Comparison of Probabilistically Evaluated COF with the Base Shear Distribution of IBC and Japanese Code

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Abstract

In the earthquake resistant design concept, frame structures are usually designed with a column over design factor (COF) to ensure the preferable failure mode during earthquakes. Initially, in the present study, the failure modes of multistory ductile frame structures are investigated probabilistically by applying the first order reliability method (FORM). The base shear distribution of the International Building Code (IBC-2006), which adopted the seismic design provisions of ASCE 7-05, and the \( \text{Ai} \) distribution of the Building Standard Law of Japan are taken into account in this study. Based on the investigations, the target values of the COF that probabilistically ensure the preferable entire beam hinging failure mode prior to story collapse are evaluated and compared. The study has been conducted for different reliability levels and the corresponding COF values are presented in this paper.

Keywords: frame structure; story mechanism; failure probability; reliability level; column overdesign factor

1. Introduction

An earthquake is a natural phenomenon that poses a huge threat for any building that is not designed and constructed properly to withstand it. Earthquakes cause thousands of lives to be lost and huge amounts of property damage in the world every year. The most destructive earthquakes are caused by seismic waves that reach the earth surface at areas where man-made structures are located. For a building to remain safe during an earthquake, the philosophy of the strong column weak beam has been widely accepted by structural designers and researchers. In this seismic design concept, it is assumed that the yielding of all beams in flexure will occur prior to the possible yielding of columns, which is considered to be the preferable failure mode (Anderson and Gupta 1972; Park and Pauly 1975; Lee 1996). In fact, this mode provides higher ductility, and a better distribution of inelastic deformation and energy dissipation among the structural elements and thus before the collapse, the building can absorb a large amount of energy.

To ensure that a frame structure collapses according to the preferable beam-hinging pattern as shown in Fig.1., the columns of the structure that receive forces from the beams of a building structure are generally designed with a column over design factor (COF) value greater than one to make the columns relatively stronger than the beams. Many studies have been conducted so far by researchers in search of the dominant collapse modes of frames and design of the strong column weak beam frames. Hibino and Ichinose (2005) presented a numerical study on the effect of the column-to-beam strength ratio on the seismic energy dissipation of beams and columns in fishbone-type steel moment frames. The major parameters considered were the number of stories, the strengths of the columns, the strengths of the beams and the ground motion. Findings of the study show that with the increase of the beam to column strength ratio, the energy contributing to the story mechanism decreases. Nakashima and Sawaizumi (1999) simplified a frame structure into a fishbone shaped model to perform dynamic analysis with earthquake motion as input and indicated that the necessary COF value that ensures beam hinging responses increases steadily with the increase of the ground motion amplitude. Medina and Krawinkler (2005) studied a family of regular frames to evaluate the strength demands relevant to the seismic design of the columns and indicated that the potential of plastic hinging in the columns is high for frames designed according to the strong column weak beam requirements of current code provisions. Kawano et al. (1998) presented basic information on the COF for forming the weak-beam type of plastic mechanisms in steel reinforced concrete frames. Kuwamura et al.
(1989) conducted Monte Carlo simulations of the static and dynamic performance of a six story rigid plane frame. The simulation results showed that the randomness in the yield strengths has a predominant influence on the failure mechanism and consequently on the system ductility.

Most of these studies used a deterministic approach for specific structures, so the occurrence probability of the undesirable failure mode and the risk of failure of the structure remain unknown. However, from practical experience we observe that many structures collapse unexpectedly according to some undesirable failure modes, because of the uncertainties associated with the member strength and the earthquake loads. Therefore, it is difficult to absolutely ensure that the structure will collapse according to the preferable failure mode; a better strategy may be the probabilistic approach, i.e., to ensure an occurrence probability of the preferable failure mode larger than the probabilities of the undesirable modes. The COF ensuring probabilistic priority of beam hinging failure mode to story mechanism needs to be determined.

The probabilistic approach was applied by Kuwamura et al. (1989) for specific structures with specific earthquake input.

