Abstract: A feedback-dominance based adaptive back-stepping (FDBAB) controller is designed to drive a container ship to follow a predefined path. In reality, current, wave and wind act on the ship and produce unwanted disturbances to the ship control system. The FDBAB controller has to compensate for such disturbances and steer the ship to track the predefined (or desired) path. The difference between the actual and the desired path along which the ship is to sail is defined as the tracking error. The FDBAB controller is built on the tracking error model which is developed based on Serret-Frenet frame transformation (SFFT). In addition to being affected by external disturbances, the ship has more outputs than inputs (under-actuated), and is inherently nonlinear. The back-stepping controller in FDBAB is used to compensate the nonlinearity. The adaptive algorithms in FDBAB is employed to approximate disturbances. Lyapunov’s direct method is used to prove the stability of the control system. The FDBAB controlled system is implemented in Matlab/Simulink. The simulation results verify the effectiveness of the controller in terms of successful path tracking and disturbance rejection.

Keywords: under-actuated, nonlinear, environmental disturbance, path following, Serret-Frenet frame transformation (SFFT), ship steering.

1. Introduction

Container ships are common means of commercial inter-modal freight transport, and carry most sea-going non-bulk cargo in modern world. Container ship capacity is measured in twenty foot equivalent unit (TEU). The biggest container ships can carry over 23 000 TEU. Therefore, they are large in size and mass. Nowadays the container ship is the most cost-effective way to ship manufactured goods over oceans. A path following control system is critical to keeping the ships staying on the desired path despite the presence of environmental disturbances such as wind, current, wave, tide, and so on [1]. However, the under-actuation and nonlinearity of the ship propose a difficulty in controller design [2]. Typically, a container ship has three outputs in a two-dimensional space. They are the displacement or velocity outputs along surge, sway and yaw directions respectively. However, it only has one torque input. Therefore, the ship is under-actuated. In addition, the ship is modeled by nonlinear kinematics and kinetics equations. Thus, it is inherently nonlinear. The control system has to overcome the challenges of under-actuation and nonlinearity. It also has to approximate the external disturbances and reduce the effects of the disturbances in real time.

Back-stepping control has been broadly used in the path-following systems because it can compensate the nonlinearity and under-actuation of the container ships [3 – 9]. It requires accurate model information to achieve zero tracking error. In order to build the mathematical model for ships, Serret-Frenet transformation has been studied. The Serret-Frenet formulas describe the kinematic properties of a moving particle, and the geometric properties of a curve in a three-dimensional space. It has been adopted by the researchers to develop the kinematic model of a vehicle using the distance and angle tracking errors. Serret-Frenet transformation was introduced in [6] for the design of back-stepping control, but the external disturbances were disregarded in [6]. The back-stepping controllers in [7,9] were not robust against disturbances either. In [8], integral actions were added to the controller to compensate for a constant bias of environmental disturbances.

The disturbance observer can estimate the environmental disturbances in a ship’s path-following system. The disturbance observer based back-stepping controller in [10 – 16] were demonstrated to compensate for constant environmental disturbances. In [17], a disturbance observer was developed to approximate time-invariant and time-variant disturbances. In [18], a sliding-mode disturbance observer was used to compensate for various disturbances. It was theoretically proved in [18] that the path-following
Adaptive control has been successful in identifying system parameter variations and external disturbances [13,19–22]. In [13], the adaptive controller was built upon a disturbance observer which estimated time-variant disturbances. However, the controller has complicated configuration with at least ten controller parameters being adjusted. The adaptive back-stepping controller (ABC) was reported in [19–22]. The ABC in [19] was robust against both random noise and time-invariant external disturbance. The ABC in [20] can drive the ship to sail along a straight path in the presence of slightly varying disturbances. In [21], the ABC was constructed on the basis of the approximations of both time-invariant and time-variant disturbances. It steers the ship to follow non-curved and curved paths. However, at least nine controller parameters have to be adjusted in the ABC [21]. A robust ABC was developed in [22] to follow the non-curved path. The ABC is robust against the disturbances that change with time.

