Modelling limit stress of a seam roof ahead of a working face

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Abstract. Solving of boundary problem of geomechanic state of a roof working developed along the seam is introduced in the article. The rock mass works under the condition of a plane-strain deformation with marginal seam zones are in a limit stress state. The problem is solved by boundary element method using Coulomb – Mohr and Mohr – Kuznetsov strength indices.

1. Introductions

Quantitative evaluation of a stress state of a rock mass with hard rock enclosing a coal seam and its workings with limit stress zones on its sides is an important and challenging research and production issue as different geodynamic phenomena take place in such zones [1]. The solutions of several problems on limit stress state of a coal seam sides with hard walls together with the presence of a soft layer in workings are introduced in [2].

Depending on strength characteristics of a coal seam and enclosing wall rocks the transition of this rock into a limit state during the seam development can take place in different ways. In one case a limit state of a seam roof with its subsequent failure can take place under relatively small size of a goaf. In other cases, before the roof failure, the size of a goaf can be of rather significant volume and could result in rock heaving in a seam soil that in case of a roof failure can cause accidents while ventilating mine openings or bring about a failure of the operating mining equipment [3]. Such cases lead to intentional roof breaking to prevent sizable overhanging of a roof [4].

The researching results of a coal mass state for several characteristics of a working seam structure and size where the seam roof goes into limit state ahead of the face are presented in the article.

2. Problem setting and solutions

It is known that in the seam sides, during deep mining, limit stress zones with inelastic deformations are formed. And one can define only approximate dimension of such zones [5].

This research adopts a model of a massif where the presence of limit stress marginal zones in a coal seam with the size of LOT is taken into an account.

Solving the problem, we assume:

1) the path of the workings and its dimensions along x-axis are vastly superior then the dimensions in its cross-sectional plane 0yz, hence we may assume that rock mass and slots in the vicinity of the workings are under the state of plane strain.

2) a seam strength is significantly below the strength of its enclosing rock;
3) compressive normal stresses are positive.

Further, the following task can be set. In a rock mass modelled by weightless plane there is a rectangular cross-section working with the dimensions \( b \times h \) which was made in a depth \( H \) along the seam with the thickness of \( m = h \). Support reaction \( p \) is in effect in the roof and sill of the working. The massif is loaded with gravitational pressure \( \gamma H \) (\( \gamma \) – average volume density ofoverlaying rock), and \( \lambda \gamma H \) is pressure on sides (\( \lambda \) – coefficient of lateral thrust) (figure 1).

While solving the problem we define stress field in the vicinity of the working, providing that limit stress zones of a seam are formed prior.

A defined problem demands solving of several other problems. Firstly, we set a problem on the state of the massif in a limit stress zone where the sliding lines and three section appear (up and down from the y-axis) (see figure 1). Unique features of a seam state reveal themselves on each of them. Section VCN (V1C1N1) – is a section of a limit uniaxial compression with the length \( L_k \). Sections V1BCNGDC1 and VCDG1N1C1B with the dimension \( L_{pr} \) are Prandtl zones, and section GDG1P1P with the dimension \( L_p \) – is a section of seam inelastic deformation.

The seam experiences uniaxial compression in the area of protrusion in a shape of prism \( VBV_1 \). Principal stress \( \sigma_1 \) has an effect along the edge of seam outcorp forming an angle \( \varepsilon \) with system of isogonal sliding lines direction. It is equal to limit seam strength on uniaxial compression \( \sigma_0 \).

Considering that along the seam contacts with neighboring rocks the rock discontinuity is possible and it is manifested in the form of a seam sliding motion, so both on the seam and on its contacts with side rocks there will exist two limit state simultaneously – normal (for a seam) and unique (for the seam contacts) [5-7]. It can be demonstrated by Mohr’s stress circle (figure 2). It is seen from the figure that the first area element of sliding surface with normal \( \nu \) according to the Coulomb-Mohr criteria is in the contacting point of a stress circle and the rectilinear envelope 1 of the seam limit state with the angle of gradient \( \rho \) to X-axis.
Figure 2. Scheme of defining angles ε, ω, φ, θ in limit stress zones.

