Finite $\beta$ Effects on Short Wavelength Ion Temperature Gradient Modes

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Abstract

The Electromagnetic effect is studied on the short wavelength branch of the ion temperature gradient mode in the linear regime for the first time using a global gyrokinetic model. The short wavelength ion temperature gradient mode growth rate is found to be reduced in the presence of electromagnetic perturbations at finite plasma \( \beta \). The effect on real frequency is found to be weak. The threshold value of \( \eta_i \) is found to be increase for the mode as the magnitude of \( \beta \) is increased. The global mode structure of the short wavelength branch of the ion temperature gradient mode is compared with the conventional branch. The magnetic character of the mode, measured as the ratio of mode average square values of electromagnetic potential to electrostatic potential, is found to increase with increasing values of the plasma \( \beta \). The mixing length estimate for flux shows that the maximum contribution still comes from the long wavelengths modes. The magnitude of flux decreases with increasing \( \beta \).

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1. INTRODUCTION

It is now well known that magnetically confined plasmas bear the prospects of a future fusion reactor. The progress in this endeavor, however, is hamstrung by technological and physics issues. With the available technology, the success of such a goal depends on the understanding the nature of the instabilities and controlling them. The Larmor radius scale instabilities, which are also named as microinstabilities, cause anomalous transport leading to expulsion of heat and particles from the system. This is undesirable and has to be avoided or at least reduced. These microinstabilities feed on the free energy available in the inhomogeneities in temperature and density of the particles. The ion temperature gradient (ITG) [1–5] mode, trapped electron mode (TEM) [6–11], and universal drift instabilities [12–14], are some of the examples of such unstable modes at the ion scale while the electron temperature gradient mode (ETG) [15–18] is another class of instabilities at the electron scale. Finite $\beta$ plasmas also give rise to electromagnetic instabilities such as microtearing modes (MTM) [19–27] and kinetic ballooning modes (KBM) [28–31]. Intermediate to these scales, there exists a class of instabilities driven by strong ion temperature gradient. This mode occurs on the shorter scale than the conventional ITG mode mentioned above and therefore, is named as the short wavelength ion temperature gradient (SWITG) mode [32–40]. Usually the ion temperature gradient driven mode exists for $k_{\perp}\rho_i \leq 1$. However, when the gradient scale lengths are very short one can observe another branch of ITG mode which becomes unstable at scales $k_{\perp}\rho_i > 1$.

