Novel Type I Compactifications

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Abstract

We argue that there are two distinct classes of type I compactification to four dimensions on any space. These two classes are distinguished in a mysterious way by the presence (or absence) of a discrete 6-form potential. In simple examples, duality suggests that the new class of compactifications have reduced numbers of moduli. We also point out analogous discrete choices in M, F and type II compactifications, including some with $G_2$ holonomy. These choices often result in spaces with frozen singularities.
1 Introduction

The moduli space of string compactifications is an intricate and complex object. Even compactifications with sixteen supersymmetries contain surprising amounts of new physics \[1\]. The goal of this letter is to describe new classes of type I compactifications to four dimensions, or lower. These compactifications are quite mysterious because they involve discrete choices that are very hard to measure. This makes it difficult to analyze the low-energy physics, and leaves much to be understood. However, we shall see that combinations of dualities do suggest that these new compactifications exist, and that they generically have reduced numbers of moduli.

There are also discrete choices in type II, M and F theory compactifications to four dimensions or lower, which we also describe. The geometries involved in these compactifications typically contain “frozen singularities” generalizing the kind encountered in \[1–4\]. Our study of discrete choices is by no means complete. Rather, the particular discrete choices that we describe have a natural origin in orientifold physics.

The basic mechanism that we want to consider is most easily described in compactifications to three dimensions. These compactifications typically have a number of abelian gauge fields. Let us take one such gauge field, $A$. In three dimensions, we can dualize this gauge field to obtain a scalar field $\phi$, $d\phi = * F$, (1)

where $F = dA$ is the field strength. However, this scalar field actually has a compact moduli space; it takes values in $U(1)$. To see this, we can partially rewrite the action in term of $\phi$:

$$\frac{1}{g^2} \int F \wedge * F = \frac{1}{g^2} \int F \wedge d\phi = \frac{1}{g^2} \int d(F \phi)$$ (2)

$$= \frac{1}{g^2} \int_{S^2} F \langle \phi \rangle.$$ (3)

We denote the zero-mode expectation value of $\phi$ by $\langle \phi \rangle$. The integral of $F$ over $S^2$ measures the first Chern class of the bundle and so is integer. Shifting $\phi$ by $2\pi g^2$ therefore has no effect on the path-integral, and leaves the physical theory unchanged.

What will concern us is how the physics depends on the modulus $\langle \phi \rangle$. In perturbation theory, nothing depends on $\langle \phi \rangle$ because the shift symmetry requires that only derivatives of $\phi$ appear in any low-energy effective action. However, non-perturbative instanton cor-
rections do depend on $\langle \phi \rangle$ since the $n$-instanton action is weighted by a factor,

$$e^{in\langle \phi \rangle/g^2}.$$  (4)

Although the abelian theory is free and so trivial, we could also imagine starting with a non-abelian theory, for example, N=8 $SU(2)$ Yang-Mills. On the Coulomb branch of this theory, we are again left with a single light photon. However, the moduli space is now $(\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$ because there are 7 scalars transforming in the adjoint, and we have to mod out by the Weyl group. What is key for us is that there are two fixed points on $S^1$ at 0 and $\pi$. Indeed, this very system appears in the study of D2-branes and O2-planes, where the physics at the two fixed points can be very different [5–8]; see also [9, 10]. This is a germane example in field theory of the kind of phenomena that we shall encounter in string theory. Namely, a discrete modulus which is undetectable in perturbation theory does, nevertheless, strongly alter the low-energy physics.

