Spacetime Supersymmetry and Duality in String Theory

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I discuss the role of spacetime supersymmetry in the interplay between strong/weak coupling duality and target space duality in string theory which arises in string/string duality. This can be seen via the construction of string soliton solutions which in \( N = 4 \) compactifications of heterotic string theory break more than \( 1/2 \) of the spacetime supersymmetries but whose analogs in \( N = 2 \) and \( N = 1 \) compactifications break precisely \( 1/2 \) of the spacetime supersymmetries. As a result, these solutions may be interpreted as stable solitons in the latter two cases, and correspond to Bogomol’nyi-saturated states in their respective spectra.
1. Introduction

The construction of soliton solutions in string theory is intimately connected with the presence of various dualities in string theory (for recent reviews of string solitons, see [1,2]). Most of the solitonic solutions found so far break half of the spacetime supersymmetries of the theory in which they arise. Examples of string-like solitons (i.e. with one Killing direction) in this class are the fundamental string solution of [3] and the dual string solution of [4], which are interchanged once the roles of the strong/weak coupling $S$-duality and target space $T$-duality are interchanged.

In this talk I will first summarize the basic features of $S$-duality, $T$-duality and string/string duality in heterotic string theory. Then I will discuss newly constructed [5] classes of string-like soliton solutions, making connections between the solution-generating subgroup of the $T$-duality group and the number of spacetime supersymmetries broken in $N = 4, D = 4$ compactifications of $N = 1, D = 10$ heterotic string theory, as well as the natural realization of these solutions in $N = 1$ and $N = 2$ four-dimensional compactifications. For simplicity, I will restrict myself to solutions in the gravitational sector of the string (i.e. all Yang-Mills fields will be set to zero).

For an interesting discussion of six-dimensional string/string duality see [6]. New and exciting connections between the various dualities in heterotic string theory and type II string theory can be found in [7].

2. $S$ Duality

We adopt the following conventions for $N = 1, D = 10$ heterotic string theory compactified to $N = 4, D = 4$ heterotic string theory: (0123) is the four-dimensional spacetime, $z = x_2 + ix_3 = r e^{i\theta}$, (456789) are the compactified directions, $S = e^{-2\Phi} + ia = S_1 + iS_2$, where $\Phi$ and $a$ are the four-dimensional dilaton and axion. $S$ duality generalizes strong-weak coupling duality, since $g = e^\Phi$ is the string loop coupling parameter. In $N = 4, D = 4$ heterotic string theory $S$ duality corresponds to the group $SL(2, \mathbb{Z})$. In other words, the four-dimensional theory exhibits an invariance under

$$S \rightarrow \frac{aS + b}{cS + d},$$

where $a, b, c, d$ are integers and $ad - bc = 1$.

There is now considerable evidence [8, 13, 14, 15] in favor of $S$ duality also being an exact symmetry of the full string theory. One obvious attraction to demonstrating $S$ duality exactly in string theory is that it would allow us to use perturbative string techniques in the strong-coupling regime.
In the absence of nontrivial moduli and Yang-Mills fields, the low-energy four-dimensional bosonic effective action in the gravitational sector of the heterotic string has the form
\[ S_4 = \int d^4x \sqrt{-g} \left( R - \frac{g^{\mu\nu}}{2S_1^2} \partial_\mu S \partial_\nu S \right). \] (2.2)

A solution of this action is given by
\[ ds^2 = -dt^2 + dx_1^2 + \text{Re}S(dx_2^2 + dx_3^2) \]
\[ S = -\frac{1}{2\pi} \sum_{i=1}^{N} n_i \ln \frac{(z - a_i)}{r_{i0}}, \] (2.3)

where \( N \) is an arbitrary number of string-like solitons each with arbitrary location \( a_i \) in the complex \( z \)-plane and arbitrary winding number \( n_i \) respectively. One can replace \( z \) by \( \bar{z} \) in \( S \), thereby changing the orientation of the windings. There is also an \( SL(2, \mathbb{R}) \) symmetry manifest in the low-energy action, which is broken down to \( SL(2, \mathbb{Z}) \) in string theory via axion quantization and from which the above solution can be generalized further. Note that the \( x_1 \) Killing direction gives the above solution the structure of a parallel multi-string configuration. Each string is interpreted as a macroscopic fundamental string \([3]\). For dynamical evidence for this identification see \([17]\).

3. \( T \) Duality

\( T \) duality in string theory is the target space duality, and generalizes the \( R \to \alpha'/R \) duality in compactified string theory. For \( N = 4, D = 4 \) compactifications of heterotic string theory, \( T \)-duality corresponds to the discrete group \( O(6, 22; \mathbb{Z}) \) and is known to be an exact symmetry of the full string theory \([18–25]\).

