Avoiding Side Effects in Complex Environments

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Abstract

Reward function specification can be difficult, even in simple environments. Realistic environments contain millions of states. Rewarding the agent for making a widget may be easy, but penalizing the multitude of possible negative side effects is hard. In toy environments, Attainable Utility Preservation (AUP) avoids side effects by penalizing shifts in the ability to achieve randomly generated goals. We scale this approach to large, randomly generated environments based on Conway’s Game of Life. By preserving optimal value for a single randomly generated reward function, AUP incurs modest overhead, completes the specified task, and avoids side effects.

1 Introduction

Reward function specification can be difficult, even when the desired behavior seems clear-cut. Rewarding progress in a race leads an agent to collect checkpoint reward, instead of completing the race [15]. We want to minimize the negative side effects of misspecification: from a manufacturing robot which breaks expensive equipment, to content recommendation systems which radicalize their users, to potential future AI systems which negatively transform the world [4, 21].

Side effect avoidance poses a version of the “frame problem”: each action can have many effects, and it is impractical to explicitly penalize all of the bad ones [5]. For example, a housekeeping agent should clean a dining room without radically rearranging furniture, and a manufacturing agent should assemble widgets without breaking equipment. A general, transferable solution to side effect avoidance would ease reward specification: the agent’s designers could just positively specify what should be done, as opposed to negatively specifying what should not be done.

Breaking equipment is bad because it hampers future optimization of the true objective (which includes our preferences about the factory). That is, there often exists a reward function \( R_{true} \) which fully specifies the agent’s task within its deployment context. In the factory setting, \( R_{true} \) might encode “assemble widgets, but don’t spill the paint, break the conveyor belt, injure workers, etc”.

We want the agent to preserve optimal value for this true reward function. While we can accept suboptimal actions (e.g. pacing the factory floor), we cannot accept the destruction of value for the true task. By avoiding negative side effects which decrease value for the true task, the designers can correct any misspecification and eventually achieve low regret for \( R_{true} \).

Despite being unable to directly specify \( R_{true} \), we demonstrate a method for preserving its optimal value anyways. In Turner et al. [25]’s toy environments, preserving optimal value for many randomly generated reward functions often preserves the optimal value for \( R_{true} \). In this paper, we generalize this approach to combinatorially complex environments and evaluate it in the chaotic and challenging SafeLife test suite [27]. We show the rather surprising result that by preserving optimal value for a single randomly generated reward function, AUP preserves optimal value for \( R_{true} \) and thereby avoids negative side effects.

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2 Prior work

AUP avoids negative side effects in small gridworld environments while preserving optimal value for randomly generated reward functions [25]. Penalizing decrease in (discounted) state reachability achieves similar results [14]. However, this approach has difficulty scaling: naively estimating all reachability functions is a task quadratic in the size of the state space.

In the supplementary material, proposition 4 shows that preserving initial state reachability [10] bounds the maximum decrease in optimal value for $R_{\text{true}}$. Unfortunately, due to irreversible dynamics, initial state reachability often cannot be preserved.

Everitt et al. [9] frame reward misspecification as the composition of a corruption function with $R_{\text{true}}$. Shah et al. [24] exploit information contained in the initial state of the environment to infer which side effects are negative; for example, if vases are present, humans must have gone out of their way to avoid them, so the agent should as well. Christiano et al. [8] infer human preference information from solicited trajectory comparison. Hadfield-Menell et al. [13] consider the provided reward function to only suggest the designer’s true preferences on the training distribution.

Robust optimization selects a trajectory which maximizes the minimum return achieved under a feasible set of reward functions [19]. However, we do not assume we can specify the feasible set. In constrained MDPs, the agent obeys constraints while maximizing the observed reward function [2, 1, 28]. Exhaustively specifying constraints is difficult.

In the multi-agent setting, empathic deep Q-learning preserves optimal value for another agent in the environment [6]. Schaul et al. [22] demonstrate a value function predictor which generalizes across both states and goals.

Safe reinforcement learning focuses on avoiding catastrophic mistakes during training [18, 11, 3, 7], while this work only considers the consequences of the learned policy.

3 Approach

Consider a Markov decision process (MDP) $\langle S, A, T, R, \gamma \rangle$ with finite state space $S$, finite action space $A$, transition function $T : S \times A \rightarrow \Delta(S)$, reward function $R : S \times A \rightarrow \mathbb{R}$, and discount factor $\gamma$. We assume the agent may take a no-op action $\emptyset \in A$. We refer to $V^*_R(s)$ as the optimal value or attainable utility of reward function $R$ at state $s$.

