Modeling the formation of acoustic resonant waves in a closed space

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Abstract. An analysis is made and a method for calculating resonant acoustic waves, which allows determining the effect of sound pressure at any point in a room with different finishes for enclosing structures, is developed. The method allows taking into account the type of finishing material for vertical and horizontal structures.

1. Introduction (problem statement)
Modern cities are full of different sounds, which vary from the rustling of leaves to the sound of an airplane taking off. Traffic noise, household noise, audio and video advertising, people talking, special machinery and special signals are the main components of acoustic pollution in urban areas. The noise indicators often exceed the standard permissible values, which adversely affects the health of the population [1,2]. Noise from city streets enters the building. There are institutions in which the norms provide for the ventilation of individual rooms at a certain time. Through the open windows, the noise wave enters the rooms and then it turns into a resonant wave after being reflected from the walls, increasing its impact on the hearing organs.

Acoustic phenomena on the territory of urban development and within it are described using complex mathematical models, the basis of which are the equations for the propagation of sound waves in free / closed space. The behavior of sound waves in the room is determined by: the initial characteristics of the noise source, the geometric parameters of the room, the acoustic characteristics of the enclosing structures and the presence / absence in the room of objects that scatter the sound wave. This leads to the need to develop an appropriate computing apparatus for analyzing and modeling the formation of resonant waves inside a closed room, taking into account the finishing materials used.

2. Literature review
The modern spectrum of ways to study the behavior of acoustic sound waves is wide. In relation to the aspect of technospheric safety, sound waves are considered in the structure of the generated city noise. Shubin I.L. and others, in their work [3], devoted to assessing the influence of reflected sound energy on the noise mode of residential development, considered existing methods for calculating the reflected sound wave, the basis of which is the specular reflection of sound, for tracking of which it is necessary to build systems of imaginary (secondary) noise sources. Due to the laboriousness of the...
methods and the low accuracy, the need arose for new solutions to the problem of reflected sound in urban areas. The authors used a diffuse model of sound reflection from obstacles with a realization in the form of the Kuttruf integral equation. On its basis, the necessary algorithms and computer programs for calculating the influence of reflected sound on the noise situation in residential buildings were developed. This made it possible to increase the reliability of designing noise protection facilities in residential buildings.

Benov D.M., Nikolov N.D. and others [4] considered the propagation of acoustic waves in the city. These phenomena are described using a complex mathematical apparatus based on the classical equations of sound wave propagation in space. In order to facilitate the computational process, the authors developed a number of programs for a personal computer, replacing traditional methods of computation, such as nomograms, tables, etc. sources of noise. The work of the proposed system is based on the creation of a general digital model, which includes separate models for an urbanized area, with the construction of visual maps with noise sources.

Antonov A.I., Batsunova A.V. and others [5] authors consider the reflected sound wave in confined spaces and identify the main factors affecting its distribution, including: spatial and temporal characteristics of the noise source, room sizes, which generally determine the density of the reflected sound wave, acoustic characteristics of the enclosing structures, the presence of objects scattering the acoustic wave in space. The authors conducted studies of the distribution of the sound wave from pulsed noise sources.

Zhogoleva O., Ledenev V.I. and others [6] have developed a method for calculating noise in apartments with cellular planning systems. The uniqueness of this method consists in taking into account the redistribution of energy in the premises of the apartment as a system of closed volumes having an acoustic connection between them.

In order to minimize noise exposure from urban noise in the territory of residential and industrial areas, researchers are developing to reduce the level of sound noise screens and their design in the works [7-9].

Researchers in the works [10-12] highlight the balance relations of the technosphere with the components of the biosphere. They consider the motor transport infrastructure of the city from the standpoint of biosphere compatibility and its impact on the ecological situation in urban areas.

3. Formulating goals and setting work objectives

The purpose of the work is to analyze and determine the level of the influence of finishing materials on the formation and propagation of acoustic resonant waves from external noise sources in a closed space. Among the main tasks, we can highlight: to describe a mathematical model of the propagation of acoustic and electromagnetic waves, to consider the propagation of acoustic resonant waves in the premises received from motor urban streams, to determine the limitations for the mathematical model of propagation of resonant waves depending on the type of finishing materials for enclosing structures.

