Single Higgs boson production at the ILC in the left-right twin Higgs model

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In this work, we analyse three dominant single SM-like Higgs boson production processes in the left-right twin Higgs model (LRTHM): the Higgs-strahlung (HS) process $e^+e^- \rightarrow Zh$, the vector boson fusion (VBF) process $e^+e^- \rightarrow \nu \bar{\nu}h$ and the associated produced with top pair process $e^+e^- \rightarrow t\bar{t}h$ for three possible energy stages of the International Linear Collider (ILC), and compared our results to the expected experimental accuracies with various accessible Higgs decay channels. The following observations have been obtained: (i) In the reasonable parameter space, the LRTHM can generate significant contributions to these processes with polarized beams; (ii) Among various Higgs boson decay channels, the $b\bar{b}$ signal strength is most sensitive to the LRTHM due to the high expected precision. For the $t\bar{t}h$ production process, only for larger $M$ and lower values of the scale $f$, the absolute value of $\mu_{b\bar{b}}$ can deviate from the SM prediction by over 8.7% and thus may be observable at the future ILC at $\sqrt{s} = 1$ TeV; (iii) The ILC Higgs data can give strong limit on the scale parameter $f$.

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I. INTRODUCTION

On July 4th, 2012, a neutral Higgs boson with a mass around 125 GeV was discovered at CERN’s Large Hadron Collider (LHC) by both the ATLAS and CMS collaborations [1, 2], whose properties appear to be well consistent with those expected of the Standard Model (SM) [3, 4]. However, the SM suffers from the so-called little hierarchy problem [5] and cannot provide a dark matter (DM) candidate, which is actually a sound case for new physics (NP) beyond the SM. On the other hand, the Higgs-like resonance with mass about 125 GeV can also be well explained in many NP models where the Higgs is a pseudo-Goldstone boson. Here we focus on the left-right twin Higgs model (LRTHM) [6, 7], which can successfully solve the little hierarchy problem and also predicts a good candidate for weakly interacting massive particle (WIMP) dark matter.

As we know, the precision measurements of the Higgs boson are rather challenging at the LHC, since various Higgs couplings to SM fermions and vector bosons still have large uncertainties based on the current LHC data [8]. Thus the most precise measurements will be performed in the clean environment of a future high energy $e^+e^-$ linear collider, such as the International Linear Collider (ILC) [9, 10]. Furthermore, a unique feature of the ILC is the presence of initial state radiation (ISR) and beamstrahlung, which can help to improve the measurement precision. The ILC is planned to operate at three stages for the center-of-mass (c.m.) energy: 250 GeV, 500 GeV and 1 TeV. At the different energy stages, the Higgs-strahlung (HS) process $e^+e^- \rightarrow Zh$, the vector boson fusion (VBF) process $e^+e^- \rightarrow \nu\bar{\nu}h$, and the associated with top pair process $e^+e^- \rightarrow t\bar{t}h$ are three main production channels for the Higgs boson, which are very important for studying the properties of Higgs boson and testing NP beyond the SM [11]. These processes have been studied in the context of the SM [12, 13] and various of NP models, such as the MSSM [14], the little Higgs models [15] and other composite Higgs models [16].

The LRTHM predicted the existence of the new heavy gauge bosons, top partner, neutral and charged scalars at or below the TeV scale, which can produce rich phenomenology at the high energy colliders [17–23]. In the LRHM, the couplings of the electroweak gauge bosons to top quarks and the couplings of the SM-like Higgs boson to ordinary particles are corrected at the order $O(v^2/f^2)$. Besides, the new particles, such as the heavy neutral gauge boson and top partner, can also contribute to some Higgs boson production processes. On the other hand, the decays $h \rightarrow gg, \gamma\gamma$, and $Z\gamma$ all receive contributions from the modified Higgs couplings and the new heavy particles, which has been studied in our recent work [24]. The aim of this paper is to consider the processes $e^+e^- \rightarrow Zh$, $e^+e^- \rightarrow \nu\bar{\nu}h$ and $e^+e^- \rightarrow t\bar{t}h$ in the LRTHM, and see whether the effects of this model on these processes can be detected in the future ILC experiments.

