Generalized parton distributions: Status and perspectives

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Abstract. We summarize recent developments in understanding the concept of generalized parton distributions (GPDs), its relation to nucleon structure, and its application to high–\(Q^2\) electroproduction processes. Following a brief review of QCD factorization and transverse nucleon structure, we discuss (a) new theoretical methods for the analysis of deeply–virtual Compton scattering (\(t\)-channel–based GPD parametrizations, dispersion relations); (b) the phenomenology of hard exclusive meson production (experimental tests of dominance of small–size configurations, model–independent comparative studies); (c) the role of GPDs in small–\(x\) physics and \(pp\) scattering (QCD dipole model, central exclusive diffraction). We emphasize the usefulness of the transverse spatial (or impact parameter) representation for both understanding the reaction mechanism in hard exclusive processes and visualizing the physical content of the GPDs.

Keywords: Generalized parton distributions, high–\(Q^2\) electroproduction, QCD factorization

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INTRODUCTION

Generalized parton distributions (GPDs) have established themselves as a key concept in the study of hadron structure in QCD. On the experimental side, they enable a unified description of several high–\(Q^2\) electroproduction processes (inclusive, exclusive) on the basis of QCD factorization. On the theoretical side, they provide us with information about the transverse spatial distribution of quarks/gluons and have given rise to new ideas of “quark/gluon imaging” of the nucleon. Studies of GPDs are the subject of completed and on-going experiments (HERA, HERMES, JLab 6 GeV) and are an essential part of the physics program of future facilities for \(ep = eA\) scattering (JLab 12 GeV, EIC). The purpose of this article is to summarize the status and future perspectives of this field, with emphasis on the interplay between experimental data and theoretical/phenomenological concepts; for more detailed reviews, see Ref. [1].

QCD FACTORIZATION IN HIGH–\(Q^2\) ELECTROPRODUCTION

The basis for a partonic description of hadron structure is the idea of factorization in high–\(Q^2\) electroproduction processes. At \(Q^2 \gg R_{\text{had}}^2\) the virtual photon has a transverse resolution much smaller than the typical hadronic size, and the scattering process takes place with a quasi–free parton in the nucleon. This is well–known in inclusive DIS (see Fig. 1a), where the cross section can be factorized into that of the parton–level process, involving transverse distances \(1 = Q\), and the distributions of partons in the nucleon (PDFs), governed by long–distance dynamics \(R_{\text{had}}\). The latter are formally defined...
as the expectation values of a certain quark bilinear operators in the nucleon state, $\langle N | \bar{\psi}(0) : : \psi(z) N \rangle \mid_{z = 0}$, and can be interpreted in space–time as the densities of quarks with longitudinal momentum $xP$ in a fast–moving nucleon (momentum $P$).

The same reasoning applies to certain exclusive production processes at high $Q^2$, specifically deeply–virtual Compton scattering (DVCS) and exclusive meson production (see Fig. 1b and c), where the final–state photon (meson) is produced in the reaction with a single parton in the nucleon. The amplitudes can be factorized into the quark–level production amplitude and the GPDs, describing the coupling of the active parton to the nucleon. They depend on the momentum fractions of the initial and final parton, $x$, $\xi$ ($2\xi$ is the fractional longitudinal momentum transfer to the nucleon), as well as the transverse momentum transfer to the nucleon, $\Delta t$, or, alternatively, the invariant momentum transfer, $t$. The GPDs are formally defined as the transition matrix elements $iN^G : : N \mid \bar{\psi}(0) : : \psi(z) N \rangle$, of the quark (or gluon) bilinear operators and combine aspects of the parton densities with those of the nucleon elastic form factors; in fact, they contain both as limiting cases. QCD factorization guarantees that they are universal, process–independent characteristics of the nucleon, which can be probed using different partonic scattering processes (e.g. different meson production channels); this fact is essential not only for experimental studies of GPDs, but also for our ability to calculate them using non-perturbative QCD methods such as lattice simulations. (For a discussion of QCD radiative corrections and the scale dependence of GPDs, see Ref. [1].)

