A Rapid Plotting Algorithm of Geodetic Line within Arch Height Error Threshold Limits under Mercator Projection

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Abstract. In view of the problem that the interpolated distance parameters and the plotting precision of the geodetic line cannot be quantitatively regulated and adaptively matched, this paper extends the application of the solution method for constant line segment by considering the angle of the constant line and the geodetic line to the quantitative evaluation and control process of the error of the geodetic line plotting. Using the strategy of limiting the overall length error of the geodetic line plotting by the local arch height error threshold limits, a quantitative control model of interpolated distance parameters within the geodetic line plotting arch height error threshold limits is also established. A rapid plotting algorithm of the geodetic line with arch height error threshold under Mercator projection is proposed in this paper. The experimental results show that the algorithm can realize the rapid plotting of geodetic lines in any length and any latitude and longitude range, and the precision is controlled within the specified threshold range.

1. Introduction
Geodetic line is the shortest curve between two points on the earth spheroid. Its characteristic of the shortest distance on the earth spheroid determines its wide application prospect in the long-distance economical navigation of aircraft and ships. Meanwhile, the uniqueness of the geodetic line between two points is determined by the fact that geodetic lines, the shortest distance between points, do not intersect each other on the ellipsoid. It is crucial to accurately depict all kinds of “straight lines” such as linear territorial sea baselines in the delimitation of oceans. It should be pointed out that because of the characteristics of constant line plotting as a straight line, the Mercator projection is widely used in the field of navigation, and because the behavior of maritime delimitation is usually carried out on the chart plane, the Mercator projection, which is an important mathematical basis of the chart, is also widely used in the field of maritime delimitation. Therefore, it is of great theoretical and practical significance to analyze and demonstrate the relationship between efficiency index and precision threshold in the course of map projection plane plotting, and to establish a fast plotting model of geodetic line with limited precision threshold, thereby improving the economical and efficient navigation of aircrafts and ships and guaranteeing fairness in maritime delimitation.
The essence of geodetic line plotting lies in the accurate expression of geodetic line function equation on projection plane. However, as geodetic line has the dual characteristics of space curvature and disturbance rate, the values of which are different everywhere on the earth spherop, it is difficult to precisely simulate the geodetic line function equation by using grid data model \cite{1,2,3}. In this scenario, scholars try to apply differential geometry to the study of precise geodetic plotting. Referred to the "straight replace curve" increasing interpolated method of the plane curve painting, an equal-lying-spacing geodetic line plotting algorithm without restrictions was proposed in \cite{3}, and achieved the precise plotting of the line elements, surface elements on the earth spherop under the given interpolated distance parameters. However, the setting principle of interpolating distance parameters in the algorithm and the degree of influence on the geodesic plotting error are not clear. Combined with the precision requirement of maritime demarcation, the idea of limiting the geodetic inset distance parameter to the minimum resolution of the two points on the paper chart was put forward in \cite{4}. However, the cumulative error of the overall length of the geodetic line plotting is not discussed. In \cite{6}, the precision of the geodesy theme solution was analyzed and demonstrated, but just like \cite{3} and \cite{4}, the algorithm also does not specify the numerical correlation between the geodetic plotting error and the interpolated distance parameter, and it is difficult to realize the rapid plotting of the geodetic line under the condition of precision threshold limit.

In order to solve the problem that the interpolated distance parameters and plotting precision can not be quantitatively regulated and adaptively matched in the execution of the existing geodetic line plotting algorithm in vector data, combined with the application background of the Mercator projection plane of the geodetic line plotting, the derived solution of the constant line segments considering the angle between the constant line and the geodetic line is extended to the geodetic line plotting error quantitative evaluation and control process. In this paper, a rapid plotting algorithm for geodetic line defined by arch height error threshold under the condition of Mercator projection is proposed, and the rapid geodetic line plotting under the condition of precision threshold is realized. Finally, the accuracy and efficiency of the proposed algorithm are proved by experimental verification.

2. Problems of the equal-lying-spacing geodetic line plotting algorithm without restrictions

The equal-lying-spacing geodetic line plotting algorithm without restrictions is the increasing interpolated of geodetic line under the condition of preset interpolating distance parameters, and its solution process does not involve the complex geodetic line differential equation solution, which has high algorithmic efficiency. However, the interpolated distance parameters in the equal-lying-spacing geodetic line plotting algorithm without restrictions will not be changed once determined, and they can not be changed throughout the whole process of geodetic line plotting, which has a great influence on the accuracy, efficiency and effect of the geodetic line plotting.

