IT’S DUST: SOLVING THE MYSTERIES OF THE INTRINSIC SCATTER AND HOST-GALAXY DEPENDENCE OF STANDARDIZED TYPE I A SUPERNOVA BRIGHTNESSES

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Submitted to The Astrophysical Journal

ABSTRACT

The use of Type Ia Supernovae (SNe Ia) as cosmological tools has motivated significant effort to: understand what drives the intrinsic scatter of SN Ia distance modulus residuals after standardization, characterize the distribution of SN Ia colors, and explain why properties of the host galaxies of the SNe correlate with SN Ia distance modulus residuals. We use a compiled sample of ~1450 spectroscopically confirmed, photometric light-curve of SN Ia and propose a solution to these three problems simultaneously that also explains an empirical 1σ detection of the dependence of Hubble residual scatter on SN Ia color. We introduce a physical model of color where intrinsic SN Ia colors with a relatively weak correlation with luminosity are combined with extrinsic dust-like colors (E(B−V)) with a wide range of extinction parameter values (RV). This model captures the observed trends of Hubble residual scatter and indicates that the dominant component of SN Ia intrinsic scatter is from variation in RV. We also find that the recovered E(B−V) and RV distributions differ based on host-galaxy stellar mass and this explains the observed correlation (γ) between mass and Hubble residuals seen in past analyses as well as an observed 4.5σ dependence of γ on SN Ia color. This finding removes any need to prescribe different intrinsic luminosities to different progenitor systems. Finally we measure biases in the equation-of-state of dark energy (w) up to |Δw| = 0.04 by replacing previous models of SN color with our dust-based model; this bias is larger than any systematic uncertainty in previous SN Ia cosmological analyses.

Subject headings: supernovae, cosmology

1. INTRODUCTION

Studies in the last decade of research in cosmology with Type Ia supernovae (SNe Ia) have forewarned that the measurements of the equation-of-state of dark energy (w) will soon hit a systematic floor. Yet, such measurements (B14; Betoule et al. 2014, S18; Scolnic et al. 2018, B19b: Brout et al. 2019, Jones et al. 2019) continually reach better levels of both statistical and systematic precision. This is due to the improvement of systematic uncertainties in survey and camera design, but also due to the possibility afforded from significantly larger samples to understand systematics in the analysis. In the most recent analyses (S18, B19b), it has been found that systematic uncertainties in understanding the intrinsic scatter of standardized SN Ia brightmesses is of a similar level or larger than uncertainties due to external, photometric calibration. As calibration uncertainties have been dominant in past systematic error budgets, this moment marks a transition from a need to understand external issues independent of the supernovae to a need to also better understand SN Ia physics.

With current cosmological analyses of SNe Ia requiring mmag-level control of systematics, uncertainty over how to understand the intrinsic scatter of standardized SN Ia brightnesses, which is on the 0.1 mag level, is problematic. Practically, intrinsic scatter is measured as the excess scatter of SN Ia distance residuals to a best-fit cosmology after accounting for measurement noise. A holistic understanding of SN Ia intrinsic scatter and its underlying characterization has remained elusive, but its size has been found to depend on a wide variety of measurement components: redshift (e.g., B14), wavelength range of the photometric observations (e.g., Mandel et al. 2011), host-galaxy properties (e.g., Uddin et al. 2017), and spectroscopic features (e.g., Fakhouri et al. 2015). Furthermore, Scolnic & Kessler (2016) showed that the relative amounts of chromatic versus achronmic components the intrinsic scatter models were directly linked to the intrinsic SN Ia color population and reddening law; however, this study was unable to discriminate between different models.

After the discovery of the accelerating universe (Riess et al. 1998; Perlmutter et al. 1999), there were two commonly used light-curve fitters: MLCS2k2 (Jha et al. 2007) and SALT2 (Guy et al. 2010), that diverged in their approach to color and intrinsic scatter. MLCS2k2 attempted to model color based on dust with the possibility that each SN could have its own extinction law, and assumed that a large amount of the intrinsic scatter was in color. The SALT2 model, on the other hand, was agnostic to any physical properties of the SN color and its relation to the intrinsic scatter. Cosmological analyses have since favored the SALT2 model due to its native spectral-model to account for k-corrections and updated calibration, and it has been used in most recent cosmology analyses including the Joint Light-Curve Analysis (JLA: B14), Pantheon (S18), the Dark Energy Survey 3 Year Sample (DES3YR: Brout et al. 2019, B19a), and the Foundation + Pan-STARRS1 photometric analysis (Jones et al. 2019). However, despite the fact that MLCS2k2 has not been used in recent cosmological analyses, papers such as Scolnic et al. (2014b, 2018); Mandel et al. (2017) have attempted to bridge the gap between SALT2 and MLCS2k2 methods by modeling a connection between the underlying population of color, dust, and reddening laws.

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Still, SN Ia analyses that attempt to model dust using a cosmological sample have typically made the simplistic assumption that there is a single total-to-selective extinction parameter, $R_V$, that can be fixed at a single number. $R_V$ is defined as $A_V/(A_B-A_V)$, where $A_V$ is the extinction in the V ($\lambda \sim 5500$ Å) band, and $A_B$ is the extinction in the blue ($\lambda \sim 4400$ Å) band. As $R_V$ varies for different dust grain sizes and composition, and galaxies have different dust properties, it is well known that different galaxies and different regions within galaxies exhibit a wide range of $R_V$ values. In fact, while the Milky Way galaxy has an $R_V$ on average $\sim 3.1$, it has a distribution of at least $R_V = 0.2$ (Schlafly et al. 2016). Additionally, different parts of the LMC and SMC have been found to have $R_V$ values with a range of $R_V \sim 2-5$ (Gao et al. 2013; Yanchulova Merica-Jones et al. 2017). Furthermore, Salim et al. (2018) study the dust attenuation curves of 230,000 individual galaxies in the local universe, using GALEX, SDSS, and WISE photometry calibrated on the Herschel ATLAS, and they find quiescent galaxies, which are typically high-mass, have a mean $R_V = 2.61$ and star-forming galaxies, which are lower-mass on average, have a mean $R_V = 3.15$.

$R_V$ has also been measured through large SN sample statistics and detailed studies of individual SNe, though often with varying sets of assumptions. Cikota et al. (2016) compiled 13 various studies of SN Ia samples from the literature which determined a range of $R_V$ values from $\sim 1$ to $\sim 3.5$. Cikota et al. (2016) itself determined $R_V$ from nearby SNe and for 21 SNe Ia observed in Sab-Sbp galaxies and 34 SNe in Sbc-Scp they find $R_V = 2.71 \pm 0.58$ and $R_V = 1.70 \pm 0.38$ respectively. While so many past analyses have recovered $R_V \leq 2$ for studies of individual SNe (e.g. Wang et al. 2005; Krisciunas et al. 2006), these were often SNe Ia with high $E(B-V)$, and it was postulated $R_V$ may decrease with $E(B-V)$. However, Nobili & Goobar (2008) found from a sample of modestly reddened ($E(B-V) < 0.25$ mag) SNe Ia, a small value of $R_V \sim 1$ and more recently, Amanullah et al. (2015) analyzed high-quality UV-NIR spectra of 6 SNe and found that SNe with high reddening indicated $R_V$‘s ranging from $\sim 1.4$ to $\sim 2.8$ and SNe with low amounts of reddening also indicated $R_V$‘s of $\sim 1.4$ and $\sim 2.8$. Importantly, Amanullah et al. (2015) stressed that the observed diversity in $R_V$ is not accounted for in analyses that measure the cosmological expansion of the universe.

