Finite-Time Control of Wing-Rock Motion for Delta Wing Aircraft Based on Whale-Optimization Algorithm

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ABSTRACT

The rise of wing-rock motion in delta-wing aircraft has an adverse effect on the manoeuvrability of aircraft and it may result in its crash. This study presents a finite-time control design to tackle the dynamic motion due to the Wing-Rock effect in delta-wing aircraft. The control design is developed based on the methodology of Super Twisting Sliding Mode Control (STSMC). The Lyapunov stability analysis has been pursued to ensure asymptotic convergence of errors and to determine the finite time. The design of STSMC leads to the appearance of design parameters, which have a direct impact on the dynamic performance of the controlled system. To avoid the conventional tuning of these parameters and to have an optimal performance of the proposed controller, a modern optimization technique has been proposed based on Wale Optimization Algorithm. A comparison study between optimal and non-optimal finite-time super twisting sliding mode controllers has been established and their effectiveness has been verified via numerical simulation using MATLAB programming format.

ARTICLE INFO

Article History:
Submitted/Received 14 Mar 2020
First revised 26 Apr 2021
Accepted 06 Jul 2021
First available online 26 Aug 2021
Publication date 01 Dec 2021

Keyword:
Delta wing aircraft,
Finite-time control,
Super-twisting sliding mode control,
Stability analysis,
Whale optimization algorithm,
Wing-Rock motion.
1. INTRODUCTION

The wing rock is a self-sustaining limit cycle oscillation (LCO), which occurs due to nonlinear coupling between unsteady aerodynamic forces and the dynamic response of the aircraft. Many aircrafts having slender plan-form may experience self-induced oscillatory rolling motion when operating at high angles of attack. This oscillatory motion is commonly known as "wing rock". The wing rock motion also arises from nonlinear aerodynamic mechanisms arising at high angles of attack (Saleh et al., 2010; Al-Qassar et al., 2021a; Al-Qassar et al., 2021b; Castellanos et al., 2021).

In general, the source of wing rock phenomenon in the aircraft could be caused by aerodynamic conditions during flight or mechanical hysteresis due to cable stretching, dry friction, backlash, and hydraulic oil compressibility. Wing rock can also be caused by inherent mechanical hysteresis in the aircraft. For example, in certain commercial aircraft, the occurrence of wing rock is due to backlash in the power transmission system. The phenomenon of wing rock occurs only under certain flight conditions, e.g., altitude of 3000 ft and Mach number of 0.6 for the aircraft under study. The precise knowledge and shape of the waveform of wing rock are difficult to obtain from the aircraft. The limit cycle is characterized by low amplitude within the range of 0.1 to 0.5 degrees and a period of 1.5 to 3.0 seconds in yaw oscillation. By reducing the magnitude of backlash in these sources, the problem of wing rock or limit cycle oscillation was significantly reduced.

A wide range of studies has also been conducted by researchers on aircraft exhibiting wing rock phenomenon resulting from aerodynamic hysteresis. Some of these aircraft that have been reported to experience wing rock include; F-16 Fighting Falcon, F-18 Hornet F-14 Tomcat, F-4 Phantom, A-4 Skyhawk, Gnat Trainer, and Torando. Owing to the different types of aircraft and different sources of the primary physical mechanism responsible for the nonlinearities, the scope and problem of wing rock are also not very well understood. In general, it is believed that wing rock motion is excited by flow asymmetries, evolved by negative roll-damping and sustained by the nonlinear aerodynamic of roll damping. Figure 1 shows the schematic of the subsonic flow field over the top of the delta wing. As indicated in the figure, the wing-rock is generated due to asymmetry of leading-edge vortices, asymmetry of fore-body vortices and dynamic stall.

![Figure 1. Schematic of the subsonic flow field.](https://example.com/figure1.png)
Wing rock motion may also be initiated either with a side-slip or during a zero side-slip flight with flow asymmetries over the flight of aircraft at a high angle of attack. For a study conducted on F-4 Phantom, it was observed that when operated at a high angle of attack the aircraft could undergo divergence behaviour in pitch and yaw known as "nose slice". Preceding the nose slice, the aircraft would experience wing rock motion.

Wing rock depends on the details of the configuration geometry of the aircraft. To suppress the wing rock on all types of aircraft, the primary mechanism responsible for the wing rock must be identified. Due to the complexity of flow fields for different aircraft, the identification of the exact causes and the source of the primary mechanism could be difficult. To eliminate the aircraft configuration dependent effects, research has been devoted to the slender delta wing model.

Wing rock motion is not acceptable from the operational and safety point of view. The problem is a concern to a pilot because it may have an adverse effect on aircraft manoeuvrability during landing approach or during a dogfight in a combat situation and it may lead to its crash (Ignatyev, 2018). The severity of wing rock may degrade the performance of weapon aiming control and accuracy.

