Outer Resonances and Effective Potential Analogy in Two-Dimensional Dielectric Cavities

Jinhang Cho\textsuperscript{a}, Inbo Kim\textsuperscript{a}, Sunghwan Rim\textsuperscript{a}, Geo-Su Yim\textsuperscript{b}, Chil-Min Kim\textsuperscript{a,∗}

\textsuperscript{a}Acceleration Research Center for Quantum Chaos Applications, Sogang University, Seoul 121-742, South Korea
\textsuperscript{b}Department of Physics, Pai-Chai University, Daejeon 302-735, South Korea

Abstract

Outer resonances are studied as one type of quasinormal modes in two-dimensional dielectric cavities with refractive index $n > 1$. The outer resonances can be verified as the resonances which survive only outside the cavity in the small opening limit of the dielectric disk. We have confirmed that the outer resonances universally exist in deformed cavities irrespective of the geometry of cavity and they split into nearly degenerate states in slightly deformed cavity. Also we have introduced an extended interpretation of the effective potential analogy for the outer resonances. Since most outer resonances in the dielectric cavities have quite high leakages, they would affect to the broad background in the density of states. But, for TE polarization case, relatively low-leaky outer resonances exist and it presents the possibility that they can interact with the inner resonances.

Keywords: outer resonance, microcavity, effective potential

PACS: 05.45.Mt, 03.65.Sq, 42.55.Sa

1. Introduction

Dielectric microcavities have been attracted much attention over the past decade owing to their useful applicability as a prototype model of mesoscopic open systems and a versatile element of hybrid optoelectronic circuits\textsuperscript{[1]}. The applicability of microcavities is based on high quality factor achieved by so-called whispering gallery modes (WGMs). The WGMs are formed in simple geometries with rotational symmetry like spherical, cylindrical, and disk shapes resulting from complete confinement of light by total internal reflection. But their isotropic emission lacks directionality,
so directional emission from slightly deformed cavity has been intensively studied \cite{2,10}. Recently the issue of uni-directionality is resolved by the geometries of spiral \cite{7,9} and limaçon \cite{10}.

Since the analytical analysis for the deformed microcavities combined with quantum and classical aspects is almost impossible, various semiclassical approaches have been applied to researches on their characteristics. Scar theory \cite{11}, which is originally developed in the context of closed billiard system, is widely believed to be suitable to explain the origin of the localized wave patterns of numerically and experimentally observed resonances in microcavities \cite{2,5,12,13}. However, recent findings suggest that the scar-like localized resonance patterns (i.e., quasicar \cite{14,17}) are the effects of the openness of dielectric cavity rather than that of unstable periodic orbits by the scar theory \cite{18,19}. For this reason, it is necessary to examine closely the characteristics of system according to the openness. Especially, our interest has been focused on basic differences between open cavities and closed billiards since a link between them remains a problem yet to be established. As a part of the research on the openness of dielectric cavity, in this report, we have investigated the outer resonances which do not exist in the billiard systems and their characteristics through the well-known effective potential analogy.

The paper is organized as follows: Firstly, we obtain the outer resonances in two-dimensional dielectric circular disk and describe its properties in Sec. \ref{sec2}. In Sec. \ref{sec3} we apply the effective potential analogy to the outer resonances and introduce that extended interpretation is needed to describe the properties of them. Finally, in Sec. \ref{sec4} the general existence of the outer resonances in two-dimensional cavities is verified through the deformations of cavity shape.