On the one hand, the equal-lying-spacing geodetic line plotting algorithm without restrictions can not achieve adaptive matching of geodetic line plotting accuracy and efficiency, and it is difficult to achieve its fast and efficient plotting in the navigation diagram. As shown in Figure 1, \(w_{PQ}\) and \(w_{PQ}'\) are two geodetic lines with the same starting point \(P\), and \(w_{PQ}\) is a part of \(w_{PQ}'\) (i.e. \(Q\) is a point on the geodetic line \(w_{PQ}'\)). Under the condition of the same preset interpolated distance parameter \(\Delta S\), the geodetic lines \(w_{PQ}\) and \(w_{PQ}'\) can be expressed as polylines \(w_{PQ} = \{T_i | 0 \leq i \leq n | n = \frac{S_{PQ}}{\Delta S}\}\) and \(w_{PQ}' = \{T_i | 0 \leq i \leq n | n = \frac{S_{PQ}'}{\Delta S}\}\) ( \(n \geq n'\) ) respectively after processing by the equal-lying-spacing geodetic line plotting algorithm without restrictions.
Fig. 1 Effect map of part (whole) geodetic line interpolated under the same interpolated distance parameters

With reference to the statistical law of the difference between the spherical distance and its cut plane distance\textsuperscript{7}, when the interpolated distance parameter \(\Delta S\) is small, the length \(d_{r,Ti}\) of the polyline segment formed by adjacent interpolated points \(T_i\) and \(T_{i+1}\) is approximately equal to the length of the geodesic line \(w_{Ti,T_{i+1}}\). Thus, the plotting errors of the length of the geodetic line \(w_{PQ}\) and \(w'_{PQ}\) can be approximately expressed as \(n\delta_{AS}\) and \(n'\delta_{AS}\) (\(\delta_{AS}\) can obtained by table lookup according the value of \(\Delta S\)). That is to say, the length error of the equal-lying-spacing geodetic line plotting algorithm without restrictions is directly proportional to the length of the geodetic line. Based on the above analysis, under the assumption that the plotting length error \(n\delta_{AS}\) of geodetic line \(w_{PQ}\) is not greater than the plotting length limit error range \(\delta_{\Omega}\) (the value of \(\Delta S\) is relatively large), the plotting length error \(n'\delta_{AS}\) \((n' \geq n)\) of the geodetic line can not be quantitatively controlled within the plotting length limit error range \(\delta_{\Omega}\). On the contrary, if the plotting length error \(n'\delta_{AS}\) of the geodetic line \(w'_{PQ}\) is limited to meet the requirement of plotting length limit error range \(\delta_{\Omega}\), the value of \(\Delta S\) is too small to get a high efficiency of geodetic line plotting.

On the other hand, in order to balance the contradiction between the accuracy and efficiency of plotting geodetic line, a geodetic interpolated distance parameter \(\Delta S\) calculation model for different scale (denominator of scale is \(l\)) is proposed in [4] based on the limit resolution \(d_{chart}\) of paper chart recognized by human eyes (0.01cm in figure). Namely, \(\Delta S = ld_{chart}/100\). Although the calculation model realizes the adaptive adjustment and matching of interpolated distance parameters under the condition of plotting accuracy to a certain extent, it does not establish the quantitative regulation and adaptive matching model of interpolated distance parameters and plotting accuracy, and can not guarantee the uniqueness of the geodetic plotting effect under different application conditions. As shown in Figure 2, \(\Delta S\) and \(\Delta S'\) are the interpolated distance parameters of geodetic lines obtained from the calculation model in [4] under different scale of nautical charts, the geometric shapes of polylines plotting by the same geodetic line \(w_{PQ}\) on different scale of nautical charts are quite different. The results of geodetic line measurement and analysis are uncertain, which makes it difficult to satisfy the application requirements of fairness in marine delimitation.
3. Calculation model of geodetic line plotting arch height error

