Baryogenesis via Lepton Number Violation and Family Replicated Gauge Group

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Abstract

We developed a previous model fitting all quark and lepton – including neutrino quantities in the region of the Large Mixing Angle-MSW solar solution – order of magnitudewise by only six adjustable parameters (Higgs vacuum expectation values) to also give an agreeing prediction for the amount of baryogenesis produced in the early time of cosmology. We use Fukugita-Yanagida scheme and take into account now also the effect of the renormalisation equation for the Dirac neutrino sector from Planck scale to the see-saw scale. The present version of our model with many approximately conserved (gauge) charges distinguishing various left- and right-Weyl particles has the largest matrix element of the mass matrix for the three flavour see-saw neutrinos being the off-diagonal elements associated with the second- and third-proto-flavour and gives the ratio of baryon number density to the entropy density to $2.59^{+17.0}_{-2.25} \times 10^{-11}$ which agrees perfectly well as do also all the fermion masses and their mixing angles order of magnitudewise.

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1 Introduction

Recently we have improved a rather specific model [1] seeking to fit /explain quark and lepton masses and mixing angles using as the reason for the large mass ratios and usually small mixing angles, except two neutrino mixing angles, approximately conserved quantum numbers invented for that purpose into our model. The approximately conserved quantum numbers – which are really assumed to be gauged, although that may not be so crucial – are supposed to be broken by various Higgs fields. We could get a good fit of all the masses and mixing angles. However, this previous version of our model fails by providing too little baryogenesis to be produced in the early Big Bang. To calculate the baryogenesis we use the Fukugita-Yanagida scheme for producing a $B - L$ excess at the time when the temperature of the Universe passes the scale of the see-saw neutrino masses; in fact the see-saw mechanism for neutrinos masses is incorporated into our model in as far as we have three right-handed neutrinos having Majorana masses acquired via a certain Higgs vacuum expectation value (VEV), $\langle \phi_{B-L} \rangle$.

In such model(s) we can play around with the system of charges, but most importantly we have played with developing the charge assignments of the Higgs fields breaking the (gauge) group so that these charges are only approximately conserved. It is the detailed quantum numbers of the speculated Higgses that we used to develop to fit more and more experimental informations. In the last improvement we got the model to predict Large Mixing Angle-MSW [2] (LMA-MSW) solution rather than Small Mixing Angle-MSW (SMA-MSW) solution by replacing the two of our Higgs fields which are able to cause transitions between first and second generation quarks and leptons by a new pair which actually turned out to have more elegant quantum numbers.

As we are now fitting all the masses and mixing angles, the constraints on our model are so tight that we hardly can change it anymore. However, we shall see below – and that is the main point of the present article – that we could still change the quantum numbers of the Higgs field delivering the over all scale of the see-saw neutrino masses.

The difficulty of getting enough baryons with our type of model hangs together with that even the largest element in the (Dirac) neutrino matrix is suppressed compared to unity by a factor $10^{-2}$. Only our up-type quark and right-handed neutrino mass matrix have order of unity elements. Models with two Weinberg-Salam Higgs doublets can on the contrary have a $\tan \beta$ that is large and give suppressions to the down-type quark and charged lepton masses so that the Yukawa couplings for these (lepton and down-type quark) particles could be as large as of order unity. That is to say two Higgs doublet models may in general tend to get stronger Yukawa couplings and thereby easier CP violation since the latter comes only in one-loop accuracy.

It means that we are in need for possible enhancements for baryogenesis. The possibility for that which it actually turns out that we can obtain very elegantly in our model is to make the mass difference between some of the see-saw neutrinos smaller than in the previous model. In fact the CP violating asymmetry parameter is essentially inversely proportional to the mass difference between the see-saw neutrino producing the asymme-
try in its decay and another see-saw neutrino. Due to the self-energy contribution almost
degenerate neutrinos can even enhance the asymmetry by orders of magnitude compared
to what it would be with the right-handed neutrino mass ratios of order unity. It would
therefore be a progress in the direction of producing enough baryons to put two of our
see-saw neutrinos almost degenerate in mass. It is very natural to obtain this situation
by letting an off-diagonal element in the see-saw neutrino mass matrix dominate. That
may be done by adjusting the quantum numbers of that Higgs field $\phi_{B-L}$ which gives the
masses and the mass scale of the see-saw neutrinos so that it causes the transition from
one flavour to another.

This article is organised as follows: in the next section, we present general assumptions
for the model building, and we review our model – the family replicated gauge group
model. In section 3 we define our notation for the fermion mass terms in the Lagrangian
and their mixing angles. Then, in section 4 the renormalisation group equations of all
sectors are presented. The calculation is described in section 5 and the results for the
fermion masses and mixing angles are presented in section 6. In section 7 we discuss the
Fukugita-Yanagida scheme of baryon number production, and in section 8 the wash-out
of the produced $B-L$ excess is approximated and our results of the calculation are given
in section 9. A discussion on the proton decay goes into section 10. Finally, section 11
contains our conclusions.

2 Model with many mass protecting charges

2.1 General feature

The main point of the model we use here is that the “small hierarchies” of the quark
and lepton masses are due to approximately conserved quantum numbers [3] preventing
the masses from being different from zero, i.e., that the Yukawa couplings which are
in Standard Model mysteriously small should really be understood as being so due to
that they need breaking of some charge conservation. In fact we consider these Yukawa
couplings just some effective couplings representing at the fundamental level more com-
plicated vertex diagrams involving attachments to the VEVs of Higgs fields breaking the
charge conservations in question (see Fig. 1).

We summarise here our philosophy to build a model that can predict, rather say fits
by it having the all fermion quantities including not only the neutrino oscillations but
also baryogenesis in following ingredients:

- All fundamental coupling constants and masses are of order unity.

All gauge and Yukawa couplings are of order one at the Planck scale, in fact, we
take the fundamental scale as the Planck one. From this point of view we treat
them as random numbers, and partly to be able to “explain” $CP$ violation
we assume that all Yukawa couplings are also complex random numbers. Also
the masses of particles should be of order Planck scale unless they are mass protected, so as to have masses determined by the VEVs of Higgs fields say.

- Only VEVs are allowed far from order one.

Thus also the scalar field masses corresponding to the VEVs are allowed to be very small, since otherwise it would be inconsistent to have $M_{H}^{2} = \lambda \langle \phi_{WS} \rangle$ unless $\lambda$ has huge value.

This assumption could gain supports from

- the well-known weak scale being extremely small compare to the Planck scale.
- dynamical symmetry e.g. Nambu-Jona-Lasinio [4] can lead to equation for the logarithm of the VEV scale.
- in superconductivity VEVs tend very small compared to the atomic dimensionality argument.

- There are a priori several conserved charges and the various Weyl components of quarks and leptons have usually different charges, i.e., quantum numbers.

Since we let Weyl components of different families have in general different quantum numbers of these kinds, they are to be considered horizontal, but they should better not bring together into multiplets different families because that would make the remarkably large inter-family mass ratios difficult to incorporate (unless the group is strongly broken).

- Incorporate the see-saw mechanism [5].

A new scale must be introduced into the model. In our case we assume a new scale about $10^{10}$ GeV. This scale becomes crudely the masses of right-handed (Majorana) neutrinos, and therefore $B - L$ quantum charge being broken at that energy scale.

Figure 1: Diagram giving the effective Standard Model Dirac Yukawa coupling. Here Weinberg-Salam Higgs field is denoted as $\phi_{WS}$. 

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Note that we do not assume the existence of Supersymmetry or Grand Unification. However, our prediction came out that $SU(5)$ mass relations are order of magnitude-wise valid due to the diagonal elements of different mass matrices need same quantum numbers being broken in our specific model.

### 2.2 Our specific model of many conserved charges

We have already investigated a model \([1, 6]\) which can predict all quark and charged lepton quantities including baryogenesis\(^1\). This model is of the type described in the foregoing subsection with the further good restriction that the quantum numbers for the Weyl particles are so chosen that they obey the anomaly cancelation conditions required for them to be imagined gauged charges. It is namely easily seen that our gauge group has no anomalies – neither gauge nor mixed ones – because it has the Standard Model group plus a $B - L$ separately for each family. In fact our model has the specific group

$$
\times_{i=1,2,3} (SMG_i \times U(1)_{B-L,i}) ,
$$

where $SMG_i \equiv SU(3)_i \times SU(2)_i \times U(1)_i$ denotes the Standard Model gauge group for each generation $(i = 1, 2, 3)$; $\times$ means the Cartesian product. However, it is unlikely to be important to have this special group, provided we have sufficiently many conserved charges separating in quantum number the various Weyl fields. Actually the important thing is that one by means of the quantum numbers can construct some Higgs fields as we propose them for the breaking of the group to the Standard Model with much the same powers of fields being needed in the various mass matrix elements. The latter may in fact not be difficult to obtain with other groups, at least the non-abelian part of our proposed group is totally irrelevant for the fitting.

We should emphasize that due to the non-zero neutrino masses using the see-saw picture, it is necessary to introduce a right-handed neutrino, or preferably several – we have three in our model – having Majorana masses. In our model we associate the see-saw mass scale with a gauged $B - L$ charge. We let the latter come from the diagonal subgroup of the Cartan product of a $U(1)_{B-L,i}$ $(i = 1, 2, 3)$ for each family.

Note that this family replicated gauge group, eq. (1), is the maximal gauge group under the following assumptions:

1. Considering only part of the gauge group acting non-trivially on the known 45 Weyl fermions of the Standard Model and the additional three heavy see-saw (right-handed) neutrinos, \(i.e.,\) our gauge group is a subgroup of $U(48)$.

2. Avoid gauge transformations transforming a Weyl state from one irreducible representation of the Standard Model group into another irreducible representation,

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\(^1\)The model \([6]\) provided only the SMA-MSW solution. A main point of present article is to calculate baryogenesis with LMA-MSW solution predicting mass matrices including running effects of all sectors (see sections \([4, 7-9]\).
i.e., there is no place for the quarks and leptons occupying the same irreducible representation under our gauge group.

(3) No gauge nor mixed anomalies due to the renormalisability requirement.

2.3 Description of quantum numbers of mass protected particles in model

In addition to the practically any non-mass-protected particles, which we have already assumed to be present with Planck scale masses, we have as is easily seen some mass protected particles. The Weyl particles without chiral partners in our model are the well-known Weyl components of the quarks and leptons in the Standard Model plus three see-saw (right-handed) heavy neutrinos, which are all easily seen to be mass protected by our gauge group eq.(4). These 48 Weyl particles have lower mass than the Planck scale. They fall into three families each consisting of the 16 Weyl particles of a usual Standard Model generation plus one see-saw particle. In this way we can label these particle as proto-left-handed or proto-right-handed $u$-quark, $d$-quark, electron etc. To get the quantum numbers under our model gauge group for a given proto-irreducible representation, we proceed in the following way: We note the generation number $i$ ($i = 1, 2, 3$) of the particle for which we want quantum numbers and we look up, in the Standard Model, what are the quantum numbers of the irreducible representation in question and what is the $B - L$ quantum number. Then we take trivial quantum numbers for the two generations $j$ different from $i$ and put the index $i$ to the gauge group’s names, so we put the representations of the groups $SU(3)_i$, $SU(2)_i$ and $U(1)_i$ equal to the just asked quantum numbers.

