Conversion from Free-Form Surface to Ruled Surface of Centrifugal Compressor Impellers

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Abstract: At present, there are several conversion methods exist, which are based on cutter fixing at a point on the blade surface to swing around the blade surface, searching for a position where the error between the cutter and the free-form surface is the smallest. They rarely take into account the benefit from searching cutter position along the blade's tangential direction, and ignore the possible optimal cutter position from such search. In this paper, we propose a new cutter search method that allows this tangential swing in addition to meridional plane swing of the cutter in searching for the optimal position of the cutter. The mathematics of the method is described, and two different centrifugal compressor impellers with free-form blade surfaces are employed as test cases. The results show that this method can well convert the free-form surfaces of the impeller blades into ruled surfaces.

1. Introduction
Turbocharger has been widely used in transportation and other fields. For one of its core components centrifugal compressor impeller, the requirements of mechanical performance, noise, and design-to-production time are more and more stringent. The impeller is made mainly by casting and machining, but casting technology is not only of low processing efficiency, but also produces impellers of poor quality, and it is struggling to meet the requirements of modern turbocharging technology. So, more and more impellers are now made by machining [1]. The commonly used machining methods include five-axis point-milling and five-axis flank milling. The point milling technology is suitable for machining impellers with free-form surfaces, as it uses a cylindrical ball head milling cutter to remove materials from blade surfaces. The cutter typically starts from the shroud of the blade, and moves step by step toward the hub after finishing the previous step, until the entire surface of the blade is finished [2], as shown in Figure 1a. This method preserves the geometric accuracy of the impeller and associated benefits. However, the processing time is long which leads to the high manufacturing cost and inhibits its wider application.

The flank milling technology employs a cutter with a long conical cutting edge to sweep on blade surfaces, as shown in Figure 1b. [3,4]. It is used to process centrifugal compressor impellers composed of ruled blade surfaces or surfaces formed with straight lines [5]. In the final sweeping, the milling cutter sweeps one blade surface once to complete this side of the blade and the cutter then moves...
on to the other blade surface to complete the milling of it similarly. Compared with the multiple cutter passes required by point milling technology, flank milling technology greatly reduces the number of the passes hence processing time and cost, and it also has better blade surface finish than point milling technology.

In its design process, impeller blade shape is controlled span-wisely as much as possible to improve the aerodynamic and mechanical performances, and the resulting blade shape is often complex freeform surfaces [12]. These impellers can only be made by point milling or casting process. Commercial CAM software have provided flank milling option for machining ruled surface, but it is not suitable for free-form surface impellers. The simple flank milling method provided usually leads to reduction of compressor aerodynamic efficiency because the ruled surfaces obtained may depart considerably from the optimised, free-form geometry. Finding a method to convert an existing free-form surface of impeller into a ruled surface with minimum geometric deviation for CAM software is therefore highly desirable, and offers a solution to this problem [6,7].

Chen and Pottmann [8] made a pioneering research on transformation and processing of free-form surfaces. They used some discrete straight line segments to fit and approximate free-form surfaces, and then used a surface in the form of b-spline tensor product to approximate these segments, and finally formed a surface with straight line elements.

Wang and Elber [9] proposed a method of segmenting straight surface approximation for flank milling, to solve the problem of approximating the free-form surfaces with drastic variation of curvature by a single straight-line surface.

By applying Klein mapping, Zhou et al [10] converted a ruled surface in Euclidean space into a curve on a Dual Unit Sphere (DUS). The kinematic ruled surface approximation algorithm is set up based on the novel definition of a spline on the DUS and the corresponding spline interpolation algorithm. The algorithm was applied to approximate a free-form blade surface to a ruled surface to reduce the manufacturing cost.

Tsay et al [11] developed a flank cutting technology for centrifugal compressor impellers by using the B-spline curve interpolation, ruled surface construction, and coordinate transformations.

Bi et al [12] proposed a new flank milling conversion control method. When converting a surface into a ruled surface, important manufacturing constraints such as the smoothness of cutter motion and the ductility of ruled surface are considered. This method has fast milling speed and good approximation effect. The design and machining experiments of turbocharger impeller show the effectiveness of the method.

