Using auxiliary marginal distributions in imputations for nonresponse while accounting for survey weights, with application to estimating voter turnout

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Abstract

The Current Population Survey is the gold-standard data source for studying who turns out to vote in elections. However, it suffers from potentially nonignorable unit and item nonresponse. Fortunately, after elections, the total number of voters is known from administrative sources and can be used to adjust for potential nonresponse bias. We present a model-based approach to utilize this known voter turnout rate, as well as other population marginal distributions on demographic variables, in multiple imputation for unit and item nonresponse. In doing so, we ensure that the imputations produce design-based estimates that are plausible given the known margins. We introduce and utilize a hybrid missingness model comprising a pattern mixture model for unit nonresponse and selection models for item nonresponse. Using simulation studies, we illustrate repeated sampling performance of the model under different assumptions about the missingness mechanisms. We apply the model to examine voter turnout by subgroups using the 2018 Current Population Survey for North Carolina. As a sensitivity analysis, we examine how results change when we allow for over-reporting, i.e., individuals self-reporting that they voted when in fact they did not.

Key Words: missing; multiple imputation; pattern mixture; selection.

1 Introduction

When data suffer from unit nonresponse (no values are observed for some units) or item nonresponse (some values are observed and some values are missing for some units), analysts generally need to make strong assumptions about the reasons for missingness. For example, they may need to assume data are missing at random (Rubin, 1976; Mealli and Rubin, 2015) or that they follow a specific nonignorable missingness mechanism (Linero and Daniels, 2018). Such assumptions are inescapable with missing data.

One way to reduce reliance on strong assumptions is to utilize auxiliary information from external data sources. Here, we consider sets of population percentages or totals for categorical variables. For example, we may have accurate estimates of population percentages of demographic variables
from censuses, large-sample surveys, or administrative databases. To motivate how such auxiliary information can help with missing data, suppose we have data from a survey that includes a question on whether or not one votes in an election. The survey suffers from missingness on vote, which we want to (multiply) impute to facilitate estimation of the turnout rate. Among survey respondents, 70% indicate that they voted in the election. However, using population size estimates and post-election administrative data, we know that only 50% of voters turned out. Assuming respondents do not misreport, this auxiliary marginal information suggests that the missingness for vote is nonignorable; that is, people who did not vote are more likely not to tell their voting status. Knowing the margin, we can impute values for the missing votes so that the completed data estimates of turnout are closer to 50% than 70%. Without this margin, we would likely impute the missing votes using a missing at random (MAR) assumption—which would perpetuate the nonresponse bias—or have to make some heroic, unverifiable assumption about the vote distribution of the nonrespondents.

In this article, we present methods for leveraging auxiliary marginal distributions with application to estimating turnout in the 2018 U.S. Congressional election in North Carolina using the 2018 Current Population Survey (CPS). To do so, we build on the missing data with auxiliary margins (MD-AM) framework developed by Akande et al. (2021), which we review in Section 2. Specifically, because the CPS relies on a complex sampling design—as do many large-scale government and social surveys—we extend the MD-AM framework to account for survey weights from unequal probability sampling designs. The basic idea is to generate multiple imputations so that the completed data result in plausible design-based estimates of the known margins. This strategy was introduced by Akande and Reiter (2022), who illustrated it for stratified simple random samples subject to item nonresponse only. We leverage and extend this strategy to surveys with weights generated by other complex sampling designs and subject to both item and unit nonresponse. In doing so, we introduce a hybrid missing data mechanism that uses a pattern mixture model formulation for unit nonresponse and a selection model formulation for item nonresponse. Using simulation studies, we show that using a hybrid missingness MD-AM model can result in more reasonable survey-weighted inferences than approaches that do not utilize the margins. We then use a hybrid missingness MD-AM model to estimate voter turnout for subgroups in the 2018 CPS. More broadly, the CPS analysis is illustrative of a general, multiple imputation strategy for using auxiliary margins to handle nonignorable missing data in surveys with complex sampling designs.

The remainder of this article is organized as follows. In Section 2, we review the MD-AM framework of Akande et al. (2021) for simple random samples, explaining how it can leverage auxiliary margins to handle unit and item nonresponse. In Section 3, we present an MD-AM model that uses the hybrid missingness mechanism and accounts for survey weights. We also describe methods for estimating the parameters of this model and generating multiple imputations of missing values, and we summarize results of simulation studies evaluating the model fitting procedure. The simulation studies are described in detail in the supplementary material. In Section 4, we use a hybrid missingness MD-AM model to estimate turnout in demographic subgroups using the 2018 CPS in North Carolina. We also assess the sensitivity of results to potential reporting error in the vote responses. In Section 5, we provide some concluding remarks.
2 MD-AM Modeling with Simple Random Samples

When presenting the MD-AM framework for simple random samples, Akande et al. (2021) describe a two step process. In step 1, the analyst specifies a joint distribution for the survey variables and for the indicators for nonresponse that are identifiable from the observed data alone. This model typically uses default assumptions about the missingness, such as MAR or itemwise conditionally independent nonresponse (ICIN, Sadinle and Reiter 2017). In step 2, the analyst, informed by the available margins, adds parameters to the model while ensuring that the model as a whole can be identified as in Sadinle and Reiter (2019). Adding these parameters weakens the assumptions about the missingness mechanisms. In this section, we review the MD-AM framework for simple random samples, beginning with general notation that we use throughout.

2.1 Notation

Let $\mathcal{D}$ comprise the data that were planned to be collected from a survey of $i = 1, \ldots, n$ individuals, and let $\mathcal{A}$ comprise the auxiliary margins. Let $X = (X_1, \ldots, X_k)$ represent the $p$ variables in both $\mathcal{A}$ and $\mathcal{D}$, where each $X_k = (X_{1k}, \ldots, X_{nk})^T$ for $k = 1, \ldots, p$. Let $Y = (Y_1, \ldots, Y_q)$ represent the $q$ variables in $\mathcal{D}$ but not in $\mathcal{A}$, where each $Y_k = (Y_{1k}, \ldots, Y_{nk})^T$ for $k = 1, \ldots, q$. We disregard variables in $\mathcal{A}$ but not in $\mathcal{D}$ as they are not of primary interest. We assume that $\mathcal{A}$ contains only sets of marginal distributions for variables in $X$, e.g., available in some external database.

For each $k = 1, \ldots, p$, let $R_{ik}^x = 1$ if individual $i$ would not respond to the question on $X_k$ in the survey and $R_{ik}^x = 0$ otherwise. Similarly, for each $k = 1, \ldots, q$, let $R_{ik}^y = 1$ if individual $i$ would not respond to the question on $Y_k$ in the survey and $R_{ik}^y = 0$ otherwise. Let $R^x = (R_1^x, \ldots, R_p^x)$ and $R^y = (R_1^y, \ldots, R_q^y)$ be the vectors of item nonresponse indicators for variables in $X$ and $Y$ respectively, where each $R_k^x = (R_{ik1}^x, \ldots, R_{ink}^x)^T$ and $R_k^y = (R_{ik1}^y, \ldots, R_{ink}^y)^T$. Let $U_i = 1$ when individual $i$ in $\mathcal{D}$ does not respond to the survey, so that we do not have their responses. Let $U_i = 0$ when observe at least some values of the survey variables for individual $i$. Let $U = (U_1, \ldots, U_n)$.

