ABSTRACT

We have used the hydrodynamical adaptive mesh refinement code ENZO to investigate the dynamical evolution of the gas at the centre of dark matter haloes with virial velocities of $\sim 20 - 30 \, \text{km s}^{-1}$ and virial temperatures of $\sim 13000 - 30000 \, \text{K}$ at $z \sim 15$ in a cosmological context. The virial temperature of the dark matter haloes is above the threshold where atomic cooling by hydrogen allows the gas to cool and collapse. We neglect cooling by molecular hydrogen and metals, as may be plausible if $\text{H}_2$ cooling is suppressed by a metagalactic Lyman–Werner background or an internal source of Lyman–Werner photons, and metal enrichment has not progressed very far. The gas in the haloes becomes gravitationally unstable and develops turbulent velocities comparable to the virial velocities of the dark matter haloes. Within a few dynamical times, it settles into a nearly isothermal density profile over many decades in radius losing most of its angular momentum in the process. About 0.1–1 per cent of the baryons, at the centre of the dark matter haloes, collapse into a self-gravitating, fat, ellipsoidal, centrifugally supported exponential disc with scalelength of $\sim 0.075 - 0.27 \, \text{pc}$ and rotation velocities of 25–60 km s$^{-1}$. We are able to follow the settling of the gas into centrifugal support and the dynamical evolution of the compact disc in each dark matter halo for a few dynamical times. The dynamical evolution of the gas at the centre of the haloes is complex. In one of the haloes, the gas at the centre fragments into a triple system leading to strong tidal perturbations and eventually to the infall of a secondary smaller clump into the most massive primary clump. The formation of centrifugally supported self-gravitating massive discs is likely to be an important intermediary stage en route to the formation of a massive black hole seed.

Key words: black holes physics – methods: numerical – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Supermassive black holes (SMBHs) were invoked as the central engine powering quasi-stellar objects (QSOs) soon after the first QSOs were discovered (Salpeter 1964; Zel’Dovich & Novikov 1964). Early predictions that SMBHs are ubiquitous and may be found in many if not most galaxies as the remnants of dead QSOs (Lynden-Bell 1969) have stood the test of time. SMBHs are now believed to reside in most if not all galactic bulges and their masses correlate tightly with the stellar velocity dispersion and the mass of the bulges of their host galaxies (Ferrarese & Merritt 2000; Gebhardt et al. 2000). The luminosity functions of active galactic nuclei across the electromagnetic spectrum and its evolution with redshift have been used to constrain the growth history of SMBHs (e.g. Merloni, Rudnick & Di Matteo 2004; Shankar, Weinberg & Miralda-Escude’ 2007; Merloni & Heinz 2008). Little is known, however, about how the first (super)massive black holes came into being. The discovery of very luminous, bright QSOs at $z \geq 6$ appears to suggest that at least some of the most massive black holes were already in place when the Universe was less than 1 billion years old (Fan 2001; Fan, Carilli & Keating 2006; Haiman 2006).

A wide range of generic pathways to a SMBH is possibly ranging from direct collapse of gas into rather massive seed black holes to Eddington-limited accretion on to stellar mass black holes and the dynamical evolution of dense star clusters (Rees 1978, 1984). The tight relation between galactic bulges and their central mass suggests that they have formed and grown in a connected way. In the well-established $\Lambda$ cold dark matter ($\Lambda$CDM) paradigm of structure formation, dark matter (DM) haloes grow by hierarchical merging. The last decade has seen extensive and detailed modelling of the buildup of galaxies and SMBHs in this context using semi-analytic descriptions as well as numerical simulations (Efstathiou & Rees 1988; Carlberg 1990; Haehnelt & Rees 1993; Kauffmann...
was originally developed by Greg Bryan to perform adaptive uses an director ENZO 0.74, ≳(comoving) kpc for simulation A and 250 2009 The Authors. Journal compilation 0.72. We further assume a and no nested grids. In these exploratory simulations, we capability to employ nested grids which we describe in the code. The nested grids are introduced 2009 RAS, MNRAS 0.9 and +858–871 and H = and e cooling /Ω1 by guest. Downloaded from https://academic.oup.com/mnras/article-abstract/393/3/858/967568 2007, 2008). Coppi & Larson 1999; Abel, Bryan & Norman 2000, 2002; Bromm, et al. 2004; Fabbiano et al. 2006). There is thus plenty of motiva- tion to consider seriously the possibility that the growth of SMBHs has started from massive seed black holes with masses substan- tially larger than those forming as stellar remnants (see Begelman, Volontieri & Rees 2006 for a recent discussion).

