Fluid Flow Model with Mean Microtubule Pressure through Porous Media

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Authors’ contributions
This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

ABSTRACT
We discussed in this paper a fractional model arising in flow of three different incompatible fluids through a porous medium with mean microtubule pressure. The method adopted for obtaining the solution is the regular perturbation technique for the analytical solution and for the transformation of the boundary conditions. The results are in decent agreement with the findings of researched work reviewed in this paper.

Keywords: Regular perturbation; microtubule pressure; fluid flow; porous medium.

1. INTRODUCTION
Microtubule pressure is termed the difference in pressure across the interface between two immiscible fluids which emanates from the forms of the microtubule. These microtubule forces are surface tension and interfacial tension [1]. Microtubule pressure plays a vital role in the description of fluid flow in a porous media. The perfect description and visualization of microtubule pressure is the spontaneous wicking mechanism, where the liquid flows without any external aided force [2]. Such phenomena according to [3] are closely related in one hand to

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the surface tension between the surrounding media such as air and on another hand, liquid-solid and air-solid surface energies.

A porous media is a solid containing void spaces (pores) either connected or unconnected, dispersed within it in either a random or regular manner [4]. These pores contain various kinds of liquid such as air, water and oil. e.t.c. Various examples can be mentioned where porous media are employed as technological tools. Examples are soil science, the porous media (soil) contains and transport nutrient and water to the plant. In Hydrology, the porous media is a water bearing and sealing layer. In the field of Engineering, porous media is applied as filter on catalyst and (reservoir rock) storing natural gas and crude oil [1].

Several research work has centered on fluid flow phenomena of two incompatible fluid through a porous media employing different mathematical techniques. [5] and [6] came up with microtubule imbibition equations, based on Hagen- Poiseuille (H.P) flow in a cylindrical tube model, on whose bases the foundation for the imbibition model was developed [7]. [8] to [12] used different mathematical approaches to study the flow in a porous media. As a result of the complex nature of porous media, there have been modifications in the classical LW equation by considering different structural characteristics of porous media.

[13] Derived the equation of microtubule rising rate in sandstone samples based on three-dimension straight pores structure network model which is composed of a repeat microtubule element with a step change. [14] Derived solutions by the application of the Sumudu transform and the Fourier sine transform. [15] Proposed a new cylindrical model to describe microtubule imbibition in a porous medium, composing of periodic succession of a single hollow spherical element. Benavente et al. [16], modified the LW equation by the introduction of the correction factors of the pores shape and tortuously to describe the pore shape of natural rock. [17] Derived an analytic solution for the microtubule rise of liquids in a cylindrical tube in terms of height as a function of time. [18] and [19] introduced the fractional theory into the microtubule imbibition model through the improved H-P equation and Modified Laplace-Young L-Y equation which strongly considering the shape and the size of the pores, the tortuousity of random porous media and the initial wetting phase saturation [20].

Microtubule imbibition model was derived from the work of [7] by considering the resistance of fluids pores media. [21] Tackled geometry description of the microtubule pressure. The state equation is formulated using notions from algebraic topology and cast in terms of measures of the macroscale state. Synchrotron-based X-ray micro-computed tomography (CT) and high resolution pore-scale simulation is applied to examine the uniqueness of the proposed relationship.

[22] Studied a new numerical approach to model microtubule effect in porous media. A pressure enhanced pace is locally introduced in the consideration of finite element discretization of the Darcy’s equation, furthermore a variation multi-scale stabilization method is selected to take into account the subrig effects on the finite element solutions. Both converges and implementation are first validated with the method of manufactured solution (MMS). A good convergences and correlation between experimental and numerical results was shown. Also, [23] investigated a circular motion of generalized second grade fluid through a porous media. They employed the method of fractional partial differential equations, solutions in the form of fox’s H-functions was obtained using the Hankel transform and the Laplace transform of the sequential fractional derivatives.

The overall goal of this work is to use a regular perturbation technique and obtain an expression which is an extension of the work of [14] for a fluid flow of three incompatible fluid through a porous media with mean microtubule pressure.

2. FORMALISM

The velocities of seepage of water ($U_w$), oil ($U_o$), and gas ($U_g$) which is an extension of [11] is given as

$$U_w = \frac{K_w k \partial \rho_w}{\mu_w}$$ \hspace{1cm} (1)
\[ U_o = -\frac{k_k k}{\mu} \frac{\partial P_o}{\partial x} \]  
\[ U_g = -\frac{k_k k}{\mu} \frac{\partial P_g}{\partial x} \]  

And the respective equations of continuity are stated as
\[ \psi \frac{\partial S_w}{\partial t} + \frac{\partial U_w}{\partial x} = 0 \]  
\[ \psi \frac{\partial S_o}{\partial t} + \frac{\partial U_o}{\partial x} = 0 \]  
\[ \psi \frac{\partial S_g}{\partial t} + \frac{\partial U_g}{\partial x} = 0 \]

where \( K \) is the permeability of the medium, \( k_w, k_o \) and \( k_g \) are respectively the relative permeability of water, oil and gas that are function of the saturation of water \( (S_w) \), oil \( (S_o) \) and gas \( (S_g) \).

