Numerical Simulation of the Flow of Nano-Eyring-Powell Fluid through a Curved Artery with Time-Variant Stenosis and Aneurysm

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(Received : November 26, 2018)

In this paper, the flow of blood through a curved vessel having stenosis and aneurysm is investigated. To evaluate the impact of stenosis and aneurysm in a curved channel, the curvilinear coordinates are used to formulate a suitable geometry. The flow and heat transfer are investigated in the presence of nanoparticles that play a significant role in blood flows through arteries and they are gaining popularity in hematological treatment. The dynamical behavior of blood flow is modeled by using Eyring-Powell fluid model and the coupled partial differential equations are formulated to study the blood rheology. The flow, and heat and mass transfer equations are numerically solved by using finite difference scheme. The effect of some significant parameters on blood flow through a curved channel with stenosis and aneurysm is discussed and displayed in graphs. The pattern of blood flow is also depicted through geometrical patterns.

Key Words: Blood flow / Nanoparticles / Curved artery / Stenosis / Aneurysm

1. INTRODUCTION

The flow of blood in a curved vessel with stenosis is an important topic for researchers due to the fact that many cardiovascular diseases are mainly identified from the blood flow and mechanical conduct of the walls of the arteries; in most of the cases, circulatory disorders cause deaths. Since the pioneer work of Mann, et al.1), this problem has been the subject of many scientific investigations. The identification of atherosclerosis at initial stages of the disease, which implies just negligible contraction of the blood vessel lumen, is of high clinical intrigue. Stenosis is characterized as a partial constriction of the artery due to high cholesterol, fats, and abnormal development of tissues2). It is amongst the most common irregularities in circulation of blood that significantly alters the blood flow once it is developed and leads to the evolution of cardiovascular diseases, for instance, heart attack and stroke. It happens due to stenosis, the tapering of coronary arteries to the point that they cannot transport adequate blood to the heart muscle so that it can work proficiently. The fluid dynamical aspects play a significant role as the stenosis keeps on increasing. Many studies are conducted to investigate the physiology and depth of the stenosis, variation in flow resistance, and the shear stress of wall along the axial direction3-6).

The combined occurrence of stenosis and aneurysms is common in most vessels. It is now largely acknowledged that the evolution and advancement of stenosis is closely related to the study of fluid transport of post-stenotic blood flow. At high flow rates, aneurysm in the coronary arteries arises, conceivably because of the wall shear stress resulting the constriction7, 8). Yet, less attention is directed towards the inspection of the behavior of flow of blood through vessels with both aneurysm and stenosis. As indicated by Moore, et al.9), aneurysms or post-stenotic enlargements are as often discovered in coronary arteries. In this manner, it is more valuable to examine the hemodynamics in an artery with both aneurysm and stenosis. This inspired Pincombe and Mazumdar10) to study the impacts of post-stenotic enlargement on blood moving in veins. In that study, to depict the blood rheology, they used the Bingham plastic fluid model. Later, Pincombe, et al.11) examined the fully developed flow of Casson fluid from a vein with multiple contractions and dilations. Wong, et al.12) also presented a study on the blood flow from a vein having multiple aneurysms and stenosis. Further, Zaman, et al.13) examined the blood flow from a contracted and dilated vessel with a catheter placed in it.
The convective transport of nanofluids has gained attention of researchers owing to their vast range of benefits in natural and material science, and in chemical and thermal engineering. In base fluid the existence of nanoparticles can raise the fluid flow and heat transfer in liquids. These particles have several uses in micro-manufacturing, metallurgical and chemical sectors, and in medical sciences such as treatment of cancer using thermal therapy\(^{24}\). Thermal conductivity and diffusivity are some important thermophysical characteristics of nanofluids\(^{15-17}\). Various significant researches have been conducted to explore the properties of nanofluids under certain fluid and boundary conditions\(^{18-21}\). The blood circulatory system is a complex arrangement of blood vessels that transport vital nutrients in the body and removes waste products. This system is also responsible for exchange of gases, macromolecules, and ions. Its ability to carry variable payloads to tissues is tremendously related to drug delivery. Blood-mediated nanoparticle transport is a rising field in the development of therapeutics and diagnostics\(^{22}\). On the other hand, the presence of nanomaterials within blood circulatory system can cause injurious effects on interaction with various blood components. It can trigger pathophysiological activities and modify physicochemical properties of nanoparticle. Therefore, blood compatibility is obligatory for the safe transport of drug-loaded nanoparticles through blood\(^{23, 24}\).

