An ease-off flank modification method for high contact ratio spiral bevel gears with modified curvature motion

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Abstract
In order to solve the problem of the design and fabrication of high contact ratio spiral bevel gears with seventh-order transmission error (TE), an ease-off flank modification method is proposed based on the modified curvature motion method. In this paper, firstly, based on the predesigned seventh-order transmission error, the polynomial coefficients of transmission error curve can be obtained. Secondly, the pinion target tooth surface is obtained by modifying the pinion auxiliary tooth surface along the meshing line with a predesigned modification curve, and the pinion auxiliary tooth surface is obtained with the predesigned seventh-order transmission error through using the gear as a virtual cutter. Thirdly, a new method called modified curvature motion (MCM) method is proposed to improve the adjustability of spiral bevel gear by modifying parts of machine-tool settings in the process of gear manufacturing. Finally, an optimization model solved by using the improved NSGA-II algorithm is proposed to solve the adjustments of pinion machining settings, and we carry out TCA and LTCA to verify the feasibility of the tooth modification method. The results keep in line with the preconditions that transmission error is seventh-order curve and the contact path is located in the middle of the tooth surface. The proposed flank modification methodology can serve as a basis for developing a general technique of flank modification for spiral bevel gears.

Key words: Spiral bevel gear, Flank modification, Ease-off, Modified curvature motion, Adjustability

1. Introduction

Spiral bevel gears find their widely use in transmit rotations and torques in the intersecting axes, such as helicopters, automobiles and so on. Spiral bevel gears have a great many advantages such as transmission stability, low noise levels and high loading capacity.

Tooth surface modification is an important technique for spiral bevel gears to reduce vibration, noise and it can help to reduce the probability of edge contact. Meanwhile, it is also an effective method to improve the transmission efficiency. Nishino (2009) developed a computerized tool to predict the running performances of hypoid gears. Tooth surface modification can be divided into two parts: profile modification and lengthwise modification. Kolivand and Kahraman (2010) proposed an ease-off based method for loaded tooth contact analysis of hypoid gears having local and global surface deviations. Shih and Fong (2007, 2010) proposed a spiral bevel gear tooth surface modification method based on ease-off, and the sensitivity matrix and linear regression method were used to identify the NC axis motion coefficient. Stadtfeld (2000a, 2000b) developed the tooth contact optimization method and the higher order tooth surface modification method called the Universal Motion Concept (UMC) by means of ease-off topography. Fan (2011) developed a method of optimizing the face cone element for spiral bevel and hypoid gears using an ease-off method. Although ease-off has been studied in detail, and the sensitivity matrix and linear regression method were used to identify the NC axis motion coefficient. Stadtfeld (2000a, 2000b) developed the tooth contact optimization method and the higher order tooth surface modification method called the Universal Motion Concept (UMC) by means of ease-off topography. Fan (2011) developed a method of optimizing the face cone element for spiral bevel and hypoid gears using an ease-off method.

To cut down the transmission errors caused by misalignment, and improve the meshing performance of spiral bevel gears, the transmission error of spiral bevel gears is usually designed as second-order parabolic (Litvin et al., 1991, 2002a, 2002b, 2004). A lot of results indicate that the high-order transmission error can greatly reduce gear vibration and noise. Stadtfeld (2000b) enforced the fourth-order transmission error to reduce running noise in Gleason Works.
Wei et al. (2003) proposed a high-order transmission error that required only modifying the roll coefficients. Su et al. (2013) proposed a seventh-order transmission error to improve the stability of spiral bevel gears. For spiral bevel gears with second-order transmission error, traditional methods can meet requirements of reducing loaded transmission error and it has the advantages of machining easily. But for spiral bevel gears with higher-order transmission error, traditional method is both low productivity and low precision. To get high contact ratio spiral bevel gears with high precision and high productivity, a more efficient technology is needed. With the extensive application of the high-speed and high-precision CNC machine tool system, it is possible to grind high precision gears by more efficient and convenient method. Wang and Fong (2005a, 2005b) proposed modified radial motion method for face-milling spiral bevel gears, and it adjusted tooth surface curvature via the cutter-head diameter and the roll ratio modifications. On the basis of modified radial motion method, we proposed modified curvature motion method.

