Isospin Distillation with Radial Flow: a Test of the Nuclear Symmetry Energy

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We discuss mechanisms related to isospin transport in central collisions between neutron-rich systems at Fermi energies. A fully consistent study of the isospin distillation and expansion dynamics in two-component systems is presented in the framework of a stochastic transport theory. We analyze correlations between fragment observables, focusing on the study of the average N/Z of fragments, as a function of their kinetic energy. We identify an EOS-dependent relation between these observables, allowing to better characterize the fragmentation path and to access new information on the low density behavior of the symmetry energy.

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Heavy ion reactions with exotic nuclei at Fermi energies can be used to study the properties of the symmetry term at densities below and around the saturation value. In fact, this regime of densities is important for studies of the structure of exotic nuclei, of the neutron star crust, and for supernova explosions, where a key issue is the clustering of low-density matter \cite{1}. In central collisions at 30-50 MeV/u, where the full disassembly of the system into many fragments is observed, one can study specifically properties of liquid-gas phase transitions occurring in asymmetric matter in the presence of radial flow. For instance, in neutron-rich matter, when two phases co-exist, one expects to observe an isospin distillation: fragments (liquid) appear more symmetric with respect to the initial matter, while light particles (gas) are more neutron-rich \cite{2,3,4,5}. The amplitude of this effect depends on specific properties of the isovector part of the nuclear interaction, namely on the value and the derivative of the symmetry energy at low density. Hence the analysis of the isotopic content of the reaction products allows to get information on the low-density isovector Equation of State (EOS) \cite{7}.

This investigation is interesting in a more general context: In heavy ion collisions the dilute phase appears during the expansion of the interacting matter. Thus we study effects of the coupling of expansion, fragmentation and distillation in a two-component (neutron-proton) system. In a statistical model, the effect of the expansion on fragment emission was studied earlier in Ref.\cite{8}. Here we present a fully consistent dynamical study, based on microscopic transport approaches widely tested in heavy ion collisions \cite{4,5,11,12,13}. This leads us to single out the isospin signal as a good tracer of the reaction mechanism and to suggest new correlation observables to probe the symmetry term at sub-saturation density.

Recently, some efforts have been devoted to the study of the isotopic content of pre-equilibrium emission, looking in particular at the emitted neutron to proton ratio as a function of the kinetic energy \cite{2,10}. In this Letter we propose to extend this type of investigation to fragments.

Correlations between fragment charges and velocities have recently been observed, providing information on the interplay between thermal and entrance channel (collective) effects in the fragmentation mechanism \cite{14,15,16}. The study of the correlations between fragment isotopic content and kinematical properties should allow to get a deeper insight into the reaction path and to study more in detail the effects of different EOS’s and, in particular, of the symmetry energy on fragment properties \cite{17}.

Theoretically the evolution of complex systems can be described by a transport equation with a fluctuating term, the so-called Boltzmann-Langevin equation (BLE):

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\} = I_{\text{coll}}[f] + \delta I[f],$$

where $f(r,p,t)$ is the one-body distribution function, $H(r,p,t)$ is the one-body Hamiltonian and $\delta I[f]$ represents the fluctuating part of the two-body collision integral \cite{18,19}. Here we will follow the approximate treatment to the BLE presented in Ref.\cite{12}, the Stochastic Mean Field (SMF) model, that consists in the implementation of stochastic spatial density fluctuations.

Calculations have been performed using a Boltzmann-Nordheim-Vlasov (BNV) code (TWINGO), where the test particle method is used to solve Eq.\cite{11}. We adopt a soft EOS, with compressibility modulus $K = 200$ MeV and, for the density ($\rho$) dependence of the symmetry energy, we consider two representative parameterizations, $E_{\text{sym}}(\rho)/A \equiv C_{\text{sym}}(\rho)\rho^2, I \equiv (N - Z)/A$ : one with a rapidly increasing behaviour with density, roughly proportional to $\rho^2$ (asysoft) and one with a kind of saturation above normal density (asy stiff, $SKM^*$) (see Ref.\cite{2,13} for more detail). The two parameterizations obviously cross at normal density. The symmetry energy at densities below the normal value is larger in the asysoft case, while above normal density it is higher in the asystiff case. Hence in the low-density regime, that is
in the region of interest for our analysis of spinodal instabilities in central collisions, isospin effects are expected to be stronger in the asystiff case.

