CPT, DISSIPATION, AND ALL THAT

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Abstract
A phenomenological paradigm for the study of CPT-violating effects in the neutral kaon system is presented. Besides the familiar direct and indirect breakings, it encodes possible phenomena leading to irreversibility and dissipation, that could originate from quantum effects at Planck’s scale. These new effects can be experimentally probed with great accuracy, in particular at φ-factories.
1. INTRODUCTION

One of the most striking properties that can be derived from the general structure of quantum field theory is the invariance under the combination of charge conjugation ($C$), parity reflection ($P$), and time reversal ($T$) transformations. This result is based on a rigorous formulation of the theory, that incorporates in form of assumptions (the Wightman axioms) very general physical requirements, that any sensible quantum field model should satisfy; these include: existence of a (cyclic) vacuum state, positivity of energy (spectral condition), covariance under the action of the restricted Poincaré group.

A further condition is necessary in order to guarantee the validity of Einstein’s causality; it is usually called locality (or microcausality) and essentially states that the field variables commute (or anticommute) at space-like separated points. Actually, this assumption can be substituted by the weak locality condition, a much less restrictive requirement: it states the commutativity in the two-point (more in general, $n$-point) correlation functions for space-like separated points. Together with the (Wightman) axioms mentioned before, this very mild assumption is equivalent to the invariance under $CPT$ transformations: this is the content of the $CPT$ theorem. (For a precise formulation and discussions, see [1-3].)

In view of this result, $CPT$-invariance seems unavoidable in any sensible model based on the relativistic quantum theory of fields. Since our present description of elementary particle physics phenomena is based on the standard model (and field extensions of it), the symmetry under $CPT$ transformations appears automatic, at least from the theoretical point of view.

This situation can be summarized by saying that $CPT$-invariance is almost a “kinematical” property of relativistic quantum field theory.[4] In view of this, any experimental observation of a violation of the $CPT$-symmetry would clearly represent a strong evidence of unconventional physics, and signal the need of a profound modification of the present interpretation of high energy phenomena in terms of local, relativistic fields.

General consequences of $CPT$-invariance are the equality of mass and lifetime of particles and their relative antiparticles; and indeed, most of the experimental effort in testing the symmetry under $CPT$ transformations has been devoted to check this equality for various elementary particles.[5] Although some of the quoted bounds coming from these tests look impressive (in particular, the one involving the $K^0$-$\bar{K}^0$ mass difference), one should keep in mind that they can be translated into direct bounds on possible $CPT$-violating effects only with the help of additional considerations.[6-9]

In order to improve the present experimental limits on $CPT$-invariance, some theoretical guidance is clearly needed; in particular, one should provide indications on what phenomena should be looked at and consequently on what kind of experiment should be carried out and at which accuracy. In this respect, the best solution would be the construction of a microscopic theory, valid at the fundamental level; this would then allow the calculation from first principles of the phenomenological parameters, with predictions about size and quality of the various signals, and further permit a direct comparison of the results coming from different experiments.

Unfortunately, such a predictive theory does not yet exist; nevertheless, various treatments of $CPT$-violating effects have been proposed in the literature. Although inspired by more fundamental dynamics (strings, branes), they are all phenomenological in nature.
Despite their limited predictive power, they can be of great help in analyzing data from present and future experiments.

2. TOWARDS A UNIFIED PARADIGM FOR CPT-VIOLATION

As already said, within standard quantum field theory there seems to be little space for interactions violating CPT-symmetry. However, there are technical assumptions in the CPT theorem that, once released, could open the possibility to CPT-violating effects.

One of these implicit conditions requires the field variables to have a finite number of components, and therefore to transform according to finite-dimensional representations under the action of the restricted Lorentz group. Thus, infinite component field models can violate CPT-invariance.[10]

The first example was presented in the early developments of quantum field theory: indeed, Majorana wrote down a model of infinite-dimensional wave equation without negative-energy solutions of non-negative square mass, i.e. without antiparticles.[11] Since then, other more realistic theories have been constructed† and their quantization studied.[13, 14] Unfortunately, these models either have an infinite mass degeneracy with respect to spin, or violate locality. More precisely, one can show that (composite) systems with an infinite degeneracy in the mass spectrum do not admit a simple description in terms of local fields, even with infinite components; therefore, the correct treatment of these models requires either the weakening of the assumption of strict locality, or the replacement of the field variables by some sort of extended objects.[10]

This brings up another possible way to circumvent the assumptions of the CPT theorem: construct field theories which present some sort of non-locality. This possibility is not as exotic as it might appear at first: after all, we are quite accustomed to the quantization of Yang-Mills theories in non-local (Coulomb like) gauges; furthermore, a rigorous treatment of quantum gauge theories might even require a reformulation of the Wightman axioms. In any case, the non-local properties that these theories contain are generally not enough to produce CPT-violating effects.[3]

Nevertheless, non-local field models without CPT-symmetry have been explicitly constructed; they are Lorentz covariant and are quantized in the standard way, but contain non-local field operators, due to an unconventional interplay between charge conjugation and half-integer spin.[15] Unfortunately, the kind of multiplets these theories describe does not seem part of our phenomenological world.

