Non-reciprocal transport of Exciton-Polaritons in a non-Hermitian chain

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We consider theoretically the dynamics of exciton-polaritons in a zigzag chain of coupled elliptical micropillars subjected to incoherent excitation. The driven-dissipative nature of the system along with the naturally present polarization splitting inside the pillars gives rise to the non-reciprocal dynamics, which eventually leads to the non-Hermitian skin effect, where all the modes of the system collapse to one edge. As a result the polaritons propagate only in one direction along the chain, independent of the excitation position, and the propagation in the opposite direction is suppressed. The system shows fair robustness against the typical disorder present in modern samples. Finally, using the bistable nature of the polaritons we show that information encoded in the bistability can be transferred only one way. This paves the way for compact and robust feedback-free one dimensional polariton transmission channels without the need for external magnetic field, which are compatible with proposals for polaritonic circuits.

Introduction.— Non-reciprocal elements, where the transfer of a signal is favoured only in one direction, are an essential part of information processing. However, designing such components in optical circuits is far from trivial due to the time-reversal invariance of the Maxwell’s equations. Magnetic materials can be used to achieve on-chip optical isolation but they are limited by large external magnetic field.

Other ways to achieve optical non-reciprocity, such as time-varying fields and optical nonlinearity are technically difficult to scale down to the micro- or nanoscale.

The optoelectronic system of exciton-polaritons is commonly discussed for applications in optical information processing. Polaritonic switches, amplifiers, and routers have already been realized. Mechanisms to connect elements, particularly without feedback, remain essential for further development of this field.

Significant motivation has been drawn from the field of topological photonics, where chiral edge states have been suggested for directionally dependent connections. Theoretically, robust polaritonic edge states can be achieved in many ways, such as: by using the combination of lifting the spin degeneracy and TE-TM splitting; by realizing Hofstadter’s butterfly through nonlinear interactions; by using two staggered honeycomb lattices; and by Floquet engineering. The scheme in Refs. has been realized experimentally under high external magnetic field using superconducting coils. Apart from the need for magnetic field, which implies a bulky system, chiral edge states in this scheme appear in counter-propagating pairs, which can cause unwanted feedback. To circumvent this problem, mechanisms to switch off one of the edge states, to make both the edge states co-propagating, or to use the polariton lifetime for input/output isolation, have been considered. However, topological schemes remain ultimately inefficient for information transport. The plasmonic or exciton-polaritonic zigzag Su-Schrieffer-Heeger (SSH) chain shows robust topological protection of edge-localised states, however no transport. In 2D systems, chiral topological states can propagate along the edge of the system, however, they require a large-sized bulk to separate along with

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any chance of non-reciprocity.

In this Letter we propose a scheme for non-reciprocal propagation of exciton-polaritons in a quasi-1D geometry, which relies on inherent non-Hermiticity of the system. We make use of the recent experimental development of elliptical micropillars [32, 39], where significant polarization splitting can be controllably engineered, and use the driven-dissipative nature of the system to demonstrate a robust propagation in a chain of micropillars even in the presence of disorder. This happens due to the non-Hermitian skin effect, where the usual bulk-boundary condition breaks down and all the modes of the system become localised at one edge of the chain [44, 45]. One of the key advantages of the system is that, unlike two dimensional topological polaritons, here robust propagation can be obtained through a one dimensional chain of micropillars without the need for an external magnetic field. This is particularly promising for scalability, allowing for a more compact mechanism of coupling for future polaritonic devices. We also demonstrate the transport of binary information between localized sites where non-linearity allows a bistable behaviour.