QCD factorization is formally justified is the $Q^2 \rightarrow \infty$ limit and assumes that the production process takes place predominantly over transverse distances $1/Q$. In processes at $Q^2 \approx$ few GeV$^2$ there can be significant contributions from finite–size configurations, leading to power ($1=Q^2$, higher twist) corrections to the amplitude, in particular in meson production (see below). Combining small–size and finite–size contributions in a consistent approach is an important problem for phenomenological studies.
The concept of GPDs has proved to be a most useful tool in the general effort to describe nucleon structure in terms of QCD degrees of freedom. The most interesting aspect is that the GPDs describe the transverse spatial distribution of quarks and gluons in the nucleon. This is seen most easily in the special case of zero longitudinal momentum transfer, $\xi = 0$, where the GPD represents the “transverse form factor” of the QCD operator measuring the density of partons with longitudinal momentum fraction $x$. Its Fourier transform $\Delta_{\xi} b$ describes the distribution of these partons with respect to their transverse displacement, $b$, from the center–of–momentum of the nucleon (impact parameter representation, see Fig. 2a) [2]. It corresponds to a set of tomographic parton images of the nucleon at fixed longitudinal momentum fraction $x$ (Fig. 2b). This representation provides a natural framework for visualizing the partonic structure and discussing its relation to long–distance hadronic dynamics. At $x > 0.3$ on “sees” the valence quarks (and gluons), which are distributed over transverse distances $b$ $1$ fm. At $x < M_\pi = M_N = 0.15$ the pion cloud makes a distinct contribution to the partonic structure, extending up to transverse distances $b = M_\pi$ [3]. At even smaller $x$ the dominant partons are radiatively generated singlet quarks and gluons; the area of their transverse distribution $b^2$ grows logarithmically with $1/x$ (effective Regge slope).

The transverse coordinate representation also lends itself to the discussion of polarization effects [4, 5]. There are 4 helicity components of the quark GPDs, corresponding to the operators measuring the sum/difference of quark helicities (cf. unpolarized/polarized PDFs) and their nucleon helicity non-flip/flip matrix elements (cf. Dirac/Pauli and axial/pseudoscalar form factors). The Pauli form factor–type GPD $(E)$ has an interesting interpretation in the transverse nucleon spin basis; it describes the distortion of the longitudinal motion of the quarks in a transversely polarized nucleon (Fig. 2c) [4]. Likewise, for a longitudinally polarized nucleon, one can form combinations of the unpolarized and polarized GPDs corresponding to definite quark helicity states $(H \bar{H})$, and study the spatial distribution of quarks polarized along or opposite to the nucleon spin.

**FIGURE 2.** (a) Impact parameter representation of the GPDs ($\xi = 0$). (b) Tomographic parton images of the nucleon at fixed longitudinal momentum fraction $x$. (c) Distortion of the longitudinal motion of partons by transverse polarization of the nucleon.
Another reason for interest in the GPDs is that their moments are related to fundamental static properties of the nucleon. Integration over $x$ with a weighting factor $x^n$ projects out the spin–$n$ component of the non–local quark bilinear operator. The spin–2 component coincides with the quark part of the QCD energy–momentum (EM) tensor, whose matrix elements cannot be measured directly with local electroweak probes. In particular, the EM tensor is related to the angular momentum operator; this is the basis of the Ji sum rule which expresses the total quark angular momentum in terms of the second moments of the unpolarized GPDs $E$ and $H$ \cite{6,7}. The components of the EM tensor also provide information about the QCD forces acting on a quark in the nucleon at rest (pressure, shear forces) \cite{8}; these relations have recently been illustrated by a calculation within the chiral quark–soliton model of the nucleon \cite{9}.

Moments of the GPDs, as matrix elements of local operators, can also be calculated in lattice QCD \cite{10,11}. Present simulations are limited to $n=4$, allowing for first conclusions about the correlation of $x$– and $t$–dependence in non-singlet GPDs (singlets require inclusion of disconnected diagrams). Accurate lattice results would have the potential to constrain future GPD parametrizations.

**GPDs IN eN SCATTERING**

Quantitative studies of GPDs draw from several sources of information. Important basic information comes from the parametrizations of PDFs and elastic nucleon form factors, which, respectively, constrain the GPDs in the zero momentum–transfer limit ($\xi = 0 \; \& \; t = 0$) and their first moments as functions of $t$. The “new” information about the correlation of the $x;\xi$ and $t$–dependence, which \textit{e.g.} determines the transverse profile of the nucleon and its change with $x$ in Fig. 2b, comes from measurements of high–$Q^2$ exclusive processes, in particular DVCS and meson production (see Fig. 1b and c).

