Pixel and multipole space correlation analysis of CMB with foregrounds

Pavan K. Aluri$^{1,2}$ and Pranati K. Rath$^2$

$^1$ Indian Institute of Science Education and Research Bhopal, Bhopal - 462023, India
$^2$ Indian Institute of Technology Kanpur, Kanpur - 208016, India

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ABSTRACT

In this paper we study the presence of any extended foreground correlated regions in a cleaned CMB map. To estimate the cosmological parameters of interest, it is customary to employ a galactic mask on a cleaned map to omit the galactic region which remain contaminated even after cleaning. This study shows that there are residual foreground correlations present even outside a galactic mask, which could potentially be responsible for some of the large scale anomalies found in CMB. Here, we defined two correlation statistics, one in pixel space and the other in multipole space. Pixel correlation statistic was used to convey a pictorial impression of the level of residual foregrounds present in a cleaned map, while the multipole correlation statistic was used to identify any polluted multipoles. Using the pixel statistic, we find that there are notable spatially-extended, foreground-correlated features in the estimated CMB map. One interesting feature we found is that, the anomalous CMB cold spot region in the southern galactic hemisphere is highly correlated with synchrotron emission templates. Using the multipole statistic, we find that the CMB quadrupole is contaminated by synchrotron.

Key words:

1 INTRODUCTION

Cosmic microwave background radiation (CMBR) is one of the basic foundations in establishing the inflationary Big Bang model, which is based on the Cosmological Principle, as our current standard model of cosmology. The $\Lambda$CDM model or the cosmic concordance model, which assumes that the energy density of the universe is now dominated by dark energy, followed by dark matter, fits the observations quite well (Riess et al. 1998; Perlmutter et al. 1999; Larson et al. 2011).

There have been many studies in the recent cosmology literature which found large scale anomalies in the CMB radiation. These studies include directional anisotropies such as a preferred direction for our universe as suggested by the anomalous alignment of quadrupole and octopole of CMBR (de Oliveira-Costa et al. 2004; Copi et al. 2004; Ralston & Jain 2004; Hajian et al. 2005; Land & Magueijo 2006a; Copi et al. 2006; de Oliveira-Costa & Tegmark 2006; Samal et al. 2008) and power anomalies such as the low quadrupole power, ecliptic north-south hemispherical power asymmetry and the even-odd multipole power asymmetry (Bennett et al. 2003a; Efstathiou 2003, 2004; Copi et al. 2007; Eriksen et al. 2004a, 2007; Hansen et al. 2009; Land & Magueijo 2005b; Gurzadyan et al. 2007; Kim & Naselsky 2010; Aluri & Jain 2012). Since CMB is a key to understanding our universe, the presence of these anomalies and their origin need to be investigated thoroughly. They may lead us to new physics and may provide new information about the early universe.

An important aspect of the CMB studies is to efficiently extract the cosmic CMB amidst the froth of all components (CMB, foreground, noise, etc.) registered by the detector. Even after cleaning the raw data, some residual foregrounds are still present in the cleaned maps (Bielewicz, Gorski & Banday 2004; Slosar & Seljak 2004; Abramo et al. 2006; Rakic et al. 2006; Bunn & Bourdon 2008; Aluri et al. 2011). A standard practice in CMB analysis is to exclude the heavily contaminated galactic plane, and other smaller foreground contaminated regions and point sources from the full-sky data, before using it in estimating any cosmological parameters of interest. This is achieved by applying a foreground mask such as the KQ85 mask employed by the WMAP team in their seven year temperature data analysis (Gold et al. 2011).