Since the low $R_V$ values ($< 2$) are not found in studies of the Milky Way, this has motivated various SN Ia studies to ascribe the dust to circumstellar dust around the progenitor at the time of the explosion (Wang 2005; Goobar 2008). However, an alternative interpretation could be that the low $R_V$ values are caused by dust in the interstellar medium (Phillips et al. 2013). This understanding has been supported by (Bulla et al. 2018a), which constrained the location of the dust that caused the reddening in the SN Ia spectra to be, for the majority of the SNe that they observed, on scales of the interstellar medium, rather than circumstellar surroundings. This could be due to cloud-cloud collisions induced by the SN radiation pressure (Hoang 2017) which produce small dust grains (Gao et al. 2015; Nozawa 2016).

While accounting for dust remains a challenge for current and future photometric cosmology analyses, this pursuit has often been done in parallel to the search for correlations between measured supernova luminosity after standardization and host-galaxy properties. Global and local properties of SN Ia host galaxies such as stellar mass, star formation rate (SFR), stellar population age, and metallicity have all been shown to correlate with the distance modulus residuals after standardization (Hicken et al. 2009a; Sullivan et al. 2010; Lampeitl et al. 2010; Childress et al. 2013; Rose et al. 2019). This correlation is often parameterized as a step function in host-galaxy stellar mass and is now commonplace in SN Ia cosmology analyses despite the lack of understanding of its physical underpinning or convincing evidence for exactly which host-galaxy property is most influential on SN Ia luminosity (e.g. Jones et al. 2018a; Scolnic et al. 2020). To explain this correlation, recent studies have suggested a potential relation between the luminosity of the SN and the progenitor, which can be related to the age of the galaxy, or the local environment of the galaxy (Childress et al. 2013; Rigault et al. 2013; Roman et al. 2018). However, as the aforementioned galaxy properties are all directly linked to dust properties, it is likely that the lack of dust modeling in SN Ia cosmology is related to the correlations between host galaxy properties and standardized luminosities.

In this analysis, we show that there are clear limitations in SN Ia standardization techniques with a single color luminosity correlation, but that these limitations can be addressed by inclusion of dust modeling with variation in $R_V$. This paper relies heavily on the work of Mandel et al. (2017), which follows closer to the framework of MLCS2k2 and developed a hierarchical Bayesian model to build a more rich understanding of SN color. Mandel et al. (2017) only used low-redshift data, did not account for selection effects, and assumed a fixed $R_V$ extinction parameter; here we use a much larger dataset across a wide redshift range and use survey simulations to forward-model what is done in Mandel et al. (2011), though with additional features to explain discrepancies seen between simulations and data.

In Section 2, we present the data compilation, light-curve fitting and discrepancies between the data and a simple understanding of SN color. In Section 3, we discuss how to differentiate between past models of SN color and our new dust-based color model. In Section 4, we show how the new model can explain the commonly seen correlation between distance modulus residuals and host-galaxy properties. In Section 5, we assess the impact on recovered cosmological parameters, and in Sections 6 & 7, we discuss further studies and conclusions.

2. DATA SAMPLE, DISTANCE MODULI, AND DESCRIPTION OF SN IA COLORS

2.1. Data

We use a compilation of publicly available, spectroscopically classified, photometric light curves of SNe Ia that have been used in past cosmological analyses and that have been calibrated to the SuperCal system (Scolnic et al. 2015). The low-redshift (low-z) SNe used here are made up of, in part, by those used in B19b which are from CSP (Stritzinger et al. 2010) and CfA3-4 (Hicken et al. 2009b,a, 2012). At low-z, we also include the recently released 180 low-z SNe from the Foundation sample (Foley et al. 2018). At high-z, we include SNe from PS1 (Rest et al. 2014; Scolnic et al. 2018), SDSS (Sako et al. 2011) and SNLS (B14) as was done in the Pantheon analysis. Finally, we include data from the recently released DES 3-year sample (Brout et al. 2019), hereafter DES3YR. The redshift distribution of SNe Ia used in this work can be found in the top panel of Figure 1.

This analysis relies largely on the host galaxy mass esti-
magnitudes provided by past analyses. We adopt the same masses released in the Pantheon sample, and references therein, for SDSS, PS1, SNLS, CSPDR2, and CfA. For DES3YR masses, we use the updated masses provided by Smith et al. (2020); Wiseman et al. (2020). For the Foundation sample, we utilize masses derived in Jones et al. (2018b).

2.2. Light-curve fits and Distance Modulus Determination

We fit the SNe with the SALT2 model as presented in Guy et al. (2010) and updated in B14. In SALT2, the SN Ia flux at phase ($p$) and wavelength ($\lambda$) is given as

$$F(SN, p, \lambda) = x_0 \times [M_0(p, \lambda) + x_1 M_1(p, \lambda) + \ldots] \times \exp[c CL(\lambda)],$$  

(1)

where the parameter $x_0$ describes the overall amplitude of the light-curve, $x_1$ describes the observed light-curve stretch, and $c$ describes the observed color of each SN. $M_0$, $M_1$, $CL$ are global model parameters of all SNe Ia: $M_0$ represents the average spectral sequence (SED); $M_1$ is the SED variability; and $CL$ is the average color correction law. The light-curve fits assume Fitzpatrick (1999) for Milky Way reddening. The mean observed $c$ and $x_1$ for the data, binned over redshift, is shown in the bottom panels of Figure 1.

Distances are inferred following the Tripp estimator (Tripp 1998). The distance modulus ($\mu$) to each candidate SN Ia is obtained by:

$$\mu = m_B + \alpha_{SALT2\times 1} - \beta_{SALT2} - M$$  

(2)

where $m_B$ is peak-brightness based off of the light-curve amplitude ($\log_{10}(x_0)$) and where $M$ is the absolute magnitude of a SN Ia with $x_1 = c = 0$. $\alpha_{SALT2}$ and $\beta_{SALT2}$ are the correlation coefficients that standardize the SNe Ia and are determined following Marriner et al. (2011), in a similar process to what is done in B14. In recent analyses with the Tripp estimator (S18, B19b), there is often additional additive terms $\delta_{\text{bias}}$, the correction for distance biases calculated from survey simulations and $\delta_{\gamma}$, the correction due to the host-galaxy mass correlation; these additional corrections are not applied because new treatments for both of these terms are introduced in following sections.

Distance uncertainties are computed from the uncertainties in the light-curve fit parameters and their covariance ($C$):

$$\sigma^2 = C_{m_B, m_B} + \alpha_{SALT2}^2 C_{x_1, x_1} + 2\alpha_{SALT2} C_{m_B, x_1} - 2\beta_{SALT2} C_{m_B, c} - 2\alpha_{SALT2} \beta_{SALT2} C_{x_1, c} + \sigma^2_{\text{Vpec}} + \sigma^2_{\gamma} + \sigma^2_{\text{Lens}} + \sigma^2_{\text{Int}},$$  

(3)

where $\sigma_{\text{Vpec}}$ is the distance modulus uncertainty due to peculiar velocities (250 km/s), $\sigma_\gamma$ is the distance modulus uncertainty due to the measured redshift uncertainty, $\sigma_{\text{Lens}}$ is the additional uncertainty from weak gravitational lensing (0.055$\sigma$), and $\sigma_{\text{Int}}$ is determined such that the reduced $\chi^2$ relative to a best fit cosmology is 1.

