Tensions in the flavour sector

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Abstract. Flavour physics is currently well described by the Standard Model except for some measurements which could be signalling new physics. We briefly summarize the status of tensions in rare exclusive semi-leptonic $B$ decays and in $R_K/D(\ast)$ ratios. We also address the issue of the long standing tension in the $|V_{cb}|$ and $|V_{ub}|$ exclusive/inclusive determinations.

1 Introduction

In a general scenario of optimal agreement within the Standard Model (SM), the flavour physics sector exhibits some measurements which present non-significant but intriguing tensions with SM predictions. Here we briefly discuss $B$ decays mediated by the parton $b \to s l^+ l^-$ decays at the lowest order in the SM. Being rare decays, they are particularly susceptible to new physics effects. We also briefly address the discrepancy between the $R_K$ and $R_{D(\ast)}$ measured ratios and the corresponding SM predictions, which may hint at lepton non-universality. The last issue succinctly examined is the determination of the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $|V_{cb}|$ and $|V_{ub}|$, which strongly affects the identification of new physics. The SM does not predict the values of the CKM matrix elements and their precise measurement allows a powerful check of the unitarity of the CKM matrix. A long standing tension remains between the inclusive and exclusive determinations.

2 Exclusive rare semileptonic $B$ decays

LHCb has provided the most precise measurements of the branching fractions of the $B^+ \to K^{(*)+} \mu^+ \mu^-$ and $B^0 \to K^{(*)0} \mu^+ \mu^-$ decays $[1\text{--}3]$. All these measurements are below the SM prediction. The large $q^2$ region is the domain of election for lattice QCD and unquenched calculations of form factors have been performed. The more recent ones are for $B \to K l l$ decays by HPQCD $[4, 5]$, for $B_s \to K \nu l$ and $B \to K l l$ decays by Fermilab/MILC collaboration $[6, 7]$ and for $B_s \to \phi l^+ l^-$ and $B \to K^* l^+ l^-$ decays by the Cambridge collaboration $[8, 10]$. Also in the light-cone-sum rule formalism, recent results have been presented, for $B \to \rho$, $B_q \to \omega$, $B \to K^*$ and $B_s \to \phi$ form factors $[11]$.

Branching fraction measurements alone are not sufficient to exploit the opportunities given by exclusive rare semileptonic $B$ decays, which present several asymmetries and angular observables that can be studied as functions of the dimuon invariant mass squared, $q^2$. Evidence of a non-vanishing
value of the isospin asymmetry $|V_{ud}|$ has not been confirmed in an update with larger statistics [11]. All CP asymmetries measured so far in these decays are consistent with zero [13–15], as predicted by the SM.

The angular distributions of the $B^+ \to K^+ \mu^+ \mu^-$ and $B^0 \to K^0 \mu^+ \mu^-$ decays are described by two parameters, $F_H$, which is a measure of the contribution from (pseudo)scalar and tensor amplitudes to the decay width in the approximation that muons are massless, and $A_{FB}$, the forward-backward asymmetry of the dimuon system. In the SM, $A_{FB}$ is zero and $F_H$ highly suppressed, and their measured values are compatible with the SM expectations [2, 16]. LHCb reports also the latest measurements of $A_{FB}$ and $F_H$, the fraction of longitudinal polarisation, for the $B^0 \to K^0 \mu^+ \mu^-$ channel as a function of the dimuon invariant mass, which agree with SM predictions [17].

A broad peaking structure has been observed in the dimuon spectrum of $B^+ \to K^+ \mu^+ \mu^-$ decays in the kinematic region where the kaon has a low recoil against the dimuon system [18]. The mean and width of the resonance are compatible with the properties of the $\psi(4160)$. The resonant decay and the interference contribution make up 20% of the yield for dimuon masses above 3770 MeV/$c^2$. This contribution is larger than theoretical estimates [18].

