Plasma \( \beta \) Dependence of Density, Temperatures, and Magnetic-field Correlations of Mirror Structures: Observation and Theory

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Received 2019 November 14; revised 2020 March 3; accepted 2020 March 23; published 2020 May 12

Abstract

The mirror structures identified by the depressed or enhanced magnetic field associated with the enhanced or depressed plasma density are widely observed in the solar system plasma. These structures are generated by the mirror instability as a result of sufficiently large temperature anisotropy of \( T_\perp > T_\parallel \). Here, \( T_\perp \) and \( T_\parallel \) are, respectively, to be the temperatures perpendicular and parallel to the magnetic field and \( \beta = p/(B^2/2\mu_0) \). Two important observed characteristics are the uneven density-magnetic field compressibility, defined as \( C_{p,B} = (\delta p/\rho)/(\delta B/B) \), among the various mirror events, and the anticorrelation between the temperatures and magnetic field. This study first shows two mirror structures observed in the magnetosheath with distinct \( \beta \) and \( C_{p,B} \). Specifically, \( \beta \sim 6.67, C_{p,B} \sim -0.23 \) and \( \beta \sim 1.44, C_{p,B} \sim -0.67 \). The linear kinetic theory is adopted to derive the phase relations for the mirror instability which show an inverse relation between \( C_{p,B} \) and \( C_{T,B} = (\delta T_\perp/T_\perp)/(\delta B/B) \), which are negative for various parameter values, and \( \beta \). While the correlation \( C_{T,B} = (\delta T_\perp/T_\perp)/(\delta B/B) \) may be negative or positive for the mirror unstable and stable parameter regimes, respectively. The theoretical analyses are consistent with the observational results that \( \delta T_\perp \delta B < 0 \) for both events and \( \delta T_\parallel \delta B < 0 \) is more pronounced for the high \( \beta \) case. The statistical analysis results are summarized by the relationships between \( C_{p,B}, C_{T_\perp,B}, C_{T_\parallel,B}, \) and \( \beta \) for six mirror wave events that show high agreements between the observations and theory.

Unified Astronomy Thesaurus concepts: Space plasmas (1544); Planetarv magnetosphere (997); Solar wind (1534); Alfvén waves (23); Plasma physics (2089)

1. Introduction

Nonlinear waves with anticorrelated density and magnetic field perturbations are widely observed in the solar wind, planetary magnetosheaths, and heliosheath etc. (e.g., Constantinescu et al. 2003; Joy et al. 2006; Burlaga et al. 2007; Soucek et al. 2008; Tsurutani et al. 2011; Schmid et al. 2014). These structures have the typical size of \( 10 \sim 20 \text{~ion~inertial~lengths} \) (e.g., Horbury & Lucek 2009) are usually associated with the temperature anisotropy of \( T_\perp > T_\parallel \) and have been interpreted as the evolutionary results of mirror instabilities (e.g., Russell et al. 1987). Here, \( T_\perp \) and \( T_\parallel \) are the temperatures perpendicular and parallel to the magnetic field, respectively. Alternative theories for the nonlinear structures with anticorrelated density and magnetic field have also been proposed; in particular, the dispersive MHD model may give rise to the slow mode solitons similar to the observed mirror structures (Baumgärtel 1999; Stasiewicz 2004a). These fluid mode solitons are purely nondissipative and do not require the presence of temperature anisotropy, in contrast to the mirror instability, which is nonisotropic and cannot be described by the adiabatic fluid formulation. In particular, the linear mirror instability criterion derived based on the double-adiabatic or Chew–Goldberger–Low (CGL) MHD model (Chew et al. 1956) is inconsistent with the criterion, \( T_\perp/T_\parallel > (1 + \beta_\perp)/\beta_\parallel \), obtained from the kinetic theory (e.g., Vedenov & Sagdeev 1958; Abraham-Shrauner 1967; Southwood & Kivelson 1993; Ferrière & Andréi 2002).

