Adaptive Interval Type-2 Fuzzy Tracking Control of PV Grid-Connected Inverters

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ABSTRACT Fuzzy logic systems with approximation capabilities provide effective control for nonlinear and uncertain systems. Due to the characteristics of photovoltaic (PV) and the PWM method, a grid-connected PV system is a considerably nonlinear system with unpredictable parameters. In this study, a new adaptive interval type-2 fuzzy approximation-based controller (AIT2FAC) was developed to control a three-phase grid-connected PV system. The proposed controller can be implemented without any prior knowledge of the system mathematical model. In the presence of both parametric and modeling uncertainty, the developed controller can achieve the control objectives. The proposed controller utilizes the principle of input-output feedback linearization and the approximation capability of fuzzy systems to track prescribed reference values. The proposed AIT2FAC controller is capable of handling system uncertainties due to the interval type-2 fuzzy logic system capability to cope with a high level of uncertainty. Lyapunov analysis is used to determine the closed-loop system stability and the updating laws of the proposed controller parameters. The effectiveness of the designed controller to achieve the required tracking is validated for different operating cases, including system disturbances, modelling, and parameter uncertainties. For evaluation, the proposed type-2 fuzzy controller is compared to a type-1 fuzzy controller in terms of some performance measures. The comparison results demonstrate that the proposed type-2 fuzzy controller has better tracking performance than the type-1 fuzzy controller in terms of the settling time, the maximum overshoot, the integral absolute error (IAE) and the integral time of absolute error (ITAE).

INDEX TERMS Interval type-2 fuzzy, adaptive control, feedback linearization, PV grid inverter.

I. INTRODUCTION

Developing effective controllers for nonlinear systems with uncertain parameters is not an easy task. When dealing with unknown nonlinear systems, an adaptive approximation-based control technique is a sensible solution. To control uncertain nonlinear systems, adaptive fuzzy control (AFC) approaches based on the approximation property of fuzzy logic systems have been proposed [1]–[4] and applied to many power systems and drive applications [5]–[9]. Researchers initially focused on type-1 fuzzy logic systems (T1FLSs) and their applications. The T1FLSs have strong control capabilities in different applications where they prove very useful in integrating uncertainty in situations where membership functions (MFs) can be calculated exactly by a single numerical value for a fuzzy group. However, there are more challenging cases where it is extremely hard to find the exact numerical value of the MFs for unknown systems. Generating the fuzzy rule base that is used in fuzzy systems is a difficult task since the information needed is vague. Therefore, the creation of unsuitable fuzzy rules may lead to confusion in MFs as well. Then, we can reasonably say that, regardless of the effectiveness of T1FLSs in many applications, there are other situations where they would not be enough to reach the required degree of accuracy or efficiency. To overcome this drawback, the type-2 fuzzy logic system (T2FLS) was first introduced in 1975 by Zadeh [10] and then type-2 fuzzy logic controllers (T2FLCs) were introduced by Karnik and Mendel [11], Karnik et al. [12], and Liang and Mendel [13].
The capability of the T1FLS to cope with a high level of uncertainty is limited since the degrees of MFs are crisp. The MFs used in T2FLS are a combination of two T1FLS MFs: the upper MF (UMF) and the lower MF (LMF), as shown in Figure 1 [14]. Hence, the T2FLS MF becomes three-dimensional (3D) with an extra degree of freedom to take care of uncertainties in the controlled system. The third dimension is called “Footprint of Uncertainty” (FOU) and is the function value at each point in a two-dimensional space. As shown in Figure 1, the MF can have more than one response to the same input variable [15]. T2FLS without uncertainties performs as T1FLS. The grades of the MF of the type-2 fuzzy set are also fuzzy in themselves, so the type-2 fuzzy set improves the fuzziness that helps the system handle vague data in a relatively accurate way [14].

FIGURE 1. T2FLS membership functions.

T2FLS has been utilized in a wide range of applications and has achieved remarkable success in managing higher levels of uncertainty [16]. Those applications include image processing, pattern recognition [17], and vehicle classification [18]. Moreover, T2FLS has also proven successful in clinical diagnosis, differential diagnosis, and nursing evaluation in the health field [19].

