Impurity effect of Λ hyperon on shape-coexistence nucleus $^{44}\text{S}$ in the energy functional based collective Hamiltonian

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The non-relativistic Skyrme energy density functional (EDF) based collective Hamiltonian, that takes into account dynamical correlations related to the restoration of broken symmetries and fluctuations of quadrupole collective variables, is applied to quantitatively study the impurity effect of Λ hyperon on the collectivity of $^{44}\text{S}$. Several Skyrme forces for both the nucleon-nucleon (NN) and Λ-nucleon (ΛN) interactions are used. The influence of pairing strengths on the polarization effect of Λ hyperon is also examined. It is found that although these Skyrme forces with different pairing strengths give somewhat different low-lying spectra for $^{44}\text{S}$, all of them give similar and generally small size of Λ reduction effect (within 5%) on the collective properties.

1. Introduction

Since the first discovery of Λ hypernuclei by observing cosmic-rays in emulsion chambers, hypernuclei – which are nuclei with one or more of the nucleons being replaced with hyperons – have been used as a natural laboratory to study hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions, properties of hadrons in nuclear environment, and in particular the impurity effect of hyperon in nuclear medium. Due to the absence of Pauli’s principle between the nucleon and the Λ particle, a Λ hyperon can probe deeply into the interior of nuclear medium and have important influences on its properties, including softening the equation of state modifying the shape and size of finite nucleus, changing the nuclear binding and thus the driplines of neutrons and protons as well as the fission barrier heights in heavy nuclei. The facilities built at J-PARC will provide an opportunity to perform hypernuclear γ-ray spectroscopy study with high precision by improving the quality of the secondary mesonic beam. These facilities offer useful tools to study the low-lying states of hypernuclei.

In recent years, both the non-relativistic Skyrme-Hartree-Fock (SHF) and the relativistic mean-field (RMF) approaches have been applied to study

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the polarization effect of Λ hyperon on the deformation of atomic nuclei. It has been found that, generally, the shape polarization effect of the Λ hyperon is not evident, but with several exceptions, including $^{13}_A\Lambda$, $^{23}_A\Lambda$, and $^{29,31}_A\Lambda$. In these studies, however, the predicted energy surface is shown to be somewhat soft, in which case a large shape fluctuation effect of collective vibration might be expected. Furthermore, symmetry (e.g., translation, rotation, particle number) is usually spontaneously broken in the single-reference (SR) EDFs. The symmetry restoration becomes particular important for studying the spectrum of low-lying states.

Recently, the low-lying states of Λ hypernuclei have been studied with the antisymmetrized molecular dynamics (AMD) model\textsuperscript{16} in which the projection techniques and generator coordinate method (GCM) have been implemented to take into account the above deficiencies of static mean-field approaches. As the gaussian overlap approximation of GCM, the collective Bohr Hamiltonian with parameters determined by the self-consistent mean-field calculations is much simple in numerical calculations, and has achieved great success in description of the low-lying states of normal nuclei.\textsuperscript{17} In view of these facts, most recently, we have constructed a five-dimensional collective Hamiltonian (5DCH) with the parameters derived from the SHF+BCS calculations for the nuclear core $^{25}_M\Lambda$Mg and calculated the corresponding low-spin excitation spectra.\textsuperscript{18} The SGII force\textsuperscript{19} for the NN effective interaction and the No.1 set in Ref.\textsuperscript{20} for the ΛN interaction were adopted. It has been shown that the Λ hyperon stretches the ground state band and reduces the $B(E2 : 2^+_1 \rightarrow 0^+_1)$ value by $\sim 9\%$, mainly by softening the potential energy surface towards the spherical shape. Similar conclusion has also been found in the AMD study for $^{25}_M\Lambda$Mg quite recently.\textsuperscript{21}

In this paper, we apply the same framework of Ref.\textsuperscript{18} to quantitatively evaluate the impurity effect of Λ hyperon on the collectivity of shape-coexistence nucleus $^{44}_S$S with several different Skyrme forces for both the NN and ΛN effective interactions as well as different pairing strengths for nucleons.

