Introduction to Non-linear Longitudinal Beam Dynamics

H. Damerau
CERN

Introduction to Accelerator Physics

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Outline

• Introduction

• Linear and non-linear longitudinal dynamics
  • Equations of motion, Hamiltonian, RF potential

• Longitudinal manipulations
  • Bunch length and distance control by multiple RF systems
  • Bunch rotation

• Synchrotron frequency distribution
  • Effect on longitudinal beam stability

• Summary
Introduction
Signals generated by radio-frequency systems in particle accelerators are of the form $V \sin(h \omega_{\text{rev}} t)$

- Resonance effect: large voltage with little effort

- Inherently non-linear
- Linear longitudinal beam dynamics only an approximation

Effect of non-linearity on beam?

Tools to describe and analyse non-linearity

Use non-linearity to improve beam conditions
Non-linear longitudinal dynamics
Example: LHC-type beam in the CERN PS

- Non-linear RF allows to control all longitudinal parameters
  → Number of bunches, bunch length and emittance, longitudinal stability

Triple splitting at $E_{\text{kin}} = 2.5$ GeV

Split in four at flat top energy

Inject $4+2$ bunches

Eject 72 bunches

$\gamma_{\text{tr}}$

2nd injection

$E_{\text{kin}} = 2.5$ GeV

$E_{\text{kin}} = 2.5$ GeV

26 GeV/c

$h = 7$

$h = 21$

$h = 84$

Cycle time [s]
Example: LHC-type beam in the CERN PS

- Non-linear RF allows to control all longitudinal parameters
  → Number of bunches, bunch length and emittance, longitudinal stability
Inject 4+4 bunches

Controlled blow-ups

$h = 9$

$h = 21$

$\gamma_{tr}$

$h = 84$

Eject 48 bunches

→ RF manipulation from 8 bunches in $h = 9$ to 12 in $h = 21$

→ Transition crossing

→ RF voltage reduction during acceleration

→ Splitting at the flat-top

→ Bunch shortening (rotation) before extraction
Inject 4+4 bunches

Controlled blow-ups

Where profit from non-linear RF?

→ RF manipulation from 8 bunches in \( h = 9 \) to 12 in \( h = 21 \)
→ Transition crossing
→ RF voltage reduction during acceleration
→ Splitting at the flat-top
→ Bunch shortening (rotation) before extraction
Applications

- Introduce extra non-linearity
  - Bunch lengthening in double-harmonic RF system to reduce peak current (space charge)
    \[ V_1 \sin(h_1 \omega_{\text{rev}} t + \phi_1) + V_2 \sin(h_2 \omega_{\text{rev}} t + \phi_2) \]
  - Short and long bunches with multi-harmonic RF systems
    \[ \sum_n V_n \sin(h_n \omega_{\text{rev}} t + \phi_n) \]
  - Adapt bunch-to-bunch distance

- Profit from non-linearity for beam stabilization
  - Stabilize beam using higher-harmonic RF
  - Controlled longitudinal emittance blow-up
Linear longitudinal beam dynamics
Interaction between particles and RF

Simple accelerator model:

Energy dependent phase advance, $\phi$:

$$\phi_{n+1} = \phi_n + 2\pi \hbar \eta \frac{\Delta E_n}{\beta^2 E}, \quad \eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$$

Phase dependent energy gain, $\Delta E$:

$$\Delta E_{n+1} = \Delta E_n + qVg(\phi_{n+1})$$

Works for arbitrary shape of acceleration amplitude $g(\phi)$
Linear longitudinal beam dynamics

- Usual longitudinal beam dynamics already non-linear, since RF system usually provides sinusoidal amplitude
- **Linear** longitudinal beam dynamics?
Linear longitudinal beam dynamics

- Construct Hamiltonian from equations of motion

\[
\frac{d}{dt} \phi = \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)
\]

\[
\frac{d}{dt} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{qV}{2\pi \phi}
\]

- Hamiltonian constant on trajectory
  → ‘Energy conservation’

\[
q = \phi \quad p = \frac{\Delta E}{\omega_{\text{rev}}}
\]

\[
H(p, q) = T(p) + W(q)
\]

\[
H(p, q) = H_{\text{trajectory}}
\]
Linear longitudinal beam dynamics

\[ \frac{d}{dt} \phi = \frac{h \eta \omega_{rev}}{pR} \left( \frac{\Delta E}{\omega_{rev}} \right) \]

\[ \frac{d}{dt} \left( \frac{\Delta E}{\omega_{rev}} \right) = \frac{qV}{2\pi} \phi \]

