A New Nucleosynthesis Constraint on the Variation of $G$

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Big Bang Nucleosynthesis can provide, via constraints on the expansion rate at that time, limits on possible variations in Newton’s Constant, $G$. The original analyses were performed before an independent measurement of the baryon-to-photon ratio from the cosmic microwave background was available. Combining this with recent measurements of the primordial deuterium abundance in quasar absorption systems now allows one to derive a new tighter constraint on $G$ without recourse to considerations of helium or lithium abundances. We find that, compared to today’s value, $G_0$, $G_{BBN}/G_0 = 1.01^{+0.20}_{-0.15}$ at the 68.3% confidence level. If we assume a monotonic power law time dependence, $G \propto t^{-\alpha}$, then the constraint on the index is $-0.004 < \alpha < 0.005$. This would translate into $-3 \times 10^{-13} \text{ yr}^{-1} < (\dot{G}/G)_{\text{today}} < 4 \times 10^{-13} \text{ yr}^{-1}$.

The predictions of the light element abundances from big bang nucleosynthesis (BBN) have long served as a powerful probe of the early Universe. These predictions depend on a number of parameters such as the number of light particle species in equilibrium at that time, the baryon to photon ratio ($\eta$), nuclear reaction rates, and the gravitational constant, $G$. BBN constraints on cosmological parameters have been traditionally limited by uncertainties in the observed primordial abundances of the light elements as well as degeneracy among the independent parameters. In particular, without a tight independent constraint on the baryon density, variation in the predicted abundance of light elements could be largely compensated, at least within observational uncertainties, by varying $\eta$.

Recently, WMAP has provided an independent measure of $\eta$ allowing one to use the baryon density as an independent constraint on BBN instead of inferring its value using the apparent agreement between light element abundance predictions and observations. In this vein, prior to the WMAP results other CMB experiments had sufficiently constrained $\eta$ to allow tests for the consistency of BBN.

At the same time recent measurements of the primordial deuterium abundance ($D/H$) using quasar absorption by intervening high redshift systems has dramatically altered its significance in comparing BBN predictions with observations.

Combining these two new observations gives one a powerful new handle to use in order to constrain cosmological parameters using BBN predictions. Here we explore the impact of these developments on one’s ability to constrain the variability of $G$. Variation of the gravitational constant was originally postulated by Dirac and remains a key component of many theories that seek to resolve various hierarchy problems in particle physics. Varying the gravitational constant has significant impact during BBN.

While previous analyses (for example) attempted to utilize all BBN abundance predictions as a way to limit the effect of uncertainties in $\eta$, as can be seen from the results of [6], there is good reason to believe that utilization of deuterium alone, now that its primordial value has been tightly constrained, might be sufficient to yield a stronger constraint. This is because deuterium production is particularly sensitive to the value of the expansion rate during BBN, which in turn depends upon the value of $G$. While helium is also sensitive to this rate, existing systematic uncertainties in its primordial value confuse the situation. In addition, while earlier analyses were performed before it was empirically demonstrated that 3 light neutrino species exist, this residual uncertainty has now been removed.

It has been known for almost three decades that deuterium is only produced in significant quantities in the big bang [3, 8]. This, coupled with the fact that deuterium depends sensitively on $\eta$, makes it an excellent probe of the Universe at the time of BBN (for example see [5]). The detection of deuterium in a number of high redshift quasar absorption systems allows for a precise determination of the primordial deuterium abundance. Kirkman et al. [10] give as the best current estimate of the primordial deuterium abundance $D/H = 2.78^{+0.34}_{-0.17} \times 10^{-5}$ based on the observation of five quasar absorption systems. This corresponds to log ($D/H$) = $-4.556 \pm 0.064$ where the errors are assumed to be Gaussian.

