THE ONE, THE MANY, AND THE QUANTUM

Ulrich Mohrhoff
Sri Aurobindo Ashram, Pondicherry 605002, India
ujm@auroville.org.in

Abstract

The problem of understanding quantum mechanics is in large measure the problem of finding appropriate ways of thinking about the spatial and temporal aspects of the physical world. The standard, substantival, set-theoretic conception of space is inconsistent with quantum mechanics, and so is the doctrine of local realism, the principle of local causality, and the mathematical physicist's golden calf, determinism. The said problem is made intractable by our obtruding onto the physical world a theoretical framework that is more detailed than the physical world. This framework portrays space and time as infinitely and intrinsically differentiated, whereas the physical world is only finitely differentiated spacewise and timewise, namely to the extent that spatiotemporal relations and distinctions are warranted by facts. This has the following consequences: (i) The contingent properties of the physical world, including the times at which they are possessed, are indefinite and extrinsic. (ii) We cannot think of reality as being built “from the bottom up”, out of locally instantiated physical properties. Instead we must conceive of the physical world as being built “from the top down”: By entering into a multitude of spatial relations with itself, “existence itself” takes on both the aspect of a spatially differentiated world and the aspect of a multiplicity of formless relata, the fundamental particles. At the root of our interpretational difficulties is the “cookie cutter paradigm”, according to which the world’s synchronic multiplicity is founded on the introduction of surfaces that carve up space in the manner of three-dimensional cookie cutters. The neurophysiological underpinnings of this insidious notion are discussed.
“I feel that the real joke that the eternal inventor of enigmas has presented us with has absolutely not been understood as yet.”

Albert Einstein [1, p. 411]

1 Introduction

Quantum mechanics is an incredibly successful theory. No experiment, no observation, has ever given the lie to it. But if it is not lying, what does it tell us about the world? A fairly typical answer is: Nothing; quantum mechanics concerns statistical regularities in the behavior of measuring instruments; any attempt to go beyond the “brute facts”, to give a realistic or epistemological account of how it is that the statistical regularities predicted by quantum mechanics come out the way they do, is idle metaphysics. Considering the widely divergent and more or less bizarre accounts that abound, this resigned attitude appears justified. But is it?

In this essay I examine the metaphysical presuppositions that stand in the way of making sense of quantum mechanics and trace them to their neurophysiological underpinnings. Because these underpinnings to a considerable extent determine the nature of the phenomenal world and consequently prejudice our thinking about the physical world, rejecting those presuppositions is no easy task. But it is worth the effort, for at the end we shall find that the mathematical elegance and simplicity of quantum mechanics is matched by the depth and transparency of its ontological message.

The problem of understanding quantum mechanics is in large measure the problem of finding appropriate ways of thinking about the spatial and temporal aspects of the physical world. Section 2 contrasts the salient features of phenomenal space with the standard mathematical description of physical space. It is argued that the standard description has empirically unwarranted features that stand in the way of finding the right interpretation. An alternative, essentially relationalist way of thinking is proposed.

In the sections that follow the relevant empirical findings are progressively taken into account. Section 3 takes account of the existence of noncomposite entities, a.k.a. the fundamental particles of matter. Its chief conclusion is that these entities must be thought of as formless. The still prevalent notion that a fundamental particle has a (pointlike) form is an unwarranted (in fact, illegitimate) importation from the phenomenal world into the world of physics. The form of a material object is the set of its internal spatial relations. An object that lacks internal spatial relations also lacks a form.

Section 4 takes account of the behavior of indistinguishable particles. It is argued that a fundamental particle is not something to which existence is contingently attributable. The basic reality of matter is a single entity, and this entity cannot be qualified as anything more particular or determinate than “existence itself”. Considered in relation to each other, fundamental particles are instances of “existence itself”. Considered out of relation to other particles, each fundamental particle is “existence itself”. The individuation or multiple instantiation of this entity is discussed.
Section 5 takes account of the behavior of electrons in two-slit interference experiments. This behavior is Nature’s corroboration of the relationalist conception of physical space developed in Secs. 2 and 3. It is argued that the standard, substantival, set-theoretic conception of space is as inconsistent with quantum mechanics as absolute simultaneity is with special relativity. So, therefore, is the doctrine of local realism, the principle of local causality, and the mathematical physicist’s golden calf, determinism.

Section 6 deals with two frequent mistakes, the error of the instrumentalist, who rejects all efforts to make ontological sense of quantum mechanics, and the error of the quantum realist, who considers probability one sufficient for the existence of an element of reality. While the instrumentalist errs by ignoring the possibility of a quantum world with extrinsic properties, the quantum realist errs by ignoring that the properties of the quantum world, including the times at which they are possessed, are extrinsic: They “dangle” from what happens or is the case in the rest of the world. The contingent properties of the quantum world exist precisely to the extent that their values are indicated.

Section 7 pinpoints the sense in which the relative positions of the world’s material constituents are indefinite. This involves reference to objective probabilities, which can be assigned only to counterfactuals. The very possibility of assigning objective probabilities to the possible results of an unperformed measurement entails that the possessed values of quantum-mechanical observables are extrinsic. The extrinsic nature of the world’s contingent properties thus follows directly from their indefiniteness. Hence unless we are willing to take seriously the extrinsic nature of quantum-mechanical properties, we shall not be in a position to make proper sense of the indefiniteness that is the hallmark of quantum mechanics, and hence of quantum mechanics itself.

While even a macroscopic object (defined in Sec. 7) has a position only because of the facts from which this can be inferred, the dependence of its position on position-indicating facts is a qualitative (ontological or existential) dependence, not a quantitative one. For the quantitative purposes of physics, it is legitimate to ignore this dependence, to consider macroscopic positions as intrinsic, and to apply to them classical causal concepts. This is fortunate, for otherwise quantum mechanics would be inconsistent, inasmuch as its very formulation presupposes a classical domain of positions that can be thought of as being factual per se. Causality nevertheless is emergent; it is not part of the ontological foundation. All the determining that goes on in the physical world is the determining of probabilities associated with possibilities. There aren’t any causally determined facts. The quantum formalism concerns probabilistic correlations among facts – diachronic correlations between facts indicating properties of the same system at different times and synchronic correlations between facts indicating properties of different systems in spacelike separation, – and it is these correlations that explain why causal explanations work to the extent they do. Trying to causally explain the correlations would be putting the cart in front of the horse. All of this is explained in Sec. 8.

The seemingly intractable problem of understanding quantum mechanics is largely due to our obtruding onto the physical world a theoretical framework that is more detailed than the physical world, in that it portraits space and time as infinitely and
intrinsically differentiated. Section 9 shows that the physical world is only finitely differentiated spacewise and timewise, namely to the extent that spatiotemporal relations and distinctions are warranted by facts. As a consequence, we cannot think of reality as being built “from the bottom up”, on an infinitely and intrinsically differentiated space, out of locally instantiated physical properties. Instead we must conceive of the physical world as being built “from the top down”: By entering into a multitude of spatial relations with itself, “existence itself” takes on both the aspect of a spatially differentiated world and the aspect of a multiplicity of relata – the fundamental particles. By allowing the spatial relations to change, it takes on the further aspect of a temporally differentiated world.

The final section homes in on the idea that is at the root of our interpretational difficulties, traces its philosophical fallout, and examines its neurophysiological underpinnings. This is the “cookie cutter paradigm”, according to which the world’s synchronic multiplicity is founded on the introduction of surfaces that carve up space in the manner of three-dimensional cookie cutters.

2 The Reality of Spatial Continuity

The problem of understanding quantum mechanics is in large measure the problem of finding appropriate ways of thinking about the spatial and temporal aspects of the physical world. The way we think about space is obviously indebted to our awareness of phenomenal space, the three-dimensional expanse which contains both sensory perceptions (a.k.a. qualia or introspectible properties) and visual images. One of the things that quantum mechanics is trying to tell us is that the way we are accustomed to think about space is more appropriate for dealing with the spatial aspect of the phenomenal world than it is for dealing with the spatial aspect of the physical world. In the following sections I will successively introduce the relevant empirical data and examine their implications. In the present section I focus on what we can learn by paying attention to our direct awareness of phenomenal space and by contrasting it with the standard mathematical representation of physical space.

Physicists routinely represent space as a transfinite set of point individuals in one-to-one correspondence with triplets of real numbers. This mathematical representation lacks certain features that are vital for interpreting quantum mechanics correctly, and it possesses certain empirically unwarranted features that stand in the way of finding the correct interpretation. The features that are lacking are the quality of continuous spatial extension and a unity that goes beyond the unity of a set, defined by Cantor as “a Many that allows itself to be thought of as a One”. The empirically unwarranted features are the multiplicity that is inherent in the set $\mathcal{R}^3$ of all triplets of real numbers and the intrinsic distinctness of the members of $\mathcal{R}^3$.

1Neuroscience is increasingly backing the notion that experience and imagination share the same space. There is mounting evidence that visual perception and visual imagery compete for the same processing mechanisms, and that the neural processes which produce visual percepts and those which produce visual images are to some extent the same.
Consider the visual image of a line. This is in an obvious sense continuously extended in one spatial dimension. If the line is unbounded, we can set up a one-to-one correspondence between the set $\mathbb{R}$ of real numbers and points on the line; if it is bounded, we can set up a one-to-one correspondence between any finite interval of $\mathbb{R}$ and points on the line. What is important is that these points do not exist in advance of any one-to-one correspondence that we may set up. They are introduced into the line by our setting up such a correspondence. The line is in an obvious sense divisible into segments, but nothing in its image warrants the notion that it is intrinsically multiple, let alone that it is a concatenation of point individuals with the cardinality of the real numbers. While between two distinct real numbers $a$, $b$ there exists a nondenumerable set $S$ of real numbers, between the points that correspond to $a$ and $b$ there exists a perfectly continuous and intrinsically undifferentiated line segment $L$. $S$ and $L$ are entirely different things. $S$ possesses something that $L$ lacks, namely the multiplicity of a nondenumerable set and the distinctness of its members. $L$ possesses something that $S$ lacks, namely the quality of continuous spatial extension and the unity of an intrinsically undivided whole.

Unfortunately, we are so accustomed to conflating the continuity of phenomenal space with the discreteness of $\mathbb{R}^3$ that it has become almost impossible for us to tease them apart. We tend to visualize the reals as a continuous line without realizing that the very act of visualization introduces a qualitative element that is not warranted by the mathematical construction of the reals. Conversely, in our attempt to get a conceptual grip on physical space we seize on the reals as a set that appears to contain sufficiently many elements to “fill” a continuous line and thus to possess its continuity. We even deprive ourselves of the words that are needed to distinguish between $S$ and $L$, as when we use the adjectives “continuous” and “discrete” to qualify sets. In what follows, all words starting with the letters “c-o-n-t-i-n-u” will signify (possession of) that continuity which is an obvious pre-theoretical feature of our visual percepts and images, and “discrete” will refer to that discreteness which is an obvious property of every “Many that allows itself to be thought of as a One”.

Continuity, so defined, is a feature of objects in phenomenal space (that is, of visual percepts and visual images), and it is not a feature of any mathematical set. Is continuity a feature of objects in physical space? Does it exist in the physical world? These questions are quite analogous to the following: Does the turquoise of a tropical lagoon exist in physical space (that is, is it an objective property of the lagoon)? Few would venture an affirmative answer. Qualia belong to phenomenal space; they are correlated with certain physical quantities but are themselves not physical. But the continuity of phenomenal space is as much a qualitative feature of our visual percepts as is the sensation of turquoise. It eludes mathematical description just as definitely as does the

---

2 The phraseology which depicts space as some kind of container in which objects are situated is common enough, but it must not be taken literally. Phenomenal space contains neither in the set-theoretic sense of “containment” nor in the sense in which a closed surface $b$ encompasses its interior. Neither of these senses is applicable to an unbounded continuous space. Hence if we say of something that it exists or is situated “in” space, what we mean is that it is spatially extended, or that it is spatially related to something else, or that it stands out from a spatial expanse like the visual image of a point.

3 An accurate physical correlate of color sensations is given by the triplets of measurable quantities known as “integrated reflectances”.

---
color of the lagoon. One theorist who was acutely aware of this was Hermann Weyl [7, p. 23]:

\[ \ldots \] it ought to be emphasized just how little mathematics can claim to capture the phenomenal \textit{anschauliche} nature of space: nothing in geometry concerns what makes phenomenal space what it distinctively \textit{is}.\ldots Our conceptual theories are capable of penetrating only one aspect of space, and only the most superficial and formal at that.

A triplet of real numbers is not the same as a point. The difference between two real numbers is not the same as the distance between two points. Distances possess, in addition to their values, a purely qualitative aspect, and nobody lacking our pre-mathematical grasp of phenomenal space is in a position to know it, anymore than Mary, confined from birth to a black-and-white room, was in a position to know color [8].

It might be concluded that we should think of continuity in the same way as we think of color sensations – as an appearance or a secondary quality, rather than as an actual feature of the physical world: Continuity is a property of percepts and images in phenomenal space; the real thing, physical space, is discrete and cardinally equal to \( R^3 \). However, in view of the interpretational difficulties associated with quantum mechanics, it may be worth a try to consider the continuity we find in phenomenal space, and the undifferentiated unity that goes with it, as objective features of the physical world, and to determine how, this being so, the multiplicity of \( R^3 \) and the distinctness of its members relate to the physical world.

It is worth noting, to begin with, that if we conceive of physical space as a set of points corresponding one-to-one to the members of \( R^3 \), the individual points of space are not visualizable. If we imagine a point, we imagine something that stands out from an otherwise undifferentiated spatial expanse. If we think of this point as one of the points of space, then this particular point of space possesses a quality that is not present at those points of space that make up the surrounding expanse. Hence what we imagine is not a point of space but this quality situated or instantiated at a particular point of space. In other words, what we imagine is a pointlike \textit{object}. We can visualize pointlike objects, but we cannot visualize an individual point of space. The points of space are not objects but nonvisualizable \textit{positions} at which visualizable objects may be situated or visualizable qualities may be instantiated.

Positions in phenomenal space can only be defined relatively, as spatial relations that hold among visual percepts or images. We have neither empirical nor theoretical reasons to doubt that the same is true of physical space: Positions are relatively defined, as spatial relations that hold either among material objects or among the points of space. The latter alternative, however, introduces into the physical world a spatial multiplicity and a degree of spatial differentiation that not only are empirically unwarranted but also violate the Identity of Indiscernibles, a principle of analytic ontology which says that two things cannot have exactly the same properties. Since no objective differences correspond to the differences between the triplets of real numbers by which we label the points of space, these points have identical properties and therefore cannot be thought of as distinct individuals.
The points of space being positions at which material objects may be situated or physical qualities may be instantiated, and positions being relatively defined, the points of space are relatively defined. We do not have, on the one hand, a set of preexistent positions (cardinally equal to the reals) and, on the other hand, a set of spatial relations that hold among these positions. The positions are the spatial relations, or else they are defined by them: The exist, as distinct locations, by virtue of the distinguishing relations. The relata owe their existence to the relations, and the relations owe their existence to the material objects or instantiated physical properties to which they are attributable.

It follows that the synchronic multiplicity of the world does not go beyond the multiplicity of material objects (or locally instantiated physical properties) existing at any one time, and the multiplicity of their relations. It does not include the nondenumerable multiplicity of the “points of space”. The function of this multiplicity – the multiplicity of coordinates – is not to label “vacant” positions but to quantify the undifferentiated spatial relations that exist between material objects. (The actually existing spatial relations, which spatially differentiate the world, are not themselves spatially differentiated.)

This is relationism, the doctrine that space is a family of spatial relations holding among the world’s material constituents, rather than an additional constituent of the world. The alternative to relationism is substantivalism. According to the latter, there is such a thing as “space itself”, and this has, by itself, a definite number of dimensions and a metric. The dimensionality of physical space, however, is fully determined by the system of spatial relations that hold among material objects; it is the number of coordinates needed to specify those relations. There is no need to attribute it to a separate constituent of the world.

