THE ROLE OF MAGNETIC HELICITY IN STRUCTURING THE SOLAR CORONA

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Received 2016 July 22; revised 2016 December 1; accepted 2016 December 7; published 2017 January 20

ABSTRACT

Two of the most widely observed and striking features of the Sun’s magnetic field are coronal loops, which are smooth and laminar, and prominences or filaments, which are strongly sheared. Loops are puzzling because they show little evidence of tangling or braiding, at least on the quiet Sun, despite the chaotic nature of the solar surface convection. Prominences are mysterious because the origin of their underlying magnetic structure—filament channels—is poorly understood at best. These two types of features would seem to be quite unrelated and wholly distinct. We argue that, on the contrary, they are inextricably linked and result from a single process: the injection of magnetic helicity into the corona by photospheric motions and the subsequent evolution of this helicity by coronal reconnection. In this paper, we present numerical simulations of the response of a Parker (1972) corona to photospheric driving motions that have varying degrees of helicity preference. We obtain four main conclusions: (1) in agreement with the helicity condensation model of Antiochos (2013), the inverse cascade of helicity by magnetic reconnection in the corona results in the formation of filament channels localized about polarity inversion lines; (2) this same process removes most complex fine structure from the rest of the corona, resulting in smooth and laminar coronal loops; (3) the amount of remnant tangling in coronal loops is inversely dependent on the net helicity injected by the driving motions; and (4) the structure of the solar corona depends only on the helicity preference of the driving motions and not on their detailed time dependence. We discuss the implications of our results for high-resolution observations of the corona.

Key words: Sun: corona – Sun: filaments, prominences – Sun: magnetic fields

1. INTRODUCTION

A well-known feature of the solar magnetic field is the filament channels observed at photospheric polarity inversion lines (PILs). These magnetic structures, situated in the upper chromosphere and lower corona, underlie and support the cool plasma that comprises prominences and filaments (Martin 1998; Gaizauskas 2000). Filament channels are regions of highly sheared magnetic field, containing large amounts of free energy that ultimately is converted into kinetic and thermal energy of the plasma, as well as nonthermal particle energies, when filament channels erupt and drive coronal mass ejections (CMEs). The shear is a manifestation of magnetic helicity stored in the filament channels, which are known as dextral if they have negative helicity and sinistral if they have positive helicity. Observations indicate that dextral (sinistral) filament channels dominate in the northern (southern) hemisphere (e.g., Martin et al. 1992; Rust & Kumar 1994; Zirker et al. 1997; Pevtsov et al. 2003) This hemispheric helicity preference has also been observed in quiet-Sun magnetic fields (Pevtsov & Longcope 2001), sigmoids (Rust & Kumar 1996), active-region magnetic fields (Seehafer 1990), CMEs, and sunspot whorls (Pevtsov et al. 2014). The strength of the preference ranges from about 55% in active-region filaments (Martin 1994) to over 80% in quiescent filaments (Pevtsov et al. 2003) and does not seem to change with solar cycle (Hale 1927; Martin 1994; Hagino & Sakurai 2002). Although these properties of solar magnetic structures are well established and accepted, how they are manifested in filament channels remains obscure at best. Indeed, how filament channels are formed in the first place is one of the major long-standing mysteries of heliophysics.

A second, seemingly unrelated, feature of the solar magnetic field is the large-scale loops observed in the corona. These loops have been observed at high resolution in XUV and X-ray images from the Transition Region and Coronal Explorer (TRACE), such as the one in Figure 1. On the quiet Sun, at least, they are seen to be very smooth and laminar, with little evidence for tangling or braiding (Schrijver et al. 1999; Schrijver 2007). As a result, there appears to be next to no mutual magnetic helicity associated with such loops wrapping around one another in the corona. This is puzzling, however, given the chaotic patterns of convection on the Sun’s surface. Such flows would seem destined to braid the loops around one another as the loop footpoints are randomly shuffled across the surface (Parker 1972). The observed motions are highly complex (Schmieder et al. 2014), with convective cells appearing randomly throughout the photosphere and occurring over a broad range of scales, from of the order of minutes for granules to days for supergranules (Hirzberger et al. 2008). Clearly, some process is responsible for smoothing out the loops that otherwise would become braided around one another, leaving them laminar and untangled.

These aspects of prominences or filaments, on the one hand, and of coronal loops, on the other, raise complementary questions about the role and fate of magnetic helicity in the corona. Regarding the helicity that does not manifest itself in braided coronal loops, where does it go? Concerning the helicity that does manifest itself in sheared filament channels, where does it come from? Is there a connection between the two, and if so, how does this connection come about?

Both the ultimate origin and ultimate disposition of the coronal magnetic helicity are widely recognized and broadly accepted. The helicity originates from the photosphere, due to a combination of injection by flux emergence from below the surface and injection by footpoint motions on the surface after emergence. Emerging flux in solar active regions frequently
laminar structure, with little to no braiding or tangling. Taken from Figure 13 of
the region, i.e., at the PIL. This is precisely what is observed as
carries electric current and associated magnetic free energy and
helicity into the corona, as shown by numerous studies (e.g.,
Leka et al. 1996; Tian et al. 2008; Malanushenko et al. 2011,
2014; Kazachenko et al. 2015). These regions occasionally
present transient, braided loop structures. In the absence of
significant flux emergence, observed shearing and twisting
footpoint motions have been shown to inject substantial
amounts of helicity into the corona (e.g., Liu & Schuck 2012).
Together, these photospheric processes supply ample amounts
of magnetic helicity to the corona. Because its Lundquist
number is very high, the corona is believed to shed essentially
all of its helicity into the heliosphere by ejecting magnetic
fluxes, rather than dissipating the helicity in situ. Conse-
quently, the Sun injects helicity into the corona virtually
everywhere over its surface, then somehow concentrates it into
filament channels at PILs, where it is subsequently carried
away by solar eruptions.

A new model, magnetic helicity condensation (Antiochos
2013), explains simultaneously the formation of sheared
filament channels and the smoothness of quiet-Sun coronal
loops. It is based on the well-established inverse cascade, from
small scales to large scales, of magnetic helicity in turbulent
magnetohydrodynamic (MHD) systems (Biskamp 1993). In the
helicity condensation model, photospheric convection imparts
helicity to the coronal magnetic field, and the helicity is then
transported throughout the corona by reconnection, which is
well known to conserve helicity (Woltjer 1958; Taylor 1974, 1986;
Berger 1984). If the surface convection imparts the same sense of twist to adjacent flux tubes, they are
able to undergo component reconnection at their contact point.
This component reconnection produces a single flux tube with
an axial flux equal to the sum of the two original axial fluxes,
and encircled by the same twist field present in the two original
tubes. In this way, the helicity, in the form of twist, inverse-
cascades to ever larger scales. The PIL forms a natural
boundary of the flux system: when the twist reaches this
boundary, it cannot proceed further, since all of the flux has
already reconnected. The end result of the process is a mostly
axial and untangled field throughout the interior of the region,
and a highly sheared field concentrated near the boundary of
the region, i.e., at the PIL. This is precisely what is observed as
a filament channel. The smooth internal field corresponds to the
laminar coronal loops observed routinely on the quiet Sun and
shown in Figure 1. In this way, the helicity condensation model
provides a natural mechanism for the simultaneous formation
of highly sheared filament channels and laminar, untangled
coronal loops. These two, seemingly unrelated features of the
solar atmosphere are actually created by a single process
(Antiochos 2013).

