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Robustness of the thermal Hall effect close to half-quantization in α-RuCl₃

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A key feature of quantum spin liquids is the predicted formation of fractionalized excitations. They are expected to produce changes in the physical response, providing a way to observe the quantum spin liquid state. In the honeycomb magnet α-RuCl₃, a quantum spin liquid has been proposed to explain the behaviour observed on applying an in-plane magnetic field \( H_{\parallel} \). Previous work reported that the thermal Hall conductivity took on a half-integer quantized value and suggested this as a signature of a fractionalized Majorana edge mode predicted to exist in Kitaev quantum spin liquids. However, the temperature and magnetic-field range of the half-quantized signal and its association with Majorana edge modes are still under debate. Here we present a comprehensive study of the thermal Hall conductivity in α-RuCl₃ showing that approximately half-integer quantization exists in an extended region of the phase diagram, particularly across a plateau-like parameter regime for \( H_{\parallel} \) exceeding 10 T and temperature below 6.5 K. At lower fields, the thermal Hall conductivity exhibits correlations with complex anomalies in the longitudinal thermal conductivity and magnetization, and is suppressed by cooling to low temperatures. Our results can be explained by the existence of a topological state in magnetic fields above 10 T.

A prime candidate for experimentally accessible quantum spin liquids is the Kitaev model, which has an exactly solvable ground state with itinerant and localized Majorana fermions. Although they are charge-neutral, itinerant Majorana fermions carry heat and are therefore expected to contribute to thermal transport. Applying a magnetic field gaps the bulk Majorana bands, leading to a topologically protected chiral edge current. This edge state carries a thermal Hall conductivity per layer divided by temperature, namely, \( k_{\text{thermal}} \approx \frac{\pi k_B T}{m} \), where \( k_B \) and \( h \) are the Boltzmann and Planck constants, one-half of that of an equivalent electronic edge state in the quantum Hall effect due to the fractionalized nature of the Majorana fermion.

The search for material candidates of the Kitaev model has focused on Mott insulators with \( 5d^1 \) Ir⁺⁺ and \( 4d^1 \) Ru⁺⁺ having strong spin–orbit coupling on a honeycomb lattice. Here α-RuCl₃ is a prime candidate that shows antiferromagnetic zigzag order below the antiferromagnetic ordering temperature \( T_N \approx 7.5 \) K. This magnetic order is suppressed with an in-plane critical magnetic field of \( H_{C1} \approx 7 \) T (refs. 24–25), where the magnetic moment is not yet fully saturated, revealing a region of the phase diagram where a quantum spin liquid may arise. A thermal Hall effect with a magnitude close to the half-quantized value \( k_{\text{thermal}} \approx \frac{\pi k_B T}{m} \) (corresponding to \( k_{\text{XX}} \approx \frac{\pi k_B T}{m} \) per atomic plane) was reported in the spin quantum liquid region with an in-plane magnetic field \( H_{\parallel} \) along the \( a \)-axis (perpendicular to the Ru–Ru bond direction)⁴, which was discussed as a signature of the Majorana edge state expected for the Kitaev quantum spin liquid.

To establish the presence of an edge state, however, the robustness of the half-quantized thermal Hall effect should be demonstrated over a reasonably wide range of magnetic fields and temperatures. Several studies report a plateau-like region in a field at around 5 K with a magnitude close to \( k_{\text{thermal}} \approx H / T \) (refs. 26–27). These studies, however, are limited down to \( T \approx 3.5 \) K and the plateau behaviour as a function of \( T \) is not as clear as it is in the field. The plateau-onset fields differ between studies, ranging from \( H_{\parallel} = 7.8 \) T (ref. 26) to 9.9 T (ref. 27). In preparing this manuscript, we noticed that another study reported a broad \( k_{\text{thermal}} \approx H / T \) dome with a height appreciably smaller than \( k_{\text{thermal}} \approx H / T \), which was interpreted as not supporting the existence of a half-quantized plateau. Recently, anomalies in the magnetocaloric effect, specific heat and magnetic Grüneisen parameter were reported at around \( H_{\parallel} = 10 \) T (refs. 28–29) in the spin liquid region (above \( H_{\text{C2}} \)), suggesting that the phase diagram may be more complex. The question of the relationship between these anomalies and the thermal Hall signature may be of fundamental importance to understand the nature of the possible field-induced quantum spin liquid, and more specifically, the possible topological edge state.

