Probing Black-Hole Physics in the Laboratory Using High Intensity Femtosecond Lasers

G. Schäfer and R. Sauerbrey

Theoretisch-Physikalisches Institut und Institut für Optik und Quantenelektronik
Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, D-07743 Jena, Germany

(April 24, 2021)

Abstract

It is shown how laboratory experiments performed with high intensity femtosecond lasers can probe the physics of black holes in the near-horizon regime. The acceleration generated by the high intensity laser ranging from $10^{13}g$ to more than $10^{18}g$ is identified with the gravitational acceleration at stretched horizons. In the black-hole’s asymptotic region, the stretched-horizon-reflected light shows a measurable universal phase acceleration of $c^4/4GM$.

PACS numbers: 04.70.Bw, 97.60.Lf, 52.40.Nk, 52.50.Jm

One of the major unresolved problems in astrophysics is the unique identification of black holes. Due to the limited spatial resolution of astronomical observations a large mass concentration alone is not sufficient. Processes showing the existence of an event horizon are, however, clear signatures of a black hole. In this letter we describe a scenario where the investigation of light emission from stretched horizons of black holes can be simulated in intense femtosecond laser-plasma interaction experiments and simultaneously leads to a clear signature for a black hole.

Physics near the event horizon of black holes is known to be extreme [1]. The nearer to the event horizon matter and radiation are located the stronger the gravitational attraction
becomes that pulls them to the horizon. In the event-horizon limit the accelerations grow unboundedly. This contrasts sharply with extended bodies which at their surfaces, according to general relativity, can reach gravitational accelerations of about $10^{12}g$ only [2], where $g$ denotes the acceleration at the surface of the earth ($g \approx 10\text{m/s}^2$). Recently, laboratory experiments employing table-top high intensity femtosecond lasers have produced large accelerations over a wide range exceeding $10^{18}g$, and this even for macroscopic bodies with masses on the order of picograms [3], [4]. In this letter we shall consider highly accelerated laboratory plasmas in correspondence with plasmas which may exist near the event horizon of a rotating black hole immersed in a homogeneous magnetic field. This analogy leads to a novel possibility for exploring the near-horizon regime of black holes by means of a phase sensitive analysis of the emitted radiation, and thus, to uniquely identify black holes.

In the following discussion it will be sufficient to consider the geometry of uncharged and non-rotating black holes only. An object at rest in the gravitational field of a black hole experiences a proper acceleration as given, in Schwarzschild coordinates, by $b = (GM/r^2)(1 - r_s/r)^{-1/2}$, where $G$, $M$, and $r_s$ denote the Newtonian gravitational constant, the black-hole mass, and the Schwarzschild radius (radius of event horizon), respectively, [1]. $r_s$ is given by $r_s = 2GM/c^2$, where $c$ denotes the speed of light. If the coordinate distance $\epsilon$ from the event horizon is much smaller than $r_s$, i.e. $0 < \epsilon \ll 0.1r_s$, the acceleration reads $b = c^2/2(r_s\epsilon)^{1/2}$. Surfaces with radii $r = r_s + \epsilon$ are called stretched horizons. At stretched horizons the gravitational acceleration is finite. In terms of the geometric length $l$ of the coordinate distance $\epsilon$, $l = 2(r_s\epsilon)^{1/2}$, the acceleration simply reads, $b = c^2/l$.

