NUCLEAR SPIN IN DIRECT DARK MATTER SEARCH

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Abstract
The Weakly Interacting Massive Particles (WIMPs) are among the main candidates for the relic dark matter (DM). The idea of the direct DM detection relies on elastic spin-dependent (SD) and spin-independent (SI) interaction of WIMPs with target nuclei. The importance of the SD WIMP-nucleus interaction for reliable DM detection is argued. The absolute lower bound for the detection rate can naturally be due to SD interaction. An experiment aimed at detecting DM with sensitivity higher than $10^{-5}$ event/day/kg should have a non-zero-spin target.

1 Introduction

The lightest supersymmetric (SUSY) particle (LSP) neutralino is assumed to be the Weakly Interacting Massive Particle (WIMP) and the best dark matter (DM) candidate. It is believed that for heavy enough nuclei this spin-independent (SI) interaction of DM particles with nuclei usually makes the dominant contribution to the expected event rate of its detection. The reason is the strong (proportional to the squared mass of the target nucleus) enhancement of SI WIMP-nucleus interaction. Nevertheless there are at least three reasons to think that SD (or axial-vector) interaction of the DM WIMPs with nuclei could be very important. First, contrary to the only one constraint for SUSY models available from the scalar WIMP-nucleus interaction, the spin WIMP-nucleus interaction supplies us with two such constraints (see for example [1] and formulas below). Second, one can notice [2, 3] that even with a very sensitive DM detector (say, with a sensitivity of $10^{-5}$ events/day/kg) which is sensitive only to the WIMP-nucleus scalar interaction (with spin-less target nuclei) one can, in principle, miss a DM signal. To safely avoid such a situation one should have a spin-sensitive DM detector, i.e. a detector with non-zero-spin target nuclei. Finally, there is a complicated nuclear spin structure, which, for example, characterized by the so-called long $q$-tail form-factor behavior. Therefore for heavy target nuclei and heavy WIMP the SD efficiency to detect a DM signal is much higher than the SI efficiency [4]. However, simultaneous study of both spin-dependent and spin-independent interactions of the DM particles with nuclei significantly increases the chance to observe the DM signal.
Figure 1: Due to the expected annual modulation signature of the event rate (1) only the Sun-Earth system is a proper setup for the successful direct DM detection.

2 Two constraints for SUSY due to the spin

One believes to detect directly a DM particle $\chi$ via its elastic scattering on a target nucleus $(A, Z)$. The nuclear recoil energy $E_R (E_R \approx 10^{-6} m_\chi \approx \text{few keV})$ is measured by a proper detector (Fig. 1). The differential event rate depends on the distribution of the relic DM particles in the solar vicinity $f(v)$ and the cross section of LSP-nucleus elastic scattering [5]–[12]:

$$\frac{dR}{dE_R} = N \frac{\rho_\chi}{m_\chi} \int_{v_{\text{min}}}^{v_{\text{max}}} dv f(v) v \frac{d\sigma}{dq^2}(v, q^2), \quad E_R = \frac{q^2}{2 M_A}. \quad (1)$$

Here, $N = \mathcal{N}/A$ is the number density of target nuclei. $\mathcal{N}$ and $A$ stand for the Avogadro number and the atomic mass in AMU, respectively. $M_A$ denotes the nuclear mass. $v_{\text{max}} = v_{\text{esc}} \approx 600 \text{ km/s}$, $v_{\text{min}} = (M_A E_R/2 \mu^2 A)^{1/2}$, the DM density $\rho_\chi = 0.3 \text{ GeV cm}^{-3}$. The neutralino-nucleus elastic scattering cross section for spin-non-zero ($J \neq 0$) nuclei is a sum of the coherent (spin-independent) and axial (spin-dependent) terms [4, 13, 14, 15]:

$$\frac{d\sigma^A}{dq^2}(v, q^2) = \frac{\sum |M|^2}{\pi v^2 (2J + 1)} = \frac{S_{\text{SD}}^A(q^2)}{v^2(2J + 1)} + \frac{S_{\text{SI}}^A(q^2)}{v^2(2J + 1)}$$

$$= \frac{\sigma_{\text{SD}}^A(0)}{4 \mu_\chi^2 v^2} F_{\text{SD}}^2(q^2) + \frac{\sigma_{\text{SI}}^A(0)}{4 \mu_\chi^2 v^2} F_{\text{SI}}^2(q^2). \quad (2)$$
It is useful to separate the zero-momentum transfer cross sections and introduce the normalized-to-unity \( F_{SD,SI}(0) = 1 \) nonzero-momentum-transfer nuclear form-factors:

\[
F_{SD,SI}(q^2) = \frac{S_{SD,SI}^A(q^2)}{S_{SD,SI}^A(0)}.
\] (3)

