Study of $CP$ Violation in Flavor Tagged and Untagged $D^0 \rightarrow K^-\pi^+$ Decays

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We review $CP$-violating observables in $D^0 \rightarrow K^-\pi^+$ decays and evaluate the $CP$ asymmetry difference between the tagged and untagged decays. We note that this commonly neglected difference is not zero in principle and can be significant in future $B$ factory experiments. We also construct an expression to extract the strong phase difference between $\bar{D}^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^-\pi^+$ decays, independently of existing experimental methods.

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The violation of combined charge conjugation and parity ($CP$) symmetry in the quark sector through the weak interaction was predicted by the Cabibbo-Kobayashi-Maskawa (CKM) mechanism and has been experimentally observed in $K$- and $B$-meson systems. On the other hand, $CP$ violation in charm meson system has not been observed yet and is expected to be small within the standard model (SM). Therefore, search for $CP$ violation in charm meson system naturally provides a window for new physics beyond the SM.

Recent experimental $CP$ asymmetry measurements in charm meson decays adapted $D^0 \rightarrow K^-\pi^+$ (referred to as “untagged”) and $D^+ \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+_\text{soft}$ (referred to as “tagged”) decays as control samples to correct for asymmetries due to different reconstruction efficiencies between positively and negatively charged tracks.

According to Ref., untagged $D^0 \rightarrow K^-\pi^+$ reveals $CP$ asymmetry resulting from the interference between the decays with and without $D^0-\bar{D}^0$ mixing even with no direct $CP$ violation, which has been considered in Refs. Other measurements used both untagged and tagged decays with the common assumption that the difference in $CP$ asymmetries between the tagged and untagged decays is zero.

Also, one of the obstacles in interpreting the experimental $D^0-\bar{D}^0$ mixing measurements in the decay $\bar{D}^0 \rightarrow K^-\pi^+$ is the appearance of the phase difference between $D^0 \rightarrow K^-\pi^+$ and $\bar{D}^0 \rightarrow K^-\pi^+$ decays due to the strong interaction. A direct experimental way to extract this strong phase difference has been utilizing a quantum-coherent production of $D^0\bar{D}^0$ pairs from $\psi(3770)$ and their measurements of the strong phase difference are $\cos\delta = 1.15^{+0.19}_{-0.17} - 0.08$, $\sin\delta = 0.56^{+0.33}_{-0.20}$.

$\delta = (18^{+11}_{-15})^\circ$ [13] and $\cos\delta = 1.02\pm0.11\pm0.06\pm0.01$ [14]. Further precise measurements are highly desired for interpretation of the recent experimental observations of $D^0-\bar{D}^0$ mixing in the decay $\bar{D}^0 \rightarrow K^-\pi^+$ [15, 17].

In this paper, we compute $CP$ violation contributions in untagged and tagged decays, testing the validity of the aforementioned assumption in the $CP$ measurements [7-11]. We also propose a model independent method to extract the strong phase difference between $D^0 \rightarrow K^-\pi^+$ and $\bar{D}^0 \rightarrow K^-\pi^+$ decays.

The time evolution of the $D^0-\bar{D}^0$ system can be described by the Schrödinger equation

\[ i\frac{\partial}{\partial t} \left( \begin{array}{c} D^0(t) \\ \bar{D}^0(t) \end{array} \right) = \left( M - \frac{i}{2} \Gamma \right) \left( \begin{array}{c} D^0(t) \\ \bar{D}^0(t) \end{array} \right) \]

(1)

where $M$ and $\Gamma$ are Hermitian matrices associating with the transitions, $D^0 \rightarrow D^0$ and $D^0 \rightarrow \bar{D}^0$. Our mass $(|D_{1,2}|)$ and flavor $(|D^0, \bar{D}^0|)$ eigenstates of neutral $D$ mesons are expressed as [18]

\[ |D_{1,2}| = p|D^0| \pm q|\bar{D}^0| \]

