HUREWICZ THEOREM FOR EXTENSION DIMENSION

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Abstract. We prove a new selection theorem for multivalued mappings of C-space. Using this theorem we prove extension dimensional version of Hurewicz theorem for a closed mapping \( f : X \to Y \) of \( k \)-space \( X \) onto paracompact C-space \( Y \): if for finite CW-complex \( M \) we have \( \text{e-dim} Y \leq [M] \) and for every point \( y \in Y \) and every compactum \( Z \) with \( \text{e-dim} Z \leq [M] \) we have \( \text{e-dim}(f^{-1}(y) \times Z) \leq [L] \) for some CW-complex \( L \), then \( \text{e-dim} X \leq [L] \).

1. Introduction

The classical Hurewicz theorem states that for a mapping of finite-dimensional compacta \( f : X \to Y \) we have

\[
\dim X \leq \dim Y + \dim f, \quad \text{where} \quad \dim f = \max \{\dim(f^{-1}(y)) \mid y \in Y\}.
\]

There are several approaches to extension dimensional generalization of Hurewicz theorem [3],[6],[1],[7],[8],[9].

Using the idea from [3] we improve Theorem 7.6 from [1]:

**Theorem 3.1.** Let \( f : X \to Y \) be a closed mapping of a \( k \)-space \( X \) onto paracompact C-space \( Y \). Suppose that \( \text{e-dim} Y \leq [M] \) for a finite CW-complex \( M \). If for every point \( y \in Y \) and for every compactum \( Z \) with \( \text{e-dim} Z \leq [M] \) we have \( \text{e-dim}(f^{-1}(y) \times Z) \leq [L] \) for some CW-complex \( L \), then \( \text{e-dim} X \leq [L] \).

The notion of extension dimension was introduced by Dranishnikov [4]: for a CW-complex \( L \) a space \( X \) is said to have **extension dimension** \( \leq [L] \) (notation: \( \text{e-dim} X \leq [L] \)) if any mapping of its closed subspace \( A \subset X \) into \( L \) admits an extension to the whole space \( X \).

To prove Theorem 3.1 we need an extension dimensional version of Uspenskij’s selection theorem [11]. In section 2 we prove Theorem 2.8 on selections of multivalued mappings of C-space. Then Theorem 2.9 helps us to prove Theorem 2.9 — a needed version of Uspenskij’s theorem.

Filtrations of multivalued maps are proved to be very useful for construction of continuous selections [10],[1][2]. And we state our selection theorems in terms of

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of filtrations. Note that Valov [12] used filtrations to prove a selection theorem for mappings of finite $C$-spaces.

Let us recall some definitions and introduce our notations. A space $X$ is called a $k$-space if $U \subset X$ is open in $X$ whenever $U \cap C$ is relatively open in $C$ for every compact subset $C$ of $X$. The graph of a multivalued mapping $F: X \to Y$ is the subset $\Gamma_F = \{(x, y) \in X \times Y : y \in F(x)\}$ of the product $X \times Y$.

We denote by $\text{cov}X$ the collection of all coverings of the space $X$. For a cover $\omega$ of a space $X$ and for a subset $A \subseteq X$ let $\text{St}(A, \omega)$ denote the star of the set $A$ with respect to $\omega$. We say that a subset $A \subset X$ refines a cover $\omega \in \text{cov}X$ if $A$ is contained in some element of $\omega$. A covering $\omega' \in \text{cov}X$ strongly star refines a covering $\omega \in \text{cov}X$ if for any element $W \in \omega'$ the set $\text{St}(W, \omega')$ refines $\omega$.

**Definition 1.1.** A topological space $X$ is called $C$-space if for each sequence $\{\omega_i\}_{i \geq 1}$ of open covers of $X$, there is an open cover $\Sigma$ of $X$ of the form $\bigcup_{i=1}^{\infty} \sigma_i$ such that for each $i \geq 1$, $\sigma_i$ is a pairwise disjoint collection which refines $\omega_i$.

If the space $X$ is paracompact, we can choose the cover $\Sigma$ to be locally finite and every collection $\sigma_i$ to be discrete.

**Definition 1.2.** A multivalued mapping $F: X \to Y$ is said to be strongly lower semicontinuous (briefly, strongly l.s.c.) if for any point $x \in X$ and any compact set $K \subset F(x)$ there exists a neighborhood $V$ of $x$ such that $K \subset F(z)$ for every $z \in V$.

