Relativistic Hard-Scattering and Tsallis Fits to $p_T$ Spectra in $pp$ Collisions at the LHC

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Abstract
Motivated by the good Tsallis fits to the high-$p_T$ spectra in $pp$ collisions at the LHC, we study the relativistic hard-scattering model and obtain an approximate analytical expression for the differential hard-scattering cross section at $\eta \sim 0$. The power-law behavior of the transverse spectra, in the form of $d\sigma/dp_T^2 \propto 1/p_T^n$, gives a power index $n$ in the range of 4.5-5.5 for jet production as predicted by pQCD, after the dependencies of the structure functions and the running coupling constant are properly taken into account. The power indices for hadron production $n$ are slightly greater than those for jet production.

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1 Introduction

The spectra of the transverse momentum of produced particles in nuclear collisions provide useful information on the dynamics of the colliding systems. These spectra are often described by the Tsallis distribution [1] (see also [2, 3, 4]) in the form

$$\frac{E d\sigma}{d^3p} = \frac{A}{\left(1 + \frac{m_T}{m}\right)^n},$$

(1)

where $A$ is a normalization constant, $T$ the 'temperature' parameter, and $n$ the power index, for produced hadrons with a mass $m$ and transverse mass $m_T$.

For $pp$ collisions at $\sqrt{s} = 7$ TeV, the $p_T$ spectra from 0.5 GeV to 181 GeV can be described well by a power index $n = 6.60$ [5]. The good Tsallis fits to
the $p_T$ spectra over such a large range of $p_T$ with only three parameters raise interesting questions. Why are there so few degrees of freedom in the spectra over such a large $p_T$ domain? Mathematically, the power index $n$ is related to the parameter $q=1 + 1/n$ in Tsallis non-extensive statistical mechanics [1]. What is the physical meaning of $n$? If $n$ is related to the power index of the parton-parton scattering law, then why is $n \sim 7$ and not $n \sim 4$ as predicted naively by pQCD? In addition to the power law $1/p_T^n$, does the differential cross section contain other additional $p_T$-dependent factors? Are the power indices for jet production different from those for hadron production? Do multiple parton collisions play any role in modifying the power index $n$? Does the hard scattering process contribute significantly to the production of low-$p_T$ hadrons?

As the relativistic hard-scattering model is the proper model for the high-$p_T$ distribution of jets and hadrons in high-energy collisions [6]-[11], we seek answers to these questions from the relativistic hard-scattering model.

2 Approximate Hard-Scattering Integral

Approximate expressions for the hard-scattering integral were obtained previously for simplifying cases [6, 8, 9]. We would like to work out an approximate analytical expression using the saddle point method [9, 11]. We consider the collision of $A$ and $B$ in the center-of-mass frame at an energy $\sqrt{s}$ with $c$ coming out at $\eta \sim 0$ in the reaction $A + B \rightarrow c + X$. Upon neglecting the intrinsic transverse momentum and rest masses, the differential cross section in the lowest-order parton-parton elastic collisions is given by

$$\frac{E_c d^3 \sigma(AB \rightarrow cX)}{d^3 c} = \sum_{ab} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{E_c d^3 \sigma(ab \rightarrow cX')}{d^3 c}, \quad (2)$$

where the parton-parton invariant cross section is related to $d\sigma(ab \rightarrow cX')/dt$ by

$$E_c \frac{d^3 \sigma(ab \rightarrow cX')}{d^3 c} = \hat{s} \frac{d\sigma(ab \rightarrow cX')}{dt} \delta(\hat{s} + \hat{t} + \hat{u}), \quad (3)$$

and

$$\hat{s} = (a + b)^2, \quad \hat{t} = (a - b)^2, \quad \hat{u} = (b - c)^2. \quad (4)$$

We write out the momenta in the infinite momentum frame,

$$a = (x_a \frac{\sqrt{s}}{2}, \mathcal{O}_T, x_a \frac{\sqrt{s}}{2}), \quad b = (x_b \frac{\sqrt{s}}{2}, \mathcal{O}_T, -x_b \frac{\sqrt{s}}{2}),$$

$$c = (x_c \frac{\sqrt{s}}{2} + \frac{c_T^2}{2x_c \sqrt{s}}, \mathcal{O}_T, x_c \frac{\sqrt{s}}{2} - \frac{c_T^2}{2x_c \sqrt{s}}).$$

