Can we build a sensible theory with broken charge and colour
symmetries?

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Abstract: Charge and colour symmetries are broken using the vevs of scalar fields, but it is shown the theory possesses a photon-like massless particle and electroweak interactions almost identical to those of the Standard Model. We consider experimental signatures of this theory.

1 Introduction

In the Standard Model (SM) gauge symmetry breaking is achieved through a non-zero vacuum expectation value (vev) acquired by a scalar field. This field has no colour or electric charge. So, the QCD and electromagnetic symmetries, $SU(3) \times U(1)$, are preserved and we are left with massless gluons and photons. However, in supersymmetric models there are several other scalars, partners of the fermionic fields, which have colour and electric charge. It is then possible that there are minima of the scalar potential with non-zero vevs for some charged or coloured fields. The requirement that the lowest of these minima does not have this type of vev was first explored by Frére et al \cite{1} to constrain the extensive parameter space of supersymmetric models. The study of these charge and colour breaking (CCB) minima was further developed by many authors \cite{2}.

In this article we wish to explore an alternative point of view: is it possible to construct models where charged and coloured fields have non-zero vevs and, at the same time, obtain a massless gauge field that resembles the normal photon? In fact, we build such a model and show that its electroweak sector is almost identical to the SM’s. Nevertheless, the “new” photon and Z boson have also some mixing with the $SU(3)$ gauge fields. In the strong sector, the differences with the SM are larger - five gluons are massive, four of these have electric charge and the quarks have integer electric charge, which depends on their colour. Hence we obtain, in this context, the quark charge assignments of the Han-Nambu model \cite{3}. Models with Han-Nambu quarks have been obtained before from the spontaneous symmetry breaking of gauge groups larger than the SM’s \cite{4}. In our approach, we start we the SM gauge group and the integer charged quarks arise because the colour group $SU(3)$ is broken.

This article is structured as follows: in the next section we build the model and analyse the gauge bosons’ masses, mixings, their interactions with fermions and self-couplings. In section \cite{4} we compare the predictions from this model with the SM ones, with special emphasis put on results from two-photon physics. These will be shown to be better explained by a theory with
integer charged quarks. We close with a general discussion and conclusions.

2 Model building

Our starting point is the gauge group $SU(3) \times SU(2) \times U(1)$ which will be spontaneously broken to $SU(2) \times U(1)$ via the Higgs mechanism.

2.1 Breaking the gauge symmetry

Let us consider a general theory with a gauge group $G$ with $N$ generators $T^a$ and $n$ scalar fields $\phi_i$, each with a vacuum expectation value described by the vector $|v_i>$. Then, as is well known, the covariant derivatives are given by

$$D_\mu \phi_i = \partial_\mu \phi_i + \frac{i}{2} \sum_{a=1}^N g_a T^a A^a_\mu \phi_i,$$  \hspace{1cm} (1)

where $g_a$ is the gauge coupling corresponding to the generator $T^a$ and $A^a_\mu$ its respective gauge field. When the fields $\phi_i$ acquire vevs, $|v_i>$, mass terms for the gauge fields are produced with a mass matrix given by (no sum in $\{a,b\}$)

$$M_{ab}^2 = \frac{g_a g_b}{2} \sum_{i=1}^n <v_i| T^a T^b + T^b T^a |v_i>.$$  \hspace{1cm} (2)

In the SM there is a single $SU(2)$ doublet scalar field. This produces the known spectrum of gauge boson masses: massive $W$’s and $Z^0$, massless gluons and photons.

Let us now consider the Minimal Supersymmetric Standard Model (MSSM). Gauge symmetry breaking is achieved by two colourless Higgs doublets $H_1$ and $H_2$ with vevs $v_1/\sqrt{2}$ and $v_2/\sqrt{2}$ respectively. Among the many possible ways of causing CCB (see ref. [5] for a detailed review) we have chosen the case where the scalar partners of the top quark $\{t_L, t_R\}$ acquire non-zero vevs. The field $t_L$ is an $SU(3)$ triplet which is part of the $SU(2)_L$ doublet $Q_L = (t_L, b_L)$, with hypercharge $y_Q = 1/6$. The field $t_R$ is a $3$ of $SU(3)$ and singlet of $SU(2)_L$ with hypercharge $y_t = -2/3$. We denote by $q/\sqrt{2}$ and $t/\sqrt{2}$ the vevs of $t_L$ and $t_R$ respectively, both with colour index $3$. This means we are effectively breaking the colour symmetry along a particular direction. In the Standard Model these vevs would have an electric charge $\pm 2/3$ and thus, in principle, the electromagnetic gauge invariance would be broken. However we will see that this is not the case and the model still maintains an unbroken $U(1)$ subgroup.

Because we are considering vevs carrying a particular colour index, it is convenient to alter slightly the usual notation: let $Q_L$ be written as

$$Q_L = \begin{pmatrix} t^1_L \\ t^2_L \\ t^3_L \\ b^1_L \\ b^2_L \\ b^3_L \end{pmatrix} \quad \text{vacuum} \rightarrow \begin{pmatrix} 0 \\ 0 \\ q/\sqrt{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$  \hspace{1cm} (3)
We write the $Q_L$ covariant derivative as
\[ D_\mu Q_L = \partial_\mu Q_L + i g' Y B_\mu + \frac{i}{2} g \sigma'_i Q_L W^i_\mu + \frac{i}{2} g_3 \lambda'_a Q_L G^a_\mu, \quad (4) \]
where $\{g', g_3\}$ are the $U(1)$, $SU(2)$ and $SU(3)$ gauge couplings and $\{B_\mu, W^i_\mu, G^a_\mu\} (i = 1, 2, 3, a = 1, \ldots 8)$ are their respective gauge fields. In this representation the hypercharge matrix $Y$ is, for a field of hypercharge $y$, given by $Y = y \mathbb{1}_6$, and the generalised Pauli and Gell-Mann matrices $\sigma'_i, \lambda'_a$ are
\[ \sigma'_1 = \begin{pmatrix} 0 & 1 \bar{3} \\ 1 \bar{3} & 0 \end{pmatrix}, \quad \sigma'_2 = \begin{pmatrix} 0 & -i \bar{3} \\ i \bar{3} & 0 \end{pmatrix}, \quad \sigma'_3 = \begin{pmatrix} 1 \bar{3} & 0 \\ 0 & 1 \bar{3} \end{pmatrix}, \quad \lambda'_a = \begin{pmatrix} \lambda_a & 0 \\ 0 & \lambda_a \end{pmatrix}, \quad (5) \]
with $\mathbb{1}_n$ the $n \times n$ unit matrix and $\lambda_a$ the usual Gell-Mann matrices. Likewise, for the field $t_R$, we have
\[ t_R = \begin{pmatrix} t_1^R \\ t_2^R \\ t_3^R \end{pmatrix}, \quad \text{vacuum} \rightarrow \begin{pmatrix} 0 \\ 0 \\ t/\sqrt{2} \end{pmatrix}, \quad (6) \]
and its covariant derivative has an expression similar to eq. (4), without the $g$ term and with the matrices $\lambda_a$ replaced by $-\lambda_*^a$. At this point we can use eq. (2) and write down the gauge bosons’ mass matrices. Since the Gell-Mann matrices $\{\lambda_1, \lambda_2, \lambda_3\}$ have zeros in their third row and column the gluons $G^i_\mu, G^2_\mu$ and $G^3_\mu$ remain massless. On the other hand $G^i_\mu$ to $G^7_\mu$ acquire a mass given by
\[ M^2_{G_{4\ldots7}} = \frac{1}{4} g^3_3 (q^2 + t^2). \quad (7) \]
Our choice of colour breaking implies that there is no mixing among any of these seven fields, or between them and the electroweak gauge fields. Like in the SM the fields $W^i_\mu$ and $W^2_\mu$ mix to form the charged weak bosons $W^{\pm}$ with mass given by
\[ M^2_{W^\pm} = \frac{1}{4} g^2 (v^2 + q^2), \quad (8) \]
with $v^2 = v_1^2 + v_2^2$. Finally the only non-trivial mass matrix is the one involving the gauge bosons $W^3_\mu, B_\mu$ and $G^8_\mu$, namely $^1$.
\[ [M^2_{G^8}] = \begin{pmatrix} \frac{g^2}{4} (v^2 + q^2) & -\frac{g g'}{4} (v^2 - 2 y Q q^2) & -\frac{g g_3}{2 \sqrt{3}} q^2 \\ -\frac{g g'}{4} (v^2 - 2 y Q q^2) & \frac{g^2}{4} (v^2 + 4 y Q q^2 + 4 y t^2) & \frac{g g_3}{\sqrt{3}} (y Q q^2 - y t^2) \\ -\frac{g g_3}{2 \sqrt{3}} q^2 & \frac{g g_3}{\sqrt{3}} (y Q q^2 - y t^2) & \frac{g^2}{3} (q^2 + t^2) \end{pmatrix}. \quad (9) \]
The determinant of this matrix is
\[ \det[M^2_{G^8}] = \frac{1}{12} \left(y Q + y t + \frac{1}{2}\right)^2 g^2 g^2 g^2 g^2 v^2 q^2 t^2. \quad (10) \]
If either $q$ or $t$ are zero we immediately have a massless neutral gauge boson. Obviously, $q = t = 0$ is the SM case. However, it is interesting to point out that the quantum numbers of $\{t_L, t_R\}$ in
\[ ^1 \text{This study is valid for the case of } m \text{ scalar higgs-like doublets each with a different vev } v_i, \text{ simply replacing } v^2 \text{ by } \sum_{i=1}^{m} v_i^2 \text{ in the formulae.} \]
the MSSM are exactly such that this determinant is zero - with the values of the hypercharges given before we have \( y_Q + y_t = -1/2 \). The relevance of obtaining a massless particle will be evident in the next chapter, but we can anticipate that, despite the fact it results from mixing with a gluon, this particle has interactions that make it virtually identical to the photon.

