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To cite this article: Andrea Mammarella and Massimo Mannarelli 2018 J. Phys.: Conf. Ser. 981 012010

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Meson properties in asymmetric matter

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Abstract. In this work we study dynamic and thermodynamic (at $T = 0$) properties of mesons in asymmetric matter in the framework of Chiral Perturbation Theory. We consider a system at vanishing temperature with nonzero isospin chemical potential and strangeness chemical potential; meson masses and mixing in the normal phase, the pion condensation phase and the kaon condensation phase are described. We find differences with previous works, but the results presented here are supported by both theory group analysis and by direct calculations. Some pion decay channels in the normal and the pion condensation phases are studied, finding a nonmonotonic behavior of the decay width as a function of $\mu_I$. Furthermore, pressure, density and equation of state of the system at $T = 0$ are studied, finding remarkable agreement with analogue studies performed by lattice calculations.

1. Introduction
The properties of strongly interacting matter in an isospin and/or strangeness rich medium are relevant in a wide range of phenomena including the astrophysics of compact stars and heavy-ion collisions. It is known that depending on the value of the isospin chemical potential, $\mu_I$, and on the value of the strangeness chemical potential, $\mu_S$, three different phases can be realized: the normal phase, the pion condensed ($\pi c$) phase and the kaon condensed ($K c$) phase [1, 2, 3]. The realization of a mesonic condensate can drastically change the low energy properties of matter, including the mass spectrum and the lifetime of mesons.

Previous analysis of the meson condensed phases by QCD-like theories were developed in [4, 5]. Pion condensation in two-flavor quark matter was studied in [2, 6] and in three-flavor quark matter in [3]. In particular, the phase diagram as a function of $\mu_I$ and $\mu_S$ was presented in [3]. Finite temperature effects in $SU(2)_L \times SU(2)_R$ chiral perturbation theory ($\chi$PT) have been studied in [7, 8, 9]. One remarkable property of quark matter with nonvanishing isospin chemical potential is that it is characterized by a real measure, thus the lattice realization can be performed with standard numerical algorithms [10, 11]. The $\pi c$ phase and the $K c$ phase have been studied by NJL models in [12, 13, 14] and by random matrix models in [15]. All these models find results in qualitative and quantitative agreement, and in particular, the phase diagram of matter has been firmly established. However, regarding the low energy mass spectrum in three-flavor quark matter, we found that it was only studied in [3]. Our results are in disagreement with those of [3], the most relevant difference is in the mixing between mesonic states. Regarding the pion decay, previous works focused on density and temperature effects in standard decay channels [16, 17, 18], but not all the decay channels have been considered. The thermodynamic properties of the meson condensed phases have been studied by LQCD
simulations in [19, 20]. Previously, various results on the πc phase were derived in [9] by an NJL model. In particular, the equation of state (EoS) of the NJL model was presented. Recently in [21, 22, 23] a perturbative analysis of QCD at large isospin density has been presented. Those pQCD results are consistent with LQCD for $\mu_I \gtrsim 3m_\pi$ [23], where $m_\pi$ is the pion mass. At smaller values of $\mu_I$ it seems that pQCD underestimates the energy density and is not able to capture the condensation mechanism. However, for small values of $\mu_I$ (and $\mu_S$), χPT can be used. Although the χPT isospin density of the system has been determined several years ago [2], to the best of our knowledge a careful study of the EoS of imbalanced matter has never been derived within this framework.

In this article we briefly review how to include chemical potentials in χPT [24, 25, 26, 27, 28], then we describe some phenomenological properties related to this inclusion, like the existence of the different phases already listed, the meson masses and mixing. We will also show how the inclusion of chemical potentials affects pion decay channels [29]. Finally we will study the effect of nonzero isospin and strangeness chemical potentials on the pressure, density and state equation of the system [30].

