An electrovortex flow around two fully submerged electrodes

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Abstract. Electrovortex flow (EVF) of liquid metal in a container with submerged electrodes and bifilar current supply relates to the problem of melt stirring in electrometallurgical aggregates like welding, electroslag remelting, direct current (DC) electrical arc furnaces (EAF) and others. Maximum values of electromagnetic (EM) force and flow velocity are developing in the region between electrodes. Sophisticated 3D flow patterns are forming around them. In presenting paper, the examples of measured electrical potential and 3D self-magnetic field of supplied DC in comparing with numerical calculation results as well as with quasi-2D model of EM fields are discussed. Several numerically computed 3D EVF patterns are presented for two regimes: i) laminar and ii) turbulent, which is obtained with LES (Large Eddy Simulation) approach. There is good qualitative and quantitative correspondence between experimentally obtained data and numerical computations results as well as with previously published results.

1. Introduction

EVF between submerged electrodes have been studied in 1970–1980 for purposes of physical modelling of electrometallurgical technologies. In considered model, a spatially distributed electrical current with density \( J \) interacts with self-magnetic fields \( B \) and generates EM force \( F = J \times B \). This force field is non-potential and drives conductive liquid circulation in the confined container [1].

Due to essentially 3D pattern, the main investigation methods were concerned with EM force field estimations, prognoses of flow patterns character and flow observation on a model’s free surface. The measurements of velocity along symmetry axis were performed with fiber optical sensor in [2]. A review of various flow patterns between two electrodes and basic flow properties are presented in [3].

The development of innovative technologies is the cause of renewed interest to EVF at present time. Current paper considers laboratory scale 3D system of relatively simple common configuration with two planes of symmetries and bifilar DC supply.

3D self-magnetic EM field is researched with the following approaches:
• measurements at experimental setup;
• analytical calculations using developed quasi 2D model;
• numerical computations with application of commercial package *ANSYS Maxwell*.

3D EVF patterns are analyzed with the following approaches:
• numerical study of flow laminar and turbulent regimes using *ANSYS FLUENT* package; LES model of turbulence is used;
• flow patterns obtained during observation of liquid metal free surface.

2. Scheme of experimental setup and computational model

The scheme of model is shown in figure 1(a). The photo of experimental setup is presented in figure 1(b) – mirror of liquid metal reflects surrounding fragments of laboratory.

Cold (melting point is 10.5°C) liquid metal (*Galinstan*) is placed in a cylindrical container (diameter $D = 220$ mm) of low depth ($H = 20$ mm). Top surface of liquid is free.

Two copper cylindrical electrodes (diameter $d = 20$ mm) are built in bottom-up through the container bottom and are fully submerged into liquid – flat top ends of electrodes are at the same level with liquid free surface. Electrode axes are located on a plane of bath symmetry and on equal opposite distances from the bath axis. The distance between electrode axes $2x_e = 60$ mm.

Bifilar power supply scheme is used so that electrical current circuit is closed through a liquid metal between cylindrical walls of electrodes. Total current $I$ can be changed in the interval from 100 A till 900 A.

Electrical conductivity for liquid metal (*Galinstan*) and copper are $\sigma_m = 3 \cdot 3.3 \cdot 10^6 S/m$ and $\sigma_e = 5.8 \cdot 10^7 S/m$.

![Figure 1](image)

3. Governing MHD (magnetohydrodynamics) equations

The stationary EVF is described [1] with transfer equation for vorticity (1) and continuity equation (2)

\[
\nabla \times (W \times V) = \frac{\eta}{\rho} \Delta W + \frac{1}{\rho} \nabla \times (J \times B) 
\]

\[
\nabla \cdot V = 0
\]

where $V$ is velocity and $W = \nabla \times V$ is vorticity for liquid metal. Dynamic viscosity $\eta$ and density $\rho$ are constants.

EM field is described with *Maxwell* equations (3)–(6) and *Ohm’s low* (7)

\[
\nabla \times B = \mu_0 J
\]
\[ \nabla \cdot B = 0 \]  (4)
\[ \nabla \times E = 0 \]  (5)
\[ \nabla \cdot E = 0 \]  (6)
\[ J = \sigma E \]  (7)

where \( E \) is electrical field intensity; \( \mu_0 \) is magnetic constant and electrical conductivity \( \sigma \) for liquid metal \( \sigma_m \) and electrodes \( \sigma_e \) are constants.

The following assumptions are taken into account:
- there is no influence of fluid flow on EM fields \( V \times B \ll J/\sigma \) (electrodynamics approximation);
- free surface deformation is not essential and don’t influence the distribution of EM field;
- thermogravity convection is neglected in comparison with strong EM stirring.

The Cartesian coordinate system \( x, y, z \) with orts \( I_x, I_y, I_z \) are used. The origin is placed at the bath bottom centre so, that \( z \)-axis directs to the free surface, \( x \)-axis – to the right electrode and \( y \)-axis – depthward from an electrodes plane. There are two plane of symmetries geometry of model: \( y = 0 \) is the electrodes’ plane and \( x = 0 \) is the orthogonal plane.

