Nonlinear optical isolators and circulators with dynamic nonreciprocity

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On-chip optical nonreciprocal devices are vital components for integrated photonic systems and scalable quantum information processing. Nonlinear optical isolators and circulators have attracted great attention because of their fundamental interest and their important advantages in integrated photonic circuits. However, optical nonreciprocal devices based on Kerr or Kerr-like nonlinearity are subject to dynamical reciprocity when the forward and backward signals coexist simultaneously in a nonlinear system. Here, we theoretically propose a method for realizing on-chip nonlinear isolators and circulators with dynamic nonreciprocity. Dynamic nonreciprocity is achieved via the chiral modulation on the resonance frequency due to coexisting self- and cross-Kerr nonlinearities in an optical ring resonator. This work showing dynamic nonreciprocity with a Kerr nonlinear resonator is an essential step toward integrated optical isolation.

I. INTRODUCTION

Optical nonreciprocal components, such as optical isolators and circulators, can isolate detrimental backscattering fields from the signal source and thus are vital for photon-based information processing in both the classical and quantum regimes [1–5]. Their realization relies on breakdown of the Lorentz reciprocity. Magneto-optical effect [6, 7] is commonly used to realize optical isolators and circulators but limited to bulk optics. Spatiotemporal modulation of optical systems has successfully demonstrated the capability of achieving optical nonreciprocity [8, 9]. A chiral atom-cavity system with spin-momentum locking can conduct magnetic-free single-photon isolation and reach the quantum regime [10–17]. An ensemble of hot atoms provides a useful platform to achieve all-optical isolation when unidirectional control fields are used to induce susceptibility-momentum locking in atoms [18–26]. Susceptibility-momentum locking has become a new toolkit for realizing quantum nonreciprocity [27]. By using the macroscopic Doppler shift in a unidirectional moving atomic lattice, all-optical isolators and unidirectional reflectionless are obtained [28–30]. Alternatively, spinning resonators and optomechanical resonators are also suitable for magnetic-free optical isolation [31–33]. In spite of a great progress in non-magnetic optical nonreciprocal devices, realization of integrated all-optical isolation on a solid-state platform is still desired but very challenging.

Nonlinearity in solid-state optical material like silicon has ever been a promising candidate for breaking the Lorentz reciprocity without magnetic fields or complicate spatiotemporal modulation [36]. Thus, nonlinear devices had attracted intense study for realization of integrated isolators and circulators because they can be integrated on a chip with silicon-based materials and is bias-free [37–40]. However, an optical nonreciprocal device based on Kerr or Kerr-like nonlinearity is subject to dynamic reciprocity, which was derived from a nonlinear Helmholtz equation only including the cross-Kerr nonlinearity of material but excluding the self-Kerr nonlinearity [41]. Because of dynamic reciprocity, this type of nonlinear nonreciprocal devices can not work as optical isolators when the forward and backward fields coexist [37–40] [42]. The problem of dynamic reciprocity is the major challenge of using a nonlinear platform for integrated nonlinear optical isolators. Other types of nonlinear optical isolators using quantum nonlinearity [43] or nonlinearity in a parity-time-symmetry-broken system [44, 45] are also subject to dynamic reciprocity. Therefore, bypassing dynamic reciprocity with a nonlinear optical device becomes highly desired for integrated optical isolation. Dynamic reciprocity has been widely accepted as a basic knowledge in nonlinear optics. To avoid the problem of dynamic reciprocity, pulsed signals are used in nonlinear optical isolators. In this way, the opposite-propagating signals are temporal separate, allowing isolation of the pulsed backscattering field [39, 40]. Nevertheless, the dynamic reciprocity still limits its application in the circumstance of continuous signals.

Some novel mechanisms have been proposed for bypassing dynamic reciprocity in nonlinear optical isolators. These methods exploit either chiral Kerr-type nonlinearity in atomic medium [19, 22, 24], nonlinearity-induced spontaneous symmetry breaking [46, 47], and unidirectional parametric nonlinear processes [48, 49]. Nevertheless, circumventing the problem of dynamic reciprocity in a solid-state platform with Kerr nonlinearity is still desirable. More to the point, this can change the concept related to dynamic reciprocity.