Here we present comprehensive measurements of \( k_{\text{thermal}} \) and the magnetic susceptibility (d\( M/dH \)) in a \( T \)-range from 150 mK to 9 K and \( H_{\parallel} \) up to 13 T (\( H_{\parallel} \) along the \( a \)-axis) on an α-RuCl₃ single crystal, which reveals that \( k_{\text{thermal}} \approx H / T \) stays close to \( k_{\text{thermal}} \approx H / T \) over a reasonably wide range of both temperature and magnetic field (below \( T \approx 6.5 \) K and \( H_{\parallel} \approx 10 \) T up to at least 13 T), hinting at the presence of a protected half-quantized plateau. Below \( T \approx 10 \) K, no signature of a plateau is observed and \( k_{\text{thermal}} \approx H / T \) is suppressed to zero on lowering the temperature below 6.5 K, which appears to correlate with the multiple high-field (\( H_{\parallel} > H_{\text{C2}} \)) anomalies present in both \( k_{\text{thermal}} \) and \( dM/dH \).

Magnetic-field-induced anomalies in thermal conductivity \( k_{\text{XX}} \). The temperature dependence of \( k_{\text{XX}} \) reproduces the data from previous reports very well and displays a sharp minimum at \( T_\text{XX} \approx 7.5 \) K, which moves to lower temperatures as \( H_{\parallel} \) increases and eventually fades out above \( H_{\text{C2}} \approx 7.1 \) T, tracing the extent of the antiferromagnetically ordered region (Fig. 1a,c). As a function of \( H_{\parallel} \), the magnitude of \( k_{\text{XX}} \) first decreases to a broad minimum at around 7 T and then rapidly increases (Fig. 1b). The data below 2 K reveal additional features in \( k_{\text{XX}}(H_{\parallel}) \): the broad minimum splits into two at \( H_{\text{C2}} = 6.05 \) T and \( H_{\text{C2}} = 7.10 \) T. These fields coincide with two sharp peaks in \( dM/dH \) (Fig. 1c), which correspond to two reported transitions: from zigzag-ordered phase I to intermediate-ordered phase II and from phase II to the high-field paramagnetic phase (Fig. 1c). The suppression of \( k_{\text{XX}} \) at the magnetic-phase transitions is consistent with the interpretation that \( k_{\text{XX}} \) is dominated by phonons that scatter off magnetic excitations.

At higher in-plane magnetic fields and below 5 K, further anomalies can be seen in \( k_{\text{XX}} \), namely, a broad maximum at \( H_{\parallel} \approx 10 \) T.
flanked by broad minima at \(H_{C1} \approx 8.8\ T\) and \(H_{C2} \approx 10.7\ T\) (Fig. 1b). Features corresponding to \(H_{C1}\) and \(H_{C2}\) can be clearly identified as a broad shoulder and a sudden drop in \(dM/dH\), respectively (Fig. 1c and Extended Data Fig. 1). We argue that these high-field anomalies in \(k_{xx}\) arise from crossovers or very weak phase transitions.

In analogy to the minima in \(k_{xx}\) at \(H_{C1}\) and \(H_{C2}\), increased phonon scatterings by soft magnetic excitations can explain the minima at \(H_{C1}\) and \(H_{C2}\). However, the anomalies at \(H_{C1}\) and \(H_{C2}\) appear broader and weaker than those at the well-defined phase transitions at \(H_{C1}\) and \(H_{C2}\) (Fig. 1b). They are enhanced on cooling to 500 mK but subsequently suppressed below 500 mK, indicative of a reduction in phonon scatterers. This suggests that \(H_{C1}\) and \(H_{C2}\) may not be well-defined phase transitions like \(H_{C1}\) and \(H_{C2}\) but highly probable crossovers with soft but finite energy excitations or alternatively very weak transitions having only a limited number of low-energy excitations involved.

