K and B Physics: a way beyond the Standard Model

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ABSTRACT

I review some aspects of K and B physics both in the context of the standard model and in some cases in a scenario which is rather different from the standard model. I discuss, in particular, where we are likely to see deviations from the standard model in the near future before new colliders are built.

1. The Three Family Standard Model.

In the title, which was given to me by the organizers, I interpret the word way as in path. However it also appears that the title somehow implies that new physics means the standard model is wrong. What if it is right? What would be the path in this case? It is like the story of a visitor to a remote part of Ireland who gets lost. He comes to a cross road where he sees an old man sitting. After passing the time of day with the old man he then asks for directions to the village he wants to visit. The old man thinks for a while and then remarks “If I wanted to go to there, I would not start from here.”

1.1. Unitarity Constraints.

The standard model, with three families has the interaction Lagrangian

\[ \mathcal{L}_I = -\frac{g}{\sqrt{2}}(\bar{u} \gamma \bar{c} \bar{t})L \gamma_\mu W^{\mu}V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + h.c. \]  

where \( g \) is related to the Fermi constant by

\[ g^2 / 8m_W^2 = G_F / \sqrt{2} \]  

Here \( V \), the CKM matrix is a unitary matrix connecting the mass and weak eigenstates. It is important to note that it contains information about the left hand quarks only. The right hand sector is quite independent of it. For \( n \) doublets of quarks the CKM matrix is the smallest unitary matrix which can account for all of the mixing among the families and at the same time for CP violation. Remember, for a unitary matrix with \( n \) rows and columns the number of parameters is 1 for \( n = 2 \), the Cabibbo

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angle, and 4, consisting of three angles and a complex phase for \( n = 3 \), the CKM matrix. In general there are \((n-1)^2\) parameters which divide up into \(n(n-1)/2\) real angles and \((n-1)(n-2)/2\) phases. Lack of unitarity of the \(3 \times 3\) matrix would be a sure sign of something beyond the standard model. At the same time, extra families obviously introduce a much larger number of parameters, e.g., for \( n = 4 \) there would now be 9 parameters in total, with 6 real angles and 3 phases.

1.2. The CKM Matrix and Model Dependencies.

We have the CKM matrix, in the Wolfenstein form:

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \approx \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4) \tag{3}
\]

The present limits are \(|V_{us}| \equiv \lambda \approx \sin \theta_c = 0.2205 \pm 0.0018\), \(|V_{cs}| = 1.02 \pm 0.18\), \(|V_{cd}| = 0.204 \pm 0.017\) and \(|V_{ud}| = 0.9744 \pm 0.0010\). There has been new activity in the determination of \(|V_{cb}|\), which now has the value \(0.040 \pm 0.003 \pm 0.002\), or equivalently, from the definition in Eq. (3) \(A = 0.90 \pm 0.10\). A number of attempts to deduce the value of \(V_{cb}\) in a model independent way have recently been given. There has also been criticism of some of these attempts. A recent phenomenological fit to the data with a parametrization of the Isgur-Wise form factor \(\xi(y)\) of the form \(\xi(y) = 1 - \rho^2(y-1) + c(y-1)^2\), finds \(\rho = 1.10 \pm 0.10\) and \(c = 0.67 \pm 0.25\). Some of the earlier fits violate the unitarity constraints on the slope of \(\xi(y)\).

The elements of the CKM matrix, with \(\eta\) denoting the amount of CP violation lead to a (unitary) triangle in the standard model. The size of the expected CP violation parameters \(\varepsilon, \varepsilon'\) are given by the area of the triangle (in units of \(2A\lambda^3\)). Thus, an accurate determination of a non-zero value for \(\eta\) will be a sure sign of CP violation in the mixing matrix. Such an effect is usually called indirect CP violation and the standard model has both direct and indirect modes of CP violation, as I shall discuss below.

![Fig. 1. The unitary triangle in the standard model.](image)

The present limits on the allowed values of \(\eta, \rho\) are still dependent on the mass of the top quark, and until this is known, we do not know even if the triangle has the angle \(\gamma < \pi/2\) or \(\pi/2 \leq \gamma < \pi\). It is only possible to give a region of validity,
depending on the uncertainties in our knowledge of $|V_{ub}/V_{cb}| = 0.09 \pm 0.04$, the uncertainty in the bag constant $B_K = 2/3 \pm 1/6$, the $B - \bar{B}$ mixing parameter $x_d \equiv \Delta M/\Gamma = 0.708 \pm 0.085 \pm 0.071$, and the values of the $b$ quark mass and the decay constant $f_B$. For example, in a recent review, Pich used the values $m_b = 4.6 \pm 0.1 \text{GeV}$ and $f_B = (1.7 \pm 0.4) f_\pi$. More likely, $m_b = 4.9 \pm 0.1 \text{GeV}$ since the constituent quark and current quark masses are not that different for the heavy quarks. A lower value of $f_B = (1.35 \pm 0.2) f_\pi$ would represent the consensus among quark models and factorization models and perhaps even the latest lattice results (but see below). Many of the possibilities are given in Ref. 17.

2. CP Violation and Rare Decays of Neutral K Mesons.

CP violation in the $K$ system has been with us since 1964, when the effect was first seen and a full description can be found in books. Here I shall briefly describe the relevant notation.

