D-term chaotic inflation in supergravity

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Even though the chaotic inflation is one of the most popular inflation models for its simple dynamics and compelling resolutions to the initial condition problems, its realization in supergravity has been considered a challenging task. We discuss how the chaotic inflation dominated by the D-term can be induced in supergravity, which would give a new perspective on the inflation model building in supergravity.

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Cosmic inflation has been one of the most successful early universe scenarios for more than 25 years, with the ever-growing supports from the observations of the cosmic microwave background anisotropies and the large scale structure of the universe [1, 2, 3]. Among many types of inflation models proposed so far, chaotic inflation is special for its amelioration of the initial condition problems [4] and it would be of considerable interest to realize chaotic inflation in a sensible particle physics theory.

One of the leading theories as an extension of the minimal standard model is supersymmetry [5] which gives the attractive solutions to the hierarchy problem of the standard model as well as the unification of three gauge couplings. In particular, in the early universe, its local version supergravity would govern the dynamics of the universe, while there has been criticisms for the implementation of chaotic inflation in supergravity. This is simply because the scalar potential coming from the F-term has an exponential dependence on the Kähler potential. This prevents the scalar fields from acquiring the amplitudes larger than the reduced Planck scale \( M_P \approx 2.4 \times 10^{18} \text{GeV} \) and also spoils the flatness of an inflaton potential (so called \( \eta \)-problem). In order to circumvent this difficulty [6], Kawasaki, Yanagida and one of the present authors (M.Y.) introduced the Nambu-Goldstone-like shift symmetry [7], so that the imaginary part of the an inflaton field does not suffer from the exponential growth. Though such a shift symmetry is motivated from the string theory and is similar to, for example, the Heisenberg symmetry [8], it may be difficult to associate it with the low energy effective theory of particle physics such as grand unified theory (GUT). It would be then worth seeking an alternative supergravity chaotic inflation model without such a symmetry possibly absent in the effective field theory.

The scalar potential in supergravity also consists of, in addition to the F-term, the D-term which does not have an aforementioned dangerous exponential factor. A plausible possibility to realize the supergravity chaotic inflation then would be to consider the inflation models where the D-term dominates over the F-term. In the conventional D-term inflation models [9, 10, 11, 12], the energy density is sourced by the Fayet-Iliopoulos (FI) term \( \xi \) in D-term and the slope of an inflaton potential is induced by the one-loop corrections. Since the one-loop corrections cannot exceed the tree level potential energy density of order \( \xi^2 \ll M_P^4 \), the inflation cannot start from the Planckian energy scale. This in turn implies that chaotic inflation is not compatible with the standard D-term inflation models because the universe would collapse before such an inflation energy scale is reached unless the universe started with an open geometry. It would be then intriguing to consistently incorporate chaotic inflation in a D-term dominated inflation model, possibly without making use of the FI term dominance or one-loop corrections.

In this paper, we present a chaotic inflation model in supergravity where the D-term dominates the inflation energy density. The D-term is helpful for the realization of chaotic inflation by allowing the (beyond) Planckian energy density of order \( \xi^2 \ll M_P^4 \), the inflation cannot start from the Planckian energy scale. This in turn implies that chaotic inflation is not compatible with the standard D-term inflation models.

We introduce four superfields \( \Phi_i \) (\( i = 1, 2, 3, 4 \)) charged under \( U(1) \) gauge symmetry and (global) \( U(1)_R \) symmetry. The charges \( Q_i, Q_i^R \) of the superfields are given in Table I which ensure our toy model is anomaly free [13, 14]. Then, the general (renormalizable) superpotential for these fields is given by

\[
W = a \Phi_1^2 \Phi_3 - b \Phi_2 \Phi_3 + c \Phi_2 \Phi_4^2,
\]

