A Cognitive Analysis of Students’ Mathematical Communication Ability on Geometry

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Abstract. This study aims to analyze the difficulties of mathematical communication ability of students in one of secondary school on “three-dimensional space” topic. This research conducted by using quantitative approach with descriptive method. The population in this research was all students of that school and the sample was thirty students that was chosen by purposive sampling technique. Data of mathematical communication were collected through essay test. Furthermore, the data were analyzed with a descriptive way. The results of this study indicate that the percentage of achievement of student mathematical communication indicators as follows 1) Stating a situation, ideas, and mathematical correlation into images, graphics, or algebraic expressions is 35%; 2) Stating daily experience into a mathematical language / symbol, or a mathematical model is 35%; and 3) Associating images or diagrams into mathematical ideas is 53.3%. Based on the percentage of achievement on each indicator, it can be concluded that the level of achievement of students’ mathematical communication ability is still low. It can be caused the students were not used to convey or write their mathematical ideas systematically. Therefore students’ mathematical communication ability need to be improved.

1. Introduction
Mathematics learning in schools plays an important role to prepare students to have cognitive abilities such as problem solving, reasoning, mathematical communication, and critical thinking through improving the quality of learning in order to face global developments. This is in line with the formulation of the objectives of mathematics learning namely: a) learning to communicate (mathematical communication); b) learning to reason (mathematical reasoning); c) learning to solve problems (mathematical problem solving); d) learning to associate ideas (mathematical connections); and e) learning to present ideas (mathematical representation) [1]. The other objectives of mathematics learning are also as follows: (1) understanding the mathematical concepts, explaining the correlation interconcept and applying the concept or algorithm flexibly, accurately, efficiently and appropriately in problem solving, (2) solving problems, and 3) communicate ideas with symbols, tables, diagrams, or other media to clarify circumstances or problems [2]. Based on the purpose of learning mathematics, the ability of mathematical communication is one of the cognitive abilities that are required to be mastered by students.

The ability of mathematical communication is a very important part in mathematics and mathematics education and it is the way that students share ideas or opinions and to clarify understanding [1]. This is supported that the reason for the importance of communication in mathematics learning is that mathematics is not only a tool for thinking, finding patterns, or solving
problems, but also as a tool to communicate various idea clearly, precisely, and summarily, as well as a vehicle interaction between the students and students with teachers [3].

Communication is the process of sending information, ideas, emotions, abilities, and others through the use of symbols like words, pictures, numbers, and so forth [4]. The definition of mathematical communication capability is the ability of a person to write mathematical statements, to write reasons or explanations of every mathematical argument that he uses to solve mathematical problems, using terms, tables, diagrams, notations, or mathematical formulas appropriately, and examine or evaluate the others mathematical thoughts [5]. Armed with these abilities, the students will be easier to convey mathematical ideas, so that the mathematical difficulties faced by the students can be easily understood by themselves and others.

The ability of this mathematical communication can take the form of oral and written. Oral skills for example, can be seen when students express ideas and thoughts about mathematics to friends or teachers in the classroom. While the communication of writing can be the use of tables, drawings, diagrams, or other mathematical symbols that he stated in writing about a mathematical problem. With mathematical communication, students will be easier in solving mathematical problems that he encountered because of mathematical communication ability supporting other math abilities.

However, in fact the students' mathematical communication ability is still not as expected. Some studies have found poor students' mathematical communication ability. A study found that in the case of mathematical communication ability to determine the volume of water in the bathtub, among 37 students who participated, only four students answered almost correctly, while the other students answered incorrectly and there were four students who did not answer the question [6]. Most students are less able to propose an idea or a mathematical situation into images, symbols or mathematical models and solve them. In addition, another study also found that the mathematical communication ability of grade VIII students is still low, that is the average of only about 12% of the class only students who can complete the mathematical communication tests provided [7]. The result of other study in the preliminary observation also found that in general the mathematical communication ability of grade VII students is still low, as evidenced by: a) The ability of students in conveying the mathematical idea about the concept of the volume space is still worse; b) the students' ability to use mathematical symbols / notations of the concept of spatial volume is not accurate; and c) the student's ability to explain the building of a three-dimensional space into a contextual and appropriate description is lacking, such as determining the diagonal of the sides, the diagonal of the chamber, and the diagonal plane of the pencil box in the shape of a beam [8]. The results of this study indicate that many students do not have good mathematical communication ability. Therefore, in this study researcher want to analyze the answers of students in answering the questions on the matter of three-dimensional space to find out what kind of mistakes students made in answering mathematical communication test.