The eigenvalues of matrix (11) are 0 and \((A \pm \sqrt{A^2 - 4B})/2\), with

\[
A = \frac{g^2 + g^2}{4} v^2 + \left( \frac{g^2}{4} + g^2 y_Q + \frac{g_3}{3} \right) q^2 + \left[ \left( y_Q + \frac{1}{2} \right)^2 g^2 + \frac{g^2}{3} \right] t^2
\]

\[
B = \frac{1}{4} \left[ \frac{g^2}{3} (g^2 + 2g^2) + \frac{1}{4} \left( y_Q + \frac{1}{2} \right)^2 g^2 q^2 \right] [q^2 t^2 + v^2 (q^2 + t^2)] . \tag{11}
\]

We want to identify one of these massive eigenvalues with the \( Z \) boson, denoted by \( \tilde{Z} \). The other one which, for physical reasons to be explained shortly, will be heavier than the \( Z \), we call \( \tilde{G} \). The eigenstates \( \tilde{A}_\mu \), corresponding to the zero eigenvalue, \( \tilde{Z}_\mu \) and \( \tilde{G}_\mu \) are given by the unitary transformation

\[
\tilde{A}_\mu = a_1 B_\mu + b_1 W^3_\mu + c_1 G^s_\mu
\]

\[
\tilde{Z}_\mu = a_2 B_\mu + b_2 W^3_\mu + c_2 G^s_\mu
\]

\[
\tilde{G}_\mu = a_3 B_\mu + b_3 W^3_\mu + c_3 G^s_\mu , \tag{12}
\]

with

\[
\frac{b_1}{a_1} = \frac{g'}{g}
\]

\[
\frac{c_1}{a_1} = \frac{\sqrt{3}}{2} \frac{g'}{g_3} (2y_Q + 1)
\]

\[
\frac{b_{2,3}}{a_{2,3}} = -\frac{g}{g'} \frac{(2y_Q + 1) [2y_Q g^2 + (2y_Q + 1) t^2] g^2 \frac{4}{3} g_3^2 (q^2 + t^2) - 4 m_{2,3}^2}{(2y_Q + 1) g^2 q^2 + \frac{4}{3} g_3^2 (q^2 + t^2) - 4 m_{2,3}^2}
\]

\[
\frac{c_{2,3}}{a_{2,3}} = \frac{2 \sqrt{3}}{3} \frac{g_3}{g'} (2y_Q + 1) g^2 q^2 + \frac{4}{3} g_3^2 (q^2 + t^2) - 4 m_{2,3}^2 . \tag{13}
\]

Notice that the coefficients corresponding to the massless field depend only on the gauge couplings, as opposed to the remaining two, which depend on the vevs. Clearly the SM limit cannot be obtained from eq. (13) by taking the limit \( q \to 0 \), \( t \to 0 \). In fact in this limit the initial matrix (11) has a two-fold degenerate zero eigenvalue.

### 2.2 Gauge interactions of fermions

We obtain the gauge interactions of a fermionic field \( F \) by replacing, in the free lagrangean, the partial derivatives by covariant ones, so that

\[
\bar{F} \sigma \gamma^\mu F \rightarrow \bar{F} \sigma \gamma^\mu F + i g' y_F \bar{F} F B + i \frac{g}{2} \bar{F} \sigma_i F W^i + i \frac{g_3}{2} \bar{F} \lambda_a F G^a , \tag{14}
\]

where \( y_F \) is the field’s hypercharge. If the field is an \( SU(2) \) (\( SU(3) \)) singlet the \( g \) (\( g_3 \)) term is not present. The expression above is written for the case \( F \) is a triplet of \( SU(3) \), for a field in the 3 representation we would have to replace \( \lambda_a \) by \(-\lambda_a^* \), as we observed earlier for the scalar fields. Already we observe that the interactions of the \( W \)’s remain unchanged - these fields do not mix
with others, so we obtain here the same vertices as in the SM (we have seen the W mass is now given by a different expression, but that does not affect the form of the vertices). Likewise the first seven gluons have the same gauge interactions they did previously - again they do not mix with any other field, and the fact that four of them now have mass does not affect their vertices. Where the differences with the SM appear are in the interactions of the “photon”, “Z0” and the eighth gluon - for the case F is a left lepton doublet L = (νeL , eL) (of hypercharge 1/2; we wrote down that of the first lepton family, but this calculation is valid for the remaining two) or a right singlet, eR (of hypercharge −1, if we are thinking of electrons), the lepton’s gauge interactions will be

\[ \nu_{eL} : - \frac{1}{2} (\bar{\nu}_e \gamma^\mu \nu_e)_L (g' B_\mu - g W_\mu^3) \]

\[ e_L : - \frac{1}{2} (\bar{e} \gamma^\mu e)_L (g' B_\mu + g W_\mu^3) \]

\[ e_R : - g' B_\mu . \]

(15)

Now, we can invert eq. (12) so that

\[ B_\mu = a_1 \tilde{A}_\mu + a_2 \tilde{Z}_\mu + a_3 \tilde{G}_\mu \]

\[ W_\mu^3 = b_1 \tilde{A}_\mu + b_2 \tilde{Z}_\mu + b_3 \tilde{G}_\mu \]

\[ G^3_\mu = c_1 \tilde{A}_\mu + c_2 \tilde{Z}_\mu + c_3 \tilde{G}_\mu , \]

and so the couplings of the fields \( \tilde{A}, \tilde{Z}, \tilde{G} \) to leptons are given by

\[ \nu_{eL} : - \frac{1}{2} g a_i (g' / g - b_i / a_i) \]

\[ e_L : - \frac{1}{2} g a_i (g' / g + b_i / a_i) \]

\[ e_R : - g' a_i . \]

(16)

For quarks the situation is more complex, we must take into account their colour and the fact that the gauge fields will interact differently with them based on it. So we have, for instance, for an up-type left quark (hypercharge 1/6),

\[ \bar{u}_L \phi u_L \rightarrow \left( \frac{g'}{6} B + \frac{g}{2} W^3 \right) (\bar{u}_L^1 u_L^1 + \bar{u}_L^2 u_L^2 + \bar{u}_L^3 u_L^3) + \frac{g_3}{2 \sqrt{3}} (\bar{u}_L^1 u_L^1 + \bar{u}_L^2 u_L^2 - 2 \bar{u}_L^3 u_L^3) G^8 . \]

(17)

In the \( g_3 \) term we see that the gauge interactions with the quarks will depend on their colours, even for the photon and Z0! Thus, for the interactions between \( \tilde{A}, \tilde{Z}, \tilde{G} \) and the quarks we will have

\[ u_L : g' a_i \left[ \frac{1}{6} + \frac{1}{2} \frac{b_i}{a_i} g' + \frac{g_3}{2 \sqrt{3} g'} \frac{c_i}{a_i} \left( \begin{array}{c} 1 \\ 1 \\ -2 \end{array} \right) \right] \]

\[ u_R : g' a_i \left[ \frac{2}{3} + \frac{g_3}{2 \sqrt{3} g'} \frac{c_i}{a_i} \left( \begin{array}{c} 1 \\ 1 \\ -2 \end{array} \right) \right] \]

\[ d_L : g' a_i \left[ \frac{1}{6} - \frac{b_i}{2 a_i} g' + \frac{g_3}{2 \sqrt{3} g'} \frac{c_i}{a_i} \left( \begin{array}{c} 1 \\ 1 \\ -2 \end{array} \right) \right] \]

\[ d_R : g' a_i \left[ -\frac{1}{3} + \frac{g_3}{2 \sqrt{3} g'} \frac{c_i}{a_i} \left( \begin{array}{c} 1 \\ 1 \\ -2 \end{array} \right) \right] , \]

(18)
where we used an obvious notation to indicate the \( g_3 \) terms (and them alone) act differently on the quarks, depending on their colours. It would seem all we have done up to this point does not require supersymmetry at all - could we not obtain the same results by adding scalar fields with the quantum numbers of \( Q_L \) and \( t_R \) to the SM? The answer is no as such a theory would have mixing terms between quarks and leptons. For instance, \( t_R \bar{Q}_L \bar{L}_c \) - the tilde denotes fermionic fields, \( \bar{L} \) is the lepton fermionic doublet, \( \bar{L} = (\bar{\nu}_L, \bar{\tau}_L) \) (for the third generation). This term involves the charge conjugate of \( \bar{L} \). It does not occur in a supersymmetric theory because one cannot build SUSY lagrangians using simultaneously a field and its charge conjugate. This is the same argument that implies the existence of a minimum of two Higgs doublets in the MSSM. Remember that the SUSY Yukawa term involving the leptonic superfield \( \bar{L} \) - which contains \( \bar{\nu}_L \) - is

\[
\lambda \tau_L H_1 \tau_R^r \quad \text{(with } y_L = -1/2, y_{\tau R} = +1). 
\]

Supersymmetry is thus necessary if we want to avoid mixing between quarks and leptons. However, another type of mixing occurs, between the third colour component of the bottom quark and the charginos, and between the top quark, the neutralinos and the eighth gluino. Such mixings are common when CCB occurs (see for instance [6] for mixing between \( \chi^\pm, \chi^0 \) and the leptonic sector) and would in principle affect the electromagnetic interactions of the third quark generation. At the very least such mixing could be used to impose constraints on the SUSY parameter space. We hope to address these questions in a forthcoming paper [7]. Another reason to work in the framework of supersymmetry is the well-known fact that adding scalars to the SM usually spoils asymptotic freedom, as the new fields have positive contributions to the \( \beta \)-function of the strong coupling constant. In a supersymmetric theory, however, the field content is such that asymptotic freedom is preserved so that, above the energy scale CCB occurs, the strong coupling runs with a supersymmetric \( SU(3) \) \( \beta \)-function.

