Di-neutron correlations and decay dynamics of nuclei near and beyond drip line

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Abstract. Possible experimental probes for the strong di-neutron correlations in halo nuclei $^6$He and $^{11}$Li are discussed in the Coulomb breakup reactions. We then study a nucleus beyond the drip line $^{26}$O through the direct two-neutron decay. The excited first $2^+$ state of $^{26}$O is also discussed. To this end, we use consistently a three-body model taking into account the coupling to the continuum. Calculated results are compared with the recent experimental data from RIBF (Radioactive Ion Beam Factory) in RIKEN.

1. Introduction
One of the most important issues in quantum many-body physics is to clarify the nature of correlations beyond the independent particle picture. In nuclear physics, the pairing correlation has been well established as a strong many-body correlation which leads to the superfluid phase characterized by phenomena such as the even-odd staggering of binding energy, the moment of inertia of rotating deformed nuclei, the difference of the excitation energy spectra between even-even and odd-even nuclei, the pair transfer reactions and also the fission barrier [1, 2, 3].

With the pairing correlation, one may naively expect that two nucleons forming a pair are located at a similar position inside a nucleus. A spatial structure of two valence neutrons was shown to be strongly clusterized in $^{210}$Pb [4]. Subsequently, it was argued that two neutrons may be bound in a nucleus even though they are not bound in the vacuum [5].

The strong localization of two neutrons inside a nucleus has been referred to as the di-neutron correlation. It has been nicely demonstrated in Ref. [6] that an admixture of configurations of single-particle orbits with opposite parity is essential to create the strong di-neutron correlation. This implies that the pairing correlation acting only on single-particle orbits with same parity may not be sufficient in order to develop the di-neutron correlation, and the pairing model space needs to be taken sufficiently large so that both positive parity and negative parity states are included.
Although the di-neutron correlation exists even in stable nuclei, it is therefore more enhanced in weakly bound nuclei because the admixtures of single-particle orbits with different parities are easier due to the couplings to the continuum spectra [7, 8]. Three-body model calculations have revealed that a strong di-neutron correlation indeed exists in weakly-bound Borromean nuclei, such as $^{11}$Li and $^6$He [9, 10, 11, 12, 13, 14]. For instance, Fig. 1 shows the two-particle density for the $^{11}$Li and $^6$He nuclei obtained with the three-body model calculation with a density dependent contact pairing interaction [12]. One can see that the densities are concentrated in the region with small opening angles, that is nothing but the di-neutron correlation. It has been shown that the di-neutron correlation exists also in heavier neutron-rich nuclei [15, 16] as well as in infinite neutron matter [17]. The di-proton correlation, which is a counter part of the di-neutron correlation, has also been shown to exist in the proton-rich Borromean nucleus, $^{17}$Ne [18].

From these studies, the di-neutron correlation seems to have been theoretically established. However, it is not straightforward to probe it experimentally. In this contribution, we discuss how one can probe the di-neutron correlation. To be more specific, we shall discuss the Coulomb breakup, and the two-nucleon radioactivity as prominent probes for the correlation.

2. Electric dipole strength of Borromean nuclei and di-neutron correlations

Let us first discuss the Coulomb breakup reactions of Borromean nuclei, $^{11}$Li and $^6$He. The experimental breakup cross sections in Refs. [19, 20], especially those for the $^{11}$Li nucleus, show a strong concentration in the low excitation region, reflecting the halo structure of these nuclei. Moreover, the experimental data for $^{11}$Li are consistent only with the theoretical calculation which takes into account the interaction between the valence neutrons, strongly suggesting the existence of the di-neutron correlation in this nucleus (see also Ref. [21]).

