CHAOTIC CUTTLESH: KING OF CAMOUFLAGE WITH SELF-EXCITED AND HIDDEN FLOWS, ITS FRACTIONAL-ORDER FORM AND COMMUNICATION DESIGNS WITH FRACTIONAL FORM

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(Communicated by Miguel Sanjuan)

Abstract. There are many works on self-excited and hidden attractors. However, the relationship between them is less investigated. In this study we present a system which can have both hidden self-excited attractors. Dynamical properties of the chaotic system are studied using the equilibrium points and Eigenvalues analysis, Lyapunov exponents and bifurcation plots. Since fractional order models are more interesting in engineering applications, the fractional order version of the proposed system is derived using Adomian decomposition method. Bifurcation and stability analysis of the fractional order model shows the existence of chaotic oscillations. To demonstrate the engineering importance of the fractional order model, we have designed a digital communication system with SCSK (Symmetric Chaos Shift Keying) modulation method separately for both self-excited attractor and hidden attractor, the bit error rate performance was compared.

2010 Mathematics Subject Classification. Primary: 37D45, 34A08, 68P30; Secondary: 94A14.
Key words and phrases. Hidden attractors, self-excited attractors, fractional order chaotic systems, chaos-based communication designs, SCSK modulation.

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1. **Introduction.** Time varying behavior of a system is modeled with a set of differential equations. Of these equations those which don’t satisfy the homogeneity and superposition properties cause a system to be known as nonlinear dynamical system. Some time-series which seem random are actually results of deterministic nonlinear systems which are called chaotic systems [7]. Chaotic attractors are characterized by strong dependence on initial conditions, and sensitivity to the changes in the system’s parameters, presence of strong harmonics in the signals, fractional dimension of space state trajectories and the presence of a stretch direction, represented by a positive Lyapunov exponent. Behaviors of a dynamical system can be studied by using the concept of phase space. It is a theoretical space where every state of the system mapped to a unique spatial location. An attractor is a region of phase space that “attracts” all nearby points as time passes. The dynamic response of the system is depicted with its attractors. There are various classes of attractors; the first one is called point attractor or equilibrium point (fixed point), a solution which does not change in time. Instead of settling in point, if the system settles in a cycle, then it is called a limit cycle attractor or periodic attractor.

In 1971, David Ruelle and Floris described the strange attractor as it is the one in which we can see recognizable shapes in phase space, but the system never follows exactly the same trajectory through the phase space [15]. Recently many researchers have proposed two significant classifications of chaotic attractors named as self-excited (SA) and hidden attractors (HA) [5, 10]. HA are important in most of the engineering applications as they allow unpredicted responses [11]. Various works [5] of Kuznetsov and Leonov provided a platform to understand hidden attractors and its influence on real time systems. Chaotic attractors with hidden oscillations can be seen in systems with no equilibrium [12], with only stable equilibria [9] and with curves of equilibria [13].

Three famous categories of HA plus systems with SA was discussed in [8]. A hyperchaotic system which behaves like a cuttlefish was discussed in [14]. The paper is organized as follows: In section 2 we introduce the new chaotic system with SA and HA. Dynamical properties of the new system are explored in section 3 using Lyapunov exponents, equilibrium points and the stability of equilibrium, bifurcation of the system and basin of attraction. In section 4 the fractional order model of the system is obtained and the dynamical behavior of the fractional order system are investigated in section 5. Finally in section 6 we derive a new digital modulation scheme (symmetric chaos shift keying) using the fractional order system and the performance analysis of the proposed modulations system is presented for various frequencies.

2. **Chaotic cuttlefish.** In this section we introduce a new system named as chaotic cuttlefish (color changing fish popularly known as king of camouflage) for the reason that the system changes between self-excited and hidden oscillations depending on the value of parameter \( b \). Table 1 shows the chaotic cuttlefish with its parameters.

