Qualitative spatial reasoning on topological relations by combining the semantic web and constraint satisfaction

Yandong Wang⁷, Mengling Qiao⁷, Hui Liu⁷ and Xinyue Ye⁷

⁷State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, Wuhan, China; ⁸Collaborative Innovation Center of Geospatial Technology, Wuhan University, Wuhan, China; ⁹Wuhan Land Resource and Planning and Information Center, Wuhan, China; ¹⁰Department of Geography, Kent State University, Kent, OH, USA

ABSTRACT
Qualitative spatial reasoning on topological relations can extract hidden spatial knowledge from qualitatively described topological information, which is of significant importance for decision-making and query optimization in spatial analysis. Qualitative reasoning on spatial topological information based on semantic knowledge and reasoning rules is an efficient means of reducing both the known relations and the corresponding rules, which can result in enhanced reasoning performance. This paper proposes a qualitative reasoning method for spatial topological relations based on the semantic description of reasoning rules and constraint set. Combined with knowledge from the Semantic Web, the proposed method can easily extract potential spatial results consistent with both unique and non-unique rules. The Constraint-Satisfaction-based approach, describing constraint set with semantic expressions, is then used together with an improved path consistency algorithm to verify the consistency of the unique-rules-based and non-unique-rules-based reasoning results. The verification can eliminate certain reasoning results to ensure the reliability of the final results. Thus, the task of qualitative spatial reasoning on topological relations is completed.

1. Introduction
Qualitative reasoning is a knowledge processing method that does not require quantitative numerical calculations. Compared with quantitative descriptions, qualitative descriptions are more consistent with general cognition and knowledge. Accurate numbers are necessary when quantitatively expressing spatial relations (e.g. 30°W; 30°N). However, from the perspective of habitual expression, there is often no need to describe spatial attributes so accurately. Instead, the necessary information can be well described in a qualitative form (Jiang and Yao 2006). Directional relations can be expressed using terms such as southeast and north, and distance relations can be expressed using terms such as far and near (Du, Feng, and Guo 2015; Du, Wang, and Qin 2006; Kwan and Ding 2008; Philip, Abdelmoty, and El-Geresy 2014; Yao and Thill 2006).

Common descriptions of spatial information are often purely qualitative. The extraction of hidden spatial knowledge from such qualitatively described information through qualitative reasoning is of significant importance for decision-making and query optimization in spatial analysis. In the case of finding a suitable location for a special factory, Zhao, Murayama, and Zhang (2005) verified that the result obtained by traditional approach in Geographic Information System (GIS) with clear result boundary may obtain no areas satisfying the constraints, which could not provide help for decision-makers. However, the result obtained by the field-based fuzzy spatial reasoning approach showed better decision-making reliability than the traditional one.

Based on the descriptions of logical knowledge, artificial intelligence, and the Semantic Web, researchers have conducted studies to evaluate various aspects of qualitative spatial reasoning. Spatial knowledge models have been built based on various descriptions of logic (Croitoru and Compatangelo 2007; Cui, Cohn, and Randell 1993; Parsia and Sirin 2004), such as a model for the expression of spatial knowledge based on traditional mathematical models (Clementini and Felice 1995; Egenhofer and Franzosa 1991). In addition, related logical rules are defined based on current spatial knowledge models and are then combined with reasoning tools to perform qualitative reasoning (Batsakis 2013; Batsakis and Petrakis 2010; Grutter and Bauer-Messmer 2007). Also, related studies have been conducted to thoroughly improve the precision of these spatial knowledge models (Long and Li 2013).
Researchers have also studied highly efficient qualitative spatial reasoning algorithms based on artificial intelligence techniques (e.g., path consistency algorithms). Nam and Kim (2015) presented an algorithm working on a mixture of 9 cone-shaped directional (CSD−9) relations and 8 region connection calculus (RCC8) topological relations. To achieve highly efficient reasoning methods, continuous improvement of the efficiency of these reasoning algorithms is essential. Nebel and Renz (2001) conducted experiments on various algorithms used for qualitative reasoning, and the results showed that large RCC8 instances could be solved using the maximal tractable subsets of RCC8 they identified. Furthermore, Renz and Nebel (1999) extended Bennett’s encoding of RCC8 in modal logic and proved path-consistency in their fragment is sufficient for deciding consistency. In addition, these spatial reasoning methods are also used to solve related issues in other fields, like the geographic problems of land use and soil classification in GIS field (Du, Liang, and Sun 2012; Holt and Benwell 1999).

As a result of the increasingly widespread application of the Semantic Web (Parsia and Sirin 2004), ontologies are used to describe spatial entities and the relationships between them. Batsakis and Petrakis (2010) realized the capability of ontologies to reasoning out spatial-temporal dynamic information. Grutter and Bauer-Messmer (2007) realized to express RCC in Web Ontology Language (OWL) making reasoning with these two formalisms possible. Stocker and Sirin (2009) realized spatial reasoning using a path consistency algorithm, and subsets that could be processed separately were proposed to improve the reasoning efficiency. Christodoulo, Petrakis, and Batsakis (2012) expressed topological relations basing on semantic rules and achieved reasoning on topological relations using a reasoning engine. The Semantic Web technology implements reasoning by using description logic to express concepts and relations. However, spatial relations are very complex, and description logic is not very suitable for solving complex problems. Therefore, spatial reasoning based on the Semantic Web has some limitations.

In our study, by combining the Semantic Web and Constraint Satisfaction Problems (CSPs), a method of qualitative spatial reasoning on topological relations is proposed. The method implements reasoning based on the semantic description of both unique and non-unique reasoning rules. The corresponding reasoning results are subsequently verified by the use of CSPs. The verification following the unique-rules-based (or non-unique-rules-based) reasoning can reduce both the known relations and the corresponding rules to obtain enhanced reasoning performance. Besides, an improved path consistency algorithm is introduced in our study. In an experimental example, a campus layout problem is considered to verify the feasibility of the proposed method.

