Columnar Fluctuations as a Source of Non-Fermi-Liquid Behavior in Weak Metallic Magnets

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It is shown that columnar fluctuations, in conjunction with weak quenched disorder, lead to a $T^{3/2}$ temperature dependence of the electrical resistivity. This is proposed as an explanation of the observed non-Fermi-liquid behavior in the helimagnet MnSi, with one possible realization of the columnar fluctuations provided by skyrmion lines that have independently been proposed to be present in this material.

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Weak metallic ferromagnets, which display long-range order only at temperatures on the order of 10 K, provide a rich testing ground for fundamental aspects of condensed-matter physics. A prototypical example is MnSi, which displays long-range magnetic order below a critical temperature $T_c \approx 29$ K \cite{1}. This low $T_c$ implies that the (in this case ferromagnetic) exchange interaction between the electrons has been renormalized down from the atomic scale ($\approx 10^5$ K) by a factor of almost $10^4$, and thus has to compete with spin-spin interactions of relativistic nature that are negligible in magnets with higher values of $T_c$. In MnSi, a prominent role is played by the Dzyaloshinsky-Moriya (DM) interaction \cite{2,3}, which leads to a term $M \cdot (\nabla \times M)$ in the free energy density, with $M$ the magnetization. Such a term can only occur in systems that are not inversion symmetric, such as MnSi which crystallizes in the non-centrosymmetric B20 structure, and leads to a helical ground state in the ordered phase, rather than a ferromagnetic one \cite{4}. The DM interaction derives from the spin-orbit interaction, which is weak on the atomic energy scale. As a result, the pitch wave number $q$ of the helix is small compared to the inverse atomic length scale; in MnSi, $2\pi/q \approx 180$ Å. Crystal-field effects weakly pin the helix wave vector in the $\langle 1,1,1 \rangle$ directions. Upon the application of hydrostatic pressure, $T_c$ decreases until it vanishes at a critical pressure $p_c \approx 14.6$ kbar \cite{5}, see Fig. 1\textsuperscript{1} Partial magnetic order has been observed in the paramagnetic phase below a temperature $T_0$ \cite{6,7}. In a larger region within the paramagnetic phase, below a temperature of about 10 K and at pressures up to about 2$p_c$, the electrical resistivity $\rho$ displays a $T^{3/2}$ temperature dependence over almost three decades from a few mK to almost 10 K \cite{6}. This is in striking contrast to Fermi-liquid theory, which predicts a $T^2$-dependence of $\rho$ at low temperatures. The boundary of the partial order and the boundary of the non-Fermi-liquid (NFL) behavior meet the helical phase boundary at roughly the location of a tricritical point ($T_{TCP} \approx 10$ K) that separates a line of second-order or very weakly first-order transitions at higher temperatures from a line of strongly first-order transitions at lower ones \cite{7}. In a magnetic field $H$ the magnetization acquires a homogeneous component in addition to the helix, which leads to the formation of a conical phase \cite{8}. The amplitude of the helix vanishes at a critical field $H_{c2}$, above which one has a non-chiral field-polarized state. The NFL behavior in the disordered phase persists for $H > 0$ up to the boundary of the field-polarized ferromagnetic state \cite{6}. In MnSi, the relevant field scale is on the order of $0.1 - 0.5$ T, see Fig. 2\textsuperscript{1} In an intermediate field region within the ordered phase near $T_c$ one observes the so-called A-phase, which was believed to represent a single helix perpendicular to the field direction \cite{10}, but more recently has been interpreted in terms of spin textures known as skyrmions \cite{11}. All of these observations are

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{figure1.png}
\caption{Schematic phase diagram of MnSi in the $T$-$p$ plane showing the helical phase, the partial-order region, and the NFL region. The transition to the helical phase is second order (solid line) above a tricritical temperature $T_{TCP} \approx 10$ K, and first order (dashed line) below. Non-Fermi-liquid (NFL) behavior characterized by an electric resistivity $\rho \propto T^{3/2}$ is observed in the paramagnetic phase below about 10 K up to a pressure of at least 2$p_c$. The boundary of the NFL region is not sharp. The data in the inset are from Ref.\cite{6}.}
\end{figure}
A very interesting aspect of helical magnets is that they provide electronic analogs of liquid crystals. The helical phase is analogous to the ordered phase in cholesteric liquid crystals, and the helical Goldstone modes are very similar to the ones found in both cholesteric and smectic liquid crystals. The partial magnetic order in the paramagnetic phase is reminiscent of order found in the blue phases of cholesterics, and various analogs of blue-phase physics have been invoked to explain the line labelled $T_0$ in Fig. 1. These analogies ultimately break down due to lattice effects in solids at very small energy scales; how small depends on the system. Order in the spin sector (e.g., helical magnets) is advantageous in this respect compared to order in the charge sector (e.g., density stripe phases), since the electronic spin couples to the ionic lattice only via the spin-orbit interaction, which is much weaker than the Coulomb interaction.