The centres of high-redshift DM haloes with virial tempera- tures ≥10^4 K, not yet significantly enriched with metals, have been identified as a promising environment to form such massive seed black holes. If H2 cooling is suppressed in these haloes either due to an external ultraviolet (UV) background or more likely espe- cially in the later stages of the collapse due to internal sources of UV radiation (cf Wise & Abel 2007b) early fragmentation should not occur favouring the formation of a compact massive rotationally supported disc (Oh & Haiman 2002; Bromm & Loeb 2003; Volonteri et al. 2003; Koushiappas, Bullock & Dekel 2004; Begelman et al. 2006; Lodato & Natarajan 2006; Rees & Volonteri 2007; Volonteri, Lodato & Natarajan 2008). This is rather different from the situation in the lower mass haloes studied extensively as possible sites for the formation of the ‘first’ stars where H2 cooling is the dominant cooling mechanism (e.g. Abel et al. 1998; Bromm, Coppi & Larson 1999; Abel, Bryan & Norman 2000, 2002; Bromm, Coppi & Larson 2002, Bromm & Larson 2004; O’Shea & Norman 2007, 2008).

We use the publicly available code enzo to perform adaptive mesh simulations of the dynamical evolution of the gas in three DM haloes with virial velocities between ~19 and ~30 km s⁻¹ and virial temperatures between ~13 000 and ~30 000 K at z ~ 15 in a cosmological context. The work is similar in spirit to that of Wise, Turk & Abel (2008, hereafter W08), who simulated two haloes somewhat less massive than our haloes. W08 find that the gas at the centre of their haloes does not reach rotational support and does not fragment. Our simulations have a similar setup but unlike W08 we do not push for resolution in the central region where a very small fraction of the gas can collapse to very high density but rather follow the dynamical evolution of a significant fraction of the gas settling into a rotationally supported disc for several dynamical times. The paper is structured as follows. In Section 2, we describe the details of the numerical simulations. In Section 3, we summarize some basic formulae that we use to characterize the properties of the haloes. In Section 4, we describe the results of our numerical simulations and in Section 5 we summarize our conclusions. Throughout this paper, we assume a standard ΛCDM cosmology with the following parameters, based on the Wilkinson Microwave Anisotropy Probe (WMAP) first-year data (Spergel et al. 2003), Ω_m,0 = 0.26, Ω_λ = 0.0463, σ_8 = 0.9 and h = 0.72. We further assume a spectral index of primordial density fluctuations of n = 1.

2 THE SETUP OF THE NUMERICAL SIMULATIONS

2.1 The adaptive mesh refinement code ENZO

We have used the publicly available adaptive mesh refinement (AMR) code ENZO. ENZO was originally developed by Greg Bryan and Mike Norman at the University of Illinois (Bryan & Norman 1995, 1997; Norman & Bryan 1999; O’Shea et al. 2004). The gravity solver in ENZO uses an N-body particle mesh technique (Efstathiou et al. 1985; Hockney & Eastwood 1988). Additional finer meshes can then be added in regions of high density to calculate the dy- namics of the DM particles more accurately.

The hydrodynamics solver employs the piecewise parabolic method combined with a non-linear Riemann solver for shock capturing. The Eulerian AMR scheme was first developed by Berger & Oliger (1984) and later refined by Berger & Colella (1989) to solve the hydrodynamical equations for an ideal gas. Bryan & Norman (1997) adopted such a scheme for cosmological simulations. In addition, there are also modules available which compute the radiative cooling of the gas together with a multispecies chemical reaction network. There are two versions of the chemistry solver available, one with six species (H, H+, He, He++, e⁻ and e⁺) and one with nine species (same as before plus H2, H2⁺ and H⁺). As stated previously, the simulations conducted here do not include H2 cooling and chemistry. Our simulations make extensive use of ENZO’s capability to employ nested grids which we describe in the next section.

2.2 Nested grids and initial conditions

Initial conditions were generated with the initial conditions generator supplied with the ENZO code. The nested grids are introduced at the initial conditions stage. We have first run exploratory DM only simulations with coarse resolution, setting the maximum refine- ment level to 4. These DM only simulations have a root grid size of 128^3 and no nested grids. In these exploratory simulations, we have identified the most massive halo at a redshift of 10 and then rerun the simulations, including the hydrodynamics module. We also introduce nested grids at this point. The nested grids are placed around the region of interest, as identified from the coarse DM simulation. We have used four levels of nested grids in our simulations with a maximum effective resolution of 1024^3. The introduction of nested grids is accompanied by a corresponding increase in the DM resolution by increasing the number of particles in the region of interest. We only use the highest resolution nested grid to refine fur- ther thus economizing our computational output. This corresponds to a region of 625 h⁻¹ (comoving) kpc for simulation A and 250 h⁻¹ (comoving) kpc for simulations B and C. The total number of particles in our simulation is 4935 680, with 128^3 of these in our highest resolution region. The nested grids are distributed as follows in root
grid sizes, $L_0 = 128^3, L_1 = 32^3, L_2 = 24^3, L_3 = 16^3$. Table 1 gives the details of the simulations discussed here.

