Plasmon resonant light scattering on spheroidal metallic nanoparticle embedded in a dielectric matrix

N. I. Grigorchuk

Bogolyubov Institute for Theoretical Physics, NAS of Ukraine - 14-b Metrologichna Str., Kyiv, 03143, Ukraine

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Abstract – The efficiency of light scattering on metal nanoparticles with an excitation of plasmon resonance electron vibrations is calculated. The behavior of the light scattering in a region of the surface plasmon resonance is studied in detail. A simple universal formula for light scattering cross-section by a metal nanoparticle with different shape embedded in any dielectric media is obtained. It is shown that the distance between the doublet peaks makes it possible to estimate the degree of oblateness or prolateness of a nanoparticle. The sensitivity to the shape and size of a metal nanoparticle, as well as to the scattering angle is illustrated for the Au nanoparticle.

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Introduction. – The physical understanding of the light scattering processes is based so far on the conception presented by Lord Rayleigh [1] that the small spherical particle behaves as a point electrical dipole. The general theory of light scattering by a spherical particle was developed by Mie [2,3]. In the classical electrodynamics it is supposed that the light scattering results from the polarization of the scattered particle when it is illuminated with a beam of light. Because the spatially uniform field causes only a dipole polarization, the nanoparticle scatters the light as a vibration dipole [4].

The Rayleigh treatment is limited to the nonresonant scattering [5]; the Mie solution to the problem is exact and applies to spheres of arbitrary size, but it is mainly a numerical solution that cannot be accomplished without resorting to a robust code [6]. The problem is more complicated for nonspherical metal nanoparticles (MNs), when several plasmon modes are excited.

As is well known, the plasmon resonance light scattering (PRLS) occurs when the incident light has an energy close to the energy excitation of the surface plasmon oscillations [7,8]. As a result, local electromagnetic (EM) fields near the particle can be many orders of magnitude higher than the incident fields, and the incident light around the resonant-peak wavelength is scattered very strongly. The exact position, shape and intensity of the localized nanoparticle surface plasmon resonance (SPR) is determined by such factors as particle morphology (size and shape), dielectric environment (surrounding medium, supporting substrate, coating), and interparticle coupling (e.g., the state of aggregation). The nanoparticle SPR sensors and biosensors [9] are based on modifications of any of those factors. Resonance light scatterers are employed in the different near-field scanning optical microscopies, fibers, and nanowires. They are important for various modern applications, e.g., field concentration for nanopatterning with nanowires [10], plasmonic nanolithography [11] and near-field scanning optical microscopies [12], astigmatic optical tweezers [13], and finally nonlinear optics in nanowires [14]. The single-particle scattering spectroscopy based on the localized surface plasmon resonance spectra of single metal nanoparticles is well developed now [15].

The problem of scattering of electromagnetic waves by nonspherical MNs has been examined by several authors [16–18]. Using the Hertz-vector representation of the electromagnetic field of a point dipole and its decomposition into plain waves, a general theory of light scattering was developed [16], which permits one to calculate the optical properties of large metal particles, the sizes of which are much greater than the electron mean free path. Basing on this theory, the resonances in surface-enhanced Raman scattering for silver particles arranged in two-dimensional grating have been interpreted in terms of the particle dipolar interactions and effects of finite particle size [17]. In [18] the numerical
point-matching method was used to show that the peaks in the absorbance curve of a nonspherical plasma particle may differ significantly from that of a spherical particle.

In contrast, we present here an analytical treatment of the resonant light scattering from nonspherical MNs, any of whose sizes can be much less than the electron mean free path inside the particle material. The main novelty of this letter is that a simple formula for the principal components of the polarizability tensor is proposed. It enables one to calculate the light scattering or the absorption processes for metallic nanoparticles embedded in any dielectric media through the components of the conductivity tensor. These last-mentioned components have been calculated by us earlier in ref. [19] using the kinetic approach. This approach permits one to account correctly the effect of particle surface, when the particle sizes are less than the electron mean free path.