The T2FLC has found its way into different applications [20]–[28], to name a few. The implementation of T2FLC to control a DC-DC buck converter is presented, and the obtained results indicate that the T2FLC offers better performance than the T1FLC. For photovoltaic systems, the interval T2FLC (IT2FLC) based on the maximum power point tracking (MPPT) method is proposed in [21]. In order to handle the rules’ uncertainties during high weather conditions and variations, the work in [22] proposes to implement the MPPT using a T2FLC. The IT2FLC is used as the MPPT algorithm [23] for a single-phase grid-connected PV system. In the field of hybrid electric vehicles, the T2FLC was proposed [24] to handle energy management efficiently. The direct and indirect adaptive T2FLC for nonlinear systems was introduced in [25], [26] to handle unpredictable internal disturbances and data uncertainties. In another work [14], a new type-2 fuzzy-PID interval controller with a low pass filter (T2FPIDF) was introduced for the automatic generation control (AGC) of interconnected power systems. Type-2 fuzzy neural network (T2FNN) is presented in [29], [30], where in [29] proportional-derivative (PD) T2FNN is proposed to control DC-link voltage of a three-phase AC-DC PWM rectifier. The findings show that the proposed controller outperforms traditional PD and PI controllers.

An Adaptive fuzzy controller for grid connected inverter systems using type-1 fuzzy is presented in [31], [32]. The authors believe that the tracking performance of the controller developed in [31] can be significantly improved by employing IT2FLS. Moreover, to the best of the authors’ knowledge, there is no published work that describes the application of the adaptive fuzzy controller using type-2 fuzzy to the grid connected inverter system. In this paper, an adaptive interval type-2 fuzzy-based tracking control (AIT2FAC) is proposed for a PV grid-connected three-phase inverter system to overcome the nonlinearity and uncertainty associated with such a system [31]. The PV grid-connected inverter systems are highly nonlinear uncertain systems. These nonlinearities and uncertainties can cause power quality issues, losses, output harmonics, and system implementation issues if they are not addressed with fast-acting inverter control. The type-2 fuzzy system is utilized to approximate the nonlinearities and compensate for the uncertainties inherited in the PV systems. The type-2 fuzzy controller is proposed in this paper to handle system uncertainty presented in [31] in a more accurate way. To evaluate the superiority of the proposed adaptive interval type 2 over the type-1 fuzzy logic system [31] controllers, a comparison study was conducted between both controllers.

The following is a summary of the paper’s contributions:

- The paper proposes an interval type-2 fuzzy-based tracking control for PV grid-connected inverters.
- The proposed controller’s implementation does not require prior knowledge of the system’s mathematical model. The tracking error, derivatives of the reference signal, and fuzzy approximations of unknown nonlinear functions are all that is needed.
- The proposed controller achieves excellent tracking efficiency under various operating conditions.
- The tracking performance of the proposed controller outperforms that of the type-1 fuzzy logic system controller [31].

The rest of the paper is organized as follows: Section II presents the IT2FLS. Section III discusses the proposed AIT2FAC for nonlinear systems. Section IV presents the design of the AIT2FAC for a PV grid-connected three-phase inverter. Simulation results along with a comparison with adaptive T1FLS are presented in section V. The conclusions are drawn in section VI.

II. INTERVAL TYPE-2 FUZZY LOGIC SYSTEMS

Due to the mathematical complexity of T2FLSs, a simpler form called interval type-2 fuzzy logic systems (IT2FLS) has been proposed [33]. A type-2 fuzzy set in the universal set X is defined as $\tilde{A}$, which is described by a type-2 MF $\mu_\tilde{A}(x)$ as in (1) [25], [33]:

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x) dx = \int_{x \in X} \left[ \int_{u \in J_x} [ f_x (u) / u] \right] dx \quad (1)$$

where $\mu_{\tilde{A}}(x)$ is described as a secondary MF and is a type-1 fuzzy set in [0, 1], and $f_x (u)$ is the amplitude of a secondary MF, where $0 \leq f_x (u) \leq 1$. The secondary MF domain is
called the primary membership of \( x \). In (1), \( J_x \) is the primary membership of \( x \) and \( u \) is a fuzzy set in \([0,1]\) rather than a crisp point in \([0,1]\), \( f_x(u) = 1 \), \( y \in J_y \subseteq [0,1] \), \( \mu_A(x) \), can be called an interval type-2 MF. Equation (1) can then be written as [25].