The light-cone variable $x_c$ of the produced parton $c$ is

$$x_c = \frac{c_0 + c_z}{\sqrt{s}}. \quad (5)$$

The constraint of $\hat{s} + \hat{t} + \hat{u} = 0$ gives

$$x_a(x_b) = x_c + \frac{c_T^2}{(x_b - \frac{c_T^2}{x_c \sqrt{s}}) s}. \quad (6)$$
In this special case with \( c \) coming out at \( \theta_c = 90^\circ \), we have
\[
x_c = \frac{c_T}{\sqrt{s}}, \quad x_a(x_b) = x_c + \frac{x_c^2}{x_b - x_c}, \quad \text{and} \quad x_a = x_b = 2x_c.
\] (7)

We have therefore
\[
E_C \frac{d^3\sigma(AB \rightarrow cX)}{dc^3} = \sum_{ab} \int dx_b dx_a G_a(A) G_b(B) x_a x_b \delta(x_a - x_a(x_b)) \frac{d\sigma(ab \rightarrow cX')}{dt}. \]

We define
\[
G_a(x_a) = x_a G_a(A), \quad G_b(x_b) = x_a G_b(B). \] (8)

After integrating over \( x_a \), we obtain
\[
E_C \frac{d^3\sigma(AB \rightarrow cX)}{dc^3} = \sum_{ab} \int dx_b \frac{G_a(x_a(x_b)) G_b(x_b)}{\pi(x_b - c_T^2/x_c^3)} \frac{d\sigma(ab \rightarrow cX')}{dt}. \] (9)

To integrate over \( x_b \), we use the saddle point method to write
\[
G_a(x_a(x_b)) G_b(x_b) = e^{f(x_b)}, \] (10)

and expand \( f(x_b) \) about its minimum at \( x_b=0 \). We obtain
\[
\int dx_b e^{f(x_b)} g(x_b) \sim e^{f(x_{b0})} g(x_{b0}) \sqrt{\frac{2\pi}{-\partial^2 f(x_b)/\partial x_b^2 |_{x_b=x_{b0}}}}. \] (11)

For simplicity, we assume \( G_a/A \) and \( G_b/B \) to have the same form. At \( \theta_c \sim 90^\circ \) in the CM system, the minimum value of \( f(x_b) \) is located at
\[
x_{b0} = x_{a0} = 2x_c. \] (12)

We get
\[
E_C \frac{d^3\sigma(AB \rightarrow cX)}{dc^3} \sim \sum_{ab} B[x_{a0} G_a(A) [x_{b0} G_b(B)] \frac{d\sigma(ab \rightarrow cX')}{dt}], \] (13)

where
\[
B = \frac{1}{\pi(x_b - c_T^2/x_c^3)} \sqrt{\frac{2\pi}{-\partial^2 f(x_b)/\partial x_b^2 |_{x_b=x_{b0}}}}. \] (14)

For the case of \( G_a(x_a) = x_a G_a(A) = A_a(1 - x_a)^{\alpha_a} \), we find
\[
E_C \frac{d^3\sigma(AB \rightarrow cX)}{dc^3} \sim \sum_{ab} A_a A_b (1 - x_{a0})^{\alpha_a} + \frac{\delta}{\pi \sqrt{x_c (1 - x_c)}} \frac{d\sigma(ab \rightarrow cX')}{dt}. \] (15)

If the basic process \( ab \rightarrow cX' \) is \( gg \rightarrow gg \), the cross section at \( \theta_c \sim 90^\circ \) [12] is
\[
\frac{d\sigma(gg \rightarrow gg)}{dt} \sim \frac{9\pi\alpha_s^2}{16c_T^4} \left[ \frac{3}{2} \right]^3. \] (16)

If the basic process \( ab \rightarrow cX' \) is \( qq' \rightarrow qq' \), the cross section at \( \theta_c \sim 90^\circ \) [12] is
\[
\frac{d\sigma(qq' \rightarrow qq')}{dt} = \frac{4\pi\alpha_s^2}{9c_T^4} \frac{5}{16}. \] (17)

In either case, the differential cross section varies as \( d\sigma(ab \rightarrow cX')/dt \sim \alpha_s^2/(c_T^2)^2 \).
3 Parton Multiple Scattering

With increasing collision energies, we probe regions of smaller $x$, where the parton density increases rapidly. The number of partons and the total hard-scattering cross section in $pp$ collisions increases with increasing collision energies [7]. The presence of a large number of partons in the colliding system leads to parton multiple scattering in which a projectile parton may make multiple hard scatterings with target partons. It is of interest first to explore how the power index may be affected by the multiple scattering process.