**Definition 1.3.** Let $L$ be a $CW$-complex. A pair of spaces $V \subset U$ is said to be $[L]$-connected (resp., $[L]_c$-connected) if for every paracompact space $X$ (resp., compact metric space $X$) of extension dimension $e\dim X \leq [L]$ and for every closed subspace $A \subset X$ any mapping of $A$ into $V$ can be extended to a mapping of $X$ into $U$.

An increasing sequence of subspaces $Z_0 \subset Z_1 \subset \cdots \subset Z$ is called a filtration of space $Z$. A sequence of multivalued mappings $\{F_k : X \to Y\}$ is called a filtration of multivalued mapping $F : X \to Y$ if $\{F_k(x)\}$ is a filtration of $F(x)$ for any $x \in X$.

**Definition 1.4.** A filtration of multivalued mappings $\{G_i : X \to Y\}$ is said to be fiberwise $[L]_c$-connected if for any point $x \in X$ and any $i$ the pair $G_i(x) \subset G_{i+1}(x)$ is $[L]_c$-connected.

### 2. Selection theorems

The following notion of stably $[L]$-connected filtration of multivalued mappings provides a key property of the filtration for our construction of continuous selections.

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1We consider only increasing filtrations indexed by a segment of the integral series.
Definition 2.1. A pair \( F \subset H \) of multivalued mappings from \( X \) to \( Y \) is called stably \([L]\)-connected if every point \( x \in X \) has a neighborhood \( O_x \) such that the pair \( F(\overline{O}_x) \subset \bigcap_{z \in O_x} H(z) \) is \([L]\)-connected.

We say that the pair \( F \subset H \) is called stably \([L]\)-connected with respect to a covering \( \omega \in \text{cov}X \), if for any \( W \in \omega \) the pair \( F(W) \subset \bigcap_{x \in W} H(x) \) is \([L]\)-connected.

A filtration \( \{F_i\} \) of multivalued mappings is called stably \([L]\)-connected if every pair \( F_i \subset F_{i+1} \) is stably \([L]\)-connected.

Clearly, any stably \([L]\)-connected pair of multivalued maps of a space \( X \) is stably \([L]\)-connected with respect to some covering of \( X \).

We denote by \( Q \) the Hilbert cube. We identify a space \( Y \) with the subspace \( Y \times \{0\} \) of the product \( Y \times Q \) and denote by \( \text{pr}_Y \) the projection of \( Y \times Q \) onto \( Y \).

Definition 2.2. For a subspace \( Z \subset Y \times Q \) we say that \( Y \) projectively contains \( Z \). We say that a multivalued mapping \( F: X \to Y \) projectively contains a multivalued mapping \( G: X \to Y \times Q \) if for any point \( x \in X \) the set \( \text{pr}_Y \circ G(x) \) is contained in \( F(x) \).

Lemma 2.3. Let \( L \) be a finite CW-complex. If a topological space \( Y \) contains a compactum \( K \) of extension dimension \( e\text{-dim}K \leq [L] \) such that the pair \( K \subset Y \) is \([L]_c\)-connected, then \( Y \) projectively contains a compactum \( K' \) of extension dimension \( e\text{-dim}K' \leq [L] \) such that \( K \) lies in \( K' \) and the pair \( K \subset K' \) is \([L]\)-connected.

Proof. There exists \( AE([L])\)-compactum \( K' \) of extension dimension \( e\text{-dim}K' \leq [L] \) containing the given compactum \( K \). Clearly, the pair \( K \subset K' \) is \([L]\)-connected. Since \( e\text{-dim}K' \leq [L] \), there exists a mapping \( p: K' \to Y \) extending the inclusion of \( K \) into \( Y \).

It is easy to see that there exists a mapping \( q: K' \to Q \) such that \( q^{-1}(0) = K \) and \( q \) is an embedding on \( K' \setminus K \). Now define an embedding \( j: K' \to Y \times Q \) as \( j = p \times q \). Since \( q^{-1}(0) = K \), the mapping \( j \) coincide with \( p \) on \( K \) which is inclusion on \( K \).

Definition 2.4. We say that a filtration \( F_0 \subset F_1 \subset \ldots \) of multivalued mappings from \( X \) to \( Y \) projectively contains a filtration \( G_0 \subset G_1 \subset \ldots \) of multivalued mappings from \( X \) to \( Y \times Q \) if for any point \( x \in X \) and any \( n \) the set \( \text{pr}_Y \circ G_n(x) \) is contained in \( F_n(x) \).

