Convective flow speed of particles in a vibrated powder bed

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Abstract

When a powder bed is vibrated, the particles may move like a fluid, and various flow patterns and surface shapes can exist, depending on the vibration conditions — particularly the frequency and amplitude. A two-dimensional (2D) bed is often used so that the flow pattern in a vibrated powder bed can be directly visualized. Because three-dimensional (3D) fluidization phenomena are complicated, 2D observation is not sufficient. However, it is difficult to observe the internal flow of a 3D fluidized bed. A unique method for doing this is positron emission particle tracking (PEPT), a technique derived from the same physical phenomena as in positron emission tomography (PET). In the most common version of the PEPT technique, the flow in a bed is analyzed by following the behavior of a single radioactively labeled particle. We adopted the PEPT technique to analyze flow patterns in a cylindrical vibrated powder bed. From the particle trajectory data obtained with PEPT, some characteristic internal flow structures that cannot be understood from observing surface particle motion were successfully obtained. However, it is difficult to clarify the detailed mechanisms driving such flow patterns from experimental data alone. Therefore, we also carried out numerical simulations based on the discrete element method (DEM). Because the DEM is a Lagrangian method, its computational time depends on the number of particles, so that simulations were restricted to 2D. In order to compare the simulated results to the experimental ones, the average speeds of particles were obtained. Consequently, it was found that the simulated average speed of particles depends only on the amplitude of the applied vibration. In general, good agreement between simulated and experimental results was obtained, but agreement became poorer under conditions of high frequency and low vibration strength.

Nomenclature

| Symbol | Description |
|--------|-------------|
| A      | amplitude of vibration [m] |
| \( F_{pp} \) | particle-particle interaction force acting on a particle [N] |
| f      | frequency of vibration [Hz] |
| \( F_t \) | tangential soft sphere interaction at contact [N] |
| I      | Inertial moment of particle [kgm²] |
| g      | gravitational acceleration [m s⁻²] |
| k      | spring constant [N m⁻¹] |
| \( m_p \) | particle mass [kg] |
| \( r_p \) | radius of particle [m] |
| t      | time [s] |

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Fluidized beds, in which the fluid is a gas such as air, are often utilized in powder engineering processes. To control the powder flow, vibration is sometimes applied to a fluidized bed, particularly when the powder is cohesive and therefore prone to agglomeration. Although fine powders (below about 50 μm in diameter) are of increasing industrial interest, it is difficult to process such powders because of strong cohesiveness between particles. To avoid such problems, vibration that can break agglomerates and spread powders uniformly, is often utilized.

However, it is well-known that vibration alone can give rise to convective flow of powder without a fluidization gas/liquid. Such a powder state is called a ‘vibrated powder bed’. The vibrated powder bed has been studied by many researchers. Faraday [1] first observed that a convective flow occurs when vibration is applied to a powder bed. Various powder flow patterns appear, depending on vibration conditions, frequency and amplitude. The flow pattern is also affected by particle size, bed size, and aspect ratio. Furthermore, it has been established that not only the convective flow pattern, but also the powder bed surface shape may change according to vibration conditions. Laroche et al [2] observed that the center region of the powder bed rises, and the surface becomes mountain-shaped. Evesque et al [3] observed an avalanche occurring on the oblique surface of the powder bed. Because it is difficult to observe the interior flow of a powder bed, a so-called two-dimensional (2D) powder bed is often utilized for visualization, in which the depth is small compared to the horizontal and vertical dimensions. The depth of container is generally tens of times larger than the size of a particle. Here, we need to be reminded that three-dimensional movement of particles is strongly restricted in the 2D bed.