Owing to the highly nonlinear nature of the flight dynamics, the problem of the wing rock is not very well understood. No satisfactory method has been developed to solve the problem. During the aerodynamic design stage of the aircraft, consideration can be taken to minimize the occurrence of the wing rock, such as the design with a slender body and highly swept wings, use of the roll damper, fore-body jet blowing, etc. These conventional control methods may not be effective to control wing rock occurring at high angles of attack.

The wing rock phenomenon is characterized by high nonlinearity and, hence, advanced and nonlinear controller has to be developed to cope with this complex nonlinearity. In what follows, literature of recent control researches that fix and solve the wing-rock problem in aircraft have been presented.

Some researchers proposed the recurrent dynamic RBF (Radial Basis Function) network control methodology to control the wing rock motion in WTI/F-16 test-bed aircraft with delta-wing configuration. The recurrent dynamic RBF network, in the proposed neural control scheme, works as the identifier, which approximates the unknown nonlinearities in the physical system according to input-output data gathered from the information of wing rock motion.

Castellanos et al., (2021) presented L1 adaptive control design to solve the stabilization problem due to the wing rock phenomenon. The proposed controller could guarantee the closed-loop stability in the presence of model and actuator dynamic uncertainties. It has been shown that the order of the chosen filter, within the structure of L1 adaptive control, has a direct effect on the performance of the controlled system.

Ignatyev, (2018) proposed a neural network adaptive controller (NNAC) to suppress the wing-rock motion for the scaled 3-DOF aircraft model mounted on a rig and maneuverers inside wind tunnel.

Gonzalez and Ra’ul, (2005) presented direct adaptive control to stabilize and control strict feedback systems. The semi-global asymptotic stability has been proven under limited knowledge of system dynamics. The slender delta-wing rock phenomenon has been applied as a case study of this work.

Some researchers proposed dynamic recurrent radial basis function (DRRBF) for modelling the mechanical hysteresis, which is the main cause of wing-rock behaviour in
the aircraft. The adaptive control law has been developed to minimize the cost function such as to guarantee the stability of the controlled system based on the Lyapunov theorem.

Guglieri and Satori, (2013) proposed Sliding Mode Control (SMC) to suppress the effect of wing rock behaviour by minimizing a cost function in terms of roll angular error and commanded input. The robustness of the proposed controller has been verified in the presence of parametric disturbances.

Humaidi and Hameed, (2017) have presented three versions of controllers based on model reference adaptive control (MRAC). These controllers are $\sigma$-modified MRAC, weighted $\sigma$-modified MRAC and the classical MRAC. The proposed controllers are developed to control the rolling motion of Delta-Wing aircraft due to rock-wing dynamics subjected to unmatched uncertainty. The numerical simulation showed that the weighted $\sigma$-modified controller outperforms the others in terms of control effort, robustness characteristics, and the accuracy of tracking error.

Liu and Su, (2004) presented a fuzzy logic control (FLC) design based on variable universe of discourse. The wing-rock motion has been as a case study and the proposed FL controller showed both high tracking precision and robustness against parametric uncertainty.

Ajel et al., (2021) presented control design of roll motion for a vertical take-off and landing unmanned air vehicle (VTOL-UAV) in the hovering flight phase based on Model Reference Adaptive Control (MRAC).

Wu et al., (2017) has designed robust controller composed of RBF-based neural network and extended state observer (ESO). The study considered the input saturation problem by synthesizing an auxiliary system. The computer simulation showed the effectiveness of proposed wing rock control based on backstepping control law and improved ESO.

Roshanian and Rahimzadeh, (2020) presented the design of robust Model Reference Adaptive Control (MRAC) for single DOF motion of wing-rock dynamics. The general structure has been developed for stable adaptive laws, which guaranteed the asymptotic stability of controlled system with uniform bounded tracking of robust controller.

Humaidi and Alaq, (2019) presented a comparison study in performance between two nonlinear control strategies for angle control in roll channel for delta-wing aircraft subjected to wing rock behaviour. The first control scheme is based on sliding mode backstepping control, while the other is based on observer-based sliding mode control. The performances of the proposed controllers have been verified and a comparison has been conducted in terms of robustness and transient characteristics.

The super-twisting sliding mode control (STSMC) is a developed version of SMC. The STSMC is characterised by low chattering in control signal as compared SMC and it enables zeroing of state and its derivative in finite time; i.e., the state trajectories can reach the equilibrium point in finite time. In addition, the STSMC needs only output or sliding variable and there is no need of the knowledge of state derivatives, which leads to less computation effort. Also, the exact convergence is guaranteed, and the singularity can be avoided with this control strategy (Humaidi & Alaq, 2019; Humaidi & Hameed, 2018; Feng & Fei, 2018; Alaq et al., 2021; Al-Dujail et al., 2020; Hameed et al., 2019).