The analysis of high–$Q^2$ exclusive processes generally proceeds in two steps. One first investigates the reaction mechanism and tries to ascertain that a description based on short–distance dominance and QCD factorization is applicable; this is done by testing certain qualitative implications of the approach to the short–distance regime ($Q^2$–scaling, \textit{etc.}) which do not depend on the specific form of the GPDs. One can then attempt to extract quantitative information about the GPDs by analyzing the kinematic dependences of the leading–twist (leading in $1=Q^2$) observables. The details of this program vary considerably between DVCS and meson production, because of the different complexity and reaction dynamics of the two processes.

\textit{Deeply–virtual Compton scattering.} DVCS is generally regarded as the most promising channel for probing GPDs in the valence quark regime. In this process one expects short–distance dominance to set in already at $Q^2 \approx$ few GeV$^2$, based on the formal analogy of DVCS to inclusive DIS and the experience with other exclusive two–photon processes, such as $\gamma \gamma \rightarrow \pi^0$ studied in $e^+ e^-$ annihilation. In the measured $N(e,\gamma^* e)N$ cross section the DVCS process interferes with the Bethe–Heitler (BH) process, whose amplitude is purely real and calculable in terms of the nucleon elastic form factors (see Fig. 3a); this circumstance amplifies the DVCS amplitude and allows one to measure it — and thus the GPDs — at the amplitude level, by isolating the interference term through polarization observables.
Recent measurements of the beam spin–dependent BH–DVCS interference cross section in the JLab Hall A experiment [12] indicate an early approach to $Q^2$–scaling (see Fig. 3b), in accordance with theoretical expectations, suggesting that a description based on leading–twist QCD factorization should be applicable already at $Q^2$ few GeV$^2$. Present experimental efforts focus on separating the BH–DVCS interference and the DVCS$^2$ term in the beam spin–independent cross section, separating the nucleon helicity–components of the GPDs through measurement of target spin asymmetries, and measuring neutron GPDs through quasi–free neutron DVCS with nuclear targets [7].

The methods for analyzing DVCS observables at the leading–twist level and extracting information about GPDs are well developed and have been extensively discussed in the literature. Recently, two important new tools were added to the arsenal. One is GPD parametrizations based on the idea of a $t$–channel partial–wave expansion of the hard exclusive amplitude. In the so–called dual parametrization [13], inspired by the analogy with dual amplitudes in hadron–hadron scattering, the GPD $viz.$ hard exclusive amplitude is represented as a formal series of $t$–channel resonance exchanges, which is then resummed and parametrized in terms of functions of the complexity of usual parton densities (“forward–like” functions; see Ref. [14] for a recent summary). In the approach of Ref. [15], based on the analogy with Regge theory, a similar representation is derived using complex angular momentum techniques. These $t$–channel representations have several advantages over the traditional double distribution (spectral) GPD parametrization [16]: (a) they diagonalize the QCD evolution equations for GPDs by using the conformal moments of the QCD operator; (b) they suggest a natural high–energy (small–$\xi$) expansion of the hard exclusive amplitudes. The representation of Ref. [15] has successfully been applied to fit the HERA DVCS data at NLO accuracy [17]; for a summary of the status of LO fits based on the dual parametrization see Ref. [18]. A simple GPD model based on the dual parameterization describes the bulk of the JLab spin–dependent cross section and asymmetry data [19]. However, as a general GPD parametrization the $t$–channel representation is likely to be less useful in the large–$x_B$ (large–$\xi$) region.
The other new tool are s–channel dispersion relations for the DVCS amplitude [20]; see also Ref. [17]. They allow one to restore the real part of the DVCS amplitude from the imaginary part, which is directly accessible in the spin–dependent BH–DVCS interference cross section, and an energy–independent subtraction constant. In the GPD description, the imaginary part is proportional to the GPD at the transition points \( x = \xi \), while the real part is given by a certain integral of the GPD over \( x \). The existence of a dispersion relation thus implies that there is a “hidden” relation between the values of the GPDs at \( x = \xi \) and elsewhere; this property is in a subtle way related to the fact that moments of the GPD, as matrix elements of local spin–n operators, are required by Lorentz invariance to be polynomials of degree \( n \) in \( \xi \) (“polynomiality condition”). The subtraction constant turns out to be related to the D–term [21], a distinct component of the unpolarized GPDs required by polynomiality, which is located in the region \( \xi < x < \xi \) and drops out in the zero momentum–transfer limit (\( \xi = 0 \)), and thus has no analog in the usual parton densities. If confirmed by further studies, these recent findings would have profound implications for the GPD analysis of DVCS data in the valence quark regime (\( x_B > 0.1 \)). They would mean that in measurements at fixed \( Q^2 \) only the GPDs at \( x = \xi \) (as functions of \( \xi \) and \( t \)) and the D–term (as a function of \( t \) alone) are observable; whatever information from the GPDs at \( x \notin \xi \) enters in the amplitude could be reconstructed from the dispersion relation. The main task of experiments then would be to provide accurate and comprehensive data on the imaginary part of the DVCS amplitude through measurements of the spin–dependent interference cross section.

