Cattaneo–Christov theory for a time-dependent magnetohydrodynamic Maxwell fluid flow through a stretching cylinder

Saeed Islam¹, Abdullah Dawar¹, Zahir Shah² and Adnan Tariq³

Abstract
This research work explores the thermal and mass transport phenomena for a time-dependent Maxwell fluid flow in the presence of Cattaneo–Christov concept. The Maxwell fluid flow is analyzed through a stretching cylinder and sheet. Brownian motion, non-uniform heat source/sink, thermophoresis, and variable thermal conductivity are operated in this study. A theoretical analysis of the modeled system of equations is explored with the help of HAM. Impacts of fixed constraints on velocity, thermal, and concentration functions are offered graphically. It is concluded that the velocity profile heightens quickly for Newtonian fluid equated to non-Newtonian fluid (Maxwell) via curvature parameter while the temperature and concentration distributions increase quickly for non-Newtonian fluid as equated to the Newtonian fluid via curvature parameter. The presence of Maxwell and magnetic parameters increases the size of the trapping bolus.

Keywords
Maxwell fluid, MHD, Cattaneo–Christov concept, cylinder, sheet, HAM

Date received: 29 March 2021; accepted: 11 June 2021

Handling editor: James Baldwin

Introduction
Owing to the various uses throughout the fields of engineering and manufacturing equipment including insulation of nuclear reactors, heat exchangers, refrigerators, polymer processes, and plastics extrusion, research teams are now putting an excessive amount of focus on mass and heat transfer observation. The basic mathematical associations of Fourier’s and Fick’s principles are being used to explain the process of mass and heat transfer rate throughout a particular channel due to concentration and temperature variations, respectively. Computational and experimental investigations are achieved to study the characteristics of fluid flow and thermal transmission over the cylinder. Na and Pop¹ examined thermal performance in viscous flow field across a flowing cylinder in the trajectory and reverse of free flow. Kumari and Nath² studied a magnetohydrodynamic (MHD) time-dependent stagnation point viscous fluid flow. Toh et al.³ introduce numerical analysis to the Newtonian fluid flow and thermal transmission system in micro-channels. According to this analysis, when the Reynolds number is low, the viscosity of liquid declines, and thermal energy increases. The analysis of Oldroyd-B nanofluid

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over an extending surface with Cattaneo–Christov theory (CCT) was presented by Khan et al. The numerical investigation of incompressible Eyring-Powell nanofluid through a bi-directional extending sheet with CCT was offered by Wubshet. The stagnation point of nanofluid through a bi-directional extending sheet with convective investigations of incompressible Eyring-Powell stretching surface were investigated by Gireesha et al. The special effects of Hall current on MHD flow of dusty fluid through a stretching surface were investigated by Girish et al. of Hall current on MHD flow of dusty fluid through a stretching surface were investigated by Girish et al. The incompressible Maxwell fluid flow through a bi-directional extending surface with convective investigations was presented by Mahanthesh et al. Consequences of Joule heating and Hall current viscous dissipative flows of nanofluid were Mahanthesh et al. The Maxwell fluid flow with homogeneous-heterogeneous reactions (HHR) and CCT between spiraling disks was presented by Ahmed et al. The Maxwell fluid flow with homogeneous-heterogeneous reactions (HHR) and CCT between spiraling disks was presented by Ahmed et al. The Hall Effect on the convective flow of micropolar fluid with CCT was investigated by Shah et al. In another article, Shah et al. analyzed the MHD second-grade thin film flow with CCT over a time-dependent stretching sheet. The carbon nanotubes nanofluid between rotating disks was introduced by Bhattacharyya et al. Researchers got interests in non-Newtonian fluids due to their diverse uses in the field of engineering to analyze the transportation thermal and solutal behaviors. To define the behaviors of various non-Newtonian fluids, researchers recommend different models. Furthermore, various numerical and analytical techniques are applied to solve highly nonlinear and complicated constitutive equations of different models. The buoyancy effect on MHD Oldroyd-B, Jeffrey, and Maxwell nanofluids through cone with variable properties were presented by Raju et al. The Maxwell and Carreau nanofluids with different physical phenomena were offered by Hsiao, Ahmed et al. offered the Maxwell nanofluid flow with a thermal sink/source over a rotating disk. Shah et al. presented the thermal conduction in a Casson ferrofluid through an extending surface. The MHD and viscous dissipative flow of micropolar nanofluid through an extending sheet was analyzed by Hsiao. The MHD Maxwell fluid through an extending surface was considered by Abel et al. The Oldroyd-B nanofluid flow with mass and thermal stratification conditions was inspected by Waqas et al. The Oldroyd-B fluid flow with CCT and HHR under the impact of nonlinear thermal conduction was studied by Irfan et al. The MHD Williamson nanofluid flow through a nonlinear extending plate was presented by Dawar et al. The MHD non-Newtonian nanofluid flow through two dissimilar geometries with Joule heating was presented by Dawar et al. The MHD Jeffrey fluid flow with Hall and ion slip influences inspected by Krishna, Khader and Sharma analyzed the radiative flow of micropolar fluid with a magnetic effect over a stretching sheet. The MHD flow of Casson fluid with heating influence was presented by Tassaddiq et al. Further related studies are mentioned in Abel et al., Waqas et al., Irfan et al., Rajagopal, and Alamri et al.

