Propagation of the Alfvén Wave and Induced Perturbations in the Vicinity of a 3D Proper Magnetic Null Point

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Abstract

The aim of the present work is to study the propagation of the Alfvén wave around a 3D proper magnetic null point and its accompanying perturbations. In this line, the shock-capturing Godunov-type PLUTO code is used to solve the magnetohydrodynamic (MHD) equations. It is found that the Alfvén wave propagates toward the null point at the fan plane and the wave–wave interaction could be the main reason for the Alfvén wave energy dissipation, while, at other planes including the spine axis, the Alfvén wave spreads toward the spine axis and accumulates along it. Furthermore, the fast magnetoacoustic wave moves toward the null point at the fan plane and also at two other planes including the spine axis. The fast magnetoacoustic wave also refracts around the null point without any significant accumulation along the spine axis. Finally, the slow mode moves toward the null point at the fan plane. It is illustrated that, at the $x,z$ plane, in addition to the refraction of the slow wave around the null point, there is an accumulation of the slow mode along the spine axis, while, at the other plane including the spine axis, the slow magnetoacoustic wave refracts around the null point. Moreover, it is found that the 3D structure results in the high amplitude of MHD wave energy in comparison with the 2.5D structure. Finally, it is found that the Alfvén wave gives its energy to the induced fast and slow magnetoacoustic waves and they have more time to heat the plasma.

Unified Astronomy Thesaurus concepts: The Sun (1693); Magnetohydrodynamics (1964); Alfvén waves (23); Solar coronal waves (1995)

1. Introduction

Three-dimensional (3D) magnetic null points are in abundance in the solar corona (Longcope et al. 2003; Close et al. 2005). The structure of the magnetic field in this case involves two characteristics that are the fan plane and the spine axis. It must be noted that a pair of magnetic field lines approach (keep out) the null point with opposite directions along the spine axis. However, 2.5D X-point magnetic reconnection models were studied in previous works by Sabri et al. (2019, 2020). It must be considered that results for 2D and also 2.5D structures are no longer applicable in such a 3D magnetic field topology. Identifying the area in which the main dynamical phenomena and energy release may take place is a nontrivial problem. There is an observational proof that magnetic reconnection at the presence of 3D null points might be important in some solar flares (Fletcher et al. 2001). In addition, magnetic reconnection around a 3D null point in Earth’s magnetotail has recently been observed by the Cluster spacecraft (Xiao et al. 2006). While current density growth at the presence of the 3D magnetic null point has also been detected in a laboratory plasma (Bogdanov et al. 1994).

Due to the structure of the emission in the solar corona, observational studies showed the presence of 3D magnetic null points (Filippov 1999; Masson et al. 2009). Then, to comprehend the plasma heating process, it would be better to pursue real 3D magnetic field structures instead of simple 2D and 2.5D structures. MHD wave propagation in the vicinity of magnetic null points has been investigated extensively in the 2D regime. That studies gave an initial physical concept of the interaction of MHD waves with the magnetic null points. For a more complete understanding, we try to evaluate the key results of wave propagation around a 3D model. The importance of 3D magnetic null points is not limited to astrophysical plasmas but they play a key role in the Earth’s magnetosphere and also some laboratory plasmas.

Furthermore, the dissipation of Alfvén waves is one of the key mechanisms of the non-radiative heating of the astrophysical plasmas, such as in the solar corona and the solar wind. Because of the strong magnetic field in such plasmas, Alfvén waves can transfer both mechanical and electromagnetic energy far away. Due to the small dissipation terms in corona (resistivity and viscosity), inhomogeneities in the background magnetic structure are important gradients for the wave dissipation. So, interactions between waves and the inhomogeneities can transform the wave energy to small scales, where it can be significantly dissipated before leaving the system. In 2D magnetic field structures, two main dissipation mechanisms have been extensively discussed: phase mixing and resonant absorption (Mok & Einaudi 1985; Steinolfson 1985; Davila 1987; Hollweg 1987a; Califano et al. 1992, 1990; Lee & Roberts 1986). The phase-mixing effect bends wavefronts due to the differences of phase velocity in the transversal direction to the wave propagation. Resonant absorption focuses the wave energy in a narrow layer where local frequency is equal to the characteristic frequency.

Tsiklauri & Nakariakov (2002) and Tsiklauri et al. (2003) studied the evolution of both Alfvénic and magnetoacoustic waves in the presence of a 1D inhomogeneous background...
investigations that are reported by Wedemeyer-Bohm & van der Voort (2009), indicated that the rotational motions, which can be observed in the lower layers of the solar atmosphere, generated magnetic twister-like motions in the transition region and corona. Instead, the tornado-like motions were described as torsional Alfvén waves.

