Mathematical Modelling of a Laboratory Drying Process: Case Study for Experimental Design Project

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Abstract — The drying process was chosen as a case study for the experimental design project. This design project is related to a heat and mass transfer laboratory for undergraduate students. The drying process was performed at different operating variables such as sample drying temperatures, air velocities, and sample particle size. Many runs were performed for each operating variable and the work was twice repeated for consistency. Each experimental run was continued until no further mass change was observed. The mass of material, wet and dry bulb temperature and air velocity were collected as a function of time. Many mathematical formulas were applied. The kinetics and the model of the drying process were estimated. The heat and mass transfer coefficients were calculated and related to the air temperature, moisture content, velocities, and the size of the sample. It was found that the drying process of wet sand followed the proposed model by Wang and Singh. Many other drying relations were studied as shown in the entire paper. This is a non-ending design project work.

Keywords — drying process, heat and mass transfer, mathematical modelling.

I. INTRODUCTION

The experimental design project is defined as a non-ending work for undergraduate students by varying the operating conditions such as flow rate, temperature, pressure, pump speed, etc. to be more familiar with the process. The design of experiment (DOE) is defined as a systematic technique to study the factors that affect the process and the output of the process [1], [2]. The experimental design project is an application of DOE. The experimental design project includes software applications for calculations, designing, application in the engineering field, comparing with literature work, impact on the environment, etc. This work is essential for undergraduate engineering students to enhance their experience using available laboratory equipment as an integration for today’s industrial reality. The students need to apply all the theoretical knowledge gained in the classroom. The students investigate open-ended process engineering design projects and engage in creative problem-solving and decision-making activities. Many case studies of design projects related to process engineering labs were written [3]-[11]. The main objective of this study is to investigate the effect of drying temperature, air velocity, and particle size of wet material on the drying rate constant, heat and mass transfer coefficient, kinetics, and evaluate the ability of some mathematical models to quantify the drying behaviour on the temperatures ranges usually encountered in tray dryers.

The drying is a simultaneous heat and mass transfer process. Drying is the removal of a relatively small amount of water from solids, liquids, or gases. Water may be removed from solids mechanically by presses and centrifuges or thermally by vaporization. Thermal vaporization is emphasized in this work. Hot air is utilized to dry wet silica sand. It is a complicated process that involves simultaneous heat and mass transfer, accompanied by physicochemical transformations. When a solid dries, two simultaneous processes occur: heat is transferred to evaporate liquid, and mass is transferred as a liquid or vapor within the solid and as a vapor from the surface. These factors govern the rates of these processes to determine the drying rate. The surface of the sand is exposed to a drying medium such as hot dry air passing over the surface. Immediately after contact between the wet solid and the drying medium, the solid temperatures adjust until it reaches a steady state. The solid temperature and the rate of drying may increase or decrease to reach the steady state condition. At steady state, the temperature of the wet solid surface is the wet bulb temperature of the drying medium. The temperature within the drying solid also tends to equal the wet bulb temperature of the gas, but the lag in movement of mass and heat result in some deviation. Once the stock temperatures reach the wet bulb temperature of the gas, they are quite stable, and the drying rate also remains constant. This is the constant rate drying period that ends when the solid reaches the critical moisture content. Beyond the point, the surface temperature rises, and the drying rate falls rapidly. There may be another change in the drying rate. If this occurs, it is referred to as the second falling rate period. The falling rate periods can take a far longer time than the constant rate period even though the moisture removal may be less. The drying rate approaches zero at some equilibrium moisture content which is the lowest moisture content obtainable with the solid under the drying conditions used. The drying rate is calculated by measuring the weight of a drying material as a function of time. The mathematical modelling of a dryer is based on differential energy and mass equations, empirical and complementary equations. The mass transfer coefficient is a basic parameter in the project and analysis of the drying process. The numerical solution of the differential equations depends on knowledge of the mass transfer coefficient. The drying process can be described by the following equations [12]:

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\[
\frac{dW}{dt} = -AK_H \left( H_s - H_g \right)
\]  
(1)

To measure the saturated humidity:

\[
\log P^* = 8.107 - \frac{175.0286}{235 + T_{wet,29}}
\]

(2)

Saturated humidity \( H_s = \frac{M_W \text{water} \cdot P^*}{M_W \text{air} \cdot (p-p^*)} \)  
(3)

