Polarization observables in double neutral pion photoproduction

The CBELSA/TAPS Collaboration

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Abstract. Measurements of target asymmetries and double-polarization observables for the reaction \( \gamma p \rightarrow p \pi^0 \pi^0 \) are reported. The data were taken with the CBELSA/TAPS experiment at the ELSA facility (Bonn University) using the Bonn frozen-spin butanol (\( C_5H_9OH \)) target, which provided transversely polarized protons. Linearly polarized photons were produced via bremsstrahlung off a diamond crystal. The data cover the photon energy range from \( E_\gamma = 650 \text{ MeV} \) to \( E_\gamma = 2600 \text{ MeV} \) and nearly the complete angular range. The results have been included in the BrGa partial wave analysis. Experimental results and the fit agree very well. Observed systematic differences in the branching ratios for decays of \( N^* \) and \( \Delta^* \) resonances are attributed to the internal structure of these excited nucleon states. Resonances which can be assigned to \( SU(6) \times O(3) \) two-oscillator configurations show larger branching ratios to intermediate states with non-zero intrinsic orbital angular momenta than resonances assigned to one-oscillator configurations.

1 Introduction

Studying the excitation spectrum of the nucleon is an important tool to gain a better understanding of the non-perturbative regime of QCD. Final states originating from the successive decay of intermediate resonances play an important role in understanding the baryon spectrum [1]. At higher excitation energies (\( W \gtrsim 1.9 \text{ GeV} \)), where the so-called missing resonances are expected, the neutral multi-meson cross sections for photo-induced reactions exceed those for the production of single neutral mesons. This emphasizes the importance of multi-meson final states. The missing resonances are states which are predicted by quark models [2,3] and by approximations of QCD on a lattice [4], but have not (yet) been found experimentally. One possible explanation is that instead of three constituent quarks participating in the internal baryon dynamics, two quarks cluster in a diquark, and the quark-diquark system forms an underlying structure (for a review on those models see e.g. Ref. [5]). This would reduce the number of degrees of freedom and the number of resonances. Alternatively, the reason might be in an
inadequate choice of the dynamical constituents. Alternative approaches to the physics of strong interactions generate hadron resonances from the interaction of their decay products [3]. It is unclear, however, if this approach is capable of explaining why so many baryon resonances expected by the quark model remain unobserved. The hadrogensesis conjecture suggests that it should be possible to generate the full spectrum of excited hadrons from hadrons of lower masses [4], but again it is still unknown, whether the missing resonances would be generated dynamically or whether they would be absent. These questions cannot be answered presently. Yet in any case, an experimental search for the missing resonances seems rewarding.

A very different interpretation of the high-mass spectrum of baryon resonances was given by Glozman [5], who started from the observation that many baryon resonances form parity doublets, i.e., pairs of resonances with similar masses, same total angular momentum J but opposite parities. At low masses, this symmetry is heavily broken: the nucleon with J = 1/2+ is about 600 MeV lighter than its parity partner N(1535)1/2−. Obviously, chiral symmetry is broken. Glozman argues that chiral symmetry breaking might not play a role in the high-mass spectrum of hadron resonances. Then, all resonances should have parity partners. Possibly, an extended symmetry even leads to mass-degenerate quartets of resonances of N++s and Δ−s with opposite parities like N(1900)3/2+, N(1895)3/2−, Δ(1920)3/2+, and Δ(1930)3/2−. The consequences of this conjecture were discussed intensely, see reviews [9,10,11]. In [12] it is pointed out that spin-parity partners naturally emerge when the (squared) masses increase with M°2 ∝ L + N (L is the total orbital angular momentum, N the radial excitation quantum number) – as predicted within AdS/QCD [13,14] and an empirical mass formula [15] – and not with M°2 ∝ L + 2N which follows from the harmonic oscillator potential. In this case, resonances with J = L + 3/2 on the leading Regge trajectory (like Δ(1232)3/2+, Δ(1950)7/2+, Δ(2420)11/2+, ..., ) should have no parity partners. Indeed, a search for Δ− states with J°2 = 7/2+ in the 1900 to 2200 MeV region found a positive-parity resonance at (1917±4) MeV and a negative-parity resonance at (2176±40) MeV. A negative parity partner of the Δ(1950)7/2+ was not observed [16]. A confirmation of the result and an extension to N++s in the fourth resonance seems promising.

When the wave function of an excited baryon is expanded into wave functions of a harmonic oscillator (with two oscillators λ and ρ), three classes of wave functions result. One class has wave functions in which the excitation energy oscillates between the two oscillators. For an excitation with two units of orbital angular momentum, either lρ = 2, λρ = 0 or lρ = 0, λρ = 2. In a second class of wave functions both oscillators are simultaneously excited, lρ = 1, λρ = 1. The majority of resonances belongs to a third class in which the wave function is given by the coherent sum of both components. Detailed calculations show that the harmonic-oscillator wave function with defined orbital angular momentum and total quark spin is often the leading part of the full wave function with only small admixtures of higher-order terms [3]. Therefore, the classes of harmonic-oscillator wave functions correspond to classes of baryon resonances. Recent studies on double-pion photoproduction [18,19] have indicated that the first class of resonances decays preferentially into a ground-state baryon and a ground-state meson, e.g. Nπ or Δ(1232)π. The third class of resonances based on a mixture of single- and two-oscillator excitations also decays into Nπ or Δ(1232)π, but decays were observed in addition in which one particle was an excited hadron: Decays like those into N(1520)π or Nσ were seen [18]. This was interpreted as evidence that the component in the wave function in which both oscillators are excited simultaneously, first de-excites into an excited hadron and a ground-state hadron. In this step, one of the two excited oscillators de-excites while the other oscillator remains excited. In the final step, the second oscillator releases its energy as well, so that finally three ground-state hadrons are produced [18,19].

This interpretation may explain why some of the expected resonances might be missing: Resonances of the second class, with both oscillators being simultaneously excited, should then decay only in two steps, never in a single decay process. Hence, they also cannot be produced in a single step process: They are neither produced in photo- nor in pion-induced reactions. Still, mixing of states could nevertheless lead to a small production rate but mostly, the calculated mixing angles are small [3], and the production strength of these resonances is expected to be small.

The study of sequential decays of high mass resonances into multiple mesons thus offers three chances, i) to search for missing resonances, ii) to study parity doublets in the high-mass region, and iii) to study the internal structure of baryons through the dynamics of sequential decays. Clearly, high statistics is required for further progress and, even more importantly, the measurement of new polarization observables.

In this paper, new data on polarization observables for the reaction
\[
\gamma p \rightarrow p\pi^0\pi^0, \tag{1}
\]
taken with a transversely polarized target and a linearly polarized photon beam, are presented. The photoproduction of two neutral pions is particularly well suited to study sequential decays. It suffers much less from non-resonant contributions than the production of π+π− pairs. In reaction (1), there is no diffractive ρ production, no Δ++-Kroll-Ruderman term (direct Δ++π− production), and contributions from t-channel processes or Born terms are suppressed compared to e.g. ρπ− photoproduction. The p2π0-channel is hence very well suited for the investigation of baryon resonances.