In past, numerous experimental investigations and theoretical studies on the flow of blood from stenosed vessels have been conducted realizing that the tendency of periodic variations in the flow of blood cannot be dismissed. In major part of the existing literature, the flow of blood is considered to be Newtonian in nature. Whereas, this Newtonian nature of the flow of blood is adequate only for the flow with high shear, such as the case of flow from larger vessels. However, when the shear rate is low, it is not of much impact such as the flow in downstream of stenosis or in smaller vessels; blood displays shear-dependent viscosity in small vessels and requires a limited yield stress before the flow can begin, thus making the modelling of non-Newtonian important to accurately study the nature and behavior of blood flow. From previous studies, it is concluded that blood flowing at 0.3 m/s for tubes of 0.8 mm diameter with a hematocrit of 38 %, exhibits shear-dependent viscosity\(^{25}\). Lately, many investigations have been conducted to study the blood flow from stenosed arteries\(^{26, 27}\). The mathematical models used to describe the flow of blood are the Power Law\(^{28}\), Casson\(^{11}\), Herschel-Bulkley\(^{29}\), Bingham\(^{10}\), micropolar\(^{30}\), Sisko\(^{31}\), and Eyring-Powell\(^{32}\) fluid models. Eyring-Powell nanofluid is the best example of human blood at low shear rates less than 100 s\(^{-1}\).\(^{33}\) Moreover, Cho and Kensey\(^{34, 35}\) compared the experimental blood viscosity data with predictions from several non-Newtonian models and concluded that the Eyring-Powell models fit the experimental data well in the entire range of the shear rate.

All the above studies have been conducted for flow in straight channel or tubes. Whereas, most of the physiological ducts have curved shapes. The studies dealing with the data identified with the progression of blood course from stenosed curved vessels are limited. The similar study of the blood flow in curved channel with both stenosis and aneurysm was carried out by Zaman, et al.\(^ {36}\), but they used viscous fluid to model the blood flow. Due to complex nature of blood, it cannot be modeled accurately by Newtonian fluid. Hence, the Eyring-Powell fluid model is used to investigate the flow of blood through stenosis and aneurysm in the presence of nanoparticles.

### 2. PHYSIOLOGICAL MODELING OF BLOOD FLOW IN A CURVED ARTERY

Assume the unsteady, laminar, and incompressible flow of blood in an inflexible, curved artery with two abnormal sections, an aneurysm and a stenosis, and the blood is flowing through these diseased segments. The artery of length \(L\) is looped in a circle of radius \(R\) from the center \(O\). The geometry of diseased segments can be defined mathematically as:

\[
R(x) = \begin{cases} 
\frac{\delta_i^*}{\delta_i^*} \left( 1 - \frac{x}{R_i^*} \right) & \text{if } x \in [0, \frac{\delta_i^*}{\delta_i^*}], \\
\frac{\delta_i^*}{\delta_i^*} & \text{otherwise}
\end{cases}
\]

where \(R(x)\) formulates the geometry of the upper wall of the curved artery, and the lower wall is defined by \(-R(x)\). In above equation, \(R_i^*\) symbolizes the radius of the normal vessel, \(\alpha = \tan \Psi\) measures the constriction of an artery, \(\Psi\) is the tapered angle, \(\sigma_i^*\) symbolizes the length of \(i\)th abnormal region from the origin, and \(\delta_i\) is the length of diseased segment. A graphical model of curved artery with stenosis and aneurysm is presented in Fig. 1, where \(\delta_i^*\) represents the critical height of the \(i\)th diseased segment occurring at two explicit locations defined as:

\[
x = \sigma_i^* + \frac{\delta_i}{2} \quad \text{and} \quad x = \sigma_i^* + \frac{\delta_i}{2}
\]

where \(\delta_i^*\) assumes the positive value for stenosis and negative value for aneurysm.

