Analysis of the main characteristics of tsunamis based on data from deep-ocean stations

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Abstract. A tsunami in the ocean, according to one idea, is a long wave with a practically unchanged period and a relatively slow attenuation. According to others, the dispersion affects the evolution of a tsunami, the wave period increases with time, and the amplitude attenuation is more rapid. This study investigates the transformation of tsunamis from source to coast, including the change in amplitude and duration of the head waves with distance. It is shown that the dispersion does affect the evolution of the tsunamis. The moment of dispersion manifestation depends not only on the magnitude of the earthquake that caused each tsunami, but also significantly on the depth of the ocean in the focal region.

1. Introduction

The question of whether a tsunami is a long or dispersive ocean wave does not currently have a definitive answer. There is no doubt that dispersion has an impact on the evolution of a tsunami. If the long wave propagates with the conservation of the period and relatively slow attenuation, then the dispersive wave propagates with an increasing period and faster decay. It is not clear when dispersion begins to occur, which characteristics of the tsunami source this moment depends on. Knowledge of the characteristics of the rolling wave is necessary when assessing the degree of danger of a tsunami, when solving problems of tsunami zoning of the coast and in long-term tsunami forecasting. The tsunami run up onto the coast (heights of possible flooding and flooding duration) is determined not so much by the main component in the tsunami spectrum but by the amplitude and duration of the head or maximum wave.

The dependences of the wavelength on the earthquake magnitude were studied in [1, 2] using limited information regarding earthquakes and tsunamis. Tsunamis near Japan for the period 1931–1969 were considered. The relation connecting the tsunami wavelength \( \lambda \) with the characteristic horizontal size of the source \( R \) was derived as \( \lambda \sim 2.8 R \).

The characteristic size of the tsunami source \( R \) was estimated as \( R = 0.5 \sqrt{L_{\text{max}} \cdot L_{\text{min}}} = \sqrt{S/\pi} \), where \( L_{\text{max}} \) and \( L_{\text{min}} \) are large and small horizontal dimensions, respectively, and \( S \) is the square of the focal area.

It is of interest to consider earthquakes and tsunamis of other regions, as well as tsunamis propagating in the ocean.
According to the established ideas, a tsunami wave is considered as a long wave in shallow water, the length of which is much greater than the water depth. Such a wave propagates without changing the duration (period) of the head wave, and the amplitude decreases with distance $r$ as $1/\sqrt{r}$.

According to other concepts, tsunamis are affected by dispersion. As a result, the wave period increases and the amplitude decreases as $1/r$. Despite the large number of computational works on tsunamis, both without and with dispersion, the effects of dispersion on real tsunami waves and the conditions for the applicability of the models have not been studied.

In [3], nonlinear and dispersion effects at sufficiently large distances from the source were considered. An expression for the so-called dispersion length, i.e., the distance after which the wave is affected by the dispersion, is given by:

$$L_{disp} \sim 0.06 \cdot A^3/D^2$$  \hspace{1cm} (1)

Estimates of the dispersion lengths made for some tsunamis give values of 8–48 thousand km, which are comparable to (and exceed) the size of the ocean. According to these estimates, the effect of dispersion on tsunami propagation is not significant. In [3], some facts of the effect of dispersion on tsunamis are noted.

Dispersion effects for tsunami propagation are discussed in [4]. The Cauchy-Poisson problem on the excitation of waves from an instantaneous uniform elevation of a section of the seabed of radius $R$ by an amount $a_0$ was considered. Using the stationary phase method, an asymptotic expression is obtained that describes the waveform:

$$\zeta(r, t) \approx \frac{a_0 R}{ct} \text{ch}^{-1}\left(\frac{2\pi}{\sqrt{\tau/D}}\right) \sqrt{\frac{t}{2\pi}} \cdot f_1\left(\sqrt{\frac{2\pi}{\tau}}\right) \cos\left(\frac{ct}{3D} \sqrt{\frac{2\pi}{\tau}}\right),$$  \hspace{1cm} (2)

where $t$ is the travel time of the wave front, $c = \sqrt{gD}$ is the propagation velocity of long waves, $f_1$ is the Bessel function and $\tau$ is the relative time counted from the moment of wave arrival at the observation point.

