Investment projects implementation with production facilities location taking into account the environmental pollution

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Abstract

When implementing an investment projects a problem of production facilities location with taking into account the environmental pollution arises. A mathematical model of the industrial enterprises location in the region is formalized and studied in this paper. It is necessary to locate objects polluting the environment in such way as to maximize the total income of the players. The total income is calculated as income from the activities of enterprises, minus the funds that are spent on the recover damage to the environment. This problem is formalized as a non-cooperative game with n players that exploit common sources – natural objects. Numerical example is solved.

1 Introduction

Let us define a Cartesian coordinate system. The axis OX is directed to the east, the axis OY is directed strictly to the north. In the region under consideration there are natural objects $A_1, \ldots, A_m$. Each object is characterized by a pair of coordinates $(x_j, y_j)$. Participants in the competitive process are objects: "pollutants" $B_1, \ldots, B_n$ and "contaminated" $A_1, \ldots, A_m$. The number of players is $n$, and the number of natural objects is $m$.

The basics of the approaches and methodologies applied here can also be seen in [1-50].

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2 Main results

The set of the player \(i\) strategies:

\[
D_i = \{u_i = (x_i, y_i), \rho \leq \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \leq \bar{\rho}, \forall j \in m, j \neq i, i \in n \}
\]

\[
0 \leq x_i \leq x_{\max}, 0 \leq y_i \leq y_{\max}.
\]

where \(\rho(\bar{\rho})\) – is the minimum (maximum) distance between objects.

For each players strategy profile the players payoff functions are defined. The income of the player is equal to the value of its payoff function.

Let us take the income function for the \(i\) player as follows:

\[
H_i(u) = \sum_{j=1, j\neq i}^{m} \frac{L_{ij}}{\rho(i, j)} - \sum_{j=1, j\neq i}^{m} \frac{Q_{ij} W_i}{2\pi \rho^2(i, j)}, i = 1, n,
\]

where \(m\) is the number of players, \(m\) is the number of natural objects; \(\rho(i, j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\) is the distance between the industrial enterprise \(i\) and the natural object \(j\).

The income from the activity of the enterprise \(i\), received by the player \(i\) is:

\[
\frac{L_{ij}}{\rho(i, j)},
\]

where \(L_{ij} \geq 0\) is the amount of the loss, depending on the distance \(\rho(i, j)\) between the objects \(i\) and \(j\). The greater the distance between the enterprise and the natural object, the lower of the income amount of the \(i\) player will receive.

The amount of resources that the player \(i\) spends on compensating for environmental damage to the natural object \(j\):

\[
\frac{Q_{ij} W_i}{2\pi \rho^2(i, j)}.
\]

\(Q_{ij}\) is the weighting factor. It defines environmental damage that object \(i\) causes to object \(j\). \(W_i\) is the amount of harmful substances that the object \(i\) emits into the environment.

If the object \(i\) does not harm the object \(j\), then \(Q_{ij} = 0\).

The function \(H_i(u)\) in the domain of \((\rho \leq \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \leq \bar{\rho})\) is continuous and has partial derivatives of the first and second order that are continuous in this region. Hence, it is smooth in the domain of the job.

3 Numerical example

Let’s consider a numerical example.
A noncooperative game $G = (N, \{X_i\}_{i \in N}, \{H_i\}_{i \in N})$ is considered where $N = 3$ is the number of players, $X_i$ is the set of player $i$ strategies, $H_i$ is the player $i$ payoff function. Let the number of natural objects be $m = 5$. Let the region have an area of 15 square kilometers. We set $\pi = 3$. Let the natural objects $A_1, A_2, A_3, A_4, A_5$ in the region be arranged as follows:

Table 1: Location of natural objects.

| Natural objects | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|-----------------|-------|-------|-------|-------|-------|
| Coordinate $x$  | 2     | 5     | 9     | 14    | 8     |
| Coordinate $y$  | 3     | 9     | 6     | 1     | 13    |

