Rotons in gaseous Bose-Einstein condensates irradiated by a laser

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A gaseous Bose-Einstein condensate (BEC) irradiated by a far off-resonance laser has long-range interatomic correlations caused by laser-induced dipole-dipole interactions. These correlations, which are tunable via the laser intensity and frequency, can produce a ‘roton’ minimum in the excitation spectrum—behavior reminiscent of the strongly correlated superfluid liquid helium II.

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According to the celebrated Bijl-Feynman formula\(^1\) for the excitation spectrum of helium II

\[ E(k) \leq \frac{\hbar^2 k^2}{2m S(k)}, \]

the peculiar "roton" minimum at \( k \approx 2\pi/r_0 \), where \( r_0 \) is the average atomic separation, is due to a corresponding peak in the static structure factor \( S(k) \equiv \langle \rho_k^2 \rangle / N \). Here \( N \) is the number of atoms of mass \( m \), \( \langle 0 | \rho \rangle \) the ground state of the system, and \( \rho_k \equiv \sum_q c_q^+ c_{q+k} \) the density fluctuation operator. \( S(k) \) is the Fourier transform of the pair correlation function and hence provides a measure of the degree of pair (2nd order) correlation between the atoms. The existence of strong pair correlations in helium II may at first seem surprising since it remains a liquid even at temperatures approaching absolute zero precisely because of weak interatomic interactions (in combination with a small atomic mass)\(^2\). However, despite their apparent weakness these interactions are very effective because the density of the liquid state is such that the average atomic separation, \( r_0 = 4.44\text{Å} \), is close to the minimum of the attractive interatomic potential well at 3Å.

Contrast this now with an ultra-cold alkali atom gas in which the Bose-Einstein condensed fraction can be very nearly 100 \%\(^3\). The interactions in \(^{87}\text{Rb}\), for example, are repulsive and characterized by an s-wave scattering length, \( a \approx 5.5\text{nm} \). This is between one and two orders of magnitude smaller than the average atomic spacing at typical densities. Steinhauer et al\(^5\) recently measured the bulk excitation spectrum of a \(^{87}\text{Rb}\) BEC and found excellent agreement with Bogoliubov theory\(^3\) (appropriate for a degenerate almost ideal Bose gas). There was no roton minimum, a consequence of the diluteness with respect to \( a \). Indeed, since Eq.\(^1\) becomes an equality within the Bogoliubov theory\(^3\) one sees the pair correlation is small compared to helium II. Significant pair correlation might exist...
in gaseous BECs at the very small scale of $a$, but this is fairly inaccessible in such a delicate system.

A marvellous feature of atoms though, is that their interactions can be manipulated using external fields, allowing us to microscopically engineer the macroscopic properties of a many-body system. Thus the experiment of Inouye et al \[6\] took advantage of a Feshbach resonance to change the s-wave scattering length using magnetic fields. We have recently proposed the use of off-resonant lasers to induce long-range dipole-dipole interactions whose characteristic length is the laser wavelength. These interactions can cause laser induced self-“gravity” in a BEC, leading to 3-dimensional self-trapping and correlations of a gaseous BEC are changed when excitation spectrum and, by virtue of (1), the internal fields, allowing us to microscopically engineer the macroscopic properties of a many-body system.

Consider a BEC confined by a potential $H_{\text{trap}} = \frac{m}{2} \omega_r^2 (x^2 + y^2) + \frac{m}{2} \omega_z^2 z^2$ into a very elongated cigar shape ($\omega_r \gg \omega_z$), irradiated by a far-off-resonance plane-wave laser (Fig. 1 inset). The laser polarization is along the long z-axis of the condensate to suppress collective (“superradiant”) Rayleigh scattering \[9\] or coherent atomic recoil lasing \[10\] that are forbidden along the direction of polarization. The far-off-resonance condition, together with the small extent of the BEC along the laser propagation direction, enables us to treat the electromagnetic field inside the BEC in the Born approximation (field at each point is the sum of the incident plus once-scattered fields). Then the dipole-dipole potential between two atoms of separation $r$, induced by far-off resonance light of intensity $I$, wave-vector $\mathbf{k}_L = k_L \hat{\mathbf{y}}$ (along the y-axis), and polarization $\hat{\mathbf{e}} = \hat{\mathbf{z}}$ (along the z-axis) is \[11\]

$$U_{dd}(r) = \frac{I \alpha^2 (\omega) k_L^3}{4 \pi c \varepsilon_0^2} V_{zz} (k_L, r) \cos (k_L y). \quad (2)$$