### 2.3 The photon

Let us now apply these formulae to the massless field \( \tilde{A}_\mu \), using the values for \( \{a_1, b_1, c_1\} \) given in eq. (13): because \( b_1/a_1 = g'/g \), we see from eqs. (14) that \( \tilde{A} \) does not couple to the neutrinos. Plus, from the same expressions we see that it does couple identically with the left and right electrons. That is a further incentive to identify \( \tilde{A} \) with the photon - the value of its coupling with the electrons must therefore be the electric charge \( e \), that is, \( g' a_1 = e \). With the results from (13) we can therefore write

\[
a_1 = \frac{g}{\sqrt{g^2 + g'^2 + x^2}} \quad \text{with } x = \frac{2 g g'}{\sqrt{3} g_3}. \tag{19}
\]

We see \( a_1 \) is a generalisation of the cosine of the Weinberg angle, involving the \( SU(3) \) coupling as well. As for the couplings of \( \tilde{A} \) with the quarks we obtain, from eqs. (15) and (13),

\[
\begin{align*}
  u_L, u_R : & \quad e \left[ 2 + \left( \begin{array}{c} 1 \\ -2 \end{array} \right) \right] \\
  d_L, d_R : & \quad e \left[ -1 + \left( \begin{array}{c} 1 \\ -2 \end{array} \right) \right].
\end{align*} \tag{20}
\]

If \( \tilde{A} \) is indeed the photon then eq. (20) is an expression of a very interesting phenomenon: the electric charge of the quarks depends on their colour. In particular, for up-type quarks, the two first colours have charge +1 and the third is neutral. For down-type quarks the situation is reversed: \( d^1, d^2 \) are chargeless and \( d^3 \) has charge -1. We can resume these results in group theory...
terms, by noting that in this theory with broken colour and charge the electric charge operator is no longer \( Q = T_3 + Y \) but rather
\[
Q = T_3 + Y + C_8,
\]
with \( C_8 \) the “eighth colour”, \( C_8 = \lambda_8/\sqrt{3} \). Notice how this new definition of the electric charge does not affect the charge assignments of the leptons - they are colourless and thus the \( \lambda_8 \) term does not come into play. Perhaps more interestingly, it does away with fractional electric charges altogether: the quarks’ charges depend on their colours but are integer numbers. We recognise these as being identical to Han-Nambu quarks \([3, 4]\) even though they are being obtained in a different context.

2.4 The \( Z^0 \)

So far we have been able to ignore the values of the vevs in the theory, because the coefficients \( \{a_1, b_1, c_1\} \) do not depend on them. But trying to identify one of the two massive eigenstates as the \( Z^0 \) will require the determination of \( \{v, q, t\} \). There are two possibilities, the \( Z^0 \) can be the lighter or heavier of the two states. For reasons to be addressed in the next section it is more reasonable to assume \( m_\tilde{Z} < m_\tilde{G} \) and that is the case we discuss. The results for the other possibility are not qualitatively different from those presented here. From eqs. (16) we can write the couplings of \( \tilde{Z} \) to the leptons as
\[
\begin{align*}
g_{Z\nu} &= \frac{1}{2} (gb_2 - g'a_2) \\
g_{ZV} &= -\frac{1}{4} (gb_2 + 3g'a_2) \\
g_{ZA} &= \frac{1}{4} (gb_2 - g'a_2)
\end{align*}
\]
(22)
where we have split the coupling of the \( \tilde{Z} \) to charged leptons in an axial coupling (the terms that multiply the \( \gamma^5 \) matrix) and a vector coupling, as is usual. We immediately see that \( g_{ZA} = g_{Z\nu}/2 \). In the SM at tree level we have an identical expression, \( g^{SM}_{ZA} = g^{SM}_{Z\nu}/2 \). In terms of the unit electric charge \( e \), the \( W \) and \( Z \) masses \( M_W, M_Z \), we have
\[
\begin{align*}
g^{SM}_{Z\nu} &= \frac{e M_Z^2}{2 M_W \sqrt{M_Z^2 - M_W^2}} \\
g^{SM}_{ZV} &= \frac{e (3 M_Z^2 - 4 M_W^2)}{4 M_W \sqrt{M_Z^2 - M_W^2}}
\end{align*}
\]
(23)
So the \( \tilde{Z} \) has leptonic couplings identical to those of the \( Z^0 \) provided we have
\[
\begin{align*}
a_2 &= -\frac{e}{g'} \frac{\sqrt{M_Z^2 - M_W^2}}{M_W} \\
b_2 &= \frac{e}{g} \frac{M_W}{\sqrt{M_Z^2 - M_W^2}}
\end{align*}
\]
(24)
In the SM these formulae reduce to the known results \( -\sin\theta_W, \cos\theta_W \) - here, due to eq. (19) it is no longer true that \( a_2^2 + b_2^2 = 1 \). At this point we can determine \( \{v, q, t\} \) by using as inputs the experimental values of \( \{e, g_3, M_W, M_Z\} \). For given values of the vevs \( g \) is given by \( g^2 = 4 M_W^2/(v^2 + q^2) \), \( g' \) is determined by eq. (19) and from eq. (17) we calculate \( m_\tilde{Z} \). Following
a simple minimisation procedure we can choose the vevs so that the leptonic couplings of the \( \tilde{Z} \) are identical to those of the \( Z \), requiring conditions (24). This procedure \(^2\) gives values of \( m_\tilde{Z} \) that, in the vast majority of cases, verify \( |m_\tilde{Z} - M_Z| < 10^{-3} \) GeV. The minimisation process is dependent on the initial guess for the \( t \) vev - for the same values of \( \{e, g_3, M_W, M_Z\} \) different solutions \( \{v, q, t\} \) may be found. However, they all share a common trait, the fact that \( \sqrt{v^2 + q^2} \simeq 246 \) GeV, as may be seen in fig. (1). The points in this plot correspond to solutions obtained with different initial guesses for \( t \) (from 100 to 500 GeV in steps of 100), with random values of \( \{e, g_3, M_W, M_Z\} \) taken in the intervals \( 0.3093 \leq e \leq 0.3173, 1.063 \leq g_3 \leq 1.1757, 80.39 \leq M_W \leq 80.47 \) GeV, \( 91.18 \leq M_Z \leq 91.20 \) GeV. These intervals correspond to the current experimental values and uncertainties for these quantities (at the scale \( M_Z \) for the couplings).

Usually the larger the initial guess for \( t \) the larger the number of solutions found (for the largest value tried, \( t_{\text{initial}} = 2000 \) GeV, about 87% of the parameter space considered yielded a solution. We emphasise this is \textit{not} an exhaustive study of all possibilities involved, but the parameter space we considered is realistic and the solutions found, credible. It should also be noted that the difference between the quantity plotted in fig. (1), \( \sqrt{v^2 + q^2} \), and the SM vev,

\[
v^{SM} = \frac{2 M_W}{e M_Z} \sqrt{M^2_Z - M^2_W},
\]

is smaller than \( 10^{-3} \) GeV for all the points considered. This is reassuring since this quantity is experimentally fixed by the lifetime of the muon (at tree level at least). This also implies the coupling \( g \) is the same in our theory and in the SM (not so for \( g' \), though).

Having chosen \( \{v, q, t\} \) to “fix” the leptonic couplings we now look at the couplings of \( \tilde{Z} \) to quarks. In the SM we have

\[
\begin{align*}
g^{SM}_{Zv_a} &= \frac{e}{12} \frac{8 M^2_W - 5 M^2_{\tilde{Z}}}{M_W \sqrt{M^2_Z - M^2_W}} \\
g^{SM}_{Zv_d} &= -\frac{e}{12} \frac{4 M^2_W - M^2_{\tilde{Z}}}{M_W \sqrt{M^2_Z - M^2_W}} \\
g^{SM}_{Za} &= -\frac{e}{12} \frac{M^2_W}{M_W \sqrt{M^2_Z - M^2_W}} \\
g^{SM}_{Zd} &= -g^{SM}_{Za}
\end{align*}
\]

Using eqs. (18) and the fact the coefficients \( \{a_2, b_2\} \) are now given by the expressions (24) we obtain, for the axial couplings, \( g_{Za_u} = g^{SM}_{Za_u} \) and \( g_{Za_d} = g^{SM}_{Za_d} \). For the vector couplings the results is not as simple since there is dependence on the colour \( i \) of the quark in question,

\[
\begin{align*}
g_{Zv^i_a} &= g^{SM}_{Zv_a} + \frac{g_3}{2\sqrt{3}} c_2 \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \\
g_{Zv^i_d} &= g^{SM}_{Zv_d} + \frac{g_3}{2\sqrt{3}} c_2 \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}
\end{align*}
\]

2.5 The gluons

Five gluons are massive in this theory. One of them, which we call \( \tilde{G} \), due to its mixing with \( \tilde{\gamma} \) and \( \tilde{Z} \) couples directly to the leptons. First of all we must ask what are the masses of these gluons - as explained earlier we have two possibilities, \( m_{\tilde{G}} < m_{\tilde{Z}} \) and \( m_{\tilde{G}} > m_{\tilde{Z}} \). Since \( \tilde{G} \) could in principle be produced in \( e^+e^- \) collisions we assume that the first possibility is ruled out by

\(^2\)Doing this we are effectively equaling our tree-level predictions for the \( \tilde{Z} \) couplings to the tree-level results of the SM.
experiment. In fact, given LEP2’s results and operational energy we must probably require $m_{G} > 200$ GeV, and this not counting possible interference effects. For the moment let us not worry about absolute experimental bounds and simply consider the case $m_{G} > m_{Z}$. As we showed in the last section, requiring this model to correctly reproduce the SM’s tree-level predictions for electroweak interactions constrains $v^2 + q^2 \simeq (246 \text{ GeV})^2$. It is the value of the $t$ vev that thus will set the scale for the mass of the gluons. Therefore the fact that different sets of solutions $\{v,q,t\}$ may be obtained, for the same input values of $\{e,g_3,M_W,M_Z\}$, for different initial guesses for $t$, gains added importance. In fig. (3) we plot the ratio of the neutrino, axial electron and vector electron couplings for $G$ and $Z^0$ couplings. We see that the $G$ couplings are of the same order of magnitude than the $Z^0$ ones, and mostly independent of either the input values $\{e,g_3,M_W,M_Z\}$ and the $t$ vev - the difference in signs is due to the orthogonality of the coefficients $\{a_i,b_i,c_i\}$.

