Observer-Based Backstepping Fault-tolerant Control for Spacecraft with Reaction-wheel Failures

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Abstract. In this paper, nonlinear satellite system with actuator faults is considered. To solve the problem of attitude stabilization of satellite under external disturbances and actuator faults, an observer is designed firstly. And the output of the observer is proved to be uniformly ultimately bounded. By using the estimated information of the observer, a backstepping fault-tolerant controller is derived. The proposed controller could achieve attitude stabilization in the presence of actuator faults. Finally, a simulation of attitude stabilization is carried out to illustrate effective of the fault-tolerant controller.

1 Introduction
With the deepening of deep space exploration missions, makes it put forward higher requirements for spacecraft guidance, navigation and control technology. To this end, the concept of fault-tolerant control system design was proposed[1]-[3]. As a popular technology, adaptive control has been used in spacecraft control systems widely [[4]-[6]]. One of the advantage of adaptive control is that it could identify uncertain parameters in real-time. In [[7]], for rendezvous and proximity problem of spacecrafts, the author designed an adaptive controller under input saturation. In [[8]], under the gain actuator fault, the author proposed an adaptive controller to deal with it. To solve the problem of actuator transient response, paper [[9]] proposed a control scheme based on technology of variable structure control. But the control schemes mentioned above were not proved to achieve attitude stability in a finite time. To solve this problem[[10]], the author combined adaptive control technique and adding a power integrator to solve the problem of attitude tracking control. In [[11]], an SMC-based control approach is derived under actuator outage fault. The proposed controller could stabilized the attitude quickly. In [[12]], an SMC scheme is proposed. Under the roll or yaw faults, attitude rapid maneuvering was achieved.

Although the adaptive control technology is robust to uncertainties, but it is under the assumption that the uncertainty is bounded. However it will increase the conservation of controller. Under this situation, controller based on fault diagnosis scheme was investigated. In [[13]], a nonlinear unknown input observer was proposed to observe the actuator faults under external disturbances and gyro drift. It could achieve the isolation of actuator faults. Similarly, paper [[13]] designed an observer to detect the time-vary thruster faults. The observer could observe the fault torque quickly. However both of them did not derive controllers for spacecraft. In [[15]], the author proposed a fault-tolerant control scheme based on an observer. The controller could achieve attitude stabilization under gain fault of actuators. In [[16]], an sliding-mode observe is designed to identify the uncertain parameters.

In the following of this paper, attitude kinematics and dynamics of the spacecraft is derived. And problem formulation is given. In section 3, an observer is designed to observer the fault torque of actuators and external disturbances. The boundness of the observer is proved by Lyapunov stability theory. In section 4, a backstepping controller is proposed based on the observer. And the system state...
is uniform ultimately bounded. Simulation is carried out in section 5.

2 Kinematics and dynamics model

2.1 Mathematical model of rigid spacecraft
MRPs is used to describe the spacecraft attitude in this paper. \( n \in \mathbb{R}^3 \) is defined as the Euler principal axis, \( \varphi \in \mathbb{R} \) is the rotation angle around \( n \). Define \( \omega \) as the angular velocity of satellite. The mathematical model of satellite attitude kinematics are given by

\[
\sigma_i = \frac{q_i}{1 + \sigma_0}, \quad i = 1, 2, 3, \quad \dot{\sigma} = F(\sigma)\omega
\]

\[
F(\sigma) = \frac{1}{4} \left\{ (1 - \sigma^T \sigma) I_{3 \times 3} + 2[\sigma^T] + 2\sigma \sigma^T \right\}
\]

where \( q_i \) and \( q_i, \; i = 1, 2, 3 \) denote the unit quaternions. Assumed that the satellite equipped \( N (>3) \) reaction-wheels. The satellite’s dynamics equation is given as follows

\[
J \dot{\omega} = -\omega^T J \omega + \tau + d
\]

where \( J \) represents the satellite inertial matrix, \( \tau \) represents the control torque, \( d \) represents the disturbance torque, \( \omega^T \) represents the skew symmetric matrix of \( \omega \).