The gas humidity can be found from Psychometric chart. The heat transfer rate from the moving air to the solid can be calculated from the following equation:

\[
Q = h A (T_g - T_s)
\]

(4)

The heat transfer required to vaporize the liquid from the solid phase to the air stream is given by:

\[
q = KA \left( H_s - H \right) L
\]

(5)

At equilibrium, the rates of heat transfer given by equations (4) and (5) must be equal. Hence,

\[
H_s - H = \frac{h(T_g - T_s)}{K L}
\]

(6)

It is necessary to remove free moisture from the surface and the moisture from the interior of the solid. The change in moisture content with time for a material can be determined, and a drying curve may be drawn. The form of the drying rate curve varies with the structure and type of materials. The three characteristic drying phases are usually apparent in each drying curve. Two of the three phases of drying occur in every drying curve. After a preliminary period during which the temperature of the material adjusts itself to the drying conditions, each curve has a horizontal segment, which pertains to the first major drying period. This period, which may be very short if the initial moisture content of the solid is less than a certain minimum value, is called the constant-rate period. It is characterized by a rate of drying which is independent of the moisture content. During this period, a continuous film of water that acts as if the solid was not present covers the solid. If the solid is non-porous, the water removed during this period is mainly superficial water found on the surface of the solid. In a porous solid, most of the water removed in the constant-rate period is supplied from the interior of the solid.

To calculate the rate of drying during the constant rate period, either the mass transfer equation or the heat transfer equation may be used:

\[
\frac{dw}{dt} = \frac{hA(T - T_s)}{L} = KA \left( H_s - H \right)
\]

(7)

where \( T \) = air temperature (°C).

The heat transfer coefficient depends on the air velocity and the direction of flow of the air. It has been determined from the following equation:

\[
h = CG^{0.8}
\]

(8)

For air flow parallel to the plane surface, the value of \( C \) is 14.5 for the heat transfer coefficient expressed in W/m²·K [13]. It is important that the air velocity is sufficient. At a high velocity, the condition of the gas stream is not affected appreciably by the evaporation of liquid and the heat transfer by convection is large compared with that by radiation and conduction from the surroundings. Other equations are used in this work.

II. EQUIPMENT SETUP AND EXPERIMENTAL PROCEDURE

Tray dryer was used to proceed with this work. The equipment is controlled by a power outlet which when turned on; the air fan speed and heater power can be controlled to the desired values (Fig. 1). Hot-air stream passes over the surface of the wet sand, providing some of the heat of evaporation and acting as a medium by which water vapor is carried away from the solid. Aspirating psychometric device was used to measure the wet and dry bulb temperatures before and after the location of the dried sample. Digital anemometer was used to measure the speed and the temperature of the air at the outlet of the dryer. Drying rate experiments can be simply done by measuring the weight change during drying. The material to be dried is placed on the tray. The tray is suspended from a balance and exposed to air flow in the drying tunnel. Silica sand was dried in an oven for about three hours and then inserted to the sieve shaker to distribute the sand according to its particle size. Two particle sizes were used in this work 710 µm, and 1 mm. The silica sand samples were collected and stored in a dry container. A specific dry sample was weighed and placed in the tray of known dimensions. Cover the sand sample with water and reweigh to find the mass of wet sand. Different drying temperatures and air flow rates were performed. The wet and dry bulb temperatures, air velocity, and mass of the sand were measured every ten minutes until no further mass change will observe. Many runs were performed to achieve the objectives of this work.

III. RESULTS AND DISCUSSION

The mass of wet silica sand sample was reduced with drying time as shown in Fig. 2. The drying rate which represents the change in mass of silica soil with time (\( \Delta W/\Delta t \)) can be plotted with drying time on a histogram curve (Fig. 3). These graphs showed a reduction of the mass of sand with time due to the evaporation of water from the wet sand.
The mass of silica sand during the drying process followed the following polynomial model which best fits the drying experimental data.

\[
\text{Mass of sand} = 9E^{-0.05} t^3 - 0.0078 t^2 - 1.5048t + 699.84
\]  
(9)

This will cause a decrease in the value of moisture content with drying time. The moisture content in the sample is diffused or evaporated to the drying medium existing in the dryer over time and the sample continues drying.

