Cognition, Optimism and the Formation of Age-Dependent Survival Beliefs: An Application∗

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Abstract

This paper develops a structural life-cycle model to investigate how psychologically biased survival beliefs affect saving behavior. [TBC]

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1 Introduction

[TBC]

We revisit the question in Groneck, Ludwig, and Zimper (2016) (GLZ) and investigate how strongly biases in subjective survival beliefs affect life-cycle asset holdings. While our approach is similar to that previous work—we compare asset holdings simulated using a calibrated rational expectations model to calibrated models with biased survival beliefs—four major differences are worth emphasizing. First, GLZ derive a functional form of the dependence of cognition (ambiguity) on age from a model of biased Bayesian learning. While cognition in their work is allowed to change, a parameter of relative optimism (respectively pessimism) is constant in age. GLZ estimate the parameters of this functional form directly on the data of subjective survival beliefs. Here, we instead extract the trends of psychological measures from the data. Second, GLZ apply the theory of non-additive Choquet expected utility (CEU) by employing neo-additive capacities on the basis of Chateauneuf et al. (2007), which can be interpreted as linear approximations to inverse-S shaped probability weighting functions for interior objective survival probabilities. Here, we instead employ the fully non-linear framework. The main difference is that the linear model with cognitive biases predicts subjective survival beliefs that are bounded away from zero also in very old-age (when objective survival rates converge to zero), whereas the non-linear PWFs also forces subjective survival beliefs to converge to zero, which may have significant implications for simulated old-age savings behavior and asset holdings. Third, as shown in GLZ psychologically biased survival beliefs of the functional form considered here will lead to dynamically inconsistent behavior if agents of age \( h \) do not anticipate that their cognitive strength will decreases and their motivational attitudes will change in future ages \( t < h \). GLZ then analyze both naive and sophisticated agents, who anticipate the deviation from the plan of their own futures self, and show that the difference between both types is small. To simplify, we here consider only naive agents.\(^1\) Finally, since GLZ do not explore information on how cognitive and motivational measures influence survival belief formation and since they do not employ non-linear PWFs, they

\(^1\)Results for sophisticated agents shown are virtually identical.
cannot differentiate between a base bias and the effects of changes in cognitive and psychological attitudes. In contrast, we are explicit about this distinction.

The remainder of this paper is organized as follows. Section 2 presents the life-cycle model. Section 3 contains information on calibration. Section 4 presents the main results. Section 5 concludes the analysis.

2 The Life-Cycle Model

The theoretical background for the structural model is rank dependent utility theory (Quiggin 1981; Quiggin 1982), which is identical to *cumulative prospect theory* (CPT), whenever (i) CPT is restricted to the domain of gains and (ii) the CPT decision maker faces objective probabilities (Tversky and Kahneman 1992).

2.1 Risk and Time

We assume that agents start their economic life at age 0 (which is biological age 20) and die for sure at age $T$. Each model period has length of one calendar year. Agents retire at the age of $t_r$ and receive exogenous stochastic labor income during the working period and a deterministic stream of pension income during retirement. In each period, agents may survive to the next period with objective probability $\psi_{t,t+1}$. We assume rational expectations with respect to earnings risk, and psychologically biased beliefs with respect to survival risk.

2.2 The Rank Dependent Utility Model

Consider an agent of age $h \geq 0$ and fix some $T \geq h$ with the interpretation that the agent possibly lives until the maximal age of $T$. We construct the additive probability space $(\Omega, \mathcal{F}, \psi)$ such that the state space is given as $\Omega = \{1, \ldots, T\}$ and the $\sigma$-algebra $\mathcal{F}$ is given as the powerset of $\Omega$. We interpret $D_t \equiv \{t\}, t \in \Omega$ as the event in $\mathcal{F}$ that the agent dies at the end of age $t$. Observe that $D_t \cup \cdots \cup D_T$ stands for the event in $\mathcal{F}$ that the agent of age $h < t$ survives until (at least) the beginning of age $t$. 
We interpret $\psi$ as the objective (unconditional) probability measure that comprehensively governs the agent’s mortality risk. Denote by $\psi^h$ the conditional probability measure of an agent who has reached age $h < T$; that is, for all $A \in \mathcal{F}$,

$$
\psi^h (A) \equiv \frac{\psi (A \cap (D_h \cup \cdots \cup D_T))}{\psi (D_h \cup \cdots \cup D_T)} .
$$