We find that for the collision of a parton $a$ with a target of $A$ partons in sequence without centrality selection, the differential $c_T$ distribution is given by

$$
\frac{d\sigma^{(\text{tot})}}{dc_T} = A \alpha_s^2 \int db \ T(b) \ 
+ \frac{A(A-1) 16 \pi \alpha_s^4}{2} \frac{\ln \{c_T \}}{c_T^2} \int db |T(b)|^2 \ 
+ \frac{A(A-1)(A-2)}{6} 936 \pi^2 \alpha_s^6 \frac{\ln \{\ln (c_T^2/p_0^2)\}}{c_T^2} \int db |T(b)|^3,
$$

where the terms on the right-hand side correspond to collisions of the incident parton with one, two and three target partons, respectively. The quantity $A$ is the integral of the parton density (structure function) over the parton momentum fraction. This result shows that without centrality selection in minimum-biased events, the differential cross section will be dominated by the contribution from a single parton-parton scattering that behaves as $\alpha_s^2/c_T^4$ for the production of partons in the highest $p_T$ region, in line with previous analyses on the multiple scattering process in [13, 14, 15]. Multiple scatterings with $N > 1$ scatterers contribute to terms of order $\alpha_s^{2N} \cdot [\ln (C_T/Np_0)]^{N-1}/c_T^{2+2N}$ [11].

4 The Power Index in Jet Production

From the results in the above sections, the approximate analytical formula for hard-scattering invariant cross section $\sigma_{\text{inv}}$, for $A + B \rightarrow c + X$ at $\eta \sim 0$, is

$$
E_c d^3\sigma(AB \rightarrow cX) \propto \frac{\alpha_s^2 (1 - x_{a0}(c_T))^{g_a + \frac{1}{2}} (1 - x_{b0}(c_T))^{g_b + \frac{1}{2}}}{c_T^{2} \sqrt{c_T/s} \sqrt{1 - x_c}}. \quad (19)
$$

The power index $n$ has the value $4+1/2$ in the above analytical expression. One can plot $\ln \sigma_{\text{inv}}$ as a function of $\ln c_T$, and the slope in the linear section gives the value of $n$, and the variation of $\ln \sigma_{\text{inv}}$ at high $\ln c_T$ gives the value of $g_a$ and $g_b$. On can also extract the value of the power index $n(x_c)$ by considering a fixed $x_c$ and looking at two different energies as suggested by Arleo et al. [10]

$$
\frac{\ln \sigma_{\text{inv}}(\sqrt{s_1}, x_c)/\sigma_{\text{inv}}(\sqrt{s_2}, x_c)}{\ln[s_2/s_1]} \sim n(x_c) - \frac{1}{2}, \quad (20)
$$

4
We alternatively analyze the $p_T$ spectra by using a running coupling constant

$$\alpha_s(Q(c_T)) = \frac{12\pi}{27\ln(C + Q^2/\Lambda_{QCD}^2)},$$

(21)

where $\Lambda_{QCD}$ is chosen to be 0.25 GeV to give $\alpha_s(M_Z^2) = 0.1184$ [16], and the constant $C$ is chosen to be 10, both to give $\alpha_s(Q/\Lambda_{QCD}) \sim 0.6$ in hadron spectroscopy studies [17] and to regularize the coupling constant for small values of $Q(c_T)$. We identify $Q$ as $c_T$ and search for $n$ by writing the invariant cross section for jet production as

$$E_c \frac{d^3\sigma(AB \rightarrow cX)}{d c^3} = \frac{Aa^2_s(Q^2(c_T))(1 - x_{a0}(c_T))^g_a + \frac{1}{2}}{c_T^2 \sqrt{1 - x_c}} \frac{(1 - x_{b0}(c_T))^g_b + \frac{1}{2}}{x_c^2},$$

