Gauge invariance and weak forces in an isotropic medium

A.I. Rez\textsuperscript{1}, V.B. Semikoz\textsuperscript{2}

Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences (IZMIRAN), 142190 Troitsk, Moscow Region, Russia

Abstract

We demonstrate that the neutrino kinetic equation derived by the standard Bogolyubov method is formally gauge non-invariant and give a recipe how to recast it to the gauge invariant form recovering the standard Lorentz form weak force term which apparently conserves the lepton current. The analogy with the QED plasma case is traced which in asymptotic regions implies substitution of electric charge $e$ by the induced electric charge $e_{\nu}^{ind}$ of neutrino within the phase factor connecting neutrino gauge invariant and non-invariant distribution functions.

Key words: Neutrino kinetic equations, Gauge invariance

PACS: 13.10.+q, 13.15.+g, 14.60.Pq

1 Introduction

The neutrino Relativistic Kinetic Equation (RKE) is a useful tool to describe many phenomena in astrophysics and cosmology. In particular, neutrinos play the most important role for a supernova (SN) burst or in the lepton asymmetry formation before the primordial nucleosynthesis in the early universe. The usual motivation to use the RKE approach for neutrino propagation in a dense matter is stipulated by the account of neutrino collisions: within a SN neutrinosphere or in the hot lepton plasma of the early universe before neutrino decoupling.

However, in addition to collision integrals there are self-consistent weak interaction terms in the neutrino RKE [1] that are \textit{linear over the Fermi constant}

\textsuperscript{1} rez@izmiran.rssi.ru
\textsuperscript{2} semikoz@izmiran.rssi.ru
$\sim G_F$ and analogous to the Lorentz force terms for charge particles in the standard Boltzman RKE which in turn are linear over the electric charge $\sim q$ ($q = -|e|$ for electrons).

In the standard kinetics these self-consistent electromagnetic fields are well-known to play a very crucial role. In collisionless, or Vlasov approximation, such kinetic equations describe, e.g. thermonuclear plasmas in laboratory and stars for which an energy exchange between electromagnetic waves (eigen modes) and charged particles proceeds faster than via the direct particle collisions with all following issues in collisionless plasma: instabilities, heating, etc.

One expects that the self-consistent weak interaction ($\sim G_F$) could lead for neutrinos to some analogous collective interaction effects, e.g. to neutrino driven streaming instability in an isotropic plasma [2], to generation of magnetic fields in the early universe or in a SN [3,4].

Recently neutrino RKE has been rederived along different ways in [2,4,5] and the goals of this letter are: (i) to establish the conformity of these RKE’s with the standard Bogolyubov approach (Bogolyubov-Born-Green-Kirkwood-Yvon (BBQKY) chains of kinetic equations) used in [1]; (ii) to elucidate the physical sense of the ponderomotive weak force appeared in effective Lorentz form meanwhile the neutrino density matrix is gauge invariant with respect to the corresponding electron current transformation.

To this end, in section 2 we find that the formal lepton current nonconservation in the master RKE [1] stems from the absense of gauge invariance (see eq. (4) below) for the usual in quantum statistics definition of the gauge non-invariant distribution $f(\vec{x}_1, \vec{x}_2, t) = Tr(\hat{\rho}(t)\hat{\Psi}^+(\vec{x}_2)\hat{\Psi}(\vec{x}_1))$.

Then in section 3 we suggest the recipe of gauge invariance restoration which allows to derive the Lorentz form of the weak force term in RKE [2,5] and after that in the main section 4 we find the phase transformation (12) which connects the gauge invariant distribution for neutrinos with the gauge noninvariant one.

The above transformation is fully analogous to the one used in the case of QED plasma, being based on the same way of inclusion of interaction: with the self-consistent e/m field $A_\mu$ in QED plasma, and with the self-consistent electron four-current $J_\mu$ for neutrinos. As a result, these transformations turn out to be equivalent with the accuracy of a change of charges $e \to e'^{\text{ind}}_\nu$, where $e'^{\text{ind}}_\nu$ is the induced electric charge of neutrino [6](see section 4). This analogy is completed in section 5 where we recast the weak force term into a form of radiative damping force due to a plasmon emission by neutrino.