where we set the constants \( a, b, c \) to be real and positive for simplicity and a non-renormalizable term \( \Phi_1^4 \Phi_2^2 \) is omitted since it does not change the dynam-
ics essentially. Taking the canonical Kähler potential, $K(\Phi_i, \Phi^*_i) = \sum_i |\Phi_i|^2$, and the minimal gauge kinetic function, $f_{ab}(\Phi_i) = \delta_{ab}$, leads to the scalar potential consisting of the F-term $V_F$ and D-term $V_D$

$$V = V_F + V_D,$$

$$V_F = e^K \left[ 2a \phi_1 \phi_3 + \phi_1^* \phi_2^* \right]^2 - b \phi_3^2 + c \phi_4^2 + \phi_4^* \phi_2^* - 3|W|^2,$$

$$V_D = \frac{g^2}{2} \left| \phi_1^2 + 2|\phi_2|^2 - 2|\phi_3|^2 - |\phi_4|^2 \right|^2,$$

where $g$ is the gauge coupling of the $(U(1)$ symmetry), and we take the vanishing FI term for simplicity. Here and hereafter we set the reduced Planck scale $M_0$ to be unity.\(^{1}\)

The minimum of the F-term (the F-flat condition) is given by

$$V_F = 0 \iff \left\{ \begin{array}{l} a \phi_1^2 - b \phi_3^2 = 0 \quad \& \quad \phi_3 = \phi_4 = 0. \\
\phi_1 = \phi_2 = 0 \quad \& \quad -b \phi_3 + c \phi_4^2 = 0. \end{array} \right.$$  (3)

On the other hand, the minimum of the D-term (the D-flat condition) is given by

$$V_D = 0 \iff |\phi_1|^2 + 2|\phi_2|^2 = 2|\phi_3|^2 + |\phi_4|^2.$$

Hence, the global minimum of the potential is given by

$$\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0.$$

However, when the universe starts around the Planck scale, if $|\phi_1| \gg 1$ or $|\phi_2| \gg 1$, and $|\phi_3|, |\phi_4| \lesssim 1$ for example, the almost F-flat condition ($a \phi_1^2 \approx b \phi_2^2, \phi_3 = \phi_4 = 0$) is first realized due to the exponential factor $e^K$ of the F-term. Consequently, the potential is mostly dominated by the D-term and chaotic inflation can take place.

Because the system is invariant under the following transformation

$$\phi_1 \leftrightarrow \phi_4, \quad \phi_2 \leftrightarrow \phi_3, \quad a \leftrightarrow c, \quad Q \rightarrow -Q,$$

the dynamics is essentially the same even if we interchange $\phi_1$ and $\phi_2$ by $\phi_3$ and $\phi_4$. Thus, we concentrate on the case that $|\phi_1| \gg 1$ or $|\phi_2| \gg 1$, and $|\phi_3|, |\phi_4| \lesssim 1$.

Now, we investigate the dynamics in details. Despite the $e^K$ factor of F-terms, due to the presence of the relatively rather small but non-vanishing D-terms, the actual inflation trajectory is slightly deviated from the exact F-flat direction and given by solving the equations (1) $\partial V / \partial \phi_i^\ast = 0$ ($i = 2, 3, 4$) or (2) $\partial V / \partial \phi_3 = 0$ ($i = 1, 3, 4$), depending on the magnitude of $b/a$, as clarified later. The solution (named M1) of the first equations is given by $\phi_2 = \phi_2(\phi_1), \phi_3 = \phi_4 = 0$ and that (named M2) of the second equations is given by $\phi_1 = \phi_1(\phi_2), \phi_3 = \phi_4 = 0$.

