Texture Zero Neutrino Models and Their Connection with Resonant Leptogenesis

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Abstract

Within the low scale resonant leptogenesis scenario, the cosmological CP asymmetry may arise by radiative corrections through the charged lepton Yukawa couplings. While in some cases, as one expects, decisive role is played by the $\lambda_\tau$ coupling, we show that in specific neutrino textures only by inclusion of the $\lambda_\mu$ the cosmological CP violation is generated at 1-loop level.

With the purpose to relate the cosmological CP violation to the leptonic CP phase $\delta$, we consider an extension of MSSM with two right handed neutrinos (RHN), which are degenerate in mass at high scales. Together with this, we first consider two texture zero $3 \times 2$ Dirac Yukawa matrices of neutrinos. These via see-saw generated neutrino mass matrices augmented by single $\Delta L = 2$ dimension five ($d = 5$) operator give predictive neutrino sectors with calculable CP asymmetries. The latter is generated through $\lambda_{\mu,\tau}$ coupling(s) at 1-loop level. Detailed analysis of the leptogenesis is performed. We also revise some one texture zero Dirac Yukawa matrices, considered earlier, and show that addition of a single $\Delta L = 2$, $d = 5$ entry in the neutrino mass matrices, together with newly computed 1-loop corrections to the CP asymmetries, give nice accommodation of the neutrino sector and desirable amount of the baryon asymmetry via the resonant leptogenesis even for rather low RHN masses($\sim$few TeV – $10^7$ GeV).

Keywords: CP violation; Resonant Leptogenesis; Neutrino mass and mixing; Renormalization.

PACS numbers: 11.30.Er, 98.80.Cq, 14.60.Pq, 11.10.Gh.

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1 Introduction

Problem of neutrino masses and generation of the baryon asymmetry of the Universe, together with the dark matter problem and naturalness issues, call for some reasonable extension(s) of the Standard Model (SM). Perhaps simplest and most elegant simultaneous resolution of the first two puzzles is by the SM extension with the right handed neutrinos (RHN). This, by the $\Delta L = 2$ lepton number violating interactions generates the neutrino masses via celebrated see-saw mechanism [1, 2], accommodating the atmospheric and solar neutrino data [3], and gives an elegant possibility for the baryogenesis through the thermal leptogenesis [4] (for reviews see Refs. [5–7]).

Motivated by these, we consider the minimal supersymmetric standard model (MSSM) augmented by two degenerate RHNs. Note that the degeneracy in the RHN mass spectrum offers an elegant possibility of resonant leptogenesis [8–10] (see [11–15] for recent discussions on resonant leptogenesis). This framework, as it was shown in [15–17], with specific forms of the Yukawa couplings, allows to have highly predictive model. In particular, in [18] all possible two texture zero $3 \times 2$ Dirac type neutrino Yukawa couplings have been considered. Those, via see-saw generated neutrino mass matrices augmented by a single $d = 5$, $\Delta L = 2$ operator, gave consistent neutrino scenarios. As it was shown, all experimentally viable cases allowed to calculate the cosmological CP violation in terms of a single known (from the model) leptonic phase $\delta$. In the subsequent work [15], the quantum corrections, primarily due to the $\lambda_{\tau}$ Yukawa coupling, have been investigated and, confirming earlier claim of Refs. [16], it was shown that the cosmological CP asymmetry arises at 1-loop order. Demonstrated on a specific fully consistent neutrino model [15], this was shown to work well and opened wide prospect for the model building for the low scale resonant leptogenesis.

The goals of this work are following. First we give detailed and conscious derivation of the loop induced leptonic cosmological CP violation showing the necessity of inclusion of the charged lepton Yukawa couplings. Proof includes analytical expressions and is extended by inclusion of the $\lambda_{\mu}$ coupling which as it turns out in specific neutrino scenarios is the only relevant source of the cosmological CP violation within considered scenarios with the RHN masses $\lesssim 10^7$ GeV. We apply obtained result to specific neutrino textures. While in Refs. [19–22], [16], [17], [23–25] the textures relating the cosmological CP violation to the leptonic $\delta$ phase (being still undetermined from the construction) have been discussed, in [18] we have proposed models, which not only give such relations, but also predict the values of the $\delta$ (the leptonic Dirac phase) and $\rho_{1,2}$ (two leptonic Majorana phases) and consequently the cosmological CP violation. From the constructions of [18] we consider viable neutrino models built by two texture zero $3 \times 2$ Yukawa coupling generated see-saw neutrino mass matrices augmented by the single $\Delta L = 2$, $d = 5$ operator. For all these neutrino models, applying obtained all relevant corrections, we investigate the resonant leptogenesis process, which has not been performed before. Along with the cases where crucial is $\lambda_{\tau}$ coupling, we have ones for which the leptonic asymmetry originates due to the $\lambda_{\mu}$ Yukawa coupling. Such possibility has not been presented before in the literature. We also revise textures of [17] and consider their improved versions by addition of single $d = 5$ entry to the neutrino mass matrix, making them consistent and also viable for the baryogenesis. The details of the calculation of the

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3This setup with the SUSY scale $M_S \sim$ few TeV guarantees the natural stability of the EW scale.
4The approach with texture zeros has been put forward in [19], which successfully relates the phase $\delta$ with the cosmological CP asymmetry [19–22], [16], [17], [23–25], [18], [15].
5Studies of [17] included only $\lambda_{\tau}$’s 2-loop effects in the RG of the RHN mass matrix, which give parametrically more suppressed cosmological CP violation in comparison with those evaluated in [15].
contribution to the leptonic asymmetry from the right handed sneutrino decays are given as well. These include new corrections corresponding to the muon lepton soft SUSY breaking terms. Also, refined and more accurate expressions for the decay widths and absorptive parts, relevant for the CP asymmetries, are used.

Although in this work we are using the results of the loop induced cosmological CP violation (summarized in section 2 and in Appendixes A, B) for specific texture zero models, the application can be extended to any model with two (quasi) degenerate RHNs.

The paper is organized as follows. In section 2 after defining the setup with two degenerate RHNs, we give details of the calculation of the loop induced cosmological CP violation. Mainly we follow the method of Ref. [15] proving inevitable emergence of the cosmological CP violation via charged lepton Yukawas at 1-loop level, confirming earlier result of [46] (which took into account \( \lambda_\tau \) coupling). We also include the contribution due to the \( \lambda_\mu \) which has not been considered before. In section 3 first we list all possible two texture zero \( 3 \times 2 \) Yukawa matrices, considered in [18]. The see-saw neutrino mass matrices, obtained from these Yukawa textures, are augmented by the addition of single \( \Delta L = 2, \ d = 5 \) mass terms to certain zero entries. This makes the list of the phenomenologically viable and predictive neutrino mass matrices. From them we pick up those which involve complexities and have potential for the CP asymmetry. With the updated neutrino data, we give updated results of the corresponding neutrino models which are highly predictive and determine cosmological CP violating phases in terms of the \( \delta \) phase. In section 4 applying results of the previous sections we determine cosmological CP violation for each considered model and use them for calculating of the baryon asymmetry. The latter is generated via resonant leptogenesis. We demonstrate that successful scenarios are possible for the low RHN masses (in a range few TeV – \( 10^7 \) GeV). In section 5 we revise textures of Ref. [17] and make model improvements of the obtained neutrino mass matrices by adding the single \( \Delta L = 2, \ d = 5 \) mass terms to certain non-zero entries (in a spirit of Sect. 3). This makes the neutrino scenarios compatible with the best fit values of the neutrino data [3] and also proves to blend well with the leptogenesis scenarios. We stress that in the \( P_4 \) neutrino texture scenario (discussed in Sect. 3) and also in the texture \( B_2' \) (considered in Sect. 5), for successful leptogenesis to take place crucial role is played by the \( \lambda_\mu \) Yukawa coupling which via 1-loop correction generates sufficient amount of the cosmological CP asymmetry. Such possibility has not been considered in the literature before. (The general expressions for the corresponding corrections are presented in Sect. 2). Sect. 6 includes discussion and outlook where we also summarize our results and highlight some prospects for a future work. Appendix A includes some expressions, details related to the renormalization group (RG) studies and description of calculation procedures we are using. In Appendix B the contribution to the net baryon asymmetry from the decays of the scalar components (RHS) of the RHN superfields is considered in detail. These analyses also include new corrections due to \( \lambda_\mu \) and corresponding soft SUSY breaking trilinear \( A_\mu \) coupling (besides \( \lambda_\tau, A_\tau \) and other relevant couplings).

## 2 Loop Induced Calculable Cosmological CP Violation

Before going to the calculations we first describe our setup. The framework is the MSSM augmented with two right-handed neutrinos \( N_1 \) and \( N_2 \). This extension is enough to build consistent neutrino sector accommodating the neutrino data [3] and also to have a successful leptogenesis scenario.
The relevant lepton superpotential couplings are given by:

\[ W_{\text{lept}} = l^T Y_e \text{diag} e^c h_d + l^T Y_\nu N h_u - \frac{1}{2} N^T M_N N, \quad (2.1) \]

where \( h_d \) and \( h_u \) are down and up type MSSM Higgs doublet superfields respectively and \( l^T = (l_1, l_2, l_3) \), \( e^c T = (e^c_1, e^c_2, e^c_3) \), \( N^T = (N_1, N_2) \). We work in a basis in which the charged lepton Yukawa matrix is diagonal and real:

\[ Y_e^\text{diag} = \text{Diag}(\lambda_e, \lambda_\mu, \lambda_\tau). \quad (2.2) \]

Moreover, we assume that the RHN mass matrix \( M_N \) is strictly degenerate at the GUT scale, which will be taken to be \( M_G \approx 2 \times 10^{16} \text{ GeV} \). Therefore, we assume:

\[ \text{at } \mu = M_G : \quad M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(M_G). \quad (2.3) \]

This form of \( M_N \) is crucial for our studies. Although it is interesting and worth to study, we do not attempt here to justify the form of \( M_N \) (and of the textures considered below) by symmetries. Our approach here is rather phenomenological aiming to investigate possibilities, outcomes and implications of the textures we consider. Since \( (2.3) \) at a tree level leads to the mass degeneracy of the RHN’s, it has interesting implications for resonant leptogenesis \[16,17,22\] and also, as we will see below, for building predictive neutrino scenarios \[17,18\].

For the leptogenesis scenario two necessary conditions need to be satisfied. First of all, at the scale \( \mu = M_{N_{1,2}} \) the degeneracy between the masses of \( N_1 \) and \( N_2 \) has to be lifted. And, at the same scale, the neutrino Yukawa matrix \( Y_\nu \) - written in the mass eigenstate basis of \( M_N \), must be such that \( \text{Im}[(Y_\nu^\dagger Y_\nu)_{12}] \neq 0 \). [These can be seen from Eq. \[4.1\] with a demand \( \epsilon_{1,2} \neq 0 \]. Below we show that both of them are realized by radiative corrections and needed effect already arises at 1-loop level, with a dominant contribution due to the \( Y_e \) Yukawa couplings (in particular from \( \lambda_\tau \) and in some cases from \( \lambda_\mu \)) in the RG.

As it was shown \[46,17,15\], within considered setup, radiative corrections are crucial for generating cosmological CP violation. In particular, the needed asymmetry is generated at 1-loop level due to \( \lambda_\tau \) Yukawa coupling provided that the condition \( (Y_\nu)_{31}(Y_\nu)_{32} \neq 0 \) is satisfied \[15\]. Here, to be more generic and to not limit the class of the models, we also include the effects of the \( \lambda_\mu \) Yukawa coupling in the calculation\[7\]. Thus, in this section we present details of these calculations. We will start with radiative corrections to the \( M_N \) matrix. RG effects cause lifting of the mass degeneracy and, as we will see, are important also for the phase misalignment (explained below).

At the GUT scale, the \( M_N \) has off-diagonal form with \( (M_N)_{11} = (M_N)_{22} = 0 \) [see Eq. \[2.3\]]. However, at low energies, RG corrections generate these entries. Thus, we parameterize the matrix \( M_N \) at scale \( \mu \) as:

\[ M_N(\mu) = \begin{pmatrix} \delta_N^{(1)}(\mu) & 1 \\ 1 & \delta_N^{(2)}(\mu) \end{pmatrix} M(\mu). \quad (2.4) \]

\[6\]Degeneracy of \( M_N \) can be guaranteed by some symmetry at high energies. For concreteness, we assume this energy interval to be \( \geq M_G \) (although the degeneracy at lower energies can be considered as well).

\[7\]In Sections 4 and 5, among other neutrino scenarios, we consider ones for which such corrections are crucial for generation of the needed amount of Baryon asymmetry.
While all entries of the matrix \( M_N \) run, for our studies will be relevant the ratios \( \frac{(M_N)_{11}}{(M_N)_{12}} = \delta_N^{(1)} \) and \( \frac{(M_N)_{22}}{(M_N)_{12}} = \delta_N^{(2)} \) (obeying the RG equations investigated below). That’s why \( M_N \) was parametrized in a form given in Eq. (2.4). With \( |\delta_N^{(1,2)}| \ll 1 \), the \( M \) (at scale \( \mu = M \)) will determine the masses of RHNs \( M_1 \) and \( M_2 \), while \( \delta_N^{(1,2)} \) will be responsible for their splitting and for complexity in \( M_N \) (the phase of the overall factor \( M \) does not contribute to the physical CP). As will be shown below:

\[
\delta_N^{(1)} = (\delta_N^{(2)})^* \equiv -\delta_N.
\] (2.5)

Therefore, \( M_N \) is diagonalized by the transformation

\[
U_N^T M_N U_N = M_N^{\text{Diag}} = \text{Diag} \left( M_1, M_2 \right), \quad \text{with} \quad U_N = P_N O_N P_N',
\]

\[
M_1 = |M| (1 - |\delta_N|), \quad M_2 = |M| (1 + |\delta_N|),
\] (2.6)

where

\[
P_N = \text{Diag} \left( e^{-i\eta/2}, e^{i\eta/2} \right), \quad O_N = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array} \right), \quad P_N' = \text{Diag} \left( e^{-i\phi M/2}, ie^{-i\phi M/2} \right),
\]

\[\text{with} \quad \eta = \text{Arg} (\delta_N), \quad \phi_M = \text{Arg} (M).\] (2.7)

In the \( N \)'s mass eigenstate basis, the Dirac type neutrino Yukawa matrix will be \( \tilde{Y}_\nu = Y_\nu U_N \). In the CP asymmetries, the components \( (\tilde{Y}_\nu^{(1)} \tilde{Y}_\nu)_{21} \) and \( (\tilde{Y}_\nu^{(1)} \tilde{Y}_\nu)_{12} \) appear [see Eq. (4.1)]. From (2.6) and (2.7) we have

\[
\left[ (\tilde{Y}_\nu^{(1)} \tilde{Y}_\nu)_{21} \right]^2 = - \left[ (O_N^T P_N^* Y_\nu^\dagger Y_\nu P_O N_2)_{21} \right]^2, \quad \left[ (\tilde{Y}_\nu^{(1)} \tilde{Y}_\nu)_{12} \right]^2 = - \left[ (O_N^T P_N^* Y_\nu^\dagger Y_\nu P_O N_2)_{22} \right]^2.
\] (2.8)

Therefore, the CP violation should come from \( P_N^* Y_\nu^\dagger Y_\nu P_O \), which in a matrix form is:

\[
P_N^* Y_\nu^\dagger Y_\nu P_O = \left( \frac{(Y_\nu^\dagger Y_\nu)_{11}}{(Y_\nu^\dagger Y_\nu)_{21}} |(Y_\nu^\dagger Y_\nu)_{12}|^{\eta - \eta'} e^{i(\eta' - \eta)} \right), \quad \text{with} \quad \eta' = \text{Arg}[(Y_\nu^\dagger Y_\nu)_{21}].
\] (2.9)

We see that \( \eta' - \eta \) difference (mismatch) will govern the CP asymmetric decays of the RHNs. Without including the charged lepton Yukawa couplings in the RG effects we will have \( \eta' \simeq \eta \) with a high accuracy. It was shown in Ref. [13] that by ignoring \( Y_e \) Yukawas no CP asymmetry emerges at \( \mathcal{O}(Y_e^4) \) order and non-zero contributions start only from \( \mathcal{O}(Y_e^6) \) terms [14]. Such corrections are extremely suppressed for \( Y_e \lesssim 1/50 \). Since in our consideration we are interested in cases with \( M_{1,2} \lesssim 10^7 \text{ GeV} \) leading to \( |(Y_\nu)_{ij}| < 7 \cdot 10^{-4} \) (well fixed from the neutrino sector and the desired value of the baryon asymmetry), these effects (i.e. order \( \sim Y_e^6 \) corrections) will not have any relevance. In Ref. [17] in the RG of \( M_N \) the effect of \( Y_e \), coming from 2-loop corrections, was taken into account and it was shown that sufficient CP violation can emerge. Below we show that including \( Y_e \) in the \( Y_\nu \)'s 1-loop RG, will induce sufficient amount of CP violation. This mainly happens via \( \lambda_\tau \) and in particular cases (which are considered below) from \( \lambda_\mu \) Yukawa couplings. Thus, below we give detailed investigation of \( \lambda_{\tau,\mu} \)'s effects.

