Relative Spacetime Transformations in Gamma-ray Bursts

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Abstract

GRB 991216 and its relevant data acquired from the BATSE and the Rossi X-Ray Timing Explorer and Chandra satellites are used as a prototypical case to test the theory linking the origin of gamma-ray bursts (GRBs) to the process of vacuum polarization occurring during the formation phase of a black hole endowed with electromagnetic structure. The relative spacetime transformation paradigm is presented. It relates the observed signals of GRBs to their past light cones, defining the events on the worldline of the source that is essential for the interpretation of the data. Since GRBs present regimes with unprecedentedly large Lorentz factors, and also sharply varying with time, particular attention is given to the constitutive equations relating the four time variables: the comoving time, the laboratory time, the arrival time, and the arrival time at the detector corrected by the cosmological effects. This paradigm is at the very foundation of any possible interpretation of the data of GRBs.

Subject headings: black hole physics — gamma rays: bursts — supernovae: general

In recent years, a large variety of very accurate experimental data, ranging from gamma rays all the way to the radio band, has been obtained for the afterglows of gamma-ray bursts (GRBs), following their first discovery by the BeppoSAX satellite (see, e.g., Costa 2001 and references therein). In the theoretical models of GRBs, there are currently three topics under debate:

1. The “internal shock model,” introduced by Rees & Mészáros (1994), has many aspects that have been developed by Paczyński & Xu (1994), Sari & Piran (1997), Fenimore (1999), and Fenimore et al. (1999). The underlying assumption of this model is that all the variations of GRBs in the range \( \Delta t \sim 1 \) ms up to the overall duration \( T \) on the order of 50 s are determined by the “inner engine.” The difficulties of explaining the long timescale bursts by a single explosive model have led us to create a class of models assuming an inner engine with prolonged activity (e.g., Piran 2001 and references therein).

2. The “external shock model,” also introduced by Mészáros & Rees (1993), is less popular today. It relates the GRBs’ light curves and time variations to interactions of a single, thin blast wave with clouds in the external medium. There is the distinct possibility, within this model, that the “GRBs’ light curves are tomographic images of the density distribution of the medium surrounding the sources of GRBs” (Dermer & Mitman 1999; see also Dermer, Chiang, & Böttcher 1999, Dermer 2000, and references therein). In this case, the structure of the burst does not come directly from the inner engine.

3. In order to decrease the energy requirements of GRBs, the effect of beaming has been advocated (see, e.g., Mao & Yi 1994 and Davies et al. 1994). The possibility of inferring its existence from changes in the power-law index of the afterglow is generally considered attractive (see, e.g., Mészáros & Rees 1997a; Rhoads 1997, 1999; Mészáros, Rees, & Wijers 1998; Panaitescu, Mészáros, & Rees 1998; Dermer & Chiang 1999; Sari, Piran, & Halpern 1999; Panaitescu & Mészáros 1999; Halpern et al. 2000; Gou et al. 2001).

For the astrophysical nature of the system originating GRBs, a binary system of merging neutron stars has been proposed (see, e.g., Eichler et al. 1989; Narayan, Paczynski, & Piran 1992; Mészáros & Rees 1992a, 1992b). Problems occur (1) in the general energetics that cannot exceed \( \sim 3 \times 10^{52} \) ergs, (2) in explaining the longer bursts (see Salmonson, Wilson, & Mathews 2001 and Wilson, Mathews, & Marronetti 1996), and (3) in the observed location of the GRBs’ sources in star-forming regions (see Bloom, Kulkarni, & Djorgovski 2000). Alternatively, novel classes of astrophysical systems have been postulated, including black hole–white dwarf (Fryer et al. 1999) and black hole–neutron star binaries (Paczynski 1991; Mészáros & Rees 1997b) as well as hypernovae (see Paczynski 1998), failed supernovae and collapsars (see Woosley 1993 and MacFadyen & Woosley 1999), and supranovae (see Vietri & Stella 1998, 1999).

We take a somewhat intermediate approach by studying the GRBs emitted by the process of vacuum polarization around a black hole endowed with electromagnetic structure: the EMBH model. Such a model has the advantage that all of its basic intermediate theoretical background, starting with the process of gravitational collapse itself, has been developed. The model can therefore make precise predictions that can be compared with the observations.