We will focus on central collisions, $b = 2$ fm, in symmetric reactions between systems having three different initial asymmetries: $^{112}$Sn + $^{112}$Sn, $^{124}$Sn + $^{124}$Sn, $^{132}$Sn + $^{132}$Sn, with $(N/Z)_{en} = 1.24, 1.48, 1.64$, respectively. The considered beam energy is 50 MeV/u. 1200 events have been run for each reaction and for each of the two symmetry energies adopted. The first two reactions, $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn have been widely investigated both from the experimental and theoretical point of view [6, 13, 20, 21].

We first focus on some general properties of the fragmentation events, as described by the SMF model. Along the fragmentation path a bubble-like configuration is formed, where the initial fragments are located [4]. The average fragment multiplicity is approximately equal to 6 for the reactions considered here [13]. Along the reaction path, several nucleons are emitted at the early stage (pre-equilibrium emission) and/or are evaporated while fragments are formed. Primary fragments are identified by applying a coalescence procedure to the matter with density larger than $\rho_{\text{cut}} = 1/5 \rho_0$ (liquid phase). The remaining particles are considered as belonging to the gas phase. The correlation between fragment charge and c.m. position, as calculated at the “freeze-out” time, is displayed in Fig.1 (top left panel) for the $^{124}$Sn + $^{124}$Sn system (asystiff interaction). The “freeze-out” time is defined as the time when fragments are completely formed and their multiplicity does not evolve anymore. One observes that fragments are located on a shell of about 10-15 fm radius and consist of two components. The upper component can be associated with events where one or two larger fragments survive, as a memory of the entrance channel. The fragment charge shows a decreasing trend with the distance. The second component can be related to more explosive events, where the system breaks up into a larger number of smaller fragments. In simulations of very central collisions ($b = 0$ fm) only this second component is present. Since we are mostly interested in this component, fragments with charge larger than 10 (dashed line in the figure) will not be considered in the following analysis.

Another feature observed at this beam energy is the existence of a radial collective flow [14], as evidenced in Fig.1 (top right panel) where the correlation between fragment velocity and radial distance is shown. As a consequence of the fact that large fragments are preferentially located at a closer distance, one expects to see a decrease of the average fragment velocity with the fragment charge. This feature has been experimentally observed in central collisions [16].

The aim of this Letter is to investigate correlations between fragment asymmetry and velocity (or kinetic energy). The idea is that fragmentation originates from the break-up of a composite source that expands with a given velocity field. Since neutrons and protons experience different forces, one may expect a different radial flow for the two species. In this case, the N/Z composition of the source would not be uniform, but would depend on the radial distance from the center or mass or, equivalently, on the local velocity. This trend should then be reflected in the fragment asymmetries as a function of their kinetic energy. The existence of such correlations can be qualitatively seen in Figure 1 (bottom panels) where the fragment asymmetry, $I$, is plotted as a function of the radial distance and velocity. It is observed that the asymmetry decreases with the radial distance, indicating a different proton/neutron radial distribution in the fragmenting source (bottom left). Since radial distance and velocity are correlated (top right), this implies correlations between fragment asymmetry and velocity, as seen in the bottom right part of Fig.1. However, in both cases, fluctuations are rather large.

Let us now discuss the isotopic content of fragments and emitted nucleons, as obtained with the two iso-EOS considered. In the following we will restrict our analysis to fragments with charge between 3 and 10 (intermediate mass fragments (IMF)). The average N/Z of emitted nucleons (gas phase) and IMF’s is presented in Fig.2 as a function of the initial $(N/Z)_{\text{in}}$ of the three colliding systems. Generally the gas phase is seen to be more neutron-rich in the asystiff relative to the asystiff case, while IMF’s are more symmetric. The difference between the asymmetries of the gas and liquid phases increases with the $(N/Z)_{\text{in}}$ of the system, and is always larger in the asystiff case, due to the larger value of the asymmetry energy at low density [13]. In the asystiff case, however, due to the rather low value of the symmetry energy, this difference is negative for the neutron-poor system ($^{112}$Sn + $^{112}$Sn). In fact, in this case Coulomb