The NFL behavior observed in MnSi is extremely remarkable. Fermi-liquid theory is very general, and there are not many established examples of it breaking down. One is one-dimensional systems, where the concept of a Fermi liquid must be replaced with that of a Luttinger liquid. In higher dimensions, NFL behavior is usually associated with quantum critical points. If critical soft modes couple to the electron density, NFL behavior can result. For instance, the NFL linear $T$-dependence of the resistivity in the normal phase of high-$T_c$ superconductors has been proposed to be due to a hidden quantum critical point. In the case of MnSi this is not a viable explanation. Not only is the transition at low temperatures first order and hence there are no critical soft modes, but also the NFL region extends to pressures much too far from the critical pressure $p_c$ for the system to be in a critical regime. (There is a quantum critical point at nonzero field that one might invoke, but the second objection still applies.) Still, soft modes coupling to the relevant degrees of freedom are the only known mechanism for producing NFL behavior, and hence an obvious question is the possible existence of generic soft modes that exist in an entire region of the phase diagram.

We now show that the observed NFL behavior in MnSi can be explained by columnar spin textures and their fluctuations, which have an unusual dispersion relation. In addition, weak quenched disorder is necessary to explain the observations. To motivate this suggestion we stress again that helical magnets are in many respects electronic analogs of liquid crystals. Columnar phases are known to exist in the latter, and it is natural to expect them in helical magnets as well. Indeed, a particular realization of columnar order, namely, spin textures known as skyrmions, have been proposed to exist in MnSi both in a magnetic field (in the A-phase), and in zero magnetic field. For our purposes the particular realization of the columnar order is not important, and neither is the existence of long-range order in the columnar phase; our conclusions remain valid as long as columns exist.
Let us assume columnar order forming a two-dimensional hexagonal lattice with lattice constant $a$, as shown in Fig. 3. Let the columns point in $z$-direction, and we denote the direction perpendicular to $z$ by $\perp$. Elasticity theory [24] shows that fluctuations with a wave vector $k$ in the $\perp$ direction will have the dispersion of ordinary phonons, i.e., the frequency squared will scale linearly with $k_z^2$. However, since the energy of the lattice cannot change under a uniform rotation of the columns, the frequency cannot depend on $k_z^2$, and the lowest power of $k_z$ allowed is $k_z^4$, which can be seen as follows.

Consider an equilibrium state that is a hexagonal lattice of lines, Fig. 3(a), described by two-dimensional lattice vectors $\mathbf{R}_n = (X_n, Y_n)$. The fluctuations of the lattice can be expressed in terms of a two-dimensional displacement field $\mathbf{u}(\mathbf{x})$, with the fluctuating lines given by

$$\mathbf{r}_n(z) = (X_n + u_x(\mathbf{R}_n, z), Y_n + u_y(\mathbf{R}_n, z), z).$$  \hfill (1)

The free energy $F$ associated with the harmonic fluctuations of the lattice can be expressed in terms of the derivatives of $\mathbf{u}$ as follows [22]

$$F = \frac{1}{2} \int d\mathbf{x} \left[ B(\partial_x u_x + \partial_y u_y)^2 + C(\partial_x u_x - \partial_y u_y)^2 
+ (\partial_x u_y + \partial_y u_x)^2 
+ K \left( (\partial_z u_x)^2 + (\partial_z u_y)^2 \right) \right],$$  \hfill (2)

where $B$, $C$, and $K$ are elastic constants. The absence of a tilt modulus proper, which would multiply a term quadratic in $\partial_z \mathbf{u}$, is due to rotational invariance. The diagonalization of the quadratic form given by Eq. (2) yields two eigenvalues which, in Fourier space, have the form

$$\lambda_{1,2} = \alpha_{1,2} k_z^2 + K k_z^4,$$  \hfill (3)

where $\alpha_1 = C$, $\alpha_2 = B + C$. The frequency of columnar fluctuations must therefore be of the form

$$\omega_{\text{col}}(k) = \sqrt{c_z k_z^2 + c_\perp a^2 k_z^4}.$$  \hfill (4)

Here $c_z$ and $c_\perp$ are elastic constants, and generically $c_z \approx c_\perp$ apart from a factor of $O(1)$. The application of a field $H$ that provides a restoring force for rotations of the lattice leads to a tilt modulus proper that is proportional to $H^2$, and hence to a term proportional to $k_z^2$ in Eq. (4) whose prefactor is proportional to $H^2$.

Since the displacement vector, and hence the columnar lattice, are two-dimensional there are two such modes; one compression mode akin to longitudinal phonons, and one shear mode akin to transverse phonons. When the lattice melts the transverse mode will disappear, but the longitudinal one will survive as long as columns exist.