Typical selection cuts are applied on the observed data sample as was done in B19b: we require fitted color uncertainty < 0.05, fitted stretch uncertainty < 1, fitted light-curve peak date uncertainty < 2, light-curve fit probability (from SNANA) > 0.01, and Chauvenaut’s criterion is applied to distance modulus residuals, relative to the best fit cosmological model, at 3.5$\sigma$. In total, after selection cuts, there are 1445 SNe in this sample.

2.3. Key Pillars of the Complexity of the Colors of SNe Ia

The complexity of the SN Ia color model is readily apparent after a simple Tripp standardization. Here, three critical features are presented in the observed dataset that must be explained by models of SN Ia color and intrinsic scatter.

- The distribution of observed SN Ia colors is shown in the top of Fig. 2. There is a clear asymmetry, with an excess of red SNe in comparison with blue SNe, that is inconsistent with a symmetric Gaussian distribution.

- The relation between the root-mean-square (RMS) scatter of distance modulus residuals (with mean residual removed in each bin) as a function of SN Ia color is shown in the middle panel of Fig. 2. There is a 11$\sigma$ dependence relative to a flat line, where the redder SNe Ia ($c > 0.1$) exhibit nearly twice as much scatter ($\sim 0.18$ mag) as the bluest SNe Ia ($c < -0.1$), which exhibit $\sim 0.1$ mag scatter. This effect remains if any single survey is removed from the sample.

- The relation between Hubble residual binned distance biases and SN Ia color is shown in the bottom panel of Fig. 2. There is a $\sim 7.8\sigma$ dependence relative to a straight line. As shown in Fig 2, the recovered $\beta_{\text{SALT2}}$ of the data is $3.05 \pm 0.06$. 

![Figure 1](image-url)
The relation of increased scatter as a function of color has not been analyzed in a previous analysis. This paper is motivated by quantifying these observed features and building a model that can address all of them simultaneously.

2.4. Using Survey Simulations to Evaluate SN Ia Color and Intrinsic Scatter Models

For every model presented in this paper, 100 realizations of dataset-sized simulations are run. SNANA (Kessler et al. 2009) is used to simulate realistic samples of SNe Ia. These simulations account for observing cadence, observing conditions, noise properties, selection effects, cosmological effects, and astrophysical effects. A general description of the simulation methodology can be found in Kessler et al. (2019) and the survey specific simulation details for SDSS and SNLS are described in Kessler et al. (2013); PS1, CSP, and CfA are described in S18; DES3YR is described in B19b and Foundation is described in Jones et al. (2018b).

We define three metrics based on the three panels of Fig. 2 which are pseudo $\chi^2$ evaluations that assess agreement between simulations that assume an SN Ia model and the data. The first metric is defined as $\chi^2_c$ for the color in histogram of data ($N_{data}$) and survey simulations ($N_{sim}$) such that

$$\chi^2_e = \sum_j (N_{data} - N_{sim})^2 / \sigma^2_j,$$

and is determined in bins of color ($j$) where $\sigma_j$ is determined by Poisson statistics.

A second metric, the agreement in total Hubble diagram scatter (RMS) between data ($\text{RMS}_{data}$) and survey simulations ($\text{RMS}_{sim}$), is defined as $\chi^2_{\text{RMS}}$ over color bins such that

$$\chi^2_{\text{RMS}} = \sum_i (\text{RMS}_{data} - \text{RMS}_{sim})^2 / \sigma^2_i,$$

and is determined in bins of color $i$ and where $\sigma_i$ are the errors determined from 100 realizations of the simulated dataset. We use RMS instead of intrinsic scatter as a metric because, for intrinsic scatter, the sensitivity of the different components of the error modeling is difficult to track.

A third metric is the agreement in distance modulus residuals between data ($\Delta \mu_{data}$) and survey simulations ($\Delta \mu_{sim}$) which can be expressed as $\chi^2_{\Delta \mu}$ over color bins such that

$$\chi^2_{\Delta \mu} = \sum_i (\Delta \mu_{data} - \Delta \mu_{sim})^2 / \sigma^2_{\mu_i},$$

and is determined in bins of color $i$ and where $\sigma_{\mu_i}$ are errors derived from the data itself.

3. EVALUATING MODELS OF TYPE IA SUPERNOVAE COLORS AND INTRINSIC SCATTER

3.1. Previous Models of Intrinsic Scatter and Associated Intrinsic Color Populations

Recent studies have focused on two models of intrinsic scatter, which to first order, can both be described by two parameters: the magnitude of chromatic and achromatic scatter. The two models are the ‘G10’ scatter model (Guy et al. 2010) which prescribes 70% of the intrinsic scatter to coherent variation and 30% to chromatic (wavelength dependent) variation and the ‘C11’ scatter model (Chotard 2011) which prescribes only 25% of the intrinsic scatter to coherent variation but 75% to chromatic variation. Both of these models were trained on data: for C11, it was trained on spectra from the SNFactory (Aldering et al. 2002) and for G10, it was trained during the creation of the SALT2 model on a large subset of the light curves used in this analysis (Guy et al. 2010, B14).

These scatter models cannot be used in survey simulations to predict color distributions or the trends of Fig. 2 without an associated color population and a $\beta_{\text{SALT2}}$ as defined in Eq. 2. For both the G10 and C11 scatter model, Scollnic & Kessler (2016), hereafter SK16, determined the underlying color population such that when it was combined with measurement noise, the color scatter from the scatter model, and selection effects, the observed color distribution matched that seen for the data in the top panel of Fig. 2. The underlying population was described by an asymmetric gaussian, with three free parameters. The value of $\beta_{\text{SALT2}}$ was determined by finding what input $\beta_{\text{SALT2}}$ in the simulations would yield an output $\beta_{\text{SALT2}}$ consistent with that found in the data from the methodology outlined in Section 2.2.

The number of parameters that describe the framework for one of these scatter models is six: two parameters for the spectral and coherent scatter, three parameters for underlying population, and the value of $\beta_{\text{SALT2}}$. However, in order to explain inconsistencies between the low-z targeted sample and the high-z samples, SK16 determined the underlying population for each separately. Therefore, in total, a description of the full sample is described by 9 parameters.
For the simulations with G10 and C11, a single input $\beta_{\text{SALT2}}$ value is used for each one: $\beta_{\text{SALT2}} = 3.1$ and $\beta_{\text{SALT2}} = 3.8$ for G10 and C11 respectively. As explained in past analyses (Scolnic et al. 2014b; Scolnic & Kessler 2016; Kessler & Scolnic 2017), applying the 1D fitting procedure from Marriner et al. (2011) recovers an observed $\beta_{\text{SALT2}} \sim 3.1$ for both the G10 and C11 cases. Due to the larger amount of color scatter in the C11 model, the associated underlying color population of C11 appears much more like a sharp dust-like exponential distribution (Scolnic et al. 2014b) than the one for G10. While it is unclear how to apply a physical interpretation to the G10+SK16 model, one possible interpretation for the C11+SK16 model is that there are two color-luminosity relations: one that relates the dust-like color to luminosity, and another with no luminosity correlation ($\beta = 0$) for the intrinsic color distribution. The populations used for the samples in Pantheon (Low-z, PS1, SNLS, SDSS) can be found in SK16, for Foundation in Jones et al. (2018b), and for DES3YR in B19b.

