Bullet Engraving Automated Comparison Optimization Method Based on Second Moment Invariant

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Abstract. In the research of bullet engraving comparison method at home and abroad, the traditional method is through visual observation by microscope, comparing line-type engraving of two bullets to distinct whether the line match, which is inefficient and having big error. In this paper we proposed method based on moment invariant to identify the bullet engraving, which eliminate the actual measurement error (translation error and rotation error). Finally we get the high performance result of bullet engraving identifying based on second moment invariant.

Keywords: Bullet engraving; second moment; similarity; translation error; rotation error.

1. Introduction
Bullet trace inspection is an important criminal investigation technique which used to detect crimes involving guns. In the practice of public security, the inspection of the warhead traces is visually observed through a microscope, and the line-shaped traces on the two warheads are compared to see if the thickness distribution of these lines is consistent. Nowadays, the detection technology at home and abroad is mostly based on qualitative analysis\cite{1}\cite{2}, these methods are extremely inefficient and have large errors. Because many situations are "paradoxical", it is almost impossible to compare multiple bullets (for example, dozens or hundreds) and the factors that are subject to human interference are large, and the bullet samples are not easy to store and are prone to rust damage. In order to improve the accuracy of warhead trace inspection and analysis, our article adopts relevant methods to reduce the error and noise caused by measurement errors and traces on the data. In this paper, the method of applying invariant moments is proposed to directly obtain the results of the similarity of different warhead traces. By comparing with the method of similarity, an optimal scheme is presented, namely the method based on second-order moment invariant, in order to quantitatively obtain the true shape of bullet marks. The 22 warhead traces in the real database\cite{7} were compared in pairs, and the top 5 warheads with the similarity of each warhead from high to low were obtained.

2. Error Handling of Bullet Marks
2.1. The Mechanism of Error
According to the standards for measuring bullet traces: 1) The reference plane of the measurement is taken as the XOY plane of the spatial rectangular coordinate system fixed on the measuring equipment;
2) The center line of the bullet cylinder is as parallel to the reference plane as possible; 3) The Y-axis is as parallel as possible Y the direction of scratches. It can be determined that the ideal position of the warhead placed in the space rectangular coordinate system is shown in Figure 1. With the center line of the warhead cylinder as the Y axis, the direction is the direction of the bullet, the XOY plane is parallel to the reference plane, and the Z axis is perpendicular to the XOY plane. However, the posture of the warhead is manually adjusted, so the position of the warhead in the spatial Cartesian coordinate system cannot be accurate, and there is a rotation error. Among them, the Y axis cannot be accurately parallel to the direction of the scratches. The axis rotation error and the cylinder center line cannot be accurately parallel to the reference plane caused by the rotation error around the XOY plane. The possible positions of the warhead in the case of incorrect placement are shown in Figure 2. In this case, the space coordinates (x, y, z) of the point are deviated from the actual value. In the actual bullet measurement process, there will be translation error and rotation error. Translation error refers to the error caused by the translation of the ideal state and actual state of the bullet. As shown in Figure 3, the rotation error refers to the ideal state and actual state of the bullet. The angle error below is shown in Figure 4.

![Figure 1](image1.png)  ![Figure 2](image2.png)

**Figure 1.** The ideal state of the bullet.  **Figure 2.** The state of the bullet in the actual situation.

![Figure 3](image3.png)  ![Figure 4](image4.png)

**Figure 3.** Schematic diagram of bullet translation error.  **Figure 4.** Schematic diagram of bullet rotation error.

![Figure 5](image5.png)  ![Figure 6](image6.png)

**Figure 5.** Refined schematic diagram of bullet position change  **Figure 6.** Schematic diagram of eliminating rotation error

### 2.2. Methods to Reduce Errors

#### 2.2.1. Processing method of translation error

As shown in Figure 3, there will be translation errors in the actual bullet trace processing process, and the position change of the bullet is abstracted as the change before and after the cylinder center-line [3]. As shown in Figure 5, suppose the coordinates of the center O of the bottom surface of the warhead at the ideal position are (0,0,0), and the coordinates of C are (0,L,0) (where L is the height of the cylindrical part of the warhead is a certain value), the actual position of the coordinate of the center O is (x',y',z'), the coordinate of C is (x,y,z). Here the plane C'O'C'' is perpendicular to the plane C'O'C', and set the value of angle C'O'C'' is α, the value of angle C'O'C'' is β. Since the point C'' and O'' are on the plane X'O'Y', and the axis coordinate of the plane in the original space rectangular coordinate system is Z', the coordinates of the point C'' and C''' in the new space rectangular coordinate system can be set to be
\((x_{c'}, y_{c'}, z_{c'})\) and \((x_{c''}, y_{c''}, z_{c''})\) respectively. For the coordinates \((x_{c''}, y_{c''}, z_{c''})\) of \(C''\), it satisfies the condition (1)(2):

\[
(1)|\vec{a'}\vec{c'}|=|\vec{a''}\vec{c''}|(2)\cos\alpha = \frac{\vec{a'}\vec{c'}\cdot\vec{a''}\vec{c''}}{|\vec{a'}\vec{c'}||\vec{a''}\vec{c''}|}
\]

