Aharonov–Bohm effect in the non-Abelian quantum Hall fluid

Lachezar S. Georgiev\(^1\) and Michael R. Geller\(^2\)

\(^1\)Institute for Nuclear Research and Nuclear Energy, 72 Tsarigradsko Chaussee, 1784 Sofia, Bulgaria
\(^2\)Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602-2451

(Dated: November 8, 2005)

The \(\nu = 5/2\) fractional quantum Hall effect state has attracted great interest recently, both as an arena to explore the physics of non-Abelian quasiparticle excitations, and as a possible architecture for topological quantum information processing. Here we use the conformal field theoretic description of the Moore–Read state to provide clear tunneling signatures of this state in an Aharonov–Bohm geometry. While not probing statistics directly, the measurements proposed here would provide a first, experimentally tractable step towards a full characterization of the 5/2 state.

PACS numbers: 71.10.Pm, 73.43.–f, 03.67.Lx

The quantum Hall fluid has become a paradigm of strongly correlated quantum systems \(^1\). A combination of two-dimensional confinement and strong magnetic field leads to rich phenomena driven by electron-electron interaction and disorder. As such, the use of traditional theoretical techniques such as many-body perturbation theory has had only limited success, and, in an approach pioneered in 1983 by Laughlin \(^3\), some of the most important advances have been made by correctly guessing the many-particle wave function. The states described by Laughlin, and their generalizations \(^4,5,6\), have Hall conductances \(\sigma_{xy}\) given (in units of \(e^2/h\)) by fractions with odd denominators only. The charged excitations, which have fractional charge \(4,5,6\) and statistics \(7\), are abelian anyons. In 1987, however, evidence for an even-denominator quantized Hall state at \(\nu = 5/2\) was discovered in the first excited Landau level \(8\), and the state is now routinely observed in ultrahigh-mobility systems \(3,4,9,10\). Motivated in part by this surprising result, Moore and Read (MR) introduced the ground-state trial wave function \(\Psi\)

\[
\Psi_{\text{MR}}(z_1, z_2, \ldots, z_N) = \text{Pf} \left( \frac{1}{z_i - z_j} \right) \prod_{i < j} (z_i - z_j)^2 \tag{1}
\]

for \(N\) (even) electrons in a partially occupied Landau level with complex coordinates \(z_i\), where the first term is the Pfaffian and the standard Gaussian factor is suppressed. Exact diagonalization studies \(13,14\) indicate that the exact ground state at \(\nu = 5/2\) is close to \(\Psi\).

Moore and Read also constructed excited-state wave functions. By identifying \(\Psi\) with a two-dimensional chiral conformal field theory (CFT) correlation function

\[
\Psi_{\text{MR}}(z_1, z_2, \ldots, z_N) = \langle \psi(z_1) \psi(z_2) \cdots \psi(z_N) \rangle \tag{2}
\]

of charge 1 fermion fields \(\psi \equiv \exp(i \sqrt{\kappa} \phi) \cdot \psi_M\), where \(\phi\) is a \(u(1)\) boson \(15\) and \(\psi_M\) a neutral Majorana fermion, MR proposed CFT-based excited states of the form

\[
\langle \psi_{\text{qh}}(\eta_1) \cdots \psi_{\text{qh}}(\eta_{2n}) \psi(z_1) \cdots \psi(z_N) \rangle. \tag{3}
\]

Here \(\psi_{\text{qh}} \equiv \exp(i \sqrt{\kappa} \phi) \cdot \sigma\) is the fundamental charge 1/4 quasihole field of the CFT, with \(\sigma\) the chiral spin field of the critical Ising model.

The most spectacular prediction of Moore and Read is that the quasiparticle excitations of the 5/2 state are non-Abelian: An excited state of \(2n\) quasiholes has degeneracy \(2^n - 1\), and the braiding of their worldlines generates elements from the orthogonal group \(SO(2n)\) acting on the degenerate multiplet. This property follows from the correlation function \(\Psi\), and also from an alternative picture of the state \(\Psi\) as a BCS condensate of \(l = -1\) pairs of composite fermions \(16\), which supports exotic non-Abelian vortex excitations \(17,18\).

