Noise Thermometry with Two Weakly Coupled Bose-Einstein Condensates

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Here we report on the experimental investigation of thermally induced fluctuations of the relative phase between two Bose-Einstein condensates which are coupled via tunneling. The experimental control over the coupling strength and the temperature of the thermal background allows for the quantitative analysis of the phase fluctuations. Furthermore, we demonstrate the application of these measurements for thermometry in a regime where standard methods fail. With this we confirm that the heat capacity of an ideal Bose gas deviates from that of a classical gas as predicted by the third law of thermodynamics.

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The generation of two independent matter-wave packets by splitting a single Bose-Einstein condensate (BEC) is a well established technique [1, 2, 3] in the field of atom optics. New phenomena arise if the two separated parts can still coherently interact in analogy to Josephson junctions in condensed matter physics [4] and superfluid Helium Josephson weak links [5]. An advantage of the realization of weakly coupled BEC in a double-well potential [6] is the possibility to observe the phase difference between the two macroscopic wave functions directly. Our experimental investigation of this relative phase reveals that it is not locked to zero but exhibits fluctuations. Two fundamental types of fluctuations are discussed in the literature, quantum fluctuations [7] and thermally induced fluctuations [8]. In this letter we report on the experimental investigation of thermal fluctuations of the relative phase arising from the interaction of the BEC with its thermal environment, which is always present.

The essential prerequisite for the investigation of these thermally induced phase fluctuations is the ability to prepare a BEC adiabatically in a symmetric double-well potential and to adjust its temperature. In our experiments this is achieved by splitting a single $^{87}\text{Rb}$ BEC produced and trapped in an optical dipole trap by slowly ramping up a barrier in the center. The tunneling coupling is adjusted by the barrier height and its strength can be deduced from numerical simulations of the BEC in the trap using the model described in [9]. The temperature of the BEC is adjusted by holding the cloud in the trap, where due to fluctuations of the trap parameters energy is transferred to the atoms. Once the final temperature is reached a standing light wave is ramped up generating a barrier in the center, leading to an effective double-well trapping potential (upper part of Fig. 1a).

When the potential is switched off the matter-wave packets start to expand, overlap and form a double-slit interference pattern which depends on their relative phase as indicated in the lower part of Fig. 1a. Repeating the interference measurements reveals that this relative phase is not constant but fluctuates around zero. The general behavior of these phase fluctuations is connected to two parameters: the temperature of the system randomizing the phase and the tunneling coupling of the two matter-wave packets stabilizing the phase. The results depicted in Fig. 1b show that the phase fluctuations become more pronounced as the temperature is increased since the fluctuations outweigh the stabilizing effects. From this point of view it is expected - and also experimentally observed (Fig. 1c) - that keeping the tem-
perature constant and increasing the tunneling coupling leads to a reduction of the fluctuations. A measure for the fluctuations is the coherence factor $\alpha = \langle \cos \phi \rangle$ which is directly connected to the visibility of the ensemble averaged interference fringes.

Through the barrier averaged interference fringes.

The dynamics can be described in terms of a two mode approximation by assuming weak coupling between the localized modes of the BEC. This corresponds to treating the BEC in the double-well potential as two separated matter-wave packets connected via tunneling through the barrier. In the following we will use the acronym for bosonic Josephson junction (BJJ) to describe this system. Within the two mode approximation the dynamics of the BJJ can be described by two conjugate variables, the atom number difference between the matter-wave packet on the left (l) and on the right (r) $\Delta n = (N_l - N_r)/2$ and their relative phase $\phi = \phi_l - \phi_r$. The Hamiltonian governing the evolution of the two conjugate variables in the limit of small $\Delta n$ is given by

$$H = \frac{E_c}{2} \Delta n^2 - E_j \cdot \cos \phi,$$

(1)