Taking into account the behaviour of the wave at the front and in front of it, as studied in [4], the duration of the first period of the wave is estimated as $T = 3.91 \cdot D^{2/3} \cdot c^{-2/3}$.

The aim of the work is to discover whether tsunami waves in the ocean are long waves or waves with dispersion. We determine what is the behaviour of a tsunami during its propagation in the ocean, how the amplitude and period of the head wave change, what is the limit to which the tsunami can be viewed as a long wave, and, starting with which, the dispersion influences the tsunami behaviour.

2. Estimation of dispersion length (time)

We derive the expression for the dispersion length (time), based on considerations other than those in [3].

In equation (2), the functions defining the shape of the wave packet are the envelope (Bessel function $f_1\left(\sqrt{\frac{2\pi}{\tau}}\right)$) and $\cos\left(\frac{ct}{3D} \sqrt{\frac{2\pi}{\tau}}\right)$. The moving speed along the axis of relative time $\tau$ of the first zero of the Bessel function (equal to ~4) is higher than the moving speed of the second zero of the cosine. At the initial stage, the zero of the Bessel function is to the left of the second zero of the cosine. At the initial stage, the zero of the Bessel function is to the left of the second zero of the cosine. Then, moving with time, the zero of the Bessel function takes a position to the right of several zeros of the cosine.

For the moment the dispersion begins to appear, we take the instant of coincidence of the positions of the first zero of the Bessel function and the second zero of the cosine.

From the equalities $\sqrt{\frac{2\pi R}{\tau D}} = 4 \pi / \sqrt{\frac{3}{2}} D^{1/2} \sigma$ and $\frac{3\pi}{2} / \sqrt{\frac{3}{2}} D^{1/2}$, excluding $\tau$, we obtain

$$\tau_{disp} = \frac{3\pi}{2} \cdot \frac{3\pi}{2} \cdot \frac{R^3}{cD^2} \approx 0.22 \cdot \frac{R^3}{cD^2}.$$  \hspace{1cm} (3)
By analogy with the concept of the dispersion length, the resulting relation in equation (3) is deemed the dispersion time. At this time, the duration of the first period is
\[ T = \frac{2.36R}{c}. \]

The derived relations were obtained under conditions of constant basin depth and instantaneous uniform displacement of a portion of the basin bottom. However, these relationships are used to analyse the behaviour of natural tsunamis in the ocean.

To estimate the size of the source of a tsunami, the relations between the magnitude earthquake \( M \) and the sizes of the source of the tsunami were used [5]:
\[ \log L_{\text{max}} = 0.5 M - 1.9; \log L_{\text{min}} = 0.5 M - 2.2. \]

For estimates of the dispersion times in equation (3), half the length of the lesser axis of the source is taken as
\[ R = \frac{10^{0.5M-2.2}}{2} \]
and the depth of the ocean is taken as constant at \( D = 4400 \) m.

For example, for the Tohoku tsunami (2011, \( M = 9.1 \)), \( R = 97 \) km, \( t_{\text{disp}} = 800 \) min and \( L_{\text{disp}} = 10500 \) km. Accordingly, for the Simushir Tsunami (2006, \( M = 8.3 \)), \( R = 39 \) km, \( t_{\text{disp}} = 50 \) min and \( L_{\text{disp}} = 670 \) km. These estimates are very different from the estimates in equation (1).

3. Statement of problem

According to tsunami data obtained in the ocean by DART stations [6], the crest amplitude and the period of the first wave were measured. The dependencies of these characteristics on the tsunami travel time to the registration point were plotted. The arrival time of the first crest to the measurement point was taken as the travel time. The duration of the first period was normalised to the characteristic time in the focus (normalisation period)
\[ T_{\text{norm}} = \frac{R}{c_0}, \]
where \( R \) is the half length of the minor axis of the tsunami source and \( c_0 \) is the speed of long waves at a depth of \( D_0 \) in the epicentre of the tsunami source: \( c = \sqrt{gD_0} \).

