Are redshift-space distortions actually a probe of growth of structure?

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We present an impact of coupling between dark matter and a scalar field, which might be responsible for dark energy, on measurements of redshift-space distortions. We point out that, in the presence of conformal and/or disformal coupling, linearized continuity and Euler equations for total matter fluid significantly deviate from the standard ones even in the sub-horizon scales. In such a case, a peculiar velocity of total matter field is determined not only by a logarithmic time derivative of its density perturbation but also by density perturbations for both dark matter and baryon, leading to a large modification of the physical interpretation of observed data obtained by measurements of redshift-space distortions. We reformulate galaxy two-point correlation function in the redshift space based on the modified continuity and Euler equations. We conclude from the resultant formula that the true value of the linear growth rate of large-scale structure cannot be necessarily constrained by single-redshift measurements of the redshift-space distortions, unless one observes the actual time-evolution of structure.

Introduction.

The current cosmological observations, such as type Ia supernovae [1, 2] and cosmic microwave background [3], indicate the presence of dark matter and dark energy, which have not been identified yet. The existence of dark matter is also well established by astrophysical observations, which indicate dark matter as a non-luminous and pressure-less fluid with small dispersion velocity [4–7]. The dark energy is responsible for explaining the late-time accelerated expansion of the Universe, and numerous attempts to identify it have been intensively proposed in many literatures. One such candidate is to introduce a scalar degree of freedom as a new contribution to energy-momentum tensor or modification in a gravitational sector (see for reviews e.g., [8, 9]).

When the ordinary matter, baryon, directly couples with such a scalar degree of freedom, it induces the fifth force. While the fifth force between baryonic matter is tightly constrained by the solar-system experiments [10], this is not true for the dark force that is active only between dark matter since the solar-system experiments do not probe such an interaction. Then, the natural arena for probing such interactions is cosmology. When additional interaction only between the cold dark matter (CDM) is present, the growth rate of the CDM density perturbations would be generically different from that of the baryon density perturbations. We then expect that observing the growth of the CDM density perturbation provides us with rich information about such an interaction.

In the standard treatments, the linear growth rate of large-scale structure is mainly obtained by observing galaxy peculiar velocity field through measurements of redshift-space distortions (RSDs) in galaxy survey. On large scales, where the linear perturbation theory is valid, the galaxy peculiar velocity field is considered to be identical to the velocity field of the total matter. Based on the continuity equation, the matter velocities should be given by the logarithmic time derivative of the density field, that is, the linear growth rate, \( f_m(a) \). Galaxy maps produced by estimating distances from observed radial velocities, which include components from both the Hubble flow and peculiar velocities driven by the clustering of matter, show an anisotropic galaxy distribution. Due to such an effect, the galaxy power spectrum on large scales is known to be enhanced by the factor \((1 + \beta \mu^2)^2\) (named “Kaiser formula”), where \(\beta \equiv f_m/b_k\) with \(b_k\) being the linear galaxy bias factor and \(\mu\) is the cosine of the angle between the line of sight and the Fourier momentum [11]. Although there is a degeneracy between the growth rate and the linear bias factor, this degeneracy can be in principle broken by using e.g., higher-order statistics [12] and cross-correlations between other observables [13] by which the linear bias factor alone can be constrained. Hence, it is widely believed that measurements of RSDs even at single redshift allow direct constraints on the growth rate. Moreover, several attempts show that the relation between the peculiar velocity and the growth rate for each species, which is based on the continuity equation, is valid even for the wide range of cosmological scenarios including modified theories of gravity (see e.g., [14]). However, as we will show below, this relation is not necessarily correct in more general situations. In this Letter, we would like to address how the above Kaiser formula is modified when the CDM couples with the scalar field.

Setup.

