Minimal Theory of Isomerism- Q.Q and Other Interactions

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Abstract. We perform shell model calculations using a quadrupole-quadrupole interaction (Q.Q) in a single $j$ shell space. We show that this one-parameter interaction is a good predictor of where nuclear isomerism occurs and where it does not occur. The limitations of this interaction are also discussed. We then include other interactions, e.g. in the $f_{7/2}$ shell those obtained from the (local) spectrum of a two particle system and then from a two hole system. In the $g_{9/2}$ where there is insufficient empirical data from the two hole system and so various empirical interactions are used.

1. Introduction

Study of a nuclear isomer in the different region of nuclear chart attracted much experimental and theoretical investigations \[1, 2, 3, 4, 5]\). In the present work we consider two types of isomeric states. In one case, we have the spin-gap isomer state of angular momentum $J$ for which there are no states of angular momentum ($J-1$) or ($J-2$) below the isomeric state. These isomers can have very long half-lives. They cannot decay by E1, M2 or E2 transitions. Then there is the second kind of isomer, where there is a, say, a state of angular momentum ($J-2$) below the $J$ state but the energy difference is so small that the transition is hindered; e.g. for E2 transitions the rate goes as $(E_i - E_f)^5$; for $M_1$ and $E_1$ as $(E_i - E_f)^3$.

While admirable progress has been made in the calculations of larger and larger spaces for quantitative shell model properties \[6, 7, 8]\, it should not be forgotten that much insight can be gained by doing simple calculations with simple interactions. We here discuss, in part, the quadrupole-quadrupole interaction (Q.Q) which has only one parameter i.e. the overall strength. This parameter can be adjusted to give the correct excitation energy. We will show that the Q.Q interaction is a good predictor of where isomeric states can be found. It should be said that the idea of using matrix elements from experiment came from earlier works of deShalit and Talmi \[9, 10]\.
We then consider, in addition, other empirical (fitted) interactions. In the f\textsubscript{7/2} shell we can use the spectrum of the two-particle system \textsuperscript{42}Sc from the experimental data for the effective interactions to make predictions of isomerism in \textsuperscript{43}Sc, \textsuperscript{43}Ti and \textsuperscript{44}Ti. We then use, the local hole-hole spectrum of \textsuperscript{54}Co to make predictions of the isomerism in \textsuperscript{53}Fe, \textsuperscript{53}Co and \textsuperscript{52}Fe. In the g\textsubscript{9/2} shell, there is insufficient experimental data about the two hole system \textsuperscript{98}In so empirical two-body matrix elements are obtained by other means \cite{11, 12, 13}.

For a neutron and proton in a single j shell, the states of even angular momentum J have isospin T= 1 whilst the odd ones have T=0 zero. The T=0 two-body matrix elements can only be obtained from the np system- not from nn or pp. They are usually not as well known as the ones with T=1. Because of charge independence the spectra of T=1 states for the np, nn and pp systems are nearly identical and this is a big help in obtaining the corresponding two-body matrix elements extracted from the energy-levels. We will later show the striking differences in T=0 matrix elements obtained in 1964 \cite{14, 15, 16, 17} and the ones obtained in 2006 \cite{18} when better data were available. Our aim is to see differences in the energies with earlier and new set of effective interactions.

Isomeric states played a very prominent role in the ground breaking book, elementary theory of nuclear shell structure, by Mayer and Jensen \cite{19}. They are cited as evidence that there is indeed a shell structure in nuclei. For updated reading and broadening the scope the review articles by Heyde and Wood \cite{20} on shape isomers and by Draculis, Walker and F.G. Kondev \cite{21} on isomers in heavier nuclei are recommended. The latter also discusses medical applications of nuclear isomers.

