On the Study of Quasi-degraded Channel to CoMP-NOMA System

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Abstract

In this paper, quasi-degraded channel is studied for downlink CoMP-NOMA system. In quasi-degraded channel, given the users’ target rates and the maximum transmit power of each base station, CoMP-NOMA can achieve the same minimal total transmission power as Dirty paper coding (DPC). This paper provides the mathematical condition for the channel to satisfy quasi-degradation. And closed-form expression for the optimal precoding vector of CoMP-NOMA under quasi-degraded channel is provided. A novel greedy algorithm with superior performance is proposed by applying quasi-degraded channel, which reveals the applicable value of quasi-degraded channel.

I. SYSTEM MODEL

Consider a two-cell downlink N-NOMA system as shown in Fig. 2. Two BSs, termed BS 1 and BS 2, are located at \((-D, 0)\) and \((D, 0)\) represented by Cartesian coordinate, respectively. The coverage area of each BS \(i\) \((i = 1, 2)\) is denoted by a big disc \(D_i\) centered at the BS \(i\). The radius of the disc \(D_i\) is given by \(R_0\). A CoMP user, termed user 0, which is far from both considered BSs, is uniformly and randomly distributed in the intersection area of two big discs. The CoMP user is served cooperatively by the two BSs, and the cooperative schemes considered in this paper are termed joint beamforming scheme and BS selection scheme which...
will be presented later. In addition to the CoMP user, each BS individually serves a NOMA user which is close to the BS, by occupying the same resource block allocated to the CoMP user. Particularly, the NOMA user associated with BS $i$ ($i = 1, 2$) is denoted by user $i$ and is randomly and uniformly distributed in a small disc centered at BS $i$ with radius $R_i$.

Each BS is equipped with $N$ antennas and each user is equipped with a single antenna. The channel between BS $i$ ($i = 1, 2$) and user $j$ ($j = 0, 1, 2$) is modeled by $h_{ij} = \frac{g_{ij}}{d_{ij}^{\alpha}}$, where $g_{ij}$ is the small scale Rayleigh fading, $d_{ij}$ is the distance between BS $i$ and user $j$, and $\alpha$ is the corresponding large scale path loss exponent.

The transmitted signal by BS $i$ ($i = 1, 2$) is given by:

$$x_i = w_{i0}s_0 + w_{ii}s_i, i = 1, 2$$  

(1)

where $s_i$ ($i = 0, 1, 2$) is the signal intended for user $i$, $w_{i0}$ ($i = 1, 2$) is the beamforming vector for the CoMP user, $w_{ii}$ ($i = 1, 2$) is the beamforming vector for user $i$.

Note that, $w_{i0}$ might be a zero vector, according to the values of $w_{i0}$ ($i = 1, 2$), the transmission scheme can be classified into the following two different types:

- Joint transmission (JT), where both base stations transmit information to the CoMP user, i.e., $w_{i0} \neq 0, i = 1, 2$;
- Non-joint transmission (NJT), where only one base station transmits information to the CoMP user, i.e., $\exists i, w_{i0} = 0$.

The CoMP user treats the signals of NOMA users as interferences, thus the signal to interference plus noise ratio (SINR) of the CoMP user to decode its own signal is:

$$\text{SINR}_{0 \rightarrow 0} = \frac{||h_{10}^H w_{10}||^2 + ||h_{20}^H w_{20}||^2}{||h_{10}^H w_{11}||^2 + ||h_{20}^H w_{22}||^2 + \sigma^2}$$  

(2)
where $\sigma^2$ is the noise power, $\text{SINR}_{j\rightarrow j'}$ denotes the SINR when user $j$ decodes the signal of user $j'$.

Different from the the CoMP user, if $w_{i0} \neq 0$, $i = 1, 2$, the NOMA user (user 1 or 2) first decode the signal of the CoMP user with the following SINR:

$$\text{SINR}_{j\rightarrow 0} = \frac{||h_{ij}^Hw_{j0}||^2}{||h_{ij}^Hw_{ij}||^2 + \sigma^2}, j = 1, 2, i = j.$$  \hfill (3)

If the CoMP user’s signal can be successfully decoded, user $j$ ($j = 1, 2$) carries out successive interference cancellation (SIC) to remove the signal of the CoMP user, and then decodes its own signal. If $w_{i0} = 0$, $i = 1, 2$, user $i$ will decode its own signal directly. In the above two cases, the SINR when the NOMA user decodes its own signal can be expressed as:

$$\text{SINR}_{j\rightarrow j} = \frac{||h_{ij}^Hw_{ij}||^2}{\sigma^2}, j = 1, 2, i = j.$$  \hfill (4)

Thus, the achievable rates of the users are given by:

$$R_0 = \min \left\{ \log (1 + \text{SINR}_{0\rightarrow 0}), \min_{j = 1, 2} \log (1 + \text{SINR}_{j\rightarrow 0}) \right\} \hfill (5)$$

$$R_j = \log \left\{ 1 + \text{SINR}_{j\rightarrow j} \right\}, j = 1, 2.$$  \hfill (6)

Note that user 0’s achievable rate depends on not only its own channel condition, but also the NOMA users’ channel conditions.

### A. Formulation of CoMP-NOMA

Given the target rate of each user and the largest transmit power of each base station, the total transmission power minimization problem of CoMP-NOMA can be formulated as follows:

$$\min_{w_{10}, w_{20}, w_{11}, w_{22}} ||w_{10}||^2 + ||w_{20}||^2 + ||w_{11}||^2 + ||w_{22}||^2$$  \hfill (7a)

s.t. \hspace{1cm} R_j \geq r_j, j = 0, 1, 2 \hfill (7b)

$$||w_{10}||^2 + ||w_{11}||^2 \leq P_{max},$$  \hfill (7c)
\[ \|w_{20}\|^2 + \|w_{22}\|^2 \leq P_{\text{max}}. \]  
(7d)

where (7b) means that the achievable rate of user \( j \) \((j = 0, 1, 2)\) must be larger than a given target rate \( r_j \), (7c) and (7d) mean that the transmission power of each base station should not exceed a given maximum \( P_{\text{max}} \).