To examine the characteristics of blood flow in a curved...
channel with stenosis and aneurysm, non-Newtonian Eyring-Powell fluid is considered in the existence of nanoparticles, the stress tensor of fluid model considered for this study can be defined as

\[
\tau_{ij} = \mu A + \frac{1}{\beta d} \sinh\left(\frac{1}{d} \gamma \right) A
\]

(3)

where \(\tau_{ij}\) symbolizes the stress tensor, \(\mu\) is the viscosity coefficient, and \(\beta\) and \(d\) represent the fluid parameters; the rate-of-strain-tensor is obtained from \(A = \nabla V + (\nabla V)^t\) where \(V\) represents the velocity vector and \(V\) is the differential operator.

The second invariant of rate-of-strain-tensor \(\gamma\) is expressed as

\[
\dot{\gamma} = \frac{1}{2} \dot{\tau} \dot{A}
\]

(4)

To obtain the simplified form of equation of momentum, the Taylor approximation of second order of \((\sinh)^{-1}\) function is defined as

\[
\sinh^{-1}\left(\frac{1}{d} \gamma \right) \approx \frac{1}{2} \sinh\left(\frac{1}{d} \gamma \right) \approx \frac{1}{6} \left(1 - \frac{1}{d} \gamma \right)
\]

(5)

On employing the above approximation, Eq. (3) becomes

\[
\tau = \left(\mu + \frac{1}{\beta d} \right) A - \frac{1}{6 \beta d^2} \gamma^2 A
\]

(6)

To derive the flow equations in curved artery, the velocity vector \(V\) is defined as \(V = (u(r,x,t), v(r,x,t))\), where \(u\) and \(v\) are the radial and axial velocity components in curvilinear coordinates \((r, x)\), chosen in such a way that the \(r - axis\) is measured from central line and is normal to the \(x - axis\) which lies along the centerline of the curved channel with scaling factors \(h_1 = 1\) and \(h_2 = \frac{R}{r + R}\). The differential operator \(V\) is defined in curvilinear coordinates as \(V = \left(\frac{\partial}{\partial r} \cdot \frac{R}{r + R}, \frac{\partial}{\partial x}\right)\).

In an axisymmetric flow, the components of rate-of-strain-tensor are now attained as:

\[
A_r = 2\left(\frac{\partial u}{\partial r}\right) A_r, A_z = A_z = 2\left(\frac{\partial v}{\partial r}\right)
\]

(7)

Thus, the components of stress term become:

\[
\tau_{rr} = 2\left(\mu + \frac{1}{\beta d} \right) \left(\frac{\partial u}{\partial r}\right) \frac{1}{r + R} + \frac{1}{(r + R)^2} \left(\frac{\partial v}{\partial r}\right) \frac{1}{r + R}
\]

\[
\tau_{zz} = 2\left(\mu + \frac{1}{\beta d} \right) \left(\frac{\partial v}{\partial r}\right) \frac{1}{r + R} + \frac{1}{(r + R)^2} \left(\frac{\partial u}{\partial r}\right) \frac{1}{r + R}
\]

\[
\tau_{rz} = 2\left(\mu + \frac{1}{\beta d} \right) \left(\frac{\partial u}{\partial r}\right) \frac{1}{r + R} + \frac{1}{(r + R)^2} \left(\frac{\partial v}{\partial r}\right) \frac{1}{r + R}
\]

(8)

Considering all the above assumptions and further assuming that the free stream is kept at uniform ambient temperature \(T_\infty\) and the constant temperature \(T_0\) and concentration \(C_0\) are maintained at the walls of the channel; the continuity, momentum, temperature, and concentration equations are obtained as:

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial v}{\partial x} = 0
\]

(10)

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial v}{\partial x} = \frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{R^*}{r + R^*} \frac{\partial \tau_{zz}}{\partial x} + \frac{R^*}{r + R^*} \frac{\partial \tau_{rz}}{\partial x}
\]

(11)

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial v}{\partial x} = \frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{R^*}{r + R^*} \frac{\partial \tau_{zz}}{\partial x} + \frac{R^*}{r + R^*} \frac{\partial \tau_{rz}}{\partial x} + \rho g a (T - T_0) + \rho g a (C - C_0)
\]

