Supplementary Material for

JSFit: a method for the fitting and prediction of J- and S-shaped concentration-response curves

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Supplementary Material includes the following: The observed effects (three replications) of ionic liquid ([epy]Cl) of different concentrations against Vibrio Qinghaiensis sp.-Q67 at different times (Table S1). Original forms of five biphasic models with their corresponding interpretation of each parameter (Table S2). Model coefficients and GoFs of training set models and the effects predicted by the training set models at four concentrations (Table S3). Outliers analysis of observed effect on the basis of confidence intervals relative error (Table S4).The concentration-response curves of [epy]Cl against Vibrio Qinghaiensis sp.-Q67 at different exposure times of 0.25, 2, 4, 6, 8, 10, and 12 h. (Fig. S1). The concentration-response relationships of ionic liquid ([epy]Cl) against Vibrio Qinghaiensis sp.-Q67 (Fig. S2). Fitting CRCs of different category data of different types of chemicals against Q67 (Fig. S3). The goodness-of-fit parameters, $R_{adj}^2$ (a), RMSE (b), and AIC (c), of two models (M5 and model 4) in five different times (Fig. S4).
Table S1: The observed effects (three replications) of ionic liquid ([epy]Cl) of different concentrations against *Vibrio Qinghaiensis* sp.-Q67 at different times

