Hints on 5d Fixed Point Theories from Non-Abelian T-duality

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- 5d fixed point theories arise in the infinite bare coupling limit of N=1 SUSY gauge theories with very specific gauge groups and matter content  \((\text{Seiberg’96; Intriligator, Morrison, Seiberg’97})\)
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- Search for $AdS_6$ backgrounds: Brandhuber and Oz’s quite unique (Passias’12)
In this talk:

• New $AdS_6$ solution through non-Abelian T-duality
  (Y.L., O Colgain, Rodriguez-Gomez, Sfetsos, PRL (2013))

• Hints on the associated dual CFT
  (Y.L., O Colgain, Rodriguez-Gomez, arXiv:1311.4842)
Non-Abelian T-duality in AdS/CFT

- In 4 dim: (Sfetsos & Thompson’10)

  \[ AdS_5 \times S^5 \quad \xrightarrow{\text{NATD}} \quad AdS_5 \times H_2 \times M_4 \quad (H_2 \rightarrow S^2) \]
  (Gaiotto & Maldacena geometries for N=2 SCFTs)

  \[ AdS_5 \times T^{1,1} \quad \xrightarrow{\text{NATD}} \quad AdS_5 \times H_2 \times M_4 \quad (H_2 \rightarrow S^2) \]
  (Sicilian quivers (N=1 SCFTs)
  (Benini, Tachikawa, Wecht))

- Klebanov & Strassler
  \[ \xrightarrow{\text{NATD}} \quad \text{New geometries in massive IIA} \]
  Confining quarks, domain walls,
  Seiberg duality,…

  (Itsios, Nuñez, Sfetsos & Thompson’13)
  (Nuñez & colab‘13,14)

martes 5 de agosto de 2014
• In 5 dim: \( (Y.L., \text{ O Colgain, Rodriguez-Gomez, Sfetsos'12; Y.L., O Colgain, Rodriguez-Gomez'13}) \)

\[
AdS_6 \times S^4 \quad \xrightarrow{\text{NATD}} \quad \text{New } AdS_6 \text{ geometry in IIB}
\]

Dual CFT quiver with two nodes
Outline

1. 5d fixed point theories
2. The D4-D8 system
3. Non-Abelian T-duality back in the 90’s
4. Non-Abelian T-duality as a solution generating technique
5. The non-Abelian T-dual of Brandhuber & Oz
6. Hints on the 5d dual CFT
7. Conclusions and open issues
1. 5d fixed point theories

5d gauge theories are non-renormalizable:

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[g^2] = M^{-1} \rightarrow g^2 E \rightarrow \text{UV completion}
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• 5d SYM with maximal SUSY defined in the UV in terms of the (2,0) 6d theory.

In fact equivalent (Douglas’10; Lambert, Papageorgakis, Schmidt-Sommerfeld’10)
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- 5d SYM with minimal SUSY can be at fixed points for specific gauge groups and matter content, where they can exhibit interesting phenomena such as exceptional global symmetry groups  (Seiberg’96; Intriligator, Morrison, Seiberg’97)
• 5d fixed point theories are intrinsically strongly coupled
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Passias’12: • Unique SUSY solution in massive IIA:
Near horizon of the D-brane system giving rise to \( Sp(N) \) with specific matter content
(Brandhuber, Oz’99)*

• Non-existence of \( AdS_6 \) solutions in other SUGRAs not completely excluded, but strongly suggested

* And orbifolds thereof (Bergman, Rodríguez-Gómez’12)
2. The D4-D8 system

5d SUSY fixed points with $E_{N_f+1}$ global symmetry can be obtained in the infinite bare coupling limit of N=1 SYM with gauge group Sp(N), one antisymmetric hypermultiplet and $N_f < 8$ fundamental hypermultiplets (Seiberg’96)
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From the D4-D4 sector:
- Vector multiplet with Sp(N) gauge symmetry
- Massless hyper in the antisym. of Sp(N)