In this paper, an ease-off flank modification method for high contact ratio spiral bevel gears with modified curvature motion is proposed, which can produce the high-precision and high reliability gears. The TCA results keep in line with the preconditions that transmission error is seventh-order curve and the contact path is located in the middle of the tooth surface. The proposed flank modification methodology can serve as a basis for developing a general technique of flank modification for spiral bevel gears.

2. Function-oriented design based on ease-off

The function-oriented design of spiral bevel gears is to develop the topography of pinion tooth surface under the required by the predesigned functions. Firstly, the transmission error curve is predesigned. Secondly, the pinion auxiliary surface is obtained based on the predesigned transmission error. Finally, the pinion target surface is acquired through modifying pinion auxiliary surface along the meshing line with predesigned modification curve.

2.1 Predesigned meshing performance

2.1.1 Predesigned transmission error curve

We design a seventh-order TE, the geometric shape and parameters are shown in Fig. 1, where \( \varphi_1 \) is the pinion rotation angle, and \( \delta \) is the amplitude of TE. There are five control points on the transmission curve: points A and E are the entrance and exit meshing point, respectively. Points B and D are two peaks of transmission error curve. Point C is the reference point. \( \varphi_i \) \((i=A,B,\ldots,E)\) are the pinion rotation angles at five meshing points, and their corresponding amplitude of transmission error are \( \delta_i \). Furthermore, \( \lambda_1 \) and \( \lambda_2 \) are used to control the relationships between the peak points and the reference point.

\[
\begin{align*}
\varphi_A &= \varphi_C - C_R T Z / 2 \\
\varphi_B &= \varphi_C - \lambda_1 T Z \\
\varphi_C &= \varphi_C \\
\varphi_D &= \varphi_C + \lambda_2 T Z \\
\varphi_E &= \varphi_C + C_R T Z / 2
\end{align*}
\]

The seven-order transmission error curve is defined as

\[
\begin{align*}
\varphi_2 &= \varphi_20 + Z_2 / Z_1 (\varphi_1 - \varphi_10) + \delta (\varphi_1) \\
\delta (\varphi_1) &= c_0 + c_1 \varphi_1 + c_2 \varphi_1^2 + c_3 \varphi_1^3 + c_4 \varphi_1^4 + c_5 \varphi_1^5 + c_6 \varphi_1^6 + c_7 \varphi_1^7
\end{align*}
\]

where \( c_i \) \((i=1,2,\ldots,7)\) are the polynomial coefficients of transmission error curve. \( \varphi_20 \) and \( \varphi_10 \) denote the original rotation angles of the gear and pinion, respectively, while \( Z_2 \) and \( Z_1 \) are the tooth numbers, respectively.

2.1.2 Predesigned contact pattern

The contact path may have a great effect on the meshing quality of spiral bevel gears. The design of the contact path is implemented on the rotary projection of the pinion tooth surface, which can be shown in Fig. 2. In the design of the contact path, straight line type is used to reduce sensitivity to misalignment and improve the meshing performance.

The modification parameters along the contact path are as follows: (1) \( \eta \) is the direction angle of the contact path. (2) \( C_R \) is the contact ratio; (3) \( \delta \) is the amplitude of transmission error; (4) \( \lambda_i (i=1,2) \) is pinion rotation angle control.
parameters on peak points. The modification parameters along the meshing line are as follows: (1) \( a \) is half the length of the major axis of the contact ellipse; (2) \( a' \) is the parabolic control parameter.

\[
d \in \frac{1}{2} \text{length of major axis of contact ellipse; } a' \text{ is the parabolic control parameter.}
\]

2. Pinion auxiliary tooth surface

Based on the predesigned parameters, the polynomial coefficients of TE curve are calculated by Eqs. (1-2) as shown in Fig. 2, point \( M \) is the reference point. The equation of contact path is as follows:

\[
y = (x - x_M) \tan \eta + y_M
\]

where \((x_M, y_M)\) is the coordinates of the reference point \( M \).

