ADVERTISING GAMES ON NATIONAL BRAND AND STORE BRAND IN A DUAL-CHANNEL SUPPLY CHAIN

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Abstract. This paper investigates a dual-channel supply chain, where one national brand manufacturer has both online and retail channels. The retailer is assumed to sell the national brand as well as his store brand to customers. The following five scenarios are considered: Centralized case, Stackelberg-manufacturer (SM) game, Stackelberg-retailer (SR) game, Nash-manufacturer (NM) game and Nash-retailer (NR) game. The paper derives the conditions under which the supply chain members would like to participate in cooperative advertising. The results show that in the Stackelberg games, the leader in Stackelberg game will reduce its investment in cooperative advertising when it gets a lower marginal profit from the cooperative advertising; In addition, the dual-channel supply chain can get a higher profit if it is dominated by the member whose marginal profit from cooperative advertising is higher. In the Nash games, in order to increase the whole supply chain’s profit, the member who has a higher marginal profit in the cooperative advertising should give up the decision power on cost-sharing rate voluntarily. In addition, if there exists a leader in the supply chain, the cooperative advertising will be higher. Furthermore, the introduction of store brand will trigger the manufacturers antipathy for the low profit.

1. Introduction. In order to enhance competitiveness in today’s business world, retailers start to pay attention to their own brand product (called store brand or private label) distributed through exclusive channel [17]. Store brand is becoming more and more popular among large wholesalers or retailers, such as Wal-Mart, Carrefour, CR Vanguard and Tesco [27]. Store brands take an increasingly important share of the retail market (25% of US supermarket sales) and are rated among the top three brands in 70% of supermarket product categories [32]. Smart
retailers know that store brand products are a foundation for consumer loyalty and strong profits. In fact, according to the Private Label Manufacturers Association, the sales revenue of store brand outpaced national brands in 2013 and set market share record across all U.S. retail channels.

The introduction of store brand has a significant impact on both manufacturers and retailers, which becomes focus of theoretical research. The store brand is always competing with national brand. In the context of brand competition, cooperative advertising plays a significant role in marketing strategy [50]. Cooperative advertising is an arrangement between a manufacturer and a reseller where the reseller’s expenditures on local promotional activities are partly paid by the manufacturer [22]. Many manufacturers are using this strategy to coordinate channel distributions. For example, in the business market, Apple, Dell, and HP are actively utilizing this strategy to stimulate market sales.

With the rapid development of e-commerce, more and more companies begin to engage in online sales. For example, Lenovo, IBM, Samsung, Sony and Estee Lauder have had their own direct sale channels [10, 37]. According to Commerce Department estimates (http://www.electran.org) and Internet Retailer (http://www.Internet-retailer.com), U.S. e-commerce sales totaled $225.5 billion in 2012, up from $194.7 billion in 2011, representing a nearly 16 percent gain. The China e-commerce report pointed out that e-commerce develops rapidly and the sales totaled 20.8 trillion yuan in 2015, grew nearly 27% on year.

In multichannel environments, consumers can move easily among different channels [11]. Consumers can obtain services from one retailer and place their business with another. In a dual-channel supply chain, some consumers would like to gather information and enjoy service provided by the retailer, and then switch to online channel to complete the purchase, which is regarded as cross-channel free-riding behavior [11, 18, 19]. Such phenomenon happens when online price is lower than offline price. Van Baal and Dach [39] pointed out that more than 20% of consumers are free riders in a dual-channel supply chain. Such free-riding behaviors will intensify the contradictions between two channels. An in-depth understanding of the impact of free-riding behavior is essential for service providers [12]. Therefore, the study takes consumers’ cross-channel free riding behavior into account.

In the competitive environment, the supply chain has become an important topic [49]. And the chain members should be regarded as strategic partners with a long-term cooperative relationship [13]. While channel conflict can undermine the cooperative relations in the intermediated channel and lower the profits of all parties. Therefore, some coordination schemes should be used to regulate the relationship among the supply chain’s members [23]. Supply chain coordination can help improve the performance of the whole channel and channel members. In reality, the manufacturer usually uses non-price incentive methods, such as cooperative advertising, to alleviate the channel competition [40]. Cooperative advertising is a cost sharing and promotion mechanism for the manufacturer to affect retail performance [47].

It is worthwhile to note that many companies establish direct channels and implement national advertising and local advertising at the same time. Kurata et al. [28] pointed out that most products, which are distributed through direct channel, are national brands. For instance, L’Oreal sells their products through the official website and Watsons. At the same time, Watsons also sells its own brand’s skincare products, which competes with L’Oreal’s. In addition, Lock&Lock sells its products
through online channel and traditional offline retail stores such as Wal-Mart. Mainstays, which is a brand under Wal-Mart, also produces cups. No matter in Watsons’ store or Wal-Mart, it is common to see the advertising promotion for both the national brand and the store brand. Moreover, HP sells its computer in the online store and bricks-and-mortar computer stores (e.g., COMPUSA). Also, COMPUSA often sells original generic brand computers [28]. Therefore, it is worthwhile to study the advertising strategies in a dual channel supply chain, which can provide relevant management information for supply chain members.

This research addresses strategic decisions of the manufacturer and the retailer in a dual-channel supply chain with national brand and store brand. Our aim is to improve understanding of how the manufacturer and the retailer should manage their advertising decisions. This paper tries to formulate the following research questions: (1) Considering the impact of store brand and consumers’ free-riding behaviors, is it profitable for chain members to cooperate in advertising? (2) What is the impact of bargaining power on chain members’ advertising decisions and profits in the dual-channel environment? (3) Under what conditions does a manufacturer decide to open a direct channel and under what conditions does a retailer decide to introduce his own brand? Based on our results, the manufacturer and the retailer can optimize their respective decision-making. The results and the modeling approach are useful references for managerial decisions and administrations.

The remainder of our paper is organized as follows. Section 2 provides a summary of the relevant literature and Section 3 presents assumptions and notations. Section 4 considers the Stackelberg games and the Nash games. The comparison between Stackelberg and Nash game models is shown in section 5. Section 6 illustrates the effect of introduction of advertising competition, store brand and direct channel. In Section 7, experiments are conducted to analyze the sensitivity of the optimal profits with respect to parameters. And the conclusions are given in Section 8. We give all the proofs in Appendix A.

2. Literature review. This research is closely related to advertising strategies in supply chains, store brand phenomenon and dual-channel supply chain management. To highlight our contributions, we just review the literature that is representative and particularly relevant to our study.

2.1. Advertising strategies in supply chains. The research on advertising strategies has gained substantial attention in recent years, where competitive advertising and cooperative advertising are mainly concerned [31]. The research on competitive advertising is limited and mainly focuses on empirical study. Wang and Wu [41] presented an empirical study on the Lanchester model of combat for competitive advertising decisions. Chaabane et al. [6] investigated the effect of competitive advertising within store flyers on both manufacturers and retailers. They found that the increase of the number of competing brands does no harm to manufacturers and it may enhance recognition of brands, especially for well-known brands. The two studies on competitive advertising are relevant to our study; however, the backgrounds of these studies differ.

Another stream of literature mainly analyzes the cooperative advertising. For example, Yue et al. [46] studied the cooperative advertising and coordination of supply chain. They showed that if the manufacturer provides more discounts to consumers, the retailer would increase the investment of cooperative advertising
under a certain cost-sharing rate. Although the study enumerated cooperative advertising in a two-echelon supply chain similar to that in our model, our model consider the dual-channel mode. Ahmadi-Javid and Hoseinpour [1] pointed out that the sales-volume functions considered in Huang et al. [20] and Xie and Ai [43] have the unfortunate property of becoming negative for some values of the decision variables. So they improved the demand function by considering the negativity problem as suggested by Yue et al. [46]. They concluded that the advertising investment depends on marginal profits when the manufacturer is a leader. Recently, Jorgensen and Zaccour [22] surveyed the literature on cooperative advertising in supply chains using game theoretic methods. Aust and Buscher [3] provided a comprehensive review and a systematic analysis of articles on cooperative advertising, especially the mathematical models in supply chain management. Similar to the above two studies, we also use game theoretic methods to discuss the cooperative strategies in the supply chain. But they only considered one kind of product in the supply chain; in this aspect, these models differ from ours.

All the above papers just focus on advertising in the single retail channel, except Chen [9]. They focused on the impact of price schemes and cooperative advertising mechanisms on dual-channel supply chain competition, which is closely related to our study. However, our contributions are in addressing the advertising strategies in dual-channel supply chain with different games and considering the impact of consumers’ cross-channel free-riding behavior.

2.2. Store brand phenomenon. A growing body of analytical and empirical research on store brand exists. Most of them focus on the competition between national brand and the store brand [33]. There is limited literature that takes both cooperative advertising and store brand into consideration. Karray and Zaccour [26] showed that the manufacturer could profit from the introduction of store brand under a specific range of the retailer’s advertising efficiency and of the price competition intensity. Furthermore, Karray and Martín-Herrán [24] considered the relationship between pricing and advertising decisions. They showed that the retailer’s optimal reaction to competitive advertising effects in the channel depends on the price competition level between the store and the national brands, and the strength of the competitive advertising effects. Amrouche et al. [2] also considered a marketing channel where a retailer sells manufacturer’s brand as well as his own store brand. They suggested that investing in building up some equity for each brand would reduce the price competition. However, none of these research papers study the impact of the direct selling channel. With regard to the store brand in dual-channel supply chain, Kurata et al. [28] formulated a Nash game to discuss the pricing policies under the competition between a national brand and a store brand. They found that brand loyalty development is more important than channel loyalty development.

None of the aforementioned research papers, however, studies the advertising strategies considering national brand and store brand in a dual-channel supply chain. This paper, therefore, is distinct from the literature above in the following three aspects. Firstly, we assume that the national brand is sold through online channel as well as the retail channel, and the store brand is sold only in the retail store. Secondly, we pay attention to the issue that the retailer advertises for both the national brand and the store brand in the retail store. Last but not least, we analyze the advertising investment of the supply chain members.
2.3. Dual-channel supply chain management. Recent topics arising in Supply Chain Management (SCM) are typically focusing on dual-channel issues. In particular, their studies relate to the optimal operations and channel conflict, which is closely related to our study. Dan et al. [15] studied the optimal decisions on retail services and prices in a dual-channel supply chain. They showed that retail services would influence channel member’s pricing strategies strongly. By contrast, we studied the optimal advertising decisions in the supply chain. Additionally, various mechanisms for channel coordination in the dual-channel context have been proposed in recent decades. For example, Chen et al. [7] showed that a wholesale price contract with a two-part tariff or a profit-sharing agreement can coordinate the dual-channel supply chain. Shang and Yang [34] used profit-sharing contracts to coordinate the two- and three-stage dual-channel supply chains. However, all these papers do not take the advertising strategy into consideration. Wu and Chuang [42] modeled for understanding the fundamentals of the diffusion of electronic supply chain management with multiple stages among trading partners. Recently, some researchers also introduce dual-channel framework into other areas, such as green supply chain and tourism supply chain management [21, 45].