(12)

\[
\left(\frac{\partial T}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial T}{\partial x}\right) = \frac{k}{\rho c_p} \left(\frac{\partial T}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial T}{\partial x}\right) + \tau_0 \left(\frac{\partial C}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial C}{\partial x}\right)
\]

(13)

\[
\left(\frac{\partial C}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial C}{\partial x}\right) = \tau_0 \left(\frac{\partial C}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial C}{\partial x}\right) + D_\text{t} \left(\frac{\partial T}{\partial r} + \frac{u}{r} + \frac{R^*}{r + R^*} \frac{\partial T}{\partial x}\right)
\]

(14)

where, \(\rho\) is the density of the fluid, \(T\) is the temperature and \(C\) is the concentration specie, \(\alpha_t\) is the thermal expansion coefficient, \(\alpha_c\) is the thermal expansion with nanoconcentration, \(C_P\) is the specific heat at constant pressure, \(\theta_0\) is the constant of heat generation or absorption, \(k\) is the thermal conductivity, \(D_t\) is the coefficient of thermo-diffusion, \(D_b\) is the molecular diffusivity, and \(\tau = \frac{(\rho C)}{(\rho C)}\) is the heat capacity ratio of nanofluids.
Now introducing the non-dimensional variables as

\[
\begin{align*}
\tau = \frac{r \cdot t}{R_i}, \quad \tau_c = \frac{r \cdot \lambda_i}{R_i \cdot U_i}, \quad \frac{r \cdot U_i}{U_i}, \quad \frac{U_i}{U_i}, \quad \frac{R_i}{R_i}, \quad \frac{R_i}{R_i}, \quad \frac{\rho \cdot n}{\mu}, \quad \frac{\rho \cdot n}{\mu}, \\
\theta = \frac{\theta - \theta_T}{\theta_L - \theta_T}, \quad \phi = \frac{\phi - \phi_T}{\phi_L - \phi_T}, \quad \frac{\theta - \theta_T}{\theta_L - \theta_T}, \quad \frac{\phi - \phi_T}{\phi_L - \phi_T}, \quad \frac{n - n_L}{n_L - n_T}, \quad \frac{n - n_L}{n_L - n_T}
\end{align*}
\]

Where, \( r, x, u, v, t, p \) represent the dimensionless form of the radial coordinate, axial coordinate, radial velocity, axial velocity, time, and pressure respectively, \( \kappa \) is the dimensionless radius of the curved artery, \( \theta \) and \( \phi \) are the dimensionless temperature and concentration species, \( \text{Re} \) is the Reynolds number, \( \Theta \) is the dimensionless heat generation or absorption parameter, \( Gr_r \) is the thermal Grashof number, \( Gr_c \) is the nanoparticle Grashof number, \( Pr \) is the Prandtl number, \( M \) and \( N \) are the Eyring-Powell fluid parameters, \( \text{Sc} \) is the Schmidt number, \( N_b \) is the Brownian motion parameter, and \( N_t \) is the thermophoretic parameter.

Using the above defined dimensionless variables in Eqs. (10)-(14), dropping the bars, and assuming that the length of stenosed region and radius of artery are comparable \( R_b / \lambda_i \approx 0(l) \) and the maximum height of the stenosis is less in comparison to the radius of the channel \( \delta = \frac{\delta^*}{R_i} \ll 1 \), the suitable equations for the unsteady flow of Eyring-Powell fluid in curved artery with two diseased sections can be defined as:

\[
0 = \frac{\partial p}{\partial r} \tag{16}
\]

From above equations, it can be seen that \( p \) is independent of \( r \) whereas, \( v, \theta, \) and \( \varphi \) are the functions of \( (r, t) \). The axial pressure gradient equation proposed by Burton\(^{37} \) can be written as

\[
-\frac{\partial p}{\partial x} = A_0 + A_1 \cos(2\pi \omega t), \quad t > 0 \tag{20}
\]

Where, \( A_0 \) is used for the mean pressure gradient and \( A_1 \) symbolizes the maximum value of the periodic component, responsible for diastolic and systolic pressures. The dimensionless form of above equation can be obtained as

\[
-\frac{\partial p}{\partial x} = B(1 + \text{e} \cos(ct)) \tag{21}
\]