The fluctuations in CMBR are denoted as $\Delta T(\hat{n}) = T(\hat{n}) - \bar{T}$, where $\bar{T}$ is the mean temperature of CMB over the sky, and $\Delta T(\hat{n})$ represent the anisotropies around that
mean temperature in the direction \( \hat{n} \). We are observing CMB anisotropies as projected onto a sphere. Hence, they are conventionally expanded in terms of spherical harmonics, \( Y_{lm}(\hat{n}) \), as
\[
\Delta T(\hat{n}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{n})
\]
where \( a_{lm} \)'s are the coefficients of expansion. The spherical harmonics are suitable for expanding a function defined on a sphere as they form a complete basis. The unit vector \( \hat{n} = (\theta, \phi) \) denotes the co-ordinates on the sphere. The zero mean Gaussian random fluctuations of CMBR in multipole space have the statistical properties, \( \langle a_{lm} \rangle = 0 \) and \( \langle a_{lm} a_{l'm'}^\ast \rangle = C_l \delta_{mm'} \), where \( C_l \) is the two point correlation function in \( l \)-space, which is also called the power spectrum, and the angular brackets denote an ensemble average.

In this paper, we define two correlation statistics which can shed some light on the residual foreground features present in the cleaned data. The first one is defined in pixel space which is useful to map-out large scale foreground features/correlations hidden in a cleaned CMB map, and the second one is defined in multipole space. The multipolar expansion coefficients, \( a_{lm} \)'s, are used for a variety of purposes in cosmological analysis. This multipole correlation coefficient can reveal how polluted a particular multipole is and caution us of it’s usage and interpretation.

The CMB foregrounds are not yet well characterized. Though, thermal dust, free-free and synchrotron emission from our galaxy are agreed to be the dominant foregrounds, there are also some anomalous non-CMB signals reported to be present in the data that contribute at a sub-dominant level. Spinning dust is one such example (de Oliveira-Costa et al. 1997; Leitch et al. 1997; Dobler & Finkbeiner 2008a; Gold et al. 2011). An anomalous “haze” component was also found to be present in the CMB data by Dobler & Finkbeiner (2008b). A non-acoustic signal was found by Jiang et al. (2010) in the V − W difference map from the WMAP’s V and W band foreground-reduced maps. Also, Diego et al. (2010) found an anomalous ecliptic signal in the combination map \( V + W − 2Q \) from WMAP’s foreground-reduced CMB maps at Q, V and W channels, which was speculated to be zodiacal light contamination. Straylight contamination to CMB anisotropies were discussed by Burigana et al. (2001, 2006). Compensating dust filled local voids were proposed by Inoue & Silk (2006, 2007), as an explanation to some of the observed CMB anomalies. In addition, the presence of anomalous foreground components, other than the generally recognized foregrounds were also found in the WMAP data (Bonaldi et al. 2007; La Porta et al. 2008).

In view of these earlier reports, these correlation coefficients may be quite useful in assessing residual contamination in the cleaned maps.

2 PIXEL AND MULTIPOLe SPACE CCC’S

Correlation is one of the basic statistical measures which is employed for a wide variety of purposes. We try to understand the CMB foreground residuals in two ways, one in pixel space and the other in multipole space, by defining suitable cross-correlation coefficients (CCC).

Using the pixel space CCC, we try to understand the residual foreground contamination present in the cleaned data “pictorially”. We do so by cross-correlating a clean CMB map with various foreground templates and generate an all sky correlation map. This will help us in understanding the efficiency of the employed method for cleaning the raw data, and also equips us with a knowledge of the level of known and unknown contamination present in the recovered cosmic signal.

The correlation coefficient in pixel space is computed in small patches across the whole sky, using a large pixel from a low resolution HEALPix (Gorski et al. 2005) sky discretization grid as a mask, on both the CMB map and the foreground template, which are available at higher resolution. The CCC in pixel space is defined as
\[
R_{p}^{CF} = \frac{\sum_{p' \in p}(C_{p'} - \bar{C}_{p})(F_{p'} - \bar{F}_{p})}{\sqrt{\sum_{p' \in p}(C_{p'} - \bar{C}_{p})^2 \sum_{p' \in p}(F_{p'} - \bar{F}_{p})^2}},
\]
where \( C \) stands for the CMB signal and \( F \) stands for the foreground signal. The subscript, \( p \), denotes the pixel indices for the low resolution correlation map and \( p' \) stands for the pixel indices of the high resolution CMB/foreground maps. \( \bar{C} \) and \( \bar{F} \) are the mean values of CMB and foreground signal, respectively, for the larger pixel region (\( p \)). We used CMB and foreground full sky maps at HEALPix resolution of \( N_{side} = 512 \) and generated the correlation maps at \( N_{side} = 8 \). In this way, we have \((512/8)^2 = 4096 \) pixels available to compute the correlation coefficients robustly in each patch of the sky.

