Simultaneous polarization squeezing in polarized $N$ photon state and diminution on a squeezing operation

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We study polarization squeezing of a pure photon number state which is obviously polarized but the mere change in the basis of polarization leads to simultaneous polarization squeezing in all the components of Stokes operator vector except those falling along or perpendicular to the direction of polarization state, is observed. We use the most general definition of polarization squeezing and discuss the experimental feasibility of the result. We also observe that a squeezing operation like non-degenerate parametric amplification of the state does not reveal simultaneous squeezing in all Stokes operator vectors and decreases in this sense.

INTRODUCTION

Products of quantum fluctuations in two non commuting observables satisfy the uncertainty relation but the individual fluctuations can be reduced and this gives the well known concept of squeezing. In classical optics, the polarization state of light beam can be visualized as direction of a Stokes vector in the Poincare sphere and is determined by the four Stokes parameters $S_0, S_1, S_2, S_3$. For a monochromatic unidirectional light traveling along z-direction, the classical Stokes parameters $S_0$ and $S = S_1, S_2, S_3$ are defined as

$$S_{0,1} = \langle \mathcal{E}_x \mathcal{E}_x^* \rangle \pm \langle \mathcal{E}_x \mathcal{E}_y^* \rangle, \quad S_2 + i S_3 = 2 \langle \mathcal{E}_x \mathcal{E}_y \rangle,$$

where $\mathcal{E}_{x,y}$ are the components of analytic signal for the electric field. For perfectly polarized light

$$S_0^2 = S_1^2 + S_2^2 + S_3^2,$$

and the point $(S_1, S_2, S_3)$ is on a sphere of radius $S_0$, called the Poincare sphere.

Quantum mechanical analogue of Stokes parameters can also be defined to characterize quantum nature of polarization. These are observables and can be associated with hermitian operators $\hat{S} = \hat{S}_{1,2,3}$ and defined as

$$\hat{S}_{0,1} = \hat{a}_x^\dagger \hat{a}_x \pm \hat{a}_y^\dagger \hat{a}_y, \quad \hat{S}_2 + i \hat{S}_3 = 2 \hat{a}_x^\dagger \hat{a}_y,$$

where $\hat{a}_{x,y}$ and $\hat{a}_{x,y}^\dagger$ are annihilation and creation operators, respectively for the two orthogonal linear polarization modes $x$ and $y$. These Stokes operators obey the commutation relations

$$[\hat{S}_0, \hat{S}_j] = 0, \quad [\hat{S}_j, \hat{S}_k] = 2i \sum_l \epsilon_{jkl} \hat{S}_l,$$

following the SU(2) algebra. Here, $\epsilon_{jkl}$ is Levi-Civita symbol for $(j, k, l = 1, 2, \text{or} 3)$. The obvious uncertainty relations for the fluctuation in Stokes operators are,

$$V_j V_k \geq \langle \hat{S}_j \rangle^2, \quad V_j \equiv \langle \hat{S}_j^2 \rangle - \langle \hat{S}_j \rangle^2.$$

Polarization of optical fields is a well known and old concept in classical optics and there is a very good correspondence between Stokes parameters and Stokes operators. The Stokes parameters involve coherence functions of order (1,1) and it has been realized that these are insufficient to describe polarization completely and, e.g., $S = 0$ does not represent only unpolarized light. These parameters still remain important mainly due to the non-classicalities associated with polarization, viz., polarization squeezing [5–10] and polarization entanglement [11].

The Stokes operators are the relevant continuous variables for the system describing polarization. Squeezed radiation states in quantum optics are identified by the property that their quantum fluctuations are reduced below the standard quantum limit in one of the quadrature components. Similar to this concept, polarization squeezing is defined using the commutation relations followed by Stokes operators and uncertainty products. Existence of a minimum quantum limit for product of uncertainties in measurement of two different Stokes operators leads to the concept of polarization squeezing discussed in the next section, where one of the uncertainties $V_j$ and $V_k$ can be reduced below $| \langle \hat{S}_i \rangle |$ at the expense of the other, e.g. in $\langle \hat{S}_i \rangle$.

These properties are of paramount importance in the real world applications on different scales as quantum Stokes operators and non-classical polarization can be used for quantum information protocols and quantum communication. It is convenient to study polarization squeezing because it is easy to measure the stokes parameters using linear optical elements and thus polarization squeezing is easy to experimentally observe. The direct measurement schemes for measuring these parameters are developed methods which preserve quantum noise property. Looking forward to such applications, it is desirable to devise methods for generation of states with appreciable polarization squeezing.