### 2.3 Following the collapse of the gas and dark matter with ENZO

As mentioned previously, ENZO uses adaptive grids to provide increased resolution where it is required. For the simulations discussed in this paper, we have used four refinement criteria implemented in ENZO: DM overdensity, baryon overdensity, Jeans length and cooling time. The first two criteria introduce additional meshes when the overdensity of a grid cell with respect to the mean density exceeds 3.0 for baryons and/or DM. The third criterion is the Truelove criterion (Truelove et al. 1997) which in its original form states that at least four grid cells should be used to resolve the Jeans length to ensure that no artificial fragmentation takes place. The cooling time is often shorter than the dynamical time and the cooling time refinement criterion helps to ensure that the Jeans mass is properly resolved. As other authors (e.g. O’Shea & Norman 2008; W08), we are conservative here and set this criterion to 16 grid cells in our simulations. The fourth criterion ensures that the cooling time in a given grid cell is always longer than the sound-crossing time of the cell. We also set the MinimumMassForRefinementExponent parameter to −0.2 making the simulation super-Lagrangian and therefore reducing the criteria for refinement as higher densities are reached (O’Shea & Norman 2008). We furthermore set the MinimumPressureSupportParameter equal to 10 as we have restricted the maximum refinement level in our simulations (e.g. Kuhlen & Madau 2005). With the MinimumPressureSupport option, the code assumes in each computational cell the minimum temperature necessary to make the cell Jeans stable at the highest refinement level.

We first advance our simulation forward in time using a maximum refinement level of 4. If we would have run the simulation with a high maximum refinement level from the outset, the simulation would have stalled once the virial temperature of the halo has reached 10000 K in the attempt to follow the dynamical evolution of a small dense region before a sufficiently massive DM halo had formed. We are effectively delaying the collapse by limiting the resolution. We use a friends of friends (FoF) algorithm to keep track of how our halo is progressing through time. Once a sufficiently massive halo (corresponding to a DM particle number of about 30000 particles in simulation A and 120000 particles in simulations B and C) has formed we halt the simulation. We then restart the simulation with the maximum level of refinement set to 18 (16 in simulations B and C). We will refer to this point as the collapse redshift.

The simulation then continues at a much higher resolution albeit at a significantly slower speed. We allow the simulation to evolve until the code no longer tracks the hydrodynamic evolution of the gas accurately due to a lack of resolution at the highest densities caused by restricting the highest allowed refinement level. The ENZO code issues warnings alerting the user when the code begins to produce spurious results due to a lack of resolution. At this point, we terminate the simulation.

We have also followed the recommendation in Lukic et al. (2007) for the starting redshift of our simulations. As a result, the initial redshift is quite a bit higher than that of other simulations described in the literature. Experimenting with the value of the initial redshift suggests, however, that this increased initial redshift has little effect on our results.

### 3 Characteristic Formulae

As mentioned before, we use a FoF algorithm to identify DM haloes in our simulation outputs. We adopt a linking length of 0.2 for all of our analysis (see Jenkins et al. 2001; Lukic et al. 2007). We compute physical properties of the virialized haloes using standard formulae (see e.g. Mo & White 2002). For convenience, we summarize some of the formulae below. We calculate characteristic properties for a sphere for which the mean enclosed density is 200 times the mean cosmic value $\bar{\rho}$. The characteristic ‘virial’ radius is then related to the mass as

$$R_{200} = \left[ \frac{GM}{100\Omega_m(z)H^2(z)} \right]^{1/3},$$

while the circular velocity, $V_c$, can be written as

$$V_c = \left[ \frac{GM(r)}{r} \right]^{1/2},$$

where $H(z)$ is Hubble’s constant at redshift $z$ and $\Omega_m(z)$ is the density parameter of non-relativistic matter. $M$ is the mass of our halo as determined by FoF group finder. Rewriting the equation for the virial velocity in terms of the mass, $M$, only gives

$$V_c = M^{1/3} \left[ 100\Omega_m(z) GH(z) \right]^{1/3}.$$
The Hubble constant and density parameter are related to their present-day values by

\[ H(z) = H_0 E(z) \]  (4)

and

\[ \Omega_m(z) = \frac{\Omega_m(1 + z)^3}{E^2(z)} \]  (5)

where \( E(z) \) is given by

\[ E(z) = \left( \Omega_{\Lambda,0} + (1 - \Omega_{m,0} - \Omega_{\Lambda,0})(1 + z)^3 + \Omega_{m,0}(1 + z)^7 \right)^{1/2} \].  (6)

We can now define the virial temperature of the halo as

\[ T_{\text{vir}} = \frac{\mu V^2}{2k} = 3.24 \times 10^4 \left[ \frac{V_c}{30 \, \text{km} \, \text{s}^{-1}} \right]^2 \, \text{K}, \]  (7)

where \( \mu = 0.6m_p \), with \( m_p \) being the proton mass, is the mean molecular weight.