4. **Analytical solution for EM field using quasi 2D approximation**

The approximations for the case of \( \sigma_e \gg \sigma_m \) are following:
- \( z \)-component of current density in liquid is equal to zero;
- considered functions depend on coordinates \( x, y \):

\[
J_x = \frac{\partial \varphi}{\partial x} = -\frac{\partial \psi_1}{\partial y}, \quad J_y = \frac{\partial \varphi}{\partial y} = \frac{\partial \psi_1}{\partial x}, \quad J_z = 0 \tag{8}
\]

\[
\frac{\partial \varphi}{\partial z} = \frac{\partial \psi_1}{\partial z} = 0 \tag{9}
\]

where \( \varphi = \varphi(x, y) \) is electrical potential and \( \psi_1 = \psi_1(x, y) \) is electrical stream function in planes \( z = \text{const} \) inside liquid layer \( 0 \leq z \leq H \).

Complex potential \( Q = Q(u) = \varphi + i \psi_1 \) of complex variable \( u = x + iy \) for model, shown in figure 1(a), can be written as follows:

\[
Q = \frac{A}{2\pi} \ln \left| \frac{U - 1}{U + 1} \right| \tag{10}
\]
\[
\varphi = \frac{A}{2\pi} \ln \sqrt{\frac{x + 1}{x - 1}} \tag{11}
\]
\[
\psi_1 = \frac{A}{2\pi} \left( \text{arctg} \frac{y}{x + 1} - \text{arctg} \frac{y}{x - 1} \right) \tag{12}
\]

where \( \varphi_0 \) is the calibration point potential.

Then from equation (3) and expressions (8),(9) we can get related expressions for self-magnetic field of the current:

\[
B_x = (b + cz) \frac{\partial \psi_1}{\partial x} \tag{13}
\]
\[
B_y = (b + cz) \frac{\partial \psi_1}{\partial y} \tag{14}
\]
\[
B_z = \psi_1(c - 1) \tag{15}
\]
and corresponding expressions for EM force vector and its curl:

\[
F = J \times B = F_x I_x + F_y I_y + F_z I_z = \left| 1 - c \right| \nabla \psi_1 \frac{1}{2} - \left| b + cz \right| \nabla \psi_1 \right|^2 I_z
\]  \hspace{1cm} (16)

\[
\nabla \times F = -\frac{\partial F_z}{\partial y} I_x + \frac{\partial F_z}{\partial x} I_y
\]  \hspace{1cm} (17)

Here all sizes are related to \( x_e \), the constant \( A \) in equations (10)–(12) is proportional to total current, constants \( b \) and \( c \) in equations (13)–(15) will be checked experimentally. As it seen from equation (16), only \( z \)-component of EM force is non-potential; and horizontal gradients of \( F_z \) define the curl of EM force (17). Note that the maximum values of \( J \), \( B \), \( F \) and \( \nabla \times F \) in liquid metal can be found in the electrode plane \( y = 0 \) near side wall of electrodes and between them [2, 3]. Thus main flow pattern is generated in central region as well. Consequently, the influence of external boundary to EM field in central area of bath is neglected and therefore external boundary is not included into complex potential in equation (10).

5. Comparison of measured and computed EM field

EM field for 3D model (figure 1(a)) has been numerical computed with ANSYS Maxwell package. At experimental setup (figure 1(b)) the electric potential has been measured for two planes:

- for electrodes plane \( y = 0 \) (electrodes axes lie in this plane) between electrodes for coordinates \(-20 \text{ mm} < x < 20 \text{ mm} \) and \( 0 < z < 20 \text{ mm} \);
- for orthogonal plane \( x = 0 \) and coordinates \(-20 \text{ mm} < x < 20 \text{ mm} \) and \( 0 < z < 20 \text{ mm} \);

The step between measurements points is equal \( 1 \text{ mm} \) for all coordinates \( x, y \) and \( z \).

5.1. Electrical potential

Comparison of measured (figure 2(a)) and calculated (figure 2(b)) results for electrical potential distribution shows, that analytical solution using quasi 2D approximation is enough for precise estimations of EM field at middle horizontal plane \( z = 10 \text{ mm} \) between electrodes.

![Figure 2](image)

**Figure 2.** Distribution of electrical potential between electrodes at plane \( z = 10 \text{ mm} \):

a) measurements results; b) results of calculation (red line) with quasi 2D approximation – equation (11) and constant values \( \psi_0 = 0.0157 \) and \( A = 5000 \)

5.2. Induction of magnetic field

Measured (figure 3 (a)) and numerically computed (figure 3 (b)) results for \( y \)-component of magnetic field induction are of good qualitative correspondence. Magnetic field is stronger near electrodes in zone of bath bottom (\( z = 0 \)).
The quantitative distinctions may be concerned with the following reason:

- measured electrical conductivity of Galistan $\sigma_m$ in experimental setup is for 10% smaller than declared in material certificate and thus is smaller than $\sigma_m$ values, which is used in numerical computations.