The Hamiltonian from the equations can be written as

\[ H \left( \phi, \frac{\Delta E}{\omega_{rev}} \right) = \frac{1}{2} \frac{h \eta \omega_{rev}}{pR} \left( \frac{\Delta E}{\omega_{rev}} \right)^2 - \frac{1}{2} \frac{qV}{2\pi} \phi^2 \]

\[ = \frac{1}{2} \frac{pR}{\eta \omega_{rev}} \dot{\phi}^2 - \frac{1}{2} \frac{qV}{2\pi} \phi^2 \]

\[ \eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2} \]
Linear longitudinal beam dynamics

\[ H \left( \phi, \frac{\dot{\phi}}{\omega_S} \right) = \frac{1}{2} \left( \frac{\dot{\phi}}{\omega_S} \right)^2 + \frac{1}{2} \phi^2 = T + W \]

→ Particles move on circular trajectories in $\phi-\dot{\phi}/\omega_S$ phase space
→ RF potential is parabolic, $W(\phi) \sim \phi$
→ Hamiltonian is constant on these trajectories
Linear longitudinal phase space

- Simple model
- Circular trajectories
- All particles have same synchrotron frequency
- Normalized bucket area: $A_b = \pi r^2 = \pi^3$

→ Harmonic oscillator
Non-linear longitudinal beam dynamics
Introduce most simple non-linearity

RF amplitude function

\[ V\phi \rightarrow V\sin\phi \]

\[
\frac{d}{dt} \phi = \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)
\]

\[
\frac{d}{dt} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{qV}{2\pi} (\sin\phi - \sin\phi_S)
\]

\[
H\left(\phi, \frac{\Delta E}{\omega_{\text{rev}}}\right) = \frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos\phi - \cos\phi_S + (\phi - \phi_S) \sin\phi_S \right]
\]

with \( \phi = \phi_S + \Delta\phi \) this becomes

\[
H\left(\Delta\phi, \frac{\Delta E}{\omega_{\text{rev}}}\right) = \frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos(\phi_S + \Delta\phi) - \cos \phi_S + \Delta\phi \sin \phi_S \right]
\]

→ Standard longitudinal beam dynamics → Lectures F. Tecker
Introduce most simple non-linearity

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos(\phi_S + \Delta \phi) - \cos \phi_S + \Delta \phi \sin \phi_S \right] \]

using \[
\cos(\phi_S + \Delta \phi) = \cos \phi_S \cos \Delta \phi - \sin \phi_S \sin \Delta \phi
\]

\[ \simeq \cos \phi_S \left( 1 - \frac{1}{2} \Delta \phi^2 \right) - \sin \phi_S \Delta \phi \]

this Hamiltonian simplifies to

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) \simeq \frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 - \frac{1}{2} \frac{qV}{2\pi} \cos \phi_S \Delta \phi^2 \]
In the centre of the bucket, particles move on elliptical trajectories in $\Delta \phi$-$\Delta E$ phase space.

Hamiltonian is constant on these trajectories.

In the bucket centre, particles oscillate with the synchrotron frequency, $\omega_s = 2\pi f_s$.

$$\omega_s^2 = -\frac{h\eta \omega_{rev} qV \cos \phi_s}{2\pi pR}$$

$$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$$
Longitudinal emittance

• Compare two particles on the same trajectory
  1. No phase deviation       2. No energy deviation

\[
H \left( \Delta \phi = 0, \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2
\]

\[
H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} = 0 \right) = -\frac{1}{2} \frac{qV}{2\pi} \cos \phi_s \Delta \phi^2
\]
Longitudinal emittance

- Compare two particles on the same trajectory
  1. **No phase deviation**  
  2. **No energy deviation**

![Diagram of longitudinal emittance](image)

\[ H \left( \Delta \phi = 0, \frac{\Delta E}{\omega_{rev}} \right) = \frac{1}{2} \frac{\hbar \eta \omega_{rev}}{p R} \left( \frac{\Delta E}{\omega_{rev}} \right)^2 \]

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{rev}} = 0 \right) = -\frac{1}{2} \frac{q V}{2 \pi} \cos \phi_S \Delta \phi^2 \]

