Optimal power flow with enhancement of voltage stability and reduction of power loss using ant-lion optimizer

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Abstract: In this work, the most common problem of the modern power system named optimal power flow (OPF) is optimized using the novel meta-heuristic optimization algorithm ant lion optimizer (ALO). ALO is inspired by the hunting process of ant-lions in the natural environment. ALO has a fast convergence rate due to the use of roulette wheel selection method. For the solution of the optimal power flow problem, standard 30 bus IEEE system is used. ALO is applied to solve the suggested problem. The problems considered in the OPF problem are fuel cost reduction, voltage profile improvement, voltage stability enhancement, minimization of active power losses and minimization of reactive power losses. The results obtained with ALO is compared with other methods like firefly algorithm (FA) and particle swarm optimization (PSO). Results show that ALO gives better optimization values as compared with FA and PSO which verifies the strength of the suggested algorithm.

Subjects: Intelligent Systems; Power Engineering; Systems & Controls

Keywords: optimal power flow; voltage stability; power system; ant-lion optimizer; constraints

1. Introduction
At the present time, the optimal power flow (OPF) is a very significant problem and most focused objective for power system scheduling and operation (Bouchekara, Abido, Chaib, & Mehasni, 2014). The OPF is the elementary tool which permits the utilities to identify the economic operational and many secure states in the system (Duman, Güvenç, Sönmez, & Yörükeren, 2012; Niknam, Narimani, Jabbari, &Malekpour, 2011). The OPF problem is one of the utmost operating desires of the electrical power system (Carpentier, 1962). The prior function of OPF problem is to evaluate the optimum
operational state for Bus system by minimizing each objective function within the limits of the operational constraints like equality constraints and inequality constraints (Bouchekara, Abido, & Boucherma, 2014). Hence, the optimal power flow problem can be defined as an extremely non-linear and non-convex multimodal optimization problem (Abou El Ela & Abido, 2010).

From the past few years too many optimization techniques were used to solve the optimal power flow (OPF) problem. Some traditional techniques are used to solve the proposed problem have been suffered from some limitations like converging at local optima, not suitable for binary or integer problems and also have the assumptions like the convexity, differentiability, and continuity (Bouchekara, 2013). Hence, these techniques are not suitable for the actual OPF situation (AlRashidi & El-Hawary, 2009; Frank, Steponavice, & Rebennack, 2012a). All these limitations are overcome by meta-heuristic optimization methods like genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO), differential evolution algorithm (DEA) and harmony search algorithm (HSA) (Frank, Steponavice, & Rebennack, 2012b; Yildiz, 2012).

In this paper, a newly introduced meta-heuristic optimisation technique named ant-lions optimizer (ALO) is implemented to solve the optimal power flow problem. The ALO technique is a biological and sociological inspired algorithm. This technique is follows the hunting process of the antlions. Key steps of hunting ants such as ants random walk, builds traps, trapping of ants, grasping foods, and rebuilding the traps are applied. The capabilities of ALO are finding the global solution, fast convergence rate due to the use of roulette wheel selection, can handle continuous and discrete optimization problems.

According to no free lunch theorem state that single Meta-heuristic algorithm is not best for every problem so we considered ant-lions optimizer for continues optimal power flow problem. In this work, the ALO is applied to standard 30 bus IEEE test system (Lee, Park, & Ortiz, 1985) to solve the OPF (Bakirtzis, Biskas, Zoumas, & Petridis, 2002; Belhadj & Abido, 1999; Bouktir, Labdani, & Simani, 2005; Ongsakul & Tantimaporn, 2006; Soliman & Mantawy, 2012) problem. There are three objective cases considered in this paper that have to be optimize using ant-lion optimizer (ALO) technique are fuel cost reduction, voltage stability improvement, and voltage deviation minimization. The result shows the optimal adjustments of control variables in accordance with their limits. particle swarm optimisation (PSO) and firefly algorithm (FA) are the most popular algorithms in swarm algorithms. So, the results obtained using ALO technique has been compared with particle swarm optimisation (PSO) and firefly algorithm (FA) techniques. The results show that ALO gives better optimization values as compared with different methods which prove the strength of the suggested method.

2. Ant-lion optimizer technique
The ALO technique reflects the intellectual activities of antlions in catching ants in the environment. The ALO algorithm inspired by the interface of antlions and ants inside the pit. To model such interfaces, ants have to travel over the exploration space, and antlions are permitted to pursue them and become stronger using traps (Mirjalili, 2015).

2.1. Operators of ALO algorithm
As ants travel randomly in search space when finding the prey, a random walk is selected for demonstrating ants’ effort and it is given by Equation (1) (Mirjalili, 2015):

$$ X(t) = \left[ 0, \text{cumsum}(2r(t - 1) - 1), \text{cumsum}(2r(t - 2) - 1), \ldots, \text{cumsum}(2r(t - n) - 1) \right] $$

where cumsum computes the cumulative sum, n is the maximum No. of iteration, t is the step of ants random walk (iteration), and r(t) is a stochastics function defined as follows (Mirjalili, 2015):

$$ r(t) = \begin{cases} 
1 & \text{rand} > 0.5 \\
0 & \text{rand} \leq 0.5 
\end{cases} $$
where $t$ is the step of ants random walk and rand represent a random number created by constant circulation in the interval of $[0, 1]$ (Mirjalili, 2015).