In section 6 we give conclusions and in Appendix we remind some well-known
properties of self-consistent electromagnetic fields in plasma.

2 Neutrino RKE for gauge non-invariant Wigner distribution

Neglecting electron spin from the quantum Liouville equation one finds in the Vlasov approximation the neutrino Relativistic Kinetic Equation (RKE) as the classical equation for the gauge non-invariant distribution function \( \tilde{f}^{(\nu)}(q, \vec{x}, t) \) [1],

\[
\frac{\partial \tilde{f}^{(\nu)}(q, \vec{x}, t)}{\partial t} + \vec{x} \frac{\partial \tilde{f}^{(\nu)}(q, \vec{x}, t)}{\partial \vec{x}} + q \frac{\partial \tilde{f}^{(\nu)}(q, \vec{x}, t)}{\partial q} = 0 ,
\]

where \( \vec{x} = \vec{v} = q/q \) is the velocity of massless neutrino, the derivative \( \dot{q} \) is given by

\[
\dot{q} = G_F \sqrt{2c_V} \left[ \nabla n^{(e)}(\vec{x}, t) - \nabla (\vec{n} \vec{j}^{(e)}(\vec{x}, t)) \right] .
\]

Here \( j^{(e)}_{\mu}(\vec{x}, t) = (n^{(e)}(\vec{x}, t); j^{(e)}(\vec{x}, t)) = \int (d^3p/(2\pi)^3)(p_\mu/\varepsilon_p) f^{(e)}(\vec{p}, \vec{x}, t) \equiv \vec{j}^{(e)}(\vec{x}, t)/e \) is the four-vector of the electron current density divided on the electron charge \( e = -|e| \); \( c_V = 2\xi \pm 0.5 \) is the weak vector coupling (upper sign for electron neutrinos), \( \xi = \sin^2 \theta_W \simeq 0.23 \) is the Weinberg parameter.

Obviously, due to the second term in (2) the RKE (1) does not obey the neutrino current conservation law,

\[
\frac{\partial j^{(\nu)}_{\mu}(\vec{x}, t)}{\partial x_\mu} \neq 0 ,
\]

where \( j^{(\nu)}_{\mu}(\vec{x}, t) = \int d^3q(q_\mu/q) \tilde{f}^{(\nu)}(q, \vec{x}, t)/(2\pi)^3 \) is the neutrino four-current in medium.

First, note that RKE (1) can be derived from the canonical equation \( \partial \tilde{f}^{(\nu)}/\partial t + \{H, \tilde{f}^{(\nu)}\} = 0 \) with use of the neutrino Hamiltonian in a medium [2]

\[
H = H_0 + V_{eff} = \sqrt{(\vec{Q} - G_F \sqrt{2c_V} \vec{j}^{(e)}(\vec{x}, t))^2 + G_F \sqrt{2c_V} n^{(e)}(\vec{x}, t)} ,
\]

Here the kinematical momentum \( \vec{q} \) of massless neutrino is connected with the canonical one, \( \vec{Q} \), as

\[
\vec{q} = \vec{Q} - G_F \sqrt{2c_V} \vec{j}^{(e)}(\vec{x}, t) ,
\]
and canonical definitions \( \vec{n} = \vec{q}/q = \partial H/\partial Q \equiv \partial H/\partial \vec{q}, \dot{q}^i = -\partial H/\partial x^i = \nabla H \) lead to (1).

The origin of the non-conservation of the lepton current seen from (1) is the absence of the invariance of the Hamiltonian \( H \) (and, hence, of the neutrino RKE) with respect to the gauge transformation of the electron current

\[
\dot{j}_{\mu}^{(e)}(\vec{x}, t) \to \dot{j}_{\mu}^{(e)}(\vec{x}, t) - \partial_{\mu} \chi(\vec{x}, t),
\]

where an arbitrary function \( \chi(\vec{x}, t) \) should also obey d’Alambert equation \( \partial_{\mu} \partial^{\mu} \chi(\vec{x}, t) = 0 \).

