ERRATUM

The Influence of Stress Gradient on the Pull-in Phenomena of Microelectromechanical Switches
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2006 Journal of Physics: Conference Series 34 1117-1122

Received: 13 June 2011
Published: 16 June 2011

A bug was discovered in the code used for the finite difference method of solving the nonlinear differential quadrature of MEMS switches incorporating stress gradient in the original article. This erratum presents the abstract, figures 4 and 6 and conclusion for the corrected results.

Abstract
The electrostatic pull-in instability of a cantilever beam MEMS switches with considering the linear positive and negative stress gradient is studied in this paper. The nonlinear differential equation of the pull-in instability is solved by central finite-difference method. The stress gradient is modelled as built-in moment acting on the cantilever. Unlike the previous linear models, it is shown that the response of the pull-in instability to the stress gradient is nonlinear. The presented model with the numerical solution can offer an efficient tool for design and optimization of cantilever-like MEMS switches.

![Figure 4: Pull-in voltage versus stress gradient (Positive and Negative).](image)
Conclusions
Using central finite-difference method, the electrostatic pull-in instability of a cantilever-like MEMS switches incorporating linear positive and negative stress gradient was investigated. The model approximated the stress gradient as built-in moment that effectively changed the initial gap between the cantilever and the substrate. One of the important findings of this paper is that the response of pull-in instability in cantilevers to the linear stress gradient is highly nonlinear, and therefore, it has to be included in the design of MEMS switches. It can be also used as a material properties characterization tools for measuring the amplitude of stress gradient.
The Influence of Stress Gradient on the Pull-in Phenomena of Microelectromechanical Switches

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Abstract. The electromechanical behaviour of cantilever beam MEM switches incorporating linear residual stress gradient is studied in this work. Using Finite-difference method (FDM), behavioural models that predict the electrostatic deflection and pull-in voltage of such structures have been established. The model accounts for the effect of residual stress gradient on the mechanical behaviour of the cantilever MEM switches, and results have been validated via comparison with experimental data. The completed model offers an efficient means of design, analysis and optimization of cantilever-based electrostatically actuated MEMS devices. It can also be utilized for material property measurement and analysis.

1. Introduction
The electrostatic actuation principle [1] is one of the most common actuation methods of modern Microsystems. Electrostatically actuated micromachined cantilever beams are used in a wide range of applications such as MEM switches. Accurate behavioral models of electrostatically actuated microstructures, as a means of analysis and optimizations, are therefore critical to the design of such devices. The problem of modeling electrostatically actuated cantilevers is not restricted to the complexity of elastic-electrostatic interaction, but also includes the reality of negative effects of the microfabrication process. Since one end of the cantilever is free, any residual stress within the film is released. In the actual switches will exhibit some curvature in the initial configuration due to stress gradients and process variations. This stress gradient is due to the different deposition condition encountered at the bottom and top layers of the cantilevers, or to the use of multiple layers each with a different residual stress component [2]. It is shown that Pull-in voltage estimation becomes inadequate when stresses and – more importantly – stress gradients are present in the structural material. The influence of stress gradients on the behavior of cantilever beams is small unless the stress gradients are large. The change in buckling amplitude only becomes significant once the stress gradient is large enough to cause significant curling i.e. a deflection on several microns of the tip of the cantilever. Since such curling is not observed in mumps cantilevers, it can be safely assumed that the influence of stress gradients on the buckling amplitude of cantilever beams is negligible. J. De Coster et al investigated the influence of stress gradients on the characteristics of electrostatic actuators [3].

In this article the influence of stress gradient on the cantilever beam MEM switch is investigated.

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2. Nonlinear Distributed Electromechanical Coupled Model

The cantilever MEMS switch shown schematically in Figure 1 is constructed from a single elastic beam to a fixed frame. The cantilever beam is used in various MEMS applications ranging from micromirrors [4] to inertial sensors [5]. Consider an electrostatic cantilever beam MEM switch formed from elastic conducting bodies with general shapes. The surface of the conducting bodies forms a free space capacitor, \( C \), with a variable gap. An energy source applies a voltage difference \( V \) across the capacitor electrodes forming an electrostatic force. In response, the bodies deform and mechanical strain energy is increases. The related mechanical stress that develops is the generalized restoring force.