### 2. Outer resonances in Dielectric disk

The solutions in the open system can be obtained in two perspectives according to the boundary condition at infinity as follows: In scattering perspective, the wavefunctions, called *scattering states*, are composed of incoming plane waves and outgoing scattered waves. The wavenumbers $k$ are real and are shown as the smoothly continuous peak spectra structure. In emission perspective, the wavefunctions, called *resonances* or *quasinormal modes* (QNMs), satisfy the purely outgoing wave condition at infinity. The wavenumbers $k$ are complex with negative imaginary parts due to the leakage. The real value and the imaginary value of $k$ correspond to wavelength $\lambda$ ($\text{Re}[k] = 2\pi/\lambda$) and lifetime $\tau$ ($\text{Im}[k] = -1/(2c\tau)$), respectively. The discrete wavenumbers on the complex space in the emission perspective can be connected to the spectrum peaks in the scattering perspective.
The Helmholtz equation of the dielectric disk as a typical model of two-dimensional integrable open system is given by

$$\nabla^2 \psi(k, r, \phi) + k^2 \epsilon(r) \psi(k, r, \phi) = 0, \quad (1)$$

$$\epsilon(r) = 1 + (n^2 - 1) \Theta(R - r), \quad (2)$$

where $k$ is the wavenumber, $\psi(k, r, \phi)$ is the wavefunction, $n$ is the refractive index of the cavity, $R$ is the radius of the disk, and $\Theta$ is the unit step function. We assume that the refractive index outside the cavity is unity and $n > 1$. On account of the rotational symmetry, one can choose the solutions to be angular momentum eigenstates. The exact wavefunctions for scattering states are found to be

$$\psi_m(k, r, \phi) = \begin{cases} 
  I_m(k) J_m(nkr), & 0 \leq r \leq R \\
  H_m^{(2)}(kr) + S_{mm}(k) H_m^{(1)}(kr), & r > R 
\end{cases} \quad (3)$$

where $m$ is angular momentum quantum number, $k$ is real wavenumber, the $S$-matrix is diagonal in the angular momentum basis

$$S_{lm}(k) = -\frac{H_m^{(2)}(kR) - n \frac{d}{dn} \left(\frac{nkr}{R}\right) H_m^{(2)}(kR)}{H_m^{(1)}(kR) - n \frac{d}{dn} \left(\frac{nkr}{R}\right) H_m^{(1)}(kR)} \delta_{lm}, \quad (4)$$

and $I_m(k)$ is the mode strength amplitude \([20]\). The prime denotes the derivative with respect to $r$.

In Eq. (3) for $r > R$, the Hankel function of the second kind corresponds to an incident wave. To obtain the QNMs in the emission perspective, we reduce Eq. (3) to be

$$\psi_m(k, r, \phi) = \begin{cases} 
  I_m(k) J_m(nkr), & 0 \leq r \leq R \\
  S_{mm}(k) H_m^{(1)}(kr), & r > R 
\end{cases} \quad (5)$$
because the QNMs satisfy purely outgoing boundary condition at infinity. In this stage, we should note that the wavenumber $k$ is extended from the real space to the complex space, i.e., the solution has a leakage in the emission perspective.

Considering the boundary matching conditions for TM polarization at $r = R$, we obtain the requirement

$$ I_m(k) J_m(nkR) = S_{mm}(k) H_m^{(1)}(kR), $$
$$ I_m(k) n J'_m(nkR) = S_{mm}(k) H_m^{(1)'}(kR). $$

For having a non-trivial solution in the homogeneous system, the determinant

$$ D = \begin{vmatrix} J_m(nkR), & -H_m^{(1)}(kR) \\ n J'_m(nkR), & -H_m^{(1)'}(kR) \end{vmatrix} $$

should be vanished. Using the recursion relations for Bessel and Hankel functions of the first kind, the resonance condition is obtained as follows,

$$ n J_{m+1}(nkR) H_m^{(1)}(kR) = J_m(nkR) H_m^{(1)}(kR). $$

By solving this equation, one can obtain complex wavenumbers $k_r$ for resonances and normalized wavefunctions

$$ \psi_m(k, r, \phi) = e^{-im\phi} \begin{cases} A_m J_m(nk_r r), & 0 \leq r \leq R \\ H_m^{(1)}(k_r r), & r > R \end{cases} $$

where $A_m$ is the normalized amplitude

$$ A_m \equiv \frac{I_m(k)}{S_{mm}(k)} = \frac{H_m^{(1)}(k_r R)}{J_m(nk_r R)}. $$

We plot several resonances obtained numerically from the boundary matching condition (8) for the dielectric disk with a refractive index $n = 2.0$ in Fig. 1(a). It is shown that the resonances are separated into two groups; One group (black triangles) is composed of resonances with relatively small absolute value of imaginary part and the other group (red circles) have quite large absolute values of imaginary part.

