The anomalous diffusion in high magnetic field and the quasiparticle density of states

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Abstract: We consider a disordered two-dimensional electronic system in the limit of high magnetic field at the metal-insulator transition. Density of states close to the Fermi level acquires a divergent correction to the lowest order in electron-electron interaction and shows a new power-law dependence on the energy, with the power given by the anomalous diffusion exponent \( \eta \). This should be observable in the tunneling experiment with double-well GaAs heterostructure of the mobility \( \sim 10^4 cm^2/V/s \), at fields of \( \sim 10 T \) at temperatures of \( \sim 10 mK \) and voltages of \( \sim 1 \mu V \).

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1 Introduction

One-parameter scaling theory of localization predicts that, at least in the absence of time-reversal breaking fields and interactions, there is a single independent critical exponent, the one for the localization length, which characterizes the metal-insulator transition as a critical point [1]. In particular, the exponent which governs the diffusion behavior and the eigenstates correlations at the mobility edge is set by one-parameter scaling hypothesis to η = 2 − d, where d is the dimensionality of the system. In strong magnetic field however, the observation of integer quantum Hall effect signals a breakdown of one-parameter scaling in two-dimensional (2d) electron system [2]. Within each disorder-broadened Landau level (LL) there is an unique value of the energy where the extended states reside and the system undergoes a metal-insulator transition when the Fermi level is at the mobility edge. Chalker and co-workers [3] demonstrated another surprising feature of the localization transition in strong fields: right at the mobility edge the diffusion constant develops a dependence on the frequency and the wave-vector, \( D(q^2/\omega) \propto (q^2/\omega)^{-\eta/2} \) for large \( q^2/\omega \), with a non-zero value of the exponent \( \eta \). Equivalently, the wave-functions at the mobility edge are neither truly extended nor localized, but instead are fractals with the generalized dimension \( d_{\text{eff}} = 2 - \eta \) [4]. The value of the exponent \( \eta \) is expected to be an universal quantity which characterizes the strong-field limit and independent of the details of a disorder potential [5].

The result that the exponent \( \eta \) is non-trivial in high magnetic field has been conjectured from an exact inequality satisfied by the two-particle spectral function in the lowest LL and by using the hypothesis of scale-invariance at the mobility edge [3]. Subsequently, the conclusion has been confirmed in numerical calculations [3],[4],[5]. Experimentally however, the anomalous diffusion exponent has yet to be observed, and in that context it is important to find measurable consequences of this new scaling behavior at the mobility edge. Shimshoni and Sondhi [7] have argued that the temperature dependence of Coulomb drag between two electron layers at the transition yields the information on the exponent \( \eta \). Brandes et al.
have suggested that \( \eta \) should appear in the power of the temperature dependence of the energy loss rate of hot electrons due to phonon emission and of the quasiparticle lifetime due to electron-phonon interaction. The quasiparticle lifetime determined by electron-electron interactions however, is not altered by the anomalous diffusion exponent in any significant way \([8], [9]\). In this paper we demonstrate that when the Fermi level is at the mobility edge, Coulomb interactions between electrons in the lowest LL strongly suppress the quasiparticle density of states (DOS) close to the Fermi level. In certain range of energies DOS now exhibits a power-law dependence on the energy (at \( T=0 \)) or on the temperature (at \( T \neq 0 \)) with power \( \eta/2 \), and our perturbative calculation suggests that right at the Fermi level DOS vanishes. This implies a weak zero-bias anomaly in the I-V characteristics of the tunneling into the electronic layer which would provide a direct information on the anomalous diffusion exponent. We discuss the experimental conditions under which this effect could be observed.