Here, we check whether inflation indeed occurs along this field trajectory. For this purpose, we first evaluate the mass terms of the fields $\phi_3$ and $\phi_4$ along these trajectories M1 and M2, $V_{ij} |_{M} \phi_i^\ast \phi_j$ with $V_{ij} = \partial^2 V / (\partial \phi_i^\ast \partial \phi_j)$. The suffix $M$ represents the evaluations along either the trajectory M1 or M2. Then, the mass matrix of the fields $\phi_3$ and $\phi_4$, $V_{ij} |_{M}$, is given by

$$V_{33} |_{M} \simeq e^K \sum_i (4a^2|\phi_i|^2 + b^2), \quad V_{34} |_{M} = 0 \quad (i = 1, 2, 4),$$

$$V_{44} |_{M} \simeq 4e^K c^2 |\phi_2|^2, \quad V_{44} |_{M} = 0 \quad (i = 1, 2, 3).$$  (7)

Both of these masses are much larger than $H^2 \simeq g^2(|\phi_1|^2 + 2|\phi_2|^2)/2$ unless the constants $a, b, c$ are exponentially small (typically $e^{-g^{-1}}, g = O(10^{-6})$), which makes $\phi_3$ and $\phi_4$ quickly go to the zeros. As a result, we can safely set $\phi_3$ and $\phi_4$ to be zero and we can discuss the dynamics of the inflaton based on the following potential,

$$V_{\text{eff}}(\phi_1, \phi_2) \equiv V(\phi_1, \phi_2, \phi_3 = 0, \phi_4 = 0).$$  (8)

By use of the $U(1)$ gauge symmetry, we can, for instance, make the field $\phi_1$ real without loss of generality, so that the Im$\phi_2$ rapidly goes to the zero because the effective mass squared of the imaginary part of $\phi_2$ is given by $m_{\text{eff}}^2 \simeq b^* e^K$. We therefore consider the following effective potential by redefining the fields $\phi_i \equiv \sqrt{2} \text{Re} \phi_i$ ($i = 1, 2$, and we take $\phi_1$ to be positive for definiteness) and $b' \equiv \sqrt{2} b$.

$$V_{\text{eff}}(\phi_1, \phi_2) = \frac{1}{4} e^K (a \phi_1^2 - b' \phi_2^2)^2 + \frac{g^2}{8} (\phi_1^2 + 2 \phi_2^2)^2$$  (9)

with $K = (\phi_1^2 + \phi_2^2)/2$ and the canonical kinetic terms.

Now, we would like to discuss the dynamics of the inflation based on the above potential. First of all, we consider the region where $B \equiv b'/a \gg \phi_1(1) > 1$ is satisfied. As shown later, in this region, the field $\phi_1$ plays the role of the inflaton while the inflationary trajectory is almost determined by the condition (M1) $\partial V_{\text{eff}} / \partial \phi_2 = 0$, which is equivalent to

$$a \phi_1^2 - b' \phi_2^2 = e^{-K} \frac{g^2(\phi_1^2 + 2 \phi_2^2) \phi_2}{2 b' - 1 \phi_2 (a \phi_1^2 - b' \phi_2)} \simeq e^{-K} \frac{2}{b'} g^2 (\phi_1^2 + 2 \phi_2^2) \phi_2.$$  (10)

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1 The qualitative features of our chaotic inflation model would not be affected even if the non-renormalizable terms, such as $\lambda \phi^4 (\phi/M_0)^2$, appear in the potential as long as $\lambda \ll 1$, that is, the effective cut-off scale is larger than the reduced Planck scale. Such a small $\lambda$ associated with the breaking of the R-symmetry used in our toy model to prohibit the non-renormalizable terms is natural in 't Hooft’s sense.
Here note that combining $a\phi_1^2 - b'\phi_2 = O(e^{-K})$ with $B = b'/a \gg \phi_1$ leads to $\phi_1 \gg \phi_2$. The F-term contribution to the potential is estimated as

$$V_F = \frac{\frac{1}{2}b'\phi_1(a\phi_1^2 - b'\phi_2) - \frac{1}{2}b\phi_2(a\phi_1^2 - b'\phi_2)}{2\phi_1^2 (a\phi_1^2 - b'\phi_2)} g^2(\phi_1^2 + 2\phi_2^2)$$