(2)

where $p$ and $q$ are complex numbers with the convention $CP|D^0| = -|\bar{D}^0|$ and $CP|\bar{D}^0| = -|D^0|$ under $CP$ conservation. Note that we adopt the convention used in Ref. [18]. The time evolution of the mass eigenstate is given by $|D_{i}(t)| = e^{-im_{i}t-\frac{i}{2}\Gamma_{i}t}|D_{i}|$, ($i$=1,2) where $m_{i}$ and $\Gamma_{i}$ are the mass and width of $|D_{i}|$. From these, one usually defines mixing parameters $x \equiv (m_{1} - m_{2})/\Gamma = \Delta m/\Gamma$ and $y \equiv (\Gamma_{1} - \Gamma_{2})/2\Gamma = \Delta\Gamma/2\Gamma$ where $\Gamma \equiv (\Gamma_{1} + \Gamma_{2})/2$, in order to describe the time evolution of the $D$ meson system and $CP$ asymmetries conveniently.

For the study of $D^0 \rightarrow f$ decay, one defines decay amplitude of an initially produced $D^0/\bar{D}^0$ into the final state $f/\bar{f}$ to be $A_{f}/\bar{A}_{f}$ for Cabibbo-favored (CF) decays and $A_{f}/\bar{A}_{f}$ for doubly Cabibbo-suppressed (DCS) decays, respectively, where $f/\bar{f}$ stands for $K^-\pi^+/K^+\pi^-$.
where $\sqrt{R_D}$ is the magnitude of the ratio of DCS to CF decay amplitudes and $\delta$ is CP conserving strong phase difference between the two decay amplitudes. $R_M$ and $\phi$ are the magnitude and argument of $q/p$, where $R_M \neq 1$ indicates $CP$ violation in the mixing and $\phi \neq 0$ (or $\phi \neq \pi$) implies $CP$ violation in the interference of the mixing and decay. With the relations given in Eq. (3), we have expressions of decay rates, expanded up to the order of $x$ and $y$ ($|x|, |y| \ll 1$), to be

$$
\Gamma[D^0(t) \to f] = e^{-\Gamma_1}|A_J|^2 \{1 + \\
\Gamma_2 \sqrt{R_D R_M} [y \cos(\delta - \phi) + x \sin(\delta - \phi)]}, \\
\Gamma[D^0(t) \to \bar{f}] = e^{-\Gamma_1}|A_J|^2 \{1 + \\
\Gamma_2 \sqrt{R_D R_M} [y \cos(\delta + \phi) + x \sin(\delta + \phi)]}, \\
\Gamma[D^0(t) \to f] = e^{-\Gamma_1}|A_J|^2 \{R_D + \\
\Gamma_2 \sqrt{R_D R_M} [y \cos(\delta - \phi) - x \sin(\delta - \phi)]}, \\
\Gamma[D^0(t) \to \bar{f}] = e^{-\Gamma_1}|A_J|^2 \{R_D + \\
\Gamma_2 \sqrt{R_D R_M} [y \cos(\delta + \phi) - x \sin(\delta + \phi)]},
$$

and they are our fundamental relations in the construction of various $CP$ asymmetries described below. Throughout this paper, we assume no direct $CP$ violation in the decays, $|A_J/A_f| = 1$ and $|A_J/A_f| = 1$. Furthermore, we also set $R_M$ to be 1, neglecting small $CP$ violation in mixing. Thus, $CP$ violation considered in this paper is effectively $CP$ violation in the interference of the mixing and decay.

