Unruh’s detector in the presence of Lorentz symmetry breaking

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Abstract

We investigate the quantum field theory of a Lorentz non-invariant model with a massive nonlinear dispersion relation in Minkowski space. The model involves some non-causal signals in the form of wave packets propagating with super-luminal group velocities. To avoid the problems with causality we characterize the causal sector of the theory by a cutoff condition excluding all super-luminal group velocities. It is argued that in the causal theory satisfying the energy positivity condition an Unruh’s detector moving with a constant velocity with respect to the preferred frame does not detect any particle. But in a causal theory violating energy positivity the registered of field quanta occurs. We comments on the origin of this particle creation.

1 Introduction

The Lorentz symmetry is supposed to be a fundamental symmetry of physics. It contains two distinct sub-groups, rotation and boost. The former has not any ambiguity since it can be checked and has been checked. But boost transformations is not clear very well. Since their parameters are unbounded, the Lorentz group is non-compact. This non-compactness imposes an upper-bound for testing the Lorentz symmetry experimentally. It means that the Lorentz symmetry may be a symmetry in low energy physics only. This aspect is clearly reflected in the theoretical apparat of Quantum Field Theory where a short distance cutoff is always necessary to remove the divergencies or even to avoid the triviality problem [2]. Such a cutoff may break the Lorentz symmetry\textsuperscript{1}. Quantum Gravity and String Theory predict also some breaking of the Lorentz symmetry [3, 4, 5]. This arises as a result of the existence of a minimum length [6]. In addition, some cosmological observations suggest some deviation from conventional energy-momentum’s dispersion relation of particles predicted by spacial relativity e.g.: events involving gamma radiation with energies beyond 20TeV from distant sources such as Markarian 421 and Markarian 501 blazars [7] and studies of the evolution of air showers produced by ultra high-energy hadronic particles suggest that pions live longer than expected [8]. These experimental data, if confirmed (observation uncertainty is still too large), might indicate that Lorentz invariance is violated\textsuperscript{2}. This, however, may be one possibility among others [13, 14], but

\textsuperscript{1}This argument is not appropriate for any theory, for example for loop quantum gravity, see [1].

\textsuperscript{2}In this context one should also mention those experiments including resonant cavity tests and clock-comparison experiments in [9, 10, 11]. But the results do not indicate any Lorentz violating effects and merely improve the constraints on the parameters of the Lorentz violating standard model extension [12].
taking it as a serious one, so one has to deal with the modification of the dispersion relations due to Lorentz violation [16].

In this paper we study the Lorentz symmetry breaking in the context of Quantum Field Theory in a classical background. Specifically we study the response of an Unruh’s detector in Minkowski space. If such a detector moves with a constant acceleration with respect to some inertial system, it detects a thermal radiation (Unruh’s effect) [17]. The same detector however does not detect any particle if it moves with a constant velocity. This arises as a consequence of the exact Lorentz invariance of the vacuum of Minkowski space. One may ask how this latter result is changed in the presence of the Lorentz symmetry breaking? In the remainder of this paper we will try to answer this question. In section 2 we describe a model of Lorentz symmetry breaking. Unruh’s detector will be discussed in section 3 and the effect of symmetry breaking will be demonstrated. In the last section we will close this paper with some concluding remarks.

2 The Lorentz symmetry breaking model

We begin with the massive version of Jacobson and et al.’s model for breaking the boost subgroup of the Lorentz group [18]. In this model, the Lorentz invariance is broken using a dynamical coupling of a real scalar field $\varphi$ with a preferred frame. The Lagrangian of this model is given by

$$ L_\varphi = \frac{1}{2}(\nabla^a \varphi \nabla_a \varphi + \alpha^2 (D^2 \varphi)^2 - m^2 \varphi^2) $$

(1)

where $\alpha$ is a real constant with a very small value and $D^2$ is defined by

$$ D^2 \varphi = -D^a D_a \varphi = -q^{ac} \nabla_c (q^b_a \nabla_b \varphi) $$

(2)

with

$$ q_{ab} = -\eta_{ab} + u_a u_b $$

(3)

where $\eta_{ab} = diag(1, -1, -1, -1)$ is the four dimensional Lorentzian metric and the vector $u_a$ is a unit future directed time-like vector field. It can be interpreted as the four velocity of a preferred inertial observer. The rest frame of this observer may be considered as the mathematical realization of a preferred frame (aether). Here we assume this preferred frame be the rest frame of a preferred inertial observer thus we set $u_a = (1, 0, 0, 0)$ in the preferred frame.

