Renormalization Conditions and the Effective Potential of the Massless $\phi^4$ Theory

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We point out that there is a missing portion in the two-loop effective potential of the massless $O(N) \phi^4$ theory obtained by Jackiw in his classic paper, Phys. Rev. D 9, 1686 (1974).

PACS number(s): 11.10.Gh

I. INTRODUCTION

The effective potential in quantum field theory plays a crucial role in connection with the problem of the spontaneous symmetry breaking. In this field there are three classic papers [1,2,3]. Coleman and Weinberg [1] were the first ones to calculate the higher-order effective potential of a scalar field at one loop level by summing up an infinite number of Feynman graphs. Jackiw [2] has used the Feynman path-integral method to obtain a simple formula for the effective potential. He has succeeded in representing each loop order containing an infinite set of conventional Feynman graphs by finite number of graphs using this algebraic method which can formally be extended to the arbitrary higher-loop order. In Ref. [3] the functional integral is explicitly evaluated using the steepest descent method at two-loop level. Higher-loop calculations with this method are very difficult.

The purpose of this paper is to show that there is a missing portion in the two-loop effective potential of the massless $O(N) \phi^4$ theory obtained by Jackiw [2]. In this paper we employ the dimensional regularization method [4] instead of the cutoff regularization method used in Ref. [2] and for the sake of brevity we confine ourselves to the case of single component theory ($N = 1$).

II. REVIEW OF THE CALCULATION

The Lagrangian for a theory of a self-interacting spinless field $\phi$ is given as

$$\mathcal{L}(\phi(x)) = \frac{1}{2} \delta Z \partial_\mu \phi \partial^\mu \phi - \frac{m^2 + \delta m^2}{2} \phi^2 - \frac{\lambda + \delta \lambda}{4!} \phi^4,$$

(1)

where the quantities $\phi$, $m$, and $\lambda$ are the renormalized field, the renormalized mass, and the renormalized coupling constant respectively, whereas $\delta Z$, $\delta m^2$, and $\delta \lambda$ are corresponding (infinite) counterterm constants. We will confine ourselves to the massless theory ($m = 0$). The effective potential is most suitably defined, when the effective action $\Gamma[\phi_{cl}]$, being the generating functional of the one-particle-irreducible (1PI) Green’s functions $(\Gamma^{(n)}(x_1, \ldots, x_n))$, is expressed in the following local form (the so-called derivative expansion):

$$\Gamma[\phi_{cl}] = \int d^4x \left[ -\mathcal{V}(\phi_{cl}(x)) + \frac{1}{2} \delta Z(\phi_{cl}(x)) \partial_\mu \phi_{cl}(x) \partial^\mu \phi_{cl}(x) + \cdots \right],$$

(2)

where $\phi_{cl}(x)$ is the vacuum expectation value of the field operator $\phi(x)$ in the presence of an external source. By setting $\phi_{cl}(x)$ in $\mathcal{V}(\phi_{cl}(x))$ to be a constant field $\hat{\phi}$, we obtain the effective potential $V_{\text{eff}}(\hat{\phi})$

$$V_{\text{eff}}(\hat{\phi}) \equiv \mathcal{V}(\phi_{cl}(x))|_{\phi_{cl}(x)=\hat{\phi}}.$$

(3)

Following the field-shift method of Jackiw [2] for the calculation of the effective potential, we first obtain the shifted Lagrangian with the constant field configuration $\hat{\phi}$

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\[
\mathcal{L}(\phi'; \phi(x)) = \frac{1 + \delta Z}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} \left( \delta m^2 + \frac{\lambda + \delta \lambda}{2} \phi^2 \right) \phi^2 \\
- \frac{\lambda + \delta \lambda}{6} \phi^3 - \frac{\lambda + \delta \lambda}{4!} \phi^4 .
\] (4)

The Feynman rules for this shifted Lagrangian are given in Fig. 1.

\[
\begin{align*}
\begin{array}{c}
\rule{0.25\text{cm}}{0.25\text{cm}} \\
= & \frac{i\hbar}{(1 + \delta Z)\hbar^2 - \delta m^2 - (\lambda + \delta \lambda)\phi^2 / 2}, \\
\begin{array}{c}
\begin{array}{c}
\end{array}
\end{array}
\end{align*}
\]

FIG. 1. Feynman rules of the shifted Lagrangian, Eq. (4).

