Engineering Optimization Method of Orbit Transfer Strategy for All-electric Propulsion Satellites

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Abstract. Nowadays, all-electric propulsion satellites have shown very good performance in missions such as geostationary communication and deep space exploration, and have been widely concerned by major aerospace companies around the world. Because the excellent performance of the electric thruster can enable the satellite to carry more payloads. While the use of electric thrusters for orbit transfer saves fuel consumption significantly, it also leads to the problem of long orbit transfer time. To solve this problem, a simplified low-thrust orbit transfer strategy is designed, which is simple in form, easy to store, while having few parameters to be optimized. In order to optimize the simplified transfer strategy, a bidirectional stochastic gradient descent method suitable for on-board calculation is proposed, with a simple algorithm process and small computation. It can be known from actual calculations that after 600 times of orbit extrapolation, it can converge to the optimal solution. Finally, we can draw a conclusion that the algorithm is feasible and effective by numerical simulation.

1. Introduction
All-electric propulsion satellites have attracted much attention in the international market in recent years, due to the use of electric propulsion systems instead of chemical propulsion systems, all-electric propulsion satellites are able to consume less fuel to obtain the same speed increment[1], due to the excellent specific impulse of the electric thrusters. As a result, all-electric propulsion satellites have greater advantages in terms of payload carrying capacity and in-orbit lifetime[2]. However, the thrust of electric thrusters is generally extremely low, and it takes a considerable time before they can provide large velocity increments. Therefore, the optimization of orbit raising strategy of all-electric propulsion satellites has become a very important problem. The optimal strategy calculation problem is to calculate a temporal sequence of thrust direction changes from the rocket-satellite separation to target orbit, so that the orbit transfer time is the shortest. The obtained temporal sequence usually contains thousands or even tens of thousands of data points recording the optimal thrust direction at different orbit arcs, which is not conducive to on-board storage. Therefore, an engineering simplification of the strategy that is easy to store and convenient to compute is needed.

The orbit transfer optimization of all-electric propulsion satellites belongs to the category of low-thrust orbit transfer optimization problem[3]. At present, the main solutions available include direct method and indirect method. The direct collocation method and shooting method are presented to solve the low-thrust transfer trajectory optimization problem[4-5]. Aziz proposes a direct optimization method that is more applicable for dealing with multi-circle orbit transfer problem[6]. Zhang et al. apply the indirect method to solve the fuel optimal low-thrust transfer problem[7], which is transformed into a two-point boundary value problem by the Pontryagin maximum principle. Mazzini uses the orbit
averaging technology to solve the problem of low-thrust transfer[8]. Meng et al. propose a gradient-based hybrid continuation method[9], which can speed up the solution of nonlinear programming problems related to multiple shooting. Morante et al. develop a hybrid approach to design an orbit optimization method that consists of two steps[10].

Considering the practical engineering application scenarios, such as computation time, storage capacity, and other limitations, there are some relevant studies. A feedback control method based on Lyapunov's guidance law is proposed to implement the on-orbit autonomous orbit transfer of satellites[11-12], providing a feasible guidance solution for practical space missions. Wang and Li et al. present a guidance law based on Global Navigation Satellite System (GNSS) orbit determination[13], which can realize autonomous orbit transfer by obtaining accurate orbit elements through Extended Kalman filter. Zhang et al. propose a closed-loop guidance strategy easy to implement in engineering[14], using polynomial fitting optimal trajectory as the reference orbit stored on the satellite. The above methods consider engineering applications of all-electric propulsion satellites orbit transfer strategies, but less consideration is given to orbit transfer time optimization and geostationary orbit satellite fixation point.

This paper focuses on the on-orbit calculation and optimization of the orbit transfer strategy for GEO all-electric propulsion satellites, with the following main contributions: 1) the characteristics of the optimal strategy for low-thrust orbit transfer are analyzed, based on which a simplified orbit raising strategy for all-electric propulsion satellites is proposed, with few parameters to be optimized and suitable for engineering applications; 2) an on-orbit optimization method based on bidirectional stochastic gradient descent for the strategy previously mentioned is proposed, which is simple and effective, while adapting to the on-orbit calculation conditions. In the end, the feasibility and effectiveness of the algorithm are proved by numerical simulation.

2. Problem Description

2.1. Coordinate systems and dynamic model
This paper involves the J2000 Earth-Centered Inertial (ECI) coordinate frame, the Earth-Centered Earth-Fixed (ECEF) coordinate frame, and the Radial-Transverse-Normal (RTN) reference frame. J2000 Earth-Centered Inertial Coordinate System is defined with the Earth's Mean Equator and Equinox at 12:00 Terrestrial Time on 1 January 2000. The x-axis is aligned with the mean equinox. The z-axis is aligned with the Earth's rotation axis or celestial North Pole and the y-axis is rotated by 90° East about the celestial equator. The Earth-Centered Earth-Fixed Coordinate’s z-axis extends through true north, which does not coincide with the instantaneous Earth rotational axis. The x-axis intersects the sphere of the earth at 0° latitude and 0° longitude. In the RTN reference frame, R is parallel with the radial vector, N is parallel with the orbit normal and T completes the orthonormal frame. These three coordinate systems are shown in figure 1.