Though, the correlation map provides a measure of the level of CMB and foreground correlations, one may not be able to readily infer the correlations due to the large pixel size at \( N_{side} = 8 \). In order to convey a direct impression of the level of correlations in various parts of the sky, we upgrade the low resolution correlation map at \( N_{side} = 8 \) to \( N_{side} = 512 \) and smooth this upgraded map with a beam FWHM of 450’. This way we will be able to identify correlations present in the cleaned CMB map on scales slightly larger than the pixel size of a HEALPix map at \( N_{side} = 8 \), which is approximately equal to \( \sqrt{4\pi/12 \times 8^2 \times 180^2/\pi} \approx 439.7 \) arcmin. The smoothing does alter (decrease) the actual value of the correlation coefficient in each pixel, but aid us in gaining a better pictorial understanding of the correlations.

We also define a correlation coefficient in multipole space as
\[
R_{l}^{CF} = \frac{\sum_{m} a_{lm} a_{l'm'}^{\ast} \delta_{nm}}{\sqrt{\sum_{m} |a_{lm}|^2 \sum_{m} |a_{l'm'}^{\ast}|^2}},
\]
where \( a_{lm} \)'s are the coefficients of spherical harmonic decomposition of either of the CMB or the foreground maps. Residual correlations are quantified using the multipole space CCC.

The pixel correlation statistic, \( R_{p} \) and the multipole correlation statistic, \( R_{l} \), will serve as complementary measures to understand CMB residuals. While the correlation maps are created to help us perceive the residual correlations visually, we can also make a quantitative estimate of

1 http://healpix.jpl.nasa.gov/
the correlation coefficient in pixel space. However it is very
tedious and computationally exhaustive when trying to estimate
the significance of the observed correlation, region by region. Hence, we use multipole CCC to make a quantitative
assessment of the level of CMB and foreground correlations.

3 DATA SETS AND SIMULATIONS

We used IPSE cleaned CMB map (Tegmark et al. 2003, Saha et al. 2006, 2008) obtained using WMAP’s seven year raw data (hereafter IPSE7 map) and the ILC cleaned CMB map (hereafter ILC7) (Bennett et al. 2003b, Eriksen et al. 2004b), which is also obtained using WMAP’s seven year temperature data, as reference CMB maps in this analysis. For foregrounds, we use pre-launch Planck Sky Model (PSM) foreground templates (2) the total foreground (intensity) maps from WMAP’s five year data release produced using the procedure described by de Oliveira-Costa & Tegmark (2006), the seven year MEM foreground maps from WMAP team (Bennett et al. 2003b, Gold et al. 2011), and also the spinning dust template and the steep synchrotron like component obtained through an MCMC analysis by the WMAP team (Gold et al. 2009, 2011). We also made use of the 408MHz map (Haslam et al. 1982) dominated by synchrotron radiation, which is a composite full-sky map stitched by combining various surveys, and the model M8 dust map at 94GHz by Finkbeiner (2003). These foreground templates except for the PSM templates and the total foreground maps are available at the LAMBDA site (4).

We then used an ensemble of pure and IPSE/ILC cleaned CMB maps in this study to estimate the significance of the observed correlations, and also to estimate the significance of some of the anomalous features found in the CMB sky. The two cleaning methods are briefly described below. For more details the reader is referred to the original articles.