**B. Formulation of Dirty paper coding (DPC)**

By utilizing the known information of each user at the transmitter, DPC can effectively avoid inter-user interferences, which generally achieves the optimal performance. Particularly, in the considered DPC in this paper, at BS 1, user 0’s information if encoded before user 1’s information, which ensures that user 1 can avoid the interference from user 0. Similarly, at BS 2, user 0’s information if encoded before user 2’s information, which ensures that user 2 can avoid the interference from user 0. Thus the power minimization problem of DPC can be given by:

\[
\min_{w_{10}, w_{20}, w_{11}, w_{22}} \|w_{10}\|^2 + \|w_{20}\|^2 + \|w_{11}\|^2 + \|w_{22}\|^2 
\]

s.t.
\[
\log(1 + \text{SINR}_{0\rightarrow 0}) \geq r_0, 
\]
(8b)
\[
R_j \geq r_j, \quad j = 1, 2 
\]
(8c)
\[
\|w_{10}\|^2 + \|w_{11}\|^2 \leq P_{\text{max}}, 
\]
(8d)
\[
\|w_{20}\|^2 + \|w_{22}\|^2 \leq P_{\text{max}}. 
\]
(8e)

It can be found that compared to problem (7), there are two fewer constraints in problem (8). Thus, the minimal transmission power of the DPC scheme provides a lower bound of the minimal transmission power of the CoMP-NOMA scheme. However, due to the high complexity, it is difficult to implement DPC in practical systems. Moreover, superposition coding used in CoMP-NOMA is much easier to implement than DPC. Thus, an interesting question is that whether CoMP-NOMA can achieve the same minimal transmission power as DPC under some channel conditions. The channel makes this true is called quasi-degraded channel of CoMP-NOMA.

**C. Quasi-degraded channel condition**

In the following, we would like to derive the expression of the condition that the quasi-degraded channel should meet, which is called quasi-degraded channel condition \([1]\). To this end, it is necessary to obtain the closed-form optimal solution of problem (8).
Without loss of generality, it is assumed that \(|h_{20}| \geq |h_{10}|\). It can be found that the optimal solution of problem (8) subjects to a specific format as shown in the following lemma.

**Lemma 1.** The optimal solution of problem (8) can be expressed as follows:

\[
\begin{align*}
    w_{10}^{D*} &= \sqrt{P_{10}} h_{10} / ||h_{10}||, \\
    w_{20}^{D*} &= \sqrt{P_{20}} h_{20} / ||h_{20}||, \\
    w_{11}^{D*}(x) &= \sqrt{P_{11}(x)} \frac{(I - x h_{10} h_{10}^H / ||h_{10}||^2) h_{11}}{||(I - x h_{10} h_{10}^H / ||h_{10}||^2) h_{11}||}, \\
    P_{11}(x) &= \frac{\sigma^2 \epsilon_1 \left(||h_{11}||^2 - (2x - x^2) ||h_{10} h_{11}^H||^2 / ||h_{10}||^2\right)}{\left(||h_{11}||^2 - x ||h_{10} h_{11}^H||^2 / ||h_{10}||^2\right)}, \\
    w_{22}^{D*}(y) &= \sqrt{P_{22}(y)} \frac{(I - y h_{20} h_{20}^H / ||h_{20}||^2) h_{22}}{||(I - y h_{20} h_{20}^H / ||h_{20}||^2) h_{22}||}, \\
    P_{22}(y) &= \frac{\sigma^2 \epsilon_2 \left(||h_{22}||^2 - (2y - y^2) ||h_{20} h_{22}^H||^2 / ||h_{20}||^2\right)}{\left(||h_{22}||^2 - y ||h_{20} h_{22}^H||^2 / ||h_{20}||^2\right)}.
\end{align*}
\]

where \(0 < y \leq \epsilon_0 / (1 + \epsilon_0), 0 < x \leq \epsilon_0 / (1 + \epsilon_0)\) are undetermined coefficients which will be given later.

**Proof:** Please refer to Appendix A.

In Lemma 1, there are four undetermined variables, i.e., \(P_{10}, P_{20}, x,\) and \(y\). Next, we need to find the optimal \(P_{10}, P_{20}, x,\) and \(y\), which are denoted by \(P_{10,opt}, P_{20,opt}, x_{opt}\) and \(y_{opt}\), respectively. According to the values of \(P_{10}\) and \(P_{20}\), the solution of (8) can be classified into three cases:

- **Case I:** \(P_{20} > 0\) and \(P_{10} = 0\);
- **Case II:** \(P_{20} > 0\) and \(P_{10} > 0\);
- **Case III:** \(P_{20} = 0\) and \(P_{10} > 0\).

In the following, first, the optimal solution for each case will be given, then, the optimal solutions of the three cases will be compared to determine the global optimal solution of problem (8).

For notational simplicity, define the following functions:

\[
F_{20} (x) = \frac{\sigma^2 \epsilon_0 \epsilon_0 ||h_{20} h_{22}^H||^2}{||h_{20}||^2} + \frac{\sigma^2 \epsilon_0 ||h_{10} h_{11}^H||^2}{||h_{20}||^2 (A - Bx)^2},
\]

\[
F_{10} (y) = \frac{\sigma^2 \epsilon_0 \epsilon_0 ||h_{10} h_{11}^H||^2}{||h_{10}||^2} + \frac{\sigma^2 \epsilon_0 \epsilon_2 ||h_{20} h_{22}^H||^2}{||h_{10}||^2 (C - Dy)^2}.
\]
where \( A = ||h_{11}||^2 \), \( B = ||h_{10}^H h_{11}||^2 / ||h_{10}||^2 \), \( C = ||h_{22}||^2 \), \( D = ||h_{20}^H h_{22}||^2 / ||h_{20}||^2 \), \( \hat{w}_{22} = w_{22}^D (\epsilon_0 / (1 + \epsilon_0)) \), and \( \hat{w}_{11} = w_{11}^D (\epsilon_0 / (1 + \epsilon_0)) \).

In the following, the optimal solution for case with \( P_{20} > 0 \) and \( P_{10} = 0 \) is firstly provided.

**Theorem 1.** Given \( ||h_{20}|| > ||h_{10}|| \), problem (8) is feasible under the case where \( P_{20} > 0 \) and \( P_{10} = 0 \) if and only if:

\[
P_{11}(0) < P_{max} \text{ and } F_{20}(\tilde{x}_B) \leq P_{max} - P_{22}(\epsilon_0 / (1 + \epsilon_0))
\]

where,

\[
\tilde{x}_B = \begin{cases} 
    \frac{\epsilon_0}{1 + \epsilon_0}, & \text{if } P_{11} \left( \frac{\epsilon_0}{1 + \epsilon_0} \right) \leq P_{max} \\
    \frac{A}{2B}, & \text{else if } B = \frac{\sigma^2 \epsilon_1}{P_{max}} \\
    \frac{B - ABP_{max} / \sigma^2 \epsilon_1 - \sqrt{B(A-B)(AP_{max} / \sigma^2 \epsilon_1) - 1}}{B - B^2 P_{max} / \sigma^2 \epsilon_1}, & \text{otherwise.}
\end{cases}
\]

