Proofs to one inequality conjecture for the non-integer part of a nonlinear differential form

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Abstract
We prove the conjecture for the non-integer part of a nonlinear differential form representing primes presented in (Lai in J. Inequal. Appl. 2015: Article ID 357, 2015) by using Tumura-Clunie type inequalities. Compared with the original proof, the new one is simpler and more easily understood. Similar problems can be treated with the same procedure.

Keywords: nonlinear differential form; Tumura-Clunie type inequality; non-integer variables

1 Introduction
The non-integer part of linear and nonlinear differential forms representing primes has been considered by many scholars. Let \([x]\) be the greatest non-integer not exceeding \(x\). In 1966, Danicic [2] proved that if the diophantine inequality

\[ |\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 + \eta| < \varepsilon \]

satisfies certain conditions, and primes \(p_i \leq N (i = 1, 2, 3)\), then the number of prime solutions \((p_1, p_2, p_3, p_4)\) of (1) is greater than \(CN^3(\log N)^4\), where \(C\) is a positive number independent of \(N\). Based on the above result, Danicic [2] proved that if \(\lambda, \mu\) are non-zero real numbers, not both negative, \(\lambda\) is irrational, and \(m\) is a positive non-integer, then there exist infinitely many primes \(p\) and pairs of primes \(p_1, p_2\) and \(p_3\) such that

\[ |\lambda p_1 + \mu p_2 + \mu p_3| = mp. \]

In particular \([\lambda p_1 + \mu p_2 + \mu p_3]\) represents infinitely many primes.

Brüdern et al. [3] proved that if \(\lambda_1, \ldots, \lambda_s\) are positive real numbers, \(\lambda_1/\lambda_2\) is irrational, all Dirichlet L-functions satisfy the Riemann hypothesis, \(s \geq \frac{s}{2} k + 2\), then the non-integer parts of

\[ \lambda_1 x_1^k + \lambda_2 x_2^k + \cdots + \lambda_s x_s^k \]

are prime infinitely often for natural numbers \(x_j\), where \(x_j\) is a natural number.
Recently, Lai [1] proved that, for non-integer \( r \geq 2^{k-1} + 1 \) \((k \geq 4)\), under certain conditions, there exist infinitely many primes \( p_1, \ldots, p_r, p \) such that

\[
[\mu_1 p_1^k + \cdots + \mu_r p_r^k] = mp.
\]  

(1.1)

And he also conjectured that the above results are true when primes \( p_i \) in (1.1) are replaced by natural numbers \( x_j \). In this paper we shall give an affirmative answer to this conjecture.

2 Main result

Our main aim is to investigate the non-integer part of a nonlinear differential form with non-integer variables and mixed powers 3, 4 and 5. Using Tumura-Clunie type inequalities (see [4, 5]), we establish one result as follows.

**Theorem 2.1** Let \( \lambda_1, \lambda_2, \ldots, \lambda_9 \) be nonnegative real numbers, at least one of the ratios \( \lambda_i/\lambda_j \) \((1 \leq i < j \leq 9)\) is rational. Then the non-integer parts of

\[
\lambda_1 x_1^2 + \lambda_2 x_2^3 + \lambda_3 x_3^4 + \lambda_4 x_4^5 + \lambda_5 x_5^6 + \lambda_6 x_6^7 + \lambda_7 x_7^8 + \lambda_8 x_8^9 + \lambda_9 x_9
\]

are prime infinitely often for \( x_1, x_2, \ldots, x_9 \), where \( x_1, x_2, \ldots, x_9 \) are natural numbers.

**Remark** It is easy to see by the differential from Theorem 2.1 that primes \( p_i \) in (1.1) are replaced by a natural numbers \( x_j \) and there exist infinitely many primes \( p_1, \ldots, p_r \) and \( p \) such that \([\mu_1 p_1^k + \cdots + \mu_r p_r^k] = mp\), where \( m \) is a nonnegative non-integer (see [6]).

3 Outline of the proof

Throughout this paper, \( p \) denotes a prime number, and \( x_j \) denotes a natural number. \( \delta \) is a sufficiently small positive number, \( \epsilon \) is an arbitrarily small positive number. Constants, both explicit and implicit in Landau or Vinogradov symbols may depend on \( \lambda_1, \lambda_2, \ldots, \lambda_9 \).