The helicity condensation model was initially simulated by
Zhao et al. (2015), who injected magnetic helicity into a plane-
parallel Parker corona (Parker 1972). In the plane-parallel
representation, the boundaries of the region of imposed
twisting flows serve as the boundaries of the flux system, in
analogy with the PILs in coronal-field configurations. Those
authors found that photospheric motions that inject the same
helicity everywhere form filament channels at the flux-system
boundaries. Furthermore, randomizing the photospheric motions while keeping the same helicity injection rate did not
qualitatively affect the accumulation of twist flux. Zhao
et al. (2015) also tested the effect of injecting helicity of
opposite signs on adjacent flux tubes, and found that their fields
could not reconnect due to the twist components being co-
aligned.

In subsequent work (Knizhnik et al. 2015, hereafter KAD15),
we rigorously tested the helicity condensation model. We found
not only that it qualitatively produced results consistent with the
properties of filament channels, but that the inverse cascade of
magnetic helicity due to reconnection produces a twist flux at the
boundary of the flux system that agrees quantitatively with the
predictions of the helicity condensation model. Based on this
result, we estimated that with the helicity preference observed on
the Sun, filament channels will form in about a day or so, in
agreement with observations of their formation (Martin 1998;
Gaizauskas 2000). We showed that helicity condensation agreed
both qualitatively and quantitatively with observed properties of
filament channels, and that the process produced a much less
twisted coronal-loop region away from the flux-system bound-
aries. Those results were obtained for a 100% helicity preference,
however, meaning that all of the helicity injected into the corona
was of the same sign. An obvious question to be raised is: what
happens if a fraction of the injected helicity has the opposite sign?
Indeed, this is a more realistic scenario since, as described above,
the corona has a hemispheric helicity preference, rather than an
exclusive rule, so that some helicity of the non-preferred sign is
injected into the corona at all times.

It is reasonable to expect that injecting helicity of the
opposite sign into the corona would simply slow down the
helicity condensation process. However, this result is not as
straightforward as it may seem. The simulations of Zhao et al.
(2015) demonstrated that adjacent flux tubes have difficulty
reconnecting if they are twisted in opposite senses. As a result,
the twist was unable to inverse-cascade to larger scales, as is
required to form the sheared filament channels and smooth
coronal loops. Even if reconnection between the adjacent flux
tubes is eventually achieved as a result of some instability—
such as ideal kinking—driving the interaction, the twist-flux
cancellation is expected to be far from perfect. Substantial
residual twist, even braiding, could remain in what otherwise
would have been a region of smooth, laminar coronal loops.
Therefore, it is important to test whether sheared filament
channels and smooth coronal loops form when both signs of
magnetic helicity are injected into the corona.
We noted above that the observed photospheric flows are highly complex. From the standpoint of helicity injection, the important flows are those that have significant vorticity, twisting the field internal to individual flux tubes or braiding flux tubes around one another (Longcope et al. 2007). Compression of the field by converging flows that have negligible vorticity, in contrast, does not impart any helicity to the field, so these flows cannot be responsible for the shear observed in filament channels. Therefore, it suffices to model the helicity injection into the corona with simple twisting motions, as has been done by many authors (e.g., Wilmoth-Smith et al. 2010; Rappazzo et al. 2013). Vortical flows have been observed below the surface through helioseismic measurements (Duvall & Gizon 2000; Gizon & Duvall 2003; Komm et al. 2007; Seligman et al. 2014), and at the photosphere on the scales of both granules (e.g., Bonet et al. 2008, 2010; Vargas Domínguez et al. 2011, 2015) and supergranules (Brandt et al. 1988; Attie et al. 2009).

In this paper, we investigate the effect of varying helicity preference on the structure of the closed-field corona. We performed helicity-conserving numerical simulations that inject helicity into a plane-parallel Parker corona, as we did in KAD15. Extending our previous work, the fraction of helicity of each sign, and thus the net preferred helicity overall, that is injected into the corona was varied. We report on three cases: (1) 100% of the injected helicity is positive (100% net preference); (2) 75%/25% of the injected helicity is positive/negative (50% net preference); and (3) 50%/50% of the injected helicity is positive/negative (0% net preference). To make the simulations as realistic as possible, we also randomized the pattern of helicity injection in a second set of simulations. This randomization introduces additional braiding, along with twisting, directly into the magnetic field. We compare the simulations with a fixed pattern of helicity injection to those with a randomized pattern.

The paper is organized as follows. In Section 2 we discuss the setup and initialization of our numerical simulations. In Section 3 we describe how magnetic helicity is injected into the domain, and how the helicity preference is employed. In Section 4 we discuss the results of our simulations, exploring the formation of filament channels and the smoothness of coronal loops for various helicity preferences, and compare the simulations with fixed and randomized patterns of helicity injection. We discuss the implications for understanding coronal magnetic structure in Section 5.

2. NUMERICAL MODEL

The numerical model used in this study was described previously in KAD15. We solve the equations of MHD using the Adaptively Refined Magnetohydrodynamics Solver (ARMS; DeVore & Antiochos 2008) in three Cartesian dimensions. The equations have the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$  

(2.1)

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B},$$  

(2.2)

$$\frac{\partial T}{\partial t} + \nabla \cdot (T \mathbf{v}) = (2 - \gamma)T \nabla \cdot \mathbf{v},$$  

(2.3)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}).$$  

(2.4)

Here \(\rho\) is mass density, \(T\) is temperature, \(P\) is thermal pressure, \(\gamma\) is the ratio of specific heats, \(v\) is velocity, \(\mathbf{B}\) is magnetic field, and \(t\) is time. We close the equations via the ideal gas equation,

$$P = \rho RT,$$  

(2.5)

where \(R\) is the gas constant.

ARMS uses finite-volume representations of the variables to solve the system of equations. Its Flux Corrected Transport algorithms (DeVore 1991) provide minimal, though finite, numerical dissipation, which allows reconnection to occur. As a result, to a very good approximation, ARMS conserves the magnetic helicity in the system.