In a similar way, we can consider a string moving in four dimensions. The string couples to a 2-form gauge-field $B$ with field strength $H = dB$. We can dualize $H$ to get a scalar,

$$d\phi = *H.$$  

That this scalar is also $U(1)$-valued follows by essentially the same argument given above after we note that

$$\int_{S^3} H \in \mathbb{Z}.$$  

This compact scalar is always present in any compactification of closed string theory from ten dimensions to four, with constant dilaton. We can also understand the appearance of this scalar from the ten-dimensional perspective. If we dualize $H$ in ten dimensions, we obtain a 6-form gauge-field, $A_6$, coupling to the NS 5-brane,

$$dA_6 = *H.$$  

From this perspective, the expectation value $\langle \phi \rangle$ corresponds to

$$\int_{\mathcal{M}_6} A_6,$$

where $\mathcal{M}_6$ is the compactification space.

As an example, consider type IIB string theory compactified on $T^6$. There are 70 scalars parametrizing the moduli space $E_7(7)/SU(8)$. At least two of these scalars – those coming from dualizing the NS-NS and RR 2-forms – arise in the manner we have described.
String perturbation theory is insensitive to the values of these compact moduli. Wrapped Euclidean 5-branes, however, will contribute to amplitudes in the four-dimensional effective action. Each of these BPS instanton contributions is weighted by a factor of the form appearing in equation (4). For a discussion of such effects, see [11, 12]. We can also consider higher dimensional theories. For example, the 3-form potential $C_3$ coupling to a membrane moving in five dimensions is dual to a compact scalar. This modulus is always present in compactifications of M theory to five dimensions.

Typically, in cases of low or no supersymmetry, potentials will be generated for these moduli. For most of our discussion, this should not play a role.

2 Novel Orientifolds

The first case of interest to us is that of string theory compactified to four dimensions on a space $\mathcal{M}_6$ with volume $V_6$. Since we will be dualizing various interactions, we measure this volume with respect to the Einstein frame metric. Recall that type I string theory can be viewed as an orientifold of type IIB by world-sheet parity, $\Omega$. Under the action of $\Omega$, the NS-NS B-field, RR 0-form $C_0$, and 4-form $C_4$ are inverted

$$B_2 \rightarrow -B_2, \quad C_0 \rightarrow -C_0, \quad C_4 \rightarrow -C_4.$$  \hfill (5)

However, this leaves a discrete choice of turning on a half-unit of $B_2$ through a 2-cycle and a half-unit of $C_4$ through a 4-cycle. The $B_2$ flux results in a type I compactification without vector structure [13, 14]. The case of $C_4$ flux for a $T^4$ compactification has been recently studied in [15].

Since $B_2$ is inverted, its dual potential $A_6$ is also inverted by $\Omega$. We therefore have the freedom of turning on a half-unit of $A_6$ flux through $\mathcal{M}_6$. Therefore, there are 2 classes of type I compactification to four dimensions. From a purely four-dimensional perspective, we have a compact scalar $\phi$ dual to $B_2$. Normalizing $\phi$ to be dimension one with conventional kinetic terms, we see that $^1$

$$\phi \sim \phi + \frac{\alpha'}{\sqrt{V_6}}.$$  \hfill (6)

Under the $\mathbb{Z}_2$ projection, there are two invariant values: 0 and $\frac{\alpha'}{2\sqrt{V_6}}$. The first is a conventional type I compactification; the second is a mysterious new choice.

We might worry that we generate an anomaly for this second choice. However, string perturbation theory is completely insensitive to this discrete modulus so any such anomaly would have to be very subtle. We will see in the case of $T^6$ that an anomaly is unlikely.

$^1$Modulo factors of 2 and $\pi$. 

2.1 Toroidal compactification

Is there really any difference in the low-energy physics? This is a natural second concern. Let us turn to the case of compactification on $T^6$. We will use a combination of dualities to convert this compactification into something less mysterious. Our starting point is IIB on $T^6/\Omega$ with $A_6$ flux through the torus. If we T-dualize along $T^6$, the $A_6$ potential remains invariant.

