Symmetries and dynamics in an AC-driven self-assembled quantum dot lens

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Theoretical results for a single electron in multi-level system given by a lens-shape self-assembled quantum dot in the presence of an intense harmonic electric field are presented. A non-perturbative Floquet approach is used to study the dynamical localization of the particle when going beyond the two-level approach by introducing the full spectral level structure. It is discussed the role of the different quasi-energy sidebands as the parameters of the system change. It is found that the contribution of different drive harmonics is controlled by fine tuning of field intensity. It is also shown that avoided crossings in the quasi-energy spectrum are correlated with the spectral force of the sidebands and dynamical state localization.

I. INTRODUCTION

There is intense activity on the experimental and theoretical understanding of the dynamical evolution of quantum systems exposed to strong time-dependent external fields [1, 2]. The topic has acquired further relevance in connection with the practical operation of devices subjected to oscillating electrical and magnetic fields at the nanoscale. Examples include the shift of resonances in heterostructures as ac-fields are applied (the ac-Stark effect) [3], the behavior of electronic bands in spatially periodic systems [4, 5, 6, 7], and the production of currents in an ac-driven quantum dot [8]. One important effect in these systems is the strong dynamical suppression of tunneling at suitable values of applied ac-field. The coherent destruction of tunneling that appears, known as dynamical localization in the literature, has been well studied in two-level systems as coming from the destructive interference introduced by the drive [2, 9], whenever there is a crossing of quasi-energy levels in the spectrum. Dynamical localization has been proposed as a tool to control the spatial location of a particle in a two-well potential [2], and to selectively control the tunneling in a multiple-well system [10].

Typical growth conditions of semiconducting quantum dots result in dots with lens geometry, and an analysis of this spatial symmetry on the electronic structure is of interest [11, 12]. In this work we explore the problem of periodic driving force and dynamical localization in a realistic level structure that describes self-assembled quantum dots in semiconductors. This requires that we extend the Floquet formalism to self-assembled quantum dots with lens shape. We also establish the importance of incorporating the multi-level structure of a real system and identify in this complex level structure the conditions for dynamical localization.

We find the realistic lens shape to be crucial in the description of the dynamics, as the spatial non-separability of the state plays a relevant role. Consideration of the multilevel structure present in typical quantum dots is a vital requirement for the correct description of the dynamical response of carriers. It is essential to go well beyond the consideration of only two active levels to fully describe the Floquet quasi-energy and the time evolution of electrons in realistic quantum dots. Even for weak driving forces or frequencies, the description of the dynamics requires the inclusion of many different states in order to achieve a fully converged description of the time evolution. The quantum lens geometry makes for a complex and rich theoretical description of the problem [13]. Interestingly, we show that the real dot system allows the generation of higher harmonics of the driving frequency, with intensity that is fully dependent on the amplitude of the drive, which could be used for its generation. The phenomenon of dynamical localization is shown to remain for suitable values of driving field, with strongly diminished localization at high intensity fields.

II. FORMALISM AND RESULT.

We consider a typical self-assembled quantum dot (SAQD) with lens symmetry of circular cross section of radius $a$ and maximum height $b$ which is harmonically driven by an electric field along the axial symmetry $z$ of the lens with intensity $F$ and frequency $\omega$, $F = F \sin \omega t \hat{z}$. Assuming that the electron is described by an isotropic band with
effective mass $m^*$, the dynamics of the system is governed by the time-dependent Schrödinger equation

$$\hat{L}\Psi(r, t) = \left( -\frac{\hbar^2}{2m^*} \nabla^2 - eF \mathbf{z} \cdot \mathbf{r} \sin \omega t - i\hbar \frac{\partial}{\partial t} \right) \Psi(r, t) = 0, \quad (1)$$

where, in spherical coordinates, we have $\hat{L} = \mathbf{z} \cdot \mathbf{r} = r \cos \theta$. The space of functions where the operator $\hat{L}$ of Eq. (1) is defined, corresponds to those spatio-temporal functions which are bounded functions defined in the real space $\mathcal{R}_3$ of the lens domain and are also periodic functions on time with period $\tau = 2\pi/\omega$. The solution of Eq. (1) can be obtained following the standard Floquet theory [2] where $\Psi(r, t)$ is written as

