Non-zero helicity extinction in light scattered from achiral (or chiral) small particles located at points of null incident helicity density

Manuel Nieto-Vesperinas

Instituto de Ciencia de Materiales de Madrid, Consejo Superior de Investigaciones Científicas, Campus de Cantoblanco, Madrid E-28049, Spain

E-mail: mnieto@icmm.csic.es

Received 28 December 2016, revised 28 February 2017
Accepted for publication 7 March 2017
Published 2 May 2017

Abstract
Based on a recent unified formulation on dichroism and extinction of helicity on scattering by a small particle, dipolar in the wide sense, magnetodielectric or not, chiral or achiral, we show that such extinction is enhanced not only at resonances of polarizabilities, but also due to interference between left and right circularly polarized components of the incident wave, which contributes with appropriate parameters of the illuminating field, even if the particle is achiral and is placed at points of the incident field at which the local incident helicity density is zero.

This phenomenon goes beyond standard circular dichroism (CD), and we analyze it in detail on account of the values of several quantities involved in the process, both of the incident light and the particle. In addition, this interference produces a term in the helicity extinction that remarkably yields information on the real parts of the electric and/or magnetic polarizabilities, which are not provided by CD, and of which the helicity extinction phenomenon may be considered a generalization.

Keywords: extinction rate of helicity, twisted light scattering by dipolar magnetodielectric particles, resonant polarizabilities, scattered helicity

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent times, the concept of circular dichroism (CD) [1–3] has been extended to extinction by scattering (or diffraction), transmission, and/or absorption by nanostructures that may or may not be chiral [4–8]. It has also been extended to procedures to enhance its weak signal from absorbing molecules, which has been proposed by the following: enhancing the helicity of the illuminating field [9] by interposing a resonant particle, either chiral or achiral, between the molecule near field and the detecting tip [10–12]; reinforcing CD from nanostructures by creating near field hot spots between sets of plasmonic nanoparticles according to the choice of incident polarization [13]; by making thermal-controlled chirality in a hybrid THz metamaterial with VO₂ inclusions [14]; or by fostering the interplay between electric and magnetic dipoles of the excited molecule [15]. A helicity optical theorem (HOT) has been recently established [16] showing that dichroism phenomena are particular effects resulting from a fundamental law of electrodynamics: the conservation of electromagnetic helicity [9, 17–22]. This also lends sufficient conditions to produce chiral fields by scattering [8], and provides answers to long-standing questions on the interplay between the chirality of fields and matter [23, 24].

The helicity of quasi-monochromatic, i.e. time-harmonic, light waves, which are those addressed in this paper, is equivalent to their chirality [25]. The latter is a term employed in [9], and having subsequently became of widespread use in the literature, we shall also consider here. As stated in [16], for these time-harmonic fields both magnitudes
differ only by the square of the wavenumber. However, for
general time-dependent fields both quantities have a different
physical nature and hence are not equivalent. This distinction
is important in matter. Also, as noticed in [18], helicity has
dimensions of angular momentum whereas chirality does not.

In this paper we exploit the equations for the extinction
of helicity and energy that we established in a previous work,
where a unified formulation of helicity extinction and
dichroism beyond the CD concept was presented. Hence, we
now show that CD may be generalized to 3D polarized fields,
for which we introduce a helicity extinction factor \( g \), a particular case that is the standard CD dissymmetry factor. In
addition, we further analyze the extinction of helicity on the
scattering of 3D polarized fields possessing a longitudinal
component, and whose projection in the plane transversal to
the propagation direction has elliptic polarization, namely, is
the sum of a left circular (LCP) and a right circular (RCP)
wave. This helicity extinction may be generated not only as in
CD by the cross electric-magnetic polarizability that character-
izes the particle chirality, or by the incident helicity
density, but also by an interference factor that mixes the LCP
and RCP components. This is a phenomenon in which the
above mentioned cross-polarizability plays no role, and
whose existence was already shown in [8]. We shall study it
in detail here.

In this way, we discuss how \( g \) assesses the helicity
extinction in comparison with that of energy. We analyze this
under different values of the polarizabilities of a particle that
we initially assume to be of rather general characteristics;
namely, bi-isotropic, magnetodielectric, and chiral, (we shall
later relax this latter property) in the resonant regions of its
polarizabilities. In addition, we analyze this extinction for
different local values of the incident helicity density, and
assess the contribution of the aforementioned interference to
this helicity extinction in comparison with that of particle
chirality and incident helicity density, as well as the polariz-
ability resonances that we have chosen in this study to
enhance these effects.

Among the illuminating fields whose electric and magnetic
vectors fulfill the conditions leading to this interference effect,
discussed in sections 2 and 3, we shall use a Bessel beam,
which has been well studied and is known to be accessible and
employed in many experiments. We shall give some of its
details in section 4, specially in connection with its functional
contribution to the densities of incident energy and helicity, as
well as to the aforementioned interference phenomenon
between LCP and RCP components. Then in section 5 we shall
illustrate how the discussed quantities depend on the transversal
position of the particle within the incident beam.

2. 3D polarized fields with LCP and RCP transversal
polarization

We address the spatial parts \( \mathbf{E} \) and \( \mathbf{B} \) of the electric
and magnetic vectors of quasi-monochromatic fields in their com-
plex representation. Their scattering in a medium of refractive
index \( n = \sqrt{\varepsilon \mu} \) by a particle that generally consider
magnetodielectric, chiral, bi-isotropic, and dipolar in the wide
sense [28, 29], is thus characterized by its polarizabilities. For
a sphere for example, these are as follows:

\[
\alpha_e = i \frac{3}{2k} a_1, \quad \alpha_m = i \frac{3}{2k} b_1, \quad \alpha_{em} = i \frac{3}{2k} c_1, \quad \alpha_{me} = i \frac{3}{2k} d_1 = -\alpha_{em}, \quad \text{and} \quad k = \omega/c = 2\pi n /\lambda,
\]

where \( a_1, b_1 \) and \( c_1 = -d_1 \) stand for the electric,
magnetic, and magnetoelectric first Mie coefficients,
respectively [30].