![Figure 1. Coordinate systems.](image)

The magnitude of the acceleration produced by the electric thruster acting on the satellite is very small, therefore, it is appropriate to use the Gauss form of the Lagrange Planetary Equation to describe the dynamic model of the all-electric propulsion satellite orbit transfer[15]
where

\[ p = a(1 - e^2), \quad r = \frac{p}{1 + e \cos \theta} \]  

(2)

\( a, e, i, \Omega, \omega, \theta \) is the six classic orbit elements, \( p \) is the semi-focal chord of satellite orbit, \( r \) is the distance from the satellite centroid to the earth's center, \( \mu \) is the earth's gravitational constant. \( F_r, F_i, \) and \( F_n \) are the accelerations generated by the combined force acting on the satellite in the three coordinate axis directions of RTN reference frame respectively, including thrust acceleration and perturbation acceleration.

### 2.2. Problem modeling

Assuming that the initial orbit elements are \( (a_0, e_0, i_0, \Omega_0, \omega_0, \theta_0) \), the orbit elements of target orbit are \( (a_f, e_f, i_f, \Omega_f, \omega_f, \theta_f) \), the acceleration term in the equation can be divided into the acceleration provided by the electric thruster and the perturbation acceleration, that is

\[
F = \frac{T}{m} \mathbf{u} + F_d
\]

(3)

where, \( F = (F_r, F_i, F_n) \) is the combined acceleration vector of the satellite in the RTN reference frame, \( m \) is the mass of the satellite, and \( \mathbf{u} \) is the unit vector of the thrust, \( F_d \) is the perturbation acceleration of the satellite, including the non-spherical gravitational perturbation of the earth, the gravitational perturbation of the sun and the moon, the solar pressure acceleration, and the atmospheric drag acceleration, \( T \) is the magnitude of the combined thrust of the electric thrusters working on the satellite. The conical earth shadow model[16] is used, and the electric thrusters can be ignited when the solar visibility factor is greater than 0.1, otherwise the electric thrusters are switched off.

Assuming that \( \alpha \) represents the angle between the projection of the thrust unit vector \( \mathbf{u} \) on the orbital plane and the R-axis, and \( \beta \) represents the angle between the thrust unit vector \( \mathbf{u} \) and the orbit plane, then \( \mathbf{u} \) can be written as

\[
\mathbf{u} = (\cos \alpha \cos \beta, \sin \alpha \cos \beta, \sin \beta)
\]

(4)

During the orbital maneuver, the mass of the satellite decreases with the consumption of propellant, and the changing rate is

\[
\dot{m} = -\frac{T}{I_{sp}g_0}
\]

(5)

where \( I_{sp} \) is the specific impulse of the electric thruster, and \( g_0 \) is the acceleration due to gravity. The problem is described in mathematical language as follows:
Performance index function

$$\min J = \int_{t_0}^{t_f} \mathrm{d}t$$  \hspace{1cm} (6)

Dynamic equation

$$\begin{cases}
\dot{x} = M \left( \frac{T}{m} \mathbf{u} + \mathbf{F}_d \right) + \mathbf{D} \\
\dot{m} = -\frac{T}{I_{so} g_0}
\end{cases}$$  \hspace{1cm} (7)

where \( x = (a, e, i, \Omega, w, \theta) \), \( M \) in the first differential equation is a 6x3 matrix, and \( D \) is a 6x1 matrix, which is the matrix form description of the Gauss form of the Lagrange Planetary Equation[14].

Initial conditions and terminal conditions

$$\begin{cases}
x(t_0) = x_0, & x(t_f) = x_f \\
m(t_0) = m_0, & m(t_f) \geq m_f
\end{cases}$$  \hspace{1cm} (8)

where \( t_0 \) and \( t_f \) are the initial and end time, \( m_0 \) is the mass in initial orbit, and \( m_f \) is the expected minimum mass of the satellite after the orbit transfer.

3. Engineering optimization method

3.1. Simplified low-thrust orbital transfer strategy

The initial and target orbit elements of the satellite are shown in Table 1, with an initial mass of 1600 kg and carrying two ion electric thrusters equipped with gimbaled booms, which can generate a combined thrust force of 400 mN and a specific impulse of 2000 s, taking into account the effects of J2 perturbation and the earth’s shadow.