The level of rotational support is often characterized by the dimensionless angular momentum parameter \( \lambda \) which can be written as (e.g. Bullock et al. 2001)

\[ \lambda = \frac{|\overline{L}|}{\sqrt{2} M V_c r}, \]  (8)

where \( |\overline{L}| \) is the angular momentum inside a sphere of radius \( r \) containing mass \( M \) and \( V_c \) is the virial velocity of the halo. The mean value of \( \lambda \) is \( \sim 0.04 \) for typical haloes in cosmological simulations (Barnes & Efstathiou 1987; Bett et al. 2007). We have included the above properties for the three virialized haloes in Table 1 for reference.

Also of interest is the dynamical time which we express in terms of the enclosed mass \( M(r) \) at radius \( r \),

\[ t_{\text{dyn}} = \left( \frac{3\pi}{16G \rho} \right)^{1/2} \sim 2.09 \times 10^5 \left( \frac{r}{1 \, \text{pc}} \right)^{3/2} \left[ \frac{10^9 \, M_\odot}{M(r)} \right]^{-1/2} \text{yr.} \]  (9)

### 4 RESULTS OF THE NUMERICAL SIMULATION

#### 4.1 The global dynamics of the collapse of the gas and dark matter

We first discuss the global dynamical evolution of the gas and dark matter in the three haloes we have simulated. Note that the dynamical evolution in simulation C is similar to that in simulation A and we will thus show only a subset of the plots for simulation C in the following. We have simulated boxes with 5 and 2 \( h^{-1} \) Mpc on a side, respectively. Figs 1 and 2 show the location of the haloes in simulations A and B within the typical characteristic web of filaments and sheets which is already well developed at \( z \sim 15 \) (simulation C looks very similar in this respect). The haloes are located at the intersection of several filaments. In the top row of Figs 1 and 2, we zoom in on the haloes which are approximately spherical and have a virial radius of about 1.5 and 0.5 kpc. In Fig. 3, we focus on the later stages of the evolution of the gas at the centre of simulation C. From one panel to the next, we either change the output time or the density/length scale. We have selected time zero as the time at which we restart the simulation with increased resolution, with a maximum refinement level of 18 for simulation A and 16 for simulations B and C. Recall that up to now we had run the simulation limiting the refinement to four levels in order to allow the halo to build up. We will discuss possible limitations of this approach later. Once the resolution is increased, the inner part of the haloes starts to collapse.

As discussed by W08, the collapse is highly turbulent and dynamically complex. In each halo, we are able to follow the collapse of the gravitationally unstable gas at the centre of the halo into a centrifugally supported disc (or disc-like object) and follow the dynamical evolution of the disc for several dynamical times. In simulation A, this disc has a mass of \( 2 \times 10^4 \, M_\odot \). In the second of our haloes, the gas at the centre of the halo fragments into three gravitationally bound clumps with masses of a few times \( 10^4 \, M_\odot \) and a mass ratio of approximately 3:1:1. The clumps tidally distort each other and subsequently undergo a violent dynamical interaction. One of the smaller clumps eventually merges with the most massive clump and forms a disc of \( \sim 1 \times 10^4 \, M_\odot \). The second small clump is tidally stripped of most of its mass and has a mass of a few times \( 10^3 \, M_\odot \) at the end of the simulation. The (temporary) formation of multiple systems appears, however, not to be generic. Neither simulation A with its five times more massive halo nor simulation C with its halo of similar mass shows the formation of a similar multiple system. The disc forming in simulation C has a mass of \( \sim 1 \times 10^5 \, M_\odot \).

#### 4.2 The onset of gravitational instability, mass inflow and the evolution of the density profile

The gas at the centre of our haloes attains an almost isothermal temperature profile with a characteristic temperature of 6000–7000 K, the temperature below which atomic cooling due to hydrogen cooling becomes inefficient. The temperature rises gently to 9000–10000 K at larger radii. The left-hand panels in Fig. 4 show the temperature profile of the gas in the haloes and its evolution for simulation A (top panel) and simulation B (bottom panel), respectively.