The paper is organized as follows: In Section 2, a short description of the CBELSA/TAPS experiment is given. In Section 3, the selection criteria are discussed to obtain a nearly background-free sample of events for reaction (1). Section 4 is devoted to the determination of the polariz-
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2 Experimental setup

The CBELSA/TAPS experiment (setup shown in Fig. 1) is located at the Electron Stretcher Accelerator ELSA [20] at the University of Bonn. Electrons with an energy of 3.2 GeV impinged on a diamond crystal (thickness 500 µm) and produced linearly polarized photons with a maximal polarization of 66 % at 850 MeV via coherent bremsstrahlung [21]. The deflection of the electrons in a magnetic dipole field was measured with a tagging system consisting out of 480 scintillating fibers and 96 scintillating bars, covering the energy range between 2 % and 88 % of the incident electron energies.

The tagged photons hit the Bonn frozen-spin target placed at the center of the Crystal Barrel calorimeter. The target material was butanol (C₄H₉OH) and the protons in the hydrogen nuclei were transversely polarized. During the data taking the 2 cm long target cell was cooled to 50–60 mK. A holding coil produced a magnetic field of about 0.6 T. Relaxation times of several hundred hours were reached with this so-called frozen-spin technique. Every two to three days a re-polarization of the target was necessary. This lead to a mean polarization of about 74 %.

For background studies the target cell containing the butanol was replaced with a carbon foam target. Using the same cryostat (then operating at about 1 K) and the carbon foam target at about the same area density as the carbon foam target at about the same area density as the target center and covered the lowest polar angle range from 12° down to about 1°. Combined, the two calorimeters covered the polar angle range of 1° to 156° and the full azimuthal angle $\varphi$, thus covering about 95 % of the full solid angle in the laboratory frame.

Charged-particle identification was possible by a scintillating fiber detector (513 fibers in three layers) directly surrounding the target, 180 plastic scintillation counters in two layers in front of the first 90 CB, and plastic scintillators in front of the TAPS crystals.

A CO₂-Cherenkov detector was placed between the CB and TAPS calorimeters to identify and suppress electromagnetic background at the trigger level.

The flux of incoming photons was measured by taking into account the coincidence between the Gamma-Intensity-Monitor (GIM), a 4 × 4 lead glass matrix at the end of the beam line, and the tagging system. Since the GIM efficiency decreased at high rates ($\gg$ 1 MHz), the FluMo detector, placed right in front of the GIM, measured $e^+e^-$-pairs from photon conversion in a lead foil at low rate, thus allowing to monitor the rate-dependent GIM efficiency.

Trigger conditions demanded at least two energy deposits in the calorimeters and no veto signal from the cherenkov detector. If there was no energy deposit in the TAPS calorimeter or the first 90 crystals of the CB, a charged hit in the inner detector was required since the signals of the central part of the Crystal Barrel calorimeter could not be used in the first-level trigger. In this case, two energy deposits in the CB were required, identified by a fast cluster encoder in the second-level trigger. In case of only one energy deposit below 27.5°, at least one additional hit was required in the second-level trigger.

For further details on the experimental setup see Ref. [27].

3 Data selection

Both pions in the reaction $\gamma p \to p\pi^0\pi^0$ predominantly decay into two photons, leading to four photons in the final state. Therefore, events with exactly one charged and four neutral detector hits were selected. The charged hit was then treated as a proton. Hits in the charge sensitive fiber detector or the CB-scintillators in forward direction alone without a corresponding calorimeter hit were considered proton candidates as well. This was necessary to also reconstruct protons with low kinetic energy, which did not deposit sufficient energy in the calorimeters or hit an insensitive region between the calorimeter modules.

A time coincidence was required between the measured final state particles and the scattered electron in the tagger. Random time background in the tagger was subtracted by means of a side-band subtraction.

The kinematics of the reaction $\gamma p \to p\pi^0\pi^0$ can be described by five independent kinematic variables. One choice of these variables makes use of two planes inherent
to the process: the reaction plane spanned by the incoming photon and one of the outgoing particles $p_1$, and the decay plane spanned by the other two outgoing particles $p_2$ and $p_3$, as illustrated in Fig. 3. The kinematic variables then are the energy of the incoming photon $E_\gamma$, the scattering angle in the CMS $\cos \vartheta$, the invariant mass $m_{23}$ of the particles $p_2$ and $p_3$, the angle $\phi_{23}$ between the reaction plane and the decay plane, and the angle $\theta_{23}$ between the $z$-axis and the particle $p_2$.

Since the two pions are indistinguishable, two choices for the particles $p_1(p_2p_3)$ remain, namely: $p(\pi^+\pi^0)$ and $\pi^0(\pi^+\pi^-)$. The polar angle $\vartheta$ is influenced by the Lorentz boost to the laboratory frame. Hence, the difference of the proton hit to the calculated proton angles was used which should vanish. A Gaussian distribution was fitted to the angular distributions to determine the cut limits and events within $2\sigma$ were retained. Additionally, the mass of the calculated proton has to agree with the nominal proton mass. Here, a Novosibirsk function [28] was fitted and events within the $2.27\%$ and $97.73\%$ quantiles were selected, thus retaining $95.45\%$ of the events just like with the $\pm 2\sigma$ cut in case of the angular differences. The four photons in the final state were grouped pair-wise to form the two pions. A Novosibirsk function was fitted to both pion mass spectra and events were selected within the same quantiles mentioned above. Fig. 3 shows the four cut variables on a logarithmic scale before the kinematic cuts were applied and after all but the cut on the shown variable.

The width of the distributions varied with the kinematic variables, e.g. the width of the missing mass depended on the energy of the incoming photon. Hence, all the above-mentioned fits were done separately for different beam energies and polar angles of the $4\gamma$-system. Approximate limits for the cuts are given in Table 1.

Table 1. Approximate limits of the kinematic cuts.

| variable         | lower limit | upper limit |
|------------------|-------------|-------------|
| $\Delta \varphi$ | $168^\circ \ldots 175^\circ$ | $185^\circ \ldots 192^\circ$ |
| $\Delta \vartheta$ | $-11^\circ \ldots -2^\circ$ | $2^\circ \ldots 10^\circ$ |
| $m_{\pi,\text{miss}}$ | $840 \text{ MeV}$ $\ldots$ $910 \text{ MeV}$ | $990 \text{ MeV}$ $\ldots$ $1140 \text{ MeV}$ |
| $m_{\gamma\gamma}$ | $105 \text{ MeV}$ $\ldots$ $120 \text{ MeV}$ | $145 \text{ MeV}$ $\ldots$ $155 \text{ MeV}$ |

3.1 Kinematic cuts

The kinematics of the initial state of the reaction were known (photon of known energy along the $z$-axis, target proton at rest). Thus, the four-momentum of one final-state particle could be calculated using energy and momentum conservation.

In order to suppress background events, several kinematic constraints had to be fulfilled for an event to be retained. In the center-of-mass system (CMS) the final-state proton and the four-photon system depart back-to-back. Therefore, the difference of the azimuthal angle $\varphi$ of the proton to the $4\gamma$-system should be $180^\circ$ (coplanarity).

3.2 Kinematic fit

After applying these kinematic cuts combinatorial background remained, stemming from the three possibilities to combine four photons into two pions. Of course, mostly only one of those combinations survived the mass cuts but in about 10% of the cases there were more than one. A kinematic fit chooses the combination with the highest...
confidence level (see below), thus eliminating the combinatorial background. The control spectra of the kinematic fit (pull distribution and confidence level) were determined from a very broad cut event sample ($\gtrsim 5\sigma$). A detailed description of the kinematic fit can be found in [29]. Due to the finite detector resolution, energy and momentum conservation were not fulfilled exactly in the data. The concept of the fit is to vary the measured parameters within their uncertainties to exactly fulfill the constraints of the fit. Energy and momentum conservation as well as the masses of the final state particles can be used as constraints. Since the proton was not always stopped in the calorimeters and the energy deposit of protons differs from that of photons, the proton’s measured energy did not correspond to its true energy. Hence, the proton was not fitted but treated as a missing particle and only the three mass constraints remained for the fitted hypothesis $\gamma p \rightarrow p_{\text{miss}} \pi^0 \pi^0$. The kinematic fit provides a handle on the systematic effects, the so-called pull distribution, which is the difference of the fitted to the measured variables normalized by the uncertainties of the difference. The pull distributions of the uncut events should follow a standard normal distribution (vanishing mean and unit standard deviation).

