Spin-polarized tunneling into helical edge states: Asymmetry and conductances

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Abstract – We consider tunneling from the spin-polarized tip into the Luttinger liquid edge state of quantum spin Hall system. This problem arose in the context of the spin and charge fractionalization of an injected electron. Renormalization of the dc conductances of the system is calculated in the fermionic approach and scattering states formalism. In the lowest order of the tunneling amplitude we confirm previous results for the scaling dependence of conductances. Going beyond the lowest order we show that the interaction affects not only the total tunneling rate, but also the asymmetry of the injected current. The helical edge state forbids the backscattering, which leads to the possibility of two stable fixed points in the renormalization group sense, in contrast to the Y-junction between the usual quantum wires.

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Introduction. – Topological insulators (TI) is a class of materials apparently important for future electronic devices [1,2]. Being an insulator in the bulk, the TI necessarily has conducting states on its edge. The spin-orbit interaction leads to electrons spin-momentum locking in the edge states creating so-called helical states. Backscattering on the non-magnetic impurities in the same channel of such edge states is absent due to TI time-reversal symmetry. It allows the creation of devices with a lossless electron transport.

Electrons on the edge of 2D TI are confined to one spatial dimension (1D). One-dimensional electron transport near the Fermi energy is described by the well-known Tomonaga-Luttinger liquid (TLL) model [3]. The TLL corresponds to strongly correlated electron matter, and the 1D helical edge states add new peculiarities to it. This explains the large experimental and theoretical interest in helical TLLs, with several suggested ways of probing their strongly correlated nature. Investigating the helical edge states was theoretically proposed by means of a magnetic field in [4–6], by the specific spatial construction of a helical state in [7–9] and by quantum dots in [10,11].

Another natural way of probing —tunneling from the tip into the edge state— attracted a lot of attention. There is experimental evidence [12] and a theoretical explanation [13] that the electron injection even into usual TLL may be characterized by charge fractionalization resulting in the currents imbalance. For helical TLL probing by the tip was considered theoretically in papers [14–19]. Experimentally it was confirmed that the TI edge state is a quantum spin Hall state [20], the helical behavior of the edge state was observed in [21], the STM experiments were reported in [22,23].

The transport properties of the TLL model may be studied in two main approaches. The bosonization approach [3] takes into account the electron interaction exactly, the impurities are regarded as perturbation. The fermionic approach treats the interaction perturbatively and studies the renormalization of impurities/junctions in the S-matrix formalism of asymptotic electronic states. Transport in the quantum point contact is described by the conductances (transparencies) of junctions. The problem with one impurity in a quantum wire can also be interpreted as the point contact of two wires. The latter case was considered within the TLL model in bosonization [24–26] and fermionic [27,28] approaches. The conclusion of both approaches is that the repulsive interaction renormalizes the bare conductance to zero (total reflection case), whereas the attractive interaction leads to perfect transmission in the low-temperature limit.

Further studies of the usual TLL in the bosonization approach addressed the Y-junction with equal interaction
constant [29] and arbitrary interaction in three wires [30]. The corner junction of helical TLLs was considered for two TIs in [31–33].

In the fermionic approach the renormalization group (RG) equations for the $S$-matrix characterizing the junction for an arbitrary number of quantum wires was presented in [34,35]. Conductances of the Y and X junctions were analyzed in the first order of the interaction.

A partial summation of the perturbation series was performed in [36] in order to recover the exact scaling exponents for conductance for one impurity. The results were used for the RG analysis of the Y-junction [37–39] and generalized to an arbitrary number of wires [40]. The contact of two helical TLLs and tunneling from an unpolarized spinful wire into the helical edge state were considered in [41]. The agreement between bosonization and fermionic approaches has been obtained when the comparison was available.

In the present letter we focus on the spin-polarized tunneling into the helical edge state. This problem was previously studied within the bosonization approach [15], with the setup consisting of a point contact between fully spin-polarized tip and the helical edge state (see fig. 1). Generally, spin polarization in the tip differs from the spin quantization axis in the TI edge state, and the angle between the spin-polarized tip and the helical edge state (see fig. 1).