We set up a model coronal field that is initially straight and uniform between two plates, as shown in Figure 2. In this model, straight flux tubes represent coronal loops whose apex is located in the center of the domain and with the boundaries representing the photosphere. Our domain size is \([0, L_x] \times [-L_y, L_y] \times [-L_z, L_z]\), where \(x\) is taken normal to the photosphere (the vertical direction) and we set \(L_x = 1\), and \(L_y = L_z = 1.75\). At all six sides, we use zero-gradient conditions, and the four side walls have open boundary conditions. Closed boundary conditions are employed at the top and bottom, where the magnetic field is line-tied at the high-\(\beta\) photosphere. The footpoints of the field lines do not move in response to magnetic forces, but do respond to imposed boundary flows to mimic driving at the plasma-dominated photosphere.

As in our previous work, we set the initial, uniform values in our dimensionless simulations to \(\rho_0 = 1\), \(T_0 = 1\), \(P_0 = 0.05\), and \(B_0 = \sqrt{4\pi} \). These choices set the gas constant, \(R = 0.05\), the Alfvén speed, \(c_A = B_0/\sqrt{4\pi \rho_0} = 1\), and the plasma beta, \(\beta_0 = 8\pi P_0 / B_0^2 = 0.1\). \(\beta < 1\) corresponds to a magnetically dominated plasma, which is generally true of the corona. The results discussed below will be given in simulation time, which is normalized to the time required for an Alfvén wave at unit

![Figure 2. Setup of the numerical simulations. Black lines represent the initial vertical magnetic field; color shading on the top and bottom plates represents magnitude of velocity. To emulate the random photospheric convection in our numerical experiments, the local sense of rotation (clockwise/counterclockwise) of individual convective cells shown in this figure can be set randomly, and the global hexagonal pattern of the cells collectively can be rotated randomly about its center. See the text for details.](image-url)
speed ($c_{A0} = 1$) to travel the distance separating the top and bottom plates ($L_s = 1$).

As before, we set up our convective cells in a hexagonal pattern, with 84 cells on the top and bottom plates. Each individual cell has the same spatial and temporal profiles as described in KAD15. The angular velocity of each cell is given by

$$\Omega(r, t) = \begin{cases} -\Omega_0 g(r)f(t) & \text{if } r \leq a_0 \\ 0 & \text{if } r > a_0 \end{cases}$$

where

$$g(r) = \left( \frac{r}{a_0} \right)^4 - \left( \frac{r}{a_0} \right)^8$$

and

$$f(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi t}{\tau} \right) \right]$$

with $\Omega_0 = 7.5$ and $r$ the cylindrical radial coordinate with respect to the center of the convective cell. The flow is confined to a circle of radius $a_0 = 0.125$, and the magnitude of the flow is ramped up and back down over a period $\tau = 3.5$. As demonstrated in KAD15, this velocity profile for the convective cells conserves the normal magnetic field distribution, $B_n$, on the photospheric boundaries.

The hexagonal pattern shown in Figure 2 also includes a central region where no velocity is imposed. This region, also modeled in Knizhnik et al. (2015), is meant to demarcate the internal boundary of the flux system, similar to how coronal holes on the Sun mark a boundary between open and closed flux. Thus, while this internal region contains no open flux, it is meant to mimic the boundary between flux systems that exists near coronal holes.

The simulation mesh for this study is specified, as in KAD15, such that we resolve very finely that part of the domain where these photospheric flows are imposed and the coronal magnetic field is influenced by the surface stresses. We use $4 \times 14 \times 14$ elemental blocks to span the simulation domain, each containing $8 \times 8 \times 8$ uniform, cubic grid cells. In the highly resolved portion of our simulation volume, which included the convective cells, the lanes between them, the untwisted region in the interior, and a buffer region around the outer perimeter of the pattern, we applied two additional levels of refinement, such that each rotation was covered by 32 grid points across its diameter. Closer to the side walls, the grid was allowed to coarsen by two levels, such that the grid spacing near the walls was a factor of four larger than in the interior. With this grid distribution, the ratio of the smallest grid spacing to the height of the box was about 0.001, resulting in a magnetic Reynolds number $R_m \sim 10^3$.

The key difference from our previous work, described in detail below, is that the sense of rotation of each cell, as well as the angular orientation of the entire pattern of cells, is changed randomly during the course of the simulations. The top plate mirrors the bottom plate at all times. These variations are meant to model more faithfully the random nature of the Sun’s surface convection. Below, we describe how the relative fraction of cells rotating in the opposite sense affects the amount of magnetic helicity injected into our simulations.

3. MAGNETIC HELICITY INJECTION

Magnetic helicity is a topological quantity describing linkages in the magnetic field, such as twist, shear, and writhe. In a volume $V$ bounded by a surface $S$, which need not be a magnetic flux surface, the relative magnetic helicity is given by (Finn & Antonsen 1985)

$$H = \int_V (A + A_P) \cdot (B - B_P) \, dv.$$  \hspace{1cm} (3.1)

Here $B = \nabla \times A$ is the magnetic field in the volume $V$, generated by the vector potential $A$, and $B_P = \nabla \times A_P$ is a current-free field ($\nabla \times B_P = 0$) satisfying $B_P \cdot \hat{h}_S = B \cdot \hat{h}_S$. The rate of change of the helicity in Equation (3.1) is given in ideal MHD by

$$\frac{dH}{dt} = 2 \oint_S [(A_P \cdot B) - (A_P \cdot B)v] \cdot dS.$$  \hspace{1cm} (3.2)

The first term represents the effects of motions on the boundary with velocity $v$, while the second term represents the emergence or submergence of magnetic field through the boundary. As a result, the magnetic helicity in our simulation changes only due to motions on or through the boundary. In highly conducting ($R_m \gg 1$) plasmas, such as the corona, magnetic helicity is conserved even in the presence of a small localized resistivity that enables magnetic reconnection (Woltjer 1958; Taylor 1974, 1986; Berger 1984). Since no new flux is being injected at our top and bottom boundaries, and the side boundaries are sufficiently far away that no flux leaves the system, the rate of change of magnetic helicity in our simulation reduces to

$$\frac{dH}{dt} = 2 \oint_S (A_P \cdot B) \cdot dS.$$  \hspace{1cm} (3.3)

The magnetic helicity $H_0$ injected into a single flux tube (i.e., one top/bottom pair of rotation cells) over one cycle is obtained by integrating Equation (3.3) from $t = 0$ to $t = \tau$, employing Equations (2.6)–(2.8) and the vector potential for the uniform, current-free initial field,

$$A_P = \frac{B_0}{2} (\hat{y}z - \hat{z}y).$$  \hspace{1cm} (3.4)

If the sense of rotation is clockwise, the resulting positive helicity injected is

$$H_0 = 2 \times 10^{-2}.$$  \hspace{1cm} (3.5)

