Trajectory Planning of a Planar Cable-Driven Robot for Industrial Detection

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Abstract. This paper presents a planar two-degree-of-freedom cable-driven robot for detecting the surface defect of the boiler pipes. The mechanism is first described and inverse solution laws of position, velocity and acceleration are studied based on the dynamic model of the cable-driven robot. A linear trajectory planning approach is developed, which is based on the position discrete differential and trapezoidal speed planning. An example trajectory is presented, then simulating and analyzing the change law of the length and velocity of the driving cable based on the point-to-point linear trajectory planning algorithm. The research shows that the length of the cable is continuously changed, which ensures that the end-effector of cable-driven robot can run smoothly, continuously, and accurately during the industrial detection.

1. Introduction

For the urgent need for automatic detection of surface defects in boiler pipes, the analysis of the characteristics of large detection intervals in boiler pipes and complex environments has led to the innovative introduction of a detection platform with the cable-driven parallel robot, and the inspection and positioning of pipelines in defective areas through the robot end-effector platform and industrial camera. Cable-driven robot is a parallel structure robot that converts the motion of the drive motor to the movement of the end-effector by using the cable as the medium. The cable is used to replace the traditional rigid components, which makes its working area broadened and its installment simplified and its mechanical structure simple. These characteristics have attracted widespread attention in the academic community. According to the number of cables m and the number of freedoms n of the mobile platform, the cable-driven robot can be divided into under-constrained (m<n+1), full-constrained (m=n+1), and redundant constraints(m>n+1). For the suspension planar cable-driven robot with a two-degree-of-freedom in this paper, its gravity and the combined force of two passive restraining cables can be used to control an external force of the platform movement of the end camera, so only when m=n, the cable can be used to achieve the complete constraint of the platform. However, since the cable can only provide pulling force and cannot provide thrust, it is necessary to keep the cable tension greater than zero during the platform movement[1]. Secondly, during the end-effector movement, the length and angle of the cable vary with the position of the platform. With these changes, these non-linear characteristics increase the difficulty of planning the trajectory.

Recently, cable-driven robot have drawn many attentions in many fields. It is applied in the fields of coordinate detection[2], cranes[3], large radio telescopes[4] [5], medical rehabilitation robots[6], wind
tunnel vehicle systems[7], film and television photography[8] and other fields. This paper focuses on the kinematics and point-to-point linear planning of planar cable-driven robots. Roberts systematically analyzed the inverse kinematics of a cable-driven parallel robot[9]. Maicr pointed out that the inverse kinematics formulas were used for trajectory planning analysis[10]. Gosselin et al. have studied the point-to-point dynamic trajectory planning of a planar cable-driven robot[11], using higher-degree polynomials and trigonometric functions to implement trajectory planning with high complexity. Based on this, this paper proposes a method of combining position discrete differentiation and trapezoidal velocity planning to implement the robot's point-to-point linear trajectory planning.

This paper first establishes the kinematics model of the robot and derives the inverse kinematics rules. Then it studies the trajectory planning algorithm of the planar cable-driven parallel robot. The linear equations of the robot are implemented by using the position equation and ladder-type speed planning algorithm based on the required trajectory planning. Design a number of continuous linear trajectories, and simulate to analyze the change rules of the length and position velocity of the cable when the robot performs trajectory motion. The simulation verifies the correctness of the constructed robot kinematics model and the continuous controllability of point-to-point motion.

2. Robot architecture

For a cable-driven robot, its design on structure is different for its different uses. The robot studied in this thesis which is used for in-process quality control of boiler pipes in power plants is a flat cable traction robot with two translational degrees of freedom. It mainly includes infrastructure, reel, camera terminal platform, cable driving unit, pull cable sensor, etc. for the structure of the robot. The system has two cable driving units, that is to say, the two upper cables drive the retracting and retracting of the cables simultaneously, which control the end camera platform to move in the plane XY direction, while the lower two cables are provided by roll ripe sensor, which is used to obtain the accurate position information of the terminal platform and provide a constant restraint force to the terminal platform, so as to prevent the platform from swinging laterally.

