Learning Based Hybrid Beamforming for Millimeter Wave Multi-User MIMO Systems

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Abstract—Hybrid beamforming (HBF) design is a crucial stage in millimeter wave (mmWave) multi-user multi-input multi-output (MU-MIMO) systems. However, conventional HBF methods are still with high complexity and strongly rely on the quality of channel state information. We propose an extreme learning machine (ELM) framework to jointly optimize transmitting and receiving beamformers. Specifically, to provide accurate labels for training, we first propose an fractional-programming and majorization-minimization based HBF method (FP-MM-HBF). Then, an ELM based HBF (ELM-HBF) framework is proposed to increase the robustness of beamformers. Both FP-MM-HBF and ELM-HBF can provide higher system sum-rate compared with existing methods. Moreover, ELM-HBF cannot only provide robust HBF performance, but also consume very short computation time.

Index Terms—Beamforming, millimeter wave, MIMO, machine learning, fractional programming (FP).

I. INTRODUCTION

RECENTLY, hybrid beamforming (HBF) design for millimeter wave (mmWave) multi-user multi-input multi-output (MU-MIMO) systems has been receiving increasing research attention due to the advantages of using less fully digital (FD) beamformers and providing high beamforming gain to overcome the severe pathloss at affordable hardware cost [1]–[4]. Generally, the main challenges of the HBF optimization problem for mmWave MU-MIMO systems are the non-convex constraints of analog beamformers and the inter-user interference. Some optimization based methods have been proposed to optimize the hybrid beamformers by using following FD methods: block diagonalization zero-forcing (BD-ZF) [4], [5], minimum mean square error (MMSE) and weighted MMSE (WMMSE) [3]. For example, to design hybrid beamformers, a WMMSE based orthogonal matching pursuit (OMP) method is proposed in [3] and a BD-ZF based exhaustive search method is proposed in [6]. However, above approaches either require perfect channel state information (CSI), or high computational capability.

Recent development in machine learning (ML) provides a new way for addressing problems in physical layer communications (e.g., direction-of-arrival estimation [7]). HBF design [6], [8] and channel estimation [9]). ML based techniques have several advantages such as low complexity when solving non-convex problems and the ability to extrapolate new features from noisy and limited training data [10]. In [6], a convolutional neural network (CNN) framework is first proposed to optimize hybrid beamformers for mmWave MU-MIMO systems, in which the network takes the imperfect channel matrix as the input and produces the analog and digital beamformers as outputs. However, this work is only feasible for single stream transmission and the exhaustive search HBF algorithm is with extremely high computational complexity. Furthermore, methods in [6], [8] using multiple large dimensional layers constructions may consume tremendous computation time in training phase, which is impractical with the hardware constraint (e.g., limited computational capability and memory resources) of mobile terminals.

From above observations, we propose an extreme learning machine (ELM) framework with easy implementation to jointly optimize transmitting and receiving beamformers. The main contributions are summarized as follows:

- **New HBF optimization algorithms**: We decouple the HBF optimization problem to two sub-problems. Unlike the work in [3] that minimizes MMSE, we first propose an fractional programming (FP) based FD beamforming algorithm that directly maximizes the system sum-rate. With FD beamformers, we then propose a low-complexity majorization-minimization (MM) based algorithm to jointly optimize hybrid beamformers. Finally, the convergence and computational complexity of proposed algorithms are analyzed. We show that above proposed methods can achieve higher sum-rate than conventional FD beamforming and HBF methods.

- **Robust and low-complexity ML based HBF design**: We propose an easily implemented ML based HBF framework (i.e., ELM-HBF) to jointly estimate the precoders and combiners. Different from conventional HBF methods, of which the performance strongly relies on the quality of CSI, our learning based approach can achieve more robust performance since ELM are effective at handling the imperfections and corruptions in the input channel information. We show that, for 64 transmit antennas, ELM-HBF can achieve 50 times faster prediction time than WMMSE-OMP-HBF [3], and 1200 times faster training time than CNN-HBF [6].

