Inflationary Spectra from Lorentz Violating Dissipative Models

Julian Adamek, David Campo and Jens C. Niemeyer
Institut für Theoretische Physik und Astrophysik,
Universität Würzburg,
97074 Würzburg, Germany

Renaud Parentani
Laboratoire de Physique Théorique,
CNRS UMR 8627,
Université Paris-Sud 11,
91405 Orsay Cedex, France

The sensitivity of inflationary spectra to initial conditions is addressed in the context of a phenomenological model that breaks Lorentz invariance by dissipative effects above some threshold energy $\Lambda$. These effects are obtained dynamically by coupling the fluctuation modes to extra degrees of freedom which are unobservable below $\Lambda$. Because of the strong dissipative effects in the early propagation, only the state of the extra degrees of freedom is relevant for the power spectrum. If this state is the ground state, and if $\Lambda$ is much larger than the Hubble scale $H$, the standard spectrum is recovered. Using analytical and numerical methods, we calculate the modifications for a large class of dissipative models. For all of these, we show that the leading modification (in an expansion in $H/\Lambda$) is linear in the decay rate evaluated at horizon exit, and that high frequency superimposed oscillations are not generated. The modification is negative when the decay rate decreases slower than the cube of $H$, which means that there is a loss of power on the largest scales.

I. INTRODUCTION

Today’s picture of cosmological evolution assumes that all large scale structures we observe today developed from primordial fluctuations on top of a homogeneous and isotropic state of the early universe. These primordial fluctuations naturally arise in the context of inflation. As a consequence of the accelerated expansion, short wavelength vacuum fluctuations are amplified as they exit the horizon scale. After the end of inflation, these fluctuations re-enter the horizon and eventually undergo gravitational collapse. Depending on the total number of e-folds of inflation, the structures we observe today may originate from fluctuations with extremely small initial wavelengths as defined in the homogeneous frame. In fact, unless we fine-tune the number of e-folds, the relevant scales were all well beyond the Planck scale at the onset of inflation [1].

Inflation, therefore, effectively acts as a spacetime microscope, offering the tempting opportunity to probe very high energies by looking for signatures in the primordial perturbation spectrum [2, 3]. Let us denote the scale at which new physics becomes important by $\Lambda$. Depending on the nature of the dominant new physics (which may or may not be of gravitational origin), $\Lambda$ might be the Planck scale, the string scale, or below. Demanding that the theory yields the usual results in the infrared generically gives rise to a suppression of the corrections by some power of $H_p/\Lambda$, where $H_p$ is the Hubble scale at the time when the mode $p$ under consideration left the horizon. There are thus two ways of detecting the new physics in the perturbation spectrum. First, if the correction contains a sharply defined phase factor which is a function of $H_p/\Lambda$, an oscillatory feature extending over a wide range of the power spectrum may be produced [4, 5, 6]. Second, even if oscillations are absent, there is still a possibility that the largest scales we observe today correspond to sufficiently large $H_p/\Lambda$ that the new physics lead to a distinctive suppression or enhancement of the low-l power spectrum at a detectable level. While we do not find effects of the first kind in our model, we conjecture that it generically predicts a large scale modification of the power spectrum.

In the absence of clear predictions from a fundamental theory, several types of phenomenological approaches have been proposed. If Lorentz invariance is retained at all energies, see e.g. [7], there is no possibility to introduce deviations from the standard relativistic propagation and thus very little hope to get any signatures. It is therefore interesting to consider the breaking of Lorentz invariance in the ultraviolet (UV) sector, and in fact this was done in essentially all approaches.

In the simplest of these models, scalar or tensor perturbation modes are prescribed to be created in their adiabatic vacuum at some fixed initial time, where the initial conditions are specified either on a spacelike surface common to all modes [8], or when the physical momentum $P = p/a$ of each mode with comoving momentum $p$ satisfies $P = \Lambda$ [4, 5, 6]. These models predict a spectrum with superimposed oscillations whose amplitude is a power of $H_p/\Lambda$ which depends on the degree of non-adiabaticity of the initial state. The validity of this conclusion was questioned in [9] on the basis that...
the modulation of the corrections artificially follows from the sharp, and thus non-adiabatic, character of imposing the initial state at a given instant.

In another approach [2,3], Lorentz invariance is broken by introducing deviations from the relativistic dispersion relation above a UV scale Λ:

\[ \frac{\Omega^2}{P^2} = 1 \pm \left( \frac{P}{\Lambda} \right)^n + \mathcal{O}\left(\left( \frac{P}{\Lambda} \right)^{n+1}\right), \]

where Ω and P are the proper frequency and the proper momentum as measured in the preferred frame which is assumed to coincide with the cosmological frame [24]. It has been understood that the standard predictions are robust [12], i.e. the modifications of the spectra scale as a power of \( H_p/\Lambda \), provided the initial state is the asymptotic vacuum and the modes evolve adiabatically. Under these conditions, dispersive models generically predict no superimposed oscillations [13].

The alternative possibility that Lorentz invariance is broken by dissipative effects has received much less attention so far. To be realized while maintaining unitarity, one must introduce additional, unobservable degrees of freedom, hereafter called Ψ, which couple to the observable field φ in the UV sector [14]. In this paper we aim to compute the modifications of the spectrum induced by such dissipative effects.

In order to obtain a local equation for the effective propagation of φ after tracing out Ψ, a simple class of models for the propagation of Ψ and its interaction with φ is analyzed in detail. This class is characterized by the “decay rate” Γ of the φ modes in the preferred frame, which – in analogy to eq. (1) – we parameterize by

\[ \Gamma = \left( \frac{P}{\Lambda} \right)^n + \mathcal{O}\left(\left( \frac{P}{\Lambda} \right)^{n+1}\right). \]

If \( H/\Lambda \ll 1 \), we argue in Sec. III that the power spectrum of a wide range of dissipative models can be effectively described by a simplified model characterized by such a decay rate.

The paper is organized as follows. The model is presented in Sec. III. After introducing the settings in Sec. IIIA, we derive the effective equation of motion of the φ mode in Sec. IIIB. In Sec. IIIC the power spectrum is presented in terms of a double integral of a noise kernel governed by Ψ and the retarded Green function of φ. From an approximate expression of the Green function (Sec. IIIA), we derive analytic expressions for the power spectrum in Sec. IIIB and IIIC at zero and high temperatures, respectively. The numerical scheme and the results are presented in Sec. IV.

II. MODEL AND POWER SPECTRA

A. The model

In slow-roll inflation, the background is a flat Friedman-Lemaître-Robertson-Walker universe with the usual Friedmann-Lemaître-Robertson-Walker metric

\[ ds^2 = -dt^2 + a^2(t) dx^2 = a^2(\eta) (-d\eta^2 + d\mathbf{x}^2), \]

and the variation of \( H = \partial_\eta \ln a \) is governed by the slow-roll parameter \( \varepsilon = -\partial_\eta H/H^2 \ll 1 \). Since the background is homogeneous, the fields decompose into Fourier modes labeled by the comoving wave vector \( \mathbf{p} \). Moreover, when the cosmological and preferred frames coincide (see [11]), the action splits into disconnected sectors, \( S = \int d^3 p S(p) \).