For instance, if we want to find the quantum numbers of the proto-left-handed bottom quark, we note that the quantum numbers of the left-handed bottom quark in the Standard Model are weak hypercharge $y/2 = 1/6$, doublet under $SU(2)$ and triplet under $SU(3)$, while $B - L$ is equal to the baryon number $= 1/3$. Moreover, ignoring mixing angles, the generation is denoted as number $i = 3$. The latter fact means that all the quantum numbers for $SMG_j$ $j = 1, 2$ are trivial. Also the baryon number minus lepton number for the proto-generation number one and two are zero: only the quantum numbers associated with proto-generation three are non-trivial. Thus, in our model, the quantum numbers of the proto-left-handed bottom quark are $y_3/2 = 1/6$, doublet under $SU(2)_3$, triplet under $SU(3)_3$ and $(B - L)_3 = 1/3$. For each proto-generation the following charge quantisation rule applies

$$t_i^3 + d_i^2 + y_i^2 = 0 \pmod{1},$$

where $t_i$ and $d_i$ are the triality and duality for the $i$'th proto-generation gauge groups $SU(3)_i$ and $SU(2)_i$ respectively.

Combining eq. (2) with the principle of taking the smallest possible representation of the groups $SU(3)_i$ and $SU(2)_i$, it is sufficient to specify the six Abelian quantum numbers $y_i/2$ and $(B - L)_i$ in order to completely specify the gauge quantum numbers of the fields, i.e. of the Higgs fields and fermion fields. Using this rule we easily specify the fermion
representations as in Table 2.3. However, as already mentioned in section 2.2 these non-abelian $SU(3)_i$ and $SU(2)_i$ are really not important for our fit since they just follow the weak hypercharges like “slaves”.

Note that fermion quantum numbers for each proto-generation gauge group $SMG_i \times U(1)_{B-L,i}$ are obtainable as considering it a subgroup of $SO(10)$, i.e. our gauge group eq. (1) is really a subgroup of $SO(10)^3$. However, we avoid the not well-agreeing accurate predictions of simplest grand unification by not having neither $SU(5)$’s nor $SO(10)$’s: the mass spectra, and also proton decay.

|          | $SMG_1$ | $SMG_2$ | $SMG_3$ | $U_{B-L,1}$ | $U_{B-L,2}$ | $U_{B-L,3}$ |
|----------|---------|---------|---------|-------------|-------------|-------------|
| $u_L, d_L$ | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $u_R$ | 0 | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $d_R$ | $-\frac{1}{3}$ | 0 | 0 | 0 | 0 | 0 |
| $e_L, \nu_{eL}$ | $-\frac{1}{2}$ | 0 | 0 | $-1$ | 0 | 0 |
| $e_R$ | $-1$ | 0 | 0 | $-1$ | 0 | 0 |
| $\nu_{eR}$ | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $c_L, s_L$ | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $c_R$ | 0 | $\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $s_R$ | 0 | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $\mu_L, \nu_{\mu L}$ | 0 | $-\frac{1}{3}$ | 0 | 0 | $-1$ | 0 |
| $\mu_R$ | 0 | $-1$ | 0 | 0 | $-1$ | 0 |
| $\nu_{\mu R}$ | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $t_L, b_L$ | 0 | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ |
| $t_R$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ |
| $b_R$ | 0 | 0 | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ |
| $\tau_L, \nu_{\tau L}$ | 0 | 0 | $-\frac{1}{3}$ | 0 | 0 | $-1$ |
| $\tau_R$ | 0 | 0 | $-1$ | 0 | 0 | $-1$ |
| $\nu_{\tau R}$ | 0 | 0 | 0 | 0 | $-1$ | 0 |
| $\omega$ | $\frac{1}{6}$ | $-\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $\rho$ | 0 | 0 | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 |
| $W$ | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $T$ | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 | 0 | 0 |
| $\chi$ | 0 | 0 | 0 | 0 | $-1$ | 1 |
| $\phi_{WS}$ | 0 | $\frac{2}{3}$ | $-\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ |
| $\phi_{B-L}$ | 0 | 0 | 0 | 0 | 1 | 1 |

Table 1: All $U(1)$ quantum charges for the proto-fermions and Higgs fields in the model.
2.4 Higgses breaking the family replicated gauge group to the Standard Model

The gauge group \( \times_{i=1,2,3}(SMG_i \times U(1)_{B-L,i}) \) is at first spontaneously broken down at one or two orders of magnitude below the Planck scale, by 5 different Higgs fields, to the gauge group \( SMG \times U(1)_{B-L} \) which is the diagonal subgroup of the original one:

\[
\times_{i=1,2,3}(SMG_i \times U(1)_{B-L,i}) \rightarrow \left( \times_{i=1,2,3}(SMG_i \times U(1)_{B-L,i}) \right)_{\text{diagonal}} \equiv SMG \times U(1)_{B-L} .
\]  

(3)

This diagonal subgroup is further broken down by yet two more Higgs fields — the Weinberg-Salam Higgs field \( \phi_{WS} \) and another Higgs field \( \phi_{B-L} \) — to \( SU(3) \times U(1)_{em} \). The VEV of the \( \phi_{B-L} \) Higgs field is taken to be about \( 10^{10} \) GeV and is designed to break the gauged \( B-L \) quantum number. In other words the VEV, \( \langle \phi_{B-L} \rangle \), gives the see-saw scale (All \( U(1) \) quantum charged for the proto-fermions and Higgs fields are presented in Table 2.3).

We should emphasise here that the quantum numbers of the \( \phi_{B-L} \) has been changed relative to our previous publication due to the baryogenesis point of view. The point is that baryogenesis calculated the way described below with the “old” quantum charge of \( \phi_{B-L} \):

\[
\vec{Q}_{\phi_{B-L}} \bigg|_{\text{old}} = (0, 0, 0, 0, 0, 2)
\]  

(4)

would produce insufficient amount of baryon minus lepton number, whereas we shall see that quantum numbers making the off-diagonal elements \((2, 3)\) and \((3, 2)\) of the right-handed neutrino mass matrix be the dominant ones as in Table 2.3:

\[
\vec{Q}_{\phi_{B-L}} = (0, 0, 0, 1, 1) .
\]  

(5)

3 Fermion masses and their mixing angles

In this section we define our notations for the mass terms of the charged fermion sectors and the neutrino sectors. These define also the mixing angle unitary matrices for quark sector and neutrino sector, respectively.

3.1 Charged fermion masses and their mixing angles

The full Lagrangian of our model is that of a gauge theory with scalars and Weyl particles of which many are the ones at the Planck scale with which we do not go into details. We shall therefore leave it to the readers imagination and only present the notation for the Weinberg-Salam Yukawa part effective Lagrangian of the Standard Model form

\[
- \mathcal{L}_{\text{charged-fermion-mass}} = \overline{Q_L}Y_U\Phi_{WS}U_R + \overline{Q_L}Y_D\Phi_{WS}D_R + \overline{L_L}Y_L\Phi_{WS}E_R + h.c.
\]  

(6)
Here $\Phi_{WS}$ is the Weinberg-Salam Higgs field, $Q_L$ denotes the three $SU(2)$ doublets of left-handed quarks, $U_R$ denotes the three singlets of right-handed up-type quarks and $Y_U$ is the three-by-three Yukawa coupling matrix for the up-type quarks. Similarly $Y_D$ and $Y_E$ are the Yukawa coupling matrices for the down-type quarks and charged leptons respectively. The $SU(2)$ doublets $\Phi_{WS}$ and $Q_L$ can be represented as 2 component column vectors and we then define:

$$\tilde{\Phi}_{WS} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \phi_{WS}^\dagger$$

and

$$\overline{Q_L} = \begin{pmatrix} U_L \\ D_L \end{pmatrix} = \begin{pmatrix} U_L \\ D_L \end{pmatrix},$$

where $\overline{U}_L$ are the $CP$ conjugates of the three left-handed up-type quarks.

The effective Yukawa couplings $Y_E$, for example, are obtained from identifying the lowest order vertex in eq. (6) with a vertex Fig. 1.

After electroweak symmetry breaking the Weinberg-Salam Higgs field gets a VEV and we obtain the following mass terms in the Lagrangian:

$$- \mathcal{L}_{\text{charged-fermion-mass}} = \overline{U}_L M_U U_R + \overline{D}_L M_D D_R + \overline{E}_L M_E E_R + h.c.$$  

where the mass matrices are related to the Yukawa coupling matrices and Weinberg-Salam Higgs VEV by:

$$M = Y \frac{\langle \phi_{WS} \rangle}{\sqrt{2}}.$$  

We have chosen the normalisation from the Fermi coupling constant:

$$\langle \phi_{WS} \rangle = 246 \text{ GeV}.$$  

In order to obtain the masses from the mass matrices, $M_U$, $M_D$ and $M_E$, we must diagonalise them to find their eigenvalues. In particular we can find unitary matrices, $V_U$ for the up-type quarks, $V_D$ for the down-type quarks and $V_E$ for the charged leptons:

$$V_U^\dagger M_U V_U = \text{diag} \left( m_u^2, m_e^2, m_t^2 \right),$$

$$V_D^\dagger M_D V_D = \text{diag} \left( m_d^2, m_s^2, m_b^2 \right),$$

$$V_E^\dagger M_E V_E = \text{diag} \left( m_e^2, m_\mu^2, m_\tau^2 \right).$$

The quark mixing matrix is then defined with these unitary matrices as [7]:

$$V_{\text{CKM}} = V_U^\dagger V_D.$$  

This $V_{\text{CKM}}$ unitary matrix is parameterised as follows:

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{23}s_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{23}s_{13} \end{pmatrix}.$$  

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$  

9
where \(c_{ij} \equiv \cos \theta_{ij}\) and \(s_{ij} \equiv \sin \theta_{ij}\) for the generation labels \(i, j = 1, 2, 3\) and \(\delta_{13}\) is a \(CP\) violating phase in the quark sector.

### 3.2 Neutrino masses and their mixing angles

The left-handed neutrinos are massless in Standard Model because of non-existence of right-handed neutrinos and the Weinberg-Salam Higgs field having only weakhypercharge \(y/2 = 1/2\), and not \(y/2 = 1\) as is required to give Majorana neutrino a mass. Thus, in order to let left-handed neutrinos be massive particles as is very strongly indicated by experiments, we assume the existence of the three very heavy right-handed neutrinos, which are mass-protected so as finally get masses from the “new” Higgs field, \(\phi_{B-L}\), at an energy scale of about \(10^{10}\) GeV in our present model.

We use the gauged \(B-L\) charge to mass-protect the right-handed neutrinos; in fact we use the total – diagonal – one in fact we break \(U(1)_{B-L,1} \times U(1)_{B-L,2} \times U(1)_{B-L,3} \supset U(1)_{B-L}\) at a much higher energy scale, say about \(10^{18}\) GeV.

The assumption of the existence of three right-handed Majorana neutrinos at a high scale gives rise to the addition of Majorana mass terms to the Lagrangian:

\[
- \mathcal{L}_{\text{neutrino-mass}} = \bar{\nu}_L M^D \nu_R + \frac{1}{2} (\nu_L^c)^c M_L \nu_L + \frac{1}{2} (\nu_R^c)^c M_R \nu_R + \text{h.c.}
\]

\[= \frac{1}{2} (\nu_L^c)^c M n_L + \text{h.c.} \tag{17}\]

where

\[n_L \equiv \left( \frac{\nu_L}{(\nu_L^c)^c} \right), \quad M \equiv \left( \begin{array}{cc} M_L & M^D \nu_R \\ M^D \nu_L & M^D_R \end{array} \right). \tag{18}\]

Here \(M^D\) is the left-right transition mass term – Dirac neutrino mass term – and \(M_L\) and \(M_R\) are the isosinglet Majorana mass terms of left-handed and right-handed neutrinos, respectively.