What about the metric? This too need not be attributed to a separate constituent of the world. As was pointed out by Riemann [7, p. 101] [9, p. 752], unless physical space is inherently discrete (and therefore a separate constituent of the world with inherent properties), metric relations are extrinsic to it. On the relationist view of space, metric relations exist only between material objects, and the assignment of values to such relations is based on the behavior of material objects, rather than on properties intrinsic to space. Since the behavior of material objects finds formal expression in laws that make reference to distances, it is possible that the metric properties of the world and the laws of physics are individually underdetermined. What is observable, or factual, is the behavior of material objects. To describe it, we use both a metric and a set of dynamical laws, neither of which is separately observable. Hence, in principle, the same behavior is describable in relation to different metrization schemes, by alternative sets of physical laws. This means that the metric properties of the world are partly based on a conventional choice between alternative sets of laws, as was stressed by Poincaré [10].

If we embrace relationism, we cannot attribute continuity to physical space as if this were a separate constituent of the world – a three-dimensional expanse existing independently of its material “content”. If there is anything in the physical world to

---

4The qualifier “locally instantiated” does not imply that locations exist in advance of instantiation.

5Arguably, the specific lawful behavior responsible for the metric properties of the world consists in the invariant periodicities that are associated with the inertial masses of particles [11, 12].
which we can attribute the continuity that we find in phenomenal space, it is the spatial
relations that exist between material objects. (As the following section will show, it is not
necessary to attribute continuity separately to the forms of spatially extended objects,
for such forms are simply sets of spatial relations.) Moreover, since the synchronic
multiplicity of the world is limited to the multiplicity of material objects existing at any
one time and the multiplicity of their relations, each relation possesses not only this
continuity but also the unity of an undifferentiated line segment.

Let me amplify. Given Cartesian coordinates, the spatial relation between two ma-
terial objects \( A \) and \( B \) is essentially the distance \( D(AB) \) between \( A \) and \( B \). \( D(AB) \) is
quantitatively determined by the algebraic differences between the coordinates of \( A \) and
\( B \). But the distances between material objects also have a qualitative character. They
are not just numbers; they are spatial. We tend to think of \( D(AB) \) as a quantity, and
we tend to attribute to this “quantity” the multiplicity of some interval of the so-called
“real line”. That is, we tend to take it for granted that there are as many places between
the two objects as there are real numbers between 0 and \( d(AB) \), the value of \( D(AB) \). In
reality there are as many places in the material world as there are material objects. The
places at which objects may be located do not actually exist unless objects are actually
located there. Thus if we consider the distance \( D(AB) \), we are looking at something
that possesses the quality of spatial extension as well as a value \( d(AB) \), but that is
destitute of multiplicity. Unless there are other, appropriately situated objects, there
are no places between \( A \) and \( B \).

It thus is a mistake to think of \( D(AB) \) as intrinsically multiple. There is no multi-
plicity that would qualify as inherent in a single spatial relation. There are no points or
places between \( A \) and \( B \) unless other material objects are situated between \( A \) and \( B \). It
takes a third object \( C \) to introduce another two distances \( D(AC) \) and \( D(CB) \) such that
\( d(AB) = d(AC) + d(CB) \). The same equation does not hold among the three distances
\( D(AB) \), \( D(AC) \) and \( D(CB) \). None of these distances is the sum of anything. \( D(AB) \)
is not a quantity that is “made up” of quantities; it is a relation that possesses (i) the
qualitative property of undifferentiated continuous extension and (ii) the value \( d(AB) \).
It interposes no places between \( A \) and \( B \). If anything interposes a location between
the location of \( A \) and the location of \( B \), it is another material object \( C \).

To thinkers from Aristotle to Kant and Gauss it appeared self-evident that points
on a line are extrinsic to the line, in the sense that they are additional features not
contained in its image. They considered the line itself and, by implication, space itself
is an inherently undivided “whole” existing in an anterior relationship to limits and
divisions. “Space is essentially one”, Kant wrote \[3, p. 25\], “the manifold in it…
arises entirely from the introduction of limits.” Kant was right in saying that what is
continuous is intrinsically one. But he was wrong in attributing the world’s synchronic
multiplicity to the introduction of limits. Synchronous multiplicity owes its reality to
the spatial relations that exist between objects. Therefore there exists no such tension
of contrast as that between the unity of a three-dimensional continuous expanse and
a multiplicity of divisions that somehow appear in it. Continuity and unity are not
properties of “space itself” but pertain to (are instantiated with) each spatial relation
between a pair of material objects. The continuous unity of \( D(AB) \) thus in no way
conflicts with the spatial multiplicity of the world, for the former belongs to each spatial relation while the latter belongs to the totality of spatial relations that exist in the world.

We have assumed the existence of Cartesian coordinates. Since the central purpose of this article is to find a way of thinking about the world’s actual multiplicity that is consistent with quantum mechanics (or a way of thinking about quantum mechanics that is consistent with the world’s actual multiplicity), this assumption is justified by the fact that quantum mechanics presupposes the use of Cartesian coordinates. Specifically, every quantization scheme that leads to Hilbert space vectors and Weyl operators presupposes the “flat” metric that goes with Cartesian coordinates \[14, 15, 16\]. To see why this should be so, we must bear in mind that in quantum mechanics coordinates do not represent positions but values that are available for attribution to the positions of material objects. Since these positions are relative positions, a useful coordinate system has to be riveted to some material object (e.g., the nucleus of an atom, the center of mass of a composite object, a macroscopic part of a macroscopic apparatus). A coordinate system \(C_A\) riveted to a given object \(A\) represents the “space” of values that are potentially attributable to the positions of objects \(B_i\) relative to \(A\), but it does not represent the “space” of values that are potentially attributable to the positions of the objects \(B_i\) relative to each other. The appropriate value “space” for the positions of the objects \(B_i, i \geq 0\), relative to \(B = B_0\) is not \(C_A\) but \(C_B\).

This would be of no consequence if the relative positions of material objects had definite values, for then \(C_B\) could be obtained by simply translating (and maybe rotating) \(C_A\). But since the relative position of each pair of material objects is to some extent “fuzzy” – a notion that will be substantiated and made precise in what follows, – there isn’t any point transformation that takes us from \(C_A\) to \(C_B\). It follows that the coordinate “space” presupposed by quantum mechanics can have nothing to do with curvature. This “space” of attributable values is always riveted to a single material object \(A\), whereas a test for curvature involves at least two different, not determinately related, value “spaces”. (To detect curvature near \(A\), one needs to know not only the positions of nearby objects \(B_i\) relative to \(A\) but also the positions of the objects \(B_i\) relative to each other.) Each individual value “space” is therefore necessarily and trivially flat.

In this section arguments supporting the following notions have been presented:

- Physical space is not a set of point individuals but a system of spatial relations that hold among material objects.

- Like the sensation of turquoise, continuous extension is a qualitative feature of our visual percepts. Unlike that sensation, it is an objective feature of the physical world (that is, it is an objective feature of every spatial relation).

---

\[6\] This assumption [of replacing classical canonical coordinates by corresponding operators] is found in practice to be successful only when applied to the dynamical coordinates and momenta referring to a Cartesian system of axes and not to more general curvilinear coordinates” – Dirac \[17, p. 114\].
• The relative position of any pair of material objects (or the distance between the two objects) lacks multiplicity; it possesses the quality of continuous spatial extension (in one dimension) and the undifferentiated unity of an unsegmented line in phenomenal space.

• The spatial multiplicity of the world is the multiplicity of spatial relations that hold among material objects. It is not due to the introduction of boundaries.

• A clear distinction has to be made between physical space and the set $\mathcal{R}^3$ of values that are potentially attributable to the relative positions of material objects. The quality of continuous spatial extension is not attributable to this set, nor is the concept of curvature applicable to it.

As will become clear in what follows, the conceptual framework staked out so far is eminently suited to the task at hand – making sense of quantum mechanics.

3 Are Fundamental Particles Pointlike?
The Ontology of Synchronic Multiplicity

Let us now take account of the relevant empirical findings. We begin with the existence of simple or noncomposite entities, which we will refer to as “fundamental particles”. According to the current standard model of elementary particle physics, the particles that are fundamental are the quarks and the leptons, but all we need to know or assume at this point is that noncomposite objects exist. Combined with the conclusions reached in the previous section, this warrants the following claims:

• Any composite material object (with the possible exception of the universe as a whole) is made up of a finite number of fundamental particles and the spatial relations that hold among them. To the particles it owes its finite multiplicity; to the spatial relations it owes its spatial extension.

• There are not two kinds of spatially extended things, such as regions of space and a material stuff that occupies or fills them. The only kind of spatial extension in existence is the spatial extension that is attributable to possessed spatial relations. There is no material stuff that occupies or fills space. What is spatially extended either is a spatial relation or has spatial relations among its constituents. A composite object “occupies” space only in the sense that it is partly constituted by the spatial relations between its material constituents.

• There are not two kinds of synchronic multiplicity, such as a multiplicity of material things and a multiplicity of self-existing locations. There is only the multiplicity of fundamental particles and the multiplicity of their spatial relations. The actually existing locations are the (relatively defined) positions that are attributable to fundamental particles and composites thereof.
In respect of any given object \( O \), one may distinguish two kinds of spatial relations: those \textit{internal} to \( O \), which hold between its material constituents, and those \textit{external} to \( O \), which hold between \( O \) or its material constituents and objects having no constituents in common with \( O \). By definition, a fundamental particle lacks internal spatial relations. In the minds of most physicists an object lacking internal spatial relations is pointlike, but such an object may just as well be formless. It must be emphasized that there can be no direct evidence of either the formlessness or the pointlike form of a fundamental particle. What can be experimentally ascertained about a specific type of particle is the absence of evidence of internal structure. If absence of evidence is interpreted as evidence of absence – the standard model does this in respect of the quarks and the leptons, – this may be construed either as the possession of a pointlike form or as the nonexistence of any form. On the latter view, the forms of all the beasts and baubles in this world resolve themselves into the spatial relations that obtain among their constituent parts. Ultimate parts have no parts, so they also have no form. On the former view there exists, in addition to the forms that resolve themselves into spatial relations, another type of spatial form, namely the pointlike form of a fundamental particle. It is obvious which is the more parsimonious view.

There are further reasons for rejecting the notion that a fundamental particle has a pointlike form, besides the empirical inaccessibility of such a form and Occam’s principle of theoretical parsimony. Since space is not a storehouse of preexistent positions, positions need to be realized or brought into being. They may be realized individually – by the presence of a material object with (ideally) the form of a point – or they may be realized in pairs, as relative positions. But if they are realized individually, we still need to specify quantitatively how they are related to each other. A single object or locally instantiated property can define “here” qualitatively, but it is not enough to quantify (attribute a numerical value to) “here”. For this purpose we need to know how “here” is quantitatively related to “elsewhere”. And this is all we need to know. For the quantitative purposes of physics, the existence of position-marking forms is irrelevant. Why then should Nature go to the trouble of investing the ultimate spatial relata with (pointlike) forms, when it is enough to realize positions \textit{in pairs}, as spatial relations? The fact that in the phenomenal world we do not and cannot encounter formless entities is no argument. As we shall see in Sec. 10, this is a consequence of the neurophysiological basis of the phenomenal world, and thus has no bearing on the nature of the physical world.

Attributing to a fundamental particle the form of a point is not only gratuitous but also inconsistent with the relationist conception of space. To see this, consider a world that contains a single fundamental particle. If this had a pointlike form, then there would exist, in addition to the particle, a surrounding spatial expanse. But if the spatial aspect of the world is a family of spatial relations between material objects, all that is spatially extended is the spatial relations between material objects. A world that contains a single fundamental particle contains no spatial relations, and therefore

\footnote{There is indirect evidence: As is shown in this section, the formlessness is entailed by the relationist conception of space, which in turn is entailed by the behavior of electrons in two-slit experiments, as I shall argue in Sec. 5.}
it contains nothing that is spatially extended. There is no surrounding spatial expanse. And so there is no pointlike form either, as this implies the existence of such an expanse.

I think it is high time that we recognize the notion that a fundamental particle has a (pointlike) form for what it is – an unwarranted importation from the phenomenal world into the world of physics. In the phenomenal world, the existence of spatial relations presupposes the existence of spatially related forms (visual percepts or images) to which the spatial relations are attributable. Forms are phenomenologically prior to spatial relations. In the physical world the converse is true. Spatial relations are ontologically prior to forms. The form of an object $O$ is the set of $O$’s internal spatial relations. A noncomposite object lacks internal relations. Hence “the form of a noncomposite object” is a contradiction in terms.

In the days when an atom was still largely thought of as a miniature solar system, Werner Heisenberg, if I remember rightly, argued that if atoms are to explain what the phenomenal world looks like, they cannot look like anything in the phenomenal world – an insight that we haven’t fully assimilated yet. In the phenomenal world, in which objects are bundles of qualia, every object necessarily has a form. If the same were true of the physical world, a noncomposite object would have to be pointlike. But the stuff that the physical world is made of is very different from qualia. The physical world is made of formless particles and the spatial relations that hold among them.

Let us complete the list of points made in this section:

- Spatial forms are sets of spatial relations. A composite object possesses both a position (consisting in its external spatial relations) and a form (consisting of its internal spatial relations). An object that lacks internal spatial relations also lacks a form.

4 The Identity of Fundamental Particles

In what way does a material object $O$ differ from its form? What does $O$ possess in excess of the spatial relations that make up its form? The obvious answer is, the formless entities to which we attribute those spatial relations. And what is it that these entities contribute to $O$ over and above their spatial relations? The obvious answer is, existence pure and simple. The difference between a set $F$ of spatial relations and a material object $O$ whose internal spatial relations make up $F$ is that $O$ exists while $F$ by itself does not. What $O$ has in excess of its form is the existence that its formless material constituents bestow on its internal spatial relations.

An actually existing composite object $A$ possesses other properties besides the property of existence. A composite object therefore is something to which existence is contingently attributable; an object with the properties of $A$ may or may not exist. A relative position is likewise definable independently of its existence, so it, too, may or may not exist. A fundamental particle, on the other hand, is not something to which existence is contingently attributable. Divested of its external spatial relations and the dynamical parameters that contribute to determine the evolution of these relations, a fundamental particle is

---

*Fundamental particles are usually thought of as possessing “by themselves” a mass, a spin, and
Figure 1: Scattering of two particles at right angles. If the alternative processes on the left-hand side of this symbolic equation are distinguishable experimentally, the equation holds for probabilities. In this case the diagram on the right-hand side gives an incomplete picture of what actually happens. If the alternative processes are indistinguishable experimentally, the equation holds for amplitudes. In this case the diagram on the right-hand side gives the complete picture of what actually happens, while the diagrams on the left-hand side are overcomplete: They involve a distinction that Nature doesn’t make.

A fundamental particle, therefore, either is “existence itself” or is an instance of “existence itself”.

So which is it – “existence itself” or an instance of it? The answer is an unequivocal “both”. To arrive at this conclusion, we will have to take account of another empirical finding. Let us consider a scattering experiment with particles of the same type – say, two protons. Let us assume that there are two incoming particles, one (N) moving northwards and one (S) moving southwards, and two outgoing particles, one (E) moving eastwards and one (W) moving westwards. Anyone unfamiliar with quantum mechanics will expect the following to be the case: either N is the same particle as W and S is the same particle as E, or N is the same particle as E and S is the same particle as W. Yet nothing could be further from the truth. Quantum mechanics tells us in no uncertain terms that neither of the incoming particles is identical with either of the outgoing particles.

