Spatial fluctuations of the pairing potential in disordered superconductors

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Abstract

We study the effect of site diagonal, non-magnetic, disorder on the a pairing amplitude in an extended Hubbard model with the intersite attraction. Analyzing fluctuations of a pairing potential we discuss the instability of mixed solutions, ’s + d’ and ’s + id’, in presence of disorder. The influence of disorder on extended s- and d- wave superconductors appear to be comparable but in certain regions of the phase diagram, even weak disorder can change the symmetry of the order parameter.

Key words: exotic pairing, doping, disorder
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1 Introduction

The study of disordered superconductors with exotic pairing is of general interest. D-wave pairing plays an important role in the superconductivity of the cuprates [1] while p-wave in strontium ruthenate (Sr$_2$RuO$_4$) [2]. Depending on symmetry of order parameter, the effect of disorder is dramatically different on the superconducting states which are described as having isotropic, s-wave, or anisotropic, extended s-, d- and p-wave, order parameter symmetry [3–13]. The works on disorder effect on superconductors are stimulated by the recent experiments [14–19].

In this paper we analyze an extended Hubbard model with intersite attraction, whose phase diagram includes both s- and d-wave regions and introduce disorder into the problem by allowing the site energies, $\varepsilon_i$, to be independent random variables.

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In the case of anisotropic $s$– and $d$–wave pairing there is no strict equivalent of the Anderson Theorem [3,5,9] which governs the response of isotropic $s$–wave superconductors to randomness in the crystal potential. However, a weaker statement, which may be similarly useful, can be formulated [7,11,12]. Here we will calculate the ensemble average $< \Delta_{ij} \Delta_{il}^* >$ where $\Delta_{ij}$ is the pairing amplitude in the cases where the sites, $i, j$ and $l$, are nearest neighbours to lowest order in the fluctuations of the site energies $\delta \varepsilon_i = \varepsilon_i - \varepsilon_0$ about their mean $\varepsilon_0$. Our results imply that the disorder does not induce large fluctuations in the pairing potentials and hence for systems of large coherence length $\xi$ the amplitude of pairing potential $|\Delta(ij)|$ may rather to be the same for all bonds in $x$ and $y$ directions [9,11,12]. Eventually, we compare the corresponding standard square deviation of fluctuating potentials $\sigma(\delta \Delta_{ij})$ and $\sigma(\delta \varepsilon_i)$ (Fig. 1). Their ratio $\Gamma = (\sigma(\delta \Delta_{ij})/\sigma(\delta \varepsilon_i))^2$ will be a criterion of pairing potential fluctuations leading finally to the destruction of superconducting phase. We shall be also interested what is the effect of disorder on anisotropic $s$– and $d$–wave superconductors, where they coexist, and if the disorder favour any of particular solution.

![Fig. 1. The examples of Gaussian distribution for $\delta \varepsilon_i$ with a mean square deviation $\sigma(\delta \varepsilon_i) = \sqrt{< \varepsilon_i^2 >}$ and the expected distribution of a pairing potential $\delta \Delta_i$ with corresponding $\sigma(\delta \Delta_{ij}) = \sqrt{< |\Delta_{ij}|^2 >}$. Fluctuating potentials $\delta \varepsilon_i$ and $\delta \Delta_{ij}$ are in the units of $\sqrt{< \varepsilon_i^2 >}$](image)

The paper is organized as follows. In Sec. 2 we present the model in the clean limit, introduce the Hamiltonian and approximations used in paper. Here we also discuss briefly the phase diagram of the clean system. In Sec. 3 we include disorder induced spatial fluctuations of a pairing potential. Here we discuss the formalism used in our analysis and show the results on the disordered effect on anisotropic superconductors. Finally we investigate the stability of various solutions in presence of disorder. Section 4 contains conclusions and remarks.
The model in clean limit.

Here we use a single band, extended, Hubbard model with an intersite attraction. It is defined by the Hamiltonian [20]:

\[
H = \sum_{ij\sigma} (\varepsilon_{ij} + t_{ij}) c_{i\sigma}^+ c_{j\sigma} + \sum_{ij\sigma\sigma'} \frac{W_{ij}}{2} n_{i\sigma} n_{j\sigma'} - \mu \sum_{i\sigma} c_{i\sigma}^+ c_{i\sigma},
\]

(1)

where \( c_{i\sigma}^+ \) and \( c_{i\sigma} \) are usual, fermionic operators which create and annihilate, respectively, an electron with spin \( \sigma \) at the lattice site labeled by \( i \), \( t_{ij} \) is an electron hopping integral, \( n_{i\sigma} \) is the operator of particle number of spin \( \sigma \) at site \( i \), \( \varepsilon_i \) is the site energy, varying from site to site in random fashion, at the site \( i \) with mean value \( \varepsilon_0 = \langle \varepsilon_i \rangle = 0 \) and \( W_{ij} \) is the interaction potential of two electrons with opposite spins on neighbour sites \( i, j \). Finally, \( \mu \) denotes the chemical potential.

