ADC non-linear error corrections for low-noise temperature measurements in the LISA band

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Abstract. Temperature fluctuations degrade the performance of different subsystems in the LISA mission. For instance, they can exert stray forces on the test masses and thus hamper the required drag-free accuracy. Also, the interferometric system performance depends on the stability of the temperature in the optical elements. Therefore, monitoring the temperature in specific points of the LISA subsystems is required. These measurements will be useful to identify the sources of excess noise caused by temperature fluctuations. The required temperature stability is still to be defined, but a figure around 10 µK Hz⁻¹/² from 0.1 mHz to 0.1 Hz can be a good rough guess. The temperature measurement subsystem on board the LISA Pathfinder mission exhibits noise levels of 10 µK Hz⁻¹/² for f >0.1 mHz. For LISA, based on the above hypothesis, the measurement system should overcome limitations related to the analog-to-digital conversion stage which degrades the performance of the measurement when temperature drifts. Investigations on the mitigation of such noise will be here presented.

1. Introduction

LISA is a space-based gravitational wave detector with the primary scientific goal of observing GWs from astronomical sources in the frequency range from 0.1 mHz to 0.1 Hz [1]. However, the detection of GWs requires differential measurements of distance at the pico-meter level between two bodies separated by 5 million kilometres. This fact implies challenging requirements in the test masses’ (TM) residual acceleration noise in order to ensure GW detection with high signal-to-noise ratio. Otherwise GW signals will be buried in the random TM motion associated to the forces acting on them. The LISA top level requirement, in terms of TM acceleration noise, is [2]

\[ S_{ba,LISA}(\omega) \leq 3 \times 10^{-15} \left( 1 + \left( \frac{\omega}{2\pi} \frac{\text{mHz}}{8} \right)^4 \right)^2 \left( 1 + \left( \frac{0.1 \text{mHz}}{\omega/2\pi} \right) \right)^{1/2} \text{m s}^{-2} \text{Hz}^{-1/2} \]

from 0.1 mHz to 0.1 Hz. One of the noise sources that can jeopardise the LISA performance is temperature fluctuations affecting key subsystems of the mission. More specifically, thermal stability is necessary to, first, ensure that the acceleration noise caused by temperature
fluctuations\(^1\) in the TMs is kept below the top level requirement —see Eq. (1); and second, to ensure the stability of the optical elements properties of the interferometric systems\(^2\) [3].

The very demanding requirements for the \textit{LISA} mission led to recognise the need of a technology demonstration mission: the so-called \textit{LISA} Pathfinder mission [4]. The main goal of LPF is to check if the residual acceleration in Eq. (1) can be achieved, i.e., it must verify the ability to set a TM in purely gravitational free fall to the level required by Eq. (1), although in LPF the requirements have been relaxed one order of magnitude below the \textit{LISA} requirements both in amplitude and in frequency.

The \textit{LISA} Technology Package\(^3\) (LTP) includes a temperature measurement subsystem (TMS) in order to assess whether or not the required thermal stability is met, and to provide information of the actual coupling between temperature fluctuations and acceleration noise. At this moment the LTP TMS flight model version has been already designed and tested, and is compliant with the LTP requirements [5] —see Fig. 1. The TMS must exhibit noise levels of \(10^{-5} \text{K Hz}^{-1/2}\) from 1 mHz to 30 mHz in order to measure the expected temperature fluctuations in the LTP\(^4\) with a good signal-to-noise ratio. However, such system exhibits extra noise in the submilli-Hertz range when the measured temperature drifts \((\sim \mu \text{K s}^{-1})\). Although this is not a problem for the LTP, investigations are needed to overcome such limitation in view of \textit{LISA}: the measurement bandwidth (MBW) stretches down to 0.1 mHz, the floor noise level requirement is one order of magnitude more demanding and, therefore, weaker temperature drifts can deteriorate the performance of the TMS.