One arrives at this conclusion by considering the probability $p(E,W)$ of the assumed final state (one eastbound particle and one westbound particle), given the assumed initial state. If the particles are of different types – say, a proton and a neutron in, a proton and a neutron out – $p(E,W)$ is the sum of two probabilities, the probability that N is various types of charge. These properties, however, are more appropriately understood as dynamical parameters characteristic of the evolution of spatial relations, rather than as intrinsic properties of individual particles. They tell us nothing about what a particle is in itself, or how it behaves out of relation to other particles.
the same particle as $E$ and $S$ is the same particle as $W$, and the probability that $N$ is the same particle as $W$ and $S$ is the same particle as $E$, in agreement with the dictates of common sense:

$$p_c(E, W) = |\langle EW|NS \rangle|^2 + |\langle WE|NS \rangle|^2.$$

$\langle EW|NS \rangle$ and $\langle WE|NS \rangle$ are the respective probability amplitudes associated with the alternatives ($N \rightarrow E, S \rightarrow W$) and ($N \rightarrow W, S \rightarrow E$), in obvious notation. The two possibilities that contribute to $p(E, W)$ are illustrated in Fig. 1. If the particles are of the same type, the probability for scattering at right angles is given by

$$p(E, W) = |\langle EW|NS \rangle + \langle WE|NS \rangle|^2.$$

For bosons $\langle EW|NS \rangle = +\langle WE|NS \rangle$, and $p(E, W)$ is twice as large as $p_c(E, W)$:

$$p_b(E, W) = 4|\langle EW|NS \rangle|^2 = 2p_c(E, W).$$

For fermions $\langle EW|NS \rangle = -\langle WE|NS \rangle$, and $p_f(E, W) = 0$. Both results are inconsistent with the notion that the two particles possess permanent identities. A fortiori they are inconsistent with the idea that a particle possesses the property of “being this very particle”, known to philosophers as “thisness” or “haecceity”.

Before and after the scattering, the two particles possess distinguishing characteristics: They travel in opposite directions, and they are in different places (relative to the laboratory frame). But at the time of scattering no such distinguishing characteristics exist, nor is it possible to causally link a particular incoming particle to a particular outgoing particle, nor can there be anything (such as thisness) that makes up for the missing causal identifiers. All of this follows from the experimentally confirmed scattering probabilities predicted by quantum mechanics. How many things, then, exist at the time of scattering? Two absolutely indistinguishable things? Or a single thing with the capacity for twofold instantiation?

According to the Identity of Indiscernibles, there cannot be two absolutely indistinguishable things. Seen from the laboratory frame, the two particles are absolutely indistinguishable at the time of scattering. In particular, their positions relative to the laboratory frame are identical. In spite of this, however, the two particles remain in possession of a nontrivial relative position, and this is sufficient for them to be two things even at the time of scattering. By a “nontrivial relative position” I mean a relative position that is not equivalent to an exactly vanishing distance. (Owing to the fuzziness of all possessed relative positions, no relative position is trivial in this sense.) The existence of a relation implies the existence of two relata. The spatial relation that exists between the two particles thus warrants their twoness without making them discernible in the laboratory frame. This permits either of two conclusions. If we consider the possession of distinguishing properties relative to the laboratory frame necessary for the discernibility of the two particles, then at the time of scattering there exist two indiscernible

---

9If particles were impenetrable bits of stuff, it would be impossible for them to have identical positions. But formless entities obviously can have identical positions relative to a reference object or frame, and this is equally true of objects that are made up of formless entities.
particles, and the Identity of Indiscernibles fails. If on the other hand we consider the existence of a not exactly vanishing distance between the two particles sufficient for their being discernible, then all particles are discernible, and the principle holds.

In either case, the reason why the two particles at the time of scattering are two things is their nontrivial spatial relation. Hence intrinsically (that is, considered out of relation to each other) the two particles are not two things. They are identical, and this not in the weak sense of exact similarity but in the strong sense of numerical identity. Hence the unequivocal “both”: Intrinsically (out of relation to each other) either particle is “existence itself”, and extrinsically (that is, by virtue of the spatial relation between them), they are two instances of “existence itself”.

What has been established so far is that particles of the same type, considered out of relation to each other, are numerically identical. If type conversions are possible, this conclusion can be extended to particles of different types. Suppose that the two particles in our scattering experiment are of different types (e.g., a proton and a neutron in, a proton and a neutron out) but that particles of the first type can be converted into particles of the second type (a proton into a neutron and vice versa). Further suppose that $N$ and $E$ are of type 1, and that $S$ and $W$ are of type 2. Then the probability of this scattering event is given by

$$p(E_1, W_2) = |\langle E_1 W_2 | N_1 S_2 \rangle + \langle W_2 E_1 | N_1 S_2 \rangle|^2,$$

where the indices specify the types to which the incoming and outgoing particles belong. Once again it is impossible to say whether a particular incoming particle is the same as or different from a particular outgoing particle. And the reason this is so is not that we have no means of knowing which of the alternatives depicted in Fig. 2 represents what actually happens. The reason this is so is that the distinction we make between the alternatives is a distinction that Nature does not make. Nothing in the physical world corresponds to the conceptual difference between these alternatives. It would

Figure 2: Same as Fig. 1, except for the possibility of type swapping. Since the alternative processes on the left-hand side (with and without type swapping) are indistinguishable experimentally, the diagram on the right-hand side gives the complete picture of what actually happens, while the diagrams on the left-hand side again involve a distinction that Nature doesn’t make.
therefore be incorrect to interpret the specified scattering event by affirming that the particles scatter both with and without type swapping, and it would be equally incorrect to interpret it by affirming that the particles scatter neither with nor without type swapping. Both interpretations involve a conceptual distinction that Nature does not make. That distinction exists solely in our minds.\[10\]

The same argument that took us from \( p(E, W) \) to the numerical identity of particles of the same type (considered out of relation to each other), now takes us from \( p(E_1, W_2) \) to the numerical identity of all particles of the same basic type (considered out of relation to each other). What is characteristic of a basic particle species is that its members cannot be converted into members of a different basic species. How many basic types of particle exist depends on the theory. According to the standard model, a member of one of the two species of particles known respectively as hadrons and leptons cannot be converted into a member of the other species (the same applies to bosons and fermions), while in the so-called grand unified theories hadrons and leptons are mutually convertible, and in supersymmetric theories “once a fermion, always a fermion” is no longer true either. In these theories, being a hadron, lepton, boson, or fermion is an accidental property of something that by itself is neither hadron nor lepton nor boson nor fermion; there exists just one basic type of particle. But whether or not the final theory (assuming that there will be one) permits conversions between all particle types, the property of belonging to a particular type of particle can be thought of as accidental or contingent, and all existing fundamental particles can be thought of as being intrinsically one and the same entity – “existence itself”.

Moreover, as was pointed out in note 8, particle species are distinguished by what is essentially a set of dynamical parameters governing the evolution of spatial relations. As attributes of an individual particle, considered out of relation to other particles, these parameters are meaningless. Hence if we consider a fundamental particle as it is in itself, we must mentally strip it not only of its spatial relations but also of the type to which it belongs. This too entails that intrinsically all fundamental particles are identical in the strongest possible sense: But for their spatial relations, they are one and the same entity. The basic reality of matter, accordingly, is not a multitude of fundamental particles but a single entity, and this entity cannot be qualified as anything more particular or determinate than “existence itself”. Henceforth I shall omit the coy quotation marks and represent this entity by the symbol \( \mathcal{E} \).

A fundamental particle is not just like \( \mathcal{E} \); each fundamental particle is \( \mathcal{E} \). At bottom the world has exactly one fundamental material constituent, namely \( \mathcal{E} \). The individuation or multiple instantiation of \( \mathcal{E} \) is a consequence of the realization, or the coming into existence, of spatial relations. \( \mathcal{E} \) has the capacity to enter into spatial relations with itself, and this is what gives it the aspect of a multitude of relata. But what exists at either end of each spatial relation, considered in itself, is identically the same entity \( \mathcal{E} \). At bottom all there is is \( \mathcal{E} \) and spatial relations between \( \mathcal{E} \) and itself. The spatial relations owe their existence to \( \mathcal{E} \) – they exist because they are relational determinations.

\[10\] As my aim here is to make sense of standard quantum mechanics, I take the nonexistence of hidden variables for granted. This warrants the logical step from a set of interfering alternatives to the objective unreality of whatever it is that renders the alternatives distinct to our minds.

16
of $\mathcal{E}$, – and $\mathcal{E}$ owes its individuation or multiple instantiation to the existence of spatial relations: A multiplicity of relations implies a multiplicity of relata, even if intrinsically each relatum – each fundamental particle of matter – is $\mathcal{E}$.

Instantiation is traditionally conceived as running parallel to predication: What gets instantiated is a predicable universal; the resulting instance is an impredicable individual. This way of thinking suggests that what is responsible for the instantiation is something that is present in the individual but absent from the universal, and this idea is at the root of the Platonic-Aristotelian dualism of Matter and Form and its subsequent transformations, including the idea that physical qualities are instantiated by the “points of space”. The individuation that takes us from $\mathcal{E}$ to the fundamental particles of matter is something else altogether. A fundamental particle is not two things – (i) instantiated existence and (ii) something (such as a “point of space” or a part of Plato’s matter-space) that does the instantiating. In a fundamental particle there isn’t anything that is distinct from $\mathcal{E}$ and of which existence is predicable. There is nothing that acquires “$\mathcal{E}$-ness” the way a bounded portion of Platonic-Aristotelian matter acquires Being or actuality. There is nothing present in a fundamental particle that is absent from $\mathcal{E}$.

Only existence can instantiate existence, for an effective instantiator must exist in advance of the instantiation, and the only “thing” that exists in advance of the instantiation of $\mathcal{E}$ is $\mathcal{E}$. But this is the same as saying that the only way to instantiate existence is to relate it to itself. Only in this way can the proper logical dependences be implemented: The instances of existence exist because the instantiating spatial relations exist; the instantiating relations exist because they are properties of $\mathcal{E}$ (or because $\mathcal{E}$ has assumed them, or because $\mathcal{E}$ has entered into spatial relations with itself); and $\mathcal{E}$ exists because it is “the one independent reality of which all things are an expression” (a dictionary definition of “the absolute” [18]).

The relationship between $\mathcal{E}$ and its instances thus is as close and as intelligible as can be. It is not some mystical relation between formless Matter and immaterial Form. It is identity plain and simple. The spatial multiplicity of the world is not based on something that is intrinsically multiple like Plato’s matter or a set of point-individuals with the cardinality of the reals. It is a multiplicity of relations that entails a multiplicity of relata. But only the relata qua relata are many. Intrinsically (out of relation to other particles) each relatum is $\mathcal{E}$. The “expression” of the one independent reality $\mathcal{E}$ is effected by means of relations between $\mathcal{E}$ and itself. Physical properties are relational – they are either spatial relations or dynamical parameters governing the evolution of spatial relations, – and since existence is contingently attributable to them, they may be thought of as universals. But $\mathcal{E}$ cannot be thought of as another universal. Material things are made of relations, and these owe their existence not to a predicable universal but to the existence that they relate.
5 The Heart of Quantum Mechanics

In the previous section we addressed the “the miraculous identity of particles of the same type”, which Misner et al. [19, p. 1215] regard as “a central mystery of physics”. In this section we examine a phenomenon which according to Feynman et al. [20, Sec. 1–1] “has in it the heart of quantum mechanics”.

Let \( R \) be some bounded spatial region. We tend to think that the world can be divided into things or parts that are inside \( R \) and things or parts that are outside \( R \). We tend to think that spatial distinctions like the distinction between the inside of \( R \) and the outside of \( R \) are real \textit{per se}, and that \textit{a fortiori} they are real for everything that exists in space. If this were the correct way of thinking about space, any object \( O \) would at any time satisfy one of the following three conditions: (i) \( O \) is wholly inside \( R \); (ii) \( O \) is wholly outside \( R \); (iii) \( O \) has two parts, one inside \( R \) and one outside \( R \). Once again reality does not comply with the dictates of common sense. Quantum mechanics tells us in no uncertain terms that sometimes all of these propositions are false.

The paradigm example is a two-slit interference experiment with electrons [20, Chap. 1]. Anyone unfamiliar with quantum mechanics expects one of the following propositions to be true of each electron: (i) it goes through the first slit, (ii) it goes through the second slit, (iii) it consists of parts that go through different slits. Yet whenever the experimental arrangement is such that interference fringes are observed, the electron does none of this. It doesn’t go through a particular slit and it doesn’t get divided by its passage through the slits. Albert [23, p. 11] draws from this the conclusion that “[e]lectrons seem to have modes of being, or modes of moving, available to them which are quite unlike what we know how to think about”.

The behavior of electrons becomes intelligible if we reject the substantival, set-theoretic conception of physical space. The three propositions just considered are based on the assumption that the two slits are distinct \textit{per se}. One of the things that the interference fringes are trying to tell us is that our conceptual distinction between the regions defined by the slits does not exist for the electron. Hence the distinction cannot be real \textit{per se}. In other words, it cannot be intrinsic to space. Hence the substantival, set-theoretic conception of physical space, which entails the distinction, cannot be the right way of thinking about physical space.

The electron is able to go through the two slits \textit{indiscriminately} (that is, without going through a particular slit and without being divided) because the conceptual distinction between “disjoint parts of space” (which is inherent in the set-theoretic conception of space) does not always correspond to something that is physically real. When it is appropriate to add amplitudes rather than probabilities, the distinction we make be-

---

11There is nothing so obvious that a philosopher cannot be found to deny it, as Lockwood [21, p. 45] observed. It seems to be the same with physicists. Bohmian mechanics [2] tries to salvage as much of common sense as is consistent with the empirical data, by attributing to each particle a definite but observationally meaningless position. By introducing unobservable (and hence arbitrary) causes for stochastic events, this theory makes each electron go through a definite slit.

12As will become clear in the next section, this is the case when the following two conditions are fulfilled: (i) There isn’t any matter of fact about the alternative taken, and (ii) the quantum system under consideration, \( S_1 \), is not correlated (“entangled”) with another quantum system \( S_2 \) in such a way
tween the corresponding alternatives is a distinction that Nature does not make. We can say that the electron went through both slits if “both slits” stands for a single positional attribute – the opening made up of the two slits. But we cannot make this equivalent to two propositions (“the electron went through the first slit” and “the electron went through the second slit”) since this involves a distinction that has no counterpart in the physical world. The distinction exists solely in our minds. But if a distinction exists solely in our minds, the same is true of any notion that implies the distinction. The notion which makes the two alternatives in a two-slit experiment distinct for us, and the behavior of electrons incomprehensible, is the notion that physical space is intrinsically and infinitely differentiated. The notion that the multiplicity of \( R^3 \) is inherent in physical space (and that, consequently, the individual points of space or space-time can be regarded as carriers of physical properties) is as inconsistent with quantum mechanics as the notion of absolute simultaneity is with special relativity. This notion perhaps more than any other is what prevents us from making sense of quantum mechanics.

If space were an intrinsically and infinitely differentiated constituent of the physical world, then any spatial region, however small, would per se be distinct from any other (disjoint) spatial region, and every material object \( O \) would consist of as many spatial parts as the space it occupies. The parts of space would define the parts of \( O \). If \( O \) has only a finite number of material constituents, these would necessarily be pointlike, and for any partition \( \{d^3R\} \) of space into infinitesimal regions the following would necessarily be the case: Each of the material constituents of \( O \) is located inside a particular region \( d^3R \). In other words, on a substantival, set-theoretic account of physical space, the positions of things are necessarily definite. In reality – so the behavior of the electrons tells us – space is a system of spatial relations between material objects, and these spatial relations are (more or less) indefinite. The behavior of electrons in two-slit interference experiments is Nature’s corroboration of the relationist conception of physical space developed in Secs. 2 and 3.

