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Analysis of Nanoparticle Additive Couple Stress Fluids in Three-layered Journal Bearing

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Abstract. The present theoretical study investigates the load capacity and friction coefficient in a three-layered journal bearing lubricated with nanoparticle additive couple stress fluids. The couple stresses effects are analyzed based on Stokes micro-continuum theory. The nondimensional pressure and shear stress expressions are derived using modified Reynolds equation. The nondimensional load capacity increases and the coefficient of friction decreases using nanoparticle additive lubricants with couple stress effects. The three-layered journal bearing performance characteristics are improved with increase in both (i) surface adsorbent fluid film layer thickness and (ii) dynamic viscosity ratio of surface to core layer.

1. Introduction
Analysis of load capacity and coefficient of friction in hydrodynamic lubrication are significantly influenced by structure and properties of fluid film. Szeri [1] offered a new approach to the friction reduction in hydrodynamic bearing using a composite-film journal bearing. The composite film configuration consists of immiscible high-viscosity and low-viscosity layers adjacent to bearing and journal surfaces respectively. Tichy [2] developed a rheological model applicable for thin films that considers high viscosity layer adhered to solid surfaces. Meurisse and Espejel [3] derived generalized Reynolds equation for surface layer model in thin film lubrication. Higher load capacity and lower friction coefficient of journal bearing are obtained with increase in viscosity and thickness of surface layer.

Lubricants with additives are used in hydrodynamic lubrication to improve the bearing performance characteristics. The couple stress fluid takes into account the properties of lubricants with additives. Stokes [4] proposed a simplified couple stress fluid model based on micro-continuum theory. Lin [5] and Mokhiamer et al. [6] analyzed the effects of couple stresses on finite journal bearing characteristics. Li and Chu [7] presented the influence of porous media and couple stress models on thin film lubrication of journal bearing. Both surface adsorbent layer and couple stress fluid models improve the performance characteristics of journal bearing. The influence of surface adsorbent layer using the couple stress fluid model is of considerable interest in the analysis of hydrodynamic lubrication.

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Nanoparticle additive lubricants have higher viscosity and hence improve the load carrying capacity of fluid film bearings. Shenoy et al. [8] presented the effect of nanoparticle additive lubricants on the static performance characteristics of an externally adjustable fluid film bearing. Nair et al. [9] investigated the performance characteristics of hydrodynamic journal bearing with nanoparticle additive lubricants. Theoretical models used to approximate the viscosity of nanofluids have been derived from the work of Einstein based on spherical particle suspensions in viscous fluids. Einstein’s formula is applicable to relatively low particle volume fraction. Brinkman [10] extended the Einstein equation to a more generalized form considering moderate particle volume fraction. Batchelor [11] considered the effect due to Brownian motion of spherical shape nanoparticle additives to predict the viscosity of nanofluids. Recent experimental [12] and empirical [13] models provide a comparison of earliest works [10,11] on viscosity of nanofluids.

The motivation of this theoretical study is to analyze improvements in the bearing performance characteristics (nondimensional load capacity and coefficient of friction). The purpose of this analysis is to investigate a three-layered film structure (adsorbent and core layers) of nanoparticle additive lubricants with couple stress fluid properties. The objective is to present one-dimensional (infinitely long bearing approximation [1]) analysis of nanoparticle additive couple stress fluids in three-layered journal bearing. A modified form of Reynolds equation is derived for couple stress fluids. Reynolds boundary conditions are used to solve the pressure distribution. The nondimensional pressure and shear stress expressions obtained. The parameters used to analyze the nondimensional load capacity and coefficient of friction are: (i) surface adsorbent fluid film layer thickness ($\Delta$); (ii) dynamic viscosity ratio of surface to core ($\beta_s$); (iii) nanoparticle volume fraction ($\psi$); and (iv) couple stress parameter ($\lambda$). The proposed model is applicable for thin film lubrication in which the film is of the order of several molecular lengths thickness [2].

2. Analysis

The schematic of three-layered journal bearing is shown in Figure 1. Journal bearing is lubricated with viscous fluid which forms a film that separates the load-bearing surfaces [1]. The three-layered film in journal bearing is described through adsorbent layer at surfaces of higher viscosity than core layer. The adsorbent layers (film region I: $0 \leq y \leq \delta$ and film region III: $h - \delta \leq y \leq h$) at surfaces of higher viscosity than core layer ($\beta_s = \frac{\mu_s}{\mu_c} \mu_c = \mu_n$ and $\beta_n = \frac{\mu_n}{\mu_f}$) and the core layer (film region II: $\delta \leq y \leq h - \delta$) are modeled using couple stress fluids.