The data for the analysed earthquakes and tsunamis are given in Table 1.

| No. | Geographic location of the tsunami centre date | Coordinates of the epicentre (latitude/longitude) | Magnitude | Half-width of the minor axis of the tsunami focus, \( R, \) km | Depth in epicentre, \( D_0, \) m | Normalisation period, \( T_{\text{norm}}, \) min |
|-----|---------------------------------------------|-----------------------------------------------|-----------|-------------------------------------------------|----------------|-----------------|
| 1   | Simushir 2009.01.15                         | 46.857, 155.154                               | 7.4       | 13.8                                            | 6960           | 0.88            |
| 2   | Tsunami 2011.03.09 Alaska                    | 38.435, 142.842                               | 7.5       | 15.5                                            | 1470           | 2.14            |
| 3   | Simushir 2018.01.23 Alaska                   | 56.046, 149.073                               | 7.9       | 24.5                                            | 4560           | 1.93            |
| 4   | Simushir 2006.11.15                         | 46.592, 153.266                               | 8.3       | 38.8                                            | 3520           | 3.48            |
| 5   | Simushir 2007.01.13                         | 46.243, 154.524                               | 8.1       | 30.8                                            | 6650           | 2.01            |
| 6   | Chile 2014.04.01                             | -19.610, -70.769                              | 8.2       | 34.6                                            | 2110           | 4.01            |
| 7   | Peru 2007.08.15                              | -13.386, -76.603                              | 8.0       | 27.5                                            | 140            | 12.36           |
| 8   | Chile 2015.09.16                             | -31.573, -71.674                              | 8.3       | 38.8                                            | 300            | 12.0            |
| 9   | Tohoku 2011.03.11                            | 38.297, 142.373                               | 9.1       | 97.5                                            | 970            | 16.66           |
The study carried out is of a preliminary, qualitative nature. Statistical processing of the measurement results was not carried out.

The belonging of a tsunami to dispersive or non-dispersive waves was estimated by two indicators: the degree of attenuation of the amplitude and the degree of increase in the duration of the period of the head wave depending on the travel time. However, it seems that the second indicator is more reliable, since the first indicator depending on the focusing, scattering properties of the topography of the ocean floor can differ significantly from the theoretical one.

4. Results and discussion
The analysed events are divided into three series. The results of the analysis are presented below. The first series includes tsunamis with a small earthquake magnitude \( M = 7.4–7.9 \), but with a relatively large ocean depth at the epicentre \( D_0 = 1470–6960 \) m (Figure 1). The second series includes tsunamis with a high earthquake magnitude \( M = 8.1–8.3 \) and a large ocean depth at the epicentre \( D_0 = 3520–6650 \) m (Figure 2). The third series contains tsunamis with a high earthquake magnitude \( M = 8.0–9.1 \) but with a small ocean depth at the epicentre \( D_0 = 140–2110 \) m (Figure 3).

In the left column of all figures the dependencies of the amplitude of the head wave from the tsunami travel time are shown. The dependencies \( 1/\sqrt{t} \) and \( 1/t \) are depicted in all the figures.

In the right column the dependencies of the period of the head wave, normalised by equation (4), on the tsunami travel time are shown. The dependencies \( t^{1/3} \) are depicted in these figures.

The icons in the figure indicates the different directions of registration points relative to the tsunami focus. The oblique cross is the dispersion time and the normalised period of the first wave at this time.

![Figure 1](image.png)

**Figure 1.** Dependences of amplitude and normalised period of the first wave of the tsunami on travel time: a, b – Simushir, 2009.01.15; c, d - Tsunami 2011.03.09; e, f – Alaska, 2018.01.23.
In the first series for all three events, the results of period measurements fall well on the $t^{1/3}$ time dependence, which is characteristic for dispersive waves.

The results of the measurements of the amplitude of the first two events fall well on the dependence on the time $1/t$, in the last event the amplitude attenuation in the east direction is faster than $1/\sqrt{t}$, whereas in the west, it is faster than $1/t$.

The dispersion times calculated by equation (3) for the first two events are small and go beyond the field of the figure. For the third event, $t_{disp} \approx 13$ min., which is consistent with the results of measurements, even though equation (3) is derived under conditions of constant depth of the basin.