Such an experiment typically features several macroscopic objects like an electron gun (a heated tungsten wire in a metal box with a hole, the wire being at a negative voltage with respect to the box), a thin metal plate with two slits in it, and an array of detectors (Geiger counters or electron multipliers connected to loudspeakers). What these objects have in common is that their relative positions can be treated as definite. The only objects with pertinently indefinite positions (relative to any macroscopic part of the apparatus) are the electrons coming from the electron gun. We tend to visualize the position \( P \) of such an electron (relative to the apparatus) as a cloud or a smudge (a “wave packet” or, worse, two “wave packets”). It is important to understand that this picture cannot be taken to represent a state of affairs in physical space. The “space”

\[13\] Why this so is explained in Sec. 7. Sometimes an exception is made for the slit plate. If the transverse momentum of the slit plate is so sharp that by measuring it one can infer the slit taken by the electron, the positional indefiniteness of the slit plate (relative to the rest of the apparatus) rules out the existence of an interference pattern \[21\] Sec. 1–8]. The correct explanation of the obliteration of the interference pattern, however, is not the positional indefiniteness of the slit plate but the ensuing correlation of the transverse momentum of the slit plate with the transverse position of the electron.
containing the cloud is the set $C$ of values that are available for attribution to the relative position $P$. This value “space” must not be conflated with the physical space $S$, which contains $P$. It is a set of triplets of real numbers. It does not possess the quality of spatial extension known to us from percepts and images in phenomenal space. Outside of phenomenal space, this quality is attributable solely to the possessed spatial relations that make up $S$.

The “point of contact” between $C$ and $S$ is *detectors*. By a “detector” I mean anything that is capable of indicating the presence of a material object, such as an electron, in a particular spatial region $R_i$, or its passing through a particular section $\sigma_i$ of a plane. The regions $R_i$ or the sections $\sigma_i$ are physically realized or realizable with the help of macroscopic boundaries (that is, boundaries made up of a large number of material constituents the spatial relations between which can be treated as definite). These regions or sections define values (“inside $R_i$” or “through $\sigma_i$”) that are potentially attributable to $P$. They therefore “exist” both in $S$ and in $C$. But only if a detector clicks (that is, only if there is a matter of fact about the particular region containing the electron or the particular section crossed by the electron) is the corresponding value actually attributable to $P$, as will become clear in what follows.

If the substantival, set-theoretic conception of physical space is inconsistent with quantum mechanics, so is the doctrine called local realism, according to which, “… all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another…. We have geometry: a system of external relations of spatiotemporal distance between points…. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated…. And that is all…. All else supervenes on that” [24, p. X]. What quantum mechanics is trying to tell us is that reality is not built “from the bottom up”, on an infinitely and intrinsically differentiated space, out of locally instantiated physical properties. There are no points on which a world of such properties can be built. Reality instead is built “from the top down”: By assuming a multitude of spatial relations, $E$ takes on not only the aspect of a spatially differentiated world but also the aspect of a multiplicity of fundamental particles.

If the doctrine of local realism is an exploded myth, so is the principle of local action (a.k.a. local causality), according to which the “local matters of particular fact” at a point $x$ are influenced only by the “local matters of particular fact” in the infinitesimal neighborhood of $x$. Local action therefore cannot be the solution to the perceived problem of action at a distance. Nor does this require a solution, since it is a pseudo-problem arising from erroneous assumptions. For one thing, it presupposes the existence of separate interacting entities $A$ and $B$, as well as the existence of separate regions of space containing $A$ and $B$. But $A$ and $B$ aren’t separate entities, nor are there separate regions of space. If $A$ and $B$ are fundamental particles, they both are $E$, and if they are composite objects, the ultimate material constituents of $A$ are numerically identical with the ultimate material constituents of $B$. Physical space, on the other hand, isn’t something that by itself has parts; it is a system of spatial relations between material objects. The spatial differentiation of the physical world is a property of its material content, not of a substantively conceived space. Two locations are therefore
never separate “by themselves”. Only material objects can be spatially separated, and for two objects A and B to be spatially separated, it is necessary that their relative position \( P(A, B) \) is sufficiently large compared to its indefiniteness. But this is not enough: The position of B relative to A, as seen from the laboratory frame, \( P_L(A, B) = P(B, L) - P(A, L) \), must likewise be sufficiently large compared to its indefiniteness.14

The following often cited statement [25] is therefore unfounded: “An essential aspect of [the] arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects ‘are situated in different parts of space’.” There are no “different parts of space”.

For another thing, the very attempt to explain (causally or otherwise) the fundamental behavior of matter appears misconceived. Explanation begins with the fundamental behavior of matter, which can only be described, and which in part is described by quantum mechanics. We have space, which is a system of relations between \( \mathcal{E} \) and itself. In classical physics we have dynamical laws, which spell out how these relations evolve in time. In quantum physics we have mathematical condensations of statistical regularities. The idea that particles act on particles, or that the distribution and motion of matter “here” and “now” is causally related to the distribution and motion of matter “there” and “then”, or that measurement results are so related, adds nothing of physical significance to (classical) laws that simply describe how spatial relations are temporally related, or to (quantum-mechanical) laws that, as will become clear in what follows, describe statistical correlations between property-indicating facts. And specifically, any attempt to explain the statistical regularities in terms of causal strings that criss-cross space or space-time involves the principle of local action and therefore is inconsistent with quantum mechanics.

6 Quantum-Mechanical Properties Are Extrinsic

Quantum mechanics is, if nothing else, a tool for calculating probabilities. It represents the possible values \( q_k^i \) of observables \( Q^k \) as projection operators \( P Q^k = q_k^i \) on some Hilbert space \( \mathcal{H} \). The projection operators that jointly represent the range of possible values of a given observable are mutually orthogonal. If one defines the “state” of a system as a probability measure on the projection operators on \( \mathcal{H} \) resulting from a preparation of the system [26, 27, p. 92–94], one finds [26, 28, p. 132] that every such probability measure has the form \( p(P) = \text{Tr}(WP) \), where \( W \) is a unique density operator [that is, a unique self-adjoint, positive operator satisfying \( \text{Tr}(W) = 1 \) and \( W^2 \leq W \)]. \( \text{Tr} \) signifies the trace defined by the formula \( \text{Tr}(X) := \sum_i \langle i | X | i \rangle \) for any orthonormal basis \( \{ | i \rangle \} \). If \( W^2(t) = W(t), W(t) \) projects on a one-dimensional subspace of \( \mathcal{H} \) and thus is

14 \( L \) marks the origin of the laboratory frame. While \( P(A, B) \) corresponds to a single conditional probability distribution – the distribution of \( P(B, L) \) conditional on \( A \)'s having a numerically exact position in the laboratory frame, \( -P_L(A, B) \) corresponds to the difference between the distributions of \( P(B, L) \) and \( P(A, L) \). (A word of caution: As we shall see below, the probabilities that one may use to quantitatively describe indefinite relative positions are distributed over counterfactuals. This rules out naive realistic interpretations of position probability distributions and wave functions, such as Schrödinger’s original interpretation of \( \psi \) as some bizarre kind of real jelly.)
equivalent – apart from an irrelevant phase factor – to a state vector $|\psi(t)\rangle$ or a wave function $\psi(r, t)$, $r$ being any point in the system’s configuration space.

It is tempting to attribute the truth of quantum-mechanical probability assignments to an underlying state of affairs. It is equally tempting to assume that this state of affairs is somehow represented by the state vector or the density operator. Instead of jumping to such conclusions, however, one should heed van Kampen’s warning: “Whoever endows $\psi$ with more meaning than is needed for computing observable phenomena is responsible for the consequences. . . .” One should not lose sight of the fact that the state vector or the density operator is what one gets if one defines the “state” of a system as a probability measure on the projection operators representing the possible properties of the system. The idea that what by definition is a tool for assigning probabilities to possibilities also describes an actual state of affairs, is simply a category mistake.

Quantum mechanics being inconsistent with local realism, it is impossible to interpret $\psi(r, t)$ as a local quality – something that “need[s] nothing bigger than a point at which to be instantiated”. The fact that we live in a relativistic world, moreover, commits us to treating space and time on an equal footing, as far as this is consistent with the obvious qualitative differences between the two. Specifically, if space is a system of spatial relations, then time is a system of temporal relations (or, rather, then space-time is a system of spatiotemporal relations), and if space isn’t intrinsically and infinitely differentiated (or, rather, if the world isn’t infinitely differentiated spacewise), then time isn’t intrinsically and infinitely differentiated either (that is, the world isn’t infinitely differentiated timewise). In other words, if space isn’t a set of self-existent points, time can’t be a set of self-existent instants. Hence $\psi(r, t)$ not only isn’t something that exists at “the point $r$” but also isn’t something that exists at “the instant $t$”. Once this is understood, there doesn’t seem to be much point in construing the state vector as an actual state of affairs, for the principal reason for doing so has always been to protect the golden calf of determinism, which requires local realism in general and the local reality of $\psi(r, t)$ in particular. That probabilities are nonlocal in space and in time makes perfect sense: The probability for an object to be found inside a region $R_i$ at a time $t$ is not something that exists inside the region $R_i$ or at the time $t$. A state of affairs that is nonlocal in time and space, on the other hand, is not something that we know how to make sense of.

That it makes no sense to construe the state vector as an actual state of affairs does not mean that one cannot make ontological sense of quantum mechanics. However, in order to do so one must steer clear of two diametrically opposed mistakes. The first is the error of the instrumentalist, who regards as idle metaphysics any attempt to give a realistic or epistemological account of how it is that the statistical regularities predicted by quantum mechanics come out the way they do. For the instrumentalist, the results of measurements are not possessed properties, for there are no “quantum objects” that could possess them; there is no “quantum world” to which the results of measurements could be attributed.

The instrumentalist throws out the baby with the bath water. Suppose that we perform a series of position measurements, and that every position measurement yields exactly one result (that is, each time exactly one detector clicks). In this case we are
entitled to infer the existence of an entity $O$ that persists through time, to think of the clicks given off by the detectors as matters of fact about the successive positions of this entity, to think of the behavior of the detectors as position measurements, and to think of the detectors as detectors. The case for instrumentalism is that the position-indicating clicks are not only sufficient but also necessary for the existence of the positions indicated by the clicks. That is why Bohr [30, 31] insisted on the necessity of describing quantum phenomena in terms of the experimental arrangements in which they are displayed. But this does not mean that there is no object to which the indicated positions can be attributed. What it means is that the contingent properties of the quantum world are extrinsic rather than intrinsic, in the sense that the existence of a fact indicating that $O$ is (has the property) $q$ is necessary for $O$’s being $q$: The properties of the quantum world are what can be inferred from property-indicating facts (including the outcomes of laboratory experiments) or from measurement results (if “measurement result” is taken in the general sense of “property-indicating fact”). This tells us something significant about the quantum world, rather than controverts the existence of a quantum world.

On this interpretation, the number of objects that exist at any one time is as extrinsic as are the properties that can be attributed to them. It is only because every time the same number of detectors click (in the previous example, one) that there exists, at the times of the position measurements, the same determinate number of objects. The quantum world lacks intrinsic properties, including the property of containing a determinate number of objects. But, again, this is not the same as saying that there is no quantum world. There is something to which a particular number of spatial relations and a particular number of corresponding relata can be attributed, provided that the attribution is warranted by facts. This something is $E$. The quantum world is $E$ plus whichever properties are warranted by the facts, including the number of times that $E$ is instantiated.

The opposite mistake is the error of the quantum realist, who endorses the following “sufficiency condition” [32, p. 72]: “If we can predict with certainty, or at any rate with probability one, the result of measuring a physical quantity at time $t$, then at the time $t$ there exists an element of reality corresponding to the physical quantity and having a value equal to the predicted measurement result”. To begin with, $p (R_i, t) = 1$ does not mean that the probability of finding $O$ in $R_i$ is one at the time $t$. Instead, it means that the probability of finding $O$ in $R_i$ at the time $t$ is one. There is a significant difference between these two interpretations. Since probabilities are not things that exist in time or at times, one cannot speak of a probability as having such and such a value at such and such a time. One can only speak of the probability of finding $O$ in $R_i$ at the time $t$. This probability exists neither inside $R_i$ nor at the time $t$. The insufficiency of Redhead’s “sufficiency condition” hinges on the nonexistence of the particular time $t$ in the absence of an actual measurement performed at the time $t$, as I proceed to show.

A particular region $R_i$ recall, does not exist unless it is physically realized with the help of a macroscopic boundary, as defined in the previous section. And it does not exist for $O$ (that is, the difference between “$O$ is inside $R$ at $t$” and “$O$ is outside $R$ at $t$” has

\[^{15}\text{A property } q \text{ attributable to } O \text{ is a contingent property of } O \text{ if it is not necessarily possessed by } O.\]
no physical reality) unless there is a matter of fact about $O$’s whereabouts with respect to $R$ (“inside” or “outside”) at the time $t$ (that is, unless there is an actual event or state of affairs, such as the click of a detector, from which the truth of either “inside” or “outside” can be inferred). Since the spatial distinctions that we make in our heads are not physically real per se, they need to be realized in order to be physically real – they need a physical “hook” from which they can “dangle”. This hook is to be sought among the actual events and states of affairs that happen or obtain, and specifically among those that indicate the value of a relative position (from a given range of values such as “inside $R_i$”, $i = 1, \ldots, n$). The possessed values of relative positions dangle from position-indicating facts. And so do the actually existing spatial distinctions, since they are contingent on the possessed values of relative positions.

An immediate consequence of the extrinsic nature of contingent properties is that the position of one object $A$ relative to the laboratory frame may be definite with respect to a given partition of this frame (there may be a matter of fact about the particular region containing $A$), while the position of another object $B$ may be indefinite with respect to the same partition (there may not be any matter of fact indicating the particular region containing $B$). In other words, a particular spatial distinction may be real for one object and not real for another. The particular region $R$ may exist for one object and not exist for another.

The particular time $t$, likewise, does not exist unless it is physically realized. Since temporal distinctions are not real per se, they too need to be realized; they too need a physical hook from which they can dangle. Once again the hook is to be found among the actual events and states of affairs that happen or obtain, but now specifically among those that indicate not only the possessed value of some observable but also the time at which this is possessed. The times at which the extrinsic properties of the quantum world are possessed are themselves extrinsic: They dangle from facts indicating both that a property is possessed and when it is possessed. An immediate consequence of the extrinsic nature of the times at which properties are possessed, is that a particular time $t$ may be real for one object and not real for another object. For instance, it may be real for a laboratory clock but not real for the quantum system at hand. Where $O$ is concerned, the particular time $t$ exists iff the possession by $O$ of some property $q$ at the time $t$ is factually warranted (that is, iff there is a property $q$ such that the inference “$O$ is $q$ at $t$” is warranted by facts). Otherwise $t$ does not exist for $O$, and $O$ cannot be said to exist at the particular time $t$.

In nonrelativistic quantum mechanics, where the number of objects or subsystems is constant, we feel that we may imagine a particle as existing also between the times at which it has factually warranted properties, and we feel that we may imagine it as permanently possessing such invariable properties as a mass or an electric charge. If a region $R$ is surrounded by an impenetrable barrier, and if the presence of $O$ in $R$ at a time $t$ is factually warranted, we further feel that we may imagine $O$ as being inside $R$ also before and after the time $t$. But let us be clear about the meanings of the temporal referents “between the times at which it has factually warranted properties” and “before and after the time $t$”. Since time is not a set of instants, this cannot mean “at any or every instant between those times” or “at any or every instant before or after $t$”. It can
only mean “during the continuous, undifferentiated time spans between those times” or “during the continuous, undifferentiated time spans before and after \( t' \). Are these legitimate temporal referents?

