Abstract

We consider the classical theory of a gravitational field with spin 2 and (Pauli-Fierz) mass $m$ in flat spacetime, coupled to electromagnetism and point particles. We establish the law of light propagation and calculate the amount of deflection in the background of a spherically symmetric gravitational field. As $m$ tends to zero, the deflection is shown to converge to $3/4$ of the value predicted by the massless theory (linearized General Relativity), even though the spherically symmetric solution of the gravitational field equations has no regular limit. This confirms an old argument of van Dam and Veltman on a purely classical level, but also shows its subtle nature.

Notation and Conventions

We work in flat Minkowski space with metric $\eta = \text{diag}(1, -1, -1, -1)$. Spacetime indices, running from 0 to 3 are greek, whereas space indices, running from 1 to 3 are latin. The Minkowski inner product is denoted by $\eta(p, q) = p \cdot q = p^\mu q_\mu$, and $p \cdot p = p^2$. Partial differentiation with respect to $x^\mu$ is either written as $\partial_\mu$ or sometimes simply as $, \mu$. Throughout we use units in which the velocity of light is set to 1.
Introduction

According to an old and well known argument of van Dam and Veltman [2, 3], which we will sketch below, the theory of massive spin 2 fields in flat Minkowski spacetime does not approximate the strictly massless theory (linearized General Relativity (GR)), in the limit as the mass tends to zero. (See also [4] and the more comprehensive account in [1].) This means that there exist corresponding observables in both theories which distinguish the massless limit of the massive theory from linearized GR. One such observable, so it is claimed, is the amount of deflection of a light ray in the background of a, say, rotationally symmetric, static gravitational field. More precisely, in the limit the mass tends to zero, the deflection angle predicted by the massive theory tends to 3/4 of the angle predicted by linearized GR.

As we shall see below, the argument given by van Dam and Veltman is entirely based on the properties of free propagators, whose structure is determined by Poincaré invariance. The aim in this paper is to improve our understanding of the classical aspects of this limit, which is known to be non trivial for several reasons, but has not been explored in all details in the mentioned references. In particular, we wish to understand how certain observables can have a smooth limit as the mass tends to zero, even though they refer to solutions which diverge in that limit. The deflection of light is an example of such a case, whose derivation from first principles we shall consider in this paper.

Let us now turn to the argument proper of van Dam and Veltman: In momentum space, the free propagators are given by

\[ P_{\mu\nu\alpha\beta}(p) = \frac{1}{2}(\pi_{\mu\alpha}\pi_{\nu\beta} + \pi_{\mu\beta}\pi_{\nu\alpha}) - \frac{i}{3}\pi_{\mu\nu}\pi_{\alpha\beta} \]

\[ \frac{p^2 - m^2 + i\epsilon}{p^2 - m^2 + i\epsilon} \]  

\[ \text{massive case (1)} \]

\[ P_{\mu\nu\alpha\beta}(p) = \frac{1}{2}(\pi_{\mu\alpha}\pi_{\nu\beta} + \pi_{\mu\beta}\pi_{\nu\alpha}) - \frac{i}{2}\pi_{\mu\nu}\pi_{\alpha\beta} \]

\[ \frac{p^2}{p^2 + i\epsilon} \]  

\[ \text{mass-less case (2)} \]

where \( \pi_{\mu\nu} \) is the induced Riemannian metric on the mass-hyperboloid \( p^2 = m^2 \) in momentum space:

\[ \pi_{\mu\nu}(p) := \frac{p_\mu p_\nu}{m^2} - \eta_{\mu\nu}. \]

We now consider two systems with (Fourier transformed) energy-momentum tensors \( T^{\mu\nu} \) and \( T'_{\mu\nu} \). If we assume these tensors to be conserved, \( 0 = p^\mu T^{\mu\nu}(p) = p^\mu T'_{\mu\nu}(p) \), the one-graviton interaction takes the form (we write \( T = T' \) etc.)

\[ \kappa T^{\mu\nu}(p) P_{\mu\nu\alpha\beta}^m(p) T'_{\alpha\beta}(-p) = \kappa \frac{T^{\mu\nu}(p)T'_{\mu\nu}(-p) - \frac{1}{3}T(p)T'(-p)}{p^2 - m^2 + i\epsilon} \]  

\[ \text{massive case (4)} \]
in the massive case, and
\[
\kappa_0 T^{\mu\nu}(p) P_{\mu\nu\alpha\beta}(p) T'_{\alpha\beta}(-p) = \kappa_0 \frac{T^{\mu\nu}(p) T'_{\mu\nu}(-p) - \frac{1}{3} T(p) T'(-p)}{p^2 + i\varepsilon}
\] (5)
in the massless case, with gravitational constants \( \kappa \) and \( \kappa_0 \) respectively.

