DETECTABILITY OF GRAVITATIONAL RADIATION FROM PROMPT AND DELAYED STAR COLLAPSE TO A BLACK HOLE

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We consider the emission of gravitational waves in the two proposed models for the collapse of a massive star to a black hole: the prompt collapse, in which nearly all the star collapses to a black hole in a dynamical time scale, and the delayed collapse, in which a light black hole, or a neutron star, which subsequently accretes matter, forms due to the fall-back achieving, in the neutron star case, the critical mass for black hole formation. Recent simulations strongly support this last scenario. We show that, due to the slowness of fall-back, in the delayed collapse the main burst of gravitational radiation is emitted depending on the parameters, mass and angular momentum, of the initial, light, black hole. We estimate, under different assumptions, the detectability of the emitted gravitational waves showing that such kind of collapse is not particularly suited for detection by forthcoming interferometric detectors. Detectors with high sensitivity at frequencies greater than $\sim 4 \div 5 \text{ kHz}$ would be better suited for this kind of sources. We calculate also the final mass distribution function of single black holes.

1. Introduction

Gravitational collapse of a stellar core to a black hole has been studied since many years. Efforts have been also devoted to the calculation of the gravitational radiation emitted in this process. Most studies have been based on a perturbative approach, others on the numerical solution of the full Einstein equations. All these papers consider the collapse of a “naked” stellar core, described by a “dust” of particles or, at most, by a polotropic equation of state, without taking into account the presence of the outer layers of the star, which are involved in the process. In particular, depending on the ratio between the energy released in the final explosion of a massive star and the binding energy of the ejected material, two different kinds of collapse have been outlined: the prompt collapse, in which a large fraction of the star collapses on a dynamical time-scale forming a massive black hole, and the delayed collapse, in which a low mass black hole or, alternatively, a neutron star forms at the beginning and later accretes matter, due to fall-back. If a neutron star is the initial outcome of the collapse, fall-back pushes its mass above the critical mass and the formation of a black hole takes place. In both cases, this light black hole continues to slowly accrete matter until the final mass is reached.

In the gravitational wave community the prompt collapse has been considered
for a long time as representative of realistic collapse processes. The estimated mass (several solar masses) of the first black hole candidates (like Cygnus X−1) has led to the idea that black holes should often be born with a "typical" mass of \( \sim 10M_\odot \). This assumption appears no more justified now: we know that the evolution of massive stars in binary systems (to which all observed black hole candidates belong) is different from that of single stars; in addition, more refined observational techniques have allowed to find black hole candidates of few solar masses. In the light of these results and of the recent numerical simulations which we will describe, the prompt birth of such massive black holes (\( \sim 10M_\odot \) or more) should be considered as a very rare event.

In this paper, we will discuss the delayed collapse model, the expected mass distribution of isolated black holes and the detectability of the emitted signal, by forthcoming interferometric detectors. Our main aim is to understand how our perspectives of detection change with respect to the "naive" prompt collapse. The plan of the paper is as follows. In Sec.2 we will shortly describe the delayed collapse scenario. In Sec.3 we will estimate the contribution of fall-back to the total gravitational emission in the delayed collapse discussing the consequences. In Sec.4 we will discuss the detectability of the emitted signal by forthcoming interferometric detectors, and compare with the prompt collapse which could happen for progenitor stars above about 40\( M_\odot \), if stellar winds were much less important in the evolution of massive stars than it is currently believed. In Sec.5 we will derive the theoretical final mass distribution function for isolated black holes, using the results of recent simulations of the collapse of massive single stars. Finally, in Sec.6 the results and their implications will be discussed.