In addition to its intrinsic interest as a system to explore non-Abelian quantum mechanics, Das Sarma et al. \(19\) have proposed to use a pair of antidots at \(\nu = 5/2\) to construct a topological quantum NOT gate, building on an intriguing idea by Kitaev to use the transformations generated by non-Abelian anyon braiding for fault-tolerant quantum computation \(20\). Unfortunately, the braiding matrices generated by \(\Psi\) are not computationally universal. But by generalizing the pairing present in \(\Psi_{\text{MR}}\) to clusters of \(k > 2\) particles, Read and Rezayi \(21\) have proposed a hierarchy of ground and excited states—CFT correlators of parafermion currents reducing to \(\Psi\) and \(\Psi\) when \(k = 2\). These parafermion states are already computationally universal at \(k = 3\).

Computation with non-Abelian quasiparticles will be incredibly challenging experimentally. Demonstrating that the actual 5/2 state is in the universality class of \(\Psi\), and that the quasiparticles are indeed non-Abelian, are necessary first steps. Direct interferometric and thermodynamic probes of the non-Abelian statistics have been proposed recently \(22,23\). Here we propose an even simpler test of the MR state, in a similar antidot geometry. Although the tunneling measurement proposed below does not directly probe the non-Abelian nature of the excitations, it can distinguish between the tunneling of ordinary electrons and that of a bosonic charge 1 excitation we call \(\kappa\), which is allowed by the CFT and which has lower scaling dimension. And the existence of two inequivalent charge 1 excitations is itself a fingerprint of the non-Abelian nature of the fundamental quasihole \(\psi_{\text{qh}}\). Experimental observation of the electron and \(\kappa\) tunneling channels would give confidence in the MR state, the powerful CFT approach, and, by extension, the \(k > 2\) parafermion hierarchy necessary for universal topological quantum computation.
need to consider. We emphasize that $\kappa$ is an allowed excitation of the Pfaffian state satisfying the parity rule [24, 30]. The scaling dimension of $\kappa$ is 1, and if it tunnels it will dominate electron tunneling in the low-temperature limit, leaving a clear experimental signature.

We turn now to a calculation of the source-drain current

$$I(V, T, \varphi) = I_0(V, T) + I_{AB}(V, T) \cos \left( \frac{\mu}{\Delta \epsilon} + \varphi \right)$$

as a function of voltage $V$ and temperature $T$, which according to our analysis can be decomposed into flux-independent and period-one oscillatory Aharonov–Bohm (AB) components. Here $\mu$ is the mean electrochemical potential of the contacts, $\Delta \epsilon \equiv 2 \pi e/L$ is the noninteracting level spacing on the antidot with edge velocity $v$ and circumference $L$, and $\varphi$ is the AB flux in units of $h/e$.

The Hamiltonian in the strong-antidot-coupling regime is

$$H = H_L + H_R + \delta H,$$

with $\delta H = \sum_{i=1,2} \left( \Gamma_i B_i + \Gamma_i^* B_i^\dagger \right)$.  \hfill (5)

The Hamiltonians for the uncoupled right- and left-moving edge states of length $L_{\text{sys}}$

$$H_R = \frac{2 \pi v}{L_{\text{sys}}} \left( L_0 - \frac{c}{24} \right) \quad \text{and} \quad H_L = \frac{2 \pi v}{L_{\text{sys}}} \left( L_0 - \frac{c}{24} \right)$$

are given by the zero modes of the CFT stress tensors $T(z)$ and $\bar{T}(\bar{z})$,

$$L_0 = \oint \frac{dz}{2 \pi i} z T(z) \quad \text{and} \quad \bar{L}_0 = \oint \frac{d\bar{z}}{2 \pi i} \bar{z} \bar{T}(\bar{z})$$

which satisfy the Virasoro algebra with central charge $c = 3/2$ [29, 50, 51]. Then

$$L_0 = \frac{1}{2} \hat{J}_0^2 + \sum_{n=1}^{\infty} J_{-n} J_n + \sum_{n=1}^{\infty} (n - \frac{1}{2}) \psi^M_{-n} + \frac{1}{2} \psi^M_n,$$

and similarly for $\bar{L}_0$. The Laurent mode expansion of the Majorana field with antiperiodic boundary conditions on the cylinder ($z = e^{2 \pi i n z/L_{\text{sys}}}$) is

$$\psi_M(z) = \sum_{n \in \mathbb{Z}} \psi^M_{n} z^{-n} \quad \text{with} \quad \psi^M_{n} = \oint \frac{dz}{2 \pi i} z^{n-1} \psi_M(z).$$

