Surface and bulk polaritons in a PML-type magnetoelectric multiferroic with canted spins: TE and TM polarization

V Gunawan and R L Stamps

School of Physics M013, University of Western Australia, 35 Stirling Highway, Crawley, Western Australia 6009, Australia

E-mail: slamev01@student.uwa.edu.au (V Gunawan)

Received 29 July 2010, in final form 3 September 2010
Published 18 February 2011
Online at stacks.iop.org/JPhysCM/23/105901

Abstract
We present a theory for surface polaritons on ferroelectric–antiferromagnetic materials with canted spin structure. A small uniform canted moment is allowed, resulting in a weak ferromagnetism directed in the plane parallel to the surface. Surface and bulk polariton modes for a semi-infinite film are calculated for the case of transverse electric (TE) and transverse magnetic (TM) polarization. Example results are presented using parameters appropriate for BaMnF4. We find that the surface modes are non-reciprocal for the TE polarization due to the magnetoelectric interaction, and the non-reciprocity can be controlled by an applied electric field. Example results for attenuated total reflection (ATR) are calculated. The magnetoelectric interaction also gives rise to ‘leaky’ surface modes in the case of TM polarization. These are pseudo-surface waves that exist in the pass band, and dissipate energy into the bulk of the material. We show that these pseudo-surface mode frequencies and properties can be modified by temperature and the application of external electric or magnetic fields.

1. Introduction

Magnetic polaritons are electromagnetic waves that travel in a material with dispersion and properties modified through coupling to magnetic excitations [1]. Polaritons can display a number of interesting and useful properties, including localization to surfaces and edges. They can also display non-reciprocity [1–4], whereby the propagation frequency may not be symmetric under direction reversal: i.e. \( \omega(k) \neq \omega(-k) \). Surface polaritons at optical frequencies have received much attention in recent years, and appear in a number of different applications including detectors [5], biosensors [6] and microscopy [7].

Theoretical treatments for ferromagnetic polaritons [2, 8] and simple antiferromagnets [1, 4, 9] were given several years ago. The experimental study of bulk magnetic polaritons dates from the 1980s, when bulk magnetic polaritons were studied using transmission through FeF2 [10] and the dispersion of magnetic polaritons in CoF2 was determined using far-infrared reflectometry [11]. Attenuated total reflection (ATR) experiments have been used to study surface magnetic polaritons on FeF2 [12].

A most interesting class of polaritons are in multiferroic materials, where magnetoelectric interactions couple magnetic and electric responses [3, 13, 14]. A focus of theoretical work has been on bulk modes in linear magnetoelectric coupled media [3, 13]. Surface modes have also been discussed for the case of no applied external magnetic or electric fields [14, 15]. Whereas there are numerous reports from experimental studies of magnetoelectric effects in the older literature [16–19], no measurement of surface polaritons has been reported for multiferroics.

Modification and control of multiferroic surface polaritons through external electric and magnetic fields is an intriguing prospect. A material that has been studied is FeTiO3 [20]. It was shown, through first principle calculations, that it is possible to have a magnetoelectric coupling and canting of the magnetic sub-lattices. A magnetoelectric coupling is believed to operate in the multiferroic BaMnF4 [21]. The canted condition associated with the magnetoelectric coupling in this material has been reported by Venturini [22]. Assuming a linear magnetoelectric coupling, Tilley [21] showed that susceptibilities can have poles at both magnon and phonon frequencies. Bulk polaritons in BaMnF4 were...
studied theoretically by Barnas [3], who showed that the bulk transverse electric (TE) polariton is not affected by magnetoelectric coupling.

In the present paper we discuss in detail how temperature, and electric and magnetic fields affect surface modes in canted spin multiferroics with linear magnetoelectric coupling. In order to make contact with previous work on bulk polaritons, we provide numerical predictions for BaMnF4. A discussion of parameters for BaMnF4 is presented at the start of section 5. We allow for canting of magnetic sub-lattices, and show that canting is very important for understanding and manipulating surface mode frequencies. The appropriate magnetoelectric coupling for BaMnF4 is of the type PML, where polarization of the ferroelectric \( P \) causes the sub-lattice magnetization of the antiferromagnetic (with antiferromagnetic vector \( L \)) to be canting, resulting in a weak ferromagnetism \( M \). By directly including the canting angle in the calculation of the susceptibility, we can detect the effect of magnetoelectric coupling in the TE modes. Moreover, the canting angle gives rise to non-reciprocity for the TE modes.

We also show that the transverse magnetic (TM) field polarization surface polariton excitations are in fact pseudo-surface modes characterized by complex propagation wavevectors whose imaginary parts are proportional to the strength of the magnetoelectric interaction. On the other hand, transverse electric surface polaritons are pure surface modes and display non-reciprocity. Most significantly, we show that the non-reciprocity can be controlled through an applied electric field.

The paper is organized as follows. The geometry and energy density of the system are defined in section 2, where we also discuss the canting angle in relation to the magnetoelectric coupling. In section 3, susceptibilities are derived using dynamic equations. The electromagnetic problem is solved in section 4 and results given in section 5 for surface and bulk modes on BaMnF4. Effects on surface mode properties due to possible modifications of material parameters are also discussed. Conclusions are given in section 6.

2. Geometry

The geometry is sketched in figure 1. We consider a semi-infinite multiferroic film that fills the half space \( z < 0 \). The magnetic component of the multiferroic is a two sub-lattice antiferromagnet with uniaxial magnetic anisotropy. The two magnetic sub-lattices are allowed to cant in the \( x-z \) plane with canting angle, \( \theta \). We assume symmetric canting such that \( |\mathbf{M}_a| = |\mathbf{M}_b| = M_c \). The canting generates a weak ferromagnetisation which is perpendicular to the spontaneous polarization. Both the weak ferromagnetic moment and spontaneous polarization are constrained to lie in \( x-y \) plane, parallel to the surface. The magnetic easy axis is out-of-plane, along the \( z \) direction. An external electric field is applied parallel to the spontaneous polarization, and an external magnetic field is applied along the weak ferromagnet moment.

We assume that the surface modes propagate on the surface along the \( \hat{y} \) direction. There are two polarizations to be considered for the polaritons problem: TE, with the electric field parallel to the surface; and TM with the magnetic field parallel to the surface. For TE polarization, the electric field lies on the surface in the \( \hat{x} \) direction while the magnetic field has \( H_x \) and \( H_y \) components. The TM polarization has the magnetic field in the \( \hat{x} \) direction while the electric field has \( E_y \) and \( E_z \) components.