| Concentration (mol/L) | 0.25 | 2     | 4     | 6     | 8     | 10    | 12    |
|-----------------------|------|-------|-------|-------|-------|-------|-------|
| 0.0005816             | -0.009 | 0.007 | 0.038 | 0.051 | 0.044 | 0.019 | -0.010 |
|                       | -0.044 | -0.066 | -0.043 | -0.042 | -0.066 | -0.088 | -0.081 |
|                       | -0.062 | -0.085 | -0.102 | -0.103 | -0.200 | -0.192 | -0.078 |
| 0.0008143             | -0.015 | -0.019 | -0.008 | -0.004 | -0.013 | -0.035 | -0.051 |
|                       | -0.005 | -0.030 | -0.008 | -0.015 | -0.035 | -0.062 | -0.082 |
|                       | -0.055 | -0.032 | -0.037 | -0.025 | -0.007 | -0.042 | -0.040 |
| 0.001241              | 0.052  | -0.015 | -0.024 | -0.029 | -0.042 | -0.058 | -0.074 |
|                       | 0.058  | -0.047 | -0.032 | -0.057 | -0.071 | -0.086 | -0.099 |
|                       | 0.035  | -0.025 | -0.044 | -0.047 | -0.027 | -0.062 | -0.122 |
| 0.001784              | 0.103  | 0.006  | -0.029 | -0.045 | -0.070 | -0.086 | -0.096 |
|                       | 0.133  | 0.023  | 0.003  | -0.002 | -0.030 | -0.053 | -0.084 |
|                       | 0.085  | -0.029 | -0.078 | -0.091 | -0.088 | -0.125 | -0.138 |
| 0.002637              | 0.178  | -0.018 | -0.090 | -0.120 | -0.151 | -0.176 | -0.191 |
|                       | 0.186  | -0.048 | -0.139 | -0.179 | -0.215 | -0.236 | -0.237 |
|                       | 0.159  | -0.006 | -0.079 | -0.098 | -0.111 | -0.152 | -0.146 |
| 0.003878              | 0.282  | 0.001  | -0.159 | -0.201 | -0.279 | -0.315 | -0.344 |
|                       | 0.290  | 0.027  | -0.137 | -0.181 | -0.249 | -0.282 | -0.306 |
|                       | 0.267  | 0.033  | -0.114 | -0.156 | -0.175 | -0.263 | -0.291 |
| 0.005816              | 0.435  | 0.204  | 0.036  | -0.070 | -0.130 | -0.185 | -0.232 |
|                       | 0.441  | 0.193  | 0.066  | -0.041 | -0.096 | -0.147 | -0.208 |
|                       | 0.380  | 0.173  | 0.062  | -0.002 | -0.043 | -0.116 | -0.208 |
| 0.008531              | 0.589  | 0.456  | 0.451  | 0.264  | 0.185  | 0.059  | -0.074 |
|                       | 0.588  | 0.431  | 0.451  | 0.268  | 0.227  | 0.078  | -0.034 |
|                       | 0.544  | 0.369  | 0.416  | 0.297  | 0.214  | 0.098  | -0.019 |
| 0.01241               | 0.723  | 0.600  | 0.704  | 0.634  | 0.526  | 0.424  | 0.295  |
|                       | 0.720  | 0.595  | 0.713  | 0.655  | 0.567  | 0.488  | 0.380  |
|                       | 0.674  | 0.523  | 0.684  | 0.635  | 0.566  | 0.472  | 0.334  |
| 0.01784               | 0.772  | 0.673  | 0.804  | 0.817  | 0.788  | 0.693  | 0.588  |
|                       | 0.772  | 0.677  | 0.810  | 0.819  | 0.789  | 0.687  | 0.580  |
|                       | 0.741  | 0.613  | 0.795  | 0.824  | 0.800  | 0.735  | 0.608  |
| 0.02637               | 0.801  | 0.690  | 0.846  | 0.887  | 0.909  | 0.897  | 0.856  |
|                       | 0.800  | 0.718  | 0.857  | 0.892  | 0.913  | 0.903  | 0.867  |
|                       | 0.779  | 0.683  | 0.849  | 0.896  | 0.921  | 0.919  | 0.893  |
| 0.03878               | 0.849  | 0.798  | 0.906  | 0.934  | 0.965  | 0.981  | 0.978  |
|                       | 0.844  | 0.805  | 0.907  | 0.935  | 0.965  | 0.982  | 0.983  |
|                       | 0.828  | 0.775  | 0.898  | 0.930  | 0.958  | 0.973  | 0.976  |
| model | Original form | Parameters |
|-------|--------------|------------|
| M1    | \( y = \frac{k + \gamma x}{1 + e^{\theta x^\delta}} + d \) | \( k \): untreated control  
\( d \): expected response at infinite concentration  
\( \gamma \): the initial rate of increase at low concentration  
\( b \): the way in which response decreases with concentration  