Recently, Bogomolny et al. obtained similar results in two-dimensional dielectric disk with $n = 1.5$. They separate these resonances into two groups by the line of

$$ \text{Im}[kR] \sim -\frac{1}{2n} \ln \frac{n+1}{n-1}. $$
and called the low-leaky modes “internal resonances (Feshbach resonances)” and the high-leaky modes “external whispering gallery modes” or “outer resonances (shape resonances)” [21, 22]. We will call the low-leaky modes inner resonances and the high-leaky modes outer resonances in this paper.

The wave intensity patterns of resonances and their sectional views are illustrated in Fig. 2. Figure 2(a) and 2(b) are an inner resonance and an outer resonance which have the same angular momentum quantum number \( m = 12 \) and similar inside wavelength \( \lambda_m \sim 0.325 \). Figure 2(a) corresponds to the eigenstate which has radial quantum number \( l = 2 \) in the closed billiard system. But the mode in Fig. 2(b) can not be found in the corresponding billiard eigenstates and its wave intensity inside the cavity is almost zero (not exactly zero). Another outer resonance for \( m = 8 \) is shown in Fig. 2(c). It also has similar wave intensity pattern to that for \( m = 12 \), except for the angular nodal numbers.

To investigate the correspondence to the closed system in more detail, we checked up the tracing behavior of resonance mode as shown in Fig. 3. In the limit \( n \to \infty \) (small opening limit), \( k_r R \) of an inner resonance becomes zero but \( nk_r R \) converges to some constant real value as shown in Figs. 3(a) and 3(b). i.e., the mode becomes a bound state without leakage and the wave is confined inside the cavity boundary. Consequently, we can say that the bound state corresponds to a eigenmode of the billiard with a specific real value \( k \).

But, strictly speaking, the billiard is not the same with the small opening limit of cavity. Commonly used term, billiard indicates a non-leaky quantum system with Dirichlet boundary condition. For inner resonance in TM polarization, a recent work shows that the real part of \( nk_r R \) for a given \( m \neq 0 \) in the small opening limit approaches zeros of \( J_{m-1}(nk_r R) \) which are different from the eigenvalues for the same \( m \) in billiard [23]. Moreover, it is recently reported that \( nk_r R \) of an inner resonance with \( m = 0 \) corresponding to the ground state of billiard becomes zero in the small opening limit [24]. It has been known that the modes disappearing in the closed system, so called “zero modes”, exist. In Ref. [25, 26], the zero modes were studied in one- and three-dimensional systems. We will deal with the zero modes in two-dimensional cavity in another report.

In contrast to the tracing behavior of the inner resonance, \( k_r R \) of the outer resonances is retained to some constant complex value as \( n \to \infty \). Dettmann et al. show that \( k_r R \) values of outer resonances in the small opening limit become zeros of Hankel function of the first kind [24]. But Re\([nk_r R]\) of them diverges to \( \infty \) (Figs. 3(c) and 3(e)) and Im\([nk_r R]\) to \( -\infty \) (Figs. 3(d) and
i.e., the leakage of the modes inside the cavity becomes infinity. In Ref. [21], the “external whispering gallery modes” arising from complex zeros of the Hankel functions were obtained in the semiclassical limit by using Langer’s formula. In their formulas, we can also see that the imaginary part of $nk_rR$ becomes $-\infty$ in the limit $n \to \infty$. Thus the outer resonances in the small opening limit exist only outside the cavity and can be considered as the poles of scattering amplitude for the rigid cylinder [27].

