Quantum coherent dynamics of a $\Lambda$-system driven by a thermal environment: Enhancing coherence lifetimes via incoherent driving

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We present a theoretical study of quantum coherent dynamics in a three-level $\Lambda$ system driven by a thermal environment. By solving the nonsecular Bloch-Redfield master equations, we obtain analytical results for the ground-state population and coherence dynamics and classify the dynamical regimes of the incoherently driven $\Lambda$-system as underdamped and overdamped depending on whether the ratio $\Delta / |r f(p)|$ is greater or less than one, where $\Delta$ is the ground-state energy splitting, $r$ is the incoherent pumping rate, and $f(p)$ is a dimensionless function of the transition dipole alignment parameter $p$. In the underdamped regime, we observe long-lived coherent dynamics that lasts for $\tau_c \approx 1/r$, even though the initial state of the $\Lambda$-system contains no coherences in the energy basis. In the overdamped regime for $p = 1$, we observe the emergence of coherent quasi-steady states with the lifetime $\tau_c = 1.34(r/\Delta^2)$, which have low von Neumann entropy compared to the conventional thermal states. Our results suggest that thermal excitations can enhance the lifetimes of initially created coherent superpositions, making thermal driving a potentially useful coherence-enhancing tool for quantum information science.

Introduction. Long-lived quantum states with low entropy are an indispensable resource for quantum sensing, quantum information processing, and precision timekeeping [1–3]. The lifetime of quantum superposition states is limited by their interaction with an external environment [4–6], whose fluctuations affect the relative phases between the different components of the superposition, leading to the production of high-entropy mixed states (such as thermal states), which are much less useful for quantum information processing than their low-entropy counterparts [5]. Identifying and suppressing decoherence in atomic and molecular systems interacting with external environments is therefore a central goal of quantum information science, which motivated the development of decoupling techniques for solid-state qubits [3, 7–11], engineering decoherence-free subspaces [12, 13], and the search for decoherence-minimizing clock transitions and magic trapping conditions for atomic [14, 15] and molecular [16] qubits.

While decoherence can originate from a variety of different mechanisms, one of the most common scenarios involves the interaction of a quantum system with a thermal bath, represented as a collection of noninteracting bosonic field modes [4]. This description is widely used to describe decoherence and dissipation in quantum optical systems using the Lindblad and Bloch-Redfield quantum master equations [6]. Here, the bath is represented by a thermal electromagnetic field such as incoherent light or blackbody radiation [17, 18].

For a prototypical two-level system weakly coupled to a thermal bath, the coherence initially present in the system decays on the timescale $\propto \lambda^{-2}$, where $\lambda$ is the system-bath coupling constant [18, 28]. While this result might lead one to expect that interactions with a thermal environment always lead to an irreversible decay of quantum coherence, this is not the case: Multiple studies have shown that thermal driving alone can generate noise-induced Fano coherences [19–29] in multilevel quantum systems (qudits) due to the interference of different incoherent transition pathways. Early theoretical studies of Fano coherences focused on coherent population trapping and resonance fluorescence of trapped ions [19, 20]. More recent theoretical work explored their potential role in suppressing spontaneous emission from three-level atoms [30], in enhancing the efficiency of quantum heat engines [23], in biological processes induced by solar light [24, 31–36] and in negative entropy production [37]. We have explored the dynamical evolution of Fano coherences in a model three-level $V$-system as a function of the excited-level splitting and the radiative decay rate [25–28]. Recently, vacuum-induced Fano coherences have been detected experimentally in a cold ensemble of Rb atoms [38].

A three-level $\Lambda$-system consisting of two nearly degenerate ground levels radiatively coupled to a single excited level [see Fig. 1(a)] is another prototypical qudit [39], which serves as a fundamental building block of complex quantum optical systems and quantum heat engines [23, 24]. However, despite its profound significance, the thermally driven $\Lambda$-system has only been explored in the regime of degenerate ground levels ($\Delta = 0$) [22], leaving open the question of whether thermal environments can induce and sustain coherent ground-state dynamics in realistic atomic and/or molecular $\Lambda$-systems with $\Delta > 0$.

Here, we study the quantum dynamics of the $\Lambda$-system driven by a thermal environment represented by isotropic incoherent light (i.e. blackbody radiation). By solving nonsecular quantum master equations, we obtain analytic results for the time evolution of noise-induced Fano coherences between the ground levels of the $\Lambda$-system and establish the existence of two distinct dynamical regimes, where the coherences exhibit either underdamped oscillations or quasi-steady states with low entropy compared to the conventional thermal states. We further show that Fano coherences induced by incoherent driving can extend the lifetime of quantum coherent superpositions, making thermal driving a
potentially useful tool for quantum information science.