Hartree–Fock–Gorkov equation of motion for Gorkov Greens \( 2 \times 2 \) functions \( G(i, i; \omega) \) yields:

\[
\sum_i \begin{bmatrix}
(\omega - \varepsilon_i + \mu)\delta_{il} - t_{il} - W_{il} (\langle c_{i\downarrow}^+ c_{l\downarrow} \rangle - n_l) - W_{il} (\langle c_{i\uparrow}^+ c_{l\uparrow} \rangle + n_l) \\
W_{il} (\langle c_{i\uparrow}^+ c_{l\uparrow} \rangle + n_l) - (\omega + \varepsilon_i - \mu)\delta_{il} + t_{il} + W_{il} (\langle c_{i\downarrow}^+ c_{l\downarrow} \rangle + n_l)
\end{bmatrix} G(l, j; \omega) = \mathbf{1} \delta_{ij},
\]

(2)

where we assumed a paramagnetic state \( n_l = n_{l\uparrow} = n_{l\downarrow} \).

The charge \( n_i \) on the site \( i \) and pairing potential \( \Delta_{il} = W_{il} \langle c_{i\downarrow} c_{l\uparrow} \rangle \) for neighbour sites \( i, l \) can be expressed by following equations:

\[
n_i = -\frac{2}{\pi} \int_{-\infty}^{\infty} d\omega \ \text{Im} \ G_{11}(i, i; \omega) \frac{1}{e^{\beta\omega} + 1},
\]

\[
\Delta_{il} = -\frac{W_{il}}{\pi} \int_{-\infty}^{\infty} d\omega \ \text{Im} \ G_{12}(i, l; \omega) \frac{1}{e^{\beta\omega} + 1},
\]

(3)

where \( \beta = \frac{1}{k_B T} \) and \( k_B \) is the Boltzmann constant.
For a pure system we take the lattice Fourier transform of Eq. 2 with \( \varepsilon_i = 0 \) and find:

\[
\begin{bmatrix}
\omega - \epsilon_k' + \mu' & \Delta_k \\
\Delta_k^* & \omega + \epsilon_k' - \mu'
\end{bmatrix} \mathbf{G}^0(\mathbf{k}; \omega) = \mathbf{1},
\]

where \( \mu' \) is shifted chemical potential \( \mu' = \mu + W n \), and the Hartree–Fock kinetic energy \( \epsilon_k \) is given by

\[
\epsilon_k' = \epsilon_k - \frac{W}{N} \sum_k n_k \gamma_k,
\]

where

\[
n_k = < c_k c_k^+ > = \sum_{ij} < c_i^+ c_j > e^{i(R_j - R_i)k}.
\]

Following Ref. [20] we will neglect the Fock term \( \frac{W}{N} \sum k n_k \gamma_k \) (Eq. 5). Thus, for two dimensional lattice and the electron hopping defined between nearest neighbour sites only, we get

\[
\epsilon_k' \approx \epsilon_k = \sum_i t_{ij} e^{i(R_j - R_i)k} = -t \gamma_k,
\]

where

\[
\gamma_k = 2(\cos k_x + \cos k_y).
\]

As can be readily shown the order parameter \( \Delta_k \) satisfies the following gap equation:

\[
\Delta_k = \frac{1}{N} \sum_q W_{k - q} \Delta_q \tanh \left( \frac{\beta \omega}{2} \right),
\]

where

\[
W_{k - q} = |W| |\gamma_{k - q} - \eta_{k - q}| = |W| \left( \frac{\gamma_{k - q} + \eta_{k - q}}{4} + 2\sin(k_x)\sin(q_x) + 2\sin(k_y)\sin(q_y) \right)
\]

and

\[
\eta_k = 2(\cos k_x - \cos k_y),
\]

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while $E_q$ denotes quasi-particle energy:

$$E_q = \sqrt{\bar{\epsilon}_q^2 - \Delta_q^2}, \quad \bar{\epsilon}_q = \epsilon_q - \mu'.$$

(12)

Generally, the singlet type of solution (Eq. 9) can be written as

$$\Delta_k = \Delta_s \gamma_k + \Delta_d \eta_k$$

(13)

for the real type solution if $s$ and $d$ parts of $\Delta(k)$ have the same phase and

$$\Delta_k = \Delta_s \gamma_k + i \Delta_d \eta_k$$

(14)

for the complex solution. Pairing amplitudes: $\Delta_s, \Delta_d$ are defined as real numbers, corresponding to $s$- and $d$-wave components respectively.