To define AUP’s pseudo-reward function, the designer provides a finite reward function set $R \subset \mathbb{R}^S$, hereafter referred to as the auxiliary set. This set does not necessarily contain $R_{\text{true}}$. Each auxiliary reward function $R_i \in R$ has a learned Q-function $Q_i$.

AUP penalizes average change in ability to optimize the auxiliary reward functions. The motivation is that by not changing optimal value for a wide range of auxiliary reward functions, the agent also does not decrease optimal value for $R_{\text{true}}$.

Definition (AUP reward function [25]). Let $\lambda \geq 0$. Then

$$R_{\text{AUP}}(s, a) := R(s, a) - \frac{\lambda}{|R|} \sum_{R_i \in R} \left| Q^*_i(s, a) - Q^*_i(s, \emptyset) \right| .$$

(1)

The regularization parameter $\lambda$ controls penalty severity. In practice, the learned auxiliary $Q_i$ is a stand-in for the optimal Q-function $Q^*_i$.

3.1 Example AUP trajectory

To an approximation, Wainwright and Eckersley [27]’s SafeLife evolves according to the transition rules of Conway’s Game of Life [12]. Cells endure, spawn, or die depending on how many living neighbors they have. In the eight cells surrounding the agent, no cells spawn or die – the agent can disturb dynamic patterns by merely approaching them.

Figure 1 compares AUP with Schulman et al. [23]’s Proximal Policy Optimization (PPO) in a simple scenario. While PPO optimizes the primary reward $R$, AUP also preserves the optimal value for a single auxiliary reward function ($|R| = 1$).
Figure 1: The agent (●) receives 1 primary reward for entering the goal (■). The agent can move in the cardinal directions, destroy cells in the cardinal directions, or do nothing. Walls (□) are not movable. The right end of the screen wraps around to the left. (a): The learned trajectory for the misspecified primary reward function \( R \) destroys fragile green cells (●). (b): Starting from the same state, AUP’s trajectory preserves the green cells.

It is important to note that we did not hand-select an informative auxiliary reward function to induce the trajectory of fig. 1b. Instead, the auxiliary reward was the output of a one-dimensional observation encoder, corresponding to a continuous Bernoulli variational autoencoder [17] trained through random exploration (see section 5).

3.2 Theoretical results

Our theorems provide intuition about how the AUP penalty term works. Proofs and additional results are in appendix B.

**Definition.** Let \( D \) be a continuous distribution over reward functions bounded \([0, 1] \), with probability measure \( F \). The **attainable utility distance** between state distributions \( \Delta, \Delta' \in \Delta(S) \) is

\[
d_{AU}(\Delta, \Delta') := \int_D \left| \mathbb{E}_{\Delta} [V^*_R(s)] - \mathbb{E}_{\Delta'} [V^*_R(s')] \right|dF(R).
\] (2)

**Theorem 1.** \( d_{AU} \) is a distance metric on \( \Delta(S) \).

Viewing the designer as sampling auxiliary reward functions from distribution \( D \), the AUP penalty term is the Monte Carlo integration of

\[
\frac{\lambda}{|R|} \sum_{R_i \in R} \left| Q^*_i(s, a) - Q^*_i(s, \emptyset) \right| = \lambda \gamma \cdot \frac{1}{|R|} \sum_{R_i \in R} \left| \mathbb{E}_{T(s, a)} [V^*_i(s_a)] - \mathbb{E}_{T(s, \emptyset)} [V^*_i(s_{\emptyset})] \right|.
\] (3)

Insofar as the Monte Carlo integration approximates \( d_{AU} \), the attainable utility distance sheds light on the attainable utility penalty term.

**Theorem 2** (Movement penalties are small). Let \( \Delta \neq \Delta' \). Suppose that all states in the support of \( \Delta \) can deterministically reach in one step all states in the support of \( \Delta' \), and vice versa. Then \( 0 < d_{AU}(\Delta, \Delta') < 1 \).

In general, we only have \( 0 \leq d_{AU}(\Delta, \Delta') < \frac{1}{1-\gamma} \) (corollary 8 in appendix B.2).

The intuitive notion of “power” corresponds to the ability to achieve goals in general, which can be formalized as average optimal value. For example, resources increase average optimal value, while immobility decreases it.

**Definition** (Average optimal value [26]). \( V^*_\text{avg}(s) := \mathbb{E}_D[V^*_R(s)] \).

AUP significantly penalizes change in expected power compared to inaction. In fig. 1b, AUP heavily penalized the destruction of green cells. Destroying these cells reduces power.

**Theorem 3** (Power-shift penalties are large). \( d_{AU}(\Delta, \Delta') \geq \left| \mathbb{E}_\Delta [V^*_\text{avg}(s)] - \mathbb{E}_{\Delta'} [V^*_\text{avg}(s')] \right| \).