In order to create a new interpretative paradigm, we consider a “prototypical” GRB case, which we then apply to the observations of other GRBs. Since some of the best data, from BATSE\(^1\) to the Rossi X-Ray Timing Explorer (RXTE; Corbet & Smith 2000), as well as the remarkable accuracy of the Chandra (Piro et al. 2000) satellite, are available for GRB 991216, we use it as our prototype. In addition, (1) it is one of the strongest observed GRBs; (2) it radiates mainly in X-rays and gamma rays, and less than 3% is emitted in optical and radio bands; and (3) a precise value of the slope of the energy emission during the afterglow as a function of time,

\[ D \sim \frac{\Delta t}{T} \]

\( D \) is the characteristic time, the laboratory time, the arrival time, and the arrival time at the detector corrected by the cosmological effects. This paradigm is at the very foundation of any possible interpretation of the data of GRBs.

\(^1\) See http://gammaray.msfc.nasa.gov/~kippen/batserbr.
related to the mass and the electromagnetic parameters of the GRBs. It is useful to parameterize the baryonic mass and with the interstellar medium (ISM) leads to the different remnant, left over from the gravitational collapse of the protostar, subsequent interaction of this pulse with the baryonic matter of the outside the EMBH horizon formed of an optically thick plasma in this theory is the definition of the dyadosphere (Ruffini 1998; M. B. todocoulou & Ruffini 1971) via the vacuum polarization process to the extractable electromagnetic energy of an EMBH (Chris- todorou et al. 2001a; C. L. Bianco, P. Chardonnet, F. Fraschetti, R. Jantzen, R. Ruffini, & S.-S. Xue 2001, in preparation). Neglecting $r_{ds}$, the solution of equation (5).

$$t_u = \frac{t - \frac{v_0}{c} t - \frac{1}{2} a t^2 - \cdots}{c},$$

is in general highly nonlinear.

If and only if $v$ is constant and $v \approx c$, can equation (5) be rewritten, neglecting $r_{ds}$, as

$$t_u \approx t \left(1 - \frac{v}{c}\right) = t \frac{\left(1 - v/c\right)(1 + v/c)}{(1 + v/c)} \approx \frac{t}{2\gamma^2}. $$

It is clear that the knowledge of $t_u$, which is indeed essential for any physical interpretation of GRB data, depends on a definite integral whose integration limits extend from the gravitational collapse to the time $t$ relevant for the observations (see eq. [5]). Such an integral is not generally expressible as a simple linear relation or even by any explicit analytic relation since we are dealing with processes with variable Lorentz factors of unprecedented magnitude and time variability. Most studies have adopted an approximation of the kind given in equation (7) (see, e.g., Fenimore, Madras, & Nayakshin 1996). We instead use equation (5). The adoption of equation (7) misses a crucial feature of the GRB process and leads to a subversion of the spacetime relations in GRBs, with a wide range of consequences: all theoretical computations on the power-law indices of the afterglow are affected. Specific illustrative examples pointing out these differences are shown in the following paragraphs (see Ruffini et al. 2001a for details).

The bookkeeping of the four different times and the corresponding space variables must be done carefully in order to keep the correct causal relation in the time sequence of the events involved. This will also have important consequences in the supernova-GRB correlation (see Ruffini et al. 2001c).

The second set of constitutive equations are the full nonlinear relativistic hydrodynamic equations of energy and momentum conservation, which are to be solved together with the rate equation for the $e^\pm$ plasma. The computations carried out semi-analytically in Rome have been validated by the full numerical computations performed using Wilson’s codes at Livermore (see Ruffini et al. 1999a, 1999b, 2000, 2001a).
Fig. 1.—Theoretically computed Lorentz factor for the parameter values $E_{\text{iso}} = 9.57 \times 10^{52} \text{ergs}$ and $B = 4 \times 10^{-3}$ given as a function of the radial coordinate in the laboratory frame. The corresponding values in the comoving time, laboratory time, and arrival time are given in Table 1. The different eras, indicated by roman numerals, are illustrated in the text, while the points 1, 2, 3, 4, and 5 mark the beginning and end of each of these eras. The point $P$ marks the maximum of the afterglow flux (see Ruffini et al. 2001b). At point 4, the transparency condition is reached.