\[
\tilde{A} = \int_{x \in X} \mu_A(x) \, dx = \int_{x \in X} \left[ \int_{u \in J_y \subseteq [0,1]} 1/u \right] \, dx
\]

The type-2 fuzzy set is in a region known as a footprint of uncertainty (FOU), which is bounded by a UMF denoted as \( \overline{\mu}_A(x) \) and an LMF denoted as \( \underline{\mu}_A(x) \) [12]. Equation (2) can then be written as

\[
\tilde{A} = \int_{x \in X} \left[ \int_{u \in [\underline{\mu}_A(x), \overline{\mu}_A(x)]} 1/u \right] \, dx
\]

Like T1FLS, T2FLS contains a rule base, a fuzzifier, an inference engine, and an output processor, as shown in Figure 2 [33]. The inference engine combines rules to provide a mapping from input T2FLSs to T2FLSs as output. To complete this step, type-2 set unions and intersections must be computed. The output processor consists of a type reducer that produces a type-1 fuzzy output and a defuzzifier to obtain a single crisp output value [33].

\[
\tilde{r} = \overline{\mu}_{F_1}(x_1) * \cdots * \overline{\mu}_{F_p}(x_p)
\]

The Type-2 interval consequent set centroid is \( \tilde{y} \in Y_i \), and for any value \( y \in Y_{cos} \), \( y \) can be expressed as

\[
y = \frac{\sum_{i=1}^{M} y_i^i}{\sum_{i=1}^{M} f_i^i}
\]

Moreover, \( y_i \) is the minimum associated only with \( y_i^i \), and \( y_r \) is the maximum associated only with \( y_r^i \). Hence, \( y_l \) and \( y_r \) can be expressed as a fuzzy basis function (FBF) as [13], [25].

\[
y_l = \frac{\sum_{i=1}^{M} f_i^i y_i^i}{\sum_{i=1}^{M} f_i^i} = \sum_{i=1}^{M} \frac{y_i^i}{f_i^i} \hat{y}_l^i = y_l^T \hat{y}_l
\]

\[
y_r = \frac{\sum_{i=1}^{M} f_i^i y_i^i}{\sum_{i=1}^{M} f_i^i} = \sum_{i=1}^{M} y_r^i \hat{y}_r^i = y_r^T \hat{y}_r
\]

where, \( \hat{y}_l^i = \frac{f_i^i}{\sum_{i=1}^{M} f_i^i}, \hat{y}_r^i = \frac{f_i^i}{\sum_{i=1}^{M} f_i^i}, \hat{y}_l = [\hat{y}_l^1, \ldots, \hat{y}_l^M], \hat{y}_r = [\hat{y}_r^1, \ldots, \hat{y}_r^M], y_l^T = [y_l^1, \ldots, y_l^M] \) and \( y_r^T = [y_r^1, \ldots, y_r^M] \).

Finally, the defuzzified crisp value from an IT2FLS is obtained as [25]

\[
y(x) = \frac{y_l + y_r}{2} = \xi^T \Theta
\]

where, \( \xi^T = \frac{1}{2} [\xi_l^T \xi_r^T], \Theta^T = [\Theta_l^T \Theta_r^T], \Theta_l^T = [y_l^T y_r^T] \) and \( \Theta_r^T = [y_r^T y_l^T] \).

**III. ADAPTIVE INTERVAL TYPE-2 FUZZY APPROXIMATION CONTROLLER DESIGN**

Consider the multi-input multi-output nonlinear (MIMO) system given by

\[
\dot{x} = f(x) + g(x) u
\]

where \( f(x) \) and \( g(x) \) are the system’s unknown nonlinear functions. The following are the main assumptions used throughout the paper.

**Assumption 1:** System (11) is a MIMO system with an equal number of inputs and outputs.

**Assumption 2:** The state vectors are measurable.

**Assumption 3:** The reference tracking signals \( y_{ref1}, \ldots, y_{refp} \) and their derivatives up to order \( r_p \) are smooth and bounded.