![FIG. 1: (Color online) Top panels: (left) Correlations between fragment charge $Z$ and radial distance from the system center of mass for the reaction $^{124}$Sn + $^{124}$Sn, at $b = 2$ fm, E/A = 50 MeV/u (asystiff interaction). (right) Correlations between fragment velocity and radial distance. Bottom panels: Correlations between fragment asymmetry and radial distance (left) or fragment velocity (right). The dashed line corresponds to the system initial asymmetry.](image-url)
effects dominate and protons are preferentially emitted.

Now we move to discuss the correlations between fragment isotopic content and kinematical properties. As a measure of the isotopic composition of the IMF’s we will consider the sum of neutrons, $N = \sum_i N_i$, and protons, $Z = \sum_i Z_i$, of all IMF’s in a given kinetic energy bin (here taken as $1.5 \text{ MeV/u}$), in each event. Then we take the ratio $N/Z$ and we consider the average over the ensemble of events. This observable is plotted in Fig.3 for the three reactions and the two iso-EOS considered and is seen to be rather sensitive. For the neutron-poor system, the $N/Z$ decreases with the fragment kinetic energy, especially in the asystiff case. Here the symmetry energy is relatively low at low density \[13\] and the Coulomb repulsion pushes the protons towards the surface of the system. Hence, more symmetric fragments acquire larger velocity. The same effects are responsible for the relatively neutron-poor pre-equilibrium emission in this case (see Fig.2). The decreasing trend is less pronounced in the asysoft case because now Coulomb effects are counterbalanced by the larger attraction of the proton symmetry potential. In systems with larger initial asymmetry, the decreasing trend is inverted, due to the larger neutron repulsion in neutron-rich systems. Due to the balance between the asymmetry of the liquid phase, that is larger in the asystiff case, and the value of the symmetry energy at low density, that is larger in the asysoft case, the last effect is of similar amplitude in the two EOS’s.

In order to isolate isospin effects, it is advantageous to compare results for systems with different initial asymmetry \[6\]. \[10\]. Hence we construct the “double” $N/Z$ ratio, as a function of the asymmetry of the fragment kinetic energy, by taking the ratio of the $N/Z$ of a neutron-rich relative to a neutron-poor reaction.

The double $N/Z$ ratio was already investigated for the pre-equilibrium emission, where experimental data have been compared to the predictions of the BUU transport model \[9\]. A larger value of the double ratio was observed in the asysoft case, corresponding to the larger amount of emitted neutrons in neutron-rich systems, especially at high kinetic energy. This is qualitatively in agreement with our findings, as seen in Fig.2. In Fig.4 (top panel) we show the double $N/Z$ ratio constructed for the pair of reactions $^{124}\text{Sn} + ^{124}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$ (circles) and the pair $^{132}\text{Sn} + ^{132}\text{Sn}$ and $^{112}\text{Sn} + ^{112}\text{Sn}$ (squares), respectively. Conversely to what was observed for the pre-equilibrium emission \[9\], the fragment double $N/Z$ ratio is larger in the asystiff case. This reflects the fact that fragments originate from the composite system that survives the pre-equilibrium emission, the asymmetry of which is larger in the asystiff case, especially in neutron-rich systems. \[13\] From this point of view, fragments bear a complementary information with respect to the pre-equilibrium emission. Hence opposite effects are expected and observed in Fig.4 for fragment emission. The differences between the two iso-EOS’s increase with the very neutron-rich reaction $^{132}\text{Sn} + ^{132}\text{Sn}$, as expected, however the sensitivity to the iso-EOS appears rather small (less than 5% ).