This paper makes the following contributions. Firstly, this paper considers both cooperative advertising and competitive advertising in the dual-channel supply chain. At the same time, we also study the impact of advertising competition on the supply chain’s decisions and profits. Secondly, we take consumers’ cross-channel free-riding behavior into consideration, while the aforementioned literature ignores this point. Thirdly, this paper formulates and compares four different game models according to different bargaining power between supply chain members. We try to find whether or not the supply chain members are willing to join in the cooperative advertising strategy. Our results can provide decision-makers with some new managerial insights under dual-channel environment.

3. Model setup and notations. Consider a two-echelon supply chain, where a national brand manufacturer sells the products through online channel and retail channel, but the store brand produced by an original equipment manufacturer is only available at the retail store. In the following content, the manufacturer refers to the national brand manufacturer. Figure 1 shows a framework of the supply chain. The manufacturer and the retailer, respectively, advertise for the national brand and the store brand. The retailer also advertises for the national brand, and the manufacturer will be responsible for a certain percentage of the advertising expense. Hence, the cooperative advertising refers to the advertising for national brand in the retail channel while the local advertising refers to retailer’s local advertising for store brand. According to different status of members in the supply chain that proposed by Zhang and Xie [48], and Chen and Yang [8], we analyze four game models: Stackelberg-manufacturer (SM), Stackelberg-retailer (SR), Nash-manufacturer (NM) and Nash-retailer games (NR).

3.1. Notations. Symbols and notations used through the paper are discussed as follows.

- \(a_i\) Baseline sales for the national brand-online, national brand-retail and store brand respectively (\(i = 0, 1, 2\))
- \(A_0\) Manufacturer’s national advertising expenditure
- \(A_1\) Cooperative advertising expenditure
- \(A_2\) Retailer’s local advertising expenditure on store brand
In our framework, the decision-making authority of cooperative advertising and the participation rate in cooperative advertising depend on different type of games.

3.2. Model description. In a dual-channel supply chain, the competition is inevitable between the existent retailer and the manufacturer’s direct channel [7]. Both the manufacturer and the retailer want to capture consumers through advertising. Therefore, the competition of advertising between two channels exists.

National advertising is beneficial to enhance the brand image of the manufacturer and consumers’ acceptance of the product. Therefore, national advertising has a positive influence on the demand for national brand in both the direct channel and the retail channel. Due to the competition between national brand and store brand, the national advertising has a negative effect on the demand for store brand.

The cooperative advertising is favorable to increase the sales volume of national brand in the retail channel. In multichannel environments, some consumers will engage in cross-channel free-riding. Traditional retailers report that customers visit their stores to have the possibility of examining physically the goods. If satisfied, many consumers go home and buy the product online. Therefore, the increment of sales volume in retail channel will be partly transferred to direct channel. Due to the competition between two channels, the cooperative advertising has a competitive effect on the demand in direct channel. In addition, the local advertising for store...
brand will increase the demand of store brand, but decrease the demand of national brand.

In this work, we concentrate on the impact of cooperative and competitive advertising. In order to simplify the analysis and focus on our core issue, we assume that the market prices are exogenous variables. Similar assumption can be found in Jorgensen et al. [22]. Such demand function is mainly used for convenience and tractability. Further, this assumption is also rational in practice because brand competition is more and more obvious nowadays. Take H&M and Coca Cola for example, they focus on the improvement of their brand image while their product prices are very stable at the meantime [29]. Here, we set \( \rho_0 = p_0 - c_1, \rho_1 = w - c_1, \rho_2 = p_1 - w, \rho_3 = p_2 - c_2 \). Obviously, \( \rho_0 > \rho_1 \).

1. The demand function for national brand in the direct channel is as follows.
\[
d_0 = a_0 + \theta_0 \sqrt{A_0} - (\beta_0 - \lambda) \sqrt{A_1} - \varphi_0 \sqrt{A_2}. \tag{1}
\]

2. The demand function for national brand in the retail channel is as follows.
\[
d_1 = a_1 + \theta_1 \sqrt{A_0} + (\beta_1 - \lambda) \sqrt{A_1} - \varphi_1 \sqrt{A_2}. \tag{2}
\]

3. The demand function for store brand in the retail channel is as follows.
\[
d_2 = a_2 - \theta_2 \sqrt{A_0} - \beta_2 \sqrt{A_1} + \varphi_2 \sqrt{A_2}. \tag{3}
\]

Such a static model is used because the cooperative advertising is intended to generate short-term sales [20]. So we consider one-period sales response volume functions as mentioned above. Such demand functions are tractable and widely used in advertising strategies and supply chain management literature (see, for example, literature [20] and [26]). There are four decision variables in our model. The manufacturer decides the national advertising expenditure \( A_0 \), while the retailer decides the local advertising expenditure on store brand \( A_2 \). The decision power of cooperative advertising expenditure \( A_1 \) and the cost-sharing rate \( t \) depends on chain members’ bargaining power. The cost of local promotional activities for the national brand (e.g., features, displays, merchandising efforts) is denoted as \( A_1 \). If the manufacturer and the retailer together join in a cooperative advertising program, then the leader will decides the cost-sharing rate while the follower decides the cooperative advertising expenditure. The cost-sharing rate also refers to the manufacturers participation rate in the cooperative advertising. Thus, the manufacturer should pay \( tA_1 \) and the retailer should pay \( (1 - t)A_1 \) to promote the national brand in retail shop.

As in Huang et al. [20] and Xie and Wei [44], we assume that the national advertising, cooperative advertising and local advertising for store brand influence sales in different ways. In our model, the parameter \( \theta_i (i = 0, 1, 2) \) represent the national advertising sensitivity coefficient. They describe the efficiency of national advertising in stimulating or decreasing sales. The higher the value of \( \theta_i \) is, the more influence the national advertising has on the demand. The description of parameter \( \beta_i \) and \( \varphi_i (i = 0, 1, 2) \) are similar to that of \( \theta_i \). The effectiveness of advertising on sales has been estimated by Stewart and Pavlou [36]. And the form of \( \sqrt{A_0} \) describes the diminishing effectiveness of advertising on sales. In addition, the parameter \( \lambda \) is denoted as the cross-channel free-riding coefficient and describes the shift between two channels with regards to the cooperative advertising. The higher the value of \( \lambda \) is, the more cross-channel free-riding consumers will be engaged in.
We add some assumptions to the preceding demand functions:

**Assumption 1.** \( \theta_0 > \beta_0 \land \varphi_0, \beta_1 > \theta_1 \land \varphi_1 \land \lambda, \varphi_2 > \theta_2 \land \beta_2. \)

Assumption 1 guarantees that in each channel, the self-advertising sensitivity, in absolute value, is higher than the cross-advertising sensitivity. The similar assumption can also be found in Desai [16] and Karray and Zaccour [26]. In addition, the cooperative advertising will certainly increase the demand of national brand in the retail channel, thus \( \beta_1 > \lambda. \)

**Assumption 2.** The national brand and store brand are homogeneous products. That means they operate in a highly competitive market.

Based on the aforementioned demand functions, the profit functions for supply chain members are formulated respectively as follows:

1. The profit function for the manufacturer is formulated as Eq. (4).
   \[
   \pi_m = \rho_0 d_0 + \rho_1 d_1 - A_0 - t A_1. \tag{4}
   \]

2. The profit function for the retailer is formulated as Eq. (5).
   \[
   \pi_r = \rho_2 d_1 + \rho_3 d_2 - (1-t)A_1 - A_2. \tag{5}
   \]

3. The profit function for the dual-channel supply chain is formulated as Eq. (6).
   \[
   \pi = \pi_m + \pi_r = \rho_0 d_0 + (\rho_1 + \rho_2) d_1 + \rho_3 d_2 - A_0 - A_1 - A_2. \tag{6}
   \]

Eqs. (4) and (5) can be rewritten as follows:

\[
\begin{align*}
\pi_m &= \rho_0 a_0 + \rho_1 a_1 + (\rho_0 \beta_0 + \rho_1 \theta_1) \sqrt{A_0} + \left[\rho_1 (\beta_1 - \lambda) - \rho_0 (\beta_0 - \lambda)\right] \sqrt{A_1}
\quad - (\rho_0 \varphi_0 + \rho_1 \varphi_1) \sqrt{A_2} - A_0 - t A_1, \tag{7}
\end{align*}
\]

\[
\begin{align*}
\pi_r &= \rho_2 a_1 + \rho_3 a_2 + (\rho_2 \beta_1 - \rho_3 \beta_2) \sqrt{A_2} + \left[\rho_2 (\beta_1 - \lambda) - \rho_3 \beta_2\right] \sqrt{A_1}
\quad + (\rho_3 \varphi_2 - \rho_2 \varphi_1) \sqrt{A_2} - (1-t) A_1 - A_2. \tag{8}
\end{align*}
\]

The following assumption should be made to ensure the validity of models as Eqs. (7) and (8).

**Assumption 3.** \( \rho_1 (\beta_1 - \lambda) - \rho_0 (\beta_0 - \lambda) \geq 0, \rho_2 \theta_1 - \rho_3 \theta_2 \geq 0, \rho_2 (\beta_1 - \lambda) - \rho_3 \beta_2 \geq 0 \) and \( \rho_3 \varphi_2 - \rho_2 \varphi_1 \geq 0. \)

Assumption 3 guarantees both the manufacturer’s and the retailer’s participation in cooperative advertising, see, e.g., Karray and Zaccour, [25]. If Assumption 3 cannot be satisfied, either the manufacturer or the retailer will obtain a negative profit, and the cooperative advertising among channel members will not be achieved.

We first consider the centralized case where the common goal of supply chain upstream and downstream members is to maximize the system performance. In this case, both the manufacturer and the retailer operate as an entirety and cooperate to determine the advertising strategies.

**Lemma 3.1.** In the centralized model, the optimal solutions are as follows:

\[
\begin{align*}
A_0^* &= \frac{(\rho_0 \beta_0 + \rho_1 \theta_1 + \rho_2 \beta_1 - \rho_3 \beta_2)^2}{4}, \\
A_1^* &= \frac{[(\rho_1 + \rho_2) (\beta_1 - \lambda) - \rho_0 (\beta_0 - \lambda) - \rho_3 \beta_2]^2}{4}, \\
A_2^* &= \frac{(\rho_3 \varphi_2 - \rho_0 \varphi_0 - \rho_1 \varphi_1 - \rho_2 \varphi_1)^2}{4}.
\end{align*}
\]
The result of the centralized model is a benchmark for the decentralized cases. In reality, the supply chain members are usually independent in the decentralized model and make their own decisions to maximize their profits. According to different bargaining power of the supply chain members, we mainly analyze Stackelberg games and Nash games in the following sections.