Due to mass-protection by the Standard Model gauge symmetry, the left-handed Majorana mass terms, \(M_L\), are negligible in our model with a fundamental scale set by the Planck mass. Then, naturally, the effective light (left-left transition) neutrino mass matrix can be obtained via the see-saw mechanism \([6]\):

\[M_{\text{eff}} \approx \frac{M^D}{M_R^{-1}} (M^D)^T. \tag{19}\]

In the framework of the three active neutrinos model, the flavour eigenstates \(\nu_\alpha (\alpha = e, \nu, \tau)\) are related to the mass eigenstates \(\nu_i (i = 1, 2, 3)\) in the vacuum by a unitary matrix \(V_{\text{MNS}}\):

\[|\nu_\alpha \rangle = \sum_i (V_{\text{MNS}})_{\alpha i} |\nu_i \rangle. \tag{20}\]
Here $V_{\text{MNS}}$ is the three-by-three Maki-Nakagawa-Sakata (MNS) mixing matrix \( \mathbf{8} \) which is parameterised by

$$
V_{\text{MNS}} = \begin{pmatrix}
\tilde{c}_{12}\tilde{c}_{13} & \tilde{s}_{12}\tilde{c}_{23} - \tilde{c}_{12}\tilde{s}_{23}\tilde{s}_{13}e^{i\tilde{\delta}_{13}} & \tilde{s}_{12}\tilde{c}_{13} \\
\tilde{s}_{12}\tilde{s}_{23} - \tilde{c}_{12}\tilde{c}_{23}\tilde{s}_{13} & \tilde{s}_{12}\tilde{s}_{23} - \tilde{c}_{12}\tilde{c}_{23}\tilde{s}_{13}e^{i\tilde{\delta}_{13}} & \tilde{s}_{23}\tilde{c}_{13} \\
\tilde{s}_{12}\tilde{s}_{23} - \tilde{c}_{12}\tilde{c}_{23}\tilde{s}_{13} & -\tilde{c}_{12}\tilde{s}_{23} & \tilde{s}_{23}\tilde{s}_{13}e^{i\tilde{\delta}_{13}}
\end{pmatrix}
\times
\begin{pmatrix}
\tilde{c}_{12} & 0 & 0 \\
0 & e^{i\varphi} & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(21)

where $\tilde{c}_{ij} \equiv \cos \theta_{ij}$ and $\tilde{s}_{ij} \equiv \sin \theta_{ij}$ are the neutrino mixing parameters and $\tilde{\delta}_{13}$ is a neutrino $CP$ violating phase. Note that, due to the existence of Majorana neutrinos, we have two additional $CP$ violating Majorana phases $\varphi$ and $\psi$, which are also included in the MNS unitary mixing matrix.

In order to get predictions for the neutrino masses from the effective mass matrix, $M_{\text{eff}}$, we have to diagonalise this matrix using a unitary matrix, $V_{\text{eff}}$, to find the mass eigenvalues:

$$
V_{\text{eff}}M_{\text{eff}}V_{\text{eff}}^\dagger = \text{diag}(m_{1}^2, m_{2}^2, m_{3}^2).
$$

(22)

With the charged lepton unitary matrix $V_{e}$, eq. (14), we can then find the neutrino mixing matrix:

$$
V_{\text{MNS}} = V_{e}V_{\text{eff}}^\dagger.
$$

(23)

Obviously, we should compare these theoretical predictions with experimentally measured quantities, therefore we define:

$$
\Delta m_{\odot}^2 \equiv m_{2}^2 - m_{1}^2,
$$

(24)

$$
\Delta m_{\text{atm}}^2 \equiv m_{3}^2 - m_{2}^2,
$$

(25)

$$
\tan^2 \theta_{\odot} \equiv \tan^2 \theta_{12},
$$

(26)

$$
\tan^2 \theta_{\text{atm}} \equiv \tan^2 \theta_{23},
$$

(27)

$$
\tan^2 \theta_{\text{chooz}} \equiv \tan^2 \theta_{13}.
$$

(28)

Note that since we use the philosophy of order of magnitudewise predictions (see section 3) with complex order one coupling constants, our model is capable of making predictions for these three phases, the $CP$ violating phase $\tilde{\delta}_{13}$ and the two Majorana phases; put simply, we assume that all these phases are of order $\pi/2$, i.e. essentially maximal $CP$ violations.

### 4 Renormalisation group equations

From the Planck scale down to the see-saw scale or rather from where our gauge group is broken down to $SMG \times U(1)_{B-L}$ we use the one-loop renormalisation group running of
the Yukawa coupling constant matrices and the gauge couplings \[9\] including the running of Dirac neutrino Yukawa coupling:

\[
16\pi^2 \frac{dg_1}{dt} = \frac{41}{10} g_1^3 , \\
16\pi^2 \frac{dg_2}{dt} = -\frac{19}{16} g_2^3 , \\
16\pi^2 \frac{dg_3}{dt} = -7 g_3^3 , \\
16\pi^2 \frac{dY_U}{dt} = \frac{3}{2} \left( Y_U (Y_U)^\dagger - Y_D (Y_D)^\dagger \right) Y_U + \left\{ Y_S - \left( \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right) \right\} Y_U , \\
16\pi^2 \frac{dY_D}{dt} = \frac{3}{2} \left( Y_D (Y_D)^\dagger - Y_U (Y_U)^\dagger \right) Y_D + \left\{ Y_S - \left( \frac{1}{4} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right) \right\} Y_D , \\
16\pi^2 \frac{dY_E}{dt} = \frac{3}{2} \left( Y_E (Y_E)^\dagger - Y_\nu (Y_\nu)^\dagger \right) Y_E + \left\{ Y_S - \left( \frac{9}{2} g_1^2 + \frac{9}{4} g_2^2 \right) \right\} Y_E , \\
16\pi^2 \frac{dY_\nu}{dt} = \frac{3}{2} \left( Y_\nu (Y_\nu)^\dagger - Y_E (Y_E)^\dagger \right) Y_\nu + \left\{ Y_S - \left( \frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) \right\} Y_\nu , \\
Y_S = \text{Tr} \left( 3 Y_U^\dagger Y_U + 3 Y_D^\dagger Y_D + Y_E^\dagger Y_E + Y_\nu^\dagger Y_\nu \right) ,
\]

where \( t = \ln \mu \) and \( \mu \) is the renormalisation point.

However, below the see-saw scale the right-handed neutrino are no more relevant and the Dirac neutrino terms in the renormalisation group equations should be removed, and also the Dirac neutrino Yukawa couplings themselves are not accessible anymore. That means that, from the see-saw scale down to the experimental scale (1 GeV), the only leptonic Yukawa \( \beta \)-functions should be changed as follows:

\[
16\pi^2 \frac{dY_E}{dt} = \frac{3}{2} \left( Y_E (Y_E)^\dagger \right) Y_E + \left\{ Y_S - \left( \frac{9}{2} g_1^2 + \frac{9}{4} g_2^2 \right) \right\} Y_E .
\]

Note that the quantity, \( Y_S \), must be also changed below the see-saw scale:

\[
Y_S = \text{Tr} \left( 3 Y_U^\dagger Y_U + 3 Y_D^\dagger Y_D + Y_E^\dagger Y_E + Y_\nu^\dagger Y_\nu \right) .
\]

Further, we should evolve the effective neutrino mass matrix considered as a whole running as an irrelevant or nonrenormalisable 5 dimensional term \[10\] from the see-saw scale, set by \( \langle \phi_{B-L} \rangle \) to our model, to 1 GeV:

\[
16\pi^2 \frac{dM_{\text{eff}}}{dt} = (-3g_2^2 + 2\lambda + 2Y_S) M_{\text{eff}} - \frac{3}{2} \left( M_{\text{eff}} (Y_E Y_E)^T M_{\text{eff}} \right) ,
\]

where \( Y_S \) defined in eq. (38) and in this energy range the Higgs self-coupling constant running equation is

\[
16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 - \left( \frac{9}{5} g_1^2 + g_2^2 \right) \lambda + \frac{9}{4} \left( \frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2^2 + g_2^4 \right) + 4Y_S \lambda - 4H_S ,
\]

12
with
\[ H_s = \text{Tr} \left\{ 3 \left( Y^d \tilde{Y}^s \right)^2 + 3 \left( Y^d \tilde{Y}^d \right)^2 + \left( Y^e \tilde{Y}^e \right)^2 \right\}. \] (41)

The mass of the Standard Model Higgs boson is given by
\[ M_H^2 = \lambda \langle \phi_{WS} \rangle^2 \]
and, for definiteness, we take \( M_H = 115 \text{ GeV} \) at weak scale.

In order to run the renormalisation group equations down to 1 GeV, we use the following initial values:

\[
\begin{align*}
U(1) : & \quad g_1(M_Z) = 0.462, \quad g_1(M_{\text{Planck}}) = 0.614, \\
SU(2) : & \quad g_2(M_Z) = 0.651, \quad g_2(M_{\text{Planck}}) = 0.504, \\
SU(3) : & \quad g_3(M_Z) = 1.22, \quad g_3(M_{\text{Planck}}) = 0.491.
\end{align*}
\] (42) (43) (44)

\section{Method of numerical calculation}

According to our philosophy – at the Planck scale, all coupling constants are complex numbers of order unity – we evaluate the product of mass-protecting Higgs VEVs required for each mass matrix element and provide it with a random complex number of order one as a factor. This means that we assume essentially maximal \( CP \) violation in all sectors, including the neutrino sector. Since the exact values of all these coupling constants are not known, our model is only able to provide its results order of magnitudewise.

In this way, we simulate a long chain of fundamental Yukawa couplings and propagators making the transition corresponding to an effective Yukawa coupling in the Standard Model. In the numerical computation we then calculate the masses and mixing angles time after time, using different sets of random numbers and, in the end, we take the logarithmic average of the calculated quantities according to the following formula:

\[
\langle m \rangle = \exp \left( \frac{\sum_{i=1}^{N} \ln m_i}{N} \right).
\] (45)

Here \( \langle m \rangle \) is what we take to be the prediction for one of the masses or mixing angles, \( m_i \) is the result of the calculation done with one set of random number combinations and \( N \) is the total number of random number combinations used. The calculations are done using a Monte Carlo method by putting in for the order of unity couplings random complex numbers of order unity. Strictly speaking, we just put such random number factors on the different mass matrix elements. We then interpret the matrix elements estimated as the products of a random number of order unity and the Higgs VEVs (suppression factors) as the running Yukawa couplings at the scale where the Higgs fields \( W, T, \rho, \omega \) and \( \chi \) roughly all have their VEVs namely about one order of magnitude below the Planck scale. We ignore running of couplings over this single order of magnitude as not important. We then run down case after case of random numbers the couplings to get the observable quantities at the experimental scale, which we take to be 1 GeV as convention so that our masses are running ones at \( \mu = 1 \text{ GeV} \). At first we get the different results for each set of
random numbers, but then we average over the logarithm of these quantities and present the exponents of the average log’s, as “logarithmic averages”. In estimating the quality of our fits of the adjustable VEVs we define a quantity which we call the goodness of fit (g.o.f.) reminiscent of the chi-square using again the logarithms

\[ \text{g.o.f.} \equiv \sum \left[ \ln \left( \frac{\langle m \rangle}{m_{\text{exp}}} \right) \right]^2. \]  

(46)

It should be kept in mind that, in our calculations, we use the renormalisation group \( \beta \)-functions to run Yukawa couplings down to the experimentally observable scale 1 GeV. This is because we took the charged fermion masses to be compared to “measurements” at the conventional scale of 1 GeV, except for the top quark. We used the top quark pole mass instead \( M_t \):

\[ M_t = m_t(M) \left( 1 + \frac{4 \alpha_s(M)}{\pi} \right), \]  

(47)

where we set \( M = 180 \) GeV as an input, for simplicity.