$$< g^2(\phi_1^2 + 2\phi_2^2) \ll V_D = \frac{g^2}{2} (\phi_1^2 + 2\phi_2^2)^2$$

(11)

for $\phi_1 \gg 1$. Thus, the potential is dominated by the D-term during inflation. We also consider the mass matrix of the fields $\phi_1$ and $\phi_2$, $V_{ij}|_{M1}(V_{ij} \equiv \partial^2 V_{\text{eff}}/\langle(\partial_{\phi_i}\partial_{\phi_j})\rangle)$, which is given by

$$V_{11}|_{M1} \simeq 2a^2\phi_1^2 e^K + 2a\frac{\phi_2^2(\phi_1^2 + 2\phi_2^2)}{2a} + \phi_2 (1 + 2\phi_1^2)$$

$$V_{12}|_{M1} \simeq -ab'\phi_1 e^K + 2\frac{\phi_2^2(\phi_1^2 + 2\phi_2^2)}{2a} \phi_1 \phi_2 (1 + 2\phi_1^2)$$

$$V_{22}|_{M1} \simeq \frac{1}{2}b'^2 e^K - 2g^2(\phi_1^2 + 2\phi_2^2)\phi_2 + 2g \phi_1 \phi_2$$

up to the order of $O((e^K)^0)$.\(^2\) The effective mass squared $\lambda$ of the fields $\phi_1$ and $\phi_2$ is given as the solutions of the following equation,

$$\lambda^2 - (V_{11} + V_{22})\lambda + V_{11}V_{22} - V_{12}^2 = 0, \quad (13)$$

where

$$V_{11} + V_{22}|_{M1} \simeq e^K \left(2a^2\phi_1^2 + \frac{1}{2}b'^2\right) \simeq \frac{1}{2}b'^2 e^K,$$

$$V_{11}V_{22} - V_{12}^2|_{M1} \simeq \frac{3}{4}g^2b'^2\phi_1 e^K$$

(14)

up to the order of $O(e^K)$. Here, we have used the approximation that $a\phi_1^2 - b'\phi_2 = O(e^{-K})$, $B = b'/a \gg \phi_1$, and $\phi_1 \gg \phi_2$. The effective squared masses are then approximately given by

$$\lambda \simeq \frac{1}{2}b'^2 e^K, \quad 3\frac{g^2\phi_1^2}{\phi_2} \ll H^2 \simeq V_D/3, \quad (15)$$

where $H$ is a Hubble parameter. The inflaton field in the chaotic inflation corresponds to this effectively massless mode. This light mass squared vanishes for $g = 0$ as expected, reflecting the exact flat-exponential.

Since $V_{22} \gg V_{11}$ and $\phi_1 \gg \phi_2$, the inflationary trajectory is given by the minimum of the field $\phi_2$, $\partial V/\partial \phi_2 = 0$, which enables us to write the minimum of $\phi_2$ as a function of $\phi_1$, $\phi_2^m = \phi_2^m(\phi_1)$. The field trajectory governing the inflation dynamics therefore can be parameterized by the field $\phi_1$ which we call an inflaton.\(^3\) Then, by inserting the above relation into the effective potential, we define the reduced potential $V_{11}(\phi_1)$ as

$$V_{11}(\phi_1) \equiv V_{\text{eff}}(\phi_1, \phi_2^m(\phi_1)) \left(\simeq \frac{g^2}{8} \phi_1^4\right). \quad (16)$$

As explicitly shown in Ref. [16], when there is only one massless mode and the other modes are massive, the generation of adiabatic density fluctuations as well as the dynamics of the homogeneous mode is completely determined by the reduced potential $V_{11}(\phi_1)$. Indeed, the equation of motion for the homogeneous mode of the inflaton $\phi_1$ along the rolling direction (M1) is approximated as

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 + \frac{\partial V_{\text{eff}}}{\partial \phi_1}|_{M1} = 0 \quad (17)$$

where the dot represents time derivative. Thus, the dynamics of the inflation with the inflaton $\phi_1$ can be estimated by using the reduced potential $V_{11}(\phi_1)$ as long as the dynamics rolls down along the minimum of $\phi_2$ (M1).