\( g \): no simple interpretation |
| M2    | \( y = 1 - \frac{k(1 + \gamma x)}{1 + (1 + 2\gamma EC_{50}) \cdot (x \cdot EC_{50})^\delta} \) | \( \gamma \): the initial rate of increase at low concentration  
\( k \): untreated control  
\( C \): expected response at infinite concentration  
\( b \): no simple interpretation |
| M3    | \( y = C + \frac{d - c + \gamma \exp(-1/x^\omega)}{1 + \exp(b(\ln(x) - \ln(e)))} \) | \( \gamma \): the initial rate of increase at low concentration  
\( \alpha \): the rate of the hormetic effect manifests itself  
\( c \): no simple interpretation  
\( \omega \): expected response at infinite concentration  
\( b \): the steepness of the curve after the maximum hormetic effect  
\( e \): the lower bound on the EC50 level  
\( \omega \): expected response at infinite concentration |
| M4    | \( y = \frac{\text{min} - \alpha + ((\omega - \text{min}/(1 + (\text{EC}_{\text{up}} \cdot x)^{\beta_{\text{up}}}))}{1 + (x / \text{EC}_{\text{up}})^{\beta_{\text{up}}}} \) | \( \text{EC}_{\text{up}} \): the concentration at the midpoint of the rising slope  
\( \beta_{\text{up}} \): the steepness of the rising (positive) slope  
\( \omega \): expected response at infinite concentration  
\( \alpha \): untreated control  
\( \text{EC}_{\text{dn}} \): the concentration at the midpoint of the falling slope  
\( \beta_{\text{dn}} \): the steepness of the falling (negative) slope  
\( \alpha \): untreated control |
| M5    | \( y = \min + \left( \alpha - \text{min}/(1 + 10^{(x - \text{EC}_{\text{up}})/\beta_{\text{up}}}) \right) \)  
\( + \left( \omega - \text{min}/(1 + 10^{(x - \text{EC}_{\text{dn}})/\beta_{\text{dn}}}) \right) \) | \( \beta_{\text{up}} \): the steepness of the rising (positive) slope  
\( \omega \): expected response at infinite concentration  
\( \text{EC}_{\text{up}} \): the concentration at the midpoint of the rising slope  
\( \beta_{\text{dn}} \): the steepness of the falling (negative) slope  
\( \text{EC}_{\text{dn}} \): the concentration at the midpoint of the falling slope  
\( \text{min} \): the minimum effect that would be approached by the downslope  
\( \alpha \): untreated control  
\( \omega \): expected response at infinite concentration |
| Time (/h) | E<sub>m</sub>   | EC<sub>mid1</sub> | H<sub>1</sub> | E<sub>max</sub> | EC<sub>mid2</sub> | H<sub>2</sub> | RMSE  | R<sup>2</sup> | q<sup>2</sup>ext | Pred.1 | Pred.2 | Pred.3 | Pred.4 | q<sup>2</sup>ext |
|----------|----------------|------------------|-------------|--------------|-----------------|-------------|-------|--------|-------------|-------|-------|-------|-------|--------|
| 12       | -23.675        | 5.295E-03        | 2.634       | 1.461        | 1.950E-04       | 0.745       | 0.0397| 0.9978 | -0.0357     | -0.2337| 0.0310| 0.8165| 0.9884|
| 10       | -1.658         | 4.673E-03        | 2.398       | 1.091        | 5.646E-03       | 1.663       | 0.0592| 0.9954 | -0.0232     | -0.2134| 0.1375| 0.8804| 0.9940|
| 8        | -3.197         | 9.318E-03        | 2.172       | 1.014        | 4.701E-03       | 3.409       | 0.0510| 0.9966 | -0.0159     | -0.1626| 0.2377| 0.9081| 0.9989|
| 6        | -1.191         | 6.952E-03        | 2.111       | 0.962        | 5.768E-03       | 3.620       | 0.0238| 0.9992 | -0.0127     | -0.1229| 0.3284| 0.8997| 0.9954|
| 4        | -0.460         | 6.645E-03        | 1.636       | 0.957        | 5.875E-03       | 37.961      | 0.0223| 0.9993 | -0.0144     | -0.0832| 0.5747| 0.8661| 0.9676|
| 4*       | -0.793         | 7.093E-03        | 1.972       | 0.937        | 5.663E-03       | 5.107       | 0.0243| 0.9990 | -0.0110     | -0.0947| 0.6798| 0.8709| 0.9988|

* The concentrations corresponding to four predictive effects, Pred. 1, Pred. 2, Pred. 3 and Pred. 4, are 0.0008143, 0.002637, 0.008531 (0.01241 for 4 h*), and 0.02637 mol/L, respectively.
Table S4  Outliers analysis of observed effect on the basis of confidence intervals relative error

| Model (Threshold) | Time (h) | Concentration (mol/L) | Observed Effect | Predicted Effect | Predicted Effect Range | $RE_{CI}$ | Outlier | $p_{out}$ |
|-------------------|----------|------------------------|-----------------|------------------|------------------------|----------|---------|----------|
| HM6 (1.635)       | 10       | 0.0005816              | -0.192          | -0.024           | [-0.129, 0.081]         | 1.595    | 0       | 0        |
|                   | 8        | 0.0005816              | -0.200          | -0.006           | [-0.091, 0.079]         | 2.277    | 1       | 0.028    |
|                   | 6        | 0.0005816              | 0.051           | -0.005           | [-0.055, 0.049]         | -1.146   | 0       |          |
|                   | 6        | 0.0005816              | -0.103          | -0.005           | [-0.055, 0.049]         | 1.970    | 1       | 0.028    |
|                   | 6        | 0.001784               | -0.002          | -0.067           | [-0.117, -0.018]        | -1.305   | 0       |          |
|                   | 4        | 0.0005816              | -0.102          | -0.007           | [-0.102, -0.007]        | 1.984    | 1       | 0.028    |
|                   | 4        | 0.001784               | 0.003           | -0.054           | [-0.102, -0.007]        | -1.212   | 0       |          |
|                   | 10       | 0.0005816              | -0.192          | -0.068           | [-0.123, -0.013]        | 2.250    | 1       | 0.028    |
|                   | 10       | 0.0005816              | 0.019           | -0.068           | [-0.123, -0.013]        | -1.599   | 0       |          |
|                   | 8        | 0.0005816              | -0.200          | -0.046           | [-0.092, -0.001]        | 3.393    | 1       | 0.056    |
|                   | 8        | 0.0005816              | 0.044           | -0.046           | [-0.092, -0.001]        | -1.982   | 1       |          |
|                   | 6        | 0.0005816              | 0.051           | -0.028           | [-0.059, 0.003]         | -2.567   | 1       |          |
|                   | 6        | 0.0005816              | -0.103          | -0.028           | [-0.059, 0.003]         | 2.421    | 1       |          |
|                   | 6        | 0.001784               | -0.002          | -0.049           | [-0.080, -0.019]        | -1.524   | 0       | 0.056    |
|                   | 6        | 0.001784               | -0.092          | -0.049           | [-0.080, -0.019]        | 1.369    | 0       |          |
|                   | 6        | 0.002637               | -0.179          | -0.132           | [-0.166, -0.098]        | 1.371    | 0       |          |
|                   | 4        | 0.0005816              | 0.038           | -0.033           | [-0.064, -0.001]        | -2.269   | 1       | 0.056    |
|                   | 4        | 0.0005816              | -0.102          | -0.033           | [-0.064, -0.001]        | 2.231    | 1       |          |
|                   | 4        | 0.001784               | 0.003           | -0.045           | [-0.077, -0.014]        | -1.556   | 0       |          |
|                   | 4        | 0.002637               | -0.140          | -0.096           | [-0.129, -0.065]        | 1.401    | 0       |          |
| HM7-1 (1.692)     | 10       | 0.0005816              | -0.192          | -0.068           | [-0.123, -0.013]        | 2.257    | 1       | 0.028    |
|                   | 10       | 0.0005816              | 0.019           | -0.068           | [-0.123, -0.013]        | -1.597   | 0       |          |
|                   | 8        | 0.0005816              | -0.200          | -0.046           | [-0.091, -0.001]        | 3.392    | 1       | 0.056    |
|                   | 8        | 0.0005816              | 0.043           | -0.046           | [-0.091, -0.001]        | -1.975   | 1       |          |
|                   | 6        | 0.0005816              | 0.051           | -0.028           | [-0.059, 0.003]         | -2.569   | 1       |          |
|                   | 6        | 0.0005816              | -0.103          | -0.028           | [-0.059, 0.003]         | 2.423    | 1       |          |
|                   | 6        | 0.001784               | -0.002          | -0.049           | [-0.080, -0.018]        | -1.525   | 0       | 0.056    |
|                   | 6        | 0.001784               | -0.091          | -0.049           | [-0.080, -0.018]        | 1.370    | 0       |          |
|                   | 6        | 0.002637               | -0.179          | -0.131           | [-0.166, -0.098]        | 1.376    | 0       |          |
|                   | 4        | 0.0005816              | 0.038           | -0.024           | [-0.059, 0.010]         | -1.800   | 1       |          |
|                   | 4        | 0.0005816              | -0.102          | -0.024           | [-0.059, 0.010]         | 2.237    | 1       | 0.056    |
|                   | 4        | 0.001784               | 0.003           | -0.048           | [-0.082, -0.013]        | -1.456   | 0       |          |
|                   | 4        | 0.002637               | -0.139          | -0.094           | [-0.129, -0.060]        | 1.299    | 0       |          |
| HM7-2 (1.692)     | 10       | 0.0005816              | -0.192          | -0.019           | [-0.133, 0.095]         | 1.513    | 0       |          |
|                   | 8        | 0.0005816              | -0.200          | -0.009           | [-0.095, 0.077]         | 2.218    | 1       | 0.028    |
|                   | 6        | 0.0005816              | 0.051           | -0.013           | [-0.076, 0.049]         | -1.032   | 0       |          |
|                   | 6        | 0.0005816              | -0.103          | -0.013           | [-0.076, 0.049]         | 1.432    | 0       | 0.028    |
|                   | 6        | 0.001784               | -0.179          | -0.074           | [-0.137, -0.012]        | 1.660    | 1       |          |
|                   | 4        | 0.0005816              | -0.102          | -0.002           | [-0.084, 0.081]         | 1.213    | 0       |          |

$RE_{CI}$ refers to confidence intervals relative error, which is used to measure deviation degree of observed effect.

$p_{out}$ is outliers ratio of observed effect at the corresponding time.
Fig. S1  The concentration-response curves of [epy]Cl against *Vibrio Qinghaiensis* sp.-Q67 at different exposure times of 0.25, 2, 4, 6, 8, 10, and 12 h. At a concentration of 0.004 mol/L, the effect decreases with time.
Fig. S2  The concentration-response relationships of ionic liquid ([epy]Cl) against *Vibrio Qinghaiensis* sp.-Q67 where “----” is the OCIs, “—” the FCIs, “——” the fitting CRCs, “∞” the
experimental scatters, “hill(T)” represents S-shaped CRC at the time of T (T=0.25 or 2) h, and “Y(T)” represents J-shape CRC at the time T (T=4, 6, 8, 10, or 12) h by the model Y.
Fig. S3 Fitting CRCs of different category data of different types of chemicals against Q67. “X_Y_Z” represents CRC of “Z” (Z=S, Hdata 1, Hdata 2 or Hdata 3) category data of chemical “X” with molder “Y” (Y=S, HM5, HM6 or HM7_1).
Fig. S4  The goodness-of-fit parameters, $R^2_{adj}$ (a), R MSE (b), and AIC (c), of two models (M5 and HM5) in five different times.