### 5.1 Mass matrices

With the system of quantum numbers in Table 2.3 one can easily evaluate, for a given mass matrix element, the numbers of Higgs field VEVs of the different types needed to perform the transition between the corresponding left- and right-handed Weyl fields. The results of calculating the products of Higgs fields needed, and thereby the order of magnitudes of the mass matrix elements in our model, are presented in the following mass matrices:

- the up-type quarks:
  \[ M_U \simeq \frac{\langle \phi_{\text{WS}} \rangle^\dagger}{\sqrt{2}} \begin{pmatrix} \omega^3 W(T^\dagger)^2 & \omega^2 \rho W(T^\dagger)^2 & \omega \rho T^3 \\ \omega (\rho^\dagger)^3 W(T^\dagger)^2 & W(T^\dagger)^2 & T^3 \\ \omega^2 \rho W^2(T^\dagger)^4 & W^2(T^\dagger)^4 & WT \end{pmatrix} \]  
  (48)

- the down-type quarks:
  \[ M_D \simeq \frac{\langle \phi_{\text{WS}} \rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W(T^\dagger)^2 & \omega^2 \rho W(T^\dagger)^2 & \omega \rho T^3 \\ \omega \rho^\dagger W(T^\dagger)^2 & W(T^\dagger)^2 & T^3 \\ \omega^2 \rho W^2(T^\dagger)^4 & W^2(T^\dagger)^4 & WT \end{pmatrix} \]  
  (49)

- the charged leptons:
  \[ M_E \simeq \frac{\langle \phi_{\text{WS}} \rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W(T^\dagger)^2 & \omega^2 (\rho^\dagger)^3 W(T^\dagger)^2 & \omega \rho^\dagger W^4 T^4 \chi \\ \omega (\rho^\dagger)^3 W(T^\dagger)^2 & W(T^\dagger)^2 & WT^4 \chi \\ \omega^2 (\rho^\dagger)^3 W^2(T^\dagger)^4 & W^2(T^\dagger)^4 & WT \end{pmatrix} \]  
  (50)

- the Dirac neutrinos:
  \[ M_{\nu}^D \simeq \frac{\langle \phi_{\text{WS}} \rangle^\dagger}{\sqrt{2}} \begin{pmatrix} (\rho^\dagger)^3 W(T^\dagger)^2 & (\rho^\dagger)^3 W^2 T^2 & (\rho^\dagger)^3 W^4 T^4 \chi \\ (\rho^\dagger)^3 W(T^\dagger)^2 & W^{3 T^2} & W^{3 T^2} \chi \\ (\rho^\dagger)^3 W^2(T^\dagger)^4 & W^2(T^\dagger)^4 & W^{2 T^2} \chi \end{pmatrix} \]  
  (51)
and the Majorana (right-handed) neutrinos:

$$M_R \simeq \langle \phi_{B-L} \rangle \begin{pmatrix} (\rho^\dagger)^6 \chi^\dagger & (\rho^\dagger)^3 \chi^\dagger/2 & (\rho^\dagger)^3/2 \\ (\rho^\dagger)^3 \chi^\dagger/2 & \chi^\dagger & 1 \\ (\rho^\dagger)^3/2 & 1 & \chi \end{pmatrix} \label{52}$$

In order to get the true model matrix elements, one must imagine that each matrix element is provided with an order of unity factor, which is unknown within our system of assumptions and which, as described above, is taken in our calculation as a complex random number, later to be logarithmically averaged over as in eq. \ref{45}.

The off-diagonal elements of the right-handed neutrino mass matrix (eq. \ref{52}) are divided by a factor 2 because the symmetric (Majorana) mass matrix gives rise to the same off-diagonal term twice, i.e., we avoid the overcounting of the corresponding Feynmann diagrams. However, the element which couples with only one Higgs field should not be multiplied by an extra factor 1/2.

Note that the quantum numbers of our 6 Higgs fields are not totally independent. In fact there is a linear relation between the quantum numbers of the three Higgs fields $W$, $T$ and $\chi$:

$$\vec{Q}_\chi = 3 \vec{Q}_W - 9 \vec{Q}_T \label{53},$$

where the 6 components of the charge vector $\vec{Q}$ correspond to the 6 columns of Table 2.3. Thus the Higgs field combinations needed for a given transition are not unique, and the largest contribution has to be selected for each matrix element in the above mass matrices.

Furthermore, we should mention that we do not assume Supersymmetry, so that we can make use of an expectation value of a Higgs field and of the Hermitian conjugate one without any restriction; they are numerically equal. If we supersymmetrised the model we would need to double the Higgs fields and we would loose predictive power, because we would get more parameters.

### 5.2 Renormalisation group effect on Dirac neutrino Yukawa coupling

In the previous work \cite{1} we ignored the running of the Yukawa couplings for the neutrino sector from the Planck scale down to the see-saw scale and only used the running of the effective dimension five operator (eq. \ref{39}), but in the present article we use also the running of the Dirac Yukawa couplings for the neutrinos above the see-saw scale according to eq. \ref{35}. By putting in the various values we may estimate that the most important term is the top-Yukawa containing term, $Y_S$, which makes the overall size of the neutrino Dirac Yukawa matrix increase towards the ultraviolet. Since our predictions are \textit{a priori} made at the Planck scale, this means that inclusion of this running makes the predictions for these Yukawa couplings a bit – about a factor $\sqrt{2}$ or less – smaller at lower energy scale.
6 Results for the quantities of quarks and leptons

Using the three charged quark-lepton mass matrices and the effective neutrino mass matrix together with the renormalisation group equations we made a fit to all the fermion quantities in Table 2 varying just 6 Higgs fields VEVs. We averaged over \( N = 10,000 \) complex order unity random number combinations (see eq. (45)). These complex numbers are chosen to be the exponential of a number picked from a Gaussian distribution, with mean value zero and standard deviation one, multiplied by a random phase factor that has smoothly distributed phase. We varied the 6 free parameters and found the best fit, corresponding to the lowest value for the quantity g.o.f. defined in eq. (46), with the following values for the VEVs:

\[
\langle \phi_{WS} \rangle = 246 \text{ GeV} , \quad \langle \phi_{B-L} \rangle = 1.23 \times 10^{10} \text{ GeV} , \quad \langle \omega \rangle = 0.245 , \\
\langle \rho \rangle = 0.256 , \quad \langle W \rangle = 0.143 , \quad \langle T \rangle = 0.0742 , \quad \langle \chi \rangle = 0.0408 ,
\]

(54)

where, except for the Weinberg-Salam Higgs field and \( \langle \phi_{B-L} \rangle \), the VEVs are expressed in Planck units. Hereby we have considered that the Weinberg-Salam Higgs field VEV is already fixed by the Fermi constant. The results of the best fit, with the VEVs in eq. (54), are shown in Table 2 and the fit has g.o.f. = 3.38 (see the definition in eq. (46)).

We have 11 = 17 – 6 degrees of freedom – predictions – leaving each of them with a logarithmic error of \( \sqrt{3.38/11} \approx 0.55 \), i.e., we can fit all quantities with a typical error of a factor \( \exp \left( \sqrt{3.38/11} \right) = 1.74 \) of the experimental value.

Unlike in older versions of the model, the first and second family sub-matrix of \( M_D \) is now dominantly diagonal. In previous versions of the model this submatrix satisfied the order of magnitude factorisation condition \( (M_D)_{12} \cdot (M_D)_{21} \approx (M_D)_{11} \cdot (M_D)_{22} \); thus the down quark mass \( m_d \) received two contributions (off-diagonal as well as diagonal) of the same order of magnitude as the up quark mass \( m_u \). This extra off-diagonal contribution to \( m_d \) of course improved the goodness of the fit to the masses of the first family, since phenomenologically \( m_d \approx 2 m_u \). However, in the present version of the model with the \( \omega \) and \( \rho \) Higgs fields, the off-diagonal element \( (M_D)_{21} \) becomes smaller and we are left with a full order of magnitude degeneracy of the first family masses, even including the down quark. Our best fit values for the charm quark mass \( m_c \) and the Cabibbo angle \( V_{us} \) are smaller than in our previous fits to the charged fermion masses [6, 11]. Nonetheless, as mentioned above, our present best fit agrees with the experimental data within the theoretically expected uncertainty of about 64% and is, therefore, as well as can expect from an order of magnitude fit.

6.1 Neutrino oscillation parameters

The charge current interactions from the Sudbury Neutrino Observatory (SNO) [12] have provided an important signal confirming the existence of the solar neutrino mass [13, 4, 13, 14, 17]: SNO detected a flux of \( \nu_\mu \) neutrino or \( \nu_\tau \) neutrino among solar neutrinos after
traveling from the core of the Sun to the Earth. Combination of the SNO results with previous measurements from other experiments confirms the standard solar model \[18\], whose predictions of the total flux of active \(^8\)B neutrinos in the Sun agree with the SNO and Super-Kamiokande \[17\] data. Furthermore, the measurement of the \(^8\)B and hep solar neutrino fluxes shows no significant energy dependence of the electron neutrino survival probability in the Super-Kamiokande and SNO energy ranges. In fact, global analyses \[19, 20, 21, 22\] of solar neutrino data, including the first SNO results and the day-night effect \[23\], which disfavoured the SMA-MSW solution at the 95% C.L., have confirmed that the LMA-MSW solution gives the best fit to the data and that the SMA-MSW solution is very strongly disfavoured and only accepted at the 3\(\sigma\) level.

Furthermore, the combination of the results from atmospheric neutrino experiments \[24\] and the CHOOZ reactor experiment \[25\] constrains the first- and third-generation mixing angle to be small, i.e. the 3\(\sigma\) upper bound is given by \(\tan^2 \theta_{\text{chooz}} \lesssim 0.06\). This limit was obtained from a three flavour neutrino analysis (in the five dimensional parameter space – \(\theta_\odot\), \(\theta_{\text{chooz}}\), \(\theta_{\text{atm}}\), \(\Delta m^2_\odot\) and \(\Delta m^2_{\text{atm}}\)), using all the solar and atmospheric neutrino data and based on the assumption that neutrino masses have a hierarchical structure, i.e.

|               | Fitted       | Experimental |
|---------------|--------------|--------------|
| \(m_u\)       | 5.2 MeV      | 4 MeV        |
| \(m_d\)       | 5.0 MeV      | 9 MeV        |
| \(m_e\)       | 1.1 MeV      | 0.5 MeV      |
| \(m_c\)       | 0.70 GeV     | 1.4 GeV      |
| \(m_s\)       | 340 MeV      | 200 MeV      |
| \(m_\mu\)     | 81 MeV       | 105 MeV      |
| \(M_t\)       | 208 GeV      | 180 GeV      |
| \(m_\tau\)    | 7.4 GeV      | 6.3 GeV      |
| \(m_\nu\)     | 1.11 GeV     | 1.78 GeV     |
| \(V_{us}\)    | 0.10         | 0.22         |
| \(V_{cb}\)    | 0.024        | 0.041        |
| \(V_{ub}\)    | 0.0025       | 0.0035       |
| \(\Delta m^2_\odot\) | \(9.0 \times 10^{-5}\) eV\(^2\) | \(4.5 \times 10^{-5}\) eV\(^2\) |
| \(\Delta m^2_{\text{atm}}\) | \(1.8 \times 10^{-3}\) eV\(^2\) | \(3.0 \times 10^{-3}\) eV\(^2\) |
| \(\tan^2 \theta_\odot\) | 0.23         | 0.35         |
| \(\tan^2 \theta_{\text{atm}}\) | 0.83         | 1.0          |
| \(\tan^2 \theta_{\text{chooz}}\) | \(3.3 \times 10^{-2}\) | \(\lesssim 2.6 \times 10^{-2}\) |
| g.o.f.       | 3.38         | –            |

Table 2: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass. Note that we use the square roots of the neutrino data in this Table, as the fitted neutrino mass and mixing parameters \(<m>\), in our goodness of fit (g.o.f.) definition, eq. (46).
\[ \Delta m^2_{\odot} \ll \Delta m^2_{\text{atm}} \]  

In Table 2 presented solar neutrino data, however, come from a global two flavour analysis, which means that the first- and third-generation mixing angle is essentially put equal to zero, \( i.e., \) the dependence of \( \theta_{13} \) on the solar neutrino parameters have been ignored. In principle, of course, we have to fit to neutrino parameters from a three flavour analysis.

