Collision of atoms with formation of Feshbach resonance in the case of the coupled decay channels with the presence of laser radiation.

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Abstract. Resonance scattering with formation of Feshbach resonance in the presence of laser radiation in the case of the decays in open and closed channels is considered. The presence of the resonant laser radiation leads to additional channels of the elastic and inelastic scattering. The cross sections of the elastic and inelastic scattering are obtained which show that they are determined by two interconnected phases of resonance scattering and interference phenomena are observed.

1. Introduction

The Feshbach resonance is displayed when the energy of a bound molecular state in the closed channel is close to the energy of decaying state in the open channel [1-3]. In this case a weak coupling may lead to a strong mixing between the channels. The laser radiation couples the molecular state with the state of the incident atom. In this case a quasibound state of colliding atoms is formed in the open channel. The intermediate bound state has a finite lifetime and can decay in the initial channel or in other channels at interaction with continuum. The energy difference may, in particular, be controlled by means of a magnetic field. An alternative way to control the Feshbach resonance is optical method.

Figure 1. Diagram of formation of Feshbach resonance in the field of laser radiation.
From this point of view the investigation of the Feshbach resonance with formation of a quasibound state (molecule) in the field of laser radiation is of certain interest.

In the present paper we consider collision of atoms with formation of the Feshbach resonance in the field of laser radiation which couples molecular levels. The resonance between the laser field and discrete molecular levels is described by quasi-energy wave functions. On the other hand the one-photon transition from the upper level to other (inelastic) channel (Fig.1) where the molecule decays, under action of laser radiation with the interaction $\Omega_{k}$, into two excited atoms which fly away to infinity.

2. The cross sections for the collision of atoms in the laser field.

Laser radiation at the frequency $\omega_{1}$ couples by the one-photon resonance with the Rabi frequencies $\Omega$ the stable state $|1\rangle$ with a quasibound state $|2\rangle$ (Fig.1). Laser radiation at the frequency couples the upper quasistationary state with the second continuum (inelastic channel) with the Rabi frequency $\Omega_{E}$. Interaction $U_{E}$ couples the upper state with the first continuum (initial channel).

$$H = H^{(1)} + H^{(2)}$$

$$H_{1} = E_{i} |1\rangle\langle1| + E_{2} |2\rangle\langle2| + \Omega(t) |2\rangle\langle1| + \Omega^{\dagger}(t) |1\rangle\langle2|$$

$$H_{2} = \int E[|E\rangle_{11}<E| + |E\rangle_{22}<E|]dE + \int (\Omega_{E}(t) |E\rangle_{1}<1| + \Omega_{E}^{\dagger}(t) |1\rangle\langle E| +$$

$$+ \Omega_{E}^{\dagger}(t) |E\rangle_{2}<2| + \Omega_{E}^{\dagger}(t) |2\rangle\langle E| + U_{E} |E\rangle_{1}<2| + U_{E}^{\dagger} |2\rangle\langle E|]dE$$

We choose as basis wave functions of the discrete spectrum the quasienergy wave functions $\varphi_{k}(t)$ and $\varphi_{j}(t)$ of the Hamiltonian obtained in the one-photon resonance approximation with adiabatic turning-on the periodic perturbations $\Omega(t)$ and $\Omega^{\dagger}(t)$:

Here

$$\tilde{E}_{k} = E_{k} + \mu_{k} + \omega,$$

$$|\varphi_{k}(t)\rangle = e^{-i(\tilde{E}_{k} - \omega)t}\left(\alpha_{k}|1\rangle + \beta_{k}e^{-i\omega t}|2\rangle\right)k = 1,2$$

$$\mu_{12} = \frac{1}{2}\sqrt{\nu^{2} + 4|\Omega|^{2}},$$

$$\alpha_{12} = \left(\frac{\mu_{21}}{\mu_{21} - \mu_{12}}\right)^{1/2}, \quad \beta_{12} = \pm \frac{\Omega}{|\Omega|}\left(\frac{\mu_{21}}{\mu_{21} - \mu_{12}}\right)^{1/2}$$