2.2 Illustrative Model Specification

We illustrate the ideas underpinning the MD-AM framework using a $\mathcal{D}$ comprising two binary variables, $X_1$ and $X_2$. We have auxiliary information for both from $\mathcal{A}$; thus, this illustrative example does not include any $Y$ variables (whereas our 2018 CPS analysis does use $Y$ variables). We suppose $X_1$ suffers from item nonresponse and $X_2$ is fully observed, so that $R_{ik}^x$ is the vector of item nonresponse indicators for $X_1$. For units with $U_i = 1$, we do not observe $(X_{1i}, X_{2i}, R_{1i}^x)$. We do not concern ourselves with $R_2^x$ since $X_2$ is fully observed for cases with $U_i = 0$. $U$ is fully observed for all records in $\mathcal{D}$. Figure 1 displays the observed data and the auxiliary information on $X_1$ and $X_2$. We use this same example to introduce the hybrid missingness MD-AM model in Section 3. Akande et al. (2021) recommend using selection models for all missingness indicators. With their modeling strategy, we write the joint distribution of $(X_1, X_2, R_1^x, U)$ as a product of sequential conditional distributions, using selection models for the unit and item nonresponse indicators.

$$P(X_1, X_2, R_1^x, U) = P(R_1^x|X_1, X_2, U)P(U|X_1, X_2)P(X_1|X_2)P(X_2).$$

(1)

The joint distribution in (1) can be fully parameterized with fifteen parameters plus the constraint that probabilities sum to one. However, $\mathcal{D}$ alone provides enough information to identify only six
Figure 1: Illustrative scenario for the MD-AM model in simple random samples. \( D \) comprises two binary variables, \( X_1 \) and \( X_2 \). \( X_2 \) is fully observed; \( X_1 \) suffers from item nonresponse; and, some units do not respond. Here, “✓” represents observed values, “?” represents missing values.

\[
\begin{array}{cccc|c}
X_1 & X_2 & R_1^x & U \\
\checkmark & \checkmark & 0 & 0 \\
? & \checkmark & 1 & \\
? & ? & ? & 1 \\
\checkmark & \checkmark & \end{array}
\]

\( D \)

\[
\begin{array}{cccc}
A \\
\end{array}
\]

parameters. This is evident in the six observable quantities in Figure 1, namely the probabilities for \( U \) and \( (R_1^x \mid U = 0) \), the two probabilities for \( (X_2 \mid R_1^x = r, U = 0) \) where \( r \in \{0, 1\} \), and the two probabilities for \( (X_1 \mid X_2 = x, R_1^x = 0, U = 0) \) where \( x \in \{0, 1\} \). Thus, we need to make some identifying assumptions about the missingness in order to estimate the joint distribution.

The auxiliary information about the marginal distributions of \( X_1 \) and \( X_2 \) add constraints on the joint distribution in (1). The MD-AM framework leverages this additional information. We begin with step 1: specify an identifiable model based on \( D \) alone with at most six parameters. Using selection models for the nonresponse indicators, one natural choice is

\[
\begin{align*}
X_2 & \sim \text{Bernoulli}(\pi_{x_2}), \quad \text{logit}(\pi_{x_2}) = \alpha_0 \\
X_1 \mid X_2 & \sim \text{Bernoulli}(\pi_{x_1}), \quad \text{logit}(\pi_{x_1}) = \beta_0 + \beta_1 X_2 \\
U \mid X_1, X_2 & \sim \text{Bernoulli}(\nu_0) \\
R_1^x \mid U, X_1, X_2 & \sim \text{Bernoulli}(\pi_R), \quad \text{logit}(\pi_R) = \gamma_0 + \gamma_1 X_2.
\end{align*}
\] (2) (3) (4) (5)

This default model has unit nonresponse following a missing completely at random mechanism (MCAR) mechanism and item nonresponse in \( X_1 \) following an ICIN mechanism.

In step 2, we add one term related to \( X_1 \) and one term related to \( X_2 \) to the specifications in (4) – (5). One approach is to add both terms to the model for unit nonresponse in (4), leaving the model for item nonresponse as in (5). With slight re-use and abuse of notation, we replace (4) with

\[
U \mid X_1, X_2 \sim \text{Bernoulli}(\pi_U), \quad \text{logit}(\pi_U) = \nu_0 + \nu_1 X_1 + \nu_2 X_2.
\] (6)

This is an example of an additive nonignorable model \cite{Hirano2001, Deng2013, Schifeling2015, Si2016} for unit nonresponse. It includes the MCAR mechanism as a special case (\( \nu_1 = \nu_2 = 0 \)), but allows for nonignorable mechanisms as well (either \( \nu_1 \neq 0 \) or \( \nu_2 \neq 0 \)). Thus, the data can inform whether MCAR is plausible, or if not allow for unit nonresponse to depend on \( X_1 \) or \( X_2 \).

There is a key identifying assumption in (6): there is no interaction between \( X_1 \) and \( X_2 \). With only marginal information on \( X_1 \) and \( X_2 \), we have no way to identify, for example, a differential effect of \( X_1 \) on \( U \) at \( X_2 = 1 \) versus \( X_2 = 0 \). If we had the joint margin of \( (X_1, X_2) \), we could add their interaction to (6). Nonetheless, using \( A \) has allowed us to weaken substantially the strong MCAR assumption in (4).

As an alternative to (6), we could specify an additive nonignorable model for \( R_1^x \), which would allow the mechanism for item nonresponse in \( X_1 \) to depend on the value of \( X_1 \) itself. In doing so,
however, we have to remove the direct dependence of $U$ on $X_1$, as the margin for $X_1$ only provides enough information to estimate one additional parameter. Thus, again re-using notation for the regression parameters, we could replace (4) and (5) with the alternative specification,

$$U | X_1, X_2 \sim \text{Bernoulli}(\pi_U), \; \text{logit}(\pi_U) = \nu_0 + \nu_1 X_2$$

$$R_1^* | X_1, X_2, U \sim \text{Bernoulli}(\pi_R), \; \text{logit}(\pi_R) = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2.$$  

As this example suggests, analysts can use $A$ to enrich the unit nonresponse model or item nonresponse model. Akande et al. (2021) suggest that analysts use the auxiliary margins to help model unit nonresponse when the unit nonresponse rate is higher than the item nonresponse rate or when the primary concern is about nonignorable unit nonresponse. Similarly, analysts can enrich the item nonresponse model when item nonresponse is a bigger concern than unit nonresponse. Researchers can fit multiple model specifications and compare their results as a sensitivity analysis.

In the CPS analysis described in Section 4, unit nonresponse is more substantial than item nonresponse. Hence, when describing our adaptation of the MD-AM framework for handling unequal survey weights, we focus on using $A$ to enrich modeling of unit nonresponse.

### 3 MD-AM with Hybrid Missingness Model and Weights

The MD-AM model in Section 2 is intended for simple random samples. In complex designs, it can result in biased estimates of finite population quantities, as the modeling and imputation steps do not account for unequal probabilities of selection. Akande and Reiter (2022) describe how to modify the MD-AM model to account for the special case of stratified simple random sampling subject to item nonresponse only. To do so, they use a rejection sampler: one proposes imputations of the missing values, computes design-based estimates of totals for variables with known margins, and accepts or rejects the proposals according to how close in distribution the estimated totals are to the known marginal total. Their model employs selection models for the item nonresponse indicators.

We attempted to apply the selection modeling approach in [1] and the strategy in Akande and Reiter (2022) to account for more general unequal probability sampling designs with both item and unit nonresponse. However, the acceptance rate of the rejection sampler is generally too low to be practically useful, and it can be difficult to generate imputations that result in plausible design-based estimates for arbitrary survey weights. We instead propose a hybrid missingness model that uses a pattern mixture model for unit nonresponse and a selection model for item nonresponse. Using the example in Section 2, we specify a model for $U$, a model for $(X_1, X_2 | U)$, and a model for $(R_1^* | X_1, X_2, U)$, as we now describe.