As mentioned before in connection with infinite component fields, the conclusions of the CPT theorem can be avoided by considering models not formulated in terms of field variables. String theories have emerged as the most promising candidates for unifying all

† The explicit construction of these models also clarifies a point often misunderstood in the literature: CPT-invariance and the spin-statistics connection are independent and separate issues. For a recent discussion, see [12].
fundamental interactions, gravity included; it is therefore of high interest to investigate whether CPT-invariance holds or not in such models. Indeed, these theories involve extended objects as fundamental dynamical variables, which in the low energy regime reduce to an infinite tower of field-like degrees of freedom. Therefore, at least some of the requirements that makes CPT-invariance unavoidable in quantum field theories do not hold anymore.

Despite these considerations, the general structure of the theory could easily accommodate CPT-invariance,[16] and explicit investigations also support this possibility.[17] However, the knowledge of the vacuum structure of string theory is not precise enough to draw definite conclusions. Indeed, it has been proposed that in some string model a mechanism for spontaneous breaking of Lorentz invariance can occur and, as a consequence, lead to possible CPT-violating effects.[18] This phenomenon could originate from certain type of interactions among tensor fields that are known to be present in string field theory; if these interactions destabilize the vacuum and produce non-trivial expectation values for some of these tensor fields, then Lorentz symmetry is spontaneously broken.

Even assuming that this mechanism is really at work, it is nevertheless difficult to predict its effects in the low energy regime, at the level of particle physics phenomena. At the present stage of string theory knowledge, only an effective, phenomenological recipe can be given: add by hand to the Lagrangian of the standard model CPT and Lorentz violating terms, with the additional requirement of maintaining as much as possible its fundamental properties, like gauge invariance and perturbative renormalizability.[19-21] In this way, one obtains a phenomenological extension of the standard theory that, although inspired by the fundamental theory of strings, is effectively formulated in terms of ordinary field theory. The model contains many unconstrained parameters, so that in this formalism full predictivity is lost; nevertheless, at least to a certain extent, one can still relate and classify CPT-violating effects in different elementary particle systems.

For instance, the fermion sector of the standard model Lagrangian can be augmented by terms of the form:

$$\mathcal{L}_f^{CPT} = v^\mu \bar{\psi} \gamma_\mu \psi + a^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi ;$$

(2.1)

similarly, to the gauge sector one can add Chern-Simons like terms (for simplicity, in the abelian case):

$$\mathcal{L}_g^{CPT} = p^\mu \epsilon_{\mu\nu\lambda\rho} A^\nu F^{\lambda\rho} .$$

(2.2)

In these expressions, $v^\mu$, $a^\mu$ and $p^\mu$ are (dimensionfull) phenomenological constants; they are fixed vectors in space-time (i.e. constant external fields), so that invariance under (active) Lorentz and CPT transformations is broken by hand.

The new, additional pieces in (2.1) and (2.2) produce modifications of various observables in many physical situations; the experiment can therefore provide sensible bounds at least on some of the components of $v^\mu$, $a^\mu$ and $p^\mu$. In particular, $v^\mu$ can be constrained by experiments on neutral mesons systems,[22] while stringent bounds on $p^\mu$ are given by the observation of radio waves coming from sources at cosmological distance.[19]

The dynamics of extended objects (strings and branes) could give origin to other CPT-violating phenomena, that can not be described in terms of simple modifications of the basic standard model Lagrangian, as in (2.1) and (2.2): these effects are thus distinct
from those discussed so far. The basic idea is that, due to quantum fluctuations and the appearance of virtual black holes, space-time should be topological non-trivial at Planck’s scale, and therefore it should be properly described in terms of a “foam” structure;[23] as a consequence, this could lead to possible loss of quantum coherence, which in turn could be accompanied by \( CP \) and \( CPT \) violating effects.[24-29]

The proper framework to analyze these phenomena is that offered by the study of open quantum systems.[30-32] Quite in general, these systems can be modeled as being subsystems \( S \) in interaction with suitable, large environments \( E \). The total system \( S + E \) is closed and therefore its dynamics can be analyzed using standard quantum mechanical techniques: it is realized by unitary operators, \( U(t) = e^{-iH_{\text{tot}}t} \), generated by the total hamiltonian \( H_{\text{tot}} \), the sum of the hamiltonian \( H_S \) describing the dynamics of \( S \) in absence of the environment, the hamiltonian \( H_E \) of \( E \) and the interaction hamiltonian \( H_{SE} \).

Nevertheless, the time evolution of the subsystem \( S \) alone, obtained by eliminating the environment degrees of freedom, usually develops dissipation and irreversibility.