From the D4-D8 sector:
- Massless hypers in the fundamental of $SO(2N_f)$
A D4-brane probe in the D8-O8 background metric
(Brandhuber, Oz’99; Ferrara, Kehagias, Partouche, Zaffaroni’98)

\[ ds^2 = H_8^{-1/2} (-dt^2 + dx_1^2 + \cdots + dx_8^2) + H_8^{1/2} dz^2 \]

\[ H_8(z) = c + 16 \frac{z}{l_s} - \sum_{i=1}^{8} \frac{|z - z_i|}{l_s} - \sum_{i=1}^{8} \frac{|z + z_i|}{l_s} \]

(\( z_i \): locations of the 16 D8-branes)

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In the field theory limit (\( l_s \to 0 \) + gauge coupling fixed):

\[ \phi = \frac{z}{l_s^2} \text{ must be constant } \Rightarrow \text{ Region near } z = 0 \text{ (location of the O8\textsuperscript{-} plane)} \]
Then: 
\[
\frac{1}{g^2} = \frac{c}{l_s} + 16\phi - \sum_{i=1}^{8} |\phi - m_i| - \sum_{i=1}^{8} |\phi + m_i|
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with \( \phi = \frac{z}{l_s^2} \), \( m_i = \frac{z_i}{l_s^2} \).

This reproduces the effective gauge coupling of the 5d Sp(N) gauge theory with 16 fundamental hypers with masses \( m_i \) and one massless antisym. hypermultiplet \( \text{(Seiberg'96)} \).
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Taking \( N_f \) massless hypermultiplets:

\[ \frac{1}{g^2} = \frac{1}{g_{cl}^2} + 16\phi - \sum_{i=1}^{N_f} |\phi - m_i| - \sum_{i=1}^{N_f} |\phi + m_i| \]

one gets:

\[ \frac{1}{g^2} = \frac{1}{g_{cl}^2} + (16 - 2N_f)\phi \]
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- For \( N_f > 8 \), \( g^2 \) becomes negative at some point in the moduli space → Sick theory
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at the origin of the Coulomb branch

Here the global symmetry of the theory: \( SU(2) \times SO(2N_f) \times U(1) \)
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The field theory calculation can be generalized to other gauge groups and matter content \( (\text{Intriligator, Morrison, Seiberg’97}) \), which lack however an AdS/CFT description
The near horizon geometry of the D4-D8 system is a fibration of $AdS_6$ over half-$S^4$ with an $S^3$ boundary at the position of the O8-plane, preserving 16 SUSYs.

$$ ds^2 = \frac{W^2 L^2}{4} \left[ 9 ds^2(AdS_6) + 4 ds^2(S^4) \right] $$

$$ F_4 = 5 L^4 W^{-2} \sin^3 \theta \, d\theta \wedge \text{Vol}(S^3) $$

$$ e^{-\phi} = \frac{3 L}{2 W^5} , \quad W = (m \cos \theta)^{-\frac{1}{6}} $$

$$ m = \frac{8 - N_f}{2\pi l_s} $$

$\theta \in [0, \frac{\pi}{2}]$
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• $SO(5)$ symmetry broken to $SO(4) \sim SU(2) \times SU(2)$:

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- $SO(2, 5) \leftrightarrow$ Conformal symmetry
Passias’12: Analyzed the constraints imposed by SUSY on the geometry and fluxes of $AdS_6 \times M_4$ warped backgrounds in massive IIA → Brandhuber & Oz only possible background
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We will see: New $AdS_6$ solution in Type IIB through non-Abelian T-duality*

* Also through Abelian T-duality, describing the same fixed point theory
3. Non-Abelian T-duality back in the 90’s

Rocek and Verlinde’s formulation of Abelian T-duality for ST in a curved background (Buscher’88):

\[
S = \frac{1}{4\pi\alpha'} \int \left( g_{\mu\nu} \, dX^\mu \wedge \ast dX^\nu + B_{\mu\nu} \, dX^\mu \wedge dX^\nu \right) + \frac{1}{4\pi} \int R^{(2)} \phi
\]

In the presence of an Abelian isometry: \( \delta X^\mu = \epsilon \, k^\mu \) / 

\[
\mathcal{L}_k g = 0, \quad \mathcal{L}_k B = d\omega, \quad i_k d\phi = 0
\]

i) Go to adapted coordinates: \( X^\mu = \{ \theta, X^\alpha \} \) such that \( \theta \rightarrow \theta + \epsilon \) and \( \partial_\theta (\text{backgrounds}) = 0 \)
ii) **Gauge the isometry:**
\[ d\theta \rightarrow D\theta = d\theta + A \]