We distinguish between two different kinds of transformations according to the work [12] and [19], namely the observer Lorentz transformation and the particle Lorentz transformation. The former corresponds to a transformation of an inertial system while the latter corresponds to a transformation of particles or localized fields within a fixed inertial system.

Since the Lagrangian (1) is a space-time scalar under observer transformations, the theory exhibits observer Lorentz symmetry. But this Lagrangian is not invariant under the particle boost transformation

$$ \varphi'(x) = \varphi(\Lambda^{-1} x) $$

(4)

where $\Lambda$ is the boost transformation matrix. Hence the Lagrangian (1) violates the particle Lorentz invariance. This is taken as acceptable form of Lorentz violation for physical systems$^3$.

In the present context any particle Lorentz transformation may be imagined as a transformation of a physical system (particles or localized field) within the rest frame of the preferred observer, i.e. the aether.

$^3$Particle boost transformation are sometimes called active Lorentz transformation [20]
We note that the symmetry under space-time translations, considered here as particle (active) transformations
\[ \varphi'(x) = \varphi(x - a), \]
where \( a \) is a real four-vector, is valid.

In the preferred frame the equation of motion takes the form
\[ \Box \varphi - \alpha^2 q^{ab} q^{cd} \nabla_a \nabla_b \nabla_c \nabla_d \varphi + m^2 \varphi = 0. \]
(6)
The positive frequency solution of (6) are
\[ u_k(x) \propto \exp(-ik_a x^a) \]
where \( k^a \) and \( x^a \) are 4-vectors and \( k^a \) satisfy the modified dispersion relation
\[ \eta^{ab} k_a k_b - \alpha^2 q^{ab} q^{cd} k_a k_b k_c k_d - m^2 = \omega^2 - k^2 + \alpha^2 k^4 - m^2 = 0 \]
(8)
The 4-vector solutions of this dispersion relation at sufficiently high frequencies become space-like (at the massless limit all solutions are space-like). These solutions violate the energy positivity condition \( (\omega \geq |k|) \) and can introduce some instabilities in the quantized theory [15]. Moreover, this nonlinear dispersion relation means that the group velocity of the wave, namely,
\[ v_g = \frac{d\omega}{dk^2} = \frac{1 - 2\alpha^2 k^2}{\sqrt{1 - \alpha^2 k^2 + m^2/k^2}}, \]
(9)
is not a constant and, at sufficiently high frequencies, can be greater than the velocity of light \( (c = 1) \). This leads to non-local wave packets which violate causality. Since the quantization of such wave packets is obscure one can define a causal sector of the theory by the condition that all super-luminal group velocities are excluded. This naturally imposes a cutoff \( k_c \) on the momentum space and ensures us that all wave packets propagate with a sub-luminal (or luminal) velocity. In this paper, we assume \( \alpha \ll 1 \) and take the cutoff bound as \( k_c = \frac{\sqrt{m}}{\alpha} \).

One can see that for all \( |k| \) below \( k_c \) the wave-vector \( k^\mu \) is time-like and the group velocity does not exceed 1. Thus this causal sector satisfies the energy positivity condition. Furthermore, if \( k^2 > \frac{1 + \sqrt{1 + 4\alpha^2 m^2}}{2\alpha^2} > \frac{m}{\alpha} \) the frequency \( \omega \) will take an imaginary value. It causes the mode solutions become singular. When imaginary frequency exist, the energy spectrum of the field theory is unbounded from below, a condition which is generally believed to lead to instability [22]. Moreover, the existence of such imaginary frequencies can cause some problems concerning the unitarity of the time-evolution operator. But this region of the momentum space is above the chosen cutoff and therefore these imaginary frequency modes are automatically eliminated.

