Numerical simulations of hot accretion flows around black holes have shown the existence of strong wind. Those works focused only on the region close to the black hole and thus it is unknown whether or where the wind production stops at large radii. To address this question, we have recently performed hydrodynamic (HD) simulations by taking into account the gravitational potential of both the black hole and the nuclear star cluster. The latter is assumed to be proportional to \( \sigma^2 \ln(r) \), with \( \sigma \) being the velocity dispersion of stars and \( r \) the distance from the center of the galaxy. It was found that when the gravity is dominated by nuclear stars, i.e., outside a radius \( R_A = GM_{BH}/\sigma^2 \), winds can no longer be produced. That work, however, neglects the magnetic field, which is believed to play a crucial dynamical role in the accretion and thus must be taken into account. In this paper, we revisit this problem by performing magnetohydrodynamic (MHD) simulations. We confirm the result of our previous paper, namely that wind cannot be produced in the region \( R > R_A \). Our result, combined with recent results of Yuan et al., indicates that the formula describing the mass flux of wind, \( M_{\text{wind}} = M_{\text{BH}}(r/20r_s) \), can only be applied to the region where the black hole potential is dominant. Here \( M_{\text{BH}} \) is the mass accretion rate at the black hole horizon and the value of \( R_A \) is similar to the Bondi radius.

Key words: accretion, accretion disks – black hole physics – hydrodynamics
\[
\frac{dv}{\rho dt} = -\nabla p - \rho \nabla \psi + \frac{1}{4\pi}(\nabla \times B) \times B, \tag{2}
\]
\[
\rho \frac{d(e/\rho)}{dt} = -\rho \nabla \cdot v + \eta J^2, \tag{3}
\]
\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta J). \tag{4}
\]

Here, \(\rho, p, v, \psi, e, B,\) and \(J(=e/4\pi\nabla \times B)\) are density, pressure, velocity, gravitational potential, internal energy, magnetic field, and the current density, respectively. \(d/dt(=\partial/\partial t + v \cdot \nabla)\) denotes the Lagrangian time derivative.

We adopt an equation of state of an ideal gas, \(p = (\gamma-1)e,\) and set \(\gamma = 5/3.\)

The gravitational potential \(\psi\) can be expressed as
\[
\psi = \psi_{BH} + \psi_{\text{star}}. \tag{5}
\]

The black hole potential \(\psi_{BH} = -GM_{BH}/(r - r_s),\) where \(G\) is the gravitational constant, \(M_{BH}\) is the mass of the black hole, and \(r_s\) is the Schwarzschild radius. As in Paper I, we assume in this paper that the velocity dispersion of nuclear stars is a constant for varying radius. This seems to be the case for many AGNs. So the potential of the star cluster is \(\psi_{\text{star}} = \sigma^2 \ln(r) + C,\) where \(\sigma\) is the velocity dispersion of stars and \(C\) is a constant. So we have
\[
R_A = GM_{BH}/\sigma^2. \tag{6}
\]

We set \(\sigma^2 = 10\) and \(G = M_{BH} = 1\) to define our units in the present work. In these units we have \(R_A = 0.1.\) For a typical physical value of \(\sigma \sim (100-400) \text{ km s}^{-1}\) (e.g., Kormendy & Ho 2013), \(R_A \sim (10^3-10^4)r_s.\) Figure 1 in Paper I shows the distribution of gravitational force.

In the above equations, the final terms in Equations (3) and (4) are the magnetic heating and the dissipation rate mediated by a finite resistivity \(\eta.\) The exact form of \(\eta\) is the same as that used in Stone & Pringle (2001). In the code we adopt, the energy equation is an internal energy equation; numerical reconnection inevitably results in loss of energy from the system. By adding the anomalous resistivity \(\eta,\) the energy loss can be captured in the form of heating in the current sheet (Stone & Pringle 2001).

In this paper, time is expressed in units of the orbital time at the center of the torus.