Using \( M_N \)'s RG given in Eq. (A.3) (of Appendix A.1), for \( \delta_N^{(1,2)} \), which are the ratios \( \frac{(M_N)_{11}}{(M_N)_{12}} \) and \( \frac{(M_N)_{22}}{(M_N)_{12}} \), [see parametrization in Eq. (2.4)], we can derive the following RG equations:

\[
16\pi^2 \frac{d}{dt} \delta_N^{(1)} = 4(Y_\nu^\dagger Y_\nu)_{21} + 2\delta_N^{(1)} [(Y_\nu^\dagger Y_\nu)_{11} - (Y_\nu^\dagger Y_\nu)_{22}] - 2(\delta_N^{(1)})^2 (Y_\nu^\dagger Y_\nu)_{12} - 2\delta_N^{(1)} \delta_N^{(2)} (Y_\nu^\dagger Y_\nu)_{21}
\]
were in second lines of (2.10) and (2.11) are given 2-loop corrections depending on $Y_e$. Dots there stand for higher order irrelevant terms. From 2-loop corrections we keep only $Y_e$ dependent terms. Remaining contributions are not relevant for us\footnote{Omitted terms are either strongly suppressed or do not give any significant contribution to either the CP violation or the RHN mass splittings.} From (2.10) and (2.11) we see that dominant contributions come from the first terms of the r.h.s. and from those given in the second rows. Other terms give contributions of order $O(Y_e^8)$ or higher and thus will be ignored. At this approximation we have
\begin{equation}
\delta^{(1)}_N(t) \simeq \delta^{(2)}_{N*}(t) \equiv -\delta_N(t) \simeq -\frac{1}{4\pi^2} \int^{t_G}_t dt \left( Y^\dagger Y (1 - \frac{1}{16\pi^2} Y e^\dagger Y e) Y^\dagger \right)_{21} \tag{2.12}
\end{equation}
where $t = \ln \mu$, $t_G = \ln M_G$ and we have used the boundary conditions at the GUT scale $\delta^{(1)}_N(t_G) = \delta^{(2)}_N(t_G) = 0$. For evaluation of the integral in (2.12) we need to know the scale dependence of $Y^\dagger Y$ and $Y_e$. This is found in Appendix A.1 by solving the RG equations for $Y^\dagger Y$ and $Y_e$. Using Eqs. (A.5) and (A.6), the integral of the matrix appearing in (2.12) can be written as:
\begin{equation}
\int^{t_G}_{t_M} dt \frac{\bar{r}_\tau(t)(1 - \frac{\lambda^2}{16\pi^2})}{\int_{t_M}^{t_G} \kappa(t) dt} \frac{\bar{r}_\mu(M)(1 - \frac{\lambda^2}{16\pi^2})}{\int_{t_M}^{t_G} \kappa(t) dt} \approx \kappa(M) Y^{\dagger \nu G} \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \bar{r}_\mu(M) & 0 \\ 0 & 0 & \bar{r}_\tau(M) \end{array} \right) Y^{\dagger \nu G} \tag{2.13}
\end{equation}
where
\begin{align}
\bar{r}_\tau(M) &= \frac{\int_{t_M}^{t_G} \kappa(t) r_\tau(t)(1 - \frac{\lambda^2}{16\pi^2}) dt}{\int_{t_M}^{t_G} \kappa(t) dt}, \\
\bar{r}_\mu(M) &= \frac{\int_{t_M}^{t_G} \kappa(t) r_\mu(t)(1 - \frac{\lambda^2}{16\pi^2}) dt}{\int_{t_M}^{t_G} \kappa(t) dt}, \\
\kappa(M) &= \frac{\int_{t_M}^{t_G} \kappa(t) dt}{\int_{t_M}^{t_G} \kappa(t) dt}, \\
\bar{r}_\tau(M) &= \frac{\int_{t_M}^{t_G} \kappa(t) r_\tau(t)(1 - \frac{\lambda^2}{16\pi^2}) dt}{\int_{t_M}^{t_G} \kappa(t) dt}, \\
\bar{r}_\mu(M) &= \frac{\int_{t_M}^{t_G} \kappa(t) r_\mu(t)(1 - \frac{\lambda^2}{16\pi^2}) dt}{\int_{t_M}^{t_G} \kappa(t) dt}, \\
\kappa(M) &= \frac{\int_{t_M}^{t_G} \kappa(t) dt}{\int_{t_M}^{t_G} \kappa(t) dt}
\end{align}
and we have ignored $\lambda_e$ Yukawa couplings. For the definition of $\eta$-factors see Eq. (A.6). The $Y^{\dagger \nu G}$ denotes corresponding Yukawa matrix at scale $\mu = M_G$. On the other hand, we have:
\begin{equation}
(Y^\dagger Y^\nu |_{\mu = M}) \simeq \kappa(M) Y^{\dagger \nu G} \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & r_\mu(M) & 0 \\ 0 & 0 & r_\tau(M) \end{array} \right) Y^{\dagger \nu G}. \tag{2.16}
\end{equation}
(Derivations are given in Appendix A.1)

Comparing (2.13) with (2.16) we see that difference in these matrix structures (besides overall flavor universal RG factors) is in the RG factors $r_{\tau,\mu}(M)$ and $r_{\tau,\mu}(M)$. Without the $\lambda_{\tau,\mu}$ Yukawa couplings these factors are equal and there is no mismatch between the phases $\eta$ and $\eta'$ [defined in Eqs. (2.7) and (2.9)] of these matrices. Non zero $\eta' - \eta$ will be due to the deviations, which we parameterize as
\begin{align}
\xi_\tau &= \frac{\bar{r}_\tau(M)}{r_\tau(M)} - 1, \\
\xi_\mu &= \frac{\bar{r}_\mu(M)}{r_\mu(M)} - 1.
\end{align}

\begin{equation}
16\pi^2 \frac{d}{dt} \delta^{(2)}_N = 4(\bar{Y}_e^\dagger Y_e)_{12} + 2\delta^{(2)}_N \left( (\bar{Y}_e^\dagger Y_e)_{22} - (\bar{Y}_e^\dagger Y_e)_{11} \right) - 2(\delta^{(2)}_N)^2 (\bar{Y}_e^\dagger Y_e)_{21} - 2\delta^{(1)}_N \delta^{(2)}_N (\bar{Y}_e^\dagger Y_e)_{12} \tag{2.10}
\end{equation}
The values of $\xi_\mu$ and $\xi_\tau$ can be computed numerically by evaluation of the appropriate RG factors. Approximate expressions can be derived for $\xi_{\tau,\mu}$, which are given by:

$$\xi_\tau \simeq \left[ \frac{\lambda_\tau^2(M)}{16\pi^2} \ln \frac{M_G}{M} + \frac{1}{3} \frac{\lambda_\tau^2(M)}{(16\pi^2)^2} \left[ 3\lambda_\tau^2 + 6\lambda_\tau^2 + 10\lambda_\tau^2 - (2\epsilon_\tau^a + \epsilon_\tau^b) g_{a\mu}^2 \right]_{\mu = M} \left( \ln \frac{M_G}{M} \right)^2 \right]^{1-\text{loop}}$$

$$- \left[ \frac{\lambda_\tau^2(M)}{16\pi^2} \right]^{2-\text{loop}},$$

(Eq. 2.18)

where one and two loop contributions are indicated. Derivation of approximate expression of $\xi_\tau$ [Eq. (2.18)] is given in Appendix A.1 of Ref. [15]. Eq. (2.19) can be derived in a similar way. As we see, non-zero $\xi_{\tau,\mu}$ are induced already at 1-loop level [without 2-loop correction of $\lambda_\tau^2/16\pi^2$ in Eq. (2.14)]. However, inclusion of 2-loop correction can contribute to the $\xi_{\tau,\mu}$ by amount of $\sim 3 - 5\%$ (because of the 1-loop factor suppression) and we have included it.

Now we write down quantities which have direct relevance for leptogenesis calculations. Using Eq. (2.13) in (2.12) and then applying Eq. (A.3) [for expressing $Y_{\nu G}$’s elements with corresponding entries of $Y_{\nu}(M)$], with definitions of Eqs. (2.15) and (2.17), we obtain:

$$|\delta_N(M)| e^{i\eta} = \frac{1}{4\pi^2} \kappa(M) \left[ ((Y_{\nu}^\dagger Y_{\nu})_{21} |e^{i\eta'} + \xi_\tau(Y_{\nu})_{31}(Y_{\nu})_{32}|e^{i(\phi_{31} - \phi_{32})} + \xi_\mu(Y_{\nu})_{21}(Y_{\nu})_{22}|e^{i(\phi_{21} - \phi_{22})}) \right]_{\mu = M}$$

(Eq. 2.20)

where $\phi_{ij}$ denotes the phase of the matrix element $(Y_{\nu})_{ij}$ at scale $\mu = M$. Eq. (2.20) shows well that in the limit $\xi_{\tau,\mu} \to 0$, we have $\eta = \eta'$, while the mismatch between these two phases is due to $\xi_{\tau,\mu} \neq 0$. With $\xi_{\tau,\mu} \ll 1$, from (2.20) we derive:

$$\eta - \eta' \simeq \xi_\tau|(Y_{\nu})_{31}(Y_{\nu})_{32}| \sin(\phi_{31} - \phi_{32} - \eta) + \xi_\mu|(Y_{\nu})_{21}(Y_{\nu})_{22}| \sin(\phi_{21} - \phi_{22} - \eta').$$

(Eq. 2.21)

We stress, that the 1-loop renormalization of the $Y_{\nu}$ matrix plays the leading role in generation of $\xi_{\tau,\mu}$, i.e. in the CP violation. [This is also demonstrated by Eq. (2.18).] When the product $(Y_{\nu})_{31}(Y_{\nu})_{32}$ is non-zero, the leading role for the mismatch between $\eta$ and $\eta'$ is played by $\xi_\tau$. However, for the Yukawa texture, having this product zero, important will be contribution from $\xi_\mu$. [As we will see on working examples, this will happen for $T_B$ of Eq. (5.1) and texture $B_2$ of Eq. (5.2).]

The value of $|\delta_N(M)|$, which characterizes the mass splitting between the RHN’s, can be computed by taking the absolute values of both sides of (2.20):

$$|\delta_N(M)| = \frac{\kappa_N}{4\pi^2} \left| (Y_{\nu}^\dagger Y_{\nu})_{21} + \xi_\tau(Y_{\nu})_{31}(Y_{\nu}^*)_{32} + \xi_\mu(Y_{\nu})_{21}(Y_{\nu}^*)_{22} \right| |(Y_{\nu}^\dagger Y_{\nu})_{21}| \mu = M \ln \frac{M_G}{M},$$

with $\kappa_N = \frac{\bar{\kappa}(M)}{\kappa(M) \ln \frac{M}{M}}$.

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Note that since RG equations for $M_N$ and $Y_{\nu}$ in non-SUSY case have similar structures (besides some group-theoretical factors) the $\xi_{\tau,\mu}$ would be generated also within non-SUSY setup.
These expressions can be used upon the calculation of the leptogenesis, which we will do in sections 4 and 5 for concrete models of the neutrino mass matrices.

3 See-Saw via Two Texture Zero $3 \times 2$ Dirac Yukawas Augmented by Single d=5 Operator. Predicting CP Violation

Within the setup with two RHNs, having at the GUT scale mass matrix of the form (2.3), we consider all two texture zero $3 \times 2$ Yukawa matrices. As given in [18], there are nine such different matrices:

\[
T_1 = \begin{pmatrix} \times & 0 \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 = \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} \times & \times \\ \times & 0 \\ \times & 0 \end{pmatrix},
\]

\[
T_4 = \begin{pmatrix} 0 \\ \times & \times \\ 0 \\ \times \\ \times \times \end{pmatrix}, \quad T_5 = \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & 0 \end{pmatrix}, \quad T_6 = \begin{pmatrix} \times & \times \\ \times & 0 \\ \times & 0 \end{pmatrix},
\]

\[
T_7 = \begin{pmatrix} \times & \times \\ 0 & 0 \\ \times \times \end{pmatrix}, \quad T_8 = \begin{pmatrix} \times & \times \\ 0 & 0 \\ \times \times \end{pmatrix}, \quad T_9 = \begin{pmatrix} \times \times \\ 0 & 0 \end{pmatrix},
\]

(3.1)

where "×"s stand for non-zero entries. From these textures one can factor out phases in such a way as to make maximal number of entries be real. As it was shown in [18], phases can be removed from all textures besides $T_4, T_7$ and $T_9$. Thus, here we pick up only $T_{4,7,9}$ textures, which lead to cosmological CP violation and have potential to realize resonant leptogenesis [16], [17] (due to quasi-degenerate $N_1$ and $N_2$ states). Therefore, we can parametrize these three textures as:

TEXTURE $T_4$

\[
T_4 = \begin{pmatrix} 0 & 0 \\ a_2e^{i\alpha_2} & b_2e^{i\beta_2} \\ a_3e^{i\alpha_3} & b_3e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ a_2 & b_2 \\ a_3 & b_3e^{i\phi} \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix},
\]

(3.2)

with

\[
\omega = \alpha_2 - \beta_2 + \rho, \quad y = \beta_2 - \rho, \quad z = \alpha_3 - \alpha_2 + \beta_2 - \rho, \quad \phi = \alpha_2 - \alpha_3 + \beta_3 - \beta_2.
\]

(3.3)

TEXTURE $T_7$

\[
T_7 = \begin{pmatrix} a_1e^{i\alpha_1} & b_1e^{i\beta_1} \\ 0 & 0 \\ a_3e^{i\alpha_3} & b_3e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ 0 & 0 \\ a_3 & b_3e^{i\phi} \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix},
\]

(3.4)

with

\[
\omega = \rho + \alpha_1 - \beta_1, \quad x = \beta_1 - \rho, \quad z = \alpha_3 - \alpha_1 + \beta_1 - \rho, \quad \phi = \alpha_1 - \alpha_3 - \beta_1 + \beta_3.
\]

(3.5)
TEXTURE $T_9$

$$T_9 = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 e^{i\phi} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix},$$

(3.6)

Where

$$\omega = \alpha_1 - \beta_1 + \rho, \quad x = \beta_1 - \rho, \quad y = \alpha_2 - \alpha_1 + \beta_1 - \rho, \quad \phi = \alpha_1 - \beta_1 - \alpha_2 + \beta_2.$$  

(3.7)

The phases $x, y$ and $z$ can be eliminated by proper redefinition of the states $l$ and $e^c$. As far as the phases $\omega$ and $\rho$ are concerned, because of the form of the $M_N$ matrix (2.3), they too will turn out to be non-physical. As we see, in textures $T_4, T_7$ and $T_9$ there remains one unremovable phase $\phi$ (i.e. in the second matrices of the r.h.s. of Eqs. (3.2) (3.4) and (3.6) respectively). This physical phase $\phi$ is relevant to the leptogenesis [17] and also, as it was shown in [18], it can be related to phase $\delta$, determined from the neutrino sector. As will be shown on concrete neutrino models, this will remain true after inclusion of specific single $d = 5$ operator. Integrating the RHN’s from the superpotential couplings of Eq. (2.1), using the see-saw formula, we get the following contribution to the light neutrino mass matrix:

$$M_{\nu}^{\text{ss}} = -\langle h_u^0 \rangle^2 Y_{\nu} M_N^{-1} Y_{\nu}^T.$$  

(3.8)

For $Y_{\nu}$ in (3.8) the textures $T_{4,7,9}$ should be used in turn. All obtained matrices $M_{\nu}^{\text{ss}}$, if identified with light neutrino mass matrices, will give experimentally unacceptable results. The reason is the number of texture zeros which we have in $T_i$ and $M_N$ matrices. In order to overcome this difficulty, in Ref. [18], the following single $d = 5$ operator was included for each case:

$$O_{ij}^5 \equiv \frac{\tilde{d}_5 e^{ix_5}}{2 M_s} l_i h_u l_j h_u$$  

(3.9)

where $\tilde{d}_5, x_5$ and $M_s$ are real parameters. (3.9), together with (3.8) will contribute to the neutrino mass matrix. This will allow to have viable models and, at the same time because of the minimal number of the additions, we will still have predictive scenarios. The operators (3.9) can be obtained by another sector in such a way as to not affect the forms of $T_{4,7,9}$ and $M_N$ matrices (one detailed example was presented in [15]). See Sect. 4 for more discussion on a possible origin of the (3.9) type operators. Above we have written the Yukawa textures in the form:

$$Y_{\nu} = P_1 Y_{\nu}^R P_2,$$  

(3.10)

where $P_1, P_2$ are diagonal phase matrices and $Y_{\nu}^R$ contains only one phase. Making the field phase redefinitions:

$$l' = P_1 l, \quad N' = P_2 N, \quad (e')^c = P_1^* e_c$$  

with $P_1 = \text{Diag}(e^{ix}, e^{iy}, e^{iz}), \quad P_2 = \text{Diag}(e^{i\omega}, e^{i\rho})$  

(3.11)

the superpotential coupling will become:

$$W_e = (l')^T Y_e^\text{diag}(e')^c h_d, \quad W_{\nu} = (l')^T Y_{\nu}^R N' h_u - \frac{1}{2} (N')^T M_N' N'$$  

(3.12)
with:
\[ M'_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M e^{-i(\omega + \rho)}. \]  
(3.13)

Now, for simplification of the notations, we will get rid of the primes (i.e. perform \( l' \rightarrow l, \ e'^\gamma \rightarrow e^\gamma\)) and in Eq. (3.8) using \( Y^R \) instead of \( Y_e \), from different \( T_{4,7,9} \) textures we get corresponding \( M^e_{\nu} \), and then adding the single operator (3.9) terms to zero entries of (3.8), one per \( M^e_{\nu} \), obtain the final neutrino mass matrices. Doing so, one obtains the neutrino mass matrices [18]:
\[ P_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad P_3 = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad P_4 = \begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \]  
(3.14)

where each type of texture originate as:
\[ P_1 - \text{type} : \ M^{(12)}_{T_4}, \quad P_2 - \text{type} : \ M^{(13)}_{T_4}, \quad P_3 - \text{type} : \ M^{(23)}_{T_7}, \quad P_4 - \text{type} : \ M^{(23)}_{T_9} \]

where subscript for \( M \) indicates which Yukawa texture the see-saw part [of Eq. (3.8)] came from, while superscript denotes the non-zero mass matrix element arising from the addition of the \( d = 5 \) operator of type (3.9). Since within our setup we are deriving neutrino mass matrices, we are able to renormalize them from high scales down to \( M_Z \). With details given in the Appendix A of Ref. [15], we here write down \( P_{1,2,3,4} \) textures at scale \( M_Z \) and give results already obtained in [18].