We have integrated both sets of constitutive field equations given in Ruffini et al. (1999a, 1999b, 2000) and Bianco, Ruffini, & Xue (2001) for the source GRB 991216. Correspondingly, we have obtained the parameter values presented in Ruffini et al. (2001b): $E_{\text{iso}} = 10^{53} \text{ergs}$ and $B = 4 \times 10^{-3}$. These values correspond to any of the following pairs of values for the EMBH mass and charge-to-mass ratio $(\mu, \xi) = (22.3, 0.1), (10.0, 0.15)$, and $(5.5, 0.2)$.

Crucial to any GRB data interpretation is the relation of the Lorentz factor to the radial coordinate of the source in the laboratory frame and the corresponding values of the above four time parameters. In Figure 1, the $\gamma$ factors for the different eras are given as a function of the radial coordinate of the source in the laboratory frame. Correspondingly, we present in Figure 2 the relation between the laboratory time and the detector arrival time for the source GRB 991216. The highly nonlinear behavior is obvious, and the different results obtained from the use of equations (6) and (7) are clearly visible. Details are given in Ruffini et al. (2001a).

In Table 1, for each successive “era” and for one very significant event, we give the initial and final values of the Lorentz factor, the four time parameters mentioned above, as well as the radial coordinates in the laboratory frame. We then have the following:

Era I.—The pair-electromagnetic plasma, initially at $\gamma = 1$, expands away from the EMBH horizon and from the dyadosphere as a pulse (the PEM pulse). In the comoving frame, the thickness of the pulse increases during the expansion, but the Lorentz contraction in the laboratory frame exactly balances this expansion so that, in the laboratory frame, a constant thickness approximation can be adopted for the burst (Ruffini et al. 1999a, 1999b). The expansion of the PEM pulse occurs in a region of very low baryonic contamination with density $\rho_b \ll 10^{-7} \text{g cm}^{-3}$ (Ruffini 2001). The final Lorentz factor approaches $\gamma = 1$, the relation between $t$ and $t^\prime$ asymptotically goes to $\gamma = t^\prime$. Details are given in Ruffini et al. (2001a).

Era II.—While the PEM pulse is still optically thick, it reaches the remnants left over by the gravitational collapse of the progenitor star. The engulfment of this baryonic material induces by conservation of energy and momentum a drastic reduction in the $\gamma$ factor (Ruffini et al. 2000). The amount of baryonic matter in the remnant has been fixed by the determination of parameter $B$ in the fitting of the afterglow data (see Ruffini et al. 2001b). Since these data contain important direct information on the progenitor star, we report in Table 2 some specific values of the parameters corresponding to selected values of the EMBH masses: they include the radius and thickness of the remnant as well as the density of baryonic matter.

| Point | $r$ (cm) | $\tau$ (s) | $t$ (s) | $t_\mu$ (s) | $t^\prime_d$ (s) | $\gamma$ |
|-------|---------|----------|--------|-------------|----------------|--------|
| 1     | $1.610 \times 10^9$ | 0.0 | 0.0 | 0.0 | 0.0 | 1.00 |
| 2     | $7.659 \times 10^8$ | $1.985 \times 10^{-2}$ | $2.580 \times 10^{-1}$ | $1.846 \times 10^{-1}$ | $3.692 \times 10^{-3}$ | 48.38 |
| 3     | $9.153 \times 10^8$ | $2.292 \times 10^{-2}$ | $3.089 \times 10^{-1}$ | $2.780 \times 10^{-1}$ | $5.559 \times 10^{-3}$ | 11.38 |
| 4     | $9.692 \times 10^8$ | $14.23$ | $3.295 \times 10^2$ | $6.805 \times 10^{-2}$ | $1.361 \times 10^{-1}$ | 239.6 |
| $P$   | $4.863 \times 10^8$ | $7.784 \times 10^8$ | $1.653 \times 10^8$ | $1.186$ | $23.72$ | 160.2 |
| 5     | $2.958 \times 10^8$ | $1.082 \times 10^8$ | $9.989 \times 10^8$ | $1.2195 \times 10^7$ | $2.439 \times 10^7$ | 2.7  |