A fourth order Ginzburg–Landau (G–L) energy density is assumed to describe the dielectric contribution to the energy:

\[
F_e = \frac{1}{2} \xi_1 P_x^2 + \frac{1}{2} \xi_2 P_y^2 + \frac{1}{4} \Delta_1 (P_x^2 + P_y^2) + \frac{1}{4} \Delta_2 (P_x^4 + P_y^4) - P_x E_y. \tag{1}
\]

The first and second terms on the right-hand side of equation (1) represent the energy density for the \( y \) component of the polarization with \( \xi_1 \) and \( \xi_2 \) dielectric stiffnesses. The third and fourth terms represent the contribution of \( x \) and \( z \) polarization components with dielectric stiffnesses \( \Delta_1 \) and \( \Delta_2 \). The final term is the external electric field applied parallel to the spontaneous polarization \( P_y \).

We restrict our theory to long wavelengths, so that spatial dispersion is neglected. The magnetic contribution to the energy density is then assumed to be of the form:

\[
F_M = -\lambda (\mathbf{M}_a \cdot \mathbf{M}_b - \frac{K}{2} [\nabla (\mathbf{M}_a \cdot \hat{z})^2 + \nabla (\mathbf{M}_b \cdot \hat{z})^2]) - (\mathbf{M}_a + \mathbf{M}_b)_0 \cdot \mathbf{H}_0. \tag{2}
\]

The first term on the right-hand side of equation (2) is an exchange energy with a strength \( \lambda < 0 \). The second term represents the anisotropy energy with anisotropy constant \( K \) and the last term is the Zeeman energy from an external magnetic field. Although the easy axis is out-of-plane, we assume that a canted antiferromagnet produces a negligible magnetization along the \( z \) direction and therefore ignore demagnetization effects.

The magnetoelectric coupling is assumed to be of the Dzyaloshinskii–Moriya (DM) form, appropriate to materials such as FeTiO3 [20] and BaMnF4 [3, 13, 21]. As in [21], we
suppose a PML coupling of the form

\[ F_{\text{ME}} = -\alpha P_y M_x L_z \]
\[ = -\alpha P_y \langle (\vec{M}_y \times \vec{M}_z) - \alpha \langle (M_y)_z (M_z)_x \rangle - (M_z)_x (M_x)_z \rangle \]  

(3)

where

\[ M_x = (\vec{M}_x + \vec{M}_y)_z = 2 M_x \sin \theta \]  

(4)

\[ L_z = (\vec{M}_y - \vec{M}_z)_z = 2 M_y \cos \theta \]  

(5)

and \( \alpha \) is the magnetoelastic coupling constant. The component \( M_x \) is the transverse component of magnetization and represents the weak ferromagnetism. The component \( L_z \) is the magnitude of the longitudinal component of the magnetization. The first term in equation (3) represents the polarization induced DM interaction in our model, and we assume that this term is responsible for generating canting in the magnetic sub-lattices.

The canting angle \( \theta \) is determined by minimizing the magnetic and magnetoelastic energies with respect to \( \theta \). Minimizing, we arrive at the condition

\[ H_n \cos \theta - \frac{1}{2} K M_s \sin 2 \theta + 2 \alpha P_y M_x \cos 2 \theta + \lambda M_s \sin 2 \theta = 0. \]  

(6)

In the absence of an external magnetic field, equation (6) simplifies to

\[ \tan(2 \theta) = \frac{4 \alpha P_y}{K - 2 \lambda}. \]  

(7)

Note that a positive magnetoelastic constant describes a weak ferromagnetism \( M_s \) aligned along \(-x\).

The canting angle \( \theta \) depends on the equilibrium magnitude of \( P_y \), and this is found by minimizing the dielectric and magnetoelastic energies. Requiring \( \frac{\partial}{\partial P_y} (F_E + F_{\text{ME}}) = 0 \) results in

\[ \zeta_1 P_y + \zeta_2 P_y^3 = 2 \alpha M_x^2 \sin 2 \theta - E_y = 0. \]  

(8)

Here, \( \zeta_1 \) depends on temperature according to \( \zeta_1 = \xi (T - T_c) \), where \( T_c \) represents ferroelectric Curie temperature. Likewise, the magnitude of \( M_s \) depends on temperature. While a G–L expansion is appropriate for the ferroelectric, a mean field approximation for the magnet results in a Brillouin function \( B(\eta) \), valid for low temperatures and appropriate to the case when \( T_N < T_c \):

\[ M_x = M_x(0) B(\eta) \]  

(9)

where

\[ \eta = \frac{3 \mu_0 S}{k_B T} \left[ -\lambda M_s \cos 2 \theta + K M_s \cos^2 \theta + 2 \alpha P_y \sin 2 \theta + H_0 \sin \theta \right]. \]  

(10)

The spontaneous polarization, magnetization and canting angle are calculated by solving simultaneously equations (6), (8) and (9). The solution for general angles is found numerically using root finding techniques for coupled transcendental equations.

3. Dynamic susceptibility

In order to solve the electromagnetic boundary value problem for the surface and bulk polariton modes, we need constitutive relations for the dielectric and magnetic responses. We consider a linear response and calculate the permittivity and permeability using equations of motion derived from equations (1), (2) and (3). We start by obtaining the magnetic and electric susceptibility from the equations of motion for the magnetic and electric response, these are given by the magnetic torque equation and the Landau–Khalatnikov equation of motion, as in [14, 15, 24]. The magnetic and electric susceptibilities provide information about the resonant response of the spin and electric dipoles. The dynamic susceptibilities can be obtained from the magnetic and electric equations of motion without damping. The magnetic torque equations are of the usual form [1, 4, 23]:

\[ \dot{M} = \gamma \vec{M} \times \left( \frac{\partial}{\partial M} (F_M + F_{\text{ME}}) \right) \]  

(11)

and the dielectric response is obtained from [14, 15, 24]

\[ \vec{P} = -\frac{\partial}{\partial P} (F_E + F_{\text{ME}}) \]  

(12)

where \( \gamma \) and \( f \) are the gyromagnetic ratio and the inverse of the phonon mass.

For a small amplitude response, the magnetization and polarization can be written as

\[ \vec{M}_d = (m_x^d, m_y^d, M_z + m_z^d) \]  

(13)

\[ \vec{M}_b = (m_x^b, m_y^b, M_z + m_z^b) \]  

(14)

\[ \vec{P} = (p_x, p_y, p_z) \]  

(15)

where \( m_x^d, m_y^b \) and \( p_i \) represent the dynamic components proportional to \( e^{-i\omega t} \).