In the present work, we will study the PRLS effect on the angle-dependent scattering cross-section. We calculate and discuss the spectra for the scattering efficiencies. In particular, we illustrate results for gold nanoparticles.

The model neglects absorption within the particles and it is assumed that the whole extinction involves only the scattering. We will take into account the dissipation ($\epsilon'' = \text{Im} \epsilon \neq 0$), when the scattering can strongly deviate from the classical Rayleigh scattering.

**Formulation of the problem.** – Let us consider the scattering of electromagnetic waves on a metallic nanoparticle, the sizes of which $d$ are much smaller than the considered wavelength $\lambda \sim c/\omega$, or $kd \ll 1$ (Rayleigh scattering), where $c$ is the light speed, $\omega$ is the wave frequency and $k$ is the wave number. Then the EM field around the MN can be considered as a homogeneous one. For nanoparticles much smaller than the wavelength of light, the EM field is uniform across a particle and all conduction electrons move in phase producing only dipole-type oscillations.

Another size limitation to the MNs is connected with the mean free path of electrons $l$. We will consider MNs much smaller than $l$, when the collision of the conducting electrons with the particle surface become important as an additional relaxation process.

The particle in a uniform periodic field gains an electrical $d$ and magnetic $M$ moment, which vary over time. The scattering wave can be described as a result of irradiation from those varying moments. The field of the scattered wave in a medium with dielectric constant $\epsilon_m$, at distances $r$ large compared to the wavelength $\lambda$, has the form [20]

$$\mathbf{H}_s = \left( \frac{\omega}{c} \sqrt{\epsilon_m} \right)^2 \frac{1}{r} (\mathbf{n} \times d + \mathbf{n} \times (M \times \mathbf{n})),$$

where $\mathbf{n}$ is the unit vector directed along the scattering direction. The values of $d$ and $M$ should be taken at the time moment $t - r/c$. Further, we will consider only the particles and media with magnetic permeability $\mu = \mu_m = 1$. This allows us to neglect the magnetic dipole term in eq. (1).

The dipole electric moment for a spherical metallic particle embedded in a medium with dielectric constant $\epsilon_m$, can be written as

$$d = \epsilon_m \alpha \mathbf{E},$$

where $\alpha = (\alpha' + i\alpha'')$ is the polarizability of the MN and $\mathbf{E}$ is the amplitude of an external electric field.

The intensity of the light irradiation in a space angle $d\Omega = \sin \theta d\varphi d\theta$ averaged over the wave period, at a distance $r$ far away from the MN, can be presented as

$$dI_s = \frac{1}{8\pi\epsilon_m} \left| \frac{c}{\epsilon_m} |\mathbf{H}_s|^2 r^2 \right| d\Omega,$$

where $1/2$ comes from the averaging over the wave period.

The differential scattering cross-section can be obtained by dividing the energy scattered by MN by the energy flux density of an incident wave (known as the Poynting vector)

$$S = \frac{c}{8\pi} \sqrt{\epsilon_m} |\mathbf{E}|^2.$$

As a result, using eqs. (1), (3), one obtains [3,6]

$$dC_s = \frac{dI_s}{S} = \left( \frac{\omega}{c} \sqrt{\epsilon_m} \right)^4 |\alpha|^2 \sin^2 \theta d\Omega,$$

where $\theta$ is the angle between the scattering direction $\mathbf{n}$ and the direction of the electric field of an incident wave.

