Analysis of three wheeled electric vehicle with increased stability on the road

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Abstract. The using of electric vehicles is a huge challenge in order to avoid the pollution and to take care of environment too. The designed three-wheeled motorcycle with electric induction motor actuation provides the advantages of a motorcycle with the ones of a car. The yaw, pitch and roll movements of the front side of this vehicle offer the pleasure of riding. Meantime, the stability is improved due to its rear side resembling with a car. By using the finite element analysis of a welded assembly, the paper is a study applied to the main assembly with the most important functioning requirements demanded by combing the movements specified above. The solutions of the nonlinear mathematical model written for the dynamic conditions have the main goal of providing the values for the forces acting when a curve trajectory has to be followed.

1. Introduction

 Nowadays, there are many concerns about the way we have to improve the environment quality, so the green energy could be a solution. The electric vehicles technologies were developed rapidly and their technical performances are studied for continuous assessment on a remarkable growing market.

 The actuation system uses electric induction motors supplied by voltage inverter and the right possibility of battery type choosing, such as Lithium – Ion battery accumulator, is an important option influencing the quality of transportation such as speed level and maximum distance as independent driving.

 Due to the advantage of minimum pollution, such vehicles are allowed on very narrow streets downtown especially. Among these electric vehicles, there are three wheeled vehicles combining the advantages of a car with those of motorcycles: the comfort of a car and the pleasure of driving a motorcycle are the most significant. The less space in traffic as well as for parking in congested cities are other opportunities.

 The narrow three wheeled vehicles need additional degrees of freedom for leaning taking into account the lateral stability. Moreover, they need rapid manoeuvring and safety driving avoiding the slippery on the road surfaces, so that the rear wheels have to be very carefully controlled from the steering point of view.

 The state-of-the art regarding the three-wheeled vehicles concerns with improving lateral stability solutions, such as controller design, as well as the dynamic mathematical modelling used for dynamic forces computation.
The paper [1] describes the main advantages of a three-wheeled vehicle, which combines performances of a motorcycle and a car. It is an electric vehicle—a good example for green energy, it is necessary the minimum space for parking, it is very useful for one or two persons. The mathematical model affords the study of dynamic stability, taking into account the three motions: longitudinal, lateral and tilt. Meantime, there are two driven wheels in front of the vehicle and one non-driven wheel in the rear. Generally speaking, there are six degrees of freedom for a body: three translational movements (longitudinal, lateral and vertical along the three axes) and three rotational movements around these axes (yaw, pitch and roll).

The authors have written the mathematical dynamic model for this vehicle, based on its degrees of freedom together with the forces acting during the movement. The variations of forces acting on the tires were included in the model. Finally, the most important thing is the achievement of dynamic stability, so that a controller was designed based on the dynamic objectives of the entire system.

A simplified vehicle dynamic model is presented in paper [2], taking into account the angular deviation of the front wheel when the trajectory is a curve, so the authors have analysed the errors of forces and mechanical couples for such movement. The entire analysis is focused on the stability control as result of mathematical model. The main goal was to find out an optimal value for slip ratio between the lateral and longitudinal directions by maximizing the yaw moment. The tire forces and geometry are considered too.

The paper [3] presents a non-linear mathematical model for a vehicle with nine degrees of freedom. Five of them are considered for the following motions: lateral, longitudinal, rotation about z axis (yaw), roll and pitch. On each wheel is acting a vertical load as function of weight shift and pitch motions. Consequently, the authors have ignored the vertical motion in z axis direction. The latter four degrees of freedom are taken from the vehicle wheels implying their dynamics. Finally, the model was used for applying the technique control, starting from the two degrees of freedom vehicle model. In order to stabilize the motion, this 2 DOF (degrees of freedom) model was taken as theoretical model response of the yaw moment.

The authors of [4] were focused on the estimation of disturbances forces with origin in road conditions especially for curbs and these one depend on vehicle speed. It is proposed an extended Kalman filter whose role is to estimate the lateral and longitudinal velocity along with the tire forces. The same principle could be used to estimate the tire / road friction coefficient.

The mathematical model for the vehicle dynamics is extended to a hybrid one composed of two-track model at the front axle and a single-track model at the rear axle. It is the best solution for a vehicle with three wheels. The model is written in terms of ordinary differential equations with external disturbances from the road acting on the longitudinal vehicle direction.