2 Quasiparticle density of states

Consider a 2d electron layer in a magnetic field strong enough so that all electrons are polarized and in the lowest LL. The LL mixing due to the disorder potential or interactions is neglected and we work in units in which the magnetic length \( l = (\hbar c/eB)^{1/2} \) is unity. If \( \psi_n(\vec{r}) \) are the lowest LL eigen-functions of a particular realization of the disorder, the Hamiltonian in this basis is given by

\[
H = \sum_n E_n a_n^{\dagger} a_n + \frac{1}{2} \sum_{n,m,p,q} \langle m, n | v_c | q, p \rangle a_m^{\dagger} a_n^{\dagger} a_p a_q,
\]

where \( v_c = e^2/r \) is the Coulomb interaction and we measure energy from the Fermi level.

First we assume \( T = 0 \), and calculate the self-energy correction to the single-particle Green’s function \( G_m(\omega) = (\omega - E_m - \Sigma_m(\omega))^{-1} \) within the perturbation theory. The real part of the disorder-averaged self-energy at energy \( E \),

\[
\Sigma_E(\omega) = N_0^{-1}(E) \sum_m \delta(E - E_m) \Sigma_m(\omega),
\]

(2)
determines to the lowest order in the interaction, the quasiparticle DOS via [10]:

\[ N(E) = N_0(E)(1 + \frac{d}{dE} \text{Re}\Sigma_E(E))^{-1}. \]  

(3)

\(N_0(E)\) is the average DOS of non-interacting electrons in the lowest LL and in the random potential and angular brackets denote the averaging over disorder. Consider first the exchange contribution to the self-energy:

\[ \Sigma_{\text{exc}}(E) = -\frac{1}{N_0(E)} \int_{-\infty}^{0} dE' \frac{d^2q}{(2\pi)^2} S(E, E', q) v_{\text{scr}}(\omega, q), \]  

(4)

where the disorder-averaged two-particle spectral density is defined by [10]

\[ S(E, E', q) \equiv \int d^2\vec{r} \exp(i\vec{q}\vec{r}) \left( \sum_{n,m} \delta(E - E_n)\delta(E' - E_m)\psi^*_n(\vec{r})\psi^*_m(0)\psi_m(\vec{r})\psi_n(0) \right). \]  

(5)

For small momentum and energies close to the mobility edge the two-particle spectral density takes the familiar diffusive form [3], [10]:

\[ S(\nu, q) = \frac{N_0(0)}{\pi} \frac{D(\nu, q)q^2}{\nu^2 + D^2(\nu, q)q^4}, \]  

(6)

and we assume the generalized diffusion constant \(D(\nu, q) = \hbar D f(q^2/cN_0(0)\nu), \nu = |E - E'|,\) \(c\) and \(D\) are constants and \(f(y) = 1\) for \(y < 1\) and \(f(y) = y^{-\eta/2}\) for \(y > 1\) [3]. It is essential for our discussion that even though we are calculating the lowest order contribution to the average self-energy we include the effect of screening on Coulomb interaction in eq. 4. By definition, the dynamically screened Coulomb interaction is given by:

\[ v_{\text{scr}}(\omega, q) = v_c(q)(1 + v_c(q)\Pi(\omega, q))^{-1} \]  

(7)

and we assume that the polarization function has it’s standard RPA form:

\[ \Pi(\omega, q) = \frac{N_0(0)D(\omega, q)q^2}{-i\omega + D(\omega, q)q^2}. \]  

(8)

As will be shown shortly, this assumption is by no means essential for our results. To calculate the correction to the quasiparticle DOS, we will need only the static limit of the
screened Coulomb interaction. Thus, as long as the polarization function has a diffusive form, the effects beyond simple RPA approximation which would be represented by a more complicated diffusion constant, exactly cancel out in the calculation. Important point is that the static part of the screened interaction is short ranged. Using the equations 4, 6, 7, 8 after some calculation we find, for $E \approx 0$:

$$\frac{d}{dE} \text{Re} \Sigma_{E}^{exc}(E) = \frac{F_{exc}(x, E)}{2\pi g} \left( \frac{|E|}{\Delta} \right)^{-\frac{\eta}{2}},$$

(9)

where the function $F_{exc}(x, E)$ is given by an integral

$$F_{exc}(x, E) = x \int_{(|E|/\Delta)^{1/2}}^{1} \frac{dt}{t^{1-\eta}(t + x)}.$$

