Optimal Two-Sided Market Mechanism Design for Large-Scale Data Sharing and Trading in Massive IoT Networks

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Abstract—The development of the Internet of Things (IoT) generates a significant amount of data that contains valuable knowledge for system operations and business opportunities. Since the data is the property of the IoT data owners, the access of the data requires the permission from the data owners, which gives rises to a potential market opportunity for the IoT data sharing and trading to create economic values and market opportunities for both data owners and consumers. In this work, we leverage optimal mechanism design theory to develop a monopolist matching platform for data trading over massive IoT networks. The proposed mechanism is composed of a pair of matching and payment rules for each side of the market to optimize the payoffs under welfare-maximization and the profit maximization schemes. We characterize the optimal mechanism with a class of threshold matching rules under welfare and profit-maximization and study three matching behaviors including separating, bottom pooling, and top pooling. We use HealthyGo as a case study to illustrate the optimal threshold matching rules and corroborate the analytical results.

I. INTRODUCTION

As the number of cyber and physical devices are connected to the existing Internet infrastructure, the Internet of Things (IoT) provides connectivity and network solutions to emerging applications such as smart homes, enterprises, and cities [1]. Each application consists of a large number of IoT objects, and they are designed to perform heterogeneous tasks, including sensing, auction, data collection, storage, and processing. Data collected for fulfilling the individual tasks are often either deleted or stored and locked down in independent data silos, as shown in Fig. 1. The data generated in the IoT system has a significant amount of valuable knowledge hidden, such as behaviors, life patterns, and habits, which can be used to improve the human lives as well as the IoT system itself. Enterprises can benefit from big data analytics to reduce cost, help product development, and optimize marketing strategies. The data owners, however, generally lack knowledge and techniques to conduct knowledge discovery from their data, and each owner can only gain access to her data which has little value with respect to knowledge discovery [2]. On the other hand, the enterprises or other data consumers have abilities to discover the hidden knowledge accurately and efficiently, but they may not have a sufficient amount of targeted data or the rights to access them due to privacy concerns. As a result, data sharing over the IoT network is important to create economic values and market opportunities for both data owners and consumers.

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Since the data is the property of the IoT data owners, post-primary usages of the data require the permission from the data owners, leading to a potential market opportunity for the IoT data trading, instead of the free data sharing. The difference in the roles in dealing with data can divide the market participants into two distinct classes, i.e., the data owners (sellers) and data consumers (buyers). Therefore, a two-sided market (e.g., [3]) is a suitable market model for IoT data trading. Data consumers have different requirements on the features of the data traded (including category, history, quality, and quantity of the data), the owner’s engagements with the IoT system (including the usage frequency of IoT products), and the user’s personal information (including life styles and income). On the other hand, the data owners have different preferences for the rewards, including cash and coupons, offered by the data consumers for trading the data and different tolerance levels of privacy violation that might occur in the data trading. The data consumers’ objective is to buy valuable data that satisfy their requirements while the data owners’ goal is to sell the data to those service providers who offer the highest rewards based on their preferences.

For example, HealthyGo is a healthy food provider, who is keen to know whether there are external factors that influence the healthy food consumption such as workout patterns, sleep quality, weather, and temperature through data analysis. Therefore, HealthyGo is looking for IoT data of household products that can collect such data. HealthyGo has barriers to obtain such data in an efficient and low-cost way. On the other hand, a family has a smart refrigerator with built-in sensors that can monitor the food consumption, a smart thermostat that controls the room temperature, and
smart fitness trackers that provide heart rate monitoring, sleep tracking, and sports tracking. However, generally the data collected has little value after the primary usage. Therefore, a data trading platform is important to enable mechanisms to take advantages of both the data owner and the data consumers and provides motivations of participating while ensuring the interests of all parties in the IoT system.

Matching intermediary platforms are usually introduced to tie together multiple distinct classes of participants (i.e., users and service providers in this paper) in a market. The business of a matching intermediary is to match participants from multiple sides of a market. In the two-sided market, the platforms play a role to enable and encourage interactions between two sides of participants by appropriately charging each side for offering matching rules. There can be multiple platforms or a monopolist platform in a market.