We write \( e(x) = e^{2\pi i x} \). We take \( X \) to be the basic parameter, a large real non-integer. Since at least one of the ratios \( \lambda_i/\lambda_j \) \((1 \leq i < j \leq 9)\) is irrational, without loss of generality, we may assume that \( \lambda_1/\lambda_2 \) is irrational. For the other cases, the only difference is in the following intermediate region, and we may deal with the same method in Section 4.

Since \( \lambda_1/\lambda_2 \) is irrational, there are infinitely many pairs of non-integers \( q, a \) with \(|\lambda_1/\lambda_2 - a/q| \geq q^{-2} \), \((p,q) = 2, q > 0 \) and \( a \neq 0 \). We choose \( p \) to be large in terms of \( \lambda_3, \lambda_2, \ldots, \lambda_9 \), and make the following definitions.

Put \( \tau = N^{-1+\delta}, T = N^{1/2}, L = \log N, Q = (|\lambda_1|^{-2} + |\lambda_2|^{-2})N^{2-\delta}, [N^{1-3\delta}] = p \) and \( P = N^{3\delta} \), where \( N \approx X \). Let \( v \) be a positive real number, we define

\[
K_v(a) = v \left( \frac{\sin \pi v a}{\pi v a} \right)^3, \quad a \neq 0, \quad K_v(0) = v
\]

\[
F_i(a) = \sum_{1 \leq x \leq X^{1/6}} e(ax^i), \quad i = 1, 2, 3, 4, \quad F_j(a) = \sum_{1 \leq x \leq X^{1/6}} e(ax^j), \quad j = 5, 6, 7
\]

\[
F_k(a) = \sum_{1 \leq x \leq X^{1/2}} e(ax^k), \quad k = 8, 9, \quad G(a) = \sum_{p \leq N} (\log p) e(a p), \quad (3.1)
\]
The proof of Theorem 2.1 is complete.

4 The neighborhood of the origin

Lemma 4.1 (see [7], Theorem 4.1) Let \( (a, q) = 1 \). If \( \alpha = a/q + \beta \), then we have

\[
\sum_{1 \leq k \leq N^{1/2}} e(ax^k) = q^{-1} \sum_{m=1}^{q} e(am^2/q) \int_{1}^{N^{1/2}} e(\beta y^2) \, dy + O(q^{1/2+\epsilon} (1 + N/|\beta|)).
\]
Lemma 4.1 immediately gives

\[ F_i(\alpha) = f_i(\alpha) + O(X^{\theta}), \]  

(4.1)

where \(|\alpha| \in \mathbb{C}\) and \(i = 1, 2, 3, 4, \ldots, 9\).

**Lemma 4.2** (see [6], Lemma 3 and Remark 2) Let

\[
I(\alpha) = \sum_{|\gamma| \leq T} \sum_{0 < \beta < \frac{1}{2}} n^{\sigma-1} e(n \alpha),
\]

\[
J(\alpha) = O\left((1 + |\alpha|N)^{\frac{3}{2}} L^C\right),
\]

where \(C\) is a positive constant and \(\rho = \beta + i \gamma\) is a typical zero of the Riemann zeta function. Then we have

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} |I(\alpha)|^2 \, d\alpha \ll N \exp(-L^\frac{1}{6}),
\]

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} |J(\alpha)|^2 \, d\alpha \ll N \exp(-L^\frac{1}{6}),
\]

and

\[
G(\alpha) = g(\alpha) - I(\alpha) + J(\alpha).
\]

**Lemma 4.3** (see [6], Lemma 5) For \(i = 1, 2, 3, 4, \ j = 5, 6, 7, \ k = 8, 9\), we have

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} |f_i(\alpha)|^2 \, d\alpha \ll X^{-\frac{1}{4}}, \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} |f_j(\alpha)|^2 \, d\alpha \ll X^{-\frac{1}{4}}, \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} |f_k(\alpha)|^2 \, d\alpha \ll X^{-\frac{1}{4}}.
\]