For slowly-moving massive objects the leading component of the energy-momentum tensor is \( T_{00} \) and we get \( \frac{2}{3} \kappa T^{00}(p) T'_{00}(-p) \) in the massive and \( \frac{1}{2} \kappa_0 T^{00}(p) T'_{00}(-p) \) in the massless case. To identify \( \kappa \) and \( \kappa_0 \) one requires that both cases lead to Newton's law of attraction with the same Newtonian constant \( G \) (taking the limit \( m \to 0 \) in the massive case). This gives
\[
\kappa = \sqrt{\frac{3}{4}} \kappa_0 = \sqrt{\frac{12}{\pi}} G.
\] (6)

On the other hand, if we consider the interaction of light and slowly-moving matter, there are no trace-terms in (4) and (5) due to the tracelessness of the electromagnetic energy-momentum tensor. But now (4) implies that the massive theory, in the limit \( m \to 0 \), leads to an interaction of light and matter which is weaker by a factor \( 3/4 \) as compared to the massless theory. Accordingly, in that limit, light deflection comes out smaller by the factor \( 3/4 \) as compared to the massless theory, i.e., linearized General Relativity. This is often taken as sufficient justification to rigorously argue that present-day observations exclude a finite graviton mass.

Here one should stress that the flatness of spacetime enters in an essential way. In fact, it was recently argued that there is no discontinuous behaviour at \( m = 0 \) if the cosmological constant \( \Lambda \) is non-zero. For negative \( \Lambda \) (anti-de Sitter spacetime) a smooth limit of propagator residues was shown in [8]. For positive \( \Lambda \) (de Sitter spacetime) the situation is again different, since there is no quantum theory of spin 2 fields in the mass range \( 0 < m^2 < \frac{3}{4} \Lambda \) obeying unitarity and certain locality requirements [4, 7]. Here a discontinuity occurs at \( m^2 = \frac{3}{4} \Lambda \).

## 1 Massive spin-2 fields in Minkowski space

We briefly review the theory of massive spin-2 fields in flat spacetime. It is represented by a symmetric tensor field \( h_{\mu\nu} \). The space of such fields still represents the Poincaré group in a highly reducible fashion. A projection into
an irreducible subspace of pure spin-2 and mass \( m \) is given by

\[
\begin{align*}
\Box + m^2 h_{\mu \nu} &= 0 \quad (7) \\
\partial^\mu h_{\mu \nu} &= 0 \quad (8) \\
h_{\mu}^\mu &= 0 \quad (9)
\end{align*}
\]

1.1 Static rotationally symmetric solution

We seek a solution of (7,8,9) which is static, i.e. \( \partial_0 h_{\mu \nu} = 0 \) and rotationally symmetric in the following sense:

\[
D(R)_{\mu}^\rho D(R)_{\nu}^\sigma h_{\rho \sigma}(D(R)^{\lambda}_{\sigma} x^\tau) = h_{\mu \nu}(x^\lambda) \quad \forall R \in SO(3),
\]

where, in a 1+3 - split matrix notation,

\[
D(R) = \begin{pmatrix}
1 & 0^\top \\
0 & R
\end{pmatrix}.
\]

For physical reasons (finite energy around spatial infinity) we also require the asymptotic fall-off condition \( \lim_{r \to \infty} h_{\mu \nu}(x) = 0 \), where \( r \) is the 3-dimensional radius. The 00-component of (7) has the one-parameter (\( b \)) family of solutions

\[
h_{00}(\bar{x}) = -b f(r) := -b \frac{\exp(-mr)}{r},
\]

where the minus sign is introduced for later convenience. The 0-component of (8) and (10) imply \( h_{0i} = ax^i/r^3 \), which contradicts (7) unless \( a = 0 \); hence \( h_{0i} = 0 \). To determine the spatial components, we first remark that any rotationally symmetric two-tensor in space is of the form

\[
h_{ij}(\bar{x}) = f_1(r) \delta_{ij} + f_2(r) \frac{x^i x^j}{r^2}.
\]