The SD structure function \( S_{SD}^A(q) \) contains the isoscalar \( S_{00} \), isovector \( S_{11} \) and the interference \( S_{01} \) terms:

\[
S_{SD}^A(q) = a_0^2 S_{00}(q) + a_1^2 S_{11}(q) + a_0 a_1 S_{01}(q).
\] (4)

Here the isoscalar \( a_0 = a_n + a_p \) and isovector \( a_1 = a_p - a_n \) effective coupling constants are used (see (9)). For \( q = 0 \) the nuclear SD and SI cross sections take the forms

\[
\sigma_{SI}^A(0) = \frac{4 \mu_A^2 \sigma_{SI}(0)}{(2J + 1)} = \frac{2 \mu_s^2}{\mu_p^2} \sigma_{SI}^A(0),
\] (5)

\[
\sigma_{SD}^A(0) = \frac{4 \mu_A^2 S_{SD}(0)}{(2J + 1)} = \frac{4 \mu_A^2 (J + 1)}{\pi J} \left\{ a_p \langle S_p^A \rangle + a_n \langle S_n^A \rangle \right\}^2.
\] (6)

Here, \( \mu_A = \frac{m_\chi M_A}{m_\chi + M_A} \) is the reduced \( \chi \)-nucleus mass and \( \mu_s^2 = \mu_p^2 \) is assumed. The dependence on effective neutralino-quark couplings \( C_q \) and \( A_q \) in the underlying (SUSY) theory

\[
\mathcal{L}_{eff} = \sum_q (A_q \cdot \bar{\chi} \gamma_\mu \gamma_5 \chi \cdot q \gamma^\mu \gamma_5 q + C_q \cdot \bar{\chi} \chi \cdot q q) + ...
\] (7)

and on the spin \( (\Delta_q^{(p,n)}) \) and the mass \( (f_q^{(p,n)}) \) structure of nucleons enter into these formulas via the zero-momentum-transfer proton and neutron SI and SD cross sections:

\[
\sigma_{SI}^p(0) = \frac{4 \mu_p^2 \sigma_0^p}{\pi}, \quad \sigma_{SI}^n(0) = \frac{4 \mu_n^2 \sigma_0^n}{\pi}, \quad \sigma_{SD}^{p,n}(0) = \frac{12 \mu_{p,n}^2 a_{p,n}^2}{\pi};
\] (8)

\[
c_0^{p,n} = \sum_q C_q f_q^{(p,n)}, \quad a_p = \sum_q A_q \Delta_q^{(p)} \quad a_n = \sum_q A_q \Delta_q^{(n)}.
\] (9)

The factors \( \Delta_q^{(p,n)} \), which parametrize the quark spin content of the nucleon, are defined as \( 2 \Delta_q^{(p,n)} s^\mu \equiv \langle p, s | \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q | p, s \rangle_{(p,n)} \). The \( \langle S_p^A \rangle \) is the total spin of protons (neutrons) averaged over all \( A \) nucleons of the nucleus \( (A, Z) \):

\[
\langle S_p^A \rangle \equiv \langle A | S_p^A | A \rangle = \langle A | \sum_i s_{p(n)}^i | A \rangle
\] (10)

The mean velocity \( \langle v \rangle \) of the relic DM particles of our Galaxy is about 300 km/s = \( 10^{-3}c \). For not very heavy \( m_\chi \) and \( M_A \) one can use the SD matrix element in zero momentum transfer limit [15, 16]

\[
\mathcal{M} \propto \langle A | a_p S_p + a_n S_n | A \rangle \cdot s_\chi.
\] (11)

