Phase Randomization and Doppler Peaks in the CMB Angular Power Spectrum

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(January 19, 2022)

Abstract

Using the Boltzmann equation with a Langevin-like term describing the stochastic force in a baryon-photon plasma, we investigate the influence of the incoherent electron-photon scattering on the subhorizon evolution of the cosmic microwave radiation. The stochastic fluctuation caused by each collision on average is found to be small. Nevertheless, it leads to a significant Brownian drifting of the phase in the acoustic oscillation, and the coherent oscillations cannot be maintained during their dynamical evolution. As a consequence, the proposed Doppler peaks probably do not exist.

PACS numbers: 98.70.Vc, 98.80.Es, 05.40.+j

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The anisotropies of the cosmic microwave background (CMB) provide a key to the understanding of the origin of primordial fluctuations and the thermal history of the early universe. The spectrum of the CMB anisotropy on large angular scales has been found to be consistent with the inflationary scenario of the early universe [1]. It is generally believed that many cosmological parameters can be determined from the fine structures in the power spectrum of CMB anisotropy. Among them, the possible existence of Doppler peaks – the peaks in the CMB anisotropy spectrum on an angular scale of about one degree or less has attracted much attention [2].

It has been shown theoretically that the amplitude and the position of these Doppler peaks are functions of the spatial curvatures, mass density, reionization time, cosmological constant etc. Thus, a precision measurement of the Doppler peaks may provide us an effective tool to determine various cosmic parameters. However, the observed amplitudes of the CMB anisotropy on degree-scale have not so far been very conclusive in determining the existence of the Doppler peaks. Indeed, some observations seems to exhibit high amplitudes as expected in a Doppler-peak scenario, while others show no peak amplitudes [2]. One may assert at this stage that the expected peaks have not yet been clearly identified in current data, but would be determined by a new generations of the CMB anisotropy observations.

In this letter, we shall take a different approach to reexamine the theoretical foundation of the prediction of the Doppler peaks. In particular, we shall argue based on a stochastic Boltzmann equation that the coherence of acoustic oscillations in the baryon-photon plasma will be largely disturbed, and may even be totally erased if the stochastic force of the incoherent scattering is included. In the standard theory of the CMB evolution [3], the acoustic oscillations in the baryon-photon plasma on subhorizon scales are treated coherently, i.e. different modes are frozen at different phases of their oscillation. The position of the Doppler peaks is then calculated by the phase at the recombination. However, the inclusion of correlation due to the stochastic term in the baryon-photon plasma will lead to a phase randomization. The coherence of the oscillations could be maintained if there is a mechanism for providing a negative entropy current to prevent the decoherence due to the
phase randomization (like a laser). Unfortunately, no such mechanism exists in the epoch of recombination, while the existence of a stochastic force term in the kinetic equation is inevitable in a system with dissipation. We shall show that the phase randomizations introduced by the incoherent electron-photon scattering are, indeed, substantial, and as a result, the Doppler peaks will be erased. Instead, one expects a large dispersion of the amplitudes in the CMB power spectrum due to different realizations on subhorizon scales, reflecting the stochastic nature of the phases. We suggest that the dispersed observations of the CMB angular power spectrum on degree-scale, though very coarse at this stage, are in support of our view.

Let us use conventional notations in the theory of the CMB anisotropy [3]. Since the coherent oscillations of the Doppler peaks act only on subhorizon scales, the choice of gauge is irrelevant for our calculation. The evolution of the photon distribution function $f(t, x, p)$ is calculated by the Boltzmann equation

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{dx_i}{dt} \frac{\partial f}{\partial x^i} + \frac{dp_0}{dt} \frac{\partial f}{\partial p_0} + \frac{d\gamma^i}{dt} \frac{\partial f}{\partial \gamma^i} = C[f]. \tag{1}$$

where $\gamma_i$ is the direction cosines of $p_i$ with respect to the corresponding spatial coordinate. The left hand side of eq.(1) describes the free-streaming, and the right hand side is the collision term given by

$$C[f] = \int dq dq' dp' W(p, q, p', q') \times \{ g(t, x, q') f(t, x, p') [1 + f(t, x, p)] - g(t, x, q) f(t, x, p) [1 + f(t, x, p')] \} \tag{2}$$

where $g$ is the electron distribution function, and the collision rate $W$ is determined by the Compton scattering between electron and photon from state $(q', p')$ to $(q, p)$.

