Numerical analysis of inertance pulse tube cryocooler with a modified reservoir

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Abstract. Pulse tube cryocoolers are used for cooling applications, where very high reliability is required as in space applications. These cryocoolers require a buffer volume depending on the temperature to be maintained and cooling load. A miniature single stage coaxial Inertance Pulse Tube Cryocooler is proposed which operates at 80 K to provide a cooling effect of at least 2 W. In this paper a pulse tube cryocooler, with modified reservoir is suggested, where the reverse fluctuation in compressor case is used instead of a steady pressure in the reservoir to bring about the desired phase shift between the pressure and the mass flow rate in the cold heat exchanger. Therefore, the large reservoir of the cryocooler is replaced by the crank volume of the hermetically sealed linear compressor, and hence the cryocooler is simplified and compact in size. The components of the cryocooler consist of a connecting tube, aftercooler, regenerator, cold heat exchanger, flow straightener, pulse tube, warm heat exchanger, inertance tube and the modified reservoir along with the losses were designed and analyzed. Each part of the cryocooler was analysed using SAGE v11 and verified with ANSYS Fluent. The simulation results clearly show that there is 50% reduction in the reservoir volume for the modified Inertance pulse tube cryocooler.

1. Introduction

Cryogenic coolers are required for a wide range of applications such as cooling infrared sensors for improved resolutions, cooling of superconducting electronics for advanced digital filters, night vision surveillance and missile detection. The demand for efficient Cryocoolers has led to perfections in major problem like reliability, efficiency, vibration and heat rejection. The use of high-temperature superconductors has encouraged much research to reduce the cost of cryocooler and to increase lifetime goal by 3-5 years or higher. Stirling type Cryocoolers generally uses oscillatory flows provided by a rotary or linear compressor, regenerative heat exchangers and a cold displacer. The cold displacer puts the gas in acceptable phase with the pressure oscillations, thus separating the cooling and heating effects of the cryocooler cycle. In Gifford-McMahon (GM) and Stirling cycle Cryocoolers, the cold displacer is a piston that moves at cryogenic temperatures. Generally, there is a very large temperature difference across the cold displacer but a relatively small pressure difference. Another kind of Stirling cryocooler, namely the pulse tube refrigerator was first described by Gifford and Longsworth in 1959. Unlike Stirling or Gifford McMahon, Pulse Tube Cryocoolers (PTC) have no moving parts at the cold end. Here displacer is replaced by a pulse tube. The function of pulse tube is to insulate the processes at
its two ends. That is, it must be large enough that gas flowing from the warm end traverses only part way through the pulse tube before flow is reversed. The absence of cold moving parts has allowed it to solve some of the problems associated with Cryocoolers such as vibration, lifetime and reliability.

1.1. Pulse tube cryocoolers
The pulse tube development started with Basic Pulse tube refrigerator (BPTR) typically operated at a pulse rate of few hertz and could reach a temperature of 120 K in a single stage. The modification on basic pulse tube made it suitable for lower cryogenic temperature started with the Orifice type followed by the double inlet orifice type cooler and then the inertance pulse tube cryocooler. This was first reported by Kano [1] and states that the variables can be controlled are flow impedance, the reservoir volume, the charge pressure and the frequency. Detailed analysis was carried out by de Boer [2] using a simple turbulent flow model. He concludes that the inertance Pulse tube is superior to the Orifice type at certain higher frequencies. To achieve higher performance in Pulse Tube with better reliability the key point is the phase shift imparting devices that contributes towards phase shift between mass flow and pressure amplitude, i.e. to improve the effect of pulse tube, inertance tube and reservoir.