**Longitudinal emittance, \( \varepsilon_l \)**

- Surface occupied by particles in longitudinal phase space
- Preserved in physical \([\pi \Delta \tau \Delta E] = \text{eVs}\)
More non-linearity: multi-harmonic RF

RF amplitude: \[ V \sin \phi \rightarrow V[\sin \phi + r \sin(n\phi + \phi_1)] \]

- Example case \( n = 2 \) and \( r = 0.5 \)

→ Local voltage gradient decreased
→ Bunch is stretched
→ Lower peak current

→ Local voltage gradient increased
→ Bunch is compressed
→ Higher peak current
Example application: space charge in PSB

RF amplitude: \( V \sin \phi \rightarrow V[\sin \phi + r \sin(n\phi + \phi_1)] \)

\( \rightarrow \) Space charge \( \propto \) instantaneous current

- Inverted gradient at bucket centre
- Flattened bunch with reduced peak current \( \rightarrow \) Space charge reduction at low energy
Long and short bunches simultaneously

- Example BESSY VSR

- Depending on user of synchrotron radiation: need long or short bunches

- Do long and short bunches simultaneously!

- $4 \times 0.5$ GHz NC (existing)
- $4 \times 1.5$ GHz supercond.
- $4 \times 1.75$ GHz supercond.
**Bunch length modulation**

- Future 3-harmonic RF system for BESSY VSR

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**RF voltage sum**

![Graph of RF voltage sum]

- RF voltages sum up or cancel

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**RF Potential**

![Graph of RF potential]

- Short deep or bathtub potential well

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**Longitudinal phase space**

![Graph of longitudinal phase space]

- Short or stretched bucket
• 300 mA average current
→ High-current single bunches
  → short (0.8 mA) & long (10 mA)
→ Special high-current density bunches

开设 Two electron storage ring in one

Thanks to longitudinal beam dynamics trick
Example: adjust bunch spacing

- Was used at CERN PSB-to-PS to transfer 2 bunches at once
- Circumference ratio $C_{PS}/C_{PSB} = 4$

→ Ratio virtually moved to $2/7$: use $h_{RF} = 2 + 1$

1. Add $h_1$ component such that bunches approach to 245 ns (small spacing) → big spacing becomes 327 ns
2. Synchronize on $h_1$ to the PS
3. Trigger extraction kicker in-between the small spacing
4. Eject two bunches per ring at a distance of 327 ns
Introduce general non-linearity

Replace \( V \sin \phi \rightarrow V g(\phi) \rightarrow \text{arbitrary amplitude} \)

Equations of motion

\[
\frac{d}{dt} \phi = \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right) \\
\frac{d}{dt} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{qV}{2\pi} \left[ g(\phi) - g(\phi_S) \right]
\]

The Hamiltonian describing the system becomes

\[
H \left( \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 - \frac{qV}{2\pi} \left[ \int g(\phi) d\phi - g(\phi_S) \phi \right]
\]

\[
\eta = \frac{1}{\gamma_{\text{tr}}^2} - \frac{1}{\gamma^2}
\]
**Arbitrary RF waveform**

\[
H\left(\phi, \frac{\Delta E}{\omega_{\text{rev}}}\right) = \frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left(\frac{\Delta E}{\omega_{\text{rev}}}\right)^2 - \frac{qV}{2\pi} \left[ \int g(\phi) d\phi - g(\phi_S)\phi \right]
\]

Using

\[
\dot{\phi} = \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left(\frac{\Delta E}{\omega_{\text{rev}}}\right)
\]

The Hamiltonian can be rewritten, with RF potential \(W(\phi)\)

\[
H(\phi, \dot{\phi}) = \frac{1}{2} \left(\frac{\dot{\phi}}{\omega_S}\right)^2 + W(\phi)
\]

\[
W(\phi) = \frac{1}{\cos \phi_S} \left[ \int g(\phi) d\phi - g(\phi_S)\phi \right]
\]
Longitudinal beam manipulations using non-linearity
→ Calculate aspect ratio of bucket trajectories

\[ H \left( \Delta \phi = 0, \frac{\Delta E}{\omega_{\text{rev}}} \right) = \frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{p R} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 \]

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} = 0 \right) = -\frac{1}{2} \frac{q V}{2 \pi} \cos \phi_S \Delta \phi^2 \]