The mass eigenstates $K_L$ and $K_S$ can be written, in the Wu – Yang phase convention, in terms of the two CP eigenstates $K_1$ and $K_2$, corresponding to $CP = +1$ and $CP = -1$, respectively, as

$$
K_L = \frac{K_2 + \epsilon K_1}{\sqrt{1 + \epsilon^2}} \\
K_S = \frac{K_1 + \epsilon K_2}{\sqrt{1 + \epsilon^2}}
$$

(4)

where, in a CP – invariant world, the CP eigenvalues are

$$
K_1 = \frac{K^\circ - \bar{K}^\circ}{\sqrt{2}} \quad K_2 = \frac{K^\circ + \bar{K}^\circ}{\sqrt{2}}
$$

(5)

2.1. Direct and Indirect CP Violation.

The parameter $\epsilon$ can be related to the quantities $\eta_{+-}$ and $\eta_{oo}$ which are defined in terms of the ratio of the amplitudes for decays into CP – forbidden and CP – allowed $2\pi$ combinations as follows:-

$$
\eta_{+-} = \frac{\langle \pi^+\pi^- \mid T \mid K_L \rangle}{\langle \pi^+\pi^- \mid T \mid K_S \rangle} \\
\eta_{oo} = \frac{\langle \pi^0\pi^0 \mid T \mid K_L \rangle}{\langle \pi^0\pi^0 \mid T \mid K_S \rangle}
$$

(6)

When all the complications of phases are fixed up (see Rosner for a very complete discussion on this) the quantities $\eta_{+-}$ and $\eta_{oo}$ can be written in terms of two small quantities $\epsilon$ and $\epsilon'$:-

$$
\eta_{+-} = \epsilon + \epsilon' \quad \eta_{oo} = \epsilon - 2\epsilon'
$$

(7)

If the world were CP invariant then both types of $\eta$ quantities would be zero. The parameter $\epsilon$ is a measure of the CP violation coming from the mixing in the mass matrix and is called indirect CP violation. The parameter $\epsilon'$ give a measure of the CP
violation in the decay amplitudes, which is called direct CP violation. A non-zero value for this leads to the relation

\[ | \eta_{oo} / \eta_{+-} |^2 \approx 1 - 6 \text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) \]  

(8)

At the moment there are two conflicting values for \( \frac{\varepsilon'}{\varepsilon} \) with the CERN experiment NA31, reporting \((2.3 \pm 0.7) \times 10^{-3}\) and the Fermilab experiment E731 finding a null result \((0.60 \pm 0.69) \times 10^{-3}\). The first of these would indicate that CP violation takes place in both ways, through the mass matrix and in the decay amplitudes, while the second has only the indirect, mass matrix source. This latter case could be explained in a number of ways, including the superweak model which was set up to give CP violation only through the mass mixing.

The phases of \( \varepsilon \) and \( \varepsilon' \) are determined by the final state interactions in the \( \pi \pi \) system and are nearly equal to one another and to \( \pi/4 \). In the standard model, \( \varepsilon \) is calculated using box diagrams and \( \varepsilon' \) comes from penguin diagrams. The \( \Delta S = 2 \) Lagrangian from the box diagrams give an effective 4–quark operator which is usually calculated in the vacuum insertion approximation,

\[ \langle \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} | K^0 \rangle = (8/3) B_K f_K^2 m_K^2 \]  

(9)

Here \( B_K \) is the bag constant, which parameterizes the vacuum approximation and which is the subject of considerable uncertainty. In the discussion above on the CKM parameters I used the central value of 2/3. The range of values in the literature range from 2/3 \pm 0.1 in 1/N calculations through a much smaller value in hadronic sum rule calculations, slightly higher values in QCD sum rules of 0.54 \pm 0.22 and 0.87 \pm 0.20 in a lattice calculation. On the other hand, the decay constant is well known, \( f_K = 159.8 \pm 1.4 \text{MeV} \). In the standard model the penguin and box diagrams differ slightly in phase and it is expected that \( \varepsilon' / \varepsilon \approx 0.5 \times 10^{-2} \), i.e., there should be some direct CP violation.

2.2. Possible Origin of CP Violation.

A number of new physics models have been discussed in the literature and I do not want to repeat that here. Instead, I will discuss a radical new proposal on the origin of CP violation. This will indeed be beyond the standard model if it is proven viable. There is a very different signal from the standard model – and even most non–standard models.

In a recent paper Chardin and Rax revive an old idea that was dismissed before the discovery of CP violation. In an early article by Morrison it was pointed out that the anti–gravity theory violated CPT invariance, the Eötvös experiment, and energy conservation – not a good start for any theory! A little later, Good discussed the same problem in the context of \( K \) physics. He showed that the \( K_L - K_S \) mass difference is a very sensitive measure of any new physics that would cause a difference in the mass, weight or potential energy of the system. In particular, since \( \delta M_K = 3.522 \times 10^{-6} \text{eV} \) and the potential energy of a \( K \) at the earth is about 0.4 eV this imposes very stringent limits on the change of the mass matrix. For example, if the
neutral $K$ mass matrix is written as:
\[
\begin{pmatrix}
M_K + V(1 + \delta) & iW \\
iW & M_K + V(1 - \delta)
\end{pmatrix}
\]

where $V$ is the gravitational potential and $\delta$ represents the fraction which changes sign in anti–gravity, then $\delta$ has to be extremely small since $\varepsilon = V\delta / \Delta M \approx \gamma p$. Here $\gamma$ is the usual Lorentz factor and $p$ depends on the type of field causing the violation. This can be an important effect since experiments have been done up to a value of $\gamma = 100$. The values of $p$ are 0, 1, 2 for scalar, vector and tensor fields, respectively. Thus a vector field will have a limit for $\delta$ relative to a scalar field that is down by $\gamma^2$.