Where \( c = \frac{A_1}{A_0} \), \( c = \frac{R_b \omega}{U_i} \), and \( \theta = \frac{A_0 \omega^3}{R_b U_i} \). Now substituting \( -\frac{\partial p}{\partial x} \) in Eq. (17) gives

\[
\frac{r \cdot \text{Re} \cdot \frac{\partial \theta}{\partial t}}{\kappa} = B(1 + \text{e} \cos(ct)) + \left( \frac{r \cdot \kappa \cdot \frac{\partial \psi}{\partial \psi}}{\kappa} \right) \left( \frac{r \cdot \kappa \cdot \frac{\partial \psi}{\partial \psi}}{\kappa} + \frac{1}{\kappa} \frac{\partial \psi}{\partial \psi} \right) \left( \frac{1}{\kappa} \frac{\partial \psi}{\partial \psi} + \frac{1}{\kappa + \kappa} \frac{\partial \psi}{\partial \psi} \right) \tag{22}
\]

Now the boundary conditions related to the flow can be defined as:

\[
\begin{align*}
v(r, t) &= 0 \quad \text{on} \quad R_b, \quad v(r, t) = 0 \quad \text{on} \quad R_i, \quad v(r, 0) = 0, \\
\theta(r, t) &= 0 \quad \text{on} \quad R_b, \quad \theta(r, t) = 0 \quad \text{on} \quad R_i, \quad \theta(r, 0) = 0, \\
\varphi(r, t) &= 0 \quad \text{on} \quad R_b, \quad \varphi(r, t) = 0 \quad \text{on} \quad R_i, \quad \varphi(r, 0) = 0.
\end{align*}
\]  
\text{otherwise} \tag{23}
\]

Where, the dimensionless form of the geometry of the channel is obtained as

\[
R(x) = \left\{\begin{array}{ll}
(\text{ax}+1) & \text{if } \frac{1}{2} \left[ 1 + \text{cos} \left( \text{ar} \cdot \frac{1}{2} \right) \right], \\
\text{otherwise}
\end{array}\right.
\]  
\tag{24}
\]

The boundary conditions in Eq. (23) are defined at a moving frame of reference which makes the problem complicated. Therefore, a new coordinate transformation is introduced as

\[
\xi = \frac{r}{R(x)} \tag{25}
\]

Now the boundary conditions in Eq. (23) can be rewritten as

\[
\begin{align*}
v(\xi, t) &= 0 \quad \text{on} \quad \xi = 0, \quad v(\xi, 0) = 0, \\
\theta(\xi, t) &= 0 \quad \text{on} \quad \xi = 0, \quad \theta(\xi, 0) = 0, \\
\varphi(\xi, t) &= 0 \quad \text{on} \quad \xi = 0, \quad \varphi(\xi, 0) = 0, \tag{26}
\end{align*}
\]

On carrying the transformation defined in Eq. (25) into Eqs. (18), (19), and (22), the momentum, energy, and concentration equations, respectively, can be rewritten as:
The important dimensionless physical quantities, flow rate and wall shear stress, can be expressed as:

\[
\frac{xR^{2} + \kappa}{\kappa} \frac{d \nu}{\delta} - M \left( \frac{1}{R^{2} \frac{R^{2}}{\delta}} + \frac{1}{R \frac{\delta}{R^{2}}} \frac{1}{R^{2} \frac{\delta}{R^{2}}} \right) + \frac{v}{R^{2} \frac{R^{2}}{\delta}} + \frac{v}{R \frac{\delta}{R^{2}}} = 0
\]  

(27)

On employing the transformation defined in Eq. (25) into Eqs. (30) and (31), the expression for flow rate and wall shear stress can be rewritten as:

\[
\frac{v}{R^{2} \frac{R^{2}}{\delta}} \left( \frac{1}{R^{2} \frac{R^{2}}{\delta}} + \frac{1}{R \frac{\delta}{R^{2}}} \right) + \frac{v}{R^{2} \frac{R^{2}}{\delta}} + \frac{v}{R \frac{\delta}{R^{2}}} = 0
\]  

