Study on fuzzy duration of serial coupled iterative design tasks in product development

Qi Hua Tian, Ci Zhang¹, Wei Zhu, Jia Kang Huang, Yu Rong Zhang and Xiang Man Zhou

College of Mechanical and Power Engineering, China Three Gorges University, Yichang 443002, China

¹Email: zc889919@163.com

Abstract. The existing serial coupling iterative design in product development has the uncertainty of the task duration, and it is difficult to solve the task duration. Based on the Markov chain method and the fuzzy theory, a fuzzy duration model of serial coupled iterative design task was constructed, and the fuzzy durations of each stage of product design task were obtained. Taking the body design process of an electric vehicle as an example, this method was applied for analysis. The research shows that the execution time of each stage task is prolonged under serial coupling iteration, and the longer the serial coupling iteration task execution chain is, the more the task execution time increases. This study has a certain reference function for complex serial coupled iterative design planning.

1. Introduction

Product development is a series of design activities to meet customer needs. The coupling and complexity of development tasks and the diversity and dynamics of development environment lead to the complexity of design tasks. In complex coupled design tasks, the ambiguity of task execution time not only increases the management difficulty of a single phase task, but also makes the delivery time difficult to control when the product design is completed. With the development of society, people put forward more and higher requirements on product function and performance, which requires more scientific product design and planning. Complex product design projects contain a large number of design tasks, reasonably controlling the completion time of each stage of design task is not only conducive to the reasonable allocation of design tasks, but also convenient for better management of the completion time of product development tasks, so as to reduce the risk management cost of delayed delivery of customer orders. Therefore, how to reasonably express the uncertain task completion time is worth further study.

Serial coupled iteration is one of the common patterns in complex product design tasks. Design Structure Matrix (DSM) has a good advantage in solving the analysis of design task problems in the process of complex product development [1]. The construction of serial coupled iterative model through this method has been used by many scholars in the Design and analysis of complex products. For example, Wang Mingqi et al obtained the coupling sub-matrix by combining the design structure matrix and path search algorithm, and used the serial iteration model to calculate and solve the execution time of the design process at each stage [2]. Yang Baosen et al studied the relationship between serial iteration and risk management cost of product development schedule [3]. Tian Qihua et al in order to solve the problem of rework of coupling design tasks, a serial coupling design task...
assignment model is constructed, which simplifies the task assignment process [4]. Smith R P et al used markov chain method to analyze small-scale serial coupling design tasks, solved and obtained the optimal execution sequence of tasks [5]. And so on. The above scholars have deeply studied and applied the serial coupling iterative model and obtained the corresponding results. However, they are based on the assumption that the completion time of a single design task is a fixed value, which is difficult to reflect the execution of tasks with uncertain duration in complex product development.

Fuzzy theory is an effective method to solve uncertain problems [6]. At present, many scholars have applied this method to the design and development of complex products. For example, Liu Haotien in the selection of prototype products, considering the engineering characteristics and the factors involved in product development, a fuzzy multi-criteria decision method is proposed to select the best prototype products [7]. Lv yanjie et al a task assignment model based on fuzzy theory is proposed to solve the problem of large subjectivity of task assignment in the process of complex product design [8]. Shen Ze et al based on fuzzy mathematics theory, the product design DNA analysis aided design system was designed and developed to identify and classify tasks [9]. And so on. The above scholars applied the fuzzy theory to the analysis of the problem in which the numerical information is relatively difficult to be determined under complex conditions and obtained a good analysis result.

Based on the existing serial iteration model, this paper introduces the fuzzy theory method to describe the dynamic process of design task, studies the fuzzy duration model of serial coupling design task based on fuzzy theory, and solves the model through markov chain correlation principle and fuzzy number operation to obtain the fuzzy duration of each stage.

2. Description of serial coupled iteration model

Serial iteration is one of the classical iteration patterns in product design. In serial iteration, the number of tasks executed each time is one. Assuming that resources are sufficient in the process of task execution, there is no delay, and the task execution time does not consider emergencies, the execution time can be assumed to be a fixed value. After the completion of the task in one stage, the task in the next stage includes the task in this stage and the rework task in the previous phase. And the rework probability will change with the number of rework times. This paper assumes that the rework probability will not change after the first rework. Based on the above assumptions, the design structure matrix DSM can be used to describe serial iteration [10]. Where, diagonal elements represent the time required to perform a task alone. Off-diagonal elements represent the size of the rework probability, and the rework probability value represents the strength of the rework probability of a task at a certain stage. The higher the value, the greater the likelihood of rework.

