On the computation of multigluon amplitudes

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Abstract

A computational algorithm based on recursive equations is developed in order to estimate multigluon production processes at high energy hadron colliders. The partonic reactions $gg \rightarrow (n - 2)g$ with $n \leq 9$ are studied and comparisons with known approximations are presented.

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1 Introduction

High-energy hadron colliders will be valuable tools for developing particle physics in the next decade, especially with the planned run of the Large Hadron Collider (LHC), which will operate at $\sqrt{s} = 14$ TeV, the highest energy achieved so far. Multijet final states are of great importance at high energies, and in fact events with very high jet multiplicities, up to six jets, have already been observed and measured at the TEVATRON collider [1].

On the other hand, the estimate of the multijet production cross sections as well as their characteristic distributions is a formidable task from the theoretical point of view [2]. Perturbation theory based on Feynman graphs, the ‘brute-force’ method, runs into computational problems, since the number of graphs contributing to the $n$-gluon amplitude grows like $n!$, jeopardising the computational efficiency. In order to simplify the computation, colour decomposition [3, 4, 5] has been used extensively in the past. According to this scheme, the $n$-gluon amplitude can be written as:

$$\mathcal{M} = 2ig^{n-2} \sum_{P(2,\ldots,n)} Tr(t^{a_1} \ldots t^{a_n})C(1,\ldots,n)$$  \hspace{1cm} (1)

where $t^a$ are the generators of the $SU(N)$ group in the fundamental representation, $g$ is the gauge coupling constant and $C(1,\ldots,n)$ are gauge invariant subamplitudes that do not contain any explicit information on the colour structure. The computational complexity is now due to the large number of gauge invariant subamplitudes, which is roughly proportional to $n!$. Moreover the colour summation leads to a colour matrix with dimensionality $(n-2)! \times (n-2)!$ [6], another source of computational inefficiencies.

On the other hand recursive methods [7, 6, 8, 9, 10] have been used extensively in order to overcome the computational obstacles of the ‘brute-force’ method. Following this line of thinking we present in this paper results on $gg \to (n-2)g$ with $n$ up to $n = 9$. The main ingredients we have used, in order to make the computation as simple and efficient as possible, are

1. The amplitude is computed using recursive equations which result in a computational cost growing asymptotically as $3^n$, to be compared with the $n!$ growth in the ‘brute-force’ method [11].

2. Colour and helicity structure are appropriately transformed so that a replacement of the usual summation with Monte Carlo integration is made possible.

2 The Recursive Equations for QCD
2.1 The Generating Function

Let us assume that $\phi(x)$ represents a field with arbitrary quantum numbers. The generating function of all connected Green functions is given by the standard formula [12]

$$Z[J] = -\Gamma[\phi] + \int d^4 x \phi(x) J(x), \quad (2)$$

subject to the condition that $\phi$ is a solution of the field equations

$$\frac{\delta \Gamma}{\delta \phi} = J. \quad (3)$$

$\Gamma[\phi]$ is the effective action which at tree order is given by

$$\Gamma[\phi] = \int d^4 x \mathcal{L}, \quad (4)$$

where $\mathcal{L}$ is the Lagrangian of the theory. For a process with $n$ external particles the source, $J(x)$, is given as an expansion over the free asymptotic states,

$$J(x) = \sum_{l=1}^{n} a_l e^{-ip_l \cdot x}, \quad (5)$$

or in momentum space by

$$J(q) = (2\pi)^4 \sum_{l=1}^{n} a_l \delta(q - p_l), \quad (6)$$

The standard LSZ reduction formula relates the amplitude with the $n$-th derivative of the generating function

$$\mathcal{A}(p_1, p_2, \ldots, p_n) = \lim_{\text{on-shell}} \frac{\partial^n Z}{\partial a_1 \ldots \partial a_n} \left[ Z^{(2)}(p_l) \right]^{-1} \quad (7)$$

where $[Z^{(2)}(p)]^{-1}$ is the inverse propagator and the on-shell limit is understood. Now it is not difficult to prove that the field equations acquire the following solution [10, 13]

$$\phi(q) = (2\pi)^4 \sum_{m=1}^{2^{n-2}} b_m \delta(q - P_m) \quad (8)$$

where $P$ are all possible sums of the $n$ external momenta, $p_i$, $i = 1, \ldots, n$. For example, $p_1 + p_2, p_1 + p_4 + p_n$, etc. Their number is simply determined by the binomial sum

