End state of gravitational collapse and the related cosmic censorship conjecture continue to be amongst the most important open problems in gravitational physics today. My purpose here is to bring out several aspects related to gravitational collapse and censorship, which may help towards a better understanding of the issues involved. Possible physical constraints on gravitational collapse scenarios are considered. It is concluded that the best hope for censorship lies in analyzing the genericity and stability properties of the currently known classes of collapse models which lead to the formation of naked singularities, rather than black holes, as the final state of collapse and which develop from a regular initial data.

Keywords: gravitational collapse, naked singularities, black holes

1. Introduction

One of the most outstanding problems in the relativistic astrophysics and gravitation theory today is the final fate of a massive star, which enters the state of an endless gravitational collapse once it has exhausted its nuclear fuel. What will be the end state of such a continual collapse which is entirely dominated by the force of gravity? The conjecture that such a collapse, under physically realistic conditions, must end into the formation of a black hole is called the cosmic censorship hypothesis (CCH). Despite numerous attempts over the past three decades, such a conjecture remains unproved and continues to be a major unsolved problem, lying at the foundation of the theory and applications in black hole physics.

Considering the failure of many attempts to establish the censorship conjecture, it would seem natural to arrive at the conclusion that what is really necessary here is to understand better and in a more extensive manner the actual dynamical gravitational collapse process within the framework of general theory of relativity. Such efforts have taken place over the past decade or so, and the conclusion that is emerging is that the final fate of a continual collapse would be either a black hole (BH), or a naked singularity (NS), depending on the nature of the regular initial

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data, from which the collapse develops evolving from an initial spacelike surface. For a discussion on various aspects of this problem and for some recent reviews, we refer to [1-4], and also [5] for a detailed technical discussion on CCH).

We need to clarify here what we mean by occurrence of a naked singularity developing as end point of collapse. At times, the non-existence of trapped surfaces till the formation of the singularity in collapse is taken as the signature that the singularity is naked. However, this need not be the case (see e.g. [1-2] for details). What we mean by the development of a naked singularity in collapse is that there exist families of future directed non-spacelike curves, which in the past terminate at the singularity. No such families exist originating from the singularity when the end product of collapse is a black hole. In the case of a black hole forming, the resultant spacetime singularity will be hidden inside an event horizon of gravity, remaining unseen by external observers. On the other hand, if the collapse ends in a naked or visible singularity, there is a causal connection between the region of singularity and faraway observers, thus enabling in principle a communication from the super dense regions close to singularity to faraway observers.

My purpose here is to examine and discuss several issues involved here which clarify the implications of some of the work in this area done so far, and to see where we stand as far as the censorship hypothesis is concerned. We also point out, by drawing on examples from the models analyzed so far, that it is not possible to rule out the occurrence of naked singularities in collapse just based on simple physical reasonings, and that the problem is deeper. The point of view that emerges is as far as the occurrence is concerned, both black holes and naked singularities appear to be basic properties consequent from the dynamics of Einstein equations, emerging in a natural manner and as a logical consequence of the general theory of relativity. However, the crucial issue is that of genericity and stability of naked singularities. It would appear that the real hope for censorship lies in investigating in detail the stability properties of the collapse models which develop into naked singularity.

In the next section, we consider several physical conditions that one may like to impose on a physically realistic gravitational collapse scenario, and we point out the status of CCH vis-a-vis such constraints. In Section 3 we discuss three further possibilities, which I think offer a more serious avenue as far as the CCH is concerned. In the concluding section, it is emphasized that it might help to try to get a better insight into the phenomenon of naked singularity formation, that is, we may try to understand better why actually naked singularities develop in gravitational collapse.

2. Physical Constraints on Gravitational Collapse

As noted above, in any general discussion on CCH, it is stated that naked singularities do not form in collapse under physically realistic conditions. However, the precise physical conditions under which the censorship is supposed to be holding are usually not specified. The advantage then is that even if a certain set of physical
conditions did not work towards proving CCH, one still has the option to try out another set of physical constraints to continue further efforts.

If by such a procedure we are to prove censorship in some suitable form, we have to eventually arrive at a proper and appropriate mathematical formulation of the censorship conjecture we want to establish. As of today, a suitable formulation of CCH is a major problem in itself. Many natural looking physical conditions have been proposed and tried out, these being indicated as a remedy to rule out naked singularities. This is with the hope to arrive ultimately at a suitable formulation of CCH.