(22)

$$E_T (GeV)$$

|          | 10^{-6} | 10^{-5} | 10^{-4} | 10^{-3} | 10^{-2} | 10^{-1} | 10^{0}  | 10^{1}  |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|
| $E_T$ (GeV) | 50  | 100 | 200 | 300 | 400 | 500 | 600 | 700 |

Figure 1: (Color online) Comparison of the relativistic hard-scattering model results for jet production, Eq. (22) (solid curves), with experimental $d\sigma/d\eta E_T dE_T$ data from the D0 Collaboration [21], for hadron jet production within $|\eta| < 0.5$, in $\bar{p}p$ collision at (a) $\sqrt{s} = 1.8$ TeV, and (b) $\sqrt{s} = 0.63$ TeV.

The exponential index $g_a = g_b$ for the structure function of a gluon varies from 6 to 10 in different structure functions [18, 19, 20]. We shall take $g_a = 6$ from [18]. As shown in Fig. 1, D0 $d\sigma/d\eta E_T dE_T$ data [21] for hadron jet production within $|\eta| < 0.5$ can be fitted with $n = 4.39$ for $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV, and with $n = 4.47$ for $\bar{p}p$ collisions at $\sqrt{s} = 0.630$ TeV. In other comparisons with the ALICE data for jet production in $pp$ collisions at $\sqrt{s} = 2.76$ TeV at the LHC within $|\eta| < 0.5$ [22], the power index is $n = 4.78$ for $R = 0.2$, and is $n = 4.98$ for $R = 0.4$ (Table I). The power index is $n = 5.39$, for CMS jet differential cross section in $pp$ collisions at $\sqrt{s} = 7$ TeV at the LHC within $|\eta| < 0.5$ and $R = 0.5$ [23]. This latter $n$ value is slightly greater than the expected value of $n = 4.5$. 

5
Except for the CMS data at 7 TeV that may need further re-examination, the power indices extracted for hadron jet production as listed in Table I are in approximate agreement with the value of $n=4.5$ in Eq. (19) and with previous analysis of Arleo et al. [10], indicating the approximate validity of the hard-scattering model for jet production in hadron-hadron collisions, with the predominant $\alpha_s^{2}/\epsilon_T^4$ parton-parton differential cross section as predicted by pQCD.

| Collaboration | $\sqrt{s}$ | $R$ | $|\eta|<0.7$ | $n$ |
|--------------|------------|-----|-------------|-----|
| D0           | $\bar{p}p$ at 1.80 TeV | 0.7 | 4.39        |
| D0           | $\bar{p}p$ at 0.63 TeV | 0.7 | 4.47        |
| ALICE        | $pp$ at 2.76 TeV | 0.2 | 4.78        |
| ALICE        | $pp$ at 2.76 TeV | 0.4 | 4.98        |
| CMS          | $pp$ at 7 TeV | 0.5 | 5.39        |

Table 1: The power index for jet production in $\bar{p}p$ and $pp$ collisions

5 Phenomenological Modifications for Hadron Productions

Equation (22) describes jet production. To apply Eq. (22) for the case of hadron production, it is necessary to take into account additional effects. Jets undergoes fragmentation and hadronization to produce the observed hadrons. From the fragmentation function for a parent parton jet to fragment into hadrons [24], an observed hadron $p_T$ of transverse momentum $p_T$ can be estimated to arise (on the average) from the fragmentation of a parent jet $\bar{c}_T$ [11],

$$\bar{c}_T = 2.3p_T.$$  

Furthermore, the power law $1/p_T^n$ appropriate for high $p_T$ needs to be regularized for low $p_T$. We can regularize the factor $1/p_T^n$ as $1/(1 + m_T/m_{T0})$ by a linear dependence on the transverse mass, $m_T = \sqrt{m^2 + p_T^2}$, where $m$ is the pion mass. With such a regularization, we examine empirically the power index $n$ in the hadron production process $A+B \rightarrow p+X$ by modifying Eq. (22) from the case for jet production to the case for hadron production as

$$d^3\sigma(AB \rightarrow pX)/dydp_T \propto \alpha_s^2(\bar{c}_T)(1-x_{a0}(\bar{c}_T))^{\bar{g}_a+1/2}(1-x_{b0}(\bar{c}_T))^{\bar{g}_b+1/2}\left[1+m_T/m_{T0}\right]^n\sqrt{1-x_c},$$  

