On the thermodynamics of moving bodies

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Abstract. We model the situation in which a photon detector moves with constant velocity in a Minkowski heat-bath by considering an Unruh-DeWitt detector, with variable energy gap $\hbar \omega$, moving on a brane, at a non-zero (Unruh) temperature $T$ due to acceleration in an orthogonal direction. We compute the angular response for a 2-brane. At low velocity we find agreement with the standard Doppler shift formula in the limit $\hbar \omega \gg T$ in which the photon gas is classical; otherwise there is a discrepancy. At relativistic velocities our result disagrees with the standard formula even when $\hbar \omega \gg T$, and above a critical velocity there is no response from the detector within a ‘backward’ cone. We discuss potential implications for observations of the cosmic microwave background radiation. [Talk given by JGR at this 1st Mediterranean Conference in Classical and Quantum Gravity].

1. Introduction

One might suppose that the formulation of thermodynamics for systems in uniform relative motion should follow in a straightforward way from its formulation in the rest-frame of a heat-bath at temperature $T$ and the kinematics of special relativity. Planck [1] and Einstein [2] evidently thought so in 1907 when they proposed that a body in uniform motion with velocity $v$ should have temperature $T \sqrt{1 - v^2}$. The history up to 1921 is summarized by Pauli in his book of that year on relativity [3], and the logic is presented in detail in Tolman’s 1934 book [4]. Nevertheless, the conclusion of Planck and Einstein was disputed in the 1960s and the current consensus, which is sometimes traced back to a 1976 paper of Israel [5], appears to be that the term “temperature” should be reserved to mean the temperature in the rest frame because the thermal properties of a body in motion cannot be characterized by a temperature understood in any other way (see e.g. [6], but also e.g. [7] for a different view).

The frame-dependence of thermodynamic quantities acquired importance following the 1965 discovery of the cosmic microwave background radiation (CMBR) because one needs to take into account the (constant) velocity $v$ of a photon detector relative to the rest-frame of this radiation. Fortunately, there is no need to use any dubious temperature Lorentz transformation: given a gas of particles at temperature $T$ in its rest frame, the distribution of these particles in any other inertial frame may be unambiguously determined. The result for massless particles, which is especially simple, was derived in the context of the CMBR by Peebles and Wilkinson (PW) [8] (see also [9]): the distribution is anisotropic in a moving frame but is still thermal for fixed
direction, specified by unit vector $\mathbf{n}$, with an effective temperature

$$T_{\text{eff}} = \frac{T}{\gamma}, \quad \gamma = \frac{1}{\sqrt{1 - v^2}}.$$  

(1)

Applied to photons of the CMBR, this implies a direction-dependent photon density

$$n_{\text{PW}}(\omega, \mathbf{n}) d\omega d\Omega = \frac{\omega^2}{8\pi^3} \left(e^{\hbar\omega/T_{\text{eff}}} - 1\right) d\omega d\Omega,$$

(2)

where we have set Boltzmann’s constant and the speed of light to unity. This result, which may also be deduced by an application of the relativistic Doppler effect, is implicit in Pauli’s book although he did not express it in terms of an effective temperature. An integration over solid angle yields the ‘integrated’ number density

$$n_{\text{PW}}(\omega) d\omega = \frac{\omega T}{4\pi^2 \hbar \gamma v} \log \left[ \frac{1 - e^{-\hbar\omega(1-v)T}}{1 - e^{-\hbar\omega(1+v)T}} \right] d\omega.$$  

(3)

Note the factor of $v$ in the denominator, which ensures that the $v \to 0$ limit is non-zero; naturally, this limit yields the usual Planckian distribution.

Any practical application of these results also depends on an understanding of the behaviour of photon detectors. The purpose of this paper is to point out a feature of this behaviour that leads to an apparent correction to the formula (2). We say “apparent” because the effect may be attributable to the detector, but it is no less real for that and must be taken into account when interpreting observational results. As photon polarization will play no part in our analysis, apart from doubling the number of states, we shall replace photons by ‘scalar photons’, i.e. particle excitations of a massless scalar field; these have only one polarization state so there will now be an extra factor of $1/2$ on the right-hand side of (2). Also, the term “photon gas” can be understood to refer to any gas of neutral massless particles at zero chemical potential.

The behaviour of a photon detector moving inertially in a photon heat bath is relevant in the context of motion near the horizon of a black brane. Consider first a Schwarzschild black hole in thermal equilibrium with radiation at the Hawking temperature. A static photon detector will register a thermal spectrum at the local temperature determined by the Tolman law for thermal equilibrium. As shown by Unruh [10], the local temperature near the horizon has a purely kinematic origin that is due to the constant proper acceleration needed to keep the detector at a fixed distance $r$ from the horizon, which is locally indistinguishable from the acceleration horizon of the 2-dimensional Rindler spacetime. By ‘promoting’ the initial black hole spacetime to a $(4 + p)$-dimensional black $p$-brane spacetime, we arrive instead at a detector in a flat $(2 + p)$-dimensional spacetime with metric

$$ds^2 = -r^2 d\eta^2 + dr^2 + \sum_{i=1}^{p} dy_i^2.$$  

(4)

The temperature still varies with $r$ in the way required by the Tolman law: $T = \hbar/(2\pi r)$, which equals the Unruh temperature $T_U = \hbar a/(2\pi)$ because the proper acceleration of a static observer in this spacetime is $a = 1/r$. This is true of all static observers, so each flat $p$-surface at fixed $r$, which we may view for kinematic purposes as a $p$-brane, is accelerating uniformly with proper acceleration $a = 1/r$ [11]. Although the $r$-dependence of the temperature implies an anisotropy in the thermal distribution of photons arriving at a static detector, the distribution of photons arriving from within the subspace at fixed $r$, which is a $(p+1)$-dimensional Minkowski spacetime, will be isotropic.
What this shows is that a flat $p$-brane undergoing constant uniform proper acceleration in a direction orthogonal to its Minkowski worldvolume can be used to model a $(p+1)$-dimensional Minkowski heat bath. The same general idea underlies the GEMS approach to black hole thermodynamics: the black hole spacetime is globally and isometrically embedded in a higher-dimensional flat spacetime; it is then found that a static observer in the embedded spacetime is accelerating in the higher-dimensional flat spacetime, just such that the Unruh formula yields the local temperature in thermal equilibrium [12]. At spatial infinity, we have essentially the accelerating brane set up described above. It should be appreciated that there is no necessity to ascribe physical reality to the extra dimension(s): we may view the brane set-up as just a physically possible model in which to investigate the behaviour of photon detectors in a heat bath. It may be that other physical phenomenas can distinguish between a ‘real’ extra dimension (as in the case of a detector restricted to a fixed distance from a black $p$-brane horizon) and a ‘fictitious’ extra dimension (as in the GEMS program); e.g. the Coulomb force between point charges (although we are free to replace point sources by line sources in the extra dimension, if this is fictitious, such that the Coulomb force is the same as in the lower dimension). However, we consider only photons in a photon gas; if we restrict the photon detector to detect photons arriving isotropically, in the rest-frame of the gas, from directions ‘on the brane’ to which we confine the motion of the detector, then we expect the behaviour of the detector to be independent of other factors that are unobservable from its perspective.