An additional criterion for the quality of the fit is the so-called confidence level (CL) which is the integral over the $\chi^2$ probability density distribution starting at the given $\chi^2$. For independent parameters with normally distributed statistical uncertainties, the CL-distribution should be flat for events which fulfill the hypothesis (no background). A rise to low values of the CL is due to background events where the fit has a large $\chi^2$.

Apart from polarizable protons in the hydrogen nuclei, the butanol data also contains events on protons bound in carbon and oxygen nuclei which are subject to initial Fermi motion. Thus, the hypothesis of an initial proton at rest led to a steady rise to low values in the CL for those events (cf. black line in Fig. 4). For control purposes, the CL of the kinematic fit was investigated with data taken on a 5 cm long hydrogen target. The resulting CL distribution is shown in Fig. 4 as a blue line and shows the expected flat behavior with the rise at small CL values. Additionally, the CL distribution is shown for butanol events from which the contributing carbon events have been subtracted. Also here the expected flat behavior is observed. A cut on the CL of $\gtrsim 0.1$ was applied to reduce the number of background events in the selected data.

The reaction $\gamma p \rightarrow p\pi^0\eta$ has the same 4$\gamma$ final state and therefore, it is possible for a photon combination from such a reaction to mimic the two pion reaction. The hypothesis $\gamma p \rightarrow p_{\text{miss}}\pi^0\pi^0$ was also fitted and events which had a higher CL for that anti-hypothesis than for the $\gamma p \rightarrow p_{\text{miss}}\pi^0\pi^0$ hypothesis were discarded.

After the cuts described above, the selected data sample contained about 254000 events in the energy range of 650–2600 MeV shown in section 4.4.

### 3.3 Background contribution

It is not possible to estimate the (potentially polarized) background contamination of the selected events from the fit of $\gamma p \rightarrow p_{\text{miss}}\pi^0\pi^0$ since signal and background cannot be separated in the fitted pion mass. Therefore, a less constrained kinematic fit with the hypothesis $\gamma p \rightarrow p_{\text{miss}}\pi^0\gamma\gamma$ was employed. The determined amount of background $x$ will then not be used on an event-to-event basis in the maximum-likelihood fit (cf. section 4.2) for extracting the observables. Instead, it will be used as a first order correction of the observables and as a source of the systematic uncertainty, cf. section 4.3.

Fig. 4. Confidence level distribution of hydrogen data (blue), butanol data (black) and butanol with carbon data subtracted (red) for the hypothesis $\gamma p \rightarrow p_{\text{miss}}\pi^0\pi^0$. In this plot, only very broad data selection cuts were applied to the data.

Fig. 5. Example of the mass distribution of the nearest neighbors. The signal (red) is described by the sum of two Gaussian functions (the narrower one with negative amplitude), the background function (dashed blue) is a second order polynomial. The dip on the pion peak is due to the fact that the fit uses the best 2$\gamma$ combination for the fitted pion. The data is binned here for presentational purposes only.
The invariant mass of the unfitted (and uncut) photon pair in the hypothesis $\gamma p \rightarrow p_{\text{miss}} \pi^0 \gamma \gamma$ showed a peak at the pion mass on a background distribution, cf. Fig. 6. The fit uses the best $2\gamma$ combination for the fitted pion, which results in a dip on the pion peak.

The amount of background can vary significantly depending on the kinematic point. To allow for variations in the 5-dimensional phase space, the background contribution was determined with methods of a multivariate sideband subtraction [39]. This method, also known as the Q-factor method, assigns a quality factor to every event which gives the probability for this event to originate from the signal sample. For a given event, the nearest neighbors in the 5-dimensional phase space were taken. The corresponding $2\gamma$-mass distribution was then fitted with an unbinned maximum-likelihood fit by a sum of two normal distributions (peak with dip) and a polynomial background. For the fit to reliably converge the number of nearest neighbors in the 5-dimensional phase space was increased until there were 300 events in the background region ($m_{\gamma\gamma} < 100$ MeV or $> 170$ MeV) or 15000 nearest neighbors in total were reached. From the ratio of the background function to the function describing the signal the amount of background at the position of the seed event (at the position of the vertical line in Fig. 5) was estimated. An example of such a fit is shown in Fig. 6.

Originally, it was planned to provide the PWA group with the data on the polarization observables on an event-to-event basis and therefore also determine the background (and dilution factor, cf. section 4.1) on an event-to-event basis. But since the data also contain beam asymmetry contributions from bound nucleons (with in principle unknown values of the beam asymmetries, cf. section 4.1), at this time only binned data were provided and the averaged background within these bins ($x_j = (1 - Q_{\text{bin}}^0)$, where $x$ is the background in a one- or multi-dimensional bin) was used for the determination of the systematic uncertainty, cf. section 4.3. In addition, a correction of the observables has been performed as described below.

The background was determined to be at the order of 1–2.5% for incident photon energies below 1500 MeV decreasing to below 0.5% for higher energies, as shown in Fig. 6 and 7. Only for the lowest $m_{\gamma\gamma}$ significantly higher background of up to $\approx 5\%$ was found, see Fig. 6.

The observables shown in section 4.3 were corrected by a factor $(1 - x)^{-1}$, where $x$ denotes the amount of background, since the background events will lower the value of the resulting polarization observable, assuming
the background to be unpolarized. A possible polarization of the background events will be treated in the section on the systematic uncertainties (cf. section 4.3).

The background contribution was somewhat over-estimated with the method described here. A test with simulated \( p\gamma - \) and \( p\pi^0\pi^0\)-final states, the former serving as the background contribution, revealed the determined background contribution to be about 50% higher than expected from the simulated values. The effect of using the higher value is still covered by the systematic uncertainty on the observables were already somewhat overestimated, the correlation contributions were omitted in \( \Delta BG_{syst} \), which is (roughly) proportional to the determined background contribution (cf. Eq. (11)).

Correlations between the chosen nearest neighbors were found to be small: The set of nearest neighbors of a typical event has an intersection only with about 1% of all events and in this case the mean cardinality of the intersecting event has an intersection only with about 1% of all events found to be small: The set of nearest neighbors of a typical event has an intersection only with about 1% of all events. The mean cardinality of the intersecting event was found to be small: The set of nearest neighbors of a typical event has an intersection only with about 1% of all events.