4 SafeLife

In Conway’s Game of Life, cells are alive or dead. Depending on how many live neighbors surround a cell, the cell comes to life, dies, or retains its state. Even simple initial conditions can evolve into
complex and chaotic patterns, and the Game of Life is Turing-complete when played on an infinite grid [20]. SafeLife turns the Game of Life into an actual game. An autonomous agent moves freely through the world, which is a large finite grid. There are many colors and kinds of cells, many of which have unique effects (see fig. 2).

![Image](a) append-spawn (b) prune-still-easy

Figure 2: Trees (●) are permanent living cells. The agent (●) can move crates (●) but not walls (●). The screen wraps vertically and horizontally. (a): The agent receives reward for creating gray cells (●) in the blue areas. The goal (●) can be entered when some number of gray cells are present. Spawners (●) stochastically create yellow living cells. (b): After the agent removes some number of red cells, the goal turns red (●) and can be entered.

Wainwright and Eckersley [27] score side effects as the degree to which the agent perturbs green cell patterns. Over an episode of \( T \) time steps, side effects are quantified as the Wasserstein 1-distance between the configuration of green cells had the state evolved naturally for \( T \) time steps, and the actual final configuration. As the primary reward function \( R \) is indifferent to green cells, this proxy measures the safety performance of learned policies. If the agent never disturbs green cells, it achieves a perfect score of zero. By construction, minimizing side effect score preserves \( R_{true} \)'s optimal value, since \( R_{true} \) encodes our preferences about the existing green patterns.

As shown in table 1, Turner et al. [25] evaluated AUP on toy environments. In contrast, SafeLife vigorously challenges modern reinforcement learning algorithms.

| AI safety gridworlds [16] | SafeLife [27] |
|--------------------------|---------------|
| Dozens of states         | Millions of states |
| Deterministic dynamics   | Stochastic dynamics |
| Handful of preset environments | Randomly generated environments |
| One side effect per level | Many side effect opportunities |
| Immediate side effects   | Chaos unfolds over time |

Table 1: SafeLife is ideal for testing side effect avoidance.

5 Experiments

5.1 Comparison

Method. The agent can move in the cardinal directions, spawn/destroy a living cell in the cardinal directions, or do nothing. We have four conditions: PPO, AUP, AUP\(_{proj}\), and \( \text{Naive} \). Each condition is PPO trained on a different reward signal for five million (5M) time steps. See supplemental material for architectural and training details.

- **PPO** Trained on the primary SafeLife reward function \( R \), without a side effect penalty.
AUP. For the first 100K time steps, the agent randomly explores to collect observation frames. These frames are used to train a continuous Bernoulli variational autoencoder with a $Z$-dimensional latent space and encoder network $E$.

If $Z = 1$, the auxiliary reward is the output of the encoder $E$. Otherwise, we draw linear functionals $\phi_i$ uniformly randomly from $(0, 1)^{Z}$. The auxiliary reward function $R_i$ is defined as $\phi_i \circ E : \mathcal{S} \rightarrow \mathbb{R}$. For each of the $|\mathcal{R}|$ auxiliary reward functions, we learn a Q-value network for 1M time steps.

The learned $Q_R$, define the penalty term of eq. (1). The agent learns $R_{\text{AUP}}$ for 3.9M steps, during which time $\lambda$ is linearly increased from .001 to $\lambda^\infty$.

AUP$_{proj}$ AUP, but the auxiliary reward function is a random projection from the downsampled observation space to $\mathbb{R}$, without using a variational autoencoder.

Naive trained on the primary reward function $R$ minus (roughly) the $L_1$ distance between the current state and the initial state. The agent is penalized when cells differ from their initial values. Wainwright and Eckersley [27] found that an unscaled $L_1$ penalty produced the best results.

While a good benchmark for ideal behavior in certain static tasks, penalizing state change often fails to avoid crucial side effects. State change penalties do not differentiate between moving a box and irreversibly wedging a box in a corner [14].

The default settings are: $N_{env} = 8$ randomly generated environments in the curriculum, $Z = 1$ latent space dimension, $|\mathcal{R}| = 1$ auxiliary reward function, and $\lambda^\infty = .01$ final $R_{\text{AUP}}$ penalty severity. The discount rate $\gamma = .97$.

We evaluate the conditions in the append-spawn (fig. 2a) and prune-still-easy (fig. 2b) tasks. Furthermore, we include two easier variants of append-spawn: append-still (no stochastic spawners) and append-still-easy (no stochastic spawners, fewer green cells).

