Numerical Analysis of Wind Wake Superposition and Boundary Layer Interaction.

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Abstract. Understanding the wake impacts to a wind farm is critical for layout planning and turbine design. Wind Wake Models estimates wake development through a single turbine which is then aggregated with a wake variety of methods to calculate the combined wake profile for a wind farm. This study considered downstream wake interactions, and the energy content in the wake profile as the wake developed for three rows and five downstream turbines. This study also looks at the influence of different atmospheric stratifications to wind farm wake superposition. Our findings for the wake and boundary superimposition model determine that the streamwise velocity component range from 8.8 to 9.0 m s⁻¹ (98 to 100 %) for the stable ABL ranges and 8.92 to 9.0 m s⁻¹ (99 to 100 %) for the unstable ABL ranges. This study further determines that the minimum streamwise velocity for the stable and unstable ABL are 8.97 and 8.96 m s⁻¹, and the mean normalized velocity deficit for the stable and unstable ABL are 0.972 and 0.975. The wake deficit for the superimposition of a coupled wake boundary model in the stable case surpasses that of the unstable case due to lower turbulence.

1. Introduction
Understanding the wake effects of a wind farm is critical for layout planning and turbine design. The wake model is a simple model for determining wind speed loss and wake expansion in a wind farm. Wind Wake Models usually quantify the aggregate wake profile by estimating the wake generated by a single turbine, which is then aggregated using several techniques. Based on simple mathematical models, present wake superposition models do not capture the developing flow dynamics that occur when wakes interact and combine with neighboring wakes. Accordingly, we can establish that the accuracy of wake superposition models is lacking under certain flow conditions. However, the accuracy of wake superposition models can be improved considering the spanwise and streamwise non-homogeneous nature of turbulence intensity and the boundary layer's effects in all three directions.

Abkar and Porté-Agel [1] demonstrated that the horizontal and vertical wake expansion coefficients could vary due to ground or thermal stratification effects. It is, therefore, viable to consider the nonhomogeneous nature of turbulence while estimating the wind farm flow. The Gaussian Wake Models consider the incoming boundary-layer flow properties, assuming identical vertical and horizontal wake expansion [2]. A critical issue that needs to be addressed is the consideration of the atmospheric properties as the wake expands, interacts, and superimposes to the surrounding wakes. Wind farm wake superposition's complexity, therefore, requires the estimation of boundary aspects in the wind farm for beyond one wind turbine span. Likewise, it is viable to consider the impacts of turbulence interaction with wake and the progressive depletion downstream wind speed as the individual wake combines to larger wind-farm wakes. This study aims to superpose a coupled 3D Gaussian wake model while considering the atmospheric boundary layer to estimate the overall wake deficit. To conserve the total
momentum deficit for the entire wind farm, this study considers the spanwise and streamwise non-homogeneous nature of turbulence intensity in all three directions and the ABL in the linear wake summation.

The study considered the downstream wake interactions and the wake profile development for three rows and five downstream turbines. The wind farm inlet is at \( X/D \approx 0.01 \) upstream of the first column of turbines, while the outlet is at \( X/D \approx 138 \). We placed the turbines equidistantly at a downstream distance at \( X/D = 30D \). The 3D Gaussian Wake Model parameters are obtained from the estimates by Bastankhah and Porté-agel [2] and boundary parameters as indicated in Table A1. The computational domain for the 3D wakes superposition model is set up considering 10 Vestas V80-2 MW wind turbines with a hub height of \( z_h = 80 \text{m} \) and an approximate rotor diameter of \( D_0 = 102 \text{m} \).

2. Methodology and Numerical Setup

2.1. The boundary-layer simulation set up

Panofsky and Dutton [3] established that the horizontal variations do not obey Monin-Obukhov similarity theory and suggested that the introduction of stability parameter, leading to:

\[
I_u(h) = \frac{\sigma_u(h)k_0}{u_*(0) \left[ \ln \left( \frac{Z_h}{Z_0} \right) - \psi_m \left( \frac{Z_h}{L} \right) \right]} \approx \frac{\phi k_0}{\ln \left( \frac{Z_h}{Z_0} \right) - \psi_m \left( \frac{Z_h}{L} \right)}
\]

(1)

We further estimated the added turbulence based on the relationship in equation (2) proposed by Crespo and Hernandez [22] on experimental results.

\[
I_+ = 0.73a^{0.8325}f_0^{0.0325} \left( \frac{x}{D_0} \right)^{0.32}
\]

(2)