Here a new feedback-dominance based adaptive back-stepping (FDBAB) controller is established. It steers the container ships to sail along non-curved and curved paths in spite of external disturbances. The FDBAB controller consists of adaptive laws and the feedback-dominance backstepping controller (FDBC) technique. The FDBC is designed to overcome nonlinearity and under-actuation of a container ship. In addition, adaptive laws are designed to estimate environmental disturbances. Since the cargo ships are large in size and mass, the current, wind and wave disturbances have minimal influence on the steering of the cargo ships, and thus could be taken as constants. In order to reduce the complexity of the controller design, a tracking error model is derived based on Serret-Frenet frame transformation (SFFT). FDBAB is then built on the tracking error model. The major innovative contribution of this study is the development of an FDBAB controller for the ship to track the reference (or the desired) path. The major advantage of the new FDBAB controller over the controllers in [10–22] is its relatively simple configuration with only six tuning parameters.

The mathematical modeling of container ships is demonstrated in Section 2. The FDBAB controller is established in Section 3 where the Lyapunov based stability proof is derived as well. The simulation results are presented in Section 4. The concluding remarks and future research work are explained in Section 5.

2. Tracking error model of container ships

In this paper, we suppose the ship is moving in a planar surface of Earth. Thus, it only has three degree of freedom (DOF) motions which are surge, sway and yaw. The schematic of the ship in a horizontal plan is shown in Fig. 1. In this figure, the surge velocity is given by \( u \), the sway velocity is given by \( v \), the yaw velocity is represented by \( r \), and the total speed is given by \( u_t = \sqrt{u^2 + v^2} \). The parameters \( x \) and \( y \) stand for the linear positions of the ship in a horizontal plane, \( \psi \) represents the yaw angle, \( \beta \) represents the drift angle that is given by \( \beta = \arctan \left( \frac{u}{v} \right) \), and \( \delta \) represents the rudder deflection.

\[
\begin{align*}
\dot{x} &= u_0 \cos \psi - v \sin \psi \\
\dot{y} &= u_0 \sin \psi + v \cos \psi \\
\dot{\psi} &= r \\
\dot{u} &= 0 \\
\dot{v} &= \frac{m_{11}}{m_{22}} u_0 r - \frac{d_{22}}{m_{22}} v + \frac{1}{m_{22}} \tau_v E \\
\dot{r} &= -\frac{m_{11} - m_{22}}{m_{33}} u_0 v - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_r E + \frac{1}{m_{33}} \tau
\end{align*}
\]  

(1)

As shown in (1), the first order ordinary differential equations (ODEs) of \( x, y \) and \( \psi \) describe the kinematics of the ship, and the ODEs of \( u, v \) and \( r \) describe the kinetics of the ship. In (1), \( m_{11}, m_{22} \) and \( m_{33} \) represent the moment of inertia, \( d_{22} \) and \( d_{33} \) are damping coefficients, and \( d_{22} = d_{v1} + d_{r2} |v| + d_{v3} v^2 \) and \( d_{33} = d_{r1} + d_{r2} |r| + d_{r3} r^2 \), \( d_{v1} \) and \( d_{r1} \) are linear damping coefficients, \( d_{v2}, d_{v3}, d_{r2} \) and \( d_{r3} \) are nonlinear damping coefficients, \( \tau_v E \) and \( \tau_r E \) represent the environmental disturbances, and \( \tau \) is the torque input.
From (1), the three outputs of the ship are $x$, $y$ and $\psi$, and the single control input is $\tau$. There are more outputs than inputs. Thus, the system is under-actuated. The control goal is to steer the ship with the torque input such that the outputs $x$, $y$ and $\psi$ remain on the desired path. The control design on (1) is challenging due to the nonlinearity and under-actuation. If the distance between the ship and the desired path is minimized, the ship will follow the desired path. Thus, the path-following error model for the ships to be derived for the path-following control system of ships. The error model is based on SFFT [6,23,24].

The actual position of a ship and the predefined path which the ship shall follow are illustrated in Fig. 2. In this figure, point $P$ is the closest point along the predefined path to point $O_b$, which is the center of the ship. The length measured from $O_b$ to $P$ is marked as the cross error $z_c$. The error of the yaw angle is given by $\psi_s^* = \psi - \psi_d + \beta$, where $\psi_d$ is the reference yaw angle. Once the two errors $z_c$ and $\psi_s^*$ are regulated to be zero, the ship would be on the predefined path. In addition, as the total velocity $u_t$ is tangent to the curve of the path, the ship would track the desired path.