Section 2 provides a detailed introduction of the relevant background information related to the topological relation models and reasoning rules, the Semantic Web, and the CSPs. Section 3 introduces the proposed path consistency algorithm for spatial relations and the proposed method of qualitative spatial reasoning. Section 4 verifies the proposed method through an example. Section 5 discusses and summarizes the proposed method.

2. Background

2.1. Topological relation models and reasoning rules

Expressions of topological relations serve as the basis of reasoning on spatial data. Researchers have conducted extensive studies on the expression of topological relations and have proposed multiple topological relation models. The point-set topology-based "n-intersection" model (Egenhofer and Franzosa 1991; Long and Li 2013) and RCC8 model (Randell, Cui, and Cohn 1992) are two representative models.

RCC8 model is a means of expressing qualitative spatial knowledge (Cohn et al. 1997; Renz 2002). It is used to express the relation between two regions based on two axioms:

$$\forall x, C(x, x)$$  \hspace{1cm} (1)

$$\forall x, y, [C(x, y) \rightarrow C(y, x)]$$  \hspace{1cm} (2)

where $x$ and $y$ represent two different spatial regions, $C$ represents a reflexive and symmetric relation, and $C(x, y)$ represents a set of dyadic relations between region $x$ and region $y$.

The topological relations expressed in the RCC8 model consist of 8 completely united and mutually disjoint sets of basic relations. The 8 basic relations are DC (disconnected), EC (externally connected), PO (partially overlapping), EQ (equal), NTPP (non-tangential proper part), NTPPi (the inverse of non-tangential proper part), TPP (tangential proper part), and TPPi (the inverse of tangential proper part) (Randell, Cui, and Cohn 1992; Renz 2002). The RCC8 model provides visual topological explanations of topological structures, and Figure 1 illustrates the 8 basic relations. The topological relations described by the RCC8 model have the following characteristics: (1) mutual inverse, e.g., the relationship between NTPP and NTPPi is reciprocal, as is that between TPP and TPPi; (2) symmetry, e.g., the relationships between regions $x$ and $y$ in the DC, EC, PO, and EQ relations are all symmetrical; (3) transitivity, e.g., the relationships between regions $x$ and $y$ in the NTPP, NTPPi, TPP, TPPi, and EQ relations are transitive (Egenhofer and Franzosa 1991).
Randell, Cui, and Cohn (1992) deduced a composition table based on the RCC8 model (Table 1), serving as a basis for reasoning for the hidden topological relations. In Table 1, we suppose there are three spatial objects $X$, $Y$, and $Z$. The first row of the table represents the possible relations between $X$ and $Y$, and the first column of the table represents the possible relations between $Y$ and $Z$. The other items are the possible relations between $X$ and $Z$. Table 1 shows that the model defines a total of 64 relations. Among these 64 relations, 27 of them are definite, i.e. each of these 27 relations is a basic relation with only one unique reasoning result, which is called a unique relation in this paper. The other 37 relations are indefinite, i.e. each of these 37 relations allows multiple basic relations, which are called non-unique relations in this paper.

A reasoning rule consists of a head representing a reasoning consequent and a body representing a reasoning antecedent, each of which is a set of atomic formulas. If each atomic formula within the body holds, then the fact described by the head can be extrapolated (Horrocks et al. 2004). For example, two relations “I hasParent($x_1$, $x_2$)” and “I hasBrother($x_1$, $x_3$)” are predefined, we can use ontology to describe that $x_1$ is the parent of $x_2$ and $x_1$ has a brother $x_3$. According to the description of ontology, we can define the body including the relations of “I hasParent($x_2$, $x_1$)” and “I hasBrother($x_1$, $x_3$)”, then we can reason out that “I hasUncle($x_1$, $x_2$)” meaning $x_2$ is the uncle of $x_1$ is the head part. Rule-based qualitative spatial reasoning is the process of obtaining hidden spatial relations through reasoning rules based on known spatial relations (Batsakis 2013; Batsakis and Petrakis 2010). The topological relation characteristics of the RCC8 model and the knowledge contained in the composition table described in this section provide an objective basis for the establishment of spatial reasoning rules. In our paper, the RCC8 model is used to qualitatively reasoning out spatial topological relations by combining with the Semantic Web technology and the CSPs.

2.2. Semantic Web

The Semantic Web is an extension of the current World Wide Web. It provides meaning to data and services, making them to be understood and used correctly (Berners-Lee, Hendler, and Lassila 2001). The OWL is the recommended markup language for semantic publication and the sharing of ontologies on the World Wide Web. OWL provides a framework for describing the classes of concepts involved in Web applications and the relations between these concepts (Mcguinness and Harmelen 2004). The Semantic Web Rule Language (SWRL) is a language for describing rules based on a combination of the OWL Description Logic (DL) and OWL Lite sublanguages of OWL with the Unary/Binary Datalog RuleML sublanguages of the Rule Markup Language (Horrocks et al. 2004).

Currently, a number of semantically expressed reasoning rules have been defined, and certain researchers have described these reasoning rules using SWRL, whereas others have described those using Rule Interchange Format (RIF) (Nikkila et al. 2013). In this paper, we choose to describe reasoning rules using SWRL. SWRL describes knowledge using a highly abstract grammar and is closely related to semantics, meaning that it can be used to define semantic relations. Therefore, information

![Figure 1](image_url). The 8 basic relations of the RCC8 model.

| Relation | DC($x, y$) | EC($x, y$) | PO($x, y$) | TPP($x, y$) | TPP$i$($x, y$) | NTPP($x, y$) | NTPP$i$($x, y$) | EQ($x, y$) |
|----------|------------|-----------|-----------|-------------|-------------|-------------|-------------|-----------|
| DC($y, z$) | * | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| EC($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| PO($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| TPP($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| NTPP($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| TPP$i$($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| NTPP$i$($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |
| EQ($y, z$) | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP | DC, EC, PO, TPP, NTPP |