In the language of density matrices, the reduced dynamics is obtained by tracing over the environment variables:

\[
\rho_S(0) \rightarrow \rho_S(t) = \text{Tr}_E \left[ U(t) \rho_{S+E} U^\dagger(t) \right],
\]

where the density matrix \( \rho_S \) describes the state of \( S \), while \( \rho_{S+E} \) represents the initial state of the total system; in absence of initial correlations between \( S \) and \( E \), a situation commonly encountered in many physical applications, it can be written in factorized form:

\[
\rho_{E+S} = \rho_S \otimes \rho_E.
\]

In general, the dynamics given in (2.3) is very complicated and can not be explicitly described; however, when the interaction between system and environment is “weak”, a condition usually very well satisfied in practice, it becomes free from memory effects and can be realized in terms of linear maps. In addition, these transformations are seen to satisfy very basic physical requirements, like forward in time composition (semigroup property), probability conservation, entropy increase, complete positivity. They form a so-called quantum dynamical semigroup.

This description is very general and has been applied to successfully model a large variety of physical situations; in particular, it has been used to study quantum statistical systems,[30-32] to analyze dissipative effects in quantum optics,[33-35] to describe the interaction of a microsystem with a macroscopic measuring apparatus.[36, 37] As mentioned before, it is also the natural framework to study phenomena leading to violation of \( CPT \)-symmetry induced by dissipative effects.[38]

More in general, effective time evolutions based on quantum dynamical semigroups can also accommodate more “standard” \( CPT \)-breaking terms, of the kind induced by the terms in (2.1) and (2.2);[39, 40] a unified, phenomenological description of \( CPT \)-violating phenomena is therefore at hand.
3. PHENOMENOLOGY OF CPT-VIOLATING EFFECTS IN THE NEUTRAL KAON SYSTEM

One of the most promising venues to look for CPT-violating effects is certainly the system of neutral kaons: the sophistication of present (and planned) kaon experiments is so high that systematic and very accurate searches are possible. Although a predictive, microscopic theory for such effects is presently unavailable, a phenomenological approach based on quantum dynamical semigroups can nevertheless provide a sufficiently general framework for a meaningful interpretation of such non-standard phenomena.

The evolution and decay of the neutral kaon system can be effectively modeled by means of a two-dimensional Hilbert space.[41] The states of definite CP,

\[ |K_1\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle + |\bar{K}^0\rangle \right], \quad |K_2\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle - |\bar{K}^0\rangle \right], \quad (3.1) \]

constitute an orthonormal basis in this space. In this basis, a kaon state can be conveniently described by means of a density matrix \( \rho \),

\[ \rho = \begin{pmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{pmatrix}, \quad \rho_4 = \rho_3^*; \quad (3.2) \]

this is a positive hermitian operator, i.e. with real, positive eigenvalues, and constant trace (in case of unitary evolutions).

The evolution in time of the kaon system can then be formulated in terms of a linear equation for \( \rho \); it takes the general form:[38, 39]

\[ \frac{\partial \rho(t)}{\partial t} = -iH_{\text{eff}} \rho(t) + i\rho(t)H_{\text{eff}}^\dagger + L[\rho(t)] . \quad (3.3) \]

The first two terms on the r.h.s. of this equation are the standard quantum mechanical ones: they contain the effective hamiltonian \( H_{\text{eff}} = M - i\Gamma/2 \), which includes a non-hermitian part, characterizing the natural width of the kaon states.

The entries of this matrix can be expressed in terms of the complex parameters \( \epsilon_S \), \( \epsilon_L \), appearing in the eigenstates of \( H_{\text{eff}} \),

\[ |K_S\rangle = \frac{1}{(1 + |\epsilon_S|^2)^{1/2}} \begin{pmatrix} 1 \\ \epsilon_S \end{pmatrix}, \quad |K_L\rangle = \frac{1}{(1 + |\epsilon_L|^2)^{1/2}} \begin{pmatrix} \epsilon_L \\ 1 \end{pmatrix}, \quad (3.4) \]

and the four real parameters \( m_S, \gamma_S \) and \( m_L, \gamma_L \), the masses and widths of the states in (3.4), characterizing the eigenvalues of \( H_{\text{eff}} \): \( \lambda_S = m_S - \frac{i}{2}\gamma_S \), \( \lambda_L = m_L - \frac{i}{2}\gamma_L \).[42] It proves convenient to use also the following positive combinations: \( \Delta \Gamma = \gamma_S - \gamma_L \), \( \Delta m = m_L - m_S \), as well as the complex quantities: \( \Gamma_\pm = \Gamma \pm i\Delta m \) and \( \Delta \Gamma_\pm = \Delta \Gamma \pm 2i\Delta m \), with \( \Gamma = (\gamma_S + \gamma_L)/2 \). One easily checks that CPT-invariance is broken when \( \epsilon_S \neq \epsilon_L \), while a nonvanishing \( \epsilon_S = \epsilon_L \) implies violation of CP symmetry. For these reasons, in order to parametrize these so-called “indirect” violations, the constants \( \epsilon_K = (\epsilon_S + \epsilon_L)/2 \) and \( \delta_K = (\epsilon_S - \epsilon_L)/2 \) are sometimes introduced.
The various paradigms for CPT-violation mentioned in the previous section can provide, at least in principle, some informations on these parameters. For instance, the effective approach to CPT-violation motivated by a possible “stringy” spontaneous symmetry breaking mechanism can have some bearings on the parameter $\delta_K$, via contributions induced by the first term in (2.1); it predicts a dependence of $\delta_K$ on differences of the parameters $v^\mu$ pertaining to different quark flavours and on the kaon 4-velocity.\cite{22}