In 2013, new angular observables denoted as $P'_{4,5,6,8}$, that are free from form-factor uncertainties at leading order [19], have been proposed in the $B^0 \to K^+ \mu^+ \mu^-$ channel. The LHCb experiment has performed the first measurement of these angular observables using data collected in 2011 and announced a 3.7σ local discrepancy in one of the $q^2$ bins (4.30–8.68 GeV/$c^4$) for the angular observable $P'_{4}$ [20]. This discrepancy has been confirmed in a new LHCb analysis employing the complete LHCb Run 1 dataset recorded in $pp$ collisions at centre-of-mass energies of 7 and 8 TeV during 2011 and 2012, respectively, and corresponding to an integrated luminosity of 3.0 fb$^{-1}$ [21]. A deviation from the SM prediction in Ref. [22] has been observed in each of the 4.0<$q^2<$6.0 GeV/$c^4$ and 6.0<$q^2<$8.0 GeV/$c^4$ bins at a level of 2.8 and 3.0 standard deviations, respectively. The LHC analysis [21] also presents a complete set of observables, for the first time, based on the full angular distribution. Correlations between the different observables are computed to allow the results to be included in global fits of $b \to s$ data. A global analysis of the CP-averaged angular observables determined from the maximum likelihood fit indicates differences with the presently-available SM predictions at the level of 3.4 standard deviations [21]. These analyses have prompted a large number of theoretical investigations, searching for NP in several frameworks or assessing the effects of non-perturbative corrections on the SM predictions, which may change the significance of the discrepancy (see e.g. [23–28] and references within.)

# 3 Exclusive decays into heavy leptons

Exclusive semi-tauonic $B$ decays were first observed by the Belle Collaboration in 2007 [29]. Subsequent analysis by Babar and Belle [30–32] measured branching fractions above, although consistent with, the SM predictions. The ratio of branching fractions (the denominator is the average for $l \in \{e, \mu\}$)

$$R_{B^{(*)}} \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_{\tau})}{\mathcal{B}(B \to D^{(*)}l\nu_{l})}$$

(1)

is typically used instead of the absolute branching fraction of $B \to D^{(*)}\tau\nu_{\tau}$ decays to cancel uncertainties common to the numerator and the denominator. These include the CKM matrix element $|V_{ub}|$ and several theoretical uncertainties on hadronic form factors and experimental reconstruction effects. In 2012–2013 Babar has measured $R_{D^{(*)}}$ by using its full data sample [33, 34], and reported a significant excess over the SM expectation, confirmed in 2015 by LHCb [35]. In 2016 such excess has been
confirmed also by the Belle Collaboration, which has performed the first measurement of $R_{D^*}$ using the semileptonic tagging method, giving [36]

$$R_{D^*} = 0.302 \pm 0.030 \pm 0.011$$

where the first error is statistic and the second one is systematic. By averaging the most recent measurements [32–36], the HFAG Collaboration has found [37]

$$R_D = 0.397 \pm 0.040 \pm 0.028 \quad R_{D^*} = 0.316 \pm 0.016 \pm 0.010$$

where the first uncertainty is statistical and the second is systematic. $R_D$ and $R_{D^*}$ exceed the SM value $R_{SM}^D = 0.300 \pm 0.008$ given by the HPQCD Collaboration [38] and the SM phenomenological prediction $R_{SM}^{D^*} = 0.252 \pm 0.003$ [39] by 1.9σ and 3.3σ, respectively. The combined analysis of $R_D$ and $R_{D^*}$, taking into account measurement correlations, finds that the deviation is 4σ from the SM prediction. Other recent SM predictions are available for $R_{SM}^D$, that is $R_{SM}^D = 0.299 \pm 0.011$ by FNAL/MILC Collaboration [40] and $R_{SM}^{D^*} = 0.299 \pm 0.003$ [41]. They are also below data, and in agreement with older $R_{SM}^D$ determinations [42, 43].

Most recently, the Belle collaboration has reported the first measurement of the $\tau$ lepton polarization in the decay $\bar{B} \rightarrow D^* \tau^- \bar{\nu}$ as well as a new measurement of in the hadronic $\tau$ decay modes which is statistically independent of the previous Belle measurements, with a different background composition [44]. The preliminary results give [44]

$$R_{D^*} = 0.276 \pm 0.034^{+0.029}_{-0.026}$$

where the first errors are statistical and the second ones systematic. This result is consistent with the theoretical predictions of the SM in Ref. [39] within 0.6σ standard deviations.