In the present paper, we first discuss the significance of dif-
ferent criteria for polarization squeezing in the next section. 
In section 3, We use the most general criterion \[12\] for polar-
ization squeezing to show that in case of a polarized \( N \) photon 
number state, all the three orthogonal components of Stokes 
operator vector may be squeezed and as a matter of fact, al-
most all components are squeezed. We consider the polarization 
squeezing along a general component of Stokes operator 
vector and show that there is polarization squeezing unless 
the component is either along or perpendicular to the direc-
tion of polarization of light. In section 4, we show that after 
a squeezing operation like parametric amplification applied to 
a photon number state, squeezing is observed only in certain 
Stokes operators in specific conditions.

**POLARIZATION SQUEEZING**

First definition of polarization squeezing is due to Chirkin 
et al. \[5\] in terms of variances of Stokes operators for a given 
state and for an equally intense coherent state, written as
\[
V_j < V_j(\text{coh}) = \langle \hat{S}_0 \rangle,
\]
\( j \), i.e., \( \hat{S}_j \) is squeezed if \( V_j \) is less than \( V_j \) 
for equally intense coherent light which gives the value \( \langle \hat{S}_0 \rangle \) of the variance. This definition has been used by some authors \[6\].

Heersink et al. \[7\] defined polarization squeezing using the 
uncertainty relations \[5\] in the form
\[
V_j < |\langle \hat{S}_j \rangle| < V_k, \quad j \neq k \neq l,
\]
for squeezing of \( \hat{S}_j \). This definition has also been used by 
some authors \[8\].

Luis \[9\] considered various criteria for polarization squeeze-
ning and compared their stringency. He finally gave the crite-
ron for polarization squeezing of a component of \( \hat{S} \) along a 
unit vector \( n \) as
\[
V_n < |\langle \hat{S}_{n\perp} \rangle|,
\]
here \( \hat{S}_{n\perp} \) is component of \( \hat{S} \) along unit vector \( n \perp \) which 
is perpendicular to \( n \). For suitable orthogonal components \( n \) 
and \( n \perp \), he discussed the order of stringency of the various 
criteria and it can be represented as
\[
V_n < \langle \hat{S}_{n\perp} \rangle^2/\langle \hat{S}_0 \rangle < |\langle \hat{S}_{n\perp} \rangle| < \langle \hat{S}_0 \rangle.
\]

The authors finally have written the criterion for polari-
sation squeezing \[12\] in the form
\[
V_n \equiv \langle \Delta \hat{S}_n^2 \rangle < |\langle \hat{S}_{n\perp} \rangle|_{\text{max}}
= \sqrt{\langle \hat{S} \rangle^2 - \langle \hat{S}_n \rangle^2},
\]
arguing that for a given component \( \hat{S}_n \) there are infinite 
directions \( n \perp \) and consideration of the maximum possible 
value of \( |\langle \hat{S}_{n\perp} \rangle| \).

It can be seen that each of Eqs. \([5] - [9] \) and \([12] \) describes 
a non-classicality in quantum optics but only \([11] \), \([8] \) and \([10] \) 
are related to uncertainty relations. We shall use the crite-
rium \([10] \) for polarization squeezing which is most general and 
based on the actual uncertainty relations. We may also define 
squeezing factor \( S_n \) and degree of squeezing \( D_n \) by writing
\[
S_n = \frac{V_n}{\sqrt{|\langle \hat{S} \rangle^2 - \langle \hat{S}_n \rangle^2|}}, \quad D_n = 1 - S_n.
\]

Non-classicalities appear when \( 1 > S_n > 0 \) and the degree of 
squeezing \( D_n \) lies between 0 and 1. If \( \rho \) is density operator of 
radiation given in the diagonal Sudarshan-Glauber representa-
tion \[13\] by
\[
\rho = \int d^2 \alpha \ d^2 \beta \ P(\alpha, \beta) |\alpha, \beta \rangle \langle \alpha, \beta|.
\]