### 2.2. Initial Conditions

For the initial condition, we assume a rotating equilibrium torus embedded in a non-rotating, low-density medium. We assume that the torus has constant specific angular momentum \(L\) and assume a polytropic equation of state, \(p = A\rho^\gamma,\) where \(A\) is a constant. The density distribution of the torus is
\[
\rho = \rho_c \times \left\{ \max \left[ \Psi(R_0, \pi/2) - \psi(r, \theta) - L^2/(2r \sin \theta)^2, 0 \right] / A[\gamma/(\gamma - 1)] \right\}^{1/(\gamma - 1)}, \tag{7}
\]

where \(R_0\) is the density maximum (center) of the torus (Nishikori et al. 2006) and \(\rho_c\) is the density at the center of the torus. In this paper, we assume \(\rho_c = 1\) and \(A = 0.4.\)

The ambient medium in which the torus is embedded has density \(\rho_0\) and pressure \(p_0/\rho_0.\) The mass and pressure of the ambient medium are negligibly small; so we choose \(\rho_0 = 10^{-4}.\)

### 2.3. Models

The initial magnetic field is generated by a vector potential, i.e., \(B = \nabla \times A.\) In models A1 and A2, the initial magnetic field has a dipolar configuration (the same as that in Stone & Pringle 2001). We take \(A\) to be purely azimuthal with \(A_\phi = \rho^2/\beta\) and \(\beta = 200.\) The only difference between models A1 and A2 is that the resolution of model A1 is twice that of model A2. We find that the results for models A1 and A2 are almost the same. So the resolution in model A2 is sufficient for our problem. In order to study the dependence of results on the initial magnetic field configuration, we investigate model A3. In this model, the initial magnetic field has a quadrupolar configuration with \(A_\phi = \rho^2/\beta r \cos \theta\) and \(\beta = 100.\) Table 1 summarizes the models.

For the initial condition in models A1, A3, and A4, over most of the central regions of the initial torus, we have six grids for one wavelength of the fastest growing model. Therefore, the fastest growing model of MRI is marginally resolved in our simulations.

In this paper, the simulations are two-dimensional (hereafter 2D). According to the antidynamo theorem (Cowling 1933; Sadowski et al. 2015), the turbulence induced by MRI cannot be self-sustaining. Therefore there cannot be a true steady state, and the quasi-steady state of the simulations is only transient. In this paper, we still perform 2D simulation because, on one hand, we can simulate a larger radial dynamical range, and on the other hand, previous works have indicated that for many problems the results from 2D and 3D simulations are often quite similar (e.g., see the short review in Yuan et al. 2012b, in the case of the radial profile of inflow and outflow rates). Still, for our present study of wind from accretion flows, it is necessary to carefully examine whether the results from 2D simulation are consistent with those from 3D simulation.

In order to answer this question, we have carried out a 2D MHD simulation of accretion flows close to a black hole. The domain of the simulation is \(2r_r-400r_r.\) In this simulation, only the gravity of the black hole is taken into account. Using the trajectory approach as described in Yuan et al. (2015), we have calculated the mass flux of wind, which is \(~52\%\) of the total outflow rate calculated by Equation (9). For comparison, the mass flux of wind calculated in Yuan et al. (2015), which is based on 3D MHD simulation data of accretion flow, is \(~60\%\) of the total outflow rate calculated by Equation (9). Such a good consistency suggests that the present 2D study should be a good approximation.

### 2.4. Numerical Method

We use the ZEUS-2D code (Stone & Norman 1992a, 1992b) to solve Equations (1)–(4). The polar range is \(0 \leq \theta \leq \pi.\) We adopt

| Model | Initial Magnetic Field | Resolution | Computational Domain |
|-------|------------------------|------------|----------------------|
| A1    | dipolar                | 294 \times 160 | 0.03–4 |
| A2    | dipolar                | 147 \times 80  | 0.03–4 |
| A3    | quadrupolar            | 294 \times 160 | 0.02–4 |
| A4    | dipolar                | 294 \times 160 | 0.002–0.4 |
a non-uniform grid in the radial direction \((\Delta r)_{i+1}/(\Delta r)_i = 1.037\). The distributions of grids in the \(\theta\) direction in the northern and southern hemispheres are symmetric about the equatorial plane. The resolution at \(\theta\) in the northern hemisphere is same as that at \(\pi - \theta\) in the southern hemisphere. In order to resolve the accretion disk well around the equatorial plane, the resolution is increased from the north–south rotational axis to the equatorial plane with \((\Delta \theta)_{i+1}/(\Delta \theta)_i = 0.9826\) for \(0 \leq \theta \leq \pi/2\) and \((\Delta \theta)_{i+1}/(\Delta \theta)_i = 1.0177\) for \(\pi/2 \leq \theta \leq \pi\). We use axisymmetric boundary conditions at the poles, and outflow boundary conditions at the inner and outer radial boundaries.