For the enterprise of each player there is a permissible set of points where it is possible to build it, that is, each player $i$ has an acceptable set of strategies:

Table 2: Admissible positions $(B_1, B_2, B_3)$ for placing the company’s player 1.

| Industrial enterprise 1 | $B_1$ | $B_2$ | $B_3$ |
|-------------------------|-------|-------|-------|
| Coordinate $x$          | 7     | 1     | 9     |
| Coordinate $y$          | 8     | 2     | 10    |

Table 3: Admissible positions $(C_1, C_2, C_3, C_4)$ for placing the company’s player 2.

| Industrial enterprise 2 | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-------------------------|-------|-------|-------|-------|
| Coordinate $x$          | 6     | 11    | 5     | 8     |
| Coordinate $y$          | 4     | 15    | 3     | 15    |

Table 4: Admissible positions $(D_1, D_2)$ for placing the company’s player 3.

| Industrial enterprise 3 | $D_1$ | $D_2$ |
|-------------------------|-------|-------|
| Coordinate $x$          | 4     | 6     |
| Coordinate $y$          | 12    | 1     |
Table 5: The amount of loss $L_{ij}$ of player 1.

| Industrial enterprise 1 | $B_1$ | $B_2$ | $B_3$ |
|-------------------------|-------|-------|-------|
| Natural object $A_1$    | 10    | 1     | 13    |
| Natural object $A_2$    | 4     | 11    | 8     |
| Natural object $A_3$    | 5     | 12    | 6     |
| Natural object $A_4$    | 13    | 15    | 14    |
| Natural object $A_5$    | 9     | 15    | 6     |

Table 6: The amount of loss $L_{ij}$ of player 2.

| Industrial enterprise 2 | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-------------------------|-------|-------|-------|-------|
| Natural object $A_1$    | 5     | 17    | 2     | 15    |
| Natural object $A_2$    | 7     | 10    | 8     | 9     |
| Natural object $A_3$    | 4     | 13    | 6     | 12    |
| Natural object $A_4$    | 11    | 16    | 13    | 18    |
| Natural object $A_5$    | 13    | 3     | 14    | 1     |

Table 7: The amount of loss $L_{ij}$ of player 3.

| Industrial enterprise 3 | $D_1$ | $D_2$ |
|-------------------------|-------|-------|
| Natural object $A_1$    | 8     | 3     |
| Natural object $A_2$    | 1     | 7     |
| Natural object $A_3$    | 5     | 4     |
| Natural object $A_4$    | 10    | 6     |
| Natural object $A_5$    | 2     | 9     |

Table 8: The weighting coefficient $Q_{ij}$, which determines the environmental damage caused by an industrial enterprise 1 to natural object $j$.

| Industrial enterprise 1 | $B_1$ | $B_2$ | $B_3$ |
|-------------------------|-------|-------|-------|
| Natural object $A_1$    | 1.15  | 2.75  | 1.45  |
| Natural object $A_2$    | 1.5   | 1.95  | 2.15  |
| Natural object $A_3$    | 1     | 1.15  | 1.05  |
| Natural object $A_4$    | 2.2   | 1.8   | 2.9   |
| Natural object $A_5$    | 1.9   | 2.6   | 1.4   |
Table 9: The weighting coefficient $Q_{ij}$, which determines the environmental damage for player 2.

| Industrial enterprise 2 | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-------------------------|-------|-------|-------|-------|
| Natural object $A_1$    | 2.4   | 1.96  | 1.34  | 2.05  |
| Natural object $A_2$    | 1.67  | 1.02  | 1.73  | 1.09  |
| Natural object $A_3$    | 2.45  | 1.75  | 1 | 2.05  |
| Natural object $A_4$    | 1.85  | 2.3   | 1.6   | 1.31  |
| Natural object $A_5$    | 1.1   | 2.7   | 1.32  | 1.09  |

Table 10: The weighting coefficient $Q_{ij}$, which determines the environmental damage for player 3.

