The structure of a single sharp quantum Hall edge probed by momentum-resolved tunneling

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Momentum-resolved magneto-tunnelling spectroscopy is performed at a single sharp quantum Hall edge. We directly probe the structure of individual integer quantum Hall (QH) edge modes, and find that an epitaxially overgrown cleaved edge realizes the sharp edge limit, where the Chklovskii picture relevant for soft etched or gated edges is no longer valid. The Fermi wavevector in the probe quantum well probes the real-space position of the QH edge modes, and reveals inter-channel distances smaller than both the magnetic length and the Bohr radius. We quantitatively describe the lineshape of principal conductance peaks and deduce an edge filling factor from their position consistent with the bulk value. We observe features in the dispersion which are attributed to fluctuations in the ground energy of the quantum Hall system.

Momentum-resolved tunneling has been used to measure both the dispersion of electronic excitations, as well as their momentum-resolved density of states or spectral functions. Spectral functions of two-dimensional (2D) and one-dimensional (1D) systems have been experimentally measured, and the dispersion relations of 1D systems have shown evidence for spin-charge separation. In the quantum Hall regime, Kung et al. fabricated a pair of coplanar 2D systems laterally coupled through a tunnel barrier, and were able to map out the dispersion of the integer quantum Hall (QH) edge by probing one QH system with another, qualitatively matching the expected sharp-edge dispersion. Deviations in the data from a simple dispersion picture have been cited as evidence of new interaction effects. This Letter implements a different tunneling geometry consisting of two orthogonal quantum wells in order to provide new information about the sharp QH edge as well as its spectral function. In our geometry we can measure for the first time the real-space positions of the edge states at a sharp edge, and we present a lineshape analysis which fits the anticipated spectral functions in the system. We demonstrate how the Chklovskii screening picture does not apply at a sharp edge, but we have instead experimentally realized the type of sharp edge first envisioned by Halperin in his original work on edge states. We observe an additional step in the dispersion which is attributed to fluctuations in the quantum Hall ground energy. Although single QH edge tunneling was investigated previously in the high field limit in a different kind of device the low field limit where momentum resolved conductance resonances are evident had remained unexplored until now.

In our cleaved-edge overgrown structure, two orthogonal GaAs quantum wells (QW) intersect in a T-junction with a tunnel barrier at the intersection (Inset, Fig. 1). A magnetic field \( B \) perpendicular to the first well QW identifies this as the quantum Hall effect system under study, while the second well functions as the probe quantum well or QW. The first well is \( w^\perp = 150 \) Å wide with an electron density of \( n^\perp = 1.9(1.7) \times 10^{11} \) cm\(^{-2} \) for Sample I (II), and a mobility of \( \mu^\perp \approx 2 \times 10^{5} \) cm\(^2\) Vs. The second orthogonal well is grown after cleaving this sample in situ in the growth chamber in the (110) plane and overgrowing a \( b = 50 \) Å wide and 0.3 eV high \( Al_{0.33}Ga_{0.67}As \) tunnel barrier. This \( w^\parallel = 200 \) Å quantum well is then grown with a density of \( n^\parallel \approx 2.3 \times 10^{11} \) cm\(^{-2} \) and an estimated mobility of \( \mu^\parallel \approx 1 \times 10^{5} \) cm\(^2\) Vs. The tunnel junction is extended typically 20 \( \mu \)m along the cleaved edge. The QW’s are separately contacted with ohmic indium contacts and the tunnel current \( I \) is studied under applied bias \( V \) in a \( ^3 \)He cryostat at temperatures of 360 mK. No temperature dependence was observed up to 1 K.

Fig. 1 shows the differential tunnel conductance \( dI/dV \) measured using lock-in techniques while sweeping the \( B \)-field. At zero bias we observe well-developed peaks in \( dI/dV \) at certain values of the magnetic field, and we resolve up to four of these peaks (denoted with \( n = 0, 1, 2, 3 \) from right to left). Their width and height above the background is larger for those at higher \( B \)-field, with the peaks showing a slightly asymmetric shape with a steeper slope at the high \( B \) side. The zero-bias conductance is strongly suppressed above 5 T.

