Experimental realization of a relativistic fluxon ratchet

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We report the observation of the ratchet effect for a relativistic flux quantum trapped in an annular Josephson junction embedded in an inhomogeneous magnetic field. In such a solid state system mechanical quantities are proportional to electrical quantities, so that the ratchet effect represents the realization of a relativistic-flux-quantum-based diode. Mean static voltage response, equivalent to directed fluxon motion, is experimentally demonstrated in such a diode for deterministic current forcing both in the overdamped and in the underdamped dynamical regime. In the underdamped regime, the recently predicted phenomenon of current reversal is also recovered in our fluxon ratchet.

I. INTRODUCTION

A particle in a periodic potential lacking spatial reflection symmetry, known as a ratchet potential, exhibits a net unidirectional motion in the absence of a net driving force. This static response to an oscillating force is known as the ratchet effect. The net unidirectional motion exhibited in ratchet potentials is the key feature potentially interesting for applications. In Josephson junction systems, various realizations of ratchet effect have been investigated recently.

Recently we proposed a single flux quantum in a long annular Josephson junction embedded in an inhomogeneous magnetic field as an experimentally accessible solid state example of a relativistic particle in a ratchet potential. Here we briefly review the experimental demonstration of the ratchet effect in such a solid state system in the overdamped regime and we report new data in the underdamped regime.

II. THEORY

An inhomogeneous magnetic field generates a potential for a flux quantum trapped in a long annular Josephson junction [see Fig. 1(a)]. With a suitable choice for the width $W(x)$ of a control line on the top of the junction [see Fig. 1(a)] a sawtooth-like magnetic field can be generated.

Using the perturbative approach, the equation of motion for the center of mass $\xi(t)$ of a flux quantum trapped in the junction can be found. The result is the equation of a relativistic particle subjected to an inhomogeneous force $F(\xi)$ with a sawtooth-like ratchet potential $U(\xi)$ [see Fig. 1(b)] and an homogeneous forcing $\eta$:

$$
\frac{4}{\pi} \left( 1 - \frac{\xi^2}{2} \right)^{\frac{3}{2}} \xi + \frac{4\alpha}{\pi} \frac{\xi}{\sqrt{1 - \xi^2}} = \eta + F(\xi). \quad (1)
$$

Here the strength of the ratchet potential $U(\xi)$ is controlled by the control current $I_{\text{CONTROL}}$ and the homogeneous forcing $\eta$ by the current $I$ feeding the junction. The measured static voltage $V$ is proportional to the spatial mean $u = \langle \xi \rangle$, of the fluxon velocity. In physical units, $V = \Phi_0 u / L$ with limiting velocity $u = v$.

![Fig. 1: (a) A long annular Josephson junction with a control line generating a sawtooth-like magnetic field. (b) The effective potential experienced by a flux quantum trapped in the junction when the control current is turned on.](image-url)
III. EXPERIMENTS

The samples were fabricated with the geometry shown in Fig. 1(a). The Nb/Al₂O₃/Nb annular junctions had a circumference \( L = 1200 \, \mu \text{m} \) and width \( W = 20 \, \mu \text{m} \). The control lines were electrically insulated from the junctions by a 300 nm thick SiO₂ layer. Here we report junctions with normalized lengths \( l = \lambda_J / \lambda_J \approx 10 \), and \( l \approx 20 \).

A. Overdamped regime

Figure 2(a) shows the measured current-voltage (force-velocity) curve of the fluxon trapped in the junction with \( l \approx 10 \) when a current is fed into the control line. As expected for an asymmetric potential, two different depinning current \( I_d^+ \) and \( I_d^- \) are recovered. The curve is recorded at \( T=6.5 \, \text{K} \). This temperature, the dissipation parameter \( \alpha \) is large enough to overcome inertial effects and the overdamped regime with a nonhysteretic current-voltage curve is achieved. In this regime the dynamics is described by the noninertial version of Eq. (I).

The ratchet effect generates a net transport velocity, \( u_R \), in response to an ac forcing. In our system such a velocity is proportional to the measured voltage

\[ V_R = \langle V \rangle \text{ averaged over the period of an alternating current fed into the junction. For ac forcing we use an } \text{“adiabatic” square wave, i.e., with a period much larger than the typical response time of the system, of the order of } 150 \, \text{ps. The measured ratchet voltage as a function of the amplitude of the ac forcing is plotted in Fig. 3(b). A comparison of experimental data with theory is also shown.} \]

The three typical regions expected from the ratchet effect are clearly exhibited: the static region I where the fluxon is at rest (pinned), the active region II where the fluxon motion is strictly unidirectional, and the overdriven region III where fluxon motion is unidirectional in the mean. From the time domain snapshots in Fig. 3(c), we also notice a strong rectification of the input current (diode effect) in the active region and a weak rectification effect in the overdriven region.

B. Underdamped regime and current reversal effect

As noted above, results in Fig. 2 were recorded in the overdamped regime we achieved at \( T=6.5 \, \text{K} \). As the thermal bath temperature is lowered, the dissipation parameter \( \alpha \) is reduced more and more, so that an underdamped regime with dynamics described by Eq. (I) can be achieved. The numerically calculated current-voltage curve for this regime, shown in Fig. 3(a) is now fully hysteretic. The ratchet voltage calculated for an adiabatic triangular ac current forcing is shown in Fig. 3(b). Experimentally, such an inertial regime is achieved at \( T=4.2 \, \text{K} \) for our fluxon. Both the measured current-voltage [Fig. 3(a)] and the measured ratchet voltage [Fig. 3(b)] fully agree with numerical predictions shown in Fig. 4. The time domain snapshots in Fig. 3(c) suggest that the strongest rectification of the input current (diode effect) is again achieved in the active region. Moreover,
with respect to the noninertial regime, a substantial rectification is also obtained in the overdriven region.