TABLE 1
Lorentz Factors for Selected Events and Their Spacetime Coordinates
requirements. The detailed spectral distribution depends on the flux. We have neglected the spreading due to off-axis emis-

sions, and momentum is radiated away in the afterglow, mainly in the optical and radio emission (see Halpern et al. 2000). We have used the "fully radiative case" condition (see, e.g., Piran 2001). This definition is assumed in order to distinguish with and . A contrast with existing slopes in the literature (see, e.g., Vietri 1997) are presented in Ruffini et al. 2001a). In Figure 1, we show how, during this era, the Lorentz factor first coasts to a constant value and then rapidly decreases, going from \( \gamma = 239.6 \) to \( \gamma = 2.7 \). Most important is the point \( P \) where \( \gamma = 160.2 \) corresponds to the peak of the afterglow (see Ruffini et al. 2001b). Beyond this point \( P \), the slope of the afterglow flux, as a function of arrival time, approaches the power-law index \( n = -1.6 \) in perfect agreement with the observations of RXTE and Chandra (see Ruffini et al. 2001b). The final Lorentz factor and spacetime parameters are given for point 5 in Table 1. It is important to emphasize that this power-law index results from the combination of three critical assumptions: (1) the emission occurring in a "fully radiative" regime, (2) the condition of spherical symmetry, and (3) the constancy of the ISM density. Earlier results relevant to this treatment can be found in Sari (1997) and in Dermer et al. (1999) (see, for comparison and contrast, Ruffini et al. 2001a).

Era V.—This is the transition to the relativistic and nonre-
lativistic regimes. This era is more complex. It contains two successive suberas, one with a power-law index of the energy emitted in the afterglow as a function of the detector arrival time \( n = -1.36 \), corresponding to a still relativistic era (\( 1.1 \leq \gamma \leq 2.7 \)), and a final one approaching the pure Newtonian regime, with \( n = -1.45 \) and \( \gamma < 1.1 \). A contrast with existing slopes in the literature (see, e.g., Vietri 1997) are presented in Ruffini et al. 2001a). No data of GRB 991216 are available for checking the theoretical predictions of this last era.

In conclusion, we see from Table 1 and Figure 1 the remarkable and perfectly reasonable results that a motion of the pulse corresponding to a displacement of \( 9.692 \times 10^{13} \) cm will correspond to an arrival time interval of \( 1.360 \times 10^{-1} \) s, leading to what has been called apparent superluminal behavior. Similarly, on a larger scale, a displacement of the pulse by \( 2.958 \times 10^{17} \) cm will correspond to an increment of \( 2.439 \times 10^{3} \) s in arrival time, leading again to apparently superluminal behavior.

From the above results, we are ready to express the relative spacetime transformation (RSTT) paradigm: the necessary condition for interpreting the GRB data, given in terms of the arrival time at the detector, is the knowledge of the entire worldline of the source from the gravitational collapse. In order to meet this condition, given a proper theoretical description and the correct constitutive equations, it is sufficient to know the energy of the dyadosphere and the mass of the remnant of the progenitor star (see Ruffini et al. 2001b). The application of this RSTT paradigm will have important consequences for the interpretation of the burst structure (IBS), leading to a new paradigm (the IBS paradigm; see Ruffini et al. 2001b), as well as for the GRB-supernova correlation (Ruffini et al. 2001c).

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| M \( (M_{\odot}) \) | \( \xi \) | \( r_{n} \) \( (\times 10^{12} \text{ cm}) \) | \( r_{\text{gap}} \) \( (\times 10^{12} \text{ cm}) \) | \( \Delta_{\text{gap}} \) \( (\times 10^{12} \text{ cm}) \) | \( \rho \) \( (\text{g cm}^{-3}) \) |
|-----------------|------|-----------------|-----------------|-----------------|-----------------|
| 22.3 \ldots \ldots | 0.10 | 1.67 | 8.36 | 1.67 | 0.30 |
| 10.0 \ldots \ldots | 0.15 | 1.37 | 6.86 | 1.37 | 0.55 |
| 5.5 \ldots \ldots | 0.20 | 1.17 | 5.8 | 1.17 | 0.90 |

* In the laboratory frame.
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