4. Stackelberg games and Nash games. To analyze our model, we adopt a game-theoretical approach. Game theory has been an important tool to analyze members’ interactive optimization problems in supply chain management. In this section, two Stackelberg and two Nash game-theoretic models, i.e., SM game, SR game, NM game and NR game are discussed. Different from the centralized case, under the four game models, each chain member will act according to its personal situations and related members’ response.

4.1. SM game. In Stackelberg game, the member who has more bargaining power acts as a leader and the other one acts as a follower. In this subsection, we firstly introduce the case that the manufacturer is a leader and the retailer is a follower, which is denoted as SM game. The SM scenario arises in markets where manufacturers’ sizes are larger than their retailers. The supply chain consists of P&G and Watsons can be regarded as one of the examples of SM game. P&G, one of the world’s largest daily consumer goods company, is the leader in the washing products and personal care products markets. Watsons, who has its own brand in personal care products markets, is also one of the retailers of P&G. There is no doubt that P&G has more bargaining power than Watsons in the supply chain. Similar to Zhang and Xie [48], we also assume that the game follower should not decide the cooperative participation rate. The sequence of events is as follows:

(1) The manufacturer announces the participation rate \( t \) in the cooperative advertising and the national advertising expenditure \( A_0 \), respectively, in order to maximize profit \( \pi_m \).

(2) In response to the advertising strategy of the manufacturer, the retailer determines the cooperative advertising expenditure \( A_1 \), and local advertising expenditure \( A_2 \) to maximize his profit \( \pi_r \).

Lemma 4.1. There exist SM equilibrium, where

\[
A_0^{S,M} = \frac{(\rho_0 \theta_0 + \rho_1 \theta_1)^2}{4}, \quad \pi_0^{S,M} = \frac{2[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)] - \rho_2(\beta_1 - \lambda) + \rho_3 \beta_2}{2[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)] + \rho_2(\beta_1 - \lambda) - \rho_3 \beta_2},
\]

\[A_1^{S,M} = \left[\frac{2(\rho_1 + \rho_2)(\beta_1 - \lambda) - 2\rho_0(\beta_0 - \lambda) - \rho_3 \beta_2}{16}\right]^{2}, \quad A_2^{S,M} = \frac{(\rho_4 \varphi_2 - \rho_5 \varphi_1)^2}{4}.
\]

Denote \( q = [\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)] / [\rho_2(\beta_1 - \lambda) - \rho_3 \beta_2] = B_1 / B_2 \), then \( q > 1 \). It means the ratio of the manufacturer’s marginal profit over the retailer’s marginal profit from the cooperative advertising. If \( q < 1 \), the retailer can gain more marginal profit from the cooperative advertising than the manufacturer. Conversely, the manufacturer can gain more marginal profit than the retailer.

Corollary 1. (1) If \( 0 < q \leq 1/2 \), then \( t^{S,M} = 0 \); (2) If \( q > 1/2 \), then \( t^{S,M} > 0 \).

Corollary 1 presents the precondition that the manufacturer will share the advertising expenditure of the retailer in a dual-channel supply chain. That is, only when \( q > 1/2 \) will the manufacturer share the advertising cost with the retailer. Meanwhile, we can also find that the marginal profit gained by the manufacturer
from cooperative advertising should be more than half of that gained by the retailer. Otherwise, the manufacturer would drop out of the cooperation, and the retailer has to bear all the advertising for national brand in retail store.

**Corollary 2.** (1) $\partial \sqrt{A_0^{S_M^*}}/\partial \theta_0 > \partial \sqrt{A_0^{S_M^*}}/\partial \theta_1$; (2) $\partial t^{S_M^*}/\rho_1 > 0$, $\partial t^{S_M^*}/\rho_2 < 0$, $\partial t^{S_M^*}/\rho_3 > 0$; (3) If $\beta_0 < \lambda$, then $\partial t^{S_M^*}/\rho_0 > 0$; if $\beta_0 \geq \lambda$, then $\partial t^{S_M^*}/\rho_0 < 0$; (4) If $\rho_0 \rho_2/\rho_3 \beta_2 > (\rho_0 - \rho_1)/(\beta_1 - \beta_0) > 0$, then $\partial t^{S_M^*}/\lambda > 0$; if $(\rho_0 - \rho_1)/(\beta_1 - \beta_0) \geq \rho_0 \rho_2/\rho_3 \beta_2 > 0$ or $\beta_1 < \beta_0$, then $\partial t^{S_M^*}/\lambda < 0$.

From Lemma 4.1 and Corollary 2(1), we find that the optimal national advertising is increasing in $\theta_0$ and $\theta_1$, and $\theta_0$ has a greater influence on national advertising. If the national advertising sensitivity coefficient in direct channel increases, namely, the national advertising can bring more sales volume, then the optimal national advertising investment will increase. In addition, the national advertising is positive with $\rho_0$ and $\rho_1$. A high marginal profit strongly induces the manufacturer to invest more to the brand.

Corollary 2(2) shows that the higher marginal profit of the manufacturer in retail channel can always induce him to increase the cost-sharing rate. Similarly, if $\rho_3$ is higher, the manufacturer will improve the cost-sharing rate to strengthen the competitiveness of national brand in the retail store. Conversely, when the manufacturer realizes that $\rho_2$ is higher, then the manufacturer will cut down the cost-sharing rate, so as to transfer a part of the retailers profit to the upstream of the supply chain.

Contrastively, the relationship between $t^{S_M^*}$ and $\rho_0$ is uncertain. When $\beta_0 < \lambda$, then the optimal participation rate is increasing in $\rho_0$. In this case, the number of consumers who engage in cross-channel free-riding is large enough to make up for the reduction of sales volume due to the channel competition. In other words, the cooperative advertising can stimulate the sales volume in the direct channel. Therefore, a high marginal profit in direct channel can motivate the manufacturer to take more cooperative advertising cost, but a low marginal profit in direct channel can lead to the reduction of cost-sharing rate. Additionally, when $\beta_0 \geq \lambda$, the optimal cost-sharing rate is decreasing in $\rho_0$.

Combined with Corollary 2(4), we can know that only when $\frac{\rho_0 \rho_2}{\rho_3 \beta_2} > \frac{\rho_0 - \rho_1}{\beta_1 - \beta_0} > 0$ can the increase of cross-channel free-riding coefficient inspire the manufacturer to increase the cost-sharing rate. Otherwise, the optimal cost-sharing rate is decreasing in $\lambda$.

**Corollary 3.** (1) $A_1^{S_M^*}$ is increasing in $t$; (2) If $\beta_0 < \lambda$, then $\partial \sqrt{A_1^{S_M^*}}/\partial \rho_0 > 0$; otherwise, $\partial \sqrt{A_1^{S_M^*}}/\partial \rho_0 \leq 0$; (3) If $\rho_2 < 2(\rho_0 - \rho_1)$, then $\partial \sqrt{A_1^{S_M^*}}/\partial \lambda > 0$; otherwise, $\partial \sqrt{A_1^{S_M^*}}/\partial \lambda \leq 0$; (4) If $\rho_3 < 2 \rho_0$, then $\partial \sqrt{A_1^{S_M^*}}/\partial \beta_0 < \partial \sqrt{A_1^{S_M^*}}/\partial \beta_2$; otherwise, $\partial \sqrt{A_1^{S_M^*}}/\partial \beta_0 \geq \partial \sqrt{A_1^{S_M^*}}/\partial \beta_2$; (5) If $\rho_2 < \rho_3$, then $\partial \sqrt{A_2^{S_M^*}}/\partial \varphi_2 > -\partial \sqrt{A_2^{S_M^*}}/\partial \varphi_1$; otherwise, $\partial \sqrt{A_2^{S_M^*}}/\partial \varphi_2 \leq -\partial \sqrt{A_2^{S_M^*}}/\partial \varphi_1$. 


Corollary 3(1) shows that the increase of cost-sharing rate can effectively motivate the retailer to increase investment in cooperative advertising. Meanwhile, we can find that when $\beta_0 < \lambda$, the optimal cooperative advertising expenditure is increasing in $\rho_0$. In this case, the manufacturer would like to increase the cost-sharing rate. Such behavior will effectively motivate the retailer to invest more in the cooperative advertising. On the other hand, when $\beta_0 \geq \lambda$, the optimal cooperative advertising expenditure is decreasing in $\rho_0$. Corollary 3(3) shows that the relationship between $A_1^{SM^*}$ and $\lambda$ depends on $\rho_0$, $\rho_1$ and $\rho_2$. When $\rho_2 < 2(\rho_0 - \rho_1)$, the retailer will increase the cooperative advertising expenditure with the increase of cross-channel free-riding coefficient. Conversely, when $\rho_2 \geq 2(\rho_0 - \rho_1)$, the optimal cooperative advertising expenditure is decreasing in $\lambda$.

From Lemma 4.1, we conclude that the optimal local advertising for national brand is negative with $\beta_0$ and $\beta_2$. Corollary 3(4) indicates that when the store brands marginal profit is less than twice of marginal profit gained by the manufacturer from direct channel, then $\beta_0$, rather than $\beta_2$, has a greater influence on the optimal local advertising for national brand. In this case, the retailer will give more consideration to the competition between two channels when he makes decisions. When $\rho_3 = 2\rho_0$, $\beta_0$ and $\beta_2$ have the same sensitivity to the optimal cooperative advertising. When $\rho_3 > 2\rho_0$, the retailer will pay more attention to the degree of the competition between national brand and store brand. The closer features and quality they have, the stronger the substitutability, and the fiercer competition between them.

With Lemma 4.1, we know that the optimal local advertising expenditure is decreasing in $\varphi_2$ while increasing in $\varphi_1$. With Corollary 3(5), we can know that when $\rho_2 < \rho_3$, namely, the retailer can gain higher profit from store brand than national brand, then $\varphi_2$ has a greater influence on $A_2^{SM^*}$ than $\varphi_1$. In contrast, if $\rho_2 > \rho_3$, the retailer will consider more according to the degree of the competition between national brand and store brand.

**Corollary 4.** (1) If $\beta_0 < \beta'$, then $\partial \pi_{m^*}^{SM}/\partial \beta_0 < 0$; otherwise, $\partial \pi_{m^*}^{SM}/\partial \beta_0 \geq 0$; (2) If $\beta_2 < \beta''$, then $\partial \pi_{r}^{SM}/\partial \beta_2 < 0$; otherwise, $\partial \pi_{r}^{SM}/\partial \beta_2 \geq 0$.

where $\beta' = (2\rho_1 + \rho_2)/(\beta_0 - \lambda) - \rho_2 \rho_3/\rho_3$, $\beta'' = (\rho_1 + \rho_2)/(\beta_0 - \lambda) - \rho_0 \rho_3/\rho_3$.