Trivially, the net helicity $H_S$ injected into $N$ such flux tubes all twisted in the clockwise sense, as in KAD15, is

$$H_S = NH_0.$$  \hspace{1cm} (3.6)

In this paper, we generalize to cases in which $N_r/N_c$ cells rotate clockwise/counterclockwise and inject positive/negative helicity, with $N_r + N_c = N$. The net helicity injected into the corona then becomes

$$H_S = (N_r - N_c)H_0.$$  \hspace{1cm} (3.7)

The case studied in KAD15 has $N_r = N_c = 0$, so Equation (3.7) reduces to Equation (3.6). If, on the other hand, $N_r = N_c$, equal numbers of cells rotate in each sense and the net injected helicity vanishes, $H_S = 0$. In our simulations described below, we allowed $N_r$ and $N_c$ to vary from cycle to cycle.
cycle. The net helicity injected into the corona after \( M \) cycles therefore is
\[
H_\Sigma = \sum_{i=1}^{M} (N_{+,i} - N_{-,i}) H_0,
\]
where \( N_{+,i}/N_{-,i} \) is the number of cells that rotate clockwise/counter-clockwise during cycle \( i \).

In each simulation, we assign a probability \( k \) that any individual top/bottom pair of cells injects positive helicity and a probability \( 1 - k \) that the pair injects negative helicity. A random number \( \kappa_i \in [0, 1] \) is generated for each pair of cells \( j \in [1, N] \) during each cycle, and \( \kappa_i \) is compared with \( k \) to determine whether the sense of rotation is clockwise or counterclockwise over that cycle. The helicity \( H_j \) injected by the \( j \)th pair of cells is
\[
H_j = H_0 \times \begin{cases} 
+1 & \text{if } \kappa_j \leq k; \\
-1 & \text{if } \kappa_j > k.
\end{cases}
\]

On average, the expectation is that, during each cycle, a fraction \( 2k - 1 \) of the maximum positive helicity \( NH_0 \) will be injected into the corona,
\[
\langle H_\Sigma \rangle = (2k - 1)NH_0.
\]
Throughout the remainder of the paper, we will refer to this fraction,
\[
p = 2k - 1,
\]
as the helicity preference, \( p \). It is the net fractional helicity associated with \( k \).

For this paper, the cases \( k = 0.75 \) and \( k = 0.5 \) were simulated to complement the case \( k = 1 \) previously presented in KAD15. For reference, the expectation values of the net helicity injected per cycle are
\[
\langle H_\Sigma \rangle = pNH_0 = NH_0 \times \begin{cases} 
1.0 & \text{if } k = 1.0; \\
0.5 & \text{if } k = 0.75; \\
0.0 & \text{if } k = 0.5.
\end{cases}
\]

Hereafter, we refer to these simulations as having helicity preferences \( p = 100\% \), \( p = 50\% \), and \( p = 0\% \), respectively. In every case, we use the precise number of cells injecting each sign of helicity during each cycle to evaluate \( H_\Sigma \) in Equation (3.8). That prediction is compared to the instantaneous value \( H(t) \) calculated directly from the volume integral of Finn & Antonsen (1985) for the relative magnetic helicity in the simulation, as described in KAD15.

The helicity preference \( p \) introduces one aspect of randomness into our simulations through the assignment of a clockwise/counter-clockwise sense of rotation (and positive/negative helicity) to each pair of rotation cells during each cycle of rotation. In order to emulate the stochastically shifting spatial pattern of convection on the solar surface and investigate its effect on coronal structure, we also introduce a second aspect of randomness into a separate set of simulations. After each cycle of rotations, we displace the entire hexagonal cellular pattern shown in Figure 2 by a randomly chosen angle \( \theta \in [0^\circ, 60^\circ] \) about its central vertical axis \((y, z) = (0, 0)\). For simplicity, the same angular displacement is applied to both the top and bottom plates, so that the top/bottom pairs of rotation cells remain aligned as before. Now, however, the random displacement means that the rotation cells will, in general, encompass parts of multiple neighboring flux tubes that were twisted during the previous cycle of rotations. The ensuing cycle therefore introduces braiding, as well as twisting, into the coronal magnetic field between the plates. As we will show, however, this displacement-induced braiding has no effect on the rate of helicity accumulation in the corona, and has only a minor influence on the smoothness of the induced magnetic structure. This result concurs with the qualitative conclusions of Zhao et al. (2015) from a much simpler simulation setup and is analyzed quantitatively here for the first time.

### 4. RESULTS

In this section, we first describe the results of our simulations with both fixed and randomized patterns and for the various helicity preferences. Then, we analyze those results in the context of filament-channel formation and the smoothness of coronal loops.

#### 4.1. Fixed Cellular Pattern

The first set of simulations holds the cellular pattern fixed in the orientation shown in Figure 2, randomizing only the sense of rotation of the individual cells as described in Section 2. To compare the \( p = 50\% \) simulation most consistently with the \( p = 100\% \) case presented in KAD15, we ran it for twice as many cycles (42 versus 21). As shown by Equation (3.8), the expectation value for the net injected helicity is the same. The \( p = 0\% \) case, in contrast, accumulates a net helicity only due to statistical fluctuations away from its average value of zero. For that case, therefore, we simply ran the simulation for 21 cycles. All three simulations were then extended for five additional cycles without imposing any rotational motions, to allow transients to die down and the system to relax toward a quasi-equilibrium final state.

As an example, the top row of Figure 3 shows the azimuthal component of the velocity on the bottom plate, \( V_\phi(x = 0, y, z) \), during the first (left) and second (right) cycles of the \( p = 50\% \) case. For both the \( p = 50\% \) and \( p = 0\% \) cases, each individual cell does not necessarily preserve its sense of rotation from one cycle to the next. The sense of rotation is assigned randomly at each cycle, as given above in Equation (3.9). For \( p = 100\% \), all cells rotate in the same sense in the first and second—indeed, throughout all—cycles.

Figure 4 shows the analytically expected (solid line) and numerically calculated (filled circles) helicities for each simulation. The orange (\( p = 100\% \)) curve is the same as that presented in KAD15. After 21 cycles, the rotation cells have injected \( H = 36 \) units of helicity. The red (\( p = 50\% \)) curve shows that approximately the same \( H = 36 \) units are injected over twice the time (see Equation (3.12)) for the 50% preference. The blue (\( p = 0\% \)) curve shows, as expected, that almost no net helicity is accumulated over its first 21 cycles. All of the numerically calculated curves match very well the analytical values at each cycle, demonstrating that our simulations conserve helicity to a very high degree of accuracy. Therefore, the evolution of the magnetic field in our simulations is due predominantly to convection and reconnection, rather than to numerical diffusion that would dissipate helicity.
4.2. Randomly Displaced Patterns

The second set of simulations is identical to the first, except that we also displace the entire hexagonal cellular pattern through a random angle after each cycle as described in Section 2. In these setups, different flux tubes wrap around each other, creating a braided field, in addition to being twisted by the rotation cells. We ran each simulation for the same number of cycles as in the fixed-pattern cases and for the same values of \( p \).