We examine below several such physical constraints on a realistic gravitational collapse scenario, and the implications they have towards determining the final fate of collapse. In particular, the motivation is to rule out NS as final state by imposing such conditions. It turns out that one still does not succeed in ruling out NS with the help of such conditions considered so far. But the advantage of such an analysis is, firstly, it clarifies the situation as to what such conditions can possibly achieve. Secondly, it serves as a pointer to something deeper we should look for if we are to establish CCH. Finally, this also implies that in fact NS do develop in wide classes of gravitational collapse scenarios under realistic physical conditions. We also try to get an insight here why many of such conditions have not worked, or are unlikely to work towards establishing CCH, and why we must explore further more subtle alternatives. Eventually, we point out that the hope for CCH appears to lie in a detailed genericity and stability analysis only of the available collapse models which result in a naked singularity formation.

2.1. A suitable energy condition must be obeyed

This is one of the basic conditions assumed in the classical gravity description, and should be satisfied by the matter fields constituting the star at least till the collapse has proceeded to such an advanced stage so as to enter a phase governed by quantum gravity, that is, till the classical description starts breaking down in one way or the other.

In fact, if one allowed for completely arbitrary matter fields, it will be quite easy to produce naked singularities. For example, start with a geometry allowing families of future directed non-spacelike geodesics, which are future endless, but terminate in the past at the singularity. Then define the matter fields to be given by,

\[ T_{ij} = \frac{1}{8\pi} G_{ij}. \]  

Hence, it is obvious that one must consider scenarios where matter fields do satisfy reasonable physical conditions. One would hope that a suitable energy condition would be one of these, as all observed classical fields do obey such a condition. A further motivation would be energy conditions have been used extensively in the
singularity theorems in general relativity, which predict the existence of singularities in gravitational collapse and cosmology.

One would like to see if CCH is obeyed once we have assumed matter fields to satisfy suitable energy conditions. It turns out, however, that there are several classes of collapse models, where in fact collapsing matter does satisfy a proper energy condition, but the collapse does end in a naked singularity (henceforth, when additional references are not given then [1-4] and [5] may be looked up for details).

Actually, there are classes of collapse models where satisfying the energy condition appears to be aiding the naked singularity formation as the end state of collapse, in turn making it physically more interesting and serious. An example of this is the spherically symmetric self-similar collapse of a perfect fluid. The general form of metric is,

$$ds^2 = -e^{2\nu(r,t)}dt^2 + e^{2\psi(r,t)}dr^2 + r^2 S^2(r,t)(d\theta^2 + \sin^2\theta d\phi^2) \quad (2)$$

where the metric functions depend on $X = t/r$ due to self-similarity. One can work out the outgoing null geodesics from the naked singularity, which turn out to be related to the density and pressure distributions in the spacetime via the Einstein equations. These are then given by,

$$r = D(X - X_0)^{\frac{2}{H_0}} \quad (3)$$

Here $H_0$ is the limiting value of the quantity $H = (\eta + p)e^{2\psi}$, $\eta$ and $p$ correspond to density and pressure, and $D > 0$ is constant of integration. The weak energy condition is then equivalent to the statement that $H_0 > 0$, which in turn ensures, from the above geodesics equation, that families of null geodesics, as opposed to single isolated curves, come out from the naked singularity at $t = 0, r = 0$, which is a node in the $(t, r)$ plane.

It has to be noted that the Einstein equations as such do not require or impose an energy condition on matter distributions. It is a criterion motivated on purely physical grounds. This then suggests another possibility, namely that if some how in the later stages of collapse the energy conditions are violated through whatever agency, there may then be a hope to preserve CCH. What we mean is, for example in the models discussed above, in the equation for null geodesics if we violate energy conditions that would be corresponding to a negative value of $H_0$, and then there are no outgoing null geodesics from the singularity and CCH is essentially preserved. It may be noted here that for a quantum fluid or a quantum field, which may have important role to play in the very late stages of collapse, the weak energy condition need not be obeyed, and it may be worth speculating if this some how may help save cosmic censorship.

