Decay of the Higgs boson $h \rightarrow \tau^-\tau^+ \rightarrow \pi^-\nu_\tau \pi^+\bar{\nu}_\tau$ for a non-Hermitian Yukawa interaction

ALEXANDER YU. KORCHIN

NSC Kharkiv Institute of Physics and Technology, 61108 Kharkiv, Ukraine

V.N. Karazin Kharkiv National University, 61022 Kharkiv, Ukraine

VLADIMIR A. KOVALCHUK†

NSC Kharkiv Institute of Physics and Technology, 61108 Kharkiv, Ukraine

The differential rate of the decay of the Higgs boson ($h$) to a pair of $\tau$ leptons with their subsequent decay in the $\tau^- \rightarrow \pi^-\nu_\tau$ and $\tau^+ \rightarrow \pi^+\bar{\nu}_\tau$ channels is studied. The Yukawa interaction between the Higgs boson and the $\tau$ leptons is assumed to include scalar ($S$) and pseudoscalar ($PS$) couplings. Angular distributions of the pions in the $h \rightarrow \tau^-\tau^+ \rightarrow \pi^-\nu_\tau \pi^+\bar{\nu}_\tau$ decay are considered. For real values of the $S$ and $PS$ couplings, this decay is known to be a source of information on $CP$ violation in the $h\tau\tau$ interaction. In the present paper, the main attention is paid to a possible non-Hermiticity of this interaction. Influence of non-Hermiticity on the distribution of the angle between planes of the $\tau^- \rightarrow \pi^-\nu_\tau$ and $\tau^+ \rightarrow \pi^+\bar{\nu}_\tau$ decays, and distribution of the polar angle of one of the pions are analyzed. Asymmetries sensitive to parameters of $CP$ violation and non-Hermiticity of $h\tau\tau$ interaction are proposed.

1. Introduction

In framework of the Standard Model (SM), the fermion masses are generated by the Yukawa interaction between the Higgs field and fermion fields. Measurement of the corresponding couplings is needed for identification of the particle $h$ with the Higgs boson. In the SM, the Higgs boson is the $CP$-even scalar particle. However, there exist many models with a more complicated structure of the Higgs sector in which both the $CP$-even and $CP$-odd scalar particles can exist, as well as particles which do not have definite $CP$ parity (see recent review [1] and references therein). Therefore, it

† Deceased
is possible that the observed Higgs boson $h$ \cite{2,3} is a mixture of $\mathcal{CP}$-even and $\mathcal{CP}$-odd states, although other possibilities are not excluded. Thus, clarification of the $\mathcal{CP}$ properties of the Higgs boson is a necessary step in investigation of the mechanism which breaks electroweak symmetry and generates the particle masses. The present status of the LHC measurements of the $\mathcal{CP}$ properties of the Higgs-boson interactions with vector bosons and fermions is reviewed in Ref. \cite{4}.

In the SM, the source of violation of the $\mathcal{CP}$ symmetry is unremovable phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix \cite{5,6}. Moreover, the existing data indicate that this phase is the dominant source of $\mathcal{CP}$ violation in the flavor changing processes. However, model calculations show that $\mathcal{CP}$ violation in the SM is too small to explain the matter-antimatter asymmetry in the Universe \cite{4,8}. There should be other sources of $\mathcal{CP}$ violation beyond the CKM mechanism. Thus, the search for new sources of $\mathcal{CP}$ violation is one of the main directions in the particle physics. One of possibilities in this search is the Higgs boson decay $h \to \tau^- \tau^+$.

The study of the $\mathcal{CP}$ properties and violation of the $\mathcal{CP}$ symmetry in the Higgs sector, using the correlations between the spins of $\tau$ leptons created in the Higgs-boson decay, has been carried out in a series of papers, e.g. \cite{9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}.

