Optimal physicians schedule in an Intensive Care Unit

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Abstract. In this paper, we consider a case study for the problem of physicians scheduling in an Intensive Care Unit (ICU). The objective is to minimize the total overtime under complex constraints. The considered ICU is composed of three buildings and the physicians are divided accordingly into six teams. The workload is assigned to each team under a set of constraints. The studied problem is composed of two simultaneous phases: composing teams and assigning the workload to each one of them. This constitutes an additional major hardness compared to the two phase’s process: composing teams and after that assigning the workload. The physicians schedule in this ICU is used to be done manually each month. In this work, the studied physician scheduling problem is formulated as an integer linear program and solved optimally using state of the art software. The preliminary experimental results show that 50% of the overtime can be saved.

1. Introduction
The personnel scheduling problems consist on the assignment of some tasks to a set of workers under a number of constraints that can be conflicting in some circumstances. These constraints make the actual scheduling problem harder than the simple assignment problem. From practical point of view, the preparation of the personnel schedule is a hard task that requires taking in consideration a plenty of rules such as: workers preferences, assignment of the night shifts, weekends, restrictions on the number of consecutive shifts, days off and vacation periods. The personnel scheduling problems are of practical interest since they are met in several working environments. In the other hand, these scheduling problems are of theoretical interest because of their hardness. Subsequently, during the last decades these scheduling problems have received a lot of attention from researchers community where a satisfactory literature have been provided and several approaches have been proposed ([1, 2, 3 and 4]).

In the health care sector, the personnel scheduling problems present a particular interest since several services (emergency rooms, intensive care unit) should be ensured continuously twenty four hours per day, seven days a week to preserve human life. Thus, several authors have addressed the personnel scheduling problems and most of them treated the case of nurse scheduling ([5, 6, 7, and 8]). In addition, there are several differences between the nurses scheduling and the physicians scheduling in terms of preferences, requirements, specialties…

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Some works on physicians scheduling problem have been proposed in literature. The Authors in ([9, 10, 11 and 12]) tackled the emergency rooms scheduling problem, and ([13 and 14]) treated the same scheduling problem in operating rooms. More recent works ([15, 16 and 17]) considered the physician scheduling problem and provided a master scheduling for the hospital (operating rooms, emergency rooms…).

In this work, a case study in an Intensive Care Unit in a local hospital is considered. In this ICU the proposed physicians schedule is carried out manually each month under several particular restrictions and rules. The performing of such a schedule is a time consuming one with critical distribution from physician’s side. To overcome all the manual schedule drawbacks an integer linear programming model is proposed and preliminary experiments are conducted and compared with the existing ones.

The paper is organized as follows: in Section 2, we give a brief description of the studied ICU. The mathematical formulation is presented in section 3. In addition, an experimental analysis of the proposed integer linear program is presented in section 4. Finally, we conclude and we present some directions for future research.

2. Description of the studied ICU

The considered ICU is composed of three buildings according to the provided service. These buildings are:

1. **Building 1: The general ICU building** which is the main building for the ICU department.
2. **Building 2: Maternity & burn ICU building** which concern the maternity and burn cases.
3. **Building 3: Mobile ICU for accidents & emergency**, where a team of physicians is located and intended to intervene outside in cases of accidents, in order to treat severe cases.

The available set of physicians is divided into six teams, where each one is assigned to a building. The required number of physicians, for each one of these buildings depends on the building itself and on the season (summer, fall, winter, spring). The determination of such a number is based on the previous collected data. The workload per day is composed of two shifts (day and night shifts), with duration of 12 hours for each one. The week starts Sunday until Thursday and the weekend is for Friday and Saturday. In addition, the physicians schedule is performed each month manually under the following constraints.

- Each physician is assigned to one team.
- The regular workload for each physician is 208 hours per month.
- The night shift for all the buildings will be served by one team only.
- All the team members are assigned to the same location at the same time.
- Building 1 is the main ICU building which serves mostly the severe cases. So, for the comfort of the patient, the same team should serve for the whole week days (Sunday – Thursday).
The work in building 1 is intense. So, each physician shouldn’t work two consecutive weeks at that building.
Each physician should have two consecutive days as a week vacation.
At the weekend, the team assigned for the day shift in building 1 will also be assigned for the night shift (24 hours working).
The team working on Saturday or Friday at building 1 must have a day-off before and a day-off after.
If the physician worked the weekends in buildings 2 or 3, he should work Friday and Saturday.
If the physician worked at the night shift today, he shouldn’t work tomorrow morning.