Thus, we have a model in which a detector that is moving with constant velocity $v$ with respect to the rest-frame of a heat bath, at temperature $T$, is equivalent to a ‘braneworld’ detector that is ‘drifting’ with constant velocity $v$ on a brane that is itself accelerating in an additional dimension with proper acceleration $a = 2πT/\hbar$. The motion of the detector in the higher-dimensional spacetime is therefore one of ‘acceleration with drift’, which is a stationary motion. Stationary motions are those in which the worldline is an orbit of a timelike Killing vector field, which has the consequence that the detector’s excitation spectrum is time independent. We wish to compare the known thermal response of a ‘braneworld’ detector undergoing pure constant proper acceleration in the embedding spacetime with the time-independent response of a similar detector that is undergoing an ‘acceleration with drift’. We certainly expect to find a difference because the stationary motions of ‘pure’ acceleration and ‘acceleration with drift’ are distinct stationary motions according to the classification (for 4-dimensional Minkowski spacetime) of [13], which is based on the local curvature, torsion and hypertorsion of the detector’s worldline. There are six distinct stationary motions in all; the others are inertial motion, circular motion, a ‘null’ case and a ‘generic’ case. A Lorentz covariant version of this classification follows from consideration of the detector’s relativistic acceleration, jerk and snap [14].

We shall consider a simple two-state Unruh-DeWitt ‘monopole’ detector [10, 15] with variable energy gap $\hbar\omega$. The response of an Unruh-DeWitt detector to acceleration is known to have an ‘artificial’ dimension dependence: the thermal distribution of ‘photons’ seen by a detector undergoing constant proper acceleration in an odd-dimensional spacetime is apparently of Fermi-Dirac type rather than Bose-Einstein type [16]. Although this is indeed ‘artificial’, for reasons explained in [17, 18], it is relevant here because we would need to embed the 4-dimensional Minkowski braneworld of a 3-brane in a 5-dimensional Rindler spacetime: the $p = 3$ case of (4).

We avoid this problem by going down one dimension: we consider instead the 3-dimensional Minkowski braneworld embedded in 4-dimensional Rindler spacetime; i.e. we consider the $p = 2$ case of (4). This means that we can make use of results obtained previously for the response of an Unruh-DeWitt detector in a 4-dimensional Minkowski spacetime.

The integrated response, i.e. the result after integration over solid angle, has been extensively investigated for all types of stationary motion in a 4-dimensional Minkowski spacetime [13, 19], although only numerical results are known for ‘acceleration with drift’. Here, however, we need the angular-dependence of the excitation spectrum. We undertake this computation.
for ‘acceleration with drift’ in section 2, and our results are potentially of interest in the wider context of the problem of understanding the behaviour of particle detectors undergoing stationary motion; we comment further on this in the conclusions. Our result must be compared to the analog of (2) for a three-dimensional (braneworld) spacetime (i.e. $p = 2$), which is

$$n_{\text{PW}}(\omega, \varphi) d\omega d\varphi = \frac{\omega}{4\pi^2} \left( e^{\hbar \omega/T_{\text{eff}}} - 1 \right) d\omega d\varphi, \quad T_{\text{eff}} = T \frac{\sqrt{1 - v^2}}{1 - v \cos \varphi},$$

where $\varphi$ is the angle to the direction defined by the detector’s velocity. This assumes a single polarization state, as appropriate to ‘scalar photons’. Experimentally, one would deduce the ‘photon’ number density from the excitation rate of the detector. These quantities have the same dimension for $D = 3$ and we shall see that they are proportional for $v = 0$; we use this fact to fix the constant of proportionality. We then find a result that agrees with (5) for small drift velocity in the ultra-violet (UV) regime $\hbar \omega \gg T$, which is where we may think of the photons in the photon gas as classical particles. This is encouraging because it shows that the classical Doppler-shift effect is indeed reproduced by our accelerating braneworld set-up.

Despite this agreement in the UV regime, we find a discrepancy with (5) in the infra-red (IR) regime $\hbar \omega \ll T$. In the context of our brane set-up, there is a simple likely explanation for this discrepancy. As shown in [21, 22], there is an ergo-region outside the Rindler horizon at non-zero drift velocity and the analogy with rotating black holes leads us to expect spontaneous emission of radiation from this ergo-region, in addition to the thermal radiation from the horizon. We discuss this in section 3. There is also a more obvious reason for the discrepancy that does not rely on the accelerating brane picture. Induced emission is important to the detector’s response in the IR limit, and the angular dependence of the photon distribution due to the motion of the detector implies an angular dependence in the induced emission rate that is not captured by the classical Doppler shift. This observation applies as much to motion in a 4-dimensional Minkowski heat bath as to a 3-dimensional one, which is one indication that our results in 2 + 1 dimensions are likely to have implications for the detection of photons in the CMBR (for which $v \sim 0.002$); we discuss this in section 4.

For a relativistic drift velocity, the disagreement between the formula (5) and our result is significant even in the UV limit. In fact, above a critical velocity $v_c \sim 0.22$ we find that there is no response from the detector to photons arriving within some angle $\varphi = \pi$; in other words, there is a ‘cone of silence’ in place of a wake as the detector moves through the photon heat bath.

2. Detector response

In this section we compute the response of an Unruh DeWitt detector undergoing the stationary motion that we refer to as “acceleration with drift” in a four-dimensional Minkowski spacetime with metric

$$d\mathbf{X} \cdot d\mathbf{X} = -dX_0^2 + dX^2 + dY^2 + dZ^2.\quad (6)$$

The worldline of the detector in this spacetime is given by

$$X_0 = a^{-1} \sinh (a \gamma \tau), \quad X = a^{-1} \cosh (a \gamma \tau), \quad Y = \gamma v \tau, \quad Z = 0, \quad (7)$$

where $\gamma = 1/\sqrt{1 - v^2}$. The 4-acceleration $A$ is non-zero for any $v$. For $v = 0$ we have the well-known trajectory of a detector undergoing constant proper acceleration $|A| = a$. When $v \neq 0$ we have “acceleration with drift” in which uniform motion with velocity $v$ in the $Y$ direction is superposed on constant proper acceleration $|A| = \gamma^2 a$ in the $X$ direction. For pure acceleration, the worldline has non-zero curvature but zero torsion, whereas the torsion is also non-zero for acceleration with drift [13]. Equivalently, the relativistic jerk [11, 14] is zero for
pure acceleration but not for acceleration with drift. We therefore expect a $v$-dependence of the detector’s response to the motion.