Alfvén wave propagation in inhomogeneous magnetized plasma can generate magnetoacoustic waves by the nonlinear magnetic pressure gradients (Nakariakov et al. 1997; Verwichte et al. 1999; Botha et al. 2000; Tsiklauri et al. 2001; Thurgood & McLaughlin 2013; Sabri et al. 2018). The wave couples to the density and other compressive perturbations in the plasma that result in the nonlinear wave dissipation. The Alfvén waves, when coupled to compressive modes, can play an important role in coronal heating. Compressive modes via the resonant creation are able to decay the Alfvén wave energy. Mikhalyaev & Bembotov (2014) analytically pursued the resonant excitation of compressive waves by nonlinear coupling of two torsional Alfvén waves propagating in the opposite directions. Therefore, the interaction of nonlinear torsional Alfvén waves and compressive magnetoacoustic waves were inevitable.

In this work, the role of a 3D magnetic null point where the magnetic filed disappears has been investigated. A key question is how do external disturbances behave as they move and dissipate in extreme inhomogeneities around a 3D magnetic null point. The behavior of MHD waves at the vicinity of the 3D magnetic null points has only been partially investigated. In this paper, we would like to study the Alfvén wave dynamic near the 3D magnetic null point. As a matter of fact, the main aim of keeping the same initial conditions in comparison with our previous studies in 2.5D (Sabri et al. 2018, 2020) is to study only the 3D effect on the problem.

McLaughlin et al. (2008) studied the behavior of the Alfvén wave near the 3D magnetic null point via WKB approximation. It was suggested that the Alfvén wave moved at the equilibrium Alfvén speed along the magnetic field lines and focused along the fan and spine of the magnetic null point as 2D results.

In the 3D structure, Alfvén wave coupling to the fast and slow magnetoacoustic modes was investigated by Shelyag et al. (2016), Cally & Goossens (2008), and Felipe (2012) without considering a magnetic null point or strong magnetic field gradients. They demonstrated that mode coupling has the strong dependence on the relative orientation of the wavevector, magnetic and gravitational fields. McLaughlin et al. (2008) made a WKB analysis for a linear 3D magnetic null point and found that fast magnetoacoustic wave energy focused at the null point with a current density generation. Current density excitation by the propagation of the Alfvén wave in the vicinity of a 3D magnetic null point was also studied by Sabri et al. (2021). Thurgood & McLaughlin (2012) also performed numerical simulation in the presence of a 3D magnetic null point and recognized a coupling between the fast magnetoacoustic wave and Alfvén wave. McLaughlin et al. (2008) and Thurgood & McLaughlin (2012) considered zero plasma beta so the only energy accumulation way was an enhance in the current density.

The aim of this paper is to study the nonlinear generation of magnetoacoustic waves by propagation of the nonlinear Alfvén wave, their interactions, and transport and transfer of the energy in 3D background magnetic field structure. Defining the specific dissipation mechanism is not our aim of study while we focus on the propagation of the waves. Since coronal
plasma is dominated by the magnetic field, a low-$\beta$ assumption instead of the incompressibility is more suitable. We consider the propagation and dissipation of the Alfvén wave in 3D inhomogeneous equilibrium magnetic field structure with low-$\beta$ assumption. We used the MHD equations while neglecting the gravity and viscosity terms that is common in investigation of low-$\beta$ coronal plasma. In a compressible plasma both Alfvén and magnetoacoustic perturbations can propagate then we will peruse the behavior of three different models. With considering an initial Alfvén wave, there are no initially fast or slow magnetoacoustic waves. As a matter of fact, it is found that the propagation of nonlinear Alfvén waves excites magnetoacoustic waves (see also Murawski 1992a; 1992b). These indirectly generated waves propagate across the magnetic field lines and accumulate around the null point.

The remainder of this paper is organized as follows. In Section 2, we discuss the model used in the simulation of the Alfvén wave propagation toward the magnetic null point. In Section 3, we have the numerical results and discussion. Finally, in Section 4, we provide a brief conclusion and outlook.

2. Model

We use magnetohydrodynamic solver PLUTO code to solve the resistive MHD equations (Mignone et al. 2007):

$$\rho \left[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = \left( \frac{1}{\mu} \nabla \times B \right) \times B - \nabla P,$$

$$\frac{\partial B}{\partial t} = \nabla \times (V \times B) + \eta \nabla^2 B,$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0,$$

$$\frac{\partial P}{\partial t} + (V \cdot \nabla) P = -\gamma P \nabla \cdot V + (\gamma - 1) \frac{\eta}{\mu} \nabla \cdot \nabla \times B + \frac{(\gamma - 1) \eta}{\mu} (\nabla \times B)^2,$$

$$\nabla \cdot B = 0,$$

where $\rho$, $V$, $B$, and $P$ respectively represent the mass density, plasma velocity, magnetic field, and plasma pressure. The constants $\mu$ and $\gamma$ are the magnetic permeability and the ratio of the specific heats, where we have $\mu = 4\pi \times 10^{-7} \text{Hm}^{-1}$, $\eta = 10^{-4}$ and $\gamma = 5/3$. It must be noted that due to a parameter study, we have estimated the numerical resistivity to be less than $10^{-6}$. The equations are solved in nondimensional form that scaled to typical coronal parameters.

