Schrödinger representation of SU(2) Skyrmion

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(Dated: February 3, 2005)

Exploiting the SU(2) Skyrmion Lagrangian with second-class constraints associated with Lagrange multiplier and collective coordinates, we convert the second-class system into the first-class one in the Batalin-Fradkin-Tyutin embedding through introduction of the Stueckelberg coordinates. In this extended phase space we construct the “canonical” quantum operator commutators of the collective coordinates and their conjugate momenta to describe the Schrödinger representation of the SU(2) Skyrmion, so that we can define isospin operators and their Casimir quantum operator and the corresponding eigenvalue equation possessing integer quantum numbers, and we can also assign via the homotopy class \( \pi_4(SU(2)) = \mathbb{Z}_2 \) half integers to the isospin quantum number for the solitons in baryon phenomenology. Different from the semiclassical quantization previously performed, we exploit the “canonical” quantization scheme in the enlarged phase space by introducing the Stueckelberg coordinates, to evaluate the baryon mass spectrum having global mass shift originated from geometrical corrections due to the \( S^3 \) compact manifold involved in the topological Skyrmion. Including ghosts and anti-ghosts, we also construct Becci-Rouet-Stora-Tyutin invariant effective Lagrangian.

PACS numbers: 12.39.Dc; 14.20.-c; 11.10.-z; 11.10.Ef; 11.10.Lw; 11.15.-q; 11.30.-j
Keywords: Skyrmion; baryons; Schrödinger representation; BRST symmetries

I. INTRODUCTION

It is well known that baryons can be obtained from topological solutions, known as SU(2) Skyrmions, since homotopy group \( \Pi_3(SU(2)) = \mathbb{Z} \) admits fermions [1, 2, 3]. Using collective coordinates of isospin rotation of the Skyrmion, Witten and coworkers [1] have performed semiclassical quantization having static properties of baryons within 30% of the corresponding experimental data. The hyperfine splittings for the SU(3) Skyrmion [4] has been also studied in the SU(3) cranking method by exploiting rigid rotation of the Skyrmion in the collective space of SU(3) Euler angles with full diagonalization of the flavor symmetry breaking terms [5]. Callan and Klebanov [6] later suggested an interpretation of baryons containing a heavy quark as bound states of solitons of the pion chiral Lagrangian with mesons. In their formalism, the fluctuations in the strangeness direction are treated differently from those in the isospin directions. Moreover, exploiting the standard flavor symmetric SU(3) Skyrmion rigid rotator approach [7], the SU(3) Skyrmion with the pion mass and flavor symmetry breaking (FSB) terms has been studied to investigate the chiral symmetry breaking pion mass and FSB effects on the ratio of the strange-light to light-light interaction strengths and that of the strange-strange to light-light [8]. However, due to the geometrical constraints involved in the SU(2) group manifold of the Skyrmion, the pairs of the collective coordinates and their momenta were not yet canonical conjugate ones on quantum level and thus these quantizations could be only semiclassically performed.

On the other hand, the Dirac method [9] is a well known formalism to quantize physical systems with constraints. In this method, the Poisson brackets in a second-class constraint system are converted into Dirac brackets to attain self-consistency. The Dirac brackets, however, are generically field-dependent, nonlocal and contain problems related to ordering of field operators. To overcome these problems, Batalin, Fradkin and Tyutin (BFT) [10] developed a method which converts the second-class constraints into first-class ones by introducing Stueckelberg fields. This BFT embedding has been successively applied to several models of current interest [11]. Recently, to show novel phenomenological aspects [12], the compact form of the first-class Hamiltonian has been constructed [13] for the O(3) nonlinear sigma model, which has been also studied to investigate the Lagrangian, symplectic, Hamilton-Jacobi and BFT embedding structures [14]. The Becci-Rouet-Stora-Tyutin (BRST) symmetries [15] have been also constructed for constrained systems [11] in the Batalin-Fradkin-Vilkovisky (BFV) scheme [16].