Despite these pessimistic arguments, the idea has recently been discussed and revived. The latter argument can be stated by using the dimensions of the system, the size of the $K$ and the time needed for mixing via weak interactions $\delta\tau \approx h / \Delta M c^2$. If the quark anti–quark separation in a time $t \approx \delta\tau$ is larger than the size of the $K$ then a large $K_S$ component could be regenerated. An estimate of this can be given: $gt^2 \approx \varepsilon h / M_K c$ which gives $t \approx 1.7\Delta\tau$. This suggests that the CP–violating parameter could be written as $\varepsilon \approx g\Delta\tau^2 h / M_K c$. Any type of long range mechanism for CP–violation will have $\varepsilon' = 0$ but this one has a distinctive difference from, say, the superweak model, in that the similar parameter for CP–violation in B mesons will scale down by $10^{-3}$ from the values of $\varepsilon_K$. Thus, indirect CP–violation in $K$ mesons and a detection of CP in B physics will certainly tell whether there is anything to this proposal.

2.3. Rare K Decays: Present and Future.

Here, I shall review, briefly, the possibilities of using the rare decays of $K$ mesons to either get bounds on some of the quantities discussed above or to see something new. Most of the experimental limits are still well above the expected standard model predictions.

- $K^+ \rightarrow \pi^+\nu\bar{\nu}$: the present limit from BNL–E787 at $5 \times 10^{-9}$ is now about an order of magnitude greater than the expected rate. This mode would measure both $\rho$ and $\eta$. Until it is seen at the expected rate, it can only signal new physics! However, the chances of this happening are now quite slim – when I gave this talk the limit was about an order of magnitude larger.

- $K^0_L \rightarrow \mu\bar{\mu}$: the measured branching fraction of $(7.3 \pm 0.4) \times 10^{-9}$ seems to be understood in terms of the process $K_L \rightarrow \gamma\gamma \rightarrow \mu\bar{\mu}$.

- $K^0_L \rightarrow \pi^0\nu\bar{\nu}$: the expected branching fraction is $< 10^{-11}$ which is well below an old experiment which has been retroactively analyzed to give a branching ratio limit of $7.6 \times 10^{-3}$.

- $K^0_{L,S} \rightarrow \gamma\gamma$: the expected ratio of $|\varepsilon'| / \varepsilon \approx 0.02$.

- $K^0_L \rightarrow \pi^0e^+e^-$: the present limit is $5.5 \times 10^{-9}$. The theoretical calculations of this mode have been controversial. Decays via the CP–violating mode are expected to dominate and give a limit of about $10^{-11}$ or even an order of magnitude smaller. There is some doubt about whether the direct and non direct CP–violation parts are of the same order of magnitude (in which case there could be significant interference). A further controversy revolves around the $2\gamma$ CP–conserving intermediate state $K_L \rightarrow$
\[ \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^- \]. Here the estimates are all over the map from the largest \(10^{-10}\) to \(10^{-11}\), to \(10^{-12}\), to \(10^{-13} \sim 10^{-15}\). In fact from a recent NA31 result\(^{[35]}\) we now have 
\[ Br(K_L \rightarrow \pi^0 \gamma \gamma) = (1.7 \pm 0.2 \pm 0.2) \times 10^{-6} \] so that \(K_{L,S} \rightarrow \pi \gamma \gamma \rightarrow \pi^0 e^+ e^- \leq 4.5 \times 10^{-13} \); the CP–violation mode is in fact the dominant one.

For future prospects in \(K\) physics we shall have to wait for a new facility such as the proposed KAON factory in Canada. An indication of the type of mass limits possible in such a facility has been recently given\(^{[43]}\). From the LEP talk at this meeting we see that the limits on the Higgs Scalar already exceeds any of the limits expected at KAON.

### 3. B Physics

Although \(K\) meson physics has been an important source of information leading to the formulation of the standard model, \(B\) meson physics is much newer and full of promise. This can be seen in the relevant numbers of papers with the letter \(K\) (1064) or \(B\) (1762) in their title as listed in the SLAC database (by July 17, 1992). (Not all of these deal with the mesons - some refer to algebras!).