We consider a static ($V_0 = 0$) background plasma with a magnetic field given by

$$B_0 = [x, \epsilon y, -(1+\epsilon) z],$$

in which the eccentricity parameter $\epsilon$ defines the direction of predominantly field lines. It must be noted that, in this study, $\epsilon = 1$ is applied. It results in a proper magnetic null point that has azimuthal symmetry about the spine axis without any preferred direction of field lines.

In order to control the solenoidal constraint for the magnetic field, the constrained transport (CT) formulation with the face-centered magnetic field is used. Furthermore, this magnetic field can be expressed in terms of a vector potential as follows:

$$A_0 = [\gamma y, -\epsilon xz, (1-\epsilon) xy].$$

The specific quantitative values of the initial equilibrium are to be consistent with the typical parameters of the solar coronal plasma: $\rho_0 = 10^{-12} \text{kg m}^{-3}$, plasma $\beta = 0.02$, and $C_s = 0.129 \text{ Mm s}^{-1}$. It must be noted that by defining the three main parameters $\rho_0$, $L_0 = 10^8 \text{ cm}$, and $V_0 = 10^8 \text{ cm s}^{-1}$ which constitute a characteristic density, length scale, and velocity, all other physical quantities of the system are made dimensionless.

Open boundaries are applied that prevent reflection from the boundaries. The simulation domain has been considered as $(-6, 6) \times (-6, 6) \times (-3, 3) \text{ Mm}$ with $1500 \times 1500 \times 600$ grid points. Furthermore, a stretched grid has been applied to gather the majority of the grid points near the magnetic null point. Therefore, we set $1200 \times 1200$ grid points in the numerical domain $(-3, 3) \times (-3, 3) \text{ Mm}$ with an effective resolution of $\delta x \approx \delta y \approx \delta z \approx 1/100 \text{ Mm}$.

Whereas we want to study about the behavior of the MHD waves around the 3D magnetic null point, we introduced a magnetic-flux-based coordinate system in which each direction described a definite MHD wave mode. The $A_0$ direction corresponds the Alfvén wave, longitudinal perturbations along $B_0$ associates with slow mode, and $C = A_0 \times B_0$ direction allocates to the fast mode.

3. Numerical Results and Discussion

3.1. How the Alfvén Pulse acts

We initially consider a single wave pulse at a finite distance from the magnetic null point. At $t = 0$ the profile

$$V \cdot A = v_0 = A_0 \sin \left[ \frac{\pi}{2} \left( \frac{x^2 + y^2}{r_0^2} - r_1 \right) \right] \frac{B_x}{(B_y)^2 + (B_z)^2},$$

is kicked at a definite distance from the magnetic null point, where we have set $V \cdot C = V \cdot B = 0$, and $A_0 = 1$ as the initial amplitude of the circular Alfvén pulse. The typical values of the initial pulse are defined as $r_0 = 1 \text{ Mm}$ and $r_1 = 5 \text{ Mm}$.

The previous 2D studies gave an initial grounding about the physics of null points. It must be noted that we pursued the behavior of this initial Alfvén pulse in 2.5D magnetic structure Sabri et al. (2018, 2020). In 2.5D structure, it is found that the Alfvén wave propagates toward the magnetic null point and accumulates toward the separatrices without crossing them. Moreover, the propagation of the Alfvén wave around a 2.5D magnetic null point results magnetoacoustic wave perturbations and plasma flows.

A nonlinear Alfvén wave around the 2D magnetic null point, due to the nonlinear magnetic pressure gradient (ponderomotive force), results in excitations of the magnetoacoustic waves (Thurgood & McLaughlin 2013, Sabri et al. 2018, 2019). Sabri et al. (2018) found that generations of the compressible magnetoacoustic waves impact on the dynamics of the main Alfvén wave by variation of the plasma density (phase mixing).

If magnetic null points have 3D structure in reality, then it could be important to extend our previous studies for a 3D structure. In this work, the behavior of Alfvén waves around a 3D magnetic null point and its following effect on plasma parameters are studied. Numerous works have studied the MHD waves behavior around a 3D equilibrium magnetic structure, but most of them focused on identifying the regions where magnetic reconnection takes place rather than the
behavior of MHD wave propagation (Pontin & Galsgaard 2007; McLaughlin et al. 2009; Thurgood et al. 2017, 2018).

Snapshots of the propagating Alfvén wave with the velocity component $v_A$ are shown in Figure 1. According to panel (a) of Figure 1, the Alfvén wave separates into two waves with opposite propagation directions. Due to the increasing Alfvén speed further away from the null point, a wave that propagates outward is more intense than the incoming wave. This property is called anisotropy and is a natural characteristic of any MHD turbulence in the presence of the magnetic field inhomogeneity. This strong turbulence anisotropy in the presence of the coronal magnetic null point results in drastic wave energy dissipation via wave–wave interaction. This indicates that the fan plane, where there are increasingly steep gradients, is a likely region for plasma heating events. As it is evident in panel (c) of Figure 1, there is a refraction of the wave around the null point. This could be related to the fact that there is a coupling between MHD waves. In fact, refraction is one of the main characteristics of the magnetoacoustic waves. Since the Alfvén speed is zero at the null point, it is expected that the Alfvén wave does not go through the null point. According to panel (d) of Figure 1, at the fan plane, the Alfvén wave accumulates around the null point and deposits its energy there. It would be better to say that the asymmetry in panel (d) of Figure 1 could be related to the accumulation of the wave and the deposit of its energy there.

