Research Article

FDA-MIMO Beampattern Synthesis with Hamming Window Weighted Linear Frequency Increments

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1. Introduction

Phased array (PA) has been widely applied because it can electronically scan to desired direction with high directional gains [1]. Given that the beam steering of phased array shares the same gain over all the range cells at a fixed angle, it is hard for phased array to deal with range-dependent target indication as well as range-dependent interference suppression. To address the current need for more degrees of freedom, a flexible array called frequency diverse array (FDA) is proposed [2]. By applying a tiny frequency offset across the array elements, the FDA generates a transmit beampattern which is range-angle-dependent and time-variant [3]. It gives more degrees of freedom in the range dimension [4].

Owing to its ability of range-angle-dependent, the FDA has exceptional application potential in radar and navigation [5–7]. Many studies about FDA have been presented in recent years [8–17]. Many researchers take their exploration in getting rid of the disadvantages of FDA beampattern such as range-angle coupling [18–20] and time-variant [21, 22]. Taking use of the range-angle property, many methods are proposed for the FDA in range-angle two-dimensional localization [23, 24] and mainlobe deceptive jamming suppression [25, 26]. Furthermore, the FDA is utilized in combination with advanced radar systems such as multiple-input multiple-output (MIMO) radar [27, 28] and cognitive radar [29, 30] for performance improvement. In [18], a new FDA scheme with logarithmically increasing frequency offset is proposed, and the target can be located at a single maximum. A uniformly spaced symmetric FDA with nonuniform frequency sets is proposed to form a range-angle-decoupled beampattern [19]. By employing particle swarm optimization algorithm to optimally determine the frequency offsets and element spacing, a pencil beampattern design approach is proposed, which also has lower sidelobes in range-angle dimensions [20]. To achieve a time-invariant beampattern, an approach for the FDA which uses time-dependent frequency offset (TDFO) is presented [21]. In [22], a FDA scheme using time-modulated optimized frequency offset (TMOFO) is proposed to realize time-invariant spatial fine focusing beampatterns. For target range-angle localization, a localization system which combines subarray-based FDA...
and full-band FDA is proposed [23]. In [24], a method for FDA localization using a two-stage estimator is presented, and the localization performance is analyzed and discussed. Taking advantage of the FDA-MIMO radar, a subspace projection-based sample selection method is proposed to alleviate the mainlobe deceptive jamming suppression problem [25]. An approach utilizing nonhomogeneous sample detection in FDA-MIMO is proposed to suppress the deceptive jamming in the mainlobe [26]. Combining FDA with MIMO, the performance of joint range, angle, and Doppler estimation is analyzed [27]. Based on FDA-MIMO, the Cramér-Rao lower bound (CRLB) and mean square error (MSE) expressions are derived [28]. By exploiting the strengths of cognitive radar with situational awareness and FDA with range-angle-dependent beampattern, a cognitive FDA radar with situational awareness is proposed [29]. A cognitive transmit beamforming scheme which uses FDA-MIMO is presented to enhance the low probability of intercept (LPI) capability [30]. For joint radar and communication applications, a range-angle-dependent beampattern design method which adopts FDA with null depth control is proposed [31].

According to the analysis in [32, 33], most of the methods about the beampattern of FDA do not take the signal propagation delay and time-range relationship into consideration. The consideration about time-range relationship is essential [33], and the study in [32] did make an outstanding contribution in the correction of time-range relationship and frequency-phase relationship. However, after our newly derivation about FDA in [17], we find that the correction in [32] is important but not comprehensive. The signal model cannot be corrected just dependent on subtracting the delay caused by propagation distance from time, and the range-angle-dependent transmit beampattern is unconvincing for the FDA radar, leading to the invalidity of the existing range-angle-decoupled beampattern design methods. Furthermore, we also find that the method which is presented in [34, 35] is an effective measure to eliminate the time parameter in the transmit-receive beampattern. The key point of the elimination is the adoption of a series of mixers and matched filters at a receiver. Similarly, the employment of FDA-MIMO radar and multiple matched filters at the receiver also can produce a beampattern without the influence of time parameter [25, 26]. Besides, the transmit beampattern is supposed to be the equivalent transmit beampattern at the receiver, which is range-angle-dependent and time-invariant.

