A numerical model of forced convection condensation on a horizontal tube in the presence of noncondensables

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Abstract. This paper presents the results of validation a model in which the flow of a vapour–air mixture is described by the Navier–Stokes equations for laminar regime or by the RANS equations for turbulent regime and the condensate film is considered using a one-dimensional model. The data available from the literature was employed to verify the proposed model for forced convection condensation with and without noncondensables. The numerical results are in good agreement with the literature experimental data. A description is presented of the details of the numerical implementation of the algorithm developed. The results of the addition test to validate the assumptions and simplifications used in the model are also presented.

1. Introduction

In recent years, interest in designing more efficient heat exchangers, including condenser units, has grown significantly due to increased requirements for energy efficiency of various technological processes, the use of thermal and waste heat and other factors. Modern condensing heat exchangers consist of tube bundles, including hundreds or thousands of tubes. Generally, while designing these facilities, various semi-empirical engineering methods \cite{1} are used, the application of which is extremely difficult if the geometric and operating parameters of the installation are different from the conditions in which the calculation methods were developed. The progress of computer technology has radically expanded the range of possibilities for studying the phenomena of heat and mass transfer in such systems. At present, the most effective approach is the one in which the movement of a steam–air mixture inside a condenser unit is modelled using single-phase hydrodynamic equations, while the liquid film simulations are performed using simplify models describing its behaviour on the walls. The film models in this case are similar to the models used in many earlier studies (i.e. \cite{2–4}) based on the fundamental study of Nusselt \cite{5}. Unfortunately, the use of more detailed models of two-phase flow (for example, models for tracking and locating the free surface) is limited by difficulties in modelling processes with significantly different time and spatial scales and also by problems with heat and mass transfer models incorporated in it.
This paper presents the results of testing a model in which the flow of a vapor–air mixture is described by the Navier–Stokes equations for laminar regime or by RANS equations for turbulent regime and the condensate film is considered using a one-dimensional model of a film. The model is verified on the experimental data on the forced convection condensation of steam from pure steam and a steam–air mixture flowing vertically downward over a single horizontal tube. Similar approaches have been used in recent papers [6, 7]. In [6], the authors use a more detailed model of the condensate film; it requires the reconstruction of the grid during the solution, which limits the further use of their model for more complex geometry. In [7], the authors use boundary layer equations which also impose some restrictions on the expansion of the region of applicability of the model. Also, in [6, 7] there are no detailed comparison of the numerical results with a significant amount of available experimental data for steam condensation from the steam–air mixture. A detailed review of the current state of the problems of studying heat transfer during film condensation of steam can be found in [1].

2. Mathematical model

2.1. Problem statement

Simulation of forced convection condensation of steam from a steam–air mixture flowing vertically downward over a single horizontal tube was performed. The scheme of the computing domain is shown in figure 1, where $T$ corresponds to temperature, $u$ to velocity, $c$ to mass fraction, $g$ to gravitation acceleration, $\theta$ to angle from stagnation point and $d_T$ to tube diameter. Subscripts “0” and “w” correspond to the free stream parameters and wall parameters respectively.

Figure 1. Problem statement.

2.2. Governing equations

To describe the external flow, a single-phase model for a mixture of steam and air was used. The basic equations are as follows:
\[ \text{div}(\rho_m u_m) = 0 \]
\[ \text{div}(\rho_m u_m u_{m,x} - \rho_m v_{\text{eff}} \nabla u_{m,x}) = -\frac{\partial p_m}{\partial x} + (\rho_m - \rho_{m0}) g \]
\[ \text{div}(\rho_m u_m u_{m,y} - \rho_m v_{\text{eff}} \nabla u_{m,y}) = -\frac{\partial p_m}{\partial y} - (\rho_m - \rho_{m0}) g \]
\[ \text{div}(\rho_m u_m c_r - \rho_m D_{\text{eff}} \nabla c_r) = 0 \]
\[ \text{div}(\rho_m u_m c_v - \rho_m D_{\text{eff}} \nabla c_v) = 0 \]
\[ \text{div}(m_v H_v + m_a H_a - \lambda_{\text{eff}} \nabla T_m) = 0 \]
\[ m_v = \rho_m u_m c_v - \rho_m D_{\text{eff}} \nabla c_v, \quad m_a = \rho_m u_m c_a - \rho_m D_{\text{eff}} \nabla c_a \]
\[ p_m = p + \rho_{m0} g y, \quad v_{\text{eff}} = v_m + v_t, \quad \lambda_{\text{eff}} = \lambda_m + \frac{\rho_v c_v v_t}{Pr}, \quad D_{\text{eff}} = D_m + \frac{v_t}{Sc} \]

where \( u \) denotes velocity vector, \( \rho \) density, \( p \) pressure, \( \nu \) kinematic viscosity, \( \lambda \) thermal conductivity, \( D \) the diffusion coefficient, \( H \) enthalpies, \( Pr \) the Prandtl number and \( Sc \) the Schmidt number. Subscript “\( m \)” corresponds to the mixture parameter, “\( a \)” to the air parameters, “\( v \)” to the vapour parameters, “\( t \)” to turbulence parameters.