\( p_w, p_o \) and \( p_g \) Represent pressure of water, oil and gas respectively. \( U_w, U_o \) and \( U_g \) are respectively the kinematic viscosities of water, oil and gas and \( \psi \) is the porosity of porous medium. From the definition of phase saturation [11], we state that
\[ S_w + S_o + S_g = 1 \]

The microtubule pressure \( (p_c) \) which is flowing phases across their interfaces is given as
\[ p_c = p_o - (p_w + p_g) \]

Following [12] and its extension, we get
\[ K_w = s_w \]  
\[ K_o = s_o \]  
\[ K_g = s_g \]  
\[ K_o = 1 - (S_w + S_g) \]

If we put equations (1) (2) and (3) into equations (4), (5) and (6) respectively, the results is stated as
\[ \psi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} (K_w k \frac{\partial P_w}{\partial x}) \]  
\[ \psi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x} (K_o k \frac{\partial P_o}{\partial x}) \]  
\[ \psi \frac{\partial S_g}{\partial t} = \frac{\partial}{\partial x} (K_g k \frac{\partial P_g}{\partial x}) \]
Eliminating \( \frac{\partial p_g}{\partial x} \) from equations (10), the result is

\[
\psi \frac{\partial s_w}{\partial t} = \frac{\partial}{\partial x} \left( K_w \frac{\partial p_c}{\partial x} \right)
\]

Brining equations (11), (12), (13) and (7) together, the result is

\[
\frac{\partial}{\partial x} \left( K_w k \frac{\partial p_c}{\partial x} \right) - K(K_w - K_g) \frac{\partial p_o}{\partial x} - K(K_w - K_g) \frac{\partial p_g}{\partial x} = 0
\]

Integrating equation (14), the expression takes the form

\[
\frac{K_w k}{\mu_w} \frac{\partial p_c}{\partial x} - K(K_w - K_g) \frac{\partial p_o}{\partial x} - K(K_w - K_g) \frac{\partial p_g}{\partial x} = -C
\]

where \( C \) is the constant of integration. The minus sign is assumed.

Equation (16) is put into equation (13) and the expression is

\[
\psi \frac{\partial s_w}{\partial t} = \frac{\partial}{\partial x} \left( \frac{k_w}{\mu_w} \frac{\partial p_c}{\partial x} \right) + \frac{C}{K_w k \left( 1 - \frac{k_w \mu_w}{\mu_o k_w} \right)} \frac{\partial p_o}{\partial x} - \frac{\partial p_g}{\partial x}
\]

Pressure of oil \( p_o \) can be defined as

\[
p_o = \frac{p_o + (p_w + p_g)}{2} + \frac{p_o - (p_w + p_g)}{2} = \bar{p} + \frac{1}{2} p_c
\]

where \( \bar{p} \) is the constant mean pressure.
Equation (18) is put into equation (15) and the value of $C$ is given as

$$C = \left( k \frac{k_w}{\mu_w} - \frac{k_g}{\mu_g} \right) \frac{\partial P_g}{\partial x} - k \left( \frac{K_w}{\mu_w} - \frac{k_o}{\mu_o} \right) \frac{\partial P_c}{\partial x}$$  

Equation (19) is substituted into equation (16) and after simplification, results in

$$\psi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left\{ \left( -\frac{3k}{2} \frac{k_0}{\mu_0} + \frac{k}{2} \frac{k_w}{\mu_w} \right) \frac{\partial P_c}{\partial S_w} \frac{\partial S_w}{\partial x} + \beta_1 \right\}$$

Equation (20) can be written as

$$\psi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left\{ \beta_1 \frac{\partial S_w}{\partial x} + \beta_2 \right\}$$

Where $\beta_1 = \left( -\frac{3k}{2} \frac{k_0}{\mu_0} + \frac{k}{2} \frac{k_w}{\mu_w} \right) \frac{\partial P_c}{\partial S_w}$.

And $\beta_2 = \frac{k_g \frac{\partial P_g}{\partial x} - k \left( \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w} - \frac{k_o}{\mu_o} \right) \frac{\partial P_g}{\partial x}}{(1 - \frac{k_o}{\mu_o} \frac{k_w}{k_w})}$ are assumed constant.

Equation (21) further reduces to

$$\psi \frac{\partial S_w}{\partial t} - \beta_1 \frac{\partial^2 S_w}{\partial x^2} = 0$$

With boundary conditions of motion for saturation constraints as

$$S_w(x,0) = 0$$
$$S_w(0,t) = 1$$
$$\frac{\partial S_w'}(0,t)}{\partial x} = 0$$

Following [24] and [25], a regular perturbation expansion of the form
\[ S_w(x, t) = \theta(x) e^{i \omega t} \]  

With the modified boundary condition

\[ \theta(0, t) = e^{-i \omega t}, \quad \theta'(0, t) = e^{-i \omega t} \]  

Substitute equation (23) into (22), the simplified results gives

\[ \theta''(x) + \frac{\psi w \theta(x)}{\beta_1} = 0 \]  

The resulting solution after applying the boundary condition of equation (24) is

\[ \theta(x) = \cos \left( \sqrt{\frac{\psi \omega}{\beta_1}} x \right) + \sin \left( \sqrt{\frac{\psi \omega}{\beta_1}} x \right) \]  

### 3. CONCLUSION

The paper presented fluid flow model of three incompatible fluids through porous media mean microtubule pressure. The analytical solution adopted a regular perturbation technique and obtained an expression which is in line with the work of [14] but an extension which is the main aim of the paper.

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### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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