The gear tooth surface is treated as a virtual cutter to create pinion auxiliary surface under the predesigned TE, as shown in Figs. 3-4. The coordinate systems \( S_1 \) and \( S_2 \) are rigidly attached to the pinion and the gear cutter, respectively. The gear tooth surface is as follows:

\[
\begin{align*}
\mathbf{r}_2 &= r_2(s_g, \theta_g, \phi_g) \\
\mathbf{n}_2 &= n_2(s_g, \theta_g, \phi_g) \\
f_2(s_g, \theta_g, \phi_g) &= 0
\end{align*}
\]

where \( s_g \) and \( \theta_g \) are the surface parameters, \( \phi_g \) is the gear cradle angle, \( \mathbf{r}_2 \) is the position vector of gear, \( \mathbf{n}_2 \) is the unit normal vectors. \( f_2 \) denotes the meshing equation between the cutter and the gear.

Pinion auxiliary surface is shown as follows:

\[
\begin{align*}
\mathbf{r}_1' &= M_{1b}(\phi_1)M_{b2}(\phi_2)\mathbf{r}_2 \\
\mathbf{n}_1' &= L_{1b}(\phi_1)L_{b2}(\phi_2)\mathbf{n}_2 \\
f_{12}(s_g, \theta_g, \phi_g, \phi_1) &= 0
\end{align*}
\]
where $\phi_1$ is pinion rotation angle, $\phi_2$ is gear cutter rotation angle, $f_{12}$ indicates meshing equation between the gear cutter and the pinion. The matrices $M_{h2}$ and $M_{h1}$ are the homogeneous coordinate transform matrices from $S_2$ to $S_h$, from $S_h$ to $S_1$, respectively. $L_{h1}$ and $L_{h2}$ are the upper-left 3x3 sub-matrix of $M_{h1}$ and $M_{h2}$, respectively.

\[
\begin{align*}
M_{h2} &= \begin{bmatrix}
\cos \Sigma & \sin \Sigma \sin \phi_2 & -\sin \Sigma \cos \phi_2 & 0 \\
0 & -\cos \phi_2 & -\sin \phi_2 & 0 \\
-\sin \Sigma & \cos \Sigma \sin \phi_2 & -\cos \Sigma \cos \phi_2 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \\
M_{h1} &= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi_1 & \sin \phi_1 & 0 \\
0 & -\sin \phi_1 & \cos \phi_1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\] (6)

(7)

2.3 Pinion target tooth surface

In order to improve the quality of high contact ratio spiral bevel gears, the pinion double crowned is proposed. The contact area of pinion tooth surface is the same as the original design. However, we only modify non-contact area in the following discussion, which means that more material is removed in non-contact areas, and the contact path cannot be extended to these areas. This proposed method can significantly improve the quality of high contact ratio spiral bevel gears.

Pinion auxiliary surface is modified along the meshing line with a predesigned modification curve to control the contact area. The modification curve along the meshing line is shown in Fig. 5. The modification along meshing line is carried out with parabola $I$ inside the designed contact area $B$. Nevertheless, the modification in the outside area $A$ and $C$ is carried out with parabola $II$. As is shown in Fig. 6, the pinion target tooth surface is obtained.

The equations of modification curve along meshing line are as follows:

\[
\delta_j = \begin{cases}
\delta_1 = \frac{\delta_1}{a^2} l^2 - a \leq l \leq a \\
y_{12} = \left(\frac{\delta_1}{a^2} l^2 + a^2\right)^2 + a^2, l < a \text{ or } l > a
\end{cases}
\] (8)

where $\delta_1$ is the elastic deformation.