$$\sum_{l=1}^{n-1} \binom{n}{l} = 2^n - 2.$$
The coefficients $b_m$ satisfy non-linear equations which can be solved iteratively leading to recursive equations. Since we are interested in multigluon amplitudes, we study the derivation of these equations in the case of QCD. The Lagrangian for QCD is given by:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

$$\mathcal{L}_0 = -\frac{1}{4} (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) - \frac{1}{2\xi} (\partial^\mu A^a_\mu)^2$$

$$\mathcal{L}_{int} = -\frac{g}{2} f^{abc} (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) A^{b\mu} A^{c\nu} - \frac{g^2}{4} f^{ace} f^{bde} A^a_\mu A^b_\nu A^c_\rho A^d_\sigma$$

where $\xi$ is the gauge fixing parameter. We shall take the value $\xi = 1$ for the rest of this paper. In momentum space the effective action, $\Gamma = \int d^4x \mathcal{L}$, is written as:

$$\Gamma = -\frac{1}{2} \int \frac{d^4q}{(2\pi)^4} A^a_\mu(-q)\delta^{ab} q^{\mu\nu} q^2 A^b_\nu(q)$$

$$+ \frac{ig}{3!} \int \left( \prod_{i=1}^3 \frac{d^4q_i}{(2\pi)^4} \right) V^{abc}_{\mu\nu\lambda}(q_1, q_2, q_3) A^{a\mu}(q_1) A^{b\nu}(q_2) A^{c\lambda}(q_3) (2\pi)^4 \delta^4(\sum_{i=1}^3 q_i)$$

$$- \frac{g^2}{4!} \int \left( \prod_{i=1}^4 \frac{d^4q_i}{(2\pi)^4} \right) G^{abcd}_{\mu\nu\rho\lambda}(q_1, \ldots, q_4) A^{a\mu}(q_1) A^{b\nu}(q_2) A^{c\rho}(q_3) A^{d\lambda}(q_4) (2\pi)^4 \delta^4(\sum_{i=1}^4 q_i) ,$$

where $V^{abc}_{\mu\nu\lambda}(q_1, q_2, q_3)$ and $G^{abcd}_{\mu\nu\rho\lambda}(q_1, q_2, q_3, q_4)$ are the usual three- and four-point vertices in QCD [12].

For $n$-gluon scattering the source is given by

$$J^a_\mu = (2\pi)^4 \sum_{i=1}^n \epsilon_\mu(p_i, \lambda_i) \delta^{aa} \delta(q - p_i)$$

where $\{p_i^{\mu}, \lambda_i, a_i\}$ are the momenta, helicities and colours of the external gluons. All momenta are taken to be incoming. The field equations acquire a solution of the form

$$A^a_\mu(q) = (2\pi)^4 \sum_{m=1}^{2^n - 2} b^a_\mu(m) \delta(q - P_m)$$

where $P_m$ are as in Eq.(8). The form of the solution, Eq.(14), suggests that a natural ordering of the momenta can be done in the following way: for the number $m$, lying between 1 and $2^n - 2$ we take its binary representation, which is an $n$-dimensional vector, $\{m_i\}, i = 1, \ldots, n$, with entries either 0 or 1, and we define

$$P^\mu_m = \sum_{i=1}^n m_i p^{\mu}_i .$$
In the usual perturbative sense, one performs an expansion in terms of the coupling constant

\[ b_\mu^a(m) = \sum_{k=1}^{n-1} b_\mu^a(m,k) g^{k-1} \]  

(15)

and the field equations take the form:

\[ b_\mu^a(m,k) = \frac{i}{2P_m^2} \sum \delta_m|m_1,m_2 \delta_k|k_1,k_2 \hat{V}^{a_1,a_2}_{\mu_1\mu_2}(P_m,P_{m_1},P_{m_2}) b_{\mu_1}^{a_1}(m_1,k_1) b_{\mu_2}^{a_2}(m_2,k_2) \]

\[ - \frac{1}{6P_m^2} \sum \delta_m|m_1,m_2,m_3 \delta_k|k_1,k_2,k_3 \hat{G}^{a_1 a_2 a_3}_{\mu_1 \mu_2 \mu_3}(P_m b_{\mu_1}^{a_1}(m_1,k_1) b_{\mu_2}^{a_2}(m_2,k_2) b_{\mu_3}^{a_3}(m_3,k_3), \]