In light of the overhead declared studies, we anticipated the analytical study of time-dependent non-Newtonian Maxwell fluid with Cattaneo–Christov theory through an extending cylinder. The non-uniform heat source/sink, variable thermal conductivity, Brownian motion, and thermophoresis phenomena are operated. Section 2 represents the mathematical modeling of the proposed study. Section 3 deals with the analytical solution of the present model. Section 4 grants the results and physical discussion on the embedded parameter. Section 5 displays the concluding remarks of the present analysis.

Problem formulation

The Maxwell fluid flow through a stretching cylinder of radius $R$ is considered here. A magnetic field $B = (0, 0, B_0)$ of strength $B_0$ is applied normal to the fluid flow. The variable properties are taken into account to analyze the heat transmission of the fluid flow. Cattaneo–Christov theory, thermophoresis, and Brownian motion are also utilized. Assume that $u$ and $w$ are the components of velocity along $z$– and $r$– directions correspondingly. $z$ is the axis of cylinder where $r$ is normal to $z$ as described in Figure 1. Furthermore, it is also assumed that $u_w(t, z) = \frac{\alpha z}{1 - \alpha t}$ is the unsteady stretching velocity of cylinder where $\alpha$, $\gamma$ are positive constants $T$, $T_w$, and $T_s$ are the temperature, surface temperature, and free stream temperature.

![Figure 1. Geometrical representation of the flow problem.](image)
\[ C, C_w, \text{ and } C_\infty \text{ are the concentration, surface concentration, and free stream concentration. According to all these assumptions, the leading equations are}^{34,35}: \]
\[ \frac{\partial (ru)}{\partial z} + \frac{\partial (rw)}{\partial r} = 0, \]  
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + w \frac{\partial u}{\partial r} + \lambda \left( \frac{\partial^2 u}{\partial r^2} + 2u \frac{\partial^2 u}{\partial r \partial z} + 2w \frac{\partial^2 u}{\partial z^2} \right) \]
\[ = \nu \left( \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) - \frac{\sigma B_e^2 u^2}{\rho}, \] 
\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} + w \frac{\partial T}{\partial r} + \lambda_1 \left( \frac{\partial^2 T}{\partial r^2} + u \frac{\partial^2 T}{\partial r \partial z} + 2w \frac{\partial^2 T}{\partial z^2} \right) \]
\[ + \frac{1}{\rho c_p} \frac{\partial}{\partial r} \left( K(T) \frac{\partial T}{\partial r} \right) + \frac{1}{\rho c_p} \frac{\partial}{\partial r} \left( \frac{k_f U_x}{r} \right) + \frac{1}{\rho c_p} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right)^2, \] 
\[ \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial z} + w \frac{\partial C}{\partial r} + \lambda_2 \left( \frac{\partial^2 C}{\partial z^2} + u \frac{\partial^2 C}{\partial r \partial z} + 2w \frac{\partial^2 C}{\partial z^2} \right) \]
\[ + \frac{1}{\rho c_p} \frac{\partial}{\partial r} \left( \frac{k_f U_x}{r} \right) + \frac{1}{\rho c_p} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right)^2, \] 
\[ \text{with boundary conditions}^{35}: \]
\[ u(t, z, r) = u_w(t, z) = \frac{az}{(1 - \alpha t)}, \quad w(t, z, r) = 0, \quad T = T_w, \quad C = C_w \text{ at } r = R, \]
\[ u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } r \to \infty. \]

Using (6), (2)–(5) are reduced as:

\[ (1 + 2\gamma \xi)\chi'' + 2\gamma \chi\chi'' - \frac{S}{2} \xi \chi'' - S\chi' - \chi^2 + \chi \chi'' - \frac{7}{4}\xi B_S^2 \chi'' - \frac{\beta}{4} \xi^2 \chi'' + 2\xi B_S \chi' \]
\[ - 2\xi B_S H^2 - S\xi \chi' \chi'' + S\xi \chi \chi'' + 3S\beta \chi \chi'' + 2\beta \chi \chi'' - \frac{\gamma^2}{1 + 1 + 2\gamma \xi} \chi^2 \chi'' - \beta \chi^2 \chi'' - M \chi' = 0, \] 
\[ (1 + 2\gamma \xi)\psi' + \text{Pr} \left( \chi' \psi' - \frac{S}{2} \xi \psi' \right) + (1 + 2\gamma \xi) \left( \psi \psi'' + \psi^2 \right) \xi + \]
\[ 2\gamma \psi' + 2\gamma \psi \xi \psi' + A^* \chi' + B^* \psi + N (1 + 2\gamma \xi) \psi^2 \]
\[ + N \left( 1 + 1 + 2\gamma \xi \right) \psi' - \]
\[ \text{Pr} \frac{3}{4} S^2 \xi \psi' - \frac{3}{2} S \chi \psi' - \frac{S}{2} \xi \psi' \chi' + \frac{1}{4} \xi S^2 \psi'' - S \xi \chi \psi'' + \psi'' \chi^2 + \psi \chi \chi' = 0, \] 