3. Mathematical formulation
In this section, the physician scheduling problem is formulated as an integer linear programming. For that aim we introduce the following notations and definitions.

\( N \): The number of available physicians during a month.

\( I \): set of physicians. \( I = \{1, 2...N\} \)

\( J \): set of shifts. \( J = \{1, 2\} \)

\( K \): set of buildings. \( K = \{1, 2, 3\} \)

\( L \): set of days. \( L = \{1, 2... 28\} \)

\( M \): set of Team. \( M = \{A, B, D, G, E, H\} \).

\( R_{kj} \): the minimum required number of physicians in building \( k \) for the shift \( j \).

The considered decision variables are:

\[ x_{ijklm} = \begin{cases} 1 & \text{if physician } i \text{ is assigned to shift } j \text{ on building } k \text{ for day } l \text{ and team } m \\ 0 & \text{otherwise} \end{cases} \]

\[ y_{im} = \begin{cases} 1 & \text{if physician } i \text{ is assigned to team } m \\ 0 & \text{otherwise} \end{cases} \]

\[ U_{il} = \begin{cases} 1 & \text{if physician } i \text{ has two consecutive days off (within a week) starting from day } l \\ 0 & \text{otherwise} \end{cases} \]

\[ v_{ihm} = \begin{cases} 1 & \text{if physicians } i \text{ and } h \text{ belong to the team } m \\ 0 & \text{otherwise} \end{cases} \]

The objective function presenting the total over time is given by the following expression:
\[
Z = \sum_{i=1}^{N} \sum_{j=1}^{2} \sum_{k=1}^{3} \sum_{m=1}^{28} 12 x_{ijklm} - \sum_{i=1}^{N} \sum_{j=1}^{2} \sum_{m=1}^{28} (12 x_{i22lm} + 12 x_{i23lm}) - 208N
\]

where,

208N is the minumum load for the physicians(each one with 208 hours per month).

\[\sum_{i=1}^{N} \sum_{j=1}^{2} \sum_{k=1}^{3} \sum_{m=1}^{28} (12 x_{i22lm} + 12 x_{i23lm}) \] we substract this expression since the night shift for all the buildings will be served by only one team.

\[\sum_{i=1}^{N} \sum_{j=1}^{2} \sum_{k=1}^{3} \sum_{m=1}^{28} 12 x_{ijklm} \] the total loadwork with repetition presented in the latter expression.

1- Each physician \(i\) is assigned exactly to one team:

\[
\sum_{m=1}^{6} y_{im} = 1 \quad \forall i = 1,\ldots, N
\]

2- The number of team members is from three to six members:

\[
3 \leq \sum_{i=1}^{N} y_{im} \leq 6 \quad \forall m = 1,\ldots,6
\]

3- The relation between the decision variables \(x\) and \(y\):

\[
x_{ijklm} \leq y_{im} \quad \forall i = 1,\ldots, N, \forall j = 1,\ldots,2, \forall k = 1,\ldots,3, \forall l = 1,\ldots,28, \forall m = 1,\ldots,6
\]

4- The regular workload for each physician is 208 hours per month:

\[
\sum_{j=1}^{2} \sum_{k=1}^{3} \sum_{m=1}^{28} 12 x_{ijklm} - \sum_{i=1}^{N} \sum_{j=1}^{2} \sum_{m=1}^{28} (12 x_{i22lm} + 12 x_{i23lm}) \geq 208 \quad \forall i = 1,\ldots, N
\]

5- Each physician \(i\) should work one shift per day at building 2 and 3:

\[
\sum_{j=1}^{2} x_{ijklm} \leq 1 \quad \forall i = 1,\ldots, N, \forall k = 1,\ldots,3, \forall l = 1,\ldots,28, \forall m = 1,\ldots,6
\]

6- Each physician \(i\) should work one shift per day at building 1: (Sunday – Thursday):

\[
\sum_{j=1}^{2} x_{ijklm} \leq 1 \quad \forall i = 1,\ldots, N, \forall k = 1, \forall l = 1,\ldots,28/\{6,7,13,14,20,21,27,28\}, \forall m = 1,\ldots,6
\]

7- The night shift for all the buildings will be served by one team only.

\[
x_{i23lm} = x_{i22lm} = x_{i23lm} \quad \forall i = 1,\ldots, N, \forall l = 1,\ldots,28, \forall m = 1,\ldots,6
\]

