Implications of Recent Data
on Non-mesonic Decay of Light $\Lambda$-hypernuclei

Namyoung Lee$^a$, Hong Jung$^b$, and Dongwoo Cha$^a$

$^a$ Department of Physics, Inha University, Inchon 402-751, KOREA
$^b$ Department of Physics, Sookmyung Women’s University, Seoul 140-742, KOREA

Abstract

We analyze the recent data on the non-mesonic decays of light $\Lambda$-hypernuclei up to $^{12}\Lambda$C using the phenomenological model of Block and Dalitz. Fitting the spin-isospin dependent $\Lambda N \to NN$ reaction rates to six data points, we predict the remaining data in reasonable consistency. We find that despite the short-range nature of the $\Lambda N \to NN$ interaction, the non-mesonic decay of $p$-shell hypernuclei seems to be strongly induced by the $p$-shell neutrons. Also, the recent data indicate that the $\Delta I = \frac{1}{2}$ rule, well-proved at the hadronic level, may not be sacred in the nuclear medium and the $\Delta I = \frac{3}{2}$ interactions seem to be needed to describe the non-mesonic decays of $\Lambda$-hypernuclei.
The structure of hypernuclei has been understood somewhat successfully in terms of empirical hyperon-nucleon potentials and mean field theories as in the case of ordinary nuclei \[1\]. However, the decay rates of hypernuclei have revealed intriguing features \[1-4\]. The $\Lambda$-hypernucleus, whose ground state is stable with respect to strong interactions, decays only via weak interactions and so far $\Lambda \rightarrow N\pi$ (so called pi-mesonic decay) and $\Lambda N \rightarrow NN$ (two-body non-mesonic decay) have been considered as its main decay mechanisms although there have been some reports on the importance of the three-body non-mesonic process $\Lambda NN \rightarrow NNN$ \[3\].

The pi-mesonic decays ($\Lambda \rightarrow p\pi^-$ and $\Lambda \rightarrow n\pi^0$) account for most of the decays of free $\Lambda$-hyperons whose decay width is

$$\Gamma_\Lambda \approx 2.50 \mu\text{eV}. \quad (1)$$

In the rest frame of the $\Lambda$, the energy release is $m_\Lambda - m_N - m_\pi \approx 35 \text{ MeV}$ and the momentum of the emitted nucleon is about 100 MeV. Also, the ratio of the decay widths for the two channels is $\Gamma(\Lambda \rightarrow p\pi^-)/\Gamma(\Lambda \rightarrow n\pi^0) \approx 1.8$, which means that $\Delta I = \frac{1}{2}$ amplitude dominates over the $\Delta I = \frac{3}{2}$ amplitude as is in the free decays of $K$-mesons. This is the well-known $\Delta I = \frac{1}{2}$ rule, although it is yet to be understood why it should hold true at the hadronic level.

When the $\Lambda$-hyperon is bound in the nuclear medium, its pi-mesonic decay mode becomes suppressed because the allowed phase space is smaller due to the small energy release from the bound $\Lambda$-hyperon and the emitted nucleons are Pauli-blocked. Therefore, the non-mesonic decay mode whose energy release is about $m_\Lambda - m_N \approx 175 \text{ MeV}$, much larger than in the pi-mesonic case, becomes more important as the hypernucleus becomes heavier. However, contrary to the pi-mesonic decay for which we have a well-known effective Hamiltonian satisfying the $\Delta I = \frac{1}{2}$ rule, we do not have such a reliable effective Hamiltonian for the non-mesonic decay of $\Lambda$-hypernuclei yet. The main reason is that there are no experimental data on the free $\Lambda N \rightarrow \Lambda N$ reaction. So far most of the theoretical studies have been based on meson-exchange models with the $\Delta I = \frac{1}{2}$ rule assumed. However, their predictions for
the partial decay widths, $\Gamma_n$ for $\Lambda n \to nn$ and $\Gamma_p$ for $\Lambda p \to np$, are in general not quite consistent with the experimental data, although they are strongly model dependent [1,6].

Decades ago, Block and Dalitz [2] analyzed the measurements of the non-mesonic decays of $^4\Lambda$H and $^4\Lambda$He in terms of the spin and isospin dependence of the $\Lambda N \to NN$ interaction without referring to any specific effective Hamiltonian. Assuming that the interaction is local and the $\Lambda$ decay induced by different nucleons is incoherent, they expressed the non-mesonic decay rates of hypernuclei in terms of the elementary reaction rates for $\Lambda N \to NN$ with definite angular momentum and the mean nucleon density at the $\Lambda$ position. By fitting the elementary reaction rates to the available empirical data of hypernuclear decays, one could do meaningful predictions on other non-mesonic decay rates of hypernuclei.