On the other hand, the additional piece $L[\rho]$ in the evolution equation (3.3) takes care of a different kind of phenomena leading to possible violations of CP and CPT symmetries; it encodes effects leading to dissipation and irreversibility that cannot be reproduced by the standard quantum mechanical treatments, like those just mentioned.

The explicit form of the linear map $L[\rho]$ can be uniquely fixed by taking into account the basic physical requirements that the complete time evolution, $\lambda_t : \rho(0) \mapsto \rho(t)$, generated by (3.3) needs to satisfy; the one-parameter family of maps $\lambda_t$ must transform density matrices into density matrices and have the properties of increasing the von Neumann entropy, $S = -\text{Tr}[\rho \ln \rho]$, of obeying the semigroup composition law, $\lambda_t[\rho(t')] = \rho(t + t')$, for $t, t' \geq 0$, of being completely positive. If one expands the $2 \times 2$ matrix $\rho$ in terms of Pauli matrices, $\sigma_1, \sigma_2, \sigma_3$, and the identity, $\sigma_0$, $\rho = \sigma_\mu \rho_\mu$, $\mu = 0, 1, 2, 3$, $L[\rho]$ can be represented by a symmetric $4 \times 4$ matrix $[L_{\mu\nu}]$, acting on the column vector with components $(\rho_0, \rho_1, \rho_2, \rho_3)$. It can be parametrized by the six real constants $a, b, c, \alpha, \beta, \text{ and } \gamma:\cite{39}$

$$[L_{\mu\nu}] = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix},$$

with $a, \alpha$ and $\gamma$ non-negative. These parameters are not all independent; to assure the complete positivity of the time-evolution generated by (3.3), they have to satisfy the following inequalities:

$$2 R \equiv \alpha + \gamma - a \geq 0, \quad RS - b^2 \geq 0,$$

$$2 S \equiv a + \gamma - \alpha \geq 0, \quad RT - c^2 \geq 0,$$

$$2 T \equiv a + \alpha - \gamma \geq 0, \quad ST - \beta^2 \geq 0,$$

$$RST - 2bc\beta - R\beta^2 - Sc^2 - Tb^2 \geq 0.$$  \hspace{1cm} (3.6)

As already mentioned, although the basic general idea behind such an approach to the kaon dynamics is that quantum phenomena at Planck’s scale produce loss of phase-coherence, it should be stressed that the form (3.3), (3.5) of the kaon time-evolution is quite independent from the microscopic mechanism responsible for the dissipative effects; indeed, in view of the properties mentioned above, the evolution of any quantum irreversible process can be effectively modeled using quantum dynamical semigroups. In this respect, the equations (3.3), (3.5) should be regarded as phenomenological in nature, and therefore quite suitable to experimental test: any signal of non-vanishing value for some of the parameters in (3.5) would certainly attest in a model independent way the presence of non-standard, dissipative effects in kaon physics.

Among the physical requirements that the evolution (3.3), (3.5) satisfies, complete positivity is perhaps the less intuitive. Indeed, it has not been enforced in many analysis,
in favor of the more obvious simple positivity. Simple positivity is in fact generally enough
to guarantee that the eigenvalues of the kaon density matrix \( \rho(t) \) remain positive at any
time; this requirement is obviously crucial for the consistency of the formalism, in view of
the interpretation of the eigenvalues of \( \rho(t) \) as probabilities.

Complete positivity is a stronger property, since it assures the positivity of the density
matrix describing the states of a larger system, obtained by coupling in a trivial way the
neutral kaon system with another arbitrary finite-dimensional one. Although trivially
satisfied by standard quantum mechanical (unitary) time-evolutions, the requirement of
complete positivity seems at first a mere technical complication. Nevertheless, it turns
out to be essential in properly treating correlated systems, like the two neutral kaons
coming from the decay of \( \phi \)-meson (a common physical situation at \( \phi \)-factories); it assures
the absence of unphysical effects, like the appearance of negative probabilities, that could
occur for just simply positive dynamics.[43]

An example is provided by the extended kaon dynamics originally discussed in [24], and
further developed in [44, 45]; there, the generator of the dissipative part of the evolution
is taken of the form (3.5), but with \( a = b = c = 0, \alpha \neq \gamma, \) and \( \alpha \gamma \geq 2 \), to assure simple
positivity. One can check that the total time evolution is no longer completely positive: the
inequalities (3.6) are clearly violated; and indeed, in the case of correlated kaons, one can
easily evidentiate physical inconsistencies for such simplified dynamics: some expectation
values of the corresponding two-kaon density matrix become negative.[43] The only way to
avoid this serious drawback is to restore complete positivity, by letting \( \alpha = \gamma \) and \( \beta = 0, \)
a consequence of (3.6) for a vanishing \( a. \)† Let us stress that these considerations are not
purely academic: the need of complete positivity in the neutral kaon dynamics can be
actually probed using experimental set-ups at \( \phi \)-factories (for a complete discussion, see
[48]).