The scalar product for two solutions of equation of motion (6) is defined as
\[ (\varphi_1, \varphi_2) = -i \int_S d\Sigma_\alpha \{ \varphi_1(x)(g^{ab} \overrightarrow{\nabla}_b - \alpha^2 q^{ab} q^{cd} \overrightarrow{\nabla}_b \overrightarrow{\nabla}_c \overrightarrow{\nabla}_d ) \varphi_2^*(x) \} \]
(10)
where
\[ \varphi_1 \overrightarrow{\nabla}_a \overrightarrow{\nabla}_b \overrightarrow{\nabla}_c \varphi_2 = +\varphi_1 \overrightarrow{\nabla}_a \overrightarrow{\nabla}_b \overrightarrow{\nabla}_c \varphi_2 - \varphi_2 \overrightarrow{\nabla}_a \overrightarrow{\nabla}_b \overrightarrow{\nabla}_c \varphi_1 \\
-\overrightarrow{\nabla}_a \varphi_1 \overrightarrow{\nabla}_b \overrightarrow{\nabla}_c \varphi_2 + \overrightarrow{\nabla}_a \varphi_2 \overrightarrow{\nabla}_b \overrightarrow{\nabla}_c \varphi_1 \\
+\overrightarrow{\nabla}_b \varphi_1 \overrightarrow{\nabla}_a \overrightarrow{\nabla}_c \varphi_2 - \overrightarrow{\nabla}_b \varphi_2 \overrightarrow{\nabla}_a \overrightarrow{\nabla}_c \varphi_1 \\
-\overrightarrow{\nabla}_c \varphi_1 \overrightarrow{\nabla}_b \overrightarrow{\nabla}_a \varphi_2 + \overrightarrow{\nabla}_c \varphi_2 \overrightarrow{\nabla}_b \overrightarrow{\nabla}_a \varphi_1. \]
(11)
With this definition we have $\nabla_a J^a = 0$. As a consequence the scalar product (10) becomes time independent. With respect to (10) the orthonormal positive frequency mode solutions of (6) are

$$u_k(x) = \frac{1}{\sqrt{2k_0 - 4\alpha^2 q_0 b^2(q_0 k^2 k^4)}} \exp(-ikx).$$

(12)

In the casual sector the field $\varphi(x)$ is taken to expanded in terms of $u_k(x)$ and $u_k^*(x)$ with $|k|$ not exceeding the cutoff $k_c$, namely

$$\varphi(x) = \int_{|k| \leq k_c} (a_k u_k(x) + a_k^* u_k^*(x)) d^3k.$$  

(13)

3 Quantization

To quantize the field represented by (13) we begin with the following general remarks. Since the particle boost transformations (4) are not symmetry transformations of the Lagrangian (1), at the quantum level these transformations cannot unitarily be implemented in a given representation. However as mentioned above, the Lagrangian (1) is invariant under the space-time translations (5). Thus space-time translations can unitarily be implemented. Because these symmetry transformations leave the vector field $u_a$ invariant, one can consider $u_a$ as a super-selection quantity that takes a particular value, namely $u_a = (1, 0, 0, 0)$ within a corresponding unitary class of a given representation. This unitary representation will be called the preferred representation of the model.

The unitary class of the preferred representation can be obtained in terms of the Fock space representation as follows. According to the invariance of the Lagrangian (1) under the space-time translations (5) one can obtain a canonical energy-momentum tensor which is conserved for solutions of the equation of motion:

$$T^{\mu\nu} = \frac{\partial L}{\partial (\partial_\mu \varphi)} \partial^\nu \varphi + \frac{\partial L}{\partial (\partial_\mu \partial_\rho \varphi)} \partial_\rho \partial^\nu \varphi - \partial_\rho \left( \frac{\partial L}{\partial (\partial_\mu \partial_\rho \varphi)} \right) \partial^\nu \varphi - L \eta^{\mu\nu} \quad (\mu, \nu = 0, \ldots, 3)$$

(14)

and

$$\partial_\mu T^{\mu\nu} = 0$$

(15)

for all solutions of equation (6). The integral of $T^{0\mu}$ over the space-like hypersurface $t = \text{const.}$ represents the time-conserved energy-momentum four-vector

$$P^\mu = \int T^{0\mu} d^3x.$$  

(16)

To quantize this model, we note that the space-time translations (5), as the symmetry of the model, can unitarily be implemented. This means that $P^\mu$ can be considered as the generators of space-time translations, yielding

$$[P^\mu, \varphi(x)] = i\partial^\mu \varphi(x)$$

(17)

\footnote{Note that the complex form of the Lagrangian (1) is invariant under global $U(1)$ transformation obviously. So as a consequence of Noether’s theorem a conserved current $j^a$ exists. The form of this current $j^a$ corresponds to the form of current $J^a$ introduced in (10).}
which determines the operator nature of $\varphi(x)$. In particular, the equations (17) guarantee that time translation is a unitary transformation.