The set of dynamic equations after linearization are

\[ -i \omega m_x = (\omega_o \cos \theta + 2 \omega_{\text{me}} \sin \theta) l_x \]  

(16)

\[ -i \omega m_y = 2 \gamma M_x h_y \sin \theta + \omega_o \cos \theta + 2 \omega_{\text{me}} \cos \theta) l_y \]  

(17)

\[ -i \omega m_z = 2 \gamma M_y h_z \sin \theta + (\omega_o + 2 \omega_{\text{me}} \cos \theta) m_z \]  

(18)

\[ -i \omega l_x = 2 \gamma M_x h_y \cos \theta - (\omega_o \cos \theta + 2 \omega_{\text{me}} \sin \theta) m_y - 2 \omega_{\text{ex}} \cos \theta) m_y \]  

(19)

\[ -i \omega l_y = (2 \omega_{\text{ex}} \cos \theta - \omega_o \cos \theta - 4 \omega_{\text{me}} \sin \theta) m_x + (2 \omega_{\text{ex}} \sin \theta + 4 \omega_{\text{me}} \cos \theta \sin \theta + \omega_o) l_x + 4 \gamma \omega M_x^2 \cos \theta \sin \theta + 2 \gamma M_y h_z \cos \theta \]  

(20)

\[ -i \omega l_z = -2 \omega_{\text{ex}} \sin \theta + 2 \omega_{\text{me}} \cos \theta \sin \theta + \omega_o \]  

(21)

\[ \frac{\omega^2}{f} p_x = \Delta_1 p_x - e_x \]  

(22)

\[ \frac{\omega^2}{f} p_y = (\zeta_1 + 3 \zeta_2) p_y - 2 \alpha M_x (m_x \cos \theta + l_z \sin \theta) - e_y \]  

(23)

and

\[ \frac{\omega^2}{f} p_z = \Delta_1 p_z - e_z \]  

(24)
where \( I_i = m_i^0 - m_i \) and \( M_i = m_i^0 + m_i \). The notation used above is in units of frequency and defined by \( \omega_0 = \gamma \omega_M \) as the magnetic anisotropy, \( \omega_{ex} = \gamma \omega_M \) as the exchange, \( \omega_{me} = \gamma \alpha \omega_M \) as the magnetoelectric coupling and \( \omega_s = \gamma \omega_M \) as the external magnetic field. There are two groups of coupled equations from the set of dynamic equations above. The first group consists of equations (17), (18) and (19) and their solution gives the magnetic susceptibilities \( \chi^m \), \( \chi^e \) and \( \chi^{me} \). These give the magnetic dynamics, these susceptibilities are associated with \( \omega \) above is in units of frequency and defined by

\[ \omega = \frac{\omega_0}{\omega_1 + \omega_2} \]

The magnetic field and is given by \( \omega_1 = \omega_{me,TE} + \omega_{ex,TE} \).

The second group of equations are equations (16), (20), (21) and (23) and the uncoupled equation (24). These give the magnetic susceptibility \( \chi^m \), the electric susceptibility \( \chi^e \), the magnetoelectric susceptibilities \( \chi^{me} \) and \( \chi^{em} \). Together with the electric susceptibility \( \chi^e \) given by the electric dynamic equation (22), which is not coupled to the magnetic dynamics, these susceptibilities are associated with TE modes.

For the TM modes, the magnetic dynamics are coupled to the electric dynamics through the ME susceptibilities \( \chi^{me} \) and \( \chi^{em} \) defined by

\[ \tilde{m} = \chi^m \tilde{h} + \chi^{me} \tilde{e} \quad \text{and} \quad \tilde{p} = \chi^e \tilde{e} + \chi^{em} \tilde{h}. \] (32)

The relevant magnetic susceptibilities for TM modes are given by

\[ \chi^m = \frac{1}{2\pi} \alpha_M (\omega_s \cos^2 \theta + \omega_{me} \sin 2\theta) \]

and

\[ \chi^e = f \left( \frac{C_{ey}}{(\omega_{ex} - \omega_{m}^2)} + \frac{C_{my}}{(\omega_{ex} - \omega_{m}^2)} \right). \] (34)

where \( \alpha_M = \gamma 4\pi M_s \). The electric susceptibilities are

\[ \chi^{me} = \frac{f}{(\omega_{ex} - \omega_{m}^2)} \] (35)

and

\[ \chi^{em} = \frac{f}{(\omega_{ex} - \omega_{m}^2)} \] (36)

The frequency \( \omega_{ex} \) is the phonon frequency along the \( \hat{z} \) direction. The magnetoelectric susceptibility is

\[ \chi_{xy}^{me} = \chi_{xy}^{em} = \frac{C_{me}}{(\omega_{ex} - \omega_{m}^2)} + \frac{C_{me}}{(\omega_{ex} - \omega_{m}^2)}. \]

The frequencies \( \omega_{ex} \) and \( \omega_{m} \) are defined as \( \omega_{ex}^2 = \omega_{m}^2 + \delta \) and \( \omega_{m}^2 = \omega_{m}^2 - \delta \), where \( \delta \) is expressed in the form

\[ \delta = \frac{1}{2} \left[ \left( \omega_{ex} - \omega_{m}^2 \right)^2 - 4\Omega_{o,TE}^2 \right] \]

where

\[ \Omega_{o,TE} = \gamma \omega_0 \cos 2\theta \] (38)

with \( \gamma_C = \frac{\gamma}{2\alpha^2 M_s^2} \). \( \omega_{o,TE} \) is the frequency of the soft phonon along the spontaneous polarization and \( \omega_{o,TE} \) is the magnetic resonance frequency given by

\[ \omega_{o,TE}^2 = \omega_{o,TE}^2 + \Omega_{o,TE}^2. \] (39)

Here \( \omega_{o,TE} \) is the canting antiferromagnet resonance frequency,

\[ \omega_{o,TE}^2 = \omega_{o,TE}^2 + \Omega_{o,TE}^2. \] (40)

\( \omega_{o,TE} \) is related to the magnetoelectric interaction,

\[ \Omega_{o,TE}^2 = 8\omega_{me}^2 + 2\omega_{me} \omega_{ex}^2 \sin 2\theta \] (41)

and \( \omega_{o,TE} \) is related to the external magnetic field,

\[ \Omega_{o,TE}^2 = \omega_{o,TE}^2 + 6\omega_{me} \omega_{ex} \cos \theta \] (42)