**Exclusive meson production.** In meson production the quantum numbers of the produced meson determine the spin (unpolarized/polarized), charge parity (\( q \bar{q} \)), and flavor of the GPD, allowing one to probe the individual GPD components more selectively than in DVCS. However, because of the additional interaction required in the formation of the meson the partonic process is more complicated, and detailed study of the reaction mechanism is necessary before a factorized description can be applied.

QCD factorization for meson production relies on the fact that for \( Q^2 \! \! \! \! \downarrow \sim \infty \) the meson is predominantly produced in a small–size configuration (transverse size \( 1=Q \)), whose coupling to the target is weak (“color transparency”) and can be computed perturbatively. The approach to the small–size regime with increasing \( Q^2 \) can be verified experimentally in a model–independent manner and constitutes a crucial test of the reaction mechanism. The \( t–\)slope of the differential cross section (more precisely, the \( \Delta^2 \) slope) measures the transverse area of the interaction region, reflecting the size of the target and that of the dominant configurations of the produced meson. As \( Q^2 \) increases and small–size meson configurations become more important, one expects the \( t–\)slope to decrease and eventually stabilize (see Fig. 4a). Exactly this behavior is seen in the HERA vector meson production data (see Fig. 4b), where the exponential \( t–\)slope of \( \rho^0 \) and \( \phi \) production changes from the “soft” value of \( B \sim 10 \text{GeV}^2 \) at \( Q^2 = 0 \) to a value of \( B \sim 5 \text{GeV}^2 \) at \( Q^2 = 20 \text{–} 30 \text{GeV}^2 \); the latter value is close to the slope of \( J=\psi \) production, which is practically \( Q^2–\)independent because the \( J=\psi \) is produced in small–size configurations even at \( Q^2 = 0 \). These observations attest to the approach to the small–size regime in vector meson production at high \( Q^2 \). The limiting value of the \( t–\)slope for \( Q^2 \downarrow \sim \infty \) then reflects the size of the target only and can be associated with the \( t–\)dependence of the GPD (see Fig. 4c).
In vector meson production at \( Q^2 \gtrsim 4 \text{ GeV}^2 \) the data indicate substantial contributions from finite–size configurations in the produced meson. This is confirmed by theoretical calculations which account for the finite meson size by including the intrinsic quark transverse momentum in the partonic scattering process (higher–twist corrections) [23, 24, 25]. They describe well the high–energy data (quark transverse momentum in the partonic scattering process (higher–twist corrections) [22]. (c) In the small–size regime, the measured \( t \)–dependence can be identified with that of the gluon GPD.

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In the regime where small–size configurations dominate, meson production data can be used to extract information about the GPDs. The analysis of absolute cross sections is generally challenging, as the theoretical predictions depend on the detailed treatment of the hard scattering process (effective scale, higher–order QCD corrections; see Ref. [28] for a recent discussion) as well as on the GPD parametrization/model. One way of simplifying the analysis is to consider ratios of cross sections in which the uncertainties associated with the hard scattering process cancel, and one is left only with ratios of the the GPDs or their integrals. An example is the transverse target asymmetry in \( \gamma_L p \rightarrow \rho^0 p \), which is sensitive to the Pauli form–factor type GPD \( E \) figuring in the angular momentum sum rule [1]. A similar asymmetry can be studied in \( \gamma_L p \rightarrow K^+ \Lambda \) by measuring the recoil polarization of the produced \( \Lambda \) [29]; this process probes the \( p \rightarrow \Lambda \) decays.
FIGURE 5. The longitudinal cross section for (a) exclusive $\phi$, (b) exclusive $\rho^0$ production (adapted from Ref. [25]). The curve/error band show the GPD–based model calculation of Ref. [25].

transition GPDs, which can be related to the usual flavor–diagonal GPDs in the proton using $SU(3)$ flavor symmetry (see Ref. [29] for other interesting “ratio observables”).