Let us consider the following invertible metric transformation [15],

\[
\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi,
\]

(1)
where \( g_{\mu \nu} \) is the original frame metric, and \( A(\phi, X) \) and \( B(\phi, X) \) are respectively called conformal and disformal factors, which are functions of the scalar field \( X \) and its kinetic term \( X \equiv -g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi /2 \). Here and hereafter, \( \phi \) is a generic scalar field, and we do not specify it though we are responsible for dark energy is the most interesting. The action is given by

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} (R[g] - 2\Lambda) + L_\phi[g, \phi] \right] + S_m(2)
\]

where \( L_\phi \) represents a Lagrangian for scalar field and \( S_m \) a total matter action. For simplicity, we consider the canonical scalar field: \( L_\phi = -\frac{1}{2} (\partial \phi)^2 - V(\phi) \) and assume the scalar field does not modify the gravitational sector, i.e., the absence of kinetic braiding \([16]\). As for the matter sector, we assume that the baryon is minimally coupled for simplicity while the CDM couples with the scalar field through the barred metric \( \bar{g}_{\mu \nu} \) defined in (1). The total matter action is thus given by

\[
S_m = S_b + S_c = \int d^4x \left[ \sqrt{-g} \mathcal{L}_b[g_{\mu \nu}, \psi_b] + \sqrt{-g} \mathcal{L}_c[g_{\mu \nu}, \psi_c] \right] (3)
\]

where \( S_b \) and \( S_c \) represent the actions for baryon and CDM, respectively. Due to the non-minimal coupling between the dark matter and the scalar field, baryonic matter and dark matter do not move in the same way.

The variation with respect to the metric \( g_{\mu \nu} \) leads to the Einstein equations as usual,

\[
G_{\mu \nu} + \Lambda g_{\mu \nu} = \frac{1}{M_{Pl}^2} \left( T^{(b)}_{\mu \nu} + T^{(c)}_{\mu \nu} + T^{(\phi)}_{\mu \nu} \right). \quad (4)
\]

Here and hereafter, \( T^{(I)}_{\mu \nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_I}{\delta g^{\mu \nu}} \) and \( T^{(\phi)}_{\mu \nu} = -\frac{2}{\sqrt{-g}} \frac{(\delta g_{\mu \nu}/\delta \phi)}{\delta \phi} \). The superscript \( I \) represents \( b, c \) or \( m \) for baryon, dark matter and total matter, respectively. The combination of the energy-momentum tensor for total matter \( T^{(m)}_{\mu \nu} := T^{(b)}_{\mu \nu} + T^{(c)}_{\mu \nu} \) and the scalar sector \( T^{(\phi)}_{\mu \nu} \) is conserved as \( \nabla^\mu (T^{(m)}_{\mu \nu} + T^{(\phi)}_{\mu \nu}) = 0 \). The energy-momentum conservation for baryon also takes the familiar form, \( \nabla^\mu T^{(b)}_{\mu \nu} = 0 \). On the other hand, the energy-momentum tensors for dark matter and scalar field no longer satisfy the conservation law individually, and it rather takes the following form, \( \nabla^\mu T^{(c)}_{\mu \nu} = -\nabla^\mu T^{(\phi)}_{\mu \nu} \).

The scalar equation is given by

\[
\Box \phi - V_\phi = Q, \quad (5)
\]

where \( Q \), which characterizes the coupling between CDM and the scalar field, is defined as

\[
Q \equiv -\frac{1}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_c)}{\delta \phi} = \nabla_\mu W^\mu - Z, \quad (6)
\]

with

\[
Z = \frac{1}{2A} \left\{ \left( A_\phi + \frac{A X (A_\phi - 2B X)}{A - A X + 2B X X^2} \right) T^{(c)}_{\mu \nu} + \left( B_\phi + \frac{B X (A_\phi - 2B X)}{A - A X + 2B X X^2} \right) \nabla_\mu \phi \nabla_\nu \phi \right\}, \quad (7)
\]

\[
W^\mu = \frac{1}{2A} \left[ 2BT^{(c)}_{\mu \nu} \partial_\mu \phi - \frac{A - 2B X}{A - A X + 2B X X^2} \right] \nabla_\nu \left( A T^{(c)}_{\nu \beta} + B X T^{(c)}_{\nu \beta} \right), \quad (8)
\]

where \( U_\phi = \partial U/\partial \phi, \ U_X = \partial U/\partial X \) for \( U = A, B \). By the use of Eq. (5), the energy-momentum conservation for CDM and total matter can be recast as

\[
\nabla^\mu T^{(c)}_{\mu \nu} = -Q \partial_\nu \phi. \quad (9)
\]