2.2 Model of reaction wheel faults
When faults occur, the torque generated by reaction-wheels could be modeled as follows

\[
\tau_i = e_i(t) \tau_{\omega} + \tau_{\rho}
\]

Further, Eq.(5) could be modified as

\[
\tau_i = \tau_{\omega} + \tau_{\rho}
\]

where \( \tau_{\rho} = [e_i(t) - 1] \tau_{\omega} + \tau_{\rho} \) denotes the fault torque.

Assumption 1 There exists a constant \( \tau_{\max} \) that makes following inequality holds \( |\tau_{\rho}| \leq \tau_{\max} \).

3 Design of observer
In this paper, fault torque is not estimated for one single reaction-wheel. Design the input of observer as \( v(t) \). If the observer is stable, the following equation holds

\[
\tau(t) = \tau_{\omega}(t) + v(t)
\]

if \( T_i(t) = \text{diag} \left( \tau_{\omega}, \tau_{\omega}, \tau_{\omega} \right) \), then \( \tau(t) = T_i(t) \rho(t) \), where \( \rho(t) = [\rho_1(t) \quad \rho_2(t) \quad \rho_3(t)] \), the following equation holds

\[
T_i(t) \rho(t) = \tau_{\omega}(t) + v(t)
\]

According to the dynamics model(4), the observer designed as follows

\[
J \dot{\hat{\omega}} = -\omega^T J \omega + \lambda \omega - \hat{\omega} + \tau + v(t)
\]

\[
v(t) = K_1 v(t - T) + K_2 \left[ \omega(t) - \hat{\omega}(t) \right] + K_3 \left[ \omega(t - T) - \hat{\omega}(t - T) \right]
\]

where \( K_1, K_2 \) and \( K_3 \) are gain matrices of the observer, \( T \) denotes the sampling-time. \( \lambda \) is a positive
defined matrix to be designed.
From Eq.(4) Eq.(6) and Eq.(9), equation of estimation error could be derived
\[
\dot{J}_\omega = \left(-\omega^J \dot{J}_\omega \omega^J\right) - A\dot{\omega} + \tau_c + u_f(t) - v(t)
\]  
(11)
where \(\dot{\omega} = \omega - \dot{\omega}\) and \(u_f(t) = \tau_c + d\).

The following assumptions are made to prove the stability of the observer:
\textbf{Assumption 2.} \(u_f(t)\) is bounded and the following inequality holds \(u_f(t) - K, u_f(t-T) \leq k_d\).

\textbf{Assumption 3.} The nonlinear term \(\omega^J\dot{\omega}\) satisfies \(\left|\omega^J\dot{\omega}\right| \leq \eta \|\omega\|, \eta > 0\) is an unknown constant\([18]\).

\textbf{Assumption 4.} The angular velocity information is available, and it is continuously differentiable, so \(\|\omega(t-T)\| \leq \xi \|\dot{\omega}(t)\|\) for an unknown \(\xi > 0\).

4 Design of backstepping fault-tolerant controller
Through the observer designed before, then an adaptive backstepping controller will be designed for achieve the attitude stabilization under actuator failures and disturbances. First, variables are defined as
\[
x_i = \int \sigma dt, \ x_2 = \sigma, \ x_3 = \omega
\]  
(12)

Then the dynamics model could be rewritten as
\[
x_i = x_2
\]  
(13)
\[
\dot{x}_2 = F(x_1)x_2
\]  
(14)
\[
J\dot{x}_3 = -x_3^J Jx_3 + \Theta(t)\tau_c - A(t)\tau_c + d
\]  
(15)
where \(\Theta(t) = \text{diag}\left([\rho_1, \rho_2, \rho_3]\right), A(t) = \text{diag}\left([\delta_1, \delta_2, \delta_3]\right)\), \(\delta_i = \dot{\rho}_i - \rho_i\). To derive the backstepping controller, making the following transformation
\[
z_i = x_i, z_2 = x_2 - a_i, z_3 = x_3 - a_2
\]  
(16)
Where \(a_i\) and \(a_3\) are virtual control variables.

\textbf{Step 1} According to Eq.(16)
\[
z_i = \dot{z}_i = z_2 = z_2 + a_i
\]  
(17)
Consider a Lyapunov function as \(V_i = 1/2 \cdot z_i^T z_i\). And design \(a_i = -c_i x_i\), \(c_i\) is a constant to be designed, then the time derivative of \(V_i\) could be obtain \(\dot{V}_i = z_i^T z_i = z_i^T (z_i - c_i z_i) = -c_i \|z_i\|^2 + z_i^T z_i\).