Other parameters in equations (33)–(36) are defined as

\[ C_{mx} = \frac{(\omega_{ex} - \omega_{m}^2)}{(\omega_{ex}^2 - \omega_{m}^2)} \] (43)

\[ C_{ex} = \delta \frac{(\omega_{ex}^2 - \omega_{m}^2)}{(\omega_{ex}^2 - \omega_{m}^2)} \] (44)

\[ C_{ey} = \frac{(\omega_{ex}^2 - \omega_{m}^2)}{(\omega_{ex}^2 - \omega_{m}^2)} \] (45)
For the TE modes, the constitutive relation connecting the attenuation factor of the sample and vacuum, respectively. An implicit relation for the constitutive equations connecting the susceptibilities is given by

\[ C_{\text{me}} = C_{\text{ef}} \alpha_{0} \cos 2\theta (\alpha_{0} \alpha_{\text{me}} \sin \theta) / (\alpha_{\text{eff}}^2 - \alpha_{0}^2) \]  

(47)

where \( C_{\text{ef}} = \frac{1}{\mu_{0} M_{s} f} \).

From the expression of the susceptibilities above, it can be seen that the applied magnetic field directly influences the susceptibilities through \( \omega_{2} \). By way of contrast, the applied electric field changes the susceptibilities indirectly by affecting the magnitude of the spontaneous polarization.

4. Theory for bulk bands and surface modes

4.1. Dispersion relation for TE modes

Since there is no dynamic ME coupling in TE modes, the dispersion relation for bulk polaritons can be calculated from the macroscopic electromagnetic wave equation

\[ \nabla^2 \tilde{H} - \nabla (\nabla \cdot \tilde{H}) - \frac{\varepsilon}{\varepsilon_{0}} \beta^2 (\mu_{0} \gamma \sin \theta) = 0. \]  

(48)

For the TE modes, the constitutive relation connecting \( \tilde{B} \) with \( \tilde{H} \) is defined by \( \tilde{\mu} = 1 + 4\pi \chi_{m} \). The solution of the wave equation for the bulk modes is of the form

\[ \tilde{H} = e^{i(k_{x}x + k_{z}z - \omega c t)} \]  

(49)

and the implicit expression for the bulk mode frequency is

\[ \mu_{y} k_{y}^2 = \epsilon_{z} \omega^2 / c^2 (\mu_{0} \mu_{z} - \mu_{y}^2). \]  

(50)

Equation (50) has two zeros, one for \( \epsilon_{z} = 0 \) and the other for \( f(\mu) = \mu_{0} \mu_{z} - \mu_{y}^2 = 0 \). It also has two resonance poles from \( \epsilon_{x} \) and \( \epsilon_{y} \) from the condition \( \mu_{x} = 0 \). As a result, there are three bands of bulk polariton modes.

The dispersion relation for surface modes is obtained by assuming solutions in the form

\[ \tilde{H} = e^{i\beta_{z} z} e^{i(k_{x}x + k_{y}y - \omega c t)} \quad \text{for} \quad z < 0 \]  

(51)

\[ \tilde{H} = e^{-i\beta_{z} z} e^{i(k_{x}x + k_{y}y - \omega c t)} \quad \text{for} \quad z > 0 \]  

(52)

where \( \beta \) and \( \beta_{0} \) are positive real attenuation constants for the sample and vacuum, respectively. An implicit relation for the attenuation factor \( \beta \) of the medium is derived by substituting equation (51) into the wave equation (48):

\[ \mu_{z} \beta^2 = \mu_{y} k_{y}^2 - \epsilon_{z} \omega^2 / c^2 (\mu_{0} \mu_{z} - \mu_{y}^2). \]  

(53)

An explicit relation for the attenuation constant \( \beta_{0} \) (in a vacuum) is given by

\[ \beta_{0}^2 = k_{y}^2 - \left( \frac{\omega}{c} \right)^2. \]  

(54)

An implicit surface mode dispersion relation is calculated by requiring continuity of tangential \( \tilde{H} \) and normal \( \tilde{B} \) at the interface \( z = 0 \). Using equations (53) and (54), we find

\[ \mu_{z} \beta + (\mu_{0} \mu_{z} - \mu_{y}^2) \beta_{0} + \mu_{y} k_{y} = 0. \]  

(55)

The dispersion implied by equation (55) describes magnetic surface polaritons for the weak ferromagnet. From symmetry considerations, we know that magnetic surface polaritons can be non-reciprocal [25]. The direction of \( k_{y} \) matters in equation (55) and non-reciprocal propagation arises, as we will discuss later.

A key point is that the non-reciprocity of the surface modes depends strongly on the canting angle. If the canting angle is zero, then \( \mu_{z} \rightarrow 0 \) and \( \mu_{y} \rightarrow 1 \), and the dispersion relation in equation (55) becomes

\[ \beta + \mu_{0} \beta_{0} = 0. \]  

(56)

In that case, the surface modes are reciprocal under reversal of \( k \).

4.2. Dispersion relation for TM modes

The electric and magnetic dynamics are coupled for the TM polarization, and the dispersion relations for the bulk modes are obtained by solving the electromagnetic Maxwell equations

\[ \nabla \times \tilde{E} = -\frac{1}{\varepsilon} \frac{\partial \tilde{B}}{\partial t} \quad \text{and} \quad \nabla \times \tilde{H} = \frac{1}{c} \frac{\partial \tilde{D}}{\partial t} \]  

(57)

where the fields \( \tilde{D}, \tilde{E}, \tilde{B}, \tilde{H} \) are connected through the constitutive equations

\[ D_{x} = \mu_{0} H_{y} + 4\pi \chi_{m} E_{y} \quad \text{and} \quad D_{z} = \epsilon_{x} E_{z} + 4\pi \chi_{m} H_{y}. \]  

(58)

with permeability and dielectric functions defined as \( \mu_{x} = 1 + 4\pi \chi_{m}^{x} \) and \( \epsilon_{y} = 1 + 4\pi \chi_{m}^{y} \), respectively.