The increase in temperature would increase the weight loss and the drying rate of the sample due to more heat in the heat transfer process. The moisture content \( (M) \) at any time of drying \( (\%) \), was calculated according to the following equation.

\[
M = \frac{w_i - w_d}{w_i} \times 100\%
\]  
(10)

Considering the sand particle is a spherical particle, then the drying rate constant \( (K_H) \) is:

\[
(K_H) = 10.66045 \text{ Kg dry air} /\text{m}^2.\text{min}
\]

The saturated and gas humidity were calculated and plotted with the drying rate of silica sand as shown in Fig. 5 and 6.

The difference between the saturated humidity and gas humidity is plotted versus the drying rate. A straight line was obtained with a slope of 538.46 as shown in Fig. 7. The slope represents the drying rate constant multiplied by the area as shown in (1).

\[
K_H \cdot A = 538.46
\]  
(11)

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Given the complexity of the occurring phenomena during the product drying, mathematical models in the form of empirical or semi-empirical relations to describe the drying were proposed. The equations of these models express the evolution of the reduced (ratio) moisture content $M_R$ according to time. These formulas contain constants which are adjusted to make the theoretical results match with the experimental drying curves. Consequently, they are valid only in the field of experimental investigation for which they were established.

Table I shows some mathematical equations reported in the literature and are frequently used to fit empirical correlations for describing the drying behaviour of natural products. Many models were available. The following models of $M_R$ represent the amount of moisture remaining in the solid samples reported to the initial moisture content.

The moisture ratio can be defined from the following equation:

$$MR = \frac{M-M_e}{M_0-M_e}$$  \hspace{1cm} (13)

where, $M_o$ is initial moisture content (dry basis), $M_e$ is equilibrium moisture content and $M$ is the moisture content at a given time on a dry basis. The equilibrium moisture contents ($M_e$) were determined by drying the sand until no further change in weight was observed for the silica sand samples in each treatment and drying conditions [23\textendash}24].

The reduction of moisture ratio with drying time was used to analyse the experimental drying data as shown in Fig. 10. The reduced moisture content relationship followed the polynomial model of order 2 which best fits the drying experimental data:

$$M_{R}=2E-05t^2-0.0119t +1.0069$$  \hspace{1cm} (14)

| Table I: Mathematical Models for Drying Curves |
|-----------------------------------------------|
| Model          | Model names | Model          | Equation number | Reference |
|----------------|-------------|----------------|-----------------|-----------|
| 1              | Lewis       | $M_R = \exp (-Kt)$ | (13)           | [14]      |
| 2              | Page        | $M_R = \exp (-k t^2)$ | (14)           | [15]      |
| 3              | Henderson and Pabis | $M_R = a \exp (-k t^2)$ | (15)           | [16]      |
| 4              | Logarithmic | $M_R = a \exp (-k t) + b$ | (16)           | [17]      |
| 5              | Wang and Singh | $M_R = 1 + b + a t^2$ | (17)           | [18]      |
| 6              | Midilli et al. | $M_R = a \exp (-k t^2) + b t$ | (18)           | [19]      |
| 7              | Modified Page | $M_R = \exp (-k t^2)$ | (19)           | [20]      |
| 8              | Two term    | $M_R = a \exp (-k t^2) + b \exp (-k_2 t)$ | (20)           | [21]      |
| 9              | Offset modified Page | $M_R = \exp (-k t^2) + (a-1) t^2$ | (21)           | [22]      |
This model followed Wang and Singh model as shown in Table I.

\[
dM/dt = 100\exp(-\frac{0.4}{MR})
\]

Fig. 10. Moisture ratio vs. drying time.

Fig. 11 reveals a characteristic behaviour of silica sand during drying (i.e., structural deformation due to drying, hardening of the particles external layer in contact with drying air, etc.). The drying kinetic of silica sand that does not exhibit structural changes such as shrinkage or hardening, reveals three stages: (i) constant drying period, (ii) falling rate period, and (iii) final falling rate [25], [26]. While for high water content agro-food particles, more than four stages are detected mostly during the falling rate period. The curve followed polynomial model order 6 which best fits the drying experimental data as shown in Fig. 11. Fluctuations in the relative humidity of the drying air due to the variations in the air flow velocity, temperature, and humid conditions can be observed.

