Detection of spin bias in four-terminal quantum-dot ring

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In this work, we show that in a four-terminal quantum-dot ring, via introducing a local Rashba spin-orbit interaction the spin bias in the transverse terminals can be detected by observing the change in current in the longitudinal probes. It is found that the Rashba interaction, quantum interference in this system becomes spin-dependent and the opposite-spin currents induced by the spin bias can present different magnitudes, so charge currents emerge. Besides, the current direction relies on both the magnitude and spin polarization direction of the spin bias. We believe that this method provides an electrical but practical scheme to detect the spin bias (or the spin current).

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I. INTRODUCTION

The field of spintronics has become a significant concern of both experimental and theoretical communities for the purpose of realizing the quantum information processing.\textsuperscript{1,2} Under such a topic, researchers devoted themselves to the fabrication of various nano-devices to efficiently generate and manipulate the spin current by magnetic means including the application of strong magnetic field or ferromagnetic electrodes to achieve the spin injection,\textsuperscript{3} or by the optical approach consisting of employing the polarized light.\textsuperscript{4} Recently, the Rashba spin-orbit (SO) interaction is recommended to realize the spin manipulation in the low-dimensional structures, because it offers a simple manner to control the electron spin.\textsuperscript{5} Albeit these existed works, on the other hand, the quantitative detection of spin current is still at its infancy, because that the occurrence of the pure spin current is usually not accompanied by any electric signal for direct measurement.\textsuperscript{12}

As mentioned in the previous literature, there have been a variety of proposals to focus on the measurement of spin current. Kato et al. first experimentally observed the spin Hall current via the magneto-optical Kerr effect in GaAs semiconductor systems.\textsuperscript{13} Other suggestions to detect spin current include measuring the induced mechanical deformation of the macroscopic sample or measuring the induced spin torque.\textsuperscript{14} Recently, a report showed that the spin current can be detected in a double-QD system when the presented Coulomb interaction destroys the symmetry of spin-up and spin-down current.\textsuperscript{15,16} However, the schemes described above are comparatively complicated, thus any simple one to achieve the detect the spin current is still desirable.

It is distinct that for a spin current flowing through a structure, a spin-dependent chemical potential (spin bias) is usually induced that is the driving force of spin current, thus we can measure the spin bias instead of spin current. With such an idea, in this work we also adopt the local Rashba SO interaction we suggest to electrically measure the spin bias in virtue of the quantum interference in a QD ring. Its key point is that the local Rashba interaction gives rise to the spin-dependent quantum interference and the electron transmission through the QD ring is then spin-polarized. Therefore, the spin polarization in this QD ring breaks the symmetry of the motion of spin-up and spin-down electrons, and nonzero spin-bias–induced charge current correspondingly come up.

II. MODEL AND FORMULATION

The considered four-QD ring is illustrated in Fig.\textsuperscript{1}. We assume that in the transverse leads (lead-1 and -3) there exist the spin bias $V_s$, i.e., the spin-dependent chemical potentials for the spin-up and spin-down electrons are $\mu_{\uparrow}\neq \mu_{\downarrow}=\varepsilon_F+\sigma eV_s$ with the Fermi level of the system $\varepsilon_F$ and $\sigma=\pm 1$ (or $\uparrow, \downarrow$) being the spin index. We additionally insert two normal metallic probes (lead-2 and -4) longitudinally to observe the electric signal change of them affected by the spin bias, so as to ascertain the existence of spin bias. The single-electron Hamiltonian in this structure can be written as

$$H_s = \frac{\mathbf{p}^2}{2m_e} + V(\mathbf{r}) + \sum_{\alpha} \left\{ \alpha (\hat{\sigma} \times \mathbf{p}) + (\hat{\sigma} \times \mathbf{p}) \alpha \right\},$$

where the potential $V(\mathbf{r})$ confines the electron to form the structure geometry, namely, the leads, QDs and the connections. The last term in $H_s$ denotes the local Rashba SO coupling on QD-2. For the analysis of the electron properties, we have to second-quantize the Hamiltonian, which is composed of three parts:

1. $H_{\text{QD}}$: Quantum interference and the electron transmission through the QD ring.
2. $H_{\text{leads}}$: Spin-dependent chemical potential in each terminal.
3. $H_{\text{leads-QD}}$: Connections between QDs and leads.