#### 3.1. \(f_B\) and CP Violation Asymmetries.

One important constant which is not known at the moment is \(f_B\). There are a number of attempts to calculate this; if we write \(f_B = n f_\pi\) then the question at the moment is whether \(n \approx 1\) or \(2\)? In the table we summarize the results from different types of calculations.

| \(f_B\) | \(f_{B_s}\) | \(f_D\) | \(f_{D_s}\) | Reference | Method             |
|--------|---------|-------|---------|------------|-------------------|
| 1.1    | 1.4     | 1.8   |         |            | hyperfine int.    |
| 0.8    | 1.2     | 1.33  | 1.8     |            | lattice           |
| 0.9    | 1.1     | 1.4   | 1.6     |            | lattice           |
|        |         | 1.4   | 1.7     |            | lattice           |
| 0.9    | 1.3     | 1.1   | 1.6     |            | lattice           |
| 0.6    | 0.6     | 0.9   | 1.0     |            | lattice           |
| 1.7    | 1.9     | 1.4   | 1.5     |            | lattice           |
| 1.2    | 1.6     | 1.8   | 2.2     |            | lattice           |
| 1.4    |         | 1.7   | 2.0     |            | lattice           |
| 1.4    | 1.5     | 1.3   | 1.6     |            | lattice           |
|        | 1.5     | 1.7   | 2.0     |            | sum rules         |
|        | 0.9     | 1.2   | 1.5     |            | sum rules         |
| 1.5    |         | 1.7   | 2.0     |            | sum rules         |
| 1.1    | 1.4     | 1.7   | 2.1     |            | sum rules         |

This summarizes the present state of a number of calculations of the ratios of the decay constants in units of \(f_\pi\); in some references only a few of the decay constants are available. For comparison, I also show the appropriate constants in \(D\) decays.
To get some of these values, I used $D_s = 276$ from the estimated result $\pm 45 \pm 44$ since in some cases only the ratios of the coupling constants were calculated. The lattice results in the table are now a few years old. More recent lattice calculations tend to get larger values for $f_B/f_\pi$, viz., $f_B/f_\pi = 2.3$ (ref. [45]), $1.4 < f_B/f_\pi < 1.9$ (ref. [46]), and $f_B/f_\pi = 1.5$ (ref. [47]). However, there are large errors and e.g., in the latter case the quoted result is $f_B = 195 \pm 10 \pm 30 - 60$ where the first two are roughly the statistical and systematic errors and the last error only has a minus sign, representing the difference in two methods used.

It is important for CP–violation (for recent reviews see, e.g. [21, 59] and [60]) that we get a good estimate on $f_B$. For example, the important mixing ratios, $x_d$ and $x_s$ are significantly affected; in the standard model they arise from the box diagrams dominated by $t$ quark exchange:

$$x_d = \frac{\Delta M}{\Gamma} = \frac{G_F^2}{6\pi^2} |V_{td}|^2 M_W^2 m_b B_B f_B^2 \eta_B \tau_B E(x_l)$$

with a similar equation for $x_s$. Here, $E(x)$ is a known function[61,62] shown below in Section 3.3, $\eta_B$ is a QCD correction, $B_B$ is the appropriate bag constant and $\tau_B$ is the lifetime. As shown below, the mixing ratios also enter into the CP–violating asymmetries.

### 3.2. An Ambiguity in the Standard Model.

In this section we consider the CP–violating asymmetries in some detail, since it has recently been shown that there could be an important ambiguity in the standard model.

The asymmetry in the decay $B \rightarrow \psi K_S$ is defined as

$$A(\psi K_S) = \frac{\Gamma(B^0 \rightarrow \psi K_S) - \Gamma(\bar{B}^0 \rightarrow \psi K_S)}{\Gamma(B^0 \rightarrow \psi K_S) + \Gamma(\bar{B}^0 \rightarrow \psi K_S)}$$

In an actual experiment, the semi-leptonic decays tag the identity of the particle-antiparticle, since $B^0 \rightarrow l^- + ...$ while $\bar{B}^0 \rightarrow l^+ + ....$. When time ordered, there are four possible combinations:

1. $B^0, \bar{B}^0 \rightarrow l^-(t_1), \psi K_S^0(t_2), t_1 > t_2$
2. $B^0, \bar{B}^0 \rightarrow l^+(t_1), \psi K_S^0(t_2), t_1 > t_2$
3. $B^0, \bar{B}^0 \rightarrow l^-(t_2), \psi K_S^0(t_1), t_2 > t_1$
4. $B^0, \bar{B}^0 \rightarrow l^+(t_2), \psi K_S^0(t_1), t_2 > t_1$

The asymmetry $A(\psi K_S)$ is then defined by:

$$A(\psi K_S) = \frac{(1) - (2) + (3) - (4)}{(1) + (2) + (3) + (4)}$$

In the standard model, the angles $\beta$ and $\alpha$ of Fig. 1 are related to the asymmetries of the decays of the $B$ into final states $\psi K_S$ and $\pi^+\pi^-$, respectively in the following
\[
\sin(2\beta) = -\left(\frac{1 + x^2}{x}\right) A(\psi K_S)
\]
\[
\sin(2\alpha) = -\left(\frac{1 + x^2}{x}\right) A(\pi^+\pi^-)
\]
(13)

where the mixing fraction is \( x = \Delta M / \Gamma = 0.708 \pm 0.085 \pm 0.071 \).

In the superweak model, there is no direct CP–violating effect, \( \varepsilon' = 0 \), nevertheless, \( B - \bar{B} \) mixing can give rise to a non-zero asymmetry. Since \( CP(\pi^+\pi^-) = 1 \) and \( CP(\psi K_S) = -1 \) the superweak model would predict a change in sign between the two final states.\(^6\)\(^7\) That is, if \( \alpha = \beta \), then the standard model would have the same (negative) asymmetries in both cases whereas the asymmetries have opposite signs in the superweak model.