The same consideration is given to the coordinates \((x_{c''}, y_{c''}, z_{c''})\) of the point \(C''\), and the following conditions are met (3)(4), based on the above conditions (1)-(4), the solution points \(C'\) and coordinates \(C''\) are established, and the corresponding equations are:

\[
\begin{align*}
\{&x - x_0 - t_x (y - y_0) + z_0 (z - z_0) \} \\
&\{y - y_0 \} \\
&\{z - z_0 \}
\end{align*}
\]

This completes the adjustment of the bullet so that its cylindrical center line is parallel to the y axis, which is the translation error situation shown in Figure 3. \(\eta = \omega' = (x_{c'}, y_{c'}, z_{c'})\) represents the position translation vector of the two warheads from the ideal position to the converted coordinates. Therefore, the position parameters (data) of the warhead in the ideal state only need to remove the influence \(\eta\) of the three components of the translation vector from the adjusted data parameters. The adjusted data information satisfies: the final adjusted position data parameter the first adjusted position parameter equal to the state data parameter adjusted in the first step.

2.2.2. Processing method of rotation error. As shown in Figure 4, the rotation error mainly considers the small changes in the observation area of the warhead caused by manual adjustment. In fact, the coordinates of the observation point in the y-axis direction do not change with the rotation, that is, the rotation error only affects the x-axis and z-axis. The change of the coordinate component in the segments related to the arc \(\theta\) can be converted based on the above conditions (5) (6) by establishing an equation set, and realize the overall conversion of the entire strip area. Secondly, consider the change of the original coordinate \(PQ\) relationship after the position \(\theta\) conversion angle in the schematic diagram of the rotation error change in Fig. 6. Correspond to the fixed coordinates \(y_0\) of any y-axis direction, the selected \(y = y_0\) section is on this plane, and the real coordinates of the point \(P\) and \(Q\) are \(P' = (x_p, y_0, z_p), Q' = (x_q, y_0, z_q)\), after change position the point \(P\) and \(Q\) are change to \(P'' = (x_p', y_0, z_p'), Q'' = (x_q', y_0, z_q')\), there \(x, y_0\) and \(z\) three points satisfy the following relationship:

\[
|PQ|^2 = |P'Q'|^2 \quad \cos \theta = \frac{PQ\cdot PQ'}{|PQ||P'Q'|} \quad (5)
\]

Similar to the first part, the corresponding equations based on the above conditions can be solved to obtain the first position conversion to new coordinate parameters \(P'' = (x_{p''}, y_0, z_{p''})\). In fact, any point corresponding to the arc \(PQ\) can be converted based on the above conditions (5) (6) by establishing an equation set, and realize the overall conversion of the entire strip area. Second, consider the coordinate conversion from the position \(P''Q''\) converted in the first step to the ideal position. Suppose the actual bandwidth \(PQ\) is \(2d\), then the coordinates of the point \(Q\) are \(Q = (d, y_0, \sqrt{r^2-d^2})\), here \(r\) is half of the diameter of the bottom surface of the warhead \((7.90\text{mm})\), then the translation transformation vector of the second step is \(\bar{Q}Q\), and its value is \(\bar{Q}Q = (d - x_q, 0, \sqrt{r^2-d^2} - z_q)\). Therefore, according to the vector \(\bar{Q}Q\), the coordinate relationship obtained by the first conversion can be directly transformed in one step, that is, the final adjusted state data parameters satisfy: The final adjusted state data parameter equal to the state data parameter adjusted in the first step \(PQ\). In this way, the error caused by the original rotation is converted to the ideal position after one-step coordinate rotation transformation and
one-step coordinate translation transformation. In summary, for the warhead in any position and posture, the position adjustment and one-step translation conversion should be performed first, and then the posture adjustment and one-step translation conversion should be performed for the rotation error, and finally the current position of any warhead should be changed.