The position of the second area with normal η is calculated according to Mohr-Kuznetsov criteria as a result of crossing with the stress circle of rectilinear envelope 2 of the limit state along the surface of weakness making an angle of gradient ρ' with X-axis. The area with normal η coincides with seam contacting with surrounding rock. Angles ρ and ρ' are angles of internal friction, K and K' are the cohesion coefficients, respectively for the seam and its contact with side rocks, s0 – is a strength limit on the seam uniaxial compression.

Angle θ between the areas with normal between ν and η we calculate in figure 2

$$\theta = \frac{\pi}{4} + \frac{1}{2} (\rho - \rho') - \frac{1}{2} \arcsin \left[ \frac{\sin \rho'}{\sin \rho} \left( 1 - \frac{c}{s_*} \right) \right],$$

where s* and s*' values are defined according to the formulas

$$s_* = c + \frac{s_1 + s_3}{2}, \quad s'_* = c' + \frac{s_1 + s_3}{2}.$$

Resulted from figure 2 and they are presented as reduced stress in a seam and on the surface of weakness, respectively [2, 5]. Values c and c' are expressed through K, ρ, K', ρ' using the formulas (figure 2).

The angles ε and ω are formed by the sliding surface with normal ν and primary stress σ1, σ3. As it is seen from figure 2 they are calculated by the formulas

$$\varepsilon = \frac{\pi - \rho}{4}, \quad \omega = \frac{\pi + \rho}{4}.$$
Two systems of sliding lines are formed in Prandtl zones (figure 1). The first system it is the family of logarithmic spirals and the second system is a bunch of radial lines.

Angle $\Delta \phi$ of the sectors and radial lines length are defined according to [5]

$$\Delta \phi = \phi - \theta, \quad r = r_0 \cdot e^{-\Delta \phi \cdot \frac{r}{2}},$$

where $r_0$ – is an interval $VB(V_iB)$ adjacent to the area of protrusion $VBV_i$, $r$ – border (marginal) interval of Prandtl sector equal to the interval $VC$ (or $V_iC_i$).

Stresses in Prandtl zones along the respective radial line are constant and they change exponentially along the logarithmic spirals [5]. For example, the stress in points $C$ and $D$ is defined as

$$\sigma_C = \sigma_0 \cdot e^{2\Delta \phi \cdot \frac{r}{2}}, \quad \sigma_D = \sigma_C \cdot e^{2\Delta \phi \cdot \frac{r}{2}}. \quad (1)$$

In the areas of limit stress zone positioned to the right of the sliding line $DG$ (figure 1) to define angles $\theta$ it is necessary to know reduced stress in junction points on a slope contacts with enclosing rocks. To define them it is recommended to use the function of reduced stresses $\sigma_r$ along $Y$-axis that change exponentially [5]

$$\sigma_r = \sigma_{0r} \cdot e^{k \left(\frac{r-b}{2}\right)},$$

where $\sigma_{0r}$ – reduced stress on the edge of the seam and $k$ parameter are defined according to the formulas [5].

$$\sigma_{0r} = \frac{\sigma_0}{2 \cdot \sin \rho}, \quad k = \frac{2}{m} \cdot \tan \theta \cdot \frac{r}{2} \cdot \rho.$$ 

Normal and shear stresses in limit stress zones on the contact of the seam with a massif are expressed by evident relations arise from figure 2

$$\sigma_\eta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cdot \cos 2\phi, \quad \tau_\eta = \frac{\sigma_1 - \sigma_3}{2} \cdot \sin 2\phi,$$

which results in obtaining formulas for calculating primary stresses.

$$\sigma_1 = \sigma_\eta \left(1 + \sin \rho\right) - c, \quad \sigma_3 = \sigma_\eta \left(1 - \sin \rho\right) - c,$$

$$\sigma_\eta = \sigma_\eta \left(1 - \sin \rho \cdot \cos 2\phi\right) - c, \quad \tau_\eta = \sigma_\eta \cdot \sin \rho \cdot \sin 2\phi,$$

where $\phi$ – is an angle between $\sigma_1$ and a normal towards the surface of weakness.