With the progress in the tokamak fusion research, tokamaks are now able to operate in advanced scenarios. Some of such scenarios offer very strong gradients. For example in the region of internal transport barriers, the gradients can be very strong [41]. In H mode plasma as well, the profiles turn very steep, resulting in very strong gradients [42]. Apart from other modes, the shorter wavelength mode might be very unstable with $k$ spectrum extending well beyond the conventional modes [43] for parameters relevant to tokamak experiments. The current and future tokamak machines confine plasma at a higher pressure which is expressed in $\beta$. Therefore, the modification of the mode properties in the presence of finite $\beta$ has to be taken into consideration. The $\beta$ effect turns out to be very important for both electrostatic and electromagnetic modes. While the electromagnetic instabilities arise and become stronger with increasing $\beta$, the same finite-$\beta$ effect suppresses the electrostatic
instabilities. In the presence of fast ions high $\beta$ leads to the reduction in the profile stiffness observed in many experiments and confirmed in subsequent gyrokinetic studies \cite{44}. The role of electromagnetic perturbations on drift modes has been studied extensively. While electromagnetic perturbation are observed to give rise to instabilities such as kinetic ballooning mode (KBM) \cite{28,31}, tearing and microtearing modes \cite{19,26}, etc., same is found to stabilize some other drift modes such as ITG mode, trapped electron mode, universal drift modes, etc \cite{13,45,48}. The stabilization effect of the electromagnetic perturbation on the ITG mode can be attributed to the stabilization effect of the field line bending induced by the electromagnetic perturbations. Although, electromagnetic effects on ITG mode are well known, the effect of electromagnetic perturbation on the SWITG mode is not known so far. We know that the SWITG mode is inherently an ion mode \cite{37}, and magnetic shear has stabilizing effect \cite{36}. The dependence of the mode on $T_e/T_i$, toroidicity, magnetic shear, safety factor, etc., are studied in Refs \cite{38,39}. The role of $EXB$ shear has been studied in Ref. \cite{36}. Thus the properties of the SWITG mode have been extensively studied in the electrostatic limit. However, in the presence of $\beta$ or electromagnetic perturbations, which corresponds to more realistic scenario of tokamak experiments, the properties of the SWITG mode have not been investigated so far. To complement this gap in the knowledge of SWITG, in the present work we, explore the properties of the SWITG mode in the presence of finite $\beta$. Also note that, for high $n$ modes the local calculations might be appropriate; but for low and intermediate $n$ modes, the mode can span over several mode rational surfaces with mode width multiple times the Larmor radius. The global profile effects become important in such cases. It is clear from previous simulations \cite{39} that although the SWITG modes appears at high $n$, the mode is quite global and local calculations overestimate the growth rates and real frequencies in certain cases. This underscores the importance of the use of a global code in the study of the SWITG mode. For this purpose we use the electromagnetic version EM-GLOGYSTO \cite{31,49,50} of the global gyrokinetic linear model GLOGYSTO \cite{51,53}. This code has been extensively used to study electrostatic and electromagnetic modes. For example, in the electrostatic limit the code is used to study ITG \cite{4,52}, TEM \cite{10}, SWITG \cite{39}, modes while in the electromagnetic limit KBM \cite{31,49,50}, and MTM \cite{24,54} modes are studied in detail. The present manuscript is arranged as follows. Section II describes the model briefly. In Section III, the profiles and parameters used in the present simulations are described. Section IV discusses the results observed in the presence of finite $\beta$. Eventually,
the results are summarized in the Section V.

2. THE SIMULATION MODEL

The EM-GLOGYSTO code is a global spectral code that calculates the real frequency and growth rates of unstable modes for a given equilibrium using Nyquist method and also gives the eigen mode structure. Both ion and electron species are considered fully gyrokinetic. The equilibrium considered is circular and axisymmetric and can include Shafranov shift. The code can treat trapped and passing particles separately and calculates finite Larmor radius (FLR) effect to all orders. It includes all kinetic effects including Landau damping physics. The model includes both transverse and compressional perturbation in the electromagnetic limit. In the present study, however, we consider $\phi$ and $A_\parallel$ components only.

Before proceeding to present the results we briefly summarize the simulation model in this section. A greater detail of the model can be found in Refs. [51, 52]. For electromagnetic calculations readers are referred to Refs. [31, 49]. The perturbed density for a species $j$, can be expressed as sum of adiabatic and nonadiabatic parts as follows.

$$\tilde{n}_j(r; \omega) = -\left(\frac{q_j N}{T_j}\right) \left[\hat{\varphi} + \int dk \exp(ik \cdot r) \int dv f_{Mj} \frac{f_{Mj}}{N}(\omega - \omega^*_j) \left(\frac{\omega - \omega^*_j}{\omega - \omega^*_j}ight) \hat{\varphi}(k; \omega) J_0^2(x_{Lj})\right]$$

In the above equation $q_j$ and $T_j$ are the charge and temperature for the species $j$, $N$ stands for the equilibrium density. The diamagnetic drift frequency is given by $\omega^*_j = \omega_n j \left[1 + q_2 \left(\frac{v^2}{v^2_{thj}} - 3\right)\right]$ where $\omega_n j = (T_j \nabla_n \ln N k_\theta)/(q_j B)$, $\nabla_n = -r B_p \frac{\partial}{\partial \psi}$, and $\eta_j = (d \ln T_j)/(d \ln N)$, $v_{thj}$ is the thermal velocity of species $j$. The Bessel function $J_0(x_{Lj})$ with $x_{Lj} = k_\perp \rho_{Lj}$, incorporates the full finite Larmor radius effect. Note that here $m$ and $n$ are poloidal and toroidal wave numbers, $q(s)$ is the safety factor, $k_\theta$ is the poloidal wave vector, $B_p$ is the poloidal magnetic field, $f_{Mj}$ is a local Maxwellian for species $j$ of mass $m_j$ and is given by

$$f_{Mj}(\varepsilon_j, \psi) = \frac{N(\psi)}{(2\pi T_j(\psi))^{3/2}} \exp\left(-\frac{1}{2} m_j v^2\right)$$