As explained in the introduction, we will interpret a photon heat-bath in a 3-dimensional Minkowski spacetime as a 2-brane at the Unruh temperature associated with an orthogonal acceleration. This set-up is achieved by the following family of embeddings of a 3-dimensional worldvolume with coordinates $(\tau, y, z)$ [14]:

\[
X_0 = a^{-1} \sinh [a\gamma(\tau + vy)], \quad X = a^{-1} \cosh [a\gamma(\tau + vy)], \\
Y = \gamma(y + v\tau), \quad Z = z, \quad (\gamma = 1/\sqrt{1 - v^2})
\]  

(8)

A computation of the induced metric shows that the embedded spacetime is indeed Minkowski and also that the coordinates $(\tau, y, z)$ are cartesian. This Minkowski worldvolume can be viewed as the congruence of worldlines parametrized by position $(y, z)$ on the brane. Each such worldline defines a stationary motion of acceleration with drift, with proper time $\tau$. The embedding with non-zero $v$ is obtained from that with $v = 0$ by a worldvolume Lorentz boost in the $y$ direction. Although the induced metric is not affected by this boost, the extrinsic geometry of the worldlines of the congruence is affected by it. In particular, the torsion (equivalently, relativistic jerk) of the worldlines of the congruence is zero only for $v = 0$. When account is taken of the Unruh temperature on the brane due to the acceleration, this fact can be seen as the kinematical equivalent of the fact that the rest frame of a heat bath provides a ‘preferred’ reference frame.

We begin with some generalities. We assume a ‘monopole’ detector [15] with (variable) energy gap $\hbar\omega$ coupled to a free quantum scalar field $\Phi(X)$. The interaction is

\[
S_{int} = g \int d\tau m(\tau)\Phi(X(\tau)),
\]  

(9)

where $m(\tau)$ is the detector’s monopole moment operator, and $X(\tau)$ is the detector’s worldline, parametrized by proper time $\tau$. The coupling constant $g$ is dimensionless (this is a simplifying feature of four spacetime dimensions) and may be assumed small. The probability of excitation and accompanying emission of a massless particle of 4-momentum $\hat{P} = \hbar(k, k)$ into solid angle element $d\Omega$ may then be computed using first-order perturbation theory. Integrating over photon energy, and omitting a small dimensionless constant of proportionality, one has [23, 20]

\[
\frac{dP}{d\Omega} = \int_0^\infty dk \int_{-\infty}^\infty d\tau_+ \int_{-\infty}^\infty d\tau_- \exp \left(2i\hbar\omega\tau_- - i \hat{P} \cdot \Delta X(\tau, \tau') \right),
\]  

(10)

where

\[
\Delta X(\tau, \tau') = X(\tau) - X(\tau'), \quad \tau_\pm = \frac{1}{2}(\tau \pm \tau').
\]  

(11)

Henceforth, we choose units such that $\hbar = 1$, which means that all dimensionful quantities have dimensions of mass to some power.

2.1. Integrated response

From the brane perspective that we will eventually adopt, it would make no sense to integrate the detector’s response over solid angle. However, for the general purpose of understanding the physics of detector response it is instructive to consider this integral. From (10) we have ($\hbar = 1$)

\[
\int d\Omega \frac{dP}{d\Omega} = \int d\tau_+ \hat{P}, \quad \hat{P} = 16\pi^3 \int_{-\infty}^\infty d\tau_- e^{2i\omega\tau_-} G^+,
\]  

(12)

where $\hat{P}$ is the integrated excitation rate, determined by

\[
G^+ = \left[4\pi^2 |\Delta X(\tau, \tau')|^2 \right]^{-1}.
\]  

(13)
which is the Wightman function evaluated for points on the detector's worldline specified by proper times $\tau$ and $\tau'$. The integral should vanish for inertial motion, for which $G^+ = -1/(16\pi^2\tau^2)$. The integrand has a pole at $\tau_- = 0$ but the usual ‘$i\epsilon$’ prescription will push this pole below the real axis so that it does not lie within the contour completed in the upper half of the complex $\tau_-$-plane (we assume $\omega > 0$). With this prescription, we recover the expected result that the detector remains in its ground state if it is moving inertially.

It has been proposed [24] that the response of an inertial detector in a heat bath at temperature $T$ is given by

$$\dot{P}_T \propto \int_{-\infty}^{\infty} d\tau_- e^{2i\omega\tau_-} G^+_T(\tau_-), \quad (14)$$

where $G^+_T$ is the thermal Wightman function at temperature $T$, again evaluated for points on an inertial worldline specified by proper times $\tau$ and $\tau'$:

$$G^+_T = -\frac{1}{16\pi^2} \sum_{n=-\infty}^{\infty} \frac{1}{(\tau_- - in\gamma/2T)^2 + (n\gamma v/2T)^2}. \quad (15)$$

Now there are poles inside the contour even for inertial motion and one gets a non-zero result that depends on the detector’s velocity with respect to the heat bath. The ‘scalar photon’ number density may be read off from this result, and this turns out to be half the photon number density (3) [24, 6]. This computation appears to support the proposition that a photon detector moving with constant velocity through a heat-bath will register an angular spectrum given (for a 4-dimensional Minkowski spacetime) by (2). However, the formula (14) has not been derived from first principles.

We return to the conventional formula (12) for the integrated excitation rate, which is derived from first principles. This is time independent for any stationary motion since $G^+$ is then a function only of $\tau_-$. For “acceleration with drift” one finds that [13]

$$G^+ = -\frac{a^2}{16\pi^2 \left[ \sinh^2 \xi - v^2 \xi^2 \right]}, \quad \xi = a\gamma\tau_- , \quad (16)$$

as can be shown directly using (7). This yields a non-zero integrated excitation rate. One can arrive at the same result by first performing the $k$ integral in (10). To this end, we choose spherical angles, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, so that

$$P_0 = k , \quad P_X = k \cos \theta , \quad P_Y = k \sin \theta \cos \phi , \quad P_Z = k \sin \theta \sin \phi . \quad (17)$$

We then find that

$$\frac{dP}{d\Omega} = -\frac{a}{2\gamma} \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\tau_+ e^{2i(\omega/a\gamma)\xi} \frac{\Gamma_\theta \sinh \xi - v\xi \cos \phi \sin \theta}{\left[ \Gamma_\theta \sinh \xi - v\xi \cos \phi \sin \theta \right]^2} \quad (18)$$

where

$$\Gamma_\theta = \cosh (a\gamma\tau_+) - \cos \theta \sinh (a\gamma\tau_+) . \quad (19)$$

Integrating over solid angle, we recover the integrated excitation rate in terms of $G^+$.