The motivation of this paper is to quantize “canonically” the SU(2) Skyrmion model in the BFT embedding by constructing quantum operator commutators of the canonical coordinates and their conjugate momenta. In the Schrödinger representation of this system, we will study the quantum mechanical characteristics to yield the energy spectrum of the topological Skyrmion, with the geometrical global shift originated from the compactness of the target

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II. FIRST-CLASS HAMILTONIAN OF SU(2) SKYRMION

Now we start with the SU(2) Skyrmion Lagrangian of the form

\[ L_0 = \int d^3r \left[ -\frac{f^2}{4} \text{tr}(l_\mu l^\mu) + \frac{1}{32\epsilon^2} \text{tr}[l_\mu, l_\nu]^2 \right], \quad (2.1) \]

where \( l_\mu = U^\dagger \partial_\mu U \) and \( U \) is an SU(2) matrix satisfying the boundary condition \( \lim_{r \to \infty} U = I \) so that the pion field vanishes as \( r \) goes to infinity. In the Skyrmion model, since the hedgehog ansatz has maximal or spherical symmetry, it is easily seen that spin plus isospin equals zero, so that isospin transformations and spatial rotations are related to each other and spin and isospin states can be treated by collective coordinates \( a_\mu = (a_0, \vec{a}) \) \((\mu = 0, 1, 2, 3)\) corresponding to the spin and isospin rotations

\[ A(t) = a_0 + i \vec{a} \cdot \vec{\tau}, \quad (2.2) \]

which is the time dependent collective variable defined on the SU(2)_F group manifold and is related with the zero modes associated with the collective rotation. Here \( \tau_i \) \((i = 1, 2, 3)\) are the Pauli matrices. With the hedgehog ansatz and the collective rotation \( A(t) \in SU(2) \), the chiral field can be given by \( U(\vec{x}, t) = A(t)U_0(\vec{x})A^\dagger(t) = e^{i\epsilon\vec{a} \cdot \vec{r}} \) where \( R_{ab} = \frac{1}{2} \text{tr}(\epsilon_{abc} \tau_b A^\dagger) \) and the Skyrmion Lagrangian can be written as

\[ L_0 = -M_0 + 2i\dot{a}_\mu \dot{a}_\mu + a_4 a_\mu \dot{a}_\mu, \quad (2.3) \]

where \( a_4 \) is the Lagrange multiplier implementing the second-class constraint \( a_\mu \dot{a}_\mu \approx 0 \) associated with the geometrical constraint \( a_\mu a_\mu - 1 \approx 0 \) and \( M_0 \) and \( I \) are the static mass and the moment of inertia given as

\[ M_0 = \frac{2\pi f_\pi}{e} \int_0^\infty dz \ z^2 \left( \left( \frac{d\theta}{dz} \right)^2 + 2 + 2 \left( \frac{d\theta}{dz} \right)^2 + \frac{\sin^2 \theta}{z^2} \right) \left( \sin^2 \theta \right), \quad (2.4) \]

\[ I = \frac{8\pi}{3e^3 f_\pi} \int_0^\infty dz \ z^2 \sin^2 \theta \left[ 1 + \left( \frac{d\theta}{dz} \right)^2 + \frac{\sin^2 \theta}{z^2} \right], \quad (2.5) \]

with the dimensionless quantity \( z = e f_\pi r \).

From the Lagrangian (2.3) the canonical momenta conjugate to the collective coordinates \( a_\mu \) and the Lagrange multiplier \( a_4 \) are given by

\[ \pi_\mu = 4I \dot{a}_\mu + a_\mu a_4, \]

\[ \pi_4 = 0. \quad (2.6) \]

Exploiting the canonical momenta (2.6), we then obtain the canonical Hamiltonian

\[ H = M_0 + \frac{1}{8\epsilon^2} (\pi_\mu - a_\mu a_4)(\pi_\mu - a_\mu a_4). \quad (2.7) \]

The usual Dirac algorithm is readily shown to lead to the pair of second-class constraints \( \Omega_i \) \((i = 1, 2)\) as follows

\[ \Omega_1 = \pi_4 \approx 0, \]

\[ \Omega_2 = a_\mu \pi_\mu - a_\mu a_\mu a_4 \approx 0. \quad (2.8) \]

to yield the corresponding constraint algebra with \( \epsilon^2 = -\epsilon'^2 = 1 \)

\[ \Delta_{kk'} = \{\Omega_k, \Omega_{k'}\} = \epsilon^{kk'} a_\mu a_\mu. \quad (2.9) \]