2.2. The collapse must develop from regular initial data

This is one of the most important physical constraints necessary for any possible version of CCH. If we are to model realistic collapse scenarios of matter clouds such
as gravitationally collapsing massive stars, then the densities, pressures and other physical quantities must be finite and regular at the initial surface from which the collapse develops. That is, the initial spacelike surface should not admit any density or curvature singularities in the initial data so as to represent collapse from regular matter distribution.

Generally, this is ensured by imposing the usual differentiability conditions on the functions involved, together with requirements of finiteness and regularity. It is known by now that regular distributions of initial densities and pressures (for example, finite and suitably differentiable on the initial surface) do give rise to both naked singularities and black holes, depending on the nature of the regular initial data from which the collapse evolves (see e.g. [4] for more details). It turns out that given such an initial data, there are still sufficient number of free functions available to choose in Einstein equations, subject to weak energy condition and a suitable matching to the exterior of the collapsing cloud, so that the evolution can end in either of the BH/NS outcomes as desired.

At times, more stringent requirements are imposed on the initial data, e.g. asking for a complete smoothness of densities and pressures. Usually, there are two motivations for this. One could be the requirements while doing numerical evolutions, where smoothness (which is the same as demanding the analyticity of these functions) simplifies the analysis considerably. At other times, it is argued that astrophysically reasonable initial data must be analytic. In the case of collapse of a dust cloud, this amounts to demanding analyticity of the density function. The initial density $\rho(r)$ then must contain no odd powers in $r$, and we have,

$$\rho(r) = \rho_0 + \rho_2 r^2 + \rho_4 r^4 + \ldots$$

at the initial surface $t = t_i$, which gives an analytic density profile. It is known, however, that even in the case of smooth density profiles with only even terms non-vanishing, the marginally bound dust evolution can end in a naked singularity (for example, when $\rho_2 \neq 0$), which is gravitationally strong, i.e. sufficiently fast divergence of curvatures does take place in the limit of approach to the singularity.

### 2.3. Singularities from realistic collapse must be gravitationally strong

This has been one of the most useful physical requirements, which was explored rather thoroughly in order to develop a formulation for CCH. The idea has been that any singularity that will develop from a realistic collapse has got to be physically serious in various aspects, including powerful divergences in all important physical quantities such as densities, pressures and curvatures etc, at least at the classical level. A typical condition for the singularity to be gravitationally strong is, in addition to the divergences such as above, the gravitational tidal forces must diverge and all physical volumes are crushed to zero size in the limit of approach to the NS. A sufficient condition for this to happen is,

$$R_{ij} V^i V^j \propto \frac{1}{k^2}$$
where $k$ is the affine parameter along the non-spacelike geodesics coming out from the singularity, with $k = 0$ at the NS, and $V^i$ is the tangent vector to these curves emanating from NS. Another criterion for strength was given recently by Nolan\textsuperscript{6}.

The singularity developing within the black hole formed out of the standard dust cloud collapse as investigated by Oppenheimer and Snyder is gravitationally strong in this sense. Now, if one could establish that whenever naked singularities formed in gravitational collapse, they are always gravitationally weak, in the sense of important divergences such as above not being present in the limit of approach to the singularity, then such singularities could be removable from the spacetime, and one may be able to extend the spacetime through the same\textsuperscript{5}. Such removable naked singularities should no longer be regarded as physically genuine, and one has then established CCH in some form such as NS could develop in gravitational collapse, however, they would be always gravitationally weak and removable.

This possibility has been investigated thoroughly, and it is known now that gravitationally powerfully strong naked singularities actually do result from collapse from regular initial data (including smooth analytic density profiles), for several reasonable forms of matter such as dust, perfect fluids, Vaidya radiation collapse, and several other forms of matter satisfying suitable energy conditions. At such naked singularities, the densities, curvature scalars such as the Kretschmann scalar and gravitational tidal forces diverge most powerfully as characterized above, which is as powerfully strong as the divergences observed at physical singularities such as the big-bang in cosmology.