### 2.2. Angular response

We now focus on the formula (18), which gives the angular response of the detector integrated over time. Consider first the $v = 0$ case. We have, from (18) and (19),

$$\left. \frac{dP}{d\Omega} \right|_{v=0} \propto \frac{1}{\tau_+} \left[ \Gamma_\theta(\tau_+) \right]^{-2} \frac{dP}{d\Omega} \bigg|_{v=0} \quad (20)$$
where

\[
\frac{d\mathcal{P}}{d\Omega} \bigg|_{\Omega=0} = -\frac{a}{2\gamma} \int_{-\infty}^{\infty} d\xi \frac{e^{2i(\omega/a\gamma)\xi}}{\sinh^2 \xi},
\]

(21)

The integrand of the \(\tau_+\) integral is to be identified with an excitation rate but this is \(\tau_+\)-dependent because of the \([\Gamma_0]^{-2}\) factor. As shown in [20, 25], this factor arises because the detector has a time-dependent velocity \(dX/dT = \tanh(a\gamma\tau_+)\) relative to an inertial ‘laboratory’ frame. The two frames coincide at \(\tau_+ = 0\), so the time-independent excitation rate in the detector’s frame is just \((d\mathcal{P}/d\Omega)_{\Omega=0}\). The same logic applied for \(\nu \neq 0\) yields the excitation rate

\[
\frac{d\mathcal{P}}{d\Omega} = -\frac{\pi a}{\gamma} \int_{-\infty}^{\infty} d\xi \frac{e^{2i(\omega/a\gamma)\xi}}{[\sinh \xi - v_k \cos \varphi \sin \theta + i\varepsilon]^2},
\]

(22)

where we have now made explicit the \(i\varepsilon\) prescription.

To compute the integral in (22), we complete the contour in the upper-half complex \(\xi\)-plane. For non-zero \(u \equiv v \cos \varphi \sin \theta\) it encloses a finite number of poles at

\[
\xi = i\varepsilon, \quad \sin x = ux, \quad x > 0, \quad (u \equiv v \cos \varphi \sin \theta).
\]

(23)

There is a unique pole at \(x = x_0(u)\) when \(u > u_1 \approx 0.13\), and thus the residue calculus yields

\[
\frac{d\mathcal{P}}{d\Omega} = \frac{\pi a}{\gamma} c \left( x_0(u) \right) e^{-\frac{2\omega x_0(u)}{a\gamma}},
\]

(24)

where

\[
c \left( x \right) = \frac{2\omega u - 2\omega \cos x + a\gamma \sin x}{a\gamma \left( u - \cos x \right)^3}.
\]

(25)

At \(u = u_1\) there is an extra double pole, which splits into two poles at \(x = x_1 > x_0\) and \(x = x'_1 > x_1\) for \(u < u_1\). As long as \(u > u_2\), where \(u_2 < u_1\) is a second critical value of \(u\), there will be at most three poles. As \(u\) decreases further, new pairs of poles appear at a sequence of critical value values \(u_\ell\) of \(u\), and for any non-zero \(v\) one finds that

\[
\frac{d\mathcal{P}}{d\Omega} = \frac{\pi a}{\gamma} \left\{ c \left( x_0 \right) e^{-\frac{2\omega x_0}{a\gamma}} + \sum_{\ell=1}^{\ell_{\text{max}}} \theta(u_\ell - u) \left[ c \left( x_\ell \right) e^{-\frac{2\omega x_\ell}{a\gamma}} + c \left( x'_\ell \right) e^{-\frac{2\omega x'_\ell}{a\gamma}} \right] \right\},
\]

(26)

where \(2\ell_{\text{max}} + 1\) is the number of roots of \(\sin x = ux\). For \(u \to 0\) (e.g. as a result of \(v \to 0\)) \(\ell_{\text{max}} \to \infty\) and the poles in the contour move to \(\xi = \pi ki\), thus reproducing the \(v = 0\) result. Despite the step functions, \(d\mathcal{P}/d\Omega\) is a smooth function of \(u\) because the poles of poles that that appear as \(u\) decreases though a critical value \(u_\ell\) give contributions of opposite signs that cancel at \(u = u_\ell\). In the ultraviolet (UV) limit, the first term in (26) dominates, exponentially, so that (24) gives the asymptotic behaviour.

So far we have considered only positive \(u\), but \(u\) becomes negative when \(3\pi k > \varphi > \frac{\pi}{2}\). The equation \(f(u) \equiv \sin x - ux = 0\) then has no solution for \(u\) below a certain critical value \(u_\ast\), determined as follows. For \(u\) slightly above the critical value \(u_\ast\), there are two roots, which become a double root at \(u = u_\ast\). At this point \(u = u_\ast\) also solves \(f'(u) = \cos x - u = 0\). Combining both equations, we find the critical value at

\[
\tan x_\ast = x_\ast, \quad x_\ast \approx 4.4934, \quad u_\ast = \cos x_\ast \approx -0.2172.
\]

(27)

Since \(u = v \cos \varphi \sin \theta\), this critical point exists only at velocities \(v > v_c = |u_\ast| \approx 0.2172\). At any velocity \(v > v_c\), there is no pole of the integrand of (22) within the contour whenever

\[
-\sin \theta \cos \varphi > \frac{v_c}{v}.
\]

(28)
In other words, there is no response from the detector in a ‘backward’ cone, with forward axis defined by the detector velocity, of angle $\alpha$ such that

$$\cos \alpha = \frac{v_c}{v}. \quad (29)$$

The angle $\alpha$ goes to zero as $v \rightarrow v_c$, while $\alpha \rightarrow \arccos(v_c) \approx 0.57\pi \approx 77.5^\circ$ as $v \rightarrow 1$. The cone closes at $v = v_c$, and for $v < v_c$ the detector has a response in all directions. For $u$ slightly above $u_*$, the equation $\sin x - ux = 0$ has two roots $x_1, x_2$ and and the detector response is of the form

$$\frac{d\dot{P}}{d\Omega} = \pi a \frac{\sin (x_1) e^{-\frac{2\omega x_1}{a\gamma}} + \sin (x_2) e^{-\frac{2\omega x_2}{a\gamma}}}{\gamma}. \quad (30)$$

Again it should be noted that there is no discontinuous behaviour: the two terms give contributions of opposite signs which cancel as $u \rightarrow u_*$. 