$$\psi(r, t) = e^{-i\varepsilon t/\hbar} \varphi(r, t), \quad (2)$$

which allows to rewrite Eq. (1) as an eigenvalue problem with the same operator $\hat{L}$ for a real-valued eigenenergy $\varepsilon$ (called quasi-energy) and eigenfunction $\varphi(r, t)$ which fulfills the periodic condition $\varphi(r, t) = \varphi(r, t + \tau)$. The periodic time part of the function is expanded as

$$\varphi(r, t) = \sum_{n=-\infty}^{\infty} \frac{e^{i n \omega t}}{i^n \sqrt{2\pi/\omega}} u_n(r). \quad (3)$$

Furthermore, the function $\varphi_n = e^{i n \omega t} \varphi$ is also a solution with quasi-energy $\varepsilon_n = \varepsilon + n\hbar\omega \quad [1, 2]$. These additional solutions, a consequence of the time periodicity, have been called replicas or sidebands. By subtracting a suitable desired convergence. Along this paper it is used a lens domain with ratio $b/a$ of $0.71$, 25 energy levels in the expansion of $u_n(r)$, and for each energy level, the index $n$ for the replicas in Eq. (3) was taken as $n = 0, \pm 1, \pm 2, \ldots, \pm 20$.

Starting from an initial state at $t = 0$, the carrier particle is induced to explore the complete spectrum of the system when the ac-field is connected. How this occurs as a function of time can be analyzed by following the evolution of an initial electron state $f_o(r) = f(r, t = 0)$, which is written as a linear combination of Floquet states

$$f(r, t) = \sum_{\{P\} \in \text{FBZ}} A_P \Psi_P(r, t), \quad (4)$$

with coefficients $A_P$ fixed by the initial conditions. The summation is taken on the First Brillouin Zone (FBZ), including all the replicas [1]. At $F = 0$ the label $P$ indicates the index of quasi-energy $\varepsilon_{N,m}$ with mth replica and level $N$. For simplicity we consider that the initial state has well-defined $z$-component of the angular momentum in such a way that only states with the same $m$ are considered. Then, Eq. (4) can be cast in the following way

$$f_m(r, t) = \sum_N \Delta_N(t) \Phi_{N,m}^{(b/a)}(r), \quad (5)$$

where

$$\Delta_N(t) = \sum_{\{P\} \in \text{FBZ}} \sum_{n=-\infty}^{\infty} A_P C_{n,N,m}(P) \frac{e^{i(n\omega t - \pi/2)}}{\sqrt{2\pi/\omega}} e^{-i\varepsilon(P) t/\hbar} \quad (6)$$

The function $\Delta_N(t)$ contains all the dynamical information due to the presence of the ac electric field. Thus, $P_N(t) = |\Delta_N(t)|^2$ gives, at a given time $t$, the probability of finding the system in the state $\Phi_{N,m}^{(b/a)}$. The typical definition of dynamical localization considers that the carrier does not evolve away from its initial spatial state or configuration, as the effective tunneling amplitude from site to site is suppressed [2]. Here, we monitor dynamical localization by the corresponding quantum probability of finding the particle in its initial state as a function of time. In what follows we assume that the initial state $f_o(r)$ is the zero-field ground state $\Phi_{N,m}^{(b/a)}(r)$ with $N = 1$ and $m = 0$. 

In order to quantify the degree of localization, we study the Oscillation Amplitude (OA) as a function of the field intensity, where the OA is defined as $OA = P_1(t = 0) - P_{\text{min}}$, where $P_{\text{min}}$ is the minimum value that $P_1(t)$ takes over a long interval of time (100 time units in our case) [14]. In Fig. 1 and 2 the OA is plotted as a function of the dimensionless parameter $eFa/\hbar \omega$ where $\Omega = \hbar \omega/E_o$ is a dimensionless frequency with $E_o = \hbar^2/(2m^*a^2)$ as a unit of energy. At zero field the OA is naturally always zero and increases with the field strength, since the latter induces mixing of the states and it forces the system to explore larger regions of the eigenvalue spectrum of the system away from the initial state. The OA reaches the maximum value ($OA = 1$) at certain value of the reduced field intensity
FIG. 3: a) Small window of the FBZ for the quasi-energies as a function of the reduced field intensity. b) Intensity sidebands \( \rho_P(n) \) for \( P = 2 \) and \( P = 13 \) with \( n = 0 \). c) Idem as b) but now with \( n = -3 \). Calculations are done for \( b/a = 0.71 \) and \( \Omega = 100 \).