The calculations of $\mathbf{d}$ are especially simplified for the case in which the particle sizes are small as compared to a certain “wavelength” $\delta \sim c/(\sqrt{|\epsilon_m|} \omega)$, which corresponds to the frequency $\omega$ in the particle bulk. In this case, one can calculate the polarizability of the MN using the formulae obtained for an external uniform static field. For a MN in host, they look like [6,21,22]

$$\alpha_j(\omega) = \frac{V}{4\pi \epsilon_m(\omega) + \omega L_j(\omega) - \rho_m(\omega)}$$

where $V$ is the volume of the MN, $L_j$ are the geometrical factors that are also known as the depolarization coefficients, and $\epsilon$ is the frequency-dependent dielectric constant of the particle material, which for the case of frequencies close to the electron plasma oscillations in metal may be approximated as

$$\epsilon(\omega) \simeq 1 - \left( \frac{\omega_p}{\omega} \right)^2 + 4\pi \sigma(\omega) \frac{\omega}{\omega}.$$

Here $\omega_p = (4\pi ne^2/m)^{1/2}$ is the plasma frequency of electron oscillations in metal, $e$ and $m$ are, respectively, the charge and mass of the electron, $n_e$ is the concentration of electrons, and $\sigma(\omega)$ is the optical conductivity, the real part of which in the long-wave limit takes the form

$$\sigma(\omega) \simeq \frac{\nu}{4\pi} \left( \frac{\omega_p}{\omega} \right)^2,$$

where $\nu = 1/\tau$ is the electron collision frequency.
Neglecting the dissipation in eq. (5), one obtains for the polarizability of the spherical MN in a vacuum, the well-known Clausius-Mossotti relation

$$\alpha(\omega) = \frac{3}{4\pi} V \left[ \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} \right]$$

(8)

Using eq. (6), the squared expression (5) for a spherical MN embedded in a dielectric medium with $\varepsilon_m = 1$, is then given by

$$|\alpha|^2 = \left( \frac{3}{4\pi} V \right)^2 \frac{\omega^4 p + (\frac{4}{3} \omega \varepsilon(\omega))^2}{(\omega^2 - \omega_p^2)^2 + (\frac{4}{3} \omega \varepsilon(\omega))^2}$$

(9)

where $\omega_p = \omega_d / \sqrt{3}$ is the surface plasmon frequency.

The overall scattering cross-section for a spherical MN in the long-wavelength limit, as is evident from eq. (4), is expressed as [6]

$$C_s = \frac{8}{3} \pi \left( \frac{\omega}{c \sqrt{\varepsilon_m}} \right)^4 |\alpha|^2$$

(10)

The dependence of the cross-section on the frequency of light is realized as well through the polarization. Note, that the scattering cross-section is proportional to the square of the particle volume.

In the case of nanoparticles of a spheroidal shape, the dipole moment of the MN can be presented in the form [22]

$$\mathbf{d} = \alpha_{\parallel} \varepsilon_m \mathbf{E} + (\alpha_{\parallel} - \alpha_{\perp}) \varepsilon_m (\mathbf{m} \cdot \mathbf{E}) \mathbf{m},$$

(11)

where $\mathbf{m}$ is the unit vector directed along the spheroid revolution axis. In view of eqs. (3) and (1), the squared dipole moment (11) is written as

$$\mathbf{d}^2 = \alpha_{\parallel}^2 \varepsilon_m \mathbf{E}^2 + (\alpha_{\parallel}^2 - \alpha_{\perp}^2) \varepsilon_m (\mathbf{m} \cdot \mathbf{E})^2,$$

(12)

or, taking into account that $\mathbf{m} \cdot \mathbf{E} = E \cos \phi$, where $\phi$ is the angle between the spheroid axis of revolution and the direction of an electric field $\mathbf{E}$, as

$$\mathbf{d}^2 = (\alpha_{\parallel}^2 \cos^2 \phi + \alpha_{\perp}^2 \sin^2 \phi) \varepsilon_m E^2.$$

(13)

The differential scattering cross-section for a spheroidal MN with account for eqs. (13), (4) and (2), is found to be

$$dC_s = \left( \frac{\omega}{c \sqrt{\varepsilon_m}} \right)^4 \left[ |\alpha_{\parallel}|^2 \cos^2 \phi + |\alpha_{\perp}|^2 \sin^2 \phi \right] \sin^2 \theta \, d\Omega,$$