**Basic equations.**

We work on a spatially flat FLRW metric in Newtonian gauge,

\[
ds^2 = -[1 + 2\Phi(t, x)] dt^2 + a^2(t)[1 - 2\Psi(t, x)] dx^2, \quad (10)
\]

and define the background and perturbations of the energy-momentum tensor for the baryon, the dark matter and the total matter as

\[
T^{(0)}_{\mu \nu} = -\rho(t) \left[ 1 + \delta(t, x) \right], \quad (11)
\]

\[
T^{(0)}_{\mu \nu} = -\rho(t) \partial_\mu \partial_\nu t, \quad (12)
\]

and (otherwise) = 0 \(^1\). Based on these equations, we can find relations as

\[
\delta_m = \omega_c \delta_c + \omega_b \delta_b, \quad (13)
\]

\[
v_m = \omega_c v_c + \omega_b v_b, \quad (14)
\]

where \( \omega_1 = \rho_1/\rho_m \). We also split the scalar field as \( \phi(t, x) \rightarrow \phi(t) + \delta \phi(t, x) \). The background part of the Einstein equation gives

\[
H^2 = \frac{1}{3M_{Pl}^2} \left( \rho_c + \rho_h + \Lambda + \frac{1}{2} \dot{\phi}^2 + V \right), \quad (15)
\]

\[
3H^2 + 2\dot{H} = \frac{1}{M_{Pl}^2} \left( \Lambda - \frac{1}{2} \dot{\phi}^2 + V \right). \quad (16)
\]

The background equation of motion for \( \phi \) (5) yields

\[
\ddot{\phi} + 3H \dot{\phi} + V_\phi = -Q_0, \quad (17)
\]

\(^1\) Note that the pressureless feature of the CDM is robust at least at first order of perturbations even if we take other definitions of energy momentum tensor such as \( T^{(c)}_{\mu \nu} = -(2/\sqrt{-g}) \delta (\sqrt{-g} \mathcal{L}_c) / \delta \gamma^{\mu \nu} \) and \( T^{(c)}_{\mu \nu} = -(2/\sqrt{-g}) \delta (\sqrt{-g} \mathcal{L}_c) / \delta g^{\mu \nu} \).
and the energy-momentum conservation for baryon and CDM lead to the background equations

\begin{align}
\rho_b + 3H\rho_b &= 0, \\
\rho_c + 3H\rho_c &= Q_0\dot{\phi},
\end{align}

where $Q_0$ is a background value of $Q$. We can rewrite $Q_0$ from the definition (6)-(8) together with (19),

\begin{equation}
\frac{\dot{\phi}}{\rho_c} Q_0 = \frac{1}{2\Omega_0} \log \left[ \frac{(2A - A_X \phi^2) + B_X \phi^4}{A - B\phi^2} \right],
\end{equation}

where $A$, $A_X$, $B$ and $B_X$ are evaluated at background fields.

In deriving perturbed equations, we use the quasi-static approximation, which is applicable when the wavelength of perturbations is well inside the sound Horizon of the scalar field, $k^{-1} \ll c_s/(aH)$, where $c_s$ is the sound speed of the scalar field. Although the sound speed of the scalar field generally differs from unity in our setup [17], we assume $c_s = \mathcal{O}(1)$ for simplicity. Then we can neglect time-derivative terms of perturbations while keeping spatial derivative terms \(^2\), and we obtain the linearized perturbed Einstein equations in the Fourier space,

\begin{equation}
\frac{k^2}{a^2} \Psi = \frac{k^2}{a^2} \Phi = -4\pi G\rho_m \delta_m.
\end{equation}