2. Star Collapse to a Black Hole

The fate of a massive star (\( m_{\text{prog}} > 9 \ M_\odot \)) is the core collapse with the formation of a compact object: a neutron star or a black hole. Numerical simulations show that the actual final product, and also the way in which it is formed, depend on many factors, among which the mass and the angular momentum of the progenitor, the explosion energy, the high density matter equation of state (EOS) and also the way in which the physics is implemented (regarding, for instance, neutrino physics or angular momentum transport). Moreover, results cannot be considered conclusive until fully relativistic 3D simulations will be performed. It has been shown that, for a wide range of progenitor masses and explosion energies, the shock cannot expell all the matter outside the collapsing core, so that part of the helium mantle and heavy elements may slow down below the escape velocity and be accreted by the just formed neutron star, with a timescale of minutes to hours. If the neutron star mass grows above a critical value, a black hole forms and we have a delayed collapse. On the other hand, if the explosion completely fails, or if it is too weak, a black hole immediately forms: this is the prompt collapse. According to recent simulations (starting from non-rotating progenitors) by Woosley & Weaver and by Fryer &
Kalogera, typical collapses are always delayed, the prompt ones occurring only for very massive progenitor stars \((M > 40 M_\odot)\) if stellar winds are negligible, an assumption which appears rather unlikely. The EOS of high density matter plays a basic role in the determination of neutron stars limiting mass, i.e. the critical mass for black hole formation, \(m_{\text{min}}\). For conventional EOS the neutron star maximum mass ranges between \(1.7 M_\odot\) and \(2.2 M_\odot\) (e.g. \([4, 8, 10, 14]\)), with a 10÷20% increase if rotation is taken into account. On the other hand, if \(\pi\) or \(K\) condensation, or formation of quark matter, takes place at very high densities (e.g. \([18, 19, 22]\)), the EOS is softened and this reduces the maximum mass that the pressure of degenerate matter can sustain. In this case, the critical neutron star mass is \(\sim 1.5 M_\odot\). The masses of 26 neutron stars, measured in pulsars, are compatible with a gaussian distribution with \(\overline{m} = 1.35 \pm 0.04\), and then are consistent with that previous limit \([21]\). Moreover, the lack of evidence for neutron stars with mass near the maximum derived from conventional EOS is reinforced by the lack of evidence for a pulsar as a remnant of the supernova SN1987 A whose progenitor had a mass of \(\sim 18 M_\odot\), which should have left behind a remnant mass of about \(1.5 M_\odot\) \([14, 22]\). However, no definite conclusion can be still drawn \([23, 24]\). Given the uncertainties in the neutron stars equation of state, in the following we will always refer to two different values of the black hole minimum mass: \(m_{\text{min}} = 1.5 M_\odot\), representative of soft equations of state (soft EOS), and \(m_{\text{min}} = 2M_\odot\) for standard, with no phase transition, equations of state (conventional EOS).

3. Gravitational Radiation from Fall-Back

In this section, we want to estimate the contribution of fall-back to the emission of gravitational waves in the delayed collapse of a massive star to a black hole.

There are many studies on the capture by a black hole of finite-size shells of matter \([6, 9, 10]\). A general result is that the amount of gravitational radiation emitted is always smaller than that emitted by the capture of a pointlike particle with the same mass of the shell. This is a consequence of destructive interference of the radiation emitted by different parts of the infalling extended matter. These results are confirmed also by the recent, more realistic, calculations by Papadopoulos & Font \([12]\). In the case of an axisymmetric irrotational shell of matter with mass \(\mu\), much smaller than the mass \(m\) of the black hole, they find that the efficiency in the emission of gravitational waves decreases as a function of the radial width \(L\) of the shell as follows:

\[
\epsilon_s = \frac{\Delta E}{m c^2} = 8 \cdot 10^{-3} \left( \frac{\mu}{m} \right)^2 \left( \frac{m}{M_\odot} \right)^{2.4} \left( \frac{L}{1 \text{ km}} \right)^{-2.4} \tag{1}
\]