(14)

where the square of the diagonal components of the polarizability tensor can be presented in Lorenzian form, namely

$$|\alpha_{\parallel}|^2 = \left( \frac{V}{4\pi L_{\parallel}} \right)^2 \frac{(1 - \xi_m)\omega^2 - \omega^2_{\parallel})^2 + (2\omega\gamma_{\parallel})^2}{(\omega^2 - \omega_{\parallel})^2 + (2\omega\gamma_{\parallel})^2},$$

(15)

Here

$$V = \frac{4}{3}\pi R_{\parallel} R_{\perp}^2,$$

$$\xi_m = \frac{\varepsilon_m}{\varepsilon_m + L_{\parallel}(1 - \varepsilon_m)},$$

(16)

and

$$\omega_{\parallel}^2 = \frac{L_{\parallel}(1)}{\varepsilon_m + L_{\parallel}(1 - \varepsilon_m)} \omega_{\parallel}^2,$$

(17)

$$\gamma_{\parallel} = \gamma_{\parallel}^{(\parallel)}(\omega) = \frac{2\pi L_{\parallel}(1)}{\varepsilon_m + L_{\parallel}(1 - \varepsilon_m)} \sigma_{\parallel}^{(\parallel)}(\omega)$$

(18)

represents the half-width of the resonance curve for light polarized along $\parallel$ or across $\perp$ the rotation axis of the spheroid; $\sigma_{\parallel}$, $\sigma_{\perp}$ are the corresponding components of the conductivity tensor, and $L_{\parallel}$, $L_{\perp}$ are the components of the depolarization factor of the spheroid [20]

$$L_{\parallel} = \frac{1}{2} (1 - L_{\parallel}),$$

$$L_{\parallel} = \frac{1 - \varepsilon_p^2}{2\varepsilon_p^2} \left[ \ln \left( 1 + \frac{\varepsilon_p}{1 - \varepsilon_p} \right) - 2\varepsilon_p \right], \quad \text{for a prolate spheroid,}$$

$$L_{\parallel} = \frac{1 + \varepsilon_p^2}{\varepsilon_p^2} (\varepsilon_p - \arctan \varepsilon_p), \quad \text{for an oblate spheroid,}$$

where $\varepsilon_p = \sqrt{1 - (R_{\perp}/R_{\parallel})^2}$ is the eccentricity of the spheroid. The spheroid semiaxes $R_{\parallel}$ and $R_{\perp}$ can be easily expressed through the radius of a sphere $R$ of an equivalent volume via the relations

$$R_{\parallel} = R \left( \frac{R_{\parallel}}{R_{\parallel}} \right)^{1/3}, \quad R_{\perp} = R \left( \frac{R_{\parallel}}{R_{\parallel}} \right)^{-2/3}.$$

(19)

In the case of a MN of spherical shape, $L_{\parallel} = L_{\perp} = 1/3$, and eqs. (16)–(18) can be rewritten as

$$\xi_m = \frac{3\varepsilon_m}{2\varepsilon_m + 1},$$

(20)

$$\omega_{\parallel}^2 = \omega_{\parallel}^{2 \text{ph}},$$

(21)

$$\gamma_{\parallel} = \frac{2\pi}{2\varepsilon_m + 1} \sigma_{\parallel}^{\text{ph}}(\omega)$$

(22)

and at $\varepsilon_m \to 1$, eq. (15) transforms exactly into eq. (9).

In accordance with eq. (14), the scattered light intensity oscillates between $dC_{\text{min}} = 0$ and $dC_{\text{max}} = 4\varepsilon^2$, where $A$ is a constant. The maximal differential cross-section is reached at $\theta = \pi/2$. As the particle diameter $d$ is decreased, the scattered intensity drops as $d^6$.