(28)

\[
\frac{v}{R^{2} \frac{R^{2}}{\delta}} \left( \frac{1}{R^{2} \frac{R^{2}}{\delta}} + \frac{1}{R \frac{\delta}{R^{2}}} \right) + \frac{v}{R^{2} \frac{R^{2}}{\delta}} + \frac{v}{R \frac{\delta}{R^{2}}} = 0
\]  

(29)

The important dimensionless physical quantities, flow rate and wall shear stress, can be expressed as:

\[
Q = \chi \frac{R}{R^{2}} \int r \nu(r, t) dr
\]  

(30)

\[
\tau_{w} = \tau_{w} \left( 1 + M - M N \left( \frac{\delta \nu}{\delta x} - \frac{v}{R^{2} \frac{R^{2}}{\delta}} \right) \right) \left( \frac{\delta \nu}{\delta r} - \frac{v}{R^{2} \frac{R^{2}}{\delta}} \right) - \Theta
\]  

(31)

On employing the transformation defined in Eq. (25) into Eqs. (30) and (31), the expression for flow rate and wall shear stress can be rewritten as:

\[
Q = \chi \frac{R}{R^{2}} \int \nu(x, t) dx
\]  

(32)

\[
\tau_{w} = \tau_{w} \left( 1 + M - M N \left( \frac{\delta \nu}{\delta x} - \frac{v}{R^{2} \frac{R^{2}}{\delta}} \right) \right) \left( \frac{\delta \nu}{\delta r} - \frac{v}{R^{2} \frac{R^{2}}{\delta}} \right)
\]  

(33)

3. RESULTS AND DISCUSSION

To understand the flow behavior in a curved channel in the presence of nanoparticles, the momentum, energy, and concentration equations are solved numerically by using finite difference scheme, also called forward in time and central in space (FTCS). The impact of important flow parameters on blood flow through a curved artery with two diseased regions, one contracted and another dilated, is discussed by graphically displaying the various rheological properties such as velocity profiles, flow rate, and wall shear stress. The heat and mass transfer are also investigated during the blood flow in the existence of nanoparticles that play a significant role in blood flows through arteries and have gained popularity due to significance in hematological treatment. For the conciseness of the study, some of the variables are kept constant while studying the impact of other variables. The constant values assumed for the variables are \( M = 0.2, N = 0.1, \kappa = 3, B = 1.4, c = 1, \text{Re} = 0.5, \text{Gr}_{r} = 2, \text{Gr}_{c} = 2, \Theta = 2, \text{Pr} = 3, \text{Sc} = 1, \text{N}_{t} = 1, \text{N}_{b} = 1, \Psi = 0, \delta^{*} = 0.1, \sigma_{1} = 0.2, \text{and} \sigma_{2} = 1.5.

The throats of stenosis and aneurysm are defined at \( x = 0.7 \), and \( x = 2 \), respectively.

Figures 2 (a-g) depict the axial velocity profiles in a curved artery at the throat of stenosis against the radial distance for different values of \( \Psi, M, \kappa, \text{Gr}_{r}, \text{Gr}_{c}, \Theta, N_{t}, \text{and} N_{b} \), respectively. These figures demonstrate that at the peak value of stenosis, the velocity is maximum near the centerline, whereas it reduces to zero near the walls of the channel, which confirms the no slip condition. Figure 2a shows the velocity profiles for three distinct types of tapered arteries, converging \( \Psi < 0 \), normal \( \Psi = 0 \), or diverging \( \Psi > 0 \). It can be clearly noted that the velocity of flow is maximum in a diverging artery and minimum when it is converging. Figure 2b shows that increasing the non-Newtonian parameter reduces the velocity of the blood flow, which validates the results of the studies\(^{26,27}\). It is detected from Fig. 2c that the velocity shows different behavior below and above the centerline for increasing values of the radius of the channel. It decreases at the bottom of the channel and increases at the centerline and upper part of the artery. Figures 2 (d,e) are designed to demonstrate the influence of thermal and nanoparticle Grashof numbers, referring to the ratio of thermal buoyancy force to the viscous force and nanoparticle buoyancy force to the viscous force, respectively. The velocity of the blood flow increases as \( \text{Gr}_{r} \) increases and decreases as \( \text{Gr}_{c} \) increases, which means increase in thermal buoyancy force increases the blood flow, whereas increase in nanoparticle buoyancy force reduces it. Figures 2 (f,g) demonstrate the impact of thermophoresis and Brownian motion parameters on axial velocity. The magnitude of the blood flow rises on rising the thermophoresis parameter and decreases by enhancing the Brownian motion parameter. This behavior refers to the phenomenon of reduction in concentration of nanoparticles as the size of nanoparticles increases on increasing \( N_{b} \), dominating the heat transfer conduction, which leads to the deceleration of the axial velocity.

