Connection between Fisher Information and Wave Mechanical Interpretations of Universe

Esra Russell\textsuperscript{1,2},* Oktay K. Pashaev\textsuperscript{1}

\textsuperscript{1}Department of Mathematics, Izmir Institute of Technology, 35430 Gulbahcekdagi/Urla, Izmir, Turkey.
\textsuperscript{2}Kapteyn Astronomical Institute, University of Groningen, PO Box 800, 9700 AM Groningen, The Netherlands.

Accepted .... Received ...; in original form ...

ABSTRACT

In this study, we model the dark matter and baryon matter distribution in the Cosmic Web by means of highly nonlinear Schrödinger type and reaction diffusion wave mechanical descriptions. The construction of these wave mechanical models of the structure formation is achieved by introducing the Fisher information measure and its comparison with highly nonlinear term called the quantum potential in the wave equations. Strikingly, the comparison of the nonlinear term and the Fisher information measure provides a dynamical distinction between lack of self-organization and self-organization in the dynamical evolution of the cosmic components. Mathematically equivalent to the standard cosmic fluid equations, these approaches make it possible to follow the evolution of the matter distribution even into the highly nonlinear regime by circumventing singularities. In addition, these wave formalisms are extended to two-fluid descriptions of the coupled dark matter and baryon matter distributions in the linear regime, in the Einstein de Sitter Universe (EdS) to construct toy models of the cosmic components in this relatively simple Universe model. Based on these two different wave mechanical formalisms, here fully analytical results for the dark matter and baryon distributions are provided. Also, numerical realizations of the emerging weblike patterns are presented from the nonlinear dynamics of the baryon component corresponding to soliton-like solutions. These soliton-like solutions might represent a proper description of filamentary structures even in the linear regime.

Key words: methods: analytical –cosmology: theory, dark matter, large-scale structure of Universe

1 INTRODUCTION

The large scale structure of the Universe is marked by prominent filamentary features embedded within a weblike network, the Cosmic Web (Bond et al. 1996). Extensive N-body simulations are used to model and understand its complex and intricate dynamical structure. The N-body simulations are based on semi-analytical models and the two well known theoretical methods. These methods are classified into two broad classes: The Eulerian and the Zel’dovich approximations.

While the Eulerian approximation provides an accurate description of the gravitational instability in the linear regime, the Zel’dovich approximation is an exact solution of the fluid equations as long as particle trajectories do not cross each other (Zeldovich 1972). When the trajectories cross, the velocity field becomes multi-valued by causing singularities in the density field. To solve this singularity problem, the adhesion theory is proposed by Kofman & Shandarin (1988). In the adhesion approximation, when shell crossing occurs, the particles are assumed to stick to each other by introducing an artificial viscosity term in the Burger’s equation. In the special case, when the viscosity term tends to zero, structures formed in the adhesion model are infinitely thin and the adhesion approximation reduces to the Zel’dovich approximation outside of mass concentration. As is seen, these analytical models are not enough to describe full nonlinear evolution of the structure formation of the Universe. That is why we may need to find a full analytical formalism to understand its complex structure.

As an alternative approach, Spiegler (1980) show the correspondence between the fluid structure equations and the nonlinear wave equations. Approximately a decade after this work, Widrow & Kaiser (1993) are the first to apply the Schrödinger representation to the problem of the cosmological structure formation for cold dark matter (CDM).
develop an advanced nonlinear numerical method known as the Schrödinger Method (SM) to follow the nonlinear evolution of the dark matter field by introducing an alternative particle mesh code. This new numerical model describes the matter as a Schrödinger field obeying the coupled classical Schrödinger and Poisson equations. This code is later modified by Davies & Widrow (1997). Another extension of the SM is done by Coles (2002). Coles (2002) suggests that the nonlinear Schrödinger equation is a good candidate to model the CDM. However, Coles (2002) points out that this nonlinear equation presents some difficulties to model the CDM. Later on, Szapudi & Kaiser (2003) introduce an elegant nonlinear Schrödinger cosmological dark matter perturbation theory in the correspondence limit that gives a formalism equivalent to the collisionless Boltzmann (or Vlasov) equations.

Following up on the work of Coles (2002), Coles & Spencer (2003) demonstrate a wave mechanical approach to treat the singularity problem of the Zel’dovich approximation. This approach is similar to the adhesion approach. Coles & Spencer (2003) obtains a nonlinear dynamical term which is analogous to the infamous quantum pressure (or quantum potential) and, suggest that this nonlinear term has the same effect as the viscosity term of the adhesion theory. As a result of this, the wave mechanical approach avoids the singularities of the density field. In their study, Coles & Spencer (2003) also investigate the effect of the quantum pressure term in the gravitational instability. Based on this study of Coles & Spencer (2003), Short & Coles (2006b) propose a different approach of self gravitating CDM called the free particle approximation. In this approach, quantum pressure and the gravitational potential are neglected. Short & Coles (2006b) transform the usual hydrodynamical equations of motion into a linear Schrödinger equation. Also they show that the free particle approximation is useful in the mildly nonlinear regime and it has the same result as the adhesion approximation. Another important alternative work on the Schrödinger approach in order to interpret the dark matter evolution is done by Johnston et al. (2010). Here, the wave mechanical solutions of the equations of motion for the cosmological homogeneous background evolution of a spherical dark matter overdensity are first obtained, in which the effect of the so called quantum potential is neglected. The reason of ignoring the quantum potential is explained by an assumption that on large scales this nonlinear quantum potential term becomes unimportant. Then Johnston et al. (2010) obtain the boundary conditions satisfied by the wave function that is analyzed from the quantum mechanical point of view. Following this, in the concept of quantum mechanics, they find the equations governing the evolution of multiple fluids and then solve them numerically in such a system. Note that Davies & Widrow (1997); Coles & Spencer (2003); Short & Coles (2006b); Johnston et al. (2010) use the Madelung transformation of Madelung (1927) to obtain Schrödinger equation from the cosmological fluid dynamical equations.