We explain these observations with momentum conserved tunneling between the 2D probe contact and the edge states of the QH system. To build up a spatially intuitive picture, we will express all dispersions in terms of an orbit center coordinate, \( X \). Translational invariance in \( y \) guarantees that \( k_y \) is conserved upon tunneling, or identically that \( X = k_y l_y^2 \) for \( X \) and \( k_y \), the orbit center coordinate in \( x \) is conserved, where \( l_y^2 = \hbar/eB \) is the squared magnetic length. The probe contact dispersion is \( E^\parallel(k_y, k_z) = \frac{\hbar^2}{2m^\parallel}(k_y^2 + k_z^2) \), or in terms of the orbit center coordinate, \( E^\parallel(X, k_z) = \frac{\hbar^2}{2m^\parallel}[(\frac{X}{l_y})^2 + k_z^2] \).
To find the corresponding expression for the QH system we introduce the $y$-translationally invariant Landau gauge $A = xB\hat{y}$ for the magnetic field $B = B\hat{z}$ with $x = 0$ in the center of the probe QW. The wave function can then be separated into product form $\Psi_{n,X}(x,y) = \exp(-iy\hat{a}^\dagger)\psi_{n,X}(x)$, with the $x$-component $\psi_{n,X}(x)$ obeying the Schrödinger equation:

$$\left[\frac{\hbar^2}{2m_e^*} + \frac{1}{2} m_e^* \omega_z^2 (x-X)^2 + \Phi(x,V)\right] \psi_{n,X}(x) = E^\parallel_n(X)\psi_{n,X}(x)$$

(1)

The dispersion curve $E^\parallel_n(X)$ for the $n^{th}$ Landau band is calculated for an infinitely sharp step function edge potential at the left wall of the barrier: $\Phi(x,V) = eV + \Delta$ for $x \leq -(b + \frac{m_e^*}{\omega_z^2})B$; $= \infty$ for $x > -(b + \frac{m_e^*}{\omega_z^2})B$. $V$ is the applied voltage and $\Delta$ is the ground energy difference between the two systems. The resulting dispersions are plotted in Fig. 2 for two different magnetic fields for $V = 0$. We note that the orbit center $X$ can be inside the barrier since the orbit itself always remains outside as a skipping orbit. We neglect spin splitting, a point which will be discussed later in this paper, as well as the $B$ dependence of the dispersion $E^\parallel(X,k_z)$, which can be shown to be negligible.

The resonance condition is achieved when the outermost Fermi point in the probe intersects the Landau band dispersion. The real space position probed by the Fermi point is its orbit center $X_F(B) = \hbar k^\parallel/F_e B = k^\parallel F_e B_0^{1/2}$, one cyclotron radius to the left of the probe QW center. The orbit center of the $n^{th}$ Landau band at the Fermi energy $\xi_n$ is defined by the condition $E^\parallel_n(\xi_n) = E_F$. When these two coincide, $\xi_n = X_F$, the resonance gives rise to the $n^{th}$ experimentally observed conductance peak. With the probe Fermi momentum $|k^\parallel_F| = 1.2 \times 10^8$ m$^{-1}$, the distance of this orbit center from the barrier can be determined $X^\parallel(B) = |X_F(B)| - b = \frac{m_e^*}{\omega_z^2}$ (top axis in Fig. 1) and is listed for each measured resonance in Table I.