Theory. The quantum dynamics of the $\Lambda$-system driven by isotropic incoherent radiation is described by the Bloch–Redfield (BR) quantum master equations for the density matrix in the eigenstate basis [22, 40]

$$\dot{\rho}_{g_1g_1} = -r_i \rho_{g_1g_1} + (r_1 + r_2) \rho_{ee} - p \sqrt{r_1 r_2} \rho_{g_1g_2}^R$$

$$\dot{\rho}_{g_1g_2} = -i \rho_{g_1g_2} \Delta - \frac{1}{2} (r_1 + r_2) \rho_{g_1g_2} + p (\sqrt{r_1 r_2} + \sqrt{r_1 r_2}) \rho_{ee} - \frac{p}{2} \sqrt{r_1 r_2} (\rho_{g_1g_1} + \rho_{g_2g_2})$$

(1)

where $\rho_{g_1g_1}$ are the ground-state populations, $\rho_{g_1g_2} = \rho_{g_2g_1}^R + i \rho_{g_2g_2}^R$ is the coherence between the ground states $|g_1\rangle$ and $|g_2\rangle$ [see Fig. 1 (a)] with the real and imaginary parts $\rho_{g_1g_2}^R$ and $\rho_{g_1g_2}^R$, $\gamma_i$ is the radiative decay rate of the excited state $|e\rangle$ into the ground state $|i\rangle$, $r_i = n \gamma_i$ is the incoherent pumping rate, $\bar{n}$ is the average occupation number of the thermal field, $p = (\mu_{g_1e} \cdot \mu_{g_2e})/\mu_{g_1e} \mu_{g_2e}$ is the transition dipole alignment factor, and $\mu_{ij}$ is the transition dipole matrix element between the states $i$ and $j$.

We consider a symmetric $\Lambda$-system ($r_1 = r_2 = r$) driven by a suddenly turned on incoherent light. This restriction drastically simplifies the solution of the BR equations without losing the essential physics [25, 26]. Equations (1) rely on the Born-Markov approximation, which is known to be very accurate for quantum optical systems [6]. Significantly, we do not assume the validity of the secular approximation, which cannot be justified for nearly degenerate energy levels [25–28, 40–46]. This approximation is equivalent to setting $p = 0$ in Eqs. (1), which eliminates the population-to-coherence coupling terms and hence Fano coherences (see below). Several authors have shown that BR equations [25, 40, 42–46] and related Lindblad-form master equations [47] generally provide a more accurate description of open quantum system dynamics than secular rate equations.

To solve the BR equations (1) we recast them in matrix form $\mathbf{x}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{d}$, where $\mathbf{x}(t) = (\rho_{g_1g_1}, \rho_{g_1g_2}^R, \rho_{g_2g_1}^R)^T$ is the state vector in the Liouville representation, $\mathbf{A}$ is the matrix of coefficients on the right-hand side of Eq. (1), $\mathbf{d}(s)$ is the driving vector, and $\mathbf{x}_0$ defines the initial conditions for the density matrix [48]. The solutions of the matrix BR equation $\mathbf{x}(t) = e^{\mathbf{A} t} \mathbf{x}_0 + \int_0^t d \tau e^{\mathbf{A} (t-s)} \mathbf{d}(s)$ may be expressed in terms of the eigenvalues of $\mathbf{A}$, which determine the decay timescales of the different eigenmodes of the system [48]. The general features of the solutions can be understood without finding the eigenvalues by considering the discriminant of the characteristic equation [48]

$$D = B^3 + \left[ C - \frac{3}{2} A (B + A^2) \right]^2$$

(2)

where $A = \frac{1}{2} (5 r + 2 \gamma)$, $B = \frac{1}{3} [\Delta^2 + r^2 + (2 - p^2) r (3 r + 2 \gamma)] - A^2$, and $C = \frac{1}{2} (3 r + 2 \gamma) [\Delta^2 + (1 - p^2) r^2] + A^3$. Three dynamical regimes can be distinguished depending on the sign of $D$ using the analogy with the damped harmonic oscillator [26]. In the underdamped regime ($D > 0$), $\mathbf{A}$ has one real and two complex conjugate eigenvalues, giving rise to an exponentially decaying and two oscillating eigenmodes. In the overdamped regime ($D < 0$) all of the eigenvalues are real and negative and thus the eigenmodes decay exponentially. In the critical regime ($D = 0$) all eigenvalues are real and degenerate.