IPSE cleaning procedure

In this method a set of CMB raw maps from a multi-
frequency mission, say, with $n_c$ channels, are linearly combined in multipole space. This method can be used to estimate CMB signal entirely internally by knowing the beam and noise properties of each channel from a multichannel experiment, without recourse to any external information or foreground modeling (Tegmark et al. 2003, Saha et al. 2006, 2008). The clean CMB spectral coefficients are estimated by taking a linear combination of the raw maps as

$$a_{lm}^{\text{clean}} = \sum_{i=1}^{n_c} \hat{w}_i a_{lm}^{i} / B_l^i ,$$

where $\hat{w}_i$ are the weights with which the $a_{lm}^i$’s of individual raw maps are combined and $B_l^i$ are the symmetrized beam transfer functions of the $i^{th}$ channel. The weights are chosen such that the variance of the cleaned map in multipole space ($C_l^{\text{clean}} = \sum_m |a_{lm}^{\text{clean}}|^2 / (2l + 1)$) is minimum, and

that the weights add up to unity so that the CMB signal remains untouched. Thus the weights are given by

$$\hat{W}_l = [\hat{w}_1 \hat{w}_2 \ldots \hat{w}_n]$$

$$= \frac{e_0^2 C_l^{-1}}{e_0 e_l} ,$$

(5)

where $e_0 = [1 1.1 \ldots 1]^T$ and $C_l$ is the empirical covariance matrix constructed from the raw map $a_{lm}$’s, whose elements are given by

$$\hat{C}_{l,ij} = \frac{1}{2l + 1} \sum_{m=-l}^{l} \frac{a_{lm}^i a_{lm}^j}{B_l^i B_l^j} .$$

(6)

The clean CMB power is then given by

$$\hat{C}_{l}^{\text{clean}} = \frac{1}{e_0 e_l} .$$

(7)

In order to improve the CMB signal estimation, the spatial variation of foreground contamination is taken into account and this cleaning procedure is implemented in an iterative fashion in various parts of the sky. For this purpose, the CMB sky is divided into disjoint regions depending on the level of foreground contamination.

ILC cleaning procedure

The ILC cleaning procedure is similar to IPSE procedure in the sense that, this procedure also estimates the CMB signal by taking a weighted combination of the raw maps. But this linear combination is done in pixel space (Bennett et al. 2003b, Eriksen et al. 2004b). All the raw maps used in CMB estimation are smoothed to a common resolution (here, to one degree) and the clean map is obtained by linearly combining the smoothed raw maps as

$$T_{\text{clean}}(p) = \sum_{i=1}^{n_c} \hat{w}_i T_i(p)$$

(8)

where $T_i(p)$ is the observed sky temperature at pixel $p$ in the $i^{th}$ frequency band, and $\hat{w}_i$ are the weights with which the $n_c$ raw maps are combined. Here also the raw maps are chosen such that the variance of the cleaned map is minimum, but in pixel space ($\text{var}(T_c) = \langle T_c^2 \rangle - \langle T_c \rangle^2$), is minimum. The weights are given by

$$\hat{w}_i = \sum_{j=1}^{n_c} C_{ij}^{-1} \left/ \sum_{i,j=1}^{n_c} C_{ij}^{-1} \right. ,$$

(9)

where $C_{ij}$ are the elements of the map-to-map covariance matrix given by

$$C_{ij} = \frac{1}{N_{\text{pix}}} \sum_{p=1}^{N_{\text{pix}}} (T_i(p) - \bar{T}_i) (T_j(p) - \bar{T}_j) .$$

(10)

Here $\bar{T}_i$ is the mean temperature over the sky (or a part of the sky, if one is doing an iterative cleaning in disjoint regions of varying levels of foreground contamination in channel $i$ and $N_{\text{pix}}$ is the total number of pixels in the sky (or part of the sky under consideration). These weights have to add up to one so that the primordial CMB signal remains untouched. Here also the raw maps are cleaned iteratively.
by partitioning the sky depending on the level of foreground contamination.

Between these two methods, the IPSE cleaning has the advantage that the clean CMB map thus obtained has the same resolution of that channel which has the highest resolution (here W-band) among the various channels used for observation.

To simulate the raw data, a random realization of CMB signal based on $\Lambda$CDM model is combined with the dust, free-free and synchrotron emission templates from pre-launch Planck Sky Model (PSM) and Gaussian random noise corresponding to a WMAP’s frequency channel. Thus we generate five raw maps with appropriate beam smoothing and noise properties corresponding to the five frequency channels in which WMAP makes the measurements. These are then cleaned using the IPSE and ILC procedures as outlined earlier.