As an example, the bottom row of Figure 3 shows \( \vec{V}_\phi \) on the bottom plate for the case of a fixed pattern (top) and a random pattern (bottom) with \( p = 50\% \) during the first (left) and second (right) cycles. Red/yellow (blue/teal) represent clockwise/negative (counterclockwise/positive) velocity.

**Figure 3.** \( \vec{V}_\phi \) on the bottom plate for the case of a fixed pattern (top) and a random pattern (bottom) with \( p = 50\% \) during the first (left) and second (right) cycles. Red/yellow (blue/teal) represent clockwise/negative (counterclockwise/positive) velocity.

**Figure 4.** Helicity for fixed-pattern simulations. Solid curves are the analytically expected helicity based on the number of convective cells injecting positive and negative helicity during each cycle; filled circles are the numerically calculated values. The orange, red, and blue curves represent the cases of \( p = 100\% \), \( p = 50\% \), and \( p = 0\% \), respectively.

**Figure 4.** Helicity for fixed-pattern simulations. Solid curves are the analytically expected helicity based on the number of convective cells injecting positive and negative helicity during each cycle; filled circles are the numerically calculated values. The orange, red, and blue curves represent the cases of \( p = 100\% \), \( p = 50\% \), and \( p = 0\% \), respectively.

4.2. Randomly Displaced Patterns

The second set of simulations is identical to the first, except that we also displace the entire hexagonal cellular pattern through a random angle after each cycle as described in Section 2. In these setups, different flux tubes wrap around each other, creating a braided field, in addition to being twisted by the rotation cells. We ran each simulation for the same number of cycles as in the fixed-pattern cases and for the same values of \( p \).

As an example, the bottom row of Figure 3 shows \( \vec{V}_\phi (x = 0, y, z) \) during the first (left) and second (right) cycles of the \( p = 50\% \) case. Like the corresponding fixed-pattern cases, in the random \( p = 50\% \) and \( p = 0\% \) cases, each individual cell does not necessarily maintain its sense of rotation. Unlike the fixed-pattern cases, however, the pattern itself is displaced by a random angle after each cycle. Except for the \( p = 100\% \) case, the randomized patterns (Figure 3, bottom row) exhibit different distributions of color than the fixed patterns (Figure 3, top row). We used different sequences of random numbers \( n_j \) to set the clockwise/counterclockwise sense of rotation of the individual rotation cells in the two sets of simulations. The random angular displacements of the
cellular pattern between the first and second cycles are evident by comparing the left and right columns for each helicity preference.

Figure 5 shows the analytically expected (solid line) and numerically calculated (filled circles) helicities for the various cases. Although the average helicities (Equation (3.12)) injected into the corona are identical for each value of \( p \), the precise helicities (Equation (3.8)) actually injected differ between the fixed and randomized cases due to statistical fluctuations. Thus, the curves in Figures 4 and 5 are slightly different for \( p \neq 100\% \). The orange (\( p = 100\% \)) curve shows that the helicity injected for the 100% preference is identical for the fixed and randomized patterns, as expected. The red (\( p = 50\% \)) curve shows that this case injects slightly more helicity in twice the time. The blue (\( p = 0\% \)) curve shows that, as before, almost no net helicity is injected in this case. In all cases, we again find excellent agreement between the numerically calculated and analytically expected helicities.

4.3. Formation of Filament Channels

Figure 6 shows the azimuthal component of the magnetic field in the horizontal midplane, \( B_\phi (x = 0.5, y, z) \), halfway through the first cycle of twist for each simulation. At this early stage, each case exhibits the characteristic hexagonal pattern of rotation cells. For \( p = 100\% \), every cell injects the same sign of \( B_\phi \), so adjacent flux tubes always have oppositely directed twist fields and are able to reconnect readily. For the \( p = 50\% \) and \( p = 0\% \) cases, in contrast, adjacent twist fields sometimes are in the same direction. On average, this is true half the time in the \( p = 0\% \) case, suppressing reconnection between adjacent flux tubes whose twist fields are parallel rather than antiparallel.

The effect of the helicity preference on the formation of filament channels can be seen clearly in the final-time \( B_\phi \) maps in Figure 7. The \( p = 100\% \) case has accumulated oppositely signed bands of twist flux at the outer and inner boundaries of the hexagonal pattern, as described previously in KAD15. These bands result from the inverse-cascade of twist flux from small to large scales due to reconnection, collecting at the boundaries to form filament channels according to the helicity-condensation model (Antiochos 2013). The \( p = 50\% \) case has been run out twice as long in order to accumulate roughly the same helicity, and it has acquired similar bands of twist at the outer and inner boundaries of the hexagonal pattern. Thus, despite the one-third (25%/75%) of twist fields on neighboring flux tubes that are parallel rather than antiparallel in this simulation, sufficient reconnection has occurred to enable the helicity to condense at the flux-system boundaries here, as well. The shapes of the filament channels differ slightly in the \( p = 100\% \) and \( p = 50\% \) cases. The contrast is most evident in the randomized-pattern simulations, where the twist flux has a very uniform, circular appearance in the \( p = 100\% \) case, while its structure is more ragged, especially at the inner boundary, in the \( p = 50\% \) case. At the largest scales, however, these two cases yield qualitatively identical outcomes: the twist flux forms two bands of opposite polarity at the boundaries of the hexagonal pattern of rotations. In the corona, such bands would manifest themselves as extended, sheared filament channels.

The sharply contrasting \( p = 0\% \) case, on the other hand, displays a very different final-time appearance. No long, coherent bands of twist flux have accumulated at either the outer or inner boundaries of the hexagonal pattern. Instead, there are localized concentrations of twist flux dispersed across the interior of the pattern, as well as at its boundaries. Because zero net helicity is injected into this system, on average, zero net twist flux is available to be transported by reconnection to the hexagonal boundaries where it can accumulate. Turning this argument around, if the net condensed twist flux were finite, then the net helicity would be finite as well. We demonstrated this result analytically in KAD15. Consequently, the helicity-condensation process does not form filament channels in the case of a 0% helicity preference, where positive and negative helicities are injected more or less equally.

