Palatini $f(R)$ gravity and k-inflation within variation of strong coupling scenario

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Abstract

We show that upon applying Palatini $f(R)$, characterized by an $\alpha R^2$-term, within a scenario motivated by a temporal variation of strong coupling constant, then one gets a quadratic kinetic energy. We do not drop this term, but rather study two extreme cases: $\alpha \ll 1$ and $\alpha \gg 1$. In both cases one can generate an inflationary paradigm, legitimately called thus a k-inflation, which fits the Planck 2018 data.

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1 Introduction

In [1], we adopted a model of variations of constants in order to generate an inflationary scenario, where the strong coupling was assumed to vary in time encoded in a scalar field representing this variation. Although current geophysical and astronomical data preclude any variation of constants, be it strong coupling [2], or Higgs vev [3], or electric charge [4], however no data precludes variation in very early times. In [5], a connection between variation of constants and inflation was suggested, whereas in [6] this idea was pursued further into a concrete model shown to be able to accommodate data in some variants involving multiple inflaton fields. Alternatively, the single inflaton model was shown in [1] to be viable provided one changes the gravitational sector and assumes $f(R)$ gravity. The addition of an $\alpha R^2$ term in the pure gravity Lagrangian changed the potential into an effective one, but also led to a quadratic kinetic energy term which was dropped in [1] on the ground that it involved an $\alpha$-coupling which could be argued to be small perturbatively, and this allowed to derive formulae for the spectral index $n_s$ and the scalar-to-tensor ratio $r$ contrasted with planck data 2018 [7] separately or combined with other experiments [8].

The aim of this work is to study the effect of the quadratic kinetic energy term. For this, we take two extreme cases. The first case corresponds to $\alpha \gg 1$, which makes the scalar field non-canonical per excellence. Many studies were carried out to refine the inflationary scenario within the framework of scalar fields possessing a non-canonical kinetic term [9]. We find that with or without a non-minimal coupling to gravity (non-MCtG) the model can fit the data. However, one can not get closed forms of the “canonical” potential except in some cases which we illustrate in order to show the “plateau”-form of the potential in terms of the “canonical” field which rolls slowly during inflation. The second case corresponds to

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the perturbative regime where we restrict the analysis to first order in $\alpha$. Our model in this case parallels the well known constant-roll k-inflation \cite{10}, and we prove that within a given limit corresponding to vanishing non-MCtG with $\alpha$ small, $\ell$ large the model is viable, and we check this numerically.

The paper is organized as follows. In section 1, we introduce the model and illustrate how the quadratic kinetic energy appears. In section 2, we study the case of large $\alpha$ computing the spectral parameters to be contrasted with data. Section 3 is devoted to the study of the “canonical” potential shape when $\alpha > > 1$. In section 4, we analyse the perturbative regime where $\alpha$ is small, whereas in section 5 we prove its viability. We end up with a summary and conclusion in section 6.

2 Analysis of the basic model

Our starting point is the general four dimensional action:

$$S = S_\phi + S_g + S_{g\phi}$$  \hspace{1cm} (1)

where $S_\phi$ is the varying strong coupling constant action given by \cite{6}:

$$S_\phi \equiv \int d^4x \sqrt{-g} \mathcal{L}_\phi = \int d^4x \sqrt{-g} \left\{-\frac{1}{4} f(\phi) g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)\right\}$$  \hspace{1cm} (2)

where $f(\phi) = \frac{1}{\ell^2 \phi}$, and $V(\phi) = \frac{V_0}{\ell^2}$ with $\phi$ embodying the strong coupling constant variation $g(x) = g_0 \phi(x)$ and $\ell$ is the Bekenstein length scale, and $V_0 = \frac{(G_0)^2}{4}$ encodes the gluon field strength vacuum expectation value (vev) at inflation temperature $T$, whereas $S_g$ is the pure gravity Lagrangian including the Einstein-Hilbert action to which is added an $f(R)$ gravity term taken in our case as a quadratic function of the Ricci scalar $\alpha R^2$, and we include also a coupling term $S_{g\phi}$ between gravity and the field $\phi$. Adopting units where the Planck mass $M_{pl}$ is equal to one, we have:

$$S_g = \int d^4x \sqrt{-g} \left[\frac{1}{2} (R + \alpha R^2)\right]$$  \hspace{1cm} (3)

$$S_{g\phi} = -\xi R \phi^2$$  \hspace{1cm} (4)

with $R$ the Ricci Scalar constructed from the metric $g_{\mu \nu}$. Note that the form of the potential in Eq. (2) is not put by hand, but rather is dictated by the physical assumption of a varying strong coupling constant, where gauge and Lorentz invariance impose this form originating from the gluon condensate \cite{5,6}.

We start by making a change of variable absorbing the function $f$ in order to get a “canonical” kinetic energy term. Thus we introduce the field $h$ defined as $\phi = \exp(\ell h)$, so that to get the action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} F(R) + \frac{1}{2} G(h) R - \frac{1}{2} g^{\alpha \beta} \partial_\alpha h \partial_\beta h - V(h)\right]$$  \hspace{1cm} (5)

where

$$V(h) = V_0 \exp(-2\ell h), G(h) = -\xi \exp(2\ell h), F(R) = R + \alpha R^2$$  \hspace{1cm} (6)

Instead of using at this stage the 1\textsuperscript{st}-order cosmological perturbation theory, by perturbing the metric $(g_{\mu \nu} \rightarrow g_{\mu \nu} + \delta g_{\mu \nu})$ and keeping terms of first order in the perturbations, we anticipate that the $\alpha R^2$ would contribute involved terms upon this metric change, so we follow \cite{11,12} and introduce an auxiliary field $\psi$ and an action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} G(h) R + \frac{1}{2} \left\{F(\psi) + F'(\psi)(R - \psi)\right\} - \frac{1}{2} g^{\alpha \beta} \partial_\alpha h \partial_\beta h - V(h)\right]$$  \hspace{1cm} (7)
The equation of motion of $\psi$ gives $R = \psi$ provided $F''(\psi) \neq 0$. We change variable again $\psi \rightarrow \lambda$ such that $(\lambda = F'(\psi) = 1 + 2\alpha \psi)$, so we get

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( \lambda + G(h) \right) R - \frac{1}{2} \left( \psi \lambda - F(\psi(\lambda)) \right) - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right] \quad (8)
\]

We carry out a conformal transformation on the metric

\[
g_{\alpha\beta} \rightarrow \Upsilon^2 g_{\alpha\beta} \quad : \quad \Upsilon^2 = \lambda + G(h)
\]

then we get in the “Metric” formulation, where the Christoffel symbols are defined in terms of the metric and thus are not independent and the corresponding affine connection is defined to be the Levi-Civita one, the following [13]:

\[
S^{\text{Metric}} = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{1}{2} \tilde{R} - \frac{3}{4} \tilde{g}^{\mu\nu} \nabla_\mu (\lambda + G(h)) \nabla_\nu (\lambda + G(h)) - \frac{1}{2} \frac{1}{\lambda + G(h)} \tilde{g}^{\alpha\beta} \partial_\alpha h \partial_\beta h - \tilde{V}(h, \lambda) \right] \quad (10)
\]

\[
\tilde{V}(h, \lambda) = \frac{V(h) + W(\lambda)}{(\lambda + G(h))^2} \quad (11)
\]

where

\[
W(\lambda) = \frac{1}{2} \left( \psi \lambda - F(\psi(\lambda)) \right) \quad (12)
\]

We see that in the “Metric” formulation, we get a kinetic energy term for $(\lambda + G(h))$, and the field $\lambda$ is dynamic, i.e. its equation of motion cannot be solved algebraically.

For simplicity, then, we restrict the study from now on to the “Palatini” formulation, where the Christoffel symbols are considered independent and are to be determined dynamically. Remembering here that the pure gravity is not represented by a simple $R$-term, then the connection will be different from the Levi-Civita one. Under this formulation, we get:

\[
S^{\text{Palatini}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \tilde{R} - \frac{1}{2} \frac{1}{\lambda + G(h)} \tilde{g}^{\alpha\beta} \partial_\alpha h \partial_\beta h - \tilde{V}(h, \lambda) \right] \quad (13)
\]

where again $\tilde{V}(h, \lambda)$ is given by Eq. [11], and where eq. [12] is again valid.