2.4. The matter fields must be sufficiently general

If NS formed in the collapse for certain special forms of matter only, such as dust or collapsing radiations, that would not be much of interest. For example, the role of pressures cannot be underestimated in realistic collapse and so one would like to know if matter with pressures will necessarily give rise to a black hole only on undergoing gravitational collapse. If such was the case, one could then rule out matter fields giving rise to NS as special or unphysical towards formulating CCH, even if they satisfied an energy condition or the collapse developed from regular initial data.

It is now known, however, that naked singularities are not special to any particular form of matter field. One can study the collapse for a general form of matter, the so called type I matter fields (all the known physical forms of matter, such as dust, perfect fluids, massless scalar fields etc are included in this class) subject to weak energy condition. The result is, given an arbitrary but regular distribution of matter on the initial surface, there are always evolutions available from this initial data which would either result in a black hole or naked singularity, depending on the allowed choice of free functions available from Einstein equations. More specifically, in spherically symmetric collapse with a type I general matter field, given the distribution of density and the radial and tangential pressure profiles on the initial
surface, from which the collapse develops, one can then choose the free function describing the velocities of the in-falling shells in such a manner so as to have a black hole or a naked singularity as the final end product, depending on this choice (see [4] and references therein for details).

2.5. The collapsing cloud must obey a realistic equation of state

It is conjectured at times that even though naked singularities may develop for general matter fields, they must go away once a physically reasonable and realistic equation of state is chosen for the collapsing cloud.

This is a very difficult argument to formulate as it turns out. Firstly, naked singularities do form in collapse of several well-known equations of state, such as dust, perfect fluids, or in-flowing radiation shells. Secondly, it is extremely difficult to make any guesses as to what might be the state of matter, or the realistic equation of state within a collapsing body such as a massive star which is in its advanced stages of collapse. Thirdly, the collapsing cloud may not have a single equation of state, which might actually be changing as the collapse evolves. There have been speculations, for example, that strange quark matter may be a good approximation to the collapsing star in its final stages, and the collapse was then examined in a Vaidya geometry, which again results in BH/NS phases as usual. In other words, such a choice of equation of state also does not remove naked singularities. At the other extreme, there are also arguments such as those given by Hagerdorn and Penrose, that the equation of state, in the very final stages of collapse much closely approximate that of dust. In other words, at higher and higher densities, matter may behave more and more like dust. The point is, if pressures are not negative then they also may contribute positively to collapse just to add to the dust effect, and may not alter the conclusions arrived at in the dust case. In such a case, the dust collapse situation, which has been investigated rather thoroughly, would imply that both BH/NS phases would clearly develop in gravitational collapse, depending on the initial density and velocity distributions.

The point is, while there are several widely used and familiar equations of state available which result in the formation of naked singularities as final fate of collapse, there is still not a single equation of state available so far which definitely ensures that the end product will be necessarily a black hole only. Under the situation, one cannot help hazarding a guess that the crux of the matter may not lie in the equation of state or the form of matter collapsing, and over-emphasizing that particular option might amount eventually to barking under a wrong tree as far as the search for CCH is concerned, given the severe uncertainties on the state of the matter in the very late stages of collapse, as described above.

2.6. All radiations from naked singularity must be infinitely red-shifted

In certain sub cases of dust collapse resulting in naked singularity, it is seen that the red shift along the null geodesics coming out from the NS diverges in the limit.
of approach to NS. This has given rise to the possibility that even if NS forms in
collapse, no energy could escape from the same. In that sense, NS may be invisible to
external observers for all practical purposes. Of course, it has to be noted that even
if true, this does not save CCH in the actual sense, because after all and basically
CCH is about the question of principle in the general theory of relativity, namely
whether singularities forming in gravitational collapse are causally connected to an
external observer or not via nonspacelike trajectories.

In any case, it may be good to explore such a possibility, because it may give
some information on the structure of NS at least in certain special models, and if
ture generally, then it will provide some kind of a physical formulation for CCH.
However, in my own perception, it will be extremely difficult to establish in general
that no energy can come out from NS. There could be several reasons for this.
Firstly, it may be quite tricky to apply the conventional definition of red shift,
which corresponds to a regular source and observer, to emissions from a naked
singularity. Secondly, even if there was no escape of energy along null geodesics,
the possibility of mass emission via timelike or non-spacelike non-geodetic families
of paths coming out from the naked singularity remains open. In the case of such a
violent event being visible, particles escaping with ultra relativistic velocities cannot
be ruled out from the neighborhood of NS.

