Coulomb excitation of the $|T_2| = \frac{1}{2}$, $A = 23$ mirror pair

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Background: Electric-quadrupole (E2) strengths relate to the underlying quadrupole deformation of a nucleus and present a challenge for many nuclear theories. Mirror nuclei in the vicinity of the line of $N = Z$ represent a convenient laboratory for testing deficiencies in such models, making use of the isospin-symmetry of the systems.

Purpose: Uncertainties associated with literature E2 strengths in $^{23}$Mg are some of the largest in $T_z = \frac{1}{2}$ nuclei in the sd-shell. The purpose of the present work is to improve the precision with which these values are known, to enable better comparison with theoretical models.

Methods: Coulomb-excitation measurements of $^{23}$Mg and $^{23}$Na were performed at the TRIUMF-ISAC facility using the TIGRESS spectrometer. They were used to determine the $E2$ matrix elements of mixed $E2/M1$ transitions.

Results: Reduced $E2$ transition strengths, $B(E2)$, were extracted for $^{23}$Mg and $^{23}$Na. Their precision was improved by factors of approximately six for both isotopes, while agreeing within uncertainties with previous measurements.

Conclusions: A comparison was made with both shell-model and $ab$ initio valence-space in-medium similarity renormalization group calculations. Valence-space in-medium similarity-renormalization-group calculations were found to underpredict the absolute $E2$ strength - in agreement with previous studies.

I. INTRODUCTION

Electric-quadrupole (E2) transitions strengths are a powerful probe of nuclear structure, relating directly to the underlying quadrupole deformation of the nucleus. Simultaneously, they present a challenge to valence-space based theoretical models, with significant contributions to E2 strength arising from particle-hole excitations out of the model space. In the vicinity of the line of $N = Z$, mirror nuclei (nuclei with inverted numbers of protons and neutrons) are an excellent laboratory for nuclear physics, with isospin symmetry enforcing analogous structures for both nuclei. Studies of transition strengths in isobaric analogue transitions have been employed for a huge range of nuclei, from low-mass systems such as $^7$Be and $^7$Li [1], through the $f_{7/2}$ shell (e.g. Ref. [2]), and extending into the upper-$fp$ and $g_{9/2}$ shell model spaces (e.g. Ref. [3]).

Within the $sd$-shell, one is able to compare modern $ab$ initio techniques such as the valence-space in-medium similarity renormalization group (VS-IMSRG) to calculations utilising exceptionally successful empirical shell-model interactions such as the USDB [1]. Systematic studies of deficiencies in such models require, however, high-quality experimental data. In this work, we build on our previous studies of $^{22}$Mg [3] and $^{21}$Na [4] by presenting an improved experimental measurement of the low-lying $E2$ strength in the $|T_2| = \frac{1}{2}$, $A = 23$ mirror pair, $^{23}$Mg and $^{23}$Na. Prior to the present work, the $B(E2)$ value between the ground and first-excited state in $^{23}$Mg [3, 4] was the most imprecisely measured of all
900 of 42.9 MeV, while the \( ^{23} \text{Na} \) beam provided by OLIS \cite{13} was used to determine the surface ionized contamination originating from \( ^{23} \text{Na} \). Shown in the inset is the total time structure arising from the laser-ionization in the present measurement with the fitted area indicated.

\[ T_z = -\frac{1}{4}, \text{sd-shell nuclei} \]  
A detailed systematic study, comparing VS-IMSRG and shell-model calculations to the available data within the sd-shell is the subject of a separate publication \cite{10}.

The precision to which \( E^2 \) strengths are determined in odd-mass sd-shell nuclei is often limited by the fact that decays are of a mixed \( E^2/M1 \) nature. When the decay is dominated by \( M1 \) strength, as is the case in \( ^{23} \text{Mg} \) and \( ^{23} \text{Na} \), the leading uncertainty in determining the \( E^2 \) strength is typically the mixing ratio \( \delta \) between \( E^2 \) and \( M1 \) contributions determined, for example, from the angular correlations between emitted \( \gamma \) rays. By performing a Coulomb excitation measurement, rather than determining the \( E^2 \) strength from the decay properties, this source of uncertainty can be largely eliminated, allowing for a higher level of precision.

