Non-local pure spin current injection via quantum pumping and crossed Andreev reflection

Colin Benjamin and Roberta Citro

Dipartimento di Fisica "E. R. Caianiello", Universita degli Studi di Salerno, and Laboratorio Regionale SuperMat, I.N.F.M.
Via S. Allende, I-84081 Baronissi (SA), Italy

(Dated: June 25, 2018)

A pure spin current injector is proposed based on adiabatic pumping and crossed normal/Andreev reflection. The device consists of a three-terminal ferromagnet-superconductor-semiconductor system in which the injection of a pure spin current is into the semiconductor which is coupled to the superconductor within a coherence length away from the ferromagnet enabling the phenomena of crossed normal /Andreev reflection to operate. Quantum pumping is induced by adiabatically modulating two independent parameters of the ferromagnetic lead, namely the magnetization strength and the strength of coupling between the ferromagnet and the superconductor. The competition between the normal/Andreev reflection and the crossed normal/Andreev reflection, both induced by pumping, leads to non-local injection of a pure spin current into the semiconductor. The experimental realization of the proposed device is also discussed.

PACS numbers: 73.23.Ra, 5.60.Gg, 72.10.Bg

Keywords: quantum pumping, spin current injector, Andreev reflection

I. INTRODUCTION

In the past few years there has been a lot of interest in the field of spintronics, which aims at creating devices based on the spins of electrons. Conventional electronics deals with charge transport and it is based on number of charges and their energy but the performance of charge based conventional electronics is limited in speed and dissipation. On the contrary spintronics is based on direction of spin and spin coupling and it is capable of much higher speed at very low power. Spin transport in addition is much more resilient to impurities than charge transport since spins won't flip until and unless there are magnetic impurities. Recent studies have shown that spin coherence persists for hundreds of nanoseconds over hundreds of microns and further spin transport is largely insensitive to temperature. Further spin based electronics promise greater integration between the logic and storage devices and the generation, manipulation and detection of spin currents have been the object of intense theoretical research in recent years. Many spintronics devices, such as the spin valve and magnetic tunnelling junctions, are associated with the flow of spin polarized charge currents. Spin polarized currents coexist with charge currents and are generated when an imbalance between spin up and spin down carriers is created, for example, by using magnetic materials or applying a strong magnetic field or by exploiting spin-orbit coupling in semiconductors. More recently, there has been an increasing interest in the generation of pure spin current without an accompanying charge current. The generation of a pure spin-current is only possible if all spin-up electrons flow in one direction and equal amount of spin-down electrons flow in the opposite direction. In this case the net charge current $I_{\text{charge}} = I_{+1} + I_{-1}$ vanishes while a finite spin current $I_{\text{spin}} = I_{+1} - I_{-1}$ exists, because $I_{+1} = -I_{-1}$, where $I_{+1}$ or $I_{-1}$ are the electron current with spin up or spin down.

One of the main problems with spintronics is the difficulty in generating and transporting a spin current, i.e., spin injection into a semiconductor. The ohmic injection from ferromagnet has low efficiency because of the conductivity mismatch and almost all of the spin polarization is lost at the interface. Therefore pure spin currents generation can be the most efficient tool for spin injection. One of the ways of generating pure spin currents is through the use of quantum pumping. In fact an experimental realization of a quantum pumping procedure has already shown to work and generate a pure spin current. In our work a pure spin current injection method is presented based on the principles of quantum pumping. We propose a novel device made of a three terminal hybrid structure, in which a ferromagnet and a semiconductor kept at a small distance apart (less than the superconducting coherence length), are contacted with barriers of strength $V_1$ and $V_2$ to a superconductor (see Fig.1). The quantum pumping mechanism is incorporated by adiabatic variation of the magnetization strength in the ferromagnetic lead and the strength of the contact barrier. It should be noted that there is no voltage bias applied to either the ferromagnet, the semiconductor or the superconductor, all being kept at the same chemical potential. Adiabatic modulation of two independent parameters is the only mechanism by which a pumped current is generated locally in the ferromagnetic lead and more importantly nonlocally in the semiconducting lead. As the main result of our study we find that at a particular value of the magnetization strength and the contact barrier strength a pure spin current can be generated in the semiconducting lead. This effect makes possible the use of our device as a pure...
spin current injector and offers a possible solution to spin injection problems. The distinguishing characteristic of our proposal is non-locality, while previous proposals generated pure spin currents in the semiconductor locally our work creates the same non-locally.