Influenced by the numerical changes of the curvature and disturbance rate of the earth spheroid, the numerical variation of the elevation error of the geodetic line plotting arch height error varies from different interpolated points, which needs to be obtained by the complex geodesic differential equation, which is difficult to achieve and the calculation complexity is higher. Taking into account the relatively small value of interpolating distance parameter $\Delta S$ in the equal-lying-spacing geodetic line plotting algorithm without restrictions, and combined with the partial earth line length in the small range of geodesy to the earth spheroid to calculate approximate equivalence and the analysis conclusion of the error can be controlled, the radius at the pointcut of the earth spheroid is proposed in this paper to be an alternative to the spherical radius of the earth spheroid (within 200km range, the difference between the length of the earth line and the length of the large arc is less than 0.5m), and the calculation model of the geodetic line plotting arch height error $h_i$ is established by replacing the spherical surface with the spherical sphere. As shown in Figure 3, point $B$ represents the midpoint of the coordinates after the conversion of the longitude and latitude coordinates of adjacent interpolated points $T_i$ and $T_{i+1}$. Point $A$ represents the intersection of the extension line and the ball surface of the ball center $O$ to point $B$, and the length of the polyline $AB$ ($AB = OA - OB$) in the spherical surface is the geodetic line plotting arch height error $h_i$.

According to the calculation principle of the radius at the pointcut of the earth spheroid, the latitude coordinates of the midpoint between the adjacent interpolated points $T_i$, $T_{i+1}$ is obtained as $\phi_i = \frac{B_i + B_{i+1}}{2}$. Substitute it into the formula:

$$ r_e = \frac{a \sqrt{1 - e^2 \sin^2 \phi_i + e^2 \sin^2 \phi_i}}{\sqrt{1 - e^2 \sin^2 \phi_i}} $$

(1)
where \( r \) represents the radius at the pointcut of the earth spheroid at latitude \( \varphi \), \( a \) represents the long half axis of the earth spheroid, \( e \) represents the first eccentricity of the earth spheroid. On the spherical surface with the radius at the pointcut of the earth spheroid \( r \) as the radius, the calculation of the length of the polyline segment \( d_{iT_{i+1}} \) between the adjacent interpolated points \( T_i, T_{i+1} \) involves the conversion calculation of longitude and latitude coordinates \( (B_i, L_i) \) with corresponding position elevation \( H_i \) (since each interpolated point \( T_i \) is on the earth spheroid, the corresponding position elevation \( H_i = 0 \)) and the space right angle coordinate \((X_i,Y_i,Z_i)\), the calculation formula is:

\[
\begin{align*}
X_i &= r_i \cos B_i \cos L_i \\
Y_i &= r_i \cos B_i \sin L_i \\
Z_i &= r_i \sin B_i \\
d_{iT_{i+1}} &= \sqrt{(X_i - X_{i+1})^2 + (Y_i - Y_{i+1})^2 + (Z_i - Z_{i+1})^2} \tag{2}
\end{align*}
\]

On the spherical surface with the radius at the pointcut of the earth spheroid \( r \) as the radius, the solution of the geodetic line plotting arch height error \( h \) is reduced to the calculation of the length of the polyline segment \( AB \) \((AB = OA - OB)\) inner the sphere, so that the calculation of the length \( OB \) becomes the key of the solution of the geodetic line plotting arch height error \( h \). In \( \triangle OT_iB \), there is a right triangle relationship, i.e. \( OB = \sqrt{(OT_i)^2 - (BT_i)^2} \). By replacing the known conditions, i.e. \( OA = OT_i = r_i \), \( BT_i = \frac{d_{iT_{i+1}}}{2} \), with the triangle relationship described above, the calculation model of the geodetic line plotting arch height error is defined as:

\[
h_i = r_i - \sqrt{r_i^2 - \left(\frac{d_{iT_{i+1}}}{2}\right)^2} = r_i - \sqrt{r_i^2 - \frac{(X_i - X_{i+1})^2 + (Y_i - Y_{i+1})^2 + (Z_i - Z_{i+1})^2}{4}} \tag{3}
\]

where the geodetic line plotting arch height error \( h_i \) is the local description of the degree of proximity in the vertical height of the earth line for the polyline segments between the adjacent interpolated points \( T_i \) and \( T_{i+1} \). Under the condition that the geodetic line \( w_{PQ} \) is fixed, the geodetic line plotting arch height error \( h_i \) is inversely proportional to the number of interpolated points \( n \), and the interpolating distance parameter \( \Delta S \) is in positive proportional relationship. Ideally, when \( h_i \to 0 \), the geodetic line \( w_{PQ} \) coincides fully with the polyline \( w_{PQ} \) composed of the interpolated points.