The dynamics of nonlinear Alfvén waves has been actively investigated both analytically and numerically by, e.g., Cohen & Kulsrud (1974), Nakaraikov et al. (1997), and Sabri et al. (2018). The nonlinear evolution of the Alfvén wave leads to the gradients in the horizontal direction in the background configuration. These background variations significantly distort the Alfvén wave that loses its initial circular shape and accumulates along the separatrices. It is interesting that it does’ take place in the 3D structure. In the 2.5D structure, it was found that the Alfvén wave lost its initial circular shape and accumulated along the separatrices (Sabri et al. 2018). In this structure with a 3D magnetic null point, maybe because of wave–wave interaction, the wave dissipates soon and it does not experience more nonlinear effects. However, it needs to be explored further. Interestingly, the Alfvén wave moves in the fan plane and accumulates around the magnetic null point, which is not possible for the Alfvén wave in 2D magnetic structure.

Our model enables us to investigate where the MHD mode coupling takes place and, in particular, if this is in the vicinity of the null point or not. In according to panels (a) and (b) of Figure 1, the wave focuses around the null point without any refraction effect, it could be deduced that no mode coupling to the fast magnetoacoustic wave occurs. But panel (c) of Figure 1 shows the refraction of some part of the wave around the null point that is one of the main properties of the magnetoacoustic waves. It could be related to coupling of the MHD waves. Note that this behavior, which somehow shows the coupling of the waves, was not shown in the case of propagation of Alfvén wave around a 2.5D magnetic null point (Sabri et al. 2018).

Another point is that the incoming wave never arrives at the null point as is observable in panels (c) and (d) of Figure 1. Since the Alfvén speed is zero at the null point, then it could be asserted that the wave that accumulates around the null point without crossing the null is the Alfvén wave. From Figure 1, it is clear that the wave becomes weaker over time, so that at $t = 1.4s$ (panel (d)), the wave is narrow with a very weak amplitude, meaning that the wave has lost energy supply around the null point.

In addition to the evolution of the Alfvén wave at the fan plane, the behavior of the Alfvén wave at two other planes that include the spine axis is also illustrated. At first, evolution of the Alfvén wave at the $xz$ plane is depicted in Figure 2. Panels (a) and (b) of Figure 2 show accumulation of the wave along the spine axis and also around the null point. It must be noted that magnetic field lines along $z$-axis that near or relinquish the
magnetic null point are separated by the spine axis. This means that there is coupling between the MHD waves. Similar to panel (d) of Figure 1, which illustrates the accumulation of the Alfvén wave around the null point, panel (c) of Figure 2 shows the same behavior. In other words, according to panel (c) of Figure 2, the Alfvén accumulates along the spine axis that is one of the main properties of these waves. Above all, besides the coupling of the waves at the middle of the simulation time, finally there is just the Alfvén wave.

Finally, the behavior of Alfvén wave at the $y,z$ plane that again includes the spine axis is depicted in Figure 3. In panel (a) of Figure 3, in addition to the accumulation of the Alfvén wave along the spine axis, refraction of the wave around the null point happens. But again, there is just accumulation of the wave along the spine axis without any refraction around the null point (panel (c) of Figure 3).

In the proceeding subsection, the plan is to study the generation of the fast and slow magnetoacoustic waves due to the propagation of the nonlinear Alfvén wave. Considering this point that the fast waves are transverse and slow waves are mainly longitudinal, the behavior of fast and slow magnetoacoustic waves will be explored.

### 3.1.1. Induced Perturbations

Propagation of the nonlinear Alfvén wave leads to the creation of secondary waves. This is due to nonlinear coupling of the transverse variables to the longitudinal ones in the MHD equations. So, total pressure perturbation generates the longitudinal (slow magnetoacoustic) waves and also fast waves that are polarized in the direction perpendicular to the flux surfaces.

Creation of the normal/perpendicular perturbations can be ascribed to the nonlinear ponderomotive force, related to the gradients of the perturbed perpendicular Alfvén speed (Nakaraikov et al. 1997). On the other hand, generation of the parallel flows may be associated with the nonlinear ponderomotive force that is induced by the gradients of perturbed parallel Alfvén speed. Modification of the local Alfvén speed happens due to the ponderomotive excitation of magnetoacoustic waves that affects the Alfvén wave itself and gives rise to wave steepening. Now we pursue fluid velocity perturbations in the other perpendicular directions $\hat{B}$ and $\hat{C}$.

