Modeling of wave responses to time-dependent dislocation source in layered media

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Abstract

This paper presents synthesis of acoustic-emission (AE) wave propagation in multi-layer materials and simulation of AE wave responses at free surface. In particular, the AE source is modelled as an arbitrary-orientation dislocation over an inclined-to-surface fault within one layer or at the layer-to-layer interface, while the materials are assumed as multi-layer media, each of which is homogeneous, isotropic and linearly elastic. With the use of the integral transformation approach, the three-dimensional wave propagation in the materials is solved in transformed or frequency-wavenumber domain. Subsequently, a closed-form solution for wave responses at free surface is found, which can then be converted in time-space domain. Numerical examples are finally provided for illustration.

Keywords: acoustic emission, wave propagation and scattering, dislocation source

1. Introduction

Acoustic emission (AE) is a naturally-occurring phenomenon in materials, in which the rapid release of energy from localized damage such as cracking generates elastic stress waves within the materials. Analysis of AE recordings at...
free surface can help detect damage location and severity in nondestructive testing and evaluation for materials. Since AE testing is not reproducible due to the source nature, exemplified as sudden and sometimes random formation of a crack, modeling of AE source and wave propagation is essential. Synthesis and simulation of wave responses with the AE modeling have been widely used in structural health monitoring of nuclear power, aerospace and materials processing (e.g., Hamstad, 1986, Yoshinoriet al., 2005, Barsoum et al., 2009, Kalicka, 2009 and Spasova et al., 2007).

Previous research (e.g., Freund, 1998, Grosse and Ohtsu, 2008) suggests that AE is typically considered as time-dependent dislocation over a finite fault area, or simply the finite dislocation source. The finite dislocation source can be modeled as the summation (or integration in the limit) of point sources, each of which accounts for the evolutionary dislocation over a discretized sub-area triggered at different time instant. Therefore, truthfully characterizing the point source is a key in understanding of nature of the finite dislocation source.

The mechanism of the above point source is typically modeled as the product of a source time function characterizing the dislocation growth, a factor combining nine couples of impulsive forces that is equivalent to the unit dislocation, and a scaling factor or magnitude (=final dislocation x material rigidity x fault area). Each couple (or dipole) can be represented mathematically by two impulsive forces acting in opposite directions, with an infinitesimal separation distance either along or perpendicular to the impulse direction or, in the limit, by the derivative of the impulse with respect to the separation-distance parameter. The combination of the nine couples are theoretically equivalent to any kind of a dislocation (normal and shear) in an arbitrary orientation from an elastodynamic approach, which uses the generalized Betti reciprocal relation by introducing a time parameter, as first described in Burridge and Knopoff (1964). Alternatively and more generally, Bakus et al. (1976a,b) use the concept of stress glut to derive the same results. That derivation uses the assumption that the indigenous sources are considered to be the result of a localized, transient failure of the linearized elastic constitutive relation, and leads to the stress glut as a function of the dislocation quantity (for details, see Dhalen and Tromp, 1998).

While the above point-source characterization based on equivalent forces is useful in solving wave motion equations, which have been well developed and widely used in study of seismology and fracture mechanics (e.g., Freund, 1998 and Grosse, 2003), the source description can be directly given with a dislocation form and subsequently used for finding wave motion. In addition, the equivalent forces are valid only for describing a dislocation within a medium, which is not applicable to generalized dislocations at the layer-to-layer interface, exemplified as debonding damage in reality.

Built upon the aforementioned studies as well as pertinent others (Kennett, 1983, Hamstad et al., 2001), this paper presents a continuum-mechanics model for AE wave propagation in layered materials, which is generated by time-dependent dislocation over a finite fault area buried within one of the layers or at the layer-to-layer interface.

2. Wave propagation with a point, impulsive dislocation

In this study, the AE source is described as an impulsive dislocation rupturing at a point with a given fault area, while the materials are modelled as vertically non-homogeneous or layered media with each layer being isotropic, homogeneous and linearly elastic, as schematically shown in Fig. 1. The AE waves can then be obtained by solving force-free wave equations of motion with the dislocation. In doing so, it is also required of pertinent conditions that are the continuity conditions at each layer-to-layer interface and boundary conditions at both surfaces.