As shown in the Supplemental Material [48], $D = \frac{6}{\sqrt{37}} \sum_{k=0}^\infty d_k (p, \Delta/\gamma) \bar{n}^k$. Figure 1(b) illustrates the different dynamical regimes of the incoherently driven $\Lambda$-system obtained by solving the equation $D = 0$. We observe that the regions of positive $D$ are separated from those of negative $D$ by the critical line $\Delta/\gamma = f(p) \bar{n}$, where $f(p)$ is a universal function of $p$ [48], which gives the slope of the line. Thus, the overdamped regime is realized for $\Delta/\gamma < f(p) \bar{n}$ and the underdamped regime for $\Delta/\gamma > f(p) \bar{n}$.

In the limit $\bar{n} \to 0$ we obtain $D = \frac{6}{\sqrt{37}} (\Delta/\gamma)^2 (\Delta^2 + 4 r^2) > 0$. Thus, as shown below, the $\Lambda$-system always exhibits underdamped oscillatory dynamics under weak thermal driving, in marked contrast with the weakly driven $V$-system, where the underdamped regime is realized only for $\Delta/\gamma > 1$ [25–27]. This is because the ground states of the $\Lambda$-system are not subject to spontaneous decay, unlike the excited states of the $V$-system. As a result, the two-photon coherence lifetime of the $V$-system scales as $1/\gamma$, whereas that of the $\Lambda$-system as $1/r$ (as follows from Eq. (1) for $p = 0$).
The eigenvalues $\lambda_j$ of matrix $A$ correspond to the decay rates of the corresponding eigenmodes [48]

$$\lambda_j = -A + \alpha_j \frac{B}{T} - \beta_j T$$  \hspace{1cm} (3)

where $T = \sqrt{E + \sqrt{D}}$, $E = [C - \frac{3}{2}A(B + A^2)]$, $\omega = -\frac{1 + \sqrt{3}}{2}$, and the quantities $A$, $B$, and $C$ are defined below Eq. (2) and $\alpha_j$, $\beta_j$ are the cube roots of unity with the values $(\alpha_1, \beta_1) = (1, 1)$, $(\alpha_2, \beta_2) = (\omega, \omega^2)$, and $(\alpha_3, \beta_3) = (\omega^2, \omega)$. We next consider the various limits of coherence dynamics defined by the sign of $D$.

In the weakly driven regime ($r/\gamma \ll 1$) the general expressions (3) can be simplified to give $\lambda_1 = -(3r + 2\gamma)$ and $\lambda_{2,3} = -rQ(\frac{\Delta}{\gamma}) \pm i\Delta$, where $Q(x)$ is a dimensionless function of $x = \Delta/\gamma$ [48]. The analytical solutions of the BR equations in the underdamped regime are ($P = \frac{2r\gamma}{\sqrt{\Delta\gamma^2 + 4\gamma^2}}$) [48]

$$\rho_{g_1g_1}(t) = \frac{1}{(3r + 2\gamma)[r + \gamma + \frac{r}{2}e^{-2\gamma t}]}$$ \hspace{1cm} (4)

$$\rho_{g_1g_2}^R(t) = P[2r\gamma e^{-2\gamma t} - (2\gamma \cos \Delta t + \Delta \sin \Delta t)e^{-rQ(\frac{\Delta}{\gamma})t}],$$ \hspace{1cm} (5)

with the imaginary part of Fano coherence $\rho_{g_1g_2}^I(t) = P[\Delta e^{-2\gamma t} + (\gamma \sin \Delta t - \Delta \cos \Delta t)e^{-rQ(\frac{\Delta}{\gamma})t}]$.

Figure 1(c) shows the ground-state population and coherence dynamics of the incoherently driven $\Lambda$-system initially in the incoherent mixture of the ground states ($\rho_{g_1g_1}(0) = 1/2$) obtained by numerical solution of the BR equations. We observe that incoherent driving produces quantum beats due to Fano coherences in the absence of initial coherence in the system. The coherent oscillations decay on the timescale $[rQ(\frac{\Delta}{\gamma})]^{-1}$ in agreement with the analytical result (5). As the energy gap between the ground levels narrows down, the function $Q(\frac{\Delta}{\gamma})$ decreases from 1 to 1/2 [48] and the coherence lifetime increases by a factor of two. This is because the incoherent excitations $|g_i\rangle \leftrightarrow |e_i\rangle$ interfere more effectively at small $\Delta/\gamma$ [29], as signaled by the non-negligible population-to-coherence coupling term $-p\sqrt{r_1r_2}\rho_{g_1g_2}^R$ in Eq. (1) in the limit $\Delta/\gamma \ll 1$.

