On the decoherence of primordial fluctuations during inflation

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Abstract
We study the environment-induced decoherence of cosmological perturbations in an inflationary background. Splitting our spectrum of perturbations into two distinct sets characterized by their wavelengths (super and sub-Hubble), we identify the long-wavelength modes with our system and the remainder with an environment. We examine the effects of the interactions between our system and the environment. This interaction causes the long-wavelength modes to decohere for realistic values of the coupling and we conclude that interactions due to backreaction are more than sufficient to decohere the bulk of the system within 60 e-foldings of inflation. This is shown explicitly by obtaining an analytic solution to a master equation detailing the evolution of the density matrix of the system.

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1. Introduction

As is well known, temperature fluctuations in the CMB and the inhomogeneities that seed structure formation in the universe share a common origin. Both are a result of the scalar metric perturbations produced during inflation. However, these perturbations are of a purely quantum mechanical nature while no cosmological systems of interest (CMB anisotropies, clusters, etc) display any quantal signatures. Presumably, for this to be the case, the primordial density perturbations underwent a quantum-to-classical transition some time between generations during inflation and recombination, when structure first became apparent.

Decoherence is a much studied process (see [1] for a comprehensive review). Although not all conceptual issues have been resolved, it is understood that it can occur whenever a
quantum system interacts with an ‘environment’. In other words, this effect can be said to pervade open systems due to the difficulty of creating a truly closed, macroscopic quantum system. Along with its ubiquity, it is also known to be a practically irreversible process, since the loss of quantum correlations in the system is accompanied by an increase in entropy.

Early studies of the classicalization of primordial perturbations focused on intrinsic properties of the system (see, for example, [2, 3]). This was made possible by the application of ideas of quantum optics to the theory of cosmological perturbations. Primordial density fluctuations (the scalars as well as the tensors) evolve into a peculiar quantum state—a squeezed vacuum state [4, 5]. By studying the large squeezing limit of these states, it was found that quantum perturbations become indistinguishable from a classical stochastic process. In other words, quantum expectation values in a highly squeezed state are identical to classical averages calculated from a stochastic distribution, up to corrections which vanish in the limit of infinite squeezing (see, [40], however). The authors of [3] refer to this as ‘decoherence without decoherence’ while [6] endows the phenomenon with the more technical epithet ‘quantum non-demolition measurement’. We emphasize that these works focused on the classical properties of the states and not on the coherence properties of the system.

As is well understood, in order to study true classicalization, one must consider two distinct aspects of a system. First the quantum states must evolve, in some limit, into a set of states analogous to classical configurations. The second is that these resultant states interfere with each other in a negligible fashion. This last property constitutes decoherence and is equivalent to the vanishing of the off-diagonal elements of the density matrix.

A truly closed gravitational system is a practical impossibility (unless one considers the totality of the universe to constitute the system as in, for example, quantum cosmology). Since the gravitational interaction has infinite range and couples to all sources of energy, interactions with some sort of environment are an inevitability. As such, environmentally induced decoherence must also be present and would play an important role in the classicalization of primordial density fluctuations.

The purpose of the present paper is to determine precisely the effects by the ‘inflationary environment’ (we will elucidate this notion below) on cosmological perturbations and to study the resultant decoherence. Other authors have also examined this problem (see, for example, [6–10])—however, we are the first to present an exact analytic expression for the density matrix with a realistic environment–system interaction.

The paper is organized as follows. In the following section, we review some basic properties of decoherence of which we will make use. After reviewing the quantum theory of cosmological perturbations in section 3, we make clear our concept of the environment and motivate some realistic interactions in section 4. Subsequently, we develop necessary formalism which, in section 6, we make use of to demonstrate the classical nature of the system and calculate the decoherence time scale.

2. Decoherence

In the present section, we intend to present an extremely (but, hopefully, not exceedingly) terse account of the theory of decoherence. The physics of classicalization is elegant and subtle and a thorough exposition of its finer points would bring us too far afield from the purpose of this paper. We confine our attention solely to the cardinal features and disregard any peripheral aspects. The reader unsatisfied by our presentation is encouraged to consult any of a number of excellent reviews of which we mention but a few [1, 11, 12].

From an operational perspective, the process of decoherence usually refers to the disappearance of off-diagonal elements of the density matrix. These elements (phase
relations) represent the interference of states inherent in any quantum system. Evidently, their disappearance is an integral part of a quantum-to-classical transition.

Having mathematically defined decoherence, we now turn to the physical processes responsible for it. At the heart lies the concept of the open system and the near impossibility of forming a macroscopic closed state. Virtually, all realistic systems must interact with an environment of some sort where, by environment, we refer to degrees of freedom which interact with degrees of freedom in our system but which are not witnessed by some observer intent only on the evolution of the system. This leads to the first important characteristic of decoherence—its *ubiquity*.

Next, we come upon the concept of entangled states. Initially, if we disregard all correlations between system and environment, our composite wavefunction (system + environment) can be expressed as the outer product of the system and environment states (more generally, it will be the outer product of ensembles of states, as is the case when one makes use of density matrices). Though initially factorizable, interactions between the environment–system pair rapidly change this: the total state evolves from the form

\[ |\Psi_1\rangle = \left(\sum_i \alpha_i |\phi_{\text{system}}^i\rangle\right) \otimes \left(\sum_j \beta_j |\phi_{\text{environment}}^j\rangle\right) \]  

\( (1) \)



to

\[ |\Psi\rangle = \sum_{i,j} \gamma_{ij} |\phi_{\text{system}}^i\rangle |\phi_{\text{environment}}^j\rangle, \]  

\( (2) \)

where \( (2) \) represents an entangled state and, as such, is non-factorizable in this basis. Entanglement is key to the whole process for the following reason—an entangled state produces a density matrix which is non-factorizable. The operational equivalent of an observer ignoring the environmental degrees of freedom is to trace out (partial trace) these degrees of freedom. Due to the orthogonality of the environment states, the observer is left with a density matrix which diagonalizes as the states entangle (the fact that the decoherence rate is related to the rate of entanglement has been used to estimate decoherence times. See, for example, [6, 13]). An interesting property of classicalization follows from this—the interference terms are still present, but are unobservable by a ‘local’ observer (local in the sense that he only observes the system).

