Sampled-data control of pan-tilt platform using discrete-time high gain observer

Hameed Ullah¹, Fahad Mumtaz Malik¹, Anjum Saeed¹, Zeeshan Ali Akbar¹ and Sajjad Hussain¹

¹National University of Sciences and Technology, DEE CE&ME, Islamabad, Pakistan

Abstract. In this paper we have presented the design of sampled-data control algorithm for Pan-tilt platform using Discrete-time High Gain Observer (DHGO). The Pan-Tilt platform is a 2-axis serial robotic manipulator having a nonlinear dynamic model. The design of the control algorithm is carried out in 2 steps. In 1st steps, a continuous time state feedback control law based on feedback linearization is designed which is subsequently discretized. In the second stage, the states of the system are estimated using DHGO. To evaluate the performance of the suggested algorithm extensive simulations are carried out in Matlab Simulink.

1. Introduction

Pan-tilt platform (PTP) is a complex combination of electro-mechanical components used to carry high-precision devices such as camera or range finding sensors. The Schematic of a Pan-tilt platform has 2 degrees of freedom (DOF) and is composed of a base, a rotatable pan mechanism and a tilt mounted on the pan to orient the device in the desired direction. Pan-tilt platform also known as a gambal has potential applications in inspections, monitoring, unmanned systems and so on [1]. In all these applications, the gimbal is required to follow a profile of the pan and tilt angles termed as the reference angles/ trajectories. However, because of the presence of nonlinearities in the system dynamic model and other parametric uncertainties, achieving high precision stabilization or trajectory tracking is difficult. Furthermore, these systems are prone to external disturbances such as high-frequency vibrations, wind, and sudden shocks etc., thus a sophisticated control algorithm, which will ensure the desired performance specification irrespective of the system model parametric uncertainties and external disturbances, is required.

The solution turns out to be output feedback linearization based control algorithm. Output feedback linearization, which converts non-linear systems into linear systems, is a powerful technique to achieve the desired performance specification. However, to implement this control algorithm all the system parameters and states needs to be known, which practically is not possible. To solve this problem high gain observer (HGO) has been extensively used as an essential tools for the design of feedback control for non-linear system [2]. In literature two approaches have been reported for the implementation of the sampled data control algorithm. In the first approach, the control algorithm in continuous time is designed by means of the plant continuous-time model after that it is discretized and implemented while in the 2nd approach the controller is designed in the discrete-time by means of the system discrete time model [3]. In this study, we will opt for the first approach.

This work presents the design of a sampled data HGO based on the feedback controls for 2DOF pan-tilt platform. To achieve the aforementioned goal first a dynamic model for the system is presented. Then a continuous time control algorithm is defined for the system using a continuous time
model. Using Forward Difference (FD) method the continuous time control algorithm is discretized and is then implemented using the 2 DOF model of the gamble.

The paper is organized as, the mathematical modelling of 2DOF system is explained in section 2, followed by the design of continuous-time feedback linearization based control and HGO in section 3 and 4 respectively. The discretization of continuous time control algorithm is discussed in section 5. While in section 6 results and in section 7 conclusion are discussed.

2. Mathematical modelling
The mathematical modelling of moving objects is accomplished by attaching frames to its links. The frame assignment for our system is as shown in fig. 1. Frame 0, also known as the base frame, in case of gamble it is attached to the unmanned aerial vehicle (UAV). The motion of first rotary joint represented by frame 1. Frame 2 is attached to the second joint. It is important to note that the rotation of the 1st link is about z-axis by an angle “Θ1” as known as pan angle and the actuator torque “τ1” cause this rotation. So, the dynamic equation for the first joint mathematically given as [4]-[6],

\[
\dot{\Theta}_1 = \frac{-2l_2x\dot{\Theta}_1\dot{\Theta}_2\cos\Theta_2\sin\Theta_2+2l_2\dot{\Theta}_1\dot{\Theta}_2\cos\Theta_2\sin\Theta_2+\tau_1}{(l_2x\sin^2\Theta_2+l_2z\cos^2\Theta_2+l_1z)}
\]  

(1)

Where \(l_{2x}\) and \(l_{2z}\) are the moments of inertia for link-2 about x and z-axis respectively and \(l_{1z}\) is the moment of inertia for link-1 about z-axis.

While the rotation of the 2nd link is about y-axis by an angle “Θ2” known as tilt angle and the actuator torque “τ2” is responsible for the rotation of the second joint. So, the dynamic equation for the second joint mathematically given as [4]-[6],

\[
\dot{\Theta}_2 = \frac{l_2y\dot{\Theta}_1^2\cos\Theta_2\sin\Theta_2-l_2z\dot{\Theta}_1^2\cos\Theta_2\sin\Theta_2+\tau_2}{l_{2y}}
\]  

(2)

Where \(l_{2y}\) is the moment of inertia for link-2 about y-axis.