As a notational convention, we write for the objective probability that an agent of current age $h$ survives until (at least) the beginning of age $t > h$

$$
\psi_{h,t} \equiv \psi^h (D_t \cup \cdots \cup D_T) = \sum_{k=t}^{T} \psi^h (D_k) . \tag{1}
$$

Now assume a probability weighting function $\omega : [0, 1] \to [0, 1]$ that is increasing and satisfies $\omega (0) = 0$ and $\omega (1) = 1$. Then the rank-dependent utility (RDU) of risky consumption prospect $c$ of an $h$-old agent with respect to the probability weighting function $\omega$ is given as follows:

$$
RDU^h (c, \omega) \equiv \sum_{t=0}^{T-h} \mathbb{E}_h [U (c^{T-t})] \left[ \omega \left( \psi^h (D_T) + \cdots + \psi^h (D_{T-t}) \right) - \omega \left( \psi^h (D_T) + \cdots + \psi^h (D_{T-t+1}) \right) \right] \tag{2}
$$

where $\mathbb{E}_h [U (\cdot)] \in \mathbb{R}_+$ denotes a continuous expected utility function satisfying

$$
\mathbb{E}_h [U (c^T)] \geq \ldots \geq \mathbb{E}_h [U (c^h)] , \tag{3}
$$

where it is assumed that rational expectations $\mathbb{E}_h$ are taken with respect to earnings risk.

We next assume that the expected utility with respect to risky consumption prospect $c^t$ is additively separable with exponential raw time discount factor $\beta \in (0, 1]$, i.e.,

$$
\mathbb{E}_h [U (c^t)] = \mathbb{E}_h \left[ \sum_{s=h}^{t} \beta^{s-h} u (c_s) \right] , \tag{4}
$$

where $u : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and strictly concave per period utility.

\footnote{The standard convention $\omega (\psi_n + \psi_{n+1}) = 0$ applies.}
function. Using the notational convention (1), we can transform (2) as

\[ RDU^h (c, \omega) = \sum_{t=0}^{T-h} E_h \left[ U \left( c^{T-t} \right) \right] \left[ \omega \left( \psi_{h,T-t} \right) - \omega \left( \psi_{h,T-t+1} \right) \right] \]

\[ = \sum_{t=0}^{T-h} E_h \left[ \sum_{s=h}^{T-t} \beta^{s-h} u \left( c_s \right) \right] \left[ \omega \left( \psi_{h,T-t} \right) - \omega \left( \psi_{h,T-t+1} \right) \right] \]

\[ = u \left( c_h \right) + E_h \left[ \sum_{t=h+1}^{T} \omega \left( \psi_{h,t} \right) \beta^{t-h} u \left( c_t \right) \right]. \] (5)

Finally, assume that the probability weighting function \( \omega (\cdot) \) is given as a Prelec (1998) probability weighting function \( \nu (\cdot) \), denoted by \( \nu^h_{h,t} \).

### 2.3 Endowments

There are discrete shocks to labor productivity in every period \( t = 0, 1, \ldots, t_r - 1 \), which are required in the model to achieve a realistic calibration. We denote shocks by \( \eta_t \in E, E \) finite, which are i.i.d. across households of the same age. By \( \eta^t = (\eta_1, \ldots, \eta_t) \) we denote a history of shocks. Let \( E \) be the powerset of the finite set \( E \). \( E_{t-r} \) are \( \sigma \)-algebras generated by \( E, E, \ldots \). We assume that there is an objective probability space \( (\times_{t=0}^{t-r-1} E_{t-r}, \pi) \) such that \( \pi_t(\eta^t | \eta^h) \) denotes the probability of \( \eta^t \) conditional on \( \eta^h \).