Before doing this, we set up conventions, which are used below. Since we work in the basis in which charged lepton Yukawa matrix is diagonal and real, the lepton mixing matrix \( U \) is related to the neutrino mass matrix as:
\[ M_{\nu} = P U^+ M_{\nu}^{\text{diag}} U^+ P \]  
(3.15)

where \( M_{\nu}^{\text{diag}} = (m_1, m_2, m_3) \) (\( m_{1,2,3} \) are light neutrino masses) and the phase matrices and \( U \) are:
\[ P = \text{Diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3}), \quad P' = \text{Diag}(1, e^{i\omega_1}, e^{i\omega_2}), \]  
(3.16)
\[ U = \begin{pmatrix} c_{13} s_{12} & c_{13} s_{12} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -s_{23} s_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix}, \]  
(3.17)

where \( s_{ij} \equiv \sin \theta_{ij} \) and \( c_{ij} \equiv \cos \theta_{ij} \). For normal and inverted neutrino mass orderings (denoted respectively by NH and IH) we will use notations:
\[ \Delta m^2_{\text{sol}} = m_2^2 - m_1^2, \quad \Delta m^2_{\text{atm}} = m_3^2 - m_2^2, \quad m_1 = \sqrt{m_3^2 - \Delta m^2_{\text{atm}}} \quad m_2 = \sqrt{m_3^2 - \Delta m^2_{\text{atm}}} \]  
(3.18)
\[ \Delta m^2_{\text{atm}} = m_2^2 - m_3^2, \quad \Delta m^2_{\text{sol}} = m_2^2 - m_1^2, \quad m_1 = \sqrt{m_3^2 + \Delta m^2_{\text{atm}}} \quad m_2 = \sqrt{m_3^2 + \Delta m^2_{\text{atm}}} \]  
(3.19)

As far as the numerical values of the oscillation parameters are concerned, since the bfv’s of the works of Ref. [3] differ from each other by few %’s, we will use their mean values:
\[ \sin^2 \theta_{12} = 0.308, \quad \sin^2 \theta_{23} = \begin{cases} 0.432 & \text{for NH} \\ 0.591 & \text{for IH} \end{cases}, \quad \sin^2 \theta_{13} = \begin{cases} 0.02157 & \text{for NH} \\ 0.0216 & \text{for IH} \end{cases}, \]  
10
\[ \Delta m_{sol}^2 = 7.48 \cdot 10^{-5} \text{ eV}^2, \quad \Delta m_{atm}^2 = |m_3^2 - m_2^2| = \begin{cases} 2.47 \cdot 10^{-3} \text{ eV}^2 & \text{for NH} \\ 2.54 \cdot 10^{-3} \text{ eV}^2 & \text{for IH} \end{cases} \quad (3.20) \]

In models, which allow to do so, we use the best fit values (bfv) given in (3.20). However, in some cases we also apply the value(s) of some oscillation parameter(s) which deviate from the bfv’s by several \( \sigma \).

**P₁ Neutrino Texture**

This texture, within our scenario, can be parameterized as:

\[
M_\nu(M_Z) = \begin{pmatrix}
0 & d_5 & 0 \\
d_5 & 2a_2b_2 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu3} \\
0 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu3} & 2a_3b_3e^{i\phi}r_{\nu3}^2
\end{pmatrix} \bar{m} \quad (3.21)
\]

where,

\[
\bar{m} = -r_m v^2(M_Z) / M \cdot e^{-i(\omega + \rho)} \quad (3.22)
\]

| \( \delta \) | \( \rho_1 \) | \( \rho_2 \) | works with |
|---|---|---|---|
| ±0.0879121 | ±3.11851 | ±3.03949 | NH, \( \sin^2 \theta_{23} = 0.451 \), \( \sin^2 \theta_{12} = 0.323 \) and best fit values for remaining oscillation parameters, \((m_1, m_2, m_3) = (0.00694406, 0.0110914, 0.0509217)\), \(m_{\beta\beta} = 0\) |

Table 1: Results from \( P_1 \) type texture. Masses are given in eVs.

and RG factors \( r_m \) and \( r_{\nu3} \) are given in Eqs. (A.17) and (A.18) of Ref. [15]. (For notations and definitions see also Appendix A.2 of the present paper.) The entries depending on \( a_i, b_j \) in (3.21) arise from the \( T_4 \) texture [given in (3.2)] by the see-saw mechanism. The entry \( d_5 \) comes from the \( (3.9) \) type operator \( \bar{d}_5 e^{i\tau_5} l_1 l_2 h_u h_u \). Since, as we see from Eqs. (3.2) and (3.3), the phase \( x \) is undetermined, we can select it in such a way as to set (3.21)'s \( d_5 \) entry to be real. Therefore, we still have single physical phase \( \phi \). It will be related to the phase \( \delta \) and will govern the leptogenesis process (discussed in Sect. 4). Due to the texture zeros, it is possible to predict the phases and values of the neutrino masses in terms of the measured oscillation parameters. In particular, the conditions \( M_{\nu}^{(1,3)} = 0 \) and \( M_{\nu}^{(1,1)} = 0 \), using (3.15)–(3.17), give:

\[
\frac{m_1}{m_3} c_{12}^2 + \frac{m_2}{m_3} s_{12}^2 e^{i\rho_1} = -i s_{13} e^{i(\rho_2 + 2\delta)} \quad (3.23)
\]

and

\[
- \left( \frac{m_1}{m_3} - \frac{m_2}{m_3} e^{i\rho_1} \right) t_{23}s_{12}c_{12} - s_{13}e^{i(\rho_2 + \delta)} + s_{13}e^{-i\delta} \left( \frac{m_1}{m_3} c_{12} + \frac{m_2}{m_3} s_{12} e^{i\rho_1} \right) = 0. \quad (3.24)
\]

These two complex equations with the input of five oscillation parameters allow to calculate all neutrino masses and predict three phases \( \delta, \rho_1 \) and \( \rho_2 \). Without providing here further analytical relations [followed from Eqs. (3.23), (3.21) and given in [18]], in Table II we summarize the results. [Only normal hierarchical (NH) neutrino mass ordering scenario works for the \( P_1 \) type texture.]
\[
M_\nu(M_Z) = \begin{pmatrix}
0 & 0 & d_5 \\
0 & 2a_2b_2 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu_3} \\
d_5 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu_3} & 2a_3b_3e^{i\phi}r_{\nu_3}^2
\end{pmatrix} \bar{m} \tag{3.25}
\]

This texture’s \(a_i, b_i\) entries are also obtained from the \(T_4\) texture \([3.2]\) via the see-saw mechanism and by addition of the \(d = 5\) operator \(\frac{\delta e^{i\phi}}{M_\nu}l_3h_uh_u\). By proper adjustment of the phase \(x\) [remaining undetermined in \((3.2)\) and \((3.3)\)], we can set \(d_5\) entry of \((3.25)\) to be real. The two conditions \(M_\nu^{(1,1)} = 0\) and \(M_\nu^{(1,2)} = 0\) give relation of Eq. \((3.23)\) and

\[
- \left(\frac{m_1}{m_3} - \frac{m_2}{m_3}e^{i\rho_1}\right)s_{12}c_{12} + s_{13}c_{23}e^{i\delta} - s_{13}c_{23}e^{-i\delta} \left(\frac{m_1}{m_3}c_{12}^2 + \frac{m_2}{m_3}s_{12}^2e^{i\rho_1}\right) = 0. \tag{3.26}
\]

which allow to predict neutrino masses and three phases \(\delta, \rho_{1,2}\). Results are given in Table 2. For inputs the best fit values (bfv) of the oscillation parameters are taken from Eq. \((3.20)\). For more details we refer the reader to [18].

| \(\delta\) | \(\rho_1\) | \(\rho_2\) | works with |
|-----------|-----------|-----------|------------|
| \(\pm 1.71006\) | \(\mp 2.79206\) | \(\mp 1.47308\) | NH and bfv’s of oscillation parameters, \((m_1, m_2, m_3) = (0.00471158, 0.0098488, 0.0506656), \ m_{\beta\beta} = 0\) |

Table 2: Results from \(P_2\) type texture. Masses are given in eVs.

**P_3 Neutrino Texture**

Using the see-saw formula \((3.8)\) for the \(T_7\) texture \((3.4)\) and including the \(d = 5\) operator \(\frac{\delta e^{i\phi}}{M_\nu}l_3h_uh_u\), we obtain the \(P_3\) neutrino texture:

\[
M_\nu(M_Z) = \begin{pmatrix}
2a_1b_1 & 0 & (a_3b_1 + a_1b_3e^{i\phi})r_{\nu_3} \\
0 & 0 & d_5 \\
(a_3b_1 + a_1b_3e^{i\phi})r_{\nu_3} & d_5 & 2a_3b_3e^{i\phi}r_{\nu_3}^2
\end{pmatrix} \bar{m} \tag{3.27}
\]

Since the phase \(y\) is not fixed in \((3.4)\) and \((3.5)\), without loss of any generality the \(d_5\) entry of \((3.27)\) can be set to be real. The conditions \(M_\nu^{(1,2)} = 0\) and \(M_\nu^{(2,3)} = 0\), similar to previous cases, allow to predict \(m_{1,2,3}\) and \(\delta, \rho_{1,2}\). Without giving the expressions (being lengthy and presented in Ref. [18]), we proceed to give numerical results, which for NH and inverted hierarchical (IH) neutrino mass orderings are summarized in Table 3.

| \(\delta\) | \(\rho_1\) | \(\rho_2\) | works with |
|-----------|-----------|-----------|------------|
| \(\pm 1.53714\) | \(\pm 0.0867342\) | \(\pm 3.20236\) | NH and bfv’s of oscillation parameters, \((m_1, m_2, m_3) = (0.0588907, 0.0595224, 0.077543), \ m_{\beta\beta} = 0.059436\) |
| \(\pm 1.58066\) | \(\mp 0.114316\) | \(\pm 3.06301\) | IH and bfv’s of oscillation parameters, \((m_1, m_2, m_3) = (0.0696426, 0.0701776, 0.0488354), \ m_{\beta\beta} = 0.0692588\) |

Table 3: Results from \(P_3\) type texture. Masses are given in eVs.
$P_4$ Neutrino Texture

This texture is obtained by applying the see-saw formula (3.8) to the $T_9$ texture (3.6) and including the $d = 5$ operator $\frac{\delta}{M_*} l_3 l_\beta h_u h_u$. Doing these we obtain the $P_4$ neutrino texture:

$$M_\nu(M_Z) = \begin{pmatrix}
2a_1 b_1 & (a_2 b_1 + a_1 b_2 e^{i\phi}) & 0 \\
(a_2 b_1 + a_1 b_2 e^{i\phi}) & 2a_2 b_2 e^{i\phi} & d_5 \\
0 & d_5 & 0
\end{pmatrix} \tilde{m} \quad (3.28)$$

In this case the phase $z$ is not fixed [see Eqs. (3.6) and (3.7)] and we can use this phase freedom to take $d_5$ entry of (3.28) matrix as a real parameter. The conditions $M_\nu^{(1,3)} = M_\nu^{(3,3)} = 0$ will give two complex (i.e. four real) equations, which contain three phases $\delta, \rho_{1,2}$ and one of the neutrino masses (remember that two measured parameters $\Delta m^2_{sol} = m^2_2 - m^2_1$ and $\Delta m^2_{atm} = |m^2_3 - m^2_2|$ leave undetermined values of the neutrino masses). Therefore, as for previous cases, with input of five measured oscillation parameters (which are: $\Delta m^2_{sol}, \Delta m^2_{atm}$ and $\{\theta_{12}, \theta_{23}, \theta_{13}\}$) from the conditions given above we predict all light neutrino masses and three phases $\delta, \rho_{1,2}$. Still referring to [18], for analytical expressions, in Table 4 we give the numerical results obtained for this texture $P_4$ for NH and IH cases. The value of $s^2_{23}$ we are using is deviated from the bfv, because the conditions $M_\nu^{(1,3)} = M_\nu^{(3,3)} = 0$ do not allow to use bfv’s. Note that in NH, case 2 and for IH the values of $s^2_{23}$ are less deviated from bfv, but the NH’s case 1, as it turns out, is preferred for obtaining needed amount of the baryon asymmetry. Without the latter constraint, just for satisfying the neutrino data, we could have used smaller values of $s^2_{23}$, but this would give higher values of neutrino masses which would not satisfy the current cosmological constraint $\sum_i m_i < 0.23$ eV (the limit set by the Planck observations [26][7]). Upon leptogenesis investigation we will use NH, case 1 given in Table 4.

| $\delta$ | $\rho_1$ | $\rho_2$ | works with |
|----------|----------|----------|------------|
| NH, case 1 | $\pm 1.62446$ | $\pm 0.129186$ | $\pm 3.05085$ | NH and $\sin^2 \theta_{23} = 0.6$ and bfv’s for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.044819, 0.0456458, 0.0674799)$, $m_{\beta\beta} = 0.0454757$ |
| NH, case 2 | $\pm 1.59508$ | $\pm 0.0647305$ | $\pm 3.09629$ | NH and $\sin^2 \theta_{23} = 0.551$ and bfv’s for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.0707692, 0.0712957, 0.0869084)$, $m_{\beta\beta} = 0.0712444$ |

| $\delta$ | $\rho_1$ | $\rho_2$ | works with |
|----------|----------|----------|------------|
| $\pm 1.56553$ | $\pm 0.0733633$ | $\pm 3.19198$ | IH and $\sin^2 \theta_{23} = 0.441$ and bfv’s for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.0820116, 0.0824663, 0.065274)$, $m_{\beta\beta} = 0.0817407$ |

$^\dagger$Tighter upper bound can be obtained by considering additional combined datasets [27]. However, bound also depends on the theoretical framework and can be relaxed (see e.g. 2nd Ref. of [3], where as demonstrated in Table II, the scenario with extra $A_{\text{lens}}$ parameter yields more relaxed bounds). Thus, upon our calculations we use the constraint $\sum_i m_i < 0.23$ eV.
Table 4: Results from $P_4$ type texture. Masses are given in eVs.