3.2. Evaluating Past SN Ia Scatter Models

As expected, because the SK16 populations were determined so that simulations would reproduce the observed color distribution of the data, simulations based on C11+SK16 and G10+SK16 show excellent agreement with the data (Figure 4): $\chi^2$ of 9.0 and 9.5 respectively (12 bins). The mean observed $c$ and $x_i$ for the simulations, binned over redshift, is shown in the bottom panels of Figure 1 and is in similarly good agreement with the data. However, the agreement between data and simulations for both the RMS (Fig. 5a) and mean Hubble residuals (Fig. 5b) is comparatively poor.

For the RMS of Hubble residuals (Fig. 5a), it is clear that neither G10+SK16 nor C11+SK16 produce the trend observed in the data. We do see non-linear behavior predicted from the simulations for the C11+SK16 model, which prescribes more scatter due to SN Ia chromatic variation and achieves a $\chi^2_{\text{RMS}} = 35$, whereas G10+SK16, which prescribes little color variation, achieves a $\chi^2_{\text{RMS}} = 68$. The relatively flat dependence of the RMS on color as predicted from the G10+SK16 model shows that the trend in the data cannot be explained by lower signal-to-noise for SNe with redder colors.

The agreement between data and simulations for mean Hubble residuals (Fig. 5b) is somewhat better for G10+SK16 ($\chi^2_{\Delta \mu} \sim 12$) but worse for C11+SK16 ($\chi^2_{\Delta \mu} \sim 29$). As discussed in SK16, and used for the motivation of Kessler & Scolnic (2017), both models do predict the upturn in mean Hubble residuals for blue colors. Such distance modulus biases arise due to the combination of asymmetric color distributions with color scatter and selection effects.

3.3. Parameterization of a new dust-based color model

We discuss in Fig. 2 a simple and more physical understanding of the trends seen: the redder colors can be explained by dust extinction, the high RMS for red SNe Ia could be explained by variations in the extinction parameter, and Hubble residual biases for the blue and red SNe can be explained by different respective color-luminosity relations. Here, we follow Mandel et al. (2011) and Mandel et al. (2017), which build on the work of Jha et al. (2007) to create a model of SN color based on two components: 1) an intrinsic color component ($c_{\text{int}}$) related to luminosity by a correlation coefficient $\beta_{\text{SN}}$ and 2) a dust-component ($E_{\text{dust}}$) described by an exponential distribution of reddening values related to luminosity by the extinction ratio $R_V$. The observed color $c_{\text{obs}}$ can be expressed as

$$c_{\text{obs}} = c_{\text{int}} + E_{\text{dust}} + \epsilon_{\text{noise}}. \quad (7)$$

where $\epsilon_{\text{noise}}$ is measurement noise. We expand on the model from Mandel et al. (2011) by allowing $R_V$ to be described by a Gaussian distribution to reflect that a range of values are seen in the literature, rather than a single value. In total, the model has seven fundamental parameters:

- $\bar{c}$: the mean of the intrinsic color distribution described by a symmetric Gaussian.
- $\sigma_c$: the 1-sigma width of the intrinsic color distribution described by a symmetric Gaussian.
- $\beta_{\text{SN}}$: the correlation between intrinsic color and luminosity.
- $\sigma_{\beta_{\text{SN}}}$: the 1-sigma width of the Gaussian distribution from which the correlation between intrinsic color and luminosity is drawn for each SN.
- $R_V$: the center of the Gaussian distribution from which $R_V$ values are drawn for each SN.
- $\sigma_{R_V}$: the 1-sigma width of the parent Gaussian $R_V$ distribution.
- $\tau_{\text{e}}$: the parameter describing the exponential distribution from which $E_{\text{dust}}$ reddening values are drawn.

To set a ‘reddening-free’ color, it is assumed that the intrinsic colors of SNe Ia can be determined by:

$$P(c_{\text{int}}) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-(c_{\text{int}}-\bar{c})^2/2\sigma_c^2}. \quad (8)$$
The reddening for each SN is described by $E_{\text{dust}}$ from Eq. 7 and is related to the extinction of the SN by the standard equation

$$A_V = R_V \cdot E_{\text{dust}}$$  \hspace{1cm} (9)$$

where $E_{\text{dust}}$ corresponds to $E(B-V)$.

The reddening values $E_{\text{dust}}$ are drawn from an exponential distribution following Mandel et al. (2017) with probability density

$$P(E_{\text{dust}}) = \begin{cases} \tau_{E}^{-1} \cdot e^{-E_{\text{dust}}/\tau_{E}}, & E_{\text{dust}} > 0 \\ \tau_{E}^{-1} \cdot \tau_{E}, & E_{\text{dust}} \leq 0 \end{cases}$$  \hspace{1cm} (10)$$

where $\tau_{E}$ is a parameter in the model described above.

In addition, we draw from distribution of possible values for $R_V$:

$$P(R_V) = \frac{1}{\sqrt{2\pi} \sigma_{R_V}} e^{-(R_V - \bar{R}_V)^2 / 2\sigma_{R_V}^2}$$  \hspace{1cm} (11)$$

where $\bar{R}_V$ is the center of the Gaussian distribution of $R_V$, $\sigma_{R_V}$ is the width, and where individual $R_V$ values below 0.5 are not allowed.

Finally, similar to Eq. 11, values for $\beta_{SN}$ are drawn for each SN using model parameters $\beta_{SN}$ and $\sigma_{\beta_{SN}}$ such that:

$$P(\beta_{SN}) = \frac{1}{\sqrt{2\pi} \sigma_{\beta_{SN}}} e^{-\beta_{SN}^2 / 2\sigma_{\beta_{SN}}^2}$$  \hspace{1cm} (12)$$

In total, the change in observed peak brightness of a SN due to color can be expressed as $\Delta m_B$:

$$\Delta m_B = \beta_{SN} \epsilon_{\text{int}} + (R_V + 1) E_{\text{dust}} + \epsilon_{\text{noise}}$$  \hspace{1cm} (13)$$

where each observed parameter is unique to each SN. The coefficient $R_V + 1$ is used rather than $R_V$ as in Eq. 9, because to measure the change in $m_B$, the extinction parameter $R_V = R_V + 1$ is needed.

To describe one survey with this model, seven parameters are required. If one is to solve for parameters to describe all ‘rolling’ and ‘targeted’ surveys separately, then one additional parameter is needed: a separate $\tau_{E}$ for each survey design. In total, this makes 8 parameters. In contrast, as discussed previously, the G10+SK16 or C11+SK16 models require 9 parameters when ‘rolling’ and ‘targeted’ samples are accounted for separately. Thus, the dust-based framework described here has fewer free parameters than those used in past cosmological analyses.

Grouping of rolling versus targeted subsamples is chosen because rolling surveys (DES, PS1, SNLS, SDSS and Foundation) have no preferential host-galaxy selection and Targeted surveys (CfA samples and CSP) preferentially targeted SNe in brighter galaxies. The split between rolling versus targeted surveys is similar to splitting between high versus low-redshift, except for the Foundation Survey. As discussed in Foley et al. (2018) and Jones et al. (2019), Foundation follows-up objects discovered by rolling surveys, and the sample properties look in-between a high-z rolling survey and a low-z targeted survey. As the Foundation sample is not large enough to discriminate between the designations according to our metrics, we leave it as a rolling survey.

3.4. Results for the New Color Model

The parameters described in Section 3.3 can be fit from the photometric data itself using the three metrics (Eqs. 4, 5, & 6) and the requirement that after running the fitting method from Marriner et al. (2011) on simulations, we recover to within 1σ, a value consistent with the $\beta_{SN}$ observed from the data. To determine the model parameters properly with forward modeling would likely require both an Approximate Bayesian Computing technique (ABC, e.g. Jennings et al. 2016) in combination with very large simulations of SNe Ia with flat parameter distributions to be later re-weighted via importance sampling. Since we do not have this computationally expensive infrastructure set up, we present results for a plausible set of model parameters (Table 1) achieved through coarse minimization with human supervision to find the lowest $\chi^2$ across the three metrics. Uncertainties on parameters are not computed, but we show the power of the constraints from the three metrics to break degeneracies between model parameters in Appendix Fig. 9.