Theorem 2.5. For a finite CW-complex \( L \) any fiberwise \([L]_c\)-connected filtration of strongly l.s.c. multivalued mappings of paracompact space \( X \) to a topological space \( Y \) projectively contains stably \([L]\)-connected filtration of compact-valued mappings.
Proof. For a given fiberwise $[L]_c$-connected filtration $F_0 \subset F_1 \subset \ldots$ of strongly l.s.c. multivalued mappings we construct stably $[L]$-connected filtration $G_0 \subset G_1 \subset \ldots$ of compact-valued mappings $G_n : X \to Y \times Q^n$ as follows: successively for every $n \geq 0$ we construct a covering $\omega_n = \{W^n_\lambda\}_{\lambda \in \Lambda_n} \in \text{cov}X$ and a family of subcompacta $\{K^n_\lambda\}_{\lambda \in \Lambda_n}$ of $Y \times Q^n$, and define the mapping $G_n$ by the formula

$$G_n(x) = \cup\{K^n_\lambda \mid x \in W^n_\lambda\}.$$ 

First, we construct $G_0$, i.e. the covering $\omega_0$ and the family $\{K^0_\lambda\}_{\lambda \in \Lambda_0}$. Since $F_0$ is strongly l.s.c., there exists a locally finite open covering $\omega_{-1} = \{W^{-1}_\lambda\}_{\lambda \in \Lambda_{-1}} \in \text{cov}X$ and a family $\{M^{-1}_\lambda\}_{\lambda \in \Lambda_{-1}}$ of points in $Y$ such that $W^{-1}_\lambda \times M^{-1}_\lambda \subset \Gamma_{F_0}$ for any $\lambda \in \Lambda_{-1}$. Denote by $H_0$ a multivalued mapping taking a point $x \in X$ to the set $H_0(x) = \cup\{M^{-1}_\lambda \mid x \in W^{-1}_\lambda\}$. Note that $H_0(x)$ is contained in $F_0(x)$ and consists of finitely many points. By Lemma 2.3 for any $x \in X$ there exists a compactum $\hat{H}_0(x) \subset F_1(x) \times Q$ of extension dimension $\text{e-dim}\hat{H}_0(x) \leq [L]$ such that the pair $H_0(x) \subset \hat{H}_0(x)$ is $[L]$-connected. Since $F_1$ is strongly l.s.c., any point $x \in X$ has a neighborhood $\mathcal{O}_0(x)$ such that the product $\mathcal{O}_0(x) \times \hat{H}_0(x)$ is contained in $\Gamma_{F_1} \times Q$. Since $X$ is paracompact, we can choose neighborhoods $\mathcal{O}_0(x)$ in such a way that the covering $\mathcal{O}_0 = \{\mathcal{O}_0(x)\}_{x \in X}$ strongly star refines $\omega_{-1}$. Let $\omega_0 = \{W^0_\lambda\}_{\lambda \in \Lambda_0}$ be a locally finite open cover of $X$ refining $\mathcal{O}_0$. For every $\lambda \in \Lambda_0$ we fix a point $x_\lambda$ such that $W^0_\lambda \subset \mathcal{O}_0(x_\lambda)$ and put $M^0_\lambda = H_0(x_\lambda)$. For every $\lambda \in \Lambda_0$ we fix $\alpha(\lambda) \in \Lambda_{-1}$ such that $\text{St}(W^0_\lambda, \mathcal{O}_0) \subset W^{-1}_{\alpha(\lambda)}$ and put $K^0_\lambda = M^{-1}_{\alpha(\lambda)}$.