| Industrial enterprise 3 | $D_1$ | $D_2$ |
|-------------------------|-------|-------|
| Natural object $A_1$    | 2.9   | 1.25  |
| Natural object $A_2$    | 1.05  | 1.64  |
| Natural object $A_3$    | 2.1   | 1.36  |
| Natural object $A_4$    | 1.9   | 1.82  |
| Natural object $A_5$    | 1.08  | 1.6   |

Table 11: Amount of harmful substances $W_i$, discarded enterprise $i$ to favorites

| Industrial enterprise 1 | 60 |
|-------------------------|----|
| Industrial enterprise 2 | 15 |
| Industrial enterprise 3 | 35 |

Table 12: The influence of the choice of strategies of players 2 and 3 on the income of the player 1, when using 1 strategy.

| Player positions 3/Player positions 2 | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|--------------------------------------|-------|-------|-------|-------|
| $D_1$                                | 0.17  | 0.89  | 0.25  | 0.32  |
| $D_2$                                | 1.01  | 1.82  | 1.54  | 1.76  |

We calculate the value of the payoff function of each player according to the formula (1) taking into account the data specified above. We get the following payoff matrices for players 1, 2 and 3, where the lines are player’s
Table 13: The influence of the choice of strategies of players 2 and 3 on the income of the player 1, when using 2 strategy.

| Player positions 3/Player positions 2 | C₁  | C₂  | C₃  | C₄  |
|--------------------------------------|-----|-----|-----|-----|
| D₁                                  | 1.54| 1.82| 1.01| 1.76|
| D₂                                  | 0.25| 0.89| 0.17| 0.32|

Table 14: The influence of the choice of strategies of players 2 and 3 on the income of the player 1, when using 3 strategy.

| Player positions 3/Player positions 2 | C₁  | C₂  | C₃  | C₄  |
|--------------------------------------|-----|-----|-----|-----|
| D₁                                  | 0.32| 0.25| 0.89| 0.17|
| D₂                                  | 1.76| 1.54| 1.82| 1.01|

Table 15: The influence of the choice of strategies of players 1 and 3 on the income of the player 2, when using 1 strategy

| Player positions 3/Player positions 1 | B₁  | B₂  | B₃  |
|--------------------------------------|-----|-----|-----|
| D₁                                  | 0.74| 0.85| 0.96|
| D₂                                  | 0.32| 0.46| 0.57|

Table 16: The influence of the choice of strategies of players 1 and 3 on the income of the player 2, when using 2 strategy.

| Player positions 3/Player positions 1 | B₁  | B₂  | B₃  |
|--------------------------------------|-----|-----|-----|
| D₁                                  | 0.46| 0.57| 0.32|
| D₂                                  | 0.85| 0.96| 0.74|

Table 17: The influence of the choice of strategies of players 1 and 3 on the income of the player 2, when using 3 strategy.

| Player positions 3/Player positions 1 | B₁  | B₂  | B₃  |
|--------------------------------------|-----|-----|-----|
| D₁                                  | 0.85| 0.74| 0.96|
| D₂                                  | 0.46| 0.32| 0.57|
Table 18: The influence of the choice of strategies of players 1 and 3 on the income of the player 2, when using 4 strategy.

| Player positions 3/Player positions 1 | B1 | B2 | B3 |
|--------------------------------------|----|----|----|
| D1                                   | 0.46 | 0.57 | 0.32 |
| D2                                   | 0.85 | 0.96 | 0.74 |

Table 19: The influence of the choice of strategies of players 1 and 2 on the income of the player 3, when using 1 strategy.

| Player positions 1/Player positions 2 | C1 | C2 | C3 | C4 |
|--------------------------------------|----|----|----|----|
| B1                                   | 0.65 | 0.45 | 0.74 | 0.12 |
| B2                                   | 1.63 | 1.6 | 1.7 | 1.5 |
| B3                                   | 1.25 | 0.96 | 1.32 | 0.8 |

