Quantum spherical spin-glass with random short-range interactions

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Abstract

In the present paper we analyze the critical properties of a quantum spherical spin glass model with short range, random interactions. Since the model allows for rigorous detailed calculations, we can show how the effective partition function calculated with help of the replica method for the spin glass fluctuating fields $Q_{\alpha\gamma}(\vec{k}\omega_1\omega_2)$ separates into a mean field contribution for the $Q_{\alpha\alpha}(0;\omega;\omega)$ and a strictly short range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k}\omega_1\omega_2)$. Here $\alpha, \gamma = 1..n$ are replica indices. The mean field part $W_{MF}$ coincides with previous results. The short range part $W_{SR}$ describes a phase transition in a $Q^3$-field theory, where the fluctuating fields depend on a space variable $\vec{r}$ and two times $\tau_1$ and $\tau_2$. This we analyze using the renormalization group with dimensional regularization and minimal subtraction of dimensional poles. By generalizing standard field theory methods to our particular situation, we can identify the critical dimensionality as $d_c = 5$ at very low temperatures due to the dimensionality shift $D_c = d_c + 1 = 6$. We then perform a $\epsilon'$ expansion to order one loop to calculate the critical exponents by solving the renormalization group equations.

PACS numbers: 64.60.Cn; 75.30.M; 75.10.N
1. Introduction

Since the formulation of the renormalization group theory to explain the critical behaviour and scaling properties of phase transitions, it emerged the natural question of how this theory would apply to phase transitions in quantum systems. In these systems, time plays an essential role through the equations of motion of the operators even in equilibrium quantum statistical mechanics, then a natural conjecture was that there would be a dimensional shift from the space dimension \( d \) to the effective \( D = d + 1 \), and that scaling behaviour in the critical region would require the introduction of a new critical dynamical exponent \( z \) \cite{[1], [2]}. This is evident when the quantum mechanical partition function is written as a functional integral in terms of fields that are functions of position and imaginary time \( \tau \) variables, where \( 0 \leq \tau \leq \beta \) and \( \beta = \frac{1}{T} \) is the inverse temperature in units with \( \hbar = k_B = 1 \), as phase transitions occur in infinite systems when the correlation length \( \xi \) becomes infinite at the critical temperature \( T_c \). If \( T_c > 0 \), the "length" \( \beta_c \) in the imaginary time direction is finite and the associated correlation length \( \xi^z > \beta_c \), then the transition would be classical in \( d \) space dimensions. However, if quantum fluctuations drive the critical temperature to \( T_c = 0 \), at this point the time length \( \beta_c \) is infinite and a new transition with a dimensional shift \( D = d + 1 \) is expected at a quantum critical point (QCP) \cite{[3]}. Physical realizations of quantum phase transitions occur in strongly correlated systems\cite{[4]} and other physical systems as described extensively in ref.(\cite{[5]}). A particular class of systems that present a quantum critical point are quantum spin glasses like the insulating \( LiHo_2Y_{1-x}F_4 \) \cite{[6]}, that is well represented by the \( M \)-component spin-glass model in a transverse field\cite{[7]} or
by the spin-glass model of $M$-components quantum rotors\[5, 8\]. In the limit $M \to \infty$ the quantum rotors model\[9\] reduces to the quantum spherical model for a spin glass that has been studied before\[10\] in the mean field limit of infinite range interactions, following the classical spin glass theory of Sherrington-Kirkpatrick (SK)\[11\]. It was also shown that the effective action of the quantum spherical spin glass is invariant\[14\] under the Becchi-Rouet-Stora-Tyutin supersymmetry and consequently the spin glass order parameter vanishes while replica symmetry (RS) is exact. The $p$-spin quantum spherical model was also studied by using the boson operators representation for the harmonic oscillator\[12, 13\] and it was shown that the systems with $p = 2$ and $p \geq 3$ belong to different universality classes. For $p \geq 3$ there is replica symmetry breaking (RSB) and the system belongs to the same universality class as the SK model, with a finite order parameter $Q \neq 0$ below the critical temperature.

Since the formulation of (SK) spin-glass theory with infinite range interactions, the natural question was asked of how finite range, random interactions would modify the critical properties of spin glasses and which would be the critical exponent associated to them. To answer this question, renormalization group calculations were performed above criticality\[15\] in an expansion in $\epsilon = 6 - d$ for short range interactions, from where emerges $d_c = 6$ as the critical dimensionality of the classical spin-glass. Renormalization group calculations for long range interactions decaying as $r^{-(d+\sigma)}$ in the classical spin-glass were also performed\[16, 17\]. Below the critical temperature there is replica symmetry breaking (RSB) and a non-vanishing order parameter that is in fact a matrix in replica space, then a more difficult renorma-
tion group in replica space should be performed, as it is discussed in detail in ref. ([18]). Here also the critical dimensionality appears to be \( d_c = 6 \), thus completing the description of the short-range classical spin glass below the critical temperature.