Figure 1. Example of a 2×2 order serial iteration model.

|     | A   | B   |
|-----|-----|-----|
| A   | 4.0 | 0.3 |
| B   | 0.2 | 6.0 |

Take a 2×2 order serial iteration model as an example, as shown in Figure 1. In A single task will take time to 4, task B independent completion time is 6, coupling relationship between A and B task, when the task in the task B before execution, due to the information communication process between A and B task, task A message in the task B are incompatible, had to rework probability is 0.2, the probability of end directly after the completion of the task is 0.7; Similarly, when task B is executed before task A, the probability of having to do it again later is 0.3, and the probability of finishing the task directly after completion is 0.8. It can also be seen from the above analysis that the serial iteration DSM contains two important information, namely rework probability and task execution duration. When task A is executed first, the execution of tasks in each stage can be represented by Markov Chain (MC) as shown in Figure 2.
In a model that is executed in a completely serial iterative manner, tasks are executed sequentially, the previous task is completed, and the next task is started. A single phase contains only one task, the task completion time of a single stage is the time it takes to complete a task. Therefore, time can be obtained as a specific value. However, there are many variables in the task execution process, and task execution cannot be performed in a completely serial iterative manner, but there will be problems of communication and coupling between task information. In a single stage, not only one task is executed, but also some rework tasks that previously performed the task, which makes it difficult to determine the task execution time of a single stage as a specific value.

Considering the actual development process, a single task completion time will be affected by many factors, such as: product development environment changes and different designers experience lead to blocked information exchange, different stages to complete the quality differences lead to rework, etc. Therefore, the task completion time cannot be represented by a certain value, that is, there is uncertainty. Due to the subjectivity of human brain thinking, the uncertain value is usually expressed by interval, that is, the minimum value and maximum value, but this will make the expression of construction duration ambiguous. Aiming at this kind of problem, this paper introduces the fuzzy method to describe it.

3. Construction and solution of fuzzy duration model

For the imprecise duration value, it can be expressed by the number of intervals \([a, b]\) in the fuzzy theory, where \(a\) and \(b\) are the minimum and maximum required completion time respectively. Fuzzy number is an extension form of interval number [11], which is defined as follows:

Let \(A \in S(R)\), \(R\) is a real number field, in \(R\) there is a closed interval \([a, b]\), and it is satisfied that:

1. on \([a, b]\), \(\hat{A}(x) \equiv 1\);
2. on \((-\infty, a)\), \(\hat{A}(x)\) is an increasing function, right continuous, and \(0 \leq \hat{A}(x) \leq 1\);
3. on \((b, +\infty)\), \(\hat{A}(x)\) is a decreasing function, left continuous, and \(0 \leq \hat{A}(x) \leq 1\).

Then the fuzzy number \(\hat{A}\) can be expressed as \(\hat{A} = ([a, b], L_d, R_d)\), where \(L_d\) is the left continuous increasing function and \(R_d\) is the right continuous decreasing function. According to the above definition, a bounded triangular fuzzy number can be expressed as \(\hat{A} = ([a, m, b], L_d, R_d)\), of which \(m\) represents the most likely completion time.

The 2 × 2 order serial iterative model in the above section is illustrated as an example. The completion of the design task needs to go through two stages: Stage 1: including a single task; Stage 2: including a single task and increment from previous task rework. Because the task execution time is uncertain, the task execution time can be expressed by fuzzy numbers, which are \(\hat{A} = ([a_A, m_A, b_A], L_A, R_A)\) and \(\hat{B} = ([a_B, m_B, b_B], L_B, R_B)\). The execution of the serial coupling design task is shown in Figure 3.
During task execution, the task rework probability is affected by the number of rework iterations, and the way the duration value is expressed does not affect its value. In Figure 3, the rework probability value does not change. Under the fuzzy theory, the task execution duration is no longer a specific value, but a fuzzy number with membership relationship. Taking task A as an example, the task duration is $\tilde{A}$, the minimum execution duration is $a_A$, the most likely execution duration is $m_A$, and the maximum execution duration is $b_A$. The corresponding membership relationship can be expressed as follows:

$$\tilde{A}(x) = \begin{cases} 0 & x > b_A \text{ or } x < a_A \\ \frac{x - a_A}{m_A - a_A} & a_A \leq x \leq m_A \\ \frac{b_A - x}{b_A - m_A} & m_A \leq x \leq b_A \end{cases}$$

(1)

Definition, $L_A(x) = (x - a_A) / (m_A - a_A)$, $R_A(x) = (b_A - x) / (b_A - m_A)$.