Table 1. The initial and target orbital elements of the satellite.

| Orbit elements                  | Initial value | Target value |
|---------------------------------|---------------|--------------|
| Semi-major axis(km)             | 17169.8       | 42165        |
| Eccentricity                    | 0.6087        | 0            |
| Orbit inclination(°)            | 28.5          | 0            |
| Right ascension of the ascending node(°) | 0             | Unconstrained |
| Argument of perigee(°)          | 0             | Unconstrained |
| True anomaly(°)                 | 180           | Unconstrained |

The curve of thrust direction angle solved using the indirect method proposed in [17] is shown in figure 1.
The optimal transfer time is 138.1 days. The curve of the thrust direction angle $\alpha$ in the orbit plane with true anomaly and the curve of the thrust direction angle $\beta$ out of the orbital plane with true anomaly and $u$ ($u$ is the sum of true anomaly and argument of perigee) are shown in figure 2, where the red curves represent the early stage of orbit transfer and the blue part represents the end of orbit transfer, and each individual curve represents one orbit cycle. The entire orbit transfer process has experienced 271 orbit periods in total.

Analyzing figures 1 and 2, the thrust direction angles $\alpha$ and $\beta$ can be summarized as follows:

1) $\alpha$ is mainly distributed around 90 in the early stage of the orbit transfer;
2) In the late stage, $\alpha$ changes continuously with the true anomaly, and it is $-90^\circ$ at perigee and $90^\circ$ at apogee. The shape of the curve is close to a sine curve.
3) In an orbit period, $\beta$ is negative in the ascending arc, positive in the descending arc, reaches the extreme value at the ascending node and descending node, and is 0 when $u$ is $90^\circ$ and $270^\circ$.
4) $\beta$ changes continuously and periodically with $u$, and changes symmetrically with respect to the descending node.

Therefore, combining the above analysis and engineering implementation requirements, considering the strategy as simple and feasible as possible, the change strategy of angle $\alpha$ and $\beta$ in each orbit period can be designed to shorten the orbit transfer time as follows:

\[
\alpha = \begin{cases} 
\frac{\pi}{2}, a \leq a_c \\
\frac{\pi}{2}(1 + 2\cos\left(\frac{\theta}{2}\right)), a > a_c 
\end{cases}
\]  
\tag{9}

\[
\beta = \frac{\pi}{2}b\cos(u)
\]  
\tag{10}

Among them, $a_c$ is the demarcation point of the orbit transfer stage. $u = \omega + \theta$, and $b$ is the amplitude of $\beta$, which mainly affects the rate of decrease of inclination.

In order to simplify the calculation, the parameters $a_c$ and $b$ to be optimized are designed to be constant values in the orbit transfer strategy. By optimizing $a_c$ and $b$, the sub-optimal simplified strategy can be obtained.

The performance index function is redesigned to reflect the orbit transfer time and the distance between the satellite and the target orbit at the end of the orbit transfer. The design is as follows...
\[
\min J = \int_0^T \left[ \frac{a(t) - a_f}{a_f} + \left( e(t) - e_f \right)^2 + \left( i(t) - i_f \right)^2 \right] dt + \\
\zeta \left\{ \left[ \frac{a(t_f) - a_f}{a_f} \right] + \left[ e(t_f) - e_f \right]^2 + \left[ i(t_f) - i_f \right]^2 \right\}
\]

(11)

where \( \zeta \) is the gap factor, which is used to reflect the distance from the target orbit at the end of the transfer stage.

The entire orbit transfer process can be encapsulated into a function whose input is the variable to be optimized and output is the performance index, as follows

\[
J = f(a, b)
\]

(12)

The function is iterated through fast orbit extrapolation. When it reaches the target orbit or the state exceeds the set boundary, the iteration is considered to be over, and the performance index is calculated and output.

4. Bidirectional stochastic gradient descent method

Due to the influence of the shadow area and perturbation, it is difficult to obtain the analytical formula for the gradient of the satellite’s orbit elements. Therefore, it is not suitable to adopt the traditional numerical optimization method. Intelligent optimization methods can be used to optimize the problems in this paper. However, general intelligent optimization methods, such as evolutionary algorithms and swarm intelligence algorithms, need to generate a large number of individuals and perform multiple iterations, which requires a large amount of calculation. Therefore, this paper designs an intelligent optimization algorithm based on gradient information, which can achieve effective optimization through a small amount of calculation, and is suitable for on-board computing.