Apart from such technical difficulties, it is also to be noted that the classical
possibilities such as above regarding the probable light or particle emission, or oth-
erwise, from a naked singularity may not perhaps offer a serious physical alternative
eventually one way or the other. The reason is, in all physical situations, the clas-
sical general relativity should break down once the densities and curvatures are
sufficiently high so that quantum or quantum gravity effects should become impor-
tant in the process of an endless collapse. Such quantum effects would come into
play much before the actual formation of the classical naked singularity, which itself
may possibly be smeared out by quantum gravity. The key question then is that of
the possible visibility, or otherwise, of these extreme strong gravity regions, which
develop in any case, in the vicinity of the classical naked singularity. It is then the
causal structure, that is, the communicability or otherwise, of these extreme strong
gravity regions that would make the essential difference as far as the physical con-
sequences of a naked singularity are concerned, rather than aspects such as classical
red shift\textsuperscript{10}.

3. Other Alternatives

As we noted above, none of the physical conditions, such as those discussed above,
are quite able to effectively rule out naked singularities, which in turn may lead us
to some possible formulation of CCH, either physical or a mathematical one. With
each of the constraints such as above, we have counter-examples which obey such a
physical constrain but produce NS as end state of dynamical collapse.

This brings us to three further possibilities which are under active current in-
vestigation as of today towards a possible formulation of CCH, and which I think may offer a better hope for CCH. We now briefly discuss these below.

3.1. **Will quantum gravity remove naked singularities?**

It is sometimes argued that after all the occurrence of singularities is a classical phenomena, and that whether they are naked or covered should not be relevant - quantum gravity will any way remove them all. But this is missing the real issue it would appear. It is possible that in a suitable quantum gravity theory the singularities will be smeared out (though this has been not realized so far, and also there are indications that in quantum gravity also the singularities may not go away). However, in any case, the real issue is whether the extreme strong gravity regions formed due to gravitational collapse are visible to faraway observers or not. Because collapse certainly proceeds classically till the quantum gravity starts governing the situation at the scales of the order of Planck length, that is till the extreme gravity configurations have developed due to collapse. It is the visibility or otherwise of such regions that is under discussion.

The point is, classical gravity implies necessarily existence of strong gravity regions, where both classical and quantum gravity come into their own. In fact, as pointed out by Wald [2], if naked singularities do develop in gravitational collapse, then in a literal sense we come face-to-face with the laws of quantum gravity whenever such an event occurs in the universe. Then collapse phenomena has the potential to provide us with a possibility of actually testing the laws of quantum gravity.

In the case of a black hole developing in the collapse of a finite sized object such as a massive star, such strong gravity regions have got to be necessarily hidden behind an event horizon of gravity, which would be well before the physical conditions became extreme. Then the quantum effects, even if they caused qualitative changes closer to singularity, will be of no physical consequences. This is because no causal communications are then allowed from such regions. On the other hand, if the causal structure were that of a NS, communications from such a quantum gravity dominated extreme curvature ball would be visible in principle, either directly or via secondary effects such as shocks produced in the surrounding medium.

3.2. **Should one consider all naked singularities produced by matter fields to be unphysical?**

There has been a suggestion\textsuperscript{11} that all naked singularities, whenever they are produced by matter fields such as dust, perfect fluids etc should be rejected as being only ‘matter singularities’, which should have nothing to do with pure gravity. From such a perspective, the NS caused by massless scalar fields will be of course worrisome, which is included in the type I matter fields discussed above. While realistic stars are not made up of matter fields such as massless scalar field, may be in the very final stages of collapse such matter forms may have important role to play in
It must be admitted, however, that not all will be comfortable with rejecting outright the logical consequences of collapse studies involving matter forms such as dust, perfect fluids, and such other fields. After all, the classic gravitational collapse scenario, really at the foundation of black hole physics and its chief motivator, is the homogeneous dust collapse model, as studied by Oppenheimer and Snyder\textsuperscript{12}. Now, in the same models, when one puts in a density perturbation at the center, then a naked singularity results, rather than a black hole. The structure of the event and apparent horizons then change drastically so as to expose the singularity to an external observer. Now, all realistic stars will have a higher density at the center, falling off as some rate as one moves away from the center. In that sense, one may want to regard the NS developing due to this density gradient at least as physical as the black hole. After all, general relativists have worked with dust and perfect fluids for several decades, and could be quite comfortable with the logical outcomes available within those collapse scenarios\textsuperscript{13}. Again, if one considers arguments such as those given above in Sec. 2.5 in favour of equations of state such as dust in the final phases of collapse, one may like to take the outcomes of such a collapse physically more seriously.

