Influence of the nucleon-nucleon collision geometry on the determination of the nuclear modification factor for nucleon-nucleus and nucleon-nucleus collisions

Jiangyong Jia\textsuperscript{1, 2}

\textsuperscript{1}Department of Chemistry, Stony Brook University, Stony Brook, NY 11794, USA
\textsuperscript{2}Physics Department, Brookhaven National Laboratory, Upton, NY 11796, USA

(Dated: August 3, 2009)

The influence of the underlying nucleon-nucleon collision geometry on evaluations of the nuclear overlap function ($T_{AB}$) and number of binary collisions ($N_{coll}$) is studied. A narrowing of the spatial distribution of the hard-partons with large light-cone fraction $x$ in nucleons leads to a downward correction for $N_{coll}$ and $T_{AB}$, which in turn, results in an upward correction for the nuclear modification factor $R_{AB}$. The size of this correction is estimated for several experimentally motivated nucleon-nucleon-nucleus and nucleon-nucleus collisions, and are much larger at the LHC energy of $\sqrt{s}=200$ GeV than for the RHIC energy of $\sqrt{s}=\text{5.5}$ TeV. The implications for experimental measurements are also discussed.

PACS numbers: 25.75.-q

I. INTRODUCTION

In experiments at the Relativistic Heavy Ion Collider (RHIC), modification of hard-scattering processes is usually quantified via the nuclear modification factor. For collision between nucleus A and B (A-B collision), it is defined as

$$R_{AB} = \frac{1}{N_{coll}} \frac{dN_{AB}}{d^3p_T} = \frac{1}{N_{coll}} \frac{dN_{AB}}{d^3p_T}$$

where $T_{AB}$ is the nuclear overlap function calculated as the convolution of the thickness functions $T_{A,B}(\vec{b}) = \int \rho_{A,B}(\vec{b}, z) dz$ for A and B,

$$T_{AB}(\vec{b}) = \int d\vec{s} T_{A}(\vec{s}) T_{B}(\vec{s} - \vec{b}),$$

and $(T_{AB})$ is the average nuclear overlap function for the corresponding central bin

$$\langle T_{AB} \rangle = \frac{\int T_{AB}(\vec{b})(1 - e^{-\sigma_{nn}^{inel} T_{AB}(\vec{b})}) d\vec{b}}{\int (1 - e^{-\sigma_{nn}^{inel} T_{AB}(\vec{b})}) d\vec{b}} = \langle N_{coll} \rangle / \sigma_{nn}^{inel}.$$  \hfill (3)

In Eqs. 1 and 3 $(N_{coll})$ is the average number of inelastic nucleon-nucleus (n-n) collisions with inelastic n-n cross section $\sigma_{nn}^{inel}$. At the nominal RHIC energy of $\sqrt{s}=200$ GeV, $\sigma_{nn}^{inel} = 42 mb = 4.2 fm^2$. The estimation of $N_{coll}$, $T_{AB}$ and other geometrical quantities are usually obtained either numerically via an optical Glauber approach or statistically via a Monte-Carlo Glauber approach (see Appendix A). More details on the Glauber model can be found in [1].

The definition of $T_{AB}$ in Eq. 2 ignores the fact that nucleons are extended object and the multiplicity of a n-n collision also depends on the impact parameter $b_{nn}$. This can be accounted for by introducing a n-n overlap function $t(\vec{b}_{nn})$ with normalization of $\int d\vec{b}_{nn} t(\vec{b}_{nn}) = 1$, which generalizes Eq. 2 to Wong’s formula [2]:

$$T_{AB}(\vec{b}_{AB}) = \int d\vec{b}_{AB} d\vec{b}_{B} T_{A}(\vec{b}_{A}) T_{B}(\vec{b}_{B}) t(\vec{b}_{AB} - \vec{b}_{A} + \vec{b}_{B})$$

$$= \int d\vec{b}_{nn} T_{A}(\vec{s}) T_{B}(\vec{s} - \vec{b}_{AB} + \vec{b}_{nn}) t(\vec{b}_{nn}).$$ \hfill (4)