4 Resonant Leptogenesis

Expression for $\delta_N(M)$ with effects of $\lambda_{\mu,\tau}$ and ignoring $\lambda_e$, is given by Eq. \eqref{2.20}. The CP asymmetries $\epsilon_1$ and $\epsilon_2$ generated by out-of-equilibrium decays of the quasi-degenerate fermionic components of $N_1$ and $N_2$ states respectively are given by \cite{9}, \cite{10}\cite{11}

$$\epsilon_1 = \frac{\text{Im}[(\hat{Y}^\dagger_{11}\hat{Y}_{12})_{22}]^2}{(\hat{Y}^\dagger_{11}(\hat{Y}_{12}(\hat{Y}^\dagger_{21})_{22}) (M_2^2 - M_1^2)} M_1\Gamma_2$$

$$\epsilon_2 = \epsilon_1(1 \leftrightarrow 2) . \quad (4.1)$$

Here $M_1, M_2$ (with $M_2 > M_1$) are the mass eigenvalues of the RHN mass matrix. These masses, within our scenario, are given in \eqref{2.6} with the splitting parameter given in Eq. \eqref{2.22}. For the decay widths, here we will use more accurate expressions \cite{5}:

$$\Gamma_{N_i} = \frac{M_i}{8\pi}(\hat{Y}^\dagger_{11}\hat{Y}_{12})_{ii} \left( 1 - 4\frac{M_3^2}{M_i^2} \right)^{\frac{1}{2}} + \frac{1}{2} + \frac{c_\beta^2}{\gamma} \left( 1 - \frac{M_3^2}{M_i^2} \right)^2 \right), \quad (4.2)$$

where $M_S$ is the SUSY scale and we assume that all SUSY states have the common mass equal to this scale. $s_\beta$ and $c_\beta$ are short hand notations for $\sin \beta$ and $\cos \beta$ respectively. $N_i$ decays proceed via $N_i \rightarrow h_u l_i$ and $N_i \rightarrow h_u l_i$ channels. Upon derivation of \eqref{4.2} we took into account that $h_u$ is a linear combination of the SM Higgs doublet $h_{SM}$ and the heavy Higgs doublet $H$: $h_u \simeq s_\beta h_{SM} + c_\beta H$. Mass of the $h_{SM}$ has been ignored, while the mass of the $H$ has been taken as $M_S$. Moreover, the imaginary part of $[(\hat{Y}^\dagger_{11}\hat{Y}_{12})_{22}]^2$ will be computed with help of \eqref{2.8} and \eqref{2.2} with the relevant phase given in Eq. \eqref{2.21}. Using general expressions \eqref{2.22} and \eqref{2.22} for the given neutrino model we will compute $\eta - \eta'$ and $|\delta_N(M)|$. With these, since we know the possible values of the phase $\phi$ [see Eqs. \eqref{4.6}, \eqref{4.8}, \eqref{4.10}, \eqref{4.12}], and with the help of the relations \eqref{4.7}, \eqref{4.9}, \eqref{4.10}, \eqref{4.11} we can compute $\epsilon_1, \epsilon_2$ in terms of $|M|$ and $a_2$ or $a_1$ (depending on the texture we are dealing with). Recalling that the lepton asymmetry is converted to the baryon asymmetry via sphaleron processes \cite{28}, with the relation $n_b^f / n_\pi \simeq -1.48 \times 10^{-3} (\kappa_f(1)\epsilon_1 + \kappa_f(2)\epsilon_2)$ we can compute the baryon asymmetry. The notion $n_b^f$ is used for the baryon asymmetry created through the decays of the fermionic components of $N_{1,2}$ superfields. The net baryon asymmetry $n_b$ receives the contribution from the decays of the scalar components $\tilde{N}_{1,2}$. The latter contribution we denote by $\tilde{n}_b$. The computation of it (being suppressed in comparison with $n_b^f$) will be discussed in Appendix \cite{B}. For the efficiency factors $\kappa_f(1,2)$ we will use the extrapolating expressions \cite{5} (see Eq. (40) in Ref. \cite{5}), with $\kappa_f(1)$ and $\kappa_f(2)$ depending on the mass scales $\tilde{m}_1 = \frac{\nu^2(M)}{M_1}(\hat{Y}^\dagger_{11}\hat{Y}_{11})$ and $\tilde{m}_2 = \frac{\nu^2(M)}{M_2}(\hat{Y}^\dagger_{22}\hat{Y}_{22})$ respectively.

Within our studies we will consider the RHN masses $\simeq |M| \lesssim 10^7$ GeV. With this, we will not have the relic gravitino problem \cite{29}, \cite{30}. For simplicity, we consider all SUSY particle masses to be equal to $M_S < |M|$, with $M_S$ identified with the SUSY scale, below which we have just SM. As it turns out, via the RG factors, the asymmetry also depends on the top quark mass.

\footnote{In Appendix \cite{B} we investigate the contribution to the baryon asymmetry via decays of the scalar components of the RHN superfields. As we show, these effects are less than 3.4%.}
It is remarkable that within some models the observed baryon asymmetry
\[
\left( \frac{n_b}{s} \right)_{\text{exp}} = (8.65 \pm 0.085) \times 10^{-11}
\] (4.3)
(the recent value reported by WMAP and Planck [26]), can be obtained even for low values of the MSSM parameter \( \tan \beta = \frac{v_u}{v_d} \) (defined at the SUSY scale \( \mu = M_S \)).

Below, we perform analysis for each of these \( P_{1,2,3,4} \) cases (and for revised models of Ref. [17] discussed in Sect. 5) in turn and present our results. As an input for the top’s running mass we will use the central value, while for the SUSY scale \( M_S \) we will consider two cases:

\[ m_t(m_t) = 163.48 \text{ GeV}, \]

Case (I) : \( M_S = 10^3 \text{ GeV} \), Case (II) : \( M_S = 2 \times 10^3 \text{ GeV} \). (4.4)

Procedure of our RG calculation and used schemes are described in Appendix A.3. As it was shown in [18], for neutrino mass matrix textures \( P_{1,2,3,4} \), we will be able to relate the cosmological phase \( \phi \) to the CP violating phase \( \delta \). We will introduce the notation:

\[
A_{ij} = U_{i1}^* U_{j1} m_1 + U_{i2}^* U_{j2} m_2 e^{i \rho_1} + U_{i3}^* U_{j3} m_3 e^{i \rho_2},
\] (4.5)

which will be convenient for writing down expressions for the \( \phi \) and for expressing neutrino Dirac type Yukawa couplings in terms of one independent coupling element. (The latter will be selected by the convenience.)

**For \( P_1 \) Texture**

For this case, using the form of the \( M_\nu \) [given by Eq. (3.21) and derived within our setup] in the relation (3.15) and equating appropriate matrix elements of the both sides, we will be able to calculate the phase \( \phi \) [18], [15]:

\[
\phi = \text{Arg} \left[ \left( \frac{A_{23}}{\sqrt{A_{22} A_{33}}} \pm \sqrt{\frac{A_{23}^2}{A_{22} A_{33}} - 1} \right)^2 \right].
\] (4.6)

Note, all elements at right hand side of Eq. (4.6) are known and therefore the phase \( \phi \) is calculable in this case.\(^{12}\) Moreover, expressing \( a_3, b_{2,3} \) in terms of \( a_2 \) (taking \( a_2 \) to be an independent variable) and other known and/or predicted parameters, we will have:

\[
a_3 = \frac{a_2}{r_{13} |A_{22}|} \left| A_{23} \pm \sqrt{A_{23}^2 - A_{22} A_{33}} \right|, \quad b_2 = \frac{|A_{22}|}{2 |m| a_2}, \quad b_3 = \frac{|A_{33}|}{2 |m| a_3 r_{13}^2}.
\] (4.7)

As we see from Eqs. (4.6) and (4.7), there is a pair of solutions. When for the \( a_3 \) in (4.7) we are taking the ” + ” sign, in (4.6) we should take the sign ” − ”, and vice versa. (The same applies to the cases of textures \( P_{2,3,4} \).) For this case, the baryon asymmetry via the resonant leptogenesis has been investigated in Ref. [15]. In this work, for the decay widths we use more refined expressions of Eq. (4.2). Because of this, the values of \( \tan \beta \) (given in Table 5) are slightly different. Since in this model \( (Y_\nu)_{31} \) and \( (Y_\nu)_{32} \) are non-zero, according to Eq. (2.20) the mismatch \( \eta - \eta' \) (e.g. CP

\(^{12}\)Same will be true also for textures \( P_2, P_3 \) and \( P_4 \).
asymmetry) is mainly arising due to \( \xi_\tau \). However, in numerical calculations we have also taken into account the contribution of \( \xi_\mu \). The results are given in Table 5 (for more explanations see also caption of this table). While in the table we vary the values of \( M \) and \( \tan \beta \), the cases with I and II correspond respectively to the cases (I) and (II) of Eq. (4.4) (i.e. \( M_S = 1 \) and 2 TeV resp.). For the definition of the RG factors given in this table see Appendix A.2 of Ref. [15]. For finding maximal values of the Baryon asymmetries (given in Tab5) we have varied the parameter \( a_2 \). As we see, the value of the net baryon asymmetry \( n_b \) slightly differs from \( n_f \). This is due to the contribution from \( \tilde{n}_b \) [coming from the right handed sneutrino (RHS) decays], which is small (less than 3.4% of \( n_f \)). Details of \( \tilde{n}_b \)'s calculations are discussed in Appendix B.

| Case  | \( M (\text{GeV}) \) | \( \tan \beta \) | \( r_{\bar{m}} \) | \( r_{v_3} \) | \( \kappa_N \) | \( 10^5 \times \xi_\tau \) | \( 10^{11} \times \frac{n_f}{s}_{\text{max}} \) | \( 10^{11} \times \frac{n_b}{s}_{\text{max}} \) |
|-------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| (I.1) | \( 3 \cdot 10^3 \) | 1.72 | 0.8868 | 0.9714 | 1.206 | 6.106 | 8.29 | 8.57 |
| (I.2) | \( 10^4 \) | 1.619 | 0.832 | 0.9523 | 1.2322 | 5.303 | 8.34 | 8.6 |
| (I.3) | \( 10^5 \) | 1.664 | 0.7482 | 0.9203 | 1.1807 | 4.821 | 8.36 | 8.6 |
| (I.4) | \( 10^6 \) | 1.719 | 0.682 | 0.8923 | 1.1345 | 4.381 | 8.37 | 8.6 |
| (I.5) | \( 10^7 \) | 1.773 | 0.629 | 0.8676 | 1.0971 | 3.937 | 8.37 | 8.6 |
| (II.1) | \( 6 \cdot 10^3 \) | 1.701 | 0.8689 | 0.9678 | 1.175 | 5.897 | 8.294 | 8.57 |
| (II.2) | \( 10^4 \) | 1.615 | 0.8464 | 0.9599 | 1.1994 | 5.365 | 8.334 | 8.59 |
| (II.3) | \( 10^5 \) | 1.625 | 0.7629 | 0.9283 | 1.1669 | 4.755 | 8.36 | 8.6 |
| (II.4) | \( 10^6 \) | 1.678 | 0.6974 | 0.9008 | 1.1243 | 4.321 | 8.36 | 8.6 |
| (II.5) | \( 10^7 \) | 1.731 | 0.645 | 0.8765 | 1.0894 | 3.887 | 8.36 | 8.6 |

Table 5: Texture \( P_1 \), normal hierarchy: Baryon asymmetry for various values of \( M \) and for minimal (allowed) value of \( \tan \beta \). With neutrino oscillation parameters and results given in the Table 1 and computed from Eq. (4.6) \( \phi = \pm 1.264 \). For all cases \( r_{v_3} \approx 1 \).

For \( P_2 \) Texture

With a pretty similar procedure, for this case we get:

\[
\phi = \text{Arg} \left[ \left( \frac{A_{33}}{\sqrt{A_{22}A_{33}}} \mp \sqrt{A_{23}^2 - A_{22}A_{33}} \right)^2 \right]. \tag{4.8}
\]

Expressing \( a_3, b_{2,3} \) in terms of \( a_2 \) and other parameters (yet known or predicted in this scenario), we will have:

\[
a_3 = \frac{a_2}{r_{v_3} |A_{22}|} \left| A_{23} \pm \sqrt{A_{23}^2 - A_{22}A_{33}} \right|, \quad b_2 = \frac{|A_{22}|}{2|\bar{m}|a_2}, \quad b_3 = \frac{|A_{33}|}{2|\bar{m}|a_3 r_{v_3}^2} \tag{4.9}
\]

Results for this case are presented in Table 6.
Table 6: Texture $P_2$, normal hierarchy: Baryon asymmetry for various values of $M$ and for minimal (allowed) value of $\tan \beta$. With neutrino oscillation parameters and results given in the Table 2 and computed from Eq. (4.8) $\phi = \pm 1.1$. For all cases $r_\nu^3 \simeq 1$.

For $P_3$ Texture

| Case | $M$(GeV) | $\tan \beta$ | $r_{\bar{m}}$ | $r_{\nu u}$ | $\kappa_N$ | $10^5 \times \xi_r$ | $10^{11} \times (n_\ell / s)_{\text{max}}$ | $10^{11} \times (n_b / s)_{\text{max}}$ |
|------|----------|---------------|----------------|--------------|-------------|-----------------|---------------------------------|---------------------------------|
| (I.1) | $3 \cdot 10^4$ | 7.158 | 0.904 | 0.9761 | 1.0076 | 76.29 | 8.49 | 8.59 |
| (I.2) | $10^4$ | 6.802 | 0.8717 | 0.9635 | 0.9983 | 64.79 | 8.508 | 8.6 |
| (I.3) | $10^5$ | 6.922 | 0.82 | 0.9417 | 0.9819 | 59.11 | 8.51 | 8.6 |
| (I.4) | $10^6$ | 7.074 | 0.7789 | 0.9225 | 0.9692 | 53.92 | 8.51 | 8.6 |
| (I.5) | $10^7$ | 7.227 | 0.7467 | 0.9056 | 0.96 | 48.65 | 8.51 | 8.6 |
| (II.1) | $6 \cdot 10^3$ | 7.146 | 0.8852 | 0.9723 | 0.9986 | 75.06 | 8.5 | 8.6 |
| (II.2) | $10^4$ | 6.85 | 0.8725 | 0.9672 | 0.9954 | 67.24 | 8.5 | 8.6 |
| (II.3) | $10^5$ | 6.858 | 0.8229 | 0.946 | 0.9802 | 59.44 | 8.51 | 8.6 |
| (II.4) | $10^6$ | 7.003 | 0.7835 | 0.9274 | 0.9684 | 54.17 | 8.51 | 8.6 |
| (II.5) | $10^7$ | 7.151 | 0.7524 | 0.9109 | 0.9597 | 48.87 | 8.51 | 8.6 |

Table 7: Texture $P_3$, normal hierarchy: Baryon asymmetry for various values of $M$ and for minimal (allowed) value of $\tan \beta$. With neutrino oscillation parameters and results given in the Table 3 and computed from Eq. (4.10) (for NH case) $\phi = \pm 2.92$. For all cases $r_\nu^3 \simeq 1$.  

17
Table 8: Texture $P_3$, inverted hierarchy: Baryon asymmetry for various values of $M$ and for minimal (allowed) value of $\tan \beta$. With neutrino oscillation parameters and results given in the Table 3 and computed from Eq. (4.10) (for IH case) $\phi = \pm 3.124$. For all cases $r_\nu \simeq 1$.

(For notations and definitions see also Appendix A.2 of the present paper.)

\[
\phi = \text{Arg} \left[ \left( \frac{A_{13}}{\sqrt{A_{11}A_{33}}} \pm \sqrt{ \frac{A_{13}^2}{A_{11}A_{33}} - 1 } \right) \right]^2 .
\]

Expressing $a_3, b_{1,3}$ in terms of $a_1$ and other fixed parameters, we will have:

\[
a_3 = \frac{a_1}{r_\nu} \frac{1}{|A_{11}|} \begin{vmatrix} A_{13} \pm \sqrt{A_{13}^2 - A_{11}A_{33}} \end{vmatrix}, \quad b_1 = \frac{|A_{11}|}{2|m|a_1}, \quad b_3 = \frac{|A_{33}|}{2|m|a_3 r_\nu^2}
\]

Results for this texture for cases of NH and IH neutrinos are presented in Tables 7 and 8 respectively.

**For $P_3$ Texture**

For this case cosmological phase is given by:

\[
\phi = \text{Arg} \left[ \left( \frac{A_{12}}{\sqrt{A_{11}A_{22}}} \pm \sqrt{ \frac{A_{12}^2}{A_{11}A_{22}} - 1 } \right) \right]^2 .
\]

Expressing $a_1, b_{1,2}$ in terms of $a_2$ and other known and/or predicted parameters, we will have:

\[
a_1 = \frac{|A_{11}|}{|A_{12} \pm \sqrt{A_{12}^2 - A_{11}A_{22}}|} a_2, \quad b_1 = \frac{|A_{11}|}{2|m|a_1}, \quad b_2 = \frac{|A_{22}|}{2|m|a_2}
\]

In this scenario, since $(Y_\nu)_{31}$ and $(Y_\nu)_{32}$ are zero, according to Eq. (2.20) the mismatch $\eta - \eta'$ (e.g. CP asymmetry) is arising due to $\xi_\mu$. Since the latter is suppressed by $\lambda_\mu^2$, as it turns out large values of the $\tan \beta$ are required and only in NH case needed amount of the Baryon asymmetry can be generated. Results are given in Table 9.
Table 9: Texture $P_4$, normal hierarchy: Baryon asymmetry for various values of $M$ and for minimal (allowed) value of $\tan\beta$. With neutrino oscillation parameters and results given in the Table 4, NH, case 1, and $\phi$ computed from Eq. (4.12) (for NH case) $\phi = \pm 2.872$.