It is the purpose of this paper to analyze the critical properties of a quantum spherical spin glass with random short range interactions by using renormalized perturbation theory with dimensional regularization and minimal subtraction of dimensional poles [19]. This task is far from trivial as the structure of the resulting field theory differs from standard theories. To start with, the quantum rotor model without disorder is covariant in space and time [5], the frequency appearing just as a new momentum component to form a \((d+1)\)-dimensional vector with modulus \( k^2 + \omega^2 \), giving thus the value of the exponent \( z = 1 \), but this is not the case in the disordered model. The disorder has no dynamical fluctuations and if average over disorder restores space translational invariance, this is not necessarily so in the time direction. Consequently, we are forced to consider an effective action in terms of a spin glass field that depends on position \( \vec{r} \) and on two imaginary times \( \tau_1 \) and \( \tau_2 \) (not on the two times difference). This carries on the need to formulate in this paper new rules for the calculation of diagrams in the loop expansion [19]. We find accordingly that under renormalization the relation between space and imaginary time changes and as a result the exponent \( z \) differs from unity.

The quantum spherical model for a spin glass is the ideal testing ground for these ideas, as its formulation in terms of functional integrals allows for rigorous analytic calculations. As a difference with the infinite range quantum spherical spin glass [14], in the case of short range interactions we have to use
the replica method to derive the effective action, and this we do in Sect.2. In Sect.3 we derive the renormalized perturbation theory to order one loop in an expansion in $\epsilon' = D-6 = d-5$, then our critical space dimension is effectively $d_c = 5$. We calculate the critical exponents by solving the renormalization group equations in the critical region. We leave Sect.4 for discussions. The detailed and far from trivial calculations are described in the Appendix to keep the natural flow of calculations in the paper.
2. The model

We consider a spin glass of quantum rotors with moment of inertia $I$ in the spherical limit with Hamiltonian

$$H_{SG} + \mu \sum_i S_i^2 = \frac{1}{2I} \sum_i P_i^2 - \frac{1}{2} \sum_{i,j} J_{ij} S_i S_j + \mu \sum_i S_i^2$$  \hspace{1cm} (1)$$

where the spin variables at each site are continuous $-\infty < S_i < \infty$ and we introduced the canonical momentum $P_i$ with commutation rules:

$$[S_j, P_k] = i\delta_{j,k}$$  \hspace{1cm} (2)$$

The sum in eq. (1) runs over sites $i, j = 1..N$. The bond coupling $J_{ij}$ in eq. (1) is an independent random variable with the gaussian distribution

$$P(J_{ij}) = e^{-J_{ij}^2 / 2V_{ij}} \sqrt{\frac{1}{2\pi J^2 V_{ij}}}$$  \hspace{1cm} (3)$$

and $V_{ij} = V(\vec{R}_i - \vec{R}_j)$ is a short range interaction with Fourier transform at low momentum $k$

$$V(k) \approx 1 - k^2$$  \hspace{1cm} (4)$$

The chemical potential $\mu$ is a Lagrange multiplier that insures the mean spherical condition

$$- \frac{\partial \langle \ln W \rangle}{\partial (\mu)} = \sum_i \int_0^\beta d\tau \langle S_i^2 \rangle = \beta N$$  \hspace{1cm} (5)$$

and $\beta = 1/T$ is the inverse temperature. We work in units where the Boltzmann constant $k_B = \hbar = 1$ and $W$ is the quantum partition function

$$W = Tr e^{-\beta (H_{SG} + \mu \sum_i S_i^2)}$$  \hspace{1cm} (6)$$
That can be expressed as a functional integral\cite{14,20,21}

\[ W = \int \prod_i DS_i \exp (-A_O - A_{SG}) \] (7)

where the non interacting action \( A_O \) is given by

\[ A_O = \int_0^\beta d\tau \sum_i \left( \frac{I}{2} \left( \frac{\partial S_i}{\partial \tau} \right)^2 + \mu S_i^2(\tau) \right) \] (8)

and the interacting part

\[ A_{SG} = \frac{1}{2} \sum_{i,j} J_{ij} \int_0^\beta d\tau S_i(\tau)S_j(\tau) \] (9)

The free energy may be calculated with the replica method

\[ F = -\frac{1}{\beta N} \lim_{n \to 0} \frac{W_n - 1}{n} \] (10)

where \( \langle W^n \rangle_{\alpha\alpha} = W_n \) is the partition functional for \( n \)-identical replicas, configurationally averaged over the probability distribution of \( J_{ij} \) in eq.(3).

It is shown in the Appendix that \( W_n \) may be expressed as a functional over fluctuating spin glass fields \( Q_{\alpha\gamma}(\vec{k},\omega,\omega') \), where \( \omega = \frac{2\pi m}{\beta} \) is a discrete Matsubara frequency for finite temperature and \( \alpha, \gamma = 1..n \) are replica indices. The result obtained in eq.(10) is that the partition functional separates into two parts

\[ W_n = W_{MF}W_{SR} \] (11)

where \( W_{MF} \) in eq.(17) is the mean field functional for the fields \( Q_{\alpha\alpha}(0,\omega,-\omega) \) already obtained in ref.(14) that determines the critical temperature \( T_c(I) \), while \( Z_{SR} \) depends on the spin glass fluctuations \( Q_{\alpha\neq\gamma}(\vec{k},\omega,\omega') \) for short range interactions and determines the critical behaviour. We remark that
these fields depend naturally on two independent times (frequencies) and not on the difference of two times, because the disorder is not time correlated and it restores translational invariance in space, but not in time\[8\]. We obtain from eq.(49)

\[
W_{SR} = \int \prod_{\alpha \neq \gamma} \mathcal{D}Q_{\alpha \gamma}(\vec{k}, \omega, \omega') \exp(-A_{SR}\{Q\})
\]