It is important to mention that the nonlinear quantum potential term also arises in the different studies that are slightly different due to its derivation. For example, the nonlinear quantum pressure emerges out of exact solutions of the full Wheeler-De Witt equation Blaut & Kowalski-Glikman (1998). This is especially important given the fact that the cosmic wave function describing the quantum state of the Universe satisfies the cosmological kinetic Wheeler-De Witt equation. Apart from this, Lani (1999) show that the quantum pressure in the causal interpretation of quantum mechanics can be reduced to the Lane-Emden equation that is discussed extensively in the theory of stellar evolution Chandrasekhar (1939); Capozziello et al. 2011; Boubaker & Van Gorder 2012; Boubaker & Bhrawy 2012; Chavanis & Harko 2012; Herbst & Monomial 2012. Moreover, da Rocha & Nottale (2003) suggest a solution to the cosmological problem of the formation and evolution of gravitational structures on many scales by using a gravitational Schrödinger equation. Its solutions give probability densities that quantitatively describe precise morphologies in position space and in velocity space da Rocha & Nottale 2003. Finally the theoretical predictions are successfully checked by a comparison with observational data, and it is found that matter is self-organized in accordance with the solutions of the gravitational Schrödinger equation da Rocha & Nottale 2003. In addition, Lee & Koh (1996); Peebles (2000); Goodman (2003); Arbev et al. (2003); Bohmer & Harko (2007) suggest that dark matter haloes may be the self-gravitating Bose-Einstein condensates (BCEs) with short range interactions by a single wave function $\psi(r,t)$. Bohmer & Harko (2007) also point out that this wave function obeys the Gross-Pitaevskii-Poisson system. Following these studies, Rinder-Daller & Shapire 2012 mention that phase-space density of light bosons as dark matter candidates may form BCEs. Therefore they show that vortex formation in haloes can be described as a fluid by obeying the cubic nonlinear Schrödinger equation (NLSE) which is also known as Gross-Pitaevskii equation. Although this concept of light bosons obeying BEC as dark matter candidates is not well studied yet, there are some other recent results on structure formation studies involving BEC as cold dark matter Guzmán & Urena-López 2003; Fukuyama et al. 2008; Marsh & Ferreira 2011; Harko 2011a, b). Woo & Chiueh (2009) indicate that quantum pressure results from quantum stress and this pressure acts against gravity for the light bosons in the high resolution simulations. In the same study, they confirm that low-mass halos are indeed suppressed quantum stress even when the small scale fluctuations are abundant in the initial power spectrum. Also, Chavanis (2012) obtains the quantum potential in the cosmological fluid equations. However by assuming the Thomas-Fermi limit (large mass bosons). Chavanis (2012) ignores its contribution in the equations.

In addition, all these studies neglect the effect of the nonlinear term the so called quantum pressure (or quantum potential) in different type of Schrödinger equations arisen from the astrophysical processes. As is aforementioned, the reason to neglect this is shown as the scale of interest, which is a large scale state, not a microscopic state of quantum mechanics. Also, Coles & Spencer (2003) and Johnston et al. 2010 point out that this nonlinear term is dependent on the amplitude of the wave and the density field slowly varies in large scale structures. On the other hand, from the quantum mechanical point of view, the quantum potential term is not dependent on the density of the field but only upon its form. As a result of this property, this nonlinear term may show a strong effect on the motion of a particle where the density is very low. This form dependence leads to strong effects even for small particles that are separated by large distances Bohm et al. 1993; Greenberger 1994; Hiley et al. 2000. Because of these strong effects, in the concept of quantum mechanics Bohm et al. 1993 and Greenberger 1994.
pressure function is demonstrated by $p\phi = \dot{x}$, where $\phi = \nabla x$, and the Poisson equation, $(1994)$ suggest that the quantum potential may be interpreted as information potential. The word information stands for an action that brings the order and self organization. In the case of analogy between particles and an $N$-body system of the Universe, the self organization nature of this nonlinear term involves a nonlocal correlation of motion of all the bodies in the collective density field [Bohm et al. 1993; Greenberger 1994; Hiley et al. 2000; Hiley & Maroney 2000]. Therefore, $N$-bodies may be controlled by a pool of information that is encoded in the wave function. Note that this pool of information guides the system of bodies rather that affecting them mechanically [Hiley et al. 2000; Hiley & Maroney 2000]. Therefore, ignoring of emergence of the quantum potential as a self organization may limit our way to obtain a full understanding of evolution of the large scale structure of the Universe, since it indicates an important role in the large scale evolution of the Universe rather than being an unimportant microscopic parameter. Consequently, it seems that the main reason of skipping this term from the wave equations is caused by mathematical tractability rather than its physical interpretation.

Separately from the cosmological perspective, Parwani & Pashaev (2007) relate the quantum potential/pressure term with the positive signed Fisher Information Measure of information theory. They also show that depending on the choice of the enthalpy function which is related with the equation of state, one may obtain the cubic NLSE or other modified NLSEs for barotropic compressible fluids. On the other hand, Frank (2009) proves that a negative sign of FI leads to maximizing the FI which is equivalent to the Shannon measure in the concept of biological systems. From the dynamical point of view, Cabezas & Fath (2002) relates the Fisher information with dynamical behavior of a system based on the sign of Fisher information. According to this, positive FI indicates loss of self organization while negative one demonstrates strong self-organization in a dynamical system. Apart from this, Madelung (1964) indicate that the linearization of any fluid equation can be done by a choice of density distribution in the form of a real or an imaginary exponential transformations. These transformations lead to the two different dynamical wave interpretations of the same fluid, which are the Schrödinger type and reaction diffusion systems.

In this paper, progressing by previous studies, we derive fully analytical wave mechanical approaches of the dark matter and baryon field components of the Universe in a two-fluid formalism in the linear regime. To do this, first we introduce the cosmological Newtonian fluid dynamical equations in comoving coordinates in order to give the evolution of the Universe in terms of expanding background. Following this, comoving matter fluid equations are scaled by using velocity potential and scaled density parameters. Then we show that matter fluid obeys a Schrödinger type nonlinear wave form which has contributions from baryon and dark matter fields and includes the infamous quantum pressure term by applying the Madelung transformation [Madelung 1927]. After introducing the Fisher information measure in the Lagrangian functional of the matter component that obey the Schrödinger type equation, it is shown that two different wave forms arise for the two-fluid formalisms depending on sign given by the relation of the Fisher measure of the system and the guiding force (or quantum potential) of the same system. As a result, it is indicated that the Newtonian fluid dynamical equations of the incompressible cosmological two component fluid can be transferred into nonlinear Schrödinger equations as well as Reaction diffusion/heat equations based on the study of Madelung (1964). Then we provide full solutions of these equations with the nonlinear potential term in the EdS Universe in the linear regime. Moreover, it is also showed that the solution of these formalisms provides the Zel’dovich approach for the Schrödinger wave description of the dark matter component in the linear regime. On the other hand, the baryon component presents soliton-like perturbative solutions from the Reaction Diffusion type wave description.