Table 1: Two-body matrix elements of the Q.Q interaction. We have chosen χ’ =1 corresponding to χ’ = χ \textsuperscript{b4}.

| J      | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| f\textsubscript{7/2} | -1.9184 | -1.5530 | -0.8952 | -0.0914 | 0.6395 | 1.0049 | 0.6396 | 0.8952 |     |     |
| g\textsubscript{9/2} | -2.9178 | -2.5642 | -1.9010 | -1.0168 | -0.0442 | 0.8400 | 1.4147 | 1.4147 | 0.5305 | -1.5916 |

We will be dealing mainly with high spin isomers. In the single j shell it is easy to determine the maximum J. For identical particles put a given particle in the highest m state consistent with the Pauli principle. For example, for \textsuperscript{43}Sc the one proton is in m=7/2 state and the two neutrons in 7/2 and 5/2. This adds up to 19/2 which is indeed J\textsubscript{max} for \textsuperscript{43}Sc. It is also J\textsubscript{max} for \textsuperscript{43}Ti, \textsuperscript{53}Co and \textsuperscript{53}Fe. For \textsuperscript{44}Ti we have two protons and two neutrons one with m=7/2 and one with m=5/2. This is responsible to generate J = 12\textsuperscript{+}.

Table 2: Two body matrix elements used in the f\textsubscript{7/2} shell.

| J        | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|
| MBZ(1964)| 0.0000 | 1.036 | 1.509 | 2.248 | 2.998 | 1.958 | 3.400 | 0.617 |
| MBZE (2006)\textsuperscript{42}Sc | 0.0000 | 0.6110 | 1.5803 | 1.4904 | 2.8153 | 1.5100 | 3.2420 | 0.6163 |
| hole-hole \textsuperscript{54}Co | 0.0000 | 0.9369 | 1.4457 | 1.8215 | 2.6450 | 1.8770 | 2.9000 | 0.1974 |
| Q.Q      | 0.0000 | 0.3655 | 1.0232 | 1.8270 | 2.5579 | 2.9233 | 2.5580 | 1.0232 |
Table 3: Two-body matrix elements used in the $g_{9/2}$ shell.

|     | 0   | 1    | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|-----|------|-----|------|------|------|------|------|------|------|
| CCGI| 0.000 | 0.829 | 1.710 | 1.877 | 2.217 | 2.046 | 2.383 | 1.913 | 2.527 | 0.915 |
| Qi et al. | 0.000 | 1.220 | 1.458 | 1.592 | 2.283 | 1.882 | 2.549 | 1.930 | 2.688 | 0.626 |
| $g_{9/2}$ Q.Q | 0.000 | 0.3536 | 1.0168 | 1.8990 | 2.8736 | 3.7618 | 4.3325 | 4.3325 | 3.4483 | 1.3262 |
| INTd | 0.000 | 1.1387 | 1.3947 | 1.8230 | 2.0283 | 1.9215 | 2.2802 | 1.8797 | 2.4275 | 0.7500 |

2. The Q.Q and other interactions in the single j shell

The interaction we use is $\chi Q.Q = -\chi \sqrt{5} [(r^2 Y^2)_i (r^2 Y^2)_j]^0$. Two body matrix elements were constructed using harmonic oscillator radial wavefunctions. These have one parameter, the oscillator length $b$, which is approximately equal to $A^{1/6} \text{ fm}$. In evaluating energies, unless specified otherwise, we set $\chi b^4$ to 1 MeV. Alternately, one can say that the energy is in units of $\chi b^4$. Results for two-body matrix elements of this interaction are shown in table 1.

We shall also be showing results from other interactions for comparison. In the early 1960’s empirical two body matrix elements were taken from the spectrum of $^{42}\text{Sc}$ in order to do calculations in the $f_{7/2}$ region. We cite the works of Bayman et al. [15], McCullen et al. [16] and Ginocchio and French [17]. However, at that time (1964) the T=0 two body matrix elements were not well determined. The T=1 states also occurred in $^{42}\text{Ca}$ and $^{42}\text{Ti}$ and so were much better known. Despite these deficiencies we show in the second column of table 2 the results of the old MBZ (1964) interaction. This should be compared with the results of the newer MBZE (2006) interaction shown in the 3rd column, as performed by Escuderos, Zamick and Bayman [18]. The latter calculation is basically the same as the 1964 one except that the input parameters were better known in 2006. Also, shown are matrix elements from the two hole system $^{54}\text{Co}$. These are appropriate for nuclei in the upper part of the $f_{7/2}$ shell (had one used the same interaction, the spectrum of holes would be the same as that for particles). In the first 3 rows of table 2 the ground state energy has been set to zero. To make a better comparison with Q.Q we also added a constant (1.9184 MeV) to the matrix elements in the first row of Table 1 and show this in the last row of Table 2. Adding a constant will not affect the level spacings. In table 3 we have shown the two-body matrix elements used for the $g_{9/2}$ shell calculations.