Equation (7) now reduces to two coupled ODEs for \( f_1 \) and \( f_2 \), which may be decoupled by introducing the new function \( \tilde{f}_1 := 3f_1 + f_2 \). One obtains

\[
\begin{align*}
(\Delta - m^2) \tilde{f}_1 &= 0,

(\Delta - m^2 - \frac{6}{r^2}) f_2 &= 0,
\end{align*}
\]

which, under the given fall-off conditions, have the unique 2-parameter set of solutions

\[
\begin{align*}
\tilde{f}_1(r) &= c_1 \frac{\exp(-mr)}{r}, \\
f_2(r) &= c_2 \frac{\exp(-mr)}{r} \left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right).
\end{align*}
\]
Condition (8) and (9) now imply \( c_2 = -\frac{1}{2}c_1 \) and \( c_1 = -b \) respectively, thereby projecting out a unique one-parameter set of solutions, which, in terms of the function \( f \) defined in (12), can be written in the simple form

\[
h_{\mu\nu}(\vec{x}) = -b \left( \frac{f(r)}{0} \frac{\partial}{\partial \vec{r}} \right) \left( \delta_{ij} f(r) - \frac{1}{m^2} \partial_i \partial_j f(r) \right).
\]

We note that the \( \partial_i \partial_j f \) part is of a form that would be pure gauge in the massless theory. However, there is no gauge freedom in the massive theory and the prefactor, \( m^{-2} \), causes this term to diverge as \( m \to 0 \).

A slightly more geometric way to write the solution (18) is as follows: Let \( \vec{n} = \vec{x}/r \) be the radial unit vector; we define the spatial projection tensors \( \rho_{ij} := n_i n_j \) in radial direction and \( \tau_{ij} := \delta_{ij} - \rho_{ij} \) in the orthogonal direction, tangential to the spheres of constant \( r \). In terms of these, the spatial part of (18) takes the form

\[
h_{ij}(\vec{x}) = -\frac{bf(r)}{2} \left[ \tau_{ij} + \frac{1 + mr}{(mr)^2} (\tau_{ij} - 2\rho_{ij}) \right],
\]

which clearly separates the trace part \( \propto \tau \), which stays finite for \( m \to 0 \), and the trace free part \( \propto \tau - 2\rho \), which diverges as \( m \) tends to zero.

### 1.2 Lagrangian formulation and matter couplings

The previous equations (7,8,9) are equivalent to the Euler-Lagrange-equations of the following action:

\[
S_g = \frac{1}{4} \int d^4x \left[ h_{\mu\nu,\lambda} h^{\mu\nu,\lambda} - 2h^{\mu\lambda,\chi} h_{\mu\nu,\chi} + 2h^{\mu\nu,\chi} h_{\mu,\chi} h_{,\nu} - h_{,\mu} h_{,\nu} - m^2(h_{\mu\nu} h^{\mu\nu} - h^2) \right].
\]

The mass term is sometimes called the Pauli-Fierz term. A straightforward calculation of the variational derivative \( E^{\mu\nu} := \delta S_g / \delta h_{\mu\nu} \) (keeping in mind the symmetry \( h_{\mu\nu} = h_{\nu\mu} \)) shows that the conditions \( E^{\mu\nu} = 0, \partial_\mu E^{\mu\nu} = 0 \), and \( E^\mu_\mu = 0 \) indeed imply (7,8,9) and vice versa, given that \( m \neq 0 \).

The coupling to some specific form of matter is described by an interaction term

\[
S_{\text{int}} = -\frac{\kappa}{2} \int d^4x h_{\mu\nu} T^{\mu\nu},
\]

where \( T^{\mu\nu} \) is the energy-momentum tensor of the matter. The possibility of an \( hT \)-coupling is excluded by an argument given in section (1.4). Taking the
divergence of $\delta(S_g + S^{\text{int}})/\delta h_{\mu\nu}$ we get
\[ m^2 \partial_\mu (h^{\mu\nu} - \eta^{\mu\nu} h) = -\kappa \partial_\mu T^{\mu\nu}. \] (22)

Let us assume energy-momentum conservation for the matter field
\[ \partial_\mu T^{\mu\nu} = 0. \] (23)