The model.— We start by considering exciton-polaritons in a pair of coupled elliptical micropillars (Fig. 1(a)) described by the following set of driven dissipative Schrödinger equations in the time-binding limit

\[ i\hbar \frac{\partial \psi_{l}^{\sigma_{+}}}{\partial t} = (\varepsilon + d\varepsilon/2) \psi_{l}^{\sigma_{+}} + J\psi_{l}^{\sigma_{-}} + \Delta_{T} e^{+2i\theta_{l}} \psi_{l}^{\sigma_{+}}, \]

\[ i\hbar \frac{\partial \psi_{r}^{\sigma_{-}}}{\partial t} = (\varepsilon + d\varepsilon/2 - i\Gamma) \psi_{r}^{\sigma_{-}} + J\psi_{l}^{\sigma_{-}} + \Delta_{T} e^{-2i\theta_{l}} \psi_{l}^{\sigma_{-}}, \]

\[ i\hbar \frac{\partial \psi_{r}^{\sigma_{+}}}{\partial t} = (\varepsilon - d\varepsilon/2 - i\Gamma) \psi_{r}^{\sigma_{+}} + J\psi_{l}^{\sigma_{+}} + \Delta_{T} e^{+2i\theta_{l}} \psi_{l}^{\sigma_{+}}, \]

\[ i\hbar \frac{\partial \psi_{l}^{\sigma_{-}}}{\partial t} = (\varepsilon - d\varepsilon/2) \psi_{l}^{\sigma_{-}} + J\psi_{l}^{\sigma_{+}} + \Delta_{T} e^{-2i\theta_{l}} \psi_{l}^{\sigma_{+}}. \]

Here exciton-polariton modes in each pillar are described by a wave function having two circular polarization components, \( \psi_{l}^{\sigma_{\pm}} \), where \( l \) and \( r \) represent the left and right pillars, respectively. We allow the left and right pillars to have different energies, where \( \varepsilon \) is their average energy and \( d\varepsilon \) is their energy difference. \( \Gamma \) is the dissipation due to the finite lifetime of polaritons, which is compensated using an incoherent excitation for modes \( \psi_{l}^{\sigma_{\pm}} \) and \( \psi_{r}^{\sigma_{\pm}} \) (the compensation of losses was previously discussed in Ref. [43]). Each pillar is coupled with its neighbour by the Josephson junction term \( J \). \( \Delta_{T} \) is the polarization splitting inside each pillar which is naturally present due to the anisotropy of the structure [44], and \( \theta \) represents the angle of polarization splitting [43]. Next, we shift to a rotating frame by redefining the wavefunctions \( \psi \rightarrow \psi \exp(-i\theta t/\hbar) \) such that the effective onsite energies become \( \pm d\varepsilon/2 \). Due to the presence of \( \Gamma \), the dynamics of \( \psi_{l}^{\sigma_{-}} \) and \( \psi_{r}^{\sigma_{+}} \) is much faster compared to that of \( \psi_{l}^{\sigma_{+}} \) and \( \psi_{r}^{\sigma_{-}} \). As a result, on the timescale of \( \psi_{l}^{\sigma_{+}} \) and \( \psi_{r}^{\sigma_{-}} \), \( \psi_{l}^{\sigma_{-}} \) and \( \psi_{r}^{\sigma_{+}} \) can be approximated as stationary states given by,

\[ \psi_{l}^{\sigma_{-}} = \frac{J\psi_{r}^{\sigma_{+}} + \Delta_{T} e^{-2i\theta_{l}} \psi_{l}^{\sigma_{+}}}{(d\varepsilon/2 - i\Gamma)}, \]

\[ \psi_{r}^{\sigma_{+}} = \frac{J\psi_{l}^{\sigma_{-}} + \Delta_{T} e^{+2i\theta_{l}} \psi_{r}^{\sigma_{-}}}{(d\varepsilon/2 - i\Gamma)}. \]