Such columnar spin fluctuations will couple to the electronic charge degrees of freedom and contribute to the electrical resistivity $\rho$, just as ordinary magnons in antiferromagnets (which obey $\omega(\mathbf{k}) = \sqrt{c \mathbf{k}^2}$), contribute to $\rho$ [23]. In a material with no imperfections, this results in $\rho \propto T^3$, which is subleading compared to the Fermi-liquid $T^2$ contribution. In real systems, quenched disorder is always present. The MnSi samples where the NFL behavior is observed are clean enough to be, for experimentally realizable temperatures, in the weak-disorder or ballistic regime [26] where $T \tau > 1$, with $\tau$ the elastic mean-free time that enters the Drude formula for the residual ($T = 0$) resistivity, $\rho_0 = m_e/\pi e^2 \tau$, with $m_e$ the electronic effective mass, $n$ the electron density, and $e$ the electron charge. In this regime the electrical resistivity due to scattering by columnar fluctuations can be calculated perturbatively by adapting a theory developed in Ref. [21]. In the current case, we are interested in scattering by excitations with a dispersion relation given by Eq. (4). These are generalized phase fluctuations, so they couple to the conduction electrons via the gradient of the fluctuation variable. The formalism of Ref. [28] then applies, the only difference being that the resonance frequency of the fluctuations is given by Eq. (4). The leading diagrams are shown in Fig. 4. The conductivity tensor is anisotropic, but the temperature dependence is the same for all components. The result of the calculation is

$$\delta \rho = \rho_0 f(a_k, \lambda/\epsilon_F) (T/\epsilon_F)^{3/2}. \quad \hfill (5)$$

Here $\epsilon_F$ is the Fermi energy, $k_F$ is the Fermi wave number, and $\lambda$ is the energy scale associated with the magnetic order, e.g., the Stoner splitting between electron bands that results from helical or skyrmionic order involving one or more helices. The prefactor $f$, which depends on dimensionless combinations of these parameters, is model dependent. It tends to depend on $a_k$ to a negative power. Since $a$ is given by the inverse pitch wave number in any spin texture that involves helices, and since the latter is small compared to $k_F$ on account of the weakness of the spin-orbit interaction, the prefactor of the $T^{3/2}$ dependence tends to be much smaller in model calculations than what is experimentally observed. The same is true for calculations of the light scattering cross-section in the blue phases of liquid crystals [29]. In that case, the scattering is believed to be hugely enhanced by mode-mode coupling effects [30, 32], and a similar enhancement is likely to occur in analogous magnetic systems. In a magnetic field $H$, a term proportional to $k_z^2$ with a prefac-

![FIG. 4: Leading diagrams for the conductivity in the ballistic or weak-disorder limit. Dotted lines represent the propagator of the columnar fluctuations, and crossed dashed lines represent the quenched impurities. The dots represent current vertices.](image-url)
tor proportional to $H^2$ appears under the square root in Eq. (4). With parameter values appropriate for MnSi, simple considerations show that the $T^{3/2}$ behavior will still hold for temperatures $T \gtrsim (\mu_B H)^2/k_B T_c$. For fields $H \approx 0.1$ T, see Fig. [2] and with $T_c \approx 30$ K, this yields a lower bound for the NFL region of less than 1 mK, which is consistent with the observations.

This remarkable result suggests the following explanation of the observed NFL behavior in MnSi. The DM interaction leads to chiral spin textures of helical nature everywhere in the phase diagram. In the ordered phase with no magnetic field the ground state is a single helix, which has a lower free energy than columnar order. In the disordered phase, short-ranged columnar order (possibly of skyrmion type, but this is not essential for our argument) is present at low temperatures; the disordered phase is a columnar chiral fluid. It is divided into a chiral liquid and a chiral gas; the former is the partial order phase reported in Refs. [7, 8] and it is separated from the latter by a first-order phase transition that ends in a critical point [13]. The columnar soft mode is present in both the chiral liquid and the chiral gas as explained above. In samples that are sufficiently clean to be in the weak-disorder or ballistic limit, it leads to the observed NFL behavior by scattering the conduction electrons. At temperatures higher than a few K, the short-ranged columnar order is gradually destroyed and the NFL behavior fades. The upper temperature limit of the ballistic regime also contributes to a crossover to a different $T$-dependence of the resistivity. The weakness of the spin-orbit interaction translates into a small prefactor of the $T^{3/2}$-contribution to the resistivity in any bare theory; the quantitative observations require anomalously large fluctuations in the disordered phase. The same is true for the blue phases in liquid crystals, where theories yielding such an enhancement effect have been put forward [30–32]. In the light of the overall strong analogies between smectic and cholesteric liquid crystals on one hand, and helical magnets on the other, it is plausible to expect a similar enhancement effect have been put forward [30–32]. In the light of the overall strong analogies between smectic and cholesteric liquid crystals on one hand, and helical magnets on the other, it is plausible to expect a similar enhancement effect have been put forward [30–32]. In the light of the overall strong analogies between smectic and cholesteric liquid crystals on one hand, and helical magnets on the other, it is plausible to expect a similar enhancement effect have been put forward [30–32]. In the light of the overall strong analogies between smectic and cholesteric liquid crystals on one hand, and helical magnets on the other, it is plausible to expect a similar enhancement effect have been put forward [30–32]. In the light of the overall strong analogies between smectic and cholesteric liquid crystals on one hand, and helical magnets on the other, it is plausible to expect a similar enhancement effect have been put forward [30–32].