Inductive step of our construction is similar to the first step. Suppose that a covering $\omega_{n-1} = \{W^{n-1}_\lambda\}_{\lambda \in \Lambda_{n-1}} \in \text{cov}X$ and a family $\{M^{n-1}_\lambda\}_{\lambda \in \Lambda_{n-1}}$ of compacta in $Y \times Q^{n-1}$ are already constructed such that $\text{e-dim}M^{n-1}_\lambda \leq [L]$ and the product $W^{n-1}_\lambda \times M^{n-1}_\lambda$ is contained in $\Gamma_{F_{n}} \times Q^n$ for any $\lambda \in \Lambda_{n-1}$. Denote by $H_n$ a multivalued mapping taking a point $x \in X$ to the compactum $H_n(x) = \cup\{M^{n-1}_\lambda \mid x \in W^{n-1}_\lambda\}$. Note that $H_n(x)$ is contained in $F_n(x) \times Q^n$ and has extension dimension $\text{e-dim}H_n(x) \leq [L]$. By Lemma 2.3 for any $x \in X$ there exists a compactum $\hat{H}_n(x) \subset F_{n+1}(x) \times Q^{n+1}$ of extension dimension $\text{e-dim}\hat{H}_n(x) \leq [L]$ such that the pair $H_n(x) \subset \hat{H}_n(x)$ is $[L]$-connected. Since $F_{n+1}$ is strongly l.s.c., any point $x \in X$ has a neighborhood $\mathcal{O}_n(x)$ such that the product $\mathcal{O}_n(x) \times \hat{H}_n(x)$ is contained in $\Gamma_{F_{n+1}} \times Q^{n+1}$. Since $X$ is paracompact, we can choose neighborhoods $\mathcal{O}_n(x)$ in such a way that the covering $\mathcal{O}_n = \{\mathcal{O}_n(x)\}_{x \in X}$ strongly star refines $\omega_{n-1}$. Let $\omega_n = \{W^n_\lambda\}_{\lambda \in \Lambda_n}$ be a locally finite open cover of $X$ refining $\mathcal{O}_n$. For every $\lambda \in \Lambda_n$ we fix $\alpha(\lambda) \in \Lambda_{n-1}$ such that $\text{St}(W^n_\lambda, \mathcal{O}_n) \subset W^{-1}_{\alpha(\lambda)}$ and put $K^n_\lambda = M^{-1}_{\alpha(\lambda)}$.

To show that the pair $G_{n-1} \subset G_n$ is stably $[L]$-connected, we prove that the pair $G_{n-1}(W^n_\lambda) \subset \cap\{G_n(x) \mid x \in W^n_\lambda\}$ is $[L]$-connected for any $W^n_\lambda \in \omega_n$. By the construction of $G_n$, the set $K^n_\lambda$ is contained in $\cap\{G_n(x) \mid x \in W^n_\lambda\}$. We
know that the pair $H_{n-1}(x_{\alpha(\lambda)}) \subset \widehat{H}_{n-1}(x_{\alpha(\lambda)}) = M_{n(\alpha)}^{n-1} = K^n_{\lambda}$ is $[L]$-connected. Therefore it is enough to show the following inclusion:

$$G_{n-1}(W^n_\lambda) = \bigcup \{ K^n_{\beta} \ | \ W^n_\lambda \cap W^{n-1}_\beta \neq \emptyset \} \subset \bigcup \{ M^{n-2}_\nu \ | \ x_{\alpha(\lambda)} \in W^{n-2}_\nu \} = H_{n-1}(x_{\alpha(\lambda)})$$

which follows from the fact that $W^n_\lambda \cap W^{n-1}_\beta \neq \emptyset$ implies $x_{\alpha(\lambda)} \in W^{n-2}_\nu$ (note that $M^{n-2}_{\alpha(\beta)} = K^{n-1}_\beta$). By the choice of $\alpha(\lambda)$ we have $W^n_\lambda \subset \mathcal{O}_{n-1}(x_{\alpha(\lambda)})$. Then $W^n_\lambda \cap W^{n-1}_\beta \neq \emptyset$ implies $\mathcal{O}_{n-1}(x_{\alpha(\lambda)}) \cap W^{n-1}_\beta \neq \emptyset$ and $x_{\alpha(\lambda)} \in \mathcal{O}_{n-1}(x_{\alpha(\lambda)}) \subset \text{St}(W^{n-1}_\beta, \mathcal{O}_{n-1}) \subset W^{n-2}_\alpha$.

**Definition 2.6.** For a space $Z$ a pair of spaces $V \subset U$ is said to be $Z$-**connected** if for every closed subspace $A \subset Z$ any mapping of $A$ into $V$ can be extended to a mapping of $Z$ into $U$.

**Definition 2.7.** A pair $F \subset H$ of multivalued mappings from $X$ to $Y$ is called **stably $Z$-connected** if every point $x \in X$ has a neighborhood $O_x$ such that the pair $F(O_x) \subset \cap_{z \in O_x} H(z)$ is $Z$-connected.