We note that the quantization rule based on (17) is an application of the correspondence principle and is more general than the standard canonical quantization, because it uses only the symmetry transformation of the theory. Therefore, it is applicable to all kind of theories admitting the space-time translation invariance such as the theory considered in this paper. In the present context, the application of the quantization rule (17) is urgent, because a priori it is not known whether the standard canonical quantization scheme is appropriate in the presence of the cutoff.

Now taking (13) and (17) together we get [23]

$$[a_{\vec{k}}, a_{\vec{k}'}^\dagger] = [a_{\vec{k}}, a_{\vec{k}'}^\dagger] = 0, \quad [a_{\vec{k}}, a_{\vec{k}'}] = \delta^3(\vec{k} - \vec{k}')$$

which are the usual form of commutation relations for $a_{\vec{k}}$ and $a_{\vec{k}}^\dagger$. Thus we can consider $a_{\vec{k}}$ and $a_{\vec{k}}^\dagger$, as annihilation and creation operators of a particle with momentum $\vec{k}$ and energy $\omega$, respectively. One can define a no particle state as

$$a_{\vec{k}} | 0 > = 0, \forall | \vec{k} | \leq k_c$$

and a one-particle state as

$$| \vec{k} > = a_{\vec{k}}^\dagger | 0 >$$

which is a eigenstate of energy-momentum operator:

$$P^\mu | \vec{k} > = k^\mu | \vec{k} > .$$

A general n-particle state may be constructed in the standard manner.

## 4 Unruh’s detector

We consider now an Unruh’s detector [24], understood here as a point-like detector moving along a general trajectory $x = x(\tau)$ interacting with the scalar field (13). The interaction between detector and the field is as follows:

$$\delta \mathcal{H} = \beta \hat{M}(\tau) \varphi(x(\tau))$$

where $\beta$ is coupling constant, $\tau$ is detector proper time and $\hat{M}(\tau)$ is a quantum operator that describes the detector’s monopole moment. Let the detector ground state be $|E_0\rangle$ and its possible states form $|E\rangle$. In this sense, the absorption of a particle changes the state of the detector from $|E_0\rangle$ to $|E\rangle$ (such that $\Delta E = E - E_0 > 0$) and $\Delta E < 0$ for emission of a particle with respect to the detector. Transition amplitude $\mathcal{M}$ from an initial state $|E_i\rangle | i \rangle$ to a final state $|E_f\rangle | f \rangle$ at the first order perturbation is ($| i \rangle$ and $| f \rangle$ are scalar field states)

$$\mathcal{M}(i, f) = \mathcal{M}(E_i, \tau_i; E_f, \tau_f) = \beta E_{i,f} \int^{\tau_f}_{\tau_i} d\tau' e^{-i(E_f - E_i)\tau'} \langle f | \varphi(x(\tau')) | i \rangle$$

\footnote{For a linear theory without a (momentum) cutoff (17) is equivalent with the standard form of the canonical method [23].}
where \( E_{i,f} = \langle E_f | \hat{M}(0) | E_i \rangle \) (\( \hat{M}(\tau) = e^{i\hat{H}_0 \tau} \hat{M}(0) e^{-i\hat{H}_0 \tau} \)) and \( \hat{H}_0 \) is the free Hamiltonian of the detector. Then probability of this transition is

\[
P_{i,f} = \beta^2 \left| E_{i,f} \right|^2 \int_{\tau_i}^{\tau_f} d\tau' \int_{\tau_i}^{\tau_f} d\tau'' e^{-i(E_f - E_i)(\tau'' - \tau')} iG(\tau'', \tau')
\]