3. Moment Invariant

The moment invariant moment function has a wide range of applications in image analysis [4], such as pattern recognition, target classification, target recognition and direction estimation, image coding and reconstruction, etc. The feature comparison of images is carried out by examining the moments of the regions on the image plane. For a digital image function \( f(x,y) \), if it is segmented continuous and only a finite number of points on the XY plane is not zero, it can be proved that its various steps exist. The moment of the area is calculated using all the points that belong to the area, so it is not affected by noise. The \( m_{pq} \) is defined as (7), there can prove \( m_{pq} \) is determined by \( f(x,y) \), otherwise \( m_{pq} \) determine \( f(x,y) \).

\[
m_{pq} = \sum_{x} \sum_{y} x^p y^q f(x,y) \quad (7)
\]

\[
\mu_{pq} = \sum_{x} \sum_{y} (x-x_0)^p (y-y_0)^q f(x,y) \quad (8)
\]

where \( x_0 \) and \( y_0 \) is the gray scale promotion corresponding to the barycenter coordinates of the binary image defined in equation (7) and (8). The Normalized central moment of \( f(x,y) \) can be expressed as

\[
\eta_{\gamma} = \frac{\mu_{pq}}{\mu_{00}}, \text{ there } \gamma = \frac{p + q}{2}, p + q = 2,3,\ldots
\]

The first-order moment is related to the shape, the second-order moment shows the extent of the curve around the average value of the straight line, and the third-order moment is a measurement of the symmetry of the average value. Hu[5] uses the second and third order central moments to construct seven invariant moments (the first four are written here), which can keep the image translation, scaling, and rotation unchanged. The specific definitions are as follows:

\[
\phi_1 = \eta_3 + \eta_0 \quad \phi_2 = (\eta_3 - \eta_0)^2 + 4\eta_1 \quad \phi_3 = (\eta_3 - 3\eta_1)^2 + (3\eta_0 - \eta_1)^2 \quad \phi_4 = (\eta_3 + \eta_1)^2 + (\eta_1 + \eta_0)^2
\]

\[
\phi_5 = (\eta_3 - 3\eta_1)(\eta_3 + \eta_1)(\eta_0 + \eta_1)^2 - 3(\eta_0 + \eta_1)(\eta_1 + \eta_0)^2 \\
\phi_6 = (3\eta_3 - 3\eta_1)(\eta_3 + \eta_1)(\eta_0 + \eta_1)^2 - (\eta_0 + \eta_1)(\eta_1 + \eta_0)^2
\]

Studies have shown [5] that only the description of two-dimensional objects based on the invariant moments of the second-order moment is truly independent of rotation, translation and scale. Higher-order moments are very sensitive to errors in the imaging process, small deformations and other factors, so the corresponding invariant moments cannot be used for effective object recognition. Among these invariant moments, only two \( \phi_1, \phi_2 \) are based on second-order moments, and the rest \( \phi_3, \ldots, \phi_6 \) are based on third-order moments. Therefore, select these two second-order \( \phi_1, \phi_2 \) moment invariant to process the data.

4. Bullet Mark Comparison Method Based on Invariant Moments

In the practice of public security, it is necessary to judge whether the two warheads were fired by the same gun based on the marks on the warheads. The judgment standard is based on the characteristics of the marks [6]. Among the lines on the image, a relatively bright thick line represents the scratch. The key is to find the width and depth of the scratch as shown in Figure 7 and Figure 8. The width of the scratch can be calculated by using edge detection to calculate the upper and lower lines of the scratch to obtain the size of the trace area. The depth of the scratch is reflected in the brightness of the scratch, and a larger brightness indicates a deeper depth.
4.1. Data Prepossessing

4.2. Trace Comparison based on Second-order Moment Invariant

Based on the theory of moment invariant the vertical coordinates $z_{kj}$, $k=1,2,3,4$, $j=1,2,\ldots,46384$ in the first scratch data of a warhead are calculated as follows:

**Step 1**: For the $i$-th fixed $y$-axis coordinate, compose the corresponding $z$-coordinate values into a column, where $i=1,2...756$ to the following $564 \times 756$ order matrix construct by $z_{i}$:

$$z_{i} = \begin{bmatrix} z_{i1} & z_{i2} & \cdots & z_{iN} \\ z_{i1} & z_{i2} & \cdots & z_{iN} \\ \vdots & \vdots & \ddots & \vdots \\ z_{i1} & z_{i2} & \cdots & z_{iN} \end{bmatrix}$$

**Step 2**: Perform the following normalization processing on the above data: For any element $z_{ij}^{p}$ in the matrix $Z_{i}$, convert it to

$$z_{ij}^{p} = \frac{z_{ij}^{p} - \min_{1 \leq i \leq 756} |z_{ij}^{p}|}{\max_{1 \leq i \leq 756} |z_{ij}^{p}| - \min_{1 \leq i \leq 756} |z_{ij}^{p}|}$$

In this way, any scratch data of any warhead is converted to the interval $[0,1]$. Based on the above normalization operation, we convert the digital information of the scratch into the grayscale value of the image, Then compare the bullet mark information based on the image's invariant moment theory.