3. Feedback linearization of 2-DOF gimbal
Feedback linearization is one of the most important and widely used techniques for controlling nonlinear system. The basic idea of feedback linearization (FBL) is to choose appropriate control inputs which will linearize the nonlinear system by cancelling out the nonlinearities of the system, so that feasible linear control techniques could also be used for these systems [7]. The 1st stage for feedback linearization is to write the model of the system in state space form. To generate state-space form of nonlinear system explained in equation (1) and (2), we will consider the joint angles as the state variable and the motor torques as the input to the system, given as follow,

\[x_1 = \Theta_1, x_3 = \Theta_2, u_1 = \tau_1, u_2 = \tau_2\]

Substituting these values in equation (1) and (2), results in the state space model for gamble, given as

\[
\dot{x}_1 = x_2, \dot{x}_2 = \frac{-2l_2x_2x_4\sin x_3\cos x_3+2l_2x_2x_4\sin x_3\cos x_3+u_1}{l_2x_2\sin^2 x_3+l_2z\cos^2 x_3+l_1z},
\]

\[
\dot{x}_3 = x_4, \dot{x}_4 = \frac{l_2y x_2^2\sin x_3\cos x_3-l_2y x_2^2\sin x_3\cos x_3+u_2}{l_2y}
\]  

(3)

3.1. Controller Design
In compact form equation (3) can be written as,
\[ x_1 = x_2, \dot{x}_2 = f_1(x) + g_1(x)u_1, \dot{x}_3 = x_4, \dot{x}_4 = f_2(x) + g_2(x)u_2 \] (4)

Where \( f_1(x), f_2(x), g_1(x) \) and \( g_2(x) \) are defined as follow,

\[
\begin{align*}
    f_1(x) &= \frac{2(I_2z - I_2x)x_2x_4 \sin x_3 \cos x_3}{I_2x \sin^2 x_3 + I_2z \cos^2 x_3 + I_1z}, \\
    f_2(x) &= \frac{(I_2x - I_2z)x_2^2 \sin x_3 \cos x_3}{I_2y}, \\
    g_1(x) &= \frac{1}{I_2x \sin^2 x_3 + I_2z \cos^2 x_3 + I_1z}, \\
    g_2(x) &= \frac{1}{I_2y}.
\end{align*}
\]

The state feedback control input for system model based on feedback linearization approach [7] and [8] can be defined as,

\[
\begin{align*}
    u_1 &= (-f_1(x) + \nu_1)g_1^{-1}(x), \\
    u_2 &= (-f_2(x) + \nu_2)g_2^{-1}(x)
\end{align*}
\] (5)

Putting this control inputs in equation (4) will results in linear counterpart of the nonlinear system, as

\[
\begin{align*}
    \dot{x}_1 &= x_2, \\
    \dot{x}_2 &= \nu_1, \\
    \dot{x}_3 &= x_4, \\
    \dot{x}_4 &= \nu_2
\end{align*}
\] (6)

Where \( \nu_1 \) and \( \nu_2 \) are known as the auxiliary inputs and mathematically given as

\[
\begin{align*}
    \nu_1 &= -35.569x_1 - 11.968x_2 - x_3 - 0.175x_4, \\
    \nu_2 &= -1.005x_1 - 0.175x_2 - 30.181x_3 - 11.031x_4
\end{align*}
\]

4. Continuous-time Design

Let a multiple input multiple output (MIMO) nonlinear system mathematically defined [12] as

\[ \dot{x} = Ax + B\phi(x, u), \quad y = Cx \] (7)

where \( x \in \mathbb{R}^n \) represents the state vector, \( u \in \mathbb{R}^r \) represents the control input whereas \( y \in \mathbb{R}^r \) is the output. Where \( A \) is the \( n \times n \) matrix, \( B \) is the \( n \times r \) matrix and \( C \) is the \( r \times n \) matrix are given by

\[
A = \text{block diag}[A_1, \ldots, A_r], \quad B = \text{block diag}[B_1, \ldots, B_r],
\]

\[
B_j = [0 \ldots 1]_{n_j \times 1} \quad \text{and} \quad C = \text{block diag}[C_1, \ldots, C_r], \quad C_j = [1 \ldots 0]_{1 \times n_j}
\]

Where \( 1 \leq j \leq r \) and \( n = n_1 + \ldots + n_r \).