The equation of motion of $\lambda$ can be solved algebraically to give it in terms of the field $h$ and its derivatives, so $\lambda$ is not a new degree of freedom

\[
\lambda = \frac{1 + G(h) + 8\alpha V(h) + 2\alpha G(h)(\partial h)^2}{1 + G(h) - 2\alpha(\partial h)^2} \quad (14)
\]

Substituting (Eq. [14]) in (Eq. [13]), we get (dropping the “Palatini” superscript and the \sim over the metric):

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} \frac{1}{(1 + G(h))(1 + 8\alpha U)} \tilde{g}^{\alpha\beta} \partial_\alpha h \partial_\beta h - \frac{1}{2} \frac{1}{(1 + G(h))^2(1 + 8\alpha U)} (\partial^\alpha h \partial_\alpha h)^2 - \bar{U} \right] \quad (15)
\]

where

\[
\bar{U} = \frac{V(h)}{(1 + G(h))^2} = \frac{V_0 \exp (-2\xi h)}{(1 - \xi \exp (2\xi h))^2} \quad (16)
\]

In order to get a “canonical” kinetic energy term, we again make the change of variable $(h \rightarrow \chi)$ by

\[
\frac{dh}{d\chi} = \pm \sqrt{(1 + G(h))(1 + 8\alpha U)} \quad (17)
\]
to get finally

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi + \frac{\alpha}{2} (1 + 8a\bar{U})(\partial^\alpha \chi \partial_\alpha \chi)^2 - U \right] \]  \hspace{1cm} (18)

where

\[ U = \frac{\bar{U}}{(1 + 8a\bar{U})} = \frac{V_0}{8aV_0 + (e^{\eta} - \xi e^{3\eta})^2} \]  \hspace{1cm} (19)

We see here that the effect of \( \alpha R^2 \)-term is manifested in two ways. First, it helps in getting a “flat” effective potential \( U \). Actually, regardless of the form of \( \bar{U} \), we see that the \( \alpha R^2 \)-term leads, when \( \bar{U} \) increases indefinitely, to an effective potential with a flat portion \( (U \sim (8\alpha)^{-1}) \). Second, the \( \alpha R^2 \)-term leads to the appearance of squared kinetic energy \( (\partial^\alpha \chi \partial_\alpha \chi)^2 \).

In [1], \( \alpha \) was taken to be small in a way to neglect the quadratic kinetic energy term. In fact, upon perturbing the metric then the \( (\alpha \delta g) \)-term would give higher order terms, whereas the \( \alpha(\partial^\beta \chi \partial_\beta \chi)^2 \) would give, in the slow-roll inflationary era, contributions of order \( \alpha^4 \) which is subdominant compared to the \( \alpha \)-correction in \( U \). Thus, in [1], one could apply the shortcut “potential method”, using \( U \) as an effective potential. We intend now to refine this analysis, and consider the effect of the quadratic kinetic energy, keeping first order in \( \alpha \) when \( \alpha \) is small, and studying, in addition, the case where \( \alpha \) is large.

### 3 k-inflation: case \( \alpha >> 1 \)

Under the assumption

\[ 1 \ll \alpha(1 + 8a\bar{U})(\partial^\alpha \chi \partial_\alpha \chi) \]  \hspace{1cm} (20)

our k-inflation model features a single scalar field with the action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + \frac{\alpha}{2} (1 + 8a\bar{U})(\partial^\alpha \chi \partial_\alpha \chi)^2 - U \right] \]  \hspace{1cm} (21)

Introducing the “standard” field \( \varphi \) defined by

\[ \frac{\partial \varphi}{\partial \chi} = \left[ 2\alpha(1 + 8a\bar{U}) \right]^{\frac{3}{4}} \]  \hspace{1cm} (22)

we get a “standard” form for the k-inflation Lagrangian

\[ \mathcal{L} = \frac{R}{2} + p(\varphi, X) : p(\varphi, X) = X^2 - U(\varphi), X = \frac{1}{2} \partial^\alpha \varphi \partial_\alpha \varphi \]  \hspace{1cm} (23)