The location of ants are kept and used during optimisation in the given matrix (Mirjalili, 2015):

$$M_{\text{Ant}} = \begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1d} \\
A_{21} & A_{22} & \cdots & A_{2d} \\
\vdots & \vdots & \ddots & \vdots \\
A_{n1} & A_{n2} & \cdots & A_{nd}
\end{bmatrix}$$

(2)

where $M_{\text{Ant}}$ = the matrix for storing the location of every ant, $A_{ij}$ = the value for $j^{th}$ variable (dimension) of $i^{th}$ ant, $n$ = the No. of ants and $d$ = the total No. of variables.

For calculating individual ant, a fitness function is used in optimisation and subsequent matrix saves the fitness value of each ants (Mirjalili, 2015):

$$M_{\text{OA}} = \begin{bmatrix}
f(A_{11}, A_{12}, \ldots, A_{1d}) \\
f(A_{21}, A_{22}, \ldots, A_{2d}) \\
\vdots \\
f(A_{n1}, A_{n2}, \ldots, A_{nd})
\end{bmatrix}$$

(3)

where $M_{\text{OA}}$ = the matrix for storing the each ant fitness, $A_{ij}$ = the value of $j^{th}$ variable of $i^{th}$ ant, $n$ = the total No. of ants and $f$ = the objective function.

So we suppose that ants, as well as the antlions, are hide anywhere in the search area. So as to store their locations and fitness values, the following matrices are used:

$$M_{\text{Antlion}} = \begin{bmatrix}
AL_{11} & AL_{12} & \cdots & AL_{1d} \\
AL_{21} & AL_{22} & \cdots & AL_{2d} \\
\vdots & \vdots & \ddots & \vdots \\
AL_{n1} & AL_{n2} & \cdots & AL_{nd}
\end{bmatrix}$$

(4)

where $M_{\text{Antlion}}$ = the matrix for storing the location of individual antlion, $AL_{ij}$ = the value of $j^{th}$ variable of $i^{th}$ antlion, $n$ = No. of ants and $d$ = the No. of variables.

$$M_{\text{OAL}} = \begin{bmatrix}
f([AL_{11}, AL_{12}, \ldots, AL_{1d}]) \\
f([AL_{21}, AL_{22}, \ldots, AL_{2d}]) \\
\vdots \\
f([AL_{n1}, AL_{n2}, \ldots, AL_{nd}])
\end{bmatrix}$$

(5)

where $M_{\text{OAL}}$ = the matrix for storing the fitness of individual antlion, $AL_{ij}$ = the value of $j^{th}$ variable of $i^{th}$ antlion, $n$ = No. of ants and $f$ = the objective function.

2.2. Random walk of ants

Each of the behaviors is mathematically modeled as (Mirjalili, 2015).

The random walks of ants is calculated by Equation (6):
where $a_i = \text{minimal of the random walk of } i^{th} \text{ variable}, b_i = \text{maximum of random walk in } i^{th}\text{ variable}.$

2.3. Trapping in antlions pits
The trapping in ant-lion’s pits is calculated by Equations (7) and (8):

\[ c^t_i = \text{Antlion}^t_j + c^t \] (7)

\[ d^t_i = \text{Antlion}^t_j + d^t \] (8)

2.4. Sliding ants towards antlion
The sliding ants towards ant-lion calculated by Equations (9) and (10):

\[ c^t = \frac{c^t}{I} \] (9)

\[ d^t = \frac{d^t}{T} \] (10)

where $I = \text{ratio}, c^t = \text{minimal of total variables at } t^{th} \text{ iteration}, \text{ and } d^t = \text{vector containing the maximum of total variables at } t^{th} \text{ iteration}.$

2.5. Catching prey and re-building the pit
Hunting prey and re-arranging the pits calculated by Equation (11):

\[ \text{Antlion}^t_j = \text{Ant}^t_i \text{ if } f(\text{Ant}^t_i) > f(\text{Antlion}^t_j) \] (11)

where $t = \text{the current iteration}, \text{ Antlion}^t_j = \text{the location of chosen } j^{th} \text{ antlion at } t^{th} \text{ iteration}, \text{ and } \text{Ant}^t_i = \text{the location of } i^{th} \text{ ant at } t^{th} \text{ iteration}.$

2.6. Elitism
Elitism of ant-lion calculated using roulette wheel by Equation (12):

\[ \text{Ant}^t_i = \frac{R^t_A + R^t_E}{2} \] (12)

where $R^t_A = \text{the random walk nearby the antlion chose by means of the roulette wheel at } t^{th} \text{ iteration}, \text{ and } R^t_E = \text{the random walk nearby the elite at } t^{th} \text{ iteration}, \text{ Ant}^t_i = \text{the location of } i^{th} \text{ ant at } t^{th} \text{ iteration}.$ (Mirjalili, 2015).

3. Optimal power flow problem formulation
As specified before, OPF is Optimized power flow problem which provides the optimal values of control (independent) variables by minimizing a predefined objective function with respect to the operating bounds of the system (Bouchekara, Abido, Chaib, et al., 2014). The OPF problem can be mathematically expressed as a non-linear constrained optimization problem as follows (Bouchekara, Abido, Chaib, et al., 2014):

Minimize $f(a, b)$ (13)

Subject to $s(a, b) = 0$ (14)
And \( h(a, b) \leq 0 \)  

where \( a = \text{vector of state variables}, b = \text{vector of control variables}, f(a, b) = \text{objective function}, s(a, b) = \text{different equality constraints set}, h(a, b) = \text{different inequality constraints set}. \)