Table 20: The influence of the choice of strategies of players 1 and 2 on the income of the player 3, when using 2 strategy.

| Player positions 1/Player positions 2 | C1 | C2 | C3 | C4 |
|--------------------------------------|----|----|----|----|
| B1                                   | 0.45 | 0.74 | 0.12 | 0.65 |
| B2                                   | 0.96 | 1.32 | 0.8 | 1.25 |
| B3                                   | 1.6 | 1.7 | 1.5 | 1.63 |

strategy 1, the columns are the strategies of player 2, the matrices are the strategies of player 3:

\[
\begin{bmatrix}
(0.444; 3.931; 1.007) & (2.326; 2.565; 0.186) & (0.654; 4.220; 2.633) & (0.836; 3.759; 1.487) \\
(5.339; 4.515; 0.697) & (6.309; 3.178; 2.525) & (3.501; 3.674; 2.323) & (6.101; 4.658; 2.044) \\
(1.154; 5.100; 1.146) & (0.902; 1.784; 2.478) & (3.210; 4.766; 1.936) & (0.613; 2.615; 1.239) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
(2.640; 1.700; 1.201) & (4.757; 4.739; 1.735) & (4.025; 2.284; 2.135) & (4.600; 6.946; 4.537) \\
(0.867; 2.444; 1.975) & (3.085; 5.352; 2.562) & (0.589; 1.589; 3.336) & (1.109; 7.845; 4.003) \\
(6.348; 3.028; 0.320) & (5.554; 4.126; 3.523) & (6.564; 2.830; 4.270) & (3.643; 6.047; 4.350) \\
\end{bmatrix}
\]

3.1 The Nash Equilibrium

In the book [2] it is presented the construction of the Nash equilibrium strategy profile. Since in our problem each of the three players has a finite number of strategies, and each competitive strategy profile corresponds to
the set of the income functions values of the players, then in this game there is at least one Nash equilibrium strategy profile in the mixed strategies. Let us find the equilibrium strategy profile in this example. Let us recall the definition of the equilibrium strategy profile.

**Definition** A set of the strategies $u^* = (u_1^*, \ldots, u_k^*)$, $(u_i^*) \in D_i$ is called a Nash equilibrium strategy profile if for any strategy $u_i \in D_i$ the following inequality is valid

$$H_i(u^*) \geq H_i(u^*|u_i), i = 1, k;$$

where $(u^*|u_i) = (u_1^*, \ldots, u_{i-1}^*, u_{i+1}^*, \ldots, u_k^*)$.

From the definition of equilibrium it follows that an agent $i$ one-sided deviation can only lead to a decrease in his income.

In this problem, the Nash equilibrium strategy profile is searched as follows: in the first step let us fix the strategy 1 of player 2 (column 1) and strategy 1 of player 3 (matrix 1). Then let us go through all the values (three) of the payoff function of player 1 and choose the largest of them.

In the second step let us fix strategy 2 of player 2 (column 2) and strategy 1 of player 3 (matrix 1). Similarly, let us find the largest value of the payoff function of player 1.

Thus, fixing the strategies of 2 and 3 players, let us find the greatest value of the payoff function of the player 1 in each column of each matrix.

After that let us search the largest value of the payoff function of player 2. Let us fix player’s strategy 3 (matrix) and player’s strategy 1 (fixed-matrix string). Further let us compare the values player’s payoff function 2 (on matrix columns). Let us find the largest value in each row of both matrices.

Similarly, by fixing the strategies of player 1 (row) and player 2 (column) let us choose the largest value of the payoff function of player 3 by its two strategies (matrices).

Finally, let us examine the intersection of the chosen win values for each player. This is the Nash equilibrium strategy profile.

Following this algorithm, let us obtain the following Nash equilibrium point: $(4.600; 6.946; 4.537)$.