1.2. Phase shifters in pulse tube cryocoolers
We are aware of the fact that the cooling effect of the PTC depends on the phase shifting phenomenon. There are different mechanisms for doing so, like, use of inertance along with the orifice as in use of inertance in orifice [3], use of a double inlet pulse tube [4] to adjust the phase shift between the pressure and the mass flow in the pulse tube by the use of an additional bypass orifice between the hot end and the transfer line at inlet. The above are in reference to the modification of phase shifters. In addition to this, there are methods like modifying the compressor pattern like in case of step piston [5], sharing of compression space between two cryocoolers as in case of Tandem type coolers [6], introducing an additional compressor of lower capacity as in case of Active Phase Control [7]. These techniques are adopted to decrease the size constrain of the cryocooler reservoir volume. Wang [8] introduced an improved method to using the crank space volume as the reservoir. Where the volume of reservoir is 300 cc and the compressor backside take up a volume of 850 cc. The current work aims to support that with an appropriate design such that the phase shift could be controlled so as to reduce the capacity of the reservoir when we use backside of compressor as the reservoir.

2. Analysis of modified reservoir IPTC
The objective of present work is to reduce the reservoir capacity by making use of the crank case pressure fluctuations, which is at 180-degree (approximately) phase difference. To check the feasibility, an electrical circuit, SAGE and computational fluid dynamics model is proposed. The design is being done for a cryocooler that give at least 2 W cooling power at 80 K. For the purpose of modeling the additional fluctuation, in the reservoir is assumed to be a pressure source as in case of Electrical Circuit Analogy(ECA) and in CFD. The SAGE model includes conduction and convection losses at regenerator and DC flow circulation between compressor and crank case volume through the piston leakage.

2.1. Electrical circuit analogy
In harmonic approximation, the oscillatory variables are considered to be sinusoidal with the time. As in case of oscillating pressure, (P) is analogous to AC electric voltage, and oscillatory volumetric velocity (U) is analogous to AC electric current. These electric circuits analysed as the same case of complex variable analysis to account for the amplitudes and phases of the
oscillatory variables. The electrical equivalence of tank is \( C = \frac{V}{\gamma P_m} \) where \( V \) is the volume of the tank, \( P_m \) is the mean pressure (i.e. the average pressure), and \( \gamma \) is the ratio of isobaric to isochoric specific heats. As in an electrical circuit, the complex impedance of a compliance is \( Z_c = \frac{l}{j\omega C} \), where \( j = \sqrt{-1} \) and \( \omega = 2\pi f \), \( f \) the frequency of the oscillations. Inertia of the moving gas contributes an inertance(equivalent to inductance) \( L = \rho l/A \) where \( l \) is tube length and \( A \) cross-sectional area [3].

2.1.1. Modified reservoir electric circuit model

The electrical circuit analogy is proposed and a network analysis has been developed was drawn for Inertance Pulse Tube Cryocooler (IPTC) as shown in Figure 1a. and for Modified reservoir Inertance Pulse Tube Cryocooler as in Figure 1b. As every component in the electrical equivalent circuit is analogous to the different mechanical phenomena involved in the present system. The input parameter variation does not alter the characteristics of the system. In the proposed model, an auxiliary pressure input is applied along with the conventional input. This variation is changing the net input to the system and thus the overall mass flow through the modified reservoir. Net input will be the phasor sum of conventional and auxiliary pressure inputs. According to electric circuit theory, elements in the equivalent circuit do not change with the changes in input voltage. This is evident from the fact that \( R, L \) and \( C \) depends only on material property and dimensions of the elements. Similarly, the elements in the equivalent circuit model for modified reservoir type are only related with the material properties and system design. Hence, the impedance characteristics can remain same for the proposed model to that of Inertance pulse tube cryocooler (IPTC). Notations \( P_1 \) and \( P_2 \) indicate the primary and reverse pressure oscillations equivalent to AC electric voltage.

![Figure 1.](image)

**Figure 1.** (a) Circuit diagram for Inertance tube and reservoir volume for IPTC. (b). Circuit diagram for Modified Reservoir IPTC where \( P_2 \) is the reverse fluctuation in reservoir space.