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + \left( \frac{1}{2} \partial^\alpha \varphi \partial_\alpha \varphi \right)^2 - U \right] \]  \hspace{1cm} (24)

The spectral index \( n_s \) and the tensor-to-scalar ratio are given now by [14]:

\[ n_s - 1 = \frac{1}{3} (4\eta - 16\epsilon), \quad r = 16c_s \epsilon \]  \hspace{1cm} (25)

where

\[ \epsilon = \frac{1}{2} 3^\frac{3}{2} \left( \frac{U(\varphi)}{U^2} \right)^\frac{1}{2}, \quad \eta = 3^\frac{1}{2} \frac{(U(\varphi), \varphi)}{U^2(U(\varphi))^\frac{1}{2}} \]  \hspace{1cm} (26)

\[ c_s^2 = \frac{p(\varphi, X), X + p(\varphi)}{2X(\varphi, X)^2} = \frac{1}{5} \]  \hspace{1cm} (27)

where the comma (,) means differentiation with respect to what follows it. However, one should note that in order to compute the derivative with respect to the “standard” field \( \varphi \),
one should differentiate $U$ with respect to $h$, which is known from Eq. (19) and using Eqs. (17, 22), so to get

$$\frac{dh}{d\varphi} = \left(\frac{(1 - \xi e^{2\ell h})^2 + 8\alpha V_0 e^{-2\ell h}}{2\alpha}\right)^{\frac{1}{4}}$$

(28)

$$U_{,\varphi} = \frac{dU}{dh} \frac{dh}{d\varphi}$$

(29)

$$(U_{,\varphi})_{,\varphi} = \frac{d^2U}{dh^2} \left(\frac{dh}{d\varphi}\right)^2 + U_{,\varphi} \frac{dh}{d\varphi} \frac{d}{dh} \left(\frac{dh}{d\varphi}\right)$$

(30)

The input parameters are $(V_0, \ell, \alpha, \xi)$ and the initial values of the “original” inflaton field $h$ at the start of inflation. However, one can show analytically that the model is able to fit the data for some regions in the parameter space. Actually, we get for $\xi = 0$ the following analytic formulae

$$1 - n_s = \frac{8e^{2\ell h} (2\alpha^{3/4}(e^{2\ell h} + 8\alpha V_0)^3 - 42^{1/4}\alpha}\ell^2 V_0^2(e^{2\ell h} - 8\alpha V_0)(1 + 8\alpha e^{-2\ell V_0})^{3/4}}{3^{4/5}\alpha^{13/12}V_0^{7/3}(e^{2\ell h} + 8\alpha V_0)^{11/3}}$$

$$r = \frac{16e^{2\ell h}\ell^{4/3}}{3^{3/10}\alpha^{1/3}V_0^{1/3}(e^{2\ell h} + 8\alpha V_0)^{2/3}}$$

(31)

Thus, enforcing the Bekenstein hypothesis $\ell > 1$, which means in our adopted units the absence of any length scale shorter than Planck length, we can look for regimes corresponding to $\alpha >> 1$, and so we get

$$0 \leq 1 - n_s = \mathcal{O}(\alpha^{15/3}) + \mathcal{O}(\alpha^{11/4}) = \mathcal{O}(1/\alpha)$$

(32)

$$0 \leq r = \mathcal{O}(1/\alpha)$$

(33)

