Abstract

The standard protocol for teleportation of a quantum state requires an entangled pair of particles and the use of two classical bits of information. Here, we present two protocols for teleportation that require only one classical bit. In the first protocol, chained XOR operations are performed on the particles before one of them is removed to the remote location where the state is being teleported. In the second protocol, three entangled particles are used. In a variant scheme, Bob’s particle is distributed to him right in the beginning (as in the standard case) and 1.5 classical bits are required for teleportation.

Introduction

In the standard teleportation protocol[1], an unknown quantum state (of particle X) is teleported to a remote location using two entangled particles (Y and Z) and two classical bits of information. This has been interpreted as the disembodied transfer of an unknown quantum state from one place to a remote location or the exchange of quantum information. (One may also consider the question of transfer of state mode in teleportation[4].) The question of the dependence between the amount of classical bits of information and the conditions necessary for teleportation to occur has not been addressed before. Here we present new teleportation protocols that require only one classical bit of information by altering the conditions under which the protocols proceed.

The proposed protocols exploit the property that chained XOR transformations correlate alternate qubits. In contrast to the standard protocol
where the operations are done on particles X and Y alone, the first protocol requires operations involving all the three particles. In experiments on teleportation, the entangled particles are generated while the experiment is being performed\cite{2, 3, 5}, therefore, this condition is not too restrictive. In the second protocol, three entangled particles are used. In a variant of the second protocol, there is no need to transfer Bob’s particle to him in the middle of the sequence of steps. In both these protocols, the unknown state may be recovered back by Alice after the entangled particle has been transmitted to the remote location.

The First Protocol

Alice starts with the pure entangled state $|00\rangle + |11\rangle$ representing particles Y and Z that may be assumed to have been created by a broker. (We leave out normalizing constants in this and other expressions.) Alice wishes to send to Bob the unknown qubit $|\phi\rangle$ associated with X. Without loss of generality, $|\phi\rangle = a|0\rangle + b|1\rangle$, where $a$ and $b$ are unknown coefficients. The initial state of the three particles is:

$$a|000\rangle + b|100\rangle + a|011\rangle + b|111\rangle$$

The protocol consists of the following four steps:

1. Apply chained XOR transformations:
   1a. XOR the states of X and Y.
   1b. XOR the states of Y and Z.

   The particle Z is now transferred to Bob at the remote location.

2. Apply H on the state of X.

3. Measure the state of X and transfer information regarding it to Bob.

4. Apply appropriate operator on Z to complete teleportation of the unknown state.

Note Steps 1b and 2 may be exchanged. However, doing so will preclude the remote transfer of particle Z.
Proof Consider the \( \text{XOR} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \) operator on the first two qubits (X and Y) (Step 1a). This leads to the state:

\[ a|000\rangle + b|110\rangle + a|011\rangle + b|101\rangle \]

The next XOR operation on the qubits Y and Z (Step 1b) gives us the state:

\[ a|000\rangle + b|111\rangle + a|010\rangle + b|101\rangle \]

This second XOR operation makes the qubits X and Z to become fully entangled. This is a consequence of the property that chained XOR transformations correlate alternate qubits. (The power of chaining may be seen by considering the compound state of \( a|0\rangle + b|1\rangle \) and the entangled bits \( |0000\rangle + |1111\rangle \) on which chained XOR transformations (on the first and second, followed by second and third, and so on) are applied. This gives:

\[ a|00000\rangle + b|11111\rangle + a|01010\rangle + b|10101\rangle \]

which is characterized by symmetry and the correlations across the first, the third, and the fifth qubits.)

The application of \( H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \) operator on the first qubit (Step 2), gives us:

\[
\begin{align*}
& a(|00\rangle + |10\rangle) + b(|011\rangle - |111\rangle) \\
& + a(|010\rangle + |110\rangle) + b(|001\rangle - |101\rangle)
\end{align*}
\]

Simplifying, we obtain:

\[
\begin{align*}
& |00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a|0\rangle + b|1\rangle) \\
& + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|0\rangle - b|1\rangle) \\
& = |0\rangle(|0\rangle + |1\rangle)(a|0\rangle + b|1\rangle) + |1\rangle(|0\rangle + |1\rangle)(a|0\rangle - b|1\rangle)
\end{align*}
\]
Alice now measures the first two qubits (X and Y) (Step 3). The state of the remaining qubit (Z) collapses to one of the two states:

\[ a|0\rangle + b|1\rangle \text{ or } a|0\rangle - b|1\rangle. \]

The information of the first qubit (X) is enough to determine which of the two operators

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}, \quad \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]

should be applied to Z to place it in the state \(|\phi\rangle\) (Step 4). The measurement of the second qubit (Y) does not provide any useful information because the qubits Y and Z are uncorrelated.