4. Main part

A mathematical model of the propagation of acoustic and electromagnetic waves is described by the well-known wave equation:

\[ -\nabla^2 U(x, y, z, \tau) + a \cdot d^2 U(x, y, z, \tau) \over d\tau^2 = 0, \]

where \( U(x, y, z, \tau) \) is the output function, \( x, y, z \) - are spatial coordinates, \( \tau \) - time, \( X_L, Y_L, Z_L \) - are given values, \( a \) - is a given parameter (if acoustic processes are considered, then \( a=1/c^2 \), \( c \) - is the speed of sound in the medium, if electromagnetic then \( a=e\mu \), where \( e, \mu \) - dielectric constant and magnetic permeability of the resonator medium, respectively).

The boundary conditions for equation (2) will be given by predetermined ratio
\[ U(x = 0, y, z, \tau) = U(x = X_L, y, z, \tau) = 0, \]
\[ U(x, y = 0, z, \tau) = U(x, y = Y_L, z, \tau) = 0, \]
\[ U(x, y, Z_L, \tau) / \partial z = \alpha(x, y, \tau), \]
\[ \frac{\partial U(x, y, z = 0, \tau)}{\partial z} = 0, \]

where \( \alpha(x, y, \tau) \) is the input action. Assuming that the input can be represented as

\[ \alpha(x, y, \tau) = \sum_{i,\nu} I_{i,\nu} (\tau) \cdot \sin(\psi_i \cdot x) \cdot \sin(\Omega_{i,\nu} \cdot y), \]

where \( I_{i,\nu} \) – is amplitude, \( \psi_i = \frac{\pi \cdot i}{X_L}, \quad \Omega_{i,\nu} = \frac{\pi \cdot \nu}{Y_L}, \quad (i, \nu = 1, \infty). \)

Let us find the reaction of the object under consideration, described by equation (1) and boundary conditions (2) to each component of series (3). We will look for this reaction in the form

\[ U_{i,\nu}(x, y, z, \tau) = \rho_{i,\nu}(z, \tau) \cdot \sin(\psi_i \cdot x) \cdot \sin(\Omega_{i,\nu} \cdot y), \]

where \( \rho_{i,\nu}(z, \tau) \) are the functions to be defined.

Substituting the component of the series (4) in (1) and transforming, we get

\[ (\psi_i^2 + \Omega_{i,\nu}^2) \cdot \rho_{i,\nu}(z, \tau) - \frac{\partial^2 \rho_{i,\nu}(z, \tau)}{\partial z^2} + \alpha \cdot \rho_{i,\nu}(z, \tau) / d \tau^2 = 0. \]

Transforming (5) the Laplace transform, with zero initial conditions, we get the following result

\[ (\psi_i^2 + \Omega_{i,\nu}^2 + a \cdot s^2) \cdot \rho_{i,\nu}(z, s) - \frac{\partial^2 \rho_{i,\nu}(z, s)}{\partial z^2} = 0, \]

where \( \rho(z, s) \) – is the Laplace-transformed function \( \rho(z, \tau). \)

Find the solution to equation (6). This solution will be sought in the form

\[ \rho_{i,\nu}(z, s) = C_{1,i,\nu} \cdot e^{\beta_{1,i,\nu} z} + C_{2,i,\nu} \cdot e^{-\beta_{2,i,\nu} z}, \]

where \( C_{1,i,\nu} , C_{2,i,\nu} \) are functions determined from the boundary conditions,

\[ \beta_{1,i,\nu} = (\psi_i^2 + \Omega_{i,\nu}^2 + a \cdot s^2)^{1/2}. \]

Using the boundary conditions, we obtain the following relations:

\[ C_{1,i,\nu} \cdot \beta_{1,i,\nu} - C_{2,i,\nu} \cdot \beta_{2,i,\nu} = 0, \quad \rightarrow \quad C_{1,i,\nu} = C_{2,i,\nu}, \]

\[ C_{1,i,\nu} \cdot \beta_{1,i,\nu} \cdot (e^{\beta_{1,i,\nu} z_0} - e^{-\beta_{1,i,\nu} z_0}) = I(s)_{i,\nu}. \]

