Local Axisymmetric Instability Criterion in the Thin, Rotating, Multicomponent Disk.

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Abstract

Purely gravitational perturbations are considered in a thin rotating disk composed of several gas and stellar components. The dispersion relation for the axisymmetric density waves propagating through the disk is found and the criterion for the local axisymmetric stability of the whole system is formulated. In the appropriate limit of two-component gas we confirm the findings of Jog & Solomon (1984) and extend consideration to the case when one component is collisionless. Gravitational stability of the Galactic disk in the Solar neighborhood based on the multicomponent instability condition is explored using recent measurements of the stellar composition and kinematics in the local Galactic disk obtained by Hipparcos satellite.

Key words: Galaxy: evolution – Galaxy: kinematics and dynamics – Galaxy: solar neighborhood – ISM: kinematics and dynamics – stars: kinematics

1 Introduction

It was first shown by Safronov (1960) that a thin gaseous rotating disk can become unstable against local axisymmetric perturbations under the action of its own gravity. Quantitatively the instability criterion is expressed in terms of Toomre’s $Q$:

$$Q \equiv \frac{\kappa c}{\pi G \Sigma},$$

with $\kappa$ being the epicycle frequency, $c$ being the speed of sound, and $\Sigma$ being the surface density of the disk. Instability arises when $Q < 1$.

Disks composed of stars behave very similarly to the gaseous disks for long wavelength because epicyclic motion of stars is the same as that of the gas. But collisionless disks of stars have no pressure to resist short wavelength density perturbations and thus the density waves in this case are different from simple sound waves present in the gaseous disks in this regime (Toomre 1964). When the wavelength becomes shorter than the typical amplitude of the star’s epicyclic motion the gravitational effect of the perturbation averages out and stars perform almost pure epicyclic motion with frequency $\kappa$. Instability can occur though for some intermediate wavelength which leads to a close similarity of the corresponding instability conditions.

However, real disks of spiral galaxies do not consist of only one component which could be characterized by a single value of the velocity dispersion. Indeed, a significant part of the ISM in galactic disks is usually concentrated in the form of cold gas with characteristic sound velocity $\sim 5 – 10$ km s$^{-1}$. At the same time stars usually have larger velocity dispersions $\sim 20 – 35$ km s$^{-1}$.

The problem of the treatment of such two-component disks was first addressed by Jog & Solomon (1984a, JS). They assumed galactic disk to consist of two fluid components: one hot, playing the role of the stellar population, and one cold representing the usual ISM, and they neglected the fact that in real galaxies hot component is collisionless and provides no damping. Though this could be important shortcoming of the analysis for the short wavelength it turns out to be a good approximation for the stability purposes as we will see later in this paper. Jog & Solomon (JS) have demonstrated that even relatively small amounts of the cold gas ($\sim 10\%$ by mass) can very effectively destabilize the whole system.

The two-fluid stability criterion was applied to a variety of disks by Jog & Solomon (1984b), Elmegreen (1995), and Jog (1996). They assumed galactic disk to consist of two fluid components: one hot, playing the role of the stellar population, and one cold representing the usual ISM, and they neglected the fact that in real galaxies hot component is collisionless and provides no damping. Though this could be important shortcoming of the analysis for the short wavelength it turns out to be a good approximation for the stability purposes as we will see later in this paper. Jog & Solomon (JS) have demonstrated that even relatively small amounts of the cold gas ($\sim 10\%$ by mass) can very effectively destabilize the whole system.

The two-fluid stability criterion was applied to a variety of disks by Jog & Solomon (1984b), Elmegreen (1995), and Jog (1996). They showed that our Galaxy is stable against local axisymmetric perturbations to the extent of the accuracy of the parameters and mass distribution models assumed. Theory can be further refined by including the effects of the finite thickness of the disk on its stability (Toomre 1964) and by taking into account the vertical motions in the disk (Romeo 1992). It was shown that the instability con-
condition remains the same provided the unperturbed surface density is multiplied by an appropriate reduction factor.