The higher of $\beta_0$ means that the higher efficiency of cooperative advertising in decreasing online channel’s sales. However, Corollary 4(1) illustrates that the increase of $\beta_0$ is not always detrimental to the manufacturer. When $\beta_0 > \beta'$, the increase of $\beta_0$ is beneficial to the manufacturer. If $\beta_0 < \beta'$, the manufacturer’s profit is decreasing in $\beta_0$. Corollary 4(2) presents the relationship between the retailer’s profit and $\beta_2$. The increase of $\beta_2$ means that the cooperative advertising has greater negative effect on the demand of store brand. In other words, the competition between national brand and store brand in retail store will be fiercer, but both of their sales can increase the retailers profit. If $\beta_2 > \beta''$, we can find that the fiercer competition between national brand and store brand in retail channel will result in the higher profit for the retailer. However, when $\beta_2 < \beta''$, the increase of $\beta_2$ will reduce the profit of the retailer.

4.2. **SR game.** We now model the manufacturer-retailer relationship as a Stackelberg game where the retailer has more bargaining power, and is a leader. The SR scenario arises in markets where the retailer’s sizes is large compared to his supplier. Today, large retailers like Wal-Mart and Carrefour, who are powerful in the market,
have greater bargaining power in the supply chain [35]. They can maintain their margin on sales while squeezing profit from their suppliers [30]. As Buratto et al. [5] assumed, the retailer can choose the cost-sharing rate that the manufacturer must undertake in the SR case. Then the manufacturer, as a follower, determines the national advertising and the local advertising for national brand.

**Lemma 4.2.** There exist SR equilibrium, where

\[
A_0^{SR} = \frac{(\rho_0 \theta_0 + \rho_1 \theta_1)^2}{4}, \quad t^{SR} = \frac{2[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)]}{4[\rho_2(\beta_2 - \lambda) - \rho_3(\beta_3 - \lambda)] + [\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)]},
\]

\[
A_1^{SR} = \frac{(2\rho_2 + \rho_1)(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda) - 2\rho_3(\beta_3 - \lambda)}{16}, \quad A_2^{SR} = \frac{(\rho_3 \beta_2 - \rho_2(\beta_1 - \lambda))^2}{4}.
\]

**Corollary 5.** (1) If \( q < 2 \), then \( 0 < t^{SR} < 1 \); (2) If \( q \geq 2 \), then \( t^{SR} = 1 \).

From Corollary 5, we see that when \( q \) is relatively small (\( q < 2 \)), both the manufacturer and the retailer is willing to participate in the cooperative advertising. With \( q \) increasing and falling in \( q \geq 2 \), the retailer will require the manufacturer to pay the entire cost of cooperative advertising. In this case, the marginal profit that the manufacturer derived from the cooperative advertising is more than twice of the retailer’s.

### 4.3. NM game

With the improvement of retailers’ market position, none of the members is dominant in the supply chain. In this case, both the supply chain members have equal bargaining power and make their decisions simultaneously. This situation is called Nash game and the solution to this structure is Nash equilibrium. This scenario occurs in a market in which there are relatively small to medium-sized manufacturers and retailers [30]. Different from the previous studies, we further consider two Nash games according to the decision-making power of cost-sharing rate. We analyze two possible scenarios: NM game (the manufacturer decides the cost-sharing rate) and NR game (the retailer decides the cost-sharing rate). This is reasonable because the cost-sharing rate may be decided by the manufacturer or the retailer. In reality, before the retailer dominates the market over manufacturer, both NM and NR game may arise. The consideration of the two cases can help analyze the possible situations more comprehensively.

In this section, we consider the NM game where the cost-sharing rate is decided by the manufacturer, and the cooperative advertising is decided by the retailer. According to Dai and Chao [14], in case the supplier’s capacity is unlimited, a unique Nash equilibrium always exists. Hence, we can derive the following Nash equilibrium for NM game.

**Lemma 4.3.** There exist NM equilibrium with

\[
A_0^{NM} = \frac{(\rho_0 \theta_0 + \rho_1 \theta_1)^2}{4}, \quad t^{NM} = 0, \quad A_1^{NM} = \frac{[\rho_2(\beta_1 - \lambda) - \rho_3(\beta_2)]^2}{4},
\]

\[
A_2^{NM} = \frac{(\beta_2 - \beta_1)^2}{4}.
\]

Denote \( \lambda' = \frac{\rho_1 - \rho_0}{\rho_2(\beta_1 - \beta_2)}, \lambda'' = \frac{\beta_1 - \beta_2}{\rho_2(\beta_1 - \beta_2)} \).

**Corollary 6.** (1) \( \frac{\partial \pi_m^{NM}}{\partial \beta_0} < 0; \frac{\partial \pi_m^{NM}}{\partial \beta_2} < 0; \frac{\partial \pi_m^{NM}}{\partial \lambda} < 0 \); (2) If \( 0 < \lambda < \lambda' \), then \( \frac{\partial \pi_m^{NM}}{\partial \lambda} > 0 \); if \( \lambda \geq \lambda' \), then \( \frac{\partial \pi_m^{NM}}{\partial \lambda} \leq 0 \); (3) If \( 0 < \lambda < \lambda'' \), then \( \frac{\partial \pi_m^{NM}}{\partial \lambda} > 0 \); if \( \lambda \geq \lambda'' \), then \( \frac{\partial \pi_m^{NM}}{\partial \lambda} \leq 0 \).
In NM game, the manufacturer’s profit will decrease in $\beta_0$ and the retailer’s profit will decrease in $\beta_2$. The higher efficiency of cooperative advertising in decreasing online channel’s sales (store brand’s sales), the lower profit the manufacturer (retailer) can get. However, from Corollary 4 we can know that in SM game, the increase of $\beta_0$ is not always detrimental to the manufacturer and the increase of $\beta_2$ will not always do harm to the profit of the retailer.

Corollary 6(2) shows that the increasing number of consumers who engage in cross-channel free-riding is not always beneficial to the manufacturer. If $\lambda \geq \lambda'$, the manufacturer’s profit will decrease with the increase of cross-channel free-riding coefficient. Corollary 6(3) illustrates that when $0 < \lambda < \lambda''$, the increase of cross-channel free-riding coefficient can improve the profit of the whole supply chain. On the other hand, consumers’ free-riding behavior will certainly lower the profit of the retailer. Therefore, when $\lambda > max\{\lambda', \lambda''\}$, the increasing number of consumers who engage in cross-channel free-riding is detrimental to the manufacturer and the retailer.

**Corollary 7.** (1) If $(\rho_0^2 + \rho_1^2)/[\rho_2(\beta_1 - \lambda) - \rho_3\beta_2] < 1$, then $A_{0N}^M < A_{1N}^M$; otherwise, $A_{0N}^M \geq A_{1N}^M$;

(2) If $[\rho_2(\beta_1 - \lambda) - \rho_3\beta_2]/(\rho_3\varphi_2 - \rho_2\varphi_1) < 1$, then $A_{1N}^M < A_{2N}^M$; otherwise, $A_{1N}^M \geq A_{2N}^M$.

In NM game, the manufacturer does not provide any advertising subsidy and is just responsible for implementing national advertising, while the retailer must bear all the cooperative advertising investment. When $(\rho_0^2 + \rho_1^2)/[\rho_2(\beta_1 - \lambda) - \rho_3\beta_2] < 1$, the retailer’s optimal advertising investment in national brand will be greater than the manufacturer’s national advertising investment. Furthermore, if $[\rho_2(\beta_1 - \lambda) - \rho_3\beta_2]/(\rho_3\varphi_2 - \rho_2\varphi_1) \geq 1$, meanwhile, the retailer’s optimal advertising investment in national brand will be not only greater than the national advertising but also greater than the local advertising for store brand.

**4.4. NR game.** In this section, we consider the NR game where the cost-sharing rate is decided by the retailer and the cooperative advertising expenditure is decided by the manufacturer.

**Lemma 4.4.** There exist NM equilibrium with

$$A_{0N}^* = \frac{(\rho_0^2 + \rho_1^2)^2}{4}, \quad t_{NR}^* = 1, \quad A_{1N}^* = \frac{[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)]^2}{4},$$

$$A_{2N}^* = \frac{(\rho_3\varphi_2 - \rho_2\varphi_1)^2}{4}.$$ 

**Corollary 8.** (1) If $(\rho_0^2 + \rho_1^2)/[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)] < 1$, then $A_{0N}^* < A_{1N}^*$; otherwise, $A_{0N}^* \geq A_{1N}^*$;

(2) If $[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)]/(\rho_3\varphi_2 - \rho_2\varphi_1) < 1$, then $A_{1N}^* < A_{2N}^*$; otherwise, $A_{1N}^* \geq A_{2N}^*$.

In NR game, the retailer requires the manufacturer to provide the full amount of advertising subsidies and is just responsible for local advertising. When $(\rho_0^2 + \rho_1^2)/[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)] > 1$, the manufacturer will invest more advertisement in the direct channel than that of retail channel. On the other hand, when $[\rho_1(\beta_1 - \lambda) - \rho_0(\beta_0 - \lambda)]/(\rho_3\varphi_2 - \rho_2\varphi_1) = 1$, the optimal cooperative advertising is equal to the local advertising.
5. Comparative analysis. Base on the optimal decisions derived above, this section provides the comparison on optimal decisions and profits derived in the previous subsection. Some implications on advertising policies for the dual-channel supply chain are presented.

5.1. Comparative analysis of two Stackelberg games. The comparisons on the optimal decisions and the optimal profits between SM and SR games are concluded in the following propositions.

**Proposition 1.**
(1) $A_{0}^{SM*} = A_{0}^{SR*}$, $A_{2}^{SM*} = A_{2}^{SR*}$;
(2) $t^{SM*} \leq t^{SR*}$, $\pi_{m}^{SM*} \geq \pi_{m}^{SR*}$, $\pi_{r}^{SM*} \leq \pi_{r}^{SR*}$;
(3) If $q < 1$, then $A_{1}^{SM*} < A_{1}^{SR*}$, $\pi_{m}^{SM*} < \pi_{m}^{SR*}$; if $q \geq 1$, then $A_{1}^{SM*} \geq A_{1}^{SR*}$, $\pi_{m}^{SM*} \geq \pi_{m}^{SR*}$.

Proposition 1(1) presents that the optimal national advertising and local advertising for store brand remain the same in SM and SR games. In other words, the optimal national advertising and local advertising for store brand are not affected by the change of leadership. Proposition 1(2) indicates that the cost-sharing rate in SR game is larger than that in SM game. That is because in SR game, the retailer is a leader so he is entitled to decide the cost-sharing rate. Naturally, the retailer will require the manufacturer to share more cost. Meanwhile, it can be observed that the manufacturer can earn higher profit in SM game while the retailer’s profit status becomes better in SR game. Thus, we can conclude that in the Stackelberg game, the supply chain member can obtain higher profit when it is a leader rather than a follower. The result is consistent with the earlier research.

Proposition 1(3) indicates that when the manufacturer’s marginal profit from the cooperative advertising is less than that of the retailer, the manufacturer’s subsidy of cooperative advertising in SM game is less than that in SR game. Similarly, the retailer would reduce his investment in cooperative advertising if his marginal profit from the cooperative advertising is lower. In other words, the leader in Stackelberg game will reduce its investment when it gets a lower marginal profit from the cooperative advertising. Moreover, it is presented that the dual-channel supply chain can get a higher profit if it is dominated by the member whose marginal profit from cooperative advertising is higher.