Aoki et al [4] visualized the powder circulations occurring in the vibrated powder bed by using colored particles as tracers, and they reported that the number of circulation cells changes depending on vibration strength, bed height, and other factors. Some attempts have been made to define the relationship between convective flow patterns and vibration conditions. Hamada et al [5] devised a flow regime map for the number of circulation cells plotted on axes of vibration strength and frequency. However, this flow regime map is for a 2D powder bed. There are some reports (Clement et al[6]; Rajcheebach et al[7]) that have insisted that convective flow patterns in a 3D powder bed differ from those in a 2D powder bed. Erichs et al [8] measured the 3D velocity distribution in a powder bed using magnetic resonance imaging (MRI). The diameter of the column used in the experiment was 16.3 mm and the height of the bed height was 27 mm. The density and average diameter of the particles were 726 kg m$^{-3}$ and 1 mm, respectively. They obtained the velocity distribution of particles in the vertical sectional area and successfully visualized the particles drifting upward in the central region. Numerical simulations have also been utilized, particularly using the Discrete Element Method, and it was determined from the simulated results that particle-particle repulsion and frictional forces affect convective flow (Taguchi [9]; Lee [10]). They used a bed whose depth was equal to the particle size, i.e. a 2D bed. Numerical and experimental approaches have been independently utilized in most of the past studies and the comparison between the two approaches has rarely been conducted. Furthermore, the applicability of two-dimensional simulation to the three-dimensional bed analysis has not been examined sufficiently. Since it is important to analyze a complicated phenomenon with a simple model as a first step, the two-dimensional simulation is thus meaningful.
This study is aimed at clarifying the fundamental mechanism of the powder flow in a vibrated powder bed. In order to achieve the aim, we use dimensional analysis. When the dimensional analysis is conducted, all dominant parameters need to be listed. However, there are many parameters in this problem, e.g. the properties of the particles, the conditions of vibration and the size and shape of column. We adopted the positron emission particle tracking (PEPT) technique (Leadbeater et al [11]) to visualize convective flow patterns. From the particle trajectory data obtained with PEPT, some characteristic internal flow structures that cannot be understood from observing surface particle motion were successfully obtained [12]. Since it is difficult to clarify the detailed mechanisms driving such flow patterns from experimental data, we also carried out numerical simulations based on the discrete element method (DEM). The two approaches were used to examine the effects of changes in bed size.

### 2. Experiment

#### 2.1. Principle of PEPT

The PEPT technique for measurement of flow utilizes the same physical principles as the medical imaging technique of positron emission tomography (PET), and the equipment used is similar. Whereas PET is used to detect and image the spatial distribution of a substance labeled with a radionuclide, PEPT visualizes powder behavior by tracking a single particle that contains or is coated with a radioisotope. The principle of PEPT is briefly introduced here; further detail is given by Leadbeater et al [11]. PEPT detects gamma rays emitted as a result of positron-emitting radio isotopes placed between two detectors as shown in figure 1. The emission of a positron takes place during radionuclide $\beta^+$ decay. Each emitted positron rapidly annihilates with an electron, producing a pair of back-to-back $\gamma$-photons. The pair of photons is detected by two position-sensitive detectors and if the time between detections is less than a certain interval it is assumed that both originated from the same annihilation. The simultaneous detection of both $\gamma$-photons then defines a line of response (LoR) along the photon trajectory. The annihilation site and therefore the position of the radioisotope source is then assumed to

![Figure 1. Principle of PEPT measurement.](image)

### Table 1. Physical properties used in simulations.

| Parameter                                          | Value    |
|----------------------------------------------------|----------|
| Diameter of particle (No particle distribution) [$\mu$m] | 210      |
| Density of particle [kg m$^{-3}$]                  | 2,630    |
| Friction coefficient between particles [-]         | 0.3      |
| Friction coefficient between particle and wall [-] | 0.3      |

### Table 2. Sizes of containers and powder beds.

| Case  | Width [mm] | Height [mm] | Bed height [mm] | Number of particles (#) |
|-------|------------|-------------|-----------------|-------------------------|
| 1 (1/1 size) | 139.6      | 105         | 75              | 259,195                 |
| 2 (1/2 size) | 70.35      | 52.5        | 37.5            | 64,799                  |
| 3 (1/4 size) | 34.65      | 26.25       | 18.75           | 16,200                  |
| 4 (1/8 size) | 17.85      | 12.6        | 9.375           | 4,049                   |