## 2 MODIFIED FLUID DYNAMICAL EQUATIONS

To obtain the nonlinear Schrödinger type wave equations to model the dark matter and baryon components, we take our first starting point as the Newtonian equations for a matter fluid in terms of comoving coordinates in an expanding Universe. Then the cosmological fluid equations in comoving coordinates can be written as the continuity,

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla x [(1 + \delta) \mathbf{v}] = 0, \quad (1)$$

the Euler,

$$\frac{\partial \mathbf{v}}{\partial t} + Hv + \frac{1}{a} \mathbf{v} \nabla x \mathbf{v} = -\frac{1}{a} \nabla x \phi - \frac{1}{a \rho_a (1 + \delta)} \nabla x p, \quad (2)$$

and the Poisson equation,

$$\nabla x^2 \phi = 4 \pi Ga^2 \rho_a (1 + \delta), \quad (3)$$

where $x$ is the comoving coordinate expanding with the scale factor $a(t)$, the Hubble parameter $H = H(t)$ is defined by $H = \dot{a}/a$ where the dot denotes a derivative with respect to time $t$. The peculiar velocity field $\mathbf{v} = \mathbf{v}(x, t)$ is given by $\mathbf{v} = \alpha \mathbf{x}$ and $\phi = \phi(x, t)$ is the peculiar Newtonian gravitational potential. The density contrast $\delta = \delta(x, t) = \delta + 1 = \rho / \rho_a$ where $\rho = \rho(x, t)$ is the matter density field, while $\rho_a = \rho_a(t)$ is the density contribution in the homogeneous background. The pressure function is demonstrated by $p(x, t)$. The relation between pressure and the density $\rho(x, t)$ is given by the equation of state,

$$p = \omega \rho.$$

$$\quad (4)$$
where $\omega$ is the dimensionless adiabatic parameter and characterizes the constitute of the medium. The cosmological perfect fluid shows a barotropic process in which the pressure is only a function of density $p = p(\rho)$. The total energy of the perfect fluid that fills the Universe is defined by the enthalpy function which can be written in terms of enthalpy potential $V_{\text{enth}}(\rho)$ and is given by Parwani & Pashaev (2007) as,

$$
\epsilon(\rho) = \int_{\rho_0}^{\rho} \frac{dp}{\rho}, \quad \epsilon(\rho) = \frac{dV_{\text{enth}}(\rho)}{d\rho}.
$$

(5)

### 3 EINSTEIN DE SITTER UNIVERSE

The Einstein de Sitter (EdS) Universe is a matter dominated Friedmann model with a flat geometry since its critical density is equal to unity $\Omega = 1$. Due to $\Omega = 1$, the EdS Universe expands for ever. In this universe model, the expansion factor $a(t)$ is proportional to,

$$
a(t) \propto t^{2/3} \propto D(t),
$$

(6)

where $D(t)$ is the linear density growth factor and indicates the structure formation of the universe.

### 4 SCALING THE FLUID DYNAMICAL EQUATIONS

In addition to these basic definitions and mathematical descriptions, it is sensible to describe the evolution in terms of the expansion factor $a(t)$ or, even more convenient and appropriate, in terms of the linear density growth factor $D(t)$ (Short & Coles 2006b) in order to describe the evolution against an expanding background. Hence, the linear growth factor follows from solving the second order ordinary differential equation (Peebles 1980),

$$
\ddot{D} + 2H\dot{D} - \frac{3}{2} \Omega H^2 D = 0,
$$

(7)

where the dot presents derivative with respect to $t$ and, at some initial time $t_i$, $D_i = D(t_i) = 1$ and the critical density is $\Omega = 1$ in the EdS Universe. With respect to the time variable $D(t)$, we define a scaled peculiar velocity $v'$,

$$
v' \equiv \frac{dx}{dD} = \frac{\mathbf{v}}{a\dot{D}}.
$$

(8)

We also introduce the comoving velocity potential $\phi_v$ for the velocity $v'$,

$$
v' \equiv \nabla_x \phi_v,
$$

(9)

and the scaled density $\chi$ as follows,

$$
\chi(x,t) \equiv \delta + 1 = \rho/\rho_u.
$$

(10)

As a result, the velocity flow characterized by the velocity potential into the Euler and continuity equations are scaled. In addition, the scaled Euler equation is integrated once in terms of comoving coordinates in order to obtain the Bernoullie equation. Hence the scaled continuity and the scaled Bernoullie equations become,

$$
\frac{\partial \chi}{\partial D} + \nabla_x (\chi \nabla_x \phi_v) = 0,
$$

(11)

$$
\frac{\partial \phi_v}{\partial D} + \frac{1}{2} (\nabla_x \phi_v)^2 = -V_{\text{eff}} - A^2(D)\epsilon(\chi),
$$

(12)

where $\epsilon(\chi)$ is the scaled enthalpy. The time dependent function $A$ is defined as $A(D) \equiv 1/a\dot{D}$ in which scale factor $a$ is equivalent to the linear growth factor $D$ in the EdS Universe. Then the dispersion term $A$ becomes dependent on the growth factor $D$ only. Apart from this, the effective potential $V_{\text{eff}}$ includes contributions from the matter velocity potential $\phi_v$ and the gravity potential $\phi$,

$$
V_{\text{eff}} = \frac{3}{2f^2D^2}\Omega \phi_v + \frac{1}{a^2D^2} \phi = \frac{3\Omega}{2f^2D} (\phi_v + \theta_g).
$$

(13)

Here, the linear velocity growth factor $f(\Omega)$, also known as the Peebles factor (Peebles 1980) is given as,

$$
f = \frac{a \dot{D}}{aD} = \frac{\dot{D}}{H\dot{D}}.
$$

(14)
and the scaled gravity potential $\theta_g$ defined as,

$$\theta_g = \frac{2\phi}{3\Omega a^2 D H^2}. \quad (15)$$

In the following section, we introduce the two-fluid wave mechanical approach and discuss the emergence of two different wave formalisms for each cosmic component.

5 EFFECTIVE POTENTIAL IN THE EDS UNIVERSE AT THE LINEAR REGIME $\delta \ll 1$

As is aforementioned in the section 4 in its definition (13), the effective potential have the total contribution form of the peculiar gravitational function,

$$V_{eff} = V_m = \frac{3\Omega \phi_{v,m}}{2f^2 D} + \frac{\phi_m}{a^2 D^2},$$

where the effective potential $V_m$ includes contributions from the dark matter velocity potential $\phi_{v, dm}$ and the matter gravity potential $\phi_m$. Jones (1999) presents an analytical model for nonlinear clustering of the baryon material in a Universe where the gravitational field is dominated by dark matter and this baryon matter flow is dissipative and is driven by the dark matter potential. Therefore, here it is assumed that the potential function is dominated by the dark matter component as dark matter creates ever deeper potential wells in which baryon matter will fall. This means that in the total peculiar gravitational potential function $\phi_m = \phi_b + \phi_{dm}$, the baryon component has a very small contribution $\phi_b << \phi_{dm}$ and we can say that the total gravitational field is dominated by the dark matter component $\phi_m \approx \phi_{dm}$. As the baryon component is driven by the dark matter gravitational field, we can assume that the velocity field of the baryon matter follows the dark matter velocity field. This means that the velocity potentials of the dark and baryon matter components are approximately equal $\phi_{v, dm} \approx \phi_{v, dm}$. Under these assumptions the effective potentials become,

$$V_b \approx V_{dm} = \frac{3\Omega \phi_{v, dm}}{2f^2 D} + \frac{\phi_{dm}}{a^2 D^2}. \quad (16)$$