The term $U_j$ represents the guiding center propagator for the passing particles where,

$$iU_j = \sum_{p,p'} J_p(x^\sigma_{Lj}) J_{p'}(x^\sigma_{Lj}) \exp(\iota(p - p')(\theta - \bar{\theta}_\sigma)), \quad (2)$$
with \( x_{ij}^\sigma = k_\perp \xi_j, \xi_j = v_{dj}/\omega_t, v_{dj} = (v_{i_1}/2 + v_{i_2})/(\omega_{ej} R), \mu = v_{i_1}^2 / 2B, \omega_t = \sigma |v_{i_1}|/(q(s) R), \sigma = \pm 1 \) (sign of \( v_{i_1} \)), \( \varepsilon_j = 1/2 m_j v_j^2 \). The perpendicular and parallel wave-vectors are given as \( k_\perp = \sqrt{\kappa^2 + k_\theta^2}, k_\parallel = [nq(s) - m] / (q(s) R) \). Also note that \( \bar{\theta} \) is given by \( \tan \bar{\theta} = -\kappa/k_\theta \). The Bessel functions contain the effect of \( \nabla B \) and curvature drifts through arguments \( x_{ij}^\sigma = k_\perp v_{dj}/\omega_t \). Thus the Bessel functions in Eq.(2) take into effect the coupling of the flux surfaces and also coupling of the neighboring poloidal modes. Note that the argument of the Bessel functions \( J_p \) in Eq.(2), \( x_{ij}^\sigma = k_\perp \xi_j \) also takes into account the effect of transit frequency \( \omega_t \). The quasi-neutrality condition then gives

\[
\sum_j \tilde{n}_j(r; \omega) = 0, \tag{3}
\]

In the case of electrostatic fluctuations only, this leads to an eigenvalue problem, with \( \omega \) and \( \tilde{\varphi} \) being eigenvalues and eigenvectors and can be solved in Fourier space. For fully gyrokinetic ions and electrons with only passing particles we have:

\[
\sum_{k'} \sum_{j=i,e} \tilde{M}_{k,k'}^j \tilde{\varphi}_{k'} = 0. \tag{4}
\]

For an axisymmetric system, one can fix the toroidal mode number \( n \), and thus \( k = (\kappa, m) \) for the wave vector represents the radial and poloidal wave numbers \( \kappa \) and \( m \), respectively. Hence, \( k = (\kappa, m) \) and \( k' = (\kappa', m') \). With the inclusion of the electromagnetic perturbations, but neglecting \( \delta B_\parallel \), Eq.(1) above is modified as \( \text{[31, 49]} \)

\[
\tilde{n}_j(r; \omega) = -\left( q_j N / T_j \right) \left[ \tilde{\varphi} + \int d\mathbf{k} \exp(i \mathbf{k} \cdot \mathbf{r}) \int dv f_{Mj} (\omega - \omega_j^*) (i \mathbf{u}_j) [\tilde{\varphi}(\mathbf{k}; \omega) - v_{\parallel} \tilde{A}_\parallel(\mathbf{k}; \omega)] J_0^2(x_{Lj}) \right],
\]

where \( \tilde{A}_\parallel \) is the parallel component of the vector potential. The perturbed parallel current density can be written as,

\[
\tilde{j}_{\parallel j}(r; \omega) = -\left( q_j^2 T_j / T_j \right) \left[ \int d\mathbf{k} \exp(i \mathbf{k} \cdot \mathbf{r}) \int v_{\parallel} dv f_{Mj} (\omega - \omega_j^*) (i \mathbf{u}_j) [\tilde{\varphi}(\mathbf{k}; \omega) - v_{\parallel} \tilde{A}_\parallel(\mathbf{k}; \omega)] J_0^2(x_{Lj}) \right],
\]