In their cutter search mode in current methods, the cutter moves along the streamline direction of the blade to swing from side to side in the meridional plane searching for the optimal cutter position, where the error between the cutter and the free-form blade surface is the smallest. One such example is given in Figure 2. However, the cutter can also swing in the tangential direction, and this may bring some benefit for the optimal cutter position. This paper presents a new cutter search method, which allows the cutter to perform both meridional and tangential swings to find its best position. The paper
is organised as follows: first, the principles of the method and mathematic implementation are introduced; then two different centrifugal compressor impellers for dissimilar turbocharging applications are studied and the results shown; finally, conclusions are drawn with a further discussion of the method.

![Cutter swing search for the best position in two-dimensional plane](image)

**FIG. 2** Cutter swing search for the best position in two-dimensional plane

2. **Principles of the new conversion method**

The proposed method converts a free-form 3D blade surface into a surface formed by straight lines so that the surface can be flank milled. The principles of the method can be described in the following steps.

1) The three-dimensional data points in cartesian coordinate system that make up the free-form surface blades are exported from the blade geometry generator such as Bladegen software from Ansys Inc., and process the data into cylindric coordinate system.

\[
z = Z
\]

\[
r = (X^2 + Y^2)^{1/2}
\]

\[
\theta = \arccos(X/r)
\]

Where \((X, Y, Z)\) is the 3D coordinates of the data points of the impeller in the cartesian coordinate system, and \((z, r, q)\) is the corresponding data in the cylindric coordinate system.

2) The suction and pressure sides of the blade are exported with \(m\) quasi-streamlines equally spaced along blade span respectively, and the \(i\)th quasi-streamline, \(i = 1 \sim m\), is composed of \(n\) data points from leading edge to trailing edge. **Figure 2** shows a blade suction surface in the meridional plane. The distance between the two adjacent 3D data points on each quasi-streamline is calculated, and from it the total distance between the leading edge and the trailing edge, and the percentage distance of all the data points \(t_i\) on the curve.

3) Two quasi-streamlines are selected as the guide lines for the cutter search. They are named as the Outer guide line and Inner guide line respectively, **Figure 3**. The data point on the outer guide line is the fixed or pivoting point of the cutter rotation when the conversion is carried out, so there will be no converting error at this point. And when the cutter moves along the outer guide line to complete the conversion of entire blade surface, there will be no converting error at this line. In general, shroud or a quasi-streamline close to the shroud is selected as the outer guide line, because blade geometry in the shroud region is critical to both aero and mechanical performance of the impeller and one would wish the conversion error there as small as possible. On the other hand, the hub streamline is not selected as the inner guide line, because fillets will be added later to the hub for mechanical reason, and there is no point to demand minimum converting error now unless the fillet is already added to the blade. For this reason, a quasi-streamline above the hub is better selected as the inner guide line. The data point on the inner guide line serves as the center of the cutter rotation, **Figure 3**. Both the inner and outer guide lines are extended beyond the leading and trailing edges so that the two edges may be better
converted.

FIG. 3 Meridional plane of blade suction surface

4) The traditional flank milling cutter search method is to perform a two-dimensional swing search in an approximately parallel two-dimensional plane of the impeller surface, as shown in Figure 2. This cutter search method ignores the possibility of the optimal cutter position in the tangential direction. The proposed new search method to find the best cutter path is illustrated in Figure 4. The search starts from trailing edge, and moves toward leading edge. To enable the search at both the meridional and tangential directions, at one position C1 on the outer guide line, where the cutter is pivoted, the cutter rotates around this point in 3D fashion to form a series of concentric conics with different radius. A maximum search radius is prescribed, together with the last finished point on the inner guide line, they define the search center C2 on the inner guide line. The search for the best straight-line surface approximation is carried out at the discrete points on these concentric cones.

FIG. 4 Three-dimensional rotating search method of cutter

5) The fitting error between the m original quasi-streamlines and the cutter at each cutter position is calculated, and an averaged error for all the quasi-streamlines is computed. This average error for all the cutter positions is then compared to find the optimal position of the cutter within the search range.

6) Once the best cutter position is found for point C1, the point moves on along the outer guide line to the next position, and steps 4) and 5) are repeated until one side of blade, including half of the leading edge, is completed.