Physical observables of the neutral kaon system can be computed from the density
matrix \( \rho(t) \); its time evolution is given as a solution of the equation (3.3), (3.5) for a given
initial state \( \rho. \) On the basis of rough dimensional estimates,[44, 45, 38] the parameters
\( a, b, c, \alpha, \beta \) and \( \gamma \) appearing in (3.5) are expected to be very small (at most of order
\( m_K^2/M_P \sim 10^{-19} \) GeV, with \( m_K, M_P \) the kaon and Planck’s mass), roughly of the same
order of magnitude of \( \epsilon_S \Delta \Gamma \) and \( \epsilon_L \Delta \Gamma \); therefore, one can use an expansion in all these
small parameters, so that approximate expressions for the entries of \( \rho(t) \) can be explicitly obtained.[39, 40] In particular, one can work out the contributions \( \rho_L \) and \( \rho_S \) that
correspond to the physical “long lived” and “short lived” neutral kaon states:

\[
\rho_L = \left[ \left( \epsilon_L + \frac{2C}{\Delta \Gamma} \right)^2 + \frac{\gamma}{\Delta \Gamma} - 8 \left( \frac{C}{\Delta \Gamma} \right)^2 - 4 \Re \left( \frac{\epsilon_L C}{\Delta \Gamma} \right) \epsilon_L + \frac{2C}{\Delta \Gamma} \right] \left( \epsilon_L + \frac{2C}{\Delta \Gamma} \right),
\]

(3.7a)

† As a result of recent investigations on the effective D-brane dynamics, a non-linear
time evolution is suggested as a possible alternate way of avoiding these pathologies.[46]
in this respect, we note that the correct treatment of non-linear dynamics for correlated
systems is still an unsolved problem.[47]
and

\[
\rho_S = \left[ \begin{array}{cc}
1 & \epsilon_S^* + \frac{2C^*}{\Delta T^+} \\
\epsilon_S + \frac{2C}{\Delta T^+} & \epsilon_S + \frac{2C}{\Delta T^+} - \frac{\gamma}{\Delta T^+} - 8 \left| \frac{C}{\Delta T^+} \right|^2 - 4 \Re \left( \frac{2C^*}{\Delta T^+} \right) \end{array} \right],
\] (3.7b)

where

\[
C = c + i \beta .
\] (3.8)

Note that the states \( \rho_L \) and \( \rho_S \) are mixed: the matrices (3.7) are not projectors; only in absence of the contribution in (3.5) one recovers the matrices \( |K_L\rangle\langle K_L| \) and \( |K_S\rangle\langle K_S| \).

As a further remark, let us point out that in writing down the evolution equation (3.3), an arbitrary phase convention for the states \( |K^0\rangle, |\overline{K}^0\rangle, \) and \( |K_S\rangle, |K_L\rangle \) has been implicitly fixed. We remark that physical observables, being the result of a trace operation (see below), are by definition independent from any phase convention. Provided a complete set of observables are computed, one can always consistently re-express a result obtained in a given phase convention, into any other phase choice. Nevertheless, once a phase choice is adopted and a given approximation used, one should consistently stick within this convention, paying attention not to generalize conclusions drawn from only a limited number of physical observations. In particular, as already observed, the constants \( a, b, c, \alpha, \beta \) and \( \gamma \) appearing in the dissipative part (3.5) of the dynamical equation (3.3) are expected to be tiny. Since the dependence of the map \( L[\rho] \) on these parameters is linear, the latter, being small in one phase convention, remain small in any other phase choice. As a consequence, the generality of the discussion in the next sections is by no mean affected by the selection of any particular phase convention.

4. SINGLE NEUTRAL KAON OBSERVABLES

The evolution equation (3.3), (3.5) provides the most general linear dynamics for the neutral kaon system, encoding \( CP \) and \( CPT \)-violating effects. These are the so-called “indirect” effects: further “direct” violations can occur. They are related to specific observables, typically those associated with the decays of the kaons into pion states, and into semileptonic states. In the language of density matrices, these final decay states are described by suitable hermitian operators; taking the trace of these operators with \( \rho(t) \) allows computing the explicit time dependence of various experimentally relevant observables.[44, 45, 39, 40, 49]

For instance, the operators \( O_{+-} \) and \( O_{00} \) that describe the \( \pi^+\pi^- \) and \( 2\pi^0 \) final states have the form:

\[
O_{+-} \sim \left[ \begin{array}{cc}
1 & r_{+-}^* \\
r_{+-} & \left| r_{+-} \right|^2
\end{array} \right], \quad O_{00} \sim \left[ \begin{array}{cc}
1 & r_{00}^* \\
r_{00} & \left| r_{00} \right|^2
\end{array} \right].
\] (4.1)