If the spheroidal MN is embedded in a medium with $\varepsilon_m \to 1$, then eqs. (16)–(18) are reduced to

$$\xi_m = 1,$$

(23)

$$\omega_{\parallel}^2 = L_{\parallel}(1) \omega_{\parallel}^2,$$

(24)

$$\gamma_{\parallel} = 2\pi L_{\parallel}(1) \sigma_{\parallel}(\omega),$$

(25)

and eq. (15) takes the form

$$|\alpha_{\parallel}|^2 = \left( \frac{V}{4\pi L_{\parallel}(1)} \right)^2 \frac{\omega_{\parallel}^2 + (2\omega\gamma_{\parallel})^2}{(\omega^2 - \omega_{\parallel}^2)^2 + (2\omega\gamma_{\parallel})^2}.$$
On resonance, the square of the polarizability tensor is

\[ |\alpha_{(p)}|^2 = \left( \frac{V}{4\pi L_{(1)}} \right)^2 \left( 1 + \frac{\xi_m^2 \gamma^2_{(1)}}{4 \gamma^2_{(1)}} \right), \] (27)

with \( \gamma \) taken at \( \omega = \omega_{(1)} \). The contribution of nonresonant frequencies is so small that their input into the scattering cross-section can be neglected.

Using eqs. (14) and (27), one can see that the main contribution to the scattering cross-section is given by plasma resonances, when the incident photon frequency is resonant with the collective excitation of the conduction electrons. It is known as the localized surface plasmon resonance. Near the surface plasmon resonance, light may interact with the particle over a cross-sectional area larger than the geometric cross-section of the particle because the polarizability of the particle becomes very high in this frequency range.

The principal components of the conductivity tensor (entering in \( \gamma \)) in the high-frequency limit, for the case when the mean free path of the electrons exceeds the particle size, can be presented as [19]

\[ \sigma_{(p)}(\omega) = \frac{9}{32\pi} \left( \frac{\omega_{pl}}{\omega} \right)^2 v_F \left( \eta^H_{c}(\epsilon_p) / \rho_H(\epsilon_p) \right), \] (28)

where \( v_F \) is the Fermi velocity, \( R_{(p)} \) is the spheroid semi-axis directed across the spheroid rotation axis, and \( \eta^H_{c}(\epsilon_p) \) and \( \rho_H(\epsilon_p) \) are some smooth functions depend only on the spheroid eccentricity \( \epsilon_p \). The explicit form of these functions can be found in ref. [19]. Below, we present only asymptotic expressions for these functions, for spherical MNs or spheroidal ones with large and small aspect ratio:

\[ \eta^H_{c} = \begin{cases} \frac{3}{8}, & R_{(p)} \ll R_{(1)}, \\
\frac{3}{4}, & R_{(p)} \approx R_{(1)}, \\
1 \frac{R_{(1)}}{\sqrt{2}} R_{(p)} \gg R_{(1)}, & R_{(p)} \gg R_{(1)}. \end{cases} \] (29)

and eq. (30), one can estimate the radius of a MN for which the absorption cross-section becomes comparable with the scattering cross-section. It is most easily done for a spherical MN in vacuum. In the high-frequency limit, we have

\[ R_{HF} \simeq \left( \frac{3}{2} \right)^{3/4} \left( c v_F \right)^{1/4} \sqrt{\frac{c}{\omega_{pl}}}. \] (33)

For instance, \( R \approx 100 \text{ Å} \) for a spherical Au nanoparticle at plasmon frequency \( \omega = \omega_{pl}/\sqrt{3} \).

In the case of low frequencies, we have

\[ R_{LF} \simeq \frac{c}{\omega_{pl}} \sqrt{\frac{6c}{v_F \sqrt{\epsilon_m}}}. \] (34)

The latter formula gives \( R \approx 7860 \text{ Å} \) for a spherical Au nanoparticle embedded in a medium with \( \epsilon_m = 1 \).

