I provide a brief, and therefore necessarily biased, review of the application of effective field theory to electromagnetic processes on the deuteron. Electron-deuteron scattering is discussed for virtual photon momenta up to 700 MeV, and photon-deuteron scattering is examined for incident photon energies of order the pion mass.

1. INTRODUCTION

Effective field theory (EFT) is a technique commonly used in particle physics to deal with problems involving widely-separated energy scales. It facilitates the systematic separation of the effects of high-energy physics from those of low-energy physics. In strong-interaction physics the low-energy effective theory is chiral perturbation theory (χPT) \[\text{[1]}\]. Here the low-energy physics is that of nucleons and pions interacting with each other in a way that respects the spontaneously-broken approximate chiral symmetry of QCD. Higher-energy effects of QCD appear in χPT as non-renormalizable contact operators. In this way the EFT provides a long-wavelength approximation to the amplitudes for various hadronic processes. It yields amplitudes which can be thought of as expansions in the ratio of nucleon or probe momenta (denoted here by \(p\) and \(q\)) and the pion mass to the scale of chiral-symmetry breaking, \(\Lambda_{\chi\text{SB}}\). \(\Lambda_{\chi\text{SB}}\) is of order \(m_\rho\), and so such an expansion in \(Q\), with

\[
Q \equiv \frac{p}{\Lambda_{\chi\text{SB}}} \cdot \frac{q}{\Lambda_{\chi\text{SB}}} \cdot \frac{m_\pi}{\Lambda_{\chi\text{SB}}}
\]

provides a controlled way to calculate hadronic processes, as long as \(p\) and \(q\) are not too large. In light nuclei the typical momentum of a bound nucleon is of order \(m_\pi\) or less, and so we should be able to calculate the response of such nuclei to low-energy probes using χPT, thereby providing systematically-improvable, model-independent descriptions of these reactions. In this paper I will describe some recent work in this direction. Section 2 outlines the expansion which χPT gives for these processes, while Sections 3 and 4 sketch this expansion’s use for electron-deuteron scattering and Compton scattering on the deuteron.
Consider an elastic scattering process on the deuteron whose amplitude we wish to compute. If \( \hat{O} \) is the transition operator for this process then the amplitude in question is simply \( \langle \psi | \hat{O} | \psi \rangle \), with \( |\psi\rangle \) the deuteron wave function. In this section, we follow Weinberg, and divide the formulation of a systematic expansion for this amplitude into two parts: the expansion for \( \hat{O} \), and the construction of \( |\psi\rangle \).

Chiral perturbation theory gives a systematic expansion for \( \hat{O} \) of the form

\[
\hat{O} = \sum_{n=0}^{\infty} \hat{O}^{(n)},
\]

where we have labeled the contributions to \( \hat{O} \) by their order \( n \) in the small parameter \( Q \) of Eq. (1). This expansion is partially motivated by the idea of a long-wavelength limit, where distances smaller than \( 1/\Lambda_{\chi SB} \) are not resolved.

Equation (2) is an operator statement, and the nucleon momentum operator \( \hat{p} \) appears on the right-hand side. Thus, one might worry about the unboundedness of this quantum-mechanical operator. However, once we take matrix elements of \( \hat{O} \) to construct the physical amplitude for the process in question the only quantities which appear are objects like \( \langle \psi | \hat{p} | \psi \rangle \). For weakly-bound systems like the deuteron these expectation values are generically small compared to \( \Lambda_{\chi SB} \).

The procedure to construct \( \hat{O}^{(n)} \) begins with writing down the vertices appearing in the chiral Lagrangian up to order \( n \). One then draws all of the two-body, two-nucleon-irreducible, Feynman graphs for the process of interest which are of chiral order \( Q^n \). The rules for calculating the chiral order of a particular graph are:

- Each nucleon propagator scales like \( 1/Q \);
- Each loop contributes \( Q^4 \);
- Graphs in which both particles participate in the reaction acquire a factor of \( Q^3 \);
- Each pion propagator scales like \( 1/Q^2 \);
- Each vertex from the \( n \)th-order piece of the chiral Lagrangian contributes \( Q^n \).

In this way we see that more complicated graphs, involving two-body mechanisms, and/or higher-order vertices, and/or more loops, are suppressed by powers of the small parameter \( Q \).