2.3. Braneworld detector response

We now want to apply our results to the braneworld approach to motion in a Minkowski heat-bath. We therefore consider a detector restricted to detect photons arriving from directions within the brane. Recalling that $\theta$ is the angle that a photon’s 3-momentum makes with the $X$-axis, we see that we must set $\theta = \pi/2$. For convenience, we define

$$\frac{d\dot{F}}{d\phi} \equiv \frac{d\dot{P}}{d\Omega} \bigg|_{\theta = \pi/2} = -\frac{a}{2\gamma} \int_{-\infty}^{\infty} d\xi \frac{e^{2i(\omega/\gamma)\xi}}{[\sinh \xi - v\xi \cos \phi + i\varepsilon]^2}. \quad (31)$$

The result of this integration is obtained from the previous formulas (24) and (26) by setting $\theta = \pi/2$, so that $u = v \cos \phi$. From these formulas we may deduce the detector response for relativistic velocities, but we postpone this analysis until we have considered the low velocity limit.

For $v \ll 1$, it is convenient to return to (31) and expand the integrand in powers of $v$. This yields

$$\frac{d\dot{F}}{d\phi} = -\frac{a}{2\gamma} \sum_{n=0}^{\infty} (n+1) \int d\xi \frac{e^{\frac{2i\xi}{\gamma}} (v \xi \cos \phi)^n}{[\sinh \xi + i\varepsilon]^{n+2}}. \quad (32)$$

Again completing the contour in the upper half complex $\xi$- plane, we find poles within the contour at $\xi = \pi ki$ for positive integer $k$, and the residue calculus yields

$$\frac{d\dot{F}}{d\phi} = \frac{2\pi\omega}{\gamma^2} \sum_{k=1}^{\infty} \sum_{n=0}^{\infty} \frac{p_n}{n!} \left(\frac{(-1)^{k+1} 2\pi \ell \omega u}{a\gamma}\right)^n e^{-\frac{2\pi\ell\omega}{a\gamma}}, \quad (33)$$

where $p_n$ is a polynomial in $a/\omega$, of degree $n+1$ for $n > 0$, with $p_n(0) = 1$; in particular,

$$p_0 = 1, \quad p_1 = 1 - \frac{a\gamma}{k\pi\omega} + \frac{a^2\gamma^2}{4\omega^2}. \quad (34)$$

We can compute exactly the linear terms in the velocity by keeping only the $n = 0, 1$ terms in the sum over $n$ in (33). Using the Unruh formula $2\pi T = |A| = \gamma^2 a$, we find that

$$\frac{d\dot{F}}{d\phi} = \frac{2\pi\omega e^{-\omega/T}}{e^{\omega/T} - 1} + \frac{2\pi\omega^2 v \cos \phi e^{-\omega/T}}{T (1 + e^{-\omega/T})^2} \left[1 - \frac{2T}{\omega} + \frac{\pi^2 T^2}{\omega^2} - \frac{2T}{\omega} e^{-\omega/T}\right] + O(v^2). \quad (35)$$

As anticipated in the introduction, the excitation rate for $v = 0$ reproduces the thermal number density (5) up to an overall factor (which includes the coupling constant of the interaction of
the detector with the field). This allows us to fix the factor relating the excitation rate and the number density:

\[ n(\omega, \varphi) = \frac{1}{8\pi^3} \frac{d\mathcal{F}}{d\varphi} = \frac{\omega}{4\pi^2(\omega^2/T - 1)} + \frac{\omega^2 v \cos \varphi e^{-\omega/T}}{4\pi^2 T (1 + e^{-\omega/T})^2} \left[ 1 - \frac{2T}{\omega} + \frac{\pi^2 T^2}{\omega^2} - \frac{2T}{\omega} e^{-\omega/T} \right] + \mathcal{O}(v^2). \]  

In the same approximation, the excitation rate implied by (5) is

\[ n_{PW}(\omega, \varphi) = \frac{\omega}{4\pi^2(\omega^2/T - 1)} + \frac{\omega^2 v \cos \varphi e^{-\omega/T}}{4\pi^2 T (1 - e^{-\omega/T})^2} + \mathcal{O}(v^2), \]  

Thus, we find agreement in the UV part of the spectrum where \( \omega \gg T \). Otherwise there is a discrepancy already at linear order in \( v \), and this discrepancy is significant in the IR limit, as we anticipated in the introduction on the grounds that an angular-dependent induced emission rate becomes important in this limit.

For relativistic velocities we return to formulas (24) and (26) evaluated at \( \theta = \pi/2 \). For \( v > |u_*| \approx 0.2172 \) we find, for negative \( u \), the same “cone of silence” within which the detector has no response because the integrand of (31) has no pole within the contour. This cone is now defined by \( \varphi > \varphi_0 \), with \( \cos \varphi_0 = u_*/v \). The “cone of silence” is obviously not a feature of the PW formula. Consider now \( u \rightarrow 1 \), which implies both \( \varphi \rightarrow 0 \) and \( v \rightarrow 1 \). In this case there is a single pole which is located at \( x_0 = \sqrt{6}/v \); the leading behaviour is given by

\[ n(\omega, \varphi) = \frac{1}{8\pi^3} \frac{d\mathcal{F}}{d\varphi} \approx \frac{T}{16\pi^3} \frac{\sqrt{3}}{\sqrt{2}} \frac{\exp\left(-\frac{\sqrt{3}\pi\omega}{\sqrt{T}} \sqrt{1 - v \cos \varphi} \right)}{(1 - v \cos \varphi)^{3/2}}. \]  

This is significantly different from the ultra relativistic limit of the PW-type formula (5).

