Study on Design Method of Cold-formed Steel Beam with No Axis of Symmetry Section

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Abstract. The paper mainly describes the detailed study resulted on bending carrying capacity of two kinds of cold-formedsteel beam with abroach and asymmetric section. This paper has completed 9 groups tests of torsion bending, and totally 12 specimen based on the current theory, deducing the critical moment of asymmetric section under lateral loading, providing the formula of coefficient of integral stability, and compared with tests results determined the simplified formula for engineering design.

1. Introduction
In recent years, Container building has been widely used in earthquake relief, construction line, military station, industrial housing, commercial buildings and other projects due to its advantages of green environmental protection, recycling, quick disassembly and installation, and so on. Accompanying functional requirements, structural requirements increased. After Technical specification for modular freight container building has been issued in 2013[1], Container building is becoming more and more research hotspot.

At present, China is in a period of rapid development of steel structure housing. The housing industry has been developing for many years, and there are cold-formed steel structure system[1], layered prefabricated support steel structure industrial building system[2] and other residential structure forms. At present, the research on the overall stability of cold-formedSteel mainly focuses on the members with biaxial symmetric section, uniaxial symmetric section and polar symmetric section. Cold-formed steel beams are mostly open-section, and the section form is becoming more and more complex. There are few researches on the mechanical properties of this type of section members at home and abroad, and few researches on the mechanical mechanism of container building beams with this section as the main component.

Based on this, this paper takes the cold-formed steel beam without symmetry axis as the research object, studies the stable load of such cold-formed steel beam without symmetry axis by means of experiment and theoretical analysis, and puts forward the corresponding formula for calculating the overall stability coefficient.

2. Section design of cold-formed steel beam without symmetry axis
Container building bottom frame beam (hereinafter referred to as the bottom beam) and Container...
building top frame beam (hereinafter referred to as the top beam) are the main components in the Container building structural system. In order to ensure that the connection between the beam and the wallboard and floor meet the requirements of Structural requirements and construction convenience, the transformation of c-section is adopted. Therefore, there are two cold-formed steel beam sections without symmetry axis, as shown in the following figure 1.

![Figure 1-1. Bottom beam section size](image)

![Figure 1-2. Top beam section size](image)

In order to better understand the mechanical properties of cold-formed steel beams without symmetry axis, the stability bearing capacity of these two kinds of cross-sectional beams under the action of concentrated forces in mid-span is studied in this paper.

3. Study on stability of cold-formed steel beam without symmetry axis

The test specimen of cold-formed steel beam without symmetry axis are divided into bottom frame beam and top frame beam. The main parameters of the specimen are shown in table 1. All specimen are provided with stiffening ribs at the loading position and the support. The numbering rules of specimen are shown in figure 2. The total number of specimen is 12. The geometric size of section of all specimen is measured at three sections in the middle and both ends of the research section, and the average value of the results measured three times is taken as the actual section size of specimen. The number of each plate is a1, a2…a9. The average of the actual measurements is shown in table 2.

![Figure 2. the numbering rules of specimen](image)

| Specimen number | Plate thickness/mm | Loading position | Quantity |
|-----------------|--------------------|------------------|----------|
| 1               | BFB4               | 4                | Midspan  | 2        |
| 2               | BFB3.5             | 3.5              | Midspan  | 2        |
| 3               | BFB3               | 3                | Midspan  | 2        |
| 4               | TFB4               | 4                | Midspan  | 2        |
| 5               | TFB3.5             | 3.5              | Midspan  | 2        |
| 6               | TFB3               | 3                | Midspan  | 2        |
3.1. Research on initial parameters of cold-formed steel beam without symmetry axis

According to GB/T228.1, part 1: tensile testing of metallic material: methods at room temperature, material test results are summarized in table 3:

| Type       | Yield Strength /Mpa | E/10^6 Mpa | Elongation /% |
|------------|---------------------|------------|---------------|
| Bottom beam| 344.61              | 2.11       | 31.08         |
| Top beam   | 305.73              | 2.10       | 33.80         |

Some initial defects may occur in the process of fabrication. The cold-formed steel beam without symmetry axis studied in this paper has many bending times, and the beams with two section forms have 9 bending times, which will produce relatively large initial distortion defects, as shown in figure 3. Therefore, measurement is required.

