THE FAST COLLISIONLESS RECONNECTION CONDITION AND THE SELF-ORGANIZATION OF SOLAR CORONAL HEATING

DMITRI A. UZDENSKY

Department of Astrophysical Sciences, Princeton University, Peyton Hall, Princeton, NJ 08544; uzdensky@astro.princeton.edu

Received 2007 May 19; accepted 2007 August 30

ABSTRACT

I propose that solar coronal heating is a self-regulating process that keeps the coronal plasma roughly marginally collisionless. The proposed self-regulating mechanism is based on the interplay of two effects. First, plasma density controls coronal energy release via the transition between the slow collisional Sweet-Parker regime and the fast collisionless reconnection regime. This transition takes place when the Sweet-Parker layer becomes thinner than the characteristic collisionless reconnection scale. I present a simple criterion for this transition in terms of the upstream plasma density \( n_e \), the reconnecting \( B_0 \) and guide \( B \) magnetic field components, and the global length \( L \) of the reconnection layer: \( L \leq 6 \times 10^9 \text{ cm} \left( n_e / 10^{10} \text{ cm}^{-3} \right)^{-3} (B_0 / 30 G)^{4/3} (B_0 / B)^2 \). Next, coronal energy release by reconnection raises the ambient plasma density via chromospheric evaporation, and this in turn temporarily inhibits subsequent reconnection involving the newly reconnected loops. Over time, however, radiative cooling gradually lowers the density again below the critical value and fast reconnection again becomes possible. As a result, the density is highly inhomogeneous and intermittent but, statistically, does not deviate strongly from the critical value, which is comparable with the observed coronal density. Thus, in the long run the coronal heating process can be represented by repeating cycles that consist of fast reconnection events (i.e., nanoflares), followed by rapid evaporation episodes, followed by relatively long periods (\( \leq 1 \text{ hr} \)) during which magnetic stresses build up and the plasma simultaneously cools down and precipitates.

Subject headings: magnetic fields — MHD — Sun: corona — Sun: flares — Sun: magnetic fields

1. INTRODUCTION

In this paper I address some aspects of solar coronal heating (see Aschwanden et al. 2001b and Klimchuk 2006 for recent reviews) in the context of Parker’s nanoflare model (Parker 1972, 1983, 1988). A more concise version of this work is presented in Uzdensky (2006, 2007).

Since the main heating process in the nanoflare model is magnetic reconnection, I first discuss what we have learned about reconnection in the past 20 years or so (see § 2). I argue that even though we still do not have a complete picture of reconnection, there now appears to be some consensus in the magnetic reconnection community about some of its fundamental aspects. One of the main goals of this paper is to use this emerging knowledge to shed some new light on the old coronal heating problem. In this paper I purposefully adopt a rather conservative approach: I invoke only those very few results that seem to be relatively firmly established and try not to rely on details of reconnection physics that are still under vigorous debate. Specifically, there is now strong evidence coming from numerous numerical simulations and some laborious laboratory experiments that there are two main modes of reconnection: slow Sweet-Parker reconnection taking place in collisional plasmas for which classical resistive MHD applies; and fast Petschek-like reconnection in collisionless plasmas (in § 2.1). The transition between these two regimes seems to be rather sharp. An approximate condition for this transition can be formulated as a relationship between the global length \( L \) of the reconnecting system and the electron collisional mean-free path inside the layer, \( \lambda_{\text{e, mfp}} \) (see § 2.2). Furthermore, this condition can be cast in terms of the plasma density \( n_e \), the reconnecting magnetic field \( B_0 \), and \( L \); that is, one can define a critical density \( n_c(L, B_0) \) below which reconnection switches to the fast regime. In the strong guide field case, \( B_x \gg B_0 \), the condition is modified; that is, the critical density for transition to fast reconnection is suppressed by a factor of order \( (B_x / B_0)^{2/3} \) (see § 2.4). In § 2.5 I address various alternative ideas and offer caveats that may complicate the simple picture given above.

I then apply these results to the active solar corona (in § 3), viewed within the nanoflare model. My main point here is that the corona should be regarded as a self-regulating machine keeping itself (in a statistical sense) around marginal collisionality. This conclusion comes from the interplay between the way the plasma density controls reconnection via the collisionless reconnection transition and the way the coronal magnetic energy release due to reconnection in turn controls the ambient gas density via chromospheric evaporation. The coronal heating process is then highly intermittent and inhomogeneous; it can be thought of as a sequence of characteristic energy circulation cycles that occur simultaneously in a broad range of spatial, temporal, and energy scales. Each such elementary cycle consists of several phases: (1) a fast reconnection event (a nanoflare), causing (2) an evaporation episode filling the loop with hot dense plasma, followed by (3) a longer period during which the magnetic stresses build up and the plasma density goes down due to slow radiative cooling (and thermal conduction). It is the collisionless reconnection condition that makes this scenario (in particular, the last phase) possible. Indeed, an important point here is that even if a current sheet is formed, it will stay in the slow Sweet-Parker state if the density is large enough. This will continue for some time, until the reconnecting magnetic field becomes strong enough and/or the ambient plasma density becomes low enough for the system to transition to the fast collisionless reconnection regime. This scenario enables a nontrivial amount of free magnetic energy to be accumulated before a sudden release. The amount of energy, along with the characteristic timescale between nanoflares and the characteristic coronal density, is determined (statistically)
by the balance between the rate at which current sheets are created and amplified by the photospheric footpoint motions and the efficiency of radiative cooling. The collisionless reconnection condition plays a key role in this process as a mediator and ultimately governs the statistical distribution of the coronal reconnection events (flares).

In § 4 I discuss some of the open questions that in my view need to (and can) be addressed in the near future, in order to see whether the physical picture put forward in this paper is correct and what modifications should be made to improve it. This section is mostly targeted toward researchers doing numerical simulations of reconnection and also toward experimentalists and observers. Finally, I present my conclusions in § 5.

I also would like to make a clarifying remark about the terms “collisional” and “collisionless” that are used many times throughout this paper. Often, by collisionless one means a regime in which “collisional” and “collisionless” that are used many times through-

2. FAST COLLISIONLESS RECONNECTION CONDITION

2.1. The Existence of Two Reconnection Regimes

Magnetic reconnection research started 50 years ago with the Sweet-Parker theory (Sweet 1958; Parker 1957) for solar flares. As was realized immediately at that time, this relatively simple and elegant theory could not reproduce the very short (~10^3 s) reconnection timescales required by flare observations; instead, it predicted reconnection times of order a few weeks or months. After several years, however, Petschek (1964) proposed a modification to the classic Sweet-Parker theory, which apparently resulted in a much higher reconnection rate, thus eliminating the main contradiction with observations. He realized that the main bottleneck in the Sweet-Parker reconnection model is the need to have a reconnection layer that is both thin enough for the resistivity to be important and thick enough for the plasma to be able to flow out. The result of this compromise is the famous Sweet-Parker scaling for the thickness \( \delta_{\text{SP}} \) of the reconnection layer and for the reconnection velocity \( v_{\text{rec}} \):

\[
\frac{\delta_{\text{SP}}}{L} = \frac{v_{\text{rec}}}{V_\Lambda} = \sqrt{\frac{\eta}{L V_\Lambda}} \equiv S^{-1/2},
\]

where \( L \) is the global length of the reconnection layer, \( \eta \) is the magnetic diffusivity, and \( V_\Lambda \) is the Alfvén speed corresponding to the reconnecting (in-plane) magnetic field component. A problem arises since the Lundquist number in the solar corona is usually very large (10^{12} or greater), the layer is very thin, and the resulting reconnection rate is very small. Furthermore, Petschek (1964) proposed that this difficulty can be circumvented if the reconnection region has a certain special structure: the famous Petschek configuration, with four slow-mode shocks attached to a small Sweet-Parker central diffusion region. As he showed, this structure yields an additional geometric factor that could lead to a much faster reconnection rate.

I would like to emphasize the special importance of Petschek’s idea for reconnection in astrophysical systems, including the solar corona. As I discuss below, there are several local physical effects (e.g., the Hall effect or anomalous resistivity) that can indeed broaden the reconnection layer and prevent it from collapsing down to the very small Sweet-Parker thickness \( \delta_{\text{SP}} \). However, each of these processes comes with its own microscopic physical scale, e.g., the ion gyroradius \( \rho_i \) or the ion collisionless skin depth \( d_i \equiv c/\omega_{pi} \). These scales are determined by the local values of the basic plasma parameters inside the reconnection layer, such as magnetic field, density, or temperature. The important point is that they know nothing about the overall global system size \( L \). Astronomical systems are often astronomically large, however. That is, \( L \) is typically much larger than any microscopic physical scale \( \delta \) and larger than the Sweet-Parker layer thickness (which is a hybrid length scale). Therefore, any simple Sweet-Parker–like analysis would give a reconnection rate \( v_{\text{rec}}/V_\Lambda \) scaling as \( \delta/L \ll 1 \); this would not be rapid enough to be of any practical interest. Thus, we come to conclusion 1: irrespective of the actual microphysics inside the layer, Petschek’s mechanism, or a variation thereof, is absolutely necessary for a sufficiently fast astrophysical reconnection, including solar flares (and even micro- and nanoflares).

Because Petschek’s model was able to reproduce the very short observed flare timescales, it quickly became popular, such that people believed in it for the next 20 years. With the advent of computer simulations, however, the original Petschek model came under fire. In particular, several numerical studies (e.g., Biskamp 1986; Scholer 1989; Ugai 1992, 1999; Ma & Bhattacharjee 1996; Uzdensky & Kulsrud 1998, 2000; Erkaev et al. 2000, 2001; Biskamp & Schwarz 2001; Birn et al. 2001; Malyshkin et al. 2005; Cassak et al. 2005, 2006) showed that in resistive MHD with uniform resistivity (and, by inference, with resistivity that is a smooth function of plasma parameters, e.g., Spitzer resistivity; see Biskamp & Schwarz 2001) Petschek’s mechanism fails and Sweet-Parker scaling applies instead. The basic physical reason for this, as has been elucidated analytically by Kulsrud (2001; see also Uzdensky & Kulsrud 2000 and Malyshkin et al. 2005) is that the transverse (i.e., perpendicular to the current sheet) magnetic field component, which is necessary to sustain Petschek’s shocks, is swept away by the Alfvénic flow out of the layer so rapidly that it cannot be regenerated by the nonuniform resistive merging of the reconnecting magnetic field component.

In addition, strong evidence for the existence of a slow Sweet-Parker reconnection mode in collisional resistive-MHD plasmas has been obtained from recent laboratory studies in the Magnetic Reconnection Experiment (MRX; Yamada et al. 1997) in the high-collisionality regime (Ji et al. 1998; Trintchouk et al. 2003; Kuritsyn et al. 2006).

This enables us to draw conclusion 2: In the collisional regime, when classical resistive MHD applies, Petschek’s fast reconnection mechanism does not work; slow Sweet-Parker reconnection process takes place instead.