where \(\alpha, \gamma = 1..n\) are replica indices and

\[
A_{SR}\{Q\} = \sum_{\alpha \neq \gamma} \sum_{\omega_1 \omega_2} \int d\vec{k} \left[ \frac{\mu - \mu_c}{\mu_c} + k^2 + s^2(\omega_1^2 + \omega_2^2) \right] Q_{\alpha \gamma}(\vec{k}, \omega_1, \omega_2) Q_{\alpha \gamma}(-\vec{k}, -\omega_1, -\omega_2)
\]

\[
+ \frac{\lambda}{3!} \sum_{\alpha \neq \gamma \neq \delta} \sum_{\omega_1 \omega_2 \omega_3} \int dk_1 dk_2 Q_{\alpha \gamma}(k_1, \omega_1, \omega_2) Q_{\gamma \delta}(k_2, -\omega_2, \omega_3) Q_{\delta \alpha}(-k_1 - k_2, -\omega_3, -\omega_1)
\]

(13)

Having in mind a renormalization group calculation, the frequency term in the non-interacting inverse propagator is affected by the coefficient \(s\), as it will turn out that momentum and frequency renormalize differently and they cannot be kept both equal to unity. The infinite volume limit was taken in eq.(13), but for the moment the temperature is kept finite and the sums are over discrete Matsubara frequencies. In all the following work is implicit that \(Q_{\alpha \gamma}\) means \(Q_{\alpha \neq \gamma}\), while \(Q_\alpha(\omega)\) means \(Q_{\alpha \alpha}(0, \omega, -\omega)\).

We now proceed with the renormalization group calculation using dimensional regularization and minimal subtraction of dimensional poles\[19\], to one loop order. In eq.(13) we kept only the terms \(O(Q^3)\) because the terms \(O(Q^4)\) would be irrelevant close to the critical dimensionality of a \(Q^3\)-theory, as there is no change in the sign of \(\lambda\) for the gaussian probability distribution of the random bonds\[15\]. To analyze the value of the critical dimensionality we consider separately the case of finite temperature than that of \(T = 0\). In
both cases the vertex functions that present divergencies needing renormalization are the inverse propagator $\Gamma^{(2)}$, the three point vertex function $\Gamma^{(3)}$ and the two point vertex function with one insertion $\Gamma^{(2,1)}$. To one loop order they are given by the diagrams in fig.(2). At this point it is important to distinguish between the system temperature $T$ and the critical parameter $t = \frac{\mu - \mu_c}{\mu_c}$ that measures the approach to criticality.

We start by analyzing the transition at finite temperature $T$. The action in eq.(13) must be dimensionless, then dimensional analysis tells us that, for $\Lambda$ an inverse length

$$[k] = \Lambda \quad [Q] = \Lambda^{-d/2-1} \quad [\lambda] = \Lambda^{3-d/2}$$

and the critical dimensionality is $d_c = 6$, as corresponds to a classical system.

The vertex functions calculated with the usual rules in $\varphi^3$-field theory \[19, 22\] are

$$\Gamma^{(2)}(\vec{k}, \omega_1, \omega_2) = \Gamma^{(0)}(\vec{k}, \omega_1, \omega_2) -
(n - 2)\frac{1}{2} \lambda^2 \sum_\omega \int d\vec{p} G_0(\vec{p}, \omega, \omega_1) G_0(\vec{k} - \vec{p}, \omega_2, -\omega)$$

(15)

where

$$\Gamma^{(0)}(\vec{k}, \omega_1, \omega_2) = t + k^2 + s^2(\omega_1^2 + \omega_2^2) =
= G_0^{-1}(\vec{k}, \omega_1, \omega_2)$$

(16)

and
\( \Gamma^{(3)}(\vec{k}_1, \vec{k}_2, \omega_1, \omega_2, \omega_3) = \lambda + (n - 3) \lambda^3 \sum_\omega \int d\vec{p} G_0(\vec{p}, \omega_1, \omega) G_0(\vec{k}_1 + \vec{p}, -\omega, \omega_2) G_0(\vec{k}_1 + \vec{k}_2 + \vec{p}, -\omega, \omega_3) \) 

(17)

The theory will be renormalized at the critical point \( t = 0 \). To get away from the critical point we should consider a perturbation expansion in \( t \) by means of the insertion\[19\]

\[ \Delta A = \frac{1}{2!} \sum_{\gamma, \nu} \sum_{\omega_1, \omega_2} \int d\vec{q} t(\vec{q}) \int d\vec{p} Q_{\gamma \nu}(\vec{p}, \omega_1, \omega_2) Q_{\nu \gamma}(\vec{q} - \vec{p}, -\omega_2, -\omega_1) \]

(18)

that leads to a third singular vertex function \( \Gamma^{(2,1)} \) with two external legs and one insertion shown in fig.(2)(bottom).

\[ \Gamma^{(2,1)}(\vec{k}, \vec{q}, \omega_1, \omega_2) = 1 + (n - 2) \lambda^2 \sum_\omega \int d\vec{p} G_0(\vec{p}, \omega, \omega_1) G_0(\vec{q} - \vec{p}, -\omega_1, -\omega) G_0(\vec{k} + \vec{p}, \omega, \omega_2) \]

(19)

At finite temperature \( T \) and critical \( t = 0 \), the sums over Matsubara frequencies have only one singular term with \( \omega = 0 \) and the vertex functions are singular when \( \omega_i = 0 \), then we recover the transition for classical spin glasses described by an expansion in \( \epsilon = 6 - d \).\[15\]