These ‘hidden’ interference terms lead us to our next point. By tracing out the environmental degrees of freedom, an observer throws away all the correlation terms, leading to a decrease in the amount of information available in the system—hence, this leads to an increase in the entropy from which we can conclude that decoherence is a practically irreversible process.

The system being decohered, it can only be found in a much smaller subset of the states that were previously allowed—this is what prevents us, in part, from seeing ‘Schrödinger’s Cat’ states at a macroscopic level. The states that diagonalize the density matrix of the system are referred to as pointer states [14], and these states remain in the subset of physical states after decoherence. If the evolution of the system is dominated by the self-Hamiltonian of the system, the pointer basis is composed of the eigenstates of the self-Hamiltonian while, if the interaction dominates, the eigenstates of the interaction form the basis [15]. Pointer states are also those states for which the production of entropy during decoherence is minimized (predictability sieve) [16].

Finally, we conclude with a heuristic view of decoherence. Neglecting certain interacting degrees of freedom in a theory will generally lead to an apparent loss of unitarity. Thus,
one should expect a flow of probability out of the system which, in turn, manifests itself as a vanishing of certain elements of the density matrix.

3. Quantum perturbations in an inflationary universe

3.1. The action for quantum perturbations

We provide in this section an overview of the quantum theory of cosmological perturbations in an inflationary background. For a more in-depth treatment, the reader is referred to [17] or [18].

The classical action for an inflationary model is given by (in this and in what follows, we set $G = \hbar = 1$)

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right). \tag{3}$$

If the potential $V(\phi)$ for the matter scalar field $\phi$ is sufficiently flat and if, in addition, initial conditions are chosen for which the kinetic and spatial gradient terms in the energy density are negligible, this action leads to a period of inflation during which the spacetime background is close to de Sitter

$$dx^2 = \left( \frac{a}{\eta} \right)^2 \left( -d\eta^2 + (dx^i)^2 \right), \tag{4}$$

where $\eta$ is conformal time.

During the course of inflation, any pre-existing classical fluctuations are diluted exponentially. However, quantum fluctuations are present at all times in the vacuum state of the matter and metric fluctuations about the classical background spacetime. Their wavelengths are stretched exponentially, become larger than the Hubble radius $H^{-1}(t)$ and re-enter the Hubble radius after inflation ends. These fluctuations are hypothesized to be the source of the currently observed density inhomogeneities and microwave background anisotropies. In order for this hypothesis to be correct, the fluctuations must decohere.

The quantum theory of linear fluctuations about a classical background spacetime is a well-established subject (see, e.g., the reviews [17] or [18]). If the matter has no anisotropic stress (which is the case if matter is described by a collection of scalar fields), then a gauge (coordinate system) can be chosen in which the metric including its (scalar metric) fluctuations $\psi$ can be written as

$$dx^2 = \left( \frac{a}{\eta} \right)^2 \left( -(1 + 2\psi(x, \eta)) \, d\eta^2 + (1 - 2\psi(x, \eta))(dx^i)^2 \right), \tag{5}$$

and the matter including its perturbation $\delta \phi$ is

$$\phi \rightarrow \phi + \delta \phi(x, \eta). \tag{6}$$

The quantum theory of cosmological perturbations is based on the canonical quantization of the metric and matter fluctuations about the classical background given by $a(\eta)$ and $\phi(\eta)$. Since the metric and matter fluctuations are coupled via the Einstein constraint equations, the scalar metric fluctuations contain only one independent degree of freedom. To identify this degree of freedom, we expand the action (3) to second order in $\delta \phi$ and $\psi$, and combine the terms by making use of the so-called Mukhanov variable [19, 20]

$$v = a(\eta) \left[ \delta \phi + \frac{\phi'}{H} \psi \right]. \tag{7}$$

1 We are not considering the vector and tensor metric fluctuations. In an expanding background, the vector perturbations decay, and the tensor fluctuations are less important than the scalar metric modes.
in terms of which the perturbed action $S_2$ takes on a canonical form (the kinetic term is canonical) and the perturbations can hence readily be quantized:

$$ S_2 = \frac{1}{2} \int d^4x \left[ v^2 - v, v, + \frac{z''}{z} v^2 \right], \quad (8) $$

where $z = \frac{a''}{a}$ and a prime indicates a derivative with respect to $\eta$. This action contains no interaction terms: it represents the evolution of a free scalar field with a time-dependent square mass

$$ m^2 = -\frac{z''}{z}, \quad (9) $$

propagating in a flat, static spacetime. This action leads directly to a well-defined quantum theory via the canonical commutation relations.

The Hamiltonian corresponding to the above action $S_2$ can be written in second quantized form

$$ H = \int d^3k \left[ k (a^+ k a_k + a^+ -k a_{-k} + 1) - i \frac{z'}{z} (a^* k a_{-k} - \text{h.c.}) \right]. \quad (10) $$

The first term in the brackets represents back-to-back harmonic oscillators, in phase such that the system has no net momentum. The second term leads to the ‘squeezing’ of the oscillators on scales larger than the Hubble radius $H^{-1}(t)$ (on these scales the second term in (10) dominates over the spatial gradient terms coming from the first term in the equation of motion for $v$). On these scales, the squeezing leads to an increase in the mode amplitude

$$ v_k(\eta) \sim z(\eta) \sim a(\eta), \quad \text{(11)} $$

where the second proportionality holds if the equation of state of the background geometry does not change in time. We take this to be the case in our subsequent analysis.