II. EXPERIMENTAL DETAILS

\( ^{23} \text{Mg} \) and \( ^{23} \text{Na} \) were investigated through Coulomb excitation using the TIGRESS facility \cite{11} at TRIUMF ISAC. \( ^{23} \text{Mg} \) nuclei were produced by the impinging of 480-MeV protons onto a SiC ISAC target. The Mg atoms produced were then selectively laser ionized using three step resonant excitation (285.3 nm-880.8 nm-291.6 nm) into an auto-ionizing state and extracted. \( ^{23} \text{Na} \) contamination was suppressed by the use of the ion-guide laser ionization source (IG-LIS) \cite{12}. A repeller plate is held at 40 V to suppress the extraction of surface-ionized contaminants by factors of up to \( 10^6 \). \( ^{23} \text{Na} \) ions were produced by the surface ion source of the TRIUMF offline ion source (OLIS) \cite{13}. The beams were then accelerated by the TRIUMF ISAC accelerator chain and delivered to TIGRESS. The \( ^{23} \text{Mg} + ^{23} \text{Na} \) cocktail beam had an energy of 42.9 MeV, while the \( ^{23} \text{Na} \) beam provided by OLIS was provided at energies of both 42.9 MeV and 39.4 MeV. The total beam intensity for the \( ^{23} \text{Mg} \) portion of the experiment was maintained at roughly \( 3 \cdot 10^5 \) particles per second - this includes a component from the remaining \( ^{23} \text{Na} \) contamination. The \( ^{23} \text{Na} \) beam intensity was maintained at approximately \( 6 \cdot 10^7 \) particles per second. The beams were then impinged onto a 0.44-mg/cm\(^2\) thick, \text{nat} Ti target at the center of the TIGRESS array. Scattered beam and target-like nuclei were detected in an S3-type \cite{14} silicon detector, mounted 31-mm downstream of the target position. Gamma rays were detected using the TIGRESS array, which for the present measurement comprised fourteen clover-type HPGe detectors. The HPGe detectors were operated in their withdrawn configuration, with the face of the detectors 14.5 cm from the target and the BGO suppression shields forward, providing the best possible peak-to-background ratio and Doppler-correction.

While the use of IG-LIS heavily suppresses extraction of \( ^{23} \text{Na} \), a degree of contamination remains which was monitored in two ways. First, a Bragg detector was used to provide an instantaneous measure of the beam composition. While the composition is being determined in this way experimental data cannot be acquired. For the second method, the 10 kHz signal used to synchronize the laser ionization system was used, with every second pulse triggering the generation of a ramping waveform, which could then be digitized. The amplitude of the digitized waveform thereby gave a proxy for the time of the detection relative to the laser-ionization pulse and could thus be used to distinguish laser-ionized beam components which had a 10 kHz pulsed structure from the continuous surface ionized contaminants. This method allowed for a continuous determination of contamination, allowing to monitor for sudden changes in the ISAC target behavior. Based on these analyses, the \( ^{23} \text{Na} \) contribution to the beam cocktail was determined to be 15.2(9) % of the total, with the uncertainty being predominantly systematic and arising from the choice of fitting region. Figure \ref{fig1} shows the laser timing distribution, the tail of which was fit with an exponential and baseline to determine the relative contributions to the beam cocktail.

III. ANALYSIS

The data were unpacked using the GRSISort \cite{15} software package, built in a ROOT \cite{16} framework. Gamma-ray events were Doppler corrected event-by-event on the basis of the beam and target kinematics determined from the hit location in the annular silicon detectors and whether the detected particle had beam-like or target-like kinematic properties. Gamma-ray spectra for \( ^{23} \text{Na} \) at 39.4 MeV, and the \( ^{23} \text{Mg} + ^{23} \text{Na} \) cocktail beam are shown in Fig. 2 and Fig. 3 respectively. Relative \( \gamma \)-ray detection efficiencies for TIGRESS were determined using a standard suite of \(^{152} \text{Eu} \), \(^{133} \text{Ba} \) and \(^{60} \text{Co} \) sources. \( ^{23} \text{Na} \) data were split into forty-eight groups: twelve angular bins for both beam-like and target-like detection, repeated
FIG. 2. Doppler-corrected γ-ray spectra on the basis of $^{23}$Na (red) and $^{48}$Ti (black) kinematics for a $^{23}$Na beam energy of 39.4 MeV. Top: Detection of a target-like recoil ($^{48}$Ti) in the downstream annular silicon detector. Bottom: Detection of a beam-like recoil ($^{23}$Na) in the downstream annular silicon detector. The additional width of the $^{23}$Na peak in the top figure arises from the wide angles at which the scattering occurs, leading to significant slowing in the target material. Other lines in the titanium corrected (black) spectra arise from isotopes of titanium with a lower natural abundance than $^{48}$Ti (73.8%).

FIG. 3. As Fig. 2 but for a cocktail $^{23}$Mg ($\approx 85\%$) and $^{23}$Na ($\approx 15\%$) beam at an energy of 42.9 MeV.