*Represents the universal relation.
expressed using SWRL incidentally contains rich semantic information (Mattheus et al. 2005). Reasoning rules described using SWRL can provide semantic support for spatial reasoning and can enhance the reasoning capability of a reasoning system. SWRL offers rich forms of expression and supports embedded self-defined rules, which greatly facilitates the expansion of a knowledge base to increase its richness for reasoning (Horrocks et al. 2004; Mattheus et al. 2005). Describing reasoning rules using SWRL is advantageous for expressing the information contained in the RCC8 composition table in the form of rules (Batsakis, Antoniou, and Tachmazidis 2014; Marc-Zwecker et al. 2013). For example, Marc-Zwecker et al. (2013) implemented the negation, conjunction and disjunction of RCC8 spatial relationships based on reification, and SWRL was used to cope with the loss of OWL properties such as symmetry or transitivity. In this paper, we construct rules by organizing the existing qualitative spatial knowledge offered by the RCC8 model using SWRL.

2.3. CSPs and path consistency algorithms

CSPs which have obvious advantages in complex problems are one focus of research in the field of artificial intelligence. Such a problem consists of a set of variables and a set of constraints (θ) on those variables. The variable set is a set of n variables (v₁, v₂, ⋯, vₙ), whereas the constraint set is a set of conditions that must be satisfied by the variable set (Simonis 2005). The objective of a CSP is to search for a solution to satisfy all of the constraints. There are many common and simple problems can be modeled as a CSP such as the Sudoku puzzles (Back, Fogel, and Michalewicz 1997). Sudoku puzzles are special cases of Latin Squares. The objective is to fill a 9 × 9 grid with digits so that each column, each row, and each of the nine 3 × 3 sub-grids that compose the grid contains all of the digits from 1 to 9. Each cell in the grid is a variable vᵢ, vⱼ ∈ {v₁, v₂, ⋯, v₉_i}. The constraints of a Sudoku puzzle include that the digits of each column, each row, and each of the nine 3 × 3 sub-grids are from 1 to 9 and do not allow to appear twice.

In a CSP, if each constraint restricts the relations among entities such that the values assigned to any two variables must satisfy a certain binary relation, then the problem is called a binary CSP (Egenhofer and Sharma 1993; Renz and Nebel 2007). Based on this definition, qualitative spatial reasoning can be regarded as a binary CSP because most spatial relations are pairwise relations between spatial objects, which satisfy the definition of a binary relation, and any further unknown spatial relations are then extrapolated based on the given spatial relations.

A CSP is solved by assigning values to the variables such that they satisfy the constraints. Thus, when the values assigned to all variables satisfy all constraints, the CSP is called a consistent CSP (Mackworth 1977; Papadias and Egenhofer 1996) and the assigned values of the variables constitute the solution to the problem. CSPs are classical nondeterministic polynomial complete problems. The complexity of solving a CSP is thus exponentially related to the scale of the problem (Renz and Nebel 2007).

In current studies, binary CSPs are solved through the direct application of a path consistency algorithm (Egenhofer and Sharma 1993). In a sense, the path consistency algorithm is a pre-processor for solving CSPs. The path consistency algorithms cannot solve the CSPs totally, but they can eliminate the local inconsistencies that cannot participate in any global solutions to reduce the complexity of solving a CSP (Mackworth and Freuder 1985).

In a path consistency algorithm, triads are established in the form of <i, j, k>, and the operation shown in Equation (3) is iterated for all triads until a fixed value of M(i,j) is obtained. This iterative procedure is performed to eliminate the relations that result in conflicts. If M(i,j) is empty, then the path to establish the triad in question is inconsistent (Du, Liang, and Sun 2012).

∀I, M(i,j) ← M(i,j) ∩ M(i,k) ∪ M(k,j) (3)

where initially, I ← {(i,k)|1 ≤ i ≤ k ≤ n}, the symbol ← denotes the assignment operation. M is defined as an n × n matrix. M(i, j), M(i, k), and M(k, j) represent the relationships between objects i and j, between objects i and k, and between objects k and j, respectively. The symbols ∩ and ∪ denote the set composition and intersection operations, respectively. When Equation (3) is iterated, it is easy to calculate L, where L = M(i, j) ∩ M(i, k) ∪ M(i, j) ∩ M(i, k) ∪ M(k, j). If M(i,j) ≠ L, then M(i, j) ← L, M(j, i) ← INVERSE(L) and I ← I ∪{(i,j)}, where the symbol U denotes the union operation. Then, L = M(j,k) ∩ M(i,j) ∩ M(i,k) is calculated. If M(j,k) ≠ L, then M(j,k) ← L, M(k, j) ← INVERSE(L), and I ← I ∪{(j,k)}. The iteration will terminate when M(i,j) = L.

3. Methods

3.1. Semantically supported method of qualitative spatial reasoning on topological relations

From Section 2.1, we know that the 64 total relations in Table 1 include 27 definite relations and 37 indefinite relations. Corresponding to these definite relations and indefinite relations, we define unique rules and non-unique rules, respectively, to be used in the semantically supported method of qualitative spatial reasoning described below.

The semantically supported qualitative spatial reasoning method consists of six steps (Figure 2): (1) semantic description of the reasoning rules by converting the information in the RCC8 composition table into
reasoning rules expressed using SWRL, (2) semantic description of the constraint set for the considered problem by completing the semantic definitions of the constraints, (3) reasoning based on the unique rules, (4) verifying the consistency of the unique-rule-based reasoning results, (5) reasoning based on the non-unique rules to extrapolate the remaining relations between spatial objects, and (6) verifying the consistency of the non-unique-rule-based reasoning results.