4 Data analysis

The cross section of double pion photoproduction using a linearly polarized beam (polarization degree \( \delta_\text{beam} \)) with angle \( \phi_\text{beam} \) to the \( x \)-axis of the reaction plane and a transversely polarized target (polarization degree \( \Lambda \)) with angle \( \phi_\text{target} \) to the \( x \)-axis of reaction plane can be written in the form (cf. [31]):

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \cdot \left\{ 1 + d \Lambda T \sin(\beta - \varphi) + d \Lambda \sin(\beta - \varphi) \cdot P_x + d \Lambda \sin(\beta - \varphi) \cdot P_y + \delta_\text{t} \sin(2\alpha - 2\varphi) \cdot I_{\text{eff}}^\text{t} + \delta_\text{t} \cos(2\alpha - 2\varphi) \cdot I_{\text{eff}}^\text{c} + d \Lambda \delta_\text{t} \sin(\beta - \varphi) \sin(2\alpha - 2\varphi) \cdot P_x^c + d \Lambda \delta_\text{t} \sin(\beta - \varphi) \sin(2\alpha - 2\varphi) \cdot P_y^c + d \Lambda \delta_\text{t} \cos(\beta - \varphi) \cos(2\alpha - 2\varphi) \cdot P_x^c + d \Lambda \delta_\text{t} \cos(\beta - \varphi) \cos(2\alpha - 2\varphi) \cdot P_y^c \right\},
\]

(2)

where \( d\sigma_0/d\Omega \) denotes the unpolarized cross section, \( P_x \) and \( P_y \) are target asymmetries, \( I_{\text{eff}}^\text{t} \) and \( I_{\text{eff}}^\text{c} \) beam asymmetries, and \( P_x^c, \ P_y^c, \ P_x^e, \ P_y^e \) double polarization observables.

The photon polarization plane and the target polarization vector are oriented with respective angles \( \alpha \) and \( \beta \) to the \( x \)-axis in the laboratory frame. The reaction plane is rotated by an angle \( \varphi \) relative to the laboratory frame, as depicted in Fig. 8. The observables were determined using a butanol target which contained polarizable free protons as well as unpolarized bound nucleons in the carbon and oxygen nuclei, i.e. \( I_{\text{eff}}^\text{e} = dl^\text{e} + (1 - d)I_{\text{bound}}^\text{e} \). The target polarization was effectively reduced by the so-called dilution factor \( d \) which is the fraction of polarizable free protons. The measured beam asymmetries \( I_{\text{eff}}^\text{t} \) and \( I_{\text{eff}}^\text{c} \) are a mixture of the beam asymmetries from free protons and those from the bound nucleons.

If the kinematic variable \( \delta^* \) is integrated out, half of the polarization observables in Eq. (2) vanish, namely \( P_x \), \( \Gamma_{\text{eff}} \), \( P_y^c \), and \( P_x^e \). When integrating over \( \phi^* \) and \( \theta^* \), the kinematic mimics the situation of single-meson photoproduction and the remaining observables are often identified with the single-meson polarization observables [32]:

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \cdot \left\{ 1 + d \Lambda T \sin(\beta - \varphi) - d \Lambda \delta_\text{t} \sin(2\alpha - 2\varphi) \cdot P_x^c - d \Lambda \delta_\text{t} \delta_\text{c} \cos(\beta - \varphi) \sin(2\alpha - 2\varphi) \cdot P_x^c \right\},
\]

(3)

where \( P_y \) becomes \( T \), \( \Gamma_{\text{eff}}^\text{c} = -\delta_\text{e} \), \( P_x^c = -H \), and \( P_x^e \).

4.1 Dilution factor

The contributions from the bound nucleons were determined by placing the carbon foam target of density close to that of the carbon and oxygen nuclei in the butanol target within the cryostat. The azimuthal angle difference between measured proton and the four photons (coplanarity) was used to calculate the dilution factor. Since the bound nucleons are subject to Fermi motion their coplanarity spectrum is broadened compared to the free protons, as shown in Fig. 9.

Differences in the photon flux (\( \Phi \)) of butanol and carbon data were accounted for by an energy-dependent weighting factor which is the ratio of the measured photon fluxes: \( \Phi = \Phi_{\text{But}}/\Phi_{\text{C}} \). Additional small differences in the target area density were treated by a global scaling factor \( s = 1.008 \pm 0.009 \), which was determined from the ratio of the butanol to the carbon events in the outer parts of the coplanarity spectrum (below 155° or above 205°, corresponding to \( \approx 5 \sigma \), cf. Fig. 9).

In single-meson photoproduction the double polarization observable denoted as \( P \) here is identical to the recoil asymmetry, usually denoted as \( P \). In double meson photoproduction this is in general not the case.

\( \text{Fig. 8. Definition of the relevant angles in the laboratory frame and the reaction frame. Details see text.} \)
The dilution factor number a hyper-sphere in the 5-dimensional phase space. Then the neighbors from the butanol data were taken which defines is described in section 3.3. Here, a number basis using the nearest neighbor method similar to what ble to determine the dilution factor on an event-to-event axis.

\[ \vartheta \cos \]  

are shown. The insert shows the same spectrum on a linear axis.

\[ \vartheta \cos \]  

To keep the volume of the hyper-sphere (and thus the size already scaled. Events with 650 MeV < \( E_\gamma \) < 2600 MeV are shown. The insert shows the same spectrum on a linear axis.

Using the global scaling factor \( s \), it was then possible to determine the dilution factor on an event-to-event basis using the nearest neighbor method similar to what is described in section 3.3. Here, a number \( n_B \) of nearest neighbors from the butanol data were taken which defines a hyper-sphere in the 5-dimensional phase space. Then the number \( n_C \) of events from the carbon target within the same hyper-sphere were determined. The dilution factor is then calculated as

\[ d = \frac{n_B - w \cdot s \cdot n_C}{n_B}. \]

To keep the volume of the hyper-sphere (and thus the size chosen such that 10 events (before weighting) from the carbon data were inside.

To show the variation of the dilution factor with the kinematic variables, the event-based dilution factors were sorted into bins. Fig. 9 shows an example of such a histogram binned in \( E_\gamma \) and \( \cos \vartheta_\rho \).

The event-based results are consistent with a previously performed 2-dimensional binned analysis, which did not extend to the highest \( E_\gamma \) presented here.

### 4.2 Event-based maximum-likelihood fit

The polarization observables were determined by an event-based maximum-likelihood fit to the azimuthal distribution of the events. Therefore, it was not necessary to bin the data in the azimuthal angle \( \varphi \) but only in the kinematic variables. The azimuthal variation of the polarized cross section was expanded in a Fourier series

\[ f_{\text{phy}} = \frac{d\varOmega}{d\varOmega_{\varphi}/d\varTheta_{\varphi}} = 1 + \sum_{k=1}^{3} [a_k \sin(k \varphi) + b_k \cos(k \varphi)], \]

where the coefficients \( a_k \) and \( b_k \) were determined from Eq. 2 and include only the eight polarization observables as free parameters. They are given by:

\[ a_1 = + d_{\text{PP}} \sin(\varphi) - d_{\text{Y}} \cos(\varphi) \]
\[ - \frac{1}{2} \delta_{d\varOmega/d\varTheta_{\varphi}} [(P_x^e - P_y^e) (\cos(2\varphi) \cos(\varphi) + \sin(2\varphi) \sin(\varphi))] \]
\[ - (P_x^e + P_y^e) (\sin(2\varphi) \cos(\varphi) - \cos(2\varphi) \sin(\varphi))] \]
\[ b_1 = + d_{\text{P}} \cos(\varphi) + d_{\text{Y}} \sin(\varphi) \]
\[ + \frac{1}{2} \delta_{d\varOmega/d\varTheta_{\varphi}} [(P_x^e - P_y^e) (\sin(2\varphi) \cos(\varphi) - \cos(2\varphi) \sin(\varphi))] \]
\[ + (P_x^e + P_y^e) (\cos(2\varphi) \cos(\varphi) + \sin(2\varphi) \sin(\varphi))] \]
\[ a_2 = - \delta_{d\varOmega/d\varTheta_{\varphi}} \cos(2\varphi) + \delta_{d\varOmega/d\varTheta_{\varphi}} \sin(2\varphi) \]
\[ b_2 = + \delta_{d\varOmega/d\varTheta_{\varphi}} \sin(2\varphi) + \delta_{d\varOmega/d\varTheta_{\varphi}} \cos(2\varphi) \]
\[ a_3 = - \frac{1}{2} \delta_{d\varOmega/d\varTheta_{\varphi}} [(P_x^e + P_y^e) (\cos(2\varphi) \cos(\varphi) - \sin(2\varphi) \sin(\varphi))] \]
\[ - (P_x^e - P_y^e) (\sin(2\varphi) \cos(\varphi) + \cos(2\varphi) \sin(\varphi))] \]
\[ b_3 = + \frac{1}{2} \delta_{d\varOmega/d\varTheta_{\varphi}} [(P_x^e + P_y^e) (\sin(2\varphi) \cos(\varphi) + \cos(2\varphi) \sin(\varphi))] \]
\[ + (P_x^e - P_y^e) (\cos(2\varphi) \cos(\varphi) - \sin(2\varphi) \sin(\varphi))]. \]

Additionally, potential detector asymmetries were expanded in a Fourier series as well:

\[ f_{\text{det}} = 1 + \sum_{k=1}^{\infty} [c_k \sin(k \varphi) + d_k \cos(k \varphi)]. \]

Those two series had to be multiplied and normalized, leading to

\[ f_{\text{rel}} = \frac{f_{\text{phy}} \cdot f_{\text{det}}}{\int f_{\text{phy}} f_{\text{det}} d\varphi} = \frac{f_{\text{phy}} \cdot f_{\text{det}}}{1 + \frac{1}{2} \sum_{k=1}^{3} (a_k b_k + b_k d_k)}. \]

Since the series representing the physical signal \( f_{\text{phy}} \) stops at \( k = 3 \), the coefficients can only be influenced from

\[ 4 \quad \text{A global normalization factor of (2\pi)}^{-1} \text{only leads to a global shift in the log-likelihood and can therefore be omitted.} \]
coefficients of the detector series \( f_{\text{det}} \) up to twice that number. Therefore, the series expansion of \( f_{\text{det}} \) could be stopped at \( k = 6 \). The detector efficiency coefficients \((c, d)\) decouple from the physics coefficients \((a, b)\) by changing the polarization directions: \( \Lambda \mapsto -\Lambda \) or \( \delta \mapsto -\delta \).

To account for random time background in the data, the likelihood function \( L \) also contained a function of form Eq. 3 called \( f_{\text{tbg}} \) for those events. The coefficients of this function are determined with events in a side-band region. Thus, the following expression was minimized:

\[
- \ln L = - \sum_{i=1}^{N_{\text{sig}}} \ln (\xi f_{\text{sig}}(\varphi_i) + (1 - \xi) f_{\text{tbg}}(\varphi_i)) - \sum_{j=1}^{N_{\text{tbg}}} \ln (f_{\text{tbg}}(\varphi_j)),
\]

where \( \xi = \frac{N_{\text{sig}} - R N_{\text{sig}}}{N_{\text{sig}}} \) and \( R = \frac{1}{20} \) is the ratio of the time cut width to the side-band width.\(^5\)

Together with the eight observables and twelve detector asymmetry coefficients in the signal part, there are the same number of parameters in the time background part, leading to a total of 40 free parameters.

### 4.3 Systematic uncertainty

The main sources of the systematic uncertainty are the uncertainty of the beam and target polarization values, the uncertainty of the dilution factor, and the amount of background events in the data sample. The relative uncertainties of the first three contributions are directly proportional to the absolute value of the polarization observable. In order not to under-estimate the systematic uncertainty for very small observables, the relative uncertainties of the first three contributions are multiplied with a convolution of the absolute value of the corrected (see below) measured observable \( \mathcal{O}/(1 - x) \) with a Gaussian which has the statistical uncertainty \( \sigma_s \) of the observable as its width:

\[
\mathcal{O}' := \int_{-5\sigma_s}^{5\sigma_s} \left| \frac{\mathcal{O}}{1 - x} - \omega \right| \cdot \frac{1}{\sqrt{2\pi\sigma_s^2}} \cdot \exp \left( -\frac{\omega^2}{2\sigma_s^2} \right) \, d\omega. \tag{8}
\]

In the numerical integration, a precision of \( 10^{-6} \) can be achieved with integration limits of \( \pm 5\sigma_s \).

\(^5\) This follows directly from the product-to-sum identities, e.g. \( \cos x \cos y = \frac{1}{2} (\cos(x - y) + \cos(x + y)) \).

\(^6\) Different polarization settings are already needed, in order to extract the eight observables from the six coefficients \((a, b)\). This effectively disentangles the double polarization observables from the target polarization observables in \( a_1 \) and \( b_1 \).

\(^7\) For example, an observable of absolute value \( 0 \pm 0.1 \) would have a systematic uncertainty from the polarization uncertainties of 0.

The systematic uncertainty due to the amount of background \( \Delta \mathcal{O}_{\text{BG}} \) is independent of the value of the polarization observable. The true observable \( \mathcal{O} \) is related to the measured one \( \langle \mathcal{O} \rangle \) and the possible background observable \( \mathcal{O}_{\text{BG}} \) by

\[
\mathcal{O} = (1 - x) \cdot \mathcal{O} + x \cdot \mathcal{O}_{\text{BG}}, \tag{9}
\]

where \( x \) is the amount of background. Dividing the measured observable by \((1 - x)\) is a first order correction but a possible contribution of polarized background remains.

The uncertainty due to the background can be estimated by the expected value of the quadratic difference of the corrected measured observable \( \frac{\mathcal{O}}{1 - x} \) to the true value \( \mathcal{O}_S \):

\[
(\Delta \mathcal{O}_{\text{BG}})^2 = E \left[ \left( \frac{\mathcal{O}}{1 - x} - \mathcal{O}_S \right)^2 \right]. \tag{10}
\]

Since nothing is known about the background observable, a uniform distribution between \(-1\) and 1 was assumed (principle of maximum entropy), leading to the following expression for the systematic uncertainty:

\[
(\Delta \mathcal{O}_{\text{BG}})^2 = \frac{1}{2} \int_{-1}^{1} \left( \frac{x}{1 - x} \right)^2 \mathcal{O}_{\text{BG}}^2 \, d\mathcal{O}_{\text{BG}} = \left( \frac{x}{1 - x} \right)^2 \frac{1}{3}. \tag{11}
\]

Systematic effects due to limited acceptance in the 5D phase space were investigated by Monte Carlo events with polarization weights from a BnGa PWA solution. The observables from generated events (with perfect acceptance) were compared to the reconstructed events (taking the detector acceptance into account). Examples of such comparisons are shown in Fig. 14. The mean of the distribution of differences mostly vanished within 1σ of the statistical uncertainty of the difference of the MC event sample. Therefore, no significant deviation due to the acceptance was found.