Although, the conditions for this protocol are more restrictive than for the standard protocol, the situation with respect to Alice and Bob is symmetric in the end. This means that if Bob were to apply the H operator on Z instead of Alice applying it on X, the state \(|\phi\rangle\) can be recovered by Alice (after Bob supplies the necessary one classical bit to her to know which transformation to apply to her state). This supports the point of view that this protocol be considered a case of true teleportation.

**The Second Protocol**

In this protocol there are a total of four particles: X, Y, Z, and U. Of these, U is at the remote location with Bob in the closing stages of the protocol (in a variant scheme described later, U remains with Bob right from the beginning). The three particles Y, Z and U are in the pure entangled state \(|000\rangle + |111\rangle\).

The initial state of the four particles is:

\[ a|000\rangle + b|100\rangle + a|011\rangle + b|111\rangle \]

The protocol consists of the following six steps:

1. Apply chained XOR transformations on the particles available to Alice:
   1a. XOR the states of X and Y.
   1b. XOR the states of Y and Z.
2. Apply H on the state of X.
3. Measure the state of X and Y.
4. Apply appropriate operators (described in the proof) on Z and U. U is now transferred to Bob at the remote location.

5. Apply H operator on Z.

6. Measure Z and transmit one classical bit of information to Bob to complete the teleportation of $|\phi\rangle$ to him.

**Proof** Consider the XOR operator on the first two qubits (X, Y) (Step 1a). This leads to the state:

$$a|0000\rangle + b|1100\rangle + a|0111\rangle + b|1011\rangle$$

The next XOR operation on the qubits Y and Z (Step 1b) gives us the state:

$$a|0000\rangle + b|1110\rangle + a|0101\rangle + b|1011\rangle$$

The application of $H$ operator on the first qubit (Step 2), gives us:

$$a(|0000\rangle + |0100\rangle) + b(|0110\rangle - |1110\rangle)$$
$$+a(|0101\rangle + |1101\rangle) + b(|0011\rangle - |1011\rangle)$$

Simplifying, we obtain:

$$|00\rangle(a|00\rangle + b|11\rangle) + |01\rangle(a|01\rangle + b|10\rangle)$$
$$+|10\rangle(a|00\rangle - b|11\rangle) + |11\rangle(a|01\rangle - b|10\rangle)$$

Alice measures the first two qubits (X and Y) (Step 3). The state of the remaining qubits (Z and U) collapses to one of the four states:

$$a|00\rangle + b|11\rangle, a|01\rangle + b|10\rangle, a|00\rangle - b|11\rangle, a|01\rangle - b|10\rangle.$$

Alice uses the information in the first two qubits to determine which of the four operators

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

should be applied on Z and U so that they are in the compound state $a|00\rangle + b|11\rangle$.

Alice applies H on the state of Z, that is on $a|00\rangle + b|11\rangle$, to give:
\[ a|00⟩ + a|10⟩ + b|01⟩ − b|11⟩ \]
\[ = |0⟩(a|0⟩ + b|1⟩) + |1⟩(a|0⟩ − b|1⟩). \]

Alice measures Z, and, based on her measurement, transmits one classical bit of information to Bob, enabling him to use the appropriate operator on U to obtain the unknown \(|φ⟩\).

**Variant of Protocol 2**

In this variant, particle U is distributed to Bob in the beginning of the protocol. Therefore, this corresponds to the standard condition of teleportation. The protocol proceeds in exactly the same way as before until Step 4 where one may require a transformation on U with probability half (when at the end of Step 3 the states \(a|01⟩ + b|10⟩\) or \(a|01⟩ − b|10⟩\) were obtained). In this case a classical bit of information is sent to Bob to apply to his particle
\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix},
\]
the appropriate operator in this case, so that the joint state of Z and U is \(a|00⟩ + b|11⟩\). Since this will happen only in 50\% of the cases, its computational burden is one-half bit. With the further requirement of one classical bit in Step 6, one needs a total of 1.5 classical bits. This is better than the 2 classical bits of the standard scheme.

**Conclusion**

In any visualization of a teleportation experiment, it is convenient to generate the entangled pair at about the same time as the teleportation is sought to be done. In such a case, the additional pre-processing (the second XOR on Y and Z) may not be a problem. The first protocol works by making X and Z entangled and then using the gate H to expand this entanglement into the two Bell states that are teleported to Z. In the second protocol, three entangled particles are required to teleport the unknown state to the remote location.

The power of the protocols springs from their ability to transform \(a|0⟩ + b|1⟩\) into \(a|00⟩ + b|11⟩\). This process may be generalized further for several interesting effects. The symmetry of \(a|00⟩ + b|11⟩\) from the point of view of Alice and Bob means that the unknown state is jointly shared and it may be recovered back by Alice.

The reduction from the four Bell states to two in the teleported state might make it easier to implement and verify the protocols (although the use
of three entangled particles in the second protocol would impose additional burden). This reduction is evidently optimal, because otherwise it would become possible to transfer information faster than the speed of light.

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References

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