The solution of the Schrödinger equation has the form

$$\Phi_{\lambda}^{(j)}(t) = \sum_{k}e^{i\tilde{E}_{k}t}a_{k}^{\dagger}(\lambda)\varphi_{k}(t) + \int dE\left[\frac{P}{\lambda - E} + z^{(j)}(\lambda)\delta(\lambda - E)\right]$$

$$\times \left\{W_{ik}(E) |E\rangle_{1} + e^{-i\omega t}W_{2k}(E) |E + \omega\rangle_{2}\right\}a_{k}^{\dagger}(\lambda):$$

$$W_{ab}(E) = \left\{\begin{array}{ll}
\alpha_{a}E_{k} + \beta_{a}U_{E}, & a = 1, \\beta_{a}E_{k}^{\dagger}, & a = 2 \end{array}\right. \quad k = 1,2$$

$$\left\langle \Phi_{\lambda}^{(j)}(t) |\Phi_{\lambda'}^{(j)}(t)\right\rangle = \delta(\lambda - \lambda')\delta_{jj}$$
Here \( \frac{2\pi}{z_j^{(j)}(\lambda)} \) \((j = 1, 2)\) are the eigenvalues of the Hermitian reaction matrix:

\[
K(\lambda) = -2\pi W(\lambda)[\lambda I - \Lambda - \Delta(\lambda)]^{-1}W^*(\lambda),
\]

where

\[
\Lambda_{_{ij,k}} = \overline{\lambda}_i \delta_{_{i,j}},
\]

with eigenvectors \( X^{(j)}_{\alpha}(\lambda) \),

\[
Z^{(j)}_{\alpha}(\lambda) = \frac{1}{2} \left[ \frac{1}{\sum_{\mu} \frac{|\tilde{W}_{\mu}(\lambda)|^2}{\lambda - \overline{\lambda}_\mu}} - \frac{1}{\sum_{\mu} \frac{|\tilde{W}_{2\mu}(\lambda)|^2}{\lambda - \overline{\lambda}_\mu}} \right]^{-1/2}
\]

\[
\left\{ \begin{array}{c}
\sum_{\mu} \frac{1}{\lambda - \overline{\lambda}_\mu} \left[ \frac{|\tilde{W}_{\mu}(\lambda)|^2 + |\tilde{W}_{2\mu}(\lambda)|^2}{\lambda - \overline{\lambda}_\mu} \right] \\
\sum_{\mu} \frac{1}{\lambda - \overline{\lambda}_\mu} \left[ \frac{|\tilde{W}_{\mu}(\lambda)|^2 - |\tilde{W}_{2\mu}(\lambda)|^2}{\lambda - \overline{\lambda}_\mu} + 4 \sum_{\mu} \frac{\tilde{W}_{\mu}^*(\lambda)\tilde{W}_{2\mu}(\lambda)}{\lambda - \overline{\lambda}_\mu} \right]^{-1/2}
\end{array} \right.
\]

and

\[
\sum_{\alpha} X^{(j)*}_{\alpha}(\lambda)X^{(j)}_{\alpha}(\lambda) = \frac{\delta_{_{jj\mu}}}{\pi^2 + z_j^{(j)}(\lambda)^2}, \quad \alpha, j = 1, 2.
\]

\[
|X^{(j)}_{\mu}(\lambda)|^2 = \frac{1}{\pi^2 + z_j^{(j)}(\lambda)^2} \left\{ 1 - z_j^{(j)}(\lambda) \sum_{\mu} \frac{|\tilde{W}_{2\mu}(\lambda)|^2}{\lambda - \overline{\lambda}_\mu} \right\}^2
\]