As it follows from figure 2 there is a link between angles $\phi, \theta, \varepsilon,$ and two more important relations

$$\varphi = \frac{\pi}{2} - (\varepsilon + \theta), \quad \left(K \cdot \cot \rho + \frac{\sigma_0}{2}\right) \sin \rho = \frac{\sigma_0}{2}, \quad \left(K \cdot \cot \rho + \frac{\sigma_1 + \sigma_3}{2}\right) \sin \rho = \frac{\sigma_1 - \sigma_3}{2}. \quad (2)$$
The transformation of the relations (2) allows us to get the connection between \( K \) and \( \sigma_0 \), and the limit state according to Coulomb-Mohr strength theory

\[
\sigma_0 = \frac{2K \cdot \cos \rho}{1 - \sin \rho} , \quad \sigma_1 = \beta_n \sigma_3 = \sigma_0 ,
\]

(3)

where \( \beta_n \) – the parameter of bulk stress is calculated according to the following formula.

\[
\beta_n = \frac{1 + \sin \rho}{1 - \sin \rho} .
\]

(4)

As it follows from the formula (3) the left side which is called equivalent stresses \( \sigma_i \) according to Coulomb-Mohr theory is constant on all the sections of limit stress zone. Hence the diagram of conventional stresses \( \sigma_i - \varepsilon_i \) (\( \varepsilon_i \) - equivalent deformations) corresponds to Prandtl diagram (line 0-1-2 in figure 3). Section 0-1 of the diagram is an area of a seam elastic deformation, and its slope ratio to X-axis is equal to a seam elasticity module \( E \).

**Figure 3.** Prandtl and Linkov diagrams on deformed seam conventional stresses.

Under real conditions, a seam can partially deform on the sections of limit stress zones and in its turn lessen the limit strength to the value of residual strength \( \sigma_{ost} \) [1]. So, to evaluate a massif stress state Linkov idealized diagram of a seam deformation is used (line 0-1-3-4 in figure 3)[1]. There is a section 1-3 of a seam weakening (drop down) and section 3-4 – its residual strength. Tangent of an angle \( \delta \) numerically equals to weakening (dropout) module \( M \).

As it is assumed from the problem of a massif with a seam working where the seam is deforming according to Linkov diagram, the first section of limit stress zone FN (figure 1) corresponds to a horizontal section 3-4 of Linkov diagram. On the second section, NG stresses are also changed according to the formula (1) but substituting \( \sigma_0 \) for \( \sigma_{ost} \). On the third section of this GP zone the parameters \( \sigma_{ost} \) and \( c \) in each point change proportionally to equivalent stresses on weakening section 1-3 of the diagram \( \sigma_i - \varepsilon_i \). If \( n \) – is number of points on sections 1-3 and GP and \( k \) is a number of points from 3 to \( K \) then

\[
\sigma_{ost} = \frac{\sigma_k}{2 \sin \rho} \frac{1}{2 \sin \rho} \left[ \sigma_{ost} + (\sigma_0 - \sigma_{ost}) \cdot \frac{k}{n} \right] , \quad c = \frac{1}{2} \left( \frac{1}{\sin \rho} - 1 \right) \left[ \sigma_{ost} + (\sigma_0 - \sigma_{ost}) \cdot \frac{k}{n} \right] .
\]

All abovementioned functions describe fully the seam state in limit stress zones allowing us to formulate boundary states on the seam contact with surrounding rocks.
The condition of excessive maximum primary stresses \( \sigma_1 \) acting in the seam over \( \sigma_1 \) values of its roof can be taken as a transition of a seam roof into a limit state [5]. The condition forms the formula of seam rock strength index \( f_k \) that corresponds to the initial transition into a limit state [5]

\[
f_k \leq \frac{k_a \gamma H - \beta_k^2 \cdot p}{10 \cdot (1 + \beta_k)},
\]

where \( k_a \) is a coefficient of accumulation of stresses equals to the relation between maximum primary stress in a seam to \( \gamma H \) value, \( \beta_k \) is a parameter of a bulk strength of a seam roof calculated from the formula (4) with the substitution of \( \rho \) for \( \rho_k \) and \( \phi_k \) is an angle of a roof rock internal friction.