When a driving voltage is applied between the electrodes, the electrostatic pressure deflects the beam. The mechanical bending strain energy \( U_m \) of the beam and the electrical co-energy \( U_e^* \) stored between the upper and lower electrode of the beam are given by:

\[
U_m = \int_0^L \left( -\frac{d^2 u}{dx^2} - z \frac{du}{dx} \right) dA = \int_0^L \frac{EI}{2} \left( \frac{d^2 u}{dx^2} \right)^2 \ dx
\]

\[
U_e^* = \frac{1}{2} \int_0^L \varepsilon_0 w V^2 \left( \frac{g - u(x)}{2} \right) \ dx
\]

where \( z \) is the coordinate in the load direction with origin in the centroid of the cross section, \( \varepsilon_0 \), \( w \), and \( g \) represent the permittivity of air, the width of the beam, and the initial gap between the upper and lower electrodes. \( u(x) \) is the deflection function and \( V \) is the applied voltage between the movable/ground plates on the fixed substrate. The total potential energy \( U \) of the system can be expressed as

\[
U = U_m + U_e
\]

The variation of total energy is zero at the equilibrium position.

\[
\delta U = \delta U_m + \delta U_e = \delta U_m - \delta U_e^* = 0
\]

\[
\delta U = \delta \int_0^L \frac{EI}{2} \left( \frac{d^2 u}{dx^2} \right)^2 \ dx - \delta \int_0^L \varepsilon_0 w V^2 \left( \frac{g - u(x)}{2} \right) \ dx = 0
\]

The \( \delta u \) is an arbitrary function, so to satisfying equation (5):

\[
\frac{d^2}{dx^2} \left( \frac{EI}{2} \frac{d^2 u}{dx^2} \right) = \frac{\varepsilon_0 w V^2}{2(g - u(x))^2}
\]

\( \tilde{E} \) is dependent on the beam width \( w \) and film thickness \( t \). A beam is considered wide when \( w \geq 5t \). Wide beams exhibit plane-strain conditions, and therefore, \( \tilde{E} \) becomes the plate modulus \( E/\left(1-v^2\right) \), where \( E \) is the young’s modulus. A beam is considered narrow when \( w < 5t \). In this case, \( \tilde{E} \) simply becomes the young’s modulus, \( E \). \( I \) is the effective moment of inertia of the cross-section and is \( wt^3/12 \) which is wide relative to thickness and width.

![Poly-silicon cantilever](image)
A uniform magnetic field cannot drop abruptly to zero at an edge. In actual situation, there is always a “fringing field” existing, and a more realistic situation including “fringing field” modification is necessary. The first order fringing-field correction \([6], [7]\) is expressed as:

\[
 f_f = 0.65 \frac{g - u(x)}{w},
\]

Therefore, the nonlinear electromechanical coupled equation would be

\[
 \tilde{E} \tilde{I} \frac{d^4 u}{dx^4} = -\varepsilon_0 V^2 w \left( 1 + 0.65 \frac{g - u(x)}{w} \right)
\]

3. Stress Gradients Consideration

Nonuniform stresses in the film thickness create built-in moments, which in released cantilevers cause them to curl out of plane. The stress gradient is modelled as a bending moment applied to the initially flat cantilever causing an initial deflection \(\delta_0(x)\). Figures 2 and 3 schematically demonstrate the modelled positive and negative stress gradient in cantilever beam respectively.

