Modelling and control of a nonlinear magnetostrictive actuator system

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Abstract. This paper explores the implementation of a feedforward control method to a nonlinear control system, in particular, Magnetostrictive Actuators (MA) that has excellent properties of energy conversion between the mechanical and magnetic form through magnetostriction effects which could be used in actuating and sensing application. MA is known to exhibit hysteresis behaviour and it is rate dependent (the level of hysteresis depends closely on the rate of input excitation frequency). This is, nonetheless, an undesirable behaviour and has to be eliminated in realising high precision application. The MA is modelled by a phenomenological modelling approach via Prandtl-Ishlinskii (P-I) operator to characterise the hysteresis nonlinearities. A feedforward control strategy is designed and implemented to linearize and eliminate the hysteresis by model inversion. The results show that the P-I operator has the capability to model the hysteretic nonlinearity of MA with an acceptable accuracy. Furthermore, the proposed control scheme has demonstrated to be effective in providing superior trajectory tracking.

1. Introduction

The Magnetostrictive Actuator (MA) is a solid state magnetic actuator which requires a magnetic field to be driven. It is quite similar to a typical DC motor as it consists of magnets and coils. Nonetheless, it does not need as much power as the conventional DC motor whilst its mechanical strain is typically low. In the recent years, the modelling and control of hysteresis in smart materials have gained considerable attention due to its high precision and nanoscale accuracy. This type of actuator is of particular interest in robotics and new emerging applications that necessitates precision.

Although MA has its inherent advantages namely compact in size, excellent bandwidth (operating frequency) and high stiffness, it also displays a strong hysteresis behavior [1]. Magnetic hysteresis is a condition where there exists a lack of detection property during magnetisation and zero magnetisation process [2]. The presence of hysteresis nonlinearities in such smart materials may leads to uncertainties in its modeling which in turn may also lead to unstable and low performances of the controlled plant. Therefore, it is worth to note that the reduction and elimination the hysteretic behavior is non-trivial in order to make full use of the MA in high precision applications. Typically, there are two approaches of hysteresis modeling i.e. differential based and operator based methods.
Operator based approach is gaining popularity due to its simplicity where it is based on the summation of superposition of elementary relays [3,4]. Moreover, to address the issue of modelling unknown parameters particularly the hysteresis phenomenon, parameter identification or system identification appears to be promising candidate [5,6].

Hitherto, different means of modelling and control of MA have been investigated. Tan et al. utilised a Preisach operator to model the hysteresis of an MA whilst employing inverse compensation algorithms and a robust control framework [3]. The Prandtl-Ishlinskii (P-I) operator has been utilized in modelling the hysteresis of a piezo-actuated nano-positioner [7]. A feed forward compensator and sliding mode controller was used to reduce the hysteresis effect and the inversion error, respectively. The present study aims at adopting the Prandtl-Ishlinskii (P-I) operator as the hysteresis model and to explore feed forward control strategy namely hysteresis linearization and model inversion in compensating of the inherent hysteresis effect in MA.

2. Theoretical Background
The modeling and control an unconventional actuator is often a challenging task, due to its intrinsic hysteresis behaviour. In the present study, a classical Prandtl-Ishlinskii (P-I) hysteresis operator is used to model the MA. This model is selected as it has been demonstrated to characterise hysteretic nonlinearities reasonably well and it is a class of linear model that has invertible properties which is desirable for feedforward control design.

The input-output relationship of the MA based on P-I hysteresis operator is given as

\[ y(t) = c_0u(t) + \int_0^t p(r)F_r[u](t)dr \]  

with \( u(t) \) is the input function and \( y(t) \) is the output response, while \( c_0 \) and \( p(r) \) are the constants as such that \( p(r), c_0 > 0 \). While \( F_r[u](t) \) is the play operator where the initial condition is given as

\[ F_r[u](0) = \max\{u(0)-r, \min\{u(0)+r,0\}\} \]  

and for \( \forall t > 0 \)

\[ F_r[u](t) = \max\{u(t)-r, \min\{u(t)+r,F_r[u](0)\}\} \]  

where \( r_i = \frac{i}{n+1}\|u(k)\|_e \) being the threshold vector where its dimension is the same as \( p(r) \). In discrete form, the above input-output relationship may be rewritten as follows

\[ y(k) = c_0u(k) + \sum_{i=1}^n p_iF_{r_i}[u](k) \]  