In this notation, Eq. 2 is obtained for the special case when the n-n overlap function is a delta function: $t(\vec{b}_{nn}) = \delta(\vec{b}_{nn})$. Eq. 4 shows that the nuclear overlap function $T_{AB}$ (and thus $N_{coll}$) depends on not only the nuclear impact parameter $\vec{b}_{AB}$, but also the n-n impact parameter $\vec{b}_{nn}$ through $t(\vec{b}_{nn})$. In other words, the $b_{AB}$ and $b_{nn}$ are correlated. Because of this correlation, Eq. 4 in general is not the same as Eq. 2 at fixed $b_{AB}$. However if the nuclear thickness function varies linearly within the length scale of nucleon size, Eq. 4 is a good approximation since the integral of the first term of its Taylor expansion vanishes due to spherical symmetry of the nucleon,

$$\int d\vec{b}_{nn} T_{B}(\vec{s} - \vec{b}_{AB} + \vec{b}_{nn}) t(\vec{b}_{nn})$$

$$\approx \int d\vec{b}_{nn} \left[ T_{B} + \nabla T_{B} \cdot \vec{b}_{nn} \right] t(\vec{b}_{nn}) = T_{B}(\vec{s} - \vec{b}_{AB}).$$

This approximation is rather precise for central A-B collisions, but breaks down in peripheral collisions. Furthermore one can show that Eq. 4 (hence Eq. 2) obeys the following sum rule

$$\int d\vec{b}_{AB} T_{AB}(\vec{b}_{AB})$$

$$= \int d\vec{s} d\vec{b}_{nn} d\vec{b}_{AB} T_{A}(\vec{s}) T_{B}(\vec{s} - \vec{b}_{AB} + \vec{b}_{nn}) t(\vec{b}_{nn})$$

$$= \int d\vec{b}_{AB} T_{A}(\vec{s}) T_{B}(\vec{s} - \vec{b}_{AB})$$

$$= AB / \sigma_{AB}^{geo},$$  \hfill (5)

where $\sigma_{AB}^{geo}$ is the total A-B geometrical cross-section. This equation implies that the integral of $T_{AB}$, i.e. for
b) Influence of this sensitivity on the interpretation of the n-n overlap function for hard-scattering processes. The primary goal of this paper is to investigate the sensitivity of the n-n overlap function for fixed b_{nA} to different assumptions about n-n overlap function for hard-scattering processes. The influence of this sensitivity on the interpretation of the nuclear modification factor R_{AB} is also discussed.

II. RESULTS AND DISCUSSION

To help visualizing the level of correlation between b_{AB} and b_{nn}, we plot the b_{nn} distributions for various ranges of b_{AB} in Au-Au collisions in Figure 2. These distributions are calculated using the MC Glauber approach (see Appendix A). A n-n total inelastic cross section of σ_{nn}^{inel} = 42 mb is used in the calculation, which corresponds to a geometrical radius of r_n = √(σ_{nn}^{inel}/π)/2 = 0.5781 fm for nucleons. A n-n collision is considered to occur when their distance in the xy-plane is less than d_{max} = 2r_n = 1.156 fm. The individual distributions in Figure 2 have been re-scaled to match each other at b_{nn} = 2.5 fm. For minimum bias Au-Au events, the distribution increase linearly with b_{nn}. This is so because all possible xy positions of a given nucleon in one nucleus relative to the other nucleus are uniformly sampled (consistent with the sum rule Eq. 3). Thus the corresponding b_{nn} distribution has the same shape as that for pure n-n collisions. By contrast, the distributions for centrality selected Au-Au events are not linear functions of b_{nn}. For peripheral Au-Au collisions, the n-n distribution has a concave-like shape, implying that the selection of large b_{AB} results in a bias towards peripheral n-n collisions (large b_{nn}). For central Au-Au collisions, the n-n distribution has a convex-like shape, implying that the requirement of a small b_{AB} value leads to a bias towards slightly central n-n collisions.

