SUPERCONDUCTIVITY IN ANYON FLUID AT FINITE TEMPERATURE AND DENSITY*

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Abstract

The boundary effects in the screening of an applied magnetic field in a charged anyon fluid at finite density ($\mu \neq 0$) and temperature ($T \neq 0$) are investigated. By analytically solving the extremum equations of the system and minimizing the free energy density, we find that in a sample with only one boundary (the half plane), a total Meissner effect takes place; while the sample with two boundaries (the infinite strip) exhibits a partial Meissner effect. The short-range modes of propagation of the magnetic field inside the fluid are characterized by two temperature dependent penetration lengths.

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Since the claim by Laughlin and his collaborators [1], [2] that fractional statistics could play a crucial role in high-$T_C$ superconductivity, a significant work has been done to investigate the superconducting characteristics of the charged anyon fluid in two spatial dimensions.

The anyon superconductivity at $T = 0$ has been investigated by many authors [2]-[8]. In this case, the anyon superconductivity appears due to the exact cancellation between the bare and induced Chern-Simons terms in the effective action of the theory.

The possible realization of anyon superconductivity at $T \neq 0$ has also been extensively investigated [3]-[12]. At finite temperature, based on non-vanishing correction to the induced Chern-Simons coefficient, some authors (see, ref. [6]) have concluded that the superconductivity is lost at $T \neq 0$. In contrast with this result, in refs. [7], [11] it was argued that the non-vanishing correction to the induced Chern-Simons coefficient is numerically negligible at $T < 200 \, ^oK$. On the other hand, the development of a pole $\sim (\frac{1}{k^2})$ at $T \neq 0$ in the polarization operator component $\Pi_{00}$, characteristic of the Debye screening in plasmas, was found [7], [11] as the main reason for the lack of a total Meissner effect in the charged anyon fluid at finite temperature. In these papers it was discussed how the appearance of this pole leads to a partial Meissner effect with a penetration which appreciably increases with temperature. Independently, in ref. [5], it was claimed that the anyon model fails to provide a good superconducting behavior at finite temperature. The reason is that a long-range mode was obtained inside the infinite bulk, which vanishes only at $T = 0$.

In the present paper, working in the self-consistent field approximation [3], [7], [11], we show that the finite temperature superconducting behavior of the charged anyon fluid depends on the sample boundary conditions. This result is obtained by analytically solving the field equations of the system and the stability conditions derived from the free energy density, subject to two different set of boundary conditions: a half infinite planar sample with an external magnetic field applied in the boundary ($x = 0$), and an infinite strip with external magnetic field applied in the two boundaries ($x = 0$ and $x = L$).

For the half plane we find that the external magnetic field cannot penetrate the bulk (total Meissner screening). In this case the external magnetic field is damped within the
anyon fluid by two characteristic lengths, corresponding to two short-range eigenmodes of propagation.

In the case of an infinite strip, it is shown that a partial penetration occurs (partial Meissner screening). That is, the applied magnetic field propagates, within the anyon fluid, through one long-range and two short-range modes of propagation.

To understand the genesis of these results, one must take into account that in this model
the zero component of the Maxwell and Chern-Simons gauge fields, $A_0$ and $a_0$ respectively, enter in the field equations in the same foot as the electromagnetic and Chern-Simons field strengths. Accordingly, $A_0$ and $a_0$ become physical, and, as we will prove, their asymptotic behaviors (which are inherently linked to the sample boundary conditions) affect the magnetic screening properties within the bulk. In this sense, the model exhibits a kind of Aharonov-Bohm effect. The importance of the boundary conditions in 2+1 dimensional models has been already stressed in ref. [13].