The parameters are shown in Table 1. We find a mean reddening-free color of $\bar{c} = -0.078$ with an intrinsic color distribution of $\sigma_{\bar{c}} = 0.044$ and a mean intrinsic color-luminosity correlation coefficient of $\beta_{SN} = 1.8$ with small variation of $\sigma_{\beta_{SN}} = 0.3$. The recovered $\beta_{SN}$ is smaller than the traditional $\beta_{SALT2} \sim 3$ found when assuming a single correction for the full SN Ia color and dust population simultaneously, and shows a relatively weak correlation between intrinsic color and luminosity. We find the $R_V$ distribution for the dust component is described by $\bar{R}_V = 2.0$ and $\sigma_{R_V} = 1.4$. The value of $\sigma_{R_V} = 1.4$ indicates a wide range of $R_V$, though with a set-floor of $R_V = 0.5$. Because a single color-luminosity relation is applied in the fit, but there is a range of $R_V$ values, the measured $R_V$ variation dominates the scatter of distance modulus residuals, contributing 0.093 to $\sigma_{\text{int}}$, the majority of the total $\sigma_{\text{int}}$ (0.105). On the other hand, the measured variation in the intrinsic color-luminosity relation ($\sigma_{\beta_{SN}} = 0.3$) contributes 0.039
Fig. 5.— a) The zero-mean RMS of the Hubble residuals relative to ΛCDM versus the observed color c of the SNe Ia. The data is shown in black points, and the predictions from simulations of G10+SK16 and C11+SK16 models are shown in orange and purple dotted lines respectively. The model created for this work, labeled BS20, is shown in green. Inset: same as main figure but for intrinsic scatter term σint instead of RMS. b) Binned Hubble Diagram residuals versus color. Biases are seen in the observed data (black points) and predicted by the scatter models (solid/dotted lines). For the BS20 model used here, there is no split on host-mass.

to the total σint.

The results of simulations with our model are presented in Fig. 4 and Fig. 5. We find a βSALT2 = 3.06 ± 0.01 when analyzed identically to the observed dataset, which is consistent with that of the observed dataset (βSALT2 = 3.04 ± 0.06). In Fig. 4, we show that the BS20 model results in observed SN Ia color distribution similar to that of the data (χ2 ∼ 7). Furthermore, as shown in Fig. 5a, this model captures the increased RMS scatter for the redder SNe (χ2RMS ∼ 8), which is attributed to variation of RV. This is further demonstrated by the inset of Fig. 5a, which derives the magnitude of the intrinsic scatter (σint) after removing additional uncertainty from the SALT2 error-snake, and shows that the distribution of dust and extinction laws can account for the majority of the observed intrinsic scatter. Finally, as shown in Fig. 5b, the BS20 model results in excellent agreement with observed Hubble residual biases (χ2∆µ ∼ 10).

In comparing χ2 values between the different color scatter models for the three metrics in Table 2, the advancement of the BS20 model is clear, and with one less parameter, the improvement cannot be simply attributed to additional model complexity.

4. THE DEPENDENCE OF THE HOST-MASS CORRELATION WITH SNE IA LUMINOSITY ON COLOR

4.1. Observed trends of color metrics based on Host-Galaxy Stellar Mass

Many studies have found correlations between the the Hubble residuals and various host-galaxy properties (Hicken et al. 2009a; Lampeitl et al. 2010; Sullivan et al. 2010; Childress et al. 2013; Rigault et al. 2013; Roman et al. 2018; Rose et al. 2019). Here, we focus on the host-galaxy stellar mass as it is the most commonly used, most accessible, and often yields some of the strongest correlations with Hubble residuals. In the top panel of Fig. 6a, the RMS versus SALT2 color plot as shown in Fig. 5a is remade, but for the high and low host-mass subsamples separately. For the ‘dust-free’ blue SNe (c ∼ −0.1), there is little difference between the RMS for SNe in low and high-mass hosts. However, the RMS increases with redder SN colors, and much more significantly for SNe in low-mass hosts.

As shown Fig. 6b, when splitting the dataset into high and low host-mass subsamples, there is a distinct difference of the color dependence in the biases of Hubble residuals. For the ‘dust-free’ blue SNe, the slope of the color-luminosity relation as well as the absolute Hubble residual biases for SNe in low and high-mass hosts are identical. For the redder SNe however, there are distinctly different color-luminosity relations and there is as much as a ∼ 0.15 mag difference in Hubble residuals. Overall, the subsamples are discrepant at greater than 5σ (χ2/Nbin = 57/10) relative to each other.

Pursuing this further, we follow recent works like B19b and define γ as the mean difference in Hubble residuals given a
The typical $\gamma$ dependence of color are weakly correlated if at all (e.g. Sullivan et al. 2010), Kessler & Scolnic (2017), the bias corrections are applied. If we apply corrections based on a dust-based explanation. We note that the trend seen in the residuals and mass for different colors are all indicative of a our model shows that redder colors can be described by dust, the recovered is 0 $\gamma$ split in host galaxy properties:

$$\delta \gamma = \gamma \times [1 + e^{(M_{\text{host}} - M_{\text{sun}})/0.01}]^{-1} - \frac{\gamma}{2} \quad (14)$$

where $M = \log(M_{\text{host}}/M_{\text{sun}})$ and a log host-mass step location ($M_{\text{step}}$) of 10 is assumed. We determine $\gamma$ for the sample in discrete color bins. This is shown in the bottom panel of Fig. 6a. As expected from the observations in Fig 6b, for ‘dust-free’ SNe Ia that are bluer than the intrinsic color $c$, $\gamma = 0.003 \pm 0.029$, consistent with 0. However, for redder SNe, there is a significant $\gamma = 0.083 \pm 0.011$ as well as a 4.5$\sigma$ increasing trend where $\Delta \gamma \sim 0.72 \pm 0.14 \times c$, showing that the typical $\gamma$ values around 0.06 mag recovered in previous analyses are driven by the red SNe in the sample.

While many studies have shown that host-mass and SN color are weakly correlated if at all (e.g. Sullivan et al. 2010), the dependence of $\gamma$ itself on color has not been studied. As our model shows that redder colors can be described by dust, the difference between observed correlations between Hubble residuals and mass for different colors are all indicative of a dust-based explanation. We note that the trend seen in the bottom of Fig. 6a is largely insensitive to whether distance bias corrections are applied. If we apply corrections based on Kessler & Scolnic (2017), the $\gamma$ recovered is $0.0 - 0.02$ mag lower per bin than that shown, which is discussed at length in Smith et al. (2020). The trend with RMS is not affected by these corrections because the RMS measured per bin is calculated after a mean offset is removed, thereby effectively doing a similar correction as Kessler & Scolnic (2017).