We modify step 1 in the MD-AM framework to reflect the alternate factorization. Again re-using some notation for regression parameters, we have the default model,

$$U \sim \text{Bernoulli}(\pi_U)$$

$$X_2 | U \sim \text{Bernoulli}(\pi_{x_2}), \; \text{logit}(\pi_{x_2}) = \alpha_0$$

$$X_1 | X_2, U \sim \text{Bernoulli}(\pi_{x_1}), \; \text{logit}(\pi_{x_1}) = \beta_0 + \beta_1 X_2$$

$$R_1^* | X_1, X_2, U \sim \text{Bernoulli}(\pi_{R_1}), \; \text{logit}(\pi_{R_1}) = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2.$$  

This is essentially the same model proposed for step 1 in Section 2, re-organized here to clearly show the pattern mixture modeling for $U$. We have to assume $R_1^*$ is conditionally independent of $U$.
given \( D \) to enable identification of \((\gamma_0, \gamma_1)\), as we have no information to distinguish distributions of \( R^*_i \) for unit respondents and nonrespondents.

Following step 2 of the MD-AM framework, we next add terms related to \( X_1 \) and \( X_2 \) to leverage the information in the known margins. We dedicate both terms to unit nonresponse modeling by adding an indicator variable for unit nonresponse, which we label as \( U \), to (10) and (11). We have

\[
X_2 | U \sim \text{Bernoulli}(\pi_{x_2}), \quad \text{logit}(\pi_{x_2}) = \alpha_0 + \alpha_1 U \tag{13}
\]

\[
X_1 | X_2, U \sim \text{Bernoulli}(\pi_{x_1}), \quad \text{logit}(\pi_{x_1}) = \beta_0 + \beta_1 X_2 + \beta_2 U. \tag{14}
\]

Here, (13) implies that the distribution of \( X_2 \) differs for unit nonrespondents and unit respondents, and \( \alpha_1 \) controls the strength of nonresponse bias. Specifically, \( \text{logit}(\pi_{x_2}) = \alpha_0 \) for unit respondents, and \( \text{logit}(\pi_{x_2}) = \alpha_0 + \alpha_1 \) for unit nonrespondents. As a result, with larger \( |\alpha_1| \), the distribution of \( X_2 \) is less homogeneous across unit nonrespondents and unit respondents. Similar interpretations can be applied for (14). Essentially, we can view (14) as a model with different intercepts for unit respondents and unit nonrespondents, but with slopes preserved.

For the hybrid missingness MD-AM model, one key identifying assumption is no interactions with \( U \). This is unavoidable when using only univariate margins. With bivariate margins, analysts can add interaction effects. For example, when the joint margin of \((X_1, X_2)\) is known, analysts can add an interaction between \( U \) and \( X_2 \) in (14).

### 3.1 Incorporating Survey Weights

We now incorporate survey weights into the hybrid missingness MD-AM model. Our overarching goal is to facilitate valid design-based estimation, which is the preferred approach to finite population inference for many practitioners. Let \( N \) denote the population size from which the \( n \) survey units in \( D \) are sampled. For \( i = 1, \ldots, N \), let \( \pi_i \) be the probability that unit \( i \) is selected to be in the sample, and let \( w^d_i = 1/\pi_i \) be its design weight (also known as its sampling weight).

#### 3.1.1 Using Margins for Probabilistic Constraints

In finite population inference, often we seek to estimate the population totals or means of survey variables. For example, we may seek to estimate the total of some variable \( X_k \), which we write as \( T_{X_k} = \sum_{i=1}^{N} X_{ik} \). Analysts can estimate \( T_{X_k} \) with the Horvitz and Thompson (1952) estimator,

\[
\hat{T}_{X_k} = \sum_{i \in D} w^d_i X_{ik}. \tag{15}
\]

Finite population central limit theorems ensure that when \( n \) is large enough, for fully observed data we have

\[
\hat{T}_{X_k} \sim N(T_{X_k}, V_{X_k}) \tag{16}
\]

with some variance \( V_{X_k} \). Analysts can estimate \( V_{X_k} \) using design-based principles (Fuller, 2009).

However, we cannot compute \( \hat{T}_{X_k} \) directly when units in \( D \) are missing values of \( X_k \), either due to unit or item nonresponse. Nonetheless, (16) still holds for the unobserved value of \( \hat{T}_{X_k} \). Furthermore, we know \( T_{X_k} \) (or the population percentage, \( T_{X_k}/N \)) for any \( X_k \) with a margin in \( \mathcal{A} \). Thus, as suggested in Akande and Reiter (2022), we should impute the missing values for \( X_k \) so that any completed data set produces a reasonable \( \hat{T}_{X_k} \) based on (16).
We implement this probabilistic constraint in the illustrative model with binary $X_1$ and $X_2$ as follows. For all $i \in D$ and for all $k$, let $X_{ik}^* = X_{ik}$ when $R_{ik}^x = 0$, and let $X_{ik}^*$ be an imputed value when $R_{ik}^x = 1$ or $U_i = 1$. In addition to following (12) – (14), the imputations of missing $(X_1, X_2)$ should satisfy,

$$
\sum_{i \in D} w_i^d X_{i1}^* \sim N(T_{X_1}, V_{X_1}), \quad \sum_{i \in D} w_i^d X_{i2}^* \sim N(T_{X_2}, V_{X_2}).
$$

(17)

When values of $X_k$ are missing, the design-based estimate of $V_{X_k}$ also is not directly computable. We therefore consider $V_{X_k}$ as a parameter set by the analyst to reflect how closely $\hat{T}_{X_k}$ should match $T_{X_k}$ in any completed data set. When we set $V_{X_k}$ to be relatively small, we encourage imputations that result in $\hat{T}_{X_k}$ close to $T_{X_k}$. We note, however, that with small $V_{X_k}$ it can be challenging computationally to find imputations that satisfy (17) and adequately explore the space of plausible imputations of the missing values (Tang, 2022). On the other hand, when we let $V_{X_k} \to \infty$, we effectively impose no probabilistic constraint on $\hat{T}_{X_k}$, in which case the MD-AM model is not identifiable.

We suggest setting $V_{X_k}$ to be plausibly close to what its design-based estimate would have been absent missing data. To do so, analysts can use the following procedure. First, generate a completed data set by imputing $X_{ik}$ for units with $R_{ik}^x = 1$ using a common MAR mechanism, such as via multiple imputation by chained equations (Azur et al., 2011) or, if available, by using the imputations made by the agency responsible for collecting $D$. Second, compute the survey-weighted estimate of variance of $\hat{T}_{X_k}$ with this completed data set. We denote this estimate as $\hat{V}_{X_k}$ and use it for $V_{X_k}$. For the survey-weighted estimation, we assume that the analyst uses respondent weights that are adjusted for unit nonresponse, for example, by the agency responsible for collecting $D$.

### 3.1.2 Weights for Unit Nonrespondents

Data analysts do not always have access to design weights for survey nonrespondents; indeed, federal agencies routinely do not provide any weights for survey nonrespondents. Therefore, we develop methods for scenarios where weights are unavailable for unit nonrespondents. For convenience in notation, when using design weights in estimates, we set $w_i^d = 0$ whenever $U_i = 1$.