Another important aspect of the Yukawa interaction is Hermiticity of the Lagrangian. In the SM, the Lagrangian of the interaction between fermions and scalar fields satisfies the symmetry with respect to the gauge transformations $SU(2)_L \times U(1)_Y \times SU(3)_c$ and, in addition, it is Hermitian. The latter requirement is imposed on the Lagrangian. In contrast to other terms in the Lagrangian which are naturally Hermitian, the Yukawa interaction “acquires” Hermiticity which may not be necessary. This aspect has been raised in \cite{26}.

Note that Ref. \cite{26} also suggested a modification of the SM electroweak interaction to the case of a non-Hermitian interaction between the Higgs fields and fermions. The consideration there was restricted to one generation of the fermions. It was shown that for positive values of the Yukawa couplings, the fermions get the positive mass and the interaction between the Higgs boson and fermions violates $\mathcal{CP}$ symmetry without additional Higgs fields. It seems therefore important to investigate further this mechanism in the Higgs-boson decays.

Let us mention that various aspects of non-Hermitian field theories have been studied in Refs. \cite{32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50}. Influence of non-Hermiticity of the Yukawa interaction on the photon polarization parameters in the $h \to \gamma \gamma$ and $h \to \gamma Z$ decays has been addressed in \cite{51,52}, and on the forward-backward lepton asymmetry
in the $h \to \gamma \ell^+ \ell^- \ (\ell = e, \mu, \tau)$ decays in [53, 54].

In the present paper, we investigate the decay of the Higgs boson to a pair of the $\tau$ leptons with their consequent decay through the $\tau \to \pi \nu_\tau$ channel, namely the $h \to \tau^- \tau^+ \to \pi^- \nu_\tau \pi^+ \bar{\nu}_\tau$ process. The case of a non-Hermitian interaction of the Higgs boson with the $\tau$ leptons is considered.

In Sec. 2 the full angular distribution of the pions is obtained. Then we derive the distribution of the angle between the $\tau^- \to \pi^- \nu_\tau$ and $\tau^+ \to \pi^+ \bar{\nu}_\tau$ decay planes, and the distribution of the polar angle of one of the pions in the helicity frame. The influence of non-Hermiticity of the $h\tau\tau$ interaction on the pion distributions is calculated and analyzed. In Sec. 3 the conclusions are presented.

2. Angular distributions of pions

We assume that the interaction of the Higgs boson ($h$) with the $\tau$ leptons is determined by the Lagrangian which includes scalar ($S$) and pseudoscalar ($PS$) parts

$$\mathcal{L}_{h\tau\tau} = -\frac{m_\tau}{v} h \psi_\tau (a_\tau + i b_\tau \gamma_5) \psi_\tau,$$

(1)

where $\psi_\tau$ is the field of the fermion, $v = \left(\sqrt{2} G_F \right)^{-1/2} \approx 246$ GeV is the vacuum expectation value of the Higgs field, $G_F = 1.1663787(6) \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant [55], $m_\tau$ is the fermion mass and $a_\tau, b_\tau$ are complex parameters ($a_\tau = 1$ and $b_\tau = 0$ correspond to the SM). Eq. (1) can be considered as a phenomenological parametrization of effects of new physics [11, 12, 26]. For the real-valued parameters $a_\tau, b_\tau$, the interaction (1) is Hermitian, however, we are interested in the case of non-Hermitian interaction with complex-valued parameters $a_\tau, b_\tau$. At the same time, the Higgs interaction with the $W^\pm$ and $Z$ bosons is chosen Hermitian as in the SM.