The enthalpies of the mixture components are calculated by the following relationships:
\[ H_v = c_{pv} T_m + \Delta h_v, \quad H_a = c_{pa} T_m \]

(2)

where \( \Delta h_v \) is latent heat at free stream parameters.

The system of equations (1) is valid for both turbulent and laminar regimes. In the latter case, the coefficient of turbulent viscosity \( \nu_t \) is zero.

For \( D_m \), a convenient empirical formula [8] is used:
\[ D_m = 0.216 \cdot 10^{-4} \left( T / 273.15 \right)^{1/8} \left( 1.01 \cdot 10^4 / P \right) \]

(3)

The system (1) must be supplemented by boundary conditions. Below, only the boundary conditions on the cylinder surface are discussed because on the other boundary of the computation domain the standard free stream conditions were used. In the present study the boundary condition on the tube simulate the flow of condensate film along the cylinder wall. If one creates a list of the boundary faces of the boundary control volumes (CVs) connected with the tube and sorts them in order of increasing the angle \( \theta \), one obtains a “curvilinear coordinate system” \((\zeta, \eta)\), where \( \zeta = d\theta/2 \). In what follows, when describing a film, this coordinate system will be used. The value \( \theta \) corresponds to the \( \eta \)-coordinate of the wall in that coordinate system. In this coordinate system, simple one-dimension equations are solved to describe condensate film flow.

The main assumptions of the model are:
1. The film is considered thin compared to the dimensions of the grid cells. This means that, for the external steam–air mixture flow, the boundary of the tubes coincides with the boundary of the film.
2. The film flow is laminar.
3. In the equations of momentum and energy conservation of the film, inertial terms can be neglected (in fact, the Nusselt model is used).

As in the Nusselt theory, a mass balance for the condensate film gives:
\[ \frac{d}{d\zeta} \left( \delta \bar{u}_j \right) = \frac{m_f}{\rho_f} \delta \theta, \quad \bar{u}_j = \frac{1}{\delta \theta} \int u_{j,\eta} d\eta \]

(4)

A momentum equation for streamwise film velocity:
\[
\frac{d}{d\eta} \left( \mu_f \frac{du_f}{d\eta} \right) - \frac{\partial p_m}{\partial \zeta} + g_\zeta \left( \rho_f - \rho_{m0} \right) = 0
\]

(5)

\[
u_f (0) = 0, \quad \mu_f \frac{du_f}{d\eta} \bigg|_{\eta=0} = \tau_{sm}
\]

(6)

where \( m_f \) denotes condensation mass flux, \( \mu \) dynamic viscosity and \( \tau \) viscosity stress. Subscripts “f” and “sm” correspond to film and to surface mixture respectively.

Integrating equation (5), one can relate the film thickness to the average velocity and velocity on the interphase surface:

\[
u_f (\eta) = \frac{g_\zeta \left( 1 - \frac{\rho_{m0}}{\rho_f} \right)}{\nu_f} \left( \delta \eta - \frac{\eta^2}{2} \right) + \frac{\tau_{sm}}{\rho_f \nu_f} \eta
\]

(7)

where subscript \( s \) denotes film surface.

An energy equation:

\[
\frac{d}{d\eta} \left( \frac{\lambda_f}{\nu_f} \frac{dT_f}{d\eta} \right) = 0
\]

(8)

\[
T_f (0) = T_w, \quad T_f (\delta) = T_{sat}
\]

(9)

where subscript “sat” correspond to saturated parameters.

To calculate the temperature at the interphase boundary, the mass fraction of the vapour and the mass flux, one can use the interface balance equation:

\[
m_f \Delta h_f (T_{sat}) = \left[ \frac{\lambda_f}{\nu_f} \frac{dT_f}{d\eta} \right]_{\eta=0} - q_{sm}
\]

(10)

\[
m_f = \frac{j_{sv}}{1 - c_{vs}}
\]

\[
p_{vs} = p_{sat} (T_{sat}), \quad r_{vs} = \frac{p_{vs}}{p_m}, \quad c_{vs} = \frac{r_{vs} M_v}{r_{vs} M_v + (1 - r_{vs}) M_u}
\]

where \( M \) is molar mass, \( q_{sm} \) the heat flux due to thermal conductivity to the interface surface, and \( j_{sv} \) the diffusion flux of steam to the film surface, \( r_{vs} \) the molar fraction of vapor.