Since fitting the data requires $(0 < 1 - n_s << 1$ and $0 < r << 1$), we expect that for $\alpha$ quite large, one can fit the data. Fig. 1 indeed, shows that there are acceptable points, colored in blue, for the following scanning:

$$\ell \in [1, 2], \alpha \in [100, 250], e^{\ell h} \in [1, 10], V_0 \in [20, 40], \xi = 0.$$
Figure 1: Kinematically derived inflation, in a model of varying strong coupling constant 
\( g(x) = g_0 \phi(x) \), with \( f(R) \) gravity via \( \alpha R^2 \)-term and non-MCtG \(-\xi R \phi^2 \)-term. The blue points correspond to the limit where \( \alpha >> 1 \), and we imposed \( \xi = 0 \). The red points correspond to the limit \( \alpha >> 1 \) and \( \alpha \overline{U} >> 1 \) with \( \xi \neq 0 \). The green points correspond to the limit \( \alpha << 1 \) with \( \xi = 0 \). The models are contrasted to Planck 2018, separately or combined with other experiments, contour levels of spectral parameters \((n_s, r)\). All acceptable points correspond to \( \ell > 1 \).

4 The “plateau” shape: case \( \alpha >> 1 \)

From Eq. [19] we see that in the limit where
\[
1 << 8\alpha \overline{U} \Leftrightarrow (1 - \xi e^{2\ell h})^2 e^{2\ell h} << 8\alpha V_0
\]
the effective potential shows a “plateau” form \( U(h) \sim \frac{1}{\alpha} \), and our objective in this section is to study the shape of this plateau in terms of the “canonical” field \( \varphi \) which is the field to roll slowly along the effective potential.

Actually, one would like, starting from the known potential \( U(h) \) given in Eq. [19] to find an analytic expression of the potential in terms of the “canonical” field \( \varphi \). However, it is not possible in general to do this, as we can not carry out analytically the following integral, originating from Eqs. [17,22], let alone invert it to express \( h \) in terms of \( \varphi \):
\[
\varphi = (2\alpha)^{1/4} \int \frac{dh}{[(1 - \xi e^{2\ell h})^2 e^{2\ell h} + 8\alpha V_0 e^{2\ell h}]^{1/4}}
\]
Even in the case of MCtG \( (\xi = 0) \), and although one can in principle carry out the above integration but the resulting expression involving hypergeometric functions is not invertible.

However, in the limit of Eq. (35), one can carry out analytically the integration and get
\[
\epsilon^{2\ell h} = \frac{\sqrt{V_0}}{2} (\ell \varphi)^2
\]
and we see that the effective potential is given as
\[
U(\varphi) = \left[ 8\alpha + \frac{\ell^4 \varphi^4}{4} \left( 1 - \xi \frac{V_0}{4 \ell^4 \varphi^4} \right)^2 \right]^{-1}
\]
In the left part (A) of Fig. 2, we plot the shape of effective potential, and find that it has one local maximum (minimum) at $\varphi_0 = \sqrt{4/(V_0 \xi)} \ell^{-1} (\varphi_0 / \sqrt{3})$. We see that the limit of Eq. (35) is equivalent to

$$\ell^2 \varphi^2 \left| 1 - (\varphi / \varphi_0)^4 \right| \ll \sqrt{32 \alpha}$$

(39)

Thus, we see that as long as the field, during its slow rolling along the plateau from $\varphi = 0$, does not meet the local minimum, then the slow roll condition is satisfied and the inflationary solution is consistent. In the right part (B) of Fig. 2, we draw the same plateau in the case of MCtG. However, the solution is not viable for $\ell > 1$.

As a matter of fact, one can compute the observable parameters ($n_s$, $r$) using the effective potential expression in this limit (Eq. 38), and we find with the combination ($y = \xi V_0 \varphi^4 \ell^4$)

$$1 - n_s = \ell^{4/3} 3^{-4/5} \frac{48 - 112y + 33y^2}{(8 - 8y + 3y^2/2)^{2/3}}$$

(40)

$$r = 2^{2/3} 3^{-3/10} 8^{4/3} \varphi^4 \frac{(16 - 16y + 3y^2)^{1/3}}{512 \alpha + \ell^4 \varphi^4 (-4 + y)^2}$$

(41)

We see here that for $\xi = 0$ one can not meet $0 < 1 - n_s \sim 4 \sqrt{3} \ell^{4/3} \ll 1$ for $\ell > 1$, whereas for $y \sim 4/33(14 \pm \sqrt{97}$ (roots of the numerator of $(1 - n_s)$) and having $\alpha$ quite large, one can satisfy $(0 < 1 - n_s \ll 1, 0 < r \ll 1)$. The red points in Fig (1) represent acceptable points generated upon scanning the parameters as follows.