This might seem strange even for T-duality on one circle which gives type IIA on

$$\left(T^5 \times S^1/\mathbb{Z}_2\right)/\Omega,$$

or type I’. At first, it appears that we have converted a $\mathbb{Z}_2$-valued field into a $U(1)$-valued field because $\mathbb{Z}_2\Omega$ would appear to leave $A_6$ invariant. Fortunately, this is not the case. The $\mathbb{Z}_2\Omega$ action lifts to an M theory orientifold which acts on the M theory 3-form, $C_3$, by inversion. Tracing the action on the dual potential shows that this particular component of $A_6$ is also inverted.

We now dualize along all six circles of the torus giving IIB on,

$$T^6/\Omega(-1)^{F_L}\mathbb{Z}_2,$$

with $A_6$ flux. We can now S-dualize the background. Typically, commuting S-duality with an orientifold action is problematic. By the criterion described in [10], S-duality should be permitted in this situation. This action leaves $\Omega(-1)^{F_L}\mathbb{Z}_2$ invariant, but converts $A_6$ to the RR 6-form, $C_6$. We shall see further evidence for this equivalence when we discuss F theory compactifications. We can now T-dualize on four circles of the torus giving IIB on,

$$\left(T^2/\mathbb{Z}_2 \times T^4\right)/\Omega(-1)^{F_L},$$

with $C_2$ flux through $T^2/\mathbb{Z}_2$. This is a situation which is dual to a compactification of type I with no vector structure by the following duality chain: type I on $T^2$ with no vector structure can be viewed as IIB on $T^2/\Omega$ with $B_2$ flux. Dualizing on both circles of $T^2$ gives IIB on,

$$T^2/\Omega(-1)^{F_L}\mathbb{Z}_2,$$

with $B_2$ flux. S-dualizing this configuration gives the desired orientifold with $C_2$ flux. This chain of reasoning, while not water tight, suggests that our new orientifold sits in the same moduli space as a compactification with no vector structure. The rank of the gauge group is therefore reduced by 8.
Two comments are in order: what we currently lack is a way of characterizing the effect of the $A_6$ flux. In the case of $B_2$ flux, we could describe the effect in terms of an exotic Stiefel-Whitney class $\tilde{w}_2$, for the $\text{Spin}(32)/\mathbb{Z}_2$ bundle. This naturally only involves two-cycles of the compactification space $\mathcal{M}_6$.

For the case of $A_6$ flux, we need an analogous statement. A few facts might be helpful toward this end: unfortunately, the cohomology of $B(\text{Spin}(32)/\mathbb{Z}_2)$ contains no exotic ‘$w_6$’ class [18]. Interestingly, however, the cohomology of $B\text{SO}(32)$ does contain a $w_6$. Since the perturbative type I gauge group is actually $\text{O}(32)/\mathbb{Z}_2$, the correct characterization of the resulting type I compactification might well involve an obstruction preventing a lift of an $\text{SO}$-bundle to a $\text{Spin}$-bundle. Clearly, a better understanding of the physics of $A_6$ is needed.

As a second comment, we note that in the field theory example mentioned in the introduction, $\langle \phi \rangle$ could be detected by monopole instantons. These instantons modify BPS couplings in the effective action, like the 4 photon $F^4$ interaction, in a way that can be determined by supersymmetry [19,20]. This is not the case in our type I construction. The natural probes for $\langle \phi \rangle$ are Euclidean NS5-branes wrapping $\mathcal{M}_6$. However, these configurations are projected out of the spectrum by the orientifold action. This is the primary reason that the effect of $A_6$ flux is difficult to determine.

### 2.2 Variants of F theory

If we start with type I on an elliptically-fibered space, we can imagine T-dualizing along each cycle of the torus fiber to obtain a IIB orientifold on,

$$\mathcal{M}_6/\Omega(-1)^F\mathbb{Z}_2,$$

where the $\mathbb{Z}_2$ acts on both cycles of the fiber. This is the orientifold limit of F theory compactified on an eight-dimensional space $\mathcal{M}_8$ [21].