Real galaxies always contain more than two isothermal components. It is now widely believed that ISM in the galactic disk is subdivided into a number of components with different temperatures and, thus, different dynamical properties (McKee & Ostriker 1977). The same is true about the stellar population. Stars in the galactic disk are being constantly scattered by giant molecular clouds and transient spiral arms which steadily increase their velocity dispersions (Spitzer & Schwarzschild 1951, 1953; Barbanis & Woltjer 1967; Carlberg & Sellwood 1985). It means that the stars of different ages have different velocity dispersions and thus dynamically should be treated separately.

Recent results of the Hipparcos satellite (ESA 1997) provided us with a wealth of information about the local stellar kinematics. The proper motions of different star populations were accurately measured and the resulting velocity dispersions were accurately measured and the resulting velocity dispersions were accurately measured and the resulting velocity dispersions and thus dynamically should be treated separately.

Since here we are interested in axisymmetric perturbations only it is always supposed that \( \partial \phi / \partial \theta = 0 \) in our analysis. We assume that all the first order dependent quantities vary like \( \exp[i(kr - \omega t)] \), where \( \omega \) is the angular frequency and \( k = 2\pi/\lambda \) is the wavenumber. For the local analysis we employ the WKB approximation (or tight-winding approximation) which requires that \( kr \gg 1 \) and allows us to neglect terms proportional to \( 1/r \) compared to the terms proportional to \( k \). With all these simplifications equations (4) and (5) reduce to

\[
- \omega u_i - 2 \Omega v_i = -ik(\Phi_1 + h_{1i}), \quad -i\omega v_i - 2B u_i = 0, \quad -i\omega \Sigma_{1i} + ik \Sigma_{0i} u_i = 0,
\]

for \( i \)-th gas component.

where \( v_r \) is the circular velocity at current distance from the galactic center and \( \sigma_{rj} \) and \( \sigma_{\phi j} \) are the velocity dispersions in \( r \) and \( \phi \) directions correspondingly. These velocity dispersions are related by \( \sigma_{rj}^2/\sigma_{\phi j}^2 = AB^2/\kappa^2 \), where \( B \) is Oort’s \( B \) constant (Binney & Tremaine 1987). We will later use simply \( \sigma \) to denote the velocity dispersion in the \( r \)-direction \( \sigma_r \).

For the description of collisional components we assume usual hydrodynamical description with isotropic pressure. To describe the star-like components we use a different approach which is rooted in the kinetic treatment of the collisionless systems as described elsewhere (Toomre 1964; Binney & Tremaine 1987). We will see in §4 that this accurate treatment shows that fluid approach really does quite a good job in describing the stability of the collisionless systems with multiple components as it does in one-fluid case, though some quantitative differences exist.

Equations governing the motion of the gas components in our coordinates are Euler’s and continuity equations:

\[
\begin{align*}
\frac{\partial v_i}{\partial t} + (v_i \nabla) v_i &= -\frac{1}{\Sigma} \nabla P_i - \nabla \Phi, \\
\frac{\partial \sigma_{ri}^2}{\partial t} + \nabla (\Sigma_i v_i) &= 0,
\end{align*}
\]

for \( i \)-th component.

We assume that pressure of each component \( P_i = K \Sigma_i \) and introduce specific enthalpy

\[
\frac{\dot{h}_i}{\dot{c}_i} = \frac{\dot{\gamma}_i}{\dot{\gamma}_c} = 1 - \frac{B^2}{\kappa^2}.
\]

We linearize equations (3) and (4) by assuming that \( v_i = u_i, v_{\phi i} = v_\phi, h_i = h_0 + h_{1i}, \Sigma_i = \Sigma_{0i} + \Sigma_{1i}, \) and \( \Phi = \Phi_0 + \Phi_1 \).

