Comparison of the interaction between scalar fields as dark energy with neutrinos and put constraint on neutrino mass in these models

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Abstract. In this paper, we considered the interaction neutrino with three scalar fields (phantom, quintessence and quintom model) and put constraint on total mass of neutrinos and comparison the interaction constant $\beta$ in these models. The data where used in this paper are: supernova type Ia (pantheon catalog), CMB and BAO data. For each model, we first investigate the results obtained from Pantheon data, then survey the CMB and BAO data, and finally, the total data from these catalogs. It seems that using a combination of data produces more favorable results. For combination data, we find that the total mass of neutrino $\sum m_\nu < 0.121\,eV$ (95\% Confidence Level (C.L.) for quintom model and $\sum m_\nu < 0.19\,eV$ (95\% Confidence Level (C.L.) for phantom model and $\sum m_\nu < 0.124\,eV$ (95\% Confidence Level (C.L.) for quintessence model. These results are in good agreement with the results of Planck 2018 where the limit of the total neutrino mass is $\sum m_\nu < 0.12\,eV$ (95\% C.L., TT,TE,EE+lowE+lensing+BAO).
1 Introduction

Observational evidence shows that the universe is currently experiencing an accelerated phase due to the unknown component of "dark energy". [1]- [2]- [3]- [4]. Recently accelerated expansion of the universe has been criticized by some but all indications [5] suggests that expansion. If we want to explain this issue by general relativity in four dimensions, we have two ways to describe this recent cosmic expansion: one of them is to modify the gravitational part of Einstein’s equations [6]- [7], and the other is to modify the universe’s contents. The universe is represented by the introduction of the dark energy component with negative pressure. The simplest candidate for the cosmological constant dark energy is the equation of state parameter \( \omega = -1 \), which fits well with the observational evidence. Unfortunately, the cosmological constant has several serious problems, such as the need for fine-tuning and lack of dynamics. [8]- [9]- [10]. If the cosmological constant is the reason for the cosmic current acceleration, we must find a mechanism that can obtain a very small amount of it that is consistent with the observational evidence. If the cosmological constant has dynamics like a field, it can be gradually reduced. There are other candidates for dark energy, such as scalar fields. A canonical scalar field is called the quintessence [11], (a phantom field is a scalar field with negative kinetic energy) [12], Tachyon field is derived from string theory [13]- [14], a scalar field with generalized kinetic energy called K-Essence field [15]- [16] and Chaplygn gas [17] are the types of scalar fields that can be named. The quintessence and phantom fields in a combined model are another candidate for dark energy, called the quintom model [18]- [19]- [20]. As we know, in the quintessence model of dark energy, the equation of state parameter always remains greater than -1. Also in the phantom model, the equation of state parameter is always less than -1. An important feature of the Quintom model is that in this model the equation of state parameter can cross the phantom boundary. Because of this, the Quintom model seems to be an interesting candidate for dark energy. Discovering the rapid expansion of the universe is also a major challenge for particle physics. For the first time in the observation of neutrino flavor fluctuations, the mass of neutrinos showed that at least two states of neutrinos have mass. While laboratory attempts at particle physics to measure the absolute mass of neutrinos have always faced great challenges, cosmological observations are
more prone to measuring the absolute mass of neutrinos. Because mass neutrinos can play an important role in cosmic microwave anisotropy (CMB) and large-scale structure (LSS) at different periods of cosmic evolution, some recent studies using cosmic observations have attempted to limit the total Neutrino mass as well as the effective number of degrees of relativity have freedom ($N_{eff}$). Also, the cosmological implications of the interaction of dark energy and dark matter have been extensively studied, however, we are interested in highlighting the role of neutrinos. In this work, we implement a cosmological model proposed by observations to limit the total neutrino mass. [21], [22] Scalar field plays the role of dark energy and the accelerating expansion of the universe to justify them. Using the Pantheon, CMB and BAO data, we put constraint on the ($\sum m_\nu, N_{eff}, h, \lambda$). In the next section we will briefly discuss Friedmann’s modified equations of neutrino-dark energy.

2 Data

All observational data where used in this paper are the type Ia supernovae (SN) observation (Pantheon compilation), CMB and BAO data.

2.1 Pantheon data

The data used in this paper are Type Ia supernova, for a total of 1,048 supernova (pantheon catalogue) data. The constant luminosity of supernovae is a good tool for studying some cosmic phenomena. Therefore, they are usually used as standard candles. By examining their behavior, we can talk about the expansion of the universe as well as the speed of dark flow.