4.2. Dust Modeling Explains Mass Step

We repeat the process as described in Section 3 for determining the underlying dust-based color model, with now for the low and high-mass host-galaxy subamples separately. The fitted parameters are given in the ‘Mass-split’ grouping of Table 1. Parameters that are intrinsic to the SNe Ia are fixed for both host-galaxy subsamples while the dust distributions are allowed to vary for each subsample. We find that for SNe in low-mass hosts, $R_V = 2.5$ with $\sigma_{R_V} = 2.2$, whereas for SNe in high-mass hosts, $R_V = 1.5$ with $\sigma_{R_V} = 0.8$. After accounting for selection effects, the distribution shifts such that the average observed $R_V$ for the detected SNe in the sample is 2.94 and 1.55 for low-mass hosts and high-mass hosts respectively. In these simulations, 6% of all the detected SNe have simulated $R_V$ values greater than 5. The dust distribution for SNe in high-mass hosts that are discovered in rolling surveys can be described with $\tau_V = 0.11$ whereas for low-mass hosts it is $\tau_V = 0.09$ and similarly for targeted surveys the SNe can be described with $\tau_V = 0.18$ whereas for low-mass hosts it is $\tau_V = 0.16$. Therefore, we find that the amount of dust is slightly higher for high-mass hosts in comparison to low-mass hosts, but that $R_V$ is significantly lower for high-mass hosts.

We show in Fig. 6b that simulations with these separate dust models do indeed each recover the trends in Hubble residuals, and consequently the trend seen in the bottom panel of Fig. 6a. Therefore, we conclude that modeling different dust properties for different galaxy populations can fully explain the net $\gamma \sim 0.06$ mag offset seen in past analyses as well as the $\gamma$ dependence on observed SN Ia color. Furthermore, the wider $R_V$ distribution for low-mass hosts explains why in the top panel of Fig. 6, the amount of scatter is significantly higher for low-mass hosts versus high-mass hosts.

As shown from the data, applying a single offset ($\gamma$) as has been done in past analyses, is incorrect. Furthermore, it has been unclear in past analyses why there should be any ‘step’ behavior (Sullivan et al. 2010). Here, it is shown that the past step is an artifact of improper fitting, and arises because of significantly different $R_V$ distributions for different types of galaxies.
As shown in Fig. 7, we see the same effect with simulations as we do for data when simulating a sample of 10,000 SNe with realistic proportions and distributions of SNe Ia from each survey. Here, the simulations of ‘datasets’ are based on the BS20 model, but bias corrections are determined from the other models.

To determine cosmological parameters, we use CosmoMC (Lewis & Bridle 2002) and combine with CMB (Planck Collaboration et al. 2018) constraints. In Table 5, the biases in cosmological parameters are given when simulated SNe Ia datasets use different models of SN Ia color than the model used for the determination of distance bias correction. We find that if the ‘true’ model of SN Ia color is the dust-based model presented in Section 3.3, but the bias corrections are based on the G10+SK16 or C11+SK16 models, the propagated bias in $w$ will be -0.025 and -0.040 respectively. Again, this bias is larger than any other systematic uncertainty reported in recent cosmological analyses.

In Table 5 we also show the differences in $w$ for the real data when we apply bias corrections based on simulations using the three separate models of color: G10+SK16, C11+SK16 and BS20. Relative to BS20 bias corrections, there are changes in recovered $w$ for G10+SK16 and C11+SK16 of -0.033 and -0.041 respectively, which is consistent with simulations. Interestingly, as shown in Fig. 5a, while C11 and BS20 better match the trend in the data, they produce the largest differences in $w$ of $\sim 0.04$.

### 6. DISCUSSION

#### 6.1. The Dependence Between $R_V$ and Host Galaxy Properties

That the mass correlation can be explained by separate dust properties is now the only direct explanation for the correlation between host-mass and distance modulus residuals. This possibility was briefly discussed in Mandel et al. (2017), which showed if one changed the dust distribution ($\tau_R$) for the SNe in low and high-mass subsamples, one could remove 1/3 of the magnitude of $\gamma$, but not the whole effect. We follow this idea from Mandel et al. (2017), but add that the $R_V$ distribution as well should be different for these subsamples. This can then explain the full $\gamma$ as well as its color dependence. Our dust explanation aligns well with the observations in Burns et al. (2018) that at low-$z$, the host-mass correlation with SN Ia luminosity is larger in the optical than in the NIR, where the correlation is consistent with 0. This should be the case if the correlation is tied to reddening, as the corresponding extinction ratio of $R_V$ in the NIR is smaller. Furthermore, the range of $R_V$ values is in good agreement with studies of $R_V$ from individual SNe like in Amanullah et al. (2015). While the model shows that a fraction of SNe should have $R_V > 5$, we find that this is only 6% after accounting for selection effects.

This analysis makes a strong prediction that SNe in lower-mass galaxies have on average, higher $R_V$ values with a wider distribution than SNe in higher-mass galaxies. As there are very few measurements of $R_V$ in the interstellar medium of galaxies beyond the Milky Way, LMC and SMC, it is difficult to find evidence that this trend would hold for galaxies themselves. Salim et al. (2018), which measured the dust attenuation curves of 230,000 individual galaxies in the local universe, found that quiescent galaxies, which are typically high-mass, have a mean $R_V = 2.61$ and star-forming galaxies, which are lower-mass on average, have a mean $R_V = 3.15$. 

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5. IMPACT ON RECOVERY OF COSMOLOGICAL PARAMETERS

To understand the impact of these different models of SN Ia color on the recovery of cosmological parameters, both data and simulations are used. Before measuring cosmological parameters, we apply bias corrections following the methodology of Marriner et al. (2011) and B14 using large simulations with the three color models (G10+SK16, C11+SK16, BS20) to measure the dependence of distance biases with redshift, which are then applied as corrections to the dataset or a simulated dataset. Bias corrections following Kessler & Scolnic (2017) are not used because they have so far been only been designed to work given a $\beta_{\text{SALT2}}$ and a variation in $c$, but not $R_V$, nor variation thereof. Therefore, we apply bias corrections that assume a single $\beta_{\text{SALT2}}$ and follow the same formalism that was used in the JLA analysis and we do not split by host-mass. This is so a self-consistent comparison can be made against the impact of the G10+SK16 and C11+SK16 scatter models.

The impact of the bias corrections on the data is shown in Fig. 7. The most noticeable differences between the corrections of BS20 versus the other scatter models are at $z < 0.1$ and $z > 0.8$, where selection has the greatest influence. Here, the differences in recovered distance modulus can change by up to $\sim 0.05$ mag at low or high-$z$ depending on which color model is used. This difference is larger than any other systematic in past cosmology analyses (e.g., B19b).
This trend is in general agreement with our prediction.

The observation that global properties of the galaxy can impact the dust measured from the SNe is supported by Phillips et al. (2013) and Bulla et al. (2018b), which found the dust responsible for the observed reddening of SNe Ia appears to be predominantly located in the interstellar medium of the host galaxies and not in the circumstellar medium associated with the progenitor system. It’s also supported by Childress et al. (2013) which showed that color of SNe Ia is strongly tied to the metallicity of the host galaxy. For a future analysis, it is encouraged to repeat this same exercise but instead of using stellar mass to use metallicity, specific star formation rate, or local color; improved estimates of the dust distribution parameters would likely be obtained. For example, as shown in Sullivan et al. (2010), when measuring a single color-luminosity coefficient $β$ for different samples, there is an even bigger difference when splitting the sample for specific star formation rate than there is for mass. As our model constrains both the amount of dust and the properties of dust itself, it is likely that different galaxy properties (e.g., distance to host and inclination, Holwerda et al. 2015; Galbany et al. 2012) will yield complementary insight about both of these components. We stress that our analysis does not limit the use of host-galaxy information in cosmological studies with SNe Ia, but rather, proposes a new path forward.