The procedure for estimating the MD-AM model, which we describe in section 3.2, requires the availability of weights for all units. When the design weights for survey respondents are available as part of $D$, we need to create weights for the survey nonrespondents. To do so, we treat all nonrespondents equally and let each have the same weight. We generate weights for nonrespondents so that the sum of the weights for all $n$ units in $D$ equals $N$. Therefore, when design weights are available, the weights that we use for analysis are as follows:

$$
\begin{align*}
    w_i = \begin{cases}
    w_i^d & \text{if } U_i = 0 \\
    \frac{N - \sum_{i \in D} w_i^d}{\sum_{j \in D} U_j} & \text{if } U_i = 1.
    \end{cases}
\end{align*}
$$

(18)

### 3.1.3 Using Adjusted Weights

It is a common practice for design weights to be adjusted for unit nonresponse so that the respondents weight up to the full population (Valliant et al., 2013). Indeed, adjusted weights, which we denote as $w_i^a$, are often the only weights available to analysts. Weight adjustment methods include
but are not limited to weighting class adjustment, propensity score adjustment, poststratification and raking (Valliant et al. 2013; Lohr 2010; Si and Zhou 2021). If we use adjusted weights in [18] we may end up with very small or even negative values of $w_i$ because the sum of the $w_i^a$ for the unit respondents already might be close to or even exceed $N$.

We avoid this undesirable outcome by down-weighting each $w_i^a$ to attempt to get close to $w_i^d$. In some instances, enough information about the weighting adjustment procedures is provided to enable analysts to do so exactly. Often, however, this is not the case. Absent information to allow reverse-engineering of the design weights, analysts can use an ad hoc and simple technique as follows. As before, set $w_i^a = 0$ whenever $U_i = 1$. For all $i \in D$, we create and use for estimation,

$$w_i^* = \begin{cases} w_i^a \times \left(1 - \frac{\sum_{j \in D} U_j}{n}\right), & \text{if } U_i = 0 \\ \frac{\sum_{j \in D} w_i^a}{n}, & \text{if } U_i = 1. \end{cases}$$ (19)

Here, (19) down-weights the adjusted weights for unit respondents by the unit response rate, and assigns the remaining weight evenly to the unit nonrespondents. After this re-assignment step, $\sum_{i \in D} w_i^* = \sum_{i \in D} w_i^a$, which should be a reasonable estimate of $N$.

Analysts could create weights for unit nonrespondents by instead requiring $\sum_{i \in D} w_i^* = 0$ whenever $U_i = 0$. For all $i \in D$, we create and use for estimation,

$$w_i^* = \left(1 - \frac{\sum_{j \in D} U_j}{n}\right),$$

which should be imputed as 1 when $w_i^a = 1$ and 0 otherwise.

3.2 Estimation Strategy for Illustrative Model

In this section, we illustrate a general estimation strategy for the hybrid missingness MD-AM model using the model defined by (12)–(14). The strategy can be extended to handle additional variables by replicating the various sampling steps. We present the sampler when design weights for unit respondents are published. When those weights are not available, we substitute $w_i^a$ for $w_i$. We assume $T_{X_1}$ and $T_{X_2}$ are available in $A$, and that the analyst has specified $V_{X_1}$ and $V_{X_2}$.

We use normal approximations for the distributions of the estimated coefficients of the logistic regressions to simplify computations (Raghunathan et al. 2001). Drawing from the exact posterior distribution can be achieved via data augmentation using Pólya-Gamma latent variables (Polson et al. 2013), although with large enough samples the normal approximations often are reasonable.

Let $n_U$ be the number of unit nonrespondents in $D$. We get starting values for the sampler by imputing $X_{1i}$ for units with $R_{1i} = 1$ under the assumption of MAR. We then compute maximum likelihood estimates for all parameters in the logistic regressions from this completed data set, which we use as starting values. The sampler proceeds as follows from any iteration $t$.

**IM1.** Draw a value $\hat{T}_{X_2}^{(t+1)} \sim N(T_{X_2}, V_{X_2})$.

**IM2.** Calculate the number of times $X_{2i}^{(t+1)}$ should be imputed as 1 when $U_i = 1$ so that the weighted sum of $X_2^{(t+1)}$ is as close to $\hat{T}_{X_2}^{(t+1)}$ as possible; denote this number as $n_{2U}$.

$$n_{2U} = \left[ \frac{\hat{T}_{X_2}^{(t+1)} - \sum_{i \in D} w_i X_{i2}^{(t+1)} I(U_i = 0)}{\sum_{i \in D} w_i I(U_i = 1)/n_U} \right].$$ (20)

**IM3.** Let $\hat{\alpha}_0$ and $V_{\hat{\alpha}_0}$ denote the maximum likelihood estimate and the variance of $\alpha_0$ based on the $U_i = 0$ observations. Sample $\alpha_0^{(t+1)}$ from its approximate posterior distribution, $N(\hat{\alpha}_0, V_{\hat{\alpha}_0})$. 8
IM4. Calculate the proportion of unit nonrespondents that should be imputed as 1 for $X_2$. Denote this number as $\hat{p}_2 = \frac{n_{2U}}{n_U}$. Then, set $\alpha_1^{(t+1)}$ so that $\mathbb{E}[\alpha_0^{(t+1)} + \alpha_1^{(t+1)}] = \text{logit}(\hat{p}_2)$.

IM5. Draw imputations of $X_2$ for unit nonrespondents from $X_2|U = 1 \sim \text{Bernoulli}\left(1/(1 + \exp(-\alpha_0^{(t+1)} - \alpha_1^{(t+1)}))\right)$.

IM6. Draw a value $\hat{T}_{X_1}^{(t+1)} \sim N(T_{X_1}, V_{X_1})$.

IM7. Calculate the number of times $X_1^{(t+1)}$ should be imputed as 1 when $U_i = 1$ so that the weighted sum of $X_1^{(t+1)}$ is as close to $\hat{T}_{X_1}^{(t+1)}$ as possible; denote this number as $n_{1U}$. Specifically,

$$n_{1U} = \left\lfloor \frac{\hat{T}_{X_1}^{(t+1)} - \sum_i^n w_i X_1^{(t)} I(U_i = 0)}{\sum_i^n w_i I(U_i = 1)} \right\rfloor.$$  \hspace{1cm} (21)

IM8. Let $\hat{\beta}$ and $V_{\hat{\beta}}$ denote the maximum likelihood estimates and the covariance matrix of $\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$ based on the $U_i = 0$ observations. Sample $\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}^{(t+1)}$ from its approximate posterior distribution $N(\hat{\beta}, V_{\hat{\beta}})$.

IM9. Calculate the proportion of unit nonrespondents that should be imputed as 1 for $X_1$. Denote it as $\hat{p}_1 = \frac{n_{1U}}{n_U}$. Then, set $\beta_2^{(t+1)}$ so that $\mathbb{E}[\beta_0^{(t+1)} + \beta_1^{(t+1)} X_2 + \beta_2^{(t+1)}] = \text{logit}(\hat{p}_1)$.

IM10. Draw imputations of $X_1$ for unit nonrespondents from $X_1|U = 1 \sim \text{Bernoulli}\left(1/(1 + \exp(-\beta_0^{(t+1)} - \beta_1^{(t+1)} X_2 - \beta_2^{(t+1)}))\right)$.

IM11. Let $\hat{\gamma}$ and $V_{\hat{\gamma}}$ denote the maximum likelihood estimates and the covariance matrix of $\begin{bmatrix} \gamma_0 \\ \gamma_1 \end{bmatrix}$ based on $U_i = 0$ observations. Sample $\begin{bmatrix} \gamma_0 \\ \gamma_1 \end{bmatrix}^{(t+1)}$ from its approximate posterior distribution, $N(\hat{\gamma}, V_{\hat{\gamma}})$.