Substituting the above expressions into Eqs. (1) and (2), the dynamics of the slow components of the system can be expressed by the following eigen value equation

\[ i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_{l}^{\sigma_{+}} \\ \psi_{r}^{\sigma_{-}} \end{pmatrix} = \begin{pmatrix} H_{ll} & H_{lr} \\ H_{rl} & H_{rr} \end{pmatrix} \begin{pmatrix} \psi_{l}^{\sigma_{+}} \\ \psi_{r}^{\sigma_{-}} \end{pmatrix}, \]

where,

\[ H_{ll} = d\varepsilon/2 - \frac{J^{2}}{(d\varepsilon/2 - i\Gamma)} - \frac{\Delta_{T}^{2}}{(d\varepsilon/2 - i\Gamma)}, \]

\[ H_{rr} = -d\varepsilon/2 - \frac{J^{2}}{(d\varepsilon/2 - i\Gamma)} - \frac{\Delta_{T}^{2}}{(d\varepsilon/2 - i\Gamma)}, \]

\[ H_{lr} = -J\Delta_{T} \begin{pmatrix} e^{2i\theta_{l}} & e^{-2i\theta_{l}} \\ e^{-2i\theta_{l}} & e^{2i\theta_{l}} \end{pmatrix}, \]

\[ H_{rl} = -J\Delta_{T} \begin{pmatrix} e^{-2i\theta_{l}} & e^{2i\theta_{l}} \\ e^{2i\theta_{l}} & e^{-2i\theta_{l}} \end{pmatrix}. \]

Due to the non-Hermitian nature of the system \( H_{lr} \neq H_{rl} \) we can make \( H_{rl} = 0 \) while keeping \( H_{lr} \neq 0 \). This will give rise to the non-reciprocal hopping between the micropillars. Indeed, such a condition can be obtained by setting [43]

\[ d\varepsilon \neq 0, \quad \text{and} \quad (\theta_{r} - \theta_{l}) = \arctan(-2\Gamma/d\varepsilon). \]

The onsite energies can be manipulated by adjusting the size of the micropillars, whereas the angle of polarizations can be controlled by the orientation of the pillars [43]. It should be noted that for the above mentioned excitation scheme effectively \( \sigma_{+} \) polaritons hop from the left micropillar to the right one. It is also possible to do the same for the \( \sigma_{-} \) polaritons by interchanging the components subject to incoherent excitation [43].

We construct the chain with the aforementioned pairs of micropillars (see Fig. 1(b)). Since the non-reciprocal transmission is also flipping the spin polarization, we use one pair to transport \( \sigma_{+} \) polarization rightward to a \( \sigma_{-} \) state, which is then coupled with a regular (bidirectional) coupling to another pair that transports the \( \sigma_{-} \) state to a \( \sigma_{+} \) state. This gives a unit cell of four micropillars (see Fig. 1(c)). Even though the connection between the pairs is bidirectional, the non-reciprocity within each pair ensures non-reciprocity of the whole chain. In Fig. 2 the mode profiles of a normal chain and the non-reciprocal chain are shown. For the normal chain the modes are distributed over all sites while for the case of the non-reciprocal chain all the modes are located at one edge. Surprisingly, even though the non-reciprocal coupling is
from left to right, all the modes are localized at the left end of the chain. This is known as the non-Hermitian skin effect \[40, 51\], which occurs in non-Hermitian systems with non-reciprocity and can not be reproduced in Hermitian systems. To obtain the non-Hermitian skin effect, it is not necessary to fully switch off one of the non-diagonal terms; a small anisotropy between them is sufficient for collapsing the modes to one edge (see supplementary Movie 1).

**Band structure and pulse propagation.**— To illustrate that our results do not depend on the tight-binding approximation, we now move to a continuous model in space using the following driven-dissipative Gross-Pitaevskii equation in the circular polarization basis,

\[
\frac{i\hbar}{\partial t} \psi_{\sigma} = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(x,y) + i \left( P_{\sigma} (x,y) - \Gamma(x,y) \right) \right] \psi_{\sigma} + g P_{\sigma} (x,y) + I_{NL}^{\sigma} \psi_{\sigma} + V_T(x,y,\pm \theta) \psi_{\sigma}.
\]