Without introducing any new loop-expansion parameter, which is eventually set to be unity, we will use \(\hbar\) as a loop-counting parameter \([5]\). This is the reason why we have kept all the traces of \(\hbar\)’s in the Feynman rules above in spite of our employment of the usual “God-given” units, \(\hbar = c = 1\). In addition to the above Feynman rules, Fig. 1, which are used in constructing two- and higher-loop vacuum diagrams, we need another rule (Fig. 2) solely for a one-loop vacuum diagram which is dealt with separately in Jackiw’s derivation of his prescription and is essentially the same as that of Coleman and Weinberg \([1]\) from the outset.

\[
\begin{align*}
\begin{array}{c}
\end{array}
\end{align*}
\]

FIG. 2. Feynman rule for one-loop vacuum diagram.

Using the rules, Fig. 1 and Fig. 2, and including the terms of zero-loop order, we arrive at the formal expression of the effective potential up to two-loop order:

\[
\begin{align*}
V_{\text{eff}}(\phi) = & \left[ \frac{\delta m^2}{2} \phi^2 + \frac{\lambda + \delta \lambda}{4!} \phi^4 \right] + \left[ \text{Diag. 1} \right] + \left[ \text{Diag. 2} \right] + \left[ \text{Diag. 3} \right].
\end{align*}
\] (5)

The last three (bracketed-) terms on the right-hand side in the above equation appear in Fig. 3.

\[
\begin{align*}
- \frac{\hbar}{i} \left[ \frac{1}{2} \bigcirc + \frac{1}{2^3} \bigcirc \bigcirc + \frac{1}{3!2} \bigcirc \bigcirc \bigcirc \right]
\end{align*}
\]

FIG. 3. [Diag. 1] + [Diag. 2] + [Diag. 3].

For the purposes of renormalization we first expand the counterterm constants in power series, beginning with order \(\hbar\):

\[
\begin{align*}
\delta m^2 = \hbar \delta m_1^2 + \hbar^2 \delta m_2^2 + \cdots , \\
\delta \lambda = \hbar \delta \lambda_1 + \hbar^2 \delta \lambda_2 + \cdots , \\
\delta Z = \hbar \delta Z_1 + \hbar^2 \delta Z_2 + \cdots .
\end{align*}
\]

In what follows we will use the following notation for the effective potential up to the \(L\)-loop order:

\[
\begin{align*}
V_{\text{eff}}^{[L]}(\phi) = \sum_{i=0}^{L} \hbar^i V_{\text{eff}}^{(i)}(\phi).
\end{align*}
\]

2
The zero-loop part of the effective potential is given as
\[ V_{\text{eff}}^{(0)}(\phi) = \frac{\lambda}{4!} \phi^4. \] (6)

The one-loop part of the effective potential is readily obtained as
\[ V_{\text{eff}}^{(1)}(\phi) = \frac{3}{2} \frac{\delta m_1^2}{\sqrt{\phi}} + \frac{\delta \lambda_1}{4!} \phi^4 - \frac{1}{8(4\pi)^2} \lambda^2 \phi^4 + \frac{\lambda^2}{8(4\pi)^2} \left[ \frac{3}{32} + \frac{\gamma}{16} + \frac{1}{16} \ln \left( \frac{\lambda \phi^2}{4\pi M^2} \right) \right], \] (7)

where \( \gamma \) is the usual Euler constant and \( M \) is an arbitrary constant with mass dimension. The \( \epsilon \) poles in this equation are readily cancelled out by choosing the counterterm constants \( \delta m_1^2 \) and \( \delta \lambda_1 \) as follows:
\[ \delta m_1^2 = a_1, \quad \delta \lambda_1 = \frac{3\lambda_2}{(4\pi)^2} + b_1, \] (8)