Let us consider the decay of $h$ to a pair of $\tau$ leptons with their consequent decay to the $\tau^- \to \pi^- \nu_\tau$ and $\tau^+ \to \pi^+ \bar{\nu}_\tau$ channels. The differential decay rate of $h \to \tau^- \tau^+ \to \pi^- \nu_\tau \pi^+ \bar{\nu}_\tau$ in the Higgs boson rest frame can be written as

$$d^3 \Gamma(h \to \tau^- \tau^+ \to \pi^- \nu_\tau \pi^+ \bar{\nu}_\tau) \quad \frac{d^3\Gamma(h \to \tau^- \tau^+)}{d \cos \theta_- d \cos \theta_+ d\chi} = \Gamma(h \to \tau^- \tau^+) \times \left(\text{BR}(\tau \to \pi \nu_\tau)\right)^2 \frac{d^3 W}{d \cos \theta_- d \cos \theta_+ d\chi}.$$

(2)

Here, $\Gamma(h \to \tau^- \tau^+)$ is the Higgs boson decay width which in the leading
order is given by

$$
\Gamma(h \rightarrow \tau^- \tau^+) = m_h \beta^2 \frac{G_F m^2_\tau}{4 \sqrt{2} \pi} (|a\tau|^2 \beta^2_{\tau} + |b\tau|^2),
$$

(3)

where $m_h$ is the Higgs boson mass, $\beta_{\tau} = \sqrt{1 - 4m^2_\tau / m^2_h}$ is the velocity of the $\tau$ lepton in the rest frame of the Higgs, $\text{BR}(\tau \rightarrow \pi\nu_\tau)$ is the branching of the $\tau$ decay through the $\tau \rightarrow \pi\nu_\tau$ channel. Further, the total angular distribution for the $h \rightarrow \tau^- \tau^+ \rightarrow \pi^-\nu_\tau \pi^+\bar{\nu}_\tau$ decay has the form of

$$
d^3W = \frac{1}{8\pi} \left( 1 - \cos \theta_- \cos \theta_+ - 2 \Re(a\tau b^*_\tau) \right) \beta_{\tau} \sin \theta_- \sin \theta_+ \sin \chi
$$

(4)

where $\theta_- (\theta_+)$ is the angle between the direction of the $\pi^- (\pi^+)$ meson motion in the $\tau^- (\tau^+)$ lepton rest frame and the direction of the $\tau^- (\tau^+)$ lepton motion in the $h$ boson rest frame, and $\chi$ is the angle between the planes of the $\tau^- \rightarrow \pi^-\nu_\tau$ and $\tau^+ \rightarrow \pi^+\bar{\nu}_\tau$ decays in the $h$ boson rest frame (see Fig. 1).

It is useful in parameterization of Eq. (4), instead of parameters $a_{\tau}, b_{\tau}$, to introduce the parameters (angles) $\phi_{\text{CP}}$ and $\phi_H$ defined as

$$
tan \phi_{\text{CP}} \equiv \frac{|b\tau|}{|a\tau|},
$$

(5)

$$
2 \Re(a\tau b^*_\tau) / |a\tau|^2 + |b\tau|^2 = \sin 2\phi_{\text{CP}} \sin \phi_H,
$$

(6)

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{helicity_angles.png}
\caption{Definition of helicity angles $\theta_-, \theta_+, \text{and } \chi$ in the sequential decay $h \rightarrow \tau^- \tau^+ \rightarrow \pi^-\nu_\tau \pi^+\bar{\nu}_\tau$.}
\end{figure}
\[ \frac{2 \text{Re}(a_\tau^* b_\tau^*)}{|a_\tau|^2 + |b_\tau|^2} = \sin 2\phi_{\text{CP}} \cos \phi_H. \] (7)

As a result, Eq. (4) takes the form of
\[
\frac{d^3W}{d \cos \theta_- d \cos \theta_+ d\chi} = \frac{1}{8\pi} \left( 1 - \cos \theta_- \cos \theta_+ \right.
\left. - \frac{\beta_\tau \sin 2\phi_{\text{CP}} \sin \phi_H}{\beta_\tau^2 \cos^2 \phi_{\text{CP}} + \sin^2 \phi_{\text{CP}}} (\cos \theta_- - \cos \theta_+) \right.
\left. - \left( \frac{\beta_\tau^2 \cos^2 \phi_{\text{CP}} - \sin^2 \phi_{\text{CP}}}{\beta_\tau^2 \cos^2 \phi_{\text{CP}} + \sin^2 \phi_{\text{CP}}} \cos \chi \right. \right.
\left. \left. + \frac{\beta_\tau \sin 2\phi_{\text{CP}} \cos \phi_H}{\beta_\tau^2 \cos^2 \phi_{\text{CP}} + \sin^2 \phi_{\text{CP}}} \sin \chi \right) \sin \theta_- \sin \theta_+ \right). \] (8)