If the $\Lambda$-system is initialized in a coherent superposition of its ground states ($\rho_{g_1g_2}(0) = 1/2$) the dynamics under incoherent driving can be obtained by adding to Eq. (5) a term $\rho_{g_1g_2}^I = \frac{1}{2} \cos \Delta t e^{-rt}$, which arises from the coherent initial condition [48]. From Fig. 1(d) we observe that, in the absence of Fano interference ($p = 0$), the initially excited coherent superposition decays on the timescale 1/r as expected due to incoherent transitions to the excited state $|g_i\rangle \rightarrow |e_i\rangle$ followed by spontaneous decay to the vacuum modes of the electromagnetic field. Remarkably, incoherent driving in the presence of Fano interference ($p = 1$) leads to a two-fold enhancement of the lifetime of the initial coherent superposition caused solely by the noise-induced contribution (5).

We now turn to the overdamped dynamics of the strongly driven $\Lambda$-system defined by the condition $\Delta/r < f(n)$. Expanding the eigenvalues of $A$ in $1/n \ll 1$ we find that $\lambda_i$ do not depend on $\Delta/\gamma$ for $p < p_c$, where $p_c \simeq 1$ [27, 48]. Remarkably, when the transition dipoles are nearly perfectly aligned ($p > p_c$), the scaling changes dramatically to $\lambda_2 = -0.75\frac{r^2}{\gamma^2} (\Delta/\gamma)^2$, giving rise to a coherent quasi-steady state with the lifetime $\tau_c = 1.34r/\Delta^2$ that increases without limit as $\Delta \rightarrow 0$. This point is illustrated in Figs. 2(a) and (b), where we plot the population and coherence dynamics of the $\Lambda$-system strongly driven by incoherent light. We observe that the coherences rise quickly from zero to an intermediate “plateau” value, where they remain for $t = \tau_c$ before eventually decaying back to zero. Our analytical results for coherence dynamics are in excellent agreement with numerical calculations as shown in Figs. 2(a) and (b) (see the Supplemental Material [48]). They reduce to prior $\Delta = 0$ results [22] in the limit $\Delta \rightarrow 0$, where the quasisteady states shown in Figs. 2(a) and (b) become true steady states.

To understand the physical origin of long-lived Fano coherences in the $\Lambda$-system, we use the effective decoherence rate model [27]. As shown in Fig. 2(c) the decay of the ground-state population $\rho_{g_1g_1}$ is accompanied by a steady growth of the population inversion $\rho_{ee} = \rho_{g_1g_1}$, which drives coherence generation. We observe that in the quasi-steady state the time evolution of the population difference is identical to that of $\rho_{g_1g_2}^R$ and that $\rho_{g_1g_2}^I$ is time-independent. Neglecting the terms proportional to $\gamma$ in Eq. (1), which is a good approximation in the strong pumping limit, and setting the left-hand side of the resulting expression to zero, we obtain $\rho_{g_1g_2}^I = -(\Delta/r)\rho_{g_1g_2}^R$ [48]. This leads to a sim-
The overdamped and underdamped regimes of coherence with an external magnetic field, providing access to both ∆ excited and underexcited states. The magnetic field is parallel to the propagation vector of the x-polarized incoherent light, which defines the quantization axis. (b) Ground-state coherence dynamics of He* excited by x-polarized incoherent light with the parameters γ = 10^8 s⁻¹, n = 10⁻³, and ∆/γ = 10⁻².

**Figure 3.** (a) Schematic diagram of the A-system configuration to detect Fano coherences in He* atoms. The 3S₃/₂ ↔ 3P₀ transitions are indicated by double-headed arrows. The magnetic field is parallel to the propagation vector of the x-polarized incoherent light, which defines the quantization axis. (b) Ground-state coherence dynamics of He* excited by x-polarized incoherent light with the parameters γ = 10^8 s⁻¹, n = 10⁻³, and ∆/γ = 10⁻².

Simplified equation of motion for ρᵣᵣ₂, valid at t > 1/r, is consistent with the analytical result derived above. There are two distinct contributions to the overall decoherence rate in Eq. (6) which are similar to that identified in our previous work on the V-system [27]: (i) the interplay between coherence-generating Fano interference and incoherent stimulated emission [the term r(1 − p)] and (ii) the coupling between the real and imaginary parts of the coherence due to the unitary evolution [the term ∆²/γ]. The first mechanism does not contribute in the limit p → 1, explaining the formation of the long-lived coherent quasi-steady state shown in Fig. 2, which decays via mechanism (ii) at a rate ∝ r/∆².