In the stability analysis and design development of FTSTSMC for wing-rock dynamic control, some design parameters arise. These parameters can be tuned such as to improve the performance of controlled system dynamics. One way to solve the problem of their setting is to use modern optimization technique to have optimal performance of proposed controller. This study suggests whale optimization algorithm.
(WOA) to adjust the design parameters in terms of dynamic performance optimality. The WOA mimics the social behaviour of humpback whale population, developed in 2016 by scholars Mirjalili and Lewis. The WOA is inspired by the hunting behaviour of humpback whales and their hunting using bubble-net strategy. This WOA simulates shrinking encircling, spiral update of position, and random hunting mechanisms. The algorithm includes three stages: encircling prey, bubble net attack and search for prey. This optimization technique is characterized to solve complex optimization problems and it is used in many applications due to fast convergence speed, simple structure, requires less operator and offering better balance capability between exploration and exploitation phases. Owing to its optimal performance and efficiency, the applications of the algorithm have extensively been utilized in multidisciplinary fields (Mirjalili & Lewis, 2016; Rana et al., 2020; Hu et al., 2016; Yan et al., 2018).

Based on our previous studies (Eftekhari & Al-Obaidi, 2019; Eftekhari et al., 2020; Al-Obaidi et al., 2021; Al-Qassar et al., 2021a; Al-Qassar et al., 2021b), in the current study, the main contribution can be highlighted by the following points:

- Design of FTSTASMC to develop the control law for wing-rock motion control in Delta-Wing Aircraft.
- Conduct Lyapunov stability analysis of wing-rock dynamic controlled by FTSTSMC.
- Development of WOA to adjust the design parameters towards optimal control performance.
- Establish a comparative study between non-optimal and optimal versions of FTSTSMC via computer simulation.

2. THE DYNAMIC MODEL OF WING ROCK MOTION

To describe the dynamic model of wing-rock motion, the experimental data, which are gathered based on wind tunnel simulation, are fitted to mimicking the behavior of wing-rock phenomenon. The models gathered from experimental data-fitting results in nonlinear models of second-orders that differ from each other in the nonlinearity terms.

The following equation of motion describes model of wing rock dynamics in case of one degree of freedom (1-DOF) (Hsu et al., 1985; Guglieri, 2012):

\[ \ddot{\phi} = \left( \rho U_\infty^2 S_b / 2 I_{xx} \right) C_I + D u \]  

(1)

where \( \rho \) and \( \phi \) represent the air density and roll angle, respectively. \( U_\infty \) represents the speed of free air stream, \( S \) denotes wing area of aircraft. The coefficients \( b \) defines the chord of the wing, \( I_{xx} \) defines the wing moment of inertia around the roll span axis. The parameters \( u \) and \( D \) represent the position and effectiveness rolling of differential ailerons, respectively. The parameter \( C_I \) can be expressed by (Hsu et al., 1985; Elzebda et al., 1989):

\[ C_I = a_1 \phi + a_2 \dot{\phi} + a_3 |\phi| \dot{\phi} + a_4 |\dot{\phi}| \dot{\phi} + a_5 \phi^3 \]  

(2)

where the coefficients \( a_1, a_2, a_3, a_4 \) and \( a_5 \) are unknown constants. Combining Equation (1) and Equation (2) to have:

\[ \ddot{\phi} = b_1 \phi + b_2 \dot{\phi} + b_3 |\phi| \dot{\phi} + b_4 |\dot{\phi}| \dot{\phi} + b_5 \phi^3 + b_6 u \]  

(3)

According to Equation (3), the state variable can be established by setting \( x_1 = \phi \), \( x_2 = \dot{x}_1 = \dot{\phi} . This gives the following state variable Equation (4),

\[ \begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= b_1 x_1 + b_2 x_2 + b_3 |x_1| x_2 + b_4 |x_2| x_2 + x_2^3 + b_6 u 
\end{align*} \]  

(4)
where the coefficients $b_1, b_2, b_3, b_4$ and $b_5$ are also unknown constants.