We consider a scalar field φ. The action for the rescaled mode \( \Phi_p = a \phi_p \) is

\[ S_\Phi (p) = \int d\eta \, \Phi_p^\dagger [ -\partial_\eta^2 - \omega_p^2 ] \Phi_p, \]

where the conformal frequency is given by

\[ \omega_p^2 (\eta) = p^2 - \frac{\partial^2 f}{f}. \]

For the scalar field φ, one has \( f = a \). The same is true for tensor modes, whereas for density perturbation modes \( f = \sqrt{2} a \). Given the simplicity of these substitutions, we limit our discussion to the scalar field in this work.

The power spectrum of φ is given by the Fourier transform of the two-point correlation function at equal times

\[ \mathcal{P}_p (\eta) \equiv \int \frac{d^3 \mathbf{x}}{(2\pi)^3} e^{ip\mathbf{x}} \langle \phi (\eta, \mathbf{x}) \phi (\eta, 0) \rangle = \frac{1}{2a^2(\eta)} \int d^3 \mathbf{p'} \langle \{ \Phi_p (\eta), \Phi_p' (\eta) \} \rangle, \]

where \( \langle \cdot \rangle \) and \( \{ \cdot, \cdot \} \) denote the quantum expectation value and the anticommutator, respectively.

In the Bunch-Davies vacuum (the adiabatic vacuum for \( P = p/a \to \infty \)), the power spectrum is simply

\[ \mathcal{P}_p (\eta) = \frac{1}{a^2(\eta)} |\Phi^\text{in}_p (\eta)|^2. \]

Here, \( \Phi^\text{in}_p \) denotes the unit Wronskian positive frequency solution of \( [\partial_\eta^2 + \omega_p^2] \Phi_p = 0 \). Evaluated at late time \( (p/a \ll H_p) \), one obtains the standard expression

\[ \mathcal{P}_p^0 = \frac{H_p^2}{2p^3}, \]

where \( H_p \) is the value of H when the \( p \)-mode exits the Hubble scale.

The goal of this paper is to compute (numerically where necessary) the modifications of this power spectrum due to interactions with some additional field Ψ.
inducing dissipative effects as parametrized in eq. (2). To this end, we use the model introduced in [14] whose essential feature is that \( S_\Phi + S_{\text{int}} \), the action of \( \Psi \) plus that governing the \( \Phi - \Psi \) interactions, breaks Lorentz invariance in the UV sector. As a result, similarly to the dispersive models of eq. (1), the propagation of \( \Phi \) remains unaffected in the low-energy sector, whereas it is no longer Lorentz invariant in the UV, even in the vacuum.

There is of course a lot of freedom to choose \( S_\Phi \) and \( S_{\text{int}} \). But as explained in the Introduction, our aim is to obtain simple equations for the effective propagation of \( \Phi \) after having traced over \( \Psi \). From this point of view, \( \Psi \) is introduced only to give rise to dissipative effects while preserving unitarity. Since Gaussian models are the simplest and yet do the job, we work with quadratic actions. For further discussion concerning the general character of these actions, see [14] and Sec. II F below.

Because of Gaussianity, the action still splits into disconnected sectors:

\[
S = \frac{1}{2} \int d^3p \ (S_\Phi (p) + S_\Psi (p) + S_{\text{int}} (p)) .
\] (9)

The action \( S_\Phi \) is given in eq. (4), and we use

\[
S_\Phi (p) = \int dt \int dk \ \Psi_{p,k}^\dagger \left[ -\partial_t^2 - (\pi \Lambda k)^2 \right] \Psi_{p,k} ,
\] (10)

\[
S_{\text{int}} (p) = \int d\eta \ g_p(\eta) \int dk \ \left( \Phi_p \partial_\eta \Psi_{p,k} + \text{h.c.} \right) .
\] (11)

In the action (10), the proper frequency of the \( \Psi_k \) is dimensionalized by \( \Lambda \) which is the only constant (proper) scale. These frequencies are chosen to remain constant as the universe expands because this guarantees that the \( \Psi_k \) are not excited by the cosmological expansion. Moreover, the \( \Psi_k \) carry no spatial momentum. They are thus at rest with respect to the cosmological frame.

In eq. (11), the coupling is bilinear, so that the model is indeed Gaussian. Hence, we can integrate out the \( \Psi_k \). Moreover, the index \( k \) is chosen to be continuous so that the Poincaré recurrence time of the system is infinite in order to effectively obtain dissipation [23]. The time dependent coupling \( g_p \) will be chosen so as to produce the desired dependence on \( P/\Lambda \) in the decay rate \( \Gamma \), see Sec. II B. In order to obtain an effective equation of motion of \( \Phi \) which is local in time, see eq. (13) below, we have used a derivative coupling.

**B. The effective equation of motion of \( \Phi \)**

Since our model is Gaussian, all equations of motion are linear in the field amplitude. Hence, they can be treated as equations for the field operators in the Heisenberg picture.

The equation for \( \Psi_k \) is

\[
\left[ \partial_t^2 + \Omega_k^2 \right] \Psi_{p,k} = - \partial_t \left( g_p \Phi_p \right) ,
\] (12)

where \( \Omega_k \equiv \pi \Lambda |k| \). Its general solution is

\[
\Psi_{p,k}(t) = \Psi_{p,k}^0(t) - \int dt' \ G_k^\Psi(t, t') \partial_t \left( g_p \Phi_p \right) .
\] (13)

Here, \( \Psi_{p,k}^0 \) obeys the homogeneous equation, and the retarded Green function is given as

\[
G_{k}^\Psi(t, t') = \frac{1}{2\pi} \int \frac{d\Omega}{\Omega_k^2 - \Omega^2 - i\varepsilon \Omega} .
\] (14)

Similarly, the equation for \( \Phi \) reads

\[
\left[ \partial_{\eta}^2 + \omega_p^2(\eta) \right] \Phi_p = \int dk \ g_p \partial_\eta \Psi_{p,k} .
\] (15)

Inserting eq. (13) into the r.h.s. yields

\[
\left[ \partial_{\eta}^2 + \omega_p^2(\eta) \right] \Phi_p = \int dk \ g_p \partial_\eta \Psi_{p,k}^0 - \frac{g_p}{\lambda} \partial_\eta \left( g_p \Phi_p \right) ,
\] (16)

where we have used

\[
\int dk \ \partial_t G_k^\Psi(t, t') = - \frac{\delta(t - t')}{\lambda} .
\] (17)

It is this equation which has motivated our choice of the action (10) and (11). Indeed, in general one would have obtained a non-local equation, whereas here, the effective equation of motion of \( \Phi_p \) is simply

\[
\left[ \partial_{\eta}^2 + 2\gamma_p \partial_\eta + \omega_p^2 + \partial_\eta \gamma_p \right] \Phi_p = g_p \int dk \ \partial_\eta \Psi_{p,k}^0 .
\] (18)

The term \( 2\gamma_p \partial_\eta \) gives rise to dissipative effects. They are governed by the decay rate (in conformal time)

\[
\gamma_p(\eta) = \frac{(g_p(\eta))^2}{2\Lambda} .
\] (19)

The time dependence of the coupling \( g_p \) is fixed by the following conditions. We first demand that the scale \( \Lambda \) be the proper energy at which interactions between \( \Phi \) and \( \Psi \) appear, irrespective of the comoving momentum label \( p \). We also impose \( g_p \rightarrow 0 \) for low momenta \( P \ll \Lambda \), so that the \( \Phi_p \) decouple from \( \Psi \) and propagate freely. This is required by particle physics observations which put severe constraints on possible violations of Lorentz invariance, see e.g. [10].