### 3.2. Properties of squeezed states

There exists an extensive literature on squeezed states. We refer the reader to [21, 22] for the mathematical properties of squeezed states. For their physical interest, we direct the reader to [23].

The evolution of a state of a system governed by the Hamiltonian (10) can be described by the following evolution operator:

$$ U = S(r_k, \phi_k) R(\theta_k), \quad \text{(12)} $$

where

$$ S_k(\eta) = \exp \left[ \frac{r_k(\eta)}{2} (e^{-2\phi_k(\eta)} a_{-k} a_k - \text{h.c.}) \right] \quad \text{(13)} $$

and

$$ R(\theta_k) = \exp \left[ -i\theta_k (a^+ k a_k + a^+ -k a_{-k}) \right], \quad \text{(14)} $$

where $S(r_k, \phi_k)$ is the two-mode squeeze operator, $R(\theta_k)$ is the rotation operator and the real number $r$ is known as the squeeze factor. The rotation operator and the phase $\theta_k$ play no important role in what follows hence we ignore them from now on.

The action of the squeezing operator on the vacuum results in squeezed vacuum states $S_k(\eta)|0\rangle = |k\rangle$

$$ = \sum_{n=0}^{\infty} \frac{1}{\cosh(r_k(\eta))} \left( -e^{2\phi_k(\eta)} \tanh(r_k(\eta)) \right)^n |n, k; -n, -k\rangle, \quad \text{(15)} $$

where $k$ denotes momentum.
The behaviour of the squeezing parameter $r_k$ is completely determined by the background geometry. The evolution of the squeezing parameters is typically very complicated, but an exact solution is known in the case of a de Sitter background [5]:

$$r_k = \sinh^{-1}\left(\frac{1}{2k\eta}\right),$$  \hspace{1cm} (16)

$$\psi_k = -\frac{\pi}{4} - \frac{1}{2} \arctan\left(\frac{1}{2k\eta}\right),$$  \hspace{1cm} (17)

$$\theta_k = k\eta + \tan^{-1}\left(\frac{1}{2k\eta}\right),$$  \hspace{1cm} (18)

where the vacuum state being operated upon corresponds to the Bunch–Davies vacuum.

Although squeezed states do not provide a basis (as they are overcomplete), they do form an orthogonal set of states:

$$\langle l|k \rangle \simeq \delta^l_k.$$ \hspace{1cm} (20)

This follows from the properties of many particle states with the approximation improving in the limit of large squeezing.

An important property of squeezed states of which we will make use is the fact that the number of particles in such a state can be expressed entirely in terms of the squeezing parameter via

$$\langle k|N_k|k \rangle = \sinh^2(r_k),$$ \hspace{1cm} (21)

where $N_k$ is the number operator for the $k$-mode. Physically, squeezed states represent states which have minimal uncertainty in one variable (high squeezing) of a pair of canonically conjugate variables—the uncertainty in the other is fixed by the requirement that the state saturates the Heisenberg uncertainty bound. For those states of cosmological interest, we take the squeezing to be in momentum.

For our application, the squeezing parameter will be quite large. As shown in [24],

$$r_k \approx \ln \left(\frac{a(t_2)}{a(t_1)}\right),$$ \hspace{1cm} (22)

where $a(t_1)/(a(t_2))$ is the scale factor at first (second) Hubble crossing. For current cosmological scales, $r_k \approx 10^2$.

3.3. The hidden sector

An essential ingredient in the theory of decoherence is the presence of unobserved, ‘hidden’ degrees of freedom: their interaction with our system degrees of freedom causes the delocalization of the phase relations. In this section, we show that de Sitter space naturally provides us with a hidden sector and that the borderline between the visible and invisible in our theory is naturally given by the Hubble scale.

Although de Sitter space is geodesically complete, a geodesic observer will be subject to the effects from both a particle horizon and an event horizon [25, 26]. That the latter constitutes a true event horizon can best be seen by examining the behaviour of null geodesics
in Painleve–de Sitter coordinates (see, for example, [27]), which remain finite across the horizon, in contrast to static coordinates. Specifically, we have

$$ds^2 = -\left(1 - \frac{r^2}{l^2}\right)dt^2 - 2r^2 \frac{dt}{l} dr + dr^2 + r^2 d\Omega^2. \quad (23)$$

Here, \(l = 1/H\) denotes the de Sitter radius. Clearly, setting \(r = l\) causes our timelike coordinate to become spacelike (the characteristic feature of an event horizon). Timelike observers that cross from \(r - |\epsilon|\) to \(r + |\epsilon|\) find themselves incapable of getting back, trapped outside of a sphere of radius \(l\).

Now, if one transforms to the coordinates typically used when discussing inflation (the so-called planar coordinates) and examines the behaviour of timelike geodesics, one finds that all timelike observers originating within the horizon must eventually cross.

The zero-point fluctuations induced by the horizon [28] can be thought of as the seeds for metric perturbations [29, 30]. Heuristically, the horizon can be thought of as a source of thermal radiation with a temperature \(H/2\pi\) (in complete analogy with the black hole case). This radiation then produces gravitational metric perturbations, with the same spectrum, which are stretched out by subsequent cosmological evolution and ultimately lead to the formation of structure in the post-inflationary universe.

Note, however, that this naive picture is not quite correct—the equation of state of the produced radiation is not thermal [31], and including the effects of gravitational backreaction leads to corrections to the thermal spectrum (this is also true in the black hole case [32]). However, our ensuing discussion in no way relies on strict thermality.