This study is aimed at clarifying the fundamental mechanism of the powder flow in a vibrated powder bed. In order to achieve the aim, we use dimensional analysis. When the dimensional analysis is conducted, all dominant parameters need to be listed. However, there are many parameters in this problem, e.g. the properties of the particles, the conditions of vibration and the size and shape of column. We adopted the positron emission particle tracking (PEPT) technique (Leadbeater et al [11]) to visualize convective flow patterns. From the particle trajectory data obtained with PEPT, some characteristic internal flow structures that cannot be understood from observing surface particle motion were successfully obtained [12]. Since it is difficult to clarify the detailed mechanisms driving such flow patterns from experimental data, we also carried out numerical simulations based on the discrete element method (DEM). The two approaches were used to examine the effects of changes in bed size.
lie along or near this line shown. All of the measured LoRs should ideally intersect at the position of the particle if there is just a single radioactive particle within the field of view, as in the case in figure 1. However, in a practical measurement, some of the detected events are corrupt, due to scattering and other events. A PEPT algorithm is used to reject these corrupt events and to determine the location of the tracer particle based on the remaining events, the most commonly used approach being the Birmingham method \[13, 14\]. Since the PEPT tracer is mobile, the tracer position must be estimated by using the average of a part of the LoR time evolution. Therefore, there is a tradeoff between the measurable velocity of the particle and the accuracy of the tracer position. Parker \textit{et al}[15] stated that a tracer moving at 5 m s\(^{-1}\) can be located every millisecond to an accuracy of better than 2 mm. In this study, the speed of the particles is much smaller than 5 m s\(^{-1}\), giving a location inaccuracy of about 1 mm.

2.2. Tracer particle
When a relatively large particle (3–5 mm in diameter) is used as a tracer particle, it can be activated directly in the beam from a cyclotron. On the other hand, the amount of activity the tracer picks up per unit time is proportional to its cross-sectional area, and running the irradiation for longer is not effective because of the short half-life. Hence, small particles need to be made radioactive by other means. In this study, smaller silica sand particles (300 \(\mu\)m in diameter) were used, so that the particles cannot be activated directly. Furthermore, their low surface area prevents effective sorption-based methods for attaching the radioisotope. Therefore, a gamma alumina particle, of closely similar density and with \(^{18}\)F ions physically adsorbed onto its porous surface, was used as a tracer. This combination of the particle and tracer has been already applied in the different study and gave good results \[16\].

2.3. Experiment conditions
An acrylic cylinder was used as the container. The inner diameter was 140 mm. Toyoura sand weighing 1.8 kg was filled to 75 mm in height. The median diameter and diameter distribution were 210 \(\mu\)m and 110–300 \(\mu\)m.
respectively. The sand density was 2630 kg m$^{-3}$. The vibration was vertically added to the bottom wall. The waveform was sinusoidal.

3. Numerical simulation

3.1. Numerical method
Various experimental parameters, such as the size and aspect ratio of the powder bed, are considered in the present analyses. Numerical simulation is a convenient way to explore this parameter space. The DEM developed by Cundall and Strack [17] was adopted in this study. The motion equations are shown as follows:

$$m_p \frac{d\mathbf{v}}{dt} = \mathbf{F}_{pp} - m_p \mathbf{g}$$

(1)

$$I \frac{d\omega}{dt} = |\mathbf{F}_\tau| r_p$$

(2)

where $m_p$, $I$, $\omega$ and $r_p$ are the mass, velocity, inertia moment, angular velocity and radius of particle, respectively. $t$ and $g$ indicate time and gravitational acceleration. The particle-particle interaction force, $F_{pp}$, and $F_p$, are modeled with a linear spring and a dashpot system [17].

(normal direction)

$$\mathbf{F}_{pp} = k_p \mathbf{x}_n - \eta_n \frac{d\mathbf{x}_n}{dt}$$

(3)

(tangential direction)

$$\mathbf{F}_\tau = k_\tau \mathbf{x}_\tau - \eta_\tau \frac{d\mathbf{x}_\tau}{dt} |\mathbf{F}_\tau| < \mu_\tau |\mathbf{F}_{pp}|$$

(4)
In this model, the shape of a particle is spherical. In order to obtain the displacement, i.e. the values of $x_n$ and $x_l$, the overlap of two contacting particles is calculated from the positions of the centers of the two particles.