If the above condition holds, the optimal solution of problem (8) for the case with \( P_{20} > 0 \) and \( P_{10} = 0 \) can be given by:

\[
P_{10,opt} = 0,
\]

\[
P_{20,opt} = F_{20}(x_{opt}),
\]

\[
y_{opt} = \frac{\epsilon_0}{1 + \epsilon_0},
\]

\[
x_{opt} = \begin{cases} 
    \tilde{x}_B, & \tilde{x}_B < \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2 / ||h_{10}||^2} \\
    \tilde{x}_A, & \tilde{x}_A \geq \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2 / ||h_{10}||^2} \\
    \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2 / ||h_{10}||^2}, & \text{else}
\end{cases}
\]

where,

\[
\tilde{x}_A = \begin{cases} 
    \frac{A \sqrt{\epsilon_0}}{B \sqrt{\epsilon_0}}, & \text{if } A \sqrt{\epsilon_0} < 1 \\
    0, & \text{else}
\end{cases}
\]

\[
P_A = \left( P_{max} - P_{22}(\epsilon_0 / (1 + \epsilon_0)) \right) - \frac{\sigma^2 \epsilon_0 + \epsilon_0 ||h_{20}^H w_{22}||^2}{||h_{20}||^2} \\
\]

\[
\left( \frac{\sigma^2 \epsilon_0 \epsilon_1 ||h_{10}^H h_{11}||^2}{||h_{10}||^2} \right).
\]

**Proof:** Please refer to Appendix B.

**Corollary 1.** Given \( ||h_{20}|| > ||h_{10}|| \), if problem (8) has no power constraint, i.e., \( P_{max} = \infty \), then the optimal solution of problem (8) can be expressed as:

\[
P_{10,opt} = 0,
\]
\[ P_{20,\text{opt}} = F_{20}(x_{\text{opt}}), \quad (26) \]

\[ x_{\text{opt}} = \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2/||h_{10}||^2}. \quad (27) \]

The following theorem provides the optimal solution of the case where both \( P_{10} \) and \( P_{20} \) are larger than zero.

**Theorem 2.** Given \( ||h_{20}|| > ||h_{10}|| \), problem is feasible under the case where \( P_{10} > 0 \) and \( P_{20} > 0 \) if and only if:

\[ P_{\text{max}} - P_{11} \left( \frac{\epsilon_0}{1 + \epsilon_0} \right) > 0, \quad P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)) > 0, \quad (28) \]

and

\[ ||h_{10}||^2 \left( P_{\text{max}} - P_{11} \left( \frac{\epsilon_0}{1 + \epsilon_0} \right) \right) + ||h_{20}||^2 \left( P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)) \right) \geq F_{20} \left( \frac{\epsilon_0}{1 + \epsilon_0} \right) ||h_{20}||^2, \quad (29) \]

If the above conditions holds, then the optimal solution of problem (8) can be expressed as:

\[ P_{10,\text{opt}} = F_{20} \left( \frac{\epsilon_0}{1 + \epsilon_0} \right) \frac{||h_{20}||^2/||h_{10}||^2 - ||h_{20}||^2/||h_{10}||^2 (P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0))]}{P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0))}, \quad (30) \]

\[ P_{20,\text{opt}} = P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)), \]

\[ x_{\text{opt}} = \frac{\epsilon_0}{1 + \epsilon_0}, \]

\[ y_{\text{opt}} = \frac{\epsilon_0}{1 + \epsilon_0}. \quad (31) \]

**Proof:** Please refer to Appendix C.  

From Theorem 1 and 2, a necessary condition for the feasibility under the first two cases can be easily obtained, as highlighted in the following.

**Corollary 2.** When \( P_{20} > 0 \), problem (8) is feasible only if:

\[ F_{22}(\epsilon_0/(1 + \epsilon_0)) < P_{\text{max}}. \quad (32) \]

**Theorem 3.** \( ||h_{20}|| > ||h_{10}|| \), problem is feasible under the case where \( P_{10} > 0 \) and \( P_{20} = 0 \) if and only if:

\[ F_{22}(0) < P_{\text{max}} \quad \text{and} \quad F_{10}(\tilde{y}_B) \leq P_{\text{max}} - P_{11}(\epsilon_0/(1 + \epsilon_0)), \quad (33) \]
where

\[
\hat{y}_B = \begin{cases} \frac{\epsilon_0}{1+\epsilon_0}, & \text{if } P_{22} \left( \frac{\epsilon_0}{1+\epsilon_0} \right) \leq P_{\text{max}} \\ C/2D, & \text{else if } D = \frac{a^2 \epsilon_2}{P_{\text{max}}} \\ \frac{D-CDP_{\text{max}}/\sigma^2 \epsilon_2}{D-D^2P_{\text{max}}/\sigma^2 \epsilon_2}, & \text{otherwise} \end{cases}
\]  

If (33) is satisfied, the optimal solution of problem (8) for the case with \( P_{10} > 0 \) and \( P_{20} = 0 \) can be given by:

\[
\begin{align*}
P_{10,\text{opt}} &= F_{10}(y_{\text{opt}}), \\
P_{20,\text{opt}} &= 0 \\
x_{\text{opt}} &= \frac{\epsilon_0}{1+\epsilon_0}, \\
y_{\text{opt}} &= \hat{y}_B.
\end{align*}
\]

**Proof:** Please refer to Appendix D.

Until now, the optimal solution for each case is obtained. Obviously, directly comparing the optimal values of the three cases, the optimal solution for problem (8) can be obtained. However, this method provides little insight of the problem. Indeed, there is a more elegant way to determine the optimal solution of problem (8), which can provides more insight. Interestingly, the optimal solutions of the three cases have the following relationships.

**Proposition 1.** If problem (8) is feasible when \( P_{20} > 0 \), then the optimal solution of problem (8) should satisfy \( P_{20} > 0 \).

**Proof:** Please refer to Appendix E.

The above proposition means that if one of the first two cases is feasible, then the optimal solution must be one of the first two cases. In other words, the optimal solutions of the first two cases must be better than that of the third case.

The following corollary, which can be easily obtained from the proof for Proposition 1, is useful for deciding the optimal solution belongs to which case.

**Corollary 3.** When \( F_{22}(\epsilon_0/(1+\epsilon_0)) < P_{\text{max}} \), if there is a feasible solution of case III, then there exist a better solution which belongs to Case I or II.

One important application of the above corollary is that: when \( F_{22}(\epsilon_0/(1+\epsilon_0)) < P_{\text{max}} \) and there is no feasible solution of Case I nor II, it can be concluded that problem (8) is infeasible.
Further, the relationship between the optimal solutions of the first two cases can also be clearly given by the following proposition.