Note a coupling of the spin of \( \chi \), \( s_\chi \), to the spin carried by the protons and the neutrons. The uncertainties arising from electroweak and QCD scale physics are incorporated in the
Table 1: Zero momentum spin structure of nuclei in different models. The measured magnetic moments used as input are enclosed in parentheses. From [17].

| Nucleus | Model | $S_p$ | $S_n$ | $\mu$ (in $\mu_N$) |
|---------|-------|-------|-------|---------------------|
| $^{19}$F ($L_J = S_{1/2}$) | ISPSM, Ellis–Flores [18, 19] | 1/2 | 0 | 2.793 |
| | OGM, Engel–Vogel [20] | 0.46 | 0 | (2.629)$_{\text{exp}}$ |
| | EOGM ($g_A/g_V = 1$), Engel–Vogel [20] | 0.415 | −0.047 | (2.629)$_{\text{exp}}$ |
| | EOGM ($g_A/g_V = 1.25$), Engel–Vogel [20] | 0.368 | −0.001 | (2.629)$_{\text{exp}}$ |
| | SM, Pacheco-Strottman [21] | 0.441 | −0.109 | |
| | SM, Divari et al. [22] | 0.4751 | −0.0087 | 2.91 |
| $^{25}$Na ($L_J = P_{3/2}$) | ISPSM | 1/2 | 0 | 3.793 |
| | SM, Ressell-Dean [15] | 0.2477 | 0.0198 | 2.2196 |
| | OGM, Ressell-Dean [15] | 0.1566 | 0.0198 | (2.219)$_{\text{exp}}$ |
| | SM, Divari ar al. [22] | 0.2477 | 0.0199 | 2.22 |
| $^{27}$Al ($L_J = D_{5/2}$) | ISPSM, Ellis–Flores [18, 19] | 1/2 | 0 | 4.793 |
| | OGM, Engel–Vogel [20] | 0.25 | 0 | (3.642)$_{\text{exp}}$ |
| | EOGM ($g_A/g_V = 1$), Engel–Vogel [20] | 0.333 | 0.043 | (3.642)$_{\text{exp}}$ |
| | EOGM ($g_A/g_V = 1.25$), Engel–Vogel [20] | 0.304 | 0.072 | (3.642)$_{\text{exp}}$ |
| | SM, Engel et al. [16] | 0.3430 | 0.0296 | 3.584 |
| $^{13}$Ge ($L_J = G_{9/2}$) | ISPSM, Ellis–Flores [18, 19] | 0 | 0.5 | −1.913 |
| | OGM, Engel–Vogel [20] | 0 | 0.23 | (−0.879)$_{\text{exp}}$ |
| | IBFM, Iachello et al. [23] and [14] | −0.009 | 0.469 | −1.785 |
| | IBFM (quenched), Iachello et al. [23] and [14] | −0.005 | 0.245 | (−0.879)$_{\text{exp}}$ |
| | TFFS, Nikolaev–Klapdor-Kleingrothaus, [24] | 0 | 0.34 | — |
| | SM (small), Ressell et al. [14] | 0.005 | 0.496 | −1.468 |
| | SM (large), Ressell et al. [14] | 0.011 | 0.468 | −1.239 |
| | SM (large, quenched), Ressell et al. [14] | 0.009 | 0.372 | (−0.879)$_{\text{exp}}$ |
| | “Hybrid” SM, Dimitrov et al. [25] | 0.030 | 0.378 | −0.920 |
| $^{127}$I ($L_J = D_{5/2}$) | ISPSM, Ellis–Flores [19, 26] | 1/2 | 0 | 4.793 |
| | OGM, Engel–Vogel [20] | 0.07 | 0 | (2.813)$_{\text{exp}}$ |
| | IBFM, Iachello et al. [23] | 0.464 | 0.010 | (2.813)$_{\text{exp}}$ |
| | IBFM (quenched), Iachello et al. [23] | 0.154 | 0.003 | (2.813)$_{\text{exp}}$ |
| | TFFS, Nikolaev–Klapdor-Kleingrothaus, [24] | 0.15 | 0 | — |
| | SM (Bonn A), Ressell–Dean [15] | 0.309 | 0.075 | 2.775 {2.470)$_{\text{eff}}$ |
| | SM (Nijmegen II), Ressell–Dean [15] | 0.354 | 0.064 | 3.150 {2.7930)$_{\text{eff}}$ |
| $^{131}$Xe ($L_J = D_{3/2}$) | ISPSM, Ellis–Flores [18, 19] | 0 | −0.3 | 1.148 |
| | OGM, Engel–Vogel [20] | 0.0 | −0.18 | (0.692)$_{\text{exp}}$ |
| | IBFM, Iachello et al. [23] | 0.000 | −0.280 | (0.692)$_{\text{exp}}$ |
| | IBFM (quenched), Iachello et al. [23] | 0.000 | −0.168 | (0.692)$_{\text{exp}}$ |
| | TFFS, Nikolaev–Klapdor-Kleingrothaus, [24] | −0.186 | — | |
| | SM (Bonn A), Ressell–Dean [15] | −0.009 | −0.227 | 0.980 {0.637)$_{\text{eff}}$ |
| | SM (Nijmegen II), Ressell–Dean [15] | −0.012 | −0.217 | 0.979 {0.347)$_{\text{eff}}$ |
| | QTDA, Engel [4] | −0.041 | −0.236 | 0.70 |