For ease in describing the solution procedure, the following displacement and traction vectors are defined:

\[ \ddot{u}(x, y, z, t) = u_x \vec{e}_x + u_y \vec{e}_y + u_z \vec{e}_z \quad , \quad \vec{f}(x, y, z, t) = \tau_{x} \vec{e}_x + \tau_{y} \vec{e}_y + \tau_{z} \vec{e}_z \]  

(1)

where \( \vec{e}_x, \vec{e}_y, \vec{e}_z \) are respectively orthogonal unit vectors in the x, y and z directions, \( u \) the displacement, and \( \tau \) the stress.

The wave motion can then be found by solving the following equations in force-free layer \( i \)

\[ \tau_{jx} + \tau_{jy} + \tau_{jz} = \rho \ddot{u}_j \quad j = x, y, z \quad ; \quad i=1,2,\ldots,n \]  

(2)

where \( \rho \) is the density and the prime in subscript denotes the partial derivative.
The constitutive relationship between displacements and stresses is expressed as:

$$
\tau_{jk} = \lambda \delta_{jk} (u_{x,x} + u_{y,y} + u_{z,z}) + \mu (u_{x,k} + u_{k,k}) \quad j, k = x, y, z \quad i = 1, 2, \ldots, n \quad (3)
$$

in which $\delta_{jk}$ is Kronecker delta, equal to one if $j = k$ and zero otherwise, and $\lambda$ and $\mu$ are the Lame’s elastic constants and can be found as $\lambda = \rho (\alpha^2 - 2\beta^2)$ and $\mu = \rho \beta^2$ in terms of $P$ and $S$ wave speeds ($\alpha$ and $\beta$) and density ($\rho$).

The continuity condition at each layer-to-layer interface requires that displacement and traction vectors be continuous across $z = z_i$:

$$
\vec{u}(x, y, z_i^-, t) = \vec{u}(x, y, z_i^+, t) \quad \vec{t}(x, y, z_i^-, t) = \vec{t}(x, y, z_i^+, t) \quad (4)
$$

where $z_i^-$ and $z_i^+$ represent respectively the negative and positive sides of interface $z = z_i$.

The boundary conditions at one free surface and another fixed or free one are

$$
\vec{t}(x, y, z = 0, t) = 0 \quad \text{and} \quad \vec{t}(x, y, z^+, t) = 0 \quad \text{or} \quad \vec{u}(x, y, z^+, t) = 0 \quad (5)
$$

or boundary condition with infinite depth of layer $n$, i.e., radiation condition that no propagating waves come from the place where $z$ is infinity.
The impulsive dislocation at a point \((x_s=0, y_s=0, z_s)\) with fault area \(A\) can be described as

\[
\ddot{u}(x_s', y_s', z_s', t) - \ddot{u}(x_i', y_i', z_i', t) = \dot{\delta}(t) = \left[ \Delta u_x \ddot{e}_x + \Delta u_y \ddot{e}_y + \Delta u_z \ddot{e}_z \right] \delta(t) \tag{6}
\]

where \(\Delta u_i = [\Delta u_x (\cos \theta_i n_{x_s'} + \sin \theta_i n_{y_s'}) + \Delta u_y n_{y_s'}] \delta(x) \delta(y) \delta(z - z_s)\), \(i = x, y, z\), i.e., dislocation component \(\Delta u_i\) in the global \(x-y-z\) coordinates is expressed in terms of normal and shear dislocation components \((\Delta u_x, \Delta u_y, \Delta u_z)\) with shear slip direction \((\theta, \varphi)\) in local \(x'-y'-z'\) coordinate system shown in Fig. 2.