In the context of the present study, the nonlinear aspect of the wave propagation is taken under consideration. However, there are various forces connected to the nonlinear interaction of the Alfvén wave (Suzuki 2011; Thurgood & McLaughlin 2013) on compressive flows (Vahseghani Farahani et al. 2011) and on itself (Vahseghani et al. 2012). Regarding the null point, Thurgood & McLaughlin (2013) focused on the nonlinearly induced fast magnetoacoustic waves due to the ponderomotive force (Verwichte et al. 1999). This force leads to the variations of magnetic pressure and the plasma density of the form

$$ F = (\nabla \times \hat{B}) \times \hat{B} = (\hat{B} \cdot \nabla)\hat{B} - \frac{1}{2} \nabla (\hat{B} \cdot \hat{B}). \quad (9) $$

The first term on the right-hand side in the above equation is a magnetic tension, while the second term corresponds to a magnetic pressure gradient. Both the magnetic tension and the magnetic pressure gradient lead to an isotropic wave that propagates in both the longitudinal and transverse directions (Thurgood & McLaughlin 2013). The effect of nonlinear interaction of compressive torsional waves has not been studied in detail yet. In particular, the effects regarding the transverse profile of the torsional wave need to be investigated in more detail.

Propagation of the Alfvén wave through an inhomogeneous plasma causes gradients in the magnetic field lines, which also causes the local Alfvén speed to depend on the specific field line its following. In Figure 4, the perpendicular component of the wave velocity (fast wave) at the fan plane is shown. Initially, there is no fast magnetoacoustic wave because the initial pulse is Alfvénic. Shortly after the start of the simulation, however, the fast magnetoacoustic wave is excited, as can be seen at $t = 0.02$ s panel (a) of Figure 4. The initial form of the produced magnetoacoustic wave is circular just like the primitive Alfvén pulse. It is evident that the fast mode moves toward the null point and then focuses its energy there. Refraction happens at $t = 0.8$ s (panel (c) of Figure 4), which also results in the almost asymmetrical behavior of the wave. Finally, the fast magnetoacoustic wave accumulates around the null point and gives its energy there. In other words, the fast magnetoacoustic wave focuses around the null point and gives its energy around the null point at the fan plane. The
accumulation of the wave and the losing of its energy could be the reason for the asymmetry in the last panel of Figure 4. Since the Alfvén speed is zero at the null point, then the Alfvén wave does not pass through the null point. In according to panel (d) of Figure 4, due to the coupling of the MHD waves, the last accumulation of the wave without any refraction could be associated with the Alfvén wave while not going through the null point. Interestingly, it means that, in 3D topology, there is the coupling of the Alfvén and fast magnetoacoustic waves in the nonlinear regime, which did not happen in the 2.5D structure (Sabri et al. 2018). The 3D magnetic null points are also possible locations of the heating effects. It is interesting to note that the fast magnetoacoustic wave lost its energy around the null point at an earlier time than the increase of nonlinearity and the impact of it on the wave. Thus, we can say that wave—wave interaction (turbulence) leads to the MHD wave dissipation in the 3D inhomogeneous magnetic structure. As we know, MHD waves have been divided into two waves with opposite directions of propagation and MHD waves are always accompanied with the turbulence effect. When the intensity of the outgoing wave is so much higher than the incoming wave, the fast magnetoacoustic wave dissipation takes place and results in the wave dissipation.

In addition to the fan plane, behavior of the fast magnetoacoustic wave at two other planes including the spine axis is also pursued. At first, the propagation of fast magnetoacoustic wave at the $x_z$ plane is plotted in Figure 5. As is evident in panels (a)–(c) of Figure 5, there is not any significant accumulation along the spine axis. In fact, most of the fast magnetoacoustic wave refracts around the null point and accumulates around the null point without any dominant accumulation along the spine axis.

Furthermore, the behavior of the fast magnetoacoustic wave propagation at the $y_z$ plane is also depicted in Figure 6. Panels (a)–(c) of Figure 6 show that the fast magnetoacoustic wave moves toward the null point and accumulates around the null point due to the refraction effect. In fact, there is not any accumulation of the fast wave along the spine axis at the $y_z$ plane and whole of the fast wave focuses around the null point and gives its energy to the null point.

Creation of the slow magnetoacoustic waves is due to the solar atmospheric conditions. This wave is a consequence of the finite plasma-$\beta$ regime. Under such conditions, the coupling between slow and fast magnetoacoustic modes becomes important. Figure 7 shows the evolution of the parallel speed $V_p$, i.e., a proxy of the induced slow magnetosonic wave traveling around the null point. Clearly, initially (at $t = 0$) there is no parallel velocity perturbation, due to the initial Alfvénic pulse. Figure 7 shows the presence of the slow magnetoacoustic wave since the parallel flows are nonzero toward the null point. The wave propagates in the fan plane independently of the driving Alfvén wave. It moves along the null point and wraps around it. Since the proper 3D magnetic null points are transversely inhomogeneous, the net ponderomotive force is nonzero and it leads to further excitations in this direction that are observable in panels (b), (c), and (d) of Figure 7. This is an important feature that has not been reported in previous studies.