2.2 CMB data

Cosmic background radiation anisotropies are caused by a variety of factors. Careful examination of these anisotropies has become an important tool for testing cosmological theories. The current fluctuations of cosmic microwave background radiation are the result of the evolution of the early fluctuations of this radiation under various physical effects. The main point in examining the fractures is that these fractures are due to the initial conditions of the catheters; Therefore, useful observations in the study of cosmic microwave background radiation anisotropies are their only statistical features and not the fluctuations themselves. Since the temperature fluctuations of cosmic background radiation are a function of fences on the sphere, the best way to study them is to expand them in terms of spherical coordinates.

2.3 BAO data

In cosmology, baryon acoustic oscillations (BAOs) are fluctuations in the density of visible baryon matter (normal matter) in the universe caused by sound density waves in the early plasma of the early universe. Just as supernovae provide a "standard candle" for astronomical observations, BAO material clustering provides a "standard ruler" for the length scale in cosmology. They can pass before the plasma cools to a point that turns into neutral atoms (recombination period), which stops the plasma density waves from propagating. They have been "frozen" in place. The length of this standard ruler (approximately 490 million light-years in the universe today) can be measured by observing the large-scale structure of matter using astronomical surveys. Have the nature of dark energy (which causes the world to expand rapidly).
3 The Models

3.1 Phantom model

In this section, we study all three scalar fields separately. We start with the phantom dark energy. Evidence from observational data suggests that the universe is currently in a narrow band near parameter $\omega = -1$ that may be well below this value (which lie in the so-called phantom regime), an area called the Phantom Age. Phantom dark energy is the non-canonical scalar field which is very similar to canonical scalar fields (such as quintessence) except that it has negative kinetic energy. The Lagrangian of this model is:

$$L_\sigma = +\frac{1}{2}\partial\sigma^2 - V_\sigma$$  \hspace{1cm} (3.1)

where $V_\sigma = V_0 \exp^{-\lambda\sigma}$ is a self-interacting potential. Assuming a flat space in FLRW metric filled with baryons, radiation, dark matter, dark energy and neutrinos. We start with the Friedmann equations as:

$$3H^2 = \kappa^2 \left[ \rho_m - \frac{1}{2}\dot{\sigma}^2 + V(\sigma) \right] + \kappa^2 \rho_\nu$$  \hspace{1cm} (3.2)

where $\rho_m$ is matter density and $\rho_\nu$ is density of neutrino.

$$2\dot{H} + 3H^2 = \kappa^2 \left[ -\omega \rho_m + \frac{1}{2}\dot{\sigma}^2 + V(\sigma) \right] + \kappa^2 \omega_\nu \rho_\nu$$  \hspace{1cm} (3.3)

The evolution equations for their energy densities are:

$$\dot{\rho}_\sigma + 3H\rho_\sigma(1 + \omega_\sigma) = -\beta \rho_\nu(1 - 3\omega_\nu)\dot{\sigma}$$  \hspace{1cm} (3.4)

$$\dot{\rho}_\nu + 3H\rho_\nu(1 + \omega_\nu) = \beta \rho_\nu(1 - 3\omega_\nu)\dot{\sigma}$$  \hspace{1cm} (3.5)

$$\dot{\rho}_m + 3H\rho_m = 0$$  \hspace{1cm} (3.6)

respectively, where $\beta$ stands for the coupling constant between neutrino and dark energy.

In standard model of particles, neutrinos have no mass and regarded as relativistic particles, but with this coupling constant, they become massive and turn into non-relativistic particles that have oscillations. Since the dark energy is modeled as a scalar field $\sigma$ its energy density and pressure are given by

$$\rho_\sigma = -\frac{1}{2}\dot{\sigma}^2 + V(\sigma), \quad p_\sigma = -\frac{1}{2}\dot{\sigma}^2 - V(\sigma)$$  \hspace{1cm} (3.7)

where $V(\sigma)$ denotes the potential of the scalar field. In this paper, we consider the interactions between dark energy and neutrino as the following evolution equation in addition, from the resulting of equations of the scalar field:

$$\ddot{\sigma} = -\lambda V(\sigma) - \frac{3}{2}H\dot{\sigma}(1 + \omega_\sigma) - \frac{3HV}{\dot{\sigma}}(1 + \omega_\phi) + \beta \rho_\nu(1 - 3\omega_\nu)$$  \hspace{1cm} (3.8)
where as before \( \dot{a} \) means differentiation with respect to the coordinate time \( t \). The EoS of the scalar field is now given by:

\[
\omega = \frac{1}{2} g^2 + \frac{V(\sigma)}{\frac{1}{2} g^2 - V(\sigma)}
\]  

(3.9)

The above equations are a nonlinear set of second-order differential equations that can only be solved analytically for certain cases. Therefore, they can be examined numerically. To simplify the equations, we can introduce a number of new variables. Turned the second-order differential equations into a set of first-order equations. There are several reasons for this, including: 1-systems of the first order for the numerical solution is very simple and convenient. On the other hand, it allows us to study the behavior of the system in phase space. 2. In the numerical solution of first-order equations, unlike high-order equations, which require more than one condition for each equation, only one initial condition is required. 3- Most importantly, the first-order dynamics can be described on the phase space and it is possible to check the stability of the system.