The exact condition of resonance depends on the band offset $\Delta$ between the two systems. Both dispersions are fully determined at a given $B$, but differing QW ground energies and stray electric fields at the junction may shift the ground energy of one 2D system relative to the other, or slightly alter the density of QW$^\perp$ at the junction. To accommodate such effects we empirically shift the calculated QH dispersion by an energy $\Delta$ until it intersects with the Fermi point of the probe, satisfying $E^\parallel_n(\xi_n) = E^\parallel(X_F, k_z = 0) = E_F$ as in Fig. 2. The resulting offset of $\Delta = 2.2$ meV is accounted for principally by the ground energy difference between the two square wells $\Delta_{QW} = \frac{\hbar^2}{2m_e^*}(\frac{1}{w_0^2} - \frac{1}{w^2}) = 1.4$ meV. We attribute the remaining difference to stray electric fields across the barrier. The consistency of this fit is our first confirmation that we are reasonably within the sharp edge limit.

Due to the abruptness of the edge potential, the Chklovskii picture of (in)compressible strips is not valid at this experimentally realized sharp edge. According to Chklovskii, et al. [12] alternating compressible and incompressible strips will form at the edge of a QH system if the inter-edge spacing is much greater than both the minimum screening distance (the Bohr radius $a_0 = \frac{2\hbar e^2}{m^* e^2} = 10$ nm in GaAs) and the wavefunction width (the magnetic length $l_0 = \sqrt{\hbar/eB}$ in Table I). The wavefunctions shown in the right of Fig. 2 each have nodes at the barrier, with the $n = 1$ branch wavefunction having one additional node and extending further to the left than the $n = 0$ branch. Due to the sharp confinement, the wave functions of these two branches share their rightmost node and therefore completely overlap, making the interedge spacing less than $l_0$ and $a_0$ in violation of the Chklovskii criterion. Even in the case of a single edge mode (Fig. 2 left) no compressible strip is expected to form since the edge state is within $l_0$ and $a_0$ of the hard wall. Just as compressible strips are not allowed to form at a sharp edge, the same length scale and screening arguments forbid edge reconstruction, whereby Coulomb interactions would cause a strip of charge to separate from the edge.

The fixing of the resonance condition determines the Fermi energy at the QH edge, which can be expressed in terms of an edge filling factor. Analogous to Ref. [4], our simplest estimate for $\nu_{edge}$ assumes broadened spin-degenerate Landau bands giving a flat density of states $g(E) = \frac{2eB}{\hbar \omega_c} \frac{1}{m^*} = \frac{m^*}{\pi \hbar^2}$. The edge filling factor is then

$$\nu_{edge} = 2\frac{E_{F\text{ edge}}}{\hbar \omega_c}$$

(2)

Fig. 2 shows the cases for $n = 0$ and 1 peaks, giving $\nu_{edge}$ estimates $2.0$ and $3.8$, respectively, in fair agreement with the bulk filling factor at these fields, $\nu = 2.3$ and $4.1$ (see also Table I). Later we will see evidence that the density of states $g(E)$ is not so flat as this simplified picture, yet the above estimate offers another consistency check of the sharp edge picture.

We can also explain the lineshape of the prominent zero-bias conductance peaks. The tunnel conductance at $V = 0$ is proportional to the number of states at the Fermi energy that overlap in momentum space.

$$\frac{dl}{dV} \sim \sum_{k_y,k_y'} |t_{k_y,k_y'}(B)|^2 A_{\text{QHE}}(k_y,E_F)A_{\text{probe}}(k_y',E_F)$$

(3)

Solving the Schrödinger equation for the transmission and assuming perfect momentum conservation, we find that the transmission probability $|t(B)|^2 \sim B \delta_{k_y,k_y'}$ is roughly proportional to the magnetic field for low fields ($l_0 > b$), contributing to the smaller peak height observed at lower $B$. To quantify the lineshape we first
We assume $A_{QHE}^\delta(k_x, E_F) \sim \sum \delta(E_F - E_n(k_y))$ for the spectral function of the quantum Hall system. In the probe contact the component of the Fermi circle in the $k'_y$ direction results in a van Hove-like singularity in the probe spectral function at the Fermi point $k'_y = -k_F$:

$$A_{probe}^\delta(k'_y, E_F) \sim 1/\sqrt{E_F - \hbar^2 k'^2_y/2m^*}.$$ The resulting calculated $dI/dV$ reproduces the asymmetry observed in the experimental conductance of Fig. 1 (dashed line). As in Ref. 15 we can replace the $\delta$-function in $t_{k_x,k'_y}(B)$ with a gaussian of full width $\Delta k = 1.6 \times 10^7 m^{-1}$ to account for small non-$k$-conservation, giving an excellent fit to the experimental data (dotted line). Differentiating the probe orbit center equation yields $\Delta B = B \Delta k/|k_F|$, meaning the linewidth of the resonances narrows with decreasing $B$ as observed.

The structure of the edge near zero bias can also be examined by sweeping the $V$ for a series of fixed $B$ (Fig. 1). A tunnel bias $V$ shifts the dispersion curves in energy with respect to each other, and from Fig. 2 one can see that an increasing negative bias $V$ raises the Fermi point of the probe past the successive Landau branches, with corresponding conductance peaks labelled $n = 0, 1, 2, 3$ in Fig. 1. Within a given Landau branch, increasing $B$ shifts the resonance more towards negative $V$, allowing one to map out the entire Landau band dispersion. This behavior is explained in further detail in Ref. 9. Fig. 4 shows such a scan of $dI/dV$ where the peak positions in $V$ vs. $B$ map out the low-energy dispersion of the edge modes in $E$ vs. $k$ with the maxima indexed as in Fig. 1. Most notable is the step in the $n = 0$ dispersion curve which does not cross the zero bias line continuously, but instead at $B = 3.5$ T shows a splitting of $\Delta V \sim 4$ mV.

We explain this feature with a fluctuating ground energy in the quantum Hall edge. As shown in the inset of Fig. 5, the band diagram corresponding to the peak condition changes discontinuously as the Fermi energy near the edge jumps between Landau levels $n = 0$ and 1 for $\nu_{edge} < 2$ and $\nu_{edge} > 2$ respectively. Upon increasing $B$ such that $\nu_{edge} < 2$, the peak condition requires an additional voltage $\Delta V = -\hbar \omega_c/e = -6$ meV at 3.5 T. The observed jump of $-4$ meV can be attributed to disorder broadening which narrows the mobility gap in the density-of-states. Whereas in standard soft QH edges the compressible strips screen any bulk ground energy oscillations from reaching the outermost edge, the observation of a step at $n = 0$ demonstrates that sharp edges are unable to screen. Note that the measured dispersion curves for $n = 1, 2, 3$ show no step at $B = 3.5$ T because at larger negative bias the edge depletes and the resulting smooth edge potential gives rise to compressible strips that do screen the bulk oscillations. A second important observation is that the $\nu_{edge} = 2$ jump occurs at the edge at a $B$-field where we also expect a $\nu = 2$ jump in the bulk. This is a second indication that the edge filling factor $\nu_{edge}$ is close to the bulk value $\nu$.

The additional shoulder at $B = 4.1$ T on the high $B$-field side of the $n = 0$ peak can not be explained within this model. Translated into the orbit-center coordinate $X$ the shoulder is separated from the main peak by $\sim 3$ nm. Recalling the length scale comparisons above, this short distance rules out that it is a signature of either standard edge reconstruction or the Chklovskii (in)compressible strip picture. Instead of resulting from real-space structure, it may result from structure in the energy spectrum. For example, it could possibly be an artifact of the previously described $\hbar \omega_c$ jump in the chemical potential near $\nu = 2$ ($B = 3.5$ T), or it may be a signature of the exchange enhanced spin-split gap.

