Influence of the thrust bearing on the natural frequencies of a 72-MW hydropower rotor

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Abstract. The thrust bearing is an essential element of a hydropower machine. Not only does it carry the total axial load but it also introduces stiffness and damping properties in the system. The focus of this study is on the influence of the thrust bearing on the lateral vibrations of the shaft of a 72-MW propeller turbine. The thrust bearing has a non-conventional design with a large radius and two rows of thrust pads. A numerical model is developed to estimate natural frequencies. Numerical results are analyzed and related to experimental measurements of a runaway test.

The results show the need to include the thrust bearing in the model. In fact, the vibration modes are substantially increased towards higher frequencies with the added properties from the thrust bearing. The second mode of vibration has been identified in the experimental measurements. Its frequency and mode shape compare well with numerical results.

1. Introduction

A rotating machine needs to operate outside a safety margin from a natural frequency. Resonance can occur and high vibrations are expected if a natural frequency gets excited. This could cause damage and shutdown of the machine. The characteristics of a rotor, i.e. the natural frequencies and their associated modes therefore need to be identified. Unfortunately, the natural frequencies of hydropower machines, especially old ones, are generally not known in advance. There are two ways to estimate them: experimentally and numerically.

Estimating these frequencies experimentally would require an exciter which would make the shaft vibrate over a range of frequencies. This can be done on a test rig but it can hardly be performed on a full-scale machine. Besides, the results from a test rig do not scale well. Alternatively, the shaft is naturally shaken when passing through regimes such as start-ups, shutdowns and runaways. For these non-magnetized regimes, a single source of excitation at the turbine could reveal resonance frequencies.

In order to accurately determine natural frequencies of a machine, numerical models can also be used. They have to integrate as many as possible of the following elements which come into play: bearings, brackets, supports structure, rotor unbalance, magnetic pull, added mass and damping due to the water in the turbine. All of these features add some stiffness and damping in the system and therefore influence the natural frequencies. However, it is difficult to integrate all of them in a
numerical model as many are unknown. In particular, the thrust bearing of a hydropower shaft is an important component of the system. In fact, the total axial load of the machine is carried by a thin oil film. Considering the radius of the thrust pads at which the reaction forces are applied, reaction moments are created when the shaft bends. There is then a coupling between lateral and angular motions. Axial stiffness and damping are converted into rotational stiffness and damping in the system. It is common practice to neglect thrust bearings in lateral vibration calculations [1], [2]. Standard designs of thrust pads are contained within a restricted radius with the result that the impact of the bearing on lateral vibrations is often assumed to be negligible. However, Luneno et al. [3], who studied the influence of a combi-bearing, showed that this may cause significant errors in the calculation of the system’s natural frequencies. The same authors confirmed these results experimentally [4].

The present paper aims at estimating the natural frequencies of a 72-MW propeller turbine in operation at Hydro-Québec. The influence of an odd design of the thrust bearing which, unlike usual designs, has a larger radius span and two rows of pads is investigated. A numerical model is developed to estimate lateral vibrations of the shaft. Results are compared with and without the thrust bearing to determine the influence of the terms introduced in the system. Results from experimental measurements of a runaway test are also analyzed in order to correlate possible natural frequencies.

2. Model and methods

2.1. Numerical model

The rotor is described with 22 nodes and is modeled using Timoshenko beam elements (see Table 1 and Figure 1). Lumped masses with directional inertias are added at nodes 1 and 22 to account for the turbine and generator, respectively. The rotational speed is 94.7 rpm. The shaft is supported by two radial guide bearings: the turbine guide bearing (TGB) at node 7 and the generator guide bearing (GGB) at node 19. Both are multi-lobe bearings with fixed segments. The thrust bearing (TB) is located at node 18. The bearing brackets and supports are not considered in this study.

| Table 1. Dimensions of the machine |
|-----------------------------------|
| **Shaft**                         | **TGB**                          | **TB**                          |
| D_{in} [mm]                       | Diameter [mm] 1233.93             | Inner row                       |
| D_{out} [mm]                      | Length [mm] 457.2                 | r_{in0} [mm] 584.2              |
| Length [m]                        | Clearance [µm] 203                | r_{in1} [mm] 939.8              |
| **Turbine**                       | Pads 8                           | Pivot radius, r_{in1} [mm] 782.57|
| Mass [kg]                         | GGB                              | Pad angle [°] 40                 |
| Diametrical inertia [kg m^2]      | Diameter [mm] 2692.4              | Outer row                       |
| Polar inertia [kg m^2]            | Length [mm] 200.02                | r_{out0} [mm] 990.6             |
| **Generator**                     | Clearance [µm] 381                | r_{out1} [mm] 1333.5            |
| Mass [kg]                         | Pads 20                          | Pivot radius, r_{out1} [mm] 1174.75|
| Diametrical inertia [kg m^2]      |                                  | Pad angle [°] 20                 |
| Polar inertia [kg m^2]            |                                  |                                  |
|                                  | 213232                           |                                  |
|                                  | 16500000                         |                                  |
|                                  | 33000000                         |                                  |
2.2. Resolution method