We follow Carroll (1992) by assuming that one element in \( E \) is zero (zero income). This feature gives rise to a self-imposed borrowing constraint and thereby to continuously differentiable policy functions. (Self-imposed) borrowing constraints are required to generate realistic paths of life-time consumption, savings and asset accumulation. Continuous differentiability is convenient when we model a sophisticated agent, where the derivative of the policy function enters the first-order condition. In addition, we assume productivity to vary by age. Accordingly, \( \phi_t \) denotes age-specific productivity which is estimated from the data and results in a hump-shaped life-cycle earnings profile.

After retirement at age \( t_r \) households receive a lump-sum pension income, \( b \).

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3Note that we require of the per period utility function \( u(c_s) \geq 0 \) for all \( s \) to ensure that condition (3) is satisfied.
As shocks to labor income, retirement income is modeled in order to achieve a realistic calibration. Pension contributions are levied at contribution rate \( \tau \). To achieve a self-imposed borrowing constraint and continuous policy functions also during the retirement period, we assume that there is a small i.i.d. probability of default of the government on its pension obligations. Accordingly, \( \eta_t \in E^r = [1, 0] \) during retirement. Correspondingly, let \( E^r \) be the powerset of the finite set \( E^r \). \( E^{T-t_r+1} \) are \( \sigma \)-algebras generated by \( E^r, E', ... \) and \( (\times_{t=t_r}^T E^r, \pi^r) \) is the objective probability space in the retirement period.

We further assume that income risk is first-order Markov (and zero income shocks are i.i.d.) so that we can generally write \( \pi_t(\eta_t | \eta^{t-1}) = \pi_t(\eta_t | \eta_{t-1}) \). This assumption is convenient below where we specify (and solve) the recursive formulation of the problem.

Collecting elements, income of a household of age \( t \) is given by

\[
y_t = \begin{cases} 
\eta_t \phi_t w (1 - \tau) & \text{for } t < t_r \\
\eta_t b & \text{for } t \geq t_r.
\end{cases}
\]

We abstract from private annuity markets, an assumption which can be justified by the observed small size of private annuity markets. We also assume a fixed interest rate, \( r \). The dynamic budget constraint therefore writes as

\[
a_{t+1} = a_t (1 + r) + y_t - c_t. \tag{6}
\]

### 2.4 Government

We assume a pure PAYG public social security system. Denote by \( \chi \) the net pension benefit level, i.e., the ratio of pensions to net wages. The government budget is assumed to be balanced each period and is given by

\[
\tau w \sum_{t=0}^{t_r-1} \phi_t N_t = b \sum_{t=t_r}^T N_t = \chi (1 - \tau) w \sum_{t=t_r}^T N_t. \tag{7}
\]

In addition, accidental bequests—arising because of missing annuity markets—are taxed away at a confiscatory rate of 100%. Revenue from this source is used for government consumption which is otherwise neutral. Also, in the unlikely
event of default of the government on its pension obligations, the government
uses contributions to the pension system for otherwise neutral government con-
sumption.

2.5 Recursive Formulation and Dynamic Inconsistency

We can rewrite the household problem recursively which is convenient for the
computational solution and for the illustration of dynamic inconsistency of the
problem. Let the value function in any period \( t \geq h \) be given by

\[
V^h_t(a_t, \eta_t) = \max_{c_t, a_{t+1}} \left\{ u(c_t) + \beta \frac{\nu^h_{h,t+1}}{\nu^h_{h,t}} \mathbb{E}_t V^h_{t+1}(a_{t+1}, \eta_{t+1}) \right\}
\]

Maximization of the above subject to the resource constraint (6) gives the first-
order condition of naive RDU decision makers as

\[
u^h_{h,t+1} \mathbb{E}_t [u_c(c_{t+1})] = \beta (1 + r) \frac{\nu^h_{h,t+1}}{\nu^h_{h,t}} \mathbb{E}_t [u_c(c_{t+1})]
\]