With the quasi-neutrality condition Eq.(3), and Ampere’s law

\[
\frac{1}{\mu_0} \nabla_\perp^2 \tilde{A}_\parallel = -\sum_j \tilde{j}_{\parallel j}
\]
We eventually arrive at a linear system of equations as follows

\[ \sum_{k'} \sum_{j=i,e} \mathcal{N}_{k,k'}^{j} \left( \tilde{\varphi}'_{k'} \left( \tilde{A}_{|| k'} \right) \right) = 0 \]  

This forms the core of the simulation model used in the present study. Note that Eq. (5) above represents a complex eigenvalue equation for the (complex) eigen frequency \( \omega \), which is found numerically based on Nyquist theorem. More details can be found in Ref. [53].

RESULTS AND DISCUSSION

1. Parameters and Profiles

For the present study, we consider the following profiles and parameters.

**Parameters:**
- B-field: \( B_0 = 1.0 \) Tesla
- Temperature: \( T_0 = T(s_0) = 7.5 \) keV
- Major Radius: \( R = 2.0 \) m
- Minor Radius: \( a = 0.5 \) m
- \( \tau(s) = \frac{T_i(s)}{T_e(s)} = 1 \)
- \( \epsilon_n = \frac{L_{n0}}{R} = 0.1 \)
- \( \epsilon_T = \frac{L_{T0}}{R} = 0.04 \)
- \( \delta s_n = 0.35 \)
- \( \delta s_T = 0.2 \) at \( s = s_0 \)
- \( \delta s = 1.25 + 0.67 s^2 + 2.38 s^3 - 0.06 s^4 \)

The corresponding profiles for density and temperature for ions and electrons are shown in Fig. 1. The left panel shows temperature while the right panel shows the densities for the species.

FIG. 1. Temperature profiles for electron and ion (left panel) and density profile (right panel).
The left panel of Fig.2 shows the $\eta_i$ profile. It is clear that it peaks at $s_0 = 0.6$. The $\eta_i$ profile is important, as this parameter determines the instability drive for the ITG and SWITG modes. The safety factor and shear profiles are depicted in the right panel of Fig.2.

![FIG. 2. $\eta_i$ profile (left panel) and safety factor and shear profiles (right panel).](image)

2. Dispersion relation

In Fig.3 the real frequencies and growth rates are plotted for different values of the toroidal mode number $n$. The real frequency and growth rates are normalized by $\omega_{d0} = v_{th,j}\rho_L/a^2$. These calculations are carried out for different values of $\beta$, namely, 0.0001 (red curve), 0.0005 (green curve) and 0.001 (blue curve). It is clear from the figure in the left panel that the real frequency increases with respect to the toroidal mode number $n$. The real frequencies increase up to toroidal mode number $n \approx 20$. Beyond this point the real frequencies stay virtually constant with $n$. This behavior is typical of the SWITG mode [37]. It is also evident

![FIG. 3. Real frequency (left panel) and growth rate (right panel) with respect to toroidal mode number for different $\beta$ values. The $\beta$ values considered are 0.0001 (red curve), 0.0005 (green curve) and 0.001 (blue curve).](image)
that the real frequencies are weakly affected by the increase in \( \beta \). The right panel of Fig. 3 shows the corresponding growth rates with increasing toroidal mode number \( n \). The growth rate increases initially with respect to \( n \) and peaks around \( k_{\theta} \rho_{Li} = 0.5 \) \((n \approx 9)\) and then falls off with increasing \( n \). This is the conventional ITG mode: with increasing toroidal mode number the resonance between mode frequency and magnetic drift enhances and therefore the growth rate increases. Beyond the peak the resonance becomes weaker and growth rate decreases. Also with increase in the toroidal mode number the finite Larmor radius effect comes in to play, because for a given ion Larmor radius with increasing toroidal mode number the wavelength decreases. However, with further increases in the toroidal mode number the real frequency turns to saturate and the growth rate again starts to increase. This is the regime of the SWITG mode. The growth rate increases up to \( k_{\theta} \rho_{Li} = 1.22 \) \((n \approx 21)\) after which the growth rate again starts to decrease giving rise to the second hump which is the characteristics of the SWITG mode. The peak growth rates for both conventional ITG and SWITG mode are of similar strength. Before discussing the electromagnetic effects on the mode it is an appropriate place to revisit the theory of the SWITG mode following Ref. [37]. Upon integrating the perturbed distribution function with respect to velocity in the limit \( \omega_n > \omega > (\omega_{di} + k_{||}v_{||}) \), one can write the perturbed ion density in the electrostatic limit as follows [37]