where \( \lambda \) is the relaxation time, \( \nu \) is the kinematic viscosity, \( \rho \) is the density, \( c_p \) is the specific heat, \( \lambda_1 \) and \( \lambda_2 \) are the thermal and concentration relaxation times respectively, \( D_B \) and \( D_T \) are diffusion coefficients of Brownian motion and thermophoresis, \( K(T) = k_\infty (1 + \alpha \psi) \) is the variable thermal conductivity where \( k_\infty \) is the free stream conductivity, \( \epsilon \) is the small conductivity parameter, and \( \psi \) is the dimensionless temperature parameter, \( A^* > 0 \) and \( B^* > 0 \) are the heat generation parameters, \( A^* < 0 \) and \( B^* < 0 \) are the heat absorption parameters.
\[
(1 + 2\gamma \xi) \Phi'' + Le Pr \left( \Phi' \frac{S}{2} \xi \Phi' \right) + 2\gamma \Phi' + \frac{Nt}{Nb} (1 + 2\gamma \xi) \psi'' + 2\gamma \psi' - \\
Le Pr \beta_1 \left( \frac{3}{4} S^2 \xi \Phi'' - \frac{3}{2} S \xi \Phi' \right) + \frac{1}{4} S^2 \xi^2 \Phi'' - S^2 \xi \Phi' \chi + \Phi'' \chi^2 + \Phi' \chi' \right) = 0,
\]

with transformed boundary conditions:

\[
\chi(0) = \chi'(\infty) = \psi(\infty) = \Phi(\infty) = 0, \\
\chi'(0) = \psi(0) = \Phi(0) = 1.
\]

Here, the unsteadiness parameter is indicated by \(S = \frac{\xi}{\eta} \), magnetic parameter is demarcated by \(M = \frac{\alpha B(1-\alpha)}{\eta} \), curvature parameter is denoted by \(\gamma = \frac{1}{B} \sqrt{\frac{1}{\alpha(1-\alpha)}} \), Maxwell parameter is signified by \(\beta = \frac{\alpha \beta}{c^2} \), Prandtl number is indicated by \(Pr = \frac{c_p}{\alpha} \), thermal and mass relaxation times parameters are represented by \(\beta_1 = \frac{\alpha \beta}{c^2} \) and \(\beta_2 = \frac{\alpha \beta}{c^2} \) respectively, Lewis number is indicated by \(Le = \frac{\alpha l}{Db} \), Brownian motion parameter is denoted by \(Nb = \frac{(\rho c_p) D_B (C_r - C_s)}{v} \), thermophoresis parameter is signified by \(Nt = \frac{(\rho c_p) D_B (T_r - T_s)}{v} \).

The skin friction, local Nusselt number, and Sherwood number are defined as:

\[
\sqrt{Re \text{C}_f} = -\frac{\text{Nu}_s}{\sqrt{Re}} = -\frac{\text{Sh}_s}{\sqrt{Re}} = -\Phi'(0).
\]

**HAM solution**

The linear operators and initial guesses are defined as:

\[
L_x[\chi] = \chi'' - \chi', \quad L_{\Phi}[\psi] = \psi'' - \psi, \quad L_{\Phi}[\Phi] = \Phi'' - \Phi,
\]

\[
\chi_0(\xi) = 1 - e^{-\xi}, \quad \psi_0(\xi) = e^{-\xi}, \quad \Phi_0(\xi) = e^{-\xi},
\]

with

\[
L_x \left[ R_1 + R_2 e^\xi + R_3 e^{-\xi} \right] = 0, \\
L_{\Phi} \left[ R_6 e^\xi + R_7 e^{-\xi} \right] = 0, \quad L_{\Phi} \left[ R_6 e^\xi + R_7 e^{-\xi} \right] = 0,
\]

where \(R_1 - R_7\) are called arbitrary constants.