There remains the problem of constructing a deuteron wave function which is consistent with the operator \( \hat{O} \). The proposal of Weinberg was to construct a \( \chi PT \) expansion as per Eq. (2) for the \( NN \) potential \( V \), and then solve the Schrödinger equation to find the deuteron (or other nuclear) wave function. Recent calculations have shown that the \( NN \) phase shifts can be understood, and deuteron bound-state static properties reliably computed, with wave functions derived from \( \chi PT \) in this way. The difficulty is that

\[\text{The ideas presented in this section can easily be extended to inelastic processes, or to other light nuclei.}\]
there is no *a priori* reason within χPT to iterate V to all orders, since the loop corrections
which are responsible for generating the deuteron bound state are, in principle, higher-
order effects in the EFT. In marked contrast to χPT in the one-nucleon sector, Weinberg’s
application of χPT to nuclei has power-counting only for the potential, and not for the
NN scattering amplitude. The most notable attempt to remedy this deficiency was made
by Kaplan, Savage, and Wise, in Ref. [7]. I will not discuss these issues further here, but
instead refer to Ref. [8] for a survey of the state of play. In this work I focus on the χPT
expansion for the operators ˆO and use wave functions found in one of three ways:

1. The full chiral wave function of Ref. [5] (or equivalently Ref. [4]). This implements
   Weinberg’s proposal to use the chiral expansion for V, and includes terms in the
   potential up to chiral order Q^3.

2. In the long-wavelength approximation it is valid to employ a wave function that
   is the solution to the Schrödinger equation with a potential which is the sum of
   one-pion exchange and a short-distance interaction of range R < 1/m_π [9,10]. The
   short-distance potential should be tuned so as to reproduce the deuteron binding
   energy B and the asymptotic wave-function normalizations A_S and A_D. Wave
   functions generated using potentials with different values of R will then differ from
   each other only for r < R.

3. Similarly, potential-model wave functions, e.g. the Nijm93 wave function [11], can
   be used for |ψ⟩. Historically these have been the “wave functions of choice” for
   χPT calculations of reactions on the deuteron. While not entirely consistent, this
   “hybrid” approach has led to successful calculations of processes including, but not
   limited to, γd → π^0d [12] and pp → dν_e [13]. Ref. [14] contains a thorough review
   of results obtained via this method.

3. ONE PHOTON: DEUTERON ELECTROMAGNETIC FORM FACTORS

In this section I will discuss some aspects of electron-deuteron scattering in χPT (see [10]
for a more thorough treatment). As is well-known, the interaction of a deuteron with an
electromagnetic current is naturally described in terms of three nuclear response functions:
the charge (F_C), magnetic (F_M), and quadrupole (F_Q) form factors. These form factors
can be extracted from electron scattering measurements using differential cross-section
and tensor-polarization data. They can also be computed systematically in χPT. The
Q-expansion of the deuteron charge operator is:

\[ \hat{Q} = e \left[ 1 + \left( \text{relativistic corrections} - \frac{1}{6} q^2 r_N^2 \right) \right] + O(Q^3), \]  (3)

where the first term is the leading contribution, which comes only from the charge of a
point proton. Corrections to this O(ε) result arise at order eQ^2 from the finite-size of the
nucleon and from relativistic effects. The latter can be systematically computed in χPT
and scale as 1/M^2 (M is the nucleon mass), but I have not written them explicitly here.
At O(εQ^3) χPT gives a two-body contribution due to the exchange of pions in Q.

This picture of the deuteron charge is similar to that obtained from systematic 1/M
expansions in non-relativistic potential models (see e.g. [15]), apart from the somewhat
unusual expedient of expanding out the nucleon’s charge form factor in powers of \( q^2 r_N^2 \). Thus, it is reasonable to ask what is gained when \( \chi \)PT is employed here.

Firstly, and perhaps most importantly, one gains the ability to include two-body contributions to \( \hat{Q} \) systematically. In \( \chi \)PT such meson-exchange currents can be included as they arise order-by-order in the \( Q \)-expansion. This is a critical advantage in reactions where two-body mechanisms are important, as we shall see in Section 4.