Next, we evaluate the primordial density fluctuations in the longitudinal gauge. The equation of motion for the perturbation $\delta \phi_1$ of each real field is given by [17]

$$\delta \phi_1 + 3H\dot{\delta} \phi_1 - \frac{\nabla^2}{a^2} \delta \phi_1 + \sum_j \frac{\partial^2 V_{\text{eff}}}{\partial \phi_j \partial \phi_1}|_{M1} \delta \phi_j$$

$$= -2\frac{\partial V_{\text{eff}}}{\partial \phi_1}|_{M1} \Phi + 4\dot{\phi}_1 \dot{\Phi} \quad (18)$$

where $\Phi$ is the gravitational potential. We hereafter use the same symbol $\phi_1$ for both the homogeneous mode and the full field for notational brevity unless stated otherwise.

We are interested only in the adiabatic density fluctuations characterized by the condition

$$\frac{\delta \phi_1}{\phi_1} = \frac{\delta \phi_2}{\phi_2} \iff \delta \phi_2 = \frac{d\phi_2^m(\phi_1)}{d\phi_1} \delta \phi_1 \quad (19)$$

where we have used $\phi_2^m = \phi_2^m(\phi_1)/\phi_1$. Since the relation $\partial V_{\text{eff}}(\phi_1, \phi_2^m(\phi_1))/\partial \phi_2 = 0$ holds for any $\phi_1$ in the relevant region, we find

$$\frac{d}{d\phi_1} \left[\frac{\partial V_{\text{eff}}(\phi_1, \phi_2^m(\phi_1))}{\partial \phi_2}\right]|_{M1} = 0 \quad (20)$$

Taking into account this relation and

$$\frac{d^2 V_{11}}{d\phi_1^2} = V_{11} + 2\frac{d\phi_2^m}{d\phi_1} V_{12} + \left(\frac{d\phi_2^m}{d\phi_1}\right)^2 V_{22}|_{M1}$$

\(^3\) Note here that the effectively massless field trajectory parameterized by the inflaton field $\phi_1$ is different from the $\phi_1$ direction with the mass $V_{11} \gg H^2$.\(^2\)
the gravitational potential is described only by the homogeneous mode of the effectively massless, we can see from the mass matrix that the reduced potential coincides with that of the single field in a toy model. It could also indicate that, once a model possesses a F-flat direction lifted by a D-term, the chaotic inflation could be induced without so much restrictions on the model parameters besides those from the cosmic perturbations.

Finally we give a few comments on our model. The standard procedure shows that the gauge coupling $g$ should be $g \sim 10^{-6}$ in order to explain the primordial density fluctuations. This value of the gauge coupling is much smaller than the standard gauge couplings. However, this may not be a problem because the gauge symmetry may be a hidden gauge symmetry, or the gauge coupling could be suppressed, for instance, by considering the extra dimensions. Next, even though we presented a toy model of the quartic potential chaotic inflation, the leading order polynomial can be different by an appropriate choice of the non-minimal gauge kinetic function (for instance, a form $f = 1 + d_i | \phi |^2$ ($d_i$: const) could lead to a quadratic potential). Note this is in contrast to the standard D-term inflation with the dominance of the FI term where a non-minimal gauge kinetic function spoils the flatness of the inflaton potential. As for the reheating, it may require the spontaneous breaking of the gauge symmetry after inflation because the inflaton cannot directly decay into the standard particles due to the charge conservation. Such a breaking can occur, for instance, by the introduction of the FI term or a Higgs-like field, and such modifications of our simple toy model presented in this paper would lead to a variety of inflation models with the potentially rich phenomena. Further study will be given in the forthcoming paper.