In analogy to eq. (1), in order to cover the general case we classify dissipative effects according to the lowest order of \( P/\Lambda \):

\[
\frac{\gamma_p}{P} = \frac{\Gamma}{P} = \left( \frac{P}{\Lambda} \right)^n \left( 1 + \mathcal{O}\left( \frac{P}{\Lambda} \right) \right) .
\] (20)

The first series coefficient can always be set to unity by a redefinition of \( \Lambda \). From this equation one can already conclude that in cosmology, as the proper momentum
\[ P = p/a \text{ redshifts, the modes go from a strongly dissipative regime } \Gamma/P = O(1) \text{ for } P \gtrsim \Lambda, \text{ to an underdamped regime where } \Gamma/P \ll 1. \] Using eq. (19), we see that the \( n \)-th coupling function should be taken as

\[ g_n = \sqrt{2p\Lambda} \left( \frac{P}{\Lambda} \right)^{n/2} = \sqrt{2p\Lambda} \left( \frac{p}{na\Lambda} \right)^{n/2}. \] (21)

In this we follow the same approach as previously employed in dispersive models. First, we replace the relativistic relation by an effective equation of motion, eq. (18), where the dissipative effects are chosen from the outset, and second, we determine the modifications of the power spectrum induced by this replacement. In this paper we have no ambition to put forward “privileged” (or “inspired”) dispersive/dissipative models that could be derived from first principles. Yet another way to position this approach is to state that we follow a bottom-up rather than a top-down route to new physics.

C. The power spectrum in dissipative settings

The general solution of eq. (18) is

\[ \Phi_p(\eta) = \Phi_p^d(\eta) + \int d\eta' G_p^\Phi(\eta, \eta') \int dk g_k \partial_\eta \Psi_{p,k}^0, \] (22)

where \( \Phi_p^d(\eta) \) and \( G_p^\Phi \) are the homogeneous solution and the retarded Green function, respectively.

The homogeneous solution \( \Phi_p^d \) decays as

\[ \Phi_p^d(\eta) \propto \exp \left( -\int_{\eta_{\text{fin}}}^\eta d\eta' \gamma_p(\eta') \right), \] (23)

where the initial time \( \eta_{\text{fin}} \) fixes the moment when \( \Phi \) and \( \Psi \) start to interact. Since we do not want to fine-tune the number of e-folds, we assume that \( \eta_{\text{fin}} \) is located deep in the ultraviolet regime

\[ \frac{P_{\text{fin}}}{\Lambda} = \frac{p}{a(\eta_{\text{fin}})\Lambda} \gg 1. \] (24)

In this case, the \( \Phi_p^d \) does not contribute to any observable at late time, implying that only the state of the \( \Psi_k \) is relevant \[26\]. Therefore, the power spectrum of super-horizon modes is insensitive to the initial state of \( \Phi_p \).

Let us establish this important property in more detail. If the initial state factorizes, i.e. \( \Psi_{p,k}^0 \) and \( \Phi_p^d \) are not initially correlated, the anti-commutator of \( \Phi \) reads

\[ \langle \{ \Phi_p(\eta), \Phi_p(\eta') \} \rangle = \langle \{ \Phi_p^d(\eta), \Phi_p^d(\eta') \} \rangle + \int_{\eta_{\text{fin}}}^\eta d\eta_1 \int_{\eta_{\text{fin}}}^{\eta'} d\eta_2 G_p^\Phi(\eta, \eta_1)G_p^\Phi(\eta', \eta_2)N_{p,p'}(\eta_1, \eta_2), \] (25)

where we introduced the so-called noise kernel

\[ N_{p,p'}(\eta_1, \eta_2) \equiv \delta^3(p-p')N_p(\eta_1, \eta_2) = g_p(\eta_1)g_p(\eta_2)\int dkdk' \langle \{ \partial_\eta_1 \Psi_{p,k}^0, \partial_\eta_2 \Psi_{p',k'}^0 \} \rangle, \] (26)

whose properties will be specified below. We have simply assumed that the state of \( \Psi_k \) is homogeneous. Because of the decay given in eq. (23), we immediately conclude that the first term in eq. (25) will be exponentially damped. Hence, the anti-commutator, and thus the power spectrum \[4\], are entirely given by the term which is driven by the noise kernel.

In other words, if inflation lasts long enough and if dissipation is sufficiently efficient in the UV (this requirement will be discussed in more detail below), the equation

\[ P_p = \lim_{\eta_{\text{fin}} \to -\infty} \frac{1}{2a^2(\eta_{\text{fin}})} \times \int_{\eta_{\text{fin}}}^{\eta_{\text{fin}}} d\eta_1 \int_{\eta_{\text{fin}}}^{\eta_{\text{fin}}} d\eta_2 G_p^\Phi(\eta, \eta_1)G_p^\Phi(\eta, \eta_2)N_{p,p'}(\eta_1, \eta_2), \] (27)

is exact, and we may furthermore take \( \eta_{\text{fin}} \) to \( -\infty \). This equation replaces the usual expression of eq. (7), governed by the norm of the free mode \( \Phi_p^0 \), and valid both for relativistic and modified dispersion relations.

In view of eq. (27), we see that dissipation affects the structure of the equations much more profoundly than dispersion does. We also understand that the introduction of \( \Psi \) could not have been avoided, since \( \Psi \) determines both the noise kernel \( N_p \) (through its anti-commutator) and the decay rate \( \gamma_p \) (through its retarded Green function, see eqs. (13, 16)). These must be related to each other by a fluctuation-dissipation relation, see [14] for a brief review in the present context. This explains why, unlike dispersion, one cannot treat dissipative effects by simply introducing an imaginary term in the dispersion relation.

The remarkable property of dissipation when it is introduced by coupling to some dynamical degrees of freedom is that \( P_p \) is independent of all their properties if \( H_p \ll \Lambda \) and if they are in their ground state. Moreover, in this case, as we will show, the spectral power \[27\] agrees with the standard value given by eq. (8).

D. The noise kernel

The definition of the model is complete once we specify the state of the \( \Psi_k \). For simplicity, we only consider thermal states. Recall that the proper frequencies of \( \Psi_k \) are time independent so that the proper Hamiltonian of \( \Psi_k \) has stationary eigenstates.