Comparing the above equation with the hadron transverse momentum distributions in $pp$ collisions at the LHC from the CMS [25], ATLAS [26], and ALICE Collaborations [27] in Fig. 2(b), we find $n = 5.69$ and $m_{T0} = 0.804$ GeV for $\sqrt{s} = 7$ TeV, and $n = 5.86$, and $m_{T0} = 0.634$ GeV for $\sqrt{s} = 0.9$ TeV (Table II). If we introduce $q = 1 + 1/n$ and $T = m_{T0}/q - 1$, then we get a distribution that contains the Tsallis distribution of Eq. (1) as a factor. The difference is the additional $p_T$-dependencies on $\alpha_s^2(\bar{c}_T)$, $x_{a0}(\bar{c}_T)$, $x_{b0}(\bar{c}_T)$, and $x_c$. 

6
Figure 2: (Color online) Comparison of the experimental \( \langle E_p d^3N/dp^3 \rangle_\eta \) data for hadron production in \( pp \) collisions at the LHC with the relativistic hard-scattering model results (solid and dashed curves) (a) using Eq. (24), with a quadratic \( m_{T} \) dependence of the regulating function, and (b) using Eq. (25), with a linear \( m_{T} \) dependence of the regulating function.

Equation (24) is not the only way we can regularize the \( 1/p_T \) behavior. The gluon exchange propagator in the Feynman diagrams involves the quantity \( 1/p_T^2 \). We can regularize \( 1/p_T^2 \) by \( 1/(1+m_{T}^2/m_{T0}^2) \), with a quadratic dependence on \( m_{T}^2 \).

We can therefore alternatively modify Eq. (22) from the case of jet production to the case for hadron production as

\[
d^3\sigma(AB \rightarrow pX) \propto \alpha_s^2(\bar{c}T)(1-x_{a0}(\bar{c}T))^{n_a+1/2}(1-x_{b0}(\bar{c}T))^{n_b+1/2} \left[ 1 + m_{T}^2/m_{T0}^2 \right]^{n/2} \sqrt{1-x_c}
\]

By comparing the above equation (25) with experimental \( \langle E_p d^3N/dp^3 \rangle_\eta \) data for hadron production in \( pp \) collisions at the LHC from the CMS[25], ATLAS[26], and ALICE Collaborations[27], we find \( n=5.45 \) and \( m_{T0}=1.09 \) GeV for \( \sqrt{s}=7 \) TeV, and \( n=5.49 \) and \( m_{T0}=0.837 \) GeV for \( \sqrt{s}=0.9 \) TeV (See Fig. 2(a)). We list the parameters that describe the \( p_T \) distributions in Table II.

Comparing the results from the two different ways of expressing the power-law behaviors, we find that the agreements of the data with the theoretical curves are nearly the same above \( p_T \sim 3 \) GeV/c, but the theoretical results with the linear \( m_{T} \) dependence of Eq. (24) are less than the experimental ALICE data for \( p_T \sim 2 \) GeV/c but greater than the experimental data for \( p_T \lesssim 0.5 \) GeV/c. On the other hand, the quadratic \( m_{T}^2 \) expression of Eq. (25), that is a more natural regularization from the field theory point of view involving gluon propagators, leads to a better agreement in the lower \( p_T \) region.
For pp collisions at the LHC, the above comparisons indicate that the power index extracted from hadron spectra has the value of $n \sim 6$. The power indices for hadron production are slightly greater than the power indices of $n \sim 4-5$ extracted from jet transverse differential cross sections. Considering the differences between jets and hadrons, we infer that the fragmentation and showering processes increase slightly the value of the power index $n$ of the transverse spectra.