![Figure 1. Geometry of three-layered journal bearing](image)
In the present one-dimensional (infinitely long bearing approximation [1]) analysis, the variation of pressure across the fluid film is assumed to be negligible and pressure in the three-layered journal bearing is a function of sliding direction. Pressure distribution is obtained by using Reynolds boundary conditions.

The simplified momentum equation for couple stress fluid is (0 ≤ y ≤ δ for j = 1, δ ≤ y ≤ h − δ for j = 2 and h − δ ≤ y ≤ h for j = 3)

\[ \frac{1}{\eta} \frac{dp}{dx} = \frac{\mu_j d^2 u_j}{\eta dy^2} - \frac{d^4 u_j}{dy^4} \]  

(1)

The boundary conditions for velocity at the bearing surface, at the interface of bearing adsorbent and core layer, at the interface of core and journal adsorbent layer, and at the journal surface respectively are

\[ y = 0 : u_1 = 0 \text{ and } \frac{d^2 u_1}{dy^2} = 0 \]  

(2)

\[ y = \delta : u_1 = u_2 = u_{12}, \mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy}, \frac{d^2 u_1}{dy^2} = 0 \text{ and } \frac{d^2 u_2}{dy^2} = 0 \]  

(3)

\[ y = h − \delta : u_2 = u_3 = u_{23} \text{ and } \mu_2 \frac{du_2}{dy} = \mu_3 \frac{du_3}{dy}, \frac{d^2 u_2}{dy^2} = 0 \text{ and } \frac{d^2 u_3}{dy^2} = 0 \]  

(4)

\[ y = h : u_3 = u_j \text{ and } \frac{d^2 u_3}{dy^2} = 0 \]  

(5)

The nanoparticle additive fluid dynamic viscosity increases with the particle volume fraction. Based on Batchelor [11] model, the dynamic viscosity ratio of nano to base fluid is

\[ \beta_n = \frac{\mu_n}{\mu_f} = (1 + 2.5\psi + 6.5\psi^2) \]  

(6)

Integrating (1) using the boundary conditions in (2)-(5), the non-dimensional velocity distribution in bearing adsorbent layer (0 ≤ Y ≤ Δ), in core layer (Δ ≤ Y ≤ H − Δ), and in journal adsorbent layer (H − Δ ≤ Y ≤ H) respectively are expressed as

\[ 0 ≤ Y ≤ \Delta: U_1 = U_{12} \frac{Y}{\Delta} + \frac{1}{2\beta_n \beta_n} \frac{dp}{d\theta} Y(Y - \Delta) + \lambda_1^2 \frac{1}{2\beta_n \beta_n} \frac{dp}{d\theta} C_1 \]  

(7)

\[ \Delta ≤ Y ≤ H - \Delta: U_2 = U_{12} + (U_{23} - U_{12}) \left( \frac{Y - H + \Delta}{\Delta} \right) + \lambda_1^2 \frac{1}{2\beta_n \beta_n} \frac{dp}{d\theta} (Y - \Delta) (Y - H + \Delta) \]  

(8)

\[ H - \Delta ≤ Y ≤ H: U_3 = U_{23} + (1 - U_{23}) \left( \frac{Y - H + \Delta}{\Delta} \right) + \frac{1}{2\beta_n \beta_n} \frac{dp}{d\theta} (Y - H) (Y - H + \Delta) + \lambda_1^2 \frac{1}{2\beta_n \beta_n} \frac{dp}{d\theta} C_3 \]  

(9)

where

\[ C_1 = \left[ 1 + \frac{\sinh(\frac{Y - \Delta}{\lambda_n}) - \sinh(\frac{Y}{\lambda_n})}{\sinh(\frac{\Delta}{\lambda_n})} \right], \quad C_2 = \left[ 1 + \frac{\sinh(\frac{Y - H + \Delta}{\lambda_n}) - \sinh(\frac{Y - H}{\lambda_n})}{\sinh(\frac{H - \Delta}{\lambda_n})} \right], \quad C_3 = \left[ 1 + \frac{\sinh(\frac{Y - H}{\lambda_n}) - \sinh(\frac{Y - H + \Delta}{\lambda_n})}{\sinh(\frac{\Delta}{\lambda_n})} \right] \]  