Thus we simulated an ensemble of 800 IPSE cleaned CMB maps and a set of 600 ILC cleaned CMB maps, and also used 4000 pure CMB maps generated with 1° beam smoothing. We also used filled CMB maps, meaning, the cleaned CMB maps (IPSE/ILC cleaned) are first masked using the KQ85 mask and the galactic region is then filled with the corresponding pure CMB realization with appropriate noise. These are generated so as to get an estimate of the CCC’s as due to foreground residuals which are solely outside the galactic cut. Thus five sets of simulated data (pure CMB maps and cleaned/filled IPSE/ILC maps) were used to compute significances of the quantities of interest in this study.

4 RESULTS

First, we present our results from pixel correlation studies. Later, the multipole correlation analysis of the cleaned CMB maps with foregrounds is presented.

All the maps considered here are smoothed to a common 1° resolution. Some of the maps used here were upgraded to HEALPix $N_{\text{side}} = 512$, as they are available at other resolutions. The MEM and MCMC foreground maps from WMAP’s seven year data release are available at $N_{\text{side}} = 128$ and $N_{\text{side}} = 64$, respectively.

4.1 Pixel correlation studies

As an initial step towards these correlation studies, we first correlate the cleaned CMB maps obtained using the IPSE ($IPSE_7$ map) and the ILC ($ILC_7$ map) procedures. Both the CMB maps, the original correlation map generated at HEALPix $N_{\text{side}} = 8$ and the smoothed, upgraded correlation map at $N_{\text{side}} = 512$ are shown in Fig. 1. As expected, these two maps are highly correlated. Except for some regions in and close to the galactic plane, the CCC is close to one. The KQ85 galactic mask (Gold et al. 2011) used in the seven year temperature data analysis by the WMAP science team is also shown in Fig. 1.

We next discuss the spatial correlations between a cleaned CMB map and foregrounds. Here we cross-correlate full-sky maps. While computing parameters of interest from CMB, one generally excludes the most contaminated galactic region. We expect significant correlation between the foregrounds and the recovered CMB signal in this region. However, outside the masked region, the foreground residuals are assumed to be negligible. From here on we will only present the smoothed, upgraded versions of the correlation maps.

In Fig. 2, the correlations between the $IPSE_7/ILC_7$ CMB maps, and the foreground maps of galactic thermal dust, free-free and synchrotron emission, from pre-launch PSM and WMAP seven year MEM foreground maps, are shown. Here we also used a CMB map generated using the recently introduced kurtosis minimization, instead of variance minimization, in pixel space (Saha 2011). The clean map obtained so was referred to as Gauss map. The spatial correlations between the Gauss map and the three principal foregrounds from the pre-launch PSM and seven year WMAP’s MEM foreground maps are also shown.

In Fig. 3, correlation of both the $IPSE_7$ and $ILC_7$ maps, with the M8 dust template by Finkbeiner (2003), and the Haslam’s 408 MHz map dominated by synchrotron emission (Haslam et al. 1982) are shown. The Haslam’s synchrotron map, M8 dust map and the extinction corrected Hα map as free-free emission template, are used as priors in generating the MEM foreground maps at WMAP’s frequencies. In the same figure, correlation of the two clean maps with the anomalous emissions viz., spinning dust map and a steep synchrotron like component from an MCMC analysis by the WMAP team using seven years data (Gold et al. 2011) are presented. Finally, the correlation of $IPSE_7/ILC_7$ maps with the total foreground (intensity) maps by de Oliveira-Costa & Tegmark (2006), produced from WMAP’s five year data, are shown in Fig. 4.