At Belle II a better understanding of backgrounds tails under the signal and a reduction of the uncertainty to 3% for $R_{D^*}$ and 5% for $R_D$ is expected at 5 ab$^{-1}$.

While $R_B$ is defined as the ratio of branching fractions of decays that occur at tree level in the SM at the lowest perturbative order, the observable $R_K$ is defined as the ratio of branching fractions of rare decays. At LHCb, $R_K$ has been measured to be [45]

$$R_K \equiv \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

where the first error is statistical and the second one is systematic. The measurement was performed across the dilepton invariant-mass-squared range $1 \text{ GeV}^2 < m_{ll}^2 < 6 \text{ GeV}^2$. This result is 2.6σ deviations away from the SM prediction $R_{SM}^K = 1.0003 \pm 0.0001$ [46]. The impact of radiative corrections has been estimated not to exceed a few % [47].

The alleged breaking of lepton-flavour universality suggested by most of the data is quite large, and several theoretical models have been tested against the experimental results. A welcome feature of measurements in the $\tau$ sector is the capacity of putting stringent limits on new physics models (see e.g. [48–53]).

4 |$V_{cb}$| and |$V_{ub}$| determinations

The inclusive and exclusive semi-leptonic searches rely on different theoretical calculations and on different experimental techniques which have, to a large extent, uncorrelated statistical and systematic uncertainties. This independence makes the comparison of |$V_{cb}$| and |$V_{ub}$| values from inclusive and
exclusive decays an interesting test of our physical understanding (see e.g. \[54\]–\[57\] and references therein). Recent results of inclusive and exclusive determinations of \(|V_{cb}|\) are collected in Table 1. The most precise estimates of \(|V_{cb}|\) come from lattice determinations in the \(B \to D^*\) channel, followed by determinations based on inclusive measurements. They all stay below 2\% uncertainty. We observe a tension between exclusive and inclusive determinations by comparing the latest inclusive determination \[54\] and the latest \(B \to D^*\) FNAL/MILC lattice result \[59\], which amounts to about 3\(\sigma\). The tension lessens by comparing the same inclusive determination with the (considerable less precise) exclusive determination based on the sum rule calculation of the \(B \to D^*\) form factor \[60\]–\[62\]. In the \(B \to D\) channel, where the uncertainty has recently decreased, an inclusive/exclusive discrepancy is also observed \[41\]. It is also possible to determine \(|V_{cb}|\) indirectly, using the CKM unitarity relations together with CP violation and flavor data, excluding direct informations on decays. The indirect fits provided by the UTfit collaboration \[66\] and the CKMfitter collaboration \[67\] are also reported in in Table 1. Indirect fits prefer a value for \(|V_{cb}|\) that is closer to the (higher) inclusive determination.

The parameter \(|V_{ub}|\) is the less precisely known among the modules of the CKM matrix elements. The CKM-suppressed decay \(B \to \pi l\bar{v}\) with light final leptons is the typical exclusive channel used to extract \(|V_{ub}|\). It is well-controlled experimentally and several measurements have been performed by both BaBar and Belle collaborations \[68\]–\[74\]. Recently, the measurements of branching ratios of \(B \to \rho/\omega l\bar{v}\) decays and the computation of their form factors have been refined, and estimates of \(|V_{ub}|\) have been inferred by these decays as well. Another channel depending on \(|V_{ub}|\) is the baryonic semileptonic \(\Lambda_b^0 \to p\mu^-\bar{\nu}_\mu\) decay. At the end of Run I, LHCb has measured the probability of this decay relative to the channel \(\Lambda_b^0 \to \Lambda^+\mu^-\bar{\nu}_\mu\) \[75\]. This result has been combined with the ratio of form factors computed using lattice QCD with 2+1 flavors of dynamical domain-wall fermions \[76\], enabling the first determination of the ratio of CKM elements \(|V_{ub}|/|V_{cb}|\) from baryonic decays \[75\]. The value of \(|V_{ub}|\) depends on the choice of the value of \(|V_{cb}|\). By taking the inclusive determination \(|V_{cb}|_{\text{incl}} = (42.21 \pm 0.78) \times 10^{-3}\), the value \(|V_{ub}| = (3.50 \pm 0.17_{\text{exp}} \pm 0.17_{\text{FF}} \pm 0.06_{V_{ub}}) \times 10^{-3}\) is obtained \[77\], where the errors are from experiment, the form factors, and \(|V_{cb}|\), respectively. By taking instead