Then equations (3) and (4) reduce in the first order to

\[
\begin{align*}
\frac{\partial u_i}{\partial \tau} + \frac{\partial u_i}{\partial \phi} - 2\Omega v_i &= -\frac{1}{r} \frac{\partial}{\partial r} (\Phi_1 + h_{1i}), \\
\frac{\partial v_i}{\partial \tau} + \Omega \frac{\partial v_i}{\partial \phi} - 2B u_i &= -\frac{1}{r} \frac{\partial}{\partial \phi} (\Phi_1 + h_{1i}), \\
\frac{\partial \Sigma_{1i}}{\partial \tau} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma_{0i} u_i) + \frac{\partial \Sigma_i}{\partial \phi} + \frac{\partial u_i}{r} \frac{\partial}{\partial \phi} &= 0,
\end{align*}
\]

where

\[
B = -\frac{1}{2} [\Omega + \frac{\partial (\Omega r)}{\partial r}]
\]

is the Oort’s \( B \) constant.

We work in non-rotating cylindrical system, \( r, \phi, z \), such that \( z \)-axis coincides with the rotation axis of the disk and \( \phi \) increases in the direction of rotation. We start first with the equations for the fluid components and then make some correction to take collisionless components into account. We will usually use index \( i \) to describe gas components and index \( j \) for stellar components.

All the components are spatially concentrated in the thin disk and the effects of the finite disk thickness will usually be disregarded. All the motions are assumed to occur only in the plane of the disk.

We suppose that there are \( n_g \) gaseous components contributing to the mass of the Galaxy, each characterized by the sound velocity \( c_i \) and surface density \( \Sigma_{0i} \), and \( n_c \) collisionless components with \( \Sigma_{1i} \), being the surface density of the \( j \)-th component. Each collisionless component in the unperturbed state is assumed to have a Schwarzschild distribution function, that is for \( j \)-th component

\[
f(v_r, v_\phi) = \frac{\Sigma_{1j}}{2\pi \sigma_{rj} \sigma_{\phi j}} \exp \left\{ -\frac{v_r^2}{2\sigma_{rj}^2} - \frac{[v_\phi - v_c]^2}{2\sigma_{\phi j}^2} \right\}, \quad (2)
\]
These equations have to be supplemented with Poisson equation
\[ \nabla^2 \Phi = 4 \pi G \left( \sum_{i=1}^{n_g} \sum_{j=1}^{n_s} \delta(z) \right), \]
where \( \delta(z) \) is the Dirac delta function arising from our assumption of infinitely thin disk. With \( \Sigma_{1i,j} \) and \( \Phi_i \) being proportional to \( \exp[i(kr - \omega t)] \) the relation between the perturbations of the potential and surface densities becomes (Toomre 1964)
\[ \Phi_i = \frac{-2 \pi G}{k} \left( \sum_{i=1}^{n_g} \sum_{j=1}^{n_s} \Sigma_{1i,j} \right). \]

The sound speed for each gas component is defined as
\[ c_j^2 = \frac{\partial P_j}{\partial \Sigma_{0j}}, \]
and in this case the perturbation of enthalpy reduces to
\[ h_{ij} = c_j^2 \frac{\Sigma_{1ij}}{\Sigma_{0j}}. \]

3 DISPERSION RELATION AND STABILITY CRITERION

Equations (14)-(12), (11), and (10) form a closed set of linear equations and we now solve them to get the dispersion relation.

From equations (10) and (11) we can relate perturbation of the radial velocity of each component \( u_i \) to the potential perturbation \( \Phi_1 \) using equation (12):
\[ u_i = -\frac{\omega k}{\Delta} \left( \Phi_1 + c_j^2 \frac{\Sigma_{1ij}}{\Sigma_{0j}} \right), \]
where
\[ \Delta = \kappa^2 - \omega^2, \]
and \( \kappa^2 = -4 \Omega B \) is the epicyclic frequency.

Now we are in position to treat the star-like components. Since they are supposed to be collisionless they cannot create any pressure and it means that for \( j \)-th stellar component sound velocity \( c_j = 0 \). But because of the epicyclic motion at any given point in the disk there are stars from different parts of the perturbed structure and it leads to important cancellation effects. They were first calculated by Toomre (1964) and it can be shown that each stellar component is described by the Jeans equations which are pretty similar to the usual hydrodynamic equations with important difference being that instead of equation (10) we have
\[ u_k = -\frac{\omega k}{\Delta} \Phi_1 F \left( \frac{\omega}{\kappa}, \frac{k^2 \sigma_j^2}{\kappa^2} \right), \]
where the reduction factor \( F \) is given by the expression (Binney & Tremaine 1987)
\[ F(s, \chi) = \frac{2}{\chi} \left( 1 - s^2 \right) e^{-s} \sum_{n=1}^{\infty} \frac{I_n(\chi)}{1 - s^2/n^2}, \]
and \( I_n \) are the Bessel functions of order \( n \).