2.3. Solution algorithm

The results presented in this paper were obtained using the in-house CFD code ANES [9], developed in the Department of Engineering Thermophysics of the Moscow Power Engineering Institute. In this
study, ANES was used as a convenient and versatile software environment for the numerical solution of the conservation equations. The key point of the solution is describing the condensate film. Equations for a film are solved before the iteration of the equations of hydrodynamics. The surface of the tube is divided into one-dimensional CVs. The width of the CV $\Delta \zeta$ depends on the external grid. All variables of the film are specified for the centre of the CV, except for the flow rate per unit length of the condensate $\Gamma$, which is specified on the faces of the CV.

To start the calculation, one needs to set the value of $\Gamma(\zeta=0)$. In current calculations, it is assumed that $\Gamma(\zeta=0) = 0$. Note that the calculation does not require the film thickness at $\zeta = 0$ (in the Nusselt model this value is non-zero for $\zeta = 0$).

The CV is computed sequentially using the following iterative algorithm:

1. Set the initial value of the film thickness $\delta$. For internal CV, this is just the value from the previous CV. For the first CV, the value calculated by the Nusselt solution was taken.
2. Calculate the mass flux at the phase boundary solving the nonlinear energy balance equation, written through the temperature at the film surface.
3. Calculate the flow rate per unit length of the condensate at the right boundary of the CV.
4. Calculate the new film thickness solving the nonlinear equation.
5. If convergence occurs, i.e. the film thickness does not vary from iteration to iteration, then the step of calculation is over and go to next hydrodynamics iteration. Otherwise, set new film thickness and go to step 2.

### 2.4. Mesh

The calculations were performed on two types of grids: a staggered grid in the polar coordinate system (SG) and an unstructured Cartesian grid with cut cells (UCGCC). The first grid was used as a reference (it has uniform CVs near the tube wall) for testing the solution obtained on the second grid (that type more suitable for complex geometry). This is because the use of cut cells leads to the so-called ‘small cell problem’. Near the embedded boundary, the grid cells may be orders of magnitude smaller than regular Cartesian grid cells (figure 2), which leads to oscillations of the parameters along the wall. This is, for instance, typically a problem for turbulent flow modelling [10]. Figure 3 shows a comparison of the film thickness distributions obtained on both types of grid. The parameters
corresponded to the laminar condensation of slow-moving pure steam $u_0 = 0.5$ m/s, $p_0 = 20$ kPa, $\Delta T = 10$ K, $d_T = 12.5$ mm. The figure also shows a smoothed solution obtained on the UCGCC and a distribution corresponding to the Nusselt theory. The film thickness at 0, obtained from the Nusselt solution, was chosen as the scale. Integral parameters differ by no more than 7%.

2.5. Addition test

It is well known that when liquid flows around a cylinder, unsteady effects can be observed. It is also known that at Reynolds numbers of about 400, low levels of turbulence start to appear within the vortices; only for high values of Re (say $10^4$), does the turbulence spread out of the vortices and turbulent wake is observed [11]. For saturated steam at atmospheric pressure and when the cylinder diameter is equal to 12.5 mm, the Reynolds numbers 400 and $10^4$ correspond to a velocity of flow equal to 0.66 m/s and 16.5 m/s respectively. This range of velocity are important in practice. Because of these circumstances, it is necessary to test the suitability of the stationary formulation of the problem in the computational domain containing only half of the cylinder, as well as the impact of non-use or the use of turbulence models on the results. These tests were performed. The calculation results were compared with the results of the measurement of position of point of separation for moderate Reynolds numbers (from $10^2$ to $10^4$) from [12]. Good agreement was obtained. The calculation results showed that, at Reynolds numbers above 300, one should use a turbulence model. In our calculations, the SST turbulence model [13] was used. Also, a comparison was carried out between the pressure distribution on the surface of a cylinder in the presence and absence of suction from [14] and the results of our simulation. (See, for instance, the results for Reynolds number equal to 8300 in figure 4.) Additionally, all of these calculations were also performed in the unsteady problem formulation in the computer domain containing a full tube. In general, the use of this formulation led to a better coincidence with experimental data, but the difference was small and was located in the rear region of the cylinder. When the angle $\theta$ exceeds 90°, the film thickness is significant and so this region makes a substantially lower contribution to the overall heat transfer coefficient during condensation; therefore, the stationary formulation described earlier was chosen as a base.