A different scenario emerges when \( T \) is near zero. For sufficiently low \( T \) the frequency sums may be replaced by integrals

\[ \sum_\omega \rightarrow \beta \int_{-\infty}^{\infty} d\omega \]

(20)
and now all the frequencies contribute to the renormalization process and the vertex functions in eq. (15), eq. (17) and eq. (19) will be singular at a new effective dimension $D_c = d_c + 1 = 6$, the new critical space dimensionality becoming $d_c = 5$, as predicted [1, 2, 5, 3].
3. Results

In the following we present results for the critical properties in an expansion in $\epsilon' = 5 - d$, to one loop order. The new features that emerge from the calculation are that the frequencies renormalize differently than the momenta, then the exponent $z$ differs from unity and from the exponent $\eta$, depending also on the dimensionality through the $\epsilon'$ expansion. The integrals over momentum and frequency of eq. (15), eq. (17) and eq. (19) are calculated in the Appendix at a space dimensionality $d$, when they converge$^{19, 22}$ and the singularities appear as dimensional poles in $\epsilon'$. We obtain for the singular parts, to leading order in the coupling constant

$$\Gamma^{(2)}_{\alpha\gamma}(\vec{k}, \omega_1, \omega_2) = k^2 + s^2(\omega_1^2 + \omega_2^2) + (n - 2)\frac{1}{6s\epsilon'}u_0^2[k^2 + 3s^2(\omega_1^2 + \omega_2^2)] = [1 + (n - 2)\frac{1}{6s\epsilon'}u_0^2\{k^2 + s^2(\omega_1^2 + \omega_2^2)[1 + \frac{n - 2}{3\epsilon'}u_0^2]\}]$$

(21)

$$\Gamma^{(3)}_{\alpha\gamma\delta} = u_0k\epsilon'/2[1 + (n - 3)u_0^2\frac{1}{s\epsilon'}]$$

(22)

$$\Gamma^{(2,1)}_{\alpha\gamma} = 1 + (n - 2)u_0^2\frac{1}{s\epsilon'}$$

(23)
where we introduced the bare dimensionless coupling $u_0$ through

$$\lambda^2 \beta S_{d+1} = u_0^2 \kappa'$$  \hfill (24)$$

and $S_{d+1}$ is the surface of the unit sphere in $d + 1$ dimensions. In $\Gamma^{(3)}$ and $\Gamma^{(2,1)}$ the external momenta and frequencies were taken at the symmetry point $k_1^2 = k_2^2 = -2k_1 \cdot k_2 = \omega_i^2 = \kappa^2$, where $\kappa$ is the scale parameter. In order to cancel the dimensional poles we must introduce a renormalized, dimensionless, coupling $u$ and renormalized vertex functions by means of renormalization of the field $Q_{\alpha \gamma}$ and of the insertion $Q^2_{\alpha \gamma}$ through the functions $Z_Q$ and $Z_{Q^2}$. The correction to the frequency term $\omega_i^2$ in $\Gamma^{(2)}$ in eq.(21) is different than the contribution to $k^2$, then besides the field renormalization $Z_Q$ that keeps the coefficient of $k^2$ equal to unity it is necessary to renormalize also the frequency coefficient $s(u)$. All together we obtain

$$\Gamma_R^{(2)}(u) = Z_Q(u) \Gamma^{(2)}(u_0, s)$$
$$\Gamma_R^{(3)}(u) = Z_Q^2(u) \Gamma^{(3)}(u_0)$$
$$\Gamma_R^{(2,1)}(u) = Z_{Q^2}(u) \Gamma^{(2,1)}(u_0)$$

\hfill (25)$$

where, in the interesting limit $n = 0$

$$u_0 = u[1 + \frac{5}{s\epsilon'} u^2] \quad (a)$$
$$Z_Q(u) = 1 + \frac{1}{3\epsilon'} u^2 \quad (b)$$
$$Z_{Q^2}(u) = 1 + \frac{2}{\epsilon'} u^2 \quad (c)$$
$$s^2 = 1 + \frac{2}{3\epsilon'} u^2 \quad (d)$$

\hfill (26)
From eq. (26-a) we calculate the $\beta$-function

$$
\beta(u) = \kappa \frac{\partial u}{\partial \kappa} \bigg|_{\lambda} = -\frac{\epsilon'}{2} u \left[ 1 - \frac{5}{\epsilon'} u^2 \right]
$$

(27)

that vanishes at the trivial fixed points $u^* = 0$, stable for $\epsilon' < 0$, and $u^{*2} = \frac{1}{5} \epsilon'$, stable for $\epsilon' > 0$. To obtain the critical exponents we have to solve the renormalization group equations[19] for the vertex function $\Gamma_R(\vec{k}, s\omega_i, t, u, \kappa)$ near criticality, where $t = \frac{\mu - \mu_c}{\mu_c}$. Now we have to take into account also the dependence of $s$ on $\kappa$ through the coupling $u$, so calling $y_i = s\omega_i, i = 1, 2$, we obtain the renormalization group equation at the fixed point $\beta(u^*) = 0$

$$
\left[ \kappa \frac{\partial}{\partial \kappa} + \gamma_s^* \sum_i y_i \frac{\partial}{\partial y_i} - \theta t \frac{\partial}{\partial t} - \eta \right] \Gamma^{(2)}_R(\vec{k}, y_i, t, \kappa) = 0
$$