where \( a_1 \) and \( b_1 \) are unspecified but finite constants at this stage. One may put \( a_1 \) (and \( a_2 \) below) to be zero from the beginning because the theory is massless. In our dimensional regularization scheme the pole part of \( \delta m_1^2 \) vanishes, but this is not the case in the cutoff regularization method. Besides \( \delta m_1^2 \) and \( \delta \lambda_1 \), there is another counterterm constant. It is \( \delta Z_1 \). In Jackiw’s calculation, \( \delta Z_1 \) is set to be zero. This is matched to the standard condition for the defining the scale of the field
\[ Z|_{\phi=0} = 1. \] (9)

In the massless theory, however, the above condition is afflicted by the infrared singularity, as remarked by Coleman and Weinberg \[ [6] \]. (In fact, this singularity cannot be seen in \( Z^{(1)} \), the one-loop order contribution to \( Z \). The infrared singularity appears for the first time in the two-loop order \[ [6] \].)

Now let us determine \( \delta Z_1 \) so as to meet the following modified condition which avoids the infrared singularity:
\[ Z|_{\phi^2=M^2} = 1. \] (10)

To this end, we use the following relation \[ [6] \]
\[ Z|_{\phi^2=M^2} = \left. \frac{\partial \tilde{\Gamma}^{(2)}(p^2)}{\partial p^2} \right|_{p^2=0, \phi^2=M^2}, \] (11)

where \( \tilde{\Gamma}^{(2)}(p^2) \) is the (momentum-conserving) 1PI two-point Green’s function in the shifted theory. The right-hand side of Eq. (11) is calculated as
\[ 1 + \frac{\hbar \lambda}{6(4\pi)^2} + \hbar \delta Z_1, \]
from which we find
\[ \delta Z_1 = -\frac{\lambda}{6(4\pi)^2} \equiv c_1. \] (12)

Note that this wave function renormalization constant \( \delta Z_1 \) is free of \( \epsilon \) singularity. But in a higher-loop order the wave function renormalization constant \( \delta Z_n \) may have the \( \epsilon \) singularity.

The two-loop part of the effective potential is obtained as
\[
V_{\text{eff}}^{(2)}(\phi) = \frac{\delta m_2^2}{2} \phi^2 + \frac{\delta \lambda_2}{4!} \phi^4 - \frac{\lambda^2}{8(4\pi)^2} \phi^4 + \frac{\lambda^2}{8(4\pi)^2} \left[ \frac{3}{32} + \frac{\gamma}{16} + \frac{1}{16} \ln \left( \frac{\lambda \phi^2}{4\pi M^2} \right) \right] + \frac{\lambda^2}{8(4\pi)^2} \left[ \frac{3}{32} + \frac{\gamma}{16} + \frac{1}{16} \ln \left( \frac{\lambda \phi^2}{4\pi M^2} \right) \right] + \frac{\lambda^2}{8(4\pi)^2} \left[ \frac{3}{32} + \frac{\gamma}{16} + \frac{1}{16} \ln \left( \frac{\lambda \phi^2}{4\pi M^2} \right) \right].
\] (13)
In the above equation, $A$ is a constant whose value is defined in Eq. (A2). Notice that the so-called “dangerous” pole terms such as $[\partial^2/\epsilon^n]\ln^n[\lambda^2/(4\pi M^2)]$, $(l = 0, 2, 4; \, m = 1, 2; \, n = 1, 2)$, in the above equation, which cannot be removed by terms of counterterm constants ($\delta m^2 \hat{\phi}^2/2$ and $\delta \lambda \hat{\phi}^4/(4!)$), have been completely cancelled out among each other. The counterterm constants $\delta m^2$ and $\delta \lambda_2$ are determined as
\[
\delta m^2 = \frac{a_1 \lambda}{(4\pi)^2\epsilon} + a_2, \\
\delta \lambda_2 = \frac{9\lambda^3}{(4\pi)^4\epsilon^2} - \frac{3\lambda^3}{(4\pi)^4\epsilon} + \frac{6b_1 \lambda}{(4\pi)^2\epsilon} - \frac{6c_1 \lambda^2}{(4\pi)^2\epsilon} + b_2, \tag{14}
\]
where $a_2$ and $b_2$ are also unspecified but finite constants.