We shall now extrapolate this equation for \(\mu > m\) and compare to the efficiency we expect from core collapse which, for a maximally rotating core, is \(\epsilon_c \sim 7 \cdot 10^{-4}\) in the axisymmetric case \([6]\). For instance, assuming that the initial mass of the newly formed black hole is \(m = 2 M_\odot\), and that the mass of the shell is \(\mu = 10 M_\odot\), we find that the condition \(\epsilon_s < 0.1 \cdot \epsilon_c\) requires \(L > 100 \text{ km}\). This condition is largely verified for...
massive stars. Then, we can conclude that the main burst of gravitational radiation is emitted at the formation of the black hole while subsequent accretion of matter gives no important contributions. To this respect, two possibilities can occur. First, the Fe core of the star has a mass greater than $m_{\text{min}}$. In such a case the black hole initial mass is equal to the core mass. Second, the Fe core is lighter than $m_{\text{min}}$. In this case, a neutron star is produced at the beginning. It then accretes matter until the critical mass is reached, so that the black hole initial mass is equal to the minimum one. From the core masses given by Woosley & Weaver, we see that for soft EOS ($m_{\text{min}} = 1.5 \, M_\odot$) all stars with initial mass greater than $18 \, M_\odot$ produce cores that immediately collapse to a black hole of mass greater than $m_{\text{min}}$, but lower than $\sim 1.8 \, M_\odot$. The same happens according to Fryer. For conventional EOS ($m_{\text{min}} = 2 \, M_\odot$) black holes are formed, after fall-back on a neutron star, from the collapse of progenitors of mass $m_{\text{prog}} > 26 \, M_\odot$ (following Fryer & Kalogera; this value is reduced to $m_{\text{prog}} \sim 20 \, M_\odot$). Then, all black holes form with the same mass $m_{\text{min}}$, so that their initial mass distribution function, for the delayed collapse scenario, can be written as a δ-function: $f(m) = \delta(m - m_{\text{min}})$.

We will consider $m_{\text{min}} = 1.5 \, M_\odot$ and $m_{\text{min}} = 2 \, M_\odot$.

3.1. The rate of black hole formation

Let us now estimate the expected event rate for collapses. We assume a Galactic rate for Supernovae of type II given by $R_{SNI} = 0.02 \, yr^{-1}$. Such rate is the sum of the rate of collapses to neutron stars and, in the delayed scenario, of the rate of collapses to black hole (a supernova explosion is produced by all delayed collapses, independently of $m_{\text{min}}$). For instance, for soft EOS, we have seen that a black hole is produced by progenitor stars with mass $18M_\odot \leq m_{\text{prog}} \leq 40M_\odot$. Their formation rate, $R_{bh}$, is a fraction $\lambda$ of the rate of formation of neutron stars $R_{ns}$, that are generated by progenitors with mass $9M_\odot \leq m_{\text{prog}} \leq 18M_\odot$. Of course, this ratio depends on the initial mass function of the progenitor stars: $\lambda = 0.43$ for a Salpeter law with exponent $\alpha = -2.35$, while $\lambda = 0.32$ for a Scalo law with $\alpha = -2.7$. Then, we can write

$$R_{SNI} = R_{ns} + R_{bh} = R_{ns}(1 + \lambda),$$

from which we can get $R_{ns}$ and $R_{bh}$. In Tab. we tabulate the galactic rates of black hole formation in the different cases, considering both delayed ($R_{bh}$) and prompt ($R_{bh,p}$) collapses. Moreover, the total expected rate within the Virgo cluster, $R_{@20\,Mpc}$, is given in the last column. From Tab. we see that the fraction of delayed collapses producing a black hole is in the range $10\% \div 43\%$ of those leading to a neutron star. These percentages increase to $18\% \div 60\%$ if prompt collapse, for stars more massive than $40 \, M_\odot$, is also taken into account.

\(^{c}\)The upper limit takes into account also the possible stabilizing effect of rotation.
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| W&W m\textsubscript{prog}/m relation | $R_{bh}$ [yr\textsuperscript{-1}] | $R_{bh,p}$ [yr\textsuperscript{-1}] | $R_{20\text{Mpc}}$ [yr\textsuperscript{-1}] |
|-----------------------------------|-------------------------------|-------------------------------|----------------------------------|
| Salpeter, $m_{min} = 1.5 M_{\odot}$ | $8.5 \cdot 10^{-3}$ | $3.4 \cdot 10^{-3}$ | $5.1 (2.0)$ |
| Salpeter, $m_{min} = 2 M_{\odot}$ | $2.8 \cdot 10^{-3}$ | $2.7 \cdot 10^{-3}$ | $1.7 (1.6)$ |
| Scalo, $m_{min} = 1.5 M_{\odot}$ | $6.6 \cdot 10^{-3}$ | $1.9 \cdot 10^{-3}$ | $3.8 (1.1)$ |
| Scalo, $m_{min} = 2 M_{\odot}$ | $1.9 \cdot 10^{-3}$ | $1.5 \cdot 10^{-3}$ | $1.2 (0.9)$ |
| F&K m\textsubscript{prog}/m relation | | | |
| Salpeter, $m_{min} = 1.5 M_{\odot}$ | $8.5 \cdot 10^{-3}$ | $3.4 \cdot 10^{-3}$ | $5.1 (2.0)$ |
| Salpeter, $m_{min} = 2 M_{\odot}$ | $6.3 \cdot 10^{-3}$ | $3.1 \cdot 10^{-3}$ | $3.8 (1.9)$ |
| Scalo, $m_{min} = 1.5 M_{\odot}$ | $6.6 \cdot 10^{-3}$ | $1.9 \cdot 10^{-3}$ | $3.8 (1.1)$ |
| Scalo, $m_{min} = 2 M_{\odot}$ | $4.8 \cdot 10^{-3}$ | $1.8 \cdot 10^{-3}$ | $2.9 (1.1)$ |