### Table 1. Status of exclusive and inclusive \(|V_{cb}|\) determinations

| Exclusive decays | \(|V_{cb}| \times 10^{-2}\) |
|------------------|-----------------------------|
| \(B \to D^* l\bar{v}\) Flag 2016 \[58\] | \(39.27 \pm 0.49_{\text{exp}} \pm 0.56_{\text{latt}}\) |
| FNAL/MILC 2014 (Lattice \(\omega = 1\)) \[59\] | \(39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{latt}} \pm 0.19_{\text{QED}}\) |
| HFAG 2012 (Sum Rules) \[60\]–\[62\] | \(41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}\) |

| \(B \to D l\bar{v}\) Global fit 2016 \[41\] | \(40.49 \pm 0.97\) |
| Belle 2015 (CLN) \[40\], \[63\] | \(39.86 \pm 1.33\) |
| Belle 2015 (BGL) \[38\], \[40\], \[63\] | \(40.83 \pm 1.13\) |
| FNAL/MILC 2015 (Lattice \(\omega \neq 1\)) \[40\] | \(39.6 \pm 1.7_{\text{exp+QCD}} \pm 0.2_{\text{QED}}\) |
| HPQCD 2015 (Lattice \(\omega \neq 1\)) \[38\] | \(40.2 \pm 1.7_{\text{latt+stat}} \pm 1.3_{\text{syst}}\) |

| Inclusive decays | \(|V_{cb}|\) |
|------------------|-------------|
| Gambino et al. 2016 \[64\] | \(42.11 \pm 0.74\) |
| HFAG 2014 \[65\] | \(42.46 \pm 0.88\) |

| Indirect fits | \(|V_{cb}|\) |
|---------------|-------------|
| UTfit 2016 \[66\] | \(41.7 \pm 1.0\) |
| CKMfitter 2015 (3\(\sigma\)) \[67\] | \(41.8_{-0.7}^{+0.97}\) |
the higher value of the exclusive determination $|V_{cb}| = (39.5 \pm 0.8) \times 10^{-3}$, given by PDG 2014 [78], the LHCb reports $|V_{ub}| = (3.27 \pm 0.23) \times 10^{-3}$. The latter result, together with other recent exclusive determinations of $|V_{cb}|$, have been reported in Table 2. Let us observe that the values obtained for $B \to \rho/\omega l\nu$ appear to be systematically lower than the ones for $B \to \pi l\nu$. Indirect determination of $|V_{ub}|$ by the UTfit [66] and the CKMfitter [67] collaborations have also been reported in Table 2. At variance with the $|V_{cb}|$ case, the results of the global fit prefer a value for $|V_{ub}|$ that is closer to the (lower) exclusive determination.

Table 2. Status of exclusive $|V_{ub}|$ determinations and indirect fits [54]

| Exclusive decays | $|V_{ub}| \times 10^{-3}$ |
|------------------|--------------------------|
| $B \to \pi l\bar{\nu}$ | |
| FLAG 2016 [58] | $3.62 \pm 0.14$ |
| Fermilab/MILC 2015 [80] | $3.72 \pm 0.16$ |
| RBC/UKQCD 2015 [81] | $3.61 \pm 0.32$ |
| HFAG 2014 (lattice) [65] | $3.28 \pm 0.29$ |
| HFAG 2014 (LCSR) [65, 82] | $3.53 \pm 0.29$ |
| Imsong et al. 2014 (LCSR, Bayes an.) [83] | $3.32^{+0.26}_{-0.22}$ |
| Belle 2013 (lattice + LCSR) [74] | $3.52 \pm 0.29$ |
| $B \to \omega l\bar{\nu}$ | |
| Bharucha et al. 2015 (LCSR) [11] | $3.31 \pm 0.19_{\text{exp}} \pm 0.30_{\text{th}}$ |
| $B \to \rho l\bar{\nu}$ | |
| Bharucha et al. 2015 (LCSR) [11] | $3.29 \pm 0.09_{\text{exp}} \pm 0.20_{\text{th}}$ |
| $\Lambda_b \to p\mu\nu\bar{\mu}$ | |
| LHCb (PDG) [79] | $3.27 \pm 0.23$ |
| Indirect fits | |
| UTfit (2016) [66] | $3.74 \pm 0.21$ |
| CKMfitter (2015, 3$\sigma$) [67] | $3.71^{+0.17}_{-0.20}$ |