Now, from (12), (11) and (10) we can eliminate \( u_i \) to get
\[ \Sigma_{1i} = -\frac{k^2 \Sigma_{0i}}{\Delta + k^2 c_j^2} \Phi_1, \]
for \( i \)-th gas component and
\[ \Sigma_{1j} = -\frac{k^2 \Sigma_{0j}}{\Delta} \Phi_1 F_j, \]
for \( j \)-th stellar component and \( F_j = F(\omega/\kappa, k^2 \sigma_j^2/\kappa^2) \).

We can then substitute (21) and (22) into (10) to obtain finally the desired dispersion relation:
\[ 2 \pi G k \sum_{i=1}^{n_g} \frac{\Sigma_{0i}}{\kappa^2 + k^2 c_j^2 - \omega^2} + 2 \pi G k \sum_{j=1}^{n_s} \frac{\Sigma_{0j}}{\kappa^2} F_j = 1. \]

If only 2 fluid components are present in the disk this dispersion relation clearly reduces to the one derived by JS as it should. If one of the components is collisionless the dispersion relation is different and it was first considered by Romeo (1992). We discuss the difference between the two cases in §4.

The multicomponent disk is unstable if \( \omega^2 < 0 \) because in this case oscillatory behavior of the density changes to an exponential growth. The dispersion relation (22) has an infinite number of solutions with \( \omega^2(k) > 0 \) for all \( k \), which are clearly stable. But there is also one mode of oscillations in the system which has a single solution with \( -\infty < \omega^2/\kappa^2 < 1 \) and this mode could be unstable.

To find the condition for instability we note that as \( \omega^2 \to -\infty \) at a fixed wavenumber \( k \) LHS of the equation (22) tends to zero, and it monotonically increases to \( +\infty \) as \( \omega^2 \to \kappa^2 \). This property of a steady growth is important because if (22) has no solution for negative \( \omega^2 \) it implies that the LHS of (22) at \( \omega^2 = 0 \) must be less than 1 and vice versa. So the axisymmetric instability arises in multicomponent disk if and only if
\[ 2 \pi G k \sum_{i=1}^{n_g} \frac{\Sigma_{0i}}{\kappa^2 + k^2 c_j^2} + 2 \pi G k \sum_{j=1}^{n_s} \frac{\Sigma_{0j}}{\kappa^2} \Psi_j > 1, \]
where
\[ \Psi_j = F \left( 0, \frac{k^2 \sigma_j^2}{\kappa^2} \right) = \frac{2 \kappa^2}{k^2 \sigma_j^2} e^{-k^2 \sigma_j^2/\kappa^2} \sum_{n=1}^{\infty} I_n \left( \frac{k^2 \sigma_j^2}{\kappa^2} \right). \]

Since
\[ \sum_{n=1}^{\infty} I_n(\chi) = \frac{1}{2} \left[ e^\chi - I_0(\chi) \right], \]
we can rewrite (24) as
\[ \Psi_j = 1 - \frac{1 - e^{-\chi_j}}{\chi_j} I_0(\chi_j) = 1 - \frac{1 - e^{-k^2 \sigma_j^2/\kappa^2}}{k^2 \sigma_j^2/\kappa^2} I_0(k^2 \sigma_j^2/\kappa^2). \]

Suppose again that there are only two fluid (collisional) components in the disk then the instability condition (23) reduces to the one derived by JS as it should.