![Fig. 2 Actual position of a ship and a predefined curved path](image)

The tracking error model based on Fig. 2 and SFFT is represented by

$$
\begin{align*}
\dot{z}_c &= u_t \sin \psi_s^* \\
\dot{\psi}_s^* &= \left(1 - \frac{m_{11} u_0^2}{m_{22} u_t^2}\right) r - \frac{\kappa u_t}{1 - \kappa z_c} \cos \psi_s^* - \frac{u_0}{u_t^2} \frac{d_{22}}{m_{22}} v - \frac{u_0}{u_t^2} \frac{1}{m_{22}} \tau_{vE} \\
\dot{v} &= \frac{m_{11}}{m_{22}} u_0 r - \frac{d_{22}}{m_{22}} v + \frac{1}{m_{22}} \tau_{vE} \\
\dot{\tau} &= \frac{m_{11} - m_{22}}{m_{33}} u_0 v - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_{rE} + \frac{1}{m_{33}} \tau
\end{align*}
$$

(2)

where $\kappa$ is the curvature of the desired path.

Define

$$
\begin{align*}
b &= 1 - \frac{m_{11} u_0^2}{m_{22} u_t^2} \\
f_1 &= -\frac{\kappa u_t}{1 - \kappa z_c} \cos \psi_s^* - \frac{u_0}{u_t^2} \frac{d_{22}}{m_{22}} v, \\
f_2 &= -\frac{m_{11}}{m_{22}} u_0 r - \frac{d_{22}}{m_{22}} v, \\
f_3 &= \frac{m_{11} - m_{22}}{m_{33}} u_0 v - \frac{d_{33}}{m_{33}} r.
\end{align*}
$$

Then (2) can be rewritten as

$$
\begin{align*}
\dot{z}_c &= u_t \sin \psi_s^* \\
\dot{\psi}_s^* &= \left(1 - \frac{m_{11} u_0^2}{m_{22} u_t^2}\right) r - \frac{\kappa u_t}{1 - \kappa z_c} \cos \psi_s^* - \frac{u_0}{u_t^2} \frac{d_{22}}{m_{22}} v - \frac{u_0}{u_t^2} \frac{1}{m_{22}} \tau_{vE} \\
\dot{v} &= \frac{m_{11}}{m_{22}} u_0 r - \frac{d_{22}}{m_{22}} v + \frac{1}{m_{22}} \tau_{vE} \\
\dot{\tau} &= \frac{m_{11} - m_{22}}{m_{33}} u_0 v - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_{rE} + \frac{1}{m_{33}} \tau
\end{align*}
$$

As shown in (7), there are two outputs for the ship. They are cross error $z_c$ and yaw angle error $\psi_s^*$. There is one single input $\tau$. The system is still under-actuated. We aim at regulating the cross error $z_c$ and the yaw angle error $\psi_s^*$ to zero with the design of input $\tau$. Moreover, two environmental disturbances $\tau_{vE}$ and $\tau_{rE}$ have to be estimated by adaptive laws. Then the estimated disturbance will be rejected in the controller. As a result, the ship will track a reference path as expected.

3. FDBAB controller

In order to design the FDBAB controller, we have to make four assumptions as follows.

**Assumption 1** The system states $x$, $y$, $\psi$, $u$, $v$, $r$ and $u_t$ are measurable.

**Assumption 2** Functions $f_1$, $f_2$ and $f_3$ in (4), (5) and (6) are known and differentiable.

**Assumption 3** The environmental disturbances $\tau_{vE}$ and $\tau_{rE}$ are constants and bonded: $|\tau_{vE}| \leq \tau_{vE} \max < \infty$ and $|\tau_{rE}| \leq \tau_{rE} \max < \infty$, where $\tau_{vE} \max$ and $\tau_{rE} \max$ are positive real numbers.