There are several indicators of mathematical communication ability such as: a) expressing ideas or opinions, situations and mathematical relations both orally and in writing, and describing them visually in the form of real objects, images, graphs, and algebraic expressions; b) represents real objects, images, and diagrams in the form of mathematic ideas and / or mathematical symbols; c) declare or create mathematical models of daily events or other mathematical problems in the language of mathematical symbols, terms, and structures; d) construct conjectures, arguments, formulate definitions and generalizations; e) reexpress a mathematical description or paragraph in her/his own language [9],[1]. As in this research used some indicators of mathematical communication as follow:

a. Stating a situation, ideas, and mathematic correlation into images, graphics, or algebraic expressions.
b. Stating daily experience into a mathematic language / symbol, or a mathematic model.
c. Associating images or diagrams into mathematic ideas.

2. Experimental Method
This research conducted by using quantitative approach with descriptive. The population in this research was all students in one of secondary school and the sample was thirty students that was chosen by purposive sampling technique. Data of mathematical communication were collected
through essay test for 40 minutes duration. Topic of the essay test was Three-dimensional space. Furthermore, the data were analyzed with a descriptive way.

3. Result and Discussion

The results of this study indicate the percentage of student mathematical communication indicators achievement as following: 1) Stating a situation, idea, and mathematical relation into the form of images, graphics, or algebraic expressions is 35%; 2) Stating daily experience into a mathematic language / symbol, or a mathematic model is 35%; and 3) Associating images or diagrams into mathematical ideas is 53.3%. Based on the percentage of achievement on each indicator, it can be concluded that the level of achievement of students’ mathematical communication ability is still low with the average percentage of each indicator is 41.1%. Some students’ answers are analyzed as follows:

1. First communication question, a block has a square base with a circumference of 60 cm and a height of 20 cm. Draw in Cartesian graphic shape how high water in a block if it is filled with water as much as 900 cm$^3$, 1350 cm$^3$, or 2700 cm$^3$.

   Figure 1 shows that the student cannot yet make the right data for the y-axis because it appears that the data on the x-axis and the y-axis are the same. Students should write water volume data on the x-axis and water level data on the y-axis, and write the starting point 0 on the intersection of x-axis and the y-axis of the graph. Similar in Figure 2 it appears that the student only wrote the number 20 on the y-axis and was not supplemented with other data, so that the dashed lines drawn by students was meaningless.

![Figure 1. Student’s answer for communication question indicator 1](image1)

![Figure 2. Student’s answer for communication question indicator 1](image2)

Figure 3 shows that the data written by the student is correct, but the data is not put in its right place. The data on the x-axis should be placed on the y-axis, and vice versa. Because the question is how high is the water level in a block with a certain water volume, then based on the student's answer in Figure 3, the graph shows the volume of water with a certain height inside a beam. This is certainly contrast with the commands in question.
Figure 3. Student’s answer for communication question indicator 1

Figure 4 shows that student create y-axis data, but do not write data on the x-axis. In addition, the student’s drawings are also not a graph although there are x-axis and y-axis. From all the students’ answers, the researcher draws some conclusions: 1) the student does not make the complete data for the x-axis and the y-axis according to the questions needed, including the starting point 0 on the intersection of x-axis and the y-axis; Data should be on the x-axis or y-axis, 3) the student does not write a description on each axis, thus making the reader confuse what the graph is displaying.

Figure 4. Student’s answer for communication question indicator 1

2. The second communication question with the indicator state daily experience into the form of a mathematic language/symbol, or mathematic model, as follows: A pyramid-shaped table decoration has a rectangular base with a circumference of 42 cm. The size of the base of the pyramid is 3/4 times the length of the pyramid base, and the pyramid height is 2/3 the length of the pyramid base. Make a mathematic model of the decoration size to determine the volume of the decoration.