These examples demonstrate that the helicity preference plays a major role in the organization of the twist flux and the formation of filament channels. The \( p = 50\% \) case forms similarly strong, although rather more azimuthally asymmetric, bands of twist flux over twice the time as the \( p = 100\% \) case. As is argued below, the timescale for filament-channel formation is inversely proportional to the average net helicity preference, i.e., to \( p \). This dependence is supported further by the absence of filament-channel structure in the \( p = 0\% \) case, whose predicted timescale for channel formation is infinite.

4.4. Accumulation of Twist Flux

The results above demonstrate that there are major qualitative and quantitative contrasts between the results for different helicity preferences, but more minor differences between the fixed and random patterns for a given helicity preference. We begin the quantitative analysis of our simulations by calculating the positive twist flux \( \Phi_{tw}^+ \) through the \( z = 0 \) plane,

\[
\Phi_{tw}^+ = \int_0^L dx \int_0^L dy \int_0^L dz B_{\phi}^+(x, y, z = 0),
\]

where \( B_{\phi}^+ \) is the positive component of the magnetic field.
where the corresponding positive twist field $B_{tw}^+$ is

$$B_{tw}^+ = \frac{1}{2} (B_0 + |B_0|) \geq 0.$$  \hfill (4.2)

The twist flux $\Phi_{tw}^+$ is plotted in Figure 8 for both the fixed (solid curves) and random (dashed curves) patterns. All six simulations exhibit a brief initial phase of ideal evolution, of about one rotation cycle in duration, in which twist flux is injected into and stored in individual, noninteracting flux tubes. At this stage, the sense of rotation of adjacent cells is irrelevant to the accumulation of twist flux. As the twisting continues, however, the flux tubes expand laterally to compress the volume between them. This forms and strengthens electric current sheets between neighboring tubes that have antiparallel twist fields. Reconnection between such tubes commences during subsequent twist cycles. This process, together with the randomization of the sense of rotation of individual cells (for $p = 100\%$) and of the orientation of the cellular pattern (for the random
cases), causes the curves to deviate increasingly from one another at later times.

The two cases with nonzero net helicity preferences, $p = 100\%$ (orange) and $p = 50\%$ (red), show relatively small differences between the fixed and random patterns for fixed $p$. Over the full duration of the simulations, each preference accumulates essentially the same twist flux. The slightly larger values for the $p = 50\%$ case reflect the slightly larger magnetic helicities accumulated in those simulations (Figures 4 and 5) compared to the $p = 100\%$ case. All four of these simulations eventually accumulate twist flux at a rate per cycle that is in good agreement with the calculation by KAD15 (their Equation (4.13) and Figure 11),

$$\Delta \Phi_{nw} = \frac{1}{2} \frac{\Delta \langle H_s \rangle}{\Phi_N} = \frac{p}{2} \frac{H_0}{\Phi_0}$$

where each of the $N$ twisted flux tubes contains $\Phi_0$ units of magnetic flux. The expression in Equation (4.3) assumes that the twist flux $\Phi_{nw}$ condenses at the outer boundary of the flux system, which occurs in our simulations with $p = 100\%$ and $p = 50\%$. In a spirit similar to KAD15 (their Equation (4.28)), we calculate the filament-channel formation time $\tau_{fc}$ over which

Figure 7. Twist field $B_\phi$ (color shading) in the midplane ($x = 0.5$) at the end of the fixed-pattern (left) and random-pattern (right) simulations. From top to bottom are the $p = 100\%$, $p = 50\%$, and $p = 0\%$ cases. Red/yellow (blue/teal) represent clockwise/negative (counterclockwise/positive) twist.
a critical amount of twist magnetic flux $\Delta \Phi_{\text{tc}}$ accumulates,

$$\tau_{\text{tc}} = \frac{\Delta \Phi_{\text{tc}}}{\Delta \Phi_{\text{tw}}} = \frac{2}{p} \left( \frac{\Phi_0}{H_0 \tau_0} \right) \Delta \Phi_{\text{tc}},$$

where $\tau_0$ is the duration of one twist cycle. Equations (4.3) and (4.4) quantitatively express the observed factor-of-two differences in twist-flux accumulation rates and filament-channel formation times between our $p = 100\%$ and $p = 50\%$ cases shown in Figure 8. They also predict how these quantities should change for other helicity preferences $p$.

Our last two simulations have zero net helicity preference, $p = 0\%$. For this case, the predicted accumulated twist flux $\Delta \Phi_{\text{tw}}$ vanishes and the filament-channel formation time $\tau_{\text{tc}}$ is infinite. We observe in these simulations (Figure 8) that the fluctuations in the twist flux are relatively large, and the average amount of flux saturates after about 10 cycles have elapsed. Thereafter, the average seems to be statistically quasi-steady, increasing or decreasing randomly according to the cycle-to-cycle variations of the sign of twist in individual rotation cells (in both simulations) and of the orientation of the cellular pattern (in the random-pattern simulation only). Evidently, these simulations have reached a roughly steady-state balance between the rates of twist-flux injection by the twisting motions and extraction via a combination of untwisting motions and reconnection between antiparallel twist fields.

4.5. Smoothness of Coronal Loops

We have seen that, when it is effective, the helicity-condensation process transports twist via reconnection to the boundaries of the flux system, where it condenses. This leaves the interior of the system relatively smooth and untangled. The final configuration then corresponds to a corona with strong shear concentrated at its PILs and laminar coronal loops in interior regions away from its PILs. This result can be seen clearly in Figure 7. In the $p = 100\%$ and $p = 50\%$ cases with nonzero net helicity preferences, at a glance, the interior of each flux system seems very smooth, with little twist evident. A careful comparison of the two cases reveals that the annular region between the filament channels is somewhat more inhomogeneous for $p = 50\%$, with localized, small-amplitude twists of both signs accumulating in the interior. As might be anticipated, these concentrations are somewhat less noticeable for the simulations with randomly displaced patterns than for their fixed-pattern counterparts.

The $p = 0\%$ case is strikingly different in appearance from the $p = 100\%$ and $p = 50\%$ cases. For the fixed pattern especially (bottom row, left column of Figure 7), small-scale, coherent concentrations of twisted field are present throughout the interior of the hexagonal flow region. In addition, the magnitude of the accumulated twist is significantly smaller than in the $p = 100\%$ and $p = 50\%$ cases. This is due to both the random untwisting of previously twisted field lines in successive cycles and the zero net twist flux that can accumulate globally and be transported to the flux-system boundaries. The local twist concentrations that are formed appear and disappear transiently as the system evolves. Each such concentration has a lifetime of the order of one rotation period of the convection cells. Taking the rotation period to be of the order of a day, or the lifetime of a typical supergranule, these concentrations of twist should easily survive for timescales long enough to be detected remotely. The lack of such observations indicates that the photosphere likely injects helicity with a significant preference.