**Assumption 4** The denominator in (2) is positive: $1 - \kappa z_c = \delta^* > 0$ [25].

The block diagram of an FDBAB controlled container ship system is illustrated in Fig. 3. SFFT uses the measured position and velocity signals from the container ship to generate the cross error and yaw angle error signals to the FDBAB controller. The FDBAB controller outputs the torque control signal to the container ship.
We estimate the constant environmental disturbance \( \tau_{ve} \) in (9) with an estimation variable \( \hat{\tau}_{ve} \). The estimation error \( \hat{\tau}_{ve} = \tau_{ve} - \hat{\tau}_{ve} \).

The derivative of (15) is given by

\[
\dot{\hat{\tau}}_{ve} = -\hat{\tau}_{ve}
\]  

with \( \hat{\tau}_{ve} = 0 \).

We derive the adaptive law \( \dot{\hat{\tau}}_{ve} \) by defining a Lyapunov function that includes the estimation error variable \( \hat{\tau}_{ve} \) as

\[
V_2 = V_1 + \frac{p_1}{2} \hat{\tau}_{ve}^2 + \frac{p_1}{2q_1} \hat{\tau}_{ve}^2 > 0
\]  

for any \( z_2 \neq 0 \). In (17), \( p_1 \) and \( q_1 \) are positive real numbers.

Then the derivative of \( V_2 \) is given by

\[
\dot{V}_2 = -c_1 z_1^2 u_t \sin \psi_e^* + z_1 z_2 u_t \frac{\sin \psi_e^*}{\psi_e^*} + \]

\[
p_1 z_2 \dot{\hat{\tau}}_{ve} - \frac{p_1}{q_1} \hat{\tau}_{ve} \dot{\hat{\tau}}_{ve}
\]

Substituting (19) into (18), we obtain

\[
\dot{V}_2 = -c_1 z_1^2 u_t \sin \psi_e^* + z_1 z_2 u_t \frac{\sin \psi_e^*}{\psi_e^*} + \]

\[
p_1 z_2 \left( br + f_1 + \frac{u_0}{u_t^2 m_{22}} \tau_{ve} \right) + p_1 c_1 u_t z_2 \sin \psi_e^* - \frac{p_1}{q_1} \hat{\tau}_{ve} \dot{\hat{\tau}}_{ve}
\]

Equation (20) can be further developed as

\[
\dot{\hat{\tau}}_{ve} = -c_1 z_1^2 u_t \sin \psi_e^* + z_1 z_2 u_t \frac{\sin \psi_e^*}{\psi_e^*} + \]

\[
p_1 z_2 \left( br + f_1 + \frac{u_0}{u_t^2 m_{22}} \tau_{ve} \right) + p_1 c_1 u_t z_2 \sin \psi_e^* \left( z_2 + \alpha_1 \right) - \frac{p_1}{q_1} \hat{\tau}_{ve} \dot{\hat{\tau}}_{ve}
\]

Since \( \alpha_1 = -c_1 z_1 \) and \( z_3 = r - \alpha_2 \), (21) can be rewritten as

\[
\dot{\hat{\tau}}_{ve} = -c_1 z_1^2 u_t \sin \psi_e^* + z_1 z_2 u_t \frac{\sin \psi_e^*}{\psi_e^*} \left( 1 - p_1 c_1^2 \right) + \]

\[
p_1 z_2 \left( b(z_3 + \alpha_2) + f_1 + \frac{u_0}{u_t^2 m_{22}} \tau_{ve} + c_1 u_t z_2 \sin \psi_e^* \right) - \frac{p_1}{q_1} \hat{\tau}_{ve} \dot{\hat{\tau}}_{ve}
\]
Choose parameter $p_1$ and the virtual control $\alpha_2$ as

$$p_1 = \frac{1}{c_1}$$

$$\alpha_2 = -\frac{1}{b} \left( f_1 + c_2 z_2 + \frac{u_0}{u^2_t} \frac{1}{m_{22}} \hat{r}_{vE} \right)$$

(23)

(24)

where parameter $c_2$ is a positive real number.