Figure 5 shows that student has written the things that he/she get from the question, but in the last two steps, students make unreasonable mistakes, so that the answers written by the students cannot be understood. The student should simply write the formula to find the volume of pyramid, then substitute the size of the width and height of the pyramid in the length of the line (p).

Figure 5. Student’s answer for communication question indicator 2
Figure 6 shows that there is first step that students make a mathematic model of the question given. The student wants to find the p value first, but he/she does not continue to the end. From the students’ answers, the researcher concludes that students do not get accustomed to make mathematic models of a description question or daily problem that is not yet known directly in size. Students usually face the mathematic questions which have definite information convey in it.

![Figure 6](image_url)

**Figure 6.** Student’s answer for communication question indicator 2

3. The final question is a matter with indicators linking images or diagrams into mathematical ideas. The students were asked to write down their ideas about the volume of that three dimensional space with their own explanations.

Perhatikan gambar bangun ruang berikut.

Hal apa yang dapat kamu simpulkan mengenai volume bangun ruang tersebut?

Figure 7 shows that the student states that the volume of three-dimensional space on the problem is five times the volume of the small cube. Here the researcher judges that the student's answer is closed to true. It's just that the students did not explain in more detail what the size of the small cube in question. If the small cube in question is a cube with a 4-piece rib size, then the student's answer can be accepted. The student's answer would be perfect if he directly wrote the volume for a small cube he meant, i.e. $4^3 = 64$ units of volume, so for the overall volume of three-dimensional space is $5 \times 64 = 320$ units of volume.

![Figure 7](image_url)

**Figure 7.** Student’s answer for communication question indicator 3

Figure 8 shows that the student declares that the three-dimensional space in question consists of three 3D spaces, but he does not mention any three-dimensional space that shape them and the size of each three-dimensional space, so that he can determine the overall volume of the three-dimensional space. Students should be able to write that the space consists of 2 beams of p = 4, l = 4, and t = 8 and 1 cube with 4-piece ribs, or consist of 2 cubes with 4 ribs length and 1 beam with size p = 12, l = 4, and t = 4.
Figure 8. Student’s answer for communication question indicator 3

Figure 9 also shows that student has not been able to express mathematical ideas based on the image given. The student’s answer in Figure 9 shows that he understood the three dimensional space on the question consists of several cubes with a 4-membered rib, but he could not correctly write the overall volume of the three-dimensional space, so the answer that he gave did not match to the supposed answer.

Figure 9. Student’s answer for communication question indicator 3

Based on the student’s answer in Figure 10, the researcher observed that the student wanted to state that the space in question consisted of 2 pieces of beam and 1 cube, so that the volume of the three-dimensional space was matched with the student’s answer. But in the answer, the students did not explain how the sizes of blocks and cubes are based on the question, so he did not get the results of how much the overall volume of that three-dimensional space is based on the question.

Figure 10. Student’s answer for communication question indicator 3

From some students’ answers on problem no.3, researcher draws the conclusion that students have not been able to express the mathematical ideas well from the picture or diagram given. This certainly reflects how far students' understanding of the problem.

The mistakes that students make in answering those questions reflect that students have difficulties in communicating mathematically. This can be caused students are not used to convey ideas or mathematical ideas and write them in the form of graphs, tables, diagrams, or mathematical models of a problem systematically. One effort that can be done by teachers in the classroom to improve students’ mathematical communication is to familiarize the students to openly expose mathematical ideas on the topics being studied, and to create discussion forums so that students can ask each other and listen to each other's answers in the class. This is supported that give attention carefully to the questions of friends in a group can also help students to construct math knowledge more and to set up more effective answer strategies [10]. Furthermore, after the students are used to convey their ideas, teachers should also train students to write the ideas systematically for the expected mathematical communication indicators can be achieved maximally.
4. Conclusion
Based on the results of student achievement and analysis of each indicator of mathematical communication, it can be concluded that students’ mathematical communication ability is still low with average achievement of each indicator is 41.1%. Thus, it is necessary to increase the ability of students’ mathematical communication by developing teacher competence in educating and choosing the appropriate learning approach and model to be applied in class so that students can develop their mathematical communication ability well.

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