The final state of the untagged decay is the sum of the $CP$ decay $D^0 \to f$, the DCS decay $D^0 \to f$, the $CP$ decay following $D^0$, $\bar{D}^0$ mixing $D^0 \to \bar{D}^0 \to f$, and the $CP$ decay following $D^0$, $\bar{D}^0$ mixing $D^0 \to D^0 \to f$. Thus, the time-integrated decay rates for the untagged case are

$$
\Gamma_t^{untag} = \int_0^{\infty} dt \{\Gamma[D^0(t) \to f] + \Gamma[\bar{D}^0(t) \to f] + \\
\Gamma[D^0(t) \to \bar{f} + \Gamma[\bar{D}^0(t) \to \bar{f}].
$$

The $CP$ asymmetry in this case is defined as

$$
A_{CP}^{untag} = \frac{\Gamma_t^{untag} - \Gamma_t^{untag}}{\Gamma_t^{untag} + \Gamma_t^{untag}}.
$$

Note that our definition of $A_{CP}^{untag}$ has an opposite sign from the one in Ref. [12]. The expression for $A_{CP}^{untag}$ can be evaluated using the relations shown in Eq. (4):

$$
A_{CP}^{untag} = 2\sqrt{R_D} y \sin \delta \sin \phi.3
$$

Note that the factor 2 is not present in Ref. [12].

For tagged analysis, the decay $D^+ \to D^0(\to f)\pi^0_{soft}$ is the sum of the $CP$ decay $D^0 \to f$ and the $CP$ decay following $D^0$, $\bar{D}^0$ mixing $D^0 \to D^0 \to f$. The time-integrated decay rates for the tagged decays are

$$
\Gamma_t^{tag} = \int_0^{\infty} dt \Gamma[D^0(t) \to f], \\
\Gamma_t^{\bar{f}} = \int_0^{\infty} dt \Gamma[\bar{D}^0(t) \to \bar{f}].
$$

The $CP$ asymmetry in the tagged decays is defined as

$$
A_{CP}^{tag} = \frac{\Gamma_t^{tag} - \Gamma_t^{\bar{f}}}{\Gamma_t^{tag} + \Gamma_t^{\bar{f}}}
$$

and this asymmetry is expressed as

$$
A_{CP} = -\sqrt{R_D} (x \cos \delta - y \sin \delta) \sin \phi.
$$

Therefore, in general $A_{CP}^{tag} \neq A_{CP}^{untag}$ and the difference is

$$
A_{CP}^{tag} - A_{CP}^{untag} = -\sqrt{R_D} (x \cos \delta + y \sin \delta) \sin \phi.
$$

Using present world averages [11], the difference in $A_{CP}$ is estimated to be at most $0.01 \times 10^{-2}$ at 95% confidence level, which can be neglected for the current experimental sensitivities [7, 11]. For example $A_{CP}^{K^+K^-} = (-0.32 \pm 0.21 \pm 0.09) \times 10^{-2}$ [11], where the first uncertainty is statistical and the second is systematic. The sensitivity of $A_{CP}^{K^+K^-}$ at the super-B factory currently under construction [20], however, is expected to be $0.03 \times 10^{-2}$ (statistical) and $0.01 \times 10^{-2}$ (systematic). The difference in Eq. (11) will thus become significant in the future $CP$ asymmetry measurements.

In the absence of direct $CP$ violation and neglecting the small $CP$ violation in mixing, the strong phase difference between $D^0 \to K^-\pi^+$ and $D^0 \to K^-\pi^+$ decays can be obtained by taking the ratio of $A_{CP}^{tag}$ to $A_{CP}^{untag}$. The relation is

$$
\cot \delta = \frac{y}{x} \left( 1 - 2 \frac{A_{CP}^{tag}}{A_{CP}^{untag}} \right),
$$

where the strong phase can be expressed in terms of $x$, $y$, and $CP$ asymmetries only. From this equation, one