If the disturbance or non-parametric uncertainty $\zeta(t)$, represented by wind and storm gust, has been added to the dynamic model, the following extended state variable is obtained

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = b_1 x_1 + b_2 x_2 + x_2 (b_3 \left| x_1 \right| + b_4 \left| x_2 \right|) + b_5 x_1^3 + b_6 u + \zeta(t) \tag{5}
\]

**Remark 1:** In order to ensure the stability of the delta wing aircrafts, the value of $c_2$ must satisfy

\[c_2 > \delta > |\zeta(t)|.\]

### 3. FAST TERMINAL SUPER TWISTING SLIDING MODE CONTROL FOR DELTA WING AIRCRAFTS

Let the error $e$ be the difference between the actual roll angle $x_1$ and desired roll angle $x_{1d}$

\[e = x_1 - x_{1d} \tag{6}\]

The time derivative of error is given by

\[\dot{e} = \dot{x}_1 - \dot{x}_{1d} \tag{7}\]

The fast terminal STSM can be described by suggesting first-order nonlinear differential equation,

\[s = \dot{e} + \lambda e + \beta |e|^{\gamma} \text{sign}(e) \tag{8}\]

where $e \in \mathbb{R}$, $\lambda$ and $\beta > 0$ and $0 < \gamma < 1$

Equation (8) can easily prove that for $e(0) \neq 0$ and $s = 0$, the dynamics reaches $e = 0$ within finite time determined by the formula,

\[T = \frac{1}{a(1-\gamma)} \ln \left( \frac{a \left| e(0) \right|^{1-\gamma} + \beta}{\beta} \right) \tag{9}\]

Taking time derivative of Equation (8), then one can have

\[\dot{s} = \dot{e} + \lambda \dot{e} + \beta \gamma |e|^{\gamma-1} \dot{e} = \ddot{x}_1 - \ddot{x}_{1d} + \lambda \dot{e} + \beta \gamma |e|^{\gamma-1} \dot{e} \tag{10}\]

or,

\[\dot{s} = \ddot{x}_2 - \ddot{x}_{1d} + \lambda \dot{e} + \beta \gamma |e|^{\gamma-1} \dot{e} \tag{11}\]

Based on sliding control theory, the control law includes two parts: equivalent part and switching part; i.e.,

\[u = \frac{1}{b_6} (u_{eq} + u_{sw}) \tag{13}\]

The equivalent part of control can be determined by setting $s = \dot{s} = 0$. According to Equation (10), the equivalent component can be obtained

\[u_{eq} = -(b_1 x_1 + b_2 x_2 + x_2 (b_3 \left| x_1 \right| + b_4 \left| x_2 \right|) + b_5 x_1^3) + \ddot{x}_{1d} - \lambda e - \beta \gamma |e|^{\gamma-1} \dot{e} \tag{14}\]

The switch part is proposed to be

\[u_{sw} = -c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt \tag{15}\]

Hence, the control law becomes

\[u = \frac{1}{b_6} (- (b_1 x_1 + b_2 x_2 + x_2 (b_3 \left| x_1 \right| + b_4 \left| x_2 \right|) + b_5 x_1^3) + \ddot{x}_{1d} - \lambda e - \beta \gamma |e|^{\gamma-1} \dot{e} - c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt) \tag{16}\]

Accordingly, Equation (10) becomes

\[\dot{s} = -c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt + \zeta(t) \tag{17}\]

In order to ensure the asymptotic stability of Delta Wing Aircrafts based on FTSTSMC, the following candidate Lyapunov function is chosen,

\[V = \frac{1}{2} s^2 \tag{18}\]

Taking the time derivative and using Equation (18), one can obtain

\[\dot{V} = s \left( -c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt + \zeta(t) \right) \tag{19}\]

Based on the mathematical fact ($s \cdot \text{sgn}(s) = |s|$) and using the elementary linear algebra, Equation (19) can be expressed as

\[\dot{s} = \ddot{x}_2 - \ddot{x}_{1d} + \lambda \dot{e} + \beta \gamma |e|^{\gamma-1} \dot{e} \tag{11}\]
\[ \dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \int c_2 \, dt + |\zeta(t)| \, s \]  \hspace{1cm} (20)

If the last term of Equation (20) can be transformed into integration form to have

\[ \dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \int c_2 \, dt + |s| \int \zeta(t) \, dt \]  \hspace{1cm} (21)

Using Remark 1, Equation (21) can be rewritten in the following form

\[ \dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \int c_2 \, dt + |s| \int \delta \, dt \]  \hspace{1cm} (22)

or,

\[ \dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| (\int c_2 \, dt - \int \delta \, dt) \]  \hspace{1cm} (23)

4. WHALE OPTIMIZATION ALGORITHM

Whales are considered as the biggest mammals in the world, they never sleep, and they are the highly intelligent animals with emotion. The most interesting thing about the humpback whales, a species of whales, is their special hunting method. This behaviour of foraging is called "bubble-net feeding" method. The humpback whales like to hunt school of small fishes or krill near to the water surface. This foraging technique is performed by producing distinctive bubbles along ‘9’-shaped path or circular path as indicated in Figure 2. This type of feeding, bubble-net feeding, is a unique behaviour that can only be seen in such types of whales. Here, spiral bubble-net feeding manoeuvre is mathematically modelled in order to perform optimization. Their favourite preys are krill and small fish herds. Figure 2 shows this mammal. Let us now consider the three typical behaviours of Humpback whales and model them mathematically.