In summary, we have shown that chaotic inflation can take place in supergravity even without the shift symmetry where the inflaton field trajectory follows the (almost) F-flat direction lifted by the D-term. We stress that, according to our analysis demonstrated through a toy model, many of such F-flat directions could potentially cause the D-term chaotic inflation. The D-term is related to the gauge coupling and a variant of our model could lead to a possible link between a chaotic inflation model and the low energy effective theory of particle physics.

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[1] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Cambridge University Press, Cambridge, England, 2005).
land 2000); D. H. Lyth and A. Riotto, Phys. Rep. 314, 1 (1999).
[2] D. N. Spergel et al., Astrophys. J. Suppl. Ser. 170, 377 (2007).
[3] M. Tegmark et al. (SDSS Collaboration), Phys. Rev. D 69, 103501 (2004); K. Abazajian et al. (SDSS Collaboration), Astron. J. 128, 502 (2004).
[4] A. D. Linde, Phys. Lett. 129B, 177 (1983).
[5] See, for a review, H. P. Nilles, Phys. Rep. 110, 1 (1984).
[6] Early attempts to realize chaotic inflation in supergravity are given in A. S. Goncharov and A. D. Linde, Phys. Lett. 139B, 27 (1984); Class. Quantum Grav. 1, L75 (1984); H. Murayama, H. Suzuki, T. Yanagida, and J. Yokoyama, Phys. Rev. D 50, R2356 (1994).
[7] M. Kawasaki, M. Yamaguchi, and T. Yanagida, Phys. Rev. Lett. 85, 3572 (2000); Phys. Rev. D 63, 103514 (2001); also see, for the use of shift symmetry in other types of inflation models, M. Yamaguchi and J. Yokoyama, Phys. Rev. D 63, 043506 (2001); 68, 123520 (2003); M. Yamaguchi, ibid. 64, 063502 (2001); 64, 063503 (2001); M. Kawasaki and M. Yamaguchi, Phys. Rev. D 65, 103518 (2002).
[8] P. Binetruy and M. K. Gaillard, Phys. Lett. B 195, 382 (1987).
[9] E. D. Stewart, Phys. Rev. D 51, 6847 (1995).
[10] P. Binetruy and G. Dvali, Phys. Lett. B 388, 241 (1996).
[11] E. Halyo, Phys. Lett. B 387, 43 (1996).
[12] J. Rocher and M. Sakellariadou, Phys. Rev. Lett. 94, 011303 (2005); J. Cosmol. Astropart. Phys. 03, 004 (2005); 11, 001 (2006); O. Seto and J. Yokoyama, Phys. Rev. D 73, 023508 (2006).
[13] D. H. Lyth and A. Riotto, Phys. Lett. B 412, 28 (1997).
[14] R. Kallosh, L. Kofman, A. D. Linde, and V. A. Proeyen, Class. Quantum Grav. 17, 4269 (2000); P. Binetruy, G. Dvali, R. Kallosh, and V. A. Proeyen, Class. Quantum Grav. 21, 3137 (2004); H. Elvang, D. Z. Freedman and B. Kors, JHEP 0611, 068 (2006).
[15] G. Villadoro and F. Zwirner, Phys. Rev. Lett. 95, 231602 (2005); C. P. Burgess, R. Kallosh and F. Quevedo, JHEP 0310, 056 (2003).
[16] M. Yamaguchi and J. Yokoyama, Phys. Rev. D 74, 043523 (2006).
[17] J. M. Bardeen, Phys. Rev. D 22, 1882 (1980); H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. 78, 1 (1984); V.F. Mukhanov, H.A. Feldman, and R. H. Brandenberger, Phys. Rep. 215, 203 (1992).
[18] D. H. Lyth, Phys. Lett. B 419, 57 (1998).
[19] K. Kadota and M. Yamaguchi, in preparation.
[20] For a GUT model of D-term inflation, see e.g. G. Dvali and A. Riotto, Phys. Lett. B 417, 20 (1998).