As the lepton produced in the decay of the Higgs boson of the mass of 125 GeV are ultrarelativistic, one has \( \beta_\tau \approx 0.9996 \), and taking the limit \( \beta_\tau \to 1 \), we obtain
\[
\frac{d^3W}{d \cos \theta_- d \cos \theta_+ d\chi} = \frac{1}{8\pi} \left( 1 - \cos \theta_- \cos \theta_+ \right.
\left. - \sin 2\phi_{\text{CP}} \sin \phi_H (\cos \theta_- - \cos \theta_+) \right.
\left. - \left( \cos 2\phi_{\text{CP}} \cos \chi + \sin 2\phi_{\text{CP}} \cos \phi_H \sin \chi \right) \times \sin \theta_- \sin \theta_+ \right). \] (9)

For Hermitian \( h_\tau^\tau \) interaction, \( \phi_H = 0 \) or \( \phi_H = \pi \), Eq. (9) becomes
\[
\frac{d^3W}{d \cos \theta_- d \cos \theta_+ d\chi} = \frac{1}{8\pi} \left( 1 - \cos \theta_- \cos \theta_+ \right.
\left. - \cos (\chi \pm 2\phi_{\text{CP}}) \sin \theta_- \sin \theta_+ \right). \] (10)

Therefore, one of the observables with maximal sensitivity to the correlations of the \( \tau \) spins is the azimuthal angular correlation in the Higgs rest frame, which has a simple form \[28\]
\[
\frac{dW}{d\chi} = \frac{1}{2\pi} \left( 1 - \frac{\pi^2}{16} \cos (\chi \pm 2\phi_{\text{CP}}) \right). \] (11)

For a non-Hermitian \( h_\tau^\tau \) interaction \[11\], this angular correlation takes a different form of
\[
\frac{dW}{d\chi} = \frac{1}{2\pi} \left( 1 - \frac{\pi^2}{16} \cos 2\phi_{\text{CP}} \cos \chi \right.
\left. + \sin 2\phi_{\text{CP}} \cos \phi_H \sin \chi \right). \] (12)
One has to note that if it will not be possible to distinguish in experiments the events with the azimuthal angle $\chi$ from the events with the angle $2\pi - \chi$, then the resulting distribution of $\chi$ takes the form of

$$\frac{dW}{d\chi} = \frac{1}{\pi} \left( 1 - \frac{\pi^2}{16} \cos 2\phi_{\text{CP}} \cos \chi \right), \quad 0 \leq \chi \leq \pi. \quad (13)$$

Fig. 2. Distribution of the azimuthal angle $\chi$ ($0 \leq \chi \leq 2\pi$) for Hermitian interaction with $\phi_H = 0$. Solid line corresponds to the SM, dashed line $- \phi_{CP} = \frac{\pi}{8}$, dotted line $- \phi_{CP} = \frac{\pi}{4}$.

Fig. 3. Distribution of the azimuthal angle $\chi$. Solid line - SM, dashed and dotted lines correspond to non-Hermitian interaction with $\phi_H = \frac{\pi}{4}$: dashed line $- \phi_{CP} = \frac{\pi}{8}$, dotted line $- \phi_{CP} = \frac{\pi}{4}$.