Equating both sides gives

\[ \left( \frac{\Delta E}{\Delta \tau} \right)^2 = -\frac{q V}{2 \pi} E \beta^2 \hbar \omega_{\text{rev}}^2 \frac{\cos \phi_S}{\eta} \]

with emittance as \( \varepsilon_l = \pi \Delta \tau \Delta E = \text{const.} \)

\[ \Delta \tau \propto \frac{1}{\sqrt{V}} \]

→ Not efficient at all

→ 16 times more RF voltage needed to cut bunch length in half
Abrupt change of RF voltage

→ Individual particles in matched bunch oscillate but no macroscopic motion

→ Abruptly changing the RF voltage flips particles to new trajectories

→ The bunch distribution seems to rotate

→ Exchange of bunch length and momentum spread
Introduce sudden change: bunch rotation

→ Quickly exchange longitudinal phase space behind bunch
→ Increase RF voltage much faster than period of $f_s$
Introduce sudden change: bunch rotation

→ Quickly exchange longitudinal phase space behind bunch
→ Increase RF voltage much faster than period of $f_s$
Introduce sudden change: bunch rotation

→ Switch RF voltage much faster than period of $f_s$

\[
V_i \propto \left( \frac{\Delta E_i}{\Delta \tau_i} \right)^2 \quad \text{and} \quad V_f \propto \left( \frac{\Delta E_f}{\Delta \tau_i} \right)^2
\]

\[
\frac{\Delta \tau_f}{\Delta \tau_i} = \frac{\Delta E_i}{\Delta E_f} = \sqrt{\frac{V_i}{V_f}}
\]
Example: PS to SPS transfer at CERN

- Fit 14 ns long bunches into 5 ns long buckets in the SPS
  → Double-step bunch rotation

\[ \text{4σ = 14 ns} \]

\[ \text{11 ns} \]

\[ \text{4 ns} \]

\[ 80 \text{ MHz (} h = 168 \text{)} \]

\[ 40 \text{ MHz (} h = 84 \text{)} \]

Extraction

Bunch splittings

Adiabatic shortening

Bunch rotation
Example: rotation at PS-SPS transfer

→ Bunch length now proportional to $1/\sqrt{V}$ and not $1/\sqrt[4]{V}$
→ Can save enormous RF voltage

→ Bunch shortening from 14 ns to 4 ns (ratio $\sim 3.5$)
→ Starting from 100 kV at 40 MHz

→ Slow shortening would require $100 \text{kV} \cdot 3.5^4 \sim 15 \text{ MV}$
→ Installed RF voltage is only about 1.2 MV
Profiting from the non-linear rotation

Need large momentum spread for slow extraction
1. Jump RF phase such that bunch at unstable fixed point
2. Jump back
3. Let bunch rotate, switch RF off at large momentum spread

→ Non-linearly of bunch rotation helps
Example: using the non-linearity

Need large momentum spread for slow extraction

1. Jump RF phase such that bunch at unstable fixed point
2. Jump back
3. Let bunch rotate, switch RF off at large momentum spread

→ Almost constant momentum distribution after rotation
Synchrotron frequency distribution
General synchrotron frequency

- Synchrotron frequency depends on trajectory
  → Calculate average velocity for given trajectories in longitudinal phase space → **Action angle, \( J \)**

\[
J(H) = \frac{1}{2\pi \omega_S} \int \dot{\phi}(\phi) \, d\phi
\]

The angular frequency becomes

\[
\omega(H) = \frac{d}{dJ} H
\]

**General expression for \( \omega_S \)**

\[
\frac{\omega(H)}{\omega_S} = \sqrt{2\pi} \int_{\phi_l}^{\phi_u} \frac{1}{\sqrt{H/\omega_S^2 - W(\phi)}} \, d\phi
\]

(for bucket boundaries \( \phi_l \rightarrow \phi_u \))
Distribution for stationary bucket

- Single-harmonic RF in stationary bucket

\[
\frac{\omega(\Delta \phi_u)}{\omega_S} = \frac{\pi}{2K[\sin(\phi_u/2)]} \approx 1 - \frac{\phi_u^2}{16}
\]

\(K(x): 1^{\text{st}} \text{kind elliptical integral function}\)
Distribution for stationary bucket

- Single-harmonic RF in stationary bucket

\[ \frac{\omega(\Delta \phi_u)}{\omega_S} = \frac{\pi}{2K[\sin(\phi_u/2)]} \sim 1 - \frac{\phi_u^2}{16} \]

