Moderate Supersymmetric $CP$ Violation

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It is well known that supersymmetry (SUSY) gives neutron and electron electric dipole moments ($d_n$ and $d_e$) which are too large by about $10^3$. If we assume a SUSY model cannot contain fine-tunings or large mass scales, then one must require that the SUSY breaking mechanism give real soft breaking parameters, in which case the minimal SUSY model has no $CP$ violation other than from the CKM matrix (besides possible strong $CP$ violating effects). We show that in non-minimal SUSY models, a moderate amount of $CP$ violation can be induced through one loop corrections to the scalar potential, giving an effective phase of order $10^{-3}$, and thus implying $d_n$ and $d_e$ can be near their current experimental bounds naturally. This moderate amount of SUSY $CP$ violation could also prove important for models of electroweak baryogenesis. We illustrate our results with a specific model.
1 Introduction

Predictions for $CP$ violating effects in supersymmetric (SUSY) theories have often been discussed with a certain ambiguity. On the one hand, it is well known that when the complex quantities in the theory are allowed to have phases of order unity, the predicted neutron and electron electric dipole moments ($d_n$ and $d_e$) are typically too large by perhaps $10^3$ [1, 2, 3, 4, 5]. In order to avoid this, the relevant quantities are often chosen to be real, in which case the theory predicts no non-Standard Model $CP$ violation (CPV) and negligible $d_n$ and $d_e$ [6]. On the other hand, it has often been assumed that the observation of $d_n$ around the current limit of $10^{-25}\text{e cm}$ [7] could easily be accommodated by a SUSY theory with the phases somehow reduced by just the right amount. These ideas are clearly in conflict: one cannot have a theory which avoids fine-tunings by setting the SUSY parameters real, and at the same time expect $d_n$ near its current upper bound. The purpose of this paper is to describe a mechanism by which a moderate amount of SUSY $CP$ violation can naturally appear in a theory in which the soft SUSY breaking terms have been taken real.

The superpotential of the Minimal Supersymmetric Standard Model (MSSM) contains the Yukawa sector of the theory, $W_Y$, and a Higgs mixing term,

$$W_{MSSM} = W_Y + \mu H_u H_d,$$

where $H_u$ and $H_d$ are Higgs doublet superfields. If the soft breaking terms come from the superpotential, as in (3), then one can use $H_u$ and $H_d$ to rotate away the phase of $\mu$.

In order to avoid an additional hierarchy problem brought on by $\mu/M_{GUT} \ll 1$ [8], the MSSM is often extended by adding a singlet superfield $N$, whose scalar
component’s vacuum expectation value (VEV) generates the Higgs mixing term (see [3] and references therein). We refer to this model as the N+MSSM. One can use an $R$ symmetry to forbid $B$ and $L$ violating terms in $W_Y$, and to allow only cubic terms involving N, so that the superpotential can be written as

$$W_{N+MSSM} = W_Y + h N H_u H_d + a N^3.$$  \hspace{2cm} (2)

Note that we can use the Higgs and singlet N superfields to rotate away the phases of $h$ and $a$. This again assumes that the soft SUSY breaking Lagrangian can be written in the low energy supergravity (SUGRA) parametrization [10] :

$$-L_{soft} = |m_i|^2 |\varphi_i|^2 + \left( \frac{1}{2} \sum_\lambda \tilde{m}_\lambda \lambda \lambda + \tilde{A} \left[ W^{(3)} \right]_\varphi + \tilde{B} \left[ W^{(2)} \right]_\varphi + h.c. \right),$$  \hspace{2cm} (3)