We say that the pair $F \subset H$ is called **stably $Z$-connected with respect to a covering $\omega \in \text{cov}X$**, if for any $W \in \omega$ the pair $F(W) \subset \cap_{z \in W} H(x)$ is $Z$-connected.

A filtration $\{F_i\}$ of multivalued mappings is called **stably $Z$-connected** if every pair $F_i \subset F_{i+1}$ is stably $Z$-connected.

**Theorem 2.8.** Let $F : X \to Y$ be a multivalued mapping of paracompact $C$-space $X$ to a topological space $Y$. If $F$ admits infinite stably $X$-connected filtration of multivalued mappings, then $F$ has a singlevalued continuous selection.

**Proof.** Let $\{F_i\}_{i=-1}^{\infty}$ be the given filtration of $F$. Let $\{\omega_i\}_{i=-1}^{\infty}$ be a sequence of coverings of $X$ such that $\omega_{i+1}$ refines $\omega_i$ and the pair $F_i \subset F_{i+1}$ is stably $X$-connected with respect to the covering $\omega_i$. Since $X$ is paracompact $C$-space, there exists a locally finite closed cover $\Sigma$ of $X$ of the form $\Sigma = \cup_{i=0}^{\infty} \sigma_i$ such that $\sigma_i$ is discrete collection refining $\omega_i$. Define $\Sigma_n = \cup_{i=0}^{n} \sigma_i$. We will construct a continuous selection $f$ of $F$ extending it successively over the sets $\Sigma_n$.

First, we construct $f_0 : \Sigma_0 \to Y$. We define $f_0$ separately on every element $s$ of the discrete collection $\sigma_0$: take a point $p \in F_{-1}(s)$ and put $f_0(s) = p$. Since the set $s$ refines $\omega_0$, then $p \in F_0(x)$ for any $x \in s$ and therefore $f_0$ is a selection of $F_0|_{\Sigma_0}$.

Suppose that we already constructed $f_n$ — a continuous selection of $F_n|_{\Sigma_n}$. Let us define $f_{n+1}$ on arbitrary element $Z$ of discrete collection $\sigma_{n+1}$. Since $\Sigma$ is locally finite, the set $A = Z \cap \Sigma_n$ is closed in $X$. Since $f_n$ is a selection of $F_n$, then $f_n(A)$ is contained in $F_n(Z)$. Since the pair $F_n(Z) \subset \cap_{x \in Z} F_{n+1}(x)$ is
X-connected, we can extend \( f_n|_A \) to a mapping \( f_n' : Z \to \bigcap_{x \in Z} F_{n+1}(x) \). Clearly, \( f_n' \) is a selection of \( F_{n+1}|_Z \). We define \( f_{n+1} \) on the set \( Z \) as \( f_n' \).

Finally, we define \( f \) to be equal to \( f_n \) on the set \( \Sigma_n \).

**Theorem 2.9.** Let \( L \) be a finite CW-complex and \( F : X \to Y \) be a multivalued mapping of paracompact \( C \)-space \( X \) of extension dimension \( \text{e-dim} X \leq [L] \) to a topological space \( Y \). If \( F \) admits infinite fiberwise \([L]_c\)-connected filtration of strongly l.s.c. multivalued mappings, then \( F \) has a singlevalued continuous selection.

**Proof.** By Theorem 2.5, the mapping \( F' : X \to Y \times Q \) defined as \( F'(x) = F(x) \times Q \) contains a stably \([L]\)-connected filtration of multivalued mappings. By Theorem 2.8, \( F' \) has a singlevalued continuous selection \( f' \). Then the mapping \( f = \text{pr}_Y \circ f' \) is a singlevalued continuous selection of \( F \).

### 3. Hurewicz theorem

The proof of the following theorem is similar to the proof of Theorem 2.4 from [3].

**Theorem 3.1.** Let \( f : X \to Y \) be a closed mapping of \( k \)-space \( X \) onto paracompact \( C \)-space \( Y \). Suppose that \( \text{e-dim} Y \leq [M] \) for a finite CW-complex \( M \). If for every point \( y \in Y \) and for every compactum \( Z \) with \( \text{e-dim} Z \leq [M] \) we have \( \text{e-dim}(f^{-1}(y) \times Z) \leq [L] \) for some CW-complex \( L \), then \( \text{e-dim} X \leq [L] \).