4 REALISTIC TWO-FLUID CASE

Now we consider the stability in the case when we have only two fluids present in the disk in more details. Our main purpose here is to see how the fact that one of the components is in reality collisionless changes the overall dynamics of the system from the case considered by JS.
In the case of two components, one stellar and one gaseous, we assume \( \Sigma \) and \( \Sigma_s \) to be the unperturbed gas and stellar surface densities, \( c \) the gas sound speed, and \( \sigma_s \) the stellar velocity dispersion in \( r \)-direction and introduce the following dimensionless quantities

\[
Q_s = \frac{k \sigma_s}{\pi G \Sigma_s}, \quad Q_g = \frac{k \sigma_g}{\pi G \Sigma_g},
\]

\[
q = k \sigma_s / \kappa, \quad R = c_g / \sigma_s.
\]

Here \( Q_g \) is a Toomre’s \( Q \) parameter for gas, while \( Q_s \) is different from the Toomre’s \( Q \) parameter for the collisionless system, which is given by \( k \sigma_s / (3.36 G \Sigma_s) \) (Toomre 1964). It means that with our definition of \( Q \), the one-component stellar disk is gravitationally unstable when \( Q_s < 3.36 / \pi = 1.07 \).

With these definitions our instability condition (??) reduces to

\[
\frac{2}{Q_s} \frac{1}{q} \left[ 1 - e^{-q^2 I_0(q^2)} \right] + \frac{2}{Q_g} R \frac{q}{1 + q^2 R^2} > 1.
\]

If one follows the Jog & Solomon approach and treats stellar component as a fluid with sound speed equal to \( \sigma_s \), one gets the following instability condition in terms of our dimensionless variables

\[
\frac{2}{Q_s} \frac{q}{1 + q^2} + \frac{2}{Q_g} R \frac{q}{1 + q^2 R^2} > 1.
\]

In Figure 1 we show the dependences of the \( \omega^2 / \kappa^2 \) of the two-component disk \( \omega^2 \) upon the perturbed surface densities for two cases: fluid-fluid disk, G - for cold gas only, S - for stars only, and G+S - for gas with the sound speed equal to the radial velocity dispersion of stars.

Indeed, in this case \( \omega \rightarrow \kappa \) for large \( k \). Using (22) and (23) we get

\[
\frac{\Sigma_{1g}}{\Sigma_{1s}} \rightarrow \frac{\Sigma_g}{\Sigma_s} \frac{\sigma^2}{c^2} \left[ 2 e^{-q^2} \sum_{n=1}^{\infty} \frac{I_n(q^2)}{1 - \omega^2 / (\kappa^2 n^2)} \right]^{-1} \rightarrow 0.
\]

because the sum in (31) diverges as \( \omega \rightarrow \kappa \). So, in realistic gas-stellar case this ratio goes to 0 rather than increases. It is illustrated in Figure 2, where we have shown the ratio of relative perturbed surface densities for two cases: fluid-fluid...
and fluid-stellar with the same set of parameters. This decrease in the gas density perturbation with growing k leads to the dominant role of stellar component in the dynamics of the whole system for large k, which is different from the JS case.

The relative contribution to the instability per unit surface density of the component considered by JS is given by the ratio of the corresponding terms in the LHS of (30)

\[
\gamma = \frac{\Sigma_g}{\Sigma_s} \frac{1 + q^2}{1 + q^2 R^2} \tag{32}
\]

and is always larger than \(\Sigma_g/\Sigma_s\) for \(R < 1\). In the star-fluid case one gets from (29) that

\[
\gamma = \frac{\Sigma_g}{\Sigma_s} \frac{q^2}{1 + q^2 (1 + q^2 R^2) (1 - \exp(-q^2 I_0(q^2))} \tag{33}
\]

Careful study shows that for \(\sqrt{3}/2 < R < 1\) there is always a range of small q (and k) in which \(\gamma < \Sigma_g/\Sigma_s\). Of course, as q grows \(\gamma\) becomes greater than \(\Sigma_g/\Sigma_s\) for \(R < 1\) \((\gamma \rightarrow R^{-2} \Sigma_g/\Sigma_s\) as \(q \rightarrow \infty\), but for \(q \sim 1\) the relative contribution to the instability per unit surface density is greater for stars if \(\sqrt{3}/2 < R < 1\). Even in this case though gas contributes only several per cent smaller than the stars and in most astrophysically interesting cases cold material still has a serious impact on the stability of the system.