where $E_c$ accounts for the atom-atom interaction in both condensates and $E_j$ is the tunneling coupling energy resulting from the spatial overlap of the wave functions. This Hamiltonian also describes the classical motion of a particle with mass $1/E_c$ and momentum $\Delta n$ at position $\phi$ in a periodic potential. In our experiments with temperatures $T > 10$K the quantum fluctuations are small compared to the thermal fluctuations and therefore are neglected. Their influence can be estimated in the limit of small $\phi$ in which Eq. (1) can be approximated by a harmonic oscillator with the characteristic quantum mechanical energy splitting $\hbar \omega_p = \sqrt{E_c \cdot E_j}$ where $\omega_p$ is the plasma frequency, leading to the quantum mechanical fluctuations of both variables: $\langle \Delta n^2 \rangle \approx \sqrt{E_j/4E_c}$ and $\langle \phi^2 \rangle \approx 4E_c/E_j$.

The system variables $E_j$ and $E_c$ can be calculated from the experimental parameters. The trapping frequencies of the three-dimensional harmonic trap are $\omega_x = 2\pi \cdot 90(2)$Hz and $\omega_{y,z} = 2\pi \cdot 100(2)$Hz. The periodic potential of $V = V_0/2(1 + \cos(2\pi/\lambda \cdot x))$ is realized by the interference of two laser beams at a wavelength of 830nm crossing under an angle of $10^\circ$ resulting in a standing light wave with periodicity of $\lambda = 4.8(2)$µm and is ramped up to a height of $V_0/h = 500$Hz to 2500Hz. The number of atoms in the BEC fraction is chosen to be 2500(500). After the preparation of the BEC in the double-well trap the relative phase of the two matter-wave packets is measured by analyzing the double-slit interference patterns formed after time-of-flight of 5 and 6ms. The visibility of these patterns is reduced due to the short expansion time and the finite optical resolution of the imaging system. Further details of the experimental setup can be found in [10].

The relevant quantities can be calculated from these parameters using the improved two mode model [8]: $E_c/k_B$ is on the order of $20pK$ and $E_j/k_B$ is between $30pK$ and $400nK$ leading to $\hbar \omega_p/k_B$ between $25pK$ and $3nK$. Thus, both necessary conditions for the classical limit are fulfilled: $E_j \gg E_c$ leading to small quantum fluctuations of $\phi$ and $E_c \gg E_j/N$ (where $N$ is the total number of atoms in the BEC) leading to small quantum fluctuations of $\Delta n/N$. Hence, our experiment can be discussed in the classical framework where the
thermally induced phase fluctuations are closely analogous to the Brownian motion of a particle in a sinusoidal potential.

For a quantitative analysis in the thermodynamic limit at $k_B T \gg \hbar \omega_p$ the coherence factor $\bar{\alpha}$ can be calculated by a thermal average assuming a Boltzmann distribution for the relative phases

$$\alpha = \langle \cos \phi \rangle = \frac{\int_{-\pi}^{\pi} d\phi \cdot \cos \phi \cdot \exp(E_j/k_B T \cdot \cos \phi)}{\int_{-\pi}^{\pi} d\phi \cdot \exp(E_j/k_B T \cdot \cos \phi)}.$$  (2)

Eq. (2) points out that the relevant scaling parameter for thermal fluctuations is the ratio between thermal energy $k_B T$ and tunneling coupling energy $E_j$. Fig. 3 shows the experimentally obtained coherence factors as a function of this scaling parameter. Every data point represents on average 40 measurements. In these experiments the temperature of the system is changed between 49nK and 80nK by evaporatively cooling the sample to the lowest temperature and subsequently increasing the temperature by holding the atoms in the trap for different times. The temperature of the sample is measured with the standard time-of-flight expansion method. The tunneling coupling energy is varied between 0.6nK-$k_B$ and 300nK-$k_B$ by adjusting the height of the potential barrier. $E_j$ is obtained from numerical calculations using independently measured trap parameters and atom numbers. It is important to note that the recently developed improved two mode model $\mathcal{E}$ is used for these calculations because it leads to quantitative agreement between theoretical predictions and experimental measurements of dynamical quantities $\mathcal{E}$. The solid line corresponds to the theoretical prediction of the classical model (Eq. (2)) where all parameters are determined independently. It also includes the fitting error of the relative phase which arises from the finite optical resolution and leads to a reduction of the coherence factor. As shown in Fig. 3 the general behavior of the coherence is confirmed over a three orders of magnitude variation of $k_B T/E_j$. These measurements reveal that the BJJ has a higher degree of coherence than expected. This deviation might possibly be explained by an increase of the tunneling coupling resulting from the excitation of transverse modes with higher energies which are neglected by the two mode approximation.