At this point, it is interesting to note that in the linear regime the effective potential $V_{dm}$ is equal to zero. We may easily infer this from the direct linear relation between peculiar velocity $v_{dm}$ and the peculiar gravity $g$ in the linear regime (Peebles 1980),

$$v_{dm} = \frac{2f}{3\Omega H a} \frac{g}{a^2 D} = \frac{2f}{3\Omega H a} \nabla_x \phi_{dm}. \quad (17)$$

Using this relation in the expression for the dark matter velocity potential $\phi_{v, dm}$, we obtain,

$$\nabla_x \phi_{v, dm} = \frac{v_{dm}}{a D} = -\frac{2f}{3\Omega H a^2 D} \nabla_x \phi_{dm}, \quad (18)$$

from which we find the linear regime relation between dark matter velocity potential $\phi_{v, dm}$ and the peculiar potential $\phi_{dm}$,

$$\phi_{v, dm} = -\frac{2f}{3\Omega H a^2 D} \phi_{dm}. \quad (19)$$

If we rearrange the effective potential by using equation (19),

$$V_{dm} = \frac{3\Omega \phi_{v, dm}}{2f^2 D} \left( \phi_{v, dm} + \frac{2f}{3\Omega H a^2 D} \phi_{dm} \right). \quad (20)$$

The immediate conclusion is that the effective dark matter potential $V_{dm}$,

$$V_{dm} = 0. \quad (21)$$

In fact, the conclusion of $V_{eff} = 0$ stretches out much further into the quasi-linear regime, for as long as the Zel’dovich formalism still describes the motion of matter elements in the Universe. Due to the vanishing effective potential in the EdS universe and, the linear density perturbations satisfying $\delta \ll 1$, the scaled Bernouille equation (12) turn into a relatively simple form.
6 MODELING DARK MATTER AND BARYON COMPONENTS

Using the inverse of the Madelung transformation proposed by Madelung (1927), we show that the dark and baryon matter components can be presented as complex scalar fields. The Madelung transformation is given as follows,

\[ \psi(x, D) = \sqrt{x} e^{i \phi(x)/\nu}, \]

in which the wave function \( \psi(x, D) \) is a complex quantity and \( \nu \) is the adjustable parameter, and has the same dimension as velocity potential \( \phi \). Here it is important to point out that in the original work of Madelung (1927) the main purpose was to model quantum fluid. That is why he chose the adjustable parameter as the Planck constant \( \hbar \). As is seen in the Madelung wave form \( \psi \), we do not adopt the quantum scales due to our interest of scale which is the large scale structure of the Universe. As a result of this, the wave forms that will be derived here are not related with quantum scales.

The density function \( \chi \) in equation (22) satisfies the relation \( \chi = \psi^* \cdot |\psi|^2 \). Hence the modified NLSE and the Poisson system of equations in which matter density obeys in the Madelung form are given as,

\[ i\nu \frac{\partial \psi}{\partial D} + \frac{\nu^2}{2} \nabla_x^2 \psi - A^2(D)\epsilon(|\psi|^2)\psi = \mathcal{P}\psi, \]

\[ \nabla_x^2 \phi(x, D) = 4\pi G\alpha^2 \rho \cdot |\psi|^2. \]

Here the nonlinear term \( \mathcal{P} \) on the right hand side of equation (23a) has the analogy with the infamous quantum potential/pressure derived by Bohm (1952) and its form is given as,

\[ \mathcal{P} = \frac{\nu^2}{2} \nabla_x^2 |\psi|^2 \]

In this study, we focus on the information potential interpretation of this nonlinear term as is suggested by Bohm et al. (1993) and Greenberger (1994). This interpretation allows us to discuss this nonlinear dynamical term from the perspective of the information potential \( \chi \). Hence, the fluid dynamical equations (11) and (12) appear be varying this Lagrangian functional. Following up on Parwani & Pashaev (2007), the Lagrangian functional in equation (25) of the classical equation of motion can be modified by introducing the Fisher measure \( I_F \) of information theory,

\[ L = i\nu \left( \psi^* \frac{\partial \psi}{\partial D} - \psi \frac{\partial \psi^*}{\partial D} \right) + \frac{\nu^2}{2} \nabla_x \psi^* \nabla_x \psi + V_{\text{ent}h}(\psi^* \psi) - \frac{\nu^2}{8} \left( \nabla_x |\psi|^2 \right)^2 \]

As is seen, the Lagrangian varies, with respect to scaled density \( \chi \) and the velocity potential \( \phi \). Hence, the fluid dynamical equations (11) and (12) appear be varying this Lagrangian functional. Following up on Parwani & Pashaev (2007), the Lagrangian functional in equation (25) of the classical equation of motion can be modified by introducing the Fisher measure \( I_F \) of information theory,

\[ \tilde{L} = L + \frac{\lambda^2}{8} I_F, \]

where \( I_F \) is the Fisher information measure and is defined as,

\[ I_F = \int dD dD \chi \left( \nabla_x \log \chi \right)^2 = 4 \int dD dD \left( \nabla_x \sqrt{\rho} \right)^2. \]

The main reason of adding the Fisher Information to the Lagrangian is to compare the intrinsic amount of information that is contained by matter component of the Universe and the information that we observe or gain by following the description and interpretation of Fisher information based on previous studies (Frieden 2004; Frieden et al. 2002; Cabezas & Fath 2002; Frank 2009). Here Fisher’s information measure \( I_F \) reflects the amount of information of the observer and it depends on the density \( \rho \). The density function \( \rho \) appears in this context because there is uncertainty in our knowledge of observing the matter distribution of the Universe as a whole. Therefore one may adopt the principle of maximum uncertainty to constrain the probability distribution \( \rho \) characterizing the ensemble: we would like to be as unbiased as possible in its choice, consistent with our lack of information. Hence we modify the Lagrangian by adding the Fisher measure, then the new Lagrangian becomes,

\[ L = i\nu \left( \psi^* \frac{\partial \psi}{\partial D} - \psi \frac{\partial \psi^*}{\partial D} \right) + \frac{\nu^2}{2} \nabla_x \psi^* \nabla_x \psi + V_{\text{ent}h}(\psi^* \psi) - \frac{\nu^2}{8} \left( \nabla_x |\psi|^2 \right)^2 \]