where \( |\alpha, \beta \rangle \) are the coherent states defined as
\( (a_x, a_y)|\alpha, \beta \rangle = (\alpha, \beta)|\alpha, \beta \rangle \). Eq. \([6] \) then gives
\[
V_j - \langle \hat{S}_0 \rangle = \int d^2 \alpha \ d^2 \beta \ P(\alpha, \beta) [f_j(\alpha, \beta) - \langle f_j \rangle]^2, \quad (13)
\]
with
\[
\langle f_j \rangle = \int d^2 \alpha \ d^2 \beta \ P(\alpha, \beta) [f_j(\alpha, \beta)],
\]
\( f_1(\alpha, \beta) \equiv |\alpha|^2 - |\beta|^2, \quad f_2 + i f_3 = 2\alpha \beta, \)
while \( f_j \)'s are real functions of \( \alpha \) and \( \beta \) and holding of 
\( V_j < \langle \hat{S}_0 \rangle \) rules out possibility of existence of a non-negative 
weight function \( P(\alpha, \beta) \) which could be identified to a clas-
sical probability distribution. Similarly, Eq. \( [10] \) for \( j = 1 \) 
gives
\[
V_1 - \sqrt{|\langle \hat{S}_1 \rangle^2 - \langle \hat{S}_1 \rangle^2|}
= \sqrt{|\langle \hat{S}_1 \rangle^2| - |\langle \hat{S}_1 \rangle|^2 - |\langle \hat{S}_2 + i \hat{S}_3 \rangle|}
= \int d^2 \alpha \ d^2 \beta \ P(\alpha, \beta) [(f_1(\alpha, \beta) - \langle f_1 \rangle)^2 + (|\alpha| - |\beta|)^2]
< 0.
\]

which also denies the existence of a positive definite \( P(\alpha, \beta) \). 
It can be seen that if Eq. \([14] \) which represents criterion \([10] \) 
holds then Eq. \([13] \) representing criterion \([6] \) has to hold.

**POLARIZATION SQUEEZING OF PHOTON NUMBER 
STATE**

Let us now consider a polarized \( N \) photon state \( |\psi \rangle \) travel-
ing along z-direction, the polarization being given by the unit
other unit polarization vector
\[ \epsilon = \epsilon_x e_x + \epsilon_y e_y, \quad \epsilon_x = \cos \frac{\theta}{2}, \quad \epsilon_y = e^{i\phi} \sin \frac{\theta}{2}, \]  
(15)
where \( e_{x,y} \) are unit vectors along the x and y-directions. On the Poincare sphere, this polarization state can be represented by the unit vector
\[ m = \cos \theta e_x + \sin \theta (\cos \phi e_y + \sin \phi e_z). \]  
(16)
Mode orthogonally polarized to \( \epsilon \) can be represented by another unit polarization vector
\[ \epsilon_{\perp} = \epsilon_{\perp x} e_x + \epsilon_{\perp y} e_y, \quad \epsilon_{\perp x} = -\sin \frac{\theta}{2}, \quad \epsilon_{\perp y} = e^{i\phi} \cos \frac{\theta}{2}. \]  
(17)
Annihilation operators for light in this changed basis \((\epsilon, \epsilon_{\perp})\) are given by
\[ \hat{a}_{\epsilon} = \epsilon_x^* \hat{a}_x + \epsilon_y^* \hat{a}_y, \quad \hat{a}_{\epsilon_{\perp}} = \epsilon_{\perp x}^* \hat{a}_x + \epsilon_{\perp y}^* \hat{a}_y, \]  
(18)
which allows us to write the same in \((x,y)\) mode as
\[ \hat{a}_x = \epsilon_x \hat{a}_x + \epsilon_{\perp x} \hat{a}_{\epsilon_{\perp}}, \quad \hat{a}_y = \epsilon_y \hat{a}_y + \epsilon_{\perp y} \hat{a}_{\epsilon_{\perp}}. \]  
(19)
The state \(|\psi\rangle\) in \((\epsilon, \epsilon_{\perp})\) can then be written as
\[ |\psi\rangle = (N!)^{1/2} (\hat{a}_{\perp}^*)^N |\text{vac}\rangle, \]  
(20)
where \(|\text{vac}\rangle\) is vacuum state satisfying \(\hat{a}_{\epsilon} |\text{vac}\rangle = 0\). On considering the normal ordering of the operators \((13)\), expectation values of Stokes operators \((4)\) and their squares and anti-commutators can be obtained by straightforward calculations which on simplification (using \(\hat{a}_{\epsilon_{\perp}} |\psi\rangle = 0\)) give
\[ \langle \hat{S}_0 \rangle = N, \quad \langle \hat{S}_1 \rangle = N \cos \theta, \]  
\[ \langle \hat{S}_2 \rangle = N \sin \theta \cos \phi, \quad \langle \hat{S}_3 \rangle = N \sin \theta \sin \phi, \]  
\[ \langle \hat{S}_0^2 \rangle = N(N-1), \]  
\[ \langle \hat{S}_1^2 \rangle = N(N-1) \cos^2 \theta + N, \]  
\[ \langle \hat{S}_2^2 \rangle = N(N-1) \sin^2 \theta \cos^2 \phi + N, \]  
\[ \langle \hat{S}_3^2 \rangle = N(N-1) \sin^2 \theta \sin^2 \phi, \]  
\[ \langle \{\hat{S}_1, \hat{S}_2\} \rangle = 2N(N-1) \cos \theta \sin \theta \cos \phi, \]  
\[ \langle \{\hat{S}_1, \hat{S}_3\} \rangle = 2N(N-1) \cos \theta \sin \theta \sin \phi, \]  
\[ \langle \{\hat{S}_2, \hat{S}_3\} \rangle = 2N(N-1) \sin \theta \cos \phi \sin \phi. \]  
(21)