To lowest order in all small parameters, the complex constants \( r_{+-} \) and \( r_{00} \), can be written as:

\[
r_{+-} = \varepsilon - \epsilon_L + \epsilon', \quad r_{00} = \varepsilon - \epsilon_L - 2\epsilon',
\] (4.2)
where the parameters $\varepsilon$ and $\varepsilon'$ take the familiar expressions:[42]

$$
\varepsilon = \left[ \frac{\varepsilon_L + \varepsilon_S}{2} + i \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right] + \left[ \frac{\varepsilon_L - \varepsilon_S}{2} + \frac{\text{Re}(B_0)}{\text{Re}(A_0)} \right],
$$

(4.3)

and

$$
\varepsilon' = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \left[ \frac{\text{Im}(A_2)}{\text{Re}(A_0)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right] + i \left[ \frac{\text{Re}(B_0)}{\text{Re}(A_0)} - \frac{\text{Re}(B_2)}{\text{Re}(A_0)} \right].
$$

(4.4)

In these formulas, the $s$-wave phase shifts $\delta_i$ and the complex amplitudes $A_i, B_i, i = 0, 2$, have been introduced; they appear in the usual parametrization of the amplitudes for the decay of a neutral kaon into two pions:

$$
\mathcal{A}(K^0 \to \pi^+\pi^-) = (A_0 + B_0) e^{i\delta_0} + \frac{1}{\sqrt{2}} (A_2 + B_2) e^{i\delta_2},
$$

(4.5a)

$$
\mathcal{A}(K^0 \to 2\pi^0) = (A_0 + B_0) e^{i\delta_0} - \sqrt{2} (A_2 + B_2) e^{i\delta_2};
$$

(4.5b)

the indices 0, 2 refers to the total isospin. (The amplitudes for the $\bar{K}^0$ decays are obtained from (4.5) with the substitutions: $A_i \rightarrow A_i^*$ and $B_i \rightarrow -B_i^*$. ) The imaginary parts of $A_i$ signals direct $CP$-violation, while a non zero value for $B_i$ will also break $CPT$-invariance.

As a consequence, in the expressions (4.3) and (4.4), the first, second square brackets contain the $CP$ and the complex amplitudes $A_i$, respectively $CPT$, violating parameters arising both from the effective hamiltonian $H_{\text{eff}}$, as well as from their decay amplitudes.

Results similar to those presented in (4.1) are obtained for the matrices $O_{\pi^+\pi^-\pi^0}, O_{3\pi^0}, O_{3\pi^0}$, and $O_{\ell^-}$ that describe the decays into $\pi^+\pi^-\pi^0, 3\pi^0, \pi^+\ell^-\bar{\nu}$ and $\pi^-\ell^+\nu$; for explicit expressions, see Refs.[40, 49].

With the help of these matrices, one can compute the time-dependence of various useful observables of the neutral kaon system. For example, in the case of charged pions, the decay rate for a pure $K^0$ initial state $\rho_{K^0}$ is:

$$
R_{++}(t) = \frac{\text{Tr} \left[ \rho_{K^0}(t) O_{++} \right]}{\text{Tr} \left[ \rho_{K^0}(0) O_{++} \right]},
$$

(4.6)

$$
e^{-\gamma_{L} t} + R_{L}^{L} e^{-\gamma_{L} t} + 2 e^{-\Gamma t} |\eta_{++}| \cos(\Delta m t - \phi_{++}) ,
$$

where $R_{L}^{L}$ is the two-pion decay rate for the $K_L$ state:

$$
R_{L}^{L} = \left| \varepsilon_L + \frac{2C^*}{\Delta \Gamma_{-}} + r_{-} \right|^2 + \frac{\gamma}{\Delta \Gamma} - 8 \left| \frac{C}{\Delta \Gamma_{+}} \right|^2 - 4 \text{Re} \left( \frac{\varepsilon_L C}{\Delta \Gamma} \right),
$$

(4.7)

† The factor $\omega = \text{Re}(A_2)/\text{Re}(A_0)$ corresponds to the suppression due to the $\Delta I = 1/2$ rule in the kaon two-pion decays. This ratio is known to be small; therefore, in presenting the above formulas, first order terms in the small parameters multiplied by $\omega^2$ have been consistently neglected.
while the interference term is determined by the combination

$$\epsilon_L - \frac{2C}{\Delta\Gamma} + r_+ - \equiv \eta_+ = |\eta_+| e^{i\phi_+}.$$  

(4.8)

