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Testing chameleon fields with ultra cold neutron bound states and neutron interferometry

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Abstract
Chameleons can play a role in the recent acceleration of the expansion of the Universe. We present results where chameleon fields modify the terrestrial gravitational potential above a mirror and lead to a small perturbation of the energy levels of the neutrons in experimental setups such as GRANIT in Grenoble. With the current sensitivity, these experiments probe chameleon fields in a large coupling to matter regime. We also comment on the possibility of testing chameleon fields with neutron interferometry.

Keywords: European Spallation Source; Dark Energy; scalar field; neutron quantum bouncer; neutron interferometry.

1. Introduction
The recent acceleration of the expansion of the Universe has no thoroughly compelling explanation. One plausible possibility is the existence of dark energy [1], a dynamical generalization of a time-independent vacuum energy or cosmological constant. Another line of thought could be the presence of a modified law of gravity on very large cosmological scales which would not manifest itself at short distance, i.e. in the solar system and the laboratory [2]. Both explanations seem to require the use of a ubiquitous scalar degree of freedom whose vacuum expectation value now would be responsible for the acceleration of the Universe. In the dark energy case, this follows from the fact that, in analogy with an earlier period of acceleration known as primordial inflation in the very early Universe, scalar fields can have dynamical properties whereby their long time behavior is almost independent of their initial condition and their value now can be governed by the shape of their interaction potential. In such a class of models, the Ratra-Peebles one is particularly conspicuous [3]. Modified gravity has a long and tortuous history starting from the Pauli-Fierz model of massive gravity [4]. It has been known for a long time that this model describes the five helicity modes of a massive graviton: two helicity 2 ones, 2 vector-like ones and one scalar degree of freedom. The presence of a mass term implies that the propagator of gravitons has a Yukawa falloff at infinity, and therefore modifying gravity on large scales (although in a way which does not correspond to the phenomenology of an accelerating Universe). Moreover, although this theory is ghost-free, i.e. the Hamiltonian is bounded from below, on a flat space-time background; it

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becomes ghost-like in a curved background such as a cosmological one. This prevents this model from making any sense and massive gravity was therefore abandoned for a long time. It has been revived recently from a modern point of view where a ghost-free presentation has been eventually defined using a highly non-linear model [5]. When reduced to the scalar degree of freedom comprising one of the five polarizations of the massive graviton, these models reduce to the generalized Galileon models first described by Horndeski in 1974 [6]. They are the most general scalar-tensor theories whose equations of motion are second order. Moreover when reduced to a flat background, they possess a Galilean invariance which plays a fundamental role in the description of their gravitational properties in the presence of dense objects such as the sun or the moon. In both the dark energy and the modified gravity cases, the influence of the scalar degree of freedom can only be felt on very large scales provided the mass of the scalar field in the cosmological background is extremely low. The stability of this low mass to radiative corrections is a thorny issue which is not at all understood and which we will not tackle here. On the other hand, the existence of a low mass scalar field coupled to matter, either baryons or Cold Dark Matter (CDM), has a drastic influence on the physics of the solar system. Indeed, a nearly massless scalar interaction on the scale of the solar system acts as a fifth force whose coupling strength is tightly constrained by the Cassini probe [7]. If the modification of the gravitational potential in the solar system takes the form of a Yukawa potential, which is always the case as long as the scalar degree of freedom couples to matter with a coupling strength and has a mass $m$:

$$\Phi = -\frac{G_N}{r} \left( 1 + 2\beta \phi \frac{e^{-mr}}{r} \right)$$

(1)

the coupling must be such that $\beta^2 \leq 10^{-5}$. Stronger constraints are even available from the Lunar Ranging experiment [8]. Such a low coupling is unnatural unless a proper explanation can be found (the level of unnaturalness of a model is not a well defined notion although in quantum field theory one expects that couplings which are not forbidden by a symmetry should be of order one). This is where screening plays a role. Indeed, natural models of dark energy or modified gravity with a nearly massless scalar field on cosmological scales should be such that the effects of the scalar in the solar system on gravitational tests should be screened. Three types of screening have been uncovered so far and may well cover all the generic cases. They can be easily understood by expressing the scalar field $\phi$ around its background configuration $\phi_0$ (for instance its average value in the solar system) as $\phi = \phi_0 + \varphi$ and expanding the Lagrangian describing its dynamics up to second order

$$\mathcal{L} \supset -\frac{Z(\phi_0)}{2} (\partial \varphi)^2 - \frac{m^2(\phi_0)}{2} \varphi^2 - \frac{\beta(\phi_0)}{m_{\text{pl}}} \varphi \delta \rho.$$  