We conduct three trials. At the beginning of each trial, SafeLife randomly generates $N_{env}$ environments for the given task. The conditions are evaluated on the same set of random environments. The curriculum is a random sequence of these environments.

For append-still, we allotted an extra 1M steps to achieve convergence for all agents. For append-spawn, agents pretrain on append-still-easy environments for the first 2M steps and train on append-spawn for 3M steps. For AUP in append-spawn, the autoencoder and auxiliary network are trained on both tasks. $R_{\text{AUP}}$ is then pretrained for 2M steps and trained for 1.9M steps.

Results. In append-still-easy, even though AUP waits 1.1M steps to start training on $R_{\text{AUP}}$, AUP is competitive with PPO by step 1.75M (see fig. 3). By step 2.75M, AUP consistently outperforms PPO while incurring less than a twentieth of the side effects. Naive also does very well and also learns more quickly than PPO. AUP$_{proj}$ does better than PPO but worse than AUP, perhaps implying that the one-dimensional encoder provides more structure than a random projection.

In prune-still-easy, all four conditions competitively accrue reward. PPO and AUP$_{proj}$ frequently have side effects. AUP avoids side effects, but not as well as in append-still-easy. The Naive benchmark has fewer side effects than AUP. This makes sense: since these environments are static, Naive almost directly penalizes our unobserved side effect metric (change to the green cells).

append-still environments contain more green cells. Once again, AUP has far fewer side effects than PPO. AUP$_{proj}$ and Naive both flounder, earning significantly lower return than PPO. While Naive appears to have fewer side effects than AUP, this is only because Naive usually does nothing. In the supplemental material, we display episode lengths over the course of training – Naive converges to an average episode length of 843 (the maximum is 1,001). Even PPO has an average episode length of 548. In stark contrast, AUP learns effective and decisive policies with half the average length of PPO. AUP frequently attains a length of only 43 (near optimal). AUP significantly decreases episode length for most tasks – perhaps because AUP applies small movement penalties (theorem 2).

append-spawn environments contain both stochastic yellow cell spawners and more green cells. These environments challenge PPO, which has less reward and yet more side effects. AUP$_{proj}$ completely fails to learn. Naive usually fails to get any reward, as its policy erratically wanders the environment. AUP is once again superior to PPO: 131% of the reward, 46% of the side effects.
Figure 3: Learning curves with shaded regions representing ±1 standard deviation. AUP begins training on $R_{AUP}$ at step 1.1M. AUP has far fewer side effects than PPO.

Append still easy

Prune still easy

Append still

Append spawn
5.2 Hyperparameter sweep

Method. In append-still-easy, we evaluate AUP on the following settings: $\lambda^* \in \{0.1, 0.5, 1, 5\}$ and $(N_{env}, Z) \in \{8, 16, 32, \infty\} \times \{1, 4, 16, 64\}$ ($N_{env} = \infty$ means that each episode takes place in a new environment). We also evaluate PPO on each $N_{env}$ setting. For each setting, we record both the side-effect score and the return of the learned policy, averaged over three trials. We use default settings for all unmodified parameters.

![Figure 4](image.png)

Figure 4: Side effect score, averaged over three learned policies for append-still-easy. Lower score is better. Default AUP setting outlined in black.

![Figure 5](image.png)

Figure 5: Episodic reward, averaged over three learned policies for append-still-easy. Higher reward is better, although AUP only aims to match PPO. Default AUP setting outlined in black.

Results. As $N_{env}$ increases, reward tends to decrease and side effect score tends to increase. This performance degradation does not seem to be due to Attainable Utility Preservation, but rather because Proximal Policy Optimization has challenges generalizing (see fig. 4 and fig. 5). In particular, PPO accrues less reward and induces more side effects as $N_{env}$ increases. However, even when $N_{env} = \infty$, AUP ($Z = 16$) shows the potential to significantly reduce side effects without reducing episodic return.

AUP’s default configuration achieves 117% of PPO’s episodic return, without any of the side effects. The AUP penalty term might be acting as a shaping reward. This is intriguing – shaping usually requires knowledge of the desired task, whereas the auxiliary reward function is randomly generated. Additionally, once AUP begins learning $R_{AUP}$ on step 1.1M, AUP learns much more quickly than PPO did (fig. 3); this supports the shaping hypothesis. AUP imposed minimal overhead: due to its increased sample efficiency, AUP reaches PPO’s asymptotic episodic return at the same time as PPO.