Each of the criteria (26) and (34) produces some region in parameter space of the disk models in which they are stable against local axisymmetric gravitational perturbations. Unfortunately, it does not seem trivial to construct an effective analytical way of defining some effective value \(Q_{e,\text{eff}}\) as a function of \(Q_s, Q_g\) and other disk parameters so that the system is stable when \(Q_{e,\text{eff}} > 1\), as it was done for the fluid-fluid case by Elmegreen (1995). Instead we follow the approach of Jog (1996) and study the problem semianalytically. We parametrize our models here by \(1/Q_s, 1/Q_g\) and \(R\). In Figure 3 we compare the stable regions produced by each of the instability criteria.

In the JS case the marginal stability curves given by conditions \(\partial \omega^2 / \partial k = 0\) and \(\omega^2 = 0\) are symmetric with respect to the line \(Q_s = Q_g\). Indeed, if these conditions are fulfilled and an inequality in (34) changes to equality for \(Q_s = Q_1, Q_g = Q_2\) and \(q = q_{\text{crit}}\), then one can easily check that this is also true for \(Q_s = Q_2, Q_g = Q_1\) and \(q = q_{\text{crit}}/R\) provided that the instability condition is given by (34). Of course, this is not the case for the star-gas disk because any such symmetry is absent in the relation (26).

As we increase \(R\) from 0 to 1 the region of the parameter space occupied by stable models shrinks until \(R = 1\). The further increase of \(R\) beyond 1 causes reexpansion of the region occupied by stable models. In fact in the fluid-fluid case the marginal curve corresponding to some particular \(R = R_0\) coincides with the curve corresponding to \(R = 1/R_0\) which can be directly checked using (34). But these models are likely to be uncommon since they have \(c > \sigma\) which seems to be unusual in real galaxies.

The stable region in the star-gas case is in general smaller than that in the gas-gas case. The difference is especially noticeable at \(1/Q_s \approx 1\) and \(1/Q_g\), when the gas influence on the dynamics of the system is smallest and the stability condition is close to the one-fluid stellar stability criterion which is somewhat different from the fluid case. Nevertheless, the difference between two cases is quite small.
for most of the parameters (especially for small $Q_\ast$) and one can usually use the JS stability criterion in these cases.

5 APPLICATION TO THE SOLAR NEIGHBORHOOD

In this section we apply the results derived in $\S 3$ to the neighborhood of the Sun in the Galaxy.

In doing this one should realize that all the stars and gas in the Galaxy cannot be simply put into two distinct groups with some well defined velocity dispersion for stars and sound speed for gas. The reason for that is that the stars of different ages have different velocity dispersions - the older the star the larger its random motion. This random heating of stellar population is produced by the scattering of the stars by the giant molecular clouds (Spitzer & Schwarzschild 1951, 1953) and/or transient spiral density waves in the Galaxy (Barbanis & Woltjer 1967; Carlberg & Sellwood 1985).

Gas in its turn intrinsically has a multicomponent nature caused by the constant energy input from supernovae explosions and various cooling and heating processes determining its thermal equilibrium (McKee & Ostriker 1977; Kulkarni & Heiles 1987). Recent studies have enabled us to distinguish 5 phases of the ISM: molecular gas in the form of the clouds (Scoville & Sanders 1987), cold neutral medium (CM) also in the form of clouds, and 3 more or less uniformly distributed gaseous components: warm neutral medium (WNM), warm ionized medium (WIM) (Kulkarni & Heiles 1987), and hot component (Savage 1987). They have different sound speeds and surface densities which makes one treat them separately.

Unperturbed gravitational field of our Galaxy is not axisymmetric and it limits our analysis to some extent. It was also shown that the stellar distribution function can not always be represented by formula $\S 2$ because nonaxisymmetries of the Galactic gravitational field cause vertex deviations of the velocity ellipsoid (Dehnen & Binney 1998). Young stars of O and B types are likely not to have Schwarzschild distribution because they have had no time to be sufficiently scattered by the giant molecular clouds or transient spiral arms, but they probably are not important mass contributors in the local part of the Galaxy. The local approximation itself maybe questionable because most unstable waves have $\lambda$ of the order of several kpc.