The continuity and Euler equations for baryon are the standard form:

\begin{align}
\delta_b + \frac{k^2}{a^2} v_b &= 0, \\
\dot{v}_b - \Phi &= 0,
\end{align}

while those for CDM get modified as follows

\begin{align}
\dot{\delta}_c + \frac{k^2}{a^2} v_c &= \frac{\dot{\phi}}{\rho_c} (\delta Q - Q_0 \delta_c), \\
\dot{v}_c - \Phi &= \frac{Q_0}{\rho_c} (\delta \phi - \dot{\phi} v_c),
\end{align}

where the scalar field perturbation is determined by

\begin{equation}
-\frac{k^2}{a^2} \delta \dot{\phi} = \delta Q.
\end{equation}

In the quasi-static limit, the most relevant terms in $Q$ can be extracted as

\begin{equation}
\delta Q = Q_0 \delta_c + (R_1 + R_2) \phi \dot{\delta}_c + R_1 \phi \frac{k^2}{a^2} v_c + R_2 \frac{k^2}{a^2} \delta \phi, \tag{27}
\end{equation}

where

\begin{align}
R_1 &= \frac{B\rho_c}{A}, \\
R_2 &= \frac{(A - B\phi^2)(A_X - B_X \phi^2)\rho_c}{A(2A - A_X \phi^2 + B_X \phi^4)}.
\end{align}

Physical meanings of these functions are as follows: $R_1$ characterizes the strength of the modification of the CDM velocity field from the disformal coupling, and $R_2$ represents the contribution from the scalar field perturbation in $\delta Q$. We emphasize that the $R_2$ term gives the non-vanishing contributions only if the conformal and/or disformal factors depend on the kinetic term.

After eliminating the scalar field perturbations by using (26), we can rewrite the modified continuity equation in terms of the CDM density contrast and velocity field as

\begin{equation}
(1 - \Upsilon_1) \left( \dot{\delta}_c + \frac{k^2}{a^2} v_c \right) = \Upsilon_2 \left( \dot{\delta}_c - \frac{Q_0}{\phi} \delta_c \right),
\end{equation}

with

\begin{equation}
\Upsilon_1 = \frac{\phi^2}{\rho_c} \frac{R_1}{1 + R_2}, \quad \Upsilon_2 = \frac{\phi^2}{\rho_c} \frac{R_2}{1 + R_2}.
\end{equation}

In the minimal coupling case ($A = 1, B = 0$), all time-dependent coefficients are zero, $Q_0 = R_1 = R_2 = 0$. When conformal and disformal factors depend only on $\phi$, we have $Q_0 \neq 0$, $R_1 \neq 0$, and $R_2 = 0$. Thus the continuity equation is the same as the one in the minimal coupling case. One can verify this property even for a wider class of scalar-tensor theories [14]. When at least one of $A$ and $B$ depend on $X$, there arises a new contribution of $R_2$ in the continuity equation, and the CDM velocity can significantly differ from the standard case. An important implication from these equations is that the continuity equation for the total matter fluctuations, (13) and (14), is given by

\begin{equation}
\dot{\delta}_m + \frac{k^2}{a^2} v_m = \frac{\dot{\phi}}{\rho_m} \left[ \delta Q - Q_0 (\omega_b \delta_b + \omega_c \delta_c) \right]
= \omega_c \frac{\Upsilon_2}{1 - \Upsilon_1} \left( \dot{\delta}_c - \frac{Q_0}{\phi} \delta_c \right)
+ \omega_b \frac{Q_0 \phi}{\rho_m} (\delta_c - \delta_b),
\end{equation}

which differs from the standard form by the presence of the non-minimal coupling. We also found that even when the $R_2$ contribution is negligible the standard form of the continuity equation cannot be reproduced due to the second term of the right-hand-side in the second equation, which originates from the deviation of the background energy density from the standard matter (see Eq. (19)). Therefore, we conclude that there are two possibilities to break the standard relation of the continuity equation for the total matter field: One comes from the $R_2$ term in the CDM continuity equation, which appears only when