**Assumption 4:** The system matrix \( \beta(x) \) is nonsingular.

For a general MIMO nonlinear system, the input-output feedback linearizing control can be written as

\[
u = \beta^{-1}(x) [y^{(r)} - \alpha(x)]
\]

where \( y^{(r)} = [y_1^{(r)} \ldots y_p^{(r)}]^T \), \( u = [u_1 \cdots u_p]^T \), \( r_1, r_2, \ldots, r_p \) are the relative degrees of the outputs \( y_1, y_2, \ldots, y_p \) respectively, \( p \) is the number of both inputs and outputs, \( \alpha(x) \in R^{p \times 1} \) and \( \beta(x) \in R^{p \times p} \) [31].

To apply linearizing control in (12), the accurate values of the system functions \( \alpha(x) \) and \( \beta(x) \) must be accurately known. However, the parameters of the nonlinear MIMO systems may be unknown or inaccurate in practice. In this paper,
the IT2FLS is utilized to approximate the nonlinear functions \( \alpha_i(x) \) and \( \beta_{ij}(x) \), \( i = 1, 2, \ldots p \) and \( j = 1, 2, \ldots p \), where the FBF expansion \( \xi(x) \) generated using IT2FLS is utilized to construct the estimated functions \( \hat{\alpha}_i(x) \) and \( \hat{\beta}_{ij}(x) \) as

\[
\hat{\alpha}_i(x) = \theta_i^T \xi(x) \\
\hat{\beta}_{ij}(x) = \theta_{ij}^T \xi(x)
\]

where \( \theta_i, \theta_{ij} \) represent vectors of adjustable parameters.

By replacing \( \alpha(x) \) and \( \beta(x) \) in the input-output linearizing control for a general MIMO nonlinear system with their corresponding fuzzy estimates, we obtain

\[
y^{(r)} = \hat{\alpha}(x) + \hat{\beta}(x) u
\]

where

\[
\hat{\alpha}(x) = \begin{bmatrix} \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_p \end{bmatrix} \quad \text{and} \quad \hat{\beta}(x) = \begin{bmatrix} \hat{\beta}_{11} & \cdots & \hat{\beta}_{1p} \\ \vdots & \ddots & \vdots \\ \hat{\beta}_{p1} & \cdots & \hat{\beta}_{pp} \end{bmatrix}.
\]

The linearizing control law then can be formed in terms of fuzzy approximations \( \hat{\alpha}(x) \) and \( \hat{\beta}(x) \) as

\[
u = \hat{\beta}^{-1}(x)(v - \hat{\alpha}(x))
\]

and,

\[
y^{(r)} = \hat{\alpha}(x) + \hat{\beta}(x) u
\]

where

\[
y^{(r)} = \begin{bmatrix} y^{(r1)} \\ \vdots \\ y^{(rp)} \end{bmatrix}, \quad K = \text{diag} \left[ k_1, k_2, \ldots, k_p \right],
\]

\[
e = \begin{bmatrix} e_1 e_2 \cdots e_p \end{bmatrix}^{T}, \quad k_j = [k_{0j} k_{1j} \cdots k_{(r_j-1)j}] \quad \text{and} \quad e_i = [e_{i1} e_{i2} \cdots e_{i(r_i-1)}] \quad \text{and} \quad e_i = y^{(ri)} - y_i.
\]

A. CLOSED-LOOP STABILITY

Using (16) and (17) the error equation can be formed as

\[
\begin{bmatrix}
e^{(r)} \\
\vdots \\
e_p^{(r)}
\end{bmatrix} = -K e + (\hat{\alpha}(x) - \alpha(x)) + (\hat{\beta}(x) - \beta(x)) u
\]

The \( p \)th output error equation can be expressed as

\[
e^{(r)}_p = -k_p e_p + \Delta \alpha_i(x) + \sum_{j=1}^{p} \Delta \beta_{ij}(x) u_j
\]

where \( \Delta \alpha_i(x) = \hat{\alpha}_i(x) - \alpha_i(x) \) and \( \Delta \beta_{ij}(x) = \hat{\beta}_{ij}(x) - \beta_{ij}(x) \) are the fuzzy approximation errors. In state form, (19) becomes

\[
\dot{e} = A_i e + [\Delta \alpha_i(x) + \sum_{j=1}^{p} \Delta \beta_{ij}(x) u_j] b_i
\]

where,

\[
A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{(r_i-1)j} & k_{(r_i-2)j} & \cdots & \cdots & -k_{0j} \end{bmatrix}
\]