The driver of the planar cable-driven robot and terminal platform are simplified into points, and the kinematics model is obtained which is shown in Figure 1. Among them, P2 and P3 represent two cable driving units, and P2 and P4 represent two cable passive constraining force providing units which are provided by roll cable sensor. Similarly, L2 and L3 represent two driving force cables, and L1 and L4 represent two passive restraint cables. A rectangular coordinate system is established with the point P1 as the origin, a straight line from P1 to P4 as the X axis of the coordinate system, and a straight line from P1 to P2 as the Y axis of the coordinate system. Each point in the coordinates is marked as shown in Figure 1.

![Figure 1. The planar cable-driven robot’s architecture.](image)

3. Kinematic and dynamic modelling

In this thesis, through the kinematics analysis on a planar cable-driven robot, it is mainly researched on the relationship between the kinematics parameters and the cable parameters of the terminal platform. As usual, it is considered for kinematics position analysis, velocity analysis and acceleration analysis.
Though the kinematics position analysis, it mainly analyzes the forward and inverse solution to the position of the terminal platform of the cable driven robot and the length of each cable. It is assumed that the stiffness of the cable is infinite, there is no elastic deformation, and there is no consideration for the effects of bending, self-weight and other factors of the cable. In this thesis, at first, the length \( L_i \) (i=1,2,3,4) of each cable is calculated from the position information of the known terminal platform \( P \) and the inverse solution to the kinematic position. In accordance to the kinematics model shown in Figure 1, combined with the distance formula between two points in the plane, the length of each cable can be worked out as:

\[
\begin{align*}
L_1 &= \|P - P_i\| = \sqrt{(x-x_1)^2 + (y-y_1)^2} \\
L_2 &= \|P - P_i\| = \sqrt{(x-x_2)^2 + (y-y_2)^2} \\
L_3 &= \|P - P_i\| = \sqrt{(x-x_3)^2 + (y-y_3)^2} \\
L_4 &= \|P - P_i\| = \sqrt{(x-x_4)^2 + (y-y_4)^2}
\end{align*}
\]

(1)

It can be known from this that, in case that the trajectory of the terminal platform is given, the position of the terminal platform at any time can be uniquely determined.

The positive solution of kinematic position is opposite to the inverse solution process of kinematics. The position coordinate \( P \) of the terminal platform is solved from the known length value \( L_i \) of the four driving cables. In this thesis, the planar cable-driven studied has only two translational degrees of freedom, in accordance to the redundancy characteristic, only to co-establish two sets of equations (1) can it be used to solve the position coordinates of the corresponding terminal platform. As it is shown in Figure 1, let \( P_1 \) be the origin of the coordinate system, it can be obtained \( x_1 = y_1 = x_2 = y_4 = 0 \).

If the lengths of the two driving cable 2 and cable 3 are known as \( L_2 \) and \( L_3 \), the terminal platform position can be solved. In accordance to the kinematics positive solution, it can be obtained:

\[
\begin{align*}
(x-x_2)^2 + (y-y_2)^2 &= L_2^2 \\
(x-x_3)^2 + (y-y_3)^2 &= L_3^2
\end{align*}
\]

(2)

After solving the above equation, the coordinates of the terminal platform \( P \) could be obtained:

\[
\begin{align*}
x &= \sqrt{L_2^2 - \frac{(L_2^2 - L_3^2 + y_2^2)^2}{4y_3^2}} \\
y &= \frac{L_2^2 - L_3^2 + y_2^2}{2y_3}
\end{align*}
\]

(3)

In a similar way, the position information of the terminal platform \( P \) can be obtained by knowing \( L_4 \), \( L_3 \) or \( L_1 \), \( L_3 \). It can be seen from the above formula that to solve the position coordinates of the terminal platform is actually to solve the coordinates of the intersection of two circles. When two circles are tangent and there is only one intersection point, it can be known that the terminal platform is at the edge of the workspace. If the two circles intersect, the position coordinates of the terminal platform which meet the conditions could be determined in accordance to the position of the terminal platform within the working space, that is to say, the position coordinates satisfy the constraint relationship \( 0 < x < x_3, 0 < y < y_3 \).