\[3\] In case $R_M \neq 1$, the expression acquires an additional term $2\sqrt{R_D(R_M - 1)}x \sin \delta \cos \phi$, involving two small quantities, $x$ and $(R_M - 1)$.
can extract the strong phase in a model independent way by measuring the ratio of \( A_{\text{CP}}^{\text{tag}} \) to \( A_{\text{CP}}^{\text{untag}} \) experimentally. For the evaluation of the expected sensitivity on \( \delta \) (\( \sigma_\delta \)), we assign 0.007 and 0.01 for the uncertainties on \( A_{\text{CP}}^{\text{tag}} \) and \( A_{\text{CP}}^{\text{untag}} \) measurements, respectively, where the former is from the current best measurement \[21\] and the latter from our conservative assumption reflecting a conservative experimental uncertainty. We evaluate \( \sigma_\delta \) as a function of \( A_{\text{CP}}^{\text{tag}}/A_{\text{CP}}^{\text{untag}} \) by incorporating errors on \( A_{\text{CP}}^{\text{tag}}/A_{\text{CP}}^{\text{untag}} \) given above, \( x \), \( y \), and the relation between them from Ref. \[18\]. Figure 1 shows our evaluation implying that the sensitivity on \( \delta \) using the method introduced in this paper would be better than that of current measurements \[13, 14\] depending on \( A_{\text{CP}}^{\text{tag}}/A_{\text{CP}}^{\text{untag}} \). Furthermore, our evaluation shows \( \sigma_\delta \) dominates from current sensitivities of \( x \) and \( y \) except for the case \( A_{\text{CP}}^{\text{tag}}/A_{\text{CP}}^{\text{untag}} \sim 0.5 \), where the error contribution from \( x \) and \( y \) vanishes. Regardless of the sensitivity on \( \delta \), it is important to have an independent tool as a cross check on existing methods.

To conclude, we have investigated \( CP \) asymmetries of tagged and untagged \( D^0 \to K^-\pi^+ \) decays. The \( CP \) asymmetry difference between the two decays is found to be non-zero and cannot be neglected in the future super-\( B \) factory experiments. We also constructed a model independent expression for the strong phase difference in terms of \( D^0, \bar{D}^0 \) mixing parameters and \( CP \) asymmetries. This provides experimental access to the strong phase from measurements of \( CP \) asymmetries in tagged and untagged \( D^0 \to K^-\pi^+ \) decays, which is independent of existing methods \[13, 14\].

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1. N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys., 49, 652 (1973).
2. J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
3. G. Burdman and I. Shipsey, Annu. Rev. Nucl. Part. Sci. 53, 431 (2003).
4. B. R. Ko et al. (Belle Collaboration), Phys. Rev. Lett. 104, 181602 (2010).
5. M. Starič et al. (Belle Collaboration), Phys. Rev. Lett. 108, 071801 (2012).
6. B. R. Ko et al. (Belle Collaboration), J. High Energy Phys. 02 (2013) 098.
7. B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 100, 061803 (2008).
8. M. Starič et al. (Belle Collaboration), Phys. Lett. B 670, 190 (2008).
9. B. R. Ko et al. (Belle Collaboration), Phys. Rev. Lett. 106, 211801 (2011).
10. T. Aaltonen et al. (CDF Collaboration), Phys. Rev. D 85, 012009 (2012).
11. B. R. Ko et al. (Belle Collaboration), arXiv:1011.0352.
12. A. A. Petrov, Phys. Rev. D 51, 111901 (2004).
13. D. M. Asner et al. (CLEO Collaboration), Phys. Rev. D 86, 112001 (2012).
14. M. Ablikim et al. (BES III Collaboration), Phys. Lett. B 734, 227 (2014).
15. R. Aaij et al. (LHCb collaboration), Phys. Rev. Lett. 110, 101802 (2013); R. Aaij et al. (LHCb collaboration), Phys. Rev. Lett. 111, 251801 (2013).
16. T. Aaltonen et al. (CDF collaboration), Phys. Rev. Lett. 111, 231802 (2013).
17. B. R. Ko et al. (Belle collaboration), Phys. Rev. Lett. 112, 111801 (2014).
18. Y. Amhis et al. (Heavy Flavor Averaging Group), arXiv:1207.1158[hep-ex] and online update at http://www.slac.stanford.edu/xorg/hfag/charm/CHARM13/results
19. T. Abe et al., arXiv:1011.0352
20. G. Bonivcini et al., (CLEO Collaboration), Phys. Rev. D 89, 072002 (2014).