\[
\left[ 1 - z_j^{(j)}(\lambda) \sum_{\mu} \frac{|\tilde{W}_{2\mu}(\lambda)|^2}{\lambda - \overline{\lambda}_\mu} \right] + z_j^{(j)}(\lambda)^2 \left[ \sum_{\mu} \frac{\tilde{W}_{\mu}^*(\lambda)\tilde{W}_{2\mu}(\lambda)}{\lambda - \overline{\lambda}_\mu} \right]^2
\]

\[
|X^{(j)}_{\mu}(\lambda)|^2 = \frac{1}{\pi^2 + z_j^{(j)}(\lambda)^2} \left[ \sum_{\mu} \frac{\tilde{W}_{\mu}^*(\lambda)\tilde{W}_{2\mu}(\lambda)}{\lambda - \overline{\lambda}_\mu} \right]^2
\]

The sign “\(\sim\)" above the letter denotes a quantity obtained after unitary transformation of diagonalization of the interaction matrix for discrete states.

The cross sections for elastic and inelastic scattering are:

\[
\sigma_{el} = \frac{\pi}{k^2} \sum_{l} (2l + 1)|1 - S_l(\lambda)|^2,
\]

\[
\sigma_{inel} = \frac{\pi}{k^2} \sum_{l} (2l + 1) \left( 1 - |S_l(\lambda)|^2 \right),
\]
where

\[ S_{j}^{(1)}(\lambda) = \frac{X_{1l}^{(1)}(\lambda)X_{2l}^{(2)}(\lambda)S_{j}^{(1)}(\lambda) - X_{2l}^{(1)}(\lambda)X_{1l}^{(2)}(\lambda)S_{j}^{(2)}(\lambda)}{X_{1l}^{(1)}(\lambda)X_{2l}^{(2)}(\lambda) - X_{2l}^{(1)}(\lambda)X_{1l}^{(2)}(\lambda)} \times \frac{1}{S_{j}^{(2)}(\lambda)} \]  

(13)

and

\[ S_{j}^{(j)}(\lambda) = e^{2i(\delta_{j}^{+} + \eta_{j}^{(j)}(\lambda))} \quad (j=1,2) \]  

(14)

Here \( \delta_{j}^{+} \) is the phase of the potential (nonresonant) scattering and \( \eta_{j}^{(j)}(\lambda) \) is the phase caused by the resonant scattering with formation of the Feshbach resonance.

\[ \tan \eta_{j}^{(j)}(\lambda) = -\frac{\pi}{z_{j}^{(j)}(\lambda)} \quad (j=1,2) \]  

(15)

The cross sections for elastic and inelastic scattering are

\[ \sigma_{el} = \frac{4\pi}{k^{2}} \sum_{l}(2l + 1) \left| \sin \delta_{l} - \left[ A_{1}(\lambda)e^{i\eta_{j}^{(j)}(\lambda)} \sin \eta_{j}^{(+)}(\lambda) + A_{2}(\lambda)e^{i\eta_{j}^{(-)}(\lambda)} \sin \eta_{j}^{(-)}(\lambda) \right] e^{2i\lambda l} \right|^{2}, \]  

(16)

\[ \sigma_{inel} = \frac{4\pi}{k^{2}} \sum_{l}(2l + 1) \left| A_{3}(\lambda) \right|^{2} \sin^{2} \left( \eta_{j}^{(-)}(\lambda) - \eta_{j}^{(+)}(\lambda) \right) \]  

(17)

Where

\[ A_{1}(\lambda) = \frac{X_{1l}^{(1)}(\lambda)X_{2l}^{(2)}(\lambda)}{X_{1l}^{(1)}(\lambda)X_{2l}^{(2)}(\lambda) - X_{2l}^{(1)}(\lambda)X_{1l}^{(2)}(\lambda)} \]  