The helical state of the main wire is described by \( \Psi^S \) while the helical states of auxiliary wires are described by \( \Psi^F \) for short. The helical states are adiabatically connected to the reservoirs or leads such that additional scattering is absent. The helical edge state is defined by \( \Psi^G \), which is an angle between the spin-polarized tip and the helical edge state (see fig. 1). The helical state of the main wire is described by \( \Psi^S \), and the helical states of auxiliary wires are described by \( \Psi^F \) for short. The helical states are adiabatically connected to the reservoirs or leads such that additional scattering is absent. The helical edge state is defined by \( \Psi^G \), which is an angle between the spin-polarized tip and the helical edge state (see fig. 1).

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\[ \psi_{3, \text{in(out)}} \]. The S-matrix characterizes the scattering in the junction and belongs to the unitary group \( U(3) \). Our setup implies the chiral property, \( |S_{13}| = |S_{23}| = |S_{31}| \). Due to the time-reversal invariance of TI, the backscattering in the edge state without the tip is forbidden. The same is true in the case in which the polarization of the tip is parallel to the spin direction of the right or left mover, \( \xi = 0, \pi \).

In general, the chiral S-matrix is characterized by three parameters [39]. The absence of backscattering reduces the appropriate S-matrix to the following two-parametric form:

\[
S = \begin{pmatrix} r_1 & t_{12} & t_{13} \\ t_{21} & r_1 & t_{23} \\ t_{31} & t_{32} & r_2 \end{pmatrix},
\]

\[
r_1 = \sin^2 \frac{\theta}{2} \sin \xi, \quad r_2 = \cos \theta, \quad t_{12} = -\cos \theta \cos^2 \frac{\xi}{2} - \sin^2 \frac{\xi}{2}, \quad t_{21} = t_{12} e^{-\pi - \xi}, \quad t_{13} = t_{32} = \cos \frac{\xi}{2} \sin \theta, \quad t_{23} = t_{31} = \sin \frac{\xi}{2} \sin \theta.
\]

One parameter, \( \theta \), is responsible for the tunneling amplitude and \( \theta = 0 \) corresponds to the fully detached tip, in this case there is a perfect transmission through the edge state. The second parameter \( \xi \) is the above angle between the electron spin in the tip and the spin quantization axis of the helical state (see SM).

**Reduced conductances.** – In the linear response regime the conductances are defined by \( I_\rho = C_{jk} V_\rho \), where \( I_j \) and \( V_j \) are the current flowing towards in the junction and the electric potential in the \( j \)-th lead, respectively. In the dc limit one easily obtains from the Kubo formula \( C_{ij} = \delta_{ij} - Y_{ij} \), with \( Y_{ij} = |S_{ij}|^2 \). The total current conservation and the absence of current for equal voltages lead to conditions \( \sum_i C_{ij} = \sum_j C_{ij} = 0 \), which also stem from the unitarity of the S-matrix. This means that we can introduce new current and voltage combinations to show the condition explicitly [43].

We choose new combinations according to relations \( I_{\rho}^{\text{new}} = \sum_k R_{ki} I_k, \quad V_{\rho}^{\text{new}} = \sum_k R_{ki} V_k \), where the matrix

\[
R = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}
\]

has properties \( R^{-1} = R^T \), \( \det R = 1 \).

The corresponding matrix of conductances \( C^R = R^T C R \) for the above form of \( S \), eq. (2), becomes

\[
C^R = \begin{pmatrix} 1 - a & -c & 0 \\ c & 1 - b & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

with
\[
a = \frac{1}{2}(1 + \cos^2 \theta) \cos^2 \xi - \cos \theta \sin^2 \xi,
\]
\[
b = \frac{1}{4}(1 + 3 \cos 2\theta),
\]
\[
c = \frac{\sqrt{3}}{2} \cos \xi \sin^2 \theta.
\]

It is convenient to analyze the quantity \( Y^R = 1 - C^R \) below. Clearly, only two components of \( Y^R \) are independent. The chiral component of conductance, \( c \), depends on the asymmetry parameter, \( \cos \xi \), which includes the polarization angle, \( \xi \).