In the massive $O(N) \phi^4$ theory, the renormalization conditions
\[
\hat{\Gamma}^{(2)}(0) = -m^2, \quad \hat{\Gamma}^{(4)}(0) = -\lambda,
\]
are respectively translated into
\[
\left. \frac{d^2 V_{\text{eff}}(\hat{\phi})}{d\hat{\phi}^2} \right|_{\hat{\phi}=0} = m^2, \quad \left. \frac{d^4 V_{\text{eff}}(\hat{\phi})}{d\hat{\phi}^4} \right|_{\hat{\phi}=0} = \lambda. \tag{15}
\]

In our massless theory, however, we encounter the infrared singularity in the defining condition for a coupling constant. To avoid this difficulty we follow Coleman and Weinberg and require
\[
\left. \frac{d^2 V_{\text{eff}}(\hat{\phi})}{d\hat{\phi}^2} \right|_{\hat{\phi}=0} = 0, \quad \left. \frac{d^4 V_{\text{eff}}(\hat{\phi})}{d\hat{\phi}^4} \right|_{\hat{\phi}^2=M^2} = \lambda. \tag{16}
\]

Then constants $a_1$, $a_2$, $b_1$, and $b_2$ are determined order by order as follows:
\[
\begin{align*}
a_1 &= a_2 = 0, \\
b_1 &= -\frac{\lambda^2}{(4\pi)^2} \left[ 4 + \frac{3}{2} \gamma + \frac{3}{2} \ln \left( \frac{\lambda/2}{4\pi} \right) \right], \\
b_2 &= \frac{\lambda^3}{(4\pi)^4} \left[ \frac{139}{4} - 3A + 15\gamma + \frac{9}{4} \gamma^2 + \left( 15 + \frac{9}{2} \gamma \right) \ln \left( \frac{\lambda/2}{4\pi} \right) + \frac{9}{4} \ln^2 \left( \frac{\lambda/2}{4\pi} \right) \right] \\
&\quad + \frac{c_1 \lambda^2}{(4\pi)^2} \left[ \frac{19}{2} + 3\gamma + 3 \ln \left( \frac{\lambda/2}{4\pi} \right) \right]. \tag{16}
\end{align*}
\]

After disposing successfully all divergent terms in Eqs. (11) and (13) by the counterterm constants in Eqs. (8) and (14), we eventually arrive at our new result satisfying the conditions in Eq. (13):
\[
V_{\text{eff}}^{[2]}(\hat{\phi}) = \left[ \frac{\lambda}{4!} \hat{\phi}^4 \right] + \frac{h \lambda^2 \hat{\phi}^4}{(4\pi)^2} \left[ -\frac{25}{96} + \frac{1}{16} \ln \left( \frac{\hat{\phi}^2}{M^2} \right) \right] + \frac{h^2 \lambda^3 \hat{\phi}^4}{(4\pi)^4} \left[ \frac{55}{24} - \frac{13}{16} \ln \left( \frac{\hat{\phi}^2}{M^2} \right) + \frac{3}{32} \ln^2 \left( \frac{\hat{\phi}^2}{M^2} \right) \right] \\
&\quad + \frac{c_1 h^2 \lambda^2 \hat{\phi}^4}{(4\pi)^2} \left[ -\frac{25}{48} - \frac{1}{8} \ln \left( \frac{\hat{\phi}^2}{M^2} \right) \right]. \tag{17}
\]