### 3.1. Variables

#### 3.1.1. Control variables

The control variables should be so managed to fulfill the power flow equations. For the OPF problem, the set of control variables can be formulated as (Bouchekara, Abido, Chaib, et al., 2014; Bouchekara, Abido, & Boucherma, 2014):

\[
b^T = [P_G, \ldots, P_{G_{Gen}}, V_{G_1}, \ldots, V_{G_{Gen}}, Q_{C_1}, \ldots, Q_{C_{NCom}}, T_1, \ldots, T_{NTr}] 
\]

where \( P_G = \text{real power output on the generator buses excluding on the reference bus}, V_G = \text{magnitude of voltage on generator buses}, Q_C = \text{shunt VAR compensation}, T = \text{tap settings of the transformer}, NGen, NTr, NCom = \text{No. of generating units, No. of tap changing transformers and No. of shunt VAR compensation devices, respectively.} \)

#### 3.1.2. State variables

There is a need for variables for all OPF formulations for the characterization of the Electrical Power Engineering state of the system. So, the state variables can be formulated as (Bouchekara, Abido, Chaib, et al., 2014; Bouchekara, Abido, & Boucherma, 2014):

\[
a^T = [P_G, V_{L_1}, \ldots, V_{L_{NLB}}, Q_G, S_L, \ldots, S_{Nline}] \]

where \( P_G = \text{Real power output on the reference bus}, V_L = \text{magnitude of voltage on load buses}, Q_G = \text{reactive power generation of all generators}, S_L = \text{line loading or power flow}, NLB, Nline = \text{No. of PQ buses and the No. of lines, respectively.} \)

### 3.2. Constraints

There are two OPF constraints named inequality and equality constraints. These constraints are explained in the next topic.

#### 3.2.1. Equality constraints

The physical condition of the system is defined by the equality constraints of the OPF. Basically these are the load flow equations which can be explained as follows (Bouchekara, Abido, Chaib, et al., 2014; Bouchekara, Abido, & Boucherma, 2014).

##### 3.2.1.1. Real power constraints

The real power constraints can be formulated as follows (Bouchekara, 2013):

\[
P_{G_i} - P_D - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij})] = 0 
\]

##### 3.2.1.2. Reactive power constraints

The reactive power constraints can be formulated as follows (Bouchekara, Abido, & Boucherma, 2014):

\[
Q_{G_i} - Q_D - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij})] = 0 
\]

where \( \delta_i = \delta_i - \delta_j, NB = \text{total No. of buses}, P_G = \text{real power generation}, Q_G = \text{reactive power generation}, P_D = \text{active power load demand}, Q_D = \text{reactive power load demand}, B_{ij} \text{ and } G_{ij} = \text{components of the} \)
admittance matrix $Y_{ij} = (G_{ij} + jB_{ij})$ shows the susceptance and conductance among bus $i$ and bus $j$, respectively.

3.2.2. Inequality constraints
The boundaries of power system devices together with the bounds created to ensure system security are given by inequality constraints of the OPF (Abou El Ela & Abido, 2010; Bouchekara, Abido, & Boucherma, 2014).

3.2.2.1. Generator constraints. For all generators including the reference bus: voltage, real power, and reactive power outputs should be constrained by their minimum and maximum bounds as follows (Bouchekara, Abido, & Boucherma, 2014):

$$V_{G_i}^{lower} \leq V_{G_i} \leq V_{G_i}^{upper}, \quad i = 1, \ldots, N_{Gen}$$ (20)

$$P_{G_i}^{lower} \leq P_{G_i} \leq P_{G_i}^{upper}, \quad i = 1, \ldots, N_{Gen}$$ (21)

$$Q_{G_i}^{lower} \leq Q_{G_i} \leq Q_{G_i}^{upper}, \quad i = 1, \ldots, N_{Gen}$$ (22)

3.2.2.2. Transformer constraints. Tap settings of the transformer should be constrained inside their stated lowest and highest bounds as given below (Bouchekara, 2013):

$$T_{G_i}^{lower} \leq T_{G_i} \leq T_{G_i}^{upper}, \quad i = 1, \ldots, N_{Gen}$$ (23)

3.2.2.3. Shunt VAR compensator constraints. Shunt compensators need to be constrained with their lowest and highest bounds as given below (Bouchekara, Abido, & Boucherma, 2014):

$$Q_{C_i}^{lower} \leq Q_{C_i} \leq Q_{C_i}^{upper}, \quad i = 1, \ldots, N_{Gen}$$ (24)

3.2.2.4. Security constraints. These comprise the limits of the magnitude of the voltage on PQ buses and line loadings. Voltage for every PQ bus should be limited by its minimum and maximum operational bounds. Loadings over each line should not exceed its maximum loading limit. So, these limitations can be statistically stated as (Bouchekara, 2013):

$$V_{L_i}^{lower} \leq V_{L_i} \leq V_{L_i}^{upper}, \quad i = 1, \ldots, N_{Gen}$$ (25)

$$S_{l_i} \leq S_{l_i}^{upper}, \quad i = 1, \ldots, N_{line}$$ (26)

The control variables are self-constraint. The inequality constrained of state variables comprises a magnitude of load (PQ) bus voltage, active power production at reference bus, reactive power production, and loading on line may be encompassed by an objective function in terms of quadratic penalty terms. In which, the penalty factor is increased by the square of the disrespect value of state variables and is included in the objective function and any impractical result achieved is declined (Bouchekara, 2013).