Substituting (15), (23) and (24) into (22), we obtain

$$\dot{V}_2 = -c_1 z_1^2 u_t \sin \frac{\psi_e^*}{\psi_e} +$$

$$\frac{1}{c_1^2} z_2 \left( b z_3 - c_2 z_2 + c_1 u_t z_2 \sin \frac{\psi_e^*}{\psi_e} \right) +$$

$$\frac{1}{c_1^2} \hat{r}_{vE} \left( \frac{u_0}{u^2_t} \frac{1}{m_{22}} z_2 - \frac{1}{b} \hat{r}_{vE} \right).$$

(25)

Choose the adaptive law as

$$\hat{r}_{vE} = \frac{u_0}{u^2_t} \frac{q_1}{m_{22}} z_2.$$  

(26)

Substituting (26) into (25), we obtain

$$\dot{V}_2 = -c_1 z_1^2 u_t \sin \frac{\psi_e^*}{\psi_e} +$$

$$\frac{1}{c_1} z_2 \left( b z_3 - c_2 \left( c_2 - c_1 u_t \sin \frac{\psi_e^*}{\psi_e} \right) \right) =$$

$$-c_1 z_1^2 u_t \sin \frac{\psi_e^*}{\psi_e} - \frac{c_2}{c_1} z_2 \left( 1 - c_1 u_t \sin \frac{\psi_e^*}{\psi_e} \right) + \frac{1}{c_1} b z_2 z_3.$$

(27)

If we choose $c_2 > c_1 u_t$, we will have

$$-c_2 \frac{z_2}{z_2} \left( 1 - \frac{c_1 u_t \sin \frac{\psi_e^*}{\psi_e}}{c_2 \frac{\psi_e^*}{\psi_e}} \right) < 0.$$  

(28)

Consequently, $\dot{V}_2$ would be negative definite as the error variable $z_3 = 0$. The parameter $c_2$ plays a dominant role in producing a negative definite derivative of $V_2$. Furthermore, the parameter $c_2$ is derived on the availability of the feedback states $z_1$ and $z_2$. Hence, we claim that $c_2$ is selected by using the feedback-dominance technique.

### 3.3 Controller development: step three

In addition to $\tau_{vE}$, the environmental disturbance $\tau_{rE}$ needs to be estimated with an estimation variable $\hat{r}_{rE}$. The estimation errors are defined as

$$\hat{r}_{rE} = \tau_{rE} - \hat{r}_{rE}.$$  

(29)

The derivative of (29) is given by

$$\dot{\hat{r}}_{rE} = -\dot{\hat{r}}_{rE}$$

(30)

with $\hat{r}_{rE} = 0$.

To derive the adaptive laws $\dot{\hat{r}}_{rE}$, we define the Lyapunov function as

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2q_2} \hat{r}_{rE}^2 + \frac{1}{2q_3} \tau_{rE} > 0$$

(31)

for any $z_3 \neq 0$. In (31), parameters $q_2$ and $q_3$ are positive real numbers.

The derivative of $V_3$ is

$$\dot{V}_3 = \dot{V}_2 + z_3 \dot{z}_3 - \frac{1}{q_2} \hat{r}_{vE} \dot{r}_{rE} - \frac{1}{q_3} \hat{r}_{rE} \dot{r}_{rE}.$$  

(32)

Since $z_3 = r - \alpha_2$, the derivative of $z_3$ is given by

$$\dot{z}_3 = f_3 + \frac{1}{m_{33}} \tau + \frac{1}{m_{33}} \tau_{rE} - \alpha_2.$$  

(33)

Substituting (33) into (32), we obtain

$$\dot{V}_3 = -c_1 z_1^2 u_t \sin \frac{\psi_e^*}{\psi_e} - c_2 \left( 1 - \frac{c_1 u_t \sin \frac{\psi_e^*}{\psi_e}}{c_2 \frac{\psi_e^*}{\psi_e}} \right) +$$

$$\frac{1}{c_1} b z_2 z_3 + z_3 \left( f_3 + \frac{1}{m_{33}} \tau + \frac{1}{m_{33}} \tau_{rE} - \alpha_2 \right) -$$

$$\frac{1}{q_2} \hat{r}_{vE} \dot{r}_{rE} - \frac{1}{q_3} \hat{r}_{rE} \dot{r}_{rE}.$$  

(34)

The derivative of virtual control $\alpha_2$ in (24) is

$$\dot{\alpha}_2 = -\frac{1}{b} \left( \dot{f}_1 + c_2 z_2 + \frac{u_0}{u^2_t} \frac{1}{m_{22}} \hat{r}_{vE} \right).$$  

(35)

where the derivative of $z_2$ is given by (19) and the adaptive law $\hat{r}_{vE}$ is given by (26).