In this paper, based on MIMO and multiple matched filter technologies, a new FDA scheme with Hamming window weighted linear frequency increments is proposed. Different from the existing method of FDA using Hamming window-based frequency offsets whose beampattern is time-varying and symmetrical, our proposed scheme gives a solution for the problem of time variance and provides a decoupled range-angle-dependent beampattern with a narrow mainlobe and low sidelobe level. Comparing with the existing FDA system, the superiority in beampattern performance of the proposed scheme is analyzed.

The remainder of this paper is organized as follows. Section 2 introduces the theoretical analysis about the proposed framework using FDA-MIMO. The beampattern design approach and the condition for equal comparison with other existing design methods are presented in Section 3. Numerical results and discussions are shown in Section 4. Finally, conclusions are drawn in Section 5.

2. Theoretical Analysis

According to [32], the array factor of pulse FDA should be expressed as (refer to Table I in [32])

\[
AF = \text{rect} \left( \frac{t - T_p r_n/c}{T_p} \right) e^{2\pi n_f_0 (t - (r_n/c))} \sum_{n=0}^{N-1} e^{2\pi n [f_0 (nd \sin \theta/c) + n\Delta f (t - (r_n/c))]},
\]

(1)

where \(T_p\) is the pulse duration, \(c\) is the speed of light, \(r_n\) denotes the distance between the target and the \(n\)th element, \(f_0\) represents the carrier frequency, \(d\) is the interelement spacing, and \(\Delta f\) is the frequency offset unit. Note that according to (1), the term \(\text{rect} \left( \frac{t - T_p r_n/c}{T_p} \right) e^{2\pi n_f_0 (t - (r_n/c))} \sum_{n=0}^{N-1} e^{2\pi n [f_0 (nd \sin \theta/c) + n\Delta f (t - (r_n/c))]}\) is limited by \(t\in [r_n/c, r_n/c + T_p]\). The phase difference between each element and reference element (such as \(\text{rect} \left( \frac{t - T_p r_n/c}{T_p} \right) e^{2\pi n_f_0 (t - (r_n/c))} \sum_{n=0}^{N-1} e^{2\pi n [f_0 (nd \sin \theta/c) + n\Delta f (t - (r_n/c))]}\)) becomes 0. Imagine the principle of phased array, i.e., \(\Delta f = 0\); the array factor (1) becomes

\[
AF = \sum_{n=0}^{N-1} \text{rect} \left( \frac{t - T_p r_n/c}{T_p} \right) e^{2\pi n_f_0 (t - (r_n/c))},
\]

(2)

Also, the phase difference between each element and reference element (such as \(\text{rect} \left( \frac{t - T_p r_n/c}{T_p} \right) e^{2\pi n_f_0 (t - (r_n/c))} \sum_{n=0}^{N-1} e^{2\pi n [f_0 (nd \sin \theta/c) + n\Delta f (t - (r_n/c))]}\)) becomes 0, which means that the array factor of phased array is not associated with the angle parameter, resulting in angle-independent property for phased array. Obviously, it is not consistent with the working principle of phased array radar. So, we made reanalysis about the basic signal model of FDA; a more detailed analysis is reflected in [17]. After our correction, the array factor of pulse FDA can be expressed as

\[
AF^{\text{FDA}} = \text{rect} \left( \frac{t - r_0/c}{T_p} \right) e^{2\pi n_f_0 (t - (r_0/c))} \sum_{n=0}^{N-1} e^{2\pi n [f_0 (nd \sin \theta/c) + n\Delta f (t - (r_0/c))]},
\]

(3)

Note that in (3), \(t - r_0/c = t'\) is the time index within radar pulse which is limited to \(t' \in [0, T_p]\), and the actual time \(t\) contains the transmit delay \(\tau\) and the time index within radar pulse \(t'\), i.e., \(t = \tau + t'\). This is the exact understanding
about time-range relationship. When it comes to phased array, the array factor can be rewritten as

\[
A_{\text{PA}} = \text{rect}\left(\frac{t - t_0/c}{T_p}\right) e^{i2\pi f_0(t - t_0/c)} \sum_{n=0}^{N-1} e^{i2\pi f_n(d \sin \theta/c)}.
\]  

(4)

Then, the phase difference becomes \(2\pi f_0 d \sin \theta/c\) associated with the parameter of angle, which is consistent with the theory of phased array radar. In the sequel, we find that there is a misconception in the original signal model of FDA, so the correction in [32] is important but not comprehensive because the original signal model of FDA needs to be corrected rather than the scope of time only and multiple matched filters at the receiver should be adopted to create and utilize the available range-dependent property of FDA.