Surprisingly, AUP does well with a single latent space dimension ($Z = 1$). As $Z$ increases, so does AUP’s side effect score. In the supplementary material, our data show that higher-dimensional auxiliary reward functions are harder to learn, resulting in a poorly learned auxiliary Q-function. Nonetheless, for each $N_{env}$ setting, AUP’s worst configuration has significantly fewer side effects than PPO.
Surprisingly, AUP does well with a single auxiliary reward function ($|R| = 1$). We hypothesize that destroying patterns decreases power; by theorem 3, this is penalized in the limit of $|R| \to \infty$. Furthermore, we believe that decreasing power usually decreases optimal value for any given single auxiliary reward function. Since $R_{AUP}$ penalizes optimal value decrease, this might explain why AUP does well with one auxiliary reward function.

When $\lambda^* = .5$, AUP becomes more conservative. As $\lambda^*$ increases further, AUP stops moving entirely. AUP only regularizes learned policies, so AUP can still make expensive mistakes during training.

6 Discussion

We successfully scaled AUP to complex environments without providing task-specific knowledge – the auxiliary reward function was a one-dimensional variational autoencoder trained through random exploration. To the best of our knowledge, AUP is the first task-agnostic approach which avoids side effects and competitively achieves reward in complex environments.

Wainwright and Eckersley [27] speculated that avoiding side effects must necessarily decrease performance on the primary task. This may be true for optimal policies, but not necessarily for learned policies. AUP significantly improved performance on `append-still-easy` and `append-spawn`, while matching performance on `prune-still-easy` and `append-still`.

AUP enjoys moderate success on the easier tasks. This suggests that AUP works (to varying extents) for a wide range of uninformative reward functions.

While Naïve penalizes every state perturbation equally, AUP applies penalty in proportion to irreversibility. For example, the agent could move crates around (and then put them back later). AUP incurred little penalty for doing so, while Naïve was more constrained. We believe that AUP will continue to scale to useful applications, in part because it naturally accounts for irreversibility.

Future work. Off-policy learning could allow simultaneous training of the auxiliary $R_i$ and of $R_{AUP}$. Instead of learning an auxiliary Q-function, the agent could just learn the auxiliary advantage function with respect to inaction.

The SafeLife suite includes more challenging variants of `prune-still-easy`. SafeLife also includes difficult navigation tasks, in which the agent must reach the goal by wading either through fragile green patterns or through robust yellow patterns.

AUP’s excellent performance when $|R| = Z = 1$ raises interesting questions. Turner et al. [25]’s small “Options” environment required $|R| = 25$ for good performance. SafeLife environments are much larger than Options (table 1), so why does $|R| = 1$ perform so well? To what extent does the AUP penalty term provide reward shaping? Why do one-dimensional encodings provide a learnable reward signal?

Conclusion. To realize the full potential of reinforcement learning, we need more than algorithms which train policies that optimize a specified reward function. We also need to be able to specify the right reward function. Fundamentally, we face a frame problem: we often know what we want the agent to do, but we cannot list everything we want the agent not to do. AUP scales to challenging domains, incurs modest overhead, performs competitively on the original task, and avoids side effects – without explicit information as to what constitutes a “side effect”.

Broader Impact

A scalable side effect avoidance method would ease the challenge of reward specification and aid deployment of reinforcement learning in situations where mistakes are costly. Conversely, developers should carefully consider how reinforcement learning algorithms might produce policies with catastrophic impact. Developers should not blindly rely on even a well-tested side effect penalty.
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A Additional data

Figure 6: Episode length curves with shaded regions representing ±1 standard deviation. AUP begins training on $R_{AUP}$ at step 1.1M. AUP significantly decreases episode length in the append tasks.

Figure 7: Auxiliary reward curves for AUP (with a $Z$-dimensional latent space), with shaded regions representing ±1 standard deviation. Auxiliary reward is not comparable across trials, so learning is expressed by the slope of the curves.
B Theoretical results

Consider a rewardless MDP \( \langle S, \mathcal{A}, T, \gamma \rangle \) whose state space \( S \) and action space \( \mathcal{A} \) are both finite, and \( \gamma \in [0, 1) \). Reward functions \( R \in \mathbb{R}^S \) have corresponding optimal value functions \( V^*_R(s) \).

**Definition.** Let \( \mathcal{D} \) be a continuous distribution over reward functions bounded \([0, 1]\), with probability measure \( F \). The attainable utility distance between state distributions \( \Delta, \Delta' \in \Delta(S) \) is

\[
d_{AU}(\Delta, \Delta') := \int_{\mathcal{D}} \left| \mathbb{E}_\Delta \left[ V^*_R(s) \right] - \mathbb{E}_{\Delta'} \left[ V^*_R(s') \right] \right| dF(R). \tag{2}
\]

Restriction to degenerate distributions yields a distance metric over the state space.