4.1. HGO Equation

The continuous time HGO equation can be written [9], [12] as

\[ \dot{\hat{x}} = A\hat{x} + B\phi_0(\hat{x}, u) + H(y - \hat{C}\hat{x}) \] (8)

Here \( \phi_0(\hat{x}, u) \) is the nonlinear function \( \phi(\hat{x}, u) \) nominal model where \( H = \text{block diag}[H_1, \ldots, H_r] \).

\[
H_j = \left[ \frac{\alpha^j_1}{\varepsilon} \frac{\alpha^j_2}{\varepsilon^2} \ldots \frac{\alpha^j_{n_j-1}}{\varepsilon^{n_j-1}} \frac{\alpha^j_{n_j}}{\varepsilon^{n_j}} \right]_{n_j \times 1}
\]

is known as the observer gain and \( \varepsilon \) is a positive small parameter. Whereas the \( \alpha^j_i \) are the positive constants to be selected in a manner that the roots of

\[
s^{n_j} + \alpha^j_1 s^{n_j-1} + \ldots + \alpha^j_{n_j-1}s + \alpha^j_{n_j} = 0
\]

have negative real parts, for all \( j = 1, \ldots, r \).

5. Sampled-data Control

5.1. HGO Discretization

To avoid the inherent ill-conditioning in realization of HGO, caused by a small value of \( \varepsilon \), we will start by scaling the observer variables [10], let \( \varphi = E\hat{x} \) so
\[ \dot{\varphi} = \frac{1}{\epsilon} [A_0 \varphi + H_0 y + \epsilon^n B_0 (E^{-1} \varphi, u)] \]  

(9)

Where \( \dot{x} = E^{-1} \varphi \), \( E = \text{diag}[1, \epsilon, \ldots, \epsilon^{n-1}] \), \( A_0 = \epsilon E (A - HC_c) E^{-1} \), \( \dot{x} = -H_0 C \), \( EB = \epsilon^{n-1} B \) and \( H_0 = \epsilon EH \)

5.2. Discrete-time Nonlinear HGO

By using the FD method [10]-[11] the nonlinear HGO observer (9) can be discretised when \( \emptyset_0 \neq 0 \), we get

\[ q(k + 1) = A_d q(k) + B_d y(k) + T \epsilon^{n-1} \emptyset_0 (E^{-1} q(k), u(k)) \]  

(10)

Where \( x^\hat{}(k) = C_d q(k) \), \( A_d = (I + \alpha A_0) \), \( B_d = \alpha H_0 \) and \( C_d = E^{-1} \)

In the FD method, we consider that \( \alpha \) is sufficiently smaller value to make sure that the eigenvalues of \( A_d = (I + \alpha A_0) \) lies inside the unit circle. Whereas in our system of 2-DOF Gimbal of equation (4) we have that

\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, 
B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, 
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, 
\emptyset_0 = \begin{bmatrix} f_1(x) + g_1(x) u_1 \\ f_2(x) + g_2(x) u_2 \end{bmatrix}, 
H = \begin{bmatrix} \alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 \end{bmatrix}
\]

By substituting these values of \( A, B, C, \emptyset_0 \) and \( H \) in equation (9), we get \( A_0 \) and \( H_0 \) which will be further used in equation (10), we get the final equations of the discrete time High Gain Observer as

\[
\hat{x}_1[k + 1] = \left(1 - \frac{\alpha \epsilon_1}{\epsilon^3} \right) \hat{x}_1[k] + \alpha \epsilon \hat{x}_2[k] + \left(\frac{\alpha \epsilon_1}{\epsilon^3} \right) y_1[k]
\]

\[
\hat{x}_2[k + 1] = \frac{1}{\epsilon} \left[ -\frac{\alpha \epsilon_2}{\epsilon^2} \right] \hat{x}_1[k] + \epsilon \hat{x}_2[k] + \left(\frac{\alpha \epsilon_2}{\epsilon^2} \right) y_1[k] + T \epsilon^3 (f_1(\hat{x}) + g_1(\hat{x}) u_1)
\]

\[
\hat{x}_3[k + 1] = \frac{1}{\epsilon^2} \left[ 1 - \frac{\alpha \epsilon_3}{\epsilon^3} \right] \epsilon^2 \hat{x}_3[k] + \alpha \epsilon^3 \hat{x}_4[k] + \left(\frac{\alpha \epsilon_3}{\epsilon^3} \right) y_2[k]
\]

\[
\hat{x}_4[k + 1] = \frac{1}{\epsilon^3} \left[ -\frac{\alpha \epsilon_4}{\epsilon^4} \right] \epsilon^2 \hat{x}_3[k] + \epsilon^3 \hat{x}_4[k] + \left(\frac{\alpha \epsilon_4}{\epsilon^4} \right) y_2[k] + T \epsilon^3 (f_2(\hat{x}) + g_2(\hat{x}) u_2)
\]