The paper is organized as follows. In section II, we recapitulate the LRTH model and lay out the couplings of the particles relevant to our calculation. In Sec. III, we study the effects of the LRTHM on three single Higgs boson production processes and project limits on the LRTHM from the future measurements of the 125 GeV Higgs at the ILC with polarized beams. Finally, we give our conclusion in Sec.IV.

II. RELEVANT COUPLINGS IN THE LRTHM

Here we we will briefly review the essential features of this model and focus on the couplings relevant to our work. For more details one can refer to [17]. The LRTHM is gauged the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ sub-groups of the global $U(4) \times U(4)$ symmetry. Two Higgs fields $(H = (H_L, H_R)$ and $\hat{H} = (\hat{H}_L, \hat{H}_R))$ are introduced in the LRTHM, and each transforms as $(4, 1)$ and
(1, 4), respectively. With the vacuum expectation values $\langle H \rangle = (0, 0, 0, f)$ and $\langle \hat{H} \rangle = (0, 0, 0, \hat{f})$, the global symmetry is spontaneously broken down to its subgroup $U(3) \times U(3)$ and yields 14 Nambu-Goldstone bosons. The gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is eventually broken down to $U(1)_{EM}$. Six Goldstone bosons are eaten by the SM gauge bosons ($W^\pm, Z$) and by the heavy gauge bosons ($W_H^\pm, Z_H$). Here $W_H^\pm$ and $Z_H$ are three additional gauge bosons with masses of a few TeV in the LRTHM. There are left eight scalars: one SM-like Higgs boson $h$, one neutral pseudoscalar $\phi^0$, a pair of charged scalar $\phi^\pm$, and a $SU(2)_L$ doublet $h = (\hat{h}_1^+, \hat{h}_2^0)$. The lightest particle in the odd $\hat{h}_2^0$ is stable and can be treated as a candidate for DM, which has been studied in Refs. [25].

Besides the new heavy gauge bosons, a pair of vector-like quarks are also introduced to cancel the one-loop quadratic divergence of Higgs mass. The masses of the neutral heavy gauge boson, SM-like top quark and top partner are given by [17]

$$m^2_{Z_H} = \frac{e^2 C^2_W}{2 S^2_W C^2_W} (f^2 + \hat{f}^2) - M^2_Z,$$

$$m^2_t = \frac{1}{2} (M^2 + y^2 f^2 - N_t),$$

$$m^2_T = \frac{1}{2} (M^2 + y^2 f^2 + N_t),$$

with $N_t = \sqrt{(M^2 + y^2 f^2)^2 - 4 y^4 f^4 \sin^2 2x}$ and $x = v/(\sqrt{2} f)$. $S_W = \sin \theta_W$, $C_W = \cos \theta_W$, $C^2_W = \cos 2\theta_W$, and $\theta_W$ is the Weinberg angle. The values of $f$ and $\hat{f}$ are interconnected once we set $v \simeq 246$ GeV. The parameter $M$ is essential to the mixing between the SM-like top quark $t$ and its partner $T$. The value of $y$ can be determined by fitting the experimental value of the SM-like top quark mass $m_t$.