A particular time \( t' \) exists only if it is indicated by an actual event or state of affairs, and it exists for \( O \) only if it is the factually warranted time of possession, by \( O \), of a factually warranted property. Are the undifferentiated time spans \( \tau_i \) between the particular times \( t_i \), at which \( O \) has factually warranted properties \( q_i \), nevertheless legitimate temporal referents for attributions to \( O \)? The answer depends on whether the facts that indicate the possession of the respective properties \( q_i \) at the respective times \( t_i \) can be thought of as indicating the possession of the same properties during the time spans \( \tau_i \). In nonrelativistic quantum mechanics, this is the case where the property of individuated existence is concerned: If the existence of an individual object \( O \) at the times \( t_i \) is warranted by actual events or states of affairs (that is, if \( O \) is detected at the times \( t_i \)) then \( O \) also exists as an individual during the intervening time spans \( \tau_i \). And if the presence of \( O \) in \( R \) at the time \( t \) is warranted by some actual state of affairs, this state of affairs also warrants the presence of \( O \) in \( R \) during the preceding and succeeding undifferentiated time spans – but not at any particular time \( t' \) during these time spans. Unless the presence of \( O \) in \( R \) (or the possession by \( O \) of any other contingent property) at \( t' \) is factually warranted, the particular time \( t' \) does not exist for \( O \), and \( O \) cannot be said to exist at \( t' \).

Thus we can predict (as well as retrodict) with probability one that \( O \) will be (would have been) found inside \( R \) at the time \( t' \) if the appropriate measurement is (had been) performed at \( t' \), but we cannot affirm that at the particular time \( t' \) there exists an element of reality corresponding to the presence of \( O \) in \( R \). Unless the measurement is actually performed at \( t' \), \( t' \) does not exist for \( O \), so an element of reality involving \( O \) cannot exist at \( t' \). While the instrumentalist errs by ignoring the possibility of a quantum world with extrinsic properties, the quantum realist errs by ignoring the fact that the properties of the quantum world are extrinsic, and that, therefore, probability one is not sufficient for the existence of an element of reality.

In a relativistic world, in which the number of objects or subsystems is itself a variable and contingent property, we may imagine a closed system as existing also between the times at which it has factually warranted properties, and we may attribute to it its conserved quantities during these undifferentiated time spans, provided that at some time their possession is warranted by facts. But between the times at which the number of subsystems has a factually warranted value, there is only one logical subject that has a counterpart in the physical world, namely existence itself. Apart from the conserved quantities that are enduringly attributable to it, this has properties (including the number of times that it is instantiated) only if and only when they are warranted by facts.

---

16 An anonymous referee (of a different paper and a different journal) asserts that standard quantum mechanics does not include Redhead’s “sufficiency condition” but instead encompasses the “eigenstate-eigenvalue link”, according to which an element of reality corresponding to an eigenvalue of an observable exists at time \( t \) iff the system at \( t \) is “in the corresponding eigenstate of this observable”. This too is incorrect, and for exactly that same reason.
7 Indefiniteness, Macroscopic Objects, And the Emergence of Causality

The conceptual innovation due to the Copenhagen interpretation of quantum mechanics (CIQM) has been characterized by Stapp [33] in the following words: “The theoretical structure did not extend down and anchor itself on fundamental microscopic space-time realities. Instead it turned back and anchored itself in the concrete sense realities that form the basis of social life.” The first part of this statement belongs to the core of the CIQM, and it is what makes this interpretation superior to any interpretation that allows for fundamental microscopic space-time realities. There is no general consensus as to the other claims that form part of the CIQM. Bohr’s cryptic and not always consistent utterances on the subject have been invoked to support a variety of conflicting readings. Stapp’s reading restricts physics to perceived and communicable phenomena. Science, however, is driven by the desire to know how things really are, and it owes its immense success in large measure to the belief that this can be discovered. Unless there is conclusive proof to the contrary, it would be premature to relinquish this powerful “sustaining myth” [34]. But if the theoretical structure is anchored neither in microscopic space-time realities nor in concrete sense realities, then what supports it? The answer is, property- and time-indicating facts.

Loewer [35] associates with the CIQM the following claim: “An isolated quantum system evolves in conformity with a linear deterministic law (Schrödinger’s equation) unless it is measured. Measurements are governed by an indeterministic law – the collapse postulate.” This – the von Neumann-Dirac interpretation – cannot be part of the CIQM, given that the core of the CIQM rejects elementary space-time realities and therefore rejects elementary time realities. Because time is not a set of such realities (instants), a measurement does not prepare an instantaneous state of affairs that crawls predictably through an intrinsically and infinitely differentiated time until it unpredictably changes into a different instantaneous state of affairs. All that a measurement “prepares” is probabilities, and probabilities are not things that exist or evolve in time. Nor are density operators and state vectors such things, considering that they themselves are essentially (that is, by definition) probability measures on the possible outcomes of measurements. “[T]here is no interpolating wave function giving the ‘state of the system’ between measurements” [36].

Quantum-mechanical probabilities are conditional. The (standard) Born probabilities are conditional on (i) a prediction basis (the actual events or states of affairs that determine the “preparation”), (ii) the observable Q that is being measured (including its range of possible values), (iii) the time t at which this is measured, and (iv) the existence of a measurement result (that is, a matter of fact about the actual value of Q at the time t). ABL probabilities, named after Aharonov, Bergmann, and Lebowitz [37], are conditional on an inference basis rather than a prediction basis. This inference basis includes, in addition to the “preparation”, the actual events or states of affairs that determine the “retroparation” of the system [38, 39, 40, 41]. Quantum-mechanical probabilities are

\[17\] The ABL probability with which a measurement of the observable Q between the “preparation”
determined by facts (not mediatel y via an evolving, collapsible, instantaneous state of affairs but directly), and they cannot be assigned without specifying both an observable and a time at which this is measured. Hence it should be clear that the time on which quantum-mechanical probabilities, density operators, and state vectors depend is the specified time of a specified measurement, rather than the time of an evolving state of affairs. The parameter $t$ in the (Born) probability distribution

$$p(R_i, t) = \text{Tr}(W(t)P(R_i)) = \langle \psi(t)|P(R_i)|\psi(t)\rangle = \int_{R_i} d^3r \psi^*(r, t) \psi(r, t)$$

is not the time at which the probability of finding $O$ in $R_i$ has the value $p(R_i, t)$. Instead it is either the time of an actually performed measurement determining the particular region $R_i$ containing $O$ or the specified time of a counterfactually performed such measurement.

Nothing in quantum physics corresponds to the intrinsically and infinitely differentiated time through which, according to a certain folk belief, the present “moves” or “advances”. Nothing in quantum physics corresponds to the related notion of an instantaneous state that “moves” or “advances” through an ordered set of preexistent instants having the cardinality of the “real line”. And therefore nothing in quantum physics warrants the other folk belief according to which causal influences are carried towards the future by an instantaneous state of affairs (and therefore, in a relativistic world, conformally to the principle of local action). Where quantum physics is concerned, all the determining that goes on in the physical world is the determining by property-indicating facts (actually obtained measurement results) of probabilities associated with the possible results of specified measurements that are actually or counterfactually performed at specified times. What does not take place in the physical world is a determining by property-indicating facts of other property-indicating facts. There are no causal links between factually warranted properties or between the corresponding property-indicating actual events or states of affairs.

The probabilities of classical physics are subjective. They come into play whenever the exact state of a system is unknown, intractable, or irrelevant to the problem at hand. Quantum-mechanical probabilities, on the other hand, have an objective as well as a subjective application. If observable $Q$ is actually measured, the probabilities associated with the range of possible values of $Q$ are subjective; they are based on a limited knowledge that does not take account of the actual measurement result. This applies not only to Born probabilities, which take account of the system’s “preparation” only, but also to ABL probabilities, which also take account of the system’s “retroparation”. On the other hand, if no measurement is made at the time $t$, the ABL probabilities associated with $Q$’s range of possible values and the specified time $t$ are objective, in the

represented by the state $|\psi_1\rangle$ and the “retroparation” represented by the state $|\psi_2\rangle$ yields the result $q_i$, is given by the ABL formula

$$P_{ABL}(q_i) = \frac{\langle \psi_2|P_{Q=q_i}|\psi_1\rangle^2}{\sum_j \langle \psi_2|P_{Q=q_j}|\psi_1\rangle^2},$$

where the $P_{Q=q_i}$ projects on the subspace corresponding to the eigenvalue $q_i$ of $Q$.
sense that they have nothing to do with ignorance. All relevant facts have been taken into account; there is nothing for us to be ignorant of. Born probabilities in general contain a subjective element even if \( Q \) is not actually measured, for they ignore the relevant matters of fact about the properties of the system at later times. Born probabilities can be objective only if there aren’t any matters of fact about the future properties of the system. Thus objective probabilities are always assigned to counterfactuals – conditional statements that presuppose the falsity of their antecedents, which are of the form “If \( Q \) were measured”, – and if there are relevant matters of fact about the future properties of the system, they have to be calculated using the ABL formula.\(^3\)

Now is the time to make good on my promise to define what exactly I mean by an “indefinite” or “fuzzy” position. Let \( \{ R_i | i = 1, \ldots, n \} \) be some partition of space (that is, of the “space” of values available for attribution to the positions of objects relative to some reference object \( O \).) Let \( O(t) \subset R \) denote the proposition “Object \( O \) is inside region \( R \) at time \( t \)”. (This means that at the time \( t \) the position of \( O \) relative to \( O \) has the value “inside \( R \”); it does not mean that it has a definite value falling inside the range \( R \).) Let \( Q \) be the particular position observable whose range of possible values is \( \{ O(t) \subset R_i | i = 1, \ldots, n \} \). Finally, let \( Q \Rightarrow O(t) \subset R_k \) stand for the conditional “If \( Q \) is (or were) measured at the time \( t \), \( O \) is (or would be) found inside \( R_k \)”. The position of \( O \) is indefinite with respect to \( \{ R_i | i = 1, \ldots, n \} \) iff (i) the conditionals \( Q \Rightarrow O(t) \subset R_i | i = 1, \ldots, n \) are counterfactuals (\( Q \) is not actually measured) and (ii) the objective probabilities associated with these counterfactuals are positive for at least two \( i \). For instance, if there isn’t any matter of fact concerning the slit taken by an electron, the electron’s transverse position at the time of its passing the slit plate is indefinite just in case the probabilities associated with the following counterfactuals are positive: “If there were a matter of fact about the slit taken by the electron, it would indicate that the electron went through slit \( i \)” \( (i = 1, 2) \).

Note that \( O \)’s position at a specified time \( t \) may be indefinite with respect to some partition or range of possible values and definite with respect to another partition or range of possible values. Consider, for instance, a three-slit experiment incorporating a device \( D \) that is capable of indicating whether the electron went through slit \( A \) or through the union \( B \cup C \) of the remaining slits, but not capable of distinguishing between \( e_B = “the electron went through slit \( B \)” \) and \( e_C \). Suppose that \( D \) indicates that the electron went through \( B \cup C \). If the respective probabilities associated with \( e_B \) and \( e_C \) are positive and objective, the electron’s transverse position at the time of its passing

\(^3\)Kastner’s [12, 13] objection to Vaidman’s [11, 12, 16] counterfactual usage of the ABL formula has no bearing on my counterfactual usage of this formula, as may be gleaned from Kastner’s withdrawal of a paper [17] in which she raised similar objections to the way I assign ABL probabilities to counterfactuals in my forthcoming [11], admitting that her paper was “based on a misunderstanding of Mohrhoff’s use of the term ‘counterfactual’” and that “Mohrhoff’s counterfactual uses of the ABL rule correspond to special cases in which such use is valid”. The counterfactuals to which I assign ABL probabilities have antecedents of the form “if \( Q \) had been measured between \( t_1 \) and \( t_2 \) (while actually no measurement is made between the preparation at \( t_1 \) and the retroparation at \( t_2 \)”, whereas it was Kastner’s initial impression that I allow antecedents of the form “if \( Q \) had been measured between \( t_1 \) and \( t_2 \) (while actually a different, noncommuting observable \( Q’ \) was measured between \( t_1 \) and \( t_2 \)” . That my use of the ABL formula is the correct use also transpires from Cohen’s [18] analysis.
the slit plate is definite with respect to the alternative defined by $D$ but indefinite with respect to the alternative “$e_B$ or $e_C$”.

The above definition of positional indefiniteness makes reference to objective probabilities, and such probabilities, as we have just seen, can be assigned only to counterfactuals. The very possibility of assigning objective probabilities to the possible results of an unperformed measurement entails that unmeasured observables lack values, and that the possessed values of quantum-mechanical observables are extrinsic. The indefiniteness of a contingent property thus entails the property’s extrinsic nature. The indefiniteness of the physical world makes it necessary to conceive of its contingent properties as extrinsic, or supervenient on property-indicating facts. A position can be indefinite only because (i) it dangles from what happens or is the case in the rest of the world and (ii) what happens or is the case in the rest of the world may not be enough to determine its precise value. If the positions of material objects were not taken from (defined by) position-indicating facts (that is, if they were intrinsic), they would have to be taken from an intrinsically differentiated space. But an intrinsically differentiated space is an infinitely differentiated space, and such a space has no room for indefinite values. Thus unless we are willing to take seriously the extrinsic nature of quantum-mechanical properties, we shall not be in a position to make proper sense of the indefiniteness that is the hallmark of quantum mechanics, and hence of quantum mechanics itself.

One can always conceive of a partition \{$R_i\mid i = 1, \ldots, n\}$ into regions that are so small that the following is true for any specified time and any object $O$ other than $O$: The position of $O$ relative to $O$ is indefinite with respect to some subset \{$R_k(O)\mid k = 1, \ldots, m\}$ of \{$R_i\$. This is the same as saying that there is a subset \{$R_k\} of \{$R_i\$ such that the conditionals \{$Q \Rightarrow O(t) \subset R_k\mid k = 1, \ldots, m\}$ are counterfactuals and the probabilities associated with these counterfactuals are positive for at least two $k$. And this is the same as saying that no two objects ever have a definite relative position. A pair of material objects could have an exact relative position only if there existed material objects capable of indicating an exact relative position, but such objects do not exist. Facts never warrant the possession of a “sharp” relative position. However, there are objects, which I will call “macroscopic”, the relative positions of which are not manifestly indefinite. By a macroscopic object $M$ I mean an object that satisfies the following criterion: Every factually warranted inference to the position of $M$ (relative to any other macroscopic object $M$) at any specified time $t$ is predictable on the basis of factually warranted inferences to (i) the positions of $M$ (relative to $M$) at earlier times and (ii) the positions of other objects (relative to $M$) at $t$ or earlier times.

Let me say this again. Every position measurement that ever has been or will be performed on $M$ (that is, every matter of fact that has a bearing on the position of $M$ relative to another macroscopic object $M$) has a range of values \{$M \subset R_i\mid i = 1, \ldots, n\)$ between which the measurement can distinguish. If we take into account every position measurement performed on $M$ before a time $t$ and every position measurement performed on every other object before or at the time $t$, and if the result of every position measurement on $M$ made at the time $t$ is predictable on that basis via the pertinent

\[ \text{By saying that a factually warranted inference to the position of a macroscopic object is predictable,} \]
classical laws, and if this is the case for every time $t$ at which a position measurement is performed on $M$, then, and only then, $M$ is a macroscopic object. In this case nothing ever indicates a departure from what is predictable on the basis of the pertinent classical laws and earlier position-indicating facts. When I say that the positions of macroscopic objects are not manifestly indefinite, what I mean is that the indefiniteness of these positions is never evidenced by such a departure. Every matter of fact about $M$’s present position follows via the pertinent classical laws from matters of fact about $M$’s past positions and about the past and present positions of other objects.\footnote{I do not mean that the position-indicating fact is predictable, but that the position indicated by the fact is predictable. Note that several position measurements (with different ranges of values) can be performed on the same object at the same time.}

The above definition of a “macroscopic object” $M$ involves another macroscopic object $M$. This is as it should be since objects are macroscopic by virtue of their relative positions. We may introduce a “macroscopic (reference) frame” (previously called the “laboratory frame”) that is riveted to any macroscopic object, and in which the position of every other macroscopic object is not manifestly indefinite. To see that the choice of reference object is immaterial, let $\{C_i\}$, $i = 1, 2$, be two coordinate systems having for their respective origins the centers of mass of two macroscopic objects $M_i$. Even though the coordinate points of $\{C_2\}$ are somewhat fuzzy relative to $\{C_1\}$, the two sets of coordinate points are physically equivalent, not merely “for all practical purposes” but strictly, for there isn’t any actual, physical difference matching the conceptual difference between them. By definition, the relative position of a pair of macroscopic objects is not manifestly indefinite. Hence nothing ever happens or is the case that would make it possible to distinguish between the two frames.