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The Hamiltonian in the above expression, $\mathcal{H} = \mathcal{H}_c + \mathcal{H}_d + \mathcal{H}_t$. 

\[
\mathcal{H}_c = \sum_{\sigma j k} \varepsilon_{j \kappa \sigma} c_{j \kappa \sigma}^\dagger c_{j \kappa \sigma},
\]

\[
\mathcal{H}_d = \sum_{j=1, \sigma} \varepsilon_j d_{j \sigma}^\dagger d_j + \sum_{l=1, \sigma} [t_{l} d_{l \sigma}^\dagger d_{l+1 \sigma} + r_l (d_{l \sigma}^\dagger d_{l+1 \sigma} - d_{l+1 \sigma}^\dagger d_l)] - d_{l+1 \sigma}^\dagger d_l + t_3 d_{3 \sigma}^\dagger d_{\sigma} + t_4 e^{i \phi} d_{\sigma}^\dagger d_{\sigma} + \text{H.c.},
\]

\[
\mathcal{H}_t = \sum_{\sigma j k} V_{j \sigma} d_{j \sigma}^\dagger c_{j \kappa \sigma} + \text{H.c.},
\]

where $c_{j \kappa \sigma}^\dagger$ and $d_{j \sigma}^\dagger$ ($c_{j \kappa \sigma}$ and $d_{j \sigma}$) are the creation (annihilation) operators corresponding to the basis in lead-$j$ and QD-$j$. $\varepsilon_{j \kappa}$ and $\varepsilon_j$ are the single-particle levels. $V_{j \sigma}$ denotes QD-leading coupling strength. The interdot hopping amplitude is written as $t_{l \sigma} = t_l \sqrt{1 + \alpha^2 e^{-i \phi \sigma}}$ ($l = 1, 2$) where $t_l$ is the ordinary transfer integral independent of the Rashba interaction and $\alpha$ is the dimensionless Rashba coefficient with $\varphi = \tan^{-1} \alpha$. The phase factor $\phi$ attached to $t_4$ accounts for the magnetic flux through the ring. In addition, the many-body effect can be readily incorporated into the above Hamiltonian by adding the Hubbard term $\mathcal{V}_{e-e} = \sum_{j \sigma} U_{j \sigma} n_{j \sigma} n_{j \bar{\sigma}}$. 

Starting from the second-quantized Hamiltonian, we can now formulate the electronic transport properties. With the nonequilibrium Hamiltonian, we can now formulate the electronic transport properties. With the nonequilibrium Hamiltonian, we can now formulate the electronic transport properties.

\[
\mathcal{J}_j = \frac{e}{h} \sum_{j \sigma \sigma'} \int \frac{d \omega T_{j \sigma \sigma'}(\omega)}{\sqrt{\omega}} d\omega T_{j \sigma \sigma'}(\omega),
\]

where $T_{j \sigma \sigma'}(\omega) = 4 \Gamma_{j \sigma} G_{j \sigma j \sigma'}(\omega) \Gamma_{j \sigma} G_{j \bar{\sigma} j \bar{\sigma'}}(\omega)$ is the transmission function, describing electron tunneling ability between lead-$j$ to lead-$j'$. $\Gamma_{j \sigma} = \pi |V_{j \sigma}|^2 \rho_j(\omega)$, the coupling strength between QD-$j$ and lead-$j$, can be usually regarded as a constant. $G_{j \sigma}$ and $G_{\bar{\sigma}}$, the retarded and advanced Green functions, obey the relationship $[G_{j \sigma}^*] = [G_{\bar{\sigma}}^*]$. From the equation-of-motion method, the retarded Green function can be obtained in a matrix form.

\[
\begin{bmatrix}
G_{j \sigma}^{-1} = \frac{1}{\varepsilon_j - \varepsilon_{j \sigma} - \sum \lambda_{j \sigma} G_{j \bar{\sigma}}^{-1} \sigma} \int d\omega \sum \sigma \left| G_{j \sigma \sigma}^{\leq} \right| \left| G_{j \sigma \sigma}^{\geq} \right| \omega \left| G_{j \sigma \sigma}^{\geq} \right| \left| G_{j \sigma \sigma}^{\leq} \right| \sigma \sigma \mu_{j \sigma} - \omega \right)
\end{bmatrix}
\]