The new couplings expression forms which are related our calculation, are shown as [17]:

$$g^Zt\bar{T} = \frac{e C_L S_L}{2 S_W C_W}, \quad g^Zt\bar{T} = \frac{e f^2 x^2 S_W C_R S_R}{2 f^2 C^3_W},$$

$$g^Zt\bar{T} = \frac{e C_L S_L S_W}{2 C_W \cos 2\theta_W}, \quad g^Zt\bar{T} = \frac{-e C_W C_R S_R}{2 S_W \cos 2\theta_W},$$

$$g^Zt^\pm e^\mp = \frac{2 e S_W}{4 C_W \cos 2\theta_W}, \quad g^Zt^\pm e^\mp = \frac{e (1 - 3 \cos 2\theta_W)}{4 S_W C_W \cos 2\theta_W},$$

$$V_{\phi H \bar{T}} = -\frac{i y}{\sqrt{2}} S_L S_R, \quad V_{h\phi^0 Z} = i e \exp \beta \mu / (6 S_W C_W),$$

$$V_{h t \bar{T}} = -\frac{y}{\sqrt{2}} [(C_L S_R + S_L C_R) P_L + (C_L S_R x + S_L C_R) P_R],$$

$$V_{h Z_H Z_{H'}} = -\frac{e^2 f x}{\sqrt{2} S^2_W C^2_W} g_{\mu \nu}, \quad V_{h Z_H Z_{H'}} = \frac{e^2 f x}{\sqrt{2} C^2_W \cos 2\theta_W} g_{\mu \nu},$$

with

$$S_L = \frac{1}{\sqrt{2}} \sqrt{1 - (y^2 f^2 \cos 2x + M^2)/N_t}, \quad C_L = \sqrt{1 - S^2_L},$$

$$S_R = \frac{1}{\sqrt{2}} \sqrt{1 - (y^2 f^2 \cos 2x - M^2)/N_t}, \quad C_R = \sqrt{1 - S^2_R}.$$
In the LRTHM, the normalized couplings of $h f \bar{f}(f = b, c)$, $h t\bar{t}$, $h\tau^+\tau^-$, $hVV^*(V = Z, W)$, $hgg$, and $h\gamma\gamma$ are given by [17, 23]:

$$V_{hhV}/SM \equiv V_{hVV}/V_{hVV}^{SM} = 1 - \frac{v^2}{6f^2},$$  \hspace{1cm} (12)

$$V_{hf}/SM = V_{h\tau^+\tau^-}/SM = 1 - \frac{2v^2}{3f^2}, \quad V_{h\tilde{t}}/SM = CLC_R,$$  \hspace{1cm} (13)

$$V_{hgg}/SM = \frac{1}{2}F_{1/2}(\tau_i) y_t + \frac{1}{2}F_{1/2}(\tau_T) y_T,$$  \hspace{1cm} (14)

$$V_{h\gamma\gamma}/SM = \frac{4}{3}F_{1/2}(\tau_i) y_t + \frac{4}{3}F_{1/2}(\tau_T) y_T + F_1(\tau_W) y_W,$$  \hspace{1cm} (15)

with

$$F_1 = 2 + 3\tau + 3\tau(2 - \tau)f(\tau), \quad F_{1/2} = -2\tau[1 + (1 - \tau)f(\tau)],$$

$$f(\tau) = \left[\sin^{-1}(1/\sqrt{\tau})\right]^2, \quad g(\tau) = \sqrt{\tau - 1}\sin^{-1}(1/\sqrt{\tau}),$$  \hspace{1cm} (16)

for $\tau_i = 4m_i^2/m_h^2 \geq 1$. The relevant couplings $y_t$ and $y_T$ can be written as

$$y_t = S_L S_R, \quad y_T = \frac{m_t}{m_T} C_L C_R,$$  \hspace{1cm} (17)

which can be determined by the parameters $f$ and $M$. Here we have neglected the contributions from $W_H$ and $\phi^\pm$ for the $h \rightarrow \gamma\gamma$ decay, this is because their contributions are even much smaller than that for the T-quark [24]. On the other hand, the relation between $G_F$ and $v$ is modified from its SM form, introducing an additional correction $y_{G_F}$ as $1/v^2 = \sqrt{2}G_F y_{G_F}^2$ with $y_{G_F}^2 = 1 - v^2/(6f^2)$.