### 3.2 A compromise strategy profile

Let us find a compromise strategy profile in this problem. The set of compromise strategies profile $M_i = \max\{H_i(u), u \in D\}$ is defined as follows:

$$C_k = \{u \in D | \max_i(M_i - H_i(u')) \leq \max_i(M_i - H_i(u)), \forall u \in D\}.$$  

In other words, a compromise strategy profile is a strategy profile in which the largest deviation of the payoff function of one of the players from its maximum value for $i$ is not greater than the largest deviation of the $i$
payoff function to one of the players from the maximum value in any other strategy profile.

The compromise strategy profile is searched as follows in this problem: for each player let us find the maximum value of its payoff function $M_i$:

$$M_1 = 6.564; M_2 = 7.845; M_3 = 4.537. \quad (5)$$

Thus, the ideal vector is $(M_1, M_2, M_3) = (6.564, 7.845, 4.537)$.

Let us calculate the maximum values of the residuals for each strategy profile $x \in X = B \ast C \ast D$. Here $|X| = 3 \ast 4 \ast 2 = 24$. Now let us calculate $x \in X$ maximum residual value $\delta(x)$:

$$(B_1, C_1, D_1) : \delta(1) = M_1 - H_1(1) = 6.12; \quad (6)$$

$$(B_1, C_2, D_1) : \delta(2) = M_2 - H_2(2) = 5.28; \quad (7)$$

$$(B_1, C_3, D_1) : \delta(3) = M_1 - H_1(1) = 5.91; \quad (8)$$

$$(B_1, C_4, D_1) : \delta(4) = M_1 - H_1(1) = 5.728; \quad (9)$$

$$(B_2, C_1, D_1) : \delta(5) = M_3 - H_3(1) = 3.837; \quad (10)$$

$$(B_2, C_2, D_1) : \delta(6) = M_2 - H_2(2) = 4.667; \quad (11)$$

$$(B_2, C_3, D_1) : \delta(7) = M_2 - H_2(3) = 4.171; \quad (12)$$

$$(B_2, C_4, D_1) : \delta(8) = M_2 - H_2(4) = 3.187; \quad (13)$$

$$(B_3, C_1, D_1) : \delta(9) = M_1 - H_1(3) = 5.41; \quad (14)$$

$$(B_3, C_2, D_1) : \delta(10) = M_2 - H_2(2) = 6.061; \quad (15)$$

$$(B_3, C_3, D_1) : \delta(11) = M_1 - H_1(3) = 3.354; \quad (16)$$

$$(B_3, C_4, D_1) : \delta(12) = M_1 - H_1(3) = 5.951; \quad (17)$$

$$(B_1, C_1, D_2) : \delta(13) = M_2 - H_2(1) = 6.145; \quad (18)$$

$$(B_1, C_2, D_2) : \delta(14) = M_2 - H_2(2) = 3.106; \quad (19)$$

$$(B_1, C_3, D_2) : \delta(15) = M_2 - H_2(3) = 5.561; \quad (20)$$

$$(B_1, C_4, D_2) : \delta(16) = M_1 - H_1(1) = 1.964; \quad (21)$$

$$(B_2, C_1, D_2) : \delta(17) = M_1 - H_1(2) = 5.697; \quad (22)$$

$$(B_2, C_2, D_2) : \delta(18) = M_1 - H_1(2) = 3.479; \quad (23)$$

$$(B_2, C_3, D_2) : \delta(19) = M_2 - H_2(3) = 6.256; \quad (24)$$

$$(B_2, C_4, D_2) : \delta(20) = M_1 - H_1(2) = 5.455; \quad (25)$$

$$(B_3, C_1, D_2) : \delta(21) = M_2 - H_2(1) = 4.817; \quad (26)$$

$$(B_3, C_2, D_2) : \delta(22) = M_2 - H_2(2) = 3.719; \quad (27)$$

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\[(B_3, C_3, D_2) : \delta(23) = M_2 - H_2(3) = 5.015;\] (28)
\[(B_3, C_4, D_2) : \delta(24) = M_1 - H_1(3) = 2.921;\] (29)