The two phenomena on which the operation of the proposed device relies, are adiabatic quantum pumping and crossed normal/Andreev reflection. Adiabatic quantum pumping is a mean of transferring charge and/or spin carriers without applying any voltage bias by the cyclic variation of two device control parameters. The theory of adiabatic quantum pumping was put forth by P. W. Brouwer\textsuperscript{19}. In 1999, an adiabatic quantum electron pump was reported in an open quantum dot where the pumping signal was produced in response to the cyclic deformation of the confining potential\textsuperscript{15}. The variation of the dot’s shape squeezes electronic wavefunction in or out of the dot thus ‘pumping’ electrons from one reservoir to another\textsuperscript{20,21}. The AC voltages applied to the quantum dot in order to change the shape result in a DC current when the reservoirs are in equilibrium. This non-zero current is only produced if there are at least two time varying parameters in the system as a single parameter quantum pump does not transfer any charge. Later on the study of adiabatic pumping phenomenon has been extended to adiabatic spin pumping both in experiment as well as theoretical works\textsuperscript{16,17}. In the experiment on quantum spin pumping one generates a pure spin current via a quantum dot by applying an in-plane magnetic field which is adiabatically modulated to facilitate a net transfer of spin. In addition to investigations of pumping in quantum dots, theoretical ideas have been put forward for spin pumping in quantum wires\textsuperscript{22,23,24}, spin-chains\textsuperscript{25}, semiconductor heterostructures\textsuperscript{12}, magnetic barriers\textsuperscript{26}, spin-turnstile\textsuperscript{27}, in presence of a superconducting lead\textsuperscript{28,29,30}, incoherent spin pumping\textsuperscript{31}, and in carbon-nanotubes\textsuperscript{32}.

Crossed Andreev reflection\textsuperscript{33}, on the other hand, refers to the phenomenon when a spin up electron incident at the ferromagnet-superconductor interface with energy below the superconducting gap is not reflected as a spin down hole in the same ferromagnetic lead (Andreev reflection) but is reflected in the other lead which may be ferromagnetic/normal/semiconducting. For this to happen, the ferromagnetic lead must be placed at a distance less than coherence length of the superconductor from the ferromagnetic/normal/semiconducting lead, as in Fig.1. The phenomenon of crossed Andreev reflection can of course be maximally enhanced when both leads are ferromagnetic with opposite spin polarizations as shown in Ref.\textsuperscript{33} where the distance between the ferromagnetic leads (which are half-metals with opposite spin orientations) is neglected, implying an effective one-dimensional model. In our work we do not take into account the separation between the leads (the one-dimensional model) and further the leads are not half-metals, one is a ferromagnet while the other is a semiconductor.

The phenomena of crossed Andreev reflection has been explored in a large variety of systems, for details see Ref.\textsuperscript{33}. A related phenomena which can occur but only in the presence of a voltage bias is known as electron cotunneling\textsuperscript{35}. In this phenomena an electron can tunnel into the superconductor from the ferromagnet and then again tunnel out into the other lead placed at a distance less than the coherence length of the superconductor. Of course this effect will be maximally enhanced when both leads are ferromagnetic with identical spin polarizations. In the adiabatic pumping regime the probability of electron cotunneling is almost nil. It must be emphasized that in a recent work\textsuperscript{36} a spin injector was proposed based on crossed Andreev reflection and electron cotunneling. In
Ref.\textsuperscript{34}, competition between these two processes leads to a pure spin current injection into the semiconductor. In our proposed device, electron cotunneling ceases to operate. The only means of transport is through quantum pumping, and because of the competition between normal/Andreev reflection and the crossed normal/Andreev reflection at the semiconductor/superconductor interface a pure spin current injection into the semiconducting lead can be obtained.

The organization of the paper is as follows: In Sec. II we derive the pumped current in our device by a scattering matrix approach. In particular, we derive analytical expressions for the charge and spin currents in the lowest order of the contact barrier strengths. In Sec. III we present our results for the weak pumping regime as well as the strong pumping regime. In particular we will show that a pure spin current injection in the semiconductor can be obtained at particular values of the pumping parameters. Finally in Sec. IV we discuss the possible experimental realization of the proposed device and give the conclusions in Sec. V.