Further detail of HAM can be found in Liao.36–38

**Results and discussion**

The variation in velocity \(\chi'(\xi)\), temperature \(\psi(\xi)\), and concentration \(\Phi(\xi)\) functions of the Maxwell fluid due to embedded parameters is the fundamental theme of this analysis. The values of relatable factor are taken as \(\gamma = 0.1, S = 0.1, c = 0.01, \beta_c = \beta_t = \beta = 0.5, Le = Pr = 6.5, Nb = Nt = 0.7, B^* = A^* = 0.1, \) and \(M = 1.0\) for the fluid flow profiles. Impacts of embedded constraints on the flow profiles are offered with the help of Figures 2 to 22. Furthermore, in the present analysis the Maxwell fluid flow is treated through cylinder (\(\gamma \neq 0\)) and sheet (\(\gamma = 0\)).

Figures 2 to 4 display the variation in \(\chi'(\xi), \psi(\xi)\), and \(\Phi(\xi)\) due to curvature parameter \(\gamma\). Increasing curvature parameter heightens the Maxwell fluid flow profiles. Actually, the radius of the cylinder reduces with
The greater curvature parameter $\gamma$ reduces the impact of boundary in the Maxwell fluid flow. Thus, the velocity profile heightens. A similar impact of curvature parameter $g$ on temperature and concentration profiles is depicted. Furthermore, the influence of curvature parameter on Newtonian and non-Newtonian (Maxwell) fluid flows is compared. The increasing impression of $g$ on $x(\xi)$ is greater for Newtonian fluid equated to non-Newtonian (Maxwell) fluid while a contrary conduct is observed on thermal and concentration profiles. Here, the Maxwell parameter $b$ plays an important role for Newtonian and non-Newtonian fluids. Physically, with the increasing Maxwell parameter, the fluid behaves like a solid which consequently increases the non-Newtonian fluid velocity while diminishes the thermal and mass transport. Thus, the greater impact on $x(\xi)$ is depicted for Newtonian and non-Newtonian (Maxwell) fluids. Figures 5 to 7 display the variation in $\chi'(\xi)$, $\psi(\xi)$, and $\Phi(\xi)$ due to unsteadiness parameter $S$. $\chi'(\xi)$ reduces while $\psi(\xi)$ and $\Phi(\xi)$ increase with the higher $S$. Physically, $S$ has direct relation with positive constant $a$. The increasing $a$ heightens the unsteadiness parameter which consequently increases the stretching rate of the sheet and cylinder. Thus, the velocity reduces with higher unsteadiness parameter. However, this impact is reverse on thermal and concentration profiles. The stretching rate of the sheet and cylinder increase the thermal and mass profiles of the fluid flow. Figure 8 signifies the consequence of Maxwell parameter $\beta$ on $\chi'(\xi)$. Greater $\beta$ reduces $\chi'(\xi)$. Physically, with the increasing Maxwell parameter the fluid behaves like a solid which consequently increases $\chi'(\xi)$. Figure 9 signifies the influence of magnetic parameter $M$ on $\chi'(\xi)$. Higher $M$ reduces $\chi'(\xi)$. Physically, the Lorentz force produces by applying $M$ in normal direction to the fluid flow. This

**Figure 4.** Variation in $\Phi(\xi)$ via $\gamma$ when $\beta_c = \beta_t = 0$, $A' = B' = 0.1$, $Pr = Le = 6.5$, $\varepsilon = 0.1$, $Nb = Nt = 0.2$, $\beta = 0.1$, and $M = 0.4$.

**Figure 5.** Variation in $\chi'(\xi)$ via $S$ when $\beta_c = \beta_t = 0$, $A' = B' = 0.1$, $Pr = Le = 6.5$, $\varepsilon = 0.1$, $Nb = Nt = 0.2$, $\beta = 0.1$, and $M = 0.4$.

**Figure 6.** Variation in $\psi(\xi)$ via $S$ when $\beta_c = \beta_t = 0$, $A' = B' = 0.1$, $Pr = Le = 6.5$, $\varepsilon = 0.1$, $Nb = Nt = 0.2$, $\beta = 0.1$, and $M = 0.4$.

**Figure 7.** Variation in $\Phi(\xi)$ via $S$ when $\beta_c = \beta_t = 0$, $A' = B' = 0.1$, $Pr = Le = 6.5$, $\varepsilon = 0.1$, $Nb = Nt = 0.2$, $\beta = 0.1$, and $M = 0.4$. 

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Lorentz force creates resistance to the fluid flow which declines the motion of fluid particles. Thus, \( x_j(\xi) \) reduces with greater \( M \). Figures 10 and 11 depict the variation in temperature profile via heat generation (\( A^* = B^* = 0.1, Pr = Le = 6.5, \xi = 0.1, Nb = Nt = 0.2, S = 0.3, \beta = 0.1, \) and \( M = 0.4 \)).