We consider our observer to be to the left of the horizon in figure 1. In accord with our above discussion, we take our radiation to be produced at the horizon with a continuous distribution such that a non-vanishing subset of our modes have wavelengths less than \(l\) (or \(H^{-1}\)). It follows that our observer in planar coordinates, due to the event horizon, will be prevented from observing certain radiation modes. We conclude that those modes which are unobservable are those associated with physical wavelengths less than the horizon scale. Of course, gravitational redshifting will cause these modes to stretch and eventually cross the horizon. The point is that particle production is a continuous process and we expect that, at all times, a certain set of modes will be unobservable, and these modes will be associated with...
physical wavelengths less than $H^{-1}$. As a result of this, decoherence is an inevitability and we define our environment to be a set of modes whose physical momenta are greater than the Hubble scale.

Another reason for tracing out the UV sector comes about when one considers that the effects of the shortest modes produced during inflation are not observed in CMB anisotropies.

Having identified the modes of the theory which we must trace out, we ask what happens if we trace out additional modes. For example, if an observer was only interested in very low energy modes ($k \ll H$) he could ignore (or trace out) modes with ($k < H$, but not $k \ll H$)—surely this would provide an additional source of decoherence as it increases the environment. However, compare this to the case of an observer who is interested in all super-Hubble modes. The second observer would see less decoherence than the first. Decoherence is, after all, an observer dependent effect—an observer who could monitor every degree of freedom in the universe would not expect to see any decoherence. However, our goal is to determine a lower bound on the amount of decoherence as measured by any observer in the ‘out’ region of our Penrose diagram. In this case, we trace out only those modes which we must (i.e. all modes on sub-horizon scales) and take our system to be composed of the rest.

4. Interactions with the environment

Key to our investigation of decoherence is the notion of the environment. Such an environment can take on many different guises. As was stated above, we define ours in the following fashion: expanding the background fields (gravity and the inflaton) in terms of fluctuations, we identify our environment with the fluctuations whose wavelengths are less than some cutoff, while our system consists of those wavelengths greater than this cutoff. As explained above, since we are operating in a de Sitter background, the natural scale to pick for the cutoff is the Hubble scale.

In order to determine the precise form of interactions inherent to a system of cosmological perturbations, we expand (8) to the next order (recall that expanding to second order is what led to a free field theory) in the fluctuations, and express the result in terms of $v(x, \eta)$. Interactions can either be purely gravitational in nature (backreaction), or they can arise in the matter sector through $V(\phi)$, the inflaton potential.

4.1. Gravitational backreaction

To focus on the interactions due to gravitational backreaction, we must expand the gravitational action to third order in the amplitude of the perturbations and write the potential in terms of the Mukhanov variable $v$. Expanding to higher order simply introduces more complicated interactions. For our purposes, we restrict our attention to the simplest terms that arise.

In the case where the metric, including its fluctuation, is given by

$$ds^2 = a^2(\eta)[-(1 + 2\phi) \, d\eta^2 + (1 - 2\psi)(dx^i)^2]. \tag{24}$$

Note that, to first order at least, and in the case where the matter sector lacks a source of anisotropic stress, Einstein’s equations lead us to conclude that $\psi = \phi [17]$. At second order, the first-order perturbations themselves acts as sources of anisotropic stress (see, for example, [38]). However, the appearance of anisotropic stress only at second order implies that, though the second-order contributions to $\psi$ and $\phi$ are not equal, the first-order terms are equivalent. Although we make use of backreaction in what follows, we only consider the effects of the first-order terms and so our line element can be simplified to

$$ds^2 = a^2(\eta)[-(1 + 2\psi) \, d\eta^2 + (1 - 2\psi)(dx^i)^2]. \tag{25}$$
We can expand the Ricci scalar in powers of \( \psi \) to obtain
\[
R = \frac{6a''(\eta)}{a^3(\eta)(1 + 2\psi)} = \frac{6}{a^3(\eta)} \left( 1 - 2\psi + 4\psi^2 - 8\psi^3 \cdots \right),
\]
(26)
(where terms with derivatives either temporal or spatial of \( \psi \) have been ignored as they are sub-dominant) from which we can extract our term of interest, \( R^{(3)} \),
\[
R^{(3)} = -48 \frac{a''(\eta)}{a^3(\eta)} \psi^3,
\]
(27)
which is the leading order gravitational self-interaction term. Recalling definition (7) of the Mukhanov variable in a slow-roll inflationary background, our potential, expressed in terms of \( \nu \), becomes (neglecting \( \delta\phi \) when substituting \( \nu \) for \( \psi \) and we use the fact that, for our inflationary background, \( a(\eta) = 1/(H\eta) \))
\[
V = \frac{1}{16\pi M^2_{Pl}} \int d^3x \sqrt{-g} R^{(3)}
\]
(28)
\[
= \frac{1}{M^2_{Pl}} \int d^3x a^4(\eta) \frac{4}{\pi} \frac{a''(\eta)}{a^3(\eta)} \left( \frac{\mathcal{H}(\nu)}{\langle \phi \rangle a} \right)^3
\]
\[
= \frac{3}{\sqrt{2} \pi} \int d^3x \frac{H^2}{M^2_{Pl} (2\epsilon)^{3/2}} a(\eta)^3
\]
(29)
so that
\[
V = \int d^3x \lambda \nu^3,
\]
(30)
with
\[
\lambda = \frac{3}{\sqrt{2} \pi} \frac{H^2}{M^2_{Pl} (2\epsilon)^{3/2}} a(\eta) \lambda_0,
\]
(31)
and where we have used the slow roll conditions
\[
H^2 = V(\phi)/(3M^2_{Pl}), \quad 3H\dot{\phi} = -V'
\]
(32)
and
\[
\epsilon \equiv \frac{M^2_{Pl}}{2} \left( \frac{V'}{V} \right)^2,
\]
(33)
is one of the slow-roll parameters. Our dimensionful coupling is explicitly time dependent—this is to be expected since it is associated with a fixed physical scale and our theory (8) is written entirely in terms of co-moving quantities.