We note that besides a strongly attractive J=0, T=1 matrix element, Q.Q has also attractive matrix elements for the neutron-proton system in the T=0 channel, namely for J=1 and J=$J_{max}$, the latter being seven in the $f_{7/2}$ shell and nine in the $g_{9/2}$ shell. This is also a feature of the empirical two-body matrix elements in both shells. The Q.Q interaction is thus quite different from the J=0 T=1 pairing interaction which was in vogue in the early fifties, e.g. in the works of Flowers [13] and Edmund and Flowers [22].
3. The spectra with a Q.Q and other interactions.

In this section we present results of single j shell calculations of energy levels for selected nuclei in both the f_{7/2} and g_{9/2} regions. These are contained in Table 4 to 9. In Table 4, we show the spectra of even-even nuclei using the Q.Q interaction; in Table 5, odd A nuclei are considered and in Table 6, odd-odd nuclei. In Tables 7,8 and 9 we have reported the corresponding spectra, but using local interactions from experiment. For the lower part of the f_{7/2} shell we use the particle-particle spectrum of ^{42}\text{Sc} as input whilst in the upper half the hole-hole spectrum of ^{54}\text{Co}. Discussions will follow in the next section. In tables 4 to 8 we give results for \( \chi^\prime = \chi b^4 = 1 \). A reasonable fit with the experimental data in the f_{7/2} and g_{9/2} region is given for \( \chi = 0.4 \text{ MeV} \).

Table 4: Energy levels (in MeV) of even-even nuclei with a Q.Q interaction. Energy are in MeV.

| J | ^{44}\text{Ti} | ^{52}\text{Fe} | ^{48}\text{Cr} | ^{96}\text{Cd} | ^{92}\text{Pd,88}\text{Ru} |
|---|---------------|---------------|---------------|---------------|----------------|
| 0 | 0.000         | 0.000         | 0.000         | 0.000         |                 |
| 1 | 2.995         | 2.296         | 5.040         | 4.747         |                 |
| 2 | 0.570         | 0.552         | 0.867         | 0.563         |                 |
| 3 | 3.955         | 2.854         | 6.077         | 5.333         |                 |
| 4 | 1.905         | 0.925         | 2.753         | 1.557         |                 |
| 5 | 5.062         | 2.884         | 7.734         | 6.247         |                 |
| 6 | 3.468         | 1.695         | 5.352         | 3.044         |                 |
| 7 | 4.716         | 3.722         | 8.820         | 5.787         |                 |
| 8 | 5.087         | 2.647         | 5.625         | 4.817         |                 |
| 9 | 6.423         | 4.978         | 7.620         | 7.667         |                 |
|10 | 6.501         | 4.125         | 9.235         | 6.703         |                 |
|11 | 7.446         | 6.703         | 10.767        | 9.400         |                 |
|12 | 6.277         | 6.126         | 11.414        | 8.535         |                 |
|13 | 8.817         | 12.449        | 10.864        | 10.481        |                 |
|14 | 8.633         | 12.075        | 12.85         | 13.636        |                 |
|15 | 11.558        | 12.285        | 12.706        | 16.113        |                 |
|16 | 11.377        | 10.163        | 15.317        | 18.897        |                 |
|17 | 11.294        | 10.127        | 18.347        | 22.136        |                 |
|18 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
|19 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
|20 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
|21 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
|22 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
|23 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
|24 | 21.793        | 25.885        | 18.347        | 22.136        |                 |
Table 5: Energy levels (in MeV) of odd A nuclei with a Q,Q interaction.