**B.1 Main results**

**Theorem 1.** \( d_{AU} \) is a distance metric on \( \Delta(S) \).

**Proof.** For \( \Delta, \Delta', \Delta'' \in \Delta(S) \):

1. \( d_{AU}(\Delta, \Delta') \geq 0 \).
2. \( d_{AU}(\Delta, \Delta') = 0 \) iff \( \Delta = \Delta' \).
3. \( d_{AU}(\Delta, \Delta') = d_{AU}(\Delta', \Delta) \).
4. \( d_{AU}(\Delta, \Delta'') \leq d_{AU}(\Delta, \Delta') + d_{AU}(\Delta', \Delta'') \).

Properties 1 and 3 are trivially true. Property 2 follows from lemma 11. Property 4 follows from applying the triangle inequality for real numbers to the integrand. \( \square \)

**Theorem 2** (Movement penalties are small). Let \( \Delta \neq \Delta' \). Suppose that all states in the support of \( \Delta \) can deterministically reach in one step all states in the support of \( \Delta' \), and vice versa. Then \( 0 < d_{AU}(\Delta, \Delta') < 1 \).

**Proof.** \( 0 < d_{AU}(\Delta, \Delta') \) by theorem 1. Let \( R \in \mathcal{D} \). By proposition 4, \( \left| V^*_R(s) - V^*_R(s') \right| \leq (1 - \gamma) \max \left( V^*_R(s), V^*_R(s') \right) \leq 1 \). Then

\[
d_{AU}(\Delta, \Delta') \leq \int_{\mathcal{D}} \mathbb{E}_{\Delta, \Delta'} \left[ \left| V^*_R(s) - V^*_R(s') \right| \right] dF(R) \tag{4}
\]

\[
\leq \int_{\mathcal{D}} \mathbb{E}_{\Delta, \Delta'} \left[ (1 - \gamma) \max \left( V^*_R(s), V^*_R(s') \right) \right] dF(R) \tag{5}
\]

\[
\leq 1. \tag{6}
\]

Equation (5) follows because \( \mathcal{D} \) is continuous, so it cannot be the case that almost all reward functions assign 0 reward to either \( s \) or \( s' \). \( \square \)

**Definition** (Average optimal value [26]). \( V^*_\text{avg}(s) := \mathbb{E}_\mathcal{D} \left[ V^*_R(s) \right] \).

**Theorem 3** (Power-shift penalties are large). \( d_{AU}(\Delta, \Delta') \geq \left| \mathbb{E}_\Delta \left[ V^*_\text{avg}(s) \right] - \mathbb{E}_{\Delta'} \left[ V^*_\text{avg}(s') \right] \right| \).

**Proof.** Apply the reverse triangle inequality to the integrand of \( d_{AU}(\Delta, \Delta') \) and use the linearity of expectation. \( \square \)

We derive a principled motivation for preserving the reachability of the initial state, bounding decrease in \( V^*_\text{Reach} \) by how many steps it takes to return to the initial state \( s \).
Theorem 9 (Reward functions induce unique optimal value functions).

Let $\pi$ be the optimal policy for $R$. Given $\pi$, let $T_R(s) = \arg\max V^*_R(s)$ be the greedy action for each state. Let $d_R(s) = V^*_R(s) - V^*(s')$.

Proof. We first bound the maximum increase.

$$\max_{R \in [b,c]} V^*_R(s) - V^*_R(s') \leq \max_{R \in [b,c]} V^*_R(s') - \left(\frac{1 - \gamma k_1}{1 - \gamma} + \gamma k_1 V^*_R(s')\right)$$

$$\leq \frac{c}{1 - \gamma} - \left(\frac{1 - \gamma k_1}{1 - \gamma} + \gamma k_1 c\right)$$

$$= \left(1 - \gamma k_1\right) \frac{c - b}{1 - \gamma}.$$  (9)

Equation (7) holds because even if we make $R$ equal $b$ for as many states as possible, $s'$ is still reachable from $s$. The case for maximum decrease is similar.

Positive affine transformation of $D$ allows generalization of our results to other bounds, as optimal policy is invariant to positive affine transformation of the reward function.

Proposition 5. Let $D'$ be any positive affine transformation $mX + C$ of $D$.

$$d^D_{AU}(\Delta, \Delta') = m \cdot d^D_{AU}(\Delta, \Delta').$$  (10)

B.2 Additional results

Lemma 6. $d_{AU}(\Delta, \Delta') \leq E_{s \sim \Delta, s' \sim \Delta'} \left[d_{AU}(s, s')\right]$.

Lemma 7. $\forall s, s': d_{AU}(s, s') < \frac{1}{1 - \gamma}$.

