Approximate theory the electromagnetic energy of solenoid in special relativity

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Abstract. Solenoid is a device that is often used in electronic devices. A solenoid is electrified will cause a magnetic field. In our analysis, we just focus on the electromagnetic energy for solenoid form. We purpose to analyze by the theoretical approach in special relativity. Our approach is begun on the Biot Savart law and Lorentz force. Special theory relativity can be derived from the Biot Savart law, and for the energy can be derived from Lorentz for, by first determining the momentum equation. We choose the solenoid form with the goal of the future can be used to improve the efficiency of the electrical motor.

1. Introduction
The currents which arise due to the motion of charges are the source of magnetic fields. When the charges move in a wire at a certain velocity can produce a current I, the magnetic field at any point P due to the current can be calculated by adding a small part of magnetic field dB, from small segments of the wire ds, (Figure 1.1)

![Figure 1. Magnetic field dB at point P because the effect of the current on the wire](image)

These segments can be assumed as a vector quantity. It has magnitude of the length of the segment and pointing in the direction of the current flow in the wire. The source of current is very small and be written as I/ ds [1].

Let r denote as the distance from the current source (in this case is a wire) to the field point P, and \( \hat{r} \) the corresponding unit vector. The Biot-Savart law gives an expression for calculate the magnetic field contribution, d\( \mathbf{B} \), from the current source, I ds, and the equation of Biot-Savart Law is
\[ \mathcal{dB} = \frac{\mu_0 I ds \times r}{4\pi r^2} \]  

(1)

Where \( \mu_0 \) is a constant called the permeability of free space

\[ \mu_0 = 4\pi \times 10^{-7} \text{ T.m/A} \]  

(2)

Biot-Savart law is used to determine the magnetic field strength either on a straight wire, circular, solenoids or toroid. By an integral part, we can find the magnetic field strength either on a straight wire, circular, solenoids or toroid [2].

Previously some analysis related to electromagnetic theory has been done. As performed by Babson et al. Babson analyze about momentum and the electromagnetic impulses [3]. Electromagnetic Momentum can also be written in the form of relativistic, in conjunction with the speed and energy [4]. Besides momentum Faraday's law of induction can also be analyzed in the form of special relativity [5].

In this case, we just use the magnetic field strength in solenoid form. The magnetic field strength of solenoid is applied to electromagnetic force and is brought to the relativistic theory. In the last season, we must set the relativistic energy by using a momentum equation.

2. Numerical Method of Electromagnetic Force for Relativistic Theory

It was explained earlier that the Biot Savart law can be applied to a straight wire, circular, solenoids, or toroids. In this study only focused on a circular wire on the solenoid, given for most of the electronic device that works by a strong magnetic field using a solenoid wire. For example transformer, electric motor, dynamo and so on.

To analyze the electromagnetic field in relativistic, we can use two assumptions call inertial frame \( S \) and \( S' \). Assume both have a frame of reference in which the initial state \( (x = y = z = 0) \) at \( t = 0 \). While conditions for inertial frame \( S' \) has a velocity relative to frame \( S \) is parallel to the \( x \)-axis. Look at the figure bellows,

![Figure 1. Two inertial frame S and S’](image)

Inertial frame \( S \) with an overview of the time \( t \) and the coordinate axes \( (x, y, z) \), while for the inertial frame \( S' \) has reviews time \( t' \) and the coordinates \( (x', y', z') \). Relations between the two inertial frame both for time and coordinates described using Lorentz transformations. By using the Lorentz transformation we can the relationship between the inertial frame \( S \) and \( S' \) for each dimension, namely by using the constant \( \gamma \) where,

\[ \gamma = \sqrt{1 - \frac{v^2}{c^2}} \]  

(3)

Where \( v \) is the relative speed of \( S' \) with respect to \( S \).
With reviews inertial frame $S$ then we can perform an analysis of the movement of cargo with a certain speed if using a review inertial frame $S'$. For the relativistic case analysis that we use is the inertial frame $S'$ so that the charge is looked at is the charge contained in the inertial frame $S'$ moving with velocity $v$ circular [6].

In connection with the relativistic case, the number of negative charge per unit length is $\gamma N$ while the number of positive charge per unit length is $N/\gamma$, so the total number of charge is, 

$$\lambda' = q + (-q)$$

$$\lambda' = \frac{N}{\gamma} - \gamma N$$

$$\lambda' = N\left(\frac{1}{\gamma} - \gamma\right)$$

$$\lambda' = -\gamma N\left(1 - \frac{1}{\gamma^2}\right)$$

$$\lambda' = -\gamma N \frac{v^2}{c^2}$$

(4)

If the total charge is used to determine the total current, where the current total is the total amount that flows from the positive charge with a speed $v$, then $I$ can be written as a form of equation

$$I = Nv$$

(5)