Recent reports of the magnetocaloric effect\(^{22}\) and the magnetic Grüneisen parameter\(^{22}\) at temperatures above 1K also showed anomalies near \(H_{C1}\) and \(H_{C2}\) whereas the specific heat\(^{22}\) at 0.67 K reported a symmetry-breaking phase transition near 10 T, a conclusion that is contested by several other measurements\(^{25,27,28}\). In the course of preparing this manuscript, we noticed another work\(^{22}\) in which broadly similar field-dependent structures are reported in \(k_{XX}\) that are ascribed to quantum oscillations from underlying quasiparticles. That scenario, however, does not explain the disappearance of features in the low-temperature limit instead of the expected Lifshitz–Kosevitch behaviour (Extended Data Fig. 2) or the alignment of prominent minima with the well-established magnetic-phase transitions at \(H_{C1}\) and \(H_{C2}\).

**Thermal Hall conductivity close to the half-quantized value.** A strong \((\Delta\mu_{xx}/T)\) signal is resolved above \(H_{C2} = 7.1\ T\), as measured by sweeping the temperature at a fixed field (Fig. 2, \(T\) sweeps) and a magnetic field at a fixed temperature (Fig. 3, \(H\) sweeps). Great care was taken to avoid systematic errors by verifying the linear power dependence of the signal and by avoiding field hysteretic effects that may mix \(k_{xx}\) and \(k_{XY}\) (Extended Data Figs. 3 and 4 and Supplementary Discussion). Data from \(H\) and \(T\) sweeps are indeed
consistent with each other within the given error bars, which arise from random noise in thermometry (Fig. 2, black points, and Fig. 3, thick lines). Repeated measurements were performed over multiple months to eliminate the possibility of dependence on contact geometry or sample aging and produce a fully consistent dataset.

As shown in Figs. 2 and 3, $k_{\text{xy}}/T$ is larger than the half-quantized value $k_{\text{100}}/T = 0.87 \text{ mW K}^{-2} \text{m}^{-1}$ at high temperatures and at high magnetic fields (above $T = 6.5 \text{ K}$ and $H_b = 7.1 \text{ T}$) and decreases to $k_{\text{100}}/T$ and below by lowering $T$ and $H_b$, giving rise to a region with $k_{\text{xy}}/T \approx k_{\text{100}}/T$ on the $H_b-T$ plane. A colour plot of $k_{\text{xy}}/T$ across the whole phase diagram (Fig. 4) combines data from all the $H$ and $T$ sweeps, which maps out the region where $k_{\text{xy}}/T$ is within ±20% of $k_{\text{100}}/T$ (white region). We see an L-shaped white

Fig. 2 | Temperature dependence of thermal Hall effect $k_{\text{xy}}/T$. Temperature-dependent $k_{\text{xy}}/T$ data in order of increasing in-plane magnetic field from 7.6 T (top left) to 13.2 T (bottom right). The independent results of temperature sweeps at a fixed field (coloured points) and field sweeps at a fixed temperature (black points) are plotted together to demonstrate the high degree of consistency. From 10.3 T onwards, a kink is visible at around 6.5 K (red arrows); beyond this field, the low-temperature magnitude of $k_{\text{xy}}/T$ is gradually enhanced towards the half-quantized value $k_{\text{100}}/T$ (dashed line). The grey-shaded areas emphasize the temperature range below 6.5 K. The error bars represent one standard deviation.