Penalty function can be mathematically formulated as given below:

$$J_{aug} = J + \partial_P (P_{G_1} - P_{G_1}^{lim})^2 + \partial_V \sum_{i=1}^{N_{LB}} (V_{L_i} - V_{L_i}^{lim})^2 + \partial_Q \sum_{i=1}^{N_{Gen}} + \partial_S \sum_{i=1}^{N_{line}} (S_{l_i} - S_{l_i}^{max})^2$$ (27)

where $\partial_P, \partial_V, \partial_Q, \partial_S =$ penalty factors, $U_{lim} =$ Boundary price of the state variable $U.$
If $U$ is greater than the maximum value, $U_{\text{lim}}$ takings the maximum value, if $U$ is lesser than the minimum value, $U_{\text{lim}}$ takings the value of that limit. This can be shown as follows (Bouchekara, 2013):

$$
U_{\text{lim}} = \begin{cases} 
U_{\text{upper}}, & U < U_{\text{upper}} \\
U_{\text{lower}}, & U < U_{\text{lower}} 
\end{cases}
$$

(28)

4. Application and results
The ALO method implemented for the OPF solution for standard 30-bus IEEE test system and for a number of objectives with dissimilar functions. The used program is written in MATLAB R2014b computing surroundings and used on a 2.60 GHz i5 PC with 4 GB RAM. In this work, the number of search agents or number of ants is selected to be 40.

4.1. IEEE 30-bus test system
With the purpose of elucidating the effectiveness of the suggested ALO algorithm, it has been verified on the 30-bus IEEE standard test system as displays in Figure 1. The test system selected in the present work has these equipment (Bouchekara, 2013; Lee et al., 1985): six generating units, four regulating transformers and nine shunt VAR compensators.

In addition, generator cost coefficient numbers, the line numbers, bus numbers, and the upper and lower bounds for the control variables are specified in (Lee et al., 1985).

In given test system, five diverse objectives are considered for various purposes and all the acquired outcomes are given in Table 1. The very first column of this table denotes the optimal values of control variables found where:

![Figure 1. Single line illustration of 30-bus IEEE test system.](image-url)
• $P_{Gi}$ through $P_{G6}$ and $V_{Gi}$ through $V_{G6}$ signifies the power and voltages of generator 1 to 6.

• $T_{4-12}$, $T_{6-9}$, $T_{6-10}$ and $T_{28-27}$ are the transformer tap settings comprised between buses 4–12, 6–9, 6–10 and 28–27.

• $Q_{C10}$, $Q_{C12}$, $Q_{C15}$, $Q_{C17}$, $Q_{C20}$, $Q_{C21}$, $Q_{C23}$, $Q_{C24}$ and $Q_{C29}$ denote the shunt VAR compensators.

Further, fuel cost ($$/h), real power losses (MW), reactive power losses (MVAR), voltage deviation and $L_{max}$ represent the total generation fuel cost of the system, the total real power losses, the total reactive power losses, the load voltages deviation from 1 and the stability index, respectively. Other particulars for these outcomes will be specified in the next topic.

The control parameters for ALO, FA, PSO used in this problem are given in Table 1.

### Table 1. Control parameters used in ALO, FA and PSO

| Sr. no. | Parameters                      | Value     |
|---------|---------------------------------|-----------|
| 1       | Population (No. of ants) (N)    | 40        |
| 2       | Maximum iterations count (t)    | 500       |
| 3       | No. of variables (dim)          | 6         |
| 4       | Random number                   | [0, 1]    |

Case 1: Minimization of generation fuel cost

The very common OPF objective that is generation fuel cost reduction is considered in the case 1. Therefore, the objective function $Y$ signifies the total fuel cost of every generators and is calculated by Equation (29) (Bouchekara, Abido, Chaib, et al., 2014):\[
Y = \sum_{i=1}^{NGen} f_i ($/h) (29)
\]

where $f_i$ shows the fuel cost of the $i^{th}$ generator. $f_i$ may be formulated as follow:

\[
 f_i = \alpha_i + \beta_i P_{Ni} + \gamma_i P_{Ni}^2 ($/h) (30)
\]

where $\alpha_i$, $\beta_i$ and $\gamma_i$ are the cost coefficients of the $i^{th}$ generator. The cost coefficients data are specified in Lee et al. (1985).

The variation of the total fuel cost for different algorithms is presented in Figure 2. It demonstrates that the suggested method has outstanding convergence characteristics. The comparison of fuel cost obtained with diverse techniques is shown in Table 2 which displays that the results obtained by ALO are better than the other methods. The optimal ideals of control variables achieved by various methods for case 1 are specified in Table 3. By means of the same settings i.e. control variables boundaries, initial situations, and system values, the results achieved in case 1 with the ALO

### Table 2. Comparison of fuel cost obtained with different algorithms

| Method   | Fuel cost | Method description     |
|----------|-----------|------------------------|
| ALO      | 799.155   | Ant-lion optimizer     |
| FA       | 799.766   | Firefly algorithm      |
| PSO      | 799.704   | Particle swarm optimization |
| DE       | 799.289   | Differential evolution |
| BHBO     | 799.921   | Black hole based optimization |
technique are equated to different methods and it displays that the fuel cost is greatly decreased compared to the initial case (Bouchekara, 2013). Quantitatively, it is reduced from 901.951 to 799.155 $/h.