Target values of COF that ensure probabilistically the preferable entire beam hinging failure mode of frames was evaluated by Ono et al. (2000) and Zhao et al. (2002) based on triangular distribution of load along the height of the frame. However, nowadays this distribution is not commonly used. In Japan, the Ai distribution of the Building Standard Law of Japan and in USA and many other countries the load distribution of International Building Code (IBC-2006) is popular. Therefore, in this study, the base shear distribution of the IBC-2006 and Ai distribution are taken into account. Based on the investigations, the smallest values of the COF that probabilistically ensure the preferable entire beam hinging failure mode prior to story collapse are evaluated and compared. This will guide the engineers to select the COF values for frame structures under specific reliability level to probabilistically avoid the undesirable story collapse modes during earthquakes.

2. Basic Assumptions

The COF is defined for each beam-column node as the ratio of the sum of the moment capacity of the columns to the sum of the moment capacity of the beams at that node, as follows:

$$COF(k) = \frac{\sum \mu_{mcj}}{\sum \mu_{mbj}}$$

where $k$ is the $k$th node; $\mu_{mcj}$ is the mean plastic moment strength of a column connected in the $k$th node and $\mu_{mbj}$ is the mean plastic moment strength of a beam connected in the $k$th node.

For the ductile frame structures considered in this study the following basic assumptions are applied:

- Elastic-plastic frame structures are considered. The failure of a section means the imposition of a hinge and an artificial moment at that section.
- The structural uncertainties are represented by considering only the moment capacities as random variables. The coefficient of variation of the material strength is considered to be 0.1.
- Plastic moment capacities are statistically independent to one another and independent of the applied loads. All the random variables are assumed to follow the lognormal distribution.
- The external load considered is only the lateral earthquake load. The base shear distribution of the International Building Code (IBC-2006) and the Ai distribution of the Building Standard Law of Japan are taken into account. The coefficient of variation of the earthquake load is considered to be 0.8.
- The geometrical second-order and shear effects are neglected. The effect of the axial forces on the reduction of moment capacities is also neglected.
- All beam-column nodes have identical COFs, i.e., there is only one value of COF for a structure.

3. Brief Description of the Base Shear Distributions

3.1 Base Shear Distribution of IBC

The seismic base shear specified by ASCE 7-05 and adopted by the International Building Code (IBC-2006) is determined according to the following equation:

$$V = C_r W$$

where $W$ is the effective seismic weight and $C_r$ is the seismic response coefficient defined as follows:

$$C_r = \frac{S_{ds}}{R/I}$$

where $R$ is the response modification factor, $I$ is the importance factor and $S_{ds}$ is the design spectral response acceleration in the short period range.

The seismic response coefficient need not exceed the following:

$$C_r = \frac{S_{ds}}{T(R/I)} \quad \text{for} \quad T \leq T_L$$

$$C_r = \frac{S_{ds}T_L}{T^2(R/I)} \quad \text{for} \quad T > T_L$$

Fig. 1: Entire Beam Hinging Failure Mode
The value of $C_s$ shall not be less than 0.01. $T_l$ is the long period transition period (s), $S_{D1}$ is the design spectral response acceleration at a period of 1 s. $S_{DS}$ and $S_{D1}$ are defined as follows:

\[ S_{DS} = \frac{2}{3} F_v S_s \]  
\[ S_{D1} = \frac{2}{3} F_v S_1 \]

where $F_v$ and $F_s$ are the site coefficients. $S_s$ is the mapped maximum considered earthquake spectral response acceleration at short period and $S_1$ is the mapped maximum considered earthquake spectral response acceleration at a period of 1 s.

$T$ is the approximate fundamental period calculated as:

\[ T = C_i h_n^x \]

where $h_n$ is the height above the base, $C_i$ is equal to 0.028 and $x$ is equal to 0.8 for a steel moment resisting frame.

The lateral seismic force ($F_x$) at any level shall be determined from the following two equations as:

\[ F_x = C_{xx} V \]

\[ C_{xx} = \frac{w_i h_i^k}{\sum_{i=1}^{n} w_i h_i^k} \]

where $C_{xx}$ is the vertical distribution factor, $V$ is the total shear at the base of the structure, $w_i$ and $w_x$ are the portions of the total effective load located or assigned to level $i$ or $x$, $h_i$ and $h_x$ are the height from the base to level $i$ or $x$ and $k$ is an exponent related to structure period.