While there have existed many observational and theoretical studies addressing the various aspects of mirror instabilities (e.g., Leubner & Schupfer 2001; Pokhotelov et al. 2002; Passot & Sulem 2006; Borgogno et al. 2007; Hellinger 2007; Génot 2008; Horbury & Lucek 2009; Kunz et al. 2014; Rincon et al. 2015; Riquelme et al. 2015; Huang et al. 2017; Min & Liu 2018; Zhao et al. 2019), two related important issues raised earlier remain understudied, which are the density-magnetic field compressibility, \( C_{p,B} = (\delta p/\rho)/(\delta B/B) \), and the phase relationships between the temperatures and magnetic field, \( C_{T,B} = (\delta T/T)/(\delta B/B) \). In particular, some observed mirror structures appear to have relatively small absolute \( C_{p,B} \) as compared to other cases (Stasiewicz 2004b). The issue of the anticorrelation between the temperatures and magnetic field in the observed mirror structures was raised by Balikhin et al. (2009) and recently examined by Teh & Zenitani (2019) based on the double-polytropic MHD simulations, which show that only for special choices of energy closures the temperatures have anticorrelations with the magnetic field. Recent observations based on the analyses of Magnetospheric Multiscale (MMS) data in the Earth’s magnetosheath also show an anticorrelation between the temperatures and magnetic field for the MHD scale mirror structures (Zhang et al. 2018; Teh & Zenitani 2019). It is interesting to note that anticorrelated relationships between the density, temperature, and magnetic field also exist in electron mirror mode structures as observed by the MMS near the Earth’s foreshock (Yao et al. 2019).

The present study will focus on the density-magnetic field compressibility and the phase relations between the
temperatures and magnetic field of the mirror mode waves based on the observations and kinetic theory. We first show two mirror wave events observed in the Earth’s magnetosheath that have distinct $\beta$ and $C_{p,B}$ values. The study will then apply the linear Vlasov theory to examine the phase relations between the density, temperatures, and magnetic field. The analytical expressions show that both $C_{p,B}$ and $C_{T,B} = (\delta T_i/T_i)/(\delta B/B)$ are negative for various parameter regimes and have an inverse correlation with the plasma $\beta$. While for the parallel temperature the quantity $C_{T,B} = (\delta T_i/T_i)/(\delta B/B)$ may be negative for mirror unstable $\beta$ regimes and positive for mirror stable regimes. The statistical results based on the analyses of six mirror wave events observed in the magnetosheath are shown for the dependence of $C_{p,B}$, $C_{T,B}$, and $C_{T,B}$ on plasma $\beta$ which have high agreements with the theoretical predictions.

2. Mirror Wave Observations

We have identified six mirror wave events observed by the Time History of Events and Macroscale Interactions during Substorms (THEMIS) and MMS spacecraft in the magnetosheath for the analyses of the phase relations. The statistical results will be shown and compared with the theoretical predictions in the following section. For illustration, we first show two mirror events with distinct $\beta$ values. Figures 1 and 2 show the ion density, magnetic field, parallel and perpendicular temperatures, and plasma $\beta$ for the THEMIS (2008 September 12) and MMS (2016 January 16) mirror wave crossing events observed in the Earth’s magnetosheath, respectively. The THEMIS data are measured by the fluxgate magnetometer (FGM; Auster et al. 2008) and the electrostatic analyzer (McFadden et al. 2008). While the MMS data are measured by the FGM (Russell et al. 2016) and the fast plasma investigation (Pollock et al. 2016). The same MMS event is recently analyzed for its possible two-dimensional geometry by Teh (2019). The average $\beta$ values for the ions are $\beta_i = 7.4$ (1.74) and $\beta_e = 5.2$ (0.85) for the THEMIS (MMS) event. In the following calculations the combined $\beta = (2\beta_i + \beta_e)/3$ is used to characterize the plasma $\beta$ value of each case. While the average electron $\beta$ values for the THEMIS (MMS) event are $\beta_{i,e} = 1.35$ (0.28) and $\beta_{e,i} = 1.32$ (0.25). For both events the electron temperatures and anisotropies are thus relatively smaller than the corresponding ion quantities and will not be included in the following analyses. For simplicity the subscripts for ions are also removed from the symbols and expressions. There are 72 data points in both figures and, as indicated, both events have $\delta \rho B < 0$, $\delta T_i B < 0$ and $\delta T_e B < 0$ is relatively clear for the THEMIS event and less pronounced for the MMS event. Quantitative analyses show that the correlation coefficients between $\rho$, $T_i$, $T_e$ and $B$ are $r_{p,B} = -0.92$ ($-0.76$), $r_{\kappa_{T,B}} = -0.56$ ($-0.56$), and $r_{\kappa_{T,B}} = -0.66$ ($-0.02$) for the THEMIS (MMS) event. The estimated averaged perturbed quantities normalized by the corresponding mean values for the THEMIS (MMS) event are $\delta \rho / \rho = 0.18$ (0.24), $\delta B / B = 0.77$ (0.37), $\delta T_i / T_i = 0.12$ (0.14), and $\delta T_e / T_e = 0.2$ (0.12). We may define the density-magnetic field compressibility as $C_{p,B} = (\delta \rho / \rho)/(\delta B/B)$ and the estimated average $C_{p,B}$ are $-0.23$ and $-0.65$, respectively, for the THEMIS event with average $\beta \sim 6.67$ and for the MMS event with average $\beta \sim 1.44$, implying that the absolute average $C_{p,B}$ is larger for smaller average $\beta$ values. Similarly, we may define $C_{T,B} = (\delta T_i/T_i)/(\delta B/B)$ and $C_{T,B} = (\delta T_e/T_e)/(\delta B/B)$. The estimated average values for the THEMIS (MMS) event are $C_{T,B} = -0.16$ ($-0.38$) and $C_{T,B} = -0.26$ ($-0.32$). Note that we may also divide the event intervals into 3 ~ 5 subintervals and performed the analyses for each subinterval, which may have slightly different background values. The average quantities over the subinter-vals are found to be the same as the analysis results based on the entire interval with single background value. Quantitative relationships between the perturbed density, temperatures, and magnetic field will be examined by the linear Vlasov theory in the following section.