A very natural question to ask next is whether fast Petschek-like reconnection is possible in a collisionless plasma, where resistive MHD does not apply. There is now a growing consensus that the answer to this question is yes. In solar and space physics, of course, there has long been plentiful observational evidence for fast collisionless reconnection: in solar flares (e.g., Tsuneta et al. 1984, 1992; Tsuneta 1996; Masuda et al. 1994, 1995; Shibata et al. 1995; Shibata 1996; Yokoyama et al. 2001) and in the Earth magnetosphere, both in the magnetopause (e.g., Mozer et al. 2002, 2003, 2004) and in the magnetotail (Oieroset et al. 2001; Nagai et al. 2001). The evidence for fast collisionless reconnection has been further significantly strengthened by recent laboratory measurements in MRX (Ji et al. 1998, 2004; Yamada et al. 2006) and other experimental setups (e.g., Brown 1999; Egedal et al. 2007;
Bz. This laboratory measurements, however, have not been able to elucidate the special role of the Petschek mechanism in accelerating reconnection. On the other hand, over the past decade or so several theoretical and numerical studies have indicated that fast reconnection enhanced by the Petschek mechanism (or a variation thereof) does indeed take place in the collisionless regime. Moreover, it appears that there may be two mechanisms of collisionless reconnection. Physically, these two mechanisms are very different from each other; nevertheless, they both appear to lead to the establishment of a Petschek-like configuration, which enhances the reconnection rate. The two regimes in question are:

1. Hall-MHD reconnection, involving two-fluid effects in a laminar flow configuration (e.g., Mandt et al. 1994; Kleva et al. 1995; Biskamp et al. 1995, 1997; Ma & Bhattacharjee 1996; Lottermoser & Scholer 1997; Shay & Drake 1998; Shay et al. 1999, 2001; Hesse et al. 1999; Bhattacharjee et al. 1999, 2001; Birn et al. 2001; Rogers et al. 2001; Pritchett 2001; Breslau & Jardin 2003; Huba & Rudakov 2004; Cassak et al. 2005, 2006; Daughton et al. 2006).

2. Spatially localized anomalous resistivity due to plasma microturbulence that is excited when the current density exceeds a certain threshold (e.g., Coppi & Friedland 1971; Smith & Priest 1972; Coroniti & Eviatar 1977; Kulsrud 2001, 2005; Kulsrud et al. 2005). This seems to lead to a Petschek configuration with the length of the inner diffusion region on the order of the resistivity localization scale (e.g., Ugai & Tsuda 1977; Sato & Hayashi 1979; Ugaü et al. 1996, 1999; Scholer 1989; Biskamp & Schwarl 2001; Erkaev et al. 2001; Kulsrud 2001, 2005; Uzdensky 2003; Malyshkin et al. 2005). The corresponding reconnection rate is then fast enough to explain the relatively short timescales observed in solar flares (e.g., Kulsrud 2001, 2005; Uzdensky 2003).

Recent experimental evidence from MRX indicates that both anomalous resistivity (e.g., Ji et al. 2004) and the Hall effect (Ren et al. 2005; Yamada et al. 2006; Brown et al. 2006; Egedal et al. 2007) are present and may be important in collisionless reconnection. However, at present it is still not clear which one of these two mechanisms works in a given physical situation (if at all). Also not known is whether these two mechanisms can operate simultaneously in a given system and how they interact with each other.

In any case, from the point of view of our present discussion, the important thing is that one does get a Petschek-enhanced fast reconnection if the plasma is collisionless (in the sense specified below; see § 2.2). Thus, we can draw conclusion 3: Petschek-enhanced fast reconnection does happen in the collisionless regime.

To sum up, there are two regimes of astrophysical magnetic reconnection: slow Sweet-Parker reconnection in resistive MHD with classical collisional resistivity; and fast Petschek-like reconnection in collisionless plasmas. In other words, in order for reconnection to be fast, it needs Petschek’s mechanism to operate, and that in turn requires the reconnection layer to be collisionless.

In fact, whenever we observe violent and rapid energetic phenomena that we interpret as reconnection, it is always in relatively tenuous plasmas. I am not aware of any counterexamples and would be would be very interested to learn about them. Is there any evidence for fast large-scale reconnection events in collisional astrophysical environments?

2.2. The Fast Reconnection Condition: Zero Guide Field Case

How can one quantify the transition between the two regimes? For clarity, let us first consider the case with zero guide field, Bz = 0. (In the Bz ≠ 0 case some of the arguments and results presented in this section are somewhat modified; however, the main concepts and conclusions remain similar, as I show in the next section; see § 2.4.)

When there is no guide field (strictly antiparallel field merging), the condition for fast reconnection can be formulated roughly as (Ma & Bhattacharjee 1996; Kulsrud 2001, 2005; Uzdensky 2003; Cassak et al. 2005; Yamada et al. 2006).

\[
\delta_{SP} < \frac{d_i}{c} = \frac{\omega_{pi}}{\epsilon}.
\]  

What this condition means is the following. As a reconnection layer forms, its thickness δ becomes smaller and smaller. If condition (2) is not satisfied, this thinning saturates at δ = δSP; then reconnection proceeds in the slow Sweet-Parker regime. On the other hand, if condition (2) is satisfied, then various two-fluid and/or kinetic effects kick in as soon as δ drops down to about \( d_i \) or so, i.e., well before the resistive term becomes important. Then the reconnection processes necessarily involves collisionless, nonclassical resistive MHD physics. This results in a rapid increase in the reconnection rate, as has been recently documented experimentally in MRX (Yamada et al. 2006). Note that, by construction, \( \delta_{SP} \) entering in equation (2) is to be calculated using the classical Spitzer resistivity. Therefore, and since we are talking about collisional and collisionless reconnection, it is instructive to express the above condition in terms of the classical electron mean free path due to Coulomb collisions, \( \lambda_{e, mfp} \). It is pretty straightforward to show (Yamada et al. 2006) that

\[
\frac{\delta_{SP}}{d_i} \sim \left( \frac{L}{\lambda_{e, mfp}} \right)^{1/2} \left( \frac{m_e}{m_i} \right)^{1/4} \beta_e^{1/2},
\]  

Here \( \beta_e \) is the ratio of the electron pressure inside the layer to the pressure of the reconnecting magnetic field component \( B_0^2/8\pi \) outside the layer. Thus, using equation (2), we see that a reconnection layer is collisionless when

\[
L < L_c \equiv \lambda_{e, mfp} \sqrt{\frac{m_i}{\beta_e m_e}} \approx 40 \beta_e^{1/2} \lambda_{e, mfp}.
\]  

The collisionless reconnection condition in a similar form was first published by Yamada et al. (2006). Their condition actually differs from equation (4) by a factor of 2. Since the discussion here is very qualitative, I regard this difference as unessential. Moreover, to make the present discussion clearer, I systematically ignore numerical factors of order 1 everywhere in this paper.

We can go a little bit further. The mean free path \( \lambda_{e, mfp} \) can be written as

\[
\lambda_{e, mfp} \sim \frac{k_B^2 T_e^2}{n_e e^4 \log \Lambda} \approx 7 \times 10^7 \text{ cm} n_{10}^{-1} T_7^2,
\]  

where we set the Coulomb logarithm equal to 20 and where \( n_{10} \) and \( T_7 \) are the central layer electron density \( n_e \) and temperature \( T_e \), given in units of \( 10^{10} \text{ cm}^{-3} \) and \( 10^7 \text{ K} \), respectively. Combining equations (4) and (5), the criterion for fast collisionless reconnection can now be formulated as a condition on the layer’s length \( L \) in terms of the values of \( n_e \) and \( T_e \) at the center of the Sweet-Parker reconnection layer:

\[
L < L_c \equiv 40 \beta_e^{1/2} \lambda_{e, mfp} \approx 3 \times 10^9 \text{ cm} \beta_e^{-1/2} n_{10}^{-1} T_7^2.
\]  

Note that the critical layer length \( L_c \) strongly depends on the central electron temperature \( T_e \). The main cause for this high
sensitivity is the strong temperature dependence of the electron mean free path and hence Spitzer resistivity. It is therefore very important to figure out what the electron temperature inside the Sweet-Parker reconnection layer should be. In a situation in which the ambient (upstream) pressure is already high, the Sweet-Parker reconnection layer should be. In a situation in which the central electron temperature in terms of \( B_0 \) and \( n_e \) as

\[
T_e = \frac{B_0^2}{2 \kappa n_e} \simeq 1.4 \times 10^7 \text{ K} \quad (B_{1.5})^{1/3} n_{10}^{-1},
\]

where \( B_{1.5} \) is the outside magnetic field \( B_0 \) expressed in units of 30 G. On substituting this expression into equation (5), we get

\[
\beta \equiv \frac{8 \pi n_e k_B (T_e + T_i)}{B_0^2} = 1.
\]

Next, assuming that \( T_e \) inside a Sweet-Parker reconnection layer is approximately equal to (and in any case not much smaller than) the ion temperature \( T_i \), with the self-consistency of this assumption demonstrated at the end of this section, we see that \( \beta \) should generally be close to \( 1 \) (and in any case of order 1).

Then, using the pressure-balance condition (7), we can express the central electron temperature in terms of \( B_0 \) and \( n_e \) as

\[
T_e = \frac{B_0^2}{2 \kappa n_e} \simeq 1.4 \times 10^7 \text{ K} \quad (B_{1.5})^{1/3} n_{10}^{-1},
\]

where \( B_{1.5} \) is the outside magnetic field \( B_0 \) expressed in units of 30 G. On substituting this expression into equation (5), we get

\[
\frac{\lambda_{\text{c,mfp}}}{1.5} \times 10^6 \text{ cm} \quad n_{10}^{-3} B_{1.5}^4
\]

and so the fast collisionless reconnection condition can be written in the final form as

\[
L < L_c (n_e, B_0) \simeq 6 \times 10^9 \text{ cm} \quad n_{10}^{-3} B_{1.5}^4.
\]

We see that the condition \( L < L_c \) is often satisfied in the solar corona (e.g., in solar flares) and is definitely always satisfied in the Earth magnetosphere.

2.3. Plasma Temperature and Density inside the Sweet-Parker Reconnection Layer

Note that the density that enters in the above expressions is, by definition, the density at the center of the reconnection layer. In general, it is not known a priori, and we would like, instead, to get expressions involving the background (upstream) density. Equation (7) gives us one relationship between the two desired quantities (central \( n_e \) and \( T_e \)). However, we need to know both of them, and by itself equation (7) does not tell us whether the central pressure is increased (from a very low ambient value) to the required equilibrium level by raising the density or the temperature. I now argue, however, that in the regime that is relevant here, the latter is more likely to be the case; i.e., the pressure increase is mostly due to the increased temperature, whereas the density should change relatively little.

Indeed, as is evident from the above discussion, the main reason for the strong density dependence in equation (10) is the \( T_e \) dependence of the classical Spitzer resistivity, combined with the pressure balance condition (7). Therefore, what we are mostly interested in here are the central electron density and temperature in the context of the resistive Sweet-Parker theory. To determine them, we need more detailed information. Namely, we need to consider one more equation that we have not discussed up until now—the equation of energy conservation. In general, this equation should take into account ohmic heating and all the relevant loss mechanisms, such as advection, radiation, and electron thermal conduction.

Let us consider a fluid element as it travels through the reconnection layer. During its transit, the fluid element is subject to ohmic heating and to all the above loss mechanisms. The characteristic time for the advection term is, of course, just the Alfvén transit time \( \tau_A \sim L/V_A \)—the time a fluid element spends inside the layer. During this time the element is continuously heated by ohmic heating, whose rate per unit volume can be estimated within the Sweet-Parker theory as

\[
\eta' \zeta^2 \sim \eta' \left( \frac{c B_0}{4 \pi \delta} \right)^2 \sim \frac{B_0^2}{4 \pi \delta} \eta' S \sim \frac{B_0^2}{4 \pi \delta} \frac{L}{V_A},
\]

where the magnetic diffusivity \( \eta \) is related to the resistivity \( \eta' \) via \( \eta = \eta' c^2/4\pi \). Thus, during its transit, the fluid element acquires just the right amount of heat to raise its temperature to about the equipartition level given by equation (8) (Uzdensky 2003). We now need to see under what conditions one can ignore various loss mechanisms, namely, radiation and thermal conduction.