4. The interpolated distance parameter adaptive control model of the geodetic line plotting arch height error constraint

Base on the calculation model of geodetic line plotting arch height error, ideally, when \( h_i \to 0 \), the geodetic line \( w_{PQ} \) coincides fully with the polyline \( w_{PQ} \) composed of the interpolated points. Therefore, for the accurate plotting of geodetic lines under the condition of Mercator projection, the key lies in effectively combining the constant line characteristics of the polyline segments constructed by the interpolated points, and establishing the adaptive control model of the interpolated distance parameters of the geodetic line plotting arch height error constraint. As shown in Figure 4, \( w_{PQ} = \{T_i \mid 0 \leq i \leq n\} \) \((T_0 = P, T_n = Q)\) is the line formed after the geodetic line \( w_{PQ} \) being plotted, the polyline segment constructed by the adjacent interpolated points \( T_i \) and \( T_{i+1} \), sits in a straight line on the Mercator projection plane. \( A \) represents the positive azimuth of geodetic line from an interpolated point \( T_i \) to the endpoint \( Q \) (in Fig.4, the angle of the positive northbound and red dotted lines
represents the positive azimuth of geodetic line from the interpolated point \( T_0(P) \) to the endpoint \( Q \), i.e. \( A_0 \).
The geodetic line plotting arch height error is represented by \( h_i \) (e.g. \( h_0 \) is the geodetic line plotting arch height error corresponding to the polyline segment \( T_0T_1 \)).

\[
\Delta S_0 = PT_i = \frac{T_iH_i}{\sin(\angle T_iPH_i)}
\]  

where \( \angle T_iPH_i \) Represents the angle of the positive azimuth of the geodetic line from the interpolated point \( T_0(P) \) to point \( Q \) and the positive azimuth angle of the constant line segment \( T_0T_1 \). Because the interpolated distance parameters \( \Delta S_0 \), right-angle edge \( T_1H_1 \) and \( \angle T_iPH_i \) in formula (4) are each other's calculation premise, it is impossible to achieve numerical solution of interpolated distance parameters that take into account the geodetic line plotting arch height error. To this end, this paper envisages the establishment of a correlation function, which is between the current line segment \( TQ \) (from interpolated point \( T_i \) to endpoint \( Q \) ) constant line positive azimuth \( a_i \) and the geodetic line plotting arch height error limit range \( h_i \). So that the interpolated distance parameter adaptive control model with the geodetic line plotting arch height error constraint can be constructed. As shown in Figure 8, connect interpolated point \( T_0(P) \) and endpoint \( Q \), \( a_0 \) represents the positive azimuth of the constant line segment \( T_0Q \) (the angle of the heading north and cyan dotted line in Fig.5), \( \angle T_1PH_i \) is the angle between \( A_0 \) and \( a_0 \), on the constant line segment \( T_0Q \), there is a point \( T_i \) makes \( PT_i = PT_1 = \Delta S_0 \). Over \( T_i \) to \( H_1 \) is the vertical line at the direction of the positive azimuth of the geodetic line at \( T_0 \). In \( R\Delta PT_iH_i \), the following sine function relationship exists between the right-angle edge \( T_iH_1 \) and the bevel \( PT_i \), i.e. :

\[
\Delta S_0 = PT_i = \frac{T_iH_i}{\sin(\angle T_iPH_i)}
\]  

Combined with formula (4), due to \( \angle T_iPH_i \geq \angle T_iPH_i \), there exists \( T_iH_1 \geq T_iH_1 \). In addition, the geodetic line plotting arch height error \( h_0 \) of the line segment \( T_0T_1 \) in \( \Delta PT_iH_i \) is not larger than the length of the right-angle edge \( T_1H_1 \), that is \( T_iH_1 \geq h_0 \). Thus, among the right-angle edges \( T_iH_1 \), \( T_iH_1 \)
and the geodetic line plotting arch height error \( h_0 \) of the line segment \( T_0T_1 \), there exists an inequality relationship, i.e. \( T_i^1H_i^1 \geq T_iH_i \geq h_0 \).

\[
\theta_0 = |A_0 - a_0|
\]

Fig.5 The angle of constant line segment (from an interpolated point to the endpoint) and geodetic line and the numerical association with interpolated distance parameters

