A bi-objective model for optimizing replacement time of age and block policies with consideration of spare parts’ availability

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Abstract. Reliability and availability of critical systems play an important role in achieving the stated objectives of engineering assets. Preventive replacement time affects the reliability of the components, thus the number of system failures encountered and its downtime expenses. On the other hand, spare parts inventory level is a very critical factor that affects the availability of the system. Usually, the decision maker has many conflicting objectives that should be considered simultaneously for the selection of the optimal maintenance policy. The purpose of this research was to develop a bi-objective model that will be used to determine the preventive replacement time for three maintenance policies (age, block good as new, block bad as old) with consideration of spare parts’ availability. It was suggested to use a weighted comprehensive criterion method with two objectives, i.e. cost and availability. The model was tested with a typical numerical example. The results of the model demonstrated its effectiveness in enabling the decision maker to select the optimal maintenance policy under different scenarios and taking into account preferences with respect to contradicting objectives such as cost and availability.

1. Introduction

Reliability and availability of critical systems play an important role in achieving the stated objectives of engineering assets. Maintenance refers to the measures taken by the users of a product or a system to keep it in an operable condition, or to repair it in order to restore it to an operable condition [1]. Usually, these measures aim at minimizing failures and their consequences. Traditionally, maintenance is performed either due to a failure, i.e. corrective maintenance, or performed according to schedule to prevent a failure from happening, which is known as preventive maintenance. In both cases, there is a need to have spare parts, to replace the worn or faulty ones. If the spare parts are not available, the system’s downtime is extended to a longer time, resulting in unnecessary expenses and maybe even safety risks. Spare parts’ inventory level is a critical indicator that is affected by various factors such as previous demand, financial and technical asset management. Wang [2] illustrated that, generally, spare parts are managed through an inventory management system that relies on past spare parts demand regardless of the maintenance process. On the one hand, an insufficient stock level may lead to an extended system downtime. On the other hand, a high stock level results in higher holding costs. The spare parts’ demand is usually affected by the maintenance policy. Preventive maintenance with less frequent scheduled maintenance leads to less used spare parts as well as to a higher probability of corrective maintenance’s occurrence. Preventive maintenance with more frequent scheduled maintenance might lead to a higher demand of spare parts, but minimized risk of system’s breakdown.
(see Wang [3]). It is usually assumed that spare parts are always available when needed, that is the focus is mainly set on maintenance. Wang [2] emphasized that maintenance scheduling and spare parts’ management should be integrated to achieve the best results. He also cited publications [4–11] that address the mentioned integration. Further, publications that address both the optimization of maintenance scheduling and spare parts’ management are mainly based either on an age replacement policy, such as [12,13] or on a block policy, such as [14–16]. Nosoohi and Hejazi [13] presented a multi-objective model to simultaneously determine the spare parts’ number and preventive replacement times in a planning horizon. They used an age maintenance policy with three objective measures in their study, i.e. cost, corrective failure and residual lifetime. They cited research papers that considered maintenance policies based on system reliability and availability by using reliability and availability as objective measures [17–19].

In this paper, we will investigate the use of bi-objective model, i.e. cost and availability, to determine the optimal replacement time with and without spare parts availability for three preventive maintenance policies, i.e. age, block Good-As-New (GAN) and block Bad-As-Old (BAO). Compared to Nosoohi and Hejazi [13] and other research work, the main contribution of this paper is the consideration of not only of the age or block policies but also of the age, block Good-As-New (GAN) and block Bad-As-Old (BAO) policies, simultaneously.

The organization of the paper is as follows. Section 2 presents maintenance approaches. Section 3 illustrates the suggested model and its decision variables and parameters. Section 4 provides a numerical example to illustrate how the proposed model works. Finally, conclusions are provided in section 5.

