Prediction Techniques for Accelerating the Convergence of Alternating Solution Method during the Power System Dynamic Simulation Using SDC Integration Algorithm

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Abstract. The electric power of generator $P_e$ and bus voltage of network $V$ have a strong nonlinear coupling relationship with other variables, which results in bad effects on the convergence procedure for the alternating solution method during the power system dynamic simulation using the SDC integration algorithm. Deep research was conducted into this problem. The convergence procedure for $P_e$ and $V$ was illustrated during the simulation, and prediction techniques by applying the Lagrange interpolation to construct a better iterative initial value with smaller error for them was proposed. Based on the SDC simulation method, the proposed method was tested on an IEEE standard 3-generator 9-bus system on the platform of MATLAB, and results show that the prediction techniques performs quite well for the acceleration of convergence procedure for $P_e$ and $V$ and simulation efficiency of SDC integration algorithm.

1. INTRODUCTION

The Spectral Deferred Correction method, also known as SDC for short, is a kind of new integration algorithm with high precision proposed by Alok Dutt, Leslie Greengard, and Vladimir Rokhlin in 2000[1]. The algorithm first conducts an interpolation in one integration step using the Gaussian polynomials and integrations between adjacent interpolation nodes with the alternating solution method(ASM)[2] by applying the low-order integration algorithms to make sure of the numerical stability, and then implements a spectral integration using the spectral integration matrix[3] which is a high-order integration scheme for all the interpolation nodes to correct the integration results of the low-order integration, as a result, the final integration converges with high precision by the iterative correction[4]. Theoretical research claims that the SDC integration algorithm has an arbitrary convergence precision with the capability of large simulation step when solving the ordinary differential equations and it’s been applied widely to the simulations of various practical engineering problems[1], such as the ordinary differential equations as well as the differential algebraic equations.

Our research group applied the SDC integration algorithm in the simulation of the power system dynamics, and obtained a promising performance with good numerical stability and high precision of simulation capability. But we also revealed a problem during the integration of ASM for each interpolation interval that the electric power of generator $P_e$ and bus voltage of network $V$ affect the convergence procedure of the ASM due to the strong nonlinear coupling relations with other variables. Therefore, researches about improving the convergence procedure for ASM should be conducted.
Literature research has summarized properties for the network algebraic equations in the ASM and reported optimized linear solvers, such as UMFPACK[5], MKL, SuperLU[6], SuperNodal, ILU[7], and Krylov methods, to increase the simulation speed of ASM with outstanding performances. But research on how to provide better iterative initial values for the strong nonlinear coupling variables is not related. This paper illustrates the convergence procedure for \( P_e \) and \( V \) during ASM, and then constructs prediction techniques for them based on the Lagrange interpolation, aiming to accelerate the convergence procedure of ASM and to improve the efficiency of SDC integration algorithm.

2. DYNAMIC SIMULATION WITH SDC INTEGRATION ALGORITHM

The mathematical model of the power system dynamic simulation is set of differential and algebraic equations (DAE) with the differential equations describing the dynamics of the equipment and the algebraic ones indicating the network constraints, as showed in (1) with \( x, y, t \) represent algebraic variables, differential variables, and the simulation time[8].

\[
\begin{align*}
\frac{dy}{dt} &= f(t,x,y) \\
g(x,y) &= 0 
\end{align*}
\]  

The dynamic simulation of power system dynamic simulation is applying numeric integration algorithms to solve the DAE equations.