(a) Bottom beam  
(b) Top beam  

Figure 3. Schematic diagram of specimen distortion initial defects

Table 4. Measurement statistics of initial distortion defects of specimen (mm)

| Specimen number | $\Delta_{d}^{\text{max}}$ | $\Delta_{d}^{'}/t$ | Specimen number | $\Delta_{d}^{\text{max}}$ | $\Delta_{d}^{'}/t$ |
|-----------------|----------------------------|-------------------|-----------------|----------------------------|-------------------|
| BFB4a           | 6.34                       | 1.59              | TFB4a           | 5.50                       | 1.38              |
| BFB4b           | -4.00                      | -1.00             | TFB4b           | 8.12                       | 2.03              |
| BFB3.5a         | 7.00                       | 2.00              | TFB3.5a         | 5.12                       | 1.46              |
| BFB3.5b         | 5.40                       | 1.54              | TFB3.5b         | 4.82                       | 1.38              |
| BFB3a           | 8.94                       | 2.98              | TFB3a           | 5.28                       | 1.76              |
| BFB3b           | 5.02                       | 1.67              | TFB3b           | 6.40                       | 2.13              |
3.2. **Experimental parameters of cold-formed steel beam without symmetry axis were determined**

Hydraulic jack oil pump was used to load the test, the load was measured by YLR-3 pressure load sensor, and the displacement was measured by 5G105 linear displacement sensor. Data of all measurement points were recorded by DH3820 high-speed static strain test and analysis system. The beam is subjected to bending and torsion as shown in figure 4. The beam end is simply supported and the load acts on the upper flange, which is directly loaded by hydraulic jack.

![Test loading device](image)

Figure 4. Test loading device

3.3. **Research on stability bearing capacity of cold-formed steel beam**

The above test parameters are adopted for experimental research, and the maximum load obtained from the test is shown in table 5.

| Specimen number | Loading position | Thickness/mm | Measure | Average |
|-----------------|------------------|--------------|---------|---------|
| BFB4            | Midspan          | 4            | 27.03   | 27.03   |
| BFB3.5          |                  | 3.5          | 22.13   | 22.13   |
| BFB3            |                  | 3            | 17.85   | 17.85   |
| TFB4            | a                | 4            | 29.1    | 29.16   |
|                 | b                |              | 29.22   |         |
| TFB3.5          | a                | 3.5          | 25.56   | 24.75   |
|                 | b                |              | 23.93   |         |
| TFB3            | a                | 3            | 19.23   | 18.76   |
|                 | b                |              | 18.28   |         |

Throughout the whole test, no obvious local buckling occurred in the loading process of the specimen. Since the loading position did not pass through the section shear center, under the action of the concentrated load in the middle of the span, bending and torsion deformation occurred in the specimen at the beginning of loading, and the whole bending and torsion failure finally appeared. Because the eccentricity of the top frame beam is small and the web height is high, the ultimate stability bearing capacity is generally more than 5% higher than that of the bottom frame beam.

In conclusion, it can be found that the change of thickness plays a great role in the improvement of bearing capacity. The wall thickness of the beam changes from 3mm to 3.5mm, and the increase of bearing capacity is more than 16%. When the wall thickness changes to 4mm, the increase of bearing capacity is about 50%.

4. **Theoretical research on the bearing capacity of cold-formed steel beam without symmetry axis**

For the flexural members with arbitrary sections simply supported at both ends, the critical bending moment is solved by using the energy method under the action of centrality eccentric load in the middle span. The formula of the total potential energy of unidirectional bending members is directly used in literature [5]:

\[
\text{p}_u = \frac{1}{kN}
\]
When transverse load is applied on a flexural member, the total potential energy equation is obtained by adding the potential energy of transverse load into equation (1). When the lateral displacement of the member occurs \( u \), the load does not do any work during its translation, and the total potential energy does not change. The action point of the load is located above and apart from the shear center \( a \). When the section of the component twists around the shear center, the action point falls for a certain distance \( a(1-\cos \varphi) \approx (a\varphi^2)/2 \). Therefore, the potential energy of external force increased by the lateral load is:

\[
\Pi = \frac{1}{2} \int_0^L [EI_y u'^2 + EI_y \varphi'^2 + GL, \varphi'^2 + 2\beta, M, \varphi'^2 + 2M, u' \varphi \psi dz]
\]

When the load is concentrated in the middle span:

\[
V_h = -\frac{1}{2} Pa \varphi_{1/2}^2
\]

If there is also a concentrated torque applied on the beam, the potential energy of the external force due to the work done by the torque is:

\[
V_T = -\sum T_i \varphi_i
\]

When the load is concentrated in the middle span:

\[
V_T = -T \varphi_{1/2}
\]

### 4.1. Analysis of overall stability coefficient of cold-formed steel beam

For steel beams with arbitrary sections, bending and torsion failure will occur to the members when the transverse concentrated load action position does not pass through the section shear center. The distance between the load action position and the shear center is \( e \), and the transverse load is equivalent to the transverse load and concentrated torque through the shear center, as shown in FIG. 7. The end supports are simply supported at both ends. By referring to the traditional stability theory, the equation of the total potential energy when the component has small deformation is:

\[
\Pi = \frac{1}{2} \int_0^L [EI_y u'^2 + EI_y \varphi'^2 + GL, \varphi'^2 + 2\beta, M, \varphi'^2 + 2M, u' \varphi \psi dz - \frac{1}{2} Pa \varphi_{1/2}^2 - Pe \varphi_{1/2}^2]
\]