The idea of the algorithm is derived from the artificial fish swarming algorithm\[18\], which has been significantly modified to be applicable to on-board computation. This algorithm uses a single individual seeking for optimization and performs only the act of foraging. A point \( x^k \) is randomly initialized, and the value of the performance index at that point is calculated

\[
J^k = f(x^k)
\]

(13)

In order to improve the exploration efficiency, the exploration direction \( d_{rand} \) will be randomly generated to explore the front and back of the current point

\[
\begin{align*}
    x_f^k &= x^k + s_{ex} d_{rand} \\
    x_b^k &= x^k - s_{ex} d_{rand}
\end{align*}
\]

(14)

\[
\begin{align*}
    J_f^k &= f(x_f^k) \\
    J_b^k &= f(x_b^k)
\end{align*}
\]

(15)

where \( s_{ex} \) is the exploration step length.

Move to the direction where the gradient drops more or rises less, the motion direction is

\[
d_{move} = \begin{cases} 
    1, & J_f^k < J_b^k \\
    -1, & J_f^k > J_b^k \\
    rand, & J_f^k = J_b^k
\end{cases}
\]

(16)

where \( d_{move} \) represents the direction of motion, whose value is 1 for forward, -1 for backward, and \( rand \) for random forward or backward motion.

Generate the position of the next point based on the motion and exploration direction

\[
x^{k+1} = x^k + s_{move} d_{rand}
\]

(17)
where $s_{\text{move}}$ is the motion step length.

When $J_t^k > J_t^i \cap J_t^i > J_t^j$, it means that there may be a minimum point within the range of the exploration step, so a shorter step is used for random exploration of this point

$$x_{\text{ex}}^k = x^k + \frac{d_{\text{rand}}}{2}$$

(18)

where $d_{\text{rand}}$ is the random direction, $x_{\text{ex}}^k$ is the exploration point, and $J_{\text{ex}}^k$ is used to update the historical optimal value of the performance index and its position. This process only explores possible minimum points, and updates the historical optimal value without affecting the motion process of $x^k$.

Record and update the historical optimal value $J_{\text{min}}$ and its position $x_{\text{min}}$ at each step

$$J_{\text{min}} = \min(J_{\text{min}}, J_{\text{ex}}^k, J_{\text{ex}}^i, J_{\text{ex}}^j)$$

(19)

To avoid falling into local optimum, when the performance index value changes less than a certain range at each step, the historical optimum is compared with the current position performance index value, and if the historical optimum is smaller, a trend of movement toward the historical optimum is added

$$x_{k+1} = x_{k+1} + s_h \frac{x_{\text{min}} - x^k}{\|x_{\text{min}} - x^k\|}$$

(20)

The exploration and motion step length decay at the end of each step

$$s_{\text{ex}} = \lambda s_{\text{ex}}, s_{\text{move}} = \gamma s_{\text{move}}$$

(21)

where $\lambda$ and $\gamma$ is the decay factor. The specific flow chart of the algorithm is shown in figure 4.

![Figure 4. Algorithm flow chart.](image-url)
5. Simulation result
The satellite’s initial and target orbit parameters are the same as those set in Table 1, and its mass characteristics, thrust, and specific impulse are the same as those described in the previous section. The earth's non-spherical gravitational perturbation is set to 4th order, the solar light pressure reflectivity is 1, the light pressure equivalent area is 20 m², the atmospheric drag coefficient is 2.2, and the windward area is 20 m². $s_{ex}$, $s_{move}$, $s_{h}$, $\lambda$, $\gamma$, $\epsilon$ are set to 0.1, 0.3, 0.01, 0.99, 0.95, 0.001 respectively, and the simulation results are shown in figure 5-8 as follows.

![Figure 5. Curve of performance index.](image1)

![Figure 6. Trajectory of satellite orbit transfer.](image2)

![Figure 7. Curve of orbit elements with time.](image3)

![Figure 8. Curve of thrust direction with time.](image4)

The entire orbit transfer process took a total of 147.5 days. It can be seen from Figure 5 that after 100 iterations, the performance index is close to the optimal solution. Compared with the theoretical optimal solution obtained by the indirect method in the previous article, the optimal time of the orbit transfer simplified by engineering is prolonged by 6.8%. The entire calculation process has been iterated for 200 steps and experienced 600 orbital extrapolations.

6. Conclusion
This paper studies the engineering implementation and optimization methods of the all-electric propulsion satellites’ orbit transfer strategy. A sinusoidal function is used to fit the change of the thrust direction angle of each circle of the orbit, which is simple in form and easy to store. At the same time, in order to optimize the strategy as much as possible, bidirectional stochastic gradient descent method is designed, which uses less calculation to make the orbit transfer process the shortest time. Through
calculation and numerical simulation, a simplified orbit-changing strategy that can be implemented in engineering is obtained, which is close to the theoretical optimal solution, and the time is only 6.8% longer than the optimal solution. In the future, the robustness of the orbit transfer strategy in some poor engineering environments will be studied, such as unstable thrust and errors in the orbit determination of satellite. Research how to make the simplified strategy have better robustness while ensuring the optimal situation as much as possible.

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