Based on the phenomenological approach of Block and Dalitz, Cohen [3] and Schumacher [4] tried to determine the elementary decay rates using the data on $^4\Lambda$H, $^4\Lambda$He, and $^5\Lambda$He. However, Cohen used the same old data on $^4\Lambda$H and $^4\Lambda$He as in Ref. [2], and Schumacher used the controversial preliminary data on $^4\Lambda$He from the Brookhaven E-788 experiment. Now that we have more reliable experimental data on $^4\Lambda$H [7], $^4\Lambda$He [8], $^5\Lambda$He [9], $^{11}\Lambda$B and $^{12}\Lambda$C [9,10], we are in a better position to do theoretical investigations of non-mesonic decays of light $\Lambda$-hypernuclei. In this work, we reanalyze the s-shell hypernuclei ($^4\Lambda$H, $^4\Lambda$He, $^5\Lambda$He) using the recent data and also extend the approach of Block and Dalitz to p-shell hypernuclei ($^{11}\Lambda$B, $^{12}\Lambda$C).

According to Block and Dalitz [2], the non-mesonic decay width of hypernucleus $^{\Lambda}_A$Z may be written as

$$\Gamma_{nm}(^{\Lambda}_A Z) = \rho(^{\Lambda}_A Z)\bar{R}, \quad (2)$$

where $\bar{R}$ is the spin and isospin average of the elementary reaction rates $R_{NJ}$’s for $\Lambda N \to NN$ ($N = p, n$) with the total angular momentum of the $\Lambda N$ pair being $J$, and $\rho(^{\Lambda}_A Z)$ is the mean nucleon density at the $\Lambda$ position. $\rho(^{\Lambda}_A Z)$ is given by

$$\rho(^{\Lambda}_A Z) = (A - 1) \int d^3\vec{r} \rho_N(\vec{r}) |\psi_\Lambda(\vec{r})|^2, \quad (3)$$
with $\rho_N$ and $\psi_\Lambda$ being the normalized nucleon density and the $\Lambda$ wavefunction in the rest frame of the nuclear core, respectively. In this model, it is straightforward to write down the non-mesonic decay widths of light $\Lambda$-hypernuclei in terms of the four $R_{N,J}$’s:  

\[ \Gamma_{nm}(^4\Lambda H) = \frac{1}{6}\rho(4\Lambda H)(2R_{p0} + 3R_{n1} + R_{n0}) \]  

\[ \Gamma_{nm}(^4\Lambda \text{He}) = \frac{1}{6}\rho(4\Lambda \text{He})(3R_{p1} + R_{p0} + 2R_{n0}) \]  

\[ \Gamma_{nm}(^5\Lambda \text{He}) = \frac{1}{8}\rho(5\Lambda \text{He})(3R_{p1} + R_{p0} + 3R_{n1} + R_{n0}) \]  

where all the nucleons are in the s-shell and the total angular momenta of hypernuclei are 0, 0, and $\frac{1}{2}$ for $^4\Lambda H$, $^4\Lambda \text{He}$, and $^5\Lambda \text{He}$, respectively. Here, the $\Lambda$-hyperon is treated as being in the $s$-state because the decay of hypernuclei is expected to occur in their ground states.

In order to apply this model to the $p$-shell hypernuclei, we need to take into account the possibility of $\Lambda N \rightarrow NN$ reaction with the initial $\Lambda N$ pair with relative orbital angular momentum of one, that is, the non-mesonic decay induced by the $p$-shell nucleons. Thus, in general, we need to introduce six more elementary rates, $R'_{N,J}$ for $J = 0, 1,$ and 2. For $^{11}\Lambda B$ and $^{12}\Lambda C$, however, the total angular momenta are $\frac{5}{2}$ and 1, respectively, so that the $\Lambda N$ pair can have total angular momentum of one or two and there appear only two spin averaged elementary rates $R'_N \equiv (3R'_{N1} + 5R'_{N2})/8$ with $N = p, n$. To be specific, the non-mesonic decay widths of $^{11}\Lambda B$ and $^{12}\Lambda C$ are given by  

\[ \Gamma_{nm}(^{11}\Lambda B) = \frac{1}{8}\rho_s(^{11}\Lambda B)(3R_{p1} + R_{p0} + 3R_{n1} + R_{n0}) + \frac{1}{2}\rho_p(^{11}\Lambda B)(R'_p + R'_n) \]  

\[ \Gamma_{nm}(^{12}\Lambda C) = \frac{1}{8}\rho_s(^{12}\Lambda C)(3R_{p1} + R_{p0} + 3R_{n1} + R_{n0}) + \frac{1}{7}\rho_p(^{12}\Lambda C)(4R'_p + 3R'_n) \]  

where $\rho_s$ ($\rho_p$) represents the mean $s$ state ($p$ state) nucleon density at the $\Lambda$ position.