The extraction of $|V_{ub}|$ from inclusive decays requires to address theoretical issues absent in the inclusive $|V_{cb}|$ determination, since the experimental cuts, needed to reduce the background, enhance the relevance of the so-called threshold region in the phase space. Several theoretical schemes are available, which are tailored to analyze data in the threshold region, but differ in their treatment of perturbative corrections and the parametrization of non-perturbative effects. We limit to compare four theoretical different approaches, which have been recently analyzed by BaBar [84], Belle [85] and HFAG [65] collaborations, that is: ADFR by Aglietti, Di Lodovico, Ferrera and Ricciardi [86–88]; BLNP by Bosch, Lange, Neubert and Paz [89–91]; DGE, the dressed gluon exponentiation, by Andersen and Gardi [92]; GGOU by Gambino, Giordano, Ossola and Uraltsev [93, 94]. Although conceptually quite different, all these approaches lead to roughly consistent results when the same inputs are used and the theoretical errors are taken into account. The HFAG estimates [65], together with the latest estimates by BaBar [84, 95] and Belle [85], are reported in Table 3. The BaBar and Belle estimates in Table 3 refer to the value extracted by the most inclusive measurement, namely the one based on the two-dimensional fit of the $M_X - q^2$ distribution with no phase space restrictions, except for $p_T > 1.0$ GeV. This selection allow to access approximately 90% of the total phase space [95]. The BaBar collaboration also reports measurements of $|V_{ub}|$ in other regions of the phase space.

1Recently, artificial neural networks have been used to parameterize the shape functions and extract $|V_{ub}|$ in the GGOU framework [94]. The results are in good agreement with the original paper.
By comparing the results in Table 2 and 3, we observe a tension between exclusive and inclusive determinations, of the order of $2 - 3\sigma$, according to the chosen values. Belle II is expected, at about $50 \text{ ab}^{-1}$, to decrease experimental errors on both inclusive and exclusive $|V_{ub}|$ determinations up to about $2\%$.

Table 3. Status of inclusive $|V_{ub}|$ determinations [54]

| Inclusive decays | $|V_{ub}| \times 10^3$ |
|------------------|------------------------|
|                  | ADFR [86–88]           | BNLP [89–91]         | DGE [92]        | GGOU [93]       |
| HFAG 2014 [65]   | $4.05 \pm 0.13^{+0.18}_{-0.11}$ | $4.45 \pm 0.16^{+0.23}_{-0.18}$ | $4.52 \pm 0.16^{+0.13}_{-0.11}$ | $4.51 \pm 0.16^{+0.12}_{-0.11}$ |
| BaBar 2011 [84]  | $4.29 \pm 0.24^{+0.19}_{-0.19}$ | $4.28 \pm 0.24^{+0.19}_{-0.19}$ | $4.40 \pm 0.24^{+0.11}_{-0.11}$ | $4.35 \pm 0.24^{+0.10}_{-0.10}$ |
| Belle 2009 [85]  | $4.48 \pm 0.30^{+0.19}_{-0.21}$ | $4.47 \pm 0.27^{+0.19}_{-0.21}$ | $4.60 \pm 0.27^{+0.19}_{-0.11}$ | $4.54 \pm 0.27^{+0.10}_{-0.11}$ |

[84], but the values reported in Table 3 are the most precise. When averaged, the ADFR value is lower than the one obtained with the other three approaches, and closer to the exclusive values; this difference disappears if we restrict to the BaBar and Belle results quoted in Table 3. By taking the arithmetic average of the results obtained from these four different QCD predictions of the partial rate the Babar collaboration gives [84] $|V_{ub}| = (4.33 \pm 0.24_{\text{exp}} \pm 0.15_{\text{th}}) \times 10^{-3}$.

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