IM12. Draw imputations of $X_1^{(t+1)}$ when $(R_{x1} = 1, U_i = 0)$ from a $\text{Bernoulli}\left(1/(1 + \exp(-\beta_0^{(t+1)} - \beta_1^{(t+1)} X_2))\right)$.

We update $X_2$ first and use it as a predictor of $X_1$ because, in our illustrative setting, $X_2$ only suffers from unit nonresponse whereas $X_1$ also suffers from item nonresponse. When both $X_2$ and $X_1$ have item nonresponse, analysts can generate starting values for both using MICE. In general, we recommend updating variables with smaller item nonresponse rates earlier in the sequence.

This algorithm can be viewed as a generalized version of the one proposed in Pham et al. (2019), which incorporates an offset called the “calibrated-δ adjustment” calculated from auxiliary
The algorithm in [Pham et al. (2019)] handles item nonresponse only, whereas our algorithm, which we call the intercept matching algorithm, handles both item and unit nonresponse.

The intercept matching algorithm can be readily generalized to more than two variables with auxiliary margins, assuming we use the margins to add the indicator $U$ to the models for the survey variables. This can be done via repeating IM1–IM5 or IM6–IM10: generate parameter draws from their full conditional based on the $U_i = 0$ cases alone, use the auxiliary margin to adjust the intercept, and draw imputations for this variable for unit nonrespondents using the adjusted intercept. When the model for $X_k$ is a multinomial logistic regression, as is the case for some variables in the 2018 CPS hybrid missingness MD-AM model, we estimate parameters and set $U$ coefficients in the logit expression for each level of $X_k$.

The algorithm also easily incorporates variables without known margins. We add a conditional model for each $Y_k$ to the collection, without including terms for $U$. For example, for a binary $Y_k$, we follow IM3 (or IM8) to generate parameter draws based on the $U_i = 0$ cases alone. We skip IM4 (or IM9) as there is no auxiliary margin associated with $Y_k$ for the intercept adjustment. As a result, we draw imputations of $Y_k$ for unit nonrespondents from the model used for unit respondents, without changing the intercept. When $Y_k$ also suffers from item nonresponse, we follow IM11–IM12 to draw imputations for cases with $R_{ik}^Y = 1$. In general, to facilitate modeling, we recommend specifying models first for $U$ and $X \mid U$, followed by models for $Y \mid X$, $R_x$, and $R_y$.

### 3.3 Summary of Simulation Results

In the supplementary material, we present results of a series of simulations that investigate the performance of the intercept matching algorithm for fitting the hybrid missingness MD-AM model with unequal probability samples subject to unit and item nonresponse. The simulations vary the departure from ignorability for the unit and item nonresponse, and the strength of association between $X_1$ and $X_2$. To conserve space, here we summarize the main findings from those simulations.

We begin with the simulations that assume the design weights for unit respondents are known. Across all scenarios we investigated, the MD-AM model fit with the intercept matching algorithm has better repeated sampling properties than default ICIN models that do not utilize the auxiliary margins. In particular, when missingness is nonignorable, the MD-AM models generally result in multiple imputation point estimates with much lower bias than the default ICIN models. Confidence interval coverage rates for the MD-AM models generally are close to or exceed 95%, whereas those for the default ICIN model often have zero coverage due to the bias. Interestingly, the multiple imputation variance estimates with the MD-AM model are positively biased. This mirrors a finding in [Reiter (2008)], who noted that the multiple imputation variance estimator of [Rubin (1987)] has positive bias when the imputation models use more information—in this case, the auxiliary margins—than the analysis models.

When the design weights for unit respondents are not known, so that we use (19) to make analysis weights, the performance of the MD-AM model suffers compared to when we can use (3.1.1). This is because we are forced to estimate the design weights, and errors in those estimates carry through to the survey analysis. Nonetheless, the point and interval estimates from the MD-AM model continue to outperform those from the default ICIN model. In fact, for settings where the bias due to nonresponse is not extreme, the MD-AM model produces results with good repeated sampling properties. We refer the reader to the supplementary material for details.

The bottom line for the simulation studies is that the inferences from the hybrid missingness
MD-AM model with the intercept matching algorithm outperform those from default models that do not use the margins, even in the case where only adjusted weights are available. These promising results encourage us to estimate a hybrid missingness MD-AM model with the intercept matching algorithm to analyze voter turnout in the 2018 CPS data, to which we now turn.

4 Estimating Voter Turnout with 2018 CPS Data

In every Congressional and Presidential election year since 1964, the Voting and Registration Supplement has been included biennially in the November basic monthly survey of the Current Population Survey. The resulting data are regarded as one of the premier sources for studying turnout in the U. S. The large sample size and detailed demographic information enable researchers to examine voter turnout by state and within subgroups, for example, among racial and ethnic minority groups (Ansolabehere et al. 2022).

In spite of its status as a gold standard for research on turnout, the CPS suffers from unit and item nonresponse. To handle unit nonresponse, the Census Bureau adjusts the design weights of CPS survey respondents so that the resulting weighted estimates match certain demographic totals. To handle item nonresponse, the Census Bureau uses various imputation methods. Particularly relevant for turnout studies, the Census Bureau imputes all “Don’t Know,” “Refused,” and “No Response” answers to the question on turnout as non-voters. Taken together, the Census Bureau’s treatment of missingness represents very strong modeling assumptions (Hur and Achen, 2013). To be explicit, according to the Census Bureau’s missingness assumptions, unit nonresponse is conditionally independent of voting status, and item nonresponse on vote perfectly predicts voting status.

We instead handle the unit and item nonresponse with a hybrid missingness MD-AM model. We note that Akande et al. (2021) previously analyzed CPS turnout data with an MD-AM model. Their analysis uses data from the 2012 CPS, treats the CPS as a simple random sample (i.e., disregards the survey weights), and uses selection models for all nonresponse indicators rather than the hybrid missingness mechanism.

4.1 Data

We use 2018 CPS data for North Carolina downloaded from the IPUMS website (https://cps.ipums.org/cps/) and the variables described in Table 1. The data are collected via a multi-stage design, including stratification within states, probability proportional to size sampling, and oversampling of certain demographic populations. As a result, the design weights vary across sampled units. This variation can be compounded by the weighting adjustments for unit nonresponse.

The CPS reports unit nonresponse rates at the household level, that is, whether or not the household responds, rather than the individual person level. We therefore need to approximate the person level unit nonresponse rate among those eligible for the November supplement (U. S. citizens at least 18 years old). To do so, we use the strategy developed by Akande et al. (2021), which we summarize here. First, using information from the CPS data file describing why sampled households failed to respond, we exclude “Type C” households; these are deemed ineligible for the survey. Second, we estimate the average number of adult citizens per household in North Carolina. Our numerator derives from the Census Bureau’s special tabulation of the Citizen Voting Age Population (CVAP) using five-year American Community Survey (ACS) data in North Carolina,
Table 1: Description of variables used in the analysis of turnout in the 2018 CPS data, along with notation used for modeling.

| Variable | Notation | Categories |
|----------|----------|------------|
| Sex      | S        | 0 = Male, 1 = Female |
| Race     | E        | 1 = White alone, 2 = Black alone, 3 = Hispanic or Latino, 4 = Remaining people |
| Education| C        | 1 = High school or less (HS-), 2 = Some college, 3 = Bachelor’s and more (BS+) |
| Age      | A        | 1 = 18 - 29, 2 = 30 - 39, 3 = 40 - 49, 4 = 50 - 59, 5 = 60 - 69, 6 = 70 - 79, 7 = 80+ |
| Vote     | V        | 0 = Did not vote; 1 = Voted |

and our denominator is the estimated total number of households in North Carolina also from the five-year ACS data. Third, we multiply the number of eligible nonresponding households in the CPS data by the estimated average number of adult citizens per household, and round the product to the nearest person. We then append records to the observed CPS data comprising no information (other than that they reside in North Carolina and are eligible to vote).