In the existence of heat generation parameters, the thermal boundary layer gains additional energy which leads the temperature to escalate. Thus, the temperature profile rises with higher heat generation. However, the absorption parameters absorb the heat energy from the boundary layer which results in a gradual decrease in the fluid flow temperature. From these figures, we have seen a rapid increase for the case of heat generation while this impact is slow for the case of absorption. Figure 12 shows the variation in \( \psi(\xi) \) due to Prandtl number \( Pr \). The higher \( Pr \) diminishes the temperature of the fluid flow. Physically, the higher \( Pr \) declines the thermal diffusivity of the fluid which

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**Figure 8.** Variation in \( x_j(\xi) \) via \( \beta \) when \( \beta_c = \beta_l = 0 \), \( A^* = B^* = 0.1, Pr = Le = 6.5, \xi = 0.1, Nb = Nt = 0.2, S = 0.3, \) and \( M = 0.4 \).

**Figure 9.** Variation in \( x_j(\xi) \) via \( M \) when \( \beta_c = \beta_l = 0 \), \( A^* = B^* = 0.1, Pr = Le = 6.5, \xi = 0.1, Nb = Nt = 0.2, S = 0.3, \) and \( \beta = 0.1 \).

**Figure 10.** Variation in \( \psi(\xi) \) via \( A^* \) when \( \beta_c = \beta_l = 0 \), \( Nb = Nt = 0.2, Pr = Le = 6.5, \xi = 0.1, B^* = 0.1, S = 0.3, \beta = 0.1, \) and \( M = 0.4 \).

**Figure 11.** Variation in \( \psi(\xi) \) via \( B^* \) when \( \beta_c = \beta_l = 0 \), \( Pr = Le = 6.5, Nb = Nt = 0.2, B^* = 0.1, S = 0.3, \beta = 0.1, \) and \( M = 0.4 \).

**Figure 12.** Variation in \( \psi(\xi) \) via \( Pr \) when \( \beta_c = \beta_l = 0 \), \( A^* = B^* = 0.1, Le = 6.5, Nb = Nt = 0.2, S = 0.3, \beta = 0.1, \) and \( M = 0.4 \).
higher values of \( b_t \) and \( \beta_c \) reduces with greater values of \( b_t \) and \( \beta_c \) relaxation time parameters requires additional time for the transportation of mass and heat. Thus, the thermal and mass profiles reduce with greater values of \( \beta_t \) and \( \beta_c \). Figure 15 shows the change in concentration profile via Lewis number \( Le \). The concentration profile reduces with greater \( Le \). Actually, the higher values of \( Le \) reduce the thermal diffusivity and increase the Brownian diffusivity of the fluid which results reduction in \( \Phi(\xi) \). Thus, the increasing \( Le \) moderates \( \Phi(\xi) \).

Brownian diffusivity of the fluid which results reduction in \( \Phi(\xi) \). Thus, the increasing \( Le \) moderates \( \Phi(\xi) \).

Figure 15. Variation in \( \Phi(\xi) \) via \( Le \) when \( \beta_c = \beta_t = 0 \), \( A^* = B^* = 0.1, \text{Pr} = Le = 6.5, Nb = Nt = 0.2, S = 0.3, \beta = 0.1 \), and \( M = 0.4 \).

Figure 16(a) and (b) shows the streamlines for fluid flow in the absence and presence of Maxwell parameter \( \beta \). It is observed from the figures that the size of trapping bolus intensifies in the presence of \( \beta \) whereas the size of trapping bolus reduces in the absence of \( \beta \). Figure 17(a) and (b) shows the streamlines for the fluid flow in the absence and presence of magnetic parameter \( M \). It is observed from the figures that the size of trapping bolus increases in the presence of \( M \) whereas the size of the trapping bolus reduces in the absence of \( M \).

Figures 18 and 19 show the sketches of residual error for different values of \( h \) through a sheet and cylinder. The residual error for sheet converges after \( 0.0 < \text{iteration} < 6.5 \) and the residual error for cylinder converges after \( 0.0 < \text{iteration} < 4 \). Thus, we have concluded that the residual error converges quickly for cylinder as compared to sheet. Figures 20 to 22 indicate the assessment of HAM and shooting techniques for \( \chi(\xi), \psi(\xi), \) and \( \Phi(\xi) \) respectively. Here, both the techniques have quit close agreement.

Table 1 displays the numerical estimations of \( -\chi''(0) \) for increasing \( \beta \) when \( \gamma = S = M = 0 \). The increasing values of Maxwell parameter increases \( -\chi''(0) \). Furthermore, it is also established from the comparison of numerical values that the present model is valid. Table 2 displays the numerical values of \( -\psi'(0) \) for cylinder and sheet via different embedded factors when \( \beta_t = \beta_c = 0 \). The higher \( \beta, S, M, \) and \( \epsilon \) reduce \( -\psi'(0) \) whereas the higher \( \text{Pr} \) increases \( -\psi'(0) \). Furthermore, these effects are greater for cylinder as compared to the sheet. Table 3 displays the numerical estimations of \( -\Phi'(0) \) for cylinder and sheet via different embedded factors when \( \beta_t = \beta_c = 0 \). The higher \( \beta, S, \) and \( \epsilon \) reduce \( -\Phi'(0) \) whereas the higher \( Le \) increases \( -\Phi'(0) \). Furthermore, these effects are greater for cylinder as compared to the sheet.