Fig. 3 | Magnetic-field dependence of thermal Hall effect $k_{\text{xy}}/T$. a. Selection of $k_{\text{xy}}/T$ isotherms as a function of in-plane magnetic field from 250 mK to 8.5 K. A strong, complex dependence on $T$ and $H_b$ is observed, including enhancement at the highest fields towards the half-quantized value $k_{\text{100}}/T$ (dashed line), suppression on cooling at lower fields and non-monotonic structure for $0.25 < T < 4 \text{ K}$. The broad shaded lines indicate trends in the interpolated temperature sweeps at a fixed field (Fig. 2) to demonstrate consistency. b–e. Extended selection of $k_{\text{xy}}/T$ isotherms in order of decreasing temperature: 5.6–8.5 K (b), 3.5–5.6 K (c), 2.0–3.5 K (d) and 0.25–2.0 K (e). The error bars represent one standard deviation.
Fig. 4 | Overview of thermal Hall effect $k_{\text{th}}/T$ on the $H_{\parallel}$–$T$ plane.

Interpolated magnitude of $k_{\text{th}}/T$ normalized by $k_{\text{th}}/T$ across the $H_{\parallel}$–$T$ phase diagram. In the colour scheme, white represents a thermal Hall conductivity within 20% of the half-quantized value. The grey lines trace the antiferromagnetic phase transitions (for reference). The two low-temperature minima in $k_{\text{th}}/T$ at $H_{\parallel1}$ and $H_{\parallel2}$ are indicated by arrows.

Low-temperature suppression of $k_{\text{th}}/T$. What is the origin of the rapid suppression of $k_{\text{th}}/T$ to zero at low temperatures below 12 T (Fig. 4, red-coloured region)? Within this red region, weak structures are observed: three vertical streaks of minima (deeper red) at around $H_{\parallel}\approx 7.0, 9.5$ and $11.0$ T and two vertical streaks of maxima (brighter red) at around $H_{\parallel}\approx 8.5$ and $10.0$ T branching out from the white region at $T\approx 6.5$ K. Nevertheless, because of the line character of the region of $k_{\text{th}}/T$ at $H_{\parallel1} \approx k_{\text{th}}/T$, the white region (Fig. 4) below $H_{\parallel1} \approx 10$ T is probably due to accidental crossing.

If a half-quantized thermal Hall plateau arises from a chiral Majorana edge mode, it was theoretically shown that the coupling between a phonon bath and the edge mode is necessary to observe the plateau in the presence of dominant phonon conductivity. Phonons must be in the diffuse rather than the ballistic scattering regime. If phonons experience a crossover from the diffuse to the ballistic regimes on cooling or under a magnetic field, the thermal Hall signal from the Majorana edge mode should vanish due to the decoupling of phonons and edge modes, which might account for the low-temperature suppression of $k_{\text{th}}/T$. From the estimation of the phonon mean free path ($l_{\text{ph}}$) from $k_{\text{th}}$, as a function of $H_{\parallel}$ and $T$, however, we may exclude the phonon–edge mode decoupling scenario as the origin of low-temperature suppression of $k_{\text{th}}/T$ below $k_{\text{th}}/T$ from $7$ to $12$ T.

$k_{\text{XX}}$ approaches a $T^0$ power law at the highest fields and lowest temperatures (Fig. 1f), consistent with phonon transport limited by a temperature-independent scattering length. The calculated $l_{\text{ph}}$ reaches ~50 μm at the lowest temperatures, which is somewhat smaller than the sample width of ~1 mm (Fig. 1f and Extended Data Fig. 7), meaning that phonon transport at low temperatures is still marginally in the diffuse regime. Furthermore, the fact that the regions of suppressed $k_{\text{th}}/T$ coincide with the minima in $k_{\text{th}}$, that is, regions with the shortest $l_{\text{ph}}$, argues against the decoupling scenario. The low-temperature enhancement of $k_{\text{th}}/T$ to $k_{\text{th}}/T$ on increasing the magnetic field is accompanied by an increase in $l_{\text{ph}}$, the opposite of what would be expected from phonon–edge mode decoupling. We also note that the decoupling scenario appears to be contrary to a report in which the half-quantized plateau is only observed in samples with high (phonon) thermal conductivity.

