Momentum Distributions and Short-Range Correlations in Few-Nucleon Systems Using Realistic Interactions

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Abstract. We present predictions for the single-nucleon and spin-isospin projected two-nucleon momentum distributions in $^3$H and $^3$He, obtained using phenomenological potentials, as well as the most recent high-quality chiral nucleon-nucleon potentials, with and without three-nucleon force. Three-nucleon interaction contributions as well as chiral order-by-order convergence are briefly discussed. An outlook for further studies is also presented.

1. Introduction

The study of high-momentum distributions in nuclei can provide information about the short-range few-nucleon dynamics, and, ultimately, about the nature of the strong force which governs it. In fact, the picture of a nucleus as a dilute system, in which hard objects (the nucleons) are collected in a mean field, has strong limitations: the wave functions of nucleons in a medium strongly overlap, giving rise to the so-called short-range correlations (SRCs). Momentum distributions and SRCs in $^3$H and $^3$He are the subject of this contribution. In particular, we focus on single-nucleon and on the spin-isospin projected two-nucleon momentum distributions, defined as \cite{1, 2, 3}:

\begin{align}
\langle k \rangle &= \int d\mathbf{k} \int d\mathbf{k}_{\text{rel}} \Psi^\dagger(\mathbf{k}_{\text{rel}}, \mathbf{k}) P_{n/p} \Psi(\mathbf{k}_{\text{rel}}, \mathbf{k}) \quad (1) \\
\langle k_{\text{c.m.}} \rangle &= \int d\mathbf{k}_{\text{rel}} \int d\mathbf{k}_{\text{c.m.}} \Psi^\dagger(\mathbf{k}_{\text{rel}}, \mathbf{k}_{\text{c.m.}}) P_{ST} \Psi(\mathbf{k}_{\text{rel}}, \mathbf{k}_{\text{c.m.}}) \quad (2)
\end{align}

where $\mathbf{k}$ in Eq. (1) is the $n/p$ momentum and $\mathbf{k}_{\text{rel}}$ in Eq. (2) are the pair center-of-mass and the pair relative momenta, respectively. Moreover, $P_{n/p}$ and $P_{ST}$ are the projection operators on the $n/p$ nucleon or the $ST$ spin-isospin state of the $NN$ pair, and $\Psi(\mathbf{k}_{\text{rel}}, \mathbf{k})$ in Eq. (1) [or $\Psi(\mathbf{k}_{\text{rel}}, \mathbf{k}_{\text{c.m.}})$ in Eq. (2)] is the $A = 3$ nuclear wave function. The latter quantity is calculated using the Hyperspherical Harmonics (HH) method, as explained in Ref. \cite{1}. Note that the HH functions can be written in coordinate- as well as in momentum-space \cite{4}, and therefore the HH method is adapted in conjunction with any choice of the nuclear potential model, which can be either local or non-local in coordinate- or in momentum-space.
This contribution is organized as follows: in Sec. 2 we present the results for the above mentioned quantities obtained with the phenomenological Argonne $v_{18}$ (AV18) [6] two-nucleon potential model, without or with three-nucleon interaction, i.e. the Urbana IX (UIX) [7] model, or obtained with the meson-theoretic charge dependent Bonn (CDBonn) two-nucleon potential model [8], again without or with the appropriate three-nucleon force, namely the Tucson-Merlbourne (TM) [9] model. Then we will show results obtained with the most recent chiral two-nucleon potentials of Ref. [5], without and with the inclusion of the corresponding three-nucleon interactions, as constructed in Ref. [1]. Note that in Ref. [1] it has been shown that the results obtained with the AV18 and AV18/UIX potential models are in good agreement with those of Refs. [2, 3, 10]. Finally, in Sec. 3 we summarize our results and present our conclusions, with an outlook on future work.

2. Results

We present in Subsecs. 2.1 and 2.2 the results for the $A = 3$ single-nucleon and two-nucleon momentum distributions, respectively.

2.1. Single-nucleon momentum distributions

The $^3$He single-nucleon momentum distributions $n_{p/n}(k)$ of Eq. (1) are shown in Fig. 1 for all the phenomenological or meson-theoretic potential models mentioned above. From inspection of the figure we can conclude that (i) the three-nucleon interaction contributions are very small; (ii) the model dependence is quite large, as it was already found in Ref. [11] for the case of the deuteron. This model dependence can be traced back to locality vs. non-locality of the adopted models, especially in their tensor components, and, together with the small contributions of the three-nucleon interactions, will be a recurrent feature of this study.

The $^3$He single-nucleon momentum distributions obtained with the chiral potentials of Ref. [5] are shown in Fig. 2. Given the fact that the contribution from three-nucleon interaction is very small, we show results obtained with only two-nucleon interactions. Furthermore, we fix the cutoff $\Lambda$ to 500 MeV. The results with the full two- and three-nucleon interaction and with different cutoff values can be found in Ref. [1]. By inspection of the figure we can conclude that the order-by-order convergence is quite good up to $k \approx 2 - 3$ fm$^{-1}$, which is the value of the cutoff $\Lambda$.

