An Estimate of the Inclusive Branching Ratio to $\bar{B}_c$ in $\Xi_{bbq}$ Decay

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We estimate the branching ratio for the inclusive decays $\Xi_{bbq} \to \bar{B}_c^{(*)} + X_{c,s,q}$ to be approximately 1%. Our estimate is performed using non-relativistic potential quark model methods that are appropriate if the bottom and charm quarks are heavy compared to the strong interaction scale. Here the superscript ($*$) denotes that we are summing over spin zero $\bar{B}_c$ and spin one $\bar{B}_c^*$ mesons and the subscript $q$ denotes a light quark. Our approach treats the two bottom quarks in the baryon $\Xi_{bbq}$ as a small color anti-triplet. This estimate for the inclusive branching ratio to $\bar{B}_c$ and $\bar{B}_c^*$ mesons also holds for decays of the lowest lying $T_{bb\bar{q}\bar{q}}$ tetraquark states, provided they are stable against strong and electromagnetic decay.

I. INTRODUCTION

In 2017, the doubly charmed baryon $\Xi_{ccu}^{++}$ (or in the notation used in this paper $\Xi_{ccu}^{++}$) was discovered at LHCb [1]. It has been observed in the exclusive decay modes, $\Xi_{ccu}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ (the discovery mode) and $\Xi_{ccu}^{++} \to \Xi_c^+ \pi^+$ [2]. There is considerable interest in the detection of the analogous baryons containing two heavy bottom quarks $\Xi_{bbq}$, partly because it would be the first step to observing the tetraquark states, $T_{bb\bar{q}\bar{q}}$. They are thought to be stable with respect to the strong and electromagnetic interactions with masses that are around 100-200 MeV below the $\bar{B}_q \bar{B}_q$ threshold [3–5].

Recently, Gershon and Poluektov [6] proposed the inclusive decay mode $\Xi_{bbq} \to \bar{B}_c + X_{c,s,q}$ as a potential discovery channel for the doubly bottom baryon $\Xi_{bbq}$ at the LHC. They made the clever observation that $\bar{B}_c$’s that do not point back to the collision interaction point can only arise from the weak decay of a hadron with two bottom quarks. They also note that the decay chain $\bar{B}_c \to J/\psi \pi^- \to \mu^+ \mu^- \pi^-$ can be used to detect the $\bar{B}_c$ meson[1] Ordinary $\bar{B}$ mesons that do not point back to the collision point cannot be used for this purpose[2] because they can arise from the weak decay of a long lived $\bar{B}_c$ meson (via the weak decay of the anti-charm quark). The branching ratio for $\bar{B}_c$ decay to ordinary $\bar{B}$ mesons is not expected to be small and furthermore there will be many more $\bar{B}_c$’s produced at the interaction point by hadronization then there are baryons with two bottom quarks.

In this paper we make an estimate of the inclusive branching ratio, $\text{Br}(\Xi_{bbq} \to \bar{B}_c^{(*)} + X_{c,s,q})$. Here the subscript $c, s, q$ denotes the flavor quantum numbers of the inclusive final state and the superscript ($*$) denotes that we are summing over final state spin zero $\bar{B}_c$ and spin one $\bar{B}_c^*$ mesons. A $\bar{B}_c^*$ meson decays to a $\bar{B}_c$ plus a photon, so decays to the spin one state always result in a $\bar{B}_c$ in the final state.

Our method relies on treating both the bottom and charm quark as heavy compared to the scale of the non-perturbative strong interactions, $\Lambda_{QCD} \sim 200\text{MeV}$. In this limit, the two bottom quarks in the $\Xi_{bbq}$ form a small (compared with $1/\Lambda_{QCD}$) color anti-triplet

1 See [7] for a recent calculation of the branching ratio for $\bar{B}_c \to J/\psi \pi^-$. Their results imply that $\text{Br}(\bar{B}_c \to J/\psi \pi^- \to \mu^+ \mu^- \pi^-) \simeq 2 \times 10^{-4}$.

2 We thank T. Gershon for pointing this out to us.
of the bound states are non-relativistic. The state vectors are then $\Phi$ numbers. We perform the calculation in the rest frame of the decaying bottom diquark state $J/\psi$ matrix element is proportional to an overlap of wave-functions while the meson decay matrix to the calculation of the inclusive $B$ meson decay rate to $J/\psi$ is approximately twice the $b$ quark decay rate$^3$.