The electric and magnetic dipole moments, \( \mathbf{p} \) and \( \mathbf{m} \),
induced in the particle by this incident field are as follows:

\[
\mathbf{p} = \alpha_e \mathbf{E} - \alpha_m \mathbf{B}, \quad \mathbf{m} = \alpha_{me} \mathbf{E} + \alpha_{me} \mathbf{B}.
\]

(1)

Based on the angular spectrum decomposition of optical
wavefields into LCP (sign: +; the notation of [31] is fol-
lowed) and RCP (sign: −) plane wave components that we
established in [8] (though we must remark that we have
recently found that this representation was also reported in
[19]), both the incident and scattered fields may be decom-
posed into the sum of an LCP and RCP 3D wavefield. Then we
address incident fields \( \mathbf{E} \) and \( \mathbf{B} \) (which we shall subse-
quently consider to be optical beams), expressible as the sum
of 3D polarized fields whose transversal polarization is LCP
and RCP, respectively. Thus,

\[
\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\pm}(\mathbf{r}) + \mathbf{E}^{-}(\mathbf{r});
\]
\[
\mathbf{B}(\mathbf{r}) = \mathbf{B}^{\pm}(\mathbf{r}) + \mathbf{B}^{-}(\mathbf{r})
\]

\[
= -im[(\mathbf{E}^{\pm}(\mathbf{r}) - \mathbf{E}^{-}(\mathbf{r})] ;
\]

(2)

by which we express the dipolar moments as

\[
\mathbf{p}(\mathbf{r}) = \mathbf{p}^{\pm}(\mathbf{r}) + \mathbf{p}^{-}(\mathbf{r}), \quad \mathbf{m}(\mathbf{r}) = \mathbf{m}^{\pm}(\mathbf{r}) + \mathbf{m}^{-}(\mathbf{r}),
\]

(3)

with

\[
\mathbf{p}^{\pm}(\mathbf{r}) = (\alpha_e \pm n i \alpha_{me}) \mathbf{E}^{\pm}(\mathbf{r});
\]
\[
\mathbf{m}^{\pm}(\mathbf{r}) = (\alpha_{me} \mp n i \alpha_m) \mathbf{E}^{\pm}(\mathbf{r}).
\]

(4)

The 3D polarized wavefields of equation (2) are not just plane
waves or transversely polarized beams. In a XYZ-Cartesian
framework, \( \mathbf{E} \) and \( \mathbf{B} \) (see equation (2)) have, in general, a
z-component, while that in the XY-plane is elliptically polar-
ized. As shown below, we illustrate these electromagnetic
fields by the sum of two beams propagating along OZ: LCP
and RCP, respectively. Therefore, both circular polarizations
hold in the XY-plane transversal to the beam z-axis. In addi-
tion, both beams have a Cartesian component along OZ.

The relationship (2) between \( \mathbf{B} \) and \( \mathbf{E} \) is essential for the
effects that we obtain next. This is the reason why we choose,
among other possibilities, an illuminating Bessel beam in
section 4.

3. The extinction of incident helicity and energy on
scattering: beyond CD

Using a Gaussian system of units, the densities of helicity, \( \mathcal{H} \), or \( k^{-2} \times \) the chirality, and energy, \( \mathcal{W} \), (understood as
time-averaged in this work), of the incident field are [8, 19]:

\[
\mathcal{H}(\mathbf{r}) = (\epsilon/2k)[|\mathbf{E}^{\pm}(\mathbf{r})|^2 - |\mathbf{E}^{-}(\mathbf{r})|^2],
\]

(5)
and

\[ \mathcal{W}(r) = (\varepsilon / 8\pi) [\lvert E^+(r) \rvert^2 + \lvert E^-(r) \rvert^2]. \]

In what follows \( \Re \) and \( \Im \) denote real and imaginary parts, respectively.

The HOT that expresses the conservation of helicity is [16]

\[ \frac{2\pi c}{\mu} \Re \left\{ -\frac{1}{e} \mathbf{p} \cdot \mathbf{B}^* + \mu \mathbf{m} \cdot \mathbf{E}^* \right\} = \frac{8\pi c k^3}{3e} \Im [\mathbf{p} \cdot \mathbf{m}^*] + \mathcal{W}_\text{ext}. \] (7)

The left side of (7) constitutes the extinction of helicity of the incident wave on scattering with the particle. This extinction is shown in the right side of (7) to be divided into the total helicity scattered or radiated by the object (i.e. the first term on the right side) and the rate of helicity dissipation \( \mathcal{W}_\text{ext} \) (see equations (8), (11), and (12) of [16]) or converted helicity (see sections 3.2, 3.3 and 4 of [21]) upon interaction with the scattering body.

As shown by the right side of (7), as the light interacts with the particle such extinction may convey a selective dissipation of helicity \( \mathcal{W}_\text{ext} \), which adds to the total helicity of the scattered field. This latter fact agrees with [20, 21].

We should recall the analogy of the HOT with the well-known standard optical theorem (OT) for energies

\[ \frac{\omega}{2} \Im [\mathbf{p} \cdot \mathbf{E}^* + \mathbf{m} \cdot \mathbf{B}^*] = \frac{ck^4}{3n} [\varepsilon^{-1} \mathbf{p}]^2 + \mu [\mathbf{m}]^2 + \mathcal{W}_\text{ext}. \] (8)

The left side of (8) is the energy extinguished from the illuminating field, or rate of energy absorption by the object from the illuminating wave.