Plane waves are assumed for bulk traveling modes of the form \( \tilde{E}, \tilde{H} = e^{i(k_{x}x + k_{z}z - \omega c t)} \). Substitution into the Maxwell equations provides an equation for the bulk mode dispersion:

\[ \epsilon_{x} \left( \epsilon_{y} \mu_{x} \left( \frac{\omega}{c} \right)^2 - k_{x}^2 \right) - \epsilon_{z} \left( 4\pi \chi_{m}^{x} \frac{\omega}{c} \right)^2 = 0. \]  

(59)

The dispersion relation has solutions determined by the zeros of the dielectric constant \( \epsilon_{x} \) and zeros of the function \( f(\mu) = \mu_{x} \epsilon_{y} \mu_{x} - (4\pi \chi_{m}^{x})^2 \). Equation (59) diverges at the poles of \( \epsilon_{x}, \epsilon_{y} \) and \( \chi_{me} \).

The dispersion relation for surface modes is calculated by assuming surface localized plane wave solutions of the form

\[ \tilde{E}, \tilde{H} = e^{i\beta_{z} z} e^{i(k_{x}x + k_{y}y - \omega c t)} \quad \text{for} \quad z < 0 \]  

(60)

\[ \tilde{E}, \tilde{H} = e^{-i\beta_{z} z} e^{i(k_{x}x + k_{y}y - \omega c t)} \quad \text{for} \quad z > 0 \]  

(61)

where \( \beta = \beta_{0} = \beta_{z} \) are attenuation constants for the sample and vacuum respectively. Substitution of equation (60) into (57) provides an implicit relation for the attenuation constant in the material:

\[ \epsilon_{z} \left( \beta + i4\pi \chi_{m}^{x} \frac{\omega}{c} \right)^2 = \epsilon_{y} k_{y}^2 - \epsilon_{z} \epsilon_{y} \mu_{x} \left( \frac{\omega}{c} \right)^2. \]  

(62)

The attenuation constant in the vacuum, \( \beta_{0} \), is given in equation (54).
Similar to the TE modes, an implicit solution for the surface wave frequencies is found by matching the solutions in equations (60) and (61) at z = 0 using electromagnetic boundary conditions. The unique conditions are continuity of tangential \( H \), continuity of tangential \( E \) and continuity of normal \( D \). When satisfied, the following dispersion relation results:

\[
(\beta + i4\pi \chi_{\text{me}} \frac{\omega}{c}) + \varepsilon_s \beta_0 = 0.\]  

(63)

Note that the surface modes described by equation (63) should be reciprocal in the sense that \( \omega(k) = \omega(-k) \). We also see that the existence of surface modes strongly depends on the value of \( \varepsilon_s \), since the solution of equation (63) for surface modes can only be found when the value of \( \varepsilon_s \) is negative.

5. Application to BaMnF\(_4\)

We now illustrate the preceding theory for the same material, BaMnF\(_4\). Parameters appropriate for BaMnF\(_4\) were determined as follows. Measured values of the weak ferromagnetism [26], \( M_c = 18.347 \) Oe, and a canting angle [22] of 3 mrad at \( T = 4.2 \) K, inserted into the relation \( M_c = 2M_i \sin \theta \), yield the magnetization of the sub-lattices \( M_i = 3.054 \times 10^3 \) Oe.

Following the approximations by Holmes [27] for the exchange field, \( H_E = 50 \) T, and the relation \( H_E = \lambda M_c \), the exchange constant is \( \lambda = 163.72 \) erg cm\(^{-3}\) Oe\(^{-2}\). Using the measured value of the magnetic resonance frequency [28], \( \omega_r = 3 \) cm\(^{-1}\), in the relation \( \omega_r = \gamma^2 K (K - 2\lambda) M_i^2 \), we obtain the anisotropy constant \( K = 0.337 \) erg cm\(^{-3}\) Oe\(^{-2}\).

The linear magnetoelectric coupling \( \alpha \) is obtained by using the calculated spontaneous polarization [29] \( P_s = 3.45 \times 10^4 \) statC cm\(^{-2}\) in equation (7), yielding \( \alpha = 1.42 \times 10^{-5} \) cm\(^2\)/statC. The G–L constants \( \zeta_1 \) and \( \zeta_2 \) are approximated by solving simultaneously equation (8) and \( (\zeta_1 + 3\zeta_2 P_s^2) f = \omega_{cy}^2 \), the soft phonon frequency along the spontaneous polarization. With the inverse phonon mass [3] as \( f = 33.9 \) THz and transversal phonon frequency [28] \( \omega_{cy} = 7.73 \) THz, this gives \( \zeta_1 = -10.528 \) and \( \zeta_2 = 1.934 \times 10^{-8} \) cm\(^3\) erg\(^{-1}\) at \( T = 4.2 \) K using \( T_c = 1113 \) K. The susceptibility \( \chi_x \) is calculated by using the transverse phonon frequency for the \( \hat{z} \) polarization [3], \( \omega_{cz} = 33.7 \) cm\(^{-1}\).

5.1. TE modes

In previous investigations [28], the dielectric constant in the \( \hat{x} \) direction has been assumed to be independent of frequency with the value \( \varepsilon_s = 8.3 \). This assumption is valid for magnetic polaritons if the dielectric and magnetic responses lie in very different frequency ranges. As a result, only two bands for bulk magnetic polaritons can exist since only one pole and one zero are contributed by the magnetic sub-system. We also make this assumption in what follows, since BaMnF\(_4\) has a wide separation in frequency between the dielectric and magnetic resonances. At the end of this section we discuss the consequences of relaxing this assumption.

A positive value of real \( \beta \) is needed for the solution of the surface modes in equation (51). Using the above parameters, the bulk mode region is bounded by \( k = 0 \) cm\(^{-1}\) and \( k = 16 \) cm\(^{-1}\), as shown in figure 2(a), and is indicated in figure 2(b) by shading. Comparison of figures 2(a) and (b) shows that the resonance at \( \omega_m \approx 3 \) cm\(^{-1}\) is associated with a divergence of the attenuation constant \( \beta \) due to the zero of \( \mu_z \). From this zero condition, an expression for \( \omega_m \) can be derived:

\[
\omega_m = \left[ (\omega_{\text{me}}^2 \cos^2 \theta + \Omega_{\text{me,TE}}^2 + 2\omega_{\text{me}} \omega_{\text{me}} \sin 2\theta) \right]^{1/2}
\]  

(64)

with \( \omega_{\text{me}} = \gamma 4\pi M_i \).
Figure 3. ATR spectra with incident angles 30° (a) and 70° (b). In (a) two different sharp dips illustrate the non-reciprocity of the surface modes. In (b) the absence of the sharp dips indicate the absence of surface modes.

Figure 4. The condition where surface modes in $k \gg \omega/c$ exist. In (a) a solution exists using parameters $\lambda/\alpha = 1.5$, where the surface begin to overcome the bulk. In (b) the surface branch is slightly higher than bulk when the ratio increases to $\lambda/\alpha = 2$.