The analysis on kinematic speed refers to the relationship between the moving speed of the moving platform of the robot and the speed of the length change of the cable. For the mechanism studied in this thesis, since there are only \( L_2 \) and \( L_3 \) providing the cable for the driving force, this thesis mainly studies the correlation between the operating speed of the terminal platform and the speed of the change in the length of the dynamic cable, so as to provide a theoretical basis for subsequent trajectory planning. The inverse solution to kinematic velocity refers to the running speed of the terminal platform point \( P \), and the velocity of the length change of the cable. The inverse solution to kinetics speed mainly makes use of the partial derivative formula, in which, \( i = 2, 3 \), the general formula for the speed of the change in the length of the cable is:

\[
\dot{L}_i = \frac{\partial L_i}{\partial x} \cdot \dot{x} + \frac{\partial L_i}{\partial y} \cdot \dot{y}
\]

(4)
Therefore, it can be known that the speed of the length change of the power cable L2, L3 of the robot is:

$$\frac{dL_1}{dt} = \frac{\partial L_1}{\partial x} \frac{dx}{dt} + \frac{\partial L_1}{\partial y} \frac{dy}{dt}$$
$$\frac{dL_2}{dt} = \frac{\partial L_2}{\partial x} \frac{dx}{dt} + \frac{\partial L_2}{\partial y} \frac{dy}{dt}$$

(5)

The problem for the kinematics inverse solution can be expressed as:

$$\dot{L} = J \ddot{x}$$

(6)

Among which:

$$\dot{L} = \begin{bmatrix} \dot{L}_1 \\ \dot{L}_2 \end{bmatrix}, \ddot{x} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}, J = \begin{bmatrix} \frac{\partial L_1}{\partial x} & \frac{\partial L_1}{\partial y} \\ \frac{\partial L_2}{\partial x} & \frac{\partial L_2}{\partial y} \end{bmatrix}$$

Combined with the formula(4),(5), the length change rate of the cable L2, L3 can be specifically expressed as

$$\dot{L}_1 = \frac{1}{L_1} \left[ (x-x_1) \ddot{x} + (y-y_1) \ddot{y} \right] \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$\dot{L}_2 = \frac{1}{L_2} \left[ (x-x_2) \ddot{x} + (y-y_2) \ddot{y} \right] \sqrt{(x-x_2)^2 + (y-y_2)^2}$$

(7)

In the analysis of motion acceleration, the velocity and position are to solve first and second derivatives of time, respectively, and it researches on the relationship between the acceleration of the end-effector and the acceleration of the change in the length of the cable. Similar to the previous speed analysis, the inverse solution to kinematics is used to solve the acceleration of the dynamic cable L2, L3’s length. The acceleration of the cable length is based on the speed of the cable. It is continued to derive the formula to obtain the general formula for calculating the acceleration of the cable L2, L3’s length.

$$\ddot{L} = \frac{d}{dt} \left( \frac{dL}{dt} \right) = \frac{d}{dt} \left( \frac{\partial L}{\partial x} \dot{x} + \frac{\partial L}{\partial y} \dot{y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial^2 L_1}{\partial x^2} \ddot{x} + \frac{\partial^2 L_1}{\partial x \partial y} \ddot{y} \right) \dot{x} + \frac{\partial}{\partial y} \left( \frac{\partial^2 L_1}{\partial y^2} \ddot{x} + \frac{\partial^2 L_1}{\partial x \partial y} \ddot{y} \right) \dot{y}$$

$$\ddot{L} = -\left[ (x-x_1) \ddot{x} + (y-y_1) \ddot{y} \right] ^2 \left[ (x-x_1) \ddot{x} + (y-y_1) \ddot{y} \right]$$

(8)

$$\ddot{L} = \left[ (x-x_2) \ddot{x} + (y-y_2) \ddot{y} \right] ^2 \left[ (x-x_2) \ddot{x} + (y-y_2) \ddot{y} \right]$$

(9)

4. Trajectory planning algorithm

When patrolling the boiler pipes of power plants, it is necessary to link the end-effector camera with a cable to plan the motion trajectory in the required area. Based on the actual task requirements of the robot, the path to be formed by the end-effector is obtained. Corresponding variables of the cable joints are calculated with the help of kinematics. The relationship between position, speed, acceleration and time during the operation is mainly studied here, so that the system can move according to the expected trajectory and ensure the smoothness of the movement of the end-effector.

During the inspection process of the planar cable robot, the industrial camera is sent to the designated location through the point-to-point linear trajectory planning to acquire the defective pipeline image. That is, the linear trajectory position planning and speed planning of the robot need to be realized. Assume that the end-effector of the cable driven robot moves from point B1 to point B2 according to a
linear trajectory, where the coordinate position of the starting position point B1 is \((x_{b1}, y_{b1})\) and the position coordinate of the end position point B2 is \((x_{b2}, y_{b2})\).