4.2. Inflaton interactions

In addition to the gravitational backreaction terms, there are also interactions due to nonlinearities in the matter evolution equation. Consider a model of chaotic inflation with a potential of the form
\[
V = \int d^3x \sqrt{-g} \mu \phi^2,
\]
(34)
where \( \mu \) is a dimensionless coupling constant. The perturbations produced during inflation are joint matter and metric fluctuations. The matter part of the fluctuation, denoted by \( \delta\phi \), give rise to a cubic term in the interaction potential of the form
\[
V \sim \int d^3x 4\sqrt{-g} \mu \phi (\delta\phi)^3,
\]
(35)
where, in the case of slow-roll inflation, we can treat $\phi$ as a constant. Now, writing the potential in terms of the Mukhanov variable (and this time neglecting $\psi$ in the process of substitution), we have

$$V \sim \int d^3x \, 4a^4(\eta) \mu \phi \left(\frac{v}{a}\right)^3 = \int d^3x \, a(\eta) 4\mu \phi v^3,$$

so that

$$\lambda = 4\mu \phi a(\eta).$$

How do the coupling strengths of the two potentials compare? Taking the ratio of the two, we find

$$\frac{\lambda_{\text{inf}}}{\lambda_{\text{grav}}} = \frac{4\mu \phi a(\eta)}{4\sqrt{2} H^2 M_{\text{Pl}}} = \frac{4\sqrt{2} (2 \epsilon)^{3/2} \mu \phi}{3 \pi \sqrt{2}} M_{\text{Pl}} = \frac{4\sqrt{2}}{3 \pi} \frac{(2 \epsilon)^{3/2} M_{\text{Pl}}}{\phi^3}.$$ 

Since the observationally allowed value for $\phi$ at times when fluctuations relevant to current observations are generated is of the order $10^{-3} M_{\text{Pl}}$, we find that the gravitational coupling could conceivably dominate depending on the value of $\epsilon$. Since we are only interested in obtaining a lower bound on the decoherence rate, and due to the fact that the exact form of the inflaton potential (along with the initial conditions that determine $\epsilon$) is model dependent, we consider gravitational backreaction to be the main source of decoherence in what follows. Nonetheless, the above demonstrates that inflaton interactions have the potential to be important.

We couple our system to the environment by writing

$$V = \int d^3x \, \lambda v^3 = \int d^3x \, \lambda v^2 \psi,$$

where $v$ now refers only to the expansion of the Mukhanov variable in momenta less than some cutoff and $\psi$ is the same field but expanded in terms of the environment modes.

In determining the specific form of our interaction, we have neglected to include perturbation variables of order higher than 1. Generally speaking, a consistent perturbative expansion to second order would contain terms quadratic in the first-order fluctuation as well as terms linear in the second-order fluctuation—we have ignored the higher order terms since, as was shown in [39], the effects of the second-order variables on the first are quite small (approximately $1$ in $10^{-5}$) and leave the functional form of the perturbations invariant. Moreover, the perturbations that are detected through their effects on the CMB are necessarily the first-order contributions, as the second-order fluctuations (of order $10^{-10}$) are too small to be detected. Since they are unobservable, had we included them in the calculation, we would have had to trace them out which would only serve to increase the decoherence rate—as we are only trying to establish a lower bound, we are justified in neglecting them.

5. The density matrix

Having determined a candidate interaction between our system and the environment, we now face the task of deriving an appropriate master equation in order to determine the time dependence of our density matrix. Several approaches exist (for example, [33, 34]) which have been used by a number of authors—rather, we follow the method of [35] which we now review.

We assume that our system of interest is weakly interacting with some environment. The Von Neumann equation for the full density matrix ($\rho$) reads. (Note that we make use of
conformal time. This is due to the fact that our action (8) is expressed in terms of conformal time.)

\[
\frac{d \rho}{d \eta} = -i[H, \rho],
\]

(39)

where \(H\) is the total Hamiltonian of the system and can be written as

\[
H = H_0 + V,
\]

(40)

where \(H_0\) is the self-Hamiltonian and \(V\) couples the system to the environment. Note that \((\rho)\) denotes the full density matrix for the system and the environment.

Switching to the interaction representation (39) takes on the form

\[
\frac{d \rho}{d \eta} = -i[\nabla, \rho],
\]

(41)

where \(\rho = \exp(iH_0 \eta)\rho \exp(-iH_0 \eta)\),

(42)

with a similar expression for \(V\).

A perturbative solution of (41) is found to be given by the following:

\[
\rho = \rho_0 + \int_0^\eta d\tau_2 \int_0^{\tau_2} d\tau_1 [\nabla(\tau_2), [\nabla(\tau_1), \rho_0]] + \cdots.
\]

(43)

Our ultimate goal is to derive an equation of motion for the reduced density matrix. \((\rho_A = Tr B \rho, \text{where } A \text{ denotes the system quantities while } B \text{ refers to the environment. We use this notation throughout the rest of the paper.) To this end, we trace out the environmental degree of freedoms to obtain}

\[
\rho_A = \rho_A^0 - \int_0^\eta d\tau_2 \int_0^{\tau_2} d\tau_1 Tr B([\nabla(\tau_2), [\nabla(\tau_1), \rho_0]]) + \cdots.
\]

(44)

Note that the first-order term has vanished—this is due to the specific form of our system–environment coupling. Had we used a potential in which an even power of the environment field had appeared, we would have obtained a non-vanishing contribution at this order. Had this been the case, the first-order term could have been neglected on the grounds that it would lead to unitary evolution of the system—since our goal is to study the decoherence of the system (a non-unitary process), we can safely ignore such terms.