**Proposition 2.** If conditions (17) holds, then there is not feasible solution of Case II which is better than the optimal solution of Case I.

**Proof:** Please refer to Appendix F.

Further, according to the relationship between problem (7) and problem (8), the quasi-degraded channel condition can be obtained, as shown in the following theorem.

**Theorem 4.** If problem (8) is feasible, the quasi-degraded channel condition is satisfied if and only if: there exist one optimal solution of problem (8), denoted by $w_{10}^{D*}, w_{11}^{D*}, w_{20}^{D*}, w_{22}^{D*}$, satisfying the following conditions:

\[
\text{if } w_{10}^{D*} \neq 0, \quad - ||h_{11}^{H} w_{10}^{D*}||^2 + \epsilon_0 ||h_{11}^{H} w_{11}^{D*}||^2 + \epsilon_0 \sigma^2 \leq 0, \quad (39)
\]

\[
\text{if } w_{20}^{D*} \neq 0, \quad - ||h_{22}^{H} w_{20}^{D*}||^2 + \epsilon_0 ||h_{22}^{H} w_{22}^{D*}||^2 + \epsilon_0 \sigma^2 \leq 0. \quad (40)
\]

II. APPLICATION OF QUASI-DEGRADED CHANNELS

A. **H-CoMP-NOMA**

Note that, according to the previous discussion, CoMP-NOMA can achieve the same performance as the DPC scheme only when the channels are degraded. Thus, when the channels are not degraded, there is performance loss by applying CoMP-NOMA compared to DPC. Besides, when the channels are degraded, closed-form expressions for the optimal precoding vectors can be obtained. While closed-form expressions are not available when the channels are not degraded and the acquisition of precoding vectors relies on iterative algorithms, which is not efficient. Thus, in this paper, we proposed a novel hybrid CoMP-NOMA scheme as shown in Algorithm 1. In the proposed H-CoMP-NOMA scheme, CoMP-NOMA is adopted if the channel is degraded, otherwise, ZFBF is adopted.

B. **Quasi-degradation based user pairing (QDUP) for multiple CoMP users scenario**

Consider a scenario with multiple CoMP users as shown in Fig. 2. There are $3K$ users, including: (a) $K$ CoMP users, denoted by $U_{0,k}, 1 \leq k \leq K$; (b) $K$ NOMA users which are near to BS 1, denoted by $U_{1,i}, 1 \leq i \leq K$; (c) $K$ NOMA users which are near to BS 2, denoted by $U_{2,j}, 1 \leq j \leq K$. The channel between $U_{1,i}$ and BS 1 is denoted by $h_{11}^i$, the channel between
Algorithm 1: H-CoMP-NOMA scheme

Input: channel information

Output: transmission strategy

if quasi degraded channel then
    choose CoMP-NOMA transmission;
else
    choose ZFBF transmission;
end

Fig. 2: Illustration of the system model.

\[ U_{2,j} \] and BS 2 is denoted by \( h_{22}^j \), and the channels between \( U_{0,k} \) and BS 1 and 2 are denoted by \( h_{10}^k \) and \( h_{20}^k \), respectively.

The 3K users are divided into K groups. Each group consists of a CoMP user, a NOMA user of BS 1 and a NOMA user of BS 2. The index of the NOMA user of BS 1 which is paired with CoMP user \( U_{0,k} \) is denoted by \( \pi_1(k) \), and the index of the NOMA user of BS 2 which is paired with CoMP user \( U_{0,k} \) is denoted by \( \pi_2(k) \). In this paper, TDMA is applied to serve different groups, i.e., each group is allocated with an independent time slot, yielding no interference between different groups. Besides, equal power allocation is considered for each group, i.e., the largest power allocated to a group by a base station is \( P_{\text{max}}/K \). In this scenario, the power minimization problem can be formulated as follows:

\[
\min_{\pi_1, \pi_2, w_{10}^k, w_{20}^k, w_{11}^k, w_{22}^k} \sum_{k=1}^{K} \left( \|w_{10}^k\|^2 + \|w_{20}^k\|^2 + \|w_{11}^{\pi_1(k)}\|^2 + \|w_{22}^{\pi_2(k)}\|^2 \right) \quad (41a)
\]

s.t. \( R_{j}^k \geq r_j, 0 \leq j \leq 2, 1 \leq k \leq K, \quad (41b)\)

\[
\|w_{10}^k\|^2 + \|w_{11}^{\pi_1(k)}\|^2 \leq P_{\text{max}}/K, 1 \leq k \leq K, \quad (41c)
\]
Algorithm 2: Qusai-degradation based user pairing (QDUP)

Input: channel information

Output: $\pi_1(k)$, $\pi_2(k)$, $S(k)$

// $S(k) = 1$, choose CoMP-NOMA; $S(k) = 0$, choose ZFBF

for $k=1:K$ do
  Flag = 0;
  for $i=1:K$ do
    for $j=1:K$ do
      if $U_1^i$ and $U_2^j$ haven’t been paired then
        if $U_1^i$, $U_2^j$ and $U_0^k$ have quasi-degraded channels then
          $\pi_1(k) = i; \pi_2(k) = j; S(k) = 1$;
          Flag = 1;
          break;
        end
      end
      if $U_1^i$, $U_2^j$ and $U_0^k$ have orthogonal channels then
        $\pi_1(k) = i; \pi_2(k) = j; S(k) = 0$;
        Flag = 1;
        break;
      end
    end
    if Flag = 0 then
      find $\pi_1(k) = \arg\min \frac{||h_i^1 h_{10}^k||^2}{||h_i^1||^2 ||h_{10}^k||^2}$;
      find $\pi_2(k) = \arg\min \frac{||h_j^2 h_{20}^k||^2}{||h_j^2||^2 ||h_{20}^k||^2}$;
      $S(k) = 0$;
    end
  end
end

\[ ||w_{20}^k||^2 + ||w_{22}^{\pi_2(k)}||^2 \leq P_{max} / K, 1 \leq k \leq K, \] (41d)

where $R_j^k$ is the achievable rate of the user in the $k$-th group, which depends on the specific
transmission scheme applied for this group. Obviously, the task is to group user and design precoding vectors for each group.

Problem (41) is a mixed integer programming problem, finding its optimal solution is very challenging. Thus, based on quasi-degraded channel condition obtained in the previous section, a greedy algorithm with low complexity is proposed to provide a sub-optimal solution. The proposed algorithm first group users by using Algorithm 2, in each group H-CoMP-NOMA is applied.