Case 2: Voltage profile improvement

Bus voltage is considered as very essential as well as important security and service excellence indices (Bouchekara, 2013). Here the goal is to increase the voltage profile by reducing the voltage deviation of load buses from the unity 1.0 p.u. Hence, the objective function may be formulated by Equation (31) (Bouchekara, Abido, & Boucherma, 2014):

\[
Y_{\text{voltage deviation}} = \sum_{i=1}^{\text{NGen}} |V_i - 1.0|
\]  

(31)

| Table 3. Optimal values of control variables for case 1 with different algorithms |
|----------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Control variable                | Min | Max  | Initial | ALO  | FA    | PSO   |
| \( P_{G1} \)                   | 50  | 200  | 99.2230 | 177.081 | 177.760 | 177.105 |
| \( P_{G2} \)                   | 20  | 80   | 80      | 48.725  | 48.730  | 48.748  |
| \( P_{G5} \)                   | 15  | 50   | 50      | 21.312  | 21.364  | 21.318  |
| \( \nu_{4} \)                  | 10  | 35   | 20      | 21.031  | 20.260  | 20.986  |
| \( P_{G11} \)                  | 10  | 30   | 20      | 11.953  | 12.155  | 12.049  |
| \( P_{G13} \)                  | 12  | 40   | 20      | 12.000  | 12.000  | 12.000  |
| \( V_{G1} \)                   | 0.95| 1.1  | 1.05    | 1.100   | 1.100   | 1.100   |
| \( V_{G2} \)                   | 0.95| 1.1  | 1.04    | 1.088   | 1.086   | 1.088   |
| \( V_{G5} \)                   | 0.95| 1.1  | 1.01    | 1.062   | 1.058   | 1.061   |
| \( V_{G9} \)                   | 0.95| 1.1  | 1.01    | 1.070   | 1.067   | 1.070   |
| \( V_{G11} \)                  | 0.95| 1.1  | 1.05    | 1.083   | 1.070   | 1.100   |
| \( T_{11} \)                   | 0   | 1.1  | 1.078   | 1.014   | 0.998   | 0.976   |
| \( T_{11} \)                   | 0   | 1.1  | 1.069   | 0.987   | 1.023   | 0.975   |
| \( T_{15} \)                   | 0   | 1.1  | 1.032   | 1.046   | 1.028   | 1.015   |
| \( T_{15} \)                   | 0   | 1.1  | 1.068   | 0.997   | 1.007   | 0.966   |
| \( Q_{C10} \)                  | 0   | 5    | 0       | 2.805   | 2.567   | 2.353   |
| \( Q_{C12} \)                  | 0   | 5    | 0       | 2.060   | 4.314   | 5.000   |
| \( Q_{C15} \)                  | 0   | 5    | 0       | 2.254   | 3.899   | 0.000   |
| \( Q_{C15} \)                  | 0   | 5    | 0       | 4.705   | 3.190   | 0.689   |
| \( Q_{C20} \)                  | 0   | 5    | 0       | 2.144   | 2.413   | 0.003   |
| \( Q_{C20} \)                  | 0   | 5    | 0       | 2.685   | 0.021   | 5.000   |
| \( Q_{C21} \)                  | 0   | 5    | 0       | 3.892   | 2.716   | 0.000   |
| \( Q_{C21} \)                  | 0   | 5    | 0       | 2.989   | 3.224   | 0.000   |
| \( Q_{C24} \)                  | 0   | 5    | 0       | 4.121   | 3.879   | 0.000   |
| Fuel cost ($/h)                | –   | –    | 901.951 | 799.155 | 799.766 | 799.704 |
The variation of voltage deviation with different algorithms over iterations is sketched in Figure 3. It demonstrates that the suggested method has good convergence characteristics. The statistical values of voltage deviation obtained with different methods are display in Table 4 which displays that the outcomes obtained by ALO are enhanced than the other methods. The optimal values of control variables achieved by different algorithms for case 2 are specified in Table 5. By means of the same settings, the results achieved in case 2 with the ALO technique are compared to some other methods and it displays that the voltage deviation is significantly reduced compared to the initial
case (Bouchekara, 2013). It has been made known that the voltage deviation is decreased from 1.1496 to 0.1222 p.u. using ALO technique.

**Case 3: Voltage stability enhancement**

Presently, the transmission networks are enforced to work nearby their safety bounds, because of cost-effective and environmental causes. One of the significant characteristics of the network is its capability to retain continuously tolerable bus voltages to each point beneath standard operational environments, next to the rise in load, as soon as the network is being affected by disruption. The

| Control variable | Min | Max | Initial | ALO | FA | PSO |
|------------------|-----|-----|---------|-----|----|-----|
| $P_{G1}$        | 50  | 200 | 99.2230 | 176.422 | 175.763 | 175.922 |
| $P_{G2}$        | 20  | 80  | 80      | 49.012 | 48.396 | 46.389 |
| $P_{G3}$        | 15  | 50  | 50      | 21.829 | 21.492 | 21.597 |
| $P_{G4}$        | 10  | 35  | 20      | 19.974 | 20.564 | 19.396 |
| $P_{G5}$        | 10  | 30  | 20      | 14.073 | 13.163 | 17.656 |
| $P_{G6}$        | 12  | 40  | 20      | 12.001 | 13.829 | 12.000 |
| $V_{G1}$        | 0.95| 1.1 | 1.05    | 1.038 | 1.038 | 1.047 |
| $V_{G2}$        | 0.95| 1.1 | 1.04    | 1.022 | 1.023 | 1.034 |
| $V_{G3}$        | 0.95| 1.1 | 1.01    | 1.014 | 1.015 | 0.999 |
| $V_{G4}$        | 0.95| 1.1 | 1.01    | 1.006 | 1.000 | 1.005 |
| $V_{G5}$        | 0.95| 1.1 | 1.05    | 1.004 | 1.026 | 0.999 |
| $V_{G6}$        | 0.95| 1.1 | 1.05    | 1.006 | 1.050 | 1.018 |
| $T_{11}$        | 0   | 1.1 | 1.078   | 0.983 | 0.991 | 0.954 |
| $T_{12}$        | 0   | 1.1 | 1.069   | 0.939 | 0.926 | 0.969 |
| $T_{15}$        | 0   | 1.1 | 1.032   | 0.971 | 1.038 | 0.989 |
| $T_{16}$        | 0   | 1.1 | 1.068   | 0.966 | 0.964 | 0.960 |
| $Q_{12}$        | 0   | 5   | 0       | 3.051 | 2.228 | 3.948 |
| $Q_{13}$        | 0   | 5   | 0       | 3.552 | 1.976 | 1.765 |
| $Q_{14}$        | 0   | 5   | 0       | 3.925 | 0.245 | 4.844 |
| $Q_{17}$        | 0   | 5   | 0       | 4.221 | 2.092 | 3.075 |
| $Q_{25}$        | 0   | 5   | 0       | 3.230 | 4.887 | 4.687 |
| $Q_{21}$        | 0   | 5   | 0       | 4.499 | 2.177 | 4.948 |
| $Q_{22}$        | 0   | 5   | 0       | 4.485 | 4.421 | 1.623 |
| $Q_{24}$        | 0   | 5   | 0       | 4.597 | 2.846 | 3.559 |
| $Q_{29}$        | 0   | 5   | 0       | 2.479 | 2.791 | 2.034 |
| \( V_{d} \)     | –   | –   | 1.1496  | 0.1222 | 0.1474 | 0.1506 |
unoptimized control variables may cause increasing and unmanageable voltage drop causing a tremendous voltage collapse (Bouchekara, Abido, & Boucherma, 2014). Hence, voltage stability is inviting ever more attention. By using various techniques to evaluate the margin of voltage stability, Glitch and Kessel have introduced a voltage stability index called L-index depends on the viability of load flow equations for every node (Kessel & Glavitsch, 1986). The L-index of a bus shows the probability of voltage breakdown circumstance for that particular bus. It differs between 0 and 1 equivalent to zero loads and voltage breakdown, respectively.

