Inertial Instability of Accretion Disks with Relativistic Electrons

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ABSTRACT

We show that accretion disks with relativistically hot electrons which are coupled to cold ions is subject to inertial-type instability in which kinematic drag between electrons and ions produces radial acceleration of electrons.

Subject headings: Accretion Disks - Instabilities

1. Model Problem

We consider an accretion disk in which electrons are heated to relativistic energies, while ions remain cold. This condition requires that ratio of electron to ion heating in the disk is larger than \( m_e/m_i \) (\( m_e \) and \( m_i \) are electron and ion masses) and that radiation efficiency is relatively small, so that a considerable fraction of the gravitational energy released in accretion goes into heat. Ion temperature may still be much larger than electron temperature. We also assume that electrons and ions are strongly coupled so that the average velocity of the electron component is given by the velocity of ions, which, in turn, is determined by the mass encompassed by the orbit of an ion at a given radius \( r \). We also require that the sound speed \( c_s \) is much less than the orbital velocity \( c_s \ll \Omega(r)r \), where \( \Omega(r) \) is the angular velocity of the flow. Then we can approximate the flow as accretion disk with a mean velocity given at each point by the velocity of the cold ion fluid in azimuthal direction \( v^{(0)}_\phi = \Omega(r)r \). The electron component has in addition large thermal spread around the average velocity: \( \mathbf{v}_e = v^{(0)}_\phi + \mathbf{v}_T \), where electron thermal velocities \( \mathbf{v}_T \) are assumed to be close to the velocity of light \( v_T \sim 1 - \frac{1}{2T^2} \) (here \( T \) is a relativistic temperature in units of \( mc^2 \)). We stress that the assumption that the average electron velocity is \( v^{(0)}_\phi \) implies that electrons and ions are strongly interacting, i.e. there is a drag between electron and ion components. This interaction may not necessarily be through Coulomb collisions but also through collective plasma modes.

We show that such simple system is unstable toward creation of a large radial current. When the current surpasses the critical value of \( j_r \sim enc_s \) (\( e \) is a charge of an electron, \( n \) is plasma density), strong ion sound instabilities develop which would destroy the laminar flow of ions.
2. Centrifugal forces

We concentrate on the dynamical equations governing electron motion. They can be derived from relativistic Lagrangian of an electron (e.g. Landau-Lifshits, 1951).

\[
\mathcal{L} = -m_e \left( \frac{1}{\gamma} + \Phi(r) \right), \quad \gamma = \frac{1}{\sqrt{1 - \dot{v}_r^2 - \dot{v}_\theta^2 - (\dot{v}_\phi^0 + \dot{v}_\phi)^2}}
\]  
(1)

where \(v_r, v_\theta, v_\phi\) are components of the random electron velocity, \(\Phi(r) = -\frac{GM(r)}{r}\) is a gravitational potential at a radius \(r\) due to the mass \(M(r)\) and we set a speed of light to unity.

The equation of motion follows from the Lagrange equation. For the radial component

\[
\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial v_r} = \frac{\partial \mathcal{L}}{\partial r}
\]
(2)

we find

\[
\frac{\partial v_r}{\partial t} = \left(\dot{v}_\phi^0 + v_\phi\right) \frac{\partial v_\phi^0}{\partial r} \frac{(1 - \dot{v}_\phi^2 \gamma^2)}{(1 + \dot{v}_\phi^2 \gamma^2)} - \frac{1}{\gamma(1 + \dot{v}_\phi^2 \gamma^2)} \frac{\partial \Phi(r)}{\partial r}
\]
(3)

where we assumed that \(v_\theta\) and \(v_\phi\) are independent of \(r\) and \(t\). The first term here is a centrifugal acceleration felt by electrons. Averaging over random \(v_\phi\) velocities in the limit of relativistic hot electrons, \(v_r^2 \gamma^2 \gg 1\) and \(v_r^2 \sim 1\), Eq. (3) reduces to

\[
\frac{\langle \partial v_r \rangle}{\partial t} = -\frac{1}{2} \frac{\partial v_\phi^2}{\partial r} - \frac{1}{\gamma^3} \frac{\partial \Phi(r)}{\partial r}
\]
(4)

Again, the first term here is an average centrifugal acceleration felt by electrons. It is different than the centrifugal force on ions, \(\frac{v_\phi^2}{\gamma^3}\). This fact may be considered as the main result of this work.

