Spin degeneracy of Hadronic molecules in the heavy quark region

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Abstract. Hadronic molecules have been considered to appear close to the hadron-hadron threshold. For the heavy mesons, $D$ and $B$, the one pion exchange potential is enhanced by the mass degeneracy of heavy pseudoscalar and vector mesons, caused by the heavy quark spin symmetry. In this study, we investigate new hadronic molecules formed by the heavy meson $P$ ($=D$, $B$) and a nucleon $N$, being $PN$. As the interaction between $P^{(*)}$ and $N$, the pion and vector meson ($\rho$ and $\omega$) exchanges are considered. By solving the coupled-channel Schrödinger equations for $PN$ and $P^*N$, we obtain the bound and resonant states in the charm and bottom sectors, and in the infinite heavy quark mass limit. In the molecular states, the $PN - P^*N$ mixing effect is important, where the tensor force of the one pion exchange potential generates the strong attraction. In the heavy quark limit, we obtain the degeneracy of the states for $J^P = 1/2^-$ and $3/2^-$. 

1. Introduction

Study of the exotic hadron states attracts a great deal of interest in the hadron and nuclear physics. The constituent quark model has been successfully applied to explain the ordinary hadron spectra, describing the baryons as $qqq$ and mesons as $q\bar{q}$ with constituent quarks $q$ and $\bar{q}$ [1, 2]. However, recent observations of new exotic states, such as $XYZ$ in the charm and bottom regions, cannot be explained by the ordinary heavy quarkonium picture $QQ$ with $Q = c, b$ [3–6]. These states motivate us to investigate exotic structures of the hadron states. These exotic states are considered to be a multiquark state being $QQq\bar{q}$, a hadronic molecule being a meson-antimeson state, and the hybrid states being $QQg$. For $X(3872)$ [7] and $Z_b$ [8] have been considered to have a molecular component, $DD^*$ and $BB^*$, respectively, and attract our interest.

Hadronic molecule is a loosely bound state or a resonance of several hadrons appearing close to the thresholds. This is an analogous state to the deuteron or the atomic nuclei which are the bound state of protons and nucleons [9]. The deuteron is a loosely bound system with the small binding energy 2.2 MeV, and the large root mean square radius 2 fm. Similarly, the hadronic molecules in the heavy quark sector would have those properties.

In the hadron-hadron bound system, the hadron-hadron interaction plays an important role. For a heavy meson ($D, B$) and a nucleon, the strong attraction by the one pion exchange potential (OPEP) has been discussed in Refs. [10–14]. The OPEP is enhanced by the mass degeneracy between heavy pseudoscalar mesons, $P = D, B$, and heavy vector mesons, $P^* = D^*, B^*$, where the mass degeneracy are induced by the heavy quark spin symmetry
It has been known that the tensor force of the OPEP plays an important role to generate the strong attraction in nuclei. Therefore, the OPEP is expected to produce an attraction in heavy hadron systems. The attractive $\bar{D}N$ ($BN$) interaction would make it possible to give the exotic nuclear systems with heavy antiquarks ($\bar{c}$ or $\bar{b}$). These binding states are manifestly exotic states which are stable against a strong decay.

The attraction of the hadron-nucleon system shows us interesting phenomena such as the impurity effects which are not present in normal atomic nuclei. To understand phenomena caused by impurities is important to investigate hadron-nucleon interactions, properties of the hadrons in nuclear medium, and impurity effects for nuclear properties. In the strangeness sector, the impurity effects such as the formation of the high density states in the $K$ nuclei [18,19], and the shrinking of the wave functions in the hypernuclei [20,21] have been studied. The heavy hadron in nuclear systems have also been discussed [22,23].

In the present work, we focus on the two-body meson-nucleon system being $\bar{D}(s)N$ and $B(s)N$, where $D(s)$ ($B(s)$) stands for $\bar{D}$ or $D^*$ ($B$ or $B^*$). The meson-baryon interaction is given by the meson exchange potential, which induces the off-diagonal components mixing the $PN$ and $P^*N$ channels. By solving the coupled-channel Schrödinger equations, we obtain the bound and resonant states of the genuinely exotic states. We also investigate the properties of the two-body system with an infinite heavy quark mass.