Fig. (3) shows the comparison among the proposed greedy algorithm with other benchmark schemes in terms of outage performance. These benchmark schemes are:

- ZFBF/Ran algorithm: users are randomly paired and ZFBF is applied in each group;
- H-CoMP-NOMA/Ran algorithm: users are randomly grouped, and H-CoMP-NOMA is applied in each group;
- H-CoMP-NOMA/Corr algorithm: CoMP users are sequentially paired with the NOMA users with smallest channel angles, and H-CoMP-NOMA is applied in each group.

From the figure, it is shown that the proposed scheme is better than other schemes in terms of outage performance. Besides, as the number of users increases, the outage probability achieved by the proposed greedy scheme decreases, which reveals that the proposed scheme can adequately
utilize the multi-user diversity.

APPENDIX A

PROOF FOR LEMMA 1

Problem (8) can be rewritten as:

$$\min_{\mathbf{w}_{10}, \mathbf{w}_{20}, \mathbf{w}_{11}, \mathbf{w}_{22}} \quad || \mathbf{w}_{10} ||^2 + || \mathbf{w}_{20} ||^2 + || \mathbf{w}_{11} ||^2 + || \mathbf{w}_{22} ||^2$$

(42a)

s.t. \quad - || h_{10}^H \mathbf{w}_{10} ||^2 - || h_{20}^H \mathbf{w}_{20} ||^2

(42b)

+ \epsilon_0 \left( || h_{10}^H \mathbf{w}_{11} ||^2 + || h_{20}^H \mathbf{w}_{22} ||^2 + \sigma^2 \right) \leq 0,

(42c)

- || h_{11}^H \mathbf{w}_{11} ||^2 + \sigma^2 \epsilon_1 \leq 0

(42d)

- || h_{22}^H \mathbf{w}_{22} ||^2 + \sigma^2 \epsilon_2 \leq 0

(42e)

|| \mathbf{w}_{10} ||^2 + || \mathbf{w}_{11} ||^2 \leq P_{\text{max}},

(42f)

|| \mathbf{w}_{20} ||^2 + || \mathbf{w}_{22} ||^2 \leq P_{\text{max}}.

(42f)

This problem is an non-convex problem. However, as shown later, it can be transformed to a convex problem. An interesting observation is that: the directions of the optimal \( \mathbf{w}_{10} \) and \( \mathbf{w}_{20} \) should satisfy:

\[
\mathbf{w}_{10} = \frac{h_{10}}{|| h_{10} ||}, \quad \mathbf{w}_{20} = \frac{h_{20}}{|| h_{20} ||},
\]

(43)

Thus, \( \mathbf{w}_{10} \) and \( \mathbf{w}_{20} \) can be expressed as:

\[
\mathbf{w}_{10} = \sqrt{P_{10}} \mathbf{w}_{10}, \quad \mathbf{w}_{20} = \sqrt{P_{20}} \mathbf{w}_{20}.
\]

(44)

Then, problem (42) can be rewritten as:

$$\min_{P_{10}, P_{20}, \mathbf{w}_{11}, \mathbf{w}_{22}} \quad P_{10} + P_{20} + || \mathbf{w}_{11} ||^2 + || \mathbf{w}_{22} ||^2$$

(45a)

s.t. \quad - || h_{10}^H \mathbf{w}_{10} ||^2 P_{10} - || h_{20}^H \mathbf{w}_{20} ||^2 P_{20}

(45b)

+ \epsilon_0 \left( || h_{10}^H \mathbf{w}_{11} ||^2 + || h_{20}^H \mathbf{w}_{22} ||^2 + \sigma^2 \right) \leq 0,

(45c)

- || h_{11}^H \mathbf{w}_{11} ||^2 + \sigma^2 \epsilon_1 \leq 0

(45d)

- || h_{22}^H \mathbf{w}_{22} ||^2 + \sigma^2 \epsilon_2 \leq 0

(45d)

\[
P_{10} + || \mathbf{w}_{11} ||^2 \leq P_{\text{max}},
\]

(45e)
\[ P_{20} + ||w_{22}||^2 \leq P_{\text{max}}, \]  
\[ -P_{10} \leq 0, \]  
\[ -P_{20} \leq 0. \]  

It can be found that, the constraints (45b), (45e)-(45h) are convex. Intuitively, (45c) is an non-convex constraint, however, in essence, it is a convex constraint, the reasons are as follows: first, multiply \( w_{10} \) with a complex number with modulus 1, denoted by \( e^{j\phi} \), such that \( h_{11}^H w_{10} \) is a positive real number, this operation will not change the optimality of the problem, then (45c) can be transformed into a linear constraint, and hence is convex. Similarly, constraint (45d) is also convex. Thus, problem (45) is a convex problem. The Lagrangian of problem (45) is given by:

\[
L = P_{10} + P_{20} + ||w_{11}||^2 + ||w_{22}||^2 + \lambda_1 ( -||h_{10}||^2 P_{10} - ||h_{20}||^2 P_{20} + \epsilon_0 (||h_{10}^H w_{11}||^2 + ||h_{20}^H w_{22}||^2 + \sigma^2) ) \\
+ \lambda_2 ( -||h_{11}^H w_{11}||^2 + \sigma^2 \epsilon_1 ) + \lambda_3 ( -||h_{22}^H w_{22}||^2 + \sigma^2 \epsilon_2 ) \\
+ \lambda_4 (P_{10} + ||w_{11}||^2 - P_{\text{max}}) + \lambda_5 (P_{20} + ||w_{22}||^2 - P_{\text{max}}) \\
- \lambda_6 P_{10} - \lambda_7 P_{20}.
\]

where \( \lambda_i \geq 0, i = 1, 2, \cdots, 7 \) are the Lagrangian multipliers.

It is not difficult to find the following observations:

\* the optimal \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) must be larger than zero, and equality holds for the corresponding constraints;

According to the stationarity of the KKT condition, the following relationships can be established:

\[
\frac{\partial L}{P_{10}} = 1 + \lambda_4 - \lambda_6 - \lambda_1 ||h_{10}||^2 = 0, \tag{47}
\]
\[
\frac{\partial L}{P_{20}} = 1 + \lambda_5 - \lambda_7 - \lambda_1 ||h_{10}||^2 = 0, \tag{48}
\]
\[
\frac{\partial L}{w_{11}} = 2(1 + \lambda_4) w_{11} + 2 \lambda_1 \epsilon_0 h_{10} h_{11}^H w_{11} - 2 \lambda_2 h_{11} h_{11}^H w_{11} = 0, \tag{49}
\]
\[
\frac{\partial L}{w_{22}} = 2(1 + \lambda_5) w_{22} + 2 \lambda_1 \epsilon_0 h_{20} h_{22}^H w_{22} - 2 \lambda_3 h_{22} h_{22}^H w_{22} = 0. \tag{50}
\]