3. Vacuum for acceleration with drift

It is well-known that the thermal spectrum registered by an accelerating detector can be explained, in a detector independent way, as a result of a difference of the Minkowski quantum vacuum to the (Fulling) vacuum of the Rindler spacetime. Specifically, the Bogoliubov transformation that relates the particle creation and annihilation operators in the two spacetimes is non-trivial, so that the Minkowski vacuum is a thermal state in the Rindler spacetime. It would be natural to suppose that our result for ‘acceleration with drift’ has a similar explanation; after all, there is an Unruh-type effect associated with motion through a medium with refractive index \( n > 1 \) [25]. However, the Bogoliubov transformation that connects the Fulling-Rindler vacuum with the analogous vacuum in a moving frame is trivial [26], so it might seem that the photons available for detection at zero drift velocity are just the same photons that were available for detection at zero velocity, but Doppler shifted. If so, this would justify the PW formula based on the Doppler shift. However, there is more to this problem than just the Bogoliubov transformation; in particular, it was shown in [21, 22] that the stationary motion we call “acceleration with drift” leads a geometry in which there is an ergo-region outside the acceleration horizon. We present here a summary of these results and then discuss the implications. This whole analysis can be carried out for arbitrary spacetime dimension \( D \), in particular for the \( D = 4 \) case as well as for the \( D = 3 \) case that we have analyzed from the perspective of detector response.

We begin with the \((D + 1)\)-dimensional Minkowski metric in the form

\[ ds^2 = -dt^2 + dx^2 + dy^2 + |dz|^2. \]  

\( 39 \)
but where this is now defined with respect to the Rindler time gives the Rindler metric

\[ ds^2 = -r^2d\eta^2 + dr^2 + dy^2 + |d\vec{z}|^2. \]

The Rindler coordinates cover only two portions of the Minkowski spacetime, the positive (“right”) and negative (“left”) Rindler wedges,

\[ R_+ = \{ x \mid x > |t| \}, \quad R_- = \{ x \mid x < -|t| \}, \]

for positive and negative \( r \), respectively. In these regions the Killing vector field \( \partial_\eta = x\partial_t + t\partial_x \) is timelike. The worldlines with constant \( r \) correspond to constant proper acceleration \( a = 1/r \).

Consider now a complex massless scalar field \( \phi \) obeying the wave equation \( \partial^2 \phi = 0 \). The norm of any solution \( \phi \) is defined as (we follow the review article [18])

\[ ||\phi|| = -i \int d\Sigma^\mu (\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) \]

where \( \Sigma \) is the volume element on an arbitrary spacelike hypersurface. A basis of solutions of positive norm is provided by the positive frequency plane parallel waves

\[ U_k = \frac{1}{[2\omega_k(2\pi)^{n-1}]^{1/2}} e^{i(-\omega_k t + k_x x + k_y y + \vec{k} \cdot \vec{z})}, \quad \omega_k = \sqrt{k_x^2 + k_y^2 + |\vec{k}|^2}. \]

In Rindler coordinates, the solutions with positive norm are again those of positive frequency, but where this is now defined with respect to the Rindler time \( \eta \); these modes are

\[ u_k^\sigma = \frac{\theta(\sigma \eta)}{[2\Omega(2\pi)^{n-2}]^{1/2}} h_k^\sigma(r) \exp \left[ i \left( -\sigma \Omega \eta + k_y y + \vec{k} \cdot \vec{z} \right) \right], \]

where \( \sigma = \pm \) for the positive and negative Rindler wedges, respectively. Here we are using the fact that the timelike Killing vector \( \partial_\eta \) is future directed on \( R_+ \) and past directed on \( R_- \).

Introducing the coordinate \( r_\pm = \log |r| \), the equation of motion for the radial functions \( h_k^\sigma(r) \) takes the form of a one-dimensional Schrödinger equation:

\[ \left( -\frac{d^2}{dr_\pm^2} + \mu_k^2 e^{2r_\pm} \right) h_k^\sigma = \Omega^2 h_k^\sigma, \quad \mu_k = \sqrt{k_y^2 + |\vec{k}|^2}. \]

At this point it is important to note that \( \Omega \) is an arbitrary parameter that ranges from 0 to \( \infty \), independently of the values of the momenta; the reason is that for any \( \Omega > 0 \) the solution is always oscillatory for sufficiently small \( r_\pm \).

The scalar field \( \phi \) can be expanded either in terms of \( U_k, U_k^\dagger \) or in terms of \( u_k^\pm, u_k^\pm, u_k^\pm, u_k^\pm \), as both sets constitute complete sets of modes:

\[ \phi = \int d^{d-1}k \left( a_k U_k + \tilde{a}_k^\dagger U_k^\dagger \right) = \int_0^{\infty} d\Omega \int d^{d-2}k \sum_{\sigma = \pm} (b_k^\sigma u_k^\sigma + \tilde{b}_k^\sigma u_k^\sigma). \]

The linear transformation from \( a_k, a_k^\dagger \) to \( b_k^\sigma, \tilde{b}_k^\sigma \) is a Bogoliubov transformation, which mixes modes of positive and negative norm; it takes the form

\[ b_k^\sigma = \int d^{d-1}k' (\alpha_k^\sigma a_{k'} + \beta_k^\sigma \tilde{a}_{k'}). \]
The important implication is that the Minkowski vacuum state defined by
\[ a_k |0\rangle_M = \bar{a}_k |0\rangle_M = 0 \quad \forall k \]
(48)
is not equivalent to the Fulling-Rindler vacuum state defined by
\[ b_k^{(\sigma)} |0\rangle_R = \bar{b}_k^{(\sigma)} |0\rangle_R = 0 \quad \forall \sigma, k. \]
(49)
In particular, the expectation value of the Rindler particle number operator
\[ N_k = b_k^{(\sigma)\dagger} b_k^{(\sigma)} \]
in the Minkowski vacuum is
\[ \int d^{d-1}k' \beta_{kk'} \beta_{k'k}^*, \]
(50)
and as a result the Rindler observer is immersed in a thermal distribution of particles.

We now consider the effect of a drift combined with acceleration on the vacuum. The appropriate metric is obtained by a Lorentz boost in the \( y \) direction applied to the metric (41). For an observer standing at \( r_0 = a^{-1} \), the proper time is \( \tau = r_0 \eta \). We introduce new coordinates
\[ y = \gamma (\bar{y} - v r_0 \bar{\eta}) , \quad r_0 \eta = \gamma (r_0 \bar{\eta} - v \bar{y}) , \quad r_0 = a^{-1} , \]
(51)
so that
\[ ds^2 = -r^2 \gamma^2 (d\bar{\eta} - v r_0 d\bar{y})^2 + dr^2 + \gamma^2 (d\bar{y} - v r_0 d\bar{\eta})^2 + |d\vec{z}|^2 \]
(52)
\[ = -\gamma^2 (r^2 - v^2 r_0^2) d\bar{\eta}^2 + 2 \gamma^2 \frac{v}{r_0} (r^2 - r_0^2) d\bar{y} d\bar{\eta} + \gamma^2 \left( 1 - \frac{r^2 v^2}{r_0^2} \right) d\bar{y}^2 + dr^2 + |d\vec{z}|^2 . \]
This metric is stationary, with respect to the new time variable \( \bar{\eta} \), but it is not static. There is still a coordinate singularity at \( r = 0 \), which is again the acceleration horizon, but there is also a ‘static limit’ surface at \( r = vr_0 \) [22]; the region between the horizon and this static limit surface is analogous to the ergo-region of a rotating black hole. The hypersurface at \( r = r_0 \) is Minkowski spacetime in standard coordinates
\[ ds^2 \bigg|_{r=1} = -r_0^2 d\bar{\eta}^2 + d\bar{y}^2 + |d\vec{z}|^2 . \]
(53)
A static detector in this subspace has proper acceleration \( \gamma^2 a \) in the larger space, where the \( \gamma^2 \) factor is due to time dilation as a consequence of the velocity \( v \) in the \( y \) direction.