\[
\tilde{n}_i = -\frac{q_i n_o}{T_i} \tilde{\phi} + q_i \frac{\omega_{ni}(\eta_i/2 - 1)}{\omega} I_o(k_{\perp}^2 \rho_{Li}^2 \rho_{Li}) \exp(-k_{\perp}^2 \rho_{Li}^2),
\]

where \( I_o \) stands for the zeroth order modified Bessel function and \( \omega_{ni} = -(v_{thi}/L_n)(k_{\perp} \rho_{Li}) \). Then applying the quasi-neutrality condition with adiabatic electrons one arrives at the following relation

\[
\omega = \left( \frac{\tau}{\tau + 1} \right) \left( \frac{\eta_i}{2} - 1 \right) \omega_{ni} I_o(k_{\perp}^2 \rho_{Li}^2 \rho_{Li}) \exp(-k_{\perp}^2 \rho_{Li}^2),
\]

The monotonic increase in the mode frequency with toroidal mode number at lower \( n \) region (conventional ITG) and then saturation of the mode frequency at higher \( n \) (SWITG) are easily understood from the above relation using the properties of the scaled modified Bessel function \( I_o(b)e^{-b} \rightarrow 1/\sqrt{(2\pi b)} \) for large values of \( b \). For small \( k_{\perp}^2 \rho_{Li}^2 \), the mode frequency \( \omega \) behaves as \( k_{\perp} \rho_{Li} \) and for larger \( k_{\perp}^2 \rho_{Li}^2 \) it remains virtually constant. The second hump in the growth rate appears as a result of a second resonance between toroidal magnetic drift
term $\omega_{di}$ of the ions with the mode frequency which is constant at higher toroidal mode number. It is also clear from the Fig.3 that the SWITG also suffers from finite Larmor radius stabilization. This can be understood from the expression of the nonadiabatic part of ion density response in Eq.5. At very high $n$ or equivalently, high $k_\perp \rho_{Li}$, the ion magnetic drift frequency $\omega_{di}$ leads the mode frequency $\omega$. The nonadiabatic part of the perturbed ion density, in the limit $\omega_{di} >> \omega$ and large $k_\perp^2 \rho_{Li}^2$ will then decrease as $\frac{R}{L_n} I_0(k_\perp^2 \rho_{Li}^2) \exp(-k_\perp^2 \rho_{Li}^2)$. For greater detail of these calculations readers are referred to Ref [37]. It is evident from Fig.3 that, although the real frequencies are not very much affected by the increase in the value of $\beta$, the growth rates of both conventional ITG and SWITG mode suffer substantial reduction in magnitude. The reduction in the growth rates of the conventional ITG mode is well known. There have been many linear and nonlinear studies reporting the stabilization of the ITG mode by the electromagnetic perturbations. However, the role of electromagnetic perturbations on the SWITG mode is perhaps hitherto unknown. Since the advanced operating regimes where SWITG mode can be unstable also have $\beta$ values, it is important to explore the electromagnetic effect on the mode. Thus this present study shows that the SWITG mode suffers stabilization in the presence of the electromagnetic perturbation and one has to take in to account this observation as well when studying the ion heat flux induced by SWITG in real experiments. In the presence of the electromagnetic perturbation, the mode couples with the Alfen perturbations. The field lines are thus bent in the presence of $\beta$ [49, 55]. This field line bending by the electromagnetic perturbation is therefore responsible for the stabilization of the SWITG mode.