Let us consider a simple special compactification, in which the only nontrivial moduli are given by \( T = T_1 + iT_2 = \sqrt{\text{det} g_{mn}} - iB_{45} \), where \( m, n = 4, 5 \). For trivial \( S \) field, the low-energy four-dimensional bosonic effective action in the gravitational sector has the form
\[ S_4 = \int d^4x \sqrt{-g} \left( R - \frac{g^{\mu\nu}}{2T_1^2} \partial_\mu T \partial_\nu T \right). \] (3.1)

A solution of this action is given by
\[ ds^2 = -dt^2 + dx_1^2 + \text{Re}T(dx_2^2 + dx_3^2) \]
\[ T = -\frac{1}{2\pi} \sum_{j=1}^{M} m_i \ln \frac{(z - b_j)}{r_{j0}}, \] (3.2)
where $M$ is an arbitrary number of string-like solitons each with arbitrary location $b_j$ in the complex $z$-plane and arbitrary winding number $m_j$ respectively. Again, one can replace $z$ by $\bar{z}$ in $T$ and reverse the windings, and there is an $SL(2, R)$ symmetry manifest in the low-energy action which is broken down to $SL(2, Z)$, this time due to the presence of instantons, and from which the above solution can be generalized further. Note that the $x_1$ Killing direction gives the above solution the structure of a parallel multi-string configuration as well, but in this case each string is interpreted as a dual string [4].

4. String/String Duality

Note that in interchanging the $S$ field in the action (2.2) with the $T$ field in the action (3.1), one is interchanging the $S$ (fundamental) string with the $T$ (dual) string and effectively interchanging their respective couplings. In this form, the string/string duality conjecture postulates the existence of a dual string theory, in which the roles of the strong/weak coupling duality and target space duality are interchanged. It follows that the string/string duality conjecture requires the interchange of worldsheet coupling associated with $T$ duality and spacetime coupling associated with $S$ duality.

Of course the full $T$ duality group $O(6, 22; Z)$ is much larger than the $S$ duality group $SL(2, Z)$, but from the six-dimensional viewpoint, the strong/weak coupling duality of the fundamental string can be seen to emerge as a subgroup of the target space duality group of the dual string. From the ten-dimensional viewpoint, the dual theory is a theory of fundamental fivebranes ($5 + 1$-dimensional objects) [26]. However, given the various difficulties in working with fundamental fivebranes (see discussion in [3]) and the fact that the technology of fundamental string theory is reasonably well-developed, it seems natural to prefer string/string duality (in $D = 6$ or $D = 4$) over string/fivebrane duality in $D = 10$.

Both the fundamental ($S$) and dual ($T$) string break $1/2$ the spacetime supersymmetries, which can be seen either from the $N = 1, D = 10$ uncompactified theory or from the $N = 4, D = 4$ compactified theory. They also both arise in a larger $O(8, 24; Z)$ solution generating group (for an explicit $O(8, 24; Z)$ transformation that takes one from the fundamental string to the dual string see [5]). As a consequence, they both saturate Bogomol’nyi bounds and correspond to states in the spectrum of the theory [12,16].

5. Generalized Solutions and Supersymmetry Breaking

Now consider the following ansatz, in which the solution-generating subgroup of the
$O(6, 22; Z)$ $T$ duality group is contained in $SL(2, Z)^3 = SL(2, Z) \times SL(2, Z) \times SL(2, Z)$:

\[
T^{(1)} = T^{(1)}_1 + iT^{(1)}_2 = \sqrt{\det g_{mn}} - iB_{45}, \quad m, n = 4, 5,
\]
\[
T^{(2)} = T^{(2)}_1 + iT^{(2)}_2 = \sqrt{\det g_{pq}} - iB_{67}, \quad p, q = 6, 7,
\]
\[
T^{(3)} = T^{(3)}_1 + iT^{(3)}_2 = \sqrt{\det g_{rs}} - iB_{89}, \quad r, s = 8, 9
\]

are the moduli. We assume dependence only on the coordinates $x_2$ and $x_3$ (i.e. $x_1$ remains a Killing direction), and that no other moduli than the ones above are nontrivial.