In this paper, we show that a Kerr-type nonlinear microring resonator (MR) can show optical dynamic nonreciprocity. The material of this MR should include self-Kerr and cross-Kerr nonlinearity and thus is compatible with silicon. Because of the intrinsic chirality of Kerr nonlinear medium, the self-Kerr modulation (SKM) and cross-Kerr modulation (XKM) on the MR resonance frequency are different and dependent on the propagation of light. As a result, nonlinear optical
isolators and three-port quasi-circulators based on this chiral Kerr nonlinearities can be attained for continuous inputs simultaneously propagating in opposite directions. We also employ finite-difference time-domain (FDTD) simulations to validate dynamic nonreciprocity predicted by the coupled-mode theory.

II. SYSTEM AND MODEL

Our idea for realizing dynamic reciprocity with a nonlinear MR makes use of the intrinsic chirality of a nonlinear medium. A nonlinear medium like silicon possesses self- and cross-Kerr nonlinearities simultaneously. The cross-Kerr nonlinearity strength is typically twice of the self-Kerr nonlinearity [45] [50] [53]. Thus, if we design a system that the forward and backward light fields in a nonlinear medium are different in power, then the opposite-propagating fields will “see” different refractive indices of medium. For an optical system sensitive to the refractive index of medium, the opposite-input fields will have different transmissions. In this way, we can realize optical isolators and circulators with dynamic reciprocity.

The system consists of a Kerr-nonlinear MR, a bus waveguide (WG 1) with ports 1 and 4, and a drop waveguide (WG 2) with ports 2 and 3, as shown in Fig. 1. An optical attenuator with amplitude transmission $\xi$ is embedded near port 2 inside the WG 2. The incident light $\alpha_{in}$ ($\beta_{in}$) from port 1 (port 2) respectively excites the CCW (CW) mode of the MR.

III. COUPLED-MODE METHOD

Using Eq. (1), the coupled-mode equation can be written as

$$\frac{\partial}{\partial t} \begin{bmatrix} a \\ b \end{bmatrix} = i \begin{bmatrix} \Delta + \kappa_{12} & \kappa_{13} \\ \kappa_{21} & \Delta + \kappa_{23} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 4U|b|^2a - 2iU|a|^2a + \sqrt{2\kappa_{21}}|\alpha_{in}|^2 \\ 4U|a|^2b - 2iU|b|^2b + \xi \sqrt{2\kappa_{21}}|\beta_{in}|^2 \end{bmatrix}.$$ (2a)

$$b = -i(\Delta + \kappa)a - 4U|b|^2a - 2iU|a|^2a + \sqrt{2\kappa_{21}}|\alpha_{in}|^2, \quad (2b)$$

We solve Eq. (2) and find the coherent amplitudes of the resonator modes, i.e., $\alpha = (a)$ and $\beta = (b)$. Note that the KKM strength is twice of the SKM according to Eq. (2). This means that the light in the CCW mode can generate a frequency modulation to the CW mode as twice as that caused by itself with the same power. Thus, if opposite-input light fields with the same power can excite the CW and CCW mode to different power levels by introducing an attenuator to one input port, then the two input fields “see” resonator modes with different resonance frequencies and thus have different transmission. This is the key idea of our optical isolator. By including both the self- and cross-Kerr nonlinearities with different strength, our system is crucially different from that for demonstrating dynamic reciprocity. Therefore, our system can achieve dynamic nonreciprocity, allowing us to implement optical isolators and circulators.

According to the input-output relationship, we have the outputs $\alpha_{out} = \xi \sqrt{2\kappa_{12}}|\alpha_{in}|^2$, $\beta_{out} = \sqrt{2\kappa_{21}}|\alpha_{in}|^2$, $\alpha'_{out} = \alpha_{in} - \sqrt{2\kappa_{21}}|\alpha_{in}|^2$, and $\beta'_{out} = \beta_{in} - \sqrt{2\kappa_{21}}|\beta_{in}|^2$. The transmissions can be calculated as $T_{12} = |\alpha_{out}/|\alpha_{in}|^2|$, $T_{14} = |\alpha'_{out}/|\alpha_{in}|^2|$, $T_{21} = |\beta_{out}/|\beta_{in}|^2|$, and $T_{23} = |\beta'_{out}/|\beta_{in}|^2|$. In calculation of $T_{ij}$, $i$ is for the input port and $j$ for the output port.