where $\varphi_i$ are the scalar superpartners, $\lambda$ are the gauginos, and $[ \ ]_\varphi$ means take the scalar part. Here $W^{(2)}$ and $W^{(3)}$ are the quadratic and cubic pieces of the superpotential, so that in the MSSM, $W^{(3)} = W_Y$, and $W^{(2)} = \mu H_u H_d$; and in the N+MSSM, $W^{(3)} = W$. We have defined the soft breaking parameters $\tilde{A} \equiv A m_0^*$ and $\tilde{B} \equiv B m_0^*$ to include a mass scale $m_0$. The parameters $A$, $B$, their mass scale $m_0$, and the gaugino masses $\tilde{m}_\lambda$, can all be complex. These parameters contribute to $d_n$ at the order of $10^{-22} \tilde{\varphi}/\tilde{M}^2 e\text{cm}$, where $\tilde{\varphi}$ is a combination of the phases of the parameters, and $\tilde{M}^2$ is a combination of superpartner masses, normalized to the weak scale. The only known ways to make such a large $d_n$ compatible with the experimental upper bound are to fine-tune the phase $\tilde{\varphi}$ to order $10^{-3}$; have superpartner masses of order a few TeV; or somehow require all the phases to naturally be zero [11]. Both the first and second approach eliminate much of the attractiveness of SUSY [12]. For example, having large superpartner masses virtually eliminates the possibility of radiative

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breaking of \( SU_2 \times U_1 \), which was one of the major successes of SUSY. Losing this is especially undesirable now that the top mass is large enough to make it work. So we will henceforth assume that \( A, B, m_0 \) and \( \tilde{m}_\lambda \) are all real \[13\]. We do not have an explanation for how these conditions will be satisfied, but merely state that any complete SUSY model which has superpartners of order the weak scale must either satisfy these criteria, or provide an explanation for how their phases could naturally be of order \( 10^{-3} \) \[14\].

Imposing these ‘no fine-tuning criteria’ means that the only source of CPV in either the MSSM or the N+MSSM is the CKM phase \[6\]. CKM contributions to \( d_n \) and \( d_e \) from renormalization group running \[3\] and from finite effects \[3, 11\] are below \( 10^{-30} \) e cm, and are thus unobservably small. So if \( d_n \) or \( d_e \) were detected in the near future, could a SUSY theory with superpartners of order the weak scale explain them without resorting to fine-tunings?

There have also been some interesting models of baryogenesis at the electroweak scale \[15, 16, 17\], which require CPV beyond the CKM phase \[18\]. Could one construct a model with sufficient CPV for electroweak baryogenesis, while satisfying the upper bounds on \( d_n \) and \( d_e \), without fine-tunings?

With these questions in mind, we describe a mechanism by which a moderate amount of CPV can naturally arise in a non-minimal SUSY theory through loop corrections to the Higgs potential. The idea is that a phase which is unobservable at tree level can introduce an observable effective phase through loop effects. This effective phase will always be smaller than a tree level observable phase because of the usual factors of suppression associated with loops. Such a phase can make moderate contributions to \( d_n \) and \( d_e \), and may be useful in explaining the observed baryon asymmetry.
In Section II, we present an illustrative model which provides an existence proof for this moderate CPV mechanism. In that section, we show that at tree level in the scalar potential, SUSY CPV is essentially unobservable. In Section III, we show how this CPV can appear in the observable sector in the one loop effective potential. The magnitude of this CPV will be suppressed by loop coefficients, so that $d_n$ and $d_e$ can naturally be near their current experimental bounds.

2 The Model

Let us construct a model which has complex couplings only to terms which contain singlet scalar fields which have zero VEVs. We also need these particles to have no tree level couplings to quarks or leptons. This means that the Higgs scalar potential will be $CP$ conserving, as will all tree level vertices outside the neutral Higgs sector. At this order, there is no one loop contribution to $d_n$ or $d_e$. After one loop corrections to the scalar potential ($V$), a small phase can be induced into these vertices, which generates moderate $d_n$ and $d_e$. To do this in a model which is technically natural, one needs to add at least two such singlets ($N', N''$) to the N+MSSM. In order that they have zero VEVs, we impose a discrete symmetry on their superfields: $(N', N'') \rightarrow -(N', N'')$. Then the most general cubic superpotential respecting this additional symmetry is:

$$W = W_Y + hNH_uH_d + aN^3 + c'NN''^2 + c''NN''^2 + bNN'N''.$$  \hspace{1cm} (4)

One sees that the fields $N'$ and $N''$ have no direct couplings to quarks, leptons, or gauge particles. We will see below that they can each have zero VEVs, and thus their couplings will not affect the tree level minimum of the scalar potential. They do
not affect $CP$ violating observables studied to date (at tree level in $V$), so we term this sector *invisible*. This is merely nomenclature. It should be possible to detect these particles, and perhaps even to see $CP$ violating effects directly in processes in which they are produced, but they are certainly invisible when considering one loop processes involving only external quarks, leptons, and gauge bosons.