6. Simulations and results

To evaluate the performance of the suggested control algorithms simulations were carried out in the Matlab/Simulink by applying the proposed control law to a 2 DOF pan tilt platform, discussed in section 2. A comparison of DHGO based and simple feedback control is also given which shows the efficacy of the suggested control algorithm. Simulation results are presented in Fig. 2-10. The selected DHGO parameters for these simulations are, \( \epsilon = 0.8, T = 0.01, \alpha = 0.01961, \alpha_1 = 11, \alpha_2 = 30 \) and \( \alpha_3 = 12, \alpha_4 = 35.75 \). Fig. 2-5, shows the comparison of system states under simple state feedback control law with those estimated via DHGO. The error in estimating the states \( x_1 \) to \( x_4 \) via DHGO is shown in figure 6-9. From these figures it is evident that the suggested DHGO is capable to accurately track the system states. The designed control input plot of input \( u_1 \) and input \( u_2 \) are shown in Fig. 10.

![Figure 2. Comparison between system state \( x_1 \) and DHGO estimated state \( \hat{x}_1 \)](image)

![Figure 3. Comparison between system state \( x_2 \) and DHGO estimated state \( \hat{x}_2 \)](image)
Figure 4. Comparison between system state $x_3$ and DHGO estimated state $\hat{x}_3$

Figure 5. Comparison between system state $x_4$ and DHGO estimated state $\hat{x}_4$

Figure 6. Error between system state $x_1$ and DHGO estimated state $\hat{x}_1$

Figure 7. Error between system state $x_2$ and DHGO estimated state $\hat{x}_2$

Figure 8. Error between system state $x_3$ and DHGO estimated state $\hat{x}_3$

Figure 9. Error between system state $x_4$ and DHGO estimated state $\hat{x}_4$

Figure 10. Feedback control input $u_1$ and $u_2$ for DHGO

7. Conclusion
In this paper, we have discussed the design of sampled data control algorithm based on DHGO for a nonlinear 2 DOF pan tilt platform used for aerial imaging. The whole control scheme is developed in two steps, in first step HGO is design in continuous time where in the 2nd step HGO is discretised by using the FD method. The convergence of the system states shows the efficacy of DHGO based feedback linearization control. In addition to the performance recovery, another advantage of the proposed control algorithm is the reduction in implementation cost achieved by the simplification of control law through feedback linearization.

8. References
[1] Ji, Yingfeng, Ronald A. Perez, and Ryoichi S. Amano. "Modeling and Control of Underwater Pan/Tilt Camera Tracking System: Geometry Modeling and Tracking Control." ASME 2009 International Mechanical Engineering Congress and Exposition. American Society of Mechanical Engineers, 2009.
[2] Khalil, Hassan K., and Laurent Praly. "High-gain observers in nonlinear feedback control." International Journal of Robust and Nonlinear Control 24.6 (2014): 993-1015.

[3] Khalil, Hassan K. "Performance recovery under output feedback sampled-data stabilization of a class of nonlinear systems." IEEE Transactions on Automatic Control 49.12 (2004): 2173-2184.

[4] Altaf, Omair, et al. "Extended order high gain observer based stabilization of 2 DOF pan tilt platform for aerial imaging system." Proceedings of the 2018 3rd International Conference on Reliability Engineering (ICRE 2018).

[5] Craig, John J. Introduction to Robotics: Mechanics & Control, (1986).

[6] Ahmad, Naseem, et al. "Design of Adaptive SMC for EOD Robotics Manipulator Arm.".

[7] Isidori, Alberto. Nonlinear Control Systems, Springer, (1985).

[8] Khalil, Hassan K., and Jessy W. Grizzle. Nonlinear systems, vol.3 (2002).

[9] Ahrens, Jeffrey H., and Hassan K. Khalil. "High-gain observers in the presence of measurement noise: A switched-gain approach." Automatica 45.4 (2009): 936-943.

[10] Dabroom, Ahmed Mohamed, and Hassan K. Khalil. "Output feedback sampled-data control of nonlinear systems using high-gain observers." IEEE Transactions on Automatic Control 46.11 (2001): 1712-1725.

[11] Dabroom, Ahmed M., and Hassan K. Khalil. "Discrete-time implementation of high-gain observers for numerical differentiation." International Journal of Control 72.17 (1999): 1523-1537.

[12] Atassi, Ahmad N., and Hassan K. Khalil. "A separation principle for the stabilization of a class of nonlinear systems." 1997 European Control Conference (ECC). IEEE, 1997.