Proof. Because optimal value is bounded $[0, \frac{1}{1 - \gamma}]$, $d_{AU}(s, s') \leq \frac{1}{1 - \gamma}$. The equality holds iff for almost all $R \in R$, $V^*_R(s) = \frac{1}{1 - \gamma}$ and $V^*_R(s') = 0$, or vice versa. But because $D$ is continuous, $s'$ must induce positive optimal value for a positive measure set of reward functions.

Corollary 8. $d_{AU}(\Delta, \Delta') < \frac{1}{1 - \gamma}$.

Theorem 9 (Reward functions induce unique optimal value functions). $R \mapsto V^*_R$ is injective.

Proof. Given $V^*_R$ and the rewardless MDP, deduce an optimal policy $\pi^*$ for $R$ by choosing a $V^*_R$-greedy action for each state. Let $T^{\pi^*}$ be the transition probabilities under $\pi^*$.

$$V^*_R = R + \gamma T^{\pi^*} V^*_R$$

$$\left(I - \gamma T^{\pi^*}\right) V^*_R = R.$$  (12)

Definition. Let $e_s$ represent the unit vector for state $s$. The state visitation distribution induced by following $\pi$ from state $s$ is

$$f^\pi_s := \sum_{t=0}^{\infty} \gamma^t E[e_{s'} | \pi \text{ followed for } t \text{ steps from } s].$$  (13)

Lemma 10. The elements of $\{f^\pi_s | s \in S\}$ are linearly independent.
Proof. Consider the all-zero optimal value function with optimal policy \( \pi^* \). Theorem 9 implies the following homogeneous system of equations has a unique solution for \( r \):

\[
\begin{align*}
\mathbf{f}^*_{s_1}^\top r &= 0 \\
&\vdots \\
\mathbf{f}^*_{s_{|S|}}^\top r &= 0.
\end{align*}
\]

Therefore, the optimal policy \( \pi^* \) induces linearly independent \( \mathbf{f}^*_{s} \). But \( r \) is clearly the all-zero reward function (for which all policies are optimal). We conclude that the \( \mathbf{f}^*_{s} \) are independent for any policy \( \pi \).

Turner et al. [26]'s theorem 33 shows the following result for deterministic dynamics and for single states \( s \neq s' \). We generalize to the stochastic case and to distributions over states.

**Lemma 11.** If \( \Delta \neq \Delta' \), \( \mathbb{E}_\Delta \left[ V^*_R(s) \right] = \mathbb{E}_{\Delta'} \left[ V^*_R(s') \right] \mid R \sim \mathcal{D} \) = 0.

**Proof.** Let \( R \in \mathcal{D} \) (also written \( r \in \mathbb{R}^{|S|} \)), and let \( \pi^* \) be one of its optimal policies. By lemma 10,

\[
\mathbb{E}_\Delta \left[ \mathbf{f}^*_{s} \right] = \mathbb{E}_{\Delta'} \left[ \mathbf{f}^*_{s'} \right] \text{ iff } \Delta = \Delta'.
\]

Therefore, \( \mathbb{E}_\Delta \left[ \mathbf{f}^*_{s} \right] \neq \mathbb{E}_{\Delta'} \left[ \mathbf{f}^*_{s'} \right] \).

Trivially, \( V^*_R(s) = V^*_R(s') \) iff \( \mathbf{f}^*_{s}^\top r = \mathbf{f}^*_{s'}^\top r \). Since \( \mathbf{f}^*_{s} \neq \mathbf{f}^*_{s'} \), the set of satisfactory \( r \) has no interior in the subspace topology induced by \( \mathcal{D} \)'s support. This convex set has zero Lesbesgue measure; by the Radon-Nikodym theorem, it also has zero measure under continuous distributions.

\[\Box\]

### C Training details

We detail how we trained the AUP and AUP\(_{proj}\) conditions.

#### C.1 \( R_{aux} \) training

For the first phase of training, our goal is to learn \( Q_{aux} \), allowing us to compute the AUP penalty in the second phase of training. Due to the size of the full SafeLife state \((350 \times 350 \times 3)\), both conditions downsample the observations with average pooling and convert to intensity values.

Previously, Turner et al. [25] learned \( Q_{aux} \) with tabular Q-learning. They used environments small enough such that reward could be assigned to each state. Because SafeLife environments are too large for tabular Q-learning, we demonstrated two methods for randomly generating an auxiliary reward function.