From the first two equations above, it is easy to get the following relationships:

\[
\frac{\lambda_1}{1 + \lambda_4 - \lambda_6} = \frac{1}{||h_{10}||^2}, \tag{51}
\]
\[
\frac{\lambda_1}{1 + \lambda_5 - \lambda_7} = \frac{1}{\|\mathbf{h}_{20}\|^2},
\]  
(52)

While from the later two equations, the followings can be obtained:

\[
\mathbf{w}_{11} = \left( \mathbf{I} + \frac{\lambda_1 \epsilon_0}{1 + \lambda_4} \mathbf{h}_{10} \mathbf{h}_{10}^H \right)^{-1} \lambda_2 \mathbf{h}_{11} \mathbf{h}_{11}^H \mathbf{w}_{11}
\]  
(53)

\[
\mathbf{w}_{22} = \left( \mathbf{I} + \frac{\lambda_1 \epsilon_0}{1 + \lambda_5} \mathbf{h}_{20} \mathbf{h}_{20}^H \right)^{-1} \lambda_3 \mathbf{h}_{22} \mathbf{h}_{22}^H \mathbf{w}_{22}
\]  
(54)

Note that, we can make \(\mathbf{h}_{11}^H \mathbf{w}_{11}\) be a positive real number, the direction of \(\mathbf{w}_{11}\) can be expressed as:

\[
\tilde{\mathbf{w}}_{11} = \left( \mathbf{I} + \frac{\lambda_1 \epsilon_0}{1 + \lambda_4} \mathbf{h}_{10} \mathbf{h}_{10}^H \right)^{-1} \mathbf{h}_{11}
\]  
(55)

Similarly, the direction of \(\mathbf{w}_{22}\) can be expressed as:

\[
\tilde{\mathbf{w}}_{22} = \left( \mathbf{I} + \frac{\lambda_1 \epsilon_0}{1 + \lambda_5} \mathbf{h}_{20} \mathbf{h}_{20}^H \right)^{-1} \mathbf{h}_{22}
\]  
(56)

Further, according to (52) and Shermon Morrison equation, \(\tilde{\mathbf{w}}_{22}\) can be rewritten as:

\[
\tilde{\mathbf{w}}_{22} = \left( \mathbf{I} - \frac{\epsilon_0}{1 + \epsilon_0} \mathbf{h}_{20} \mathbf{h}_{20}^H / \|\mathbf{h}_{20}\|^2 \right) \mathbf{h}_{22}
\]  
(57)

let \(\mathbf{w}_{22} = \sqrt{P_{22}} \tilde{\mathbf{w}}_{22}\), and let equality holds in constraint (45d), the expression of \(P_{22}\) can be obtained.

Let \(t_x = \frac{\lambda_1 \epsilon_0}{1 + \lambda_4}\) and applying Shermon Morrison equation, it is obtained that:

\[
\tilde{\mathbf{w}}_{11} = \left( \mathbf{I} - \frac{t_x \mathbf{h}_{10} \mathbf{h}_{10}^H}{1 + t_x \|\mathbf{h}_{10}\|^2} \right) \mathbf{h}_{11}
\]  
(58)

Further, let

\[
x = \frac{t_x \|\mathbf{h}_{10}\|^2}{1 + t_x \|\mathbf{h}_{10}\|^2},
\]  
(59)

and \(\mathbf{w}_{11} = \sqrt{P_{11}(x)} \tilde{\mathbf{w}}_{11}\), and let equality holds in constraint (45c), the expression of \(P_{11}(x)\) can be obtained. Moreover, according to (51), it is obtained that \(0 < x \leq \epsilon_0 / (1 + \epsilon_0)\).

Similarly, \(\mathbf{w}_{22}\) can be expressed as:

\[
\mathbf{w}_{22} = \sqrt{P_{22}(y)} \tilde{\mathbf{w}}_{22},
\]  
(60)
where

\[ y = \frac{t_y \|h_{20}\|^2}{1 + t_y \|h_{20}\|^2}, \quad (61) \]
\[ t_y = \frac{\lambda_1 \epsilon_0}{1 + \lambda_5}. \quad (62) \]

Then, the expression of \( P_{22}(y) \) and the value range of \( y \) can be obtained as same as \( P_{11}(y) \) and \( x \).

**APPENDIX B**

**PROOF FOR THEOREM 1**

In problem (45), according to the complementary slackness of the KKT condition, we have:

\[ \lambda_6 P_{10} = 0, \lambda_7 P_{20} = 0. \quad (63) \]

Since \( P_{10} = 0 \) and \( P_{20} > 0 \) are assumed, it can be concluded that \( \lambda_7 = 0 \). By taking \( \lambda_7 = 0 \) into (52), it is obtained that:

\[ \frac{\lambda_1}{1 + \lambda_5} = \frac{1}{\|h_{20}\|^2}, \quad (64) \]

Thus \( t_y = 1/\|h_{20}\|^2 \) and the value of optimal \( y \) denoted by \( y_{opt} \) can be obtained:

\[ y_{opt} = \frac{\epsilon_0}{1 + \epsilon_0}. \quad (65) \]

Further, by noting that constraint (45b) should take mark of equality as stated in Appendix B, it is obtained that:

\[ P_{20} = F_{20}(x). \quad (66) \]

Based on the above discussions, the primal problem can be simplified to the following optimal problem:

\[
\min_x \ F_{20}(x) + P_{11}(x) \quad \text{(67a)}
\]
\[
s.t. \quad P_{11}(x) \leq P_{max}, \quad \text{(67b)}
\]
\[
P_{20}(x) \leq P_{max} - P_{22}(\epsilon_0/(1 + \epsilon_0)) \quad \text{(67c)}
\]
\[
0 < x \leq \frac{\epsilon_0}{1 + \epsilon_0}. \quad \text{(67d)}
\]

Thus, the left task is to find the optimal \( x \).
Let $G(x) = F_{20}(x) + P_{11}(x)$, and take the derivative of $G(x)$ with respect to $x$, we have

$$G'(x) = F'_{20}(x) + F'_{11}(x)$$

(68)

where

$$F'_{20}(x) = \frac{2\sigma^2 \epsilon_0 \epsilon_1 ||h_{10}^H h_{11}||^2 (A - B)(x - 1)}{||h_{20}||^2 (A - Bx)^3}$$

(69)

$$F'_{11}(x) = \frac{2\sigma^2 \epsilon_1 (A - B)Bx}{(A - Bx)^3}$$

(70)

Thus, in interval $0 < x \leq \frac{\epsilon_0}{1 + \epsilon_0}$, it is easy to have the following observations:

- $F_{20}$ decreases with $x$;
- $P_{11}$ increases with $x$;
- $G(x)$ first decreases with $x$ and then increases.