The origin of the bias can be further illustrated via peripheral n-n collisions. According to Eq. 4 the n-n impact parameter distribution for fixed b_{nA} should be

\[ f(b_{nn}, b_{nA}) = \int d\phi \frac{b_{nA}}{b_{nA} - R} \frac{d\phi}{\sigma_{nn}^{inel}} (\bar{s} - \bar{b}_{nA} + \bar{b}_{nn}) b_{nn} \]

after exchanging subscript A and B and setting T_A = δ(\bar{s}). When the distance of nucleon from the nuclear surface is of the order of nucleon size, i.e. |b_{nA} - R| ≲ r_n (R is the nuclear radius and r_n = d_{max}/2 is nucleon radius), the projectile nucleon see many more nucleons at the inner side of the target nucleus than the side close to the surface (see Figure 1b), leading to the bias shown in...
The range of the surface diffuseness of the Woods-Saxon nuclear profile bias should also be sensitive to the nuclear profile. Due to functions, which varies more rapidly towards the edge of the hard-scattering process in n-n collisions, is rather limited. In general, it is a function of the momentum distribution of partons in a nucleon. Unfortunately, information on the transverse distribution of partons in a nucleon, or the impact parameter dependence of the hard-scattering process in n-n collisions, is rather limited. In general, it is a function of $x$ and $Q^2$ and can be described by projecting the generalized parton distribution function (GPD) \[5,8\]. To estimate the magnitude of the bias, we tried several different n-n overlap functions. The default overlap function is a step function defined as

$$t_{mb}^{1}(b_{nn}) \propto \theta(b_{nn} - r_n).$$

This overlap profile has no rigorous physics foundation, other than the fact that it provides a simple and intuitive estimation of the bias. The second one assumes that the overlap function is the folding of two Gaussian profiles for hard-partons with the width $r_n$.

$$t_{hs}^{2}(b_{nn}) \propto \frac{1}{2r_n^2} e^{-\frac{b_{nn}^2}{r_n^2}} \theta(b_{nn} - 2r_n).$$

This is the n-n overlap function used by PYTHIA \[9\], except that we truncate the overlap function at $2r_n$, so the average width is narrower than the default (about 0.7$r_n$ in this case). The third one assumes that the hard-partons are uniformly distributed in a hard-sphere nucleon with radius $r_n$, the corresponding overlap function is

$$t_{hs}^{3}(b_{nn}) \propto \int d\vec{s} \sqrt{1 - \frac{s^2}{r_n^2}} \theta(s - r_n) \times \sqrt{1 - \frac{(s - b_{nn})^2}{r_n^2}} \theta(|s - b_{nn}| - r_n).$$

The last one is the dipole formula for gluons spatial distribution taken from \[10\]. It is derived from fits to $J/\Psi$ photo-production data at HERA and FNAL,

$$t_{hs}^{4}(b_{nn}) \propto m_g^2 \left( \frac{m_g b_{nn}}{2} \right)^3 K_3(m_g b_{nn}).$$

where $K_3$ is the modified Bessel function, and the mass parameter $m_g^2 \sim 1.1 GeV^2$, which depends weakly on $Q^2$ and assumed to be constant in this analysis.

With these input distributions in hand, it is fairly straightforward to evaluate the resulting differences in $N_{coll}^{n}$ and $N_{coll}$ using the MC Glauber approach. The results obtained for Au-Au collisions are shown in Figure 3; they are calculated for Eq. 8 with $\sigma_{inel}^{Au} = 42$ mb (for $\sqrt{s} = 200$ GeV). The results are plotted against either a) Au-Au impact parameter, b) $N_{part}$, or c) centrality in 10% steps (sliced according to Au-Au geometrical cross-section). Here, we shall focus the discussion on the middle panel. One can see that the $N_{coll}^{n}$ is always smaller than the nominal $N_{coll}$ value for small $N_{part}$. This is easily understood since $N_{coll}$ is calculated for a flat probability distribution while all others are calculated for a
distribution which peaks at small $b_{\text{nn}}$. The bias is sizable at $N_{\text{part}} < 20 - 50$, corresponding to > 60% centrality range. The bias for the Gaussian profile is smallest since it’s $t_{\text{nn}}$ is broader than for the other scenarios. For the more realistic case of hard-sphere and dipole overlap, the bias grows to about 5-15% for $N_{\text{part}} \sim 10 - 15$ which corresponds roughly 70-80% centrality bin. Note that the corrections all cross at $N_{\text{part}} \sim 150$ (or $b_{AuAu} \sim 8$ fm) and stay slightly above unity for central collisions. This is expected since there is no bias for minimum bias Au-Au selection, as mandated by the sum rule Eq. (5).