(10)

\[ U_{12} = F_1 - \frac{1}{\beta_n} \frac{dp}{d\theta} F_2, \quad U_{23} = F_3 - \frac{1}{\beta_n} \frac{dp}{d\theta} F_4 \]  

(11)

\[ F_1 = \frac{E_{11}^2 E_{231}}{E_{11} E_{22} - E_{12} E_{231}}, \quad F_2 = \frac{E_{13} E_{11} - E_{12} E_{232}}{E_{11} E_{22} - E_{12} E_{231}} \]  

\[ F_3 = \frac{E_{11} E_{22} - E_{12} E_{231}}{E_{11} E_{22} - E_{12} E_{231}}, \quad F_4 = \frac{-E_{11} E_{13} + E_{12} E_{231}}{E_{11} E_{22} - E_{12} E_{231}} \]  

(12)

\[ E_{11} = E_{22} = \frac{\beta_n}{\Delta} + \frac{1}{(H - 2\Delta)}, \quad E_{12} = E_{21} = -\frac{1}{(H - 2\Delta)}, \quad E_{13} = -\lambda_1 H_1^* - \lambda_n H_2^* + \frac{1}{2}(H - \Delta) \]  

(13)

\[ H_1^* = \left[ \coth(\frac{\lambda}{\lambda_n}) - \csc(\frac{\lambda}{\lambda_n}) \right], \quad H_2^* = \left[ \coth(\frac{H - 2\Delta}{\lambda_n}) - \csc(\frac{H - 2\Delta}{\lambda_n}) \right] \]  

\[ \lambda_1 = \frac{\lambda}{\sqrt{\beta_n}}, \quad \lambda_n = \frac{\lambda}{\sqrt{\beta_n}} \]  

(14)
The equation of continuity across the film is
\[ Q = \int_0^\Lambda U_1 dY + \int_{\Lambda}^{H-\Lambda} U_2 dY + \int_{H-\Lambda}^H U_3 dY \]  
(17)

Simplifying the equation of continuity across the film, yields
\[ \frac{dP}{d\theta} = \beta_n \left( \frac{G_1 - Q}{\alpha_2} \right) \]  
(18)

where
\[ G_1 = \frac{1}{2} \Delta + \frac{1}{2} (F_1 + F_3) (H - \Delta) = \frac{1}{2} H \]  
(19)
\[ G_2 = \frac{1}{2} (F_2 + F_4) (H - \Delta) + \frac{1}{12\beta_s} [2\Delta^3 + \bar{\beta}_s (H - 2\Delta)] - \frac{1}{\beta_s} [2\lambda_n^3 \Delta + \beta_s \lambda_n^3 (H - 2\Delta)] + \frac{2}{\beta_s} \lambda_n^3 H_2^* + \beta_s \lambda_n^3 H_2^* \]  
(20)

For \( \Delta = 0 \), \( G_2 \) in (20) reduces to
\[ G_2 = \frac{1}{12} H^3 - \lambda_n^2 H + 2\lambda_n^3 \tanh \left( \frac{H}{2\lambda_n} \right) \]  
(21)

For \( \lambda = 0 \), \( G_2 \) in (20) reduces to
\[ G_2 = \frac{1}{12\beta_s} \left[ 2\Delta^3 + \bar{\beta}_s (H - 2\Delta)^3 + 6\Delta (H - \Delta)^2 \right] \]  
(22)

The Reynolds boundary conditions are
\[ P|_{\theta=0} = 0, \quad P|_{\theta=\theta_r} = 0 \quad \text{and} \quad \frac{dP}{d\theta}|_{\theta=\theta_r} = 0 \]  
(23)

Integrating (18) and substituting the first boundary condition given in (23), yields the nondimensional pressure profile as
\[ P = \beta_n \left( \int_0^\theta \frac{G_1}{\alpha_2} d\theta - Q \int_0^\theta \frac{1}{\alpha_2} d\theta \right) \]  
(24)

Substitution of the Reynolds boundary conditions for nondimensional pressure at film rupture in (23) and simplifying results in \( Q \) as
\[ Q = \int_0^{\theta_r \hat{g}_n} \frac{d\theta}{\hat{g}_n \alpha_2} \]  
(25)

Substituting the pressure gradient boundary condition given in (23) in the expression for nondimensional pressure gradient in (18), results in
\[ Q = G_1|_{\theta=\theta_r} \]  
(26)

The Newton-Raphson iterative procedure is used to solve simultaneously both \( \theta_r \) and \( Q \) using (25) and (26).