**Figure 1.** Spacetime diagram for a detector at \( r = r_0 \) on a stationary trajectory of acceleration with drift velocity \( v \). There is an ergo-region for the natural time-like Killing vector field between the acceleration horizon and the static-limit surface at \( r = vr_0 \).
As \( \phi \) is a scalar field, we can obtain the solutions of the wave equation in the new coordinates from those in the original Rindler coordinates by simply making the substitution (51):

\[
\tilde{u}^{(\sigma)}_k = \frac{\theta(\sigma r)}{2\Omega(2\pi)^{n-2}} \tilde{h}^{(\sigma)}_k(r) \exp(i(-\sigma \tilde{\Omega} \tilde{y} + \tilde{k}_y \tilde{y} + \tilde{k}_z \tilde{z})) \quad (54)
\]

where

\[
a\tilde{\Omega} = \gamma(a\Omega + \sigma v k_y), \quad \tilde{k}_y = \gamma(k_y + \sigma v a\Omega),
\]

As pointed out above, \( \Omega \) and \( k_y \) take independent values, \( 0 < \Omega < \infty \) and \( -\infty < k_y < \infty \). This has the implication that a mode of positive (negative) frequency before the Lorentz boost, becomes negative (positive) frequency after the boost if \( v k_y < -\Omega \). However, the modes that were originally of positive (negative) norm still have positive (negative) norm, so the definition of the vacuum state is not affected by the boost [26]. Nevertheless, we now have a situation in which some states of positive norm have negative energy, and this implies an instability of the vacuum [22].

We can now see, within our braneworld set-up, why the detector should not detect a spectrum of particles that is simply a Doppler-shift of the thermal spectrum at zero velocity. In this extra-dimensional picture, the thermal spectrum is a consequence of the acceleration horizon, but there is also an ergo-region. In the context of black hole physics, ergo-regions are classically stable but quantum-mechanically unstable. An ergo-region is a source of spontaneous emission of radiation that decreases the angular momentum of a rotating black hole. Here we expect an angular-dependent spontaneous emission of radiation from the ergo-region that would decrease the momentum in the \( y \)-direction if this direction were to be periodically identified. The detector at \( r = a^{-1} \) should detect the photons in this radiation in addition to the photons of the thermal radiation from the horizon. This presumably explains why we find a departure from the Doppler-shift result at non-relativistic velocities, although it is hard to see how it could explain the “cone of silence” that we find above the critical, relativistic, velocity.

4. Possible Implications for the CMBR

Our local group of galaxies appears to be moving at \( v \approx 0.002 \) relative to CMBR rest frame. Using this velocity in our result for detector response in a three-dimensional Minkowski heat bath, we get a deviation from the PW-type spectrum (5) such that

\[
\frac{|n - n_{\text{PW}}|}{n} \sim 10^{-4}. \quad (56)
\]

The angular distribution of our spectrum as compared to (5) at \( v = 0.002 \) is shown in fig. 1, where one can see that the angular dependence is remarkably similar; the scale has been amplified to make the difference visible.

The energy spectrum at \( \varphi = 0 \) as compared to \( \omega n_{\text{PW}}(\omega) \) is shown in fig. 2a for \( v = 0.04 \); in this case there are seven poles contributing to the sum (26). We see that the spectra are remarkably close at this velocity, differing slightly near the peak. The energy spectra continue to be close even for much higher velocities such as \( v = 0.15 \) where only a single pole contributes to the integral and the spectrum \( n(\omega) \) is exactly given by (24) divided by \( 8\pi^3 \) (see eq.(36)). For this value of \( v \) the greatest deviation is at the peak of the spectrum and is of order 8 %, At higher velocities the difference becomes substantial: for example, for \( v = 0.8 \) the peak of the PW energy spectrum is smaller by a factor of 5 and the PW temperature (defined by the exponent of the tail) is higher by a factor of 1.8, as shown in fig 2b.

Space-time diagram for a detector at \( r = r_0 \) on a stationary trajectory of acceleration with drift velocity \( v \). There is an ergo-region for the natural time-like Killing vector field between the acceleration horizon and the static-limit surface at \( r = vr_0 \).
Figure 2. Angular distribution at $v = 0.002$, $\omega = A/2 = 0.5$. The dashed line represents the Peebles-Wilkinson spectrum. The greatest deviations are observed at $\varphi = \pi$, with a normalized value of order $8 \times 10^{-4}$, and at $\varphi \sim 0$, of order $5 \times 10^{-4}$.

Figure 3. Spectrum at $\varphi = 0$ with $v = 0.04$ (conventions as above).

Figure 4. Spectrum at $\varphi = 0$ with $v = 0.8$ (conventions as above).

Although the deviation from PW spectrum is very small at small velocities, the difference becomes appreciable if one focuses on the dipole term proportional to the detector velocity. From (5) one sees that the Doppler-shift approach predicts an angular variability of the effective temperature:

$$T_{\text{eff}}(\varphi) = T v \cos \varphi + \mathcal{O}(v^2) \quad (57)$$

In contrast, our result (36) can be interpreted as predicting a frequency-dependent angular variability:

$$T_{\text{eff}}(\varphi, \omega) = [T v \cos \varphi] F(\omega), \quad (58)$$

where

$$F(\omega) = \left(\frac{1 - e^{-\omega/T}}{1 + e^{-\omega/T}}\right)^2 \left[1 - \frac{2T}{\omega} + \frac{\pi^2T^2}{\omega^2} - \frac{2T}{\omega} e^{-\omega/T}\right] \quad (59)$$

The function $F(\omega)$ represents the influence of frequency-dependent quantum effects neglected in the Doppler-shift calculation. For the experimentally available frequency range (roughly from 50 to 500 GHz), this implies a change from the highest to the lowest frequencies by a factor of order 2 (see fig. 3).