(24)

where \( iG(\tau'', \tau') = \langle i | \varphi(x(\tau'')) \varphi(x(\tau')) | i \rangle \) is the Wightman function. For the special case in which \( |i \rangle \) corresponds to a Lorentz invariant vacuum, the transition amplitude \( M(i, f) \) vanishes for a detector moving with a constant velocity so the detection of particles is due to acceleration. Now we want to examine the case \( |i \rangle = |0 \rangle \) where \( |0 \rangle \) is the preferred vacuum introduced in the previous section. In (23), \( \langle f | \varphi(x) | i \rangle \) vanishes at the first order except for \( |f \rangle = |1_k \rangle \) so

\[
\langle 1_k | \varphi(x) | 0 \rangle = \frac{1}{(2\pi)^{3/2}} \int_{|\vec{k}'| \leq k_c} \frac{d^3k'}{\sqrt{2\omega}} \langle 1_k | \hat{a}_{\vec{k}, \vec{v}}^{\dagger} | 0 \rangle e^{-i\vec{k}.\vec{x}} e^{i\omega t} = \frac{1}{(2\pi)^{3/2}} e^{-i\vec{k}.\vec{v}} e^{i\omega t}
\]

(25)

where \( \omega^2 = k^2 - \alpha^2 k^4 + m^2 \). For a detector with constant velocity \( \vec{v} \) with respect to the preferred frame, the world line is

\[
\vec{x} = \vec{x}_0 + \vec{v}t = \vec{x}_0 + \vec{v}(1 - v^2)^{-1/2}
\]

(26)

where \( \vec{x}_0 \) and \( \vec{v} \) are constants and \( |\vec{v}| < 1 \) \((c = 1)\). In this case the transition amplitude \( M(i, f) \) takes the form

\[
M(i, f) = \frac{1}{\sqrt{4\pi \omega}} \beta E_{i,f} \int_{-\infty}^{+\infty} d\tau e^{-i(E_f - E_0)\tau} e^{i\tau(\omega - \vec{k}.\vec{v})(1 - v^2)^{-1/2}}
\]

\[
= \frac{1}{\sqrt{4\pi \omega}} \beta E_{i,f} \delta(E_f - E_0 + (\omega - \vec{k}.\vec{v})(1 - v^2)^{-1/2})
\]

(27)

In the causal sector satisfying the energy positivity condition this transition amplitude vanishes for any \( E_f > E_0 \) because in this sector all wave-vectors are time-like which means \( \omega \) is always greater than \( |\vec{k}.\vec{v}| \). Therefore no particle can be detected.

In the massless limit the situation is different. In this case the cutoff \( k_c \) approaches to zero. Thus with this choice of the cutoff there is no causal sector. But one can take a new cutoff to study the massless limit, namely \( k_c = \sqrt{3}/(2\alpha) \). This cutoff eliminates all super-luminal group velocities and all imaginary modes (For the massive case this cutoff does not ensures the energy positivity condition). However one can not satisfy the energy positivity because all wave-vectors are space-like. Thus in the massless limit we deal with a theory not exhibiting energy positivity. This violation of energy positivity leads to a non-vanishing transition amplitude for the detector. In fact, by choosing an appropriate \( \vec{v} \) the frequency \( \omega \) becomes smaller than \( |\vec{k}.\vec{v}| \). It implies a non-vanishing transition amplitude for some \( E_f > E_0 \) in (27), provided \( k \) satisfies the following inequality

\[
k^2 > \frac{1 - v^2 \cos^2 \theta + \alpha^2}{\alpha^2}
\]

(28)

where \( \theta \) is the angle between \( \vec{k} \) and \( \vec{v} \). It means that a detector moving with a constant velocity with respect to the preferred frame has a non-vanishing response, i.e. it registers quanta of the field \( \varphi \).

At this point the question arises: what is the physical interpretation of this non-vanishing response. The violation of energy positivity in the massless limit of the model leads to a stability problem because a space-like energy momentum four vector with a negative 0th component can be converted, under a suitable boost transformation, to a one with a positive 0th component. Thus we can not have a lower bound for the energy of the system in all inertial frames. It means that the vacuum state in this case is instable. This is the origin of the non-vanishing response of the detector.
5 Conclusions

As we have seen, the response of a detector computed in the preferred vacuum of the Lorentz symmetry breaking model is non-vanishing when the causal sector violates the energy positivity condition. But, it must be noticed that the detection of particles is very sensible to the form of the modified dispersion relation and the causal sector. For instance by means of a non-vanishing mass parameter one can construct a causal sector which satisfies the energy positivity condition. As it has been shown, in this case no particle creation occurs.

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