3. Vlasov Theory of Mirror Wave Phase Relations

For the analyses of the dispersion relations and the relationships among the fluid quantities and magnetic field, we adopt the same frameworks developed by Hasegawa (1975) and Ferrière & Andrei (2002), which assume an initial bi-Maxwellian velocity distribution $f_0$ with guiding-center drift approximation and the first adiabatic invariant, $\mu = m v^2 / 2B$, for the particle motions. The compressive mode derived from the linear Vlasov equation assuming $\omega / k v_\parallel |v| \ll 1$ and $\omega^2 / k^2 v_\parallel^2 |v| \ll 1$ has the dispersion relation and the mirror instability criteria

$$\frac{\omega}{k |v_\parallel|} = -i \frac{1}{2} \left( \beta_i + \beta_e \right) \frac{k^2}{k^2_0} \left( 1 + \frac{1}{2} (\beta_i - \beta_e) \right)$$

$$+ 1 + \beta_e \left( 1 - \frac{\beta_e}{\beta_i} \right) \left( 1 + \frac{1}{2} \beta_i \right) - \frac{\omega}{k |v_\parallel|} > 0$$

$$\theta = \tan^{-1} \left[ \frac{1}{2 m} \left( 1 - \frac{\beta_e}{\beta_i} + \frac{2}{\beta_i} \right) \right]^{1/2}.$$
mirror growth rate and the instability criteria without assuming $\gamma' \ll 1$. Note that the maximum $\gamma'$ occurs at $\theta = 90^\circ$ and for the two mirror events shown in Figures 1 and 2, $\gamma' \sim 0.1$, implying the validity of the approximation used in obtaining (1) from the more original dispersion relation. Although the mirror criteria are derived based on the linear theory with uniform background plasmas, the mirror condition (2) is usually examined locally. To quantify the degree of mirror instability we define a normalized parameter $m' = m/(\beta_1/\beta_0 + (1 + 1/\beta_0))$ with $m = \beta_1/\beta_0 - (1 + 1/\beta_0)$ as shown in the third panels of Figures 1 and 2. As indicated, $m$ or $m'$ may be positive or negative in some magnetic depressed and enhanced regions, respectively. For both events the absolute $m'$ are much less than