Notations: Bold lowercase and uppercase letters denote vectors and matrices, respectively. Tr(A), |A|, ||A||F, A*, A^T and A^H denote trace, determinant, Frobenius norm, conjugate, transpose and conjugate transpose of matrix A, respectively. ⊙ presents the Kronecker product. arg(a) denotes the argument/phase of vector a.
II. SYSTEM MODEL

We consider an mmWave downlink MU-MIMO system, in which a base station (BS), equipped with $N_t$ antennas and $N_{RF}$ RF radio frequency (RF) chains, is communicating with $K$ independent users (UEs). Each UE is equipped with $N_r$ antennas and $N_{RF}$ RF chains to receive $N_t$ data streams simultaneously. To guarantee the effectiveness of the communication carried by the limited number of RF chains, the number of the transmitted streams is constrained by $N_k \leq N_{RF} \leq N_t$ for the BS, and $N_k \leq N_{RF} \leq N_t$ for each UE. The received signal for the $k$-th UE is given by

$$ y_k = W_{BB,k}^H W_{RF,k}^H (H_k \sum_{i=1}^{K} F_{RF} F_{BB,k} s_i + n_k), $$

where $k \in \{1, 2, ..., K\}$, $s_i \in C^{N_t}$ is the transmitted symbol vector for the $k$-th UE such that $E(s_i^H s_i) = \frac{\rho}{KN_k} I_{N_k}$, $P$ is total transmit power, and $n_k \sim CN(0, \sigma_n^2 I_{N_t})$ is the additive white Gaussian noise (AWGN) at the $k$-th UE, $F_{BB,k} \in C^{N_t \times N_t}$ and $W_{BB,k} \in C^{N_t \times N_t}$ are the analog precoder and combiner for the $k$-th UE, respectively. Both $F_{RF}$ and $W_{RF,k}$ are implemented using analog phase shifters with constant modulus, i.e., $\|F_{RF}\|_{\infty} = 1$ and $\|W_{RF,k}\|_{\infty} = 1$ [1]. To meet the total transmit power constraint at the BS, precoding matrices $F_{RF}$ and $F_{BB,k}$ are constrained by $\sum_{i=1}^{K} \|F_{RF} F_{BB,k}\|_F^2 = KN_k$. The mmWave MIMO channel between the BS and the $k$-th UE, denoted as $H_k$, can be characterized by the Saleh-Valenzuela model [3]. From the above, the system sum-rate when transmitted symbols follow a Gaussian distribution is given by

$$ \mathcal{R} = \sum_{k=1}^{K} \log |I_{N_k} + F_{RF}^H H_k W_{BB,k} R_k W_k^H H_k F_{RF}|, $$

where $W_k = W_{BB,k} W_{RF,k}$, $F_k = F_{RF} F_{BB,k}$, and $R_k = W_k^H H_k (\sum_{n=1}^{N_k} F_n^H F_n H_k^H W_k + \rho_n W_k^H W_k)$ is the covariance matrix of total inter-user interference-plus-noise at the $k$-th UE and $\rho_n = \sigma_n^2 P / N_k$.

III. HBF DESIGN WITH FP AND MM METHOD

In what follows, we maximize the achievable system sum-rate by jointly optimizing hybrid beamformers, i.e., $\mathcal{B}_k = \{F_{RF}, F_{BB,k}, W_{RF,k}, W_{BB,k}\}, \forall k$. The optimization problem can be stated as

$$ \text{max } \mathcal{R} \quad \text{s.t. } \sum_{k=1}^{K} \|F_{RF} F_{BB,k}\|_F^2 = KN_k, $$

where $F_{RF}$ and $W_{RF,k}$ denote the feasible sets of analog beamformers which obey the constant modulus constraints for $F_{RF}$ and $W_{RF,k}$.

Obviously, the sum-rate maximization problem (P1) is non-convex and NP-hard with respect to $\mathcal{B}_k, \forall k$ due to the coupled variables in the matrix ratio term in (2) and the constant modulus constraints of analog beamformers. To make this problem tractable, we first transform problem (P1) to an easily implemented problem according to the FP theory in [11]. Then, problem (P1) can be rewritten as

$$ \text{(P2): } \max_{\mathcal{B}_k, \forall k} \mathcal{R} \quad \text{s.t. } \sum_{k=1}^{K} \|F_{RF} F_{BB,k}\|_F^2 = KN_k, $$