**Proof.** Suppose \( A \subseteq X \) is closed and \( g : A \to L \) is a map. We are going to find a continuous extension \( \tilde{g} : X \to L \) of \( g \). Let \( K \) be the cone over \( L \) with a vertex \( v \). We denote by \( C(X, K) \) the space of all continuous maps from \( X \) to \( K \) equipped with the compact-open topology. We define a multivalued map \( F : Y \to C(X, K) \) as follows:

\[
F(y) = \{ h \in C(X, K) \mid h(f^{-1}(y)) \subseteq K \setminus \{ v \} \text{ and } h|_{A} = g \}.
\]

**Claim.** \( F \) admits continuous singlevalued selection.

If \( \varphi : Y \to C(X, K) \) is a continuous selection for \( F \), then the mapping \( h : X \to K \) defined by \( h(x) = \varphi(f(x))(x) \) is continuous on every compact subset of \( X \) and because \( X \) is a \( k \)-space, \( h \) is continuous. Since \( \varphi(f(x)) \in F(f(x)) \) for every \( x \in X \), we have \( h(X) \subseteq K \setminus \{ v \} \). Now if \( \pi : K \setminus \{ v \} \to L \) denotes the natural retraction, then \( \tilde{g} = \pi \circ h : X \to L \) is the desired continuous extension of \( h \).

**Proof of the claim.** We are going to apply Theorem 2.9 to infinite filtration \( F \subset F \subset F \subset \ldots \). To do this, we have to show that \( F \) is strongly l.s.c. and that the pair \( F(y) \subset F(y) \) is \([M]_c\)-connected for every point \( y \in Y \).

First, we show that \( F \) is strongly l.s.c. Let \( y_0 \in Y \) and \( P \subset F(y_0) \) be compact. We have to find a neighborhood \( V \) of \( y_0 \) in \( Y \) such that \( P \subset F(y) \) for every \( y \in V \). For every \( x \in X \) define a subset \( P(x) = \{ h(x) \mid h \in P \} \) of \( K \). Since \( P \subset C(X, K) \) is compact and \( X \) is a \( k \)-space, by the Ascoli theorem, each
$P(x)$ is compact and $P$ is evenly continuous. This easily implies that the set $W = \{ x \in X \mid P(x) \subset K \setminus \{v\} \}$ is open in $X$ and, obviously, $f^{-1}(y_0) \subset W$. Since $f$ is closed, there exists a neighborhood $V$ of $y_0$ in $Y$ with $f^{-1}(V) \subset W$. Then, according to the choice of $W$ and the definition of $F$, we have $P \subset F(y)$ for every $y \in V$.

Fix an arbitrary point $y \in Y$. Let us prove that the pair $F(y) \subset F(y)$ is $[M]$-connected. Consider a pair of compacta $B \subset Z$ where $e\text{dim}Z \leq [M]$ and a mapping $\varphi : B \to F(y)$. Since $B \times X$ is a $k$-space (as a product of a compact space and a $k$-space), the map $\psi : B \times X \to K$ defined as $\psi(b, x) = \varphi(b)(x)$ is continuous. Extend $\psi$ to a set $Z \times A$ letting $\psi(z, a) = g(a)$. Clearly, $\psi$ takes the set $Z \times f^{-1}(y) \cap (Z \times A \cup B \times X)$ into $K \setminus \{v\} \cong L \times [0, 1)$. Since $e\text{dim}(Z \times f^{-1}(y)) \leq [L]$, we can extend $\psi$ over the set $Z \times f^{-1}(y)$ to take it into $K \setminus \{v\}$. Finally extend $\psi$ over $Z \times X$ as a mapping into $AE$-space $K$. Now define an extension $\tilde{\varphi} : Z \to F(y)$ of the mapping $\varphi$ by the formula $\tilde{\varphi}(z)(x) = \psi(z, x)$.

Corollary 3.2 (cf. Theorem 2.25 from [6]). Let $f : X \to Y$ be a mapping of finite-dimensional compacta where $e\text{dim}Y = [M]$ for finite CW-complex $M$. If for some CW-complex $L$ we have $e\text{dim}(f^{-1}(y) \times Y) \leq [L]$ for every point $y \in Y$, then $e\text{dim}X \leq [L]$.

Proof. By Theorem 6.3 from [6] for any compactum $Z$ with $e\text{dim}Z \leq e\text{dim}Y$ we have $e\text{dim}(f^{-1}(y) \times Z) \leq [L]$. Thus, we can apply Theorem 3.1.

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