Here the constraint is implemented in the Lagrangian density \( \lambda^2/8 \) (Parwani & Pashaev 2007) which minimizes the classical action and is called the Lagrange multiplier. Taking into account the modified Lagrangian (28), the nonlinear Schrödinger type equation changes its form as,

\[ i\nu \frac{\partial \psi}{\partial D} + \frac{\nu^2}{2} \nabla_x^2 \psi - A^2(D)\epsilon(|\psi|^2)\psi = \frac{\nu^2 - \lambda^2}{2} \nabla_x^2 |\psi|^2 \psi. \]
Here $\nu$ and $\lambda$ are constant parameters. Note that the wave representation of the matter Madelung representation has contributions from dark matter $\Psi_{dm}$ and baryon $\Psi_b$ components. As a result, the matter wave function $\Psi_m$ becomes,

$$\Psi_m \equiv \Psi_{dm} + \Psi_b.$$  \hfill (30)

It is crucial to mention that, the equation of state of cosmological fluid [1] is characterized by the constitute of the Universe via the adiabatic parameter $\omega$. This leads to the different enthalpy parameters for different dominant constitutes of the Universe as a result different dynamical behaviors,

- Dark Matter: $\omega = 0$, $\epsilon (\chi_{dm}) = 0$,
- Baryon Component: $\omega > 0$, $\epsilon (\chi_b) = \omega \ln \chi_b = \omega \ln |\psi_b|^2$.

Therefore, depending on what component dominates the evolution of the Universe, the nonlinear equation can be split into the two dynamical systems,

$$i\nu \frac{\partial \psi_{dm}}{\partial \tilde{D}} + \frac{\nu^2}{2} \nabla^2 \psi_{dm} = \frac{\nu^2 - \lambda^2}{2} \nabla^2 |\psi_{dm}| \psi_{dm}.$$  \hfill (31)

$$i\nu \frac{\partial \psi_b}{\partial \tilde{D}} + \frac{\nu^2}{2} \nabla^2 \psi_b - A^2(\tilde{D}) \ln |\psi_b|^2 \psi_b = \frac{\nu^2 - \lambda^2}{2} \nabla^2 |\psi_b| \psi_b.$$  \hfill (32)

As is seen in equation (29), the relation between these parameters allows us to relate the evolution of the system with the information theory. Note that the scaled density

$$\chi^2 = \left( \frac{m_{d}}{\psi_{dm}} \right)^2 - \left( \frac{m_{b}}{\psi_{b}} \right)^2 = \frac{\nu^2 - \lambda^2}{2} \nabla^2 |\psi_{dm}|$$  \hfill (33a)

and baryon component,

$$\nabla^2 \phi_b(x, \tilde{D}) = 4\pi G a^2 \rho_u |\psi_{dm}|^2,$$  \hfill (33b)

where $\nu'$ is defined as,

$$\nu' \equiv \nu^2 \left( 1 - \frac{\lambda^2}{\nu^2} \right).$$  \hfill (35)

and Madelung wave transforms are,

$$\Psi_{dm} = \sqrt{\chi_{dm}} e^{i \phi_{dm}}, \quad \Psi_b = \sqrt{\chi_{b}} e^{i \phi_{b}}$$  \hfill (36)

6.1 $\lambda < \nu$ Schrödinger type wave mechanics: loss of self-organization

If the information that we observe from the system decreases, then the Fisher information becomes smaller than the self-organization of the system. In this case, $\lambda < \nu$, we obtain the Poisson- NLS type coupled equations for dark matter dark matter, which is,

$$i \frac{\partial \psi_{dm}}{\partial \tilde{D}} + \frac{1}{2} \nabla^2 \psi_{dm} = 0,$$  \hfill (33a)

$$\nabla^2 \phi_{dm}(x, \tilde{D}) = 4\pi G a^2 \rho_u |\psi_{dm}|^2,$$  \hfill (33b)

and baryon component,

$$i \frac{\partial \psi_b}{\partial \tilde{D}} + \frac{1}{2} \nabla^2 \psi_b - \omega^2 \frac{\nabla^2 (\tilde{D})}{\nu'} \ln |\psi_b|^2 \psi_b = 0,$$  \hfill (34a)

$$\nabla^2 \phi_b(x, \tilde{D}) = 4\pi G a^2 \rho_u |\psi_b|^2.$$  \hfill (34b)
in which $\tilde{\phi}_v = \frac{\phi_v}{\sqrt{\nu'}}$, $\tilde{D} = D\sqrt{\nu'}$ and $A^2(\tilde{D})$ becomes,

$$A^2(\tilde{D}) = \frac{\nu'}{\tilde{D}}.$$  \hfill (37)

### 6.2 $\lambda > \nu$ Reaction Diffusion/Heat type Dynamics: self-organization

On the other hand, if the information that we obtain from the dynamical system becomes higher than the system’s internal self-organization ($\lambda > \nu$), then instead of a complex dynamical form, we get the time reversal pair of reaction diffusion equations of the cosmological fluid. Here we use the special transformation by Madelung (1964) and Lee & Pashaev (1998),

$$Q^+_{dm}(x, \tilde{D}) \equiv \sqrt{\chi_{dm} e^{\tilde{\phi}_v}}, \quad Q^-_{dm}(x, \tilde{D}) \equiv \sqrt{\chi_{dm} e^{-\tilde{\phi}_v}},$$  \hfill (38)

$$Q^+_b(x, \tilde{D}) \equiv \sqrt{\chi_b e^{\hat{\phi}_v}}, \quad Q^-_b(x, \tilde{D}) \equiv \sqrt{\chi_b e^{-\hat{\phi}_v}},$$  \hfill (39)

in which $\hat{\phi}_v = \frac{\phi_v}{\sqrt{\nu''}}$ and $\hat{D} = D\sqrt{\nu''}$ in which $\nu''$ is defined as,

$$\nu'' \equiv \nu^2 \left( \frac{\lambda^2}{\nu^2} - 1 \right).$$  \hfill (40)

These real functions and the wave function of the NLS equation satisfy the following relations,

$$-Q^+ Q^- = \chi = \Psi \Psi^*, \quad \frac{Q^+}{Q^-} = \left( \frac{\Psi}{\Psi^*} \right)^i,$$  \hfill (41)