(28)

where

$$
\eta = \left[ \kappa \frac{\partial}{\partial \kappa} \ln Z_Q \right]_{u=u^*}
$$

$$
\theta = \left[ \kappa \frac{\partial}{\partial \kappa} \ln Z_Q^2 \right]_{u=u^*} - \eta
$$

$$
\gamma_s^* = \left[ \kappa \frac{\partial}{\partial \kappa} \ln s \right]_{u=u^*}
$$

(29)

The solution for $\Gamma^{(2)}_R$ has the scaling form

$$
\Gamma^{(2)}_R(\vec{k}, y_i, t, \kappa) = \kappa^\eta \Phi[\vec{k}; y_i t \kappa^{\theta - \gamma_s^*}]
$$

(30)
where $\Phi$ is a function of the joint variable $y_i t \kappa^{\theta - \gamma^*}$ and dimensional analysis tells us that, for $\rho$ an inverse length:

$$
\Gamma_R^{(2)}(\mathbf{k}, y_i, t, \kappa) = \rho^2 \Gamma_R^{(2)}(\frac{\mathbf{k}}{\rho}, \frac{y_i}{\rho}, \frac{t}{\rho^2}, \frac{\kappa}{\rho}) = \rho^2 \frac{\kappa^\eta}{\rho^\nu} \Phi\left[\frac{\mathbf{k}}{\rho}, \frac{y_i t \kappa^{\theta - \gamma^*}}{\rho^{1+\theta - \gamma^*}}\right]
$$

(31)

If we choose

$$
\rho = \kappa \left(\frac{t}{\kappa^2}\right)^{\frac{1}{2+\theta}}
$$

(32)

we obtain

$$
\Gamma_R^{(2)}(\mathbf{k}, y_i, t, \kappa) = \kappa^2 \left(\frac{t}{\kappa^2}\right)^{\nu(2-\eta)} \Phi\left[\frac{\mathbf{k}}{\kappa}, \left(\frac{t}{\kappa^2}\right)^{-\nu}, \frac{y_i t \kappa^{\theta - \gamma^*}}{\rho^{1+\theta - \gamma^*}}\right]
$$

(33)

from where we identify the space correlation length exponent

$$
\xi = \left(\frac{t}{\kappa^2}\right)^{-\nu} \nu^{-1} = 2 + \theta
$$

(34)

and the time correlation length exponent

$$
\xi_t = \left(\frac{t}{\kappa^2}\right)^{-\nu_z} = \xi^z \quad z = 1 - \gamma^*_z
$$

(35)

From eq.(26), eq.(29), eq.(34) and eq.(35) we obtain the results for the critical exponents, at the non-trivial fixed point

$$
\eta = -\frac{1}{15} \epsilon'; \quad \nu = \frac{1}{2} + \frac{1}{12} \epsilon'; \quad z = 1 + \frac{1}{15} \epsilon'
$$

(36)
4. Conclusions

In the present paper we analyze the critical properties of a quantum spherical spin glass model with short range, random interactions. Since the model allows for rigorous detailed calculations, we can show how the effective partition function calculated with help of the replica method for the spin glass fluctuating fields $Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)$ separates into a mean field contribution for the $Q_{\alpha\alpha}(0,\omega,-\omega)$ and a strictly short range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k},\omega_1,\omega_2)$. Here $\alpha, \gamma = 1..n$ are replica indices. The mean field part $W_{MF}$ coincides with previous results\[14\] and a saddle point calculation provides the phase diagram in fig.(1), as it is discussed in the Appendix. We stress that $Q_{\alpha\alpha}(0,\omega,-\omega)$ is not an order parameter, as it does not vanish above the transition temperature, and the order parameter in the quantum spherical, infinite range, spin glass identically vanish\[14\]. The short range part $W_{SR}$ describes a phase transition in a $Q^3$-field theory, where the fluctuating fields depend on a space variable $\vec{r}$ and two times $\tau_1$ and $\tau_2$. This we analyze using the renormalization group with dimensional regularization and minimal subtraction of dimensional poles\[19\]. By generalizing the method in ref.(19) to our particular situation, we can identify the critical dimensionality as $d_c = 5$ at very low temperatures due to the dimensionality shift $D_c = d_c + 1 = 6$. We then perform an $\epsilon'$ expansion to order one loop to calculate the critical exponents by solving the renormalization group equations, and they are listed in eq.(36).

