Monofractal nature of air temperature signals reveals their climate variability

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(Dated: September 15, 2014)

We use the discrete “wavelet transform microscope” to show that the surface air temperature signals of weather stations selected in Europe are monofractal. This study reveals that the information obtained in this way are richer than previous works studying long range correlations in meteorological stations. The approach presented here allows to bind the Hölder exponents with the climate variability. We also establish that such a link does not exist with methods previously carried out.

PACS numbers: 05.45.Tp, 02.30.Sa, 05.45.Df

Previous works have shown the presence of long range correlations (LRC) in the trend/noise of surface air temperature data (see e.g. [6,12,13]). If such a signal is interpreted as a random walk, one can conclude to its monofractal nature. Since these studies have been performed with “bias-dependent” methods, they can not be directly applied to the “raw signal”: so-called seasonal signals contain more information than the associated trends. 

The purpose of this letter is to show that raw temperature signals display a monofractal nature. Since these studies have been performed with “bias-dependent” methods, they can not be directly applied to the “raw signal”: so-called seasonal signals contain more information than the associated trends.

We use the discrete “wavelet transform microscope” to show that surface air temperature signals display a monofractal nature. Every function \(f\) for the interval \([7,15]\) is defined as a tool for providing a multifractal formalism, which has proven to be well suited to study fractal objects (\([8,10]\)). The WT of a function \(f\) is defined as

\[
W_\psi[f](j,k) = 2^{-j} \int f(x)\psi(2^{-j}x + k)\,dx,
\]

where \(k\) is the space parameter and \(j\) the scale parameter (both take integer values). WT is well adapted to study the irregularities of \(f\), even if they are masked by a smooth behavior. If \(f\) has, at a given point \(x_0\), a local scaling/Hölder exponent \(h(x_0)\), in the sense that 

\[
|f(x) - P_{x_0}(x)| \sim |x - x_0|^h(x_0)
\]

around \(x_0\), where \(P_{x_0}\) is a polynomial of degree at most \(h(x_0)\), then with the right choice of \(\psi\), one has 

\[
W_\psi[f](j,k) \sim 2^{-j}h(x_0)
\]

for the indices \(k\) such that 

\[
2^{-j}x - k \approx x_0\]

The WLM is a transposition of the wavelet transform modulus maxima (WTMM) to the discrete setting (\([8,10]\)). Mimicking the box-counting technique, one investigates the scaling behavior of the following partition function

\[
S(q,j) = 2^j \sum_k (\sup_{j' \geq j} |W_\psi[f](j',k)|)^q \sim 2^{j\omega(q)},
\]

where \(q\) is a real number. In this framework, \(\omega\) is the Legendre transform of the singularity spectrum, defined as the Hausdorff dimension of the set of points \(x\) sharing the same Hölder exponent \(h(x)\). Monofractal functions, i.e. functions with a constant Hölder exponent \(h(x_0) = H\) are characterized by a linear spectrum: 

\[H = \partial \omega/\partial q.\]

On the contrary, a nonlinear \(\omega\) curve is the signature of functions displaying a multifractal behavior; in this case, \(h\) is not constant anymore and thus may fluctuate from one point to another.

We applied the WLM on surface air temperature data collected from \([8]\). In order to get homogeneous signals, we limited our study to daily mean temperature series with at least 50 years of data between 1950 and 2003 spread across Europe between 36\(^\circ\) (Southern Spain, Italy, Greece) and 55\(^\circ\) of latitude (Northern Ireland, Germany) and –10\(^\circ\) (Western Ireland, Portugal) and 40\(^\circ\) of longitude (Eastern Ukraine). By doing so, we were able to select 115 weather stations uniformly dispersed across the selected area. For the purpose of reducing the noise, the data \(f(t)\) were replaced with the temperature profiles \(\sum_{u=1}^n f(u)\) (see Fig. (a) and (b)). All the air temperature data display a monofractal nature: every function \(\omega\) is clearly linear with a mean coefficient of determination equal to 

\[R^2 = 0.9975 \pm 0.0028\] (see Fig. (c) and (d)). However, the value of the Hölder exponent varies from one station to another between 1.093 and 1.43 (see Fig. (d)). Let us also remark that other “bias-independent” methods give similar results for each station; we performed the WTMM (\([8]\)) as well as the \(S''\)-based multifractal formalism (\([11]\)) on the data to confirm our results.