The value of \( C_{1,i,\nu} \) is determined from the following relation

\[ I(s)_{i,\nu} = \frac{\beta_{1,i,\nu} \cdot (e^{\beta_{1,i,\nu} z_0} - e^{-\beta_{1,i,\nu} z_0})}{\beta_{1,i,\nu} \cdot (e^{\beta_{1,i,\nu} z} + e^{-\beta_{1,i,\nu} z})}. \]

The reaction of an object to each component of the series (3.39) with \( z = Z^* \) can be written in the form:

\[ U_{i,\nu}(x, y, Z^*, \tau) = (e^{\beta_{i,\nu} Z^*} + e^{-\beta_{i,\nu} Z^*}) \cdot I_{i,\nu}(s) \cdot \sin(\psi_i \cdot x) \cdot \sin(\Omega_{i,\nu} \cdot y). \]

The transfer function of the object (spatial-wave link), taking into account the generalized coordinate (G), can be written in the form:

\[ W_{i}(G, s) = \frac{e^{\beta(G, s) Z^*} + e^{-\beta(G, s) Z^*}}{\beta(G, s) \cdot (e^{\beta(G, s) Z} - e^{-\beta(G, s) Z})}, \]

where \( \beta(G, s) = (G + a \cdot s^2)^{1/2}. \)

Putting \( s = j\omega \), where \( \omega \) is the circular frequency, we get \( \beta(G, j\omega) = (G - a \cdot \omega^2)^{1/2}. \)

If the value of \( \beta(G, j\omega) \) is zero, then a resonance is observed in the system.
Using the resulting transfer function, you can determine the resonant frequencies for selected
values $G = \Psi_i^2 + \Omega_i^2$, где $\Psi_i = \frac{\pi \cdot i}{X_L}$, $\Omega_i = \frac{\pi \cdot \nu}{Y_L}$, 

$(i, \nu = 1, \infty)$, which are determined from the following relationship $(G - a \cdot \omega^2) = 0$

If the acoustic impact on the room is carried out through an open window (see Figure 1)

![Figure 1](image)

**Figure 1.** Object to be examined: the room with a window

$$-\nabla^2 U(x, y, z, \tau) + a \frac{d^2 U(x, y, z, \tau)}{d \tau^2} = 0,$$

then the boundary conditions for equation (7), in the case of absorbing walls, are written in the form:

$$U(x = 0, y, z, \tau) = U(x = X_L, y, z, \tau) = 0,$$

$$U(x, y = 0, z, \tau) = U(x, y = Y_L, z, \tau) = 0,$$

$$U(x, y = 0, z, \tau) = \alpha(x, z, \tau), x, y, z \subset S_1;$$

$$U(x, y, z, \tau) = 0, x, y, z \subset S_1,$$

$$U(x, y, z = 0, \tau) = 0.$$

The input action is carried out through the surface $S_1$ (see Fig. 1)

If the walls are reflective, and the floor is absorbing, then the boundary conditions are

$$\partial U(x = 0, y, z, \tau) / \partial x = \partial U(x = X_L, y, z, \tau) / \partial x = 0,$$

$$\partial U(x, y = 0, z, \tau) / \partial y = 0,$$

$$\partial U(x, y, z, \tau) = \alpha(x, y, \tau), x, y, z \subset S_1;$$

$$\partial U(x, y, z, \tau) / \partial y = 0, x, y, z \subset S_1,$$

$$U(x, y, z = 0, \tau) = 0.$$

Since equation (7) with boundary conditions (8) or (9) does not have an analytical solution, a
numerical model was compiled and the object’s response to a given spectrum was investigated:

$$\alpha(x, y, \tau) = A \cdot \sin(\omega \cdot \tau), x, y, z \subset S_1,$$

where $A$ is the amplitude of the input, $\omega$ is the circular frequency.

Figure 2 shows a graph of the behavior of a resonant wave with the condition of reflective vertical
enclosing structures.
Figure 2. Simulation results (Boundary condition - reflection)

Figure 3 shows a graph of the behavior of a resonant wave with the condition of absorbing vertical enclosing structures.

Figure 3. Simulation results (Boundary condition–absorption)

Figure 4 shows a graph of the behavior of a resonant wave with the condition of absorbing vertical and horizontal enclosing structures.
Figure 4. Simulation results (Boundary condition - absorption)

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6
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