Our result differs from that of Jackiw (see Eq. (3.17) in Ref. [3]) just by the underlined square-bracket term in Eq. (17) which is the missing portion in his calculation of the two-loop effective potential. After substituting the value of $c_1$ of Eq. (13), we have the final form of the effective potential up to the two-loop order as follows:
\[
V_{\text{eff}}^{[2]}(\hat{\phi}) = \left[ \frac{\lambda}{4!} \hat{\phi}^4 \right] + \frac{h \lambda^2 \hat{\phi}^4}{(4\pi)^2} \left[ -\frac{25}{96} + \frac{1}{16} \ln \left( \frac{\hat{\phi}^2}{M^2} \right) \right] + \frac{h^2 \lambda^3 \hat{\phi}^4}{(4\pi)^4} \left[ \frac{635}{288} - \frac{19}{24} \ln \left( \frac{\hat{\phi}^2}{M^2} \right) + \frac{3}{32} \ln^2 \left( \frac{\hat{\phi}^2}{M^2} \right) \right]. \tag{18}
\]
III. DISCUSSION AND CONCLUSION

Let us now apply a renormalization condition \( d^4 V_{\text{eff}}(\dot{\phi})/d\dot{\phi}^4 = \lambda \) to the two-loop effective potential with most general form of \( V_{\text{eff}}^{(2)} \) as an assumed series solution to the renormalization group equation:

\[
V_{\text{eff}}^{(2)}(\dot{\phi}) = \left[ \frac{\lambda}{4!} \dot{\phi}^4 \right] + \frac{\hbar^2 \lambda^2 \dot{\phi}^4}{(4\pi)^2} \left[ \frac{25}{96} + \frac{1}{16} \ln \left( \frac{\dot{\phi}^2}{M^2} \right) \right] + \frac{\hbar^2 \lambda^3 \dot{\phi}^4}{(4\pi)^4} \left[ \alpha_0 + \alpha_1 \ln \left( \frac{\dot{\phi}^2}{M^2} \right) + \alpha_2 \ln^2 \left( \frac{\dot{\phi}^2}{M^2} \right) \right],
\]

where \( \alpha_0, \alpha_1, \alpha_2 \) are constants. Then we readily obtain

\[
\frac{d^4 V_{\text{eff}}^{(2)}(\dot{\phi})}{d\dot{\phi}^4} \bigg|_{\dot{\phi}^2 = M^2} = \lambda + \frac{\hbar^2 \lambda^3}{(4\pi)^4} \left[ 24\alpha_0 + 100\alpha_1 + 280\alpha_2 \right].
\]

The boxed term (\( \lambda \)-cubic term) on the right-hand side of the above equation is an unwanted term. Thus it should vanish.

Next let us require the parametrization invariance of the theory. The renormalization mass, \( M \), is indeed an arbitrary parameter, with no effect on the physics of the problem. If we pick a different mass, \( M' \), then we define a new coupling constant

\[
\lambda' = \frac{d^4 V_{\text{eff}}^{(2)}(\dot{\phi})}{d\dot{\phi}^4} \Bigg|_{\dot{\phi}^2 = M'^2} = \lambda + P_1 \lambda^2 + P_2 \lambda^3,
\]

where

\[
P_1 = \frac{3\hbar}{2(4\pi)^2} \ln \left( \frac{M'^2}{M^2} \right),
\]

\[
P_2 = \frac{\hbar^2}{(4\pi)^4} \left[ (24\alpha_1 + 200\alpha_2) \ln \left( \frac{M'^2}{M^2} \right) + 24\alpha_2 \ln^2 \left( \frac{M'^2}{M^2} \right) \right].
\]

Eq. (21) is readily inverted iteratively as

\[
\lambda = \lambda' - P_1 \lambda^2 + (P_1^2 - P_2) \lambda^3 + O(\lambda^4).
\]