First, it is easy to see that, in the case of the solar corona, the radiative losses are indeed negligible on the timescales of interest \( \tau_A \). Indeed, the radiative cooling time is determined by the cooling function \( Q(T) \) via

\[
\tau_{\text{rad}} < \frac{2 c_e n k_B T}{n^2 Q(T)} = \frac{3 k_B T}{n Q(T)}.
\]

In the temperature range of interest to the solar corona, \( 10^6 \text{ K} \leq T_e \leq 10^8 \), the cooling function varies between \( 10^{-23} \) and \( 10^{-22} \) ergs s\(^{-1}\) cm\(^3\) (e.g., Rosner et al. 1978; Priest 1984, p. 88; Cook et al. 1989). Then, taking \( n = 10^{10} \text{ cm}^{-3} \), the typical radiative cooling times are of order 10 minutes (for \( T = 2 \times 10^6 \) K) and longer (up to many days for \( T = 10^8 \) K). In any case, this is much longer than the characteristic Alfvén crossing time \( \tau_A \), which is usually a few seconds.

On the other hand, the smallness of the radiative losses in other astrophysical situations cannot be taken for granted, especially in high-energy astrophysics applications, such as coronae of accretion disks around black holes and neutron stars, as well as gamma-ray bursts. If radiative losses are important, then the above estimates for the central temperature no longer apply, and correspondingly the condition for transition to the fast reconnection regime must be modified.

Next, going back to the solar corona, we need to consider the effect of electron thermal conduction on the central temperature. Note that electrons are still well magnetized throughout most of the reconnection layer (this will be even more so in the presence...
of a guide field). Therefore, the electron thermal conduction is highly anisotropic, and so we estimate the effects of parallel and perpendicular thermal conduction separately. We start with the heat conduction along the magnetic field. Since we are interested in the collisional case, \( \lambda_{e, \text{mfp}} < L \), the parallel heat transport is in the diffusive regime and can be described by a one-dimensional random walk of electrons along the magnetic field. The characteristic parallel thermal conduction timescale can then be estimated as

\[
\tau_{e, \text{cond.}} \sim \frac{\lambda_{e, \text{mfp}}}{v_{e, \text{th}}} \left( \frac{L}{\lambda_{e, \text{mfp}}} \right)^2 = \frac{L}{v_{e, \text{th}}} \frac{L}{\lambda_{e, \text{mfp}}} \sim \tau_A \sqrt{\left( \frac{m_e}{\beta m_i} \right) \frac{L}{\lambda_{e, \text{mfp}}}},
\]

where \( \tau_A \) and \( \beta \) correspond to the reconnecting field \( B_0 \). Using expression (4), we can write this expression as

\[
\tau_{e, \text{cond.}} \sim \tau_A \frac{L_{\text{ext}}}{L_{\text{e}}} \beta^{-1}.
\]

In the magnetically dominated environment of the solar corona, where the thermal pressure outside of the reconnection region is small, we expect \( \beta \sim 1 \); it could conceivably be smaller, but we never expect it to be significantly higher than 1. Therefore, we estimate that

\[
\tau_{e, \text{cond.}} \gtrsim \tau_A \frac{L}{L_{\text{e}}},
\]

Thus, we see that if we are in the regime of interest—that is, in the expected regime of applicability of the collisional Sweet-Parker theory \( (L > L_{\text{e}}) \)—then \( \tau_{e, \text{cond.}} \gtrsim \tau_A \), and hence the energy losses due to parallel electron thermal conduction are not important.

A similar line of argument can be made to show that the characteristic time for perpendicular (across the reconnection layer) electron thermal conduction is in fact automatically always comparable to the Alfvén crossing time. Indeed, the cross-field electron conduction timescale in the Sweet-Parker regime can be estimated as

\[
\tau_{e, \text{cond.} \perp} = \frac{\delta_{\perp} L_{\text{e}, \perp}}{D_{e, \perp}},
\]

where \( D_{e, \perp} \sim \rho_e^2 v_{e, \text{th}} / \lambda_{e, \text{mfp}} \) is the classical electron collisional diffusivity across the magnetic field. Using \( \beta_e \sim 1 \), we see that \( \tau_{e, \text{cond.} \perp} \) should scale as

\[
\tau_{e, \text{cond.} \perp} \sim \tau_A \frac{\delta_{\perp}^2}{D_{e, \perp}} \sqrt{\frac{m_i}{m_e} \frac{\lambda_{e, \text{mfp}}}{L}}.
\]

Upon plugging equation (3) into this expression, we immediately find that all of the factors on the right-hand side cancel, and we simply see that

\[
\tau_{e, \text{cond.} \perp} \sim \tau_A,
\]

i.e., that the two timescales are automatically comparable to each other.

Combined with our previous result regarding the effect of the parallel thermal conduction, we conclude that energy losses due to both parallel and perpendicular electron thermal conduction out of the layer are, at best, only marginally important in the collisional Sweet-Parker regime. Heat losses are still present but are not likely to cause a decrease in the central electron temperature by more than a factor of order unity. This means that the jump in the plasma pressure, required to maintain the pressure balance with the outside reconnecting magnetic field, should be attributed mostly to the increase in the plasma temperature (due to ohmic heating), whereas the density should not vary across the layer by more than a factor of a few. Thus, for our rough estimates, the density that enters in our estimate (8) for the central temperature can be taken to be the ambient plasma density. As long as we are in the collisional regime, \( L > L_{\text{e}} \), this result is consistent with the energy balance inside the layer that includes ohmic heating, heat advection by the bulk plasma outflow, and electron thermal transport. As we see in the next section, this will be true even in the presence of a guide field.

It is also interesting to check that the assumption \( T_i \sim T_e \) is not strongly violated. In principle, this could be a worry, since the collisional electron-ion energy-equilibration rate is suppressed by the large mass ratio; hence, in general, the electron and ion temperatures in the layer need not be equal. For example, ions might have been much hotter than the electrons and hence might have provided the bulk of the pressure support against the outside magnetic field. The electron temperature in this case would have been far below the equipartition value (about \( 10^7 \) K). Correspondingly, the electron mean-free path would be much lower than that given by equation (9).

The electron-ion temperature equilibration time, \( \tau_{\text{EQ}} \), is longer by a factor of \( m_i / m_e \) than the electron collision time, \( \tau_A \). Making use of the pressure balance \( c_s \sim (T_e / m_e)^{1/2} \sim V_A \), we can express \( \tau_e \) in terms of \( \lambda_{e, \text{mfp}} \) and the Alfvén speed as

\[
\tau_{e} \sim \frac{\lambda_{e, \text{mfp}}}{v_{e, \text{th}}} \sim \frac{\lambda_{e, \text{mfp}}}{c_s} \sim \frac{m_e}{m_i} \sim \frac{V_A}{V_A} \sim \frac{m_e}{m_i}.
\]

Then, we can compare \( \tau_{\text{EQ}} \) with the Alfvén crossing time, which is the characteristic time a fluid element spends inside the layer:

\[
\tau_{\text{EQ}} \sim \tau_A \frac{\lambda_{e, \text{mfp}}}{L} \sim \frac{m_e}{m_i} \sim \tau_A \frac{L_{\text{e}}}{L}.
\]

Thus, we see that, if we are in the collisional regime as defined by equation (4), the electrons and ions experience enough collisions with each other to equalize their temperatures while they transit through the layer. Thus, \( T_i \sim T_e \) should be a decent approximation in the Sweet-Parker regime.

Note that the plasma density also enters the collisionless reconnection condition via \( d_i \). The straightforward logic of the above picture dictates that this quantity is to be estimated in the collisionless regime. However, it is plausible that a corresponding estimate based on the collisionless regime also has some relevance; in this case it is interesting to compare the two. The task of estimating the central \( n_e \) in the collisionless regime is more difficult than estimating it in the Sweet-Parker regime, since there are many uncertainties here. However, it is also less important, since most of the strong density dependence in equation (10) comes from the steep temperature (and hence, indirectly, density) dependence of the Spitzer resistivity. Nevertheless, we still shall try to discuss the question of whether the density can vary significantly across a collisionless reconnection layer. If the fast collisionless reconnection is due to anomalous resistivity, then the effective \( \lambda_{e, \text{mfp}} \) is determined by wave-particle collisions and hence will be relatively short. This will result, again, in the suppression of parallel electron thermal conduction and hence in high temperature and, correspondingly, according to the pressure balance condition (7), in a relatively mild density change with respect to the upstream value (see also Uzdensky 2003). Then the
above estimates are likely to remain valid (although in this case one might have to deal with a possibility that $T_i \gg T_e$).

On the other hand, if the fast reconnection is due to the Hall effect, then we also may expect the density to be roughly uniform across the layer. This is because the ions become demagnetized on scales less than $d_i$ and hence cannot develop structures on smaller scales. The dynamics of ions inside the reconnection layer can be summarized by noting that they are pulled directly into the layer by the bipolar electric field, which accelerates them up to the Alfvén speed, and then are ejected out along the layer by the Lorentz force associated with the quadrupole out-of-plane magnetic field (e.g., Uzdensky & Kulsrud 2006). At any rate, one does not expect to see a significant variation of ion, and hence electron, density across the Hall reconnection layer (apart from the $d_i$-thick density depletion layers that run along the separatrices, as reported by Shay et al. 2001).

Finally, I would like to make a comment about the use of the term “collisionless.” As I mentioned in § 1, in the present paper this term is understood in the sense of equation (4). That is, a system is called “collisionless” when $\lambda_{e, \text{mfp}} \geq L/40$, roughly speaking. This definition is different from the condition $\lambda_{e, \text{mfp}} > \rho_e$ that is often used. Note that, when $\lambda_{e, \text{mfp}} > \rho_e$, or, equivalently, $\Omega_e > \nu_e$, then the Hall term in the generalized Ohm law formally cannot be neglected. In plasmas of interest to this paper, e.g., in the solar corona, this condition is always very well satisfied. In this sense, the coronal plasma is definitely always collisionless. Our collisionless reconnection condition (2), on the other hand, is not always satisfied. In fact, there is a large and astrophysically important region in parameter space where $\delta_{\text{SP}} > d_i$, but where the plasma is nevertheless collisionless in the sense that $\Omega_e > \nu_e$, and where therefore the Hall effect may be significant (see also Cassak et al. 2005). In the above discussion I have assumed that in this intermediate regime, reconnection still proceeds slowly, in the purely resistive Sweet-Parker mode, as suggested by numerous resistive-MHD simulations. One should be aware, however, that most of these simulations completely ignore the Hall effect; thus, it is not clear to what extent their results can be relied on in this intermediate regime. In fact, the recent numerical work by Cassak et al. (2005), who include both resistivity and Hall effect, indicates that both slow Sweet-Parker and fast Hall modes may operate in the intermediate regime, depending on the previous history of the system. In particular, they observe that, when one considers a gradual initial thinning of the reconnection layer, the system always stays in the slow regime, even if $\Omega_e > \nu_e$, as long as $\delta_{\text{SP}} > d_i$. Since it is exactly this scenario that is of interest in the context of our model, we feel we can use condition (2) with confidence.