The approach we follow is to compute the finite temperature effective action starting from the Lagrangian density

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{N}{4\pi} \varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + en_e A_0 + i \psi^\dagger D_0 \psi - \frac{1}{2m} |D_k \psi|^2 + \bar{\psi} \gamma^0 \mu \psi
$$

of a 2+1 dimensional charged fluid of non-relativistic electrons, $\psi$, coupled to two independent gauge fields, $A_\mu$ and $a_\mu$, which represent the electromagnetic field and the Chern-Simons field respectively. The covariant derivative is given by $D_\nu = \partial_\nu + i (A_\nu + e A_\nu)$, $\nu = 0, 1, 2$. The charged character of the fluid is implemented through the chemical potential $\mu$; $n_e$ is a background neutralizing “classical” charge density. From the electric charge neutrality condition, it is known that the system ground state has a non-zero expectation value of the Chern-Simons magnetic field ($\mathcal{B} = \frac{2\pi n_e}{N}$).

To investigate the linear response of the medium to an applied external magnetic field, it is enough to consider small fluctuations of the gauge potentials around the many-particle ground state. That is, we can evaluate the effective action corresponding to the Lagrangian density (1), up to second order in these small quantities,
\[ \Gamma_{\text{eff}} (A_\nu, a_\nu) = \int dx \left( -\frac{1}{4} F^2_{\mu\nu} - \frac{N}{4\pi} e^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + e n_e A_0 \right) + \Gamma^{(2)} \] (2)

\[ \Gamma^{(2)} = \int dx \Pi^\nu (x) \left[ a_\nu (x) + eA_\nu (x) \right] + \int dx dy \left[ a_\nu (x) + eA_\nu (x) \right] \Pi^{\mu\nu} (x, y) \left[ a_\nu (y) + eA_\nu (y) \right] \] (3)

\( \Gamma^{(2)} \) is the fermion contribution to the effective action in the above approximation, \( \Pi_\nu \) and \( \Pi_{\mu\nu} \) represent the fermion tadpole and polarization operators respectively. An essential point in the study of this effective theory is the calculation of these operators by using the fermion thermal Green’s function defined in the presence of the background field \( \phi \) [5], [7].

The leading behavior of these operators for static (\( k_0 = 0 \)) and slowly (\( k \sim 0 \)) varying configurations have been found in the low \( (T \ll m/\phi) \) and high \( (T \gg m/\phi) \) temperature limits by different authors (see refs. [5], [10]). These operators with the spatial momentum specialized in the frame \( k = (k_0, 0) \) are given by,

\[ \Pi_k (x) = 0, \quad \Pi_0 (x) = -n_e \] (4)

\[ \Pi_{\mu\nu} = \begin{pmatrix} \Pi_0 + \Pi_0' k^2 & 0 & \Pi_1 k \\ 0 & 0 & 0 \\ -\Pi_1 k & 0 & \Pi_2 k^2 \end{pmatrix} \] (5)

where the polarization operator coefficients in the different temperature limits are,

**Low-Temperature Limit:**

\[ \Pi_0 = \frac{\beta \phi}{\pi} \left( 1 - \frac{2}{m} e^{-\beta \phi/2m} \right) \] (6)

\[ \Pi_0' = \frac{mN}{2\pi \phi} \left[ 1 - \frac{2\beta \phi}{m} e^{-\beta \phi/2m} \right] \] (7)

\[ \Pi_1 = \frac{N}{2\pi} \left[ 1 - \frac{2\beta \phi}{m} e^{-\beta \phi/2m} \right] \] (8)

\[ \Pi_2 = \frac{N^2}{2\pi m} \left[ 1 + \frac{2}{N^2} e^{-\beta \phi/2m} - \frac{2\beta \phi}{m} \left( 1 + \frac{1}{4N^2} \right) e^{-\beta \phi/2m} \right] \] (9)
High-Temperature Limit:

\[
\Pi_0 = -\frac{m}{4\pi} \left[ \tanh \left( \frac{\beta \mu}{2} \right) + 1 \right]
\]  
(10)

\[
\Pi_0' = -\frac{\beta}{96\pi} \text{sech}^2 \left( \frac{\beta \mu}{2} \right)
\]  
(11)

\[
\Pi_1 = \frac{i\beta \hbar}{96\pi m} \text{sech}^2 \left( \frac{\beta \mu}{2} \right)
\]  
(12)

\[
\Pi_2 = \frac{1}{48\pi m} \left[ \tanh \left( \frac{\beta \mu}{2} \right) + 1 \right]
\]  
(13)