We can also introduce more “physical” combinations of the currents, \( I^{\text{phys}} = (I_a, I_b) \), and voltages, \( V^{\text{phys}} = (V_a, V_b) \), as

\[
I_a = \frac{1}{2}(I_1 - I_2), \quad I_b = \frac{1}{3}(I_1 + I_2 - 2I_3),
\]
\[
V_a = V_1 - V_2, \quad V_b = \frac{1}{3}(V_1 + V_2 - 2V_3).
\]

In terms of these, the matrix of the conductances \( I^{\text{phys}} = GV^{\text{phys}} \) is given by

\[
G = \begin{pmatrix} G_a & G_{ab} \\ G_{ba} & G_b \end{pmatrix} = \begin{pmatrix} 1 - a & -c \\ c & 2 \sqrt{3} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3(1 - b) \end{pmatrix}
\]

with the property \( 0 \leq G_{a,b} \leq 1 \), see [43].

**Renormalization group equations.** – The dc transport through the chiral Y-junction was studied in fermionic formalism in [38,39]. We sketch here the derivation of the corresponding formulas. The principal first-order interaction correction to the conductances reads [38]

\[
C_{jk} = C_{jk}^{(0)} + \frac{1}{2} \sum_{l,m} \text{Tr}[\hat{W}_{jk}^{R} \hat{W}_{lm}^{R}] g_{ml},
\]

where we defined \( C_{jk}^{(0)} = \delta_{jk} - Y_{jk} \), a matrix \( g_{ml} = g \delta_{ml} \) with interaction constants \( g \) in the Hamiltonian (1), nine \( 3 \times 3 \) matrices \( \hat{W}_{jk} = [\rho_j, \rho_k] \) and the trace operation \( \text{Tr} \) is defined with respect to the 3×3 matrix space of \( \hat{W} \)'s. The scale-dependent term \( \Lambda = \ln(\min[L, \xi]/\ell) \) with the temperature correlation length \( \xi = v_F/\ell \).

The above correction is the leading contribution, \( \sim g \Lambda \), to conductance. It was shown [43] that the higher-order corrections obey the scaling hypothesis for the conductance and are generated by a set of the differential RG equations. In its simplest form, these equations are obtained by differentiating eq. (8) over \( \Lambda \) (and then putting \( \Lambda = 0 \)) which gives

\[
\frac{d}{d\Lambda} Y_{jk}^{R} = \frac{1}{2} \sum_{l,m} \text{Tr}[\hat{W}_{jk}^{R} \hat{W}_{lm}^{R}] g_{ml}^{R},
\]

with all quantities with superscript \( R \) are defined as \( A^R = R^T A R \).
The main source of linear-in-Λ subleading corrections corresponds to a ladder series of diagrams, which can be summed analytically [43]. These corrections are independent of the scheme of regularization in the RG procedure and lead to the scaling exponents for conductances, coinciding with those obtained by the bosonization method.

The result of the ladder summation [39,40] is the RG equation similar to above equation (9) with the replacement

$$g \rightarrow g = 2(Q - Y)^{-1}.$$

The matrix $Q$ depends on the Luttinger parameters $K_j$ of individual wires and has the form

$$Q_{jk} = g_j \delta_{jk}, \quad g_j = (1 + K_j)/(1 - K_j),$$

$$K_j = [(1 - g_j)/(1 + g_j)]^{1/2}. \quad (11)$$

In our case we have $g_1 = g_2 = g$ and $g_3 = 0$. The latter equality corresponds to $g_3 \rightarrow \infty$, whose limit is easily taken in (10).

Now we are in a position to obtain RG equations of the conductances. However, the equivalent RG equations in terms of parametrization (2) have a simpler form, and we choose it below. After some algebra we obtain from (5), (9), (10) the following set of equations:

$$\frac{d\theta}{d\Lambda} = -\frac{1}{8}(1 - K) \sin \theta \frac{F_1(\theta) + F_2(\theta) \cos 2\xi}{F_3(\theta)D(\theta, \xi)},$$

$$\frac{d\xi}{d\Lambda} = \frac{1}{4}(1 - K) \frac{(1 - \cos \theta) \sin 2\xi}{D(\theta, \xi)},$$

$$F_1 = 2(1 - K) - 3K \sin^2 \frac{\theta}{2} - 3(1 - K) \sin^2 \theta,$$

$$F_2 = \sin^2 \frac{\theta}{2} \left( 2 - 3(1 - K) \sin^2 \frac{\theta}{2} \right),$$

$$F_3 = 1 - (1 - K) \sin^2 \frac{\theta}{2},$$

$$D(\theta, \xi) = K + (1 - K) \sin^2 \frac{\theta}{2} \left( 1 - \sin^2 \frac{\theta}{2} \sin^2 \xi \right). \quad (12)$$