8- Building 1 is the main ICU buildings which served mostly the severe cases. So, for the comfort of the patient, the same team should serve for the whole week days (Sunday – Thursday):

\[
x_{i11lm} = x_{i11lm} \quad \forall i = 1,\ldots, N, \forall 2 \leq l \leq 5, \forall m = 1,\ldots,6
\]
\[
x_{i18m} = x_{i1lm} \quad \forall i = 1, \ldots, N, \forall 9 \leq l \leq 18, \forall m = 1, \ldots, 6
\]
\[
x_{i115m} = x_{i1lm} \quad \forall i = 1, \ldots, N, \forall 16 \leq l \leq 19, \forall m = 1, \ldots, 6
\]
\[
x_{i122m} = x_{i1lm} \quad \forall i = 1, \ldots, N, \forall 23 \leq l \leq 26, \forall m = 1, \ldots, 6
\]

9- The work in building 1 is intense. So, each physician shouldn’t work two consecutive weeks at this building:
\[
x_{i11lm} + x_{i11(7+l)lm} \leq 1 \quad \forall i = 1, \ldots, N, \forall 1 \leq l \leq 5, \forall m = 1, \ldots, 6
\]
\[
x_{i118m} + x_{i11(14+l)lm} \leq 1 \quad \forall i = 1, \ldots, N, \forall 1 \leq l \leq 5, \forall m = 1, \ldots, 6
\]
\[
x_{i1115m} + x_{i11(21+l)lm} \leq 1 \quad \forall i = 1, \ldots, N, \forall 1 \leq l \leq 5, \forall m = 1, \ldots, 6
\]

10- The required number of physician per day should be more than \(R_{ij}\):
\[
\sum_{j=1}^{2} x_{ijklm} \geq R_{ij} \quad \forall i = 1, \ldots, N, \forall k = 1, \ldots, 3, \forall l = 1, \ldots, 28, \forall m = 1, \ldots, 6
\]

11- Each physician should have two consecutive days as a week vacation:
\[
\sum_{l=1}^{6} U_{il} \geq 1 \quad \forall i = 1, \ldots, N
\]
\[
\sum_{l=8}^{13} U_{il} \geq 1 \quad \forall i = 1, \ldots, N
\]
\[
\sum_{l=15}^{20} U_{il} \geq 1 \quad \forall i = 1, \ldots, N
\]
\[
\sum_{l=22}^{27} U_{il} \geq 1 \quad \forall i = 1, \ldots, N
\]

12- The relation between the variables \(U\) and \(x\):
\[
x_{i1klm} + x_{i2(l+1)lm} + x_{i1(l+1)lm} + x_{i2w(l+1)lm} \leq 4(1 - U_{il})
\]
\[
\forall i = 1, \ldots, N, \forall k = 1, \ldots, 3, \forall l = 1, \ldots, 28 \{/7, 14, 21, 28\}
\]
\[
\forall m = 1, \ldots, 6, \forall t = 1, \ldots, 3, \forall s = 1, \ldots, 3, \forall w = 1, \ldots, 3.
\]

13- At the weekend, the team assigned for the day shift in building 1 will also be assigned for the night shift (24 hours working):
\[
x_{i1lm} = x_{i2lm} \quad \forall i = 1, \ldots, 35, \forall l = 6, 7, 13, 14, 20, 21, 27, 28 \}, \forall m = 1, \ldots, 6
\]

14- The team working Saturday or Friday at building 1 must have day-off before and day-off after:
\[
x_{ijk(l-1)m} \leq 1 - x_{i1lm} \quad \forall i = 1, \ldots, N, \forall j = 1, \ldots, 2, \forall k = 1, \ldots, 3, \forall l = 6, 13, 20, 27 \}, \forall m = 1, \ldots, 6
\]
\[
x_{ijk(l+1)m} \leq 1 - x_{i1lm} \quad \forall i = 1, \ldots, N, \forall j = 1, \ldots, 2, \forall k = 1, \ldots, 3, \forall l = 7, 14, 21 \}, \forall m = 1, \ldots, 6
\]
\[
x_{i1(27)m} \leq 1 - x_{i11(28)m} \quad \forall i = 1, \ldots, N, \forall m = 1, \ldots, 6
\]

15- If the physician worked at the weekends in buildings 2 or 3, he should work Friday and Saturday:
\[
x_{i1lm} = x_{i1k(l+1)m} \quad \forall i = 1, \ldots, N, \forall k = 2, \ldots, 3, \forall l = 6, 13, 20, 27 \}, \forall m = 1, \ldots, 6
16- If the physician worked at the night shift today he shouldn’t work tomorrow morning:

$$x_{ijklm} + x_{i1(l+1)m} \leq 1 \quad \forall i = 1,...,N, \forall s = 1,...,3, \forall k = 1,...,3, \forall l = 1,...,27, \forall m = 1,...,6$$