Here \(m\) is the effective polariton mass, \(V(x,y)\) is the effective potential representing a zigzag chain of elliptical micropillars (See Fig.\(1\)b)), \(P_{\sigma}\) is the incoherent excitation, and \(\Gamma\) is the linear decay arising due to the finite lifetime of the polaritons. The term with the dimensionless \(g\) factor is introduced to take into account the potential created by the excitonic reservoir, which has the same profile as the incoherent excitation. \(I_{NL}^{\sigma} = (\alpha_1 - i\alpha_2) |\psi_{\sigma\uparrow}|^2 - \alpha_2 |\psi_{\sigma\downarrow}|^2\) is the nonlinear contribution, which arises due to the strong interaction between the polaritons of same (opposite) spin, represented by the coefficients \(\alpha_1(\alpha_2)\), and the nonlinear decay represented by \(\alpha_{NL}\), which corresponds to gain saturation caused by depletion of the excitonic reservoir \[53\].

\(V_T\) represents the polarization splitting inside the micropillars, which is modelled with the same spatial profile as \(V\) with an extra factor of \(\exp(\pm 2\theta)\) to take into account the orientation of each pillar \[45\].

The incoherent pumps are arranged such that the slow component of the polariton mode in each micropillar has almost zero decay. The spatial profile of the incoherent pump is shown in Figs.\(3\)a-b). Now, we have all the ingredients to calculate the band structure of the linear system \((I_{NL}^{\sigma\pm} = 0)\) under the periodic boundary condition, which is shown in Fig.\(3\)c). Being composed of four sites in one unit cell, the real part of the low energy band structure of the system is composed of four bands. It should be noted that this is an exact band structure of the system without the approximations used in Eqs. \(5-6\) and states having lower decay correspond to the approximate Hamiltonian in Eq. \(7\). Each state in the real part of the band structure is color coded according to the imaginary part of inside. (d) Mode profile, \(I_\sigma = \sum_{\sigma\pm} \int |\psi_\sigma(x,y)|^2 dy\), corresponding to the modes having lower decay (indicated with different colours) for the case of a finite chain.
this we can expect to have polariton propagation at a speed of \( v_g \) along only one direction in the micropillar chain. In Fig. 3(d) the spatial profiles of less decaying modes are plotted as a function of the propagation direction. The localization of the modes at the left edge of the chain indicates the break down of the usual bulk-boundary correspondence in Hermitian systems [50].

For the above band structure calculation, we used the elliptical micropillars with semi-major axes of 3 \( \mu \)m and 2.4 \( \mu \)m; semi-minor axes of 1.5 \( \mu \)m and 1.2 \( \mu \)m, respectively; and 2 meV potential depth. The TE-TM splitting inside the micropillars is kept at 0.2 meV, which can be adjusted by changing the ellipticity of the micropillars [44]. The decay parameter, \( \Gamma \), inside the micropillar is 0.16 meV, which corresponds to a polariton lifetime of 2 ps. The amplitude of the incoherent pumps is taken as \( P_0 = 0.45 \) meV, and the blueshift due to the excitonic reservoir is taken around 0.16 meV. Taking the angle difference of orientation between the two micropillar inside one micropillar pair around 110°, the periodicity of the lattice along the \( x \) direction becomes \( a = 12.9 \) \( \mu \)m, which corresponds to \( v_g = -0.7 \) \( \mu \)m/ps. The negative value of \( v_g \) suggests that the polaritons should propagate from right to left along the micropillar chain.

To demonstrate the non-reciprocal polariton propagation, we apply a Gaussian shaped incoherent excitation pulse in the middle of the chain. The dynamics of the polaritons can be seen in Fig. 4. Unlike a trivial chain of micropillars, where the polaritons propagate equally in both directions from the excitation spot [10], in this case they propagate only in one direction. This is expected from the band structure (see Fig. 3(c)) as the states with lower losses acquire a larger polariton population compared to the decaying states. The propagation distance can be estimated from the group velocity, \( v_g \), which is around 120 \( \mu \)m for 180 ps. Remarkably, all the polaritons in the system will be localised at the left edge of the chain, regardless of the position of the excitation spot.