The nondimensional load capacity is
\[ W = \sqrt{W_\theta^2 + W_\phi^2} \]  
(27)

where \( W_\theta = -\int_0^{\theta_r} \rho \cos \theta \ d\theta \) and \( W_\phi = \int_0^{\theta_r} \rho \sin \theta \ d\theta \)

The nondimensional shear stress in the journal bearing at \( Y = H \) is obtained as
\[ \Pi|_{Y=H} = \beta_s \beta_n \frac{dU_3}{dY}|_{Y=H} = \beta_n \left[ (1 - F_3) \left( \frac{\beta_s}{\Lambda} \right) + \left( \frac{G_1 - Q}{\alpha_2} \right) \left( F_4 \frac{\beta_s}{\Lambda} + \frac{\Lambda}{2} - \lambda_s H_1^* \right) \right] \]  
(28)
For $\Delta = 0$, $\Pi|_{Y=H}$ in (28) reduce to

$$\Pi|_{Y=H} = \beta_n \left[ \frac{1}{H} + \left( \frac{g_1 - Q}{g_2} \right) \left( -\lambda \tanh \left( \frac{H}{2\lambda} \right) + \frac{1}{2} H \right) \right]$$

(29)

For $\lambda = 0$, $\Pi|_{Y=H}$ in (28) reduce to

$$\Pi|_{Y=H} = \beta_n \left[ \frac{\beta_s}{2\Delta + \beta_s (H-2\Delta)} + \left( \frac{g_1 - Q}{g_2} \right) \frac{H}{2} \right]$$

(30)

The nondimensional friction force on the journal surface is obtained by integrating the shear stress along the journal surface as

$$F = \int_0^{\theta} \Pi \, d\theta$$

(31)

The nondimensional friction coefficient is calculated as $C_f = \left( \frac{R}{L} \right) \frac{F}{W}$. 

3. Results and Discussion

A nanoparticle additive couple stress fluid film lubricated three layered journal bearing is considered in the analysis. The parameters used in the analysis are: journal eccentricity ratio ($\varepsilon$)=0.2; dynamic viscosity ratio of surface to core layer ($\beta_s$)=10; surface (journal and bearing) adsorbent fluid film layer thickness ($\Delta$)=0.001-0.1); couple stress parameter ($\lambda$)=0.0, 0.01, 0.1 and nanoparticle volume fraction ($\psi$)=0.0-0.04. The influence of surface (journal and bearing) adsorbent fluid film layer thickness ($\Delta$) and nanoparticle volume fraction ($\psi$) on the nondimensional load capacity and coefficient of friction are analyzed. Based on the analysis presented in this theoretical study, results of nondimensional load capacity and coefficient of friction for a three-layered film are compared with a conventional film ($\Delta = 0$).

Figures 2a-2b show the non-dimensional load capacity ($W$) of a three-layered journal bearing lubricated with nanoparticle additive couple stress fluids. The variation of non-dimensional load capacity ($W$) with surface adsorbent (journal and bearing) fluid film layer thickness ($\Delta$) and nanoparticle volume fraction ($\psi$) are presented. As shown in Figure 2a, the non-dimensional load capacity is significantly increases with increase in surface adsorbent fluid film layer thickness ($\Delta$) and dynamic viscosity ratio of surface to core layer ($\beta_s$). The non-dimensional load capacity ($W$) also increases with increase in (i) increase in couple stress parameter ($\lambda$) and (ii) nanoparticle volume fraction ($\psi$). However, the configuration with lower surface adsorbent fluid film layer thickness ($\Delta<0.01$) does not generate enhancement in the non-dimensional load capacity ($W$). As shown in Figure 2b, the non-dimensional load capacity ($W$) increases with nanoparticle volume fraction ($\psi$)=0.0-0.04. The influence of increase in surface adsorbent fluid film layer thickness ($\Delta=0.0-0.1$) on the enhancement in the non-dimensional load capacity is higher compared to couple stress parameter ($\lambda=0.01-0.1$).