It is well known that the Boltzmann equation in (1) is derived under the assumption of a molecular chaos and is applicable if the fluctuations caused by the incoherent collisions are negligible. These fluctuations give rise to a stochastic force term in hydrodynamics [4], governed by the fluctuation-dissipation theorem. Similarly, these fluctuations can be taken
into account by an additional Langevin-like force term in the Boltzmann equation [3]. Eq.(1) should then be replaced by

$$\frac{df}{dt} = C[f] + r(t, x, p)$$ (3)

where the stochastic force, $r$, is characterized by its correlations:

$$\langle r(t, x, p) \rangle = 0,$$ (4)

$$\langle r(t_1, x_1, p_1) r(t_2, x_2, p_2) \rangle = \frac{1}{2} N \delta(t_1 - t_2) \times \delta(x_1 - x_2) \int dq dq' dp dp' W(q, p, q', p') \Delta(p_1)$$

$$\times \Delta(p_2) g(t_1, x_1, q) f(t_1, x_1, p)[1 + f(t_1, x_1, p')]$$ (5)

where $\langle \ldots \rangle$ is an average over the stochastic effect (i.e. over different realizations), $N$ is the total number of photons, and the function $\Delta(P)$ is defined by

$$\Delta(P) = \delta(P - q) + \delta(P - p)$$

$$-\delta(P - q') - \delta(P - p').$$ (6)

It is important to note that eqs.(3)-(6) are applicable not only in a linear approximation but also in the nonlinear region [3][6].

In principle, to study the fluctuations in the baryon-photon plasma, we should also consider the stochastic terms in the equation of baryonic matter. However, since the linear fluctuations given by independent Gaussian random "forces" are additive, the stochastic terms in the baryonic equation will increase the effects of fluctuations considered in this paper. To illustrate the main effect of electron-photon scattering in the presence of the Doppler peaks, we shall only consider the contribution from stochastic term $r$ in photon’s equation. In this case, the electron distribution $g(t, x, q)$ can be treated as an external source. For a Thomson scattering, photons do not exchange energies with electrons. The $q$-distribution of electrons do not involved in the evolution, and we need only the number density distribution of electrons $n_e = \int dqqg(t, x, q)$. In this case, the distribution of the
photon energy $p$ is also unchanged, the perturbation of the CMB can be described by the anisotropy of the brightness temperature

$$\Theta(t, x, \gamma) = \frac{1}{4\pi^2\rho_\gamma} \int fp^2 dp - \frac{1}{4},$$

(7)

where $\rho_\gamma = (\pi^2/15)T^4$ is the mean energy density of photons. In terms of $\Theta$, the photon distribution function can be approximately expressed as

$$f(t, x, p) = f_T(p)[1 + 4\Theta(t, x, \gamma)],$$

(8)

where $f_T = 1/[\exp(p/T) - 1]$.

From eqs.(1) and (8), one finds that $\Theta$ should satisfy

$$\frac{\partial}{\partial \eta}(\Theta + \Phi) + \gamma^i \frac{\partial}{\partial x^i}(\Theta + \Psi) + \dot{\gamma}^i \frac{\partial}{\partial \gamma^i}\Theta =$$

$$\tau(\Theta_0 - \Theta + \gamma_i v^i_b + \frac{1}{10}Q\Theta) + R$$

(9)

where $\Phi$ and $\Psi$ are the Newtonian potential and the space curvature perturbation, respectively. $\eta = f(1 + z)dt$ is the conformal time, $z$ the redshift, $v_b$ the baryonic (fluid) velocity, and $\tau = n_e\sigma_T$ the optical depth, $\sigma_T$ being the Thomson cross section. $Q$ is the projection operator of quadrupole, defined as $Q(\gamma, \gamma') = \sum_{m=-2}^{2} Y^*_m(\gamma)(1/4\pi) \int d\Omega Y_{2m}(\gamma')$. Obviously $QQ=Q$. The isotropic component $\Theta_0(\eta, x)$ is given by $\Theta_0 = O\Theta$, where $O \equiv (1/4\pi) \int d\Omega$ is the monopole projection operator, and $OO=O$. The term $\Theta_0$ appearing on the right-hand side of eq.(9) indicates that without an external driving force (such as $v_b$) the isotropic state is the “equilibrium” state in the kinetic evolution of the Thomson scattering.