\( K(x) \): 1st kind elliptical integral function

\[ \frac{\Delta \omega}{\omega_S} \]

\[ \frac{\omega_S}{\omega_S} \text{ vs. trajectory duration} \]

\[ \frac{\omega_S}{\omega_S} \text{ vs. surface encircled by trajectory} \]

→ Different synchrotron frequencies of particles in bunch
→ Total spread \( \Delta \omega / \omega_S \) depends on filling factor of bucket
Example: Emittance control with noise

- Noise excitation of bunch by band-width limited noise
  → Controlled longitudinal blow-up in the PSB

1. Choose upper frequency to **cover synchrotron frequency at bunch centre**
2. Choose lower frequency to **match target emittance**
3. Excite

---

D. Quartullo
Analogies: pendulums mounted on a bar

- All particles have the same resonance frequency
  - Easy to excite macroscopic oscillation

- Resonance frequencies of individual particles vary
  - Difficult to excite macroscopic oscillation

→ Large synchrotron frequency spread increases stability
Bucket filling ratio

Smaller or larger bunch or bucket? What is more stable?

- Easy to excite
  → Prone to instability

- Large $f_S$ spread
  → More stable
Example: stabilization with lower voltage

→ Acceleration of protons in the CERN PS ($E_{\text{total}} = 3.4 \rightarrow 26 \text{ GeV}$)
Example: stabilization with lower voltage

→ Acceleration of protons in the CERN PS (3.4 → 26 GeV total)

- Same principle also applied in SPS and LHC
  → Prevent bucket filling to decrease
Additional non-linearity by double RF

→ RF system at twice the main frequency and at half amplitude

\[ \frac{V_2}{V_1} = \frac{1}{2}, \ \frac{h_2}{h_1} = 2 \]

- Both RF systems in phase
  → Important increase in synchrotron frequency spread
  → Improves stability
Additional non-linearity by double RF

→ RF system at twice the main frequency and at half amplitude

\[ \frac{V_2}{V_1} = \frac{1}{3}, \frac{h_2}{h_1} = 3 \]

\[ V_1 \sin(h_1 \phi) + V_2 \sin(h_2 \phi) \]

• Both RF systems in phase
  → Important increase in synchrotron frequency spread
  → Improves stability
Additional non-linearity by double RF

→ RF system at twice the main frequency and at half amplitude

$V_2/V_1 = 1/4, \ h_2/h_1 = 4$

$V_1 \sin(h_1 \phi) + V_2 \sin(h_2 \phi)$

→ Local regions of bunch with no $f_S$ gradient
→ Again prone to instability
→ Reduce voltage of 2\textsuperscript{nd} harmonic RF system
→ Improving stability depends on appropriate voltage ratio
Two RF systems in counter-phase?

→ 2nd RF twice frequency, half amplitude in counter-phase

\[ \frac{V_2}{V_1} = \frac{1}{2}, \frac{h_2}{h_1} = 2 \]

![Graph showing RF systems in counter-phase](image)

- Large frequency spread at bunch centre with perfectly adjusted phases
  → Minor phase offset causes locally unstable regions
  → Works only for very short bunches
  → Electron accelerators
Example: damping observations in the PS

- Quadrupolar coupled-bunch oscillations at flat-top
- Main RF system: \( h_1 = 21, \) 10 MHz, 4 out of 18 bunches
- Higher-harmonic RF system: \( h_2 = 84, \) 40 MHz

Both RF systems in phase:
\[ \rightarrow \] Highest peak current, but most stable
Summary

• Longitudinal beam dynamics
  → Everything non-linear

• Longitudinal manipulations
  → Tricks to adjust length and distance of bunches
  → Do more with less RF

• Synchrotron frequency spread
  → More RF voltage may result in less stability
  → Higher peak density may be more stable
  → Improve stability and control emittance
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Spare slides
Stationary bucket in normalized coordinates

→ RF bucket properties become independent from accelerator parameters

→ Significant simplification of equations, easy to use

Example of stationary bucket

→ Bucket height
  \[ \frac{\phi_B}{\omega_S} = 2 \text{ rad} \]

→ Bucket area
  \[ \frac{A_B}{\omega_S} = 16 \text{ rad}^2 \]

→ Exception: conservation of longitudinal phase space