We carried out 2D simulations for the following two reasons. First, the aim of this study was to clarify the detailed mechanism of convective flow patterns in a vibrated powder bed, for which 2D is more suitable than 3D. Since it is necessary to know whether the two-dimensionally simulated flow behavior agrees with the three-dimensionally simulated one, a comparison with the experiment is needed. When the result simulated two-dimensionally disagrees with the experimental one, we can easily understand that the movement of particles in the experiment is three-dimensional. Furthermore, the difference between simulated and experimental results corresponds to the difference between 2D and 3D movements. This would give a clue to understanding the mechanism of the 3D flow structure. Second, when the DEM simulations are implemented under the same conditions as those of the experiment in the previous study (Kogane et al [12]), the number of particles becomes too large for the process to be implemented on a PC. The simulation code used was developed by the authors and has been already applied to many problems in various powder processes (e.g. [18, 19]).

### 3.2. Simulation conditions

The physical properties of particles used in the present simulations are shown in table 1. These properties correspond to those of our previous experiment [12]. It was reported that friction coefficients significantly affect the powder bed flow pattern [6, 10], but this effect was not examined in the present work, in which both particle-particle and particle-wall friction coefficients were set to 0.3. Here, it is difficult to obtain the exact value of the friction coefficient because the practical sand shape is not a perfect sphere but we used a spherical shape in our simulation model. 0.3 was commonly taken for the friction coefficient in various past studies with the DEM [20–22]. Therefore, the value was assumed to be 0.3.

Table 2 shows the sizes of the containers and powder beds. The sizes in Case 1 correspond to the cross section through the center line of the cylindrical container used in the experiment.
Because the significance of the powder bed size has not been sufficiently clarified previously, four powder beds were simulated in this study. In order to examine the similarity, the powder bed dimensions were changed with the aspect ratio held constant. The particle numbers were calculated from the bed size and the packing fraction for a fixed bed when randomly packed.

A vertical vibration was applied to the powder bed, and the vibration frequency changed under the condition of a constant value of the dimensionless acceleration, as defined in equation (6):

\[ \Gamma = \frac{A \omega^2}{g} \]  

where \( A \) is the vibration amplitude and \( g \) the gravitational acceleration. From equation (6), the amplitude becomes large when the frequency is small, because \( \omega = 2\pi f \) and the \( \Gamma \) value is constant. In such a case, the particles sometimes jump violently, then collide with the upper calculation boundary. This is not realistic. When the larger region in the vertical direction, the realistic simulation can be conducted. However, this is not efficient because we focus on the scaling relationship.

4. Results

4.1. Flow structures visualized with PEPT

Three typical flow structures of powder were obtained with the PEPT in our previous study [12] as shown in figures 2 to 4. Figures 2 to 4 shows the trajectories of a single tracer particle determined using PEPT. Figure 2 shows the trajectory at \( f = 60 \) Hz, \( \Gamma = 10 \). A doughnut-shaped circulation structure can be seen. Figure 2(b) shows that the flow pattern is almost axi-symmetric in the part of main circulation flow. Figure 3 shows the trajectory at \( f = 60 \) Hz, \( \Gamma = 2 \). The top surface became oblique as shown in figure 3(d). This flow structure was observed at low vibration frequency and low dimensionless acceleration. The flow structure was clearly three-dimensional as shown in figure 3(b), and lost much of its axi-symmetry.
Figure 4 shows the trajectory at $f = 100$ Hz, $\Gamma = 10$. The doughnut-shaped circulation became smaller and the outer circulation became larger compared with the case of figure 2. Figures 4(b) and (c) indicate that the flow is again almost axi-symmetric, as is the case in figure 2.

4.2. Space- and time-averaged velocity vectors

4.2.1. Influence of vibration frequency

To show the convective structures of the powder, the space- and time-averaged velocity vectors were calculated.