When \(z_i = 0\), \(\dot{V}_i = -c_i \|z_i\|^2\), \(z_i\) is asymptotically convergence, that is \(\lim_{i \to \infty} z_i(t) = 0\).

\textbf{Step 2} The time derivative of \(z_i \) is
\[
z_i = \dot{z}_i = \dot{z}_2 = \dot{z}_3 = z_i + a_3
\]  
(18)
Consider Lyapunov candidate as \(V_2 = V_1 + 1/2 \cdot z_2^T z_2\). Design \(a_2\) as \(a_2 = F^{-1}(x_2)(-z_2 + c_i z_2 - c_2 z_2)\), Then \(\dot{V}_2 = -c_i \|z_2\|^2 + c_i \|z_2\|^2 + z_2^T F(x_2) z_2\), if \(z_2 = 0\), \(\dot{V}_2 = -c_i \|z_2\|^2 - c_2 \|z_2\|^2\). So \(z_i\) and \(z_2\) are asymptotically convergence, that is \(\lim_{i \to \infty} z_i(t) = 0\).

If the controller could make \(\tau_c\) satisfy \(\lim_{i \to \infty} z_i(t) = 0\), then \(\lim_{i \to \infty} z_i(t) = \lim_{i \to \infty} z_2(t) = 0\) also satisfied.

\textbf{Theorem 2} For satellite dynamics system determined by Eq.(2) and Eq.(4) with actuator failures. Under observer determined by Eq.(9) and Eq.(10), design the controller as follows
\( r_c = \text{Sat}(\mu, \tau_{\text{max}}) \)  

\[
\mu(t) = \Theta(t)^{-1} \left\{ \frac{dF^{-1}(x_1)}{dt} \left( z_1 + c_1 x_1 + c_2 z_2 \right) + F^{-1}(x_2) \left( \dot{z}_1 + c_1 \dot{x}_2 + c_2 \dot{z}_2 \right) \right\} + \frac{\alpha_T z_1}{\alpha_T + \tau_{\text{max}}} \right\}
\]

\[
x_c J x_c - c_1 z_1 - G_c x_c - F^T(x_c) z_2 - \frac{\alpha_T z_1}{\alpha_T + \tau_{\text{max}}} + \varepsilon \exp(-\beta t)
\]

\[
\dot{x}_c = -G_c x_c - \frac{\|\Theta(t)\| \|r_c - \mu\|}{\|x_c\|} x_c - \Theta(t)(r_c - \mu)
\]

where \( \alpha_T = \tau_{\text{max}} + \Delta \), \( \varepsilon \) is a small constant, \( K_1, K_2, \beta \) and \( c_i \) are gains of the controller. If the following inequalities satisfied  
\( c_1 - 1 > 0, G_c - G_c^2/2 - 1/2 > 0 \). Then, the close loop of attitude control system is stable. Furthermore, there exists \( \varepsilon_1 > 0 \), \( \varepsilon_2 > 0 \) and \( T > 0 \), for an arbitrary \( t > T \),  
\( \|\sigma(t)\| \leq \varepsilon_1 \) and  
\( \|\omega(t)\| \leq \varepsilon_2 \) holds.

**Proof.** From Eq.(15)

\[
\dot{z}_1 = J^T \left[ -x_c J x_c + \Theta(t) r_c + d \right] + \frac{dF^{-1}(x_1)}{dt} \left( z_1 + c_1 x_1 + c_2 z_2 \right) + F^{-1}(x_2) \left( \dot{z}_1 + c_1 \dot{x}_2 + c_2 \dot{z}_2 \right)
\]

Where

\[
\frac{dF^{-1}(x_1)}{dt} = \frac{16(1 + x_c^T x_c)^2}{(1 + x_c^T x_c)^3} \frac{dF(x_1)}{dt} - 64(1 + x_c^T x_c) x_c^T F(x_1) x_c F^T(x_1)
\]

\[
\frac{dF(x_1)}{dt} = \frac{1}{2} \left[ -x_c^T \left[ F(x_2) x_c \right] I_3 + \left[ F(x_2) x_c \right] + F(x_2) x_c x_c^T + x_c x_c^T F^T(x_2) \right]
\]