Finally, in fig. (4) we plot the ratio of the quark couplings of $G$ and $Z^0$. As before we see there is little variation in the strength of the couplings with the value of either $t$ or the input values we considered. In this figure we do not consider the terms proportional to $c_3$ in eq. (15), those who distinguish between the quarks’ colour - if this gluon is not confined then we would expect that (see section 3) the $c_3$ contributions will sum to zero in any $q\bar{q}$ reaction; if the gluon is confined then those contributions will be of interest. In fig. (5) we plot the ratio of $g_3 c_3/2\sqrt{3}$ to the $Z$ vector couplings of the $u$ and $d$ quarks and see, without surprise, that these “pure colour” couplings are much stronger than the “pure electroweak” ones we plotted in fig. (4).

2.6 Gauge boson self-couplings

In a non abelian theory gauge bosons can interact between themselves due to the presence in the lagrangean of cubic and quartic terms of the form

$$-g f_{abc}(\partial_{\mu}A_{\nu}^{a})A_{\mu}^{b\nu}A^{c\nu} - \frac{g^2}{4} f_{abc} f_{ade} A_{\mu}^{b} A_{\nu}^{c} A_{\xi}^{d\mu} A_{\nu\xi}^{e}$$

(28)

for a gauge group with structure constants $f_{abc}$ and a gauge coupling $g$. The Feynman rules for the vertices corresponding to these terms are

**Cubic:** $g f_{abc} [(r-q)_{\lambda} g_{\mu\nu} + (q-p)_{\nu} g_{\lambda\nu} + (p-r)_{\mu} g_{\nu\lambda}]$

**Quartic:** $-i g^2 [f_{abc} f_{ade} (g_{\mu\nu} g_{\rho\rho} - g_{\mu\rho} g_{\nu\rho}) + f_{ade} f_{abc} (g_{\lambda\nu} g_{\mu\rho} - g_{\mu\lambda} g_{\nu\rho}) + f_{abd} f_{ace} (g_{\lambda\mu} g_{\nu\rho} - g_{\mu\nu} g_{\rho\lambda})]$, (29)

where $\{p,q,r\}$ are the 4-momenta of each particle in the vertex and the greek letters their Lorentz indices. In the SM the mixing between $B_{\mu}$ and $W_{\nu}^{3}$ produces such vertices as $\gamma W^{+} W^{-}$, $Z^0 W^{+} W^{-}$, $\gamma \gamma W^{+} W^{-}$, $\gamma Z^0 W^{+} W^{-}$, etc. There are however no vertices mixing $SU(2) \times U(1)$ gauge fields with $SU(3)$ ones.

---

3 Remember that the $Z^0$ makes its presence “felt” in $e^+ e^-$ collisions even below its peak.

4 There are no $c_3$ contributions to the axial couplings.
by eq. (12) changes that situation – every time we consider terms involving the fields $W^3_\mu$ or $G^8_{\mu}$ in eq. (29) vertices involving both gluons and electroweak gauge bosons appear. Their respective couplings are given by the product of $g$ ($g_3$) and the coefficients $b_i$ ($c_i$). Let us observe however that once again our theory perfectly reproduces the electroweak SM results - having chosen the vevs so that eq. (24) is satisfied we conclude that the $\tilde{Z}$ coupling in any gauge boson vertex is

$$gb_2 = e \frac{M_W}{\sqrt{M_Z^2 - M_W^2}} = g (\cos \theta_W)^{SM}$$

(30)

exactly the same as the analogous $Z^0$ coupling. Likewise for $\tilde{A}$, the coupling will be $gb_1 = g' a_1 = e$, just like for the photon. Thus the cubic and quartic vertices involving only the electroweak gauge bosons remain unchanged. We now list all the new self-interaction gauge boson vertices and their couplings (in listing the quartic couplings we leave out a factor of $-i$).

From $SU(2)$ we find a single new cubic vertex, $\tilde{G} W^+ W^-$, with coupling obviously given by $gb_3$. As for quartic vertices, with two $W^3$ fields in the quartic terms of (29) we obtain three vertices with $\tilde{G}$ fields,

$$\tilde{G} \tilde{G} W^+ W^- \iff (gb_3)^2$$

$$\tilde{G} \tilde{\gamma} W^+ W^- \iff e gb_3$$

$$\tilde{G} \tilde{Z} W^+ W^- \iff -g^2 b_3 (\cos \theta_W)^{SM} .$$

(31)

For $SU(3)$ self-couplings, the group’s structure constants are well known, their non-zero values given by

$$f_{123} = 1$$

$$f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2}$$

$$f_{845} = f_{807} = \frac{\sqrt{3}}{2} ,$$

and all permutations of these indices bearing in mind that $f_{abc}$ is entirely antisymmetric. Notice that $G^8$ (and thus $\tilde{\gamma}$, $\tilde{Z}$, $G$) does not couple directly to the massless gluons (there are no non-zero $f_{81i}, f_{82i}$ or $f_{83i}$ structure constants). The main difference between the SM cubic gluon vertices and our own is the fact five gluons are now massive. However, for vertices not involving $G^8$ the vertices themselves are unchanged (the propagators for $G^{1...7}$, of course, are now those of massive spin-1 particles). Three new vertices appear,

$$\tilde{G} G G \iff \frac{\sqrt{3}}{2} g_3 c_3$$

$$\tilde{\gamma} G G \iff \frac{\sqrt{3}}{2} g_3 c_1 = e$$

$$\tilde{Z} G G \iff \frac{\sqrt{3}}{2} g_3 c_2 .$$

(33)

Notice that the $\tilde{\gamma} G G$ vertex implies the $4...7$ gluons have electric charge in this theory - the interaction with $\tilde{\gamma}$ changes the gluons’ colour as well, though, in agreement with the formula (21) for the electric charge, now containing a colour contribution. From this point forward we will name the massive $4...7$ gluons by $G^+_1$, $G^+_2$ and the massless $1...3$ ones by $G$. Also notice that, with $M_G > 76$ GeV as we found in fig. (1), the decay $\tilde{Z} \rightarrow G^+_i G^-_i$ is kinematically forbidden.

---

5We emphasise that $(\cos \theta_W)^{SM}$ is a SM quantity, it is not given, in our theory, by $g/\sqrt{g^2 + g'^2}$, due to eq. (11).
- again, the existing experimental data could be used to find lower bounds on $M_G$. We must emphasise that because $G^8$ does not couple directly to massless gluons there are no vertices of the form $\tilde{\gamma} GG$ or $\tilde{Z} GG$, interactions that would certainly have been observed already - an extra argument to support the identification $\tilde{\gamma} \equiv \gamma$ and $\tilde{Z} \equiv Z^0$.

Given the structure constants (32) we may have quartic vertices with one $G^8$ field - these will necessarily include a massless gluon $G$ and two charged gluons. We therefore have three new vertices,

$$\tilde{\gamma} G_{i}^{+} G_{i}^{-} \iff \frac{\sqrt{3}}{4} g_3^2 c_1 = \frac{g_3}{2} e$$

$$\tilde{Z} G_{i}^{+} G_{i}^{-} \iff \frac{\sqrt{3}}{4} g_3^2 c_2$$

$$\tilde{G} G_{i}^{+} G_{i}^{-} \iff \frac{\sqrt{3}}{4} g_3^3 c_3 .$$

Finally, the vertices with two $G^8$ “legs” produce

$$\tilde{\gamma} \tilde{\gamma} G_{i}^{+} G_{i}^{-} \iff \frac{3}{4} g_3^2 c_1^2 = \frac{1}{2} e^2$$

$$\tilde{\gamma} \tilde{Z} G_{i}^{+} G_{i}^{-} \iff \frac{3}{4} g_3^2 c_1 c_3 = \frac{\sqrt{3}}{2} g_3 c_2 e$$

$$\tilde{Z} \tilde{Z} G_{i}^{+} G_{i}^{-} \iff \frac{3}{4} g_3^2 c_2^2$$

$$\tilde{Z} \tilde{G} G_{i}^{+} G_{i}^{-} \iff \frac{3}{4} g_3^2 c_2 c_3$$

$$\tilde{G} \tilde{G} G_{i}^{+} G_{i}^{-} \iff \frac{3}{4} g_3^2 c_3^2 .$$

Throughout this section we showed our broken QCD theory predicts the existence of vertices inexistent in the SM - this, like the existence of massive gluons and integer quark charges, would be an unequivocal way of distinguishing both models.

3 Testing the theory

In this theory the leptons have the same charges and couplings they do in the SM, for sensible choices of the vevs involved. For the most their electroweak gauge interactions are thus identical in both theories. The only exceptions are a new interaction in the broken $SU(3)$ theory - the vertex $e^+ e^- \tilde{G}$, already discussed - and two-photon physics. Other potential differences are found in the strong sector.

