Calculation of Inter-Column Slabs Bearing Capacity by Kinematic Method

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Abstract. The method of bearing capacity calculation of the inter-column slabs of flat slab frame system is developed by a kinematic way based on the ultimate equilibrium method. The basis of the study is the kinematic method of the limit equilibrium method and its application to calculate the load-bearing capacity of the inter-column slab in flat slab structural system, as well as to select the area of the working reinforcement. Attention is also paid to establishing the location of the lines of plastic hinges formation.

1. Introduction

The condition of the housing stock and providing citizens with housing is an urgent and, at the same time, the least regulated problem of the Ukrainian economy. The availability of housing for the general public and the provision of socially disadvantaged groups is one of the priority areas of public policy in the field of construction. Over the years, the housing queue and the low availability of population compared to European countries - testifies to the increasing relevance of affordable housing at the regional and national level.

There are many reasons for this problem in construction. Among them - the use of imperfect structural systems and time-consuming technological processes of work, low level of mechanization of technological processes of production of building structures and materials, high level of material intensity of buildings, the use of imperfect and outdated architectural and planning decisions and others.

One of the solutions to this problem is the use of a flat slab structural system [1-3].

2. Flat slab structural system

The solution of the existing problem of providing affordable housing to the population is possible by improving the technology of building residential buildings based on the introduction of constructive systems, among which the most attractive is the pre-fabricated, flat slab system with the minimum number of standard sizes of prefabricated constructions (Fig. 1).

Interconnecting overlapping in buildings with applied frame (Fig. 1) consist of three types of prefabricated reinforced concrete slabs: overcolumned (pos. 5), inter-column (pos. 1) and middle (pos. 2). The thickness of all slabs is 160 mm, their size, in order to unify formwork, adopted the same -
3000 × 3000 mm. Vertical bearing elements of the frame are prefabricated concrete columns (pos. 4) with a section size of 400 × 400 mm, and partially reinforced concrete rigidity diaphragms (pos. 3).

The frame, designed for the construction of residential buildings of 16 stories high, in areas with a seismicity of up to 9 points, is quickly mounted and endowed with considerable simplicity in the production of individual elements.

The spatial rigidity and stability of the applied frame of buildings is mainly due to solid reinforced concrete rigidity diaphragms.

The overlapping disk of a precast flat slab system, it turns out that the loading on the columns is transmitted in the following sequence (Fig. 2): from the middle slab (MS) the load is transmitted to four adjacent inter-columned slabs (ICS); inter-columned slabs transfer the load on the overcolumned slabs (OCS); the overcolumned slabs transfer the load to the columns. With such a load transfer scheme, each slab will have its own destruction scheme, based on which its analysis is realized.

3. Kinematic method for bearing capacity calculation

Replacement frame calculation is provided in many sources. This method of calculation has its advantages and disadvantages, but the main problem is that such a scheme does not correspond to the actual work of the frame under load. This is especially true for overlapping.

The elastic grid theory can be applied to the calculation of non-intersecting slabs that are locally based on many regularly spaced columns. As experience has shown, the calculation of flat slab overlapping using the theory of elastic grids in the practice of design is of little use due to the complexity of calculations. In addition, this technique becomes unsuitable for the calculation of flat slab overlapping in the system of reinforced concrete frame, because it does not take into account the partial clamping of the slab on the supports, and considers the elastic slab as supported on points.

Kinematic schemes of the intercolumn slab destruction provide two cases of plastic hinges formation: the first - the formation of a hinge in the middle of the slab (Fig. 3,a); the second is the formation of plastic hinges at the break points of the reinforcing bars at a distance of a2 from the supports (Fig. 3,b).

To determine the internal forces in the cross-section of the slab it is proposed to use the boundary equilibrium method implemented in the kinematic way. On the basis of this method the equation of virtual works equality from external \( q \) and internal \( M \) efforts on possible slab movements (Fig. 4) is presented in the form:
\[ W_{Ed} = W_{rd} \]  \hspace{1cm} (1)

Figure 2. The Load redistribution schemes between overlapping elements: 1 - loading the column; 2 - distribution of load from intercolumn slabs to overcolumned slabs; 3 - distribution of load from middle slabs to intercolumn slabs; 4 - columns.

Figure 3. Two possible kinematic schemes for the destruction of the intercolumn slab: a) first scheme, b) second scheme.

According to the applied kinematic scheme, the destruction of the intercolumn slab occurs because of the formation in it of a linear plastic hinge from the bottom in the middle of the slab [4]. In this case, it is loaded over the whole area by uniformly distributed loads \( q \) and triangular loading on both opposite sides with a maximum value of ordinate \( q \cdot l / 2 \) in the middle of the span (triangular loading on the slab is transferred (Fig. 4) from its two adjacent middle slabs at the moment of separation disks in the limit state) [5].
The following designations are introduced on the calculated kinematic scheme of the intercolumn slab fracture:

- $l$ – slab spans in both directions (it is taken into account that in practice the slab has the same dimensions in two directions $l$);
- $M_1$ – bending moment in span;
- $M_2 = 50$ kN\cdot m/m – bending moment that arises at the joint of inter-columned and overcolumned slabs (value is obtained from experimental investigation);
- $f$ – virtual deflection of the intercolumn slab in the stage of its destruction;
- $\phi$ – is the virtual rotation angle of the formed discs of the slab in the stage of its destruction.

According to the calculation scheme as a result of rotation of disks 1 and 2 (moments $M_1$ and $M_2$) virtual work is carried out (Fig. 1):

$$W_{ed} = W_M = M_1 \cdot 2\phi \cdot l + M_2 \cdot l \cdot \phi.$$  \hspace{1cm} (2)

In equation (1) the moments $M_1$ and $M_2$ are distributed per meter, that is, their unit is kN\cdot m/m.