2.1. SDC integration algorithm

Taking the trapezoidal rule as the low-order integration algorithm, we implemented the following differential scheme to solve the differential equations in ASM while applying the SDC integration algorithm to simulate the dynamic of power system.

\[
y_{i+1}^{k+1} = y_i^{k+1} + \frac{\Delta t_i}{2} \left[ f(t_i, y_i^{k+1}) + f(t_{i+1}, y_{i+1}^{k+1}) \right] + l_i^{k+1}(\tilde{y}^{k}) - \frac{\Delta t_i}{2} \left[ f(t_i, y_i^{k}) + f(t_{i+1}, y_{i+1}^{k}) \right] 
\]  

where \( i = 0,1,2,\ldots, n - 1 \) with \( n \) represents the interpolation nodes dividing the simulation step into \( n \) intervals, \( t_i \) is the time for node \( i \), \( \Delta t_i = t_{i+1} - t_i \), \( k \) is the iterative counts, and \( \tilde{y} \) is the numerical calculation of \( y \). \( l_i^{k+1}(\tilde{y}^{k}) \) is calculated by the history value though the spectral integration[1].

It’s clear that there is an additional high-order correction item compared to the traditional differential scheme, making (2) being an integration scheme with high-order truncation error.

2.2. Iterative procedure for ASM

2.2.1. Nonlinearity

The ASM integrates the interval \( \Delta t_i \) by solving the differential equations one by one using (2) while fixing \( x \) variables unchanged, and the solving the network algebraic equations while fixing \( y \) variables unchanged. Iterations are required until all the variables converge to the desired precision. The iteration is actually a kind of fixed point iteration with super-linear convergence [9]. But there are variables converge slower than others due to the strong nonlinear coupling as following.

1) Electric power of generator \( P_e \). As the most important linkage between the electrical dynamics and mechanical dynamics, \( P_e \) is coupled with various variables in the generator system. It’s decided by the bus voltage \( V \) and input current \( I \), and then fed back to rotor motion.

2) Bus voltage of network \( V \). Transformation between d-q coordinates and x-y coordinates is required during the generator interface equation, because the bus voltage in generator is represented in d-q coordinates while the network algebraic equations need them in x-y. The transformation matrix is constructed by trigonometric function of power angle \( \delta \), making \( V \) variables strong nonlinear coupled.

2.2.2. Influence on convergence

Here we present the 3-generator 9-bus system as shown in figure 1 to illustrate the influence of the nonlinear coupling on the procedure of convergence for ASM.
The mathematical model for the generators in the system is 6-order detailed sub-transient model. A three-phase ground fault occurs to the transmission line between B4 and B6 with a ground impedance of 0.1+0.1j at t=1.0s and disappears at t=1.1s. Execute the simulation on the platform of MATLAB using the SDC integration algorithm with termination time of 20s.

Figure 1. Structure of 3-gen 9-bus system

Figure 2 shows the dynamic response of electric power for G1. And figure 3 illustrates the convergence procedure for different variables with err-dw, err-cur, and err-pacc representing power angle $\delta$, $V$, and $P_e$, respectively. We can see that $\delta$ converges to the prescribed error $10^{-10}$ quite well at the 4th iteration, while $P_e$ and $V$ converges at the 6th iteration due to a larger iterative initial error.

Seeing that all the variables have a nearly the same error decaying rate, we can accelerate the convergence procedure by constructing a better iterative initial value for them.

3. PREDICTION TECHNIQUES

The iterative initial value adopts the results at the previous step during the traditional simulation, and the error becomes larger under a larger step size. Here we consider the prediction techniques to construct a better iterative initial value using the Lagrange interpolation[10] for $P_e$ and $V$.

3.1. Lagrange interpolation prediction

Assuming function $y(x)$ satisfy the Lipschitz continuity condition, we can construct an interpolation polynomial $\mathcal{L}(x)$ by $n + 1$ interpolation nodes

$$\mathcal{L}(x): = \sum_{j=0}^{n} \ell_j(x) \ast y(x_j)$$  \hspace{1cm} (3)

Where $\ell_0(x), \ell_1(x), \ldots, \ell_n(x)$ are the basis for each nodes $x_0, x_1, \ldots, x_n$ defined by

$$\ell_j(x) = \prod_{0 \leq k \leq n, k \neq j} \frac{x-x_k}{x_j-x_k}$$  \hspace{1cm} (4)

Then (4) has a truncation error of order $n$. 

