Calculation and visualization of relative position uncertainty between adjacent wellbores

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Abstract. In directional drilling engineering of cluster wells and infill wells, it is important to scientifically assess the risk of wellbore collisions between wells being drilled and adjacent wells that have already been drilled. At present, the calculation results of adjacent well distance scanning and separation factors often used in drilling sites still cannot scientifically evaluate the risk of wellbore collision. This paper proposes to characterize the relative position measurement error between the reference well and the comparison well by the size of the combined error ellipsoid with the effect of well diameters of reference well and comparison well. And, the calculation model of the combined ellipsoid is established. This paper also introduces the 3D visualization method of relative position measurement error between adjacent wells based on OpenGL. The example calculating shows that the calculation results and 3D views of the relative position measurement error of the adjacent wells can be used to judge the reliability of the calculation result of the pedal curve separation factors, and provide a more scientific basis for the implementation of anti-collision operations in directional drilling. So the research has important application value.

1. Introduction
With the increasing requirements for cost, efficiency, safety and environmental protection, multiple wells and “well factory” operation modes have broad application prospects in the efficient production of China’s remaining oil and gas reserves. With the decrease of the distances between wellheads and of the adjacent well distances, the risk of collision in adjacent wells increases, and the difficulty of collision prevention is increasing. The methods for assessing the risk of collision in adjacent wells mainly include: calculation of adjacent well distance scanning [1-4], analysis of wellbore position uncertainty [5-8] and calculation of separation factors [9-14]. Since the calculation model of the separation factors with the effect of adjacent well distances and wellbore position uncertainty, using the calculation results of the separation factors to guide the wellbore anti-collision operation is a commonly used method at the current drilling site. However, the calculation model of the separation factor often used in commercial software cannot scientifically evaluate the risk of collision in adjacent wells. It has an overestimation or underestimation of the risk of collision in adjacent wells.

In order to more scientifically evaluate the risk of adjacent well collisions, this paper proposes a method for calculating and visualizing the relative position uncertainty between adjacent wells. The calculation results combined with the calculation results of the separation factors can provide a more
scientific basis for implementing wellbore anti-collision in directional drilling. It has important practical significance.

2. The pedal curve separation factors

Since the concept of the separation factor was proposed, there have been many methods for calculating the separation factors, such as the traditional method, the pedal curve method, and the zooming method. At present, the pedal curve separation factor is used in the Landmark software. This method is also recommended by the SPE Wellbore Positioning Technical Section (WPTS).

![Image](image_url)

**Figure 1.** The pedal curve separation factor overestimates the risk of wellbore collision

As shown in Fig.1, the point M represents a reference point; the point P represents a scanning point; the line AB is tangent to the combined error ellipsoid, the tangent point is B; the straight line AB is perpendicular to the straight line MP, and the intersection point is A. The pedal curve separation factor $f_{cs}$ can be expressed as [9]:

$$f_{cs} = \frac{R_S - r}{e}$$  \hspace{1cm} (1)

where, $R_S$ represents the radius of the 3D closest approach scanning; $r$ represents the sum of the radius of the reference well and the comparison well; $e$ represents the distance between point M and point A. It can be seen from Fig.1 that, when the reference well is close to the comparison well, when the point W and the point A coincide, the pedal curve separation factor is equal to 1, and the combined error ellipsoid and the circle of radius $r$ still do not intersect. Therefore, in the case shown in Fig.1, the pedal curve separation factor is overestimating the risk of collision between the reference well and the comparison well.

In the case shown in Fig.2, it can be seen from Eq. (1) that the pedal curve separation factor is greater than one at this time, but the comparison well trajectory has intersected the combined error ellipsoid. Therefore, in this case, the pedal curve separation factor underestimates the risk of collision between the reference well and the comparison well.
3. Calculation of relative position measurement error between adjacent wells

It can be seen from Fig. 1 and Fig. 2 that the pedal curve separation factor commonly used by commercial software cannot scientifically evaluate the risk of adjacent well collision. As shown in Fig. 3, we consider the size of the combined error ellipsoid with the effect of the well diameter of the reference well and the comparison well as the magnitude of the relative position measurement error between the reference well and the comparison well.