### III. NUMERICAL RESULTS AND DISCUSSIONS

In the LRTHM, the tree-level Feynman diagrams of the processes $e^+e^- \rightarrow Zh$, $e^+e^- \rightarrow \nu\bar{\nu}h$, and $e^+e^- \rightarrow t\bar{t}h$ are shown in Figs. 1-2. We can see that the modified couplings of $hXX$ and the

![Feynman diagrams](image)

**FIG. 1.** Feynman diagrams for the Higgs-strahlung process $e^+e^- \rightarrow Zh$ (left) and $WW$-fusion process $e^+e^- \rightarrow \nu\bar{\nu}h$ (right).
additional particles ($Z_H$ and $T$-quark) can contribute these processes at the tree level. Obviously, the heavy $T$-quark, the neutral scalar $\phi^0$ and the charged Higgs bosons $\phi^\pm$ can contribute these processes at the loop level. However, their contributions are negligible and can be safely neglected since (i) $T$-quark is heavy and meanwhile the couplings of $h T \bar{T}$ is very small, (ii) the couplings of $h \phi^0 \phi^0$ and $h \phi^+ \phi^-$ are both suppressed largely by a factor of $v^2/(2 f^2)$. Thus, we only focus on these contributions at the lowest-order in this work.

In our numerical calculations, the SM-like Higgs boson mass is fixed as 125 GeV, the SM input parameters involved are taken from [26]. There are two LRTHM parameters: $f$ and $M$. The indirect constraints on $f$ come from the $Z$-pole precision measurements, the low energy neutral current process and high energy precision measurements off the $Z$-pole. The mixing parameter $M$ is constrained by the $Z \to b \bar{b}$ branching ratio and oblique parameters [17, 18]. By combined the Higgs data from the LHC and electroweak precise measurements, we vary them in the ranges:

$$600 \text{GeV} \leq f \leq 1500 \text{GeV}, \quad 0 \leq M \leq 150 \text{GeV}. \quad (18)$$

From Fig.2(e) we can see that the neutral scalar $\phi^0$ mass can also contribute the cross section. However, this contribution is very small due to the suppressed couplings of $h \phi^0 Z$ and $\phi^0 t \bar{t}$, and thus we can safely take its mass as $m_{\phi^0} = 150$ GeV. All the numerical studies are done using CalcHEP package [27].

FIG. 2. Feynman diagrams for the process $e^+ e^- \to t \bar{t} h$ in the LRTHM.
A. The production cross section with polarized beams

With the longitudinal polarization of the initial electron and positron beams, the cross section of a process can be expresses as [28]

\[
\sigma(P_e^-, P_e^+) = \frac{1}{4} [(1 + P_e^-)(1 + P_e^+)\sigma_{RR} + (1 - P_e^-)(1 - P_e^+)\sigma_{LL} \\
+ (1 + P_e^-)(1 - P_e^+)\sigma_{RL} + (1 - P_e^-)(1 + P_e^+)\sigma_{LR}],
\]

(19)

where \( P_e^- (P_e^+) \) is the polarization degree of the electron (positron) beam. \( \sigma_{LR} \) stands for the cross section for completely left-handed polarized \( e^- \) beam \( (P_e^- = -1) \) and completely right-handed polarized \( e^+ \) beam \( (P_e^+ = 1) \), and other cross sections \( \sigma_{LL}, \sigma_{RR}, \) and \( \sigma_{RL} \) are defined analogously.