(18)

\[ A_{2}(\lambda) = -\frac{X_{2l}^{(1)}(\lambda)X_{1l}^{(2)}(\lambda)}{X_{1l}^{(1)}(\lambda)X_{2l}^{(2)}(\lambda) - X_{2l}^{(1)}(\lambda)X_{1l}^{(2)}(\lambda)} \]

\[ A_{3}(\lambda) = \frac{X_{l}^{(1)}(\lambda)X_{l}^{(2)}(\lambda)}{X_{1l}^{(1)}(\lambda)X_{2l}^{(2)}(\lambda) - X_{2l}^{(1)}(\lambda)X_{1l}^{(2)}(\lambda)} \]

Expressions (16) and (17) show that the cross-sections are determined by two coupled phases of resonance scattering, and interference phenomena are observed. From the expression (16) for the elastic resonance scattering the interference phenomena may be observed. The cross-sections for the elastic scattering can be represented in the form.

\[ \sigma_{el} = \sigma_{pot} + \sigma_{el}^{nonres} + \sigma_{el}^{nonres} \mathfrak{R}_{L}(\lambda), \]  

(19)

where the interference term has the form

\[ \mathfrak{R}_{L}(\lambda) = \left| A_{1}^{(L)}(\lambda) \right|^{2} F_{1}(\lambda) + \left| A_{2}^{(L)}(\lambda) \right|^{2} F_{2}(\lambda) + 2\left| A_{3}^{(L)}(\lambda) \right|^{2} F_{3}(\lambda) \]  

(20)

\[ \sigma_{pot} = \frac{4\pi}{k^{2}} \sum_{l \neq L}(2l + 1) \sin^{2} \delta_{l}, \quad \sigma_{nonres} = \frac{4\pi}{k^{2}} (2L + 1) \sin^{2} \delta_{L}, \]  

(21)

\( \sigma_{nonres} \) is the scattering cross-section for angular momentum \( L \) in the absence of resonant state.

\[ F_{j}(\lambda) = \frac{\chi_{L}^{2} + 2\xi_{L}^{(l)}(\lambda) - 1}{1 + \xi_{L}^{(l)}(\lambda)^{2}} \quad (j=1,2) \]  

(22)
\[ F_i(\lambda) = \frac{\chi_L^2 (1 + \xi_2^{(L)}(\lambda) \xi_{2,1}^{(L)}(\lambda)) + \chi_L \xi_1^{(L)}(\lambda) \xi_{2,1}^{(L)}(\lambda)(\xi_1^{(L)}(\lambda) + \xi_{2,1}^{(L)}(\lambda))}{(1 + \xi_1^{(L)}(\lambda))(1 + \xi_{2,1}^{(L)}(\lambda))} + \xi_2^{(L)}(\lambda) + \xi_{1,2}^{(L)}(\lambda) - (\xi_1^{(L)}(\lambda) + \xi_{2,1}^{(L)}(\lambda)) - 1}{(1 + \xi_1^{(L)}(\lambda))(1 + \xi_{2,1}^{(L)}(\lambda))} \]

with \( \chi_L = -\cot \delta_L \)

Figure 2. The functions \( F_i(x) \) (i=1,2,3) with \( x = \frac{\lambda - L}{\Gamma_1^{(2)}} \) for \( \Gamma_1^{(2)} = 1, \Gamma_2^{(2)} = 2, \Gamma_1^{(1)} = 1.5, \Gamma_2^{(1)} = 0.5, \Omega = 1, \nu = 0 \) (all quantities are in units \( \Gamma_1^{(2)} \)) at the value \( \chi_L = 3 \).
The functions $F_i(x) \ (i=1,2,3)$ (entering the interference term (20)) with 

$$x = \frac{\lambda - \tilde{\lambda}}{\Gamma_1^{(2)}}$$

are plotted in Fig.2.

3. Conclusion

We obtained cross-sections of resonance scattering with formation of Feshbach resonance. As distinct from the case of two-photon resonance in absence of the decay channel via interaction $\Omega_c$ [4] and from the case of elastic scattering at one-photon resonance [5], the cross-sections are determined by two coupled phases of resonance scattering which pass in respective limits to the corresponding expressions reported in [4,5].

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