Using this approach, we generate 913 unit nonrespondents to add to the 2013 unit respondents; thus, we have \( n = 2926 \) in \( D \) with 31% unit nonresponse. Two observations in the CPS data have all variables missing except their survey weights. We treat them as unit nonrespondents, without modifying their weights. Item nonresponse rates among respondents are modest among sex (0%), race (2%), education (4%), and age (4%). Item nonresponse among respondents for vote is more substantial at 18%.

We use auxiliary margins for several of the variables in Table 1. For vote, we use the total ballots counted voter-eligible population (VEP) turnout rate for North Carolina in the 2018 election, which is 49% (http://www.electproject.org/2018g). For sex and race, we utilize auxiliary margins for the VEP from the 2018 ACS for North Carolina. Specifically, we have 52% female and 48% male; we have 69.9% with white race alone, 21.8% with black race alone, 3.9% with Hispanic, and 4.4% comprising people not in these categories. The standard errors of these statewide ACS estimates are small enough that we can treat them as known population margins.

4.2 Hybrid missingness MD-AM model

For any individual \( i \in D \), let \( A_i \) denote their age, \( S_i \) denote their sex, \( E_i \) denote their race/ethnicity, \( C_i \) denote their education, and \( V_i \) denote their vote. We use this notation instead of \( X_{ik} \) and \( Y_{ik} \) to make it easier to identify the variables. Note that \( \{G, E, V\} \in A \) and \( \{A, C\} \notin A \). For any individual \( i \in D \), let \( (U_i, R^E_i, R^C_i, R^A_i, R^V_i) \) represent the indicators for unit nonresponse and item nonresponse for race/ethnicity, education, age, and vote, respectively.

Following the MD-AM framework, we first specify an identifiable model for the collection of survey variables and nonresponse indicators without using auxiliary margins. Table 2 summarizes the sequence of conditional models before using the margins. Exploratory analysis of the missing data patterns reveals that whenever an individual’s age is missing, that person’s vote also is missing. Hence, when specifying the default model for \( R^A \), we cannot include a term for \( V \); there is not sufficient information to identify a coefficient of vote. We set \( R^V_i = 1 \) whenever \( R^A_i = 1 \). In the model for \( V \), we include interactions between \( S \) and \( E \), \( S \) and \( C \), and \( S \) and \( A \) as we are especially interested in estimating voter turnout in these subgroups. Overall, the collection of models specifies fewer parameters than can be identified using the CPS data alone; however, the simplified models...
Table 2: Hybrid missingness MD-AM model in the analysis of turnout in the 2018 CPS data. Abbreviations for models include “B” for Bernoulli distribution, “MR” for multinomial logistic regression, “LR” for logistic regression. For the predictors, “I” stands for an intercept. Categorical predictors with \( d \) levels are modeled with a series of \( d - 1 \) indicator variables. Interaction effects between any two variables \( A \) and \( B \) represented by \( A : B \).

| Variable | Model | Predictors |
|----------|-------|------------|
| \( U \)  | B     | I          |
| \( S \)  | B     | I          |
| \( E \)  | MR    | I + S      |
| \( C \)  | MR    | I + S + E  |
| \( A \)  | MR    | I + S + E + C |
| \( V \)  | LR    | I + S + E + C + A + S : E + S : C + S : A |
| \( R^E \)| LR    | I + S + C + A + V |
| \( R^C \)| LR    | I + S + E + A + V |
| \( R^A \)| LR    | I + S + E + C |
| \( R^V \)| LR    | \( (R^A = 0) : I + S + E + C + A \) |
|          |       | \( (R^A = 1) : R^V = 1 \) |

explain the data reasonably well while avoiding the need to estimate extra parameters, especially in the multinomial models.

Following step 2 of the MD-AM framework with hybrid missingness, we add main effects for \( U \) to the models for \( S \), \( E \), and \( V \). We do not add \( U \) to the models for \( A \) and \( C \) since we do not utilize auxiliary margins on age and education.

As the CPS file from IPUMS includes only adjusted weights, we apply the algorithm in (19) to create the analysis weights. We use non-informative priors for all model parameters and normal approximations for the coefficients of all regressions. We employ the intercept matching algorithm for 10000 iterations and discard the first 5000 iterations as burn-in. Using a non-optimized code in \( R \) and a standard laptop, it takes about 14 hours to generate the 10000 draws. Evaluations of trace plots of model parameters suggest that the sampler converges. Among the 5000 posterior samples, we retrieve every 100th posterior sample to create \( L = 50 \) multiple imputation data sets, and do survey-weighted multiple imputation inference using those completed data sets.

4.3 Results

We begin by examining the completed-data marginal distributions of the variables other than vote, i.e., \( S \), \( E \), \( C \) and \( A \). Table 3 displays the multiple imputation point estimates, \( \bar{q}_L \), and corresponding multiple imputation standard errors using the 50 completed data sets generated under the hybrid missingness MD-AM model. Survey-weighted estimates are computed with the “survey” package (Lumley, 2021) in \( R \), assuming each completed data set is a probability proportional to size with replacement sample. This is a common estimation strategy for analyzing complex surveys (Lohr, 2010). For comparisons, we also generate 50 completed data sets using a default implementation of MICE with only the unit respondents. Estimates for the MICE-completed data sets are based on the CPS adjusted weights on the IPUMS file. Finally, we also display weighted estimates based on complete cases.

Overall, estimates are similar across all the survey-weighted analyses. This is not surprising, as
Table 3: Estimated marginal distributions of sex, age, and race based on 50 imputations generated from the hybrid missingness MD-AM model and default MICE. The entries under the column labeled Q are the available population percentages used as auxiliary margins. The entries in the columns labeled CC are survey-weighted estimates using the complete cases.

|        | MD-AM | MICE | CC |
|--------|-------|------|----|
|        | \(q\) | \(\bar{q}_L\) | SE | \(\bar{q}_L\) | SE | Est | SE |
| Male   | .48   | .478 | .017 | .470 | .011 | .466 | .013 |
| Female | .52   | .522 | .017 | .530 | .011 | .534 | .013 |
| White  | .699  | .699 | .013 | .696 | .011 | .709 | .012 |
| Black  | .218  | .219 | .013 | .220 | .010 | .213 | .011 |
| Hispanic | .039 | .038 | .005 | .035 | .004 | .033 | .005 |
| Rest   | .044  | .044 | .005 | .049 | .005 | .046 | .005 |
| (0, 29] | .213  | .216 | .010 | .205 | .011 |
| (29, 39] | .160 | .141 | .008 | .144 | .009 |
| (39, 49] | .189 | .161 | .009 | .150 | .009 |
| (49, 59] | .162 | .173 | .009 | .177 | .009 |
| (59, 69] | .147 | .161 | .008 | .168 | .009 |
| (69, 79] | .092 | .103 | .007 | .111 | .008 |
| > 79   | .036  | .043 | .004 | .045 | .005 |
| HS-    | .372  | .375 | .011 | .366 | .012 |
| Some College | .297 | .300 | .011 | .306 | .012 |
| BA+    | .331  | .325 | .011 | .328 | .012 |

the CPS adjusts design weights so that the survey-weighted estimates for some of these demographic variables approximately match their known demographic margins. We point out that, for \(S\) and \(E\) where we have auxiliary margins, the multiple imputation standard deviations for the MD-AM model are biased high. The true standard deviation for each of these two \(\bar{q}_L\) is close to zero, as the model essentially forces each of these two \(\bar{q}_L\) to match their corresponding known margins.