The equation of motion for the assembled system is:

\[ M\ddot{x} + (C + \Omega G)x + Kx = f(t) \]  \tag{1}

where \( M \) is the mass matrix, \( K \) the stiffness matrix, \( C \) the damping matrix, \( G \) the gyroscopic matrix, \( \Omega \) the rotational speed and \( f \) the force vector.

Unbalance is considered in this study. It occurs when the center of mass is not aligned with the center of rotation and it induces speed-dependent forces at the turbine and generator masses in the form:

\[ f_{unb} = m \cdot e \cdot \Omega \cdot \Omega \]  \tag{2}

The balancing grade is typically chosen to be \( e\Omega = 6.3 \text{ mm/s} \) for hydropower units \[5\]. Since the rotor is vertical, there is no load on the bearings due to the weight of the machine. A simplified approach is considered to model the bearings and solve the system \[6\]. The rotor is assumed to be rigid and the bearing load, coming from the unbalance forces, is assumed to be static, i.e. constant in magnitude and direction. In fact, knowing the bearing positions relative to the unbalance positions enables us to deduce the bearing loads and, consequently, the bearing properties. The bearing coefficients are recalculated for each rotational speed since the unbalance forces varies according to equation (2). The unbalance forces are therefore not directly included in the equation of motion. Equation (1) becomes:

\[ M\ddot{x} + (C + \Omega G)x = 0 \]  \tag{3}

By taking the state vector \( q^T = \{x^T \ \dot{x}^T\} \), the equation in state space becomes:

\[ \dot{q} = \begin{bmatrix} 0 & I \\ -M^{-1}[K] & -M^{-1}(C + \Omega G) \end{bmatrix} q \]  \tag{4}

The eigenvalue problem is formed by assuming a solution in the exponential form \( q(t) = ye^{\lambda t} \). The eigenvalues \( \lambda_i \) and eigenvectors \( y_i \) obtained are usually complex. The undamped natural frequency \( \omega_i \) of the \( i \)-th mode and the associated damping ratio \( \xi_i \) can be calculated:
The damped natural frequency is given by the expression:

$$\omega_i = |\lambda_i|$$

$$\xi_i = \frac{-Re(\lambda_i)}{|\lambda_i|}$$

The damped natural frequency is given by the expression:

$$\omega_{di} = \omega_i \sqrt{1 - \xi_i^2}$$

2.3. Thrust bearing

As previously stated, thrust bearings are usually discarded in the modeling of rotor dynamics. However, interest is sparked by the fact that the machine under investigation here has an odd design. The thrust bearing is composed of two rows of thrust pads: an inner row with eight pads and an outer row with 16 (see Figure 2). In many hydropower applications, the pads of the thrust bearing rest on a spring bed to allow tilt but this is not the case here. Each pad is allowed to tilt around a rigid pivot point.

![Figure 2. Thrust bearing](image)

The weight of the shaft is carried entirely by the fluid film of the thrust bearing. The total load acting on the thrust bearing can be separated into the load carried by the inner row of pads ($F_{z(in)}$) and that carried by the outer row of pads ($F_{z(out)}$).

$$F_z = F_{z(in)} + F_{z(out)}$$

Numerical simulations are performed for each row separately. All the pads of each row are considered identical and equidistant such that the load is evenly distributed on each pad of a row. By assuming small vibrations, the axial forces can be linearized around a static equilibrium position:

$$F_{z(in)} = F_{z(in)0} + \left(\frac{\partial F_{z(in)}}{\partial z_{(in)}}\right)_0 \Delta z_{(in)} + \left(\frac{\partial F_{z(in)}}{\partial \dot{z}_{(in)}}\right)_0 \Delta \dot{z}_{(in)}$$

$$F_{z(out)} = F_{z(out)0} + \left(\frac{\partial F_{z(out)}}{\partial z_{(out)}}\right)_0 \Delta z_{(out)} + \left(\frac{\partial F_{z(out)}}{\partial \dot{z}_{(out)}}\right)_0 \Delta \dot{z}_{(out)}$$

The axial force becomes:

$$F_z = F_{z(in)0} - k_{z(in)} \Delta z_{(in)} - c_{z(in)} \Delta \dot{z}_{(in)} + F_{z(out)0} - k_{z(out)} \Delta z_{(out)} - c_{z(out)} \Delta \dot{z}_{(out)}$$
with $k_{zi}(in)$, $c_{zi}(in)$, $k_{zi}(out)$ and $c_{zi}(out)$ being the total stiffness and damping in the axial direction for the inner and outer row of pads, respectively.