II. THEORY

In the device shown in Fig. 1, we consider the pumping of charge/spin carriers by adiabatic modulation of the magnetization strength ($h_{ex} = h_0 + h_p \sin(wt)$) in the ferromagnet and of the contact barrier strength ($V_1 = V_0 + V_p \sin(wt + \phi)$) at the ferromagnet-superconductor interface. To study the current pumped in such a system we apply the scattering matrix approach\textsuperscript{3,35-38}. To calculate the scattering amplitudes we start by writing the wave function for an electron with spin $\sigma$ incident at the ferromagnet-superconductor interface which is given by:

$$
\Psi_F(x = 0) + \Psi_{Sm}(x = 0) = \Psi_{Sc}(x = 0),
$$

$$
\frac{d\Psi_F}{dx}(x = 0) - \frac{d\Psi_{Sc}}{dx}(x = 0) = \frac{2mV_1}{\hbar^2} \Psi_{Sc}(x = 0),
$$

and,

$$
\frac{d\Psi_{Sm}}{dx}(x = 0) - \frac{d\Psi_{Sc}}{dx}(x = 0) = \frac{2mV_2}{\hbar^2} \Psi_{Sc}(x = 0).
$$

Solving the system of equations arising from the above boundary conditions, all the scattering amplitudes are obtained. Similarly, for the injection of a hole from the ferromagnetic lead, one can determine the following scattering amplitudes $S^{he}_{p,F}$ (for Andreev reflection), $S^{he}_{p,F}$ (for normal reflection), $S^{he}_{p,SmF}$ (for crossed Andreev reflection) and $S^{he}_{p,SmF}$ (for crossed normal reflection) in the semiconductor. The explicit expression for the scattering amplitudes of the crossed/normal Andreev reflection can be found in Ref.\textsuperscript{40}. In the same manner one can derive the scattering amplitudes for electron or hole injection in the semiconducting lead, hence $\epsilon = 0$. 

The exchange field in the ferromagnet, $\Delta$ being the superconducting gap and $E_F$ is the Fermi energy. In Eq. (1), $S^{he}_{p,F}$ is the amplitude for Andreev reflection, $S^{he}_{p,SmF}$ the amplitude for crossed Andreev reflection, $S^{ee}_{p,F}$ the amplitude for normal reflection, and finally $S^{ee}_{p,SmF}$ the amplitude for crossed normal reflection. $u$ and $v$ are the superconducting coherence factors. In similar fashion one can write the wavefunction for an electron injected with spin $\sigma$ in the semiconductor. In the Andreev approximation\textsuperscript{33}, we take $k^+ = k^- = k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$. 

The scattering amplitudes for electron/hole with spin $\sigma$ injected from either leads are calculated by applying the boundary conditions that are set up by matching the wave-functions at the interface and from current conservation at ferromagnet-superconductor and semiconductor-superconductor interfaces:

$$
\Psi_F(x = 0) + \Psi_{Sm}(x = 0) = \Psi_{Sc}(x = 0),
$$

$$
\frac{d\Psi_F}{dx}(x = 0) - \frac{d\Psi_{Sc}}{dx}(x = 0) = \frac{2mV_1}{\hbar^2} \Psi_{Sc}(x = 0),
$$

and,

$$
\frac{d\Psi_{Sm}}{dx}(x = 0) - \frac{d\Psi_{Sc}}{dx}(x = 0) = \frac{2mV_2}{\hbar^2} \Psi_{Sc}(x = 0).
$$
Due to the cyclic variation of external parameters $X_1$ and $X_2$, the adiabatic pumped electron current with arbitrary spin $\sigma$ in the semiconductor is given by:

\[
I_{\sigma,Sm}^e = \frac{wq_e}{\pi} \int_0^\tau \frac{dN_{\sigma,Sm}^e}{dX_1} \frac{dX_1}{dt} + \frac{dN_{\sigma,Sm}^e}{dX_2} \frac{dX_2}{dt}
\]

wherein, $\tau = 2\pi/w$ is the cyclic period, $w$ is the pumping frequency and $q_e$ represents the electronic charge. $\frac{dN_{\sigma,Sm}^e}{dX_j}$ is the electronic injectivity in the semiconducting lead given at zero temperature by

\[
\frac{dN_{\sigma,Sm}^e}{dX_j} = \frac{1}{2\pi} \sum_{\beta=F,Sm} \Im[S_{Sm,\sigma}^{ee,\sigma,\beta} \partial X_j S_{Sm,\sigma}^{ee,\sigma,\beta} + S_{Sm,\sigma}^{eh,\sigma,\beta} \partial X_j S_{Sm,\sigma}^{eh,\sigma,\beta}], \text{ with } j=1,2
\]