The maximum surface gravitational acceleration of an extended object is reached for $\epsilon = r_s/8$ (Buchdahl limit). This object might be a neutron star with roughly two solar masses. Although no extended object can exist for radii smaller the $9r_s/8$ [2], matter (plasma) can be kept at rest between the Schwarzschild and Buchdahl radii of a black hole. Any mechanism that keeps the plasma at rest at a stretched horizon will lead to large accelerations in the plasma and consequently to a characteristic modification of the phase of an electromagnetic...
wave emitted or reflected from this plasma. The easiest way to keep plasma at rest in this regime is to assume that the black hole is electrically charged. In an astrophysical setting, however, this charge may not be large enough [5]. Much stronger electric fields can be obtained by, e.g., a rotating black hole immersed in a homogeneous magnetic field $B$, [6]. Such an electric field corresponds to a charged black hole with net electric charge equal to zero and with charge density at the event horizon’s north and south poles of $\sigma_{BJ} = -BJ/2\pi r_s^2 c$. In order to achieve sufficient charge densities a small angular momentum $J$, i.e. $cJ/GM^2 \lesssim 0.1$, can be sufficient. The assumed geometry of an uncharged and non-rotating hole is then still an excellent approximation. Averaged over sufficiently long times, the charge density $\sigma_{BJ}$ is neutralized by a charge density $\sigma_Q$ resulting from infalling charges, i.e. the temporal average of the total charge density will vanish, $\sigma_Q + \sigma_{BJ} = 0$. Because of stochastic processes such as pair creation which keep the black hole’s magnetosphere filled with plasma [1], and turbulences in the plasma, we may, however, expect also large charge fluctuations $\delta \sigma_Q$ in the stretched horizon regime, with $|\delta \sigma_Q| << |\sigma_{BJ}|$. For fluctuations corresponding to $M = 2M_\odot$, $B = 10^{10}$ gauss, and $cJ/GM^2 = 0.03$ ($M_\odot$ is the solar mass) the resulting repulsive electric force cancels the gravitational force acting on the incoming plasma ions, at a stretched horizon located at $l = 0.5$ cm; i.e. $b = 10^{18}$ g. The temporal duration of the fluctuations must exceed $l/c \approx 20$ ps to stop and halt the plasma long enough for the effect to be measured. The magnetic field exceeding $10^{10}$ gauss, needed for $\sigma_{BJ}$, may be supplied by an accreting plasma [7]. Of course, if $l$ is increased, corresponding to larger stretched horizons, the charge fluctuation, magnetic field or angular momentum of the black hole may be considerably smaller to achieve the same effect.

It was shown recently that high intensity femtosecond lasers produce hot, dense matter of solid density and keV-electron energies [8]. Internal pressures approaching Gigabars lead to a rapidly expanding plasma that experiences accelerations on the order of $b \approx 10^{19}$ m/s$^2$ $\approx 10^{18}$ g. Similar accelerations are caused by the radiation pressure that also reaches about 1 Gigabar at an intensity of $I \approx 3 \times 10^{18}$ W/cm$^2$, [4]. These accelerations actually dominate
the motion of plasma material during the laser pulse duration of $\tau_L \approx 100$ fs. For an expanding plasma the final velocity reached by the plasma corresponds to the ion acoustic velocity of $v_i \approx 10^7...10^8$ cm/s and is small compared to the speed of light. Due to the short laser-pulse duration and the rapid heating of the plasma, however, this velocity is reached in a very short time and thus the acceleration may be estimated to $b \approx v_i/\tau_L \gtrsim 10^{18}$ m/s$^2$.

Such accelerations have recently been measured by spectral analysis of chirped laser pulses reflected from the plasma surface [3], [4].

For a direct measurement of the acceleration of the plasma surface (more precisely, the location of the critical electron density), amplitude and phase of the reflected laser pulse or a temporally delayed probe pulse have been experimentally determined. Modifications in the reflected laser light of an ultrashort laser pulse may be caused by kinematic effects, i.e. movement of the reflecting surface as well as by self-phase modulation induced by rapidly changing optical properties of the reflecting surface due to fast heating and ionization of the plasma. We consider laser pulses of pulse durations of $\tau_L \approx 100$ fs and intensities ranging from $10^{16}...10^{20}$ W/cm$^2$. For typical aluminium plasma parameters ($Z \approx 10; k_BT_e \approx 1$ keV; $M_i = 27$ amu) we obtain for intensities in the $10^{16}$ W/cm$^2$ range and a high contrast ratio ($> 10^9$) of the laser pulse $L \approx v_i \tau_L \approx 20$ nm as an upper limit for the plasma scale length. Therefore, $L/\lambda << 1$ holds for typical laser wavelength of $\lambda \approx 800$ nm. The reflected wave can therefore be obtained using the Fresnel approximation for steep density gradients, and the kinematics of the plasma surface is described by a “moving mirror” [9] of temporally varying reflectivity [10].