Fick’s equation of diffusion was used to calculate the effective diffusivity, considering a constant moisture diffusivity, infinite slab geometry, and uniform initial moisture distribution [27], [28]:

\[
MR = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp\left(-\frac{(2n+1)^2 \pi^2}{4L^2} D_{eff} t\right)
\]  

(15)

The effective diffusion coefficient was calculated by

\[
\text{slope} = \frac{D_{eff} \pi^2}{4L^2}
\]

(18)

The plot of Ln MR as a function of drying time is shown in Fig. 12.

From Fig. 12, the slope of the straight line is equal to \(\pi^2 \cdot D_{eff} / 4L^2\). Solving this equation,

\[
D_{eff} = 0.0003666 \text{ m}^2/\text{s}
\]

(19)

The variation of the effective diffusivity coefficient \(D_{eff}\) for each air temperature and drying air velocity is shown in Fig. 13 and 14.

\[
D_{eff} \text{ is the effective moisture diffusivity (m}^2/\text{s)} \text{ and L is the size of the sand particle, n = 1, 2, 3, the number of terms taken into consideration, t is the time of drying in seconds. For long drying periods, by neglecting the higher order terms in the previous equation, it can be simplified to only the first term of the series and written in logarithmic [29].}
\]

\[
M_R = \frac{8}{\pi^2} \exp\left(-\frac{D_{eff} n^2 t}{4L^2}\right)
\]

(16)

This equation assumed that the movement of moisture is one dimensional, without volume change; a constant diffusivity, a uniform distribution of moisture, and an outer negligible resistance [30]. The equation (25) can be written as follows:

\[
\ln \left(\frac{M-M_e}{M_0-M_e}\right) = \ln \left(\frac{8}{\pi^2}\right) - \frac{\pi^2 D_{eff}}{4L^2} t
\]

(17)

The variation of effective diffusivity with surface temperature at constant air velocity of 1.02 m/s.

Fig. 10. Moisture ratio vs. drying time.

Fig. 11. Drying rate as a function or normalized moisture content.

Fig. 12. Ln MR with drying time of silica sand.

Fig. 13. The variation of effective diffusivity with surface temperature at constant air velocity of 1.02 m/s.

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Fig. 14. The variation of effective diffusivity with air velocity at constant temperature of 52 °C.

The effect of temperature on the diffusivity of moisture is considerable. Indeed, \( D_{\text{eff}} \) values increase with increasing temperature. Similarly, for a given temperature, \( D_{\text{eff}} \) increases with increasing the air velocity as shown in the following equations.

\[
D_{\text{eff}} = 1 \times 10^{-05} \text{ (surface temperature) } - 0.0003 \quad (20)
\]

\[
D_{\text{eff}} = 0.0135 \times \text{(air velocity)}^2 + 0.0667 \times \text{(air velocity)} - 0.0279 \quad (21)
\]

As the temperature increases, the rate of evaporation also increases. This is related to an increase in kinetic energy of the surface molecule which removes them from a liquid phase to vapor phase. This is in agreement with many authors who reported the influence of temperature on the diffusivity of moisture [32]-[35]. The effective moisture diffusivity is one of the important parameters to be considered for analysis and optimization of dehydration process [36], [37]. Correct simulation of different drying processes required a precise value of moisture diffusivity. The moisture diffusivity varies due to numerous factors such as type of dried material properties and process parameters. The effect of coefficients of heat and mass transfer and the thermo-physical characteristics on the drying kinetics has been investigated [38]. The mechanism of heat and mass transfer during drying process of an unsaturated porous medium placed in horizontal channel which submitted to a forced convection laminar flow had been investigated [39]. The studied includes the space time variability effect of heat and mass transfer coefficients on space-time evolution of temperature, water saturation and gas-pressure in the porous medium, in the solid-fluid interface and in the channel during the drying process. Fourier number is a ratio of the current time to time to reach steady state, it represents the dimensionless time in temperature curves, used in explicit finite difference stability criterion. Equation (17) can be evaluated numerically for Fourier number,

\[
F_0 = D_{\text{eff}} \times \frac{t}{4L^2} \quad (22)
\]

This equation can be rewritten as follows:

\[
MR = \left( \frac{8}{\pi^2} \right) \exp \left( -\pi^2 \frac{F_0}{MR} \right) \quad (23)
\]

Simplifying equation (22):

\[
F_0 = -0.101 \ln(MR) - 0.0213 \quad (24)
\]

The effective moisture diffusivity can also be calculated using the following equation:

\[
D_{\text{eff}} = \frac{F_0}{(2\pi^2)} \quad (25)
\]

Fourier number can be calculated from the previous equation. The experimental data can be used to plot the analogous Fourier number (\( F_0 \)) as a function of time to describe the drying kinetics of silica sand as shown in Fig. 15.