2. Maintenance approaches

Maintenance approaches are classified as reactive or preventive. For more details on maintenance approaches and their development see [1,20,21]. Reactive maintenance can be either run-to-failure in cases where the failure has little or no effect or breakdown maintenance due to failures of critical components [1]. On the other hand, preventive maintenance is defined by [9] as “any task designed to prevent failures or mitigate their effects”. Preventive maintenance can be either time-based or condition-based. With time-based policies, critical components are replaced at pre-specified intervals regardless of their conditions. The time for action is usually determined based on experience, suggestion by the original equipment manufacturer (OEM) or optimized by minimizing maintenance total costs using life distributions generated from failure data [1].

Age and block policies, which were first proposed by [7], are widely used for optimizing the renewal intervals for components. When using age policy renewal is done either at a predetermined fixed age (running or calendar time since its last renewal) or when it fails, whichever event occurs sooner. However, with block renewal policy the time of preventive renewal is never interrupted or changed due to failures that occur between the planned renewals. The cycle time is fixed and there may be any number of failures between the planned renewals. Using block policy one has two options, either to replace the component with a new one, i.e. restoring the item to a standard condition Good-As-New (GAN), or to repair the component with minimum repair, i.e. Bad-As-Old (BAO). These models differ in their impact on the hazard rate function. For example, with both age and block-GAN models the time variable in the hazard function is restored to zero, whereas with the block-BAO model the time variable in the hazard function is assumed not be affected. [8,9].

Condition based maintenance is a strategy where the decision to perform maintenance is reached by observing the “condition” of the system and/or its critical components. It is applied to items which fail due to aging or accidentally [22] and implies the use of surveillance and monitoring techniques to monitor the condition of critical components [20,23].

3. Model

The proposed model considers both the unit cost and the availability to optimize the maintenance interval under three time-based preventive maintenance policies, namely age, block-GAN and block-BAO. The model considers the possibility of having available spare part in its formulation. This formulation represents a generalized model, where the model without available spare parts can be seen as special case as will be illustrated later.
3.1. Model parameters and decision variables
The optimization model parameters are defined as follows: $C_{f1}$ and $C_{f2}$ are the cost of fixing failure with and without the availability of spare parts and are used in the case of age and block-GAN models. $C_{p1}$ and $C_{p2}$ are the cost of performing preventive maintenance with and without spare parts. $C_{r1}$ and $C_{r2}$ are the costs of repairing failure with and without spare parts used in the case of block-BAO. In this case, the failed component is replaced by a reconditioned one or restored to its “as bad as old” state. $T_{f1}$ and $T_{f2}$ are the time required to fix failure with and without the availability of spare parts and are used in the case of age and block-GAN models. $T_{p1}$ and $T_{p2}$ are the time needed to perform preventive maintenance with and without spare parts respectively. $T_{r1}$ and $T_{r2}$ are the time required to repair failure with and without spare parts.

$P(t)$ defines the cumulative probability of having spare parts, $F(t)$ is the failure cumulative distribution function. $R(t)$ defines the component reliability function, $N(t)$ represents the expected number of failure in block-GAN model and $H(t)$ is the cumulative hazard function of the failure distribution. The decision variable of the model is the maintenance interval $t$ for all the three maintenance policies.

3.2. Model objectives and equations
For the age policy, the integration of the reliability function $R(t)$ over the interval $[0, t]$ leads to the estimated expected cycle length and the up-time duration. Jardine [24] showed that the expected cycle length is equal to (the length of the preventive cycle times the probability of preventive cycle) plus (expected length of a failure cycle times the probability of a failure cycle). Sherwin [9] illustrated that when a failure occurs, it happens before the end of the preventive interval $t$. We need the conditional mean of the distribution given that event. Knowing that the mean value of any continuous function is by definition the first moment of area of the function relative to the origin and over appropriate intervals. This conditional mean combines the expected value and the probability of that event, as illustrated in equation (1):

\[ \text{In most cases, the expected cycle length} \]
\[ = t \cdot R(t) + \int_0^t u \cdot f(u) \, du \quad \text{[= \int_0^t R(u) \, du for most distribution forms]} \]  \hfill (1)

The unit cost function is defined as the expected total cost over the expected cycle length and the availability is defined as the up-time duration over the total cycle length. The model objectives used in this study are:

- For age policy:
  \[ \min C_{age}(t) = \frac{C_{f1} \times F(t) \times P(t) + C_{f2} \times F(t) \times (1 - P(t)) + C_{p1} \times R(t) \times P(t) + C_{p2} \times R(t) \times (1 - P(t))}{\int_0^t R(t)} \]  \hfill (2)

- For block-GAN policy:
  \[ \max A_{GAN}(t) = \frac{\int_0^t R(t)}{t} \]  \hfill (3)

- For block-BAO policy:
  \[ \min C_{BAO}(t) = \frac{C_{f1} \times H(t) \times P(t) + C_{f2} \times H(t) \times (1 - P(t)) + C_{p1} \times P(t) + C_{p2} \times (1 - P(t))}{t} \]  \hfill (4)

  \[ \max A_{BAO}(t) = \frac{\int_0^t R(t)}{t} \]  \hfill (5)

where $C_{age}(t)$ and $A_{age}(t)$ represent the unit cost and availability equations for age policy, $C_{GAN}(t)$ and $A_{GAN}(t)$ represent the unit cost and availability for block-GAN and $C_{BAO}(t)$ and $A_{BAO}(t)$ represent the unit cost function and availability function for block-BAO model. It is worth noting that
setting \( P(t) = 0 \) converts the model to the traditional one, which does not consider the availability of spare parts. Thus, this model is a generalized form of the traditional one.

### 3.3. Solving the bi-objective function

Many methods in the literature are available for solving a set of multiple objective. In this work, we apply the weighted comprehensive criterion method [25,26], a scalariazen technique which combines the set of bi-objective functions into one single objective function in order to minimize the deviation from the optimal value of each objective as defined in the below equations. The weighted comprehensive criterion method is adopted due to its simplicity and to the fact that a set of Pareto optimal solutions can be hereby obtained. Different optimal solutions can be obtained by controlling the importance weights \( \tau_1 \) and \( \tau_2 \), where \( \tau_1 + \tau_2 \) must equal to 1.0. Pareto diagram can be obtained by changing the values of \( \tau_1 \) and \( \tau_2 \).

\[
\begin{align*}
\min E^{Age} &= \tau_1 \times \frac{C^{Age}(t) - C_{opt}^{Age}}{C_{opt}^{Age}} + \tau_2 \times \frac{A_{opt}^{Age} - A^{Age}(t)}{A_{opt}^{Age}} , \\
\min E^{GAN} &= \tau_1 \times \frac{C^{GAN}(t) - C_{opt}^{GAN}}{C_{opt}^{GAN}} + \tau_2 \times \frac{A_{opt}^{GAN} - A^{GAN}(t)}{A_{opt}^{GAN}} , \\
\min E^{BAO} &= \tau_1 \times \frac{C^{BAO}(t) - C_{opt}^{BAO}}{C_{opt}^{BAO}} + \tau_2 \times \frac{A_{opt}^{BAO} - A^{BAO}(t)}{A_{opt}^{BAO}} ,
\end{align*}
\]

\( C_{opt} \) and \( A_{opt} \) represent the ultimate optimal value for the cost objective and the availability objective when solved separately.

### 4. Numerical example

To test the suggested model, we assume to have a component with a lifetime described by a Weibull distribution with following characteristics: a shape parameter (\( \beta = 1.6 \)), a scale parameter (\( \eta = 10500 \)),

\[
F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}, \quad R(t) = 1 - F(t) \quad \text{and} \quad H(t) = \int_0^t \frac{\beta}{\eta} t^{\beta-1} .
\]

The calculation of \( N(t) \) is done based on the modified version of the mean value theorem for integral (modified-MeVTI) proposed by Sasongko and Mahatma [27]. The probability of having spare parts is assumed to be described by an exponential distribution as in [13]. \( P(t) = 1 - e^{\eta w} \), where \( w \) represents the work status calibration factor and \( v \) represents the ratio of the spare parts’ number to the number of planned maintenance [13], \( v \) is set to 1.0. The costs and times parameters needed for the model are listed below.