These equations are analyzed in the next section. Notice that the asymmetry parameter $p = \cos \xi$ is now determined not only by the initial polarization angle, but also by the interaction strength and tunneling amplitude.

**Spin-polarized tunneling.**

**Lowest-order calculation.** Let us first consider the limit of weak tunneling into the helical edge state. This means that both tunneling from the tip and the backscattering in the edge state due to this tunneling are approximately zero. In terms of the S-matrix (2) we write $|r_2|^2 \simeq 1$ and $|r_1|^2 \simeq 0$. The first condition gives $\theta \simeq 0$ or $\theta \simeq \pi$, and the second condition states that if $\theta \simeq \pi$ then $\xi \simeq 0$ or $\xi \simeq \pi$. These three seemingly unconnected regions form a neighborhood of the fixed point (FP) $A$, as is discussed below.

Confining ourselves to the linear-in-$\theta$ terms, we get from eqs. (12)

$$\frac{d\theta}{d\Lambda} = -\frac{(1 - K)^2}{4K} \theta, \quad \frac{d\xi}{d\Lambda} = 0. \quad (13)$$

Hence, the parameter $\theta$, which is related to the barrier transparency, is renormalized to zero with the scaling law $\theta = \theta_0 \exp(-\nu \Lambda/2)$, and the tunneling exponent $\nu = (1 - K)^2/2K$. The asymmetry ratio $t_L/t_R = \tan \xi/2$ is not renormalized.

This result is in exact agreement with the earlier work [15], where the renormalization of the tunnel amplitudes $t_L$ and $t_R$ was studied in the bosonization approach. Equation (9) of the above paper obtains the currents flowing to the right $I_R$ and to the left $I_L$ ends of the edge state. One has

$$I_L + I_R = I_0, \quad I_L - I_R = -K I_0 \cos \xi. \quad (14)$$

In our notation (6) the current $I_0$ equals $I_0$, and the difference of the currents $I_L - I_R$ is $2I_0$. For zero bias in the edge state, $V_0 = 0$, we have $I_0 = G_0 V_0$, $I_a = G_a V_0$, and obtain from eqs. (5), (7):

$$2I_0 = -I_0 \cos \xi. \quad (15)$$

This equation coincides with (14) apart from the factor $K$, which is absent in the dc limit in our setup with the Fermi leads. The absence of this factor in the similar setup was also obtained in the work [18] in the bosonization approach.

Now we consider the next terms in the expansion of the RG equations in $\theta$. The quadratic-in-$\theta$ term appears only in the RG equation for the asymmetry angle:

$$\frac{d\xi}{d\Lambda} = \frac{1 - K}{8K} \theta^2 \sin 2\xi. \quad (16)$$

The solution of the equation shows that the asymmetry angle is renormalized according to the law

$$\tan \xi = \tan \xi_0 e^{-(\theta^2 - \theta_0^2)/2(1 - K)}. \quad (17)$$

Since $\theta$ is renormalized to zero (see eq. (13)), we find the whole renormalization of $\xi$ insignificant for small bare tunneling amplitude $\theta_0$. At the same time the renormalization of $\xi$ in (16) is the first-order interaction effect for small interaction $\frac{1 - K}{K} \approx g \ll 1$, whereas the tunneling exponent for $\theta$ in eq. (13) is given by the second order of the interaction, $\frac{1 - K^2}{K} \approx g^2$.