Note that the invariance of the one-particle neutrino motion equation in a medium under the same gauge transformation (4) is equivalent to the neutrino current conservation too since in the integrand of the action \( S = \int (L_0 + L_{int}(\vec{x}, t))d^4x \) there appears an additional (second) term

\[
G_F j_{\mu}^{(\nu)}(\vec{x}, t) \dot{j}_{\mu}^{(e)}(\vec{x}, t) \to G_F j_{\mu}^{(\nu)}(\vec{x}, t) j_{\mu}^{(e)}(\vec{x}, t) - G_F \dot{j}_{\mu}^{(\nu)}(\vec{x}, t) \partial_{\mu} \chi(\vec{x}, t),
\]

which does not contribute to the action, or such gauge transformation should not influence the motion equation coming from the extremum, \( \delta S = 0 \), exactly due to \( \partial \dot{j}_{\mu}^{(\nu)}(\vec{x}, t)/\partial x_{\mu} = 0 \).

3 Neutrino RKE for the gauge invariant Wigner distribution

The recipe of gauge invariance restoration is the same as in QED plasma [7]: we should recast (1) for the gauge invariant Wigner distribution function

\[
f^{(\nu)}(\vec{q}, \vec{x}, t) = \tilde{f}^{(\nu)}(\vec{Q}, \vec{x}, t),
\]

where the latter obeys the same RKE (1) but with the substitution of the kinematical momentum \( \vec{q} \) by the canonical one (3),

\[
\frac{\partial \tilde{f}^{(\nu)}(\vec{Q}, \vec{x}, t)}{\partial t} + \vec{n} \frac{\partial \tilde{f}^{(\nu)}(\vec{Q}, \vec{x}, t)}{\partial \vec{x}} + \dot{Q} \frac{\partial \tilde{f}^{(\nu)}(\vec{Q}, \vec{x}, t)}{\partial \vec{Q}} = 0.
\]

Accounting for (2), (3), the total time derivative \( d\vec{j}^{(e)}(\vec{x}, t)/dt = \partial \vec{j}^{(e)}(\vec{x}, t)/\partial t + (\vec{n} \nabla)\dot{\vec{j}}^{(e)}(\vec{x}, t) \) with the identity

\[
(\vec{n} \nabla)\dot{\vec{j}}^{(e)}(\vec{x}, t) - \nabla(\vec{n} \dot{\vec{j}}^{(e)}(\vec{x}, t)) \equiv -[\vec{n} \times \nabla \times \dot{j}^{(e)}(\vec{x}, t)],
\]
and using the recipe (5) one can easily check that the RKE above takes the form \[2\]

\[\frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial t} + \vec{n} \cdot \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{x}} + F^{(V)}_{j\mu}(\vec{x}, t) \frac{q^\mu}{\varepsilon_q} \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial q_j} = 0, \quad (6)\]

where the antisymmetric tensor \(F^{(V)}_{jk}(\vec{x}, t)\) entering the effective Lorentz force is given by the weak vector current,

\[F^{(V)}_{j0}(\vec{x}, t)/G_F \sqrt{2} c_V = -\nabla_j n^{(e)}(\vec{x}, t) - \frac{\partial j^{(e)}_j(\vec{x}, t)}{\partial t},\]

\[F^{(V)}_{jk}(\vec{x}, t)/G_F \sqrt{2} c_V = \epsilon_{jkl}(\nabla \times j^{(e)}(\vec{x}, t)), \quad (7)\]

and in accordance with (3) we changed the derivative \(\partial/\partial \vec{Q} \to \partial/\partial \vec{q}\).

Obviously, the tensor (7) (and thus the whole RKE (6)) is invariant with respect to the transformation (4) and obeys the continuity equation, or the neutrino current \(j^{(\nu)}_\mu(\vec{x}, t) = \int (d^3q/(2\pi)^3)(q_\mu/q)f^{(\nu)}(\vec{p}, \vec{x}, t)\) is conserved,

\[\frac{\partial j^{(\nu)}_\mu(\vec{x}, t)}{\partial x_\mu} = 0. \quad (8)\]

So far we did not obtain any new formulae. However, we argue that the above formal recipe (5) is a simple consequence of the gauge invariance under the transformation (4). In the next section we try to elucidate the physical sense of such invariance in plasma and its connection with the important definition of the gauge invariant Wigner distribution in SM (5).