A. Mode structures for ITG and SWITG

Figure 4 shows the mode structures for electrostatic and electromagnetic potentials $\phi$ and $A_\parallel$ for $\beta = 0.0001$. The toroidal mode number corresponding to the mode structure is $n = 9$. It is clear from the figure that for both $\phi$ and $A_\parallel$ the mode structures exhibit ballooning character. Similarly, Fig.5 depicts the mode structure for $\phi$ and $A_\parallel$ for $\beta = 0.001$ for toroidal mode $n = 9$. Similar to the modes in Fig.4, the modes exhibit ballooning structure which is characteristics of ITG mode. It is evident that mode structure $A_\parallel$ is more elongated in poloidal direction compared to electrostatic potential $\phi$. Figs.6 and 7 show the mode structures of $\phi$ and $A_\parallel$ for SWITG mode for $\beta = 0.0001$ and $\beta = 0.001$, respectively. The
FIG. 4. Mode structure of $\phi$ (left panel) and $A_\parallel$ (right panel) for $n = 9$ corresponding to the ITG mode $\beta = 0.0001$.

FIG. 5. Mode structure of $\phi$ (left panel) and $A_\parallel$ (right panel) for $n = 9$ corresponding to the ITG mode $\beta = 0.001$.

toroidal mode number for these figures is $n = 21$. The figures clearly show that the mode is ballooning in nature and more localized both radially and poloidally.
In this section we investigate the effect of $\beta$ on the real frequency and growth rates of SWITG mode and compare with the conventional ITG mode. The real frequency and growth rate are calculated for increasing values of $\beta$ and are shown in Fig.8. The real frequency versus
$\beta$ are shown in the left panel and the growth rates with respect to $\beta$ are shown in the right panel. It is clear from the figure that with increasing $\beta$ the real frequency increases weakly with $\beta$. In contrast to this the growth rate decreases with increasing $\beta$. However, at higher $\beta$ growth rate of SWITG mode decreases slowly with respect to $\beta$ compared to that of ITG mode. Thus we can conclude that the SWITG mode suffers $\beta$ stabilization like its long wavelength counterpart. The relative strength of the electromagnetic to the electrostatic character is shown in Fig.9. This is expressed as the ratio $\langle A_{\parallel} \rangle^2 / \langle \phi \rangle^2$. It is clear from the figure that the ratio increases almost linearly with increasing $\beta$. It is also observed that the value of the $\langle A_{\parallel} \rangle^2 / \langle \phi \rangle^2$ ratio for SWITG is lower compared to that of the conventional ITG mode.

FIG. 8. Real frequency (left panel) and growth rates (right panel) for the ITG and SWITG mode with respect to $\beta$

FIG. 9. Ratio of $\langle A_{\parallel} \rangle^2 / \langle \phi \rangle^2$ with respect to $\beta$ for the ITG and SWITG mode
C. $\eta_i$ scan

For temperature gradient modes, such as the SWITG mode, $\eta$, which is the ratio of density and temperature gradient scale lengths, is a very important parameter. When $\eta$ is above a certain value the SWITG mode becomes unstable. Beyond the threshold value the growth rate increases monotonically with increasing $\eta$. To explore this physics in the presence of electromagnetic perturbation we calculate the mode frequency and growth rate for increasing values of $\eta_i$ for both the ITG and the SWITG modes for different values of $\beta$. The results for the ITG mode are shown in Fig.10 and the corresponding results for the SWITG mode are shown in Fig.11.