The pole and zero frequencies of the bulk modes, $\omega_p$ and $\omega_z$, are related to the zeros of $\mu_y$ and $f(\mu) = \mu_y \mu_z - \mu_z^2$. These give

$$\omega_p = \left[\omega_{2am}^2 \cos^2 \theta + \Omega_{mc,TE}^2 + (2\omega_m \omega_s \cos 2\theta)\right]^{1/2}$$

and

$$\omega_z = \frac{1}{\sqrt{2}}\left[\Omega_{mp}^2 + (\Omega_{mp}^2 - 4\omega_m^2 \omega_p^2)^{1/2}\right]^{1/2}$$

where

$$\Omega_{mp}^2 = \omega_m^2 + \omega_p^2 + 4\omega_s^2 \sin^2 \theta.$$  

These three frequencies, $\omega_m$, $\omega_p$, and $\omega_z$, divide the bulk region into the shaded regions shown in figure 2(b). Note that the lowest two bands overlap, and are deformed strongly near the resonance $\omega_m$.

Surface modes, which are indicated by ‘SP’ in figure 2(b), can exist in the gaps between $\omega_m$ and $\omega_p$. The surface branches are non-reciprocal with respect to propagation, with $\omega(k) \neq \omega(-k)$. The negative branch begins from the intersection of the lightline with the $\omega_m$ resonance, and is indicated by ‘-L’. The branch terminates in the left middle bulk band. The positive branch begins at the lightline with frequency

$$\omega = \left[\epsilon_x \omega_{2am}^2 - \omega_{2am}^2 \cos^2 \theta + \Omega_{mc,TE}^2 \right]^{1/2}$$

which is slightly higher than $\omega_m$, and then terminates at the upper bulk band.

It is also shown in figure 2(b), that the positive and negative branches of surface modes intersect with the 30° ATR lines at frequencies slightly above and below 3.05 cm$^{-1}$. However, there is no intersection between the surface modes and the 70° ATR lines.

A method for detecting the surface modes at THz frequencies is ATR [1] and has been used to study surface
The ATR spectra at 30° angle of 70° negative surface mode branches. The ATR for an incidence are the intersection points discussed above for the positive and polaritons traveling in opposite directions and the frequencies of surface modes. The two sharp dips correspond to surface with d and the wavevector r of the sample, and r is defined as

\[ R = \frac{k_z^2(1 + e^{-2\beta d}) - i \beta_0(1 - e^{-2\beta d})}{k_z^2(1 + e^{-2\beta d}) + i \beta_0(1 - e^{-2\beta d})} \]  

where d represents the distance between the prism and the sample, and r is defined as

\[ r = \frac{\beta_0 - \kappa}{\beta_0 + \kappa} \]  

with

\[ \kappa = \frac{\mu_s \beta - \mu_y k_y}{\mu_s \mu_z - \mu_y^2}. \]  

Here, the wavevector

\[ k_y = (\epsilon_p)^{1/2} \frac{\omega \sin \theta}{c} \quad \text{and} \quad k_z = (\epsilon_p)^{1/2} \frac{\omega \cos \theta}{c} \]  

represents propagation along and normal to the surface, where \( \epsilon_p \) is the prism dielectric constant, and \( \theta \) is the incidence angle.

Calculated ATR results for the BaMnF_4 are presented in figures 3(a) and (b) for the incidence angles 30° and 70°. The ATR spectra at 30° illustrate nicely the non-reciprocity of surface modes. The two sharp dips correspond to surface polaritons traveling in opposite directions and the frequencies are the intersection points discussed above for the positive and negative surface mode branches. The ATR for an incidence angle of 70°, shown in figure 3(b), does not allow coupling to surface modes. The shallow dips represent bulk modes.

It is interesting to note that the surface modes do not exist in the region where the wavevector \( k \gg \frac{\omega}{c} \). In this region, the surface dispersion relation in equation (55) can be re-written in the form

\[ \mu_y \mu_z - (\mu_y + 1)^2 = 0 \]  

for \( k = \pm \infty \). In terms of \( \omega_m, \omega_p \) and \( \omega_s \), equation (73) becomes

\[ (\omega_m^2 - \omega^2)(\omega_p^2 - \omega^2) - [2\omega_s \omega \sin \theta + (\omega_m^2 - \omega_p^2)]^2 = 0 \]  

where \( \omega_s = (\omega_m^2 \cos^2 \theta + \Omega_{me}^2)^{1/2} \) represents the pole frequency of permeabilities without external field. This pole is located slightly below \( \omega_m \), so that \( \omega_s < \omega_m < \omega_p \). Assuming that the asymptotic frequency for the surface is located in the region above the pole frequency \( \omega_p \), for \( k = +\infty \), the second term in equation (74) is always larger than the first term. Hence, equation (74) is never satisfied and the surface modes do not exist at large \( k \).

A surface mode at \( k = -\infty \) may exist for some values of \( \omega_s, \omega_m \) and \( \omega_p \). For a solution to exist in this region, the function \( f(\mu) \) of equation (74) should cross zero. This gives a condition from equation (74) at the frequency \( \omega = \omega_p \) for a surface mode solution to exist. The requirement for \( f(\mu) \) to vanish means that

\[ \omega_p \leq \omega_o \cos 2\theta + 2\omega_m \sin 2\theta. \]  

For BaMnF_4 parameters are such that there are no surface modes with \( k \gg \frac{\omega}{c} \). We note that the condition equation (75) can be satisfied by changing, for example, the exchange parameter so that \( \lambda/\alpha = 1.5 \). An example is the dispersion relation presented in figure 4(a). In this case a surface mode is able to exist at the top of the middle bulk band, for arbitrarily small wavelengths.

5.2. Effect of applied field for TE modes

In principle, application of an external field can be used to modify the surface mode frequencies by changing the canting angle. Since the electric part is uncoupled from the magnetic part, an application of an electric field has a negligible effect on the polariton frequencies. However, an interesting result is obtained if the external electric field is applied opposite
to the spontaneous polarization. At a certain value for weak ME coupling (for example, $-5 \times 10^{9} \text{ V m}^{-1}$), which is quite high, the polarization can be flipped into the opposite direction. As a consequence, the canting will also flip into the opposite direction. Hence the position of surface branches flip, as can be seen by the dotted lines in figure 2(b). Note that we assumed here that the voltage is below the breakdown voltage.