5.2. Comparative analysis of two Nash games. This section shows the comparisons on equilibrium solutions and optimal profits between NM game and NR game.

**Proposition 2.**
(1) $A_{0}^{NM*} = A_{0}^{NR*}$, $A_{2}^{NM*} = A_{2}^{NR*}$; (2) $t^{NM*} < t^{NR*}$;
(3) If $q < 2$, then $\pi_{m}^{NM*} > \pi_{m}^{NR*}$; if $q \geq 2$, then $\pi_{m}^{NM*} \leq \pi_{m}^{NR*}$;
(4) If $q < 1/2$, then $\pi_{r}^{NM*} > \pi_{r}^{NR*}$; if $q \geq 1/2$, then $\pi_{r}^{NM*} \leq \pi_{r}^{NR*}$;
(5) If $q < 1$, then $A_{1}^{NM*} > A_{1}^{NR*}$, $\pi_{m}^{NM*} > \pi_{m}^{NR*}$; if $q \geq 1$, then $A_{1}^{NM*} \leq A_{1}^{NR*}$, $\pi_{m}^{NM*} \leq \pi_{m}^{NR*}$.

From Proposition 2(1), we see that the optimal national advertising and local advertising for store brand remains the same in NM and NR games. Proposition 2(2) indicates that the cost-sharing rate in NM game is lower than that in NR game. This is reasonable because the retailer will increase the cost-sharing rate when he obtains the decision-making power. As is shown in Proposition 2(3), only when $q < 2$ can the manufacturer gain higher profit in NM game; otherwise, the manufacturer will earn a high profit in NR game. That is to say, under Nash game, it
Proposition 4. (1) If \( q \geq 1/2 \), then the marginal profit gained by the manufacturer from cooperative advertising is more than half of that gained by the retailer. Thus, it is not always beneficial for a manufacturer to get a decision right on the cost-sharing rate. Proposition 2(3) indicates that, when \( q \geq 1/2 \), it is not always profitable for a manufacturer to get a decision right on the cost-sharing rate. Conversely, when \( q < 1 \), the optimal cooperative advertising in NM game is greater than that in NR game. And if \( q < 1 \), the profit in NM game is higher than that in NR game for the whole supply chain.

In summary, if \( q < 1/2 \), for both the manufacturer and the retailer, NM game is superior to NR game. In this case, the chain members can gain higher profit if the cost-sharing rate is decided by the manufacturer. Thus, to increase the whole profit, the manufacturer, whose marginal profit from cooperative advertising is lower than half of that gained by the retailer, should take on the responsibility on deciding the cost-sharing rate. Conversely, when \( q > 2 \), NR game is better than NM game. In other words, if the marginal profit of the manufacturer gained from \( A_1 \) is more than twice of that retailer gains, it is better for the retailer to decide the cost-sharing rate.

From managerial insight, if none of the members can dominate the supply chain, then the member who has a higher marginal profit in the cooperative advertising should give up the decision power on cost-sharing rate voluntarily. Such strategy can help the manufacturer and the retailer together enjoy the premium of the supply chain profit.

5.3. Comparative analysis of five models. By comparing the optimal advertising decisions for the manufacturer and the retailer under the centralized model and four games, we can obtain the following propositions:

**Proposition 3.**
1. \( A_0 > A_0^{S_m} = A_0^{R} = A_0^{N_m} = A_0^{N_r} \);
2. \( A_1^* > \max \{ A_1^{S_m}, A_1^{R}, A_1^{N_m}, A_1^{N_r} \} \);
3. \( A_2^* < A_2^{S_m} = A_2^{R} = A_2^{N_m} = A_2^{N_r} \);
4. \( \pi^* > \max \{ \pi^{S_m}, \pi^{R}, \pi^{N_m}, \pi^{N_r} \} \).

Proposition 3 shows that both manufacturer’s national advertising expenditure and retailer’s local advertising expenditure on store brand remain the same in four games. It also indicates that the national advertising expenditure and cooperative advertising expenditure in the centralized case is higher than that in the decentralized cases. However, retailer’s local advertising expenditure on store brand in the centralized case is lower than that in the decentralized cases. It means that if the supply chain members are integrated as a whole system to make decisions, both the manufacturer and the retailer will devote attention to the promotion of national brand. To some extent, it may suppress the development of store brand. However, the centralized case can bring higher profit for the supply chain. This results from the effect of ‘double marginalization’. And the result is consistent with the previous literature. Moreover, how to allocate the profits between the supply chain members depends on their bargaining powers.

**Proposition 4.**
1. If \( q \geq 1/2 \), then \( A_1^{S_m} > A_1^{N_m} \), \( t^{S_m} > t^{N_m} \); otherwise, \( A_1^{S_m} = A_1^{N_m} \), \( t^{S_m} = t^{N_m} \);
2. If \( 0 < q < 1 \), then \( A_1^{S_r} > A_1^{N_r} \), \( t^{S_r} > t^{N_r} \); otherwise, \( A_1^{S_r} = A_1^{N_r} \), \( t^{S_r} = t^{N_r} \);
3. \( A_1^{S_m} \geq A_1^{N_r} \), \( t^{S_m} \leq t^{N_r} \); (4) \( A_1^{S_r} \geq A_1^{N_m} \), \( t^{S_r} \geq t^{N_m} \).
From Proposition 4(1), we know that SM game is quite different from NM game. In NM game, the manufacturer is reluctant to share any local advertising costs. That is because in NM game, the manufacturer and the retailer determine their strategies independently and simultaneously. The advertising subsidy policy of the manufacturer will have no effect on the retailer’s local advertising investment, thus it will not affect the sales volume of national brand in the retail channel. Obviously, it would be completely meaningless for the manufacturer to share advertising cost with the retailer. When \( q > 1/2 \), the optimal cooperative advertising in SM game is greater than that in NM game; when \( 0 < q \leq 1/2 \), the cost-sharing rate and optimal cooperative advertising in SM game is the same as that in NM game. Proposition 4(2) presents that when \( q < 2 \), the cooperative advertising in SR game is higher than that in NR game, but when \( q \geq 2 \), the manufacturer has to be responsible for all of the cooperative advertising cost in both two games. Proposition 4(3) shows that the cooperative advertising is always higher in SM game than that in NR game. And the cost-sharing rate in SM game is lower than that in NR game. Proposition 4(4) indicates that both the cooperative advertising and the cost-sharing rate in SR game are always higher than that in NR game.

Ahmadi-Javid and Hoseinpour [1] pointed out that the advertising investment depends on marginal profits when the manufacturer is a leader. Our results further conclude that marginal profits play a decisive role in chain member’s advertising investment decisions. In the dual-channel cases, no matter the leader or follower will make the advertising decisions according to the marginal profits.

Conclusively, if there exists supply chain leader, then the cooperative advertising will be higher. The disparity in bargaining power between the two chain members will impel the members to devote attention to the development of national brand.

**Proposition 5.** Comparing SM, SR, NM and NR games, we have

1. If \( q > 1/2 \), then \( \pi_{SM}^* > \pi_{NM}^* \), \( \pi_{SR}^* > \pi_{NR}^* \); otherwise, \( \pi_{SM}^* = \pi_{NM}^* \), \( \pi_{SR}^* = \pi_{NR}^* \);
2. If \( q < 1/2 \), then \( \pi_{SM}^* \geq \pi_{NM}^* \), \( \pi_{SR}^* \geq \pi_{NR}^* \); and if \( q < 1/2 \), then \( \pi_{SM}^* > \pi_{NM}^* \), \( \pi_{SR}^* > \pi_{NR}^* \), if \( q \geq 1/2 \), then \( \pi_{SM}^* \leq \pi_{NM}^* \);
3. If \( q < 2 \), then \( \pi_{SM}^* > \pi_{NM}^* \), \( \pi_{SR}^* > \pi_{NR}^* \); otherwise, \( \pi_{SM}^* = \pi_{NM}^* \), \( \pi_{SR}^* = \pi_{NR}^* \);
4. If \( q < 2 \), then \( \pi_{SM}^* > \pi_{NM}^* \), \( \pi_{SR}^* > \pi_{NR}^* \); and if \( q < 2 \), then \( \pi_{SM}^* < \pi_{NM}^* \), \( \pi_{SR}^* < \pi_{NR}^* \), if \( q \geq 2 \), then \( \pi_{SM}^* \geq \pi_{NM}^* \).

Proposition 5(1) proves that SM game is superior to NM game for both of supply chain members in terms of their profits. With Propositions 1 and 2, we know that when \( q \leq 1/2 \), the SM equilibrium is the same as NM equilibrium. In this case, the manufacturer quits the cooperative advertising. That is, the manufacturer will give up the priority of decision, thus the SM game is equivalent to a NM game. Therefore, the manufacturer and the retailer will prefer SM game to NM game. In other words, the cooperative advertising strategy in dual-channel supply chain is beneficial to the manufacturer and the retailer. Similarly, Proposition 5(3) shows that the SR case can earn higher profit than the NR case can. Combined with 5(1) and (3), Propositions 5(2) and (4) illustrate that the profit of the leader in Stackelberg game is not less than those in both of the two Nash games; moreover, the profit of the supply chain is higher in Stackelberg games than that in Nash games.
Table 1. Equilibrium advertising strategies under four games considering advertising competition in retail channel

| Games | \( A_1^* \) | \( A_2^* \) |
|-------|-------------|-------------|
| SM    | \( \frac{2B_1+B_2}{4-4(p_1+p_2-p_3)\gamma} \) | \( \frac{\rho_2\varphi_2-\rho_3\varphi_1}{2-2(p_3-p_2)\gamma} \) |
| SR    | \( \frac{B_1+2B_2}{4-4(p_1+p_2-p_3)\gamma} \) | \( \frac{\rho_2\varphi_2-\rho_3\varphi_1}{2-2(p_3-p_2)\gamma} \) |
| NM    | \( \frac{B_2}{2-2(p_2-p_3)\gamma} \) | \( \frac{\rho_2\varphi_2-\rho_3\varphi_1}{2-2(p_3-p_2)\gamma} \) |
| NR    | \( \frac{B_1}{2(1-\rho_1\gamma)} \) | \( \frac{\rho_2\varphi_2-\rho_3\varphi_1}{2-2(p_3-p_2)\gamma} \) |

In brief, SM game is superior to the two Nash games when \( q \leq 1/2 \), as the manufacturer and the retailer can gain higher profit in SM game. When \( q \geq 2 \), SR game is better than two Nash games. Considering Propositions 1 to 5, we can conclude that from the perspective of the manufacturer or the retailer, they will certainly choose the Stackelberg game if they possess sufficient bargaining power. Furthermore, the member who has a higher marginal profit in the cooperative advertising is willing to be the follower because of the incremental profit.