One could readily notice spatially extended correlations across the CMB sky. Some of these features appear very prominent. Some of our immediate observations are

- Large positive correlation of the cleaned CMB maps with synchrotron maps roughly in the direction of the CMB cold spot (Vielva et al. 2004; Cruz et al. 2006; Rudnick et al. 2007; Embraces et al. 2008; Cruz et al. 2008; Bridges et al. 2008; Masina & Notari 2009; Bernardi 2009; Smith & Huterer 2010; Bremer et al. 2010; Granett et al. 2010; Vielva et al. 2011). The cold spot as seen in the IPSE cleaned, 1° smoothed CMB map from WMAP seven year data is shown in Fig. 6.
- In the correlation map of CMB with dust M8 model template by Finkbeiner (2003), one can notice the wing like feature at the north pole to be present also in the correlation map of pre-launch PSM dust template and the $IPSE_7/ILC_7$ maps, which reveals that the PSM dust template is generated using the dust M8 model template. This feature is however absent in the MEM dust correlation map. This also reveals the present ambiguity in the estimation (and separation) of the actual level of a foreground component in CMB measurements.
- Significant correlations are also seen with 408 MHz Haslam map, especially the cold spot region, and the spur like region near the south pole which could be related to the redder regions along the ecliptic found by Diego et al. (2010). Using the $V + W - 2Q$ combination map from foreground re-
produced WMAP band maps, they found an anomalous signal along the ecliptic plane.

- Extended foreground correlated regions away for the galactic plane are also seen with spinning dust and steep synchrotron like component templates from WMAP’s MCMC analysis.

- The finger like structure in the correlation maps with the total foreground (intensity) maps estimated by de Oliveira-Costa & Tegmark (2006) on the southern right quadrant of the CMB sky. These finger like extensions, of which the southern cold spot is also a part of, were pointed out by WMAP team in their seven year analysis (Bennett et al. 2011). The large positive correlation in the direction of CMB cold spot fades away in the correlation map with total foreground map at $W$-band. So, it suggests that the cold spot might be originating from a foreground component(s) at low frequencies, which could be synchrotron.

- Interestingly enough, we don’t find such a consistent identifiable correlation for the large cold spot (Bennett et al. 2011), present slightly to the right from the center of the CMB map in galactic co-ordinates, with any of the foregrounds considered here.

There may be many features which we have not identified here. This spatial correlation analysis can help us understand the residual foregrounds better. This correlation analysis can be readily extended to include correlations of $Q$ and $U$ polarization maps of CMB and foreground. In Fig. 5, CMB $Q$ and $U$ maps from foreground reduction by template fitting and WMAP’s seven year MCMC analysis as correlated with polarized synchrotron and dust maps from...
Figure 2. The correlations of the IPSE7, ILC7 and the Gauss maps with galactic dust (first column), free-free (second column) and synchrotron (third column) emissions are shown here. The first two rows are the correlation maps of IPSE7 with the pre-launch PSM and MEM foreground maps. The next two and the last two rows are also the correlations maps with PSM and MEM foreground maps, but with ILC7 and Gauss maps, respectively. These maps are originally generated at $N_{\text{side}} = 8$, but were upgraded to $N_{\text{side}} = 512$ and smoothed eventually with a circular beam function of 450' FWHM for easy visual perception of the residual foregrounds still present in the cleaned data. One can readily see significant correlations even outside the KQ85 mask. A notable feature in these maps is a large positive correlation of the IPSE7/ILC7 CMB maps in the direction of the anomalous cold spot with synchrotron emission (last column).
WMAP’s seven year MCMC analysis are shown. We notice that the stokes $Q$ and $U$ maps of CMB and foregrounds are largely anticorrelated across the sky. Also there is a lot of difference in the correlation pattern when maps from different estimation procedures are used for correlation. To understand this difference we correlated the CMB and foreground temperature maps obtained from template cleaned and MCMC analysis and found that the correlation pattern is almost consistent, except for the galactic plane. This again reflects out poor understanding of (polarized) CMB and foreground component estimation and separation. Also, the polarization measurements in WMAP’s observations are dominated by noise.