**Figure 1.** The upper panel shows the charge form factor of the deuteron for several different wave functions, all of which have the same tail, as well as the same one-pion exchange part. The solid line is the result for the Nijm93 potential [11]. The lower panel depicts the tensor polarization observable \( T_{20} \) for the same four wave functions, compared to data below \( q = 1 \) GeV. Data are taken from Ref. [16].

Secondly, on an aesthetic level \( \chi \)PT leads to what might be considered a more elegant formulation—one where the amount of effort required to obtain accurate answers is commensurate with the physics of the long-wavelength limit. In particular, it is straightforward to compute wave functions and electromagnetic operators consistently, and many of the issues which arise due to the complicated form of modern \( NN \) potentials do not occur in a \( \chi \)PT computation of electron-deuteron scattering. In fact, it is somewhat surprising how well a very simple picture of the deuteron describes the charge and magnetic form factors. In the upper panel of Fig. 1 I have displayed the charge form factor computed with the leading-order charge of Eq. (3) and a variety of wave functions obtained with the simple OPEP plus short-distance potential described in the previous section [10]. One sees that the sensitivity to the physics at distances smaller than \( R \) is slight, appearing only at momentum transfers of order 600 MeV. Furthermore, for a range of values of
the result for $F_C$ from these simple wave functions is close to that of the much more sophisticated Nijm93 wave function.

A third role of $\chi$PT is to point to physics not currently included in potential-model descriptions of electron-deuteron scattering. For instance, a survey of the results for deuteron static properties found using the same wave functions employed in the calculations for $F_C$ reveals that the deuteron quadrupole moment, $Q_d$, is particularly susceptible to short-distance physics. It varies from 0.258 fm$^2$ when the radius of the short-distance potential, $R$, is 2.5 fm to 0.269 fm$^2$ when $R = 1.5$ fm. Such sensitivity of $Q_d$ to short-range dynamics will be no surprise to those familiar with $NN$ potential models. However, in an EFT it suggests the presence of a two-nucleon one-quadrupole-photon counterterm which modifies $Q_d$ [10,17]. This is physics “beyond the nuclear standard model”, since such mechanisms are not included in the usual calculations of $Q_d$. An analysis of the counterterm required to shift $Q_d$ to its experimental value of 0.286 fm$^2$ suggests that it is “natural”, i.e. of a size consistent with the scales in the problem. This counterterm, with its value adjusted to give the experimental value of $Q_d$ for each different wave function, is easily included in the calculation of the tensor polarization observable $T_{20}$. The lower panel of Fig. 3 then shows that (a) the residual sensitivity to the short-distance behaviour of the wave function is very small, and (b) using only the $O(e)$ charge operator and the $Q_d$ counterterm we can describe almost all of the existing $T_{20}$ data out to $q = 700$ MeV.

4. TWO PHOTONS: COMPTON SCATTERING ON THE DEUTERON

In this section I will discuss Compton scattering on the deuteron in $\chi$PT. (Ref. 18 contains a full treatment.) This reaction has been the subject of recent experiments [19,20], one goal of which was to extract the electromagnetic polarizabilities of the neutron.

In the case of the proton these electric and magnetic polarizabilities, $\alpha$ and $\beta$, have been extracted from Compton scattering data [21]:

$$\alpha_p + \beta_p = 13.23 \pm 0.86 + 0.20 \pm 0.45 \times 10^{-4} \text{fm}^3; \quad \alpha_p - \beta_p = 10.11 \pm 1.74 + 1.22 - 0.86 \times 10^{-4} \text{fm}^3, (4)$$

These numbers agree well with the predictions of $\chi$PT at leading loop order ($O(e^2Q)$) [1]: $\alpha_p = 12.2 \times 10^{-4} \text{fm}^3$ and $\beta_p = 1.2 \times 10^{-4} \text{fm}^3$. At this order $\chi$PT also predicts $\alpha_n = \alpha_p$ and $\beta_n = \beta_p$. $\alpha_n$ and $\beta_n$ are difficult to extract from experiments. In particular, their difference is largely unconstrained by data. $\alpha_n - \beta_n$ will play a role in Compton scattering on deuterium, and one might hope to measure it there. However, if a reliable extraction of this difference is to be made from $\gamma d$ data a formalism must be employed in which the contribution of two-body currents to the Compton cross section is under control.

