Luminosity and physics considerations on HL-LHC–and HE-LHC–based \( \mu p \) colliders

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Abstract – Construction of the future muon collider tangential to the Large Hadron Collider will give opportunity to realize \( \mu p \) collisions at multi-TeV center-of-mass energies. Using the nominal parameters of high-luminosity and high-energy upgrades of the LHC, as well as the design parameters of muon colliders, it is shown that \( L_{\mu p} \) of the order of \( 10^{33} \text{cm}^{-2}\text{s}^{-1} \) is achievable for different options with \( \sqrt{s_{\mu p}} \) from 4.58 TeV to 12.7 TeV. Certainly, the proposed \( \mu p \) colliders have a huge potential for clarifying QCD basics and searches for new physics.

Introduction. – It is known that lepton-hadron collisions play a crucial role in our understanding of matters structure (proton form-factors, quark-parton model, EMC effect and so on). The first electron-proton collider HERA explored this structure further and provided parton distribution functions (PDFs) for the LHC and Tevatron. Construction of a TeV (or multi-TeV) scale lepton-hadron collider is mandatory both for clarifying QCD basics, which is responsible for 98% of the mass of the visible part of our Universe, and to provide PDFs for adequate interpretation of incoming data from HL/HE-LHC \([1,2]\) and FCC/SppC \([3,4]\).

On the other hand, while the electro-weak part of the Standard Model (SM) has been completed with the discovery of the Higgs boson at the LHC, this is not the case for the QCD part: the confinement hypothesis should be clarified experimentally. In this respect, energy frontier lepton-hadron colliders will play a role analogous to that of the LHC, which clarify the Higgs mechanism hypothesis.

Today, linac-ring–type ep colliders are considered as the sole realistic way to (multi-)TeV scale in lepton-hadron collisions (see the review \([5]\) and references therein) and LHeC \([6]\) is the most promising candidate. However, the situation may change in the coming years: \( \mu p \) colliders can come forward depending on progress in muon production and cooling issues.

TeV energy muon-proton colliders \([7,8]\) were proposed two decades ago as alternatives to linac-HERA–and linac-LHC–based ep/\( \gamma p \) colliders (see review \([9]\) and references therein). Two years later an ultimate \( \sqrt{s} = 100 \text{TeV} \mu p \) collider (with an additional 50 TeV proton ring in the \( \sqrt{s} = 100 \text{TeV} \mu \text{on collider tunnel} \) was suggested in \([10]\). It should be noted that luminosities of \( \mu p \) collisions in \([7]\) and \([8]\) were overestimated by 3 and 1 order of magnitudes, respectively (see subsects. 3.2 and 2.3 in \([10]\).)

Recently, FCC- and SppC-based energy frontier muon-hadron colliders have been proposed in \([11]\) and \([12]\), respectively. In order to complete the picture and keeping in mind the approved high-luminosity and proposed high-energy upgrades of the LHC (HL-LHC and HE-LHC, respectively), the corresponding \( \mu p \) colliders have to be considered as well.

In this paper we propose the construction of Muon Colliders (MC) tangential to the LHC (see fig. 1) to allow for multi-TeV center-of-mass energy \( \mu p \) collisions. The nominal parameters of HL-LHC, HE-LHC and MC are presented in the following section. Center-of-mass energies and luminosities for different \( \mu p \) collider options are estimated in the third section. Then, in the fourth section we briefly discuss physics at these colliders. Finally, in the last section we present our conclusions and recommendations.