Discussion of results. – Using eq. (14), we can calculate the scattering efficiency of the MN, which we define as the ratio of the differential scattering cross-section to the geometrical cross-sections of the individual particles

\[ Q_s = \frac{dC_s}{G}, \] (35)

where

\[ G = \pi \left( \frac{c}{\omega_{pl}} \right)^2 \] (36)

is the geometrical cross-section.

In fig. 1, we present the dependence of the scattering efficiency \( Q_s \) for gold nanoparticles of different shapes in vacuum as a function of the incident-light frequency, calculated using eq. (14) at \( \phi = \pi/3 \) and \( \theta = \pi/2 \). The
frequencies were normalized by a plasmon resonance frequency $\omega = \omega_{pl}/\sqrt{3} \equiv \Omega$ for a spherical MN in vacuum. We have taken $\nu_{Pl} \simeq 3.39 \times 10^{15} \text{s}^{-1}$ [23] and $n_e \simeq 5.9 \times 10^{22} \text{cm}^{-3}$ [24], which, it is assumed, are appropriate values for gold particles. The volume of a spheroidal MN was taken equal to the volume of an equivalent sphere with radius $R = c/\omega_{pl}$. The Fermi velocity was estimated by the formula

$$v_F = \frac{2\pi \hbar}{m} \left( \frac{3 n_e}{8\pi} \right)^{1/3}. \quad (37)$$

The particle scattering effects depend on the size and shape of the individual particles. The shape factor of the spheroidal particles was parameterized by the $R_\perp/R_\parallel$ ratio. Oblate spheroids correspond to the $R_\perp/R_\parallel > 1$ case (pancake shaped), spherical particles to $R_\perp/R_\parallel = 1$, and prolate spheroids to $R_\perp/R_\parallel < 1$ (cigar or needle-like shaped). The action of the size factor usually is studied at a fixed ratio of $R_\perp/R_\parallel$, with changing the volume of the MN.

The maximal cross-section is observed in a spherical nanoparticle in vacuum (curve 1) at a frequency which corresponds to the plasmon resonance. When the symmetry of the particle decreases, the number of resonance peaks increases. If the shape of the particle deviates from a spherical one, the 3-fold degeneracy dipole mode splits into two (for spheroid) or three (for ellipsoid) modes giving rise to corresponding scattering peaks [25]. In the case of a spheroid, the electrical field incident at $\phi \neq 0^\circ$ (or $90^\circ$) excites the oscillations of two dipole modes along and across the spheroid revolution axis. The observed peaks in fig. 1 are induced by the two orthogonally directed dipoles, that is why we have the splitting of the PRLS into a doublet with a transverse mode (perpendicular ($\perp$) to the spheroid revolution axis) and a longitudinal mode (parallel ($\parallel$) to the spheroid revolution axis).

The relative height of the peaks in the doublet can be controlled by varying the incidence angle of the electromagnetic wave. As the incidence angle decreases, the $\perp$-peaks of the scattering efficiency decrease and the $\parallel$-peaks increase. On the contrary, if the incidence angle increases, then the $\perp$-peaks of scattering efficiency increase and $\parallel$-peaks decrease. At the angle $\theta = 0^\circ$, the $\perp$-peaks disappear in correspondence with eq. (15), and the intensity of the $\parallel$-peaks reaches a maximum. Otherwise, at an angle of $\pi/2$, the $\parallel$-peaks disappear, and the height of the $\perp$-peaks remains maximum.