In this work, we focus on massive IoT networks with a sufficiently large number of owners and consumers. We develop a mechanism design approach and propose a two-sided matching market model for the IoT data trading as a sustainable business model that captures the potential market opportunity in the IoT data sharing mechanism. As illustrated in Fig. 2, data owners have IoT data generated during the primary usages of different IoT products to sell, and the data consumers have different requirements on the data demand. The data consumers offer rewards to the data owners for trading their data. A monopolist matching platform matches the data owners and the data consumers based on their information and preferences by charging both sides with an appropriate price. The price charged for offering matching rules generates profits for the platform. In the IoT data market framework, the matching rule determines the datasets that achieve the highest utility of the data consumers. Since the matching is reciprocal, the data traded (according to the matching rule) by the data owners, on the other hand, determines the level of rewards they can receive from the data consumers. In other words, the matching can generate welfare over the IoT network. Our framework considers the interactions between a large number of data buyers and sellers, and study the matching rules under profit and welfare maximization schemes [4], respectively. We study a class of threshold matching rules and show that under certain assumptions both the profit-maximizing and the welfare-maximizing matching rules have optimal threshold matching rules.

Our main contribution of this work is summarized as follows:

- We leverage optimal mechanism design theory to develop an incentive-compatible optimal matching mechanism for data trading over massive IoT networks. The developed mechanism is composed of a pair of matching and payment rules for each side of the market to optimize the payoffs under profit and welfare maximization schemes.
- We develop a monopolist trading platform with two matching stages for massive IoT networks with a large number of devices. The framework captures the types of the data owners and consumers by their characteristics and valuations, which are private information unknown to the platform.
- We characterize the optimal mechanism with a class of threshold matching rules under welfare-maximization and the profit-maximization, and study three matching behaviors including separating, bottom pooling, and top pooling. We use a case study to show the numerical analysis of the optimal threshold matching rules.

A. Related Work

There is a rich literature on the economic analysis and pricing schemes of the market model for data collection in IoT networks [5]–[7] based on a variety of approaches, including smart data pricing scheme in [8] and auction-based pricing such as sealed-bid auctions [9]. Also, utility maximization-based pricing schemes have also been studied [10]–[12]. For example, in [13], an optimal dynamic spectrum reservation contract has been designed for mission-
critical IoT systems. Devices can make an advance payment at the time of reservation and receive a rebate if the reservation is released. Devices are incentivized to reveal the true application type and it has been shown that the incentive compatible mechanism leads to an efficient utilization of the spectrum as well as a greater profitability of the IoT network operator.

The IoT data market model in this work is a two-sided matching model with a monopolist intermediary matching platform. The trading is made directly between the original data owners and the final data consumers. The role of the platform is to provide matching rules to both sides of the market. The goal of the market model is to establish optimal matching rules under the profit-maximization for the platform and the welfare-maximization for the massive IoT network. Also, unlike the auction-based model, the model does not describe competitions on the side of data owners, thereby leading to a convenient monopolist matching rule that matches sellers and buyers in an efficient way. Furthermore, the reciprocal nature of the matching also prevents the monopolist platform from appropriating the entire surplus of the market to himself [14].

B. Organization of the Paper

The rest of the paper is structured as follows. Section II describes the IoT data market framework. We first describe the market structure, followed by elaborating the mathematical model. Section III presents the matching mechanism by considering the welfare-maximizing and the profit-maximizing schemes. In Section IV we present optimal matching rules by considering a class of threshold matching rules with the theoretical analysis. A case study is conducted in Section V. Finally, Section VI presents the concluding remarks and future research directions.

II. IOT DATA MARKET

In this section, we present the framework of the IoT data trading market, by starting with describing the market structure and followed by the mathematical model of the market.

A. Market Structure

In the scenario of IoT, the data market has two sides: the seller side ($S$) constituting of (market) participants that are the IoT data owners, and the buyer side ($B$) constituting of participants who are the IoT data consumers. We consider a massive IoT networks, in which there is a sufficiently large number of participants on both sides of the market, and each participant has no market power. As a result, the population of each side of the market is modeled by a unit-mass continuum of participants indexed by $i \in [0, 1]$. A monopolistic market platform (platform) operates as a matching intermediary to providing matching rules with different prices that match the owners from side $S$ and consumers from side $B$. The pricing of matching rules in this market considers the second-degree price discrimination [14], [15], in which the price of matching rules varies subject to the number of participants involved [15]. In this IoT data market model, the matching rule determines the benefits of the consumers in the trading. Since the matching is reciprocal, the data sold by the owners, on the other hand, determines the rewards they can receive from the data consumers for trading their data. Our model takes the two sides of the market into account and studies the matching rules under profit-maximization and welfare-maximization [4], respectively.

Each participant $i$ from side $K \in \{S, B\}$ has private information about her willingness to pay for the quality of matching rules offered by the platform as well as her market attractiveness as perceived by the participants from the opposite side $L \neq K$. The platform has to internalize the existence of such private information and provide incentives in terms of trading matching quality and pricing rules for the participants when designing the matching rules. We consider two matching stages. In the first matching stage, the platform matches a group of owners and a group of consumers based on the data categories that the owners have and the consumers require. In the second matching stage, the responsibility of the platform is to design optimal matching rules under welfare-maximizing and profit-maximizing schemes, respectively, by considering the willingness-to-pay and the market attractiveness reported by the participants.