The stochastic term in eq.(9) is $R = (1/4\pi^2\rho_r) \int rp^3 dp$, and its correlation function is given by

$$\langle R(\eta_1, x_1, \gamma_1)R(\eta_2, x_2, \gamma_2) \rangle =$$

$$\frac{3}{8\pi^4} \frac{N}{16\pi^4\rho_r^2} \tau \delta(\eta_1 - \eta_2)\delta(x_1 - x_2)$$

$$\times \left[ \frac{4}{3} 4\pi \delta(\gamma_1 - \gamma_2) - \frac{1}{2}(1 + \cos^2 \beta) \right]$$

$$\times \int p^4 dp[f(\eta_1, x_1, p, \gamma_1)$$

$$+ 2f(\eta_1, x_1, p, \gamma_1)f(\eta_2, x_2, p, \gamma_2) + f(\eta_2, x_2, p, \gamma_2)]$$

(10)
where $\beta$ is the angle between $\gamma_1$ and $\gamma_2$. Because eq. (9) is linear in $\Theta$, it reduces to eq. (1) by taking an average over the stochastic effect. Thus, the calculation based on eq. (1) is actually only for $\langle \Theta \rangle$ but not for $\Theta$, i.e. the fluctuations due to the stochastic terms are entirely overlooked in eq. (1). In the incoherent processes, the linear fluctuations, $\delta \Theta = \Theta - \langle \Theta \rangle$, caused by stochastic force $R$ may not in general play a very important role. However, these fluctuations lead to “forgetting history”. Thus, the existence and maintenance of coherence should be seriously reconsidered.

Let us calculate the linear fluctuations, $\delta \Theta(\eta, x, \gamma)$, at a given time $\eta$ around a solution to eq. (1). It is governed by

$$
\left\{ \frac{\partial}{\partial \eta} + \gamma^i \frac{\partial}{\partial x^i} + \gamma^i \frac{\partial}{\partial \gamma^i} + \tau [1 + \frac{1}{10} Q] \right\} (\delta \Theta - \delta \Theta_0) = R'
$$

(11)

where $R' = (1 - O)R$. The projection operator arises from the isotropic term $\Theta_0$ in eq. (9).

The solution to eq. (12) can be generally expressed as

$$
\delta \Theta(\eta, x, \gamma) - \delta \Theta_0(\eta, x, \gamma) = \int_0^\infty d\lambda e^{-\lambda \tau [1 + (1/10) Q]} \int_0^\lambda d\lambda' \tau [1 + Q'] \int d\Omega \left\{ \delta \Theta(\eta - \lambda, x - \int_0^\lambda d\lambda' \gamma', \gamma - \int_0^\lambda d\lambda' \dot{\gamma}') \right\}
$$

(12)

Obviously, both sides of eq. (13) become zero upon an integration $(1/4\pi) \int d\Omega$ because $O(1 - O) = 0$.

The correlation functions of fluctuations, such as $\langle \delta \Theta(\eta_1, x_1, \gamma_1) \delta \Theta(\eta_2, x_2, \gamma_2) \rangle$, can be calculated from eqs. (10) and (12). For a flat universe, the light path is a straight line in comoving coordinates $x$, and thus $\dot{\gamma}^i = 0$. The fluctuation for the mode $l > 0$ is then

$$
\langle \delta \Theta_l(\eta, k) \delta \Theta_l^*(\eta, k) \rangle = \frac{1}{V^2} \int d\Omega_1 d\Omega_2 \exp\{-i k \cdot (x_1 - x_2)\}
$$