### Table 2: The power index $n$ and $m_{T0}$ for hadron production in pp collisions.

| Linear $m_T$  | Quadratic $m_T^2$ |
|----------------|-------------------|
| Eq. (24)        | Eq. (25)          |
| $\sqrt{s}=7\text{TeV}$ | $\sqrt{s}=0.9\text{TeV}$ | $\sqrt{s}=7\text{TeV}$ | $\sqrt{s}=0.9\text{TeV}$ |
| $n$  | 5.69 | 5.86 | 5.45 | 5.49 |
| $m_{T0}$ (GeV) | 0.804 | 0.634 | 1.09 | 0.837 |

### 6 Conclusions and Discussions

Using the saddle point integration method, we obtained an approximate analytical expression for the differential hard-scattering cross section at $\eta \sim 0$ with a power index of $4+1/2$ in pQCD, in approximate agreement with the experimental data for jet production. The power indices for hadron production is empirically found to be slightly greater than those for jet production.

With the regularization of both the power law $1/p_T^4$ and the running coupling constant $\alpha_s$ for small values of $p_T$, the hard-scattering model has been applied to extrapolate to hadron production in the low-$p_T$ region in Fig. 2. It should be noted that in this low-$p_T$ region, the hard-scattering cross section will be modified by the intrinsic $p_T$ of the partons [9], the parton recombination effects [28], and the small $x$ dependencies of the structure functions. Nevertheless, the extrapolation of the hard-scattering results to the low-$p_T$ region as obtained here in Fig. 2 indicates indeed that the hard-scattering process can contribute substantially to the production of particles in the low-$p_T$ region.

Regarding the Tsallis distribution which motivates the present investigation, we can conclude that the successes of representing the transverse spectra at high-$p_T$ by a Tsallis distribution arises from (i) the simple power-law behavior of the parton-parton scattering cross section, $\alpha_s^2/c_T^4$, with a power index of 4, (ii) the few number of the degrees of freedom in the hard-scattering model, and (iii) the power index of 4 that is not significantly modified by the multiple scattering process at high $p_T$ in minimum biased measurements. The $\alpha_s^2/p_T^4$ power law lays the foundation for Tsallis/Hegedorn-type transverse momentum distributions, and the few degrees of freedom in the Tsallis distribution is a reflection the few degrees of freedom in the underlying hard-scattering model. There are additional $p_T$ dependencies due to the parton structure function, the running coupling constant, and the parton momentum integration, which lead to a slightly larger power index. Furthermore, in going from the parton measurements in terms of jets to hadron measurements in terms of fragmented hadron products, there are additional showering and fragmentation processes.
which give rise to a greater value of the power index. The Tsallis distribution is flexible enough to adjust the power index to accommodate the different and changing environment, yielding a non-statistical description of the distribution.

Because of its non-statistical nature, the parameters in a Tsallis distribution can only be supplied and suggested from non-statistical means, such as the QCD basic parton-parton scattering power index and the QCD multiple scattering shadowing effects. It also is limited in its application to the transverse degree of freedom, as there is no way to generalize the Tsallis parameters across the three-dimensional space from transverse to longitudinal coordinates. For a more fundamental description, it is necessary to turn to the basic parton model for answers. The underlying relativistic hard-scattering model has a greater range of applications and a stronger theoretical foundation.

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References

[1] C. Tsallis, J. Stat. Phys. 52, 479 (1988), and Eur. Phys. J. A40, 257 (2009); cf. also C. Tsallis, Introduction to Nonextensive Statistical Mechanics (Berlin 2009: Springer).

[2] M. Rybczyński, Z. Wlodarczyk and G. Wilk, Nucl. Phys. (Proc. Suppl.) B97, 81 (2001); F. S. Navarra, O. V. Utyuzh, G. Wilk, and Z. Wlodarczyk, Phys. Rev. D67, 114002 (2003); G. Wilk and Z. Wlodarczyk, J. Phys. G38 065101 (2011); G. Wilk and Z. Wlodarczyk, Eur. Phys. J. A48, 161 (2012); G. Wilk and Z. Wlodarczyk, Cent. Eur. J. Phys. 10, 568 (2012); G. Wilk and Z. Wlodarczyk, Eur. Phys. J. A40, 299 (2009); M. Rybczyński, Z. Wlodarczyk, and G. Wilk, J. Phys. G39, 095004 (2012).

[3] T. Wibig, J. Phys. G37, 115009 (2010); K. Ürmösy, G. G. Barnaföldi and T. S. Biró, Phys. Lett. B701, 111 (2012), and B718 125 (2012); J. Cleymans and D. Worku, J. Phys. G39, 025006 (2012); J. Cleymans and D. Worku, Eur. Phys. J. A48, 160 (2012).