Here $\alpha(\omega)$ is the isotropic, dynamic, polarizability of the atoms at frequency $\omega = c k_L = 2 \pi c / \lambda_L$. The pre-factor can be expressed in terms of the Rayleigh scattering rate, $\gamma_R$, as $I \alpha^2 k_L^3 / (4 \pi c \varepsilon_0^2) = (3/2) \hbar \gamma_R$. $V_{zz}$ is the component of the retarded dipole-dipole interaction tensor generated by the linearly $\hat{\mathbf{z}}$-polarized laser light

$$V_{zz} = \frac{1}{k_L^3 r^3} \left[ (1 - 3 \cos^2 \theta) \left( \cos k_L r + k_L r \sin k_L r \right) - \sin^2 \theta k_L^2 r^2 \cos k_L r \right] \quad (3)$$

$\theta$ being the angle between the interatomic axis and the z-axis. The far-zone ($k_L r \gg 1$) behavior of (2) along the z-axis is proportional to $-\sin(k_L r) / (k_L r)^2$ and many atoms ($400$ at densities of $8 \times 10^{14}$ atoms/cm$^3$) may lie within the

![Figure 1: The total (s-wave+dipole-dipole) 1D interatomic potential $U_{tot}^z(z)$ (FT of (2)), for $w_r = 3.5 \lambda_L$. A repulsive contact term $4E_R a(k_L w_r)^2(1 + 4I/3)|\delta(k_L z)|$ is not shown. Inset: The laser beam and condensate geometry.](image)
characteristic interaction volume \((\lambda_1^2)\) of this attractive long-range potential. As for the electron gas and charged Bose gas, mean-field (here Bogoliubov) theory applies in this high density regime [12].

The laser (dynamically) induced dipole-dipole potential is distinguished from the static field \((r^{-3})\) case [13, 14] by a longer range and a huge enhancement of atomic polarizability around a resonance. For example, in \(^{87}\text{Rb}\) atoms are magnetically trapped in the maximally stretched \([5s^2S_1/2, F = 2, M = 2]\) state. A laser polarized along \(\hat{z}\) is then \(\pi\)-polarized and only \(\Delta M = 0\) dipole transitions are allowed. If the light is detuned by, say, \(\delta = 2\pi\times(6.5\text{GHz})\) (i.e. 1134 natural line widths) below the D1 line \((795.0\text{nm})\) then only virtual transitions to the \([5p^2P_1/2, F = 2, M = 2]\) state need be considered. We calculate \(\alpha \approx 5.0 \times 10^{-35}\text{cm}^2/\text{V}\) (cf. the static value 5.3 \times 10^{-39}\text{cm}^2/\text{V}).

In terms of the condensate density \(n(\mathbf{r})\) at zero temperature, we account for atom-atom interactions using a mean-field energy functional of the form \(H_{dd} + H_{sc}\), where \(H_{dd} = (1/2) \int n(\mathbf{r}) U_{dd}(\mathbf{r} - \mathbf{r}') n(\mathbf{r}') \, d^3r \, d^3r'\), and \(H_{sc} = (1/2) (4\pi \hbar^2 / m) \int n(\mathbf{r})^2 \, d^3r\) is due to short-range interactions, which are described as usual, by a delta function pseudo-potential (we take here the repulsive case for which \(a > 0\)). By working with the bare dipole-dipole interaction we assume the Born approximation also for atom-atom scattering by this long-range part of the total potential. We note that the short-range (static) part of the laser-induced dipole-dipole interaction can cause a shift in \(a\). For the laser intensities and detunings considered here this shift is small according to existing estimates [14].

In a radially tight trap it is reasonable to assume a cylindrically symmetric ansatz for the density profile of radial width \(w_r\): \(n(\mathbf{r}) = N (\pi w_r^2)^{-1} n^z(z) \exp \left[-(x^2 + y^2) / w_r^2\right]\), where \(N\) is the total number of atoms and \(n^z(z)\) is normalized to 1 and left general. Denoting the FT of the atomic density by \(\tilde{n}(\mathbf{k}) = \int \exp[-i\mathbf{k} \cdot \mathbf{r}] \, n(\mathbf{r}) \, d^3r\), then we have \(H_{dd} = (1/2) (2\pi)^{-3} \int \tilde{U}_{dd}(\mathbf{k}) \tilde{n}(\mathbf{k}) \tilde{n}(-\mathbf{k}) \, d^3k\), where the FT of the dipole-dipole potential [2], \(U_{dd}(\mathbf{k}) = \int \exp[-i\mathbf{k} \cdot \mathbf{r}] U_{dd}(\mathbf{r}) \, d^3r\), is the real part of