Then, at temperature \( T \), the noise kernel \[28\] is

\[ N_p = g_p(\eta_1)g_p(\eta_2) a(\eta_1) a(\eta_2) \times \frac{2T}{\Lambda} \partial_\eta \coth(\pi T (t_1 - t_2)). \] (28)

This directly follows from the fact that the (free) fields can be decomposed as

\[ \Psi_{p,k}^0(t) = \hat{a}_{p,k} \psi_k(t) + \hat{a}_{p,k}^\dagger \psi_k^\dagger(t), \] (29)
where \( \hat{a}_{p,k} \) satisfy canonical commutation relations and where the isotropic mode functions
\[
\psi_k = \frac{1}{\sqrt{2\Omega_k}} e^{-i\Omega_k t},
\]
have unit Wronskian \( W[\psi_k] = 2\text{Im}(\psi_k \partial_k \psi_k^*) = 1 \). We have also used the fact that in the thermal states one has
\[
2\langle \hat{a}_{p,k}^\dagger \hat{a}_{p,k} \rangle_{T+1} = \coth \left( \frac{\Omega_k}{2T} \right). \tag{31}
\]

At high temperature, as usual, the kernel becomes local
\[
\lim_{T \to \infty} N_p = 4T \gamma_p(\eta_1) a(\eta_1) \delta(\eta_1 - \eta_2), \tag{32}
\]
where we used \( \delta(t_1 - t_2) = \delta(\eta_1 - \eta_2)/a(\eta_1) \) and eq. [18]. At zero temperature (vacuum) instead, one gets
\[
\lim_{T \to 0} N_p = \frac{4}{\pi} \sqrt{\gamma_p(\eta_1) \gamma_p(\eta_2)} a(\eta_1) a(\eta_2) \partial_\eta \left( \text{PV} \frac{\partial}{(t_1 - t_2)} \right), \tag{33}
\]
where the singular behavior should be interpreted as the derivative of the Cauchy principal value.

### E. Retarded Green function

To compute the power spectrum \([27]\), we need the retarded Green function of eq. \([18]\). It satisfies the boundary conditions
\[
G^\Phi_p (\eta' = \eta) = 0, \quad \partial_\eta G^\Phi_p |_{\eta' = \eta} = 1. \tag{34}
\]

Therefore, it can be written as
\[
G^\Phi_p (\eta, \eta') = -\theta (\eta - \eta') \frac{2\text{Im} \left( \varphi_p (\eta) \varphi_p^* (\eta') \right)}{W[\varphi_p]_{\eta' | \eta}}, \tag{35}
\]
where the mode function \( \varphi_p (\eta) \) may be any homogeneous solution of eq. \([18]\) that has a nondegenerate Wronskian \( W[\varphi_p] \).

We introduce the function
\[
\mathcal{I}_p (\eta, \eta_0) \equiv \int_{\eta_0}^\eta \gamma_p (\eta') d\eta', \tag{36}
\]
which gives the amount of dissipation from \( \eta_0 \) to \( \eta \). It will play a crucial role in what follows. Using it, we can get rid of the friction term in eq. \([18]\), by writing
\[
\varphi_p (\eta) = e^{-\mathcal{I}_p (\eta; \eta_0)} \chi_p (\eta). \tag{37}
\]
Indeed, \( \chi_p \) obeys
\[
\left[ \partial_\eta^2 + \omega_p^2 (\eta) - \gamma_p^2 (\eta) \right] \chi_p (\eta) = 0. \tag{38}
\]
Taking into account the time dependence of the Wronskian of \( \varphi_p \), eq. \([35]\) can be rewritten as
\[
G^\Phi_p (\eta, \eta') = -2\theta(\eta - \eta') \text{Im} \left[ \chi_p (\eta) \chi_p^* (\eta') \right] e^{-\mathcal{I}_p (\eta; \eta')}, \tag{39}
\]
when the constant Wronskian of \( \chi_p \) was chosen to unity. It should be noticed that only the decay accumulated from \( \eta \) to \( \eta' \) appears in \( G^\Phi_p \). The fact that \( \eta_0 \) must drop out can be seen from eq. \([34]\). In fact, the second equality replaces the equal time commutation relation in the presence of interactions, see \([14]\) for further details.

Returning to eq. \([27]\), the presence of the two functions \( \mathcal{I}_p \) evaluated both until \( \eta_\text{fin} = 0 \) limits the past history that is relevant to the power spectrum of super-horizon modes. To characterize this relevant domain, we define the time \( \eta^*_p \) by the moment where
\[
\mathcal{I}_p (\eta_\text{fin} = 0, \eta^*_p) = 1. \tag{40}
\]

Times earlier as \( \eta^*_p \) play no significant role in the power spectrum. In other words, \( \mathcal{I}_p \) can be considered as an “optical depth”.

Having established these features, we can now explain why, if \( H \ll \Lambda \), any dissipative model exhibiting dissipation above \( \Lambda \) behaves as if it belonged to the class of models we just considered.

### F. General properties of dissipative models

The models we studied are based on several simplifying assumptions. First, they are Gaussian; second, the frequency of the \( \Psi_k \) is constant; and third, a derivative was introduced in the action \( S_{\text{int}} \) in order to get a local equation for \( \Phi \). Nevertheless, the features we obtained are more general: they will be found in all dissipative models respecting minimal assumptions that we now clarify.

Before listing these conditions, it should be noticed that when dealing with nonlinear interactions, it is no longer convenient to work with the mode operator \( \Phi_p \) as we just did. Instead, it is appropriate to study the effective evolution in terms of the two-point correlation functions of \( \Phi_p \), see \([14]\). In particular, it can be shown that the expectation value of the anti-commutator of \( \Phi \), the l.h.s. of eq. \([25]\), always obeys a linear integro-differential equation with a source \([17]\). This also applies to non-derivative, bilinear couplings, so that the following discussion includes both cases.

Adopting this language, we can transpose the two conditions we used in Section \[II C \]: First, the dissipative effects should be strong enough so as to erase the contribution of the homogeneous solution of this integro-differential equation. In this we recover, in the language of two-point correlation functions, the neglect of the homogeneous solution \( \Phi^\Phi_p \) of eq. \([18]\). Second, the state of the entire system must be spatially homogeneous. When both conditions are met, the expectation value of the anti-commutator of the Fourier mode \( \Phi_p \) is driven by a \( p \)-dependent (c-number) source through the above-mentioned integro-differential equation. This implies that the power spectrum will be given by eq. \([27]\) in any (unitary) dissipative model, Gaussian or not. In other words, the power spectrum is always governed by
Let us begin with the kernel $N_p$, which encodes the properties of the state of the system. In Gaussian models, it is simply given by the expectation value of the anti-commutator of the r.h.s. of eq. (18). In non-Gaussian models, it must be computed order by order in a loop expansion. This calculation might turn out to be difficult, but (in renormalizable theories) $N_p$ is a well defined kernel which is given by the real part of the (renormalized and time-ordered) self-energy of $\Phi_p$ (see for instance Appendix B in [18]). Therefore, when $N_p$ has been computed, it will “drive” the power spectrum as indicated in eq. (18).

Let us briefly discuss the modifications one encounters when the proper frequencies $\Omega_k(t)$ of the $\Psi_k$ depend on time. In this case, their state will be parametrically excited. However, if the variation of $\Omega_k(t)$ is slow enough, this amplification will be exponentially suppressed by virtue of the adiabatic theorem. (A similar situation is expected when dealing with non-Gaussian models.) Then, if the $\Psi_k$ are initially in (or close to) their ground state, $N_p$ will essentially be the noise of the adiabatic vacuum of the $\Psi_k$.