Figures 3 (a-d) illustrate the impact of \( \Psi, \Theta, \kappa, \text{and} N_{t} \), on dimensionless temperature profiles at the throat of stenosis in a curved artery. Figure 3a depicts the temperature profiles for three distinct types of tapered arteries. In a diverging artery, the magnitude of the blood flow is maximum which results in higher temperature as compare to the non-tapered or converging arteries. The velocity of the blood flow increases as \( \text{Gr}_{r} \) increases and decreases as \( \text{Gr}_{c} \) increases, which means increase in thermal buoyancy force increases the blood flow, whereas increase in nanoparticle buoyancy force reduces it. Figures 2 (d,e) demonstrate the impact of thermophoresis and Brownian motion parameters on axial velocity. The magnitude of the blood flow rises on rising the thermophoresis parameter and decreases by enhancing the Brownian motion parameter. This behavior refers to the phenomenon of reduction in concentration of nanoparticles as the size of nanoparticles increases on increasing \( N_{b} \), dominating the heat transfer conduction, which leads to the deceleration of the axial velocity.

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increases, the temperature rises above the centerline and reduces in the lower part of the artery. Figure 3d shows the impact of thermophoretic parameter on blood temperature, which significantly reduces the blood temperature as the value of $N_t$ rises.

Figures 4 (a-e) are designed to display the variation in concentration of the nanoparticles in blood flow at the throat of stenosis for several values of $\Psi$, $\Theta$, $\kappa$, $N_t$, and $N_b$. The behavior of concentration profiles is quite similar to the temperature profiles; the concentration of nanoparticles is maximum at the centerline and minimum at the walls of the artery. Figure 4a displays the concentration profiles for three distinct types of tapered arteries. In a diverging artery, the concentration of nanoparticles is maximum which is the result of higher temperature, whereas the concentration in non-tapered or converging arteries is less due to low temperature of the blood. Figure 4b is shows the variation in concentration for change in heat absorption/generation parameter. It

![Graphs showing the variation in fluid velocity and concentration profiles](image)

Fig. 2 Variation of fluid velocity in a curved artery at the throat of stenosis for (a) converging $\Psi < 0$, normal $\Psi = 0$, or diverging $\Psi > 0$ artery, (b) Eyring-Powell (non-Newtonian) parameter, (c) nanoparticle Grashof number, (d) thermal Grashof number, (e) Radius of the channel, (f) thermophoretic parameter, and (g) Brownian motion parameter.
Fig. 3 Variation of temperature in a curved artery at the throat of stenosis for (a) converging $\Psi < 0$, normal $\Psi = 0$, or diverging $\Psi > 0$ artery, (b) heat absorption/generation parameter, (c) Radius of the channel, and (d) thermophoretic parameter.

Fig. 4 Variation of concentration in a curved artery at the throat of stenosis for (a) converging $\Psi < 0$, normal $\Psi = 0$, or diverging $\Psi > 0$ artery, (b) heat absorption/generation parameter, (c) Radius of the channel, (d) thermophoretic parameter, and (e) Brownian motion parameter.
is evident that larger the value of $\Theta$, higher the temperature and the concentration. From Fig. 4c the behavior of concentration profile is observed to be changing at the centerline for different values of the radius of the channel. Like the temperature profiles, as the radius increases, the concentration rises above the centerline and reduces in the lower part of the artery. Figure 4 (d,e) display the impact of thermophoretic and Brownian motion parameters on nanoparticle concentration. The thermophoretic parameter significantly enhances the concentration, whereas increase in the value of $N_b$ reduces the blood concentration due to increase in the size of nanoparticles.