For the case of TE polarization, we can obtain the resonance positions by using the boundary matching condition [28] which is given as

$$nJ_m(nkR) H^{(1)}_{m+1}(kR) - J_{m+1}(nkR) H^{(1)}_m(kR) = \frac{m}{kR} \left(n - \frac{1}{n}\right) J_m(nkR) H^{(1)}_m(kR).$$ (12)

The existence of outer resonances for TE polarization has been confirmed by tracing the high-leaky modes represented by red circles in the resonance position plot (Fig. 1(b)). Most of them are far from the inner resonances and the wave patterns are similar to that of outer resonances in TM. But, differently from TM case, we can see a group of the relatively low-leaky outer resonances having $\text{Im}[k_rR] \sim -1.0$ and their inside wave intensities are relatively strong as shown in Fig. 2(h). Some of them have been reported as “additional TE modes” related with the existence of the Brewster angle in Ref. [23]. The existence of these low-leaky outer resonances has great significances as follows: Firstly, density of states without the considering of low-leaky outer resonances will bring a mismatch between theoretical and experimental results. Secondly, inner resonances can interact with the low-leaky outer resonances according to the change of system parameters, since the low-leaky outer resonances are in the position among the inner resonances as shown in Fig. 4 when we just trace $k_rR$ of the resonances toward $n \to 1.5$. Therefore these outer resonances deserve to be considered in the study of the resonance dynamics for inner resonances in TE case like avoided resonance crossing or exceptional points.

3. Effective potential analogy for Outer resonances

The effective potential is a well-known analogy to explain the characteristics of resonance modes in a symmetrical dielectric sphere or disk [28, 30]. The radial part of Helmholtz equation of the
dielectric disk can be written in the form of

\[- \left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right] \psi(r) + V_{eff}(r) \psi(r) = E \psi(r), \tag{13} \]

where the effective potential is

\[ V_{eff}(r) = k^2 \left[ 1 - n^2(r) \right] + \frac{m^2}{r^2}. \tag{14} \]

Here, the wavenumber \( k \) and the energy \( E \) defined as \( k^2 \) are real because the analogy is rooted to the scattering perspective. Due to the interplay between the dielectric potential with refractive index and the repulsive centrifugal potential, the effective potential has the form of metastable well as shown in Fig. 5(a). The classical turning points are defined by the condition \( E - V_{eff}(r) = k^2 n^2 - m^2/r^2 = 0 \) and a classically allowed or classically forbidden region is represented as positive or negative value of \( E - V_{eff}(r) \), respectively. Using this condition, turning points on the boundary lead the relations for the maximum and the minimum possible value of \( k \) that the wave can be trapped in the well for a given \( n \) and \( m \) as follows,

\[ k_T^2 = \left( \frac{m}{R} \right)^2, \tag{15} \]

\[ k_B^2 = \left( \frac{m}{nR} \right)^2, \tag{16} \]

where \( R \) is the radius of the disk.

For a given \( m \), the top of the potential well at the disk boundary \( R = 1.0 \) is fixed to \( k_T^2 \) and independent of the variation of \( k \) and \( n \). The bottom of the potential well at the boundary meets with \( k_B^2 \) when \( E = k_B^2 \) and it has the dependence on \( k \) and \( n \). If \( k \) is fixed and \( n \) increases, the depth of the potential well may further deepen. In the case that \( n \) is fixed, if \( k^2 \) is larger than \( k_B^2 \), the bottom moves downward. On the contrary, if \( k^2 \) becomes smaller than \( k_B^2 \), it moves upward, and eventually the potential well becomes very shallow.

Figure 6(a) shows the inner and the outer resonance positions for \( \text{Re}[k_r R] \) vs \( m \). One can easily find the inner resonances above the dotted line \( \text{Re}[kR] = m \), namely above-barrier resonances and the inner resonances in the trap range \( (m/n < \text{Re}[kR] < m) \), namely below-barrier resonances. They are represented by black triangles in Fig. 6. In general, wave oscillates in classically allowed region and diffuses in classically forbidden region. For the below-barrier resonances, the waves are well trapped in the potential well. The below-barrier resonances decay only by tunneling via the effective potential barrier and the absolute values of \( \text{Im}[k_r R] \) are very small, i.e., high-Q
modes are formed. But we must take notice that the absolute values of imaginary part of the above-barrier resonances are also comparatively small although they are in non-trap region, because the potential depth is more deepen as \( \text{Re}[k_rR] \) increase.