3. RESULTS

3.1. Mass Inflow Rate

Following Stone et al. (1999), we define the mass inflow and outflow rates, \(M_{\text{in}}\) and \(M_{\text{out}}\) as follows:

\[
M_{\text{in}}(r) = 2\pi r^2 \int_0^{\pi} \rho \min (v_r, 0) \sin \theta d\theta,
\]

\[
M_{\text{out}}(r) = 2\pi r^2 \int_0^{\pi} \rho \max (v_r, 0) \sin \theta d\theta.
\]

The net mass accretion rate is

\[
M_{\text{acc}}(r) = M_{\text{in}}(r) + M_{\text{out}}(r).
\]

Note that the above rates are obtained by time-averaging the integrals rather than integrating the time averages.

Figure 1 shows the time-averaged (from \(t = 130\) to \(136\) orbits) and angle-integrated mass inflow rate \(M_{\text{in}}\) (solid line), outflow rate \(M_{\text{out}}\) (dashed line), and the net rate \(M_{\text{acc}}\) (dotted line) in model A1. They are defined in Equations (8)–(10), respectively.

radius. According to this criterion, our simulation of model A1 has reached a steady state within \(r \sim 0.6\). We note that the flow in model A1 is convectively unstable (see Section 3.3). The accretion timescale is roughly equal to one turnover time of local convective eddies. It may require many turnover times for the convection to reach a steady state. Thus it will be interesting to run our simulations for several times longer in the future to check whether the results will change.

3.2. Does Strong Wind Exist in a CAAF?

The significant mass outflow rate shown in Figure 1 does not mean the existence of strong real outflow (wind) because it may be due to the turbulent motion. To study whether wind exists, we first look directly at the velocity field shown in Figure 2. We see that turbulent eddies occupy the whole domain and it is hard to find systematic winds.

To investigate this problem precisely, following Yuan et al. (2015), we use the trajectory method to study the motion of the virtual particles. The details of this approach can be found in Yuan et al. (2015). To get the trajectory, we first need to choose some virtual “test particle” in the simulation domain. These are, of course, not real particles, but some grids representing fluid
Figure 3. Trajectory of gas for model A1. The black dots located at $r = 0.3$ are starting points of the “test particle.” Different colors denote the trajectories of a “test particle” starting from different angles $\theta$. It is clear that the “test particle” crosses the starting radius many times. From this figure we can see that the real wind trajectories, i.e., the trajectories which extend from $r = 0.3$ to large radius and never cross $r = 0.3$ again, are very few. Winds are very weak.

elements. Their locations and velocities at a certain time $t$ are obtained directly from the simulation data. We can then obtain their location at time $t + \delta t$ from the velocity vector and $\delta t$. The trajectory of particles gives a truer reflection of their motion than the streamline, which is crucial for us in investigating whether real wind exists. The trajectory is equivalent to the streamline only for strictly steady motion, which is not the case for accretion flow since it is always turbulent.

The trajectory approach can easily tell us which particles are real outflows (i.e., winds) and which are undergoing turbulent motions. Combining this with the information on density and velocity fields from the simulation data, we can then obtain the various properties of wind such as the mass flux, angular distribution, and velocity (see Yuan et al. 2015 for details). Figure 3 shows the trajectory of some gas particles starting from $r = 0.3$ in model A1. From this figure we can see that the real wind trajectories, i.e., the trajectories that extend from $r = 0.3$ to large radius and never cross $r = 0.3$ again, are very few. This implies that the mass flux of wind is very small. Our quantitative calculation confirms this result. For example, we find that at $r = 0.4$ the ratio of mass flux of winds to the total outflow rate calculated by Equation (9) is only 0.2%. This means that there is almost no wind. This result is consistent with that found in Paper I. As a comparison, in the case of BHAF, Yuan et al. (2015) find that this ratio is 60%.