Indirect explanations of $γ$ have suggested that SNe from different progenitor systems have different luminosities, and the progenitor system can be potentially linked to the age of the host galaxy (Childress et al. 2013). However, any model that assumes that the luminosity depends on progenitors does not predict the key observation in our analysis that the magnitude of $γ$ depends on color. A progenitor-based explanation has motivated studies by Rigault et al. (2013), Childress et al. (2014), Jones et al. (2015), Jones et al. (2018a), and Roman et al. (2018), which focus on the local specific star formation rate, local mass, and local color. Some of these studies seem to indicate that measuring the local color produces the highest correlation with measured SN Ia luminosity. In light of our model-based SN Ia color model, a simple explanation is that the local host color yields insight about the amount of dust and/or dust properties at the position of the SN. Our model does not differentiate whether the dust is in the circumstellar surrounding which is still linked to the progenitor or in the interstellar medium which is not linked to the progenitor, but we can rule out a luminosity dependence on the progenitor system.

Relatively, many studies have found correlations between spectral features and Hubble residuals (Fakhouri et al. 2015; Siebert et al. 2020). Interestingly, Wang et al. (2009) split a sample of 158 SNe Ia based on whether their spectra indicate ‘normal velocity’ or ‘high velocity’ features, and find $R_V = 2.36 ± 0.07$ and $1.57 ± 0.07$ for the two subsamples respectively. Pan et al. (2015) show that the velocity of spectral features correlates with the mass of the host galaxies, such that high-mass host galaxies regularly have high-velocity SNe, so one would expect low $R_V$ to be found for high-mass hosts. This is in great agreement with the results of our study, though we note that Foley & Kasen (2011) show that different $R_V$ from Wang et al. (2009) depend on using SNe with very red colors $E(B-V) > 0.4$. As velocity features has typically been thought of indicative of properties of the progenitor and circumstellar surrounding, it is unclear at what level things are causally connected versus correlated.

6.2. Application of BS20 Model in Future Analyses

While we have shown that biases in $w$ from our model relative to previous models would have been the largest systematic uncertainty of previous analyses, there is a clear path to utilize this model for future analyses. In order to do so optimally, there are three necessary improvements. First, a full ABC fit to solve for the intrinsic and extrinsic parameters, broken by survey, redshift range, or targeted versus un-targeted is needed. This could be facilitated by the recent advancements by Fippin (Hinton & Brout 2020). Future work can fully constrain and characterize systematic uncertainties on all 9 dust and hyper-parameters using a combination of the

| SN Ia Color Model Data | Host Dust Model Data | SN Ia Color Model 1D BiasCor | Host Dust Model 1D BiasCor | SN Ia + Dust | $w_{BS20}$ | $w_{BS20}$ |
|------------------------|---------------------|-----------------------------|-----------------------------|-------------|----------|----------|
| BS20                   | BS20                | C11 + SK16 Parent           | No Host Dust                | 3.06 ± 0.01 | -0.040   |
| BS20                   | BS20                | G10 + SK16 Parent           | No Host Dust                | 3.08 ± 0.01 | -0.025   |
| Real Data              | Real Data           | C11 + SK16 Parent           | No Host Dust                | 3.11 ± 0.01 | 0.000    |
| BS20                   | BS20                | G10 + SK16 Parent           | No Host Dust                | 3.05 ± 0.06 | -0.041   |
| Real Data              | Real Data           | BS20                        | BS20                        | 3.04 ± 0.06 | -0.033   |
| Real Data              | Real Data           | BS20                        | BS20                        | 3.05 ± 0.06 | 0.000    |

*Datasets are based on large simulations of ~10,000 SNe Ia. Each dataset (row) is a unique statistical realization.

*Bias Correction samples are large simulations of >1,000,000 SNe Ia.

*$Δw = w_{fit} - w_{BS20}$: this is relative to the last row (BS20) of each dataset grouping.
Thus halving the difference between them, but still found it to be one of the largest at $\sigma_w = 0.017$. As the sensitivity of cosmological parameters to different scatter models is so large, we emphasize that this issue cannot be ignored in any future cosmological analysis. This statement is true for analyses of $w$ and for analyses of $H_0$ as well. Dhawan et al. (2020) estimates biases due to scatter models to be on the level of $0.5 - 1.0\%$ in $H_0$. As the $H_0$ measurement has different systematic sensitivity than $w$ due to the comparison of SNe in calibrator galaxies versus Hubble flow galaxies, we recommend these two samples to have similar demographics of blue and red SNe. A full systematics treatment, as done in Dhawan et al. (2020), should be done using the new dust-based SN Ia color model described in this paper. Furthermore, we note that past discussions (e.g., Rigault et al. 2013; Jones et al. 2018a) about potential biases in $H_0$ should be reconsidered in light of this paper’s findings.

7. CONCLUSION

In this paper, we introduced a new, physical, two-component color model of SNe Ia with an intrinsic component modeled as a simple symmetric Gaussian that correlates with SN Ia luminosity and an extrinsic component that can be modeled by a dust distribution that is tied to extinction by a wide $R_V$ distribution. This model has fewer free parameters than previous models of SN Ia color and a more physical motivation that better matches the data. Our findings suggest that the dominant component of observed SN Ia intrinsic scatter is from $R_V$ variation of the dust around the SN. We also show that there is a $4.5\sigma$ dependence on color of the correlation of host-mass with distance modulus residuals. Strikingly, this shows that previously observed host-galaxy property correlations with SN Ia luminosity are driven by the redder SNe of the sample. This also suggests a dust-based explanation for the host-galaxy property correlations. By allowing our model to have different parameters for the dust distributions of SNe in high-mass versus low-mass host-galaxies, we show that the correlation between distance modulus residuals and host-galaxy stellar mass can be attributed to correlations between host-galaxy properties and $R_V$.

By finding that the previously seen host-galaxy correlation with SN Ia luminosity after standardization is actually due to differences of dust, and not due to possible variation in the luminosity based on progenitor systems, we find that that there is a tremendous amount of leverage to continue to improve cosmological analyses by studies of larger samples, measurements covering larger wavelength ranges, more host galaxy properties examined and improved dust models. Our study shows that so many disparate analyses of SNe Ia are actually intricately connected, and unifying these studies will provide tremendous improvements to measurements of the expansion of the universe.

8. ACKNOWLEDGEMENTS

We thank Rick Kessler, Adam Riess, Saurabh Jha, The Goobar Research Group, David Jones, Mat Smith, Doug Finkbeiner, Eddie Schlafly, Charlie Conroy, Antonella Palmesi, and Sam Hinton for very useful discussions. We are appreciative of Rick Kessler for his ever-useful S/NANA package. DB acknowledges support for this work was provided by NASA through the NASA Hubble Fellowship grant HST-HF2-51430.001 awarded by the Space Telescope Science Institute, which is operated by Association of Universities for Research in Astronomy, Inc., for NASA, under contract
from simulations, we do see similar trends with color when splitting on Section 4. While there is no dependence of the RMS of distance modulus residuals on is important to note that besides the BS20, G10+SK16 and C11+SK16 models, none of the other models are fit to match the data. case because as some variants may have a good χ in Fig. 10a for our compiled dataset and from this, we expect similar trends with compute a Hubble residual step when splitting on δκ at SN Ia stretch step location (\(x_1\)) of -0.5 is assumed, albeit \(x_{1\text{top}}\) values between -0.5 and +0.5 provide good discrimination between sub-samples according to our three metrics. We determine \(κ\) for the sample in discrete color bins (Fig. 10d). When deriving \(δκ\) for the full sample with a single \(x_{1\text{top}}\) split, we find \(δκ = 0.032 ± 0.011\) mag, roughly half the size of the step when splitting by host stellar mass. As shown in Fig. 10d, similarly to host mass, the magnitude of \(κ\) depends on color; there is a 3.9σ deviation relative to a single step.