In terms of the optimal values of the adjustable parameters \( \theta_i^* \) and \( \theta_{ij}^* \), the minimum fuzzy approximation error \( w_i \) [1] is

\[
w_i = [\hat{\alpha}_i(x|\theta_i^* - \alpha_i(x))] + \sum_{j=1}^{p} \left[ \hat{\beta}_{ij}(x|\theta_{ij}^* - \beta_{ij}(x)) \right] u_j
\]

Then (20) becomes

\[
\dot{e} = A_i e + [w_i + \varphi_{ai}^T \xi(x)] + \sum_{j=1}^{p} \varphi_{aij}^T \xi(x) u_j
\]

where \( \varphi_{ai} = (\theta_i - \theta_i^*) \) and \( \varphi_{aij} = (\theta_{ij} - \theta_{ij}^*) \) are the parameter errors, and the derivatives of these parameter errors are given by \( \dot{\varphi}_{ai} = \dot{\theta}_i \) and \( \dot{\varphi}_{aij} = \dot{\theta}_{ij} \) [31].

To study the closed-loop stability and find the updating laws of the adjustable parameters \( \theta_i \) and \( \theta_{ij} \), the next positive Lyapunov function is formed as a quadratic function of the relevant errors, mainly the tracking error (20) and the parameter errors \( \varphi_{ai} \) and \( \varphi_{aij} \)

\[
V_i = \frac{1}{2} e_i^T P_i e_i + \frac{1}{2} \gamma_i \varphi_{ai}^T \varphi_{ai} + \sum_{j=1}^{p} \frac{1}{2} \gamma_{aij} \varphi_{aij}^T \varphi_{aij}
\]

where \( P_i \) is a unique positive definite solution of the Lyapunov equation (24) with an arbitrary positive definite matrix \( Q_i \) and \( \gamma_i \) and \( \gamma_{aij} \) are design parameters.

\[
A_i^T P_i + P_i A_i = -Q_i
\]

The time derivative of (23) can be found as

\[
\dot{V}_i = -\frac{1}{2} e_i^T Q_i e_i + \frac{1}{\gamma_i} \varphi_{ai}^T \dot{\varphi}_{ai} + \frac{1}{2} \gamma_i \varphi_{ai}^T P_i e_i + \frac{1}{2} \gamma_{aij} \varphi_{aij}^T \dot{\varphi}_{aij} + e_i^T P_i b_i \dot{w}_i + e_i^T P_i b_i \dot{w}_i
\]

by choosing the parameters’ updating laws as

\[
\dot{\theta}_i = -\gamma_i e_i^T P_i b_i \xi(x)
\]

\[
\dot{\theta}_{ij} = -\gamma_{aij} e_i^T P_i b_i \xi(x) u_j
\]

Then, substituting the parameters’ updating laws (25) becomes

\[
\dot{V}_i = -\frac{1}{2} e_i^T Q_i e_i + e_i^T P_i b_i \dot{w}_i
\]

The closed-loop stability can be achieved using (28). This can be shown as follows: Assuming the norm of the minimum fuzzy approximation error is such that \( \|w_i\| \leq \sigma_i \) and taking the norm of (28) to get

\[
\dot{V}_i \leq -\frac{1}{2} \lambda_{\min}(Q_i) \|e_i\|^2 + \sigma_i \lambda_{\max}(P_i) \|e_i\|
\]
where \( \lambda_{\min} (Q_i) \) and \( \lambda_{\max} (P_i) \) are the minimum and maximum eigenvalues of the indicated matrices and \( \| \cdot \| \) stands for the Euclidean norm [32]. Rewriting (28-a) as

\[
\dot{V}_i \leq -\frac{1}{2} (1 - \beta_i) \lambda_{\min} (Q_i) \| e_i \|^2 + \left[ \frac{\beta_i}{2} \lambda_{\min} (Q_i) \| e_i \| - 2\sigma \lambda_{\max} (P_i) \right] \| e_i \| \tag{28-b}
\]

where \( 0 < \beta_i < 1 \). Provided that

\[
\| e_i \| \geq \frac{4\sigma \lambda_{\max} (P_i)}{\beta_i \lambda_{\min} (Q_i)} = r_i
\]

then (28-b) becomes

\[
\dot{V}_i \leq -\frac{1}{2} (1 - \beta_i) \lambda_{\min} (Q_i) \| e_i \|^2 \tag{28-d}
\]