Another important quantity needed to investigate the behavior of the anyon fluid is its free-energy

\[
\mathcal{F} = \frac{1}{2} \int \left[ \int_{-L_z/2}^{L_z/2} dy \int_0^{L_x} dx \left\{ (E^2 + B^2) + \Pi_0 (eA_0 + a_0)^2 + \Pi_0' (eA_0 + a_0) \frac{\partial^2}{\partial x^2} (eA_0 + a_0) \\
+ \Pi_1 [(eA_0 + a_0) \frac{\partial}{\partial x} (eA_2 + a_2) - (eA_2 + a_2) \frac{\partial}{\partial x} (eA_0 + a_0)] + \Pi_2 (eA_2 + a_2) \frac{\partial^2}{\partial x^2} (eA_2 + a_2) \right\} \right]
\]  
(14)

To study the linear response of the anyon fluid to an applied external magnetic field we have to solve the extremum equations derived from the effective action (2). This formulation is known in the literature as the self-consistent field approximation [7], [11]. The corresponding Maxwell and Chern-Simons extremum equations are respectively,

\[
\partial_\nu F^{\nu \mu} = eJ_{\mu}^{\mu}\ind
\]  
(15)

\[
-\frac{N}{4\pi} \epsilon^{\mu \nu \rho} f_{\nu \rho} = J_{\mu}^{\mu}\ind
\]  
(16)

Here, \( f_{\mu \nu} \) is the Chern-Simons gauge field strength tensor, defined as \( f_{\mu \nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \), and \( J_{\mu}\ind \) is the current density induced by the anyon system at finite temperature and density. Their different components are given by
\[ J^0_{\text{ind}}(x) = \Pi_0 [a_0(x) + eA_0(x)] + \Pi_0' \partial_x (E + eE) + i\Pi_1 (b + eB) \tag{17} \]

\[ J^1_{\text{ind}}(x) = 0, \quad J^2_{\text{ind}}(x) = i\Pi_1 (E + eE) + \Pi_2 \partial_x (b + eB) \tag{18} \]

In the above expressions the notation: \( E = f_{01}, E = F_{01}, b = f_{21}, B = F_{21} \) was used. Eqs. (17)-(18) play the role in the anyon fluid of the London equations in BCS superconductivity. When the induced currents (17), (18) are substituted in eqs. (15) and (16), one finds, after some manipulation, the following set of independent differential equations,

\[ \omega \partial_x^2 B + \alpha B = \gamma [\partial_x E - \sigma A_0] + \tau a_0 \tag{19} \]

\[ \partial_x B = \kappa \partial_x^2 E + \eta E \tag{20} \]

\[ \partial_x a_0 = \chi \partial_x B \tag{21} \]

The coefficients appearing in these differential equations depend on the components of the polarization operators through the relations,

\[ \omega = \frac{2\pi}{N} \Pi_0', \quad \alpha = ie^2 \Pi_1, \quad \tau = -e \Pi_0, \quad \chi = -\frac{2\pi}{eN}, \quad \sigma = \frac{e^2}{\gamma} \Pi_0, \quad \eta = -\frac{ie^2}{\delta} \Pi_1 \]

\[ \gamma = 1 - e^2 \Pi_0' - \frac{2\pi i}{N} \Pi_1, \quad \delta = 1 + e^2 \Pi_2 + \frac{2\pi i}{N} \Pi_1, \quad \kappa = -\frac{2\pi}{N\delta} \Pi_2. \tag{22} \]

The extremum equations (19)-(21) are not essentially different from those found for the anyon effective theory at finite temperature by other authors \[7, 11\]. Distinctive of these equations is the appearance of the nonzero constant coefficients \( \sigma \) and \( \tau \). They are related to the Debye screening which is a property of the charged medium. It is a peculiar fact that in the anyon fluid these coefficients appear linked to the magnetic field (see eq. (19)). As a consequence, the zero components of the gauge potentials, \( A_0 \) and \( a_0 \), play a nontrivial role in the magnetic field solution of eqs. (19)-(21).