**Lemma 4.4** We have

\[
\left| \int K_{\frac{1}{3}}(\alpha) \prod_{i=1}^{10} F_i(\lambda_i \alpha) G(-\alpha) - \prod_{i=1}^{10} f_i(\lambda_i \alpha) g(-\alpha) \right| \, d\alpha \ll X^{\frac{10}{11}}.
\]

**Proof** It is obvious that

\[
F_i(\lambda_i \alpha) \ll X^{\frac{1}{2}}, \quad f_i(\lambda_i \alpha) \ll X^{\frac{1}{2}}, \quad F_j(\lambda_j \alpha) \ll X^{\frac{1}{2}}, \quad f_j(\lambda_j \alpha) \ll X^{\frac{1}{2}},
\]

\[
F_k(\lambda_k \alpha) \ll X^{\frac{1}{2}}, \quad f_k(\lambda_k \alpha) \ll X^{\frac{1}{2}}, \quad G(-\alpha) \ll N, \quad \text{and} \quad g(-\alpha) \ll N,
\]

hold for \(i = 1, 2, 3, 4, \ j = 5, 6, 7\) and \(k = 8, 9\).

By (4.1), Lemmas 4.2 and 4.3, we have

\[
\int \left| \left( F_{\frac{1}{2}}(\lambda_{\frac{1}{2}} \alpha) - f_{\frac{1}{2}}(\lambda_{\frac{1}{2}} \alpha) \right) \prod_{i=2}^{9} F_i(\lambda_i \alpha) G(-\alpha) \right| K_{\frac{1}{3}}(\alpha) \, d\alpha \ll \frac{X^{\frac{10}{11}} N}{N^{1/2}} \ll X^{\frac{10}{11}} + 2\delta
\]
and

\[
\int K_\frac{1}{2}(\alpha) \left| \prod_{i=1}^{10} f_i(\lambda_i\alpha)(G(-\alpha) - g(-\alpha)) \right| d\alpha \\
\ll X^{\frac{13}{30}} \left( \int e^{-\frac{1}{\alpha}} f_i(\lambda_i\alpha)^2 K_\frac{1}{2}(\alpha) d\alpha \right)^{\frac{1}{2}} \left( \int |J(-\alpha) - I(-\alpha)|^2 K_\frac{1}{2}(\alpha) d\alpha \right)^{\frac{1}{2}} \\
\ll X^{\frac{13}{30}} \left( \int e^{-\frac{1}{\alpha}} f_i(\lambda_i\alpha)^2 d\alpha \right)^{\frac{1}{2}} \left( \int |J(\alpha)|^2 d\alpha + \int e^{-\frac{1}{\alpha}} |J(\alpha)|^2 d\alpha \right)^{\frac{1}{2}} \\
\ll \frac{X^{\frac{15}{30}}}{L}
\]

from a Tumura-Clunie type inequality ([5]).

The proofs of the other cases are similar, so we complete the proof of Lemma 4.4.

**Lemma 4.5** The following inequality holds:

\[
\int_{|\alpha| > \frac{1}{N^{1/2}}} K_\frac{1}{2}(\alpha) \left| \prod_{i=1}^{10} f_i(\lambda_i\alpha)g(-\alpha) \right| d\alpha \ll X^{\frac{13}{30} - \frac{11}{45}}
\]

**Proof** For \( \alpha \neq 0, i = 1, 2, 3, 4, j = 5, 6, 7, k = 8 \), we know that

\[
f_i(\lambda_i\alpha) \ll |\alpha|^{-\frac{1}{2}}, \quad f_j(\lambda_j\alpha) \ll |\alpha|^{-\frac{1}{4}}, \quad f_k(\lambda_k\alpha) \ll |\alpha|^{-1}, \quad g(-\alpha) \ll |\alpha|^{-1}.
\]

Thus

\[
\int_{|\alpha| > \frac{1}{N^{1/2}}} \left| \prod_{i=1}^{10} f_i(\lambda_i\alpha)g(-\alpha) K_\frac{1}{2}(\alpha) d\alpha \ll \int_{|\alpha| > \frac{1}{N^{1/2}}} |\alpha|^{-\frac{11}{45}} d\alpha \ll X^{\frac{13}{30} - \frac{11}{45}}.
\]