We find that (see the appendix)

\[
Tr B(\nabla(x_1, \eta_1)\nabla(x_2, \eta_2)\rho) = \frac{8 \pi^2 a^2(\eta)}{V H^5} \delta(\eta_1 - \eta_2) \delta(x_1 - x_2).
\]

(45)

In light of the fact that this equation was derived in the limit of small time intervals, we can approximate the integral in (44) by the product of the integrand with the time interval, \(\eta\).

Bringing \((\rho_0)\) to the left-hand side and dividing both sides by time allows us to write

\[
\frac{\rho - \rho_0}{\eta} = \frac{d \rho}{d \eta},
\]

(46)

in the limit of small \(\eta\).

As the initial time \((\eta = 0)\) is arbitrary, we conclude that our equation for the reduced density matrix may be written as (in terms of physical time)

\[
\frac{d \rho(t)}{dt} \simeq -a(t) \frac{8 \pi^2 a^2(t)}{V H^5} \int d^3 x [v^2, [v^2, \rho(t)]]
\]

(47)
where the details have been relegated to the appendix. $V$ is a normalization volume, $\mathcal{H} = a(t)H$, with $H$ the physical Hubble scale, and we note that in order to obtain condition (45), it was necessary to eliminate non-local terms by coarse-graining over scales of order $\mathcal{H}$ in both time and space.

The differential equation (47) is the master equation for our system. In order to proceed, we obtain a matrix representation in the basis of squeezed states. Again, we point out that these do not form a true basis for the Hilbert space (note, however that the use of an overcomplete basis poses no difficulties when it comes to obtaining representations of the density matrix [11]). However, in the limit of large squeezing, squeezed states become orthogonal to other states in the system. Since squeezed states are the ‘natural’ states of the system, we view all other states as being spurious and truncate our Hilbert space so that it contains only the former. Furthermore, as our interactions are small compared to (10), we identify the squeezed states as our pointer basis [36].

Finding a matrix representation of equation (47) is a relatively simple affair—due to the nature of the squeezed states, the expectation value operator $v_{n,n}^2$ must be diagonal in this (discrete) basis of states. This, along with the identities [21]:

$$S(r_k, \varphi_k)a_{\pm k}S^\dagger (r_k, \varphi_k) = a_{\mp k}\cosh(r_k) + a_{\pm k}^\dagger e^{2i\varphi_k} \sinh(r_k)$$

and

$$S^{-1}(r_k, \varphi_k) = S^\dagger (r_k, \varphi_k) = S(-r_k, \varphi_k),$$

renders the calculation relatively straightforward. Note that $\langle k | N_k | k \rangle = \sinh^2(r_k)$, where $N_k$ is the number operator [21]. With this in mind, we find that (47) reduces to

$$\frac{d\rho_{ij}}{dt} \simeq -a(t)\frac{128\pi^2}{V^2} \frac{\lambda^2(t)}{\mathcal{H}^5} \left( \frac{\sinh^4(r_i)}{k^2_i} + \frac{\sinh^4(r_j)}{k^2_j} - 2\frac{\sinh^2(r_i) \sinh^2(r_j)}{k_i k_j} \right) \rho_{ij},$$

where, for simplicity, we have replaced the $\cosh^2(r)$ terms with $\sinh^2(r)$ since we are interested in the limit of large $r$.

The combination $\frac{\sinh^2(r_k)}{V} = n_i(t) = a^2(t)n_i(0)$ is to be interpreted as the particle density, a quantity which is finite in the thermodynamic limit. Clearly, the decoherence rate increases as the difference between the two momenta increases. For this reason, we take our states of interest to have approximately the same momenta, and the above reduces to

$$\frac{d\rho_{ij}}{dt} \simeq -128\pi^2 a^2(t)\lambda^2(0) \frac{V^2}{H^5} \frac{k^2_i k^2_j}{k^2_i - k^2_j} \rho_{ij},$$

in terms of physical time and co-moving momenta and volume. Note that (51) is strictly only valid in the regime of $k_i \approx k_j$.

A few things are immediately obvious:

1. The diagonal elements suffer no loss of coherence. This actually could have been surmised much earlier from equation (44) by noting that the trace over the system degrees of freedom must vanish.

2. The rate of decoherence grows extremely rapidly. In fact, in order to decohere the system within 60 e-foldings (approximately the minimal time permissible for the duration of inflation), the initial particle density ($n_0$) can be as low as $10^{-25}$ particles per Hubble volume².

3. The particular time $t = 0$ for a pair of modes should be taken to correspond to the time that the shortest of the pair (the higher energy mode) crosses the horizon.

² In arriving at this estimate, we have considered the case where $k_i \approx k_j$, $k_j \approx H, H \approx 10^{-3} M_{Pl}, \epsilon \approx 10^{-2}$. 
So far, we have argued that a certain sector of the theory is unobservable (thus justifying a minimal amount of tracing), determined an interaction between our visible and invisible sectors, and obtained a lower bound on the parameters of the theory such that decoherence takes place within 60 e-foldings of inflation. The question remains: in a realistic cosmological model, are the parameters of the theory such that decoherence can take place during the inflationary period, and be caused by the leading order gravitational backreaction term? In other words, is the bound we found satisfied in conventional models?

In order to answer that question, we must obtain the number density of particles in a typical super-Hubble mode at first Hubble crossing.