Upon employing equations (1)-(4), the extinction of incident helicity (see equation (7)) \( (2\pi c/\mu) \Re \left\{ -\frac{1}{e} \mathbf{p} \cdot \mathbf{B}^* + \mu \mathbf{m} \cdot \mathbf{E}^* \right\} \), which henceforth we denote as \( \mathcal{W}_\text{ext} \), is expressed as [8]:

\[ \mathcal{W}_\text{ext}(r) \equiv \frac{2\pi c}{\mu} \left\{ \Im [\mathbf{p}^+(r) + i\mu \mathbf{m}^+(r)] \cdot \mathbf{E}^{\ast+}(r) \right\} \]

\[ - \left[ \mathbf{p}^-(r) - i\mu \mathbf{m}^-(r) \right] \cdot \mathbf{E}^{\ast-}(r) \]

\[ + 2\Re \{ \alpha_e - n^2 \alpha_m \} \Im [\mathbf{E}^-(r) \cdot \mathbf{E}^{\ast+}(r)] \right\} \]

\[ = \frac{2\pi c}{\mu} \left\{ \frac{2k}{e} \Re [\alpha_e + n^2 \alpha_m] \mathcal{H}(r) \right\} \]

\[ + 16\pi \sqrt{\frac{\mu}{e}} \Re [\alpha_m] \mathcal{W}(r) + 2\Re [\alpha_e - n^2 \alpha_m] \]

\[ \times \Im [\mathbf{E}^-(r) \cdot \mathbf{E}^{\ast+}(r)] \}; \] (9)

whereas equations (1)-(4) yield for the extinction of incident energy (see equation (8)) \( \omega/2 \Im [\mathbf{p} \cdot \mathbf{E}^* + \mathbf{m} \cdot \mathbf{B}^*] \), which we write as \( \mathcal{W}_\text{ext} \) [8] (note that a slightly different notation is used for this quantity in [8]):

\[ \mathcal{W}_\text{ext}(r) \equiv \frac{\omega}{2} \left\{ \Im [\mathbf{p}^+(r) + i\mu \mathbf{m}^+(r)] \cdot \mathbf{E}^{\ast+}(r) \right\} \]

\[ + \left[ \mathbf{p}^-(r) - i\mu \mathbf{m}^-(r) \right] \cdot \mathbf{E}^{\ast-}(r) \]

\[ + 2\Re \{ \alpha_e - n^2 \alpha_m \} \Im [\mathbf{E}^-(r) \cdot \mathbf{E}^{\ast+}(r)] \right\} \]

\[ = \frac{\omega}{2} \left\{ \frac{8\pi c}{k} \Re [\alpha_e + n^2 \alpha_m] \Re [\mathbf{E}^-(r) \cdot \mathbf{E}^{\ast+}(r)] \right\} \]

\[ + 4k \sqrt{\frac{\mu}{e}} \Re [\alpha_m] \mathcal{H}(r) + 2\Re [\alpha_e - n^2 \alpha_m] \]

\[ \times \Re [\mathbf{E}^-(r) \cdot \mathbf{E}^{\ast+}(r)] \}. \] (10)

In (9) and (10) \( r \) is the position vector of the center of the particle immersed in the illuminating wavefield. Equations (9) and (10) are fundamental as they establish the connection of the extinction of helicity \( \mathcal{W}_\text{ext} \) and energy \( \mathcal{W}_\text{ext} \) of the incident wave with the densities of incident helicity \( \mathcal{H} \) and energy \( \mathcal{W} \) and with the chirality of the dipolar particle, characterized by \( \alpha_m \).

They remarkably show how the incident \( \mathcal{H} \) and \( \mathcal{W} \) contribute to \( \mathcal{W}_\text{ext} \) and \( \mathcal{W}_\text{ext} \) with their roles exchanged with respect to the polarizability factors in the corresponding term where they appear.

Notice from the right side of (7) that \( \mathcal{W}_\text{ext} \) contains both the total scattered helicity and the incident helicity dissipation (or conversion). Similarly, from (8) one sees that \( \mathcal{W}_\text{ext} \) contains the total scattered energy as well as the dissipation of incident energy in the particle. In particular, if both \( \mathcal{W}_\text{ext} \) and \( \mathcal{W}_\text{ext} \) are zero, \( \mathcal{W}_\text{ext} \) and \( \mathcal{W}_\text{ext} \) represent the total scattered helicity and energy, respectively.

Since we are interested in the rate of helicity extinction, we observe in (9) that \( \mathcal{W}_\text{ext} \), apart from being due to the incident helicity density \( \mathcal{H} \) coupled with the dissipative part of electric and magnetic polarizabilities, is generated by a coupling of the incident energy density \( \mathcal{W} \) with the particle chirality through \( \Re [\alpha_m] \). Moreover, of special importance is that, as shown by the third term \( \Re [\alpha_e - n^2 \alpha_m] \Im [\mathbf{E}^- \cdot \mathbf{E}^{\ast+}] \) in equation (9), placing the small particle at a position \( r_0 \) in the illuminating wave, an incident field with no helicity density at \( r_0 \), may give rise to an extinction rate of helicity upon interaction with the particle. This is not only due (as well-known) to the particle chirality through the term with \( \Re [\alpha_m] \mathcal{W} \), but also (and this is the new feature addressed in this work) because of the interference coupling factor \( \mathbf{E}^- \cdot \mathbf{E}^{\ast+} \), i.e. a non-zero \( \mathcal{W}_\text{ext} \) will be generated at \( r_0 \) even if the incident helicity \( \mathcal{H}(r_0) = 0 \) and the particle is not chiral \( \alpha_m = 0 \). Moreover, since \( \Re [\alpha_e] \) and \( \Re [\alpha_m] \) are usually larger than their imaginary counterparts at non-resonant \( \lambda \), this interference term acquires special importance for molecules [8, 16].

In this respect, and in contrast with the above argument for \( \mathcal{W}_\text{ext} \), if \( \mathcal{H} = 0 \) there is no analogous reasoning for a non-zero energy extinction \( \mathcal{W}_\text{ext} \) (equation (10)) if the incident electromagnetic energy \( \mathcal{W} \) is null, since this would convey that \( \mathbf{E}^- = \mathbf{E}^+ = \mathcal{H} = 0 \), and thus \( \mathcal{W}_\text{ext} = 0 \), as it should.

However, in (9) \( \Re [\alpha_e] \) and \( n^2 \Re [\alpha_m] \) appear in a subtraction, and thus compete with each other in their contribution.
to the last term of (9), which becomes zero when \( \alpha_e = n^2 \alpha_m \), namely when the particle is dual [16].