Fig. 15. Fouriers number with drying rate of silica sand.

It is clearly shows that a linear evolution is obtained indicating that there is no deviation from Fickian behaviour. The activation energy can be modified from Arrhenius equation. From drying kinetic constant rate \( K \) and the ration of output power for tray dryer to sand sample weight \( m/P \) instead of air temperature. The following equation can be used [40].

\[
K = K_0 \exp \left( -\frac{E_a m}{P} \right) \quad (26)
\]

The correlation between effective diffusion coefficient and (\( m/P \)) is used for calculation of the activation energy.

\[
D_{\text{eff}} = Do \times \exp \left( -\frac{E_a m}{P} \right) \quad (27)
\]

where \( k \) is the drying rate constant obtained by using the best model (1/min), \( K_0 \) is the pre-exponential constant (1/min), \( E_a \) is the activation energy (W/g), \( m \) is the mass of raw sample (g) and \( D_0 \) is the pre-exponential factor (m²/s). The effective moisture diffusivity can be related with temperature by a simple Arrhenius equation as follows [41]-[44].

\[
D_{\text{eff}} = D_0 \exp \left( -\frac{E_a}{RT} \right) \quad (28)
\]

\( D_{\text{eff}} \) = effective moisture diffusivity, m/\( \text{h}^2 \);

\( Do = \) constant equivalent to the diffusivity at infinitely high temperature, m²/h;

\( E_a \) = activation energy, kJ/kg mol;
with the Reynolds and Prandtl numbers.

Equation (27) can be presented in linearized form as:

\[ \ln(Def) = \ln(Do) - \left( \frac{Ea}{R} \right) \left( \frac{1}{T} \right) \]  

(29)

The activation energy, \( Ea \) was determined by plotting \( \ln(Def) \) vs \( 1/T \) as shown in Fig. 16.

From the slope of the straight line, the activation energy is equal to 29.44 KJ/mole. The activation energy is the relative ease of moisture migration within the product and a lower value indicates high moisture diffusivity [45]. This means that 29.44 KJ/mole energy is required for the moisture diffusion and subsequent evaporation from the surface of the silica sand. The air temperature influenced the mass transfer process which is controlled by diffusion. It is clear that the mass transfer coefficient is related to the moisture content of the solid material and the air temperature.

Heat transfer coefficient can be calculated by measuring the Nusselt number (Nu). The empirical correlation is used to relate the Nusselt number with the Reynolds and Prandtl numbers [46].

\[ \text{Nu} = C(\text{Re} \times \text{Pr})^n \]

where \( C \) and \( n \) are constants.

Mass transfer within solid products was studied by many authors. Most of the work was based on diffusivity model. Reynolds number (Re), Prandtl number (Pr), Schmidt number (Sc) can be calculated from the following equations.

\[ \text{Re} = \frac{\rho v L}{\mu} \]  

(30)

\[ \text{Pr} = \frac{\mu C_p}{K} \]  

(31)

\[ \text{Sc} = \frac{\mu}{\rho D} \]  

(32)

The calculated values of drying rate constant, Reynolds number, Prandtl number, and Schmidt number are shown in Table II.

Since the value of \( G \) is within the range of 0.68–8.1 and the temperature of dryer is in the range of 45–150 °C, the following equation can be used to determine the heat transfer coefficient [47].

\[ h_H = 0.0304 G^0.8 \]  

(33)

\[ h_H = 0.0253 \text{ KW/m}^2 \times \text{K} \]  

(34)

The Lewis relationship can be used to determine the mass transfer coefficient \( Km \) as follows:

\[ \frac{h_H}{K_m} = Cp \]  

(35)

An increasing in air velocity would increase the rate of weight loss due to increasing in drying rate which accelerating the evaporation process [48], [49]. The mass transfer coefficient \( Km \) is equal to 0.025424 Kg/m². S. The heat transfer coefficient is change with air velocity for two sample particle sizes of 710 µm and 1 mm as shown in Fig. 17.