### 4 Gauge invariant Wigner distribution and induced electric charge of neutrino

Let us remind the definitions and the physical sense of the gauge invariant Wigner distribution functions in QED plasma [8],

\[f^{(e)}(\vec{p}, \vec{x}, t) = \hat{f}^{(e)}(\vec{p} + e\vec{A}(\vec{x}, t), \vec{x}, t) = \int d^3y e^{i\vec{p}\cdot\vec{y}} f^{(e)}(\vec{x} - \vec{y}/2, \vec{x} + \vec{y}/2, t), \quad (9)\]

where the gauge invariant distribution function in the coordinate representation \(f^{(e)}(\vec{x}_1, \vec{x}_2, t)\) is connected with the gauge non-invariant \(\hat{f}^{(e)}(\vec{x}_1, \vec{x}_2, t) = Tr \left( \hat{\rho}(t) \hat{\Psi}^{(e)}(\vec{x}_2) \hat{\Psi}^{(e)}(\vec{x}_1) \right)\) by the important phase factor [7]:
\[ f^{(e)}(\vec{x}_1, \vec{x}_2, t) = \exp \left[ ie(\vec{x}_2 - \vec{x}_1) \int_0^1 d\xi \vec{A} (\vec{x}_2 + \xi (\vec{x}_1 - \vec{x}_2), t) \right] \times \]
\[ \times \tilde{f}^{(e)}(\vec{x}_1, \vec{x}_2, t). \]

Namely due to this phase factor the distribution (10) is invariant under the standard gauge transformation (with an arbitrary gauge function \( \chi(\vec{x}, t) \) obeying the d’Alambert equation),

\[ \hat{\Psi}^{(e)}(\vec{x}_1) \rightarrow e^{-ie\chi(\vec{x}_1, t)} \hat{\Psi}^{(e)}(\vec{x}_1), \]
\[ \hat{\Psi}^{(e)+}(\vec{x}_2) \rightarrow e^{+ie\chi(\vec{x}_2, t)} \hat{\Psi}^{(e)}(\vec{x}_2), \]
\[ \vec{A}(\vec{x}_2 + \xi (\vec{x}_1 - \vec{x}_2), t) \rightarrow \vec{A}(\vec{x}_2 + \xi (\vec{x}_1 - \vec{x}_2), t) \]
\[ - \frac{\partial \chi(\vec{x}_2 + \xi (\vec{x}_1 - \vec{x}_2), t)}{\partial \vec{x}_2}, \]

or, equivalently, this arbitrary phase \( \chi(\vec{x}, t) \) cancels in (10). Such invariance is crucial for macroscopic physics since it provides the physical sense of the Wigner function (9) and the conservation of the macroscopic electric current.

Really, as in the case of neutrino RKE (1), the kinetic equation for the gauge non-invariant distribution of charged particles \( \tilde{f}^{(e)}(\vec{x}_1, \vec{x}_2, t) \) derived from the quantum Liouville equation by the same Bogolyubov method does not obey electric current conservation. This is because the force term depends on the electromagnetic potentials \( A_{\mu}(\vec{x}, t) \) which do not enter as combinations expressed via field strengths, \( \vec{E}, \vec{B} \) [8]. The recasting of such RKE for the gauge-invariant distribution (9) allows to obtain the usual Lorentz form of the force term in the standard Boltzman equation for charged particles [8]:

\[ \frac{\partial f^{(e)}(\vec{p}, \vec{x}, t)}{\partial t} + \vec{v} \frac{\partial f^{(e)}(\vec{p}, \vec{x}, t)}{\partial \vec{x}} + e (\vec{E}(\vec{x}, t) + [\vec{v} \times \vec{B}(\vec{x}, t)]) \frac{\partial f^{(e)}(\vec{p}, \vec{x}, t)}{\partial \vec{p}} = 0, \]

for which, of course, the electric current
\[ j^{(e)}(\vec{x}, t) = \int d^3p (p_\mu/\varepsilon_p) f^{(e)}(\vec{p}, \vec{x}, t)/(2\pi)^3 \]

is conserved, \( \partial j^{(e)}_\mu / \partial x_\mu = 0 \).

Note that through the whole text we use exactly this standard RKE for electrons neglecting their mutual weak interactions.