\[
\begin{align*}
\text{\( c_{f1} \)}} & = 70,000 \\
\text{\( c_{f2} \)}} & = 105,000 \\
\text{\( c_{pl} \)}} & = 15,000 \\
\text{\( c_{r1} \)}} & = 55,000 \\
\text{\( c_{r2} \)}} & = 75,000 \\
\text{\( T_{f1} \)}} & = 12 \\
\text{\( T_{f2} \)}} & = 18 \\
\text{\( T_{pl} \)}} & = 2 \\
\text{\( T_{r1} \)}} & = 12 \\
\text{\( T_{r2} \)}} & = 8
\end{align*}
\]

In this numerical example, we present four different cases. In the first case, we consider \( w = 1 \) in the spare parts availability function. In the second case, we consider \( w = 2 \) which means lighter operation conditions. In the third case, we consider \( w = 1 \) again, but \( c_{pl} = 20000 \) instead of 15000, which results in lower \( C_{f1}/C_{pl} \). Finally, in the last case, we consider \( P(t) = 0 \), i.e. we assume no spare parts to be available. Both objectives are considered equally important in all the three numerical examples.

Table 1 provides the optimal solution for solving the bi-objective model in the four different cases. When considering the cost, for all the cases, block-BAO is the most cost effective policy, followed by age, and Block-GAN. However, when considering the availability measure, the first preferred policy is age, followed by block-GAN and block-BAO. This order was obtained in the other cases, too, and can be justified by taking into account that block-BAO is the most efficient, i.e. least expensive, solution but also the less effective one with respect to the number of failures and consequently system breakdowns. For case 2, the availability results are the highest of all cases because \( P(t) = 86.4\% \) for case 2, while for case 1 \( P(t) = 63.2\% \). In other words, the spare parts’ availability leads to higher availability result. Compared to case 1, in case 3 the ratio of \( C_{f1}/C_{pl} \) decreased from 4.67 to 3.5, which resulted in larger replacement periods. Finally, case 4 shows the effect of spare parts’ unavailability, thus, the lowest availability results are obtained with the three maintenance policies.
Table 1. Bi-objective optimal results

| Age | Block-GAN | Block-BAO | Age | Block-GAN | Block-BAO |
|-----|-----------|-----------|-----|-----------|-----------|
| \( t \) | 7.9056 | 8.4050 | 7.5855 | \( C(t) \) | 8.1512 | 8.722 |
| \( A(t) \) | 99.8587 | 99.8481 | 99.8337 | \( E(t) \) | \( 4.14 \times 10^{-6} \) | \( 5.71 \times 10^{-6} \) | \( 1.07 \times 10^{-6} \) |

| Case 2: \( w = 2, c_p = 15,000, P(t) = 86.4\% \) |

| Age | Block-GAN | Block-BAO | Age | Block-GAN | Block-BAO |
|-----|-----------|-----------|-----|-----------|-----------|
| \( t \) | 7.166 | 7.941 | 6.737 | \( C(t) \) | 8.877 | 11.554 |
| \( A(t) \) | 99.88133\% | 99.87443\% | 99.86476\% | \( E(t) \) | \( 1.39 \times 10^{-7} \) | \( 1.52 \times 10^{-7} \) | \( 6.72 \times 10^{-6} \) |

5. Conclusions

In this study, a bi-objective model was developed to determine the optimal replacement time for three preventive maintenance policies (age, block good as new, block bad as old) with consideration of spare parts’ availability. The weighted comprehensive criterion method (WCCM) was used to deal with two conflicting objectives, namely cost and availability. The results demonstrated that the suggested model’s use enables the decision maker to select the optimal maintenance policy taking into account different scenarios as well as contradicting objectives such as cost and availability, longer preventive maintenance interval, or less spare parts’ inventory. As future work, it is suggested to investigate the joint optimization of production planning, inventory, and maintenance optimization and sustainability. Testing the proposed model’s behavior in real applications and with other maintenance strategies such as condition monitoring could be part of the future work as well.

6. References

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