For the initial conditions \([0,1,0]\) and parameter values as shown in Table 1, the 2D phase plains of the system are displayed for the self-excited and hidden attractor case as shown in Fig. 1 and 2 respectively. In all our further investigations we use the parameters \( b = 0.6, a = 1.493 \) for the self-excited case and \( b = -0.8, a = 1.493 \) for the hidden attractor case.
Table 1. Chaotic cuttlefish

| Chaotic system | System name | Parameters | Type of System |
|----------------|-------------|------------|----------------|
| $\dot{x} = y$  | SA          | $a = 0.6 - 1.5$ | Self-excited attractor ($b > 0$) |
| $\dot{y} = -4x - 4yz$ | HA          | $a = 0.6 - 1.5$ | Hidden attractor ($b \leq 0$) |
| $\dot{z} = x^2 + y^2 + b z^2 - a$ | | | |

Figure 1. Self-excited attractor

Figure 2. Hidden attractor

3. Dynamic properties of the cuttlefish system.

3.1. Equilibrium points: The cuttlefish system is given by the $x = 0; y = 0; z = \pm \sqrt{a/b}$ for parameter $b$. The system has no equilibrium when $b \leq 0$. It has an unstable equilibrium $E_{1,2} = (0, 0, \pm 1.414)$ when $a = 1, b = 0.5$. The eigenvalues of the system at $E_1$ are $\lambda_1 = -0.8286, \lambda_2 = -4.8274, \lambda_3 = 1.414$ where $\lambda_3$ is the unstable node and the eigenvalues of the system at $E_2$ are $\lambda_1 = 0.8286, \lambda_2 = 4.8274, \lambda_3 = 0$. The characteristic equations at $E_1$ is $\lambda^3 + 4.242\lambda^2 - 3.99\lambda - 5.65$ and at $E_2$ is $\lambda^3 - 5.656\lambda^2 - 3.99\lambda$. All the principal minors must be positive for stable system according to Routh-Hurwitz criterion. They are,

$$\Delta_1 = \delta_1 > 0, \Delta_2 = \begin{vmatrix} \delta_1 & \delta_0 \\ \delta_3 & \delta_2 \end{vmatrix} > 0, \Delta_3 = \begin{vmatrix} \delta_1 & \delta_0 & 0 \\ \delta_3 & \delta_2 & \delta_1 \\ 0 & 0 & \delta_3 \end{vmatrix} > 0$$ (1)
where $\delta_0 = 1, \delta_1 = 4.242, \delta_2 = -3.99, \delta_3 = -5.65$ for equilibrium $E_1$ and for $E_2$ the values are $\delta_0 = 1, \delta_1 = -5.656, \delta_2 = 4, \delta_3 = 0$. It is clear that $\Delta_2 < 0$ and hence the chaotic cuttlefish system is unstable and shows chaotic oscillations.

3.2. **Lyapunov exponents and Kaplan-Yorke dimension:** Lyapunov exponents of a chaotic system defines the convergence and divergence of the states. [19]. We use the well-known Wolf’s algorithm [19] to calculate the finite time Lyapunov exponents (LEs) calculated for a time length of 40000s. The initial conditions used are $[0,1,0]$ for both the cases. Table II shows the Lyapunov exponents (LEs) and the Kaplan-Yorke (KY) dimension of the chaotic cuttlefish system.

| Attractor   | LEs          | KY dimension |
|-------------|--------------|--------------|
| Self-excited| $L_1=0.0144, L_2=0, L_3 = -0.0421$ | 2.342        |
| Hidden      | $L_1=0.01865, L_2=0, L_3 = -0.0372$ | 2.501        |