3.1.1. Semantic description of the reasoning rules
The first step of the method proposed in this study is to semantically describe the reasoning rules. In this step, the reasoning rules are defined based on the information contained in the RCC8 composition table and the characteristics of the RCC8 topological relations. The reasoning rules are described using SWRL and are classified into reverse reasoning rules, unique reasoning rules, and non-unique reasoning rules as follows:

Reverse reasoning rules: Figure 1 shows the set of 8 basic topological relations expressed by the RCC8 model. The inverse and symmetric attributes of these basic relations are used to define 8 corresponding reverse rules. For instance, the corresponding reverse rule of DC(x, y) is DC(y, x), and that of NTPP(x, y) is NTPP(y, x).

Unique reasoning rules and non-unique reasoning rules: In Section 2.1, these 27 basic relations with one unique reasoning result are referred to as unique relations. These 27 relations are defined as unique reasoning rules in this study, and the remaining 37 relations that allow multiple basic relations are referred to as non-unique relations. These 37 non-unique relations are defined as non-unique reasoning rules in this study.

The following example illustrates the logical forms of the reasoning rules described using SWRL. There are three spatial objects: X, Y, and Z. The known topological relation between X and Y is R1(X, Y), and that between Y and Z is R2(Y, Z). Based on the given spatial knowledge, a spatial analysis is conducted to deduce that the topological relation between X and Z is R3(X, Z). The logical form of this reasoning expressed using SWRL is as follows:

\[ R_1(X, Y) \land R_2(Y, Z) \rightarrow R_3(X, Z) \]

Figure 3 shows the reverse rule base consisting of the reverse rules corresponding to the 8 basic rules defined in Figure 1.

Reverse rules are also unique rules. Figure 4 shows the complete unique rule base, containing these 27 unique reasoning rules defined in Table 1 and these 8 reverse rules.

The logical form of a non-unique reasoning rule expressed using SWRL is as follows:

\[ R_1(X, Y) \land R_2(Y, Z) \rightarrow R_3(X, Z) \lor R_4(X, Z) \]
3.1.2. Semantic description of the constraint set

From Section 2.3, we know that a CSP includes a set of constraints ($\Theta$). In this paper, the constraint set is semantically described to express the topological relations between spatial objects, and these semantic expressions can be combined with the reasoning rules to provide rich semantic information for rule-based reasoning. For instance, consider a case in which several constraint relations are defined among the following five spatial objects: dormitory ($D$), library ($L$), teaching area ($T$), playground ($P$) and office area ($O$). Figure 7 semantically describes these constraint conditions: the relation between $D$ and $P$ is EC, the relation between $D$ and $L$ is EC, the relation between $P$ and $T$ is PO, the relation between $P$ and $O$ is DC, the relation between $P$ and $L$ is

---

**Figure 3.** Reverse reasoning rules expressed using SWRL.

Figure 5 shows the non-unique reasoning rule base, containing these 37 non-unique rules as defined in Table 1. Figure 6 illustrates the use of SWRL to describe the reasoning rules used in the following example: if the topological relation between $X$ and $Y$ is NTPP and the
topological relation between $Y$ and $Z$ is DC, then we can deduce that the topological relation between $X$ and $Z$ is also DC.

---

**Figure 4.** Partial list of the unique reasoning rules expressed using SWRL.

**Figure 5.** Partial list of the non-unique reasoning rules expressed using SWRL.
3.1.3. Unique-rule-based reasoning

Unique-rule-based reasoning is the process of extrapolating hidden unique topological relationships between spatial objects using a reasoning engine based on the unique rules. During this process, the unique rules and the reverse rules are used to obtain the reverse relations of these unique topological relations:

\[ A_{ij} \leftarrow R_{ij}^\prime, \quad A_{ji} \leftarrow R_{ij}^\prime \]  \hspace{1cm} (4)
where $A$ represents an two-dimensional (2-D) matrix, and $A_{ij}$ represents the matrix element in the $i$th row and $j$th column. The initial value of $A_{ij}$ is null. $R'_{ij}$ represents the unique topological relation between variables $i$ and $j$ obtained using the unique topological relation reasoning rules, which is assigned as the value of $A_{ij}$. $A_{ji}$ represents the matrix element in the $j$th row and $i$th column. $R''_{ij}$ represents the unique topological relation between variables $j$ and $i$ that is obtained by reversing $R'_{ij}$ using the reverse rules in the unique rule base, which is assigned as the value of $A_{ji}$. The symbols $\leftarrow$ and $\curvearrowleft$ denote the assignment and reversal operations, respectively.

Based on the results of unique-rule-based results and the symmetry of the 2-D matrix, the reverse rule base is used to conveniently solve for the topological relations between pairs of spatial objects and reduce the complexity of the problem. The relations between spatial objects that are obtained in this reasoning step are unique topological relations. Thus, new unique topological relations are extrapolated based on the given topological relations.

For instance, suppose that for three spatial objects $X$, $Y$, and $Z$, the relation between $X$ and $Y$ is NTVP and the relation between $Y$ and $Z$ is DC. The unique rules can be used to extrapolate that the relation between $X$ and $Z$ is DC. Thus, a new unique topological relation (DC) is obtained.

### 3.1.4. Forming a new set of constraints
The unique-rule-based reasoning results are transformed into a new CSP. The improved path consistency algorithm proposed in Section 3.2 is used to verify the results (the relations between the spatial objects are extrapolated and verified to determine whether there are any conflicts) to ensure that the relationships between the spatial objects determined under the given constraint conditions are consistent.

Once topological relations that have thus far been obtained through reasoning have been verified to be consistent, the topological relations that satisfy the current constraint conditions can be used for further reasoning. A new set of constraints is thus derived here by combining the topological relations that are found to satisfy the constraint conditions with the existing set of constraints, thereby obtaining the conditions for subsequent reasoning.