Notice that we do not have enough freedom to rotate away all the phases in (4), and that after making the visible sector $CP$ conserving, the reparametrization invariant $b^2 e^{*} e''^*$ can be complex [19]. This is the phase which will produce CPV in the one loop scalar potential. Our first task is to be sure that this phase does not produce any CPV in the visible sector at tree level in $V$, else $d_n$ will again be too large.

All supersymmetric contributions to $d_n$ come from the mass matrices of squarks and gauginos—if the mass matrices can all be made real, the SUSY contribution to $d_n$ disappears. If they are complex, the gaugino-squark-quark couplings become complex and contribute to $d_n$ through loop diagrams [1]. Let us write the down squark mass matrix in a partially diagonalized basis:

$$
\begin{pmatrix}
\mu_{dL}^2 1 + \hat{M}_D^2 & (\hat{A}^* - h n e^{i \theta_1} \tan \beta) \hat{M}_D \\
(\hat{A}^* - h n e^{i \theta_1} \tan \beta)^* \hat{M}_D & \mu_{dR}^2 1 + \hat{M}_D^2
\end{pmatrix},
$$

(5)

where $\hat{M}_D$ is the diagonal, real, $N_F \times N_F$ quark mass matrix (where $N_F$ is the number of families), and $\mu_{q_{L,R}}^2 \sim |m_{3/2}|^2$. Here $n = |\langle N \rangle|$ (so that $hn$ takes the place of $\mu$ of the MSSM), and $\tan \beta$ is the ratio of Higgs VEVs. The angle $\theta_1$ is one of the relative phases between the three VEVs, and is defined in (10). If (as we have assumed) the soft breaking parameters are real, and the minimum of the scalar potential $V$ is $CP$ conserving, then this matrix is real.
Next we can write the chargino mass matrix, $M_{\chi^+}$,

$$
\begin{pmatrix}
\tilde{m}_W & g_2 v_2 \\
g_2 v_1 & h n e^{i \theta_1}
\end{pmatrix},
$$

(6)
in the basis of [20] (with the argument of the Higgs VEVs rotated into $\theta_1$). Here $\tilde{m}_W$ is the $SU_2$ soft breaking gaugino mass, and $g_2$ is the $SU_2$ coupling constant. Again, if the minimum of $V$ is $CP$ conserving, then this matrix is real.

Since we have added three neutral fields to the MSSM (or two to the N+MSSM), the neutralino mass matrix, $M_{\chi^0}$, is $7 \times 7$. We extend the basis of [20] with $\psi_N, \psi_{N'}$, and $\psi_{N''}$:

$$
\begin{pmatrix}
\tilde{m}_B & 0 & -g_1 v_1 / \sqrt{2} & g_1 v_1 / \sqrt{2} & 0 & 0 & 0 \\
0 & \tilde{m}_W & g_2 v_1 / \sqrt{2} & -g_2 v_2 / \sqrt{2} & 0 & 0 & 0 \\
-g_1 v_1 / \sqrt{2} & g_2 v_1 / \sqrt{2} & 0 & -h n e^{i \theta_1} & -h v_1 & 0 & 0 \\
g_1 v_2 / \sqrt{2} & -g_2 v_2 / \sqrt{2} & -h n e^{i \theta_1} & 0 & -h v_2 & 0 & 0 \\
0 & 0 & -h v_1 & -h v_2 & 3 a n e^{i (\theta_3 - 2 \theta_1)} & X & Y \\
0 & 0 & 0 & 0 & X & c'n & bn \\
0 & 0 & 0 & 0 & Y & bn & c''n
\end{pmatrix},
$$