Parameters of HL-LHC, HE-LHC and muon colliders. – In this section, we present the parameters of
Main parameters of $\mu p$ colliders. – The general expression for luminosity of the LHC-based $\mu p$ colliders is given by

$$L_{\mu p} = \frac{N_\mu N_p}{4\pi} \frac{1}{\max \left[\sigma_{x_\mu}, \sigma_{x_\mu}^*\right] \min \left[f_{r_p}, f_{r_\mu}\right]},$$

(1)

where $N_\mu$ and $N_p$ are the numbers of muons and protons per bunch, respectively; $\sigma_{x_\mu}$ and $\sigma_{y_\mu}$ are the horizontal and vertical proton (muon) beam sizes at the interaction point (IP); $f_{r_p}$ and $f_{r_\mu}$ are MC and LHC bunch frequencies. $f_c$ is expressed by $f_c = N_b f_{rev}$, where $N_b$ denotes the number of bunches and $f_{rev}$ means the revolution frequency. Some of these parameters can be rearranged in order to maximize $L_{\mu p}$ but one should note that there are main limitations due to beam-beam effects that should be kept in mind. The beam-beam parameter for proton beam is given by

$$\xi_{x_\mu} = \frac{N_\mu r_p \beta_p^*}{2\gamma_p \sigma_{x_\mu^*} (\sigma_{x_\mu} + \sigma_{y_\mu})},$$

$$\xi_{y_\mu} = \frac{N_\mu r_p \beta_p^*}{2\gamma_p \sigma_{y_\mu^*} (\sigma_{y_\mu} + \sigma_{x_\mu})},$$

(2)

(3)

where $r_p$ is the classical radius of the proton, $r_p = 1.54 \times 10^{-18}$ m, $\beta_p^*$ is the beta function of the proton beam at IP, and $\gamma_p$ is the Lorentz factor of the proton beam. $\sigma_{x_\mu}$ and $\sigma_{y_\mu}$ are horizontal and vertical sizes of the muon beam at IP, respectively. The beam-beam parameter for the muon beam is given by

$$\xi_{x_\mu} = \frac{N_p r_p \beta_\mu^*}{2\gamma_\mu \sigma_{x_\mu^*} (\sigma_{x_\mu} + \sigma_{y_\mu})},$$

$$\xi_{y_\mu} = \frac{N_p r_p \beta_\mu^*}{2\gamma_\mu \sigma_{y_\mu^*} (\sigma_{y_\mu} + \sigma_{x_\mu})},$$

(4)

(5)

where $r_\mu = 1.37 \times 10^{-17}$ m is the classical muon radius, $\beta_\mu^*$ is the beta function of muon beam at IP, and $\gamma_\mu$ is the Lorentz factor of the muon beam. $\sigma_{x_\mu}$ and $\sigma_{y_\mu}$ are the horizontal and vertical sizes of proton beam at IP, respectively.

Table 1: HL-LHC and HE-LHC design parameters [13].

| Parameter (Unit)               | HL-LHC | HE-LHC |
|-------------------------------|--------|--------|
| $\sqrt{s}$ energy (TeV)       | 14     | 27     |
| Circumference (km)            | 26.7   | 26.7   |
| Beam current (A)              | 1.12   | 1.12   |
| Protons per bunch ($10^{11}$) | 2.2    | 2.2    |
| $N_{\text{bunch}}$ per beam   | 2760   | 2808   |
| RMS bunch length (mm)         | 90     | 90     |
| Bunch spacing (ns)            | 25     | 25     |
| Norm. rms emittances ($\mu$m) | 2.5    | 2.5    |
| RMS beam sizes ($\mu$m)       | 7.1    | 9.0    |
| $L_{\text{peak}}$ per IP (cm$^{-2}$s$^{-1}$) | 5\times10$^{34}$ | 16\times10$^{34}$ |

Table 2: ERL60 $\otimes$ (HL-LHC and HE-LHC) $ep$ collider parameters [13].