If we considered the standard model, the right side of the equation would be zero, and the neutrinos are effectively uncoupled. As a result of the coupling constant \( Q \) is important. We consider an exponential potential \( V(\sigma) = V_0 e^{-\lambda \sigma} \), where \( \lambda \) is a dimensionless parameter that determines the slope of the potential. The motivation for choosing these functions have been investigated in [24]. Also we define \( \omega = \frac{P}{\rho} \). In order to simplify the field equations, we introduce following new variables,

\[
\xi_1 = \frac{\kappa^2 \rho_m}{3H^2}, \quad \xi_2 = -\frac{\kappa \dot{\sigma}}{\sqrt{6}H}, \quad \xi_3 = \frac{\kappa^2 V(\sigma)}{3H^2}, \quad \xi_4 = \frac{\rho_\nu}{3H^2}
\]  

(3.10)

In term of new variable the Friedmann equation (3.34) puts a constraint on new variables as

\[
\zeta_3 = 1 - \zeta_1 + \zeta_2^2 - \zeta_4
\]  

(3.11)

Therefore, the equations are simplified as follows:

\[
\begin{align*}
\frac{d\xi_1}{dN} &= -3\xi_1 - 2\xi_1 \frac{\dot{H}}{H^2} \\
\frac{d\xi_2}{dN} &= -\frac{3\lambda}{\sqrt{6}\kappa} \xi_3 + \frac{3}{2}(1 + \omega_\sigma)\xi_2 - \frac{3}{2}(1 + \omega_\sigma)\frac{\xi_3}{\xi_2} \\
&\quad - \frac{3\kappa Q}{\sqrt{6}}(1 - 3\omega_\nu)\xi_4 + \xi_2 \frac{\dot{H}}{H^2} \\
\frac{d\xi_3}{dN} &= -\kappa \lambda \sqrt{6}\xi_2 \xi_3 - 2\xi_3 \frac{\dot{H}}{H^2} \\
\frac{d\xi_4}{dN} &= -3(1 + \omega_\nu)\xi_4 - \frac{Q\sqrt{6}(1 - 3\omega_\nu)}{\kappa} \xi_2 \xi_4 \\
&\quad - 2\xi_4 \frac{\dot{H}}{H^2}
\end{align*}
\]  

(3.12)

Where, \( N = \ln a \). In term of the new dynamical variable, we also have,

\[
\frac{\dot{H}}{H^2} = \frac{3}{2}\left\{ 1 - \omega_m \chi_1 - \chi_2 + \chi_3 + \omega_\nu \chi_4 \right\}
\]  

(3.13)
we analyze the quintessence model. We start from Friedmann equation and acceleration equation:

\[ 3H^2 = \kappa^2 \left[ \rho_m + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] + \kappa^2 \rho_{\nu} \]  

and

\[ 2\dot{H} + 3H^2 = -\kappa^2 \left[ \omega_{\rho_m} + \frac{1}{2} \dot{\phi}^2 - V(\phi) \right] + \kappa^2 \omega_{\nu} \rho_{\nu} \]  

where \( \rho_{\nu} \) is the energy density of photons. This value of \( N_{\text{eff}} \) indicates that if it is higher than 3.046, there is a dark radiation that is other than three generations of neutrinos.

Also, there is a relation between \( N_{\text{eff}} \) and terms of the matter density, \( \Omega_m h^2 \), and the redshift of matter-radiation equality \( z_{\text{eq}} \) as

\[ N_{\text{eff}} = 3.04 + 7.44 \left( \frac{\Omega_m h^2}{0.1308} \frac{3139}{1 + z_{\text{eq}}} - 1 \right) \]  

In the early universe, there is dark radiation composed of the energy densities of photons and neutrinos. The total radiation energy density in the Universe is given by

\[ \rho_r = \rho_\gamma \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^4 N_{\text{eff}} \right] \]  

where \( \rho_\gamma \) is the energy density of photons. This value of \( N_{\text{eff}} \) indicates that if it is higher than 3.046, there is a dark radiation that is other than three generations of neutrinos.