(10)

g = \sigma_{xx}/(e^2/h)$ is the dimensionless dissipative conductance $\sigma_{xx} = e^2DN_0(0)$, the energy scale is $\Delta = \pi \Gamma/cg$, $\Gamma = 2\hbar D$ is the half-width of the disorder-broadened LL, and $x = (e^2/\epsilon d)/\Gamma$, where $d$ is the average distance between electrons. The result in eqs. 9 and 10 is the same as if we used the screened interaction in static approximation in our calculation. Thus for the present purposes, we could have set $\omega = 0$ from the beginning in the polarization function 8 so that complications related to the form of the diffusion constant in that expression cancel out. Let us now analyze the equations 9 and 10. If $x << 1$, then for $x^2 << |E|/\Delta << 1$ we have $d\text{Re} \Sigma_{E}^{exc}(E)/dE \approx (x/2\pi g(1-\eta))(|E|/\Delta)^{-1/2}$ (assuming $\eta < 1$), just like the result would be if the bare Coulomb interaction was used in the calculation. However, closer to the Fermi level, i.e., $|E|/\Delta \approx x^2$ and smaller, one obtains the novel power-law dependence on the energy: $d\text{Re} \Sigma_{E}^{exc}(E)/dE \approx (1/2\pi g\eta)(|E|/\Delta)^{-\eta/2}$.

If one would perform the previous calculation using the delta-function interaction instead of $v_{scr}(\omega, q)$ in eq.4, close to the Fermi level self-energy would again diverge with the power $\eta/2$, only with a different prefactor. This means that the same term must also exist in the Hartree contribution to the self-energy, since the two should cancel each other if the interaction is infinitely short ranged [10, 11] and electrons are completely spin-polarized. Taking both the exchange and the direct contributions into account, we finally obtain that
at zero temperature, sufficiently close to the Fermi level the correction to DOS reads:

\[
\frac{\delta N(E)}{N_0(0)} = \frac{F(x)}{2\pi \eta g} \left( \frac{|E|}{\Delta} \right)^{-\frac{\eta}{2}} \tag{11}
\]

where \( F(x) = \eta F_{\text{exc}}(x,0) - F_{\text{dir}}(x) \), and \([12, 13]\) \( F_{\text{dir}} = N_0(0) \int d^2q \tilde{\nu}_{\text{scr}}(0,q)/\pi \). The last integral gives

\[
F_{\text{dir}}(x) = 2x - 2x^2 \ln(1 + x^{-1}), \tag{12}
\]

and \( x \) is the ratio between the interaction and disorder energy scales as defined earlier.

Since we found that at \( T = 0 \) frequency dependence of the screened interaction does not affect the behavior of DOS sufficiently close to the Fermi level, at finite temperatures we may neglect it completely. The simple calculation then yields the result:

\[
\frac{\delta N(E,T)}{N_0(0)} = \frac{F(x)}{2\pi \eta g} \left( \frac{T}{\Delta} \right)^{-\frac{\eta}{2}} f\left( \frac{|E|}{T} \right) \tag{13}
\]

for \(|E|, T \approx 0\), where

\[
f(z) = \int_{-\infty}^{+\infty} \frac{e^{t+z}}{t^{\frac{\eta}{2}}(e^{t+z} + 1)^2} dt. \tag{14}
\]

The function \( f(z) = z^{-\eta/2} \) for \( z \gg 1 \) and \( f(0) = 0.92 \) (for \( \eta = 0.5 \)). Thus, at zero temperature we recover the result in eq. 11. At \( T \neq 0 \), DOS acquires a power-law dependence on the temperature instead on the energy.