In order to understand the dynamical evolution of the gas, it is illustrative to consider the thermal and turbulent velocities of the gas. In the middle panel of Fig. 4, we show the evolution of the radial velocity (black), the thermal velocity (blue) and the turbulent velocity (red) of the gas. As the collapse gets under way, the gas develops turbulent velocities comparable to the virial velocities of the halo starting in the outer parts of the halo and progressively moving inwards. The turbulent velocities are calculated by computing the root mean square velocity of the gas after subtracting the centre of mass velocity of the halo and the velocity due to the radial inflow of the gas. We also subtract an estimate of the rotation velocity (based on the specific angular momentum and the inertia tensor as described in Section 4.3) in quadrature. The turbulent velocities in the later stages of the collapse become significantly larger than the thermal velocities of the gas which we calculate as \( V_\text{th} = \sqrt{3k\ell/\mu m_H} \), where \( \mu \) is the mean molecular weight, chosen to be 1.22 and \( m_H \) is the mass of the hydrogen atom. As in the simulations of W08, the turbulent velocities are supersonic. There is a net inflow of gas with velocities about a factor of 3 smaller than the turbulent velocity component.

In the right-hand panel of Fig. 4, we compare the ratio of the inward acceleration due to gravity to the outward acceleration due to the thermal pressure (gradient) for both simulation A (upper panel) and simulation B (bottom panel). Once the DM halo has built up, the mass of the halo is above the Jeans mass and the inward gravitational acceleration comfortably exceeds the outward acceleration due to the thermal pressure. The peak which builds up...
Figure 1. A sequence of visualizations of the density distribution in simulation A. The gas is collapsing in a halo of DM mass of $2.64 \times 10^8 \, M_\odot$, virial velocity of $\sim 30 \, \text{km s}^{-1}$ and virial temperature of $\sim 32000 \, \text{K}$. At the end of the simulation, the gas at the centre of the halo has settled into a rotationally supported disc. Each plot shows a thin slice of the density distribution centred on the grid cell with the highest density in our halo of interest. Between panels either the density range or the time of the simulation output change. The colour range represents approximately an order of magnitude range in density in each plot. The scales are proper distances.

at an enclosed mass of a few times $10^4 \, M_\odot$ for simulation A and $\sim 10^5 \, M_\odot$ for simulation B is due to the formation of a centrifugally supported disc at the centre. The double peak at certain output times for simulation B is due to the presence of multiple gas clumps within the system.

In Fig. 5, we show the evolution of the enclosed gas and DM mass as a function of radius and the density profiles of the gas and DM distribution. The DM density (in units of $m_H \, \text{cm}^{-3}$) and the DM fraction are shown for the final output times only as filled diamonds. The density profiles are calculated by averaging over spherical shells centred on the densest point in the halo. Note that in simulation B the densest point is always found in the most massive of the three clumps. The haloes collapse within 12 and 7 Myr, respectively, after we have restarted the simulation with the
increased refinement level. This corresponds to about a few free-fall times for the gas in the inner few hundred parsecs which initially has pretty much constant density. As discussed already, we initially held the refinement level at a maximum of four until the halo had a mass corresponding to $\sim 30000$ ($\sim 120000$) DM particles. The gas completely decouples from the dark matter during the collapse and becomes self-gravitating. The density profile of the gas in the inner part steepens dramatically and settles into a close to $r^{-2}$ density profile over many decades in radius as expected for an isothermal collapse (e.g. Larson 1969; W08). The inner few times $10^4 \, M_\odot$ ($10^5 \, M_\odot$) collapse further and settle eventually into rotational support. In the bottom right-hand panel of Fig. 5 (simulation B), the secondary peak in the density profile is due to the secondary clump which eventually merges with the primary clump. The peak is absent in the final two output times as one of the smaller clumps has merged with the primary clump and the other smaller clump has had most of its mass stripped away. Note that the continuous accretion on to the virialized haloes from the

**Figure 2.** Same as Fig. 1 for simulation B. The gas is collapsing in a halo of DM mass of $5.37 \times 10^7 \, M_\odot$, virial velocity of $\sim 19 \, \text{km s}^{-1}$ and virial temperature of $\sim 13000 \, \text{K}$. Note how the gas fragments into three clumps at $T = 6.442 \, \text{Myrs}$ which tidally distort each other and undergo a violent dynamical interaction.
filaments occurs on much longer time-scales and thus has no visible effect.

4.3 Angular momentum loss and settling into rotational support

In Fig. 6, we show the evolution of the specific angular momentum, $|\vec{l}|$, as a function of enclosed mass for both simulations A and B.

The angular momentum vector is calculated in the usual way via the cross product $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{p}$ is the momentum vector. We then obtain the specific angular momentum vector as a function of enclosed mass centred on the centre of mass by dividing by the enclosed mass. In both haloes, there is very significant angular momentum loss. In simulation A, the angular momentum drops by a factor of 20 or more between the initial and the final output in the inner few times $10^4 M_\odot$. The evolution in simulation B is more

**Figure 3.** Same as Fig. 1 for simulation C but only for the later stages of the evolution. The gas is collapsing in a halo of DM mass of $5.15 \times 10^7 M_\odot$, virial velocity of $\sim 19.3 \text{ km s}^{-1}$ and virial temperature of $\sim 13500 \text{ K}$. Panels 9 to 15 illustrate the dynamical evolution of the rotationally supported gas at the centre of the halo for a few dynamical times.
complicated due to the complex dynamical interaction of the triple system. Initially, the angular momentum of the gas drops by a factor of 20–100 in the primary clump and then increases again by a factor of 3–5 when one of the smaller clumps merges with the primary clump.