Among them, let us find the minimum value (maximal discrepancy) for all strategies profile \(x \in X:\)
\[(B_1, C_4, D_2) : \delta(16) = M_1 - H_1(1) = 1.964,\] (30)

that is, the desired compromise strategy profile is the strategy profile \((B_1, C_4, D_2).\)
The values of the players winning features in this strategy profile are the following \((4.600; 6.946; 4.537).\)

4 Conclusion

The two principles of optimality, Nash equilibrium and compromise solution, are considered in this paper. The compromise solution in the model is the strategy profile \((B_1, C_4, D_2).\)

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References

[1] Odum Yu. Ecology. In 2 volumes. Moscow: World Publishing, 1986. P. 376.
[2] Malafeev O. A., Zubova A. F. Mathematical and computer modeling of socio-economic systems at the level of multi-agent interaction (Introduction to the problems of equilibrium, stability and reliability). SPb.: Publishing SPbSU, 2006. P. 1006.
[3] Malafeev O. A., Sosmina V. V. Management model process of cooperative three-agent interaction. Problems of mechanics and control: nonlinear dynamical systems, 2007. №39, p. 131-144.
[4] Grigorieva K. V., Malafeev O. A. The dynamic process of cooperative interaction in the multi-criteria (multi-agent) postman task. The Bulletin of Civil Engineers, 2011. №1, p. 150-156.
[5] Malafeev O. A. Managed conflict systems. Saint-Petersburg, 2000. P. 280.
[6] Malafeev O. A., Kolokoltsov V. N. Understanding game theory. New Jersey, 2010. P. 286.
[7] Malafeev O. A., Zenovich O. S., Sevek V. K. Multi-agent interaction in the dynamic problem of managing construction projects. Economic Revival of Russia, 2012. № 1, p. 124-131.

[8] Drozdova I. V., Malafeev O. A., Parshina L. G. Efficiency of options for the reconstruction of urban housing. Economic Revival of Russia, 2008. № 3, p. 63-67.

[9] Malafeev O. A., Pachar O. V. Dynamic, non-stationary task of investing projects in a competitive environment. Problems of mechanics and control: Nonlinear dynamical systems, 2009. № 41, p. 103-108.

[10] Gordeev D. A., Malafeev O. A., Titova N. D. Probabilistic and deterministic model of the influence factors on the activities of the organization to innovate. Economic Revival of Russia, 2011. № 1, p. 73-82.

[11] Grigorieva K. V., Ivanov A. S., Malafeev O. A. Static coalition model of investment of innovative projects. Economic Revival of Russia, 2011. № 4, p. 90-98.

[12] Malafeev O. A., Chernych K. S. Mathematical modeling of the company’s development. Economic Revival of Russia, 2004. №, p. 60-66.

[13] Gordeev D. A., Malafeev O. A., Titova N. D. Stochastic model of decision-making about bringing to market an innovative product. Herald of civil engineers, 2011. № 2, p. 161-166.

[14] Kolokoltsov V. N., Malafeev O. A. Mathematical modeling of systems of competition and cooperation (game theory for all), textbook. Saint-Petersburg, 2012. P. 624.

[15] Gricai K. N., Malafeev O. A. The problem of competitive management in the model of multi-agent interaction of the auction type. Problems of mechanics and control: nonlinear dynamical systems, 2007. №39, p. 36-45.

[16] Akulenkova I. V., Drozdov G. D., Malafeev O. A. Problems of reconstruction of housing and communal services of a megacity, monograph. Ministry of Education and Science of the Russian Federation, Federal Agency for Education, St. Petersburg State University of Service and Economics, Saint-Petersburg, 2007. P. 187.