In general, a possibility of measurement of the distribution (11) in experiments at the LHC or the ILC has been discussed by many authors with the aim of searching for violation of $\mathcal{C}\mathcal{P}$ symmetry in the $h \to \tau^- \tau^+$ decay.
Fig. 4. Distribution of the azimuthal angle $\chi$. Solid line - SM, dashed and dotted lines correspond to non-Hermitian interaction with $\phi_H = \frac{\pi}{2}$; dashed line - $\phi_{CP} = \frac{\pi}{8}$, dotted line - $\phi_{CP} = \frac{\pi}{4}$.

(see, for example, [28, 29]). As for the influence of a non-Hermitian interaction (11) on the form of the distribution of the observable $\chi$, this aspect has not been discussed.

In Figs. 2, 3, and 4, we show the angular distribution (12) for Hermitian and non-Hermitian interactions for a few values of the $CP$-violation parameter $\phi_{CP}$ and Hermiticity-violation parameter $\phi_H$.

It is seen from Fig. 2 that for Hermitian interaction, one can measure violation of $CP$ symmetry via the phase shift in the distribution of $\chi$ (11), if there is no background. If the interaction is non-Hermitian, then the azimuthal correlation (12) substantially differs from the SM case, as it is seen in Figs. 3 and 4, and the corresponding differences strongly depend on the parameter $\phi_H$. A model for a non-Hermitian $h\tau\tau$ interaction in which $\phi_H = \frac{\pi}{2}$ has been discussed in [26].

It would also be interesting to measure the following asymmetries:

$$A_1 \equiv \left( \frac{\pi}{2} \right) \left( dW/d\chi - \int_{\pi/2}^{3\pi/2} d\chi \right) dW/d\chi = -\frac{\pi}{8} \cos 2\phi_{CP}, \quad (14)$$

$$A_2 \equiv \left( \frac{\pi}{2} \right) \left( dW/d\chi - \int_{\pi}^{2\pi} d\chi \right) dW/d\chi = -\frac{\pi}{8} \sin 2\phi_{CP} \cos \phi_H. \quad (15)$$

If the values of these asymmetries turn out to be different from the SM prediction, then this will be a clear signal of physics beyond the SM.

Finally, we briefly discuss another observable which is sensitive to non-Hermiticity of the Yukawa interaction. This is the polar-angle correlation

$$\frac{dW}{d\cos \theta_{\pm}} = \frac{1}{2} \left( 1 \pm \sin 2\phi_{CP} \sin \phi_H \cos \theta_{\pm} \right). \quad (16)$$
It follows from Eq. (16) that for Hermitian interaction ($\phi_H = 0$ or $\phi_H = \pi$), the distribution of the observable $\cos \theta_\pm$ is uniform. Therefore, any deviation of a measured distribution from 1/2 will point to a non-Hermiticity of the Yukawa interaction and violation of the $CP$ symmetry in the Higgs boson decay to a pair of $\tau$ leptons.

3. Conclusions

In this work, we analyzed the differential rate of the decay of the Higgs boson to a pair of $\tau$ leptons with their subsequent decay into the $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ channels. The Yukawa interaction between the Higgs boson and $\tau$ leptons is assumed to include both the scalar ($S$) and pseudoscalar ($PS$) couplings. The total angular distribution of the pions in the $h \rightarrow \tau^- \tau^+ \rightarrow \pi^- \nu_\tau \pi^+ \bar{\nu}_\tau$ process is considered, as well as distribution of the angle $\chi$ between the planes spanned by the $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ decays, and distribution of the polar angle $\theta_\pm$ of the $\pi^\pm$.

For real values of the $S$ and $PS$ couplings, this decay is known to be a source of information on $CP$ violation in the $h\tau\tau$ interaction. In the present work, we concentrate on a non-Hermitian Yukawa interaction. It is shown that the distributions of the charged pions strongly depend on a parameter of non-Hermiticity of the $h\tau\tau$ interaction. Asymmetries sensitive to parameters of $CP$ violation and non-Hermiticity are proposed.