FIG. 3. The cross sections \( \sigma \) versus the scale \( f \) at the ILC with unpolarized and polarized beams for three processes with \( M = 150 \) GeV. Here the beam polarizations are tuned to be \( (e^-, e^+) = (-0.8, +0.3) \) at 250 GeV and 500 GeV as well as \( (e^-, e^+) = (-0.8, +0.2) \) at 1 TeV.
In Fig.3, we plot the production cross sections for three processes at three c.m. stages in the SM and LRTHM. We can see that the cross sections in LRTHM are always smaller than those in the SM, and the value of cross sections increases as the parameter $f$ increases, which means that the correction of the LRTHM decouples with the scale $f$ increasing. This is similar with the situation in the various of little Higgs models [29, 30]. For comparison we also show the corresponding results for unpolarized beams. We can see that the cross sections with polarized beams are larger than those with unpolarized beams, and thus make the ILC more powerful in probing such new physics effects. Note that for $\sqrt{s} = 250(500)$ GeV, the cross sections of HS process are about 240 (57) fb with the unpolarized beams, which is in accord with the result in [31]. At low energy (such as $\sqrt{s} = 250$ GeV), the HS process is dominant and the cross section can reach about 350 fb with polarized beams, while the VBF and $t\bar{t}h$ processes are more significant at high energies.

![Graphs](image.png)

**FIG. 4.** The relative correction $R = (\sigma^{LRTHM} - \sigma^{SM}/\sigma^{SM})$ versus the scale $f$ at the ILC with polarized beams for three processes.

In Fig. 4, we show the dependence of the relative correction $R = (\sigma^{LRTHM} - \sigma^{SM}/\sigma^{SM})$ on the scale $f$ for three production channels with polarized beams. For three processes, the relative corrections are all negative and the magnitude of deviation is sensitive to the scale parameter $f$. The absolute values of suppression are larger than 5% for small values of the scale $f$. While for a large value of $f$, the suppressions for three processes are only a few percent. The total SM electroweak correction for the HS production process is about 5% for $m_h = 125$ GeV and $\sqrt{s} = 250$ GeV [12]. The expected accuracies for HS and VBF processes are about 2.0 $\sim$ 2.6% for $m_h = 125$ GeV [10]. For the process $e^+e^- \rightarrow t\bar{t}h$, the values of relative corrections are sensitive to the parameters $M$ and $f$, and the magnitude of such correction becomes more sizable for larger $M$ and lower $f$. For example, for $M = 150$ GeV and $f \leq 800$ GeV, the contributions can alter the SM cross section over 8%. At the ILC with $\sqrt{s} = 1000$ GeV, an accuracy of about 6.3% can be reached with the polarized beams [10]. Thus, the LRTHM effects might be detected in the future ILC experiments.
B. The Higgs signal strengths

Considering the Higgs boson decay channels, the Higgs signal strengths can be defined as

$$\mu_i = \frac{\sigma_{LRTHM} \times BR(h \rightarrow i)_{LRTHM}}{\sigma_{SM} \times BR(h \rightarrow i)_{SM}},$$

(20)

where $i$ denotes a possible final state of the Higgs boson decay (for example $b\bar{b}$, $gg$, $c\bar{c}$, $ZZ^*$ and $\gamma\gamma$). The projected 1σ sensitivities of channels at the ILC are shown in Table 1. We can see that the $b\bar{b}$ channel is more easily accessible than other channels.

TABLE I. Projected 1σ sensitivities of channels for the ILC operating at $\sqrt{s} = 250$ GeV, 500 GeV and 1000 GeV with a corresponding integrated luminosity of 250 fb$^{-1}$, 500 fb$^{-1}$ and 1000 fb$^{-1}$, respectively [10, 32].

| $\sqrt{s}$ ($P_\ell^-, P_\ell^+$) | 250 GeV (-0.8, 0.3) | 500 GeV (-0.8, 0.3) | 1 TeV (-0.8, 0.2) |
|-------------------------------|----------------------|----------------------|-------------------|
| channel                      | HS VBF               | HS VBF ttH VBF ttH   |                   |
| $h \rightarrow bb$           | 1.1% 10.5%           | 1.8% 0.66% 35%       | 0.47% 8.7%        |
| $h \rightarrow gg$           | 9.1% -               | 14% 4.1% -           | 3.1% -            |
| $h \rightarrow c\bar{c}$     | 7.4% -               | 12% 6.2% -           | 7.6% -            |
| $h \rightarrow \tau^+\tau^-$ | 4.2% -               | 5.4% 14% -           | 3.5% -            |
| $h \rightarrow ZZ^*$         | 19% -                | 25% 8.2% -           | 4.4% -            |
| $h \rightarrow WW^*$         | 9.1% -               | 9.2% 2.6% -          | 3.3% -            |
| $h \rightarrow \gamma\gamma$ | 34% -                | 34% 23% -            | 8.5% -            |