[17] Parfenov A. P., Malafeev O. A. Equilibrium and compromise control in network models of multi-agent interaction. Problems of mechanics and control: nonlinear dynamical systems, 2007. №39, p. 154-167.
[18] Malafeev O. A., Troeva M. S. A weak solution of Hamilton-Jacobi equation for a differential two-person zero-sum game. In the collection: Preprints of the Eighth International Symposium on Differential Games and Applications, 1998. P. 366-369.

[19] Drozdova I. V., Malafeev O. A., Drozdov G. D. Modeling the processes of reconcreting the housing and communal services of a metropolis in a competitive environment, monograph. The Federal Agency for Education, St. Petersburg State University of Architecture and Civil Engineering, Saint-Petersburg, 2008. P. 147.

[20] Malafeev O. A., Boitsov D. S., Redinskikh N. D. Compromise and balance in models of multi-agent management in the corrupt network of society. A young scientist, 2014. № 10 (69), p. 14-17.

[21] Malafeev O. A., Gritsai K. N., Redinskikh N. D. Competitive management in auction models. Problems of mechanics and control: nonlinear dynamical systems, 2004. № 36, p. 74-82.

[22] Ershova T. A., Malafeev O. A. Conflict management in the model of entering the market. Problems of mechanics and control: nonlinear dynamical systems, 2004. № 36, p. 19-27.

[23] Grigorieva K. V., Malafeev O. A. Methods for solving the dynamic multicriteria mailman problem. Herald of civil engineers, 2011. № 4, p. 156-161.

[24] Malafeev O. A., Troeva M. S. Stability and some numerical methods in conflict-control systems. Yakutsk, 1999. P. 102.

[25] Shkrabak V. S., Malafeev O. A., Skrobach A. V., Skrobach V. F. Mathematical modeling of processes in agro-industrial production. Saint-Petersburg, 2000. P. 336.

[26] Malafeev O. A. On the existence of a critical value of a dynamic game. Bulletin of St. Petersburg University. Series 1. Mathematics. Mechanics. Astronomy. Saint-Petersburg, 1972. № 4, p. 41-46.

[27] Malafeev O. A., Muraviev A. I. Mathematical models of conflict situations and their resolution. Volume 1. General theory and all sorts of information, Saint-Petersburg, 2000. P. 283.

[28] Malafeev O. A., Drozdov G. D. Modeling processes in the system of urban construction management. Volume 1, Saint-Petersburg, 2001. P. 401.

[29] Malafeev O. A., Koroleva O. A. The model of corruption in contracting. Proceedings Edited by Smirnov N. V., Tamasyan G.
Sh. In the collection: Management processes and persistence. Proceedings of the XXXIX International Scientific Conference of Post-Graduate Students and Students, 2008. P. 446-449.

[30] Malafeev O. A., Muraviev A. I. Modeling of conflict situations in socio-economic systems. Saint-Petersburg, 1998. P. 317.

[31] Drozdov G. D., Malafeev O. A. Modeling of multi-agent interaction of insurance processes. Monograph. Ministry of Education and Science of the Russian Federation, St. Petersburg State University of Service and Economics, Saint-Petersburg, 2010.

[32] Zubova A. F., Malafeev O. A. The Lyapunov stability and oscillation in economic models. Saint-Petersburg, 2001. P. 101.

[33] Bure V. M., Malafeev O. A. Agreed strategy in the repeated final games of N persons. Bulletin of St. Petersburg University. Series 1. Mathematics. Mechanics. Astronomy. Saint-Petersburg, 1995. № 1, p. 41-46.

[34] Malafeev O. A. On the existence of the meaning of the pursuit game. Siberian Journal of Operation Research, 1970. № 5, p. 25-36.

[35] Malafeev O. A., Redinskikh N. D., Alferov G. V. Electric circuits analogies in economics modeling: corruption networks. Proceedings Edited by: Egorov N. V., Ovsyannikov D. A., Veremey E. I. In proceeding: 2nd International Conference on Emission Electronics (ICEE) Selected papers, 2014. P. 28-32.

[36] Malafeev O. A. Conflict-driven processes with many participants. The dissertation author’s abstract on competition of a scientific degree of physical and mathematical sciencesh, Leningrad, 1987.