A few remarks are in order at this point. The obtained power-law should be compared to the logarithmically divergent correction to DOS in \( d \geq 2 \) implied by one-parameter scaling relation \( \eta = 2 - d \) [14]. The exponent \( \eta \) is determined completely by the geometry of the extended state and is not in any sense a small parameter in the problem. The found power-law is therefore a truly distinct behavior from the standard logarithmic correction to DOS. This should be contrasted with the situations where the exponent itself is a small quantity so that the expansion in powers of this exponent to the leading order agrees with the logarithmic behavior (for a similar scenario see for instance ref. 15). Also, the assumption that the Fermi level is right at the mobility edge so that the diffusion constant is a function
of the combination $q^2/\omega$ and not only of the momentum is important. If this was not so, i.e. if the Fermi level was slightly off but close to the mobility edge, the power law in eq. 11 would be cut off below a finite energy defined by $\xi(0) \approx DE_{cut}^{-1/2}$, where $\xi(E)$ is the localization length at energy $E$ ($E = 0$ still defines the Fermi level). For the frequencies smaller than the $E_{cut}$ diffusion constant becomes a function of combination $(q\xi(0))$, and independent of frequency. With $\eta$ larger than zero the perturbative correction of the DOS then would show a crossover from the power law divergence as in eq. 11 for $E > E_{cut}$ to the standard logarithmic divergence for $E < E_{cut}$.

The divergence of the first-order correction to DOS in our problem indicates the breakdown of the perturbation theory very close to the Fermi energy. Since from the eq. 10 we saw that the power $\eta/2$ turns on for $(|E|/\Delta) \approx x^2$ and below, one might wonder whether that is in the region where higher order terms are already significant. The point where the first order correction becomes of order unity marks the region of energies where are our lowest order perturbation theory becomes insufficient: this happens at $(|E|/\Delta) \approx (F(x)/2\pi g)^2/\eta$. The function $F(x)$ behaves like $x^{\eta}/(1-\eta)$ for $x < 0.01$, peaks at $x \approx 0.2$ with the value of 0.26, and roughly stays constant for $0.2 < x < 1$. Consider first the physically more relevant regime $0.2 < x < 1$: then $x^2 \approx 0.1$ and the perturbation theory breaks down for $E/\Delta \approx 10^{-3} << x^2$. Here we assumed $g \approx 0.5$ and $\eta \approx 0.5$ [3, 5, 6]. The important point is that there is a considerable range of energies for which our simple lowest order perturbation theory is valid and the result is indeed given by eqs. 11 and 13 (see Figure 1). One may also consider the regime when $x \to 0$: in that case the breakdown occurs for $|E|/\Delta \approx x^2$, so that again right before it occurs the non-trivial power law appears, although the result is not as clear-cut as for larger $x$. Even though we can not say with certainty what happens very close to the Fermi level where the problem becomes non-perturbative, we still expect however that the perturbation theory is qualitatively correct and that DOS will indeed vanish right at the Fermi level. Standard scaling arguments would suggest that ultimately DOS still goes
to zero as a power-law, but with the exponent determined by the full interacting theory. Recent numerical calculations which include Coulomb interactions at the Hartree-Fock level find, quite surprisingly, both $\eta$ and the localization length exponent to be the same as in the non-interacting problem \[16\]. This points to the intriguing possibility that the exponent which ultimately determines the behavior of the DOS at the Fermi level is not different from the one we calculated. This issue deserves more attention in the future.

3 Tunneling

The obtained correction of the quasiparticle DOS will modify the thermodynamic quantities when the Fermi level is at the mobility edge. For instance, the specific heat will acquire a low-temperature correction: $\delta C_v/T \propto T^{\eta/2}$. However, thermodynamic quantities are difficult to measure since a typical 2d electron layer in GaAs heterostructure contains very few ($\sim 10^{11}$) particles. Recently, tunneling experiments have proved to be a new useful tool for studying 2d electron systems \[17, 18, 19\]. In what follows, we therefore consider a typical tunneling experimental setup consisting of two identical, parallel 2d electron layers separated by a semiconductor barrier. When a small voltage is applied between the layers, the tunneling current is:

$$I \propto N^2(eV,T)V$$

(15)

