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Convexity package for momentum maps on contact manifolds. (English) [Zbl 1206.53082]
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E. Lerman [Ill. J. Math. 46, No. 1, 171–184 (2002; Zbl 1021.53061)] gave a theorem analogous to the convexity theorem for Hamiltonian torus actions in contact geometry when the torus orbits are transverse to the contact distribution and asked whether the transversality condition is necessary. The authors give a “convexity package” for contact manifolds and prove the following:

Theorem. Let a torus $T$ act on a cooriented compact connected contact manifold $M$ with contact momentum map $\Psi : M \times R_{>0} \to t^*$, where $t^*$ is the dual of the Lie algebra of $T$. Assume that the action is effective and the torus has dimension greater than 2. Then:

1. Let $y_0$ and $y_1$ be any two points in $M \times R_{>0}$. If the action is transverse ($0 \not\in \text{image } \Psi$), assume that the origin is not contained in the segment $[\Psi(y_0), \Psi(y_1)]$. If the action is not transverse ($0 \in \text{image } \Psi$), assume that $\Psi(y_0)$ and $\Psi(y_1)$ are not both zero. Then there exists a path $\gamma : [0, 1] \to M \times R_{>0}$ such that $\gamma(0) = y_0$ and $\gamma(1) = y_1$ and such that $\Psi \circ \gamma : [0, 1] \to t^*$ is a weakly monotone parametrization of the (possibly degenerate) segment $[\Psi(y_0), \Psi(y_1)]$.

2. The momentum map is open as a map to its image. Consequently:

3. The momentum cone $C(\Psi) = \{0\} \cup \Psi(M \times R_{>0})$ is convex.
4. The nonzero level sets, $\Psi^{-1}(\mu)$, for $\mu \neq 0$, are connected.
5. Let $A$ be a convex subset of $t^*$. If the action is transverse, suppose that $0 \not\in A$. If the action is not transverse, suppose that $A \neq \{0\}$. Then the preimage $\Psi^{-1}(A)$ is connected.

Moreover:

6. The momentum cone $C(\Psi)$ is a convex polyhedral cone. The authors also analyze examples with $\dim T \leq 2$.

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53D20 Momentum maps; symplectic reduction
52B99 Polytopes and polyhedra

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