Figure 2. The pedal curve separation factor underestimates the risk of wellbore collision

Figure 3. Combined error ellipsoid with the effect of wellbore radius

According to the current analysis theory of wellbore trajectory measurement error, the influence of various error sources on the measurement error of the i-th measuring point of the well trajectory in the geographic coordinate system can be expressed as[6]:
where, \( N_i, E_i, V_i \) respectively represent the coordinates of the measurement point in the north, east and vertical directions in the geographic coordinate system; \( \text{var}(N_i, N_i) \), \( \text{var}(E_i, E_i) \), \( \text{var}(V_i, V_i) \) represent the variances in the north, east, and vertical directions, respectively; \( \text{cov}(N_i, E_i) \), \( \text{cov}(E_i, V_i) \), \( \text{cov}(N_i, V_i) \) represent the covariance between them, respectively. The models for calculating the covariance matrix \( \Sigma_i \) include the WdW model, the SESTEM model, and the ISCWSA model. The ISCWSA model has been recognized as an international industry standard. The process of calculating the covariance matrix \( \Sigma_i \) will not be repeated here.

Therefore, at the i-th reference point and the scan point, the influence of each error source on the wellbore trajectory measurement error can be expressed as:

\[
\Sigma_{Mi} = \begin{bmatrix}
\text{var}_m(N_i, N_i) & \text{cov}_m(N_i, E_i) & \text{cov}_m(N_i, V_i) \\
\text{cov}_m(N_i, E_i) & \text{var}_m(E_i, E_i) & \text{cov}_m(E_i, V_i) \\
\text{cov}_m(N_i, V_i) & \text{cov}_m(E_i, V_i) & \text{var}_m(V_i, V_i)
\end{bmatrix}
\]

\[\Sigma_{Pi} = \begin{bmatrix}
\text{var}_p(N_i, N_i) & \text{cov}_p(N_i, E_i) & \text{cov}_p(N_i, V_i) \\
\text{cov}_p(N_i, E_i) & \text{var}_p(E_i, E_i) & \text{cov}_p(E_i, V_i) \\
\text{cov}_p(N_i, V_i) & \text{cov}_p(E_i, V_i) & \text{var}_p(V_i, V_i)
\end{bmatrix}\]

Assuming that the measurement errors of the reference well and the comparison well are not correlated, at the i-th reference point and the scan point, the total impact of each error source on the measurement error of the reference well and the comparative wellbore trajectory can be expressed as:

\[
\Sigma_{G_i} = \Sigma_{Pi} + \Sigma_{Mi}
\]

The eigenvalues \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \) of the covariance matrix \( \Sigma_G \) of the total positional uncertainty are calculated, and the unit feature vectors \( p_{i1} \), \( p_{i2} \), and \( p_{i3} \) corresponding to the three eigenvalues are calculated. Then the three main semi-axis lengths \( a_i \), \( b_i \), and \( c_i \) of the i-th combined error ellipsoid can be obtained by the following formula:

\[
\begin{align*}
\lambda &= \lambda \sqrt{\sigma_{i1}} + r_i \\
\beta &= \lambda \sqrt{\sigma_{i2}} + r_i \\
\gamma &= \lambda \sqrt{\sigma_{i3}} + r_i
\end{align*}
\]

where, \( \lambda \) represents the minimum allowed confidence factor; \( \sigma_{i1} \), \( \sigma_{i2} \), and \( \sigma_{i3} \) represent eigenvalues of the covariance matrix \( \Sigma_{G_i} \); \( R_i \) represents the sum of the wellbore radius of the reference well and the comparison well at the i-th reference point and scanning point. The direction of the three major axes of the combined error ellipsoid is also the direction represented by the unit feature vectors \( p_{i1} \), \( p_{i2} \), and \( p_{i3} \).