The matrix form of the relationship is given as

\[
\begin{bmatrix}
  y_1 \\
  \vdots \\
  y_k
\end{bmatrix} =
\begin{bmatrix}
  u_{11} & F_{r_{12}} & \ldots & F_{r_{1n}} \\
  \vdots & \ddots & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \vdots \\
  u_{k1} & F_{r_{k2}} & \ldots & F_{r_{kn}}
\end{bmatrix}
\begin{bmatrix}
  c_0 \\
  \vdots \\
  p_n
\end{bmatrix}
\]  

From equation (1) to (5), it is apparent that the output response depends on the proper selection of number operators \((n)\) and the corresponding constants \(c_0\) and \( p(r) = [p_1, p_2, \ldots, p_n] \). Accordingly, there is a need for parameterisation or identification of the system in order to accurately estimate each of the parameter value. The following section discusses the identification method.
3. Parameter identification
The identification of the unknown parameters of the MA’s output response is basically attained using least square estimation. For a linear system, the minimisation is quite simple and fairly straightforward. However, for a nonlinear system the minimisation is often carried out via iterative numerical algorithms [6]. The performance index, \( J \) utilised in the study is the minimization of the squared error i.e.

\[
J = (y - \hat{y})^2
\]  

where \( y \) is the measured output and the \( \hat{y} \) is the estimated output.

4. Controller design
The simplest approach for hysteresis linearity is the feedforward control and it entails the manipulation of the mathematical model of the actuator itself, mainly the model inversion [8]. However, this scheme is very sensitive, i.e. any changes sensed in the model imposes a strong variation to its inversion (directly depend upon the accuracy of the model). Thus, knowledge of the model is therefore essential to ensure a successful implementation of the feedforward control strategy.

\[ u \rightarrow \Gamma^{-1} \rightarrow x \rightarrow \Gamma \rightarrow y \]

**Inverse controller**

**Hysteric nonlinearity**

\[ u \rightarrow \Gamma^{-1} \rightarrow x \rightarrow \Gamma \rightarrow y \]

**Figure 1.** Compensation of the hysteretic nonlinearity by an inverse feedforward controller [9].

The inversion of the P-I model can be derived based on the following relationship shown in figure 1 and through equations (7) to (11) where \( F_s[u](t) \), \( d_0 \), \( s \), and \( q \) are the inverse play operator, reciprocal of \( c_0 \), inverse threshold and weight, respectively. As shown in figure 1, the inverse controller is introduced prior to the plant (modelled by a hysteresis operator) to instantaneously cancel the hysteretic nonlinearity effect. The inverse controller acts as if the reciprocal of the forward P-I operator which maps input, \( u \) to intermediate output, \( x \) before being fed into the plant and later to produce the linearised output, \( y \) (at this point \( y \) essentially equals to \( u \)).

\[
x(k) = \Gamma^{-1}[u](t) = d_0 u(k) + \sum_{i=1}^{10} q_i F_s[u](k)
\]  

(7)

\[
F_s[u](0) = \max \{ u(0) - s, \min \{ u(0) + s, 0 \} \}
\]  

(8)

and for \( \forall t > 0 \)

\[
F_s[u](t) = \max \{ u(t) - s, \min \{ u(t) + s, F_s[u](0) \} \}
\]  

(9)

\[
s_j = \sum_{i=1}^{j-1} p_i (r_j - r_i)
\]  

(10)

\[
q_j = \frac{p_j}{d_0 + \sum_{j=1}^{i} p_j}
\]  

(11)
Table 1. Parameters for P-I model and its corresponding inversion.