$$
\frac{(2l + 1)^2}{64 \pi^2} \int d\Omega_1 d\Omega_2 P_l(k_0 \cdot \gamma_1) P_l(k_0 \cdot \gamma_2)
$$

$$
\int d\lambda_1 d\lambda_2 \exp\{-\tau \lambda_1 [1 + Q_1] - \tau \lambda_2 [1 + Q_2]\}
$$

\[ \int d\Omega_1 d\Omega_2 P_l(k_0 \cdot \gamma_1) P_l(k_0 \cdot \gamma_2) \int d\lambda_1 d\lambda_2 \exp\{-\tau \lambda_1 [1 + Q_1] - \tau \lambda_2 [1 + Q_2]\} \]
\[ \langle R'(\eta - \lambda_1, x_1 - \lambda_1 \gamma_1, \gamma_1) R'(\eta - \lambda_2, x_2 - \lambda_2 \gamma_2, \gamma_2) \rangle \]

where \( k_0 = k / |k| \), and \( V \) the volume being considered. Since \((\tau / k) \gg 1\), the above equation can be simplified by using eqs.(8) and (10) as

\[
\langle \delta \Theta_l(\eta, k) \delta \Theta_l^*(\eta, k) \rangle = \frac{0.531 \times 675}{\pi^{14}} \times \frac{(2l + 1)^2}{64 \pi^2} \int d\Omega_1 d\Omega_2 P_l(k_0 \cdot \gamma_1) P_l(k_0 \cdot \gamma_2) \int \tau d\lambda e^{-2\tau \lambda}(1 - O_1)(1 - O_2) \times [1 + (e^{e^{\tau/10}} - 1)Q_1][1 + (e^{e^{\tau/10}} - 1)Q_2] \times \left[ \frac{4}{3} 4\pi \delta(\gamma_1 - \gamma_2) - \frac{1}{2}(1 + \cos^2 \beta) \right] \times \frac{1}{V} \int d\mathbf{x} \langle \Theta(\eta - \lambda, x, \gamma_1) \Theta^*(\eta - \lambda, x, \gamma_2) \rangle
\]

where we used \( N = (2\zeta(3)/\pi^2)VT^3 \), \( \int p^4 dp f_T^2 = 2[\zeta(2) - \zeta(3)]T^5 = 0.844T^5 \), and \( e^{\lambda Q} = [1 + (e^\lambda - 1)Q] \). The monopole projection factors \((1 - O_1)\) and \((1 - O_2)\) can be replaced by a unity operator when \( l > 0 \). The two quadrupole projection terms \( Q_1 \) and \( Q_2 \) can be neglected because the term \( e^{-2\tau \lambda} \) picks up the main contribution in the integral from the region \( \lambda \tau < 1 \). Using the following mode decomposition

\[
\frac{1}{V} \int d\mathbf{x} \langle \Theta(\eta, x, \gamma_1) \Theta^*(\eta, x, \gamma_2) \rangle = \frac{V}{2\pi^2} \int_0^\infty k^2 dk \sum_{l'} \frac{1}{2l' + 1} \langle |\Theta_{l'}(\eta, k)|^2 \rangle \langle \Theta_{l'}(\eta, k) \rangle
\]

we have finally

\[
\langle |\delta \Theta_l(\eta, k)|^2 \rangle = 3.91 \times 10^{-5}
\]

\[
\left\{ \frac{2l + 1}{3} \frac{V}{2\pi^2} \int dk k^2 \sum_{l'} \frac{1}{2l' + 1} \langle |\Theta_{l'}(\eta, k)|^2 \rangle \right\} - \frac{2}{3} \sum_{\nu=0,\pm 2} [\delta_{\nu,0} + |(l + \nu, 0, 2, 0)|^2 + |(l + \nu, 0)|^2] \times \frac{V}{2\pi^2} \int k^2 dk \left\{ \frac{1}{2l + 1} \langle |\Theta_l(\eta, k)|^2 \rangle \right\}
\]

where the overline above \( \langle |\Theta_l(\eta, k)|^2 \rangle \) denotes an average over the region from \( \eta - (1/\tau) \) to \( \eta \), and \((l + \nu, 0, 2, 0)|l + \nu, 0)\) are the Clebsch-Gordon coefficients. For large \( l \), the second
(negative) term in the bracket eq.(16) is completely negligible. The first term is independent of $l$ and $k$. This is expected since the correlation function of the corresponding stochastic force is isotropic in $\gamma$-space and uniform in $k$-space.