Based on the trends shown in Figure 2, it is clear that the bias should grow with an increase of total n-n inelastic cross section. At the LHC energy of 5.5 TeV, the total n-n cross section is estimated to be $\sigma_{\text{inel}} \approx 72 mb = 7.2 fm^2$ [11], about 75% higher than that at the nominal RHIC energy. This corresponds to $d_{\text{max}} = 1.514 fm$ and $r_n = 0.757 fm$. The results obtained for this condition are summarized by Figure 3, which shows about 60% larger bias on $N_{\text{coll}}$ when compared to that for the RHIC energy.

As discussed before, the same bias should also exist for p-A collisions, but to a lesser extent. In Figure 4 we show the results of the calculation for deuteron-Au (d-Au) collisions at $\sqrt{s} = 5.5$ TeV. Indeed the bias is smaller compared to Au-Au (Figure 3) at the same $N_{\text{part}}$. However the available range of $N_{\text{part}}$ covered by d-Au collision is small. So the correction factor for given percentile centrality bin, sliced according to geometrical cross-section, turns out to be larger. For the $\langle N_{\text{part}} \rangle$ values for 60-100% centrality range is only about 4, for which the correction could be as large as 15%.

The results in Figure 4 underlines the importance of the nucleon-nucleon collision geometry for proper interpretation of the collision geometry in p-A and A-A collisions. Depending on the assumed hard-parton profile, the bias on the $N_{\text{coll}}$ (hence $T_{AB}$) could become sizable, especially at LHC energy where the total n-n cross-section is significantly larger, and in peripheral p-A and A-A collisions where the correlation between $b_{AB}$ and $b_{nn}$ is important. In such cases, the full formula for $T_{AB}$ (Eq. 4) and proper n-n overlap function for hard-partons are needed for proper evaluation of the nuclear modification factor. The magnitude of the bias is usually smaller than the typical systematic error quoted by the RHIC experiments [1]. Also the experimental triggering efficiency for peripheral A-A and p-A collision is low, for example, the PHENIX experiment triggers on 92% of Au-Au collisions [11] and 88% of the d-Au collisions [12], hence the relevant correction factor for experimentally accessible centrality range could be 30-50% smaller than what has been calculated here. We should point out that a related bias has been considered before by the PHENIX Collaboration [1], where a Gaussian profile was used as part of the estimation of the systematic errors on $N_{\text{coll}}$. However that study was not motivated by the narrowing of hard-parton spatial distribution, rather it was used to account for uncertainty of the nucleon matter distribution which are dominated by soft partons.

In summary, we discussed a bias in the calculation of the $N_{\text{coll}}$ and $T_{AB}$, and its influences on the nuclear modification factor $R_{AB}$ within the Glauber formalism. The bias is caused by the difference in the spatial profile between the hard-partons which contribute to the high $p_T$ yield, and soft-partons which determines the n-n inelastic cross-section. The much narrower spatial profile of the hard-partons biases the $N_{\text{coll}}$ and $T_{AB}$ to values smaller than those obtained using the uniform n-n overlap function. The magnitude of the bias is sensitive to width of the overlap function for hard-partons. A crude estimation for Au-Au collisions at RHIC energy leads to about 5% (assuming a Gaussian overlap function)-15% (assuming a hard-sphere overlap function) downward corrections for 70-80% centrality bin and becomes significantly larger for more peripheral collisions, which may account for part of the suppression seen for very peripheral Au-Au $\pi^0$ data [13]. For $N_{\text{part}} > 150$, the bias is positive, but is less than a few percent. Since the bias increases with the total n-n inelastic cross-section used in the calculation. We estimate the bias could be 60% larger at LHC energy compare to RHIC energy, and has to be properly taken into account in Glauber calculation for both Pb-Pb and p-Pb collisions.