Note: The values in parentheses are the measured magnetic moments used as input in the calculations.
factors $a_p$ and $a_n$. The nuclear matrix element $M$ in Eq. (11) is often related to the matrix element of the nuclear magnetic moment, which also consists of the matrix elements of the total proton and neutron spin operators:

$$\mu = \langle A | g_n^s S_n + g_n^l L_n + g_p^s S_p + g_p^l L_p | A \rangle.$$  

(12)

The free particle $g$-factors (gyromagnetic ratios) are (in nuclear magnetons): $g_n^s = -3.826$, $g_n^l = 0$, $g_p^s = 5.586$, $g_p^l = 1$. The nuclear magnetic moment $\mu$ is often used as a benchmark for the accuracy of the calculation of $S_p$ and $S_n$ [14, 15]. For the most interesting isotopes either $\langle S^A_p \rangle$ or $\langle S^A_n \rangle$ dominates ($\langle S^A_{n(p)} \rangle \ll \langle S^A_{p(n)} \rangle$). See, for example, Table 1.

Figure 2: Exclusion curves for the spin-dependent WIMP-proton cross section ($\sigma_{SD}^p$ as a function of the WIMP mass). DAMA/NaI-7a(f) contours for the WIMP-proton SD interaction in $^{127}$I are obtained on the basis of the positive signature of annual signal modulation [27, 28]. The scattered points are calculations of [29].
From Eqs. (6) one can conclude the spin observables in DM search give us TWO independent constraints on a SUSY model via $\sigma_{\text{SD}}^p(0)$ and $\sigma_{\text{SD}}^n(0)$, or, equivalently, via $a_p$ and $a_n$. These constraints are usually presented in the form of exclusion curves obtained with different target nuclei (Figs. 2 and 3). There is only one similar constraint from spin-independent DM search experiments (Eq. (5)). This presentation is a bit obsolete [27, 28, 29], but it allows one to compare sensitivities of different experiments.

3 Long-tail $q$-behaviour due to the spin

As $m_\chi$ becomes larger, the finite momentum transfer limit must be considered for heavier mass $M_A$ nuclei. The differential SD event rate with structure function $S_{\text{SD}}^A(q)$ (4) has now the form

$$\frac{dR_{\text{SD}}^A}{dq^2} = \frac{\rho}{m_\chi m_A} \int vModelIndex f(v) \frac{8G_F^2}{(2J + 1)v^2} S_{\text{SD}}^A(q).$$

Comparing this formula with the observed recoil spectra for different targets (Ge, Xe, F, NaI, etc) one can directly and simultaneously restrict both isoscalar and isovector neutralino-nucleon effective couplings $a_{0,1}$. These constraints will impose most model-independent restrictions on the MSSM parameter space. Another attractive feature of the SD WIMP-nucleus interaction is the $q$-dependence of SD structure function (4). The ratio of SD to SI rate in the $^{73}\text{Ge}$ detector grows with the WIMP mass [2, 3]. The growth is much greater for heavy target isotopes like xenon. The reason is the different behavior of
the spin and scalar structure functions with increasing momentum transfer. For example, the xenon SI structure function vanishes for \( q^2 \approx 0.02 \) GeV, but the SD structure function is a non-zero constant in the region (Fig. 4). As noted by Engel in [4], the relatively long tail of the SD structure function is caused by nucleons near the Fermi surface, which do the bulk of the scattering. The core nucleons, which dominate the SI nuclear coupling, contribute much less at large \( q \). Therefore the SD efficiency for detection of a DM signal is higher than the SI efficiency, especially for very heavy neutralinos.