Other interesting observables, directly measured in experiments, are the asymmetries associated with the decay into the final state $f$ of an initial $K^0$, described by the density matrix $\rho_{K^0}$, as compared to the corresponding decay into the conjugate state $\bar{f}$ of an initial $\bar{K^0}$, described by the matrix $\rho_{\bar{K}^0}$. All these asymmetries have the general form

$$A(t) = \frac{\text{Tr} \left[ \rho_{\bar{K}^0}(t) \mathcal{O}_f \right] - \left[ \rho_{K^0}(t) \mathcal{O}_f \right]}{\text{Tr} \left[ \rho_{\bar{K}^0}(t) \mathcal{O}_f \right] + \left[ \rho_{K^0}(t) \mathcal{O}_f \right]}.$$  

(4.9)

The phenomenological quantities $R^L_{+-}$ and $\eta_-$ are accessible to the experiment, so that they can be used, together with other asymmetries involving three-pion and semileptonic final states, to obtain estimates on $CPT$-violating phenomena. Of particular interest are the bounds that can be derived on the dissipative effects, encoded in (3.5); using the most recent results,[50, 5] one can indeed obtain some information on some of the non-standard parameters:

$$a = (2.5 \pm 2.6) \times 10^{-17} \text{ GeV},$$
$$c = (0.7 \pm 1.2) \times 10^{-17} \text{ GeV},$$
$$\alpha = (1.8 \pm 4.4) \times 10^{-17} \text{ GeV},$$
$$\beta = (-0.7 \pm 1.3) \times 10^{-17} \text{ GeV},$$
$$\gamma = (0.1 \pm 22.0) \times 10^{-20} \text{ GeV}.$$  

(4.15)

Unfortunately, the precision of the present experimental results on single neutral kaons is not high enough to allow a meaningful test of the hypothesis of complete positivity. Although more complete and precise data will surely be available in the future, the most promising venues for studying the consequences of the dissipative dynamics in (3.3), (3.5) are certainly the experiments on correlated neutral kaons at $\phi$-factories.

5. CORRELATED NEUTRAL KAON OBSERVABLES

The time evolution of a system of two correlated neutral kaons coming from the decay of a $\phi$-meson can be discussed using the results on the dynamics of a single kaon presented in the previous section.[40]

Since the $\phi$-meson has spin one, its decay into two spinless bosons produces an anti-symmetric spatial state. In the $\phi$ rest frame, the two neutral kaons are produced flying
apart with opposite momenta; in the basis \(|K_1\), \(|K_2\), the resulting state can thus be described by:

\[
|\psi_A\rangle = \frac{1}{\sqrt{2}} \left( |K_1, -p\rangle \otimes |K_2, p\rangle - |K_2, -p\rangle \otimes |K_1, p\rangle \right).
\] (5.1)

The corresponding density operator \(\rho_A\) is a 4 \(\times\) 4 matrix and its time-evolution can be obtained by assuming that, once produced in a \(\phi\) decay, the kaons evolve in time each according to the completely positive map generated by the equation (3.3).†

The typical observables that can be studied in such physical situations are double decay rates, i.e., the probabilities that a kaon decays into a final state \(f_1\) at proper time \(\tau_1\), while the other kaon decays into the final state \(f_2\) at proper time \(\tau_2\):

\[
G(f_1, \tau_1; f_2, \tau_2) \equiv \text{Tr}\left[ (O_{f_1} \otimes O_{f_2}) \rho_A(\tau_1, \tau_2) \right];
\] (5.2)

as before, the operators \(O_{f_1}\) and \(O_{f_2}\) are the 2 \(\times\) 2 hermitian matrices describing the final decay states.

As an example, let us consider the situation in which \(\tau_1 = \tau_2 = \tau\) and the two kaons decay into the same final state \(\pi^+\pi^-\):

\[
G(\pi^+\pi^-, \tau; \pi^+\pi^-, \tau) \sim e^{-\gamma_S \tau} \left\{ e^{-\gamma_L \tau} \left( R_{+-}^L - |\eta_{+-}|^2 \right) - e^{-\gamma_S \tau} \left[ \frac{\gamma}{\Delta \Gamma_0} + 8 \left| \frac{C}{\Delta \Gamma_+} \right|^2 - 4 \Re e\left( \frac{\epsilon_L C}{\Delta \Gamma_0} \right) \right] - e^{-\gamma_T \tau} \left( \frac{\eta_{+-} C}{\Delta \Gamma_+} e^{-i \Delta m \tau} \right) \right\}.
\] (5.3)

This time-behaviour is completely different from the one required by ordinary quantum mechanics, which predicts:

\[
G(f, \tau; f, \tau) \equiv 0,
\] (5.4)

for all final states \(f\), due to the antisymmetry of the initial state \(\rho_A\). Therefore, by studying double decay rates in a high-luminosity \(\phi\)-factory it will be possible to determine the values of the non-standard parameters \(a, b, c, \alpha, \beta, \gamma\). For instance, the long time behaviour \((\tau \gg 1/\gamma_S)\) of the three pion probability gives direct informations on the parameter \(\gamma\):

\[
G(\pi^+\pi^-\pi^0, \tau; \pi^+\pi^-\pi^0, \tau) \sim \frac{\gamma}{\Delta \Gamma} e^{-2\gamma_L \tau}.
\] (5.5)