The author wishes to thank Roy Lacey, Michael Tannenbaum and Cheuk-Yin Wong for valuable discussions. This research is supported by NSF under award number PHY-0701487.

APPENDIX A: OPTICAL VS MC APPROACH

The calculations of Glauber variables, including $\langle T_{AB} \rangle$ and $\langle N_{\text{coll}} \rangle$, are carried out either numerically using a set
FIG. 4: The ratio of $N_{\text{coll}}^{h_s}$ to $N_{\text{coll}}$ calculated for Au-Au collisions for the four n-n overlap functions described by Eq. 9-12 plotted as function of a) impact parameter $b_{AuAu}$, b) $N_{\text{part}}$ and c) centrality bins in 10% step, assuming n-n inelastic cross section of 42mb (for RHIC energy).

FIG. 5: The ratio of $N_{\text{coll}}^{h_s}$ to $N_{\text{coll}}$ calculated for Au-Au collisions for the four n-n overlap functions described by Eq. 9-12 plotted as function of a) impact parameter $b_{AuAu}$, b) $N_{\text{part}}$ and c) centrality bins in 10% steps, assuming n-n inelastic cross section of 72mb (for LHC energy).

FIG. 6: The ratio of $N_{\text{coll}}^{h_s}$ to $N_{\text{coll}}$ calculated for deuteron-gold (d-Au) collisions for the four n-n overlap functions described by Eq. 9-12 plotted as function of a) impact parameter $b_{dAu}$, b) $N_{\text{part}}$ and c) centrality bins in 10% steps, assuming n-n inelastic cross section of 72mb (for LHC energy). The Hulthen wave function for deuteron is used.
of equations like Eq. [2] ("optical approach") or statistically via Monte-Carlo method ("MC approach"). The MC approach is what has been used by RHIC experimentalist. Comparing with Optical approach, it takes into account position fluctuations of nucleons in the nucleus, which is shown to be important in peripheral collisions. In the MC approach, A-B collisions are generated randomly in the xy-plane. Within each event, nucleus A and B are populated randomly with nucleons according to the Woods-Saxon nuclear profile. All possible n-n combination between the two nucleus are considered. In the simplest version at RHIC energy, a n-n collision is considered to happen when their distance in the xy-plane is less than \( d_{\text{max}} = \sqrt{\sigma_{\text{inel}}^{nn}/\pi} = 1.156 fm \) (hard-sphere assumption, corresponding to \( t(b_{nn}) = \theta(|b_{nn}| - d_{\text{max}}) \)). The number of n-n collisions in a given event is

\[
N_{\text{coll}} = \sum_{i \in A,j \in B} t(|\vec{r}_i - \vec{r}_j|) = \sum_{i \in A,j \in B} \theta(|\vec{r}_i - \vec{r}_j| - d_{\text{max}})
\] (A1)

The \( T_{AB} \) is then calculated as \( T_{AB} = N_{\text{coll}}/\sigma_{\text{inel}}^{nn} \). The \( \langle N_{\text{coll}} \rangle \) and \( \langle T_{AB} \rangle \) are obtained by averaging over many events falling in a given centrality definition. More details can be found in [1].