The $u(1)$ current $J(z) \equiv i \partial_z \phi$ has mode expansion

$$J(z) = \sum_{n \in \mathbb{Z}} J_n z^{-n-1}, \quad \text{with} \quad J_n = \oint \frac{dz}{2 \pi i} z^n J(z).$$

Here $J_0/\sqrt{2}$ and $\bar{J}_0/\sqrt{2}$ are the usual $g = 1/2$ Luttinger liquid number currents $N_R$ and $N_L$. The operators

$$B_i \equiv \psi_L(x_i) \psi_R^\dagger(x_i), \quad i = 1, 2,$$

entering Eq. (5) are tunneling operators acting at points $x_i$ in Fig. 1. The fields $\psi_L$ and $\psi_R$ appearing in $B$ depend on the tunneling object. The tunneling amplitudes at $x_1$ and $x_2$ are

$$\Gamma_1 = \Gamma e^{i \pi (\mu/\Delta \epsilon + \varphi)} \quad \text{and} \quad \Gamma_2 = \Gamma e^{-i \pi (\mu/\Delta \epsilon + \varphi)},$$

FIG. 1: (color online). Antidot inside a $\nu = 5/2$ Hall bar with weak tunneling points at $x_1$ and $x_2$, threaded by an AB flux $\varphi$. The geometry we consider is illustrated in Fig. 1. The $\nu = 1/3$ realization of this system was considered by one of us previously [25], and a dual configuration, the double point-contact interferometer, was studied previously by Chamon et al. [23]. In the strong-antidot-coupling regime pictured, the edge states, indicated by the arrows, are strongly reflected by the antidot. This regime can be realized experimentally in two physically distinct ways: (i) The Hall fluid can be pinched off near the $x_1$, leaving large tunneling barriers or “vacuum” regions. Only electrons can tunnel through these vacuum regions. (ii) The system can begin in the weak-antidot-coupling regime, where current from the upper edge tunnels through the antidot (acting as a macroscopic impurity) to the lower one, and the temperature is then lowered. The weak tunneling regime, which permits quasiparticle tunneling, is unstable at low temperatures, as in the $\nu = 1/3$ quantum point contact [26], and the system then flows to the stable strong-antidot-coupling fixed point, which can be described by an effective weak-tunneling theory [27]. In the $\nu = 1/3$ case, this effective theory contains electron tunneling only, and is identical to that of case (i). The most dramatic illustration of this fact comes from the exact Bethe Ansatz solution the chiral Luttinger liquid model for a $\nu = 1/3$ point contact containing only charge 1/3 quasiparticles, which nonetheless describes electron-like tunneling far off resonance [28].

In principle, the effective weak-tunneling theory for case (ii) can be different than that of (i), and we will consider this possibility here. Because there is no exact solution available for the $\nu = 5/2$ quasiparticle tunneling model, we will guess the form of the effective theory. Guided by the reasonable condition that any charge 1 excitation of the CFT preserving the stability of the fixed point can potentially tunnel (the charge requirement introduced to recover the $\nu = 1/3$ result), we need to consider a cluster of at least four fundamental quasiholes $\psi_{\text{qh}}$. According to the fusion rule $\sigma \times \sigma = 1 + \psi_M$ [29, 31], the product $\psi_{\text{qh}} \times \psi_{\text{qh}} \times \psi_{\text{qh}} \times \psi_{\text{qh}} = \psi + \kappa$ yields two distinct tunneling objects, the electron/hole $\psi$, and a charge 1 boson

$$\kappa \equiv e^{i \pi \sqrt{2} \phi}:$$

whose quantum numbers are summarized in Table III. There are also excitations less relevant than the electron that we do not
is taken with respect to the Hamiltonian

$$H_0 \equiv H_R + H_L - \mu_R N_R - \mu_L N_L.$$  

The finite-temperature correlation function $X_{ij}(t)$ is computed in three steps: (i) First, it is split into products of finite-temperature correlation functions (with chirality $\pm$) of the form $\langle \psi^\dagger_\pm(x,t) \psi_\pm(x',t') \rangle_\beta$; (ii) Then these one-dimensional thermal correlation functions are obtained as zero-temperature correlation functions (after Wick rotation to imaginary time $\tau$) on a cylinder with circumference $L_T \equiv v/(k_B T)$; (iii) Finally, we map the cylinder to the complex plane by the conformal transformation $z_\pm = e^{2\pi i (\nu t x)}/L_T$ where, for primary fields with scaling dimension $\Delta$,

$$\langle \psi^\dagger_\pm(z) \psi_\pm(z') \rangle = (z - z')^{-2\Delta} \text{ for } |z| > |z'|.$$  