**AUP** We acquire a low-dimensional state representation by training a continuous Bernoulli variational autoencoder (CB-VAE) [17]. To train the CB-VAE, we collect a buffer of observations by acting randomly for \( 100,000 \) steps in each of the \( N_{env} \) environments. This gives us 100K total observations with an \( N_{env}\)-environment curriculum. We train the CB-VAE for 100 epochs, preserving the encoder \( E \) for downstream auxiliary reward training. For each auxiliary reward function, we draw a linear functional uniformly from \((0, 1)^Z\) to serve as our auxiliary reward function, where \( Z \) is the dimension of the CB-VAE’s latent space. The auxiliary reward for an observation is the composition of the linear functional with an observation’s latent representation.

**AUP\(_{proj}\)** Instead of using a CB-VAE, AUP\(_{proj}\) simply downsamples the input observation. At the beginning of training, we generate a linear functional over the unit hypercube (with respect to the downsampled observation space). The auxiliary reward for an observation is the composition of the linear functional with the downsampled observation.

After the CB-VAE has been trained, we start auxiliary reward training in the corresponding SafeLife environment. To learn \( Q_{aux} \), we modify the value function in PPO to a Q-function. Our training algorithm for phase 1 only differs from PPO in how we calculate reward. We train each auxiliary reward function for 1M steps.
C.2 $R_{AUP}$ training

In phase 2, we train new PPO agent on $R_{AUP}$ (eq. (1)) for the corresponding SafeLife task. Each step, the agent selects an action $a$ in state $s$ according to its policy $\pi_{AUP}$, and receives reward $R_{AUP}(s, a)$ from the environment. We compute $R_{AUP}(s, a)$ with the learned Q-values $Q_{aux}(s, \emptyset)$ and $Q_{aux}(s, a)$.

The penalty term is modulated by the hyperparameter $\lambda$, which is linearly scaled from $10^{-3}$ to some final value $\lambda^*$ (default $10^{-1}$). Because $\lambda$ controls the relative influence of the penalty, linearly increasing $\lambda$ over time will prioritize primary task learning in early training and slowly encourage the agent to obtain the same reward while avoiding side effects. If the value for $\lambda$ is too high – if side effects are too costly – the agent won’t have time to adapt its current policy and will choose inaction ($\emptyset$) to escape the penalty. A careful $\lambda$ schedule helps induce a successful policy that also avoids side effects.

D Hyperparameter selection

Table 2 lists the hyperparameters used for all conditions, which generally match the default SafeLife settings. Common refers to those hyperparameters that are the same for each evaluated condition. AUX refers to hyperparameters that are used only when training on $R_{AUX}$, thus, it only pertains to $AUP$ and $AUP_{proj}$. The conditions PPO and Naive use the PPO hyperparameters for the duration of their training, while AUP, AUP_{proj} use them when training with respect to $R_{AUP}$.

| Hyperparameter | Value |
|----------------|-------|
| **Common**     |       |
| Learning Rate  | $3 \cdot 10^{-4}$ |
| Optimizer      | Adam |
| Gamma ($\gamma$) | 0.97 |
| Lambda (PPO)  | 0.95 |
| Lambda (AUP)  | $10^{-3} \rightarrow 10^{-1}$ |
| Entropy Clip   | 1.0 |
| Value Coefficient | 0.5 |
| Gradient Norm Clip | 5.0 |
| Clip Epsilon  | 0.2 |
| **AUX**        |       |
| Entropy Coefficient | 0.01 |
| Training Steps | $1 \cdot 10^6$ |
| **PPO**        |       |
| Entropy Coefficient | 0.1 |
| **Policy**     |       |
| Number of Hidden Layers | 3 |
| Output Channels in Hidden Layers | (32, 64, 64) |
| Nonlinearity   | ReLU |
| **CB-VAE**     |       |
| Learning Rate  | $10^{-4}$ |
| Optimizer      | Adam |
| Latent Space Dimension ($Z$) | 1 |
| Batch Size     | 64 |
| Training Epochs | 50 |
| Epsilon        | $10^{-5}$ |
| Number of Hidden Layers (encoder) | 6 |
| Number of Hidden Layers (decoder) | 5 |
| Hidden Layer Width (encoder) | (512, 512, 256, 128, 128) |
| Hidden Layer Width (decoder) | (128, 256, 512, 512, output) |
| Nonlinearity   | ELU |

Table 2: Chosen hyperparameters.
E  Compute environment

For data collection, we only ran the experiments once. We used a single NVIDIA GTX 1080-TI. The auxiliary reward functions were trained on down-sampled rendered game screens, while all other learning used the internal SafeLife state representation. This preprocessing turned out to be computationally expensive.

| Condition | GPU-hours per trial |
|-----------|---------------------|
| PPO       | 6                   |
| AUP       | 8                   |
| AUP$_{proj}$ | 7.5               |
| Naive     | 6                   |

Table 3