Other possible complications in real galactic disks involve the presence of cosmic rays and magnetic fields which could influence the gas dynamics (Elmegreen 1987). In this paper we are primarily interested in the purely gravitational aspects of the disk instability and for this reason we neglect them at all, though it makes our consideration less realistic when applied to the galaxies. For these reasons our analysis of the stability of the Solar neighborhood should be considered only as mostly illustrative though bearing sufficient resemblance to reality.

Following Holmberg & Flynn (2000) we split the galactic disk mass between 13 major parts: 4 gaseous and 9 stellar. We neglected hot component of the gas because of its low number density, $n \sim 0.003$ cm$^{-3}$, high temperature, $T \sim 10^6$ K, (Savage 1987) and, consequently, large thickness which makes the reduction effects very important (Romeo 1992). For the same reason we neglect the stellar halo component. Parameters of the gaseous components are taken from Kulkarni & Heiles (1987) and Scoville & Sanders (1987).

Surface densities of stellar components are taken from Holmberg & Flynn (2000). We got the radial velocity dispersions based on the recent data from Hipparcos from Mignard (2000). Velocity dispersions of white and brown dwarfs have been chosen quite arbitrarily. All the parameters assumed for the mass constituents are listed in Table 1.

| i | Component | $\Sigma_i$ ($M_\odot$ pc$^{-2}$) | $\sigma_{ri}$ or $c_s$ (km s$^{-1}$) |
|---|-----------|-------------------------------|----------------------------------|
| 1 | H$_2$     | 3.0                           | 4.0                              |
| 2 | CM        | 4.0                           | 6.9                              |
| 3 | WNM       | 4.0                           | 9.0                              |
| 4 | WIM       | 2.0                           | 9.0                              |
| 5 | giants    | 0.4                           | 26.0                             |
| 6 | $M_V < 2.5$ | 0.9                          | 17.0                             |
| 7 | $2.5 < M_V < 3.0$ | 0.6 | 20.0                             |
| 8 | $3.0 < M_V < 4.0$ | 1.1 | 22.5                             |
| 9 | $4.0 < M_V < 5.0$ | 2.0 | 26.0                             |
| 10| $5.0 < M_V < 8.0$ | 6.5 | 30.5                             |
| 11| $M_V > 8.0$  | 12.3                          | 32.5                             |
| 12| white dwarfs | 4.4                          | 32.5                             |
| 13| brown dwarfs | 6.2                          | 32.5                             |

In Figure 8 we show the dependence of $\omega^2$ upon the inverse radial wavelength $k/2\pi$. Epicyclic frequency $\kappa = 36$ km s$^{-1}$ kpc$^{-1}$ is assumed throughout the calculation. Curve labeled 1 corresponds to the choice of parameters described in the Table 1. One can see that for all radial wavelength $\omega^2$ is positive that is the whole disk system is stable against local axisymmetric perturbations. To check how rigorous this conclusion is we varied some of the model parameters until the disk became unstable.

Data about the surface densities and velocity dispersions of the brown dwarfs (BDs) and white dwarfs (WDs) seem to be the most uncertain among all the model parameters, so we tried to vary them first. Curve labeled 2 has all the surface densities as listed in Table 1 but the velocity dispersion of WDs and BDs was lowered to $\sigma_r = 19.2$ km s$^{-1}$. Only if this population is so cold can it make the system unstable with all other parameters being kept unchanged. It seems inevitable that real WD and BD populations must be sufficiently hotter because only young stars can have such a low velocity dispersion (Mignard 2000).

Curve labeled 3 shows $\omega^2 - \lambda^{-1}$ dependence for the case when total surface density of WDs and BDs was raised from $10.6 M_\odot$ pc$^{-2}$ to $26 M_\odot$ pc$^{-2}$ keeping the rest of model parameters unchanged. This leads to the neutral stability of the system but such a surface density seems to be too large despite large uncertainties and claims of some authors.
that such extreme values of surface density could be common. For example, Festin (1998) found high mass density of the WD, \( \sim 2.6 \) times larger than we assume here, but his conclusions were based on a small sample of 7 sources only.