The benefits of handling item and unit nonresponse with the hybrid missingness MD-AM model become apparent when we examine voter turnout. Table 4 displays estimates of turnout by each category of the demographic variables for the three analysis strategies. Additionally, Table 4 includes the turnout estimates using the Census Bureau procedures for handling missing values. That is, we use the CPS data for unit respondents and their survey weights on the file; we use the Census Bureau’s imputations for \(S\), \(E\), \(A\), and \(C\); and, we set all missing responses for \(V\) as not voting.

Statewide, the MD-AM model estimates around 50.1% of CPS participants are voters, which is close to the population turnout of 49%. In contrast, estimates from MICE and complete cases overestimate the turnout rate overall and seemingly for most subgroups—a bias likely more than 10 percentage points in many instances. For subgroups, estimates of turnout under the MD-AM model tend to be more plausible than those under MICE and the complete case analyses. For example, college graduates are estimated to have a turnout rate of more than 80% using complete case or MICE analysis, whereas the MD-AM estimates a turnout rate around 66%.

Table 5 displays turnout for bivariate combinations of the demographic variables. Here again, the MD-AM model estimates are smaller and more plausible than those under MICE and complete
Table 4: Estimated proportion who voted in subgroups based on 50 imputations generated from the hybrid missingness MD-AM model and default MICE. Survey-weighted estimates for complete case analyses (CC) and estimates released by the Census Bureau (Census) also included. The auxiliary margin for voted in North Carolina is .49.

|                  | MD-AM  | MICE  | CC   | Census |
|------------------|--------|-------|------|--------|
|                  | $q_L$  | SD    | $q_L$| Est    | SD    | Est   | SD   |
| Full             | .501   | .016  | .630 | .012   | .524  | .012  |
| Male             | .499   | .021  | .630 | .018   | .521  | .017  |
| Female           | .504   | .020  | .631 | .017   | .526  | .016  |
| White            | .510   | .018  | .645 | .014   | .544  | .013  |
| Black            | .519   | .032  | .647 | .031   | .516  | .027  |
| Hispanic         | .390   | .061  | .494 | .070   | .386  | .061  |
| Rest             | .378   | .057  | .449 | .058   | .371  | .049  |
| (0, 29]          | .325   | .027  | .419 | .030   | .331  | .026  |
| (29, 39]         | .398   | .038  | .558 | .033   | .466  | .031  |
| (39, 49]         | .492   | .041  | .679 | .029   | .517  | .028  |
| (49, 59]         | .595   | .038  | .698 | .027   | .596  | .026  |
| (59, 69]         | .700   | .035  | .796 | .024   | .664  | .025  |
| (69, 79]         | .672   | .039  | .758 | .030   | .691  | .031  |
| > 79             | .467   | .060  | .545 | .055   | .485  | .051  |
| HS-              | .347   | .020  | .456 | .021   | .369  | .018  |
| Some College     | .519   | .023  | .653 | .022   | .552  | .021  |
| BA+              | .659   | .029  | .810 | .017   | .676  | .018  |

case analysis. For some of the most substantively interesting population subgroups, the MD-AM estimates very different turnout rates than the MICE or complete case analyses. For example, political pundits have focused considerable attention on voting behavior of non-college educated white men and women. Whereas the complete case analysis estimated they had a turnout rate of 45% in 2018, the MD-AM estimates their turnout rate at just 34%. As another example, the MD-AM model estimates that Black men had a much lower turnout rate (47.5%, which is in line with the statewide turnout rate) compared to the complete case estimates (61.8%, over 10 points higher than the statewide rate).

The results from the MD-AM model and Census Bureau imputations are somewhat similar. By imputing all item nonrespondents on vote to be non-voters, the Census Bureau essentially is forcing the overall turnout margin to be closer to 49% than the complete case results. Still, the overall turnout estimates under the MD-AM model are closer to 49%. Additionally, it seems implausible that all item nonrespondents are non-voters. In contrast, the MD-AM model imputes on average 60% of the item nonrespondents as voters—close to the complete cases turnout estimate of 63.7%, as expected from the ICIN assumption for $R^y$—and, to make the completed-data design-based estimates of turnout closely match 49%, on average 22% of the unit nonrespondents as voters.

While it is impossible to assess how accurately the MD-AM model describes the nonresponse mechanism in the CPS, we can assess how well the MD-AM model reproduces the observed data. To do so, we compare unweighted, observed percentages from $\mathcal{D}$ with estimates from replicated
Table 5: Estimated proportion who voted in detailed subgroups based on 50 imputations generated from the hybrid missingness MD-AM model and default MICE. Survey-weighted estimates for complete case analyses (CC) and estimates released by the Census Bureau (Census) also included. The auxiliary margin for voted in North Carolina is .49.

|                  | MD-AM | MICE | CC  | Census |
|------------------|-------|------|-----|--------|
|                  | $\bar{q}_L$ | SD  | $\bar{q}_L$ | Est | SD  | Est | SD |
| Male, White      | .513  | .023 | .648  | .020 | .650 | .020 | .550 | .040 |
| Male, Black      | .475  | .047 | .612  | .046 | .618 | .045 | .463 | .018 |
| Male, Hispanic   | .459  | .100 | .562  | .103 | .597 | .106 | .443 | .086 |
| Male, Rest       | .427  | .081 | .509  | .086 | .552 | .090 | .416 | .074 |
| Female, White    | .508  | .023 | .642  | .019 | .647 | .019 | .539 | .018 |
| Female, Black    | .557  | .038 | .676  | .039 | .687 | .040 | .559 | .037 |
| Female, Hispanic | .323  | .072 | .432  | .094 | .401 | .100 | .329 | .085 |
| Female, Rest     | .334  | .072 | .396  | .076 | .397 | .078 | .332 | .066 |
| Male, (0, 29]    | .313  | .038 | .405  | .042 | .408 | .043 | .317 | .036 |
| Male, (29, 39]   | .396  | .049 | .553  | .048 | .573 | .049 | .460 | .045 |
| Male, (39, 49]   | .451  | .053 | .640  | .045 | .639 | .046 | .482 | .042 |
| Male, (49, 59]   | .577  | .048 | .694  | .037 | .689 | .038 | .596 | .037 |
| Male, (59, 69]   | .714  | .049 | .813  | .034 | .810 | .035 | .656 | .037 |
| Male, (69, 79]   | .764  | .050 | .828  | .040 | .840 | .039 | .781 | .042 |
| Male, > 79       | .460  | .094 | .561  | .090 | .565 | .091 | .469 | .083 |
| Female, (0, 29]  | .337  | .038 | .433  | .041 | .432 | .043 | .344 | .037 |
| Female, (29, 39] | .401  | .050 | .562  | .045 | .569 | .046 | .472 | .042 |
| Female, (39, 49] | .530  | .048 | .710  | .037 | .714 | .038 | .545 | .037 |
| Female, (49, 59] | .594  | .046 | .702  | .039 | .711 | .037 | .596 | .037 |
| Female, (59, 69] | .687  | .043 | .780  | .034 | .781 | .034 | .671 | .035 |
| Female, (69, 79] | .596  | .054 | .700  | .044 | .694 | .044 | .619 | .044 |
| Female, > 79     | .473  | .072 | .535  | .068 | .551 | .068 | .494 | .065 |