When examining Hubble diagram residual biases in bins of color (Fig. 10e), simulations using the dust and \(R_V\) distributions that were fit in Sec. 4.2 roughly predict the residuals when splitting on \(x_1\). This indicates that \(x_1\) and \(M_{\text{host}}\) yield similar information about the dust properties. However, upon studying the mean Hubble residual bias with color, as shown in Fig. 10f, we find the information from \(x_1\) and \(M_{\text{host}}\) are complementary in potentially constraining \(R_V\) as the difference in Hubble residuals from the subsample of low \(x_1\) values and large host mass values (purple) in comparison to those from high \(x_1\) values and small mass values (orange) is larger than simply splitting on host mass (data points) as was done in Section 4. This finding is consistent with studies like Rose et al. (2019), which argue that combinations of various host-galaxy properties and light-curve parameters could further improve the standardizability of SNe Ia brightnesses, as well as with Galbany et al. (2012) who find that \(x_1\) is a good discriminator of galaxy morphology.

### APPENDIX

#### A1. Model-data Agreement and Parameter Sensitivity

Here we review variants on parameters in the various color models to show what impact it has on the three metrics. We list those variants here:

- **‘BS20’** - the main model proposed in this work.
- **‘No Dust’** - a model with a narrow intrinsic color distribution and a weak (\(\beta_{SN} = 2\)) correlation between color and luminosity.
- **‘Only Dust’** - a model with only a dust distribution and a delta function for the intrinsic color distribution.
- **‘G10+SK16’** - Described in Section 3.1
- **‘C11+SK16’** - Described in Section 3.1
- **‘C11+SK16 + \(\beta_{SN}\) variation’** - a model similar to the ‘C11+SK16’ one, except we allow the \(\beta_{SN}\) to vary to reproduce the RMS for redder colors.
- **‘BS20, \(\sigma_{\beta_{SN}} = 0\)’** - the nominal BS20 model, except \(\beta_{SN}\) values are drawn from a delta function with value \(\bar{\beta}_{SN}\).
- **‘BS20, \(R_V + 0.5\)’** - the nominal BS20 model, except we shift our \(R_V\) distribution by the full sample by 0.5.
- **‘BS20, \(\tau_E - 0.5\)’** - the nominal BS20 model, except we reduce \(\tau_E\) to describe the dust distribution by 0.05.
- **‘BS20, \(\bar{\beta}_{SN} + 0.5\)’** - the nominal BS20 model, except we increase \(\bar{\beta}_{SN}\) by 0.5.
- **‘BS20, \(\bar{\beta}_{SN} = 0\)’** - the nominal BS20 model, except we set \(\beta_{SN}\) to be 0. This effectively describes the intrinsic color distribution as color scatter, similar to what is in the C11+SK16 model.
- **‘BS20, No \(R_V\) variation’** - the nominal BS20 model, except the variation in \(R_V\) is removed.

We show the results from using these different variants in Fig. 9. We include on the bottom panel the recovered \(\beta_{SALT2}\) for each case because as some variants may have a good \(χ^2\) in the three metrics, the recovered \(\beta_{SALT2}\) is far from that the data (\(\sim 3.05\)). It is important to note that besides the BS20, G10+SK16 and C11+SK16 models, none of the other models are fit to match the data.

#### A2. Observed Correlations with SALT2 \(x_1\)

SN Ia cosmology analyses that measure correlations between SN light curve parameters and host galaxy mass regularly find a correlation between host-galaxy stellar mass and \(x_1\) (e.g., Sullivan et al. 2010; Scolnic et al. 2014a). This correlation is shown in Fig. 10a for our compiled dataset and from this, we expect similar trends with \(x_1\) that we observed with host stellar mass in Section 4. While there is no dependence of the RMS of distance modulus residuals on \(x_1\) (Fig. 10b) seen in the data or predicted from simulations, we do see similar trends with color when splitting on \(x_1\) (Fig. 10c) as we do when splitting on \(M_{\text{host}}\). We also compute a Hubble residual step when splitting on \(x_1\):

\[
δκ = κ × [1 + e^{(x_1−x_{1\text{top}})/0.01}]−1 − \frac{κ}{2}.
\]
A3. The SALT2 Color Law

In the discussion, we explain that a future analysis should retrain the color law(s) to match the data, rather than rely on our a-posteriori model selection. In Fig. 11, we derive the predicted distribution of peak rest-frame colors from our model, and compare to data. This is done by k-correcting observations to the rest-frame using the SALT2 spectral model. We show that the BS20 model better predicts the observed $(B-V)$ distribution while G10+SK16 based better predicts the observed $(U-B)$ distribution, both by similar amounts in $\chi^2$. As the BS20 model selection had little sensitivity to the rest-frame UV colors, this evaluation is not surprising. It is moderately surprising, however, that given the lack of UV sensitivity in the metrics, the BS20 does as well as it does in the UV. Still, we argue that a proper retraining of the light-curve model that incorporates flexibility for discrimination between the intrinsic SN Ia color law and dust color laws is needed in the future. Interestingly, Amanullah et al. (2015) shows that in the UV, a Fitzpatrick $R_h = 2.2$ matches observations of nearby SNe significantly better than the SALT2 color law. As such, we expect that retraining based on our model can improve the plot shown here.

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Fig. 9.— Similar to what is shown in Fig. 4 and Fig. 5, here we show the agreement between data and simulations for variants on our main BS20 Mass-split model as well as for the G10+SK16 and C11+SK16 based models. The $\chi^2$ for each metric is given, and in the bottom panel, we show the recovered $\beta_{\text{SALT2}}$ to be compared with that from the data ($\sim 3.05$). The sensitivity to these variations shows the high constraining power of the metrics.
Fig. 10.— (a) The correlation between observed SALT2 $x_1$ and host-galaxy stellar mass. The trend shows a difference in weighted average $x_1$ values (red) when split on $M_{\text{step}} = 10$. Hubble residual zero-mean RMS values are reported for subsets of the data. (b) RMS of Hubble diagram zero-mean residuals versus SALT2 $x_1$. No dependence is seen. (c) RMS of Hubble diagram residuals versus SALT2 $c$ when splitting on both $\log(M/\text{M}_\odot)$ (black) and SALT2 $x_1$ (red). (d) Host Mass step as a function of observed color now with $\gamma$ (black) and $\kappa$ (red) overlaid. Simple averages and $1\sigma$ uncertainties are shown with horizontal lines. Significance of deviation from the respective horizontal lines is reported in text. (e) Binned Hubble diagram residuals for the dataset when splitting on $x_1 = -0.5$ (points). The predictions using dust and $R_V$ distributions from Sec. 4.2 are overlaid (lines). (f) Binned Hubble diagram residuals from three sectors of the dataset (lines) corresponding to the three quadrants shown in Panel (a). Overlaid are the binned Hubble diagram residuals used when only splitting on mass.

Fig. 11.— Predicted distribution of restframe colors in $(U - B)/(1 + z)$ and $(B - V)/(1 + z)$ from BS20 and the G10+SK16 model (Nominal), where $z$ is the observed redshift of the SNe. The $\chi^2$ defined in Eq. 3 is reported. Lesser agreement in $(U - B)$ for the BS20 model motivates retraining of the light-curve model in a future study.