A non-dynamical gauge field / \[ \delta A = -d\epsilon \]

iii) **Add a Lagrange multiplier term:** \( \tilde{\theta} dA \), such that
\[ \int D\tilde{\theta} \rightarrow dA = 0 \Rightarrow A \text{ exact} \]
(in a topologically trivial worldsheet)

+ fix the gauge: \( A = 0 \) \( \rightarrow \) **Original theory**

iv) **Integrate the gauge field**

+ fix the gauge: \( \theta = 0 \) \( \rightarrow \) **Dual sigma model:**

\[ \{\theta, X^\alpha\} \rightarrow \{\tilde{\theta}, X^\alpha\} \text{ and} \]
\[ \tilde{g}_{00} = \frac{1}{g_{00}}; \quad \tilde{g}_{0\alpha} = \frac{B_{0\alpha}}{g_{00}}; \quad \tilde{g}_{\alpha\beta} = g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta} - B_{0\alpha}B_{0\beta}}{g_{00}} \]

\[ \tilde{B}_{0\alpha} = \frac{g_{0\alpha}}{g_{00}}; \quad \tilde{B}_{\alpha\beta} = B_{\alpha\beta} - \frac{g_{0\alpha}B_{0\beta} - g_{0\beta}B_{0\alpha}}{g_{00}} \]

\[ \tilde{\phi} = \phi - \log g_{00} \]  

**Buscher’s formulae**

- Conformally invariant
- Involutive transformation: \( \tilde{S} \xrightarrow{\tilde{\theta} \to \tilde{\theta} + \epsilon} S \)

- **Arbitrary worldsheets?** (symmetry of string perturbation theory): 

(a) ![Diagram](image1.png)  
(b) ![Diagram](image2.png)

**martes 5 de agosto de 2014**
Non-trivial topologies + compact isometry orbits

Large gauge transformations: \[ \int_{\gamma} d\epsilon = 2\pi n ; \ n \in \mathbb{Z} \]

To fix them:

Multivalued Lagrange multiplier: \[ \int_{\gamma} d\tilde{\theta} = 2\pi m ; \ m \in \mathbb{Z} \]
such that
\[ \int [\text{exact}] \to dA = 0 \quad + \quad \int [\text{harmonic}] \Rightarrow A \text{ exact} \]

The gauging procedure works for all genera

(Rocek, Verlinde’91)
Non-Abelian T-duality

(De la Ossa, Quevedo’93)

Non-Abelian continuous isometry: \[ X^m \to g_n^m X^n, \; g \in G \]

i) Gauge it:
\[ dX^m \to DX^m = dX^m + A^m_n X^n \]
\[ A \in \text{Lie algebra of } G \quad A \to g(A + d)g^{-1} \]

ii) Add a Lagrange multiplier term:
\[ \text{Tr}(\chi F) \]
\[ F = dA - A \wedge A \]
\[ \chi \in \text{Lie Algebra of } G, \; \chi \to g\chi g^{-1}, \text{ such that} \]
\[ \int D\chi \to F = 0 \Rightarrow A \text{ exact} \]
(in a topologically trivial worldsheet)

+ fix the gauge: \[ A = 0 \Rightarrow \text{Original theory} \]
iii) Integrate the gauge field + fix the gauge \rightarrow \text{Dual theory}
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Example: Principal chiral model with group SU(2):

Geometrically: $S^3$

$$L = Tr(g^{-1}dg \wedge *g^{-1}dg);\ g \in SU(2)$$

Invariant under:

$$g \rightarrow h_1 \ g \ h_2;\ h_1, h_2 \in SU(2)$$

Choose:

$$g \rightarrow hg;\ h \in SU(2)$$

$$\tilde{L} = \frac{1}{1 + \chi^2} \left( \delta_{ij} - \epsilon_{ijk} \chi^k + \chi_i \chi_j \right) d\chi^i \wedge *d\chi^j$$