Male, HS- .347  .026  .459  .029  .468  .030  .369  .025
Male, Some College .520  .035  .660  .032  .656  .033  .552  .031
Male, BA+ .677  .039  .835  .025  .837  .024  .699  .027
Female, HS- .348  .026  .452  .030  .460  .029  .370  .026
Female, Some College .519  .031  .648  .030  .650  .030  .552  .029
Female, BA+ .645  .034  .791  .024  .794  .024  .658  .025
White, HS- .341  .023  .445  .022  .452  .025  .374  .022
White, Some College .512  .026  .647  .024  .654  .026  .556  .024
White, BA+ .663  .030  .812  .017  .826  .018  .694  .019
White, Male, HS- .341  .031  .452  .035  .453  .035  .378  .031

Data generated from posterior predictive distributions (He and Zaslavsky 2012). These comprise the values of \( \{(A_i, S_i, E_i, C_i, V_i) : U_i = R^A_i = R^E_i = R^C_i = R^V_i = 0\} \). Using each of the 5000 draws of the parameters, we generate new values of all variables for all \( n \) units in \( D \), and compute the replicated data percentages for those generated to be respondents to all questions. We construct
Figure 2: Posterior predictive intervals for all marginal and conditional probabilities involving vote for completed data only. Crosses are observed data estimates. The first panel shows marginal probabilities of vote, sex, race/ethnicity, age and education. The second panel shows voter turnout probabilities in various sex, race/ethnicity, age and education subgroups. The third panel displays voter turnout probabilities in sex crossed with race/ethnicity, gender crossed with age and sex crossed with education subgroups.

95% intervals for marginal probabilities for the replicated data for all variables and voter turnout probabilities for subgroups. Figure 2 displays the posterior predictive intervals. All intervals contain the observed data estimates, giving us confidence in the reasonableness of the hybrid missingness MD-AM model for the observable portion of the data.

The simulation studies in the supplementary material suggest that, since design weights for unit respondents are not available, the performance of the MD-AM model in estimating voter turnout should depend on the strength of the unit nonresponse bias. The similarity of estimates in Table 3 for the MD-AM model, MICE and complete case analysis suggest practically irrelevant biases for these three variables due to unit nonresponse (after the weighting adjustments). On the contrary, the voter turnout estimated under the MD-AM model is lower than the turnout under MICE and complete case analysis, suggesting the unit nonresponse bias on vote is important. The simulation results under such scenarios in the supplementary material (namely, scenarios “b” or “d” in Table 8 in the supplement) suggest that results from the MD-AM model are more accurate than those from models that do not utilize the information in the margins.

4.4 Incorporating measurement error

Beyond the effects of item and unit nonresponse, it is well-recognized that self-reported voter turnout suffers from measurement error. More specifically, people often report that they voted when they did not in fact do so. In contrast, people who did vote rarely report that they did not (Enamorado and Imai, 2019; Jackman and Spahn, 2019; DeBell et al., 2020). Thus, some of the apparent bias in the CPS complete cases results could be the result of over-reporting of turnout.

To assess the sensitivity of results to reporting error, we layer a measurement error model on top of the hybrid missingness MD-AM model. Previous research suggests a dose-response relationship between over-reporting of vote and education: increasing education is associated with increasing rates of over-reporting (Silver et al., 1996; Karp and Brockington, 2005). Unfortunately, we do not
Figure 3: Posterior distribution of voter turnout among item nonrespondents, respondents, and unit nonrespondents for the MD-AM model. For each response type, the box to the right represents the turnout rate in the MD-AM model assuming no measurement error among respondents’ vote answers. The box to the left represents the turnout rate in the MD-AM model assuming measurement error.

We have a validated sample from the CPS to assess the over-reporting rate by educational attainment. Thus, we represent the measurement error in vote using informative prior distributions. For each individual $i \in D$ among those who have a response to the question about vote, let $Z_i = 1$ when the person is reported as a voter, and let $Z_i = 0$ when the person is reported as a nonvoter. We seek to tie each $Z_i$ to the underlying true voting status, $V_i$. Note that we define $Z_i$ only for units with $U_i = R_i^V = 0$. Let the misreporting probabilities for nonvoters given their education level be as follows.

\[
\begin{align*}
\mathbb{P}(Z_i = 1|V_i = 0, C_i = 1, U_i = R_i^V = 0) &= \theta_1 \quad (22) \\
\mathbb{P}(Z_i = 1|V_i = 0, C_i = 2, U_i = R_i^V = 0) &= \theta_2 \quad (23) \\
\mathbb{P}(Z_i = 1|V_i = 0, C_i = 3, U_i = R_i^V = 0) &= \theta_3, \quad (24)
\end{align*}
\]

where $C_i \in \{1, 2, 3\}$ per Table 1. Among nonvoters who have at most a high school education, we assume around 6% would misreport that they actually voted ($\theta_1$). Among nonvoters with some college education, we assume around 13% would misreport that they actually voted ($\theta_2$). Among nonvoters who have at least a bachelor’s degree, we assume around 19% would misreport that they voted ($\theta_3$). We treat these best guesses as measures of central tendency in Beta prior distributions. For each $\theta_c$, we assume $\theta_c \sim Beta(a_c, b_c)$, where $(a_1, b_1) = (60, 940)$, $(a_2, b_2) = (130, 870)$, and $(a_3, b_3) = (190, 810)$. These correspond to prior means of misreporting rates of $(0.06, 0.13, 0.19)$,
respectively, and prior standard deviations around 1%. In accordance with past research from other studies, we assume that anyone who truly voted is reported that they did so. We select these prior distributions based on the research findings from previous voter validation studies that have matched survey respondents to voter files (e.g., [Jackman and Spahn 2019]), accounting for differences in observed bias across election and survey type.

Adding the measurement error models and prior distributions to the MD-AM model in Section 4.2 requires two additional steps in the intercept matching algorithm: update $\theta_c$, and sample true voting status $V_i$ for individuals with $(U_i = 0, R_i^V = 0, Z_i = 1)$. The full conditionals for these updates are described in the supplementary material.

Figure 3 displays estimated voter turnout rates among respondents, item nonrespondents on vote, and unit nonrespondents before and after taking the measurement error into consideration. With the additional measurement error assumptions, the MD-AM model changes some of the reported voters to nonvoters, lowering the voting proportion among observed cases. Under the ICIN model for $R_i^V$, we therefore impute fewer voters among respondents with $R_i^V = 1$. Consequently, we need to impute more voters among unit nonrespondents to match the auxiliary margin for vote.

Tables containing results from the hybrid missingness MD-AM model with measurement error like those in Table 3, Table 4, and Table 5 are displayed in the supplementary material. Overall, despite the measurement error, the estimates are quite similar with a few estimates increasing or decreasing by a point or two, and all changes are well within one estimated standard error.

5 Concluding Remarks

The hybrid missingness MD-AM model allows analysts to take advantage of known auxiliary margins when imputing values for unit and item nonresponse in surveys with complex sampling designs. In the 2018 CPS analysis, we enriched the models to account for unit nonresponse, as it was more prevalent than item nonresponse. In other situations, analysts may prefer to enrich one or more of the models for the item nonresponse indicators. We explored adding an indicator for $V$ to the model for $R_i^V$ and removing $U$ from the model for $V$ using a rejection sampler akin to that used in Akande and Reiter (2022). However, with diverse values of weights, our sampler had a difficult time exploring the parameter space adequately. Further research is needed on computational algorithms for the version of the hybrid missingness MD-AM model that enriches item nonresponse models.

The simulation studies demonstrate that the hybrid missingness MD-AM results are more accurate when we use the unit respondents’ design weights as opposed to having to down-weight adjusted weights. This finding suggests that survey providers would help analysts by including design weights in survey data files, even when they also include adjusted weights on those files.

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