It is one thing to define macroscopic objects but quite another to show that such objects exist. There can be an unpredictable matter of fact about the position of $O$ at a time $t$ only if there are detectors with sensitive regions that are smaller than the space over which $O$’s position is distributed. (By saying that $O$’s position is distributed over a set $\{R_i\}$ of mutually disjoint regions, I mean that the prior probabilities associated with the conditionals $Q \Rightarrow O(t) \subset R_i$ are positive.) But detectors with sufficiently small sensitive regions do not always exist. There is a finite limit to the definiteness of the relative positions of material objects, and there is a finite limit to the spatial resolution of actually existing detectors. Hence there must be objects whose positions are the sharpest in existence. The position of such an object cannot be manifestly indefinite, for want of detectors capable of probing the space over which it is distributed.\footnote{Since the formal expression of the indefiniteness of an object’s position refers to counterfactuals, evidence of positional indefiniteness cannot be direct. The most direct evidence we can have is the unpredictability of position-indicating facts. What is evidenced by the unpredictability of a position-indicating actual event $e$ is a counterfactual indefiniteness: the indefiniteness that would have obtained had $e$ not occurred, other things being equal.}

\footnote{Note that the definition of a “macroscopic object” does not stipulate that events indicating departures from the classically predicted positions occur with zero probability. An object is entitled to the label “macroscopic” if no such event actually occurs. What matters is not whether such an event may occur (with whatever probability) but whether it ever does occur. We cannot be certain that a given object qualifies as macroscopic, inasmuch as not all matters of fact about its whereabouts are accessible to us. But we can be certain that macroscopic objects exist, and that the most likely reason why $M$}
While the positions of things dangle from (are supervenient on) position-indicating facts, the positions of macroscopic objects do so in a way that evinces no indefiniteness. Although no object ever follows a definite trajectory, the positions of macroscopic objects—"macroscopic positions", for short—evolve in a completely predictable fashion. Applying to macroscopic positions the formalism of quantum mechanics is therefore completely uncalled-for. It is perfectly legitimate to treat these positions as forming a self-contained system of intrinsic positions that dangle causally from each other, rather than a system of extrinsic positions that dangle ontologically from position-indicating facts. While even a macroscopic object has a position only because of the facts from which this can be inferred, the dependence of this position on position-indicating facts is a qualitative (ontological or existential) dependence, not a quantitative one. For the quantitative purposes of physics, it is legitimate to ignore this dependence, to consider macroscopic positions "in themselves" (out of relation to position-indicating facts), to treat them as facts (rather than as inferences from facts), and to apply to them classical causal concepts. It ought to be borne in mind, however, that causal concepts are emergent: Causality isn't part of the ontological foundation. As we saw earlier in this section, all the determining that goes on in the physical world is the determining of probabilities associated with possibilities. There aren't any causally determined facts. Causality, like color, lies in the mind of the beholder.

8 Interpreting the Copenhagen Interpretation

Much of what has been said in the last two sections hinges on the following questions: What constitutes a (matter of) fact? What is an actual event or state of affairs? We are now in a position to answer these questions. As we have just seen, it is legitimate to ignore the extrinsic nature of macroscopic positions, to treat them as facts. The relevant facts, actual events, or actual states of affairs either are macroscopic positions or are definable in terms of such positions. It is irrelevant that the property-indicating position of a macroscopic pointer needle dangles ontologically from facts that involve other macroscopic positions. Where the indicated property is concerned, the indicating position can be thought of as intrinsic, and hence as a position-indicating fact.

Thus there exists a "classical domain" of (macroscopic) positions that are not manifestly indefinite, and that can be thought of as intrinsic or as being factual per se. is macroscopic is the nonexistence of detectors with sensitive regions that are smaller than the space over which M’s position is distributed. (M could also be macroscopic for the unlikely reason that such detectors, though they exist, never indicate a departure from M’s classically predicted position.)

22"The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm." – Bertrand Russell.

23An apparatus pointer is not strictly a macroscopic object according to the given definition. In a typical measurement, nothing allows us to predict the pointer’s final position. However, before and after the measurement the pointer behaves as a macroscopic object, whose position can be considered independent of other position-indicating facts. Therefore the transition from the initial to the final pointer position can also be considered independent of other position-indicating facts, and thus as an actual event.
This is fortunate, for otherwise quantum mechanics would be inconsistent, as its very formulation presupposes property- and time-indicating facts. Until recently the CIQM has been the only interpretation of quantum mechanics that acknowledges the logical dependence of quantum mechanics on a classical domain, and therefore, in my opinion, it has been the only interpretation worth considering. The CIQM has been censured for being “vague, obscure, and maybe even inconsistent” by Loewer [35], but these strictures do not touch the core of the CIQM, which consists in the rejection of fundamental microscopic space-time realities and the substitution for them of property-indicating facts amenable to classical description. They address extraneous attempts to reintroduce unwarranted classical ideas, such as the notion of a quantum state that evolves in an intrinsically differentiated time.

According to Loewer, the vagueness of the conceptual muddle he passes off as the “Copenhagen interpretation” lies in the absence of (i) a clear distinction between classical and quantum systems and (ii) a clear definition of “measurement”. Its obscurity consists in its “spooky” nonlocality, in the “relationship between measurement and determinate reality”, and in our (or at least, Loewer’s) inability to understand how the position of a particle can lack a determinate value. Its possible inconsistency is that (according to Loewer) “it makes assertions about the nature of quantum-mechanical reality” and at the same time “denies that anything can be known about that reality”.

As I said, none of these strictures touch the core of the CIQM. In my forthcoming [41] I have expanded this core into a complete ontological interpretation, dubbed the “Pondicherry interpretation of quantum mechanics” (PIQM). The PIQM makes a clear distinction between the classical and quantum domains. The classical domain contains all those possessed properties that are not manifestly indefinite (including those that warrant inferences to possessed properties in the quantum domain). The quantum domain contains all other factually warranted properties. The term “measurement” is likewise clearly defined. A measurement is not something that causes the “collapse” of an evolving quantum state. Every property-indicating fact (that is, every event or state of affairs in the classical domain warranting an inference to some possessed property) qualifies as a “measurement result”. The relationship between measurement and determinate reality is equally clear. Determinate reality is the totality of factually warranted properties, while measurements provide the property-indicating facts.

As to the alleged nonlocality of the CIQM, it depends. If this is supposed to mean that “a measurement of a part of a system at one location can instantaneously change the physical situation of far distant parts of that system” [35], then it does not exist. This kind of action at a distance is one of the absurdities that follow from the spurious notion of an evolving quantum state. According to both the PIQM and the solid core of the CIQM, all that the quantum formalism tells us about is objective correlations between possibilities and statistical correlations between factually warranted properties or property-indicating facts – diachronic correlations between the factually warranted properties of the same system at different times and synchronic correlations between the factually warranted properties of different systems in spacelike separation. It says nothing whatsoever about causal connections between the correlated properties or facts. In particular, it says nothing about changes in physical situations that are caused by
measurements. On the other hand, the very fact that quantum mechanics is inconsistent with local action, as was shown in Sec. 5, implies its nonlocality. Nonlocality is a characteristic of the observed correlations (the diachronic as well as the synchronic ones) and thus independent of any particular explanation of the correlations (such as an instantaneous action at a distance).

As to the “spookiness” of the nonlocality evinced by the correlations, it is a subject for psychology or neurophysiology rather than physics, and so is Loewer’s inability to understand how the position of a particle can lack a determinate value. These nonphysical issues deserve a separate section (Sec. 10). That a sense of the miraculous accompanies all fundamental explanations is to be expected, however. Explanations begin with the fundamental behavior of matter, which can be described, but which cannot itself be explained. (If it could, it wouldn’t be fundamental.) Diachronic correlations that are not manifestly indeterministic can be passed off as causal explanations. But when we deal with synchronous correlations or diachronic correlations that are manifestly indeterministic, causal concepts are out of place. Trying to causally explain these correlations is putting the cart in front of the horse. It is the correlations that explain why causal explanations work to the extent they do. They work in the classical domain where we are dealing with macroscopic objects, and where the correlations between property-indicating facts evince no statistical variations (dispersion). If we go beyond this domain, it becomes clear that even where no statistical variations are in evidence, the correlations between facts are statistical rather than causal.

Loewer’s claim that the CIQM both “makes assertions about the nature of quantum-mechanical reality” and “denies that anything can be known about that reality” is based on the following claims, which he attributes to the CIQM: “The right way to understand quantum mechanics is not as a true description of physical reality but rather as an instrument for predicting the outcomes of laboratory experiments. There is no coherent interpretation of the quantum-mechanical formalism as describing an unobservable reality that is responsible for those experimental results. That reality is forever beyond our ken.” The PIQM denies not merely the possibility of such an interpretation but the very existence of an unobservable reality that is responsible for the experimental results. There is no reality beyond our ken. Quantum mechanics is an instrument for assigning conditional probabilities to possible property-indicating facts on the basis of actual property-indicating facts. Under certain conditions, specified above, these conditional probabilities are objective and indicative of an objective indefiniteness. This has implications concerning the actual spatiotemporal differentiation of the physical world. And all this is part of the true and complete description of physical reality that quantum mechanics affords. If there is anything that is incomplete, it is the physical world, but its incompleteness exists only in relation to a conceptual framework that is more detailed than the physical world, as I proceed to show.

24Saying that quantum mechanics affords a true and complete description of physical reality is obviously very different from claiming that the state vector is such a description.
9 The Spatiotemporal Differentiation of the Physical World

Because there exists a finite limit to the spatial resolution of actually existing detectors, any finite region $R$ contains at most a finite number of regions $R_k$ the distinctness of which is physically realized. Accordingly, for any material object $O$ located within $R$ at most a finite number of alternative positions ("inside $R_k$") are available as possible attributes. Hence there exists in our minds (that is, we can conceive of) a finite partition \{\{R_i\}\} of the macroscopic frame that exists only in our minds. The elements of \{\{R_i\}\} are so small that there aren’t any detectors capable of realizing their conceptual distinctness. Nothing in the physical world corresponds to the distinctions we make between those regions.

Suppose that \{\{R_i\}\} is a partition of $R$ at the limit of resolution achieved by actually existing detectors. How should we visualize this partition? Certainly not as a set of sharply bounded regions! Sharp boundaries imply exact positions, and exact positions imply the existence of detectors with infinitesimal sensitive regions, in contradiction to the finite actual differentiation of $R$. Since no detector ever has a sharply bounded sensitive region, no object ever possesses an exact position, and since no object ever possesses an exact position, no detector ever has a sharply bounded sensitive region. Even the boundaries of macroscopic detectors are fuzzy. But they are fuzzy only in relation to a finer partition of $R$, and this exists solely in our minds. Thus not only the mental picture of sharply bounded regions $R_i$ but also the mental picture of fuzzily bounded regions $R_i$ is more detailed than the finitely differentiated reality it is supposed to represent. The boundaries of these regions are neither sharp nor fuzzy! The notion that “sharp” and “fuzzy” are jointly exhaustive terms originates in an inadequate theoretical representation of the world’s actual spatial differentiation. It involves a conception that is inconsistent with quantum mechanics – the conception of an intrinsically and infinitely differentiated space.

While no object ever has a definite position, the positions of macroscopic objects are not manifestly indefinite. How real is the indefiniteness of a position that is not manifestly indefinite? The answer is, not real at all. In a world that is spatially differentiated only to the extent that spatial relations and distinctions can be inferred from facts, no object has a sharp position. But there are objects that have the sharpest positions in existence, and the position of such an object is not fuzzy in any actual sense. For its fuzziness to be actual in some sense, the position has to be distributed, either statistically or counterfactualy, over the sensitive regions of actually existing detectors. But if it were distributed over such regions, it would not be among the sharpest positions in existence – the positions of the detectors would be sharper. Thus the positions of macroscopic objects are fuzzy only in relation to an unrealized degree of spatial differentiation. The indefiniteness of a macroscopic position exists only in relation to a theoretical framework that is more detailed than the physical world.

As positions have no physical reality unless they are attributable to material objects, so times have no physical reality unless they are attributable to possessed properties, as the times at which these are possessed. The reason this is so is that time, like
space, isn’t something that is intrinsically differentiated. It takes time-indicating facts to differentiate the world timewise. But times are indicated by factually warranted relative positions. Since these are more or less indefinite, the indicated times, too, are more or less indefinite. For instance, if there isn’t any matter of fact about the time $T_s$ at which an electron passes through the slit plate, $T_s$ lacks a definite value with respect to any partition of the interval between the time of emission by the electron gun and the time of detection behind the slit plate.\footnote{The use of a capital $T$ instead of either a lower-case $t$ or the operator symbol $\hat{T}$ is intended as a reminder that time is not a proper quantum-mechanical observable (a Hermitian operator on a Hilbert space), and that $T_s$, consequently, is not one of the possible values of such an observable. In general, neither a self-adjoint time operator conjugate to the Hamiltonian \cite{51, 52} nor a time-of-arrival operator \cite{53, 54, 55} exists. This is a consequence of the fact that quantum-mechanical probability distributions are distributions over the possible results of possible measurements that are actually or counterfactually performed at specified times. There are no quantum-mechanical probability distributions over time intervals.} However, there are macroscopic times that are not manifestly indefinite, just as there are macroscopic positions the indefiniteness of which is never evidenced by departures from what is predictable on the basis of classical laws and earlier property-indicating facts.

Let us extend our definition of the “macroscopic frame” to include the not manifestly indefinite times indicated by macroscopic positions. It follows from what has just been said that there exists in our minds (that is, we can conceive of) a partition of the macroscopic frame into finite intervals of time that are so small that nothing in the physical world corresponds to the distinctions we make between these intervals. Once again the distinctions exist solely in our minds. The physical world is temporally differentiated only to the extent that temporal relations and distinctions can be inferred from facts, and the facts warrant neither the inference of a definite temporal relation nor the partition of any finite interval of physical time into infinitely many time spans. As there is a finite limit to the spatial resolution of actually existing detectors, so there is a finite limit to the temporal resolution of actually existing clocks\footnote{There is another reason why during a finite time span $\tau$ a material object can possess at most a finite number of distinct time-indicating properties. Let $Q_C$ be an observable whose eigenkets represent the distinct time-indicating properties of a clock $C$. If $Q_C$ were measured an infinite number of times during $\tau$, $C$ would not function as a clock, for the result would always be the same \cite{56, 57, 58}.}. No property is ever possessed at a definite time. But there are clocks, which we may call “macroscopic clocks”, that indicate the sharpest times in existence, which we may call “macroscopic times”. Macroscopic times are not fuzzy in any actual sense; they are fuzzy only in relation to an unrealized degree of temporal differentiation. Like the indefiniteness of a macroscopic position, the indefiniteness of a macroscopic time exists only in relation to a theoretical framework that is more detailed than the physical world.

The seemingly intractable problem of understanding quantum mechanics is a consequence of our dogged insistence on obtruding onto the physical world a theoretical framework that is more detailed than the physical world. We have this inveterate tendency of building reality “from the bottom up”. Atomizing is the way we naturally think. As Wilson \cite{60, p. 50} put it, “[t]he descent to minutissima... is a driving impulse of Western natural science. It is a kind of instinct.” Not only do we atomize...
matter (which has its physical legitimacy) but we also atomize space and time (which is a mistake), and we tend to model the atomization of matter after the atomization of space (which is another mistake). That is, we tend to think of the parts of matter as being defined by the parts of space. In actual fact, the ultimate “parts” of matter are the fundamental particles, and these are not defined by the “parts” of space. Space isn’t something that has parts, so it cannot serve to define parts. The “parts” of matter are defined by the spatial relations that exist between them. This view, and only this, allows the spatial relations to possess indefinite values.