Notice that similar arguments exist for \( \mathcal{J} \{ \alpha_e \} \) and \( \mathcal{J} \{ \alpha_m \} \) in connection with the third term of the energy extinction, equation (10). It is also worth remarking that when \( \mathbf{E}^* \cdot \mathbf{E}^* + \mathbf{E} \cdot \mathbf{E} = 0 \), equations (9) and (10) reduce to those standard of the HOT and OT, respectively, (see section V of [16]).

Equations (9) and (10) govern a generalized dichroism phenomenon and hence account for CD as a particular case. Namely, by defining the ratio

\[
g = \frac{W_{\text{ext}}^g}{W_{\text{ext}}},
\]

which we shall name the **helicity extinction factor**. It is straightforward to see that either when the particle is dual, or when the interference terms of these equations vanish like for elliptically polarized plane waves (for which \( \mathbf{E}^* \cdot \mathbf{E}^* = 0 \) since then no longitudinal z-component exists), then choosing as usually done: \( |\mathbf{E}|^2 = |\mathbf{E}_G|^2 \), one has \( g = \sqrt{\frac{\epsilon}{\mu}} \lambda g_{\text{CD}} \), where \( g_{\text{CD}} = 2(\mathcal{W}_{\text{ext}}^g - W_{\text{ext}}^g)/\mathcal{W}_{\text{ext}}^g \) is the well-known **dissymmetry factor** of standard CD, which from (10) results in a well-known expression in terms of particle polarizabilities [1-3, 9]:

\[
g_{\text{CD}} = 4n \mathcal{R} \{ \alpha_m \} / \mathcal{J} \{ \alpha_e \} + n^2 \mathcal{J} \{ \alpha_m \}.
\]

Hence, the CD phenomenon is one of several consequences of the HOT and thus a result of the conservation of electromagnetic helicity. Namely, while standard CD is observed by illuminating the particle or structure with a LCP plane wave only, and separately with a RCP plane wave; subsequently subtracting the corresponding scattered energies as \( \sqrt{\frac{\epsilon}{\mu}} \lambda g_{\text{CD}} \); CD may be observed on a unique illumination by a wave of the form (2) with no longitudinal component along the \( \mathcal{O} \)-propagation direction (e.g. a plane wave), linearly polarized in the (transversal) \( XY \)-plane (or generally with \( |\mathbf{E}|^2 = |\mathbf{E}_G|^2 \)), and therefore whose LCP and RCP components do not interfere with each other, i.e. \( \mathbf{E} \cdot \mathbf{E}^* = 0 \). The extinction of helicity, normalized to the wavelength \( \lambda \), is identical to the above mentioned difference of LCP and RCP energies of CD.

We should remark that the HOT also accounts for the illumination of an object with those so-called superhelical fields produced by the superposition of two counterpropagating CPL plane waves of amplitudes \( E_1 \) and \( E_2 \) of opposite helicity as put forward in [9] (which on the basis of recent studies [23, 24], we prefer to call fields enhancing the dissymmetry factor). However, it is known [32] that this method is limited to particles or molecules with \( \alpha_m = 0 \), because in such configuration \( g_{\text{CD}} = 4n \mathcal{R} \{ \alpha_m \} / \mathcal{J} \{ \alpha_e \} (E_1 - E_2)/(E_1 + E_2) + n^2 \mathcal{J} \{ \alpha_m \} (E_1 + E_2)/(E_1 - E_2) \). Therefore, when \( \mathcal{J} \{ \alpha_m \} = 0 \) the usually extremely small dissymmetry factor of standard CD (often as small as \( 10^{-3} \)) for molecules, may be enhanced, as seen from this latter expression of \( g_{\text{CD}} \) just by choosing \( E_1 = E_2 \), as proposed in [9], or by making \( E_1 = -E_2 \) when \( \mathcal{J} \{ \alpha_e \} = 0 \); however, it is evident that these choices of \( E_1 \) and \( E_2 \) cannot enhance \( g_{\text{CD}} \) if both \( \mathcal{J} \{ \alpha_e \} \) and \( \mathcal{J} \{ \alpha_m \} \) are non-zero.

It is remarkable that the term \( 2\mathcal{R} \{ \alpha_e - n^2 \alpha_m \} \mathcal{J} \{ \mathbf{E} \cdot \mathbf{E}^* \} \) of (9) is the only one among those expressing the extinction of helicity (equation (9)) that provides information on the real parts of the polarizabilities. No other term of (9) contains such information. In fact, there is a well-known lack of this information in the aforementioned dissymmetry factor \( g_{\text{CD}} \) of standard CD addressed above. Therefore, this paper shows how creating experimental conditions for this interference factor to exist provides source of information on \( \mathcal{R} \{ \alpha_e \} \) and \( \mathcal{R} \{ \alpha_m \} \) through the extinction of helicity (equation (9)) and its associated extinction factor \( g \) (equation (11)).

### 4. Illustration with a Bessel beam

The contribution of \( 2\mathcal{R} \{ \alpha_e - n^2 \alpha_m \} \mathcal{J} \{ \mathbf{E} \cdot \mathbf{E}^* \} \) in (9) to the helicity extinction, while keeping \( \mathcal{R} \{ \mathbf{E} \cdot \mathbf{E}^* \} = 0 \) in (10), therefore increasing the helicity extinction factor \( g \), is next illustrated with an incident beam propagating along \( OZ \), elliptically polarized in the \( XY \)-plane, and with longitudinal component along the \( z \)-propagation direction [8]. In this case \( \partial_z \approx ik_z \), and the wavevector is written in Cartesian components as \( \mathbf{k} = (K, k_z), \) \( K = \sqrt{k_x^2 + k_y^2}, \) \( k_z = \sqrt{k_z^2 - K^2}. \)

The electric vector is expressed in terms of the vector potential \( \mathbf{A} \) [26, 27] as follows:

\[
\mathbf{E}^\pm = ik_z \mathbf{A}^\pm + \frac{i}{k_z} \nabla (\nabla \cdot \mathbf{A}^\pm),
\]

\[
\mathbf{A}^\pm (r) = \frac{1}{ik_z} (\mathbf{k} \pm i\mathbf{j}) u(r) e^{ik_zz},
\]

\[
u(r) = u_0^\pm (R, z) e^{ik_0z}, \quad R = \sqrt{x^2 + y^2}.
\]