Under the conformal map a chiral primary field transforms as $\psi_\pm(x,\tau) \rightarrow (2\pi)^{-1/2} \left[ \int_{[0,\pi] - \nu t x} \right]^{\Delta} \psi_\pm(z)$, and by going back to real time we obtain

$$\langle \psi^\dagger_\pm(x,t) \psi_\pm(0) \rangle_\beta = \frac{1}{2\pi} \text{sh}^{2\Delta} \left[ \pi (x \mp vt \pm i \varepsilon)/L_T \right],$$  

where $\varepsilon$ is a positive infinitesimal required by $\psi^\dagger_\pm$. Finally, the desired response function is

$$\langle \psi^\dagger_\pm(z) \psi_\pm(z') \rangle = (z - z')^{-2\Delta} \text{ for } |z| > |z'|.$$  

TABLE I: Some chiral conformal fields and their quantum numbers. $\Delta_c$ and $\Delta_0$ are the scaling dimensions in the charged and neutral sectors, $\Delta \equiv \Delta_c + \Delta_0$ is the total CFT dimension, and $\theta \equiv 2\pi \Delta \text{ (mod } 2\pi)$ is the statistics angle.

| field           | charge | $\Delta_c$ | $\Delta_0$ | $\Delta$ | $\theta/\pi$ |
|-----------------|--------|------------|------------|----------|--------------|
| quasihole $\psi_{qh}$ | $\frac{1}{2}$ | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\kappa$ particle | 1      | 1          | 0          | 1        | 0            |
| Majorana $\psi_m$ | 0      | 0          | $\frac{1}{6}$ | $\frac{1}{6}$ | 1            |
| electron $\psi$  | 1      | 1          | $\frac{1}{2}$ | $\frac{1}{2}$ | 1            |

where, with no loss of generality, $\Gamma$ can be taken to be real. The bare amplitude $\Gamma$ depends on microscopic details and type of tunneling object.

The tunneling current $I(V, T, \varphi)$ can be calculated by linear response theory along the lines of Refs. [24] and [25], leading to

$$I_0(V, T) = 4e\Gamma^2 \text{Im } X_{11}(\omega = eV)$$  

and

$$I_{AB}(V, T) = 4e\Gamma^2 \text{Im } X_{12}(\omega = eV),$$

where $X_{ij}(\omega)$ is the Fourier transform of the response function

$$X_{ij}(t) = -\frac{i\theta(t)}{2\pi^2} \left( \frac{\pi}{L_T} \right)^{2\Delta} \text{Im} \left\{ \text{sh} \left[ \pi (x_i - x_j + vt + \varepsilon)/L_T \right] \text{sh} \left[ \pi (x_i - x_j - vt - \varepsilon)/L_T \right] \right\}^{-2\Delta}.$$  

When $2\Delta$ is an integer, which will be the case here, the transform $X_{ij}(\omega)$ has an infinite number of poles of order $2\Delta$ and can be obtained by residue summation. Note that the local response function which determines the direct current $I_0$ could be obtained as the limit

$$X_{11}(\omega) = \lim_{\varepsilon \rightarrow 0} X_{12}(\omega).$$

$$I_0^{(el)} = \frac{e}{h} \frac{\gamma_2 \Delta \varepsilon}{240\pi^2} \left\{ 64 \left( \frac{T}{T_0} \right)^4 \left( \frac{2\pi eV}{\Delta \varepsilon} \right) + 20 \left( \frac{T}{T_0} \right)^2 \left( \frac{2\pi eV}{\Delta \varepsilon} \right)^3 + \left( \frac{2\pi eV}{\Delta \varepsilon} \right)^5 \right\},$$