We now want to discuss in more detail to what extent the gas settles into rotational support. During the turbulent collapse of the gas, it is not obvious how best to define rotation velocities. We follow W08 who use the ratio \( \frac{|\vec{I}|}{|\vec{r}|} \), where \( \vec{I} \) is the specific angular momentum vector and \( \vec{r} \) is the position vector from the centre of mass as a simple but rough measure of the rotation velocity of the gas. We also calculate a second estimate of the rotation velocity based on the angular momentum \( \vec{l} \) and the inertia tensor, \( \vec{I} \).

The nine components of the inertia tensor are given by

\[
I_{xx} = \sum_j m_j (y_j^2 + z_j^2),
\]

\[
I_{yy} = -\sum_j m_j x_j y_j,
\]

and the corresponding cyclic permutations, where the sum is over computational cells. The off-diagonal components are called the products of inertia while the diagonal components are referred to as the moments of inertia. The matrix is symmetric which guarantees that the eigenvalues are real.

The angular momentum and the inertia tensor are related as

\[
\vec{l} = \vec{I} \vec{\omega},
\]

where \( \vec{\omega} \) is the angular velocity. Using the square root of the largest eigenvalue of the inertia tensor, \( a_1 \), we then estimate the rotation velocity as

\[
V_{rot} \approx \frac{|\vec{l}|}{a_1}.
\]

In Fig. 7, we compare our two estimates of the rotation velocity \( \frac{|\vec{l}|}{|\vec{r}|} \) (red) and \( \frac{|\vec{l}|}{a_1} \) (black) to the circular velocity (blue), \( V_c(r) = \sqrt{GM(r)/r} \), for simulation A (top panel) and simulation B (bottom panel). Early on in the collapse, the gravitational potential is dominated by the dark matter halo and the gas is only slowly rotating with a ratio of rotation to circular velocity of about 1:20. As the gas in the inner part of the haloes collapses and becomes self gravitating, both the circular velocities and the rotation velocities rise. As shown in the right-hand panel, the inner few times \( 10^4 M_\odot \) reach (approximate) rotational support with \( |\vec{l}|/a_1 \) slightly larger than \( V_c \) after about 12 Myr in simulation A. In simulation B, the increase in specific angular momentum at around 6.5 Myr due to the merger of one of the small clumps with the primary clump leads to a sharp increase in the rotation velocity. As we will see in more detail later the gas nevertheless settles into a regular rotationally supported disc despite this rather violent dynamical evolution. Our estimate of the rotation velocity based on the inertia tensor exceeds the circular velocity by a factor of up to 2. We will come back to this point in Section 4.5.

Note that our two estimates of the rotation velocity trace each other, but \( \frac{|\vec{l}|}{|\vec{r}|} \) is systematically lower than \( \frac{|\vec{l}|}{a_1} \) by a factor of 1.5–3. As the gas settles into rotational support, a disc-like flattened structure should gradually form. For a turbulent collapse like the one considered here, we found this to be most easily studied by looking at the evolution of the smallest eigenvalue of the inertia tensor which for a disc should be a good proxy for the thickness of the disc. In Fig. 8, we show the ratio of the square root of the smallest eigenvalue of the inertia tensor, \( a_1 \), to the radius in which the inertia tensor is calculated as a function of the enclosed mass. The ratio reaches a minimum value at a few times \( 10^4 \) to \( 10^5 M_\odot \) for simulation A and B, respectively, as expected for the formation of a rotationally supported disc. The disc appears to fatten towards the centre and is surrounded by a more spherical infall. In simulation B, the minimum moves to higher mass values as the merger of the
initially three clumps progresses. At the end of the simulation, the minimum occurs at $\sim 1 \times 10^7 M_\odot$.

### 4.4 Formation of a self-gravitating massive fat disc

About 0.1–1 per cent of the gas in the inner part of both haloes has settled into a rotationally supported self-gravitating object with rotation velocities of 25–60 km s$^{-1}$. We now progress to inspect these structures in somewhat more detail.