The global three flavour analyses have been done by several authors \cite{26, 27, 28, 29} and they showed a significant influence of the non-zero CHOOZ angle on the solar neutrino mass squared difference and mixing angle. In \cite{29} the relatively large solar neutrino mass squared difference lying in the LMA-MSW region (with the condition \( \Delta m^2_{\odot} \gtrsim 10^{-4} \text{eV}^2 \)), the solar mixing angle and the CHOOZ reactor experiment data were analysed using the three flavour analysis method. Their works tell us that if the CHOOZ angle becomes bigger than zero then the solar mixing angle becomes smaller. This effect is more significant for the larger \( \Delta m^2_{\odot} \) values. Because of this correlation, our fit to the neutrino data should be somewhat better than that suggested by the g.o.f. value; even including the CHOOZ angle our neutrino fit is extremely good.

Experimental results on the values of neutrino mixing angles are usually presented in terms of the function \( \sin^2 2\theta \) rather than \( \tan^2 \theta \) (which, contrary to \( \sin^2 2\theta \), does not have a maximum at \( \theta = \pi/4 \) and thus still varies in this region). Transforming from \( \tan^2 \theta \) variables to \( \sin^2 2\theta \) variables, our predictions for the neutrino mixing angles become:

\[
\begin{align*}
\sin^2 2\theta_{\odot} & = 0.61 , \\
\sin^2 2\theta_{\text{atm}} & = 0.99 , \\
\sin^2 2\theta_{\text{chooz}} & = 0.12 .
\end{align*}
\]

We also give here our predicted hierarchical neutrino mass spectrum:

\[
\begin{align*}
m_1 & = 9.8 \times 10^{-4} \text{ eV} , \\
m_2 & = 9.6 \times 10^{-3} \text{ eV} , \\
m_3 & = 4.4 \times 10^{-2} \text{ eV} .
\end{align*}
\]

Compared to the experimental data these predictions are excellent: all of our order of magnitude neutrino predictions lie inside the 95\% C.L. border determined from phenomenological fits to the neutrino data, even including the CHOOZ upper bound. On the other hand, our prediction of the solar mass squared difference is a factor of 2 larger than the global fit data even though the prediction is inside of the LMA-MSW region, giving a contribution to our goodness of fit \( \approx 0.12 \).

Our CHOOZ angle also turns out to be about a factor of \( \sqrt{2} \) larger than the experimental limit at 90\% C.L., corresponding to another contribution of g.o.f. \( \approx 0.014 \). In summary our predictions for the neutrino sector agree extremely well with the data, giving a contribution of only 0.25 to g.o.f. while the charged fermion sector contributes 3.13 to g.o.f.
Note that the value of $\langle \phi_{B-L} \rangle$ presented in Table 2 does not fit very well the atmospheric neutrino mass squared difference, being about factor $\sqrt{2}$ less than the best fit reported by Super-Kamiokande collaboration [24]. That means that our model predicts "relative" degenerated mass spectra in the neutrino sector. However, if we force it to fit the value of mass squared difference as $\Delta m^2_{\text{atm}} = 2.5 \times 10^{-3} \text{ eV}^2$ and just arrange the scale, $\langle \phi_{B-L} \rangle$, of the mass squared differences – it will not significantly influence the ratio of mass squared differences – thus we get $\Delta m^2_{\odot} = 1.2 \times 10^{-4} \text{ eV}^2$ which lays still in the allowed region of the global fit of the solar neutrino analysis:

$$\langle \phi_{B-L} \rangle \big|_{\text{corrected}} = 1.0 \times 10^{10} \text{ GeV}, \quad (61)$$
$$\Delta m^2_{\text{atm}} \big|_{\text{corrected}} = 2.5 \times 10^{-3} \text{ eV}^2, \quad (62)$$
$$\Delta m^2_{\odot} \big|_{\text{corrected}} = 1.2 \times 10^{-4} \text{ eV}^2. \quad (63)$$

and the CHOOZ angle does not change $\tan^2 \theta_{\text{chooz}} = 3.3 \times 10^{-2}$ in this small deviation of the see-saw scale.

### 6.2 CP violation

Since we have taken our random couplings to be – whenever allowed – complex we have order of unity or essentially maximal CP violation so that a unitary triangle with angles of order one is a success of our model. After our fitting of masses and of mixings we can simply predict order of magnitudewise the CP violation in e.g. $K^0 - \bar{K}^0$ decay or in CKM and MNS mixing matrices in general.

The Jarlskog area $J_{CP}$ provides a measure of the amount of CP violation in the quark sector [30] and, in the approximation of setting cosines of mixing angles to unity, is just twice the area of the unitarity triangle:

$$J_{CP} = V_{us} V_{cb} V_{ub} \sin \delta, \quad (64)$$

where $\delta$ is the CP violation phase in the CKM matrix. In our model the quark mass matrix elements have random phases, so we expect $\delta$ (and also the three angles $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle) to be of order unity and, taking an average value of $|\sin \delta| \approx 1/2$, the area of the unitarity triangle becomes

$$J_{CP} \approx \frac{1}{2} V_{us} V_{cb} V_{ub}. \quad (65)$$

Using the best fit values for the CKM elements from Table 2, we predict $J_{CP} \approx 3.0 \times 10^{-6}$ to be compared with the experimental value $(2-3.5) \times 10^{-5}$. Since our result for the Jarlskog area is the product of four quantities, we do not expect the usual $\pm 64\%$ logarithmic uncertainty but rather $\pm \sqrt{4} \cdot 64\% = 128\%$ logarithmic uncertainty. This means our result deviates from the experimental value by $\ln[(2.7 \times 10^{-5})/3.0 \times 10^{-6}]/1.28 = 1.7$ "standard deviations".

The Jarlskog area has been calculated from the best fit parameters in Table 2, it is also possible to calculate them directly while making the fit. So we have calculated $J_{CP}$.
for $N = 10,000$ complex order unity random number combinations. Then we took the logarithmic average of these $10,000$ samples of $J_{CP}$ and obtained the following result:

$$J_{CP} = 3.0 \times 10^{-6},$$

(66)

in good agreement with the values given above.

### 6.3 Neutrinoless double beta decay

Another prediction, which can also be made from this model, is the electron “effective Majorana mass” – the parameter in neutrinoless beta decay – defined by:

$$|\langle m \rangle| \equiv \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right|,$$

(67)

where $m_i$ are the masses of the neutrinos $\nu_i$ and $U_{ei}$ are the MNS mixing matrix elements for the electron flavour to the mass eigenstates $i$. We can substitute values for the neutrino masses $m_i$ from eqs. (58-60) and for the fitted neutrino mixing angles from Table 2 into the left hand side of eq. (67). As already mentioned, the $CP$ violating phases in the MNS mixing matrix are essentially random in our model. So we combine the three terms in eq. (67) by taking the square root of the sum of the modulus squared of each term, which gives our prediction:

$$|\langle m \rangle| \approx 2.5 \times 10^{-3} \text{ eV}.$$  

(68)

In the same way as being calculated the Jarlskog area we can compute using $N = 10,000$ complex order unity random number combinations to get the $|\langle m \rangle|$. Then we took the logarithmic average of these $10,000$ samples of $|\langle m \rangle|$ as usual:

$$|\langle m \rangle| = 3.4 \times 10^{-3} \text{ eV}.$$  

(69)

This result does not agree with the central value of recent result – “evidence” – from the Heidelberg-Moscow collaboration [31].

### 7 Baryogenesis via lepton number violation

Now we have a good model which predicts orders of magnitude for all the Yukawa couplings including the see-saw particles, so it is natural to ask ourselves whether this model can predict also the right amount of $Y_B$ – the ratio of the baryon numbers density relative to the entropy density – using the Fukugita-Yanagida mechanism [32]. According to this mechanism the decays of the right-handed neutrinos by $CP$ violating couplings lead to an excess of the $B - L$ charge (baryon number minus lepton number), the relative excess in the decay from Majorana neutrino generation number $i$ being called $\epsilon_i$. This excess is then immediately – and continuously back and forth – being converted partially
to a baryon number excess, although it starts out as being a lepton number $L$ asymmetry, since the right-handed neutrinos decay into leptons and Weinberg-Salam Higgs particles.

It is a complicated discussion to estimate to what extent the $B - L$ asymmetry is washed-out later in the cosmological development, but in our approximation below we agree within a factor 3 with the baryon number excess left to fit the Big Bang development at the stage of formation of the light elements primordially (nucleosynthesis). The “experimental” data of the ratio of baryon number density to the entropy density is obtained by recent measurement of cosmic microwave background radiation [33]:

$$Y_B \big|_{\text{exp}} = \left( \frac{8.5}{-1.0} \right) \times 10^{-11}. \quad (70)$$

Recently, baryogenesis calculations had been done by several authors [34] using different models based on this scenario, and they used “usual” range of $Y_B$:

$$Y_B \big|_{\text{exp}} = (1.7 - 8.1) \times 10^{-11}. \quad (71)$$

In the following subsection, we review briefly the already known Fukugita-Yanagida mechanism.

### 7.1 Fukugita-Yanagida scenario for lepton number production

The $SU(2)$ instantons [35], rather say, Sphaleron [36] guaranteed the rapid exchanges of the minus baryon number and lepton number in which though $B - L$ is conserved in the time of Big Bang, even when the temperature was above the weak scale. In fact the three right-handed neutrinos decay in the $B - L$ violating way, at the temperature about the see-saw mass scales. This means that the baryon number is violated. From our assumption – all fundamental coupling constants are of order of unity and we treat them as complex random numbers at the Planck scale – it is clear that the model has not only $C$ violation but also $CP$ violation. Out-of-equilibrium condition comes about during the Hubble expansion due to the excess of the three type of the right-handed Majorana neutrinos caused by their masses.

These statements lead to that all Sakharov conditions [37] are fulfilled in our model: (1) baryon number violation, (2) $C$ and $CP$ violation, (3) departure from thermal equilibrium.

### 7.2 Entropy density in cosmology

In order to investigate the quantity, $Y_B$, we need the expression for the entropy of Planck radiation which is given by

$$s_i = \frac{2\pi^2}{45} g_{*i} T^3, \quad (72)$$

where $g_{*i}$ is the total number of effectively massless degrees of freedom of the plasma at the temperature of the right-handed neutrino, and the index $i$ denotes the number of
copiously existing right-handed neutrinos at the time in question. For the estimate of the excess of baryon number coming from a certain right-handed neutrino $i$, we should compare to the entropy density at the temperature being approximately equal to the mass of the corresponding right-handed neutrino.

The $g_{*i}$ are obtained as follows: There are 14 bosons and 45 well-known Weyl fermions plus $i$ Majorana particles when the temperature is high enough compared to $10^2$ GeV:

$$g_{*i} = \sum_{j=\text{bosons}} g_j \left(\frac{T_j}{T}\right)^4 + \frac{7}{8} \sum_{j=\text{fermions}} g_j \left(\frac{T_j}{T}\right)^4$$

Standrad Model see-saw particles

$$= 28 + \frac{7}{8} \cdot 90 + \frac{7}{4} \cdot i$$

here $T_j$ denotes the effective temperature of any species $j$. When we have coupling as at the stage discussed between all the particles $T_j = T$.

### 7.3 CP violation in decays of the Majorana neutrinos

A right-handed neutrino, $N_R$, decays into a Weinberg-Salam Higgs particle and a left-handed lepton or into the CP conjugate channel: These two channels have different lepton numbers $\pm 1$. If there were no CP violation they would have the same branching ratio. However, all Yukawa couplings are complex order unity random numbers, therefore the partial widths do not have to be equal in the next-to-leading-order of perturbation theory: CP violation in the decay of right-handed neutrinos.