The paper is organized as follows. In Sec. 2 I describe the model and how it predicts different phases. In Sec. 3 we show how group theory tools can be used to calculate the mass eigenstates and the meson mixing in the condensed phases and we list and discuss the results obtained. In Sec. 4 we discuss the impact of chemical potentials on charged pion decays. In Sec. 5 we study the thermodynamic properties of the system. Finally, in Sec. 6, we summarize the results.

2. Model

2.1. Lagrangian and definitions

In this section we briefly review the model that we are going to use in the following. It is the one described in [3]. The general $O(p^2)$ Lorentz invariant Lagrangian density describing the pseudoscalar mesons can be written as

$$\mathcal{L} = \frac{F_0^2}{4} \text{Tr}(D_\mu \Sigma D^{\mu} \Sigma^\dagger) + \frac{B_0^2}{4} \text{Tr}(X \Sigma^\dagger + \Sigma X^\dagger),$$

(1)

where $\Sigma$ corresponds to the meson fields, $X = 2B_0(s + ip)$ describes scalar and pseudoscalar external fields and the covariant derivative is defined as

$$D_\mu \Sigma = \partial_\mu \Sigma - \frac{i}{2} [v_\mu, \Sigma] - \frac{i}{2} [a_\mu, \Sigma],$$

(2)

with $v_\mu$ and $a_\mu$ the external vectorial and axial currents, respectively. The Lagrangian has two free parameters $F_0$ and $B_0$, related to the pion decay and to the quark-antiquark condensate, respectively, see for example [24, 25, 26, 27, 28].

The Lagrangian density is invariant under $SU(N_f)_L \times SU(N_f)_R$ provided the meson field transforms as

$$\Sigma \to R \Sigma L^\dagger,$$

(3)

and the chiral symmetry breaking corresponds to the spontaneous global symmetry breaking $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{L+R}$. In standard χPT, the mass eigenstates are charge eigenstates as well. Thus mesons are particles with a well defined mass and charge. The presence of a medium can change this picture. In particular, if the vacuum carries an electric charge, then the mass eigenstates will not typically be charge eigenstates. The effects of a medium can be taken into account by considering appropriate external currents in Eq. (1).

At vanishing temperature the ground state is determined by maximizing the Lagrangian density with respect to the external currents. The pseudoscalar mesons are then described as
oscillations around the vacuum. We use the same nonlinear representation of [3] corresponding to
\[ \Sigma = u\bar{\Sigma}u \quad \text{with} \quad u = e^{iT\cdot\phi/2}, \] (4)
where \( T_a \) are the \( SU(N_f) \) generators and \( \bar{\Sigma} \) is a generic \( SU(N_f) \) matrix to be determined by maximizing the static Lagrangian. The reasoning behind the above expression is that under \( SU(N_f) \times SU(N_f) \) mesons can be identified as the fluctuations of the vacuum as in Eq. (3) with \( \theta_a^R = -\theta_a^L = \phi_a \).

In the following we will assume that \( a_\mu = 0, p = 0, X = 2GM, \) where \( M \) is the \( N_f \times N_f \) diagonal quark mass matrix and \( G \) is a constant, that with these conventions is equal to \( B_0 \).

Moreover, we will assume that \( v^\nu = 2\mu_0^\nu \), meaning that the vectorial current consists of the quark chemical potential, with \( \mu \) a \( SU(N_f) \) matrix in flavor space. Its explicit expression is:
\[ \mu = \text{diag}(\mu_u, \mu_d, \mu_s) = \text{diag}\left(\frac{1}{3}\mu_B + \frac{1}{2}\mu_I, \frac{1}{3}\mu_B - \frac{1}{2}\mu_I, \frac{1}{3}\mu_B - \mu_S\right) = \frac{\mu_B - \mu_S}{3}I + \frac{\mu_I}{2}\lambda_3 + \frac{\mu_S}{\sqrt{3}}\lambda_8, \] (6)

It is important to remark that this model only holds for \( |\mu_B| \lesssim 940 \text{ MeV}, |\mu_I| \lesssim 770 \text{ MeV} \) and \( |\mu_S| \lesssim 550 \text{ MeV} \) [3].