Independent measurements have been performed for the lowest temperatures ($T = 15nK$) to test for thermal equilibration. The measurements of $\alpha$ were compared for different $E_j$ for two ramping schemes. The first scheme was ramping up the barrier in 1.3s and the second scheme was holding the atoms for 1s in the trap and then ramping up the barrier within 0.3s. For $E_j/k_B > 1nK$ both schemes lead within the experimental errors to the same results. Thus for the fluctuation measurements the ramping in 300ms is expected to be adiabatic with respect to the response time of the BJJ given by the inverse plasma frequency and thus ensures the thermal equilibrium.

In the following we present the application of the phase fluctuation measurements for thermometry far below the critical temperature of Bose-Einstein condensation ($T_c$). The temperature of the system can be directly deduced from the variance of the phase if the tunneling coupling is known. In order to apply the phase fluctuation measurements for thermometry we introduce an empirical effective tunneling coupling $E_{j eff}$ to account for effects beyond the classical approach. For the range of 25nK < $E_j/k_B < 90nK$ we deduce from the results shown in Fig. 3 that $E_{j eff} = 1.33 \cdot E_j$. The fundamental difference between this method and previous suggestions using phase fluctuations of elongated Bose-Einstein condensates for thermometry is that the BJJ is not restricted to a quasi one dimensional situation but can be employed for all geometries. Furthermore, this method can be applied for all temperature ranges by tuning $E_c$ and $E_j$ such that thermal effects dominate and quantum fluctuations are negligible.

As a proof of applicability of this new type of thermometer we observe how the temperature of a BEC in a harmonic trap increases in time (see Fig. 4), which reveals clearly the effect of quantum statistics below the critical temperature. In these experiments the lowest temperatures ($T < T_c/3$) can only be measured with the phase fluctuation method since the thermal fraction is too small to be observed in time-of-flight measurements (less than 100 atoms with about 2500 atoms in the BEC).
fraction). For longer heating times the standard time-of-flight method can be applied and confirms the consistency of the two approaches in the overlap region. The solid line corresponds to a fitting function for the temperature where we assume a mean critical temperature of $T_c = 59nK$ (deduced from independent measurements), a temperature independent transfer rate of energy per particle and a power law for the temperature dependent heat capacity $C = (d + 1) \cdot C_{th} \cdot (T/T_c)^d$ where $C_{th}$ is the heat capacity of a classical gas. The shown excellent agreement is obtained for a heating rate of 2.3(2)nK/s for a classical gas and $d = 2.7(6)$. Thus the expected exponent $d = 3$ for an ideal Bose gas in a three-dimensional harmonic trap is experimentally confirmed. The expected increase of temperature of a classical gas is indicated by the dotted line and shows clearly the difference between the quantum and the classical behavior of ideal gases.

In summary, we have presented a quantitative analysis of thermally induced phase fluctuations in a bosonic Josephson junction. Our observations show that a universal scaling law describes the behavior of the coherence and its control leads to new applications. A method is presented for ultra-low temperature measurements, with which we have confirmed that the heat capacity of a degenerate Bose gas vanishes in the zero temperature limit as predicted by the third law of thermodynamics.

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