and for \( eFa/\hbar \omega > 11 \) it is near unity, indicating that the probability of the system remaining in the initial state is zero. However, the OA strongly decreases at some particular field intensities (for example at \( eFa/\hbar \omega = 4 \) in Fig. 1 and at \( eFa/\hbar \omega = 7.2 \) and 9.0 in Fig. 2), meaning that \( P_1(t) \) never goes to zero at those values of the field intensity, and quasi-localization of the system can be identified. At the same time and to study explicitly the behavior of \( P_1 \) on time, different panels inside Fig. 1 and 2 show the time evolution %P(1)(t)% during one hundred time units \( \tau = \hbar/E_0 \) at different values of the dimensionless field intensity. According to Eq. (6), the wave function is a multi-periodic function and presents strong oscillating behavior as seen in the inset of the figures. Notice, moreover, that at small value of reduced frequency (\( \Omega = 100 \)), the mixture of the spectrum is strong for \( eFa/\hbar \omega = 3.6 \) in Fig. 1 and for \( eFa/\hbar \omega = 4 \) a condition for quasi-localization is reached. Instead, for higher values of the frequency (\( \Omega = 300 \)), a lower slope for the increase of the OA is obtained as seen in Fig. 2. It is also reported two values for quasi-localization at \( eFa/\hbar \omega = 7.2 \) and 9.0. Notice also that the stronger the intensity the lower the values of the probability, a consequence of the strong level mixing as discussed previously.

On the other hand, the interaction among different lens levels in the quasi-energies as function of the field amplitude can be analyzed in more detail using the sideband intensity or spectral force \( \rho_P \), which amounts to the weight of the \( n \)-th sidebands on the \( \psi_P(r,t) \) Floquet state. Thus, according to the expansion of \( u_n(r) \), the spectral force is given by

\[
\rho_P(n) = \sum_N |C_{n,N,m}(P)|^2.
\]

Notice that at \( F = 0 \), \( \rho_P(n) \equiv \rho_N(n) \). On the other hand, for \( F > 0 \), the spectral force of a given \( P \) indicates the weight of the different lens states in the full Floquet expansion (3). Figure 3 a) shows a small window of the quasi-energy spectrum, where some anticrossings are present at \( \Omega = 100 \). The system parameters are the same as
in Fig. 1. In Figs. 3b) and 3c) the intensity sidebands $\rho_p(n)$ for the quasi-energies $P = 2$ and $P = 13$ have been plotted, with $n = 0$ in Figs. 3b) and $n = -3$ in Figs. 3c) respectively. We observe in the figures the effects in the spectral force $\rho_p(n)$ caused by the the anticrossing between quasi-energies $P = 2$ and $P = 13$ around $e Fa/\hbar \omega \simeq 1.8$. For values up to $e Fa/\hbar \omega \simeq 1.8$, $\rho_{P=2}(n = 0)$ and $\rho_{P=13}(n = -3)$ show a large intensity as expected due to its corresponding $F = 0$ limit, while $\rho_{P=13}(n = 0)$ and $\rho_{P=2}(n = -3)$ are barely noticeable near $F \simeq 0$. However, at the corresponding anticrossing, a strong mixing of the states takes place, and $\rho_{P=13}(n = 0)$ and $\rho_{P=2}(n = -3)$ increase rapidly while $\rho_{P=2}(n = 0)$ and $\rho_{P=13}(n = -3)$ strongly decrease for $e Fa/\hbar \omega > 1.8$, i.e., their corresponding strengths are inverted after the anticrossing. Thus, the spectral weights for $n = 0$ and $n = -3$ in this case, and in general for all the spectral contributions, are exchanged between the Floquet states at the anticrossing. Most importantly, as the field increases, so does the interlevel mixture and the weights of the various replicas become nearly identical, so that the amplitude of their contribution are similar. We emphasize that this nearly homogeneous distribution of the spectral force over many different replicas results in substantially different time evolution of the driven system. We then anticipate that time averages of physical observable could be substantially different than when only the lowest two lens levels are considered [15].