Also, there is a relation between \( N_{\text{eff}} \) and terms of the matter density, \( \Omega_m h^2 \), and the redshift of matter-radiation equality \( z_{\text{eq}} \) as

\[ N_{\text{eff}} = 3.04 + 7.44 \left( \frac{\Omega_m h^2}{0.1308} \frac{3139}{1 + z_{\text{eq}}} - 1 \right) \]  

In the following, we put constrain on total mass of neutrino by analysing of Pantheon and CMB and BA0 data. We first survey the results of the Pantheon data and then investigate the results of CMB + BA0, and finally survey the total results of these two parts. The results for the cosmological parameters are shown in Table I. Fig. I also show the parametric space at 68 \% CL and 95\%CL for some selected parameters for the different observational data sets.

From the analysis of the Pantheon data alone, for phantom we find that \( \sum m_\nu < 0.274eV \) 95\%CL and using CMB+BAO we find \( \sum m_\nu < 0.328eV \) 95\%CL and for combination of full data( Pantheon+CMB+BAO) we find \( \sum m_\nu < 0.118eV \) 95\%CL

### 3.2 Quininessence model

Quintessence is described by a canonical scalar field that is minimally coupled to gravity. Compared to other models of scalar fields such as phantoms, the simplest scenario is the scalar field. A variable calm field along a potential can accelerate the universe. This mechanism is similar to slow inflation in the early universe, but the difference is that non-relative matter (dark matter and baryon) cannot be ignored to properly discuss dark energy dynamics. Now, we analyze the quintessence model. We start from Friedmann and acceleration equation:

\[ 3H^2 = \kappa^2 \left[ \rho_m + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] + \kappa^2 \rho_{\nu} \]  

and

\[ 2\dot{H} + 3H^2 = -\kappa^2 \left[ \omega_{\rho_m} + \frac{1}{2} \dot{\phi}^2 - V(\phi) \right] + \kappa^2 \omega_{\nu} \rho_{\nu} \]  

Table 1. Observational constraints at 95\% on main and derived parameters of the \( \sum m_\nu \) scenario. The parameter \( H_0 \) is in the units of km/sec/Mpc, whereas \( \sum m_\nu \) reported in the 95\% CL, is in the units of eV.

| Dataset         | \( \Omega_b h^2 \) | \( \Omega_c h^2 \) | \( H_0 \) | \( \Omega_m \) | \( \Omega_{\text{m}h^2} \) | \( \sum m_\nu \) | \( \beta \) | \( \lambda \) | \( N_{\text{eff}} \) |
|-----------------|---------------------|---------------------|-----------|--------------|-----------------|---------------|--------|----------|----------------|
| Pantheon        | 0.0222^{+0.0053}_{-0.0051} | 0.1211^{+0.0061}_{-0.0057} | 77.05^{+0.74}_{-0.72} | 0.306^{+0.06}_{-0.062} | 0.14^{+0.01}_{-0.013} | < 0.29 | 0.0006^{+0.0006}_{-0.0006} | 0.0049^{+0.0021}_{-0.0021} | < 0.09 - 0.07 |
| CMB+BAO         | 0.0221^{+0.0062}_{-0.0064} | 0.1911^{+0.0049}_{-0.0051} | 77.03^{+0.72}_{-0.72} | 0.303^{+0.06}_{-0.062} | 0.1278^{+0.004}_{-0.0042} | < 0.33 | 0.00074^{+0.0006}_{-0.0006} | 0.006^{+0.0004}_{-0.0004} | < 0.11 - 0.11 |
| CMB+BAO+Pantheon | 0.0224^{+0.0049}_{-0.0051} | 0.1193^{+0.0031}_{-0.0033} | 68.01^{+0.7}_{-0.7} | 0.302^{+0.06}_{-0.062} | 0.1267^{+0.004}_{-0.0042} | < 0.19 | 0.00156^{+0.0005}_{-0.0005} | 8.1^{+0.6}_{-0.6} | 2.97^{+0.5}_{-0.5} |
The evolution equations for their energy densities are:

\[ \dot{\rho}_\phi + 3H\rho_\phi(1 + \omega_\phi) = -\beta\rho_\nu(1 - 3\omega_\nu)\dot{\phi} \]  

(3.18)

Figure 1: The constraints at the 95% CL two-dimensional contours for \( \sum m_\nu \) for phantom model
while the Klein-Gordon equation is:

\[ \ddot{\phi} = \lambda V(\phi) - \frac{3}{2} H \dot{\phi}(1 + \omega_\phi) - \frac{3HV}{\phi}(1 + \omega_\phi) - \beta \rho_\nu(1 - 3\omega_\nu) \]  