B. Market Model

Each participant $i \in [0, 1]$ from side $K \in \{S, B\}$ has information summarized as $\zeta = (\mu^K, \lambda^K)$, where $\mu^K$ represents the data categories, and $\lambda^K = (\gamma^K, \lambda^K) \in \Lambda^K = C^K \times V^K$ is the type of the participant $i$. The IoT data can be classified into different categories including household data (collected by household IoT products), fitness data (e.g., healthy food intake, workout information), experience data (e.g., information about road conditions collected by navigation systems, environment of a neighborhood collected by outdoor surveillance cameras). Each type $\lambda^K = (\gamma^K, \lambda^K)$ includes the characteristic $c^K \in C^K$ that includes the related information of the participant, and valuation $v^K \in V^K = [v^K, \bar{v}^K] \subset \mathbb{R}_+$.

For owner $i$ from side $S$, the data category describes the categories of the IoT data owned. The characteristic $c^K_i$ contains the data size and the statistics of the DO’s engagements with the IoT system (for example, the frequency of usage of IoT products, expense related to IoT, etc.). The valuation $v^K_i$ summarizes the owner’s willingness to pay for the quality of data trading matching rules made by the platform based on her characteristics, the estimation of the rewards for selling her data, and her tolerance of privacy violation.

For consumer $j \in [0, 1]$ from side $B$, the data category describes the categories of the IoT data wanted, and the characteristic $c^K_j$ includes the rewards offered to the matched owners. The reward $r_j(\lambda^K) : \Lambda^K \to \mathbb{R}_+$ is a function of owner
i’s type that is observed by the consumer j. A typical reward includes cash, vouchers, coupons, free products, loyalty points, and actionable advice, etc. The valuation \( v_B^j \) summarizes consumer j’s willingness to pay for the quality of data trading matching rules made by the platform by taking into account the reward she offers to the data owners.

In our model, we assume that each type \( \lambda^K_i \) of each participant i from side \( K \in \{S,B\} \) is drawn independently from a continuous distribution \( F^K(\lambda^K) = F^K(c^K_i, v^K_i) \) with density \( f^K(\lambda^K) = f^K(c^K_i, v^K_i) \). Let \( F^K_j \) with density \( f^K_j \) be the marginal distribution of \( F^K \) with respect to the valuation \( v^K_j \). A standard assumption in mechanism design is as follows.

**Assumption 1**: The virtual valuation of participant i from side \( K \),

\[
v^K_i = \frac{1 - F^K_i(v^K_i)}{f^K_i(v^K_i)}
\]

which is a continuous and non-decreasing function of \( v^K \) that measures the surplus that can be extracted from that participant [16]. For \( F^K_i \) that satisfies the virtual valuation, we call it regular.

Let \( \alpha^K_i(\lambda^K_j) : \mathcal{L}^i \rightarrow \mathbb{R}^+ \), \( K \neq L \in \{S,B\} \) model the interaction quality that the participant i from side \( K \) obtains from being matched to a participant j from the opposite side \( L \). The interaction quality \( \alpha^K_i(\lambda^K_j) \) measures the market attractiveness of the participant j from side \( L \) as perceived by the participant i from side \( K \). For example, owners with more frequent usages of IoT product induces relatively higher interaction quality (i.e., more attractive) than those with less frequent usage as seen by the consumer. The interaction quality \( \alpha^K_i(\lambda^K_j) \) enjoyed by the owner i models the profit generated by matching to consumer j by measuring the monetary value of the non-monetary reward (i.e., vouchers, coupons, free products, loyalty points, and actionable advice) \( r_i(\lambda^K_j) \) by taking into account the usefulness as seen by the owners.

The **trading matching quality** measures the cumulative interaction qualities of a set of participants. Let \( g^K \) be a Borel measurable set of participants from side \( K \in \{S,B\} \) interacted with by each participant from side \( K \neq L \in \{S,B\} \). The trading matching quality of any set \( g^K \) of participants from side \( K \neq L \in \{S,B\} \) with type profile \( \{\lambda^K_j\}_{j \in g^K} \), which is perceived by each participant i from side \( K \), is modeled as the sum of interaction qualities \( \alpha^K_i(\lambda^K_j) \). Let \( \beta^K(g^K) \), \( K \neq L \in \{S,B\} \) denote the trading matching quality defined as

\[
\beta^K(g^K) := \int_{\lambda^K \in g^K} \alpha^K_i(\lambda^K_j)dF^K(\lambda^K_j),
\]

for \( K \neq L \in \{S,B\} \). In this model, all participants from side \( K \neq L \in \{S,B\} \) agree on the value of \( \alpha^K_i(\lambda^K_j) \), hence on the value of \( \beta^K(g^K) \), for all j in side \( L \).