[4] T. S. Biró, K. Ürmösy and Z. Schram, J. Phys. G37, 094027 (2010); T. S. Biró and P. Ván, Phys. Rev. E83, 061147 (2011); T. S. Biró and Z. Schram, EPJ Web of Conferences 13, 05004 (2011).

[5] C. Y. Wong and G. Wilk, Acta Phys. Pol. B43, 2047 (2012).

[6] R. Blankenbecler and S. J. Brodsky, Phys. Rev. D10, 2973 (1974); R. Blankenbecler, S. J. Brodsky, and J. Gunion, Phys. Rev. D12, 3469 (1975); E. A. Schmidt and R. Blankenbecler, Phys. Rev. D15, 332 (1977); R. Blankenbecler, Lectures presented at Tübingen University, Germany, June 1977, SLAC-PUB-2077 (1977).
[7] T. Sjöstrand and M. van Zijl, Phys. Rev. D36, 2019 (1987); R. Corke and T. Sjöstrand, JHEP 1001, 035 (2010); R. Corke and T. Sjöstrand, JHEP 1103, 032 (2011).

[8] C. Y. Wong, Introduction to High-Energy Heavy-Ion Collisions, World Scientific Publisher, 1994.

[9] C. Y. Wong and H. Wang, Phys. Rev. C58, 376 (1998).

[10] F. Arleo, S. Brodsky, D. S. Hwang, and A. M. Sickles, Phys. Rev. Lett. 105, 062002 (2010).

[11] C. Y. Wong and G. Wilk, Phys. Rev. D87, 114007 (2013).

[12] R. Gastman and T. T. Wu, The Ubiquitous Photon, Clarendon Press, Oxford, 1990.

[13] K. Kastella, Phy. Rev. D36, 2734 (1987).

[14] G. Calucci and D. Treleani, Phys. Rev. D41, 3367 (1990), Phys. Rev. D44, 2746 (1990); Int. Jour. Mod. Phys. A6, 4375 (1991); Phys. Rev. D49, 138 (1994); Phys. Rev. D50, 4703 (1994); Phys. Rev. D63, 116002 (2001); A. Accardi and D. Treleani, Phys. Rev. D64, 116004 (2001).

[15] M. Gyulassy, P. Levai, I. Vitev, Nucl. Phys. B594, 371 (2001).

[16] J. Beringer et al., (Particle Data Group), Phys. Rev. D86, 010001 (2012).

[17] C. Y. Wong, E. S. Swanson, and T. Barnes, Phy. Rev. C, 65, 014903 (2001).

[18] D. W. Duke and J. F. Owens, Phy. Rev D30, 49 (1984).

[19] S. Chekanov et al., (ZEUS Collaboration), Phy. Rev. D67, 012007 (2003).

[20] S. Chekanov et al., (ZEUS Collaboration), Eur. Phys. J. C42, 1 (2005).

[21] B. Abbott et al. (D0 Collaboration), , Phys. Rev. D 64, 032003 (2001).

[22] B. Abelev et al., (ALICE Collaboration), Phys. Lett. B722, 262 (2013).

[23] S. Chatrchyan et al. (CMS Collaboration), Phys. Rev. Lett. 107, 132001 (2011) [arxiv:1106.0208]; also arxiv:1212.6660 (2013).

[24] J. Binnewies, B. A. Kniehl and G. Kramer, Z. Phys. C65, 471 (1995).

[25] V. Khachatryan et al. (CMS Collaboration), JHEP 02, 041 (2010) and Phys. Rev. Lett. 105, 022002 (2010); V. Khachatryan et al. (CMS Collaboration), JHEP 08, 086 (2011) , [arxiv:1104.3547].

[26] G. Aad et al. (ATLAS Collaboration), New J. Phys. 13, 053033 (2011).

[27] K. Aamodt et al. (ALICE Collaboration), Phys. Lett. B693, 53; Eur. Phys. J. C 71, 1594 (2011) and 1655 (2010).

[28] R. C. Hwa and C. B. Yang, Phys. Rev. C67, 034902 (2003).