Hence, in analogy with the QED plasma definition (10) we should reformulate (5), which leads to the neutrino current conservation in final RKE (6), for the gauge invariant distribution in the coordinate representation, \( f^{(\nu)}(\vec{x} - \vec{y}/2, \vec{x} + \vec{y}/2, t) = \int d^3q e^{-iq\vec{y}} f^{(\nu)}(\vec{q}, \vec{x}, t)/(2\pi)^3 \).

To this end, comparing the neutrino canonical momentum (3) with the well-known \( \vec{p} = \vec{P} - e\vec{A}(\vec{x}, t) \) in the case of QED plasma, we find the important
(weak) phase factor that connects the gauge invariant distribution \( f^{(\nu)}(\vec{x}_1, \vec{x}_2, t) \) with the gauge non-invariant \( \tilde{f}^{(\nu)}(\vec{x}_1, \vec{x}_2, t) \),

\[
f^{(\nu)}(\vec{x}_1, \vec{x}_2, t) = \exp \left[ iG_F \sqrt{2}c_V (\vec{x}_2 - \vec{x}_1) \int_0^1 d\xi \tilde{j}^{(e)}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) \right] \\
\times \tilde{f}^{(\nu)}(\vec{x}_1, \vec{x}_2, t)
\]

(12)

Our goal here is the explanation of the gauge invariance of this function via the gauge transformation of electron current in medium (4) with an appropriate electromagnetic formfactor of neutrino in plasma instead of the electron charge \( e \) in the standard QED transformation (11).

Remembering the definition of the gauge non-invariant distribution \( \tilde{f}^{(\nu)}(\vec{x}_1, \vec{x}_2, t) = \text{Tr} \left( \hat{\rho}(t) \hat{\Psi}^{(\nu)}(\vec{x}_2) \hat{\Psi}^{(\nu)}(\vec{x}_1) \right) \) (compare with the electron case before (10)) we find in the completed form how the gauge invariance (4) with an arbitrary gauge \( \chi(\vec{x}, t) \) is manifested in the neutrino kinetics:

\[
\hat{\Psi}^{(\nu)}(\vec{x}_1) \rightarrow \exp \left( -iG_F \sqrt{2}c_V \chi(\vec{x}_1, t) \right) \hat{\Psi}^{(\nu)}(\vec{x}_1), \\
\hat{\Psi}^{(\nu)+}(\vec{x}_2) \rightarrow \exp \left( +iG_F \sqrt{2}c_V \chi(\vec{x}_2, t) \right) \hat{\Psi}^{(\nu)}(\vec{x}_2), \\
\tilde{j}^{(e)}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) \rightarrow \tilde{j}^{(e)}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) \\
- \frac{\partial \chi(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t)}{\partial \vec{x}_2}.
\]

(13)

It is easy to check the cancellation of \( \chi \) in the phase factor in (12).

On the one hand, this transformation provides the gauge invariance of the neutrino distribution (12) and automatically the invariance of the Wigner function \( f^{(\nu)}(\vec{q}, \vec{x}, t) \) (5) resulting in the neutrino current conservation (8).

On the other hand, in an isotropic plasma the electric current \( J_i^{(e)} = -|e| \int \tilde{j}_i^{(e)} \) is the induced one,

\[
J_i^{(e)}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) = \int \frac{d^4 Q}{(2\pi)^4} e^{-i\omega t + ik \cdot (\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2))} \Pi_{\mu\nu}(\omega, \vec{k}) A^\mu(\omega, \vec{k})
\]

i.e. the phase factor in (12) takes the form

\[
\exp \left[ -\frac{iG_F \sqrt{2}c_V}{4\pi\alpha} |e| \gamma_i \right] \int_0^1 \int \frac{d^4 Q}{(2\pi)^4} e^{-i\omega t + ik \cdot (\vec{x}_2 - \xi\vec{y})} \left( \frac{Q^2}{k^2}(\varepsilon_i - 1) k_i k_j \right)
\]
\[ + \omega^2 (\delta_{ij} - \frac{k_i k_j}{k^2}) (\varepsilon_{tr} - 1) A_j(\omega, \vec{k}) \]. \tag{14} \]

Here \( Q_\mu = (\omega, \vec{k}) \) is the plasmon four-vector; \( \varepsilon_{l,\text{tr}}(\omega, k) \) are longitudinal and transversal permittivities in an isotropic plasma; \( \vec{y} = \vec{x}_2 - \vec{x}_1 \).