Invariant under

$$\chi \rightarrow h\chi h^{-1};\ h \in SU(2)$$
• Non-involutive
• Higher genus generalization? Set to zero \( W_\gamma = P e^{\gamma} A \)
• Global properties?

\[ \chi \in \mathbb{R}^3: \text{ Global completion of } \mathbb{R}^3? \]
• Conformal invariance not proved in general
• Non-involutive
• Higher genus generalization? Set to zero \( W_\gamma = P e^{f_\gamma} A \)
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**True symmetry in String Theory?**
• Non-involutive
• Higher genus generalization? Set to zero \( W_\gamma = Pe^{f_\gamma} A \)
• Global properties?
  \( \chi \in \mathbb{R}^3: \) Global completion of \( \mathbb{R}^3 \) ?
• Conformal invariance not proved in general

**True symmetry in String Theory?**

Still, interesting as a solution generating technique

(Sfetsos, Thompson’10)
4. Non-Abelian T-duality as a solution generating technique:

Need to know how the RR fields transform

In the Abelian case: Reduce to a unique $N=2, d=9$ SUGRA (Bergshoeff, Hull, Ortín’95)

Hassan’99: Implement the relative twist between left and right movers in the bispinor formed by the RR fields:

$$\hat{P} = P\Omega^{-1}$$

$$P = \frac{e^\phi}{2} \sum_k \frac{1}{k!} F_{\mu_1...\mu_k} \Gamma^{\mu_1...\mu_k}$$

with

$$\Omega = \sqrt{g_{00}^{-1} \Gamma_{11} \Gamma^0}$$

Same thing in the non-Abelian case (Sfetsos, Thompson’10)
Interesting new solutions have been found with CFT duals

But what if NATD is not a symmetry of ST?

Some of the properties of the CFT may no longer hold after adding corrections on the inverse ‘t Hooft coupling or $1/N$
Take the $AdS_6 \times S^4$ background

$$ds^2 = \frac{W^2 L^2}{4} \left[ 9ds^2(AdS_6) + 4(d\theta^2 + \sin^2 \theta ds^2(S^3)) \right]$$

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5. The non-Abelian T-dual of Brandhuber and Oz

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• Dualize it w.r.t. one of the $SU(2)$ symmetries
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$$F_4 = 5L^4 W^{-2} \sin^3 \theta \, d\theta \wedge \text{Vol}(S^3)$$

• Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = \frac{W^2 L^2}{4} \left[ 9 \, ds^2(AdS_6) + 4 \, d\theta^2 \right] + e^{-2A} \, dr^2 + \frac{r^2 \, e^{2A}}{r^2 + e^4 A} \, ds^2(S^2)$$

$$B_2 = \frac{r^3}{r^2 + e^4 A} \, \text{Vol}(S^2) \quad e^{-\phi} = \frac{3 L}{2 \, W^5} \, e^A \sqrt{r^2 + e^4 A}$$

$$F_1 = -G_1 - m \, r \, dr \quad F_3 = \frac{r^2}{r^2 + e^4 A} \left[ -r \, G_1 + m \, e^{4A} \, dr \right] \wedge \text{Vol}(S^2)$$
• It solves the IIB equations of motion
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• SUSY preserved! First example of a non-Abelian T-dual geometry with supersymmetry fully preserved
  This is because the internal symmetry is really $SU(2) \times SU(2)_R$ and we dualize on the $SU(2)$ global symmetry
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  This is because the internal symmetry is really $SU(2) \times SU(2)_R$ and we dualize on the $SU(2)$ global symmetry
• Boundary at $\theta = \frac{\pi}{2}$ inherited.
• It solves the IIB equations of motion

• SUSY preserved! First example of a non-Abelian T-dual geometry with supersymmetry fully preserved

  This is because the internal symmetry is really $SU(2) \times SU(2)_R$ and we dualize on the $SU(2)$ global symmetry

• Boundary at $\theta = \frac{\pi}{2}$ inherited.

• What about $r$?