To solve eqs. (19)-(21) we can conveniently arrange them to obtain,
where \( a = \omega \kappa, \ d = \omega \eta + \alpha \kappa - \gamma - \tau \kappa \chi, \) and \( c = \alpha \eta - \sigma \gamma - \tau \eta \chi. \) Then the solutions for the fields \( E, \) and \( B, \) and for the potentials \( a_0 \) and \( A_0, \) can be obtained from (23), (20), (21) and the definition of \( E \) in terms of \( A_0, \) respectively. Being (23) a higher order differential equation, its solution belongs to a wider class if compared to that corresponding to the original eqs. (19)-(21). Thus, to exclude the redundant solutions we have to require that they satisfy eq. (19) as a supplementary condition. In this way we can reduce the number of independent unknown coefficients to six, which is the number corresponding to the original system (19)-(21).

Solving eq. (23) we obtain,

\[
E(x) = C_1 e^{-x\xi_1} + C_2 e^{x\xi_1} + C_3 e^{-x\xi_2} + C_4 e^{x\xi_2},
\]

(24)

where

\[
\xi_{1,2} = \left[ -d \pm \sqrt{d^2 - 4ac} \right]^{\frac{1}{2}} / \sqrt{2a}
\]

(25)

take real values at any temperature when evaluated with the typical values \( n_e = (1 \sim 5) \times 10^{14} \text{cm}^{-2}, \ m = 2 m_e \) (\( m_e = 2.6 \times 10^{10} \text{cm}^{-1} \) is the electron mass) and \( |N| = 2. \)

With the solution (24), eqs. (20), (21), and the definition \( E = -\partial_x A_0, \) we find,

\[
B(x) = \gamma_1 \left( C_1 e^{-x\xi_1} - C_2 e^{x\xi_1} \right) + \gamma_2 \left( C_3 e^{-x\xi_2} - C_4 e^{x\xi_2} \right) + C_5
\]

(26)

\[
a_0(x) = \chi \gamma_1 \left( C_1 e^{-x\xi_1} - C_2 e^{x\xi_1} \right) + \chi \gamma_2 \left( C_3 e^{-x\xi_2} - C_4 e^{x\xi_2} \right) + C_6
\]

(27)

\[
A_0(x) = \frac{1}{\xi_1} \left( -C_1 e^{-x\xi_1} + C_2 e^{x\xi_1} \right) + \frac{1}{\xi_2} \left( -C_3 e^{-x\xi_2} + C_4 e^{x\xi_2} \right) + C_7
\]

(28)

Corresponding to the magnetic field (26) we have the electromagnetic potential \( A_2 \) given by,

\[
A_2(x) = -\frac{\gamma_1}{\xi_1} \left( C_1 e^{-x\xi_1} - C_2 e^{x\xi_1} \right) - \frac{\gamma_2}{\xi_2} \left( C_3 e^{-x\xi_2} - C_4 e^{x\xi_2} \right) + C_5 x
\]

(29)

The spatial component of the Chern-Simons field is
\[ a_2(x) = \chi \left( C_1 e^{-x\xi_1} + C_2 e^{x\xi_1} + C_3 e^{-x\xi_2} + C_4 e^{x\xi_2} \right) \]  

In the above formulas we introduced the notation,

\[ \gamma_1 = \frac{\xi_1^2 \kappa + \eta}{\xi_1}, \quad \gamma_2 = \frac{\xi_2^2 \kappa + \eta}{\xi_2} \]  

(31)

The extra unknown coefficient is eliminated, as it was explained above, substituting the solutions (24), (26), (27) and (28) into eq. (19) to obtain the relation,

\[ C_5 = \frac{\tau}{\alpha} C_6 + \frac{\sigma \gamma}{\alpha} C_7 \]  

(32)

The last relation establishes a connection between the asymptotic conditions for the zero components of the gauge potentials and the asymptotic condition for the magnetic field.