### 3.1.5. Non-unique-rule-based reasoning

After the previous four steps, the unique spatial topological relations between the objects have been extrapolated. In the current step, the remaining unknown topological relations are determined based on the non-unique rule base to extract all of the relations between the spatial objects. During reasoning with the non-unique rule base, if the relation between two objects cannot be extrapolated, then the universal relation is assigned as the topological relation between those two objects (the universal relation refers to all 8 topological relations defined by the RCC8 model). The formal description of the reasoning process for this step is as follows:

$$A_{ij} \leftarrow R''_{ij} , \quad A_{ji} \leftarrow R''_{ij} \curvearrowleft$$

If $R''_{ij} = \emptyset$, then $A_{ij} \leftarrow U, A_{ji} \leftarrow U$  \hspace{1cm} (5)

where $A_{ij}$ represents the matrix element in the $i$th row and $j$th column. $A_{ji}$ represents the matrix element in the $j$th row and $i$th column. $R''_{ij}$ represents the topological relation between variables $i$ and $j$ that is obtained through reasoning, which is assigned as the value of $A_{ij}$. $R''_{ij} \curvearrowleft$ represents the topological relation between variables $j$ and $i$ that is obtained by reversing $R''_{ij}$ using the reverse rules, which is assigned as the value of $A_{ji}$. If the value of $R''_{ij}$ that is obtained through rule-based reasoning in this step is empty, then $U$ is assigned as the value of both $A_{ij}$ and $A_{ji}$, where $U$ denotes the universal relation. The symbols $\leftarrow$, $\emptyset$, and $\curvearrowleft$ denote the assignment operation, the empty set, and the reversal operation, respectively.

### 3.1.6. Completing the qualitative spatial reasoning

After the previous five reasoning steps, each element in the binary topological relation matrix has been assigned a value. Each of the spatial topological relations between spatial objects that are obtained through reasoning corresponds to one of two situations. If there is only one possible topological relation between the two objects, then it must already satisfy the constraint conditions. However, for the reasoning results obtained according to the non-unique rule base, these results are transformed into a CSP in this step to verify their consistency using the improved path consistency algorithm described in Section 3.2 and to remove any inconsistent results. Thus, the final reasoning results are obtained.

### 3.2. Path consistency algorithm for spatial relations

As stated in Section 2.3, spatial relations meet the definition of a binary CSP. The path consistency algorithm for solving CSP (Egenhofer and Sharma 1993; Renz and Nebel 2007) can thus be used to verify the consistency of spatial relations. In this study, an improved path consistency algorithm is proposed. We improve the efficiency of the path consistency algorithm by reducing the number of variables and the search range of the variables. The formal description of our algorithm is shown in Equation (6).

$$M(i, \text{UniqueList}) \leftarrow M(i, \text{UniqueList}) \cap (M(i, \text{UnList})^\prime \cup M(\text{UnList}, \text{UniqueList})) \hspace{1cm} (6)$$
In Equation (6), $\circ$ denotes the compounding of relations, $\cap$ denotes taking the intersection, and $M$ denotes a spatial relation. The variable $i$ represents the row number in the matrix. $UniqueList$ and $UnList$ are two arrays, which will be explained later.

Figure 8 presents the pseudo-code of the path consistency algorithm proposed in this study. The detailed steps are as follows:

1. Establish an $N \times N$ 2-D matrix $A$, where $N$ represents the number of spatial objects and each matrix element of $A$ represents the spatial relation between two spatial objects.
2. Read the $i$th row of the 2-D constraint matrix and record the column numbers ($j$) of all elements in the $i$th row that correspond to a unique relation (stored in the array $UniqueList$) and the column numbers ($j'$) of all elements in the $i$th row that correspond to a non-unique relation (stored in the array $UnList$).
3. Establish a triad $(i, UniqueList(n), UnList(m))$, where the topological relation between $(i, UnList(m))$ is a non-unique topological relation and the topological relation between $(i, UniqueList(n))$ is a unique topological relation.
4. Record the $S$th value of $A_{iUnList(m)}$ in the array $UnValue_{1ms}$ and record the $t$th value of $A_{iUniqueList(n)}$ in the array $UnValue_{2nt}$.
5. Execute the operation $Tag ← \cup(A_{iUniqueList(n)} \cap (UnValue_{1ms} \circ UnValue_{2nt}))$.

Let $UnValue_{1ms}$ indicate the next value. The value of $UnValue_{1ms}$ does not change. Repeat the operation in Step (5) until all elements in $A_{iUniqueList(n)}$ have been traversed, and then execute the operation in Step (7).

7. If the $Tag$ result is $FALSE$, then remove $UnValue_{1ms}$ from $A_{iUnList(m)}$.
8. Let $UnValue_{1ms}$ indicate the next value. Set the $Tag$ value as $FALSE$. Execute Steps (4)–(8) until all elements in $A_{iUnList(m)}$ have been traversed.
9. Execute the operations in Steps (3)–(8) in a loop until the established triad $(i, UniqueList(n), UnList(m))$ contains all combinations of non-unique and unique elements in the $i$th row.

10. If the operation in Step (2) has already been executed to the last row of elements in the binary constraint matrix, then exit the entire operation, otherwise, increment $i$ by one and return to Step (2).