Generally, $P_e$ and $V$ satisfy the Lipschitz continuity condition except when a fault occurs, therefore, we can construct the Lagrange interpolations for them.

3.2. Prediction for $P_e$

Let the previous k step value noted as $(t_{n-k+1}, P_e(t_{n-k+1}))\ldots, (t_n, P_e(t_n))$ where $t_j$ represents the time for each node, $(n-k+1) \leq j \leq n$, the Lagrange basis is defined by

$$L_j(t) = \prod_{0 \leq m \leq k-1, m \neq j} \frac{t-t_{n-m}}{t_j-t_{n-m}}$$

and the interpolation polynomial for $P_e$ is

$$L^P_e(t) = \sum_{j=(n-k+1)}^n [L_j(t) * P_e(t_j)]$$

therefore, the prediction value for $P_e$ at $t_{n+1}$ is

$$P_e^0(t_{n+1}) = L^P_e(t_{n+1})$$

Equation (7) provides a better iterative initial value for $P_e$ during the practical simulation.

Extract the first 4 steps after fault in figure 2 to test the prediction technique by comparing the predicted value $P_e^0$ with the exact value $P_e(t_{n+1})$ mark by red ‘*’, in figure 3. Set the following 2 kinds of simulation scenarios:

- Lagrange-2: let $k=2$ in (5), namely adopt 2 historical nodes to construct the interpolation, mark the prediction result with ‘+’ in figure 3.
- Lagrange-3: let $k=3$ in (5), namely adopt 3 historical nodes to construct the interpolation, mark the prediction result with green ‘o’ in figure 3.

![Figure 4. Prediction for $P_e$ with Lagrange Quadrature](image)

We have the following observation from figure 4.

1. The initial error is large when $P_e$ changes rapidly for the case of adopting the value of previous step as the iterative initial value (noted as original strategy).
2. The prediction technique of Lagrange-2 strategy doesn’t fit well and even worse than original strategy when the $P_e$ curve has a strong nonlinearity.
3. The prediction technique of Lagrange-3 strategy has an excellent performance for all steps because it’s actually a quadratic fitting with a higher truncation error of order 2, ensuring the prediction of a relatively high precision.

3.3. Prediction for $V$

Apply the similar procedure to construct the interpolation prediction for bus voltage $V_x, V_y$, the corresponding interpolation polynomials are

$$L^{V_x}(t) = \sum_{j=(n-k+1)}^n [L_j(t) * V_x(t_j)]$$

$$L^{V_y}(t) = \sum_{j=(n-k+1)}^n [L_j(t) * V_y(t_j)]$$

And the prediction values for $V_x, V_y$ at $t_{n+1}$ becomes

$$V_x^0(t_{n+1}) = L^{V_x}(t_{n+1})$$

$$V_y^0(t_{n+1}) = L^{V_y}(t_{n+1})$$
Following the configuration and testing method in section 2.2, the prediction curves for $V_x, V_y$ are illustrated in figure 5, separately.

We have the following observation from figure 5.

1. The initial error is large when $V_x$ and $V_y$ change rapidly for the case of adopting the value of previous step as the iterative initial value (noted as original strategy).

2. The prediction technique of Lagrange-2 strategy doesn’t fit well and even worse than original strategy when $V_x$ or $V_y$ curve has a strong nonlinearity.

3. The prediction technique of Lagrange-3 strategy has an excellent performance for all steps because it’s actually a quadratic fitting with a higher truncation error of order 2, ensuring the prediction of a relatively high precision. It’s very clear in figure 5(b) that Lagrange-3 has a much better performance than Lagrange-2.

4. NUMERICAL RESULTS

Applying the constructed iterative initial values for $P_e$ and $V$, we can make a research on the acceleration for the ASM procedure in this section.

Based on figure 3, figure 6 illustrates the convergence procedure for different variables noted as ‘-pred’ after applying the prediction techniques. We can see that the error decaying rate for different variables remains the same, but the initial error during the iteration becomes smaller. As a result, all the variables converge to the prescribed error of $10^{-10}$ at the 5th iteration, making the ASM iteration terminates ahead of time.
applying the prediction for \( P_e \) and \( V \), separately, and ‘P+V’ represents applying the prediction for \( P_e \) and \( V \) simultaneously.