In the two limiting cases: (i) quasistatic electric field \((\omega \ll k \langle v \rangle \leq k, \varepsilon_l - 1 \approx (kr_D)^{-2}, \omega^2 (\varepsilon_{tr} - 1) \to 0)\) and (ii) high-frequency electromagnetic field \((\omega \gg k \langle v \rangle, Q^2 \to 0, \varepsilon_{tr} - 1 \approx -\omega_p^2/\omega^2)\) the phase factor (14) takes the form analogous to the QED plasma result (10):

\[
\text{exp} \left[ -ie^{\text{ind}}_\nu (\vec{x}_2 - \vec{x}_1) \int_0^1 d\xi \vec{A}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) \right], \tag{15}\]

where the electromagnetic field splits (see Appendix) into the longitudinal \( \vec{A}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) = \vec{A}^{(l)}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) \) and the transversal \( \vec{A}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) = \vec{A}^{(tr)}(\vec{x}_2 + \xi(\vec{x}_1 - \vec{x}_2), t) \) correspondingly multiplied by: (i) either the \textit{quasistatic induced electric charge of neutrino} or (ii) the high-frequency one [6],

\[
e^{\text{ind}}_\nu = -\frac{|e| G_F c V}{2 \pi \alpha \sqrt{2r_D^2}}, \tag{16}\]

\[
e^{\text{ind}}_\nu = -\frac{|e| G_F c V \omega_p^2}{2 \pi \alpha \sqrt{2}}. \tag{17}\]

Here \( r_D = \sqrt{T/4\pi\alpha n_0^{(e)}} \) is the Debye radius; \( \omega_p = \sqrt{4\pi\alpha n_0^{(e)}/m_e} \) is the plasma frequency; \( T \) and \( n_0^{(e)} \) are the temperature and the mean electron density.

5 The radiation damping force in isotropic medium

Here we show that accounting for the lepton current conservation the final form of neutrino RKE (with an explicit dependence on the self-consistent electromagnetic fields \( \vec{E}, \vec{B} \)) does not depend whether we apply initial RKE (1) for the gauge non-invariant distribution function \( \tilde{f}^{(\nu)}(\vec{q}, \vec{x}, t) \) [1], or the same equation written in the completed form (6) [2].

The situation is similar to the case of standard plasma where the initial RKE for the gauge non-invariant Wigner distribution \( \tilde{f}^{(e)}(\vec{p}, \vec{x}, t) \) is often more suitable to obtain concrete results than Boltzman equation with the Lorentz force.
Nevertheless, the electric current should be written in the gauge invariant form obeying the conservation law, $\partial j^{(e)}_\mu / \partial x_\mu = 0$.

Making use in (6) of the standard connection of the induced electron current $\vec{j}^{(e)}$ with the electric field $\vec{E}$ (in the Fourier representation), $j_i(\omega, \vec{k}) = (\omega/4\pi i) [\varepsilon_{ij}(\omega, \vec{k}) - \delta_{ij}] E_j(\omega, \vec{k})$, where the permittivity tensor $\varepsilon_{ij}$ in an isotropic plasma is given by

$$\varepsilon_{ij}(\omega, \vec{k}) = \varepsilon_i(\omega, k) \frac{k_i k_j}{k^2} + \varepsilon_{tr}(\omega, k) \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right),$$

we easily obtain another form of the neutrino RKE (6),

$$\frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial t} + \vec{n} \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{x}} + e \int \frac{d^4 Q e^{-iQx}}{(2\pi)^4} \left[ F_l(\omega, k) \vec{E}_\parallel(\omega, \vec{k}) + F_{tr}(\omega, k) \right] \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{q}} = 0,$$

where $F_l(\omega, k)$ and $F_{tr}(\omega, k)$ are the neutrino electromagnetic formfactors defined in [6],

$$F_l(\omega, k) = G_F \sqrt{2} c V (\varepsilon_l(\omega, k) - 1) Q^2 / \alpha,$$
$$F_{tr}(\omega, k) = G_F \sqrt{2} c V (\varepsilon_{tr}(\omega, k) - 1) \omega^2 / \alpha;$$

where $Q = (\omega, \vec{k})$; $\vec{E} = \vec{E}_\parallel + \vec{E}_\perp$, $\vec{B}$ are the electromagnetic fields in the dispersive medium, and $\vec{E}_\parallel = k(\vec{k} \vec{E}) / k^2$.