This work aims to show an optical isolator and a three-port quasi-circulator [14] [49] [55] with dynamic nonreciprocity. Thus, we are interested in the transmissions $T_{12}$, $T_{21}$ and $T_{23}$. For the isolator, the isolation contrast and the insertion loss are defined as $\eta = (T_{12} - T_{21})/(T_{12} + T_{21})$ and $\mathcal{L} = -10\log_{10}(T_{12})$, respectively. For the quasi-circulator, the average fidelity is given by $F = \text{Tr}\left[\tilde{T}T_{id}\right]/\text{Tr}\left[T_{id}T_{id}\right]$, where $\tilde{T}$ represents the finite-difference time-domain numerical solution of the coupled-mode equations.
where $T_{id}$ is the transmission matrix for an idea three-port quasi-circulator and $T_{id} = [0 \ 1 \ 0 \ 0 \ 1]$ \cite{14,19,49,55}, and $\tilde{\mathbf{T}} = T_{ij}/\Gamma_j$ with $\Gamma_j = \sum_i T_{ij}$. The average insertion loss is defined as $\bar{L} = -10 \log_{10}(T_{12} + T_{23})/2$. Below, we will name a quasi-circulator as a circulator for simplicity.

### IV. FDTD SIMULATION

As a first-principle method, FDTD simulation can predict the behavior of an electromagnetical system very precisely \cite{56}. In this work, we also employ FDTD simulation to prove the results of the coupled-mode theory. To reduce the calculation time, we performed two-dimensional FDTD simulations which do not include the $z$ dimension. The perfect match layer (PML) boundary condition is applied for surrounding the simulation region. The cross-section of the MR and two waveguides has the same width of 200 nm. The radius of the MR is 700 nm. Taking silicon as the nonlinear medium, the refractive index of the MR and two waveguides is $n = 3.478$. The third-order nonlinear susceptibility of nonlinear medium making the MR is taken to be $\chi^{(3)} = 2.8 \times 10^{-19}$ m$^2$/V$^2$, corresponding to a nonlinear refractive index $n_2 = 2.7 \times 10^{-14}$ cm$^2$/W \cite{50}. The optical attenuator in WG 2 has an amplitude transmission $\approx 0.982$. The gaps between the MR and the two waveguides are the same, 250 nm so that $\kappa_{ex1} = \kappa_{ex2} > \kappa_{in}$. To avoid strong backscattering, the spatial grid size for simulation is set to be small enough, 5 nm. The temporal step of simulation is 0.012 fs and the total simulation time is 50 ps, allowing the system evolves to the steady state. To show dynamic nonreciprocity, two constant driving fields are incident to ports 1 and 2 at the same time. The input light fields $E$ have the same wavelength of $\lambda = 1.685$ µm and are equal.

### V. RESULTS

#### A. Time-dependent transmission

Because the response of the system is a nonlinear function of the power of the input field, it is difficult to find an analytical steady-state solution of transmission. Thus, we first study the time-evolution of the system. The time-dependent transmissions are found by solving the coupled-mode equation and FDTD numerical simulation, see Fig. 2. The former is computation-resource-efficient and can show underlying physics. It can also find solution fast. The later provides the first-principle simulation for a real device. FDTD simulation results are in reasonable agreement with the coupled-mode theory even in the details of the oscillation parts.

When solving the coupled-mode equation, two constant drivings are applied at the same time. We take $\alpha_{in} = \beta_{in} = 50 \sqrt{\kappa}$ as an example. The transmissions reach their steady-state values after some oscillation over a period of about 10$\kappa t$. The steady-state transmissions are $T_{12} \approx 0.740$, $T_{14} \approx 0.058$, $T_{21} \approx 0.040$, $T_{23} \approx 0.911$. Obviously, the forward transmission $T_{12}$ is much higher than the backward transmission $T_{21}$. Thus, an optical isolator with ports 1 and 2 is obtained. The isolation constant is $\eta \approx 0.90$. The insertion loss of the transparent direction is low, $\mathcal{L} \approx 1.31$ dB. The telecom-wavelength signal can also transmit along the direction 1 $\rightarrow$ 2 $\rightarrow$ 3, forming a three-port circulator with $\mathcal{L} \approx 0.98$ and $\mathcal{L} \approx 0.83$ dB. The results of the coupled-mode theory clearly show an optical isolator and a quasi-circulator with high dynamic nonreciprocity and low insertion loss.