Defining the measure $\epsilon_i$ for the CP violation

$$\epsilon_i \equiv \frac{\sum_{\alpha,\beta} \Gamma(N_{Ri} \to \ell^\alpha \phi^\beta_{WS}) - \sum_{\alpha,\beta} \Gamma(N_{Ri} \to \ell^\alpha \phi^\beta_{WS}^\dagger)}{\sum_{\alpha,\beta} \Gamma(N_{Ri} \to \ell^\alpha \phi^\beta_{WS}) + \sum_{\alpha,\beta} \Gamma(N_{Ri} \to \ell^\alpha \phi^\beta_{WS}^\dagger)} ,$$

where $\Gamma$ are $N_{Ri}$ decay rates (in the $N_{Ri}$ rest frame), summed over the neutral and charged leptons (and Weinberg-Salam Higgs fields) which appear as final states in the $N_{Ri}$ decays one sees that the excess of leptons over anti-leptons produced in the decay of one $N_{Ri}$ is just $\epsilon_i$. Now we shall calculate $\epsilon_i$ in perturbation theory: The total decay rate at the tree level (Fig. 2(a)) is given by

$$\Gamma_{N_i} = \Gamma_{N_i\ell} + \Gamma_{N_i\bar{\ell}} = \frac{(\tilde{M}_D^P)^\dagger M_D^P}{4\pi \langle \phi_{WS} \rangle^2} M_i ,$$

where $\tilde{M}_D^P$ can be expressed through the unitary matrix diagonalising the right-handed neutrino mass matrix $V_R$:

$$\tilde{M}_D^P \equiv M_D^P V_R ,$$

(76)
The CP violation in the Majorana neutrino decays, $\epsilon_i$, arises when the effects of loop are taken into account, and at the one-loop level, the CP asymmetry comes both from the wave function renormalisation (Fig. 2(b)) and from the vertex (Fig. 2(c)) \cite{38, 39, 40}:

$$\epsilon_i = \frac{1}{4\pi\langle\phi_{WS}\rangle^2\sum_{j\neq i}}\text{Im}[(\tilde{M}_\nu^D)^\dagger(\tilde{M}_\nu^D)^2] \left[ f\left(\frac{M_j^2}{M_i^2}\right) + g\left(\frac{M_j^2}{M_i^2}\right) \right],$$

(78)

where the function, $f(x)$, comes from the one-loop vertex contribution and the other function, $g(x)$, comes from the self-energy contribution. These $\epsilon$’s can be calculated in perturbation theory only for differences between Majorana neutrino masses which are sufficiently large compared to their decay widths, \textit{i.e.}, the mass splittings satisfy the condition, $|M_i - M_j| \gg |\Gamma_i - \Gamma_j|$: 

$$f(x) = \sqrt{x}\left[1 - (1 + x)\ln\frac{1 + x}{x}\right],$$

(79)

$$g(x) = \frac{\sqrt{x}}{1 - x}.$$  

(80)

The function $f(x)$ has for large $x$ “tricky” cancellations, so for numerical calculation we use following approximation in the region $x \gg 1$, \textit{i.e.}, hierarchical case in the right-handed sector:

$$f(x) \simeq -\frac{1}{2\sqrt{x}}.$$  

(81)

### 7.4 Off-diagonal dominant matrix elements in the see-saw neutrino mass matrix

The main modification in the model – and not only in the calculation – compared to the latest version is that we changed the quantum number assignment of the see-saw scale.
fitting Higgs field, $\phi_{B-L}$, so that it instead of giving directly and thus unsuppressed the $(3,3)$ mass matrix element, *i.e.*, a diagonal one, it gives in the present version unsuppressed size to the $(2,3)$ and $(3,2)$ mass matrix elements in this see-saw neutrino mass matrix (eq. [52]). By having such off-diagonal dominance one expects a large mixing of the second and third generation at least in the see-saw mass matrix and expect to get easily a large CHOOZ angle $\tan^2 \theta_{\text{chooz}}$. However, as we have already presented in the section 6, we still do fit the CHOOZ angle limit satisfactorily.

The gain by having this off-diagonal dominance is, however, crucial for the baryon number production in the early Universe via the Fukugita-Yanagida mechanism: By having now the two heaviest see-saw neutrinos almost degenerate in mass the interaction between these two levels become much enhanced and this causes the $C\bar{P}$ violation as expressed by the $\epsilon_i$’s for the two heaviest Majorana neutrinos to be bigger, as is easily understood by noticing the factor $f(x) + g(x)$ occurring in the expression (eq. [58]), which is in first approximation a suppression factor given by the ratio of the see-saw neutrino masses [11, 12]. With the mass difference becoming small compared to the masses themselves the term $g(x)$ can even become big compared to unity, so that one might see the possibility of these normally suppressing factors causing an enhancement. However, there are in the case with two such close Majorana neutrinos a strong cancellation between the amounts of $B-L$ produced in excess by these almost mass degenerate particles decaying. Nevertheless we can avoid significantly the suppression by having such degeneracy, therefore the baryon number production increases.

8 An approximation to the wash-out effect

The excess $B-L$ produced at first gets partly destroyed by the same or other $B-L$ violating processes. There are several processes/Feynmann diagrams that can lead to $B-L$ violation and thus wash-out an excess. Most important at the temperature scale of the right-handed neutrino masses are the resonance scattering of a Weinberg-Salam Higgs towards a left-handed lepton producing one of the Majorana neutrinos as a resonance. The rate of forming resonances this way is proportional to the width $\Gamma_i$ of the see-saw neutrino functioning as resonance from detailed balance. Since the time scale involved is given by the Hubble expansion at the time of the temperature being equal to the mass of the see-saw neutrino, a crucial parameter for the wash-out effect via the resonance process is

$$K_i \equiv \frac{\Gamma_i}{2H} \bigg|_{T=M_i} = \frac{M_{\text{Planck}}}{1.66 \langle \phi_{WS} \rangle^2 8\pi g^{1/2}_* M_i} \frac{((\hat{M}_\nu^D)^\dagger \hat{M}_\nu^D)_{ii}}{M_i} \quad (i = 1, 2, 3),$$

where $\Gamma_i$ is the width of the flavour $i$ Majorana neutrino, $M_i$ is its mass and $g_*^i$ is the number of degrees of freedom at the temperature $M_i$.

As the time goes in early cosmology, the heaviest see-saw neutrino goes out-of-thermal-equilibrium first and deploys its excess of $B-L$, then the next and so on. Thus in the hierarchical case – but, our model is not this case in as far as the two heaviest see-saw
neutrinos are approximately degenerate in mass – the excess produced by the lightest see-saw neutrino will roughly only be washed out by itself but not by the heavier neutrinos, since the latter resonances are not reachable at the temperature at that time. However, the excess from the decay of the heavier see-saw neutrinos can be washed out by the lighter one(s), and if some are degenerate they may wash out the products of each other. Now the flavours of the left-handed leptons produced in excess in the decays are not the same for the three different see-saw neutrinos. They are, however, also not orthogonal states because they are mixing almost maximally. This means that wash-out due to one see-saw neutrino of the excess caused by another one is suppressed compared to what the wash-out rate would be when a see-saw neutrino washes out its own production of excess. We shall take into account such suppressions by constructing some effective $K_i$ called $K_{\text{eff}i}$ which should mean that value of $K_i$ to use as if we had the excess of the $\nu_{R_i}$ washed out by itself, so as to obtain in practice the effect of the wash-out due to the other see-saw neutrinos included. The decay products of a given see-saw neutrino are found from the Yukawa couplings of this right-handed neutrino to a Weinberg-Salam Higgs and a left-handed lepton, and these couplings are proportional to the $M_D^\nu$-mass matrix elements. We – crudely – shall estimate $K_{\text{eff}i}$ expressions in terms of $K_i$’s which are the parameters for the wash-out of the products of the right-handed see-saw neutrinos itself, where $i = 1$ is the lightest and three ($i = 3$) the heaviest.

In order to estimate the effective $K$ factors we first introduce some normalized state vectors for the decay products:

$$|i\rangle \equiv \frac{1}{\sqrt{\sum_{k=1}^{3} \left| \left[ \hat{M}_D^\nu(M_i) \right]_{ki} \right|^2}} \left( \left[ \hat{M}_D^\nu(M_i) \right]_{1i} , \left[ \hat{M}_D^\nu(M_i) \right]_{2i} , \left[ \hat{M}_D^\nu(M_i) \right]_{3i} \right) ,$$

and further define their overlap:

$$\zeta_{ij} \equiv \left| \langle i | j \rangle \right|^2 .$$

Then we may take an approximation for the effective $K$ factors:

$$K_{\text{eff}1} = K_1(M_1) ,$$
$$K_{\text{eff}2} = K_2(M_2) + \zeta_{23} K_3(M_3) + \zeta_{21} K_1(M_1) ,$$
$$K_{\text{eff}3} = K_3(M_3) + \zeta_{32} K_2(M_2) + \zeta_{31} K_1(M_1) .$$

Strictly speaking, one shall namely not imagine the excesses to flow around as left-handed leptons only, because there is with high rate the weak instanton or Sphaleron activity converting e.g. lepton number into minus baryon number. Since, however, the instanton process at the same time convert one particle from every generation, there remains the same excess of one generation over the other even after the instanton-process. Thus we expect that the assumed suppressed wash-out effect from a different see-saw neutrino works anyway. We have for simplicity ignored the scattering processes due to exchange processes as candidates for wash-out effects, expecting that at the temperature scales relevant they will effectively be small due to cross-section going as the fourth power of Dirac neutrino Yukawa couplings rather than the square as does the resonance cross-section averaged over energy.
8.1 The dilution factors

When the $K_{\text{eff}}$’s are not small (not less than one), we have to take into account the dilution effects – wash-out effects – for the calculation. Therefore, we define the suppression factor $\kappa_i$ to be fraction of $B - L$ excess produced by the right-handed neutrino number $i$ which survives. That is to say, we define $\kappa_i$ so that the resulting relative to entropy density, $s_i$, baryon number density minus lepton number, i.e., $Y_{B-L}$ becomes

$$Y_{B-L} \equiv \left| \sum_{i=1}^{3} \kappa_i \frac{\epsilon_i}{g_{*i}} \right|.$$  \hspace{1cm} (88)

A good approximation for $\kappa_i$, the dilution factor, is inferred from refs. [41, 43]:

$$K_{\text{eff}}^i > \sim 10^6 : \quad \kappa_i = (0.1 K_{\text{eff}}^i)^{1/2} \exp \left[ -\frac{4}{3} (0.1 K_{\text{eff}}^i)^{1/2} \right],$$  \hspace{1cm} (89)

$$10 < K_{\text{eff}}^i \lesssim 10^6 : \quad \kappa_i = \frac{0.3}{K_{\text{eff}}^i (\ln K_{\text{eff}}^i)^{2/3}},$$  \hspace{1cm} (90)

$$1 \lesssim K_{\text{eff}}^i \lesssim 10 : \quad \kappa_i = \frac{1}{2 K_{\text{eff}}^i},$$  \hspace{1cm} (91)

$$0 \lesssim K_{\text{eff}}^i \lesssim 1 : \quad \kappa_i = 1.$$  \hspace{1cm} (92)

Note that these dilution factors are not smooth; therefore we used in numerical calculations instead of the eq. (91) and eq. (92) the following interpolating redefined dilution factor in the range $0 \lesssim K_{\text{eff}}^i \lesssim 10$:

$$0 \lesssim K_{\text{eff}}^i \lesssim 10 : \quad \kappa_i = \frac{1}{\sqrt{4 K_{\text{eff}}^i^2 + 1}}.$$  \hspace{1cm} (93)