and

$$I_{AB}^{(el)} = \frac{e}{h} \frac{2\gamma_2 \Delta \varepsilon (T/T_0)^3}{\pi \text{sh}^3(T/T_0)} \left\{ 2 \left( \frac{\pi eV}{\Delta \varepsilon} \right)^2 + 2 \left( \frac{T}{T_0} \right)^2 \left( 1 - 3 \text{cth}^2 \left( \frac{T}{T_0} \right) \right) \right\} \sin \left( \frac{\pi eV}{\Delta \varepsilon} \right) + 6 \left( \frac{\pi eV}{\Delta \varepsilon} \right) \frac{T}{T_0} \text{cth} \left( \frac{T}{T_0} \right) \cos \left( \frac{\pi eV}{\Delta \varepsilon} \right).$$
T

chiral Fermi liquid current \( (T_0) \) and

\[
I^{(FL)}_0 = \left( \frac{e^2}{h} \right)^2 \frac{\gamma_{FL}^2}{\pi} V \quad \text{and}
\]

\[
I^{(FL)}_\kappa = \frac{e}{\hbar} 2 \gamma_{FL}^2 \frac{\Delta \epsilon}{\pi} \left( T/T_0 \right) \sin \left( \frac{\pi e V}{\Delta \epsilon} \right).
\]  

The contribution from the \( \kappa \) channel alone is

\[
I^{(\kappa)}_\kappa = \frac{e^2}{h} 4 \gamma_{\kappa}^2 \left( \frac{T}{T_0} \right)^2 V \left[ 1 + \frac{1}{4} \left( \frac{eV}{\pi k_B T} \right)^2 \right],
\]  

and

\[
I^{(\kappa)}_\epsilon = \frac{e^2}{h} 4 \gamma_{\epsilon}^2 \left( \frac{T}{T_0} \right)^2 V \left[ 1 + \frac{1}{4} \left( \frac{eV}{\pi k_B T} \right)^2 \right],
\]  

FIG. 2: (color online) The Aharonov–Bohm current \( I_{AB}(V, T) \) for \( \Delta = 1 \) (\( \kappa \) tunneling) as a function of the applied voltage \( V \) compared to \( \Delta = 3/2 \) (electron tunneling) reduced by a factor of 100 and the chiral Fermi liquid current \( (\Delta = 1/2) \) multiplied by a factor of 10 at \( T = T_0 \).

The linear conductance \( G \equiv (dI/dV)_{V \to 0} \) for these channels takes the form

\[
G(\phi, T) = G_0(T) + \cos \left[ 2\pi \left( \frac{\mu}{\Delta \epsilon} + \phi \right) \right] G_{AB}(T),
\]

where \( G_0 \) and \( G_{AB} \) are the direct and the AB conductances, respectively. For the \( \kappa \) channel they read

\[
G^{(\kappa)}_0(T) = \left( \frac{e^2}{h} \right) 4 \gamma_{\kappa}^2 \left( \frac{T}{T_0} \right)^2,
\]

\[
G^{(\kappa)}_{AB}(T) = \left( \frac{e^2}{h} \right) 4 \gamma_{\kappa}^2 \left( \frac{T}{T_0} \right)^2 \left[ \left( \frac{T}{T_0} \right) \csc \left( \frac{T}{T_0} \right) - 1 \right].
\]
While for the electron channel we obtain

\[ G_0^{(el)}(T) = \left( \frac{e^2}{\hbar} \right) 4\gamma_{el}^2 \left( \frac{4}{15} \right) \left( \frac{T}{T_0} \right)^4, \]

\[ G_{AB}^{(el)}(T) = \left( \frac{e^2}{\hbar} \right) 4\gamma_{el}^2 \left( \frac{T}{T_0} \right)^3 \times \left[ \left( \frac{T}{T_0} \right)^2 \text{cth}^2 \left( \frac{T}{T_0} \right) + 3 \left( \frac{T}{T_0} \right) \text{cth} \left( \frac{T}{T_0} \right) \right]. \]

These conductances are compared to the Fermi liquid ones

\[ G_0^{(FL)}(T) = \left( \frac{e^2}{\hbar} \right) 2\gamma_{FL}^2, \]

\[ G_{AB}^{(FL)}(T) = \left( \frac{e^2}{\hbar} \right) 2\gamma_{FL}^2 \frac{(T/T_0)}{\text{sh}(T/T_0)} \]

and are plotted in Fig. 8. Both the electron and \( \kappa \) contributions display a pronounced maximum as a function of temperature. In the \( T \to 0 \) limit the \( \kappa \) contribution, varying as \( T^2 \), dominates the electron contribution. The temperature dependence of \( G \) in the low- and high-temperature regimes is summarized in Table II. There is also an interesting zero-temperature nonlinear regime \( k_B T \ll eV \ll k_B T_0 \), where both the direct and AB currents vary as \( I^{(\kappa)} \sim V^3 \) and \( I^{(el)} \sim V^5 \), independent of temperature. Finally, we also note that the limit of a single \( \nu = 5/2 \) quantum point contact (in the stable, strong tunneling regime) follows from our results by letting \( T_0 \to \infty \); the electron contribution in this case agrees with that for tunneling between a FL and \( \nu = 5/2 \) edge state [32].