(3.19)

We can define the energy density and pressure of the scalar field as follows:

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \]  

(3.20)

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]  

(3.21)

The resulting equation of state is as follows:

\[ \omega_\phi = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \]  

(3.22)

where \( \omega_\phi \) is a dynamically evolving parameter which can take values in the range \([-1, 1]\).

we rewrite the cosmological equations (3.25), (3.26) and (3.27) into an autonomous system of equations.

\[ \chi_1 = \frac{\kappa^2 \rho_m}{3H^2} \quad \chi_2 = \frac{\kappa \dot{\phi}}{\sqrt{6}H} \quad \chi_3 = \frac{\kappa^2 V(\phi)}{3H^2} \quad \chi_4 = \frac{\kappa^2 \rho_\nu}{3H^2} \]  

(3.23)

we can derive the following dynamical system:

\[ \frac{d\chi_1}{dN} = -3\chi_1 - 2\chi_1 \frac{\dot{H}}{H^2} \]

\[ \frac{d\chi_2}{dN} = \frac{3\lambda}{\sqrt{6}\kappa}\chi_3 - \frac{3}{2}(1 + \omega_\phi)\chi_2 - \frac{3}{2}(1 + \omega_\nu)\frac{\chi_3}{\chi_2} - \frac{3\kappa\beta}{\sqrt{6}}(1 - 3\omega_\nu)\chi_4 - \chi_2 \frac{\dot{H}}{H^2} \]

(3.24)

\[ \frac{d\chi_3}{dN} = -\kappa \lambda \sqrt{6}\chi_2 \chi_3 - 2\chi_3 \frac{\dot{H}}{H^2} \]

\[ \frac{d\chi_4}{dN} = -3(1 + \omega_\nu)\chi_4 + \frac{\beta \sqrt{6}(1 - 3\omega_\nu)}{\kappa} \chi_2 \chi_4 - 2\chi_4 \frac{\dot{H}}{H^2} \]

also

\[ \frac{\dot{H}}{H^2} = \frac{3}{2} \left\{ -1 - \omega_m \chi_1 - \chi_3^2 + \chi_3 + \omega_\nu \chi_4 \right\} \]  

(3.25)

Using the relationships mentioned to estimate the mass of neutrinos and \( N_{eff} \) in the previous section, we have: From the analysis of the Pantheon data alone, for quintessence we find that \( \Sigma m_\nu < 0.29\,\text{eV} \) 95\% CL and using CMB+BAO we find \( \Sigma m_\nu < 0.333\,\text{eV} \) 95\% CL and for combination of full data( Pantheon+CMB+BAO) we find \( \Sigma m_\nu < 0.124\,\text{eV} \) 95\% CL
Figure 2: The constraints at the 95% CL two-dimensional contours for $\Sigma m_\nu$ in Quintessence model.

Table 2: Observational constraints at 95% on main and derived parameters of the $\Sigma m_\nu$ scenario. The parameter $H_0$ is in the units of km/sec/Mpc, whereas $\Sigma m_\nu$ reported in the 95% CL, is in the units of eV.

| Dataset          | $\Omega_b h^2$ | $\Omega_c h^2$ | $H_0$  | $\Omega_m$ | $\Omega_{m h^2}$ | $\Sigma m_\nu$ | $\beta$ | $\lambda$ | $N_{eff}$ |
|------------------|----------------|----------------|--------|-------------|------------------|----------------|---------|----------|-----------|
| Pantheon         | $0.0235^{+0.0003}_{-0.0003}$ | $0.1181^{+0.0001}_{-0.0001}$ | $68.55^{+0.3}_{-0.2}$ | $0.302^{+0.002}_{-0.002}$ | $0.13^{+0.0015}_{-0.0015}$ | $<0.29$ | $-0.00094^{+0.00003}_{-0.00003}$ | $3.09^{+0.08}_{-0.09}$ |
| CMB+BAO          | $0.0239^{+0.0003}_{-0.0002}$ | $0.1169^{+0.0001}_{-0.0001}$ | $68.9^{+0.3}_{-0.2}$ | $0.291^{+0.002}_{-0.002}$ | $0.1278^{+0.004}_{-0.004}$ | $<0.33$ | $0.00054^{+0.00009}_{-0.00009}$ | $0.0003^{+0.0003}_{-0.0003}$ | $3.11^{+0.14}_{-0.11}$ |
| CMB+BAO+Pantheon | $0.0223^{+0.0003}_{-0.0002}$ | $0.1173^{+0.0001}_{-0.0001}$ | $68.7^{+0.2}_{-0.2}$ | $0.301^{+0.0015}_{-0.0015}$ | $0.1267^{+0.004}_{-0.004}$ | $<0.128$ | $0.0185^{+0.0003}_{-0.0003}$ | $3.1^{+0.03}_{-0.03}$ | $2.97^{+0.3}_{-0.3}$ |