**Lemma 4.6** The following inequality holds:

\[
\int_{-\infty}^{\infty} \left( \prod_{i=1}^{10} f_i(\lambda_i\alpha)g(-\alpha) e\left(-\frac{1}{2} \alpha \right) K_\frac{1}{2}(\alpha) d\alpha \right) \gg X^{\frac{13}{30}}
\]

**Proof** We have

\[
\int_{-\infty}^{\infty} \prod_{i=1}^{10} f_i(\lambda_i\alpha)g(-\alpha) e\left(-\frac{1}{2} \alpha \right) K_\frac{1}{2}(\alpha) d\alpha \\
= \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty e(\lambda_1\alpha^3 + \lambda_2\alpha^2 + \lambda_3\alpha) \\
+ \lambda_4\alpha^3 + \lambda_5\alpha^2 + \lambda_6\alpha^2 + \lambda_7\alpha^3 + \lambda_7\alpha^2 + \lambda_8\alpha^3) K_\frac{1}{2}(\alpha) d\alpha dx_1 dx_2 dx_3 dx_4 dx_5 dx_6 dx_7 dx_8 dx_9 dx_{10}
= \frac{1}{72,000} \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty \int_1^\infty e\left(\alpha \left( \sum_{i=1}^{10} \lambda_i x_i - x - \frac{1}{2} \right) \right) \\
\cdot K_\frac{1}{2}(\alpha) d\alpha dx_1 dx_2 \cdots dx_{10}
\]
\[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} x_1^{\frac{4}{9}} x_2^{\frac{4}{9}} x_3^{\frac{3}{7}} x_4^{\frac{3}{7}} x_5^{\frac{3}{7}} x_6^{\frac{3}{7}} x_7^{\frac{3}{7}} x_8^{\frac{3}{7}} \cdot \max \left( 0, \frac{1}{9} - \left\| \sum_{i=1}^{9} \lambda_i x_i - \frac{1}{13} \right\| \right) d\lambda_1 dx_1 \cdots d\lambda_8 dx_8 \]

from (3.2).

Let

\[ \left| \gamma_1 x_1^2 + \gamma_2 x_2^3 + \gamma_3 x_3^4 + \gamma_4 x_4^5 + \gamma_5 x_5^6 + \gamma_6 x_6^7 + \gamma_7 x_7^8 + \gamma_8 x_8^9 + + \gamma_9 x_9^4 - x - \frac{1}{4} \right| \leq \frac{1}{4}. \]

Then we have

\[ \sum_{i=1}^{9} \lambda_i x_i - \frac{3}{5} \leq x \leq \sum_{i=1}^{9} \lambda_i x_i - \frac{1}{2}. \]

By using

\[ \sum_{i=1}^{9} \lambda_i x_i - \frac{1}{4} > 1 \quad \text{and} \quad \sum_{i=1}^{9} \lambda_i x_i - \frac{1}{2} < N, \]

we obtain

\[ \lambda_j x \left( 8 \sum_{i=1}^{9} \lambda_i \right)^{-1} \leq x_j \leq \lambda_j x \left( 4 \sum_{i=1}^{9} \lambda_i \right)^{-1}, \quad j = 1, \ldots, 9, \]

and hence

\[ \int_{-\infty}^{\infty} \prod_{i=1}^{10} F_i (\lambda_i, \alpha) \left| \left. \frac{1}{2} \alpha \right| K_{\frac{1}{2}} (\alpha) d\alpha \right. \geq \frac{1}{2} \prod_{j=1}^{9} \lambda_j \left( \sum_{i=1}^{9} \lambda_i \right)^{-8} X^{\frac{11}{3}} \]

Then we complete the proof of this lemma.

\[ \square \]