A. Modelling of Encircling Prey

The Humpback whales has the capability to recognize the location of prey and they could encircle them. The algorithm whale optimization technique assumes the target prey as the current best candidate solution or close to the optimum. After defining the best search-agent, the other search-agents try to update their positions towards the best search-agent. This behaviour is represented by Mirjalili et al. (2016):

![Figure 2. Bubble-net feeding behaviour of humpback whales.](https://doi.org/10.17509/ijost.v6i3.37922)
\[ \bar{D} = \| \bar{C} \cdot \bar{X}^r(t) - \bar{X}(t) \| \]  
\[ \bar{X}(t + 1) = \bar{X}^r(t) - \bar{A} \cdot \bar{D} \]

where \( t \) denotes the current iteration, \( \bar{A} \) and \( \bar{C} \) represent the coefficient vectors, \( \bar{X} \) represents the position vector, \( \bar{X}^* \) defines the current position vector of the best solution, and \( |.| \) is the absolute value with element-wise multiplication. It is important to note that if there is an improved solution, \( \bar{X}^* \) should be updated at each iteration. The vectors \( \bar{A} \) and \( \bar{C} \) are obtained from:

\[ \bar{A} = 2 \bar{a} \cdot \bar{r} - \bar{a} \]  
\[ \bar{C} = 2 \cdot \bar{r} \]

where \( \bar{r} \) is a random vector in \([0,1]\), \( \bar{a} \) is linearly decreased over the course of iterations from 2 to 0 (in both exploration and exploitation phases). As earlier stated, humpback whales also use bubble net strategies to attack their prey. The mathematical formula is as follows.

### B. Attacking Method of Bubble-Net (Exploitation Phase).

Two approaches are pursued to describe the mathematical model of bubble-net behaviour for humpback whales:

1. Shrinking-encircling mechanism: here the value of \( \bar{a} \) is reduced by formula (26). In addition, the range of changes in \( \bar{A} \) has also been reduced by \( \bar{a} \). By setting the random value of \( \bar{A} \) within \([-1,1]\), the new location of searching agent can be defined anywhere between the location of the current best agent and the original location of the agent.

2. Spiral-update position: Firstly, one has to calculate the distance between the whale location \((X, Y)\) and the prey location \((X^*, Y^*)\). The following spiral equation is established which relates the locations of whale and prey:

\[ \bar{X}(t + 1) = \bar{D}_i \cdot e^{bl \cdot \cos(2\pi l)} + \bar{X}^*(t) \]  
\[ \bar{D} = |\bar{X}^*(t) - \bar{X}(t)| \]

where \( \bar{D}_i \) represents the distance between ith whale and prey (the best solution obtained earlier), \( b \) is a constant that defines the shape of logarithmic spiral. The random number \( l \) lies within \([-1,1]\). It has been observed that humpback whales spin or rotate around their prey along spiral-shaped paths within shrinking circles. To update the whale position during the optimization process, 50% chance has been assigned for each surrounding fence mechanism and the twisting mechanism. The mathematical model can be described by:

\[ \bar{X}(t + 1) = \begin{cases} 
\bar{D}_i \cdot e^{bl \cdot \cos(2\pi l)} + \bar{X}^*(t), & \rho \geq 0.5 \\
\bar{X}(t) - \bar{A} \cdot \bar{D}, & \rho < 0.5 
\end{cases} \]

where, \( \rho \) is a random number within \([0,1]\).

### C. The Search for a Prey (Exploration Phase)

In this scenario, the search agent is forced move far away from a reference whale using the vector \( \bar{A} \) with the random values greater than 1 or less than \(-1\). This mechanism together with \( |\bar{A}| > 1 \) emphasize exploration and allow the WOA algorithm to perform a global search. The mathematical model is as follows:

\[ \bar{D} = |\bar{C} \cdot \bar{X}_\text{rand} - \bar{X}| \]  
\[ \bar{X}(t + 1) = |\bar{X}_\text{rand} - \bar{A} \cdot \bar{D}| \]

where a random position vector \( \bar{X}_\text{rand} \) of a random whale is chosen from the current population. Other optimization techniques can be consulted and compared to WAO in the future work of this study (Humaidi et al., 2021; Al-Azza et al., 2015; Moezi et al., 2018; Mirjalili, 2016; Yue et al., 2020).

