Finite Element Analysis on the Unloading Elastic Modulus of Aluminum Foams by Unit-cell Model

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Abstract. This paper presents finite element analyses (FEA) on the unloading elastic moduli of aluminum foams under uniaxial loading and bending conditions. The unit-cell model of Gibson and Ashby was utilized in the FEA with elasto-perfectly plastic material. Compression, tension, and bending tests simulations were carried out using Abaqus software. The non-linearity effect caused by structural imperfection/buckling of unit-cell was incorporated. From the compressive test simulation, it was confirmed that the results showed good agreement with the referred experimental data. This means that the developed FE model can be regarded as an RVE model of aluminum foams. Then, the deformation characteristic and elastic modulus obtained from all simulations were compared and examined. As the results, the elastic modulus obtained from bending test demonstrated higher value than that of under uniaxial loads. This discrepancy increases gradually as the relative density decreases. The mechanism responsible for generating the discrepancy in stiffness was discussed. Furthermore, this work points out the necessity of measuring both Young’s and flexural moduli for designing an engineering structure using foam materials especially when accuracy and precision are demanded.

1. Introduction
Aluminum foams are a new class of engineering materials which belong to the group of cellular-solids. Due to its unique structural topology compared to the parent material of which it is made, aluminum foams have many novel properties and behavior in terms of physical, mechanical, thermal, electrical and acoustic properties in combination with its lightweight structure. Therefore, engineers like to use aluminum foams for structural applications which require such as weight-saving, impact energy absorption, sound and vibration control, and thermal insulation [1].

In engineering structures, aluminum foams are commonly used as the core of sandwich panel because it can improve the energy absorption capacity [2, 3]. For this reason, the panel’s deflection as well as its failure modes should be correctly predicted to avoid failure occurrence [4]. To support the design process, elastic modulus should be known since it is one of the most basic mechanical properties.
However, the elastic moduli of aluminum foams under different loading mode still have not been comprehensively understood.

Triawan et al. [5, 6] investigated the elastic behavior of Alporas aluminum foams with several different densities under compressive and bending loads. As a result, a large discrepancy in elastic moduli measured under each load was revealed. The flexural modulus, $E_f$, was found to be about two times higher than the Young’s modulus, $E$. This is unusual because the tested specimen, Alporas aluminum foam, is a highly isotropic material [7].

There are two possible reasons which could generate a discrepancy between $E_f$ and $E$ in an isotropic foam material. First is the difference in compressive and tensile moduli, and second is the presence of couple stress effect. For the first reason, as bending deformation is a combination of compressive and tensile deformations, hence, the discrepancy could be due to the difference in compressive ($E_c$) and tensile ($E_t$) Young’s moduli. With regard to this, indeed, the $E_t$ of metal foams can be different with the $E_c$. Andrews et al. [7] and Sugimura et al. [8] reported that $E_c$ of Alporas aluminum foam with density of 250 kg/m$^3$ showed 25–30% greater value than the corresponding $E_c$. Therefore, the difference in $E_c$ and $E_t$ clearly could be the reason for the discrepancy of $E_f$ and $E$.

As for the couple stress effect, Anderson and Lakes [9], and Liu and Su [10] explained that the discrepancy between $E_f$ and $E$ of foam materials could be caused by the presence of couple stress effect generated in bending deformation. As the cells of metal foams become comparable to the specimen size, the presence of couple stress effect is reported to possibly increase the bending stiffness. This effect is more pronounced in bending but less influential in uniaxial loading. Based on these theories, hence, the discrepancy in stiffness of metal foams could be due to the presence of couple stress effect. Nevertheless, there is no further explanation of the physical mechanism which can create a couple stress effect on foam materials.

In the present work, finite element analyses (FEA) were performed in order to clarify the physical mechanism of cell deformation which can cause the discrepancy in elastic moduli under bending and uniaxial loading. The unit-cell model of Gibson and Ashby [11] was used for the FEA. Compression, tension, and bending tests simulations were done using Abaqus software. The elastic modulus under each loading mode was derived from the unloading curves in which the non-linear effect from cell-structure was incorporated. The results were then examined and compared with the referred experimental data. The physical mechanism which causes the discrepancy in stiffness in aluminum foams is discussed.

2. **Unit-cell finite element model**

Figure 1 shows a unit-cell model which composes of cell-edges and cell-faces. This model is used by Gibson and Ashby [11] to describe the elastic modulus equation of foam material under uniaxial loading (tension or compression). In order to identify the physical mechanism which able to generate a discrepancy between $E_f$ and $E$, analyses utilizing this unit-cell model was firstly introduced by Triawan et al. [6]. They demonstrated a theoretical approach to estimate the elastic modulus of foam materials under bending condition. As the result, the discrepancy between $E_f$ and $E$ could be generated when the unit-cell experiences rotational deformation induced by the applied bending moment [6]. This is considered to be the physical mechanism of couple stress effect on foam materials under bending condition.