The individual systematic uncertainties are added quadratically, leading to

\[
\Delta \mathcal{O}_{\text{syst}}^2 = \left( \frac{\Delta \varphi_{\text{syst}}}{\delta \varphi} \right)^2 + \left( \frac{\Delta \Lambda_{\text{syst}}}{\Lambda} \right)^2 + \left( \frac{\Delta \delta_{\text{syst}}}{d} \right)^2 \mathcal{O}'^2 + (\Delta \mathcal{O}_{\text{BG}})^2. \tag{12}
\]

The systematic uncertainty of the dilution factor can be related to the uncertainty of the scaling factor \( s \) by

\[
\left( \frac{\Delta \delta_{\text{syst}}}{d} \right)^2 = \left( \frac{\Delta \delta_{\text{syst}}}{s} \cdot \frac{1 - d}{d} \right)^2. \tag{13}
\]

The systematic uncertainty of the scaling factor was determined from the spread of an energy-dependent determination of the scaling factor, which resulted in \( \Delta \delta_{\text{syst}}/s = 5\% \). The uncertainty from the ratio of the photon fluxes \( w \) is negligible in the dilution factor uncertainty because of the high statistics of the flux measurements.
For the target asymmetries no beam polarization is necessary. Hence, the systematic uncertainty of the beam polarization is not added in that case. Typical values for the relative polarization uncertainties are 5% (beam) and 2% (target), for the relative dilution factor uncertainty \( \lesssim 1\% \), and for \( \Delta B_{\text{syst}} \) mostly about 0.01 with only a few areas exceeding 0.03 (cf. section 5.3).

In most cases, the total systematic uncertainty amounts to \( \lesssim 0.03 \).

4.4 Results

Due to the limited statistics, a full analysis in 5-dimensional bins could not be performed. Instead, a series of 2-dimensional analyses were done where all pair-wise combinations, i.e.

\[ E_{\gamma} = \cos \theta_{\pi^0,0}, \quad E_{\gamma} - m_{\pi^0,0}, \quad \cos \theta_{\pi^0,0} - m_{\pi^0,0}, \quad \cos \theta_{\pi^0,0}, \quad \ldots \]

were investigated.

A selection of results is shown in Figs. 12-19 for the target asymmetries and in Figs. 20 and 21 for the double polarization observables together with predictions from the 2π-MAID [33], BuGa2014-02 [34] and the new fit of the BuGa PWA.

In case the observable is plotted versus \( \cos \theta_{\pi^0,0} \), this means that the two pions span the decay plane (particles 2 and 3 in Fig. 3). On the other hand, if \( \cos \theta_{\pi^0,0} \) or \( \phi_{\pi^0,0} \) are used, the proton and one of the pions are particles 2 and 3.

While some deviations occur, the BuGa2014-02 describes the data reasonably well at lower energies. But at higher energies, especially above 1700 MeV, larger discrepancies become visible. The 2π-MAID prediction is rarely consistent with the data.

In case of the observables being plotted against \( \phi^* \), several symmetry properties arise. Observables that are even under the transformation \( \phi^* \rightarrow 2\pi - \phi^* \) do not vanish while integrating over \( \phi^* \), while the others do. Additionally, the two pions are indistinguishable, which is why the observables are invariant under exchange of the pions.

Thus, if the two pions are spanning the decay plane (vectors \( p_2 \) and \( p_3 \) in Fig. 3) the observables are unchanged by the transformation \( \phi \rightarrow \phi^* + \pi \). This symmetry property is fulfilled exactly in the data, since the pions were symmetrized. Data points calculated using these symmetry properties are represented as open symbols in Figs. 14, 15, 18 and 19. The good agreement between the closed and open symbols (their difference normalized to their statistical uncertainty follows a standard normal distribution with mean \( \mu = 0.04 \pm 0.06 \) and \( \sigma = 1.01 \pm 0.04 \)) shows that the symmetry properties are indeed fulfilled in the data.

In addition to the 2-dimensional analyses, a 4-dimensional analysis was performed for the target asymmetries below 1250 MeV. Here, the beam energy \( E_\gamma \), \( \cos \theta_{\pi^0,0} \), \( m_{\pi^0,0} \) as well as \( \phi_{\pi^0,0} \) were used as kinematic variables, only \( \theta_{\pi^0,0}^{p\pi^0} \) was integrated out. Fig. 22 shows an example of this analysis where one energy bin of the target asymmetry \( P_1 \) is plotted as a function of \( \phi_{\pi^0,0}^{p\pi^0} \). The \( \cos \theta_{\pi^0,0} \) is varied in the rows while \( m_{\pi^0,0} \) is varied in the columns. Fig. 24 shows the same for a higher energy bin. The analogue result for the target asymmetry \( P_2 \) can be found in Fig. 25 and Fig. 26 respectively. In all cases the aforementioned symmetry properties are fulfilled very well.

It is easily seen that the observables can be significantly different in different bins. For example, the lowest \( \cos \theta_{\pi^0,0} \) and \( m_{\pi^0,0} \)-bin in Fig. 27 shows the opposite behavior in \( \phi_{\pi^0,0}^{p\pi^0} \) from the middle \( \cos \theta_{\pi^0,0} \) and \( m_{\pi^0,0} \)-bin.

By comparing the 4-dimensional result to the partially integrated ones (shown in Fig. 26 for the same energy bin as depicted in Fig. 22) it becomes immediately clear that information is lost in the integrated observables.

5 Partial wave analysis

The data on the target asymmetries and double polarization observables for the reaction \( \gamma p \rightarrow p\pi^0\pi^0 \) were included in the data base for the Bonn-Gatchina (BuGa) multi-channel partial-wave analysis (PWA). The formalism is described in detail in Refs. [35,36,37,38]. In addition to elastic and inelastic \( \pi N \)-scattering data, the analysis includes data on photo-induced reactions with two and three particles in the final state like \( \gamma p \rightarrow p\pi^+, n\pi^+, p\pi^0, n\pi^0, p\pi^+\pi^0, p\pi^0\eta, AK^+, \Sigma^0K^+, \Sigma^+K^+, \pi^+ \), and \( p\pi^+\pi^- \).

The inclusion of data on the latter reaction is important. We illustrate this importance using \( N(1700)3/2^- \) and \( \Delta(1700)3/2^- \) as examples. The reaction \( \gamma p \rightarrow K^+A \)**8** when plotting against \( \phi_{\pi^0,0}^{p\pi^0} \) there are only 3 independent bins (with 12 \( \phi^* \)-bins) per energy bin due to the symmetrization (indistinguishable pions).
Fig. 12. The target asymmetry $T$ as a function of beam energy $E_{\gamma}$ and $\cos \vartheta_{\pi_0\pi_0}$. The colored lines represent PWA solutions: $2\pi$-MAID in black, BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

Fig. 13. The target asymmetry $T$ as a function of beam energy $E_{\gamma}$ and $m_{\pi_0\pi_0}$. The colored lines represent PWA solutions: $2\pi$-MAID in black, BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.
Fig. 14. The target asymmetry $P_y$ as a function of beam energy $E_\gamma$ and $\phi^*_{\pi^+\pi^-}$. The open symbols make use of the symmetry properties. The colored lines represent PWA solutions: BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

Fig. 15. The target asymmetry $P_x$ as a function of beam energy $E_\gamma$ and $\phi^*_{\pi^+\pi^-}$. The open symbols make use of the symmetry properties. The colored lines represent PWA solutions: BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.
The target asymmetry $T$ as a function of beam energy $E_\gamma$ and $\cos \vartheta_p$. The colored lines represent PWA solutions: 2$\pi$-MAID in black, BuGa 2014-02 in red, new BuGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

**Fig. 16.**

The target asymmetry $T$ as a function of beam energy $E_\gamma$ and $m_{\pi^+\pi^-}$. The colored lines represent PWA solutions: 2$\pi$-MAID in black, BuGa 2014-02 in red, new BuGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