The improvements of our algorithm are described in detail below:

1. The number of unknown variables and the search range of the variables are reduced through rule-based reasoning. A dense matrix is established in path consistency algorithm. The maximal tractable subset is assigned as the values of the unknown variable in the matrix (Egenhofer and Sharma 1993; Nebel and Renz 2001). This approach can reduce the search

```
procedure PathConsistency(R[N+1][N+1])
    for i←0 to N do
        for j←0 to N do
            if R[i][j] is unique relation
                then UniqueList←j
            if R[i][j] is non-unique relation
                then UnList←j
        end for
    for m←0 to Size(UnList) do
        for n←0 to Size(UniqueList) do
            for p←0 to Size(R[i][m]) do
                for q←0 to Size(R[m][n].list) do
                    if (R[i][n] \ R[i][m].get(p) \ R[m][n].get(q))
                        then R[i][m] \ R[i][m].get(p)
                end for
            end for
        end for
    end for
end procedure
```

Figure 8. Pseudo-code of the path consistency algorithm.
range and iteration times to some extent. However, the maximal tractable subset is difficult to obtain. Alternatively, a sparse matrix is used in some studies (Du, Liang, and Sun 2012). The given variables are stored in a queue to reduce the number of repeated searches, so as to improve the search efficiency. However, unknown variables are not assigned values in the matrix, which may result in inconsistency in the reasoning results. In our method, a dense matrix is established. The efficiency of the algorithm is improved through the inclusion of rule-based reasoning. After the unique-rule-based reasoning described in Section 3.1, the unique reasoning results have been obtained, thereby increasing the number of topological relations and decreasing the number of unknown variables in the dense matrix. After the non-unique-rule-based reasoning described in Section 3.1, the non-unique reasoning results have been obtained. These non-unique reasoning results further reduce the search range and number of variables in the dense matrix, further improving the search efficiency.

(2) The number of repeated searches during consistency checking is reduced by the improved triads. In previous studies (Christodoulou, Petrakis, and Batsakis 2012; Du, Liang, and Sun 2012; Egenhofer and Sharma 1993; Nebel and Renz 2001; Renz and Nebel 1999), the triads were composed of three arbitrary spatial objects, and consistency checking was performed according to Equation (3). In our study, the triad format is \((i, \text{UniqueList}(m), \text{UnList}(m))\). We create indexes for the unique and non-unique relations between spatial objects and then use these indexes to build the triad. This reduces the numbers of iterative operations and repeated spatial-relation searches.

4. Experiment

In this paper, a campus layout planning problem is used as an experimental example to verify the feasibility of the proposed method. Appropriate planning that satisfies specified requirements is necessary for a campus layout. Let us assume that it is necessary to perform overall planning for a school. The planning results must satisfy the following constraint conditions: (1) the school should contain a teaching area, playground, and dormitory area. (2) The teaching area should include an office area and a library. (3) The office area should be disconnected from the library. (4) The dormitory area should be externally connected to the playground, and partially overlap with the teaching area for convenience in physical education and student activities. (5) The dormitory area should not be located in the teaching area, although it should be externally connected to the library for the convenience of students who wish to engage in self-study. (6) The playground should be disconnected from the office area and the library. (7) The cafeteria should partially overlap with the dormitory area and should be disconnected from the teaching area and playground. And (8) the boiler room should be adjacent to the cafeteria and included in the dormitory area.

For convenience in describing spatial objects, the following spatial objects are denoted by the following abbreviations: dormitory area \((D)\), playground \((P)\), teaching area \((T)\), library \((L)\), office area \((O)\), cafeteria \((C)\), and boiler room \((B)\). These 8 spatial topological relations are described by the object attributes in OWL. The known spatial topological relations between spatial objects can be extracted from the description of the problem, and these relations can be used as the constraint conditions for reasoning (Table 2).

The experiment is carried out using Java as a programming language, Protégé as a tool to build ontologies and Oracle 11g as a platform to implement reasoning. Protégé, a tool developed by Stanford for constructing ontology, is appropriate for us to express the ontologies and topological relations between spatial objects described above. After the ontologies are constructed, the SWRL is used to describe the ontologies to obtain the reasoning rules and to classify the reasoning rules into unique reasoning rules, non-unique reasoning rules and reverse reasoning rules based on the method mentioned in Section 3.1.1. Oracle 11g, which supports the storage, query and reasoning of the OWL data, is then used to conduct reasoning based on the method mentioned in Sections 3.1.3–3.1.6. The consistency verification of the reasoning results, unique-rule-based-reasoning results and non-unique-rule-based-reasoning results, is effective to eliminate the results that do not satisfy the

| Spatial object 1 | Topological relation | Spatial object 2 | Description |
|------------------|----------------------|------------------|-------------|
| Dormitory area   | Externally connected | Playground       | \(<D, P>: EC\) |
|                  | Externally connected | Library          | \(<D, L>: EC\) |
| Cafe teria       | Non-tangential       | Boiler room      | \(<D, B>: NTPP\) |
|                  | Disjointed           | Playground       | \(<C, P>: DC\) |
|                  | Disjointed           | Teaching area    | \(<C, T>: DC\) |
|                  | Partially overlapping| Dormitory area   | \(<C, D>: PO\) |
| Office area      | Disjointed           | Library          | \(<O, L>: DC\) |
| Playground       | Disjointed           | Office area      | \(<O, O>: EC\) |
|                  | Partially overlapping| Teaching area    | \(<O, T>: PO\) |
| Teaching area    | Non-tangential       | Office area      | \(<T, O>: NTPP\) |
|                  | Tangential           | Library          | \(<T, L>: TTPP\) |
| Boiler room      | Disjointed           | Dormitory area   | \(<T, D>: DC\) |
|                  | Externally connected | Cafeteria        | \(<B, C>: EC\) |
D is an instance of Dormitory
L is an instance of Library_L
O is an instance of Office_O
C is an instance of Cafeteria_C
B is an instance of Boiler_B
P is an instance of Playground_P
T is an instance of Teaching_T

|     | D   | L   | O   | C   | B   | P   | T   |
|-----|-----|-----|-----|-----|-----|-----|-----|
| D   | -   | EC  | DC  | PO  | NTPPi| EC  | EC  |
| L   | EC  | -   | DC  | DC  | DC  | DC  | TPP |
| O   | DC  | DC  | -   | DC  | DC  | DC  | NTPPi|
| C   | PO  | DC  | DC  | -   | EC  | DC  | DC  |
| B   | NTPPi| DC  | DC  | EC  | -   | DC  | DC  |
| P   | EC  | DC  | DC  | DC  | DC  | -   | PO  |
| T   | EC  | TPPi| NTPPi| DC  | DC  | PO  | -   |

**Figure 9.** Reasoning results for the campus spatial layout problem.

Based on the results given in Figure 9, we drew a simplified schematic diagram of the planned campus layout (Figure 10). This schematic diagram was obtained from Figure 9 based on the topological relations between each pair of spatial objects, which reflects one possible campus layout scheme and provides auxiliary decision-making support for the planning department.