| 2J | $^{43}$Sc | $^{53}$Fe, $^{53}$Co | $^{97}$Cd | $^{95}$Ag | $^{93}$Ag |
|----|-----------|---------------------|--------|--------|--------|
| 1  | 3.906     | 7.159               | 7.623  | 6.833  |
| 3  | 3.284     | 6.585               | 6.164  | 5.365  |
| 5  | 2.018     | 5.485               | 4.426  | 4.075  |
| 7  | 0.000     | 3.441               | 0.000  | 0.000  |
| 9  | 0.816     | 0.000               | 0.997  | 0.740  |
| 11 | 1.905     | 1.602               | 2.250  | 1.666  |
| 13 | 3.217     | 3.156               | 3.361  | 2.531  |
| 15 | 3.467     | 4.752               | 5.154  | 3.708  |
| 17 | 4.088     | 5.703               | 5.874  | 4.597  |
| 19 | 2.700     | 6.852               | 8.640  | 4.611  |
| 21 |           | 6.585               | 8.252  | 5.992  |
| 23 |           | 6.585               | 5.675  | 7.624  |
| 25 |           | 4.374               | 8.032  | 8.260  |
| 27 |           |                     | 10.173 | 10.775 |
| 29 |           |                     | 11.295 | 10.499 |
| 31 |           |                     | 13.139 | 13.495 |
| 33 |           |                     | 12.907 | 12.710 |
| 35 |           |                     | 14.475 | 15.045 |
| 37 |           |                     | 12.840 | 15.318 |
| 39 |           |                     |        | 18.253 |
| 41 |           |                     |        | 18.091 |
| 43 |           |                     |        | 21.424 |
| 45 |           |                     |        | 20.761 |
Table 6: Energy levels (in MeV) of Odd-Odd Nuclei with a Q.Q Interaction.

| J | $^{44}$Sc | $^{52}$Mn | $^{48}$V | $^{94}$Ag | $^{96}$Ag |
|---|-------|-------|-------|-------|-------|
| 0 | 2.982 | 5.623 | 0.000 | 3.506 |
| 1 | 0.000 | 0.000 | 0.275 | 0.000 |
| 2 | 0.474 | 0.001 | 0.631 | 0.388 |
| 3 | 0.960 | 0.558 | 1.147 | 1.037 |
| 4 | 1.786 | 0.308 | 1.885 | 1.823 |
| 5 | 2.066 | 0.588 | 2.731 | 2.694 |
| 6 | 0.472 | 0.914 | 3.667 | 3.511 |
| 7 | 1.779 | 1.426 | 0.657 | 3.779 |
| 8 | 2.989 | 2.564 | 2.217 | 0.589 |
| 9 | 3.427 | 2.682 | 3.810 | 2.580 |
| 10 | 5.254 | 4.446 | 5.555 | 4.511 |
| 11 | 4.450 | 4.407 | 6.741 | 5.727 |
| 12 | 6.412 | 9.014 | 7.544 |
| 13 | 6.520 | 9.189 | 7.409 |
| 14 | 9.079 | 9.720 | 9.013 |
| 15 | 9.263 | 11.127 | 7.245 |
| 16 | 13.312 |
| 17 | 13.460 |
| 18 | 15.805 |
| 19 | 15.437 |
| 20 | 17.745 |
| 21 | 16.507 |
Table 7: Energy levels (in MeV) of even A Nuclei with $f_{7/2}^{42}$Sc and $f_{7/2}^{54}$Co interactions.

| J | $^{44}$Ti (with $f_{7/2}^{42}$Sc) | $^{52}$Fe ($f_{7/2}^{54}$Co) | $^{48}$Cr (with $f_{7/2}^{42}$Sc) | $^{48}$Cr ($f_{7/2}^{54}$Co) |
|---|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 0 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 | 5.660 | 5.459 | 5.472 | 5.172 |
| 2 | 1.159 | 1.024 | 1.203 | 1.084 |
| 3 | 5.783 | 5.810 | 5.746 | 5.614 |
| 4 | 2.787 | 2.611 | 2.249 | 1.965 |
| 5 | 5.868 | 6.234 | 4.302 | 4.251 |
| 6 | 4.065 | 3.989 | 3.484 | 3.062 |
| 7 | 6.040 | 5.880 | 5.954 | 5.535 |
| 8 | 6.084 | 5.649 | 5.002 | 4.262 |
| 9 | 7.989 | 7.737 | 6.989 | 6.267 |
| 10 | 7.390 | 6.611 | 6.447 | 5.401 |
| 11 | 9.871 | 8.617 | 8.623 | 7.671 |
| 12 | 7.708 | 6.413 | 7.891 | 6.606 |
| 13 | | | 11.578 | 10.168 |
| 14 | | | 10.263 | 8.580 |
| 15 | | | 14.550 | 12.432 |
| 16 | | | 13.583 | 11.421 |