Considering \( \angle T_i^1PH_i^1 \) is the angle between \( A_i \) and \( a_0 \), take a note as \( \theta_0 = \angle T_i^1PH_i^1 = |A_0 - a_0| \). In combination with \( T_iH_i \geq T_i^1H_i \geq h_0 \), substitute \( h_0 \) as the value of \( T_i^1H_i \) into formula (5), then \( \Delta S_0 = \frac{h_0}{\sin \theta_0} \). Thus, the interpolated distance parameter adaptive control model with the constraint of geodetic line plotting arch height error is defined as:

\[
\Delta S_i = \frac{h_0}{\sin \theta_i}
\]  

(6)

where \( \theta_i \) represents the angle between the positive azimuth angle \( a_i \) of the constant line segment \( T_iQ \) and the geodetic line positive azimuth angle \( A_i \) (i.e. \( |A_i - a_i| \)). The meaning of formula (6) can be understood as: when the geodetic line \( w_{PQ} \) was interpolated by the interpolated distance parameter \( \Delta S_i = \frac{h_0}{\sin \theta_i} \), then the geodetic line plotting arch height error \( h_i \), of which is the line segment \( TT_i^{+1} \) connected by adjacent interpolated points \( T_i \) and \( T_i^{+1} \), can be controlled within the geodetic line plotting arch height error limit range \( h_0 \).

5. Experimental Results and Analysis

In order to verify the feasibility of the algorithm, this paper implements the equal-lying-spacing geodetic line plotting algorithm without restrictions (referred to as algorithm I) and the rapid geodetic line plotting algorithm within the arch height error limit under the condition of Mercator projection (referred to as algorithm II). The geodesy theme solution method is based on the Bessel’s algorithm applicable to the long-distance geodesy theme solution in [20]. The plotting map used in the experiment is a map of the China sea area and the adjacent sea area in Mercator projection with a reference latitude of 30 degrees north latitude. The reference spheroid is Krasovsky’s spheroid, and the experimental environment is an Inter® Core™ i3 processor with a main frequency of 3.4GHz and a memory of 2G. Two algorithms were used to carry out the geodetic line plotting experiments under the conditions of different parameter thresholds for the four sets of experimental data provided in [2], [8] and [9] (shown as Table 1).
Tab.1 Examples for geodetic line plotting

| Longitude and latitude | Krasovsky’s spheroid |
|------------------------|-----------------------|
|                        | Sample (1)            | Sample (2)            | Sample (3)            | Sample (4)            |
| start                  |                       |                       |                       |                       |
| $B_p$                  | 30°29'58.2043"        | 44°22'30.0000"        | -35°00'00.0000"       | 35°00'00.2200"        |
| $L_p$                  | 120°05'40.2184"       | 119°05'00.0000"       | 20°00'00.0000"        | 90°00'00.1100"        |
| end                    |                       |                       |                       |                       |
| $B_q$                  | 30°24'05.8354"        | 45°37'30.0000"        | -38°00'00.0000"       | -30°29'20.9600"       |
| $L_q$                  | 119°49'23.3854"       | 20°00'00.0000"        | 145°00'00.0000"       | 215°59'04.3400"       |

$S_{pq}$: 28230.936m

Tab.2 Error statistics of geodetic line plotting

| Experimental data | Statistical parameters | No interpolated | Algorithm I ($\Delta S$) | Algorithm II ($h_i$) |
|-------------------|------------------------|-----------------|---------------------------|----------------------|
| Sample (1)        | $h$ (m)                | 15.6021         | 0.0228                    | 0.0022               |
|                   | $\delta_{pq}$ (m)      | 0.0170          | 0.0003                    | 0.0003               |
| Sample (2)        | $h$ (m)                | 992.4088        | 0.0336                    | 0.0069               |
|                   | $\delta_{pq}$ (m)      | 11.7028         | 0.0003                    | 0.0003               |
| Sample (3)        | $h$ (m)                | 1904656.9964    | 0.0127                    | 0.0022               |
|                   | $\delta_{pq}$ (m)      | 1031095.4910    | 0.1024                    | 0.0337               |
| Sample (4)        | $h$ (m)                | 3931822.28613   | 0.0238                    | 0.0022               |
|                   | $\delta_{pq}$ (m)      | 3234544.4016    | 0.1788                    | 0.0022               |