FIG. 4. Fit of the γ-ray peaks observed in TIGRESS corresponding to the de-excitation of the first-excited state in $^{23}$Mg and the analogue state in the stable contaminant and mirror nucleus, $^{23}$Na. These data were coincident with events from the first four rings of the downstream annular silicon detector, corresponding to angles of $19.5^\circ \rightarrow 25.8^\circ$. This fitting method can be used for all cases where the beam-like particle was detected. See the text for details of the analysis for target-like particle detection.
FIG. 5. Low-lying levels in $^{23}\text{Mg}$ and $^{23}\text{Na}$ relevant to the present analysis. The $5/2^+ \rightarrow 3/2^+$ transition (red) was investigated and other transitions were included within the GOSIA analysis. Gray transitions indicate mixed $E2/M1$. Data taken from Ref. [9].

FIG. 6. $\chi^2$ surface resulting from the GOSIA2 analysis of $^{23}\text{Mg}$ from which transition and diagonal matrix-elements were extracted.

FIG. 7. $\chi^2$ surface resulting from the GOSIA2 analysis of $^{23}\text{Na}$ from which transition and diagonal matrix-elements were extracted.

Extracted matrix elements are summarized in Tab. I, along with other properties derived from the present results. We compare the present results with those calculated from two theoretical models. VS-IMSRG calculations were performed using the EM1.8/2.0 interaction [19, 20], which was generated by SRG evolution [21] of the chiral N3LO NN interaction of Entem and Machleidt [22], and adding a non-locally regulated N2LO 3N interaction with the low energy constants adjusted to reproduce the triton binding energy and the $^4\text{He}$ matter radius. Calculations are performed in a harmonic oscillator basis of $\hbar \omega = 20 \text{ MeV}$ with $2n + \ell \leq e_{\text{max}} = 12$ and with a truncation on the three body matrix elements $e_1 + e_2 + e_3 \leq E_{3\text{max}} = 16$. All operators are truncated at the normal-ordered two-body level. A diagonalization was then performed using the NuShellX [23] code. Shell-model calculations were also performed in NuShellX, making use of the USDB interaction [4] with effective charges of $e_\pi = 1.36$ and $e_\nu = 0.45$.

Table II shows the present results compared to those calculated using the aforementioned models. The shell-model (USDB) calculations well reproduce the observed $B(E2)$ values. VS-IMSRG values, meanwhile, are considerably lower than the experimentally determined ones. This deficiency is consistent with that observed in our previous studies of $|T_z| = 1$ mirror pairs [5]. While the VS-IMSRG values are deficient, it should be noted that the relative $B(E2)$ strengths are better reproduced by the $ab$ initio calculations. Defining the ratio...
TABLE I. $B(E2)$ values, spectroscopic quadrupole moments and mixing ratios deduced from the present work with statistical and systematic uncertainties quoted, in that order. Where available, literature values are shown for comparison. Mixing ratios were deduced one the basis of the literature lifetimes and the presently determined $B(E2)$ values.

| Isotope | $J_i^P$ | $J_f^P$ | This Work | Ref. |
|---------|---------|---------|-----------|-----|
| $^{23}\text{Na}$ | $\frac{1}{2}^+$ | $\frac{1}{2}^+$ | $0.252 \pm 0.003 \pm 0.004$ | 0.237$^{+0.014}_{-0.015}$ |
| $B(E2; \frac{1}{2}^+ \to \frac{3}{2}^+)$ e$^2$fm$^4$ | 106 $\pm$ 3 $\pm$ 3 | 93 $\pm$ 12 |
| $^{23}\text{Mg}$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $0.285 \pm 0.015 \pm 0.004$ | 0.23$^{+0.07}_{-0.10}$ |
| $B(E2; \frac{1}{2}^+ \to \frac{3}{2}^+)$ e$^2$fm$^4$ | 135$^{+15}_{-14}$ $\pm$ 4 | 86 $\pm$ 58 |
| $Q_{\lambda}(\frac{1}{2}^+)$ e | $-0.22^{+0.28}_{-0.22}$ $\pm$ 0.04 | $-0.22^{+0.25}_{-0.22}$ $\pm$ 0.04 |
| $\delta_{E2/M1}$ | 0.0038 $\pm$ 0.0004 | 0.0034$^{+0.0004}_{-0.0003}$ |

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Fig. 5 was created using the SciDraw scientific figure preparation system [21]. The codes imsrg++ [25] and nutbar [26] used in this work make use of the Armadillo library [27].

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