The pairing parameters $\Delta_s, \Delta_d$ (Eqs. 13, 14) may be determined from Eq. (9). Namely:

$$\Delta_s = -\frac{W}{N} \sum_q \frac{\gamma_q}{8E_q} \tanh\left(\frac{\beta E_q}{2}\right) \Delta_q,$$

$$\Delta_d \alpha = -\frac{W}{N} \sum_q \frac{\eta_q}{8E_q} \tanh\left(\frac{\beta E_q}{2}\right) \Delta_q,$$

(15)

where and $\alpha = 1$ for a real solution $(s + d)$ while $\alpha = i$ for a complex one $(s + i d)$.

The above set of equations for pairing potentials (15) should be completed by the equation for the chemical potential $\mu$ (Eq. 3). It can be written as:

$$n - 1 = -\frac{2}{N} \sum_q \frac{\bar{\epsilon}_q}{2E_q} \tanh\left(\frac{\beta E_q}{2}\right).$$

(16)

The gap equations (Eqs. 15) can be used to calculate the superconducting critical temperature $T_C$ after the linearization. The two separate equations read:

$$1 = -\frac{W}{N} \sum_q \frac{\gamma_q^2}{8\bar{\epsilon}_q} \tanh\left(\frac{\beta C \bar{\epsilon}_q}{2}\right),$$

$$1 = -\frac{W}{N} \sum_q \frac{\eta_q^2}{8\bar{\epsilon}_q} \tanh\left(\frac{\beta C \bar{\epsilon}_q}{2}\right),$$

(17)
where $\beta_C = \frac{1}{k_B T_C}$.

In the Fig. 2 we present $T_C$ results for two dimensional lattice versus band filling $n$. The interaction parameter $W$ was chosen as $W/D = -0.3, -0.5, -0.7$, where $D = 8t$ denotes a bandwidth. Because of the assumed form of

![Fig. 2](image.png)

Fig. 2. The superconducting critical temperature $T_C$ versus band filling $n$ for s- and d-wave pairing (dashed and full lines respectively) for three values of intersite attraction $|W|/D = 0.3, 0.5, 0.7$

a dispersion relation $\varepsilon_k$ (Eq. 7) the model (Eq. 1) possesses the particle–hole symmetry, therefore $T_C$ for band–filling $n$ and $2 - n$ is the same. The results show that s–wave type of superconductivity exists for low electron or hole concentration while d–wave type exists close to half–band filling [20]. For a large enough electron interaction parameter $W/D > 0.2$ the curves $T_C = T_C(n)$ for s- and d-wave pairing cross and regions of s- and d-wave superconductivity are not separated. The pairing parameters, $\Delta^s$ and $\Delta^d$ at zero temperature ($T = 0$ K) should be calculated from the full set of equations Eqs. (15). For the real type solution (Eq. 13) they are plotted for the same three values of the electron attraction $W/D = -0.3, -0.5, -0.7$ in Fig. 3a. Note that in all cases there are small but visible regions of mixed ’s + d’ solutions. In Fig. 3b we have plotted the same for the complex solution (Eq. 14) of mixed ’s + id’ pairing. Clearly, the regions with possible mixed solution in case of ’s + id’ pairing are much larger than for ’s + d’ mixture. We will consider these two solutions more carefully.