Recently, the scenario of purely phonon-driven thermal Hall was proposed based on the discovery of a large thermal Hall effect in non-magnetic SrTiO$_3$, and the correlation between the $T$ dependence of $k_{\text{XX}}$ and $k_{\text{th}}$ in RuCl$_3$ (ref. 23). In addition, in the magnetic insulator Ba$_2$Cu$_3$Sb$_2$O$_{9}$, a thermal Hall angle ($\tan(\theta_H)\approx 10^{-2}$), similar to that of RuCl$_3$, was claimed to arise from the phonon thermal Hall effect alone. The observed suppression of $k_{\text{th}}/T$ at low temperatures and its correlation with $k_{\text{XX}}$ are in line with a phonon-driven scenario. Both show an overall increase on increasing the field, with dip-like suppressions at $H_{\parallel1}$ and $H_{\parallel2}$. Although a phonon–only scenario could explain these correlations between $k_{\text{XX}}$ and $k_{\text{th}}$, it is not clear how it would naturally explain the observed plateau-like behaviour of $k_{\text{th}}/T$ close to $k_{\text{th}}/T$.

The identification of possible topological phase transitions into and out of the half-quantized state still remains an open question. For the former, unveiling the nature of high-field different sample-mounting geometries (Extended Data Fig. 3) and different sequences of data acquisition ($H$ and $T$ sweeps). Recent studies indeed report the region of $k_{\text{th}}/T\approx k_{\text{th}}/T$ at higher fields than the original study, although the onset field is also higher than the original study and present study. Sample dependence is beyond the scope of this study but should be carefully checked in the future.

The most intriguing feature in Fig. 4 is the notable broadening of the white region at low temperatures with an increasing field above ~10 T, giving rise to a large, triangular white area that extends down to at least 2 K at 13 T. Does the observation of a region with $k_{\text{th}}/T\approx k_{\text{th}}/T$ on the $H_{\parallel}$–$T$ plane have a physical implication and support the presence of the topologically protected half-quantized plateau? The enhancement of $k_{\text{th}}/T$ could have different origins, for instance, through (non-quantized) topological magnon or phonon transport. In such scenarios, a crossing may give rise to an accidental $k_{\text{th}}/T\approx k_{\text{th}}/T$ line on the $H_{\parallel}$–$T$ plane. In contrast, a quantized, topologically protected $k_{\text{th}}/T\approx k_{\text{th}}/T$ plateau should exhibit insensitivity to changes in both magnetic field and temperature across and in the extended area in $H_{\parallel}$ and $T$.

The extended nature of $k_{\text{th}}/T$ ($H_{\parallel}$) at $k_{\text{th}}/T$ above 10 T represented by the white triangular plane in Fig. 4 argues against an accidental crossing scenario and hints at a half-quantized plateau and hence a Majorana edge state in α-RuCl$_3$. The corresponding $T$-sweep data (Fig. 2) shows that for fields greater than 10.3 T, a kink-like singularity in the $T$ dependence of $k_{\text{th}}/T$ appears at ~6.5 K (Fig. 2, red arrows). Notably, at the kink, $k_{\text{th}}/T$ is always ~$k_{\text{th}}/T$, implying that the magnitude of $k_{\text{th}}/T$ has special importance in the system. On lowering $T$, the kink is followed by an almost $T$-independent $k_{\text{th}}/T\approx k_{\text{th}}/T$ region, reminiscent of an incipient half-quantized $T$ plateau and then by a rapid decrease to zero. The width of the $T$-plateau-like region gradually expands on increasing the field from 10.3 T, giving rise to the white triangle in Fig. 4.