In the above expression, $g_{j \sigma}$ is the Green function of QD-$j$ unperturbed by the other QDs and in the absence of Rashba effect. $g_{j \sigma} = [(z - \varepsilon_{j \sigma}) \lambda_{j \sigma} + i \Gamma_{j \sigma}]^{-1}$ with $z = \omega + \text{i} 0^+$ and $\lambda_{j \sigma} = \sum_{j \sigma} \varepsilon_j^\dagger \varepsilon_j n_{j \sigma}$ resulting from the second-order approximation of the Coulomb interaction. $\langle n_{j \sigma} \rangle$ can be numerically resolved by the formula $\langle n_{j \sigma} \rangle = -\frac{i}{\pi} \frac{d}{d \omega} G_{j \sigma}^{\leq}$ where $G_{j \sigma}^{\leq} = \sum_{\sigma'} G_{j \sigma \sigma'}^\dagger G_{j \sigma \sigma'}^{\leq}$ and $G_{j \sigma}^{\leq} = \sum_{\sigma'} G_{j \sigma \sigma'}^\dagger G_{j \sigma \sigma'}^{\geq}$ is the Fermi distribution function in lead-$j$ with the step function of $\theta(x)$.

### III. NUMERICAL RESULTS AND DISCUSSIONS

We now proceed on to calculate the currents in the longitudinal terminals, lead-2 and lead-4 in this case. Before calculation, the QD-lead couplings are assumed to take the uniform values with $\Gamma_{j \sigma} = \Gamma$, and we consider $\Gamma$ as the energy unit (Its order is meV approximately for some experiments based on GaAs/GaAlAs QDs, as mentioned in the previous works). The structure parameters are for simplicity taken as $t_{l \sigma} = t_3 = t_4 = \Gamma$, and $\varepsilon_F$ is viewed as the energy zero point of this system. Besides, to carry out the numerical calculation, we choose the Rashba coefficient $\alpha = 0.4$ which is available in the current experiment. 

We first show the linear-transport results. It is known that in the linear regime, the current flow is proportional to the linear conductance, i.e., $J_m = G_{m} \cdot V_s (m = 1, 3)$, where the linear conductance

\[
G_m = \frac{e^2}{h} \left| \theta(T_{m \sigma \sigma} + T_{m \bar{\sigma} \bar{\sigma}}) \right| + \sigma(T_{m \sigma \sigma} + T_{m \bar{\sigma} \bar{\sigma}}) \right|_{\omega = \varepsilon_F} (3)
\]

obeys the Landauer-Büttiker formula. Consequently, in this case, by only investigating the properties of the linear conductance the spin-bias driven
charge current can be clarified. From such a formula, one can readily find that in the absence of any spin-dependent fields the electron transmission is irrelevant to the electron spin. Hence the opposite-spin currents driven by the spin bias flow through this ring with the same magnitude and opposite directions, leading to the result of zero $g_{nm}$ and the failure of measuring the spin bias [see the dashed line in Fig.2(a)].

On account of the recent researches, they show that in the low-dimensional systems, such as the QD structures, in the electron transport process introducing a local Rashba interaction could bring out the spin polarization, which helps to manipulate the electron spin via the electric means. Thereupon, we introduce a local Rashba interaction to QD-2 of this structure to try detecting the spin bias in the transverse leads. As shown in Fig.2(a), in the presence of Rashba SO coupling and the absence of magnetic field, there indeed emerge the nonzero currents in the longitudinal probes driven by the spin bias when the QD levels are separate from the energy zero point (i.e., $\epsilon_0 \neq 0$). An additional interesting phenomenon is that in the whole regime the amplitude of $J_2$ is the same as that of $J_4$ accompanied by their opposite directions. Such a result means that by building a closed circuit between lead-2 and -4 the spin bias of this system can be detected by observing the current flowing between the longitudinal probes. Besides, it is found that the direction of charge current is related to the separation of QD levels from the Fermi level, namely, in the case of $\epsilon_0 > 0$ the value of $J_2$ is less than zero and $J_2 > 0$, whereas $J_2 > 0$ and $J_2 < 0$ under the condition of $\epsilon_0 < 0$.