Our computation of $\Gamma(\Xi_{bbq} \rightarrow \bar{B}_c^{(*)} + X_{c,s,q})$ does not include decay products from an excited (radial or orbital) $\bar{B}_c$ (or $B_c^*$) mesons. We will calculate the decay rates to the first radially excited $\bar{B}_c$ and $B_c^*$ mesons and show they are suppressed, and then argue that decays to the other excited states are suppressed as well.

Our calculation of the inclusive decay rate of a $\Xi_{bbq}$ baryon to $\bar{B}_c$ and $B_c^*$ mesons is similar to the calculation of the inclusive $B$ meson decay rate to $J/\psi$ $^9$. One important difference is that the baryon decay is not color suppressed. Another difference is that the baryon decay matrix element is proportional to an overlap of wave-functions while the meson decay matrix element is proportional to the $J/\psi$ wave function at the origin.

II. THE DECAY RATE

In this section, we outline the calculation of the $\Phi_{bb} \rightarrow \bar{B}_c + c + s$ invariant matrix element $\mathcal{M}(\Phi_{bb}(0, \gamma) \rightarrow \bar{B}_c(k) + c(p_{c}, \alpha) + s(p_s, \beta))$, where greek letters denote the color quantum numbers. We perform the calculation in the rest frame of the decaying bottom diquark state $\Phi_{bb}$, which is a color anti-triplet and has spin one. We assume that the relative momentum of the bound states are non-relativistic. The state vectors are then

$$|\bar{B}_c(k, s, m_s)\rangle = \frac{\sqrt{2E_{\bar{B}_c}(k)}}{\sqrt{3}} \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}_{\bar{B}_c}(p) C_{s1m_2}^{s1m_2} |b\left(\frac{m_bk}{m_b + m_c} + p, \delta, s_1\right)\rangle |c\left(\frac{m_ck}{m_b + m_c} - p, \delta, s_2\right)\rangle$$

$$|\Phi_{bb}(0, \gamma, m)\rangle = \frac{1}{2} \sqrt{2m_{\Phi_{bb}}} \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}_{\Phi_{bb}}(p) \epsilon^{\gamma\alpha\beta} C_{1m_2}^{s1m_2} |b(p, \alpha, s_1)\rangle |b(-p, \beta, s_2)\rangle$$

(2.1)

where repeated indices are summed over and the state $|\bar{B}_c(k, s, m_s)\rangle$ corresponds to a $\bar{B}_c$ meson if $s = 0$ and a $B_c^*$ meson if $s = 1$. The bound states have been normalized such that $\langle \bar{B}_c(k_1, s_1, m_{s_1}) | \bar{B}_c(k_2, s_2, m_{s_2}) \rangle = 2E_{\bar{B}_c} \delta^{s_1s_2} \delta^{m_1m_2}(2\pi)^3 \delta^3(k_1 - k_2)$ and similarly for the $\Phi_{bb}$ state. The state vectors on the right hand side of (2.1) have no hidden normalization factors, and are just the appropriate creation operators acting on the vacuum. The functions $\tilde{\psi}_{\bar{B}_c}(p)$ and $\tilde{\psi}_{\Phi_{bb}}(p)$ are the wavefunctions for the relative momentum of the quarks in the bound states and, in the non-relativistic limit, have support when $p$ is much less than the masses of the bound quarks.

The weak Hamiltonian that induces the decay is\textsuperscript{4}

$$H = \frac{4G_F}{\sqrt{2}} V_{cs}^* V_{cb} [C_1 O_1 + C_2 O_2]$$

(2.2)

where

$$O_1 = \left[ c^\alpha \gamma^\mu P_L b_\alpha \right] \left[ \bar{s}_\beta \gamma_\mu P_L c_\beta \right] \quad O_2 = \left[ c^\alpha \gamma^\mu P_L b_\alpha \right] \left[ \bar{s}_\alpha \gamma_\mu P_L c_\beta \right].$$

(2.3)

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\textsuperscript{3} A more accurate estimate (which we will use) that applies the operator product expansion and heavy quark methods can be found in [8].