† This assures that the resulting evolution map \(\rho_A \mapsto \rho_A(t)\) is completely positive and of semigroup type; further, this dynamics is independent from the particular situation under study and can be easily generalized to systems containing more than two particles. Although other possibilities are conceivable, this choice is the most natural one. In fact, it is very hard to produce linear dynamical maps for a system of two particles not in factorized form without violating very basic physical principles. Indeed, if one requires that after tracing over the degrees of freedom of one particle, the resulting time evolution for the remaining one be completely positive, of semigroup type and independent from the initial state of the first particle, than the only natural possibility is a factorized dynamics.
Similarly, the small time behaviour of the ratio of semileptonic probabilities,

$$\frac{G(\ell^\pm, \tau; \ell^\pm, \tau)}{G(\ell^\pm, \tau; \ell^\mp, \tau)} \sim 2 a \tau, \quad (5.6)$$

is sensible to the parameter $a$.

However, much of the analysis at $\phi$-factories is carried out using integrated distributions at fixed time interval $\tau = \tau_1 - \tau_2$. One then deals with single-time distributions, defined by:

$$\Gamma(f_1, f_2; \tau) = \int_0^\infty dt \ G(f_1, t + \tau; f_2, t), \quad \tau > 0. \quad (5.7)$$

Starting with these integrated probabilities, one can form asymmetries that are sensitive to various parameters in the theory. A particularly interesting example is given by the following observable, involving two-pion final states,

$$A_{\varepsilon'}(\tau) = \frac{\Gamma(\pi^+ \pi^-, 2\pi^0; \tau) - \Gamma(2\pi^0, \pi^+ \pi^-; \tau)}{\Gamma(\pi^+ \pi^-, 2\pi^0; \tau) + \Gamma(2\pi^0, \pi^+ \pi^-; \tau)}; \quad (5.8)$$

it is used for the determination of the ratio $\varepsilon'/\varepsilon$. Indeed, one can show that, to first order in all small parameters:

$$A_{\varepsilon'}(\tau) = 3 \Re\left(\frac{\varepsilon'}{\varepsilon}\right) \frac{N_1(\tau)}{D(\tau)} - 3 \Im\left(\frac{\varepsilon'}{\varepsilon}\right) \frac{N_2(\tau)}{D(\tau)}, \quad (5.9)$$

where the $\tau$-dependent coefficients $N_1, N_2$ and $D$ are now functions of $\varepsilon$ and of the dissipative parameters $c, \beta$ and $\gamma$ (for explicit expressions, see [52].)

The clear advantage of using the asymmetry $A_{\varepsilon'}$ to determine the value of $\varepsilon'/\varepsilon$ in comparison to the familiar “double ratio” method,[53, 54] is that, at least in principle, both real and imaginary part can be extracted from the time behaviour of (5.9). Due to the presence of the dissipative parameters however, this appears to be much more problematic than in the standard case; a meaningful determination of $\varepsilon'/\varepsilon$ is possible provided independent estimates on $c, \beta$ and $\gamma$ are obtained from the measure of other independent asymmetries. This is particularly evident if one looks at the large-time limit ($\tau \gg 1/\gamma_S$) of (5.9):

$$A_{\varepsilon'}(\tau) \sim 3 \Re\left(\frac{\varepsilon'}{\varepsilon}\right) \frac{|\varepsilon'|^2 + 2 \Re\left(\varepsilon C/\Delta \Gamma_+\right)}{|\varepsilon|^2 + D}$$

$$- 6 \Im\left(\frac{\varepsilon'}{\varepsilon}\right) \frac{\Im\left(\varepsilon C/\Delta \Gamma_+\right)}{|\varepsilon|^2 + D}, \quad (5.10)$$

where

$$D = \frac{\gamma}{\Delta \Gamma} - 4 \left|\frac{C}{\Delta \Gamma_+}\right|^2 + 4 \Re\left(\frac{\varepsilon C}{\Delta \Gamma_+}\right) - 4 \Re\left(\frac{\varepsilon_L C}{\Delta \Gamma}\right); \quad (5.11)$$

only when $c = \beta = \gamma = 0$, the expression in (5.10) reduces to the standard result: $A_{\varepsilon'} \sim 3 \Re\left(\varepsilon'/\varepsilon\right)$. Therefore, if the non-standard, dissipative parameters in (3.5) are
found to be non-zero, even neglecting the contribution from the imaginary part, the actual value of $\text{Re}(\varepsilon'/\varepsilon)$ could be significantly different from the measured value of the quantity $A_{\varepsilon'}/3$.\[52\]

In conclusion, dissipative effects in the dynamics of both single and correlated neutral kaon systems could affect the precise determination of various relevant quantities in kaon physics; dedicated experiments, in particular those involving correlated kaons, will certainly provide stringent bounds on these dissipative effects, thus clarifying their role in generating $CP$ and $CPT$-violating phenomena.

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