2.2. Ground state and different phases
To find the ground state we have to substitute (4) in the Lagrangian (1). It is not necessary to use a complete \( SU(3) \) parametrization for \( \bar{\Sigma} \), but is sufficient:
\[ \bar{\Sigma} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{pmatrix}, \] (7)
because \( \bar{\Sigma} \) has to be orthogonal to the chemical potential in the \( SU(3) \) generator space [29]. Substituting (7) in (1) and maximizing, we find three different vacua, that implies that there are three ground states and therefore three different phases [3]:

- Normal phase:
  \[ \mu_I < m_\pi, \quad \mu_S < m_K - \frac{1}{2}\mu_I, \] (8)
  characterized by
  \[ \alpha_N = 0, \quad \beta_N \in (0, \pi), \quad \bar{\Sigma}_N = \text{diag}(1,1,1). \] (9)

- Pion condensation phase:
  \[ \mu_I > m_\pi, \quad \mu_S < \frac{-m_\pi^2 + \sqrt{(m_\pi^2 - \mu_I^2)^2 + 4m_K^2\mu_I^2}}{2\mu_I}, \] (10)
  characterized by
  \[ \cos \alpha_\pi = \left(\frac{m_s}{\mu_I}\right)^2, \quad \beta_\pi = 0, \quad \bar{\Sigma}_\pi = \begin{pmatrix} \cos \alpha_\pi & \sin \alpha_\pi & 0 \\ -\sin \alpha_\pi & \cos \alpha_\pi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{3} + \frac{2}{3}\cos \alpha_\pi I + i\lambda_2 \sin \alpha_\pi + \frac{\cos \alpha_\pi - 1}{\sqrt{3}}\lambda_8. \]
• Kaon condensation phase:

\[
\mu_S > m_K - \frac{1}{2}\mu_I \quad \mu_S > \frac{-m_\pi^2 + \sqrt{(m_\pi^2 - \mu_I^2)^2 + 4m_K^2\mu_I^2}}{2\mu_I} ,
\]

characterized by \( \cos \alpha_K = \left( \frac{m_K}{\mu_I + \mu_S} \right)^2 \), \( \beta_K = \pi/2 \).

\[\Sigma_K = \begin{pmatrix}
\cos \alpha & 0 & \sin \alpha \\
0 & 1 & 0 \\
-\sin \alpha & 0 & \cos \alpha
\end{pmatrix} = \frac{1 + 2\cos \alpha_K}{3} I + \frac{\cos \alpha_K - 1}{2\sqrt{3}} \left( \sqrt{3}\lambda_3 - \lambda_8 \right) + i\lambda_5 \sin \alpha_K .\]

Note that the kaon condensation can only happen for

\[\mu_S > \bar{\mu}_S = m_K - \frac{m_\pi}{2}.\]

3. Meson Masses and Mixing

3.1. Mixing

As we have seen in Sec. 2.2 the ground state in the condensed phases is not diagonal and thus has SU(3) charges that cause symmetry breaking. It is useful to study the breaking pattern to learn something about the possible meson mixing in the condensed phases.

The starting Lagrangian has an \( SU(3)_L \times SU(3)_R \) symmetry, broken to \( SU(3)_V \) by the quark masses. The introduction of chemical potentials via the external current (5) further reduces this symmetry to \( U(1)_L \times U(1)_R \times U(1)_L \times U(1)_R \), meaning that the \( \lambda_3 \) and \( \lambda_8 \) terms in (5) break isospin and hypercharge conservation. When the system enters one of the condensed phases the vacuum acquires a charge and thus there is no symmetry left.