\[
\tilde{U}_{dd}(\mathbf{k}) = \frac{1}{2} \frac{\alpha^2}{\epsilon_0^2 c} \left( \frac{k_x^2 - k_y^2}{k_x^2 + (k_y - k_L)^2 + k_z^2 - k_L^2 - i\eta} + \frac{k_x^2 - k_y^2}{k_x^2 + (k_y + k_L)^2 + k_z^2 - k_L^2 - i\eta} \right) \frac{2}{3}. \tag{4}
\]

The principal value of the radial integration in \(H_{dd}\) can be evaluated analytically so that the dipole-dipole energy reduces to a one dimensional functional along the axial direction \(H_{dd} = (N^2 / 2) \int n^z(z) n^z(z') U_{dd}^z(z - z') \, dz \, dz' = (N^2 / 4\pi) \int \tilde{n}^z(k_z) \tilde{n}^z(-k_z) U_{dd}^z(k_z) \, dk_z\), where \(\tilde{n}^z(k_z)\) is the FT of the axial density \(n^z(z)\). The one-dimensional (1D) axial potential that appears in this expression has the form

\[
\tilde{U}_{dd}^z(k_z) = \frac{1}{4\pi \epsilon_0 c} Q(w_r, k_z), \quad Q(w_r, k_z) = \frac{2}{3} \frac{1}{k_L^2 w_r^2} \left[ \frac{k_x^2 - k_y^2}{k_x^2} \right] \exp \left( k_z^2 - 2k_L^2 w_r^2 / i\eta \right) \times \sum_{j=0}^{\infty} \left( k_L w_r \right)^{2j} 2^j j! \Re \left\{ E_{j+1} \left[ \left( \frac{k_x^2 - k_y^2}{k_x^2} \right) \frac{w_r^2}{2} \right] \right\}. \tag{5}
\]

where \(\Re\{E_{j+1}[z]\}\) is the real part of the generalized exponential integral [15]. The FT of the total (s-wave plus dipole-dipole) 1D reduced interatomic potential is

\[
\tilde{U}_{tot}^z(k_z) = 4 E_R a \left( (k_L w_r)^{-2} + iQ(w_r, k_z) \right). \tag{6}
\]

where \(E_R = \hbar^2 k_L^2 / 2m\) is the photon recoil energy of an atom and \(I\) is the dimensionless ‘intensity’ parameter

\[
I = \frac{1}{8\pi \epsilon_0^2 c \hbar^2 a} \tag{7}
\]

It is emphasized that the radial degree of freedom is contained in \(w_r\) via the radius \(w_r\). The coordinate space potential, \(U_{tot}^z(z)\), is shown in Fig. [1].
We now have the essential ingredients to compute the excitation spectrum of the BEC as it is the FT of the effective interatomic interaction potential that appears in the Bogoliubov dispersion formula \[3\]. Since the influence of radial excitations upon the low-energy spectrum can be largely frozen out under tight radial confinement, we shall consider only axial phononic excitations and assume that the system is infinite along this \( \hat{z} \) direction. In terms of the phonon momentum \( p_z = \hbar k_z \), the axial Bogoliubov spectrum is (cf. Eq. (1))

\[
E_B = \sqrt{c_z^2 p_z^2 + \left( \frac{p_z^2}{2m} \right)^2} = p_z^2 / (2mS(k_z)) \tag{8}
\]

where \( c_z^2 = \pi n(0) w_r^2 \tilde{U}_{\text{tot}}(k_z)/m \). \( n(0) \) is the central density in the cigar, so \( \pi n(0) w_r^2 \) is the linear density along the cigar. For the linear parts of the spectrum, \( c_z \) can be interpreted as the speed of sound in the gas. Shining a 795.0nm laser upon a \(^{87}\)Rb BEC of density \( n(0) = 8 \times 10^{20} \text{atoms/m}^3 \) and radius \( w_r = 3.5\lambda_L = 2.78\mu \text{m} \), a ‘roton’ minimum appears when \( I \geq 0.051 \) (i.e. \( I \geq 0.506\text{W/cm}^2 \)), although the dispersion relation is considerably altered far before this. The change in the dispersion relation could be observed using Bragg spectroscopy as performed in \[5\]. Fig. 2 plots the Bogoliubov dispersion for \( I = 0.057 \) (\( I = 0.565\text{W/cm}^2 \)).