According to the two-point equation and the distance formula between the two points, the analytical formula of the straight line B1B2 and the distance between the points B1 and B2 are obtained as:

\[
y = \frac{(x - x_{b1})(y_{b2} - y_{b1})}{x_{b2} - x_{b1}} + y_{b1}
\]

\[
\text{Dis} = \| B_2 - B_1 \| = \sqrt{(x_{b2} - x_{b1})^2 + (y_{b2} - y_{b1})^2}
\]

Represent points B1 and B2 through a vector expression, then point B21 can be expressed as:

\[
B_{21} = B_2 - B_1 = x_{b2}i + y_{b2}j - (x_{b1}i + y_{b1}j)
\]

Unit directional vector can be expressed as: \(n_{b1} = \frac{B_{21}}{\text{Dis}}\).

When the end-effector runs along a straight trajectory, it is assumed that the speed of uniform velocity is \(v\) and the total running time is \(T\). At any time \(t\) in the operation, the position vector expression of the end-effector is \(P_t = x_ti + y_tj\), namely the kinematic expression of the trajectory planning is \(P_t = P_1 + n_{b1}vt, t \in [0, T]\).

In order to ensure the smoothness of the end-effector’s motion trajectory of the cable driven robot, combined with the non-linear characteristics of cable driving, the trapezoidal speed control and position discrete differentiation method were used to realize the point-to-point speed planning of the end-effector. The trapezoidal speed control method has been widely used because of its simple structure and convenient calculation. The standard trapezoidal speed curve is shown in the Figure 2:

![Figure 2. The trapezoidal speed control.](image)

It can be seen that its corresponding output distance \(s\) is a piecewise function, that is:

\[
0 \leq t \leq \frac{v}{a} : s(t) = \frac{1}{2}at^2
\]

\[
\frac{v}{a} < t \leq T - \frac{v}{a} : s(t) = vt - \frac{v^2}{2a}
\]

\[
T - \frac{v}{a} < t \leq T : s(t) = \frac{2avT - 2v^2 - a^2(T - T)^2}{2a}
\]

Differenntiate the straight line B1B2 according to the formula(10), and calculate the difference \(\Delta/2\), \(\Delta \beta\)between the lengths of the dynamic cables L2 and L3 in the segments. According to the trapezoidal speed plan of the end-effector of the formula, inversely solve the speed of each segment, and then acquire the rules of movement time and speed from the starting position B1 to the end position B2, and the data is stored in the control system buffer to realize the point-to-point linear trajectory of the end effector driven by the cable.

5. Simulation
In this thesis, the cable-driven robot is mainly based on point-to-point linear trajectories during field in-process quality control operations. Though the kinematics model previously established for the system
structure, the software Matlab is used to calculate the simulation experiment for the robot's point-to-
point linear trajectory planning examples. To set the end point P of the flat cable robot in the XOY
parallel plane, and take point P1(200mm,1300mm)as the starting point and end point in the linear
trajectory planning, which passes the points(1800,1250),(400,1150),(1000,700),(200,700) and 5-
segment straight path can be obtained through trajectory planning, which is shown in Figure 3. Position
can be planned through the use of trapezoidal speed planning algorithm for speed planning, inverse
kinematics and position discrete differentiation. Supposing the initial speed as V0=0 mm/s, the
maximum speed as 200 mm/s, and the acceleration, uniform speed, and deceleration time are equal,
taking the first straight track as an example.

![Figure 3. 5-segment straight path of end-effector.](image)

The solution to the path change of the end-effector as shown in Figure 4, and the cable length and
speed change of the power cable L2, L3 as shown in Figure 5,6. During the simulation, the period
throughout the trajectory is 39.1 s.

![Figure 4. The path change of the end-effector.](image)

![Figure 5. The cable length’s change of L2, L3](image)
6. Conclusion
It can be seen from the simulation results above that, the path of the end-effector changes, the change of the cable length and the speed of the driving cable are relatively stable, which indicates that the trajectory planning algorithm can be used to achieve the smooth operation of the point-to-point linear motion of the end-effector and meet the requirements of the planar cable-driven inspection robots on their motion performance.

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