[37] Malafeev O. A., Neverova E. G., Nemnyugin S. A., Alferov G. V. Proceedings Edited by: Egorov N. V., Ovsyannikov D. A., Veremey E. I. Multicriteria model of laser radiation control. In proceedings: 2nd International Conference on Emission Electronics (ICEE) Selected papers, 2014. P. 33-37.

[38] Kolokoltsov V. N., Malafeev O. A. Dynamic competitive systems of multi-agent interaction and their asymptotic behavior (Part II). Herald of civil engineers, 2011. № 1, p. 134-145.

[39] Malafeev O. A. Stability of solutions to multicriteria optimization problems and conflict-controlled dynamic processes. Saint-Petersburg, 1990.
[40] Malafeev O. A., Redinskikh N. D., Alferov G. V., Smirnova T. E. Corruption in the models of the first price auction. In the collection: Management in marine and aerospace systems (UMAS-2014) 7th Russian multiconference on problems of management: Conference materials. GNC RF OAO “Concern” CNII “Electrical Appliance”, 2014. P. 141-146.

[41] Malafeev O. A., Nemnyugin S. A., Alferov G. V. Charged particles beam focusing with uncontrollable changing parameters. Proceedings Edited by: Egorov N. V., Ovsyannikov D. A., Veremey E. I. In proceedings: 2nd International Conference on Emission Electronics (ICEE) Selected papers, 2014. P. 25-27.

[42] Malafeev O. A., Redinskikh N. D., Smirnova T. E. Network model of investing projects with corruption. Management processes and sustainability, 2015. V. 2, № 1, p. 659-664.

[43] Pichugin Yu. A., Malafeev O. A. On assessing the risk of bankruptcy of a firm. In the book: Dynamic Systems: Steadiness, Management, Optimization, Theses of reports, 2015. P. 204-206.

[44] Alferov G. V., Malafeev O. A., Maltseva A. S. Game-Theoretic model of inspection by anti-corruption group. In proceeding: AIP Conference Proceedings, 2015. P. 450009.

[45] Malafeev O. A., Redinskikh N. D., Gerchiu A. L. Optimization model for the location of corrupt officials in the network. In the book: the construction and operation of energy-efficient buildings (theory and practice taking into account the corruption factor) (Passivehouse) Kolchedantsev L. M., Legalov I. N., Badin G. M., Malafeev O. A., Aleksandrov E. E., Gerchiu A. L., Vasilev U. G., collective monograph. Borovichi, 2015. P. 128-140.

[46] Malafeev O. A., Redinskikh N. D., Gerchiu A. L. Project investment model with possible corruption. In the book: the construction and operation of energy-efficient buildings (theory and practice taking into account the corruption factor) (Passivehouse) Kolchedantsev L. M., Legalov I. N., Badin G. M., Malafeev O. A., Aleksandrov E. E., Gerchiu A. L., Vasilev U. G., collective monograph. Borovichi, 2015. P. 140-146.

[47] Malafeev O. A., Chernych K. S. Mathematical modeling of the company’s development. Economic Revival of Russia, 2005. № 2, p. 23.

[48] Malafeev O. A., Muraviev A. I. Mathematical models of conflict situations and their resolution. Saint-Petersburg, 2001. Vol. 2, Mathemat-
ical foundations of modeling the processes of competition and conflicts in socio-economic systems, p. 294.

[49] Kefeli I. F., Malafeev O. A. Mathematical Principles of Global Geopolitics. Saint-Petersburg, 2013. P. 204.

[50] Malafeev O. A., Redinskikh N. D., Parfenov A. P., Smirnova T. E. Corruption in the models of the first price auction. In the collection: Institutes and the mechanism of innovative development: world experience and Russian practice, 2014. P. 250-253. Collection of scientific articles of the 4th International Scientific and Practical Conference. Managing editor: Gorochov A. A.