It is necessary to pay attention here to the fact that the oblate- and prolate-shaped MNs differ essentially in spectral manifestations. In the case of prolate-shaped MNs, the low-energy peaks ($\omega < \Omega$) correspond to parallel ($\parallel$) oscillations of the electrons to the spheroid rotation (major) axis, and the high-energy ones ($\omega > \Omega$) correspond to perpendicular ($\perp$) oscillations to the spheroid rotation axis (see fig. 1). In the case of oblate-shaped MNs, vice versa, the low-energy peaks result from dipole oscillations of the electrons perpendicular to the spheroid rotation (minor) axis, and the high-energy ones correspond to oscillations parallel to the spheroid rotation axis. As the aspect ratio of the nanospheroids increases, the separation between two plasmon bands becomes more pronounced.

The shape of a MN is rigidly related to the frequency of the resonance absorption [3,6]. With increasing the Au particle prolateness, the $\parallel$-component of the surface plasmon resonance peak (at $\omega < \Omega$) shifts to the red side\(^1\), whereas the $\perp$-component (at $\omega > \Omega$) of the plasmon peak shifts to the blue side of the spectrum (compare curves 2 and 3 with spheroid aspect ratio $R_\perp/R_\parallel = 0.75$ and 0.625). For more and more oblate-shaped MNs, the $\perp$-component of the peak (at $\omega < \Omega$) shifts to the red side\(^2\), and the $\parallel$-component (at $\omega > \Omega$) shifts to the blue side of the spectrum (compare curves 4 and 5 with $R_\perp/R_\parallel = 1.25$ and 1.5). One can see from fig. 1 that the prolate MNs (for the above selected aspect ratios) exhibit a stronger spectral tunability than oblate ones. This corresponds to the results obtained in [27]. The total area under both the $\parallel$ and the $\perp$ cross-section curves does not change with altering the spheroid aspect ratio and corresponds to the area under curve 1 for spherical MN.

The resonance frequencies of peaks 1–5 also get redshifted as the dielectric function of the surrounding medium increases.

It was found that for fixed angles $\theta$ and $\phi$, the PRLS intensity drastically depends on the particle radius. Larger particles have a larger scattering cross-section. For spherical particles the size of the particle does not affect the

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1 In that polarization, such tendency has been found earlier experimentally for Ag nanoparticles [26].

2 This correlates well with earlier calculations for Au nanoparticles [27].
resonance position, but affects only the scattering intensity. The resonance curves are sharper for larger particle radii and tend to broaden for smaller particle radii.

Figure 2 shows how the scattering efficiency changes for gold nanoparticles of different shapes (but equal volumes) with varying the scattering angle at fixed field polarization. At $\phi = \pi/2$ only the $\parallel$-polarization of electrons is actual and we have the maximal scattering efficiency for a prolate-shaped gold nanoparticle and the minimal efficiency for an oblate one. If one chooses $\phi = 0^\circ$, then the $\perp$-polarization of electrons plays a dominant role and we get the maximal scattering efficiency for an oblate-shaped gold nanoparticle and the minimal efficiency for a prolate one.

Conclusions. – We have developed a theory of light scattering phenomena for metal nanoparticles embedded in any dielectric media. It allows to determine the cross-section of light scattering at various polarizations of the incident electromagnetic wave. The obtained analytic formulas make it possible to evaluate the enhancement of light scattering intensity at plasmon frequencies for nanoparticles with arbitrary spheroidal aspect ratio, and to find the scattering rate depending on the light polarization, the angle of the scattering direction and the volume of the MN.

We have obtained a simple expression for the principal components of the polarization tensor. It enables one to account properly the effect of the nanoparticle surface, when the electron free path is much higher than the particle sizes. The effects of the particle geometry and the surrounding medium are also accounted for. The case, when the frequency of a laser beam is close to plasmon frequencies in a spheroidal MN is studied in detail.

We found that the cross-section rate of a scattering light in spheroidal MNs as a function of frequency has two maxima of different intensities, in contrast to the spherical MNs, where a single maximum is observed. By the distance between the doublet peaks, one can estimate the degree of oblateness or prolateness of the MN. The intensity of the doublet peaks can be controlled by the variation of the incidence angle of a laser beam relative to the rotation axis of the spheroid.

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