2. Interactions

As the $P(s)N$ interaction, the meson exchange potential is employed. In particular, the OPEP appears in the process $PN \leftrightarrow P^*N \leftrightarrow PN$, while the $\pi$ exchange is absent in $PN \leftrightarrow PN$, because the $PP\pi$ vertex violates parity conservation. Therefore, the OPEP is enhanced by the small mass splitting of $P$ and $P^*$ mesons. In the heavy quark sector, the mass differences of the mesons are small, e.g., $m_{D^*-}-m_{\bar{D}} \approx 140$ MeV for charm and $m_{B^*-}-m_{B} \sim 45$ MeV for bottom, while $m_{\rho}-m_{\pi} \approx 600$ MeV, and $m_{K^*}-m_{K} \approx 400$ MeV in the up, down and strange sectors have the large mass difference. Therefore, the pion coupling will be important in the heavy meson sectors.

The $\pi$, $\rho$ and $\omega$ exchange potentials are obtained by the meson exchange scattering amplitudes. The amplitudes are given by the effective Lagrangians for heavy mesons and light mesons, and for nucleons and light nucleons [11,12,24]. The Lagrangian for the heavy mesons and the light mesons ($\pi,V=(\rho,\omega)$), based on the HQS, are

$$\mathcal{L}_{\pi HH} = ig_\pi \text{Tr} \left[ H_b \gamma_\mu \gamma_5 A_{\mu \alpha}^a H_a \right],$$

$$\mathcal{L}_{V HH} = -i \beta \text{Tr} \left[ H_b \nu^\mu (\rho_{\mu})_{\alpha\beta} H_a \right] + i \lambda \text{Tr} \left[ H_b \sigma^{\mu\nu} F_{\mu\nu} (\rho)_{\alpha\beta} H_a \right],$$

where the heavy meson fields $H$ and $\bar{H}$ are given by

$$H_a = \frac{1+\gamma^5}{2} \left[ P_{a\mu} \gamma^\mu - P_a \gamma^5 \right],$$

$$\bar{H}_a = \gamma^0 \bar{H}^\dagger_a \gamma^0,$$

which are expressed by the linear combination of the heavy pseudoscalar ($P$) and vector ($P^*$) meson fields. The heavy meson fields are normalized as

$$\langle 0 | P(p_\mu) \rangle = \sqrt{p_0},$$

$$\langle 0 | P^*_\mu(p_\mu,\alpha) \rangle = \varepsilon(\alpha)_\mu \sqrt{p_0},$$

where $\varepsilon(\alpha)_\mu$ is the polarization vector of $P^*$ with polarization $\alpha = \pm, 0$. $\nu^\mu$ is the four-velocity of a heavy quark, where we take the static approximation $\nu^\mu = (1, \vec{0})$. The subscripts $a,b$ are
The nucleon mass is given by

\[ m_N = \frac{940}{3} \text{MeV}, \]

and the vector meson exchange potentials are given by

\[ V_{\mu} = \sqrt{2} \left( \frac{(\rho^0 + \omega)/\sqrt{2}}{\rho^-} \right) \mu, \]

with the pion decay constant \( f_\pi = 92.3 \text{ MeV} \) and \( g_V = m_\rho/f_\pi \). The coupling constants of the Lagrangians are summarized in Table 1.

The Lagrangians for nucleons and light mesons are given by the Bonn model [11, 12] as

\[ \mathcal{L}_{\pi NN} = ig_{\pi NN} \bar{N}_b \gamma^5 N_a \tau_{ba}, \]

\[ \mathcal{L}_{\nu NN} = g_{\nu NN} \bar{N}_b \left( \gamma^\mu (\hat{V}_\mu)_{ba} + \frac{\kappa}{2m_N} \sigma_{\mu\nu} \partial^\nu (\hat{V}_\mu)_{ba} \right) N_a. \]