5. Summary

This paper proposes a semantically supported method of qualitative spatial reasoning. Based on semantic descriptions of reasoning rules, the possible spatial topological relations between objects are first qualitatively extrapolated. Then, the task of checking the consistency of the potential results obtained through reasoning is transformed into a semantically described CSP. Finally, a semantically supported path consistency algorithm is used to solve the CSP to verify the potential reasoning results and eliminate inconsistent results to yield the final reasoning output. The contributions of this paper are twofold: (1) a semantically supported method of qualitative spatial reasoning on topological relations that combines Semantic Web technology and qualitative spatial reasoning by means of semantic descriptions of reasoning rules and a set of constraints and (2) an improved path consistency algorithm. There are still some shortcomings in our method. It is unknown whether the method proposed in this study is applicable for large amounts of spatial data, and the universality of the method must yet be verified. In addition, various types of spatial relations, such as topological relations, distance relations, and direction relations, should be combined to perform reasoning in future studies. Furthermore, range relations, topological relations, and distance relations from the perspective of three-dimensional (3-D) space may be incorporated to improve the

**Figure 10.** Schematic diagram of the campus layout results.
current reasoning method for more accurate expression of the relations among spatial objects. The improvement of more accurate expression of fine reasoning on indoor spatial relations based on 3-D constraint conditions may also advantageous to provide support for various applications, such as indoor navigation.

**Funding**

This work is funded by the National Natural Science Foundation of China [grant number 41271399], the China Special Fund for Surveying, Mapping and Geo-information Research in the Public Interest [grant number 201512015] and the National Key Research Program of China [grant number 2016YFB0501400].

**Notes on contributors**

Yandong Wang is a professor of Wuhan University, specializing in GIS and Cartography. His research interests include space information intelligent service, GIS theory and engineering application, and spatio-temporal information knowledge mining.

Mengling Qiao is a PhD candidate at Wuhan University. Her current researches focus on space information intelligent service and urban structure detection.

Hui Liu is a researcher in GIS field. He obtained his master degree in Wuhan University. Now he works at Wuhan Land Resource and Planning and Information Centre. His research interests are surveying and mapping.

Xinyue Ye is a professor of Kent State University, specializing in GIS and cartography. His research interests are urban crime analysis, urban expansion, and spatio-temporal information mining.

**ORCID**

Xinyue Ye [http://orcid.org/0000-0001-8838-9476]

**References**

Batsakis, S. 2013. "Reasoning over 2D and 3D Directional Relations in OWL: A Rule-based Approach." In RuleML 2013: Theory, Practice, and Applications of Rules on the Web, edited by L. Morgensten, P. Stefaneas, F. Lévy, A. Wyner, and A. Paschke, 37–51. Heidelberg: Springer.

Batsakis, S., G. Antoniou, and I. Tachmazidis. 2014. "Representing and Reasoning over Topological Relations in OWL." Paper Presented at the 4th International Conference on Web Intelligence, Mining and Semantics, Thessaloniki, Greece, June 2–4.

Batsakis, S., and E. G. Petrakis. 2010. "SOWL: Spatio-temporal Representation, Reasoning and Querying over the Semantic Web." Paper Presented at the 6th International Conference on Semantic Systems, Graz, Austria, September 1–3.

Back, T., D. B. Fogel, and Z. Michalewicz. 1997. *Handbook of Evolutionary Computation*. Bristol: Institute of Physics Publishing.

Berners-Lee, T., J. Hendler, and O. Lassila. 2001. "The Semantic Web." *Scientific American* 284 (5): 28–37.

Croitoru, M., and E. Compatangelo. 2007. "Ontology Constraint Satisfaction Problem Using Conceptual Graphs." In *Research and Development in Intelligent Systems XXIII*, edited by M. Bramer, F. Coenen, and A. Tuson, 231–244. London: Springer.

Cui, Z., A. G. Cohn, and D. A. Randell. 1993. "Qualitative and Topological Relationships in Spatial Databases." *Advances in Spatial Databases* 692: 296–315.

Clementini, E., and P. D. Felice. 1995. "A Comparison of Methods for Representing Topological Relationships." *Information Sciences – Applications* 3 (3): 149–178.

Christodoulou, G., E. G. Petrakis, and S. Batsakis. 2012. "Qualitative Spatial Reasoning Using Topological and Directional Information in OWL." Paper Presented at the IEEE 24th International Conference on Tools with Artificial Intelligence (ICTAI), Athens, Greece, November 7–9: 596–602.

Cohn, A. G., B. Bennett, J. Goody, and G. Mark. 1997. "Qualitative Spatial Representation and Reasoning with the Region Connection Calculus." *GeoInformatica* 1 (3): 275–316.

Du, S., C. C. Feng, and L. Guo. 2015. "Integrative Representation and Inference of Qualitative Locations about Points, Lines, and Polygons." *International Journal of Geographical Information Science* 29 (6): 980–1006.

Du, Y., F. Liang, and Y. Sun. 2012. "Integrating Spatial Relations into Case-based Reasoning to Solve Geographic Problems." *Knowledge-based Systems* 33 (3): 111–123.

Du, S., Q. Wang, and Q. Qin. 2006. " Definitions of Natural-language Spatial Relations: Combining Topology and Directions." *Geo-Spatial Information Science* 9 (1): 55–64.

Egenhofer, M. J., and R. D. Franzosa. 1991. "Point-set Topological Spatial Relations." *Journal of Geographical Information Systems* 5 (2): 161–174.