5 Revising Textures of Ref. [17] and Improved Versions

In this section we revise the textures considered in the work [17]. Since some of them are excluded by the current neutrino data [3] (see also Eq. (3.20)), we apply $d = 5$ contributions (in a spirit of section 3) and achieve their compatibility with the best fit values. Together with this, we investigate resonant leptogenesis and show that one loop corrections via $\lambda_\tau$ and/or $\lambda_\mu$ are crucial. In [17], while ignoring $\lambda_\mu$ the two loop correction to $\lambda_\tau$ was taken into account and this suggested for textures A and B$_1$ specific low bounds on the values of $\tan\beta$. As demonstrated below, one loop effects of $\lambda_\tau$ (giving dominant contribution for textures A and B$_1$) and $\lambda_\mu$ (for the texture B$_2$) significantly change results.

In the setup of two degenerate RHNs, in Ref. [17] the following three possible one texture zero neutrino Dirac Yukawa couplings have been considered:

Texture A:

$$Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix},$$

(5.1)

Texture B$_1$:

$$Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & 0 \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix},$$

(5.2)

Texture B$_2$: $Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & 0 \end{pmatrix}$

where for notational consistency with the whole paper, we have shown phases $\alpha_i, \beta_j$, while assuming that the couplings $a_i, b_j$ are real. Below we will (re)investigate these textures in turn.

**Texture A**

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13 On the contrary, in Ref. [17], without writing down the phase factors, $a_i$ and $b_j$ were treated as a complex parameters.
The A Yukawa texture can be written as:

Texture A : \( Y_\nu = \begin{pmatrix} a_1e^{i\alpha_1} & 0 & 0 \\ a_2e^{i\alpha_2} & b_2e^{i\beta_2} \\ a_3e^{i\alpha_3} & b_3e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3e^{i\phi} & b_3 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 & 0 \\ 0 & e^{ip} & 0 \end{pmatrix}, \)

with \( x = \alpha_1 - \alpha_2 + \beta_2 - \rho, \ y = \beta_2 - \rho, \ z = \beta_3 - \rho, \ \omega = \alpha_2 - \beta_2 + \rho, \ \phi = \alpha_3 - \alpha_2. \) (5.3)

As we see, besides the phase \( \phi \) all phases are factored out and have no physical relevance. With the RHN mass matrix of Eq. (3.13), via the see-saw[see expression in Eq.(3.8)] we will get the light neutrino mass matrix:

\[ M^{(A)}_{\nu}(M_Z) = \left( \begin{array}{ccc} 0 & a_1b_2 & a_1b_3r_{\nu 3} \\ a_1b_2 & 2a_2b_2 & (a_2b_3 + a_3b_2e^{i\phi})r_{\nu 3} \\ a_1b_3r_{\nu 3} & (a_2b_3 + a_3b_2e^{i\phi})r_{\nu 3} & 2a_3b_2e^{i\phi}r_{\nu 3}^2 \end{array} \right) \bar{m}, \]  

(5.4)

[For definitions of \( \bar{m}, \ r_{\nu 3} \) and proper explanations see respectively Eq. (5.22) and also Eqs. (A.17), (A.18) of Ref. [15], and comments therein.] This neutrino mass texture has only two non-zero mass eigenvalues. As it was shown in [17], this for NH (\( m_1 = 0 \)) and IH (\( m_3 = 0 \)) neutrino mass patterns, gives respectively the predictive relations \( \tan \theta_{13} = \frac{\bar{m}}{m_3} s_{12} \) and \( \tan \theta_{12} = \frac{\bar{m}}{m_2} \). Both of them are in a gross conflict with the current neutrino data, which exclude this scenario.

**A’ Neutrino Texture: Improved Version**

The drawbacks coming from the A neutrino mass matrix (5.4) can be avoided by adding \( d_5 \) term to one of the entries. Here we consider this addition to the (2, 3) and (3, 2) elements of the light neutrino mass matrix, which would make the model viable. (We refer to the improved version of (5.4) as the A’ neutrino texture.) After this, the \( M_{\nu} \) will have the form:

\[ M^{(A')}_{\nu}(M_Z) = \left( \begin{array}{ccc} 0 & a_1b_2 & a_1b_3r_{\nu 3} \\ a_1b_2 & 2a_2b_2 & (a_2b_3 + a_3b_2e^{i\phi})r_{\nu 3}+d_5 \\ a_1b_3r_{\nu 3} & (a_2b_3 + a_3b_2e^{i\phi})r_{\nu 3}+d_5 & 2a_3b_2e^{i\phi}r_{\nu 3}^2 \end{array} \right) \bar{m}. \]  

(5.5)

With this modification, all masses are non-zero. One can check out, that with the fixed phase redefinitions [given in Eq. (5.3)], in general \( d_5 \) is a complex parameter. Thus, together with additional mass, we will have one more independent phase. As it turns out, only NH scenario is possible to realize. Therefore as additional independent parameters we take one of the mass and \( \Delta \rho = \rho_1 - \rho_2 \). From the condition \( M^{(1,1)}_{\nu} = 0 \) we have:

\[ \cos(2\delta - \Delta \rho) = \frac{m_2^2s_{12}^4 - m_2^2s_{12}t_{13}^4 - m_3^2t_{13}^4}{2m_2m_3s_{12}^2t_{13}^2}, \quad \rho_1 = \pi - \text{Arg} \left[ \frac{m_2^2s_{12}^2 + t_{13}^2e^{i(2\delta - \Delta \rho)}}{m_3^2} \right] \text{ with } \Delta \rho = \rho_1 - \rho_2. \]  

(5.6)

(Here and below we use shorthanded notations \( t_{ij} \equiv \tan \theta_{ij} \). From the first relation of (5.5) one can check that IH scenario can not be realized. As far as the NH scenario is concerned, it will work with low bound on the lightest neutrino mass \( m_1 \). In fact, the first relation of (5.6) gives the allowed range for \( m_1 \). For example, with bfv’s of the oscillation parameters (3.20) we have:

\[ 0.00239 \text{ eV} \lesssim m_1 \lesssim 0.00641 \text{ eV}. \]  

(5.7)

Thus, as independent parameters we will take \( m_1 \) and \( \Delta \rho \). We will select them in such a way as to get desirable baryon asymmetry. For example, with the choice

\[ m_1 = 0.005719 \text{ eV}, \quad \Delta \rho = 4.987 \]  

(5.8)
Table 10: A' Neutrino Texture, NH. Baryon asymmetry for various values of \( M \) and for corresponding minimal (allowed) values of \( \tan \beta \). With the choice given in Eqs. (5.8), (5.9) and bfv’s of \( s_{ij}^2 \). For all cases \( r_{\nu 3} \simeq 1 \).

and bfv’s of all measured oscillation parameters with help of (3.18) and (5.6) for neutrino masses and phases we are getting:

\[
(m_1, m_2, m_3) \simeq (0.005719, 0.01037, 0.05077) \text{ eV},
\]

\[
(\delta, \rho_1, \rho_2) \simeq (2.9639, 2.911, 2.076). \tag{5.9}
\]

As far as the baryon asymmetry is concerned, using (5.5) in (3.15) for the CP phase \( \phi \) and expressing couplings \( a_{1,3}, b_{2,3} \) in terms of \( a_2 \) we get

\[
\phi = \text{Arg} \left( \frac{A_{12}^2 A_{33}}{A_{13}^2 A_{22}} \right),
\]

\[
a_1 = \frac{2 |A_{12}|}{|A_{22}|} a_2, \quad a_3 = \frac{1}{r_{\nu 3}} \frac{|A_{12} A_{33}|}{|A_{22} A_{13}|} a_2, \quad b_2 = |A_{22}| \frac{1}{2 |\bar{m}| a_2}, \quad b_3 = |A_{13} A_{22}| \frac{1}{2 r_{\nu 3} |\bar{m}| a_2}. \tag{5.10}
\]

For the values of (5.8), (5.9) and bfv’s of \( s_{12,23,13}^2 \) we get

\[
\phi = -2.9297. \tag{5.11}
\]

With these, and for given values of \( M \) and \( \tan \beta \) by varying \( a_2 \) we can investigate the baryon asymmetry. Results are given in Tab. 10.

**Texture B**

The \( B_1 \) Yukawa texture can be written as:

\[
\text{Texture } B_1: \quad Y_{\nu} = \begin{pmatrix}
    a_1 e^{i \alpha_1} & b_1 e^{i \beta_1} \\
    a_2 e^{i \alpha_2} & 0 \\
    a_3 e^{i \alpha_3} & b_3 e^{i \beta_3}
\end{pmatrix} = \begin{pmatrix}
    e^{i x} & 0 & 0 \\
    0 & e^{i y} & 0 \\
    0 & 0 & e^{i z}
\end{pmatrix} \begin{pmatrix}
    a_1 & b_1 \\
    a_2 & 0 \\
    a_3 e^{i \phi} & b_3
\end{pmatrix} \begin{pmatrix}
    e^{i \omega} & 0 & 0 \\
    0 & e^{i \rho} & 0
\end{pmatrix},
\]

with \( x = \beta_1 - \rho, \quad y = \alpha_2 - \alpha_1 + \beta_1 - \rho, \quad z = \beta_3 - \rho, \quad \omega = \alpha_1 - \beta_1 + \rho, \quad \phi = \alpha_3 - \beta_3 - \alpha_1 + \beta_1. \) (5.12)
With the RHN mass matrix of Eq. (3.13), via the see-saw we will get the light neutrino mass matrix:

\[
M^{(B_1)}_{\nu}(M_Z) = \begin{pmatrix}
2a_1b_1 & a_2b_1 & (a_1b_3 + a_3b_1e^{i\phi})r_{\nu_3} \\
a_2b_1 & 0 & a_2b_3r_{\nu_3} \\
(a_1b_3 + a_3b_1e^{i\phi})r_{\nu_3} & a_2b_3r_{\nu_3} & 2a_3b_3e^{i\phi}r_{\nu_3}^{2}
\end{pmatrix} \bar{m},
\]  

(5.13)

This neutrino mass texture (referred as B_1 neutrino texture) works only for inverted neutrino mass ordering [17] (with \(m_3 = 0\)) and has two predictive relations. In particular, in terms of measured oscillation parameters we can calculate the phases \(\delta\) and \(\rho_1\). The exact expressions are:

\[
\cos \delta = \frac{m_2(t_{13}^2 - s_{13}^2) - m_1(1 + t_{12}^2 + t_{13}^2)}{2t_{23}t_{12}s_{13}(m_1 + m_2)}, \quad \rho_1 = \pi - \arg \left[\frac{(1 - t_{23}s_{13}e^{-i\delta})^2}{(t_{12} + t_{23}s_{13}e^{-i\delta})^2}\right].
\]

(5.14)

with \(m_1 = \sqrt{\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}}, \quad m_2 = \sqrt{\Delta m^2_{\text{atm}}}, \quad m_3 = 0\).  

(5.15)

Although the first expression in (5.14) excludes the possibility of using the best fit values for all oscillation parameters, it allows for keeping values of \(s_{23}^2\) and \(s_{13}^2\) within 1\(\sigma\), while confining \(s_{12}^2\) to 2\(\sigma\). Remarkably, needed baryon asymmetry can be achieved with relatively low values of \(\tan \beta\). For example,

for IH of the B_1 neutrino texture, \ with: \(s_{23}^2 = 0.604 (1\sigma), \ s_{12}^2 = 0.33 (2\sigma), \ s_{13}^2 = 0.023 (1\sigma)\)

\[\Rightarrow \delta = \pm 0.307, \quad \rho_1 = \pi \mp 0.2192, \quad \phi = \pm 3.129\]

(5.16)

(\(\Delta m^2_{\text{sol}}\) and \(\Delta m^2_{\text{atm}}\) are taken best fit values to generate baryon asymmetry of desired amount \([\frac{N_b}{s}]_{\text{max}} = 8.59 \times 10^{-11}\) in case of \(M = 3 \times 10^9\) GeV and \(M_S = 1\) TeV the value \(\tan \beta = 6.32\) is required.

**B_1’ Neutrino Texture: Improved Version**

By addition of the \(d_5\) term to (1,3) and (3,1) entries of the B_1 neutrino texture [5,13], the light neutrino mass matrix becomes:

\[
M^{(B_1')}_{\nu}(M_Z) = \begin{pmatrix}
2a_1b_1 & a_2b_1 & (a_1b_3 + a_3b_1e^{i\phi})r_{\nu_3} + d_5 \\
a_2b_1 & 0 & a_2b_3r_{\nu_3} \\
(a_1b_3 + a_3b_1e^{i\phi})r_{\nu_3} + d_5 & a_2b_3r_{\nu_3} & 2a_3b_3e^{i\phi}r_{\nu_3}^{2}
\end{pmatrix} \bar{m},
\]

(5.17)

which gives all neutrinos massive and opens up a possibility of choosing two variables such as \(m_3\) and \(\Delta \rho \equiv \rho_1 - \rho_2\) as independent ones to operate with. We refer to this (5.17) improved version as the B_1’ neutrino texture. From the condition \(M^{(2,2)}_{\nu} = 0\) we have:

\[m_1|U_{21}|^2 = |m_2(U_{22})^2 + m_3(U_{23})^2e^{i\Delta \rho}|, \quad \rho_1 = \pi - \arg \left[\frac{m_1(U_{21})^2}{m_2(U_{22})^2 + m_3(U_{23})^2e^{i\Delta \rho}}\right], \quad \text{with} \ \Delta \rho = \rho_1 - \rho_2.
\]

(5.18)

Out of the numerous values \(\Delta \rho\) and \(m_3\) can take on, we select those that are not in conflict with the observed oscillation data and at the same time together with the minimal allowed value of \(\tan \beta\)
generate baryon asymmetry of the needed amount. In case of Inverted Hierarchy both of these requirements can be satisfied. In particular:

for IH of the $B_1$' neutrino texture: $m_3 = 0.00250717$ eV and $\Delta \rho = 3.6599$ (5.19)

determine numerical values of the rest of masses, phases and eventually the neutrino double beta decay parameter:

$$(m_1, m_2, m_3) = (0.049714, 0.050461, 0.00250717) \text{ eV},$$

$$\begin{align*}
(\delta, \rho_1, \rho_2) &= (0.17303, 2.9456, -0.71436), \\
\m_{\beta\beta} &\simeq 0.019 \text{ eV}.
\end{align*}$$

As far as the baryon asymmetry is concerned, using (5.17) in (3.15), we get:

$$\begin{align*}
\phi = \text{Arg} \left( \frac{A_{12}^2 A_{33}}{A_{23} A_{11}} \right), \\
\begin{align*}
a_1 &= \frac{1}{2} \left| \frac{A_{11}}{A_{12}} \right| a_2, \\
a_3 &= \frac{1}{2r_{\nu_3}} \left| \frac{A_{23}}{A_{33}} \right| a_2, \\
b_1 &= \frac{|A_{12}|}{|\bar{m}| a_2}, \\
b_3 &= \frac{|A_{23}|}{r_{\nu_3} |\bar{m}| a_2}.
\end{align*}
\end{align*}$$

Using all these, we can calculate the baryon asymmetry. The results are given in Tab. 11. The goal of attaining needed baryon asymmetry with the minimal allowed value of $\tan \beta$ and without coming in contradiction with the experimental data can be achieved in case of Normal Hierarchy as well by selecting:

For NH of the $B_1$' neutrino texture: $m_3 = 0.0741678$ eV and $\Delta \rho = 3.2526$ (5.23)

| Case | $M$(GeV) | $\tan \beta$ | $r_{\bar{m}}$ | $r_{\nu_u}$ | $\kappa_N$ | $10^4 \times \xi_\tau$ | $10^{11} \times \left( \frac{n_i}{s} \right)_{\text{max}}$ | $10^{11} \times \left( \frac{n_i}{s} \right)_{\text{max}}$ |
|------|----------|--------------|--------------|--------------|-------------|----------------|--------------------------------|--------------------------------|
| (I.1)| $3 \cdot 10^3$ | 2.1 | 0.8928 | 0.9731 | 1.118 | 0.8134 | 8.57 | 8.62 |
| (I.2)| $10^4$ | 2.135 | 0.8499 | 0.9574 | 1.0986 | 0.7826 | 8.55 | 8.6 |
| (I.3)| $10^5$ | 2.332 | 0.7856 | 0.9316 | 1.0545 | 0.7924 | 8.56 | 8.61 |
| (I.4)| $10^6$ | 2.559 | 0.7385 | 0.9103 | 1.0209 | 0.8066 | 8.56 | 8.6 |
| (I.5)| $10^7$ | 2.822 | 0.7048 | 0.8926 | 0.9959 | 0.8242 | 8.54 | 8.59 |
| (II.1)| $6 \cdot 10^5$ | 2.118 | 0.875 | 0.9695 | 1.0933 | 0.8109 | 8.55 | 8.6 |
| (II.2)| $10^4$ | 2.119 | 0.858 | 0.9631 | 1.0876 | 0.7896 | 8.56 | 8.6 |
| (II.3)| $10^5$ | 2.302 | 0.7948 | 0.9378 | 1.0481 | 0.7932 | 8.56 | 8.6 |
| (II.4)| $10^6$ | 2.524 | 0.7484 | 0.9168 | 1.017 | 0.8067 | 8.55 | 8.59 |
| (II.5)| $10^7$ | 2.786 | 0.715 | 0.8994 | 0.9936 | 0.826 | 8.55 | 8.59 |

Table 11: $B_1$' Neutrino Texture, IH. Baryon asymmetry for various values of $M$ and for corresponding minimal (allowed) values of $\tan \beta$. With the choice given in Eqs. (5.19), (5.20) and bfv’s of $s_{ij}^2$. With $\phi = -2.9846$ and for all cases $r_{\nu_3} \simeq 1$.

give:

$$(m_1, m_2, m_3) = (0.05437, 0.0550533, 0.0741678) \text{ eV},$$

$$\begin{align*}
(\delta, \rho_1, \rho_2) &= (0.0034537, 0.25965, -2.9929), \\
\phi &= 2.2568, \\
\m_{\beta\beta} &\simeq 0.051 \text{ eV}.
\end{align*}$$
Table 12: B$_1'$ Neutrino Texture, NH. Baryon asymmetry for various values of $M$ and for corresponding minimal (allowed) values of $\tan \beta$. With the choice given in Eqs. (5.23), (5.24) and bfv’s of $s^2_{12}$. With $\phi = 2.2568$ and for all cases $r_{\nu 3} \simeq 1$ and $\frac{\bar{m}}{s} \simeq 0$.