(7)

where $g_1$ is the $U_1$ coupling constant, and $\tilde{m}_B$ is the $U_1$ gaugino mass. The angle $\theta_3$ is also defined in [11]. The cross terms $X$ and $Y$, which mix $\psi_{N'}$ and $\psi_{N''}$ with the visible sector, are proportional to the VEVs of $N'$ and $N''$, so that if these VEVs are zero, $\psi_{N'}$ and $\psi_{N''}$ decouple from the visible sector. The resulting $5 \times 5$ visible sector matrix is real, if $\sin \theta_1 = \sin \theta_3 = 0$. In that case, all of the mass matrices are real, and there is no new SUSY contribution to $d_n$.

To see if this is the case, we must consider the scalar potential $V$. We define two
Higgs doublets of the same hypercharge, and their VEVs,
\[
\langle \phi_1 \rangle \equiv \langle H_d \rangle \equiv \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle \equiv \langle H_u \rangle \equiv \begin{pmatrix} 0 \\ v_2 e^{i \xi} \end{pmatrix},
\]
and the VEVs of the singlet fields
\[
\langle N \rangle \equiv n e^{i \varphi}, \quad \langle N' \rangle \equiv n' e^{i \varphi'}, \quad \langle N'' \rangle \equiv n'' e^{i \varphi''}.
\]
It turns out in this model that if \( n' = n'' = 0 \), there are only three combinations of these VEV phases,
\[
\theta_1 \equiv \xi + \varphi, \quad \theta_2 \equiv \theta_1 - \theta_3 = \xi - 2 \varphi, \quad \theta_3 \equiv 3 \varphi,
\]
which appear in the tree level scalar potential. Elsewhere, a more general linear combination of \( \theta_1 \) and \( \theta_3 \) (with integer coefficients) can appear.

Let us write the scalar potential for our model,
\[
V = (h^2 |N|^2 + m_1^2) |\phi_1|^2 + (h^2 |N|^2 + m_2^2) |\phi_2|^2 \\
- \left[ (\bar{A}hN + 3\bar{a}hN^* + he'' N'' N''^* + he'' N'' N''^* ) (\phi_1^* \phi_2) + H.c. \right] \\
+ \lambda_1 (\phi_1^* \phi_1)^2 + \lambda_2 (\phi_2^* \phi_2)^2 + \lambda_3 (\phi_1^* \phi_1)(\phi_2^* \phi_2) + \lambda_4 (\phi_1^* \phi_2)(\phi_2^* \phi_1) \\
+ m_0^2 |N|^2 + m_0^2 |N'|^2 + m_0^2 |N''|^2 + 9a^2 |N|^4 + |c'|^2 |N'|^4 + |c''|^2 |N''|^4 \\
+ 4 |c'|^2 + |b|^2 |NN'|^2 + (4 |c''|^2 + |b|^2) |NN''|^2 \\
+ \left[ \bar{A}a N^3 + \bar{A}c' NN' + \bar{A}c'' NN'' + \bar{A}b NN'' \\
+ 2 (b^* c' + bc''^*) |N|^2 N' N''^* + 3ac' N^* N N'' + 3ac'' N N^* N'' + c' c'' N N'' + H.c. \right]
\]
where \([21]\)
\[ \lambda_1 = \lambda_2 = \frac{g_2^2 + g_1^2}{8}, \quad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \quad \lambda_4 = h^2 - \frac{g_2^2}{2}, \]  

which is just the scalar potential for the N+MSSM \[9\] plus terms which involve \(N'\) and \(N''\). The minimum of \(V\) can be written as

\[
\langle V \rangle = \langle V_{N+MSSM} \rangle + K_{20}n'^2 + K_{11}n' n'' + K_{02}n'^2 \\
+ K_{40}n'^4 + K_{22}n'^2 n'^2 + K_{04}n''^4,
\]

where \(K_{ij}\) depend upon all the other parameters. One can show that for any choice of the parameters in \(V_{N+MSSM}\), there exists a set of \(\{b', c', c''\}\) such that \(n' = n'' = 0\) is a true minimum. We will assume that this condition is satisfied, so that \(\langle V \rangle = \langle V_{N+MSSM} \rangle\).