Similar to the loss of self-organization case, the contributions of dark matter and baryon contributions in the matter density distributions due to the superposition of the components,

$$Q^+_m = Q^+_{dm} + Q^+_{db}.$$  \hfill (42)

and considering the different enthalpy parameters for different dominant constitutes of the Universe, are taken into account. Therefore, the reaction diffusion system and the Poisson equation can be split into two different dynamical forms, for the dark matter,

$$\frac{\partial Q^+_{dm}}{\partial \tilde{D}} + \frac{1}{2} \nabla^2 Q^+_{dm} = 0,$$  \hfill (43a)

$$\frac{\partial Q^-_{dm}}{\partial \tilde{D}} + \frac{1}{2} \nabla^2 Q^-_{dm} = 0,$$  \hfill (43b)

$$\nabla^2 \phi_{dm}(x, \tilde{D}) = 4\pi Ga^2 \rho_u (-Q^+_{dm} Q^-_{dm}),$$  \hfill (43c)

and the baryon components,

$$\frac{\partial Q^+_b}{\partial \tilde{D}} + \frac{1}{2} \nabla^2 Q^+_b - \frac{\omega}{\nu''} A^2(\hat{\tilde{D}}) \ln |Q^+_b| Q^-_b = 0,$$  \hfill (44a)

$$\frac{\partial Q^-_b}{\partial \tilde{D}} + \frac{1}{2} \nabla^2 Q^-_b - \frac{\omega}{\nu''} A^2(\hat{\tilde{D}}) \ln |Q^-_b| Q^+_b = 0,$$  \hfill (44b)

$$\nabla^2 \phi_b(x, \tilde{D}) = 4\pi Ga^2 \rho_u (-Q^+_b Q^-_b),$$  \hfill (44c)

in which $A(\hat{\tilde{D}})$ is defined as,

$$A(\hat{\tilde{D}}) = \frac{\nu''}{\hat{D}}.$$  \hfill (45)

so fluid equations (1), (2) or (12) can also be written as the reaction diffusion systems which are the analog of (23a).

Here we represent the decoupling reaction diffusion systems of the two different cosmic components. Above the reaction diffusion equations with negative signs (43b) and (44b) are the time reversible of the positive signed reaction diffusion equations. They are crucial for the existence of Hamiltonian structure and the integrable system [Lee & Pashaev 1998, Pashaev & Lee]
Note that the reaction diffusion systems are scaled by following up on Lee & Pashaev (1998); Pashaev & Lee (2002) and in this way the contribution of the Fisher information is hidden in the equations via transformations.

7 EXAMPLE A: DARK MATTER DYNAMICS IN EDS UNIVERSE AT $\delta \ll 1$ AND $\lambda < \nu$

Here, we provide an analytical solution in the case of loss of self-organization $\lambda < \nu$ to model dark matter as an example. In this case, the dark matter evolution can be modeled by the Schrödinger type wave mechanical approach. In the EdS Universe in terms of the linear regime, the nonlinear Schrödinger type wave equation of the dark matter component is reduced to the free particle Schrödinger equation as in the system (33),

$$i \frac{\partial \psi_{dm}}{\partial \mathcal{D}} + \frac{1}{2} \nabla_{x}^2 \psi_{dm} = 0,$$

$$\nabla_{x}^2 \phi(x, \mathcal{D}) = 4\pi G a^2 \rho_{u}.$$

To obtain an exact solution we use the exact solutions to the three dimensional time dependent Schrödinger equation by Chand & Mishra (2007). Exact solutions that Chand & Mishra (2007) used are based on the group transformation method introduced by Burgan et al. (1979). Based on this method, here we obtain the well-known solution of three-dimensional Schrödinger equation, which is given by,

$$\psi'(q, \mathcal{D}) = ABC e^{-i\mathcal{D}} \sin\left(\frac{\pi}{d_1} n_1 q_1\right) \sin\left(\frac{\pi}{d_2} n_2 q_2\right) \sin\left(\frac{\pi}{d_3} n_3 q_3\right).$$

Here the initial location $q$ is the Lagrangian coordinate of the matter element and moves along the path $x(q)$ and $\gamma(q)$ is the displacement potential field. Also, $ABC$ indicates the normalization constant for each spatial dimension while the values of $n_i$’s ($i = 1, 2, 3$) are three integer numbers and they characterize the solution and, $E$ refers to the total energy. The mathematical description of the energy is given by,

$$E = E_1 + E_2 + E_3 = E_0 \left(\frac{n_1^2}{d_1^2} + \frac{n_2^2}{d_2^2} + \frac{n_3^2}{d_3^2}\right),$$

in which $E_0 = \frac{\pi^2}{2}$. Now one may extend this into the general solution to,

$$\Psi(x, y, z, \mathcal{D}) = ABC e^{i\mathcal{D}} \left(\frac{\mathcal{D}}{E_0} - \frac{\mathcal{D}}{E_0}\right) \sin\left(\frac{\pi}{d_1} n_1 (x + \mathcal{D} \nabla \gamma(q_1))\right) \sin\left(\frac{\pi}{d_2} n_2 (y + \mathcal{D} \nabla \gamma(q_2))\right) \sin\left(\frac{\pi}{d_3} n_3 (z + \mathcal{D} \nabla \gamma(q_3))\right).$$

In Fig. 1, the evolution of the dark matter component is shown in terms of the linear growth factor $\mathcal{D}$. As is seen from Fig. 1 the scaled density $\chi_{dm}$ of the dark matter component increases with increasing growth factor $\mathcal{D}$. This indicates that dark matter density becomes prominent at the present day $\mathcal{D} = 1$ (or $z = 0$).

8 EXAMPLE B: BARYON DYNAMICS IN EDS UNIVERSE AT $\delta \ll 1$ AND $\lambda > \nu$

Similar to its fluid dynamical counterpart, there are two possible wave mechanical descriptions of the baryon component. Here we provide a solution for the case of self-organization of the baryon component $\lambda > \nu$, and as such it can be modeled by a reaction diffusion system in the EdS universe,

$$\frac{\partial Q^+_b}{\partial \mathcal{D}} + \frac{1}{2} \nabla_{x}^2 Q^+_b - A^2(\mathcal{D}) \omega \ln |Q^+_b Q^-_b| Q^+_b = 0,$$  

$$\frac{\partial Q^-_b}{\partial \mathcal{D}} + \frac{1}{2} \nabla_{x}^2 Q^-_b - A^2(\mathcal{D}) \omega \ln |Q^+_b Q^-_b| Q^-_b = 0,$$

$$\nabla_{x}^2 \phi_b(x, \mathcal{D}) = 4\pi G a^2 \rho_{u}.$$

It is a well known fact that in the linear regime and mildly linear regime the density perturbations become small $\delta \ll 1$. Taking into account this fact, we can organize the logarithmic nonlinear dispersion term $\ln |Q^+_b Q^-_b|$ in the reaction diffusion system (50). Based on the linear regime with the density perturbations $\delta \ll 1$, the series expansion of the logarithmic function around the point $\chi_b \ll 1$ for the dispersion parameter in the system (50) is obtained as,
Figure 1. Contour (left) and three-dimensional (right) plots of the evolution of the scaled dark matter density function $|\Psi(x, y, z, D)|^2 = \chi_{dm}$ based on equation (49) in terms of the growth factor $D$ by choosing parameters as $d_1 = 50$, $d_2 = 50$, $d_3 = 50$, $n_1 = 5$, $n_2 = 5$ and $n_3 = 1$.