From the above it is straightforward to observe that the problem is dynamically
inconsistent because for any future period \( t > h \) and planning ages \( h, h + 1 \) we have

\[
u^h_{h,t+1} \mathbb{E}_t [u_c(c_{t+1})] = \beta (1 + r) \frac{\nu^h_{h,t+1}}{\nu^h_{h,t}} \mathbb{E}_t [u_c(c_{t+1})]
\]

and

\[
u^1_{h+1,t+1} \mathbb{E}_t [u_c(c_{t+1})] = \beta (1 + r) \frac{\nu^1_{h+1,t+1}}{\nu^1_{h+1,t}} \mathbb{E}_t [u_c(c_{t+1})]
\]

and, generally, \( \frac{\nu^h_{h,t+1}}{\nu^h_{h,t}} \neq \frac{\nu^{h+1}_{h+1,t+1}}{\nu^{h+1}_{h+1,t}} \). Only for the nested RE model (where \( \zeta_0 = \theta_0 = 1, \zeta_1 = \theta_1 = \theta_2 = 0 \)), these ratios of beliefs are identical, because \( \frac{\nu^h_{h,t+1}}{\nu^h_{h,t}} = \frac{\psi^h_{h,t+1}}{\psi^h_{h,t}} = \psi^h_{h,t+1} \), independent of \( h \).

Sophisticated agents anticipate this dynamically inconsistent behavior of their
own future type. Accordingly, their first-order condition at any age $h$ writes as

$$u_c(c_h) = \beta(1 + r)\nu_{h+1}^h \mathbb{E}_h \left[ \Theta_{h+1}(x_{h+1}, \eta_{h+1}) u_c(c_{t+1}) + \Lambda_{h+1}(x_{h+1}, \eta_{h+1}) \right]$$

where—denoting by $m_{h+1}(\cdot) = \frac{\partial c_{h+1}(\cdot)}{\partial x_{h+1}}$ the derivative of the policy function, and by $\Delta_{h,h+1}(x_{h+2}, \eta_{h+2})$ the distance of the derivatives of value functions with respect to cash-on-hand at age $h + 2$, from the perspective of ages $h$ and $h + 1$, i.e., $\Delta_{h,h+1}(x_{h+2}, \eta_{h+2}) \equiv V_{h+2,x_{h+2}}^h(x_{h+2}, \eta_{h+2}) - V_{h+1,x_{h+1}}^{h+1}(x_{h+2}, \eta_{h+2})$—the terms

$$\Theta_{h+1}(x_{h+1}, \eta_{h+1}) = m_{h+1}(x_{h+1}, \eta_{h+1}) + \frac{\nu_{h,h+1}^{h+2}}{\nu_{h+1}^{h+1} \nu_{h+1,h+2}^{h+1}} (1 - m_{h+1}(x_{h+1}, \eta_{h+1}))$$

$$\Lambda_{h+1}(x_{h+1}, \eta_{h+1}) = \beta(1 + r) \frac{\nu_{h,h+1}^{h+2}}{\nu_{h+1}^{h+1}} (1 - m_{h+1}(x_{h+1}, \eta_{h+1})) \Delta_{h,h+1}(x_{h+2}, \eta_{h+2})$$

reflect the dynamic inconsistency of the decision maker.

### 3 Calibration

We consider three types of agents: a rational expectations agent ($RE$), and two versions of agents with psychologically biased survival beliefs, $RDU$ agents. Specifically, we consider a naive $RDU$ agent who suffers from all the psychological biases ($RDU \text{ – full}$) and a naive $RDU$ agent who does not have a base bias ($RDU \text{ – psych}$). Throughout, we denote the subjective survival belief of an agent of age $h$, to survive in some future age $t \geq h$ to some even more distance future age $t \geq s$ by $\psi_{s,t}$, with $\psi_{s,s} = 1$. For objective survival probability $\psi_{s,t} \leq 1$, with $\psi_{s,s} = 1$, we parameterize the beliefs of the three different types $i \in \{RE, RDU \text{ – full}, RDU \text{ – psych}\}$ on the basis of the baseline estimates in Grevenbrock et al. (2018).$^4$ In line with these estimates, restrict $\theta_0 = 1$ so that there is no base bias coming from the motivational attitudes. Relative to agent $RDU \text{ – full}$ we shut down for agent $RDU \text{ – psych}$ the base effect by