Figure 13. Variation in \( \psi(\xi) \) via \( \beta_t \) when \( \beta_c = 0 \), \( A^* = B^* = 0.1, \text{Pr} = Le = 6.5, Nb = Nt = 0.2, S = 0.3, \beta = 0.1 \), and \( M = 0.4 \).

Figure 14. Variation in \( \psi(\xi) \) via \( \beta_c \) when \( \beta_t = 0 \), \( A^* = B^* = 0.1, \text{Pr} = Le = 6.5, Nb = Nt = 0.2, S = 0.3, \beta = 0.1 \), and \( M = 0.4 \).
Figure 16. (a, b) Streamlines for the fluid flow in the absence and presence of Maxwell parameter.

Figure 17. (a, b) Streamlines for the Maxwell fluid flow in the absence and presence of magnetic parameter.

Figure 18. Sketch of residual error for different values of $h$ through sheet when $\beta_c = \beta_t = 0, A^* = B^* = 0.1, Pr = Le = 6.5, \varepsilon = 0.1, Nb = Nt = 0.2, S = 0.3, \beta = 0.1$, and $M = 0.4$.

Figure 19. Sketch of residual error for different values of $h$ through cylinder when $\beta_c = \beta_t = 0, A^* = B^* = 0.1, Pr = Le = 6.5, \varepsilon = 0.1, Nb = Nt = 0.2, S = 0.3, \beta = 0.1$, and $M = 0.4$. 
Conclusion

The MHD flow of Maxwell fluid with variable thermal conductivity and non-uniform heat source/sink through a stretching cylinder is analyzed here. A magnetic field is applied normal to the fluid flow. The Cattaneo–Christov theory is taken for heat and mass transmission utilization. Final remarks are recorded underneath:

Table 1. Numerical values of $-\chi'(0)$ for increasing $\beta$ when $\gamma = S = M = 0$.

| $\beta$ | Ref. 31 | Ref. 33 | Present results |
|---------|---------|---------|-----------------|
| 0.0 | 1.000000 | 1.000000 | 1.000000 |
| 0.1 | 1.026198 | 1.026163 | |
| 0.2 | 1.051899 | 1.051863 | |
| 0.3 | 1.077125 | 1.075463 | |
| 0.4 | 1.101903 | 1.101875 | |
| 0.5 | 1.126235 | 1.125674 | |

Table 2. Numerical values of $-\psi'(0)$ for cylinder and sheet when $\beta_t = \beta_c = 0$.

| $\beta$ | $S$ | $M$ | $\epsilon$ | $Pr$ | $-\psi'(0)$ |
|---------|-----|-----|---------|------|-------------|
| Cylinder | | | | | |

| $\beta$ | $S$ | $M$ | $\epsilon$ | $Pr$ | $-\psi'(0)$ |
|---------|-----|-----|---------|------|-------------|
| 0.1 | 0.1 | 1.0 | 0.1 | 6.5 | 1.915654 | 1.905633 |
| 0.2 | 1.912845 | 1.900542 | |
| 0.3 | 1.907439 | 1.899654 | |
| 0.4 | 1.815785 | 1.807799 | |
| 2.0 | 1.753784 | 1.747246 | |
| 3.0 | 1.697420 | 1.687064 | |
| 4.0 | 1.846964 | 1.836807 | |
| | | | 0.2 | 1.814984 | 1.805682 | |
| | | | 0.3 | 1.754725 | 1.743857 | |
| | | | 0.4 | 1.690807 | 1.680963 | |
| | | | 2.0 | 1.564467 | 1.550998 | |
| | | | 4.0 | 1.668643 | 1.657832 | |
| | | | 6.0 | 1.758068 | 1.743263 | |

Table 3. Numerical values of $-\Phi'(0)$ for cylinder and sheet when $\beta_t = \beta_c = 0$.