2. The Skyrme energy functional based collective Hamiltonian

The framework of Skyrme energy functionals based collective Hamiltonian for nuclear core of Λ hypernuclei has been explained in detail in Ref.\textsuperscript{18}. Here, we just present an outline. In this model, we start from the collective Hamiltonian that describes the nuclear excitations of quadrupole vibrations, 3D rotations, and their couplings\textsuperscript{22}\textsuperscript{23}\textsuperscript{24}

$$\hat{H} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}},$$

where $V_{\text{coll}}$ is the collective potential. The vibrational kinetic energy reads,

$$\hat{T}_{\text{vib}} = -\frac{\hbar^2}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \left[ \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^4 B_{\gamma} \frac{\partial}{\partial \beta} - \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^3 B_{\beta \gamma} \frac{\partial}{\partial \gamma} \right] \right. $$

$$\left. + \frac{1}{\beta \sin 3\gamma} \left[ - \frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta \gamma} \frac{\partial}{\partial \beta} + \frac{1}{\beta \sin \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta \beta} \frac{\partial}{\partial \gamma} \right] \right\} ,$$

(2)
and the rotational kinetic energy,
\[ T_{\text{rot}} = \frac{1}{2} \sum_{\kappa=1}^{3} \hat{J}_{\kappa}^2, \]
(3)
with \( \hat{J}_{\kappa} \) denoting the components of the angular momentum in the body-fixed frame of a nucleus. The mass parameters \( B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma} \), as well as the moments of inertia \( I_{\kappa} \), depend on the quadrupole deformation variables \( \beta \) and \( \gamma \).

Two additional quantities that appear in the expression for the vibrational energy, that is, \( r = B_{12}B_{3}, \) and \( w = B_{\beta\beta}B_{\gamma\gamma} - B_{3}^{2} \), determine the volume element in the collective space. The dynamics of the collective Hamiltonian is governed by seven collective quantities, that is, the collective potential \( V_{\text{coll}} \), three mass parameters \( B_{\beta\beta}, B_{\beta\gamma}, \) and \( B_{\gamma\gamma} \), and three moments of inertia \( I_{\kappa} \), all of which are determined by the constrained SHF+BCS calculations for the nuclei with and without \( \Lambda \) hyperon.

Recently, in Ref. [13], the computer code ev8 25 of SHF+BCS approach has been extended for the study of \( \Lambda \) hypernuclei, which makes it feasible to carry out the quantitative study of \( \Lambda \) effect on nuclear collectivity.

In the SHF+BCS calculations for \( \Lambda \) hypernucleus, the total energy \( E_{\text{tot}} \) can be written as the integration of three terms,
\[ E_{\text{tot}} = \int d^3r \left[ \mathcal{E}_{N}(\mathbf{r}) + T_{\Lambda}(\mathbf{r}) + \mathcal{E}_{\Lambda N}(\mathbf{r}) \right], \]
(5)
where \( \mathcal{E}_{N}(\mathbf{r}) \) is the standard nuclear part of Skyrme energy functional. 25 \( T_{\Lambda}(\mathbf{r}) = \frac{\hbar^2}{2m_{\Lambda}} \tau_{\Lambda} \) is the kinetic energy density of \( \Lambda \) hyperon. \( \mathcal{E}_{\Lambda N}(\mathbf{r}) \) is the interaction energy density between the \( \Lambda \) and nucleons given in terms of the \( \Lambda \) and nucleon densities.

Here, \( \rho_{\Lambda}, \tau_{\Lambda}, \) and \( \mathbf{J}_{\Lambda} \) are respectively the particle density, the kinetic energy density, and the spin density of the \( \Lambda \) hyperon. These quantities are given in terms of the single-particle wave-function of \( \Lambda \) and occupation probabilities, and \( W_{0}^{\Lambda}, t_{0}^{\Lambda}, t_{1}^{\Lambda}, t_{2}^{\Lambda}, t_{3}^{\Lambda} \), and \( W_{0}^{N} \) are the Skyrme parameters for the \( \Lambda N \) interaction.