| Parameter (Unit)   | HL-LHC | HE-LHC |
|--------------------|--------|--------|
| $E_p$ (GeV)        | 7      | 13.5   |
| $E_e$ (GeV)        | 60     | 60     |
| $\sqrt{s_{ep}}$ energy (TeV) | 1.3    | 1.7    |
| Bunch spacing (ns) | 25     | 25     |
| Protons per bunch ($10^{11}$) | 2.2    | 2.5    |
| Proton norm. emitt. ($\mu$m) | 2      | 2.5    |
| Electrons per bunch (10$^9$) | 2.3    | 3      |
| Electron current (mA) | 15     | 20     |
| Protons $\beta$ @ IP (cm) | 7      | 10     |
| Proton beam sizes ($\mu$m) | 4.45   | 4.17   |
| Luminosity (cm$^{-2}$s$^{-1}$) | $8 \times 10^{33}$ | $12 \times 10^{33}$ |

HL-LHC, HE-LHC and MC, which are used for the estimation of the main parameters of $\mu p$ colliders in the following section. Table 1 presents the nominal parameters for HL-LHC and HE-LHC [13]. Parameters of the proton ring upgraded for ERL60 related $ep$ colliders are given in table 2.

The main parameters of TeV-energy muon collider options [14] are listed in table 3.
Table 3: Muon collider parameters [14].

| Parameter (Unit) | Multi-TeV       |
|------------------|-----------------|
| √s energy (TeV)  | 1.5 3 6         |
| Circumference (km)| 2.5 4.5 6      |
| Muons per bunch  | 2 × 10^{12} 2 × 10^{12} 2 × 10^{12} |
| Bunch length (cm)| 1 0.5 0.2       |
| Norm. emitt. (μm)| 25 25 25        |
| β_{x,y} @ IP (cm)| 1 0.5 0.25      |
| Beam sizes (μm)  | 5.9 2.95 1.48   |
| Repetition rate (Hz)| 15 12 6       |
| Number of IPs    | 2 2 2           |
| L_{ave} (cm⁻²s⁻¹)| 1.25 × 10^{34} 4.4 × 10^{34} 12 × 10^{34} |

Table 4: Center-of-mass energies and luminosities of HL-LHC–based μp colliders.

| E_μ (TeV) | √s (TeV) | L (nominal) (10^{33} cm⁻²s⁻¹) | L (upgraded) (10^{33} cm⁻²s⁻¹) |
|-----------|----------|-------------------------------|-------------------------------|
| 0.75      | 4.58     | 0.95                          | 1.4                          |
| 1.5       | 6.48     | 0.84                          | 2.1                          |
| 3         | 9.16     | 0.57                          | 1.5                          |

Table 5: Center-of-mass energies and luminosities of HE-LHC–based μp colliders.

| E_μ (TeV) | √s (TeV) | L (nominal) (10^{33} cm⁻²s⁻¹) | L (upgraded) (10^{33} cm⁻²s⁻¹) |
|-----------|----------|-------------------------------|-------------------------------|
| 0.75      | 6.36     | 0.59                          | 1.6                          |
| 1.5       | 9        | 0.52                          | 2.8                          |
| 3         | 12.7     | 0.36                          | 1.9                          |

Keeping in mind that both LHC and MC have round beams, luminosity equations for μμ and pp are given by

\[ L_{\mu\mu} = \frac{N_\mu^2}{4\pi\sigma^2}\beta_{c\mu}, \quad (4) \]
\[ L_{pp} = \frac{N_p^2}{4\pi\sigma^2}\beta_{c\mu}. \quad (5) \]

Concerning muon-proton collisions, according to eq. (1), one should use the larger of the two transverse beam sizes and the smaller of the two collision frequency values. Keeping in mind that \( f_{c\mu} \) is smaller than \( f_{c\mu} \) by more than two orders, the following relation between \( \mu p \) and \( \mu\mu \) luminosities is found for round beams:

\[ L_{\mu p} = \left( \frac{N_p}{N_\mu} \right) \left( \frac{\sigma_{\mu\mu}}{\max \{\sigma_p, \sigma_{\mu\mu}\}} \right)^2 L_{\mu\mu}. \quad (6) \]

Using the nominal (upgraded) parameters of HL-LHC and MC from tables 1, 2 and 3, the parameters of HL-LHC–based \( \mu p \) colliders are estimated according to eq. (6) and are presented in table 4.