4 One does not miss a DM signal due to the spin

To estimate the DM detection rate we traditionally use the so-called effective scheme of MSSM (effMSSM) whose parameters are defined directly at the electroweak scale, relaxing completely constraints following from any unification assumption (see, for example [30]–[36]). Our MSSM parameter space is determined by the entries of the mass matrices of neutralinos, charginos, Higgs bosons, sleptons and squarks. The relevant definitions can be found in [31]. We have included the current experimental upper limits on sparticle and Higgs masses from the Particle Data Group. Also, the limits on the rare \( b \rightarrow s \gamma \) decay have been imposed. For each point in the MSSM parameter space (MSSM model) we have evaluated the relic density of the light neutralinos \( \Omega_\chi h^2 \) with our code [37, 38, 39] based on [40], taking into account all coannihilation channels with two-body final states that can occur between neutralinos, charginos, sleptons, stops and sbottoms. We assume \( 0.1 < \Omega_\chi h^2 < 0.3 \) for the cosmologically interesting region and we also consider the WMAP reduction of the region to \( 0.094 < \Omega_\chi h^2 < 0.129 \) [41, 42].

From Fig. 5 one sees that the SD contribution obviously dominates in the domain of large expected rates in the non-zero-spin germanium detector \( (R > 0.1 \text{ event/day/kg}) \). But as soon as the total rate drops down to \( R < 0.01 \text{ event/day/kg} \) or, equivalently, the SI (scalar) neutralino-proton cross section becomes smaller than \( 10^{-9} - 10^{-10} \) pb, the...
Figure 5: Ratio of the SD (spin) event rate to the SI (scalar) event rate in the $^{73}$Ge isotope (spin = 9/2) as a function of the total (SD+SI) event rate (left) and the scalar cross section of the neutralino-proton interaction (right). The solid vertical lines give the expected sensitivity of one of the best future projects GENIUS [43]. In the region above the horizontal line the spin contribution dominates.

SD interaction may produce a rather non-negligible contribution to the total event rate. Moreover, if the scalar cross section further decreases ($\sigma < 10^{-12}$ pb), it becomes obvious that the spin contribution alone saturates the total rate and protects it from decreasing below $R \approx 10^{-6} - 10^{-7}$ event/day/kg [44]. With only a spinless detector one can miss a signal caused by SD interaction. An experiment aimed at detecting dark matter with sensitivity higher than $10^{-5}$ event/day/kg should have a non-zero-spin target. Indeed, while the scalar cross sections governed mostly by Higgs exchange can be rather small, the spin cross section cannot be arbitrarily small because the mass of the $Z$ boson [29], which makes the dominant contribution, is well defined, provided one ignores any possible fine-tuning cancellations. Therefore, if an experiment with sensitivity $10^{-5} - 10^{-6}$ event/day/kg fails to detect a dark matter signal, an experiment with higher sensitivity should have a non-zero-spin target and will be able to detect dark matter particles only due to the spin neutralino-quark interaction.

5 Conclusion

There are at least three reasons to think that spin-dependent interaction of the DM WIMPs with nuclei could be very important. First, contrary to the only one constraint for SUSY models available from the spin-independent WIMP-nucleus interaction, the SD WIMP-nucleus interaction supplies us with two such constraints. Second, for heavy target nuclei and heavy WIMP masses the SD efficiency to detect a DM signal is much higher
than the SI efficiency. Finally, the absolute lower bound for the DM detection rate can naturally be due to SD interaction. An experiment aimed at detecting DM with sensitivity higher than $10^{-5}$ event/day/kg should have a non-zero-spin target.

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