\[
\ln |Q^+Q^-| = \ln \chi_b = (\chi_b - 1) - \frac{1}{2}(\chi_b - 1)^2 + \frac{1}{3}(\chi_b - 1)^3 - \ldots = \delta_b - \frac{\delta_b^2}{2} + \frac{\delta_b^3}{3} - \frac{\delta_b^4}{4} + \frac{\delta_b^5}{5} - \ldots
\] (51)

Here higher order terms can be omitted in the linear regime due to $\delta \ll 1$. Therefore, logarithmic nonlinearity is reduced to,

\[
\ln |Q^+Q^-| = \ln \chi_b \approx \delta_b.
\] (52)

Hence, the reaction diffusion system of the baryon component is reduced to a relatively simple form in the EdS universe,

\[
\begin{align*}
\frac{\partial Q^+}{\partial D} + \frac{1}{2} \nabla^2 Q^+ - A^2(D) \frac{\omega}{\nu''}(Q^+_b Q^-_b + 1) Q^+_b &= 0, \\
\frac{\partial Q^-}{\partial D} + \frac{1}{2} \nabla^2 Q^- - A^2(D) \frac{\omega}{\nu''}(Q^+_b Q^-_b + 1) Q^-_b &= 0,
\end{align*}
\] (53a)

This system of equations can be solved by introducing the new functions as particular solutions,

\[
Q^+_b = Q^{+}_b e^{\int \delta \frac{A^2(\eta)d\eta}{\nu''}} Q^{-}_b = Q^{-}_b e^{-\int \delta \frac{A^2(\eta)d\eta}{\nu''}}.
\] (54)

Then system of equations (53) is reduced the following form,

\[
\begin{align*}
\frac{\partial Q^+}{\partial D} + \frac{1}{2} \nabla^2 Q^+ - A^2(D) \frac{\omega}{\nu''} Q^+ Q^- &= 0, \\
\frac{\partial Q^-}{\partial D} + \frac{1}{2} \nabla^2 Q^- - A^2(D) \frac{\omega}{\nu''} Q^- Q^+ &= 0.
\end{align*}
\] (55a)

Our approach is to find the solution of (55) by following the method developed by Hirota (1971). This solution leads us to the solution of equation (53). When we apply the Hirota direct method to the reaction diffusion equation, the soliton solutions are obtained. The soliton solutions of the reaction diffusion system admit the exponentially growing and decaying components known as dissipatons. In this study to provide some illustrations, we then obtain one- and two- soliton solutions of the reaction diffusion system (53) in order to show the evolution of baryon density $\chi_b$ in the case of self-organization. The one-soliton solution of the RD system has a relatively simple form,

\[
\chi_b = \frac{\nu''}{A^2 \omega \cosh^2 (k_x x + k_y y + \tilde{\Omega}D + \beta + \eta(0))}.
\] (56)

where $k_x$ and $k_y$ are represented as amplitude of the wave while $\zeta_x$ and $\zeta_y$ are velocities of the dissipative soliton,
Figure 2. Contour plots of the evolution of the scaled baryon density function $\chi$ represented by the one-soliton solution of the reaction diffusion system \( \text{Eq.} \) \( \text{53} \) from redshift $z = 1$ to present-day redshift value $z = 0$ (from left to right).

\[
\begin{align*}
\bar{\Omega} & \equiv \frac{\sqrt{\nu''}}{2} [k_x \zeta_x + k_y \zeta_y], \\
\beta & \equiv \frac{1}{2} \ln \left[ \frac{A^2 \omega}{4 \nu'' k_x^2 + k_y^2} \right], \\
k_x & \equiv \frac{k_1^+ + k_1^-}{2}, \quad k_y \equiv \frac{m_1^+ + m_1^-}{2}, \\
\zeta_x & \equiv k_1^- - k_1^+, \quad \zeta_y \equiv m_1^- - m_1^+.
\end{align*}
\]

The scaled density function shows the perfect soliton wave shape presented as filamentary type structure. When we change the parameters $k_1^+$ and $m_1^+$ we can see the different type of soliton wave types structures which bear a striking geometric resemblance to the filaments of the Cosmic Web. The one-soliton solution provides the information of the distribution of the scaled density of the baryon component in terms of the expanding scale factor or linear growth factor $a(t) = D$, in other words, it shows the evolution of the scaled density function in the EdS Universe in Fig. 2 and Fig. 3. Figs 2 and 3 demonstrate the one-soliton solution of the scaled baryon density in which the density is plotted by choosing the adiabatic parameter $\omega = 5/3$ and the perturbation coefficients as $k_1^+ = 0.9, k_1^- = -0.5, m_1^+ = 0.3, m_1^- = -1.1, \eta^+(0) = -5$ and $\eta^-(0) = 8$. In addition to the one-soliton solution, here we provide the two-soliton solution of the scaled baryon in order to show how baryon component can show intricate structures with increasing order of soliton solutions in Fig. 4 and Fig. 5 in the case of self-organization. In Fig. 4 and Fig. 5 the perturbation parameters are chosen as $\omega = 5/3, k_1^+ = 0.52, k_1^- = -0.2, k_2^+ = 0.4, k_2^- = -0.41, m_1^+ = 0.2, m_1^- = -0.01, m_2^+ = 0.7, m_2^- = -1.6, \eta_1^+(0) = -6.5, \eta_1^-(0) = 4, \eta_2^+ = -6.5$ and $\eta_2^- = 4$. As is seen in Fig. 4 and Fig. 5 the scaled baryon density slightly increases with increasing redshift evolution. In the concept of large scale structure, this may indicate that the matter clumps merge through the bridges into the filaments. In the late time steps, these matter bridges become denser due to the merging of the matter into the high density regions/lumps in the linear regime.