The longitudinal and lateral tire forces are described in a tire specific frame with x axis aligned with the longitudinal axis of the wheel. Because it is difficult to realise the determinations of lateral tire forces at the front axle, the authors have incorporated an additional force variation producing the same effects.

The present paper is a focused approach on the advantages of using the electric motorcycle with improved stability on the road due to its three wheels running, the mathematical model of its dynamics and the finite element analysis of the welded assembly. It is assumed that the rear part of the motorcycle is characterized by the stability of four wheels car, so the centre of mass is following the imposed trajectory.

2. Technical parameters of the electric vehicle
The designed motorcycle, that is a three-wheeled vehicle, has the advantage of car comfort and the leaning stability on the road. Being a motorcycle, only for the front side has the properties of yaw, pitch and roll angular motions specific for the dynamic of vehicles with two wheels, providing high performances. The most important specific technical characteristic is the oscillatory movement of the front side, meaning it could be driven like a motorcycle.
Meantime, the rear side could be driven following the trajectory resembling with a car. Technically speaking, this particular functional improvement is achieved due to the cylindrical hinge joint H on the central plate kinematic element 1 (figure 1). This oscillatory movement with angle $\alpha$ is done around the central axes A – A’ (figure 1). This specific improvement implies some constructive design particularities of the chassis in order to assure the mechanical resistance of the framework during the movements following the curve trajectory especially. The curve trajectory riding means huge dynamic forces, like centrifugal forces depending directly on speed and on the curve radius.

![Figure 1. The schematic for the mathematical model.](image)

### 3. Mathematical model for the vehicle dynamics following the curve trajectory

The mathematical model has the aim of computing the dynamic forces during the movement and the focus will be on curve trajectory. The genuine mathematical model is written for the designed motorcycle with its construction particularities as it is presented below.

At first, we have established the position of the mass center for the rear side of the motorcycle comprising the gravity loads for the following components:

- the chassis (mass $m_1=22$ kg) with the positional coordinates for its mass center: $X_1=1.32$ m, $Z_1=0.37$ m;
- the two batteries (mass $m_2=34$ kg) and the coordinates for the mass center: $X_2=1.24$ m, $Z_2=0.45$ m;
- the electrical motor (mass $m_3=20$ kg) and the positional coordinates for its mass center: $X_3=1.02$ m, $Z_3=0.27$ m;
- the suitcases of the passengers (mass $m_4=50$ kg) and the position of the mass center: $X_4=1.80$ m, $Z_4=0.48$ m;
- the rear axle deck, whose mass is $m_5=40$ kg and its mass center position given by the coordinates: $X_5=1.44$ m, $Z_5=0.24$ m.

For the front side of the vehicle we have taken into account the following values:

- mass for the front side of the motorcycle $m_6=80$ kg and its position for the mass center $X_6=0.68$ m, $Z_6=0.48$ m;
- mass of the driver $m_7=75$ kg and $X_7=0.88$ m, $Z_7=0.85$ m;
the passenger sitting at the rear part of the saddle – mass \( m_p = 75 \text{ kg} \) and the position for its mass center is \( X_p = 1.25 \text{ m} \) and \( Z_p = 0.97 \text{ m} \).

According to the numerical values for data described above, the position of the mass center CG1 (figure 1) for the front side of the motorcycle could be established too.

We will use two axis coordinate systems one for the front side of the vehicle that is a mobile one with rotational movement \( \rho \) around OZ axis and the other one for the rear side of the vehicle that is a fix one. Consequently, the position of mass center for the front side will be computed in both systems, but for the mathematical modeling, we will consider the fix one. For instance, the coordinates for the front mass center in the fix system are: \( X_{G_{\text{front}}}^{\text{fix}}, Y_{G_{\text{front}}}^{\text{fix}}, Z_{G_{\text{front}}}^{\text{fix}} \).