The **marginal utility** of each participant i from side \( K \in \{S,B\} \) with \( \beta^K(g^K) \) is modeled by a function denoted as \( \gamma^K_i(\beta^K(g^K)) \), which is a positive, strictly increasing, and continuously differentiable function. Given any set \( g^K \) of participants from side \( L \in \{S,B\} \), the payoff that a participant i from side \( K \neq L \in \{S,B\} \) obtains from being matched to the set \( g^K \) is given by

- **Buyer side B**:

\[
\pi^K_B(g^K_i, p; \lambda^K_i) := v^K_B(\beta^K(g^K_i)) - r_i - p.
\]

- **Seller side S**:

\[
\pi^K_S(g^K_i, p; \lambda^K_i) := v^K_S(\beta^K(g^K_i)) - p.
\]

Here, \( p \) is the price paid by each participant from each side to the platform. Each participant i from side \( K \in \{S,B\} \) is required to report her type \( \lambda^K_i \) to the platform. Let \( \tilde{\lambda}^K_i = (c^K_i, v^K_i) \) be the reported type by participant i from side \( K \), as shown in Fig. 3.

### III. Matching Mechanism

In this paper, we focus on the **anonymous** mechanism design. Specifically, the set of participants from side \( K \neq L \in \{S,B\} \) that are matched to the participant i from side \( K \) and the corresponding payment \( p \) depends only on the reported type \( \tilde{\lambda}^K_i \) by i. Also, if the reported types \( \tilde{\lambda}^K_i \neq \tilde{\lambda}^K_k \) then the participant i and k are matched to the same set and the same payment \( p \) is required; in other words, the individual identities in the same side are irrelevant if they have the same type. Thus, for simplicity, the subscript indicating the identity is suppressed in the rest of the paper. Let \( h^K \subset \mathcal{L}^K \) be a Borel measurable set of types \( \lambda^K \) of participants from side \( L \in \{S,B\} \). Then, the trading matching quality (1) becomes

\[
\beta^K(h^K) := \int_{\lambda^K \in h^K} \alpha^K(\lambda^K)dF^K(\lambda^K),
\]

for \( K \neq L \in \{S,B\} \).

Our model considers the **direct-revelation** [16], [17] anonymous mechanisms \( M = \{h^K(\cdot), \rho^K(\cdot)\}_{K \in \{S,B\}} \). The rule \( h^K(\lambda^K) \in \mathcal{L}^K \), \( K \neq L \in \{S,B\} \), is the matching rule, which specifies the set of types of participants from side \( L \) that is included in the matching set of all participants from side \( K \) with the type \( \lambda^K \). The rule \( \rho^K(\lambda^K) \) specifies the payment required to all participants from side \( K \) with the type \( \lambda^K \).

The feasible matching rule is defined as follows.
Definition 1 (Feasibility): A matching rule $\bar{h}^K(\cdot)$, $K \in \{S, B\}$ is feasible if and only if the following reciprocity condition is satisfied

$$\lambda^B \in \bar{h}^B(\lambda^S) \leftrightarrow \lambda^S \in \bar{h}^B(\lambda^B), \quad (5)$$

that is, if the type $\lambda^B$ of the consumer is in the matching set of the owner with type $\lambda^S$, then the type $\lambda^S$ is also in the matching set of the consumer with type $\lambda^B$.

The payoff that a participant with type $\lambda^K = (\alpha^K, v^K)$ from side $K \in \{S, B\}$ obtains when reporting type $\hat{\lambda}^K = (\hat{\alpha}^K, \hat{v}^K)$ is defined as follows.

• **Buyer side $B$:**

$$\bar{\Pi}^B(\lambda^B, \hat{\lambda}^B; M) := v^B \cdot \gamma^B(B^B(\bar{h}^B(\lambda^B))) - r - p^B(\hat{\lambda}^B). \quad (6)$$

• **Seller side $S$:**

$$\bar{\Pi}^S(\lambda^S, \hat{\lambda}^S; M) := v^S \cdot \gamma^S(B^S(\bar{h}^S(\hat{\lambda}^S))) + r - \bar{p}^S(\lambda^S). \quad (7)$$

Denote by $\Pi(\lambda^K; M) = \Pi^K(\lambda^S, \lambda^K; M)$ the payoff that a participant with type $\lambda^K$ obtains when reporting truthfully. Based on the direct revelation principle [17], we focus on the individual rationality and incentive compatibility constraints defined as follows.