$$y \in [0.48, 0.52], \ell \in [1, 2], \varphi \in [1, 20], \alpha \in [400, 500].$$

(42)

**Figure 2:** Plateau shape in the limit of Eq. (35). Scenario (B) with $\xi = 0$ fits data provided $\ell < 1$.

5 **Constant roll k-inflation. Case $\alpha \ll 1$**

In contrast to the preceding sections, we now take the perturbative limit $\alpha \ll 1$, and we work up to first order in $\alpha$. We shall consider a specific type of k-inflation called “constant roll” k-inflation, where one slow-roll parameter ($\epsilon_2$) related to the time second derivative of the inflaton is assumed constant equaling $\beta$. Following [10], our model which has the following action:

$$S = \int d^4x \sqrt{-g}/2f(R, \chi, X)$$

(43)
where \( X = \frac{1}{2} \partial_\mu \partial^\mu \chi = \frac{\dot{\chi}^2}{2} \), \( f(R, \chi, X) = (R - 2X + 4\alpha X^2 - 2U) \) (44)

will involve the slow-roll parameters defined as

\[ \epsilon_1 \equiv \frac{\dot{H}}{H^2}, \epsilon_2 \equiv \frac{\ddot{\chi}}{H \dot{\chi}}, \epsilon_3 \equiv \frac{\dot{F}}{2HF} = 0, \epsilon_4 \equiv \frac{\dot{E}}{2HE} \] (45)

where

\[ F = f_R = 1, \quad E \equiv -\frac{F}{2X}(Xf_{,X} + 2X^2f_{,XX}) = 1 - 12\alpha X \] (46)

At the horizon crossing time instance, we have

\[ \epsilon_1 = -\frac{3}{4} \frac{\dot{\chi}^2 + \alpha \dot{\chi}^4}{U(\chi)}, \quad \epsilon_4 = \frac{6\sqrt{3}\alpha \dot{\chi}}{\sqrt{U(1 + 6\alpha \dot{\chi}^2)}} \] (47)

with real solutions given by

\[ \dot{\chi} = \frac{6(\beta + 1)(\beta + 3)4\alpha U - (81\Delta + 9\sqrt{S})^{2/3}}{3^{1/6}(\beta + 1)4\alpha \sqrt{U(81\Delta + 9\sqrt{S})^{1/3}}}, \quad \ddot{\chi} = \beta \frac{\sqrt{U}}{3} \] (48)

\[ S = (\beta + 1)^3(4\alpha)^3U^2 \left[ 81(\beta + 1)4\alpha U^2 + \frac{8}{3}(\beta + 3)^3U \right], \quad \Delta = 16(\beta + 1)^2\alpha^2 U \chi U \] (49)

The spectral parameters are given as

\[ n_s = 1 + 2 \frac{2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4}{1 + \epsilon_1} = 1 + 2 \frac{2\epsilon_1 - \beta - \epsilon_4}{1 + \epsilon_1} \] (50)

\[ r = 4 \left[ \frac{\Gamma(3/2)}{\Gamma(3/2 + \epsilon_2)} \right]^{3/2 + \epsilon_2} \frac{\sqrt{3} \dot{\chi} \sqrt{1 + 6\alpha \dot{\chi}^2}}{U} \] (51)

\[ \epsilon_2 = \frac{f_{,X} + 2Xf_{,XX}}{f_{,X} + 2Xf_{,XX}} = \frac{-1 + 2\alpha \dot{\chi}^2}{-1 + 6\alpha \dot{\chi}^2} \] (52)

The free input parameters here are \((\alpha, \xi, V_0)\) and \((\ell, h)\), which a priori determine \( \chi \), and also \( \beta \) of order unity expressing the constant roll condition. However, note that we need to express \( U_\chi \) using Eqs. (19, 17).