A general Landau theory of quantum spin glasses of M-components rotors was presented in ref.(8). Based on general properties of symmetry and invariance, the authors present an effective functional for spin glass $Q$-fields,
and at some points we make contact with their results. Our fields, as theirs, are bilocal in time, but our result for the effective functional is simpler and more tractable by standard field theory methods. It is well known than the classical, non-random spherical model is equivalent to the $M \to \infty$ limit of the $M$-vector model. The same equivalence holds between the infinite-range spherical spin glass and the infinite-range $M$-vector spin glass in the classical and in the quantum case. A particular feature of the infinite-range spherical spin glass is that it can be solved exactly without need of the replica method because annealing is exact in this model due to the internal Becchi-Rouet-Stora-Tyutin (BRST) supersymmetry and as a consequence Ward identities tell us that the order parameter identically vanish. In the case of the short-range quantum spherical spin-glass considered here, we showed that replicas are needed and that the partition functional separates exactly into a mean-field part for the replica diagonal $Q_{\alpha\alpha}(k = 0, \omega, -\omega)$ and a short range part for the fluctuating $Q_{\alpha\neq\beta}(k, \omega_1, \omega_2)$ in Eq.\(11\), while in the spin glass of $M$-components quantum rotors with short range disorder considered in Ref.\([8]\) the replica diagonal $Q_{\alpha\alpha}(\omega)$ is considered as an order parameter and a Landau functional is constructed for fluctuations diagonal in replica space around it. This leads to a theory where the time derivatives and the critical mass appear in the linear, in place of quadratic, term in the effective action. As a consequence of having different interactions, the renormalization group equations and critical exponents turn out to be $M$- independent, and the critical dimensionality obtained in Ref.\([8]\) also differs from ours. We conclude this is due to the fact that, in the case of short-range disorder considered here, the quantum spherical spin-
glass model belongs to a different universality class than the $M$-components quantum rotors model in Ref. (8).

We may ask how our results would apply to the quantum $p$-spin spherical model theories in Ref. ([12] [13]), in the case of short range disorder. In these theories the action depends on the first time derivative of the fields, then the inverse propagators would have a linear dependence on frequency (and not quadratic as it is the case here), so the results of a RG calculation remain open.

Acknowledgement

We thank W.K. Theumann for discussions. We gratefully acknowledge financial support from FAPERGS and CNPq.
Appendix

1. Effective functional

We derive here the functional $W_n$ in eq.(9). We obtain by replicating $W$ in eq.(7) and averaging over $P(J_{ij})$ in Eq.(3)

$$W_n = \int \prod_{i,\alpha} D S_{i\alpha} \exp (-\mathcal{A}_\mathcal{O} - \mathcal{A}_{SG})$$

where $\alpha = 1, 2, ..., n$ is the replica index and the free action $\mathcal{A}_\mathcal{O}$ is given by

$$\mathcal{A}_\mathcal{O} = \int_0^\beta d\tau \sum_i \sum_\alpha \left( \frac{1}{2} \left( \frac{\partial S_{i\alpha}}{\partial \tau} \right)^2 + \mu S_{i\alpha}^2(\tau) \right)$$

while the interacting part is

$$\mathcal{A}_{SG} = \frac{J^2}{4} \sum_{i,j} \sum_{\alpha\alpha'} V_{ij} \int_0^\beta d\tau \int_0^\beta d\tau' S_{i\alpha}(\tau) S_{j\alpha}(\tau) S_{i\alpha'}(\tau') S_{j\alpha'}(\tau')$$

We introduce the spin glass fields $Q_{i\alpha\gamma}(\tau, \tau')$ by splitting the quartic term by means of a Stratonovich-Hubbard transformation and we obtain

$$W_n = \int \prod_i \prod_{\alpha\gamma} D Q_{i\alpha\gamma}(\tau, \tau')$$

$$\exp \left[ -\frac{J^2}{4} \sum_{\alpha\gamma} \int_0^\beta \int_0^\beta d\tau d\tau' \sum_{i,j} Q_{i\alpha\gamma}(\tau, \tau') V_{i,j}^{-1} Q_{j\gamma\alpha}(\tau, \tau') \right] \exp [N\Lambda]$$

where

$$\exp [N\Lambda] = \int \prod_{i,\alpha} D S_{i\alpha}(\tau)$$

$$\exp \left[ -\mathcal{A}_\mathcal{O} - \frac{J^2}{2} \sum_{\alpha\gamma} \int_0^\beta \int_0^\beta d\tau d\tau' \sum_j Q_{j\alpha\gamma}(\tau, \tau') S_{j\alpha}(\tau) S_{j\gamma}(\tau') \right]$$
In eq.(41) appear the fields $Q_{j\alpha\gamma}(\tau, \tau')$ depending on two independent times $\tau, \tau'$ and not on the time difference. We define the space and time Fourier transform

$$S_{\alpha}(\vec{k}\omega) = \frac{1}{\beta \sqrt{N}} \int_0^\beta d\tau \sum_j S_{j\alpha}(\tau) \exp -i[\vec{k}.\vec{R}_j + \omega \tau]$$

(42)

$$Q_{\alpha\gamma}(\vec{k}\omega\omega') = \frac{1}{\beta^2 N} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_j Q_{j\alpha\gamma}(\tau\tau') \exp -i[\vec{k}.\vec{R}_j + \omega \tau + \omega' \tau']$$

(43)

where $\omega = \frac{2\pi m}{\beta}$ are bosonic Matsubara frequencies and we obtain from eq.(40)-eq.(43)

$$W_n = \int \prod_{\vec{k}\omega1\omega2} \prod_{\alpha\gamma} dQ_{\alpha\gamma}(\vec{k}\omega1\omega2)$$

$$\exp \left[ -\frac{(\beta J)^2}{4} \sum_{\alpha\gamma} \sum_{\vec{k}\omega1\omega2} Q_{\alpha\gamma}(\vec{k}\omega1\omega2) V(\vec{k})^{-1} Q_{\alpha\gamma}(\vec{k}, -\omega1, -\omega2) \right] \exp [N\Lambda]$$