We approximate the mirror reflectivity by the following simple function for the field amplitude, $R(t) = R_0 e^{-t/\tau_R} e^{i\phi}$, where $R_0$ is the fraction of the reflected field amplitude, $\tau_R$ the characteristic time for the decay of the reflectivity, and $\phi$ the phase angle. Measurements and calculations show [3], [11] that usually $\tau_R >> \tau_L$. Furthermore we put $\phi \approx 0$ corresponding to reflection at the critical density. The propagation pattern of the impinging field with frequency $\omega_0$ has the form $\exp\{i(\omega_0/c)[ct + x]\}$. The surface of the solid is located at $x = x_0$.

For a direct measurement of the acceleration of the plasma surface (more precisely, the location of the critical electron density), amplitude and phase of the reflected laser pulse or a temporally delayed probe pulse have been experimentally determined. Modifications in the reflected laser light of an ultrashort laser pulse may be caused by kinematic effects, i.e. movement of the reflecting surface as well as by self-phase modulation induced by rapidly changing optical properties of the reflecting surface due to fast heating and ionization of the plasma. We consider laser pulses of pulse durations of $\tau_L \approx 100$ fs and intensities ranging from $10^{16}...10^{20}$ W/cm$^2$. For typical aluminium plasma parameters ($Z \approx 10; k_BT_e \approx 1$ keV; $M_i = 27$ amu) we obtain for intensities in the $10^{16}$ W/cm$^2$ range and a high contrast ratio ($> 10^9$) of the laser pulse $L \approx v_i \tau_L \approx 20$ nm as an upper limit for the plasma scale length. Therefore, $L/\lambda << 1$ holds for typical laser wavelength of $\lambda \approx 800$ nm. The reflected wave can therefore be obtained using the Fresnel approximation for steep density gradients, and the kinematics of the plasma surface is described by a “moving mirror” [9] of temporally varying reflectivity [10].

We approximate the mirror reflectivity by the following simple function for the field amplitude, $R(t) = R_0 e^{-t/\tau_R} e^{i\phi}$, where $R_0$ is the fraction of the reflected field amplitude, $\tau_R$ the characteristic time for the decay of the reflectivity, and $\phi$ the phase angle. Measurements and calculations show [3], [11] that usually $\tau_R >> \tau_L$. Furthermore we put $\phi \approx 0$ corresponding to reflection at the critical density. The propagation pattern of the impinging field with frequency $\omega_0$ has the form $\exp\{i(\omega_0/c)[ct + x]\}$. The surface of the solid is located at $x = x_0$. 
before arrival of the laser pulse and the location of the critical surface as a function of
time is given by \( x(t) = x_0 + v_0 t + bt^2/2 \), where \( v_0 \) is an arbitrary initial expansion velocity
and \( b \) is the acceleration of the plasma front (more exactly, that of the critical density).

The electric field reflected from the accelerated mirror, for \( |v_0| \ll c \) and \( |b| t_r \ll c/2 \),
where \( t_r \equiv t - (x - x_0)/c \) is the retarded time, has now the following form \([12]\):
\[
E_R(t_r) = R(t_r)E(t_r)\exp\{i(\omega_0/c)[x_0 + (c + 2v_0)t_r + bt_r^2]\},
\]
where \( E(t_r) \) is the pulse envelope. Note that
the acceleration of the phase is twice the mirror acceleration. In the Fresnel approximation
the wave vector \( k \) of the reflected laser pulse is that of the impinging one and independent
of \( x \) \((ck = \omega_0)\).