In summary, measurement of the $h \rightarrow \tau^- \tau^+ \rightarrow \pi^- \nu_\tau \pi^+ \bar{\nu}_\tau$ decay allows one to test predictions of the SM, and can be a source of information on $CP$ violation in the Yukawa interaction and on such a fundamental property as Hermiticity of this interaction.

Acknowledgments

This work was partially conducted in the scope of the IDEATE International Associated Laboratory (LIA). A.Yu.K. acknowledges partial support by the National Academy of Sciences of Ukraine via the programs “Support for the development of priority areas of scientific research” (6541230) and “Participation in the international projects in high energy and nuclear physics” (project No. 0121U111693).

REFERENCES

[1] D. de Florian et al. Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector. *CERN Yellow Reports: Monographs*, 2:1–869, 2017.
[2] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett. B*, 716:1–29, 2012.

[3] Serguei Chatrchyan et al. Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC. *Phys. Lett. B*, 716:30–61, 2012.

[4] Steven D. Bass, Albert De Roeck, and Marumi Kado. The Higgs boson implications and prospects for future discoveries. *Nature Rev. Phys.*, 3(9):608, 2021.

[5] Nicola Cabibbo. Unitary Symmetry and Leptonic Decays. *Phys. Rev. Lett.*, 10:531–533, 1963.

[6] Makoto Kobayashi and Toshihide Maskawa. CP Violation in the Renormalizable Theory of Weak Interaction. *Prog. Theor. Phys.*, 49:652–657, 1973.

[7] Glennys R. Farrar and M. E. Shaposhnikov. Baryon asymmetry of the universe in the standard electroweak theory. *Phys. Rev. D*, 50:774, 1994.

[8] Sacha Davidson, Enrico Nardi, and Yosef Nir. Leptogenesis. *Phys. Rept.*, 466:105–177, 2008.

[9] Joseph R. Dell’Aquila and Charles A. Nelson. CP Determination for New Spin Zero Mesons by the $\bar{\tau}\tau$ Decay Mode. *Nucl. Phys. B*, 320:61–85, 1989.

[10] M. Kramer, Johann H. Kuhn, M. L. Stong, and P. M. Zerwas. Prospects of measuring the parity of Higgs particles. *Z. Phys. C*, 64:21–30, 1994.

[11] B. Grzadkowski and J. F. Gunion. Using decay angle correlations to detect CP violation in the neutral Higgs sector. *Phys. Lett. B*, 350:218–224, 1995.

[12] W. Bernreuther, A. Brandenburg, and M. Flesch. QCD corrections to decay distributions of neutral Higgs bosons with (in)definite CP parity. *Phys. Rev. D*, 56:90–99, 1997.

[13] G. R. Bower, T. Pierzchala, Z. Was, and M. Worek. Measuring the Higgs boson’s parity using tau $\rightarrow$ rho nu. *Phys. Lett. B*, 543:227–234, 2002.

[14] K. Desch, Z. Was, and M. Worek. Measuring the Higgs boson parity at a linear collider using the tau impact parameter and tau $\rightarrow$ rho nu decay. *Eur. Phys. J. C.*, 29:491–496, 2003.

[15] Małgorzata Worek. Higgs CP from $H/A0 \rightarrow$ tau tau decay. *Acta Phys. Polon. B*, 34:4549–4560, 2003.

[16] K. Desch, A. Imhof, Z. Was, and M. Worek. Probing the CP nature of the Higgs boson at linear colliders with tau spin correlations: The Case of mixed scalar - pseudoscalar couplings. *Phys. Lett. B*, 579:157–164, 2004.

[17] Andre Rouge. CP violation in a light Higgs boson decay from tau-spin correlations at a linear collider. *Phys. Lett. B*, 619:43–49, 2005.

[18] Stefan Berge, Werner Bernreuther, and Jorg Ziethe. Determining the CP parity of Higgs bosons at the LHC in their tau decay channels. *Phys. Rev. Lett.*, 100:171605, 2008.