$^4$The main parameter estimates in Grevenbrock et al. (2018) are robust in various alternative specifications.
setting $\zeta_0 = 1$. Thus, the survival beliefs of the three agent types are given by

$$
\nu_{i,s,t}^h = \begin{cases} 
\psi_{s,t} & \text{for } i = \text{RE} \\
\nu(\psi_{s,t}; \hat{c}_h, \hat{o}_h, \hat{p}_h; \zeta_0, \zeta_1, \theta_0 = 1, \theta_1, \theta_2) & \text{for } i = \text{RDU} - \text{full} \\
\nu(\psi_{s,t}; \hat{c}_h, \hat{o}_h, \hat{p}_h; \zeta_0 = 1, \zeta_1, \theta_0 = 1, \theta_1, \theta_2) & \text{for } i = \text{RDU} - \text{psych}
\end{cases}
$$

where $\nu(\cdot)$ is the non-linear Prelec (1998) probability weighting function, which depends on the age $h$ specific predicted values of the three psychological measures cognitive weakness $\hat{c}_h$, optimism $\hat{o}_h$ and pessimism $\hat{p}_h$, as well as on the parameters $\zeta_0, \zeta_1, \theta_0, \theta_1, \theta_2$.

All first-stage parameters are summarized in Table 1. The only second stage parameter is the discount factor $\beta$, calibrated in all models to match data on asset holdings and summarized in Table 2.

We assume a CRRA per period utility function with additive shifter $\Gamma \geq 0$

$$
u(c_t) = \Gamma + \frac{c_t^{1-\kappa}}{1-\kappa}
$$

where $\Gamma$ is assumed large enough so that condition (3) holds for all $t$ and $\kappa$ is the coefficient of relative risk aversion, which we fix, by reference to the literature, to $\kappa = 2$.

Parameters of the first-order Markov process for income shocks are taken from Storesletten, Telmer, and Yaron (2004), which is augmented by the small probability of zero earnings. Given the contribution rate of the pension system, $\tau = 0.124$, taken from the data, the pension benefit level $\chi$ can be computed without using the model because prices are exogenous and because pension payments only depend on objective survival rates.

Objective survival rates are estimated based on a model by Bebbington and Zitikis (2011) according to the specification

$$
\psi_{t,t+1} = 1 - \frac{A \exp(\alpha \cdot t)}{1 + s^2 (\exp(\alpha \cdot t) - 1) \frac{\Delta}{\alpha}} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2).
$$

In addition to these parameters, we estimate age-specific trends of the psychological factors $c_h, o_h, p_h$ according to the following specifications, which we use
| Parameter | Source |
|-----------|--------|
| **Technology and Prices** | |
| $w = 1$ Gross wage normalized | |
| $r = 0.042$ Interest rate | (Siegel 2002) |
| $\tau = 0.124$ Social security contribution rate | irs.gov |
| **Income Process** | |
| $\kappa = 0.97$ Persistence of income | (Storesletten, Telmer, and Yaron 2004) |
| $\epsilon = 0.68$ Variance of income | (Storesletten, Telmer, and Yaron 2004) |
| $\{\phi_t\}$ Age specific productivity | PSID |
| $\zeta = 0.005$ Probability of zero labor income | (Carroll 1992) |
| $\zeta^r = 0.001$ Probability of zero retirement income | |
| $\chi = 0.0286$ Pension benefit level | |
| **Preferences** | |
| $\kappa = 2$ Coefficient of relative risk aversion | |
| **Subjective Survival Beliefs** | |
| $\xi_0 \in \{1, 0.5457, 1\}$ for agent $\{RE, CEU – full, CEU – psych\}$ | HRS |
| $\xi_1 \in \{0, –0.0134, –0.0134\}$ | HRS |
| $\theta_0 \in \{1, 1, 1\}$ | HRS |
| $\theta_1 \in \{0, 0.0295, 0.0295\}$ | HRS |
| $\theta_2 \in \{0, –0.0583, –0.0583\}$ | HRS |
| **Age Limits and Survival Data** | |
| 0 Initial model age (age 20) | |
| $t_r = 42$ Retirement (age 62) | SCF |
| $T = 105$ Maximum human lifespan (age 125) | (Weon and Je 2009) |
| $\{\psi_{k,t}\}$ Cohort survival rates | Predictions based on HMD |
| $s = 0.41$ Logistic frailty model | |
| $\alpha = 0.13$ Logistic frailty model | |
| $A = 2.9e − 06$ Logistic frailty model | |
as inputs into the non-linear PWFs for the three agent types:

\[
\ln (35 - c_h) = \sum_{i=0}^{n_c} \beta_{c,i} h^i + \epsilon_{c,h} \tag{9a}
\]

\[
\ln (o_h) = \sum_{i=0}^{n_o} \beta_{o,i} h^i + \epsilon_{o,h} \tag{9b}
\]

\[
\ln (p_h) = \sum_{i=0}^{n_p} \beta_{p,i} h^i + \epsilon_{p,h} \tag{9c}
\]

We estimate these regressions on sample information for the three psychological measures for ages 25 – 100. Equation (9a) uses as dependent variable the natural logarithm of cognitive strength rather than weakness, so that predicted cognitive weakness \( \hat{c}_h \) does not exceed the upper bound of 35. Without the transformation to \( \ln(35 - c_h) \) this upper bound would be violated in the prediction. Also, to capture the non-linearity of cognitive weakness (strength), we choose a high degree of the polynomial and set \( n_c = 4.5 \). The trends of optimism and pessimism are more strongly linear and weaker in age so that neither a specification with a bound restriction nor a high degree polynomial is required. We accordingly set \( n_o = n_p = 1 \).

Results on the sample values and predictions for all three psychological measures are shown in Figure 1. Cognitive weakness is basically flat until the age of 60 and then increases quite strongly. Pessimism displays a week upward trend in age, optimism weak downward trend.

4 Results

Results on our experiment are presented in Table 2 and in Figure 2. The first row of Table 2 summarizes the calibration of the model. The lower discount factor in \( RDU - full \) reflects the stronger overestimation, which, ceteris paribus leads to high old-age asset holdings. Relative to the other two agent types, calibration then determines a lower discount factor. Importantly, as in GLZ, the strategy of fitting the entire asset profile from age 20 onwards imposes a lot of discipline on

\[5\] In the estimation, we use Chebyshev rather than monomial basis functions.
Figure 1: Psychological Measures: Sample Values and Prediction

(a) Cognitive Weakness  
(b) Pessimism  
(c) Optimism

Notes: This figure shows sample values and predictions of the three psychological measures according to equation (9) for $n_c = 4$ and $n_p = n_o = 1$. Source: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

calibration. E.g., the fit to old-age asset holdings of the $RE$ (or the $RDU – psych$) model could be improved by increasing $\beta_i$, but this would deteriorate the fit to young age asset holdings as agents would then save too much at a young age relative to the data.

Panel (a) of Figure 2 shows predicted survival beliefs at age 65 for the remaining life-span and Panel (b) displays asset holdings in the three models and the data, zooming in on ages 65–89. As Panel (a) indicates, psychologically biased survival beliefs feature overestimation for ages above about 82 with the effect being substantially more pronounced for $RDU – full$ than for $RDU – psych$. Before that, there is underestimation, which is rather mild for $RDU – psych$.
Table 2: Model Agent Types: Calibration and Fit

|                      | RE   | RDU – full | RDU – psych |
|----------------------|------|------------|-------------|
| Discount factor $\beta_i$ | 0.9727 | 0.9733     | 0.9708      |
| Distance $d_i$       | 16.16% | 5.52%      | 10.96%      |

Notes: This table shows the values of the calibrated discount factors $\beta_i$ and a measure of distance $d_i$ to the data on asset holdings for three agent types. Source: Own calculations, Survey of Consumer Finance (SCF), Health and Retirement Study (HRS), Human Mortality Database (HMD).