From the positive definiteness of (23) and the negative definiteness of (28-d), we conclude that the tracking error is globally ultimately bounded and the ultimate bound \( \mu_{bi} \) can be found as [34], [35]

\[
\mu_{bi} = r_i \sqrt{\frac{\lambda_{\max} (P_i)}{\lambda_{\min} (Q_i)}} \tag{28-e}
\]

IV. ADAPTIVE INTERVAL TYPE-2 FUZZY APPROXIMATION CONTROLLER DESIGN FOR GRID-CONNECTED PV INVERTER

A single-stage PV grid-connected three-phase inverter system is shown in Figure 3 [31]. The grid is assumed to be infinitely stiff, balanced, and sinusoidal with a fixed frequency.

![Three-phase grid-connected inverter](image)

FIGURE 3. Three-phase grid-connected inverter.

The state model of the system can be written as

\[
\dot{x} = \begin{bmatrix}
\omega x_2 - \frac{1}{L} x_3 \\
-\omega x_1 - \frac{1}{L} x_4 \\
\omega x_4 + \frac{1}{C} x_1 \\
-\omega x_3 + \frac{1}{C} x_2 \\
-3 \frac{v_{gd} i_{gd}}{2 C_{dc} x_5} + v_{gq} i_{gq}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
n \frac{1}{L} \\
0 \\
0 \\
0
\end{bmatrix}
\]

where \( x \in \mathbb{R}^n \) is the state vector, and \( u_1 \) and \( u_2 \) are the control inputs described by

\[
\begin{align*}
x &= \begin{bmatrix} x_1 \ x_2 \ x_3 \ x_4 \ x_5 \end{bmatrix}^T \\
u &= \begin{bmatrix} u_1 \\
u_2 \end{bmatrix}
\end{align*}
\]

where \( i_d, i_q, v_d, v_q \) are the inverter \( dq \) current and voltage respectively, \( i_{gd}, i_{gq}, v_{cd}, v_{cq} \) are the \( dq \) grid current and voltage components, \( v_{cd}, v_{cq} \) are the filter capacitor voltage \( dq \) components, \( L \) and \( C \) are the filter inductance and capacitance, and \( \omega \) is the grid angular frequency [31].

In this research work, the control purpose is to develop an AIT2FAC based on feedback linearization in which the inverter \( dq \) current components defined by \( y = [i_d \ i_q]^T \) will track specified reference current components \( y_{ref} = [i_{dref} \ i_{qref}]^T \). \( y_{ref} \) can be converted to a feedback linearizable form [34]. The relative degree of the grid-connected system can be shown as \( r_1 = r_2 = 1 \), and the feedback linearizing control law is formed as

\[
\begin{bmatrix} u_1 \\
u_2 \end{bmatrix} = \beta^{-1} (x) \begin{bmatrix} v_1 - \alpha_1 \\
v_2 - \alpha_2 \end{bmatrix}
\]

where,

\[
\alpha (x) = \begin{bmatrix} \alpha_1 \\
\alpha_2 \end{bmatrix} = \begin{bmatrix} L_f h_1 (x) \\
L_f h_2 (x) \end{bmatrix} = \begin{bmatrix} \omega x_2 - \frac{1}{L} x_3 \\
-\omega x_1 - \frac{1}{L} x_4 \end{bmatrix}
\]

\[
\beta (x) = \begin{bmatrix} L_g h_1 \\
L_g h_2 \end{bmatrix} = \begin{bmatrix} 1 \\
0 \frac{1}{L} \end{bmatrix}
\]

and \( v_1 \) and \( v_2 \) are the new input vector.

Since \( r_1 + r_2 = 2 = m < n \), the grid-connected inverter system for the inverter current control is partially linearizable. The stability of the system zero dynamics that does not transform through feedback linearization can be proved [36].