The pairing correlation between the nucleons is taken into account in the BCS approximation. The density-dependent \( \delta \)-force is adopted in the pp channel,
\[ V(\mathbf{r}_1, \mathbf{r}_2) = -g \frac{1 - \rho^{\sigma}(\mathbf{r}_1)}{2} \left[ 1 - \frac{\rho(\mathbf{r}_1)}{\rho_0} \right] \delta(\mathbf{r}_1 - \mathbf{r}_2), \]
(7)
where \( \hat{P}^{\sigma} \) is the spin-exchange operator, and \( \rho_0 = 0.16 \text{ fm}^{-3} \).
3. Results and discussion

3.1. Different Skyrme forces for particle-hole channel

Figure 1 shows the potential energy curves (PECs) from the SHF+BCS calculations with several Skyrme forces for \(^{44}\)S and \(^{45}\)Λ\(^{\Lambda}\)S, where the No.1 set of ΛN effective interactions in Ref. 20 and pairing strength \(g = 1000\) MeV fm\(^3\) are adopted. It is shown that the PECs of \(^{44}\)S by the SGII and SLy5 forces have two obvious minima in both oblate and prolate sides. The PECs by other Skyrme forces are very similar and soft along \(\beta\) direction around the spherical shape. Similar situation is also found in \(^{45}\)Λ\(^{\Lambda}\)S. Even though these Skyrme forces predict somewhat different PECs for \(^{44}\)S, the resultant polarization effects of Λ by these forces are quite similar, as shown in the panel (c) of Fig. 1. Moreover, the behavior of the difference between the PECs of \(^{44}\)S and \(^{45}\)Λ\(^{\Lambda}\)S indicates that the Λ has the effect of driving the nucleus to be of small deformation, i.e., reducing the nuclear collectivity.

In Fig. 2 we plot the ratios of spectroscopic observables for nuclear core of \(^{45}\)Λ\(^{\Lambda}\)S to those for \(^{44}\)S from the 5DCH calculations with parameters determined by the SHF+BCS calculations with several different Skyrme forces. The experimental excitation energy of \(^{2}\)\(^{+}\) state \((E(2^{+}\rangle))\) in \(^{44}\)S is indicated with a vertical dotted line. We note that the \(E(2^{+}\rangle)\) of \(^{44}\)S by the SGII, SLy5 and SIII forces are much smaller than those by SKM\(^{*}\), T6 and SkP forces, which can be understood from the
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Figure 2. (color online) The ratios of spectroscopic observables for nuclear core of $^{44}$S to those for $^{44}$S from the 5DCH calculations with parameters determined by the SHF+BCS calculations with several Skyrme forces as functions of excitation energy of $2^+_1$ state in corresponding normal nuclei.

PECs in Fig. 1 except the case of SIII force. Namely, the collective wavefunctions of $2^+_1$ state by the calculations of SGII and SLy5 forces are more concentrated on the obvious deformed minima of the PECs with larger moments of inertia. On the other hand, it shows clearly that the $B(E2 : 2^+_1 \to 0^+_0)$ is reduced by the $\Lambda$ hyperon. In particular, these different Skyrme forces give similar results of $\Lambda$ impurity effect in $^{44}$S, i.e., the $\Lambda$ impurity effect on collectivity of $^{44}$S is within 5%.

Figure 3 displays the low-spin spectra for the $^{44}$S and the nuclear core of $^{45}$S obtained by the 5DCH with the parameters determined by the SHF+BCS calculations using the SIII (NN) and No.1 (ΛN) Skyrme forces. The $B(E2)$ values are in units of $e^2$ fm$^4$. The spectrum of $^{44}$Mg is compared with the corresponding available experimental data.

Figure 3. (color online) The low-spin spectra of the ground state band for the $^{44}$S and the nuclear core of $^{45}$S obtained by the 5DCH calculations with parameters determined by the SHF+BCS calculations using the SIII (NN) and No.1 (ΛN) Skyrme forces. The $B(E2)$ values are in units of $e^2$ fm$^4$. The spectrum of $^{44}$Mg is compared with the corresponding available experimental data.
Fig. 4. (color online) The potential energy curves obtained by the Skyrme HF+BCS calculations with both the SIII and SLy4 EDFs, and different pairing strengths for (a) \(^{44}\)S and (b) \(^{45}\)\(^{-}\)S. The difference between the PECs of \(^{45}\)\(^{-}\)S and \(^{44}\)S, normalized to the spherical shape, is shown in (c).