The Coulomb breakup cross sections with the absorption of dipole photons are given by

$$\frac{d\sigma_\gamma}{dE_\gamma} = \frac{16\pi^3}{9\hbar c} N_\gamma(E_\gamma) \cdot \frac{dB(E1)}{dE_\gamma},$$

(1)

where $N_\gamma$ is the number of virtual photons, and

$$\frac{dB(E1)}{dE_\gamma} = \frac{1}{2I_i + 1} |\langle \psi_f | D | \psi_i \rangle|^2 \delta(E_f - E_i - E_\gamma),$$

(2)

is the $E1$ transition probability. In this equation, $\psi_i$ and $\psi_f$ are the wave functions for the initial and the final states, respectively, $I_i$ is the spin of the initial state, and $D_\mu$ is the operator for
the $E1$ transition. For the Borromean nuclei, assuming a three-body structure with an inert core, the $E1$ operator reads $\hat{D}_\mu = 2\frac{e}{\epsilon_c} (r_1 Y_{1\mu}(\hat{r}_1) + r_2 Y_{1\mu}(\hat{r}_2))$, where the $E1$ effective charge is given by $e_{E1} = 2Z_c e/(A_c + 2)$, with $A_c$ and $Z_c$ being the mass and charge numbers for the core nucleus, and $r_1$ and $r_2$ denote the positions of particle 1 and 2 referred to the center of mass, respectively. The dipole transition of di-neutrons is entirely the recoil effect of protons in the core and the center of mass motion is exactly removed from the dipole operator $\hat{D}_\mu$. The experimental data of the transition probabilities (2) are shown in Fig. 2 with calculated results by a three-body model in ref. [21]. We can see clearly a strong enhancement of $B(E1)$ strength just above the threshold energy due to the di-neutron correlations.

![Figure 2. (Color online) The E1 transitions in $^{11}$Li and $^6$He observed by Coulomb breakup reactions. The data are taken from ref. [19] for $^{11}$Li and from ref. [20] for $^6$He, respectively.](image)

Using Eq. (2) and the closure relation for the final state, it is easy to derive that the total $E1$ strength (the non-energy weighted sum rule) is proportional to the expectation value of $R^2$ with respect to the ground state:

$$B(E1) = \int dE_\gamma \frac{d\gamma(E1)}{dE_\gamma} = \sum_f \frac{1}{2I_f + 1} |\langle \psi_f | \hat{D} | \psi_i \rangle|^2 = \frac{3}{4\pi} e_{E1}^2 \langle \hat{R}^2 \rangle,$$

where $\hat{R} = (\hat{r}_1 + \hat{r}_2)/2$, is the center of mass coordinate for the two valence neutrons. Even though the $B(E1)$ strength distribution inevitably reflects both the correlation in the ground state and that in the final state, it is remarkable that one can extract the information which reflects solely the ground state properties after summing all the strength distribution. This implies that the average value of the opening angle between the valence neutrons can be directly extracted from the measured total $B(E1)$ value once the root-mean-square distance between the valence neutrons, $\langle r_{nn}^2 \rangle$, is available.

This quantity is related to the matter radius and $\langle R^2 \rangle$ in the three-body model [9, 21, 23],

$$\langle r_{m}^2 \rangle = \frac{A_c}{A} \langle r_{m}^2 \rangle_{A_c} + \frac{2A_c}{A^2} \langle R^2 \rangle + \frac{1}{2A} \langle r_{nn}^2 \rangle,$$

where $A = A_c + 2$ is the mass number of the whole nucleus. The matter radii $\langle r_{m}^2 \rangle$ can be estimated from interaction cross sections. Employing the Glauber theory in the optical limit, Tanihata et al. have obtained $\sqrt{\langle r_{m}^2 \rangle} = 1.57 \pm 0.04$, $2.48 \pm 0.03$, $2.32 \pm 0.02$, and $3.12 \pm 0.16$ fm for $^4$He, $^6$He, $^9$Li, and $^{11}$Li, respectively [24, 25]. Using these values, we obtain the rms neutron-neutron distance of $\sqrt{\langle r_{nn}^2 \rangle} = 3.75 \pm 0.93$ and $5.50 \pm 2.24$ fm for $^6$He and $^{11}$Li, respectively. Combining these values with the rms core - di-neutron distance, $\sqrt{\langle R^2 \rangle}$, we obtain the mean opening angle

$$\langle \theta_{nn} \rangle = 2 \tan^{-1} \left( \sqrt{\frac{\langle r_{nn}^2 \rangle}{2\langle R^2 \rangle}} \right)$$
to be 51.56$^{+1.2}_{-1.4}$ and 56.2$^{+17.8}_{-21.3}$ degrees for $^{6}$He and $^{11}$Li, respectively [26]. These values are comparable to the result of the three-body model calculation, $\langle \theta_{nn} \rangle = 66.33$ and 65.29 degree for $^{6}$He and $^{11}$Li, respectively [12], although the experimental values are somewhat smaller.