| $\beta$ | $S$ | $M$ | $e$ | $Le$ | $-\Phi'(0)$ |
|---------|-----|-----|----|-----|-------------|
| Cylinder | | | | | |

| $\beta$ | $S$ | $M$ | $e$ | $Le$ | $-\Phi'(0)$ |
|---------|-----|-----|----|-----|-------------|
| 0.1 | 0.1 | 0.1 | 0.1 | 6.5 | 0.994372 | 0.987532 |
| 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.984326 | 0.979475 |
| 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.979264 | 0.964374 |
| 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 0.347532 | 0.339895 |
| 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 0.324797 | 0.317092 |
| 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 0.309864 | 0.299663 |
| | | | | | 0.2 | 0.113564 | 0.105984 |
| | | | | | 0.3 | 0.095436 | 0.093572 |
| | | | | | 0.4 | 0.068085 | 0.094435 |
| | | | | | 2.0 | 1.356355 | 1.345096 |
| | | | | | 4.0 | 1.965468 | 1.933879 |
| | | | | | 6.0 | 2.743574 | 2.715783 |
- An increasing conduct is testified for velocity, thermal, and mass profiles via curvature parameter.
- The increasing impression of curvature parameter on velocity profile is higher for Newtonian fluid (Maxwell) as compared to non-Newtonian fluid while a contrary conduct is observed on thermal and concentration profiles.
- The velocity profile reduces while the thermal and concentration profiles increase with the higher unsteadiness parameter.
- The increasing Maxwell and magnetic parameters declines the velocity profile.
- The greater heat generation and heat absorption heighten the thermal profile while the escalating Prandtl number and thermal relaxation time factor reduce the thermal profile.
- The higher mass relaxation and Lewis number reduce the concentration profile.
- The presence of Maxwell and magnetic parameters increase the size of trapping bolus.

Author contributions
Saeed Islam: Conceptualization, methodology, formal analysis, and software, resources. Abdullah Dawar: Conceptualization, investigation, writing original draft, preparation, methodology, and software. Zahir Shah: Writing review and editing, software, visualization, writing review and editing, and validation. Adnan Tariq: Writing review and editing, software, visualization, writing review and editing, and validation.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

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Data availability
The data that support the findings of this study are available from the corresponding author upon reasonable request.

References
1. Na TY and Pop I. Flow and heat transfer over a longitudinal circular cylinder moving in parallel or reversely to a free stream. Acta Mech 1996; 118: 185–195.
2. Kumari M and Nath G. Unsteady flow and heat transfer of a viscous fluid in the stagnation region of a three-dimensional body with a magnetic field. Int J Eng Sci 2002; 40: 411–432.
3. Toh KC, Chen XY and Chai JC. Numerical computation of fluid flow and heat transfer in microchannels. Int J Heat Mass Transf 2002; 45: 5133–5141.
4. Khan WA, Irfan M and Khan A. An improved heat conduction and mass diffusion models for rotating flow of an Oldroyd-B fluid. Results Phys 2017; 7: 3583–358.
5. Ibrahim W. Three dimensional rotating flow of Powell–Eyring nanofluid with non-Fourier’s heat flux and non-Fick’s mass flux theory. Results Phys 2018; 8: 569–577.
6. Moshkin NP, Pukhnachev VV and Bozhkov YD. On the unsteady, stagnation point flow of a Maxwell fluid in 2D. Int J Non-Linear Mech 2019; 116: 32–38.
7. Gireesha BJ, Mahanthesh B, Thammanna GT, et al. Hall effects on dusty nanofluid two-phase transient flow past a stretching sheet using KVL model. J Mol Liq 2018; 256: 139–147.
8. Mahanthesh B, Gireesha BJ, Thammanna GT, et al. Nonlinear convection in nano Maxwell fluid with nonlinear thermal radiation: a three-dimensional study. Alex Eng J 2018; 57(3): 1927–1935.
9. Mahanthesh B, Shashikumar NS, Gireesha BJ, et al. Effectiveness of Hall current and exponential heat source on unsteady heat transport of dusty TiO2–EO nanoliquid with nonlinear radiative heat. J Comput Des Eng 2019; 6(4): 551–561.
10. Mahanthesh B, Shehzad SA, Ambreen T, et al. Significance of Joule heating and viscous heating on heat transport of MoS2–Ag hybrid nanofluid past an isothermal wedge. J Therm Anal Calorim 2020; 143: 1221–1229.
11. Ahmed J, Khan M and Ahmad L. Effectiveness of homogeneous heterogeneous reactions in Maxwell fluid flow between two spiraling disks with improved heat conduction features. J Therm Anal Calorim 2020; 139: 3185–3195.
12. Saleem S, Awais M, Nadeem S, et al. Theoretical analysis of upper-convected Maxwell fluid flow with Cattaneo–Christov heat flux model. Chin J Phys 2017; 55: 15–25.
13. Ellahi R, Zeeshan A, Shehzad N, et al. Structural impact of kerosene-Al2O3 nanoliquid on MHD Poiseuille flow with variable thermal conductivity: application of cooling process. J Mol Liq 2018; 264: 607–615.
14. Shah Z, Shutaywi M, Dawar A, et al. Impact of Cattaneo-Christov heat flux on non-isothermal convective micropolar fluid flow in a Hall MHD generator system. J Mater Res Technol 2020; 9(3): 5452–5462.
15. Shah Z, Alzahrani OE, Dawar A, et al. Entropy generation in MHD second-grade nanofluid thin film flow containing CNTs with Cattaneo-Christov heat flux model past an unsteady stretching sheet. Appl Sci 2020; 10: 2720.
16. Bhattacharyya A, Seth GS and Kumar R. Simulation of Cattaneo-Christov heat flux on the flow of single and multi-walled carbon nanotubes between two stretchable coaxial rotating disks. J Therm Anal Calorim 2020; 139: 1655–1670.
17. Raju CSK, Sandeep N and Malvandib A. Free convective heat and mass transfer of MHD non-Newtonian
nanofluids over a cone in the presence of non-uniform heat source/sink. *J Mol Liq* 2016; 221: 108–115.