To facilitate the derivation of the formula for calculating $W_q$, the calculation scheme shown in Fig. 4 is represented by two schemes. In the first one, the inter-column slab is loaded only with a uniformly distributed load $q$ (Fig. 5), and in the second one, the inter-column slab is loaded only with a triangular load with maximum load ordinates $q \cdot l/2$ in the middle of the span (Fig. 6).

Using the following diagrams we can write that in the stage of destruction of the intercolumn slab virtual work from the action of external load $q$ will be calculated by the equation:

$$W_{ed} = W_q = W_{1q} + W_{2q},$$ \hspace{1cm} (3)

in which $W_{1q}$ is a virtual work from a uniformly distributed load according to the first loading scheme (Fig. 5), and $W_{2q}$ is a virtual work from a triangular load according to the second loading scheme (Fig. 6).

$$W_{1q} = \int q(x) \cdot y(x) \cdot dx = q \cdot V.$$  \hspace{1cm} (4)

**Figure 4.** Design kinematic scheme of the intercolumn slab destruction in the limit equilibrium state: 1, 2 - disks of the slab.
In the equation (4) \( V \) is the volume of the prism, which formed by the turns of disks 1 and 2:

\[
V = \int y(x) \cdot dA = \frac{1}{2} l \cdot f \cdot \frac{l}{2}.
\]

After the mathematical transformations from (4), taking into account (5), we obtain:

\[
W_{Lq} = \frac{l^2 \cdot q \cdot \phi}{4}.
\]

The expression to determine the second component of virtual work in equation (3) for the intercolumn slab loaded with only triangular load:

\[
W_{2q} = 2 \cdot 2 \cdot \int_{0}^{l} q(x) \, dx \cdot y(x),
\]

where \( q(x) = q \cdot x \); \( y(x) = x \cdot \tan \phi = x \cdot \phi \).

Performing the mathematical transformations in (7) we obtain that
\[ W_{eq} = q \cdot \frac{l^3}{6} \cdot \varphi. \] 

After substitution (4) and (8) in (3) we have that external forces perform virtual work

\[ W_q = \frac{q \cdot \varphi \cdot l^3}{4} + \frac{q \cdot \varphi \cdot l^3}{6}. \]

After substitution (2) and (9) in (1) it is obtained that the equation of virtual works (1) is reduced to the following:

\[ \frac{q \cdot \varphi \cdot l^3}{4} + \frac{q \cdot \varphi \cdot l^3}{6} = 2 \cdot M_1 \cdot \varphi \cdot l + M_2 \cdot \varphi \cdot l. \]

Equation (10) gives the formula for calculating the load-bearing capacity of the inter-column floor slab, which is destroyed by the kinematic scheme shown in Fig. 4:

\[ q = \frac{24 \cdot M_1 + 12 \cdot M_2}{5 \cdot l^2}, \]

in formula (11) the value \( M_1 = M_{Ed}. \) As for the value of \( M_{Ed}, \) for the slab, the bearing capacity of which is established, will always be known the cross-sectional area of the reinforcement in the cross-section of the slab. Therefore

\[ M_{Ed} = A_s \cdot f_{yd} \cdot d_s \cdot \varphi. \]

The task of the cross-sectional area calculation of the main reinforcement per unit width of the slab section at a given load \( q \) is solved on the basis of equation (11) by the formula:

\[ A_s = \frac{M_1}{f_{yd} \cdot d_s \cdot \varphi}, \]

where \( f_{yd} \) – design value of reinforcement strength for the limit states of the first group;

\( M_2 = 50 \text{kN}\cdot\text{m}/\text{m} \) – bending moment that arises at the joint of inter-columned and overcolumned slabs;

After substituting in (13) the value of \( M_1 \) by (11) and following the simplifications, we obtained the formula for calculating the area of the main reinforcement in the intercolumn slab:

\[ A_s = \frac{q \cdot 5 \cdot l^2 - 12 \cdot M_2}{24 \cdot f_{yd} \cdot d_s \cdot \varphi} \]

Thus, as a result of theoretical studies, analytical dependences (11) and (14) were obtained to determine the bearing capacity of the inter-column slabs and to calculate the main reinforcement area in the design section, which correspond to the kinematic fracture scheme under consideration.

In a similar way, substituting expressions (5) and (4) into equation (1) for the second kinematic failure scheme (Fig. 3,b) we obtain the value of the limit load:

\[ q \left( l^2 \cdot a_2 - l \cdot a_2^2 \right) + q \frac{3l^2 \cdot a_2^2 - 4a_2^3}{6} = 2M_1k_m + M_2 \]

\[ q = \frac{12M_1k_m \cdot l + 6M_2}{a_2 \left( 9l^2 - 6l \cdot a_2 - 4a_2^2 \right)} \]

In equations (11) and (16) the value of \( q \) is the same limit value of the load on the slab in its limit state, so by equating expressions (11) and (16), we obtain that at \( a_2 = k_m \cdot l \), the value of \( M_2 = 50 \text{kN}\cdot\text{m}/\text{m} \) and the reinforcement in cross section halving at a distance from the support \( a_2 \). That is, to reduce the cross-sectional area of the main reinforcement twice is allowed at a distance \( a_2 = 0.15 \cdot l \) from the support of the slab.
4. Conclusions
In this scientific research, we have obtained the formulas of the inter-column slab load-bearing capacity of the flat slab frame structural system, which take into account the deformation compatibility with adjacent slabs. This, in turn, allows you to more accurately determine the bearing capacity of the inter-column slabs, and accordingly to more accurately calculate the area of the main reinforcement in the considered slab.

5. References
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