To demonstrate the stark difference in the amount of complex fine structure for the different helicity preferences, we plot in the left panels of Figure 9 a set of magnetic field lines from the same set of fixed points for the fixed-pattern cases of $p = 100\%$, $p = 50\%$, and $p = 0\%$. All of the field lines are rooted in the interior of the hexagonal region, which represents the “loop” portion of the corona. The bottom plane shows the magnitude of the magnetic field, which exhibits a pattern similar to that shown in Figure 7. Although only a sample of field lines is shown, these are representative of the field lines in the rest of the “loop” portion of the corona. The striking difference in the amount of complex structure in the corona is immediately evident by comparing the $p = 100\%$ and $p = 50\%$ cases with the $p = 0\%$ case.

In the last simulation, the field lines are twisted and braided around one another in a complicated fashion. In the first two simulations, in contrast, the field lines are quite smooth and laminar. There is no braiding or tangling of field lines in the upper two panels; any appearance of such intertwining is due to the combination of field-line leaning and line-of-sight effects. These images rather closely resemble topologically the smooth, laminar TRACE loops shown in Figure 1, albeit in a plane-parallel geometry. In this sense, the $p = 100\%$ and $p = 50\%$ coronal loops are like those observed on the Sun, while the $p = 0\%$ coronal loops with their tangles and braids clearly deviate quite strongly from observations. In the right panels of Figure 9, a different set of field lines is plotted for the random-pattern cases of $p = 100\%$, $p = 50\%$, and $p = 0\%$. These sample field lines are traced from the same footpoints in each case, and they too demonstrate the presence of only smooth, laminar field lines in the first two cases, and a significant amount of braiding or tangling in the last case.

As a rough measure of the braiding we can use the number of field line crossings (e.g., Berger & Asgari-Targhi 2009) Consider the randomly driven case on the right of Figure 9. For the $p = 100\%$ case we count three crossings, and roughly the same number, only two, for the $p = 50\%$ case. Note that this is
for 10 field lines; consequently, most of the field lines show no crossings. As stated by Berger & Asgari-Targhi (2009), “Observations of coronal loops ... display a well-combed set of almost parallel loops.” The field lines of the cases of nonzero helicity injection are, to a good extent, compatible with this statement. The $p = 0\%$ case, on the other hand, exhibits 17 field line crossings, and many of the field lines show multiple crossings. We conclude that the appearance of this field system is quantitatively different than what is observed for typical coronal loops.

4.6. Distribution of Twist Flux

To quantify the amount of small-scale structure in the various cases, we calculated the angle-averaged azimuthal magnetic field $\langle B_\phi(r) \rangle$ and its rms deviation $\delta B_\phi(r)$ in the midplane ($x = 0.5$). Specifically, we evaluated (for $m = 1, 2$)

$$\langle B_\phi^m(r) \rangle = \frac{1}{2\pi} \int_0^{2\pi} B_\phi^m(x = 0.5, y, z)d\phi,$$

(4.5)

$$\delta B_\phi(r) = \sqrt{\langle B_\phi^2(r) \rangle - \langle B_\phi(r) \rangle^2},$$

(4.6)

where $r = \sqrt{y^2 + z^2}$. A discrete $\rho$ grid, with the same spacing as the $y$ and $z$ grids, was adopted, and all cell-center positions ($y, z$) were grouped into corresponding $r$ intervals to calculate the integrals in Equation (4.5). Figure 10 shows $\delta B_\phi(r)$ at the end of the random-pattern simulations for each value of $p$ (color-coded). The rms deviations $\delta B_\phi(r)$ shown in Figure 10 all exhibit small-scale statistical fluctuations. The average amplitude of the fluctuations is smallest for the $p = 100\%$ helicity preference (orange) and largest for the $p = 0\%$ preference (blue), where they are nearly three times as large. A very similar trend was seen in the fixed-pattern simulations. As is evident from Figure 11, these localized fluctuations in twist field for the zero-helicity case correspond to field-line tangling that would easily be observed if it were present in the real corona.

Figure 9. A sample of “coronal loop” field lines (black) plotted at the end of the fixed-pattern (left) and random-pattern (right) simulations for $p = 100\%$ (top), $p = 50\%$ (middle), and $p = 0\%$ (bottom). The field lines are plotted inside the interior of the hexagonal region on the bottom plate, which shows magnetic field magnitude (color shading). The initial potential field is uniform and straight.
Nevertheless, there remains an obvious and substantial difference between the coronal-loop and filament-channel regions for \( p = 50\% \).

The \( p = 0\% \) case displays a strikingly different appearance from the other helicity preferences \( p \), as we found for our diagnostics described previously. The strong transverse fields \((|B_n| > |B_a|)\) that accrued at the flux-system boundaries in the \( p = 100\%\) and \( p = 50\%\) simulations are completely absent for \( p = 0\%\) in the bottom row of Figure 11. There is essentially no distinction between the structure at the boundaries and in the interior of the hexagonal region of rotation cells. Thus, there is absolutely no evidence for the formation of filament channels in this case. The figure shows that the region hosts a rather homogeneous mixture of moderately twisted \((|B_n| \approx 0.5|B_a|\) or smaller) field lines with structure at small scales. This structure is both stronger in amplitude and more homogeneous in distribution than for the \( p = 50\%\) case (middle row).

More importantly, as we noted above in Section 4.5 and showed in Figure 9, there is obvious evidence for tangling of field lines in the \( p = 0\%\) simulations. This braiding is not observed in the \( p = 100\%\) and \( p = 50\%\) cases. These findings also are consistent with our results for the twist component of the magnetic field, discussed in Section 4.3 and shown in Figure 7, which is rather randomly oriented and has small amplitude in the interior of the hexagonal region for \( p = 100\%\) and \( p = 50\%\). The concentrations are equally random in direction but noticeably larger in amplitude at smaller scales for \( p = 0\%\), consistent with the presence of braiding and tangling in that case. Finally, we point out that the contribution of these fluctuations to the global net helicity is expected to be small across all of our cases, due either to their small amplitudes, especially for \( p = 100\%\), or to their random orientations in the clockwise or counterclockwise directions, especially for \( p = 0\%\).

## 5. IMPLICATIONS FOR CORONAL STRUCTURE

The results described in the preceding section have important implications for the global structure of the solar corona. Our findings demonstrate that the magnetic helicity preference \( p \) plays key roles in determining how the corona is structured and the timescale over which that structure develops. The contrast is particularly strong between the cases with \( p = 100\% \) and \( p = 0\%\) net helicity preferences and randomly displaced cellular patterns shown at the top and bottom right, respectively, in Figures 7 and 11. For the \( 100\% \) preference, the twist flux condenses into two primary bands with opposite senses of twist at the inner and outer boundaries of the hexagonal region of rotation cells, with much less twist left in the interior. These concentrations and dilutions are reflected in the strength of the transverse field on the associated magnetic field lines, which is quite large near the two boundaries but is much smaller in the interior. For the \( 0\% \) preference, on the other hand, the twist flux does not condense into any recognizable global-scale structure, and the field lines have an essentially homogeneous distribution of intermediate lengths.