So that using \( \nabla \cdot \mathbf{A}^\pm \approx ik_z (\nabla \cdot \mathbf{A}^\pm) \) one has

\[
\mathbf{E}^\pm = e^{ik_zz}[\mathbf{k} \pm i\mathbf{j}] u + i\frac{k_z}{k} (\partial_x u \pm i\partial_y u),
\]

\[
\mathbf{B}^\pm = \mp n\mathbf{E}^\pm
\]

which fulfills both \( \nabla \cdot \mathbf{E}^\pm = 0 \) and equation (2). We shall address the Bessel function of integer order \( u_0^\pm (R, z) = e^{ik_0z} J_l(KR) \) (\( k_0 \) being constant amplitudes), which, from (12)–(16) and after a calculation using the recurrence relation \( J_{l-1}(x) + J_{l+1}(x) = (2l/x)J_l(x) \), leads to the form (2) for a Bessel beam, whose components \( \mathbf{E}^\pm \) are LCP and RCP, respectively, in the \( XY \)-plane transversal to its \( z \)-direction of propagation. Namely:

\[
\mathbf{E}^\pm (r) = e^{ik_0 z} e^{i(k_x x + k_y y)} [J_l(KR) (\mathbf{k} \pm i\mathbf{j}) + i\frac{k_z}{k} \exp(\pm i\phi) J_{l\pm 1}(KR)z].
\]

Equation (17) coincides with those of [26, 27, 33] characterizing Bessel beams. We have nevertheless undertaken a derivation of this kind of beam in order to guarantee that this field fulfills the important condition (2) (see equations (15) and (16)). From (17) we obtain

\[
\mathbf{E}^+ (r)^2 = \mathbf{E}^- (r)^2 = 2|e_0^+|^2 \pm |e_0^-|^2 J_l^2(KR) + \frac{K^2}{k_z^2} [J_{l+1}(KR)|e_0^+|^2 \pm |e_0^-|^2 J_{l-1}(KR)].
\]
On the other hand, the factor $\mathcal{R}(\mathcal{J})\{E^{-} \cdot E^{+}\}$ reduces to the contribution of the field $z$-component:

$$
\mathcal{R}(\mathcal{J})\{E^{-} \cdot E^{+}\} = \mathcal{R}(\mathcal{J})\{E_{z}^{-}E_{z}^{+}\} = -\frac{K^2}{k_{z}^2}J_{l-1}(KR)J_{l+1}(KR)\mathcal{R}(\mathcal{J})\{e_{0}^{-}e_{0}^{+}e^{-2i\phi}\}.
$$

(19)

Therefore, either of these quantities, $\mathcal{R}[\cdot]$ or $\mathcal{J}[\cdot]$, may be made arbitrarily small (or zero) depending on the choice of parameters $e_{0}$ and $e_{0}^{-}$ in the factor $\mathcal{R}(\mathcal{J})\{e_{0}^{-}e_{0}^{+}e^{-2i\phi}\}$. Since according to figure 6 in [33], this beam rotates a particle of diameter $1 \mu m$ to $6 \mu m$ placed in the inner ring of maximum intensity in about 16 s per revolution, we shall assume the signal detection time large enough for the azimuthal angle $\phi$ of the particle center position to not contribute to this factor so that we just consider the quantity $\mathcal{R}(\mathcal{J})\{e_{0}^{-}e_{0}^{+}\}$. Hence, choosing for example $e_{0}^{-}/e_{0}^{+} = \pm a \exp(ib\pi/2)$, and $a$ and $b$ being real, the value of $\mathcal{R}(\mathcal{J})\{E^{-} \cdot E^{+}\}$ will oscillate around zero as $\cos(b\pi/2)(\sin(b\pi/2))$.

Also, depending on the choice of position $\mathbf{r}$ of the particle in the beam, and thus of the argument $KR$, one will have in equations (9) and (10) the third term, whose $\mathcal{R}(\mathcal{J})\{E^{-} \cdot E^{+}\}$ factor is given by (19), being comparable, or not, to the first and second terms whose $\{[E^{-}\mathbf{r}]^2 \pm [E^{+}\mathbf{r}]^2\}$ factor is given by equation (18). This is seen by observing the factor $(K^2/k_{z}^2)J_{l-1}(KR)J_{l+1}(KR)$ in (19), which may be made either much larger or smaller than the term $J_{l}^2(KR)$, which is the dominant contribution to (18).

This latter important fact will be seen in section 5 by choosing two different positions of the particle in the beam, i.e. two distinct values of $KR$.

5. Example: Enhancements in the extinction of helicity on scattering with a resonant particle, either chiral or not

To better illustrate these effects we address them at resonant wavelengths so that there is field enhancement on interaction with the particle, which in principle we consider generally magnetodielectric and chiral. We shall later relax the latter property. We have found a particle model with these characteristics in a recent work [34], and thus we consider it useful for our illustration. Its linear dimension is not larger than 204 nm (see details of this particle, made of a composite metal (silver) dielectric in vacuum, $n = 1$, in figure 3 of [34]). Both helicity dissipation, or conversion, $\mathcal{W}_{\epsilon}^{\Delta\epsilon}$, and energy absorption $\mathcal{W}_{\epsilon}^{\Delta\epsilon}$, may occur, as previously emphasized concerning the right sides of (7) and (8). However, as stated before, in this paper we are interested in the left sides of those two optical theorems, and hence on the extinctions $\mathcal{W}_{\epsilon}^{\Delta\epsilon}$ and $\mathcal{W}_{\epsilon}^{\Delta\epsilon}$ (equations (9) and (10), respectively).