The mass transfer coefficient is change with air velocity for two silica sand particle size of 710 µm as shown in Fig. 18.

As the surface area increases (fine particle size) the rate of evaporation also increases, this means that the increasing in the surface area will increase the chance of more molecules to evaporate, i.e., the drying rate in the fine particles is slightly higher than coarse particles.
It is also found that the drying rate depends strongly on the moisture content at the heating surface. The Nusselt number (Nu) is the ratio of convective to conductive heat transfer across the boundary. For laminar flow across a spherical particle is [50]:

\[ \text{Nu}_{\text{lam}} = 0.664 \text{Re}^{1/2} \text{Pr}^{1/3} \]  

(36)

For a turbulent flow across a sphere is:

\[ \text{Nu}_{\text{turb}} = \frac{0.037 \text{Re}^{0.8} \text{Pr}^{-0.1}}{1 + 2.443 \text{Re}^{-0.1} (\text{Pr}^{-2/3} - 1)} \]  

(37)

The Nusselt number (Nu) is calculated and plotted versus the Reynolds number (Re) for silica sand particle size of 710 μm. The relationship is shown in Fig. 19. This represents the heat transfer versus gas velocity which is shown in terms of dimensionless numbers.

IV. CONCLUSION

The objective of this case study is to understanding the drying kinetics of silica sand with respect to mathematical modelling of the drying behaviour and moisture migration or transport mechanism. This is important for the description of drying behaviour, design of effective drying equipment, optimization of the drying process, and the description of heat penetration mechanism during the drying process. This is a non-ending experimental design project for undergraduate students in the heat and mass transfer laboratory. Many variables were studied; the air temperature, air velocity, and particle size of a sample. The moisture content of the sample was reduced with time. The temperature and air velocity were affecting the drying rate of the sample and also affecting the moisture content. The heat and mass transfer must be studied and understood. The heat and mass transfer coefficients were calculated and the effect of these coefficients on air velocities and particle size distribution were studied. The mathematical model for the drying process of soil samples on the basis of heat and mass transfer was studied. Different drying models were applied, and the model of Wang and Singh was applicable for the drying process of the wet sand sample. This work can be extended for other operating variables as well as the factorial design measurements to predict the influence of changing the air velocities, temperature, and particle size on the drying process of the sample. It is a good practice for the design of experiment DOE and the design and operation of dryers.

**NOMENCLATURE**

- \( \frac{dw}{dt} \): Rate of water evaporation (gram / sec)
- \( A \): Specific surface area of the dried solids (m²/kg dry solid)
- \( H_s \): Saturation humidity in the gas phase in (kg water/ kg dry air)
- \( H_g \): Water content in the gas phase in (kg water/ kg dry air)
- \( K_f \): Drying rate constant in (kg/ m².s)
- \( W \): Change in water content of the solid in (kg water/ kg dry solid)
- \( t \): Drying time in (s)
- \( M_w \): Molecular weight of water
- \( T_{w} \): Wet bulb temperature
- \( T_{dry} \): Dry bulb temperature
- \( h \): Heat transfer coefficient (w / m². °C)
- \( A \): Surface area for heat and mass transfer (m²)
- \( T_s \): Temperature of the air stream (°C)
- \( T_L \): Temperature of the liquid in the solid phase, surface temperature (°C)
- \( K_m \): Mass transfer coefficient (grams / m². sec. unit humidity difference)
- \( L \): Latent heat of evaporation of water (joule/g)
- \( G \): Mass flow rate of air in Kg/s.m²
- \( M \): Moisture content
- \( W_i \): Initial weight
- \( W_f \): Final weight
- \( M_R \): Moisture ratio
- \( M_i \): Initial moisture content (Kg of moisture/kg dry solid)
- \( M_e \): Equilibrium moisture content (Kg of moisture/kg dry solid)
- \( D_{eff} \): Effective diffusion coefficient (m²/s)
- \( P_s \): Saturated pressure
- \( C_p \): Specific heat (KJ/Kg.K)
- \( S_r \): Particle size of sand, mm
- \( Nu \): Nusselt number
- \( Re \): Reynolds number
- \( Pr \): Brandt number

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