Finally we must be sure there is no problem with spontaneous CPV. As we said, the potential depends only upon the three angles \(\theta_1-3\) (only two of which are independent). We can write

\[
\langle V \rangle = \alpha_0 - \alpha_1 \cos \theta_1 - \alpha_2 \cos \theta_2 - \alpha_3 \cos \theta_3,
\]

where the \(\alpha_i\) are functions of the magnitudes of the three VEVs. Differentiating with respect to \(\theta_1\) and \(\theta_3\), we see that one solution to \(\langle V' \rangle = 0\) is \(\sin \theta_i = 0\). Romão \[22\] showed that for this potential (i.e. \(\langle V_{N+MSSM} \rangle\)), this is the only stable minimum—that the spontaneous \(CP\) violating solution is actually a saddle point. Babu and Barr \[23\] make the interesting claim that this can be made into a minimum by large radiative corrections to the Higgs mass matrix, but these require very heavy squark masses (in which case hard CPV need not be suppressed by fine-tuning \[1\]), small
charged Higgs mass, and \( \tan \beta \sim \mathcal{O}(1) \). These conditions make satisfying the CLEO bound on \( b \to s\gamma \) nearly impossible \[24\]. It is also unlikely that a model satisfying these conditions could be consistent with such things as Grand Unification and solutions to the Dark Matter problem \[12\]. Anyway, we can certainly choose parameters such that the minimum of \( V \) is \( \mathcal{C}P \) conserving, and such that \( n' = n'' = 0 \), so that all the SUSY mass matrices (5)-(7) are real at tree level.

### 3 A Loop Induced Observable Phase

It would seem that since the tree level potential is \( \mathcal{C}P \) conserving, one could not have a one loop potential which is \( \mathcal{C}P \) violating. The important point to remember is that even though the visible sector has no CPV, there are still \( \mathcal{C}P \) violating couplings to \( N' \) and \( N'' \). Consider, for example, Figure 1, which gives a purely finite contribution to a new term in \( V \), \( \delta \lambda_5 (\phi_1^\dagger \phi_2)^2 + H.c \) (this term is not present in the tree level potential, so it must be finite). The vertices are proportional to \( c'^* \) and \( c'^{**} \), and the mixing between \( N' \) and \( N'' \) contains pieces proportional to \( b \). Thus \( \delta \lambda_5 \sim b^2 c'^* c'^{**} \), which has a reparametrization invariant phase. For \( b, c', c'' \) of order 1/2, \( \delta \lambda_5 \) can be of order \( 10^{-3} \).

Actually, the operator (to which Figure 1 contributes) is more accurately written as \( k \langle N \rangle^2 (\phi_1^\dagger \phi_2)^2 \), which gives a contribution to \( \langle \delta V \rangle \) of \( 2 |k| n^2 v_1^2 v_2^2 \cos(2\theta_1 + \text{Arg}k) \), where \( \text{Arg}k \) is just \( \theta_{CP} \equiv \text{Arg}(b^2 c'^* c'^{**}) \). We can write the general correction to \( \langle V \rangle \) as

\[
\langle \delta V \rangle = \sum_{x,y,z} \kappa_{x,y,z} \cos(x\theta_1 + y\theta_3 + z\theta_{CP}),
\]

(15)
where the $\kappa_{x,y,z}$ are real coefficients, with the subscripts $x, y, z$ taking on all integral values, though the $\kappa_{x,y,z}$ become negligible for large integers. Since our model is renormalizable and $\langle V \rangle$ is $CP$ conserving at tree level, all one loop terms in (15) with $z \neq 0$ must be finite. Note that Figure 1 gives a finite contribution to (13) with $(x, y, z) = (2, 0, 1)$.