2.4. Reconnection with a Guide Field

In the real world the zero guide field case considered in the previous section is rare. It may be encountered in some special circumstances; for example, it may be relevant to reconnection in the Earth magnetotail. Generally, however, there is some guide field, $B_0 \neq 0$, present. This is especially so in the context of the nanoflare model for heating of solar coronal loops by the braiding of individual elemental flux strands comprising a larger loop. In this case the guide field (i.e., the axial field along the general loop direction) is always present and is generally much stronger than the reconnecting field $B_0$. Another situation in which the guide field is dominant over the reconnecting magnetic field is laboratory plasma devices, such as tokamaks. Note that reconnection releases the energy of the weaker $B_0$-field; the energy of the guide field is not available. However, the presence of a guide field, especially if it is strong, may, in principle, change the structure of the reconnection layer. Therefore, in this section we consider the effect of the guide field on our collisionless reconnection condition.

The presence of the guide field modifies the collisionless reconnection condition. This is because, whereas the characteristic layer thickness in resistive MHD is not affected by the guide field and is still given by the same Sweet-Parker scaling equation (1), the characteristic scale for the collisionless reconnection layer changes. In particular, according to the published literature (e.g., Kleva et al. 1995; Rogers et al. 2001; Cassak et al. 2007), the layer thickness for Hall collisionless reconnection becomes the ion acoustic Larmor radius, $\rho_s \sim c_s/\Omega_i$, where $\Omega_i$ corresponds to the total upstream magnetic field and where $c_s \sim (T_e/m_i)^{1/2}$ is the ion-acoustic speed. This result has been recently confirmed experimentally (Egedal et al. 2007). Then the criterion for the onset of fast reconnection in the Hall-MHD regime becomes (e.g., Cassak et al. 2007)

$$\delta_{\text{SP}} < \rho_s.$$  \hspace{1cm} (20)

According to the general scheme presented in the preceding section, we first need to consider the Sweet-Parker layer. Again, we want to estimate the value of the Spitzer resistivity at the center of the reconnection layer. Therefore, we need to determine the central electron temperature. One of the difficulties here is that, if there is a guide field, then the cross-layer force-balance in the form of equation (7) is no longer applicable. This is because the pressure of the reconnecting component of the outside magnetic field, $B_0^2/8\pi$, no longer has to be balanced at the center of the layer by the plasma pressure alone. It can be partially balanced by an increase in the guide field pressure due to a modest compression. In particular, a very strong guide field, $B_0 \gg B_i$, dominates the pressure both inside and outside of the current layer and can be changed only slightly. Therefore, it essentially ensures incompressibility: $n_i \approx n_i^{\text{out}}$. The temperature then decouples from the pressure balance and has to be determined from some other considerations, namely, from the energy equation. Fortunately, most of the analysis presented in the previous section can be carried over to the strong guide field case without much change. In particular, the plasma experiences Joule heating as it travels through the layer, and as long as one is in the collisional regime, the parallel electron thermal conduction is not able to cool the layer by a large factor. Therefore, we expect the temperature inside the layer to increase roughly to the equipartition level given by equation (8). However, this equation can no longer be regarded as equality, but just as a rough estimate. For the purposes of this paper, however, this is good enough.

Now, let us discuss the right-hand side of condition (20). If the guide field is not very large compared with the reconnecting field, then one expects $\rho_s \sim d_i$, and hence criterion (2) is not changed substantially. However, in the strong guide field case, we expect to have $\rho_s \ll d_i$ (assuming, as we always do throughout this paper, that upstream $\beta \ll 1$). Then the criterion for transition to fast collisionless Hall reconnection is modified substantially. It becomes

$$\delta_{\text{SP}} < \rho_s \sim d_i \left( \frac{8\pi n_i T_e}{B_0^2} \right)^{1/2} \sim d_i \beta^{1/2} \frac{B_0}{B_i},$$  \hspace{1cm} (21)

where $\beta$ is based on the central electron temperature $T_e$ and density and on the upstream reconnecting field component $B_0$. Note that according to logic of our paper, this $\beta$ is to be estimated in the collisional regime. Nevertheless, it may also be useful to get some feeling for what $\beta$ (and hence $\rho_s$) should be immediately
after the transition to fast reconnection. As we noted above, a strong guide field makes the plasma nearly incompressible, so that \( n_e \) is roughly uniform. The question then is what one should take for the electron temperature inside the layer? There is a considerable uncertainty here. In particular, there is no good reason for \( T_i \) to be close to \( T_e \) in the collisionless regime. For example, Hsu et al. (2001) present experimental evidence for strong ion heating \( T_i \gg T_e \) inside a collisionless reconnection layer in the no guide field case in MRX, although the rise in \( T_i \) is considerably smaller in the presence of a guide field. In any case, with all these reservations, the simplest approach one can adopt is to take \( \beta_e \sim 1 \) in the above expression. Then, the condition for fast collisionless reconnection in the strong guide field case becomes

\[
\frac{\delta_{GR}}{\rho_e} \sim \left( \frac{L}{\lambda_{e,\text{mfp}}} \right)^{1/2} \left( \frac{m_e}{m_i} \right)^{1/4} \frac{B_z}{B_0} < 1, \tag{22}
\]

which, after our usual manipulations, can be rewritten as

\[
L < L_c(B_z) = \sqrt{\frac{m_i}{m_e}} \lambda_{e,\text{mfp}} \left( \frac{B_0}{B_z} \right)^2 \simeq 6 \times 10^9 \text{ cm} n_{i0}^{1/3} B_{1.5}^4 \left( \frac{B_0}{B_z} \right)^2. \tag{23}
\]

Thus, the critical global length \( L_c \) in this case becomes smaller by a factor \((B_z/B_0)^2\) and so the collisionless reconnection condition is harder to satisfy. Also note that, for fixed \( n_e \) and \( B_z \), \( L_c \) becomes very sensitive to the reconnecting field component: \( L_c \sim B_0^6 \).

### 2.5. Caveats and Alternatives

With all the above said, there are still many caveats and alternative ideas that may potentially alter the above picture. It is important to be aware of them and try to gauge their effect on the existence and the form of the fast collisionless reconnection condition. I discuss some of them in this section.

First, there are a number of ideas that may, if correct, represent a potential challenge to our conclusion 2 (that collisional resistive-MHD reconnection is slow). For example, Lazarian & Vishniac (1999) suggested that reconnection may be relatively rapid in pure resistive MHD in the presence of externally imposed three-dimensional (3D) MHD turbulence (see, however, Kim & Diamond 2001 for a dissenting view). As far as I know, this idea has not yet been tested in 3D numerical simulations, although it is definitely an interesting possibility that deserves further study. Such studies are indeed already underway (E. T. Vishniac 2007, private communication). Furthermore, even in the two-dimensional (2D) case, a resistive-MHD Sweet-Parker–like current layer may become unstable to tearing instability when its aspect ratio exceeds a certain large number (probably 100; Bulanov et al. 1978; Forbes & Priest 1983; Biskamp 1988; Lee & Fu 1986; L. M. Malyshkin 2005, private communication; Loureiro et al. 2007). Tearing may effectively result in an enhanced hyperresistivity that would dominate over the normal resistivity (e.g., Strauss 1988). What the effect of the nonlinear development of this instability will be on the overall reconnection rate is still not clear. In addition, Dahlburg et al. (1992) identified a mechanism, based on a secondary ideal MHD instability, that leads to the excitation of fully 3D turbulence within a reconnecting current layer. Finally, in a recent numerical simulation with both uniform resistivity and the Hall effect, Cassak et al. (2005) observed an interesting hysteresis phenomenon: as the resistivity was turned down, there was a transition from slow Sweet-Parker to fast Hall reconnection, in accordance with condition (2). However, as the resistivity was turned back up to a level higher than that corresponding to equation (2), the system still stayed in the fast mode. This puzzling behavior needs to be investigated further.

Second, there are recent studies relevant to conclusion 3 (that collisionless reconnection is fast). In particular, even though most of the authors agree that Hall reconnection is much faster than Sweet-Parker, there is still a significant disagreement about exactly how fast it actually is. For example, Drake and his collaborators report a universal reconnection rate of about 0.1 \( V_A \), independent of the electron microphysics ultimately responsible for breaking the field lines (e.g., Biskamp et al. 1997; Shay & Drake 1998; Shay et al. 1999, 2001; Cassak et al. 2005; see also Birn et al. 2001). On the other hand, others argue that it should not be universal but should instead be a function of the system parameters, such as the electron/ion mass ratio (e.g., Bhatcharchya et al. 2001). In addition, Daughton et al. (2006) recently raised a concern about the validity of periodic boundary conditions used in the majority of previous numerical studies. In particular, they argued that the reported very high reconnection rate may be an artifact of such boundary conditions. Furthermore, they performed 2D particle simulations with the appropriate open boundary conditions and observed that after an initial transient peak consistent with the high value reported by Shay et al. (1999, 2001), the reconnection rate went down and settled at a significantly lower level (Daughton et al. 2006). Similar conclusions were reached independently by Fujimoto (2006) using periodic 2D particle simulations, but in a very large computational domain. After a year-long debate, however, this controversy finally seems to have been resolved. It is now agreed by both sides that the electron dissipation region develops a two-scale structure with an extended high-velocity electron jet in the outflow direction; however, the dimensionless reconnection rate still remains high (\( r_{rec}/V_A \gg d/L \)), although perhaps not quite as high as previously believed (e.g., Shay et al. 2007; Karimabadi et al. 2007).

Finally, I would like to mention that the picture presented in the previous two sections, implies certain physical conditions and thus is not expected to be universally valid in all astrophysical environments. For example, in the case of weakly ionized plasma in molecular clouds (e.g., in the context of star formation), reconnection dynamics is strongly modified by recombination processes and may become significantly faster than Sweet-Parker even in the collisional regime (Zweibel 1989; Heitsch & Zweibel 2003). Also, in the case of reconnection of superstrong magnetic fields in magnetar magnetospheres (\( B_0 \approx 10^{14} \text{ G} \)), the density of the released magnetic energy is so large that the pressure inside the layer is dominated by radiation. The resulting temperature is so high that a large number of electron-positron pairs are produced; this means that the number of particles is not conserved, and so the above estimates would have to be modified (see, e.g., Thompson 1994; Lyutikov & Uzdensky 2003; Uzdensky & MacFadyen 2006).

Other issues that are worth mentioning and that may substantially complicate the simple physical picture presented in this paper include impulsive, bursty reconnection (e.g., Bhatcharchya et al. 1999, 2001; Daughton et al. 2006; Karimabadi et al. 2007) and reconnection in three dimensions (e.g., Longcope 1996).

### 3. Solar Coronal Heating

#### 3.1. General Picture

The physical picture presented in this paper has a number of interesting implications for the solar corona, which I discuss in this section. At present, the theory of magnetic reconnection has not yet reached the level of maturity that would allow us to make
accurate predictions regarding when the transition between the slow and fast reconnection modes occurs. Moreover, there is still no consensus on the exact nature of the fast collisionless reconnection mechanism or on the actual scalings of the reconnection rate in either fast or slow reconnection (see § 2.5 for more discussion). However, what is important for us in this paper is just the fact of existence of such a transition and that it has to do with the plasma collisionality. The transition does not actually have to be razor sharp, and in reality there may be not two but more different regimes with different reconnection rates and hence more than one such transition. Also, the transition condition may depend on other physical parameters such as the presence of a guide field (see § 2.4) or, perhaps, the effectiveness of radiative cooling, which we have omitted in this paper. Whatever the case, the existence of two reconnection regimes with a collisionality-related transition between them is a very important fact and we would like to exploit it in various astrophysical applications. In this section, I apply this condition (namely, eqs. [4] and [10]) to the problem of solar coronal heating. I propose that coronal heating is a self-regulating process that works to keep the corona marginally collisionless, in the sense of equations (2) and (4) (see also Uzdensky 2006, 2007).