Table 1: Galactic event rate for delayed black hole formation ($R_{bh}$), assuming a galactic type-II Supernova rate $R_{SN II} = 0.02$ yr\textsuperscript{-1}, using Woosley & Weaver\textsuperscript{13} (W&W) and Fryer & Kalogera\textsuperscript{4} (F&K) progenitor mass-remnant mass relations. “Salpeter” means that an exponent $\alpha = -2.35$ has been used in the progenitor mass distribution law, while “Scalo” corresponds to $\alpha = -2.7$. The galactic rate for prompt collapses, which could occur for $m_{prog} \in [40,120] M_{\odot}$, is also given ($R_{bh,p}$). In the last column, the total expected rate for delayed collapses within 20 Mpc, $R_{20\text{Mpc}}$, is indicated (in parenthesis the corresponding values for prompt collapses).

4. The Detectability

Gravitational signals emitted in star collapse to a black hole are characterized by an initial part, emitted during the in-fall phase and bounce, followed by an oscillating tail, which can be described as a superposition of damped sinusoids, corresponding to the black hole quasi-normal modes. The frequency and the damping time of the quasi-normal modes are a function of the black hole parameters, mass and angular momentum. For axisymmetric collapses, the main contribution (typically $\sim 90\%$ of the whole energy emitted) is given by the $l = 2$ mode, for which the frequencies and the damping times are given, as a function of the rotation parameter $a = J/(G m^2/c^2)$, in Tab.1\textsuperscript{8}. We use the waveforms calculated by Stark & Piran\textsuperscript{8}, who have computed, in the framework of the full non-linear theory, the gravitational signals emitted in the axisymmetric collapse to a black hole of a rotating core, for different values of the angular momentum. The maximum efficiency reached in their simulations is $\epsilon_{max} \simeq 7 \cdot 10^{-4}$. From the waveforms we have computed the corresponding one-sided power spectrum $f(\nu)$, i.e. the flux of energy per unit frequency which is given by

$$f(\nu) = \frac{\pi c^2 \nu^2}{2G} < h^2(\nu) >$$ \hspace{1cm} (3)

where $< h^2 >$ denotes the average of the squared gravitational signal with respect to its angular dependence. These computed energy spectra allow to evaluate the
signal to noise ratio (SNR), according to the well-known formula given by Eq. (4) below, which holds if the matched filter is applied to the data. This optimum filtering procedures implies that we know the exact waveform emitted and this is a rather optimistic hypothesis even in the case of star collapses to a black hole. We know that in the simplest cases (low rotation, no hydrodynamical effects) most of the energy is emitted in the phase of quasi-normal ringing, which can be described as a superposition of damped sinuosoids. On the other hand, if the collapse has a high degree of rotation, or if strong hydrodynamical effects take place, the emitted waveform may be no longer dominated by the quasi-normal ringing and would be less predictable. In this case, other, less optimum, data analysis procedures should be used. Then, our results have to be considered as upper limits on the SNR. The average squared SNR can be expressed as

$$\overline{SNR^2} = \frac{8G}{5\pi c^3} \int_0^{\infty} f(\nu) \frac{1}{\nu^2 S_h(\nu)} d\nu.$$  \hspace{1cm} (4)$$