Since the configuration of quantum ring, we would like to investigate the role of a local magnetic flux. Thereby, we can see that for the case of $\phi = \frac{\pi}{2}$ there appears little spin-bias-induced currents in the probes despite the adjustment of QD levels, as shown in Fig.2(b). Alternatively, when the QD levels take a typical value with $\epsilon_0 = \Gamma$, the currents present finite values except in the vicinity of $\phi = (n + 1 \frac{1}{2}) \pi$, and they oscillate out of phase. Significantly, as shown in Fig.2(c) the change of magnetic flux from $\phi = 2n\pi$ to $\phi = (2n + 1) \pi$ can vary the magnitude and direction of the spin-bias-driven charge currents in the longitudinal terminals with the critical point at $\phi = (2n + \frac{1}{2}) \pi$, and vice versa. Up to now, we can address that the presented Rashba interaction and the nonzero QD levels with respect to the zero point of energy are the two key conditions to accomplish the measure of the spin bias electrically in this structure.

For the case of the finite spin bias, the charge currents in the additional terminals can be evaluated by Eq.2. Accordingly, in Fig.3(a) we plot the current spectra vs the QD levels in the situations of $eV_s = \Gamma$ and $2\Gamma$, respectively. One can find that in such a case the current magnitudes increase with the enhancement of the spin bias. And the current spectra exhibit complicated properties, different from the linear-transport case. For the case of $|\epsilon_0| > \frac{\Gamma}{2}$, the quantitative relation between these two charge currents becomes ambiguous, especially in the case of $|\epsilon_0| > 2\Gamma$ the signs of these two charge currents can be the same as each other. Only when the QD levels shift around the Fermi level of the system (i.e., in the regime of $|\epsilon_0| < \frac{\Gamma}{2}$) the result of $J_2 = -J_4$ remains substantially. Similarly, such a phenomenon is also described by Fig.3(b). As a typical case, when taking the QD levels at $|\epsilon_0| = \Gamma$, we see that in the situation of $eV_s < \Gamma$ the charge currents in the longitudinal probes have the same magnitude and the opposite directions. However, with the enhancement of the spin bias the value of $J_4$ goes over the zero point and then shows the identical sign with $J_2$.

The calculated transmission functions are plotted in Figs.4 with $\epsilon_j = \Gamma$. They are just the integrands for the calculation of the charge and spin currents (see Eq.2). By comparing the results shown in Figs.4, we can readily see that in the absence of magnetic flux, the traces of $T_{2\uparrow,1\uparrow}$, $T_{4\downarrow,1\downarrow}$, $T_{2\uparrow,3\downarrow}$, and $T_{4\uparrow,3\downarrow}$ coincide with one another very well, so do the curves of $T_{2\uparrow,1\downarrow}$, $T_{4\uparrow,1\downarrow}$, $T_{2\uparrow,3\downarrow}$, and $T_{4\uparrow,3\downarrow}$. Substituting such integrands into the current formulae, one can certainly arrive at the result of the distinct pure spin currents in the transverse terminals. On the other hand, these transmission functions depend nontrivially on the magnetic phase factor, as exhibited in Figs.4(b) with $\phi = \frac{\pi}{2}$. In comparison with the zero magnetic field case, herein the spectra of $T_{j\uparrow,j'\uparrow}$ are reversed about the axis $\omega = \Gamma$ without the change of their amplitudes, but $T_{j\downarrow,j'\downarrow}$ only present the enhancement of their amplitudes. Similarly, with the help of Eq.2, one can understand the disappearance of spin currents in such a case. In addition, by virtue of the transmission function curves we can understand the behaviors of the charge currents with the enhancement of spin bias, i.e., when the strength of spin bias goes beyond the quantum coherence regime the current feature displayed in the linear regime disappears.