The narrow L-shaped white region below $H_{\parallel}\approx 10$ T (Fig. 4), on the other hand, represents a ‘line’ of $k_{\text{th}}/T$ ($H_{\parallel}$) at $k_{\text{th}}/T$, which is clearly demonstrated by the $T$ and $H$ sweeps in Figs. 2 and 3. Along the vertical white region in Fig. 4, $k_{\text{th}}/T$ is smoothly suppressed on cooling at 7.6 T (Fig. 2) and continuously decreases through $k_{\text{th}}/T$ with decreasing $H_{\parallel}$ in the $H$ sweeps (Fig. 3). Along the horizontal white region at around 6.5 K, $k_{\text{th}}/T$ shows a plateau in the $H$ sweep at 6.5 K (Fig. 3), but continuously decreases through $k_{\text{th}}/T$ in the $T$ sweeps up to ~10 T. It is not obvious why $k_{\text{th}}/T$ crosses $k_{\text{th}}/T$ at a constant temperature of 6.5 K. Nevertheless, because of the line character of the region of $k_{\text{th}}/T$ ($H_{\parallel}$) at $k_{\text{th}}/T$, the white region (Fig. 4) below $H_{\parallel}\approx 10$ T is probably due to accidental crossing.

At $H_{\parallel1}=7.0, 9.5$ and $11.0$ T, the $k_{\text{th}}/T$ plateau and then by a rapid decrease to zero. The $k_{\text{th}}/T$ plateau approaches a $T^0$ power law at the highest fields and lowest temperatures (Fig. 1f), consistent with phonon transport limited by a temperature-independent scattering length. The calculated $l_{\text{ph}}$ reaches ~50 μm at the lowest temperatures, which is somewhat smaller than the sample width of ~1 mm (Fig. 1f and Extended Data Fig. 7), meaning that phonon transport at low temperatures is still marginally in the diffuse regime. Furthermore, the fact that the regions of suppressed $k_{\text{th}}/T$ coincide with the minima in $k_{\text{th}}$, that is, regions with the shortest $l_{\text{ph}}$, argues against the decoupling scenario. The low-temperature enhancement of $k_{\text{th}}/T$ to $k_{\text{th}}/T$ on increasing the magnetic field is accompanied by an increase in $l_{\text{ph}}$, the opposite of what would be expected from phonon–edge mode decoupling. We also note that the decoupling scenario appears to be contrary to a report in which the half-quantized plateau is only observed in samples with high (phonon) thermal conductivity.

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anomalies may be the key question to be addressed. The high-field transition out of the half-quantized state, if it exists, must be at a field higher than 13 T. A measurement of \( k_{xy}/T \) to much higher fields is, therefore, highly desired. Alternative mechanisms leading to an enhancement of \( k_{xy}/T \), such as a topological magnon-driven or phonon-driven Hall effect, have been proposed. Our results do not exclude those scenarios but pose them a challenge to explain the plateau-like behaviour close to half-quantization.

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Methods

Samples. Large, thin (~2.5 mm × 1.3 mm × 17 μm), high-quality single crystals of α-RuCl₃ from the same growth batches as those reported in ref. were measured. The sample under investigation for thermal Hall measurements has a confirmed half-quantized Hall plateau in that report, where it was labelled as ‘sample 2’. A second sample with a similarly high thermal conductivity was used for magnetic susceptibility measurements.

Thermal conductivity was measured using a steady-state three-thermometer setup. For the temperature range of 1.8–9.0 K, Cernox CX-1050 chip thermometers (Lake Shore) were used in a ⁴He cryostat. For the temperature range of 150 mK to 3 K, ruthenium oxide chip thermometers were used in a dilution fridge. In each case, thermometers were calibrated in situ against a field-calibrated reference thermometer. A 10 kΩ NiCr resistive heater was used to apply heater power. The sample was free standing with its ‘cold’ edge mounted with Apiezon N grease onto a LiF single crystal, which was attached with silver paint onto a copper mount and cooled by the cryostat. Several copper mounts machined at different angles were used to allow for measurements in fixed tilted fields.

Geometrical error affects the measurement due to the finite contact sizes and uncertainty in determining the exact crystal dimensions. We estimate the resulting systematic uncertainty in $k_{XX}/T$ to be around 10%, which is in addition to the random error due to thermometry noise (error bars shown in Figs. 2 and 3).