### 5. COMPUTER SIMULATION

In this section, the effectiveness of optimal FTSTSMC and non-optimal FTSTSMC has been evaluated via numerical simulation.
using MATLAB programming software. The numerical simulation has used Ode45 as a numerical solver. In addition, the simulation is based on the following assumptions:

I. The roll angle can operate within the allowable range ±30 degree,

II. The uncertainty is unmatched disturbance, and it actually represents a gust wind, which can be simulated as a uniform random signal with upper and lower bounds $-5^\circ \leq \zeta(t) \leq 5^\circ$.

III. The numerical values of parameters for wing-rock motion are given as (Humaidi and Hameed, 2017):

\[ b_1 = -0.018, \quad b_2 = 0.015, \quad b_3 = -0.062, \quad b_4 = 0.009, \quad b_5 = 0.021, \quad b_6 = 0.75 \]

The optimization algorithm based on WOA results in optimal design parameters for FTSTSMC. Hence, two versions of FTSTSMC will be considered; one is based on try-and-error procedure called non-optimal FTSTMSC and the other is the optimal FTSTSMC, whose design parameters are tuned by WOA.

The optimization technique is responsible for tuning five design parameters, represented by $c_1$, $c_2$, $\lambda$, $\beta$, and $\gamma$. This study assumes the Root Mean Square Error (RMSE) to be assigned to the cost function (fitness function). In addition, minimization of cost function has been adopted to solve the optimization problem.

**Figure 3** shows the behaviour of cost function with respect to iteration. It is evident that the fitness function is decreasing monotonically. This indicates that the optimization tuning process of design parameters works properly towards error minimization. The setting of design parameters based on try-and-error procedure and WOA is listed in **Table 1**.

![Image of Figure 3](image-url)
Table 1. Non-optimal and optimal design parameters for FTSTSMC based on WOA.

| Design Constants for Non-Optimal FTSTSMC | Value | Design Constants for Optimal FTSTSMC | Value |
|------------------------------------------|-------|--------------------------------------|-------|
| $c_1$                                     | 0.05  | $c_1$                                | 0.01  |
| $c_2$                                     | 1.20  | $c_2$                                | 1.90  |
| $\lambda$                                 | 0.05  | $\lambda$                            | 0.01  |
| $\beta$                                   | 5.24  | $\beta$                              | 4.24  |
| $\gamma$                                  | 0.99  | $\gamma$                             | 0.99  |

In this study, two scenarios have been taken into account; one is based on disturbance-free case, while the other scenario considered the exertion of disturbance on the aircraft. For each case, the effectiveness of optimal FTSTSMC and non-optimal FTSTSMC has been evaluated via computer simulation. Figure 4 shows the behaviours of roll angle for the disturbance-free case based on both proposed controllers. It is evident that the WOA better enhances the performance of FTSTSM controller as compared to optimization based on conventional method. The performances of proposed controllers based on optimized and no-optimized methods are numerically evaluated and reported in Table 2. The RMS has been selected the evaluation index of performance.

![Figure 4](image-url)

Figure 4. The responses of roll motion due to optimal and non-optimal FTSTSMC.

Table 2. Transient parameters of the controlled system based on Non-Optimal FTSTSMC and Optimal FTSTSMC.

| Controller Type        | RMSE  |
|------------------------|-------|
| Non-Optimal FTSTSMC    | 0.0642|
| Optimal FTSTSMC        | 0.0541|

DOI: [https://doi.org/10.17509/ijost.v6i3.37922](https://doi.org/10.17509/ijost.v6i3.37922)

p- ISSN 2528-1410 e- ISSN 2527-8045
Figure 5 shows the change rate of roll angle. The control effort due to optimal and non-optimal FTSTSMCs is indicated in Figure 6. It is evident from the figure that the control signal in case of optimal controller is higher than that in case of non-optimal controller. However, this is the price paid by the optimal controller to enhance or improve the dynamic performance of the optimal controlled wing-rock motion. The behaviours of sliding surfaces for optimal controllers are shown in Figure 7. According to this figure, one can conclude that the sliding surface based on optimal FTSTSMC is faster than that based on non-optimal one. This shows that the sliding trajectory reaches the equilibrium point in earlier time in case of optimal controller.

Figure 5. The rate change of roll angle.

Figure 6. The control effort due to both optimal and non-optimal controllers.
In the next scenario, the disturbance due to wind gust is taken into account and the performances of optimal and non-optimal controller are evaluated based on computer simulation. The behaviour of applied disturbance is described in Figure 8. As mentioned earlier, the disturbance lies within the range $-5^\circ \leq \zeta(t) \leq 5^\circ$ and has uniform random distribution such that it is mimicking the real wind-gust. The response of roll motion based on optimal FTSTSMC in the presence of wind disturbance has been illustrated in Figure 9. The disturbance has been exerted in specified periods of time and the figure shows that the optimal controller shows good dynamic performance under the applied disturbance.