According to Parker’s nanoflare theory (Parker 1972, 1983, 1988), as long as the twisting and braiding of coronal loops by photospheric footpoint motions and flux emergence episodes keep producing thin current sheets in the corona (e.g., van Ballegooijen 1986), magnetic dissipation in these current sheets leads to continuous coronal heating. In reality, this heating, of course, is not uniformly distributed; it is instead strongly intermittent, localized in both space and time (Rosner et al. 1978, hereafter RTV78). This basic picture of coronal heating has been shown to work in several 3D MHD numerical simulations (e.g., Galsgaard & Nordlund 1996; Hendrix et al. 1996; Gudiksen & Nordlund 2002, 2005).

Now, the overall, integrated heating density, i.e., the rate of magnetic dissipation per unit volume, depends on the reconnection rate in these sheets, e.g., on whether reconnection is fast (e.g., Petschek) or slow (Sweet-Parker). This is actually a rather subtle point. Indeed, in a steady state, the energy dissipated in the corona per unit time should be equal to the power pumped into the corona magnetically from the solar surface. And it is not obvious why or how the energy-pumping rate depends on what happens in the corona. For example, if the corona were enclosed in a fixed volume, then the energy dissipation per unit volume would be fixed (as in driven MHD turbulence in a box). However, it is important to recognize that the coronal volume is not fixed! If, for example, reconnection were to suddenly slow down, more energy would be pumped in than the corona could dissipate; then, in order to accommodate this additional free magnetic energy, the corona would respond by increasing its scale height. That is, coronal magnetic structures would grow in height until, finally, the total dissipation in the corona became equal to the total input from the photosphere. Because of the increased volume, the magnetic dissipation per unit volume would decrease.

In addition, as coronal magnetic structures grow in height, especially when \( H \gtrsim R_\odot \), the amount of energy pumped into the corona by the footpoint motions may go down. This is because the work done by footpoint motions is proportional to \( \mathbf{v}_{fp} \cdot \mathbf{B}_{\text{foot}}B_{\text{vert}}/4\pi \). As the coronal structures grow in height without increasing their lateral size, the horizontal field, \( B_{\text{foot}} \), decreases whereas the vertical field component, \( B_{\text{vert}} \), does not change. Correspondingly, the overall power pumped from the photosphere into the corona goes down.

As follows from equation (10), the reconnection mode is determined by the global scale \( L \) of the reconnection layer and by the basic physical parameters characterizing the plasma in the layer (i.e., \( n_e, T_e, \) and \( B_0 \)). The typical values of \( L \) and \( B_0 \) are determined by the scale and strength of the magnetic structures emerging from the Sun and by the scale of footpoint motions (e.g., the mesogranular scale). Therefore, for the purposes of the present discussion, I regard \( L \) and \( B_0 \) as fixed and then ask what determines the coronal density and temperature.

Following this line of reasoning, let us invert equation (10) and view it as the condition for the plasma density. That is, let us introduce a scale-dependent critical density, \( n_c \), below which the reconnection process transitions from the slow collisional Sweet-Parker regime to a fast collisionless regime:

\[
  n_e(B_z \lesssim B_0) \sim 2 \times 10^{10} \text{ cm}^{-3} B_0^{4/3} L_9^{-1/3},
\]

where \( L_9 \) is the global reconnection layer length expressed in units of \( 10^9 \) km. In the strong guide field case, \( B_z \gg B_0 \), the corresponding expression becomes

\[
  n_e(B_z \gg B_0) \sim 2 \times 10^{10} \text{ cm}^{-3} B_0^{4/3} L_9^{-1/3} \left( \frac{B_0}{B_z} \right)^{2/3}.
\]

It is interesting to note that these values are close to those observed in active solar corona. I suggest that this is not a pure coincidence.

The reason for this is that there is an important feedback between coronal energy release and the coronal density. This feedback is due to the fact that the gas high in the corona actually comes from evaporation from the surface along the field lines that just underwent reconnection. In fact, it is essential to consider the coronal heating not only as the process of increasing the coronal plasma’s temperature, but also as that of increasing its density (e.g., Klimchuk 2006; Aschwanden et al. 2007). Indeed, a part of the energy released in a reconnection event is rapidly conducted along the field by fast (perhaps nonthermal) electrons to the surface and is deposited in a denser photospheric and chromospheric plasma. This leads to a localized heating at the footpoints of the postreconnected magnetic loops and to a subsequent chromospheric evaporation along them, a well-documented phenomenon in solar corona (e.g., Aschwanden et al. 2001a, 2001b; and Klimchuk 2006; see, however, Brown et al. 2000 and Aschwanden et al. 2007 for evidence that chromospheric evaporation occurs in response to chromospheric, rather than coronal, heating events). As a result, these loops become filled with a dense and hot plasma. The density rises and may now exceed \( n_c \). This shuts off any further reconnection (and hence heating) involving these post-reconnective loops until they again cool down, which occurs on a longer, radiative timescale.

Let us consider a couple of examples of how this might work. First, let us consider the case when the plasma density is relatively high, and so the radiative cooling time, \( \tau_{\text{rad}} \), is relatively short compared with the timescale on which the coronal magnetic structures change due to footpoint motions, \( \tau_{\text{fp}} \); however, the cooling time is, of course, still long compared with the fast reconnection timescale. The opposite limiting case is discussed later in this section. Let us suppose that due to field-line twisting, a reconnecting structure is set up in the corona at \( t = 0 \). Let this structure be characterized by the current sheet length \( L \) and the reconnecting field component \( B_0 \). Furthermore, let us further
suppose that, initially, the ambient density of the background plasma is higher than \(n_c(L, B_0)\). Then the reconnection layer is collisional and reconnection proceeds very slowly, in the Sweet-Parker mode. That is, there is almost no reconnection at all, and hence coronal heating is weak. The surrounding plasma gradually cools by radiation (and also, in general, by thermal conduction), and the pressure scale height gradually goes down. The gas gradually precipitates to the surface. Then the density of the plasma entering the layer decreases and at some point becomes lower than the critical density. The reconnection process then suddenly switches to the collisionless regime. Petschek-like fast reconnection ensues, and the rate of magnetic energy dissipation greatly increases. A flare commences. Some fraction of the energy released by reconnection is transported by the electron conduction along the reconnected field lines down to the base, where it is deposited in the much denser photospheric or chromospheric plasma. This in turn leads to a massive evaporation along the same field lines. As a result, the newly reconnected loops are now populated with relatively dense, hot plasma. They cool down only slowly via conduction and radiation losses, keeping their relatively high density for an appreciable length of time. If, during this time, these loops become twisted or somehow get in contact with other loops, they are now not likely to reconnect rapidly, since their plasma density is above critical. This inhibits further coronal heating in the given region. In fact, we can speculate that for any further outbursts of coronal activity in the given region to occur, one has to wait for the gas in postreconnection loops to cool down significantly, which occurs on a longer, radiative timescale.

Thus, we see that, although highly intermittent and inhomogeneous, the corona is working to keep itself roughly at about the height-dependent critical density given by equation (24) or (25). Correspondingly, the background coronal temperature should be such that results in a density scale height that is just large enough to populate the corona up to the critical density level at a given height. In this sense, coronal heating regulates itself (Uzdensky 2006, 2007).

An important remark is that loop brightness in soft X-rays and UV does not necessarily mean that this loop is actively undergoing fast reconnection at the given moment of time. Instead, it just reflects the fact that the loop has a high plasma density (since radiation intensity is proportional to \(n^2\)). This implies an episode of strong evaporation from the surface in the recent past, presumably caused by a preceding fast-reconnection coronal energy release in this loop. The nearby coronal regions that are darker clearly have lower density, and they would be ripe for reconnection from the point of view of the collisionless reconnection condition. However, they are probably filled with a magnetic field that does not have complex topology with current sheets, since the underlying reason for coronal activity is always the generation of small coronal current sheets as a result of field-line braiding driven by a complicated pattern of the footpoint motions on the solar surface.

The above radiative-cooling–dominated scenario was presented here first only because it brings out more clearly the role that plasma density plays in controlling the reconnection regime. It also highlights the role of chromospheric evaporation in controlling the plasma density. However, there is an alternative version of this picture corresponding to the opposite situation, \(\tau_{fp} < \tau_{rad}\). In this case, one can repeat the same arguments as those given above, but instead of the slow initial evolution of the plasma density due to gradual radiative cooling, invoke the slow evolution of the reconnecting magnetic field strength caused by the motion of the loop footpoints (as described by Cassak et al. 2006). In other words, one can regard the ambient plasma density \(n\) as fixed on the timescale of interest, whereas the reconnecting field component \(B_0\) gradually increases (for simplicity we also keep \(L\) fixed). Then the current sheet gradually builds up, while staying in the Sweet-Parker mode, and then rapidly switches over to the fast collisionless mode once \(B_0\) has exceeded a certain threshold. This scenario is in fact closer in spirit to the original Parker nanoflare model (Parker 1983, 1988) and also to the discussion by Cassak et al. (2005, 2006). Instead of defining \(n_c(L, B_0)\), one can just as well rewrite the fast reconnection condition in terms of a critical current-sheet strength, i.e., the critical reconnecting magnetic field component, \(B_c\), expressed as a function of \(L\) and \(n\):

\[
B_c(n, L) \approx 20 \, G \, L_0^{1/4} \, n_1^{3/4}.
\]

In the case of a strong guide field, \(B_z > B_0\), the critical reconnecting field is

\[
B_c(n, L, B_z) \approx 30 \, G \, L_0^{1/6} \, n_1^{1/2} \, B_z^{1/3}.
\]

where \(B_{z,2} \equiv B_z/(100 \, G)\).

As I show below, in the long run one can expect some sort of statistical steady state with \(\tau_{fp} \sim \tau_{rad}\. Then this second scenario is just as likely to take place as the first one. In fact, in reality they probably both occur alongside each other.

I would like to emphasize that the phenomenon described in the first part of this paper (i.e., the transition from slow to fast collisionless reconnection) is not the whole coronal heating story, although, in my view, it is an important part of it. Conceptually, it is probably about one-quarter of the whole story. The other important pieces of the puzzle are (1) the complex dynamic interaction of a large number of coronal magnetic loops, which is ultimately responsible for the generation of current sheets in the corona by footpoint motions; this requires the development of an appropriate statistical description of loops (e.g., Uzdensky & Goodman 2007); (2) chromospheric evaporation following a reconnection event and motions of plasma along the loop; and (3) radiative (and thermal-conduction) energy losses and the resulting precipitation of the material back to the solar surface. All these essential physics ingredients (or at least some effective representation of them) will eventually have to be incorporated into a single comprehensive theoretical (probably numerical) model of the solar corona.