3.3. Why Does the Inflow Rate Decrease Inward in a CAAF?

If wind is absent, what is the reason for the inward decrease of inflow rate? To answer this question, we need to examine the convective stability of the accretion flow. Hydrodynamic simulations of BHAFs (e.g., Igumenshchev & Abramowicz 1999, 2000; Stone et al. 1999; Yuan & Bu 2010) have found that the flows are convectively unstable, consistent with what has been suggested by the one-dimensional analytical study of BHAFs (Narayan & Yi 1994). The physical reason is that the entropy of the flow increases inward, which is caused by the viscous heating and negligible radiative loss. However, numerical simulations have found that a BHAF becomes convectively stable in the presence of magnetic field (Narayan et al. 2012; YBW12). In the case of a CAAF, Paper I has found that the flow is convectively unstable, the same as for a BHAF.

We now study whether a CAAF is convectively stable or not in the presence of magnetic field. We use the Høiland criterion (e.g., Tassoul 1978; Begelman & Meier 1982):

$$(\nabla s \cdot d\mathbf{r})(\nabla \cdot d\mathbf{r}) - \frac{2\gamma \psi}{R^2} [\nabla (v_r R) \cdot d\mathbf{r}] dR < 0. \quad (11)$$

In Equation (11), $R = r \sin \theta$ is the cylindrical radius, $d\mathbf{r} = dr \hat{r} + rd\theta \hat{\theta}$ is the displacement vector, $s = \ln (p) - \gamma \ln (\rho)$ is $(\gamma - 1)$ times the entropy, $g = -\nabla \psi + \hat{r} \psi^2 \hat{r} / R$ is the effective gravity, and $v_\psi$ is the rotational velocity. For a non-rotating flow, this condition is equivalent to an inward increase of entropy, which is the well-known Schwarzschild criterion.

Taking model A1 as an example, Figure 4 shows the result. The result is obtained according to Equation (11) based on the simulation data at $t = 132$ orbits at the initial center of the torus, $r = 1$. At $t = 132$ orbits, the flow has achieved a steady state since the net accretion rate averaged between $t = 130$ and 136 orbits is constant with radius (see the dotted line in Figure 1). The red regions are convectively unstable. We can see from the figure that a CAAF is mostly convectively unstable. This is different from the case of a BHAF with magnetic field. The reason should be due to the change in the gravitational potential but the detail remains unclear. This result strongly implies that the inward decrease of inflow rate is because of convection and it reminds us of the scenario of convection-dominated accretion flow (CDAF) proposed by...
Figure 5. Convective stability analysis of model A4. The result is obtained according to Equation (11) based on the simulation data at $t = 100$ orbits at the initial center of the torus, $r = 0.1$. The red region is unstable.

Narayan et al. (2000) and Quataert & Gruzinov (2000), although that model was proposed to explain the dynamics ofBHAFs rather than CAAFs.

From Figure 4, it seems that the region $r < 0.1$ is convectively unstable. Our previous work with only the gravity of the black hole shows that the flow is convectively stable (YBW12). We think the apparent discrepancy is because of the contamination of the gravitational potential by the star cluster. In model A1, in the region $r < 0.1$, it is true that the gravity of the black hole is bigger than that of the star cluster, but the gravity of the stars is not negligible. In fact, in the region $0.03 < r < 0.1$, the gravity of the black hole is bigger than that of the star cluster by at most a factor of 4. To further investigate this point, we have analyzed the convective stability of model A4. Figure 5 shows the results for the region very close to the black hole where its gravity is strongly dominant. We can clearly see that the flow is convectively stable.

3.4. Varying the Initial Configuration of Magnetic Field

In order to study whether the results depend on the initial configuration of the magnetic field, we investigate model A3. In this model, the initial configuration of magnetic field is quadrupolar. Figure 6 shows the inflow, outflow, and net rates. The inflow rate is smoother than that in Figure 1, as we have explained in Section 3.1. In addition to that, we find that all results are almost the same as those of model A1, namely the flow is convectively unstable and wind is absent. Quantitatively, using the trajectory method, we find that at $r = 0.4$ the ratio of mass flux of winds to the total outflow rate calculated by Equation (9) is 0.1%.