(2)

The background value $\phi_0$ itself depends on the environment and in particular on the background matter density. At second order, the scalar field couples to the energy density $\delta \rho$ via the coupling constant $\beta(\phi_0)$. The wave function normalisation $Z(\phi_0)$ appears in particular in models of the Galileon type where the kinetic terms of the full theory are of higher order in the derivatives of the scalar field. The mass of the scalar field $m(\phi_0)$ depends on the environment too. All in all, the three effective parameters $Z(\phi_0)$, $m(\phi_0)$ and $\beta(\phi_0)$ are enough to distinguish the main screening mechanisms.

Here, gravity is modified in as much as the coupling of $\phi$ to matter implies a modification of the geodesics which depend now on the full Newtonian potential

$$\Phi = \Phi_N + \beta \frac{\phi}{m_{\text{pl}}}$$

(3)

where $\Phi_N$ is the usual Newtonian potential satisfying the Poisson equation. The scalar force is screened by the Vainshtein mechanism [9] when $Z(\phi_0)$ is large enough that the coupling of the normalised field $\beta(\phi_0)/Z(\phi_0)$ is small enough. The chameleon mechanism [10] occurs when the mass $m(\phi_0)$ is large enough to suppress the range of the scalar force in dense environments. In particular, this implies that the field generated by the bulk of matter inside a dense body is Yukawa suppressed leaving only the contribution from a thin shell at the surface of the body reaching the less dense region outside the compact object. This suppression is the essence of the chameleon mechanism. Finally, the Damour-Polyakov screening [11] is such that $\beta(\phi_0)$ itself is small. Models of the massive gravity types with a generalized Galileon description for their scalar degree of freedom are subject to the Vainshtein mechanism. Dilatons [11] and symmetrons [12, 13, 14] are described by the Damour-Polyakov mechanism. Here we will focus on the chameleon mechanism only.
2. Chameleons

Chameleons have been introduced to model the acceleration of the expansion of the Universe [15] using a scalar field (called the chameleon) whose dynamics are governed by a potential \( V(\phi) \) which depends on a single scale \( \Lambda \)

\[
V(\phi) = \Lambda^4 f(\phi/\Lambda) \tag{4}
\]

where \( \Lambda \) is determined by the present value of the dark energy, \( \Lambda^4 = 3\Omega_{\Lambda 0} H_0^2 m_{\text{Pl}}^2 \) where \( H_0 \) is the Hubble rate now, i.e. \( \Lambda \sim 2.4 \times 10^{-12} \) GeV. Hence we require that when \( \phi \gg \Lambda \), \( f \rightarrow 1 \) which corresponds to a large class of runaway potentials. Moreover, \( f \) is assumed to be monotonic (increasing) and convex guaranteeing that the second derivative of \( V \) is positive, i.e. that the mass of the scalar field (in the absence of matter) is positive. One can choose for instance

\[
V(\phi) = \Lambda^4 \exp\left(\frac{\Lambda}{\phi}\right) \tag{5}
\]

where \( n > 0 \). When \( \phi \gg \Lambda \), this behaves like a Ratra-Peebles model

\[
V(\phi) = \Lambda^4 + \frac{\Lambda^{4+n}}{\phi^n} + \ldots \tag{6}
\]

where the leading terms are the only relevant ones. For such a model, dark energy is realised when \( \phi \sim m_{\text{Pl}} \) which corresponds to a mass of the scalar field of order \( H_0 \). This is the only case when the equation of state can be close to \(-1\). Hence this model of dark energy leads to the existence of a long range scalar force. This force can be screened in the solar system when the chameleon couples to matter. Indeed inside large (and screened) objects such as the sun, the field generated by an infinitesimal element is Yukawa-suppressed and does not reach the outer region of the compact body. Only a thin shell generates any field, which is therefore heavily depleted outside, leading to a negligible deviation from Newton’s law. It turns out that \( \Lambda_0 \) in the Ratra-Peebles potential \( \Lambda_0^{4+n}/\phi^n \) such that all the gravitational tests are evaded must be \( \Lambda_0 \leq \Lambda \). Hence chameleons can both generate the acceleration of the expansion of the Universe and satisfy the gravity tests of Newton’s law with a single scale \( \Lambda \).