  • Background perfectly smooth for all $r$
  • No global properties inferred from the non-Abelian transf.
  • Assume $r \in [0, R]$ (to avoid a continuous spectrum of fluctuations), and try to infer global properties by demanding consistency to the dual background
6. Hints on the 5d dual CFT

i) Quantization of charges:

\[ D4 \leftrightarrow D5 \ (N^\theta_5) \]
\[ D7 \ (N^\theta_7) \]
\[ D8 \leftrightarrow D7 \ (N^r_7) \]
\[ D5 \ (N^r_5) \]
6. Hints on the 5d dual CFT

i) Quantization of charges:

\[ \begin{align*}
\text{D4} & \leftrightarrow \text{D5} \quad (N_5^\theta) \\
\text{D7} & \leftrightarrow \text{D8} \quad (N_7^\theta) \\
\text{D7} & \leftrightarrow \text{D5} \quad (N_7^r)
\end{align*} \]

\[ N_5^\theta \text{ and } N_5^r \text{ depend on the large gauge transf. of } B_2: \]

\[ B_2 = \left( \frac{r^3}{r^2 + e^{4A}} - n\pi \right) \text{Vol}(S^2) \quad \text{with} \quad b = \frac{1}{4\pi^2} \int_{S^2} B_2 \in [0, 1] \]

(which seems, on the other hand, to undergo something reminiscent of the cascade in KS), in such a way that they cannot be integers for all \((r, \theta)\) unless \(n = 0\)
6. Hints on the 5d dual CFT

i) Quantization of charges:

\[ B_2 = \left( \frac{r^3}{r^2 + e^{4A}} - n\pi \right) \text{Vol}(S^2) \]

\[ b = \frac{1}{4\pi^2} \int_{S^2} B_2 \in [0, 1] \]

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This fixes the maximum value of \( r \) to \( r = \pi \)
ii) Probe the Coulomb branch:

2 directions $\leftrightarrow$ BPS D5 and D7 branes
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Fluctuations of these branes:

D5: $S_{DBI} = \int \frac{1}{g_{D5}^2} F_{\mu\nu}^2$, \quad \frac{1}{g_{D5}^2} = \frac{9 L^2 m^{-1/3} N_r^7}{128 \pi^3} \rho$

$S_{5dCS} = \frac{(2\pi)^3}{6} T_5 \int F_1 \int A \wedge F \wedge F = -\frac{N_r^7}{24 \pi^2} \int A \wedge F \wedge F$
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Consistently, we should find a wrapped brane with a tadpole given by the CS coefficient:

**D1-brane wrapped on** $r$ : 

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Consistently, we should find a wrapped brane with a tadpole given by the CS coefficient:

D1-brane wrapped on $r$: \quad $S_{CS} = -N_7^r \int A_t$

D7: Same with $N_7^r \leftrightarrow N_5^r$, D1 $\leftrightarrow$ D3 wrapped on $S^2$
ii) Baryon-like operators:

Dual to branes wrapped on the internal geometry with a tadpole proportional to the rank of the gauge group

In the D4-D8 background: D4-brane with N charge, projected out by the orbifold

In the non-Abelian dual: D1-brane with $N_7^\theta$ charge plus D3-brane (wrapped on $S^2$) with $N_5^\theta$ charge

Projected out by the dual orbifold

In any case they inform about the ranks of the dual gauge groups
iv) Putting it all together:

We seem to have two gauge groups with ranks $N_7^\theta$, $N_5^\theta$ and flavor symmetries $N_5^r$, $N_7^r$.

$N_5^\theta$ actually zero, such that the background is globally well defined.

Manifestation in the CFT of a perfectly regular background terminating at a point?
7. Conclusions and open issues

- Could a clear prescription for global properties lead to a regular background for arbitrary large gauge transformations, with non depleted gauge groups in the dual CFT?
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Depletion of the gauge group reminiscent of the cascade?
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Non-Abelian T-dual as an effective description? (Sfetsos’13)
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Depletion of the gauge group reminiscent of the cascade?

Non-Abelian T-dual as an effective description? (Sfetsos’13)

- Nature of the dual gauge groups: What is the orientifold projection in the dual theory?

\[ I_\theta \Omega \rightarrow I_\theta I_\chi \Omega : \]

Dual \( O_\rho^- \) located at \( \theta = \frac{\pi}{2}, r = 0 \)

D5-D7 system?
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Thanks!