The underlying physics being responsible for the spin dependence of the transmission functions is quantum interference, which manifests if we analyze the electron transmission process in the language of Feynman path. Note that the spin flip terms arising from the Rashba interaction do not play a leading role in causing the appearance of spin and charge currents. Therefore, to keep the argument simple, we drop the spin flip terms for the analysis of quantum interference. Based on this method, we write $T_{2\sigma,1\alpha} = |t_{2\sigma,1\alpha}|^2$ where the transmission probability amplitude is defined as $t_{2\sigma,1\alpha} = V_{2\sigma}^{\alpha}G_{2\sigma,1\alpha}^{\dagger}V_{\sigma}$ with $V_{\sigma} = V_{\sigma}\sqrt{2\pi\rho_{\sigma}(\omega)}$. With the solution of $G_{2\sigma,1\alpha}$, we find that the transmission probability amplitude $t_{2\sigma,1\alpha}$ can be divided into three terms, i.e., $t_{2\sigma,1\alpha} = t_{2\sigma,1\alpha}^{(1)} + t_{2\sigma,1\alpha}^{(2)} + t_{2\sigma,1\alpha}^{(3)}$, where $t_{2\sigma,1\alpha}^{(1)} = \frac{\pi}{4\Gamma}V_{2\sigma}^{\alpha}G_{2\sigma,1\alpha}^{\dagger}g_{1\sigma}V_{\sigma}$. 
\[ \tau_{2\alpha,1\sigma}^{(2)} = \frac{1}{D} \hat{V}_{2\sigma} g_{2\alpha} t_{2\sigma} g_{3\sigma} t_{3\alpha} g_{1\sigma} \hat{V}_{1\sigma}, \quad \text{and} \quad \tau_{2\alpha,1\sigma}^{(3)} = -\frac{1}{D} \hat{V}_{2\sigma} g_{2\alpha} t_{2\sigma} g_{3\sigma} t_{3\alpha} g_{1\sigma} \hat{V}_{1\sigma}, \]

with \( D = \text{det}\{G'\}^{-1} \prod_j g_{j\sigma}. \) By observing the structures of \( \tau_{2\alpha,1\sigma}^{(1)}, \tau_{2\alpha,1\sigma}^{(2)} \), and \( \tau_{2\alpha,1\sigma}^{(3)} \), we can readily find that they just represent the three paths from lead-2 to lead-1 via the QD ring. The phase difference between \( \tau_{2\alpha,1\sigma}^{(1)} \) and \( \tau_{2\alpha,1\sigma}^{(2)} \) is \( \Delta \phi_{2\alpha}^{(1)} = [\phi - 2\sigma \varphi + \theta_3 + \theta_4] \) with \( \theta_j \) arising from \( g_{j\sigma} \), whereas the phase difference between \( \tau_{2\alpha,1\sigma}^{(2)} \) and \( \tau_{2\alpha,1\sigma}^{(3)} \) is \( \Delta \phi_{2\alpha}^{(2)} = [\phi - 2\varphi \phi] \). It is clear that only these two phase differences are related to the spin polarization. \( T_{4\sigma,1\sigma} \) can be analyzed in a similar way. We then write \( T_{4\sigma,1\sigma} = |\tau_{4\sigma,1\sigma}^{(1)} + \tau_{4\sigma,1\sigma}^{(2)} + \tau_{4\sigma,1\sigma}^{(3)}|^2 \), with \( \tau_{4\alpha,1\sigma}^{(1)} = \frac{1}{D} \hat{V}_{4\sigma} g_{4\alpha} t_{4\sigma} g_{1\sigma} \hat{V}_{1\sigma}, \quad \tau_{4\alpha,1\sigma}^{(2)} = \frac{1}{D} \hat{V}_{4\sigma} g_{4\sigma} t_{3\sigma} g_{3\sigma} t_{2\sigma} g_{2\sigma} t_{1\sigma} g_{1\sigma} \hat{V}_{1\sigma}, \quad \text{and} \quad \tau_{4\alpha,1\sigma}^{(3)} = -\frac{1}{D} \hat{V}_{4\sigma} g_{4\sigma} t_{3\sigma} g_{3\sigma} t_{2\sigma} g_{2\sigma} t_{1\sigma} g_{1\sigma} \hat{V}_{1\sigma}. \)

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So far we have not discussed the effect of electron interaction on the occurrence of charge currents in the longitudinal probes, though it is included in our theoretical treatment. Now we incorporate the electron interaction into the calculation, and we deal with the many-body terms by employing the second-order approximation, since we are not interested in the electron correlation here. Fig.5 shows the calculated currents spectra with \( U_j = U = 3\Gamma \), respectively. Clearly, within such an approximation the spin-bias-induced charge currents remain, though the current spectra oscillate to a great extent with the shift of QD levels, as shown in Fig.5(a). On the other hand, the numerical results in Fig.5(b) tell us that when the QD levels are aligned with the zero point of energy of this structure, by the presence of Coulomb-interaction the charge currents is possible to appear with the further increase of the applied bias, different from those in the noninteracting case. Meanwhile, in the case of the QD levels separate from the energy zero point (\( \varepsilon_j = \Gamma \)), the magnitudes of the charge currents seem to be enhanced by the many-body effect. This can also be understood with the help of the discussion on the quantum interference of this structure above.