In the case of mixed solution, the order parameter (complex or real) in the lattice real space $\Delta_{ij}$ is defined as the sum of $\Delta^s_{ij}$ and $\Delta^d_{ij}$. Namely:

$$\Delta_{ij} = \Delta^s_{ij} + \alpha(-1)^{l-j}\Delta^d_{il},$$

where $l$ denotes the neighbour site ($l = j, j+1, j+2, j+3$ Fig. 4). $\alpha$ is equal to 1 and i for a real and complex solution, respectively.
Fig. 3. Amplitude of the pairing potential $\Delta$ versus band filling $n$ for $s$- and $d$- wave pairing (dashed and full lines respectively) for three values of intersite attraction $|W|/D = 0.3, 0.5, 0.7$. Note the difference around $n = 0.4$, figure (a) corresponds to the real solution ‘$s + d$’ while (b) to the complex one ‘$s + id$’ respectively.

\[ \Delta^s_{ij} = \frac{1}{2} (\Delta_{ij} + \Delta_{ij+1}) \]
\[ \Delta^d_{ij} = \frac{1}{2\alpha} (\Delta_{ij} - \Delta_{ij+1}) \]  

(19)

The free energy $F$ for a finite temperature $T$ can be calculated from the following formula [20]:

\[ F = \frac{1}{N} \sum_k \left[ -(n-1)\epsilon_k - 2k_B T \ln \left( 2 \cosh \frac{E_k}{2k_B T} \right) - \frac{|\Delta_k(T)|^2}{W} \right] \]  

(20)
Fig. 5. Free energies $F$ of mixed solutions: (a) the real combination ’$s + d$’ and (b) the complex one ’$s + id$’ for the same band filling $n = 0.4$ and the attraction parameter $|W|/D = 0.7$. Additional arrows show the minima of free energies $F$.

the corresponding derivatives determine the integral equations (Eqs. 14 and 15):

$$\frac{\partial F}{\partial \mu} = 0, \quad \frac{\partial F}{\partial \Delta \psi} = 0, \quad \psi = s \text{ or } d.$$ (21)

In Fig. 4 we have plotted the free energies $F$ of both mixed solutions: the real combination ’$s + d$’ (Fig. 5a) as well as the complex one ’$s + id$’ (Fig. 5b) versus $s$– and $d$–wave amplitudes $\Delta s$ and $\Delta d$. The band filling $n$ was chosen...
to be $n = 0.4$ and the attraction parameter $|W|/D = 0.7$ is large enough to produce the mixed solutions (Figs. 2-3). Additional arrows in the plots show the minima of free energies $F$. Clearly they show the mixed type of solutions. To check which solution is more favorable we show also the difference between free energies for two superconducting mixed states $\Delta F = F('s+d^') - F('s+id^')$ versus $s$– and $d$–wave amplitudes (Fig. 5c). One can see that in the region of coexistence $\Delta F$ is positive. Thus, the solution with $s+id$ symmetry is favored by smaller free energy $F$.

3 Disorder induced Fluctuations of pairing potential

To go further we apply the same strategy as in Refs. [9,10,21,22] we treat random site energies $\varepsilon_i$ as perturbations and we proceed by solving the Dyson equation for the Gorkov matrix of the pure superconductor evaluated at a frequency $\omega$

$$G(i,j;\omega) = G^0(i,j;\omega) + \sum_l G^0(i,j;\omega)V_l G(l,j;\omega), \quad (22)$$

where $V_l$ is the impurity potential matrix:

$$V_l = \begin{pmatrix} \varepsilon_l & 0 \\ 0 & -\varepsilon_l \end{pmatrix}. \quad (23)$$

To the lowest order in $\varepsilon_i$ we get:

$$G(i,j;\omega) = G^d(i,j;\omega) + \sum_n G^0(i,n;\omega)V_n G^0(n,j;\omega). \quad (24)$$

Following Eqs. (3) we express the interesting quantities $\Delta_{ij}$ and $\Delta_{il}$, where $l$ and $j$ are nearest neighbors of $i$, in the lowest order of $\varepsilon_i$ perturbations by means of the disordered Green function (Eq. 24) and calculate the mean square deviation of the pairing parameter as:

$$\langle \delta \Delta_{ij}\delta \Delta_{il}^* \rangle = \langle \Delta_{ij}\Delta_{il}^* \rangle - \langle \Delta_{ij} \rangle \langle \Delta_{il}^* \rangle, \quad (25)$$

where bonds $ij$ and $il$ in the pairing potentials $\Delta_{ij}$ and $\Delta_{il}$, denote bonds which can be chosen as parallel (then $j$ and $l$ coincide) as well as perpendicular.