It is evident from the above equations that light emitted from a uniformly accelerated
source shows linear chirp in frequency, i.e. a linear dependence of frequency on time, when
measured in the rest frame. Consequently, the observation of chirp from “moving mirrors”
can be used to measure acceleration. Chirp is measured using phase sensitive pulse di-
agnostics (FROG, Frequency Resolved Optical Gating) for the ultrashort laser pulse \([14]\).
Such experiments are presently in progress. First results \([1]\) indicate plasma accelerations of
\( b \approx 2 \times 10^{19} \text{ m/s}^2 \approx 2 \times 10^{18} \text{ g} \) for a \( 10^{18} \text{ W/cm}^2 \) Titanium Saphire laser of 100 fs pulse dura-
tion impinging on a carbon surface. Fig. 1 shows the result of such an experiment. From the
FROG analysis the amplitude \( E_{in}(t) \) and the phase angle \( \phi_{in}(t) \) of the incident laser pulse are
obtained. A pulse width of \( \tau_L \approx 100 \text{ fs} \) and an almost constant phase \( \phi_{in}(t) \) are evident for
the incident pulse. The reflected laser pulse clearly shows the predicted parabolic behavior of
the phase \( \phi_{out}(t) \sim t^2 \) and correspondingly a temporally broadened field amplitude \( E_{out}(t) \).

An indirect measurement of plasma acceleration of this magnitude was already obtained
from the spectral modification of chirped ultrashort laser pulses reflected from solid surfaces
\([3]\). In the following we demonstrate that such an accelerated plasma mirror in Minkowski
space shows a phase behavior of light reflected from it that is identical to the phase behavior
of light reflected from a plasma sheet at rest in Schwarzschild spacetime in the immediate
neighbourhood of the black hole’s event horizon (Fig. 2).
A linearly accelerated plasma mirror in flat spacetime (Fig. 2a) with motion of the type \( x(t) = (x_0^2 + c^2 t^2)^{1/2} \), which for \( x_0 >> ct \) means \( x(t) = x_0 + bt^2/2 \) with acceleration \( b = c^2/x_0 \), can be replaced by a plasma mirror at rest with respect to a uniformly accelerated reference frame, i.e. as being at rest in a so-called Rindler wedge [13], (Fig. 2b). In our laboratory experiment, the condition \( x_0 >> ct \) always holds, i.e. the maximum mirror velocities \( \dot{x} = c^2t/x_0 \) are always much smaller then the speed of light, indeed they are on the order of the ion acoustic velocity \( v_i \) of about \( 10^{-3}c \) [3]. In the limit, \( x_0 << ct \), the quantum thermal radiation from the mirror of temperature \( T = bh/4\pi^2ck_B \) results, where \( h \) denotes the Planck constant, [12]. This radiation is measured by an observer in inertial motion and originates from an accelerated mirror which is approaching the velocity of light.

An accelerated observer located in the Rindler wedge at \( \xi_0 \) measures a quantum thermal radiation (Unruh radiation) of temperature \( T \) as given above with \( x_0 = e^{\xi_0}/a \) (this equation relates the space coordinates of the Rindler wedge and the non-accelerated frame, at \( t = 0 \)), [14]. The Unruh radiation originates from the purely geometric (causal) properties of the event horizon of the Rindler wedge and needs no material basis (although matter falling through the horizon can be involved). It corresponds to the thermal radiation of black holes [15].