It has been recently pointed out\(^6\)\(^7\) that there could be a situation in which the standard model is indistinguishable from the superweak model (at least with regard to these asymmetries). This occurs when \( \sin(2\alpha) + \sin(2\beta) = 0 \). This means that the standard model would mimic the superweak model for any \( \rho > 0 \) and with the constraint \( \eta = (1 - \rho) \sqrt{\rho / (2 - \rho)} \).

3.3. Rare B processes - mixing

Here, we consider \( B - \bar{B} \) mixing with the spectator quark being a \( d \) quark. We have

\[
\frac{\Delta M}{\Gamma_{12}} = \left\{ \frac{[0.85 \pm 0.05]}{1.1} \frac{4 E(x)}{3 \pi x_b} \right\} \approx 60.
\]
(14)

where \( x_q = m_q^2 / m_W^2 \), the two numerical factors representing QCD corrections\(^6\)\(^8\) and

\[
E(x) = x \left\{ \frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(x-1)^2} \right\} + \frac{3}{2} \left( \frac{x}{(x-1)} \right)^3 \ln(x)
\]
\[
\approx 0.159 + 0.582x - 0.015x^2
\]
(15)

with the last approximation to the first line coming from a recent determination\(^8\) and which agrees very well with the full expression for \( 1 \leq x_t \leq 9.5 \). Notice, that as the limits on the top quark mass have risen in the past few years, care should be taken in using such approximations. For example, just a few years earlier, another approximation\(^7\) was given, \( E(x) \approx 0.75x^{7/4} \). This widely disagrees with the full form of \( E(x) \) in the present case for which \( x_t \geq 1 \).

The quantity \( \Delta M / \Gamma_{12} \) is a good one for showing something beyond the standard model. The calculation of \( \Gamma_{12} \) is well understood. It comes from cutting the box-diagrams, so it is tree–level W mediated and there does not seem to be a competing model available. If the ratio in Eq. (15) is to be made smaller then something must happen to the calculation of \( \Delta M \). Since we know that \( \Delta M / \Gamma \sim 0.7 \) it is not a simple matter in the standard (or nearly standard) model to make much of a change.
4. Inclusive Rare Decays of B Mesons.

Inclusive rare decays of the B meson, considered as decays of the form $b \rightarrow s\gamma$ and $b \rightarrow sg$ have been the subject of a great deal of interest over the past few years. In the following section, we look at the standard model calculation and estimate the reliability of the calculations.

4.1. The Standard Model.

The original calculation, based on the similar calculation in $d \rightarrow s\gamma$, suggested that if the top quark was light with $20 GeV \leq m_t \leq 60 GeV$, say, then the process $b \rightarrow s\gamma$ would be an excellent top quark “mass meter”. It was later pointed out using a modified version of the work of Shifman et al. that the short distance QCD enhancement of the $\sigma F$ operator would cause an increase in the rate for $b \rightarrow s\gamma$. However, both groups ignored terms coming from graphs with the photon attached to the down quark line and also ignored mixing effects from graphs with an internal gluon. More exact calculations came soon after.

We now have confidence that the QCD scaling of the coefficient functions of the effective operators is understood and so we are able to estimate well a number of quantities which are of interest experimentally. The most basic of these is the rare inclusive decay $b \rightarrow s\gamma$. When the quark and photon fields are on shell, the only operator which contributes is the magnetic-moment operator $O_5$, in the notation of Ref. 76, whose coefficient function is $\tilde{C}_5(m_b)$.

The width for the free quark decay $b \rightarrow s\gamma$ is given by

$$\Gamma(b \rightarrow s\gamma) = \frac{G_F^2 m_b^5}{288\pi^4} |V_{ts}V_{tb}|^2 (\tilde{C}_5(m_b))^2$$

(16)

where $\alpha = 1/137$. The dependence on $m_b$ and the CKM elements may be removed by the usual trick of normalizing to $b$-quark semileptonic decay $b \rightarrow ce\nu_e$. The width for this process is given by

$$\Gamma_{SL} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \rho \left(\frac{m_c^2}{m_b^2}\right) \chi \left(\frac{m_c^2}{m_b^2}\right)$$

(17)

where the phase-space factor $\rho$ depends on the non negligible ratio $y = m_c^2/m_b^2$:

$$\rho(y) = 1 - 8y + 8y^3 - y^4 - 12y^2 \ln y$$

(18)

and equals 0.447 for $m_c = 1.5$ GeV and $m_b = 4.5$ GeV. The factor $\chi$ is the one-loop QCD correction to the semileptonic decay:

$$\chi(y) = 1 - \frac{2\alpha_s(m_b)}{3\pi} f(y),$$

(19)

with $f(m_c^2/m_b^2) \approx 2.4$; this correction gives a modest 12 per cent suppression to the semileptonic width. Experimentally, the ratio $|V_{ts}V_{tb}/V_{cb}| \approx 1$, so the KM elements cancel.
The result for $b \to s\gamma$ normalized to semileptonic decay is

$$\frac{\Gamma}{\Gamma_{SL}} = \frac{2\alpha}{3\pi\rho\chi} (\bar{C}_5(m_b))^2$$

(20)

The branching ratio for $b \to s\gamma$ is obtained by multiplying this by the semileptonic branching ratio, which is found experimentally to be about 10 per cent. In Fig. 2 the upper line gives the branching ratio evaluated at $\mu = m_b$, and the lower line at $\mu = M_W$. One sees that QCD scaling from $M_W$ down to $m_b$ enhances the branching ratio. The magnitude of the enhancement depends on the top quark mass, and is smaller at the larger values of $m_t$. This is because the GIM suppression is weaker at large $m_t$. The GIM suppression is partially “undone” by QCD, and because there is less of a suppression to “undo” at large $m_t$, the size of the QCD effect is smaller.