The dynamic mathematical model considers the following dynamic forces acting on the motorcycle frame [5]:

- \( F_m \) – the traction force provided by the electric motor of about 2500 N
- the gravity for the front and rear side – \( G_{\text{front}}, G_{\text{rear}} \), both of them applied in the front and rear mass center computed in the fix system. The figure 1 presents \( G_{\text{front}} \) as \( G_1 \) and \( G_{\text{rear}} \) as \( G_2 \)
- the centrifugal forces are acting on the front mass center and on the rear mass center. There were computed taking into account the motorcycle speed \( v = 50 \text{ km/h} \) and the radius of the trajectory of about \( R = 84 \text{ m} \). The following notations were used: \( F_{C_{\text{front}}}^X \) - the front centrifugal force projection on OX axis and the centrifugal force acting on the rear mass center \( F_{C_{\text{rear}}}^X \) - the projection on OX axis
- \( F_a \) – the aerodynamic force applied in the front mass center computed in the fix system. We have used its value of about 10 N
- \( F_{\text{lat}} \) – the lateral force acting on the tires with its components on the front side and rear side; due to this force the tire has an oscillatory movement in XOY plane and the oscillatory angle is \( \Delta \)
- \( F_{\text{Cor}} \) – the Coriolis force with projections on the OX, OY and OZ axis applied on the hinge joint \( H \) (figure 1) whose movement is characterized by the angle \( \alpha \) around the OX axis. The projections are: \( F_{\text{Cor}}^X, F_{\text{Cor}}^Y, F_{\text{Cor}}^Z \).

The mathematical model for the vehicle dynamic comprises the following six equations:

\[
\sum F_X = F_m - F_{\text{lat}} \cdot \sin(\Delta) + F_{C_{\text{front}}}^X + F_{C_{\text{rear}}}^X + F_a + F_{\text{Cor}}^X = 0 \tag{1}
\]

\[
\sum F_Y = -F_{\text{lat}} \cdot \cos(\Delta) + F_{C_{\text{front}}}^Y + F_{C_{\text{rear}}}^Y - F_{\text{lat}}\text{rear} + F_{\text{Cor}}^Y = 0 \tag{2}
\]

\[
\sum F_Z = 2 \cdot N_2 + N_1 - G_{\text{front}} - G_{\text{rear}} + F_{\text{Cor}}^Z = 0 \tag{3}
\]

\[
\sum M_X = -(G_{\text{front}} \cdot Y_{G_{\text{front}}}^{\text{fix}} + G_{\text{rear}} \cdot Y_{G_{\text{rear}}}^{\text{fix}}) - F_{C_{\text{front}}}^Y \cdot Z_{G_{\text{front}}}^{\text{fix}} + (N_1 + 2 \cdot N_2) \cdot R \cdot (1 - \cos \alpha) - F_{C_{\text{rear}}}^Y \cdot Z_{G_{\text{rear}}}^{\text{fix}} - J_1 \cdot \Omega \cdot \cos \rho + M_{FC}^Y \tag{4}
\]

\[
\sum M_Y = F_{C_{\text{rear}}}^X \cdot Z_{G_{\text{rear}}}^{\text{fix}} + F_{C_{\text{front}}}^X \cdot Z_{G_{\text{front}}}^{\text{fix}} + G_{\text{front}} \cdot X_{G_{\text{front}}}^{\text{fix}} + G_{\text{rear}} \cdot X_{G_{\text{rear}}}^{\text{fix}} - N_1 \cdot L - F_a \cdot Z_{G_{\text{front}}}^{\text{fix}} + M_{FC}^Y = 0 \tag{5}
\]

\[
\sum M_Z = -(F_{C_{\text{rear}}}^X \cdot Y_{G_{\text{rear}}}^{\text{fix}} + F_{C_{\text{front}}}^X \cdot Y_{G_{\text{front}}}^{\text{fix}}) + F_{C_{\text{rear}}}^Y \cdot X_{G_{\text{rear}}}^{\text{fix}} + F_{C_{\text{front}}}^Y \cdot X_{G_{\text{front}}}^{\text{fix}} - F_m \cdot R + F_a \cdot Y_{G_{\text{front}}}^{\text{fix}} + F_{\text{lat}} \cdot \sin(\Delta) \cdot R - F_{\text{lat}} \cdot \cos(\Delta) \cdot L + M_{FC}^Z \tag{6}
\]