The data suggests that:

1. the prediction for \( P_e \) alone doesn’t have any obvious affection to the iteration of ASM;
2. the prediction for \( V \), \( V \) are very effective that iterations has been decreased by 73 times for the 4 SDC steps which means 22.8% increase for the ASM efficiency.
3. iterations have further decreased when applying both prediction techniques, and the increased efficiency for ASM is raised up to 25.4%.

Table 1. Number of iterations of ASM in one SDC step before prediction and after prediction

| SDC step | Before Prediction | After prediction |
|----------|-------------------|------------------|
|          | \( P_e \)         | \( P \)          | \( V \)          | \( V+P \)         |
| 1        | 65                | 65               | 61               | 61               |
| 2        | 84                | 84               | 66               | 64               |
| 3        | 84                | 84               | 56               | 53               |
| 4        | 80                | 80               | 63               | 59               |
| Summation| 316               | 315              | 243              | 235              |

As a result, we can conclude that the proposed prediction techniques for the strong nonlinear coupled variables \( P_e \) and \( V \) are very effective, it decreases the iterations for ASM and accelerates the simulation efficiency of SDC integration algorithm.

5. CONCLUSION
The paper conducts a research into the simulation efficiency of ASM iteration for SDC integration algorithm. During the ASM iteration, \( P_e \) and \( V \) have relatively larger iterative initial errors than others because of their strong nonlinear coupling with other variables, which makes ASM iteration a slow procedure. The paper illustrates the convergence procedure for different variables during simulation for the standard 3-generator and 9-bus system using SDC integration algorithm, and then proposes prediction techniques to construct better iterative initial values with smaller error for \( P_e \) and \( V \) based on the Lagrange interpolation. Numerical results show that the prediction techniques can accelerate the simulation efficiency effectively by decreasing the iterations of ASM. The proposed method has practical applications and can also be applied to other integration algorithms when the ASM is adopted.

References
[1] Dutt A, Greengard L, Rokhlin V. Spectral Deferred Correction Methods for Ordinary Differential Equations. Bit Numerical Mathematics, 2000, 40(2):241-266. differential equations." BIT Numerical Mathematics 40.2 (2000): 241-266.
[2] Xinli, Song, et al. "New mixed integral algorithm for unified dynamic power system simulations of transient, medium-term and long-term stabilities." Power Systems Conference and Exposition (PSCC), 2011 IEEE/PES. IEEE, 2011.
[3] Gottlieb, David, and Steven A. Orszag. Numerical analysis of spectral methods: theory and applications. Society for Industrial and Applied Mathematics, 1977.
[4] Jingfang, Huang, Jia, Jun, and Minion Michael. "Accelerating the convergence of spectral deferred correction methods." Journal of Computational Physics 214.2 (2006): 633-656.
[5] Khaitan, Siddhartha Kumar, James D. McCalley, and Qiming Chen. "Multifrontal solver for online power system time-domain simulation." IEEE Transactions on Power Systems 23.4 (2008): 1727-1737.
[6] Li, Xiaoye S., and James W. Demmel. "SuperLU_DIST: A scalable distributed-memory sparse direct solver for unsymmetric linear systems." ACM Transactions on Mathematical Software (TOMS) 29.2 (2003): 110-140.
[7] Chuan Fu. High-speed extended-term time-domain simulation for online cascading analysis of power system. Iowa State University, 2011.

[8] P. Kundur, N. J. Balu, and M. G. Lauby, Power system stability and control. Vol. 7. New York: McGraw-hill, 1994.

[9] Machowski J, Bialek J W, Bumby J R. Power System Dynamics. Stability and Control. West Sussex, UK: John Wiley, 2006.

[10] Crow, Mariesa L. Computational methods for electric power systems. CRC Press, 2015.