The fundamental particles exist in space only in the sense that (more or less indefinite) relative positions can be attributed to them. Considered in itself, out of relation to other material objects, a fundamental particle does not exist in space, for in itself it has neither a position nor a form. Physical space exists between the fundamental particles; they unfold it by means of their relative positions; it is spanned by them. Moreover, while multiplicity is attributable to the fundamental particles qua spatial relata, it is not attributable to the fundamental particles considered out of relation to each other. Reality, therefore, is built “top-down”: By entering into a multitude of spatial relations with itself, \( \mathcal{E} \) takes on not only the aspect of a spatially differentiated world but also the aspect of a multiplicity of fundamental particles. And by allowing the relations to change, to possess different values in succession, it takes on the further aspect of a temporally differentiated world.

There are limits to the resulting differentiations. While the sharpest relative positions and times are indefinite only relative to an unrealized degree of differentiation, the more fuzzy ones are indefinite relative to the realized degree of differentiation (that is, they are manifestly indefinite). But manifestly indefinite positions and times cannot be thought of as intrinsic. There is therefore another sense in which reality is built “from the top down”. The positions of things are defined in terms of the not manifestly indefinite relative positions of macroscopic objects, which can be thought of as being factual by themselves. The times at which properties are possessed are defined in terms of the not manifestly indefinite times indicated by macroscopic positions, which also can be thought of as being factual by themselves. The positions of the microscopic parts and the times at which they are possessed thus dangle from (are supervenient on) the property- and time-indicating positions of macroscopic wholes.

10 Stuff and Nonsense

What is it that prevents us from coming to terms with quantum mechanics without extraneous additions like hidden variables [22] or spontaneous collapses [14, 52, 53], without using “world” in the plural [54], and without implicitly or explicitly distinguishing between a mind-constructed “internal” or “empirical” reality and a mind-independent “external” or “veiled” reality [55, p. 113] [56] or dragging in consciousness or knowledge in other ways [21, 23, 57, 58, 59, 71, 72]? It is the idée fixe that the world’s synchronic multiplicity is founded on the introduction of surfaces that carve up space in the manner of three-dimensional cookie cutters. I call it the “cookie cutter paradigm”
If the physical reality of space-dividing surfaces is accepted, it needs to be accounted for, and there are at least three ways of doing so. The first is to understand those surfaces as boundaries between “full” and “empty” space (that is, as closed surfaces encompassing some kind of stuff). This is the way of the Greek atomists, who taught that atoms are filled with being while the empty space around them lacks being. The second – the most literal version of the CCP – is due to Plato. Plato’s Forms have an immaterial existence of their own, independently of their instantiation in the physical world. Insofar as they bear spatial connotations, they connote closed surfaces. Divisions in material space exist to the extent that Forms with spatial connotations are present (instantiated) in the physical world.

The third way of accounting for the existence of spatial divisions is to attribute them to space itself. On this account, all conceivable divisions of space are physically real and intrinsic to space. Reduced to one dimension this leads to the view that a set of points in one-to-one correspondence with the real numbers is intrinsic to a continuous line. This raises the issue of whether these points “make up” or “fill” the line or are separated by infinitesimal intervals. It is customary to equate real numbers whose decimal expansions converge, e.g., $0.49 = 0.5$. If we follow this practice – not everyone does [73, p. 263] – then there aren’t any infinitesimal “gaps” between the points on a line. But if we interpret real numbers as points on a continuous line, or use them to label such points, then it is legitimate to consult the visual image of a continuous line, and to demand that the properties we attribute to the real numbers be consistent with it. And arguably the practice of equating numbers with convergent decimal expansions is not consistent with the continuity of a line in phenomenal space. Be that as it may. What is clear is that the idea that all conceivable divisions of phenomenal space are intrinsic to physical space, leads naturally to the notion that physical space either contains or is identical with the set $\mathbb{R}^3$.

Physicists have long since discarded the Democritean notion that the basic material constituents are closed surfaces filled with continuous being. If they ever entertained the Platonic notion that space-dividing surfaces owe their physical reality to Forms that have a reality ante rem, they have long since discarded this notion as well. What remains to be discarded is (i) the notion that space-dividing surfaces are physically real, and (ii) the conception of space to which this notion leads if one rejects the Democritean

---

27 Consider a line segment $L$ with boundary points labeled 0 and 1, respectively. Next consider the numbers in the interval $I = (0, 1)$ that have a binary expansion of up to $n$ digits. These numbers have the general form 0.$[n]$, where $[n]$ is a string of $n$ digits (0’s or 1’s). The points corresponding to these numbers divide $L$ into $2^n$ segments of length $l_n = 2^{-n}$. In the limit $n \to \infty$ we obtain all real numbers in $I$. Let $s$ be any real number and let $(s_n)$ be a sequence of real numbers such that $s - s_n = 2^{-n}$. If $\lim_{n \to \infty} s_n = s$, so that in particular (in binary notation) $0.01 = 0.1$, the assumption that there are infinitesimal line segments between the points on $L$ corresponding to the real numbers in $I$ leads to a contradiction, for the upper and lower boundaries of those segments are defined by the same number.

28 Let $\{n\}$ denote a string of $n$ 1’s. (We again use binary notation.) If we visualize the line segment $L_n$ corresponding to the interval $(0.0\{n\}, 0.1)$, and if we visualize the right end of $L_n$ enlarged by a factor $2^m$ with every increase of $n$ by $m$, then the segment $L_{n+m}$ corresponding $(0.0\{n+m\}, 0.1)$ looks exactly the same as $L_n$. And this ought to be equally true of the segment $L_\infty$ corresponding to $(0.01, 0.1)$. 37
and Platonic accounts. Sharp space-dividing surfaces exist solely in our minds. They do not exist as the forms of material objects, for the forms of material objects are sets of more or less indefinite relative positions between formless entities, rather than bounding surfaces (Sec. 3). Nor are space-dividing surfaces intrinsic to physical space, as the behavior of electrons in two-slit interference experiments amply demonstrates (Sec. 5).

To bring home just how insidiously the CCP prevents us from making sense of quantum mechanics, let us examine some of its implications. To begin with, the idea that synchronic multiplicity depends on a partition of space into mutually disjoint regions implies the prior existence of a spatial expanse that gets partitioned or contains the partitions. The CCP thus prejudices us in favor of substantivalism, a doctrine that we found to be inconsistent with quantum mechanics. In conjunction with the CCP, substantivalism implies that not only space but each part of space is a separate constituent of the world. If the parts of matter exist by virtue of the parts of space, spatial divisions cannot arise from (processes involving) matter; they must be inherent in a preexistent space.

What transpires next depends on whether the world is or is not infinitely divided spacewise. If the division of space ends with the creation of finite bounded regions, as in the respective theories of Democritus and Plato, it seems inevitable that we follow these philosophers in attributing the existence of bounded regions to the existence of material objects, and thus conceive of existing boundaries as forms of material objects. But then the following question arises: Why can’t it happen that different material objects overlap? The answer to this question is, because the CCP defines synchronic multiplicity in terms of geometrical divisions. Suppose that there exist two bounded regions $A$ and $B$ having a finite intersection $C = A \cap B$. Then it is not the case that there exist two material objects whose respective forms are the boundaries of $A$ and $B$. Instead there exist three such objects whose respective forms are the boundaries of $A - B$, $B - A$, and $C$. The object occupying $C$ is one object (which may be a part of the object occupying $A$, or a part of the object occupying $B$, or a part of the object occupying $A \cup B$), rather than two objects (a part of the object occupying $A$ and a part of the object occupying $B$). Thus it is logically impossible for two objects to overlap (that is, for a part of one object to occupy the same region of space as a part of another object).

Material objects, however, move. This is something that bounded regions of space per se, considered out of relation to time, cannot do, and this raises a further question. We know that two material objects cannot “overlap”. If it seemed as if they did, their apparent intersection would contain a part of either object rather than a part of each object. If two identical objects came to occupy the same space, they would cease to be two objects. But this does not explain why we never see a part of one object become numerically identical with a part of another object, or two identical objects merge into one object. The obvious “explanation” of this is that material objects are not only bounded by surfaces but also “filled to capacity” with some continuous stuff.

Physics offers a different account of the apparent impenetrability of material objects: a repulsive force. The physical reason why two material objects $M$ and $N$ lacking common parts cannot come to occupy the same space is that their respective parts repel
each other. This explanation involves the spatial relations, or the distances, between the parts of $M$ and the parts of $N$, as well as a force opposing attempts to reduce those distances. It further involves the parts of $M$ and the parts of $N$, but only as the relata of those spatial relations. It does not involve them as bounded regions filled with stuff. It involves neither the forms of the parts nor any spatially extended stuff. Where physics is concerned, space-filling stuff and forms *qua* bounding surfaces are explanatorily irrelevant. They are artefacts of the CCP and the assumption that the division of space ends with finite bounded regions. In reality, as described by quantum mechanics, there are no bounding surfaces. Forms are made of spatial relations between formless parts. And if there is stuff, it consists in nothing but the spatial relata the existence of which is implied by the spatial relations.

If the CCP is combined instead with the assumption that the division of space never ends, or ends with infinitesimal regions or with a set of points cardinally equal to the reals, it leads us up a different garden path. In this case all points or infinitesimal regions are separate constituents of the world, and all physical properties are locally instantiated – they are properties of those points or infinitesimal regions. The form of an ordinary material object then consists of spatial relations between locally instantiated physical properties. If we take into account that a generic material object is composed of a finite number of noncomposite entities, we are led to conclude that the form of such an object is made up of the spatial relations between physical properties that are instantiated at a finite number of points or infinitesimal regions. It stands to reason that these locally instantiated physical properties are the characteristics of a particle species: mass, spin, and charges. Note that a pointlike form is not contained in this list of properties. Saying that a fundamental particle is pointlike is the same as saying that its properties (not including a form) are instantiated at a point or an infinitesimal region of space.

It is clear that the spatial relations between these locally instantiated properties cannot be indefinite. The CCP thus makes it impossible to understand how the position of a particle can lack a determinate value. If the synchronic multiplicity of the world conformed to the CCP, space would be intrinsically and infinitely divided, a noncomposite object could not but exist at a definite point of space or inside a definite infinitesimal region, and the distances between such objects would necessarily be sharp.\(^{29}\) If, as Albert \(^{23}\) has claimed, the behavior of electrons in two-slit experiments is “quite unlike what we know how to think about”,\(^{30}\) it is because we labor under the delusion of the CCP. If “nobody knows how it can be like that”\(^{30}\) it is because everybody is deluded by the CCP. If we could accept that the synchronic multiplicity of the world is based instead on spatial relations, we would have no reason to suppose that spatial relations must have determinate values. On the contrary, taking into account that a

\(^{29}\)Recall that the coordinates presupposed by quantum mechanics have a direct metric significance. The “points of space” being Cartesian coordinate points, their distances are determinately related to the differences between their coordinates.

\(^{30}\)“I think it is safe to say that no one understands quantum mechanics. . . . Do not keep saying to yourself, if you can possibly avoid it, ‘But how can it be like that?’ because you will go ‘down the drain’ into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that” – R.P. Feynman \(^{74}\) p. 129.
finite number of constituents is sufficient for the existence of any known material object, we have reason to suppose that the relative positions of these constituents cannot have determinate values. In fact, we know that what “fluffs out” matter is not a repulsive force but the indefiniteness of the relative positions of its constituents (in conjunction with the Pauli exclusion principle and the fact that those constituents are fermions). If it were not for this indefiniteness – other things being equal – none of the familiar objects around us would exist.

The CCP can lead to worse. This happens when the quantum state – by definition a system of probability distributions – is construed not only as an instantaneous state of affairs \( \psi(t) \) that evolves in an infinitely differentiated time but also as a local state of affairs \( \psi(r) \) that assigns physical properties to the point \( r \). This is done, for instance, by Albert [23, p. 126], who sets out with the following bundle of assumptions: “Suppose that there’s just one world. And suppose that there’s just one complete story of the world that’s true. And suppose that quantum-mechanical state vectors are complete descriptions of physical systems. And suppose that the dynamical equations of motion are always [and, presumably, everywhere] exactly right.” The first assumption excludes the many-worlds interpretation [64]; the second rules out the consistent-histories philosophy [24, 70, 77]; the third eliminates Bohmian mechanics [22] and spontaneous collapse theories [61, 62, 63]. The four assumptions together then lead to the many-minds interpretation [23, 79], or so it is suggested by its proponents. Common to all of these attempts to make sense of quantum mechanics is the idea that the dynamical equations – in the simplest case, the Schrödinger equation; in the case of spontaneous collapse theories, a modified, stochastic equation – are always and everywhere exactly obeyed, where “always” and “everywhere” stand for “at every instant of time” and “at every point of space”, respectively. Since the physical world is not infinitely differentiated spacewise or timewise, these usual acceptations of “always” and “everywhere” have no meaningful application to the physical world. For this reason alone all of the above attempts to make sense of quantum mechanics are fatally flawed.

The CCP has neurophysiological underpinnings. We are adept at recognizing three-dimensional objects in drawings that contain only outlines. (In fact, we can’t help but perceive three-dimensional objects. We always see a Necker cube as pointing either in or out.) Why is it so easy to recognize an outline as an object? The answer is, because of the way the brain processes visual information. The seminal work of Hubel and Wiesel [70] supports the following account. Visual information flows from retinal receptor cells via retinal ganglion cells to either of two lateral geniculate nuclei, and on to the primary visual cortex. The receptive field of each retinal ganglion or geniculate cell is divided into either an excitatory center and a concentric inhibitory surround (the “on center” configuration) or the reverse configuration (“off center”). (The group of retinal receptor cells from which a retinal ganglion or geniculate cell receives input is known as the cell’s receptive field.) Thus an “on center” cell responds best to a circular spot of light of a specific size, responds well to a bright line that just covers the center (since then most of the surround is not covered by the line), and does not respond at all if both center and surround are fully and equally illuminated.

When visual information reaches the visual cortex, two major transformations take
place. One leads to the fusion of input from both eyes, the other to a rearrangement of incoming information so that most of its cells respond to specifically oriented line segments. The optimal stimulus may be a bright line on a dark background or the reverse, or it may be a boundary between light and dark regions. One group of orientation-specific neurons responds best to lines with just the right tilt in a particular part of the visual field. Another group of neurons, receiving input from the first group, is less particular about the position of the line and responds best if the line is swept in a particular direction across the visual field.

These data indicate that the visual representation of a physical environment arises by way of an analysis of the visual field that is based on contrast information from boundaries between homogeneously lit regions. No sense data arrive from regions that are homogeneously colored and evenly lit. The interior of such a region is filled in on the basis of contrast information stemming from its boundary. This explains why outline drawings are readily recognized as objects: The brain adds surfaces to outlines in the same way as it adds (unperceived) colored surfaces to (perceived) changes in color and brightness across edges. It also explains why the blind spot is not perceived if it falls inside a homogeneous region (no sense data arrive from such a region anyway), and why color perception is so remarkably faithful to the reflectances of colored surfaces, and correspondingly insensitive to the spectral composition of the actual radiances of such surfaces. And most importantly, the manner in which visual information is analyzed by the visual cortex explains why the synchronic multiplicity of the phenomenal world conforms to the CCP: Unlike the physical world, the phenomenal world is constructed from boundaries, and these are filled in with qualia. A trivial consequence of this is that two objects in the phenomenal world, unlike two objects in the physical world, cannot be at the same place: The existence of two objects in phenomenal space implies the existence of at least one separating boundary.