In Fig. 9, we show the final panels of Figs 1 and 2 at a larger scale. The mass of the central object in simulation A in the left-hand panel is $\approx 2 \times 10^4 M_\odot$. The radius of the central object is $\approx 0.3$ pc with a compact inner core with a radius of $\approx 0.1$ pc. In the right-hand panel, we show the central object in simulation B at the final output...
The formation of massive compact discs

Figure 7. Left-hand panels: the circular velocity $V_c$ (blue) and our two estimates of the rotation velocity $\mathbf{T}/a_1$ (black) and $\mathbf{T}/r$ (red) are plotted as a function of radius. As the collapse proceeds, the inner part of the halo settles into centrifugal support. Right-hand panels: the ratio of $V_{\text{rot}} = \mathbf{T}/a_1$ to $V_c$, for the same six output times plotted against radius. The central region attains rotational support, after $\approx 12.6$ and $\approx 6.7$ Myr, in simulations A and B, respectively.

Figure 8. The ratio of the square root of the smallest eigenvalue, $a_3$, of the inertia tensor to the distance to the centre of the halo. Note the dip at a few times $10^4 \, M_\odot$ where the gas settles into a rotationally supported fat disc in simulation A. For simulation B, the dip at the end of the simulation occurs at a much higher mass of $\sim 1 \times 10^5 \, M_\odot$.

The mass of the central object (large clump) is $\approx 1 \times 10^5 \, M_\odot$ and the radius is $\approx 0.5$ pc. The second clump on the right-hand side has a mass of $\approx 5 \times 10^3 \, M_\odot$ and a radius of $\approx 0.1$ pc. The disc in simulation C (shown in Fig. 3) has a final mass of $\approx 1 \times 10^5 \, M_\odot$ and a radius of $\approx 0.6$ pc.

In Fig. 10, we show the rotationally supported objects face-on and edge-on. We have plotted iso-density surfaces using overdensities within a relatively narrow range (approximately an order of magnitude). These visualizations give a clear picture of the shape of the central objects that form in all three simulations. Each object...
is quite clearly a disc. The feature in the edge-on visualization of simulation A (top panel) which points north-west is the tidal tail seen at the bottom of the face-on view. There is another, less prominent, tidal tail at the top of the disc in simulation A which appears at the bottom left in the edge-on view. The disc in simulation B (middle panel) is less obstructed, the smaller clump is also visible in both visualizations. The disc in simulation C (bottom panel) is the cleanest example of a disc. Like the disc in simulation B, it has a somewhat ellipsoidal shape.

4.5 Properties of the centrifugally supported disc(-like object)

We now have a closer look at the surface mass density profile and the level of rotational support of the disc(-like) object for each simulation. As can be seen in the left-hand panel of Fig. 11, in all three cases the surface mass density profile is exponential over several scalelengths. The scalelengths are 0.075, 0.20 and 0.27 pc in simulations A–C, respectively. At the outer ‘edge’ of the disc at about 0.3–1 pc, the surface mass density profile reverts to that expected for an isothermal spherical density distribution in which the discs are embedded.

In the middle panel of Fig. 11, we compare the density structure of the discs to that of an exponential disc which is given by

$$n(r, z) = n_0 \exp \left( -\frac{2r}{R_d} \right) \text{sech}^2 \left( \frac{z}{\sqrt{2}z_0} \right),$$

with scaleheight

$$z_0 = \frac{c_s}{(4\pi G \mu m_\text{p} n_0 e^{-2r/R_d})^{1/2}},$$

where $c_s$ is the sound speed, $\mu$ is the mean molecular weight, $n_0$ is the central density, $r$ is the distance from the centre of mass and $R_d$ is the scale radius of the disc (Spitzer 1942; Oh & Haiman 2002). We show contours of the azimuthally averaged density in the $R$–$z$ plane of the discs. The discs (especially that in simulation A) are somewhat fatter than expected if they were supported by pressure only. This is not surprising considering the rather unrelaxed dynamical state of the discs. The general agreement of the density structure with that of an exponential isothermal disc is nevertheless remarkable.

In simulation B, the structure in the top right corner is due to the secondary less massive clump.

In the right-hand panel of Fig. 11, we compare the actual rotation velocities of the gas in the discs (shown by the black solid curve) to our two estimates for the rotational velocity $|\vec{\Omega}/|a_1|$ (black dot-dashed curve) and $|\vec{\Omega}/|\vec{r}|$ (red curve) and the circular velocity (blue curve). We compute the actual rotational velocities by rotating our coordinate system into the coordinate system of the disc using the matrix of eigenvectors obtained from the inertia tensor (which we then checked visually). The rotation velocity in the plane of the disc is then easily calculated using trigonometric transforms. The peak of the actual rotation velocities occurs at radii corresponding to, two to three times the scale radius of the exponential discs and range from 25 to 60 km s$^{-1}$. The actual rotation velocities agree with our estimate from the angular momentum vector and the largest eigenvalue of the inertia tensor, $|\vec{\Omega}/|a_1|$ to within 10–20 per cent, despite the fact that the latter was calculated for the enclosed mass in spheres centred on the densest cell. As already mentioned, the other estimate for the rotation velocities $|\vec{\Omega}/|\vec{r}|$ is systematically lower by a factor of 1.5–3.