where, $X_1 = V_1$, and $X_2 = h_{cx}$. $\Im$ denotes the imaginary part of the quantity in parenthesis. The first term is the injectivity of the electron due to the variation of the modulated parameter, i.e. the partial density of states (DOS) for an electron coming from either the ferromagnet or semiconductor and exiting the system as an electron in the semiconductor, and the second term is the injectivity of a hole, i.e., the DOS for a hole coming from the semiconductor or ferromagnet and exiting the system as an electron in the semiconductor. Similarly, one can calculate the pumped hole current in the semiconducting lead and it is given by the expression, with $q_e$ replaced by the hole charge $q_h$ and with $e$ replaced by $h$ in Eq.4, as below:

\[
I_{\sigma,Sm}^h = \frac{wq_h}{\pi} \int_0^\tau \frac{dN_{\sigma,Sm}^h}{dX_1} \frac{dX_1}{dt} + \frac{dN_{\sigma,Sm}^h}{dX_2} \frac{dX_2}{dt}
\]

$\frac{dN_{\sigma,Sm}^h}{dX_j}$ is the hole injectivity in the semiconducting lead given at zero temperature by

\[
\frac{dN_{\sigma,Sm}^h}{dX_j} = \frac{1}{2\pi} \sum_{\beta=F,Sm} \Im[S_{Sm,\sigma}^{eh,\beta,\sigma} \partial X_j S_{Sm,\sigma}^{eh,\beta,\sigma} + S_{Sm,\sigma}^{ee,\beta,\sigma} \partial X_j S_{Sm,\sigma}^{ee,\beta,\sigma}], \text{ with } j=1,2
\]

In the weak pumping regime i.e., $h_p \ll h_0$ and $V_p \ll V_0$, the adiabatically pumped electron current into the semiconducting lead can be written as below with $z_i = \frac{2mV_i}{k_Fh^2} (i = 0, 1, 2, p)$, $h' = h_p/E_F$ and $h = h_0/E_F$:

\[
I_{\sigma,Sm}^e = \frac{wq_e}{\pi} \sin(\phi)z_p h' \sum_{\beta=F,Sm} \Im[\partial_{\beta,0} S_{Sm,\sigma,\beta,\sigma} \partial h S_{Sm,\sigma,\beta,\sigma} + \partial_{\beta,0} S_{Sm,\sigma,\beta,\sigma} \partial h S_{Sm,\sigma,\beta,\sigma}].
\]

Similarly, the adiabatically pumped hole current into the semiconductor lead can be written as:

\[
I_{\sigma,Sm}^h = \frac{wq_h}{\pi} \sin(\phi)z_p h' \sum_{\beta=F,Sm} \Im[\partial_{\beta,0} S_{Sm,\sigma,\beta,\sigma} \partial h S_{Sm,\sigma,\beta,\sigma} + \partial_{\beta,0} S_{Sm,\sigma,\beta,\sigma} \partial h S_{Sm,\sigma,\beta,\sigma}].
\]

In the following, $q_e = -q_h$. Further, since for s-wave superconductor the scattering amplitudes satisfy the condition $(S_{Sm,\sigma}^{eh,\beta,\sigma})^* = S_{Sm,\sigma}^{ee,\beta,\sigma} (S_{Sm,\sigma}^{eh,\beta,\sigma})^* = S_{Sm,\sigma}^{ee,\beta,\sigma}$, and $(S_{Sm,\sigma}^{eh,\beta,\sigma})^* = -S_{Sm,\sigma}^{ee,\beta,\sigma}$ (where * denotes complex conjugation of the scattering amplitude), the pumped hole and electron current are exactly the same, and because of this we only derive the expressions for the pumped electron current with arbitrary spin index understanding that it is exactly equal to the pumped hole current similar. To the above expressions one can write the pumped electron/hole current with arbitrary spin index into the ferromagnetic lead. Since our focus is the use of this device as a spin injector, we confine ourselves to the semiconducting lead only.