Table 8: Energy levels (in MeV) of odd A nuclei with $f_{7/2}^{42}$Sc and $f_{7/2}^{54}$Co interactions.

| 2J | $^{43}$Sc (with $f_{7/2}^{42}$Sc) | $^{53}$Fe (with $f_{7/2}^{54}$Co) |
|---|-------------------------------|-------------------------------|
| 1 | 4.319 | 4.870 |
| 3 | 2.885 | 3.528 |
| 5 | 3.451 | 3.849 |
| 7 | 0.000 | 0.000 |
| 9 | 1.676 | 1.524 |
| 11 | 2.332 | 2.201 |
| 13 | 3.503 | 3.337 |
| 15 | 3.514 | 3.204 |
| 17 | 4.300 | 4.052 |
| 19 | 3.648 | 2.817 |
Table 9: Energy levels (in MeV) of Odd-Odd Nuclei with $f_{7/2}^{42}{\text{Sc}}$ and $f_{7/2}^{54}{\text{Co}}$ interactions.

| J  | $^{44}{\text{Sc}}$ (with $f_{7/2}^{42}{\text{Sc}}$) | $^{52}{\text{Mn}}$ (f$^{42}_{7/2}$Co) | $^{48}{\text{V}}$ (with $f_{7/2}^{42}{\text{Sc}}$) | $^{48}{\text{Mn}}$ (f$^{54}_{7/2}$Co) |
|----|---------------------------------------|-----------------------------|---------------------------------------|---------------------------------------|
| 0  | 3.055                                | 2.784                       | 5.200                                 | 5.975                                 |
| 1  | 0.427                                | 0.446                       | 0.450                                 | 0.497                                 |
| 2  | 0.000                                | 0.155                       | 0.000                                 | 0.093                                 |
| 3  | 0.762                                | 0.797                       | 0.924                                 | 0.903                                 |
| 4  | 0.719                                | 0.792                       | 0.157                                 | 0.000                                 |
| 5  | 1.279                                | 1.325                       | 0.761                                 | 0.460                                 |
| 6  | 0.381                                | 0.000                       | 0.626                                 | 5.894                                 |
| 7  | 1.275                                | 0.866                       | 1.339                                 | 0.913                                 |
| 8  | 3.099                                | 2.554                       | 2.484                                 | 1.980                                 |
| 9  | 3.392                                | 2.724                       | 2.836                                 | 2.077                                 |
| 10 | 4.801                                | 4.191                       | 4.610                                 | 3.820                                 |
| 11 | 4.635                                | 3.604                       | 4.596                                 | 3.548                                 |
| 12 |                                      |                             | 6.993                                 | 5.895                                 |
| 13 |                                      |                             | 6.910                                 | 5.493                                 |
| 14 |                                      |                             | 8.809                                 | 7.474                                 |
| 15 |                                      |                             | 9.531                                 | 7.757                                 |

4. Qualitative discussion of the Q.Q. tables

Here we will simply report whether the Q.Q interaction is able to predict a spin gap, a weak isomerism or no isomerism. These states are shown in Tables 4, 5 and 6. Note that the excitation energies are given for $\chi'' = \chi b^4 = 1.0$. We can adjust this parameter to get the (possible) isomeric state at the right energy.