When the shaft bends, the thrust collar forms an angle with the horizontal plane. A different reaction force results from opposite sides of the thrust bearing. This produces a reaction moment on the rotor acting at the intersection of the rotation axis and the $xy$ plane of the thrust bearing. The total moment is the sum of the moments created by each pad (see Figure 2).

$$M_x = -\sum_{i=1}^{8} \frac{k_{zi}(in)}{8} (r_{zi} \sin \phi_i)^2 \theta_x - \sum_{i=1}^{16} \frac{k_{zi}(out)}{16} (r_{zi} \sin \phi_i)^2 \theta_x$$

$$M_y = -\sum_{i=1}^{8} \frac{c_{zi}(in)}{8} (r_{zi} \cos \phi_i)^2 \theta_y - \sum_{i=1}^{16} \frac{c_{zi}(out)}{16} (r_{zi} \cos \phi_i)^2 \theta_y$$

(12)

(13)

where $r_{zi}$ and $r_{zi}$ are the radius of the pad pivots of the inner and outer rows respectively and $\phi$ is the angular position of the pivot of a given pad (Figure 2). $\theta_x$ and $\theta_y$ are the angle of the shaft rotational axis at the thrust-bearing elevation with the $z$-axis in the $xz$ and $yz$ planes respectively (assuming no deformation between the shaft and the collar).

Thus, the axial stiffness and damping coefficients created by the fluid film are converted into rotational stiffness ($k_{i\theta}$) and rotational damping ($c_{i\theta}$) coefficients. The equivalent coefficients at nominal speed are given by:

$$k_{i\theta_x} = k_{i\theta_y} = 4 \cdot r_{zi}^2 k_{zi}(in) + 8 \cdot r_{zi}^2 k_{zi}(out)$$

$$c_{i\theta_x} = c_{i\theta_y} = 4 \cdot r_{zi}^2 c_{zi}(in) + 8 \cdot r_{zi}^2 c_{zi}(out)$$

(14)

(15)

When a Campbell diagram is constructed, the above coefficients need to be recalculated for each rotational speed. The coefficients are calculated for a few different speeds according to equations (11), (14) and (15). Then an interpolation of the following form is made:

$$k_{i\theta_x}(\Omega) = k_{i\theta_y}(\Omega) = \frac{1}{a_0 \Omega + a_1}$$

$$c_{i\theta_x}(\Omega) = c_{i\theta_y}(\Omega) = e^{(b_0 \Omega^2 + b_1 \Omega + b_2)}$$

$a_0$, $a_1$, $b_0$, $b_1$ and $b_2$ are polynomial coefficients.

3. Experimental data/Prototype turbine

After a refurbishment of the turbine in 2014, a measurement campaign was undertaken. Data collected from proximity sensors at bearing locations were analyzed. A test conducted for a runaway is of particular interest. As the generator is not magnetized, the rotor rotates freely with the hydraulic torque. When the guide vanes close, the water flow in the turbine is highly turbulent and chaotic.
Thus, many frequencies can be found exciting the rotor at the runner position. It is of interest to
determine whether any natural frequencies could be excited during this phase.

A spectrogram of the power spectral density shows the vibrations at the GGB and TGB during a
runaway from 100% opening in Figure 3. At around 20 seconds, the generator is disconnected and the
guide vanes start to close. The synchronous response seen around the line representing the rotational
speed is strong. When the guide vanes are almost closed and the speed starts to decrease, a vibration
peak occurs at about 2.85 Hz around the time 30-35 s. This frequency is visible on the four
displacement sensors, as can be seen in Figure 4. The peak-to-peak amplitude is an order of magnitude
greater at the GGB than that at the TGB, and also slightly larger along the $x$-direction. This suggests
that it could be a natural frequency whose mode shape provides more displacement at the generator.

![Figure 3. Power spectral density of the displacement in the $x$-direction at the GGB. The black curve represents the rotational speed of the rotor.](image1)

![Figure 4. Power spectral density at $t \approx 32$ s ($\omega \approx 115$ rpm)](image2)

Among the sensors monitoring the behavior of the machine, load cells measure the force acting on
the pivot supports of each pad of the thrust bearing. With these measurements, the weight distribution
between the inner row and outer row of pads is known (see Figure 5). During the runaway, the
absolute value of the load on the thrust bearing drops rapidly as soon as the generator is disconnected. Some oscillations occur as the rotational speed decreases and the guide vanes close. As the period of interest is around 30-35 s, the value of the average load at that time is used for the thrust-bearing simulations.