From the reflection amplitudes (crossed Andreev/normal) and in the weak pumping regime (see Eq. 7), one can derive the pumped electron current, charge and spin currents in the semiconducting lead up-to first order in contact barrier strengths $z_0 = \frac{2mV_0}{k_Fh^2}$ and $z_2 = \frac{2mV_2}{k_Fh^2}$ (assuming $z_0$ and $z_2$ are small). The explicit expression for the current is

\[
I_{\sigma} = I_{\sigma,Sm}^e/I_0 = \frac{1}{1 - h^2} \left[ A + \sigma B\sigma + C\sigma \right]/D^2;
\]
where

\[ I_0 = \frac{(w_q \sin(\phi) z_q h')}{\pi}, \]
\[ D = -7 - 2(\sqrt{1 - \sigma h} + \sqrt{1 + \sigma h})(1 + 2\sqrt{1 - h^2}) + 6\sqrt{1 - h^2} - 4h^2 \]
\[ A = 16h(1 - h^2) + 14\sqrt{1 - h^2}, \]
\[ B_\sigma = \sqrt{1 - h^2}(2 + 4h^2) - 14(1 - h^2) - \sqrt{1 + \sigma h}(10 + 12h^2)\sqrt{1 - \sigma h}(-4 + 8h^2) + \sqrt{1 + \sigma h}\sqrt{1 - h^2}(5 - 36h^2), \]
\[ C_\sigma = 36h^3 - 32h + 1 \sigma h(-36h^3 + 58h) - \sqrt{1 - \sigma h}\sqrt{1 - h^2}(46h) + \sqrt{1 + \sigma h}\sqrt{1 - h^2}(60h). \]

One should notice that up to first order there are no terms involving \( z_0 \) and \( z_2 \) as the first terms involving \( z_0 \) or \( z_2 \) that appear in the expansion are of order \( O(z_i^2), i = 0, 2 \). From Eqs. (9-10) the charge and spin current pumped into the semiconducting lead are derived as below:

\[ I_{\text{spin}} = I_{+1} - I_{-1} = \frac{1}{1 - h^2}[-28(1 - h^2) + X(-4h^2 - 14 + Y(1 - 84h^2)) - W(26h - 106hY)]/D^2, \]
\[ I_{\text{charge}} = I_{+1} + I_{-1} = \frac{1}{1 - h^2}[28hY + 32h(1 - h^2) - W(6 + 20h^2 + (9 + 12h^2)Y) - X(26h + 14hY)]/D^2. \]

where, \( X = \sqrt{1 + h} + \sqrt{1 - h}, Y = \sqrt{1 - h^2}, W = \sqrt{1 + h} - \sqrt{1 - h} \) and \( D \) as in Eq. (10). At the value of the exchange field \( h = 0 \), the charge current is zero while the spin current is finite. Let us note that since we considered a one-dimensional model (see also Ref. 33) the distance \( l \) between the ferromagnet and the semiconductor does not appear explicitly in the result for the pumped current, further the width of the ferromagnetic and semiconducting leads is not taken into account. In Ref. 34 both these characteristics have been incorporated and it has been explicitly shown that crossed Andreev reflection ceases to be effective in the limit when the separation between the leads approaches the superconducting coherence length. Although the considered one-dimensional transport model we have considered is simplistic, it captures the main physics of crossed Andreev reflection.

### III. CHARACTERISTICS OF THE PUMPED CHARGE AND SPIN CURRENTS

The results for the pumped charge and spin currents both in the weak as well as strong pumping regimes are plotted in the Figs. 2-6. In all the figures the parameters are in their dimensionless form. The weak pumping regime can be described through Eqs. (9-12) and it is shown in Figures 2(a), 3(a) and 4(a), as well as through Eq. (4) as shown in Figure 6(a). In Figures 2(a), 3(a) and 4(a) the currents are scaled by a factor \( I_0 \), while in Figures 2(b), 3(b), 4(b) and in Figures 5 and 6, they are in units of \( w_q, \pi \) and are multiplied by \(-100\) for better visibility.

