Diffusion of neutrons in the toroidal nuclear electrogenerator

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Abstract. Diffusion of neutrons in the toroidal nuclear electrogenerator (nuclegen) is investigated. Important conclusion about the practically complete absence of the diffusion of neutrons and charged fission particles through the external boundary under toroidal motion is substantiated by the solution of corresponding equation diffusion with the aid of Fourier’s standard division method.

1. Introduction
It is well known, the process of formation of free neutrons inside a fission medium (thereby of formation of charged fission particles) for realization and maintenance of nuclear chain reaction is a decisive significance. Therefore the question about diffusion of neutrons and charged particles flex over the whole volume of external magnetic reflecting field which created by toroidal nuclear electrogenerator (nuclegen) is presented sufficiently important.

At critical stationary generator the coefficient of neutrons reproduction $k$ from generation to generation isn’t change ($k = 1$). Hence it follows that for any point of active zone we may write the equation of neutrons balance

$$ F - L - A - R = 0, \quad (1) $$

where $F$ is the "formation", $L$ is the "leakage", $A$ is the "absorption" and $R$ is the "reflection". Here through $R$ in relation (1) is indicated the external magnetic pressure is making in respect to the neutron flex the reflected and restored function and reflected effect of itself toroidal surface also.

2. Diffusion problem and its solution by Furier’s method
The main equation for diffusion effects (in this case neutrons diffusion) is the Fick’s law [1], [2], which establishes that resulting current (flex) of diffusive substance is proportional to the gradient of this substance and directed to the side of scope with the least density

$$ J = -D \nabla \Phi, \quad (2) $$

where $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the operator of dimensional gradient, $J$ is the resulting flex of neutrons, $\Phi = nv$ is the scalar neutron flex, $D$ is the coefficient of diffusion determined by
Here \( n \) is the number of neutrons in the unit of volume (the dimensional density), \( v \) is the average neutrons density.

Denote the leakage \( L \) in equation (1) through \( L \) too. The value \( L \) is express itself in the equality

\[
L = -D \Delta \Phi,
\]

where \( \Delta = (\partial^2 / \partial x^2, \partial^2 / \partial y^2, \partial^2 / \partial z^2) \) is the Laplace operator. If to count up that in equation (1) "formation", "absorption", "reflection" each separately are proportional to the flex \( \Phi \) then taking into account relations (2), (3) equation (1) can be written for some coefficient of proportion \( R \) in the form [3], [4]:

\[
D \Delta \Phi + R \Phi = 0,
\]

or

\[
\Delta \Phi + a^2 \Phi = 0, \quad \Phi |_{\Gamma} \in \Omega,
\]

where \( a^2 = R/D, \) \( \Phi |_{\Gamma} \in \Omega \) is the boundary condition on the \( \Gamma \) of some scope \( \Omega \) concerning the function \( \Phi \). Solve equation (4) with given boundary condition of scope \( \Omega \) into supposition that the flex \( \Phi \) is twice continuously differentiable on dimensional variables. With that end in view we introduce cylindrical coordinates \((r, \varphi, z)\). Have for the arbitrary function \( F(r, \varphi, z) \):

\[
\Delta \Phi = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial F}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 F}{\partial \varphi^2} + \frac{\partial^2 F}{\partial z^2}.
\]

Owing to the symmetry of toroid the flex doesn’t depend on angular coordinate \( \varphi \) and the diffusive problem is becoming twodimensional: \( \Phi = \Phi(r, z), \) i.e.

\[
\frac{\partial^2 \Phi(r, z)}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi(r, z)}{\partial r} + \frac{\partial^2 \Phi(r, z)}{\partial z^2} + a^2 \Phi(r, z) = 0,
\]

where \( r_1 \leq r \leq r_2, -r_0 \leq z \leq r_0, r_0 = (r_2 - r_1)/2. \) Make use of the Furier’s method variables division for the solving of equation (5) supposed that the flex \( \Phi(r, z) \) is presented in the form of two functions product as

\[
\Phi(r, z) = R(r) Z(z).
\]

Bring expression (6) in equation of diffusion (5). After termwise division on \( R(r) Z(z) \) we receive the equation

\[
\frac{1}{R(r)} \left[ \frac{d^2 R(r)}{dr^2} + \frac{1}{r} \frac{d R(r)}{dr} \right] + a^2 = -\frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2}.
\]

The left part of equation (7) depends only from \( r \) and the right — only from \( z \). Function (6) can be the solution of equation (7) as soon as in the case if both parts of this equation are constant by the arbitrary changing of variables \( r \) and \( z \), i.e. must be carried out the following relations

\[
\frac{1}{R(r)} \left[ \frac{d^2 R(r)}{dr^2} + \frac{1}{r} \frac{d R(r)}{dr} \right] = -a^2_r,
\]

\[
\frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} = -a^2_z,
\]