The particle polarizabilities have a resonance near $\lambda = 1.52 \mu m$, as shown in figure 1, where we have fitted them from their numerical values obtained in [34] to functions of $\lambda$; this enabling us to straightforwardly employ them in equations (9) and (10). We choose $K = 0.6 k$, and set $l = 1$, $e_{0}^{+} = 1$, $e_{0}^{-} = i$, i.e. $\mathcal{R}(e_{0}^{+}e_{0}^{-}) = 0$, and hence $\mathcal{R}(E^{-} \cdot E^{+}) = 0$, which is the third term of (10) for $W^{\text{ext}}$. This allows us to enhance the value of $g$ even if the incident helicity density $\mathcal{H}(\mathbf{r}) = (e/2k)([E^{-}\mathbf{r}]^2 - [E^{+}\mathbf{r}]^2)$ is very small and the particle is achiral, as shown below.

At different wavelengths, the radial coordinate $R$ of the particle center in the beam’s transversal section is adjusted to two alternative values of $KR$. One is $KR \approx 3.75$, which is near the first out-of-axis zero of $J_{l}^2(KR) = J_{l}(KR)^2$. Thus, the

![Figure 1](https://example.com/fig1.png)
contribution of this factor to equations (9) and (10) through the first term in the right side of (18) is negligible. The other alternative value is $KR \approx 2.25$, which is close to the first zero of $J_{-1}(KR) = J_0(KR)$. Hence, by virtue of (19) the contribution of the $\mathcal{J}(E^- \cdot E^+)$ factor in (9) is negligible. This is like the situation of standard CD.

These values of $KR$ also give a hint on the range $R_0$ of approximate distances between minima of the beam intensity across its section versus the size of the particle. $R_0 \approx 3.75 \lambda/2\pi = 895 \text{ nm}$ for $\lambda \approx 1.5 \mu\text{m}$, which is well above the linear size of the particle, which, as said above, is no larger than 204 nm, and thus allows for enough spatial resolution of its position since this size is well below the width $R_0$ of the circles of intensity minima and maxima in the beam section (see also [33]).

Hence, at $KR = 2.25$ one has that $|E'|^2 + |E|^2 = 8\pi \mathcal{W}$ dominates over all other parameters since it is about 2.5 (a.u.), while $\mathcal{J}(E^- \cdot E^+) = -0.019$, $\Re[\mathcal{J}(E^- \cdot E^+)] = 0$, and $|E'|^2 - |E|^2 = 0.09$. On the other hand, for $KR = 3.75$ one sees that $|E'|^2 + |E|^2$ no longer dominates since it is about 0.2 (a.u.), while $\mathcal{J}(E^- \cdot E^+) = -0.1$, $|E'|^2 - |E|^2 = 0.008$, and $\Re[\mathcal{J}(E^- \cdot E^+)] = 0$. Therefore these two choices of $KR$ convey a very small incident helicity $\mathcal{H}$.

Figure 2 exhibits the spectra of the rate of helicity extinction $\mathcal{W}_{\mathcal{H}}$, energy extinction $\mathcal{W}_{E}$, (both scaled by $10^2/2\pi c$), and helicity extinction factor $g = \mathcal{W}_{\mathcal{H}}/\mathcal{W}_{E}$, for $KR = 3.75$ (upper graph) and $KR = 2.25$ (lower graph) for a chiral particle with polarizabilities as seen in figure 1. We also show the same scaled quantities, now denoted as $\mathcal{W}_{\mathcal{H}}$, $\mathcal{W}_{E}$, and $g_{\mathcal{H}}$, for an almost achiral particle ($\alpha_{\text{me}} \approx 0$), with the same polarizabilities $\alpha_e$ and $\alpha_m$ as the former chiral particle, but whose cross electric-magnetic polarizability $\alpha_{\text{me}}$ has been somewhat artificially scaled to $1/10$ of the chiral particle $\alpha_{\text{me}}$ value (we choose the letter $\chi$ in the subscript from the Greek $\chi$ for `hand`).

As seen from equation (9) and figure 2 (above), for $KR = 3.75$, even when the particle is achiral and the incident helicity density is locally zero, namely at points $R$ fulfilling $KR = 3.75$, we confirm that the interference factor $\mathcal{J}(E^- \cdot E^+)$ may be essential to yield an appreciable helicity extinction rate $\mathcal{W}_{\mathcal{H}}$, and a resonant helicity extinction factor $|g_{\mathcal{H}}| > 1$ ($g_{\mathcal{H}} = -1.15$ at $\lambda \approx 1.53 \mu\text{m}$ in this illustration). This is one of the main results of this work, and is in contrast with standard CD in which $\mathcal{J}(\mathbf{E} \cdot \mathbf{E}^*) = 0$, and objects with zero or a purely imaginary $\alpha_{\text{me}}$, with no selective helicity dissipation, would produce no helicity extinction and therefore a zero value of $g$ in absence of incident chirality density.

In this respect we remark that the appreciable helicity extinction factors $g$ and $g_{\mathcal{H}}$ observed in figure 2 (above), may also be influenced by shifts (which depend on the particle morphology) between the resonant peaks of the helicity and energy extinction rates.

The results of figure 2 (above) should be compared with those when the factor $\mathcal{J}(E^- \cdot E^+)$ is negligible and so is the third term of equation (9). These are plotted in figure 2 (below) for $KR = 2.25$, showing that $\mathcal{W}_{\mathcal{H}}$ is extremely small compared to $\mathcal{W}_{E}$, and hence $g_{\mathcal{H}}$ is almost zero ($|g_{\mathcal{H}}| \leq 0.05$). This is in contrast with the larger values of $\mathcal{W}_{\mathcal{H}}$ and $g$ shown in figure 2 (below) for this $KR = 2.25$ when the particle is chiral, i.e. $\Re[\mathcal{J}(\alpha_{\text{me}})] = 0$, and hence in equation (9) the second term contributes, leading to significantly larger peaks of these quantities ($g \approx -1.5$ at $\lambda \approx 1.525 \mu\text{m}$) as in standard dichroism.