To construct a more sensitive observable, one can combine the two main isospin effects represented in Fig.3: 1) the decreasing trend observed for the lighter system, with a rather EOS-dependent slope; 2) the change of slope, up to the inversion of the decreasing trend, in the neutron-richer systems. Thus we propose to consider a “shifted” ratio, $(N/Z)_s$, by taking the asymmetry relative to the lowest kinetic energy bin, which is related to the slope of the curves in Fig.3, which is dependent on the iso-EOS’s considered. Thus we expect to enlarge the sensitivity of the ratio to the symmetry energy. In Fig.4, bottom panel, we present the behaviour of the double $(N/Z)_s$ ratio for the pairs of reactions considered before, and for the two iso-EOS’s. The double ratio constructed in this way is rather sensitive to the EOS employed, even for the pair of reactions with the closest $(N/Z)_m$ ($^{124}\text{Sn} + ^{124}\text{Sn}$, $^{112}\text{Sn} + ^{122}\text{Sn}$). Lower values are obtained in the asysoft case.
We can try to interpret the results in Fig. 4 in a simple model, by approximating the ratios in Fig.3 by a linear relation: \((N/Z)_i(E_{\text{kin}}) = (N/Z)_i(0) + m_iE_{\text{kin}}\), where \(i\) refers to the system. From Fig.3 we observe that \((N/Z)_i(0)\) is roughly shifted by a constant amount for soft and stiff iso-EOS's. On the other hand, the slopes in Fig.3 for the soft, relative to the stiff iso-EOS, corresponds roughly to a rotation, such that the difference \(\Delta m_{2,1}\) between two systems is almost independent of the iso-EOS. The shifted double \(N/Z\) ratio between two systems is then the ratio of the corresponding slopes and constant in energy. This is also roughly confirmed in Fig.4. Furthermore it can be written as: \(DR_{2,1} = 1 + \Delta m_{2,1}/m_1\). From this relation it is clear that the deviation of the shifted double ratio from 1 increases with the difference \(\Delta m_{2,1}\) and with the flattening of the slope \(m_1\) (notice that in our simulations \(m_1\) is always negative). The dependence on the iso-EOS then lies in the slope of the neutron-poorest system, which is most strongly affected by the symmetry energy. So, the shifted \(N/Z\) ratio allows to access directly the sensitivity of the slope \(m_1\) to the EOS. We also remark that the usual double ratio, displayed in the top panel in Fig.4, does not show a large sensitivity to the iso-EOS, because it is dominated by the differences in \((N/Z)_i(0)\), which are relatively small (about 10%).

Here we have considered primary fragments. However, the fragment properties measured in experiments are affected by secondary decay. While the curves presented in Fig.3 could be affected by secondary decay, we expect, however, the shifted double \((N/Z)\) ratio to be more robust. With the assumption that the average \(N/Z\) of final fragments, \((N/Z)_f\), is reduced by a given constant quantity \(b\) or by a constant factor \(a\) with respect to the \(N/Z\) of primary fragments, i.e. \((N/Z)_f = a(N/Z - b)\), which is reasonable for fragments from similar reactions at the same energy (i.e. similar excitation energy), the shifted double \((N/Z)\) ratio would not be affected at all by secondary decay effects.

In conclusion, we have presented a new analysis of fragmentation reactions that would allow to extract relevant information on the low-density behaviour of the symmetry energy. We propose an EOS-dependent relation between the \(N/Z\) and the kinetic energy of IMF's, which is linked to the different forces experienced by neutrons and protons along the fragmentation path, which in turn depend on the details of the isovector part of the nuclear interaction. The isospin distillation mechanism in fragment formation appears naturally coupled to the underlying expansion dynamics. As a consequence, the \(N/Z\) composition of the fragmenting source is not uniform. This analysis can be considered as complementary to the pre-equilibrium emission studies [9]. A parallel investigation of pre-equilibrium and fragment emissions would be very important for a cross-check of model predictions against experimental observables sensitive to different phases of the reaction.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{double_ratio_4}
\caption{(Color online) Top: The double \(N/Z\) fragment ratio as a function of the kinetic energy for the reactions \(^{124}\text{Sn}^{+124}\text{Sn}\) to \(^{112}\text{Sn}^{+112}\text{Sn}\) (squares) and the reactions \(^{132}\text{Sn}^{+132}\text{Sn}\) to \(^{112}\text{Sn}^{+112}\text{Sn}\) (circles). Full lines and symbols are for the asy stiff, dashed lines and open symbols for the asy soft EOS. Bottom: Same as in the top panel, but for the shifted \(N/Z\) ratio (see text).}
\end{figure}
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