(16)

where summation over repeated indices is implied and

\[ \delta_{i|i_1i_2} = \delta_{i,i_1+i_2} . \]

The initial values for the iteration are given by the source term and read

\[ b_\mu^a(m,1) = \begin{cases} 
\epsilon_\mu(P_m,\lambda_m) \delta_{a,m} & m = 2^{i-1}, i = 1, \ldots, n \\
0 & \text{otherwise} \end{cases} \]  

(17)

The amplitude is then given by

\[ A = \sum_a b^a(1,1) \cdot \hat{b}^a(2^n - 2, n - 1) \]  

(18)

where the \( \hat{\cdot} \) means that the propagator factor \( 1/P_m^2 \) has been removed.

One can further simplify Eq.(16) by defining a Lorentz scalar as

\[ b^a(m,k,\lambda) \equiv \epsilon_\mu(P_m,\lambda) b_\mu^a(m,k) \]  

(19)

where \( \lambda \) stands for the helicity of the particle and takes the values \( \lambda = \pm, L \), for transversally and longitudinally polarized gluons. Eq.(14) can be inverted with the solution

\[ b_\mu^a(m,k) = \sum_\lambda \epsilon_\mu(P_m,\lambda) b^a(m,k,\lambda) + P_{m\mu} b^a(m,k) \cdot P_m \]  

\[ \frac{1}{P_m^2} . \]  

(20)

Ward identities guarantee that all terms proportional to \( P_m \) drop out in the calculation of the physical amplitude, so that Eq.(16) can be written as

\[ b^a(m,k,\lambda) = \frac{i}{2P_m^2} \sum \delta_{m|m_1,m_2} \delta_{k|k_1,k_2} \hat{V}^{a_1,a_2}(P_m,\lambda_1,P_{m_1},\lambda_1;P_{m_2},\lambda_2) \]

\[ b^{a_1}(m_1,k_1,\lambda_1) b^{a_2}(m_2,k_2,\lambda_2) \]

\[ - \frac{1}{6P_m^2} \sum \delta_{m|m_1,m_2,m_3} \delta_{k|k_1,k_2,k_3} \hat{G}^{a_1 a_2 a_3}(\lambda,\lambda_1,\lambda_2,\lambda_3) \]

\[ b^{a_1}(m_1,k_1,\lambda_1) b^{a_2}(m_2,k_2,\lambda_2) b^{a_3}(m_3,k_3,\lambda_3) \]  

(21)
where
\[
\hat{V}^{abc}(P_i, \lambda_i; P_j, \lambda_j; P_k, \lambda_k) = \epsilon^\mu(P_i, \lambda_i)\epsilon'^\nu(P_j, \lambda_j)\epsilon''^\alpha(P_k, \lambda_k)\hat{V}^{abc}_{\mu\nu\alpha}(P_i, P_j, P_k) \tag{22}
\]
and
\[
\hat{G}^{abcd}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \epsilon^\mu(P_1, \lambda_1)\epsilon'^\nu(P_2, \lambda_2)\epsilon''^\alpha(P_3, \lambda_3)\epsilon'^\rho(P_4, \lambda_4)G^{abcd}_{\mu\nu\alpha\rho} \tag{23}
\]