Let us now turn to the effects of dissipation. In a general model, one would lose the local character of eq. (18). However, in all models (Gaussian or not), the retarded Green function of $\Phi$ obeys a linear integro-differential equation of the form

$$[\partial_\eta^2 + \omega_p^2] G^p_\eta(\eta, \eta') + \int^\eta d\eta'' D_p(\eta, \eta_1) G^p_{\eta_1}(\eta_1, \eta') = \delta(\eta - \eta') , \tag{41}$$

where the non-local kernel $D_p$ generalizes what we had in eq. (18) in that, when $D_p = \delta(\eta - \eta_1)2\gamma_p$, one recovers the usual odd term of that equation. The kernel $D_p$ is antisymmetric in the exchange of its arguments and describes dissipation. It is related to the imaginary part of the time-ordered self-energy, as $N_p$ was related to the real part, and is therefore also well-defined and computable, at least perturbatively. Moreover, as for $N_p$, if the state of the $\Psi_k$ evolves adiabatically, $D_p$ is the dissipation kernel in the adiabatic vacuum.

Concerning the power spectrum, we saw in Sec. III.B that in expanding universes with $H/\Lambda \ll 1$, only the evolution in the underdamped regime is relevant. In this low-energy, weakly dissipative regime, the non-local equation (41) can be approximated by a local one (i.e. similar to eq. (18) with an effective damping rate $\gamma_{\text{eff}}$) provided the characteristic comoving time describing the retardation effects of $D_p$ is much smaller than $\omega_p^{-1}$, or equivalently that the corresponding cosmological time is much smaller than $H_p^{-1}$. This approximation is similar to the diffusion approximation in kinetic theory. In this case, we can approximate

$$\int^\eta d\eta'' D_p(\eta, \eta_1) \varphi(\eta_1) \approx 2\gamma_{\text{eff}}(\eta) \partial_\eta \varphi(\eta) , \tag{42}$$

where $\gamma_{\text{eff}}(\eta)$ depends in general on $\eta$ and on the state of the system. (In the case one would consider the same model in Minkowski spacetime, and in its ground state, the above equation is easily obtained in the frequency representation by performing a Taylor expansion in the frequency and truncating at first order. In that case, $\gamma_{\text{eff}}$ would be constant. In an expanding universe, it becomes time dependent through the scale factor $a(t).$)

In conclusion, a general $\Phi-\Psi$ model where (i) $H/\Lambda \ll 1$, (ii) the state of $\Psi_k$ evolves adiabatically, (iii) the characteristic time of $\mathcal{D}_p$ is much smaller than $H_p^{-1}$, so that eq. (42) holds, will give the same power spectrum as that of the corresponding simplified model governed by eqs. (9-11) with the coupling $g$ matching the effective decay rate $\gamma_{\text{eff}}$ through eq. (19).

III. ANALYTICAL TREATMENT

We present some analytical expressions for the power spectrum which will facilitate the interpretation of the numerical results. They are valid for $H \ll \Lambda$ and in the slow-roll regime.

A. More properties of the retarded Green function

As we are interested in the power spectrum of super-horizon modes, we can make a first approximation by factorizing the growing mode of $\chi_p(\eta)$:

$$\chi_p(\eta) \approx \frac{i H_p a}{\sqrt{2p^3}} \eta . \tag{43}$$

By eq. (20), this is a solution of eq. (38) for $|\eta| \ll 1$. Hence, for $\eta_{\text{fin}} \rightarrow 0^-$ we have

$$G^p_\eta(\eta_{\text{fin}} \rightarrow 0^-, \eta) \approx \theta(-\eta) \frac{2H_p a(\eta_{\text{fin}})}{\sqrt{2p^3}} \times \text{Re} \left[ \chi_p(\eta) \right] e^{-\mathcal{I}_p(\eta_{\text{fin}}, \eta)} . \tag{44}$$

$G^p_\eta$ (as function of $\eta$) oscillates with a slowly varying envelope.

We now give an analytic approximation to eq. (44) valid in the slow-roll approximation and in the case of scale separation:

$$\varepsilon, \frac{H}{\Lambda} \ll 1 . \tag{45}$$

Let us first give an approximation for the envelope, given by the exponential in eq. (44). We take $\gamma_p$ to be of the form (20), i.e. $\gamma_p \propto a^{-n}$ for modes below the UV scale $\Lambda$. The term coming from the upper bound is then negligible since $\gamma_p(\eta) \rightarrow 0$ for $\eta \rightarrow 0$. Hence during inflation, the integral is dominated by the lower bound, and we may
thus estimate
\[ I_p(\eta_{	ext{fin}}, \eta) = \int_{a(\eta_{	ext{fin}})}^{a(\eta)} \frac{\gamma_p}{H a^2} \, da \approx \frac{1}{n + 1} \frac{\gamma_p(\eta)}{H(\eta)a(\eta)}, \]
where we have used the slow-roll approximation.

Let us now consider the term \( \Re \{ \chi_p(\eta) \} \). The equation of motion, eq. (35), has an oscillating solution inside the horizon. However, the term \( \gamma_p^2 \) in the effective squared frequency, i.e., the frequency shift due to dissipation, introduces some non-adiabaticity to the evolution of the mode close to the time when it leaves the ultraviolet regime. This implies that the retarded Green function receives non-adiabatic corrections for very early times and may even stop oscillating in the case where there is an overdamped regime (\( \gamma_p^2 > \omega_p^2 \)) in the UV. However, in the case of scale separation this ultraviolet behavior of the mode function occurs only where the envelope is exponentially small by a factor \( e^{-O(A/H)} \). In other words, the dispersive effects induced by dissipation are damped by dissipation itself (see also fig. 5).

Finally, notice that the noise kernel \( \mathcal{A}_p \) is proportional to \( \gamma_p^2 \) and therefore vanishes as \( \eta_1, \eta_2 \to 0^- \). In conclusion, the double integral (27) takes its value in the vicinity of \( \eta_1, \eta_2 \approx \eta_0^* \) defined by eq. (40). If \( H \ll \Lambda \), one finds that \( \chi_p(\eta_0^*) \) is well inside the horizon and at the same time sufficiently below the UV regime, such that it is justified to use a unit Wronskian free oscillator in place of \( \chi_p \) in order to estimate the power spectrum. In other words, as an approximation, we set
\[ \Re \{ \chi_p(\eta \sim \eta_0^*) \} \approx \frac{1}{\sqrt{2p}} \cos(p\eta), \]
and thus
\[ G_p^\phi(\eta_{	ext{fin}} \to 0^-, \eta \sim \eta_0^*) \approx \theta(-\eta) \frac{H a(\eta_{	ext{fin}})}{p^2} \times \cos(p\eta)e^{-\frac{\gamma_p(\eta)}{pH(\eta)a(\eta)}} \]
when we evaluate the double integral. It should be stressed that this is only done to get an analytic estimate, and that no approximation is required for the numerical treatment since we can always solve eq. (35) numerically.

### B. General properties of the power spectrum

Inserting eq. (48) into eq. (27) we have
\[ P_p = P_p^0 \int_{-\infty}^{0} d\eta_1 \int_{-\infty}^{0} d\eta_2 \frac{1}{p} \cos(p\eta_1) \cos(p\eta_2) \times e^{-\frac{\gamma_p(\eta_1)}{pH(\eta_1)a(\eta_1)}} N_p(\eta_1, \eta_2) \]  
\[ \text{The factor } a^{-2}(\eta_{	ext{fin}}) \text{ coming from the rescaling of } \phi \text{ is compensated by the two factors of } a(\eta_{	ext{fin}}) \text{ generated by the growing modes of } \chi_p. \text{ The integrand is now independent of } \eta_{	ext{fin}}. \]

In the vacuum, i.e., at vanishing temperature, it has been shown by general arguments [14, 19] that the power spectrum agrees with the standard prediction [8] for \( \Lambda \gg H \). In other words, the double integral on the right-hand side of eq. (49) evaluates to unity in this double limit. This can be shown by a lengthy calculation which will be omitted here. We only state that a naive analytic estimate can be found [20] for the magnitude of the leading order modification:
\[ \frac{d \ln \delta P_p}{d \ln (H/\Lambda)} = n + O \left( \frac{H^n}{\Lambda^n} \right), \]
with \( \delta P_p \equiv P_p - P_p^0 \).