3.3. **Bifurcation:** The control parameter $b$ is considered as the bifurcation parameter and the local maxima of state $z$ is plotted for the parameter values $a = 1.493, m = 4$. When $b \leq 0$ the system shows hidden attractors and for $b > 0$ the system shows self-excited attractors. Fig.3a shows the bifurcation of the system with parameter $b$ and Fig.3b shows the corresponding Lyapunov spectrum. It can be seen that the system shows two brief regions of chaotic oscillations for $-1 \leq b \leq -0.64$ in which the system shows hidden attractors and $0.36 \leq b \leq 1.02$ for which the system shows self-excited attractors. There are also tori regions seen for $2.18 \leq b \leq 3$ and $1.13 \leq b \leq 1.28$ in which the only positive LE becomes zero. Regions of conservative attractor can be seen for $1.28 \leq b \leq 2.17$, $-0.62 \leq b \leq -0.5$ and $-0.09 \leq b \leq 0.17$. These reports are supported by the corresponding finite time LEs. Fig.3c and 3d shows the basin of attraction of the system in the $xz$ plane for the two conditions $b = 0$ and $b = 0.2$ showing the hidden and self-excited attractor basins.

4. **Fractional order chaotic cuttlefish system.** Fractional order chaotic (FOC) systems have gained importance in the recent years for their wide range of applications in secure communication and cryptography [2]. Also it has been proved in the literatures that a higher order model can be equally to a lower order version without loss of character in fractional order. Also in real word scenarios fractional order controllers and systems are more preferred than the integer order models because of the lesser phase difference introduced and efficient controller switching’s. Grunwald-Letnikov (GL), Riemann-Liouville and Caputo definitions are commonly used for fractional-order differential operator, [20]. We use the GL, which is introduced as

$$aD^q_tf(t) = \lim_{h \to 0} \left\{ \frac{1}{h^q} \sum_{j=0}^{\left[ \frac{t}{h} \right]} (-1)^j \binom{q}{j} f(t-jh) \right\} = \lim_{h \to 0} \left\{ \frac{1}{h^q} \Delta^q_h f(t) \right\}$$

(2)

where $\Delta^q_h f(t)$ is generalized difference, $h$ is the step size, $t$ and $a$ are limits of the fractional system and $q$ is the fractional order. For calculations (2) is modified as

$$\langle t-L \rangle D^q_tf(t) = \lim_{h \to 0} \left\{ h^{-q} \sum_{j=0}^{N(t)} b_j (f(t-jh)) \right\}$$

(3)
Figure 3. a): Bifurcation plot of the chaotic cuttlefish with parameter \( b \); b): The corresponding finite time Lyapunov exponents; c): Cross section of basin of attraction of the system at \( y = 1 \) for \( a = 0.8; b = 0 \); d): Cross section of basin of attraction of the system at \( y = 1 \) for \( a = 0.8; b = 0.2 \) The notations \( P, C, T, U \) in the plots c,d denotes the regions of initial conditions leading to periodic, chaotic, tori and unbounded oscillations respectively.

Where \( h \) is the time sampling and \( L \) is the memory length.

\[
N(t) = \min \left\{ \left[ \frac{t}{h} \right], \left[ \frac{L}{h} \right] \right\} \tag{4}
\]

The binomial coefficients is calculated as,

\[
b_j = \left( 1 - \frac{a + q}{j} \right) b_{j-1} \tag{5}
\]

Using (1)-(4), the FOC cuttlefish system is obtained as,

\[
\frac{d^{t\times x}}{dt^{t\times x}} = y \]

\[
\frac{d^{t\times y}}{dt^{t\times y}} = -4x - 4yz \tag{6}
\]

\[
\frac{d^{t\times z}}{dt^{t\times z}} = x^2 + y^2 + b^2 - a.
\]
4.1. Adomian decomposition method for FOC cuttlefish system: There are three main approaches derived to solve FOC systems. They are frequency-domain method \[4,18\], Adomian Decomposition Method (ADM) \[1,6\] and Adams-Bashforth-Moulton (ABM) algorithm \[3,16\]. The proposed FOC cuttlefish system are solved by applying ADM scheme. The fractional order cuttlefish system as discrete can be given as,

\[
x_{n+1} = \sum_{j=0}^{6} C^2_j \frac{h^{j+1}}{\Gamma(1+q)} x_j \\
y_{n+1} = \sum_{j=0}^{6} C^2_j \frac{h^{j+1}}{\Gamma(1+q)} y_j \\
z_{n+1} = \sum_{j=0}^{6} C^2_j \frac{h^{j+1}}{\Gamma(1+q)} z_j
\]