Egenhofer, M. J., and J. Sharma. 1993. "Assessing the Consistency of Complete and Incomplete Topological Information." *Journal of Geographical Systems* 1 (1): 47–68.

Gott, R., and B. Bauer-Messmer. 2007. "Towards Spatial Reasoning in the Semantic Web: A Hybrid Knowledge Representation System Architecture." In *European Information Society: Leading the Way with Geo-information*, edited by S. Fabrikant and M. Wachowicz, 349–364. Heidelberg: Springer.

Holt, A., and G. L. Benwell. 1999. "Applying Case-based Reasoning Techniques in GIS." *International Journal of Geographical Information Science* 13 (1): 9–25.

Horrocks, I., P. F. Patel-Schneider, H. Boley, S. Tabet, G. Benjamine, and D. Mike. 2004. "SWRL: A Semantic Web Rule Language Combining OWL and RuleML." *World Wide Web Consortium*, May 21. [https://www.w3.org/Submission/SWRL/](https://www.w3.org/Submission/SWRL/).

Jiang, B., and X. Yao. 2006. "Location-based Services and GIS in Perspective." *Computers, Environment and Urban Systems* 30 (6): 712–725.

Kwan, M. P., and G. Ding. 2008. "Geo-narrative: Extending Geographic Information Systems for Narrative Analysis in Qualitative and Mixed-method Research." *The Professional Geographer* 60 (4): 443–465.

Long, Z. G., and S. J. Li. 2013. "A Complete Classification of Spatial Relations Using the Voronoi-based Nine-intersection Model." *International Journal of Geographical Information Science* 27 (10): 2006–2025.

Mcguinness, D. L., and F. Harmelen. 2004. "OWL Web Ontology Language: Overview." *February* 63 (45): 990–996.
Y. WANG ET AL.

Matheus, C. J., K. Baclawski, M. M. Kokar, and J. J. Letkowski. 2005. "Using SWRL and OWL to Capture Domain Knowledge for a Situation Awareness Application Applied to a Supply Logistics Scenario." In RuleML 2005: Rules and Rule Markup Languages for the Semantic Web, edited by A. Adi, S. Stoutenburg, and S. Tabet. 130–144. Heidelberg: Springer.

Marc-Zwecker, S., F. D. B. D. Beuvron, C. Zanni-Merk, and F. L. Ber. 2013. "Qualitative Spatial Reasoning in RCC8 with OWL and SWRL." Paper Presented at the International Conference on Knowledge Engineering and Ontology Development, Vilamoura, Algarve, Portugal, September 19–22.

Mackworth, A. K. 1977. "Consistency in Networks of Relations." Artificial Intelligence 8 (1): 99–118.

Mackworth, A. K., and E. C. Freuder. 1985. "The Complexity of Some Polynomial Network Consistency Algorithms for Constraint Satisfaction Problems." Artificial Intelligence 25 (1): 65–74.

Nebel, B., and J. Renz. 2001. "Efficient Methods for Qualitative Spatial Reasoning." Journal of Artificial Intelligence Research 15 (1): 289–318.

Nikkila, R., E. Nash, J. Wiebensohn, I. Seilonen, and K. Koskinen. 2013. "Spatial Inference with an Interchangeable Rule Format." International Journal of Geographical Information Science 27 (6): 1210–1226.

Nam, S., and I. Kim. 2015. "Qualitative Spatial Reasoning with Directional and Topological Relations." Mathematical Problems in Engineering 2015: 1–10.

Philip, D. S., A. I. Abdelmoty, and B. El-Geresy. 2014. "Spatial Reasoning with Place Information on the Semantic Web." International Journal of Artificial Intelligence Tools 23 (5): 1450011.

Parsia, B., and E. Sirin. 2004. "Pellet: An OWL DL Reasoner." Paper Presented at the 3th International Semantic Web Conference-poster, Hiroshima, Japan, November 7–11.

Papadias, D., and M. J. Egenhofer. 1996. "Algorithms for Hierarchical Spatial Reasoning." Geoinformatica 1 (3): 251–273.

Renz, J., and B. Nebel. 1999. "On the Complexity of Qualitative Spatial Reasoning: A Maximal Tractable Fragment of the Region Connection Calculus." Artificial Intelligence 108 (1): 69–123.

Randell, D. A., Z. Cui, and A. G. Cohn. 1992. "A Spatial Logic Based on Regions and Connection." Paper Presented at the 3rd International Conference on Principles of Knowledge Representation and Reasoning (KB1992), Cambridge, Massachusetts, USA, October 25–29: 165–176.

Renz, J. 2002. "Qualitative Spatial Reasoning with Topological Information." Chap. XVI in Lecture Notes in Computer Science. Heidelberg: Springer.

Renz, J., and B. Nebel. 2007. "Qualitative Spatial Reasoning Using Constraint Calculi." In Handbook of Spatial Logics, edited by M. Aiello, I. Pratt-Hartmann, and J. Van Benthem, 161–215. Netherlands: Springer.

Stocker, M., and E. Sirin. 2009. "Pellet Spatial: A Hybrid RCC-8 and RDF/OWL Reasoning and Query Engine." Paper Presented at the 5th International Workshop on OWL: Experiences and Directions, Chantilly, VA, USA, October 23–24.

Simonis, H. 2005. "Sudoku as a Constraint Problem." Paper Presented at the 4th International Workshop on Modelling & Reformulating Constraint Satisfaction Problems (CP 2005), Sitges, Spain, October 1.

Yao, X., and J. C. Thill. 2006. "Spatial Queries with Qualitative Locations in Spatial Information Systems." Computers, Environment and Urban Systems 30 (4): 485–502.

Zhao, Y., Y. Murayama, and Y. Zhang. 2005. "Field-based Fuzzy Spatial Reasoning Model for Geographical Information Systems: Case of Constraint Satisfaction Problem." Theory & Applications of GIS 13 (1): 21–31.