In conclusion, we have used the CFT picture of Moore and Read [12] to calculate the AB tunneling spectrum in a \( \nu = 5/2 \) antidot geometry. Observing the electron and possibly \( \kappa \) transport channels will give evidence in support of the non-Abelian nature of the 5/2 state.

We thank Ady Stern for useful discussions. LSG has been partially supported by the FP5-EUCLID Network Program of the EC under Contract No. HPRN-CT-2002-00325 and by the Bulgarian National Council for Scientific Research under Contract No. F-1406. MRG was supported by the NSF under grants DMR-0093217 and CMS-040403.

References

[1] R. E. Prange and S. M. Girvin, eds., *The Quantum Hall Effect* (Springer-Verlag, Berlin, 1990), 2nd ed.
[2] S. Das Sarma and A. Pinczuk, eds., *Perspectives in Quantum Hall Effects* (Wiley, New York, 1997).
[3] R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[4] V. J. Goldman and B. Su, Science 267, 1010 (1995).
[5] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).
[6] R. de Picciotto, M. Reznikov, M. Meiblum, V. Umansky, G. Binnin, and D. Mahalu, Nature (London) 389, 162 (1997).
[7] F. E. Camino, W. Zhou, and V. J. Goldman, Phys. Rev. B 72, 75342 (2005).
[8] R. Willett, J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. 59, 1776 (1987).
[9] W. Pan, J. S. Xia, V. Shvarts, D. E. Adams, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. 83, 3530 (1999).
[10] J. P. Eisenstein, K. B. Cooper, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 88, 76801 (2002).
[11] J. S. Xia, W. Pan, C. L. Vicente, E. D. Adams, N. S. Sullivan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. 93, 176809 (2004).
[12] G. Moore and N. Read, Nucl. Phys. B 360, 362 (1991).
[13] R. H. Morf, Phys. Rev. Lett. 80, 1505 (1998).
[14] E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. 84, 4685 (2000).
[15] This is normalized according to \( \langle \phi(z)\phi(z') \rangle = -\text{ln}(z-z') \).
[16] N. Read and D. Green, Phys. Rev. B 61, 10267 (2000).
[17] D. A. Ivanov, Phys. Rev. Lett. 86, 268 (2001).
[18] A. Stern, F. von Oppen, and E. Mariani, Phys. Rev. B 70, 205338 (2004).
[19] S. Das Sarma, M. Freedman, and C. Nayak, Phys. Rev. Lett. 94, 166802 (2005).
[20] A. Y. Kitaev, Ann. Phys. 303, 2 (2003).
[21] N. Read and E. Rezayi, Phys. Rev. B 59, 8084 (1999).
[22] A. Stern and B. I. Halperin, eprint cond-mat/0508447.
[23] P. Bonderson, A. Kitaev, and K. Shtengel, eprint cond-mat/0508616.
[24] M. R. Geller and D. Loss, Phys. Rev. B 56, 9692 (1997).
[25] C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen, Phys. Rev. B 55, 2331 (1997).
[26] K. Moon, H. Yi, C. L. Kane, S. M. Girvin, and M. P. A. Fisher, Phys. Rev. Lett. 71, 4381 (1993).
[27] This assumes, of course, that there is a stable strong-tunneling fixed point here, and that there are no other stable fixed points.
[28] P. Fendley, A. W. W. Ludwig, and H. Saleur, Phys. Rev. Lett. 74, 3005 (1995).
[29] A. Cappelli, L. S. Georgiev, and I. T. Todorov, Commun. Math. Phys. 205, 657 (1999).
[30] L. S. Georgiev, Nucl. Phys. B 651, 331 (2003).
[31] M. Milovanovic and N. Read, Phys. Rev. B 53, 13559 (1996).
[32] N. Read and E. Rezayi, Phys. Rev. B 54, 16864 (1996).