The derivative of function $f_1$ is given by

$$\dot{f}_1 = \frac{\partial f_1}{\partial z_e} \dot{z}_e + \frac{\partial f_1}{\partial \psi_e} \dot{\psi}_e + \frac{\partial f_1}{\partial v} \dot{v} =$$

$$-\kappa^2 u_t \cos \frac{\psi_e^*}{\psi_e} \frac{1}{(1 - \kappa z_e)^2} \dot{z}_e + \kappa u_t \sin \frac{\psi_e^*}{\psi_e} \frac{1}{1 - \kappa z_e} \dot{v}.$$  

(36)

where the function $\text{sgn}(v)$ is given by

$$\text{sgn}(v) = \begin{cases} 
1, & v > 0 \\
0, & v = 0 \\
-1, & v < 0 
\end{cases}.$$  

(37)

Define

$$d_{\psi} = d_{v1} + d_{v2} |v| + d_{v3} \text{sgn}(v) + 3d_{v3} v^2.$$  

(38)

Substituting (7) and (38) into (36), we have the derivative of function $f_1$ as

$$\dot{f}_1 = -\kappa^2 u_t \sin \frac{\psi_e^*}{\psi_e} \cos \frac{\psi_e^*}{\psi_e} \frac{1}{(1 - \kappa z_e)^2} + \kappa u_t \sin \frac{\psi_e^*}{\psi_e} \frac{1}{1 - \kappa z_e} \left( br + f_1 \right) -$$

$$\kappa u_t \sin \frac{\psi_e^*}{\psi_e} \frac{1}{1 - \kappa z_e} \left( br + f_1 \right) -$$

$$\kappa u_t \sin \frac{\psi_e^*}{\psi_e} \frac{1}{1 - \kappa z_e} \left( br + f_1 \right) -$$

$$\kappa u_t \sin \frac{\psi_e^*}{\psi_e} \frac{1}{1 - \kappa z_e} \left( br + f_1 \right).$$
Equation (48) shows the derivative of the Lyapunov function is negative definite. Therefore, the FDBAB system is proved to be stable in the sense of Lyapunov. From (48), we can see that the derivative of $V_3$ is uniformly continuous. Invoking Barbalat’s lemma [26], the error variables $z_1$, $z_2$ and $z_3$ will converge to zero, while $\tilde{r}_{v,E}$ and $\tilde{r}_{r,E}$ are bounded as time goes to infinity. Therefore, the cross and yaw angle errors, and the estimation errors between the actual and estimated disturbances are bounded. This proves the effectiveness of the FDBAB controller.

### 4. Simulation results

The FDBAB control system is simulated in Matlab/Simulink. From [17,27,28], the length of the ship is $230.66$ m, and the mass of it is $467.15 \times 10^5$ kg. The parameter values of the container ship are listed in Table 1 [17,27,28].

| Coefficient | Value | Unit |
|-------------|-------|------|
| $m_{33}$ | $544.55 \times 10^6$ | kg |
| $m_{22}$ | $101.34 \times 10^6$ | kg |
| $m_{11}$ | $242.48 \times 10^3$ | kg-m$^2$ |
| $d_{c1}$ | $248.37 \times 10^4$ | kg-s$^{-1}$ |
| $d_{c2}$ | $156.57 \times 10^4$ | kg-m$^{-1}$ |
| $d_{c3}$ | $528.57 \times 10^8$ | kg-m$^2$ |
| $d_{r1}$ | $0$ | kg-m$^{-2}$ |
| $d_{r2}$ | $134.98 \times 10^{11}$ | kg-m$^2$ |