The DEM is a Lagrangian approach, and instantaneous positions and velocities of particles are obtained from the simulation. The trajectories of particles can be also calculated. However, it is difficult to understand the flow patterns directly from the Lagrangian data so we transformed the Lagrangian data to an Eulerian visualization.

First, the analysis domain was divided into small regions. Using the mesh structure so obtained, the average speed of all particles in each element of the mesh was calculated. Finally, the speeds of the powder flow were time-averaged until the averaged values become steady.

Figure 5 shows the averaged velocity vectors for Case 1. A pair of convective circulations can be observed at $f = 60$ Hz. Furthermore, downward flows can be seen near the side walls. This structure qualitatively agrees with the particle trajectories obtained with the PEPT as shown in figure 2(c). As the frequency increased, the paired circulations disappeared, as indicated in figures 5(c) to (d).

Figure 6 shows the flow patterns for the powder bed that is half the size of that in Case 1. Although a similar circulation structure is observed from $f = 60$ to 150 Hz, the pair of convective circulations disappeared at $f = 180$ Hz.

Figure 6 shows a strong average circulation, down at the center and up at the walls. In fact the averaging process masks the weaker secondary circulation which was seen clearly in figure 5: up in a region close to the wall but downward at the wall itself.
Figure 7 shows the flow patterns of the powder bed that is a quarter the size of that in Case 1. From $f = 60$ to 150 Hz, a similar circulation structure is observed. However, the pair of convective circulations disappeared at $f = 180$ Hz; the downward flows near the side walls remained.

The results of the simulated flow patterns of a powder bed whose size is an eighth of that in Case 1 are shown in figure 8. Large vectors can be seen in both the upper and lower regions at $f = 60$ Hz. Such vectors are not seen in other cases. These vectors indicate the behavior of particles bouncing from the top or bottom walls. No meaningful simulation could be obtained under the conditions of figure 8(a) because of ballistic effects. Although the flow structure of the paired convective circulations from $f = 120$ to 150 Hz can be seen, the pair of convective circulations disappeared at $f = 180$ Hz.

Summarizing these results, a twin-circulation structure was commonly observed at low vibration frequencies, except in Case 4. Because the particles reached the top wall of the simulation region in Case 4, a clear convective flow pattern was not observed. Furthermore, the convective flow pattern disappeared at high vibration frequency. The threshold value of the vibration frequency beyond which the convection disappears depended on the bed size. For example, convective flow was not observed at $f = 150$ Hz for Case 1, but the flow was observed in other cases. The convective structure does not occur in high frequency conditions. In this work, the vibrational acceleration was held constant, so that higher frequencies are associated with lower amplitudes, from equation (6). Since the microscopic motion of particles is then small, particles are not able to move beyond their immediate neighbors. As a result, the particles move only up and down.

4.2.2. Influence of bed size
Figure 9 shows the relationship between the vibration frequency and simulated average particle speed with various column sizes. Here, the average speeds were obtained with the velocity fields shown in figures 5 to 8. The symbols of square, diamond, triangle, and circle indicate $\Gamma = 2, 5, 8,$ and $10$, respectively. The data includes cases for which convective motion does not occur. In all cases, the average speed of convective powder flow decreases as frequency increases. Furthermore, the average speed increases with an increase in the vibration strength: $\Gamma$. 

Figure 7. Space- and time-averaged velocity vectors of particles for Case 3.
Figure 10 shows the relationship between vibration frequency and simulated convective velocity with various vibration strengths. The symbols of circle, triangle, diamond, and square indicate Cases 1, 2, 3, and 4, respectively. The value and tendency of the average speed are almost the same in all figures. This indicates that the convective speed of the powder is independent of the column size under the present range of conditions.

4.3. Equation to estimate convective flow speed

The average speed of the powder is three-dimensionally plotted against vibration frequency and vibration strength in figure 11. The blue and red circles indicate Cases 1 and 2, and the green and yellow diamonds indicate Cases 3 and 4, respectively. From the figure, most data are on a single plane. By using this result, the following approximate equation was derived:

$$V = 14.04f^{1.7}$$

This equation is expressed by the gray plane in figure 11. Accordingly, the convective flow speed can be calculated with equation (7) regardless of the column size under the present conditions. The simulated values are distributed within approximately ± 42% of the value of equation (7).