### C. High temperature limit

In keeping with the general character of our model, we do not specify a physical motivation for the case of finite temperature, where \( \Psi \) acts as a “heat bath”. However, it serves to illustrate the mechanism how the state of the adiabatic modes is dynamically determined by the state of the \( \Psi \). In particular, if the latter are in a thermal state, \( \Phi \) thermalizes through the interaction. Furthermore, we establish that the thermal excitations of \( \Phi_p \) are effectively populated at \( \eta = \eta_0^* \) defined by eq. (40).

Let us consider the high temperature limit of eq. (49), i.e., we insert the limiting expression (32) for the noise kernel and set the \( \delta \)-function against one of the integrations. We then have
\[ P_p(T \to \infty) = P_p^0 \times 4T \int_{-\infty}^{0} d\eta \frac{\gamma_p(\eta_1)}{p} a(\eta_1) \cos(p\eta_1) e^{-\frac{\gamma_p(\eta_1)}{pH(\eta_1)a(\eta_1)}}. \]
Assuming that \( \Lambda \gg H \), we may replace the scaled cosine by its average value \( 1/2 \). Since \( \gamma_p \propto a^{-n} \) in the relevant domain, the remaining integral is now essentially a representation of the Gamma function. It evaluates to
\[ P_p(T \to \infty) = P_p^0 \times \frac{T}{\Lambda} \left( \frac{\Lambda}{H} \frac{2}{n+1} \right)^{\frac{n+1}{2}} \Gamma \left( \frac{n}{n+1} \right). \]

If we compare this result to the high temperature limit of a thermal power spectrum
\[ P_p(T) = P_p^0 \times \coth \left( \frac{\Omega^*}{2T} \right), \]
where \( \Omega^* \) denotes the proper frequency at the instant when the occupation numbers are fixed, we find that
\[ \frac{\Omega^*}{\Lambda} = \frac{2}{\Gamma(n+1)} \left( \frac{n+1}{2 \Lambda} \right)^{\frac{n+1}{2}}. \]
One verifies that $\Omega^*$ so defined coincides with the proper frequency at $\eta_p^*$:

$$\Omega^* \simeq \frac{\omega_p(\eta_p^*)}{a(\eta_p^*)}. \quad (55)$$

Note that it is below the scale $\Lambda$ (see eq. (54)), in agreement with the results of Sec. [II.A].

In conclusion, the state of $\phi$ is inherited from that of $\Psi$ at the time $\eta_p^*$.

### IV. NUMERICAL ANALYSIS

We present a numerical scheme that solves the double time integral of eq. (27) by means of Monte Carlo integration.

#### A. The numerical scheme

The procedure includes the following steps:

1. We impose the inflationary background. Specifically, de Sitter space and power law inflation are considered.
2. We specify the “relative” decay rate $\gamma_p/p$ in terms of a function of $P/\Lambda$, see eq. (20).
3. The equation of motion eq. (38) for $\chi_p(\eta)$ is solved numerically.
4. The integral $I_p(\eta, \eta')$ of eq. (36) governing the amount of dissipation is computed numerically.
5. The retarded Green function is constructed from these numerical solutions, cf. eq. (39).

#### B. Handling the singular behavior of $N$

The noise kernel (cf. eq. (28)) is singular for equal times and should be interpreted as a Cauchy principal value. Unfortunately, numerical integrators are generically incapable of calculating principal values; they usually fail to achieve convergence within a finite number of integrand evaluations. However in our case, the integrand can be rewritten in a regular form by a convenient change of variables.

We first note that the integrand of the double integral in eq. (27) is symmetric under the exchange $\eta_1 \leftrightarrow \eta_2$. Thus, if we make a change of variables to $\zeta \equiv \eta_1 + \eta_2$, $\xi \equiv \eta_1 - \eta_2$, the integrand will be symmetric around $\xi = 0$, where it has a double pole. Following the general techniques in the calculus of generalized functions [21], the integrand may be regularized by multiplication with $-\xi^2 \partial \xi \xi^{-1} = 1$ and performing an integration by parts in $\xi$. The derivative now acts on a regular (the $\xi^2$ cancels the double pole) and symmetric function. It thus gives an odd function in $\xi$ such that the remaining singular factor in $\xi^{-1}$ may be lifted. This way we obtain a regular integrand that is well-behaved within the entire domain of integration. In brief, we have

$$\mathcal{P}_p(\eta_{\text{fin}}) = \frac{1}{2a^2(\eta_{\text{fin}})} \int \frac{d\zeta d\xi}{2} G_p^\phi(\eta_{\text{fin}}, \frac{\zeta + \xi}{2}) G_p^\phi(\eta_{\text{fin}}, \frac{\zeta - \xi}{2}) N_p\left(\frac{\zeta + \xi}{2}, \frac{\zeta - \xi}{2}\right) (-\xi^2 \partial \xi \xi^{-1}). \quad (56)$$

#### C. The UV behavior of the coupling function

For numerical integration, the range of $\eta$ has to be truncated somewhere in the remote past. In order to guarantee a safe truncation, we impose that the integrand drops off exponentially. Then the cutoff can be chosen in such a way that the truncation error is negligible w.r.t. the numerical value of the integral.

The exponential behavior of the integrand is achieved by a suitable choice of the decay rate $\gamma_p$. One might be tempted to say that any positive $\gamma_p$ gives rise to an exponential behavior, simply given by $I_p$. However, the effective frequency of the mode functions depends on the damping rate as well, cf. eq. (38). If $\gamma_p$ is not bounded from above, we may run into some pathologies due to an unbounded over-damping in the UV. The reason is that dissipation is less effective in an overdamped situation. To see this, let us consider a classical oscillator with constant frequency $\omega$ and damping rate $\gamma$ in the overdamped regime, i.e. $\gamma > \omega$. It has two decaying modes with the
decay rates $\gamma \pm \sqrt{\gamma^2 - \omega^2}$. When $\gamma^2 \gg \omega^2$, the slowly decaying mode is thus $\propto \exp[-\frac{\omega^2}{2\eta} (\eta - \eta_0)]$.

Returning to our model, assuming $\gamma_p/p = (p/a\Lambda)^n$ without higher order terms, the growing WKB solution of eq. (25) for $\gamma_p \gg \omega_p$ would lead to an overall asymptotic behavior $\propto \exp(-\int_{\eta_n}^{\eta} \frac{a^2}{2\gamma_p(y')} dy')$. As the integrand drops off like $a^n$ in the remote past, the integral is generally finite in the limit $\eta_n \to -\infty$. In this case nothing guarantees that one can disregard the damped initial correlator in eq. (25) and that the power spectrum does not depend on the initial state of $\Phi$ at $\eta_n$.