6. Extensions.

6.1. Effects of the advertising competition in the retail channel. This section considers the advertising competition in a retail store. To be identified easily, we add ‘a’ to distinguish the model in this section with models in Section 4. According to Tsay and Agrawal [38], we get the demand structure as follows, where \( \gamma \) is a measure of the intensity of competition between the national brand and store brand in a retail store with respect to advertising. The larger the value of \( \gamma \) is, the greater difference between the demands for national brand and store brand in the retail channel is.

\[
d_5^a = a_0 + \theta_0 \sqrt{A_0^a} - (\beta_0 - \lambda) \sqrt{A_1^a} - \varphi_0 \sqrt{A_2^a},
\]

\[
d_1^a = a_1 + \theta_1 \sqrt{A_0^a} + (\beta_1 - \lambda) \sqrt{A_1^a} - \varphi_1 \sqrt{A_2^a} + \gamma (A_1^a - A_2^a),
\]

\[
d_2^a = a_2 - \theta_2 \sqrt{A_0^a} - \beta_2 \sqrt{A_1^a} + \varphi_2 \sqrt{A_2^a} + \gamma (A_2^a - A_1^a).
\]

The profits of the manufacturer and the retailer can be respectively expressed as follows.

\[
\pi_m^a = \pi_m + \rho_1 \gamma (A_1^a - A_2^a); \quad \pi_r^a = \pi_r + (\rho_2 - \rho_3) \gamma A_1^a + (\rho_3 - \rho_2) \gamma A_2^a.
\]

In order to ensure that each player has the motivation to participate in supply chain transaction, we give the following assumption.

Assumption 4. When \( t^a > 0 \), then \( \rho_1 \gamma < t^a \), \( \rho_2 - \rho_3 \gamma < 1 - t^a \), \( (\rho_3 - \rho_2) \gamma < 1 \).

Assumption 4 means that the supply chain members have to afford some advertising expenditure to get additional sales volume.

Lemma 6.1. There exist SM, SR, NM and NR equilibria. They are summarized in Table 1.

Denote \( A_2^* = 2\varphi_2\varphi_1\gamma + 2\varphi_1^2 \) and \( A_2^{*2} = \frac{\rho_3\varphi_3 - \rho_2\varphi_2}{2-2(p_3-p_2)\gamma} \).

The following proposition based on Lemma 6.1 is presented to show the impact of advertising competition in a retail store on supply chain members’ advertising decisions.
Proposition 6. (1) \( A_1^{aN_R} > A_1^{N_R} \); 
(2) If \( \rho_1 + \rho_2 < \rho_3 \), then \( A_1^{aSM} < A_1^{SM} \), \( A_1^{aSR} < A_1^{SR} \); If \( \rho_1 + \rho_2 \geq \rho_1 \), then \( A_1^{aSM} \geq A_1^{SM} \), \( A_1^{aSR} \geq A_1^{SR} \); 
(3) If \( \rho_2 < \rho_3 \), then \( A_1^{aNM} < A_1^{NM} \), \( A_2^{a} > A_2^{N} \); if \( \rho_2 \geq \rho_3 \), then \( A_1^{aNM} \geq A_1^{NM} \), \( A_2^{a} \leq A_2^{N} \).

Proposition 6(1) shows that in NR game, the manufacturer prefers to improve the cooperative advertising expenditure when advertising competition in a retail store exists. The result is reasonable because the manufacturer can enhance the competitiveness of the national brands in the retail channel through cooperative advertising. Proposition 6(2) presents that when the total marginal profit of national brand in the retail channel is less than the marginal profit of store brand, the cooperative advertising expenditure will decrease with respect to \( \gamma \) in two Stackelberg games; otherwise, if there is fiercer competition between cooperative advertising and local advertising, then the supply chain members will improve the cooperative advertising expenditure. Proposition 6(3) indicates that when the marginal profit of national brand for the retailer is less than the marginal profit of store brand, the retailer will reduce the investment in cooperative advertising in NM game considering the competition between cooperative advertising and local advertising. In this case, the retailer will invest more advertising in store brand under four games. This is because the retailer can gain higher profit from the store brand relative to national brand. And the retailer hopes to enhance the competitiveness of his own brand through advertising.

From the above analysis, we see that the marginal profit from each product is very important for both members, particularly when they decide whether or not to invest more money in advertising. In summary, when \( \rho_3 - \rho_1 < \rho_2 < \rho_3 \), advertising competition in the retail channel will encourage the manufacturer and the retailer to increase \( A_1 \) and \( A_2 \) in SM, SR and NR games. However, in NM game, the retailer will reduce the advertising for national brand because that the manufacturer does not offer any advertising subsidy. More discussions are presented in Numerical analysis.

6.2. Effects of the store brand’s introduction. This section considers the effect of the store brand’s introduction. When the retailer should introduce his own brand is discussed. Here, we add ‘\( b \)’ to distinguish the model without store brand with the models in Section 4. The demand structure can be as follows:

\[
d_l^b = a_0 + \theta_0 \sqrt{A_0^b - (\beta_0 - \lambda)} \sqrt{A_1^b}; \quad d_r^b = a_1 + \theta_1 \sqrt{A_0^b + (\beta_1 - \lambda)} \sqrt{A_1^b}.
\] (13)

The profits of the manufacturer, the retailer and the supply chain can be respectively expressed as follows:

\[
\pi_m^b = \rho_0 d_0^b + \rho_1 d_1^b - A_0^b - t^b A_1^b; \quad \pi_r^b = \rho_2 d_1^b - (1 - t^b) A_1^b.
\] (14)

Denote that \( q_1 = \rho_1 (\beta_1 - \lambda) - \rho_0 (\beta_0 - \lambda) / \rho_2 (\beta_1 - \lambda) = B_1 / B_3 \). It means the ratio of the manufacturers and the retailer’s marginal profit from the cooperative advertising without store brand. If \( q_1 < 1 \), the retailer can gain more marginal profit from the cooperative advertising than the manufacturer. Conversely, if \( q_1 > 1 \), the manufacturer can gain more marginal profit from the cooperative advertising than the retailer.

Lemma 6.2. There exist SM, SR, NM and NR equilibria. They are summarized in Table 2.
Table 2. Equilibrium games under four games without store brand

| Equilibria | SM game | SR game | NM game | NR game |
|------------|---------|---------|---------|---------|
| $A_1^b*$  | $(\rho \theta_0 + \rho \theta_1)^{\frac{4}{4}}$ | $(\rho \theta_0 + \rho \theta_1)^{\frac{4}{4}}$ | $(\rho \theta_0 + \rho \theta_1)^{\frac{4}{4}}$ | $(\rho \theta_0 + \rho \theta_1)^{\frac{4}{4}}$ |
| $A_2^b*$  | $\frac{(2B_1 + B_3)^2}{16}$ | $\frac{(2B_3 + B_1)^2}{16}$ | $\frac{B_2^2}{4}$ | $\frac{B_2^2}{4}$ |
| $b^*$     | $\begin{cases} 2B_1 - B_3 & \text{if } q_1 > 1/2 \\
                    0 & \text{if } q_1 < 1/2 \end{cases}$ | $\frac{2B_1}{2B_3 + B_1}$ | 0 | 1 |

Table 3. Effects of store brand’s introduction on advertising decisions

| Games | $A_1$ | $t$ | $A_{1m}$ | $A_{1r}$ |
|-------|-------|-----|----------|----------|
| SM    | $A_{1SM}^b$ | $t_{SM}^*$ | $A_{1SM}^b$ | $A_{1SM}^b$ |
| SR    | $A_{1SR}^b$ | $t_{SR}^*$ | $A_{1SR}^b$ | $A_{1SR}^b$ |
| NM    | $A_{1NM}^b$ | $t_{NM}^*$ | $A_{1NM}^b$ | $A_{1NM}^b$ |
| NM    | $A_{1NM}^b$ | $t_{NM}^*$ | $A_{1NM}^b$ | $A_{1NM}^b$ |

Table 4. Effects of store brand’s introduction on profits

| Games | $\pi_m$ | $\pi_r$ | Comparisons | Conditions |
|-------|---------|---------|-------------|------------|
| SM    | $\pi_{mSM}^b$ | $\pi_{rSM}^b$ | $\pi_{mSM}^b < (\geq) \pi_{rSM}^b$ | $a_2 < (\geq) C_1$ |
| SR    | $\pi_{mSR}^b$ | $\pi_{rSR}^b$ | $\pi_{mSR}^b < (\geq) \pi_{rSR}^b$ | $a_2 < (\geq) C_2$ |
| NM    | $\pi_{mNM}^b$ | $\pi_{rNM}^b$ | $\pi_{mNM}^b < (\geq) \pi_{rNM}^b$ | $a_2 < (\geq) C_3$ |
| NM    | $\pi_{mNM}^b$ | $\pi_{rNM}^b$ | $\pi_{mNM}^b < (\geq) \pi_{rNM}^b$ | $a_2 < (\geq) C_4$ |

From the equilibrium solutions, we can know that the introduction of store brand will affect the advertising decisions of both the manufacturer and the retailer. As a result, it will also have an effect on the profits of the manufacturer and the retailer. The following proposition shows the comparisons on equilibrium solutions and optimal profits between the models without and with store brand under four games respectively.

Denote $A_{1SM}^b = tA_{1SM}^b$, $A_{1SM}^b = (1 - t)A_{1SM}^b$. In the following propositions, $C_i (i = 1, 2, ..., 11)$ and $Z_i (i = 1, 2, 3, 4)$ are the threshold values and given in Appendix A.9.

**Proposition 7.** The comparisons of cooperative advertising and profits between the supply chain with store brand and the supply chain without store brand are shown in Tables 3 and 4.

Proposition 7 indicates that introduction of store brand tends to decrease cooperative advertising. After the introduction of store brand, the retailer will pay more attention to promote his own brand. Thus, the enthusiasm of the retailer to participate in the cooperative advertising weakens. Interestingly, the manufacturer will also decrease the cooperative advertising investment in SR game after the introduction of store brand. In SR game, retailer dominates the market over manufacturer. The relatively small market size and the competition from store brand compel the manufacturer to decrease the cooperative advertising investment. That
is because the profit increment from cooperative advertising is less than the advertising expenditure. So it would be completely meaningless to continue to raise the cooperative advertising expenditure. Proposition 7 also shows that the cost-sharing rates will be higher in Stackelberg games, and they remain the same in Nash games. In SM game, the actual investments in national brand by the manufacturer in the two channels will be higher, but they will be lower in SR game and remain unchanged in two Nash games. With store brand, the retailer would like to reduce the advertising for national brand.