Now, to study squeezing of component \(\hat{S}_n \equiv n \cdot \hat{S}\) of Stokes vector along unit vector \(n = (n_1, n_2, n_3)\), we can write down the general expressions
\[ \langle \hat{S}_n \rangle = N \langle n \cdot m \rangle, \quad \langle \hat{S}_n^2 \rangle = N(N-1) \langle n \cdot m \rangle^2 + N. \]  
(22)
The variance \(V_n\) of this general component \(\hat{S}_n\) is
\[ V_n = N[1 - \langle n \cdot m \rangle^2], \]  
(23)
and for polarization squeezing this is to be compared with maximum value of modulus of expectation value of component of \(\hat{S}\) perpendicular to \(n\), i.e., with
\[ \langle \hat{S}_n \rangle_{\text{max}} = (|\langle \hat{S} \rangle|^2 - |\langle \hat{S}_n \rangle|^2)^{1/2} = N[1 - \langle n \cdot m \rangle^2]^{1/2}. \]  
(24)
Since, \(1 - \langle n \cdot m \rangle^2 < 1\) for \(\langle n \cdot m \rangle \neq 0\), Eqs. \((13)\) and \((24)\) make it clear that, \(V_n < |\langle \hat{S}_n \rangle|\) holds unless \(\langle n \cdot m \rangle = 0, 1\) with squeezing factor and degree of squeezing is
\[ S_n = [1 - \langle n \cdot m \rangle^2]^{1/2}, \]  
\[ D_n = 1 - S_n = 1 - [1 - \langle n \cdot m \rangle^2]^{1/2}. \]  
(25)
We observe, \(S_n < 1\) for all \(n\) and therefore all components \(\hat{S}_n\) are squeezed unless \(n\) is along or perpendicular to \(m\). Thus, for any polarized \(N\) photon state, inequalities giving polarization squeezing are satisfied for all components of Stokes operators and squeezing occurs unless the component is along a direction which is perpendicular or same as the direction describing polarization of light on Poincare sphere.

Regarding the experimental observation of this effect, it should be noted that for \(n = (\cos \theta_0, \sin \theta_0 \cos \phi_0, \sin \theta_0 \sin \phi_0)\), Eq. \((13)\) gives,
\[ n \cdot \hat{S} = \cos \theta_0 (\hat{a}_{x0}^\dagger \hat{a}_x - \hat{a}_{y0}^\dagger \hat{a}_y) + \sin \theta_0 (e^{-i\phi_0} \hat{a}_{x0}^\dagger \hat{a}_y + e^{i\phi_0} \hat{a}_{y0}^\dagger \hat{a}_x) = \hat{a}_{x0}^\dagger \hat{a}_{y0} - \hat{a}_{y0}^\dagger \hat{a}_{x0}, \]  
where
\[ \epsilon_0 = \cos \frac{\theta_0}{2} e_x + e^{i\phi_0} \sin \frac{\theta_0}{2} e_y, \]  
\[ \epsilon_{0\perp} = -\sin \frac{\theta_0}{2} e_x + e^{i\phi_0} \cos \frac{\theta_0}{2} e_y. \]
Thus, to measure \(\langle n \cdot \hat{S} \rangle\) and \(\langle n \cdot \hat{S} \rangle^2\) one has to make measurements of \((\hat{N}_{x0} - \hat{N}_{x0\perp})\) and \((\hat{N}_{y0} - \hat{N}_{y0\perp})\), where \(\hat{N}_{x0}\) and \(\hat{N}_{y0\perp}\) are the photon number operators \(\hat{a}_{x0}^\dagger \hat{a}_{x0}\) and \(\hat{a}_{y0}^\dagger \hat{a}_{y0\perp}\), respectively and this can be done easily by