8.2 $B - L$ to baryon number conversion

We have presented all quantities to construct $Y_{B-L}$ in foregoing sections, moreover, we should note here that due to the electroweak Sphaleron effect, the baryon number asymmetry $Y_B$ is related to the baryon number minus lepton number asymmetry $Y_{B-L}$ [44]:

$$Y_B = \frac{8N_f + 4N_H}{22N_f + 13N_H} Y_{B-L},$$  \hspace{1cm} (94)

where $N_f$ is the number of generations and $N_H$ the number of Higgs doublets. It turns out that the final form of the baryogenesis using eq. (88) is:

$$Y_B = \frac{28}{79} \left| \sum_{i=1}^{3} \kappa_i \frac{\epsilon_i}{g_{*i}} \right|.$$  \hspace{1cm} (95)
9 Result of baryogenesis

The numerical calculation of baryogenesis will be presented in this section. All the Yukawa couplings – VEVs of seven different Higgs fields – are obtained by fitting the fermion quantities (see section 6). In order to get baryogenesis in Fukugita-Yanagida scheme, we have to calculate the following informations, i.e., the see-saw neutrino masses, $K$ factors and $CP$ violation parameters ($\epsilon_i$’s). At first, we present the see-saw neutrino masses then $K_{\text{eff}}$’s using $N = 10,000$ random number combinations and logarithmic average:

\begin{align*}
M_1 &= 2.1 \times 10^5 \text{ GeV} , \\
M_2 &= 8.8 \times 10^9 \text{ GeV} , \\
M_3 &= 9.9 \times 10^9 \text{ GeV} ,
\end{align*}

and

\begin{align*}
K_{\text{eff}_1} &= 31.6 , \\
K_{\text{eff}_2} &= 116.2 , \\
K_{\text{eff}_3} &= 114.7 .
\end{align*}

The numerical results of our best fitting VEVs also gives the $\epsilon_i$’s using same method:

\begin{align*}
|\epsilon_1| &= 4.62 \times 10^{-12} , \\
|\epsilon_2| &= 4.00 \times 10^{-6} , \\
|\epsilon_3| &= 3.27 \times 10^{-6} .
\end{align*}

The sign of $\epsilon_i$ is unpredictable due to the complex random number coefficients in mass matrices, therefore we are not in the position to say the sign of $\epsilon$’s. It turns out that we should calculate directly $Y_B$ instead of putting the results from eqs. (96)-(98) and eqs. (102)-(104) using the dilution factor formulae into eq. (95). Using the complex order unity random numbers being given by a Gaussian distribution we get after logarithmic average

\[ Y_B = 2.59^{+17.0}_{-2.25} \times 10^{-11} , \]

hereby we estimate the uncertainty in the natural exponent according ref. [11] to be $64 \% \cdot \sqrt{10} \approx 200 \%$. Dominantly the signs of $\epsilon_i$ are strongly correlated, therefore, we included in the numerical calculation using eq. (95) the correlation of signs effects correctly.

The other remark of the scale dependence for our prediction is on the baryogenesis calculation: It is obvious that the quantities, $\epsilon_i$, are approximately independent of the scale $\langle \phi_{B-L} \rangle$ from the eq. (78). In fact these quantities are influenced by only the ratio of the different mass eigenstates of the right-handed neutrino, and in addition through the Dirac neutrino Yukawa couplings which are, of course, depended on the see-saw scale via renormalisation group equations. However, a small deviation of the see-saw scale can
not contribute significantly. The number of degrees of freedom, \( g_{\ast i} \), does not depend on the see-saw scale. However, the dilution factors, rather say, the \( K_{\text{eff} i} \) factors are strongly influenced by the right-handed scale, \( i.e., \) inverse power of the right-handed neutrino masses (see eq. 82). Strictly speaking, the wash-out effect is proportional to the see-saw scale. Using the “right” value of the see-saw scale which we mention in above (eq. 61) we get the corresponding \( K_{\text{eff} i} \)’s as following

\[
\begin{align*}
K_{\text{eff}1}^{\text{corrected}} &= 37.60, \\
K_{\text{eff}2}^{\text{corrected}} &= 138.8, \\
K_{\text{eff}3}^{\text{corrected}} &= 137.1.
\end{align*}
\]

The corresponding result of baryon asymmetry is

\[
Y_B^{\text{corrected}} = 2.02 \left[ \frac{+13.3}{-1.75} \right] \times 10^{-11}.
\]

This result is a bit smaller than the result in eq. (105), however, still in allowed region of the “experimental” data in eq. (71).

10 Proton decay

It should be mentioned that our non-supersymmetric model passes without any trouble the test of not predicting proton decay that should have been observed in present experiment. Indeed we have in our model desert, except for the see-saw neutrinos and associated Higgs, \( \phi_{B-L} \), and gauge \( U(1)_{B-L} \) fields, up to about one order of magnitude below the Planck scale.

A reasonable suggestion is that the baryon decay causing bosons would have masses of the order of the scale around the VEVs of our Higgs fields, which typically are about a factor 10 below the Planck scale (see eq. (24)). We should therefore use such a boson with mass about \( 1.2 \times 10^{18} \) GeV. That is to say we could essentially use the formula for the life time of the proton \( \tau_p \) in simple \( SU(5) \) GUT with replaced mass of \( X \)-particle by \( 1.2 \times 10^{18} \) GeV. We could take for instance the expression for the \( p \to \pi^0 e^+ \) channel [46]:

\[
\tau_{(p \to \pi^0 e^+)} = \frac{1}{\Gamma (p \to \pi^0 e^+)} = 8 \times 10^{34} \text{ years} \cdot \left( \frac{0.015 \text{ GeV}^3}{\alpha_H} \right)^2 \cdot \left( \frac{M_V}{10^{16} \text{ GeV}} \right)^4,
\]

where \( \alpha_H \) is the hadronic matrix element and \( M_V \) the mass of the GUT gauge boson, and insert \( 1.2 \times 10^{18} \) GeV. Then we get crudely our prediction assuming the existence of bosons which break baryon number at this scale:

\[
\tau_{(p \to \pi^0 e^+)} \approx 10^{43} \text{ years}.
\]

However, none of the Higgs fields and gauge bosons, which we truly consider in our model would be able to cause baryon decay. In fact we have excluded by our assumption
about our gauge group the possibility of a gauge boson that could have made transitions between quarks and leptons: we excluded, namely, gauge bosons unifying different, irreducible representations of the Standard Model, and of course – it is well-known – quarks and leptons belong to different irreducible representations. Concerning the possibility that our scalar fields (Higgs fields) considered be able to cause baryon decay, it should be remarked that since they are made to break our gauge group (eq. 1) but to leave unbroken the Standard Model gauge group. Therefore, they must contain a singlet under the diagonal group in order not to break the diagonal SU(3) – QCD gauge group. Then it follows that they must under this diagonal SU(3) have triality $t = 0 \pmod{1}$ (see below eq. (2)). In turn, that means that these Higgs fields are not able to couple to neither two quarks nor to a pair of a quark and a lepton. That makes our scalar fields of no practical use for baryon decay since to look for three quarks coming together to form one point particle at a scale of $10^{18}$ GeV is too much requirement.

If we keep to truly postulated particles in our model we would have to wait for proton decay till we can get sensitive to it from Planck scale masses inserted for $M_V$. Taken this attitude of only using “the everything” that can be found at Planck scale will lead to a $10^4$ times longer proton life time prediction in our model:

$$\tau_{(p\rightarrow\pi^0\,e^+)_{\text{Planck}}} \approx 10^{47} \text{ years}. \quad (112)$$

In any case there is no chance in foreseeable future to observe proton decay according to the suggestions of our model – however presumably – this is a point that could be modified without changing our model too drastically.

11 Conclusions

We have set up a model for calculating all the mass matrices for quarks and leptons – here under especially the effective left-handed neutrino mass matrix which is obtained via the well-known see-saw mechanism using a set of three see-saw neutrinos, two of which are approximately degenerate in mass. We have fit all the masses and mixing angles within the accuracy which we theoretically expect to be $\pm 64\%$ (on a logarithmic scale) for the quantities for instance quark and lepton masses which are essentially directly given by mass matrix elements. In fact even our worst deviations of our predictions form experiment are of these to mass matrices simply related quantities: we predict the charm quark mass and the Cabibbo angle a factor two lower than experiment, while we obtain the electron and the strange mass a factor two larger. On the other hand, the squared quantities of neutrino oscillations presented in Table 2 do not even deviate a factor two. The $CP$ violating parameter – Jarlskog triangle area –, $J_{CP}$, and the baryon number relative to the entropy ratio, $Y_B$, arise as products and ratios of several mass matrix elements and thus have larger expected uncertainties, and turn out indeed to agree quite within the expected accuracy both being predicted to the low side of “data”. In conclusion we fit all the Standard Model parameters related to the mass matrices within the expected order of magnitude accuracy, i.e., we fitted 9 masses of quark and charged
leptons, 2 mass squared differences for neutrinos, 5 genuinely measured mixing angles, the Jarlskog triangle area and the baryon number to entropy density ratio. In addition, we managed to avoid violating the bounds for proton decay life time and neutrinoless double beta decay. Most interestingly we predicted the CHOOZ mixing angle – the mixing angle of the electron neutrino with the heaviest eigenmass neutrino – only about 15% above the experimental limit (completely allowed order of magnitudewise). In this way we predicted successfully 18 genuinely measured parameters not given by the Standard Model itself, and that we did with only 6 adjustable parameters – the vacuum expectation values of Higgs fields – in our model. This means that we made \( 18 - 6 = 12 \) genuine predictions in addition to avoiding limits giving potential problems for most interestingly the CHOOZ angle, the proton life time and the neutrinoless double \( \beta \)-decay, however, we also did not predict the rate of \( \mu \rightarrow e + \gamma \) nor similar suppressed processes with any problematic rate.

If one takes it as we mentioned in the earlier article [1] that the smallness of the fine structure constants in a model with our gauge group is understandable as being really of order unity except for the \( 4\pi \) in \( \alpha_i = g^2_i/4\pi \) and the corrections from the break down to diagonal subgroup, we can claim that we have understood with our model all the orders of magnitudes of the couplings and parameters of the Standard Model in terms of only 7 vacuum expectation values [10] (if we should claim the VEV of Weinberg-Salam Higgs as understood we would, of course, have to include it simply as a seventh parameters because we have not explained the hierarchy of the weak to Planck scale).

We did obtain this impressive fit order of magnitudewise in a model with the gauge group eq. (1) and by a very data inspired choice of the gauge quantum numbers of our seven Higgs fields. We have, however, good reasons to claim that the precise detailed structure of our gauge group is not important, especially in as far as we in reality only needed to use the abelian part of the extension of the Standard Model group. The presumably only important thing is that the gauge group is sufficiently large, so that it separates the various Weyl components of the quarks and the leptons, in order to accomplish the various mass matrix element suppressions needed. On the other hand, the gauge group should not be so large as to include \( SU(5) \) or other Grand unification gauge group, since that tends to give exact mass relations which become a severe problem to avoid, except for the \( \tau - b \) mass relation. Our gauge group were the biggest one represented on the supposed three heavy neutrinos and the 45 already observed Weyl quark and lepton components and avoiding such unwanted unifications. However, such a large gauge group is far from being needed although it must be considered the suggestion of the present article that a relatively large group mainly abelian or, at least, not too much “unifying” is what could fit very well the “small hierarchies” of quark and lepton masses and mixings.