III. CONCLUSIONS

We have analyzed the time evolution of an electron in self-assembled quantum dots with lens shape in the presence of intense radiation along to the rotational axis of the lens. We exploit this axial symmetry of the lens domain to solve this complex time-dependent problem. This realistic driven single-electron system has been studied over a wide range of field amplitudes. We have demonstrated that consideration of the typical two-level approximation yields an incomplete description even at moderate fields and frequencies. We have also calculated the complex quasi-energy spectra that result in this problem, and analyzed the anticrossings that appear in terms of the interaction among zero-field states and their replicas. We find that these anticrossings are associated with strong shifts in the spectral weights for the Floquet states between two quasi-energies, and that for larger field intensities the spectral weights are distributed homogeneously among a wide range of sidebands. This strong dependence indicates that the appearance of different drive harmonics in the response of the system could be easily controlled by the field strength. It is interesting to consider the possibility of utilizing such lens shape quantum dots in strong ac-fields as a source of different harmonics.

In order to study whether dynamical localization prevails under these much more complicated conditions of multi-level dynamics, we have studied the time evolution of the system prepared initially in the zero-field ground state. We find that at field intensities for which a quasi-energy anticrossing appears in the spectrum, some degree of state localization is observed; in those circumstances, the probability of finding the system in the initial state never goes completely to zero, but it reaches a minimal value. This incomplete dynamical localization has been analyzed quantitatively by means of the oscillation amplitude for different system parameters. Similar to the case of two-level systems, we find that the dynamical localization, already precarious at high frequency, disappears for lower frequency values. The analysis and results we present are important for the interpretation of experimental data, and suggest further theoretical work to assess the relevance of multi-level structures in realistic systems.

[1] J. H. Shirley, Phys. Rev. 138, B979 (1965).
[2] M. Grifoni and P. Hänggi, Phys. Rep. 304, 229 (1998).
[3] A. C. Bittencourt, G. E. Marques, and C. Trallero-Giner, Sol. Stat. Comm. 129, 57 (2004).
[4] K. F. Milfeld and R. E. Wyatt, Phys. Rev. A 27, 72 (1983).
[5] P. A. Schulz, P. H. Rivera, and N. Studart, Phys. Rev. B 66, 195310 (2002).
[6] D. F. Martinez, L. E. Reichl, and G. A. Luna-Acosta, Phys. Rev. B 66, 174306 (2002).
[7] D. Sánchez, G. Platero, and L. L. Bonilla, Phys. Rev. B 63, 201306(R) (2001).
[8] T. Brandes, R. Aguado, and G. Platero, Phys. Rev. B 69, 275386 (2004).
[9] F. Grossmann, T. Dittrich, P. Jung, and P. Hänggi, Phys. Rev. Lett. 67, 516 (1991).
[10] J. M. Villas-Bôas, S. E. Ulloa, and N. Studart, Phys. Rev. B 70, 041302 (2004).
[11] A. D. Yoffe, Advances in Physics 50, 1 (2001).
[12] A. H. Rodríguez, C. R. Handy, and C. Trallero-Giner, J. Phys.: Condens. Matter 15, 8465 (2003).
[13] A. H. Rodríguez and C. Trallero-Giner, J. Appl. Phys. 95, 6192 (2004).
[14] C. E. Creffield and G. Platero, Phys. Rev. B 66, 235303 (2002).
[15] J. M. Villas-Bôas et al., Phys. Rev. B 66, 085325 (2002).
Quasi-energy ($\hbar \omega$)

Sideband intensity $\rho_p$

$e F a / \hbar \omega$