In Table 4, we consider the even-even nuclei. For four particles ($^{44}{\text{Ti}}$) and four holes ($^{52}{\text{Fe}}$) we get identical spectra with any interaction including Q.Q. We get a prediction of a spin gap since $J = 12^+$ lies lower than $J = 11^+$ or $J = 10^+$. For the eight particle system $^{48}{\text{Cr}}$ we do not predict any isomerism. We get analogous behavior in the $g_{9/2}$ shell– a spin gap for the four hole system ($^{96}{\text{Cd}}$) but none for the eight hole case ($^{92}{\text{Pd}}$) or the eight particle case ($^{88}{\text{Ru}}$).

In Table 5, we consider even-odd and odd-even nuclei. For the three particle and three hole systems Q.Q gives a spin gap for the $J = 19/2^-$ state, which lies lower than $J = 17/2^-$ or $15/2^-$. This pertains to $^{43}{\text{Sc}}$, $^{43}{\text{Ti}}$, $^{53}{\text{Fe}}$ and $^{53}{\text{Co}}$. Q.Q also yields an spin gap for the three hole system in the $g_{9/2}$ shell $^{97}{\text{Cd}}$ and $^{97}{\text{In}}$. The $J = 25/2^+$ state lies lower than $23/2^+$ or $21/2^+$. However, no isomerism is forthcoming to the silver isotopes $^{95}{\text{Ag}}$ or $^{93}{\text{Ag}}$.

In Table 6, the odd-odd nuclei are considered and there is no isomerism with Q.Q for any of them.
5. Discussion of the tables

A spin gap in $^{52}\text{Fe}$ was found and studied by D.A. Geesaman et al. [23]. The J=12$^+$ state was below the 10$^+$. A key finding pertaining to isomers in the $g_{9/2}$ shell is contained in the work of Nara Singh et al [24]. They found a J=16$^+$ state in $^{96}\text{Cd}$ which was lower in excitation energy than the lowest J=15$^+$ and J=14$^+$ states. Thus the 16$^+$ could not decay by magnetic dipole or electric quadrupole radiation.

A popular but somewhat arbitrary definition of a nuclear isomeric state is one that lives longer than 1 ns. We adopt this definition here. In Table 10 we show data on half-lives of isomers gathered from the NNDC [25] and corresponding calculations are shown in Tables 12 and 13. Direct comparison of the calculated half-life with the experimental data is difficult because the half-lives are super sensitive to the transition energies. For comparison, in Table 10 we show very short half-lives of non-isomeric states in $^{43}\text{Sc}$ (J=15/2) and $^{44}\text{Ti}$ (J=10).

Table 10: Half-lives from the National Nuclear Data Center (NNDC).

| Nucleus | E(keV) | J       | Half life     |
|---------|--------|---------|---------------|
| $^{43}\text{Sc}$ | 2988.12 | 15/2$^-$ | 5.6 (7) ps    |
|         | 3123.73 | 19/2$^-$ | 472 (4) ns    |
| $^{44}\text{Ti}$ | 7671.4  | (10$^+$) | 1.87 (35) ps  |
|         | 8039.9  | (12$^+$) | 2.1 (4) ns    |
| $^{52}\text{Fe}$ | 6958.0  | 12$^+$    | 45.9 (6) s    |
| $^{53}\text{Fe}$ | 3040.4  | 19/2$^-$  | 2.54 (2) min  |
| $^{94}\text{Ag}$ | ?       | (7$^+$)   | 0.55 (6) s    |
| $^{95}\text{Ag}$ | 6670    | (21$^+$)  | 0.40 (4) s    |
| $^{96}\text{Ag}$ | ?       | (15$^+$, 13$^-$) | < 40 ms |
| $^{96}\text{Cd}$ | ?       | 16$^+$    | 0.29$^{+0.11}_{-0.10}$ s [12] |