The pinion auxiliary surface and target tooth surface are divided into $q$ ($q=9\times15$) meshing points, then the position and the unit normal vector of discrete point $i$ ($i=1, 2, \ldots, q$) of pinion auxiliary surface are presented as follows:

\[
\begin{align*}
p_i' &= p_i(s_i, \theta_i, \phi_i, \phi_1) \\
n_i' &= n_i(s_i, \theta_i, \phi_i, \phi_1)
\end{align*}
\] (9)

where $p'$ is the position vector of discrete points, $n'$ is the unit normal vectors of discrete points.

The position vectors of mesh point for the pinion target surface can be achieved by the following equation:

\[
p_i = p_i' + n_i' \delta_j
\] (10)
3. Adjustments of pinion machining parameters

For spiral bevel gears with second-order transmission error, traditional machining method can meet requirements and it has the advantages of ease machining. But for spiral bevel gears with higher-order transmission error, traditional machining method not only shows a low productivity, but also exerts a low precision. In order to get high contact ratio spiral bevel gears with high precision and high surface quality, a modified curvature motion method is proposed with the vertical offset ($E_{m1}$), and the increment of machine center to back ($X_{G1}$) modifications without cutter head change in pinion cutting process.

3.1 Mathematical model

The coordinate systems of pinion generator are shown in Fig. 7, in which the coordinate systems $S_p$, $S_{b1}$ and $S_1$ are rigidly connected to the cutter head, the machine tool cradle, and the pinion, respectively. What should be mentioned here in detail is that $\phi_p$ is the cradle rotation angle, $\phi_1$ is pinion rotation angle, $S_{r1}$ is the cutter radial setting, $q_1$ is the initial cradle angle setting, $E_{m1}$ is the vertical offset, $X_{b1}$ is the sliding base feed setting, $X_{G1}$ is the increment of machine center to back and $\gamma_1$ is the machine root angle. The matrix $M_{1p}$ is the homogenous coordinate transform matrix from the head cutter to the work gear.

$$M_{1p} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}$$

(11)

where

$$a_{11} = \cos \gamma_1 \cos \phi_p$$
$$a_{12} = -\cos \gamma_1 \sin \phi_p$$
$$a_{13} = \sin \gamma_1$$
$$a_{14} = S_{r1} \cos \gamma_1 \cos \phi_p \cos q_1 - X_{b1} \sin \gamma_1 - X_{G1} - S_{r1} \cos \gamma_1 \sin \phi_p \sin q_1$$
$$a_{21} = \cos \phi_1 \sin \phi_p - \cos \phi_p \sin \gamma_1 \sin \phi_1$$
$$a_{22} = \cos \phi_1 \cos \phi_p + \sin \phi_p \sin \gamma_1 \sin \phi_1$$
$$a_{23} = \cos \gamma_1 \sin \phi_1$$
$$a_{24} = E_{m1} \cos \phi_1 - X_{b1} \cos \gamma_1 \sin \phi_1 + S_{r1} \sin q_1 (\cos \phi_1 \sin \phi_p - \cos \phi_p \sin \gamma_1 \sin \phi_1) + S_{r1} \sin q_1 (\cos \phi_1 \cos \phi_p + \sin \gamma_1 \sin \phi_1 \sin \phi_p)$$
$$a_{31} = -\sin \phi_1 \sin \phi_p - \cos \phi_p \sin \gamma_1 \cos \phi_1$$
$$a_{32} = -\cos \phi_1 \sin \gamma_1 \sin \phi_p - \cos \phi_p \sin \phi_1$$
$$a_{33} = \cos \gamma_1 \cos \phi_1$$
$$a_{34} = -E_{m1} \sin \phi_1 - X_{b1} \cos \gamma_1 \cos \phi_1 - S_{r1} \cos q_1 (\sin \phi_1 \sin \phi_p + \cos \phi_p \sin \gamma_1 \cos \phi_1) - S_{r1} \sin q_1 (\sin \phi_1 \cos \phi_p - \sin \gamma_1 \cos \phi_1 \sin \phi_p)$$
$$a_{41} = a_{42} = a_{43} = 0$$
$$a_{44} = 1$$
The locus of the cutter surface is denoted as follows:

\[
\mathbf{r}_p(s_p, \Theta_p) = \begin{bmatrix} (R_p + s_p \sin \alpha_1) \cos \Theta_p \\ (R_p + s_p \sin \alpha_1) \sin \Theta_p \\ -s_p \cos \alpha_1 \end{bmatrix}
\]

(12)

\[
\mathbf{n}_p(\Theta_p) = \begin{bmatrix} \cos \alpha_1 \cos \Theta_p \\ \cos \alpha_1 \sin \Theta_p \\ -\sin \alpha_1 \end{bmatrix}
\]

(13)

The locus of the work gear tooth surface can be described as follows:

\[
\begin{align*}
\mathbf{r}_1(s_p, \Theta_p, \phi_1) &= M_{1p}(\phi_1) \mathbf{r}_p(s_p, \Theta_p) \\
\mathbf{n}_1(\Theta_p, \phi_1) &= L_{1p}(\phi_1) \mathbf{n}_p(\Theta_p)
\end{align*}
\]

(14)

3.2 Design parameters of modified curvature motion

There are three groups of design parameters which can be applied to characterize the pinion generator: the cutter head parameters, the initial machining parameters, and the polynomial coefficients of the modified curvature motion. The machine parameters and the cutter head parameters are constant during the procedure. However, the polynomial coefficients of the modified curvature motion can be adjusted to achieve a prespecified function. In order to achieve the purpose of modifying pinion surface curvature, the coefficients of \( E_{\text{ml}} \) and \( X_{G1} \) are adjusted during the procedure.

The modified spatial relationship between the pinion and the head cutter is shown in Fig. 8. The initial locus of the cutter-head center is a circular arc in the machine plane without adjustments to the pinion machine settings. To modify the adjustability of the gear set, we corrected the locus of cutter-head center into a curve.

As shown in Fig. 8, the reference point \( M \) on pinion tooth surface is selected as a control point. The initial locus and the corrected locus intersect at \( M \). The locus of the cutter-head center is adjusted from point \( A \) to point \( B \) along the unit normal vector with correction \( L \). The variations of the vertical offset \( \Delta E_{\text{ml}} \), the horizontal ordinate of the increment of machine center to back \( \Delta X_{G1} \), are changed by modifying the correction \( L \).

Assume that the variations of \( \Delta E_{\text{ml}} \) and \( \Delta X_{G1} \) are fourth-order polynomial functions of the pinion rotation angle with four unknown coefficients and shown as follows:

\[
\begin{align*}
\Delta E_{\text{ml}} &= a_1(\phi_1 - \phi_0) + b_1(\phi_1 - \phi_0)^2 + c_{11}(\phi_1 - \phi_0)^3 + d_{11}(\phi_1 - \phi_0)^4 \\
\Delta X_{G1} &= a_2(\phi_1 - \phi_0) + b_2(\phi_1 - \phi_0)^2 + c_{12}(\phi_1 - \phi_0)^3 + d_{12}(\phi_1 - \phi_0)^4
\end{align*}
\]

(15)
Where \( a_{ti}, b_{ti}, c_{ti}, \) and \( d_{ti} (i = 1, 2) \) are polynomial coefficients, \( \varphi_0 \) is the pinion rotation angle of the control point \( M \).

On the corrected position of the pinion, the variations are denoted as \( \Delta E_m1 \) and \( \Delta X_{G1x} \) respectively. The new locus of the cutter-head center in the machine plane is expressed as follows:

\[
\begin{align*}
B_x &= A_x + L_x^{(i)} n_{dx} \\
B_y &= A_y + L_y^{(i)} n_{dy}
\end{align*}
\]  

where \( L_x^{(i)} (i = 1, 2, 3, 4) \) is the correction on the discrete point position of the pitch cone.