Center-of-mass energies and luminosity values for HE-LHC–based \( \mu p \) colliders, evaluated in the same way, are given in table 5.

Concerning beam tune shifts, for round beams, eqs. (2) and (3) turn into

\[ \xi_p = \frac{N_p r_\mu^2 \beta_{p\mu}^*}{4\pi\gamma_p\sigma_p^2} \quad (7) \]
\[ \xi_\mu = \frac{N_\mu r_\mu^2 \beta_{p\mu}^*}{4\pi\gamma_\mu\sigma_p^2} \quad (8) \]

Matching transverse sizes of proton and muon beams results in

\[ \xi_p = \frac{N_p r_\mu^2}{4\pi\gamma_p N_p^2} \quad (9) \]
\[ \xi_\mu = \frac{N_\mu r_\mu^2}{4\pi\gamma_\mu N_\mu^2} \quad (10) \]

Putting the corresponding parameters from tables 1–3 into eqs. (9), (10), we obtain the tune shift values given in table 6.
According to eq. (9), this can be handled by decrease of magnitude. According to eq. (9), this can be handled by decrease of magnitude. According to eq. (9), this can be handled by decrease of magnitude. According to eq. (9)

Table 7: Achievable \( x \) Bjorken values at the LHC-based \( \mu p \) colliders.

| \( s (\text{TeV}) \) | \( Q^2 = 1 \text{ GeV}^2 \) | \( Q^2 = 25 \text{ GeV}^2 \) |
|----------------|----------------|----------------|
| 4.58           | \( 4.8 \times 10^{-8} \) | \( 1.2 \times 10^{-6} \) |
| \( \sim 6.4 \)  | \( 2.5 \times 10^{-8} \) | \( 6.1 \times 10^{-7} \) |
| \( \sim 9 \)    | \( 1.3 \times 10^{-8} \) | \( 3.1 \times 10^{-7} \) |
| 12.7           | \( 6.2 \times 10^{-9} \) | \( 1.6 \times 10^{-7} \) |

Table 6: Beam-beam tune shifts.

|                  | \( \xi_p \) | \( \xi_u \) |
|-----------------|-----------|-----------|
| HL-LHC          | Nominal  | 0.098    | 0.0096   |
|                 | Upgraded | 0.12     | 0.0096   |
| HE-LHC          | Nominal  | 0.098    | 0.0096   |
|                 | Upgraded | 0.098    | 0.011    |

One can see from table 6 that while \( \xi_p \) values are acceptable, \( \xi_u \) values should be reduced by an order of magnitude. According to eq. (9), this can be handled by decrease of \( N_u \) and/or increase of normalized emittance of the proton beam, resulting in the corresponding decrease of luminosity.

Several years ago, the AloHEP software [15,16] has been developed to estimate parameters for linac-ring-type colliders such as luminosity, beam-beam tune shift, etc. Recently, the software AloHEP has been extended to cover all types of colliding beams and collision schemes [17]. Using AloHEP, we have double-checked the parameters given in tables 4, 5 and 6, resulting in a well-consistent outcome.

**Physics search potential.** Because of high center-of-mass energy and luminosity values, the LHC-based \( \mu p \) colliders have a huge potential for the SM and BSM searches. Concerning SM physics, they will provide precision PDFs for the HE-LHC, FCC and SppC. The small \( x \) Bjorken region, which is crucial for understanding of QCD basics, can be explored down to \( 10^{-8} \). Precision Higgs physics is another important topic, which should be analyzed in detail. In table 7, we present the achievable \( x \) Bjorken values for different options of the LHC-based \( \mu p \) colliders. Let us mention that \( Q^2 = 25 \text{ GeV}^2 \) corresponds to perturbative QCD, whereas \( x_B < 10^{-6} \) implies high parton densities.