Besides the rotational deformation, unit-cell could also experience buckling deformation due to structural imperfection, such as curvatures in cell-edge or cell-face, when a compressive load is applied (Fig. 1). Buckling of unit-cell is supposed to reduce the compressive elastic modulus, $E_c$, of the foam structure which eventually results in a discrepancy with its $E_f$ [7, 8]. Moreover, as couple stress effect might be present in bending, the combination of these two effects would produce even larger discrepancy of $E_f$ and $E$. In this work, clarification on this phenomenological hypothesis has been done by performing FEA of compression, tension, bending tests using a unit-cell model.
3. Finite element analyses procedures

3.1. Compressive and tensile tests simulations
Uniaxial compression and tension tests simulations were performed using finite element code of ABAQUS 6.13. A rigid plate (also made of shell element) was used as the platen or chuck to apply a compressive or tensile load. Figure 3 shows the boundary condition of the model. Interaction between the platen and the upper nodes of the model was constrained by coupling function, so they would move together when any load is applied. Frictionless contact was applied to the contact surfaces of the model and the platen. To prevent a rigid body motion, the degrees of freedom (DOF) at x-, y-, and z-directions of the center node on the lower surface was constrained, while other nodes were constrained only in y-direction DOF.

![Figure 1. Unit-cell model in uniaxial loading conditions](image1)

![Figure 2. Unit-cell model of finite element analyses (a), meshed unit-cell model (b)](image2)
Figure 3. Boundary conditions of the unit-cell model; model under uniaxial loading $P_y$ applied by a rigid plate (a) nodes at the lower surface are fixed in x- and y-directions, denoted as $u_x$ and $u_y$ (b)

Compressive $E_c$ and tensile $E_t$ elastic moduli were determined from the unloading processes. The reaction force of the platen in $y$-direction, $P_y$, was probed to calculate the stress by dividing it with the cross-sectional area of the cell model. The strain was defined as the $y$-direction displacement divided by the total height of the model. The displacement of the platen was set to be upward (for tensile) or downward (for compressive) up to 7% strain. The unloading processes were done several times at several strain values during the loading process.

The simulations were conducted using general-static analysis in two different conditions: with and without NLGEOM. NLGEOM is a function provided in Abaqus to account for the non-linearity effect due to e.g. structural post-buckling or large deformation. Considering that the typical thickness of cell-face of actual aluminum foams is considered as a thin plate having thickness of $0.01–0.2$ mm [6], and at the same time it also contains some curvatures and corrugated parts [7], activation of the NLGEOM function in the FEA was considered to be relevant. Furthermore, in order to obtain various foam densities, the cell-face thickness was modified by changing the shell element thickness.

3.2. Bending tests simulations
Bending test simulation was done to examine the elastic modulus under bending condition. The same unit-cell model from the uniaxial loading tests simulations was used, except the platen was constrained in a different boundary condition. Pure bending moment was generated by applying a twisting angle of $\theta_z$ of $z$-direction DOF at the platen’s center node. The non-linearity effect from structural imperfection or buckling deformation was also incorporated in the calculation by activating the NLGEOM function in the analysis. Unloading processes were carried out 3 times during the loading process until the model reached 7% of strain at the compressive side. The flexural modulus $E_f$ was then extracted from the slope of the unloading curves by probing the reaction moment, $M_z$. Equation (1) which is derived from the Bernoulli-Euler beam theory for pure bending condition was used for the calculation,

$$E_f = \frac{M_z \bar{H}}{\theta_z I} \quad (1)$$

where, $H$ is the model height, $\theta_z$ is the twisting angle, and $I$ is the moment of inertia. In addition, the cell-face thickness of the specimen was varied to obtain various densities.

4. Results and discussion

4.1. Validation of finite element model
The stress-strain curves of compressive test simulations on cell models with relative densities (RD) of 0.115 and 0.093 are shown in Fig. 4. The results are compared with that obtained from experimental data reported by Triawan et al. [5, 6] who tested an aluminum foams specimen with RD of 0.096. RD is
defined as the ratio of the model’s actual density to that of its parent material. In this case, the density of model is normalized with the density of aluminum bulk of 2690 kg/m³. From the figure, it is seen that the stress-strain curves from FEA exhibited good agreement with the experiment. The plateau stresses from FEA match well with experiment, however, the slope of the 1st loading curves of FEA overestimates the experiment. This may be attributed from the deformation of free surfaces formed due to machining.

**Figure 4.** Comparison of stress-strain curves of compressive tests obtained from FEA and experiment

Figure 5 shows the $E_c$s obtained from FEA in comparison with those from experiments. First. It was found that the $E_c$ calculated from the unloading curves, named as unloading $E_c$, exhibited strong strain-dependency. As the strain increases, the unloading $E_c$ tends to decrease. Similar results obtained from compressive test experiments are reported by Sun et al. [12]. This is due to the buckling deformation of the cell’s structures under compressive loading. The unloading $E_c$s at 7% strain are then plotted against the RD in Fig. 5(b). Either with or without NLGEOM, both FEA results are able to fairly estimate the $E_c$s from experiments. This indicates that the developed unit-cell model is sufficient to be considered as a representative volume element (RVE) of an aluminum foams structure.