where $S_h(\nu)$ is the detector noise power spectrum. In Eq. (4) an average on the source-detector relative position and on the polarization of the gravitational waves emitted is also performed. As discussed in Sec.3, the delayed collapse produces black holes that, immediately after birth, have mass very close to the minimum mass. Thus, the emitted signal depends essentially on the black hole angular momentum and on the distance at which the collapse takes place. In prompt collapses, on the contrary, the SNR clearly depends on the mass of the black hole and, due to the typical sensitivity curve of ground-based interferometers, the radiation emitted by more massive black holes is more easily detectable, so that the delayed collapse process is definitely less favourable than the “naive” prompt one. In Tab. (3) we give the $SNR = \sqrt{\overline{SNR^2}}$ for delayed collapses taking place in the Galaxy (we fix $r = 10 kpc$), for different values of the minimum black hole mass and of the rotation parameter. The values of the SNR increase with the angular momentum of the

| $a$ | $n \cdot \nu$ $[kHz]$ | $\tau/n$ $[10^{-5} s]$ |
|-----|------------------|------------------|
| 0.0 | 12.0556          | 5.5472           |
| 0.2 | 12.1007          | 5.5660           |
| 0.4 | 12.2491          | 5.6230           |
| 0.6 | 12.5201          | 5.7407           |
| 0.8 | 12.9653          | 6.0061           |
| 0.9 | 13.2911          | 6.2892           |
| 0.98 | 13.6234         | 6.7170           |
| 0.9998 | 13.7137      | 6.8760           |
black hole. This is the consequence of two effects: first, the efficiency of emission increases as $a^4$; second, for very high angular momentum (say, $a > 0.8$) the bounce of the collapsing star produces a lower frequency component in the signal energy spectrum which fits better to the sensitivity curve of interferometers. On the other hand, the expected rate of detectable events is low: collapses to a black hole are detectable, essentially, only within the Local Group, with a total rate, strongly dominated by the Milky Way, which is less than $\sim 1$ event per century, see Tab.(1).

We stress, however, that higher SNR could be obtained if collapses had a higher degree of asymmetry and there are some observative indications supporting this hypothesis. Moreover, the initial core collapse to a neutron star which takes place, in the delayed case, if $m_{\text{min}} = 2 M_\odot$, could be a promising process. If it is highly asymmetric, as observations seem to indicate, a large amount of gravitational radiation could be emitted in the range of frequencies where interferometric detectors reach their best sensitivity.

We have repeated the calculation of the SNR for the advanced LIGO detector, using an approximation to its sensitivity curve given by Flanagan & Hughes. Results are given, for delayed collapses, in Tab.(4) where a distance of $r = 20\text{Mpc}$ has been assumed. Within this distance, corresponding approximately to the Virgo Cluster, the expected black hole formation rate is $\sim 1 \div 5\text{ yr}^{-1}$, but we see that the detection perspectives are not much better because the SNR is much smaller than one. The situation is different for prompt collapses. In Fig.(1) we have plotted the signal-to-noise ratio for prompt collapses as a function of the newborn black hole mass and it appears that the emitted gravitational radiation could be detectable if the rotation rate is high enough.

It should be noted, however, that according to the newtonian simulations by Rampp et al., the amount of gravitational radiation emitted in non-axisymmetric collapses is comparable to that of the axisymmetric case.

| $a$ | SNR  |
|-----|------|
| 0.42 | 6.1  |
| 2M⊙ |      |
| 0.79 | 7.9  |
| 2M⊙ |      |
| 0.94 | 25.4 |
| 2M⊙ |      |
| 1.5M⊙ | 31.9 |
| 2M⊙ |      |

Table 3: Virgo detector: signal-to-noise ratio for the delayed collapse. Different values of the rotation parameter $a$ and minimum mass for black hole formation $m_{\text{min}}$ are considered. All collapses are assumed to take place at a distance $r = 10\text{kpc}$. 
Table 4: Advanced LIGO detector: signal-to-noise ratio for the delayed collapse. Different values of the rotation parameter $a$ and minimum mass for black hole formation $m_{\text{min}}$ are considered. All collapses are assumed to take place at a distance $r = 20\, \text{Mpc}$.

| $a$     | SNR  |
|---------|------|
| 0.42    |      |
| $m_{\text{min}} = 1.5 M_{\odot}$ | 0.013 |
| $m_{\text{min}} = 2 M_{\odot}$ | 0.019 |
| 0.79    |      |
| $m_{\text{min}} = 1.5 M_{\odot}$ | 0.065 |
| $m_{\text{min}} = 2 M_{\odot}$ | 0.088 |
| 0.94    |      |
| $m_{\text{min}} = 1.5 M_{\odot}$ | 0.12  |
| $m_{\text{min}} = 2 M_{\odot}$ | 0.16  |

Fig. 1. Advanced LIGO detector: signal-to-noise ratio as a function of the black hole mass $m$ for prompt collapses. The collapse is assumed to take place at a distance $r = 20\, \text{Mpc}$. 