Figure 10 and Figure 11 shows the rate change of roll motion and control effort, respectively, under the action of disturbance.

Figure 7. The behaviors of sliding surfaces for the proposed controllers.

Figure 8. The disturbance behaviour of wind gust.
Figure 9. The roll motion of optimal FTSTSMC subjected to disturbance.

Figure 10. The rate change of roll motion under disturbance.
4. CONCLUSION

In this study, the finite control based on super-twisting sliding mode methodology has been developed to control the wing-rock motion of delta-wing aircraft. The stability analysis of FTSTSMC based on Lyapunov theorem has been conducted to show the stability of controlled dynamic and to establish the control for this unwanted motion. The WOA has been applied to tune the design parameters of FTSTSMC in order to have optimal performance of proposed controller. The optimal FTSTSMC based on WOA is compared to that based on try-and-error procedure via computer simulation within MATLAB environment. The simulated results showed the optimal controller outperforms the non-optimal in terms of transient characteristics.

5. AUTHORS’ NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. Authors confirmed that the paper was free of plagiarism.

6. REFERENCES

Ajel, A.R., Humaidi, A.J., Ibraheem, I.K., and Azar, A.T. (2021). Robust model reference adaptive control for tail-sitter vtol aircraft. *Actuators, 10*(7), 162, 1-19.

Alaq, F., Humaidi, A. J., Al-Dujaili, A., and Ibraheem, K.I. (2021). Robust super-twisting sliding control of pam-actuated manipulator based on perturbation observer. *Cogent Engineering, 7*(1858393), 1-30.

Al-Azza, A. A., Al-Jodah, A. A., and Harackiewicz, F. J. (2015). Spider monkey optimization: A novel technique for antenna optimization. *IEEE Antennas and Wireless Propagation Letters, 15*(2015), 1016-1019.

Al-Dujaili, A. Q., Falah, A., Humaidi, A. J., Pereira, D. A., and Ibraheem, I. K. (2020). Optimal super-twisting sliding mode control design of robot manipulator: Design and comparison study. *International Journal of Advanced Robotic Systems, 7*(6), 1–17.

DOI: [https://doi.org/10.17509/ijost.v6i3.37922](https://doi.org/10.17509/ijost.v6i3.37922)
p- ISSN 2528-1410 e- ISSN 2527-8045
Al-Obaidi, A. S. M., Al-Qassar, A., Nasser, A. R., Alkhayyat, A., Humaidi, A. J., and Ibraheem, I. K. (2021). Embedded design and implementation of mobile robot for surveillance applications. *Indonesian Journal of Science and Technology*, 6(2), 427-440.

Al-Qassar, A. A., Abdu-Alkareem, A. I., Hasan, A. F. Hasan, Humaidi, A. J., Ibraheem, K. I., Azar, A.T., and Hameed, A. H. (2021a). Grey-wolf optimization better enhances the dynamic performance of roll motion for tail-sitter vtol aircraft guided and controlled by STSMC. *Journal of Engineering Science and Technology (JESTEC)*, 16(3), 1932-1950.

Al-Qassar, A.A., Al-Dujaili, A.Q., Hasan, A.F., Humaidi, A.J., Ibraheem, I.K., and Azar, A.T. (2021b). Stabilization of single-axis propeller-powered system for aircraft applications based on optimal adaptive control design. *Journal of Engineering Science and Technology (JESTEC)*, 16(3), 1851-1869.

Castellanos, J. C., Suárez F. C., and Sofrony, E. J. (2021). Review of L1 adaptive control for solving the wing rock problem. *Journal of Engineering and Applied Sciences*, 16(9), 296-303.

Eftekhari, H., Al-Obaidi, A. S. M., and Eftekhari, S. (2020). The effect of spoiler shape and setting angle on racing cars aerodynamic performance. *Indonesian Journal of Science and Technology*, 5(1), 11-20.

Eftekhari, S., and Al-Obaidi, A. S. M. (2019). Investigation of a cruising fixed wing mini unmanned aerial vehicle performance optimization. *Indonesian Journal of Science and Technology*, 4(2), 280-293.

Elzebda, J. M., Nayfeh A. H., and Mook D. T. (1989). Development of an analytical model of wing rock for slender delta wings. *Journal of Aircraft*. 26(8), 737-743.

Feng, Z., and Fei, J. (2018). Design and analysis of adaptive Super-Twisting sliding mode control for a micro-gyroscope. *PLoS ONE*, 3(2018), 1-18.