The experimental results show that: ① For the processing without interpolated and algorithm I, the geodetic line plotting length error $\delta_{pq}$ is positively proportional to $S_{pq}$, which is the length of the geodetic line, and compared with the original geodetic line, the geodetic line plotting error processed by algorithm I is significantly reduced, but still shows a trend of increasing gradually. ② The geodetic line plotting arch height error $h_i$ processed by the algorithm I is in positive proportional relationship with the interpolated distance parameter $\Delta S$, but because it is not possible to establish the clear function relationship between the geodetic line plotting arch height error $h_i$ and the interpolated distance parameter $\Delta S$, the geodetic line plotting length error $\delta_{pq}$ and arch height error $h_i$ can not be solved precisely, and the value of the interpolated distance parameter $\Delta S$ can not be pre-set, which is too large or too small, will directly affect the accuracy and efficiency of the geodetic line plotting. ③ The geodetic line plotting length error $\delta_{pq}$ and arch height error $h_i$ treated by the algorithm II are positively proportional to the geodetic line plotting arch height error limit $h_i$, which is not related to
the length of the geodetic line $S_{PQ}$.

4. For algorithm II under the condition of given geodetic line plotting arch height error limit $h_1$, any length of geodetic line plotting length error $\delta_{PQ}$ and arch height error $h_1$ are quantitatively controlled within the pre-set geodetic line plotting arch height error limit $h_1$. 5. Because the center angle of the geodetic line in the samples are relatively small, the geodetic line plotting length error $\delta_{PQ}$ processed by algorithm II is significantly less than arch height error $h_1$, and the difference between the two types of error gradually decreased with the increase of the length of the geodetic line $S_{PQ}$.

In order to further verify the advantages of algorithm II in terms of the uniqueness of the plotting effect and the efficiency of the geodetic line plotting, the number of interpolated points and the time-consuming of algorithm I and algorithm II are analyzed under different parameter restrictions. The experimental conclusions are shown in Table 3.

| Experimental data | Statistical parameters | Algorithm I ($\Delta S$) | Algorithm II ($h_1$) |
|-------------------|------------------------|--------------------------|---------------------|
|                   | 700m  | 10000m | 25000m | 0.5m | 1m |
| Sample (1)        | number of interpolated points | 41 | 3 | 2 | 17 | 9 |
|                   | time-consuming (ms)   | 0 | 0 | 0 | 1 | 0 |
| Sample (2)        | number of interpolated points | 322 | 23 | 10 | 105 | 83 |
|                   | time-consuming (ms)   | 3 | 3 | 0 | 7 | 5 |
| Sample (3)        | number of interpolated points | 14485 | 1014 | 406 | 3145 | 1263 |
|                   | time-consuming (ms)   | 177 | 12 | 6 | 132 | 77 |
| Sample (4)        | number of interpolated points | 21429 | 1501 | 601 | 5813 | 2835 |
|                   | time-consuming (ms)   | 275 | 18 | 7 | 226 | 118 |

The experimental results show that: ① The number of interpolated points processed by algorithm I and algorithm II are in positive proportional to the length of the geodetic line $S_{PQ}$, and are inversely proportional to the interpolated distance parameter $\Delta S$ and the geodetic line plotting arch height error limit $h_1$. ② According to [4], the different values of the interpolated distance parameter $\Delta S$ in algorithm I can be understood as the field distance corresponding to the limit resolution of the paper chart under the conditions of different chart scales, compared with algorithm II, the geodetic plotting effect (interpolated points) in the different scale charts is different. ③ Compared with algorithm I, algorithm II involves a large number of calculation for geodetic line plotting arch height error $h_1$ in the process of operation, and algorithm I has higher algorithm execution efficiency under the condition that the interpolated points are not different a lot. ④ Under the condition of given the geodetic line plotting arch height error limit $h_1$, algorithm I can't prejudice the geodetic line plotting length error and arch height error under the condition of different interpolated distance parameters, and the
adjustment and improvement of interpolated distance parameter can't be achieved in the plotting process, so algorithm II has higher and more stable algorithm execution efficiency.

6. Conclusions
Based on the analysis of the equal-lying-spacing geodetic line plotting algorithm without restrictions, a rapid plotting algorithm of geodetic line within arch height error threshold limits under the condition of Mercator projection is proposed by extending the quantitative evaluation and control process of geodetic line plotting error, which is applying the consistent line calculation method that takes into account the angle of constant line and geodetic line. The experimental results show that the algorithm can effectively control the length error and arch height error of the geodetic line plotting, and realize the rapid plotting of the geodetic line in any length and latitude range.

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