Following the BFT embedding [10], we systematically convert the second-class constraints \( \Omega_i = 0 \) \((i = 1, 2)\) into first-class ones by introducing two Stückelberg coordinates \( (\theta, \pi_\theta) \) with Poisson bracket

\[ \{\theta, \pi_\theta\} = 1. \quad (2.10) \]
The strongly involutive first-class constraints $\tilde{\Omega}_i$ are then constructed as a power series of the St"uckelberg coordinates,

$$\tilde{\Omega}_1 = \Omega_1 + \theta, \quad \tilde{\Omega}_2 = \Omega_2 - a_\mu a_\mu \pi_\theta. \quad (2.11)$$

Note that the first-class constraints (2.11) can be rewritten as

$$\tilde{\Omega}_1 = \pi_4, \quad \tilde{\Omega}_2 = \tilde{a}_\mu \tilde{\pi}_\mu - \tilde{a}_\mu \tilde{a}_\mu \tilde{a}_4, \quad (2.12)$$

which are form-invariant with respect to the second-class constraints (2.8).

We next construct the first-class variables $\tilde{\mathcal{F}} = (\tilde{a}_\mu, \tilde{\pi}_\mu)$, corresponding to the original coordinates defined by $\mathcal{F} = (a_\mu, \pi_\mu)$ in the extended phase space. They are obtained as a power series in the St"uckelberg coordinates $(\theta, \pi_\theta)$ by demanding that they be in strong involution with the first-class constraints (2.11), that is $\{\tilde{\Omega}_i, \tilde{\mathcal{F}}\} = 0$. After some tedious algebra, we obtain for the first-class coordinates and their momenta

$$\tilde{a}_\mu = a_\mu \left( \frac{a_\sigma a_\sigma + 2\theta}{a_\sigma a_\sigma} \right)^{1/2},$$

$$\tilde{\pi}_\mu = \left( \pi_\mu + 2a_\mu a_4 \frac{\theta}{a_\sigma a_\sigma} + 2a_\mu \pi_\theta \frac{\theta}{a_\sigma a_\sigma} \right) \left( \frac{a_\sigma a_\sigma}{a_\sigma a_\sigma + 2\theta} \right)^{1/2},$$

$$\tilde{a}_4 = a_4 + \pi_\theta,$$

$$\tilde{\pi}_4 = \pi_4 + \theta,$$

and the first-class Hamiltonian

$$\tilde{H} = M_0 + \frac{1}{8T}(\tilde{\pi}_\mu - \tilde{a}_\mu \tilde{a}_4)(\tilde{\pi}_\mu - \tilde{a}_\mu \tilde{a}_4). \quad (2.13)$$

**III. SCHRÖDINGER REPRESENTATION FOR SU(2) SKYRMION**

In this section, we start with noting that the first-class coordinates and their momenta (2.13) are found to satisfy the Poisson algebra

$$\{\tilde{a}_\mu, \tilde{a}_\nu\} = 0, \quad \{\tilde{a}_\mu, \tilde{\pi}_\nu\} = \delta_\mu^\nu, \quad \{\tilde{\pi}_\mu, \tilde{\pi}_\nu\} = 0, \quad (3.1)$$

which, in the extended phase space, yield the quantum commutators as in the cases of unconstrained systems

$$[\hat{a}_\mu, \hat{a}_\nu] = 0, \quad [\hat{a}_\mu, \hat{\pi}_\nu] = i\hbar \delta_{ab}, \quad [\hat{\pi}_\mu, \hat{\pi}_\nu] = 0. \quad (3.2)$$

We emphasize here that the quantum commutators in (3.2) enable us to describe the Schrödinger representation of the SU(2) Skyrmion and to construct the quantum operator for $\hat{\pi}_\mu$

$$\hat{\pi}_\mu = -i\hbar \frac{\partial}{\partial a_\mu}, \quad (3.3)$$

1 In the case of the SU(2) Skyrmion Lagrangian without explicit inclusion of the constraint $a_4 a_\mu \dot{a}_\mu$, we can find the unusual Poisson algebra $\{\tilde{a}_\mu, \tilde{\pi}_\nu\} = \delta_\mu^\nu - \tilde{a}_\mu \tilde{a}_\nu$ for instance [11, 22], so that strictly speaking we cannot construct the quantum operator for $\tilde{\pi}_\mu$ as in (3.3), since $(\tilde{a}_\mu, \tilde{\pi}_\nu)$ are not “canonical” conjugate pair any more.
The spin operators \( \hat{S}_i \) and the isospin operators \( \hat{I}_i \) are then given by [1]

\[
\begin{align*}
\hat{S}_i &= -\frac{i\hbar}{2} \left( \partial_0 \frac{\partial}{\partial a_i} - \hat{a}_i \frac{\partial}{\partial a_0} - \epsilon_{ijk} \hat{a}_j \frac{\partial}{\partial a_k} \right), \\
\hat{I}_i &= -\frac{i\hbar}{2} \left( \partial_0 \frac{\partial}{\partial a_i} - \hat{a}_i \frac{\partial}{\partial a_0} + \epsilon_{ijk} \hat{a}_j \frac{\partial}{\partial a_k} \right),
\end{align*}
\]