1. Introducing phase shift \(\phi_0\) in the y-linearly polarized mode.
2. Rotating the plane of polarization by angle \(\frac{\phi_0}{2}\).
3. Measuring \((\hat{N}_x - \hat{N}_y)\) and \((\hat{N}_x - \hat{N}_y)^2\) in the changed basis.

**PHOTON NUMBER STATE UNDER SQUEEZING OPERATION**

Photon number state polarized in mode \((\epsilon, \epsilon_{\perp})\) with no photon in polarization mode \(\epsilon_{\perp}\) given by Eq. \((20)\) is now subjected to some nonlinear interaction like parametric amplification \((14)\). The interaction Hamiltonian for such an operation
is $H = g(\hat{a}_x^\dagger \hat{a}_y + \hat{a}_y^\dagger \hat{a}_x)$ and the annihilation and creation operator after the interaction for time $t$ can be written as
\[
\hat{a}_x(t) = (\cosh gt) \hat{a}_x - i(\sinh gt) \hat{a}_y^\dagger,
\]
\[
\hat{a}_y(t) = (\cosh gt) \hat{a}_y - i(\sinh gt) \hat{a}_x^\dagger,
\]
where $\hat{a}_x$ and $\hat{a}_y$ given by Eq. (19) with $(\varepsilon_x, \varepsilon_y)$ and $(\varepsilon_{\perp x}, \varepsilon_{\perp y})$ written in Eqs. (15) and (17). Average values of Stokes parameters and their variances are obtained as
\[
\langle \hat{S}_1 \rangle = N \cos \theta,
\]
\[
\langle \hat{S}_2 \rangle = N(c^2 + s^2) \sin \theta \cos \phi,
\]
\[
\langle \hat{S}_3 \rangle = N(c^2 + s^2) \sin \theta \sin \phi,
\]
and
\[
V_1 = N \sin^2 \theta,
\]
\[
V_2 = N(c^2 + s^2)(1 - \sin^2 \theta \cos^2 \phi) + 2c^2 s^2 [N^2(1 - \sin^2 \theta \sin^2 \phi) + N(1 + \sin^2 \theta \sin^2 \phi) + 2],
\]
\[
V_3 = N(c^2 + s^2)(1 - \sin^2 \theta \sin^2 \phi) + 2c^2 s^2 [N^2(1 - \sin^2 \theta \cos^2 \phi) + N(1 + \sin^2 \theta \cos^2 \phi) + 2],
\]
where $c = \cosh gt$ and $s = \sinh gt$, $gt$ being interaction time.

As, it is clear by having a look at the above expressions for mean values and variances of Stokes operators, it is not trivial to generalize these expressions for the Stokes operator $\hat{S}_n$ along unit vector $n$. But, it is interesting to find the extent of squeezing along the three Stokes vectors $\hat{S}_1$, $\hat{S}_2$ and $\hat{S}_3$, fitting these values in the criterion (11).

If we see squeezing in $\hat{S}_1$ operator, the squeezing factor on substituting the appropriate values and simplifying the expression is
\[
S_1 = \frac{N \sin^2 \theta}{\sqrt{N^2(c^2 + s^2)^2 \sin^2 \theta}} = \frac{\sin \theta}{\cosh 2gt},
\]
which shows squeezing in $\hat{S}_1$ component as $\sin \theta < \cosh 2gt$. Degree of squeezing is thus given by
\[
D_1 = 1 - \frac{\sin \theta}{\cosh 2gt}.
\]

We may compare the degree of squeezing in this case with $D_1 = 1 - \sin \theta$, at $gt = 0$. It shows that the squeezing in $\hat{S}_1$ increases after the parametric amplification like operation on the considered state.