Then $\partial_\mu h^{\mu\nu} = h_{;\nu}$, which, upon insertion into the Euler-Lagrange equations and taking the trace, implies that the traces of $h^{\mu\nu}$ and $T^{\mu\nu}$ must be proportional
\[ h = \frac{\kappa}{3m^2} T. \] (24)

Using (22) and (24) in the Euler-Lagrange equations they read
\[-(\Box + m^2) h_{\mu\nu} = \kappa (T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T) - \frac{\kappa}{3m^2} \partial_\mu \partial_\nu T. \] (25)

This leads indeed to the propagator (1) and establishes the background for the arguments given in the introduction.

1.3 Coupling to a point particle

The action of a free point particle of mass $m_0$ in Minkowski space is given by $-m_0$ times its arc-length. Choosing any parameter $\lambda$ to parametrize its world-line, $z^{\nu}(\lambda)$, we have
\[ S_p = -m_0 \int d\lambda \sqrt{\eta_{\mu\nu} \left( \frac{dz^{\mu}(\lambda)}{d\lambda} \frac{dz^{\nu}(\lambda)}{d\lambda} \right)}. \] (26)

Its energy-momentum tensor is
\[ T^{\mu\nu}_p(x) = m_0 \int d\tau \delta^{(4)}(x - z(\tau)) \frac{dz^{\mu}(\tau)}{d\tau} \frac{dz^{\nu}(\tau)}{d\tau}, \] (27)

where the parameter $\tau$ is the arc-length with respect to the Minkowski metric $\eta$. Since this expression is not reparametrisation invariant, we are not free to specify it otherwise. However, we can play the following trick, which turns out to be of central importance: Consider a new metric on Minkowski space, given by
\[ g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \] (28)

The arc length w.r.t. $g$ is called $s$. We consistently neglect terms of higher than linear order in $\kappa$ and may therefore replace $\tau$ by $s$ in (27), since this term
already gets multiplied by $\kappa$ in (21). In (26) we choose $\lambda = s$, replace $\eta$ by $g - \kappa h$, and expand to linear order on $\kappa$. The $\kappa$-term then just cancels the interaction term (21) and we get
\[
S_p + S_p^{\text{int}} = -m_0 \int ds \sqrt{g_{\mu\nu}(z(s))} \frac{dz^\mu(s)}{ds} \frac{dz^\nu(s)}{ds} = -m_0 \int ds,
\]
which says that the gravitationally interacting particle moves on geodesics in the metric $g$.

In order to determine the still unknown constants $\kappa$ and $b$ in $g_{\mu\nu}$ (see (18)) we compare now this equation of motion with the Newton equation. To first order in $\kappa$ this reads, writing a dot for differentiation with respect to $s$:
\[
\ddot{z}^\mu = \frac{\kappa}{2} \eta^{\mu\nu} (\partial_{\nu} h_{\alpha\beta} - \partial_{\alpha} h_{\nu\beta} - \partial_{\beta} h_{\nu\alpha}) \dot{z}^\alpha \dot{z}^\beta.
\]
In a Newtonian approximation, where we neglect terms of second and higher powers in the spatial velocities $dz^k/ds$, this reduces to
\[
\ddot{z}^0 = -\kappa h_{00,k} \dot{z}^k, \quad \ddot{z}^k = -\frac{b}{2} h_{00,k} (\dot{z}^0)^2.
\]
Introducing the Minkowski time coordinate $t := z^0$ we can rewrite the second equation, using the first one and again neglecting quadratic terms in the spatial velocities, as:
\[
\frac{d^2 z}{dt^2} = -\frac{\kappa}{2} \nabla h_{00}.
\]
Thus, recalling the fall-off condition for $h_{\mu\nu}$, we get the identification
\[
\frac{\kappa}{2} h_{00} = \Phi,
\]
where $\Phi$ is the Newtonian potential. Applied to our solution (12) we can now determine the product of the constants $\kappa$ and $b$ to be
\[
\kappa b = 2GM,
\]
where $G$ is the Newtonian gravitational constant and $M$ is the central mass, i.e., the source of the gravitational field. With this identification our metric coefficients $g_{\mu\nu}$ are now entirely determined.