where \( a^2_r, a^2_z \) are positive numbers connected by equality \( a^2 = a^2_r + a^2_z \) owing to equation (7).
3. Solution of equation on \( R(r) \)
Consider equation (8). At first we termwise its over \( r^2 R(r) \):
\[
r^2 \frac{d^2 R(r)}{dr^2} + r \frac{dR(r)}{dr} + r^2 R(r) a_r^2 = 0,
\]
and then introduce the new variable \( \rho = ra_r \). As a result we obtain the Bessel’s equation of zero order
\[
\rho^2 \frac{d^2 R(\rho)}{d\rho^2} + \rho \frac{dR(\rho)}{d\rho} + \rho^2 R(\rho) = 0.
\]
(10)
The general solution of equation (10) is presented in the form
\[
R(\rho) = C_1 J_0(\rho) + C_2 Y_0(\rho),
\]
where \( C_1, C_2 \) are the arbitrary constants, \( J_0(\rho) \), \( Y_0(\rho) \) are the Bessel’s functions of zero order first and second types determined with the help of power serieses
\[
J_0(\rho) = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k \rho^{2k}}{(k!)^2 2^{2k}},
\]
\[
Y_0(\rho) = \lim_{n \to 0} J_n(\rho) \cos n\pi - J_{-n}(\rho) \sin n\pi.
\]
Here \( J_{-n}(\rho) = (-1)^n J_n(\rho) \), \( J_n(\rho) \) is the Bessel’s function of \( n \)-th order with the constant \( a^{(n)}(\rho) \):
\[
J_n(\rho) = a^{(n)}(\rho) \rho^n \sum_{k=0}^{\infty} \frac{(-1)^k \rho^{2k}}{k!(n+1)(n+2) \cdots (n+k) 2^{2k}},
\]
\[
a^{(n)}(\rho) = \frac{1}{2^n \Gamma(n+1)}, \quad \Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx,
\]
where \( \Gamma(n) \) is the gamma-function of Euler. Note that by small \( \rho \)
\[
Y_0(\rho) \approx -\frac{2}{\pi} \ln \left( \frac{2}{\gamma\rho} \right),
\]
where \( \ln \gamma = C = 0.5772... \) is the Euler’s constant. Sometimes the function \( Y(\rho) \) is called the Neumann’s (or the Weber’s) function of zero order. If come back to the variable \( r \) then we can write as a result
\[
R(r) = C_1 J_0(a_r r) + C_2 Y_0(a_r r).
\]
(11)

4. Solution of equation on \( Z(z) \) and the general solution
Consider now equation (9). Its solution is determined easily
\[
Z(z) = C_3 \sin a_z z + C_4 \cos a_z z,
\]
(12)
where \( C_3, C_4 \) are the arbitrary constants. Take from conditions of symmetry
\[
Z = Z_0, \quad \frac{dZ(z)}{dz} \bigg|_{z=0} = 0.
\]
(13)
Obvisously then in solution (12) by conditions (13) we shall behaving \( C_3 = 0, C_4 = Z_0 \) and consequently \( Z(z) = Z_0 \cos a_z z \).
Multiply these two solutions: (11) and $Z(z)$. As a result we obtain the expression for flex (6):

$$\Phi(r, z) = \left[ C_1 J_0(a_r r) + C_2 Y_0(a_r r) \right] Z_0 \cos a_z z.$$  

Solution (14) must satisfy the main boundary condition:

$$\Phi(r, z) = 0, \quad \forall r, z \in T_\delta,$$

where

$$T_\delta = \{ r, z : \left[ r - (r_1 + r_2)/2 \right]^2 + z^2 = (r_0 + \delta)^2, \ \varphi : 0 \leq \varphi \leq 2\pi \},$$

$T_\delta$ is a $\delta$-toroid, and axial conditions of symmetry

$$\left. \frac{\partial \Phi(r, z)}{\partial r} \right|_{r=r_1+r_0} = 0, \quad \left. \frac{\partial \Phi(r, z)}{\partial z} \right|_{z=0} = 0.$$

These conditions answer the neutron flex $\Phi = (\Phi_r, \Phi_z)$ for which $\Phi_r = 0, \Phi_z = 0$. The starting equation of diffusion (4) with boundary $(\ast)$ and axial $(\ast\ast)$ conditions answer the flex $\Phi = (\Phi_r, \Phi_z, \Phi_\varphi)$, where

$$\Phi_z = 0, \quad \Phi_z = 0, \quad \Phi_\varphi = \Phi_0 = \text{const}.$$  

Parameter $\delta$ attends in condition $(\ast)$. It’s named the length of extrapolation. Neutron flex doesn’t turn into zero on the boundary as far as always there is (although insignificant) the leakage across external boundary. It means that in immediate nearness from the boundary inside of active zone must be neutrons which leave its limits. On the boundary part of neutrons leaves outside but another part diffuses inside and raises the significance of density for neutron flex. In consequence of this neutron flex on boundary be not equal zero but is supposed be equal zero from the outside on some little distance $\delta$.

5. Conclusions

Flex (15) corresponds to the absence of the leakage in stationary ($k = 1$) nuclear electrogenerator, the availability of zero parameter ($a = 0$) in equation (4) and the carrying-out of neutron balance equation: $F - A - R = 0$. The fulfillment of this condition means the important peculiarity and advantage of toroidal vacuum-nuclear systems, expressed into practically complete (without losses) circular motion of nuclear flexes inside of the toroidal active zone of nuclegen.

References

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