Applying the Einstein equivalence principle, the high-acceleration laboratory experiments can directly be transferred to stretched horizons of black holes. The mirror, at rest in the Rindler wedge, corresponds to a mirror located at a stretched horizon and the laser source, at rest in the Minkowski space, corresponds to a radially freely falling source near the stretched horizon, having zero velocity at the time \( t = 0 \). This scenario evolves as follows: In the equilibrium case, \( \sigma_Q + \sigma_{BJ} = 0 \), plasma is falling freely along the magnetic field axis. On short times scales, charge fluctuations on deeper located stretched horizons generate time-varying effective charge densities of fractions of \( \sigma_{BJ} \). Their high-frequency electric fields repel and support the infalling (mirror) plasma. The overshooting and backfalling plasma serves as a light source.
This situation is best described in radial Kruskal coordinates \((v, u)\) which in Schwarzschild spacetime play the rôle the quasi-cartesian coordinates play in Minkowski space. The important relation between Kruskal and radial Schwarzschild tortoise coordinates, \(r^* = r + r_s \ln(r/r_s - 1)\), reads, \(u^2 = v^2 + \exp(r^*/r_s)\), \([17]\). At stretched horizons we get \(u = u_0(1 + v^2/u_0^2)^{1/2}\), where \(u_0 = (\epsilon e/r_s)^{1/2}\) is the position of the stretched horizon at \(t = v = 0\) \((\epsilon = \exp(1))\). For \(v << u_0\), the phase of the electric field reflected from the stretched horizon takes the form \(\exp\{i\omega[u_0 + v_r + v_r^2/u_0]\}\), where \(\omega\) is the angular frequency in Kruskal time and \(v_r \equiv v - u + u_0\) is the retarded Kruskal time. The structure of the phase as seen by an observer at rest in Kruskal coordinates near the stretched horizon corresponds exactly to the phase of the light reflected from the accelerated plasma mirror that appears in \(E_R(t)\), for \(v_0 = 0\). To know what an observer measures far away from the stretched horizon we have to transform the phase of the wave to Schwarzschild coordinates. If \(r_0^*\) corresponds to \(u_0\), i.e. \(u_0 = \exp(r_0^*/2r_s)\), then \(v_r = u_0(1 - \exp\{-ct_r/2r_s\})\) holds with \(t_r \equiv t - (r^* - r_0^*)/c\), and for the phase propagation follows, taking into account \(t_r << 2r_s/c\) \((\geq 40\mu s,\) for two solar masses as minimum mass): \(\exp(i\omega u_0)\exp\{i(\omega u_0 c/2r_s)[t_r + ct_r^2/4r_s]\}\). In this expression, \(\omega u_0 c/2r_s\) and \(b_F \equiv c^2/2r_s = c^4/4GM\) are respectively the angular frequency and the phase acceleration measured with respect to the observer’s proper time \(t\) far away from the horizon. It is important to note that the measured acceleration is independent from the chosen stretched horizon, i.e. it is the same for all \(\epsilon \lesssim 0.1r_s\), resp. \(b \gtrsim 10^{13}g\). It is also worthwhile to mention that direct light from radially freely backfalling sources in the stretched horizon regime, i.e. light not being reflected at a stretched horizon, reveals an acceleration pattern in its phase with just the opposite sign for the acceleration.

For a neutron star with mass \(M\) the gravitational acceleration at its surface is smaller than \((8/9)^2b_F\) so, using phase measurements, a discrimination between black holes and neutron stars is possible if the mass of the object is known with sufficient precision by other means. In the neutron star case the reflecting mirror is made by its surface and the source for light are near-surface plasma processes.
The expression for the acceleration of the phase occurs also in the Unruh (purely outgoing) and Hawking radiation of a black hole with Hawking temperature of $T = b_F h / 4\pi^2 c k_B$ far away from the black hole’s event horizon. At the stretched horizons the Hawking temperature takes the blueshifted value $T_{SH} = bh / 4\pi^2 c k_B$ with $b$ as given above. The Unruh radiation originates from regions very close to the black-hole event horizon after very long times ($t \to \infty$). The radiation considered in this letter on the contrary originates from stretched horizons and, thus, appears much earlier in the black-hole formation process.

In summary we find that the application of the equivalence principle and the concept of plasma loaded stretched horizons to laboratory experiments measuring large laser induced accelerations lead to a new experimental method to identify black holes.

For critically reading the manuscript the authors thank S. M. Kopeikin. The research was supported by the Max-Planck-Gesellschaft (Grant 02160-361-TG74) and by the Training and Mobility of Researchers program (ERB 4061 PL 95-0765) of the European Community within the Superintense Laser pulse-Solid Interaction network.
REFERENCES

[1] K. S. Thorne, R. H. Price, and D. A. Macdonald (Eds.), *Black Holes: The Membrane Paradigm* (Yale University Press, New Haven, 1986).

[2] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 334.