In the mean nucleon densities $\rho(^A\Lambda Z)$ of Eq.\[3\], the nucleon wavefunction is obtained using a Woods-Saxon potential with its parameters adjusted to give the correct r.m.s. radius of the nucleus $^A-1Z$ and the $\Lambda$ wavefunction is similarly obtained by fitting the $\Lambda$ binding energy of the hypernucleus $^A\Lambda Z$. For the $p$-shell hypernuclei, we take the spin-orbit interaction into account as in Ref.\[4\]. The resulting mean nucleon densities at the $\Lambda$ position are given in the second row of Table\[4\].
The six elementary decay rates $R_{p0}$, $R_{p1}$, $R_{n0}$, $R_{n1}$, $R'_p$ and $R'_n$ can be determined uniquely from the relations similar to Eqs. (4) - (8) provided that six reliable data points are available. As for such data points, we take the three total non-mesonic decay widths of $^4\Lambda$H, $^5\Lambda$He, and $^{11}_\Lambda$B, and the two proton-induced partial decay widths of $^5_\Lambda$He and $^{11}_\Lambda$B, and finally the ratio $\Gamma_n/\Gamma_p$ of $^4\Lambda$He. In solving the simultaneous equations, we took the mean values of the data points, the bold-faced letters in the last three rows of Table I.

Our estimated values for the elementary decay rates $R_{NJ}$'s are summarized in Table II. Since we have determined all six of the necessary elementary decay rates of the model, we are now in a position to predict other decay widths. Unfortunately, however, we are left with only three experimental data to compare with up to now, namely $\Gamma_{nm}(^4\Lambda$He), $\Gamma_p(^{12}_\Lambda$C) and $\Gamma_{nm}(^4\Lambda$He). Our predictions for these decay widths are given under the heading of ‘Our model’ together with the measured values in Table II. It can be seen from the table that our predicted widths fall nicely not only within error bounds of the data but also within a few percents of the mean values except for the $^4\Lambda$He case where there is still some controversy on the measured decay width. Therefore, we conclude with reasonable confidence that the phenomenological model used in our analysis seems to work fairly well for light $\Lambda$-hypernuclei.

Once we admit that the model establishes the elementary decay rates $R_{NJ}$'s reliably from the adopted data, we can find two interesting implications from our numerical results for the six $R_{NJ}$'s shown in Table II. Firstly, $R'_n$, the spin-averaged elementary decay rate induced by the $p$-shell neutrons, turns out to be relatively quite large. This indicates that the non-mesonic $\Lambda$-hyperon decay induced by the $p$-shell nucleons (particularly by the neutrons) is not less probable than the decay induced by the $s$-shell nucleons. Certainly, this result is in contradiction to a naive expectation from the short-range nature of the $\Lambda N \rightarrow NN$ interaction that the main contribution to the decay rate should come from the $\Lambda N$ pair with zero orbital angular momentum [13]. In the present situation without a working microscopic mechanism for the non-mesonic $\Lambda$-hyperon decay, it remains to be understood how the $p$-shell nucleons contribute to the decay of $\Lambda$-hypernuclei. In particular, the three-body non-mesonic process $\Lambda nn \rightarrow nnn$ [5] may compete with the two body process $\Lambda n \rightarrow nn$ induced by the
p-shell neutrons. Secondly, if the $\Delta I = \frac{1}{2}$ rule should hold true in nuclear medium, we would have

$$\frac{R_{n0}}{R_{p0}} = 2, \quad \frac{R_{n1}}{R_{p1}} \leq 2, \quad \frac{R'_n}{R'_p} \leq 2,$$  \hfill (9)

while if $\Delta I = \frac{3}{2}$ should hold true, we would have

$$\frac{R_{n0}}{R_{p0}} = \frac{1}{2}.$$  \hfill (10)

Now, according to our results given in Table II, we have

$$\frac{R_{n0}}{R_{p0}} \approx 0.4, \quad \frac{R_{n1}}{R_{p1}} \approx 1.5, \quad \frac{R'_n}{R'_p} \approx 11.5, \quad \text{(our model prediction)}. \hfill (11)$$

