Deconstructing six dimensional gauge theories with strongly coupled moose meshes

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Abstract

It has recently been realized that five dimensional theories can be generated dynamically from asymptotically free, QCD-like four dimensional dynamics via “deconstruction.” In this paper we generalize this construction to six dimensional theories using a moose mesh with alternating weak and strong gauge groups. A new ingredient is the appearance of self couplings between the higher dimensional components of the gauge fields that appear as a potential for pseudo-Goldstone bosons in the deconstructed picture. We show that, in the limit where the weak gauge couplings are made large, such potentials are generated with appropriate size from finite one loop correction. Our construction has a number of applications, in particular to the constructions of “little Higgs” models of electroweak symmetry breaking.
1 Introduction

Field theories in higher dimensions are useful to address various issues in particle physics (for some examples see [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12]). These higher dimensional field theories have dimensionful coupling constants much like gravity in four dimensions and become strongly coupled at high energies requiring a new description of the physics. In [13] it was shown that five dimensional Yang-Mills theories compactified on a circle could be generated dynamically from asymptotically free, four dimensional theories by “deconstructing” the extra dimension. We first lattice the circle [15, 16], and the latticized action then take the form of a gauged non-linear sigma model. The gauged non-linear sigma model breaks the large, product gauge symmetry down to the diagonal subgroup and can be represented graphically as theory space (or “moose” or quiver diagram). In the continuum limit theory space becomes the actual extra dimension. The theory space structure is very rich and has many applications [18, 20, 21, 22, 23, 24].

The non-linear sigma model is non-renormalizable and breaks down at high energy, therefore simply latticizing the extra dimension does not provide an ultraviolet description of five dimensional theories. However, the non-linear sigma model can be easily UV completed in standard ways, for example with a strongly coupled theory, à la QCD. At very high energy, the theory is purely four dimensional and asymptotically free. At some scale Λ, the strongly interacting gauge group condenses, breaking chiral symmetries, and at low energies the description is in terms of non-linear sigma model fields that looks exactly like a five dimensional theory compactified on a latticized circle. Beneath the scale of the lattice spacing and above the compactification radius the theory looks five dimensional.

Generating a six dimensional theory compactified on a torus in a similar way is not as straightforward. The first step is to lattice the torus in a 2 dimensional grid of points and links, where the points are gauge groups and the links are bifundamental non linear sigma model fields. The non linear sigma model fields arise from fermion condensates triggered by strong gauge groups. Kinetic terms for these fields correspond to $\text{Tr} \, F_{\mu\nu}^2$ and $\text{Tr} \, F_{\mu\nu}^2$ terms of the continuum theory. The issue is then to generate the $\text{Tr} \, F_{56}^2$ operator which would appear as a potential for the non-linear sigma model fields in the latticized theory. Naively this appears to be at least an eight fermion operator in the ultraviolet theory and the coefficient of such term appears to be too small [14]. We demonstrate in this paper that these terms are in fact generated with sizeable coefficients from the low energy theory in the limit where the “weak” gauge couplings are made large. The scaling of couplings and the structure of radiative corrections to the effective action are crucial for understanding how this arise. We will see that the one loop finite contributions from gauge interactions in the deconstructed theory will be sufficient to generate sizeable potentials for the non-linear sigma model fields.

In section 2, we review the principles of dimensional deconstruction. In section 3 we present the relations between the parameters of the extra-dimensional theory we wish to generate and the parameters of the four dimensional non-linear sigma model from which it emerge. We also define the limit we wish to take. In section 4 and 5 we show, from
the continuum and deconstructed perspective respectively, that $\text{Tr} \ F_{56}^2$ is in fact generated with appropriate coefficient. Additionally, in section 5, we discuss the different radiative corrections that arise in the deconstructed theory and argue that we do not lose control of the qualitative dynamics of the theory, even tough we are taking the strong coupling limit. We also interpret the relation between the continuum calculation and deconstructed calculation. Conclusions and applications are then discussed.