and quite pronounced for $RDU – full$. Simulated old-age asset holdings, shown in Panel (b), come closer to the data under biased survival beliefs. Precisely, model $RDU – full$ matches well the data on asset holdings until about age 85. This finding is consistent with GLZ. Ignoring the base bias, which, as discussed previously, may reflect many alternative mechanisms, expectedly leads to a much smaller improvement of the fit to the data beyond the RE model. This is also reflected in the distance measures reported in the second row of Table 2, where we report the mean absolute distance of asset holdings of the respective agent $i$ from the data for age bin $65 – 89$, normalized by the average asset holdings in the data in that age bin.\(^6\)

**Sensitivity Analysis.** Our main results reported in Table 2 and Figure 2 are for a scenario where we assume that agents start their economic life at the age of 20 with survival beliefs formed on the basis of the non-linear PWF (8). Thus, those beliefs are formed on the basis of backcasting and forecasting the psychological measures. Shutting down backcasting, i.e., starting with biased beliefs only from age 65 onwards, does not affect our conclusions, as shown in the first part of Table 3. As additional sensitivity analyses the table reports results for sophisticated rather than naive RDU agents and for a calibration with higher risk aversion of $\kappa = 3$, also confirming our main findings.

\(^6\)Formally, the distance measure is $d_i = \frac{1}{70} \sum_{t=46}^{70} |a_t - a_t^d|$, where model age $t = 46$ corresponds to biological age 65 and model age $t = 70$ to biological age 89 and where $a_t$ are simulated assets and $a_t^d$ are assets in the data, both normalized by permanent income.
Figure 2: Life-Cycle Survival Beliefs and Asset Holdings

(a) Survival Beliefs at Age 65

(b) Assets

Notes: Predicted survival beliefs at age 65 and life-cycle asset holdings for three household types and in the data. Source: Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

Table 3: Model Agent Types: Calibration and Fit: Sensitivity Analysis

|                       | RE     | RDU − full | RDU − psych |
|-----------------------|--------|------------|-------------|
| No backcasting        |        |            |             |
| Discount factor $\beta_i$ | 0.9727 | 0.9716     | 0.9722      |
| Distance $d_i$         | 16.16% | 5.35%      | 11.23%      |
| Sophisticated RDU Agent|        |            |             |
| Discount factor $\beta_i$ | 0.9727 | 0.9719     | 0.9708      |
| Distance $d_i$         | 16.16% | 5.51%      | 10.87%      |
| High Risk Aversion, $\kappa = 3$ |        |            |             |
| Discount factor $\beta_i$ | 0.9663 | 0.9628     | 0.9627      |
| Distance $d_i$         | 10.69% | 5.71%      | 6.93%       |

Notes: This table shows the values of the calibrated discount factors $\beta_i$ and a measure of distance $d_i$ to the data on asset holdings for three agent types. Source: Own calculations, Survey of Consumer Finance (SCF), Health and Retirement Study (HRS), Human Mortality Database (HMD).
5 Conclusion

[TBC]

In sum, we can conclude that our models of biased survival beliefs result in an improvement of fit to asset data. Two open questions, however, arise. First, is it reasonable to assume that agents do act on the base bias, in which case $RDU - full$ would be the most reasonable model, or does the base bias just reflect 50–50 answering patterns that do not affect economic decisions, in which case $RDU - psych$ would be more plausible? Second, our life-cycle model postulates that psychological biases only affect formation of subjective survival beliefs leaving other parameters unaffected. This is evidently a strong simplification. For instance, according to the recent theoretical work by Gabaix and Laibson (2017), lack of cognition leads to higher pure time discounting, andBinswanger and Salm (2017) find that the association between subjective probabilities and decisions increases with an individual’s cognitive strength, whereas lower cognitive skills are more strongly associated with heuristics. Furthermore, in extended models with multiple risks, e.g., earnings risks and health risks, psychological factors will also affect the formation of beliefs with respect to these risks (Dominitz and Manski 1997; Rozsypal and Schlafmann 2017). Due to the potentially multiple ways how psychological attitudes impact on life-cycle (savings) decisions, their overall effect can, in our view, only be provided by use of structural life-cycle models such as ours, which enables researchers to explicitly take into account all these mechanisms. We leave this for future research.
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