Influential studies about the monofractal nature of air temperature data have been previously carried out (see e.g. \([6,12,13]\)). However, the approach adopted here fundamentally differs for one reason: we apply the
senting the seasonal variation and a fractional Gaussian noise (FGN), both methods will match, i.e. will detect the monofractality of the FGN. The same result is obtained if one applies the WLM on the signal where the seasonal variation has been removed (thus proceeding in the same way as for the DFA). However, this concordance is not recovered if one applies these methods on real surface air temperature time series. In this case, DFA and WLM with seasonal variation removal give similar results, while the WLM applied on the raw data leads to different outcomes. This is because information remains in the seasonal variation. This can be illustrated with a synthetic signal roughly mimicking temperature data. Let \( n \) be a FGN associated to LRC with index \( \gamma = 0.65 \),

\[
s(x) = 15 \sin \left( \frac{2\pi}{365} x - \frac{\pi}{2} - \frac{1}{20} \log(x+1) \right) + 14
\]

be a non-stationary seasonal variation and define \( f(x) = n(x) + s(x) \) (see Fig. 2(a) and (b)). DFA applied to \( f \) does not lead to a straight line (see Fig. 2(d)) and the best result for the estimation of \( \gamma \) we can hope for (knowing the expected value of \( \gamma \)) is an error of order \( 10^{-1} \). On the other hand, the WLM applied on \( f \) works properly (see Fig. 2(c)), giving a Hölder exponent equal to \( \gamma \) with an error of order \( 10^{-3} \).

A natural question arising is whether or not the observed Hölder exponent reflects the climate variability. More precisely, does the surface pressure anomalies induced on the Hölder exponents, or are these differences numerical artifacts? To test this hypothesis, we compared the map of the surface pressure anomalies from Rome with the same map where the anomalies have been replaced with the measured Hölder exponent. On these maps, each pixel, corresponding to an anomaly or a Hölder exponent (both related to a weather station), is renormalized in order to obtain values between 0 and 1. One can compute the Frobenius distance between two such maps (considered as matrices) as follows:

\[
d = \sqrt{\sum_{i,j} (x_{i,j} - x'_{i,j})^2},
\]

where \( x_{i,j} \) is a pixel of the first map, \( x'_{i,j} \) is the corresponding pixel of the second map and where the sum is taken over all pixels. In this case, the distance between these two maps is \( d_1 = 2.68 \). To check if \( d_1 \) is “large”, the “Hölder map” was randomly shuffled 10,000 times. For each realization, the distance with the original anomalies map was computed in order to get a distribution of these random distances. In this way, one can look where \( d_1 \) lies in the distribution of the distances, and one can associate a \( p \)-value to this particular distance \( d_1 \). Based on the 10,000 observations, the probability \( 1 - p \) to have a randomly shuffled map with a distance smaller than \( d_1 \) is lower than \( 10^{-4} \), which shows that the Hölder map and the pressure anomalies map are highly significantly

WLM on raw data, which is not possible with “bias-dependent methods” such as the detrended fluctuation analysis (DFA) used in 13. For the sake of comparison, let us briefly describe the DFA first introduced by in 17,18. Following 13, if \( f \) is a surface air temperature time series, the seasonal variation \( \langle f \rangle \) is defined as follows: if \( d \) is a calendar date (say June first), \( \langle f \rangle(d) \) is the average over the years in \( f \) of the values \( f(t) \) such that \( t \) corresponds to the calendar date \( d \) (June first 1950, June first 1951, ...). The corresponding trend is then defined as \( \Delta f(t) = f(t) - \langle f \rangle(d) \). To reduce the noise, the temperature profiles \( \sum_{u=1}^{t} \Delta f(u) \) are also used instead of the trend. From random walk theory (see e.g. 3), the standard deviation \( F \) of the profile in a time window of length \( t \) should behave like \( F(t) \sim t^{\gamma} \), where \( \gamma > 1/2 \) suggests the existence of LRC and is the Hölder exponent of the data. For the DFA, the best linear fit is determined on every non-overlapping segment \( \eta \) of length \( t \) and the standard deviation \( F_{\eta}(t) \) of the profile from that straight line is then computed. Finally, \( F(t) \) is defined as \( \sqrt{E[F_{\eta}^{2}(t)]} \), where \( E \) stands for the mean over all segments. By doing so, one gets rid of the influence of the possible linear trends on scales larger than \( t \). As a simple example, if one considers a signal \( f \) made of a sine (repre-
close (see Fig. 3(a)). In order to show that other methods do not give so good results, we performed the same simulation but with a map where the Hölder exponents obtained with the WLM have been replaced with the values obtained with the DFA. In this case, the distance $d_2$ between this “DFA map” and the anomalies map is 4.68, and the probability that the distance between a randomly shuffled DFA map and the anomalies map is smaller than $d_2$ is $1 - p = 0.8$. This shows that the DFA map has to be considered as random (see Fig. 3(b)). One can thus conclude that the Hölder exponents obtained via the DFA have no obvious relation with the climate variability.

As a conclusion, one can say that the trend/noise studied in $\mathbb{R}^2$ is monofractal but uniform, while the whole signal is also monofractal but not uniform. Moreover, Hölder exponent obtained here with the WLM reflects the climate variability of the station associated to the data, which is not the case with “bias-dependent” methods.

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