As expressed in equation (7), $\Gamma$ is defined by two variables, i.e. amplitude and frequency. This indicates that the average speed is not a function of the frequency alone. Figure 12 shows the relationship between the amplitude and simulated convective speed. From this figure, most data are on a straight line. By using this result, the following approximate equation was derived:

$$V = 313.39A^{1.4}$$

Using this equation, the following equation can be derived:

$$V = 313.39\left(\frac{\Gamma g}{4\pi f^2}\right)^{1.4} = 42.18f^{1.4}$$

Equation (9) is almost in accord with equation (7). Therefore, it is conjectured that simulated convective speed strongly depends on amplitude.
Figure 9. Effect of dimensionless acceleration on average speed.

Figure 10. Effect of bed size on average speed.
Figure 11. Relationship among frequency, dimensionless acceleration, and average speed.

(a) Full view of all data points

(b) From the viewpoint giving data assembly a line-like appearance

Figure 12. Effect of Amplitude on average speed.
4.4. Comparison of convective speed estimated by DEM simulation with measured speed

Figure 13 shows the relationship between average speed and frequency of vibration. Six graphs are the results at $\Gamma = 2, 4, 5, 10, 12,$ and 14, respectively. The broken line represents the average speed calculated by equation (7). The simulated results when the frequency is less than 90 Hz and dimensionless acceleration greater than 5 agree well with the experimental data in figure 13. The simulated speed is in good agreement with the experimental data at less than 75 Hz at $\Gamma = 10$ in figure 13(c). However, the experimental speed becomes larger than the simulated one at more than 80 Hz. Accordingly, the agreement between simulation and experiment is poor at high frequency. Though the simulated result at $\Gamma = 12$ and $f = 100$ Hz agrees well with the experimental one in figure 13(b), the simulated result at $\Gamma = 10$ and $f = 100$ Hz agrees poorly with the experimental one in figure 13(c). The similarity between 2D simulation and 3D experiment breaks down under conditions in which

Figure 13. Comparisons of equation (7) with experimental data.
the axi-symmetry apparent in figure 2 is weaker. In that case, the motion in the experiment becomes less ordered and more three dimensional, with a stronger circumferential component.

5. Conclusions

In order to clarify the fundamental mechanisms of power flow in a vibrated powder bed, numerical simulations based on the DEM were carried out under the following conditions: vibration frequency: 50 to 250 Hz, vibration strength: 2 to 14 [-]. From the simulated results, it was found that the average convective speed of particles in this cylindrical geometry is independent of the column size if the dimensionless acceleration is held constant. By using these results, the following approximate equation was derived for average convective speed.

$$V = 14.04 f^{1.7} A^{-2.7}$$

This equation corresponds to the following equation derived from the relationship between the amplitude and the simulated average convective speed.

$$V = 313.39 A^{1/4}$$

Therefore, it is conjectured that the average convective speed strongly depends on the vibration amplitude. The average speeds obtained using this equation were compared to the values measured using PEPT. When the frequency was less than 90 Hz and dimensionless acceleration was greater than 5, good agreement between the simulated and measured values was obtained. However, the simulated average speeds were much lower than the measured values at higher frequencies and low dimensionless accelerations. The simulations were conducted in 2D, and therefore give a poor estimate of average convective speed for situations in which the flow is three-dimensional in character. While 3D flow became dominant in the experiments at high frequency, 3D flow was not represented in the 2D simulations. As a result, the estimated average speeds were much smaller than the experimental results.

It was found that the average convective speed of particles in a vibrated powder bed can be expressed as a function of the vibration amplitude alone. Furthermore, it is almost independent of the column size. These results would contribute a lot to the dimensional analysis which will be conducted as a next step. However, the applicability of the present conclusions would be restricted to 2D flow. Accordingly, 3D analyses will be conducted in the future work.

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