Consider the square of the substitution we used to obtain our potential in terms of the Mukhanov variable

\[ v^2 = a^2(\eta) \left( \frac{\phi'}{\mathcal{H}} \right)^2 \psi^2. \]  
(52)

To determine the number of particles of the \( v \) field in terms of physically meaningful quantities, we must first quantize the Mukhanov field. However, once the theory is quantized, the expression (52) is meaningless—the left-hand side is an operator, while the right-hand side is a classical field. In light of this, we follow the usual route [37] in semi-classical gravity and replace \( v \) with its vacuum expectation value

\[ \langle v^2 \rangle = a^2(\eta) \left( \frac{\phi'}{\mathcal{H}} \right)^2 \psi^2. \]  
(53)

In the limit of large squeezing, we have that

\[ \langle v^2 \rangle = \frac{1}{2\pi^3} \int \frac{d^3k}{k} N_k(t), \]  
(54)

where \( N_k(t) \) is the number of particles in the \( k \)-mode at time \( t \), which scales in time as

\[ N_k(t) \propto a^4(t), \]  
(55)

where we now consider only physical (as opposed to co-moving as in the previous discussions) quantities. The extra factors of \( a(t) \) in the particle number appear because we are now considering the red-shifting of the momenta (see (16)). We expect the spectrum to be exponentially suppressed at high (sub-Hubble) momenta: therefore, to a good approximation, the integral in (54) can be taken to be over the infrared sector only. Furthermore, rather than performing the integral over the modes, we reparameterize and integrate over the times which these particular modes first crossed the horizon. In other words, we let

\[ k = \frac{H}{a(t)} \]  
(56)

and

\[ N_k(t) = a^4(t)N_H(0), \]  
(57)

where, as above, \( t = 0 \) denotes first Hubble crossing for a particular mode. We now have

\[ \langle v^2 \rangle \simeq \frac{2}{\pi^2} N_H(0) H^3 \int_0^{t_r} dt \ a^2(t) = \frac{H^2}{\pi^2} N_H(0)a^2(t_r), \]  
(58)

with \( t_r \) denoting the time of reheating and where we have ignored the time dependence of the Hubble scale.
Figure 2. After first Hubble crossing, all pairs of modes rapidly begin to decohere. In this figure, we plot the decoherence time in number of e-foldings on the vertical axis for a pair of modes of the same order of magnitude. ‘Epsilon’ refers to the slow-roll parameter and n corresponds to the number of particles per Hubble volume. For this particular realization, the inflationary scale was set to $\frac{H}{M_{Pl}} \simeq 10^{-4}$. What we find is that, at this energy, the bulk of the infrared phase space will easily decohere before the end of inflation. Since we have only considered one source of interaction, the actual process of decoherence will be even more efficient.

We now obtain a rough estimate of the decoherence time. During reheating, the inflaton will undergo periods when its total energy is dominated by its kinetic term. So, at the time of reheating, we can make the substitution $\dot{\phi}^2 \simeq \rho_r$ to obtain

$$\frac{N_{H}(0)H^2}{\pi^2} a^2(t_r) \simeq a^2(t_r) \rho_r \psi^2 \frac{H^2}{\pi^2}. \quad (59)$$

We identify $N_{H}H^3 = n_{H}(0)$ with the number of particles of momentum $H$ per Hubble volume and take the reheating temperature as $H$ so that $\rho_r \simeq H^4$. We can now make use of the fact that, observationally, $\psi^2 \approx 10^{-9}$, to deduce that $n_{H}(0) \approx 10^{-8}$ particles/Hubble volume at reheating. We can now relate this value to the one during inflation using the fact that, during preheating, the amplitude of the scalar perturbations increases by a factor of $1/\epsilon$ [17]. Thus, we find that the upper bound on the value of $n_{H}$ during slow-roll should be given by $10^{-8} \epsilon^2$.

For most inflationary models, this is well above the lower bound we found. In the case where $\epsilon \approx 10^{-4}$ (and taking $n_{H} = 10^{-16}$), we find that the modes will decohere approximately 20 e-foldings after crossing the horizon when the inflationary scale is $\frac{H}{M_{Pl}} \approx 10^{-4}$.

Note that by $a(t_r)$, we mean the value of the scale factor evaluated at the time of reheating ($t_r$) and not the functional form of the scale factor which is, of course, different than that of the inflationary epoch.
How general is this conclusion? In obtaining our main result (51) no assumptions were made about a particular model of inflation\(^1\). In fact, all of the information about the matter sector is buried in three parameters: namely \(H, \epsilon\) (through \(\lambda_0\)) and \(n_s(0)\). These quantities can vary significantly amongst various inflationary models.

In order to get a handle on which inflationary models could lead to total decoherence in the minimal time allotted to inflation (\(\gtrsim 60\) e-foldings), we must define what we mean by ‘total’ decoherence. Clearly, to be decohered is a relative condition—with time, quantum correlations become vanishingly small but, strictly speaking, never vanish entirely. Therefore, we take a pragmatic stance and define ‘total decoherence’ to be the case when the amplitude of the quantum correlations have been reduced to 1/100 of their initial values. Different modes cross the horizon at different times—the last to cross, immediately undergoes second Hubble crossing and so we certainly do not expect this decoherence mechanism to be effective for those highest energy modes, irrespective of the values of \(H\) and \(\epsilon\). Furthermore, correlations are between pairs of modes. The larger the difference in scales, the faster the decoherence.

We can now refine our earlier question on the generality of our conclusion to the following: how many e-foldings after Hubble crossing will it take for a pair of modes of the same order of magnitude to decohere as the slow-roll parameter and the number of particles per Hubble volume is varied? We show the result in figure 2.