The corresponding variations of \( \Delta E_m1 \) and \( \Delta X_{G1x} \) are expressed as follows:

\[
\begin{align*}
\Delta E_m1 &= B_y^{(i)} - A_y^{(i)} \\
\Delta X_{G1x} &= B_x^{(i)} - A_x^{(i)}
\end{align*}
\]  

(17)

The modified vertical offset \( E_m1 \), and the increment of machine center to back \( X_{G1} \) are denoted as follows:

\[
\begin{align*}
E_m1 &= E_m^{(0)} + \Delta E_m1 \\
X_{G1} &= (X_{G1}^{(0)} \cos \gamma_1 - \Delta X_{G1x}) / \cos \gamma_1
\end{align*}
\]  

(18)

where \( E_m^{(0)}, X_{G1}^{(0)} \) are the original machine settings of vertical offset and increment of machine center to back, respectively. Note that \( Z_{R1} \) is the distance of pinion from the pitch cone vertex to root cone vertex.

The locus of the work gear tooth surface can be described in the coordinate system \( S_1 \) as follows:

\[
\begin{align*}
\mathbf{r}_1(s_p, \theta_p, \phi_i) &= \mathbf{M}_{1p}(\phi_i) \mathbf{r}_{p}(s_p, \theta_p) \\
\mathbf{n}_1(\theta_p, \phi_i) &= \mathbf{L}_{1p}(\phi_i) \mathbf{n}_p(\theta_p)
\end{align*}
\]  

(19)

where \( s_p \) and \( \theta_p \) are the pinion tooth surface parameters. The matrix \( \mathbf{L}_{1p} \) is upper-left \( 3 \times 3 \) sub-matrix of matrix \( \mathbf{M}_{1p} \).

The meshing equation can be deduced based on differential geometry theory:

\[
f_1 = \mathbf{n}_{hp} \mathbf{V}_{hp}^{(p)} = 0
\]  

(20)

Then the position vectors and unit normal vectors of discrete point of pinion real surface are as follows:

\[
\begin{align*}
\mathbf{p}_i &= \mathbf{r}_1(s_{pi}, \theta_{pi}, \phi_{pi}) \\
\mathbf{n}_i &= \mathbf{n}_1(\theta_{pi}, \phi_{pi})
\end{align*}
\]  

(21)

The deviation between pinion real surface and target surface at discrete point can be expressed by the following equations:

\[
h_i = (\mathbf{p}_i^* - \mathbf{p}_i) \mathbf{n}_i
\]  

(22)

\[
h = [h_1, h_2, h_3, \ldots, h_k]^T
\]  

(23)

Finally, the optimization model is established to solve the correction \( L \), see Eq. (24), in which the optimization model sets correction \( L \) as variables and the least sum of square errors of the deviation between pinion real and target tooth surface as object. NSGA2 is applied to solve the model:

\[
\begin{align*}
\min f(L^{(i)}) &= h^T h \\
L^{(i)} &\in [x_1, x_2]
\end{align*}
\]  

(24)

where \( x_1 \) and \( x_2 \) are the minimum and maximum values of variables, respectively.

4. Numerical Example

In order to analyze the meshing characteristics of high contact ratio gears based on the predesigned seventh-order
transmission error with modified curvature motion, a numerical example was carried out. Taking the working surface of a spiral bevel gear as an example, the geometry parameters are listed in Tables 1-3, and the predesigned parameters of transmission error curve are given in Table 4. The predesigned parameters for modification curve along meshing line are shown in Table 5.