In conclusion we have probed the QH edge state structure at a sharp cleaved and overgrown edge. We have directly measured the real space position of the edge channel orbit centers and demonstrated that the Chklovskii picture is not valid in this system. The prominent lineshape is fully described with the spectral functions in the tunnel contacts if we include a gaussian broadening in the momentum selection rule. An edge filling factor is deduced from conductance peak positions in $B$ and agrees with the bulk value, implying uniform electron density up to the edge in these structures. Evidence for a jump in the chemical potential confirms that $\nu_{edge} = \nu$, and that this sharp edge cannot screen bulk electrostatics. The existence of chemical potential oscillations may be important for interpreting the peak lineshapes in double-edge tunneling geometries, and the extension of the bulk filling factor all the way to the sharp edge has important implications on previous tunnel experiments on different cleaved quantum Hall edge structures.

The characterization of low $B$-field momentum resolved tunneling in this device opens the door for future proposed experiments to measure fractional QH correlations at high magnetic fields in this device, with the advantage over other experiments that the edge filling factor can be determined using the methods explained here.

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| TABLE I: (Sample I) Bulk filling factor $\nu$, edge filling factor $\nu_{\text{edge}}$, as well as measured distance $X_b$ of the orbit center from the tunnel barrier for each conductance resonance, compared to the magnetic length $l_0$. |
|---|---|---|---|---|
| $n$ | 0 | 1 | 2 | 3 |
| $\nu$ | 2.3 | 4.1 | 5.9 | 7.7 |
| $\nu_{\text{edge}}$ | 2.0 | 3.8 | 5.7 | 7.6 |
| $B$ (T) | 3.44 | 1.90 | 1.33 | 1.02 |
| $X_b$ (nm) | $8 \pm 2$ | $26 \pm 3$ | $44 \pm 5$ | $62 \pm 6$ |
| $l_0$ (nm) | 14 | 19 | 22 | 25 |
FIG. 1: The differential tunnel conductance $dI/dV$ versus magnetic field $B$ at zero DC bias voltage for Sample I. The dotted and dashed curves indicate the expected resonance lineshape with and without disorder broadening. Inset: two quantum wells are arranged in a T-shape separated by a $b = 50$ Å thick tunnel barrier; a magnetic field $B$ creates quantum Hall edges in $QW^\perp (w^\perp = 150$ Å) probed by tunneling from $QW^\parallel (w^\parallel = 200$ Å).

FIG. 2: Calculated dispersions $E^\perp_n(X)$ and $E^\parallel(X, k_z)$ at the magnetic fields of the experimentally observed $n = 0$ (left) and $n = 1$ (right) zero-bias conductance peaks. The conductance peaks arise from the resonance condition where the Fermi point in the probe intersects with the Landau dispersion. The occupied part of each Landau band is in black, with the unoccupied part in grey. The conduction band potential is shown as a shaded grey background. The wavefunctions of each mode at $E_F$ are depicted with thin solid lines above the Fermi energy.

FIG. 3: $\frac{dI}{dV}$ vs. $V$ at different $B$ (0.05 T steps offset by 0.5 $\mu$S.) The conductance peak for the outermost $n = 0$ edge channel splits into two near 3.5 T, evidence that the jump in the bulk chemical potential at $\nu = 2$ is being seen at the edge. Peaks for the inner depleted $n = 1, 2, 3$ channels remain continuous at 3.5 T since the smoothly depleted edges can screen the bulk $\nu = 2$ jump. (Sample II)
\[ \frac{dI}{dV} = \sum_{n=0}^{3} \frac{QW}{360 \, \text{mK}} \]

[Diagram showing a graph with multiple peaks labeled as \( n=0, n=1, n=2, n=3 \) and a inset of a QW structure with AlGaAs and GaAs layers.]
\( B = 3.44 \, T \quad \nu = 2.3 \quad n = 1 \quad n = 0 \)

\( B = 1.90 \, T \quad \nu = 4.1 \quad n = 2 \quad n = 1 \quad n = 0 \)

\[ E = 1.0 \hbar \omega_c \quad \lambda = 8 \, \text{nm} \quad \lambda = 26 \, \text{nm} \]