**Fig. 17.**
**Fig. 18.** The target asymmetry $P_y$ as a function of beam energy $E_\gamma$ and $\phi^*_{p\gamma}$. The open symbols make use of the symmetry properties. The colored lines represent PWA solutions: BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

**Fig. 19.** The target asymmetry $P_x$ as a function of beam energy $E_\gamma$ and $\phi^*_{p\gamma}$. The open symbols make use of the symmetry properties. The colored lines represent PWA solutions: BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.
Fig. 20. The double polarization observables $\tilde{P}$ and $H$ as a function of beam energy $E_\gamma$ and $\cos \theta_{\phi\phi}$ (left), or $E_\gamma$ and $m_{\phi\phi}$ (right). The colored lines represent PWA solutions: BnGa 2014-02 in red, new BnGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

or $\gamma p \rightarrow \eta p$ can proceed only via $N(1700)3/2^-$ formation, while $\pi^+ p \rightarrow \pi^+ p$ or $K^+ \Sigma^+$ proceeds only via $\Delta(1700)3/2^-$. The two resonances can be separated in $\pi^- p$ elastic and charge exchange scattering due to their different Clebsch-Gordan (CG) coefficients to $p\pi^0$ and $n\pi^+$. There is, however, no information to separate contributions from $N(1700)3/2^-$ and $\Delta(1700)3/2^-$ in the reaction $\gamma p \rightarrow p\pi^0\pi^0$. The same is of course true for the higher mass $N^*$- and $\Delta^*$-resonances. Technically, the amplitudes for the two isospins have the tendency to become very large in the PWA and to interfere destructively. In Ref. [18] this was avoided by penalizing, i.e. limiting the photo-couplings of large amplitudes interfering destructively. With data on $\gamma p \rightarrow p\pi^+\pi^-$ in the fit, the contributions of $N^*$’s and $\Delta^*$’s can be separated due to their different CG couplings.

Pion-induced reactions are included as well, in particular the real and imaginary part of the $\pi N$ elastic scattering partial-wave amplitudes from the GWU analysis [39] or, alternatively, from the Karlsruhe-Helsinki analysis (KH) [40]. New data have been included in the BnGa data base, e.g. (double) polarization data on $\gamma p \rightarrow p\eta$ [41] and data on the reaction $\gamma p \rightarrow p\pi^+\pi^-$. Data on the latter reaction were published recently [42], where reaction cross section and nine one-dimensional histograms were reported. Data on this reaction were made available to us on a single-event basis. Thus, the data could be used in an event-by-event likelihood fit.

The BnGa PWA uses background terms – represented by $t$- and $u$-channel exchange processes and by a few mostly constant terms added to the $K$-matrix pole terms – and a large number of resonances to describe the resonant part of meson-baryon interactions. Resonances are coupled to all allowed decay channels. When branching ra-
As discussed in the introduction, baryon excitations can be divided into classes in which the excitation energy fluctuates between the $\rho$ and $\lambda$ oscillator (one-oscillator

Data sets are given a weight. Without weights, low-statistics data, e.g. on polarization observables, may be reproduced unsatisfactorily without significant deterioration of the total $L_{\text{tot}}$. For any data set, its weight is increased as long as the gain in likelihood for the data set is larger than twice the loss in the overall likelihood. The total likelihood function is normalized to avoid an artificial increase in statistics by the weighting factors. Differences in the fit quality are given as $\chi^2$ difference, with $\Delta\chi^2 = -2\Delta\ln L_{\text{tot}}$.

The main impact of the data presented here is in the observation of cascade decays into $pt\pi$ via intermediate resonances. Table 2 shows the results. In the fit, the intermediate resonances listed in the table were admitted. The large number of contributing resonances and decay modes leads to a large number of close-by solutions resulting in the uncertainties given in the table. The new numbers are compared to those from [18]. Most of the branching ratios are compatible with the previous findings within 1σ; the difference between the new and the older results exceeds 2σ only for a few exceptions.
Fig. 22. Four-dimensional determination of the target asymmetry $P_λ$ as a function of $\phi^*_p$. The $\cos θ_λ^*$ is varied within a row, the invariant mass $m_{π^0}$ within a column. Only a single energy bin is shown here. The colored lines represent PWA solutions: BuGa 2014-02 in red, new BuGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

Fig. 23. Four-dimensional determination of the target asymmetry $P_λ$ as a function of $\phi^*_p$. The $\cos θ_λ^*$ is varied within a row, the invariant mass $m_{π^0}$ within a column. Only a single energy bin is shown here. The colored lines represent PWA solutions: BuGa 2014-02 in red, new BuGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.
Fig. 24. Four-dimensional determination of the target asymmetry $P_\gamma$ as a function of $\phi_{\rho, p}^\pi$. The $\cos \vartheta_{\rho, p}$ is varied within a row, the invariant mass $m_{\pi\rho}$ within a column. Only a single energy bin is shown here. The colored lines represent PWA solutions: BuGa 2014-02 in red, new BuGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.

Fig. 25. Four-dimensional determination of the target asymmetry $P_\pi$ as a function of $\phi_{\rho, p}^\pi$. The $\cos \vartheta_{\rho, p}$ is varied within a row, the invariant mass $m_{\pi\rho}$ within a column. Only a single energy bin is shown here. The colored lines represent PWA solutions: BuGa 2014-02 in red, new BuGa 2022-02 in blue. The systematic uncertainty is shown as a gray band.
Table 2. Branching ratios (in %) for decays of nucleon and Δ* resonances. The spread of results from different solutions is used to estimate the uncertainties. x signifies forbidden transitions. Entries marked as "-" were fitted to small values and set to zero to avoid an excessive number of free parameters. The intermediate resonances on the right side of the line carry orbital excitations. Where available, the results from Ref. [18] are given as small numbers. For \(N(1710)/2^+\) two values were given in [18]; both are reproduced here. Small numbers marked with † were taken from [19]. An inclusion of the \(\Delta(1750)/1^+\) improves the fit quality only slightly, its existence cannot be established from our fits. Allowing for \(\Delta(2000)/3^+\) \(\rightarrow N(1700)/3^+\) \(\pi\) decays leads to an improvement of the fit. Due to its large uncertainty, the respective branching ratio is not given in the table.

| \(N(1535)/2^+\) | \(N(1520)/3^+\) | \(N(1650)/1^+\) | \(N(1700)/3^+\) | \(N(1675)/5^+\) | \(\Delta(1620)/1^+\) | \(\Delta(1700)/3^+\) |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| \(N(1440)/2^+\) | \(N(1460)/3^+\) | \(N(1720)/3^+\) | \(N(1680)/5^+\) | \(N(1910)/2^+\) | \(\Delta(1920)/3^+\) | \(\Delta(1905)/5^+\) |
| \(N(1710)/1^+\) | \(N(2000)/3^+\) | \(N(2100)/1^+\) | \(N(1880)/1^+\) | \(\Delta(1860)/5^+\) | \(\Delta(1900)/3^+\) | \(\Delta(1900)/5^+\) |
| \(N(1895)/1^+\) | \(\Delta(1900)/3^+\) | \(\Delta(1900)/5^+\) | \(\Delta(1900)/3^+\) | \(\Delta(1900)/5^+\) | \(\Delta(1900)/3^+\) | \(\Delta(1900)/5^+\) |