| P-I Model | Inversion of P-I Model |
|-----------|------------------------|
| $c_0 = 1.3169$ | $d_0 = 0.7593$ |
| $p_1 = 1.0679$ | $r_1 = 0.0909$ |
| $q_1 = -0.3406$ | $r_{el} = 0.1197$ |
| $p_2 = 1.0087$ | $r_2 = 0.1818$ |
| $q_2 = -0.1244$ | $r_{e2} = 0.3368$ |
| $p_3 = 1.2515$ | $r_3 = 0.2727$ |
| $q_3 = -0.0793$ | $r_{e3} = 0.6456$ |
| $p_4 = 1.1736$ | $r_4 = 0.3636$ |
| $q_4 = -0.0434$ | $r_{e4} = 1.0682$ |
| $p_5 = 0.0100$ | $r_5 = 0.4545$ |
| $q_5 = -0.0003$ | $r_{e5} = 1.5974$ |
| $p_6 = 2.1807$ | $r_6 = 0.5455$ |
| $q_6 = -0.0467$ | $r_{e6} = 2.1276$ |
| $p_7 = 0.6657$ | $r_7 = 0.6364$ |
| $q_7 = -0.0096$ | $r_{e7} = 2.8560$ |
| $p_8 = 0.5863$ | $r_8 = 0.7273$ |
| $q_8 = -0.0073$ | $r_{e8} = 3.6449$ |
| $p_9 = 1.0490$ | $r_9 = 0.8182$ |
| $q_9 = -0.0110$ | $r_{e9} = 4.4871$ |
| $p_{10} = 1.1773$ | $r_{10} = 0.9091$ |
| $q_{10} = -0.0099$ | $r_{e10} = 5.4247$ |

5. Results and discussion

Figure 2 shows the precision of the P-I model (with identified parameters) compared to the measured experimental data excited at a frequency 1 Hz and 10 Hz using aforementioned estimation method. It is apparent that the classical P-I operator has the capability to model the hysteresis with reasonable accuracy. These results were based on 10 number of play operators (see equation (4)). The corresponding identified parameters are tabulated in table 1.

Figure 2. Hysteresis curve plot of MA experimental and simulation data at 1 Hz (left) and 10 Hz (right).

Figure 3 and 4 illustrate and demonstrate the ability of the proposed hysteresis operator, viz. Prandtl-Ishlinskii model and its corresponding inversion. The theoretical results expressively show hysteretic behaviour could be characterised by the aforementioned model, and the observation further suggests that the hysteresis curve (traced by blue line in figure 3 and 4) is somewhat symmetric in nature.
Figure 3. Plot of the hysteresis curve, inversion curve and linearized output based on classical P-I model.

Figure 4. The plot input, P-I (output, inversion) (left) and linearized output (right) with sampling time of 1ms.

It could be observed that the implemented linearisation technique provides exceptional results as shown in figure 4. It could be clearly seen that the linearised output and target (input voltage and output is thought as linear when the hysteresis is eliminated) perfectly overlays on each other, suggesting that the error between them are null (in reality this is not achievable).

Figure 5. Plot of measured input, output and simulation output at 1 Hz (left) and 10 Hz (right).

Further illustration on hysteresis characterisation is given in figure 5. Figure 5 provides measured output plotted together with the estimated output using P-I. In the figure, it can be clearly noticed the
discrepancies between the reference output and the estimated output. These incongruities may be due to asymmetric inherent saturation points of the MA. As a consequence, it appears that the classical P-I model could not to a certain degree precisely characterise the nonlinearity behaviour of the MA. However, the hysteresis compensation through model manipulation is the main interest of the present study and hence, model precision is not of great importance. Furthermore, the deviation between estimated and measured hysteresis curve is not that significant (less than 0.5) as shown in figure 6.

![Figure 6](image-url)  
**Figure 6.** Estimation error between reference hysteresis and estimated ones at 1 Hz (left) and 10 Hz (right).

![Figure 7](image-url)  
**Figure 7.** The behaviour of controlled output in comparison to reference output at 1 Hz (left) and 10 Hz (right).
Figure 8. The plot of error progression for the reference input signals specified in Figure 7.

Figure 7 illustrates the trajectory tacking comparison between the linearised and desired output of the system with the inclusion of the feedforward controller. It is evident through figure 8 (tracking error) that the hysteretic nonlinearity effect is effectively compensated by the proposed controller, suggesting the robustness of the controller even in the presence of model uncertainties.

6. Conclusion
In conclusion, the inherent hysteresis behaviour that exist in MA model is successfully characterised via a phenomenological modeling approach i.e. the classical Prandtl-Ishlinskii (P-I) operator. The unknown parameters of the P-I model are addressed by means of parametric identification via the Least Square Estimation. The estimated parameters were observed to be modelled with a certain degree of accuracy, nonetheless, with the incorporation of the proposed control architecture, the estimation accuracy of the model may be neglected. Owing to the linear properties of the P-I model, a feedforward control through model inversion is implemented. The simulation results show the superior tracking are attained upon the inclusion of the proposed controller.

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