This bending moment causes an initial deflection in cantilever beam. By calculating this deflection and having the pull-in voltage, it is possible to estimate the stress gradient. The differential equation for initial state before applying voltage is:

\[
 \tilde{E} \tilde{I} \frac{d^2 u}{dx^2} = M_0
\]

\[
 \tilde{E} \tilde{I} \delta_0 = \frac{M_0 x^2}{2} + C_1 x + C_2
\]
where $\delta_0$ is the initial deflection due to stress gradient. From boundary condition of cantilever the coefficient $C_1, C_2$ are zero. Therefore the initial deflection would be

$$\delta_0 = \frac{M_0}{EI} \left( \frac{x^2}{2} \right) \quad (11)$$

By assuming the linear profile of residual stress as figure 2, 3, stress gradient is derived as bellow:

$$\sigma_r = (\nabla \sigma) z \quad (12)$$

where $(\nabla \sigma)$ is stress gradient.

$$M_0 = \int_A \sigma_r zdA = \int_A (\nabla \sigma z) \varepsilon dA = w (\nabla \sigma) \varepsilon \frac{x^3}{3} \frac{d}{dz} = \frac{wE}{12} (\nabla \sigma) \varepsilon \frac{x^3}{3} \frac{d}{dz} \quad (13)$$

$$\delta_0 = \frac{wE}{12} (\nabla \sigma) \varepsilon \frac{x^3}{2EI} \quad (14)$$

By considering stress gradient, the differential equation converts to

$$\ddot{\tilde{e}}_I \frac{d^4 u}{dx^4} = -\frac{\varepsilon_0 V^2 w}{2(g - u(x) \mp \delta_0(x)) \sqrt{1 + 0.65 \frac{g - u(x) \mp \delta_0(x)}{w}}} \quad (15)$$

A positive stress gradient causes the cantilever to bend upwards and the negative gradient acts vice versa. For the positive stress gradient, the sign in (15) is (+) and for the negative stress gradient, the sign is (-).

Now by definition stress gradient, it is possible to determine the influence of stress gradient on the pull-in voltage. By theoretical methods such as Finite difference method the pull-in voltage and deflection of cantilever can be calculated, therefore the deflection due to stress gradient is calculated and resultantly the stress gradient term can be estimated. Considering a cantilever beam made of mono-crystalline silicon subjected to a voltage, the results from proposed algorithm for cantilever beam model are compared with the result predicted from [8]. The geometric and material properties are [8]: Young’s modulus $E$ is 155.8 GPa, the Poisson’s ratio is 0.06, the length of beam $L$, is 20 mm, the width of the beam $w$, is 5 mm, the thickness $t$ is 57 μm and initial gap $g$ is 92 μm, and the permittivity of air is 8.85 PF/m. The lateral stress gradient is not explicitly specified, but is estimated at 0.02 MPa/μm. This value is used to check the influence on the pull-in voltage. Table 1 shows the gaps versus different voltage and compared with experimental results (Δ is the difference between proposed model and experimental result from [8]).

| Voltage (V) | End Gap (μm) | Experiment (μm) [8] | Δ% |
|------------|--------------|---------------------|----|
| 20         | 90.2         | 90.5                | 0.33 |
| 40         | 84.1         | 84.6                | 0.59 |
| 60         | 69.1         | 70.0                | 1.43 |
| 65         | 60           | 64.0                | 6.25 |

Figure 4 demonstrates the variation of pull-in voltage by increasing the Positive and negative residual stress. By considering the stress gradient about 0.02 MPa/μm, the deflection of the beam due to pull-in voltage is demonstrated in Figure 5.
The effects of the both positive and negative stress gradients on pull-in capacitance are demonstrated in figure 6. In the positive stress gradient form because of increasing the initial gap, the pull-in capacitance is increased and vice versa.
4. Conclusion
Understanding and accurate modelling of the pull-in behaviours is crucial for the design of electrostatically actuated MEMS devices. Behavioural formulae predicting the electrostatic deflections and pull-in voltage of micromachined cantilevers incorporating stress gradient was presented. The models developed were theoretically validated for positive and negative stress gradient and give high accuracy results. The models can be used for the design, analysis and optimization of cantilever-based electrostatically actuated MEMS and for material property measurement.

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