### Table 1 Geometry parameters of spiral bevel gears

| Items                        | Pinion | Gear |
|------------------------------|--------|------|
| Teeth number                 | 23     | 65   |
| Modulus [mm]                 | 3.9    |      |
| Pressure angle [Deg]         | 20     |      |
| Mean spiral angle [Deg]      | 25     |      |
| Shaft angle [Deg]            | 90     |      |
| Direction of rotation        | R      | L    |
| Pitch cone angle [Deg]       | 19.4861| 70.5139|
| Apex angles [mm]             | 21.777 | 71.6718|
| Root cone angle [Deg]        | 18.3282| 68.2230|
| Outer cone distance [mm]     | 134.4511|      |
| Face width [mm]              | 37     |      |
| Tooth top height [mm]        | 4.6456 | 1.9844|
| Tooth root height [mm]       | 2.7176 | 5.3788|

### Table 2 Machine settings of gear

| Items                        | Gear |
|------------------------------|------|
| Cutter diameter [mm]         | 188.4700|
| Profile angle [Deg]          | 22.5000|
| Radial setting [mm]          | 114.752|
| Initial cradle angle setting [Deg] | 48.7882|
| Roll ratio                   | 1.0599|
| Vertical offset [mm]         | 0     |
| Sliding base feed setting [mm] | 0     |
| Increment of machine center to back [mm] | 0     |
| Machine root angle [Deg]     | 68.2230|

### Table 3 Predesigned machine settings of pinion

| Items                        | Pinion |
|------------------------------|--------|
| Cutter radius [mm]           | 91.3893|
| Profile angle [Deg]          | 22.5000|
| Radial setting [mm]          | 108.6437|
| Initial cradle angle setting [Deg] | 48.5644|
| Roll ratio                   | 2.8535|
| Vertical offset [mm]         | 2.7255|
| Sliding base feed setting [mm] | 1.0942|
| Increment of machine center to back [mm] | -2.9081|
| Machine root angle [Deg]     | 18.3282|

### Table 4 Predesigned parameters of transmission error curve

| Parameters | CR | δ_1 ["] | δ_2 ["] | δ_3 ["] | δ_4 ["] | δ_5 ["] | φ_m [Deg] | 1 | 2 | 3 [Deg] |
|------------|----|----------|----------|----------|----------|----------|-----------|---|---|---------|
|            | 2.4 | -55      | -1.3     | 0        | -55      | 1        | 0.5       | 0.5| 169 |

### Table 5 Predesigned parameters for modification along meshing line

| a [mm] | a' |
|--------|----|
| 7th-order TE | 7 | 0.0001 |

#### 4.1 Pinion Target Surface

The pinion auxiliary tooth surface is acquired with the predesigned seventh-order transmission error through using the gear as a virtual cutter, and then the pinion target tooth surface is obtained by modifying the pinion auxiliary tooth surface along the meshing line with a predesigned modification curve. The target tooth surface is shown in Fig. 9-10.
Fig. 9 Pinion target surface obtained by function oriented design based on ease-off. The pinion target surface and the predesigned surface are plotted with the thick (blue), thin (black) lines, respectively.

Fig. 10 Construction of the target, predesigned surfaces. The pinion target surface and the predesigned surface are plotted with the thick (blue), thin (black) lines, respectively.

4.2 Pinion Actual Surface by Modified Curvature Motion Method

The pinion tooth surface corrections $L$ are be deduced through the NSGA2 and the corresponding parameters are listed in Table 6. Additionally, the polynomial coefficients of $E_{m1}$ and $X_{G1}$ are shown in Table 7.