With the top quark mass expected to be somewhere around 140 GeV, and including the QCD corrections, means that the sensitivity to $m_t$ is reduced compared to the original expectations. The decay becomes less useful as a top quark “mass meter”. On the other hand this lack of sensitivity to an unknown parameter of the standard model makes the decay a good place to look for signals of new physics beyond the standard model.

![Fig. 2. The effects of QCD corrections.](image)

The error bars in Fig. 2 reflect the fact that the QCD scale parameter $\Lambda_{QCD}$ is not well known; the curve is plotted for a central value of 200 MeV, with a range of ± 100 MeV. Larger values of $\Lambda_{QCD}$ correspond to larger values of the branching ratio.

The process $b \to s\gamma$ also receives a contribution from $b \to sg$. For all practical purposes, the effect is negligible. Recently a corrected calculation for $b \to sg$ was given by Misiak. Also, a correction taking into account the fact that the top quark is now thought to be much heavier than the $W$ meson shows that the branching ratio increases by as much as 14% at the higher values for $m_t$, though still within the errors coming from $\Lambda_{QCD}$ shown in Fig. 2.
A simple numerical representation of the results of the QCD corrected inclusive
decay (central value in Fig. 2) is
\[ BR(b \to s\gamma) = (2.314 + 0.5814x_t - 0.033x_t^2) \times 10^{-4} \] (21)
where, as usual, \( x_t = m_t^2/m_W^2 \). The most up-to-date limit on the branching ratio for
the inclusive decay is \( 8.4 \times 10^{-4} \).

4.2. Other Models.

I have not enough space to give more than a brief mention of models beyond the
standard model since, in many cases, there are a number of parameters that can be
adjusted. The major extensions to the standard model are:

- Extra Families. With extra families the parameters can be tuned to give rare \( b \)
decays with a branching ratio near to the present bounds. The limit on the number of
neutrino families from the LEP data which now requires that a fourth generation has
\( m_{\nu_4} > M_Z/2 \) creates a serious difficulty for these models.

- Supersymmetry. See the talk by Borzumati.

- Extra Higgs, models with non tree-level Flavor Changing Neutral Currents. After \( \varepsilon, \varepsilon'/\varepsilon \) and mixing constraints are applied, these keep the expected magnitude
of the rare \( b \) decays at the order of the standard model.

- Left–Right symmetry. These models need fine tuning. Like SUSY, there are
many variations.

5. Exclusive Rare Decays in the B System

There are limits on the exclusive decays of the \( B \) meson which come from four
groups, ARGUS and Crystal Ball at DESY, CLEO at CESR and UA1 at CERN. At the
time of writing, the best limits on the various decays were those listed in the following
table:

| Process | Limit | Experiment | Reference |
|---------|-------|------------|-----------|
| \( B \to K^*(892)\gamma \) | \( 2.4 \times 10^{-4} \) | ARGUS | 3 |
| \( B^0 \to K^*(892)^0\gamma \) | \( 0.92 \times 10^{-4} \) | CLEO | 3 |
| \( B^+ \to K^*(892)^+\gamma \) | \( 5.2 \times 10^{-4} \) | ARGUS | 3 |
| \( B^+ \to K^*(892)^+\gamma \) | \( 3.7 \times 10^{-4} \) | CLEO | 3 |
| \( B^0 \to K^*(1430)^0\gamma \) | \( 4.4 \times 10^{-4} \) | ARGUS | 3 |
| \( B^+ \to K^*(1430)^+\gamma \) | \( 1.3 \times 10^{-3} \) | ARGUS | 3 |
| \( B^0 \to K^0 e^+e^- \) | \( 1.5 \times 10^{-4} \) | ARGUS | 6 |
| \( B^+ \to K^+ e^+e^- \) | \( 5 \times 10^{-5} \) | CLEO | 7 |
| \( B^0 \to K^0 \mu^+\mu^- \) | \( 2.6 \times 10^{-4} \) | ARGUS | 8 |
| \( B^+ \to K^+ \mu^+\mu^- \) | \( 1.5 \times 10^{-4} \) | CLEO | 7 |
| \( B^0 \to K^*(892)^0 e^+e^- \) | \( 2.9 \times 10^{-4} \) | ARGUS | 6 |
| \( B^0 \to K^*(892)^0 \mu^+\mu^- \) | \( 2.3 \times 10^{-5} \) | UA1 | 8 |
5.1. Model Dependency.

Some of these limits, e.g., $B \to K^*\gamma$, are only a factor of a few times that calculated. I will examine the model dependency of the types of calculations that have been done so far. We shall see that whereas the inclusive decays are very well understood, the exclusive decays still have a lot of uncertainty.