[19] Stefan Berge and Werner Bernreuther. Determining the CP parity of Higgs bosons at the LHC in the tau to 1-prong decay channels. *Phys. Lett. B*, 671:470–476, 2009.
[20] S. Berge, W. Bernreuther, B. Niepelt, and H. Spiesberger. How to pin down the CP quantum numbers of a Higgs boson in its tau decays at the LHC. *Phys. Rev. D*, 84:116003, 2011.

[21] Roni Harnik, Adam Martin, Takemichi Okui, Reinard Primulando, and Felix Yu. Measuring CP Violation in $h \to \tau^+\tau^-$ at Colliders. *Phys. Rev. D*, 88(7):076009, 2013.

[22] Stefan Berge, Werner Bernreuther, and Hubert Spiesberger. Higgs CP properties using the $\tau$ decay modes at the ILC. *Phys. Lett. B*, 727:488–495, 2013.

[23] Stefan Berge, Werner Bernreuther, and Sebastian Kirchner. Determination of the Higgs CP-mixing angle in the tau decay channels at the LHC including the Drell–Yan background. *Eur. Phys. J. C*, 74(11):3164, 2014.

[24] Andrew Askew, Prerit Jaiswal, Takemichi Okui, Harrison B. Prosper, and Nobuo Sato. Prospect for measuring the CP phase in the $h\tau\tau$ coupling at the LHC. *Phys. Rev. D*, 91(7):075014, 2015.

[25] Stefan Berge, Werner Bernreuther, and Sebastian Kirchner. Prospects of constraining the Higgs boson’s CP nature in the tau decay channel at the LHC. *Phys. Rev. D*, 92:096012, 2015.

[26] Alexander Yu. Korchin and Vladimir A. Kovalchuk. Decay of the Higgs boson to $\tau^-\tau^+$ and non-Hermiticity of the Yukawa interaction. *Phys. Rev. D*, 94(7):076003, 2016.

[27] Xin Chen and Yongcheng Wu. Search for CP violation effects in the $h \to \tau\tau$ decay with $e^+e^-$ colliders. *Eur. Phys. J. C*, 77(10):697, 2017.

[28] Kaoru Hagiwara, Kai Ma, and Shingo Mori. Probing CP violation in $h \to \tau^-\tau^+$ at the LHC. *Phys. Rev. Lett.*, 118(17):171802, 2017.

[29] D. Jeans and G. W. Wilson. Measuring the CP state of tau lepton pairs from Higgs decay at the ILC. *Phys. Rev. D*, 98(1):013007, 2018.

[30] Xin Chen and Yongcheng Wu. Probing the CP-Violation effects in the $h\tau\tau$ coupling at the LHC. *Phys. Lett. B*, 790:332–338, 2019.

[31] Shao-Feng Ge, Gang Li, Pedro Pasquini, and Michael J. Ramsey-Musolf. CP-violating Higgs Di-tau Decays: Baryogenesis and Higgs Factories. *Phys. Rev. D*, 103(9):095027, 2021.

[32] Jean Alexandre, Carl M. Bender, and Peter Millington. Non-Hermitian extension of gauge theories and implications for neutrino physics. *JHEP*, 11:111, 2015.

[33] Jean Alexandre, Carl M. Bender, and Peter Millington. Light neutrino masses from a non-Hermitian Yukawa theory. *J. Phys. Conf. Ser.*, 873(1):012047, 2017.

[34] Jean Alexandre, Peter Millington, and Dries Seynaeve. Symmetries and conservation laws in non-Hermitian field theories. *Phys. Rev. D*, 96(6):065027, 2017.

[35] Jean Alexandre, John Ellis, Peter Millington, and Dries Seynaeve. Spontaneous symmetry breaking and the Goldstone theorem in non-Hermitian field theories. *Phys. Rev. D*, 98:045001, 2018.
[36] Philip D. Mannheim. Goldstone bosons and the Englert-Brout-Higgs mechanism in non-Hermitian theories. Phys. Rev. D, 99(4):045006, 2019.