5. Black hole final mass distribution function

In this section we calculate the black hole final mass distribution function. We do not consider the chemical evolution of the galaxy, which affects the mass distribution of compact remnants (see, e.g., Timmes et al. \[25\]). Such a theoretical distribution could be useful in statistical studies of recently formed black holes.

The black hole final mass distribution depends on the mass distribution of progenitor stars and on the relation between the mass of progenitors and that of the final remnants produced after the collapse. For the first one we assume a power law \( f(m_{\text{prog}}) \propto m_{\text{prog}}^{\alpha} \), with \( \alpha = -2.35 \) (Salpeter’s law) and \( \alpha = -2.7 \) (Scalo’s law). Regarding the relation between the mass of progenitor stars, \( m_{\text{prog}} \), and that of remnants, \( m \), we use (after conversion from baryonic to gravitational mass) the results of Woosley & Weaver \[13\] and of Fryer & Kalogera \[30\], who have performed systematic calculations of the evolution and explosion of non rotating massive stars. It must be stressed again that such evolutionary calculations are still subject to many uncertainties and this reflects in some differences between their results, obtained using different assumptions. The remnant masses they report are calculated a long time after the collapse, so that the possible fall-back is included. The distribution function we are searching for is simply given by

\[
f(m) = f(m_{\text{prog}}(m)) \cdot \frac{dm_{\text{prog}}}{dm}
\]

In Fig. (2) we plot the function \( f(m) \) obtained using a fit of the progenitor mass-remnant mass relation found by Woosley & Weaver \[13\] (see their table 3), considering different values of \( m_{\text{min}} \) and of the exponent \( \alpha \). The maximum progenitor mass considered by Woosley & Weaver is \( m_{\text{prog}} = 40 \, M_{\odot} \). According to them, for masses greater than this, the effect of stellar wind becomes relevant and strongly
modifies the star evolutionary path: a smooth convergence of remnant masses to \( \sim 1.5 \, M_\odot \), corresponding to neutron stars or low mass black holes, is expected for progenitors of mass greater than about \( 40 \, M_\odot \). As a consequence, black holes with mass greater than \( \sim 10.3 \, M_\odot \) - which is what is predicted by Woosley & Weaver for a \( 40 \, M_\odot \) progenitor star if stellar winds are neglected - should never be formed, for progenitors of solar metallicity.\(^a\) Also Fryer & Kalogera\(^3\) show that low mass remnants are produced, for progenitors more massive than about \( 40 \, M_\odot \), if stellar winds are taken into account. On the other hand, if stellar winds are neglected, they find that, for masses above about \( 40 \, M_\odot \), the star collapse takes place in the *prompt* way, i.e. a black hole of mass nearly equal to the mass of the progenitor star is produced in a dynamical timescale. In this case the black hole mass distribution function is simply proportional to \( m^{-\alpha} \). The final mass distribution function, resulting from the progenitor mass-remnant mass relation found by Fryer & Kalogera, is plotted in Fig. (3) (we refer to their “most likely” model, in which 50% of the explosion energy goes in unbinding the star). There are clear differences between the distributions functions plotted in Figs. (2,3). First of all, the expected range of masses for black holes is larger in the second case, reaching \( \sim 14.7 \, M_\odot \). Moreover, in this case the function \( f(m) \) is monotonically decreasing, while in Fig. (2) there is a minimum around \( 3 \, M_\odot \) followed by a relative maximum at \( \sim 4.5 \, M_\odot \). These differences are a consequence, obviously, of the different relations for progenitor vs. remnant masses\(^b\) and are due, mainly, to two reasons. First, explosion simulations are carried on by Woosley & Weaver in one-dimension, while

\[^a\]Black holes of mass greater than that value could be produced if the metallicity is lower.

\[^b\]In particular, the minimum in Fig. (2) is due to an increase of the slope of \( m_{\text{prog}}(m) \) for \( m_{\text{prog}} \leq 30 \, M_\odot \) while the following maximum is produced by the first term in the right-hand side of Eq. (5), which is dominant for large \( m_{\text{prog}} \).
Fryer & Kalogera simulations are two-dimensional. Second, Woosley & Weaver constrain the kinetic energy of ejected material at infinity to a constant value, about $1.2 \cdot 10^{51} \text{erg}$, while in Fryer & Kalogera this quantity decreases for increasing progenitor mass. This explains why their remnant masses are greater.