Consider a Lyapunov candidate as

\[
V_1 = V_2 + \frac{1}{2} z_c^T J z_c + \frac{1}{2} x_c^T x_c
\]

Time derivative of \( V_1 \) could be derived

\[
\dot{V}_1 = -c_1 \|x_c\|^2 - c_2 \|x_c\|^2 + z_c^T F(x_2) z_c - G_c \|x_c\|^2 - \|\Theta(t)\| \|\tau_c - \mu\|^2 - x_c^T \Theta(t) \left( \tau_c - \mu \right) + z_c^T \left[ -x_c J x_c + \Theta(t) \tau_c - A(t) \tau_c + d \right] +
\]

\[
J \left[ \frac{dF^{-1}(x_1)}{dt} \left( z_1 + c_1 x_1 + c_2 z_2 \right) + F^{-1}(x_2) \left( \dot{z}_1 + c_1 \dot{x}_2 + c_2 \dot{z}_2 \right) \right]
\]

According to the following inequalities

\[
z_c^T \Theta(t) \left( \tau_c - \mu \right) \leq \frac{1}{2} \|\Theta(t)\|^2 + \frac{1}{2} \|\tau_c\|^2, \quad -c_1 \|z_c\|^2 \leq \frac{1}{2} \|z_c\|^2 + \frac{G_c^2}{2} \|x_c\|^2
\]

\[
-x_c^T \Theta(t) \left( \tau_c - \mu \right) \leq \frac{1}{2} \|\Theta(t)\| \|\tau_c - \mu\| + \frac{1}{2} \|x_c\|^2
\]

If gains of controller selected as Theorem 2.
\[
\dot{V}_i \leq -c_1 \|z_i\|^2 - c_2 \|z_i\| - (c_3 - 1) \|z_i\|^3 - \left( G_2 - \frac{G_i^2}{2} \right) \|x_i\|^2 + \varepsilon \exp(-\beta t)
\]
\[
\leq -m \left( \|z_i\|^2 + \|z_i\|^2 + J_{\min} \|z_i\|^3 + \|x_i\|^2 + \varepsilon \right)
\]  
(28)

Where \( m = \min(c_1, c_2, (c_3 - 1)/J_{\max}, G_2 - G_i^2/2 - 1/2) \). According to the definition of \( V_i \)

\[
2V_i \geq \|z_i\|^2 + \|z_i\|^3 + J_{\min} \|z_i\|^3 + \|x_i\|^2
\]  
(29)

Furthermore

\[
\dot{V}_i \leq -2mV_i + \varepsilon
\]  
(30)

So \( V_i \) is uniform ultimately bounded. And for \( \varepsilon > \sqrt{\frac{\varepsilon}{m}} \), there exist a finite time \( T_f > 0 \), makes \( \|z_i\| \leq \varepsilon^* \) and \( \|x_i\| \leq \varepsilon^* \) holds. The proof is completed.

5 Simulation Results

In order to verify the effectiveness of the proposed controller, the controller proposed in [[17]] is compared in this paper.

**Figure 1** Attitude response of satellite  
**Figure 2** Attitude response under compared controller

The Attitude response of the backstepping fault-tolerant control method is shown in Figure 1. And Attitude response of controller proposed in [[17]] is shown in Figure 2. The Angular velocity response of the backstepping fault-tolerant control method is shown in Figure 3. And Angular velocity response of controller proposed in [[17]] is shown in Figure 4. Figure 5 shows the output signals of the observer. It could be seen in the figure, when faults happened the output signals will show the effectiveness on the system.
Figure 3 Angular velocity response of satellite

Figure 4 Angular velocity response under compared controller

Figure 5 output signals of the observer

6 Conclusion
In this paper, to solve the problem of attitude stabilization of satellite under external disturbances and actuator fault, nonlinear satellite system with actuator fault is and external disturbance is considered. An observer is developed to estimate the actuator fault torque. And the output of the observer is proved to be uniform ultimately bounded. Furthermore, by using the estimated information of the observer, a backstepping fault-tolerant controller is derived. The proposed controller could achieve attitude stabilization in the presence of actuator fault. The simulation results show the robustness to actuator faults and disturbances. And compared to a similar controller. The results show that the proposed controller could achieve attitude stabilization more effectively.

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