Experience with scalar theory has shown that the computational cost grows like $3^n$ for a $\phi^3$ theory and $4^n$ for a $\phi^4$ theory \cite{11}. This suggests that the computational cost will be lowered if we succeed in writing all equations in terms of three-vertex interactions. This can be done by introducing an auxiliary field $X$ and replacing the four-gluon part of the Lagrangian with
\[
\mathcal{L}_X = -gf^{abc}A^a_\mu A^b_\nu X^{\mu\nu \rho} - X^{\mu\nu \rho}X^{\mu\nu \rho}. \tag{24}
\]
Then Eq.\( (21) \) is written as
\[
b^{a}(m, k, \lambda) = \frac{i}{2P^2_m} \sum \delta_{m|m_1m_2}\delta_k|k_1k_2|\hat{V}^{aa_1a_2}(P_m, \lambda; P_m, \lambda_1; P_m, \lambda_2)
\]
\[
b^{a_1}(m_1, k_1, \lambda_1)b^{a_2}(m_2, k_2, \lambda_2)
- \frac{1}{P^2_m} \sum \delta_{m|m_1m_2}\delta_k|k_1k_2|H^{\mu\nu}(P_m, \lambda; P_m, \lambda_1)f^{aa_1a_2}
\]
\[
b^{a_1}(m_1, k_1, \lambda_1)X^{a_2}_{\mu\nu}(m_2, k_2)
\]
\[
X^{a}_{\mu\nu}(m, k) = -\frac{1}{4} \sum \delta_{m|m_1m_2}\delta_k|k_1k_2|H^{\mu\nu}(P_m, \lambda_1; P_m, \lambda_2)f^{aa_1a_2}
\]
\[
b^{a_1}(m_1, k_1, \lambda_1)b^{a_2}(m_2, k_2, \lambda_2) \tag{25}
\]
where
\[
H^{\mu\nu}(p_1, \lambda_1; p_2, \lambda_2) = \left(\epsilon_\mu(p_1, \lambda_1)\epsilon_\nu(p_2, \lambda_2) - \epsilon_\nu(p_1, \lambda_1)\epsilon_\mu(p_2, \lambda_2)\right). \tag{26}
\]

### 2.2 Colour and helicity treatment

Eq.\( (25) \) can be used to compute the $n$-gluon amplitude for an arbitrary momentum, colour and helicity configuration of the external particles. In order to have an estimate of the production probability, one has to sum over all colour and helicity configurations. Summation over colours is a delicate subject. If one performs the summation in a straightforward way then one has to consider something like $8^n$ configurations for the $n$-gluon scattering. In this section we show how this summation can be replaced by integration, which is then suitable for Monte Carlo computation.

As a first step a simplification of the colour structure of Eq.\( (25) \) is possible by defining the following object
\[
\hat{V}^{abc}(P_i, \lambda_i; P_j, \lambda_j; P_k, \lambda_k) = \epsilon^\mu(P_i, \lambda_i)\epsilon'^\nu(P_j, \lambda_j)\epsilon''^\alpha(P_k, \lambda_k)\hat{V}^{abc}_{\mu\nu\alpha}(P_i, P_j, P_k) \tag{22}
\]
\[ b_{AB} \equiv \sum_{a=1}^{8} t_{AB}^a b^a, \quad A, B = 1, 2, 3 \] (27)

where all other indices have been temporarily suppressed. The new objects are of course traceless three by three matrices in colour space. The interesting property of this colour representation is that it leads to a ‘diagonalization’ of the colour structure of the three-gluon vertex. More specifically, the colour part of the three-gluon vertex is now given by

\[ f^{abc} t_{AB}^a t_{CD}^{bc} t_{EF}^{ce} = \frac{-i}{4} \left( \delta_{AD} \delta_{CF} \delta_{EB} - \delta_{AF} \delta_{CB} \delta_{ED} \right) \] (28)

This colour structure is related to the colour flow occurring in the real physical process, where gluons can be represented by quark-anti-quark states in colour space and their self-interaction, as given by Eq.(28), reflects the fact that colour remains unchanged on an uninterrupted colour line.

Accordingly Eq.(23) is now transformed to

\[ b_{AB}(m, k, \lambda) = \frac{1}{2 P_m^2} \sum_{m_1 m_2} \delta_{m|m_1 m_2} \delta_{k|k_1 k_2} \hat{V}(P_m, \lambda; P_{m_1}, \lambda_1; P_{m_2}, \lambda_2) \]

\[ + \frac{i}{P_m^2} \sum_{m_1 m_2} \delta_{m|m_1 m_2} \delta_{k|k_1 k_2} H^{\mu \nu}(P_m, \lambda; P_{m_1}, \lambda_1) \]

\[ (b_{AC}(m_1, k_1, \lambda_1) b_{CB}(m_2, k_2, \lambda_2) - b_{AC}(m_2, k_2, \lambda_2) b_{CB}(m_1, k_1, \lambda_1)) \]

\[ X_{AB}^{\mu \nu}(m, k) = \frac{i}{4} \sum_{m_1 m_2} \delta_{m|m_1 m_2} \delta_{k|k_1 k_2} H^{\mu \nu}(P_{m_1}, \lambda_1; P_{m_2}, \lambda_2) \]

\[ (b_{AC}(m_1, k_1, \lambda_1) b_{CB}(m_2, k_2, \lambda_2) - b_{AC}(m_2, k_2, \lambda_2) b_{CB}(m_1, k_1, \lambda_1)) \] (29)

where

\[ \hat{V}(P_m, \lambda; P_{m_1}, \lambda_1; P_{m_2}, \lambda_2) \]

stands for the momentum part of the three-gluon vertex, defined in Eq.(22).