**Step 3**: Under a certain moment invariant, suppose that the moment invariant value $p_{k}$ of the group $p(p=1,2,\ldots,6)$ of data representing the first warhead of the gun, set $C$ represents the transposition of the subscripts of the array in a certain order, which can be recorded as

$$C = \{(p_{1},p_{2},p_{3},p_{4}), (p_{5},p_{6},p_{7},p_{8}), \ldots \}$$

Define a certain permutation in the above transposed set $C$ of the first warhead and the second warhead of a certain gun $P$, $\varphi = (\varphi_{1},\varphi_{2},\varphi_{3},\varphi_{4}) \in C$ represent the distance $D_{p}$ is $D_{p} = \sum_{i=1}^{4} |\varphi_{i} - \varphi_{i}^{p}|$, it is defined as the error $E_{p}$ between the first warhead and the second warhead moment of a gun as $E_{p} = \min_{\varphi \in C} |D_{p}|$

**Step 4**: Based on the error of the two warheads of the same gun under a certain constant moment obtained in the third step, the corresponding relationship between the four scratches of the two warheads can be determined (the smaller the error, the higher the degree of similarity).

5. Experimental Results and Analysis

Our paper uses database of question B in the 6th Graduate Mathematical Modeling Contest[7], using the correlation coefficient method and the method based on invariant second moments to measure the similarity of bullet marks in database 1 Calculate (see Table 1) to match the bullets, and then get the most effective method which the bullet matching method based on second-order invariant moments,
and apply it to the matching of the 22 bullet traces in the database 2, and get the top 5 bullets with the similarity of each bullet from high to low. It can be seen from Table 2 that the C1 edge of bullet T2 corresponds to the C3 edge of T1, and the C2 edge of T2 corresponds to the C2 edge of T1.

Table 1. Comparison of the 4 sub-edges of bullets in database 1.

| T2_1203959 | C1  | C2  | C3  | C4  |
|------------|-----|-----|-----|-----|
| T1_1203959 | 0.8124 | 0.9034 | 0.8522 | 0.9255 |
|            | 0.9073 | 0.9437 | 0.9450 | 0.9055 |
|            | 0.9470 | 0.7904 | 0.7904 | 0.8524 |
|            | 0.9467 | 0.8244 | 0.8371 | 0.9203 |

Table 2. Results of bullet and second-order moment invariant in database 1.

| Secondary order | T1 c1 c2 c3 c4 | c2 c3 c4 c1 | c3 c4 c1 c2 | c4 c1 c2 c3 |
|-----------------|----------------|-------------|-------------|-------------|
| T1_1203959      | 0.005482752   | 0.003105644 | 0.001868161 | 0.004872654 |
| T2_1203959      | 0.000060993   | 0.000050642 | 0.000013700 | 0.000036543 |
| T1_1504519      | 0.017653930   | 0.015363052 | 0.016105567 | 0.014086976 |
| T2_1504519      | 0.000131405   | 0.000088172 | 0.000100293 | 0.000080078 |
| T1_1811345      | 0.001612702   | 0.000578596 | 0.002147176 | 0.001799434 |
| T2_1811345      | 0.000038741   | 0.000033959 | 0.000067261 | 0.000035647 |

It can be seen from Table 2 that the results obtained by the second-order moment invariant method are consistent, which are exactly the same as the actual results given in [7], that is, the matching rate is 100%. It can be seen from the results that based on the method, the corresponding warhead moment (minimum distance) error is smaller, so this method is more effective.

6. Conclusion
In this paper, eliminating the errors (translation error and rotation error) of the bullet traces during the measurement process, the application of the second-order invariant moments in the bullet trace matching are studied, and compared with the similarity method. The experiment shows that method based on the second invariant moment has very small error and high matching rate. Finally, this method was applied to the Ninth National Graduate Mathematical Modeling, and the top 5 bullets with 22 rounds of warhead trace matching degree given in the question were obtained from high to low.

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