2.1.2. Analysis

To verify the feasibility of a modified reservoir, the network diagram and the simulation of the network with the help of Matlab Simulink will suffice. We have simplified our model to only modeling the inertance tube and reservoir of the modified and the existing type, which was verified using the experimental results [3]. In design, the inertance tube was copper tube with dia. 3.5 mm with a length of 3.147 m length, so that the equivalence is Inductance \( L = 691569306.1\) kg/m\(^4\) and a resistance of \( 1.10622 \times 10^{12}\) Pa-s/m\(^3\). The reservoir tank was a circular cylinder with internal volume \( V = 200\) cc. Accordingly its compliance \( C = 1.2369 \times 10^{-09}\) m\(^4\)sec\(^2\)/kg where as for 500 cc compliance \( C = 3.15289 \times 10^{-09}\) m\(^4\)sec\(^2\)/kg. In order to vary the pressure oscillations inside the compliance tank, an additional voltage source of \(.08\) MPa (pressure source) is provided in series. To study the effect of various pressure variations the
voltage source could be varied. The phase measure of voltage and current measurements are done in order to study the effect of variation in pressure and compliance volume.

2.2. SAGE analysis

The second phase of analysis was done with the help of SAGE, a software by which we can model the root model-component of a Stirling PT cryocooler which contains a number of sub-components representing piston, heat exchangers, Inertance tube, transfer line, reservoir etc. These sub-components may themselves contain sub components. The natural way to organize this in terms of sub components branching of their parent components as trees in computer science parlance. Using SAGE, we can map and optimize the model. In mapping input variables to be automatically stepped over a range of values and problem involving optimized variables, constraints and an objective function were optimized. The operating parameters such as charge pressure, regenerator parameters and frequency of operation of IPTC with modified reservoir are given in Table 1. Table 2 illustrates the dimensions and boundary conditions of heat exchangers, Regenerator, Pulse Tube, inertance tube and reservoir of the modified Cryocooler.

| Parameters                          | Values  |
|-------------------------------------|---------|
| Volume of compressor                | 5 cc    |
| Regenerator                         | SS 304, #400 |
| Connecting tube dia.                | 6 mm    |
| Outer diameter of regenerator       | 25 mm   |
| Charge Pressure                     | 3.0 MPa |
| Operating frequency                 | 50 Hz   |
| CHX temperature                     | 80 K    |
| WHX temperature                     | 300 K   |

Governing equations are:

\[ \text{Continuity} : \frac{\partial pA}{\partial t} + \frac{\partial puA}{\partial x} = 0 \]  
\[ \text{Momentum} : \frac{\partial puA}{\partial t} + \frac{\partial A}{\partial x} + \frac{\partial P}{\partial x}A - FA = 0 \]  
\[ \text{Energy} : \frac{\partial peA}{\partial t} + P \frac{\partial A}{\partial t} + \frac{\partial}{\partial x}(upeA + uPA + q) - Q_w = 0 \]

In the coaxial cryocooler model the radial thermal interaction between the pulse-tube wall and regenerator matrix has been included because the components were arranged end-to-end in series for helium flow. It means that the temperature gradients in the regenerator and pulse tube were in opposite directions and a transverse (radial) thermal connection would have resulted in the cold end of the pulse tube exchanging heat with the warm end of the regenerator and vice-versa. The effect of radial conduction between the regenerator and pulse tube wall does not seem to affect the performance much.

In modified reservoir IPTC we use crank space volume as the reservoir. There are possibilities of the gas flow in the annular region between the reservoir and cylinder liner. These unnecessary flow adds to the parasitic losses which decreases the efficiency of the system as a whole these losses were included in the SAGE model.
2.3. CFD analysis of modified reservoir configuration

The schematic diagram and the analogous circuit of the Pulse tube using Inertance and modified reservoir are presented. The single stage cryocooler consists of after cooler, regenerator, cold heat exchanger(CHX) pulse tube and warm heat exchanger(WHX). These are segmented into individual units for ease of representation. The focus is to reduce the reservoir capacity in terms of efficiency of the system.

![Figure 2. 2D axisymmetric CFD model of the modified miniature coaxial PTC](figure not on scale)

In the modified IPTC the crank space is considered as the reservoir subjected to a pressure fluctuation which is, in turn, a effect of piston movement. But here it is assumed to be an independent pressure source with appropriate phase difference so that the effect of reservoir volume could be studied. In the simulation of the cold finger, the compressor is replaced by a pressure inlet at the after cooler and variation in compressor backspace volume was replaced by a reservoir space and an additional pressure inlet at the end of reservoir. Line diagram of the model is shown in Figure 2. The geometrical dimensions and boundary conditions are described in the Tables 1 and 2.