\( k_a \) is defined out of the solution of elastic boundary value problem.

In this boundary problem boundary states are stated about a closed path that includes the roof, the soil of a working and the contact of a seam with side rocks on the area of limit zone with a length \( L_{OT} \). Thus, on horizontal sections of this maximum stress projection contour \( p_s \), \( p_v \) are equal respectively to stresses \( \sigma_1 \) and \( \tau_{rz} \). So then, in boundary integral equation of a boundary problem [1, 8, 9] integration is made about a closed contour that covers the roof, the working soil and the limit stress zone contour. There apart from the vector of distributed imaginary load the length \( L_{OT} \) is also unknown variable and to define it one can use method of successive approximations. According to this method, the \( L_{OT} \) value is being given and after that an integral equation of distributed imaginary load vectors attached to each boundary element is solved. After that summing up, the initial field stresses with the stresses resulted from the influence of defined, imaginary load made on the basis of Kelvin solution [8 - 10], can find combined stress in the points on the line 1 that crosses the seam roof (figure 1). Here upon, on the margin of elastic and limit stress zones we compare the respective stresses. In case they differ, the calculations are repeated but with other \( L_{OT} \) values to the point the stresses coincide.

The introduced approach to solving the problem is a model of geomechanic massif state enclosing a coal seam together with the mine working driven along it. It describes the parameters of a bearing pressure (maximum stress and the width of a limit stress zone) in the walls of a working in exact approximation and is introduced in [11].

3. Performed calculations and results analysis

In the framework of the described model, together with the developed program a computational experiment on defining the bearing pressure indices: accumulation of stress index, the width of a limit stress zone was performed. The following massif and working parameters were taken as initial data: \( \tau=25 \text{ kN/m}^2 \), \( \lambda=1 \), \( H=800 \text{ m} \), \( h=3 \text{ m} \), seam stress index \( f_0=1 \), \( \sigma_{os}=0.5\sigma_0 \), \( M=0.25 \text{E} \), an angle of a seam and roof rock internal friction \( \rho_p=\rho_k=20^\circ \); \( K'=0 \), \( \rho'=10^\circ \), \( p=0.5 \text{MPa} \). Other parameters were being changed during the experiment.

Figure 4 demonstrates graphic results of the method of successive approximations. Thus, in figure 4 a the diagrams built with parameters \( b=6 \text{ m} \), \( p=0.5 \text{MPa} \) introduce stress envelopes \( \sigma_2 \) and \( \tau_{yz} \) in the seam along line 1 (figure 1), limit stress zone (diagrams 1 and 3) and in elastic field of a seam (diagrams 2 and 4). Points V, N, G, P in the diagrams are the bordering sections of limit stress zone of a seam (figure 1). As it can be deduced from the figure, the stress values in elastic field and in a limit stress zone coincide in point P. A measured horizontal interval between V and P points corresponds to the volume of a limit stress zone LOT. The diagrams in figure 4 b corresponds to the primary stress \( \sigma_1 \) in a limit stress zone (figure 1) and to the elastic field of a seam (figure 2). Vertical axis referred to \( \gamma H \) in P point equals \( k_\sigma \) in a seam roof. Certain values \( L_{OT}=4.81 \text{ m} \) and \( k_\sigma=2.125 \) are deduced from diagrams 2. It can be assumed from the equation (5) that the seam roof strength indices to correspond to their transition into a limit state ahead (behind) of the working should be not higher 1.33.
The parameters of a bearing pressure in the absence of a support (p=0) are defined. They slightly differ from the previous values and equal to LOT=4.81 m, kσ=2.14. The volume of a limit stress zone did not change and the stress accumulation indices differ from each other less than 0.5 percent.