It is obvious that the third term on the left-hand side of Eq. (18) is proportional to the force of electromagnetic origin. Whereas for a point charge $e$ (when the form factors are equal to unity: $F_l = F_{tr} = 1$) this term is determined by the Lorentz force, i.e., is equal to the standard expression $e(\vec{E}(\vec{x}, t) + [\vec{n} \vec{B}(\vec{x}, t)] \partial f(\vec{q}, \vec{x}, t) / \partial \vec{q}$, for neutrinos with electromagnetic structure [6], with allowance for the constant of the weak coupling to the electric charge, $G_F \sim e^2 / M_W^2$, the third term in (18) is proportional to the radiation damping force ($\sim e^3$).

The polarization origin of such a force becomes clear after simple manipulations in (18) using the explicit expressions (19) for the form factors $F_l$ and $F_{tr}$ in an isotropic dispersive medium, for which the Fourier integrals can be completely calculated, and the considered term [1]

$$\frac{\sqrt{2} G_F c V}{e} \left( \frac{\partial^2}{\partial x_j \partial n} + n \frac{\partial^2}{\partial t \partial x_j} \right) P_n(\vec{x}, t) \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{q}_j}$$

(20)
is proportional to the second derivative of the polarization vector of the dispersive medium:

$$4\pi P_n(\vec{x}, t) = D_n(\vec{x}, t) - E_n(\vec{x}, t),$$

which is equal to the difference between the vectors of the electric induction, $D_n(\vec{x}, t) = \int d^4x' \varepsilon_{nj}(\vec{x} - \vec{x}', t - t')E_j(\vec{x}', t)$, and the electric field intensity $E_n(\vec{x}, t)$.

Note that the expression (20) for the force is also valid for anisotropic media when the permittivity tensor depends, for example, on an external magnetic field. In vacuum ($\vec{D} = \vec{E}$), the effect corresponding to plasmon emission by a moving neutrino disappears, i.e., there is no damping force in (18).

Finally, in special cases of the excitation in a dispersive medium of electrostatic waves ($\omega \ll k\langle v \rangle$) or the propagation of a high-frequency transverse wave ($\omega \gg k\langle v \rangle$), the term (20) can be represented in the form of the effective Lorentz force

$$e^\text{ind}_\nu \vec{E} \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{q}},$$

$$e^\text{ind}_\nu \left(\vec{E}_\perp + [\vec{n} \vec{B}]\right) \frac{\partial f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{q}},$$

which is proportional to the neutrino induced electric charges (16) and (17), respectively.

6 Conclusions

Thus, we proved the neutrino current conservation in a medium as the consequence of the invariance of the distribution functions (12) under the gauge transformation (13) that is similar to the standard one (11) for the charged particle distribution (10).

Second, the ponderomotive force for neutrinos given by the third term in RKE (6) is the damping (friction) force arising due to the neutrino electromagnetic structure (the electromagnetic vertex $\Gamma_\mu(\omega, \vec{k}) \sim G_F \Pi_{\mu\nu}(\omega, \vec{k})\gamma^\nu [6]$) and is stipulated by the plasmon (Cerenkov) emission in medium, $\nu \rightarrow \nu + \gamma^*$, forbidden in vacuum [1].

Third, the appearance of the same neutrino electromagnetic (loop) structure as the factor ahead the self-consistent electromagnetic fields in the phase factor.
(14) looks rather natural. In particular cases of quasistatic (longitudinal) or high-frequency (transversal) fields there is a complete analogy with the QED plasma case accomplished after substitution of the electric charge $e$ by $e^{ind}_\nu$.

Acknowledgements

This work was partly supported by the RFBR grant No. 00-02-16271.