\(^2\) We also neglect the mass $m_\phi$ of the scalar field, which could be crucial when the mass of the scalar field is large enough, i.e., $m_\phi \gtrsim k/a$. The full analysis including large mass effects will be reported in the full paper [17].
Moreover, by the use of Eq. (34) the effective linear growth rate of the CDM, \( f_c \), can significantly differ from the standard one due to the \( R_2 \) contribution as
\[
\begin{aligned}
f_c^{\text{eff}} &= f_c - \frac{\Upsilon_2}{1 - \Upsilon_1} \left( f_c - \frac{Q_0}{H\phi} \right) = f_c + \Delta f_c.
\end{aligned}
\] (35)

Therefore, the resultant galaxy power spectrum in redshift space is given by
\[
\begin{aligned}
P_{g,s}(k; t) &= \left( 1 + \beta^{\text{eff}}(t) \mu^2 \right)^2 P_g(k; t),
\end{aligned}
\] (39)

where \( P_g = \beta_g^2 P_m \) is the real-space galaxy power spectrum, \( P_m = D_2 P_0 \) is the power spectrum for the total matter density contrast, and
\[
\begin{aligned}
\beta^{\text{eff}} &= \frac{f_m^{\text{eff}}}{b_g}
\end{aligned}
\] (40)

The galaxy density contrast in the real space, \( \delta_g \), is related to the total matter density contrast \( \delta_m \) given by Eq. (13), through the standard linear bias model \( \delta_g = b_g \delta_m \) on large scales. The peculiar velocity fields of the galaxies, \( v_g \), on large scales are expected to be related to the CDM and baryon fluid velocities, and the explicit relation is determined by imposing the reasonable physical condition, e.g., momentum conservation law for each galaxy [14].

In the above investigation, we found that the effective growth rate \( f_m^{\text{eff}} \) inferred from the peculiar velocities no longer coincides with the actual growth rate \( f_m \), namely measurements of the peculiar velocity field do not necessarily provide the growth rate of clustering directly. Our example vividly demonstrates that the standard dictionary translating the RSDs measurements into the growth rate is not universal and fails for some classes of theories beyond the \( \Lambda \)CDM model. To see the impact of the breaking of the relation between the peculiar velocities and the actual growth rate, we now focus on the modification of the Kaiser formula as the simplest and most important observable effect of RSDs. The generalization to other observables related to the peculiar velocities is straightforward.

In addition to the Hubble expansion, the peculiar velocities of the galaxies relative to the Hubble expansion distort the distribution of galaxies in the 3-dimensional redshift space, and such effects must carefully be taken into account when comparing galaxy two-point correlation function with theoretical predictions [11]. The mapping of the observed redshift position \( s \) from the real space position \( x \) is given by
\[
\begin{aligned}
s = x + \frac{v_{g,z}}{aH} \hat{z},
\end{aligned}
\] (37)

where \( v_{g,z} \) is a line-of-sight component of the peculiar velocity of a galaxy and \( \hat{z} \) is a unit vector of line-of-sight. In Eq. (37), we have assumed the plane-parallel approximation, so that the line-of-sight is taken as a fixed direction, \( \hat{z} \). Recalling that the number of galaxies in the infinitesimal volume of both spaces is invariant, the overdensities in the redshift space \( \delta_{g,s} \) and the real space \( \delta_g \) are related through
\[
\begin{aligned}
\delta_{g,s} = \delta_g - \frac{1}{aH} \nabla_z v_{g,z}.
\end{aligned}
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Therefore, the resultant galaxy power spectrum in redshift space is given by
\[
\begin{aligned}
P_{g,s}(k; t) &= \left( 1 + \beta^{\text{eff}}(t) \mu^2 \right)^2 P_g(k; t),
\end{aligned}
\] (39)