![Figure 2. Time-dependent nonreciprocal transmission. (a) transmissions predicted by the coupled mode equations with parameters: $\kappa_{ex1} = \kappa_{ex2} = 0.45\kappa, \kappa_i = 0.1\kappa, \xi = 0.98, \Delta = -4.5\kappa, U = 0.001\kappa$, and $P_{in}/h\omega_{in} = 2500\kappa$. (b) transmission of the FDTD simulations with $E = 2.2 \times 10^5$ V/m.](image-url)
the coupled-mode method.

Figure 3 shows the distribution of the instantaneous electric field distribution at $t = 50$ ps as an example when the continuous light incident to ports 1 and 2 simultaneously. It can be seen that the forward light incident to port 1 exits from port 2 with a high transmission. However, the backward light from port 2 dominantly transmits to port 3. The transmission to port 1 is negligible. This FDTD simulation clearly proves dynamic nonreciprocity of the nonlinear MR and confirms a practical optical circulator.

B. Steady-state Transmission

As shown in Fig. 2, the nonlinear system can reach steady state after a long time. We can take the transmissions at $\kappa t = 30$ in the coupled-mode theory or $t = 50$ ps in FDTD simulations as steady-state transmissions. Below, we investigate the steady-state transmission versus the input power. The transmission is dependent on the detuning between the input and the MR. Specifically, we take $\Delta = -4.5\kappa$ in solving the coupled-mode equation. When $\sqrt{P_{in}/\hbar \omega_{in}} < 49 \sqrt{\kappa}$, the forward and backward transmissions are almost equal. The system is reciprocal. There is a rapid transition at $\sqrt{P_{in}/\hbar \omega_{in}} \approx 49 \sqrt{\kappa}$. We find that the forward and backward transmissions become very different after this point. For example, $T_{12} = 0.757$ and $T_{21} = 0.043$ at $\alpha_{in} = \beta_{in} = \sqrt{P_{in}/\hbar \omega_{in}} = 49 \sqrt{\kappa}$, yielding $\eta = 0.89$ and $\mathcal{L} = 1.21 \text{ dB}$. As the input power increases, the transmission $T_{12}$ exponentially decreases, while $T_{21}$ remains small. At this point of $\sqrt{P_{in}/\hbar \omega_{in}} \approx 49 \sqrt{\kappa}$, $T_{14}$ jumps to a small value from a high transmission and then increases exponentially with the input power. In contrast, $T_{23}$ remains high. As other demonstrated nonlinear nonreciprocal devices with dynamic reciprocity [37][38][40][42][46], our system shows strong dynamic nonreciprocity only for a narrow input power $49 \sqrt{\kappa} \leq \sqrt{P_{in}/\hbar \omega_{in}} \leq 52 \sqrt{\kappa}$.

Figure 4(b) shows the results of FDTD simulation versus the input field strength $E$. The four transmissions show similar dependence on the input field $E$ as Fig. 4(a). The transition point for transmissions $T_{12}$ and $T_{14}$ is at $E \approx 2.1 \times 10^7 \text{ V/m}$. The transmission $T_{12}$ ($T_{14}$) jumps up (down) from a small (large) value to a large (small) value and then exponentially decreases (increase) with the input field strength. These results provide a proof of dynamical nonreciprocity predicted by the coupled-mode theory.