**Figure 5.** The unloading elastic moduli $E_c$ exhibit strong strain-dependency behavior (a), the unloading $E_c$s obtained from FEA are comparable with those from experiments (b)

**4.2. Discrepancy in stiffness**

Figure 6 compares the deformation behavior of unit-cell model at 7% strain under compressive, tensile, and bending loading conditions. Moreover, it also compares the stress conditions of unit-cell with and without incorporating the non-linearity effect. From the figure, it can be seen that the activation of NLGEOM in the analyses has caused the unit cell model to experience severe plastic deformation which eventually decreases the unloading elastic modulus value (Fig. 5(a)). The severe plastic deformation, indicated by the red stress area, is mostly generated by buckling deformation of the cell-edges and cell-faces under compressive load, or by the thinning of the cell-edges and cell-faces under tensile load. If the NLGEOM is not activated, this non-linear deformation effect is neglected in the calculation, which thus decrease the amount of plastic deformation on the cell’s body.
The compressive and tensile tests simulations which do not account the non-linearity effect exhibit almost similar stress distribution although the applied loading mode is different (Fig. 6(a) and (c)). As a result, the unloading elastic moduli, $E_c$ and $E_t$, show exactly the same value. Having known this fact, however, from the bending test simulations, it is confirmed that the flexural moduli, $E_f$, show some discrepancies to its $E_{fs}$ or $E_{ts}$ values as denoted in Fig. 7(a). The ratio of $E_f/E_c$ (NLGEOM: off) and $E_f/E_t$ (NLGEOM: off) was found to be 1.3–1.4. This is caused by the rotational deformation of cell body under bending moment, which is believed to be the origin of couple stress effect in foam materials [6, 9, 10].

Figure 6. Stress condition of unit-cell model at 7% strain under compressive (a, b), tensile (c, d), and bending (e, f) loads. (a), (c), (e) are without NLGEOM, while (b), (d), (f) incorporate the NLGEOM.
For the simulations which incorporate the non-linearity effect (NLGEOM is activated), much larger discrepancy between $E_f$ and $E_c$ was observed. Whereas, the ratio of $E_f$ and $E_t$ did not exhibit any significant discrepancy. Comparing these results with the experimental data, the FEA somehow show reasonable fitting as plotted in Fig. 7(a). Moreover, from the deformation behavior shown in Fig. 6 (b), (d), and (f), the typical buckling deformation is clearly observed by the wavy deformed cell-faces. This has played an important role in generating the discrepancy in stiffness.

Based on the FEA results, the large discrepancy between $E_f$ and $E_c$ in aluminum foams can be concluded to be generated by the combination of rotational-deformation and non-linear deformation of the unit-cell body. Moreover, the discrepancy could depend on the density as well as the amount of strain at which the unloading process is done.

Figure 7(b) shows the fitting of FEA results on the experimental results by multiplication factor of 2.1. The plot indicates that FEA by unit-cell model is useful to explain the physical mechanism of the discrepancy between $E_f$ and $E_c$. Furthermore, since the discrepancy shows strain-dependency due to the non-linear deformation effect, this work emphasizes the importance of measuring both Young’s modulus and flexural modulus in order to precisely estimate the deflection or failure behavior of an engineering product which utilizes foam materials.

![Figure 7. The unloading elastic moduli $E_c$ exhibit strong strain-dependency behavior (a), the unloading $E_t$s obtained from FEA are comparable with the experiment results (b)]](image)

5. Conclusion
Investigation on the unloading elastic moduli of aluminum foams under compressive, tensile and bending loads was conducted by FEA. The goal is to clarify the physical mechanism which responsible in generating the discrepancy in stiffness under bending and uniaxial loading conditions. A unit-cell model is introduced and the non-linear deformation effect was considered in the analyses. Based on the results, it was concluded that the discrepancy between $E_f$ and $E_c$ of aluminum foams can be generated by the combination of rotational-deformation and non-linear deformation of the unit-cell body. Moreover, it was revealed that the discrepancy strongly depends on the relative density parameter as well as the applied strain. Furthermore, the FEA results indicate that the loading conditions has a prominent role in generating different elastic moduli in foams structure, since it could cause a different cell deformation. This is applicable not only in uniaxial loading or bending, but possibly in shear loading as well. Thus, investigation on the effect on shear deformation is suggested for further study. Lastly, this work emphasize the necessity of having both the Young’s modulus and flexural modulus in order to precisely predict the deformation and failure behaviors of an engineering structure utilizing foam materials.
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