### 3.3. Are naked singularities stable and generic?

In my own opinion, this is the key issue on which any possible future formulation and proof of the CCH would crucially depend. Even if naked singularities do develop in collapse models, if they were not generic and stable in some suitably well defined sense, that would make a good case for CCH. For example, most of the current classes of NS are within the framework of spherically symmetric collapse. While there are some indications that NS do develop in non-spherical collapse as well\textsuperscript{14}, as such non-spherical collapse remains a largely uncharted territory and it would be essential to examine it rather thoroughly. In this connection, it is also to be noted that naked singularities formed in the collapse of matter with positive energy are always ‘massless’, in a sense obvious for spherical collapse, which is yet to be made precise generically. Of course, even in that case, one still needs to worry about the extreme high densities and curvatures in that region that is visible as opposed to the BH case.

The key question one may then want to resolve here is, while we know that physically reasonable initial data do give rise to naked singularities, will the initial data subspace, which gives rise to NS as end state of collapse have a zero measure in a suitable sense? As is well-known, however, the stability theory in general relativity is a rather complicated area, because there is no well-defined formulation or criteria to test stability. Before one could test CCH, a satisfactory formulation for stability criterion has to be arrived at within the framework of general relativity. Also, the issue of what is a suitable measure in the initial data space can be a complicated one. Only after making some reasonable progress here one could then start testing
these questions for NS formation. While discussing stability and genericity, one has also to be careful on the criterion one used to test the same, because sometimes a criterion can be used which makes black holes also unstable while trying to show the instability of naked singularities.

In the absence of such well-defined criteria against which to test the available NS models, various attempts have been made to examine if NS would be stable to some kind of perturbations. These include perturbing the density profiles to include pressures, trying to see how the density gradients at various levels affect global versus the local visibility of the naked singularity, imposing symmetry conditions such as self-similarity, and then to see how the conclusions change on relaxing the self-similarity condition, study of how certain perturbations grow in the limit of approach to the Cauchy horizon, which is the first ray coming out of the naked singularity, and such others. While these attempts do not provide any definitive conclusions regarding the stability or otherwise of NS, they surely provide a good insight into the phenomena of BH/NH phases to tell us what is possible in gravitational collapse.

On the other hand, given the complexity of the Einstein field equations, if a phenomenon occurs so widely in spherical symmetry, it is not unlikely at all that the same would be repeated in more general situations as well. In fact, before the advent of well-known singularity theorems in general relativity, it was widely believed that the singularities found in more symmetric situations such as the Schwarzschild or Friedmann-Robertson-Walker cosmological models will go away once we go to general enough spacetimes. As is well-known, the singularity theorems then established that spacetime singularities occur in rather general spacetime settings without symmetry assumptions, and under a broad set of physical conditions. Thus, the singularities which manifested earlier in symmetric situations were actually indicative of a deeper phenomena. Such a possibility cannot again be ruled out in the case of occurrence of naked singularities as well.

4. Conclusions

In the above, we clarified the basic philosophy and motivation for cosmic censorship and the crucial role it plays in black hole physics. We then outlined some of the approaches that have been tried out so far to formulate or prove the same. It turns out that none of the physical constraints or natural looking physical conditions are really able to ensure the validity of CCH. In fact, one tends to conclude that naked singularities can actually develop in physically realistic gravitational collapse situations.