In this study, all the frames are considered with equal floor height of 4m and equal bay width of 8m. The response modification factor is assumed to be 8 and importance factor to be 1.0.

### 3.2 Base Shear Distribution of the Japanese Code

According to the Japanese code, the lateral shear $Q_i$ is calculated as:

\[ Q_i = C_i W_i \]  

where $W_i$ is the weight at and above level $i$. The seismic shear coefficient $C_i$ for $i$th level is determined by:

\[ C_i = Z R A C_0 \]  

where $Z$ is the seismic zone coefficient, $R$ is the design spectral coefficient, $C_0$ is the standard shear coefficient. $Z$ is assumed to be 0.9 $R_i$ to be 1.0 and $C_i$ to be 0.2.

$A_i$ is the horizontal shear distribution factor calculated as:

\[ A_i = 1 + \left( \frac{1}{\sqrt{\alpha_i}} - \alpha_i \right) \frac{2T}{1+3T} \]

where $T$ is the fundamental time period equal to 0.03*height (m) for steel structures, $\alpha_i$ is the ratio of the weight carried by the $i$th floor to the total weight of the structure.

The seismic lateral forces $F_i$ at various levels are calculated as:

\[ F_i = Q_i - Q_{i+1} \]

where, $Q_i$ and $Q_{i+1}$ are obtained from Eq. 11.

The detail information is available in respective codes.

A qualitative diagram of the distributions is shown in Fig. 2.

![Fig. 2 Qualitative Diagram of the Distributions](image)

(a) IBC-2006  
(b) Japanese code

### 4. Failure Modes Analysis

There are many failure modes for a frame structure. Among all the failure modes the collapse of one or more stories is the most dangerous. This is because story collapse leads to the collapse of the columns which is obviously more dangerous compared to beam failure or other minor failures. Therefore, the present study is based on this type of failure mode. For convenience, the story collapse modes are defined before the probabilistic evaluation so that the investigation can be carried out sequentially for each type.

In this study, the story failure modes are classified into three patterns: lower story failure pattern, middle story failure pattern and upper story failure pattern, each of which depends on the location of the failure stories, as shown in Fig. 3. The lower story failure pattern is characterized by the continuous collapse of stories from the first story of the frame; the upper story failure pattern is characterized by continuous collapsed stories from the top story of the frame; in the middle story failure pattern, the mechanism occurs in the middle stories of the frame and the stories at the top and bottom of the frame are not involved in the mechanism.
Based on the principle of virtual work, the performance function for these three story failure patterns can be established as follows:

\[
G_L(X) = 2 \sum_{j=1}^{n_c} M_{bj} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} - \sum_{j=1}^{n_c} n_j hP_j - \sum_{j=n_c+1}^{n_b} n_j hP_j
\] (15)

\[
G_M(X) = 2 \sum_{j=1}^{n_c} M_{bj} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} - \sum_{j=1}^{n_c} n_j hP_j - \sum_{j=n_c+1}^{n_b} n_j hP_j
\] (16)

\[
G_U(X) = 2 \sum_{j=1}^{n_c} M_{bj} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} - \sum_{j=1}^{n_c} n_j hP_j - \sum_{j=n_c+1}^{n_b} n_j hP_j - \sum_{j=1}^{n_c} n_j hP_j - \sum_{j=n_c+1}^{n_b} n_j hP_j
\] (17)

where \(G_L, G_M, \) and \(G_U\) are the performance functions of the lower story failure pattern, the middle story failure pattern and the upper story failure pattern respectively. \(M_{bj}\) is the moment strength of the beam of the top story, \(M_{bj}\) is the moment strength of the beam of the \(i\)th span and \(j\)th story, \(M_{cl}\) is the moment strength of an interior column, \(M_{cl}\) is the moment strength of an exterior column, \(P_j\) is the load acting on the \(j\)th story of the structure, \(n\) is the number of stories, \(n_c\) is the number of failure stories, \(n_b\) is the number of unbroken stories at the bottom of the structure, \(m\) is the number of spans, and \(h\) is the story height of the structure.