(3.4)

to yield the Casimir operator

\[
\hat{S}^2 = \hat{I}^2 = \frac{\hbar^2}{4} \left( -\frac{\partial^2}{\partial a_\mu \partial a_\mu} + 3 \hat{a}_\mu \frac{\partial}{\partial a_\mu} + \hat{a}_\nu \hat{a}_\lambda \frac{\partial}{\partial a_\mu} \partial_{a_\nu} \partial_{a_\lambda} \right).
\]

(3.5)

Note that the Casimir operator (3.5) is associated with the three-sphere Laplacian whose eigenvalue equation is of the form with quantum numbers \( l \) (= integers) [17],

\[
\left( -\frac{\partial^2}{\partial a_\mu \partial a_\mu} + 3 \hat{a}_\mu \frac{\partial}{\partial a_\mu} + \hat{a}_\nu \hat{a}_\lambda \frac{\partial}{\partial a_\mu} \partial_{a_\nu} \partial_{a_\lambda} \right) \psi_l(A) = l(l + 2) \psi_l(A).
\]

(3.6)

We can then find the eigenvalue for the Casimir operator (3.5) as follows

\[
\hat{I}^2 \psi_l(A) = \hbar^2 I(I + 1) \psi_l(A),
\]

(3.7)

where \( I = l/2 \) are the total isospin quantum numbers of baryons. Note that not all of the states discussed above are physically allowed. Since our solitons are to be fermions, we have the condition on the quantum wave function [18],

\[
\psi_l(-A) = -\psi_l(A),
\]

(3.8)

whose allowed values of \( I \) are \( I = 1/2, 3/2, \cdots \), corresponding to nucleons \( I = S = 1/2 \) and deltas \( I = S = 3/2 \) in the baryon phenomenology [1, 11]. Note that the condition (3.8) is closely related to the nontrivial homotopy class \( \pi_4(SU(2)) = Z_2 \) associated with a space-time manifold compactified to be \( S^3 = S^3 \times S^1 \) where \( S^3 \) and \( S^1 \) are compactified Euclidean three-space and time, respectively [4, 11].

Next, following symmetrization procedure [11, 19, 20] together with (2.14) and (3.3), we arrive at the Hamiltonian quantum operator for the SU(2) Skyrmion

\[
\hat{H} = M_0 + \frac{1}{8\pi} \left( -i\hbar \frac{\partial}{\partial a_\mu} + i\hbar \hat{a}_\mu \frac{\partial}{\partial a_\mu} \right) \left( -i\hbar \frac{\partial}{\partial a_\mu} + i\hbar \hat{a}_\nu \hat{a}_\lambda \frac{\partial}{\partial a_\mu} \partial_{a_\nu} \partial_{a_\lambda} \right) ;
\]

\[
= M_0 + \frac{\hbar^2}{8\pi} \left( -\frac{\partial^2}{\partial a_\mu \partial a_\mu} + 3 \hat{a}_\mu \frac{\partial}{\partial a_\mu} + \hat{a}_\nu \hat{a}_\lambda \frac{\partial}{\partial a_\mu} \partial_{a_\nu} \partial_{a_\lambda} + \frac{5}{4} \right)
\]

\[
= M_0 + \frac{1}{2\pi} \left( \hat{I}^2 + \frac{5\hbar^2}{16} \right).
\]

(3.9)

Note that the Hamiltonian quantum operator (3.9) has terms of orders \( \hbar^0 \) and \( \hbar^2 \) only, so that one can have static mass (of order \( \hbar^0 \)) and rotational energy contributions (of order \( \hbar^2 \)) without any vibrational mode ones (of order \( \hbar^1 \)). In fact, the starting Lagrangian (2.3) does not possess any vibrational degrees of freedom in itself since it has the kinetic term describing the motions of the soliton residing on the \( S^3 \) manifold. The Schrödinger representation for the SU(2) Skyrmion can be then given by the following eigenvalue equation

\[
\left[ M_0 + \frac{1}{2\pi} \left( \hat{I}^2 + \frac{5\hbar^2}{16} \right) \right] \psi_I = E_I \psi_I,
\]

(3.10)

to yield the mass spectrum of the baryons

\[
E_I = M_0 + \frac{\hbar^2}{2\pi} \left[ I(I + 1) + \frac{5}{16} \right],
\]

(3.11)

which originate from the static mass and rotational energy contributions discussed above.