The behavior of the slow magnetoacoustic wave at two other planes that include the spine axis is also investigated. At first, the propagation of the slow mode at the $x_z$ plane is illustrated in Figure 8. In panel (a) of Figure 8, the slow magnetoacoustic wave spreads along the spine axis. The point is that, in addition to the accumulation of the slow mode along the spine axis, there is refraction of the wave around the null point in panels (b), (c), and (d). But, finally the slow magnetoacoustic wave focuses along the spine axis and loses its energy along the spine axis as seen in panel (f) of Figure 8. It must be noted that the fast magnetoacoustic wave at the $x_z$ plane refracts around the null point without any significant accumulation along the spine axis. The slow mode shows the strange and different behavior.
and accumulates along the spine axis at the $x,z$ plane without any significant accumulation around the null point.

Moreover, the behavior of the slow magnetoacoustic wave at the $y,z$ plane is also pictured in Figure 9. Panels (a)–(c) of Figure 9 illustrate that slow magnetoacoustic wave at the $y,z$ plane moves toward the null point and refracts around it. On the contrary to what happens at $x,z$ plane, the slow magnetoacoustic wave accumulates around the null point and there is not any significant accumulation along the spine axis. Furthermore, it must be noted that, due to the dependency of the slow magnetoacoustic wave on the plasma $\beta$, the behavior of this mode will be more complicated than other waves. Some asymmetry along the $x$- and $z$-axes that are respectively
observable in some panels of Figures 7 and 9 could be associated with this complexity.

Above all, at the fan plane, accumulation of three whole waves happens around the null point due to the wave−wave interaction, while at two other planes, including the spine axis for the Alfvén wave, focusing of the wave takes place along the spine axis without crossing it but, for magnetoacoustic waves, most of the wave accumulates around the null point due to the refraction effect that is one of the main characteristics of the magnetoacoustic waves.

3.2. Transport and Transfer of Energy

In astrophysical plasmas, such as solar corona, 3D magnetic null points are very complicated. In such points, specifying the areas where dynamic phenomena and energy release may happen is a key and nontrivial problem. The kinetic energy is constituted by two factors; magnetic and acoustic. Evolution of the Alfvén wave kinetic energy is presented in panel (a) of Figure 10. After a finite time, the whole initial energy of the Alfvén wave is transferred. This means that a vast amount of energy has been transferred to the medium without importing any. In comparison with the 2.5D structure investigated previously by Sabri et al. (2018), the overall behavior of the Alfvén wave kinetic energy is somehow the same but there is an important difference about the amount of energy. It is so interesting that Alfvén wave kinetic energy strength in this 3D structure is around 10^6 times higher than the Alfvén wave kinetic energy for 2.5D. This means that, while every parameter is the same, in the 3D structure, the amount of the energy becomes dominant.

After the Alfvén wave has transferred its energy to the plasma medium, the Alfvén wave does not gain any further energy. This shows that, since the system is adiabatic, the only player for energy dissipation could be the nonlinearity and resistivity. The energy of the induced waves would be...
proportional to the energy loss of the Alfvén wave. Kinetic energy of the perturbations that include fast and slow magnetoacoustic waves is pictured in panel (b) of Figure 10. Since there are not any magnetoacoustic waves at first, perturbation kinetic energy is zero at the initial time step. Propagation of the Alfvén wave around the 3D magnetic null point is accompanied by the perturbations of fast and slow magnetoacoustic waves. So, the energy profile of the perturbations’ kinetic energy is similar to the Alfvén wave kinetic energy. It means that perturbations gain their energy from the Alfvén wave and eventually they give their energy to the plasma also.

Panel (c) of Figure 10 shows the time evolution of the magnetic energy. This energy also has high amplitude in comparison with the same plot for the 2.5D study (Sabri et al. 2018). It is interesting that magnetic energy in the 3D structure is around $10^{12}$ time higher than in 2.5D. This could mean that 3D structure has dominant magnetic energy as expected. The fast magnetoacoustic mode is increasingly dominated by magnetic energy while the slow mode is dominated by acoustic energy. As we see in panels (c) and (d) of Figure 10, fast and slow waves have time for enhanced energy and heat the plasma.

4. Conclusions

We have investigated the behavior of the Alfvén wave and nonlinear driving magnetoacoustic waves in the vicinity of a proper 3D magnetic null point. Nakaraikov et al. (1997) showed that Alfvén wave propagation, even with a small amplitude, excited magnetoacoustic wave propagation by transverse inhomogeneity. We consider the initial pure Alfvén wave and want to study excitation of the magnetoacoustic waves in a 3D equilibrium magnetic structure. This wave propagates along the magnetic field lines preserving its characteristics accumulating along the null point at the fan plane and spreads along the spine axis without crossing it at two other planes with the spine axis. At the same time, however, the Alfvén wave produces nonlinear perturbations where the nonlinear magnetoacoustic waves are created and directed toward the null point at the fan plane and they also refract along the null point at two other planes including the spine axis where some part of the slow magnetoacoustic wave spreads along the spine axis at the $x,z$ plane. As the initial Alfvén pulse nears the null point, it experiences steepening, which triggers magnetoacoustic waves. This is a sort of transfer of energy from the nonlinear Alfvén wave to the fast and slow magnetoacoustic waves. From this, one may deduce that the Alfvén wave gives away all of its energy without gaining any.