In the absence of the correlations, the mean opening angle is exactly $\langle \theta_{nn} \rangle = 90$ degrees. The extracted values of $\langle \theta_{nn} \rangle$ are significantly smaller than this value both for $^{11}$Li and $^{6}$He, providing a direct proof of the existence of the di-neutron correlation in these nuclei.

3. Two-neutron decays of nuclei beyond the drip lines

Figure 3 shows the calculated decay energy spectrum of $^{26}$O obtained with the $^{24}$O+$n+n$ three-body model for $^{26}$O [28]. The calculations are carried out using the Green’s function method as explained in Refs. [8, 22, 29, 30], together with a density-dependent contact neutron-neutron interaction, $v$. In this formalism, the decay energy spectrum is given by,

$$\frac{dP}{dE} = \sum_{k} \frac{|\langle \Psi_{k} | \Phi_{\text{ref}} \rangle |^{2}}{E_{k} - E} \delta(E - E_{k}) = \frac{1}{\pi} \Re \langle \Phi_{\text{ref}} | G(E) | \Phi_{\text{ref}} \rangle,$$

where $\Psi_{k}$ is a solution of the three-body model Hamiltonian with energy $E_{k}$ and $\Phi_{\text{ref}}$ is the wave function for a reference state. The reference state can be taken rather arbitrarily as long as it has an appreciable overlap with the resonance states of interest. Here, we employ the uncorrelated two-neutron state in $^{27}$F for it with the [$(1d_{3/2} \otimes 1d_{3/2})^{(I=0)}$] configuration, which is the dominant configuration in the initial state of the proton knockout reaction of $^{27}$F to produce $^{26}$O. In Eq. (6), $G$ is the correlated two-particle Green's function calculated as

$$G(E) = (1 + G_{0}(E)v)^{-1} G_{0}(E) = G_{0}(E) - G_{0}(E)v(1 + G_{0}(E)v)^{-1} G_{0}(E),$$

with the uncorrelated Green’s function, $G_{0}$, given by

$$G_{0}(E) = \lim_{\eta \to 0} \sum_{I=1}^{2} \frac{|j_{1}j_{2} \rangle \langle j_{1}j_{2}|}{\epsilon_{I} - E - i\eta}.$$

In Eq. (7), $v$ is the two-body interaction between two neutrons.

In the figure, the uncorrelated spectrum shown by the dotted line has a peak at $E = 1.498$ MeV, that is twice the single-particle resonance energy of $d$–orbit, $\epsilon_{d_{3/2}} = 749(10)$ keV for $^{25}$O [27]. With the pairing interaction between the valence neutrons, the calculations yield the decay energy of 18 keV of the ground state. The decay spectrum obtained by including only the $[d_{3/2}]^{2}$ configurations is shown by the dashed line. In this case, the peak is shifted by 0.55 MeV from the unperturbed peak at 1.498 MeV. The peak is further shifted downwards by the di-neutron correlations due to configuration mixing, and gets closer to the threshold energy.

In Ref. [27] (also in Fig. 3), a clear second peak is found at $E = 1.284^{+0.11}_{-0.01}$ MeV, which is likely attributed to the $2^{+}$ state. In Ref. [30, 28], we have investigated the $2^{+}$ state in the $^{26}$O nucleus using the three-body model. Due to the pairing interaction between the valence neutrons, the energy of the $2^{+}$ state is slightly shifted towards lower energies from the unperturbed energy, whereas the energy shift is much larger for the $0^{+}$ state due to the larger overlap between the wave functions of the two neutrons. The $2^{+}$ peak appears at 1.282 MeV, which agrees perfectly with the experimental data [28].