3.1 Hadrons

What one observes in experiments are not individual quarks of a given colour but rather mesons and baryons, which are singlets of $SU(3)$. In this theory the photon itself carries colour degrees of freedom, hence having colour cannot be synonymous of being confined. Nevertheless we continue to assume that bound states of the strong interactions are $SU(3)$ colour singlets $\Box$. In our case we have broken $SU(3)$ to a residual symmetry $SU(2)$ - if the unbroken gauge group is $SU(2)$ how can we require the hadronic bound states be singlets of $SU(3)$? After all, confinement can be interpreted as being due to a quark-quark potential that grows linearly with the quark

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*I thank Tim Jones and Carlo Becchi for discussions on this matter.*
separation $r$, $V_{qq} = \alpha r$. We argue that this is a consequence of the non-linearity of the non-abelian field equations, rather than the masslessness of the gauge boson carriers. We also remark that confinement is a non-perturbative process of which little is yet known, and that no clear demonstration exists that an unbroken $SU(3)$ gauge group is $SU(2)$ but the quark bound states are singlets of $SU(3)$ - is as much a leap in the dark as the SM one - unbroken $SU(3)$ implies confinement of quarks in $SU(3)$ singlet states. It is also worth mentioning the work by Bailin and Love [8] proposing that the colour symmetry be $SU(2)$, not $SU(3)$. Their basic argument was that baryons are states with wave functions of the form $\epsilon_{ijk} \psi^i \phi^j \chi^k$ and $\epsilon_{ijk}$ is an $SU(2)$ tensor; this structure can therefore constitute an $SU(2)$ singlet as long as the quarks $\psi, \phi, \chi$ and in the triplet representation of that group. This result is interesting to us because to reproduce the SM results from our ICQ theory all we really need is for the colour wave functions of the hadrons to have the same structure (see below). In the framework of Bailin and Love that would occur and this would fit quite well with our unbroken $SU(2)$ symmetry. Their proposal was not successful because two-loop corrections destroyed any possibility it had of predicting asymptotic freedom for the strong coupling constant $\alpha_S$. As was already explained that would not be a problem in our case, since above the CCB scale the running of $\alpha_S$ would be the “normal” supersymmetric $SU(3)$ one. However the fact remains that we would need our quarks to be in the triplet representation of $SU(2)$, when in fact they are from the start triplets of $SU(3)$.

With this hadronization hypothesis we find that the new formula for the electric charge, eq. (21), even though it changes the charge assignments for each quark, does not alter the overall electric charge of hadrons. For baryons, summing up the charge of the three quarks will correspond, each of their colours being different, to taking the trace of $\lambda_8$; the end result is thus the SM one. For mesons each quark of colour $i$ comes with an antiquark of opposite colour and so the $\lambda_8$ contributions in eq. (21) are cancelled once more. But what about the distribution of these electric charges inside the hadrons? Let us consider the example of the proton, composed of two up quarks and a down one. Because in our model the electric charge varies with the quarks’ colour we must consider the proton’s colour wave function [7].

\[
\psi^p_{\text{colour}} = \frac{1}{\sqrt{6}} \left[ u^1 u^2 d^3 - u^2 u^1 d^3 + u^2 u^3 d^1 - u^3 u^2 d^1 + u^3 u^1 d^2 - u^1 u^3 d^2 \right] .
\] (36)

From section (2.2) we know that $q(u^1) = q(u^2) = -q(d^3) = +1$ (in units of $e$) and $q(d^1) = q(d^2) = q(d^3) = 0$. We thus see that in the wave function above the first two terms correspond to a system of three unit electric charges, two positive and one negative. In the four remaining terms there is a single electric charge - of the quarks $u^1$ or $u^2$ - making up for the proton’s overall charge. This means for instance that, according to our model, in a deep elastic/inelastic scattering experiment an approaching electron would, in an electromagnetic interaction, “see” a proton as a three-charge system with probability 1/3 and as an electric monopole with probability 2/3. If we define the “proton charge wave function”, it will be given by a sum of two pieces, one describing a monopole and another a triplet of charges,

\[
\psi^p_{\text{charge}} = \frac{1}{\sqrt{3}} \left[ \psi^p_{(++)} + \sqrt{2} \psi^p_{(+)} \right] .
\] (37)

This of course corresponds to the definition

\[
\psi^p_{(++)} = \frac{1}{\sqrt{6}} \left[ u^1 u^2 d^3 - u^2 u^1 d^3 \right] .
\] (38)

For the purpose of the electric charge we need not worry about the spin wave function.
and a similar definition for \( \psi_{n}^{(+)} \) involving the four remaining terms of eq. (36). The overall proton charge thus remains the same but while in the SM that charge is distributed in three “lumps”, in our model we find it this the superposition of two different states. Of course we are looking at the proton in an extremely simplistic manner - not taking into account the sea quarks, for instance. This result seems quite remarkable though, and, for regions of transferred momentum not too high, it could be relevant. The same reasoning may be applied to the neutron, composed of two \( d \) quarks and a \( u \) one. Once again studying the colour wave function we obtain

\[
\psi_{n}^{\text{charge}} = \frac{1}{\sqrt{3}} \left[ \psi_{n}^{(0)} + \sqrt{2} \psi_{n}^{(+-)} \right].
\]  

That is to say, in a deep neutron scattering experiment the electromagnetic interaction should “see” the neutron as an electric dipole with probability \( \frac{2}{3} \) and a state composed of only neutral charges with probability \( \frac{1}{3} \). These neutral charges correspond to the terms \( \frac{u^{3}d^{1}d^{2} - u^{3}d^{2}d^{1}}{\sqrt{6}} \) in the neutron colour wave function, where all the quarks involved are neutral. The striking thing about eq. (39) is that - in this simplistic approach, at least - it predicts the neutron is never a state with three charges! We can follow this line of reasoning to the \( \pi^{0} \) meson, described by the colour wave function

\[
\psi_{\pi^{0}}^{\text{colour}} = \frac{1}{\sqrt{6}} \left( u^1 \bar{u}^1 + u^2 \bar{u}^2 + u^3 \bar{u}^3 - d^1 \bar{d}^1 - d^2 \bar{d}^2 - d^3 \bar{d}^3 \right),
\]  

(40)

(41)

which indicates the \( \pi^{0} \) should be observed as an electric dipole with probability \( \frac{1}{2} \) - not 1 as it would be in the SM. The most interesting case, though, is that of the \( \pi^{+} (\pi^{-}) \). Its colour wave function being

\[
\psi_{\pi^{+}}^{\text{colour}} = \frac{1}{\sqrt{3}} \left( u^1 d^1 + u^2 d^2 + u^3 d^3 \right),
\]  

(42)

we see that in each term there is a single quark with charge - meaning, the charge wave function will be

\[
\psi_{\pi^{+}}^{\text{charge}} = \psi_{\pi^{+}}^{(+)}.
\]  

(43)

and the \( \pi^{+} \) (likewise the \( \pi^{-} \)) should, with 100% probability, be observed as an electric monopole, never as a dipole. These are fascinating predictions that, one hopes, may be used to test the validity of this theory.

One might expect the new quark electric charge assignments to have some impact in the magnetic momenta of baryons but surprisingly that is not the case. Consider, for instance, the proton. For the calculation of its magnetic momentum it is necessary to take into account the spin part of the wave function, so we describe the proton by the (spin up) state

\[
| p \uparrow > = \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow + u \downarrow u \uparrow d \uparrow - 2 u \uparrow u \uparrow d \downarrow + \text{permutations}] \otimes \psi_{\text{colour}}
\]  

(44)

with

\[
\psi_{\text{colour}} = \frac{1}{\sqrt{6}} [1 2 3 - 2 1 3 + 2 3 1 - 3 2 1 + 3 1 2 - 1 3 2]
\]  

(45)

in the usual notation. The magnetic momentum of the proton is thus given by

\[
\mu_{p} = \sum_{i=\text{quarks}} < p \uparrow | \mu_{i} | p \uparrow >,
\]  

(46)
with \( \mu_i \) the magnetic momentum of each quark, given by

\[
\mu_u = q_u \hat{\mu}_u \sigma_u , \quad i\mu_d = q_d \hat{\mu}_d \sigma_d ,
\]

(47)

\( \sigma_u \) and \( \sigma_d \) being the spin operators of the up and down quarks, \( q_u \) and \( q_d \) their respective charges (in units of \( e \)) and

\[
\hat{\mu}_u = \frac{e}{m_u} , \quad \hat{\mu}_d = \frac{e}{m_d} . \quad (48)
\]

In the SM the charge/spin are independent of colour so the colour wave function need not concern us, \( q_u = 2/3, q_d = -1/3 \) and we obtain

\[
\mu_{p}^{SM} = \frac{1}{18} \left\{ \frac{2}{3} \hat{\mu}_u \left( \frac{1}{2} - \frac{1}{2} \right) - \frac{1}{3} \hat{\mu}_d \left( +\frac{1}{2} \right) + \frac{2}{3} \hat{\mu}_u \left( -\frac{1}{2} + \frac{1}{2} \right) - \frac{1}{3} \hat{\mu}_d \left( +\frac{1}{2} \right) \right\}
\]

\[
+ \frac{4}{9} \left\{ \frac{2}{3} \hat{\mu}_u \left( \frac{1}{2} + \frac{1}{2} \right) - \frac{1}{3} \hat{\mu}_d \left( -\frac{1}{2} \right) \right\} \right\}
\]

\[
= \frac{1}{18} \left( 8 \hat{\mu}_u + \hat{\mu}_d \right) , \quad (49)
\]

the factor of 3 taking care of the particle permutations in formula (44). In the broken QCD model we need to consider the effect \( \psi_{\text{colour}} \) has in the quarks’ charge in each term. For instance, the \( u \uparrow u \downarrow d \uparrow \) term produces six terms in the wave function,

\[
u \uparrow u \downarrow d \uparrow \longrightarrow \frac{1}{6\sqrt{3}} \left( u^1 \uparrow u^2 \downarrow d^3 \uparrow - u^2 \uparrow u^1 \downarrow d^3 \uparrow + u^2 \uparrow u^3 \downarrow d^1 \uparrow - u^3 \uparrow u^2 \downarrow d^1 \uparrow + u^3 \uparrow u^1 \downarrow d^2 \uparrow - u^1 \uparrow u^3 \downarrow d^2 \uparrow \right). \quad (50)
\]

Their contribution to the magnetic momentum is \(-\hat{\mu}_d/108\). The \( u \downarrow u \uparrow d \uparrow \) term has an identical contribution and the \( 2u \uparrow u \uparrow d \downarrow \) term, \((4\hat{\mu}_u + \hat{\mu}_d)/27\). The particle permutations of eq. (14) introduce a global multiplicative factor of 3 so that the final magnetic momentum is

\[
\mu_p = 3 \left[ \frac{1}{27} (4\hat{\mu}_u + \hat{\mu}_d) - \frac{1}{108} \hat{\mu}_d - \frac{1}{108} \hat{\mu}_d \right] = \frac{1}{18} \left( 8 \hat{\mu}_u + \hat{\mu}_d \right) , \quad (51)
\]

exactly the SM result. It is possible to generalise this calculation for the case of any baryon.