From the results explained above, it has been seen that, when a fast magnetoacoustic wave propagates near a magnetic X-type neutral point, the wave wraps itself around the null point due to refraction (McLaughlin & Hood 2004, 2006a; Thurgood & McLaughlin 2012). It is clear that the refraction of the wave focuses the energy of it toward the null point. As seen from the numerical simulation results, the wave continues to wrap around the null point, again and again. The physical significance of this is that any fast magnetoacoustic disturbance in the neighborhood of a neutral point will be drawn toward the region of zero magnetic field strength and focus all of its energy at this point. Therefore, it can be said that the propagation of the fast wave is completely dictated by the Alfvén speed profile.

The Alfvén wave energy is constituted by kinetic and magnetic components; this energy supplies the energy regarding the fast and slow magnetoacoustic waves. At early stages, the energy of the fast and slow magnetoacoustic waves possess a similar behavior although the increase in the magnetoacoustic energy is much more pronounced. This is in a sense that, in 3D structure, the energy perturbations of the magnetoacoustic wave is always increasing.

Finally, the main point of this study was to explain how the MHD waves theory around a 2D potential magnetic null point extends to a proper 3D potential magnetic null point. It is found that nonlinear effects of Alfvén wave propagation in 3D are mostly consistent with the 2D models such as generation of the fast and slow disturbances and accumulation of most of them around the magnetic null point. The main result of this study may be summarized as follows.

1. In 3D, the null point at the fan plane is a possible location for the accumulation of the MHD waves. even for the Alfvén wave that accumulates around the null point and misses its energy due to the wave–wave interaction.

2. The nonlinear Alfvén wave propagation, due to the ponderomotive force, excites longitudinal daughter disturbances. At planes other than the fan plane that include spine axis, the Alfvén wave accumulates along the spine axis while the fast magnetoacoustic wave refracts along the null point even in the presence of the spine axis. It can be concluded that the spine axis is the preferential direction of the Alfvén wave accumulation.

3. Due to the transverse inhomogeneities that result in the nonzero ponderomotive force, further excitations of the slow magnetoacoustic waves took place especially at the fan plane.

4. It was found that 3D structure results in high amplitude of the MHD wave energy in comparison with the similar.
2.5D structure. It was also found that the Alfven wave gives its energy to the induced fast and slow magneto-acoustic waves while they have more time to heat the plasma.