This is quite similar to the recently realized topological funneling of light [51]. From the intensity profile it is clear that only the slow components of the on-site modes get excited, and they are mostly localized in the micropillars with smaller dimension, which can be attributed to the fact that the fourth band in the bandstructure (see Fig. 3(c)) has its main contribution from the smaller micropillars (see the Bloch states in [45]).

**Demonstration of feedback suppression.** Due to their strong nonlinearity polaritons show bistable behavior which makes them suitable for several applications, such as solving NP-hard problems [54], realizing bistable topological insulators [55, 56], universal logic gates [57] and enhancement of dark soliton stability [58]. In this section we introduce additional resonant pumps placed at the two ends of the chain such that the end pillars, where the on-site wave-functions \( \psi^L_{\sigma_+} \) and \( \psi^R_{\sigma_+} \) are initially in their lower bistable states. The bistability curves of the end pillars are plotted in Fig. 5 (a-b), which are obtained by slowly varying the pump, \( F \). Next an incoherent pulse is introduced at the left end of the chain, which switches \( \psi^L_{\sigma_+} \) from its lower bistable state to the upper one. Due to the non-reciprocal nature of the system, polaritons cannot propagate from left to right and understandably no switching is observed for \( \psi^R_{\sigma_+} \) (see Fig. 5 (c)). Then, we introduce the same incoherent pulse at the right end of the chain. As expected \( \psi^R_{\sigma_+} \) switches instantly, but more importantly \( \psi^L_{\sigma_-} \) also switches after sometime (see Fig. 5 (d)). This can be thought of as feedback suppressed in-
formation processing where the information is encoded in the bistability and transmitted in one direction only (from right to left). The nonlinear coefficients used for the calculations are $\alpha_1 = 1 \, \mu eV \mu m^{-2}$, $\alpha_2 = 0.05 \alpha_1$ [59] and $\alpha_{NL} = 0.5 \alpha_1$ [60]. Although we have used a mechanism of bistability with resonant excitation, we expect that the non-reciprocal transport mechanism would also be compatible with non-resonantly formed bistability mechanisms [61, 62].

Discussion and conclusion.— We have presented a scheme for non-reciprocal exciton-polariton transport in a quasi-1D chain of elliptical micropillars without any external magnetic field. Due to the non-reciprocal coupling within micropillar pairs, all the states with lower losses are localized at one edge of the chain. This makes the polaritons propagate in one direction along the chain regardless of the excitation position. This non-reciprocity also protects against backscattering (see [42] for the discussion on the robustness against disorder) and allows one-way information transfer. While our theory is restricted to the semi-classical regime, it would be interesting to extend it to the quantum optical regime, where non-reciprocal blockade effects are anticipated [64]. Because of its compactness, such a chain can be extremely useful in connecting different components of future polaritonic circuits such as the polariton neural networks [65, 66].

Acknowledgment.— The work was supported by the Ministry of Education, Singapore (Grant No. MOE2019-T2-1-004). T. L. thanks E. Z. Tan for discussions.

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Supplemental material for
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Realization of the splitting angle.— Let’s consider an elliptical micropillar rotated by an angle $\theta$ as shown in Fig. S1. Modes having linear polarization along $x'$ and $y'$ will have a splitting in energy $\Delta_T$ due to the shape anisotropy. In the pump-loss free picture, the dynamics of the polaritons in the primed basis can be represented by

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi'_x \\ \psi'_y \end{pmatrix} = \begin{pmatrix} \varepsilon'_x & 0 \\ 0 & \varepsilon'_y \end{pmatrix} \begin{pmatrix} \psi'_x \\ \psi'_y \end{pmatrix}, \quad (S1)$$

where, $\Delta_T = \varepsilon'_x - \varepsilon'_y$, and $\varepsilon'_{x(y)}$ is the eigen-energy of the mode $\psi'_{x(y)}$. In order to move to the $x$ and $y$ polarization basis we make the following transformation

$$\begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \psi'_x \\ \psi'_y \end{pmatrix} \quad (S2)$$