(7)

Where \( C^2_j \) are the Adomian polynomials and \( C^0_1 = x_n, C^0_2 = y_n, C^0_3 = z_n \)

The Adomian first polynomial is obtained as,

\[
C^1_1 = C^0_2 \\
C^1_2 = -4C^0_1 - 4C^0_2 C^0_3 \\
C^1_3 = [C^0_1]^2 + [C^0_2]^2 + b[C^0_3]^2 - a
\]

(8)

The Adomian second polynomial is obtained as,

\[
C^2_1 = C^1_2 \\
C^2_2 = -4C^1_1 - 4(C^1_2 C^0_3 + C^0_2 C^1_3) \\
C^2_3 = 2C^1_1 C^0_1 + 2C^0_2 C^1_2 + b [2C^0_3 C^1_3] - a
\]

(9)

The Adomian third polynomial is obtained as,

\[
C^3_1 = C^2_2 \\
C^3_2 = -4C^2_1 - 4 \left[ C^2_2 C^0_3 + C^0_2 C^3_3 + C^2_1 C^1_3 \right] \frac{\Gamma(2q+1)}{\Gamma(q+1) \Gamma(2q+1)} \\
C^3_3 = \left[ [C^1_1]^2 + 2C^2_1 C^0_1 + [C^1_2]^2 + 2C^2_2 C^0_2 + b \left[ [C^0_3]^2 + 2C^2_3 C^0_3 \right] \right] \frac{\Gamma(2q+1)}{\Gamma(q+1) \Gamma(2q+1)} - a
\]

(10)

The Adomian fourth polynomial is obtained as,

\[
C^4_1 = C^3_2 \\
C^4_2 = -4C^3_1 - 4 \left[ \frac{C^2_2 C^0_3 + C^0_2 C^0_3 + C^2_1 C^1_3}{C^2_1 C^3_3 + C^2_2 C^1_3} \right] \frac{\Gamma(3q+1)}{\Gamma(q+1) \Gamma(2q+1) \Gamma(3q+1)} \\
C^4_3 = \left[ 2C^3_1 C^0_1 + 2C^2_1 C^1_1 + 2C^2_2 C^2_2 + 2C^2_3 C^3_3 + b \left[ 2C^0_3 C^1_3 + 2C^2_3 C^3_3 \right] \right] \frac{\Gamma(3q+1)}{\Gamma(q+1) \Gamma(2q+1) \Gamma(3q+1)} - a
\]

(11)

The Adomian fifth polynomial is obtained as,

\[
C^5_1 = C^4_2 \\
C^5_2 = -4C^4_1 - 4 \left[ C^4_1 C^0_3 + C^0_2 C^4_3 + C^2_1 C^3_3 + C^1_2 C^3_3 + C^2_2 C^2_3 + C^2_3 C^3_3 + C^1_3 C^2_3 \right] \frac{\Gamma(4q+1)}{\Gamma(q+1) \Gamma(3q+1) \Gamma(4q+1)} \\
C^5_3 = \left[ 2C^4_1 C^0_1 + 2C^3_1 C^1_1 + [C^2_2]^2 + 2C^2_2 C^0_2 + 2C^3_2 C^1_2 + [C^2_3]^2 + b \left[ 2C^3_3 C^0_3 + 2C^3_3 C^1_3 + [C^3_3]^2 \right] \right] \frac{\Gamma(4q+1)}{\Gamma(q+1) \Gamma(3q+1)} - a
\]