The assumption is that random site energies $\varepsilon_i$ in Eqs. (1,22,23,24) are independent variables; then averages $\langle \varepsilon_i \rangle$, $\langle \varepsilon_i \rangle$ being independent of the site.
index $i$ and $< \varepsilon_i \varepsilon_j > = < \varepsilon_i^2 > \delta_{ij}$ lead to

$$< \delta_i \delta_j > = \Gamma_{ij}^l < \varepsilon_i^2 >,$$  \hspace{1cm} (26)

where $l = j$ for parallel ($\Gamma_{ij}^j = \Gamma_{ij}^\parallel$) and $l = j + 1$ perpendicular bonds ($\Gamma_{ij}^{j+1} = \Gamma_{ij}^\perp$), respectively (Fig. 4). Thus, fluctuations are determined by the corresponding coefficients $\Gamma_{ij}^\parallel$ and $\Gamma_{ij}^\perp$. They can be calculated from Eqs. (3) and (22-26):

$$\Gamma_{ij}^l = \frac{\tilde{W}_i W_{ij}}{\pi^2} \frac{1}{N} \sum_q \left[ \frac{1}{N} \sum_k \int_{-\infty}^{\infty} d\omega \ \text{Im} \left( G_0^0(k;\omega) \tau_3 G_0^0(k - q;\omega) \right) \right]_{12}$$

$$\times e^{-i k (R_i - R_j)} \left[ \frac{1}{N} \sum_{k'} \int_{-\infty}^{\infty} d\omega' \ \text{Im} \left( \tilde{G}_0^0(k';\omega') \tau_3 \tilde{G}_0^0(k' - q;\omega') \right) \right]_{12}$$

$$\times e^{i k' (R_i - R_j)} e^{i q (R_j - R_l)}.$$  \hspace{1cm} (27)

After evaluation at $T = 0$ K it yields:

$$\Gamma_{ij}^l = \frac{1}{N} \sum_q \left[ \frac{W_{ij}}{N\pi} \sum_k \int_{-\infty}^{0} d\omega \ \text{Im} \left\{ G_{11}^0(k;\omega) G_{12}^0(k - q;\omega) \right\} \right.$$

$$- G_{12}^0(k;\omega) G_{22}^0(k - q;\omega) e^{-i k (R_i - R_j)}$$

$$\times \left\{ \frac{W_{il}}{N\pi} \sum_{k'} \int_{-\infty}^{0} d\omega' \ \text{Im} \left\{ G_{11}^0(k';\omega') G_{12}^0(k' - q;\omega') \right\} \right.$$

$$- G_{12}^0(k';\omega) G_{22}^0(k' - q;\omega) e^{i k (R_i - R_j)} \left\} \right.$$  \hspace{1cm} (28)

$$\times e^{i q (R_j - R_l)}.$$  \hspace{1cm} (29)

Depending on site $l$ we get the formula for $\Gamma_{ij}^\parallel$ if sites $j$ and $l$ are identical ($R_l = R_j$) and $\Gamma_{ij}^\perp$ for $l = j + 1$ (Fig. 4). After the integration over $\omega$, and $\omega'$ (Eq. 28) we get following formulae:

$$\Gamma_{ij}^\parallel = \frac{1}{N} \sum_q \left[ \frac{W_{ij}}{2N} \frac{\Delta_k \tilde{\epsilon}_k + \Delta_k \tilde{\epsilon}_{k - q}}{E_k + E_{k - q} E_k E_{k - q}} e^{i k (R_i - R_j)} \right]$$

$$\times \left\{ \frac{W_{il}}{2N} \frac{\Delta_k \tilde{\epsilon}_k + \Delta_k \tilde{\epsilon}_{k - q}}{E_k + E_{k - q} E_k E_{k - q}} e^{-i k (R_i - R_j)} \right\} e^{i q (R_j - R_l)}.$$  \hspace{1cm} (29)
Fig. 6. Fluctuation parameter $\Gamma = \Gamma^\parallel$, versus band filling $n$ for a pairing potential $\Delta_{ij}$ for three values of intersite attraction $|W|/D = 0.3, 0.5, 0.7$. Full lines correspond to the real solution ‘$s + d$’ while dashed lines to the complex one ‘$s + id$’ respectively.

$$\Gamma_{ij}^\parallel = \frac{1}{N} \sum_q \left| \frac{W_{ij}}{2N} \sum_k \frac{\Delta_k \epsilon_k + \Delta_k \epsilon_k - q}{E_k + E_k - q} e^{i(R_i - R_j)} \right|^2.$$  

In Fig. 6 we plot the coefficient $\Gamma^\parallel$ versus band filling $n$. Full and dashed lines correspond to ‘$s + d$’ and ‘$s + id$’ solutions, respectively. One can see that in case of weak interactions the results are very similar, while for large interaction ($W/D = -0.7$) where mixed solutions are present ($n \approx 0.4$) we observe a large difference. Clearly, in case of a purely real solution the fluctuations are larger. Interestingly, we find $\Gamma^\parallel = 0$ for half filled band $n = 1$. This result corresponds to the similar one in case of attractive ‘on site’ (negative $U$) interaction [9,10,21,22] and this is due to the particle hole symmetry.