For the given network with \( NB \), \( NGen \) and \( NLB \) buses signifying the total No. of buses, the total No. of generator buses and the total No. of load buses, respectively. The buses can be distinct as PV buses at the top and PQ buses at the bottom as follows (Bouchekara, Abido, & Boucherma, 2014):

\[
\begin{bmatrix}
I_L \\
I_G
\end{bmatrix} = [Y_{bus}] 
\begin{bmatrix}
V_L \\
V_G
\end{bmatrix} = 
\begin{bmatrix}
Y_{LL} & Y_{LG} \\
Y_{GL} & Y_{GG}
\end{bmatrix} 
\begin{bmatrix}
V_L \\
V_G
\end{bmatrix}
\]

(32)

where \( Y_{LL}, Y_{LG}, Y_{GL} \) and \( Y_{GG} \) are co-matrix of \( Y_{bus} \). The subsequent hybrid network of equations can be expressed as:

\[
\begin{bmatrix}
V_L \\
V_G
\end{bmatrix} = [H] 
\begin{bmatrix}
I_L \\
I_G
\end{bmatrix} = 
\begin{bmatrix}
H_{LL} & H_{LG} \\
H_{GL} & H_{GG}
\end{bmatrix} 
\begin{bmatrix}
I_L \\
I_G
\end{bmatrix}
\]

(33)

| Method     | \( L_{max} \) | Method description                  |
|------------|---------------|-------------------------------------|
| ALO        | 0.1140        | Ant-lion optimizer                  |
| FA         | 0.1184        | Firefly algorithm                   |
| PSO        | 0.1180        | Particle swarm optimization         |
| DE         | 0.1219        | Differential evolution              |
| BHBO       | 0.1167        | Black hole based optimization       |
where matrix $H$ is produced by the partially inverting of $Y_{bus}$, $H_{LL}$, $H_{LG}$, $H_{GL}$, and $H_{GG}$ are co-a matrix of $H$, $V_{G}$, $I_{G}$, $V_{L}$, and $I_{L}$ are voltage and current vector of PV buses and PQ buses, respectively.

The matrix $H$ is given by:

$$[H] = \begin{bmatrix} Z_{LL} & -Z_{LL}Y_{LG} \\ Y_{GL}Z_{LL} & Y_{GG} - Y_{GL}Z_{LL}Y_{LG} \end{bmatrix} Z_{LL} = Y_{LL}^{-1}$$  \hspace{1cm} (34)

Hence, the $L$-index denoted by $L_j$ of bus $j$ is denoted as follows:

### Table 7. Optimal values of control variables for case 3 with different algorithms

| Control variable | Min  | Max  | Initial | ALO  | FA   | PSO  |
|------------------|------|------|---------|------|------|------|
| $P_{G1}$         | 50   | 200  | 99.223  | 159.945 | 150.565 | 158.331 |
| $P_{G2}$         | 20   | 80   | 80      | 48.347  | 45.212 | 49.050 |
| $P_{G5}$         | 15   | 50   | 50      | 21.160  | 17.010 | 18.956 |
| $P_{G6}$         | 10   | 35   | 20      | 25.810  | 31.418 | 31.224 |
| $P_{G11}$        | 10   | 30   | 20      | 22.839  | 23.041 | 15.906 |
| $P_{G13}$        | 12   | 40   | 20      | 13.040  | 24.057 | 17.801 |
| $V_{G1}$         | 0.95 | 1.1  | 1.05    | 1.100   | 1.076  | 1.098 |
| $V_{G2}$         | 0.95 | 1.1  | 1.04    | 1.088   | 1.066  | 1.090 |
| $V_{G5}$         | 0.95 | 1.1  | 1.01    | 1.068   | 1.006  | 1.043 |
| $V_{G6}$         | 0.95 | 1.1  | 1.01    | 1.098   | 1.044  | 1.058 |
| $V_{G11}$        | 0.95 | 1.1  | 1.05    | 1.100   | 1.081  | 1.081 |
| $V_{G13}$        | 0.95 | 1.1  | 1.05    | 1.100   | 1.036  | 1.100 |
| $T_{11}$         | 0    | 1.1  | 1.078   | 1.035   | 0.920  | 0.900 |
| $T_{12}$         | 0    | 1.1  | 1.069   | 0.996   | 1.019  | 1.007 |
| $T_{15}$         | 0    | 1.1  | 1.032   | 1.002   | 0.954  | 1.071 |
| $T_{16}$         | 0    | 1.1  | 1.068   | 0.969   | 0.909  | 0.933 |
| $Q_{C10}$        | 0    | 5    | 0       | 1.856   | 2.673  | 3.286 |
| $Q_{C12}$        | 0    | 5    | 0       | 4.979   | 3.550  | 1.221 |
| $Q_{C15}$        | 0    | 5    | 0       | 5.000   | 3.464  | 4.601 |
| $Q_{C17}$        | 0    | 5    | 0       | 5.000   | 2.061  | 1.082 |
| $Q_{C18}$        | 0    | 5    | 0       | 5.000   | 3.515  | 0.444 |
| $Q_{C20}$        | 0    | 5    | 0       | 4.161   | 1.972  | 2.446 |
| $Q_{C21}$        | 0    | 5    | 0       | 1.669   | 2.943  | 4.753 |
| $Q_{C23}$        | 0    | 5    | 0       | 5.000   | 4.814  | 3.887 |

### Table 8. Comparison of active power transmission losses obtained with different algorithms

| Method | Active power loss | Method description         |
|--------|-------------------|---------------------------|
| ALO    | 2.891             | Ant-lion optimizer        |
| FA     | 3.307             | Firefly algorithm         |
| PSO    | 3.026             | Particle swarm optimization|
| BHBO   | 3.503             | Black hole based optimization|
Hence, the stability of the whole system is described by a global indicator $L_{\text{max}}$ which is presented by (Bouchekara, 2013):

$$L_{\text{max}} = \max (L_j) \quad j = 1, 2, \ldots, NL$$

(36)

The system is more stable as the value of $L_{\text{max}}$ is lower.

The voltage stability can be enhanced by reducing the value of voltage stability indicator $L$-index at every bus of the system (Bouchekara, 2013).

Thus, the objective function may be given as below equation:

$$Y_{\text{voltage stability enhancement}} = L_{\text{max}}$$

(37)

### Table 9. Optimal values of control variables for case 4 with different algorithms

| Control variable | Min | Max | Initial | ALO | FA | PSO |
|------------------|-----|-----|---------|-----|----|-----|
| $P_{G1}$         | 50  | 200 | 99.2230 | 51.290 | 63.292 | 51.427 |
| $P_{G2}$         | 20  | 80  | 80      | 80.000 | 80.000 | 80.000 |
| $P_{G3}$         | 15  | 50  | 50      | 50.000 | 50.000 | 50.000 |
| $P_{G4}$         | 10  | 35  | 20      | 35.000 | 35.000 | 35.000 |
| $P_{G5}$         | 10  | 30  | 20      | 30.000 | 25.730 | 30.000 |
| $P_{G6}$         | 12  | 40  | 20      | 40.000 | 32.686 | 40.000 |
| $V_{G1}$         | 0.95| 1.1 | 1.05    | 1.100 | 1.089 | 1.100 |
| $V_{G2}$         | 0.95| 1.1 | 1.04    | 1.099 | 1.084 | 1.100 |
| $V_{G3}$         | 0.95| 1.1 | 1.01    | 1.082 | 1.062 | 1.083 |
| $V_{G4}$         | 0.95| 1.1 | 1.01    | 1.089 | 1.071 | 1.090 |
| $V_{G5}$         | 0.95| 1.1 | 1.05    | 1.100 | 1.091 | 1.100 |
| $V_{G6}$         | 0.95| 1.1 | 1.05    | 1.091 | 1.084 | 1.100 |
| $T_{11}$         | 0   | 1.1 | 1.078   | 1.058 | 1.015 | 0.977 |
| $T_{12}$         | 0   | 1.1 | 1.069   | 0.972 | 0.919 | 1.100 |
| $T_{13}$         | 0   | 1.1 | 1.032   | 1.011 | 1.001 | 1.100 |
| $T_{14}$         | 0   | 1.1 | 1.068   | 0.995 | 0.993 | 0.998 |
| $Q_{C10}$        | 0   | 5   | 0       | 4.780 | 4.033 | 4.065 |
| $Q_{C12}$        | 0   | 5   | 0       | 3.026 | 4.520 | 0.000 |
| $Q_{C14}$        | 0   | 5   | 0       | 4.995 | 3.247 | 5.000 |
| $Q_{C15}$        | 0   | 5   | 0       | 4.936 | 1.836 | 5.000 |
| $Q_{C16}$        | 0   | 5   | 0       | 4.998 | 2.446 | 0.000 |
| $Q_{C17}$        | 0   | 5   | 0       | 4.998 | 2.905 | 5.000 |
| $Q_{C18}$        | 0   | 5   | 0       | 4.301 | 3.377 | 5.000 |
| $Q_{C19}$        | 0   | 5   | 0       | 5.000 | 2.033 | 0.000 |
| $Q_{C20}$        | 0   | 5   | 0       | 2.520 | 2.980 | 0.000 |
| $Q_{C21}$        | 0   | 5   | 0       | 2.891 | 3.307 | 3.026 |
| $P_{\text{Loss}}$ | -   | -   | 5.8219  | 2.891 | 3.307 | 3.026 |
The variation of the $L_{\text{max}}$ index with different algorithms over iterations is presented in Figure 4. The statistical results obtained with different methods are shown in Table 6 which displays that ALO gives improved results than the various techniques. The optimal values of control variables obtained by various methods for case 3 are display in Table 7. After implementing the ALO approach, it seems from Table 7 that the value of $L_{\text{max}}$ is considerably decreased in this case compared to initial (Bouchekara, 2013) from 0.1723 to 0.1140. Thus, the distance from breakdown point is improved.