Using the expression for a temperature perturbation

$$
\left( \frac{\Delta T}{T} \right)^2 = \frac{V}{2\pi^2} \int dk k^2 \sum_l \frac{1}{2l + 1} \langle |\Theta_l(\eta, k)|^2 \rangle, \tag{16}
$$

one has

$$
\frac{\langle |\delta \Theta_l|^2 \rangle}{2l + 1} \simeq 1.30 \times 10^{-5} |\Delta T/T|^2. \tag{17}
$$

The $l$ summation in eq.(16) should run up to $l_{\text{max}}$ where the electron-photon collision is frequent enough, i.e. to the scale about the photon mean free path $1/\tau$. Therefore, $\Delta T/T$ in eqs.(16) and (17) should not be confused with $(\Delta T/T)_{\text{obs}}$ given by observations with low resolutions, i.e. their window functions are on scales much larger than $l_{\text{max}}$. For the observation of CMR anisotropy with a resolution $l$, the fluctuation is

$$
\delta \left( \frac{\Delta T}{T} \right) = \left( \sum_{l'=0}^{l} \frac{\langle |\delta \Theta_{l'}|^2 \rangle}{2l' + 1} \right)^{1/2} \simeq 3.61 \times 10^{-3} \sqrt{\frac{l\Delta T}{T}}. \tag{18}
$$

Therefore, the fluctuations caused by the stochastic force $R$ are generally small, except for the case of a very high resolution observation.

However, the average of the fluctuations in eqs.(15) and (18) is essentially over one collision time $1/\tau$. In relation to coherent processes, we should study the cumulative effect of the fluctuations over the entire period during which the coherence is to be maintained [7]. For the Doppler peaks, the period is from the time when the perturbation enters horizon to the time of recombination, i.e. the duration of the subhorizon evolution before recombination. The cumulative effect can be easily described by the phase fluctuations of the oscillation. Use the expression $\Theta_l(\eta, k) = |\Theta_l|e^{i\phi_l}$, where $|\Theta_l(\eta, k)|$ and $\phi_l(\eta, k)$ are the amplitude and phase, respectively. The equations of $\Theta_l(\eta, k)$ can then be derived from eq.(9) as
\[
\dot{\Theta}_0 = -\frac{k}{3}\Theta_1 - \dot{\Phi} + R_0 \quad (19)
\]
\[
\dot{\Theta}_1 = k\left(\Theta_0 + \Psi - \frac{2}{5}\Theta_2\right) - \tau(\Theta_1 - V_b) + R_1 \quad (20)
\]
\[
\dot{\Theta}_l = k\left(\frac{l}{2l-1}\Theta_{l-1} - \frac{l+1}{2l+3}\Theta_{l+1}\right) - \tau\Theta_l + R_l \quad (l > 2) \quad (21)
\]

where
\[
R_l = \frac{1}{V^2} \frac{2l+1}{8\pi} \int d\Omega P_l(k_0 \cdot \gamma) \int dxe^{-ikx}R \quad (22)
\]

If the hierarchy of (21)-(23) is cut off at 2\(l\)-th order, they correspond to the equations for a system consisting of \(l\) coupled oscillators. Without the stochastic terms \(R_l\), the oscillations are coherent and the phases of the oscillations are completely fixed by initial conditions. The terms \(R_l\) lead to a phase randomization.