3.3 Quintom model

In the previous section we found that the equation of state in the quintessence model must satisfy $\omega_{de} \geq -1$, we also saw in the phantom model that the EoS of the phantom scalar field is limited to $\omega_{de} < -1$. There seems to be no way to cross the phantom barrier (i.e., the cosmological constant $\omega_{de} = -1$) using a scalar field. To cross this barrier, the Quintum
model, which allows such a passage. This dark energy scenario causes the EoS to be larger than -1 in the past and less than -1, which satisfies current observations. The simplest model is represented by a quantum Lagrangian consisting of two scalar fields, a canonical field $\phi$ (quintessence) and a phantom field $\sigma$:

$$L_{\text{quintom}} = -\frac{1}{2} \partial \phi^2 + \frac{1}{2} \partial \sigma^2 - V(\phi, \sigma)$$ (3.26)

where $V(\phi, \sigma)$ is a general potential for both the scalar fields. The kinetic energy sign is positive for the quintessence model and negative for the phantom model. The cosmological equations are given by the Friedmann and acceleration equations

$$3H^2 = \kappa^2 \left[ \rho_m + \frac{1}{2} \dot{\phi}^2 + V(\sigma, \phi) - \frac{1}{2} \dot{\sigma}^2 \right] + \kappa^2 \rho_\nu$$ (3.27)

$$2\dot{H} + 3H^2 = -\kappa^2 \left[ \omega \rho_m + \frac{1}{2} \dot{\phi}^2 + V(\sigma, \phi) - \frac{1}{2} \dot{\sigma}^2 \right] + \kappa^2 \omega_\nu \rho_\nu$$ (3.28)

and by the Klein-Gordon equations

$$\ddot{\phi} = \lambda_\phi V - \frac{3}{2} H \dot{\phi} (1 + \omega_\phi) - \frac{3HV}{\phi} (1 + \omega_\phi) - \beta \rho_\nu (1 - 3\omega_\nu)$$ (3.29)

$$\ddot{\sigma} = -\lambda_\sigma V - \frac{3}{2} H \dot{\sigma} (1 + \omega_\sigma) + \frac{3HV}{\sigma} (1 + \omega_\sigma) + \beta \rho_\nu (1 - 3\omega_\nu)$$ (3.30)

To recreate them in a dynamic system, we define the EN variables

$$\eta_1 = \frac{\kappa^2 \rho_m}{3H^2} \quad \eta_2 = \frac{\kappa \dot{\phi}}{\sqrt{6H}} \quad \eta_3 = -\frac{\kappa \dot{\sigma}}{\sqrt{6H}}$$

$$\eta_4 = \frac{\kappa^2 V(\sigma, \phi)}{3H^2} \quad \eta_5 = \frac{\rho_\nu}{3H^2}$$ (3.31)

To find the dynamics of the system governing cosmic evolution, we follow the same method

---
we used for the previous two models.

\[
\begin{align*}
\frac{d\eta_1}{dN} &= -3\eta_1 - 2\eta_1 \frac{\dot{H}}{H^2} \\
\frac{d\eta_2}{dN} &= \frac{3\lambda_0}{\sqrt{6}\kappa} \eta_4 - \frac{3}{2}(1 + \omega_\phi)\eta_2 - \frac{3}{2}(1 + \omega_\phi) \frac{\eta_4}{\eta_2} \\
&\quad - \frac{3\kappa\beta}{\sqrt{6}} (1 - 3\omega_\nu)\eta_5 - \eta_2 \frac{\dot{H}}{H^2} \\
\frac{d\eta_3}{dN} &= \frac{3\lambda_\sigma}{\sqrt{6}\kappa} \eta_4 - \frac{3}{2}(1 + \omega_\sigma)\eta_3 + \frac{3}{2}(1 + \omega_\phi) \frac{\eta_4}{\eta_3} \\
&\quad - \frac{3\kappa\beta}{\sqrt{6}} (1 - 3\omega_\nu)\eta_5 - \eta_3 \frac{\dot{H}}{H^2} \\
\frac{d\eta_4}{dN} &= -\frac{\lambda_\phi\sqrt{6}}{\kappa} \eta_2\eta_4 + \frac{\lambda_\sigma\sqrt{6}}{\kappa} \eta_3\eta_4 - 2\eta_4 \frac{\dot{H}}{H^2} \\
\frac{d\eta_5}{dN} &= -3\sqrt{6}(1 + \omega_\nu) - \frac{\beta\sqrt{6}}{\kappa} (1 - 3\omega_\nu)\eta_2\eta_5 \\
&\quad + \frac{\beta\sqrt{6}}{\kappa} (1 - 3\omega_\nu)\eta_2\eta_5 - 2\eta_5 \frac{\dot{H}}{H^2}
\end{align*}
\] (3.32)