On the other hand, both figures 2 (above and below) show that at a chosen value of $KR$, $\mathcal{W}_{\mathcal{H}}$ and $\mathcal{W}_{E}$ coincide with each other, i.e. at a given position of the particle within the beam, $\mathcal{W}_{\mathcal{H}}$ is not affected by the value of $\alpha_{\text{me}}$. This is due to the above shown almost negligible $|E'|^2 - |E|^2$, and hence small $\mathcal{H}$, for these chosen $KR$. Nonetheless when $KR = 2.25$, $\mathcal{W}_{\mathcal{H}}$, $\mathcal{W}_{E}$, and $\mathcal{W}_{\mathcal{H}}$ considerably increase through the factor $\mathcal{W}$ in (9) and (10). This is expected from the discussion in sections 4 and 5 above, since when $KR = 2.25$ the factor $|E'|^2 + |E|^2 = 8\pi \mathcal{W}$ dominates over all other factors in equations (9) and (10), while
\( \Re(\mathcal{R}\{E \cdot E^*\}) \) remains very small, again because the factor \((\kappa^2/k^2)_{J_{l-1}(KR)J_{l+1}(KR)} \) in (19) is much smaller than the first term proportional to \( J_2^* (KR) \) in (18) when \( KR = 2.25 \). Namely, we stress that in (9) and (10) \( \Re(\mathcal{R}\{E \cdot E^*\}) \) (equation (19)) can be either comparable to or much smaller than \( \mathcal{V} \) (equation (18)) according to the value of \( KR \).

Another consequence of this latter discussion is that the relative values of \( g/\mathcal{V}^{\text{ext}} \) and \( g_{\alpha m}/\mathcal{V}^{\text{ext}} \) are significantly larger in figure 2 (above) than in figure 2 (below). This once again highlights the relevance of the interference term with factor \( \Re(\mathcal{R}\{E \cdot E^*\}) \) in (9) and (10) when the terms proportional to \( \mathcal{V} \) do not dominate.

Finally, it should be recalled that, as stated in section 3, \( \Re(\alpha_{e}) \) and \( n^2 \Re(\alpha_{m}) \) appear subtracted from each other in the last term of (9). Consequently, and although not shown here for brevity, we observe that the amplitude of the peaks of both \( \mathcal{V}^{\text{ext}}_H \) and \( \mathcal{V}^{\text{ext}}_U \) increases when either \( \Re(\alpha_{m}) \) or \( \Re(\alpha_{e}) \) diminishes. Something analogous occurs with the extinction of energy (10) regarding the imaginary parts of the polarizabilities.

6. Conclusions

The concept of CD has been extended by addressing the rate of extinction of helicity \( \mathcal{V}^{\text{ext}}_H \), whose extinction factor \( g \) has been introduced and generalizes the standard CD as a symmetry factor. The parameter \( g \) monitors the rate of helicity extinction versus that of energy under different values of the polarizabilities of a generally magnetodielectric particle, either chiral or not, (i.e. for the cross electric-magnetic one \( \alpha_{m} \) ranging from large to almost zero). It also considers the local value of the incident helicity density \( \mathcal{H} \). Thus, both \( \mathcal{V}^{\text{ext}}_H \) and \( g \) assess the contribution of the remarkable interference factor \( \Im(\mathcal{E} \cdot E^*) \) to such helicity extinction in comparison with that of \( \alpha_{m} \), \( \mathcal{H} \), and the resonances of the polarizabilities that we addressed in this study in order to enhance these effects. Notice in passing that an analogous analysis may be made with the factor \( \Re(\mathcal{E} \cdot E^*) \) versus \( \mathcal{V} \), \( \alpha_{m} \), and \( \mathcal{H} \), regarding its contribution to the energy extinction rate \( \mathcal{V}^{\text{ext}} \).

When the incident fields are optical beams with LCP and RCP transversal components, the factor \( \{\mathcal{E} \cdot E^*\} \) reduces to that of interference of the longitudinal components. We have illustrated this with a Bessel beam. Interestingly, due to this interference, helicity extinction does not necessarily involve particle chirality or a non-zero local value of the incident helicity density, i.e. for \( \alpha_{m} = 0 \) and given parameters of the illuminating beam, one may find positions \( r_0 \) of the particle in the beam where this local helicity density is \( \mathcal{H}(r_0) = 0 \) while the aforementioned interference term gives rise to a non-zero extinction of helicity \( \mathcal{V}^{\text{ext}}_H \). Also, and importantly, this interference phenomenon is mediated by \( \Re(\alpha_{e}) \) and \( \Re(\alpha_{m}) \), thus yielding a source of information on these latter quantities, which was not provided by standard CD.

Finally, although we have studied these phenomena in general bi-isotropic dipolar particles, namely those magnetodielectric, whether chiral or not, the contribution of the \( 2\Re(\alpha_{e} - n^2\alpha_{m})\) \( \Im(\mathcal{E} \cdot E^*) \) term to an extinction of incident helicity (equations (7) and (9)), as well as the effect of the \( 2\Re(\alpha_{e} - n^2\alpha_{m})\) \( \Re(\mathcal{E} \cdot E^*) \) term to an extinction of incident energy (equations (8) and (10)), may also be observed in purely electric \( (\alpha_{m} = 0) \) or magnetic \( (\alpha_{e} = 0) \) particles. In this context, it will be of special importance to conduct further research and observations of these effects in high index dielectric particles that possess remarkably unique optically induced electric and magnetic dipole resonances [35, 36], and that so much interest is generating as low-loss elements of an increasingly active new area of micro and nano-optics [37, 38].

Acknowledgments

This work was supported by MINECO-FEDER, grants FIS2012-36113-C03-03, FIS2014-55563-REDIC, and FIS2015-69295-C3-1-P. The author thanks an anonymous referee for many interesting comments that contributed to the improvement of this report.