Three groups of simulation results are demonstrated. In the first group, a container ship is controlled to track a straight path with time-invariant disturbances. The forward speed of the container ship is kept at 10 m/s [29]. We select the initial conditions as zeros, i.e., $[\vec{r}(0), \vec{v}(0), \psi(0), \psi(0), r(0)] = [0 \text{ m}, 0 \text{ m}, 0 \text{ rad}, 0 \text{ m/s}, 0 \text{ rad/s}]$. The time duration for simulation is chosen as 2 000 s. The reference path is a straight line toward the north whose beginning position is $(x_0, y_0) = (0 \text{ m}, 100 \text{ m})$. In the first group of simulation results, two time-invariant disturbance torques ($\tau_{v,E} = 5 \times 10^6$ Nm and $\tau_{r,E} = 1.2 \times 10^6$ Nm) are added to the system at 1 000 s.

We tune the controller parameters for the FDBAB controller manually. The controller parameter values are given in Table 2. The parameters $c_1$, $c_2$ and $c_3$ of FDBC are purposefully selected the same as the FDBAB controller for comparison. The other three parameters $q_1$, $q_2$ and $q_3$ are not needed in FDBC.

| Coefficient | Value |
|-------------|-------|
| $c_1$ | 0.2 |
| $c_2$ | 50$c_1$ |
| $c_3$ | 500$c_1$ |
| $q_1$ | 10 |
| $q_2$ | 10 |
| $q_3$ | 10 |
The yaw angle signals $\psi$ are shown in Fig. 4. The solid line represents the yaw angle output under the control of FDBAB, while the dotted line represents the one with FDBC. The settling time is 200 s for both control systems. After the disturbances are added to the system (at $t = 1000$ s), the yaw angles are successfully driven back to zero by the controllers.

Fig. 4 Yaw angle outputs with constant environmental disturbances

The position outputs $y$ are illustrated in Fig. 5, in which the FDBAB controller effectively drives the ship to track the desired position with some overshoot in transit response. However, FDBC is unable to drive the ship back to the reference path as the disturbances $\tau_{vE}$ and $\tau_{rE}$ are present in the system.

Fig. 5 Outputs $y$ in the presence of time-invariant disturbances

Fig. 6 displays the $x$ outputs. Both FDBAB and FDBC successfully drive the ship to track the reference positions.

Fig. 6 Outputs $x$ in the presence of time-invariant disturbances

Fig. 7 shows the speed outputs $v$ in the sway direction. The settling time for both control systems is 250 s (without disturbances). After the disturbance is introduced to the system, the sway speed outputs are driven back to zero by FDBAB and FDBC.

Fig. 7 Sway speed $v$ in the presence of time-invariant disturbances

The speed outputs $r$ in the yaw direction are shown in Fig. 8. The settling time for both control systems is 100 s (without disturbances). After the disturbance is introduced to the system, the yaw speed outputs are driven back to zero by FDBAB and FDBC.

Fig. 8 Yaw speed $r$ in the presence of time-invariant disturbances

Fig. 9 shows the actual ship trajectories in the presence of time-invariant disturbances. The desired path is a straight line. The FDBAB controller is robust against the disturbances while FDBC is not. Nevertheless, the cross error is bounded for the FDBC controlled system.

Fig. 9 Ship trajectories to track a straight path in the presence of time-invariant disturbances
The control signals that drive a container ship to track a straight path with constant disturbances are displayed in Fig. 10. It is observed that the control efforts for FDBAB and FDBC are identical. However, the FDBAB controller estimates and compensates for the constant disturbances with the adaptive laws, while FDBC does not. In addition, the control signals are bounded in both transient and steady state responses, which imply that the controllers are feasible in practice.

In the second group of simulation runs, slowly time-variant disturbance torques are added to the container ship system. The duration of simulation is 10 000 s. The two disturbances that change with time are represented by $\tau_{vE} = 5 \times 10^6 \sin(2\pi ft)$ Nm and $\tau_{rE} = 1.2 \times 10^9 \sin(2\pi ft)$ Nm, where $f = 0.0001$ Hz. The time-variant disturbances are present during the entire simulation.

The yaw angle outputs $\psi$ to track a straight path are demonstrated in Fig. 11 in the presence of time-variant disturbances. It is shown in Fig. 11 that the yaw angles are driven to zero by both controllers in spite of the disturbances.