(44)

where

$$\exp [N\Lambda] = \int \prod_{\alpha} \prod_{\vec{k}\omega} dS_{\alpha}(\vec{k}\omega) \exp \left[ -\sum_{\alpha} \sum_{\vec{k}\omega} \left( \frac{\beta I \omega^2}{2} + \mu \beta \right) S_{\alpha}(\vec{k}\omega) S_{\alpha}(\vec{k} - \omega) \right]$$

$$\exp \left[ \frac{(\beta J)^2}{2} \sum_{\alpha\gamma} \sum_{\vec{k}\vec{q}\omega\omega'} Q_{\alpha\gamma}(\vec{q}\omega\omega') S_{\alpha}(\vec{k}\omega) S_{\gamma}(\vec{k} - \vec{q}, \omega') \right]$$

(45)

and for short range forces $V(k) = 1 + k^2$. The next step is to separate the term with $Q_{\alpha\alpha}(0, \omega, -\omega)$ in eq.(45) that can be introduced into the free action for $S_{\alpha}(\vec{k}\omega)$, with the result

$$W_n = W_{MF} W_{SR}$$

(46)
where \( W_{MF} \) is the mean field partition functional for the \( Q_\alpha(\omega) = Q_{\alpha\alpha}(0\omega - \omega) \) mode

\[
W_{MF} = \int \prod_{\omega\alpha} dQ_\alpha(\omega) \exp \left[ -\frac{N}{2} \sum_{\alpha\omega} \left( \frac{(\beta J)^2}{2} Q_\alpha^2(\omega) + \ln \left( \beta I \omega^2 + 2\beta \mu - (\beta J)^2 Q_\alpha(\omega) \right) \right) \right]
\]

(47)

and \( W_{SR} \) is the partition functional for the fluctuations \( Q_{\alpha \neq \gamma} \)

\[
W_{SR} = \int \prod_{\alpha \neq \gamma} \prod_{\vec{k}\omega} dQ_{\alpha \neq \gamma}(\vec{k}\omega_1\omega_2) \exp \left[ -[A_{\text{free}} + A_{\text{int}}] \right]
\]

(48)

\[
A_{\text{free}} = N \sum_{\vec{k}\omega_1\omega_2} \sum_{\alpha\gamma} Q_{\alpha\gamma}(\vec{k}\omega_1\omega_2) Q_{\gamma\alpha}(-\vec{k}, -\omega_1, -\omega_2) (\beta J)^2 \Gamma_0(\vec{k}\omega_1\omega_2)
\]

(49)

\[
A_{\text{int}} = \frac{(\beta J)^6}{3!} N \sum_{\vec{k}_1, \vec{k}_2} \sum_{\alpha\omega_1} \sum_{\gamma\omega_2} \sum_{\delta\omega_3} Q_{\alpha\gamma}(\vec{k}_1\omega_1\omega_2) Q_{\gamma\delta}(\vec{k}_2, -\omega_2, \omega_3) Q_{\delta\alpha}(-\vec{k}_1 - \vec{k}_2, -\omega_3, -\omega_1) \prod_i g_0(\omega_i)
\]

(50)

where

\[
\Gamma_0(\vec{k}\omega_1\omega_2) = 1 + q^2 - (\beta J)^2 g_0(\omega_1) g_0(\omega_2)
\]

(51)

The function \( g_0(\omega) \) in eq.(50) is the momentum independent, non-interacting two point function for the field \( S_\alpha(\omega) \)

\[
g_0(\omega) = \frac{2}{\beta I \omega^2 + 2\beta \mu - (\beta J)^2 Q_\alpha(\omega)}
\]

(52)
The variables $Q_{\alpha\alpha}(\vec{q} \neq 0, \omega_{1}\omega_{2})$ are not critical and are not coupled to the spin glass field, so we ignore them.

2. Mean Field Solution

At the saddle point of $W_{MF}$ in eq.(47) we obtain

$$2Q_{\alpha}(\omega) = g_0(\omega) \tag{53}$$

The mean spherical condition of eq.(5) reduces to

$$-\frac{1}{n} \frac{\partial}{\partial \mu} \ln W_{MF} = \beta N$$

and it gives at the saddle point

$$\int_{L_{-}}^{L_{+}} dy \sqrt{(L_{+}^{2} - y^{2})(y^{2} - L_{-}^{2})} \coth\left(\frac{\beta y}{2\sqrt{T}}\right) = 2\pi J^{2}\sqrt{I} \tag{54}$$

where

$$L_{\pm}^{2} = 2\mu \pm 2J \tag{55}$$

that is just the condition found previously by us \cite{14} for the mean field quantum spin glass and it gives $\mu$ as a function of $T$ and $I$. For high temperatures, the chemical potential $\mu \to \infty$, while $\mu = \mu_c = J$ at the critical temperature $T_c(I)$ and the critical value $I_c$ is reached when $T_c(I_c) = 0$, as it is shown in the phase diagram in fig.(1). The high $\mu$ (high temperature) solution for $Q_{\alpha}(\omega)$ in eq.(53) gives for $\Gamma_0(\tilde{k}\omega_{1}\omega_{2})$ in eq.(51), when $\mu > J$

$$\Gamma_0(\tilde{k}\omega_{1}\omega_{2}) = 1 - (J/\mu)^2 + \frac{I}{2J}(\omega_1^2 + \omega_2^2) + q^2 \tag{56}$$