References

[1] Schellman J A 1975 Circular dichroism and optical rotation Chem. Rev. 75 323–31
[2] Craig D P and Thirumamachandran T 1998 Molecular Quantum Electro dynamics: An Introduction to Radiation Molecule Interactions (New York: Dover)
[3] Barron L D 2004 Molecular Light Scattering and Optical Activity (Cambridge: Cambridge University Press)
[4] Menzel C, Helgert C, Rockstuhl C, Kley E B, Tünnermann A, Pertsch T and Lederer F 2010 Asymmetric transmission of linearly polarized light at optical metamaterials Phys. Rev. Lett. 104 253902
[5] Govorov A O, Fan Z, Hernandez P, Slocik J M and Naik R R 2010 Theory of circular dichroism of nanomaterials comprising chiral molecules and nanocrystals: plasmon enhancement, dipole interactions, and dielectric effects Nano Lett. 10 1374–82
[6] Zambrana-Puyalto X, Vidal X and Molina-Terriza G 2014 Angular momentum-induced circular dichroism in non-chiral nanostructures Nat. Commun. 5 4922
[7] Hopkins B, Podubny A N, Miroshnichenko A E and Kivshar Y S 2016 Circular dichroism induced by Fano resonances in planar chiral oligomers Laser Photonic Rev. 10 137–43
[8] Nieto-Vesperinas M 2016 Circular optical fields: a unified formulation of helicity scattered from particles and dichroism enhancement Phil. Trans. Roy. Soc. A 375 20160331
[9] Tang Y and Cohen A E 2010 Optical chirality and its interaction with matter Phys. Rev. Lett. 104 163901
[10] Guzatov D V and Klimov V V 2012 The influence of chiral spherical particles on the radiation of optically active molecules New J. Phys. 14 123009
[11] Garcia- Etxari A and Dionne J A 2013 Surface-enhanced circular dichroism spectroscopy mediated by nonchiral nanoantennas Phys. Rev. B 87 235409
[12] Alaeian H and Dionne J A 2015 Controlling electric, magnetic, and chiral dipolar emission with PT-symmetric potentials Phys. Rev. B 91 245108
[13] Wang H, Li Z, Zhang H, Wang P and Wen S 2015 Giant local circular dichroism within an asymmetric plasmonic nanoparticle trimer Sci. Rep. 5 8207
[14] Lv T T, Li Y X, Ma H F, Zhu Z, Li Z P, Guan C Y, Shi J H, Zhang H and Cui T J 2016 Hybrid metamaterial switching for manipulating chirality based on VO2 phase transition Sci. Rep. 6 23086
[15] Hu L, Tian X, Huang Y, Wang X and Fang Y 2016 Quantitatively analyzing the mechanism of giant circular dichroism in extrinsic plasmonic chiral nanostructures by the interplay of electric and magnetic dipoles Nanoscale 8 3720–8
[16] Nieto-Vesperinas M 2015 Optical theorem for the conservation of electromagnetic helicity: significance for molecular energy transfer and enantiomeric discrimination by circular dichroism Phys. Rev. A 92 023813
[17] Lipkin D M 1964 Existence of a new conservation law in electromagnetic theory J. Math. Phys. 5 696–700
[18] Cameron R P, Barnett S M and Yao A M 2012 Optical helicity, optical spin and related quantities in electromagnetic theory New. J. Phys. 14 053050
[19] Bliokh K Y and Nori F 2011 Characterizing optical chirality Phys. Rev. A 83 021805(R)
[20] Poulakakos L V, Gutsche P, McPeak K M, Burger S, Niegemann J, Hafner C and Norris D J 2016 Optical chirality flux as a useful far-field probe of chiral near fields ACS Photonics 3 1619–25
[21] Gutsche P, Poulakakos L V, Hammerschmidt M, Burger S and Schmidt F 2016 Time-harmonic optical chirality in inhomogeneous space Proc. SPIE 9756 97560X
[22] Fernandez-Corbaton I and Rockstuhl C 2016 A unified theory to describe and engineer conservation laws in light–matter interactions arXiv:1611.01007v1
[23] Coles M M and Andrews D L 2012 Chirality and angular momentum in optical radiation Phys. Rev. A 85 063810
[24] Bradshaw D S, Leeder J M, Coles M M and Andrews D L 2015 Signatures of material and optical chirality: origins and measures Chem. Phys. Lett. 626 106–10
[25] Barnett S M, Cameron R P and Yao A M 2012 Duplex symmetry and its relation to the conservation of optical helicity Phys. Rev. A 86 013845
[26] Barnett S M and Allen L 1994 Orbital angular momentum and nonparaxial light beams Opt. Commun. 110 670–8
[27] Allen L, Padgett M J and Babiker M 1999 The orbital angular momentum of light (Prog. Opt. vol 39) ed E Wolf (Amsterdam: Elsevier) pp 291–372
[28] Sersic I, van de Haar M A, Arango F B and Koenderink A F 2012 Ubiquity of optical activity in planar metamaterial scatterers Phys. Rev. Lett. 108 223903
[29] Nieto-Vesperinas M 2015 Optical torque on small bi-isotropic particles Opt. Lett. 40 3021–4
[30] Bohren C F and Huffman D R 1983 Absorption and Scattering of Light by Small Particles (New York: Wiley)
[31] Jackson J D 1998 Classical Electrodynamics 3rd edn (New York: Wiley)
[32] Choi J S and Cho M 2012 Limitations of a superchiral field Phys. Rev. A 86 063834
[33] Volle-Sepulveda K, Garces-Chavez V, Chavez-Cerda S, Arlt J and Dholakia K 2002 Orbital angular momentum of a high-order Bessel light beam J. Opt. B: Quantum Semiclassical Opt. 4 S82–9
[34] Fernandes D E and Silveirinha M G 2016 Single beam optical conveyor belt for chiral particles Phys. Rev. Appl. 6 014016
[35] Evlyukhin A B, Reinhardt C, Seidel A, Lukyanchuk B S and Chichkov B N 2010 Optical response features of Si-nanoparticle arrays Phys. Rev. B 82 045450
[36] Garcia-Etxarri A, Gomez-Medina R, Froufe-Perez L S, Lopez C, Chantada L, Scheffold F, Aizpurua J, Nieto-Vesperinas M and Saenz J J 2011 Strong magnetic response of submicron silicon particles in the infrared Opt. Express 19 4815–26
[37] Decker M and Staupe I 2016 Resonant dielectric nanostructures: a low-loss platform for functional nanophotonics J. Opt. 18 103001
[38] Kuznetsov A I, Miroshnichenko A E, Brongersma M L, Kivshar Y S and Lukyanchuk B 2016 Optically resonant dielectric nanostructures Science 354 846