At this point we may also determine the coupling $\kappa$ separately. For this we return to the field equation (22), whose 00-component for a static source and in the limit $m \to 0$ reduces to
\[
\Delta h_{00} = \frac{2}{\kappa} \kappa T_{00}.
\]
In view of (34), a comparison with the Newtonian equation \( \Delta \Phi = 4\pi G \rho \) (here \( \rho = T_{00} \)) leads to
\[
\kappa = \sqrt{12\pi G}.
\] (37)

Finally, we also remark on the fact that instead of (24) one often finds the following alternative expression for the action of a free particle (e.g. in [9]):
\[
S_p = -\frac{1}{2}m_0 \int d\lambda \eta_{\mu\nu} \frac{dz^\mu(\lambda)}{d\lambda} \frac{dz^\nu(\lambda)}{d\lambda}.
\] (38)

Like (26) it also leads to straight lines in Minkowski space, but in addition also to the condition that \( \lambda \) is (a constant multiple of) \( \tau \), the arc length measured with \( \eta \). Since one is only interested in the spacetime paths and not in their parametrisation, one may use either (26) or (38) to determine the free paths. However, we wish to point out that in the presence of an interaction of the form (21,27) the latter choice becomes inconsistent. The reason being the following: Adding the “free action” (38) with \( \lambda = \tau \) to the interaction (21) with the energy momentum tensor being given by (27), we obtain an expression like (38), where \( \eta \) is replaced by \( g \) and the parameter is still \( \tau \). But as a result of the fixed parametrisation the Euler-Lagrange equations now imply
\[
h_{\mu\nu} \frac{dz^\mu(z)}{d\tau} \frac{dz^\nu(z)}{d\tau} = 0,
\]
which in general will have no solutions. Hence we maintain that the action for the particle should be written in the square-root form (26).

### 1.4 Coupling to the electromagnetic field

The action for the free electromagnetic field in Minkowski space is given by
\[
S_{\text{em}} = -\frac{1}{4} \int d^4x \eta^{\alpha\mu} \eta^{\beta\nu} F_{\alpha\beta} F_{\mu\nu},
\] (39)
where \( F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \). The corresponding energy-momentum tensor is
\[
T_{\text{em}}^{\mu\nu} = -F^{\mu\alpha} F_{\alpha\nu} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta},
\] (40)
which we couple to the gravitational field according to (21). A priori, there is no reason why different types of matter must couple with the same constant \( \kappa \). However, if we choose different constants for different matter types, say \( \kappa_1 \) for type 1 and \( \kappa_2 \) for type 2, then the ratio of gravitational to inertial mass of type 1 would differ from the corresponding ratio of type 2 by a factor \( \kappa_1/\kappa_2 \) and hence violate the weak equivalence principle, according to which the ratio of gravitational to inertial mass is the same universal constant for all
types of matter. Hence, in order to conform with this principle, we choose the same \( \kappa \) for point particles and electromagnetism. This is also the reason why we excluded a \( hT \)-coupling in (21).

Now we can play the same trick as for the point particle and write

\[
S_{\text{em}} + S_{\text{em}}^{\text{int}} = -\frac{1}{4} \int d^4 x \eta^{\alpha\mu} \eta^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} - \frac{\kappa}{2} \int d^4 x h_{\mu\nu} T^{\mu\nu} = -\frac{1}{4} \int d^4 x \sqrt{-\det\{g\}} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} + \mathcal{O}(\kappa^2) , \tag{41}
\]

where \( g^{\mu\nu} \) and \( \det\{g\} \) are the inverse and determinant of the matrix \( g_{\mu\nu} \). To first order in \( \kappa \) we have \( g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} \) and \( \sqrt{-\det\{g\}} = 1 + \frac{\kappa}{2} h \), where \( h = h^\mu_\mu \) and indices on \( h \) are always raised and lowered with \( \eta \).

We are now in the same situation as for the point particle, since (41) is the action for the electromagnetic field on a spacetime with background metric \( g \). Without sources its equations of motion are

\[
\partial_\mu F_{\nu\lambda} = 0 \quad \partial_\mu \left( \sqrt{-\det\{g\}} g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \right) = 0 , \tag{42}
\]

which, in an eikonal approximation, imply that light rays are lightlike geodesics in the metric \( g \). Note that our decision to couple all types of matter with the universal coupling constants has the effect that light rays and particle paths are geodesics in the same metric.