We now substitute this \( \lambda \) into Eq. (11). Then the two-loop effective potential is given in terms of \( \lambda' \) and \( M' \) as follows:

\[
V_{\text{eff}}^{(2)}(\dot{\phi}) = \left[ \frac{\lambda'}{4!} \dot{\phi}^4 \right] + \frac{\hbar^2 \lambda^2 \dot{\phi}^4}{(4\pi)^2} \left[ \frac{25}{96} + \frac{1}{16} \ln \left( \frac{\dot{\phi}^2}{M'^2} \right) \right] + \frac{\hbar^2 \lambda^3 \dot{\phi}^4}{(4\pi)^4} \left[ \alpha_0 + \alpha_1 \ln \left( \frac{\dot{\phi}^2}{M'^2} \right) + \alpha_2 \ln^2 \left( \frac{\dot{\phi}^2}{M'^2} \right) \right]
\]

\[
+ \left\{ \frac{25}{32} + \frac{25}{3} \alpha_2 + \left[ -\frac{3}{16} + 2\alpha_2 \right] \ln \left( \frac{\dot{\phi}^2}{M'^2} \right) \right\} \ln \left( \frac{M'^2}{M^2} \right) \right] + O(\lambda'^4).
\]

The parametrization invariance requires that the boxed term in Eq. (22) should vanish. From this and the vanishing boxed term of Eq. (21) we obtain

\[
V_{\text{eff}}^{(2)}(\dot{\phi}) = \frac{\hbar^2 \lambda^3 \dot{\phi}^4}{(4\pi)^4} \left[ -\frac{35}{32} - \frac{25}{6} \alpha_1 + \alpha_1 \ln \left( \frac{\dot{\phi}^2}{M'^2} \right) + \frac{3}{32} \ln^2 \left( \frac{\dot{\phi}^2}{M'^2} \right) \right].
\]

This shows us that even if one has an arbitrary value of \( \alpha_1 \), the parametrization invariance still holds. This is the reason why the Jackiw’s result (Eq. (3.17) in Ref. 3) is safe from the check of the parametrization invariance. In the above equation \( \alpha_1 \) is fixed not by the parametrization invariance but by the correct two-loop calculation of the effective potential.

In summary, Jackiw used a wrong renormalization condition, Eq. (10), in the massless \( O(N) \) \( \phi^4 \) theory and obtained such an incorrect value of \( \alpha_1 \) as \( -\frac{13}{16} \), but the correct value of \( \alpha_1 \) is \( -\frac{10}{27} \) as given by our Eq. (18).
ACKNOWLEDGMENTS

This work was supported in part by Ministry of Education, Project number BSRI-97-2442 and one of the authors (J.-M. C.) was also supported in part by the Postdoctoral Fellowship of Kyung Hee University.

APPENDIX: LOOP INTEGRATIONS

In this Appendix, the momenta appearing in the formulas are all (Wick-rotated) Euclidean ones and the abbreviated integration measure is defined as

$$\int_k = M^{4-n} \int \frac{d^n k}{(2\pi)^n},$$

where $n = 4 - \epsilon$ is the space-time dimension in the framework of dimensional regularization and $M$ is an arbitrary constant with mass dimension. For the sake of completeness, we simply list one-loop and two-loop integrals needed in our calculations though they are well known. For the two-loop integrations one may refer to Ref. [8].

A. Loop integration formulas

$$S_1 \equiv \int_k \ln \left( 1 + \frac{\xi^2}{k^2 + \sigma^2} \right) = -\frac{(\xi^2 + \sigma^2)^2}{(4\pi)^2} \left( \frac{\xi^2 + \sigma^2}{4\pi M^2} \right)^{\epsilon/2} \Gamma\left( \frac{\epsilon}{2} - 2 \right) + \xi\text{-independent term},$$

$$S_2 \equiv \int_k \frac{1}{k^2 + \sigma^2} = \frac{\sigma^2}{(4\pi)^2} \left( \frac{\sigma^2}{4\pi M^2} \right)^{\epsilon/2} \Gamma\left( \frac{\epsilon}{2} - 1 \right),$$

$$S_3 \equiv \int_k \frac{1}{(k^2 + \sigma^2)^2} = \frac{1}{(4\pi)^2} \left( \frac{\sigma^2}{4\pi M^2} \right)^{\epsilon/2} \Gamma\left( \frac{\epsilon}{2} \right),$$

$$S_4 \equiv \int_{k, p} \frac{1}{(k^2 + \sigma^2)(p^2 + \sigma^2)((p + k)^2 + \sigma^2)} = \frac{\sigma^2}{(4\pi)^4} \left( \frac{\sigma^2}{4\pi M^2} \right)^{\epsilon} \Gamma^2(1 + \epsilon/2) \left( \frac{\epsilon}{1 - \epsilon}(1 - \epsilon/2)^2 \right) \left[ \frac{6}{\epsilon^2} - 3A + O(\epsilon) \right]. \quad (A1)$$