where \( P_g = b_g^2 P_m \) is the real-space galaxy power spectrum, \( P_m = D_2 P_0 \) is the power spectrum for the total matter density contrast, and
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\begin{aligned}
\beta^{\text{eff}} &= \frac{f_m^{\text{eff}}}{b_g}
\end{aligned}
\] (40)
This is a generalization of the Kaiser formula. In fact, in the minimal coupling case, we have $D_m = D_c = D_b$ and $f_m^{\text{eff}} = f_m = f_c^{\text{eff}} = f_c = f_b$, and hence Eq. (39) is reduced to the standard Kaiser formula. However, we found that in the presence of the coupling between the CDM and the scalar sector we have no longer the relation $f_m^{\text{eff}} = f_m$ as we have discussed, and it means that the RSDs are not trustable probes of growth of structure. It is notable that the RSDs cannot provide the true value of the growth rate $f_m$ even in the simple case where the conformal and disformal factors depend only on $\phi$. Since in this case, the deviation from the standard formula is proportional to $Q_0$, this effect is suppressed when the background evolution of the dark matter is almost same as the one of the baryon. On the other hand, there is a wider room for sizable modification of the standard Kaiser formula in our general setup; even when $Q_0$ or the baryonic contamination is negligibly small, $f_m^{\text{eff}}$ can differ from $f_m$ by $O(1)$. To see this clearly, let us expand the formula (40) in terms of the baryon-CDM ratio to neglect the ambiguity from the baryon contribution. The leading term gives $\beta^{\text{eff}} \approx \frac{f_m}{b_c} = \frac{f_c}{b_k} + \Delta f_c/b_k$. This immediately shows the single-redshift RSDs measurements cannot give a constraint on the linear growth rate $f_c$ unless the contributions from the couplings $\Delta f_c$ is fixed by using other observables. This fact demonstrates that one has to keep this new effect in mind when testing beyond $\Lambda$CDM theories by the RSDs measurements. Even if the growth index $\gamma \approx 0.55$ is obtained from RSDs in future galaxy survey, it is still possible that the true theory is different from the standard $\Lambda$CDM model. One way to obtain the actual growth rate of large-scale structure is to directly observe the time-evolution of structure by e.g. multiple redshift observations of galaxy power spectrum. In fact, we have a strong degeneracy between the growth of large scale structure and the redshift-dependence of the linear bias. Thus, to measure $f_m$ by multiple redshift observations, we need to fix the bias for each redshift by using other observations, i.e., cross-correlation between the clustering of galaxies and weak lensing (see, e.g., [13]). After evaluating the actual growth rate, one can compare the actual and effective growth rates to constrain the couplings between the CDM and scalar field.

**Conclusion.**

We have shown that the additional interaction mediated by the scalar field that operates only between dark matter through conformal and disformal couplings changes the continuity and Euler equations for cosmological perturbations in a non-trivial manner and investigated its impact on RSDs measurements in galaxy survey. We found that the effects of such modifications appear even at sub-horizon scales in the presence of $\phi$ and $X = -\frac{\mu^2}{2} \partial_i \phi \partial_i \phi /2)$-dependence of the conformal and/or disformal couplings. The effective linear growth rate, which is inferred from measurements of the peculiar velocities of the distributed galaxies, no longer corresponds to the logarithmic time derivative of the density perturbation and is rather characterized by both the density perturbations and their derivatives for each species in general situation. In other words, the information of the coupling is encoded in the peculiar velocity fields and the true value of the growth rate of large-scale structure cannot necessarily be constrained by the single-redshift RSDs measurements. It can be extracted by using multiple power spectra of the galaxy distribution at different redshift. This fact will play a vital role of measuring the linear growth rate $f_m$ by the RSDs measurement, and it will provide us a rich information of dark matter and dark energy.

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**Note added.**

While this paper was being completed, Ref. [18] appeared, in which the redshift space distortions in the context of interacting dark matter and vacuum energy are discussed.

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