The studies are not exhaustive but provide preliminary conclusions.

Tsunamis, excited by earthquakes with $M = 7.4–7.9$ at large depths of the ocean in the foci (1.47–6.96 km), are dispersing waves almost from the centre.

In the second series for the 2006 event (Figures 2a and b), the results of measurement of amplitude fit well with the time dependence of $1/t$, of period – by-line $t^{1/3}$ after 150 min (2.5 h). It can be assumed that at smaller times from 10 to 150 min, the amplitude decreases as $1/\sqrt{t}$ and the period remains constant.

![Figure 2. Dependences of amplitude and normalised period of the first wave of the tsunami on travel time: a, b – Simushir, 2006.11.15; c, d – Simushir, 2007.01.13.](image)

It can be assumed that for the 2007 event (Figures 2c and d), the results of amplitude measurements in the time interval 20–150 min fall on the time dependence $1/\sqrt{t}$, at $t > 150$ min. – on the time dependence $1/t$. The first period for this event is almost constant between 20 and 600 min.

The calculated dispersion times of 60 and 30 min are underestimated due to the mismatch of depths in the ocean and in the tsunami source.

The Simushir tsunami of 2006 (as a result of an earthquake with $M = 8.3$) at an ocean depth at the tsunami source of 3.52 km after 2.5 h of travel is a dispersive wave.
The Simushir tsunami of 2007 (M = 8.1) at an ocean depth at the tsunami source of 6.65 km after 2.5 h of travel, according to the degree of attenuation of the amplitude, can be attributed to the dispersive waves, according to the degree of increase of the duration of the first period, it refers rather to long waves.

In the third series for all four events, the normalised first period is almost constant over a propagation time of up to 17 h (1000 min). For that indicator, tsunamis belong to the long waves in all four events within 17 h (for South American tsunamis – from the source to the coast of the Kuril Islands and Japan, for Tohoku tsunami – from the source to the coast of South America). The duration of the first period at the long-wave stage of propagation is approximately equal to two normalising periods, for the 2014 event with a relatively large ocean depth at the epicentre of 2110 m, the duration of the first period is five normalising periods.

**Figure 3.** Dependences of amplitude and normalised period of the first wave of the tsunami on travel time: a, b - Chilean 04.04.2014; c, d - Peruvian 15.08.2007; e, f - Chilean 16.09.2015; g, h - Tohoku 11.03. 2011.
For the first three events, the dispersion times are low, apparently due to differences in the depth of the ocean and depths at the epicentres. For the latter event, the estimate of dispersion time agrees well with the results.

The studies are not exhaustive but provide some conclusions.

Tsunamis (due to earthquakes with M = 8.0–9.1) at shallow depths of the ocean in the epicentres (140–2110 m), due to the constancy of the period of the first wave, for 17 h of propagations are long waves.

5. Conclusions
A preliminary analysis confirmed that dispersion does have an effect on the evolution of a tsunami. The travel time (distance) from which the dispersion manifests itself depends not only on the magnitude of the earthquake that caused the tsunami, but also the depth of the ocean in the source of the tsunami:
- tsunamis from earthquakes with M <8 at the depths of the ocean in the focal area, comparable to and exceeding the average depth of the ocean, are dispersive ones almost from the source;
- tsunamis from earthquakes with M ≥8 at the depths of the ocean in the focal area, comparable to and exceeding the average depth of the ocean, become dispersive ones after 2.5 h from occurrence;
- tsunamis from earthquakes with M = 8.0–9.1 at the depths of the ocean in the focal area, significantly smaller than the average depth of the ocean, for 17 h remain long waves without dispersion.

It should be noted that the Simushir tsunami of 2006 and the Chilean tsunami of 2015 with the same magnitudes (8.3), but with different depths of the ocean in the sources (3520 and 300 m), are, respectively, dispersive waves and long waves.

The proposed approach to estimating the dispersion time (length) of may be more adequate than that used previously, but an adjustment is needed to take into account differences in the depths of the ocean and the focal tsunami area.

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