6. EVF patterns

6.1. Computational results

Liquid metal EVF for 3D model (figure 1(a)) has been numerical computed with ANSYS FLUENT package. The following flow regimes are considered:

- **Turbulent regime** is obtained with applications of LES model. Estimated with expression (18) value of Reynolds number is $\Re \approx 10^4$. As characteristic dimension the liquid metal height $H$ is chosen, maximum values of velocity is $v_{max} = 0.18 \text{ m/s}$. Galistan viscosity is $\eta = 2.2 \cdot 10^{-3} \text{ kg/}(\text{ m } \cdot \text{s})$ and density is $\rho = 6.361 \cdot 10^3 \text{ kg/m}^3$. Electrical current supplied over electrodes is $I_0 = 1000 \text{ A}$.

- **Laminar regime** of EVF is computed for dynamic viscosity $\eta_l = 0.22 \text{ kg/}(\text{ m } \cdot \text{s})$, which is chosen for two orders greater than Galistan viscosity. Estimated value of Reynolds number is $\Re \approx 40$, maximum values of velocity is $v_{max} = 0.07 \text{ m/s}$.

$$Re = \frac{v_{max} H}{\eta / \rho} \quad (18)$$

Laminar regime can be described with Stokes approximation (linear equation (1) without convective term) for extremely slow flows. Further development of laminar flow needs description with nonlinear equation (1) until turbulent pulsations appear. In numerical computations, the laminar flow regime of liquid metal can be reached with decrease of supplied electrical current from $I_0 = 1000 \text{ A}$ to value $I_l = 10 \text{ A}$ (100 times less). For this case non-dimensional parameter, estimated with expression (19), decreases from $S_0 = 4.2 \cdot 10^7$ (turbulent regime) to value $S_l = 4.2 \cdot 10^3$ (laminar regime), that is four orders less. The same result can be obtained with liquid viscosity increasing for two orders without variations of electrical current.

$$S = \frac{\mu_0 \rho I^2}{4\pi^2 \eta^2} \quad (19)$$

Figure 3. Distribution of magnetic field induction $B_y(x, z)$ at electrodes plane $y = 0$ (electrodes axes lie in this plane): a) measurements results; b) results of numerical computations with ANSYS Maxwell.
Computational results are illustrated as follows:
- streamlines for laminar flow regime correspond to quasi-steady state (flow time $t \approx 30 \text{ min}$) – figures 4(a), 5(a), 6(a);
- streamlines for turbulent flow regime are time-averaged for flow time period $t \approx 100 \text{ s}$ – figures 4(b), 5(b), 6(b).

3D patterns of EVF are presented for the following characteristic planes:
- at electrodes plane $y = 0$ (figure 4) maximum values of melt velocity are obtained alongside electrodes walls. At free surface between electrodes, there are liquid metal counter-moving jets, coming from electrodes to vertical axis $z$;
- at plane $x = 0$, which is orthogonal to electrodes plane $y = 0$ (figure 5), maximum values of melt velocity are obtained at free surface, where are obtained divergent jets from axis $z$;
- at horizontal middle plane $z = 10 \text{ mm}$ (figure 6) are obtained four contours of melt circulation, maximum values of melt velocity are reached around electrodes and in zone between electrodes.

Melt flows, shown in three characteristic planes, mentioned above, are forming the following integrated 3D pattern of EVF. Melt circulation is driven by EM force, which has maximum value between two electrodes. Upstream jets alongside walls of submerged electrodes transform into counter-moving jets on a free surface with further circulation both in electrodes plane and in orthogonal plane. Closing of all streams of melt is forming four contours circulation, which is observed in horizontal middle plane.

Figure 4. Streamlines at plane $y = 0$: a) laminar flow; b) turbulent flow.

Figure 5. Streamlines at plane $x = 0$: a) laminar flow; b) turbulent flow.
6.2. Qualitative comparison of computed and experimental EVF patterns

Figure 7 shows good qualitative correspondence of numerically computed (a) and experimentally observed (b) four contours circulation.

Figure 7. EVF turbulent regime of liquid metal (top view): a) computed streamlines in horizontal middle plane for model with small thickness of liquid metal; b) photo of experimentally observed circulation at the free surface [3].

7. Conclusion

• Measured values of electrical potential and magnetic field induction in central part of liquid metal bath both qualitatively and quantitatively correspond on reasonable level to results of 3D numerical computation as well as correspond to calculations using derived quasi 2D analytical solution.
• Numerically computed electrovortex flow 3D patterns have characteristic four contours of circulation in horizontal plane, which qualitatively correspond to similar experimentally obtained photos of melt circulation.
• Numerical results for turbulent regime make it possible to research of flow characteristics for large values of total supplied electrical current. Computed patterns for slow laminar regime allow
studying the details of 3D flow with relation to its driving sources – 3D field of EM force vorticity.

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