It can be obtained by integrating the equilibrium equation [6] of lateral bending of the bending member twice

\[
EI_y u'' = -M, \varphi + C_1 \psi + C_2
\]

According to the boundary conditions of simply supported at both ends, \( u''(0) = 0, \varphi(0) = 0 \), and then

\[
C_1 = 0, C_2 = 0, \text{ and then } u'' = -\frac{M, \varphi}{EI_y} \]

Substitute it into (4) and arrange it to get the expression of the total potential energy with only the torsion Angle as the variable:
When you solve it using the Rayleigh-ritz method, assume that the deformation function conforming to the boundary conditions is \( \varphi = C \sin(\pi z/l) \), there are \( \varphi' = \frac{\pi}{l} C \cos(\pi z/l) \),
\[
\varphi'' = -\frac{\pi^2}{l^2} C \sin(\pi z/l) , \varphi_{12} = C . \quad \text{when } z \leq l/2 , M_i = Pz/2 ; \quad \text{when } l/2 < z \leq l , M_i = P(l-z)/2 .
\]
Since the bending moment changes on both sides are symmetric, the total potential energy is obtained by taking the integral of the left half span of the member and substituting it into (8) to get:
\[
\Pi = \frac{1}{2} \int_0^{l/2} \left[ \left( -\frac{P^2 z^4}{4EI_y} + \frac{\pi^4 EI_y}{l^4} C \sin^2 \frac{\pi z}{l} + (Gl_y + \beta_y Pz) \frac{\pi^2}{l^3} C \sin^2 \frac{\pi z}{l} \right) dz - \frac{1}{2} P \alpha C^2 - P e C \right] \frac{1}{l} \sin^2 \frac{\pi z}{l} dz = \frac{(\pi^2 + 4\beta_y Pz)}{48\pi^2} \right] C^2 - \frac{1}{2} P \alpha C^2 - P e C
\]
Where integral expression
\[
\int_0^{l/2} \frac{C^2}{l} \sin^2 \frac{\pi z}{l} dz = \frac{(\pi^2 + 6\beta_y Pz)}{48\pi^2} \int_0^{l/2} C^2 \sin^2 \frac{\pi z}{l} dz = \frac{(\pi^2 - 4\beta_y Pz)}{16\pi^2} \frac{1}{l^2}
\]
Put into (9) to get:
\[
\Pi = \left[ -\frac{P^2 (\pi^2 + 6)}{192\pi^2 EI_y} + \frac{\pi^4 EI_y}{4l^4} + \frac{\pi^2 GI_y}{4l^4} + \frac{P(\pi^2 - 4)}{16} \right] C^2 - \frac{1}{2} P \alpha C^2 - P e C
\]
According to the principle of potential energy standing value, \( \frac{\partial \Pi}{\partial C} = 0 \), and use this formula
\[
M = Pl/4 .
\]
Buckling conditions can be obtained:
\[
\frac{(\pi^2 + 6)}{3N_y} CM^2 - \left( \beta_y (\pi^2 - 4) C - 8aC - 8e \right) M - \pi^2 C_i^n N_{\alpha} = 0
\]
\[
M_{cr} = \left[ 0.555 \beta_y - 0.756a - \frac{0.756e}{C} \right] N_y
\]
In which, \( N_y = \frac{\pi^2 EI_y}{l^2} \), \( N_{\alpha} = \frac{1}{I_{ln}} \left( \frac{\pi^2 EI_y}{l^2} + GI_y \right) \)
When \( e = 0 \), Put it into (10), It is consistent with the results in literature [5].

| Loading position | \( a_i \) | \( a_z \) | \( a_s \) | \( a_i \) |
|------------------|----------|----------|----------|----------|
| Midspan          | 0.555    | 0.756    | 0.756/C  | 1.866    |

For the cold-formed steel beam without symmetry axis under the action of the above two loads respectively, the stability coefficient of its elastic flexural and torsional buckling is calculated according to equation (11)
\[
\varphi_{bx} = \frac{M_{cr}}{W_{s} f_y}
\]
In the formula, According to equation (10), \( M_{cr} \) is the critical elastic buckling load of cold-formed steel beam without symmetric axis under the action of concentrated mid-span load is calculated, \( W_{s} = I_{s}/y \) is the wool section modulus of cold-formed thin-walled steel beam determined by
compression fiber, \( I \) is the moment of inertia of the cross section about the strong axis x, \( y \) is the distance from the section centroid to the maximum compression fiber edge, \( f_y \) is the yield strength of steel.