2 UV Completing 6 Dimensional Gauge Theory

We would like to review the framework in which extra dimensional gauge theories can be completed into QCD-like models. This was first demonstrated for five dimensional theories in [13] and extended to six dimensional theories in [14]. In the 5D case, the ultraviolet theory is an $SU(m)^N \times SU(n)^N$ gauge theory, where the $SU(n)$ gauge groups become strongly coupled at scale $\Lambda$. The theory has $N$ fermions $\chi_i$ that transform as $(m, \bar{n})$ under $SU(m)_i \times SU(n)_i$ and $N$ fermions $\psi_i$ transforming as $(n, \bar{m})$ under $SU(n)_i \times SU(m)_{i+1}$. This structure can be easily visualized in a moose diagram with $2N$ points and $2N$ links with the points representing the $SU(m)$ and $SU(n)$ gauge group alternatively and the links representing the $\chi$'s and the $\psi$ alternatively (see Fig. 1). Below the scale $\Lambda$, $\chi_i$ and $\psi_i$ form condensates. The theory can then be described by a weakly gauged non-linear sigma model and can be represented by a moose diagram with $N$ $SU(m)$ sites linked by bifundamental non-linear sigma model fields $U_i$. This non-linear sigma model is a five dimensional gauge theory compactified on a circle, with the fifth dimension latticized; the moose diagram has turned into a picture of an extra dimension.

The naive extension to the 6D case is straightforward. The moose diagram is now a mesh of alternating weak and strong groups connected by oriented links representing fermions transforming as fundamental under the gauge group the link is pointing to and as anti-fundamental under the group the link is pointing from. Below the scale of strong coupling, the strong gauge groups “disappear” from the moose diagram and we are left with $N^2$ weakly coupled gauge groups $G$ linked by non-linear sigma model fields $U_{i,j}$ and $V_{i,j}$. These link fields spontaneously break the $G^{N^2}$ gauge symmetry to the diagonal subgroup, leading to a collection of massive vector bosons. At low energy, these massive gauge bosons match part of the “KK” spectrum of a six dimensional theory compactified on a torus. This is because the kinetic terms for the link fields are the latticization of the $F_{\mu\nu}^2$ terms of a continuous six dimensional theory. However, the $F_{56}^2$ term corresponding to plaquette potential is not automatically present. In the rest of this paper, we show that in an appropriate limit, this term is in fact generated from infrared physics within this effective non-linear sigma model, with a coefficient of the appropriate size. Therefore, further discussion of the ultra-violet completion in term of QCD-like model will not be necessary.
3 Scaling

In this section we discuss the scaling of the couplings in the different regimes of the theory (see Fig. 3 and establish a dictionary between the parameters of the four dimensional and six dimensional theories. At the highest energies, the theory is a four dimensional gauged non-linear sigma model with $N^2$ gauge groups. The parameters of this theory are the breaking scale $f$, the cut-off of the theory $\Lambda$, the gauge couplings, $g_0 = g_0(\Lambda)$, that we take to be identical, and the number of sites $N^2$. The six dimensional continuum description arises at intermediate energy scales between the lattice spacing, $a$, and the radius of the extra dimension, $R$. Between those scales we can define an effective six dimensional gauge coupling, $g_6$. On it's own, this six dimensional theory would get strongly coupled at energies of order $4\pi g_6^{-1}$, but since it is part of our deconstructed theory, it is regulated at the scale of the lattice spacing $a^{-1}$. At energy scales below $R^{-1}$, the theory behaves as a four dimensional gauge theory with a gauge coupling $g_4$. The effective four dimensional degrees of freedom below this scale are a four dimensional gauge boson and two classically massless adjoint scalars, $u$ and $v$.

There are relations between the parameters that we state here that can be derived by matching the KK tower of gauge bosons in a six dimensional theory with the spectrum of massive gauge bosons of the non-linear sigma model, and by matching the low energy coupling of both theories.
Figure 3:

The cut-off of the four dimensional high energy theory is roughly $\Lambda \approx 4\pi f$, set by the scale when the non-linear sigma model fields become strongly interacting. The effective six dimensional gauge coupling is related to the high energy theory as:

$$g_6^{-1} = f \quad (1)$$

The lattice spacing is given by:

$$a^{-1} = g_0 f \quad (2)$$

This is the scale where the Kaluza-Klein modes are truncated and is where we should cut-off the six dimensional continuum theory. The circumference of the extra dimension is given by

$$R = Na = \frac{N}{g_0 f}. \quad (3)$$

Finally the low energy four dimensional gauge coupling, $g_4$ is related to the other parameters by:

$$g_4 = \frac{g_6}{R} = \frac{g_0}{N}. \quad (4)$$

As we vary the parameters in the theory, we want to keep the low energy parameters fixed. The two parameters that we choose to hold fixed are the low energy gauge coupling, $g_4$, and the radius, $R$. This means that as we scale $g_0$, we hold the ratio $g_0/N$ fixed and this imply that $g_0$, $f$ and $\Lambda$ are all fixed as well. We will push the high energy gauge couplings to the limit of perturbativity, $g_0 \to 4\pi$. In this limit, $a^{-1} \sim \Lambda$, and the theory looks six dimensional up to the cutoff.
4 Continuum Radiative Corrections

The kinetic terms for the non-linear sigma model fields, $|D_{\mu}U_{i,j}|^2 + |D_{\mu}V_{i,j}|^2$, corresponds, in the continuum language, to $\text{Tr} F_{\mu5}^2$ and $\text{Tr} F_{\mu6}^2$. The primary issue in completing the six dimensional theory (or higher dimensional theories as well), is generating the $\text{Tr} F_{56}^2$ term. We will generate this term in the deconstructed theories not from any ultraviolet physics that completes the non-linear sigma models, but from infrared effects inside the non-linear sigma model. We can get a feeling for how this can happen by considering a six dimensional gauge theory without Lorentz invariance:

$$S_6 = \int d^6x \left[ -\frac{1}{2} \text{Tr} F_{\mu\nu}^2 - \text{Tr} F_{\mu5}^2 - \text{Tr} F_{\mu6}^2 \right].$$

where the normalization is chosen such that $F_{MN}$ contains the gauge coupling $g_6$. We have set the coefficient of $\text{Tr} F_{56}^2$ to zero by hand. There is no enhanced symmetry when this coefficient vanishes and therefore it should be generated in the effective action. The question is what size the coefficient is generated with. There are two one loop diagrams that generate an $A_5^2 A_6^2$ interaction in the effective action and they are shown in Fig. 3. These diagrams are quadratically divergent in six dimensions and produce a coefficient for $\text{Tr} [A_5, A_6]^2$ of order:

$$\sim \frac{g_6^4}{64\pi^3} \Lambda_6^2.$$  

(6)

As mentioned earlier, the six dimensional theory is regulated by having two of its dimensions on a lattice, we should therefore use the effective lattice spacing, or more precisely, the top of the heavy bosons spectrum, as the cut-off of the six dimensional theory, $\Lambda_6 = 8g_6^2 f^2$. By gauge invariance, we therefore expect a term:

$$c \frac{g_6^2}{8\pi^3} \text{Tr} F_{56}^2.$$

(7)

to be generated, with $c$ of order 1. This is precisely the size that we want as $g_6 \to 4\pi$. It is important to note that we are not claiming that Lorentz invariance is exactly recovered (as we have left out all order unity factors), but merely saying that it is unnatural for the coefficient of $\text{Tr} [A_5, A_6]^2$ to be significantly smaller than $g_6^2$. Notice that this term is completely generated inside the six dimensional theory though it is cut-off sensitive. Dimensional deconstruction provides an ultraviolet completion for these theories and in the next section we explore what this ultraviolet completion says about the quadratic divergence.

$\text{Tr} F_{56}^2$ generates a KK tower as well as quartic couplings for $A_5$ and $A_6$. In the next section, we will check, directly from the deconstructed theory, that this term is generated with the appropriate coefficient by looking at the quartic coupling between the massless mode $u$ and $v$ which correspond to the zero mode of $A_5$ and $A_6$. From Eq. 7, we expect to find:

$$\int d^6xc \frac{g_6^2}{8\pi^3} g_6^2 \text{Tr} [A_5, A_6]^2 \to \int d^4xc \frac{g_6^4}{8\pi^3 N^2} \text{Tr} [u, v]^2$$

(8)
Figure 4:

Again, as $g_0 \to 4\pi$, the coefficient of $[u, v]^2$ approaches $g_4^2$ and become of the same order as the coefficient we would expect for a Lorentz invariant six dimensional gauge theory. We will also look for a tower of massive scalars corresponding to the $KK$ modes of $A_5$ and $A_6$ (denoted $u^{(n,m)}$, $v^{(n,m)}$):

$$m^2_{u^{(n,m)}} = c \frac{g_0^2}{8\pi^3} \frac{4\pi^2}{R^2} (n^2 + m^2) \quad (9)$$

In the continuum theory, we don’t expect the zero mode of $A_5$ and $A_6$ to remain exactly massless. At low energy, the theory looks four dimensional, and contains two massless scalars $u$ and $v$, as well as a massless gauge boson. This massless gauge boson will generate a potential for $u$ and $v$ that is not gauge invariant in an infinite six dimensional theory. This potential comes from infrared physics and is therefore finite and calculable\[17\]. We won’t calculate it exactly, but we can estimate it from the physical picture: It is the effective potential of the low energy theory, cutoff at the scale of the first KK mode $m_{KK}$. This effective potential can be obtained by turning on a background for $u$ and $v$, calculating the mass matrix of the gauge bosons $M(u, v)$ in this background and using the Coleman-Weinberg formula with $m_{KK}$ as the cutoff:

$$V(u, v) = \frac{3}{64\pi^2} \left( \text{Tr} \ M^2(u, v)m_{KK}^2 + \text{Tr} \ M^4(u, v) \log \frac{M^2(u, v)}{m_{KK}} \right) \quad (10)$$

For $u$ and $v$ turned on in the same direction ($\sigma_z$ for example), we get:

$$= \frac{6}{64\pi^2} \left( \frac{4g_4^2\pi^2}{R^2} (u^2 + v^2) + g_4^1(u^2 + v^2)^2 \log \left( \frac{g_4^1(u^2 + v^2)}{4\pi^2} \frac{R^2}{f^2} \right) \right) \quad (11)$$

$$= \frac{6}{64\pi^2} \left( \frac{4g_4^2g_5^2\pi^2}{N^2} (u^2 + v^2) f^2 + g_4^1(u^2 + v^2)^2 \log \left( \frac{g_4^2N^2(u^2 + v^2)}{g_5^2f^24\pi^2} \right) \right)$$

We will see in the next section, that this result is reproduced closely by the deconstructed calculation when $N$ is large. This part of the discussion is very similar to the five dimensional case \[13\].

5 Deconstructed Calculation

Having seen how the six dimensional continuum theory naturally generates a large quartic potential, we wish to find out how this arises in the deconstructed completion. In doing so
we will find that the quadratically divergent coefficient will actually arise from a finite effect in the deconstructed picture.

We will consider our theory beneath the scale of condensation where the non-linear sigma model is the appropriate description of physics. The Lagrangian for this theory is given by:

$$\mathcal{L}_{n\sigma} = \sum_{a,b}^N \left[ -\frac{1}{2} \text{Tr} F_{(a,b)}^2 + \frac{f}{4} \text{Tr} \left| D_\mu U_{(a,b)} \right|^2 + \frac{f}{4} \text{Tr} \left| D_\mu V_{(a,b)} \right|^2 \right] + \cdots. \quad (12)$$

where the ellipses represent higher derivative operators and heavy cut-off scale states. For calculational simplicity, we will consider an $SU(2)$ gauge theory, however the general arguments will apply to any non-Abelian gauge group. We want to demonstrate that a significant quartic potential is generated for the constant modes of the theory:

$$U_{(a,b)} = \exp \left( iu/Nf \right) \quad \quad V_{(a,b)} = \exp \left( iv/Nf \right).$$

At one loop there is no quadratic divergence or logarithmic divergence for these modes because the breaking of the chiral symmetries that protect these pseudo-Goldstone bosons necessitate many gauge couplings and ultraviolet physics is analytic in the couplings (see [18]). However, there is an infrared contribution to both the mass of these modes as well as the quartic coupling. These contributions can be calculated from the 1-loop Coleman-Weinberg potential. We first turn on background fields proportional to $\sigma_z$ for both $u$ and $v$. The masses squared of the gauge bosons in that background are given by:

$$M_{n,m}^2(u,v) = 4g_0^2f^2 \left( \sin^2 \frac{2n\pi}{2N} + \sin^2 \frac{2m\pi}{2N} \right) \quad (13)$$