So that the charge density is equal,

$$\lambda' = -\gamma \frac{v}{c^2} I$$

(6)

The value of the total electric field is

$$E' = \frac{\lambda'}{2\pi \varepsilon_0 r} = -\gamma \frac{v}{2 \pi \varepsilon_0 c^2} = -\gamma \frac{1}{2 \pi \varepsilon_0 c^2} \frac{v}{c^2} I$$

(7)

By setting the equation $\frac{1}{c^2} = \mu_0 \varepsilon_0$, so the Eq. (7) can be rewritten,

$$E' = -\gamma \frac{\mu_0 I}{2\pi r}$$

The electric field equations we will use to determine the electromagnetic force equation for the solenoid. Starting from Biot-Savart equation for circular wire, then developed into the form of the equation for the solenoid. If the electromagnetic force equation in the form of relativistic known, then using the momentum equation, can be determined equation to electromagnetic energy in the form of relativistic theory.

Based on the explanation before, we try to find the equation of electromagnetic energy in a relativistic theory. The first we start from Biot-Savart Law in solenoid form. At a radial distance denote by $a$ (in this case $a$ equals the radius of the wire) the long of solenoid is $l$, and the number of coils denotes by $N$, we see a magnetic field :

$$B = \frac{\mu_0 IN}{l}$$

(3)

From the equation of magnetic field strength at the solenoid, we can find the equation for the vector potential that will be used to determine the equation of the electromagnetic momentum.
3. Electromagnetic Momentum and Electromagnetic Energy

Electromagnetic Momentum is the result of multiplying the charge with a potential vector [7]. To define the vector potential in the solenoid, let’s do an analysis of the solenoid long has the winding number $N$, where the radius of solenoid denoted by $a$, and a point along the solenoid with the distance $r$, to determine potential vector we use the surface integral of $B$ which is perpendicular to the solenoid [8]. The potential vector equations obtained is,

$$A = \frac{1}{2} \mu N r l$$  \hspace{1cm} (9)

To determine the equation of the electromagnetic momentum, we can multiply the vector potential with electric charge $q$. So that we will find the equation of electromagnetic momentum in the function of $v$.

$$p = \frac{1}{2} q^2 \mu N v$$  \hspace{1cm} (10)

The electromagnetic momentum of the equation we can derive the equations of the electromagnetic energy in a relativistic theory. Electromagnetic energy can be obtained by first determining the relativistic mass (in this case the mass used is an electric charge). Electromagnetic energy in the form of relativistic is obtained by integrating the force versus distance, where the distance is the length of the solenoid is denoted by $L$. So we obtain,

$$E = \int_0^v F \cdot dL$$  \hspace{1cm} (11)

Where $F$ is a differential of momentum with respect to time, so the Eq. (11) can be rewritten as,

$$E = \int_0^v \frac{dp}{dt} \cdot v \cdot dt$$  \hspace{1cm} (12)

Eq. (12) can be solved by partial integral models. The charge that is used is relativistic charge, so we get the equation,

$$E = \int_0^v \frac{d\left(\frac{1}{2} q^2 \mu N v\right)}{dt} \cdot v \cdot dt$$

Where $\mu$ and $N$ is constants, so it can get out of the integral form
\[ E = \frac{1}{2} \mu N v \int_0^\infty \frac{d(q^2 v)}{dt} \cdot v \, dt \] (13)

Because we use the relativistic charge, so

\[ q = \gamma q_0 \] (14)

Where \( q_0 \) is the rest charge, and the value of \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \). By substituting Eq. (14) to the Eq. (13) we can obtain,

\[ E = \frac{1}{2} \mu N v \int_0^\infty \frac{d(q^2_0 v)}{dt} \cdot v \, dt \] (15)

The rest charge can be assumed as constants, so can go out from the part of integral, and we obtain,

\[ E = \frac{1}{2} \mu N v q_0^2 \int_0^\infty v \cdot d(\gamma^2 v) \] (16)

Eq. (16) can be solved by partial integral,

\[ E = \frac{1}{2} \mu N v q_0^2 \left( \int_0^\gamma \gamma^2 v^2 \, dv - \int_0^\gamma \gamma^2 v \, dv \right) \] (17)

By completing the process of the integral in Eq. (17) by changing the value of \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \) and sets the \( v = c \sin \theta \) then we will obtain the equation of the electromagnetic energy in relativistic theory

\[ E = \frac{1}{2} \mu N v q_0^2 \left( \int_0^\gamma \gamma^2 v^2 \, dv - c^2 \ln \left( \frac{1 - \frac{v^2}{c^2}}{c^2} \right) \right) \] (18)

Eq. (18) is a model of the theoretical approach to the energy equation for the relativistic electromagnetic solenoid. Further development of this theory research can be done on a toroid or some other device that uses the principle of the magnetic field.

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