The angular distribution of the emitted neutrons can be calculated with the two-particle Green’s function method [22, 29, 30]. The amplitude for emitting two neutrons with spin components of $s_{1}$ and $s_{2}$ and momenta $\vec{k}_{1}$ and $\vec{k}_{2}$ reads [29],

$$f_{s_{1}s_{2}}(\vec{k}_{1}, \vec{k}_{2}) = \sum_{j,l} e^{-i\delta_{j}} e^{i(\delta_{1}+\delta_{2})} M_{j,l,k_{1},k_{2}} \langle |\Psi_{j}(\vec{k}_{1}) \Psi_{j}(\vec{k}_{2})|^{(00)} |\chi_{s_{1}s_{2}} \rangle,$$
\[ Y_{jlm} \] is the spin-spherical harmonics, \( \chi_s \) is the spin wave function, and \( \delta \) is the nuclear phase shift. Here, \( M \) is a decay amplitude calculated to a specific two-particle final state \[ M_{j,l,k_1,k_2} = \langle jj^{(00)} | 1 - vG_0 + vG_0vG_0 - \cdots | \Psi_i \rangle = \langle jj^{(00)} | (1 + vG_0)^{-1} | \Psi_i \rangle, \] (10)

where the unperturbed Green’s function, \( G_0 \), is evaluated at \( E = e_1 + e_2 \). The angular distribution is then obtained as

\[ P(\theta_{12}) = 4\pi \sum_{s_1,s_2} \int d\vec{k}_1 d\vec{k}_2 | f_{s_1s_2}(k_1, \vec{k}_1 = 0, k_2, \vec{k}_2 = \theta_{12})|^2, \] (11)

where we have set \( z \)-axis to be parallel to \( \vec{k}_1 \) and evaluated the angular distribution as a function of the opening angle, \( \theta_{12} \), of the two emitted neutrons.

Fig. 4 shows the angular correlation. The dot-dashed line shows the distribution obtained without including the \( nn \) interaction, which is symmetric around \( \theta_{12} = \pi/2 \). In the presence of the \( nn \) interaction, the angular distribution turns to be highly asymmetric, in which the emission of two neutrons in the opposite direction (\( \theta_{12} = \pi \)) is enhanced shown by the solid line.

4. Summary
We have discussed possible experimental probes for the di-neutron correlation in neutron-rich nuclei, with which two valence nucleons are localized at a similar position in the coordinate space. In particular, we have discussed the Coulomb dissociation of Borromean nuclei, and the direct two-neutron decay of the unbound \( ^{26}\text{O} \) nucleus.

For the Coulomb dissociation of Borromean nuclei, one can use the cluster sum rule (the non-energy weighted sum rule for localized systems) to deduce the mean value of the opening angle between the valence neutrons. We have demonstrated that the mean opening angle is \( \langle \theta_{nn} \rangle = 74.5 \).
$^{+11.2}_{-13.1}$ and $65.2 \pm 11.4$ degrees for $^6$He and $^{11}$Li, respectively. These values are significantly smaller than the value for the uncorrelated distribution, that is, $\langle \theta_{nn} \rangle = 90$ degrees, clearly indicating the existence of the di-neutron correlation in these Borromean nuclei.

For the direct two-neutron decay, we have discussed the recent experimental data of the decay energy spectrum for the unbound $^{26}$O nucleus. We have shown that the decay energy spectrum can be accounted for only with the di-neutron correlation due to the mixing of many configurations including the continuum. We have also discussed the angular correlations of the emitted two neutrons. We have argued that the di-neutron correlation enhances an emission of the two neutrons in the opposite direction (the back-to-back emission), and our three-body model calculation has revealed such feature. If the enhancement of the back-to-back emission will be observed experimentally, it will provide a direct evidence for the di-neutron correlation.

Other possible probes are proposed to detect the di-neutron correlations. Those include the two-neutron transfer reactions, the nuclear breakup reaction, the $(p,d)$ scattering at backward angles, and the knockout reactions of Borromean nuclei. It would be extremely intriguing if the clear and direct evidence for the di-neutron correlation could be experimentally obtained in near future using the probes discussed in this manuscript.

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