The perturbation in (15) means that $\sin \theta_i = 0$ is no longer a solution to $\langle V(\theta_1, \theta_3)' \rangle = 0$. Since the $\kappa_{x,y,z}$ are small, the solution will lie close to this, so we can define $\theta_i = \theta_{i0} + \varepsilon_i$, where $\sin \theta_{i0} = 0$. The minimization condition can then be written in terms of the hessian, and the perturbation:

$$
\begin{pmatrix}
\frac{\partial^2 V}{\partial \theta_1^2} & \frac{\partial^2 V}{\partial \theta_1 \partial \theta_3} \\
\frac{\partial^2 V}{\partial \theta_1 \partial \theta_3} & \frac{\partial^2 V}{\partial \theta_3^2}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_3
\end{pmatrix}
\approx
\begin{pmatrix}
\sum_{x,y,z} x \kappa_{x,y,z} \sin(x \theta_{10} + y \theta_{30} + z \theta_{CP}) \\
\sum_{x,y,z} y \kappa_{x,y,z} \sin(x \theta_{10} + y \theta_{30} + z \theta_{CP})
\end{pmatrix}
$$

and solved for $\varepsilon_i$. Note that $\sin(x \theta_{10} + y \theta_{30} + z \theta_{CP}) = \pm \sin(z \theta_{CP})$. To find the effective $CP$ violating coefficient, recall that $d_n$ gets a contribution from the imaginary part of left–right squark mixing, which goes as $\sin \theta_1 \approx \varepsilon_1$. There will be finite contributions to $\varepsilon_1$ from several terms in $\delta V$, but they will be of the same order or smaller than that of $\delta \lambda_5$ from Figure 1. From (16), one finds that Figure 1 gives $\varepsilon_1 \approx |\delta \lambda_5| \frac{v^2}{\tilde{m}_g^2} \sin 2\beta \sin \theta_{CP}$. The squark mixing, gluino mediated contribution to $d_n$ due to this $\varepsilon_1$ can be written as

$$
d_n \approx 10^{-22} \text{e cm} \left( \frac{100 \text{GeV}}{\tilde{M}} \right)^2 |\delta \lambda_5| \frac{v^2}{Am_0} \sin \theta_{CP},
$$

where we have defined a SUSY mass scale

$$
\tilde{M}^2 \equiv \frac{\tilde{m}_d^4}{m_0 \tilde{m}_g}.
$$
There is a similar neutralino mediated contribution to $d_e$ which is suppressed by $m_e/m_d$ and $\alpha_w/\alpha_s$, but enhanced by the fact that sleptons tend to be lighter than squarks. If we take $A = 1$, colored superpartners $\sim 300\text{GeV}$, sleptons $\sim 150\text{GeV}$, and all other superpartners $\sim 100\text{GeV}$, one can have

$$d_n \sim 10^{-26} \sin \theta_{CP} \text{ e cm},$$

(19)

$$d_e \sim 10^{-27} \sin \theta_{CP} \text{ e cm}.$$  

(20)

These estimates depend upon the parameters and the mass scales in the theory, but the point is that the contributions entering at one loop are naturally much smaller than those from SUSY phases which contribute through tree level vertices.

## 4 Concluding Remarks

We have considered supersymmetric models which avoid excessively large contributions to $d_n$ and $d_e$ by requiring the ‘no fine-tuning criteria’ to be satisfied, i.e. that $A$, $B$, $m_0$ and $\tilde{m}_\lambda$ must be real [11]. We showed that it is possible for moderate $CP$ violating effects to be induced at one loop in models which have singlets with zero VEVs. We used an illustrative model with superpotential (4) and found that a stable minimum exists at $\langle N' \rangle = \langle N'' \rangle = 0$. This means that using the tree level scalar potential, no $CP$ violating effects would be detectable in conventional $CP$ violating observables because all of the SUSY mass matrices are real. We demonstrated that this model introduces small $CP$ violating phases into the one loop effective potential, so that one is left with a moderate contribution to $d_n$ and $d_e$. One could easily have $d_n$ and $d_e$ near their current experimental bounds in such a model, without the need
for fine-tuning or large superpartner mass scales.