We see that if we discard the small-in-$\theta$ terms in the RG equation for $\xi$, then we do not obtain the renormalization of the asymmetry at all. However the smallness-in-$\theta$ may be compensated by the less power of interaction in the RG flow of $\xi$. As a result for small interaction and tunneling amplitude the renormalization of $\theta$ and $\xi$ may be of the same order as demonstrated in fig. 2.
the RG trajectory in the form

\[ \frac{\partial p}{\partial \Lambda} = - \frac{1 - K}{4K} \theta^2 p(1 - p^2). \]  

(18)

This equation possesses three fixed points (FPs) \( p = 0, \pm 1 \), and if \( \theta^2 \) were non-vanishing at \( \Lambda \to \infty \), then we would obtain only \( p = 0 \) as a stable FP for repulsive interaction, \( K < 1 \). In the leading order of the interaction, we can combine the first equation in (13) and (18) to represent the RG trajectory in the form

\[ \theta^2 - g \log \frac{p^2}{1 - p^2} = \text{const.} \]  

(19)

**M-point.** The terms of the order \( \theta^3 \) appear only in the equation for \( \theta \):

\[ \frac{d\theta}{d\Lambda} = - \frac{(1 - K)^2}{4K} - \frac{(1 - K)(\kappa - \cos 2\xi)}{16K} \theta^3, \]  

(20)

where \( \kappa \equiv (1 - K)(2/3 + K) + 1/K \). For small interaction \( \kappa \approx 1 \) and eq. (20) becomes

\[ \frac{d\theta}{d\Lambda} = \frac{g^2}{4} \theta + \frac{g(1 - \cos 2\xi)}{16} \theta^3. \]  

(21)

First, one can see here that the \( \theta^3 \)-term is unimportant in the parallel polarization case, \( \xi = 0, \pi \); the right-hand side of eq. (16) is zero. Thus, the lines \( \xi = 0, \pi \) are RG fixed lines.

Otherwise the combination of (20) with (16) reveals the existence of a non-universal fixed point \( M \approx \sqrt{2g} \). Similar to the above eq. (16), the next-order term in the \( \theta \) expansion contains smaller power of the interaction strength. This situation was discussed for the non-chiral symmetric Y-junction in [42].

Solving RG equations for asymmetry \( p \) and \( \theta \) gives for the RG trajectory

\[ \theta^2 p - g \log \frac{1 + p}{1 - p} = \text{const.} \]  

(22)

Despite the apparently different form of eqs. (22) and (19), these equations yield numerically close curves for small \( \theta < \theta_M \). Indeed the existence of the next term in eq. (20) does not seriously affect RG flows in this limit.

Already at the level of approximate equations (20), (16) one can expect that for \( \theta \approx \theta_M \) the RG flow for the asymmetry \( \cos \xi \) may exceed the RG flow of the barrier transparency \( \theta \). We discuss this point in more detail in the next subsection.

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**Fig. 2:** (Colour online) The panel (a) shows RG flows in the plane \( (\xi, \theta) \) for \( g = 0.26 \) \((K = 0.77)\). Relative renormalization of \( \xi \) and \( \theta \) for the flow with \( \xi_0 = 0.3, \theta_0 = 0.4 \) is shown in panel (b).

**Fig. 3:** (Colour online) RG flows (blue arrows) demonstrate the simultaneous existence of the stable fixed line (blue line) at point \( A \) and stable point \( N \), depending on the initial conditions \( (\xi, \theta) \) for electron interaction \( g = 0.72 \) \((K = 0.4)\). The red line corresponds to the RG flow plotted in terms of the conductances in fig. 4(b).

**Table 1:** The position of the universal fixed points.

| \( \theta \) | \( \xi \) | \( G_a \) | \( G_b \) | \( G_{ab} \) | FP |
|---|---|---|---|---|---|
| 0 | arb. | 1 | 0 | 0 | A |
| \( \pi \) | \( \pi \) | 1 | 0 | 1 | A |
| \( \pi/2 \) | 0 | 0 | 0 | 1 | A |
| \( \pi \) | \( \pi \) | 3/4 | 3/4 | 1/2 | \( \chi^- \) |
| \( \pi \) | \( \pi \) | 3/4 | 3/4 | -1/2 | \( \chi^+ \) |
Fig. 4: (Colour online) Full scaling curves for conductances are shown in panel (a) for electron interaction $g = 0.47$ ($K = 0.6$). The RG flow corresponds to going to the point $A$, i.e., effective detachment of the tip. The panel (b) shows the RG flows for stronger electron interaction $g = 0.72$ ($K = 0.4$). The conductances are renormalized to the point $N$, i.e., effective full breaking of the junction. Bare initial conductances on both panels are the same and the qualitative change in the RG flows is caused by the change in the interaction strength.