In the LRTHM, the modifications of the $hVV (V = Z, W)$ and $hhf$ (the SM fermions pair) couplings can give the extra contributions to the Higgs boson production processes. On the other hand, the loop-induced couplings, such as $h\gamma\gamma$ and $hgg$, could also be affected by the presence of top partner, new heavy charged gauge bosons and charged scalars running in the corresponding loop diagrams. Lastly, beside the effects already seen in the HS channel due to the exchange of $s$-channel heavy neutral gauge boson $Z_H$, the exchange of top partner $T$ could also affect the production cross section for the process $e^+e^- \rightarrow t\bar{t}h$. All these effects can modify the signal strengths in a way that may be detectable with the experimental accuracies expected at the ILC.

In Fig. 5, we show the dependence of the Higgs signal strengths $\mu_i$ ($i = b\bar{b}, gg$) on the scale $f$ for the HS and VBF processes with polarized beams. One can see that the NP correction becomes smaller rapidly along with the increase of the parameter $f$. For the HS process, the contributions of the LRTHM can be detected by the measurement of the $b\bar{b}$ signal rate in the future ILC experiments. Meanwhile, it is difficult to observe these effects via the $gg$ channel due to the relative weak bound. For the VBF process, the contribution of the LRTHM can be easily detected by the measurement of the $b\bar{b}$ signal rate due to the high expected precision. Meanwhile, this contribution can also be detected by the measurement of the $gg$ signal rates in most part of the parameter spaces.

In Fig. 6, we show the dependence of the Higgs signal strengths $\mu_{b\bar{b}}$ on the scale $f$ for the process $e^+e^- \rightarrow t\bar{t}h$. From Table 1 we know that the 35% accuracy for top Yukawa couplings expected at $\sqrt{s} = 500$ GeV can be improved by 8.7% at 1 TeV. Only for $f = 600$ GeV and
ILC experiments.

and lower values of the scale \( \sqrt{s} \) difficult to observe the LRTHM effect on this process at \( \sqrt{s} = 150 \) GeV, the experimental precision limits around the SM expectation according to Table 1.

\[ M = 150 \text{ GeV}, \] the Higgs signal strengths \( \mu_i \) (i = \( b\bar{b}, gg \)) for the processes \( e^+e^- \rightarrow Zh \) (left) and \( e^+e^- \rightarrow \nu\bar{\nu}h \) (right) as a function of the model scale \( f \) at the ILC with polarized beams. The dotted red lines represent the experimental precision limits around the SM expectation according to Table 1.

\[ M = 150 \text{ GeV}, \] the absolute value of \( \mu_{bb} \) can deviate from the SM prediction by over 9%, which might be detected in the future ILC experiments.

\[ M = 150 \text{ GeV}, \] the Higgs signal strengths \( \mu_{bb} \) can reach 0.84 for \( \sqrt{s} = 500 \) GeV. Thus it is difficult to observe the LRTHM effect on this process at \( \sqrt{s} = 500 \) GeV via the \( b\bar{b} \) channel. For \( \sqrt{s} = 1 \) TeV, one can see the magnitude of such correction becomes more sizable for larger \( M \) and lower values of the scale \( f \). For example, for \( M = 150 \) GeV and \( f \leq 800 \) GeV, the absolute value of \( \mu_{bb} \) can deviate from the SM prediction by over 9%, which might be detected in the future ILC experiments.