Modes in the circular polarization basis are related to those in the linear polarization basis by

$$\begin{pmatrix} \psi_{\sigma +} \\ \psi_{\sigma -} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix}, \quad (S3)$$

or,

$$\begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} \psi_{\sigma +} \\ \psi_{\sigma -} \end{pmatrix}. \quad (S4)$$

The final transformation matrix for going to the circular polarization basis from the primed basis becomes

$$T = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta} & e^{-i\theta} \\ -ie^{-i\theta} & ie^{i\theta} \end{pmatrix} \quad (S5)$$

Finally, the Hamiltonian in the circular polarization basis can be represented by

$$H_{\sigma \pm} = T^{-1} \begin{pmatrix} \varepsilon'_x & 0 \\ 0 & \varepsilon'_y \end{pmatrix} T = \frac{1}{2} \begin{pmatrix} \varepsilon'_x + \varepsilon'_y & e^{2i\theta}(\varepsilon'_x - \varepsilon'_y) \\ e^{-2i\theta}(\varepsilon'_x - \varepsilon'_y) & \varepsilon'_x + \varepsilon'_y \end{pmatrix} = \begin{pmatrix} \varepsilon'_x + \varepsilon'_y & e^{2i\theta} \Delta_T \\ e^{-2i\theta} \Delta_T & \varepsilon'_x + \varepsilon'_y \end{pmatrix} \quad (S6)$$

It can be clearly seen that the splitting angle acts as a coupling phase between the two circularly polarized modes.

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Derivation of the non-reciprocity condition Pair 1.— Rewriting the non-diagonal terms from Eqs. (10-11) in the main text

\[
H_{lr} = -J \Delta_T \left[ \frac{e^{2i\theta_l}}{(d\varepsilon/2 - i\Gamma)} + \frac{e^{-2i\theta_r}}{(-d\varepsilon/2 - i\Gamma)} \right], \quad \text{(S7)}
\]

\[
H_{rl} = -J \Delta_T \left[ \frac{e^{-2i\theta_l}}{(d\varepsilon/2 - i\Gamma)} + \frac{e^{2i\theta_r}}{(-d\varepsilon/2 - i\Gamma)} \right]. \quad \text{(S8)}
\]

For \(H_{rl}\) to vanish,

\[
\frac{e^{-2i\theta_l}}{(d\varepsilon/2 - i\Gamma)} = \frac{-e^{-2i\theta_r}}{(-d\varepsilon/2 - i\Gamma)} \Rightarrow e^{2i(\theta_r - \theta_l)} = e^{2i \text{arctan} (-2\Gamma/d\varepsilon)} \quad \text{(S9)}
\]

\[
\therefore (\theta_r - \theta_l) = \text{arctan} (-2\Gamma/d\varepsilon). \quad \text{(S10)}
\]

To illustrate the non-reciprocal coupling, we plot \(|H_{lr}|\) and \(|H_{rl}|\) as a function of the splitting angle in Fig. S2. The coupling is always anisotropic for \((\theta_r - \theta_l) \neq n\pi/2\), where \(n\) can be zero or any integer. The non-reciprocal coupling where one of the coupling terms goes to zero are indicated by the arrows in Fig. S2(a). The same quantities are plotted in Fig. S2(b) for \(d\varepsilon = 0\), which shows the usual bidirectional coupling with \(|H_{lr}| = |H_{rl}|\) for all values of \((\theta_r - \theta_l)\). This also shows the importance of \(d\varepsilon\) in this scheme.

FIG. S2: Plot of \(|H_{lr}|\) (in red) and \(|H_{rl}|\) (in blue) as a function of \((\theta_r - \theta_l)\). For \(d\varepsilon \neq 0\) the system shows anisotropic coupling (plotted in (a)), which becomes usual bidirectional coupling for \(d\varepsilon = 0\). Parameters: \(J = 0.5\) meV, \(\Delta_T = 0.2\) meV, \(\Gamma = 0.16\) meV, \(d\varepsilon = 0.1\) meV for (a) and 0 meV for (b).