![Table 2. Dimensions and Boundary conditions.](table)

| Component       | Radius(mm) | Length(mm) | Boundary Conditions       |
|-----------------|------------|------------|---------------------------|
| After cooler    | 12.5       | 5          | Wall temperature 300K     |
| Regenerator     | 12.5       | 74.3       | Adiabatic wall            |
| Cold heat exchanger | 12.5    | 2          | Adiabatic wall            |
| Pulse tube      | 7.5        | 79.3       | Adiabatic wall            |
| Warm heat exchanger | 7.5     | 3          | Wall temperature 300K     |
| Inertance tube  | 1.75       | 3174       | wall temperature 300K     |
| Reservoir       | 35         | 51         | wall temperature 300K     |

2.3.1. Governing Equations

The governing equations for the 2-D axis-symmetric model include mass, momentum and energy conservation equation. For regenerator and heat exchangers, the properties of the solid matrix properties are taken as porous zone. Thus, the mass and momentum conservation equations are rewritten as:

\[
\frac{\partial (\epsilon \rho_f \bar{v})}{\partial t} + \nabla \cdot (\epsilon \rho_f \bar{v}) = 0 \quad (4)
\]

\[
\frac{\partial (\epsilon \rho_f \bar{v})}{\partial t} + \nabla \cdot (\epsilon \rho_f \bar{v}) = -\epsilon \nabla \rho + \Delta (\epsilon \bar{r}) + S_i \quad (5)
\]
where $\rho_f$, $\bar{v}$ and $p$ are the density, the velocity and pressure of the working fluid helium, respectively. $\bar{\tau}$ represents the stress tensor. The porosity and the momentum source term $S_i$ have different values in porous or non-porous zone, respectively. In the gas domain including the pulse tube, the inertertance tubes and the reservoir, porosity $\varepsilon$ is 1, and $S_i$ can be ignored. However, in porous zones, $\varepsilon$ is equal to the porosity of the filling mesh, and $S_i$ is given by:

$$S_i = -\left( \frac{\mu}{\alpha} \bar{v} + \frac{C_2}{2} \rho_f |\bar{v}| \bar{v} \right)$$  \hspace{1cm} (6)

where $\alpha$ is the permeability and $C_2$ is the inertial resistance factor. The energy conservation equation based on the thermal equilibrium mode is given by:

$$\frac{\partial}{\partial t} [\varepsilon \rho_f E_f + (1 - \varepsilon) \rho_s E_s] + \nabla . [\bar{v} (\varepsilon \rho_f E_f + p)] = \nabla . [k_{eff} \nabla T + (\bar{\tau} . \bar{v})]$$  \hspace{1cm} (7)

where $k_{eff}$ is the effective thermal conductivity in the porous medium, $\rho_s$ and $E_s$ are the density and total energy of the solid. The thermal equilibrium mode assumes that there is no temperature difference between the fluid and the solid matrix in the porous zone, and thus the imperfect heat transfer loss between them is ignored and the effectiveness is 1.

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**Figure 3.** 2D axisymmetric mesh for co-axial head

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**Table 3.** Mesh Parameters

| Component     | Material | $m$ | $d_w$ | $\epsilon$ | $\alpha$ ($m^2$) | $C_2$   |
|---------------|----------|-----|-------|-------------|------------------|---------|
| Regenerator   | SS304    | 400 | 25.4  | 0.686       | 55.5             | $4.8 \times 10^{-11}$ | 34189   |
| Aftercooler   | copper   | 100 | 100   | 0.691       | 222.6            | $7.7 \times 10^{-10}$ | 9234    |
| CHX           | copper   | 100 | 100   | 0.691       | 222.6            | $7.7 \times 10^{-10}$ | 6905    |
| WHX           | copper   | 100 | 100   | 0.691       | 222.6            | $7.7 \times 10^{-10}$ | 7883    |