and the Friedmann constraint

\[
\eta_4 = 1 - \eta_1 - \eta_2^2 + \eta_3^2 - \eta_5
\] (3.33)

holds. The potential used in this model is an interaction potential which is introduced as follows:

\[
V(\sigma, \phi) = V_0 \exp^{-\lambda_\phi\kappa\phi - \lambda_\sigma\kappa\sigma}
\] (3.34)

where in \(\lambda_\phi\) and \(\lambda_\sigma\) are constant. We constrain total mass of neutrino in Quintom model as follows:

for Pantheon data, we find that \(\sum m_\nu < 0.227\,\text{eV} \) 95% CL and using CMB+BAO we find \(\sum m_\nu < 0.321\,\text{eV} \) 95% CL and for combination of full data (Pantheon+CMB+BAO) we find \(\sum m_\nu < 0.121\,\text{eV} \) 95% CL.

This result is close to results of [25] the case TT, TE, EE + lowE[CamSpec] with \(\sum m_\nu < 0.38\,\text{eV} \) 95% CL and very close to results of [25] TT, TE, EE + lowE + lensing + BAO. with \(\sum m_\nu < 0.12\,\text{eV} \) at 95% CL and case TT, TE, EE + lowE + BAO with \(\sum m_\nu < 0.13\,\text{eV} \) 95% CL. In three models.

Other parameters are shown in table III.

4 Conclusion

In this paper, we used phantom, quintessence and quintom as dark energy and put constrain on neutrino mass by interaction between dark energy with neutrino. We find that the total mass of neutrino \(\sum m_\nu < 0.121\,\text{eV} \) (95% Confidence Level (C.L.) for quintom model and \(\sum m_\nu < 0.19\,\text{eV} \) (95% Confidence Level (C.L.) for phantom model and \(\sum m_\nu < 0.124\,\text{eV} \)
Figure 3 : The constraints at the 95% CL two-dimensional contours for $\sum m_\nu$ in Quintum model.

Table 3. : Observational constraints at 95% on main and derived parameters of the $m_\nu$ scenario. The parameter $H_0$ is in the units of km/sec/Mpc, whereas $\sum m_\nu$ reported in the 95% CL, is in the units of eV.

| Dataset  | $\Omega_b h^2$ | $\Omega_c h^2$ | $H_0$ | $\Omega_m$ | $\Omega_m h^2$ | $\sum m_\nu$ | $\beta$ | $\lambda_\sigma$ | $\lambda_9$ | $N_{eff}$ |
|----------|----------------|----------------|-------|------------|----------------|--------------|---------|---------------|------------|----------|
| Pantheon | 0.0221$^{+0.0009}_{-0.0008}$ | 0.1184$^{+0.0066}_{-0.0056}$ | 68.60$^{+0.00}_{-0.01}$ | 0.34$^{+0.00}_{-0.00}$ | 0.136$^{+0.0033}_{-0.0031}$ | < 0.227 | 0.00067 | < 0.0001 | < 25.7$^{+0.0}_{-0.0}$ | 0.0019 | 0.000009 | 4.14$^{+0.04}_{-0.01}$ |
| CMB+BAO  | 0.0237$^{+0.0007}_{-0.0006}$ | 0.1189$^{+0.0069}_{-0.0059}$ | 68.25$^{+0.03}_{-0.03}$ | 0.29$^{+0.00}_{-0.00}$ | 0.1128$^{+0.0044}_{-0.0043}$ | < 0.321 | 0.00077 | < 0.0002 | < 17.6$^{+0.0}_{-0.0}$ | 0.00021 | 0.000012 | 3.18$^{+0.01}_{-0.01}$ |
| CMB+BAO+Pantheon | 0.0233$^{+0.0007}_{-0.0006}$ | 0.1108$^{+0.0060}_{-0.0052}$ | 68.7$^{+0.2}_{-0.2}$ | 0.29$^{+0.01}_{-0.01}$ | 0.1267$^{+0.0062}_{-0.0062}$ | < 0.1210 | 0.00165 | < 0.0003 | < 9.5$^{+0.0}_{-0.0}$ | 0.00010 | 0.000002 | 3.97$^{+0.01}_{-0.01}$ |