We emphasize that these are mere observations and an eventual identification with known anomalous features reported in CMB studies so far. A consistent further analysis is needed before making any strong claims. This may prompt some scepticism regarding the correlation features.
Figure 4. Same as Fig. 2 and Fig. 3, but the IPSE7/ILC7 CMB maps (left/right column) are correlated with total foreground (intensity) maps obtained by de Oliveira-Costa & Tegmark (2006) at WMAP’s five frequency bands. The total foreground maps used here are estimated from WMAP’s five year data release. Notice the finger like extensions on the left side of the images in the southern hemisphere (Bennett et al. 2011).
Correlation of stokes $Q$ and $U$ maps of CMB and foregrounds (synchrotron and dust) are shown here. The left column corresponds to correlations of stokes $Q$, $U$ dust maps from WAMP's seven year MCMC analysis with CMB polarization maps from template cleaned W-band map, and MCMC analysis. The first two maps on the left correspond to correlation of stokes $Q$ maps and next two maps correspond to correlations of stoke $U$ measurements. The maps on the right correspond to correlations between MCMC polarized synchrotron maps with CMB maps from template cleaned Q-band and MCMC analysis, with same ordering as the left column.

4.2 CMB cold spot correlation with foregrounds

The cold spot correlation feature found here is an interesting finding. We do a preliminary analysis of this feature here. We generate a sky mask of $10^\circ \times 14^\circ$ in size encompassing the cold spot region. This cold spot is roughly located at $(\theta, \phi) = (147^\circ, 209^\circ)$ in galactic co-ordinates \cite{Vielva2004}. We apply this patch simultaneously on the synchrotron template and the CMB map, and compute the correlation statistic picked up a host of known features which is encouraging, such as the galactic contamination, the finger like extensions found by the WMAP team in the CMB map and a large positive correlation of the anomalous southern cold spot region with the synchrotron templates. So, the correlation maps might indicate other features that are so far not identified in the literature.
relation from that region. We find that $R_p = 0.4367$ and 0.4214, for the correlation of IPSE7 and ILC7 maps, respectively, with pre-launch PSM synchrotron template at 23GHz. In order to estimate it’s significance, we applied the same mask on 800 simulated cleaned IPSE maps and 1000 pure CMB maps, and correlated with the synchrotron template from pre-launch PSM, all at one degree smoothing. We also did this correlation analysis of the cold spot region using 600 simulated ILC cleaned maps. Further we correlated these IPSE/ILC cleaned maps with PSM dust and free-free templates, and with the MEM foreground maps as well.

The CCC histograms of the normalized frequency counts for the correlation of synchrotron, free-free and thermal dust emission templates from pre-launch PSM and WMAP’s seven year MEM templates with the simulated CMB maps (pure or IPSE/ILC cleaned) in the direction of the cold spot are shown in Fig. 4. The frequency counts of the CCC from simulated pure CMB data are peaked at zero, as expected. However, these histograms from the cold spot region would be the same for any region of the sky if CMB is truly a Gaussian random field. If we were to identify a feature as associated with (known) foregrounds, we have to use cleaned maps which have residual foregrounds. But, we find that the cleaned map histograms are also similar to pure maps’ histograms. So, we may only say that the CMB cold spot region is anomalously correlated with PSM synchrotron, and also dust templates. The pixel correlation coefficient, $R_p$, for the cleaned satellite data are shown for comparison as vertical lines in those plots. From this preliminary analysis, we find that the cold spot correlation with PSM synchrotron and dust maps are outside 2σ CL and the correlation with free-free emission is insignificant.

However, the correlation with WMAP’s MEM foreground maps turn out to be not significant. This may probably be related to the fact that the MEM maps are generated first by subtracting the ILC cleaned map from each of the raw maps from WMAP bands and then the individual foreground components in each band are estimated using maximum entropy method (MEM). So, when we are correlating the ILC cleaned map again with the MEM templates thus estimated, it might be resulting in such low value for the correlation.