The Coleman-Weinberg potential is given by:

$$V(u,v) = \frac{1}{64\pi^2} \sum_{n,m} M_{n,m}^4(u,v) \log \frac{M_{n,m}^2}{\Lambda} \quad (14)$$

The mass and quartic coupling can be extracted from the potential above:

$$m_u^2 = \frac{\partial^2}{\partial u^2} \sum_{n,m} M_{n,m}^4(u,v) \log \frac{M_{n,m}^2(u,v)}{\Lambda} \bigg|_{u,v=0} \quad (15)$$

$$\kappa = \frac{1}{4!} \frac{\partial^4}{\partial u^4} \sum_{n,m} M_{n,m}^4(u,v) \log \frac{M_{n,m}^2(u,v)}{\Lambda} \bigg|_{u,v=\phi}$$

Note that for the quartic term, we have given $u$ and $v$ a vev $\phi$ in order to avoid infrared divergence. We computed the masses and the quartic couplings numerically from these expressions. We then repeated the procedure for $u$ and $v$ proportional to $\sigma_x$ and $\sigma_y$ respectively. The masses of the gauge bosons in this background are more complicated, but we can still compute the quartic couplings numerically. The quartic couplings in this case contain a
Figure 5: Numerical calculation of the Coleman-Weinberg potential, keeping $g_0$ fixed. The Plot to the left, represent the mass as a function of $N$, fitted to a $X/N^4$ curve, where $X$ determined from the points. The set of points A on the right hand side plot shows the quartic coupling $(\lambda + \kappa_1)$ as a function of $N$ when the zero mode backgrounds are turned on in two different directions. They are fitted to a $Y/N^2 + Z/N^4$ curve, where $Y$ and $Z$ are determined from the points. The lower set of points (B), is the quartic coupling $(\kappa_1 + \kappa_2)$ when both zero mode are turned on in the same direction. They are fitted to a curve of the form $W/N^4$.

The contribution from $\text{Tr} \ [u, v]^2$ which doesn’t contribute when both $u$ and $v$ are turned on in the same direction. The results of the numerical calculation are shown in Fig. 5 and can be expressed as:

$$V_{\text{1 loop}} = m^2 \text{Tr} (u^2 + v^2) + \lambda \text{Tr} [u, v]^2 + \kappa_1 (\text{Tr} u^4 + \text{Tr} v^4) + \kappa_2 (\text{Tr} uv)^2$$  \hspace{1cm} (16)

with:

$$m^2 \sim \frac{g_0^4 f^2}{16 \pi^2 N^4} \quad \kappa_{1,2} \sim \frac{g_0^4}{16 \pi^2 N^4} \quad \lambda \sim \frac{g_0^4}{16 \pi^2 N^2} = g_4^2 \frac{g_0^2}{16 \pi^2}.$$  \hspace{1cm} (17)

We find that our numerical calculations for $m$ and $\kappa$ agree well with the expectation from the low energy effective potential of the continuum theory, Eq. 11. Also, our result for $\lambda$ agrees with the divergent contribution of the continuum theory, Eq. 8. These results should be viewed as parametric relations only, as many effects can modify the order 1 coefficients involved in these expressions. For example, the operators generated from UV physics that will be discuss next can have important effects, and the running of the couplings can change the naive relation between the high energy coupling $g_0(\Lambda)$ and $g_4$.

On the continuum side, the $\text{Tr} F_{56}^2$ term also contains $(\partial_5 A_6 - \partial_6 A_5)^2$ which should correspond on the deconstructed side to a tower of massive scalars which should also appear in the finite one loop Coleman-Weinberg potential. In Fig. 6 we show the masses of the scalars of the theory, generated by the one-loop Coleman-Weinberg potential which was computed numerically for $N = 20$. We find that a tower of massive modes is indeed generated. We also show the first 4 massive modes for $N = 40$ compared to the KK spectrum of a continuum theory. They are in reasonable agreement, given the relatively low value of $N$. 

