{B} \mathbf{E} - \mathbf{F}
\]

**FIG. 7.** The same anomaly equation leads to production of the baryonic charge \( B \) in high energy physics and to production of the linear momentum \( \mathbf{P} \) in superfluid \(^3\)He-A.

**Momentogenesis by continuous N=2 vortex**

Momentum transfer from the vacuum (superfluid) to the heat bath (matter) gives extra force on N=2 vortex

\[
\mathbf{F} = \int dV \mathbf{P} = \left(\frac{1}{4\pi^2}\right) \mathbf{B} \mathbf{E} - \mathbf{F}
\]

Due to bound states

- \( E_2 = m^2(r) c^4 + c^2p^2 \)
- \( m(r) \) = mass of Higgs-boson
- \( Z \) = mass of Z-boson
- \( c \) = velocity of light

The flow of linear momentum (transverse force on vortex):

\[
\mathbf{F}_x = \nabla \times \mathbf{E} \quad \text{for vortices}
\]

**FIG. 8.** The order parameter texture (analog of magnetic field) in the core of continuous vortex is converted into the excess of the fermionic charge (linear momentum) due to the axial anomaly in the moving continuous vortex. The moving vortex generates the time dependence of the order parameter, which is equivalent to electric field.

**Cosmic string vs Singular vortex**

Due to bound states dynamics of strings leads to baryogenesis;

- dynamics of vortices leads to baryogenesis
- cosmic string is the counterpart of the Abrikosov vortex in superconductor. It also contains the bound states of the fermions, which are important for the baryoproduction.

**Anomalous nonconservation of linear momentum in superconductors and in 3He-B vortex**

Asymmetric branch \( E(L_z) = m q L_z \)

When vortex moves along \( x \) the angular momentum changes:

\[
L_z = \mathbf{p}_F \cdot \mathbf{n}
\]

This gives the flow of levels from Fermi sea:

\[
\mathbf{n} = \frac{L_z}{\mathbf{p}_F} \quad \text{for vortices}
\]

The flow of linear momentum (transverse force on vortex):

\[
\mathbf{F}_x = \nabla \times \mathbf{E} \quad \text{for vortices}
\]

**FIG. 9.** Cosmic string is the counterpart of the Abrikosov vortex in superconductor. It also contains the bound states of the fermions, which are important for the baryoproduction.
FIG. 11. Instability of the counterflow. The counterflow generated by the rotation of the cryostat is equivalent to the nonzero chemical potential for the right electrons. When the velocity of the counterflow $v_s - v_n$ in the $\hat{z}$ direction (corresponding to the chemical potential $\mu_R$ of the chiral electrons) exceeds some critical value, the instability occurs and the container becomes filled with the $l$-texture which corresponds to the hypermagnetic field.

### Chern-Simons term leading to helical instability:

$$\frac{1}{2\pi} \mu A (\nabla \times A)$$

Chemical potential $\mu$ = $p_T l (v_s - v_n)$

Vector potential $A$ = $p_T (l + l_0)$

Magnetic field $F_{ik} = \nabla A_i \times \nabla A_k$ = $k_0 (\nabla l V_c)$

Magnetic field energy $\mu = \text{Energy of texture}$

$$\frac{1}{12\pi^2} \ln (\Lambda/T) \sqrt{g} g^{ij} \epsilon_{ij} F_{ij}$$

Metric tensor $g^{ij} = c_i^2 + c_j^2 (b^k - b^l)$

Planck energy $\Lambda = \text{gap amplitude} \Delta_0$

FIG. 12. The excess of the chiral right-handed electrons in early Universe can be effectively converted to the hypermagnetic field due to the mechanism of chiral anomaly. This is described essentially by the same equations as the counterflow instability observed in $^3$He-A.

FIG. 13. Connections of superfluid $^3$He to other branches of physics.