Now, it seems appropriate to discuss the global mass shift involved in the rotational energy contributions to the mass spectrum (3.11). In fact, in Ref. [1] the mass spectrum of the SU(2) Skyrmion was constructed in the framework of the semiclassical quantization where they ignored the effects of the geometrical constraints involved in the model.
The mass spectrum of the SU(2) Skyrmion was later improved with the Weyl ordering corrections [20, 21] in the BFT embedding, where one still could not construct well-defined quantum commutators among the collective coordinates and their conjugate momenta due to the unusual Poisson algebra of these variables. In that sense the modified Skyrmion mass spectrum in the previous BFT embedding was not rigorously constructed. However, in the above BFT embedding approach to the SU(2) Skyrmion, we recall that the quantum commutators among the collective coordinates and their conjugate momenta are canonically well defined and thus the mass spectrum has been rigorously evaluated to yield the global mass shift as in (3.11). Note that the global mass shift in the spectrum (3.11) originates from the geometrical corrections due to the characteristics of the $S^3$ compact manifold involved in the topological Skyrmion model.

**IV. BRST SYMMETRIES IN SU(2) SKYRMION**

In order to investigate the BRST symmetries [15] associated with the Lagrangian (2.3) of the SU(2) Skyrmion model, we rewrite the first-class Hamiltonian (2.14) in terms of original collective variables and St"uckelberg coordinates

$$\tilde{H} = M_0 + \frac{1}{8L}(\pi_\mu - a_\mu a_4 - a_\mu \pi_\theta)(\pi_\mu - a_\mu a_4 - a_\mu \pi_\theta) \frac{a_\nu a_\nu}{a_\nu a_\nu + 2\theta},$$

which is strongly involutive with the first-class constraints, $\{\tilde{\Omega}_1, \tilde{H}\} = 0$. Note that with this Hamiltonian (4.1), we cannot generate the first-class Gauss' law constraint from the time evolution of the constraint $\tilde{\Omega}_1$. By introducing an additional term proportional to the first-class constraints $\tilde{\Omega}_2$ into $\tilde{H}$, we obtain an equivalent first-class Hamiltonian

$$\tilde{H}' = \tilde{H} + \frac{1}{4L}\pi_\theta \tilde{\Omega}_2$$

(4.2)

to generate the Gauss' law constraint

$$\{\tilde{\Omega}_1, \tilde{H}'\} = \frac{1}{4L}\tilde{\Omega}_2, \quad \{\tilde{\Omega}_2, \tilde{H}'\} = 0.$$  

(4.3)

Note that these Hamiltonians $\tilde{H}$ and $\tilde{H}'$ effectively act on physical states in the same way since such states are annihilated by the first-class constraints.

In the framework of the BFV formalism [16], by introducing two canonical sets of ghosts and anti-ghosts, together with Lagrange multipliers

$$(C^i, \bar{P}_i), \quad (\bar{C}^i, \bar{C}_i), \quad (N^i, B_i), \quad (i = 1, 2),$$

(4.4)

and the unitary gauge choice $\chi^1 = \Omega_1$, $\chi^2 = \Omega_2$, we now construct the nilpotent BRST charge $Q$, the fermionic gauge fixing function $\Psi$ and the BRST invariant minimal Hamiltonian $H_m$

$$Q = C^i \bar{\Omega}_i + \bar{P}^i B_i, \quad \Psi = \bar{C}_i \chi^i + \bar{P}_i N^i,$$

$$H_m = \tilde{H} + \frac{1}{4L}\pi_\theta \tilde{\Omega}_2 - \frac{1}{4L}C^1 \bar{P}_2,$$