In Fig. 2(a), we plot the exact results in the regime where contact barrier strengths are neglected [see Eqs. 9-12]. The figure shows that at the value of the normalized magnetization strength \( h = 0.0 \), we have zero charge current and a finite spin current. The arrows denote the points where the charge current is zero. In the inset of Fig. 2(a) the pumped spin and charge currents are plotted by taking the contact barrier strength at the ferromagnet-superconductor junction while neglecting the contact barrier strength at the semiconductor-superconductor boundary. The pumped charge current is zero for three distinct values of the magnetization strength. In Fig. 2(b), the results for the strong pumping regime are reported. As before the contact barrier strengths are neglected in the main figure, while the results in presence of the contact barrier at the ferromagnet-superconductor junction are shown in the inset. Not much difference between the weak and strong pumping regimes is observed: in both cases at particular values of the magnetization strength a pure spin current is seen. In Fig. 3(a) we plot the pumped currents along with the pumped spin and charge currents as function of the normalized contact barrier strength \( z_0 \) at the ferromagnet-superconductor interface. The inset shows the currents when the contact barrier strength at the semiconductor-superconductor junction is taken into account. Herein, at particular values of \( z_0 \) a pure spin current in the semiconductor is observed, while in the inset there is no pure spin current. In Fig. 3(b), the currents for the strong pumping case are plotted. For this case whether or not the contact barrier strength is taken into account, there is a pure spin current. In Fig. 4(a), we plot the pumped currents along with the pumped spin and charge currents as function of the normalized contact barrier strength \( z_2 \) at the semiconductor-superconductor interface. The inset shows the currents when the contact barrier strength at the ferromagnet-superconductor junction is neglected. Herein, at a particular value of \( z_2 \) a pure spin current in the semiconductor is observed, but in the inset no such value occurs. In Fig. 4(b), the currents for the strong pumping case are plotted. In both Fig. 4(b) as well as its inset a pure spin current occurs.
FIG. 2: (Color online) The pumped charge and spin currents into the semiconducting lead as function of the dimensionless magnetization strength \( h = h_0 / E_F \) in the ferromagnet. (a) The weak pumping regime. Parameters are \( z_0 = z_2 = 0 \). In the inset parameters are \( z_0 = 2, z_2 = 0 \). (b) The strong pumping regime. Parameters are \( z_0 = z_2 = 0, z_p = 4.0, h' = 0.5 \) and \( \phi = \pi/2 \). In the inset parameters are \( z_1 = 2, z_2 = 0, z_p = 4.0, h' = 0.5 \) and \( \phi = \pi/2 \). Herein \( h' = h_p / E_F \) and \( z_i = 2mV_i / \bar{\hbar}^2 k_F \) with \( i = 0, 1, 2, p \). The arrows indicate the specific places wherein pure spin current flows in the semiconductor. \( I_\pm \) denotes the spin up and down currents.

FIG. 3: (Color online) The pumped charge and spin currents into the semiconducting lead as function of the contact barrier strength \( z_0 \). (a) The weak pumping regime. Parameters are \( z_2 = 0.0, h = 0.5 \). In the inset parameters are \( z_2 = 2.0, h = 0.5 \). (b) The strong pumping regime. Parameters are \( z_2 = 2.0, z_p = 4.0, h' = 0.45, h = 0.1 \) and \( \phi = \pi/2 \). In the inset parameters are \( z_2 = 0.0, z_p = 4.0, h' = 0.45, h = 0.1 \) and \( \phi = \pi/2 \). The arrows indicate the specific places wherein pure spin current flows in the semiconductor.
FIG. 4: (Color online) The pumped charge and spin currents into the semiconducting lead as function of the contact barrier strength $z_2$. (a) The weak pumping regime. Parameters are $z_0 = 2.0$, $h = 0.5$. In the inset parameters are $z_0 = 0.0$, $h = 0.5$. (b) The strong pumping regime. Parameters are $z_0 = 2.0$, $z_p = 4.0$, $h' = 0.45$, $h = 0.1$ and $\phi = \pi/2$. In the inset parameters are $z_0 = 0.0$, $h = 0.1$, $z_p = 4.0$, $h' = 0.45$ and $\phi = \pi/2$. The arrows indicate the specific places wherein pure spin current flows in the semiconductor.

FIG. 5: (Color online) The pumped currents as function of the strength of pumping parameters. (a) The pumped currents as function of the magnetization amplitude. Parameters are $z_1 = 0.0$, $z_2 = 2.0$, $z_p = 4.0$, $h = 0.1$ and $\phi = \pi/2$. In the inset parameters are $z_1 = 2.0$, $z_2 = 0.0$, $z_p = 4.0$, $h = 0.1$ and $\phi = \pi/2$. (b) The pumped currents as function of the amplitude of pumped contact barrier strength at the ferromagnet-superconductor junction. Parameters are $z_0 = 0.0$, $z_2 = 2.0$, $h_p = 0.8$, $h = 0.1$ and $\phi = \pi/2$. In the inset parameters are $z_0 = 0.0$, $z_2 = 0.0$, $h_p = 0.8$, $h = 0.1$ and $\phi = \pi/2$. The arrows indicate the specific places in the parameter regime wherein pure spin current flows in the semiconductor.
FIG. 6: (Color online) The pumped currents as function of the phase difference. (a) for weak pumping, the parameters are \( h = 0.8, z_1 = 4.0, z_2 = 0.0, h' = 0.1 \) and \( z_p = 0.4 \). Notice that the down spin current is almost zero and the pumped currents are almost sinusoidal. In the inset, parameters are \( h = 0.8, z_1 = 4.0, z_2 = 2.0, h' = 0.1 \) and \( z_p = 0.4 \). For very strong pumping, the parameters are \( h = 0.1, z_0 = 4.0, z_2 = 0.0, h' = 0.8 \) and \( z_p = 4.0 \). In the inset the parameters are \( h = 0.1, z_0 = 4.0, z_2 = 2.0, h' = 0.8 \) and \( z_p = 0.4 \). Notice that the sinusoidal dependence of the pumped currents is lost.