Figure 5. Dipole relative to CMBR in the experimentally available frequency range.
It is worth noting that the different terms appearing in the expression for $F(\omega)$ contain poles at $\omega = 0$ that cancel out thanks to the specific combination appearing in (59): a generic change in the coefficients in the terms appearing inside the square brackets would lead to a much higher factor. Of course, a factor of 2 variation is still large, but this variation occurs only in low frequency part of the available range; otherwise the spectrum is nearly flat. Since our results have been derived for motion in a $(2 + 1)$-dimensional Minkowski heat bath, it is possible that the analogous spectrum for $(3 + 1)$ dimensions will not be in conflict with experiment. Also, the CMBR experiment is calibrated on the dipole assuming a frequency independent dipole factor, so a small frequency dependence might show up as an anomaly elsewhere.

5. Discussion
We have studied the behaviour of a photon detector moving in a photon heat bath at constant velocity by exploiting the fact that the heat bath can be generated on a brane by acceleration in an additional direction, by virtue of the Unruh effect. The same basic idea is used in the GEMS approach to black hole thermodynamics, which gives correctly the local temperature $T$, as would be measured by a static particle detector, of various black hole spacetimes in various dimensions: a static detector undergoes constant proper acceleration $2\pi T/\hbar$ in the embedding spacetime, so the Unruh formula gives the local temperature [12]. We have exploited the fact that inertial motion on an accelerating Minkowskibrane corresponds to “acceleration with drift” in the higher dimensional spacetime, this being another of the six possible distinct stationary motions according to the classification of Letaw [13]. Thus, the behaviour of a photon detector moving inertially in a heat bath should be the same as that of a detector undergoing acceleration with drift, assuming that it is constrained to detect particles that move within the brane.

The response of an Unruh-DeWitt detector undergoing stationary motion in a Minkowski vacuum has been the subject of many previous papers, but most have concentrated on what we have referred to as the 'integrated response' obtained by integration over angles. This integration has the feature that the response is then time-independent irrespective of the frame from which it is viewed. Here we have computed the angular response for a detector undergoing acceleration with drift; this is time-independent in its own frame (necessarily for stationary motion) but not in a ‘laboratory’ frame. By restricting the angles to those “accessible” to a detector ‘on the brane’, our result can be compared with the analog of the PW formula for a $(2 + 1)$-dimensional spacetime. There is no agreement for relativistic velocities but let us postpone discussion of this fact and concentrate for the moment on non-relativistic velocities. In this case we may expand to first order in the velocity and then make the comparison. We found agreement in the UV limit $\hbar\omega \gg T$ in which the photon gas is essentially classical, and this shows at least that the Doppler-shift effect is incorporated in our approach. However, as anticipated, we found a disagreement in the IR limit.

As mentioned, there are a priori grounds for thinking that quantum effects could effectively modify the PW formula in the IR limit, but if so it should be possible to understand the physical origin of any such modification in the context of the brane set-up described above. One might suspect that it is due to some velocity dependence of the vacuum, analogous to the Unruh effect itself, which would imply the existence of photons at non-zero velocity that were absent at zero velocity (or vice versa). However, it is known that the Bogoliubov transformation from the vacuum of an accelerating detector to the vacuum of a detector accelerating with drift is trivial [26]. Nevertheless, there is an important sense in which the two vacua are not equivalent\(^1\): strictly speaking, for a detector undergoing acceleration with drift the Fulling-Rindler vacuum is not a true vacuum state; it is unstable, due to negative energy states present in the right Rindler wedge [22]. This is due to the fact that for such a detector there is an ergo-region

\(^1\) The non-equivalence of the two vacua is not surprising. In general, a detector moving at constant velocity in a non-Lorentz invariant medium is expected to see a non-trivial vacuum, and another example can be found in [20].
outside the acceleration horizon. In the context of black holes, ergo-regions are unstable against spontaneous emission. We have proposed that our result for detector response could be explained by superposing on the thermal emission from the horizon an emission of low-frequency photons from the ergo-region.

Of course, these results can be applied to the CMBR only in a fictitious universe of \((2 + 1)\) dimensions. This restriction has allowed us to make use of many standard results, and acquired intuition, concerning the behaviour of Unruh-DeWitt detectors, and to avoid known difficulties concerning motion in odd-dimensional (ambient) spacetimes. We cannot say what the analogous result should be in \((3 + 1)\) dimensions, so we can make no definite statement concerning the validity of the Peebles-Wilkinson formula as applied to the CMBR. However, we may get some idea of what to expect by substituting the observed velocity \(v \approx 0.002\) into our formula. Although this leads to a spectrum that differs little from the Doppler-shift based formula, comparison of the dipole term linear in the velocity leads to an apparent frequency dependence of the effective temperature that varies by a factor of 2 over the experimentally relevant frequency range. This indicates that it might be worthwhile to look for such frequency dependence in the CMBR data, or to analyze the implications for calibrations of assuming frequency independence.

Our results also have implications for the general understanding of particle detectors. The thermal response of an accelerating particle detector is often attributed to the different global features of Minkowski and Rindler spacetimes, and of the resulting difference between the Minkowski and Fulling-Rindler vacua. However, this cannot be the whole story because there are more distinct stationary motions than there are distinct quantum vacua, at least if two vacua are considered equivalent if the Bogoliubov transformation connecting them is trivial. We have already mentioned that the vacuum for acceleration with drift is equivalent, in this sense, to the Fulling-Rindler vacuum. It was further shown in [26] that for any stationary motion the quantum vacuum is equivalent, in this sense, to either the Minkowski vacuum or the Fulling-Rindler vacuum. In contrast, the local response of particle detectors on stationary worldlines is far more variable, being essentially different for each distinct stationary motion, and it is a challenge to understand why this is so (see e.g. [27, 28, 22]).

This issue was addressed for circular motion in [27]. The quantum vacuum for a detector undergoing uniform motion in a circle is actually equivalent to the Minkowski vacuum, despite the centripetal acceleration, so why does the detector detect particles? One may guess that the answer must have to do with the coordinate singularity at a critical radius in static co-rotating cylindrical polar coordinates, and it was shown in [27] that a co-rotating detector in the Minkowski vacuum does not detect particles if one assumes boundary conditions that remove the region of spacetime beyond the critical radius. For acceleration with drift, the resolution appears to be different. As mentioned, the appearance of an ergo-region outside the Rindler horizon is also expected to have an effect on a particle detector. However, it is not clear how this could explain the “cone of silence” that we have found above the critical velocity \(v_c \sim 0.2172\).

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