It is important to note that each of these processes comes with its own characteristic timescale. The reconnection time in the fast regime, \(\tau_{rec}\), is very short. For the purposes of our discussion here, we regard it as instantaneous. The characteristic time for material evaporation following a nanoflare is also relatively short. The two longer timescales in the problem are the characteristic timescale for generation and growth of new current sheets by photospheric footpoint motions, \(\tau_{fp}\), and the radiative cooling timescale \(\tau_{rad}\). As we show below, the system tends to adjust itself to a statistical equilibrium in which these two timescales are comparable (of order 1 hr in the solar corona). Finally, the longest timescale of all is the Sweet-Parker reconnection time. It is much longer than 1 hr, and is therefore irrelevant in our problem. That is, the energy release during the Sweet-Parker phase is not important. However, this does not mean that the existence of the slow Sweet-Parker regime is not important; it is in fact extremely important, since it allows for a strong current sheet to build up, accumulating a large amount of free magnetic energy.

This discussion brings us to one of the most interesting issues in solar coronal heating—intermittency. The key questions here
are the following: What is the distribution of (spatial, temporal, and energy) scales on which magnetic dissipation occurs? What scales are responsible for most of the energy release? In my opinion, the collisionless reconnection condition is, at the end of the day, very important for answering these questions. In particular, I think this condition plays a key role in establishing the spectrum of the energy release events, e.g., the observed power-law distribution of flare energies.

3.2. Long-Term Evolution of a Coronal Loop

To illustrate the above points, let us consider the following model problem. One can raise the question of whether the self-regulating behavior of the nonlinear dynamical system described above can exhibit an oscillatory behavior (E. N. Parker 2006, private communication), especially in light of a hysteresis-like phenomenon reported by Cassak et al. (2005). Of course, in general this issue is difficult to assess within the present model, since it first needs to be incorporated in the overall multiloop dynamics (e.g., Uzdensky & Goodman 2008, in preparation); that is, the energy release in a given fast reconnection event does not, strictly speaking, directly affect the reconnection process that produced it. However, it does lead both to the relaxation of the magnetic stress and also, importantly, to the increase in density in the postreconnected flux tube (or more precisely, in the elementary magnetic thread within the larger flux tube). This will in turn temporarily inhibit any reconnection involving this given field thread even if a new current sheet is formed. This inhibition will last for some time, until either the plasma density goes down because of the radiative cooling or the reconnecting field component in the new current sheet becomes stronger.

Let us consider a simple illustrative example that demonstrates this oscillatory behavior and the secular evolution of the system on a longer timescale. Consider a composite coronal flux tube, initially filled with a relatively low plasma density $n$. For simplicity, let us imagine that this tube consists of two separate elementary flux threads that are being wrapped around each other steadily by the sheared photospheric motions. Consequently, a current sheet gradually builds up inside the tube, in the spirit of the nanoflare model. Again for simplicity, let us assume that the length $L$ of this current sheet and the axial magnetic field strength $B_z$ in the tube stay constant at all times. Then, as the elemental flux threads are wrapped around each other, the reconnecting field component, $B_{\theta 0}$, gradually increases. Because the plasma density is low, the radiative cooling time is relatively long, and therefore we can regard the plasma density as remaining constant during this phase. On the other hand, also because the density is low, the critical field strength is also low, according to equation (27). That is, the current sheet will become collisionless relatively early, at some small value of $B_{\theta 0}$. Correspondingly, the amount of free magnetic energy attributed to the current sheet at that moment will also be small, and the resulting reconnection event will be relatively weak. After the reconnection event, the magnetic field becomes unwrapped again (almost completely, since the hysteresis behavior reported by Cassak et al. 2005, 2007, implies that fast reconnection proceeds to the end); then the footpoint-driven process of current-sheet buildup starts all over again. Now, if the plasma density in the tube under consideration were fixed at a constant small value $n_0$, then (since $L$ and $B_z$ are also kept fixed) all the subsequent fast reconnection events would be triggered at the same critical value of $B_{\theta 0} = B_{\theta 0}(n_0, L, B_z)$. Correspondingly, they would all be characterized by a relatively small amount of energy released, on the order of $V B_{\theta 0}^2(n_0, L, B_z)/8\pi$, where $V$ is the volume of the region involved in the given reconnection event. At the same time, since weaker current sheets are easier to produce, they would be following each other at a relatively rapid rate.

However, because of the mass exchange between the surface of the Sun and the corona, the ambient density is not fixed a priori. In particular, if one starts with a small initial density, the first few reconnection events will be energetically small, but, nevertheless, they will eventually lead to an increase in the plasma density via chromospheric evaporation. This will make the critical magnetic field stronger, and hence the amount of free energy stored between reconnection events and released through them larger. At the same time, such events will be more rare, since it takes longer to build up a stronger current sheet. Therefore, an increase in plasma density shifts the dominant energy-release scale toward larger energies. Thus, in this picture, the plasma density in the tube increases over time in a step-like manner: it increases by a certain amount in the aftermath of each reconnection event and stays roughly flat between them. The amount of plasma pumped into the corona as a result of each event is, roughly speaking, proportional to the magnetic energy released in this event. The latter is in turn a monotonically increasing function of density, according to equation (27). That is, keeping $L$ and $B_z$ fixed, we have $\delta n \sim B_z^2(n) \sim n$. Thus, the relative increase is then independent of density; it can be roughly estimated as follows: Consider a loop of constant cross-sectional area $L^2$ and length $L > L_0$; the loop’s volume is then $V_{\text{loop}} \sim L/L_0^2$. Assuming that the reconnection layer extends all the way along the loop, the volume $V$ of the region involved in the reconnection event is similar: $V \sim V_{\text{loop}} \sim L/L_0^2$. Then the magnetic energy released in the event is of order $E_{\text{rec}} \sim (B_z^2/8\pi )L/L_0^2$. Next let us say that a certain fraction $\epsilon < 1$ of this energy goes toward filling the entire loop with plasma evaporated from the surface. The coefficient $\epsilon$ depends on the portion of the released energy that goes to the energetic electrons and is transported by them to the photosphere times the fraction of this deposited energy that goes to material evaporation (as opposed to that immediately radiated away from the footpoints). Overall, $\epsilon$ describes the fraction of the flare energy that eventually will be radiated by postcoronal loops as soft X-rays, UV, etc., but not hard X-rays. Immediately after the evaporation episode, the energy $\epsilon E_{\text{rec}}$ is divided between the increases in the gravitational energy $\delta E_{\text{grav}}$, and in the thermal energy $\delta E_{\text{th}} \approx 3nkBT_{\text{loop}}V_{\text{loop}}$. Then the magnetic energy released in the coronal loop plasma (for simplicity of discussion, I drop the contribution to $\delta E_{\text{th}}$ associated with the increase in the gas temperature; in reality, of course, both $n$ and $T_{\text{loop}}$ should increase together). If the height of the loop is larger than the density scale height, then these two contributions are likely to be comparable, according to the virial theorem. However, in the case of a compact loop, whose height is less than the thermal scale height, the thermal energy increase will be larger than $\delta E_{\text{grav}}$. In either case, we expect $\delta E_{\text{th}} \sim \epsilon E_{\text{rec}}$, from which we immediately find

$$\frac{\delta n}{n} \sim \epsilon \frac{B_z^2}{8\pi } \frac{1}{3nkBT_{\text{loop}}} = \frac{2\epsilon}{3\beta_{\text{loop}}},$$

(28)

where $\beta_{\text{loop}} \equiv 16\pi n_0kBT_{\text{loop}}/B_z^2$ is the characteristic plasma-beta based on the critical reconnecting field. Substituting expression (27) for $B_z$, we get

$$\beta_{\text{loop}} \approx 0.07n_0T_{\text{loop}}B_{\text{c,15}}^2 \sim 0.07T_{\text{loop}}B_{\text{c,2}}^2L^{-1/3},$$

(29)

and so

$$\frac{\delta n}{n} \sim 10\epsilon T_{\text{loop}}^{-1}B_{\text{c,2}}^{2/3}L^{-1/3}.$$
There are also numerical factors of order unity that we have ignored. It is reasonable to expect that this ratio should be roughly of order unity. For now let us just assume that it is not large, i.e., $\delta n/n \lesssim 1$.

As mentioned above, in a realistic scenario the loop temperature should also increase at each step, in concert with the density. This means that to get into the coronal loop, chromospheric gas would need to be heated to a temperature that is higher than at the previous step. Then each erg of the released energy would be able to lift a smaller amount of gas, i.e., the evaporation efficiency of coronal heating would decrease over time. To illustrate this point, let us, for example, adopt the famous RTV (Rosner-Tucker-Vaiana) scaling for the loop temperature (RTV78), even though it was derived assuming steady heating and thus is not valid for the impulsive situation considered here (see below for more discussion). Recasting the RTV78 scaling in terms of the electron density instead of the pressure, we write $T_{\text{loop}} \propto 3 \times 10^6 \ K \ n_{10}^{2/3} L^{1/2}$. Substituting this into equation (30), we get

$$\frac{\delta n}{n} \sim 3\epsilon n_{10}^{-1/2} B_{z,5}^{2/3} L_9^{1/3} L_{\parallel,9}^{-1/2} \sim n^{-1/2}. \quad (31)$$

Next, the time separation between the steps also increases with the increased threshold magnetic field. In particular, let us assume that the footpoint driving twists up the field lines and thus creates and then strengthens current sheets (or perhaps, quasi-separatrix layers; see, e.g., Galsgaard & Nordlund 1996; Titov & Hornig 2002) at some constant rate. This is, of course, an oversimplification. In reality, current sheets may be created as a consequence of the nonlinear development of kinklike instabilities of twisted magnetic loops (Dahlburg et al. 2005). Then, the growth of $B_0$ with time is not likely to be a smooth function, but instead may involve sudden jumps. Here, however, we would like to avoid such complications. Thus, let us consider, as an illustration, a simple model in which the reconnecting field component increases linearly between reconnection events,

$$\frac{dB_0}{dt} = \gamma B_z, \quad (32)$$

where $\gamma = \text{const}$. Then, using equation (27), the time $\delta t$ it takes the reconnecting field to grow to a given critical value $B_c(n)$ (after a complete relaxation during the preceding reconnection event) is $\delta t = \gamma^{-1} B_c(n)/B_z \sim n^{1/2}$. Therefore, as long as the relative increase in density at each step is not large, the long-term ($t \gg \delta t$) evolution of the system can be approximately described by a differential equation,

$$\frac{dn}{dt} \sim \frac{\delta n(n)}{\delta t(n)} \sim \text{const}, \quad (33)$$

corresponding to a linear rise $n(t) \sim t$. Accordingly, the emission measure of the loop should increase as $t^2$. [Alternatively, if we neglect the evolution of the loop temperature in the above analysis and instead keep $T_{\text{loop}} = \text{constant}$, we then get $\delta n \sim n$, and hence $n(t) \sim t^2$, with the emission measure rising as $t^4$.]

This growth will continue until one of the following two processes intervenes: First, it may happen that as the density builds up, the critical value of $B_0$ will become so large (e.g., a sizable fraction of $B_z$), that the equilibrium shape of the entire loop will be affected. For example, the loop may become external-kink unstable and become helical (sigmoidal loop), which with further twisting may result in a large-scale disruption with a catastrophic energy release, i.e., a large flare. This scenario, by the way, suggests that coronal loops should gradually (on the timescale of hours) brighten up before a major disruption, because a higher plasma density (manifested as a higher emission measure) leads to a higher critical field strength and hence allows a larger amount of free magnetic energy to be accumulated without being prematurely dissipated via smaller reconnection events.