The two-body contributions to $\gamma d$ scattering can be calculated in $\chi$PT at $O(e^2Q)$ in the manner described in Section 2. The resultant graphs are just the two-body analogues of the one-nucleon graphs which yield the successful $\chi$PT $O(e^2Q)$ prediction of $\alpha_p$ and $\beta_p$. By sandwiching these two-body graphs between the wave-functions of Ref. 5, and including the one-body $O(e^2Q)$ $\chi$PT $\gamma N$ contribution, we generate the parameter-free prediction of $\chi$PT at $O(e^2Q)$ for $\gamma d$ differential cross sections for the Compton scattering process. Our result should agree with data over a range of energies from about 50 MeV up to $m_\pi$ [15]. The comparison is made in Fig. 2 for photon energies of 69 and 95 MeV. There is good agreement with the (limited) data at 69 MeV, but our results disagree with the backward-angle data at 95 MeV. This disagreement also appears in potential-model calculations of
Figure 2. Compton scattering on the deuteron at 69 and 95 MeV in chiral perturbation theory at $O(e^2 Q)$ as compared to data from Illinois [19] and Saskatoon [20] respectively.

γd scattering, unless values of $\alpha_n - \beta_n$ very different from $\alpha_p - \beta_p$ are employed [22]. In fact, comparing such computations to our $\chi$PT approach to γd scattering illustrates the utility of $\chi$PT. The calculation of Ref. [22] is significantly more complicated, but gives similar results to our Ref. [18]. These complications arise because the potential-model approach does not exploit the simplifications offered by the long wavelength of the photons.

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REFERENCES

1. V. Bernard, N. Kaiser, and U.-G. Meißner, Int. Jour. of Mod. Phys. E 4, 193 (1995), and references therein.
2. S. Weinberg, Phys. Lett. B. 295, 114 (1992).
3. S. Weinberg, Phys. Lett. B. 251, 288 (1990); Nucl. Phys. B363, 3 (1991).
4. C. Ordonéz, L. Ray, and U. van Kolck, Phys. Rev. C 53, 2086 (1996).
5. E. Epelbaum, W. Glockle, and U.-G. Meißner, nucl-th/9910064 (1999).
6. M. C. M. Rentmeester, R. G. E. Timmermans, J. L. Friar, and J. J. de Swart, Phys. Rev. Lett. 82, 4992 (1999).
7. D. B. Kaplan, M. Savage, and M. B. Wise, Phys. Lett. B. 424, 390 (1998).
8. Nuclear physics with effective field theory. II. Proceedings, INT Workshop, Seattle,
USA, *February 25-26, 1999*, edited by P. F. Bedaque, M. Savage, R. Seki, and U. van Kolck (World Scientific, Singapore, 2000).

9. T.-S. Park, K. Kubodera, D.-P. Min, and M. Rho, Nucl. Phys. **A646**, 83 (1999).

10. D. R. Phillips and T. D. Cohen, Nucl. Phys. **A668**, 45 (2000).

11. V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).

12. S. R. Beane *et al.*, Nucl. Phys. **A618**, 381 (1997).

13. T.-S. Park, K. Kubodera, D.-P. Min, and M. Rho, Astrophys. J. **507**, 443 (1998).

14. U. van Kolck, Prog. Part. Nucl. Phys. **43**, 409 (1999).

15. H. Arenhovel, F. Ritz, and T. Wilbois, Phys. Rev. **C61**, 034002 (2000).

16. D. Abbott *et al.*, nucl-ex/0002003.

17. J.-W. Chen, G. Rupak, and M. Savage, Nucl. Phys. **A653**, 386 (1999).

18. S. R. Beane, M. Malheiro, D. R. Phillips, and U. van Kolck, Nucl. Phys. **A656**, 367 (1999).

19. M. Lucas, Ph. D. thesis, University of Illinois, unpublished (1994).

20. D. L. Hornidge *et al.*, Phys. Rev. Lett. **84**, 2334 (2000).

21. J. Tonnison, A. M. Sandorfi, S. Hoblit and A. M. Nathan, Phys. Rev. Lett. **80**, 4382 (1998).

22. See, for example, M. I. Levchuk and A. I. L’vov, nucl-th/9909066.