where $\mathcal{R} = \sum_{k=1}^{K} \log |I_{N_k} + Tr(V_k) + 2Tr(F_k^H H_k W_k U_k) - Tr(V_k U_k^H R_k U_k)|$ with $V_k = W_k^H H_k (\sum_{n=1}^{K} F_n^H F_n H_k^H W_k + \rho_n W_k^H W_k)$ and $\Gamma_k = I_{N_k} + V_k$. Due to the power constraint and the non-convex constraints of analog beamformers, it is still hard to solve problem (P2) directly. Thus, we propose a two-step approach to solve problem (P2). In the first step, we mainly focus on maximizing $\mathcal{R}$ by jointly optimizing the FD beamformers (i.e., $\mathcal{D} = \{F_k, W_k, \forall k\}$). Then, problem (P2) is reformulated as

$$ \text{(P3): } \max_{\mathcal{D}, \forall k, \forall k} \mathcal{R} \quad \text{s.t. } \sum_{k=1}^{K} \|F_k\|_F^2 = KN_k, $$

Note that problem (P3) is bi-convex which can be effectively solved with alternate optimizing (AO) methods. According to AO methods, the solutions of problem (P3) can be obtained iteratively, where in iteration $i + 1$, the variables are updated as follows

$$ U_{k}^{(i+1)} = \arg \max_{U_k} \mathcal{R}(F_k, W_k, V_k, \forall k, \forall k), $$

$$ V_{k}^{(i+1)} = \arg \max_{V_k} \mathcal{R}(F_k, W_k, V_k, U_{k}^{(i+1)}, \forall k, \forall k), $$

$$ W_{k}^{(i+1)} = \arg \max_{W_k} \mathcal{R}(F_k, W_k, V_k, U_{k}^{(i+1)}, \forall k, \forall k), $$

$$ F_{k}^{(i+1)} = \arg \max_{F_k} \mathcal{R}(F_k, W_k, V_k, U_{k}^{(i+1)}, \forall k, \forall k). $$

According to some basic differentiation rules for complex-value matrices, closed-form solutions of problems (6a)-(6d) are correspondingly derived as

$$ U_{k}^{(i+1)} = \frac{\bar{F}_k}{\bar{H}_k^H} \bar{H}_k F_k, $$

$$ V_{k}^{(i+1)} = \frac{\bar{H}_k^H}{\bar{H}_k^H} \bar{H}_k W_k, $$

$$ W_{k}^{(i+1)} = \frac{\bar{H}_k}{\bar{H}_k^H} \bar{H}_k F_k \times \frac{s}{\bar{H}_k^H U_k}, $$

$$ F_{k}^{(i+1)} = \frac{\bar{H}_k}{\bar{H}_k^H} \bar{H}_k W_k U_k, $$

where the optimal multiplier $s$ is introduced for the power constraint in (3) and it can be easily obtained by bisection search. After obtaining the updated variables, we summarize the FP based FD beamforming algorithm in Algorithm 1.

With the FD beamformers derived in Algorithm 1, we then turn to optimize hybrid beamformers $\mathcal{B}_k, \forall k$. Extensive works show that minimizing the Euclidean distance between the FD beamformer and the hybrid beamformer is an effective surrogate for maximizing the sum-rate of mmWave MU-MIMO


**Algorithm 1**: FP based FD beamforming algorithm

1: Input: $H_k$, $\forall k$;
2: Output: $F_k, W_k$, $\forall k$;
3: Initialize: $F_k^{(0)}, W_k^{(0)}, V_k^{(0)}$, $\forall k$ and $i = 0$;
4: repeat
5: Update $V_k^{(i+1)}$, $\forall k$, using (7);
6: Update $W_k^{(i+1)}$, $\forall k$, using (9);
7: Update $F_k^{(i+1)}$, $\forall k$, using (10);
8: $i \leftarrow i + 1$;
9: until the stopping criteria is met.

systems [3]. In what follows, we first optimize the hybrid beamformers at the BS by minimizing the Euclidean distance between FD beamformers (i.e., $F_k, \forall k$) and hybrid beamformers (i.e., $F_{RF}F_{BB,k}, \forall k$). Letting $F_{BB} = [F_{BB,1}, ..., F_{BB,K}]$, the problem is formulated as

$$(P4): \min_{F_{RF}, F_{BB}} \sum_{k=1}^{K} ||F_k - F_{RF}F_{BB,k}||^{2}_{F}$$

subject to:

$$F_{RF} \in \mathcal{F}_{RF},$$

$$||F_{RF}F_{BB}||_{F} = KN_{a}.$$ (11)