Our intermediate case with \( p = 50\% \) preference exhibits some features of both of the previous limiting cases but, importantly, qualitatively resembles more closely the results for the \( 100\% \) preference. The bands of condensed twist flux still form, albeit twice as slowly and with intrusions of significantly less twisted field into them. The transverse field strengths are...
greater at the hexagonal-region boundaries than in the interior, although with less contrast than for the 100% preference. Extrapolating to other cases with even smaller preferences but nonzero net helicities—say, \( p = 25\% \)—we would expect these trends to continue, with a further increase in the filament-channel formation time (another doubling for \( p = 25\% \)) and in the amount and homogeneity of small-scale fluctuations in the interior of the hexagonal region.

Perhaps the clearest example of the effect of helicity preference on the structure of the closed-field corona is evident in Figure 9. The smoothness of the \( p = 100\% \) and \( p = 50\% \) coronal loops is manifestly different than the complexity of the \( p = 0\% \) coronal loops. The 0% case, shown in the bottom row of the figure, clearly exhibits numerous braided field lines. No such tangles are seen in either the 100% or 50% cases, shown in the upper rows, although the residual twist does cause some

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Figure 11. Maps of \(|B_{tr}|/|B_n|\) at the end of the fixed-pattern (left) and random-pattern (right) cases with \( p = 100\% \) (top row), \( p = 50\% \) (middle row), and \( p = 0\% \) (bottom row).
field lines to lean and thus to pass in front of others along the line of sight. Observations of the coronal magnetic field, meanwhile, invariably reveal smooth, laminar loops that closely resemble those observed in the $p = 100\%$ and $p = 50\%$ cases (see Figure 1), rather than those observed in the $p = 0\%$ case. Our simulations indicate therefore that the photosphere must inject a significant net helicity so that tangled structures such as those produced in our $p = 0\%$ case are not observed.

The simulation setups assumed in this paper are highly simplified compared to the complex photospheric polarity patterns exhibited by the Sun, illustrated by Figure 1. Nevertheless, the more-or-less homogeneous structure that we obtained for $p = 0\%$, shown in the bottom panels of Figures 7 and 11, obviously does not resemble the distinctly bimodal structure observed in the corona. In contrast, our results for both $p = 100\%$ and $p = 50\%$, shown in the other panels of those figures, do exhibit the bimodal characteristics of the corona: concentrations of twist at the flux-system boundaries in the form of highly nonpotential magnetic shear in filament channels, and generally smooth, untangled fields away from those boundaries in arcades of coronal loops. Therefore, a principal conclusion of our work is that the Sun must inject helicity into the corona with a significant hemispheric preference, favoring negative helicity forming left-handed structures in the north, and positive helicity forming right-handed structures in the south. These preferences are reflected in the observed statistics of solar filaments, sigmoid, and sunspot whorls. They also have been detected directly in the photospheric convection, although that measurement is very challenging, near the limits of observational resolution.

Our simulations are simplified in another important way compared to the Sun: there is no source of new, weakly sheared or unsheared magnetic flux in our domain, nor is there a sink of the strongly sheared flux condensing in the filament channels. Flux emergence from below the photosphere constantly injects fresh magnetic field into the corona, and CMEs regularly eject sheared magnetic field and its entrained magnetic helicity away from the Sun into the heliosphere. The characteristic timescales for these phenomena compete directly with the filament-channel formation time $\tau_c$ to establish a quasi-steady balance among these processes and the coronal magnetic structure that is observed. Such a calculation is well beyond the scope of this paper, but a first attack on the problem could be taken using global force-free modeling of the corona (e.g., Mackay et al. 2014). We point out that $\tau_c$ in Equation (4.4) is inversely proportional to the product of the net helicity preference $p$ and the angular rotation rate $\omega_0$ of the twisting motions ($H_0 \propto \omega_0 \tau_c$). If this product $p \omega_0$ is too small or too large, then the filament-channel formation time will be too long or too short compared to the emergence and ejection timescales, and the model is unlikely to replicate the Sun’s observed appearance. We anticipate that global modeling of the combined processes could provide rigorous bounds on the rotation rate $\omega_0$, to complement the narrowly constrained range of values available to the net helicity preference, $0\% \leq p \leq 100\%$.

Our simulations show that random displacements of the pattern of photospheric convection have only a secondary effect on the resulting coronal structure. This also is evident in Figures 7 and 11 by comparing the left (fixed-pattern) and right (random-pattern) columns for each helicity preference. The latter structures are somewhat smoother than the former, especially at small scales, but the large-scale organization is no different between them. This conclusion agrees with that reached by Zhao et al. (2015), who used a much simpler setup with far fewer rotation cells. The fundamental reason why the random convection pattern shows a very similar coronal end state to that of the fixed pattern is that, as shown by numerous simulations, magnetic reconnection is efficient at destroying all higher-order topological features such as braiding, leaving only the global helicity (e.g., Wilmot-Smith et al. 2010). Our results are fully in agreement with this hypothesis.

In summary, this paper presents the first simulations of the evolution of the coronal magnetic field driven by photospheric motions with varying helicity preference. Our results agree well with the helicity condensation model of Antiochos (2013), which accounts for both the formation of sheared filament channels in the corona above PILs and the smooth, laminar appearance of coronal loops elsewhere, especially on the quiet Sun. By transferring the magnetic twist injected by photospheric motions to ever larger scales, reconnection concentrates the twist at the boundaries of flux systems (i.e., at the PILs) while diluting it throughout their interiors, so long as the system has a significant net helicity preference. The remarkable implication of the model is that the global organization of the magnetic shear in the solar atmosphere is a direct consequence of local twisting of the footpoints of coronal flux tubes by surface convection. Even more remarkable, and somewhat counterintuitive, is our finding that in order for the hot closed corona—the loops—to exhibit no complex fine structure such as tangling or braiding, a great deal of helicity with a significant net preference for one sign or the other must be injected. When the photospheric driving motions have a clear helicity preference, the majority of the injected twist ends up localized around PILs. The corona is therefore a striking example on cosmic scales of a strongly self-organized system.

K.J.K. acknowledges the use of post-processing codes originally written by Benjamin Lynch and Peter Wyper. K.J.K. received funding for this work through a NASA Earth and Space Science Fellowship. The numerical simulations were performed under a grant of High-End Computing resources to C.R.D. at NASA’s Center for Climate Simulation. S.K.A. and C.R.D. were supported, in part, by grants from NASA’s Living With a Star and Heliophysics Supporting Research programs.

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