6. Conclusion

In this paper, we have studied the decoherence of cosmological fluctuations during a period of cosmological inflation, taking the effects of squeezing into account. We have determined realistic interactions for our system of perturbations and have found that, at the same order, gravitational interactions and matter (inflaton) interactions are comparable, depending on the scale of inflation and the slow-roll parameter \(\epsilon\). Furthermore, we have justified the use of the Hubble scale as a cutoff.

Having considered the leading order gravitational correction to the action of quantized cosmological perturbations, we find that super-Hubble modes decohere long before the end of inflation and we have shown that this result holds over a large range of parameters. Of course, we have only obtained a lower bound on the decoherence rate—interactions more complicated than those considered here will generally lead to much faster decoherence times [13].

For those modes that do not fully decohere during inflation, the large number of interactions present during the radiation era should rapidly complete the process. Although we do not expect decoherence to occur as quickly during the radiation era—due to the much lower energy scale of the background—the time scale involved is potentially much greater as are the number of interactions between the fluctuations and the matter sector. As a result, we would not expect to see any trace of the quantum nature of the perturbations by the time recombination came about. As exciting as it would be to observe a trace of ‘quantumness’ in the CMB, it would be hard to imagine a process that occurs as fast as (51) suggests that would be able to leave a signature observable at the current time.

7. Appendix: Tracing out the environment

In this appendix, we explicitly calculate the partial trace of equation (44).

\(^1\) Had we used the interactions arising from the matter sector (36), we would have had to commit to a specific realization.
The expansion of the Mukhanov variable in a spatially flat background takes the form
\[
v = \frac{1}{(2\pi)^{3/2}} \int d^3\vec{k} \frac{1}{\sqrt{2|k|}} (a_k e^{-ikx} + a_k^\dagger e^{ikx}),
\]
and we restrict our attention to modes within a sphere of radius $H$ in momentum space. Since our calculation will be performed in terms of co-moving quantities and we take our cutoff to correspond to a fixed physical scale, our cutoff acquires a time dependence of the form
\[
H = a(\eta)H.
\]
Our normalization conventions are as follows:
\[
k\rangle = \sqrt{2} E_k a_k^\dagger |0\rangle, \quad (k|k') = (2\pi)^3 \delta^{(3)}(k - k'),
\]
\[
[a_k, a_k^\dagger] = (2\pi)^3 \delta^{(3)}(k - k').
\]
The identity operator has the form
\[
1 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k}.
\]
For simplicity, we ignore the effects of squeezing until the very last. As our initial conditions, we do not take the environment to be in the vacuum—this would be contrary to the basic idea of the generation of inhomogeneities. We take our states to be two-particle zero-momentum states. Were we to explicitly include the effects of squeezing, we would find that our scattering amplitude $\langle i|\phi^n|j\rangle$ would scale as $a^n(\eta)$. Our approach is as follows: we calculate the scattering amplitude for a fixed particle number (2) and, at the last step, include the additional factors of $a(\eta)$ in order to embody the effects of particle production (squeezing).

Note that we must take into account squeezing since (16) tells us that all modes in de Sitter space get squeezed.

We take these states to be populated according to a distribution which falls off exponentially in the UV, with temperature parameter $T = \beta^{-1} = \frac{H}{2\pi}$. In other words
\[
\rho_{env} = C \exp(-\beta H),
\]
where this $H$ refers to the Hamiltonian. The precise form of the distribution is immaterial—after tracing, the only information that the systems retains about the environment is its ‘size’ (the cutoff scale). As another simplification, we take the energy of the state to be dominated by its momentum. Due to the nature of squeezing and in view of our comments about the distribution, this is a perfectly justifiable assumption. $C$ is a normalization constant which we determine by the condition that the trace of the left-hand side of the equation be $\rho_{sys}$, i.e.
\[
Tr_{env} \rho = \rho_{sys},
\]
\[
Tr_{env} \rho = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \langle k, -k|\rho|k, -k\rangle
\]
\[
= \frac{C}{2\pi^2} \rho_{sys} \int dE_k \frac{E_k}{2} \exp(-2\beta E_k)
\]
\[
= \frac{C}{2\pi^2} \rho_{sys} \left( \frac{1}{32\pi^2} H^2 e^{-4\pi} (1 + 4\pi) \right) \equiv \rho_{sys}.
\]
Therefore, we set $C \approx 16\pi^3 e^{4\pi} / H^2$. 
The terms on the right-hand side will all have the basic form (aside from the trace of $\rho_0$, which is the same as the above)

$$\text{RHS} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \langle k, -k|v(x)v(x')\rho|k, -k \rangle$$

$$= \frac{8\pi}{\delta(0)H^3} \int_0^\infty \frac{dk}{k(x - x')} \sin[k(x - x')/k(x - x')] e^{-2\beta E_k}$$

where the delta function arises from the normalization of the states. Since we are only interested in physics on scales much greater than $H$, we coarse-grain over time and use the relation

$$\langle e^{i\omega k(\eta - \eta')} \rangle \approx \delta(\eta - \eta') \frac{H}{\beta}.$$

Thus, we find that

$$\text{RHS} = \frac{8\pi}{\delta(0)H^3} \delta(\eta - \eta') \int_0^\infty \frac{dk}{k(x - x')} e^{-2\beta E_k}.$$

Again, as our interest lies in scales such that $\eta(x - x') \gg 1$, we perform the substitution

$$\left\langle \frac{\sin[k(x - x')]}{k(x - x')} \right\rangle = \pi \delta(\eta(x - x')).$$

Finally, we obtain

$$\text{RHS} \approx \frac{8\pi^2}{\delta(0)} \frac{\delta(\eta - \eta') \delta(x - x')}{H^3} a^2(\eta).$$

Note that we identify $\delta(0)$ with the volume of space and we have included the additional factors of $a(\eta)$ as discussed above.

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