Appendix

In an isotropic plasma the second-quantized electromagnetic fields are additive, $\hat{A}_\mu = \hat{A}^{(l)}_\mu + \hat{A}^{(tr)}_\mu$, where both the longitudinal field

$$\hat{A}^{(l)}_\mu (\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3 N_l(k)} \left( \varepsilon^{(l)}_\mu(k) \hat{a}^{(l)}(k) e^{-iqx} + \varepsilon^{* (l)}_\mu(k) \hat{a}^{(l)\dagger}(k) e^{iqx} \right),$$

and the transversal one

$$\hat{A}^{(tr)}_\mu (\vec{x}, t) = \sum_\lambda \int \frac{d^3k}{(2\pi)^3 N_{tr}(k)} \left( \varepsilon^{(tr)}_\lambda(k) \hat{a}^{(tr)}_\lambda(k) e^{-iqx} + \varepsilon^{* (tr)}_\lambda(k) \hat{a}^{(tr)\dagger}_\lambda(k) e^{iqx} \right),$$

obey the Lorentz gauge, $\partial A_\mu (\vec{x}, t) / \partial x_\mu = 0$. Here the unit polarization vectors, $\varepsilon_\mu \varepsilon^\mu = -1$, are given by $\varepsilon^{(l)}_\mu(k) = (k, \omega \hat{k})/\sqrt{Q^2}$, $\varepsilon^{(l)\dagger}_\mu(k) = (0, \varepsilon^{(l)}_\mu(k))$, and for the plasmon four-vector $Q_\mu = (\omega, \vec{k})$ obey the standard conditions for both components $Q_\mu \varepsilon^{(l,\lambda)}_\mu(k) = 0$, or $k_i \varepsilon_i^{(\lambda)} = 0$ for the transversal part only; the normalization factors $N_{l, tr}(k)$, $N_l(k) = \sqrt{Q^2 \partial \text{Re} \varepsilon_l(\omega, k)/\partial \omega}$, $N_{tr}(k) = \sqrt{2\omega[\varepsilon_{tr}(\omega, k) + \omega \partial \text{Re} \varepsilon_{tr}(\omega, k)/\partial \omega]}$ are given by the longitudinal ($\varepsilon_l(\omega, k)$) and the transversal ($\varepsilon_{tr}(\omega, k)$) permittivities correspondingly.

Note that in the Fourier representation the 3-vector parts of these four-potentials $\vec{A}^{(l, tr)}(\omega, k) = \varepsilon^{(l, tr)}(k) \hat{a}(k)/N^{(l, tr)}(k)$ obey important opposite conditions $\vec{A}^{(l)}(\omega, k) \parallel \vec{k}$ and $\vec{k} \vec{A}^{(tr)}(\omega, k) = 0$ that were used above to recast Eq. (14) into Eq. (15) which is proportional to induced electric charges of neutrino (16), (17) correspondingly.

References

[1] V.B. Semikoz, Physica 142A (1987) 157.
[2] R. Bingham, J.M. Dawson, J.J. Su, H.A. Bethe, 
Phys. Lett. A 220 (1994) 107 ; 
L.O. Silva, R. Bingham, J.M. Dawson, J.T. Mendonca, P.K. Shukla, Phys. Rev. 
Lett. 83 (1999) 2703 .

[3] P.K. Shukla et al., Plasma Phys. Control. Fusion 41 (1999) A699 .

[4] A.J. Brizard, H. Murayama and J.S. Wurtele, Phys. Rev. E 61 (2000) 4410 .

[5] L. Bento, Phys. Rev. D 61 (2000) 13004 ; 
L. Bento, Phys. Rev. D 63 (2001) 077302 .

[6] V.N. Oraevsky, V.B. Semikoz, Ya.A. Smorodinsky, Sov. Phys. JETP Lett. 43 
(1986) 709 ; 
V.N. Oraevsky, V.N. Ursov, Phys. Lett. B 209 (1988) 83 ; 
V.B. Semikoz, Ya.A. Smorodinsky, Sov. Phys. JETP 68 (1989) 20 ; 
J.C. D’Olivo, J.F. Nieves, P.B.Pal, Phys. Rev. D 40 (1989) 3679 ; 
J.F. Nieves and P.B. Pal, Phys. Rev. D 49 (1994) 1398 .

[7] S. Fujita, Introduction to non-equilibrium quantum statistical mechanics (W.B. 
Saunders Company, Philadelphia-London, 1966).

[8] A.I. Akhiezer, S.V. Peletminskii, Methods of statistical physics(Pergamon Press, 
Oxford, New York, 1981).