However, when $\Lambda \gg H$, the residual contribution of the decaying term in eq. (25) is strongly suppressed ($O[\exp(-\Lambda/H)]$) because of the dissipation between crossing and horizon exit, where the mode is in the underdamped regime. But if we want to neglect the contribution of this decaying term for all values of the ratio $\Lambda/H$, we cannot work with $\gamma_p/p = (P/\Lambda)^n$ (nor with a polynomial of finite order) since it is not bounded from above. We will instead work with a decay rate that saturates in the UV. We choose

$$\frac{\gamma_p}{p} = \kappa \tanh \left( \left( \frac{p}{a\Lambda} \right)^n \kappa^{-1} \right).$$

as a simple realization of this property. The new parameter $\kappa$ was introduced such that it only appears in the subleading terms and that the decay rate saturates in the UV when it reaches the value $\gamma_p/p \simeq \kappa$.

D. Numerical results

1. Dependence on $H/\Lambda$ in the vacuum ($T = 0$)

Let us now consider the zero temperature limit, i.e. $\Psi$ is in its vacuum state. On physical grounds, we expect that this will be the relevant case if a fundamental theory gives rise to dissipative effects in the UV sector during inflation.

The modified power spectrum is computed for $\Lambda$ ranging from $\Lambda \gg H$ down to $\Lambda \ll H$ and is compared to the standard prediction (55), both in de Sitter space (figs. 11 and 2), and in power law inflation (fig. 3).

Let us first discuss the power spectrum in de Sitter space as a function of $\Lambda$, fig. 1 coming from the high values. Note that in de Sitter, $\Lambda/H$ is time independent, so the power spectrum is scale invariant and the value of $\Lambda$ only affects the normalization of the spectrum. As expected from the analytical results, for $\Lambda \gg H$ the standard power is recovered, independently of $\kappa$. In this we corroborate the robustness of the power spectrum when the initial state is the adiabatic vacuum 12 (see also below in this section). As $\Lambda$ approaches $H$, the power spectrum is modified in a non-universal way which depends on all model parameters. The flattening of the curves for $\Lambda \ll H$ has to be attributed to the fact that our particular choice of $\gamma_p$ (cf. eq. 57) saturates to a constant in the UV and thus becomes independent of $\Lambda$ inside the horizon.

Having established the robustness in the regime $H/\Lambda \ll 1$, the signature of dissipation is contained in the first deviation with respect to the standard result. As anticipated in eq. 50, in the limit $H/\Lambda \to 0$, the deviation behaves as

$$\frac{\delta P_p}{P_p} \sim \delta_n \times \left( \frac{H_p}{\Lambda} \right)^n,$$

where the constant $\delta_n$ depends only on $n$. This has been verified for values of $1 \leq n \leq 2.5$, as defined in eq. 57, and we conjecture that it is valid for any power. For all these values, $\delta_n$ is negative which means that the spectral power is reduced w.r.t. the standard value. We were not able to probe higher values of $n$ because of the numerical difficulties to follow the sharp decrease of the modifications. However, preliminary results indicate that (as for dispersive models 13) $\delta_n$ changes sign for $n = 3$, which means that for $n > 3$, dissipation leads to an increase of the power. At present we have no explanation for this unexpected result.

In fig. 2 the difference $\delta P_p = P_p - P_p^0$ is plotted for $n = 2$ and various values of $\kappa$ using a logarithmic scale. It shows that the power spectrum becomes insensitive to the next-to-leading order terms in the expansion of $\gamma_p$, see eq. (20).
Let us now turn to power law inflation figs. 3 and 4. Fig. 4 indicates how the slow-roll parameter $\varepsilon$ impinges on the features of the modification $\delta P_p$. Surprisingly we find no dependence of $\delta_n$ on the slow-roll parameter, or if present, it must be extremely mild. This implies that the power spectrum from power law inflation, as a function of $p$ (fig. 3), is the power spectrum in de Sitter, as a function of $H_p/\Lambda$, where $H_p$ is the value of $H$ when the $p$ mode exits the horizon, up to $O(\gamma_p^2/p^2)$.

Note that the modification of the power spectrum due to dissipative effects can be reinterpreted in terms of a (scale dependent) spectral index, which is given by the slope in fig. 5. To lowest order in $\varepsilon$ and $H_p/\Lambda$, using eq. 58, we find

$$\frac{d\ln(\rho^3 P_p)}{d\ln p} = -2\varepsilon - \varepsilon n \delta_n \times \left(\frac{H_p}{\Lambda}\right)^n + \ldots$$

The first term accounts for the slow-roll evolution of the standard power spectrum, whereas the second term is due to dissipative effects. It is proportional to $\varepsilon$ and suppressed by $(H_p/\Lambda)^n \ll 1$.

For $n = 2$, the negative sign of $\delta_n$ gives rise to a downward modification of the spectral power. Moreover, the effect is enhanced at large scales due to the $p$-dependence of $H_p/\Lambda$. The effect tends to increase the spectral index (i.e. increase the slope at any given point of the curve), and the running of the spectral index should indicate a concave spectrum (the curve is bent downwards towards large scales).

2. Dissipation and non-adiabaticity

We now discuss the interplay between dissipation and non-adiabaticity in more detail. Following what was done with dispersive models 13, we plot the function

$$\sigma = \frac{\partial_{\omega_2} \omega_{\text{eff}}}{\omega_{\text{eff}}^2},$$

as a function of $x = -p\eta$ in fig. 6, where

$$\omega_{\text{eff}}^2 = \omega_p^2 - \gamma_p^2,$$

is the effective (conformal) frequency of the modes $\chi_p$, see eq. 60. We see two regions of non-adiabaticity, the usual one at the horizon $x \lesssim 1$, and a new UV feature, inside the horizon, which is due to the dispersive effects induced by the dissipation and governed by $\gamma^2$. We also plot the exponential factor $\exp[-I(\eta, \gamma)]$, see eq. 60, appearing in the retarded Green function. We see that for sufficiently large $\Lambda$, the UV feature in $\sigma(x)$ lies in the tail of the exponential. The non-adiabatic effects caused by dispersion are then completely masked by dissipation.

As $\Lambda$ decreases, the UV feature in $\sigma(x)$ begins to overlap with the exponential. For $\Lambda \simeq H$, it is well inside the region where the exponential is still $O(1)$. In that case one expects significant modifications of the power spectrum.
to the standard power\(^{P}\) and with \(\Lambda\)\(^{0}\) and the standard power spectrum for power law inflation sensitive to the value of the slow-roll parameter \(\varepsilon\).

\[\eta_{\text{initial}}\) is enforced to be that thermal state at \(\eta = \eta_{\text{initial}}\), see eq. (53).

\[\frac{\text{Deviation}}{\text{Power spectrum}} = \frac{-D_{p,\text{H}}^{\text{H0}}}{P_{\text{H0}}}\]

\[\sigma(\kappa) = 2 \quad \kappa = 0.5, 2\]

\[e^{-I}: \quad \sigma(\kappa) = 2 \quad \kappa = 0.5, 2\]

\[x \equiv -\eta \Phi\]

\[\text{Degree of non-adiabaticity} \sigma\]

\[\text{Upper panel: } \Lambda/H = 10^{3}, \text{i.e. the scales are clearly separated. Lower panel: } \Lambda/H = 10, \text{i.e. weak scale separation. The background is de Sitter, and } n = 2. \text{ (}\sigma\text{ was enhanced by a factor of } \Lambda/H \text{ for a clearer presentation.}\]

## V. CONCLUSION

Let us sum up our main results. First, in spite of the strong dissipative effects encountered in the early mode propagation, the predictions for the power spectrum converge to the standard ones in the case of scale separation, \(H_{p} \ll \Lambda\), and if the environment field \(\Psi\) is in the ground state. The power spectrum is hence a robust observable with respect to high-energy dissipative effects under these conditions. In this regard, dissipative models do not differ from dispersive ones [13]. In the region of parameter space we succeeded to explore, i.e. the power \(n \leq 2.5\), the deviations are negative, giving rise to a suppression of the power spectrum with respect to the standard one.