On the other hand, there is another, less violent outcome that is also possible. Indeed, so far we have been neglecting radiative cooling. However, as the plasma density gradually builds up due to a series of chromospheric evaporations caused by reconnection events, the radiative cooling time decreases (as $n^{-1}$). At the same time, as we have seen above, the time interval between subsequent reconnections becomes longer and longer. Eventually, a point may be reached when the amount of plasma precipitated during $\delta t$ will become equal to the amount of material injected into the loop during an evaporation episode. After that point, there will be no further net secular change in the coronal density; the system will attain a stable limit cycle behavior. To evaluate the exact conditions characterizing this state one will need to calculate the amount of material evaporated following a given small flare and the rate at which the plasma returns to the photosphere as a result of gradual cooling. This would require a detailed model of the thermal structure along the loop (e.g., along the lines of RTV78), but this can (and should!) definitely be done. For now, however, we just want to get some qualitative feeling. Therefore, let us just say tentatively that the system enters this stable cyclic regime (with no secular density gain) when the radiative cooling time becomes equal to the time between reconnection events. This yields the following condition:

$$\delta t(n) = \gamma^{-1} B_c(n)/B_z = \tau_{\text{rad}}(n). \quad (34)$$

This equation can now be viewed as a condition that determines the long-term equilibrium plasma density inside the loop, $n_*$. Substituting equation (12) for the radiative cooling time and equation (27) for $B_c$, we obtain $n_*$ as a function of the loop temperature $T_{\text{loop}}$, $\gamma$, $L$, and $B_z$:

$$n_* \simeq 2 \times 10^{10} \ \text{cm}^{-3} \ B_{z,5}^{4/9} L_9^{1/9} (\gamma \tau_{\text{rad},0})^{2/3} \left( \frac{T_{\text{loop},6}}{Q_{-22}} \right)^{2/3}, \quad (35)$$

where $Q_{-22} \equiv Q(T_{\text{loop}})/(10^{-22} \ \text{ergs cm}^{-3} \ \text{s}^{-1})$ and where, for convenience, we defined $\tau_{\text{rad},0}$ as the radiative cooling time corresponding to $n_{10} = 1$, $T_{\text{loop}} = 1 \ \text{MK}$, and $Q_{-22} = 1$: $\tau_{\text{rad},0} \approx 400 \ \text{s}$.

Correspondingly, the characteristic threshold reconnecting magnetic field is

$$B_{0,*} = B_c(n_*) = 45 \ \text{G} \ L_9^{1/9} B_{z,5}^{4/9} (\gamma \tau_{\text{rad},0})^{1/3} \left( \frac{T_{\text{loop},6}}{Q_{-22}} \right)^{1/3}, \quad (36)$$

and the characteristic time interval between subsequent reconnection events is

$$\Delta t_* = \tau_{\text{rad}}(n_*) \simeq 200 \ \text{s} \ B_{z,5}^{-4/9} L_9^{1/9} (\gamma \tau_{\text{rad},0})^{-2/3} \left( \frac{T_{\text{loop},6}}{Q_{-22}} \right)^{1/3}. \quad (37)$$
The characteristic amount of energy released in each reconnection event comprising the limit cycle is

\[ E_s \approx \frac{B_{0L}^2}{8\pi} \approx 8 \times 10^{28} \text{ ergs} \]

\[ \times L_{||}^2 L_{\perp}^{2+2/9} B_{z,2}^{10/9} (\gamma_{\text{rad},0})^{2/3} \left( \frac{T_{\text{loop},6}}{Q_{-22}} \right)^{2/3}, \]  

(38)

and hence the time-averaged rate of magnetic dissipation in the loop is

\[ H_s = \frac{E_s}{\Delta E_s} \approx 4 \times 10^{26} \text{ ergs s}^{-1} \]

\[ \times L_{||}^{2+1/9} L_{\perp}^{14/9} B_{z,2}^{14/9} (\gamma_{\text{rad},0})^{4/3} \left( \frac{T_{\text{loop},6}}{Q_{-22}} \right)^{1/3}. \]  

(39)

Note that most of the above parameters have weak dependence on \( L \), except for \( E_s \) and \( H_s \), whose strong \( L \) dependence is almost entirely due to the volume involved.

To make any further progress, we need two more relationships: (1) the cooling function \( Q(T) \) and (2) a scaling for the loop temperature \( T_{\text{loop}} \). Note that both are subject to significant uncertainties. In particular, the radiative cooling function exhibits a complicated behavior in the relevant temperature range (1–10 MK); for example, according to Cook et al. (1989), it decreases sharply between \( T = 1 \) and 3 MK, and then stays nearly flat for \( T > 3 \) MK. Therefore, in principle, one should not expect simple power-law scalings to result at all. Furthermore, there are still significant disagreements in the literature regarding \( Q(T) \) (e.g., Raymond et al. 1976; Raymond & Smith 1977; RTV78; Cook et al. 1989; Landi & Landini 1999; Aschwanden et al. 2000b). For these reasons, any realistic quantitative analysis of radiative cooling in the context of the present model is a highly nontrivial task, adding even more ambiguity to this already complicated picture. Getting into such a high level of sophistication and detail would exceed the overall level of accuracy of our model. We shall therefore leave it for a future study.

In addition to the cooling function, we need an expression for the loop temperature, \( T_{\text{loop}} \), which strongly affects plasma cooling, in terms of the other loop parameters. In principle, the temperature, or, more generally, its distribution along the loop, is determined by the hydrostatic balance in conjunction with the energy transport along the loop, which includes thermal conduction, radiative losses, and distributed heating. Such an analysis was first performed by RTV78 for the simplest case of uniform and steady heating; as a result, they derived the famous RTV scaling for the loop-top temperature: \( T_{\text{loop}}^{\text{RTV}} = 1.4 \times 10^{13} (p_L)_{1/3} \), where \( p \) is the loop pressure and \( L \) is its length (which is different and usually larger than our current-sheet length \( L_{||} \) in our model, we regard it as constant). Now, it is true that there is a lot of disagreement in the modern literature regarding the applicability of the RTV scaling to the real solar corona, and the observational support is doubtful (e.g., Porter & Klimchuk 1995; Cargill et al. 1995; Kano & Tsuneta 1995; Aschwanden et al. 2000a, 2001a). This is not surprising given that the RTV scaling was derived assuming stationary and uniform heating, and thus ignored the impulsive and intermittent nature of coronal energy dissipation (which is an intrinsic property of our model). In addition, to develop their theory, RTV78 relied on an old cooling function due to Raymond (see below), whereas somewhat different cooling functions have been used in recent years (Cook et al. 1989; Landi & Landini 1999; Aschwanden et al. 2000b).

Nevertheless, despite its significant limitations, the RTV scaling has been highly influential in solar physics. It is still a highly respected common standard, against which solar physicists often measure their theories and observations. Therefore, just to present an illustrative example, let us now combine the RTV scaling with our model. First, to be consistent, we need to adopt the cooling function due to Raymond (Raymond et al. 1976; Raymond & Smith 1977) that was used by RTV78:

\[ Q^{\text{RTV}}(T) \approx 2 \times 10^{-22} T_6^{-2/3} \text{ ergs cm}^3 \text{ s}^{-1}, \]  

(40)

applicable in the range 2–10 MK (see Appendix A of RTV78).

Substituting this cooling function into our scalings (35)–(39), we get

\[ n_s \approx 10^{10} \text{ cm}^{-3} B_{z,2}^{4/9} L_{||}^{1/9} L_{\perp}^{10/9} (\gamma_{\text{rad},0})^{2/3}, \]  

(41)

\[ B_{z,2} \approx 40 \text{ G} B_{z,2}^{5/9} L_{||}^{1/9} L_{\perp}^{5/9} (\gamma_{\text{rad},0})^{1/3}, \]  

(42)

\[ \Delta L_s \approx 160 \text{ s} B_{z,2}^{-4/9} L_{||}^{1/9} L_{\perp}^{5/9} (\gamma_{\text{rad},0})^{-2/3}, \]  

(43)

\[ E_s \approx 5 \times 10^{28} \text{ ergs} L_{||} L_{\perp}^{2+2/9} B_{z,2}^{10/9} (\gamma_{\text{rad},0})^{2/3}, \]  

(44)

\[ H_s \approx 3 \times 10^{26} \text{ ergs s}^{-1} L_{||} L_{\perp}^{2+1/9} B_{z,2}^{14/9} (\gamma_{\text{rad},0})^{4/3}. \]  

(45)

Next, recasting the RTV loop-temperature scaling in terms of the electron density instead of the pressure,

\[ T_{\text{loop}}^{\text{RTV}} \approx 3 \times 10^6 \text{ K} n_{10}^{1/2} L_{||}, \]  

(46)

and substituting it into the above scalings, we get

\[ n_s \approx 6 \times 10^{20} \text{ cm}^{-3} B_{z,2} L_{||}^{1/4} L_{\perp}^{5/4} \gamma_{-3}^{3/2}, \]  

(47)

\[ T_{\text{loop}}^{\text{RTV}} \approx 7 \times 10^6 \text{ K} B_{z,2}^{1/2} L_{||}^{1/8} L_{\perp}^{5/8} \gamma_{-3}^{3/4}, \]  

(48)

\[ B_{z,2} \approx 80 \text{ G} B_{z,2}^{5/8} L_{||}^{1/8} L_{\perp}^{5/8} \gamma_{-3}^{3/4}, \]  

(49)

\[ \Delta t_{\text{RTV}} \approx 900 \text{ s} B_{z,2}^{1/8} L_{||}^{1/4} L_{\perp}^{5/8} \gamma_{-1/4} \gamma_{-3}^{3/4}, \]  

(50)

\[ E_s \approx 2.5 \times 10^{29} \text{ ergs} L_{||}^{4/4} L_{\perp}^{2+1/2} B_{z,2}^{5/3} \gamma_{-3}^{3/2}, \]  

(51)

\[ H_s \approx 2.6 \times 10^{26} \text{ ergs s}^{-1} L_{||}^{13/8} B_{z,2}^{11/6} \gamma_{-7/4}, \]  

(52)

where, in addition, we introduced \( \gamma_{-3} \equiv \gamma_{-3} \times 10^3 \) s and used \( \tau_{\text{rad},0} = 400 \text{ s} \).

A similar exercise could be conducted, for example, for an entirely different cooling function (Cook et al. 1989; Landi & Landini 1999; Aschwanden et al. 2000b) and/or for a different scaling law (e.g., that derived by Cargill et al. [1995] or, observationally, by Kano & Tsuneta [1995]).

These estimates give us the characteristic scales of micro- and nanoflare-like energy-release events in a bright coronal loop of a fixed length \( L \), fixed cross-loop spatial scale \( L \), and a fixed axial magnetic field \( B_{z,2} \), subject to continuous braiding at a fixed rate \( \gamma \). It is of course understood that this is just an idealized model setup and that in practice using such typical values straightforwardly may not be meaningful because of the highly inhomogeneous nature of the solar corona.

\[^3\] Note that, to derive their scaling, RTV78 actually used an approximation of the Raymond cooling function: \( Q(T) \sim T^{-1/2} \).
4. DISCUSSION: WHAT NEEDS TO BE DONE

In this section I discuss several open questions relevant to the physical picture presented in this paper—questions that I would like to see answered in the near future. Correspondingly, I describe possible studies (mostly numerical) that I think ought to be performed in order to confirm, modify, or refute various elements of my model.