The new CMB constraints on $\eta$ are derived from the temperature of the CMB and a fit of the power spectrum to a set of parameters defining a cosmological model. Here we are interested in $\Omega_b h^2$ where $h \equiv H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_b = \rho_b/\rho_c$, the ratio of the baryon density to the critical density. A combined analysis including WMAP, CBI, ACBAR, 2dFGRS, and Lyman $\alpha$ data gives $\Omega_b h^2 = 0.0224 \pm 0.0009$ for $\Lambda$CDM models for the two choices of either a power law spectral index or a running spectral index [1]. Combined with the photon density, $n_r = 410.4 \pm 0.9 \text{ cm}^{-3}$, as determined from the COBE temperature measurement [11],
FIG. 1: Observational limits and theoretical expectations for \( D/H \) versus \( \eta \). The one (light shading) and 2 (dark shading) sigma observational uncertainties for \( D/H \) and \( \eta \) are shown. They do not appear as ellipses due to the linear scale in \( D/H \) but logarithmic uncertainties from the observations. The BBN predictions are shown as the solid curves where the width is the 3% theoretical uncertainties. Three different values of \( G_{BBN}/G_0 \) are shown.

gives \( \eta = (6.13 \pm 0.25) \times 10^{-10} \) where we again assume Gaussian errors.

To determine the deuterium abundance as a function of \( \eta \) and \( G \) from BBN we use the standard Kawano code [12]. We used updated reaction rates [13] as well as used the latest value for the neutron half-life [14]. Krauss and Romanelli [15] first demonstrated the need to incorporate reaction rate uncertainties using Monte Carlo techniques if one is to properly derive BBN constraints in general. The residual uncertainty in the predicted deuterium abundance is quite small. We have included this theoretical uncertainty in in the likelihood analysis as an independent Gaussian with a constant 3% uncertainty at the one sigma level [16, 17]. That is, \( \sigma_{BBN} = 0.03(D/H) \) where \( (D/H)(\eta, G/G_0) \) comes from the Kawano code as noted. The uncertainties in \( G \) today are sufficiently small [16] and have not been included. In addition, due to the short time interval associated with BBN we have assumed that \( G \) remained constant throughout the period of light element production. We have verified that our results are unchanged if we allow \( G \) to vary during this time according to \( G \propto t^{-0.03} \). Figure 1 shows the 1 and 2 sigma ellipses for the observed values of \( D/H \) and \( \eta \). Also shown are curves for the BBN predictions for \( (D/H)(\eta, G/G_0) \) for three values of \( G/G_0 \) with the 3% theoretical uncertainties from Monte Carlos of the reaction rates.

With the assumption that the observational and theoretical errors follow Gaussian distributions as discussed above, we assign a joint likelihood function \( L(D/H, \eta, G/G_0) \) where \( D/H \) is a function of \( G \) and \( \eta \). The likelihood distribution for \( G \) (which can be normalized to give the probability distribution) is found by marginalizing over \( D/H \) and \( \eta \). Applying this procedure we find \( G/G_0 = 1.01^{+0.20}_{-0.16} \) at the 68.3% confidence level, \( G/G_0 = 1.01^{+0.42}_{-0.30} \) at the 95% confidence level.

Assuming a monotonic power law time dependence \( G \propto t^{-\alpha} \) one finds the power law index \(-0.004 < \alpha < 0.005 \) at the 68.3% confidence level, \(-0.009 < \alpha < 0.010 \) at the 95% confidence level. In this case one infers \(-3 \times 10^{-13} \) yr\(^{-1} \left( G/G_0 \right)_{today} < 4 \times 10^{-13} \) yr\(^{-1} \). This is over an order of magnitude stronger than direct constraints that can be obtained on the variation of \( G \) today.

Our new constraint is about a factor of two stronger than the previous BBN constraint on \( G/G_0 \), which is significant, given that we utilize only deuterium in our analysis, and do not exploit other light element abundance predictions. In particular, as observational constraints on these elements continue to improve, along with further improvements on the deuterium abundance measurements and the baryon to photon limit one can expect BBN constraints on variability of \( G \) to continue to improve significantly.

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