\textsuperscript{4} We neglect the contribution from operators induced by penguin type diagrams.
The operators $O_{1,2}$ and coefficients $C_{1,2}$ are evaluated at a subtraction point equal to the $b$ quark mass. The invariant matrix element for the decay is then

$$\mathcal{M} = \frac{4G_F}{\sqrt{6}} V_{cb}^* V_{cb}(C_1 - C_2) \int \frac{d^3 p}{(2\pi)^3} \sqrt{E_{\bar{B}_c}(k)m_{\Phi_{bb}}} \tilde{\psi}_{\bar{B}_c}^*(|p + \frac{m_b}{m_b + m_c}k|)\psi_{\Phi_{bb}}(p)$$

$$\times \epsilon_{\gamma\alpha\beta}C_s^{(s,m_s)}C_{s_1,s_2}^{(1,m)}[\bar{u}^{(s)}(p_s,s_s)\gamma^\mu P_L\gamma^\nu(p + k,s'_2)][\bar{u}^{(c)}(p_c,s_c)\gamma^\mu P_L\gamma^\nu(p,s_2)]. \tag{2.4}$$

In eq. (2.4) the $\tilde{\psi}_{\Phi_{bb}}(p)$ wavefunction restricts $p$ to be much less than $m_b$, so we can set $u^{(b)}(p,s_2) = u^{(b)}(0,s_2)$ and $E_b(p) = m_b$. In addition, the $\bar{B}_c$ wave function restricts $|p + \frac{m_b}{m_b + m_c}k|$ to be much less than the charm quark mass, which means we can make the replacement $p + k \rightarrow (m_c/(m_b + m_c))k$ in $E_c$ and $u^{(c)}$. In the non-relativistic limit, the masses of the bound states are approximately equal to the sum of their constituent quark masses, which implies $E_c((m_c/(m_b + m_c))k) = (m_c/(m_b + m_c))E_{\bar{B}_c}(k)$. After making these replacements, eq. (2.4) becomes

$$\mathcal{M} \simeq \frac{4G_F}{\sqrt{2}} V_{cb}^* V_{cb}(C_1 - C_2) \sqrt{\frac{2(m_b + m_c)}{3m_c}} \epsilon_{\gamma\alpha\beta}C_s^{(s,m_s)}C_{s_1,s_2}^{(1,m)}I \left(-\frac{m_b}{m_b + m_c}\right)$$

$$\times \left[\bar{u}^{(s)}(p_s,s_s)\gamma^\mu P_L\gamma^\nu(p + m_c/m_b + m_c - k,s'_2)[\bar{u}^{(c)}(p_c,s_c)\gamma^\mu P_Lu^{(b)}(0,s_2)] \right] \tag{2.5}$$

where

$$I(k) = \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{\psi}_{\bar{B}_c}^*(|p + k|)\psi_{\Phi_{bb}}(p) = 4\pi \int dr r^2 \frac{\psi_{\bar{B}_c}^*(r)\psi_{\Phi_{bb}}(r)\sin(kr)}{kr}. \tag{2.6}$$

Note, the position space wavefunctions are normalized so that $\int |\psi_{\bar{B}_c}/\Phi_{bb}(r)|^2d^3r = 1$.

To determine the differential decay rate, we square the matrix element, average over initial spins and colors and sum over final spins and colors. The spin sum involving the final state $\bar{B}_c$ spins is performed using the completeness relation, $\sum_{s,m_s} C_s^{(s,m_s)}C_{s_1,s_2}^{s_1,s_2} = \delta_{s_1,s_1} \delta_{s_2,s_2}$. For the spin average over the $\Phi_{bb}$ spin magnetic quantum numbers we note that,

$$\sum_{s_1} C_{s_1,s_2}^{(1,1)}C_{s_1,s_2}^{s_1,s_2} + C_{s_1,s_2}^{(1,-1)}C_{s_1,s_2}^{s_1,-s_2} = \delta_{s_1,s_2}. \tag{2.7}$$