The outputs $y$ signals are demonstrated in Fig. 12 in the presence of time-variant disturbances. The FDBAB controller successfully drives the ship to track the reference position with a small overshoot in transit response. However, FDBC is unable to compensate for the time-variant disturbances.

The outputs $x$ signals are demonstrated in Fig. 13 in the presence of time-variant disturbances. The FDBAB controller tracks the reference positions accurately under the control of FDBAB and FDBC.

The outputs $v$ signals are demonstrated in Fig. 14 in the presence of time-variant disturbances. The FDBAB controller successfully drives the ship to track the reference position along the sway direction as the ship is tracking a straight path.

The outputs $r$ signals are demonstrated in Fig. 15 as the ship is tracking a straight path.
Fig. 15 Yaw speed outputs in the presence of time-changing disturbances

Fig. 16 demonstrates the ship trajectories in the presence of time-variant disturbances as the desired path is straight. The FDBAB controller could keep the ship staying on the straight path with very small deviations. However, FDBC fails to do so.

Fig. 16 Ship trajectories in the presence of time-variant disturbances

The control signals that drive the ship to track a straight path are presented in Fig. 17. It is shown that the control efforts for FDBAB and FDBC are identical. In addition, both control signals are bounded, implying that the controllers are realizable in practice.

Fig. 17 Control signals in the presence of time-variant disturbances

In the third group of simulation results, our goal is to drive the ship to track a circular path without environmental disturbances. The forward speed of the container ship remains constant at 10 m/s. We choose zero initial conditions, i.e., \([x(0), y(0), \psi(0), v(0), r(0)] = [0 \text{ m}, 0 \text{ m}, 0 \text{ rad}, 0 \text{ m/s}, 0 \text{ rad/s}]\). The center position of the circular reference path is \((x_0, y_0) = (0 \text{ m}, 1100 \text{ m})\). The radius of the path is 1000 m. The time duration of simulation is chosen as is 1000 s.

The yaw angle outputs \(\psi\) that are controlled by FDBABC and FDBC to track the circular path are illustrated in Fig. 18. It is observed in Fig. 18 that the yaw angle signals ascend as the container ship tracks the circular path without environmental disturbances.

Fig. 18 Yaw angle outputs to track a circular path

The outputs \(y\) are presented in Fig. 19. The two controllers successfully drive the outputs \(y\) to track its reference signal without any tracking errors.

Fig. 19 Position \(y\) signals to track a circular path

Fig. 20 displays the outputs \(x\) as the desired path is a circle. Both FDBAB and FDBC controllers drive the outputs \(x\) to its reference signals without tracking errors.

Fig. 20 Outputs \(x\) as the desired path is a circle

The sway speed signals \(v\) are presented in Fig. 21. The sway speed is nonzero as the ship turns.
The yaw speed signals $r$ are demonstrated in Fig. 22. The yaw speed is nonzero as the ship turns.

Fig. 22 Yaw speed outputs as the desired path is a circle

Fig. 23 presents the ship trajectories and its reference circle. FDBAB and FDBC controllers keep the ship staying on the circular path without tracking errors.

The control signals that drive a container ship to track the circular path are illustrated in Fig. 24. It is shown that the control efforts for FDBAB and FDBC are identical. In addition, both control signals are bounded, implying that the controllers are realizable in practice.

To conclude, the FDBAB controller is an effective control approach to drive the container ship to track both curved and non-curved reference paths. It is robust against time-invariant and slightly time-changing disturbances.

The FDBAB controller shows much better robustness and tracking performance than FDBC in the presence of disturbances.

5. Conclusions

In this paper, we originally develop an FDBAB controller for steering a cargo ship to track non-curved and curved paths. The ship is inherently nonlinear and under-actuated. The FDBAB controller successfully compensates for the nonlinearity and under-actuation of the ship, in spite of time-invariant and slowly time-variant disturbances. The stability of the control system is justified through Lyapunov’s direct method. Simulation results demonstrate that the control system is effective in path following and robust against disturbances. In the future, we are going to implement the FDBAB on a lab-use ship model. We will design an experiment and test the controller in the real world. The experimental results are expected to be consistent with the simulation results reported in this manuscript.

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