The theoretical distribution functions we have found cannot be compared with observations, which refer to black candidates belonging to binary systems. It is widely accepted that the evolution of high mass stars in binary systems is quite different from that of single stars of comparable mass. A detailed discussion on massive binary systems evolution and on the possible mechanisms and bias effects which can explain the observed black hole candidates mass distribution can be found in refs. 28 and 43.

6. Conclusions

In this paper we have considered the collapse of a massive star to a black hole, exploring the models of prompt and delayed collapse. According to recent simulations, the formation of stellar mass black holes should take place through a delayed collapse. The prompt collapse could happen for very massive progenitors (mass greater than $\sim 40M_\odot$) only if stellar winds were negligible, an assumption which appears to be not very reliable. In the delayed collapse all black holes were born with a mass equal to its minimum value, or just a little greater, and then slowly accrete matter up to their final mass. We have shown that the main burst of gravitational radiation is emitted when the black hole forms so that the gravitational energy spectrum is peaked in the range $\sim 4.5 \div 9 \text{kHz}$, depending on the initial mass and the angular momentum of the black hole. Such frequencies do not match very well with the sensitivity curve of ground-based interferometers which reach their best sensitivity in the band $\sim 60 \div 1000 \text{Hz}$. We have estimated the detectability of the emitted gravitational radiation in the delayed case, the most reliable scenario, and in the prompt case, for progenitor stars more massive than $40M_\odot$. For each collapse model, we have considered different values of black hole minimum mass and angular momentum and different laws for progenitors mass distribution. We have also derived the theoretical black hole final mass distribution function.

Delayed collapses are detectable only inside the Local Group of galaxies by interferometers of the first generation. Obviously, delayed collapses are less detectable than the prompt ones, due to the lower matching of their characteristic frequencies to the sensitivity curve of the detectors. On the other hand, the initial stage of the delayed collapse, with the formation of a neutron star following a strongly asymmetric explosion (if $m_{\text{min}} = 2 M_\odot$), could be a promising source of gravitational waves. Detection perspectives of delayed collapses are not so much better for advanced interferometers because, if distances up to the Virgo Cluster are considered (where we expect $\sim 1 \div 5 \text{ev/yr}$), the signal-to-noise ratio is much lower than one, unless a very high degree of asymmetry is produced. On the contrary, prompt collapses

$^c$Defined as the difference between the explosion energy and the binding energy of the ejecta
could be detected with large enough SNR. Delayed collapses to a black hole belong to a class of high frequency sources of gravitational waves, which comprises various processes involving compact objects, as for instance the excitation of neutron star w-modes, the coalescence of two neutron stars with the formation of a light black hole, dynamical instabilities in neutron stars, and some kinds of secular instability in neutron stars. Such sources cannot be efficiently detected by present resonant detectors and forthcoming interferometers because they are expected to emit at frequencies higher than those at which both interferometric and resonant detectors have their best sensitivity. In past years local arrays of small resonant detectors, which are particularly suited for the detection of high frequency gravitational radiation, have been proposed. A detailed study of their detection performances, considering different geometries, dimensions and materials, has been done.

The non-continuous background of gravitational waves produced by the ensemble of the star collapses to a black hole, which occurred at a higher rate in the early phases of the Universe, has recently been calculated. It would be interesting to repeat the calculation in the case of delayed collapse. In such a case we have the superposition of energy spectra nearly with the same shape and all peaked at nearly the same frequency, in the range $4.5 \div 9$ kHz (this holds also for star metallicity $Z = 0$). Roughly speaking, as most of the collapses take place at redshift $z \sim 2$, we expect to have, at the detector, a background spectrum strongly peaked somewhere in the band $1.5 \div 3$ kHz.

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