(12)
The Adomian sixth polynomial is obtained as,
\[ C_6^0 = C_2^2 \]
\[ C_6^1 = -4C_4^2 - 4 \left[ C_2^2 C_4^0 + C_2^2 C_4^1 + C_2^2 C_4^2 + C_2^2 C_4^3 + C_2^2 C_4^4 + C_2^2 C_4^5 \right] \frac{\Gamma(5q+1)}{\Gamma(q+1)^2 \Gamma(4q+1)} \]
\[ C_6^2 = \begin{bmatrix} 2C_2^2 C_4^0 + 2C_4^1 C_4^1 + 2C_4^2 C_4^0 + 2C_4^2 C_4^1 + 2C_4^2 C_4^2 \end{bmatrix} \frac{\Gamma(5q+1)}{\Gamma(q+1)^2 (4q+1)} - a \]
where \( h = t_{n+1} - t_n \) and \( \Gamma(\bullet) \) is the gamma function. The FOC cuttlefish system is numerically solved using (7) to (13). Figure 4 and 5 give the 2D phase plains of the SA and HA systems respectively.

**Figure 4.** 2D phase portraits of the self-excited fractional order chaotic cuttlefish with the commensurate fractional order of \( q = 0.998 \) and \( a = 1.493, b = 0.6, m = 4 \)

**Figure 5.** 2D phase portraits of the hidden attractor fractional order chaotic cuttlefish with the commensurate fractional order of \( q = 0.996 \) and \( a = 1.493, b = -0.8, m = 4 \)

5. Dynamic analysis of the FOC cuttlefish system.

5.1. Bifurcation with fractional order: In this section we investigate the bifurcation of the system with its fractional order. As shown in Figure 6 and 7, for SA and HA the fractional order system exhibits chaotic behavior if \( q_i > 0.996 \) and \( q_i > 0.994 \) respectively. The largest positive Lyapunov exponent \( (L_1 = 0.0272) \) of the SA becomes when against its largest integer order LE \( (L_1 = 0.0244) \) and
similarly the largest LE ($L_1 = 0.0236$) of the HA appears when against its largest integer order LE ($L_1 = 0.0215$).

Figure 6. Bifurcation of FOC cuttlefish (self-excited) system with order $q$

Figure 7. Bifurcation of FOC cuttlefish (hidden) system with order $q$

5.2. Stability analysis. Commensurate Order: FOC cuttlefish system of order $q$, the stable system is in chaos if $|\arg(eig(J_E))| = |\arg(\lambda)| > \frac{q\pi}{2}$. For the FOC cuttlefish system to be stable is $q > \frac{2}{\pi} \tan^{-1} \left( \frac{|\text{Im}(\lambda)|}{|\text{Re}(\lambda)|} \right)$. The system has two equilibrium at $E_1$ and $E_2$ and the characteristic equation for the commensurate orders $q = 0.99$ for the equilibrium point $E_{1,2}$ are given by $\lambda^{297} + 3\lambda^{199} \pm 4.242\lambda^{198} + 3\lambda^{107} \pm 8.48\lambda^{100} - 4\lambda^{99} + \lambda^{3} \pm 4.242\lambda^{2} - 4\lambda \mp 5.646$.

Incommensurate Order: The characteristic equation obtained at the equilibriums are $\det(diag[\lambda^{M_N}, \lambda^{M_{N'}}, \lambda^{M_{N''}}] - J_E) = 0$ and by substituting the values of $M$. At
equilibrium point $E_1$ is, $\lambda^{294} + \lambda^{198} + \lambda^{197} + 6.656\lambda^{196} - 1.414\lambda^{195} + \lambda^{101} + 6.656\lambda^{100} + 3.586\lambda^9 + 4.242\lambda^8 - 7.997584\lambda^7 + \lambda^3 + 4.242\lambda^2 - 3.997584\lambda - 5.656$ and at $E_2$ is $\lambda^{294} + \lambda^{198} + \lambda^{197} - 4.656\lambda^{196} + 1.414\lambda^{195} + \lambda^{101} - 4.656\lambda^{100} + 6.414\lambda^9 - 4.242\lambda^8 - 7.997584\lambda^7 + \lambda^3 - 4.242\lambda^2 - 3.997584\lambda + 5.656$. If $q_x = 0.97, q_y = 0.98, q_z = 0.99$, then $M = 100$. The approximated result of the characteristic equation is $\lambda_{294} = 0.8286$.