![Figure 1. Data plots of the THEMIS mirror wave event on 2008 September 12. The time interval includes a total of 72 data points. The first panel is the data for ion number density and magnetic field. The second panel is the temperatures including perpendicular and parallel components. The third panel is the perpendicular, parallel plasma $\beta$, and the mirror instability parameters $m = \beta_1/\beta_0 - (1 + 1/\beta_0)$ ($m > 0$ is for the instability) and $m' = m/(\beta_1/\beta_0 + (1 + 1/\beta_0))$. The fourth panel is the density, parallel temperature, and magnetic field correlations, $C_r$, and $C_T$. The intervals bounded by the green vertical lines correspond to the mirror stable regions ($m < 0$).](image-url)
1, implying small deviations from the mirror instability thresholds. For the average $\beta$ values, $m = 0.29$ (0.47) and $m' = 0.11$ (0.13) for the THEMIS (MMS) event. The results that the observed events are close to the marginal mirror conditions are consistent with the results of $\gamma' \ll 1$ and $\delta\rho/\rho$, $\delta T_\perp/T_\perp$, $\delta T_\parallel/T_\parallel$, and $\delta B/B$ being less than 1. The above analyses imply the applicability of the linear kinetic theory and the

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relations shown in (1)–(3) for the observed mirror events shown in Figures 1 and 2.

The phase relations between the density, thermal pressures, and magnetic field may be obtained by taking the zeroth and second-order moments of the perturbed distribution function (Ferrière & Andreì2002) while the temperatures and magnetic field correlations may be derived by using the ideal gas law in
the pressure equations. The relationships between the density, temperatures, and magnetic field have the following forms

$$\frac{\delta \rho}{\rho} = \left(1 - \frac{T_{\perp}}{T_{\parallel}} K\right) \frac{\delta B}{B} - iK \frac{e \delta E_i}{k_i T_{\parallel}}$$  \hspace{1cm} (4)

$$\frac{\delta T_{\perp}}{T_{\perp}} = \left(1 - \frac{T_{\parallel}}{T_{\perp}} K\right) \frac{\delta B}{B} + i(K - K_i) \frac{e \delta E_i}{k_i T_{\parallel}}$$ \hspace{1cm} (5)

$$\frac{\delta T_{\parallel}}{T_{\parallel}} = \frac{T_{\perp}}{T_{\parallel}} (K - K_i) \frac{\delta B}{B} + i(K - K_i) \frac{e \delta E_i}{k_i T_{\parallel}},$$ \hspace{1cm} (6)

for which

$$K = -\frac{1}{n_0} \int \frac{k_i v_i}{\omega - k_i v_i} f_i(\omega) \, d\omega,$$

and $K_\perp = K, K_\parallel = 1 + (\omega/k_i |k_i|^2)K$. For low-frequency limit $|z| \ll 1$, where $z = -i \omega / \sqrt{2 \omega v_i |k_i|}$,

$$K \simeq 1 + i \left(\frac{\omega}{2} \frac{\omega}{v_i |k_i|}\right)^z.$$ \hspace{1cm} (7)

The parallel electric field $\delta E_i$ in Equations (4) and (6) can be obtained from the condition of charge neutrality and for electron–proton plasma has the following form

$$\frac{\delta E_i}{k_i T_{\parallel}} = - f_E \frac{T_{\perp} \delta B}{T_{\parallel} B},$$ \hspace{1cm} (8)

where

$$f_E = \left(1 - \frac{T_{\parallel}}{T_{\perp}} \frac{T_{\parallel}}{T_{\perp}} K \right) \left(1 + \frac{T_{\parallel}}{T_{\perp}} K \right).$$

As seen in Equations (4) and (6), the quantity $f_E$ measures the relative effects of $\delta E_i$ in these relations. For low-frequency limit, $z \ll 1$ and $K_v \to 1$, the estimated $f_E$ is about 0.058 (0.1) which is negligible as compared to one for the THEMIS (MMS) event. As a result, $\delta E_i$ terms are not included in the following analyses and the perpendicular temperature-magnetic field correlation $C_{T,B}$ is identical to the density-magnetic field correlation $C_{\rho,B}$. By substituting Equation (1) into (7), the density and magnetic field relation in Equation (4) may be written as follows

$$\frac{\delta \rho}{\rho} = -\frac{1}{\beta_i} \frac{\delta B}{B} \left[1 + \cot^2 \theta \left(1 + \frac{1}{2} (\beta_i - \beta) \right) \right] \delta B.$$ \hspace{1cm} (9)