The behavior of dimensionless flow rate in a curved artery with stenosis and aneurysm is depicted in Figs. 5 (a-d). It can be clearly noted that the flow rate shows oscillatory behavior and its magnitude is less during stenosis as compare to the aneurysm. For the non-Newtonian parameter $M$ and radius of the channel $\kappa$, the flow rate is displayed along the axial distance due to stenosis and aneurysm, and for depth of the diseased segments against time at the throats of the diseased segments.

Behavior of flow rate is observed at the throats of stenosis and aneurysm over different intervals of time as their depth increases.

Figures 6 (a,b) are designed to exhibit the impact of Eyring-Powell parameter on shear stress at the upper wall of the channel. The wall shear stress is observed to be of higher for stenosis as compare to the aneurysm. Along the axial distance, the shear stress increases as the non-Newtonian parameter increases, allowing less blood to pass as depicted in Fig. 6a. However, along the varying time, the shear stress appears to be changing behavior over different intervals of time.

Figures 7 (a,b) exhibits the shear stress at the upper wall of the channel for Eyring-Powell parameter against the depth of the diseased segments. The shear stress increases as the fluid changes from Newtonian to non-Newtonian. Figure 7a illustrates that the shear stress increases as the depth of stenosis increases. On the other hand, the shear stress reduces at the throat of aneurysm as its height increases as depicted in Fig. 7b.

To get the more insight of the flow behavior, the geometric patterns, Figs. 8-10, are designed to depict the blood flow patterns at diseased segments. Figures 8a-d show the influence of flow behavior on flow profiles in a segment with stenosis, Figs. 8 (a,b), and aneurysm, Figs. 8c, d. As the non-Newtonian parameter increases from $M = 0.0$ to 0.4, the flow reduces at both the segments and a bolus of fluid appears in post-stenotic and dilated regions; the circulating region reduces as the fluid becomes non-Newtonian. Figures 9 (a-d)
Fig. 6  Dimensionless shear stress at the upper wall of the channel (a) for Eyring-Powell (non-Newtonian) parameter against axial distance due to stenosis and aneurysm and (b) for Eyring-Powell (non-Newtonian parameter against time at the throats of the diseased segments.

Fig. 7  Dimensionless shear stress at the upper wall of the channel for Eyring-Powell (non-Newtonian) parameter against the depth of the diseased segment (a) at the throat of stenosis and (b) at the throat of aneurysm.

Fig. 8  Flow patterns at the diseased segments of the artery for Eyring-Powell (non-Newtonian) parameter, respectively for $M = 0.0$ and $0.4$, in a region with (a) and (b) stenosis, and (c) and (d) aneurysm.
Fig. 9 Flow patterns at the diseased segments of the artery for different values of curvature of the channel, respectively for $\kappa = 3$ and 5, in a region with (a) and (b) stenosis, and (c) and (d) aneurysm.

Fig. 10 Flow patterns at the diseased segments of the artery for different depths of the diseased segments in a region with (a) and (b) stenosis for $\delta_1^* = 0.1$ and 0.2, and (c) and (d) aneurysm for $\delta_2^* = -0.1$ and $-0.2$, respectively.
demonstrate that the shape of the channel changes from curved to straight as radius of the channel increases allowing more fluid to pass. Figures 10 (a-d) indicate that as the depth of stenotic region increases, the flow reduces and the circulations around the fluid bolus increase in post-stenotic region and the segment with aneurysm.

4. CONCLUSION

In this study, the flow of blood in a curved channel with two diseased sections, aneurysm and stenosis has been discussed through a mathematical model. The Eyring-Powell fluid model has been used to depict the non-Newtonian behavior of blood flow. The phenomena of heat transfer and nanoparticle concentration have also been investigated due to their significance in blood rheology. The main findings of the study are:

- The non-Newtonian parameter reduces the velocity and the rate of blood flow.
- The blood flow rises with increase in thermal Grashof number, while the nanoparticle Grashof number reduces it.
- The thermophoretic parameter reduces the temperature and increases the concentration of the blood. However, the concentration reduces as the Brownian motion parameter increases the size of the nanoparticles.
- The critical depth parameter of the diseased segments increases the flow rate in a region with aneurysm and reduces it in stenotic region.
- The blood attains the higher velocity in the post-stenotic region in comparison to the stenotic region.

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