To investigate squeezing in the component $\hat{S}_2$, the expression for squeezing factor can be written as
\[
S_2 = \frac{N(c^2 + s^2)(1 - \sin^2 \theta \cos^2 \phi)}{\sqrt{N^2(c^2 + s^2)^2 \sin^2 \theta \sin^2 \phi}} + \frac{2c^2 s^2 [N^2 + N + 2 - (N^2 - N) \sin^2 \theta \sin^2 \phi]}{\sqrt{N^2(c^2 + s^2)^2 \sin^2 \theta \sin^2 \phi}}.
\]

To have a better insight of the squeezing along $\hat{S}_2$, we plot this expression with respect to $\theta$ for a fixed number of photons $N = 8$. Fig. 1 and Fig. 2 clearly indicate the occurrence of polarization squeezing in the following two cases derived from Eq. (30).

Case 1 : For plane polarized light, i.e., $\sin \phi = 0$,
\[
S_2 = (c^2 + s^2)^2 \cos \theta | + 2c^2 s^2 \left[ \frac{N^2 + N + 2}{N} \right] \sec \theta |. \ (31)
\]
This expression is arithmetic mean of the double of the two terms which gives the minimum value of $S_2$ as double of the geometric mean of the two terms as
\[
S_2(\min) = 2 \sqrt{2(c^2 + s^2)^2 \left[ \frac{N^2 + N + 2}{N} \right] c^2 s^2},
\]
and given after simplification as
\[
S_2(\min) = 2 \sqrt{\frac{N^2 + N + 2}{8N}} \sinh 4gt. \ (32)
\]
This expression shows squeezing in polarization for very small times of interaction and
\[
\cos \theta = \tanh 2gt \sqrt{\frac{N^2 + N + 2}{N}}.
\]

Case 2 : For circularly polarized light, i.e., $\sin \theta = 1$ and $\cos \phi = 0$, squeezing factor in this case observed after simplification is
\[
S_2 = \cosh 2gt + \left[ \frac{2N + 2}{N} \right] \sin^2 2gt \cosh 2gt, \ (33)
\]
and it seems to be greater than unity resulting in no squeezing for circularly polarized light.

For fixed number of photons $N$, $\phi = 0$ shows that polarization squeezing may occur for very small interaction times, e.g., $gt = 0.1$ as seen in Fig. 1 but there is no squeezing for $\phi = \pi/2$ as depicted in Fig. 2.

We also check the polarization squeezing along $\hat{S}_4$, however it does not reveal squeezing in either of the cases of circularly or plane polarized light.

**RESULT AND DISCUSSION**

We found the simultaneous polarization squeezing in almost all components of Stokes operator vector except those along or perpendicular to the direction of polarization just by changing the basis of polarization. Most importantly, it happens under no condition and gives a general result about the photon number state.
case of plane polarized light. The pattern of polarization squeezing for very small interaction times is along \( n \) or \( m \) except for which \( n, m = 0 \) or 1, where \( m \) represents polarization of pure photon number state in Poincare sphere.

However, we observe that the simultaneous polarization squeezing of all the components of Stokes operator in pure polarized photon number state is not observed after a squeezing operation. However, the state, after it is subjected to a nonlinear interaction for time \( t \) is found to be more squeezed than at \( gt = 0 \) along \( \hat{S}_1 \) irrespective of the nature of polarization. Squeezing along \( \hat{S}_2 \) is seen for plane polarized light, however, \( \hat{S}_3 \) does not exhibit any squeezing. This result is in agreement with the results reported in the previous investigation which shows simultaneous polarization squeezing in almost all components of Stokes operator vector \( \hat{S} \) except for which \( n.m = 0 \) or 1, where \( m \) represents polarization of pure photon number state in Poincare sphere.

The non-occurrence of squeezing in \( \hat{S}_2 \) at \( gt = 0 \) for circularly polarized light is clear from the fact that in this case, \( m \) is along or opposite to \( z \)-axis and is perpendicular to \( n \) which is along \( \hat{S}_2 \). Similarly, non-occurrence of squeezing in \( \hat{S}_3 \) at \( gt = 0 \) for plane polarized light is evident as \( n \) is along \( z \)-axis and \( m \) in \( x-y \) plane, while in case of circular polarization \( n \) is along \( z \)-direction and \( m \) is either along \( z \)-direction or opposite to it. The expressions for squeezing factor for \( \hat{S}_2 \) as a function of interaction time shows the occurrence of polarization squeezing for very small interaction times in case of plane polarized light. The pattern of polarization squeezing with the angles \( \theta \) and \( \phi \) describing polarization is shown in Fig.[1] and Fig.[2].

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