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**2.3.2. Solution Approaches** The CFD model uses a first order implicit unsteady pressure based segregated solver with SIMPLE Pressure – Velocity coupling. The pressure discretization employs the standard method while the second order up winding is used for density, momentum and energy terms. The convergence criterion is $10^{-3}$ for continuity and velocity and $10^{-6}$ for energy. A time step of $4 \times 10^{-4}$ s and 20 maximum iterations per time step are set for cases. The number of grid nodes is up to 13946 nodes and the simulation using ANSYS Fluent is in progress and verification of the design is to be done.
3. Simulation of modified reservoir IPTC

The cryocooler model is based as per the Tables 1 and 2 on a single stage coaxial inertance PTC is modified to improve the compactness of by using the backspace of hermetically sealed linear compressor as the reservoir. This acts as an active phase shifting device. The dimensional parameter determined are the regenerator length, inertance tube length and reservoir volume. Figure 4 shows the phase difference at the cold heat exchanger (CHX). It was found to be 49 degree for the proposed model.

![Figure 4. Phasor representation of the Cryocooler](image1)

| Component       | Phase Shift |
|-----------------|-------------|
| Compressor      | lead by 38° |
| WHX             | lead by 27° |
| Regenerator     | lag by 40°  |
| CHX             | lag by 49°  |

![Table 4. Phase shift at components](image2)

Figure 5 shows the phase difference at compressor and CHX it was found to be 38° and -49° respectively. It is to be noted that the average mass flow rate will be zero for an oscillatory wave so the mean mass flow rate was calculated as amplitude of mass divided by √2. Figure 6 shows the simulation of the variations of COP and the cooling capacity at 80 K with the variation of the reservoir volume it was found that on increasing the modified volume from 50 cc there is a steady increase upto 300 cc after that the cooling power seems to be constant. The reservoir volume is taken as 200 cc for the modified reservoir.

![Figure 5. Phase difference of mass and pressure at various location of the cooler](image3)

![Figure 6. Variations of COP/COPcarnot and cooling capacity with Reservoir volume.](image4)

The values were comparable with respect to the phase obtained by electrical circuit analogy as in Figure 4 and Table 4. Figure 5 shows the phase obtained by electrical circuit analogy as in Figure 4 and Table 4. Figure 5 shows the phase obtained by electrical circuit analogy as in Figure 4 and Table 4. Figure 5 shows the phase obtained by electrical circuit analogy as in Figure 4 and Table 4. Figure 5 shows the phase obtained by electrical circuit analogy as in Figure 4 and Table 4. Figure 5 shows the phase obtained by electrical circuit analogy as in Figure 4 and Table 4.
The single segment Inertance tube has been optimized against the cooling capacity and to obtain the basic phase shift. Figure 7 shows the variation of length of Inertance tube to the cooling capacity obtained for a reservoir capacity of 200 cc modified Inertance pulse tube. The inner diameter of Inertance tube is 3.5 mm, the optimal length was found to be 3.147 m.

The simulation results of the variation of COP and the cooling capacity at 80 K with varying the regenerator length the curve change in different with the length is shown in Figure 8. The goal is to find an acceptable point for both and thus 74.3 mm was found to be optimal with reference all other parameter concerned. The pulse tube length was constrained as the sum of length of regenerator and the two heat exchangers with a length of 79.3 mm.

Based on the above parameters, the simulation results of the net cooling capacity to the cold heat exchanger temperature. A net cooling power of 2.43 W at 80 K with an input power of 100 W is obtained. As per the simulation results, a relative Carnot efficiency of 9.13% is obtained at 80 K.

4. Conclusion
The simulation results and ECA analysis shows that Inertance pulse tube cryocooler with modified reservoir can work as efficient as an inertance pulse tube cryocooler. The verification and validation using CFD and experimentations are to be completed for more detailed analysis. Using the crankcase volume as the Inertance tube buffer volume creates the potential for DC flow circulation (through the piston leakage component), which can reduce cooling power. However, SAGE captures the DC flow effect, which we can observe by increasing the seal-leakage Gap and noting the reduced cooling power. DC flow effects are difficult to model correctly because relatively small changes of temperature or asymmetric piston seal.

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