The source of the excess noise has been seen to be due to non-idealities of the transfer curve of the analog-to-digital converter (ADC). A 16-bit successive approximation register (SAR) ADC is the only suitable option due to space qualification constraints. As shown in Fig. 1, the noise in the temperature measurement increases noticeably for \(f < 0.1 \text{ mHz}\) when the measured temperature drifts are higher than \(0.5 \mu \text{K s}^{-1}\). This paper focuses on the identification and mitigation of this problem. In Sec. 2 we review the theoretical basis of the noise introduced by a non-ideal ADC. In Sec. 3 we show how the effect of the ADC degrades the performance of TMS and how the theoretically expected noise and the experimental results agree. In Sec. 4 we describe a method to mitigate the noise coming from the ADC, the so-called \textit{dithering techniques}. The test set-up and the experimental results using the dithering technique are shown in Sec. 5. Finally, Sec. 6 highlights the most relevant results.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{LTP TMS performance. The excess noise at low frequency \((f < 1 \text{ mHz})\) appears when the measured temperature drifts ("ADC noise due to drifting temperature" trace). It vanishes when the signal is kept constant ("non-drifting temperature" trace).}
\end{figure}

\(^1\) Radiometer effect, radiation pressure or outgassing
\(^2\) Changes of the index of refraction or dilatation (and contraction) cause an error in the acceleration measurement.
\(^3\) The science module of the European Space Agency (ESA) in LPF.
\(^4\) They are \(10^{-4} \text{ K Hz}^{-1/2}\) from 1 mHz to 30 mHz.
2. Quantisation noise in SAR ADCs

In real SAR ADCs the quantisation steps are not uniform due to mismatches in their internal topology (tolerances in the capacitors, \( \delta C_k \), of the SAR array). Such non-uniformity is specified with two parameters: differential non-linearity (DNL) and integral non-linearity (INL) [6]. The latter represents the difference between the real and the ideal ADC transfer curves. The noise introduced by this error is the one limiting the temperature measurement performance when drifting input signals are present. The voltage error \( \epsilon_k \) induced by an error in the \( k \)-th bit is:

\[
\epsilon_k(v) \simeq b_k(v) \frac{\delta C_k}{C} V_{FS} = b_k(v) \frac{\delta C_k}{C} \Delta \quad (k = 0 \text{ is the LSB})
\]

(2)

where \( b_k \) is the binary digit (it is set to 0 or 1 depending on the sampled voltage), \( \delta C_k \) is \( C_{k,\text{real}} - 2^k C \), \( \Delta \) \((\simeq V_{FS}/2^N)\) is the ADC quantisation step and \( N \) is the number of bits of the ADC. LSB stands for least significant bit. Equation (2) can be expressed as [7]:

\[
|Q_k(\xi)| = \Delta \sum_{n=-\infty}^{\infty} \frac{\sin \frac{n \pi \xi}{2n+1}}{n} \sin \frac{\pi}{2n+1} \delta \left( \xi - \frac{\pi n}{2^k \Delta} \right)
\]

(3)

Equation (3) is graphically represented in Fig. 2 (center and bottom panels) where a superimposed periodic pattern in the quantisation error of the ADC with 1 LSB of amplitude and periodicity of \( 2^{k+1} \Delta \) (caused by the INL errors) is clearly seen.

![Figure 2](image_url)

**Figure 2.** Top: quantisation error functions for an ideal ADC (red) and for two non-ideal ADCs with different error bits (black and blue). Centre and bottom: error differences between the non-ideal and ideal ADCs \( [\epsilon_k(v)] \).

3. Effect of ADC non-idealities on the TMS

To estimate INL induced errors in the temperature measurements let us assume the input signal is a straight line for a certain time interval:

\[
v(t) = a + bt \quad \text{if} \quad 0 \leq t \leq t_{\text{max}}, \quad \text{otherwise} \quad v(t) = 0
\]

(4)

The difference between a real ADC (non-uniform quantisation steps) transfer curve and an ideal ADC (uniform quantisation steps) transfer curve.

We have defined the Fourier transform in the voltage domain by \( Q(\xi) \equiv \int_{-\infty}^{\infty} q(v)e^{-i\xi v}dv \). Not to be mistaken by the usual time/frequency Fourier transforms. We use special symbols for this to avoid confusion. \( \xi \) is the Fourier-conjugate variable of \( v \), and is accordingly measured in \( 2\pi \) volts\(^{-1}\).
In this case, Eq. (3) becomes (in the frequency, $2\pi$ Hz, domain)\(^7\) [8]:

$$|\tilde{q}_k(\omega)| = \Delta \sum_{n=-\infty}^{\infty} \frac{\sin \frac{\pi n k}{2k+\Delta}}{n} \frac{\sin \frac{\pi n}{2\pi \Delta}}{n} \delta \left( \omega - \frac{\pi n |b|}{2k} \right)$$