The upper story failure pattern shown in Fig.3.(c) has some similarity with the beam hinging failure mode shown in Fig.1. If the number of failure stories \(n_c\) is equal to the number of stories \(n\), then the upper collapse mode is transformed into the entire beam hinging failure mode. So the performance function of the beam hinging failure mode can be easily obtained from Eq. 17 by setting only \(n_c = n\); although it is possible to derive the Eq. 17 from Eq. 15 and 16 with some modification.

\[
G_B(X) = 2 \sum_{j=1}^{n_c} M_{bj} + 2 \sum_{j=1}^{n_c} M_{bj} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} + \sum_{j=1}^{n_c} M_{cl} - \sum_{j=1}^{n_c} n_j hP_j - \sum_{j=n_c+1}^{n_b} n_j hP_j
\] (18)

where \(G_B\) is the performance function of the beam hinging failure mode.

To make a structure designed with the same COF in all the beam column nodes, the mean values of the member strengths are assumed to have the following relationship

\[
\mu_{bj} = \mu_b; \quad \mu_{bj} = 2\mu_b; \quad \mu_{cl} = COF \times \mu_b; \quad \mu_{cl} = 2COF \times \mu_b
\] (19)

where \(\mu_b\) is the mean value of the moment strength of the beam of the top story, \(\mu_{bj}\) is the mean value of the moment strength of the beam of the \(i\)th span and \(j\)th story, \(\mu_{cl}\) is the mean value of the moment strength of an exterior column and \(\mu_{cl}\) is the mean value of the moment strength of an interior column. The mean value of the moment strength of the beam of the top story is assumed to be half of the mean value of moment strength of other beams of the lower stories. This is because the top beam has to sustain a lower load than the beams of the lower stories due to the absence of walls and other loads. The mean values of the moment strength of columns are obtained by multiplying the respective COF value. For example when COF=1.1 and the mean strength of the top beam is 104.15 KN.m, the mean strength of other beams, exterior columns and interior column will be 208.30 KN.m, 114.565 KN.m and 229.13 KN.m, respectively. Since, this study has been conducted for low-rise structures of up to six stories the columns of all the stories are considered to have the same strengths in order to simplify the calculation.

For a multi-story frame, the number of mechanisms increases with the increase of the number of stories, and the number of potential story mechanisms becomes too many.

An earlier study by Zhao et al. (2007) on the story failure modes of frame structures showed that all the lower story collapse modes and the upper story collapse modes with the highest failure stories are the most likely failure modes. Therefore, these modes are considered in the COF evaluation. In the case of a six story frame all the lower story collapse modes and the upper story collapse modes with the highest failure stories are shown in Fig.4., as these are the most likely failure modes to occur in this case.
5. Evaluation Method

In the COF evaluation, the reliability index of the preferable entire beam hinging mode \( \beta_T \) is given first to indicate the safety requirement of the structure. \( P_{\beta_T} \) is the probability corresponding to the reliability index, namely:

\[
P_{\beta_T} = \Phi(-\beta_T)
\]

(20)

The investigation of COF is conducted under a specific reliability level, which means that for a given reliability index of the entire beam-hinging failure mode, the load levels are computed using a reliability analysis method such as the first order reliability method (FORM) or method of moments (Ang and Tang 1984; Zhao and Ono 1999; 2001) to ensure that the first order reliability index becomes equal to the target reliability index \( \beta_T \) for frame structures designed with various COFs. The obtained load is then applied to compute the probabilities of the undesirable story mechanisms.

In the COF evaluation method, the following evaluation index is used:

\[
\gamma = \frac{P_{\beta_T}}{P_{\beta_1}}
\]

(21)

where \( P_{\beta} \) = the occurrence probability of the desirable beam-hinging failure mode and \( P_{\beta_1} \) = the occurrence probability of the most likely story mechanism. The probabilities of the preferable collapse mode and the undesirable collapse mode are calculated under the same load conditions; otherwise the evaluation index \( \gamma \) is meaningless.

After obtaining the abovementioned evaluation index, to ensure that the designed structure collapses according to the designed preferable failure mode, the relative occurrence rate of the most likely story mechanism \( \gamma \) must be controlled to be lower than a specific allowable level \( \gamma_0 \) as follows:

\[
\gamma = \frac{P_{\beta_T}}{P_{\beta_1}} \leq \gamma_0 \leq 1
\]

(22)

By conducting the failure mode analysis and the reliability analysis using a different COF for a frame structure, a \( \gamma \)-COF curve can be obtained and the target value of the COF for which Eq. 22 is satisfied can be determined.