[37] Jean Alexandre, John Ellis, Peter Millington, and Dries Seynaeve. Gauge invariance and the Englert-Brout-Higgs mechanism in non-Hermitian field theories. Phys. Rev. D, 99(7):075024, 2019.

[38] Peter Millington. Symmetry properties of non-Hermitian $\mathcal{PT}$-symmetric quantum field theories. J. Phys. Conf. Ser., 1586(1):012001, 2020.

[39] Andreas Fring and Takanobu Taira. Goldstone bosons in different PT-regimes of non-Hermitian scalar quantum field theories. Nucl. Phys. B, 950:114834, 2020.

[40] Jean Alexandre, John Ellis, Peter Millington, and Dries Seynaeve. Spontaneously Breaking Non-Abelian Gauge Symmetry in Non-Hermitian Field Theories. Phys. Rev. D, 101(3):035008, 2020.

[41] Andreas Fring and Takanobu Taira. Pseudo-Hermitian approach to Goldstone’s theorem in non-Abelian non-Hermitian quantum field theories. Phys. Rev. D, 101(4):045014, 2020.

[42] Jean Alexandre, John Ellis, and Peter Millington. $\mathcal{PT}$-symmetric non-Hermitian quantum field theories with supersymmetry. Phys. Rev. D, 101(8):085015, 2020.

[43] Andreas Fring and Takanobu Taira. Massive gauge particles versus Goldstone bosons in non-Hermitian non-Abelian gauge theory. arXiv:2004.00723 [hep-th].

[44] Jean Alexandre and Nick E. Mavromatos. On the consistency of a non-Hermitian Yukawa interaction. Phys. Lett. B, 807:135562, 2020.

[45] Jean Alexandre, Nick E. Mavromatos, and Alex Soto. Dynamical Majorana neutrino masses and axions I. Nucl. Phys. B, 961:115212, 2020.

[46] Andreas Fring and Takanobu Taira. ’t Hooft-Polyakov monopoles in non-Hermitian quantum field theory. Phys. Lett. B, 807:135583, 2020.

[47] Jean Alexandre, John Ellis, and Peter Millington. Discrete spacetime symmetries and particle mixing in non-Hermitian scalar quantum field theories. Phys. Rev. D, 102(12):125030, 2020.

[48] Nick E. Mavromatos and Alex Soto. Dynamical Majorana neutrino masses and axions II: Inclusion of anomaly terms and axial background. Nucl. Phys. B, 962:115275, 2021.

[49] Andreas Fring and Takanobu Taira. Complex BPS solitons with real energies from duality. J. Phys. A, 53(45):455701, 2020.

[50] Nick E. Mavromatos. Non-Hermitian Yukawa interactions of fermions with axions: potential microscopic origin and dynamical mass generation. J. Phys. Conf. Ser., 2038(10):012019, 2020.

[51] Alexander Yu. Korchin and Vladimir A. Kovalchuk. Polarization effects in the Higgs boson decay to $\gamma Z$ and test of $CP$ and $CPT$ symmetries. Phys. Rev. D, 88(3):036009, 2013.

[52] A. Yu. Korchin and V. A. Kovalchuk. Higgs Boson Decay to $\gamma Z$ and Test of $CP$ and $CPT$ Symmetries. Acta Phys. Polon. B, 44(11):2121–2128, 2013.
[53] Alexander Yu. Korchin and Vladimir A. Kovalchuk. Angular distribution and forward–backward asymmetry of the Higgs-boson decay to photon and lepton pair. *Eur. Phys. J. C*, 74(11):3141, 2014.

[54] V. A. Kovalchuk and A. Yu. Korchin. Higgs Boson Decay to Lepton Pair and Photon and Possible Non-Hermiticity of the Yukawa Interaction. *Ukr. J. Phys.*, 62(7):557, 2017.

[55] P. A. Zyla et al. Review of Particle Physics. *PTEP*, 2020(8):083C01, 2020.