From those Lagrangians in Eqs. (1)–(3), we obtain the \( P^{(*)}N \) potentials. The \( \pi \) exchange potential is given by

\[ V_{PN-PN}^\pi = -\frac{g_\pi g_{\pi NN}}{2m_N f_\pi} \frac{1}{3} \left[ \bar{\sigma}^\dagger \cdot \bar{\sigma} C(r; m_\pi) + S_\pi T(r; m_\pi) \right] \bar{\tau}_P \cdot \bar{\tau}_N, \]

and the vector meson exchange potentials are given by

\[ V_{PN-PN}^{V} = \frac{g_{\nu NN}}{2m_N f_\pi} \frac{1}{3} \left[ \bar{T} \cdot \bar{\sigma} C(r; m_V) - S_T T(r; m_V) \right] \bar{\tau}_P \cdot \bar{\tau}_N, \]

\[ V_{PN-PN}^{V} = \frac{g_{\nu NN}}{2m_N f_\pi} \frac{1}{3} \left[ 2\bar{T} \cdot \bar{\sigma} C(r; m_V) - S_T T(r; m_V) \right] \bar{\tau}_P \cdot \bar{\tau}_N. \]

The nucleon mass is given by \( m_N = 940 \text{ MeV} \). \( \bar{T} \) is the spin-one operator, \( \bar{T} = i\bar{\tau} \times \varepsilon \), and \( S_\pi \) (\( S_T \)) is the tensor operator \( S_\pi (\varepsilon) = 3(\bar{\sigma} \cdot \varepsilon)(\bar{\tau} \cdot \varepsilon) - \bar{\sigma} \cdot \bar{\sigma} \). With \( \varepsilon = \varepsilon /|\varepsilon| \) for \( \bar{\sigma} = \varepsilon \times (\bar{T}) \), where \( \bar{\tau} \) is the relative position vector between \( P^{(*)} \) and \( N \). \( \bar{\sigma} \) are Pauli matrices acting on
nucleon. $\tau_P$ ($\tau_N$) are isospin operators for $P^{(s)}$ ($N$). For the OPEP, the $PN - PN$ term is absent because the vertex of three pseudoscalar mesons violates the parity conservation. The functions $C(r; m_\alpha)$ and $T(r; m_\alpha)$ for $\alpha = \pi, \rho, \omega$ are expressed by
\begin{align}
C(r; m_\alpha) &= \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{m_\alpha^2}{\vec{q}^2 + m_\alpha^2} e^{\vec{q} \cdot \vec{r}} F(\vec{q}, m_\alpha), \\
S_C(r) T(r; m_\alpha) &= \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{-\vec{q}^2}{\vec{q}^2 + m_\alpha^2} S_C(\vec{q}) e^{\vec{q} \cdot \vec{r}} F(\vec{q}, m_\alpha),
\end{align}
where the dipole-type form factor
\begin{equation}
F(\vec{q}, m_\alpha) = \frac{\Lambda_N^2 - m_\alpha^2}{\Lambda_N^2 + |\vec{q}|^2} \frac{\Lambda_P^2 - m_\alpha^2}{\Lambda_P^2 + |\vec{q}|^2}
\end{equation}
with cutoff parameters $\Lambda_N$ and $\Lambda_P$ is introduced for spatially extended hadrons. From a quark model estimation, we use $\Lambda_B = 1.35\Lambda_N$ for $D^{(s)}$ meson and $\Lambda_B = 1.29\Lambda_N$ for $B^{(s)}$ meson, and $\Lambda_{PQ} = 1.12\Lambda_N$ for the heavy quark limit, with $\Lambda_N = 830$ MeV, as discussed in Refs. [12, 13].

The Hamiltonian is given by $H = K + V_{P^{(s)}N}$, where $K$ is the kinetic term, and $V_{P^{(s)}N}$ is the $P^{(s)}N$ potential shown above. By solving the coupled-channel Schrödinger equation, the bound state is obtained. We also obtain the resonance by analyzing the phase shift.

3. Numerical Results
In this section, we show the results for the bound and resonant states of the $\bar{D}N$ and $BN$ molecules with $J^P = 1/2^\pm, 5/2^\pm$, and the isospin $I = 0$. The obtained energy spectra are summarized in Table 2. The binding energy is shown as a real negative number. For the resonance, the energy is given by a complex value as $E_{re} - i\Gamma/2$ with a resonance energy $E_{re}$ and a decay width $\Gamma$.

We obtain the bound states both in $\bar{D}N$ and $BN$ for $J^P = 1/2^-$. In the bound states, we find that the tensor force of the OPEP in the $PN - P^*N$ mixing component plays an important role to generate the strong attraction. For the bottom sector, the $BN$ binding energy is larger than the $\bar{D}N$. The $BN$ system obtains not only the suppression of the kinetic energy but also the enhancement of the mixing effect due to the small $BB^*$ mass difference.