If we start with $A_6$ flux then we end up with $A_6$ flux through $\mathcal{M}_6$. However, there are two natural discrete fluxes in this orientifold construction: $A_6$ flux and RR $C_6$ flux through $\mathcal{M}_6$. Both give $U(1)$-valued scalars which project to $\mathbb{Z}_2$-valued scalars under the action $\Omega(-1)^F\mathbb{Z}_2$. If we dualize back to type I along the torus fiber, we see that $C_6$ corresponds to the possibility of turning on a discrete $C_4$ flux through the base of the elliptically-fibered $\mathcal{M}_6$. It is natural that both these possibilities should be on the same footing in an F theory construction.

F theory is a just a particular limit of M theory compactified on $\mathcal{M}_8$ where the volume of the $T^2$ fiber is taken to zero [22]. How are these discrete choices realized in M theory?
It is easy to trace back the origin of $A_6$ and $C_6$ in type IIB to M theory. These two potentials arise in the following way: the metric, $G$, reduced on the $T^2$ fiber gives rise to 2 Kaluza-Klein gauge-fields, $A_1$ and $A_2$.

At the orientifold point, it makes sense to discuss these two gauge-fields. In three dimensions, we can dualize each of them to a compact scalar which is $\mathbb{Z}_2$-valued under the orientifold action. This construction is reminiscent of the eight-dimensional F theory dual for the CHL string advocated in [23, 24]. We certainly do not expect all choices for these $\mathbb{Z}_2$-valued scalars to give physically distinct theories. S-duality will identify various combinations of scalar field expectation values.

2.3 More general constructions

Our preceding discussion leads us to a fairly general picture. Let us consider type II compactified to $d$ space-time dimensions on, 

$$\mathcal{M}_{10-d}/G.$$ 

For simplicity, let us assume that the covering space, $\mathcal{M}_{10-d}$, is smooth. We are interested in the spectrum of $(d-2)$-forms that arise by compactifying the type II string on the cover. These forms dualize to scalars in $d$ dimensions with a moduli space, $\mathcal{M}^\phi$. In general, the group, $G$, will not act freely on $\mathcal{M}^\phi$. Each disconnected component of $\mathcal{M}^\phi/G$ corresponds to a (classically) distinct compactification.

2.3.1 A type II example

Let us turn to an example. Consider the type II string compactified to three dimensions on, 

$$T^4/\mathbb{Z}_2 \times T^3.$$ 

Prior to orbifolding by the $\mathbb{Z}_2$, we have a variety of Kaluza-Klein gauge-fields obtained by reducing the metric $G$ and various $B$-fields along a one-cycle of $T^4$. Each can be dualized to a compact scalar in three dimensions. Under the orbifold action, there are 2 possible fixed points.

In this model, there are additional discrete choices beyond the 4 $\mathbb{Z}_2$-valued scalars coming from $G$ and the 4 from $B_2$. For the type IIA string, there are 12 additional $\mathbb{Z}_2$-valued scalars obtained by reducing $C_3$ along a 1-cycle of $T^4/\mathbb{Z}_2$ and a 1-cycle of $T^3$. The RR potential $C_5$ gives 16 scalars, while $C_7$ gives 4 more scalars. The NS potential $A_6$ gives 12 scalars.
There is a final possibility which comes about in the following way: the metric reduced on a 1-cycle of $T^3$ gives a 9-dimensional KK gauge-field. Dualizing this gauge-field gives a 6-form potential $G_6$ which couples to Kaluza-Klein monopoles with respect to the chosen 1-cycle. From each of these 3 $G_6$ potentials, we obtain 4 $\mathbb{Z}_2$-valued scalars, giving an additional 12 possibilities. There may well be more possibilities, but these 64 are the simplest $\mathbb{Z}_2$-valued scalars to describe. Again, we note that many of these choices should give the same low-energy physics.