In all of these cases we can use the quantum numbers of the \( SU(2) \) subgroups of \( SU(3) \) to label the states and to find out the ones that can mix. We need only two of them, because they are not independent. The states that can mix and the related quantum numbers are shown in table 1

| Mixing states | (T,U) |
|---------------|-------|
| \( \pi_+ , \pi_- \) | (1, 1/2) |
| \( K_+, K_- \) | (1/2, 1/2) |
| \( K_0 , \bar{K}_0 \) | (1/2, 1) |

Table 1. Mixing mesons with the corresponding T-spin and U-spin quantum numbers. These quantum numbers label the \( SU(3) \) subspace spanned by the corresponding mesonic states. The \( \pi_0 \) and the \( \eta \) do not appear because they are not U-spin eigenstates.

Unfortunately, this is not sufficient to determine if \( \pi_0 \) and \( \eta \) can mix, because they do not have a well defined U and V spin, so we have to study deeply how the ground state affects them. Let us first consider the normal phase. In the normal phase there is no operator that can induce the mixing of the mesonic states, thus the mesonic states remain unchanged but the \( Q_3 \) and \( Q_8 \) charges will induce Zeeman-like mass splittings.

In any of the condensed phases, there is an additional charge that is spontaneously induced, and the corresponding operator will lead to mixing.

Let us first focus on isospin (or T-spin). We have to consider two cases. Suppose that the vacuum has a charge that commutes with \( T^2 \), as in the \( \pi c \) phase, say the charge corresponding
to $T_2 = i(T_- - T_+)$, see Eq. (4). The $T_{\pm}$ operators can induce mixing among the charged pions and among the kaons. On the other hand, $T$-spin conservation does not allow the $|\pi_0\rangle = |T = 1, T_3 = 0\rangle$ to mix with the $|\eta\rangle = |T = 0, T_3 = 0\rangle$.

Now suppose instead that the vacuum has a charge that does not commute with $T^2$ as in the $Kc$ phase. Any operator that does not commute with isospin will commute with $U$-spin or with $V$-spin. In the $Kc$ phase $Q_3(0) \neq 0$, then the vacuum is not invariant under this charge. However, since $|T_3, U\rangle = 0$ it follows that $U$-spin is conserved. The lowering and raising operators inducing the mixing will be $U_{\pm}$. Regarding the $\pi_0$ and the $\eta$, in this case we have that $|U = 1, U_3 = 0\rangle$ and $|U = 0, U_3 = 0\rangle$ do not mix. Since $|U = 1, U_3 = 0\rangle = \frac{|\eta\rangle + \sqrt{3}|\eta\rangle}{2}$ and $|U = 0, U_3 = 0\rangle = \frac{\sqrt{3}|\eta\rangle - |\eta\rangle}{2}$, these will be the mass eigenstates.

3.2. Masses
The mass eigenstates are found diagonalizing the Lagrangian in the different phases. They present the mixing predicted by the group theory analysis of the previous subsection. They have been calculated in [29]. Here we only show plots of their $\mu_L$ dependence for different values of $\mu_S$ in figure 1. Note that these results differs from the ones of [3]. Because our results agree with the group theory reasoning explained above, we assert that the results in [3] are affected by some calculation error.

4. Pion decays
It has been shown how the introduction of chemical potentials can change meson masses and mixing. Here I will describe how it affects the charged pion decay. The processes are:

$$\pi^+ \rightarrow \ell^+ \nu_\ell,$$

$$\pi^- \rightarrow \ell^- \bar{\nu}_\ell,$$

that in the Standard Model (SM) have the decay width:

$$\Gamma^{0}_{\pi \rightarrow \ell \nu} = \frac{G_F^2 F^2_{\pi}}{4\pi} V_{ud}^2 m_{\ell}^2 m_{\pi} \left(1 - \frac{m_{\ell}^2}{m_{\pi}^2}\right)^2,$$

where $G_F$ is the Fermi constant, $V_{ud}$ is the $ud$ CKM matrix element, $m_\ell$ and $m_\pi$ are the lepton and pion masses.