Let $G'(x) = 0$, the extreme point can be obtained:

$$x_{ext} = \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2 / ||h_{10}||^2}$$

(71)

Let $\bar{x}_B$ be the largest $x$ in interval $(0, \epsilon_0/(1 + \epsilon_0)]$ which satisfies constraint (67b), obviously, $\bar{x}_B$ exists if and only if

$$P_{11}(0) < P_{\max}.$$  

(72)

If $P_{11}(0) < P_{\max}$, then the optimal solution is restricted to be located in $(0, \bar{x}_B]$. Further, take constraint (67c) into consideration, let $\bar{x}_A$ be the minimal $x$ in interval $(0, \bar{x}_B]$ which satisfies constraint (67c), and $\bar{x}_A$ exists if and only if:

$$F_{20}(\bar{x}_B) \leq P_{\max} - P_{22}(\epsilon_0/(1 + \epsilon_0))$$

(73)

Under the condition that $\bar{x}_A$ and $\bar{x}_B$ exist, it is not to derive their expressions as shown in the theorem.

Hence, the optimal solution of $x$ is restricted to be located in $[\bar{x}_A, \bar{x}_B]$. Finally, by using the relationship between $x_{ext}$ and $\bar{x}_A, \bar{x}_B$, the expression of the optimal solution can be obtained, and the proof is complete.
APPENDIX C

PROOF FOR THEOREM 2

Given $P_{10} > 0$ and $P_{20} > 0$, according to the complementary slackness of the KKT condition of problem (45), we have $\lambda_6 = 0$ and $\lambda_7 = 0$, and based on (51) and (59), we have

$$x_{opt} = \frac{\epsilon_0}{1 + \epsilon_0}, \quad y_{opt} = \frac{\epsilon_0}{1 + \epsilon_0}. \quad (74)$$

Then by applying Lemma 1, the optimal $P_{11}, P_{22}, \tilde{w}_{11}$, and $\tilde{w}_{22}$ can be determined. Then, the optimization problem (8) can be simplified to:

$$\min P_{10} + P_{20} \quad (75a)$$

$$s.t. \quad ||h_{10}||^2 P_{10} + ||h_{20}||^2 P_{20} = P_{11}(x_{opt}) \quad (75b)$$

$$0 \leq P_{10} \leq P_{max} - P_{11}(x_{opt}) \quad (75c)$$

$$0 \leq P_{20} \leq P_{max} - P_{22}(y_{opt}). \quad (75d)$$

Problem (75) is a linear programming problem with variables $P_{10}$ and $P_{20}$ and it can be easily solved. Constraints (75c) and (75d) provides the feasibility condition for the problem. Note that $||h_{20}||$ is larger than $||h_{10}||$, which means it is better to use as much $P_{20}$ as possible. Based on the observation, the optimal solution can be obtained and the proof is complete.

APPENDIX D

PROOF FOR THEOREM 3

In problem (45), according to the complementary slackness of the KKT condition, we have:

$$\lambda_6 P_{10} = 0, \quad \lambda_7 P_{20} = 0. \quad (76)$$

Since $P_{20} = 0$ and $P_{10} > 0$ are assumed, it can be concluded that $\lambda_6 = 0$. By taking $\lambda_6 = 0$ into (51), it is obtained that:

$$\frac{\lambda_1}{1 + \lambda_4} = \frac{1}{||h_{10}||^2}. \quad (77)$$

Thus $t_x = 1/||h_{10}||^2$ and the value of optimal $x$ denoted by $x_{opt}$ can be obtained:

$$x_{opt} = \frac{\epsilon_0}{1 + \epsilon_0}. \quad (78)$$

Further, by noting that constraint (45b) should take mark of equality as stated in Appendix B, it is obtained that:

$$P_{10} = F_{10}(y). \quad (79)$$
Based on the above discussions, the primal problem can be simplified to the following optimal problem:

\[
\begin{align*}
\min_x & \quad F_{10}(y) + P_{22}(y) \\
\text{s.t.} & \quad P_{22}(y) \leq P_{\text{max}}, \\
& \quad F_{10}(y) \leq P_{\text{max}} - P_{11}(\epsilon_0/(1 + \epsilon_0)) \\
& \quad 0 < y \leq \frac{\epsilon_0}{1 + \epsilon_0}
\end{align*}
\]

(80a) (80b) (80c) (80d)

Let \(Q(y) = F_{10}(y) + P_{22}(y)\), and take the derivative of \(Q(y)\) with respect to \(y\), we have

\[
Q'(y) = F'_{10}(y) + F'_{22}(y)
\]

(81)

where

\[
F'_{10}(y) = \frac{2\sigma^2 \epsilon_0 \epsilon_2 ||h_{20}^H h_{22}||^2 (C - D)(y - 1)}{||h_{10}||^2 (C - Dy)^3}
\]

(82)

\[
F'_{22}(y) = \frac{2\sigma^2 \epsilon_2 (C - D)Dy}{(C - Dy)^3}
\]

(83)

When \(0 < y < 1\), it is not hard to have the following observations:

- \(F_{10}\) decreases with \(y\) and \(F'_{10}\) increases with \(y\);
- \(P_{22}\) increases with \(y\) and \(F'_{22}\) increases with \(y\);

Thus \(Q'(y)\) increases with \(y\) when \(0 < y < 1\). Let \(Q'(y) = 0\), the extreme point can be obtained:

\[
y_{\text{ext}} = \frac{\epsilon_0}{\epsilon_0 + ||h_{10}||^2 / ||h_{20}||^2},
\]

(84)

Since \(||h_{10}|| < ||h_{20}||\), it can be concluded that \(y_{\text{ext}} > \epsilon_0/(1 + \epsilon_0)\). Thus, in interval \(0 < y \leq \frac{\epsilon_0}{1 + \epsilon_0}\), \(Q(y)\) decreases with \(y\).

Let \(\bar{y}_B\) be the largest \(y\) in interval \((0, \epsilon_0/1 + \epsilon_0]\) which satisfies constraint (67b). Then (80) is feasible if and only if \(\bar{y}_B\) exist and \(F_{10}(\bar{y}_B) \leq P_{\text{max}} - P_{11}(\epsilon_0/(1 + \epsilon_0))\) (constraint (80c)).