We consider a MIMO radar; the transmit arrays and the receive arrays are both uniform linear arrays. The numbers of transmit array elements and receive array elements are \(M\) and \(N\), respectively. The carrier frequency is \(f_0\), and the transmit interelement spacing \(d_t\) and receive interelement spacing \(d_r\) are both half wavelength. The configuration of FDA-MIMO is shown in Figure 1.

Taking the first element as the reference element, the transmitting signal of the \(m\)th element can be expressed as

\[
s_m(t) = \varphi_m(t) \exp \{j2\pi(f_0 + \Delta f_m)t\},
\]

where \(\Delta f_m\) is the frequency offset corresponding to the \(m\)th element and \(\varphi_m(t)\) is the baseband orthogonal modulation wave of the \(m\)th element which is supposed to be satisfied with

\[
\int \varphi_{m1}^*(t) \cdot \varphi_{m2}(t - \tau)e^{i2\pi(\Delta f_m - \Delta f_{m2})\tau} d\tau = 0, \quad m1 \neq m2, \forall \tau,
\]

(6)

where \(^*\) is the conjugate operation. Hence, for a far-field target is located at range \(r\) and angle \(\theta\). According to the correction analysis of FDA about time in [17] and the FDA-MIMO radar framework in [25], the receiving signal of the \(n\)th element from the \(m\)th transmit array element can be written as

\[
x_{r,m,n}(t - 2r/c) = \xi_n(t) \exp \{j2\pi(f_0 + \Delta f_m)(t - \tau_{m,n})\},
\]

(7)

where \(\xi_n\) is the complex coefficient of the signal, \(\tau_{m,n}\) is the time delay from the \(m\)th transmit array element to the \(n\)th receive element, and it can be given by

\[
\tau_{m,n} = t_0 - d_t m \sin \theta/c - d_r n \sin \theta/c,
\]

(8)

where \(t_0 = 2r/c\) is the reference time delay with respect to the reference elements of transmit and receive. Note that \(t - 2r/c = t'\) is the time index within radar pulse which is limited to \(t' \in [0, T_p]\). Then, the receiving signal of the \(n\)th receive element can be expressed as

\[
x_{r,n}(t) = \sum_{m=0}^{M-1} \xi_n \varphi_m(t) \exp \{j2\pi(f_0 + \Delta f_m)
\[
\cdot (t - \tau_0 + d_t m \sin \theta/c + d_r n \sin \theta/c)\}.
\]

(9)

In the signal processing chain at the receiver, the received signal is mixed with \(e^{i2\pi f_c t}\) in an analogue device, then mixed with \(e^{i2\pi f_0 t}\) in a digital device, and matched filtered with \(\varphi_m(t)\) also in a digital device, as shown in Figure 2. So after matched filtering, the signal becomes
In the sequel, the total signal is given by

\[
x_{x,n} \approx \xi_n \sum_{m=0}^{M-1} \exp\{j2\pi [f_m (d_x m \sin \theta/c + d_y n \sin \theta/c - \Delta f_m (2r/c)]\},
\]

where \(\xi_n\) is the complex coefficient after matched filtering. In the sequel, the total signal is given by

\[
S = \xi_n a(r, \theta) \otimes b(\theta),
\]

where \(a(r, \theta)\) and \(b(\theta)\) are, respectively, the transmit and receive steering vectors, and they are shown as

\[
a(r, \theta) = \begin{bmatrix} 1, e^{j2\pi f_0 d_x \sin \theta/c}, \ldots, e^{j2\pi f_{M-1} (M-1) \sin \theta/c} \end{bmatrix}^T,
\]

\[
b(\theta) = \begin{bmatrix} 1, e^{j2\pi f_{1} d_x \sin \theta/c}, \ldots, e^{j2\pi f_{N-1} (N-1) \sin \theta/c} \end{bmatrix}^T,
\]

where \([\cdot]^T\) denotes the transpose operation. Thus, the influence of the parameter of time can be removed after the signal processing at the receiver in this FDA-MIMO system. It can be observed from (11) that the range and angle information are both included in the transmit steering vector of FDA-MIMO, while the receive steering vector is only angle-dependent just like the phased array. If \(\Delta f_m = 0\), the FDA-MIMO system is simplified to a conventional MIMO system.