The dependence of the pumped currents on the strength of modulated parameter is another crucial indicator of the regime parameters in which the pump operates as a pure spin current injector. With this in mind in Fig. 5, we plot the pumped currents and the spin and charge currents as function of the strength of the modulated parameter, in (a) the magnetization and in (b) the contact barrier strength. A pure spin current appears at particular values of the modulated magnetization in Fig. 5(a) and at particular values of the modulated contact barrier strength in Fig. 5(b).

In Fig. 6, we plot the pumped spin and charge currents as a function of the phase difference \( \phi \). In Fig. 6(a), the weak pumping regime is shown. In this regime the pumped currents are almost sinusoidal. For the values of the parameters considered in the figure, the down spin current is almost zero and an almost pure up spin current is obtained. In Fig. 6(b), the strong pumping regime is shown. As expected, the sinusoidal dependence on the phase is lost. Furthermore in the inset of Fig. 6(b) we see that for the parameters taken into account one has pure spin current at \( \phi = \pi/2 \) and \( 3\pi/2 \). This is quite an important result since in this case both the contact barrier strengths are not neglected. A question that could arise is related to the magnitude of the pumped spin current. From the previous results one clearly sees that the pumped currents are noticeably larger in the strong pumping regime. The device thus should be ideally suitable for use in the strong pumping regime. For the parameters considered in Figure 6, taking the frequency \( w \) around 100 MHz as in Ref. [17], the order of magnitude estimate of the pumped current is around \( 10^{-11} \) Amperes, a value detectable in present day experiments. To summarize the results from all the figures and to make sense of all the parameters used, we in Table I list the physical parameters, their values (range as used in the figures), their physical meaning and materials wherein these can be realized.

| Parameter          | Symbol | Figure | Values | Physical meaning           | materials                  |
|--------------------|--------|--------|--------|-----------------------------|----------------------------|
| Contact barrier strength | \( z_i \) | 3 and 4 | 0      | ballistic contact           | sharvin contacts           |
| Contact barrier strength | \( z_i \) | 3 and 4 | \( \neq 0 \) | non reflectionless contacts | tunnel contacts            |
| Magnetization       | \( h \) | 2      | \( \sim E_F/2 \) | 50% polarized               | NiFe, Ni, Co               |
| Superconducting coherence length | \( \xi \) | 1      | \( \xi \gg l \) | crossed andreev reflection | s-wave superconductors     |

TABLE I: A comparative analysis of parameters, values and materials

In the table, above a sharvin contact is defined when strength of contact barrier is equal to zero. These type of
contacts can be realized when a point contact has a size $d$ smaller than mean free path $l$. This implies completely ballistic transport through the contact. In this case an electron is either Andreev reflected in the ferromagnet or cross Andreev reflected into the semiconductor. For non-zero contact barrier strength, the contacts are defined as tunnel contacts or non reflectionless contacts, and in this case in addition to Andreev reflection in ferromagnet and cross Andreev reflection in semiconductor there can be normal reflection in ferromagnet and crossed normal reflection in semiconductor. The finite magnetization $h$ in the proposed device can be obtained by using any type of ferromagnetic material e.g., NiFe, Cobalt or Nickel. Lastly the superconductor would ideally be of s-wave type since it has large coherence length. A high $T_c$ superconductor could also be used in the device but in that case the distance between the leads would have to be very small.