Figure 2(d) shows the time evolution of von Neumann entropy of the incoherently driven A-system calculated with (p = 1) and without (p = 0) Fano coherence. We observe that the entropy of the long-lived coherent quasi-steady state is two times smaller than that of the corresponding p = 0 thermal state due to the presence of substantial coherences in the energy basis. The low-entropy state persists for τₜ ∝ (r/∆²) before decaying to the high-entropy thermal state.

We finally turn to the question of experimental observability of Fano coherences. We suggest metastable He(2S₁/₂) atoms [49, 50] as a readily realizable A-system, in which to observe noise-induced coherent dynamics. The A-system is formed by the m = ±1 Zeeman sublevels of the metastable 3S₁ state and the nondegenerate excited ⁳P₀ state as shown in Fig. 3(a). The energy gap ∆ between the Zeeman sublevels is continuously tunable with an external magnetic field, providing access to both the overdamped and underdamped regimes of coherence dynamics. The 3S₃/₂ ↔ 3P₀ transitions are driven by a spectrally broadened laser field polarized in the x-direction. This excitation scheme allows us to (i) neglect radiative transitions involving the m = 0 ground-state Zeeman sublevel, thereby realizing an ideal three-level A-system (since He⁺ has no hyperfine structure), and (ii) bypass the p = 1 condition needed to generate Fano interference [28, 29]. Because both of the A-system transitions couple to the same polarization mode of the incoherent radiation field, the BR equations (1) can be simplified by replacing r + γ → γ [28, 29, 48].

Figure 3(b) shows ground-state Fano coherence dynamics of a He* atom driven by x-polarized incoherent light starting from a coherence-free initial state ρᵣᵣᵣ₁(0) = 1/2. As in the case of isotropic incoherent excitation considered above, the coherences exhibit quantum beats with frequency ∆ and lifetime 1/(Qr). The coherent evolution could be probed by applying a π/2 radiofrequency pulse (as part of the standard Ramsey sequence [3, 50]) to convert the coherences to the populations of the m = ±1 atomic states, which could be measured by state-selective photoionization [49, 50].

Significantly, as shown in Fig. 3(b), the Fano coherences formed under polarized incoherent excitation do not vanish in the steady state (t → ∞) unlike those in isotropic excitation. This is a result of the imbalance between polarized incoherent excitation and spontaneous emission (the former is directional whereas the latter is isotropic), leading to a breakdown of detailed balance, and the emergence of non-equilibrium steady-states [28, 29, 51]. The steady-state populations and coherences [48]

\[ \rho_i^{(SS)} = \frac{(r + \gamma)\Delta^2 + r^2\gamma}{(3r + 2\gamma)\Delta^2 + 2r^2\gamma} \]

\[ \rho_R^{(SS)} = -\frac{r^2\gamma}{(3r + 2\gamma)\Delta^2 + 2r^2\gamma} \]

deviate from the values expected in thermal equilibrium \[ \rho_i^{(SS,th)} = (r + \gamma)/(3r + 2\gamma) \]. As in the case of the V-system [29], this could be used to detect Fano coherences by measuring the deviation of steady-state populations from their equilibrium values via, e.g., state-selective photoionization [49, 50].

In summary, we have explored the quantum dynamics of noise-induced Fano coherences in a prototypical three-level A-system driven by a thermal bath. In contrast to its V-system counterpart [25–28] the weakly driven A-system always remains in the underdamped regime characterized by oscillatory coherence dynamics that decays on the timescale 1/r (for ∆/γ ≫ 1) or 2/r (for ∆/γ ≪ 1). Remarkably, the coherence time of the A-system initially prepared in a coherent superposition of its ground states is enhanced by a factor of two in the presence of Fano coherence induced by incoherent driving. This result suggests that Fano coherences may find applications in, e.g., quantum sensing [3], where a qubit’s coherence time is a key figure of merit. Similarly, the long-lived quasi-steady states that arise in the
strongly driven Λ-system [see Fig. (2)] have lower entropy than the corresponding thermal states, suggesting their potential utility for quantum information science. Finally, we propose an experimental scenario for detecting Fano coherences by driving circularly polarized $^3S_{m=\pm 1} \leftrightarrow ^3P_0$ transitions in metastable He atoms excited by $x$-polarized incoherent radiation and subject to an external magnetic field.

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