By using the fuzzy number operation in the fuzzy method, the task execution time of each stage in the serial coupling iterative model is obtained in turn. For the two-stage serial iterative model with fuzzy duration, according to Formula (1), the execution durations of task A and task B are:

$$\tilde{A} = \left( [a_A, m_A, b_A], L_A, R_A \right)$$

$$L_A(x) = (x - a_A) / (m_A - a_A)$$

$$R_A(x) = (b_A - x) / (b_A - m_A)$$

$$\tilde{B} = \left( [a_B, m_B, b_B], L_B, R_B \right)$$

$$L_B(x) = (x - a_B) / (m_B - a_B)$$

$$R_B(x) = (b_B - x) / (b_B - m_B)$$

(2)

(3)

Set the mathematical expectation values of completion time of task A and task B in stage 2 as $\tilde{T}_A$ and $\tilde{T}_B$ respectively, and the task duration can be solved in order from back to front. According to the correlation principle of Markov Chain in reference [12] and the data relationship of stage 2 in Figure 3, the following equation can be obtained:

$$\begin{cases} \hat{T}_A = \hat{T}_B \times 0.2 + \tilde{A} \\ \hat{T}_B = \hat{T}_A \times 0.3 + \tilde{B} \end{cases}$$

(4)

Substitute equations (2) and (3) into equation (4) to obtain the mathematical expected value of task B in stage 2, namely task completion time $\hat{T}_B$ in stage 2. In stage 1, only task A is executed. So the
task completion time in stage 1 is the execution duration of task A, and the task completion time in stage 1 can be calculated.

4. Case analysis
Taking the design task of an electric vehicle body as an example [13], the application of the above method in solving practical problems is explained. The design project contains multiple design tasks, and there is a close information coupling relationship between adjacent tasks. The car body appearance design is a typical serial coupling design execution mode. Among them, creative design (E1), sludge model design (E2) and digital model design (E3) were selected for the design task, and the simplified design task information relationship was shown in Table 1.

Table 1. Task information relationship of appearance coupling design for electric vehicle body.

| Design task               | Design task | E1  | E2  | E3  | a  | m  | b  |
|---------------------------|-------------|-----|-----|-----|----|----|----|
| Creative design           | E1          | 0.3 |     |     | 3  | 5  | 7  |
| Sludge model design       | E2          |     | 0.2 |     | 2  | 3  | 5  |
| Digital model design      | E3          |     | 0.4 |     | 2  | 4  | 6  |

In Table 1, a serial coupling design task relationship of 3×3 order is formed among the three design tasks, as shown in Figure 4. Among them, $A_1$, $A_2$ and $A_3$ are the execution durations of design tasks E1, E2 and E3, respectively, and all are fuzzy duration. Taking task E1 as an example, the minimum task execution duration $a_1$, the most probable task execution duration $m_1$, and the maximum task execution duration $b_1$, all of which are correspond to 3, 5 and 7, respectively.

In order to calculate the completion time of the three stages of the task, it can be solved from the back to the front. And each stage is solved separately. In the design stage 3, not only need to perform design task E3, but also the additional workload of rework of the previous two performed design tasks, and the probability that the design task E3 completes and ends directly is 0.8. Assume that the expected completion durations of tasks E1, E2, and E3 in the design stage 3 are $Q_1$, $Q_2$, and $Q_3$, respectively. From equation (4), the following equation can be listed.
According to the formula (2) and (3), the completion time of the appearance design stage 3 is $Q_3$, which corresponds to the smallest, most probable and largest task execution durations are 2.78, 5.28, 7.99, respectively.

In the appearance design stage 2, the execution of the task includes the extra workload caused by the rework of design task E2 and design task E1. The probability that the sludge model design task E2 is completed and directly enters the appearance design stage 3 is 0.7. And so on, set the expected completion time of tasks E1 and E2 in stage 2 of appearance design be $J_1$ and $J_2$, respectively. From equation (4), the following equation can be listed.

\[
\begin{align*}
Q_1 &= Q_2 \times 0.1 + Q_3 \times 0.4 + A_1 \\
Q_2 &= Q_4 \times 0.3 + Q_5 \times 0.2 + A_2 \\
Q_3 &= Q_2 \times 0.2 + A_3
\end{align*}
\]

The completion time of the appearance design stage 2 is $J_2$, which corresponds to the smallest, most probable and largest task execution durations are 2.99, 4.64, 6.32, respectively.

The appearance design stage 1 has only one design task to execute, that is, the design task E1 is completed and directly enters the appearance design stage 2. Therefore, the completion time of appearance design stage 1 is $A_1$ required for the completion of design task E1 alone, which corresponds to the smallest, most possible and largest task execution durations are 3, 5, 7, respectively.

It can be seen from the above solution results that, after introducing the fuzzy theory, the task execution time required for different stages of a serially coupled design task is longer than the original task's separate execution time, and the more stages of the task execution chain, the greater the difference between the two.

5. Conclusion

Based on Markov chain method and fuzzy theory, a serial coupling iterative model under the condition of task execution time uncertainty, and obtains the fuzzy duration of each stage task execution under serial coupling iteration. The research shows that, due to the information coupling between the tasks in each stage, the tasks will be repeated and iterated, and the execution time of the tasks in each stage will be prolonged under the serial coupling iteration, and the longer the serial coupling iteration task execution chain is, the longer the task execution time will be. This study has certain reference function for complex serial coupling iterative design planning.

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