We present in Fig. 3 the $^3$H single-nucleon momentum distributions $n_{p/n}(k)$ obtained with the phenomenological or meson-theoretic potential models mentioned above, restricting to the models where both two- and three-nucleon interactions are included. In Fig. 4 we show the same
Figure 2. Single-neutron (left panel) and single-proton (right panel) momentum distribution of $^3$He obtained with the chiral potentials of Ref. [5], from leading order (LO) up to next-to-next-to-next-to-next-to leading order (N4LO). The cutoff $\Lambda$ is fixed to 500 MeV.

momentum distributions obtained with the chiral potentials with cutoff $\Lambda = 500$ MeV. As we can see from the figures, the same conclusion drawn for the $^3$He case are valid also for $^3$H.

Figure 3. Same as Fig. 1 but for $^3$H. Only the results obtained with the two- and three-nucleon interactions AV18/UIX and CDBonn/TM are shown.

Finally, we present in Fig. 5 the ratio between the proton momentum distributions in $^3$He and in $^3$H, obtained both with phenomenological and chiral potentials, all including two- and three-nucleon interactions. Again the cutoff $\Lambda$ in the chiral potentials is fixed to 500 MeV. The experimental data are the ratio of $(e,e'p)$ cross section on $^3$He and $^3$H, measured at Jefferson Lab, using an adequate kinematical region, so that, as it has been shown in Ref. [12], this cross section ratio is supposed to be equal to the momentum distributions ratio. By inspection of the figure we can conclude that the AV18/UIX and CDBonn/TM results are quite close to each other. This is true in spite of the fact that the single-proton momentum distributions in both $^3$H and $^3$He are significantly different. On the other hand, the results obtained with the chiral potentials are quite different for $k > 400$ MeV, and present a poor order-by-order convergence even below 500 MeV, although they seem to be in better agreement with data.

2.2. Two-nucleon momentum distributions

We present here the spin-isospin projected two-nucleon momentum distributions, defined in Eq. (2), obtained with the AV18/UIX potential model. The same study with CDBonn/TM
and chiral potential models is in progress. The $^3$He and $^3$H results are shown in Fig. 6 and are compared with the results of Ref. [10]. It should be mentioned that the definition of Eq. (2) and Ref. [10] differ by a factor of $(2\pi)^3$. Furthermore, we report in Table 1 the normalization of the different distributions, i.e.

$$N_{ST} = 4\pi \int_0^{\infty} k_{rel}^2 dk_{rel} n_{ST}^{NN}(k_{rel}) ,$$

also compared with Ref. [10]. To be noticed that, in the case of $^3$H, the results of Ref. [10] are obtained using a slightly different version of the UIX potential, the so-called UX three-nucleon interaction. However, it has been shown in Ref. [10] that in the case of $^3$He, the differences are very small. By inspection of Fig. 6 we can see, for both $^3$He and $^3$H, some small differences for the smallest contributions $ST = 00$ and 11. The largest contributions, $ST = 01$ and 10, are instead in very nice agreement. These differences reflect also on the normalizations $N_{ST}$ shown in Table 1, but they are still quite small.

3. Conclusions and outlook
We have presented here a theoretical study of $A = 3$ single- and two-nucleon momentum distributions, with a large variety of nuclear potential models. All the momentum distributions have been found strongly model-dependent. However, in the ratio for the single-proton momentum distributions in $^3$He and $^3$H, this model-dependence appears only between the

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Figure 4. Same as Fig. 2 but for $^3$H.

Figure 5. Ratio between the single-proton momentum distributions in $^3$He and $^3$H. See text for more details.
Figure 6. Spin-isospin projected two-nucleon momentum distributions in $^3\text{He}$ (left panel) and $^3\text{H}$ (right panel) calculated with the present method are compared with the results of Ref. [10].

Table 1. Normalizations of the different spin-isospin projected two-nucleon momentum as defined in Eq. (3) obtained with the AV18/UIX potential model are compared with the corresponding results of Ref. [10]. The label $ST$ indicates the spin-isospin channel. See text for more details.

| $ST$  | $N(^3\text{He})$ | $N(3\text{H})$ |
|-------|------------------|----------------|
|       | Present work     | Ref. [10]      | Present work | Ref. [10] |
| 00    | 0.010            | 0.010          | 0.009        | 0.007     |
| 01    | 1.351            | 1.358          | 1.351        | 1.360     |
| 10    | 1.490            | 1.490          | 1.490        | 1.493     |
| 11    | 0.149            | 0.142          | 0.148        | 0.139     |

phenomenological AV18/UIX and CDBonn/TM potentials on one side and the chiral potentials on the other. Furthermore, the three-nucleon interaction contributions have been found extremely small. Model-dependence and small three-nucleon interaction contribution has been found also in the study of $^3\text{He}$ SRC probabilities of Ref. [1]. We expect a similar situation also for the $N_{ST}$ normalizations and SRCs in $^3\text{H}$.

This work will proceed into two directions: first of all, we will perform a systematic study of single- and two-nucleon momentum distributions for all the $A = 2 - 4$ nuclei, with a large variety of potential models, extending from the non-local chiral potentials used here from Ref. [5], to also the local ones of Refs. [13, 14, 15]. Secondly, we will study the high-momentum tail of $A = 2 - 4$ two-nucleon spin-isospin projected momentum distributions and the small range part of the $A = 2 - 4$ two-nucleon spin-isospin projected density distributions. These are crucial inputs within the Generalized Cluster Formalism (GCF) (see Ref. [16] and references therein), in order to extract the so-called nuclear contacts. Knowing these, within the GCF, the spectral functions can be constructed for a large variety of nuclei [17, 18].

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