Derivation of the non-reciprocity condition for Pair 2.— Here we exchange the components of the incoherent excitation and the evolution of the modes now become

\[
\begin{align*}
    i\hbar \frac{\partial \psi^{l}_{\sigma_+}}{\partial t} &= (\varepsilon + d\varepsilon/2 - i\Gamma)\psi^{l}_{\sigma_+} + J\psi^{r}_{\sigma_-} + \Delta_T e^{+2i\theta_l} \psi^{l}_{\sigma_-}, \\
    i\hbar \frac{\partial \psi^{l}_{\sigma_-}}{\partial t} &= (\varepsilon + d\varepsilon/2)\psi^{l}_{\sigma_-} + J\psi^{r}_{\sigma_+} + \Delta_T e^{-2i\theta_l} \psi^{l}_{\sigma_+}, \\
    i\hbar \frac{\partial \psi^{r}_{\sigma_+}}{\partial t} &= (\varepsilon - d\varepsilon/2)\psi^{r}_{\sigma_+} + J\psi^{l}_{\sigma_-} + \Delta_T e^{+2i\theta_r} \psi^{r}_{\sigma_-}, \\
    i\hbar \frac{\partial \psi^{r}_{\sigma_-}}{\partial t} &= (\varepsilon - d\varepsilon/2 - i\Gamma)\psi^{r}_{\sigma_-} + J\psi^{l}_{\sigma_+} + \Delta_T e^{-2i\theta_r} \psi^{r}_{\sigma_+}.
\end{align*}
\]
Next we move to the rotating frame by redefining the wavefunctions, \( \psi \to \psi \exp(-i\varepsilon t/\hbar) \), such that the onsite energies become \( \pm d\varepsilon/2 \). Here the dynamics of the modes \( \psi_{\sigma+}^l \) and \( \psi_{\sigma-}^r \) will be much faster (compared to those of \( \psi_{\sigma+}^l \) and \( \psi_{\sigma-}^r \)), such that they can approximated as steady states given by

\[
\psi_{\sigma+}^l = -\frac{J\psi_{\sigma+}^l + \Delta_T e^{2i\theta l}\psi_{\sigma-}^r}{(d\varepsilon/2 - i\Gamma)} \tag{S15}
\]

\[
\psi_{\sigma-}^r = -\frac{J\psi_{\sigma-}^r + \Delta_T e^{-2i\theta r}\psi_{\sigma+}^l}{(-d\varepsilon/2 - i\Gamma)} \tag{S16}
\]

In this case the non-diagonal terms of the effective Hamiltonian becomes

\[
H_{tr} = -J\Delta_T \left[ \frac{e^{-2i\theta l}}{(d\varepsilon/2 - i\Gamma)} + \frac{e^{-2i\theta r}}{(-d\varepsilon/2 - i\Gamma)} \right] \tag{S17}
\]

\[
H_{rl} = -J\Delta_T \left[ \frac{e^{2i\theta l}}{(d\varepsilon/2 - i\Gamma)} + \frac{e^{2i\theta r}}{(-d\varepsilon/2 - i\Gamma)} \right] \tag{S18}
\]

Following the steps as those for Pair 1, the non-reciprocity condition \( H_{tr} \neq 0 \) and \( H_{rl} = 0 \); is obtained for \( (\theta_r - \theta_l) = \arctan(2\Gamma/d\varepsilon) \). It should be noted that to satisfy the condition on the angle, the orientation of the pillars in Pair 2 should be opposite to those in Pair 1.