### IV. SUMMARY

In conclusion, when introducing a local Rashba interaction on an individual QD of a four-QD ring, we proposed to electrically detect the spin bias of the transverse terminals by investigating the charge currents in the two external longitudinal probes. We have found that the quantum interference in this system becomes spin-dependent by the presence of the Rashba interaction, so the opposite-spin currents driven by the spin bias show different magnitudes, leading to the emergence of the charge currents. Besides, the charge currents rely on both the magnitudes and spin polarization direction of the spin bias. Therefore, this method can provide a practical and electrical approach to detect the spin bias. On the other hand, the modulation of the QD levels and the magnetic phase factor can efficiently adjust the phases of the transmission paths, so properties of the spin bias can be shown entirely. Finally, it should be emphasized that altering the polarization directions of the spin bias, equivalent to interchange the sequence numbers of lead-1 and lead-3, can also change the polarization directions of the charge currents.

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FIG. 1: (a) Schematic of a four-QD ring structure with a local Rashba interaction on QD-2. Four QDs and the leads coupling to them are denoted as QD-\( j \) and lead-\( j \) with \( j = 1 − 4 \). Spin bias is assumed to be in lead-2 and lead-4.

FIG. 2: In (a) and (b), the linear conductances vs the QD levels are shown with the magnetic phase factor \( \phi = 0 \) and \( \frac{\pi}{2} \), respectively. The parameter values are \( \Gamma_j = \Gamma \) and \( \alpha = 0.4 \). (c) The linear conductance vs \( \phi \) with \( \varepsilon_j = \Gamma \).

FIG. 3: The charge currents in the case of finite spin bias. (a) The currents versus as functions of the QD levels with \( \varepsilon V_S = \Gamma \) and \( 2\Gamma \), respectively. (b) The change of the currents with the adjustment of spin bias strength.

FIG. 4: The spectra of transmission functions \( T_{m\sigma,m'\sigma}(m=2,4 \text{ and } m'=1,3) \) with the QD levels fixed at \( \varepsilon_j = \Gamma \). (a) and (b) Zero magnetic field case, and (c)-(d) magnetic phase factor \( \phi = \frac{\pi}{2} \).

FIG. 5: In the presence of many-body terms with \( U_j = 3\Gamma \), the currents versus \( \varepsilon_0 \) and the spin bias strength, respectively. The other parameters are the same as those in Fig 3.
(a) $\varepsilon_j = \varepsilon_0, \phi = 0$

(b) $\varepsilon_j = \varepsilon_0, \phi = \frac{\pi}{2}$

(c) $\varepsilon_j = \varepsilon_0 = \Gamma$

Conductance ($e^2/h$)
\( J_2 eV = \Gamma \), \( J_4 eV = \Gamma \), \( J_2 eV = 2\Gamma \), \( J_4 eV = 2\Gamma \)

(a) \( \varepsilon_j = \varepsilon_0, \phi = 0, \tilde{\alpha} = 0.4 \)

(b) \( \varepsilon_j = \varepsilon_0, \phi = 0, \tilde{\alpha} = 0.4 \)
(a) $\epsilon_j = \Gamma, \tilde{\alpha} = 0.4, \phi = 0$

(b) $\epsilon_j = \Gamma$

\[ \tilde{\alpha} = 0.4, \quad \phi = \frac{\pi}{2} \]

Transmission Function

$\omega$ (in units of $\Gamma$)
\(\varepsilon_j = \varepsilon_0, \phi = 0, \bar{\alpha} = 0.4, U = 3\Gamma\)

(a) \(J_2 \quad \text{eV} = \Gamma\), \(J_4 \quad \text{eV} = \Gamma\), \(J_2 \quad \text{eV} = 2\Gamma\), \(J_4 \quad \text{eV} = 2\Gamma\)

\(\varepsilon_0 \quad (\text{In units of } \Gamma)\)

\(J_2 \) and \(J_4 \quad (\text{eV}/h)\)

(b) \(\phi = 0, \bar{\alpha} = 0.4, U = 3\Gamma\)

\(\text{eV}_s \quad (\text{In units of } \Gamma)\)