The response of a nonlinear system is crucially dependent on the input field parameters such as frequency and the input power. Figure 5 shows the transmissions $T_{12}$, $T_{21}$ and $T_{23}$ as functions of the power and detuning of the incident light by solving the coupled-mode equations. Figures 6(a) and (b) present the isolation contrast $\eta$ and the insertion loss $\mathcal{L}$ for the optical isolator, respectively. Figures 6(c) and (d) show the average fidelity $\mathcal{F}$ and the average insertion loss $\mathcal{L}$ for the three-port circulator, respectively. Obviously, the transmission $T_{12}$ approaches $T_{21}$ for small detuning and weak input power because the power-dependent SKM and XKM are weak and can only cause slightly different frequency shifts to the CW and CCW modes. The system shows strong dynamic
nonreciprocity when the detuning and power are large enough, see Fig. 5(a). However, when the detuning or the input power is too large, the transparent transmission is low, implying a large insertion loss, see Fig. 5(b). Optical isolation requires trade-off between the isolation contrast and the insertion loss. An optimal point is indicated by the red star in Figs. 5 and 6, where $\sqrt{P_{\text{in}}/\hbar\omega_{\text{in}}} \approx 61\sqrt{k}$ and $\Delta \approx -5.38k$. At this optimal point, the forward and backward transmissions of the optimal point are $T_{12} \approx 0.636$ and $T_{23} \approx 0.023$, corresponding to the isolation contrast of $\eta \approx 0.93$ and the insertion loss of $\mathcal{L} \approx 1.96$ for the optical isolator. Considering the transmission $T_{23} \approx 0.933$, the system can work as a three-port circulator with the high performance of $\mathcal{F} \approx 0.93$ and $\mathcal{L} \approx 1.96$.

VI. IMPLEMENTATION

Now, we discuss the experimental implementation of our proposal. Our scheme can be implemented with MRs made from high-$\chi^3$ nonlinear materials, such as potassium titanyl phosphate [61], Si [62], SiC [63], InP [64]. We assume the intrinsic quality factor $Q_i = 1 \times 10^7$ and the resonance frequency of the nonlinear MR $\omega_0/2\pi = 193.6$ THz. The intrinsic loss of the resonator is calculated to be about $k_i \approx 2\pi \times 19.4$ MHz, and thus the total loss rate is about $\kappa \approx 10k_i \approx 2\pi \times 0.194$ GHz. We select experimentally accessible parameters: $n_0 = 1.4$, $n_2 = 5.1 \times 10^{-15}$ m$^2$/W [65, 66]. $V_{\text{in}} = 100$ $\mu$m$^3$ [67] and thus the nonlinearity strength is calculated to be $U \approx 2\pi \times 0.194$ MHz $\approx 0.001k$. Using such nonlinear MR, we can achieve an optical isolator for parameters $\{\sqrt{P_{\text{in}}/\hbar\omega_{\text{in}}} = 50\sqrt{k}, \Delta = -4.5k\}$ and $\{\sqrt{P_{\text{in}}/\hbar\omega_{\text{in}}} = 61\sqrt{k}, \Delta = -5.38k\}$ corresponding to an input power $P_{\text{in}} \approx 0.39$ $\mu$W and $P_{\text{in}} \approx 0.58$ $\mu$W, respectively.

The difference of SFM and XFM in Kerr-type nonlinear material has been exploited to demonstrate optical isolators and circulators [46], but only for very different opposite input powers. In the work [46], the backward light is 3.3 dB and 5 dB lower than the forward signal. In applications of optical sensors [5, 39, 58–60], the backward signal will has a power very close to the forward one. Our nonlinear circulator is suitable for circumstance of optical sensing. During the preparation of this paper, we noted that an experimental group [68] demonstrated a passive nonlinear optical isolator integrated on a chip by exploiting the same idea. This experiment can be an example of our system.

VII. CONCLUSION

In conclusion, we have proposed a method to realize nonlinear optical isolators and circulators based on the chirality of SKM and XKM in the nonlinear MR. We have proved dynamic nonreciprocity of this nonreciprocal device with both the coupled-mode theory and FDTD simulation. The proposed scheme paves the way to realize on-chip optical isolation, and thus can boost the integration of photonic chips.
This work was supported by the National Key R&D Program of China (Grants No. 2017YFA0303703, No. 2019YFA0308700), the National Natural Science Foundation of China (Grant Nos. 11874212, 11890704, 11690931), the Fundamental Research Funds for the Central Universities (Grant No. 021314380095), the Program for Innovative Talents and Entrepreneurs in Jiangsu, and the Excellent Research Program of Nanjing University (Grant No. ZYJH002).
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