Note that SUSY contributions to $d_n$ from three gluon operators do not affect our conclusions. Assuming that the no fine tuning criteria are satisfied, such operators will also give a negligible contribution to $d_n$. After one loop corrections to $V$ in our model, there will be small contributions to $d_n$ from these operators, but they will probably be smaller in magnitude than the quark EDM contribution [5]. Thus (13) and (20) are reasonable estimates of the natural size of SUSY CPV possible in a model such as ours.

We have discussed the issue of spontaneous CPV in Section II in the context of our model and concluded that we can easily choose the minimum of $V$ to be $CP$ conserving. It is worth noting that Maekawa [27] considered generating spontaneous CPV at one loop in the MSSM, though Pomarol [28] showed that such a model requires a $CP$ odd Higgs which is too light. Pomarol also made the interesting point that a N+MSSM model (which does not rule out $H_uH_d$, $N$, or $N^2$ terms by a symmetry) with a strictly $CP$ conserving Lagrangian might violate $CP$ spontaneously at tree level with a phase of order $10^{-2}$, and might be able to explain the $\varepsilon$ parameter as well as give $d_n$ near the current experimental bound [29]. The trouble is that the fine-tuning needed by such a model of spontaneous CPV is actually much worse than that for hard CPV because the condition which must be satisfied is of the form

$$\cos \theta = \left| \frac{X}{Y} \right| \simeq 1 - \frac{1}{2} \theta^2,$$

where $\theta$ (or $\pi - \theta$) is the relevant spontaneous CPV phase, and $X$ and $Y$ are some combination of parameters and VEVs. We need $\theta$ to be small to satisfy the bound on $d_n$, which we can achieve only if $\delta \equiv (Y - X)/Y$ is of order $\theta^2$. For example, if
we need $\theta \sim 10^{-2}$, then $\delta$ must be fine-tuned to be of order $10^{-4}$, which is completely unacceptable.

As we alluded to in the Introduction, having a moderate amount of CPV is necessary in models which generate the baryon asymmetry at the electroweak scale \[18\]. A recent interesting model of electroweak baryogenesis used CPV from the Higgs scalar mixing coefficient $\mu_{12}^2$ \[15\], which can be defined as the coefficient of the $\langle \phi_1^\dagger \phi_2 \rangle$ term in $\langle V \rangle$. It was pointed out \[16\] that $\mu_{12}^2$ can be rotated out of the Higgs potential, but the resulting phase which appears in the gaugino mass matrices was then used by \[17\]. They found that with the small phase allowed by the limit imposed by $d_n$, there is probably sufficient CPV for the observed baryon asymmetry. Our results change these conclusions in two ways. At tree level, there is no phase in the gaugino mass matrices (after imposing the no fine-tuning criteria), and no way for $\text{Arg} \mu_{12}^2$ to cause CPV. Then, using the one loop effective potential in a model such as ours, there can be an effective phase $\varepsilon_1 (\equiv \theta_1 - \theta_{10})$ introduced into the gaugino mass matrix of order $\varepsilon_1 \sim 10^{-3} \theta_{CP}$. Although this is a large suppression, $\theta_{CP}$ can be of order unity and so $\varepsilon_1$ should generate the same level of CPV as the phase used in \[17\], which was bounded by $d_n$ anyway \[30\].

From the standpoint of explaining the baryon asymmetry at the electroweak scale, or $d_n$ and $d_e$ near their current experimental bounds, a loop induced observable phase provides an attractive alternative to the fine-tuning needed in the MSSM. If $d_n$ or $d_e$ were observed in the near future, and if superpartners were determined to be of order the weak scale, SUSY model builders would have to appeal to a mechanism such as ours, which naturally explains small effective SUSY phases.

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FIGURE CAPTION

Fig 1: One loop contribution to the operator $\delta \lambda_5 (\phi_1^\dagger \phi_2)^2$. The ‘X’ indicates $N'-N''$ mixing, which is required if $\delta \lambda_5$ is to contain a reparametrization invariant phase.