3.5. Moving the Computational Domain Inward

From Figures 1 and 6, the outflow rate is very small in the region $r < 0.1$. This region is dominated by the gravity of the black hole. Previous works (Yuan et al. 2012a, 2015) have shown that in this case outflow (wind) should be strong. The apparent discrepancy between this work and our previous works is due to the fact that the region $r < 0.1$ is too close to the inner boundary where a somewhat “unphysical” boundary condition (i.e., outflow condition) is adopted. This condition means that the gas entering the inner boundary is assumed to disappear and the gradient of physical variables at the boundary is zero. However, in reality, there should be some flow entering the inner boundary, and the flows inside and outside of the boundary can interact with each other. This will significantly affect the properties of the flows around the inner boundary.

In order to illustrate this point, we have investigated model A4. In this model, our computational domain is $0.002 < r < 0.4$. The gravitational potentials from both the black hole and the nuclear star cluster are included, but obviously the former dominates. The initial condition of the magnetic field is dipolar. Figure 7 shows the trajectory of some gas particles starting from $r = 0.03$. From this figure, it is clear that strong wind is present in this case. Combined with the results presented in previous sections, this result indicates that the disappearance of wind is because of the changes in the gravitational potential.

In models A1 and A4, the setup of the model, the equations, and the potential formula we use are exactly the same. The only difference between them is that the black hole potential is more dominant in A4 while the stellar cluster potential is more dominant in A1. Therefore, the disappearance of wind is because of the changes in the gravitational potential.

4. CONCLUSION AND DISCUSSION

Numerical simulations show that strong winds exist in hot accretion flows around black holes (e.g., YBW12). The mass flux of wind follows $M_{\text{wind}} = M_{\text{BH}} (r/20r_*)$ (Yuan et al. 2015). A question is then what the value of $r$ can be, i.e., whether or where the wind production stops. In order to answer this question, we have performed HD simulations in Paper I and take into account the gravity of both the black hole and nuclear star cluster. We find that the mass inflow rate decreases inward. However, our trajectory analysis indicates that there is no wind when the potential of the star cluster dominates, i.e., beyond a certain radius $R_A \equiv G M_{\text{BH}}/\sigma^2$, with $\sigma$ being the velocity dispersion of stars. The inward decrease of inflow rate is not because of strong wind, as in the case of accretion flow in which the black hole potential dominates, but because of the convective instability of the accretion flow. In this paper, we revisit the same problem by performing more realistic MHD simulations. We find again that the conclusion remains unchanged, i.e., there is no wind beyond $R_A$. Our stability analysis again indicates that the MHD accretion flow beyond $R_A$ is convectively unstable. This is different from the case of accretion flow when the black hole potential dominates (YBW12; Narayan et al. 2012). So the
inward decrease of inflow rate is likely because of the convective motion of the flow. This result indicates that the mass flux of wind found by Yuan et al. (2015),

\[ M_{\text{wind}} = M_{\text{BH}}(r/20r_*) \]  

(12)
can only be applied to the region where the gravitational force of the black hole dominates. In the region where the star cluster potential dominates, i.e., beyond \( R_A \), no wind will be produced. In practice, the value of \( R_A \) is close to the Bondi radius \( R_B \equiv GM_{\text{BH}}/c_s^2 \) (Paper I).

What is the reason for the absence of wind beyond \( R_A \)? We speculate that it may be related to the change in slope of the gravitational potential. Such a change would change the shear of the accretion flow and thus the turbulent stress in the accretion flow. Analytical models of an accretion disk (e.g., Shakura & Sunyaev 1973) usually assume that viscous stress is proportional to the shear of the accretion disk,

\[ T_{\nu} = \rho v r \frac{d\Omega}{dr} \]  

(13)

with \( \nu = \alpha c_s^2/\Omega_k \), \( c_s \), \( \Omega \), and \( \Omega_k \) are sound speed, angular velocity, and Keplerian angular velocity, respectively. Recent 3D MHD numerical simulations show that the turbulent stress is not linearly proportional to the shear, but the dependence is stronger than that predicted by Equation (13) (Pessah et al. 2008; Penna et al. 2013). This implies that the change in the potential changes some properties of turbulence, which then change the wind production.

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