7) Above procedure is then repeated to complete the other side of the blade in the same way. Note that different inner and outer guide lines may be chosen for different sides of the blade.
3. Expression of cutter motion and fitting error calculation

3.1. Expression of cutter motion
See Figure 4, the point on the outer guide line of the cutter is the fixed point C₁, its coordinate is (C₁ₓ, C₁ᵧ, C₁ᶻ); The point C₂ on the inner guide line is the center of the cutter rotating circles, its coordinate is (C₂ₓ, C₂ᵧ, C₂ᶻ). The normal vector of the base plane of the cones formed by cutter rotation will be defined as \( \mathbf{n} = [nₓ, nᵧ, nᶻ] \). In the following expression (4), \( \mathbf{u} \) is any vector on the plane that is orthogonal to \( \mathbf{n} \), and it is defined as:

\[
\mathbf{u} = \begin{bmatrix} nᵧ \\ -nₓ \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} nₓ \\ 0 \\ -nᵧ \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0 \\ nₓ \\ nᵧ \end{bmatrix}
\]

(4)

In the following expression (5–6), \( \mathbf{v} \) is a vector product of \( \mathbf{n} \) and \( \mathbf{u} \), and is normal to both \( \mathbf{n} \) and \( \mathbf{u} \). \( \mathbf{u} \) is the unit vector of \( \mathbf{u} \), and \( \mathbf{v} \) of \( \mathbf{v} \),

\[
\mathbf{v} = \mathbf{n} \times \mathbf{u}
\]

\[
\mathbf{u} = \frac{1}{\sqrt{nₓ² + nᵧ²}} \mathbf{u}
\]

(5)

\[
\mathbf{v} = \frac{1}{\sqrt{(nₓuₓ - uₓnₓ)² + (nᵧuᵧ - uᵧnᵧ)²}}} \begin{bmatrix} nₓuₓ - uₓnₓ \\ nᵧuᵧ - uᵧnᵧ \\ nₓuᵧ - uᵧnₓ \end{bmatrix}
\]

(6)

The movement of the cutter can be expressed as:

\[
\begin{aligned}
x &= C₂ₓ + r(\mathbf{u}_x \cos T + \mathbf{v}_x \sin T) \\
y &= C₂ᵧ + r(\mathbf{u}_y \cos T + \mathbf{v}_y \sin T) \\
z &= C₂ᶻ + r(\mathbf{u}_z \cos T + \mathbf{v}_z \sin T)
\end{aligned}
\]

(7)

where \( r \) is the radius of each rotation of the cutter, \( T \) is the angle the cutter has rotated on each circle, \( T = 0 \sim 2\pi \). The cutter will be positioned at a number of points on each circle, and swipe through all the circles (Figure 4), to find one position that produces the minimum fitting or cutting error.

3.2. Evaluation of fitting error
The fitting error of the straight-line or cutter surface to the original blade surface is evaluated as the tangential distance between the two surfaces at the meridional grid of original blade. The projection of the cutter position in the meridional plane is also a straight line,

\[
\frac{r - \sqrt{C₁ₓ² + C₁ᵧ²}}{C₂ₓ - C₁ₓ} = \frac{\sqrt{C₂ₓ² + C₂ᵧ²} - \sqrt{C₁ₓ² + C₁ᵧ²}}{C₂ₓ - C₁ₓ}
\]

(8)

The quasi-stream lines of a blade surface may be expressed as functions of dimensionless meridional distance \( t \)

\[
z_i = z_i(t), \quad i = 1 \sim m
\]

(9)

\[
r_i = r_i(t), \quad i = 1 \sim m
\]

(10)
$\theta_i = \theta_i(t), \ i = 1 \sim m \quad (11)$

The intersection of the cutter projection and the $i$th quasi-streamline $(z_i, r_i)$ is first calculated by equating equations (8), (9) and (10), and an interpolation may be necessary. The known intersection at the outer guide line and the cutter position at the search circles may help to speed up the solution which produces the $t$ value for the intersection, and subsequently, the values of $z_i$, $r_i$ and $q_i$ for the intersection on the blade surface.