Though problem (P4) can be solve based on OMP and manifold optimization methods, the OMP method cannot achieve high system performance and the manifold optimization method is with extremely high complexity [6]. Thus, to jointly design the hybrid beamformers at the BS, we will solve problem (P4) based on the AO framework, where the analog precoder $F_{RF}$ is firstly optimized by by fixing $F_{BB,k}, \forall k$. Hence, (P4) can be rewritten as

$$(P5): \min_{F_{RF}} f(F_{RF}, F_{BB})$$

subject to:

$$F_{RF} \in \mathcal{F}_{RF}.$$ (12)

where

$$f(F_{RF}, F_{BB}) = \sum_{k=1}^{K} ||F_k - F_{RF}F_{BB,k}||^{2}_{F}$$

$$(a) = \sum_{k=1}^{K} \text{Tr}(F_k F_k^{H}) + f_{RF}^{2} Q_k f_{RF} - 2\text{Re}(f_{RF}^{2} e_k).$$ (13)

in which (a) follows from the identity $\text{Tr}(ABCD) = \text{vec}(A^{T})^{T}(D^{T} \otimes B)\text{vec}(C)$, $Q_k = (F_{BB,k} F_{BB,k}^{H})^{T} \otimes I_{N_{t}}$, $f_{RF} = \text{vec}(F_{RF})$, $E_k = F_k F_{BB,k}^{H}$, and $e_k = \text{vec}(E_k)$. To effectively solve non-convex problem (P5), we use an MM method. The basic idea is to transform original problem (P5) into a sequence of majorized subproblems that can be solved with closed-form minimizers. At first, according to lemma 2 in [12], we can find a valid majorizer of $f(F_{RF}, F_{BB})$ at point $F_{RF}^{(i)} \in \mathcal{F}_{RF}$ as

$$f(F_{RF}, F_{RF}^{(i)}, F_{BB}) = \sum_{k=1}^{K} \text{Re}(f_{RF}^{2}((Q_k - \lambda_k I)(F_{RF}^{(i)} - e_k))) + C,$$

where $\lambda_k$ denotes the maximum eigenvalue of $Q_k$, and the constant term $C = \sum_{k=1}^{K} \text{Tr}(F_k F_k^{H}) + \lambda_k f_{RF}^{2} f_{RF} + f_{RF}^{2} (\lambda_k I - Q_k)$. Then, according to the MM method and utilizing the majorizer in [14], the solution of problem (P4) can be obtained by iteratively solving the following problem

$$(P6): \min_{F_{RF}} f(F_{RF}; F_{RF}^{(i)}, F_{BB})$$

subject to:

$$F_{RF} \in \mathcal{F}_{RF}.$$ (15)

The closed-form solution of problem (P6) is given by

$$F_{RF}^{(i+1)} = -\exp(j \arg \frac{1}{K} \sum_{k=1}^{K} (Q_k - \lambda_k I)(F_{RF}^{(i)} - e_k)).$$ (16)

Then, we turn to design digital beamformers (i.e., $F_{BB,k}, \forall k$) at the BS with fixed $F_{RF}$. By fixing $F_{RF}$, the solution of problem (P4) without considering the power constraint in (11) is given by

$$F_{BB,k} = F_{RF}^{-1} F_k, \forall k.$$ (17)

To satisfy the power constraint in problem (P4), we can normalize $F_{BB}$ by a factor of $\sqrt{\frac{KN_{a}}{||F_{RF}||_{F}^{2}}}$. The effectiveness of the normalization step is referred to [2], [13]. With above closed-form solutions in (16) and (17), we summarize the MM based HBF algorithm in Algorithm 2. The hybrid beamformers at UEs can be designed following a similar approach to that at the BS, and details are omitted here due to space limitation. We then summarize the convergence and main complexity of Algorithm 1 and Algorithm 2 in the following theorems.