Let us take into account now the boundary conditions needed to determine the six independent unknown coefficients. Henceforth, we consider two different sample configurations: the half plane and the infinite strip.

**The half plane:**

We will consider the anyon fluid confined to a semi-infinite plane \(-\infty < y < \infty\) with boundary at \(x = 0\). The external magnetic field will be applied from the vacuum \((-\infty < x < 0)\). We restrict our solution to gauge field configurations which are static and uniform in the \(y\)-direction.

The boundary conditions for the magnetic field are \(B(x = 0) = B_0\) (\(B_0\) constant), and \(B(x \to \infty)\) finite. Because no external electric field is applied, the boundary conditions for this field are, \(E(x = 0) = 0\), \(E(x \to \infty)\) finite.

With the above conditions we obtain \(C_2 = C_4 = 0\) and \(C_1 = -C_3\), where \(C_1\) depends on the magnetic field boundary value, \(B_0\), the unknown coefficient \(C_5\) and temperature, through the relation,

\[ C_1 = \frac{B_0 - C_5}{\gamma_1 - \gamma_2} \]  

(33)

To find the remaining independent unknown coefficients \((C_6\) and \(C_7)\) we will consider the system stability condition. That is, starting from the system free energy (14) evaluated
in the field solutions (24), (26)-(30), we define the free energy density \( f = \frac{F}{A} \), where the area of the sample is given by \( A = L_1 L_2 \). Then, we find the values of \( C_6 \) and \( C_7 \) that minimize the free-energy density,

\[
\frac{\delta f}{\delta C_6} = 0, \quad \frac{\delta f}{\delta C_7} = 0 \tag{34}
\]

Considering the leading terms appearing in eq. (34) after taking the half plane limit \((L_1 \to \infty, L_2 \to \infty)\), we arrive to the following equations,

\[
C_5 + \frac{1}{2} \Pi_1 (e C_7 + C_6) = 0 \tag{35}
\]

\[
C_5 + \frac{2 \Pi_0}{\Pi_1} (e C_7 + C_6) = 0 \tag{36}
\]

From eqs. (35) and (36), together with the constraint (32), we obtain \( C_5 = C_6 = C_7 = 0 \).

The magnetic field penetration is then given by,

\[
B(x) = B_1(T) e^{-x \xi_1} - B_2(T) e^{-x \xi_2}, \quad x \geq 0 \tag{37}
\]

where the temperature dependent coefficients, \( B_1(T) \) and \( B_2(T) \), are given by,

\[
B_1(T) = \frac{\gamma_1}{\gamma_1 - \gamma_2} B, \quad B_2(T) = \frac{\gamma_2}{\gamma_1 - \gamma_2} B \tag{38}
\]

Hence, the applied magnetic field within the anyon fluid totally falls down exponentially on two essentially different scales, \( \lambda_1 = 1/\xi_1 \) and \( \lambda_2 = 1/\xi_2 \), which characterize two eigen-modes of propagation inside the fluid. Considering the obtained values for the \( C_i \) coefficients in the solution (24), we also find that the induction of an electric field inside the bulk is intimately linked to the Meissner effect in the anyon fluid. Note that this induced electric field also decays exponentially within the characteristic lengths \( \lambda_1, \lambda_2 \).