To close this section let us briefly consider the impact of the new quark charge assignments on the momentum distribution functions for partons. The proton’s structure functions \( F_1 \) and \( F_2 \) are given by

\[
F_2(x) = x F_1(x) = x \sum_{i=\text{quarks}} Q_i^2 f_i(x) , \quad (52)
\]

\( f_i(x) \) the probability distribution of finding quark \( i \) of electric charge \( Q_i \) inside the proton with a fraction \( x \) of the total momentum. In a first approximation each quark \( u \) carries, in average, as much momentum as the \( d \) ones,

\[
\int_0^1 x u(x) dx = \int_0^1 x d(x) dx \]

(53)

and so in the SM we have

\[
F_2(x) = x \left[ 2 \left( \frac{2}{3} \right)^2 u(x) + \left( \frac{1}{3} \right)^2 d(x) \right] \]

(54)
so that from eq. (53),

$$\int_0^1 F_2(x) dx = \int_0^1 x d(x) dx \simeq 0.18 \ ,$$

this value measured in deep inelastic scattering (DIS) experiments. So the momentum fraction carried by the $u$ and $d$ quarks is

$$\int_0^1 x [d(x) + 2 u(x)] dx \simeq 0.18 + 2 \cdot 0.18 = 0.54 \ .$$

Because DIS experiments probe only charged particles inside the proton this result is used as indirect evidence that about 46% of the momentum of the proton is carried by neutral particles - the gluons. In our model we know several colour components of the quarks are electrically neutral, so, how are these results affected? Due to the charge assignments we will have, instead of eq. (54),

$$F_2(x) = x [u^1(x) + u^2(x) + d^3(x)] \ ,$$

$u^1(x), u^2(x) (d^3(x))$ the probability distributions for the $u (d)$ quarks of momentum fraction $x$ and colours 1 and 2 (3) respectively. Since we need to consider the distribution probabilities for each colour the hypothesis equivalent to eq. (53) is to assume the colour component of each quark carries the same momentum in average, that is,

$$\int_0^1 x u^i(x) dx = \int_0^1 x d^i(x) dx$$

and so

$$\int_0^1 F_2(x) dx = \sum_{i=1}^3 \int_0^1 x d^i(x) dx \ .$$

Further assuming the $d$ quark distribution is independent of the colour index we will have $d^i(x) = d(x)/3$ and so we re-obtain eq. (56) - the total momentum carried by (neutral) particles other than the $u$ and $d$ quarks is still about 46%. The substantial difference between our model and the SM is that that percentage of momentum should, in first order of approximation at least, be distributed by three gluons only, not eight. The massive gluons are very heavy, as we had the opportunity to discuss in section 2.5, so the fraction of momentum they might carry must be extremely small. The open question is therefore, can the present results for the gluons' momentum distribution functions be reproduced with three massless gluons instead of eight?

### 3.2 One photon or Z processes

A quantity that might be used as evidence of FCQ is the ratio of cross sections of the processes $e^+ e^- \rightarrow q \bar{q}$ and $e^+ e^- \rightarrow \mu^+ \mu^-$, which at tree level is

$$R = \frac{\sigma(e^+ e^- \rightarrow q \bar{q})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} = 3 \sum_i Q_i^2 \ .$$

The 3 is a colour factor and the sum is over the type of quarks with masses below the energy of the experiment. Therefore the contribution of an up-type quark to this ratio is, in the SM,

---

*This is not the usual notation used - the factor of 2 affecting $u(x)$ in eq. (54) is not present as one generally considers $u(x)$ as the probability distribution of both $u$ quarks. The usual convention is to consider the $u$ quarks carry twice as much momentum as the $d$ one. We adopt this non-standard notation for the simple reason it is easier to generalise to our theory.*
\[ 3 \left( \frac{2}{3} \right)^2 = \frac{4}{3} \] and for a down quark, \( \frac{1}{3} \). Now, naively applying this formula to our broken QCD theory with the new quark charges we would obtain

\[
\begin{align*}
R_u &= \sum_{\text{colours}} Q_u^2 = 1 + 1 + 0 = 2 \\
R_d &= \sum_{\text{colours}} Q_d^2 = 0 + 0 + 1 = 1 ,
\end{align*}
\]  

but such calculation would be wrong. With the hadronization hypothesis explained in the previous section we see that in both the SM and the ICQ theory whenever a quark of a given colour is produced in a \( e^+e^- \) collision what is observed after hadronization is a superposition of colourless bound states. Whichever the final state we conclude that the probability of “finding” a quark of a particular colour is \( \frac{1}{3} \). Since individual colours are not observed the total amplitude of the process \( e^+e^- \to q\bar{q} \) is the sum of the amplitudes for each colour,

\[
T(e^+e^- \to q\bar{q}) = \sum_{i=1}^{3} T(e^+e^- \to q^i\bar{q}^i) .
\]  

The cross section \( \sigma(e^+e^- \to q\bar{q}) \) is therefore proportional to the square of the modulus of this amplitude multiplied by a factor of \( \frac{1}{3} \). Hence we obtain

\[
\begin{align*}
R_u^{SM} &= \frac{1}{3} \left| \frac{2}{3} + \frac{2}{3} + \frac{2}{3} \right|^2 = \frac{4}{3} \\
R_d^{SM} &= \frac{1}{3} \left| \frac{1}{3} - \frac{1}{3} - \frac{1}{3} \right|^2 = \frac{1}{3} ,
\end{align*}
\]  

which are the results given by eq. (60). Applying the same reasoning to the broken QCD case we obtain

\[
\begin{align*}
R_u &= \frac{1}{3} |1 + 1 + 0|^2 = \frac{4}{3} \\
R_d &= \frac{1}{3} |0 + 0 - 1|^2 = \frac{1}{3} ,
\end{align*}
\]  

and so both theories predict exactly the same value for \( R \). This quantity is not experimental evidence of fractional charges, but it is irrefutable demonstration for the existence of colour. Looking at the new definition of electric charge, eq. (21), we understand how both theories can lead to the same result for \( R \) - when we calculate the amplitude of \( e^+e^- \to q\bar{q} \) by summing over the colours of the final state we are effectively taking the trace of \( \lambda_8 \), which is zero, and are thus left with the SM result. For \( q\bar{q} \) production via a single \( Z \) boson the same argument applies: a factor of \( \frac{1}{3} \) multiplies the squared amplitude so that the cross section will be

\[
\sigma(\ldots \to q\bar{q}) \sim \frac{1}{3} \left| \sum_i \mathcal{M}(\ldots \to q^i\bar{q}^i) \right|^2 = \frac{1}{3} \left( |3g_{ZVq}^{SM}|^2 + |3g_{ZAq}^{SM}|^2 \right)
\]

\[
= 3 \left( |g_{ZVq}^{SM}|^2 + |g_{ZAq}^{SM}|^2 \right) ,
\]  

which is exactly the SM result. Having chosen the vevs of the theory so that the \( Z \) couplings to both quarks and leptons are identical to the SM tree-level ones, we find that both theories have identical (tree-level) cross sections for all processes involving a single \( Z \), or indeed a single photon.
3.3 Two-photon physics

There are two experimental results usually presented against ICQ theories, the width of the decay of the pseudoscalar mesons $\eta$ and $\eta'$ into two photons and the cross section of the process $e^+e^- \rightarrow e^+e^-$ hadrons. They have in common the fact they are both processes involving two photons and we wish to argue that this evidence against ICQ is not conclusive. For $\eta, \eta' \rightarrow \gamma\gamma$ a simple PCAC analysis yields for the width the expression [10]

$$\Gamma_{\gamma\gamma}^X = \left( \sum_{\text{colour}} <e_q^2> \right)^2 \frac{\alpha^2}{32\pi^3} \frac{m_X^3}{f_X^2}, \quad (66)$$

where $\alpha$ is the fine structure constant, $m_X$ and $f_X$ the meson’s mass and decay constant and $e_q$ the charge of its constituent quarks. The colour sum in the quarks’ squared charges yields for the pion, for example (see eq. (40)), a factor of $3(4/3 - 1/3) = 1/6$ for the SM; for our ICQ theory we would have $(1 + 1 - 1)^2/6 = 1/6$ - the same result, that is. For the $\eta_8$ meson - with a flavour wave function given by $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ - both theories give the same result, $1/18$, but for the flavour-singlet state, $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, the SM gives $4/3$, the ICQ theory $16/3$. The ICQ width is therefore 4 times larger than the SM one if one assumes nonet symmetry, that is, $f_1 = f_8 = f_\pi$. The physical mesons $\eta$ and $\eta'$ being a mixing of $\eta_1$ and $\eta_8$ this seeming discrepancy was used as evidence against ICQ theories.