The actual rotation velocities exceed the circular velocities by up to a factor of 2. This is probably due to a combination of reasons. The peak rotation velocities of a thin exponential disc is about 15 per cent larger than that of a spherical mass configuration with the same enclosed mass. More importantly, the discs in simulations B and C and probably also in simulation A have an ellipsoidal shape which should significantly raise the rotation velocities compared to the circular velocity of a spherical mass distribution. Finally, the discs have probably not yet reached centrifugal equilibrium. The gas is still settling into rotational support and has probably fallen to somewhat smaller radius initially than expected for centrifugal equilibrium and will take some time to reach centrifugal equilibrium.

4.6 Numerical limitations

Probing the collapse of gas at the centre of dark matter haloes in a cosmological context is an extremely challenging exercise when we wish to follow the collapse to very high densities. In this work, we...
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Figure 10. The density distribution of the rotationally supported fat discs from all three simulations shown ‘face-on’ (left-hand panels) and edge-on (right-hand panels). In all cases (approximate), iso-density surfaces are plotted. Note the prominent tidal tail(s) in simulation A and the secondary clump in simulation B. The mass of the disc in simulation A is \( \approx 2 \times 10^4 \, M_\odot \), while the disc in simulation B has a mass of \( \approx 1 \times 10^5 \, M_\odot \), with the smaller clump having a mass of \( \approx 5 \times 10^3 \, M_\odot \). The mass of the disc in simulation C is \( \approx 1 \times 10^5 \, M_\odot \).

have reached radii as small as 0.01 per cent of the virial radius while at the same time following the dynamical evolution of a substantial fraction of the gas in the halo. This presents a considerable challenge even for an AMR code like ENZO. The main problem is that the high refinement levels necessary to achieve such a large dynamic range mean that the code will normally grind to a halt following the dynamical evolution of whatever is the first high-density region to form. As discussed, in order to ameliorate this problem we have changed the refinement level during the simulation in order to allow our three haloes of choice to build up to their full mass. To what
level may this have affected our results? In order to address this, we have run simulations with twice the initial refinement level and 64 times the initial refinement level and found no systematic difference in the initial value of the angular momentum of the halo. While the detailed dynamical evolution is obviously different, especially in the later stages of the collapse when the initial distribution of matter in the halo is different, the qualitative behaviour of the dynamical evolution did not change. We would also like to point out that our results are in most aspects similar to those of W08 who did not change the refinement level during the simulation. This meant, however, that they were not able to follow the gas at the centre of the halo settling into rotational support. We feel that this is the most interesting aspect of our work and well worth exploring the limits of the code with some manual intervention. Obviously further work is needed to address these questions in more detail.

5 CONCLUSIONS

We have investigated the dynamical evolution of the gas in haloes with virial temperatures of between $\sim 13,000$ and $\sim 30,000$ K assuming that cooling by atomic hydrogen and helium are the dominant cooling processes. The dynamical evolution of the gas in the haloes is complex and highly turbulent with turbulent velocities approaching the virial velocity of the haloes. The gas in the inner part of our haloes collapses isothermally with a temperature of 6000–7000 K on a somewhat slower time-scale than the free-fall time and settles into a close to isothermal (\(\rho \propto r^{-2}\)) density profile. We find no signs of efficient fragmentation confirming suggestions that the isothermal collapse of gas in a dark matter halo at temperatures close to the virial temperature of the halo leads to only modest fragmentation. The inner 0.1–1 per cent of the gas in the virialized haloes loses as much as 95 per cent of its initial angular momentum during the collapse. The gas thereby collapses by a factor of 300–1000 in radius and eventually settles into a very compact rotationally supported self-gravitating disc with peak rotation velocities of 25–60 km s$^{-1}$ and ‘radii’ of 0.3–0.6 pc (0.05–0.1 per cent of the virial radius of the host halo).

The discs have an exponential surface mass density profile with scalelength in the range 0.075–0.27 pc which extends over several scalelengths. The vertical structure of the disc is somewhat more extended than expected for a purely pressure supported isothermal axisymmetric exponential disc.

Massive compact self-gravitating discs such as those found in our simulations have been suggested to evolve into massive seed black holes which later in the hierarchical buildup of galaxies will grow into the SMBHs found at the centre of the bulges of present-day galaxies. Unfortunately, we were not yet able to follow the further dynamical evolution of the discs or the gas further out in the halo.
haloes in our simulations. However, independent of whether the gas in these discs will continue to efficiently lose angular momentum and contract further or will fragment and form an ultracompact star cluster we most probably have identified an important intermediary stage en route to the formation of a massive seed black hole.

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