2.2. 3D Gaussian Wake Model simulation set up

This section of the paper discusses the derivations for the 3D Gaussian Wake Model. This study infers from the Gaussian Wake Model developed by Bastankhah and Porté-Agel [2] and assumes a self-similar Gaussian distribution for the wind speed deficit \( \Delta U/U_\infty \). The Gaussian Wake Model shown in equation (3), considers both axial distance \( X \) and radial distance \( r \).

\[
G(x, y, z) = \left\{ 1 - \frac{ct}{8 \left( \frac{\sqrt{5}}{5} + \frac{x(k_3 + I_0 k_2)}{d_0} \right)^2} \right\} \cdot \left( -\exp \left( -\frac{(zd - z_h)^2 + y^2}{2 \left( \frac{\sqrt{5}}{5} + \frac{x(k_3 + I_0 k_2)}{d_0} \right)^2} \right) \right.
\]

(3)

2.3. 3D Wake Superposition Techniques

Four different wake superposition techniques are used for quantifying wake aggregation at different spatial configurations. While a 3D Gaussian Wake Model was used for evaluating the wake development behind a single rotor, this study considered a summation of the squares of the wake velocity deficits for the wake superposition. The root-sum-square (RSS) superposition model introduced by Katic [4] considers the local thrust velocity \( u_0 \approx U_\infty \) at a location and estimates the velocity deficit \( U_w(x, y, z) \) as the square root of the quadratic sum of the upstream single turbine wake deficits as shown equation (4).

\[
U_w(x, y, z) = U_\infty - \sqrt{\sum_i \left(U_\infty - u_0(x, y, z) \right)^2}
\]

(4)

3. Results

The results for the superposition of multiple wind turbine wakes under stable and turbulent atmospheric stratification are presented and discussed in this section of the paper. This study estimated the overall wake interaction with the atmospheric boundary layer at different spans and rows. This study explains the boundary’s impact on wake superposition by comparing the quadratic summation of the wake
velocity deficits. The aggregate wake profile for the 3D Gaussian Wake Model under stable and turbulent atmospheric conditions is as shown in Figures 1 to 4.

Figures 1 to 4 show the row-wise fluctuations from the top and vertical wake profiles for the stable and unstable ABL. As seen from Figures 1 and 2, the streamwise velocity component for the stable ranges from 8.8 to 9.0 m s$^{-1}$ and unstable ABL ranges from 8.92 to 9.0 m s$^{-1}$, respectively. Along the wake centerlines, the minimum streamwise velocity component for the stable and unstable ABL was estimated to be 8.97 and 8.96 m s$^{-1}$, respectively. Figures 5 and 6 compare the normalized velocity deficit for the downstream wake expansion. The mean normalized velocity deficit for the stable and unstable ABL was estimated to be 0.972 and 0.975, respectively. The wake deficit for the stable case surpasses that of the unstable case. The wake deficits peak the oscillation at the turbine's centerline and decay at each row's extremes for both stratifications. The centerline dips at the second span fluctuation at 0.920, lesser compared to the first, which dips at 0.927 and the rest of the spans. This finding suggests that the second span has more thrust compared to the rest of the spans. A viable explanation for the higher thrust in the near-wake regions is a slower wake expansion in comparison to turbulence intensity, particularly in the wake of the first turbine row. And due to reduced shear between the wake and freestream flow, there is a more significant influence of thrust than the atmospheric boundary layer, particularly for the near-wake region.

Wind turbine wakes within farms are challenging to model because they develop and interact with neighboring wakes that contrast with the free-stream flow. We commonly classify turbine wakes regions into two categories: close wake and far wake. Depending on the turbulence level within a span, the near-wake region ranges from X/D=0 to X/D=4. The rest of the span can be considered to be the far-wake region. The rotor geometry complicates the flow in the near-wake region by propagating and developing the blade and tip vortices. As seen in Figures 3 and 4, the near-wake region is more symmetrical than previous spans because it has a lower turbulence intensity flow. These results point to a linkage between downstream wake profile fluctuations and mechanical turbulence intensity fluctuations. The effect of turbulent intensity on the wake's recovery can be seen from the horizontal and vertical wake profiles in Figures 3 and 4. A closer comparison of the two stable and unstable spans shows some significant difference in the downstream wake expansion rates. The profile fluctuations for the downstream wake profiles at the turbine centerlines depict quasi-periodic patterns for stable ABL and unstable ABL. However, the wake expands slightly faster for the unstable ABL than the stable ABL, consistent with Abkar and Porté-Agel [1]. In each turbine wake span, the velocity deficit reduces from the near-wake region further downstream as the wake mixes with turbulence. A comparison of Figures 1 and 2 shows a faster wake development rate under the unstable ABL because of faster mixing and high turbulence.