But from very early on it was recognised [1] that this argument relied too heavily on untested theoretical grounds, that is, the “nonet symmetry” hypothesis; if we had $f_1 = 2f_8$ ICQ theories would be favoured over FCQ ones. Equality of the mesons’ decay constants is tantamount to assuming equality of the wave functions of $\eta_1$ and $\eta_8$ at the origin, which would make sense if the $\eta$ and $\eta'$ were ideally mixed like the $\omega$ and $\varphi$. But the $\eta$ and $\eta'$ are far from ideally mixed, the difference in the $\eta_1, \eta_8$ binding energies being of the order of their masses. In fact more elaborate calculations found substantial differences between the several meson decay constants. For instance in ref. [2], using a Hidden Local Symmetry model [13], it was found that $f_1 = 1.4f_8$. Nonet symmetry is thus not a trustworthy tool. Chanowitz [1] deduced equations for a $\xi$ parameter ($\xi = 1$ for FCQ, $\xi = 2$ for ICQ) in terms of experimental quantities obtained from the decays $\eta/\eta' \rightarrow \gamma\gamma/\pi\pi\gamma$, equations in principle not dependent on nonet symmetry. A first evaluation of $\xi$ clearly favoured FCQ. However Chanowitz’s equations, from the very start, were less reliable if the underlying theory being tested was ICQ (a consequence of the existence of charged gluons in ICQ theories that might make for substantial mixing between glueballs and pseudoscalar mesons), other than displaying a strong dependence on the properties of the $\rho$ meson. Further, more thorough studies [4] showed the Chanowitz equations actually favoured ICQ models over a QCD-type one. Further, the best agreement between the FCQ theory and the experimental data required the introduction of an autre mass-dependent non-vector meson dominance $\gamma \rightarrow \pi^+\pi^-$ contribution. This was all the more impressive due to the fact that a separate analysis, independent of the $\rho$ contribution [5] led to the same conclusions. This seeming embarrassment for QCD did not survive more precise measurements of the decay widths of $\omega, \varphi$ mesons, shown to be in agreement with FCQ over the ICQ theory. Finally the analysis of ref. [6] concluded that the failure of the Chanowitz equations was not necessarily a failure of QCD - rather, that the equations were being misused and were not conclusive. To add to the immense complexity of the problem is the uncertainty of the glueonic content of the $\eta, \eta'$ mesons, with different groups claiming evidence for [7] or against it [8]. In short a simplistic analysis of the $\eta/\eta' \rightarrow \gamma\gamma$ decays does provide arguments against ICQ theories but more detailed studies show such clear-cut conclusions are difficult to establish. We would also add that the theory in discussion in this paper (three massless gluons four charged and one neutral massive ones) is quite different from those tested in refs. [4, 14].
The second experimental evidence rallied against ICQ theories is the cross section of the process $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ via the two-photon channel. As emphasised by Witten [18] this process is preferential to establish integer/fractional quark charges as its cross section is proportional to their fourth power - the cancellation of the colour contributions observed in section 3.2 for the photon and $Z^0$ no longer occurs. Defining the ratio

$$R_{\gamma\gamma} = \frac{\sigma(e^+e^- \rightarrow e^+e^- + \text{hadrons})}{\sigma(e^+e^- \rightarrow e^+e^- + \mu^+\mu^-)}$$

we find that, at tree level, it is given by

$$\begin{cases}
\text{FCQ} : & R_{\gamma\gamma}^F = 3 \sum_q e_q^4 \\
\text{ICQ} : & R_{\gamma\gamma}^I = \frac{1}{3} \sum_q \left( \sum_{i=\text{colours}} e_i^2 \right)^2
\end{cases} \tag{68}$$

So for an up-type quark we have $R_{\gamma\gamma}^I/R_{\gamma\gamma}^F = 9/4$ and for a down-type one, $R_{\gamma\gamma}^I/R_{\gamma\gamma}^F = 9$. This is, however, a purely perturbative analysis. Again the problem is much more complex as there is the possibility that the photon-photon interaction is not perturbative - the so-called “direct” processes - but rather a “resolved” process. At low values of their transverse momentum the photons do not interact like point particles but instead like systems with structure, that is, the photons fluctuate first into an hadronic state and the interaction occurs via a gluon or quark from “inside” the photon [18, 19]. For reviews see [10, 20]. The study of $\gamma\gamma$ interactions at the low energy regions depends thus on photon form factors and is quite complicated requiring Reggeon and Pomeron [21] parametrisations to fit, for instance, the total cross section of $\gamma\gamma \rightarrow \text{hadrons}$ [22]. These procedures have been successful in reproducing - most - experimental results up to LEP2 energies [23]. The immediate question to ask, though, is this: up to what value of typical transverse momentum should the non-perturbative processes be “needed” to describe the experimental data? We remark, for instance, that the $\gamma\gamma$ results from PETRA were explained with a perturbative ICQ theory with eight massive gluons of mass $\sim 0.3$ GeV [24], this for $P_T \sim 5$ GeV, though this match may have been accidental. Restating the problem, when should we compare data with the perturbative predictions alone and thus infer their validity? A reasonable expectation would be that the $\gamma\gamma$ production of heavy quarks would be well described by perturbative QCD, particularly in the case of the bottom quark. However, as we may observe in fig. 6, although $c\bar{c}$ production is well described by Next to Leading Order (NLO) QCD, including both direct (perturbative) and resolved processes, $b\bar{b}$ production clearly is not the theoretical prediction being a factor of 3 or so below the experimental result. In both cases, though, we see that the direct processes - the type A diagrams of ref. [26] - are not enough to describe the data, for an FCQ theory. The $b\bar{b}$ result has been used to argue for the existence of new physics, with supersymmetry brought into play to account for it [28] via a light colour octet gluino. In reference [28] another interpretation was proposed, that these results are caused by the quarks having integer electric charges. Let us admit that the cross sections we are observing at these high energies are purely perturbative ones, an expectation that does not seem absurd given the high masses of the charm and bottom quarks. To obtain the ICQ predictions, the FCQ direct cross section for $c\bar{c}$ production should be multiplied by a factor of 9/4 and the $b\bar{b}$ one by a factor of 9 (see eq. (68)). That this should constitute a very good approximation to the equivalent ICQ cross sections may be seen from the explicit expressions for $\sigma_{\text{FCQ}}(\gamma\gamma \rightarrow Q\bar{Q})$ from ref. [26]. Both the pure QED and the QCD corrections are proportional to the fourth power of the quarks’ charges so the factors we described make the transition from a

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9The high-$P_T$ production of $\pi^0$s in $\gamma\gamma$ reactions suffers from the same problem, see for example ref. [27].
**Table 1: Comparison of experimental data and FCQ and ICQ direct (perturbative) NLO QCD for $c\bar{c}$ and $b\bar{b}$ production cross sections.**

|                  | FCQ Direct | ICQ Direct | Exp. Data     |
|------------------|------------|------------|---------------|
| $\sigma(\gamma\gamma \rightarrow c\bar{c})$ | 400 - 525  | 900 - 1181.3 | 1016 ± 123    |
| $\sigma(\gamma\gamma \rightarrow b\bar{b})$ | 1.54 - 1.78 | 13.9 - 16.0 | 13.1 ± 3.1    |

FCQ to an ICQ scenario. The strong interaction corrections are however clearly different in the SM and our ICQ theory. For starters we do not expect the massive gluons to have substantial contributions to these processes, given their large masses. Therefore an approximation to the QCD corrections in our theory should involve only three massless gluons and the factor of $4/3$ in expression (2) of ref. [26] should be replaced by the appropriate $SU(2)$ Casimir, $3/4$. It is thus trivial to see that our simple approximation to $\sigma^{ICQ}(\gamma\gamma \rightarrow Q\bar{Q})$ overestimates the QCD corrections. However, also from ref. [26], we see that for high values of the center-of-mass energies the $\alpha_s$ corrections are expected to be substantially smaller than the QED contributions. Having thus argued that $9/4 \times \sigma^{FCQ}(\gamma\gamma \rightarrow c\bar{c})$ should be a good approximation to $\sigma^{ICQ}(\gamma\gamma \rightarrow c\bar{c})$ and $9 \times \sigma^{FCQ}(\gamma\gamma \rightarrow b\bar{b})$ to $\sigma^{ICQ}(\gamma\gamma \rightarrow b\bar{b})$ we gather the experimental results from LEP2 at 194 GeV from the L3 collaboration [29] (entirely compatible with the results from the OPAL collaboration [31] and with more recent data from L3 [30]) and their direct FCQ predictions. We find the results shown in table 1. We argue that the agreement between the ICQ approximate predictions and the LEP2 data is compelling enough to seriously consider the possibility that the quarks do have integer charges $^{10}$. The agreement between the ICQ prediction and the $c\bar{c}$ data is not an accident for this particular collision energy, it occurs at energies ranging from 91 to 194 GeV, as can be seen in ref. [25]. One cannot forget, however, that the data from ref. [29] were obtained assuming the experimental backgrounds were those dictated by the SM. A complete comparison of data to our ICQ predictions would have to take into account the different QCD gauge group. Also worthy of note in the $b\bar{b}$ case is that, as we already mentioned, the bottom quarks mix with the charginos in this CCB scenario. This simple analysis doesn’t take that into account - that mixing, though, depends on the supersymmetry-breaking parameters and thus one can expect sufficient freedom to reproduce the data. That complete analysis could be quite interesting in that it would presumably constrain the MSSM parameter space.

### 4 Discussion and conclusions

According to the Particle Data Group [32] the current limit on the mass of the gluon is “a few MeV”\(^\text{10}\). That, however, is a **theoretical** bound, derived from arguments that assume all gluons

\(^{10}\)This simple approximation to the ICQ cross section does not explain the $\pi^0$ production cross section excess found at LEP [27], but given the low mass of the up and down quarks one may argue the perturbative results cannot “yet” be trusted in that case.
are degenerate in mass \[33\]. That is most definitely not the case of the theory presented in this paper. Also, as others have discussed \[34\], the arguments of \[33\] neglect to take into account the quantum field theoretical aspects of the gluon-quark interactions, which casts doubt over their final conclusions. Many theories have been developed where gluons acquire mass by means of gauge symmetry breaking \[4, 35\] or the introduction of a four-vertex ghost field \[36\]. Cornwall proposed a mechanism for dynamic generation of gluon masses within a theory with unbroken colour gauge symmetry \[37\]. Recently Field \[34\] considered gluons of mass \(\sim 1\) GeV to explain the radiative decays of the \(J/\Psi\) and \(\Upsilon\) mesons. His is a convincing argument for the existence of massive gluons. Of course, the gluons discussed in this paper have masses much larger than Field’s, and our theory also includes three massless ones. Therefore it remains to be seen if the analysis of \[34\] could be reproduced by our ICQ model.