For the following calculation I will assume $\mu_S = 0$ for simplicity.

In the normal phase chemical potentials do not mix states, so the only change is that we have to replace $m_\pi$ with $m_{\pi_{\pm}} = m_\pi \mp \mu_L$ [29].

In the pion condensation phase the $\pi_+$ is massless, so it does not decay, while the $\tilde{\pi}_-$ is a mixture of $\pi_{\pm}$ so it can decay in both $\ell^+ \nu_\ell$ and $\ell^- \bar{\nu}_\ell$. The related decay widths are:

$$\frac{\Gamma_{\tilde{\pi}_- \rightarrow \ell^+ \nu_\ell}}{\Gamma_{\pi \rightarrow \ell \nu}} = \frac{|U_{21}^2 \cos \alpha + i U_{22}^*|}{2} \frac{m_{\tilde{\pi}_-} m_{\pi}^{-2}}{m_{\pi}} \left(1 - \frac{m_{\ell}^2}{m_{\tilde{\pi}_-}^2}\right)^2,$$

$$\frac{\Gamma_{\tilde{\pi}_- \rightarrow \ell^- \bar{\nu}_\ell}}{\Gamma_{\pi \rightarrow \ell \nu}} = \frac{|U_{21}^2 \cos \alpha - i U_{22}^*|}{2} \frac{m_{\tilde{\pi}_-} m_{\pi}^{-2}}{m_{\pi}} \left(1 - \frac{m_{\ell}^2}{m_{\tilde{\pi}_-}^2}\right)^2,$$

where:

$$\begin{pmatrix} \tilde{\pi}_+ \\ \tilde{\pi}_- \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \pi_+ \\ \pi_- \end{pmatrix}.$$
Figure 1. (color online) Mass spectrum of the pseudoscalar mesonic octet. Top panel, results obtained for $\mu_S = 200$ MeV. The vertical solid line represents the second order phase transition between the normal phase and the pion condensation phase. In this case the strange quark chemical potential is below the threshold value for kaon condensation, 425 MeV, thus the kaon condensed phase does not take place for any value of $\mu_I$. Middle panel, results obtained for $\mu_S = 460$ MeV. The vertical solid line represents the second order phase transition from the normal phase to the kaon condensation phase. The dashed line corresponds to the first order phase transition between the kaon condensed phase to the pion condensed phase. Bottom panel, results obtained for $\mu_S = 550$ MeV, corresponding to the largest value of $\mu_S$.

5. Thermodynamic

Because we are studying the zero temperature thermodynamics all the fluctuations can be neglected, so the pressure of the system can be found substituting $\Sigma \rightarrow \bar{\Sigma}$ in (1). In the different phases we get:

\[
\begin{align*}
 p_{N}^{LO} &= 0 \\
n_{\pi c}^{LO} &= \frac{F_0^2 \mu_I^2}{2} \left( 1 - \frac{m_{\pi}^2}{\mu_I^2} \right) \left( 1 - \frac{m_{K}^2}{\mu_K^2} \right) \\
p_{Kc}^{LO} &= \frac{F_0^2 \mu_K^2}{2} \left( 1 - \frac{m_{K}^2}{\mu_K^2} \right) \\
\end{align*}
\]

where $\mu_K = \mu_I/2 + \mu_S$. Number density and equation of state can be derived using:

\[
\begin{align*}
n_{i,LO} &= \frac{\partial p_{LO}}{\partial \mu_i} \\
\epsilon_{\pi c,LO} &= \mu_{I} n_{I} + \mu_{S} n_{S} - p,
\end{align*}
\]

where the index $i$ can represent $I=$isospin or $K=$kaon. In the $\pi c$ phase we obtain:

\[
\begin{align*}
n_{\pi c,LO} &= F_0^2 \mu_I \left( 1 - \frac{m_{\pi}^4}{\mu_I^4} \right) \left( 1 + 2 \frac{m_{\pi}^2}{\mu_I^2} - 3 \frac{m_{\pi}^4}{\mu_I^4} \right) \\
\end{align*}
\]
Figure 2. (color online) Leptonic decay rates of charged pions in normal phase and in the condensed phase normalized to the value in vacuum. The phase transition between the normal phase and the pion condensed phase corresponds to the solid vertical line. Top, results obtained for vanishing leptonic chemical potential. Bottom, results obtained assuming weak equilibrium. In this case the decay in positively charged leptons is Pauli blocked. In both plots the thick solid line represents $\Gamma_\mu^{-}/\Gamma_0$, the thin solid line represents $\Gamma_{\mu^+}/\Gamma_0$, the thick dashed line represents $\Gamma_{e^{-}/\bar{\nu}_e}/\Gamma_0$, and the thin dashed line represents $\Gamma_{e^+/\nu_e}/\Gamma_0$.

Figure 3. (color online) Energy density over the Stefan-Boltzmann limit. The lattice points have been obtained at $T = 20$ MeV; the colors correspond to the different lattice volumes considered in [19]. The orange dotted line corresponds to the pQCD results of [23]. The dashed black line corresponds to our results.

while the analogous results in the kaon condensation phase can be obtained substituting $m_\pi \to m_K$ and $\mu_I \to \mu_K$, see [20, 30]. For comparing the $\chi$PT energy density with the results obtained by LQCD simulations in [19] and by pQCD in [23], we divide it by the Stefan-Boltzmann limit, which has been defined in [19] as $\epsilon_{SB} = 9\mu_4^4/(4\pi^2)$. In Fig. 3 we report our ratio $\epsilon_{LO}/\epsilon_{SB}$ and the results of [19] and [23]. We immediately notice that the $\chi$PT curve perfectly captures the peak structure at low $\mu_I$, while it begins to depart from the LQCD results after $\mu_I \sim 2m_\pi$, indicating the breakdown of the LO approximation. An interesting result is that within our framework we can obtain an analytic expression for the position of the peak, which for the $\pi c$ phase is given by $\mu_{\text{peak}}^I = (\sqrt{13} - 2)^{1/2} m_\pi \simeq 1.276 m_\pi$ and is independent of $f_\pi$. This result is very close to the LQCD results obtained in [19], where the values $\mu_{\text{peak}}^I = \{1.20, 1.25, 1.275\} m_\pi$ have been obtained considering different spatial volumes $L^3$ with side $L = \{16, 20, 24\}$, respectively. The continuum-linearly-extrapolated value for the peak position is $\mu_{\text{peak}}^I = 1.30 m_\pi$. 


6. Conclusion
We have shown how meson physics in presence of chemical potentials can be described using Chiral Perturbation Theory. There are three different phases: a normal phase, a pion condensation phase and a kaon condensation phase. The condensed phases ground states have a charge and thus can generate mixing among mesons.

In this context we have illustrated how the mixing is influenced by model symmetries and that groups theory constraints the mixing possibilities. These results, obtained by group theory alone, is expected to hold in any theory describing meson states. Then we have described the masses of mesons in the three phases, calculated using the Lagrangian (1). These masses are in perfect agreement with the group theory reasoning.

We have also described how the charged pion decay is influenced by the chemical potentials, showing that their inclusion in the model leads to a significant asymmetry between the decay in $\ell^+\nu_\ell$ and the decay in $\ell^-\bar{\nu}_\ell$.

Finally we have derived the pressure and equation of state of the system at $T=0$, comparing them with the equivalent results obtained in lattice simulations and obtaining a very good agreement.

These results can be applied for example in the physics of compact stars, the study of the cosmic ray and the study of in medium nuclear decays.

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