The corresponding pinion actual tooth surface topologies comparison of modified curvature motion is shown in Fig. 11. Note that the construction of the actual, predesigned surfaces is shown in Fig. 12.

| $L^{(1)}$ [mm] | $L^{(2)}$ [mm] | $L^{(3)}$ [mm] | $L^{(4)}$ [mm] |
|----------------|----------------|----------------|----------------|
| -8.598E-4      | -4.0894E-4     | -1.5440E-4     | 2.9833E-4      |

**Table 7 Tooth surface curvature correction parameters**

| $\Delta E_{m1}$ | $a_t$       | $b_t$       | $c_t$       | $d_t$       |
|-----------------|-------------|-------------|-------------|-------------|
| 0.00568         | 0.03653     | -0.49318    | -2.64818    |
| $\Delta X_{G1x}$| -0.00271    | -0.01398    | 0.23077     | 0.94965     |

Fig. 11 Pinion topological error map with modified curvature motion. The pinion actual surface and the predesigned surface are plotted with the thick (red), thin (black) lines, respectively.
4.3 Tooth Contact Analysis

Take the pinion obtained from the modified curvature motion method as an example, we analyzed the meshing performance of the pinion and the gear through the TCA technology. The TCA results of spiral bevel gear are shown in Fig.13 and Table 8. MCM indicates that pinion with modified curvature motion and CMS indicates pinion with constant machining settings.
Table 8 Comparison of the meshing performance

| Parameter | Design contact ratio | Length of long half axle [mm] |
|-----------|----------------------|-------------------------------|
|           | MCM | CMS   | MCM   | CMS   |
| Predesign value |  2.4000 |   2.4236 |   7.1139 |  7.1812 |
| The actual value  |  2.4167 |   0.9833% |  1.6271% |  2.5886 |
| The percent error |   0.6958% |             |             |             |

Through analyzing data shown in Table 8 and Fig. 13, we can get the following conclusions:

The TCA results of spiral bevel gear acquired by modified curvature motion method in accordance with the preconditions: the contact path on the rotary projection of the pinion surface is a straight line. Compared with the predesigned value, the transmission error curve is seventh-order curve with an error of only 0.6305%, the symmetry error of transmission error curve is 0.8058 Sec, the percent error of contact ratio is only 0.6958%. But for spiral bevel gear with the constant machining settings, the transmission error curve is seventh-order curve with an error of 5%, the symmetry error of transmission error curve is 1.7488 Sec, the percent error of contact ratio is only 0.9833%. As is shown in Fig. 13(c), the furthest distance from predesigned contact path of gear set with modified curvature motion is 0.140mm, which is about 6.7% less than that of gear set with constant machining settings. By comparison, the meshing quality of spiral bevel gear with modified curvature motion is better than that with the constant machining settings.

Above all, the proposed modified curvature motion method can improve the meshing quality of spiral bevel gears.

In order to show the advantages of pinion double crowned, comparisons between gear set with pinion double crowned and gear set without pinion double crowned are performed. Furthermore, the contact patterns for gear tooth surface are shown in Fig. 14. Through comparison, the contact patterns for gear set with pinion double crowned is in the middle of the tooth surface, and the contact pattern mode for gear set without pinion double crowned deviates from the middle of the tooth surface. It is noting that no distortion occurred in the contact area. The simulation results proved pinion double crowned is feasible, which is helpful to improve the meshing quality of high contact ratio spiral bevel gears.

![Fig. 14 Comparison of contact pattern for gear. (a) Gear set with pinion double crowned, and (b) Gear set without pinion double crowned](image)

**5. Conclusions**

To sum up, we proposed an ease-off flank design method for high contact ratio spiral bevel gears by modified curvature motion method, which can be put into use in the phase of trial-manufacture.

Based on the second-order transmission error, a seventh-order transmission error for high contact ratio spiral bevel gears can be implemented by the modified curvature motion method with the predesigned transmission error, compared with traditional methods, the proposed modified curvature motion method can improve the quality of high contact ratio spiral bevel gears, it can be used to process high quality gears.

Compared with the conventional lengthwise curvature change method, the proposed modified curvature motion method can make full use of CNC spiral bevel gears generator, and it can give full play to the advantages of CNC spiral bevel gears generator as well. What should be noted is that the proposed method can improve the production efficiency, and at the same time, it can help to improve the quality of high contact ratio spiral bevel gears. Last but not the least, the proposed method is also applicable to other types of gears.
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