We shall first calculate the phase fluctuations raised by \(R_0\). As the coupling between the photon and the baryon is tight and the peculiar gravitational potential is weakly time-dependent in matter-dominated regime, the evolution of \(\Theta_0\) is dominated by the phase evolution \(\phi_0(\eta)\), which is approximately described by a WKB-like equation as
\[
\frac{d\phi_0}{d\eta} \simeq kc_s - \frac{1}{2(\Theta_0)} \frac{1}{kc_s} \frac{dR_0}{d\eta} , \quad (23)
\]

where \(c_s\) denotes the sound speed of the baryon-photon plasma, which is \(\sim 1/\sqrt{3}\) before recombination. The stochastically averaged solution of eq.(23) is
\[
\phi_0(\eta) = \int_{\eta_{en}}^{\eta} kc_s d\eta' + \phi_0(\eta_{en}),
\]

where \(\eta_{en}\) denotes the time when the mode \(k\) enters the horizon, and \(\phi_0(\eta_{en})\) is the initial phase. The position of the Doppler peaks for adiabatic perturbations is approximately determined by a phase relation \(\phi_0(\eta_{re}) = n\pi\), where \(n\) is integer. Eq.(23) shows also that the phase fluctuation due to \(R_0\) in the period from \(\eta_{en}\) to \(\eta_{re}\) is
\[
\delta\phi_0 \simeq \int_{\eta_{en}}^{\eta_{re}} d\eta' \frac{1}{2(\Theta_0)} \frac{1}{kc_s} \frac{dR_0(\eta')}{d\eta'} . \quad (24)
\]

Generally, the term \(R_l\) leads to a phase fluctuation at least of
\[
\delta\phi_l \simeq \int_{\eta_{en}}^{\eta_{re}} d\eta' \frac{1}{2(\Theta_l)} \frac{1}{kc_s} \frac{dR_l(\eta')}{d\eta'} . \quad (25)
\]
Considering that the derivative \((1/kc_s)d/d\eta\) in eq.(25) contributes a factor of order 1, and using eqs.(10), (15) and (17), one obtains the mean phase fluctuation as

\[
\langle (\delta \phi_l)^2 \rangle \simeq \int_{\eta_{\text{re}}}^{\eta_{\text{en}}} \frac{1}{4\langle |\Theta_l|^2 \rangle} \langle |\delta \Theta_l(\eta, k)|^2 \rangle \tau d\eta
\]

\[
\simeq 0.33 \times 10^{-5}(2l + 1) \int_{\eta_{\text{re}}}^{\eta_{\text{en}}} \frac{1}{\langle |\Theta_l|^2 \rangle} \left( \frac{\Delta T}{T} \right)^2 \tau d\eta
\]

The factor \((\Delta T/T)^2/\langle |\delta \Theta_l(\eta, k)|^2 \rangle\) is no less than 1, and therefore, the RMS of the phase fluctuation can be estimated as

\[
\sqrt{\langle (\delta \phi_l)^2 \rangle} \geq 1.8 \times 10^{-3}(2l + 1)N_c
\]

where \(N_c \equiv \tau(\eta_{\text{re}} - \eta_{\text{en}})\) is the mean number of collisions in the entire period of subhorizon evolutions of mode \(k\) before the recombination. Therefore, the behavior of the stochastic fluctuation of the phase for the \(l\)-oscillation is just like a Brownian drifting: the number of collisions corresponds to the number of steps in the random walk, and the mean shift per step is about \(1.8 \times 10^{-3}\sqrt{2l + 1}\). For Doppler peaks, we have \(l \geq 100\), and \(N \geq \tau(\eta_{\text{re}} - \eta_{\text{en}}) \geq 10^3\). Therefore, we conclude that the RMS of the Brownian phase drifting due to Thomson scattering is order 1, significant enough to disturb the coherence.

In addition to the Thomson scattering, there are other stochastic forces in the baryon-photon plasma, such as non-Thomson terms of the Compton scattering, the stochastic terms in the hydrodynamical equation of baryons. Fluctuations from different sources mostly are additive. Our estimation on the phases drifting is thus a conservative one. The proposed Doppler peaks in the CMB angular power spectrum probably do not exist. Different causal areas at the recombination can be considered as independent realizations of the stochastic force. The dispersion of the currently observed CMB anisotropies on angular scale of one degree is consistent with the scenario of the Brownian drift of the phases.

This work (Z.H.) was supported in part through the U.S. Department of Energy under Contract Nos. DE-FG03-93ER40792 and DE-FG02-85ER40213. X.P.W. is supported by a World Laboratory Fellowship.
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