The next step is to consider a colour representation so that summation over colours can be replaced by integration. We seek therefore a colour vector \( \eta^a(z) \) satisfying the following property:

\[ \int [dz] \eta^a(z) \eta^b(z) = \delta^{ab} \] (30)

with some properly defined measure \([dz]\). This can be realized by standard group-theoretical constructions. In the case of \( SU(3) \), we have to consider the 5-dimensional sphere \([14]\) parametrized by three complex numbers \((z_1, z_2, z_3)\) subject to the constraint:

\[ |z_1|^2 + |z_2|^2 + |z_3|^2 = 1 \] (31)
A useful parametrization of these three complex coordinates is as follows:

\[ z_1 = e^{i\phi_1} \cos \theta \]  
\[ z_2 = e^{i\phi_2} \sin \theta \cos \xi \]  
\[ z_3 = e^{i\phi_3} \sin \theta \sin \xi \]  

\( 0 \leq \phi_i \leq 2\pi \);  \( 0 \leq \theta \leq \frac{\pi}{2} \);  \( 0 \leq \xi \leq \frac{\pi}{2} \)

and the group invariant measure is simply given by

\[ \int \left( \prod_{i=1}^{3} dz_i d z_i^* \right) \delta \left( \sum_{i=1}^{3} z_i z_i^* - 1 \right) \]  

or, in terms of the polar variables,

\[ \left( \prod_{i=1}^{3} \int_{0}^{2\pi} d\phi_i \right) \int_{0}^{\frac{\pi}{2}} \! \! d\theta \int_{0}^{\frac{\pi}{2}} \! \! d\xi \cos \theta \sin^3 \theta \cos \xi \sin \xi \]

It is now a matter of algebra to show that the vectors

\[ \eta^a(z) = \sqrt{24} \sum_{i,j=1}^{3} z_i^*(t^a)_{ij} z_j \quad a = 1, \ldots, 8 \]  

satisfy Eq.(30).

As far as our recursive equations are concerned their structure remains unaffected and the only things to be changed are the initial conditions:

\[ b_{AB}(m, 1, \lambda) = \sum_{a=1}^{8} b^a(m, 1, \lambda) \eta^a(z) = \delta_{\lambda\lambda_m} \sqrt{6} \left( z_m z_m^* - \frac{1}{3} \delta_{AB} \right) \]  

where as usual \( m = 2i-1, i = 1, \ldots, n \), \( \lambda_m \) is the helicity and \( \vec{z}_m \) are the new continuous colour coordinates of the \( m \)-th gluon.

In the same way, summation over helicity configurations of the external gluons can be replaced by an integration over a phase variable. This is achieved by introducing the polarization vector:

\[ e^\mu_\phi(p) = \sqrt{2} \left( e^{i\phi} e^\mu(p, +) + e^{-i\phi} e^\mu(p, -) \right) . \]  

Then by integrating over \( \phi \) we obtain the sum over helicities,

\[ \frac{1}{\pi} \int_{0}^{\pi} d\phi e^\mu_\phi(p)(e^\mu_\phi(p))^* = \sum_{\lambda = \pm} e^\mu(p, \lambda)(e^\nu(p, \lambda))^* . \]
3 Results and Discussion

Using the recursive equations described so far, we study the processes $gg \rightarrow (n - 2)g$, for up to $n = 9$. The Monte Carlo integration techniques used to represent the colour and helicity summation are expected to have an influence on the variance of the squared matrix element over the extended phase space, which now includes besides the momenta, the continuous colour variables, $z_i, i = 1, 2, 3$, and the helicity phase $\phi$. In order to study the variance we have used the following computational schemes:

1. By method I we mean that colour and helicity are treated as described in the previous section.

2. In method II the helicity summation is replaced by a discrete Monte Carlo: in each ‘event’ we randomly choose a helicity configuration and then multiply by the appropriate combinatorial factor counting the total number of non-vanishing helicity configurations.

3. Finally in method III a full summation over helicity configurations has been performed.

All three methods have been used in the calculation of 3 and 4 gluon production. Moreover an ‘analytic’ approximation, the so-called SPHEL approximation [15, 16], has been used in the computation.