It is important to note that the support reaction value p = 0.5 MPa in the introduced sample corresponds to the force in hydraulic cylinders of a powered support unit in 2.4 MN (240 tons) and its width of 0.8 m. Increasing the working distance the influence of a support will be less meaningful and in further calculations that influence was not taken into account. Moreover, in significantly long distances the powered support unit is not able to provide the roof and soil backing of the working along its whole distance.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{stress_envelopes.png}
\caption{Stress envelopes $\sigma_z$, $\tau_{yz}$, $\sigma_1$ along the seam roof.}
\end{figure}

In figure 5, graphic results of a performed computational experiment are presented. According to the state (5) the dependency diagrams of the roof rock strength index $f_i$, corresponding to its stress state and the width of a face working for several values of a seam residual strength are built. In figure 5b the dependency diagrams of the stress accumulation index $k_\sigma$ in a limit seam zone and the width of a face working for several residual seam strength values are built.

Diagrams 1, 2 corresponds to Linkov diagram, diagram 1 is built when $\sigma_{ost}=0.5\sigma_0$, and diagram 2 – when $\sigma_{ost}=0.05\sigma_0$. Diagram 3 which corresponds to Prandtl diagram where $\sigma_{ost}=\sigma_0$ is given as an example.

Both series of diagrams present smooth low-arched curves. They prove that limit state of a seam roof ahead of a face working happens only in rocks with relatively small strength index even in significantly long distances of a face working. Reducing the strength limit of a seam in marginal zone change the research parameters in 20 times but not more then on 17 percent. The limit value of roof strength does not prevail over 3 units and the index for the accumulation of stresses in a bearing pressure zone is not more than 4.5 units.
4. Conclusions
1. The problem of stress field distribution in a rock massif with seam working is solved in the framework of the model of geomechanic state of anisotropic massif. The parameters of a bearing pressure in pre-contour zone of a seam are defined while solving an elastic boundary value problem where through considering limit stress zones under the boundary states an elastic-plastic problem was brought up.

2. Reducing residual strength of a seam can increase parameters of a bearing pressure and the hardness of the roof rock when they go into a limit state ahead of the face working. The influence of the support reaction on a bearing pressure in a face working is negligible.

3. In isotropic rock mass that exists in equally component stress field and enclosing a seam developing by a stope, limit state of its structurally intact monolithic roof ahead of the face manifests itself under the relatively small strength index. However, if the side pressure coefficient is less than 1 and a massif is of bedded structure then a limit state of the roof can appear due to significant horizontal stresses and stratification of the mass with its further failure directly above a goaf (mined-out space).

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References
[1] Petukhov I M and Linkov A M 1983 *Mechanics of Rock Bumps and Oubursts* (Moscow: Nedra) p 280
[2] Cherdantsev N V and Cherdantsev S V 2014 *Labour Safety in Industry* 11 41–5
[3] Krivolapov V G and Paleev 2011 *Bulletin of VOSTEC* 1 77–80
[4] Klishin V I 2002 *Adaptation of Powered Roof Supports to the Dynamic Loading States* (Novosibirsk: Nauka) p 200
[5] Fisenko G L 1976 *Limit State of Rocks Around an Excavation* (Moscow: Nedra) p 272
[6] Kuznetsov G N 1961 *Proc. VNIIMI* L 43 98–112
[7] Sokolovski V V 1990 *Statics of Granular Media* (Moscow: Nauka) p 272
[8] Lurie A I 1970 *Theory of Elasticity* (Moscow: Nauka) p 940
[9] Brebbia K, Telles F and Wrobel L 1987 *Boundary Element Methods* (Moscow: Nauka) p 525
[10] Rabotnov Yu N 1988 *Mechanics of Deformable Solids* (Moscow: Nauka) p 712
[11] Cherdantsev N V and Cherdantsev S V 2004 *Appl. Mech. and Tech. Physics* 3 141–8