Figure 1. A slice of the streamwise velocity component for the stable ABL (in m s$^{-1}$) in the X-Y plane, at different spans based on the Gaussian Wake Model Superposition.
Figure 2. A slice of the streamwise velocity component for the unstable ABL (in m s$^{-1}$) in the X-Y plane, at different spans based on the Gaussian Wake Model Superposition.

Figure 3. A slice of the streamwise velocity component for the stable and the unstable ABL (in m s$^{-1}$) in the X-Z plane, at different spans based on the Gaussian Wake Model.

Figure 4. A slice of the streamwise velocity component for the unstable ABL (in m s$^{-1}$) in the X-Z plane, at different spans based on the Gaussian Wake Model Superposition.

The mean flow profile in Figures 5 and 6 shows the quasi-periodic behavior of downstream wake development that peaks in the near-wake region for both stratifications. These findings suggest that the wake characteristics in the near-span wake region significantly depend on global turbine performance parameters (thrust coefficient) and atmospheric turbulence.
A decrease in downstream thrust velocity causes a reduction of wake deficit in the far-span region, which may explain the quasi-periodic characteristics of downstream wake development. As seen from the results, the vertical profiles are asymmetric around the hub height, especially for the near-wake region resembling a Gaussian profile. While a select study suggested that the growth of downstream turbulence is linear, the downstream wake development in Figures 5 and 6 depicts nonlinearity over several spans suggesting a strong effect of wind turbine geometry and the coefficient of thrust. However, the wake profiles within each turbine span develop into ellipsoidal shapes especially in the far-wake region because of turbulence, vertical shear profile, and boundary. A closer examination of velocity deficit peaks in Figures 5 and 6 reveals that the normalized wake deficit at the centerline has a slight asymptotic behavior. The asymptotic wake expansion characteristics depicted per span provide enough evidence that the wind flow momentum decays downstream, and the faster mixing occurs due to high turbulence in the far-wake region. The asymptotic wake expansion characteristics depicted per span occur when the ABL is fully adjusted to the wind farm flow resulting in a homogeneous flow in the far-wake region.

4. Conclusions
The wake profile findings for the superimposition of a coupled wake boundary model under stable and turbulent atmospheric conditions are:

1. The mean flow velocity shows the quasi-periodic behavior of downstream wake development, which peaks in the near-wake region for the stable and unstable ABL suggesting a strong influence of global turbine performance parameters (thrust coefficient) and atmospheric turbulence.
2. The streamwise velocity component for the stable ABL ranges from 8.8 to 9.0 m s\(^{-1}\), and for the unstable ABL ranges from 8.92 to 9.0 m s\(^{-1}\); the minimum streamwise velocity component for
the stable ABL estimated to be 8.97 and for the unstable ABL at 8.96 m s\(^{-1}\), and the mean normalized velocity deficit for the stable ABL estimated to be 0.972 and for the unstable ABL at 0.975. The wake deficit for the superimposition of a coupled wake boundary model in the stable case surpasses that of the unstable case.

3. The centerline for both stratifications' dips at the second column fluctuation at 0.920 compared to the first, which dips at 0.927 and is more significant than the preceding columns. Thus, the superimposition of a coupled wake boundary model has more wake deficit at the second column is the most dominant no matter the stratifications.

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Appendices

| Parameter                  | Units   | Common | Stable | Unstable |
|----------------------------|---------|--------|--------|----------|
| Turbine Diameter D\(_o\)  | mm      | 1100   |        |          |
| Yaw Correction Parameter   | -       | 1.88   |        |          |
| Coefficient C\(_1\)        | -       | 0.28   |        |          |
| Coefficient C\(_2\)        | -       | 0.1    |        |          |
| Temperature Theta          | Kelvin  | 279    | 288    |          |
| Coefficient A*             | -       | 1.5    | 2.1    |          |
| Coefficient B*             | -       | 5.5    | 4.6    |          |
| Gravity g                  | m s\(^{-1}\)^\(^2\) | 9.81   |        |          |
| Relative Humidity Rh       |         | 0.6    | 0.95   |          |
| Pressure P                 | hPa     | 1008   | 1037   |          |
| Liquid Water Ratio r\(_l\) |         | 0.03   |        |          |
| Length Scale L             | -       | 117    | -146   |          |
| Surface Height z\(_0\)     | -       | 9.41e-05 | 1.53e-04 |        |
| Frictional Velocity \(u_* (0)\) | -       | 0.25   | 0.32   |          |
| Turbulence Parameter Tia   | -       | 0.73   |        |          |
| Turbulence Parameter Tib   | -       | 0.8325 |        |          |
| Turbulence Parameter Tic   | -       | 0.0325 |        |          |
| Turbulence Parameter Tid   | -       | -0.32  |        |          |

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