**Theorem 1.** The convergence of Algorithm 1 is guaranteed. The main complexity of Algorithm 1 is $O(2I_{p}KN_{a}^{3})$, where $I_{p}$ is the number of iterations.

**Proof.** According to [6], \{Re$(F_{i}^{(i)}, W_{i}^{(i)}, V_{i}^{(i)}, U_{i}^{(i)})$\} is a monotonically non-decreasing sequence and it thus converges, since \{Re$(F_{i}^{(i)}, W_{i}^{(i)}, V_{i}^{(i)}, U_{i}^{(i)})$\} is upper bounded with power constraints. For Algorithm 1, the main complexity at each iteration comes from the inversion operations in (7)-(10), which is $O(2KN_{a}^{3})$.

**Theorem 2.** The convergence of Algorithm 2 is guaranteed. The main complexity of Algorithm 2 is $O(I_{out}I_{in}KN_{a}N_{RF}^{3} + N_{a}^{2}N_{RF}^{2})$, where $I_{out}$ and $I_{in}$ are the numbers of outer and inner iterations, respectively.
Proof. The proof of convergence is similar to that in [2], and omitted here for space limitation. The main complexity at each iteration of Algorithm 2 comes from finding the maximum eigenvalue of $Q_k$ and the pseudo inversion of $F_{RF}$. That is $O(l_oK(N_sN_{RF})^3 + N_s^2N_{RF}^2)$. □

Above proposed HBF algorithm (denoted as FP-MM-HBF) is iterative algorithm and still suffers from high computational complexity as the number of antennas increases. Furthermore, since the proposed HBF algorithm and existing optimization based algorithms are linear mapping from the channel matrices and the hybrid beamformers, they require a real-time computation, and are not robust to noisy channel input data. Thus, a learning based approach to address these problems is proposed in the following section.

IV. HBF DESIGN WITH ELM

In what follows, we present our ELM framework for joint hybrid precoders and combiners design, shown in Fig. 1. There is only one hidden layer in ELM, the weights of input nodes and bias for the hidden nodes are generated randomly. We assume that the training dataset is $D = \{ (x_j,t_j) | j = 1, \ldots, N \}$, where $x_j$ and $t_j$ are sample and target for the $j$-th training data. Specifically, the $j$-th training data is defined as $x_j = [\text{Re}(\text{vec}(H_{j,1})), ..., \text{Re}(\text{vec}(H_{j,K})), \text{Im}(\text{vec}(H_{j,1})), ..., \text{Im}(\text{vec}(H_{j,K}))] \in \mathbb{R}^{N_1}$, where $H_{j,k} \sim \mathcal{CN}(\Gamma_{j,k}, \Gamma_k)$ and $N_1 = 2KN_sN_c$. And $\Gamma_k$ denotes the variance of added synthetic noise, with its $(m,n)$-th entry as $\Gamma_{j,k} = \frac{|{H_j}^H|_{m,n}^2}{\text{SNR}_{\text{Train}}}$, where SNR$_{\text{Train}}$ is the SNR for the training data [6]. The target of $j$-th data is $t_j = [\text{Re}(w_{j,1}^T), \text{Im}(w_{j,1}^T), \text{Re}(w_{j,2}^T), \text{Im}(w_{j,2}^T), ..., \text{Re}(w_{j,K}^T), \text{Im}(w_{j,K}^T)] \in \mathbb{R}^{N_2}$, where $N_2 = N_sN_{RF} + K(N_sN_{RF} + N_s(N_{RF}) + N_s)$. $w_{BB} = [w_{BB,1}, ..., w_{BB,K}]$, and $w_{RF} = [w_{RF,1}, ..., w_{RF,K}]$. The beamformers in $t_j$ are obtained by Algorithms 1 and 2. According to [14], [15], the output of ELM related to sample $x_j$ can be mathematically modeled as

$$
\sum_{l=1}^{L} \beta_l g_l(x_j) = \sum_{l=1}^{L} \beta_l (w_l^T x_j + b_l) = g(x_j) \beta,
$$

where $w_l = [w_{l,1}, ..., w_{l,N}]^T$ is the weight vector connecting the $l$-th hidden node and the input nodes, $\beta = [\beta_1, ..., \beta_L]^T \in \mathbb{R}^{L \times N_s}$, and $b_l = [b_{l,1}, ..., b_{l,N}]^T$ is the weight vector connecting the $l$-th hidden node and the output nodes, and $b_1$ is the bias of the $l$-th hidden node.