Table 4. Results of Planck 2018 [25]: Constraint at 95% CL , using different observational data, whereas $\sum m_\nu$ reported in the 95% CL, is in the units of eV.

| Dataset  | $\Omega_b h^2$ | $\Omega_c h^2$ | $H_0$ | $\Omega_m$ | $\Omega_m h^2$ | $\sum m_\nu$ | $\beta$ | $\lambda_\sigma$ | $\lambda_9$ | $N_{eff}$ |
|----------|----------------|----------------|-------|------------|----------------|--------------|---------|---------------|------------|----------|
| TT-LowE  | 0.02212$^{+0.00022}_{-0.00022}$ | 0.1206$^{+0.0008}_{-0.0008}$ | 66.88$^{+0.02}_{-0.02}$ | 0.321$^{+0.003}_{-0.003}$ | 0.143$^{+0.004}_{-0.004}$ | 3411$^{+2}_{-2}$ | < 0.537 | 0.00002 | 0.00002 | 0.00002 |
| TE-LowE  | 0.02249$^{+0.00022}_{-0.00022}$ | 0.1177$^{+0.0008}_{-0.0008}$ | 68.44$^{+0.02}_{-0.02}$ | 0.301$^{+0.003}_{-0.003}$ | 0.140$^{+0.004}_{-0.004}$ | 3378$^{+2}_{-2}$ | < 0.540 | 0.00002 | 0.00002 | 0.00002 |
| TT,TE,EE-LowE | 0.02236$^{+0.00019}_{-0.00019}$ | 0.1207$^{+0.0008}_{-0.0008}$ | 67.6$^{+0.02}_{-0.02}$ | 0.316$^{+0.003}_{-0.003}$ | 0.143$^{+0.004}_{-0.004}$ | 3340$^{+1}_{-1}$ | < 0.527 | 0.00002 | 0.00002 | 0.00002 |
| TT,TE,EE-LowE+Lensing | 0.02237$^{+0.00019}_{-0.00019}$ | 0.1209$^{+0.0008}_{-0.0008}$ | 67.6$^{+0.02}_{-0.02}$ | 0.315$^{+0.003}_{-0.003}$ | 0.143$^{+0.004}_{-0.004}$ | 3340$^{+1}_{-1}$ | < 0.524 | 0.00002 | 0.00002 | 0.00002 |
| TT,TE,EE-LowE+Lensing+BAO | 0.02242$^{+0.00019}_{-0.00019}$ | 0.1195$^{+0.0008}_{-0.0008}$ | 67.6$^{+0.02}_{-0.02}$ | 0.311$^{+0.003}_{-0.003}$ | 0.142$^{+0.004}_{-0.004}$ | 3382$^{+1}_{-1}$ | < 0.520 | 0.00002 | 0.00002 | 0.00002 |

(95% Confidence Level (C.L.) for quintessence model. These results are in good agreement with the results of Planck 2018 where the limit of the total neutrino mass is $\sum m_\nu < 0.12eV$ (95% C.L.). TT,TE,EE+lowE+lensing+BAO) Also, the interaction constant, $\beta$, for three models are investigated.

- In phantom model, the value of $\beta$ for combination data (Pantheon + CMB + BAO) is 0.00156. This value indicate that the interaction between neutrino and dark energy is very small.

- In quintessence model, $\beta$ is 0.0185 which is indicate that the dark energy neutrino interaction is grater than the phantom model.

- $\beta$ value in the quintum model is about 0.000186, which shows that in the world of quintum, the interaction between neutrino and dark energy almost are the same with phantom model. Also, the results obtained in this paper indicate that the value of the equation of state in the quintum model is $\omega_\sigma = -1.04$ , $\omega_\phi = -1$. Comparing the results obtained for the equation
Figure 4: The constraints at the 68% CL two-dimensional contours for $N_{eff}$ in three models.
Figure 5: Comparison of $\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_m$ obtained values in Phantom model.
Figure 5: Comparison of $\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_m$ obtained values in Quintessence model.
Figure 5: Comparison of $\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_m$ obtained values in Quintom model.
of state of these three models, it can be concluded that for the equation of state with a value of -1 or less (Quintum and Phantom models) the amount of dark energy neutrino interaction is less than when the state equation value is greater than -1. (Quintessence model).

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