IV. EXPERIMENTAL REALIZATION

The experimental realization of the proposed device is not difficult. The phenomenon of crossed Andreev reflection has been demonstrated in two recent experiments. In the experiment of Ref. a sample geometry consisting of an aluminum bar with two or more ferromagnetic wires forming point contacts has been considered. By measuring the nonlocal resistance in the superconducting state of such structures a spin-valve signal has been observed whose sign, magnitude and decay length scale are consistent with predictions made for crossed Andreev reflection. Our suggestion is to integrate the quantum pumping mechanism (notably seen in four experiments till date, see Refs. ) into such type of set-up. This can be easily done: the magnetization can be adiabatically modulated by an external magnetic field while the strength of the contact barrier at the ferromagnet-superconductor interface can be modulated by applying a suitable gate voltage at the junction. This procedure should enable a nonlocal pure spin current generation in the semiconducting lead. The detection of the pure spin current could be achieved through a quantum spin Hall effect set-up. Apart from this method, the spin pump can be connected to a (semiconducting) ferromagnet with a known magnetization direction, or a gate-controlled bidirectional spin filter could also be used to detect the spin current.

V. CONCLUSIONS

A novel device for pure spin current generation based on the interplay of adiabatic pumping and crossed Andreev/normal reflection in a three-terminal ferromagnet-superconductor-semiconductor system has been proposed. The transfer of charges/spins in the device is achieved by adiabatic quantum pumping without any voltage bias applied. Varying the strength of the pumping parameters, namely the magnetization strength in the ferromagnet and the contact barrier strength at the ferromagnet-superconductor junction, we have shown that a pure spin current can be injected into the semiconducting lead in a completely nonlocal way. As already mentioned in the Introduction there are many different ways to inject a spin current into a semiconductor. One of the most commonly used techniques is that of using ferromagnets. This technique has been shown to be very inefficient as almost all of the spin polarization is lost at the interface. In this work we have used the method of quantum pumping to create a pure spin current in the semiconductor itself. Quantum pumping methodology has been used in many recent works to inject spin currents, however our work is novel in two respects. Firstly, it invokes non-locality. Almost all proposals which we are aware of, invoking quantum pumping to induce a spin current are local, i.e., time dependent voltages are applied directly to the semiconductor. What our proposal proves is that, though one does not touch the semiconductor, none the less one generates a pure spin current in the semiconductor. Secondly, no ferromagnet-semiconductor interface is used and therefore the problem of resistivity mismatch is avoided. As we have shown, the proposed device ideally operates in the strong pumping regime where the spin current is noticeably larger than the charge current. The experimental realization of such device as a pure spin current injector has also been discussed.

VI. ACKNOWLEDGMENTS

The authors would like to thank Prof. Costabile G. and Carapella G. for useful discussions.

* Electronic address: colin@sa.infn.it

1 S.A. Wolf et al., Science 294, 1488 (2001).
The scattering amplitudes required to calculate the pumped current in the semiconducting lead are given by:

\[ S_{e,SmF}^{he} = \frac{-2(1 + \sigma h_1)^{1/4}(1 + \sqrt{1 - \sigma h_1} + i(z_1 + z_2))}{D}, \quad S_{e,SmF}^{he} = \frac{2(z_2^2)(z_1 - i\sqrt{1 - \sigma h_1})}{(1 + iz_2)D}. \]

\[ S_{e,SmSm}^{he} = \frac{2[iz_2P + z_2P_i(z_1^2 + Q) + i\{z_1z_2P - z_1^2 - Q\}]}{(1 + iz_2)D}. \]

\[ S_{e,SmSm}^{ee} = \frac{[1 - P - z_1z_2(P + 1) - z_2(1 + Q + z_1^2) + i\{2z_2z_1 + 2 + z_2(P + 1 + z_1z_2) + 2z_2Q + z_1^2\}]}{D, \]

with \( Q = \sqrt{1 - h_1^2}, \)

\[ P = \sqrt{1 + \sigma h_1} - \sqrt{1 - \sigma h_1}, \]

\[ h_1 = h_{ex}/E_F, \]

and \( D = 1 + 3z_1z_2 + (2 + z_2^2)(Q + z_1^2) + \sqrt{1 - h_1^2} + \sqrt{1 + \sigma h_1} + iP(z_1(2 + z_2^2) + z_2). \]
46 J. E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999); Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science 306, 1910-1913 (2004).
47 P. Sharma and P. W. Brouwer, Phys. Rev. Lett. 91, 166801 (2003).
48 J. A. Folk, R. M. Potok, C. M. Marcus and V. Umansky, Science 299, 679 (2003).