González, A., Raúl, O. (2005). Adaptive control scheme for plants with time-varying structure using on-line parameter estimation. *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005 Seville*, Spain, December 12-15, 2005, 2225-2229.

Guglieri, G. (2012). A comprehensive analysis of wing rock dynamics for slender delta wing configurations. *Nonlinear Dynamics*, 69(4), 1559–1575.

Guglieri, G., and Satori, D. (2013). Design of a sliding mode control for wing rock suppression in highly-swept wing aircraft. *International Journal of Aerospace Sciences*, 2(1), 1-10.

Hameed, A. H, Al-Dujaili, A.Q., Humaidi, A. J., and Hussein, H.A. (2019). Design of terminal sliding position control for electronic throttle valve system: a performance comparative study. *International Review of Automatic Control (I.RE.A.CO.),* 12(5), 251-260.

Hsu, C., and Lan, C. E. (1985). Theory of wing rock. *Journal of Aircraft*, 22(10). 920–924.

Hu, H., Bai, Y., and Xu, T. (2016). Whale optimization algorithm with inertia weight. *Wseas Transactions on Computers*, 15(2016), 319-326.

Humaidi, A. J., and Hameed, A. H. (2017). Robust MRAC for a wing rock phenomenon in delta wing aircrafts. *AUT Journal of Modeling and Simulation*, 49(1), 113-122.

DOI: https://doi.org/10.17509/ijost.v6i3.37922

p- ISSN 2528-1410 e- ISSN 2527-8045
Humaidi, A. J., and Hameed, A.H. (2018). PMLSM position control based on continuous projection adaptive sliding mode controller. *Systems Science and Control Engineering, 6*(3), 242-252.

Humaidi, A.J., and Alaq, F. H. (2019b). Particle swarm optimization–based adaptive super-twisting sliding mode control design for 2-degree-of-freedom helicopter. *Measurement and Control Journal, 52*(9-10), 1403-1419.

Humaidi, A.J., Najem, H.T., Al-Dujaili, A.Q., Pereira, D.A., Ibraheem, I.K., and Azar, A.T. (2021). Social spider optimization algorithm for tuning parameters in PD-like interval type-2 fuzzy logic controller applied to a parallel robot. *Measurement and Control, 54*(3-4), 1-21.

Ignatyev, D.I. (2018). Neural network adaptive control of wing-rock motion of aircraft model mounted on three-degree-of-freedom dynamic rig in wind tunnel. *Progress in Flight Dynamics, Guidance, Navigation, and Control, 10*(2018), 73-86.

Liu, Z. L., and Su C. Y. (2004). Control of wing rock phenomenon with a variable universe fuzzy controller. *Proceeding of the 2004 American Control Conference Boston, Massachusetts June 30 - July 2, 2004, 1719-1724.*

Mirjalili, S. (2016). SCA: A sine cosine algorithm for solving optimization problems. *Knowledge-Based Systems, 96*(2016), 120-133.

Mirjalili, S., and Lewis, A. (2016). The whale optimization algorithm. *Advance in Engineering Software, 95*(2016), 51-67.

Moezi, S.A., Zakeri, E., and Zare, A. (2018). A generally modified cuckoo optimization algorithm for crack detection in cantilever Euler-Bernoulli beams. *Precision Engineering, 52*(2018), 227-241.

Rana, N., Abd Latiff, M. S., and Chiroma, H. (2020). Whale optimization algorithm: A systematic review of contemporary applications, modifications and developments. *Neural Computing and Applications, 32*(2020), 16245–16277.

Roshanian, J., and Rahimzadeh, E. (2020). A generalization for model reference adaptive control and robust model reference adaptive control adaptive laws for a class of nonlinear uncertain systems with application to control of wing rock phenomenon. *International Journal of Engineering, 33*(11), 2372-2383.

Saleh, M.H., Al-Qassar, A., Othman, M.Z., and Mahdi, A.S. (2010). Development of the cross-coupling phenomena of MIMO flight system using fuzzy logic controller. *Journal of Engineering Science and Technology (JESTEC), 5*(1), 85-93.

Wu, D. W., Chen, M., Gong, H. J., and Ye, H. (2014, June). Control of wing rock based on high order sliding mode disturbance observer. *Proceeding of the 11th World Congress on Intelligent Control and Automation* (pp. 873-878). IEEE.

Yan, Z., Wang, S., Liu, B., and Li, X. (2018). Application of whale optimization algorithm in optimal allocation of water resources. *E3S Web of Conferences, 53*(2018), 04019.

Yue, Z., Zhang, S., and Xiao, W. (2020). A novel hybrid algorithm based on grey wolf optimizer and fireworks algorithm. *Sensors, 20*(7), 2147.