Figures 3a-3b show the coefficient of friction ($C_f$) for a three-layered journal bearing lubricated with nanoparticle additive couple stress fluids. For the parameters considered in the study as shown in Figure 3a, minimum coefficient of friction ($C_f$) is obtained for higher values of both surface adsorbent fluid film layer thickness ($\Delta$) and dynamic viscosity ratio of surface to core layer ($\beta_s$). The influence of surface adsorbent fluid film layer thickness ($\Delta$) on reduction in coefficient of friction ($C_f$) is more than the influence of couple stress parameters ($\lambda$). The reduction in coefficient of friction ($C_f$) is significant for surface adsorbent fluid film layer thickness ($\Delta=0.1$), with increase in both dynamic viscosity ratio of surface to core layer ($\beta_s$) and couple stress parameters ($\lambda$). As shown in Figure 3b, the coefficient of friction ($C_f$) is not influenced by the increase in nanoparticle volume fraction ($\psi$)=0.0-0.04. For $\lambda=0.0$, the coefficient of friction ($C_f$) is not influenced by nanoparticle volume fraction ($\psi$), as both nondimensional pressure and nondimensional shear stress increase by the magnitude of $\beta_n$ as given in (24) and (30).
Figure 2. Non-dimensional load capacity ($\varepsilon=0.2$)

Figure 3. Coefficient of friction ($\varepsilon=0.2$)
4. Conclusion
The present study evaluates improvement in load capacity and reduction in coefficient of friction for a nanoparticle additive couple stress fluid lubricated three-layered journal bearing. The nondimensional pressure and shear stress expressions are obtained using the Reynolds boundary conditions. The nondimensional load capacity ($W$) increases significantly with increase in both surface adsorbent fluid film layer thickness ($\Delta$) and dynamic viscosity ratio of surface to core layer ($\beta_s$). The influence of couple stress parameter ($\lambda$) is greater for higher surface adsorbent fluid film layer thickness ($\Delta$). The nondimensional load capacity ($W$) also increases with increase in nanoparticle volume fraction ($\psi$). The coefficient of friction ($C_f$) decreases significantly with increase in surface adsorbent fluid film layer thickness ($\Delta$) and dynamic viscosity ratio of surface to core layer ($\beta_s$). The increase in nanoparticle volume fraction ($\psi$) does not yield improvement in the coefficient of friction ($C_f$). A three-layered journal bearing lubricated with nanoparticle additive couple stress fluids has a potential to increase the load capacity and reduce the friction coefficient.

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Nomenclature

- $C$: Radial clearance, m
- $f$: Friction force, N; $F = fC/\mu_f UR$
- $h, H$: Film thickness, m; $H = h/C$
- $L$: Length, m
- $p$: Pressure distribution, N/m$^2$; $P = pC^2/\mu_f UR$
- $q$: Volume flow rate per unit length along film thickness, m$^3$/s; $Q = q/UC$
- $R$: Journal radius, m
- $u$: Velocity component along circumferential direction
- $U$: Shaft speed, m/s; $U = \omega R$
- $w$: Static load, N; $W = wC^2/\mu_f UR^2L$
- $x$: Coordinate along circumferential direction, m; $\theta = x/R$
- $y$: Coordinate along radial direction, m; $Y = y/C$
- $\beta_n, \beta_s$: Dynamic viscosity ratios of nano to base fluid, surface to core layer respectively
- $\Delta$: Surface (journal or bearing) adsorbent fluid film layer thickness; $\Delta = \frac{\delta}{C}$
- $\varepsilon$: Journal bearing eccentricity ratio
- $\mu$: Fluid viscosity, Ns/m$^2$
- $\eta$: Material constant for couple stress, kgm/s
- $\lambda$: Couple stress parameter; $\lambda = (\sqrt{\eta/\mu})/C$
- $\psi$: Particle volume fraction
- $\theta$: Angular coordinate measured from the direction of maximum film thickness in journal bearing
- $\theta_r$: Angular extent of film rupture for journal bearing
- $\tau$: Shear stress component, N/m$^2$; $\Pi = \tau C/\mu_f U$
- $\omega$: Angular velocity of journal bearing, rad/s

Subscripts

- $c, s$: Core and adsorbent fluid film layer
- $f, n$: Base and nanoparticle additive fluid
- $r$: Extent of outlet film in journal bearing
- $1, 2, 3$: Bearing adsorbent, core and journal adsorbent fluid layers
- $12$: Fluid film interface of the bearing adsorbent and core layer
Fluid film interface between the core and journal adsorbent layer

$\varepsilon$ Along the radial direction

$\phi$ Along the tangential direction

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