Fig. 5. (color online) (Left panel) The excitation energies of \(2^+, 0^+, 2^+\) states in \(^{44}\)S and (right panel) the ratios of spectroscopic observables for nuclear core of \(^{45}\)\(^{-}\)S to those for \(^{44}\)S as functions of pairing strength \(g\) for nucleons. The experimental data for \(^{44}\)S are indicated with dashed lines in the left panel.

\(^{44}\)S are taken from Refs. 30, 31. It is shown that the spectrum of \(^{44}\)S is reproduced very well and the effect of \(\Lambda\) hyperon on the low-spin states in \(^{44}\)S is generally small.

3.2. Different pairing strengths for particle-particle channel

Figure 4 shows the PECs obtained by the SHF+BCS calculations with both the SIII and SLy4 \(^{28}\) forces, and with different pairing strengths for \(^{44}\)S and \(^{45}\)\(^{-}\)S. The No.1 (adjusted based on the SIII) of Ref. 20 and the “HP120310” (adjusted based
on the SLy4) of Ref. 32 are adopted for $\Lambda N$ effective interactions respectively. It is shown that the pairing strengths for nucleons do have influence on the topology of PECs for $^{44}\text{S}$. However, it has a negligible effect on the polarization effect of $\Lambda$ hyperon. Moreover, it is shown again that the polarization effect of $\Lambda$ hyperon in $^{44}\text{S}$ is similar for the SIII and SLy4 forces.

Subsequently, we perform the further 5DCH calculations using only the SIII force, but different pairing strengths for nucleons. Figure 5 displays the corresponding spectroscopic properties of low-spin states for $^{44}\text{S}$ and nuclear core of $^{45}\Lambda \text{S}$ as functions of pairing strength $g$ for nucleons. It shows clearly and quantitatively that the excitation energies of $2^+_1$, $0^+_2$ and $2^+_2$ states in $^{44}\text{S}$ increase monotonically and dramatically with the pairing strengths, which can be understood that the strong pairing generally drives the nucleus to be more spherical. Similar phenomenon has also been shown in the covariant EDF based 5DCH calculations. However, the impurity effect of $\Lambda$ on nuclear collectivity is not much sensitive to the pairing strength for nucleons, which varies within 5% for different pairing strengths.

4. Summary and outlook

The non-relativistic Skyrme EDF based 5DCH, that takes into account dynamical correlations related to the restoration of broken symmetries and fluctuations of quadrupole collective variables, has been applied to quantitatively study the impurity effect of $\Lambda$ hyperon on collective excitation of shape-coexistence nucleus $^{44}\text{S}$. Several Skyrme forces for both the $\text{NN}$ and $\Lambda N$ effective interactions have been used. The influence of pairing strengths on the polarization effect of $\Lambda$ hyperon has been also examined. It has been found that although these Skyrme forces give somewhat different low-lying spectra for $^{44}\text{S}$, they give similar and generally small size of $\Lambda$ polarization effect (within 5%) on the spectroscopic observables. Moreover, the pairing strength between nucleons has significant influence on the nuclear collectivity, but negligible effect on the polarization effect of $\Lambda$ hyperon.

With the Skyrme EDF based 5DCH model, the systematic study of $\Lambda$ impurity effect on collectivity of atomic nuclei in different mass region is in progress. Moreover, as pointed out in Refs. 12,13, the polarization effect of $\Lambda$ hyperon might be stronger in the calculations with covariant EDFs. The quantitative evaluation of $\Lambda$ effect on nuclear collectivity will provide a good way to constrain $\Lambda N$ effective interaction in covariant EDF. Furthermore, to calculate the low-spin spectra for the whole single $\Lambda$ hypernucleus, one has to extend the current EDF based 3DAMP+GCM or 5DCH models for odd-mass/-odd nuclei.

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