3. Beampattern Design

The frequency offsets of the conventional FDA is linearly progressively increasing across the elements, which can be expressed as

\[
\Delta f_m^{\text{Con}} = m \Delta f,
\]

where \(\Delta f\) is the frequency increment unit. The transmit beampattern of the conventional FDA is range-angle coupling, and there is periodicity in the range dimension. Therefore, it will produce many peaks at the same time, which causes difficulties in range-angle localization and jamming suppression. To create a decoupled dot-shaped beampattern in range-angle dimensions, the FDA frameworks using logarithmically increasing frequency offset and Hamming window-based frequency offset are proposed in [18, 19], respectively. Whether the frequency offsets of these methods are symmetric or not, the essence of both of them is that the nonlinear frequency offset can result in an unusual beampattern which we can make full use of. The frequency offset of the log-FDA [18] can be expressed as

\[
\Delta f_m^{\text{Log}} = \delta \cdot \log (m + 1),
\]

where \(\delta\) is constant. The frequency offset of the Hamming-FDA [19] can be expressed as

\[
\Delta f_m^{\text{Ham}} = B \left\{ 0.54 - 0.46 \cos \left[ \frac{2\pi (m - (M + 1)/2)}{M} \right] \right\},
\]

where \(B\) is the bandwidth and the number of elements \(M\) should be odd. In fact, due to the time-variant problem and time-range relationship for FDA in the transmit beampattern concluded in [17, 32, 33], it is impossible to form a useful range-angle-decoupled transmit beampattern just using the nonlinear frequency offset design approaches such as those in [18, 19]. Therefore, the same FDA-MIMO framework and matched filtering technique are also adopted when analyzing the transmit beampattern of log-FDA and Hamming-FDA to avoid the time-variant problem. It is noteworthy that the transmit beampatterns are the equivalent transmit beampatterns at the receiver. Besides, the performances of their beampatterns are equal with the corresponding transmit beampatterns at an instantaneous time analyzed in [18, 19]. The proposed FDA scheme in our work is with Hamming window weighted linear frequency increments, which can be expressed as

\[
\Delta f_m = m \Delta f \left\{ 0.54 - 0.46 \cos \left[ \frac{2\pi (m - (M + 1)/2)}{M} \right] \right\}.
\]

Figure 3: Frequency offset comparison.

It is an improved form based on the Hamming window type, and the performance will be analyzed in the following section comparing with the existing techniques. Furthermore, it outperforms the log-FDA and Hamming-FDA in terms of half-power beam width of mainlobe and sidelobe suppression.
To detect the target located at range $r_0$ and angle $\theta_0$, the steering vector can be expressed as

$$w = a(r, \theta) \otimes b(\theta)_{r=r_0, \theta=\theta_0}. \quad (17)$$

Consequently, the transmit-receive beampattern can be expressed as

$$B_{T,R} = \sum_{m=0}^{M-1} \exp \left\{ j2\pi \left( f_0 d_T m \frac{\sin \theta - \sin \theta_0}{c} - 2\Delta f_m \frac{r - r_0}{c} \right) \right\}^2 \times \sum_{n=0}^{N-1} \exp \left\{ j2\pi f_0 d_R n \frac{\sin \theta - \sin \theta_0}{c} \right\}^2. \quad (18)$$

Thus, the equivalent transmit beampattern at the receiver can be written as

$$B_T = \sum_{m=0}^{M-1} \exp \left\{ j2\pi \left( f_0 d_T m \frac{\sin \theta - \sin \theta_0}{c} - 2\Delta f_m \frac{r - r_0}{c} \right) \right\}^2. \quad (19)$$

So it is obviously observed that the transmit beampattern is range-angle-dependent and not associated with the parameter of time. In addition, the receive beampattern is given by

$$B_R = \left[ \sum_{n=0}^{N-1} \exp \left\{ j2\pi f_0 d_R n \frac{\sin \theta - \sin \theta_0}{c} \right\} \right]^2. \quad (20)$$

From (21), it is seen that the receive beampattern is only angle-dependent just like the phased array.