The corresponding $\theta$ value of the cutter, $\theta_{c,i}$ at the intersection can be calculated using the known meridional coordinates and cutter equation (7) as well as equation (3). And the fitting error at this cutter position can be calculated:

$$\text{Fitting error} = r_i (\theta_{c,i} - \theta_i), \ i = 1 \sim m. \quad (12)$$

The error expressed by equation (12) can be positive or negative. The sign of the error indicates if the blade is overcut or under cut, depending on the exact definition of $\theta$. A positive error may indicate that the blade surface is overcut or undercut.

The fitting error at all the quasi-streamlines is computed and their scalar or arithmetic mean is calculated,

$$\text{Fitting error}_m = \frac{1}{m} \sum_{i=1}^{m} r_i \| \theta_{c,i} - \theta_i \| \quad (13)$$

For a given pivot point $C_1$ of the cutter, the mean fitting error given in equation (13) for all the cutter positions are compared, the position that produces the smallest error is then selected as the best straight-line approximation to the blade surface with regard to $C_1$. Note that the error in tangential direction employed here is always larger than the error normal to blade surfaces.

4. Applications

The proposed conversion algorithm was developed into a computer program. The program can complete the flank milling cutter path planning of whole impeller blade surfaces including leading and trailing edges. It was applied to a number of centrifugal compressors, and here the results of two of them were reported. To better compare the conversion errors of different sized impellers, the errors were made dimensionless by dividing them with the radius of the impellers. As the conversion error is the main concern in this paper, only the results from impeller main blades will be discussed.

Impeller 1 is a small impeller at 46mm diameter (at hub) for passenger car turbocharger application. It has free-form blade surfaces with six main blades and six splitter blades, Figure 5. These blade surfaces were converted into ruled surface using the method outlined earlier. Figure 6 shows the conversion error of the suction and pressure surfaces. It can be seen from the figure that a good result has been achieved.
Figure 7 shows the cutter path on the pressure and suction surfaces of impeller 1. The meridional shape of the impeller blade is outlined in blue color. The concentrated lines at the leading edge are necessary for accurately converting the edge.

Figure 8 gives the conversion error when the cutter only moves on the meridional plane. Compared with Figure 6, one sees that by allowing the cutter to search in the both meridional plane and tangential direction, the new method has more than halved the converting errors.

The second impeller is larger with a tip diameter about 144 mm for turbocharging marine diesel-generator. It also has free-form blade surfaces and six main blades and six splitter blades, Figure 9. The free-form blade surfaces of its main blades were converted into ruled surface using the proposed method. Figure 10 shows the conversion error between the ruled surface blade and the original blade. It shows that the conversion error of the suction surface is quite small but is larger of the pressure surface. This impeller has a smaller trim than impeller 2, so its converting error is smaller than that of impeller 1.
FIG. 8 Converting error of impeller 1 when cutter only swings on the meridional plane

FIG. 9 3D model of 144mm impeller 2

FIG. 10 Converting error of impeller 2’s main blade

Figure 11 displays the cutter path in the meridional plan for impeller 2. It can be seen that the cutter path distribution is neat without any crossing.
Figure 12 gives the converting error when the cutter moves only on the meridional plane. Compared with Figure 10, one can see that by allowing the cutter to search in the both meridional plane and tangential direction, the proposed method produces significantly smaller converting errors.

(a) Pressure side  
(b) Suction side

FIG. 11 Cutter path for flank milling impeller 2

(a) Suction surface fitting error  
(b) Pressure surface fitting error

FIG. 12 Converting error of impeller 2 when cutter only swings on the meridional plane

5. Conclusion & Discussion
A method of using cutter rotation to search for the best position of the flank milling cutter when converting a free-form surface is proposed. Not only is the swing of the cutter on the meridional plane of the blade surface considered, but also the tangential swing of the cutter is allowed.

Mathematics of the method is described. Two turbocharger compressor impellers in different sizes and trims were studied and the results compared with cutter swinging only on the meridional plane. It shows that the new method produces smaller converting errors.

Currently, the search for the best cutter path in the meridional plane and in tangential direction is not directionally independent, but linked by search cycles. This reduces the effectiveness of the search, but the restriction can easily be removed to realise the full potential of the method. Work is currently
going on to address this issue, and initial results are promising. We will report the findings when the work is completed.

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