It can be observed that the introduction of store brand makes manufacturer’s profit drop. It results from the demand decrease of national brand. However, only when \(a_2\) is higher than the threshold shown in Table 4 can the retailer benefit from the introduction of store brand. Take SM game for example, when \(a_2\) is relatively large \((a_2 \geq C_1)\), implying that the basic market demand of the store brand is relatively large, a reasonable retailer should introduce his own brand. Thus, it is necessary for the retailer to conduct market search before he creates store brand. Otherwise, if the basic market demand of the store brand is low, then the introduction of store brand will be detrimental for both the manufacturer and the retailer. In summary, introducing a store brand is not always profitable for the retailer, but it is always harmful to the manufacturer. This conclusion seems different from the result of Karray and Zaccour [26]. They show that the store brand introduction is profit-improving for the retailer and the channel although it could harm the manufacturer’s profits. However, our result indicates that store brand introduction is not always profit-improving for the retailer. But it is reasonable because we take consumers’ free-riding behaviors into consideration.

6.3. Effects of direct channel’s introduction. This section considers the model without direct channel to show the effects of direct channel’s introduction. When the manufacturer should open his direct channel is discussed. In this case, the manufacturer implements the national advertising for national brand. We add ‘c’ to distinguish the model with aforementioned models in Sections 4. Similarly to Section 6.2, we get the demand structure as follows.

\[
d^c_1 = a_1 + \theta_1 \sqrt{A_0^c} + \beta_1 \sqrt{A_1^c} - \varphi_1 \sqrt{A_2^c}, \\
\quad \text{(15)}
\]

\[
d^c_2 = a_2 - \theta_2 \sqrt{A_0^c} - \beta_2 \sqrt{A_1^c} + \varphi_2 \sqrt{A_2^c}. \\
\quad \text{(16)}
\]

The profits of the manufacturer and the retailer can be respectively expressed as follows:

\[
\pi^c_m = \rho_1 d^c_1 - A_0^c - t^c A_1^c; \quad \pi^c_r = \rho_2 d^c_1 + \rho_3 d^c_2 - (1 - t^c) A_1^c - A_2^c. \quad \text{(17)}
\]

Denote \(q_2 = \frac{\rho_1 \beta_1}{(\rho_2 \beta_1 - \rho_3 \beta_2)} = \frac{B_4}{B_5} \).

Lemma 6.3. There exist SM, SR, NM and NR equilibria listed in Table 5.

From Table 5, we know that the introduction of direct channel has an effect on advertising decisions of the manufacturer and the retailer; therefore, it will also affect their respective profits. By comparing the single-channel with dual-channel supply chains, we have the following proposition.

Proposition 8. The comparisons of cooperative advertising and profits between the single-channel and dual-channel supply chains are shown in Tables 6 and 7.

Proposition 8 shows that the introduction of direct channel can increase the national advertising in SM, SR and NR games, which depends on the cross-channel
example, when a depends on the baseline sales for the national brand-online. Take SM game for harmful to the retailer. Whether the manufacturer should open direct channel cannot guarantee an extra profit for the manufacturer; also it is not always beneficial for the retailer when the retailer owns his brand. By comparing the profits of the manufacturer and retailer, we can conclude that the introduction of direct channel is free-riding and cooperative-advertising coefficients. In NM game, the retailer would reduce his investment in national brand.

Proposition 8 also provides the conditions under which the manufacturer should open direct channel. We also analyze whether the dual-channel supply chain is beneficial for the retailer when the retailer owns his brand. By comparing the profits of the manufacturer and retailer, we can conclude that the introduction of direct channel cannot guarantee an extra profit for the manufacturer; also it is not always harmful to the retailer. Whether the manufacturer should open direct channel depends on the baseline sales for the national brand-online. Take SM game for example, when \( a_0 \) is relatively small (0 ≤ \( a_0 < C_8 \)), implying that the basic market

| Table 5. Equilibrium solutions under four games without direct channel |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
| Equilibria | SM game | SR game | NM game | NR game |
| \( A_{c*} \) | \( \frac{(1+\theta_1)^2}{4} \) | \( \frac{(1+\theta_1)^2}{4} \) | \( \frac{(1+\theta_1)^2}{4} \) | \( \frac{(1+\theta_1)^2}{4} \) |
| \( A_{1*} \) | \( \frac{(2B_4+B_5)^2}{16} \) | \( \frac{(2B_4+B_5)^2}{16} \) | \( \frac{(2B_4+B_5)^2}{16} \) | \( \frac{(2B_4+B_5)^2}{16} \) |
| \( A_{2*} \) | \( \frac{(\rho_2\varphi_2-\rho_2\varphi_1)^2}{4} \) | \( \frac{(\rho_2\varphi_2-\rho_2\varphi_1)^2}{4} \) | \( \frac{(\rho_2\varphi_2-\rho_2\varphi_1)^2}{4} \) | \( \frac{(\rho_2\varphi_2-\rho_2\varphi_1)^2}{4} \) |
| \( t_{c*} \) | \( 2B_4-B_5 \) | \( 2B_4+B_5 \), \( q_1 > 1/2 \) | \( \frac{2B_4}{2B_4+B_5} \) | \( 0 \) |

| Table 6. Effects of direct channels introduction on advertising decisions |
|-------------------------|-----------------|
| Game type | Cooperative advertising |
| SM | \( A_{1SM}^* < A_{1SM}^* \) | \( C_{5\lambda} < \rho_0\beta_0 \) |
| SR | \( A_{1SM}^* \geq A_{1SM}^* \) | \( C_{5\lambda} \geq \rho_0\beta_0 \) |
| NM | \( A_{1SM}^* \leq A_{1SM}^* \) | \( - \) |
| NR | \( A_{1SM}^* < A_{1SM}^* \) | \( C_{7\lambda} < \rho_0\beta_0 \) |

| Table 7. Effects of direct channels introduction on profits |
|-------------------------|-----------------|-----------------|
| Game type | Manufacturers profits | Retailers Profits |
| SM | \( \pi_{SM}^* < \pi_{SM}^* \) | \( 0 \leq a_0 < C_7 \) | \( \pi_{SM}^* < \pi_{SM}^* \) | \( Z_1 < 0 \) |
| SR | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( a_0 \geq C_8 \) | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( Z_1 > 0 \) |
| NM | \( \pi_{SM}^* < \pi_{SM}^* \) | \( 0 \leq a_0 < C_9 \) | \( \pi_{SM}^* < \pi_{SM}^* \) | \( Z_2 < 0 \) |
| NR | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( a_0 \geq C_7 \) | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( Z_2 > 0 \) |

| Comparisons | Conditions | Comparisons | Conditions |
|-------------|------------|-------------|------------|
| SM | \( \pi_{SM}^* < \pi_{SM}^* \) | \( 0 \leq a_0 < C_8 \) | \( \pi_{SM}^* < \pi_{SM}^* \) | \( Z_1 < 0 \) |
| SR | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( a_0 \geq C_8 \) | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( Z_1 > 0 \) |
| NM | \( \pi_{SM}^* < \pi_{SM}^* \) | \( 0 \leq a_0 < C_9 \) | \( \pi_{SM}^* < \pi_{SM}^* \) | \( Z_2 < 0 \) |
| NR | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( a_0 \geq C_7 \) | \( \pi_{SM}^* \geq \pi_{SM}^* \) | \( Z_2 > 0 \) |
demand of the national brand-online is relatively small, the manufacturer can earn higher profit in the single channel supply chain. When $a_0 \geq C_b$ holds, it is profitable for the manufacturer to open direct channel. Thus, opening a direct channel is not always profitable for the manufacturer. The degree of customer acceptability to the direct channel should be seriously considered by the manufacturer before he opens the direct channel. In addition, from the retailers perspective, he should not blindly oppose the introduction of direct channel.

7. **Numerical analysis.** In this section, one typical numerical experiment will be conducted to corroborate and supplement the previous analysis. We make the following parameter settings: $a_0 = 400, a_1 = 500, a_2 = 300, \theta_0 = 0.7, \beta_0 = 0.3, \varphi_0 = 0.2, \theta_1 = 0.4, \beta_1 = 0.7, \varphi_1 = 0.3, \theta_2 = 0.2, \beta_2 = 0.3, \varphi_2 = 0.7, \rho_0 = 60, \rho_1 = 45, \rho_2 = 50, \rho_3 = 40$. We analyze the sensitivity of the optimal profits with respect to the parameter $\lambda$.

![Figure 2. The profits of the manufacturer versus $\lambda$](image)

Figure 2 depicts the curves of optimal profits of the manufacturer in different model with respect to $\lambda$. The most interesting problem we want to verify is whether the cross-channel free-riding phenomenon is beneficial for the manufacturer. The higher the value of $\lambda$ is, the more cross-channel free-riding consumers will be engaged in. With Corollary 5 we know that when $\lambda > 0.283$, the SR game is equivalent to a NR game. Figure 2 illustrates that with the increase of $\lambda$, the profit of the manufacturer will decrease when manufacturer is the leader. However, if the manufacturer is a follower, its profit has the tendency that becomes higher if the cross-channel free-riding coefficient increases. The conclusion seems different from the previous results, but it is reasonable. It can be demonstrated that the cross-channel free-riding phenomenon is always detrimental for the retailer (Figure 3). Thus, increase of $\lambda$ will reduce the retailer’s enthusiasm of cooperation. In SM and NM cases, the
cooperative advertising expenditure is decided by the retailer. With Lemma 3.1 and Lemma 4.2 we can easily know that the cooperative advertising expenditure in SM and NM games decreases as $\lambda$ increases. Naturally, the decrement of the national brand demand brings less profit for the manufacturer. Therefore, as the leader in supply chain, the manufacturer should take actions to reduce the cross-channel free-riding phenomenon. In addition, some incentives are also needed to raise retailer’s enthusiasm in cooperation.

Figure 3.
The profits of the retailer versus $\lambda$

Figure 3 depicts the curves of optimal profits of the retailer in different model with respect to $\lambda$. When $\lambda > 0.283$, the SR game is equivalent to a NR game. The $\lambda$ is higher means more people engage in cross-channel free-riding. It can be observed that in the four games, the optimal profit of the retailer is decreasing in $\lambda$. In other words, the cross-channel free-riding phenomenon is always harmful to the retailer. Thus, in order to reduce profit loss, the retailer needs to seriously consider the important issue of how to reduce consumers’ cross-channel free-riding behaviors.

Figure 4 depicts the curves of optimal profits of the whole supply chain in different model with respect to $\lambda$. When $\lambda > 0.283$, the SR game is equivalent to a NR game. The figure shows that the cross-channel free-riding is always detrimental to the supply chain. With the three figures above, we can conclude that in SM and NM games, the cross-channel free-riding phenomenon is detrimental to the manufacturer, the retailer and the supply chain. Though the cross-channel free-riding phenomenon can boost the manufacturer’s profit in SR and NR cases, it always decreases the profit of the retailer and the whole supply chain. Thus, both the chain members should jointly control the cross-channel free-riding phenomenon, which makes the profit of the supply chain rise up. In order to motivate the manufacturer to jointly control the cross-channel free-riding phenomenon in SR and NR cases,
the retailer should provide incentives or coordinated contract. Conclusively, some incentives should be provided by the leader, so as to motivate the follower to jointly control the cross-channel free-riding phenomenon under the dual-channel environment. Under such circumstance, the manufacturer and the retailer can together enjoy the premium of the supply chain profit created by the cooperation.