In order to avoid collinear and soft singularities the following cuts have been used:

$$p_{T_i} > 60\text{GeV}, \quad |\eta_i| < 2, \quad \theta_{ij} > 40^\circ$$

For each given multiplicity, i.e. for a given value of $n$, we use a different centre-of-mass energy, $\sqrt{s}$, since higher final-state gluon multiplicities exhibit higher energy thresholds, due to the imposed cuts. The computation within a given gluon multiplicity utilizes the same set of momenta configurations, so a positive correlation is expected. On the other hand, the variance can be used to estimate directly the computational speed. As far as the CPU time is concerned, methods I and II use the same amount of resources, whereas in method III we need 10, for $n = 5$, and 15, for $n = 6$, times more CPU time. Finally the running strong coupling constant with $N_f = 5, \Lambda_{QCD} = 200\text{ MeV}$ and a scale $Q = p_{T,max}$ has been used.

In table 1 the total partonic cross section and the corresponding variance estimate are presented. We see that method I is generally more efficient than method II since the variance is always better. Moreover, it is also better than method III if we take into account the multiplicative factor 10 ($n = 5$) and 15 ($n = 6$) needed for the computation. Colour and helicity integration do have a contribution to the variance as this can be inferred from the comparison with the variance computed in the case of SPHEL, where only
phase-space integration is involved. Of course variance reduction schemes are possible but their study goes beyond the scope of the present publication.

As a further step we have also considered differential distributions, such as the maximum and minimum gluon transverse momentum, $p_T^{\text{max}}, p_T^{\text{min}}$, and the maximum and minimum invariant two-(gluons)jet mass. In Fig.1 the four differential distributions are presented for the process $gg \to 6g$, whereas in Fig.2 the same distributions are presented for the reaction $gg \to 7g$. In all cases a qualitative agreement with SPHEL has been found. Note, however, that our computational scheme is an exact tree-order calculation of the amplitude, where no leading colour approximation nor special helicities selection have been used.

In summary a computational method based on recursive equations has been presented which enables one to compute the amplitudes for the processes $gg \to (n-2)g$ for at least up to $n = 9$. The method can be easily extended to include all partonic subprocesses \[17\]. It can provide a reliable tree-order event-generator for multi-jet production at LHC energies.

| process $\sqrt{s}$ | I         | II        | III        | SPHEL     |
|---------------------|-----------|-----------|------------|-----------|
| $2 \to 3$ 400 GeV   | $2.43 \pm 0.18$ | $2.15 \pm 0.19$ | $2.35 \pm 0.1$ | $2.43 \pm 0.06$ |
| $2 \to 4$ 600 GeV   | $(4 \pm 1) \times 10^{-4}$ | $(8 \pm 3) \times 10^{-4}$ | $(2 \pm 0.5) \times 10^{-4}$ | $(2 \pm 0.3) \times 10^{-4}$ |
| $2 \to 5$ 900 GeV   | $(4.7 \pm 0.3) \times 10^{-2}$ | $9 \pm 2 \times 10^{-6}$ | $(7.6 \pm 0.2) \times 10^{-2}$ | $(6 \pm 2) \times 10^{-6}$ |
| $2 \to 6$ 1200 GeV  | $(1.0 \pm 0.1) \times 10^{-2}$ | $(1.0 \pm 0.4) \times 10^{-6}$ | $(1.9 \pm 0.1) \times 10^{-2}$ | $(1.0 \pm 0.3) \times 10^{-6}$ |
| $2 \to 7$ 1500 GeV  | $(1.5 \pm 0.1) \times 10^{-3}$ | $(2.1 \pm 0.9) \times 10^{-8}$ | $(2.3 \pm 0.1) \times 10^{-3}$ | $(1.3 \pm 0.3) \times 10^{-8}$ |

Table 1: Total partonic cross section (in nb) and the corresponding variance with their Monte Carlo errors.
Figure 1: Differential distributions for the process $gg \to 6g$ in pb/GeV; (a) maximum transverse momentum, (b) minimum transverse momentum, (c) maximum two-jet invariant mass and (d) minimum two-jet invariant mass. Black circles correspond to the exact result whereas triangles represent the predictions of SPHEL.

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Figure 2: Same as Fig.1 for $gg \rightarrow 7g$.

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