We adopt the usual convention that the crystal $a$ axis is perpendicular to Ru–Ru bonds, the $b$ axis lies along the Ru–Ru bonds and the $c$ axis is perpendicular to the honeycomb plane. The magnetic field was applied at either 70° from the $c$ axis (⁴He) or 90° from the $c$ axis (dilution refrigerator), with the in-plane field component in the $a$ axis. The amplitude of $k_{XX}$, as well as the measured transition fields ($H_{C1}$, $H_{C2}$, $H_{D1}$ and $H_{D2}$), were confirmed to reproduce at the same values of $H_{||}$ for these two field angles (Extended Data Fig. 6); therefore, the two datasets are presented as one. The field angle was determined under an optical microscope with 1° precision. Heat was always applied along the $a$ axis.

Over the course of these measurements, the crystal was thermally cycled between room temperature and <4.2 K for 14 times; each time, identical values of $k_{XX}$ were measured, indicating that sample deterioration over time is negligible. The measurement of $k_{XX}$ is additionally in very close agreement with that previously measured in Kyoto (Extended Data Fig. 5).

Magnetization measurements above 2 K were performed using a Physical Property Measurement System vibrating sample magnetometry option (Quantum Design), and below 1 K using a custom-built Faraday force magnetometer on a dilution refrigerator (Extended Data Fig. 1). For all the magnetization data, the applied field was parallel to the crystal $a$ axis.

Data availability

The data that support the findings of this study are available from the corresponding authors upon reasonable request. Source data are provided with this paper.

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Author contributions

J.A.N.B., R.R.C., Y.M. and H. Takagi conceived the research. N.K. and H. Tanaka synthesized the single crystals. J.A.N.B. and R.R.C. designed and performed the thermal conductivity experiments. Y.M. designed and performed the magnetic susceptibility experiments. J.A.N.B., R.R.C., Y.M. and H. Takagi analysed the data and participated in the writing of the paper. All the authors contributed to the manuscript preparation.

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Competing interests

The authors declare no competing interests.

Additional information

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Extended Data Fig. 1 | Magnetization isotherms and phase transitions. Field dependences of $dM/dH$ which were used to construct the phase diagram in Fig. 1a (main text). a, Isotherms of $dM/dH$, with every curve offset by 0.003 emu/mol for clarity. Phase transitions out of the antiferromagnetic phases $H_{C1}$, $H_{C2}$ are identified by peaks in $dM/dH$, whereas $H_P$ coincides with a drop in $dM/dH$. b, Isotherms of $d^2M/dH^2$, where the feature at $H_P$ is identified as a local minimum. $H_P$ shifts only minimally in field upon heating, and is no longer resolved at 10 K. Curves are offset by $8\times10^{-4}$ emu/mol T for clarity.
Extended Data Fig. 2 | Temperature dependence of high-field features in $k_{xx}$. The amplitude of high-field features in $k_{xx}$ is expressed as $(k_{xx} - k_{bg})/k_{bg}$, where $k_{bg}$ is a smooth background fit which passes through the points where $k_{xx}$ has greatest field derivative (as discussed in Ref. 22). a, Field dependence $(k_{xx} - k_{bg})/k_{bg}$ for temperatures between 70 mK (dark blue) and 2.5 K (dark red). b, Temperature dependence of $(k_{xx} - k_{bg})/k_{bg}$ at the local minimum at ~8.6 T and the local maximum at ~9.7 T, indicated by arrows in panel a. The amplitude at constant field (dashed lines), and the amplitudes at the fields of maximum amplitude (solid lines and markers) differ slightly due to a temperature-dependent phase shift. The amplitudes initially increase upon cooling but peak around 1 K and then rapidly collapse to zero down to the lowest measured temperature, in marked contrast to the Lifshitz-Kosevitch behavior expected for conventional quantum oscillations.
Extended Data Fig. 3 | Excluding error arising from Hall contact offset. a,b, photographs of sample 2 with wires attached in different configurations 1 and 2. The latter configuration was the one used for the measurements shown in the main text. Purple dashed line are guides to the eye to help identify the contact offset. c, measured Hall contact offset expressed as $\Delta T_y/\Delta T_x$ (%) at 0 T, showing the expected inverse offsets for the two configurations. In the absence of an offset, $\Delta T_y/\Delta T_x$ would be zero (dashed line). d, isotherms of $k_{XY}/T$ at 3.0 K for both contact configurations, displaying the same features at $H_{D1}$ and $H_{D2}$. e, isotherms of $k_{XY}/T$ for configuration 1 at multiple temperatures, demonstrating the reproducibility of all the main features observed with configuration 2 (main text, Fig. 3).
Extended Data Fig. 4 | Excluding power dependence in $k_{\text{xy}}$. Repeated measurements at different levels of heater power demonstrate that the measured thermal conductivity and thermal Hall conductivity are independent of the applied heater power. 