The proposed AIT2FAC based on partial feedback linearization is employed in the PV grid-connected inverter system shown in Figure 3. The block diagram of the proposed controller is shown in Figure 4. The FBF is calculated from...
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system states and interval type-2 Gaussian MF, as shown in the block diagram. The unknown parameters of the PV grid-connected inverter system were estimated using the control laws (26) and (27), with the calculation starting from chosen initial values of $\theta_i(0)$ and $\theta_{ij}(0)$. After that, the control signals were obtained with AFAC law (16) and the PWM signals were generated. For the controlled system, the AIT2FAC law (16) can be written as

$$
\begin{bmatrix}
u_1 \\ \nu_2
\end{bmatrix} = \begin{bmatrix} \hat{\beta}_{11}(x) & \hat{\beta}_{12}(x) \\ \hat{\beta}_{21}(x) & \hat{\beta}_{22}(x) \end{bmatrix}^{-1} \begin{bmatrix} -\hat{\alpha}_1(x) + v_1 \\ -\hat{\alpha}_2(x) + v_2 \end{bmatrix}
$$

(35)

where $\hat{\alpha}_1$ and $\hat{\beta}_i$ are obtained using the fuzzy approximation (13) and (14) and the update laws (26) and (27). The matrices $A_i$, and $b_i$, $i = 1, 2$ in (20) are given by $A_1 = -k_{01}$, $A_2 = -k_{02}$ and $b_1 = b_2 = 1$, where $k_{01}$ and $k_{02}$ are design parameters selected such that the characteristic matrices $A_1$ and $A_2$ are strictly Hurwitz matrices. Moreover, due to the iterative nature of adaptation methods, the control gains should be large enough to achieve stability at startup.

To implement the proposed AIT2FAC, the interval type-2 Gaussian MF in (36) with an uncertain mean lies in $[\mu - \sigma, \mu + \sigma]$, and a fixed standard deviation $\sigma$ is used for $x_i$, $i = 1, 2, \ldots, 5$. Where the center of the corresponding MF is chosen as $x_i(0)$, and the remaining parameters are chosen arbitrarily in the constraint sets [1]. Three Gaussian fuzzy sets are utilized to generate the FBF, namely, negative (N), zero (Z) and positive (P) for each of the system states. The obtained results demonstrate that the proposed AIT2FAC provides good tracking performance for active and reactive current in the situation of unity PF control. Moreover, Figure 6 (c) shows the grid current and voltage, which shows

$$
\begin{align*}
u_A(x) &= \exp \left[ -\frac{1}{2} \left( \frac{x - m}{\sigma} \right)^2 \right], \\
m &\in [\mu - \Delta \mu, \mu + \Delta \mu]
\end{align*}
$$

(36)

The parameters of the MFs are shown in Table 1.

V. SIMULATION RESULTS

To assess the performance of the proposed controller, a three-phase grid-connected 1.4 MW PV inverter system with the parameters listed in Table 2 [37] is simulated using MATLAB/SIMULINK for different operating cases.

| Table 1. Parameters of the type-membership functions. |
| --- |
| State | Fuzzy set | N | Z | P |
| $x_1$ | $\mu_1^N = 1100$, $\Delta \mu_1^N = 30$ | $\mu_1^Z = 1400$, $\Delta \mu_1^Z = 30$ | $\mu_1^P = 1700$, $\Delta \mu_1^P = 30$ |
| $\sigma_1^N = 100$, $\sigma_1^Z = 100$, $\sigma_1^P = 100$ |
| $x_2$ | $\mu_2^N = 10$, $\Delta \mu_2^N = 0$ | $\mu_2^Z = 1$, $\Delta \mu_2^Z = 0$ | $\mu_2^P = 10$, $\Delta \mu_2^P = 0$ |
| $\sigma_2^N = 4$, $\sigma_2^Z = 4$, $\sigma_2^P = 4$ |
| $x_3$ | $\mu_3^N = 320$, $\Delta \mu_3^N = 2.5$ | $\mu_3^Z = 340$, $\Delta \mu_3^Z = 2.5$ | $\mu_3^P = 360$, $\Delta \mu_3^P = 2.5$ |
| $\sigma_3^N = 8$, $\sigma_3^Z = 8$, $\sigma_3^P = 8$ |
| $x_4$ | $\mu_4^N = 10$, $\Delta \mu_4^N = 0$ | $\mu_4^Z = 1$, $\Delta \mu_4^Z = 0$ | $\mu_4^P = 10$, $\Delta \mu_4^P = 0$ |
| $\sigma_4^N = 4$, $\sigma_4^Z = 4$, $\sigma_4^P = 4$ |
| $x_5$ | $\mu_5^N = 1200$, $\Delta \mu_5^N = 10$ | $\mu_5^Z = 1290$, $\Delta \mu_5^Z = 10$ | $\mu_5^P = 1380$, $\Delta \mu_5^P = 10$ |
| $\sigma_5^N = \sqrt{1000}$, $\sigma_5^Z = \sqrt{1000}$, $\sigma_5^P = \sqrt{1000}$ |