\[ U_\chi = \frac{dU}{dh} \frac{dh}{d\chi}, \quad \frac{dh}{d\chi} = \pm \frac{\sqrt{1 - \xi e^{2\ell h} - 8\alpha V_0 e^{-2\ell h}}}{1 - \xi e^{2\ell h}} \] (53)

and even in the case of MCtG \((\xi = 0)\), where we get an analytic expression of \( \chi \) in terms of \( h \):

\[ \chi = \int \frac{dh}{\sqrt{1 + 8\alpha V_0 e^{-2\ell h}}} = \frac{e^{-\ell h} \sqrt{8\alpha V_0 + e^{2\ell h} \log(e^{\ell h} + \sqrt{8\alpha V_0 + e^{2\ell h}})}}{\ell \sqrt{1 + 8\alpha V_0 e^{-2\ell h}}} \] (54)

however one can not invert it so \( U(\chi) \) is not obtained in a closed form.

6 Viability of the constant roll k-inflation: case \( \alpha << 1 \)

We show now the existence of viable points which fit the data. For this, we need a search strategy to reduce the number of input parameters, since our objective is limited to a proof of existence with no claim to exhaustive covering of all acceptable points, otherwise scanning over the formulae of Eqs. (47-52), which are far from simple analytical formulae, is not a trivial task.
Let us take the case of MCtG ($\xi = 0$) which with our limit case ($\alpha << 1$) leads to

$$\chi \simeq h, \dot{U} = V \simeq U, U, \chi \sim \ell U$$

(55)

If we assume the constant slow roll parameter $\epsilon_2 = \beta$ is quite small, so that to imply dropping of $\dot{\chi}$, then $\epsilon_4$ is negligible as well. In order to meet the requirements ($\alpha << 1, \ell > 1$), we shall scan over the one-dimensional sub-parameter space parameterized as

$$\alpha = \Lambda^{-n}, \ell = \Lambda^m, V_0 = \Lambda, \chi \sim h = \ell^{-1} = \Lambda^{-m}$$

(56)

with $(n, m > 0)$. Noting that $e^{\Lambda h}$ is of order $O(1)$ we get for $\Lambda$ large

$$\dot{\chi} = O(1)$$

(57)

Then in order to get the following quantities small

$$1 - n_s \approx -\frac{4\epsilon_1}{1 + \epsilon_1}, \epsilon_1 = \frac{3}{4} \frac{x^2 + \alpha \chi^4}{c}, \quad r \approx \frac{12\dot{\chi}^2}{\dot{U}}$$

(58)

we need to enforce

$$0 < n < 1, \quad 0 < m < \frac{1 - n}{4}$$

(59)

Numerically, we checked by taking, say, the following four choices that the obtained points, represented in Fig. 1 by green dots, do fit the data:

- $\Lambda = 10^6, n = 0.5, m = 0.1 \Rightarrow (1 - n_s, r) = (0.0280827, 0.112981)$
- $\Lambda = 10^{5.8}, n = 0.5, m = 0.1 \Rightarrow (1 - n_s, r) = (0.032229, 0.12977)$
- $\Lambda = 10^{6.1}, n = 0.5, m = 0.1 \Rightarrow (1 - n_s, r) = (0.0262134, 0.10542)$
- $\Lambda = 10^{4.44}, n = 0.6, m = 0.001 \Rightarrow (1 - n_s, r) = (0.0298586, 0.120145)$

(60) (61) (62) (63)

which proves the viability of the model.

7 Summary and Conclusion

We continued in this letter the work of [1] on the inflationary model generated by varying strong coupling constant, and studied here the effect of the quadratic kinetic energy term which appears upon introducing an $f(R)$ gravity, represented by an $\alpha R^2$-term in the pure gravity Lagrangian. We investigated in Palatini formalism two extreme cases corresponding first to ($\alpha >> 1$), which represents thus a highly non-canonical k-inflation, and second to ($\alpha << 1$) where we kept terms to first order in $\alpha$ and examined a specific type of the k-inflation, namely the constant roll inflation. In both cases, we showed the viability of the model for some choices of the free parameters in regards to the spectral parameters $(n_s, r)$ when compared to the results of Planck 2018 separately and combined with other experiments.

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