In contrast to the inner resonances, the outer resonances have extremely large absolute values of \( \text{Im}[k_rR] \). We have verified the probability densities of the imaginary parts of the outer resonances, the above-barrier resonances, and the below-barrier resonances obtained in the region of \( m \leq 20 \) and \( \text{Re}[k_rR] \leq 20.0 \). The aspects of distributions for three groups are certainly different. The below-barrier resonances are concentrated in the region of \( |\text{Im}[k_rR]| \sim 0.025 \) and the above-barrier resonances are concentrated in the region of \( |\text{Im}[k_rR]| \) about 10 times as large as that of below-barrier resonances. The absolute values of imaginary parts of the outer resonances are over 10 times as large as that of above-barrier resonances. In the closed system, the density of states is represented in the forms of discrete series of delta peaks positioned at the eigenvalues \( k \). While, in the case of an open cavity, the peaks positioned at the real parts of resonance spectra \( k_rR \) acquire some widths which are related with the imaginary parts of \( k_rR \). As \( |\text{Im}[k_rR]| \) increases, the peak becomes broader. Generally, the density of states in a cavity is composed of not only sharp peaks corresponding to below-barrier resonances but also broad band corresponding to above-barrier resonances. The effects of very broad peaks of the outer resonances, which have large absolute values of imaginary part, are also contained in the background.

The outer resonances in Fig. 1(a) are represented by red circles in Fig. 6(a). For a given \( m \), the number of the outer resonances can be estimated at \( \lfloor m/2 \rfloor \). Most of the outer resonances exist under the line of \( k_B \) and some of them for the case of \( m \geq 4 \) are partially located in the trap region. We have drawn the effective potentials for two of outer resonances with \( \text{Re}[k_rR] \) for \( m = 12 \) (\( \text{Re}[k_rR] = 9.67774 \)) and \( m = 8 \) (\( \text{Re}[k_rR] = 2.20483 \)) in Fig. 5(b) and 5(c). In the case of the mode for \( m = 8 \), the wave inside the boundary exists in classically forbidden region as shown in Fig. 5(c) and the intensity of the wavefunction inside the cavity is nearly zero as shown in Fig. 2(g). Hence we are apt to naively think that the effective potential analogy agrees with the wave pattern. However, for the case of \( m = 12 \) as shown in Fig. 5(b), it seems as if the wave inside the cavity should be trapped in the well. According to the effective potential analogy, the wave located in the trap region like Fig. 5(a) lives for a long time inside the cavity and its inside intensity is strongly localized in the classically allowed region as shown in Fig. 2(e). But, in fact as shown in Fig. 2(f), the outer resonance for \( m = 12 \) has very low inside intensity similar to that of \( m = 8 \), even though
the resonance is in the trap region and has the wavelength $\lambda_{in} = 0.32462$ smaller than the length of the effective potential well $l_w = 0.38002$. So we can conclude that the analogy can not be applied to the outer resonances and we need a different description of the effective potential. If we suppose that inner and outer resonances are formed inside and outside of the cavity, respectively, it can be interpreted as that the intensity of an outer resonance formed with $E$ between the cavity boundary and infinite boundary rapidly decrease inward the cavity by tunneling via the classically forbidden region of potential barrier. Therefore it does not matter whether the energy of the outer resonance is in the trap region or not.