In the above equation, $\gamma$ is the usual Euler constant, $\gamma = 0.5772156649\cdots$, and the numerical value of the constant $A$ in Eq. (A1) is

$$A = f(1, 1) = -1.1719536193\cdots, \quad (A2)$$

where

$$f(a, b) \equiv \int_0^1 dx \int_0^{1-z} dy \left( -\frac{\ln(1-x)}{1-y} \right) = \frac{z \ln z}{1-z}, \quad z = \frac{ax + b(1-x)}{x(1-x)}.$$

B. Calculation of the diagrams in Fig. 3

Diag. 1 = $-\frac{\hbar}{2(4\pi)^2} \left( \frac{\delta m^2 + (\lambda + \delta \lambda)\dot{\phi}^2/2}{1 + \delta Z} \right)^2 \left( \frac{\delta m^2 + (\lambda + \delta \lambda)\dot{\phi}^2/2}{4\pi M^2(1 + \delta Z)} \right)^{-\epsilon/2} \Gamma\left( -2 + \frac{\epsilon}{2} \right)$

$$= \hbar \left[ -\frac{\lambda^2 \dot{\phi}^4}{8(4\pi)^2} \left( \frac{\lambda}{4\pi M^2} \right)^{-\epsilon/2} \Gamma\left( -2 + \frac{\epsilon}{2} \right) \right] + \hbar^2 \left[ \frac{\lambda}{(4\pi)^2} \left( -\frac{\delta m^2 \dot{\phi}^2}{2} - \frac{\delta \lambda_1 \dot{\phi}^4}{4} + \frac{\lambda \delta Z_1 \dot{\phi}^4}{4} \right) \left( 1 - \frac{\epsilon}{4} \right) \left( \frac{\lambda \dot{\phi}^2/2}{4\pi M^2} \right)^{-\epsilon/2} \Gamma\left( -2 + \frac{\epsilon}{2} \right) \right],$$

Diag. 2 = $\hbar^2 \left[ \frac{\lambda^3 \dot{\phi}^4}{32(4\pi)^4} \left( \frac{\lambda \dot{\phi}^2/2}{4\pi M^2} \right)^{-\epsilon} \Gamma^2(1 + \epsilon/2) \left( \frac{6}{\epsilon^2} - 3A \right) \right],$

Diag. 3 = $\hbar^2 \left[ -\frac{\lambda^3 \dot{\phi}^4}{24(4\pi)^4} \left( \frac{\lambda \dot{\phi}^2/2}{4\pi M^2} \right)^{-\epsilon} \Gamma^2(1 + \epsilon/2) \left( \frac{6}{\epsilon^2} - 3A \right) \right]. \quad (A3)$
[1] S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[2] R. Jackiw, Phys. Rev. D 9, 1686 (1974).
[3] J. Iliopoulos, C. Itzykson and A. Martin, Rev. Mod. Phys. 47, 165 (1975).
[4] G. ’t Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).
[5] Y. Nambu, Phys. Lett. B26, 626 (1968).
[6] C. Itzykson and J. -B. Zuber, Quantum Field Theory (McGraw-Hill, New York, 1980): p. 455.
[7] R. Grigjanis, R. Kobes, and Y. Fujimoto, Can. J. Phys. 64, 537 (1986).
[8] J. van der Bij and M. Veltman, Nucl. Phys. B231, 205 (1984); F. Hoogeveen, Nucl. Phys. B259, 19 (1985); C. Ford, I. Jack, and D. R. T. Jones, Nucl. Phys. B387, 373 (1992); A. I. Davydychev and J. B. Tausk, Nucl. Phys. B397, 123 (1993); M. Misiak and M. Münz, Phys. Lett. B344, 308 (1995).