18. Hsiao KL. Combined electrical MHD heat transfer thermal extrusion system using Maxwell fluid with radiative and viscous dissipation effects. *Appl Therm Eng* 2017; 112: 1281–1288.

19. Hsiao KL. To promote radiation electrical MHD activation energy thermal extrusion manufacturing system efficiency by using Carreau–Nanofluid with parameters control method. *Energy* 2017; 130: 486–499.

20. Ahmed J, Khan M and Ahmad L. Stagnation point flow of Maxwell nanofluid over a permeable rotating disk with heat source/sink. *J Mol Liq* 2019; 287: 110853.

21. Shah Z, Dawar A, Khan I, et al. Cattaneo-Christov model for electrical magnetite micropolar Casson ferrofluid over a stretching/shrinking sheet using effective thermal conductivity model. *Case Stud Therm Eng* 2019; 13: 100352.

22. Hsiao KL. Micropolar nanofluid flow with MHD and viscous dissipation effects towards a stretching sheet with multimedia feature. *Int J Heat Mass Transf* 2017; 112: 983–990.

23. Abel MS, Tawade JV and Nandeppanavar MM. MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet. *Meccanica* 2012; 47: 385–393.

24. Waqas M, Khan MI, Hayat T, et al. Stratified flow of an Oldroyd-B nanoliquid with heat generation. *Results Phys* 2017; 7: 2489–2496.

25. Irfan M, Khan M and Khan WA. Impact of homogeneous-heterogeneous reactions and non-Fourier heat flux theory in Oldroyd-B fluid with variable conductivity. *J Braz Soc Mech Sci Eng* 2019; 40: 108.

26. Dawar A, Shah Z and Islam S. Mathematical modeling and study of MHD flow of Williamson nanofluid over a nonlinear stretching plate with activation energy. *Heat Transfer* 2020; 50(3): 2558–2570.

27. Dawar A, Shah Z, Tassaddiq A, et al. A convective flow of Williamson nanofluid through cone and wedge with non-isothermal and non-isosolutal conditions: a revised Buongiorno model. *Case Stud Therm Eng* 2021; 24: 100869.

28. Krishna MV. Hall and ion slip impacts on unsteady MHD free convective rotating flow of Jeffrey fluid with ramped wall temperature. *Int Commun Heat Mass Transf* 2020; 119: 104927.

29. Khader MM and Sharma RP. Evaluating the unsteady MHD micropolar fluid flow past stretching/shrinking sheet with heat source and thermal radiation: implementing fourth order predictor-corrector FDM. *Math Comput Simul* 2021; 181: 333–350.

30. Tassaddiq A, Khan I, Sooppy NK, et al. MHD flow of a generalized Casson fluid with Newtonian heating: a fractional model with Mittag–Leffler memory, *Alex Eng J* 2020; 59: 3049–3059.

31. Abel MS, Tawade JV and Nandeppanavar MM. MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet. *Meccanica* 2012; 47: 385–393.

32. Waqas M, Khan MI, Hayat T, et al. Stratified flow of an Oldroyd-B nanoliquid with heat generation. *Results Phys* 2017; 7: 2489–2496.

33. Irfan M, Khan M and Khan WA. Impact of homogeneous-heterogeneous reactions and non-Fourier heat flux theory in Oldroyd-B fluid with variable conductivity. *J Braz Soc Mech Sci Eng* 2019; 40: 521.

34. Rajagopal KR. A note on novel generalizations of the Maxwell fluid model. *Int J Non-Linear Mech* 2012; 47(1): 72–76.

35. Alamri SZ, Khan AA, Azeem M, et al. Effects of mass transfer on MHD second grade fluid towards stretching cylinder: a novel perspective of Cattaneo–Christov heat flux model. *Phys Lett A* 2019; 383: 276–281.

36. Liao SJ. Beyond perturbation: introduction to homotopy analysis method. Boca Raton: Chapman and Hall/CRC Press, 2003.

37. Liao SJ. On the homotopy analysis method for nonlinear problems. *Appl Math Comput* 2004; 147: 499e513.

38. Liao SJ. Notes on the homotopy analysis method: some definitions and theorems. *Commun Nonlinear Sci Numer Simul* 2009; 14: 983e997.