Rotational invariance implies that the decay rate is independent of the magnetic quantum number for the total spin of $\Phi_{bb}$. This means we can replace the average over its initial magnetic quantum numbers in the decay rate with the average over just the $m_1 = -1$ and $m_1 = 1$ magnetic quantum numbers. After integrating over the strange and charm momenta, the differential decay rate then becomes

$$\frac{d\Gamma(\Xi_{bb} \rightarrow \bar{B}_c^{(*)}(k) + X_{c,s,q})}{dk} \simeq \left(\frac{G_F}{3\pi^2}\right) (C_1 - C_2)^2 |V_{cb}V_{cb}|^2 |I(m_bk/(m_b + m_c))|^2$$

$$\times k^2 \left(m_{\Phi_{bb}}^2 + m_{\bar{B}_c}^2 - m_c^2 - 2m_{\Phi_{bb}}E_{\bar{B}_c}(k)\right)^2 \left(m_{\Phi_{bb}}^2 + m_{\bar{B}_c}^2 - 2m_{\Phi_{bb}}E_{\bar{B}_c}(k)\right) \tag{2.7}$$

where, as mentioned in the introduction, the superscript $(*)$ denotes that we are summing over the spin one and spin zero $\bar{B}_c$ mesons.

### III. NUMERICAL RESULTS

To evaluate the form factor $I(k)$, we need to determine the wave functions $\psi_{\Phi_{bb}}(r)$ and $\psi_{\bar{B}_c}(r)$. We do this by numerically solving the non-relativistic Schrödinger equation with
FIG. 1. The form factor $I(k)$ defined in (2.6) computed using the numerical ground state wave functions (blue) and the approximate ones (yellow) given in (3.3).

the Cornell potentials,
\[
V_{\Phi_{bb}}(r) = -\frac{2}{3} \left( \frac{0.3}{r} \right) + \frac{1}{2}(0.2\text{GeV}^2)r, \quad V_{\bar{B}_c}(r) = -\frac{4}{3} \left( \frac{0.4}{r} \right) + (0.2\text{GeV}^2)r. \tag{3.1}
\]

The relative factor of $1/2$ between $V_{\Phi_{bb}}(r)$ and $V_{\bar{B}_c}(r)$ reflects the fact that the $\Phi_{bb}$ is a color anti-triplet while the $\bar{B}_c$ is a color singlet$^5$. We took the string tension to be $0.2\text{ GeV}^2$ which fits the $b\bar{b}$ spectrum of bound states \[^{10}\]. In addition, we chose the strong fine structure constant to be $0.3$ and $0.4$ for $V_{\Phi_{bb}}(r)$ and $V_{\bar{B}_c}(r)$.

The charm and bottom quark masses are taken to be $1.5\text{ GeV}$ and $4.5\text{ GeV}$. The form factor $I(k)$ computed using the numerical ground state wavefunctions is plotted in Fig. 1 over the range of $k$ allowed in the decay,
\[
0 < k < \left[ \left( \frac{m_{\Phi_{bb}}^2 + m_{\bar{B}_c}^2 - m_c^2}{2m_{\Phi_{bb}}} \right)^2 - m_{\bar{B}_c}^2 \right]^{1/2}. \tag{3.2}
\]

The numerical solutions to the Schrödinger equation implies the radii squared of the ground state wavefunctions are $\langle r^2 \rangle_{\Phi_{bb}} = 3.2\text{ GeV}^{-2}$ and $\langle r^2 \rangle_{\bar{B}_c} = 2.8\text{ GeV}^{-2}$. It turns out that the Coulomb-like wave functions
\[
\psi_{\Phi_{bb}}(r) = \frac{1}{\sqrt{\pi}} \left( \frac{3}{\langle r^2 \rangle_{\Phi_{bb}}} \right)^{3/4} \text{Exp} \left( -\frac{\sqrt{3}r}{\sqrt{\langle r^2 \rangle_{\Phi_{bb}}}} \right),
\]
\[
\psi_{\bar{B}_c}(r) = \frac{1}{\sqrt{\pi}} \left( \frac{3}{\langle r^2 \rangle_{\bar{B}_c}} \right)^{3/4} \text{Exp} \left( -\frac{\sqrt{3}r}{\sqrt{\langle r^2 \rangle_{\bar{B}_c}}} \right). \tag{3.3}
\]

are good approximations to the numerical ones. Evaluating (2.6) using (3.3) gives the following simple analytic approximation to the form factor,
\[
I(k) = \left( \frac{\langle r^2 \rangle_{\Phi_{bb}}^{1/4} \langle r^2 \rangle_{\bar{B}_c}^{1/4}}{\langle r^2 \rangle_{\Phi_{bb}}^{1/2} + \langle r^2 \rangle_{\bar{B}_c}^{1/2}} \right)^3 \left[ 1 + \left( \frac{\langle r^2 \rangle_{\Phi_{bb}} \langle r^2 \rangle_{\bar{B}_c}}{\langle r^2 \rangle_{\Phi_{bb}}^{1/2} + \langle r^2 \rangle_{\bar{B}_c}^{1/2}} \right)^2 \right] \left( k^2/3 \right)^2. \tag{3.4}
\]

$^5$ Lattice studies indicate that the factor of one half should be extended to the non-perturbative linear part of the potential \[^{11}\].
which is also plotted in Fig. 1.