Non-perturbative RG results. Now we go beyond the weak tunneling approximation and discuss the general RG equations (12) for $K < 1$. In this rather complicated case we have five universal FPs in the $(\theta, \xi)$-plane, one interaction-dependent (i.e., non-universal) fixed point $M (\xi_M = \pi/2, \theta_M = \arccos \frac{1}{2}(\sqrt{q^2 + 6q - 3} - q)$ and the lines of fixed points ($\xi \in (0, \pi)$ and $\theta = 0$), see fig. 3 and table 1. The points $(\xi = 0, \theta = \pi), (\xi = \pi, \theta = \pi)$ and the fixed line $\theta = 0$ correspond to one point $A$ in terms of conductances.

For repulsive electron interaction the fixed line $\theta = 0$ (the point $A$) is stable and corresponds to the effective detachment of the tip from the edge state. It exists simultaneously with the stable point $N$ which corresponds to full breaking of the junction. The line in the $(\xi, \theta)$-plane which separates these two basins of attraction connects the unstable points $\chi^\pm$ with the saddle point $M$ (see fig. 3). The chiral FPs $\chi^\pm$ were discussed in detail in [29,39] and they become stable for the attractive interaction.

The whole plane $(\theta, \xi)$ is divided into two regions in fig. 3, depending on the interaction strength. One region corresponds to a basin of attraction to the point $N$, the other to the point $A$. The situation is similar to the purely tunneling case (PTC) for symmetric non-chiral Y-junction [42]. It was shown there that for PTC the points $A$ or $N$ are stable and separated by the unstable $M$-point. Any deviation from PTC, which means the backscattering in the main wire, leads to an additional RG flow driving the junction to a truly stable $N$ point while $A$ becomes of saddle point type. In our present model we have the chiral PTC junction, and any additional modification of the $S$-matrix is forbidden due to the topological character of the edge state and the absence of backscattering in the main wire.

Notice that for vanishing interaction, $g$, the FPs $A$ and $M$ merge, and the basin of attraction for the $A$-point vanish. More precisely, the $A$-point becomes of saddle point type in this limit and is stable only for the parallel polarization of the tip, $\xi = 0, \pi$. All RG trajectories in this limit obey the equation

$$\cos \xi \tan^2(\theta/2) = \text{const.} \quad (23)$$

Since the position of the $M$-point depends on the interaction strength, a qualitatively different behavior of renormalized conductances may occur even for their equal bare values at different interactions. Two examples of full scaling curves for conductances calculated for sizable interaction strength are shown in fig. 4. The values of the bare conductances are the same, but the panel (a) demonstrates RG flows going to the point $A$, whereas the panel (b) shows the system driven to the point $N$. This difference stems from the fact that changing the interaction strength shifts the separating curve between the basins of attraction of the corresponding FPs.

Conclusions. – We analyzed the simultaneous renormalization of asymmetry and total tunneling conductance from fully polarized tip into the helical edge state. In particular we showed that the RG flow for these two quantities is sensitive to the appearance of non-universal FP with asymmetry $\cos \xi_M = 0$ and tunneling conductance $G_t \approx 2g$ for small interaction, $g$.

Our method assumes the existence of Fermi leads, which are needed for the consistent definition of the $S$-matrix describing the Y-junction in terms of asymptotically free states. Experimentally it can be realized by the extended leads over the helical edge state. The helical edge state, on the other hand, is a topological object which cannot have disconnected ends. The dc current injected from the tunneling tip into the edge state may exit at another drain Y-junction. This implies the ring geometry with potentially important interference effects. The above picture of the renormalization of the single Y-junction should be valid when the temperature correlation length $\xi = v_F/T$ is smaller than the circumference of the edge state multiplied by $|r|^{\partial}$. The importance of the interference effects is then reduced [44] but the corresponding analysis is beyond the scope of this study.

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