The baryon asymmetries for cases corresponding to this NH scenario are given in Tab. 12

**Texture B$_2$**

This texture is interesting because, due to specific form of $Y_{\nu_{\tau}}$ the radiative corrections through the $\lambda_{\tau}$ coupling do not generate cosmological CP asymmetry. Thus $\lambda_{\mu}$ may be important, which we investigate below. Thus, this model (and its slight modification discussed below) serves as a good demonstration of the role of $\xi_{\mu}$ correction in emergence of needed Baryon asymmetry.

The B$_2$ Yukawa texture can be written as:

$$Y_{\nu} = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & 0 \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 e^{i\phi} \\ a_3 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix},$$

with $x = \beta_1 - \rho$, $y = \alpha_2 - \alpha_1 + \beta_1 - \rho$, $z = \alpha_3 - \alpha_1 + \beta_1 - \rho$,

$\omega = \alpha_1 - \beta_1 + \rho$, $\phi = \alpha_1 - \beta_1 - \alpha_2 + \beta_2$. (5.26)

Via the see-saw we will get the light neutrino mass matrix:

$$M^{(B_2)}_{\nu}(M_Z) = \begin{pmatrix} 2a_1 b_1 & a_1 b_2 e^{i\phi} + a_2 b_1 & a_3 b_1 r_{\nu 3} \\ a_1 b_2 e^{i\phi} + a_2 b_1 & 2a_2 b_2 e^{i\phi} & a_3 b_2 e^{i\phi} r_{\nu 3} \\ a_3 b_1 r_{\nu 3} & a_3 b_2 e^{i\phi} r_{\nu 3} & 0 \end{pmatrix} \bar{m}.$$

(5.27)

This neutrino mass texture (referred as B$_2$ neutrino texture) works only for inverted neutrino mass ordering [17] (with $m_3 = 0$) and has two predictive relations. In particular, in terms of measured oscillation parameters we can calculate the phases $\delta$ and $\rho_1$. The exact expressions are:

$$\cos \delta = \frac{m_1 t_{12}^2 t_{23}^2 - m_2 (t_{23}^2 + t_{12} s_{13}^2)}{2(m_1 + m_2) t_{12} t_{23} s_{13}}, \quad \rho_1 = \pi - \text{Arg} \left( \frac{t_{12} t_{23} - s_{13} e^{i\delta}}{t_{23} + t_{12} s_{13} e^{i\phi}} \right)^2,$$

with $m_1 = \sqrt{\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}}$, $m_2 = \sqrt{\Delta m^2_{\text{atm}}}$, $m_3 = 0$. (5.28)
From these relations one can easily check that model works only if at least two of the oscillation parameters \( \sin^2 \theta_{ij} \) are off by several \( \sigma \)'s. Taking bfv’s of the oscillation parameters would give the absolute values of the r.h.s. of expression for \( \cos \delta \) larger than one. Besides this difficulty, proper value of the baryon asymmetry (generated with help of 1-loop correction of \( \lambda_\mu \)) requires even more deviation from the bfv’s of the oscillation parameters. The root of the problem is that the value of the phase \( \phi \) is fixed so that the parameter \( \sin \phi \) (governing cosmological CP asymmetry) turns out to be too suppressed. For instance, with \( s_{12}^2 = 0.333, \ s_{23}^2 = 0.388, \ s_{13}^2 = 0.0241 \) and bfv’s of \( \Delta m_{atm}^2, \ \Delta m_{sol}^2 \), for \( M = 3 \cdot 10^3 \) GeV, with \( \tan \beta \simeq 68 \) and \( M_S = 1 \) TeV we obtain needed baryon asymmetry \( [(\frac{\Delta}{s})]_{max} \simeq 8.56 \times 10^{-11}] \), however for this case the values of \( \sin^2 \theta_{ij} \) are deviated from the bfv’s by \( (2 - 3)\sigma \).

**B_2' Neutrino Texture: Improved Version**

In order to avoid difficulties with B_2 neutrino texture we add \( d_3 \) term to the (1, 2) and (2, 1) elements of the light neutrino mass matrix. After this, the \( M_\nu \) will have the form:

\[
M^{(B_2')\nu}(M_Z) = \begin{pmatrix}
2a_1b_1 & a_1b_2e^{i\phi} + a_2b_1 + d_5 & a_3b_1r_{\nu_3} \\
 a_1b_2e^{i\phi} + a_2b_1 + d_5 & 2a_2b_2e^{i\phi} & a_3b_2e^{i\rho}r_{\nu_3} \\
a_3b_1r_{\nu_3} & a_3b_2e^{i\rho}r_{\nu_3} & 0
\end{pmatrix} \tilde{m} . \tag{5.29}
\]

With this modification, all masses are non-zero, and therefore two additional parameters \( m_3 \neq 0 \) and \( \rho_2 \) enter. We refer to this \( (5.29) \) improved version as the B_2' neutrino texture. Thus our relations will involve two more independent quantities. For convenience we take \( m_3 \) and \( \Delta \rho = \rho_1 - \rho_2 \) as such. From the condition \( M^{(3,3)}_\nu = 0 \) we have:

\[
m_1 |U_{31}|^2 = |m_2(U_{32})^2 + m_3(U_{33})^2 e^{i\Delta \rho}| , \quad \rho_1 = \pi - \text{Arg} \left[ \frac{m_2(U_{31})^2}{m_2(U_{32})^2 + m_3(U_{33})^2 e^{i\Delta \rho}} \right] \text{ with } \Delta \rho = \rho_1 - \rho_2 . \tag{5.30}
\]

From these relations the phases \( \delta \) and \( \rho_1 \) can be calculated in terms of \( m_3 \) and \( \Delta \rho \).

As it turns out, in this improved version the IH case works well for both neutrino sector and the baryon asymmetry. So, we will start with discussing the IH case. For measured oscillation parameters we take the best fit values given in \( (3.20) \) and select pairs \( m_3, \Delta \rho \) in such a way as to get needed baryon asymmetry. One such choice is:

\[
m_3 = 0.01406 \text{ eV}, \quad \Delta \rho = 3.5257 \, , \tag{5.31}
\]

which with help of \( (3.19) \) and \( (5.30) \) determine neutrino masses and phases as:

\[
(m_1, m_2, m_3) = (0.0516, \ 0.052323, \ 0.01406) \text{ eV},
\]

\[
(\delta, \rho_1, \rho_2) = (2.8528, \ 3.1385, \ -0.38724) . \tag{5.32}
\]

These for the observable \( \nu_{02}\beta\)-decay give \( m_{\beta\beta} \simeq 0.0193 \) eV.

As far as the baryon asymmetry is concerned, using \( (5.29) \) in \( (3.15) \) for the CP phase \( \phi \) and expressing couplings \( a_{2,3}, b_{1,2} \) in terms of \( a_1 \) we get

\[
\phi = \text{Arg} \left( \frac{A_{23}^2A_{11}}{A_{13}^2A_{22}} \right) ,
\]

25
These for \( \tan \beta \) (\( \theta \)) corresponding minimal (allowed) values of \( \tan \beta \). For the values of (5.31), (5.32) and bfv’s of \( \theta_{ij} \) mixing angles.

\[
a_2 = \left| \frac{A_{22}A_{13}}{A_{11}A_{23}} \right| a_1, \quad a_3 = \frac{2}{r_{i3}} \left| \frac{A_{i3}}{A_{11}} \right| a_1, \quad b_1 = \left| \frac{A_{11}}{2|m|a_1} \right|, \quad b_2 = \left| \frac{A_{23}A_{11}}{A_{13}} \right| \frac{1}{2|m|a_1}. \quad (5.33)
\]

For the values of (5.31), (5.32) and bfv’s for the \( \theta_{ij} \) angles we get

\[
\phi = 2.2301. \quad (5.34)
\]

With these, and for given values of \( M \) and \( \tan \beta \) by varying \( a_1 \) we can investigate the baryon asymmetry. Results are given in Tab. 13.

As far as the NH case is concerned, the neutrino sector can work well by certain selection of \( (m_3, \Delta \rho) \). However, in order to generate needed baryon asymmetry we need to take values of \( \sin^2 \theta_{ij} \) deviated from the bfv’s by the \( (2 - 3)\sigma \). For example, with \( (s_{12}^2, s_{23}^2, s_{13}^2) = (0.27, 0.629, 0.022) \) and \( (m_3, \Delta \rho) = (0.060651 \text{ eV, } 3.12) \) we get

for NH of the \( B_2' \) neutrino texture: \( (m_1, m_2, m_3) = (0.033671, 0.034764, 0.060651) \text{ eV, } \)

\[
(\delta, \rho_1, \rho_2) = (-0.013, -0.12393, 3.0393) \implies \phi = -2.7538, \quad m_\beta \simeq 0.032 \text{ eV.} \quad (5.35)
\]

These for \( \tan \beta = 68.1 \) and \( M = 10^6 \text{ GeV, } M_S = 1 \text{ TeV } \) give the baryon asymmetry \( (\frac{a}{s})_{\max} \simeq 8.59 \cdot 10^{-11} \).

Note that the \( B_2' \) neutrino texture coincides with the texture \( P_7 \) of Ref. [18] if all entries in (5.29) are taken to be real. As was shown in [18] the real neutrino mass texture with \( M_{\nu}^{(3,3)} = 0 \) will work for both NH and IH neutrinos (see Tab. 6 of Ref. [18]). Advantage of complex \( d = 5 \) entry [like in texture (5.29)] is that it gives good possibility for generation of the baryon asymmetry with the \( \lambda_\mu \)’s radiative correction playing the decisive role. Similar possibility has not been considered in the literature before.

Concluding, note also that the \( A' \) and \( B_1' \) neutrino textures are generalizations of the textures \( P_5 \) and \( P_6 \) (respectively), considered in [18]. The latter two had no complex phases, while \( A' \) and \( B_1' \) scenarios besides good neutrino fits give possibility for the generation of the baryon asymmetry.
6 Discussion and Outlook

In this work we have investigated the resonant leptogenesis within the extension of the MSSM by two right handed neutrino superfields with quasi-degenerate masses $\lesssim 10^7$ GeV. It was shown that in this regime the cosmological CP asymmetry arises at one loop level due to charged lepton Yukawa couplings. In particular, needed corrections may come from either of the $\lambda_{\tau}$ and $\lambda_{\mu}$ couplings. Which one is relevant from these two couplings depends on the structure of the $3 \times 2$ Dirac type Yukawa matrix $Y_{\nu}$. Aiming to make close connection with the neutrino sector, we first examined all viable neutrino models (considered earlier in Ref. [18]) based on two texture zero $Y_{\nu}$’s augmented by single $\Delta L = 2$, $d = 5$ operators. This setup is predictive and allows to relate leptonic CP violating phase $\delta$ with the cosmological CP violation. In one of such scenarios the role of the $\lambda_{\mu}$ coupling in CP asymmetry generated at quantum level has been demonstrated. We have also revised the models of Ref. [17] and considered their improved versions by including proper $\Delta L = 2$, $d = 5$ operators. This allowed to have good fit with the neutrino data and generate needed amount of the baryon asymmetry.

Without specifying their origin, in our considerations we have extensively applied the $\Delta L = 2$, $d = 5$ operators, of the form given in Eq. (3.9). Such $d = 5$ couplings can be generated from a different sector via renormalizable interactions. For instance, introducing the pair of MSSM singlet states $N$, $\bar{N}$ and the superpotential couplings

$$\lambda^{(i)} l_i N h_u + \bar{\lambda}^{(j)} l_j \bar{N} h_u - M_N N \bar{N},$$

(6.1)

it is easy to verify that integration of the heavy $N$, $\bar{N}$ multiplets leads to the operator in Eq. (3.9) with

$$\hat{d}_5 e^{ix_5} = 2\lambda^{(i)} \bar{\lambda}^{(j)}.$$

(6.2)

Important ingredient here is to maintain forms of the matrices $Y_{\nu}$, $M_N$. In [15] considering one such fully consistent extension, it was demonstrated that all obtained results (e.g. neutrino masses and mixings, and baryon asymmetry as well) can remain intact. Although the way demonstrated above is rather simple, there can be considered also alternative ways for generating those $\Delta L = 2$ effective couplings. These could be done either in a spirit of type II [31], or type III [32] see-saw mechanisms, or even exploiting alternative possibilities [33], [34] through the introduction of appropriate extra states. Details of such scenarios should be pursued elsewhere.

Throughout our studies we have studied texture zero coupling matrices, but did not attempt to explain and justify considered structures by symmetries. Our approach, being rather phenomenological, was to consider such textures which give predictive and/or consistent scenarios allowing for transparent demonstrations of the suggested mechanism of the loop induced cosmological CP violation. It is desirable to have explanation of texture zeros at more fundamental level, and exploiting flavor symmetries seems to be a good framework. We are planning to pursue this approach in a future work [35].

Since the supersymmetry is a well motivated construction, we have performed our investigations within its framework. However, it would be interesting to examine the considered models also within the non-SUSY setup. For the latter, the scenarios with low tan $\beta$ look encouraging to start with.

Finally, it would be challenging to embed considered models in Grand Unification (GUT) such as $SU(5)$ and $SO(10)$ GUTs. Due to the high GUT symmetries, additional relations and constraints would emerge making models more predictive. These and related issues will be addressed elsewhere.
Note added: After this paper was submitted to arXiv, we have been informed by F.R. Joaquim that the role of $\lambda_\tau$ coupling for the resonant leptogenesis within non-SUSY scenarios had been investigated in earlier works \[10\].

**Acknowledgments**

Z.T. thanks CERN theory division for warm hospitality and partial support during his visit there.

## A Renormalization Group Studies

### A.1 Running of $Y_\nu$, $Y_e$ and $M_N$ Matrices

RG equations for the charged lepton and neutrino Dirac Yukawa matrices, appearing in the superpotential of Eq. (2.1), at 1-loop order have the forms $[36], [37]$:

\[
16\pi^2 \frac{d}{dt} Y_e = 3 Y_e Y_e^\dagger Y_e + Y_\nu Y_\nu^\dagger Y_e + Y_e \left[ \text{tr} \left( 3 Y_e^\dagger Y_d + Y_e^\dagger Y_e \right) - c_e^a g_a^2 \right], \quad c_e^a = \left( \frac{9}{5}, 3, 0 \right), \quad (A.1)
\]

\[
16\pi^2 \frac{d}{dt} Y_\nu = Y_\nu Y_\nu^\dagger Y_\nu + 3 Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu \left[ \text{tr} \left( 3 Y_\nu^\dagger Y_u + Y_\nu^\dagger Y_\nu \right) - c_\nu^a g_a^2 \right], \quad c_\nu^a = \left( \frac{3}{5}, 3, 0 \right). \quad (A.2)
\]

$g_a = (g_1, g_2, g_3)$ denote gauge couplings of $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ gauge groups respectively. Their 1-loop RG have forms $16\pi^2 \frac{d}{dt} g_a = b_a g_a^3$, with $b_a = (\frac{33}{5}, 1, -3)$, where the hypercharge of $U(1)_Y$ is taken in $SU(5)$ normalization.