### 4.3 Multipole space correlations

In this section we assess the level of foreground residual contamination using the multipole space CCC. We cross correlate both the IPSE7 and ILC7 maps with the three principal foregrounds from pre-launch PSM and WMAP’s MEM fitting, in multipole space following Eq. 3. We also compare the data correlation coefficients with 800 simulated IPSE cleaned maps and 600 simulated ILC cleaned maps, and also with 3000 pure CMB realizations. The results are shown in Fig. 5 and Fig. 6, for correlation of CMB with dust, free-free and synchrotron emission from PSM and WMAP’s MEM analysis, respectively. A 1σ spread with the mean correlation estimated from the simulations is also plotted for each multipole. We could readily find that some of the normalized correlation statistic values for the observational data are outside the 1σ CL. The farther these data CCC’s are from the mean, the more polluted they are. We find that the quadrupole is contaminated by synchrotron.

These correlations are however between full-sky foreground maps and cleaned CMB maps. As mentioned earlier, a galactic mask is used to avoid foreground induced errors in CMB analysis. How these statistics change with masking is not straightforward. Nevertheless, this is partially addressed by filling the galactic portion of a simulated cleaned CMB map with corresponding pure map realization and appropriate noise, after applying a galactic mask (KQ85 mask). The 1σ error bars thus computed were also shown in Fig. 5 and Fig. 6. Some studies use the spectral coefficients ($a_{lm}$’s) themselves from full-sky maps, such as the directional anisotropy studies. Hence, this information from multipole CCC would be very useful in assessing their usage and interpretation.

### 5 CONCLUSIONS

In this paper, we defined two correlation statistics, one in the pixel space and the other in the multipole space to understand the foreground residuals still present in a cleaned CMB map. While the pixel correlation statistic is useful to identify, visually, any known or unknown foreground-contaminated regions, the multipole statistic will decompose the contamination in a CMB map by angular scales. We were particularly interested in determining the presence of such residuals outside the KQ85 mask used in CMB temperature analysis. Discarding a portion of the CMB sky near heavily contaminated galactic plane by applying a mask is a preventive measure in CMB analysis, to avoid foreground induced errors.

We studied the correlations with the three principal foregrounds viz., dust, free-free and synchrotron galactic emissions, and the anomalous foregrounds emissions viz., the spinning dust and the steep synchrotron like components. We also studied the clean CMB map correlations with the 408MHz Haslam map dominated by synchrotron and the model M8 dust template which are frequently used in the analysis of CMB foregrounds. For these we used the foreground maps which are available in the public domain. We find many extended spatial correlation features across the cleaned CMB sky with various foreground templates that are assumed to compromise the CMB measurements. One interesting feature we identified is that there is a large positive correlation between the CMB map and the synchrotron template roughly in the direction of the cold spot situated near the galactic south pole. A preliminary investigation of this observation was also done by applying a $10\degree \times 14\degree$ mask encompassing the cold spot on both the simulated pure maps and cleaned maps, and thus correlating that part of the sky with synchrotron template. We did similar correlation analysis with dust and free-free foreground templates also. We find a more than 2σ CL correlation with the PSM synchrotron and dust templates. As the pixel CCC has identified some of the known regions of contaminations, we may thus say with some assurance that these features could very much be residual foreground features, which are hitherto unknown.

The multipole correlation studies revealed how the various multipoles are contaminated by foregrounds. We find that some of the multipoles estimated from full sky cleaned map are contaminated. In particular, the quadrupole is...
found to be significantly contaminated by the synchrotron component. The results of this study to some extent point out the ambiguity in the foreground separation and possible contribution of residuals to any of the anomalies found in CMB data.

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Figure 7. The $p$–value estimates of the correlation of the cold spot region with various foregrounds using simulated IPSE and ILC cleaned maps, and also the pure CMB maps, are shown here. The data correlation values are highlighted as vertical lines for easy comparison.
Figure 8. Multipole correlation results for IPSE7 map with pre-launch PSM (left column) and WMAP’s MEM foreground maps (right column) are shown here. The actual data are shown as coloured dots. The mean (expected) CCC, along with the 1σ fluctuation level for each multipole as estimated from simulations (pure, clean and filled CMB maps) are also plotted. This graph reveals us how polluted an l mode is in the CMB map.
Figure 9. Same as Fig. [8], but the simulated maps are correlated with WMAP provided seven year MEM foreground maps. The data correlation coefficients are plotted as dots for each multipole, along with mean CCC and 1σ CL from simulations.

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