All the above formulas assume that cold-formed thin-walled steel beams are in elastic working stage. For stability factors \( \phi_{st} \geq 0.7 \), the calculation is carried out in accordance with the regulations for design of cold-formed thin-wall steel structures (GB 50018)\(^6\).

4.2. Overall stability analysis of cold-formed steel beam under combined action of bending and torsion

The design rules of cold-formed thin-walled steel structures in China (GB 50018) give the calculation formula of the overall stability bearing capacity of cold-formed thin-walled steel beams under the combined action of bending moment and torque.

Figure 8 shows that when the centrally-eccentric concentrated load acts on the middle span, the maximum double moment in the middle span is:

\[
B_{\text{max}} = \frac{P_e}{2k} \times \frac{sh(kl/2)}{ch(kl/2)}
\]

(12)

In the formula, \( k = \sqrt{Gl / EI_{xx}} \) is the bending and torsion characteristic coefficient of the member.

Therefore, according to equation (12) and the stability coefficient obtained above, the critical load and corresponding critical bending moment of cold-formed thin-wall steel beam under the action of centrally-eccentric concentrated load can be obtained.

Figure 8 shows the critical load of bending, torsion and buckling of cold-formed thin-walled steel beam under the action of centrally-eccentric concentrated load in the middle of the span:

\[
P_{cr} = \frac{P_{cr}}{4\phi_{cr}W_{cr}} + \frac{e}{2kW_{cr}} \times \frac{sh(kl/2)}{ch(kl/2)}
\]

(13)

In the formula, \( f_y \) is the yield strength of steel, \( \phi_{cr} \) is the elastic buckling stability coefficient of cold-formed thin-walled steel beam under the concentrated load in the midspan. At this point to make \( e = 0 \), and put \( I_x = Ar^2/3, E = 206000 \text{MPa}, G = 79000 \text{MPa} \) into the critical load expression can be obtained:

\[
\phi_{cr} = \frac{4800}{W_{cr}} \left( \beta_y - 1.362a \right) I_y + \sqrt{\left( \beta_y - 1.362a \right)^2 I_y^2 + 6.072I_y I_{cr} + 0.078I_y A_r I_{cr}^2} \right) \cdot 235
\]

(14)

When \( \phi_{cr} \geq 0.7 \), the calculation shall be carried out in accordance with the design regulations of cold-formed thin-walled steel structures (GB 50018).

4.3. No symmetry axis cold-formed steel beam stability bearing capacity calculation verification

The overall stability coefficient under the concentrated load in the middle span is obtained from equation (14), and the maximum bearing capacity is obtained by substituting equation (13), which is compared with the test results.

| Specimen number | \( p_y / kN \) | \( p_y^0 / kN \) | \( p_y / p_y^0 \) |
|-----------------|-------------|-------------|----------------|
| BFB4            | 27.03       | 28.14       | 0.96           |
| BFB3.5          | 22.13       | 20.98       | 1.05           |
| BFB3            | 17.85       | 16.77       | 1.06           |
| TFB4            | a 29.1      | 33.52       | 0.87           |
|                 | b 29.22     |             | 0.87           |
Specimen number | $p'_a / kN$ | $p'_b / kN$ | $p'_a / p'_b$
--- | --- | --- | ---
TFB3.5 | a | 25.56 | 27.76 | 0.92
 | b | 23.93 | | 0.86
TFB3 | a | 19.23 | 20.88 | 0.92
 | b | 18.28 | | 0.88

In which, $p'_a$ represent test results, $p'_b$ represents the theoretical calculation result.

It can be seen from table 7 that the test results are in good agreement with the theoretical calculation. Equation (14) proposed in this chapter can be used to calculate the overall stability coefficient of cold-formed thin-wall steel beam without symmetry axis under the action of concentrated load.

5. Conclusion

The cold-formed steel beams without symmetry axis are buckling under concentrated load. Through experimental research, it is found that, with the increase of the thickness of the specimen, the stable bearing capacity increases significantly. The stable bearing capacity of 3mm specimen is about 18kN. When the thickness increases to 3.5mm, the stable bearing capacity increases by about 25%; when the thickness increases to 4mm, the stable bearing capacity increases by more than 50%. Under the same stress condition, the lifting amplitude of the top frame beam is larger than that of the bottom frame beam. Therefore, it will bring superior economic benefits to choose the components with appropriate wall thickness by considering the stress in practical engineering.

An energy method is used to analyze the stress of cold bending thin-wall steel beam without symmetry axis under the action of eccentric concentrated load. Midspan under concentrated loads is derived for the elastic buckling critical bending moment expressions, getting the corresponding elastic buckling stability coefficient calculation formula, based on the simplification, the formula of stability factor found stability coefficient simplified formula for no symmetric axis type cold bending thin-wall steel beam stability is in conformity with the requirements of the precision for calculating the ultimate bearing capacity can be more convenient for designers to use.

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