(5)

and the main frequency component introduced by an error in the $k$-th bit is located at\(^8\):

$$\omega_{1k} = \pi \frac{2^N}{2k V_{FS}} |\dot{v}(t)|, \quad \omega'_{1k} = \pi \frac{2^N}{2k V_{FS}} |\ddot{v}(t)|$$

(6)

From Eqs. (6) some conclusion can be drawn:

- high-slope signals at the ADC input translate into high-frequency components at the output of the ADC,
- errors in the higher bits show up as noise at lower frequency than errors in the lower bits,
- consequently, high-slope inputs combined with errors in high bits may show up as excess noise in the bandwidth of interest,
- the fundamental frequency associated to a bit error varies with variations of $\dot{v}(t)$, hence the error in the bit spreads across the frequency band when $\ddot{v}(t) \neq 0$.

Figure 3 shows experimental data and the theoretical expected evolution of the power spectrum density with time —cf. Eq. (6). They are in excellent agreement.

**Figure 3.** Top: absolute value of the input signal slope, $|\dot{v}(t)|$, of a temperature measurement run. The sensitivity at the input of the ADC is $\approx 1.35$ V K\(^{-1}\), hence, temperature drifts vary between 0 and 9.3 $\mu$K s\(^{-1}\). Bottom: spectrogram of the top panel measurement. The energy of the signal is concentrated in specific frequencies which change with time precisely following $|\dot{v}(t)|$, as predicted by Eq. (6), and as shown in red dashed traces labelled by the order of the corresponding bit error (in this case from $k = 3$ to $k = 6$).

\(^7\) Dirac $\delta$-functions are the result of infinite length integration intervals in Fourier transform calculations. In any practical situation the $\delta$-functions remain sinc-functions.

\(^8\) These expressions are valid when the temperature signals can be conveniently approximated by a series of concatenated ramps with adequate slopes and lengths.
4. Mitigation of non-ideal ADC errors: dithering techniques

Two different methods have been used to mitigate the effect of the non-idealities of the ADCs on the temperature measurements. Both methods are based on the same technique: dithering signals. This technique is based on the injection of suitable analog signals to the signal of interest before quantisation in order to linearise the ADC transfer curve. In the following we briefly review the theoretical basis of this technique. If a dither voltage, \( d \), with a certain probability density function (PDF), \( p(d) \), is added (before quantisation) to the signal of interest, \( v \), the average quantisation error at the output of the ADC is [9]:

\[
\langle q(v) \rangle = \int_{-\infty}^{\infty} q(v + z) p(z) \, dz \\
\langle Q(\xi) \rangle = Q(\xi) \, P^*(\xi)
\]

(7) (8)

where \( q(v) \) [and \( Q(\xi) \)] is the quantization error of a real ADC —see Sec. 2—, \( p(d) \) is the PDF of the dither signal [and \( P(\xi) \) is the Fourier transform as defined in footnote 6] and \( \langle q(v) \rangle \) [and \( \langle Q(\xi) \rangle \)] is the quantisation error function at the output of the ADC considering we inject a dither signal. In our experiments we have tested two different dither signals: (i) Gaussian noise dither added out of the MBW and (ii) triangular wave dither of frequency 50 Hz. These signals are characterised by the following equations:

\[
\text{Gaussian noise : } p(d) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-\frac{d^2}{2\sigma^2}} \rightarrow P(\xi) = e^{-\sigma^2\xi^2/2} \\
\text{Triangular wave : } p(d) = \frac{1}{D} \rightarrow P(\xi) = \text{sinc}(D/2)\xi
\]

(9) (10)

where \( \sigma \) and \( D \) are the standard deviation of the Gaussian noise and the amplitude of the triangular wave, respectively; and \( \xi = \frac{2\pi x}{\lambda} \). Figure 4 shows the error components introduced by error bits \((k=0 \text{ and } k=3)\) and the effect of the dither signals on them (red traces).

\[\text{Figure 4. ADC quantisation error as given in Eq. (3). They are indicated by the circles on top of vertical strokes. Left: the red dashed line represents the Gaussian dither attenuation profile with } \sigma = \Delta \text{ which suffices to mitigate the error in the bit } k = 0 \text{ but not the errors in } k = 3. \text{ To do so higher values of } \sigma \text{ are required. Right: idem for the triangular wave dither of amplitude } D = 2\Delta. \text{ Note the different scales in the } x\text{-axis. The conversion to Hertz is easily done by using } \xi = \omega/|\dot{\psi}(t)|.\]