Such description of effective potential for outer resonances can be more clarified in the case of TE polarization. The effective potential analogy in two-dimensional dielectric disk can be similarly applied to TE case with one difference that $\psi(r)$ is the magnetic field instead of the electric field. In Fig. 6(b), we obtain the resonance positions for $\text{Re}[k_r R]$ vs $m$ in the case of TE polarization. Differently from TM case, the number of the outer resonances for a given $m$ can be estimated at $[(m+1)/2]$ and we can see the outer resonances located above $k_T$-line for $m \geq 3$ and below $k_T$-line for $m \leq 2$, which are the relatively low-leaky outer resonances in Fig. 1(b). $E$ of these low-leaky outer resonances existing near the $k_T$-line is above the potential barrier or has thin potential barrier than that of other outer resonances. So they can more easily tunnel in the cavity and, as a result, have relatively large inside intensity. Figure 7 shows the variation of wavefunction of a low-leaky outer resonance for $m = 17$ when effective potential is changed by $n$. $\sqrt{E}$ of the resonance is changed from (a) 18.94011 (over the line of $k_T$) to (b) 16.38559 (under the line of $k_T$). This figure well represents that the relatively strong inside intensity with oscillatory property at $n = 2.0$ becomes extremely small as the energy located below the tip of the potential barrier ($k_T^2$) at $n = 9.0$.

4. Outer Resonances in Deformed cavities

In order to show that the existence of outer resonances are generic feature of the two-dimensional cavities, we investigated the outer resonances in a spiral-shaped cavity. It is a classically complete chaotic system and has unique features different from typical deformed cavities (e.g., totally asymmetrical geometry and the presence of a notch) [7, 9, 14, 15]. The spiral cavity boundary is given by

$$r(\phi) = R \left(1 + \frac{\phi}{2\pi}\right)$$  \hspace{1cm} (17)
in polar coordinates \((r, \phi)\), with the radius of spiral \(R = 1.0\) at \(\phi = 0\), the deformation parameter \(\epsilon\), and the refractive index \(n\).

The resonance positions and wave patterns in deformed cavities can be obtained with the boundary element method (BEM) \([31]\). We have found the the spots of high-leaky solutions distributed on the region of \(\text{Im}[kR] < -1.5\), where \(\epsilon = 0.1\) and \(n = 3.0\). They have been verified as the outer resonances and two nearly degenerate states of them are illustrated in Fig. \(\Xi(a)\) and \(\Xi(b)\). The outer resonances in the dielectric disk split into nearly degenerate states in the spiral cavity by shape perturbation. The splitting of a degenerate mode for \(m \neq 0\) is a general aspect in the cavities slightly deformed from a perfect symmetric geometry like the dielectric disk. Especially, in the spiral-shaped cavity, the pairs of nearly degenerate modes have different wavelengths and \(Q\)-factors due to the broken chirality \([32]\).

For more confirmation, we obtained the resonance positions and the resonance patterns in low-\(nkR\) regime for a square-shaped cavity and a stadium-shaped cavity and easily found the outer resonances in there too as shown in Fig. \(\Xi(c)\) and \(\Xi(d)\). Hereby, we can see that the outer resonances universally exist as one group of QNMs in two-dimensional dielectric cavities whether the geometry of cavity is integrable or chaotic.

5. Conclusions

In the dielectric disk with \(n > 1\) as a two-dimensional open system, we established the existence of outer resonances besides the inner resonances. The outer resonances do not have the counterparts in billiard and disappear inside the cavity due to the infinite leakage in the small opening limit. For the finite refractive index, most of them are very short-lived resonances and the wave intensities are nearly zero inside the cavity. But, exceptionally, the outer resonances for \(\text{Re}[kR] > m\) in the TE case are relatively long-lived resonances and exist around the inner resonances in the case of low refractive index. It presents the possibility that the outer resonances can interact with the inner resonances.

Using BEM, we showed that the outer resonances universally exist as one group of QNMs in two-dimensional dielectric cavities irrespective of the geometry of cavity and, especially, exist as the nearly degenerate states in the slightly deformed cavity. The outer resonances constitute extremely broad background in the density of states because of their quite high leakage and they should be take into account in the study of the trace formula in open cavity \([22, 33]\).
Also, it is known that the effective potential analogy arising from the scattering perspective can be well applied for the description of the QNMs in the dielectric disk. However, we noticed that care must be taken to interpret effective potential depending on the class of resonances.

Acknowledgment

We would like to thank J.-W. Ryu and S.-Y. Lee for discussions. This work was supported by Acceleration Research (Center for Quantum Chaos Applications) of MEST/KOSEF.