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**References**

Bisnovatyi-Kogan, G. S. 2007, *MNRAS*, 376, 457  
Bogdanov, S. Y., Burilina, V. B., Markov, V. S., & Frank, A. G. 1994, *JETPL*, 59, 537  
Botha, G. J. J., Arber, T. D., Nakariakov, V. M., & Keenan, F. P. 2000, *A&A*, 363, 1186  
Califano, F., Chiuderi, C., & Einaudi, G. 1990, *ApJ*, 390, 560  
Califano, F., Chiuderi, C., & Einaudi, G. 1992, *ApJ*, 365, 757  
Cally, P. S., & Goossens, M. 2008, *SoPh*, 251, 251  
Cargill, P. S., & Goossens, M. 2008, *SoPh*, 251, 251  
Cranmer, S. R., & van Ballegooijen, A. A. 2005, *ApJS*, 156, 265  
Davila, J. M. 1987, *ApJ*, 317, 514  
De Pontieu, B., Carlsson, M., Rouppe van der Voort, L. H. M., et al. 2012, *ApJ*, 752, L12  
Fletcher, L., Metcalf, T. R., Alexander, D., Brown, D. S., & Ryder, L. A. 2001, *ApJ*, 544, 451  
Gruzszecki, M., Murawski, K., & Ofman, L. 2008, *A&A*, 488, 757  
Heyvaerts, J., & Priest, E. R. 1983, *A&A*, 117, 220  
Hollweg, J. V. 1981, *SoPh*, 70, 25  
Hollweg, J. V. 1987a, *ApJ*, 312, 880  
Hollweg, J. V. 1992, *ApJ*, 389, 731  
Hollweg, J. V., Jackson, S., & Galloway, D. 1982, *SoPh*, 75, 35  
Hood, A. W., Brooks, S. J., & Wright, A. N. 2002, *RSPSA*, 458, 2307  
Jess, D. B., Mathioudakis, M., Erdelyi, R., et al. 2009, *Sci*, 323, 1582  
Lee, E. M., & Roberts, B. 1986, *ApJ*, 301, 430  
Longcope, D. W., Brown, D. S., & Priest, E. R. 2003, *PhIoP*, 10, 3321  
Malara, F., Petkaki, P., & Veltri, P. 2000, *ApJ*, 533, 523  
Masson, S., Pariat, E., Aulanier, G., & Schrijver, C. J. 2009, *ApJ*, 700, 559  
Matsumoto, T., & Suzuki, T. K. 2012, *ApJ*, 749, 8  
McLaughlin, J. A., De Moortel, I., Hood, A. W., & Brady, C. S. 2009, *A&A*, 493, 227  
McLaughlin, J. A., Ferguson, J. S. L., & Hood, A. W. 2008, *SoPh*, 251, 563  
McLaughlin, J. A., & Hood, A. W. 2004, *A&A*, 420, 1129  
McLaughlin, J. A., & Hood, A. W. 2006a, *A&A*, 452, 603  
McLaughlin, J. A., Hood, A. W., & De Moortel, I. 2011, *SSRv*, 158, 205  
Mignone, A., Bodo, G., Massaglia, S., et al. 2007, *ApJS*, 170, 228  
Mikhalyaev, B. B., & Bembitov, D. B. 2014, *SoPh*, 289, 4069  
Mok, Y., & Einaudi, G. 1985, *JPlPh*, 33, 199  
Murawski, K. 1992a, *AcPPA*, 81, 335  
Murawski, K. 1992b, *SoPh*, 139, 297  
Murawski, K., & Musielak, Z. E. 2010, *A&A*, 518, A37  
Nakariakov, V. M., Mendoza-Briceno, C. A., & Ibáñez, S. M. H. 2000, *ApJ*, 528, 767  
Nakariakov, V. M., Roberts, B., & Murawski, K. 1997, *SoPh*, 175, 93  
Nakariakov, V. M., Roberts, B., & Murawski, K. 1998, *A&A*, 332, 795  
Petkaki, P., Malara, F., & Veltri, P. 1998, *ApJ*, 500, 483  
Pontin, D. L., & Galsgaard, K. 2007, *JGRA*, 112, A10303  
Sabri, S., Ebadi, H., & Poedts, S. 2020, *ApJ*, 902, 11  
Sabri, S., Poedts, S., & Ebadi, H. 2019, *A&A*, 623, A81  
Sabri, S., Poedts, S., & Ebadi, H. 2021, *ApJ*, 922, 123  
Sabri, S., Vasheghani Farahani, S., Ebadi, H., Hosseinipour, M., & Fazel, Z. 2018, *MNRAS*, 479, 4991  
Sabri, S., Vasheghani Farahani, S., Ebadi, H., & Poedts, S. 2020, *NatSR*, 10, 15603  
Seckse, D. H., Rouppe van der Voort, L., De Pontieu, B., & Scullion, E. 2013, *ApJ*, 769, 44  
Shelyag, S., Khomenko, E., de Vicente, A., & Przybylski, D. 2016, *ApJL*, 819, L11  
Similon, P. L., & Sudan, R. N. 1989, *ApJ*, 336, 442  
Steinolfson, R. S. 1985, *ApJ*, 295, 213  
Suzuki, T. K. 2011, *SSRv*, 158, 339  
Thurgood, J. O., & McLaughlin, J. A. 2012, *A&A*, 545, A9  
Thurgood, J. O., & McLaughlin, J. A. 2013, *A&A*, 555, A86  
Thurgood, J. O., & McLaughlin, J. A. 2013, *SoPh*, 288, 205  
Thurgood, J. O., Pontin, D. I., & McLaughlin, J. A. 2017, *ApJ*, 844, 2  
Thurgood, J. O., Pontin, D. I., & McLaughlin, J. A. 2018, *ApJ*, 855, 50  
Tikhonchuk, V. T., Rankin, R., Frycz, P., & Samson, J. C. 1995, *PhPl*, 2, 501  
Tsiklauri, D., Arber, T. D., & Nakariakov, V. M. 2001, *A&A*, 379, 1098  
Tsiklauri, D., & Nakariakov, V. M. 2002, *A&A*, 393, 321  
Tsiklauri, D., Nakariakov, V. M., & Rowlands, G. 2003, *A&A*, 400, 1051  
Vahseghani, Farahani, S., Nakariakov, V. M., Van Dooseelaere, T., & Verwichte, E. 2011, *A&A*, 526, A80  
Vahseghani, F. S., Nakariakov, V. M., Verwichte, E., & Van Dooseelaere, T. 2012, *A&A*, 544, A147  
Verwichte, E., Nakariakov, V. M., & Longbottom, A. W. 1999, *JPlPh*, 62, 219  
Wedemeyer-Böhm, S., & van der Voort, L. R. 2009, *A&A*, 507, L9  
Wedemeyer-Böhm, S., Scullion, E., Steiner, O., et al. 2012, *Natur*, 486, 505  
Xiao, C. J., Wang, X. G., Pu, Z. Y., et al. 2006, *PhIoP*, 2, 478  
Zaqarashvili, T. V., & Murawski, K. 2007, *A&A*, 470, 533