First, the whole picture hinges, in part, on the premise that reconnection is slow in the collisional regime. Although in the above discussion I have for definiteness assumed that it is as slow as in the classical Sweet-Parker model, I do not actually think that this needs to be the case. If the true rate of classical resistive MHD reconnection turns out to be much faster than Sweet-Parker, the overall picture would still stand, qualitatively, as long as this rate is still much smaller than the collisionless reconnection rate. The only thing that matters is that there are two regimes, slow and fast, and the transition between them is governed by the collisionality of the reconnection layer.

Nevertheless, from the point of view of my picture, studies of slow collisional (resistive MHD) reconnection are just as important as those of fast collisionless reconnection. One would like to tighten up the case for slow reconnection. Thus, I would like to encourage more studies of resistive reconnection, to really confirm that it is slow. Thus, it is important to address the following specific issues, best using numerical simulations:

1. Does the presence of small-scale 3D MHD turbulence enhance reconnection rate, as suggested by Lazarian & Vishniac (1999)? If it does, how large is the enhancement and what determines it? Could it be as fast as collisionless reconnection, in which case it would constitute a challenge to the picture presented in this paper? To address this issue, one would have to perform a 3D MHD simulation involving a regular large-scale reconnection layer with a superimposed 3D MHD turbulence. Just as importantly, one needs to investigate the possibility that 3D MHD turbulence is spontaneously generated inside the layer by the secondary-instability mechanism of Dahlburg et al. (1992, 2005) and evaluate its effect on the time-averaged reconnection rate.  

2. What is the effect of the 2D tearing instability inside the resistive Sweet-Parker layer, on the overall, time-averaged reconnection rate? It is expected that tearing instability may become important in reconnection layers that are very long, with aspect ratios $L/\delta > 100$. It may then make the reconnection process inherently time dependent, bursty. Therefore, in order to investigate this issue, one would have to perform only a 2D simulation, but with a very high resolution (corresponding to $S > 10^4$) and with a very long duration (to be able to average over many bursts).

3. The majority of existing numerical studies of resistive-MHD reconnection use a constant uniform resistivity. It is important to check whether the main results will be the same if one uses the actual Spitzer resistivity. This is especially important in the context of a magnetically dominated environment, such as the solar corona, where the temperature inside the reconnection layer may be much higher than the ambient temperature. As a result, the Spitzer resistivity may vary by a factor of a 100 across the layer, being much smaller at its center.

4. One should further pursue numerical studies of magnetic reconnection that include both resistive MHD and the Hall effect, along the lines of the recent work by Cassak et al. (2005, 2006, 2007). Of particular interest is the intermediate regime, in which $\delta_{SP} > d_i, \rho_i$ but at the same time $\Omega_e > \nu_e$. The objective of such simulations would be to observe the transition from slow to fast reconnection within one simulation run as the upstream plasma parameters, such as $B_0$ or $n$, are gradually changed, passing the critical point. Whereas the results published by Cassak et al. (2005, 2006, 2007) are already very important, the relevant parameter space is large and similar studies need to be extended to other regimes, most notably to a situation where $\beta_{upstream} \ll 1$. Also, these results need to be confirmed by other groups, preferably with a more realistic electron-to-ion mass ratio, a larger box size, and higher numerical resolution.

5. Future numerical studies of collisional reconnection should include realistic energy balance, taking into account both ohmic heating and the electron thermal conduction. The goal here would be to calculate the values of $T_e$ and $n_e$ at the center of the reconnection region. Also, one needs to study temperature equilibration between ions and electrons.

6. The effect of viscosity on resistive-MHD reconnection needs to be assessed.

7. The effect of a strong guide field on resistive-MHD reconnection and on the transition to the fast regime (e.g., Cassak et al. 2007) needs to be investigated in more detail.

8. Finally, I would like to strongly encourage further experimental (laboratory) studies of collisional reconnection, especially in the large-$S$ limit.

In addition to the above questions related to collisional reconnection, there are several important issues related to collisionless reconnection: (1) What is its physical nature of anomalous resistivity due to wave-particle interactions and under what conditions is it excited? (2) What is the structure of the Petschek-like reconnection layer for a given functional form of anomalous resistivity and what is the resulting reconnection rate in terms of the basic physical parameters? Does the anomalous resistivity spread along the separatrices (Petschek’s shocks) or is it present only in the small central region? If the latter is the case, then how does the plasma crossing the shocks get heated? (3) When does a laminar Hall effect (or, in general, two-fluid) reconnection take place? How do the Hall effect and anomalous resistivity coexist and interact with each other? (4) How rapid is two-fluid reconnection? What parameters affect the reconnection rate in this regime? (5) What is the effect of guide field on triggering and saturation of Hall-regime reconnection and on anomalous resistivity? (6) Is the collisionless reconnection process bursty and, if so, what is the time-averaged reconnection rate? (7) What is the overall partitioning of the released energy between the bulk kinetic energy, electron and ion thermal energies, and nonthermal particle acceleration?  

Finally, in order to be realistic, future numerical simulations of the solar corona should include all of the following (e.g., Miyagoshi & Yokoyama 2003; Klimchuk 2006): (1) flux emergence processes and random motions of the field-line footpoints; (2) a physically motivated prescription for the transition to fast reconnection, such as the one suggested in this paper; such a prescription would thus play the role of a subgrid model used in a large-scale MHD simulation of the corona; (3) mass exchange between the corona and the solar surface (e.g., chromospheric evaporation and plasma precipitation); (4) optically thin radiative energy losses and thermal conduction (including the contribution due to nonthermal electrons) along the magnetic field lines.

5. CONCLUSIONS

Magnetic reconnection research started 50 years ago in the field of solar physics, with the Sweet-Parker (Sweet 1958; Parker 1957) model for solar flares, followed by the Petschek (1964) theory a few years later. These studies tackled the most fundamental
The basic paradigm that emerges as a result of all these studies can be summarized as follows (see § 2.1): The starting point is the realization that there are indeed two reconnection regimes. The first one is a slow (Sweet-Parker) resistive-MHD regime that is realized in relatively dense, collisional plasmas. The second one is a fast (Petschek-like) regime that takes place in collisionless plasmas. The mechanism for the fast collisionless reconnection can be either a locally enhanced anomalous resistivity due to microinstabilities triggered when the current density exceeds a certain threshold or the Hall effect. In either case, one can formulate an approximate condition for the transition between the slow collisional and the fast collisionless regimes. If the guide field is not much larger than the reconnecting magnetic field, this condition is \( \delta_B < d_i \), where \( \delta_B \) is the thickness of the Sweet-Parker reconnection layer and \( d_i \) is the collisionless ion skin depth. One can further rewrite this condition in terms of the classical electron mean free path \( \lambda_{e,mfp} \) inside the layer as \( L < L_c \sim (m_i/m_e) \frac{1}{\lambda_{e,mfp}} \), where \( L \) is the global system size (Yamada et al. 2006). Due to the strong temperature dependence of \( \lambda_{e,mfp} \), this form of the condition highlights the need to estimate the electron temperature \( T_e \) at the center of the layer. Using considerations of pressure balance and energy conservation (see § 2.3), one can express \( T_e \) in terms of the reconnecting magnetic field \( B_0 \) and the background plasma density \( n \). Then the collisionless reconnection condition can be recast in terms of \( L, B_0, \) and \( n \) (see eq. [10] in § 2.2). In the case of a strong guide field \( B_g \gg B_0 \), the corresponding condition is \( \delta_B < \rho_i \), and this leads to an additional factor \( (B_0/B_g)^2 \) in the expression for the critical length \( L_c \) (see eq. [23]).

One of the main driving forces behind this paper is the author’s desire to bring the recently obtained knowledge about reconnection back to solar physics and to use it productively to build a better understanding of the solar corona. Although most of the present discussion is also relevant to solar flares, in this paper I focus predominantly on the problem of solar coronal heating (see § 3).

In the context of Parker’s (1983, 1988) nanoflare theory of coronal heating, magnetic energy release in the solar corona takes place in the form of many unresolved reconnection events (nanoflares). One of the most important features of this picture is the intermittent character of energy release. Random footpoint motions lead to continuous twisting of elementary magnetic strands around each other, which, in turn, leads to the formation of many small current sheets in the corona. Current sheets may form either in finite time, as suggested by Parker (1983, 1988), or exponentially in time, as was demonstrated by van Ballegooijen (1986) and later numerically by Galsgaard & Nordlund (1996); for our purposes, it does not matter which one is correct. What matters is that thin current layers do eventually form. Free magnetic energy accumulates for a while and is then suddenly released in distinct fast reconnection events. In the present paper, I suggest that the transition between the slow and fast reconnection regimes plays a key role in determining when a given nanoflare will take off and how much energy will be released (see also Cassak et al. 2005). My model can thus be regarded as an alternative to the secondary-instability mechanism proposed by Dahlburg et al. (2005). Furthermore, I argue that the fact that the fast reconnection condition involves the ambient plasma density is an important part of the story. The reason for this is that the density in the corona is not a fixed constant; in a given loop it constantly changes in response to radiative cooling and coronal evaporation caused by coronal energy-release events. The basic picture here is the following: A coronal energy release leads to an increase in density, thus making the plasma more collisional. This temporarily inhibits fast reconnection in the given region until the density decreases again (on the radiative cooling timescale) to below a certain critical value. At this point fast reconnection again becomes possible. On the longer timescale, a quasi-periodic behavior is established, characterized by repeated cycles that include fast reconnection events, followed by coronal evaporation episodes, followed by relatively long (\( \sim 1 \) hr) and steady periods during which free magnetic energy in the loop builds up and the plasma gradually cools down. Thus, coronal heating can be viewed as a self-regulating process that statistically keeps the density roughly near the critical value for the fast reconnection transition. In other words, the system constantly fluctuates around the state of marginal collisionality as defined by the collisionless reconnection condition. In the long-term statistical equilibrium, a balance is maintained in which the amount of plasma pumped into a coronal loop as a result of an evaporation episode is equal to the amount of plasma drained down to the surface during the gradual radiative cooling stage that takes place between two subsequent fast-reconnection events. The characteristic equilibrium density, the time interval between reconnection episodes, and the energy released in each such episode can be estimated in terms of the loop’s longitudinal magnetic field, its characteristic size, and the footpoint driving rate (see § 3).

Finally, I believe that the physical framework developed in this paper should also be applicable to magnetically dominated coronae of other astrophysical objects, such as other stars and accretion disks (Goodman & Uzdensky 2008, in preparation).

I am grateful to S. Antiochos, P. Cassak, J. Goodman, H. Ji, J. Klimchuk, R. Kulsrud, E. Parker, M. Shay, and M. Yamada for stimulating discussions and encouraging remarks, and to the anonymous referee for useful suggestions. This work is supported by National Science Foundation grant PHY-0215581 (PFC: Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas).

REFERENCES

Aschwanden, M. J., Nightingale, R. W., & Alexander, D. 2000a, ApJ, 541, 1059
Aschwanden, M. J., Schrijver, C. J., & Alexander, D. 2001a, ApJ, 550, 1036
Aschwanden, M. J., Poland, A. I., & Rabin, D. M. 2001b, ARA&A, 39, 175
Aschwanden, M. J., Winebarger, A., Tsiklauri, D., & Peter, H. 2007, ApJ, 659, 1673
Aschwanden, M. J., et al. 2000b, ApJ, 535, 1047
Bhattacharjee, A., Ma, Z. W., & Wang, X. 1999, J. Geophys. Res., 104, 14543
———. 2001, Phys. Plasmas, 8, 1829
Birn, J., et al. 2001, J. Geophys. Res., 106, 3715