5. Test set-up and results

The test set-up in order to check the performance of the dithering methods implemented consisted in attaching a set of thermistors inside a thermal insulator designed to screen out temperature fluctuations to the required level in the milli-Hertz range, i.e., \( < 10^{-5} \text{ K Hz}^{-1/2} \) at
the milli-Hertz [3]. A temperature control was implemented to generate temperature ramps of different slopes (from 0.5 \( \mu \text{K s}^{-1} \) to 16 \( \mu \text{K s}^{-1} \)) inside the insulator. In this manner we could observe the effect of the dither signals in the TMS under different temperature slopes.

Four configurations were tested under identical conditions. First, a 16-bit SAR ADC (AD977 of Analog Devices\(^9\)) without any dither signal was tested. Second, we tested the same ADC but injecting Gaussian noise dither before quantisation. The third configuration consisted in using the triangular wave dither signal. Finally, a higher resolution 24-bit Delta-Sigma ADC\(^{10}\) was also used for comparison. Results are shown in Fig. 5 for different slopes and configurations.

![Figure 5. Linear Spectral Density for different configurations. As can be seen in absence of dither, the noise of the ADC increases (in amplitude and bandwidth) with the slope of the input signal. Gaussian noise dither attenuates this effect although the floor noise is slightly higher than the typical one. Triangular wave dither works satisfactorily and the floor noise remains untouched. The 24-bit Delta-Sigma ADC works also well for the tested slopes.](image)

6. Conclusions

The analysis and results obtained lead to the following conclusions:

- Extra noise caused by INL errors of the ADC was seen at the lower end of the LTP MBW when drifting temperatures were measured with the LTP temperature measurement subsystem.
- For temperature slopes >0.5 \( \mu \text{K s}^{-1} \) the extra noise challenges the performance of the LTP measuring system\(^{11}\), the problem growing more severe at lower frequencies (LISA band).
- The use of dithering techniques mitigates the INL errors introduced by the ADC. However the injection of Gaussian noise dither increases the floor noise of the system in the MBW. Alternatively, the triangular wave dither does not add noise in the MBW and, therefore, results in a more efficient attenuation of INL errors. Dither technique is not foreseen to fly in LPF, but the results obtained herein are important in view of designing a more robust

\(^9\) The ADC used for the space model is the Texas Instruments ADS7809 which is based in the same structure.

\(^{10}\) Such ADC should exhibit less non-linearity problems. Space qualified version of these ADCs are not available.

\(^{11}\) The expected slopes in the LTP (in nominal conditions) are lower than 0.5 \( \mu \text{K s}^{-1} \).
TMS for the LISA mission. The dither signal to be injected in the system will depend on the expected temperature drifts in LISA and, apparently, the triangular dither signal should be able to cope with the requirements of the mission. This would avoid the use of 24-bit Delta-Sigma ADCs which they are not suitable for multiplexed systems as in the case of the TMS and, furthermore, at present they are not space qualified.

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References
[1] Bender P et al. 2000 ESA Report No ESA-SCI(2000)11
[2] The LISA International Science Team 2008 ESA-NASA Report No LISA-ScRD-Iss5-Rev1
[3] Lobo A, Nofrarias M, Ramos-Castro J and Sanjuán J 2006 Class Quantum Grav 23 5177
[4] Vitale S et al 2005 Science Requirements and Top-level Architecture Definition for the LISA Technology Package (LTP) on Board LISA Pathfinder (SMART-2), report no. LTPA-UTN-ScRD-Iss003-Rev1
[5] Sanjuán J, Lobo A, Nofrarias M, Ramos-Castro J and Riu P 2007 Rev Sci Instr 78 104904
[6] Hoeschele DF 1994 Analog-to-digital and digital-to-analog conversion techniques (New York, NY: Wiley)
[7] Wagdy MF 1996 IEEE Trans Instrum Meas 45 610
[8] Sanjuán J 2009 Development and validation of the thermal diagnostics instrumentation in LISA Pathfinder PhD dissertation
[9] Wagdy MF 1989 IEEE Trans Instrum Meas 38 850