6. Communication designs with SCSK modulation method using the FOC cuttlefish system. In this section, the novel fractional order chaotic cuttlefish system was tested communication designs with SCSK (Symmetric Chaos Shift Keying) modulation method. The communication designs with SCSK modulation method were tested separately in the SA and the HA of FOC cuttlefish system. Then, the BER performances of these communication systems were compared.

In SCSK transmitter unit, the information signal $m(t)$ is modulated (multiplied) by the chaotic signal $c(t)$ which produced by the chaotic system. The obtained modulated signal $s(t)$ is transmitted to the receiver unit.

The SCSK receiving unit contain the same chaotic system as the transmitter unit. The noisy modulated signal $r(t)$ is multiplied by the chaotic signal and integrated in the correlator. Thus, the bit energy of the signal is calculated. The information signal is obtained according to the threshold value determined using the calculated bit energy value. The SCSK transmitter unit and receiver unit block diagram is given in Figure 8 [17].

![SCSK block diagrams of transmitter unit and receiver unit](image)

Figure 8. SCSK block diagrams of transmitter unit and receiver unit [17].

The transmitter unit of the SCSK modulated communication system implemented in the Matlab-Simulink® program is given in Figure 9. An AWGN (Additive White Gaussian Noise) channel noise model added to the SCSK modulated signal and a noisy SCSK modulated signal $r(t)$ was obtained. The obtained SCSK modulated signal with noise $r(t)$ is sent to the receiver unit. The receiver unit of the SCSK modulated communication system implemented in the Matlab-Simulink® program is given in Figure 10.

The Matlab-Simulink® analysis of the communication system is performed separately for both the SA and the HA of FOC cuttlefish system. The energy per bit to noise power spectral density ratio ($E_b/N_0$) value of the AWGN channel model are chosen -10dB to 10dB in the Matlab-Simulink® analysis.

The input and output signals of the communication system analysis with 2dB $E_b/N_0$ value using the SA and HA of FOC cuttlefish system are given in Figure 11 and Figure 12, respectively. In the figures, the transmitted information signal, the SCSK modulated signal, SCSK modulated signal with noise and the retrieved signal
are given. The transmitted and retrieved signals are the same. When Figure 11 and Figure 12 are examined, it seems that it is very difficult to estimate the information signal sent from the transmitter unit. Due to these results, the fractional order chaotic cuttlefish system can be used for secure communication.

The BER (Bit Error Rate) performances of the SCSK modulated communication system using the SA and HA of FOC cuttlefish system is shown in Figure 13. According to the BER performance of the systems, the hidden attractor of fractional order chaotic cuttlefish system has a better BER performance than the self-excited attractor of fractional order chaotic cuttlefish system.

7. **Conclusions.** In this study, a new three dimensional continuous time chaotic system with cuttlefish like character was proposed and analyzed. Various dynamical properties of the system such as equilibrium points, LEs, KY dimension, and bifurcation plots in parameter space were presented. The proposed system could have both SA and HA depending on the choice of the parameter(s). Using the ADM we derived the fractional based model of the cuttlefish system. Bifurcation of
the fractional order system and stability conditions for the system were analyzed and presented. To show the importance of the proposed fractional order system in engineering applications, we designed a digital communication system with SCSK modulation method with both SA and HA. Finally BER performances of these communication systems were compared.

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Figure 12. Signals of the SCSK modulated communication system using hidden attractor of fractional order chaotic cuttlefish system (a) transmitted (b) SCSK modulated (c) SCSK modulated with noise (d) retrieved.

Figure 13. Comparison of BER performances of the SCSK modulated communication system.
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