3. Bouncing Neutrons

Chameleons have an influence on the energy levels of neutrons in the gravitational field of the earth [16]. This follows from the fact that over a mirror with a high density, the chameleon field acquires a universal profile. In the absence of a coupling to matter, the potential is \( \Phi(z) = mgz \) with \( g = 9.81 \) m/s. The interaction potential in the presence of a chameleon is

\[
\Phi(z) = mgz + \beta V_n (\Lambda z)^{\alpha_n} \tag{8}
\]

with \( V_n = (m/m_{\text{Pl}}) \Lambda (2 + n/\sqrt{2})^{2/(2+n)} \) and \( \alpha_n = 2/(2 + n) \). The Schrodinger equation becomes

\[
-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} \psi + \Phi(z) \psi(z) = E \psi(z) \tag{9}
\]

where \( m \) is the mass of the neutron, \( \psi \) the wave function (with \( \psi(0) = 0 \) on the mirror), \( E \) the energy of the neutron, and \( z \) the distance neutron-mirror. Without the chameleons, the solutions are Airy functions and the energy levels determined by the zeros of the Airy function. The shifted energy levels are given by

\[
\delta E_k = \beta V_n < \psi_k | (\Lambda z)^{\alpha_n} | \psi_k > \tag{10}
\]
where $|\psi_k>$ is the k-th level wave function. In GRANIT, the gap between two energy states such as $k = 3$ and $k = 1$ can be precisely measured with an estimated accuracy of 0.01 peV compared to the nominal energy $E_3 - E_1 = 1.91$ peV. By requiring that the chameleonic shift does not exceed the expected sensitivity, we get a bound on the coupling $\beta$ which depends on $n$. Notice that Fig. 1 plots the part of parameter space which will be covered by the GRANIT experiment. Experimental results leading to constraints not as good as the expected ones from GRANIT have already been obtained [17]. Chameleon fields will also be tested by Casimir experiments in the near future [18]. Regions below $\beta = 10^9$ could only be reached by increasing the sensitivity and/or using different experimental techniques such as neutron interferometry which will be discussed in the next section.

4. Neutron Interferometry

For the purpose of detecting chameleons, neutron interferometry can be used where the sample influencing the phase difference between neutrons along two different paths will consist of a cell with parallel plates normal to the neutron beam. In this cell the chameleon field has a bubble-like profile [19]. We assume that the transverse dimension of the cell is infinite and denote the distance between the plates by $2R$, which we set $2R = 1$ cm in Fig. 1. When the cell is filled by no gas, i.e. in vacuum, a chameleon bubble-like profile $\phi(x), -R < x < R$, will appear in the cell, inducing the potential $\beta m/m_{Pl} \phi(x)$ for the neutrons. As the pressure increases, the bubble eventually disappears implying that chameleons have an effect on neutron interferometry only at relatively low pressure. The phase shift due to the chameleon bubble is given by

$$\delta \varphi = \frac{m}{\hbar^2} \int_{-R}^{R} \beta \frac{m}{m_{Pl}} \phi(x) dx.$$  \hspace{1cm} (11)

where $m$ is the neutron mass and $k$ its energy. In vacuum the phase shift is given by

$$\delta \varphi = \frac{\sqrt{2}\beta m^2}{k m_{Pl} \hbar^2} \left( \frac{\sqrt{2} R A}{J_n(0)} \right)^{\frac{2}{3}} K_n(0).$$  \hspace{1cm} (12)
where $J_n$ and $K_n$ are Bessel functions. In Fig. 1, we have plotted the parameter space of chameleon models for a typical neutron interferometry experiment at low pressure with a sensitivity of 1 degree. One could hope that dedicated experiments could reach lower values of $\beta$.

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