[3] R. Sauerbrey, Phys. Plasmas 3, 4712 (1996). In Eq. (3) of this paper the term $k x(t)$ must be multiplied by a factor of 2, which transforms all values for velocities and accelerations to half the values given in the paper without affecting the basic physical conclusions.

[4] R. Hässner, W. Theobald, S. Niedermeier, K. Michelmann, H. Schillinger, T. Feurer, G. Schäfer, and R. Sauerbrey, in *Proceedings of the International Conference on Super-strong Fields in Plasmas* (Varenna, Italy, August 27 - September 2, 1997).

[5] R. M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984), p. 314.

[6] R. M. Wald, *Phys. Rev. D* 10, 1680 (1974).

[7] J. Frank, A. King, and D. Raine, *Accretion Power in Astrophysics* (Cambridge University Press, Cambridge, 1992), p. 238.

[8] M. D. Perry and G. Mourou, Science 264, 917 (1994).

[9] X. Liu and D. Umstadter, *Phys. Rev. Lett.* 69, 1935 (1992).

[10] Since the electron density and consequently the plasma frequency first increases during the impinging laser pulse and later decreases due to recombination, the reflectivity of the plasma surface is a rapidly varying function of time. It can be shown that for perpendicular incidence when the light is reflected from the vicinity critical surface the phase shift due to the varying electron density vanishes. Only the magnitude of the reflectivity changes. This change in reflectivity has been measured using pump-probe techniques [11]. It was shown that the reflectivity changes rapidly during the plasma generating pulse and decays on a timescale $\tau_R$ of several picoseconds.
[11] D. v. d. Linde, H. Schuler, H. Schulz, and T. Engers, in *Ultrafast Phenomena VIII*, Springer Series in Chemical Physics (Springer-Verlag, Berlin, 1993), Vol. 55, p. 280.

[12] S. A. Fulling and P. C. W. Davies, *Proc. R. Soc. London* A348, 393 (1976).

[13] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982).

[14] W. G. Unruh, *Phys. Rev. D* 14, 870 (1976).

[15] P. C. W. Davies and S. A. Fulling, *Proc. R. Soc. London* A356, 237 (1977).

[16] K. W. DeLong, R. Trebino, and D. J. Kane, *J. Opt. Soc. Am. B* 11, 1595 (1994).

[17] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco 1973), pp. 827 - 835.
FIGURE CAPTIONS

Fig. 1.

A $10^{18}$ W/cm$^2$, 100 fs Titanium Saphire laser is focused onto a fresh carbon target. The incident (dotted curves) and the reflected (drawn curves) light are analyzed with respect to the temporal behavior of amplitude (drawn lines) and phase (dashed lines). The incident light shows a pulsewidth of $\approx 100$ fs and a constant phase. The reflected light is broadened due to the large phase ($> 6\pi$) that is accumulated because of the strong acceleration of the plasma mirror by the large radiation pressure ($\approx 300$ Mbars) of the incident laser pulse.

Fig. 2.

Reference frames demonstrating the equivalence of an accelerated plasma mirror with plasma sheet at rest in the neighbourhood of the event horizon of a black hole. (a) Reflected light from an accelerated mirror as observed by an observer at rest in Minkowski space. Equidistant temporal intervals such as the period of an electromagnetic wave are transformed into time intervals with decreasing duration for increasing time. This corresponds to the linear chirp in frequency observed from a uniformly accelerated mirror (see Fig. 1). (b) In the Rindler wedge ($\eta$ time coordinate, $\xi$ space coordinate) the mirror is at rest from the time $t = \eta = 0$ on. Not shown is the past event horizon ($\eta = -\infty, \xi = -\infty$). (c) The Schwarzschild spacetime is depicted in Kruskal coordinates ($v$ time coordinate, $u$ radial space coordinate). The radially freely falling source of light rays is at rest in Kruskal coordinates over a short period of time. The light rays reflected at the plasma loaded stretched horizon are received at infinity (on graphical reasons, the source is switched off before the reflected light rays do cross it). Not shown are light rays which are directly emitted to infinity.
Laser, at rest in Minkowski space