Definition 2 (Individual Rationality): A mechanism $M$ is individually rational (IR) if $\forall \lambda^K, \hat{\lambda}^K \in \Lambda^K$,

$$\Pi^K(\lambda^K; M) \geq 0 \quad \forall \lambda^K \in \Lambda^K. \quad (8)$$

Definition 3 (Incentive Compatibility): A mechanism $M$ is incentive compatible (IC) if $\forall \lambda^K, \hat{\lambda}^K \in \Lambda^K$,

$$\Pi^K(\lambda^K; M) \geq \bar{\Pi}^K(\lambda^K, \hat{\lambda}^K; M). \quad (9)$$

A matching rule $\bar{h}^K(\cdot)$, $K \in \{S, B\}$ is implementable if there is a payment rule $\{\bar{p}^K\}_{K \in \{S, B\}}$ such that the mechanism $M$ is both IR and IC.

A. Welfare and Profit

Next, we introduce the welfare and the profit generated by market matching. The total welfare of the market participants from both sides generated by the mechanism $M$ is

$$U^W(M) = \sum_{K = S, B} \int_{\Lambda^K} v^K \gamma^K(B^K(\bar{h}^K(\lambda^K)))dF^K(\lambda^K). \quad (10)$$

The profits generated by the mechanism $m$ for the platform is given by

$$U^P(M) = \sum_{K = S, B} \int_{\Lambda^K} \bar{p}^K(\lambda^K)dF^K(\lambda^K). \quad (11)$$

The mechanism $M$ is efficient (resp. profit-maximizing) if $M^W = \arg\max_M U^W(M)$ (resp. $M^P = \arg\max_M U^P(M)$), and the matching rule $\bar{h}^K$, $K \in \{S, B\}$ of $M$ is feasible and implementable. We then have the following lemma.

Lemma 1: The mechanism $M$ is IC and IR if and only if all of the following conditions hold $\forall K \in \{S, B\}$:

• The trading matching quality $\beta^K(\bar{h}^K(\lambda^K)) = \beta^K(\bar{h}^K(\lambda^K))$ is non-decreasing in $\lambda^K$, i.e.,

$$\beta^K(\bar{h}^K(\lambda^K, v^K)) \geq \beta^K(\bar{h}^K(\lambda^K, v^K)).$$

for any $(\alpha^K, v^K)$ and $(\alpha^K', v^K)$ such that $v^K \geq v^K'$.

• The expected payoff of any two participants with the same valuation $v^K$ is the same and is irrespective of the characteristics $\lambda^K$.

• The equilibrium payoff $\Pi^K((\alpha^K, v^K); M)$ of the participants with the lowest valuation are non-negative.

• The pricing rule satisfies the following envelope formula

$$\bar{p}^K((\alpha^K, v^K)) = v^K \gamma^K(\beta^K(\bar{h}^K(\alpha^K, v^K))) \quad (10)$$

We can see that in the profit-maximizing mechanisms, the payoff of the participants with the lowest valuations $v^K$ satisfies $\Pi^K((\alpha^K, v^K); M) = 0$, $\forall K \in \{S, B\}$, due to the IR constraints. Denote the valuation associated with welfare maximization by $\theta^K_{\bar{h}^K}(v^K) := v^K$, and by $\theta^K_{\bar{h}^K}(v^K) := v^K - \bar{p}^K(\lambda^K, v^K)$ denote (virtual) valuation associated with profit maximization. Then, by the envelope formula [10], for $I = W, P$, (8) and (9) can be rewritten as

$$U^I(M) = \sum_{K = S, B} \int_{\Lambda^K} \theta^K_I(v^K) \cdot \gamma^K(B^K(\bar{h}^K(\alpha^K, v^K)))dF^K(\alpha^K) \quad (11)$$

Therefore, the platform’s responsibility includes finding an optimal matching rule $\bar{h}^K_{\bar{P}^I} \in [W, P]$, by maximizing [11] under the conditions in Lemma 1. A matching rule $\bar{h}^W_{\bar{P}^I}$ (resp. $\bar{h}^P_{\bar{P}^I}$) is welfare-optimal (resp. profit-optimal) if $\bar{h}^W_{\bar{P}^I} = \arg\min_{\bar{h}^W} U^W(M)$ (resp. $\bar{h}^P_{\bar{P}^I} = \arg\min_{\bar{P}^I} U^P(M)$).