![Figure 4](image_url)

**Figure 4.** Pressure distribution along tube surface: the dotted line shows the experimental results [14], the solid line shows calculation results. 1 denotes normal, 2 suction with 2.6 % from $u_0$, and 3 blowing with 2.6% from $u_0$. 
3. Comparison with the experimental data

The mathematical model developed was verified on the available experimental data relating to forced convection condensation on a horizontal tube. Lee and Rose [15] showed that heat transfer data for filmwise condensation from pure steam flowing vertically downward over a single horizontal tube with the expectation of the very high velocity data set for steam in good agreement with the equation from [2] that was adjusted in [16]:

$$Nu \Re^{-1/2} = 0.416 \left(1 + (1 + 9.47 \Re)^{1/2}\right)^{1/2}$$  \hspace{1cm} (11)$$

$$Nu = \frac{q_w d_f}{\lambda_f \Delta T}, \Re = \frac{u_r d_f \rho_f}{\mu_f}, F = \frac{g d_f \Delta h_f \mu_f}{\left(\lambda_f u_0^2 \Delta T\right)}$$  \hspace{1cm} (12)$$

In figure 5, the results obtained using the mathematical model presented here are compared with the calculation using equation (11) and Nusselt theory [5]. This comparison shows that the model can be used to describe the forced convection condensation of pure vapour on a horizontal cylinder.

To validate the model for the case of steam condensation from a moving steam–air mixture, the data presented in [17] was used. For instance, figure 6 shows the calculated and experimentally measured dependences of average heat flux for runs 412 to 419 from [17]. The difference between numerical results and experimental data is within 10%.

An additional study of the influence of condensation on the position of the separation point was carried out. The dependence of the position of the separation point on the dimensionless suction parameter $\xi_S = (q_w/\Delta h_w)(d_f/\rho_m \mu_p u_0)^{1/2}$ was investigated. The results were compared with the correlations from [18]:

$$\theta_s = 1.76 + 0.164 \xi_s + 0.00869 \xi_s^2$$  \hspace{1cm} (13)$$

$$\theta_s = 2.93 - 1.02 \exp(-0.147 \xi_s - 0.0127 \xi_s^2)$$  \hspace{1cm} (14)$$

with the correlation from [19]:

$$\theta_s = \cos^{-1}\left(-\left(\frac{\xi_s}{4.36}\right)^2\right), \xi_s < 4.36$$

$$\theta_s = \pi, \xi_s > 4.36$$  \hspace{1cm} (15)$$
and with experimental data from [20].

Figure 7 shows the results of the comparison. Equations (13) to (15) give the angle in radians, while figure 7 gives the angle in degrees. It can be seen that the results for the numbers $Re > 3000$ and $\xi_s < 3.0$ coincide with the equation (15). Experimental data in [20] was presented without detailed information about operation parameters for each point; one therefore cannot say that the experiment contradicts the theory, because all the experimental points lie within a cloud of simulation data. It is also interesting to note the importance of taking into consideration the speed of the film on the position of the separation point. The point of separation for run 419 from [17] ($p_0 = 101.1$ kPa, $u_0 = 1.7$ m/s, $r_v = 3.29\%$, $T_w = 320.4$ K) is shifted from 174 degrees to 138 degrees when the non-physical no-slip condition for tangential component of velocity was implemented. But this circumstance does not affect the integral characteristics such as overall heat transfer coefficient. The difference does not exceed 3%.

![Figure 7](image)

**Figure** 7. Comparison of separation angle with theory and experiment. $Re = 300$ (1), 600 (2), 1,000 (3), 1,500 (4), 3,000 (5), 6,000 (6), 12,000 (7), equation number 13 (8), 14 (9), 15 (10) and experimental data [20] with +/-10% (11)

4. Conclusion

A mathematical model of forced convection condensation on a horizontal tube in the presence of noncondensables was presented. The calculations were performed on two types of grids: a staggered grid in the polar coordinate system (SG) and an unstructured Cartesian grid with cut cells (UCGCC) using in-house CFD code ANES. Although using UCGCC leads to the so-called ‘small cell problem’, it was shown that the oscillations of the local characteristics of the film do not lead to errors in calculating its integral characteristics. The data from [15, 17] was employed to verify the current model. The numerical results are in good agreement with the literature experimental data. An additional study of the influence of condensation on the position of the separation point was carried out. It can be seen that the results for the numbers $Re > 3000$ and $\xi_s < 3.0$ coincide with the equation (15) and with the small amount of experimental data available in the literature.

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