## 2 Deflection of light

In this section we calculate the deflection of a light ray by the spherically symmetric gravitational field (18). We are interested in the amount of deflection as \( m \) tends to zero. A priori this limit is a precarious one since the metric coefficients diverge as \( m \to 0 \) (see (18)). However, we now show that the deflection has a finite limit. To this end we split (18) in a finite part,

\[
h_{\mu\nu}^f(\vec{x}) = -b \left( \frac{f(r)}{\vec{0}} \frac{\vec{0}^r}{2 \delta_{ij} f(r)} \right) , \tag{43}
\]

which has a continuous limit as \( m \to 0 \), and a diverging part,

\[
h_{\mu\nu}^d(\vec{x}) = b m^{-2} \partial_i \partial_j f(r) . \tag{44}
\]

Since we strictly stay within the linear theory and, accordingly, keep only terms linear in \( \kappa \), the contributions of \( h^f \) and \( h^d \) to the deflection add linearly, so that we can consider them separately. Expanding \( f(r) = \exp(-mr)/r \)
in powers of \( m \), we explicitly checked that no power smaller than \( 2 \) of \( m \) in (44) contributes to the deflection, so that in the limit as \( m \to 0 \) the diverging part does not contribute at all. In fact, one can argue that for each \( m \) the whole diverging part does not contribute to light deflection. The argument is as follows: Recall that under a spatial coordinate transformation \( x^i \mapsto x'^i = x^i + \kappa k^i(x^j) \) the spatial components of the metric \( g_{ij} \) change – to first order in \( \kappa \) – according to
\[
g_{ij} \mapsto g_{ij} - \kappa(k_{i,j} + k_{j,i}).
\]
Hence, choosing
\[
k_i(x^j) = \frac{1}{2} b m^{-2} \partial_i f(r),
\]
we can remove \( h^d \) from \( g = \eta + \kappa(h^f + h^d) \). The resulting coefficients \( g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}^f \) are then understood with respect to the coordinate system \( x'^i \) (we drop the dash hereafter). Since \( k^i \) falls off as \( r \to \infty \), the new coordinate axes asymptotically approach the old ones. In particular, they are again asymptotically orthogonal. Hence we may calculate the light deflection \( \delta(m) \) as usual, using the metric \( g^f = \eta + \kappa h^f \) with the identification \( \delta(m) \) is a continuous function of \( m \) through the continuous dependence of the light deflection on the background metric. Hence, in order to obtain \( \delta(0) \), we may calculate the light deflection for the limit metric
\[
\lim_{m \to 0} g^f_{\mu\nu}(\vec{x}) = \left( \begin{array}{cc}
1 - \frac{2GM}{r} & \vec{0}^+ \\
\vec{0} & -\delta_{ij} \left(1 + \frac{GM}{r}\right)
\end{array} \right).
\]
But instead of performing an explicit calculation, we merely need to compare (45) with the Schwarzschild metric in General Relativity. In isotropic coordinates the latter reads
\[
g^{\text{Schw}}(\vec{x}) = \left( \begin{array}{cc}
\left[1 - \frac{2GM}{r}\right]^2 & \vec{0}^+ \\
\vec{0} & -\delta_{ij} \left[1 + \frac{2GM}{r}\right]^4
\end{array} \right) = \left( \begin{array}{cc}
1 - \frac{2GM}{r} & \vec{0}^+ \\
\vec{0} & -\delta_{ij} \left(1 + \frac{2GM}{r}\right)
\end{array} \right) + O(G^2).
\]
As is well known, in leading (linear) order of \( G \), light deflection receives equal contributions from the time-time and space-space parts:
\[
\delta^{\text{Einst}} = \frac{2GM}{q} + \frac{2GM}{q} = \frac{4GM}{q},
\]
where \( q \) is the impact parameter. Applied to our metric this finally leads to the van Dam–Veltman value for the light deflection in the zero-mass limit:
\[
\lim_{m \to 0} \delta(m) = \frac{2GM}{q} + \frac{GM}{q} = \frac{3GM}{q} = \frac{3}{4} \delta^{\text{Einst}}.
\]
Acknowledgements: One of us (D.G.) likes to thank Norbert Straumann for discussions, in the course of which the issue of a classical understanding of the van Dam–Veltman–discontinuity came up.

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