References

[1] K.J. Vahala, Nature (London) 424 (2003) 839.
[2] J.U. Nöckel, A.D. Stone, Nature (London) 385 (1997) 45.
[3] C. Gmachl, F. Capasso, E.E. Narimanov, J.U. Nöckel, A.D. Stone, J. Faist, D.L. Sivco, A.Y. Cho, Science 280 (1998) 1556.
[4] S.-B. Lee, J.-H. Lee, J.-S. Chang, H.-J. Moon, S.W. Kim, K. An, Phys. Rev. Lett. 88 (2002) 033903.
[5] T. Harayama, T. Fukushima, P. Davis, P.O. Vaccaro, T. Miyasaka, T. Nishimura, T. Aida, Phys. Rev. E 67 (2003) 015207(R).
[6] T. Harayama, T. Fukushima, S. Sunada, K.S. Ikeda, Phys. Rev. Lett. 91 (2003) 073903.
[7] G.D. Chern, H.E. Türeci, A.D. Stone, R.K. Chang, M. Kneissl, N.M. Johnson, Appl. Phys. Lett. 83 (2003) 1710.
[8] M.S. Kurdoglyan, S.-Y. Lee, S. Rim, C.-M. Kim, Opt. Lett. 29 (2004) 2758.
[9] F. Courvoisier, V. Boutou, J.P. Wolf, R.K. Chang, J. Zyss, Opt. Lett. 30 (2005) 738.
[10] J. Wiersig, M. Hentschel, Phys. Rev. Lett. 100 (2008) 033901.
[11] E.J. Heller, Phys. Rev. Lett. 53 (1984) 1515.
[12] S.-Y. Lee, J.-W. Ryu, T.-Y. Kwon, S. Rim, C.-M. Kim, Phys. Rev. A 72 (2005) 061801(R).
[13] J. Wiersig, Phys. Rev. Lett. 97 (2006) 253901.
[14] S.-Y. Lee, S. Rim, J.-W. Ryu, T.-Y. Kwon, M. Choi, C.-M. Kim, Phys. Rev. Lett. 93 (2004) 164102.

[15] T.-Y. Kwon, S.-Y. Lee, M.S. Kurdoglyan, S. Rim, C.-M. Kim, Y.-J. Park, Opt. Lett. 31 (2006) 1250.

[16] C.C. Liu, T.H. Lu, Y.F. Chen, K.F. Huang, Phys. Rev. E 74 (2006) 046214.

[17] C.-M. Kim, S.H. Lee, K.R. Oh, J.H. Kim, Appl. Phys. Lett. 94 (2009) 231120.

[18] J. Lee, S. Rim, J. Cho, C.-M. Kim, Phys. Rev. Lett. 101 (2008) 064101.

[19] E.G. Altmann, G. Del. Magno, M. Hentschel, Europhys. Lett. 84 (2008) 10008.

[20] C. Viviescas, G. Hackenbroich, J. Opt. B 6 (2004) 211.

[21] R. Dubertrand, E. Bogomolny, N. Djellali, M. Lebental, C. Schmit, Phys. Rev. A 77 (2008) 013804.

[22] E. Bogomolny, R. Dubertrand, C. Schmit, Phys. Rev. E 78 (2008) 056202.

[23] J.-W. Ryu, S. Rim, Y.-J. Park, C.-M. Kim, S.-Y. Lee, Phys. Lett. A 372 (2008) 3531.

[24] C.P. Dettmann, G.V. Morozov, M. Sieber, H. Waalkens, Europhys. Lett. 87 (2009) 34003.

[25] R.K. Chang, A.J. Campillo (Eds.), Optical Processes in Microcavities, World Scientific, Singapore, 1996.

[26] P.T. Leung, S.Y. Liu, K. Young, Phys. Rev. A 49 (1994) 3057.

[27] J.B. Keller, S.I. Rubinow, M. Goldstein, J. Math. Phys. 4 (1963) 829.

[28] M. Hentschel, Ph.D. Thesis, Max Planck Institute for the Physics of Complex Systems, 2002.