The above relation implies that $C_{\rho,B} = (\delta \rho/\rho) / (\delta B/B)$ is negative for $\beta_i > \beta$ and has a clear inverse correlation with $\beta_i$. The absolute $C_{\rho,B}$ values decrease with increasing propagation angle $\theta$. For $\theta \geq \theta_c$ (Equation (3)), $C_{\rho,B} = -\alpha/\beta_i$, where $\alpha = (\beta_i - \beta) / (\beta_i / \beta_i)$ and $\alpha = 1$ corresponds to the mirror instability threshold. For $\theta \simeq 90^\circ$, the simple relation $C_{\rho,B} = -1/\beta_i$ is obtained. As a result, for $\theta_c < \theta < 90^\circ$ $C_{\rho,B}$ is within the range of $1/\beta_i$ and $-\alpha/\beta_i$ with $\alpha$ being greater than 1 for the unstable case. The last panels in Figures 1 and 2 show the $C_{\rho,B}$ values for the entire events, indicating the large variations in $C_{\rho,B}$ caused by the local conditions. Note the $C_{\rho,B}$ values in the figures is for $\theta = 70^\circ$ and the results for $\theta > 65^\circ$ show similar values. For comparisons with the observations, two sets of average $\beta$ values ($\beta_i = 7.4, \beta = 5.2$ and $\beta_i = 1.74, \beta = 0.85$), representing the average values for the THEMIS and MMS events, respectively, are used in Figure 3 for plotting $C_{\rho,B}$ as functions of $\theta$ (panel (a)). The points denoted by a triangle symbol on each curve correspond to $\theta = \theta_c$ and the observed values are denoted by the horizontal dashed lines. As indicated, the curve with larger $\beta$ values has smaller absolute $C_{\rho,B}$ values for all propagation angles. The observed values fall within the ranges of $\theta_c < \theta < 90^\circ$. As also shown, for $\theta > 65^\circ$ the $C_{\rho,B}$ values vary only slightly and are close to the observed values that the observed mirror structures are possibly highly oblique, consistent with the prior analysis results (e.g., Chisham et al. 1999).

Since $C_{T,B} = C_{\rho,B}$, the perpendicular temperature is anticorrelated with the magnetic field for various parameter regimes and $C_{T,B}$ has an inverse relation with the plasma $\beta$. The anticorrelations between $T_{\parallel}$ and $B$ in the observations shown in Figures 1 and 2 however are not as pronounced as the density and magnetic field. Also shown in Figure 3(a) by the horizontal dashed lines are the estimated observed $C_{T,B} = -0.16$ (0.38) for the THEMIS (MMS) event, indicating the consistent results for the inverse $\beta$ dependence and the quantitative agreements between the theory and observations at large propagation angles.

For the parallel temperature, the following relation is obtained by using (1) for $K$ and $K_i$ in (6)

$$C_{T,B} = -\frac{\beta}{\beta_i} \left(\frac{\sqrt{\pi}}{2} - \gamma^\prime\right)^\gamma$$ \hspace{1cm} (10)

where, as defined earlier, $\gamma^\prime = -i \omega / k_i v_i$ and $\gamma > 0$ is for the instability. It is clear that $C_{T,B}$ may change sign at the mirror threshold $\theta = \theta_c$ and $\gamma^\prime = 0$. For $\theta = 90^\circ$, $\gamma^\prime = -\sqrt{2 / (\pi (\beta_i / \beta_i^2)) [1 + \beta_i (1 - \beta_i / \beta_i)]}$. As a result, $C_{T,B} < 0$ occurs for $\theta_c < \theta < 90^\circ$. Also shown in the last panels of Figures 1 and 2 are the $C_{T,B}$ values for the entire events. As indicated, $C_{T,B} < 0$ for most of the event intervals while $C_{\rho,B}$ may be positive in certain enhanced magnetic field regions. In particular, the sign of $C_{T,B}$ is in accordance with the $m$ curve shown in the third panels for the mirror instability criterion. Note that in the regions where $B$ is depressed and $m > 0$, $\delta T_{\parallel} < 0$, while for the regions with enhanced $B$ and $m < 0$ the anticorrelation between $T_{\parallel}$ and $B$ is absent. The correlation coefficients between $T_{\parallel}$ and $B$ are rather small for the MMS event due to the fact that the event has low average $\beta$ values to remain unstable to the mirror instability for the entire event. Figure 3(b) shows $C_{T,B}$ versus $\theta$ for two sets of $\beta$ values and the corresponding observed average values. As indicated, for both events the estimated observed values are in good agreement with the theoretical predictions for $\theta \simeq 70^\circ$.