The application of an external static magnetic field directly changes the canting angle and thereby affects the frequencies. The canting angle as a function of applied magnetic field is shown in figure 5(a). In fact, for a sufficiently strong magnetic field, the condition of equation (73) can be satisfied and a surface mode extends to $k = -\infty$. This occurs through an increase in $\omega_{r}$ via the field dependence of $\Omega_{o,TE}$ (see equation (30)).

The frequencies $\omega_{m}$ and pole frequency $\omega_{p}$ also increase under an external magnetic field. The difference between $\omega_{m}$ and $\omega_{p}$ is given approximately by

$$\Delta^{2} \approx (\omega_{p}^{2} - \omega_{m}^{2}) \propto (\cos 2\theta - \sin 2\theta).$$

As a consequence, the middle bulk band narrows in frequency with increasing magnetic field. At a field of 15 T, the BaMnF$_{4}$ middle bulk band is sufficiently narrow that a surface mode rises above the middle band and extends to small wavelengths, as illustrated in figure 5(b).

Lastly, we consider the case where the dielectric constant depends on frequency. We suppose that the frequency dependence is given by the soft phonon mode according to

$$\omega_{p}^{2} = (\beta_{1} + P_{o}^{2} \beta_{2}) \phi$$

with the dielectric function

$$\epsilon_{s} = \epsilon_{b} \frac{\omega_{p}^{2} - \omega_{m}^{2}}{\omega_{p}^{2} - \omega_{m}^{2}}.$$  \hspace{1cm} (78)

Here $\omega_{p}^{2} = \omega_{2}^{2} + \frac{\omega_{m}^{2}}{\epsilon_{b}}$ and $\epsilon_{b} = 8.3$ is the dielectric constant of the background.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{(a) The dispersion relation without the external field is shown. The surface modes are indicated by 'SP'. The shaded regions represent bulk bands, which are limited by frequencies $\omega_{m}$, $\omega_{ez}$, $\omega_{oz}$, $\omega_{ey}$ and $\omega_{i}$. (b) The "window" where the surface modes exist is shown. (c) A narrow gap around $\omega_{i}$ is expanded. (d) The narrow gap is wider when the ME coupling is increased by a factor of 10.}
\end{figure}
5.3. TM modes

Solutions of equations (59) and (63) for TM modes in BaMnF$_4$ are plotted in figure 6(a) for the case with no applied fields present. There are two gaps in the bulk region created by the poles and zeros of equation (59). First, as illustrated in figure 6(c), a very narrow gap is located at the frequency $\omega_1$ around 41 cm$^{-1}$, created by the magnetoelectric interaction [3]. This gap is associated with zeros in $f(\mu, \epsilon) = \mu_x \epsilon_y - (4\pi \chi_{me})^2$. The pole in the bulk modes is due to a zero of the dielectric constant $\epsilon_y$. This gap is strongly dependent on the ME susceptibility, $\chi_{me}$, and disappears when $\chi_{me} = 0$. The width of the gap is approximately proportional to $(\chi_{me})^2$. Thus the gap becomes wider with larger ME coupling. This increase is illustrated in figure 6(d), where the ME coupling has been increased by a factor of ten.

A second gap exists near the magnetic frequency $\omega_m \cong 3$ cm$^{-1}$, and occurs at zero of $f(\mu, \epsilon) = \mu_y \epsilon_z - (4\pi \chi_{me})^2$ at the magnetic frequency $\omega_m$. The other boundaries for the bulk regions are determined by the attenuation constant.

Using the attenuation constants, we obtain a narrow window between transversal and longitudinal phonon frequencies, $\omega_{ey}$ and $\omega_{LY}$, associated with the pole and zero value of $\epsilon_y$ (see figure 6(b)). In this figure, since the ME coupling is weak, hence the frequency $\omega_{LY}$ is very slightly below the induced frequency $\omega_1$. Since the value of $\epsilon_y$ between these two frequencies is negative, surface modes can be obtained inside this narrow window. The surface modes start from the crossing between the lightline ($\omega = ck$) and the resonance frequency $\omega_{ey}$ and terminate at the longitudinal phonon frequency $\omega_{LY}$. The frequency $\omega_{LY}$ can be approximated as

$$\omega_{LY} = \frac{1}{2}(\epsilon_m^2 \omega_m^2 + f) + [(\epsilon_m^2 + \epsilon_m^2 + f)^2 - 4(\epsilon_m^2 \omega_m^2 + \epsilon_m^2 f)]^{1/2}$$  \hspace{1cm} (79)

and is indicated in figure 6(d). Since the surface modes terminate at the longitudinal phonon frequency, the gap around $\omega_1$ does not influence the surface modes.

Interestingly, from the expression for the attenuation constant of equation (62), and the dispersion relation of surface modes in equation (63), it was only possible to satisfy the boundary conditions with a complex $\beta$. The resulting mode is therefore not a true surface mode but instead a pseudo-surface wave [30]. Because the solution for the attenuation constant in equation (62) is complex, the attenuation constant for the material sample consists of a real and an imaginary part as illustrated in figure 7(a) and (b).

The imaginary part of $\beta$ is

$$\beta_i = -i4\pi \chi_{me} \frac{\omega}{c}.$$  \hspace{1cm} (80)

The positive value of the real part defines regions where the surface modes can exist. The existence of an imaginary part indicates that the solution in equation (60) is a pseudo-surface mode, and not purely localized to the surface. Instead, energy ‘leaks’ into the bulk. The wave is comprised of a localized component, that travels along the surface and decays into the material according to the real part of $\beta$, and a component that travels into the material with wavenumber equal to the imaginary part of $\beta$.

Values for the imaginary parts of $\beta$ are plotted as a function of frequency in figure 7(b) for the case of no applied fields. Imaginary $\beta$ depends linearly on the magnetoelectric susceptibility and becomes large near the electric resonance frequency $\omega_{ey}$. One can also see from figure 7(b), that the coupling directly influences the magnitude of the imaginary part of $\beta$. If the coupling is large, then the ME susceptibility will also be large and thereby increase the imaginary part of $\beta$.

In the case where $k \gg \frac{\omega}{c}$, the attenuation constants for the material and vacuum regions can be approximated by $\epsilon_y \beta_i^2 \approx \epsilon_y k_s^2$ and $\beta_y \approx k_y$. The dispersion relation equation (59) then reduces to

$$\epsilon_y \epsilon_y = 1.$$  \hspace{1cm} (81)

The surface modes require the permittivity $\epsilon_y$ to be negative, and the longitudinal phonon frequency polarized along $z$.