Concerning BSM physics, LHC-based \( \mu p \) colliders are comparable or essentially exceed the potential of the LHC itself in a lot of topics, such as leptoquarks related to the second family leptons, excited muon, excited muon neutrino, color octet muon, contact interactions, SUSY, RPV SUSY (especially resonant production of corresponding squarks), extended gauge symmetry, etc.

As an example, let us consider production of color octet muon (see [18] and references therein) at the HL-LHC and HE-LHC \( \mu p \) colliders. For numerical calculations, we have implemented the Lagrangian

\[
L = \frac{g}{2\Lambda} \sum_i \left\{ \bar{\nu}_l G^\alpha_{\mu\nu} \sqrt{2} \lambda_i \phi (n_L L + n_R R) + \text{h.c.} \right\} \quad (11)
\]

into the CompHEP software [19,20].

The Feynman diagram for resonant production of color octet muon at \( \mu p \) colliders is shown in fig. 2.

Assuming the compositeness scale to be equal to color octet muon mass, cross-sections for \( \mu_8 \) production at \( \mu \otimes \mu \) HL-LHC and \( \mu \otimes \mu \) HE-LHC are presented in fig. 3. Discovery mass limits for \( \mu_8 \) at muon, proton and LHC-based \( \mu p \) colliders are shown in fig. 4. It is obvious that discovery mass limits for pair production of \( \mu_8 \) at muon colliders are approximately half that of the CM energies. Discovery limit values for the HL-LHC and HE-LHC are obtained by rescaling ATLAS/CMS second-generation LQ results [21,22] using the method developed by Salam and Weiler [23]. Following ref. [24], integrated luminosity values of \( 3 \text{ ab}^{-1} \) and \( 15 \text{ ab}^{-1} \) have been used for the HL-LHC and the HE-LHC, respectively. Discovery limits for \( \mu_8 \) colliders are estimated from fig. 3 using 25 events at integrated luminosities \( 0.15 \text{ ab}^{-1} \) and \( 0.19 \text{ ab}^{-1} \) for HL-LHC and HE-LHC–based \( \mu p \) colliders, respectively. As seen in fig. 4, \( \mu p \) colliders are more advantageous than the corresponding \( pp \) and muon colliders for \( \mu_8 \) search.

Let us mention that the importance of \( \mu p \) colliders is emphasized by the results of muon \( g-2 \) factor measurement (4.2σ discrepancy with the theoretical calculations [25]) and lepton flavour violation in B-meson decays (4.0σ dis-
crepancy [26]), which could be a manifestation of the muon-related new physics.

Finally, the potential of the LHC-based \( \mu p \) collider (with \( E_p = 1 \) TeV, \( E_\mu = 7 \) TeV and \( \sqrt{s} = 5.3 \) TeV) in measuring the Higgs properties, probing the \( R \)-parity–violating supersymmetry and testing heavy new physics in the muon \( g-2 \) anomaly has been considered in a recent paper [27].

**Conclusion.** – It is shown that the construction of a future muon collider (or dedicated \( \mu \)-ring) tangential to the LHC will give the opportunity to realize \( \mu p \) colliders with multi-TeV center-of-mass energies at a luminosity of order of \( 10^{33} \) cm\(^{-2}\)s\(^{-1}\). Obviously, such colliders will essentially enlarge the physics search potential of the HL/HE-LHC for both the SM and BSM phenomena. Therefore, systematic studies of accelerator, detector and physics search aspects of the LHC-based \( \mu p \) colliders are necessary for long-term planning in the field of high energy physics.

Finally, one can consider the two-stage scenario for the LHC-based lepton-hadron colliders: the LHCl option with 9 km e-ring [28] as the first stage, already tangential to the LHC, followed by the construction of \( \mu \)-ring in the same tunnel.

**Data availability statement:** The data that support the findings of this study are available at the following URL: https://arxiv.org/abs/1905.05564v2.

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