In Fig. 2 we plot $d\Gamma/dk$ obtained using the ground state numerical wavefunctions. Integrating (2.7) over (3.2), we find that the decay rate is

$$\Gamma(\Xi_{bbq} \rightarrow \bar{B}_c^{(*)}(k) + X_{c,s,q}) = 1.5 \times 10^{10} \text{ s}^{-1}. \quad (3.5)$$

Using a total lifetime for $\Xi_{bbq}$ of 0.5 ps \footnote{The total lifetime for $\Xi_{bbq}$ is 0.5 ps.}, the branching ratio is

$$\text{Br}(\Xi_{bbq} \rightarrow \bar{B}_c^{(*)}(k) + X_{c,s,q}) \approx 8 \times 10^{-3}. \quad (3.6)$$

This branching ratio leaves out $\bar{B}_c$’s that arise from the decay of radially and orbitally excited $\bar{B}_c$ and $\bar{B}_c^{*}$ mesons. We computed the decay rate to the first radially excited $\bar{B}_c$ state with zero orbital angular momentum and found the branching ratio to be $7.3 \times 10^{-4}$, which is an order of magnitude smaller than the branching ratio to the ground state.

Decays to other radially excited $\bar{B}_c$ states will be suppressed as well. The full Hamiltonian for the $\Phi_{bb}$ system, including the kinetic terms and potential from (3.1), is almost equal to half the full Hamiltonian for the $\bar{B}_c$ one. This means the spatial wave functions for the energy eigenstates of the two Hamiltonians are almost the same, which implies $I(0) \approx 1$ for the ground state $\bar{B}_c$ mesons. In addition, it implies that the overlap integral for decays to radially excited $\bar{B}_c$ and $\bar{B}_c^{*}$ mesons will satisfy $I(0) \approx 0$, which suppresses the branching ratio to these states. Decays to orbitally excited $\bar{B}_c$ mesons are also suppressed.

A recent work on production rates for hadrons with two heavy quarks at the LHC \footnote{A recent work estimates that $\sigma(pp \rightarrow \Phi_{bb} + X) \approx 15$ nb.} estimates that $\sigma(pp \rightarrow \Phi_{bb} + X) \approx 15$ nb. Assuming most of the $\Phi_{bb}$ diquarks end up as $\Xi_{bbq}$ baryons, this implies that in an integrated luminosity of 10 fb$^{-1}$ there are around $10^8$ $\Xi_{bbq}$ baryons. Our work then implies that the decays of these baryons produce around $10^6$ $\bar{B}_c$’s that do not point back to the interaction point. About $10^2$ of them end up in the final state $\mu^+\mu^-\pi^-$, with the $\mu^+\mu^-$ arising from $J/\psi$ decay.

\footnote{About 20% end up as tetraquarks containing two bottom quarks.}
IV. CONCLUDING REMARKS

We calculated the inclusive decay rate for $\Xi_{bbq} \to \bar{B}_c + X_{c,s,q}$ to be $1.5 \times 10^{10} \text{ s}^{-1}$ (which implies $\text{Br}(\Xi_{bbq} \to \bar{B}_c^{(*)}(k) + X_{c,s,q}) \simeq 8 \times 10^{-3}$). The initial $bb$ system was treated as a tightly bound color anti-triplet diquark $\Phi_{bb}$ and we evaluated its decay rate to $\bar{B}_c + c + s$. The Schrödinger equation was solved numerically to determine the non-relativistic wavefunctions for the $\Phi_{bb}$ and $\bar{B}_c$. In reality, the relative momentum of the quarks in the $\bar{B}_c$ bound state is not truly non-relativistic. In addition, we neglected the fact that the diquark initially exists in a hadron and interactions between the final $\bar{B}_c$, $c$ and $s$ states and the soft degrees of freedom in $\Xi_{bbq}$. Despite these approximations, we expect our calculation of the decay rate to be correct at the factor of two level.

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