FIG. 5. Higgs signal strengths \( \mu_i \) (i = \( b\bar{b}, gg \)) for the processes \( e^+e^- \rightarrow Zh \) (left) and \( e^+e^- \rightarrow \nu\bar{\nu}h \) (right) as a function of the model scale \( f \) at the ILC with polarized beams. The dotted red lines represent the experimental precision limits around the SM expectation according to Table 1.

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FIG. 6. Higgs signal strengths \( \mu_{bb} \) for the process \( ee' \rightarrow t\bar{t}h \) as a function of the model scale \( f \) at the ILC with polarized beams for \( \sqrt{s} = 500 \) and 1000 GeV.


C. Global fit with the ILC data

Now we perform a global fit using the projected $1\sigma$ sensitivities for the channels in the HS and VBF processes at the ILC-250 GeV, ILC-500 GeV, and ILC-1000 GeV, respectively. The chi-square function $\chi^2$ can be defined as

$$\chi^2 = \sum_i \frac{(\mu_i - 1)^2}{\sigma^2_i},$$

where $\sigma_i$ denotes the $1\sigma$ uncertainty for the signal $i$ in Table 1. In our discussion, we take $\chi^2 - \chi^2_{\text{min}} \leq 6.18$, where $\chi_{\text{min}}$ denotes the minimum of $\chi$ which happens for the largest values of the parameters $f$ and $M$, i.e. for $M = 150$ GeV and $f = 1500$ GeV, $\chi^2_{\text{min}} = 3.33$ with the ILC-250 GeV data, and $\chi^2_{\text{min}} = 14.98$ with the ILC-500 GeV data, respectively. These samples correspond to the 95% confidence level regions (corresponding to be within $2\sigma$ range) in any two dimensional plane of the model parameters when explaining the Higgs data.

In Fig. 7 we plot the contours of $\chi^2$ for the parameters $M$ and $f$. One can see that the ILC-250 GeV, ILC-500 GeV and ILC-1000 GeV will gradually enhance the values of $f$. At the $2\sigma$ level, the value of $f$ must be larger than 1150 GeV for the ILC-250 GeV, and larger than 1300 GeV for the ILC-500 GeV, respectively. The bound for the ILC data is even much stronger than that for the LHC Higgs data [24].

IV. CONCLUSIONS

The LRTHM is a concrete realization of the twin Higgs mechanism, which provides an alternative solution to the little hierarchy problem. In this work, we have studied three Higgs boson
production processes $e^+e^- \rightarrow Zh$, $e^+e^- \rightarrow \nu\bar{\nu}h$ and $e^+e^- \rightarrow t\bar{t}h$ in the LRTHM. We calculate the production cross sections for three processes with and without the polarized beams, the relative corrections with the polarized beams for three energy stages. We also study the signal rates with the SM-like Higgs boson decaying to $b\bar{b}$ and $gg$, and perform a global fit using the ILC-250 GeV, ILC-500 GeV, and ILC-1000 GeV data. Our numerical results show that:

1. For three processes, the cross sections with polarized beams are larger than those with unpolarized beams, which are more sensitive to the LRTHM;

2. In a large part of the allowed parameter space, the LRTHM can generate significant contributions to the HS and VBF processes, which are large enough to approach the expected experimental accuracies in the $h \rightarrow b\bar{b}$ channel at the future ILC experiments with polarized beams.

3. For the process $e^+e^- \rightarrow t\bar{t}h$, we found that in certain regions of parameter space (for larger $M$ and lower values of the scale $f$), the absolute value of the Higgs signal strength $\mu_{b\bar{b}}$ can deviate from the SM prediction by over 8.7%, and thus may be observable at the future ILC for $\sqrt{s} = 1$ TeV with polarized beams $P(e^-, e^+) = (-0.8, 0.2)$.

4. The ILC Higgs data can give strong limit on the scale parameter $f$. For the ILC-250 (500) GeV, the value of $f$ must be larger than 1150 (1300) GeV at the 2$\sigma$ level.

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