We obtain ICQ from charge and colour breaking in a supersymmetric theory, though to be precise we are not really breaking electric charge symmetry - as we see from formula (21) the vevs \(v, q\) and \(t\) are neutral for the new definition of electric charge. For the old one, though, they would appear as charged. Now, we were able to determine the values of these vevs by looking at the electroweak couplings and requiring them to equal the corresponding SM values. For this scenario to be possible the SUSY-breaking parameters must be such that a CCB minimum is produced in the potential. It has been established \[5\] that CCB associated with the top Yukawa coupling is difficult to obtain (because the \(SU(3)\) D-term contributions to the potential are large and positive). One-loop studies of CCB potentials \[6, 38\] revealed commonly accepted CCB bounds could be overestimated. Considering these restrictions it is then possible that only a very small portion of SUSY parameter space could produce the pattern of CCB we are interested in. This is an interesting feature of this model as it could increase immensely the predictive power of the MSSM. Notice that CCB bounds have been considered to avoid the possibility of CCB occurring, whereas in such a study we would want to enforce it. We hope to address this subject in the future.

We have built a model that closely mimics the electroweak sector of the SM but its strong sector is very much an open question. It remains to be seen whether our spectrum of gluons is able to reproduce the plethora of QCD experimental results in existence. For instance, proton-proton collisions. We would expect that, given the large mass of five of our gluons, only the massless ones would participate in \(p - p\) processes. The partonic cross sections for those reactions would therefore not be the usual ones. But then we have to remember (section 3.1) that the gluons’ distribution functions are also different - we now have, in first approximation at least, three gluons carrying about 46% of the proton’s momentum, not eight like in the SM. The observed cross section being the convolution of the partonic cross sections and the parton distribution functions, the possibility exists that the end result, amazingly, be the same for both theories. There are however two experimental results that challenge the notion of an unbroken \(SU(2)\) colour gauge group. The first is the measurement of the gluon and quark fragmentation functions at DELPHI and determination of the ratio \(C_A/C_F\) \[34\]. For a gauge group \(SU(N)\) this ratio is given by \(C_A/C_F = 2N^2/(N^2 - 1)\). The result from \[39\] is 2.26 ± 0.16, in agreement with the predicted value for \(SU(3)\), 9/4 - but not too far from the \(SU(2)\) result, 8/3. We have to remark that the analysis of ref. \[39\] is strongly dependent on the jet algorithm used and the choice of energy scale. Also, the event selection in that work was based on the expected behaviour of three-jet systems in an unbroken \(SU(3)\) theory. Would the results be different, we ask, if the underlying theory was assumed to be an unbroken \(SU(2)\) with massive gluons? A more serious objection is the one posed by the measurements of the strong coupling \(\alpha_s\) at low energies \[12\] - the evolution of \(\alpha_s\) with the energy scale is very well described by a SM \(SU(3)\) \(\beta\)-function with the appropriate number of quark flavours. However, it is possible that admitting that the \(\beta\)-function for our ICQ model below, say, the \(M_Z\) scale, is equivalent to that of an
unbroken colour $SU(2)$ group is an oversimplification. In fact, as we can appreciate from fig. 3, for masses of $G$ as high as $\sim 130$ GeV, we have charged gluon masses lower than $M_Z$. In such a situation we would expect those gluons to have a contribution to the running of $\alpha_s$ near the weak scale, at least. We must also remember that the evolution of $\alpha_s$ should go accordingly to a supersymmetric $\beta$-function. These reduce to the SM ones at energy scales below the mass of the lightest supersymmetric partner (when all sparticles have been “integrated out” of the theory and the spectrum remaining is that of the SM). If we have a spectrum with sparticle masses lower than $M_Z$ (the experimental bounds on sneutrino or neutralino masses, for instance, are still well below $M_Z$) we would expect the $\alpha_s$ running of such a theory to be more elaborate that that dictated by a simple SM $SU(2)$ $\beta$-function. The scale at which we should start assuming that the running of $\alpha_s$ is given by a “pure” $SU(2)$ $\beta$-function is thus dependent on the particular SUSY spectrum we choose. Also, particle threshold corrections can have significant impact on the low-energy value of the coupling constants. This means further study is necessary to determine if the theory is capable or not of reproducing the low energy running of the strong coupling constant.

In conclusion, we built a theory with broken colour gauge symmetry. The vevs in this theory, according to SM expectations, have electric charge and thus the photon was expected to become massive. We saw that this isn’t so as the symmetry breaking leads to a new definition for the electric charge, with an extra contribution from $SU(3)$. This new definition does not affect the lepton charges but changes the quarks’ - they become dependent on the colour of the quark in question and are integer. As a consequence of gauge symmetry breaking five gluons gain mass, four of those are electrically charged. The neutral massive gluon is mixed with the photon and $Z$ fields, this mixing causing the appearance in the theory of new vertices, such as $G e^+e^-$. This ICQ model closely mimics the electroweak sector of the SM - it is possible to choose the vevs such that the couplings of the new photon and $Z^0$ are equal to the tree-level values of the same particles in the SM. The mass and interactions of the $W$ gauge boson are also unchanged, except for the new gauge boson self-interaction vertices mentioned earlier. The mass of the gluons was found to be strongly dependent on the choice of a particular vev - the $t$ vev, associated with the scalar field $t_R$ - and very high. In the electroweak sector the two theories have different experimental predictions only in two-photon processes. We have argued that recent LEP2 results are quite naturally explained admitting quarks have integer charges, and we must recall that the SM prediction for the cross section for $\gamma\gamma \rightarrow b\bar{b}$ is a factor of 3 below the experimental result. There are however experimental results that would seem to contradict the idea of an unbroken $SU(2)$ colour gauge group, the measurements of $C_A/C_F$ and the low energy running of $\alpha_s$. In both cases it was argued that there is a strong model dependence in the analysis of the data or the ICQ predictions. As such, the theory may still be able to accommodate these results. On theoretical grounds the weakest point in this theory seems to be the hadronization hypothesis, that hadronic bound states are singlets of $SU(3)$ even though that symmetry is now broken. As discussed, this seems no more outrageous than the SM confinement scheme, given our degree of ignorance of the non-perturbative physics that governs hadronization. Notice that we tested this model at tree-level only. The theory is obviously renormalisable so higher order corrections will be small. Given the degree of accuracy achieved at LEP loop corrections to the ICQ model may prove to be a good testing ground of the theory. Another experimental prediction is the interesting charge distribution in hadrons presented in section 3.1, that we hope to look into in the near future. The existence of the massive gluon $\tilde{G}$, with direct coupling to leptons, would of course be ideally tested in an electron-positron linear collider of high energy, like the TESLA or NLC projects. In short, and to answer this article’s title question, it seems possible to build a theory with broken charge and colour symmetries with sensible behaviour in the electroweak sector, at least. The strong sector will be the hardest test of this theory. It will be necessary to show if the different gluon spectrum can reproduce
the wealth of experimental QCD data existent. The mixing of quarks, neutralinos, charginos and gluinos existing in this model should also be carefully studied as both a possible window into the supersymmetric sector of the theory or a means of excluding it, if the changes in the bottom or top mass/interactions are too severe. The interest in considering such a theory as an alternative to the SM? If the $\gamma\gamma \rightarrow b\bar{b}$ result from LEP2 cannot truly be explained by the SM new physics will be necessary. ICQ theories were shown to fit admirably well that data - and the ICQ theory presented in this work is the simplest one there can be, as it requires a gauge group no larger than the SM’s. It also provides new insight into the supersymmetric sector and, from the CCB requirement, we hope it will have a reduced allowed parameter space and thus considerable predictive power.

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Figure 1: $\sqrt{v^2 + q^2}$ vs. $t$, for the parameter space described in the text. Each of these points corresponds to a set of vevs $\{v,q,t\}$ for which the $\tilde{Z}$ leptonic couplings are identical to those of the $Z^0$.

Figure 2: Masses of the gluons $\tilde{G}$ (crosses) and $G$ (stars) vs. $t$, for different initial guesses for $t$, 100, 200, 300, 400 and 500 GeV.
Figure 3: Ratio of $\tilde{G}$ gluon leptonic couplings to $Z^0$ ones - $g_{\tilde{G}_\nu}/g_{Z\nu}^{SM}$ (crosses), $g_{\tilde{G}_A}/g_{Z_A}^{SM}$ (stars), $-g_{\tilde{G}_V}/g_{Z_V}^{SM}$ (circles) - notice the minus sign in this last ratio.

Figure 4: Ratio of $\tilde{G}$ gluon quark couplings to $Z^0$ ones - $-g_{\tilde{G}_{A_q}}/g_{Z_{A_q}}^{SM}$ (crosses - notice the minus sign), $g_{\tilde{G}_{V_u}}/g_{Z_{V_u}}^{SM}$ (stars), $g_{\tilde{G}_{V_d}}/g_{Z_{V_d}}^{SM}$ (circles).
Figure 5: Ratio of $g_3 c_3/2 \sqrt{3}$ to $-g^{SM}_{ZV_u}$ (crosses - notice the minus sign) and $g^{SM}_{ZV_d}$ (stars).

Figure 6: Cross section for production of $c\bar{c}$ and $b\bar{b}$ pairs at LEP through the two photon channel [29].