Obviously, \(\bar{y}_B\) exists if and only if

\[
P_{22}(0) < P_{\text{max}}.
\]

(85)

If problem (80) is feasible, then \(\bar{y}_B\) is the optimal \(y\), i.e., \(y_{\text{opt}} = \bar{y}_B\).
APPENDIX E

PROOF FOR PROPOSITION 1

We use proof by contradiction to prove Proposition 1. Assume that the optimal solution belongs to the third case.

Since problem (8) is feasible when \( P_{20} > 0 \), from Corollary 2, it can be obtained that

\[
P_{22}(\epsilon_0/(1 + \epsilon_0)) < P_{\text{max}}.
\]

Thus, from Theorem 3, it can be obtained that:

\[
P_{10,\text{opt}} = F_{10}(y_{\text{opt}}), \quad (87)
\]

\[
P_{20,\text{opt}} = 0, \quad (88)
\]

\[
x_{\text{opt}} = \frac{\epsilon_0}{1 + \epsilon_0}, \quad (89)
\]

\[
y_{\text{opt}} = \bar{y}_B = \frac{\epsilon_0}{1 + \epsilon_0}. \quad (90)
\]

Let

\[
\begin{align*}
P'_{10,\text{opt}} &= \max \left\{ 0, F_{20} \left( \frac{\epsilon_0}{1 + \epsilon_0} \right) \frac{||h_{20}||^2}{||h_{10}||^2} \right. \\
&\quad \left. - \frac{||h_{20}||^2}{||h_{10}||^2} (P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0))) \right\}, \\
P'_{20,\text{opt}} &= P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)), \\
x'_{\text{opt}} &= x_{\text{opt}}, \\
y'_{\text{opt}} &= y_{\text{opt}}.
\end{align*}
\]

It can be easily proved that the tuple \((P'_{10,\text{opt}}, P'_{20,\text{opt}}, x'_{\text{opt}}, y'_{\text{opt}})\) is a feasible solution of problem (8), which belongs to Case I or II, and is better than \((P_{10,\text{opt}}, P_{20,\text{opt}}, x_{\text{opt}}, y_{\text{opt}})\). This contradicts with the assumption that \((P_{10,\text{opt}}, P_{20,\text{opt}}, x_{\text{opt}}, y_{\text{opt}})\) is the optimal solution and the proof is complete.

APPENDIX F

PROOF FOR PROPOSITION 2

The proof for Theorem 2 can be divided into three cases:

- \( x_{\text{opt}} = \bar{x}_B \);
- \( x_{\text{opt}} = \bar{x}_A \);
- \( x_{\text{opt}} = \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2/||h_{10}||^2} \).
(1) When \( x_{\text{opt}} = \tilde{x}_B \):

For this case, we have \( \tilde{x}_B < \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2/||h_{10}||^2} \). Assume that \( P_{10,\text{opt}}, P_{20,\text{opt}} \) and \( x_{\text{opt}} \) is not the optimal solution of problem \( \text{(8)} \). Thus, there must exist a optimal solution \( P'_{10}, P'_{20} \) and \( x' \) such that:

\[
P'_{10} + P'_{20} + P_{11}(x') < P_{20,\text{opt}} + P_{11}(x_{\text{opt}}),
\]

\([92]\)

notag \( P'_{10} > 0, P'_{20} > 0 \). \( [93]\)

Note that, \( x' \) must be smaller than \( \tilde{x}_B \), i.e., \( x_{\text{opt}} \). Besides, \( P'_{10} \) and \( P'_{20} \) should satisfy:

\[
||h_{10}||^2 P'_{10} + ||h_{20}||^2 P'_{20} = ||h_{20}||^2 F_{20}(x'),
\]

\([94]\)

According to \( ||h_{20}|| > ||h_{10}|| \), we have

\[
P'_{10} + P'_{20} > F_{20}(x'),
\]

\([95]\)

thus

\[
(P'_{10} + P'_{20} + P_{11}(x')) - (P_{20,\text{opt}} + P_{11}(x_{\text{opt}})) > (F_{20}(x') + P_{11}(x')) - (P_{20,\text{opt}} + P_{11}(x_{\text{opt}})) > 0
\]

\([96]\)

which is contradict with \([92]\).

(2) \( x_{\text{opt}} = \tilde{x}_A \):

Assume that \( P_{10,\text{opt}}, P_{20,\text{opt}} \) and \( x_{\text{opt}} \) is not the optimal solution of problem \( \text{(8)} \). Thus, there must exist an optimal solution \( P'_{10}, P'_{20} \) and \( x' \) such that:

\[
P'_{10} + P'_{20} + P_{11}(x') < P_{20,\text{opt}} + P_{11}(x_{\text{opt}}).
\]

\([97]\)

Note that, similar to the proof for the case when \( x_{\text{opt}} = \tilde{x}_B \), it can be proved that \( x' \) must satisfy \( x' < \tilde{x}_A \). In that case, \([94]\) also needs to be satisfied. According to the fact that \( F_{20}(x) \) is monotonically decreasing with \( x \), and \( F_{20}(\tilde{x}_A) = P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)) \), the optimal \( P_{20} \) should be \( P'_{20} = P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)) \), thus the following relationship can be obtained:

\[
(P'_{20} + P'_{10} + P_{11}(x')) - (P_{20,\text{opt}} + P_{11}(x_{\text{opt}}))
\]

\([98]\)

\[
= (P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)) + P'_{10} + P_{11}(x')) - (P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)) + P_{11}(x_{\text{opt}})
\]

\[
= \left( \frac{||h_{20}||^2 (P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)))}{||h_{10}||^2} + P'_{10} + P_{11}(x') \right)
\]

\[
- \left( \frac{||h_{20}||^2 (P_{\text{max}} - P_{22}(\epsilon_0/(1 + \epsilon_0)))}{||h_{10}||^2} + P_{11}(x_{\text{opt}}) \right)
\]
Define a new function:

\[ \tilde{G}(x) = \frac{||h_{20}||^2}{||h_{10}||^2} F_{20}(x) + P_{11}(x) \]  

(99)

By taking the derivatives of \( \tilde{G}(x) \), it can be concluded that \( \tilde{G}(x) \) is monotonically decreasing with \( x \) when \( 0 < x \leq \epsilon_0/(1 + \epsilon_0) \), thus

\[ (P'_{20} + P'_{10} + P_{11}(x')) - (P_{20,\text{opt}} + P_{11}(x_{\text{opt}})) > 0, \]  

(100)

which leads to the contradiction.

(3) \( x_{\text{opt}} = \frac{\epsilon_0}{\epsilon_0 + ||h_{20}||^2/||h_{10}||^2} \).

For this case, the optimal solution is the same as the case where there is no power constraint, as shown in Corollary 1. Thus, there is no feasible solutions which are more optimal. [?], [2], [3].

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