**Figure 7.** Attenuation constant as a function of frequency. (a) The real part of the attenuation constant is shown for two values of wavevector in the absence of any external fields. The solid line represents $k = 0$ cm$^{-1}$, while the dashed line corresponds to $k = 250$ cm$^{-1}$. (b) The imaginary part of the attenuation constant is shown for two values of ME coupling. The solid line is for $\alpha = 1.42 \times 10^{-4}$ cm$^{-2}$/statC, and the dashed line represents the coupling $\alpha = 1.42 \times 10^{-5}$ cm$^{-2}$/statC.
direction, \( \omega_{c z} \), is lower than \( \omega_{c y}' \). This means that the value of \( \epsilon_z \) is positive in regions where surface modes exist, and the requirement in equation (81) is never satisfied. Therefore, surface modes do not exist in the limit \( k \gg \frac{\omega_c}{c} \).

5.4. Effect of applied field for TM modes

We now study the influence of external fields on the band structures. Results for an electric field value of \( 5 \times 10^8 \text{ V m}^{-1} \) along the direction of spontaneous polarization (but with zero magnetic field) are shown in figure 8(b). The effect of the electric field is to shift the window where surface modes exist to higher frequencies. The upward frequency shift is due to the increase of spontaneous polarization, which directly increases the phonon frequencies \( \omega_{c y}' \) and \( \omega_{L y} \). However, the change in \( \omega_{L y} \) is smaller than the change in \( \omega_{c y}' \) (which is due to the third term in equation (79)) and so the surface mode window is narrowed. The electric field increases the canting angle slightly, as shown in figure 8(a), and the effect on magnetic resonance is negligible.

Results for a magnetic field of 10 T (with zero electric field) are shown in figure 8(c). The magnetic field increases the canting angle (see figure 5(a)) and shifts \( \omega_m \) to a lower frequency (as shown in figure 8(c)) but the effects on the bulk bands are negligible. This shift can be understood if we consider the case where the ME coupling is neglected. Then the frequency \( \omega_{m e} \rightarrow 0 \), and also \( \Omega_{m e, TM} \rightarrow 0 \). In this case, the magnetic resonance frequency will take the form

\[
\omega_m^2 = \omega_{m e}^2 + \Omega_{m e, TM}^2 \rightarrow \omega_m ( \omega_m - 2 \omega_x ) \cos^2 \theta.
\]

It can be seen from this expression that a magnetic field reduces the magnetic resonance frequency. We note that this effect is mentioned in [31].

The surface modes can be modified if the magnetic resonance frequency can be shifted into a surface wave ‘window’, as shown in figures 9(b) and 10(b). In BaMnF\(_4\), where the electric and magnetic resonance frequency are well separated around 37 cm\(^{-1}\), it is very difficult to arrange the magnetic resonance frequency inside this window. This may be possible for a suitably prepared material (or artificially...
The dispersion relation is shown in Figure 9. Dispersion relation for a material with electric and magnetic resonances near one another in frequency. (a) The dispersion relation without an external magnetic field. (b) The magnetic resonance frequency is shifted into a surface mode window with the application of a large magnetic field, $H_s = 12 \text{T}$.

Lastly, we identify the key parameters affecting surface mode frequencies. In the first case, changing the phonon mass to $f = 9 \times 10^{21} \text{statA}^2 \text{s}^2 \text{g}^{-1} \text{cm}^{-3}$ moves the magnetic resonance frequency to $1 \text{cm}^{-1}$ above the electric resonance frequency $\omega_{ey}$. The result on surface and bulk polariton bands is shown in Figure 9(a). The dielectric constant background has also been reduced to $\epsilon_\infty = 2.6$, in order to widen the surface mode window. Application of an external magnetic field lowers the magnetic resonance frequency. Application of a large external magnetic of $12 \text{T}$ places the magnetic resonance inside the window, as illustrated in Figure 9(b).

Inside the window, the magnetic resonance splits the surface mode into low and high frequency branches for each direction of propagation. The properties of the upper part are similar to that discussed in the previous section. However, the lower branch terminates at the magnetic resonance frequency, as illustrated in Figure 9(b). In this case, the requirement that the dielectric constant $\epsilon_y$ should be negative for surface modes prevents both the upper and lower branches from existing in the region where $k \gg \frac{\omega}{c}$.

For a second example, we consider if the exchange constant is $-4000 \text{erg cm}^{-3} \text{Oe}^{-2}$ and the anisotropy constant is $4 \text{erg cm}^{-3} \text{Oe}^{-2}$. The ME coupling is also changed to $40 \text{m}^2 \text{C}^{-1}$, which keeps the canting angle small. The dispersion relation at temperature $150 \text{K}$ is presented in Figure 10(a). As the temperature is increased to $250 \text{K}$, the polarization decreases while the magnetization does not change significantly. Hence, the electric resonance goes below the magnetic resonance frequency. Results are shown in Figure 10(b). As in the first example, the magnetic resonance frequency then exists inside the surface mode window and a similar splitting of the surface mode branches occurs.
6. Conclusions

We have shown how linear magnetoelectric coupling influences surface and bulk TE and TM polariton modes in a canted multiferroic. For the TE polarization, we find surface modes associated with the weak ferromagnetism that are non-reciprocal with respect to propagation direction, such that \( \omega(k) \neq \omega(-k) \). The non-reciprocity can be affected by an applied electric field through changes in the weak magnetization. For sufficiently large magnetic fields, or different material parameters, surface modes may also exist for \( k \gg \omega/c \). Application of a static magnetic field can be used to modify the middle bulk band frequencies.

For the TM polarization, a narrow Rstrahlen region forms in the bulk mode band at a frequency near the longitudinal phonon frequency (along the spontaneous polarization). Reciprocal surface excitations can exist in this region. In general, the surface excitations in this polarization are actually pseudo-surface waves that are only partially localized to the surface. The imaginary part of the decay constant is proportional to the magnetoelectric coupling.

We have also explored the possibilities of modifying the polariton band structure through changes in key material parameters. Such effects may possibly be realized in appropriate compounds, or in artificially layered heterostructures. If the magnetoelectric material has a magnetic resonance frequency above the electric resonance frequency, and the difference between those frequencies is not great, then it is possible to create surface excitations that are sensitive to temperature, electric fields and magnetic fields.

Acknowledgments

We wish to acknowledge the support of Ausaid, the Australian Research Council and DEST. Useful discussions with R E Camley and J Barnas are also acknowledged.

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