The choices of the specific quantum numbers for our seven Higgs fields represents an opportunity of a sort of discrete fitting in the sense that we adjust these quantum numbers to be able to fit the masses and mixings. Historically we developed the quantum number proposals making small changes gradually along so as to incorporate new experimental data. For example the fields \( \chi \) and \( \phi_{B-L} \) were introduced to cope with the neutrino oscillations, and previous Higgs fields (called \( S \) and \( \xi \)) were replaced by the fields \( \rho \) and \( \omega \) in order to fit the Large Mixing Angel MSW solar neutrino solution, which became
favoured strongly during the development period of our model.

Finally the new step of development in the present article is that the quantum numbers of the field $\phi_{B-L}$ got modified to make the two heaviest see-saw neutrinos become degenerate in mass only deviating by a relatively small mass difference of the order of the suppression factor associated with the VEV of the field $\chi$ being small. Thereby we could enhance the $CP$ violation in the decay of the heavier see-saw neutrinos, thus achieving a larger baryon number. The latter were needed in as far as the previous version of the model, which did not have such a degeneracy, gave too little baryon number excess.

Although in the present model we get somewhat more wash-out effect we still get closer to the from Big Bang fitting estimated baryon number density relative to entropy density, and we predict it only a factor three (see eq. (105)) below the experimental data, and that should be counted as only $\ln(8.5/2.59)/\sqrt{10 \cdot 0.64} = 0.59$ “standard deviations”!

### 11.1 How does the model function?

A few comments about how the model functions may be on its place: The diagonal elements in the four Dirac mass matrices – the two for quarks, the charged leptons, and the “neutrino Dirac mass matrix” – are suppressed to just the same degree in all these four matrices. This is so because the quantum number exchanges needed to uphold these elements are just those of a conventional Standard Model Weinberg-Salam Higgs field, however, taken as the family quantum number for that family that corresponds to the number of the diagonal matrix element in question. This gives at first an order of magnitude degeneracy of quarks and leptons in the same family at the Planck scale of running couplings, a result that is good for simulating $SU(5)$ GUT predictions – only order of magnitudewise – and for explaining the crude degeneracy of up-quark and electron mass (when extrapolated to Planck scale). However, the Charm quark and top-quark masses need to be explained as shooting up due to some special mechanism, and we managed to get them dominated by off-diagonal elements. Due to that the first family gets its mass matrix elements using $\rho$ and $\omega$ together with the Higgs fields already used to give the second family masses one tends to get just the same extra factor of the $\rho^3 \sim \omega^3$ type and for mixing angles/off-diagonal elements $\omega \rho^\dagger$. There is as a result in such models easily achievable factorisation of the mixing angles for the quarks and for the neutrinos, respectively, at the Planck scale:

$$V_{ub} \approx V_{us} V_{cb}, \quad (113)$$

$$\theta_{\text{chooz}} \approx \theta_{\odot} \theta_{\text{atm}}. \quad (114)$$

This going to first family via the second family just getting an extra $\omega^3$ or $\omega \rho^\dagger$ roughly is also behind how in our model we get much less hierarchy among the left-handed neutrinos than for the charged quarks and leptons. In fact the lightness of the right-handed neutrinos “of first generation” gives a large propagator largely being compensated by the corresponding smallness of the first family Dirac mass matrix elements. So the much less
hierarchy for the neutrinos have a rather natural explanation. This fact supports the see-saw mechanism and thereby the Fukugita-Yanagida scheme.

However, we should admit that the rather large neutrino mixing angles in our model are obtained by making it possible to fit them to be of order unity. We use e.g. for the achievement of the large atmospheric neutrino mixing angle the adjustment of the $\chi$ vacuum expectation value so as to be not much different from that of $T$. The solar mixing angle is made order unity by having $\rho \sim \omega$ with about same vacuum expectation values.

Further mass relations at the Planck scale are in our model:

$$V_{us} = \theta_c \approx (\theta_\odot)^{-\frac{1}{4}} \left( \frac{m_d}{m_s} \right)^{\frac{1}{4}},$$

$$m_b^3 \approx m_s m_c m_t.$$  \hspace{1cm} (115)

Concerning the baryon number production it should be said that due to the much larger $CP$ violation, i.e., $\epsilon_2$ and $\epsilon_3$, in the decay of the two heavier and almost degenerate see-saw neutrinos than in the decay of the lightest one, it is the heavy neutrinos that deliver the main/dominant contribution to the surviving $(B-L)$ excess and thus to the baryon number.

This of course means that it is important for our success that there were no inflation after the era of the heavy pair of degenerate see-saw neutrinos to attenuate the $B-L$ produced and add re-heating entropy diminishing $Y_{B-L}$ (or $Y_B$). It is also important for such success that we had already at this era roughly the number of see-saw neutrinos corresponding to thermal equilibrium as we used in our calculation. That means that inflation eras should already have been recovered from at the time when the temperature reached down to $10^{10}$ GeV (see eqs. [36-38]).

11.2 Developments in calculational method

It should be mentioned that we have made the present numerical calculations taking into account the renormalisation group running effects of the Yukawa couplings as well as the effective dimension five operator corresponding to two Weinberg-Salam Higgses interacting with two left-handed neutrinos giving the neutrino masses observed. Compared to our previous work we included the running between the Planck scale and the see-saw scale of the Dirac neutrino Yukawa couplings.

11.3 On the accuracy

At first one might wonder if such runnings are needed when we at the end in principle get only order of magnitude predictions. However, they can give easily factors of three (or more) and we really have managed to get agreeing results with accuracy corresponding to that the simple mass matrix elements are uncertain only by the theoretically expected
±64% logarithmically, so indeed it is important to include corrections that could be of the order of a factor two. It is quite remarkable that we are able to work/fit with this good accuracy in a model that is in principle only order of magnitude. If it is not just because we had too many discrete details, *i.e.*, quantum numbers of Higgs field, to adjust it should mean that there is indeed in nature couplings that are *a priori* of order unity in a way that in praxis means that one can count on them being of that order to the rather high accuracy we achieved. With this our ±64% accuracy in mind we may remark that we have in spite of working only “with orders of magnitude” gotten rather close to the experimental uncertainties in the quantities which we fit: For the very light quarks up, down and strange quarks one can get variations from one lattice computation to the other one that can “well” reach our uncertainty and the neutrino parameters are typically also uncertain with such uncertainties.

Of course one can still be more ambitious than we were in the present article and hope for understanding the well measured mass ratios of the charged leptons, but there is not at all as much yet to fit as if the quarks and neutrons masses and mixings had been equally well measured and thus an order of magnitude fitting is at the present time using quite a significant bit of the data that can possibly help us to dig behind the Standard Model.

At the end let us admit that our ambitious model does not have any candidate in it for dark matter nor for the LSND neutrino anomaly. That would require totally new elements in our model, as for instance a hidden sector. Combining our model with Supersymmetry would enforce us to double all the Higgs fields except W-Higgs field, and the model would loose much predictive power drastically.

**Note added**

During the completion of this article, the reference [47] appeared which treats also the β function for the Dirac neutrino Yukawa coupling. Our result (eq. 35) does not match with their result, with respect to the coefficient of weakhypercharge gauge coupling, because of the different notation: we used the GUT notation for $g_1$ coupling constant.

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References

[1] C. D. Froggatt, H. B. Nielsen and Y. Takanishi, hep-ph/0201152; to be published in Nucl. Phys. B.

[2] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369; ibid. 20 (1979) 2634;
S. P. Mikheev and A. Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913; Nuovo Cim. C 9 (1986) 17.

[3] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[4] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124 (1961) 246.

[5] T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan (1979), eds. O. Sawada and A. Sugamoto, KEK Report No. 79-18;
M. Gell-Mann, P. Ramond and R. Slansky in Supergravity, Proceedings of the Workshop at Stony Brook, NY (1979), eds. P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979).

[6] H. B. Nielsen and Y. Takanishi, Nucl. Phys. B 588 (2000) 281; ibid. 604 (2001) 405; Phys. Lett. B 507 (2001) 241.

[7] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531;
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[8] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[9] H. Arason, D. J. Castaño, B. Keszhelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. D 46 (1992) 3945.

[10] S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B 519 (2001) 238;
P. H. Chankowski and P. Wasowicz, Eur. Phys. J. C 23 (2002) 249.

[11] C. D. Froggatt, H. B. Nielsen and D. J. Smith, hep-ph/0108262.

[12] Q. R. Ahmad et al., SNO Collaboration, Phys. Rev. Lett. 87 (2001) 071301.

[13] B. T. Cleveland et al., Astrophys. J. 496 (1998) 505.

[14] J. N. Abdurashitov et al., SAGE Collaboration, Phys. Rev. C 60 (1999) 055801.

[15] W. Hampel et al., GALLEX Collaboration, Phys. Lett. B 447 (1999) 127.

[16] E. Belloti, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000.

[17] S. Fukuda et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 86 (2001) 5651.
[18] J. N. Bahcall, M. H. Pinsonneault and S. Basu, Astrophys. J. 555 (2001) 990.
[19] G. L. Fogli, E. Lisi, D. Montanino and A. Palazzo, Phys. Rev. D 64 (2001) 093007.
[20] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Peña-Garay, JHEP 0108 (2001) 014.
[21] A. Bandyopadhyay, S. Choubey, S. Goswami and K. Kar, Phys. Lett. B 519 (2001) 83.
[22] P. I. Krastev and A. Yu. Smirnov, hep-ph/0108174.
[23] S. Fukuda et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 86 (2001) 5656.
[24] Y. Fukuda et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 81 (1998) 1562; S. Fukuda et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 85 (2000) 3999; T. Toshito, talk at the 36th Rencontres de Moriond on Electroweak Interactions and Unified Theories, Les Arcs, France, 10-17 Mar 2001; hep-ex/0105023.
[25] M. Apollonio et al., CHOOZ Collaboration, Phys. Lett. B 466 (1999) 415.
[26] M. C. Gonzalez-Garcia and C. Peña-Garay, Phys. Lett. B 527 (2002) 199.
[27] M. C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay and J. W. Valle, Phys. Rev. D 63 (2001) 033005.
[28] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, hep-ph/0104221.
[29] S. M. Bilenky, D. Nicolo and S. T. Petcov, hep-ph/0112216.
[30] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039.
[31] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney and I. V. Krivosheina, Mod. Phys. Lett. A 16 (2002) 2409.
[32] M. Fukugita and T. Yanagida, Phys. Lett. B174 (1986) 45.
[33] P. Di Bari, in progress.
[34] see for example: M. Hirsch and S. F. King, Phys. Rev. D 64 (2001) 113005; F. Buccella, D. Falcone and F. Tramontano, Phys. Lett. B 524 (2002) 241; W. Rodejohann and K. R. Balaji, hep-ph/0201052; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquín and M. N. Rebelo, hep-ph/0202030; T. Fukuyama and N. Okada, hep-ph/0202214.
[35] G. ’t Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D14 (1976) 3432 [Erratum-ibid. D 18 (1976) 2199].
[36] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B155 (1985) 36.
[37] A. D. Sakharov, JETP Lett. 5 (1967) 24.

[38] M. A. Luty, Phys. Rev. D45 (1992) 455.

[39] W. Buchmüller and M. Plümer, Phys. Lett. B431 (1998) 354.

[40] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B384 (1996) 169.

[41] A. Pilaftsis, Int. J. Mod. Phys. A14 (1999) 1811.

[42] M. Fujii, K. Hamaguchi and T. Yanagida, hep-ph/0202210.

[43] E. W. Kolb and M. S. Turner, The Early Universe, Addison-Wesley, Redwood City, USA, 1990.

[44] J. A. Harvey and M. S. Turner, Phys. Rev. D42 (1990) 3344.

[45] H. Murayama and A. Pierce, Phys. Rev. D 65 (2002) 055009.

[46] D. L. Bennett and H. B. Nielsen, Int. J. Mod. Phys. A 9 (1994) 5155; ibid. A 14 (1999) 3313.

[47] S. Antusch, J. Kersten, M. Lindner and M. Ratz, hep-ph/0203233.