a, Hall temperature gradient normalized by heater power ($\Delta T_y/P$) for two independent isotherms at 3 K, one of which was performed at half the standard heater power (red markers, $\Delta T_y/T \approx 5\%$). The two curves overlap as expected. 

b, the temperature dependence of $k_{\text{xy}}/T$ at 8.64 T is compared for two $T$ sweeps, one of which was performed at half power (green markers, $\Delta T_y/T \approx 5\%$), together with full-power $H$ sweeps. All curves overlap within error. 

c, temperature dependence of $k_{\text{xy}}/T$ is compared for two $T$ sweeps at 13.2 T, one of which was performed at double the usual power ($\Delta T_y/T \approx 20\%$), together with normal-power $H$ sweeps. The data overlap within the (substantially field-enhanced) error bars.
Extended Data Fig. 5 | Reproducibility of thermal conductivity measurements. Comparison of three independent measurements of the thermal conductivity ($\kappa_{xx}$) of sample 2. Red circles: data measured in Kyoto. Solid blue circles: data taken in a dilution refrigerator in Stuttgart, open blue circles: data taken in a 4He flow cryostat in Stuttgart. Each measurement had slightly modified contact placements.
Extended Data Fig. 6 | Temperature and magnetic field-angle dependence of the thermal conductivity above 2.0 K. a, Field dependence of $\kappa_{XX}$ up to 12 T for the temperature range 2.0 K – 8.6 K. The sharp features in the field dependence of $\kappa_{XX}$ seen at low temperature rapidly disappear upon heating above 3.0 K. At high temperatures, the field dependence displays a single, broad minimum around 7 T. At 8.6 K, which is above the ordering temperature $T_N = 7.5$ K, the field dependence is dramatically weakened, although a high field upturn persists at this temperature. b, Comparison of the field dependence of $\kappa_{XX}$ at 2.0 K and 2.5 K, at field angles of 70° (markers) and 90° (lines) with respect to the c-axis. The near-perfect overlap of the data sets demonstrates the insensitivity to magnetic field angle.
Extended Data Fig. 7 | Calculated phonon mean free path across the $H_yT$ phase diagram. Contour plot of the calculated phonon mean free path $l_{\text{ph}} = \frac{3 k_{\text{xx}}}{C v}$, where $C$ is the phonon specific heat per unit volume and $v$ is the average phonon velocity. We assume a magnetic field-independent low temperature phonon specific heat with magnitude $C_{\text{ph}}/T^3 = 1.22 \text{ mJ mol}^{-1}\text{K}^{-4}$ as reported in Ref. 24. The phonon velocity is estimated using the Debye relation $v = \left( \frac{2\pi^2 k_B^3}{3 p B} \right)^{1/3} \approx 1700\text{m/s}$ where $\beta = C_{\text{ph}}/T^3$ as before. The grey lines trace the antiferromagnetic phase transitions, for reference.