Other authors (Ruiz & Takamiya 1995; Oswalt et al. 1996) claim values for WD mass density which are in agreement with what we take. Even more uncertainty is involved in determining the density of the BD. Different surveys quote values from 0.6 to 4 times what we assume in this research (Fuchs, Jahreiss, & Flynn 1998). Recent data (Reid et al. 1999) based on a large enough sample imply mass density of BD 0.005 \( M_\odot \) pc\(^{-3} \) which is about 60% of what we assume in our calculations.

Finally, the fourth curve shows the dispersion relation for the disk with the lowered sound speed in some of the gas components: in CM we set \( c_s = 5.0 \) km s\(^{-1} \) in WNM \( c_s = 7.5 \) km s\(^{-1} \), and in WIM \( c_s = 8.0 \) km s\(^{-1} \), with all other parameters unchanged. In this case disk becomes marginally unstable. Important thing to notice here is that small variations in the gas sound speed can have stronger influence on the disk stability than large changes in the stellar velocity dispersion, even though the surface density of the gas is smaller than that of stars. This is a manifestation of the crucial importance of cold material for the stability of the whole disk, which was first noted by Jog & Solomon (1984a).

It is also easy to see that variations of the gas parameters produce significant change of the most unstable wavelength compared to the variations of the stellar parameters; it is reduced from \( \sim 2 \) kpc to \( \sim 1 \) kpc by that small change in gas sound speeds.

The bottom line is that local Galactic disk seems to be stable against local gravitational axisymmetric perturbation even when allowance for a scant knowledge of some of the Galactic parameters is made, which confirms results of Jog & Solomon (1984b) and Elmegreen (1995) for two-fluid disks.

6 CONCLUSIONS

Due to the continuous interaction with the transient spiral structure and giant molecular clouds starsdiffuse in the velocity space towards higher random velocities as their age increases and it was confirmed observationally (Dehnen & Binney 1998; Mignard 2000). It raises a necessity of considering the dynamics of the Solar neighborhood taking into account its complex multicomponent structure. In this paper we studied the stability of such a system against gravitational axisymmetric perturbations in the tight-winding limit. It is possible to derive an analytic dispersion relation characterizing multicomponent thin differentially rotating disk and study its stability.

In doing so we distinguished between two types of the disk constituents: stellar and gaseous. Stellar population is dynamically different from fluid because stars form collisionless system (Binney & Tremaine 1987) while gas must be treated as a fluid. We demonstrated that the difference in the results for stability produced by two approaches is small for multicomponent disks in many astrophysically interesting cases. Some disk models though could be sensitive to the choice of the stability condition and in that case one should use correct criterion given by the equation (23).

We apply our results to the stability of the Solar neighborhood and confirm the conclusions of the previous two-fluid studies that local Galactic disk is stable against axisymmetric perturbations in the WKB limit, even taking into account uncertainties associated with determining some of the disk parameters.

Keeping in mind previous two-fluid results (JS) it is not surprising that relatively small variations of the gaseous component parameters are very important for the overall disk stability. Indeed, Figure 4 shows that decrease of the sound speed of the gas by about 1 km s\(^{-1} \) drives instability of the disk, while in the case of stellar component one needs to reduce its velocity dispersion by \( \sim 10 \) km s\(^{-1} \) to produce the same outcome. Even though the mass of the gas in the disk is smaller that the stellar mass its small random motion makes it much more susceptible to its own self-gravity than the hot stellar component.

Our study of the stability of the Solar neighborhood neglects a lot of physics such as magnetic fields or nonaxisymmetry of the Galactic gravitational field and their importance remains an open question. Measurement errors associated with determining some disk parameters also limit the applicability of multicomponent criterion because the larger the number of constituents the larger errors get accumulated. Future interferometric missions such as \emph{GAIA} and \emph{SIM} will probably solve this problem because of their anticipated accuracy. Nevertheless, even with all the simplifications and the observational uncertainties involved it seems that the Solar neighborhood is stable against purely gravitational axisymmetric perturbations.

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