8
Until now we have talked only about radiative corrections coming from infrared physics. There are also UV contributions that are not calculable, but can be estimated from spurion analysis and naive dimensional analysis. We now discuss those contributions that could potentially become large as we take the limit of strong coupling. Plaquette operators will be generated with quadratically divergent coefficient at four loops. Using $P_{ij} = U_{(i,j)}V_{(i+1,j)}U_{(i+1,j+1)}^\dagger V_{(i,j+1)}^\dagger$ as the $(i,j)$th plaquette, the effective action contains a term:

$$\delta \lambda_{ij} | Tr P_{ij}|^2 \sim 16\pi^2 f^4 \left( \frac{g_0^2}{16\pi^2} \right)^4 | Tr P_{ij}|^2$$

This results in an contribution to the quartic coupling of:

$$\delta \lambda \sim \frac{16\pi^2}{N^2} \left( \frac{g_0^2}{16\pi^2} \right)^4 = g_4^2 \left( \frac{g_0^2}{16\pi^2} \right)^3$$

where we have taken into account a factor of $(N f)^{-4}$ for the normalization of the zero modes and the fact that there are $N^2$ plaquettes. This quadratic divergence has the same $N$ scaling as the infrared contribution, and is roughly of the same size as $g_0 \to 4\pi$, though it is suppressed for $g_0 \lesssim 4\pi$. Therefore we can not calculate the exact coefficient of the quartic coupling, only say that it is not natural for it to be small.

There are also quadratically divergent $N$ loops contributions to the zero mode mass from

Figure 6: The plot on the left shows the masses of the (non zero mode) scalars, computed numerically for $N = 20$. The right-hand side plot shows a comparison between the first 4 massive modes of the deconstructed theory for $N = 40$ (on the left) and a continuum KK spectrum (on the right).
“Wilson loop operators”:

\[ \Lambda^2 f^2 \left( \frac{g_0^2}{16\pi^2} \right)^N \left| \text{Tr} \ U_{(1,1)} U_{(2,1)} \cdots U_{(N,1)} \right|^2 \]  

From the extra dimensional point of view, because they are non local, these operators should be exponentially small in the size of the extra dimension. In the deconstructed picture, for \( g_0 \lesssim 4\pi \) they are also exponentially suppressed. For any value of \( g_0 \) close to \( 4\pi \), we can increase \( N \) so that the contribution of Eq. 18 is arbitrarily small while the quartic couplings Eq. 18 and Eq. 8 take the appropriate value for a six dimensional theory. More generally, if we take \( g_0 = 4\pi/(1 + \epsilon) \), operators that stretch over \( n \) sites (“large plaquettes”), and correspond to non-local operators will have a size \( \sim e^{-4n\epsilon} \), while the local operators Eq. 18 and Eq. 8 will have size \( g_0^2(1-4\epsilon) \) and \( g_0^2(1-2\epsilon) \), which is what is expected for local operators of a six dimensional theory. This shows that, parametrically, we generate with appropriate coefficients the operators that we expect from a local six dimensional theory with departure from Lorentz invariance of order \( \epsilon \), while the operators that are non-local over distances larger than \( \sim a/\epsilon \) are exponentially suppressed. Therefore, for \( g_0 \) near \( 4\pi \), we generate a theory that parametrically looks like a six dimensional gauge theory with departure from Lorentz invariance of order \( \epsilon \), that is local at distance scales larger than \( \sim a/\epsilon \) and smaller than \( R \). We can then take \( N \) large, increasing the radius, so that there is always a range in which the theory looks like a local six dimensional theory.

The question of whether or not this can be done for realistic theory, where \( g_4 \) is not too small (and therefore \( N \) not too big) cannot really be addressed. As we already mentioned earlier, the size we derived for the various radiative corrections are to be viewed as parametric relations only. Order one coefficients for the operators generated in the UV cannot be predicted while even the computation of infrared effect can be modified, as the naive relation between \( g_0(\Lambda) \) and \( g_4 \) is affected by “power law” running of \( g_0 \).