(4.5)

with the properties $Q^2 = \{Q, Q\} = 0$ and $\{\Psi, Q\}, Q = 0$. The nilpotent charge $Q$ is the generator of the following infinitesimal transformations,

$$\delta Q a_\mu = -C^2 a_\mu, \quad \delta Q a_4 = -C^1, \quad \delta Q \theta = C^2 a_\mu a_\mu,$$

$$\delta Q \pi_\mu = C^2(\pi_\mu - 2a_\mu a_4 - 2a_\mu \pi_\theta), \quad \delta Q \pi_\theta = -C^2 a_\mu a_\mu, \quad \delta Q \pi_\theta = C^1,$$

$$\delta Q \bar{C}_i = B_i, \quad \delta Q \bar{C}_i = 0, \quad \delta Q \bar{P}_i = \bar{\Omega}_i, \quad \delta Q N^i = -\bar{P}^i,$$

(4.6)

$^2$ The BRST symmetries were constructed in the SU(2) Skyrmion Lagrangian which is different from (2.3) in the sense that the constraints $a_\mu a_\mu$ is not explicitly included [11, 22]. The BRST transformation rules for the Lagrange multiplier and its momentum $(a_4, \pi_4)$ are then missing in the Refs. [11, 22].
which in turn imply \( \{ Q, H_m \} = 0 \), that is, \( H_m \) in (4.5) is the BRST invariant.

After some algebra, we arrive at the effective quantum Lagrangian of the form

\[
L_{\text{eff}} = L_0 + L_{WZ} + L_{\text{ghost}}
\]

where \( L_0 \) is given by (2.3) and

\[
L_{WZ} = \frac{4I\theta}{1 - 2\theta} \hat{a}_\mu \hat{a}_\mu - \frac{2I}{(1 - 2\theta)^2} \hat{\theta}^2,
\]

\[
L_{\text{ghost}} = -2I(1 - 2\theta)^2(B + 2\bar{C}C)^2 - \frac{\hat{\theta}\hat{B}}{1 - 2\theta} + \hat{\theta}\hat{C} - \frac{(1 - 2\theta)^2}{2}a_4(B + 2\bar{C}C),
\]

with redefinition \( C = C^2, \bar{C} = \bar{C}^2 \) and \( B = B^2 \). This Lagrangian is invariant under the BRST transformation

\[
\delta_\epsilon a_\mu = \epsilon a_\mu C, \quad \delta_\epsilon a_4 = -2\epsilon a_4 C, \quad \delta_\epsilon \theta = -\epsilon a_\mu a_\mu C,
\]

\[
\delta_\epsilon \bar{C} = -\epsilon B, \quad \delta_\epsilon C = 0, \quad \delta_\epsilon B = 0,
\]

where \( \epsilon \) is an infinitesimal Grassmann valued parameter.

V. CONCLUSION

In conclusion, we have introduced the SU(2) Skyrmion Lagrangian having constraints of the form \( a_4 a_\mu \hat{a}_\mu \) with the Lagrange multiplier \( a_4 \) and the collective coordinates \( a_\mu \) to convert the second-class Hamiltonian into the first-class one in the BFT embedding. In this embedding by including the Stückelberg coordinates we have enlarged the phase space, where we could construct the quantum operator commutators of the “canonical” collective coordinates and their conjugate momenta and then we could describe the Schrödinger representation of the SU(2) Skyrmion model.

Constructing the isospin operators in the enlarged phase space, we have associated their Casimir quantum operator with the three-sphere Laplacian to obtain the corresponding eigenvalue equation having the integer quantum numbers. Classifying the physical states relevant to these quantum numbers via the homotopy class \( \pi_4(SU(2)) = \mathbb{Z}_2 \), we have assigned half integers to the isospin quantum number for the solitons, which are to be fermionic. Note that, different from the previous results in which the quantization was semiclassically performed, we have exploited the “canonical” quantization scheme in the enlarged phase space by introducing the Stückelberg coordinates degrees of freedom to evaluate the mass spectrum of the baryons, which has the global mass shift originated from the geometrical corrections due to the characteristics of the \( S^3 \) compact manifold involved in the topological Skyrmion.

Enlarging further the phase space by including the ghosts and anti-ghosts, we have constructed the nilpotent BRST charge in the BFV formalism to derive the effective Lagrangian invariant under the BRST transformation associated with the BRST charge. Here one notes that in constructing the BRST symmetries in the topological SU(2) Skyrmion having the second-class geometrical constraints it was crucial to introduce the Stückelberg coordinates degrees of freedom, which enable us to exploit the BFV scheme.

Acknowledgments

The author would like to acknowledge financial support in part from the Korea Science and Engineering Foundation Grant R01-2000-00015.

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