| Resonance | \(N^\pi\) | \(L\) | \(\Delta\pi\) | \(\delta_{L<0}\) | \(\delta_{L>0}\) | \(N(1440)\pi^\pi\) | \(N(1520)\pi^\pi\) | \(N(1535)\pi^\pi\) | \(N(1680)\pi^\pi\) | \(N\sigma\) | \(L\) |
|------------|---------|-----|---------|----------|----------|----------------|----------------|----------------|----------------|---------|-----|
| \(N(1535)/2^+\) | 46±6 | 0 | x | 5±3 | 2 | 6±5 | 0 | 1 | - | 1 | 5 | 0 | 4±2 | 1 |
| \(N(1520)/3^+\) | 61±3 | 2 | 10±4 | 0 | 10±3 | 2 | <1 | 2 | - | 1 | - | 1 | 2 | <2 | 1 |
| \(N(1650)/1^+\) | 52±2 | 5 | 2±1.5 | 2 | 12±8 | - | - | 0 | - | - | - | - | 6±4 |
| \(N(1700)/3^+\) | 51±2 | 12±6 | 10±10 | - | - | - | - | - | - | - | - | - | 10±8 |
| \(N(1675)/5^+\) | 80±12 | 19±3 | 2 | 4 | - | 2 | - | 1 | - | 3 | 0 | 1±1 | 3 |
| \(\Delta(1620)/1^+\) | 30±5 | 0 | x | 28±15 | 2 | 15±8 | 0 | 1 | - | 1 | - | 2 | x |
| \(\Delta(1700)/3^+\) | 22±6 | 2 | 16±15 | 0 | 8±6 | 2 | 3±2 | 2 | <1 | 1 | <1 | 1 | - | 2 | x |
| \(N(1440)/2^+\) | 66±3 | 1 | x | 10±6 | 1 | - | 1 | 1 | 2 | - | 3 | 17±6 | 1 |
| \(N(1460)/3^+\) | 17±4 | 1 | 70±16 | - | <3 | <1 | 1 | - | 0 | 2 | - | 1 | x |
| \(N(1720)/3^+\) | 14±2 | 77±5 | 2 | <3 | 22±5 | - | - | - | - | - | - | - | - | 17±6 |
| \(N(1680)/5^+\) | 12±2 | 62±15 | 6±2 | 1 | 7±3 | 0 | 4±2 | 2 | - | 1 | 20±10 | 2 |
| \(N(1910)/2^+\) | 68±8 | 3 | 8±4 | 3 | - | 3 | 1 | 2 | - | 2 | - | 1 | 8±4 | 2 |
| \(N(1920)/3^+\) | 8±4 | 18±10 | 58±14 | <4 | <4 | <5 | <4 | <5 | <2 | <4 | <5 | <2 | 18±10 | 2 |
| \(N(1900)/3^+\) | 13±4 | 3 | 20±12 | 1 | - | 3 | - | 2 | <1 | 2 | <1 | 2 | 6±2 | 1 | x |
| \(N(1900)/5^+\) | 13±4 | 3 | 20±12 | 1 | - | 3 | - | 2 | <1 | 2 | <1 | 2 | 6±2 | 1 | x |
| \(N(1950)/7^+\) | 13±2 | 3 | 20±12 | 1 | - | 3 | - | 2 | <1 | 2 | <1 | 2 | 6±2 | 1 | x |
| \(N(1950)/7^+\) | 16±1 | 3 | 20±12 | 1 | - | 3 | - | 2 | <1 | 2 | <1 | 2 | 6±2 | 1 | x |
| \(N(1950)/7^+\) | 16±1 | 3 | 20±12 | 1 | - | 3 | - | 2 | <1 | 2 | <1 | 2 | 6±2 | 1 | x |

Branching ratios (in %) for decays of nucleon and Δ* resonances. The spread of results from different solutions is used to estimate the uncertainties. x signifies forbidden transitions. Entries marked as "-" were fitted to small values and set to zero to avoid an excessive number of free parameters. The intermediate resonances on the right side of the line carry orbital excitations. Where available, the results from Ref. [18] are given as small numbers. For \(N(1710)/2^+\) two values were given in [18]; both are reproduced here. Small numbers marked with † were taken from [19]. An inclusion of the \(\Delta(1750)/1^+\) improves the fit quality only slightly, its existence cannot be established from our fits. Allowing for \(\Delta(2000)/3^+\) \(\rightarrow N(1700)/3^+\) \(\pi\) decays leads to an improvement of the fit. Due to its large uncertainty, the respective branching ratio is not given in the table.
excitations) and in which, in part of their wave function, both oscillators are excited simultaneously (mixed-oscillator excitations). In [19] it was argued that mixed-oscillator excitations prefer to de-excite in a two-step process in which first one oscillator releases its energy and than the second oscillator. Table 2 is separated into different blocks: the decay modes into the intermediate resonances carry spatial excitation, either the baryon or the meson (the $\sigma$ or $f_0(500)$). In addition, the states in the table are sorted by the kind of baryon excitation: one-oscillator or mixed-oscillator excitations. All one-oscillator excitations have a significantly higher branching ratio into the ground states $N\pi$ or $\Delta\pi$, than into the excited states $N(1520)\pi$, $N(1535)\pi$, or $N\sigma$, as depicted in Fig. 27: the black dots compared to the red squares. They decay on average with about $(60 \pm 3)\%$ into $N\pi$ or $\Delta\pi$ and with about $(6 \pm 1)\%$ into the aforementioned excited states. The residual $(34 \pm 3)\%$ are assigned to other two and multiparticle final states like, e.g. $\Delta\omega$. In contrast, resonances with mixed-oscillator excitations have branching ratios for decays into both $N\pi/\Delta\pi$ and into the excited states as shown in Tab. 2 (blue dots and green squares in Fig. 27): on average about $(30 \pm 3)\%$ go to $N\pi$ or $\Delta\pi$ and about $(17 \pm 2)\%$ go to orbitally excited states.

This finding still holds when restricted to resonances of similar mass and same quantum numbers, namely the four $N^*$ and four $\Delta^*$ resonances of positive parity with a mass around 1900 MeV. Here, the $\Delta^*$ resonances have one excited oscillator and decay into $N\pi/\Delta\pi$ with an average branching ratio of about $(44 \pm 7)\%$, while their average branching ratio to the excited states mentioned above, is about $(5 \pm 2)\%$. On the other hand, the $N^*$ resonances, which have mixed-oscillator excitations, decay into $N\pi/\Delta\pi$ with an average branching ratio of about $(34 \pm 6)\%$ and into the orbitally excited states with about $(21 \pm 5)\%$, similar to the findings without the mass restriction.

The new branching ratios are mostly compatible with the ones determined in Ref. [18]. In some cases, the new branching ratios are significantly smaller compared to the old ones. This is likely due to the interference between two resonances with the same spin-parity but different isospin. These two contributions were not properly identified in $\gamma p \rightarrow p\pi^0\pi^0$ but are under control when data with two charged pions are included in the data base.
6 Conclusion/Summary

We have reported new data on polarization and double-polarization observables for the reaction $\gamma p \rightarrow p\pi^0\pi^0$. The data were included in the BnGa PWA which contains a large data base on pion- and photo-induced reactions off protons and neutrons [44]. The consistency with earlier results is good. In particular, we confirm the conjecture that resonances in which, based on a quark model picture, a fraction of the wave function carries a simultaneous excitation of both oscillators have a significant probability to decay by sequential decays, first into an excited meson or baryon resonance, then into the three-particle final state.

This paper is dedicated to the memory of V. Nikonov.

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