IV. OPTIMAL MATCHING RULES

In this section, we present the optimal matching rules for the data market matching model. First, a fairly natural assumption is as follows.

Assumption 2: The interaction quality $\alpha^K$, the distribution of type $F^K$, and the marginal utility function $\gamma^K(\cdot)$, $\forall K \in \{S, B\}$, satisfy one of the following two:

a. Under the distribution $F^K$, $(\alpha^K(\lambda^K, v^K), \forall K \in \{S, B\}$, are weakly positively affiliated; the marginal utility function $\gamma^K(\cdot)$ is weakly concave and $\alpha^K$ is weakly increasing.

b. Under the distribution $F^K$, $(\alpha^K(\lambda^K, v^K), \forall K \in \{S, B\}$, are weakly negatively affiliated; the marginal utility function $\gamma^K(\cdot)$ is weakly convex and $\alpha^K$ is weakly decreasing.

We then have the following proposition.

Proposition 1: Under Assumption [2] both the welfare-maximizing and the profit-maximizing rules discriminate only along the valuation $v^K$, i.e., $\bar{h}^K(\alpha^K, v^K) = \bar{h}^K(\alpha^K, v^K')$, $\forall v^K, \alpha^K \neq \alpha^{K'}, K \in \{S, B\}$.

Based on Proposition 1 we re-define $\bar{h}^K(v^K) \in V^K, \forall K \in \{S, B\}$.

A. Feasible Threshold Rules

In this paper, we consider a class of feasible threshold rules [14] defined as follows.
Definition 4 (Feasible Threshold Rules): Let \( y^K_f(\cdot) : V^K \to V^K \) be a weakly decreasing threshold function such that \( \forall K \in \{A,B\}, \forall I \in \{W,P\}, \)

\[
y^K_f(v^K) = \min\{v^K : y^K_f(v^K) \leq v^K \}.
\]

(12)

A matching rule \( \tilde{h}^K \) is a feasible threshold rule if, \( \forall v^K \in V^K, K \in \{A,B\}, I \in \{W,P\}, \)

\[
\tilde{h}^K_f(v^K) = \begin{cases} [K_1(v^K), v^K], & \text{if } v^K \in [\delta^K, v^K), \\ \emptyset, & \text{otherwise} \end{cases}
\]

(13)

where \( \delta^K \in [v^K, v^K] \) is the threshold value. We say that \( (y^K_f(v^K), \delta^K) \) is the feasible threshold structure associated with the threshold rule \( \tilde{h}^K((c^K, v^K)) \).

Under the threshold rule, the participant with valuation below \( \delta^K \) are excluded, while the participant from side \( K \neq L \in \{A,B\} \) with valuation \( v^K \geq \delta^K \) is matched to any participant from side \( L \), whose valuation is within \( [y^K_f(v^K), v^K] \). The condition (12) of \( \tilde{h}^K \) means that the weakly decreasing threshold function \( y^K_f(v^K) \), \( \forall K \in \{A,B\} \), coincides with the inverse of the threshold function \( y^K_f(v^K) \), \( \forall K \neq L \). Thus, the threshold rule \( \tilde{h}^K \), i.e., the reciprocity condition [4] in Definition 1 is satisfied. Moreover, the participants of side \( K \in \{A,B\} \) with low \( v^K \) are matched only to those participants from side \( L \neq K \) with sufficiently high value of \( v^K \). Also, the matching sets are ordered across valuations, i.e., \( \tilde{h}^K_f(v^K) \subseteq \tilde{h}^L_f(v^K) \) for \( v^K < v^K \). We summarize the optimal matching rule in the sense of feasible threshold rules in the following theorem.

Proposition 2: Under Assumption 2 both the welfare-maximizing and profit-maximizing rules \( \tilde{h}^K \) have a feasible threshold structure \( (y^K_f, \delta^K), \forall K \in \{A,B\}, \forall I \in \{W,P\} \).

B. Analysis of Optimal Threshold Rules

In this subsection, we analyze the property of the optimal threshold rules given the result in Proposition 2. We first define the separation, and pooling in the sense of two-sided market in the following definitions.

Definition 5 (Separation): The I-optimal matching rule \( \tilde{h}^I_L, K \neq L \in \{A,B\} \) and \( I \in \{W,P\} \), shows separation if \( \exists \tilde{v}^K \subset V^K, y^K_f(\tilde{v}^K) \neq y^K_f(\tilde{v}^K), \forall v^K \in \tilde{v}^K, \) and \( [\delta^K, y^K_f(\delta^K_f)] \) is the separating range. The matching range is maximally separating if \( y^K_f(\cdot) \) is strictly increasing in the separating range.