17- A physician has to be assigned in only one location for the morning shift:

$$x_{11hm} + x_{i12hm} + x_{i13hm} \leq 1 \quad \forall i = 1,...,N, \forall l = 1,...,28, \forall m = 1,...,6$$

18- All the team members are assigned to the same location at the same time:

$$x_{ijklm} - x_{ijkm} \leq 1 - v_{ijhm} \quad \forall i = 1,...,N, \forall i < h \leq N, \forall j = 1,...,2, \forall k = 1,...,3, \forall l = 1,...,28, \forall m = 1,...,6$$

$$v_{ijhm} + 1 \geq y_{im} + y_{hm} \quad \forall i = 1,...,N, \forall i < h \leq N, \forall m = 1,...,6$$

$$\sum_{m=1}^{6} v_{ijhm} \leq 1 \quad \forall i = 1,...,N, \forall i < h \leq N$$

$$v_{ijhm} \leq y_{im} \quad \forall i = 1,...,N, \forall i < h \leq N$$

$$v_{ijhm} \leq y_{hm} \quad \forall i = 1,...,N, \forall i < h \leq N, \forall m = 1,...,6$$

4. Preliminary computational results

3.1 Test problems

The performance of the proposed integer linear programming model has been assessed through experimental tests. These tests focus on the month of September, where the total number of available physicians is $N=18$ and the number of the teams is $m=6$. The manual schedule for this month is displayed in the following table (Table 1).

Table 1. Results of the physician scheduled for 28 days by the manual method.
3.2 Numerical results

We report in Table 2 the obtained results after solving to optimality the proposed integer linear programming model:

Table 2. Physician scheduled for 28 days by the computerized model.

| Day | Building 1 | Building 2 | Building 3 |
|-----|------------|------------|------------|
| 1   | A E D E G E | A G D G B G | A D H D B D |
| 2   | A G D G B G | A G D G B G | A H B H E H |
| 3   | A D H D B D | A G A G D G | A B G B E B |
| 4   | A H B H E H | A G E G D G | A B G B E B |
| 5   | A B G B E B | A G A G D G | A B G B E B |
| 6   | A H H E H G H | A G A G D G | A B G B E B |
| 7   | A D D E D G D | A G A G D G | A B G B E B |
| 8   | A H G A G E G | A G A G D G | A B G B E B |
| 9   | A H G A G D G | A G A G D G | A B G B E B |
| 10  | A H G A G D G | A G A G D G | A B G B E B |
| 11  | A H A B A E A | A G A G D G | A B G B E B |
| 12  | A H G B G E G | A G A G D G | A B G B E B |
| 13  | A H D D E D | A G A G D G | A B G B E B |
| 14  | A E E E E E | A G A G D G | A B G B E B |
| 15  | A E E E E E | A G A G D G | A B G B E B |
| 16  | A E E E E E | A G A G D G | A B G B E B |
| 17  | A E E E E E | A G A G D G | A B G B E B |
| 18  | A E E E E E | A G A G D G | A B G B E B |
| 19  | A E E E E E | A G A G D G | A B G B E B |
| 20  | A E E E E E | A G A G D G | A B G B E B |
| 21  | A E E E E E | A G A G D G | A B G B E B |
| 22  | A E E E E E | A G A G D G | A B G B E B |
| 23  | A E E E E E | A G A G D G | A B G B E B |
| 24  | A E E E E E | A G A G D G | A B G B E B |
| 25  | A E E E E E | A G A G D G | A B G B E B |
| 26  | A E E E E E | A G A G D G | A B G B E B |
| 27  | A E E E E E | A G A G D G | A B G B E B |
| 28  | A E E E E E | A G A G D G | A B G B E B |

According to the obtained results (Table 2), we observe that all the constraints are satisfied in addition to new ones (compared to the 11 constraints given in section 2 for the manual schedule). Furthermore, the over time is reduced by 50%. This allows balancing the workload, which impacts the performance of the physicians and reduces the financial burden.

5. Conclusion and future directions:

In this paper, we investigated the physicians scheduling problem in an Intensive Care Unit. The schedule used to be performed manually each month. For that aim, we proposed an adequate integer linear programming model. This integer linear program is solved to optimality for one month (September) as preliminary work. The obtained results show the efficiency of the proposed model since the reduction of the over time is 50%. As future directions of this work, we plan to enlarge the experimental tests to other months. In addition, some new objective functions will be proposed and tested.

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