Local to the ‘roton’ minimum at \( k = k_{\text{roton}} \) one can write \( E = \Delta + \hbar^2(k - k_{\text{roton}})^2/2m^* \), and for the parameters above with \( I = 0.057 \) one finds \( m^* = 0.06m \). He II has \( m^* = 0.16m \) \[4\]. The static structure factor is plotted in Fig. 3. The peak in \( S(k_z) \) corresponds to the minimum in the energy spectrum. The model described here predicts that when \( I \geq 0.066 \) \( (I \geq 0.654\text{W/cm}^2) \) the minimum touches the zero energy axis. At this point the system is unstable to a periodic, supersolid-like, density modulation \[8, 16\].

The laser induced dipole-dipole potential can

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**Figure 2:** The Bogoliubov dispersion relation for \(^{87}\)Rb with \( w_r = 3.5\lambda_L \) and \( n(0) = 8 \times 10^{20} \text{atoms/m}^3 \). For pure s-wave scattering (\( I = 0 \)) the inverse healing length \( 1/\xi_0 = \sqrt{8\pi an(0)} = 1.32k_L \).

**Figure 3:** The static structure factor, \( S(k_z) \) for various laser intensities. Same parameters as Fig. 2.
lead to electrostriction (compression) of a condensate \[^7\]. In the present regime of low laser intensity/large detuning the electrostriction is negligible (on a scale set by the collapse threshold \(I = 3/2\) \[^7, 13\]). This regime also ensures the absence of two-body bound states in the 1D reduced potential shown in Fig. 1, a necessary condition for the validity of the Born approximation for atom-atom scattering by this potential. Only when \(I > 1.3\) do bound states appear.

The interaction \[^8\] arises from the forward scattering of laser photons by atom pairs. At large detunings there are two main competing processes that can heat a dense gas: A) \textbf{Light-induced transfer of pairs of colliding atoms to a quasi-molecular excited state} followed by dissociation, releasing \(\approx \hbar \delta\) into the kinetic energy of the pair \[^17\]. This is a density dependent effect whose rate can therefore be high. Even when the laser is red-detuned from an atomic resonance, when two atoms collide the energy separation between the ground state and a molecular excited state \((- C_3/r^3)\) comes into resonance at small distances. However, by choosing \(\delta\) so that the resonance point occurs between two molecular vibrational states this process is suppressed \[^17\]. Below the D1 line there are only discrete molecular vibrational states (i.e. no continuum states) so a detuning can be selected which is between these molecular resonances \[^15\], which are narrow at ultra-cold temperatures.

B) \textbf{Incoherent light scattering by single atoms} occurs at approximately the Rayleigh scattering rate which can be written \(\gamma_R = (8/3)E_R k_L a I / \hbar\). Applying the f-sum rule for the dynamic structure factor one can show \[^19\] that Rayleigh scattering transfers energy to the gas at a rate \(d E_{\text{tot}} / dt = 2E_R N \gamma_R\) which, surprisingly, is independent of the interactions between the atoms. Comparing this heating rate with the energy of the ground state of the gas, \(E_{\text{tot}} \approx H_s + H_{\text{trap}} + H_{\text{kin}},\) where \(H_{\text{kin}}\) is the kinetic energy of the atoms, one can estimate a heating time via \(\tau_{\text{heat}} = E_{\text{tot}} / (dE_{\text{tot}}/dt)\). To measure a roton the BEC must survive for longer than the roton period \(\tau_{\text{roton}} \propto 2\pi \hbar / E_R\) (cf. Fig. 2).

In conclusion, atom-atom correlations due to laser induced dipole-dipole interactions in a gaseous condensate can give a roton minimum in the Bogoliubov dispersion relation. The correlations are tunable via parameters such as radial width, laser intensity and wavelength. We thank M. Boshier, C. Eberlein, J. Steinhauer, R. Shiell, and H.T.C. Stoof for illuminating discussions, and the Engineering and Physical Sciences Research Council (EPSRC), the German-Israeli Foundation (GIF), and the EU QUACS and CQG networks for funding.

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