To validate the unity power factor tracking, simulation was carried out for the following current references $i_{dref} = 0$ A (that correspond to unity power factor) and $i_{qref} = 1400$ A. Figure 6 shows the grid-connected inverter system response with the proposed AIT2FAC. Figure 6 (a) and (b) show the tracking of the active current $i_d$ to its reference $i_{dref}$ and the tracking of the reactive current $i_q$ to its reference $i_{qref}$ respectively.
that the voltage and current are in phase, demonstrating unity PF performance.

To illustrate the system’s performance under power factor change, a step change in $i_{q\text{ref}}$ of 400 A at $t = 0.1$ s is assumed. The system response is illustrated in Figure 7. Figure 7 (b) shows the grid current components $i_q$ and $i_{q\text{ref}}$, where the results show that $i_q$ soon reaches its new reference. Figure 8 shows a phase shift between the voltage and current ($\sim 1\text{ms}$) verifying that the desired power factor of 0.988 is being tracked. Moreover, Figure 9 shows the change in power factor with $i_{q\text{ref}}$ change. Hence, the simulation results confirmed that the desired power factor can be tracked via changes $i_{q\text{ref}}$ using the proposed AIT2FAC.

For further performance evaluation, the proposed controller was tested under a change in the reference active current $i_{d\text{ref}}$. The reference active current $i_{d\text{ref}}$ is assumed to experience a 200 A step change at $t = 0.1$ s. Figure 10 (a) shows the simulation result for the current components $i_d$ and $i_{d\text{ref}}$ indicates the tracking of the desired active current change quickly. Additionally, to demonstrate the robustness of the proposed AIT2FAC against parameter changes, simulations have conducted for changes in filter inductor L. Simulation result in Figure 11 shows the grid active and reactive current components $i_d$ and $i_q$ with 10% variation in filter inductor L. The obtained results indicate the robustness of the proposed AIT2FAC with filter inductor L variation.

To evaluate the efficiency of the proposed AIT2FAC compared to other controllers, the tracking performance of the
proposed controller is compared with the type-1 fuzzy controller reported in [31]. The tracking performance of the proposed AIT2FAC and type-1 fuzzy controller is shown in Figure 12, confirming the developed AIT2FAC’s better control performance. Table 3 shows the tracking performance measures including settling time, overshoot, integral absolute error (IAE) and integral time of absolute error (ITAE) for the type-1 fuzzy controller and proposed AIT2FAC for the case of the unity power factor.

Table 3 indicates that the proposed controller outperforms the type-1 fuzzy controller in terms of tracking the required reference signal with a faster settling time and less overshoot. Additionally, according to the results, the proposed AIT2FAC has significantly lower IAE and ITAE performance indices than the type-1 fuzzy controller. Therefore, as a result, in terms of tracking performance, the proposed AIT2FAC outperforms the type-1 fuzzy controller.

VI. CONCLUSION
To address the nonlinearity and uncertainty issues of PV grid-connected inverter systems, this paper proposes adaptive interval type-2 fuzzy approximation control. The proposed controller utilizes the fuzzy system approximation capability and the input-output feedback linearization principle to approximate system unknown nonlinear functions and track specified reference values without the need for prior knowledge of the mathematical model of the system. IT2FLS’s high capability to cope with a high level of uncertainty is utilized. IT2FLS’s high capability to cope with a high level of uncertainty is utilized. The Lyapunov function was used to establish the closed-loop stability and updating law of the control parameters. The proposed controller was simulated for power factor and active current tracking applications, and the findings showed the proposed AIT2FAC usefulness with notable tracking performance. Furthermore, a comparison reveals that the proposed AIT2FAC outperforms the type-1 fuzzy controller in terms of different tracking performance measures, such as settling time, overshoot, integral absolute error (IAE) and integral time of absolute error (ITAE).

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