Using the condition that ion motion is a circular motion which satisfies \(\frac{v_\phi^2}{r} = \frac{\partial \Phi(r)}{\partial r}\) Eq. (4) can be rewritten as

\[
\frac{\langle \partial v_r \rangle}{\partial t} = -v_\phi \left( \frac{v_\phi^0}{r \gamma^3} + \frac{\partial v_\phi^0}{\partial r} \right).
\]
(5)

with \(\gamma \sim T \gg 1\). Eq. (5) shows that there is a net acceleration of electrons in radial direction.
For power law dependence of $v_{\phi}^{(0)}$ on radius and for $\gamma \gg 1$ we can neglect the first term in Eq. (5) (which is due to gravitational attraction). The net radial acceleration of electrons then is purely kinematic:

$$\langle \frac{\partial v_r}{\partial t} \rangle \approx -\frac{1}{2} \frac{\partial v_{\phi}^{(0)}}{\partial r}$$

(6)

Note, that for solid body rotation, $v_{\phi}^{(0)} = \Omega r$ with $\Omega \sim \text{const}$ Eq. (5) gives a centrifugal force $\frac{\partial v_r}{\partial t} = -\Omega^2 r$ directed inward (Henriksen & Norton, 1975, Chedia et al. 1996). Other examples include Keplerian disk around a central object of mass $M_*$ for which $v_{\phi}^{(0)} = \sqrt{\frac{GM_*}{r}}$ and radial acceleration of electrons becomes

$$\frac{\partial v_r}{\partial t} = \frac{GM_*}{2r^2}$$

(7)

Thus, in a Keplerian disk relativistic electrons are accelerated outwards. For massive disks the azimuthal velocity $v_{\phi}^{(0)}$ depends on the mass distribution in the disk. For example, if $v_{\phi}^{(0)}$ is an increasing function of radius the centrifugal acceleration of electrons will be directed towards the central object.

3. Discussion

Radial acceleration of electrons with respect to ions will result in a formation of a radial current. Current carrying plasmas are susceptible to various instabilities (e.g. Melrose 1980). Similarly to the case of plasma in electric field, if the relative motion of electrons and ions is larger than sound velocity the plasma will be unstable to low frequency ion sound instabilities with typical growth rates (Melrose 1980)

$$\Gamma \sim kc_s$$

(8)

Development of these instabilities will result in excitation of low frequency ion sound plasma turbulence. Turbulence will limit, due to the anomalous viscosity, the relative ambipolar drift between electrons and ions. In the extreme case (analogues to Dreicer critical electric field) the current in the plasma may reach its maximum value of $j_{\text{max}} \sim en_c$. This is a large global current that flows in radial direction.

There are several implications of the instability. First, there is a generation of the low frequency turbulence that would destroy the laminar flow of ions and may contribute to the angular momentum transport in the disk. Secondly, the radial current will produce azimuthal magnetic field of the order $B_\phi \sim j_{\text{max}} R$ which may play an important role both
for the magneto-rotational instabilities (Balbus & Hawley 1991) and overall magnetospheric structure of the system. Thirdly, the radial current will try to charge the central object. The implications of this current on the global structure of the disk are not clear.

Finally, we give a "physical reason" for the unusual expression for the centrifugal acceleration of electrons. As the electron is displaced outward in radial direction the loss in the potential energy in the effective potential (Landau & Lifshitz 1976) is accompanied by the change of the kinetic energy of azimuthal motion. For $\Omega(r)$ falling off slower than $r^{-1}$ the gain in the kinetic energy is larger than the loss of the potential energy, so that electron "prefers" to go inward - centrifugal force reverses.

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