In Table 4 we see clearly that with Q.Q the J=12$^+$ state for two protons and two neutrons and with two proton hole and two neutron holes is a spin gap isomer. The J=12$^+$ lies below J=11$^+$ and J=10$^+$ (6.277 vs. 7.466 and 6.501). The J=12$^+$ become isomer only when it lies below J=11$^+$ and J=10$^+$. As seen in table 10 the half life of $^{44}\text{Ti}$ is 1.87ps while that of $^{52}\text{Fe}$ is 45.9s. Clearly the J=12$^+$ state in $^{44}\text{Ti}$ is a weak isomer while the J=12$^+$ in $^{52}\text{Fe}$ is a spin gap isomer. The reason for this is that in $^{52}\text{Fe}$ the 12$^+$ state lies below the lowest 10$^+$ state but in $^{44}\text{Ti}$ J=12$^+$ is slightly above J=10$^+$. In the single j shell model $^{44}\text{Ti}$ consists of two protons and two neutrons in the $f_{7/2}$ shell while $^{52}\text{Fe}$ consists of two proton holes and two neutron holes. In the single j shell model the hole-hole interaction is the same as particle-particle interaction. Thus if the same interaction is used for the two nuclei then the J=12$^+$ states would both either be weak isomers or would both be spin gap isomers. To get around the fact that experimentally one J=12$^+$ state is a weak isomer and the other is a spin gap isomer. This is done in Table 7 where for $^{44}\text{Ti}$ we use as input the spectrum of $^{42}\text{Sc}$ while for $^{52}\text{Fe}$ we use the spectrum of $^{54}\text{Co}$. When this is done, we get a spin gap isomer for $^{52}\text{Fe}$ but not for
In the $^{52}\text{Fe}$ case we still get an isomer because the $J=12^+$ (6.413 MeV) state is lies below to $J=10^+$ (6.611 MeV) but the half life is much smaller.

There is a similar story for $^{43}\text{Sc}$ and $^{53}\text{Co}$ ($^{53}\text{Fe}$). In Table 5 Q.Q predicts a spin gap isomer but, as seen in Table 8 the local interactions predict that only for $A=53$ will there be a spin gap. The latter two are in agreement with experiment. There is a weaker isomerism in $^{43}\text{Sc}$ because the $19/2^+$ state is close to the $15/2^+$. In the $g_{9/2}$ region the $J=16^+$ state in $^{96}\text{Cd}$ is predicted to be isomeric with the Q.Q interaction, in agreement with experiment. There are no other isomerisms predicted for the even-even nuclei in Table 4. This is in accord with experiment and with calculations with more realistic interactions in larger shell model spaces.

In Table 10, we gathered experimental data of half-lives corresponding to all cases from Tables 4 to 9 where either there is a calculated spin gap isomer with Q.Q or an isomer due to a low energy transition. corresponding calculated half-lives are discussed below.

In Table 11, we show spin gap isomers as predicted by the Q.Q interaction. For these there cannot be any E2 or M1 decays. We do not attempt to calculate their lifetimes.

In Table 12, we show, for the most part, calculations of B(E2)'s and half-lives for cases where J to (J-2) transitions are allowed but the states are long lived because the energy differences are small. We also include $^{94}\text{Ag}$, although calculated lifetime is very long.

There is a previously discovered isomeric $J=21^+$ state in $^{94}\text{Ag}$ by I. Mukha et al.[26] However, with the Q.Q interaction as seen in Table 12 although the $J=21^+$ is lower than $J=20^+$, it lies above $J=19^+$. In the absence of empirical data we have given the Q.Q results for $\chi''=1$. With Q.Q, the CCGI interaction [11] and that of Qi et al. [12] the values of $\Delta E$ are smaller and the half lives are longer, but they also allow for E2 transition. However Mukha et al. [26] state that the $J=21^+$ state decays by proton emission. If we had an interaction for which $\Delta E$ was negative, however small, that would solve the problem. With the CCGI interaction [11] we are almost there.

An important point in Table 12 is that the B(E2)'s for a given nucleus with various interactions are very close for $^{43}\text{Sc}$ and $^{94}\text{Ag}$, however, the half-lives are different because of difference in the transition energies.