The hadronic matrix elements relevant to the transition $B(b\bar{q}) \to V(Q\bar{q})$, where $V$ is a vector meson, are given by

$$\langle V(k)|\bar{Q}\sigma_{\mu\nu}q^\nu b_R|B(k')\rangle = f_1(q^2)i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^*k^\lambda q^\sigma$$

$$+ \left[ (m_B^2 - m_V^2)\epsilon^*_\mu - \epsilon^* \cdot q(k' + k)_\mu \right] f_2(q^2)$$

$$+ \epsilon^* \cdot q \left[ (k' - k)_\mu - \frac{q^2}{(m_B^2 - m_V^2)}(k' + k)_\mu \right] f_3(q^2),$$

$$\langle V(k)|\bar{Q}\gamma_\mu b_L|B(k')\rangle = T_1(q^2)i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^*k^\lambda q^\sigma$$

$$+ T_3(q^2)\epsilon^* \cdot q(k' + k)_\mu + T_4(q^2)\epsilon^* \cdot q(k' - k)_\mu .$$

where $q = k' - k$.

In the static $b$-quark limit, the $b$-quark spinor has only upper component in the $B$ rest frame. Thus in the $B$ rest frame, we have the following relations between the $\gamma_\mu$ and $\sigma_{\mu\nu}$ matrix elements:

$$\langle V(k)|\bar{Q}\gamma_i b|B(k')\rangle = \langle V(k)|\bar{Q}\sigma_{0i} b|B(k')\rangle ,$$

$$\langle V(k)|\bar{Q}\gamma_i\gamma_5 b|B(k')\rangle = - \langle V(k)|\bar{Q}\sigma_{0i}\gamma_5 b|B(k')\rangle ,$$

which relate the form factors $f_{1,2,3}$ to $T_{1,2,3,4}$ as

$$f_1 = -(m_B - E_V)T_1 - \frac{(m_B^2 - m_V^2)}{m_B}T_2 ,$$

$$f_2 = -\frac{1}{2} \left[ (m_B - E_V) - (m_B + E_V)\frac{q^2}{m_B^2 - m_V^2} \right] T_1 - \frac{1}{2m_B} \left( m_B^2 - m_V^2 + q^2 \right) T_2 ,$$

$$f_3 = -\frac{1}{2}(m_B + E_V)T_1 + \frac{1}{2m_B}(m_B^2 - m_V^2)(T_1 + T_2 + T_3 - T_4) .$$

where $E_V = (m_B^2 + m_V^2 - q^2)/(2m_B)$.

Now, we wish to consider the case where $V$ is the $K^*$. The branching ratio of the exclusive process $B \to K^*\gamma$ to the inclusive process $b \to s\gamma$ can be written in terms of $f_1$ and $f_2$ at $q^2 = 0$, as

$$R = \frac{\Gamma(B \to K^*\gamma)}{\Gamma(b \to s\gamma)} \approx \frac{m_B^3(m_B^2 - m_{K^*}^2)^3}{m_B^2(m_b^2 - m_{K^*}^2)^3} \frac{1}{2} \left[ |f_1(0)|^2 + 4|f_2(0)|^2 \right] .$$

Using the static $b$ quark limit, Eqs. (26) and (27), we have $f_2(0) = (1/2)f_1(0)$. This is often referred to as a quark model result but it is clear that it is more general than that and requires only that the $b$ quark is heavy enough.
Although there is now only one form factor to calculate, this is still a controversial calculation\cite{92-95} with there being a factor of about ten uncertainty coming from the way in which the large recoil of the $K^*$ is handled.

I will review the quark model calculations to show where the ambiguity arises.

The problem comes from the momentum wave functions. These are determined by solving the Schroedinger equation of the corresponding $q\bar{q}$ system with a Coulomb plus linear potential\cite{96,97} between the quarks. For $L=0$ meson states, which we will consider here, they are chosen to be Gaussian wave functions of the form

$$\phi(p) = (\pi \beta^2)^{-3/4} e^{-\vec{p}^2/2\beta^2}, \quad (30)$$

with a variational parameter $\beta$. The formulation of the relative momentum wave function is then obviously non relativistic. The $q_3(\vec{k} + \vec{p})\bar{q}_2(-\vec{p})$ system of the $K^*$ becomes highly relativistic in the region of large recoil and the use of the above non relativistic momentum wave function for $\phi_{K^*}$ is then questionable. In Ref.\cite{96} this problem was treated by fixing the meson and quark spinor normalizations at the zero recoil point and ignoring all of the recoil dependence in the matrix element except for the momentum wave function part. The recoil momentum can be written as

$$|\vec{k}| = \sqrt{E_{K^*}^2 - M_{K^*}^2} = \sqrt{1 + \frac{t_m - q^2}{4M_B M_{K^*}} M_{K^*} M_B \sqrt{t_m - q^2}}, \quad (31)$$

where $q^2 = (P_B - P_{K^*})^2$ and $t_m \equiv (M_B - M_{K^*})^2$. Since $q^2 = t_m$ corresponds to the point of zero recoil, we have $|\vec{k}| = \sqrt{M_{K^*} M_B \sqrt{t_m - q^2}}$, the non relativistic form for $|\vec{k}|$ near this point. This non relativistic form of the recoil momentum in the momentum wave function was adopted in Ref.\cite{96} and the recoil dependence of the matrix element at large recoil was prescribed by multiplying $|\vec{k}|$ with a relativistic correction factor $1/\kappa$ ($\kappa = 0.7$ was determined by fitting the pion form factor to experiment\cite{98}).

Last year, we showed\cite{89} that it is possible to do without $\kappa$ if the recoil dependence of the spinors is taken into account. A similar result has now been given using Bjorken’s sum rule in heavy quark symmetry. Nevertheless, the problem remains that, depending on the choice of how the evaluation of $|\vec{k}|$ is performed, there can be a factor of about five in the result for the exclusive decay.