Given the above, our proposed scheme can be termed as the FDA-MIMO with Hamming window weighted linear frequency increments. Due to the fact that there is only one carrier frequency at each element, which is similar with the log-FDA and Hamming-FDA, the complexity of the proposed FDA approach is equal with the previous nonlinear frequency offset approaches.

4. Numerical Results and Discussions

The frequency offsets of conventional linear FDA, log-FDA, Hamming-FDA, and our proposed FDA are compared as shown in Figure 3. To eliminate the unfairness, the bandwidths of these methods are approximately equal. The carrier frequency is $f_0 = 10$ GHz. The element numbers of transmit and receive are $M = N = 15$.  

Figure 4: The equivalent transmit beampatterns at the receiver of (a) conventional linear FDA, (b) log-FDA, (c) Hamming-FDA, and (d) the proposed FDA.
For a target located at 20 km, 20°, the equivalent transmit beampatterns at the receiver after utilizing multiple matched filtering technique on the basis of the four FDA systems above are given in Figure 4. Figure 4(a) shows that the beampattern of the conventional FDA is coupled in range-angle dimensions, which is unfavorable for energy concentration. The other three approaches generate beampatterns with one single peak at the position of the target, facilitating the target localization and jamming suppression.

Figure 5(a) shows the comparative results of the half-power beam width of transmit beampatterns for the log-FDA, Hamming-FDA, and proposed FDA. Note that proposed FDA performs closely with the Hamming-FDA in terms of the beam width, and they both have better performance than the log-FDA. Nevertheless, the drawback of the Hamming-FDA is that its beampattern has high sidelobe points. To make it worse, the sidelobes appear at the same angle of the target, which is a disadvantage for range-dependent interference suppressing. When it comes to the profile of range, as plotted in Figure 5(b), the beampattern of the Hamming-FDA owns higher sidelobe levels, whereas the undesired range bins of the other two methods keep on a low level. It also can be observed from Figure 5(b) that the range resolution of the proposed scheme is better than that of the log-FDA. Therefore, the proposed FDA outperforms the log-FDA and Hamming-FDA in terms of the mainlobe beam width and the sidelobe levels.

It is noteworthy that the range-dependent property of FDA cannot be easily implemented so that the transmit-receive beampattern is more practical to take advantage of. According to the analysis aforementioned, the receive beampattern is range-independent just like the phased array, as shown in Figure 6. It can reinforce the energy of the desired angle. Meanwhile, the received signal from the uninterested directions can be weakened.

Taking advantage of the MIMO technique, the transmit-receive beampatterns of these three kinds of FDA are shown in Figure 7. It can be seen from Figure 7 that these three methods can generate a range-angle-decoupled beampattern. Comparing with the two previous approaches, our proposed scheme produces a more focusing beampattern which has a narrower mainlobe and well-performed sidelobe level. The comparison results of half-power beam width are also presented to demonstrate the outperformance of our proposed scheme over other two methods in range-angle occupied areas, as shown in Figure 8.

To sum up, using the appropriate FDA-MIMO system with multiple matched filters, the range-dependent property can be fully applied without the influence of time parameter. Moreover, the proposed FDA framework outperforms the log-FDA and Hamming-FDA in terms of half-power beam width of mainlobe and sidelobe suppression. As a result, it has better detection precision for the target, as well as a stronger jamming suppression ability for mainlobe jamming from the same angle but different range with the desired position. In addition, the property of range-angle-dependent makes it easy to steer a target in range-angle domains.

Figure 5: Comparative results of (a) half-power beam width of transmit beampatterns; (b) transmit beampatterns in the profile of range.

Figure 6: Receive beampattern.
5. Conclusions

In this paper, taking advantage of the MIMO technique and multiple matched filters at the receiver, the range-dependent property of the FDA radar can be fully utilized without the influence of time parameter. Furthermore, by utilizing a nonuniform frequency offset, a new FDA framework is established. The frequency offset is Hamming window weighted linearly increasing, and the array elements are uniform linearly spacing. The proposed system can generate a sharp pencil-shaped beampattern with a low sidelobe level in the range profile, which facilitates the application of target indication and jamming suppression in the aspect of range. In addition, comparing with the existing FDA in the equivalent transmit beampattern at the receiver and transmit-receive beampattern, the proposed scheme has better performance with respect to the mainlobe beam width and sidelobe levels.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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