Second, the leading deviation of the power spectrum induced by dissipation is linear in the relative decay rate \(\gamma_{p}/p\) evaluated at horizon crossing, see eq. (53). The signatures of dissipation therefore do not oscillate. In this regard as well, dissipative models behave like dispersive ones [13].

Third, we have verified that the deviations in power law inflation essentially behave as those evaluated in de Sitter space, with \(H\) replaced by \(H_{p}\) evaluated at horizon exit. As a direct consequence, any observable effects of high-energy dissipation will be more pronounced at the largest accessible scales, corresponding to the largest \(H_{p}/\Lambda\).

Fourth, in the case of \(H_{p} \ll \Lambda\), we showed how dissipation, via the function \(I_{p}\), sets the time when the “initial” state of the \(\Phi_{p}\) mode is effectively set. It is given by eq. (10) where \(P^{*} = p/a(\eta_{\text{initial}}^{*})\) is at an intermediate scale between \(H_{p}\) and \(\Lambda\). In addition, the state of \(\Phi_{p}\) coincides with that of the degrees of freedom causing the dissipation. Thus, the properties of the state of \(\Psi_{i}\) pass on to that of \(\Phi_{p}\) at \(\eta_{\text{initial}}^{*}\).

Fifth, in Sec. 1 we gave (sufficient) conditions under which a model exhibiting dissipation above the scale \(\Lambda\) can be well approximated by a Gaussian model, in the sense that both models predict the same power spectrum.

Our analysis could be extended in two directions. First, we considered a Gaussian model. If we relax this hypothesis, since dissipation grows with the coupling, it will be interesting to investigate the combined effects of...
dissipation and non-Gaussianities. Second, we calculated $\mathcal{P}_p^\phi$, the power spectrum of a test field propagating on an inflationary background. It is a challenge to construct a realistic model of inflation displaying dissipation in the UV sector.

We can nevertheless make the following observations. At the linearized level, the spectrum of scalar metric perturbations $\zeta$ is related to that of our scalar field $\phi$ by $\mathcal{P}_\zeta = \frac{\kappa}{\pi} \mathcal{P}_\phi$ [22]. This implies that the relative modification $\delta\mathcal{P}_\zeta/\mathcal{P}_\zeta$ due to some dissipative effects is the same as the one of our scalar field. Therefore, if $\Psi$ couples identically to scalar and tensor perturbations, the $S/T$ ratio should not be changed at first order. Then the absence of features in the power spectra in the regime $H/\Lambda \ll 1$ would prevent us from disentangling the new physics from a simple shift of the inflaton potential, adding yet another ambiguity to the program of the inflaton potential reconstruction.

Acknowledgments

The work of DC and JCN was supported by the Alfried Krupp Prize for Young University Teachers of the Alfried Krupp von Bohlen und Halbach Foundation. The numerical power spectra were computed using Monte Carlo integrators from the CUBA package [23]. Special thanks go to Alex Schenkel for interesting discussions and many helpful comments. We also acknowledge contributions from Tim Koslowski, Dennis Simon and many other colleagues.

[1] T. Jacobson, Prog. Theor. Phys. Suppl. **136**, 1-17 (1999).
[2] J. Martin and R.H. Brandenberger, Phys. Rev. D **63**, 123501 (2001).
[3] J.C. Niemeyer, Phys. Rev. D **63**, 123502 (2001).
[4] U.H. Danielsson, Phys. Rev. D **66**, 023511 (2002).
[5] R. Easther, B.R. Greene, W.H. Kinney, and G. Shiu, Phys. Rev. D **64**, 103502 (2001); Phys. Rev. D **66**, 023518 (2002).
[6] J.C. Niemeyer, R. Parentani, and D. Campo, Phys. Rev. D **66**, 083510 (2002).
[7] N. Kaloper, M. Kleban, A. Lawrence, S. Shenker, and L. Susskind, JHEP **0211**, 037 (2002).
[8] K. Schalm, G. Shiu, J.P. van der Schaar, AIP Conf. Proc. **743**, 362 (2005).
[9] D. Campo, J.C. Niemeyer, and R. Parentani, Phys. Rev. D **76**, 023513 (2007).
[10] T. Jacobson, Phys. Rev. D **53**, 7082 (1996).
[11] S. Kanno and J. Soda, Phys. Rev. D **74**, 063505 (2006).
[12] J.C. Niemeyer and R. Parentani, Phys. Rev. D **64**, 101301(R) (2001).
[13] J. Macher and R. Parentani, Phys. Rev. D **78**, 043522 (2008).
[14] R. Parentani, Proc. of Science (QG-Ph) 031 (2007).
[15] M.V. Libanov, V.A. Rubakov, Phys. Rev. D **72**, 123503 (2005).
[16] T. Jacobson, S. Liberati and D. Mattingly, Annals Phys. **321**, 150 (2006).
[17] J. Berges, AIP Conf. Proc. **739**, 3 (2004).
[18] D. Campo and R. Parentani, arXiv:0805.0421v1 [hep-th] (2008).
[19] R. Parentani, Class. Quant. Grav. **25**, 154015 (2008).
[20] J. Adamek, diploma thesis, Universität Würzburg (2008).
[21] D.S. Jones, *Generalised Functions*, McGraw-Hill Publishing Company Ltd., London (1966).
[22] V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger, Phys. Rept. **215**, 203 (1992).
[23] T. Hahn, Comp. Phys. Comm. **168**, 78 (2005).
[24] The description of dispersion or dissipation in empty spacetime requires to introduce a preferred frame. In condensed matter systems or in a heat bath, this frame is introduced by the medium itself. In covariant settings, the preferred frame is defined by a vector field [10]. If this field is described dynamically, it aligns itself with the cosmological frame exponentially fast in inflationary backgrounds [11].
[25] In this paper, we refrain from giving any specific interpretation to the $\Psi_k$ fields since our aim is to determine
the consequences of dissipation as opposed to justify dissipation from first principles. Nevertheless, if one wishes, the parameter $k$ may be viewed as a momentum in a flat fourth spacelike direction. One can then draw a connection to the five-dimensional models described in [15], where the equivalent of the $\Psi_k$ propagate in the bulk.

[26] The situation is a bit more subtle when there is an overdamped regime ($\gamma_p^2 > \omega_p^2$) in the UV, since the decay of the homogeneous solution may slow down considerably. This issue is addressed in detail in Sec.

[14]