8.1 Conclusion

In this study we introduce two methodologies in order to deal with the dynamical evolution of dark and baryon components of the Universe. This is achieved by obtaining the Schrödinger type and reaction diffusion dynamical forms by applying the two different Madelung transformations, based on the study of [Madelung (1964)], to the cosmological fluid dynamical equations. Following this, here the quantum potential/pressure term is named as self-organization component of cosmic components based on the suggestions of [Bohm et al. (1993); Greenberger (1994)]. Note that since this self organization component emerges out of the large scale cosmological fluid dynamical equations by using the Madelung transformation in the macroscopic scales, it is not the quantum potential. Then the Fisher information measure is introduced and compared to the nonlinear self-organization component of the systems via the lagrange constraint $\lambda$ of the Fisher measure and dispersion coefficient $\nu$. Following the studies discussing and deriving the dynamical importance of the Fisher information measure, here we show that Fisher information measures the dynamical behavior of the cosmological fluid by indicating self-organization or loss of self-organization depending on the comparison with the system’s itself. Processing from this fact, the two different dynamical forms of the dark matter and baryon components are constructed in relatively simple frameworks in the linear regime due to the vanishing effective potential. As a result, in the case of loss of self-organization $\lambda < \nu$, the dark matter wave equation is reduced to the free
Figure 3. Three-dimensional plots of the evolution of the scaled baryon density function $\chi$ represented by the one-soliton solution of the reaction diffusion system (53) from redshift $z=1$ to presentday redshift value $z=0$ (from left to right).

Figure 4. Contour plots of the baryon density in terms of the two-soliton solution based on system (53) at redshift $z=1$ and $z=0$ (from left to right).

Figure 5. Three-dimensional plots of the baryon density in terms of the two-soliton solution based on system (53) at redshift $z=1$ and $z=0$ (from left to right).
particle Schrödinger equation in the linear regime, while the baryon matter wave equation shows the special kind of nonlinear differential equation called the log-law nonlinear Schrödinger equation. On the other hand, when the Fisher information measure increases $\lambda > \nu$, the dark matter presents the heat system of wave equations while the baryon component obeys the coupled reaction diffusion system with log-law nonlinearity. Here, to provide some examples we present analytical solutions of dark and baryon matter components in different dynamical characteristics given by Fisher information cases. The dark matter Schrödinger type wave form is solved by using the particle in a box method and it is shown that there are some similarities between the Zel'dovich formalism and this dark matter wave form. To solve the coupled reaction diffusion system with log-law nonlinearity, we use a special methodology called the Hirota direct method. Due to the nature of the Hirota method, we obtain perturbative solutions of the baryon component. When we increase the order of the perturbations in the Hirota method from one-soliton solution to $N$-soliton solutions, these waves show striking similarity to the intricate structure of filamentary type features of the Cosmic Web in $2 + 1$ dimensions in the EdS Universe in the linear regime.

REFERENCES

Arbey A., Lesgourgues J., Salati P., 2003, Phys. Rev. D, 68, 023511
Blaut A., Kowalski-Glikman J., 1998, Physics Letters A, 245, 197
Bohm D., 1952a, Physical Review, 85, 166
Bohm D., 1952b, Physical Review, 85, 180
Bohm D., Hiley B. J., Holland P., 1993, Nature, 366, 420
Bohmer C. G., Harko T., 2007, Journal of Cosmology and Astroparticle Physics, 6, 25
Bond J. R., Kofman L., Pogosyan D., 1996, Nature, 380, 603
Boubaker K., Bhrawy A. H., 2012, Advances in Space Research, 49, 1062
Boubaker K., Van Gorder R. A., 2012, New Astronomy, 17, 565
Burgan J. R., Feix M. R., Fijalkow E., Munier A., 1979, Physics Letters A, 74, 11
Cabezas H., Fath B. D., 2002, Fluid Phase Equilibria, 194, 3
Capozziello S., de Laurentis M., Odintsov S. D., Stabile A., 2011, Phys. Rev. D, 83, 064004
Chand F., Mishra S. C., 2007, Pramana, 68, 891
Chandrasekhar S., 1939, ApJ, 89, 116
Chavanis P. H., 2012, A&A, 537, A127
Chavanis P.-H., Harko T., 2012, Phys. Rev. D, 86, 064011
Coles P., 2002, MNRAS, 330, 421
Coles P., Spencer K., 2003, MNRAS, 342, 176
da Rocha D., Nottale L., 2003, Chaos Solitons and Fractals, 16, 565
Davies G., Widrow L. M., 1997, ApJ, 485, 484
Frank S. A., 2009, ArXiv e-prints
Frieden B. R., 2004, Science from Fisher information : a unification
Frieden B. R., Plastino A., Plastino A. R., Soffer B. H., 2002, Phys. Rev. E, 66, 046128
Fukuyama T., Morikawa M., Tatekawa T., 2008, Journal of Cosmology and Astroparticle Physics, 6, 33
Goodman J., 2000, New Astronomy, 5, 103
Greenberger D. M., 1994, Science, 266, 147
Guzmán F., Ureña-López L., 2003, Phys. Rev. D, 68, 024023
Harko T., 2011a, Phys. Rev. D, 83, 123515
Harko T., 2011b, MNRAS, 413, 3095
Herbst R. S., Momoniat E., 2012, New Astronomy, 17, 388
Hiley B. J., Callaghan R. E., Maroney O., 2000, eprint arXiv:quant-ph/0010020
Hiley B. J., Maroney O. J. E., 2000, eprint arXiv:quant-ph/0009056
Hirot a R., 1971, Physical Review Letters, 27, 1192
Johnston R., Lasenby A. N., Hobson M. P., 2010, MNRAS, 402, 2491
Jones B. J. T., 1999, MNRAS, 307, 376
Kofman L. A., Shandarin S. F., 1988, Nature, 334, 129
Kofman L. A., Shandarin S. F., 1988, Nature, 334, 129
Lan D. M., 1999, Modern Physics Letters A, 14, 2667
Lee J.-H., Pashaev O. K., 1998, Journal of Mathematical Physics, 39, 102
Lee J.-W., Koh I.-G., 1996, Phys. Rev. D, 53, 2236
Madelung E., 1927, Zeitschrift für Physik, 40, 322
Madelung E., 1964, Die mathematischen Hilfsmittel des Physikers
Marsh D. J. E., Ferreira P. G., 2010, Phys. Rev. D, 82, 103528
Parvani R. R., Pashaev O. K., 2007, ArXiv e-prints
Pashaev O. K., Lee J.-H., 2002, Modern Physics Letters A, 17, 1601
Peebles P. J. E., 1980, The large-scale structure of the universe. Princeton University Press
Peebles P. J. E., 2000, ApJ, 534, L127
Rindler-Daller T., Shapiro P. R., 2012, MNRAS, 422, 135
Short C. J., Coles P., 2006a, Journal of Cosmology and Astroparticle Physics, 12, 12
Short C. J., Coles P., 2006b, Journal of Cosmology and Astroparticle Physics, 12, 16
Spiegel E. A., 1980, Physica D Nonlinear Phenomena, 1, 236
Szapudi I., Kaiser N., 2003, ApJ, 583, L1
Widrow L. M., Kaiser N., 1993, ApJ, 416, L71
Woo T.-P., Chiueh T., 2009, ApJ, 697, 850
Zeldovich Y. B., 1972, MNRAS, 160, 1P

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.