As stated above, our fluxon has a typical response time of the order of 150 ps, so that a response to microwave signals can be addressed. Theory predicts that when a.c. current forcing with frequency \( \nu_{RF} \) in the microwave range is used, the d.c. characteristic \( (I_{DC} - V) \) should exhibits current steps at voltages \( V_n = n\Phi_0 \nu_{RF} \), with \( n \) an integer and \( \Phi_0 \) the flux-quantum. Numerically, the voltage spacing of these microwave induced steps is

\[
\Delta V [\mu V] = 2.07 \nu_{RF} [\text{GHz}].
\]  

Dynamically, these current steps accounts for a locking of the revolution motion of the fluxon with the external drive. Moreover, the ratchet effect, i.e., the existence of a mean voltage as a response to an a.c. forcing, compels a zero-current-axis crossing in the \( I_{DC} - V \) curve. In other words, one should observe \( V \neq 0 \) at \( I_{DC} = 0 \) when the junction is irradiated with microwaves.

The mean voltage \( V(I_{DC} = 0) \equiv V_R \) induced by the a.c. current forcing normally exhibits only one polarity for a fixed sign of the static spatial ratchet potential. For example in Fig. 3(b), and Fig. 4(b) a positive polarity was recovered for the used potential. This means that the mean velocity of the fluxon was always positive (or zero, in the static region). However, recently has been predicted\[17\] that the ratchet velocity can change sign in the inertial regime. This phenomenon is known as

\[
\text{"current reversal" effect}. \text{ The word "current" is mutated from brownian motion field and means net velocity. In our system this correspond to our ratchet voltage } V_R. \text{ Current reversal (inversion of } V_R \text{ in our system) can appear for frequencies of the a.c. forcing comparable with the typical frequencies of evolution of the inertial ratchet system, or, in other words, for frequencies where locking occurs. In our system this means the microwave range. Dynamically, the current reversal corresponds to the onset of a chaotic dynamical regime.}\[17\]

Experiments with microwave forcing currents were performed on the junction with \( l \approx 20 \). The control current was chosen to have a (normally) positive ratchet voltage. The modification of the \( I_{DC} - V \) curve for increasing power of a signal at \( \nu_{RF} = 0.5 \text{ GHz} \) is shown in Fig. 5(a). As expected, a zero current axis crossing is observed \( (V_R > 0 \text{ at } P = 6 \text{ dBm}) \) with manifestation of small current steps spaced about 1 \( \mu V \), as expected from relation\[17\]. The current steps are better evident in Fig. 5(b), where a \( \nu_{RF} = 2.6 \text{ GHz} \) was used. Consistently with\[17\] here \( \Delta V \approx 5.4 \mu V \) is observed. Moreover, also somewhat that recall a chaotic behavior is envisaged in the current-voltage curves (at \( P = -20 \text{ dBm} \) and \( P = -18 \text{ dBm} \), but no current reversal is observed at the chosen frequency (is always \( V_R \geq 0 \)). Such a current reversal \((V_R < 0)\) is instead observed with \( \nu_{RF} = 2.0 \text{ GHz} \) and \( \nu_{RF} = 5.8 \text{ GHz},\[17\]

FIG. 4: Same meaning as Fig. II. Here the temperature of the thermal bath is lowered to achieve the underdamped (inertial) limit and a triangular wave was chosen as ac adiabatic forcing.

FIG. 5: Modification of the \( I_{DC} - V \) curve of the induced by a microwave signal at frequency \( \nu_{RF} = 0.5 \text{ GHz} \) (a) and \( \nu_{RF} = 2.6 \text{ GHz} \) (b). A zero current axis crossing \((V \neq 0 \text{ at } I_{DC} = 0)\) is observed, together with locked current steps.
FIG. 6: $I_{DC} - V$ curves for microwave signal at frequency $\nu_{RF}=2.0$ GHz (a) and $\nu_{RF}=5.8$ GHz (b) showing the current reversal phenomenon ($V < 0$ at $I_{DC} = 0$).

as shown in Fig. 6. The current steps corresponding to a motion synchronized in the direction opposite to the "natural" direction, i.e., the steps with negative voltage polarity in Fig. 6, are quite noisy and instable, accounting for some quite chaotic internal dynamics. The onset of a chaotic dynamics is also envisaged when current reversal occurs, as better seen in the panels at $P=-18$ dBm in Fig. 6(a), and at $P=2$ dBm in Fig. 6(b). This is in qualitative agreement with theory of inertial ratchets.

IV. CONCLUSIONS

We have considered a single flux quantum in a long annular Josephson junction embedded in an inhomogeneous magnetic field as an experimentally accessible solid state example of a relativistic particle in a ratchet potential. In such a solid state system mechanical quantities are proportional to electrical quantities, so that the ratchet effect represents the realization of a relativistic-flux-quantum-based diode. Mean static voltage response, equivalent to directed fluxon motion, has been experimentally demonstrated in such a diode for deterministic and adiabatic current forcing both in the overdamped (noninertial) and in the underdamped (inertial) dynamical regime. In the underdamped regime, also nonadiabatic current forcing, corresponding to the microwave range forcing, has been experimentally addressed, and the predicted synchronization with external drive as well as the current reversal phenomenon have been recovered in the system.

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