with the constant of proportionality being roughly the inverse of the tunneling resistance for non-interacting electrons. When the Fermi levels in both layers are tuned to the mobility edge, the tunneling conductance should exhibit a power-law anomaly as a function of the voltage ($T \ll eV$) or of the temperature ($eV \ll T$) with power $\eta$. Let us now estimate the relevant temperature (or voltage) scale over which the anomaly should become observable. DOS will start to show a power-law behavior when the correction becomes comparable to unity. This yields the temperature scale $T \approx (\pi \Gamma/cg)(F(x)/2\pi \eta g)^{2/\eta}$ as already discussed. We assume the numbers for $g \approx 0.5$ and $\eta \approx 0.5$ as before. The value of the constant $c$ is
disorder dependent and less well known. Here, we take $c \approx 60$, which was found for the white-noise random potential \cite{3}. All the numbers put together give the relevant temperatures to be $T \propto 10^{-4} \Gamma$. Note that according to our discussion, this is deep in the region where the power is determined by the screened Coulomb interaction.

In a typical experiment, GaAs samples of very high mobility (for example, $\mu = 2 \times 10^6 cm^2 V/s$ in ref. 18) are used. From the zero-field mobility we can estimate the LL broadening in the following way: assume that the scattering in the system is caused by the potential $V(\vec{r}) = \lambda \sum_i \delta(\vec{r} - \vec{r}_i)$, where $\{\vec{r}_i\}$ are 2d coordinates of uniformly distributed scatterers with the density $n_{imp}$. The zero-field mobility is $\mu = e \tau_0 / m$, $\tau_0 = \hbar / (n_{imp} \lambda^2 \pi N_0)$ is the scattering time in Born approximation and $N_0 = 2 m \pi / \hbar^2$ is 2d electron DOS in zero magnetic field. The band mass in GaAs is $m = 0.07 m_e$. In strong magnetic field, Born approximation gives the lowest LL broadening in the same potential $\Gamma \approx (\lambda^2 n_{imp} / l^2)^{1/2}$. At the magnetic field of 10T one then obtains $\Gamma \approx 6 K$, an order of magnitude smaller than the Coulomb energy scale which is around 50K (at 10T, $\hbar \omega_c \approx 160 K$). We treated the interactions as a perturbation to the disorder problem, so we need the LL broadening to be at least of the same order of magnitude as the Coulomb energy scale to be in the regime where our calculation is applicable. Presented estimate suggests that $\Gamma$ is proportional to the inverse of the square root of the mobility, hence decreasing the mobility of the sample by a factor of 100 would make $\Gamma \approx 60 K$, somewhat larger than the Coulomb energy but still sufficiently smaller than the cyclotron energy so that neglecting LL mixing is still a reasonable approximation. The temperature when the tunneling anomaly should occur is then $\sim 0.01 K$, or in terms of the voltage around $1 \mu V$.

4 Conclusion

To summarize, we demonstrated using the perturbation theory that the interaction effects in 2d electron system in high magnetic field at the metal-insulator transition cause a diverging
power-law correction of the quasi-particle density of states close to the Fermi level. This is interpreted as a precursor of the power-law vanishing of the density of states at the Fermi energy. The power in the perturbative regime is given by the anomalous diffusion exponent, so a tunneling experiment may be used to test the novel diffusion behavior in high magnetic field discussed by Chalker. An estimate of the energy scales involved shows that if a double-well GaAs heterostructure of zero-field mobility of the order of $10^4 \text{cm}^2\text{V/s}$ is used in the tunneling experiment at fields of $10T$, a zero-bias anomaly in the current-voltage characteristics should develop for temperatures of the order of $10\text{mK}$ and voltages $\sim 1\mu\text{V}$.
Captions:

Figure 1. Schematic view at the behavior of the quasiparticle density of states for small energies with the Fermi level at the mobility edge. The value of the small parameter $x^2 \approx 0.1$ marks the crossover point from the regime where the behavior is essentially determined by the bare Coulomb interaction (power 1/2) into the regime where screening becomes effective and determines the non-trivial power law (power $\eta/2$). In the shaded region the problem becomes non-perturbative.
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