Definition 6 (Pooling): The I-optimal matching rule \( \tilde{h}^I_L, K \neq L \in \{A,B\} \) and \( I \in \{W,P\} \), shows

1. bottom pooling if \( \delta^K > v^K \), i.e., all participants with valuation in a neighborhood of \( v^K \) are assigned empty matching sets;
2. top pooling if \( y^K_f(\delta^K_f) < v^K \), i.e., all participants in a neighborhood of \( v^K \) are assigned the same matching sets.

Let \( \Gamma^K(\cdot) : V^K \to \mathbb{R}_+ \) be defined as

\[
\Gamma^K(v^K) := \int_{C^I} \alpha^K((c^K, x)) dF^K_I(x).
\]

(14)

Therefore, the platform’s responsibility includes choosing a bottom pooling threshold rule with a feasible threshold structure \( (y^K_f(v^K), \delta^K) \in \{W,P\} \) by maximizing, \( \forall K \in \{A,B\} \) and \( I \in \{W,P\} \).

\[
U^I(M) = \sum_{K=3}^5 \int_{\delta^K}^{v^K} \theta^K_f(v^K) \Gamma^K(y^K_f(v^K)) dF^K(v^K)
\]

(15)

s.t. \( y^K_f(v^K) = \min\{v^K : y^K_f(v^K) \leq v^K \}, \forall v^K \in [\delta^K, v^K] \)

The following assumption is based on Myerson’s regularity condition [16] that is used to guarantee that the optimal matching rule \( \tilde{h}^K \) is maximally separating.

Assumption 3: The function \( \Theta^K_f : V^K \to \mathbb{R} \) defined as, \( K \neq L \in \{A,B\} \), \( I \in \{W,P\} \),

\[
\Theta^K_f(v^K) := \frac{f^K(v^K) \theta^K_f(v^K)}{-\frac{d}{d\alpha^I} \Gamma^K(v^K) - \frac{d}{d\alpha^L} \Gamma^K(v^K) - \frac{d}{d\alpha^I} \Gamma^K(v^K) - \frac{d}{d\alpha^L} \Gamma^K(v^K)}
\]

(16)

is strictly increasing.

Assumption 3 is interpreted as follows. Consider the case of profit maximization, i.e., \( I = P \). Let \( \theta^K_f(v^K) := v^K - \frac{F^K(v^K)}{\int_{\delta^K}^{v^K} \alpha^K((c^K, x)) dF^K_I(x)} \) be the virtual valuation of the participant of side \( K \) with valuation \( v^K \), and the denominator \( \frac{d}{d\alpha^I} \Gamma^K(v^K) \times [v^K, v^K] \) is proportional to the expected interaction quality. Thus, Assumption 3 requires that the contribution from the willingness-to-pay (i.e., valuation) of a participant from side \( K \) to the platform’s revenue grows faster than her contribution from the attractiveness (i.e., interaction quality).

Let \( D^K_f(v^K, v^K) : V^K \times V^K \to \mathbb{R} \) be the marginal surplus defined as

\[
D^K_f(v^K, v^K) = -\frac{d}{d\alpha^L} \Gamma^K(v^K) \cdot \Theta^K_f(v^K) \cdot f^K(v^K)
\]

(17)

\[
-\frac{d}{d\alpha^L} \Gamma^K(v^K) \cdot \Theta^K_f(v^K) \cdot f^K(v^K)
\]

(18)

The following theorem summarizes the optimal threshold matching rules.

Theorem 1 (Optimal threshold matching rules): Under Assumption 2 and 3, we have the following, \( \forall K \neq L \in \{A,B\} \), \( \forall I \in \{W,P\} \),

- If \( D^K_f(v^K, v^K) > 0 \), then the I-optimal matching rule is \( \tilde{h}^I_L(v^K) = V^K, \forall v^K \in V^K \).
- If \( D^K_f(v^K, v^K) < 0 \), then the I-optimal matching rule is maximally separating with
  - If \( D^K_f(v^K, v^K) > 0 \), then it shows top pooling at side \( K \) and no bottom pooling at side \( L \).
  - If \( D^K_f(v^K, v^K) < 0 \), then it shows bottom pooling at side \( K \).
- The threshold function \( y^K_f(\cdot) \) satisfies \( D^K_f(v^K, y^K_f(v^K)) = 0, \forall v^K \in [\delta^K, y^K_f(\delta^K_f)] \).