**APPENDIX B: SCALE FACTOR FOR HARD-SCATTERING PROCESS**

In this section, we derive the expression for correct scaling factor, \( N_{\text{coll}}^{\text{hs}} \), for hard-scattering process. The average hard-scattering yield in A-B event at a fixed impact parameter can be expressed as

\[
Y_{AB}^{\text{hs}} = \langle \sum_{i \in A,j \in B} t_{\text{hs}}(|\vec{b}_i - \vec{b}_j|) \sigma_{pp}^{\text{hs}} \rangle_{\text{evts}}
\] (B1)

\[
= \sigma_{nn}^{\text{hs}} \int db_{nn} f(b_{nn}, b_{AB}) t_{\text{hs}}(b_{nn})
\]

\[
= \sigma_{nn}^{\text{hs}} \int d\vec{s}\vec{b}_{nn} T_{A}(\vec{s}) T_{B}(\vec{s} - \vec{b}_{AB} + \vec{b}_{nn}) t_{\text{hs}}(\vec{b}_{nn}).
\] (B2)

where \( f(b_{nn}, b_{AB}) \) is n-n collision density in impact parameter space defined by Eq. [7] and

\[
Y_{AB}^{\text{hs}} = \sigma_{nn}^{\text{hs}} \int db_{nn} 2\pi b_{nn} t_{\text{hs}}(b_{nn}) = \sigma_{nn}^{\text{hs}} / \sigma_{\text{inel}}^{nn}
\] (B4)

where the normalization \( \int db_{nn} 2\pi b_{nn} t_{\text{hs}}(b_{nn}) = 1 \) is used.

The correct scaling factor should be the ratio of the hard-scattering rate in centrality selected A-B collisions to that of the minimum bias n-n collisions:

\[
N_{\text{coll}}^{\text{hs}} = \langle Y_{AB}^{\text{hs}} \rangle / \langle Y_{nn}^{\text{hs}} \rangle = T_{AB}^{\text{hs}} \sigma_{\text{inel}}^{nn}
\] (B5)

This equation is similar to the original definition Eq. [8] except that a different n-n overlap function is used.

A few words about \( N_{\text{coll}}^{\text{hs}} \) are in order. First, according to Eq. [5] there is no bias for minimum bias A-B event selection, i.e. \( \langle N_{\text{coll}}^{\text{hs}} \rangle = \langle N_{\text{coll}} \rangle = \frac{A B \sigma_{\text{inel}}^{nn}}{\sigma_{AB}^{pp}} \). Secondly, the n-n hard-parton overlap function \( t_{\text{hs}} \) depends on the x and Q^2, thus in general, \( N_{\text{coll}}^{\text{hs}} \) could depend on the pt^R. Last but not the least, the magnitude of the hard-scattering cross-section \( \sigma_{nn}^{\text{hs}} \) in Eq. [B5] is not important, what is important is it’s spatial distribution. In fact, \( \sigma_{nn}^{\text{hs}} \) could be arbitrarily small, and yet would still cancel between A-B and n-n (Eq. [B5]). For our MC Glauber calculation, a convenient choice is to choose \( \sigma_{nn}^{\text{hs}} = \sigma_{\text{inel}}^{nn} \), then Eq. [B1] would directly give \( N_{\text{coll}}^{\text{hs}} \).

---

[1] M. L. Miller, K. Reygers, S. J. Sanders and P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007)
[2] C. Y. Wong, Phys. Rev. D 30, 961 (1984).
[3] L. Frankfurt, M. Strikman and C. Weiss, Phys. Rev. D 69, 114010 (2004)
[4] L. Frankfurt, M. Strikman and C. Weiss, Ann. Rev. Nucl. Part. Sci. 55, 403 (2005)
[5] C. Weiss, arXiv:0902.2018 [hep-ph].
[6] V. N. Gribov, arXiv:hep-ph/0006158.
[7] E. Iancu and R. Venugopalan, arXiv:hep-ph/0303204.
[8] X. D. Ji, Phys. Rev. Lett. 91, 062001 (2003)
[9] T. Sjostrand and M. van Zijl, Phys. Rev. D 36, 2019 (1987).
[10] D. G. d’Enterria, [arXiv:nucl-ex/0302016](link)
[11] S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. C 69, 034910 (2004)
[12] S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. C 77, 014905 (2008)
[13] A. Adare et al. [PHENIX Collaboration], Phys. Rev. Lett. 101, 232301 (2008)