Cylinder vortex of superconductor in TaSe$_3$
topological ring crystals

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Abstract. Topology has many applications in modern condensed matter physics. We report
that superconducting properties are changed by topology. We have measured the magnetic
torque of the ring-shaped crystals of TaSe$_3$ by using piezoresistive cantilevers in order to
investigate the superconducting topological properties. We measured two ring samples. The
outer radius of sample A was 37.9 $\mu$m and that of sample B was 24.5 $\mu$m. We found that
the magnetic torque of the ring crystals changes periodically by increasing of the external
magnetic field. The periodicity of sample A was 1.85 Gauss and that of sample B was 4.75
Gauss, respectively. We found that the period is proportional to the circumference of the ring
crystal rather than the area enclosed by the outer circumference. In that case, it is natural
that vortices in the ring crystal were placed along the circumference. From these results, we
suggest that vortices exist as cylinder vortices in the rings at the results of topological effects
of superconductive ring-shaped crystals, and this matter give a new experimental evidence of
topological effect in superconductor.

1. Introduction
What kind of effects on the superconducting state, such as a mixed state of type II
superconductors, are caused by topology? At the mixed state vortices penetrate the
superconductor when the applied magnetic field increases beyond $H_{c1}$. Typically, Abrikosov
lattice is the most stable configuration of vortices with repulsive force interaction to each other.
However, when the system size is small, shape of superconductors, such as boundary condition,
affects penetration and arrangement of vortices [1]. Similarly, topology of systems has a potential
of affecting the superconducting properties. For example, closed systems like ring crystals [2],
vortices may not relocate to become stable like bulk superconductors, because they are trapped
globally in the hole of the crystals. Furthermore, systems like Möbius ring crystals [3], there is
the possibility that vortices cannot penetrate crystals. These speculations suggest that topology
of systems plays an important role in physical properties.

The purpose of this research is to investigate the effects of topology on superconducting
properties. The ring-shaped crystals of quasi-one-dimensional superconductor TaSe$_3$ are suitable
to investigate these topological effects, because it is expected that TaSe$_3$ causes topological effects
for its filamentary structure. Besides that, a measurement without electrode is appropriate
to investigate those effects, because this method conserves topology of systems. Therefore,
we adopted magnetic torque measurement without electrode by using micro cantilevers [4].
As a result, we found a phenomenon that the magnetic torque of the ring crystals oscillates
periodically below the transition temperature and it suggests that vortices exist as cylinder vortices in the rings by the topological effects of superconductive ring-shaped crystals.

2. Experimental

2.1. Samples

The TaSe$_3$ ring crystals have been synthesized by the Chemical Vapor Transportation (CVT) method [3, 5]. The starting material was a mixture of Ta and Se with about 5% excess Se as a transport agent. Ta and quartz ampoules were baked at around 900 $^\circ$C. After baking, the mixture was sealed in a quartz ampoule and air was evacuated from the ampoule to the order of $10^{-6}$ Torr. Next, the ampoule was placed in a furnace. The temperature of the starting-material at the end of the ampoule was raised to the reaction temperature, typically 700 $^\circ$C. The mixture was soaked at this temperature for a few hours. Using this method, thousands of ring crystals were synthesized per ampoule. For measurement, the crystal was fixed at the extremity of the cantilever by adhesive.

2.2. Magnetic Torque Measurement

In this study, we measured the magnetic torque in order to investigate the effect of topology on vortex state by using microcantilevers. This measurement is suitable for investigations of the effect of topology because we need no-electrode in this method. The magnetic torque $\tau = M \times B$ produced on superconducting samples by an applied field $B$. In a homogeneous applied field a transverse component of the sample magnetization will cause a deflection of the cantilever, which is proportional to the magnetic torque $\tau$. Here, the sample mounted at the extremity of the lever as in Figure 1, and the deflection $\Delta z$ produced by the torque $\tau$ is measured here by the change in the piezoresistance $\Delta R$.

In this study, in order to detect a small change in the resistance, we used a Wheatstone bridge circuit measured by a lock-in amplifier. We measured the magnetic torque by the magnetic field sweeping at a lower and a higher temperature than $T_c$. In case of Sample A, the angle between the applied field and the ring crystal surface was 90$^\circ$. This sample was measured in liquid $^3$He directly. In case of Sample B, the angle between the applied field and the ring crystal surface was 45$^\circ$. This sample was measured in a vacuum.

![Figure 1](image-url)  

**Figure 1.** (a) Scanning electron microscope (SEM) image of TaSe$_3$ ring crystal (Sample A) on the cantilever. (b) High magnification image of the ring crystal and the cantilever.
Figure 2. (a) Magnetic torque of Sample A as a function of the applied magnetic field and schematic drawing of the sample configuration. Arrows show the peak positions. Solid lines represent the magnetic torque measured at $T = 0.56$ K. Dotted lines represent the torque measured at $T = 4.3$ K. (b) Those of Sample B. Solid lines represent the torque measured at $T = 0.38$ K. Dotted lines represent the torque measured at $T = 10.9$ K. Magnetic torques below $T_c = 2.0$ K of both Sample A and Sample B are changed periodically.