**The infinite strip:**

Now we consider the anyon fluid confined to the strip \(-\infty < y < \infty\) with boundaries at \( x = 0 \) and \( x = L_1 = 2L \). The external magnetic field will be applied from the vacuum \((-\infty < x < 0, 2L < x < \infty)\). We again restrict our solution to gauge field configurations which are static and uniform in the \( y \)-direction.
We consider the following symmetric boundary conditions for the magnetic field: \( B (x = 0) = \overline{B} \), \( B (x = 2L) = \overline{B} \) (\( \overline{B} \) constant). The boundary conditions for the electric field are, \( E (x = 0) = 0 \), \( E (x = 2L) = 0 \). With the above boundary conditions and considering that \( L \gg \lambda_1, \lambda_2 \), we obtain that the unknown coefficients \( C_1, C_2, C_3, C_4 \) and \( C_5 \), are related through the following equations,

\[
C_1 = \frac{C_5 - \overline{B}}{\gamma_1 - \gamma_2}
\]

\[
C_2 = -C_1 e^{-2L\xi_1}, \quad C_3 = -C_1, \quad C_4 = C_1 e^{-2L\xi_2}
\]

(39)

To determine the independent unknown coefficients, \( C_6 \) and \( C_7 \), we repeat the same procedure we used in the half plane case. Taking \( L_1 = 2L \) in the free energy (14), we find that in the \( L \gg \lambda_1, \lambda_2 \) limit, the leading terms appearing in eqs. (34), are

\[
C_6 = \mathcal{K}_2 \overline{B}, \quad C_7 = \mathcal{K}_1 C_6
\]

(40)

where

\[
\mathcal{K}_1 = \left( \Pi_0 + \frac{\kappa\gamma}{2\alpha} \Pi_1 \right)^{-1} \left( \Pi_0 + \frac{e\tau}{2\alpha} \Pi_1 \right)
\]

(41)

\[
\mathcal{K}_2 = \left[ \left( \frac{\gamma_1 - \gamma_2}{\Pi} + 2 \right) \left( \frac{\kappa\gamma}{\alpha \mathcal{K}_1} + \frac{\tau}{\alpha} \right) + \frac{e\Pi_1 (\gamma_1 - \gamma_2)}{2\Pi} \left( \frac{e}{\mathcal{K}_1} + 1 \right) \right]^{-1}
\]

(42)

\[
\overline{\Pi} = \left[ e\chi (\gamma_2 - \gamma_1) + e^2 \left( \frac{1}{\xi_2} - \frac{1}{\xi_1} \right) \right] \frac{\Pi_2}{2} + \left[ e^2 (\gamma_2 - \gamma_1) + \frac{2\pi}{N} (\xi_2 - \xi_1) \right] \frac{\Pi_2}{2}
\]

(43)

From (40) and the constraint equation (32), we have that \( C_5 \) is not zero, what implies a partial Meissner effect. In this case the magnetic field inside the anyon fluid is given by

\[
B (x) = \overline{B}_1 (T) \left( e^{-x\xi_1} + e^{-(2L-x)\xi_1} \right) - \overline{B}_2 (T) \left( e^{-x\xi_2} + e^{-(2L-x)\xi_2} \right) + \overline{B} (T)
\]

(44)

where

\[
\overline{B}_1 (T) = \frac{\gamma_1}{\gamma_1 - \gamma_2} \left( \overline{B} - \overline{B} (T) \right), \quad \overline{B}_2 (T) = \frac{\gamma_2}{\gamma_1} \overline{B}_1 (T), \quad \overline{B} (T) = \mathcal{K}_2 \left( \frac{\kappa\gamma}{\alpha} \mathcal{K}_1 + \frac{\tau}{\alpha} \right) \overline{B}
\]

(45)
At \( x = L \) (i.e., in the middle of the sample), considering the large \( L \) limit, we have that
\[
B(x = L) \simeq \overline{B}(T).
\]
This can be interpreted as a partial Meissner effect, taking into account that \( \overline{B}(T) \leq \overline{B} \).