where: \( N_1 \) – the reaction applied on the front wheel; \( N_2 \) - the reaction applied on each rear wheel; \( R \) – the radius of curve; \( J_1 \) – the inertial couple of the wheel; \( L \) – the distance between the front tire and rear tire; the mechanical couple provided by the Coriolis force around the axis are \( M_{FC}^X, M_{FC}^Y, M_{FC}^Z \).
From this equation system we may compute the following unknown values: \( \rho \) – the rotational angle around OZ axis; \( \Delta \) – the angle in XY plane; \( F_{\text{lat}} \) – the lateral force; \( \Omega \) – the angular speed around the CIR of the curve trajectory; \( N_2 \) - the reaction force for the rear wheel; \( N_1 \) - the reaction force for the front wheel. The mathematical system is a nonlinear one, so we have solved it numerically with the Newton-Raphson method starting from an initial approximate solution. For the numerical set up considered above, we have obtained the following results: \( \rho = 13.72^\circ \); \( \Delta = 35.47^\circ \); \( F_{\text{lat}} = 10 \text{ N} \); \( \Omega = 616.3367 \text{ rad/s} \); \( N_2 = 3150 \text{ N} \); \( N_1 = 810 \text{ N} \).

4. The analysis of the central plate kinematic element for the hinge joint
The central plate kinematic element of the joint H (figure 1) is an important part of the complex movement of the motorcycle, so that its resistance, tensors and displacements has to be analysed when the forces computed above are acting on. At first, we have designed the assembly (figure 2) comprising seven parts assembled one from each other with linear weldment using Catia V5R17 software [6].

![Figure 2. The three dimensional model of the assembly.](image)

The three dimensional model has the following parts: 1 – right prismatic element; 2 – cylinder; 3 – left prismatic element; 4 – central plate; 5 – screw nut assembly; 6 – horizontal prismatic element.

For the analysis design we have made general analysis connections between all the elements using multi-selection of the surfaces belonging to the same part body, without handler point. The next working step was focused on applying linear weldment acting as rigid assembly.

The hint joint H (figure 1) is functionally connected to the lateral side of the motorcycle frame during the oscillatory movement with the angle \( \alpha \). Taking into account this particular designed solution, we have defined a virtual rigid body in order to compute the generative analysis results. Its main role has been to stiffening the hint joint to which it is attached for computation. Consequently, two virtual bodies have been attached on the lateral surfaces of the screw – nut assemblies, so that the virtual parts have been included in the nodes and elements section as well as in the properties section (figure 3).

On both lateral surfaces of the screw – nut assemblies, we have applied the computed centrifugal dynamic force of about 1500 N on each side. On the both screw nut assemblies, we have put the user –
defined restraints, so we have allowed only the OY axis translation. On the internal surface of cylinder 2 (figure 1) we have defined clamp.

![Figure 3](image3.png)

**Figure 3.** The tensions results for the hinge joint assembly.

![Figure 4](image4.png)

**Figure 4.** The displacements results for the hinge joint assembly.

By defining the weldment as a rigid one, we may infer that the most stressed sections are that of the basement the cylinder 2. The mesh generation was carefully monitored. For instance, the larger design
dimensions of the central plate 4 (figure 1) has the mesh with 24.875 mm size and absolute sag 3.98 mm parabolic element type. For its intermediate nodes parameters we have set jacobian 0.3 and warp 80. The central plate - 4 has almost nil displacements and tensions, having the role of making the assembly more rigid. The maximum displacement of about 0.015 mm is achieved at the lateral screw – nut assemblies, so that we may conclude it is an expected value due to the increased stiffness.

5. Conclusions
The paper aims to points out the advantages of three wheeled motorcycle that has the yaw, pitch and roll angular movement for its front side. Regarding the rear side, it resembles with a car, whose stability is increased comparing with the two wheeled vehicle. Moreover, the electric actuation based on induction motor means minimum pollution, so the using of green friendly technology with respect to the environment.

The useful advantage of combining the specific car motion with the motorcycle motion has been achieved by accordingly designing the chassis and the central welded assembly of hinge joint. The hinge joint has oscillatory movement around the longitudinal axis of the motorcycle. The proposed genuine mathematical model has developed the dynamic study when the curve is the imposed trajectory. The equations have taken into account the forces acting on the front side and on the rear side separately.

The finite element analysis was done for this welded assembly after the computation of values for the dynamic forces acting when an imposed curve trajectory was riding. The computed tensions and displacements are inside the range of requested values. The stiffness of the assembly was increased by using the virtual part considering the lateral frame that was not included in the designed assembly of the hinge joint.

For future work, we aim to apply the frequency analysis taking into account the vibrations during the movement depending on the road quality and some other constructive causes. A dynamic controller could be added for better lateral stability for the front side especially.

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