3. Results and Discussion

Results of the magnetic sweep measurement at each temperature are shown in Figure 2. Figure 2(a) is the result of Sample A, and Figure 2(b) is that of Sample B. Horizontal axis shows applied magnetic field, and vertical axis shows magnetic torques translated from measured voltages. In vertical axis, we chose the point that the magnetic torque oscillation became constant as the origin considering the effect of field cooling by the terrestrial magnetism. Both Sample A and Sample B, above $T_c$, magnetic torques were approximately constant but below $T_c$, magnetic torques oscillate as the applied magnetic field increased, then that became constant at the high magnetic field. Moreover, we discover that this oscillation was periodic.

3.1. Magnetic Periodicity

Here, we discuss the magnetic periodicity appeared in the results of measurement for TaSe₃ ring crystals. Peak positions were proportional to the applied magnetic field, therefore the magnetic torques change periodically. The average of the period of Sample A was 1.85 Gauss and that of Sample B was 4.75 Gauss. Here, characteristic values of two samples are shown in Table 1.

The magnetic periods suggest that a number of the vortices penetrate simultaneously. Then

| Area inner ($\mu$m²) | Radius outer ($\mu$m) | Width ($\mu$m) | Magnetic period (Gauss) | Number of vortices | Penetration depth ($\mu$m) |
|---------------------|----------------------|---------------|-------------------------|-------------------|--------------------------|
| Sample A 4162        | 37.9 36.4 1.5        | 1.85          | 372                     | 1.88              |
| Sample B 897         | 24.5 16.9 7.6       | 4.75          | 206                     | 1.18              |
we estimate number of vortices from $\phi/\phi_0$ and $\phi = \Delta B \cdot S$, where $\phi_0$ is the flux quantum, $\Delta B$ is average period, $S$ is the area enclosed by the inner circumference. The number of vortices of Sample A is $\phi/\phi_0 = 372$, and that of Sample B is $\phi/\phi_0 = 206$, respectively. Therefore, several hundreds of vortices penetrate simultaneously in magnetic field rather than each vortex penetrates one by one.

Next, we discuss how do these vortices place in the crystal. Hereafter we discuss with Sample A. First, we confirm Abrikosov lattice that appears on bulk type II superconductors. The area where a vortex occupies is estimated at $S = 1.19 \times 10^{-11} \text{m}^2$ when the applied magnetic field is 1.85 Gauss. Hence, if vortices placed Abrikosov lattice, the distance between two vortices is estimated at $a = 5.24 \mu\text{m}$. Since this distance $a$ is longer than width of Sample A ($w = 1.5 \mu\text{m}$), the vortices are impossible to place as Abrikosov lattice in the ring.

Then, we notice the relations between the period and the length of the crystal. We consider that vortices penetrate every period and assume that this period correspond to $H_{c1}$. Then each penetration depth $\lambda = \sqrt{\phi_0 / \pi H_{c1}}$ of Sample A is estimated at $\lambda_A = 1.88 \mu\text{m}$ and that of Sample B is estimated at $\lambda_B = 1.18 \mu\text{m}$, respectively.

From these results, we found that the ratio of $\lambda_A$ to $\lambda_B$ approximates the ratio of the ring circumference from center to outer. Furthermore, each ring circumference divided by each $\lambda$ is order of $\phi/\phi_0$. Accordingly, the magnetic period appeared in measurement results conceivably depend on ring circumferences. Therefore, we suppose that vortices of the ring crystal penetrate the area decided from the ring circumference and place in line.

In order to explain the large periodicity of the magnetic torque, we propose the cylinder vortex model. When the applied magnetic field to the ring crystal increases, vortices pass through the crystal and stay at the hole of the ring crystal by the effect of ring shape topology. When the applied field increase larger as inner magnetic field conform to the applied magnetic field, vortices penetrate the ring crystal because they cannot pass through the crystal by magnetic pressure from inner magnetic field. From above discussions, since numbers of vortices penetrate, it is expected that vortices equally penetrate whole of the ring crystal and place not Abrikosov lattice but in line. Moreover, since vortices extend along the circle direction due to strong anisotropy of crystal structure of TaSe$_3$ [6], the vortices placed in line are likely to connect each other. Hence, it is natural that vortices in the ring crystal form a cylinder vortex. The cylinder vortex model is consistent with the large periodicity of the magnetic torque.

4. Summary
In this research, we measured the magnetic torque by using piezo-electric micro cantilevers. As a result, we found that the magnetic torque changes periodically as the applied magnetic field increase in ring crystals and the period depends on the ring circumference length. These results indicate that vortices penetrate ring crystals as a cylinder vortex. The cylinder vortex is emerged by effect of the closed ring topology of the ring crystals of TaSe$_3$.

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