4. 3D visualization of relative position measurement error between adjacent wells

OpenGL is strictly defined as "the software interface of graphics hardware"[15]. It is a universal shared and open 3D graphics standard developed by SGI and other world-renowned computer companies based on SGI's GL 3D graphics library. It has good performance and is convenient and flexible. At present, OpenGL technology has been widely used in the high-tech three-dimensional stratigraphic visualization, reservoir description and fluid simulation in the petroleum industry.
In the library of OpenGL standard graphics, there is no drawing function of ellipsoid, only the drawing function of sphere. Therefore, in the implementation of the error ellipsoid, the following transformation is required: first, the sphere is drawn with a radius of 1; then, in the spherical local coordinate system, the sphere is scaled by a magnification of three half-axis lengths in the three-axis direction of the ellipsoid. In this way, the error ellipsoid in the ellipsoidal spindle coordinate system can be obtained. Assuming that the calculated points on the reference wellbore trajectory are listed as \{M_i | i=1, 2,...,n\}, the three-dimensional visualization algorithm for the relative position measurement error between the adjacent wells can be represented by the following pseudo code:

1. Reading the survey parameters (measurement depth, inclination and azimuth) of the calculated points on the reference wellbore trajectory and the combined error ellipsoid parameters (center coordinates, lengths of three main semi-axis, and unit feature vectors);
2. Calculate steps (2) to (8) for i from 1 to n cycles;
3. Draw a sphere with a radius of 1;
4. The coordinate system is scaled by the lengths of the three main semi-axis to achieve the ellipsoid drawing;
5. Coordinate system restoration;
6. Translating the center of the coordinate system to the position of the center of the error ellipsoid \((N_{ic}, E_{ic}, V_{ic})\) to achieve the translation of the ellipsoid;
7. The coordinate system is wound into the direction of the feature vectors \(p_{i1}, p_{i2},\) and \(p_{i3}\) to realize the rotation of the ellipsoid to obtain the desired error ellipsoid;
8. Coordinate system restoration.

5. Example calculation
The strength of the geomagnetic field at a certain drilling platform position is 50054.5 nT, the geomagnetic inclination angle is 46.53°, and the geomagnetic declination is 4.15°. The distance between the wellhead positions of two horizontal wells (FHW3021I well and FHW3021P well) on the platform is 25.62m, and the three-dimensional diagram of the design tracks of the two wells is shown in Fig. 4. Obviously, the FHW3021I well and the FHW3021P well have a small distance between the two wells since entering the horizontal well section, which has potential risk of wellbore collision. Therefore, it is important to predict the wellbore position measurement error and separation factor before drilling.

![Figure 4. The 3D diagram of the design track of FHW3021I well and FHW3021P well](image-url)
Taking the FHW3021I well as the reference well and the FHW3021P well as the comparison well, using the Standard ISCWSA MWD Model provided in the paper [2], the value of the confidence factor $\lambda$ is 2, and the pedal curve separation factors are shown in Fig. 5. The relative position measurement error between the adjacent wells is shown in Fig. 6. As shown in Fig. 5, the pedal curve separation factors are less than 1 when the measurement depth of reference well is greater than 560 m. As shown in Fig. 6, the entire well section of the reference well does not intersect the error ellipsoid. Comparing Fig. 5 and Fig. 6, the calculation results of the pedal curve separation factors overestimate the risk of wellbore collision between the wells FHW3021I and FHW3021P. Therefore, when assessing the collision risk of the FHW3021I well and the FHW3021P well, it is necessary to calculate the relative position measurement error between the adjacent wells.

![Figure 5](image1.png)

**Figure 5.** Variation of the pedal curve separation factors with the measurement depths of the reference well

![Figure 6](image2.png)

**Figure 6.** The 3D diagram of the relative position measurement error between the adjacent wells
6. Conclusion
The calculation model of the pedal curve separation factor used by the commercial software currently, the calculation result will overestimate or underestimate the risk of the wellbore collision under special circumstances. It does not provide a scientific basis for the implementation of anti-collision operations for directional drilling.

The relative position measurement error between a reference well and a comparison well can be obtained by calculating a combined error ellipsoid with the effect of the wellbore radius of the reference well and the comparison well. And it can implement 3D visualization based on OpenGL.

The example calculation shows that the calculation and three-dimensional visualization of the relative position measurement error between adjacent wells can be used to judge the reliability of the calculation results of the pedal curve separation factors, and provide a more scientific basis for the implementation of the anti-collision operation of the directional drilling.

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