Table 11: Spin gaps with the Q.Q interaction.

|        | $^{52}\text{Fe}(12)$ | $^{53}\text{Co}(19/2)$ | $^{97}\text{Cd}(25/2)$ | $^{96}\text{Cd}(16)$ | $^{96}\text{Ag}(15)$ |
|--------|----------------------|------------------------|------------------------|----------------------|------------------------|
| J      | 6.277                | 2.700                  | 4.374                  | 10.163               | 7.245                  |
| J-1    | 7.466                | 4.088                  | 6.585                  | 12.585               | 9.013                  |
| J-2    | 6.501                | 3.467                  | 6.585                  | 12.075               | 7.409                  |
Table 12: Selected lifetime calculations.

| Nucleus | Interaction | J_i  | J_f  | ∆E (MeV) | B(E2) e²fm⁴ | τ₁/₂ (SM) |
|---------|-------------|------|------|----------|-------------|-----------|
| ⁴³Sc    | Q.Q         | 19/2⁻| 15/2⁻| 0.768    | 5.918       | 358 ps    |
|         | ⁴²Sc/⁵⁴Co   |      |      | 0.134    | 5.919       | 2.21 µs   |
| ⁴⁴Ti    | Q.Q         | 12⁺  | 10⁺  | 0.224    | 21.85       | 45.97 ns  |
|         | ⁴²Sc        |      |      | 0.317    | 18.03       | 9.82 ns   |
|         | ⁵⁴Co        |      |      | 0.197    | 22.32       | 85.56 ns  |
| ⁹⁵Ag    | Q.Q         | 37/2⁺| 33/2⁺| 0.068    | 77.35       | 4.990 µs  |
|         | CCGI        |      |      | 0.012    | 70.15       | 0.032 s   |
|         | Qi          |      |      | 0.099    | 70.90       | 0.83 µs   |
| ⁹⁴Ag    | Q.Q         | 21⁺  | 19⁺  | 1.071    | 68.30       | 6.500 µs  |
|         | CCGI        |      |      | 0.126    | 68.85       | 0.257 µs  |
|         | Qi          |      |      | 0.290    | 68.89       | 3.98 ns   |

Table 13: Selected lifetime calculations by taking energy difference from the experimental data.

| Nucleus | Interaction | J_i  | J_f  | ∆E_{expt} (MeV) | B(E2) e²fm⁴ | τ₁/₂ (SM) | τ₁/₂ (Expt.) |
|---------|-------------|------|------|-----------------|-------------|-----------|--------------|
| ⁴³Sc    | Q.Q         | 19/2⁻| 15/2⁻| 0.136           | 5.918       | 2.041 x 10³ ns | 472 (4) ns |
|         | ⁴²Sc/⁵⁴Co   |      |      | 0.136           | 5.919       | 2.040 x 10³ ns |            |
| ⁴³Ti    | Q.Q         | 19/2⁻| 15/2⁻| 0.115           | 23.63       | 1.182 x 10³ ns | 556 (6) ns |
| ⁴⁴Ti    | Q.Q         | 12⁺  | 10⁺  | 0.369           | 21.85       | 3.759 ns | 2.1 (4) ns |
|         | ⁴²Sc        |      |      | 0.369           | 18.03       | 4.556 ns |              |
|         | ⁵⁴Co        |      |      | 0.369           | 22.32       | 3.680 ns |              |

In Table 13, we used calculated B(E2)'s but experimental transition energies. When compared with the experiment the results for ⁴⁴Ti is in reasonable agreement. We have done this only for non-spin gap isomers.

6. Summary:

In the present work we have performed shell model calculations using Q.Q interaction in \( f_{7/2} \) and then the \( g_{9/2} \) space. This interaction has one parameter, the overall strength. There is no additional parameter to determine whether a state is isomeric or not. We have compared this to other empirical interactions. We find Q.Q is a good predictor of where isomerism will occur, although, it does not always distinguish between a true spin gap and an isomer where E2 or M1 is allowed but the energy is very small. To get a weak isomer for ⁴³Sc and a spin gap for ⁵³Fe we have to use different local interactions in each case—one from the spectrum of ⁴²Sc and one from the spectrum of ⁵⁴Co.

We have predicted half-lives for non-spin gap nuclei ⁴³Sc, ⁴⁴Ti, ⁹⁴Ag and ⁹⁵Ag, and shown agreements for spin gap nuclei. Our predictions are in good agreement when we combined calculated \( B(E2) \) values with the experimental energy differences. In the case where no information is known, e.g ⁹⁷Cd, we predict a robust spin gap.
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