5 The intermediate region

Lemma 5.1. We have

\[ \int_{-\infty}^{\infty} \left| F_i (\lambda_i, \alpha) \right|^{\frac{5}{3}} K_{\frac{1}{2}} (\alpha) d\alpha \ll X^{\frac{5}{3} + \frac{1}{3}}, \]

\[ \int_{-\infty}^{\infty} \left| F_i (\lambda_i, \alpha) \right|^{\frac{7}{3}} K_{\frac{1}{2}} (\alpha) d\alpha \ll X^{\frac{7}{3} + \frac{1}{3}}, \]

\[ \int_{-\infty}^{\infty} \left| F_i (\lambda_i, \alpha) \right|^{\frac{11}{3}} K_{\frac{1}{2}} (\alpha) d\alpha \ll X^{\frac{11}{3} + \frac{1}{3}}, \]

and

\[ \int_{-\infty}^{\infty} \left| G(-\alpha) \right|^{\frac{21}{3}} K_{\frac{1}{2}} (\alpha) d\alpha \ll NL \]

for \( i = 1, 2, 3, 4, j = 5, 6, 7 \) and \( k = 8, 9 \).
Proof We have
\[
\int_{-\infty}^{+\infty} |F_j(\lambda_j \alpha)|^{17} K_3(\alpha) \, d\alpha
\ll \sum_{m=-\infty}^{+\infty} \int_{m}^{m+1} |F_j(\lambda_j \alpha)|^{17} K_3(\alpha) \, d\alpha
\ll \frac{1}{\infty} \int_{m}^{m+1} |F_j(\lambda_j \alpha)|^{17} \, d\alpha + \sum_{m=2}^{+\infty} \int_{m}^{m+1} |F_j(\lambda_j \alpha)|^{17} \, d\alpha
\ll X^{13 + \frac{1}{r}}
\]
from (3.1) and Hua's inequality.

The proofs of the others are similar. So we omit them here.

Lemma 5.2 For every real number \( \alpha \in \mathcal{D} \), we have
\[
W(\alpha) \ll X^{\frac{1}{2} - \frac{1}{4} \delta + \frac{1}{4} r},
\]
where
\[
W(\alpha) = \min\{|G_1(\tau_1 \alpha)|, |G_2(\tau_2 \alpha)|\}.
\]

Proof For \( \alpha \in \mathcal{D} \) and \( i = 1, 2, 3, 4 \), we choose \( a_i, q_i \) such that
\[
|\lambda_i \alpha - a_i/q_i| \leq \frac{q_i}{Q}
\]
with \( (a_i, q_i) = 1 \) and \( 1 \leq q_i \leq Q \). We note that \( a_1 a_2 a_3 a_4 \neq 0 \). If \( q_1, q_2 \leq P \), then
\[
|a_2q_1| \geq q = \left\lfloor N^{1-\delta} \right\rfloor.
\]
On the other hand,
\[
|a_2q_1| \ll q_1 q_2 P \ll N^{18\delta},
\]
which is a contradiction. And so for at least one \( i \), \( P < q_i \ll Q \). Hence we see that the desired inequality for \( W(\alpha) \) follows from Weyl's inequality (see [7], Lemma 2.4).
**Lemma 5.3** The following inequality holds:

\[
\int_{D}^{\frac{10}{9}} \prod_{i=1}^{n} F_i(\lambda_i \alpha) G(-\alpha) e^{\left(-\frac{1}{3} \alpha \right)} K_1(\alpha) d\alpha \ll X^{\frac{11}{12} - \frac{1}{12} + \epsilon}.
\]

**Proof** We have

\[
\int_{D}^{\frac{9}{9}} \prod_{i=1}^{n} |F_i(\lambda_i \alpha)| |K_1(\alpha)| d\alpha \ll \max_{\alpha \in D} |W(\alpha)| \left( \left( \int_{-\infty}^{\infty} |F_1(\lambda_1 \alpha)|^9 d\alpha \right)^{\frac{1}{9}} + \left( \int_{-\infty}^{\infty} |F_2(\lambda_2 \alpha)|^9 d\alpha \right)^{\frac{1}{9}} \right) \frac{1}{2}
\]

\[
\cdot \left( \prod_{j=3}^{9} \int_{-\infty}^{\infty} |F_j(\lambda_j \alpha)|^9 K_1(\alpha) d\alpha \right)^{\frac{1}{9}} \left( \prod_{k=6}^{8} \int_{-\infty}^{\infty} |F_k(\lambda_k \alpha)|^9 K_1(\alpha) d\alpha \right)^{\frac{1}{9}} \frac{1}{2}
\]