![Figure 5](image)

**Figure 5.** Forces acting on the inner and outer rows of the thrust bearing during a runaway

### 4. Results

The results are presented in the form of Campbell diagrams together with the mode shapes at nominal speed. Modes with a damping ratio over 90% are discarded. The effect of the thrust bearing in the modeling is shown by comparing results with and without this element.

#### 4.1. Without considering the thrust bearing

The Campbell diagram and critical damping ratio are presented in Figure 6 and the mode shapes in Figure 7.

![Figure 6](image)

**Figure 6.** Campbell diagram representing the first four modes without thrust bearing
The first two vibration modes are rocking modes with a rather high damping ratio, which means that when these modes are excited they should not induce large displacements. Modes 3 and 4 are the first two bending modes. Mode 3 is a backward mode whereas mode 4 is a forward mode. Both modes have a lower damping but are less likely to be excited.

4.2. Considering the thrust bearing

At nominal speed, the following values are calculated for the stiffness and damping coefficients.

**Table 2. Thrust-bearing dynamic coefficients at nominal speed.**

|                  | Stiffness  | Damping   |
|------------------|------------|-----------|
| Axial \((k_{z\text{init}}, c_{z\text{init}})\) | 1.96x10^10 N/m | 2.07x10^9 Ns/m |
| Axial \((k_{z\text{out}}, c_{z\text{out}})\) | 4.96x10^10 N/m | 2.49x10^9 Ns/m |
| Rotational \((k_{\theta}, c_{\theta})\)       | 4.81x10^10 Nm/rad | 2.90x10^9 Nms/rad |

The Campbell diagram obtained when the thrust bearing is included is shown in Figure 8.
The first natural frequency is almost unchanged whereas the second is shifted upward when the thrust bearing is taken into account. The rotational stiffness introduced changes their associated mode shapes slightly with less bending at the location of the thrust bearing. The third and fourth modes, which are flexion modes, are shifted towards higher frequencies, as shown in Table 3. The impact of the thrust bearing on these two modes of vibration is huge: their mode shapes are changed substantially with less rotation at the thrust-bearing node. Table 3 summarizes the frequencies obtained with and without the thrust bearing.

Table 3. Natural frequencies of the rotor.

| Mode  | Without thrust bearing | With thrust bearing |
|-------|------------------------|--------------------|
| Mode 1| 0.82 Hz                | 0.85 Hz            |
| Mode 2| 1.64 Hz                | 2.82 Hz            |
| Mode 3| 5.40 Hz                | 14.98 Hz           |
| Mode 4| 8.20 Hz                | 15.62 Hz           |

Whereas Luneno et al. [3] obtained variations of natural frequencies within 5% when they took into account the combi-bearing in their calculations, the variations due to the thrust bearing in the present work is much more significant. All the vibration modes are underestimated when the thrust bearing is not included. From Table 3, adding the thrust bearing increases the second mode by 72%, the third mode by 177% and the fourth mode by 90%. This huge influence is due to the size of the thrust bearing, which almost blocks the shaft in rotation at this position.

With a more realistic modeling, the natural frequency of the second mode gets closer to the experimental frequency appearing at 2.85 Hz. In both experimental and numerical analyses, the mode shape is similar with more displacement encountered at the generator. This tends to demonstrate that the highest vibration peaks at 2.85 Hz occurring during the runaway correspond to a natural frequency. Fortunately, this mode is not excited for a long period and the damping ratio is reasonably high to prevent the machine undergoing a critical vibration regime.

5. Conclusions
This work highlights the influence that a thrust bearing can have on the natural frequencies of a hydropower shaft. In this case, the natural frequencies are substantially increased with the added rotational stiffness and rotational damping coming from the thrust bearing. The most common thrust bearings in hydropower machines do not have such a large radius and are therefore usually discarded in the modeling. The present study shows that, in the case of a non-conventional thrust-bearing design with a large radius and no spring bed for the bearing pivot, the properties added to the system have a major impact. Although the bearing support has not been considered, the results show the importance of taking the thrust bearing into account in the modeling.
The highest peaks of vibrations occurring during the runaway have been identified as the second vibration mode of the rotor. Comparison between the numerical and experimental analyses of the second mode becomes much more accurate when the thrust bearing is included in the model. Further investigations need to be performed with a refined model that would include the stiffness of the bearing brackets and the support structure.

6. References
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