Comparing the predicted elementary reaction rates of ours in Eq. (11) with those of Eq. (9) and Eq. (10) it is evident that the adopted recent data favor including $\Delta I = \frac{3}{2}$ interactions in addition to $\Delta I = \frac{1}{2}$ ones in the effective Hamiltonian for the non-mesonic decay of $\Lambda$-hypernuclei. However, with the experimental error bars taken into account, one may argue that the $\Delta I = \frac{1}{2}$ rule is not excluded completely. Nevertheless, it seems quite plausible that $\Delta I = \frac{3}{2}$ amplitude can be as important as the $\Delta I = \frac{1}{2}$ amplitude in the non-mesonic decays of $\Lambda$-hypernuclei. Based on the conclusions of the present work, a microscopic approach using an effective Hamiltonian with four-fermion interactions of $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ in addition to the $\Delta I = \frac{1}{2}$ pion-exchange interactions is in progress.

In summary, we have analyzed the recent data on the non-mesonic decays of the $s$-shell hypernuclei ($^4_\Lambda$H, $^4_\Lambda$He, $^5_\Lambda$He) using the phenomenological model of Block and Dalitz and also extended our analysis to $p$-shell hypernuclei ($^{11}_\Lambda$B, $^{12}_\Lambda$C). According to the model, the non-mesonic decay widths of all the $s$-shell as well as the $p$-shell $\Lambda$-hypernuclei can be expressed in terms of only six of the elementary reaction rates $R_{NJ}$'s. Making use of the six data points from recently measured decay widths, we can determine the six elementary reaction rates uniquely. They are used in turn to predict the decay widths of the remaining data to check our model calculations. The results are certainly more than satisfactory even though there exist only three available decay widths to compare with which have been measured.
up to now. From solely the six elementary reaction rates determined by the recent data, we can deduce two interesting implications. One is that despite the short-range nature of the $\Lambda N \to NN$ interaction, the non-mesonic decay of $p$-shell hypernuclei seems to be strongly induced by the $p$-shell neutrons. The other is that the recent data favor including $\Delta I = \frac{3}{2}$ interactions in addition to $\Delta I = \frac{1}{2}$ ones in the effective Hamiltonian for the non-mesonic decay of $\Lambda$-hypernuclei. Therefore, we suggest that the $\Delta I = \frac{1}{2}$ rule, well-proved at the hadronic level, may not be sacred in the nuclear medium.

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TABLES

TABLE I. Input data for our model calculation and results from our model calculations. The second row is the mean nucleon density at the \( \Lambda \)-hyperon position calculated with the usual Woods-Saxon potential. From the third to the fifth rows, measured non-mesonic decay widths of light \( \Lambda \)-hypernuclei are given. Data points used in determining the elementary reaction rates are written in bold face numbers. ‘Our Model’ represents decay widths predicted by our calculations.

| \( \Lambda \)-hypernucleus | \( ^4_\Lambda \)H | \( ^4_\Lambda \)He | \( ^5_\Lambda \)He | \( ^{11}_\Lambda \)B | \( ^{12}_\Lambda \)C |
|--------------------------|----------------|----------------|----------------|----------------|----------------|
| Mean nucleon density     | \( \rho = 0.017 \) | \( \rho = 0.019 \) | \( \rho_s = 0.038 \) | \( \rho_s = 0.048 \) | \( \rho_s = 0.047 \) |
|                          | ( fm\(^{-3}\) ) |                |                |                |                |
| \( \Gamma_{nm}/\Gamma_\Lambda \) | 0.24 ± 0.13 a | 0.17 ± 0.05 b | 0.41 ± 0.14 c | 0.95 ± 0.17 d | 0.89 ± 0.18 d |
| Our Model                |                |                |                |                |                |

\( a \)Taken from Ref. [7].
\( b \)Taken from Ref. [8].
\( c \)Taken from Ref. [9].
\( d \)Taken from Ref. [10].
\( e \)Taken from Ref. [12].

TABLE II. The elementary reaction rates in the unit of \( \Gamma_\Lambda \) fm\(^3\) determined from six data points written by bold face letters in Table I.

| s-shell        | p-shell        |
|----------------|----------------|----------------|----------------|
| proton         | neutron        | proton         | neutron        |
| \( R_{p0} \) = 21.3 | \( R_{n0} \) = 8.8 | \( R'_p \) = 1.8 | \( R'_n \) = 20.9 |
| \( R_{p1} \) = 7.6 | \( R_{n1} \) = 11.1 |               |                |