Theorem 1 indicates that when the marginal surplus determined on the lowest valuations of any data owners and consumers, the optimal threshold matching rule shows separation. When such marginal surplus is non-negative, any data owner is matched to any data consumers.
V. Numerical Analysis

In this section, we present numerical analysis for the second matching stage to illustrate the optimal matching rules under the welfare and the profit maximization.

Example 1 (A use case): Mr. and Ms. Smith have a daughter. Their house has several household IoT devices. A smart refrigerator monitors the category, quantity, and freshness of food stored inside. A smart thermostat controls the room temperature based on the preference of the family members. They also have a fitness tracker for each family member that monitors their workout patterns and tracks their sleeping. HealthyGo is a health supplement provider. It aims to know the patterns of healthy food (e.g., a variety of vegetables, low-calorie food, etc.) consumption of families like the Smith. HealthyGo wants to figure out whether there are external factors such as workout patterns, sleeping quality, weather, and temperature that influence the healthy food consumption. Traditionally, these studies are done via focus group interview and customer survey. By trading the IoT data from thousands of families like the Smith, HealthyGo can understand the consumption patterns better through data analysis. HealthyGo can benefit from such data analysis and help new product developments and optimize the supply chain to increase the sales and reduce the cost. These advantages motivate HealthyGo to participate in the IoT data trading. As a data owner, the Smith family receives rewards from trading the IoT data with HealthyGo.

In the numerical analysis, consider the case when \( V^K \in \mathbb{R}_+ \), \( \alpha^K((c^K,v^K)) = v^K \), \( \gamma^K(x) = x \), \( \forall K \in \{S,B\} \). Suppose that all valuations \( v^K \) are drawn independently from a uniform distribution over \( V^K = [1,10] \) for \( K \in \{S,B\} \), i.e., \( F^K = \frac{v^K}{10} \). Then, \( \Gamma^K(v^L) = \frac{1}{9} \int_0^{10} x dF^K(x) = \int_0^{10} x dx \). For \( I = W \), we have

\[
D^K_W(v^L,v^K) = \frac{2}{81} > 0,
\]

and by Theorem 1 the welfare-optimal matching rule is \( h^K_W(c^K,v^K) = v^K, \forall v^K \in V^K \), a shown in Fig. 4. Fig. 6 shows the change of welfare when \( \delta^K \) changes. As can be seen, the welfare is maximized when \( \delta^K = 1 \), which coincides with the optimal \( h^K_W((c^K,v^K)) \).

For \( I = P \), we have

\[
D^K_P(v^L,v^S) = \frac{-16}{81} < 0.
\]
and
\[ D^\mathcal{K}_P(\bar{v}, \bar{\Sigma}) = \frac{-70}{81} < 0, \]
and by Theorem 1 the profit-optimal matching rule is maximally separating and shows bottom pooling at side \( L \) and no top pooling at side \( K \). The threshold function is found as
\[ y^\mathcal{K}_P(\bar{v}, \bar{\Sigma}) = \frac{10v^\mathcal{K}}{4v^\mathcal{K} - 10}, \]
and thus from \( y^\mathcal{K}_P(\delta^\mathcal{K}_P) = 10 \), we have \( \delta^\mathcal{K}_P = \frac{10}{3} \), \( \forall K \in \{S, B\} \).

Then, the corresponding profit-maximizing threshold matching rule, as shown in Fig. 5 is
\[ \tilde{h}_P(\bar{v}, \bar{\Sigma}) = \begin{cases} \min\{10v^\mathcal{K}, 10\}, & \text{if } v^\mathcal{K} \in [\frac{10}{3}, 10] \\ 0, & \text{otherwise} \end{cases} \]

Fig. 7 shows the change of profit when \( \delta^\mathcal{K} \) varies in \([1, 10]\). As shown in 7 the profit is non-negative only when \( \delta^\mathcal{K} \geq \frac{10}{3} \), which coincides with the optimal \( \tilde{h}_P(\bar{v}, \bar{\Sigma}) \) obtained above.

VI. CONCLUSION

In this paper, we have proposed a two-sided matching market framework for IoT data trading as a sustainable pricing model that incentivizes both the data owners and the data consumers. A monopolist platform has been introduced to match the data owners and the data consumers based on their knowledge and preferences. We have established a quantitative framework to model the IoT data trading market for welfare and profit maximizations. This work has characterized a class of feasible threshold matching rules. Under mild assumptions of the utility functions and the distributions of the valuations, there exist optimal threshold matching rules that maximize the welfare and the profit. We have used HealthGo IoT networks as a case study to understand the behavior of participants from both sides of the market under the optimal threshold matching rules. Our future work would extend the market design framework by introducing multiple platforms among heterogeneous IoT networks and understanding competitive behaviors among platforms.

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