Pseudoscalar meson production ($\pi^+$; $\eta$; $K^+$) in the small–size regime probes the polarized GPDs in the nucleon. A particular feature of $\pi^+$ (and to some extent also $K^+$) production is the existence of a “pole term” in the GPD, in which the QCD operator measuring the quark density is connected to the nucleon by $t$–channel $\pi^+$ ($K^+$) exchange; it gives a contribution to the amplitude proportional to the pion form factor, which is in fact the basis for measuring the latter in electroproduction experiments [30]. Better insight into the relation between the “pole” and the “non–pole” component of the GPD and their relative importance is necessary not only for improving the extraction of the pion form factor, but also for isolating the non-pole component related to the nucleon helicity structure. It could come e.g. from model–independent comparisons of $\pi^+$ (which has a pole) and $\pi^0$ (no pole) electroproduction data (see Ref. [29] for strange channels). Experimental studies of exclusive pseudoscalar meson production are challenging because the $L=T$ virtual photon cross sections have to be separated by comparing data taken at different beam energies (Rosenbluth method). There are intriguing suggestions that $\sigma_T$ in exclusive pion production above the resonance region could be described as the limit of semi-inclusive production via the fragmentation mechanism [31]; if confirmed, this could greatly aid the analysis of such processes.

GPDs IN SMALL–$x$ PHYSICS AND $pp$ SCATTERING

The notion of the transverse spatial distribution of partons conveyed by the GPDs has many important applications in small–$x$ physics and high–energy $pp$ collisions with hard processes. Measurements of the $t$–dependence of exclusive $J=\psi$ photo/electroproduction at HERA (cf. Fig. 4b) and FNAL have provided a rather detailed picture of the transverse spatial distribution of gluons with $10^{-4} < x < 10^{-2}$ [22]. In particular, these experiments have shown that the nucleon’s gluonic transverse size at $Q^2$ few GeV$^2$ is substantially smaller than its size in soft hadronic interactions at high energies, and increases less
FIGURE 6. (a) The dipole model of high-energy scattering. The dipole–nucleon scattering amplitude at fixed impact parameter, $b$, is proportional to the local gluon density in the transverse plane. (b) Hard process in $pp$ scattering.

rapidly with energy (effective Regge slope $\alpha_0 \gtrsim \alpha_{\text{soft}}$). This information provides essential input for theoretical studies of high-energy scattering in the QCD dipole model (see Fig. 6a), where the dipole–nucleon interaction is governed by the local gluon density in the transverse plane, and for modeling the initial condition of non-linear QCD evolution equations describing the approach to the black–disk regime (unitarity limit).

In high-energy $pp$ scattering with hard processes (see Fig. 6b), knowledge of the transverse spatial distribution of the partons in the colliding protons allows one to calculate the probability of the hard process as a function of the $pp$ impact parameter, $b$, and thus describe in detail the spectator interactions in such processes, which are different from minimum bias events [32]. This is particularly important in hard diffractive $pp$ scattering, where the spectator interactions determine the rapidity gap survival probability. By measuring the $p_T$ dependence of central exclusive diffraction $pp \rightarrow p + H + p$ ($H$ = high–mass system) one can even extract information about the transverse spatial distribution of gluons from the observed “diffraction pattern” [33].

FROM DENSITIES TO CORRELATIONS

A fast–moving hadron in QCD represents a complex many–body system, characterized by an equilibrium of creation and annihilation processes, capable of a wave function description as outlined by Gribov in the context of scalar field theory [34]. The GPDs summarize the single–particle structure of this many–body system, at a transverse resolution scale where one can unambiguously identify quasi–free partons (i.e., integrated over transverse momenta up to a scale $\mu^2 \lesssim R^2_{\text{had}}$). However, there is much more to be learned about this system than the single–particle densities. One example are the quantum fluctuations of the parton densities, which result from the quantum–mechanical superposition of configurations of different size, numbers of particles, etc. in the wave function, and can be revealed in hard diffractive $ep$ scattering [35]. Another interesting property are multi–parton correlations, which are probed e.g. in $pp$ scattering with multiple hard processes. There are indications of significant transverse correlations between partons, possibly related to the non–perturbative short–distance scale introduced by the spontaneous breaking of chiral symmetry in QCD (instanton size, “constituent quark” size) [36]. These correlations show up as higher–twist corrections in inclusive
DIS [37] and likely play an essential role in the $p_T$/azimuthal dependence of semi-inclusive hadron production. Studying these correlations should be the next step in the exploration of the partonic structure of the nucleon.

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