[29] B.R. Johnson, J. Opt. Soc. Am. A 10 (1993) 343.

[30] J.U. Nöckel, Ph.D. Thesis, Yale University, 1997.

[31] J. Wiersig, J. Opt. A: Pure Appl. Opt. 5 (2003) 53.

[32] J. Wiersig, S.W. Kim, M. Hentschel, Phys. Rev. A 78 (2008) 053809.

[33] S.R. Jain, Phys. Lett. A 335 (2005) 83.
Figure Captions

**Figure 1** Resonance positions in the complex $kR$ space obtained from the boundary matching conditions for (a) TM and (b) TE polarizations in the dielectric disk with $n = 2.0$. The resonances are separated into two groups by a dashed line. One group (triangles) is composed of resonances with relatively small absolute value of imaginary part (inner resonances), the other group (circles) has quite large absolute value of imaginary part (outer resonances).

**Figure 2** Wave patterns of (a) an inner resonance for $m = 12$, $k_rR = 9.61998 - i0.02300$ (TM) and three outer resonance for (b) $m = 12$, $k_rR = 9.67774 - i3.86429$ (TM) and (c) $m = 8$, $k_rR = 2.20483 - i5.50279$ (TM), and (d) $m = 8$, $k_rR = 8.67533 - i1.24009$ (TE) in the dielectric disk with $n = 2.0$. (e), (f), (g), and (h) are the sectional views of (a), (b), (c), and (d), respectively.

**Figure 3** Mode tracing behavior for an inner resonance and two outer resonance in the dielectric disk for TM polarization. Tracing Re[$nk_rR$] and Im[$nk_rR$] of (a), (b) inner resonance for $m = 12$, $l = 2$, (c), (d) outer resonance for $m = 12$, and (e), (f) outer resonance for $m = 8$, respectively.

**Figure 4** Resonance positions in the complex $kR$ space obtained from the boundary matching condition for TE polarization in the dielectric disk with $n = 1.5$. The triangular points are inner resonances and the circular points are outer resonances.

**Figure 5** Effective potential for a general inner resonance for (a) $m = 12$, $l = 2$ (Re[$k_rR$] = 9.61998) and outer resonances for (b) $m = 12$, Re[$k_rR$] = 9.67774 and (c) $m = 8$, Re[$k_rR$] = 2.20483 in the dielectric disk for TM polarization, where $n = 2.0$ and $R = 1.0$. Red solid line is Re[$k_r$]$^2 = E$, blue dotted line is $k_T^2$, and blue dashed line is $k_B^2$.

**Figure 6** Resonance positions for Re[$k_rR$] vs $m$ obtained from the boundary matching conditions for (a) TM and (b) TE polarizations in the dielectric disk with $n = 2.0$. Inner resonances and outer resonances are represented by triangular points and circular points, respectively. Dotted line is $k_T$ of effective potential well (Re[$kR$] = $m$) and dashed line is $k_B$ (Re[$kR$] = $m/n$).

**Figure 7** Effective potentials and wavefunctions for a low-leaky outer resonance ($m = 17$) in the dielectric disk with (a) $n = 2.0$ ($k_rR = 18.94011 - i1.30632$) and (b) $n = 9.0$ ($k_rR = 16.38559 - i1.62816$). Black, red, and blue solid lines are $V_{eff}$, $E$, and $|\psi(r)|^2$ (arbitrary unit), respectively.

**Figure 8** Wave patterns for outer resonances in deformed cavities for TM polarization. (a) $k_rR = 0.41394 - i1.53316$ and (b) $k_rR = 0.41708 - i1.54480$ are nearly degenerated modes in the spiral-shaped cavity with $\epsilon = 0.1$ where $n = 3.0$, respectively. (c) $k_rR = 1.97536 - i2.25493$
and (d) $k_r R = 1.43900 - i1.55748$ are the outer resonances in the square-shaped cavity and the stadium-shaped cavity with deformation parameter $L/R = 1.0$ where $n = 2.0$, respectively.
Figure 1:

Figure 2:
Figure 3:

Figure 4:
