Consistent Skyrme parametrization and its critical parameter values

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Abstract. A new Skyrme parametrization was developed based on a shortlist of Skyrme forces. These selected parameterizations, 16 in total, were found in a comprehensive study involving 240 parametrizations submitted to constraints related to nuclear matter properties. The critical parameters associated with the liquid-gas phase transition, critical density, critical pressure, and critical temperature are calculated for this new parametrization as well as its flash point.

1. Introduction
Properties of nuclear matter are reasonably well described by different versions of relativistic and non-relativistic models. The basic idea is to parameterize the NN and NNN interactions by zero range (Skyrme model) density-dependent functions to model ground state properties of finite nuclei and nuclear matter. In this perspective, the microscopic details of NN and NNN forces, such as meson exchange, are not explicitly considered and all the physically relevant information is carried by the parameters of the density-dependent phenomenological forces which include the spin, orbital angular momentum and isospin couplings. However, the parameterization of such forces is not unique and there exist, in principle, an infinite number of parameter sets, fitted to ground state properties of (doubly-or semi-magic) stable nuclei and symmetric and asymmetric nuclear matter (ANM). In this work, we presented the construction of a conventional non-relativistic Skyrme parameterization, called SkUFF, based on the set of consistent Skyrme parametrizations (CSkP), selected in Ref.[1].

2. Skyrme model
One of the advantages of the structure of the Skyrme density functional is that it allows the analytical expression of all variables characterizing infinite nuclear matter [2, 3, 4, 5, 6]. The CSkP models are: GSkI, GSkII, KDE0v1, LNS, MSL0, NRAPR, Ska25s20, Ska35s20, SKRA, Skxs20, SQMC650, SQMC700, SkT1, SkT2, SkT3 and SV-sym32 (for details see reference therein [1]). They satisfy all 11 constraints that describe the symmetric nuclear matter (SMN), the pure neutron matter (PNM), and a mixture of both related with the symmetry energy (MIX). The selection of these 16 parametrizations and their saturation properties are presented in Ref.[1].
The energy per particle of infinite nuclear matter, defined in terms of the energy density $E$ and particle number density $\rho$ of these parameterizations, is written as

$$E = \frac{\mathcal{E}}{\rho} = \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3} + \frac{t_0}{8} \rho [2(x_0 + 2) - (2x_0 + 1)H_2]$$

$$+ \frac{1}{48} t_3[2(x_3 + 2) - (2x_3 + 1)H_2] \rho^{\sigma+1} + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{5/3} (aH_{5/3} + bH_{8/3}),$$

with $a = t_1(x_1 + 2) + t_2(x_2 + 2)$, $b = \frac{1}{2} [t_2(2x_2 + 1) - t_1(2x_1 + 1)]$, and $H_n(y) = 2^{-n-1}[y^n + (1-y)^n]$. Where $y = Z/A$ is the proton fraction, and $M$ is the nucleon mass. From Eq.(1) we can obtain others nuclear matter quantities like incompressibility, symmetry energy, slope, and volumetric isospin incompressibility.

3. Results
The SkUFF parametrization is a conventional Skyrme model and has up to 9 adjusting parameters. For this calculation, it was necessary to fix observables of matter nuclear, such as binding energy, saturation density, effective mass, incompressibility, the energy of symmetry and its derivatives. We established the values of these quantities as the simple arithmetic mean of the values presented by the 16 selected models. Such numbers are arranged in the Table 1.

### Table 1. Observables values for SkUFF parametrization.

| $\rho_0$ [fm$^{-3}$] | $E_0$ [MeV] | $K_0$ [MeV] | $J$ [MeV] | $L_0$ [MeV] | $K_{\sigma}$ [MeV] | $m^*$ |
|-----------------|-------------|-------------|-----------|-------------|-------------------|-------|
| 0.163           | 15.87       | 227.85      | 32.56     | 59.18       | $-382.04$         | 0.855 |

Note that we fixed only 7 observables. Due to the type of Skyrme EoS, we can not be able to write all constants as a function of the saturations quantities only [7]. Thus, we proceed to calculate $t_0$, $x_0$, $t_1$, $x_1$, $t_3$, $x_3$ and $\sigma$ from the observables define in Table 1, and again, use an arithmetic mean for the CSkP to get the remaining data $t_2$ and $x_2$. After these procedures we find the following (Table 2) parameters for the SkUFF.

### Table 2. SkUFF parameters. The dimensions are: $t_0$ [MeV.fm$^3$], $t_1$ = $t_2$ [MeV.fm$^3$], and $t_3$ [MeV.fm$^3(\sigma+1)$]. The others are dimensionless.

| $t_0$ | $t_1$ | $t_2$ | $t_3$ | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $\sigma$ |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| -2030.31 | 193.50 | -82.81 | 12756.77 | 0.127 | 0.036 | -0.539 | 0.051 | 0.25     |

3.1. Behavior in the constraints
The values for the numerical constraints are shown in Table 3. Realize that the SkUFF model is approved in all of them and not only in those that have been fixed to find its constants. For more details, see [1].

In Figures (1) and (2) we illustrate how this new parameterization is consistent with the graphical constraints of SNM and PNM.
3.2. Application for SNM at Finite Temperature

The temperature inclusion was made using the expansion suggested by Ref. [8] for the Fermi gas. This expansion works well for $T$ approximately greater than 5 MeV. The pressure, in this case, can be written

$$P = \frac{3t_0}{8} \rho^2 + \frac{1}{8} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{8/3} (a + b) + \frac{3t_3}{48}(\sigma + 1)\rho^{\sigma+2} + T\rho + \frac{T\lambda^3}{8\sqrt{2}\gamma}\rho^2, \quad (2)$$

where $\lambda = \left[ \frac{2\pi(hc)^2}{MT} \right]^{1/2}$ is the thermal wavelength and $\gamma = 2$.

The critical values can be obtained by

$$\left( \frac{\partial P}{\partial \rho} \right) = \left( \frac{\partial^2 P}{\partial \rho^2} \right) = 0, \quad (3)$$
and is given by

\[ T_c = \frac{2}{9} \left( \frac{3\pi^2}{2} \right)^{2/3} (a + b)\rho_c^{5/3} + \frac{t_3}{16} \sigma(\sigma + 1)(\sigma + 2)\rho_c^{-1}, \]  

(4)

for the “flash” values we used this condition

\[ \left( \frac{\partial P}{\partial \rho} \right) = P = 0 \]  

(5)

that produced

\[ T_f = \frac{1}{12} \left( \frac{3\pi^2}{2} \right)^{2/3} (a + b)\rho_f^{5/3} + \frac{t_3}{16} \sigma(\sigma + 1)\rho_f^{-1}. \]  

(6)

The values for the critical and “flash” quantities are showed in Table 4.

**Table 4.** Critical and “flash” values for SkUFF. For comparison with experimental data, we showed the values from Ref.[9].

| Model   | \( T_c \) [MeV] | \( \rho_c \) [fm\(^{-3}\)] | \( P_c \) [MeV.fm\(^{-3}\)] | \( T_f \) [MeV] | \( \rho_f \) [fm\(^{-3}\)] |
|---------|----------------|-----------------|-----------------|----------------|----------------|
| SkUFF   | 16.59          | 0.054           | 0.254           | 12.67          | 0.084          |
| Ref.[9] | 17.9 ± 0.4     | 0.06 ± 0.01     | 0.31 ± 0.07     |                |                |

4. Summary

We presented the construction and the behavior of Skyrme SkUFF parameterization in nuclear matter. The SkUFF was consistent with all the constraints proposed in Ref.[1]. Besides, we studied its behavior at finite temperature, calculating its critical parameters. The calculated values for the SkUFF differ from the experimental ones by about 5% for \( T_c \) and 4% for \( P_c \). The value for the critical density was within the margin of error.

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