It then follows that more radical options, such as those listed in the previous section, must be tried out if CCH is to be preserved. We discussed these above, and it would appear that only one of them, namely that involving the stability and genericity of naked singularities can be a potentially promising candidate as far as any possible proof of CCH is concerned.
While one tries to work towards CCH along one of these or other paths, it is in fact important and quite interesting to really try to understand why do naked singularities actually develop in gravitational collapse. As we pointed out above, several important physical constraints on collapsing clouds do not appear to work towards helping CCH. It then becomes an intriguing question as to what is the physical agency that is possibly causing NS in collapse in a rather natural manner within the framework of general relativity? Some work has been done recently in that direction, and it turns out that while gravitational collapse proceeds, the shearing effects within the cloud could play a basic role to delay the formation of trapped surfaces and the apparent horizon in a natural manner. This in turn exposes the singularity to outside observers, depending on the rate of growth of shear in the limit of approach to the center. When looked at from such a perspective, one may even think that both black holes and naked singularities are rather natural consequences of gravitational collapse in classical general relativity. Perhaps one can learn a lot on gravitational collapse by examining such physical processes that could be responsible for creation of naked singularity as against a black hole. It is asked many times that how could there be any other outcome other than a black hole possible as end state of collapse, when gravity is getting stronger and stronger. Delayed formation of trapped surfaces is then the answer, and it is the physical processes like shear associated with the collapsing cloud that can achieve this in a natural manner to resolve the ‘mystery’ of naked singularity formation in gravitational collapse.

The point is, trapped surface formation is intimately connected with the question of singularities. In singularity theorems, it is the crucial assumption. For CCH, the critical question is the epoch of its formation. An intuitive characterization of CCH would be then trapped surface formation precedes the singularity formation. The event of formation of trapped surface depends upon the dynamical properties of matter and does not depend so much on the equation of state, which refers to the general character of the matter. It rather depends upon much finer and detailed properties of matter distributions. In the light of such a reality, it is likely that it may not even be possible to characterize CCH in a general form. May be it can only be studied case by case for it depends on finer structure and dynamics of the collapsing matter. The main question then is what forms first, singularity or apparent horizon (trapped surface)? For NS, it should be singularity first and for BH it should be trapped surface first. Close to such events, matter would be in super dense state, which could have very unfamiliar behaviour and quantum effects could become dominant. This is what would perhaps drive future investigation in this area. Another important aspect is that of energy carried out by null rays emanating from the singularity. This is very important from the practical point of view, because singularity could in principle be naked yet harmless.

It is then possible that the cosmic censorship conjecture does not hold classically, but may hold quantum mechanically, in some sense yet to be figured out. What may be possible then is for a star going into the final state of a naked singularity
configuration, the quantum gravity induced particle creation may take over to create a burst like emission of energy, thus clearing up the naked singularity.

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References
1. R. Penrose, in Black holes and relativistic stars, ed. R. M. Wald (University of Chicago Press, 1998).
2. R. M. Wald, gr-qc/9710065.
3. A. Krolak, Prog. Theor. Phys. Suppl. 136, p. 45 (1999)
4. P. S. Joshi, Pramana, 55, p. 529 (2000).
5. P. S. Joshi, Global aspects in gravitation and cosmology (Clarendon Press, OUP, Oxford), 1993.
6. B. C. Nolan, Phys. Rev. D 60, 024014 (1999).
7. C. J. S. Clarke, ‘The analysis of spacetime singularities’, Cambridge University Press, Cambridge, 1993.
8. T. Harko and K. S. Cheng, Phys.Lett. A 266, p. 249 (2000); S. G. Ghosh and N. Dadhich, gr-qc/0204091.
9. R. Penrose, in ‘Gravitational radiation and gravitational collapse’ (IAU Symposium No 64), ed. C. DeWitt-Morettee, Reidel, Dordrecht (1974).
10. P. S. Joshi, N. Dadhich and R. Maartens, Mod. Phys. Lett. A15 (2000) p. 991; T. Harada, H. Iguchi and K. Nakao, Prog.Theor.Phys. 107 (2002), p. 449.
11. S. W. Hawking, private communication.
12. J. R. Oppenheimer and H. Snyder, Phys. Rev. 56, p. 455 (1939).
13. W. B. Bonner, private communication.
14. P. S. Joshi and A. Krolak, Class. Quantum Grav. 13, p. 3069 (1996).
15. E. Brinis, S. Jhingan and G. Magli, Class. Quantum Grav. 17 (2000) p. 4481; P. S. Joshi, N. Dadhich and R. Maartens, Phys. Rev. D65, 101501(RC) (2002).
16. L. Witten, private communication.