I have not been able to account for the extra factor of two for the branching ratio in the sum rule approach\cite{94,95}.

5.2. Heavy Quark Symmetries.

One way to overcome the model dependencies that plague the exclusive calculations might be to try the heavy quark symmetry approach\cite{90,99-102}. It has been pointed out\cite{103} that heavy quark symmetries could relate the data on the semi-leptonic decays $D \rightarrow Ke^+\nu$ and $D \rightarrow K^*e^+\nu$ and provide information relevant to the exclusive decay $B \rightarrow K^*\gamma$. However, there is the problem of continuation of these symmetries from zero recoil momentum across the Dalitz plot to the largest recoil $q^2 = 0$. It has been suggested\cite{104} that the relations among the operator matrix elements might be valid over the full kinematic ranges even for transitions of the type $b \rightarrow s$. 

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The question has been raised whether the $s$ quark is sufficiently heavy to apply these symmetries to the $K^*$. Here, I will show that the important ingredient is that the $b$ quark is heavy and that corrections are suppressed.

The heavy quark symmetries are derived in the large mass limit. Many of the relations have been known to hold, at least in an approximate sense, in the quark model. For example, in the heavy quark model, the spin of $Q$ in $V(Q\bar{q})$ decouples from the gluon, giving

$$S^z_Q|V(Q\bar{q})\rangle = \frac{1}{2}|P(Q\bar{q})\rangle, \quad S^z_Q|P(Q\bar{q})\rangle = \frac{1}{2}|V(Q\bar{q})\rangle,$$

where $S^z_Q$ is the spin operator of the $Q$ quark, and $P$ is the scalar meson corresponding to $V$ with the same quark content of $Q\bar{q}$. The matrix relations

$$\langle V(k)|\bar{Q}\Gamma b|B(k')\rangle = 2\langle P(k)|[S^z_Q, \bar{Q}\Gamma b]|B(k')\rangle,$$

for $\Gamma$ any product of $\gamma$-matrices, then gives additional relations among the form factors

$$-T_1 = 2T_3 = -2T_4$$

These relations are also valid, to within a few percent, in the quark model.

Instead of seven, we now have two form factors, $T_1$ and $T_2$, say. We further assume that the quarks are sufficiently heavy so that in the equation of motion of the quarks we can replace the quark masses with the meson masses,

$$\langle V(k)|\bar{Q}(k' - k)\gamma_5 b|B(k')\rangle \approx -(m_B + m_V)\langle V(k)|Q\gamma_5 b|B(k')\rangle.$$  

(33)

Since $m_b \approx m_B$, and $m_B \geq m_V$, the error of using $m_V$ in place of $m_Q$ is suppressed. The static $b$ limit, $\langle V(k)|\bar{Q}\gamma_5 b|B(k')\rangle = -\langle V(k)|\bar{Q}\gamma_0\gamma_5 b|B(k')\rangle$, lets us relate $T_2$ to $T_1$;

$$\frac{m_B^2 - m_V^2}{m_B} T_2 = \frac{1}{2} \left[ (m_B + m_V)^2 - q^2 \right] T_1$$

(34)

The symmetries relate the form factors in the following ways:

$$T_1 = -\frac{2(m_B^2 - m_{K^*}^2)}{(m_B + m_{K^*})^2 - q^2} T_2 = 2T_3 = -2T_4$$

$$= \frac{-1}{(m_B + m_{K^*})} f_1 + \frac{2(m_B + m_{K^*})}{(m_B + m_{K^*})^2 - q^2} f_2 = \frac{2}{m_B - m_{K^*}} f_3$$

(35)

The presence of masses here show the effect of mass breaking of the heavy quark symmetries. On the other hand, the above discussion explicitly shows that the errors are controlled, even for a $K^*$, which normally would not be thought to be a good candidate for the heavy quark theory. A plot of individual terms in Eq. (35) (times a normalization factor $\sqrt{4m_Bm_{K^*}}$) shows that the equalities hold very well, with less
than about a 15% discrepancy, across the whole kinematic region to $q^2 = 0$. The small discrepancy reflects the error of using $m_V$ in place of $m_Q$, as noted above.

However, there still remains the troublesome problem of what to do with the overlap function. A start to solving this was proposed by Burdman and Donoghue in which the heavy quark symmetry arguments were used to relate $B \rightarrow K^* \gamma$ to the semileptonic process $B \rightarrow \rho e \bar{\nu}$ using the static $b$-quark limit and $SU(3)$ flavor symmetry. The problem with this is that the semileptonic decay vanishes at the kinematic point they use. This means that experimentally there should be no event at that point and very few in the neighbourhood, causing a large uncertainty in the measurement. Recently a new relation between the branching ratio $R(B \rightarrow K^* \gamma)$ and the $q^2$-spectrum for $B \rightarrow \rho e \bar{\nu}$ has been given. A direct measurement of $d \Gamma(B \rightarrow \rho e \bar{\nu})/dq^2$ at $q^2 = 0$ can therefore provide relevant information for $R(B \rightarrow K^* \gamma)$ since the $q^2$-spectrum for $B \rightarrow \rho e \bar{\nu}$ does not vanish at $q^2 = 0$.

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