The phenomenal world differs from the physical world not only in that it is spatially differentiated in conformity with the CCP, but also in that it is intrinsically differentiated. The visual field is differentiated not only extrinsically by differences in perceived content but also intrinsically by the retinal receptor cells – it is inherently grainy. That we are unaware of this graininess is another consequence of how the brain processes visual information. Recall that no sense data arrive from uniformly colored regions of the visual field. Such regions are filled in, and are filled in smoothly or homogeneously. The inherent graininess of boundaries is similarly glossed over. Cells that respond to specifically oriented line segments in a particular part of the visual field receive input from cells that have circular receptive fields with centers lying along a straight line. The information coming from the latter type of cell is grainy; the line segment that is perceived when the former type of cell is stimulated, is not. The phenomenal world thus is intrinsically a world of sharp and continuous boundaries filled with homogeneous con-

\[\text{31}\] If color perception is based on discontinuous color changes across edges, continuous variations in illumination across the visual field go unperceived.

\[\text{32}\] Since this involves the transition from objective brain mechanisms to subjective visual percepts, just how the filling in is accomplished is presently as impenetrable as the question of how anything material can be conscious in the first place.
tent, while the physical world is intrinsically a world of fuzzy spatial relations between relata that, but for their relations, are numerically identical. There could hardly be a greater difference between the two.

Many thinkers have been intrigued, and justifiably so, by the ability of the human mind to reproduce the physical world as faithfully as it does, or seems to do, using nothing but logic and mathematics. The success of physics is indeed astonishing, but the difficulties we face in understanding quantum mechanics reveal that we are not all that well-equipped mentally. As McGinn has stressed, “[w]e are, cognitively speaking as well as physically, spatial beings par excellence: our entire conceptual scheme is shot through with spatial notions, these providing the skeleton of our thought in general.” The trouble is that our neurophysiological make-up conditions these spatial notions to conform to the CCP. Recall from note 1 that the neural processes which produce visual percepts and those which produce visual images are to some extent the same. As a consequence, every thought about the physical world that involves visual imagery is invariably deceived by some of the neural processes on which it depends.

To conclude, in order to make sense of quantum mechanics, we must detach our spatial notions from visual imagery. We must learn to conceive of formless entities. We must disregard the intrinsic multiplicity of phenomenal space. We must think of spatial relations as ontologically prior both to forms and to the multiplicity of the corresponding relata. (Without spatial relations, all there is is existence itself. This takes on the appearance of a world of forms when it enters into spatial relations with itself.) And we must counter the inherent definiteness of visual percepts and images by resorting to probabilistic concepts and contrary-to-fact conditionals. Last but not least, we must desist from conceiving individuation along lines laid down by the CCP.

According to Strawson, the logical distinction between particular and universal, and hence between subject and predicate, is founded on spatial distinctness. “We regard $x$ and $y$ as distinct particular instances of the same universal $P$ just in so far as we acknowledge that $x$ and $y$ are at distinct places” [original emphasis]. Quantum mechanics tells us otherwise. Being at distinct places is indeed sufficient for being distinct instances of existence itself, but it is not necessary. What is necessary for being distinct instances is the existence of a spatial relation. This can be such that, relative to the laboratory frame, the two instances are not at distinct places. Nor can existence itself be thought of as a universal (Sec. 4). $E$ is not the most general predicate but the ultimate subject. Material objects owe their existence to the existence of spatial relations, and these owe their existence not to a predicatable universal but to the one existence that they manifoldly relate.

References

---

33 “Difficult though it be to imagine physics either completable or incompletable, it is perhaps even more difficult to imagine that physics should be possible at all” – von Weizsäcker [p. 174]. “The most incomprehensible thing about the world is that it is comprehensible” – Einstein [p. 112].
[1] Abraham Pais, ‘Subtle is the Lord...’: The Science and the Life of Albert Einstein (Oxford: Clarendon Press, 1982).

[2] R.A. Finke, “The functional equivalence in imagery and perception”, Psychol. Rev. 87, No. 2, 113–132, 1980.

[3] R.A. Finke and R.N. Shepard, “Visual functions and mental imagery”, in Handbook of Perception and Human Performance II (New York: John Wiley & Sons, 1986), edited by K.R. Boff, L. Kaufmann, and J.P. Thomas, pp. 37–1 to 37–55.

[4] R.N. Shepard and L.A. Cooper, Mental Images And Their Transformations (Cambridge, MA: MIT Press, 1982).

[5] Georg Cantor, in Gesammelte Abhandlungen (Berlin: Springer, 1932), edited by Abraham Fraenkel and Ernst Zermelo.

[6] Edwin H. Land, “The Retinex theory of color vision”, Sci. Am. 237, No. 6, 108–128, December 1977.

[7] Herrman Weyl, Raum Zeit Materie, 6th edition (Berlin: Springer, 1970); translation: H.L. Brose, Space-Time-Matter (London: Methuen, 1922).

[8] Frank Jackson, “What Mary didn’t know”, J. Phil. 83, 291–295, 1986.

[9] A. Grünbaum, Philosophical Problems of Space and Time (Dordrecht: Reidel, 1973).

[10] Henry Poincaré, Science and Hypothesis (New York: Dover, 1952).

[11] J. Anandan, “On the hypotheses underlying physical geometry”, Found. Phys. 10, 601–629, 1980.

[12] P.R. Silva, “A new interpretation of the de Broglie frequency?”, Phys. Essays 10, 628–632, 1997.

[13] Immanuel Kant, Critique of Pure Reason, first (German) edition, 1781.

[14] John R. Klauder, “Understanding quantization”, Found. Phys. 27, 1467–1483, 1997.

[15] John R. Klauder, “Is quantization geometry?”, Comm. Math. Theo. Phys. 1, 50–64, 1998.

[16] John R. Klauder, “Metrical quantization”, in Quantum Future (Berlin: Springer, 1999), edited by P. Blanchard and A. Jadczyk, pp. 129–138.

[17] P.A.M. Dirac, The Principles of Quantum Mechanics (Oxford: Oxford University Press, 1947).
[18] Robert Audi, *The Cambridge Dictionary of Philosophy* (Cambridge: Cambridge University Press, 1995).

[19] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (San Francisco: W.H. Freeman and Company, 1973).

[20] Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *The Feynman Lectures in Physics*, Vol. 3 (Reading, MA: Addison-Wesley, 1965).

[21] Michael Lockwood, *Mind, Brain and the Quantum* (Oxford: Basil Blackwell, 1989).

[22] David Bohm, “A suggested interpretation of quantum theory in terms of hidden variables”, *Phys. Rev.* **85**, 166–193, 1952.

[23] David Z. Albert, *Quantum Mechanics and Experience* (Cambridge, MA: Harvard University Press, 1992).

[24] David K. Lewis, *Philosophical Papers*, Vol. II (New York: Oxford University Press, 1986).

[25] Albert Einstein, “Quantum mechanics and reality”, *Dialectica* **2**, 320–324, 1948; translation in: D. Howard, “Holism, separability and the metaphysical implications of the Bell experiments”, in *Philosophical Consequences of Quantum Theory: Reflections on Bell’s theorem* (Notre Dame, Indiana: University of Notre Dame Press, 1989), edited by J. Cushin and E. McMullin, pp. 224–253.

[26] A. Cassinello and J.L. Sánchez-Gómez, “On the probabilistic postulate of quantum mechanics”, *Found. Phys.* **26**, 1357–1374, 1996.

[27] J.M. Jauch, *Foundations of Quantum Mechanics* (Reading, MA: Addison-Wesley, 1968).

[28] A.M. Gleason, “Measures on the closed subspaces of a Hilbert space”, *J. of Rat. Mech. and Analysis* **6**, 885–894, 1957.

[29] N.G. van Kampen, “Ten theorems about quantum-mechanical measurements”, *Physica* **A153**, 97–113, 1988.

[30] Niels Bohr, *Atomic Theory and the Description of Nature* (Cambridge: Cambridge University Press, 1934).

[31] Niels Bohr, *Atomic Physics and Human Knowledge* (New York: Wiley, 1958).

[32] Michael Redhead, *Incompleteness, Nonlocality and Realism* (Oxford: Clarendon, 1987).

[33] Henry Pierce Stapp, “The Copenhagen interpretation”, *Am. J. Phys.* **40**, 1098–1116, 1972.
[34] N. David Mermin, “What’s wrong with this sustaining myth?”, *Phys. Today* **49**, 11–13, March 1996.

[35] Barry Loewer, “Copenhagen versus Bohmian interpretations of quantum theory”, *Brit. J. Phil. Sci.* **49**, 317–328, 1998.

[36] Asher Peres, “What is a state vector?”, *Am. J. Phys.* **52**, 644–650, 1984.

[37] Yakir Aharonov, Peter G. Bergmann, and Joel L. Lebowitz, “Time symmetry in the quantum process of measurement”, *Phys. Rev.* **134B**, 1410–1416, 1964; reprinted in *Quantum Theory and Measurement* (Princeton, NJ: Princeton University Press, 1983), edited by John Archibald Wheeler and Wojciech Hubert Zurek, pp. 680–686.

[38] Yakir Aharonov and Lev Vaidman, “Complete description of a quantum system at a given time”, *J. Phys.* **A24**, 2315–2328, 1991.

[39] B. Reznik and Y. Aharonov, “Time symmetric formulation of quantum mechanics”, *Phys. Rev.* **A52**, 2538–2550, 1995.

[40] Lev Vaidman, “Time-symmetrized quantum theory”, *Fortschr. Phys.* **46**, 729–739, 1998.

[41] Ulrich Mohrhoff, “What quantum mechanics is trying to tell us”, forthcoming in *Am. J. Phys.*; available online as “The Pondicherry interpretation of quantum mechanics”, Eprint [quant-ph/9903051](http://arxiv.org/abs/quant-ph/9903051).

[42] R.E. Kastner, “Time-symmetrized quantum theory, counterfactuals, and ‘advanced action’”, *Stud. Hist. Phil. Mod. Phys.* **30**, 237–259, 1999.

[43] R.E. Kastner, “The three-box ‘paradox’ and other reasons to reject the counterfactual usage of the ABL rule”, *Found. Phys.* **29**, 851–863, 1999.

[44] Lev Vaidman, “Defending time-symmetrised quantum counterfactuals”, *Stud. Hist. Phil. Mod. Phys.* **30**, 373–397, 1999.

[45] Lev Vaidman, “Time-symmetrized counterfactuals in quantum theory”, *Found. Phys.* **29**, 755–765, 1999.

[46] Lev Vaidman, “The meaning of elements of reality and quantum counterfactuals: Reply to Kastner”, *Found. Phys.* **29**, 865–876, 1999.

[47] R.E. Kastner, “Comment on Mohrhoff’s ‘What quantum mechanics is trying to tell us’”, Eprint [quant-ph/0003098](http://arxiv.org/abs/quant-ph/0003098).

[48] O. Cohen, “Pre- and postselected quantum systems, counterfactual measurements, and consistent histories”, *Phys. Rev.* **A51**, 4373–4380, 1995.

[49] Peter Menzies and Huw Price, “Causation as a secondary quality”, *Brit. J. Phil. Sci.* **44**, 187–203, 1993.
[50] Bertrand Russell, “On the notion of cause”, *Proc. Aristotelean Soc.* **13**, 1–26, 1913.

[51] Wolfgang Pauli, *Encyclopaedia of Physics*, Vol. 5/1 (New York: Springer, 1958), edited by S. Flugge, p. 60.

[52] G.R. Allcock, “The time of arrival in quantum mechanics: I. Formal considerations”, *Ann. Phys. (NY)* **53**, 253–285, 1969.

[53] J. Oppenheim, B. Reznik, and W.G. Unruh, “Time as an Observable”, in *Proc. 10th Max Born Symposium, Wroclaw* (Berlin: Springer, 1998), edited by Ph. Blanchard and A. Jadczyk, pp. 204–219.

[54] J. Oppenheim, B. Reznik, and W.G. Unruh, “Minimum inaccuracy for traversal-time”, submitted to *Phys. Rev. A*, Eprint quant-ph/9801034.

[55] Y. Aharonov, J. Oppenheim, S. Popescu, B. Reznik, and W.G. Unruh, “Measurement of Time-of-Arrival in Quantum Mechanics”, *Phys. Rev.* **A57**, 4130–4139, 1998.

[56] B. Misra and E.C.G. Sudarshan, “The Zeno’s paradox in quantum theory”, *J. Math. Phys.* **18**, 756–763, 1977.

[57] C.B. Chiu and E.C.G. Sudarshan, “Time evolution of unstable states and a resolution of Zeno’s paradox”, *Phys. Rev.* **D16**, 520–529, 1977.

[58] Asher Peres, “Zeno paradox in quantum theory”, *Am. J. Phys.* **48**, 931–932, 1980.

[59] Asher Peres, “Measurement of time by quantum clocks”, *Am. J. Phys.* **48**, 552–557, 1980.

[60] Edward O. Wilson, *Consilience* (New York: Alfred A. Knopf, 1998).

[61] G.C. Ghirardi, A. Rimini, and T. Weber, “Unified dynamics for microscopic and macroscopic systems”, *Phys. Rev.* **D34**, 470–491, 1986.

[62] Philip Pearle, “Combining stochastic dynamical state-vector reduction with spontaneous localization”, *Phys. Rev.* **A39**, 2277–2289, 1989.

[63] Philip Pearle, “True collapse and false collapse,” in *Quantum Classical Correspondence* (Cambridge, MA: International Press, 1997), edited by Da Hsuan Feng and Bei Lok Hu, pp. 51–68.

[64] Bryce S. DeWitt and Neill Graham, eds., *The Many-Worlds Interpretation of Quantum Mechanics* (Princeton, NJ: Princeton University Press, 1973).

[65] Hilary Putnam, *Representation and Reality* (Cambridge, MA: MIT Press, 1988).

[66] Bernard d’Espagnat, *Veiled Reality* (Reading, MA: Addison-Wesley, 1995).
[67] John von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton, NJ: Princeton University Press, 1955).

[68] Fritz London and Edmond Bauer, “The theory of observation in quantum mechanics,” in *Quantum Theory and Measurement* (Princeton, NJ: Princeton University Press, 1983), edited by John Archibald Wheeler and Wojciech Hubert Zurek, pp. 217–259.

[69] Rudolf Peierls, “In defence of ‘measurement’”, *Physics World* 4, 19–20, January 1991.

[70] Don N. Page, “Sensible quantum mechanics: Are probabilities only in the mind?”, *Int. J. Mod. Phys.* D5, 583–596, 1996.

[71] Henry Pierce Stapp, *Mind, Matter, and Quantum Mechanics* (Berlin: Springer, 1993).

[72] N. David Mermin, “What is quantum mechanics trying to tell us?”, *Am. J. Phys.* 66, 753–767, 1998.

[73] Rudy Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (New York: Bantam Books, 1983).

[74] Richard P. Feynman, *The Character of Physical Law* (Cambridge, MA: MIT Press, 1967).

[75] Robert B. Griffiths, “Consistent histories and the interpretation of quantum mechanics”, *J. Stat. Phys.* 36, 219–272, 1984.

[76] M. Gell-Mann and J.B. Hartle, “Quantum mechanics in the light of quantum cosmology,” in *Complexity, Entropy, and the Physics of Information* (Reading, MA: Addison-Wesley, 1990), edited by W.H. Zurek, pp. 425–458.

[77] Roland Omnès, “Consistent interpretations of quantum mechanics”, *Rev. Mod. Phys.* 64, 339–382, 1992.

[78] D. Albert and B. Loewer, “Interpreting the Many Worlds Interpretation”, *Synthese* 77, 195–213, 1988.

[79] D.H. Hubel and T.N. Wiesel, “Brain mechanisms of vision”, *Sci. Am.* 241, 150–162, September 1979.

[80] C.F. von Weizsäcker, *The Unity of Nature* (New York: Farrar, Straus and Giroux, 1980).

[81] P.A. Schilpp, ed., *Albert Einstein: Philosopher-Scientist* (Evanston, IL: Library of Living Philosophers, 1949).

[82] Colin McGinn, “Consciousness and space”, *J. Consc. Stud.* 2, 220–230, 1995.

[83] P.F. Strawson, *Individuals* (London: Methuen, 1959).