The target COF requirement has been evaluated based on simple evaluation index \( \gamma \) considering only the most likely failure modes, while all other modes are ignored.

6. Comparison of the Target COFs

It has already been stated that the evaluation index is the ratio of the probabilities of the undesirable story collapse failure modes to that of the expected entire beam hinging failure mode. Therefore, if \( \gamma_0 = 1 \), the undesirable failure mode and the preferable entire beam-hinging mode have the same likelihood of occurrence, i.e., both probabilities are equal. A COF value lower than the value corresponding to \( \gamma_0 = 1 \) enhances the story collapse, i.e., the probability of the story collapse is higher than that of the entire beam-hinging mode, therefore this value is not allowed. The threshold value of COF when \( \gamma_0 = 1 \) is defined here as the target or basic COF. When the COF of the frame is higher than the target COF, the occurrence probability of the undesirable story collapse failure modes considered, i.e., most likely failure modes is less than that of the expected entire beam hinging failure mode.

Three basic frames are utilized in this analytical investigation: four story, five story and six story two bay frames. Each frame is analyzed for reliability levels 2, 3 and 4 (\( \beta_T = 2 \), \( \beta_T = 3 \) and \( \beta_T = 4 \)).

Initially the failure probabilities of the most likely failure modes are investigated by considering both distributions. It is observed that in the lower story failure modes (Mode 1 to Mode 5) the failure probabilities are higher in the case of IBC distribution than the Ai distribution of Japan. In the upper story failure mode (Mode 6) the two failure probabilities are almost the same.

Fig.5. shows a comparison of the failure probabilities of the Mode-3 of the six-story frame for the IBC and the Ai distribution under reliability level 2(\( \beta_T = 2 \)).

Fig.6.(a), Fig.6.(b) and Fig.6.(c) show the \( \gamma \)-COF curve for four story to six story frames under reliability levels 2, 3 and 4 (\( \beta_T = 2 \), \( \beta_T = 3 \) and \( \beta_T = 4 \)), respectively, based on the base shear distribution of IBC-2006.

![Fig.5. Comparison of Failure Probabilities of Mode 3 of a Six Story Frame (\( \beta_T = 2 \))]
levels 2, 3 and 4 ($\beta_T = 2$, $\beta_T = 3$ and $\beta_T = 4$) based on the base shear distribution of IBC-2006.

Table 2 shows the numerical values of target COF for four story to six story frames under reliability levels 2, 3 and 4 ($\beta_T = 2$, $\beta_T = 3$ and $\beta_T = 4$) based on the $Ai$ distribution of Japan.

It is observed that under the same reliability level, the target COF requirement increases with the increase of the number of stories and decreases with the increase of the reliability level. A similar observation is found for both distributions considered in this study.

Fig. 8 shows a comparison of the target COF for four story to six story frames under reliability levels 2, 3 and 4 ($\beta_T = 2$, $\beta_T = 3$ and $\beta_T = 4$).

The COF plays an important role in determining the building response during an earthquake. If the appropriate COF is not provided, then this can become a major reason for building damage during strong earthquakes.

From this study, which has been conducted under
some restrictive assumptions, the minimum values of the COF to probabilistically avoid the undesirable story collapse modes during an earthquake are obtained based on two important building codes. It should be noted here that these assumptions may affect the results to some extent.

7. Conclusion

The column overdesign factor (COF) requirement which probabilistically ensures the preferable entire beam hinging failure mode and probabilistically avoids the undesirable story collapse modes of the frame structure during an earthquake has been evaluated in this paper based on the base shear distribution of IBC-2006 and the $Ai$ distribution of Japan. The findings of the paper are summarized as follows:

1. For any frame structure, generally the base shear distribution of IBC-2006 provides a higher target COF value than does the $Ai$ distribution of Japan.
2. With the increase of the reliability level the relative difference of the required target COF values between the two distributions decreases.
3. Under the same reliability level the target COF requirement increases with the increase of the number of stories and decreases with the increase of the reliability level. A similar observation is found for both distributions considered in this study.

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