FIG. 10. Real frequency (left panel) and growth rates (right panel) with respect to $\eta_i$ for the ITG mode

FIG. 11. Real frequency (left panel) and growth rates (right panel) with respect to $\eta_i$ for the SWITG mode

The free energy for ITG mode and SWITG mode comes from the temperature gradient. In the presence of finite density gradient some of the free energy is used up reducing the available free energy for the mode to become unstable. It is because the temperature gradient causes plasma to convect from core to edge while density gradient leads to expansion of the plasma in the same time. Thus it is the competition between heating due to convection
and cooling due to expansion, and the net free energy available depends upon the relative strength of the density gradient and temperature gradient [55]. This is manifested in the value of $\eta_i$. Thus with increasing $\eta_i$ the free energy available to render the mode unstable also increases. This leads to the increase in the growth rate. This explains the increase in the growth rates with increasing $\eta_i$ as observed in Figs.10 and 11. It is also evident from the Figs.10 and 11 that the real frequency increases with $\eta_i$. The real frequency is proportional to the diamagnetic frequency which is proportional to the gradient. That is why the real frequency increases with increasing $\eta_i$.

D. Mixing length calculation of flux

It would be interesting to see how the electromagnetic perturbation affects the overall flux. Since this is a linear simulation, one can use a simple mixing length estimation of the transport coefficient. For the purpose we calculate the ratio $D_{ML} = \gamma/\langle k_\perp^2 \rangle$. Here the term $\langle k_\perp^2 \rangle$ stands for the mode square average of the perpendicular wave-vector while $\gamma$ is the growth rate for a given $n$. In Fig.12 we calculate the quantity $D_{ML}$ for each value of toroidal mode number and plot the same with respect to the toroidal mode number. We consider three cases of $\beta$ values, $\beta = 0.0001, 0.0005$ and $0.001$ corresponding to the results shown in Fig.3. It is observed in Fig.12 that the mixing length estimation of heat flux peaks at the longest wavelength despite the fact that the SWITG mode exhibits strongest growth rates around $n = 21$. This implies that the most of the contribution to the total flux comes from modes close to the conventional ITG. This is qualitatively in conformity with the results observed in Ref. [40]. It is to be noted that with increasing value of $\beta$ the magnitude of $\gamma/\langle k_\perp^2 \rangle$ decreases implying a reduction in the heat flux. This is consistent with the stabilization effect observed in Fig.3 and Fig.8 with respect to increasing $\beta$. Thus we conclude that increasing $\beta$ does not only reduce growth rates of the mode but also reduces the overall heat flux.

3. SUMMARY

In the present work we have carried out a systematic study of the electromagnetic effect on the SWITG mode along with the conventional ITG mode using a global linear gyrokinetic
model. Although the electromagnetic effect on the ITG mode is hitherto well known, this is the first study that investigates the electromagnetic effects on the SWITG mode. We have calculated the real frequency and growth rate for the chosen equilibrium for three different values of $\beta$. We also have shown the mode structures for both the ITG and the SWITG mode in the presence of finite $\beta$. We have found out the growth rates and real frequencies with respect to $\beta$ and with increasing $\eta_i$ for different values of $\beta$. Finally, we have estimated the mixing length estimation of the transport. The main findings are as follows.

1. The SWITG mode is stabilized by the electromagnetic effect. The real frequency of the SWITG mode is weakly affected by the electromagnetic effect.

2. The SWITG mode-structure is more localized both radially and poloidally compared to the ITG mode.

3. With increasing $\beta$ the real frequency of the SWITG mode decreases slightly while the growth rate decreases substantially.

4. For the range of $\eta_i$ values studied, the growth rate of the SWITG mode instability increases with increase in $\beta$ value.

5. The ratio of electromagnetic to electrostatic potentials increases with increasing $\beta$.

6. The mixing length estimate for transport reveals that although the linear growth rates of the SWITG mode are comparable to those of the long wavelength branch the heat flux is maximum at the long wavelength region. This implies that most of the
contribution to the ion heat flux still comes from the conventional ITG mode that occurs at $k_\theta \rho_i \leq 1.0$.

7. The mixing length estimate for transport shows that with increasing $\beta$ the heat flux decreases. This is in conformity with the linear stabilization of the mode with increasing $\beta$.

4. ACKNOWLEDGEMENT

All the work reported here are performed at Udbhav cluster at Institute for Plasma Research (IPR). This paper is dedicated to late Professor Jan Vaclavik who taught one of the authors (RG) gyrokinetic theory.

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