We have identified six mirror events in the magnetosheath that all satisfy the assumptions of low frequency and long wavelength, cold electrons, negligible finite ion gyroradius, and inertia effects, $\gamma^\prime < 1, m^\prime < 1$ and $f_E < 0$ etc. used in (1)–(5), (9) and (10) for the analyses of mirror structures. The events shown in Figures 1 and 2 have distinct average $\beta$ values for illustrating the effects of $\beta$ on the phase relations. The statistical results based on four THEMIS (2007 July 30, August 11, 2008 September 4, 12) and two MMS (2016 January 10, 16) events
are shown in Figure 4 and the two events shown in Figures 1 (2008 September 12) and Figure 2 (2016 January 16) are marked by the triangle and square symbols, respectively. Figure 4(a) show the calculated $C_{\rho B}$ and $C_{T_e B}$ values versus plasma $\beta$ for various propagation angles. The solid curves are the theoretical predictions based on (9), indicating that in all events $C_{\rho B}$ and $C_{T_e B}$ are negative and their values are bounded approximately by the curves with $60^\circ < \theta < 85^\circ$. There is an inverse dependence of the absolute $C_{\rho B}$ and $C_{T_e B}$ values on the plasma $\beta$ (or $\beta_\perp$) which are in agreement with the theoretical predictions. The observed $C_{\rho B}$ values have wider ranges of $-0.16 \sim -0.88$ as compared to the observed $C_{T_e B}$ values of $-0.16 \sim -0.55$. In particular, the observed $C_{\rho B}$ are in high agreement with the theory for all cases while the observed $C_{T_e B}$ deviate more from the theoretical predictions for low $\beta$ cases. Figure 4(b) shows the corresponding $C_{T_e B}$ versus $\beta$ for various propagation angles, indicating the good agreement between the observations and theoretical predictions (solid curves). In particular, both the theory and observations show the same tendency that $C_{T_e B}$ vary sharply in the low $\beta$ regimes and becomes nearly the same for $\beta \gtrsim 3$.

In the above analyses of phase relations the dispersion relation shown in (1) valid for low-frequency and long-wavelength limits is used. It is theoretically interesting to incorporate the FLR effects in the derivation by replacing $\gamma$ with $\gamma_{FLR} = \gamma' - (2/\pi)^{1/2}(\beta/\beta_\perp)((3/2)(r_{ci}k_i)^2)$ with the maximum growth rates occurring at $r_{ci}^2k_{i,m}^2 \sim [\beta_\perp(T_e/T_i - 1)] - 11/6$ (Hellinger 2007), where $r_{ci}^2 = \beta_\perp r_{ci}^2(T_e/T_i - 1)$, $r_{ci} = m_i v_{i,m}/eB$ and $v_{i,m} = (2k_B T_e/m_i)^{1/2}$. The modified temperatures and magnetic field relations $C_{T_e B} = -[1 + (3/2)r_{ci}^2k_i^2 - \cot^2 \theta(1 + (\beta_\perp - \beta)/2)/\beta_\perp]$ and $C_{T_e B} = -\beta_\perp/\beta_\parallel(\sqrt{\pi/2} - \gamma_{FLR})\gamma_{FLR}$ imply that $r_{ci}^2k_i^2$ may result in more negative $C_{T_e B}$ (or $C_{\rho B}$) and less negative $C_{T_e B}$. For the six mirror events shown in Figure 4, both $r_{ci}^2k_i^2$ and $r_{ci}^2k_{i,m}^2$ increase with increasing $\beta$ and are in the ranges of $0.18 \sim 0.28$ and $0.08 \sim 0.37$, respectively. In particular, for the above shown THEMIS (MMS) event with $\beta = 6.67(1.44)$ and $\theta = 71^\circ(73^\circ)$, $r_{ci}^2k_i^2 = 0.24 (0.11)$ and $r_{ci}^2k_{i,m}^2 = 0.36 (0.14)$. The FLR correction to $\gamma'$ is about $-0.028 (-0.036)$ as compared to $\gamma' \sim 0.16 (0.18)$ and the calculated $C_{T_e B}$ and $C_{T_e B}$ are about $-0.22 (-0.75)$ and $-0.21 (-0.34)$, respectively, which are mild corrections to the values of $C_{T_e B} = -0.17 (-0.68)$, $C_{T_e B} = -0.22 (-0.32)$ for $r_{ci}^2k_i^2 = 0$ shown in Figure 4.

4. Discussion and Summary

In this study we have carried out quantitative analyses of density, temperatures, and magnetic field correlations for six observed mirror wave events and compared with the linear kinetic theory of mirror instabilities. The statistical analysis results are summarized in Figure 4 and the detailed analyses are illustrated by two events with distinct plasma $\beta$ shown in Figures 1–3. The analytical expression (9) shows explicitly an inverse correlation between the density-magnetic field compressibility $C_{\rho B}$ and plasma $\beta$ as also revealed by the observations shown in Figures 3 and 4. Note that $C_{\rho B} < 0$ for various parameter regimes and the same relation is applied for the correlation between $T_\perp$ and $B$ that $C_{T_e B}$ is negative and
possesses an inverse relation with $\beta$, a result also consistent with the observations shown in Figures 3 and 4. The observed result of $\delta T_1 \delta B < 0$ has been interpreted as the breakdown of the first adiabatic invariant $\mu$ in the context of kinetic theory (Balikhin et al. 2009) and the breakdown of the adiabatic energy law, $T_\perp/B = C_\perp$, in the context of fluid models with temperature anisotropy (Hau & Wang 2007). The constancy of $\mu$ and $C_\perp$, however, are not physically the same. As shown by Ferrière & Andreï (2002), the double-adiabatic laws are incompatible with the expressions relating the thermal pressures, density, and magnetic field in the hybrid-kinetic model (see Equations (27) and (28) of Ferrière & Andreï 2002). Our theoretical analyses for the phase relations are based on the same framework developed by Ferrière & Andreï (2002), which assumes the first adiabatic invariant for the particle motions in the Vlasov equation. The result of $\delta T_1 \delta B < 0$ is shown explicitly for various $\beta$ values, which is not inconsistent with the assumption of the first adiabatic invariant. We have also attempted to incorporate the FLR effects (Pokhotelov et al. 2004; Hellinger 2007) in the phase relations and found that the corrections may yield more negative $C_{T_1,B}$, which is consistent with the analysis by Balikhin et al. (2009) on the role of FLR in the phase relation between $T_\perp$ and $B$. The FLR effects are negligible for the mirror events with wavelengths much larger than the ion scales shown in this study but may need be considered for proton scale mirror structures (Zhao et al. 2019).

Note that the results of $\delta T_1 \delta B < 0$ have been confirmed by the hybrid particle simulations (McKean et al. 1993) and the fluid simulations with FLR effects (Passot & Sulem 2008).

We have also shown that the correlation between $C_{T_1,B}$ and $\beta$ may be negative or positive in the mirror unstable and stable regimes, respectively. For the observed mirror structures, the $\beta$ values may vary in a wide range that the mirror unstable and stable regions may possibly coexist in the same event (e.g., Balikhin et al. 2009; Zhang et al. 2018). Under these circumstances, the correlation coefficients between $T_\parallel$ and $B$ may be relatively small, as evidenced in the MMS event shown in Figure 2 with relatively low $\beta$ values. For high average $\beta$ cases, which are mirror unstable for most regions like the THEMIS event shown in Figure 1, the anticorrelations between $T_\parallel$ and $B$ are more clearly shown. The comparisons between the theory and observations shown in Figures 3 and 4 show consistent results for the dependence of $C_{T_1,B}$ on plasma $\beta$.

We acknowledge V. Angelopoulos and NASA contract NAS5-02099 for the use of data from the THEMIS Mission through AIDA at Institute of Space Science, National Central University in Taiwan (data available at http://themis.ss.ncu.edu.tw/e_data_download.php). We thank the MMS team for providing the data available from the MMS Science Data Center (https://lasp.colorado.edu/mms/sdc/). This research is supported by the Ministry of Science and Technology of Taiwan (R.O.C.) under grant MOST107-2111-M-008-029-MY2 to National Central University.

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