For $J^P = 1/2^+, 3/2^\pm$ and $5/2^+$, the resonance states are found above the $PN$ threshold. In particular the states with $J^P = 3/2^-$ have the small decay width. We find the interesting structure of the resonances which is the Feshbach resonance. The states have the quasi bound states of the $P^*N$ channels.

We obtain no bound state for $I = 1$. In this channel, the OPEP does not produce the strong attraction because of the small contribution of the isospin factor $\tau_P \cdot \tau_N = 1$ for $I = 1$, while $\tau_P \cdot \tau_N = -3$ for $I = 0$.

Finally, we consider the $P_QN$ systems in the heavy quark limit, where the pseudoscalar and vector mesons, $P_Q$ and $P^*_Q$, are degenerate, namely $m_{P_Q} = m_{P^*_Q}$. We study the states for $J^P = 1/2^-, 3/2^-$. We find that both states have the same binding energy, $-37.4$ MeV as summarized in Table 3. The obtained energies of the charm, bottom and heavy quark limit sectors are compared in Table 3. The energy difference of the $J^P = 1/2^-$ and $J^P = 3/2^-$ states decreases as the heavy quark mass increases. In the heavy quark limit, these bound states are degenerate. The results show that the $J^P = 1/2^-$ and $J^P = 3/2^-$ states belong to the heavy quark spin multiplet, caused by the HQS [38, 39].

4. Summary
We have explored the possible existence of the genuinely exotic nuclei as $\bar{D}N$ and $BN$. The interaction is constructed by introducing the heavy quark spin symmetry which provides the
energy of the resonance states are given by the resonance energy $E_{re}$ and the decay width $\Gamma$ as $E_{re} - i\Gamma/2$. All values are measured from the $PN$ threshold and given in units of MeV.

| $\tilde{D}N$ | $P = -$ | $P = +$ | $BN$ | $P = -$ | $P = +$ |
|-------------|---------|---------|------|---------|---------|
| $J = 1/2$   | 2.1     | 26.8 - i65.7 | $J = 1/2$ | -23.0 | 5.8 - i3.0 |
| $J = 3/2$   | 113.2 - i8.9 | 148.2 - i5.1 | $J = 3/2$ | 6.9 - i0.05 | 31.8 - i14.4 |
| $J = 5/2$   | 176.0 - i87.4 |  | $J = 5/2$ |  | 58.4 - i24.8 |

Table 3. Comparison of the energy spectra of $\tilde{D}N$, $BN$, and $PQN$ for $J^P = 1/2^-, 3/2^-$ in Refs. [38, 39]. The energy difference of the $J^P = 1/2^-$ and $3/2^-$ states, $E_{3/2^-} - E_{1/2^-}$, is also shown. The values are given in the unit of MeV.

| $J^P$ | $\tilde{D}N$ | $BN$ | $PQN$ |
|-------|-------------|------|-------|
| $1/2^-$ | -2.1 | -23.0 | -37.4 |
| $3/2^-$ | 113.2 - i8.9 | 6.9 - i0.05 | -37.4 |
| $E_{3/2^-} - E_{1/2^-}$ | 115.3 | 29.9 | 0.0 |

mass degeneracy of the heavy pseudoscalar and vector mesons. The meson exchange potentials for $\pi$, $\rho$ and $\omega$ are employed as the $P(N)$ interaction, where those potentials induce the $P - P^*$ mixing. By solving the coupled-channel equations for $PN$ and $P^*N$, we obtain bound states and resonances in charm and bottom sectors. The tensor force of the OPEP mixing $PN$ and $P^*N$ plays the important role to produce the strong attraction. This force is enhanced by the small mass splitting of mesons due to the heavy quark spin symmetry. The mechanism is unique in the heavy hadron systems with the small mass difference of the spin partner. In the heavy quark limit, we find that the bound states for $J^P = 1/2^-$ and $3/2^-$ are degenerate. The result indicates the $J^P = 1/2^-$ and $3/2^-$ states belongs to the heavy quark spin multiplet. The obtained exotic states can be investigated in relativistic heavy ion collisions in RHIC and LHC [10, 11], J-PARC and GSI-FAIR.

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