The canonical four-dimensional bosonic action for the above compactification ansatz in the gravitational sector can be written in terms of $g_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$), $S$ and $T^{(a)}$, $a = 1, 2, 3$ as

\[
S_4 = \int d^4 x \sqrt{-g} \left( R - \frac{g^{\mu\nu}}{2T^{(1)^2}} \partial_{\mu} S \partial_{\nu} S - \frac{g^{\mu\nu}}{2T^{(2)^2}} \partial_{\mu} T^{(1)} \partial_{\nu} T^{(1)} - \frac{g^{\mu\nu}}{2T^{(3)^2}} \partial_{\mu} T^{(2)} \partial_{\nu} T^{(2)} - \frac{g^{\mu\nu}}{2T^{(3)^2}} \partial_{\mu} T^{(3)} \partial_{\nu} T^{(3)} \right).
\]

A solution of this action is given by

\[
ds^2 = -dt^2 + dx_1^2 + Re S Re T^{(1)} Re T^{(2)} Re T^{(3)} (dx_2^2 + dx_3^2)\\
S = -\frac{1}{2\pi} \sum_{i=1}^N n_i \ln \left( \frac{z - a_i}{r_{i0}} \right),
\]
\[
T^{(1)} = -\frac{1}{2\pi} \sum_{j=1}^M m_j \ln \left( \frac{z - b_j}{r_{j0}} \right),
\]
\[
T^{(2)} = -\frac{1}{2\pi} \sum_{k=1}^P p_k \ln \left( \frac{z - c_k}{r_{k0}} \right),
\]
\[
T^{(3)} = -\frac{1}{2\pi} \sum_{l=1}^Q q_l \ln \left( \frac{z - d_l}{r_{l0}} \right),
\]

where $N, M, P$ and $Q$ are arbitrary numbers of string-like solitons in $S, T^{(1)}, T^{(2)}$ and $T^{(3)}$ respectively each with arbitrary location $a_i, b_j, c_k$ and $d_l$ in the complex $z$-plane and arbitrary winding number $n_i, m_j, p_k$ and $q_l$ respectively. One can replace $z$ by $\bar{z}$ independently in $S$ and in each of the $T$ moduli, and in each of $S$ and the $T$ moduli there is an $SL(2, Z)$ symmetry manifest in the action in each of the moduli, and from which the above solutions can be generalized further. Thus one has an overall effective solution-generating group of $SL(2, Z)^4$.

It can be shown that the solutions with trivial $S$ and $1, 2$ and $3$ nontrivial $T$ fields preserve $1/2, 1/4$ and $1/8$ of the spacetime supersymmetries respectively, while the
solutions with nontrivial $S$ and $0, 1$ and $2$ nontrivial $T$ fields preserve $1/2, 1/4$ and $1/8$ spacetime supersymmetries respectively. The solution with nontrivial $S$ and $3$ nontrivial $T$ fields preserves $1/8$ of the spacetime supersymmetries for one chirality choice of $S$, and none of the spacetime supersymmetries for the other, although the ansatz remains a solution to the bosonic action in this latter case. In short, the maximum number of spacetime supersymmetries preserved in the $N = 4$ theory for a solution generating subgroup $SL(2, Z)^n$ of $O(8, 24; Z)$ is given by $(1/2)^n [5]$. 

6. Discussion

So what is the interpretation of these new solutions which break more than half the supersymmetries, since they are not expected to arise within the spectrum of Bogomol’nyi-saturated states in the $N = 4$ theory? It turns out that most of the above solutions that break $1/2, 3/4$ or $7/8$ of the spacetime supersymmetries in $N = 4$ have analogs in $N = 1$ or $N = 2$ compactifications of heterotic string theory that break only $1/2$ the spacetime supersymmetries*. Of course no solution actually preserves a higher total number of supersymmetries in the lower supersymmetric theory ($N = 1$ or $N = 2$) than in $N = 4$, but the relative number of supersymmetries preserved may be increased in truncating the $N = 4$ theory to $N = 1$ or $N = 2$ by the removal of non-supersymmetric modes. Thus a solution that preserves $1/8$ of the spacetime supersymmetries in $N = 4$ and $1/2$ of the spacetime supersymmetries in $N = 1$ actually preserves the same total amount of supersymmetry in both theories. The only difference is that in the $N = 4$ case one is starting with four times as many supersymmetries, so that a greater number of those are broken than in the $N = 1$ case.

These solutions are therefore in some sense realized naturally as stable solitons only in the context of either $N = 1$ or $N = 2$ compactifications, and should lead to the construction of the Bogomol’nyi spectrum of these theories. In these two cases, however, the situation is complicated by the absence of non-renormalization theorems present in the $N = 4$ case which guarantee the absence of quantum corrections. An exception to this scenario arises for $N = 2$ compactifications with vanishing $\beta$-function. The construction of these spectra remains a problem for future research.

* However, when at least one of the fields, either $S$ or one of the $T$ fields, has a different analyticity behaviour from the rest, no supersymmetries are preserved in $N = 1$ [6].
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