### 5.1 Comparison with Continuum Calculation

We have shown that infrared effects in the deconstructed theory generate the quartic potentials in the large \( N \) limit with a suitably large coefficient. In the previous section we showed that the quartic coupling arose as a quadratically divergent contribution in the continuum limit. One might worry that this is some sort of contradiction. In the limit we are taking, \( g_0 \to 4\pi \), the high energy regime where the theory looks four dimensional shrinks and disappears. Referring to Fig. 3, we see that in this limit the theory looks six dimensional up to the cutoff at the scale of strong coupling \( \Lambda \). The infrared contributions to the quartic couplings arise from the appearance of “Kaluza-Klein” bosons in the deconstructed theory. The quadratic divergence in the continuum theory also arise because of the increasing number of Kaluza-Klein gauge bosons. Therefore, the contributions to the quartic potential have the same origin in both the deconstructed picture and the continuum picture.
6 Conclusion

In this paper, we have demonstrated that it is possible to dynamically generate six dimensional gauge theories from four dimensional asymptotically free gauge theory. This generalizes the construction of [13]. The new element in deconstructing six dimensional (as well as higher dimensional) theories is the presence of the $\text{Tr} F_{56}^2$ term which generate a KK spectrum for $A_5$ and $A_6$ and contains quartic interactions between higher dimensional components of the gauge field, which translate in the deconstructed theory into quartic interactions between the pseudo-Goldstone bosons. This term was shown to be generated naturally with appropriate coefficient in deconstructed theories when we take the limit of strong coupling. It arise not from operators in the ultraviolet completion of the theories, but through infrared effects generated in the regime where the theory looks extra dimensional. This allows us to consider QCD-like theories as ultraviolet completions for six dimensional theories, as well as higher dimensional theories. We note however that the continuum theory that emerges is not exactly Lorentz invariant as we have no control on the exact coefficient with which $\text{Tr} F_{56}^2$ is generated.

We have shown this result from two different but equivalent perspectives. From the point of view of the 6 dimensional continuum theory, $\text{Tr} F_{56}^2$ is generated with quadratic divergences that are regulated by the deconstructed theory at the scale of the lattice spacing. As we take the high energy gauge coupling to be large, the coefficient of this term becomes of the right order. From the perspective of the deconstructed theory, $\text{Tr} F_{56}^2$ contains a $[u,v]^2$ term which is generated by finite one loop effect and is seen from numerical calculation to agree with the continuum estimate. We have also seen that a tower of massive scalars is generated by finite one loop effect, corresponding to the KK tower of $A_5$ and $A_6$.

Finally, we have shown that operators of the deconstructed theory that stretch in theory space and that correspond to non-local operators in the continuum description, can be kept small, even as we approach strong coupling, by making $N$ large. How large exactly it can be is however a question that we cannot answer since it would require precise matching to the strongly coupled theory.

One application of this result, is the possibility of having QCD-like UV completion of little Higgs model based on theory space [18, 19]. In these theories, the Higgs are the zero mode $u, v$ and they get their quartic coupling from plaquette potential. The success of these models rely on the fact that the Higgs get a small mass from radiative corrections and large quartic couplings from the plaquette that are put in at tree level. We can build realistic model where the Higgs is a fundamental of $SU(2)$ by using a moose made out of $SU(3)$ sites except one which is taken to be $SU(2) \times U(1)$. The low energy gauge group is then $SU(2) \times U(1)$, and the zero mode scalar that were in adjoint of $SU(3)$ decompose into doublets, triplets and singlets of $SU(2)$.

Generating the plaquette potential was seen as an obstacle to completing the little Higgs theories with strongly coupled dynamics. But we have shown in this paper that in the appropriate limit $g_0 \to 4\pi$, a quartic coupling of order $\lambda \sim 16\pi^2/N^2 \sim g_4^2$ is generated while the mass of the Higgs is given parametrically by $m_{\text{Higgs}}^2 \sim (g_4^2/16\pi^2)^2 \Lambda^2$. Note, that
the quartic coupling for the Higgs is of order $g^2$ much as in the MSSM and it can be kept small as we make $g_0$ large by taking the number of sites $N$ large. We should note that there are other ways of generating quartic couplings for little Higgs, either through Yukawa interactions in theory space models \cite{25, 26} or through gauge dynamics in coset models \cite{27}. We haven’t discuss how Yukawa couplings could be generated from strong dynamics. At low energy, introducing Yukawa interactions that do no spoil the ultra-violet insensitivity of the Higgs mass (see \cite{18, 19, 26}) requires the generation of operators coupling link fields and fermions. They could for example arise from four fermion operators as in ETC models. This point offers interesting avenues of investigations.

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