The RG for the RHN mass matrix at 2-loop level has the form $[37]$:

\[
16\pi^2 \frac{d}{dt} M_N = 2 M_N Y_\nu Y_\nu^\dagger - \frac{1}{8\pi^2} M_N \left[ Y_\nu^\dagger Y_\nu Y_\nu + Y_\nu^\dagger Y_\nu^\dagger Y_\nu + Y_\nu Y_\nu^\dagger Y_\nu \text{tr}(3 Y_\nu^\dagger Y_u + Y_\nu^\dagger Y_\nu) \right] + \frac{1}{8\pi^2} M_N Y_\nu Y_\nu^\dagger \left( \frac{3}{5} g_1^2 + 3 g_2^2 \right) + \text{(transpose)}, \quad (A.3)
\]

Let’s start with renormalization of the $Y_\nu$’s matrix elements. Ignoring in Eq. (A.2) the $\mathcal{O}(Y_\nu^3)$ order entries (which are very small because within our studies $|(Y_\nu)_{ij}| \leq 10^{-4}$), and from charged fermion Yukawas keeping $\lambda_\tau$, $\lambda_\mu$, $\lambda_t$ and $\lambda_b$, we will have:

\[
16\pi^2 \frac{d}{dt} \ln(Y_\nu)_{ij} \simeq \delta_{i3} \lambda_\tau^2 + \delta_{i2} \lambda_\mu^2 + 3 \lambda_b^2 - c_\nu^a g_a^2 . \quad (A.4)
\]

This gives the solution

\[
(Y_\nu)_{ij}(\mu) = (Y_{\nu G})_{ij}(\eta_{\nu}(\mu))^{\delta_{i3} \lambda_\tau^2 + \delta_{i2} \lambda_\mu^2 + 3 \lambda_b^2 - c_\nu^a g_a^2} \eta_{\nu}(\mu), \quad (A.5)
\]

where $Y_{\nu G}$ denotes Yukawa matrix at scale $M_G$ and the scale dependent RG factors are given by:

\[
\eta_{h,b,\tau,\mu}(\mu) = \exp \left( -\frac{1}{16\pi^2} \int_{t_G}^{t} \frac{L^2}{L_{\tau,b,\tau,\mu}}(t') dt' \right), \quad \eta_{a}(\mu) = \exp \left( \frac{1}{16\pi^2} \int_{t_G}^{t} \frac{L^2}{L_{a}}(t') dt' \right)
\]

\[
\eta_{\nu}(\mu) = \exp \left( \frac{1}{16\pi^2} \int_{t_G}^{t} c_\nu^a g_a^2(t') dt' \right) = \eta_{3/3}^{\nu}(\mu) \eta_{\nu}(\mu), \quad \text{with} \quad t = \ln \mu , \quad t' = \ln \mu' , \quad t_G = \ln M_G. \quad (A.6)
\]

From these, for the combination $Y_\nu Y_\nu^\dagger$ at scale $\mu = M$ we get expression given in Eq. (2.16).

On the other hand, for the RHN mass splitting and for the phase mismatch (depending on $\xi_{\tau,\mu}$ defined in Eq. (2.17)), the integrals/factors of Eqs. (2.13), (2.14), (2.15) and (2.16) will be relevant.
A.2 Relating \( M_{\nu}(M_Z) \) and \( M_{\nu}(M) \)

Details of derivations, of the results presented in this subsection, are given in Appendix A.2 of Ref. [15]. At scale \( M \), after decoupling of the RHN states, the neutrino mass matrix is generated and has the form:

\[
M_{\nu}^{ij}(M) = - \begin{pmatrix}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times 
\end{pmatrix} \frac{v_u^2(M)}{Me^{-i(\omega+\rho)}}, \tag{A.7}
\]

where ‘\( \times \)’ stand for entries depending on Yukawa couplings. After renormalization, keeping \( \lambda_t, \lambda_b \) and \( g_a \) in the RGs, the neutrino mass matrix at scale \( M_Z \) has the form:

\[
M_{\nu}^{ij}(M_Z) = \begin{pmatrix}
\times & \times & (\times) \cdot r_{\nu 3} \\
\times & \times & (\times) \cdot r_{\nu 3} \\
(\times) \cdot r_{\nu 3} & (\times) \cdot r_{\nu 3} & (\times) \cdot r_{\nu 3}^2 
\end{pmatrix} \bar{m}, \tag{A.8}
\]

with \( \bar{m} \) given in Eq. (3.22) and \( \times \) in Eq. (A.8) denotes entries determined at scale \( M \) and corresponding to those in (A.7), and RG factors \( r_{\nu 3}, r_{\bar{m}} \) are given respectively in Eqs. (A.17), (A.18) of Ref. [15].

We will also need the RG factor relating the VEV \( v_u(M) \) to the \( v(M_Z) \). Thus we define:

\[
\frac{r_{v_u}}{v(M_Z)} = s_\beta. \tag{A.9}
\]

Analytic expression for \( r_{v_u} \) derived from appropriate RGs is given by Eq. (A.20) of Ref. [15].

A.3 Calculation Procedure and Used Schemes

To find the RG factors, appearing in the baryon asymmetry and in the neutrino mass matrix renormalization, we numerically solve renormalization group equations from the scale \( M_Z \) up to the \( M_G \approx 2 \cdot 10^{16} \) GeV scale. For simplicity, for all SUSY particle masses we take common mass scale \( M_S \). Thus, in the energy interval \( M_Z \leq \mu < M_S \), the Standard Model RGs for \( MS \) coupling constants are used. However, in the interval \( M_S \leq \mu \leq M_G \), since we are dealing with the SUSY, the RGs for the DR couplings are applied. Below we give boundary and matching conditions for the gauge couplings \( g_{1,2,3} \), for Yukawa constants \( \lambda_{t,b,\tau,\mu} \) and for the Higgs self-coupling \( \lambda \).

Gauge couplings \( \alpha_a = \frac{g_a^2}{4\pi} \)

We choose our inputs for the \( MS \) gauge couplings at scale \( M_Z \) as follows:

\[
\alpha_1^{-1}(M_Z) = \frac{3}{5} c_w^2 \alpha_{em}^{-1}(M_Z) + \frac{3}{5} c_w^2 \frac{8}{9\pi} \ln \frac{m_t}{M_Z}, \quad \alpha_2^{-1}(M_Z) = s_w^2 \alpha_{em}^{-1}(M_Z) + s_w^2 \frac{8}{9\pi} \ln \frac{m_t}{M_Z}, \quad \alpha_3^{-1}(M_Z) = \alpha_s^{-1}(M_Z) + \frac{1}{3\pi} \ln \frac{m_t}{M_Z}, \tag{A.10}
\]

where logarithmic terms \( \ln \frac{m_t}{M_Z} \) are due to the top quark threshold correction [38], [39]. Taking \( \alpha_s(M_Z) = 0.1185, \alpha_{em}(M_Z) = 127.934 \) and \( s_w^2 = 0.2313 \), from (A.10) we obtain:

\[
\alpha_1^{-1}(M_Z) = 59.0057 + \frac{8s_w^2}{15\pi} \ln \frac{m_t}{M_Z}, \quad \alpha_2^{-1}(M_Z) = 29.5911 + \frac{8s_w^2}{9\pi} \ln \frac{m_t}{M_Z},
\]

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\[ \alpha_3^{-1}(M_Z) = 8.4388 + \frac{1}{3\pi} \ln \frac{m_t}{M_Z}. \] (A.11)

With these inputs we run \( q_{1,2,3} \) via the 2-loop RGs from \( M_Z \) up to the scale \( M_S \).

At scale \( \mu = M_S \) we use the matching conditions between DR – MS gauge couplings [40][41]:

\[ \begin{align*}
\alpha_1^{\text{DR}} &= \frac{1}{\alpha_1^{\text{MS}}}, \\
\alpha_2^{\text{DR}} &= \frac{1}{\alpha_2^{\text{MS}}} - \frac{1}{6\pi}, \\
\alpha_3^{\text{DR}} &= \frac{1}{\alpha_3^{\text{MS}}} - \frac{1}{4\pi}.
\end{align*} \] (A.12)

Above the scale \( M_S \) we apply 2-loop SUSY RG equations in DR scheme [36].

**Yukawa Couplings and \( \lambda \)**

At the scale \( M_S \) all SUSY states decouple and we are left with the Standard Model with one Higgs doublet. Thus, Yukawa couplings we are considering and the self-coupling are determined as:

\[ \begin{align*}
\lambda_t(m_t) &= \frac{m_t(m_t)}{v(m_t)}, \\
\lambda_b(M_Z) &= \frac{2.89\text{GeV}}{v(M_Z)}, \\
\lambda_{\tau}(M_Z) &= \frac{1.746\text{GeV}}{v(M_Z)}, \\
\lambda_{\mu}(M_Z) &= \frac{0.1027\text{GeV}}{v(M_Z)}, \\
\lambda(m_h) &= \frac{1}{4} \left( \frac{m_h}{v(m_h)} \right)^2,
\end{align*} \] (A.13)

where \( m_t(m_t) \) is the top quark running mass related to the pole mass as:

\[ m_t(m_t) = p_t M_{t}^{\text{pole}}. \] (A.14)

The factor \( p_t \) is \( p_t \approx 1/1.0603 \) [42], while the recent measured value of the top’s pole mass is [43]:

\[ M_{t}^{\text{pole}} = (173.34 \pm 0.76) \text{ GeV}. \] (A.15)

We take the values of (A.13) as boundary conditions for solving 2-loop RG equations [41], [39] for \( \lambda_t, b, \tau, \mu \) and \( \lambda \) from the \( M_Z \) scale up to the scale \( M_S \).

Above the \( M_S \) scale, we have MSSM states including two doublets \( h_u \) and \( h_d \), which couple with up type quarks and down type quarks/charged leptons respectively. Thus, Yukawa couplings we are considering at \( M_S \) are \( \approx \lambda_t(M_S)/s_\beta, \lambda_b(M_S)/c_\beta \) and \( \lambda_{\tau,\mu}(M_S)/c_\beta \), with \( s_\beta \equiv \sin \beta, c_\beta \equiv \cos \beta \).

Above the scale \( M_S \) we apply 2-loop SUSY RG equations in DR scheme [36]. Thus, at \( \mu = M_S \) we use the matching conditions between DR – MS couplings:

\[ \begin{align*}
\alpha_1^{\text{DR}} &\approx \frac{\lambda_{\text{MS}}^{t}}{s_\beta} \left[ 1 + \frac{1}{16\pi^2} \left( \frac{g_1^2}{120} + \frac{3g_2^2}{8} - \frac{4g_3^2}{3} \right) \right], \\
\alpha_2^{\text{DR}} &\approx \frac{\lambda_{\text{MS}}^{b}}{c_\beta} \left[ 1 + \frac{1}{16\pi^2} \left( \frac{13g_1^2}{120} + \frac{3g_2^2}{8} - \frac{4g_3^2}{3} \right) \right], \\
\alpha_3^{\text{DR}} &\approx \frac{\lambda_{\text{MS}}^{\tau,\mu}}{c_\beta} \left[ 1 + \frac{1}{16\pi^2} \left( -\frac{9g_1^2}{40} + \frac{3g_2^2}{8} \right) \right].
\end{align*} \] (A.16)

where expressions in brackets of r.h.s. of the relations are due to the DR – MS conversions [41].

With Eq. (A.16)’s matchings we run corresponding couplings from the scale \( M_S \) up to the \( M_G \) scale. Throughout the paper, above the mass scale \( M_S \) without using the superscript DR we assume the couplings determined in this scheme.
B Baryon Asymmetry from RHS Decays

In this appendix we give details of the contribution to the net baryon asymmetry from the right handed sneutrinos (RHS) - the scalar partners of the RHNs. Estimation of this contribution for specific textures was given in [17], while more detailed investigation was given in [15] (from the lepton couplings taking into account only $\lambda_\nu$ and $A_\tau$ in the proper RGs). Since we have seen that for some cases for the cosmological CP asymmetry decisive is the RG correction via the $\lambda_\mu$ Yukawa coupling, here we extend its calculation by taking into account also effects from $\lambda_\mu$ and $A_\mu$ into the asymmetry generated by the RHS decays.

We will consider soft SUSY breaking scalar potential

$$ V_{SB} = \bar{l}^T A_\nu \tilde{N} h_u - \frac{1}{2} N^T B_N \tilde{N} + h.c. + \bar{l} m_\nu^2 \tilde{l} + \tilde{N}^T m_N^2 \tilde{N}, \tag{B.1} $$

which will be relevant for deriving RHS masses and their couplings to the components of the $l$ and $h_u$ superfields. Using general expressions of Ref. [36] we write down 1-loop RGs for $A_\nu$ and $B_N$, which have the forms:

$$ 16\pi^2 \frac{d}{dt} A_\nu = Y_e Y_e^\dagger A_\nu + 2 \hat{A}_e Y_e^\dagger A_\nu + 5Y_\nu Y_\nu^\dagger A_\nu + A_\nu \left[ \text{tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) + 4Y_\nu^\dagger Y_\nu - c_\nu^a g_a^2 \right] $$
$$ + 2Y_\nu \left[ \text{tr}(3Y_u^\dagger \hat{A}_u + Y_\nu^\dagger A_\nu) + c_\nu^a g_a^2 M_{\hat{Y}_a} \right], \tag{B.2} $$

$$ 16\pi^2 \frac{d}{dt} B_N = 2B_N Y_\nu^\dagger Y_\nu + 2Y_\nu^T Y_\nu^* B_N + 4M_N Y_\nu^\dagger A_\nu + 4A_\nu^T Y_\nu^* M_N. \tag{B.3} $$

We parameterize the matrices $B_N$ and $A_\nu$ as:

$$ B_N = (M_N)_{12} m_B \begin{pmatrix} \delta^{(1)}_{BN} & 1 \\ \delta^{(2)}_{BN} \end{pmatrix}, \quad A_\nu = m_A a_\nu, \tag{B.4} $$

where entries $(M_N)_{12}, m_B, \delta^{(1,2)}_{BN}$ and elements of the matrix $a_\nu$ run (their RGs can be derived from the RG equations given above), while $m_A$ is a constant. The matrix $\hat{A}_e$ (similar to the structure of $Y_e$ Yukawa matrix) is

$$ \hat{A}_e = \text{Diag} (A_e, A_\mu, A_\tau). \tag{B.5} $$

Assuming proportionality / alignment of the soft SUSY breaking terms and corresponding superpotential couplings, we will use the following boundary conditions:

at $\mu = M_G$: \quad $a_\nu = Y_\nu \,$, \quad $\delta^{(1)}_{BN} = \delta^{(2)}_{BN} = 0$ \quad $\hat{A}_e = m_A \text{Diag} (\lambda_e, \lambda_\mu, \lambda_\tau)$

$$ \hat{A}_u = m_A Y_u G, \quad \hat{A}_d = m_A Y_d G. \tag{B.6} $$

Using (B.3) for $B_N$’s entries in (B.4) we have:

$$ 16\pi^2 \frac{d}{dt} \delta^{(1)}_{BN} \simeq 4(Y_\nu^\dagger Y_\nu)_{21} + \frac{8}{m_B} (Y_\nu^\dagger a_\nu)_{21}, \quad 16\pi^2 \frac{d}{dt} \delta^{(2)}_{BN} \simeq 4(Y_\nu^\dagger Y_\nu)_{12} + \frac{8}{m_B} (Y_\nu^\dagger a_\nu)_{12}. \tag{B.7} $$

For the elements of $a_\nu$, we have

$$ 16\pi^2 \frac{d}{dt} \left( \frac{(a_\nu)_{ij}}{(Y_\nu)_{ij}} \right) \simeq 2 \frac{1}{m_A} (\delta_{33} \lambda_\tau A_\tau + \delta_{i2} \lambda_\mu A_\mu) + \frac{2}{m_A} (3\lambda_i A_i + c_\nu^a g_a^2 M_{\hat{Y}_a}), \tag{B.8} $$

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which show the violation of the alignment between $a_{\nu}$ and $Y_{\nu}$ due to RG effects. At r.h.s. of (B.8) we kept $\lambda_{\mu,\tau,t,}$, gauge couplings and gaugino masses. From this we derive
\[
 a_{\nu} \simeq \begin{pmatrix}
 1 + \epsilon_0 & 0 & 0 \\
 0 & 1 + \epsilon_0 + \epsilon_{\mu} & 0 \\
 0 & 0 & 1 + \epsilon_0 + \epsilon_{\tau}
\end{pmatrix} Y_{\nu},
\]
with $\epsilon_0 = -\frac{1}{8\pi^2 m_A} \int_{t}^{t_G} dt (3\lambda_{\nu}A_{i} + c_{\nu} g_{\nu}^2 M_{\nu}^2)$, $\epsilon_{\mu,\tau} = -\frac{1}{8\pi^2 m_A} \int_{t}^{t_G} dt \lambda_{\mu,\tau} A_{\mu,\tau}$.

(B.9)

Keeping in mind that the powers of the $Y_{\nu}$ couplings can be ignored due to their smallness, the $m_B$ can be treated as a constant, and from (B.9), (B.7), (B.4) we obtain:
\[
\text{at } \mu = M: \quad B_N = m_B M \left(-\alpha_0 \delta_N (1 + \epsilon_1) \frac{1}{1 - \alpha_0 \delta_N (1 + \epsilon_2)} \right), \quad \alpha = 1 + 2 \frac{m_A}{m_B} (B.10)
\]
and
\[
\tilde{\epsilon}_1 = \frac{1}{4\pi^2 \alpha_N} \int_{t_M}^{t_G} dt \left( \frac{2m_A}{m_B} \epsilon_1 \right) Y_{\nu}^*, \quad \tilde{\epsilon}_2 = \frac{1}{4\pi^2 \alpha_N} \int_{t_M}^{t_G} dt \left( \frac{2m_A}{m_B} \epsilon_2 \right) Y_{\nu}^*,
\]
with $\tilde{\epsilon} = \text{Diag}(\epsilon_0, \epsilon_0 + \epsilon_{\mu}, \epsilon_0 + \epsilon_{\tau})$.