8. Conclusions. In order to enhance competitiveness, more and more retailers start to develop their own brand products. From the aspect of marketing, the traditional selling mode is now gradually giving way to online selling mode. This paper explores a two-echelon supply chain, where a national brand manufacturer opts to open an online channel to compete with the retailer and the retailer sells his own brand at the same time. This research considers the cross-channel free-riding phenomenon, addresses the cooperative advertising and competitive advertising strategies.

Five different scenarios are considered: Centralized case, Stackelberg-manufacturer game, Stackelberg-retailer game, Nash-manufacturer game and Nash-retailer game. Our findings demonstrate that the national brand manufacturer in a dual-channel supply chain will not always willing to take part in the cooperative advertising. In NM game, if the cross-channel free-riding coefficient is large enough, the cross-channel free-riding phenomenon is detrimental to both the manufacturer and the retailer.

By comparing the equilibria in two Stackelberg games, we conclude that the leader in Stackelberg game will reduce its investment in cooperative advertising when it gets a lower marginal profit from the cooperative advertising. The cooperative advertising strategy in the dual-channel supply chain can bring benefit to the manufacturer and the retailer. In the case that none of the members can dominate the supply chain, the decision-making power on the cost-sharing rate should be
hold by the member who has a lower marginal profit in the cooperative advertising. Moreover, from the perspective of supply chain, Stackelberg games are superior to Nash games. In addition, if there exists a leader in the supply chain, then the cooperative advertising will be higher, which is beneficial to the development of national brand. Furthermore, the effects of the introduction of store brand are illustrated. It is concluded that introducing a store brand is not always profitable for the retailer, but it is always harmful to the manufacturer.

Numerical analysis shows that the cross-channel free-riding phenomenon is profitable for the manufacturer in SR and NR cases, but it always decreases the profit of the retailer and the whole supply chain. In order to increase profits, some incentives should be provided by the leader to coordinate the supply chain.

This research can be extended in several directions in future work. For example, this paper only studies the advertising strategies. Further research can involve that the demand function is affected by both prices and advertising. Moreover, cash flow constraint can be studied when advertising investment is limited.

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Appendix A. A.1. Proof of Lemma 4.1

Proof. In a Stackelberg game, the follower responds according to its observation of the leader’s strategy. Thus, in the SM game, we begin by first solving for the reaction function of the retailer, given that the retailer has observed the decisions made by the manufacturers (on national advertising expenditure and cost-sharing rate). The leader’s decision problem is solved based on the followers response.

The first order condition can be shown as

\[
\frac{\partial \pi^{SM}_{r}}{\partial \sqrt{A^{SM}_{1}}} = \rho_{2} \theta_{1} - \rho_{3} \beta_{2} - 2(1 - t^{SM}) \sqrt{A^{SM}_{1}},
\]

\[
\frac{\partial \pi^{SM}_{r}}{\partial \sqrt{A^{SM}_{2}}} = -\rho_{2} \varphi_{1} + \rho_{3} \varphi_{2} - 2 \sqrt{A^{SM}_{2}}.
\]

Checking for optimality by calculating the second order condition, we have:

\[
\frac{\partial^{2} \pi^{SM}_{r}}{\partial \sqrt{A^{SM}_{1}}^{2}} = -2(1 - t^{SM}) < 0, \quad \frac{\partial^{2} \pi^{SM}_{r}}{\partial \sqrt{A^{SM}_{1}} \partial \sqrt{A^{SM}_{2}}} = \frac{\partial^{2} \pi^{SM}_{r}}{\partial \sqrt{A^{SM}_{2}} \partial \sqrt{A^{SM}_{1}}} = 0, \quad \frac{\partial^{2} \pi^{SM}_{r}}{\partial \sqrt{A^{SM}_{2}}^{2}} = -2 < 0,
\]

Let \( H \) be a Hessian of \( \pi^{SM}_{r} \):

\[
H = \begin{bmatrix}
-2(1 - t^{SM}) & 0 \\
0 & -2
\end{bmatrix}.
\]

Thus, the Hessian is negative definite because \( H_{11} < 0 \), and \( \text{det}(H) = 4(1 - t^{SM}) > 0 \). Hence, \( \pi^{SM}_{r} \) is jointly concave in \( \sqrt{A^{SM}_{1}} \) and \( \sqrt{A^{SM}_{2}} \).

Given the above first and second order optimality conditions, the following expression provides the retailer’s reaction function:

\[
A^{SM*}_{1} = \left[ \frac{\rho_{2}(\beta_{1} - \lambda) - \rho_{3} \beta_{2}}{2(1 - t^{SM})} \right]^{2}
\]
Using the retailer’s reaction function, we can derive the manufacturer’s optimal national advertising expenditure and cost-sharing rate. This is carried out by maximizing the manufacturers profit shown in Eq. (4), given $A_1^{SM*}$ and $A_2^{SM*}$ in Eq. (18) and (19), respectively. Thus, the optimal solution can be calculated as follows:

$$A_2^{SM*} = \left( \frac{\beta_2 \varphi_2 - \beta_2 \varphi_1}{2} \right)^2$$ (19)

A.2. Proof of Corollary 1

**Proof.** With Lemma 4.1 and Lemma 4.3, we can get the following equation.

To determine the Nash equilibrium, manufacturer and retailer’s decision

A.3. Proof of Lemma 4.3

**Proof.** To determine the Nash equilibrium, manufacturer and retailer’s decision problems are solved separately and these are as follows:

Let $\frac{\partial \pi_M^{NM*}}{\partial \sqrt{A_0^{NM}}} = 0$, then $A_0^{NM*} = \frac{(\rho_0 \theta_0 + \rho_1 \theta_1)^2}{4}$.

$$\frac{\partial^2 \pi_M^{NM*}}{\partial \sqrt{A_1^{NM}}} = -2 < 0, \quad \frac{\partial^2 \pi_M^{NM*}}{\partial \sqrt{A_1^{NM}} \partial \sqrt{A_2^{NM}}} = \frac{\partial^2 \pi_M^{NM*}}{\partial \sqrt{A_2^{NM}} \partial \sqrt{A_1^{NM}}} = 0,$$

$$\frac{\partial^2 \pi_M^{NM*}}{\partial \sqrt{A_2^{NM}}} = -2 < 0,$$

Let $H$ be a Hessian of $\pi_r^{NM*}$: $H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$.

The Hessian is negative definite because $H_{11} < 0$, and $det(H) = 4 > 0$. Hence, $\pi_r^{NM*}$ is jointly concave in $\sqrt{A_1^{NM}}$ and $\sqrt{A_2^{NM}}$ with the following equations.

$$\frac{\partial^2 \pi_r^{NM*}}{\partial \sqrt{A_1^{NM}}} = \rho_2 (\beta_1 - \lambda) - \rho_3 \varphi_2 - 2 \sqrt{A_1^{NM}} = 0,$$

$$\frac{\partial^2 \pi_r^{NM*}}{\partial \sqrt{A_2^{NM}}} = -\rho_2 \varphi_1 + \rho_3 \varphi_2 - 2 \sqrt{A_2^{NM}} = 0.$$
Thus $\pi_m^{S_m} \geq \pi_m^{N_m}$. When $q = 1/2$, then $\pi_m^{S_m} = \pi_m^{N_m}$.

For the retailer,

$$\pi_r^{S_m} - \pi_r^{N_m} = \frac{1}{16} [2\rho_1(\beta_1 - \lambda) - 2\rho_0(\beta_0 - \lambda) - \rho_1(\beta_1 - \lambda) + \rho_3\beta_2]^2 \geq 0.$$

If $q > 1/2$, then $\pi_r^{S_m} > \pi_r^{N_m}$; if $q = 1/2$, then $\pi_r^{S_m} = \pi_r^{N_m}$; if $q < 1/2$, then $t^{S_m} = t^{N_m} = 0$, $\pi_r^{S_m} > \pi_r^{N_m}$.

For the supply chain,

$$\pi_r^{S_m} - \pi_r^{N_m} = \pi_m^{S_m} - \pi_m^{N_m} + \pi_r^{S_m} - \pi_r^{N_m}$$

$$= \frac{1}{16} [2\rho_1(\beta_1 - \lambda) - 2\rho_0(\beta_0 - \lambda) - \rho_1(\beta_1 - \lambda) + \rho_3\beta_2]^2$$

If $q > 1/2$, then $\pi_r^{S_m} > \pi_r^{N_m}$; if $q \leq 1/2$, then $\pi_r^{S_m} = \pi_r^{N_m}$.

Similarly we can prove (2) to (4).

A.5. Threshold values

$$C_1 = D_1 + \frac{B_2^2 - B_3^2 + 2B_1\rho_3\beta_2}{8\rho_3}, \quad C_2 = D_1 + \frac{\rho_3\beta_2(B_1 - \rho_3\beta_2)}{4\rho_3},$$

$$C_3 = D_1 + \frac{B_4^2 - B_5^2}{4\rho_3}, \quad C_4 = D_1 + \frac{B_1\beta_2}{2}, \quad C_5 = \rho_0 - \rho_1 - \rho_2/2,$$

$$C_6 = \rho_0 - \rho_1 - 2\rho_2, \quad C_7 = \rho_0 - \rho_1,$$

$$C_8 = D_2 - \frac{[(2B_1 + B_2)^2 - (2B_4 + B_5)^2]}{16\rho_0},$$

$$C_9 = D_2 - \frac{[B_3(2B_2 + B_1) - B_4(2B_5 + B_4)^2]}{8\rho_0},$$

$$C_{10} = D_2 - \frac{B_1B_2 - B_4B_5}{2\rho_0}, \quad C_{11} = D_2 - \frac{B_1^2 - B_3^2}{4\rho_0},$$

$$D_1 = \frac{\theta_2(\rho_0\theta_0 + \rho_1\theta_1)}{2} \frac{(\rho_3\varphi_2 - \rho_2\varphi_1)^2}{4\rho_3},$$

$$D_2 = \frac{\varphi_0(\rho_3\varphi_2 - \rho_2\varphi_1)}{2} \frac{\theta_0(\rho_0\theta_0 + 2\rho_1\theta_1)}{4},$$

$$D_3 = 4\rho_0\theta_0(\rho_3\theta_1 - \rho_3\theta_2),$$

$$F_1 = \rho_0(\rho_2\beta_0 + \rho_2\beta_1 - \rho_3\beta_2) - \rho_1(2\rho_2\beta_1 - \rho_3\beta_2),$$

$$Z_1 = D_3 + B_3(2B_1 + B_2) - B_5(2B_4 + B_5),$$

$$Z_2 = D_3 + (2B_2 + B_1)^2 - (2B_5 + B_4)^2,$$

$$Z_3 = D_3/2 + B_2^2 - B_4^2, \quad Z_4 = D_3/4 + B_1B_2 - B_4B_5.$$
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