The results of this paper explicitly show something we had previously pointed out \[12\], namely, that in the charged anyon fluid the zero components of the gauge potentials become physical. Their asymptotic behavior, which through the equations of motion affect the magnetic field in the bulk, are fixed by the conditions of minimal free energy density and by the sample boundary conditions. The physical relevance of gauge potentials is not new in Field Theory. Indeed, in statistical gauge theory it is natural to expect that different asymptotic behaviors of the zero components of the gauge fields correspond to different physical situations, since it is known that non-zero constant asymptotic gauge field configurations are not gauge equivalent (under proper, periodic gauge transformations) to the trivial vacuum \[14\]. The system under study here is just an example of such a case.

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REFERENCES

[1] V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett. 59, 2095 (1987); R. B. Laughlin, Phys. Rev. Lett. 60, 2677 (1988).

[2] A. L. Fetter, C. B. Hanna and R. B. Laughlin, Phys. Rev. B39, 9679 (1989).

[3] C.B. Hanna, R.B. Laughlin and A.L. Fetter, Phys. Rev. B40, 8745 (1989), ibid. 43, 309 (1991); G.S. Canright, S.M. Girvin and A. Brass, Phys. Rev. Lett. 63, 2291, 2295 (1989); X.G. Wen and A. Zee, Phys. Rev. B41, 240 (1990); E. Fradkin, Phys. Rev. Lett. 63, 322 (1989); Phys. Rev. B42, 570 (1990); Y. Hosotani and S. Chakravarty, Phys. Rev. B42, 342 (1990); Phys. Rev. D44, 441 (1991); A. L. Fetter and C. B. Hanna, Phys. Rev. B45, 2335 (1992).

[4] Y.-H Chen, F. Wilczek, E. Witten and B. Halpering, Int. J. Mod. Phys. B3, 1001 (1989).

[5] S. Randjbar-Daemi, A. Salam and J. Strathdee, Nucl. Phys. B340, 403 (1990).

[6] J. D. Lykken, J. Sonnenschein and N. Weiss, Phys. Rev. D42, 2161 (1990); Int. J. Mod. Phys. A6, 1335 (1991).

[7] Y. Hosotani, Int. J. Mod. Phys. B7, 2219 (1993).

[8] E. J. Ferrer and V. de la Incera, Int. J. Mod. Phys. B9, 3585 (1995).

[9] P. K. Panigrahi, R. Ray and B Sakita, Phys. Rev. B42, 4036 (1990); J. Kapusta, M. E. Carrington, B. Bayman, D. Seibert and C.S. Song, Phys. Rev. B44, 7519 (1991); Y. Georgelin, M. Knecht, Y. Leblanc, and J. C. Wallet, Mod. Phys. Lett. B5, 211 (1991); Y. Leblanc, and J. C. Wallet, Mod. Phys. Lett. B6, 1623 (1992); I. E. Aronov, E.N. Bogachek, I. V. Krive and S. A. Naftulin, JETP Lett. 56, 283 (1992); Y. Kitazawa and H. Murayama, Phys. Rev. B41, 11101 (1990).

[10] S. S. Mandal, S. Ramaswamy and V. Ravishankar; Mod. Phys. Lett. B8, 561 (1994), Int. J. Mod. Phys. B8, 3095 (1994).
[11] J. E. Hetric, Y. Hosotani and B.-H Lee, Ann. Phys 209, 151 (1991); J. E. Hetric and Y. Hosotani, Phys. Rev. B45, 2981 (1992).

[12] E. J. Ferrer, R. Hurka and V. de la Incera, “High-Temperature Anyon Superconductivity”, Fredonia preprint SUNY-FRE-96-02, hep-th/9602042.

[13] S. Randjbar-Daemi, A. Salam and J. Strathdee, Int. J. Mod. Phys. B5 (1991) 845.

[14] N. Batakis and G. Lazarides, Phys. Rev. D 18, 4710 (1978); D.J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. 53, 43 (1981); A. Actor, Phys. Rev. D 27, 2548 (1983); Ann. Phys 159, 445 (1985).