![Fig. 2. Description of dislocation in an inclined-to-surface fault in local \(x'-y'-z'\) coordinates](image)

### 3. Numerical examples

For illustration, displacement wave responses are computed at surface locations 1 \((x=15 \text{ mm}, y=20 \text{ mm})\) and 2 \((x=30 \text{ mm}, y=40 \text{ mm})\) to a point, impulsive shear-dislocation source buried in a uniform half-space of 7000-series aluminum alloys (Hamstad et al., 2001). The material parameters used can be found in the second layer of Tab. 1, while dislocation parameters are selected as \(\Delta u_n = 0\), \(\Delta u_s = 0.01 \text{ mm}\), \(\theta_s = 0\), and fault area \(A = 0.001 \text{ mm}\). For comparison, the displacements at the surface are also computed for the three-layer medium given in Tab. 1.

| Layer | Physical properties of layers |
|-------|-----------------------------|
|       | \(\alpha [\text{km/m}^3]\) | \(\beta [\text{km/m}^3]\) | \(Q_{\alpha}\) | \(Q_{\beta}\) | \(p [\text{kg/m}^3]\) | \(z [\text{mm}]\) |
| 1     | 4,320                       | 2,200                       | 25             | 50             | 1670           | 0 - 0.5           |
| 2     | 6,320                       | 3,180                       | 12.5           | 25             | 2700           | 0.5 - 4.7         |
| 3     | 7,320                       | 4,150                       | 6.25           | 12.5           | 2610           | 4.7 - \(\infty\) |

Figure 3 shows the P-SV and SH wave response amplitudes at surface (i.e., \(W_s\) and \(W_t\)) versus the wave number in radial direction \(k_r\) at selected frequency 5M rad/s. The dominant peaks of P-SV wave response \(W_s\) in Fig. 3 for the uniform half-space corresponds to the propagating P- and S-wave modes at \(k_r = 791\) rad/m and \(k_r = 1572\) rad/m, while similar peaks for the three-layer medium are also found, showing the averaged propagating P- and S-wave modes over the three layers. No peak of SH wave response \(W_t\) is observed for the uniform half-space, indicating that no propagating SH-wave mode or surface waves exist for the uniform half-space. In contrast, a couple of peaks for SH wave response \(W_t\) show off for the three-layer medium in Fig. 3, corresponding to the different surface wave modes such as Love waves. Figure 4 depicts the amplitudes of x-direction displacement in the frequency domain for a uniform and a three-layer medium, indicating more complexity of wave responses in a layered medium than the uniform half space in addition to the similar overall frequency features.

Figure 5 shows the x-direction displacements at observation locations 1 and 2. It indicates that the displacement at location 1 is zero until the first P wave signal arrives at 4.0(10^-6) s which is consistent with the theoretical calculation based on the P wave speed and source-to-response distance. The S wave signals arrive later and give
rise to larger displacement amplitude than that of the P waves at 8.2(10^-6) s. The displacement at location 2 shows similar wave propagation features with later arrival times and smaller damping-related amplitudes.

The displacement wave response \( g_d \) to a time-dependent, point shear dislocation source can be found as

\[
g_d(x, y, z, t) = \frac{M_0}{2\pi} \int_{-\infty}^{\infty} m(\omega)\tilde{u}_d(x, y, z; \omega)e^{iat} d\omega
\]

where \( M_0 \) is the product of shear modulus, final average slip and slip area for shear dislocation, \( m(\omega) \) is the Fourier transformation of a source time function characterizing the dislocation growth, and \( \tilde{u}_d \) is the displacement response to a unit-impulse, point dislocation in frequency domain. For the source time function selected as a ramp function with the rise time \( T_r \), \( m(\omega) \) is given by

\[
m(\omega) = \frac{1}{\omega^2 T_r} (e^{-i\omega T_r} - 1)e^{-ia\omega}
\]
Fig. 4. The x-direction displacement amplitude spectra in a uniform and a three-layer media.

Fig. 5. The x-direction displacement wave responses at observation locations 1 and 2.
Fig. 6, accelerations vs time in a three-layer medium.
4. Conclusion

A continuum-mechanics model for AE wave propagation in layered materials has been examined. The general dislocation source is modeled and the corresponding traction discontinuity found. Wave response is solved analytically in transformed domains. Some observed wave features are shown with numerical examples. Further theoretical investigation, together with numerical validation of this study, will be performed and reported in the near future.

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