It is also possible to analyze fluctuations of each of the components of the superconducting order parameter (Eq. 17). The corresponding coefficients $\Gamma_{ij}^s$ and $\Gamma_{ij}^d$ determine fluctuations of the pairing parameter of $s$–wave symmetry $\Delta_{ij}^s$ and $d$–wave one $\Delta_{ij}^d$. Namely:

$$< \delta \Delta_{ij}^s \delta \Delta_{ij}^s > = \Gamma_{ij}^s < \epsilon_i^2 >$$
$$< \delta \Delta_{ij}^d \delta \Delta_{ij}^d > = \Gamma_{ij}^d < \epsilon_i^2 >$$  

where we have defined

$$\Gamma_{ij}^s = \frac{1}{2}(\Gamma_{ij}^\parallel + \Gamma_{ij}^\perp)$$
$$\Gamma_{ij}^d = \frac{1}{2}(\Gamma_{ij}^\parallel - \Gamma_{ij}^\perp).$$
Fig. 7. Fluctuation parameter $\Gamma = \Gamma^s, \Gamma^d$ versus band filling $n$ for $s$- and $d$-wave pairing potential (full and dashed lines respectively) (dashed and full lines respectively) for three values of intersite attraction $|W|/D = 0.3, 0.5, 0.7$. Figure (a) corresponds to the real solution 's + d' while (b) to the complex one 's + id' respectively.

The parameters $\Gamma^{s(d)}$ governing disorder induced fluctuations of the order parameter $\Delta_s(d)$ are plotted versus band filling in Fig. 7. Figure 7a corresponds to the real combination of 's + d' solution while Fig. 6b corresponds to the complex one 's + id'. Note that generally, fluctuations of 's + d' and d + id components are relatively small $|\Gamma| < 0.012$: $\Gamma^s$ and $\Gamma^d$ are of the same order. However their relative value $|\Gamma^s|/|\Gamma^d|$ depends on band filling $n$, showing that the change of symmetry is possible by disorder. In case of particle-hole symmetry at half filling both $\Gamma^s$ and $\Gamma^d$ go to zero (like $\Gamma^\parallel$ in Fig. 6).

4 Conclusions and Discussion

Solving the extended Hubbard model (1), for the appropriate system parameters, we have analyzed possible singlet solutions with s- and d-wave order parameter. Especially we have concentrated on the existence of mixed, 's + d' and 's + id', solutions. On account of that the Anderson theorem could not be applied to anisotropic superconductors we analyzed the disorder induced, spatial fluctuations of order parameter amplitudes $\Delta_s$ and $\Delta^d$. Our results show that the disorder does not induce large fluctuations in the pairing potentials and the amplitude of pairing potential $|\Delta(ij)|$ may rather be the same for all bonds in x and y directions. However, in regions of mixed solution the fluctuations are larger for the 's + d' solution, which implies that 's + id' is more stable in the presence of disorder. Interestingly, we observed zero fluctuations limit in case of particle-hole symmetry as in the case of the negative $U$ Hubbard model [9,10,21,22]. This result is obtained here at zero temperature ($T = 0$ K) in the limit of very weak disorder. Nevertheless, for stronger non-magnetic disorder
we observe a pair breaking effect strongly influencing the critical temperature [11,12]. Moreover the effect is stronger if the chemical potential passes a Van Hove singularity. In our case it is a half-filled situation $n = 1$.

Recently Ghosal et al. [23] have done self-consistent calculations of the pairing amplitude for finite cluster size and various strength of disorder. Their results, obtained away from half filling $n \neq 1$, show that in the case of strong enough disorder, large fluctuations of $\Delta_{ij}$ can lead to superconducting islands. In that situation the disorder can lead to phase fluctuations and to pseudogap phenomena [24]. To investigate these effects we need to use more sophisticated methods i.e. to go beyond the Hartree–Fock–Gorkov approximation as in Ref. [25].

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