**Supplementary movies.—** In this section we have considered a chain of 100 elliptical micropillars using the aforementioned pairs. All the parameters are kept the same as those in Fig. S2(a). The spatial profile of the modes as a function of \( (\theta_r - \theta_l) \) corresponding to the slow Hamiltonian in the tight binding limit is shown in Movie 1. Surprisingly, the modes of the system are always localized at a particular edge for all values of \( (\theta_r - \theta_l) \neq n\pi/2 \), where \( n = 0, 1, 2 \). Since \( |H_{rl}| > |H_{tr}| \) for \( 0 < (\theta_r - \theta_l) < \pi/2 \), all the modes are localized at the right edge. For \( \pi/2 < (\theta_r - \theta_l) < \pi \), the strength of the coupling becomes \( |H_{tr}| > |H_{rl}| \), which shifts all the modes from the right edge to the left one. For \( (\theta_r - \theta_l) = n\pi/2 \), where \( n = 0, 1, 2 \), there is no anisotropy between the coupling terms (\( |H_{rl}| = |H_{tr}| \)) and the modes are no longer localised at a particular edge.

In Movie 2, to check the effect of the boundary condition, we calculate the spatial profile of the modes corresponding to the same chain as above but using the periodic boundary condition. For this case, all the modes have contribution from all the sites of the chain for all values of \( (\theta_r - \theta_l) \).

In Movie 3, we once again calculate the spatial profile of the modes corresponding to the same chain as in Movie 1 but without the onsite term by putting \( d\varepsilon = 0 \). As it can be seen from Fig. S2(b), this corresponds to usual bidirectional coupling and all the modes spread through all the sites of the chain. However, when \( (\theta_r - \theta_l) \) is near \( \pi/2 \), \( J > |H_{rl}| = |H_{tr}| \), and the system behaves as a Su-Schrieffer-Heeger (SSH) chain with modes localized at two edges. For \( (\theta_r - \theta_l) = \pi/2 \), \( |H_{rl}| = |H_{tr}| = 0 \), which corresponds to isolated sites.

**Pulse propagation in a system with reciprocity.—** In this section we show propagation of an incoherent pulse in a system with reciprocity. Such a system can be easily prepared by considering a straight chain of micropillars instead of a zigzag one. The dynamics of the polaritons can be seen in Fig. S3 where the total intensity of the polaritons is plotted. Unlike the non-reciprocal chain here polaritons propagate equally in both directions from the excitations spot.

![FIG. S3: Polariton propagation under an incoherent pulsed excitation through a straight chain of micropillars where the reciprocity is not broken. From the excitation spot the polaritons propagate equally in both directions. (a-e) Intensity of the polaritons for 10 ps, 50 ps, 100 ps, 150 ps and 200 ps, respectively.](Image)
Spatial profiles of the slow decaying Bloch modes in Fig. 3 in the main text — In this section we plot the spatial profile of the slow decaying Bloch modes from the fourth band of the band structure shown in Fig. 3 in the main text. Similar to the periodic case in the tight binding model these modes are spatially distributed over all sites of the chain. They also have their main intensity located at the smaller micropillars.

Effect of spatial disorder.— Disorder is always present in realistic systems. Consequently, to take the disorder into account we add a continuous disorder potential (characterized by its root mean square value and correlation length) to the system. The robustness of the system is characterized by the quantity, $I = I_L/I_T$, where $I_L$ is the intensity at the left end pillar and $I_T$ is the total intensity of the system. In Fig. S5 $I$ is plotted as functions of time $t$, and disorder strength $V_{\text{rms}}$, where for each disorder realization an incoherent pulse is launched in the middle of the chain. The white region at initial times indicates the time taken by the polaritons to reach the left end. At larger times most of the intensity of the system is located at the left end indicating the non-reciprocal nature. For larger disorder values $I$ decreases, representing comparatively lesser polaritons reaching the left end. However, the typical disorder strength in modern samples ranges between 20-30 $\mu$eV [1, 2], for which the non-reciprocal nature of the system is unhampered.

Effect of disorder on the angle between the micropillars — In this section we introduce disorder in the angle between the micropillars and to check its robustness we calculate the quantity, $I$, defined above. In Fig. S6 $I$ is plotted as a function of time $t$ and disorder strength, where for each disorder realization an incoherent pulse is launched in the middle of the chain. From the figure it is clear that the system can survive disorder up to 25° in the splitting angle.
FIG. S6: $I$ as a function of time and disorder strength. The parameters of the pulse are kept the same as the one in Fig. (4) in the main text.

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