Since there is only one hidden layer in ELM, with randomized weights $\{w_l\}$ and biases $\{b_l\}$, the goal is to tune the output weight $\beta$ with training data $D$ through minimizing the ridge regression problem

$$(P7) \quad \beta^* = \arg \min_{\beta} \frac{1}{2} \|G\beta - T\|^2 + \frac{1}{2} \|\beta\|^2,$$

where $T = [t_1, \ldots, t_N]_{N \times N_s}$, $\lambda$ is the trade-off parameter between the training error and the regularization and

$$G = \begin{bmatrix} g(x_1) & \cdots & g_L(x_1) \\ \vdots & \ddots & \vdots \\ g(x_N) & \cdots & g_L(x_N) \end{bmatrix} \in \mathbb{R}^{L \times N}.$$

According to [2], the closed-form solution for (P7) is

$$\beta^* = G^T (\frac{1}{\lambda} + GG^T)^{-1} T.$$  

From above, it can be concluded that ELM is with very low complexity since there is only one layer’s parameters to be trained and the weight of output layer (i.e., $\beta$) is given in closed-form.

V. NUMERICAL SIMULATIONS

In this section, we numerically evaluate the performance of our proposed methods, and compare them with four state-of-the-art methods: BD-ZF-FD [5], BD-ZF-HBF [4], WMMSE-OMP-HBF [3] and CNN-HBF [6]. Uniform planar array [3] is used, and the number of cluster and array for mmWave channels are set to 5 and 10, respectively. We select $K = 3$, $N_s = 2$, $N_c = 16$ and $L = 4000$. The ELM is fed with 100 channel realizations and for each channel realization, 100 noisy channels are obtained by adding synthetic noise with different powers of SNR$_{\text{Train}} \in \{15, 20, 25\}$ dB.

Fig. 23 and Fig. 25 present the achievable sum-rate of various beamforming methods versus SNR with perfect and imperfect CSI, respectively. With perfect CSI, it shows that all the proposed methods outperform others in Fig. 24. Both FP-MM-HBF and ELM-HBF can approach the sum-rate performance of proposed fully digital beamforming scheme, i.e., FP-FD. With imperfect CSI, Fig. 25 shows that learning based methods (i.e., ELM-HBF and CNN-HBF) achieve higher sum-rate than other optimization based methods. We observe that ELM-HBF provides better performance than CNN-HBF and is very close to the performance of FP-FD. The reason is that the weight matrices of ELM are given in closed-form and are much easier to be optimized those of CNN.

Table 1 shows the computation time for different HBF methods. The computation time of a learning based method is characterized by offline training time and online prediction time. A performance comparison among two common activation functions for ELM, i.e., sigmoid function and parametric rectified linear unit (PReLU) function [2], is provided. 1000 noisy channel samples for 10 channel realizations are fed into the learning machines, and 100 noisy channel samples are used for testing. We select SNR = -8 dB, $N_c = 16$, SNR$_{\text{Train}} = \text{SNR}_{\text{Test}} = 10$ dB. We can see that ELM-HBF with different activation functions always achieve higher sum-rate than other methods. Among all methods, ELM-HBF with
PReLU consumes the shortest prediction time. For instance, as $N_t = 64$, it can achieve 50 and 10 times faster prediction time than WMMSE-OMP-HBF and BD-ZF-HBF, respectively. Moreover, CNN-HBF takes much longer training time than ELM-HBF. For instance, ELM-HBF with PReLU can achieve 1200 times faster training time than CNN-HBF.

VI. CONCLUSIONS

An ELM framework is proposed to jointly design the hybrid precoders and combiners for mmWave MU-MIMO systems. We show that proposed methods can provide higher sum-rate than existing methods. Moreover, the proposed ELM-HBF can support more robust performance than CNN-HBF and other optimization based methods. Finally, for $N_t = 64$, ELM-HBF can achieve 50 times faster prediction time than WMMSE-OMP-HBF, and 1200 times faster training time than CNN-HBF. Thus, ELM-HBF is with much lower complexity and might be more practical for implementation.

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