How does a non-trivial fixed point for one of these scalars change the low-energy physics? What we expect from arguments given in [1] is a compactification with some frozen singularities. This is manifest for one case: the $\mathbb{Z}_2$ scalars that arise from dualizing $C_7$ correspond to a choice of RR 1-form flux through $T^4/\mathbb{Z}_2$. This is in the class of compactifications considered in [2], and studied in detail in [1]. Of the 16 $A_1$ singularities on $T^4/\mathbb{Z}_2$, 8 are frozen by this flux. For more general fluxes, we again expect frozen singularities though the nature of those singularities is mysterious. The 1-form flux is particularly nice because the resulting compactification is geometric in M theory. The general case is, unfortunately, not so easy to analyze.

2.3.2 Type I in three dimensions

Let us now turn to type I on $T^7$. It is natural to expect a set of discrete choices dual to those we just found in the type II compactification. We start with IIB on, $T^7/\Omega$.

From $B_2$ reduced on a 1-cycle of $T^7$ we obtain 7 $\mathbb{Z}_2$-valued scalars. $C_4$ reduced on a 3-cycle gives 35 choices, while $C_8$ gives one scalar. Lastly, $A_6$ reduced on a 5-cycle gives 21 scalars for a total of 64 $\mathbb{Z}_2$-valued scalars.

This is a particularly nice example for the following reason. The $\mathbb{Z}_2$-valued scalar that arises by reducing $A_6$ on a 5-cycle and dualizing can be described a second way. It arises by reducing $B_2$ on the 2-cycle dual to the 5-cycle of $T^7$. However, this $\mathbb{Z}_2$ choice just corresponds to a type I compactification with (or without) vector structure! We hope that this is compelling evidence that an $A_6$ background can give a distinct anomaly free compactification which can be analyzed.

2.3.3 M theory on $G_2$ spaces

The last class of example that we shall examine involves compactifications of M theory on spaces of $G_2$ holonomy. One of the simpler cases is not really a $G_2$ compactification, but a
‘barely $G_2$’ compactification $[23,26]$ still preserving 4 supersymmetries,

$$(S^1 \times \mathcal{M}_6)/\mathbb{Z}_2.$$ 

The $\mathbb{Z}_2$ action acts without fixed points on $\mathcal{M}_6$ but inverts the circle coordinate. Reducing the M theory 3-form on $S^1$ gives a 2-form which dualizes to a compact scalar. After the $\mathbb{Z}_2$ quotient, we are left with 2 fixed points.

This case is more mysterious than the type II examples because there is no apparent change in the number of moduli. The difference (if any) between the two possible compactifications is more subtle than in prior examples.

A particularly simple way to obtain a (singular) $G_2$ compactification is by starting with a type IIA orientifold. The strong coupling description will involve M theory on a $G_2$ space. Some examples of this kind have been recently studied in $[27–29]$. Our starting point is IIA on,

$$\mathcal{M}_6/\Omega(-1)^{F_L}G,$$

where $G$ inverts 3 circles of an appropriately chosen $\mathcal{M}_6$. The potential $A_6$ is inverted by this action. In a by now familiar fashion, we see that there are 2 classes of IIA orientifold on $\mathcal{M}_6$. In the corresponding M theory model, we have an $A_6$ background through $\mathcal{M}_6$. This is essentially the ‘barely $G_2$’ case where we allow an action on $\mathcal{M}_6$ with possible fixed points. The case of $\mathcal{M}_6 = T^6$ falls into our earlier duality chain, and with non-trivial $A_6$ flux, we again expect a rank reduction of 8 in the space-time gauge group.

3 Acknowledgements

It is our pleasure to thank A. Keurentjes, A. Lawrence, J. Morgan, A. Sen, and E. Witten for helpful conversations. The work of D. R. M. is supported in part by National Science Foundation grant DMS-0074072; the work of S. S. is supported in part by an NSF CAREER Grant No. PHY-0094328, and by the Alfred P. Sloan Foundation. We would like to thank the Aspen Center for Physics for hospitality during the completion of this work.
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