Introducing eq.(56) in eq.(50), rescaling the fields $Q_{\alpha\gamma} \to \frac{1}{\beta J N}Q_{\alpha\gamma}$ and using $g_0(\omega = 0, \mu = \mu_c) = (\beta J)^{-1}$, we arrive to the effective spin glass partition functional in the main text. We took explicitly the continuum limit in real
space by replacing, for vanishing lattice constants

\[ \frac{1}{N} \sum_{\vec{k}} \to \int d\vec{k} \]

(57)

while for finite temperature the sum over Matsubara frequencies \( \omega = \frac{2\pi m}{\beta} \)
are over the discrete index \( m \). We discuss next the regions with \( T > T_c \) and \( I < I_c \).

a. **Classic Paramagnet (high temperature)**: \( \frac{\beta}{\sqrt{J}} \to 0 \)

In this limit we are in the classical region and the integral in eq.(54) can be solved exactly\(^{14}\) with the result

\[ 2(\frac{\mu}{J} - 1) = (\frac{1}{\sqrt{\beta J}} - \sqrt{\beta J})^2 \]

(58)

b. **Quantum Paramagnet (low temperature)**: \( \frac{\beta}{\sqrt{J}} \to \infty \)

In this limit \( \coth \frac{\beta y}{2\sqrt{J}} \approx 1 \) and the integral in eq.(54) can be solved in terms of elliptic integrals. For \( (\frac{\mu}{J} - 1) < 1 \) we obtain

\[ 2\pi [\sqrt{I_c J} - \sqrt{T J}] \approx -\frac{4}{3} (\frac{\mu}{J} - 1) \ln [\frac{\mu}{J} - 1] \]

(59)

Introducing eq.(58) into eq.(59) we obtain the dotted curve in fig.(1) that separates the classical from quantum paramagnetic regions.

3. Integrals in dimensional regularization In the low temperature limit the sum over frequencies are replaced by integrals as indicated in eq.(20), then we need for \( \Gamma^{(2)} \) in eq.(15), at the critical value \( t = 0 \)\(^{19}\)

\[ I_2 = \int d\omega d\vec{p} \frac{1}{p^2 + s^2(\omega_1^2 + \omega^2)} \frac{1}{|\vec{p} - \vec{k}|^2 + s^2(\omega_1^2 + \omega_2^2)} \]
\[ I_3 = \frac{S_{d+1}}{2s} \Gamma\left(\frac{d+1}{2}\right) \frac{\Gamma\left(\frac{3-d}{2}\right)}{\Gamma\left(\frac{5-d}{2}\right)} \kappa^{-(5-d)} \times \int_0^1 dx \int_0^{1-x_1} dx_2 \frac{1}{[x_1(1-x_1) + x_2(1-x_1-x_2) + s^2]^{\frac{5-d}{2}}} \]  

(60)

where \( S_{d+1} \) is the surface of the unit sphere in \((d+1)\)-dimensions. We can see that \( \Gamma\left(\frac{3-d}{2}\right) \) has a dimensional pole at \( d_c = 5 \), then calling \( \epsilon' = 5 - d \) we obtain the singular contribution in eq.(21).

To renormalize \( \Gamma^{(3)} \) and \( \Gamma^{(2,1)} \) in eq.(17) and eq.(19) we need to calculate

\[ I_3 = \int d\omega d\vec{p} \]

\[
\left[ p^2 + s^2(\omega_1^2 + \omega_2^2) \right] \left[ (\vec{p} + \vec{k}_1)^2 + s^2(\omega_1^2 + \omega_2^2) \right] \left[ (\vec{p} + \vec{k}_1^2 + \vec{k}_2^2)^2 + s^2(\omega_1^2 + \omega_2^2) \right]
\]

(61)

what we do by taking the external momenta and frequencies at the symmetry point\([19]\)

\[ k_1^2 = k_2^2 = \omega_i^2 = \kappa^2; \quad \vec{k}_1 . \vec{k}_2 = -\frac{\kappa^2}{2} \]

(62)

and performing the integral in \( d + 1 \)-dimensions as in eq.(60) with the result

\[ I_3 = \frac{S_{d+1}}{s} \frac{\Gamma\left(\frac{d+1}{2}\right)\Gamma\left(\frac{5-d}{2}\right)}{\Gamma(3)} \kappa^{-(5-d)} \times \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{[x_1(1-x_1) + x_2(1-x_1-x_2) + s^2]^{\frac{5-d}{2}}} \]

(63)

We see again the dimensional pole in \( \Gamma\left(\frac{5-d}{2}\right) \) at \( d_c = 5 \), then considering the singular part at the pole in \( \epsilon' = 5 - d \), we obtain the results in eq.(22) and eq.(23).
5. Figure Captions

Fig.1 Phase diagram in the $Tvs1/I$ plane. a) Critical line $T_c(1/I)$ (full) separating the classical paramagnetic (top) from the spin glass phase (bottom). b) Estimated line (dots) separating the classical paramagnetic (top) from the quantum paramagnetic (bottom) regions.

Fig.2 Diagrammatic representation of the vertex functions. A double line represents a propagator with two replica indices $\alpha, \gamma$, momentum $\vec{k}$ and two frequencies $\omega_1, \omega_2$. (a) Top: $\Gamma^{(2)}$; (b) Middle: $\Gamma^{(3)}$; (c) Bottom: $\Gamma^{(2,1)}$. 
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\[ \frac{T}{J} = -\frac{(1-T)^2 \ln(1-T)}{\pi/2} (\sqrt{I_c} - \sqrt{I}) \]