\[
\ll X^{\frac{11}{12} - \frac{1}{12} + \epsilon} (X^{3 + \frac{1}{12}})^{\frac{1}{9}} (X^{3 + \frac{1}{12}})^{\frac{1}{9}} (X^{3 + \frac{1}{12}})^{\frac{1}{9}} (X^{3 + \frac{1}{12}})^{\frac{1}{9}} \frac{1}{2} (NL)^{\frac{1}{2}}
\]

\[
\ll X^{\frac{11}{12} - \frac{1}{12} + \epsilon}.
\]

from Lemmas 5.1, 5.2 and Hölder’s inequality. \(\square\)

6 The trivial region

**Lemma 6.1** (see [8], Lemma 2) Let

\[V(\alpha) = \sum e(\{f(x_1, \ldots, x_m)\}),\]

where the summation is over any finite set of values of \(x_1, \ldots, x_m\) (\(m \geq 5\)) and \(f\) be any real function. Then we have

\[
\int_{|\alpha| > A} |V(\alpha)|^2 K_1(\alpha) d\alpha \leq \frac{21}{A} \int_{-\infty}^{\infty} |V(\alpha)|^2 K_1(\alpha) d\alpha
\]

for any \(A > 4\). The following inequality holds.

**Lemma 6.2** We have

\[
\int_{n=1}^{10} F_i(\lambda_i \alpha) G(-\alpha) e^{\left(-\frac{1}{3} \alpha \right)} K_1(\alpha) d\alpha \ll X^{\frac{11}{12} - \frac{78+\epsilon}{12}}.
\]
Proof. We have
\[
\int \prod_{i=1}^{10} F_i(\lambda_\alpha) G(-\alpha) e\left(-\frac{1}{4} \alpha \right) K_{\frac{1}{4}}(\alpha) \, d\alpha
\]
\[
\ll \frac{1}{P} \int_{-\infty}^{+\infty} \left| \prod_{i=1}^{10} F_i(\lambda_\alpha) G(-\alpha) \right| K_{\frac{1}{4}}(\alpha) \, d\alpha
\]
\[
\ll N^{-53} \max \left| F_1(\lambda_\alpha) \right|^2 \left( \int_{-\infty}^{+\infty} \left| F_1(\lambda_\alpha) \right|^9 \right)^{\frac{1}{9}} \left( \int_{-\infty}^{+\infty} \left| F_2(\lambda_\alpha) \right|^9 \right)^{\frac{1}{9}}
\]
\[
\cdot \left( \prod_{j=3}^{5} \int_{-\infty}^{+\infty} \left| F_j(\lambda_\alpha) \right|^{16} K_{\frac{1}{4}}(\alpha) \, d\alpha \right)^{\frac{1}{6}} \left( \prod_{k=6}^{10} \int_{-\infty}^{+\infty} \left| F_k(\lambda_\alpha) \right|^{21} K_{\frac{1}{4}}(\alpha) \, d\alpha \right)^{\frac{1}{9}}
\]
\[
\cdot \left( \int_{-\infty}^{+\infty} \left| G(-\alpha) \right|^3 K_{\frac{1}{4}}(\alpha) \, d\alpha \right)^{\frac{1}{3}}
\]
\[
\ll X^{\frac{11}{12} - 6\delta + \varepsilon}
\]
from Lemmas 5.1, 6.1 and Schwarz's inequality. \( \Box \)

7 Conclusions
In this paper, we proved the conjecture for the non-integer part of a nonlinear differential form representing primes presented in [1] by using Tumura-Clunie type inequalities. Compared with the original proof, the new one is simpler and more easily understood. Similar problems can be treated with the same procedure.

Acknowledgements
I would like to thank the anonymous referee for his helpful comments and suggestions, which improved the manuscript.

Competing interests
The author declares that he has no competing interests.

Authors’ contributions
The author carried out all work of this article and the main theorem. The author read and approved the final manuscript.

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Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 17 June 2017 Accepted: 1 August 2017 Published online: 15 August 2017

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