(B.11)

The form of $B_N$ given in Eq. (B.10) will be used to construct the RHS mass matrix. Before doing this, using Eq. (A.5) and ignoring the coupling $\lambda_{\nu}$ (as it turns out from the lepton Yukawa couplings all relevant effects are due to $\lambda_{\nu,\tau}$), for $\tilde{\epsilon}_{1,2}$ at scale $\mu = M$ we can get expressions:
\[
\tilde{\epsilon}_1(M) = \frac{1}{4\pi^2 \alpha_N} (Y_{\nu}^* \hat{K} Y_{\nu})_{21} \bigg|_{\mu = M}, \quad \tilde{\epsilon}_2(M) = \frac{1}{4\pi^2 \alpha_N} (Y_{\nu}^* \hat{K} Y_{\nu}^*)_{21} \bigg|_{\mu = M}
\]
with $\hat{K} = \frac{1}{\eta_{\nu}^2 \eta_{\nu}^2} \text{Diag} \left[ 2 \frac{m_A}{m_B} I_0, \frac{1}{\eta_{\nu}^2} \left( \frac{2m_A}{m_B} I_1^{(\mu)} + \frac{\alpha}{16\pi^2} I_2^{(\mu)} \right), \frac{1}{\eta_{\nu}^2} \left( \frac{2m_A}{m_B} I_1^{(\tau)} + \frac{\alpha}{16\pi^2} I_2^{(\tau)} \right) \right],
\]
\[
I_0 = \int_{t_M}^{t_G} dt \int_{t_M}^{t_G} dt \eta_{\nu}^2 \eta_{\nu}^2 \epsilon_0, \quad I_1^{(\mu,\tau)} = \int_{t_M}^{t_G} dt \int_{t_M}^{t_G} dt \eta_{\nu}^2 \eta_{\nu}^2 (\epsilon_0 + \epsilon_{\mu,\tau}) \eta_{\mu,\tau}^2, \quad I_2^{(\mu,\tau)} = \int_{t_M}^{t_G} dt \int_{t_M}^{t_G} dt \eta_{\nu}^2 \eta_{\nu}^2 \eta_{\mu,\tau}^2 \eta_{\mu,\tau}^2.
\]

(B.12)

Keeping the $B_N$-term in (B.11) and including the mass$^2$ term $\tilde{N}^\dagger \tilde{M}_N \tilde{N}$ coming from the superpotential, the quadratic (with respect to $\tilde{N}$'s) potential will be:
\[
V^{(2)}_{\tilde{N}} = \tilde{N}^\dagger \tilde{M}_N \tilde{N} - \left( \frac{1}{2} \tilde{N}^T B_N \tilde{N} + \text{h.c.} \right).
\]

(B.13)

With the transformation of the $N$ superfields $N = U_N N'$ (according to Eq. (2.6), the $U_N$ diagonalizes the fermionic RHN mass matrix), we obtain:
\[
V^{(2)}_{\tilde{N}} = \tilde{N}^\dagger (M_N^{\text{diag}})^2 \tilde{N}' - \left( \frac{1}{2} \tilde{N}'^T U_N^T B_N U_N \tilde{N}' + \text{h.c.} \right).
\]

(B.14)

With phase redefinition
\[
\tilde{N}' = \hat{P}_1 \tilde{N}'', \quad \hat{P}_1 = \text{Diag} \left( e^{-i\tilde{\omega}_1/2}, e^{-i\tilde{\omega}_2/2} \right), \quad \text{with } \tilde{\omega}_{1,2} = \text{Arg}[m_B(1 \mp \tilde{\alpha} |\delta_N|)]
\]

(B.15)
and by going to the real scalar components
\[ \hat{N}_1'' = \frac{1}{\sqrt{2}}(\hat{N}_1^R + i\hat{N}_1^I), \quad \hat{N}_2'' = \frac{1}{\sqrt{2}}(\hat{N}_2^R + i\hat{N}_2^I), \] (B.16)
and using (B.10), we will have:
\[ - \left( \frac{1}{2} \hat{N}^T U_N^T B_N U_N \hat{N} + \text{h.c.} \right) = - \frac{|M_{MB}|}{2} |1 - \hat{\alpha}| |\delta_N|| \left( (\hat{N}_1^R)^2 - (\hat{N}_1^I)^2 \right) \]
\[ - \frac{|M_{MB}|}{2} |1 + \hat{\alpha}| |\delta_N|| \left( (\hat{N}_2^R)^2 - (\hat{N}_2^I)^2 \right) - |M| \text{Re}(m_B \delta_e) \left( \hat{N}_1^R \hat{\bar{N}}_2^R - \hat{N}_1^I \hat{\bar{N}}_2^I \right) + |M| \text{Im}(m_B \delta_e) \left( \hat{N}_1^I \hat{\bar{N}}_2^R + \hat{N}_1^R \hat{\bar{N}}_2^I \right) \]
with \[ \hat{\alpha} = \alpha(1 + \frac{\bar{\varepsilon}_1 + \varepsilon_2}{2}), \quad \delta_e = i\alpha|\delta_N| \frac{\bar{\varepsilon}_1 - \varepsilon_2}{2} e^{-i(\bar{\omega}_1 + \bar{\omega}_2)/2} \] . (B.17)
From (B.14) and (B.17) we obtain the mass^2 terms:
\[ V_{\tilde{n}}^{(2)} = \frac{1}{2} \tilde{n}^{0T} M_\tilde{n}^2 \tilde{n}^{0}, \quad \text{with} \quad \tilde{n}^{0T} = \left( \hat{N}_1^R, \hat{N}_1^I, \hat{N}_2^R, \hat{N}_2^I \right) \]
and
\[ M_\tilde{n}^2 = \begin{pmatrix} (\tilde{M}_0^0)^2 & 0 & -|M| \text{Re}(m_B \delta_e) & |M| \text{Im}(m_B \delta_e) \\ 0 & (\tilde{M}_0^0)^2 & |M| \text{Im}(m_B \delta_e) & |M| \text{Re}(m_B \delta_e) \\ -|M| \text{Re}(m_B \delta_e) & |M| \text{Im}(m_B \delta_e) & (\tilde{M}_0^0)^2 & 0 \\ |M| \text{Im}(m_B \delta_e) & |M| \text{Re}(m_B \delta_e) & 0 & (\tilde{M}_0^0)^2 \end{pmatrix} \] (B.19)
where
\[ (\tilde{M}_0^0)^2 = |M|^2(1 - |\delta_N|^2)^2 - |m_B M| |1 - \hat{\alpha}| |\delta_N||, \quad (\tilde{M}_0^0)^2 = |M|^2(1 - |\delta_N|^2)^2 + |m_B M| |1 - \hat{\alpha}| |\delta_N|| , \]
\[ (\tilde{M}_0^0)^2 = |M|^2(1 + |\delta_N|^2)^2 - |m_B M| |1 + \hat{\alpha}| |\delta_N||, \quad (\tilde{M}_0^0)^2 = |M|^2(1 + |\delta_N|^2)^2 + |m_B M| |1 + \hat{\alpha}| |\delta_N|| \] (B.20)

The coupling of \( \tilde{n}^0 \) states with the fermions emerges from the F-term of the superpotential \( t^T Y_\nu N h_a \). Following the transformations, indicated above, we will have:
\[ (t^T Y_\nu N h_a)_F \rightarrow \tilde{h}_a t^T Y_\nu \tilde{N} = e^{-i\bar{\omega}_2/2} \tilde{h}_a t^T Y_\nu U_N \left( \rho_a e^{i(\bar{\omega}_2 - \bar{\omega}_1)/2}, \rho_d \right) \tilde{n}^0, \]
with \[ \rho_a = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & 0 \\ 0 & i \end{array} \right), \quad \rho_d = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 0 & 0 \\ 1 & i \end{array} \right) \] . (B.21)
Diagonalizing the matrix (B.19) by the transformation
\[ V_{\tilde{n}}^T M_\tilde{n}^2 V_{\tilde{n}} = (M_{\tilde{n}}^{\text{Diag}})^2, \quad \tilde{n}^0 = V_{\tilde{n}} \tilde{n}, \] (B.22)
the fermion coupling with the scalar \( \tilde{n} \) mass eigenstates will be
\[ \tilde{h}_a t^T Y_F \tilde{n} \quad \text{with} \quad Y_F = Y_\nu \tilde{V}^0 \tilde{V}_\nu, \quad \tilde{V}^0 = U_N \left( \rho_a e^{-i\bar{\omega}_2/2}, \rho_d e^{-i\bar{\omega}_1/2} \right) . \] (B.23)
The coupling with the slepton \( \tilde{l} \) is derived from the interaction term \( h_a t^T (Y_\nu M_{\tilde{N}}^* \tilde{N}^*-A_\nu \tilde{N}) \).
Going from \( \tilde{N} \) to the \( \tilde{n} \) states, one obtains:
\[ h_a t^T Y_B \tilde{n} \quad \text{with} \quad Y_B = (Y_\nu M_{\tilde{N}}^* \tilde{V}^{0*} - A_\nu \tilde{V}^0) V_{\tilde{n}} . \] (B.24)
For given values of \( M, m_{\tilde{q}} \) and \( m_A \), with help of Eqs. (B.19), (B.23) and (B.24), we will have coupling matrices \( Y_F, Y_B \) and all other quantities needed for calculation of the baryon asymmetry created via the decays of the \( \tilde{n}_{1,2,3,4} \) states.
B.1 Calculating $\frac{n_b}{s}$ - Asymmetry Via $\tilde{n}$ Decays

Due to the SUSY breaking terms, the masses of RHS’s differ from their fermionic partners’ masses. For each mass-eigenstate RHS’s $\tilde{n}_{i=1,2,3,4}$ we have one of the masses $\tilde{M}_{i=1,2,3,4}$ respectively. With the SUSY $M_S$ scale $\frac{M_S}{\tilde{M}} \lesssim 1/3$, the states $\tilde{n}_i$ remain nearly degenerate and for the resonant $\tilde{n}_i$-decays the resummed effective amplitude technique [9] will be applied. Effective amplitudes for the real $\tilde{n}_i$ decay, say into the lepton $l_{\alpha}$ ($\alpha = 1, 2, 3$) and antilepton $\bar{l}_{\alpha}$ respectively are given by [9]

$$\tilde{S}_{\alpha i} = S_{\alpha i} - \sum_j S_{\alpha j} \frac{\Pi_{ji}(\tilde{M}_i)(1 - \delta_{ij})}{\tilde{M}_i^2 - \tilde{M}_j^2 + \Pi_{jj}(\tilde{M}_i)} \, , \quad \tilde{S}^*_\alpha i = S^*_{\alpha i} - \sum_j S^*_{\alpha j} \frac{\Pi_{ji}(\tilde{M}_i)(1 - \delta_{ij})}{\tilde{M}_i^2 - \tilde{M}_j^2 + \Pi_{jj}(\tilde{M}_i)} \, ,$$

(B.25)

where $S_{\alpha i}$ is a tree level amplitude and $\Pi_{ij}$ is a two point Green function’s (polarization operator of $\tilde{n}_i$ - $\tilde{n}_j$) absorptive part. The CP asymmetry is then given by

$$\epsilon_{\alpha i}^{sc} = \frac{\sum_{\alpha} \left( |\tilde{S}_{\alpha i}|^2 - |\tilde{S}^*_{\alpha i}|^2 \right)}{\sum_{\alpha} \left( |\tilde{S}_{\alpha i}|^2 + |\tilde{S}^*_{\alpha i}|^2 \right)} \, .$$

(B.26)

With $Y_F$ and $Y_B$ given by Eqs. (B.23) and (B.24) we can calculate polarization diagram’s (with external legs $\tilde{n}_i$ and $\tilde{n}_j$) absorptive part $\Pi_{ij}$. These at 1-loop level are given by:

$$\Pi_{ij}(p) = \frac{i p^2}{8\pi} \left( 1 - \frac{M_S^2}{p^2} \right)^2 \left( Y_F^\dagger Y_F + Y_B^\dagger Y_B^* \right)_{ij} + i \frac{1}{8\pi} \left( \epsilon^2_\beta + \epsilon^2_\beta \left( 1 - \frac{M_S^2}{p^2} \right) \right) \left( Y_B^\dagger Y_B + Y_B^* Y_B^* \right)_{ij} \, ,$$

(B.27)

where $p$ denotes external momentum in the diagram and upon evaluation of (B.26), for $\Pi$ one should use (B.27) with $p = \tilde{M}_i$. In (B.27), taking into account the SUSY masses $M_S$ of all non SM states, we are using the refined expression for the $\Pi_{ij}$.

In an unbroken SUSY limit, neglecting finite temperature effects ($T \to 0$), the $\tilde{N}$ decay does not produce lepton asymmetry due to the following reason. The decays of $\tilde{N}$ in the fermion and scalar channels are respectively $\tilde{N} \to \tilde{l}h_u$ and $\tilde{N} \to \tilde{l}^* h^*_u$. Since the rates of these processes are the same due to SUSY (at $T = 0$), the lepton asymmetries created from these decays cancel each other. With $T \neq 0$, the cancellation does not take place and one has

$$\tilde{\epsilon}_i = \epsilon_i (\tilde{n}_i \to \tilde{l}h_u) \Delta_{BF} \, ,$$

(B.28)

with a temperature dependent factor $\Delta_{BF}$ given in [15][14]. Therefore, we just need to compute $\epsilon_i (\tilde{n}_i \to \tilde{l}h_u)$, which is the asymmetry created by $\tilde{n}_i$ decays in two fermions. Thus, in (B.25) we take $S_{\alpha i} = (Y_F)_{\alpha i}$ and calculate $\epsilon_i (\tilde{n}_i \to \tilde{l}h_u)$ with (B.26). The baryon asymmetry created from the lepton asymmetry due to $\tilde{n}$ decays is given by:

$$\frac{n_b}{s} \simeq -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \frac{\tilde{\epsilon}_i}{\Delta_{BF}} \eta_i = -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \epsilon_i (\tilde{n}_i \to \tilde{l}h_u) \eta_i \, ,$$

(B.29)

where an effective number of degrees of freedom (including two RHN superfields) $g_s = 228.75$ was used. $\eta_i$ are efficiency factors which depend on $\tilde{n}_i \simeq \frac{(\nu_{\alpha} \sin \beta)^2}{M} 2 (Y_F^\dagger Y_F)_{\alpha i}$, and account for temperature effects once integration of the Boltzmann equations is performed [15].

\[14\] This expression is valid with alignment $A_{\nu} = m_A Y_\nu$, which we are assuming to be true at the GUT scale and thus Eq. (B.28) can be well applicable to our estimates.

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Calculating the contribution \( \frac{\Delta n_s}{s} \) to the baryon asymmetry from the RHS decays, we have examined various values of pairs \((m_A, m_B)\) in the range of 100 GeV - few TeV. As it turned out, the ratio \( \frac{n_s}{\eta} \) is always suppressed (< 3.4 \times 10^{-2}). The results for each neutrino scenario, we have considered in this paper, for one specific choice of \((m_A, m_B)\), are given in Table 14 (see its caption for more information). The ranges for \( \frac{n_s}{\eta} \) are due to the fact that for each scenario we have considered different values of \(\tan \beta, M, M_S\). Upon the calculations, with obtained values of \(\tilde{m}_i\), according to Ref. \[45\] we picked up the corresponding values of \(\eta\) and used them in \((B.29)\). While giving the results of the net baryon asymmetry, for each case (see sections \[4\] and \[5\]), we have included corresponding contributions from \( \frac{n_s}{\eta} \) as well. As we see from the results of Tab. 14, the \( \frac{n_s}{\eta} \) is suppressed/subleading for all cases. We have also witnessed (by varying the phases of \(m_{A,B}\)) that the complexities of \(m_A\) and \(m_B\) practically do not change the results. This happens because the \(m_A\) in the \(Y_B\) coupling matrix appears in front of the \(Y_s\) [see Eq. \((B.24)\)], which is strongly suppressed. Irrelevance of the \(m_B\)’s phase can be seen from the structure of \((B.19)\). Suppression of \( \frac{n_s}{\eta} \) will always happen for the value of \(|m_B|\) in the range of 100 GeV - few TeV, because the mass degeneracy of \(\tilde{n}_i\) states is lifted in such a way that resonant enhancement of \( \frac{n_s}{\eta} \) is not realized. (Unlike the case of soft leptogenesis \[45\] which requires \(|m_B| \lesssim 10 \text{ MeV}\). Without special arrangement, such suppressed values of \(|m_B|\) seem unnatural and we have not considered them within our studies.)

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