**Case 4: Minimization of active power transmission losses**

In the case 4 the Optimal Power Flow goal is to reduce the real power transmission losses, that can be represented by power balance Equation (38) (Bouchekara, 2013):
Figure 5 display the tendency for reducing the total real power losses objective function using the different techniques. The active power losses obtained with different techniques are shown in Table 8 which made sense that the results obtained by ALO give better values than the other methods. The optimal values of control variables obtained by different algorithms for case 4 are displayed in Table 9. By means of the same settings the results achieved in case 4 with the ALO technique are compared to some other methods and it displays that the real power transmission losses are greatly reduced compared to the initial case (Bouchekara, 2013) from 5.821 to 2.891.

\[ J = \sum_{i=1}^{N_{\text{Gen}}} P_i = \sum_{i=1}^{N_{\text{Gen}}} P_{\text{Gi}} - \sum_{i=1}^{N_{\text{Di}}} P_{\text{Di}} \]  

Table 10. Comparison of reactive power losses obtained with different algorithms

| Method  | Reactive power loss | Method description              |
|---------|---------------------|---------------------------------|
| ALO     | −25.076             | Ant-lion optimizer             |
| FA      | −20.464             | Firefly algorithm              |
| PSO     | −23.407             | Particle swarm optimization    |
| BHBO    | −20.152             | Black hole based optimization  |

Table 11. Optimal values of control variables for case 5 with different algorithms

| Control variable | Min | Max | Initial | ALO | FA | PSO |
|------------------|-----|-----|---------|-----|----|-----|
| PG1              | 50  | 200 | 99.2230 | 51.328 | 71.440 | 51.644 |
| PG2              | 20  | 80  | 80      | 80.000 | 79.971 | 80.000 |
| PG3              | 15  | 50  | 50      | 50.000 | 49.999 | 50.000 |
| PG4              | 10  | 35  | 20      | 35.000 | 34.994 | 35.000 |
| PG5              | 10  | 30  | 20      | 30.000 | 29.913 | 30.000 |
| PG6              | 12  | 40  | 20      | 40.000 | 29.787 | 40.000 |
| PG13             | 0.95| 1.1 | 1.05    | 1.100 | 1.086 | 1.100 |
| PG6             | 0.95| 1.1 | 1.04    | 1.100 | 1.081 | 1.100 |
| PG11             | 0.95| 1.1 | 1.01    | 1.100 | 1.055 | 1.100 |
| PG10             | 0.95| 1.1 | 1.05    | 1.100 | 1.059 | 1.100 |
| V1              | 0.95| 1.1 | 1.04    | 1.100 | 1.059 | 1.100 |
| V1              | 0.95| 1.1 | 1.04    | 1.100 | 1.059 | 1.100 |
| V1              | 0.95| 1.1 | 1.04    | 1.100 | 1.059 | 1.100 |
| V1              | 0.95| 1.1 | 1.04    | 1.100 | 1.059 | 1.100 |
| V1              | 0.95| 1.1 | 1.04    | 1.100 | 1.059 | 1.100 |
| T11             | 0   | 1.1 | 1.078   | 0.990 | 1.027 | 0.962 |
| T12             | 0   | 1.1 | 1.069   | 1.023 | 0.934 | 1.100 |
| T15             | 0   | 1.1 | 1.032   | 0.991 | 0.983 | 0.961 |
| T16             | 0   | 1.1 | 1.068   | 0.990 | 0.959 | 0.964 |
| QC10            | 0   | 5   | 0       | 4.940 | 2.963 | 5.000 |
| QC12            | 0   | 5   | 0       | 4.997 | 1.532 | 0.000 |
| QC15            | 0   | 5   | 0       | 4.659 | 4.981 | 0.000 |
| QC11            | 0   | 5   | 0       | 5.000 | 3.212 | 0.000 |
| QC16            | 0   | 5   | 0       | 5.000 | 2.817 | 0.000 |
| QC13            | 0   | 5   | 0       | 4.999 | 3.376 | 0.000 |
| QC14            | 0   | 5   | 0       | 5.000 | 4.009 | 0.000 |
| QC17            | 0   | 5   | 0       | 4.099 | 3.654 | 5.000 |
| QC20            | 0   | 5   | 0       | 4.448 | 1.874 | 0.000 |
| QLoss (MVAR)    | −4.6066 | −25.076 | −20.464 | −23.407 |
Case 5: Minimization of reactive power transmission losses

The accessibility of reactive power is the main point for static system voltage stability margin to provision the transmission of active power from the source to sinks (Bouchekara, 2013).

Thus, the minimization of VAR losses are given by the following expression:

\[
J = \sum_{i=1}^{NGen} Q_i = \sum_{i=1}^{NGen} Q_{Gi} - \sum_{i=1}^{NGen} Q_{Di}
\]  

(39)

It is notable that the reactive power losses may not essentially positive. The variation of reactive power losses with different methods shown in Figure 6. It demonstrates that the suggested method has good convergence characteristics. The statistical values of reactive power losses obtained with different methods are shown in Table 10 which displays that the results obtained by ALO are better than the other methods. The optimal values of control variables obtained by different algorithms for case 5 are given in Table 11. It is shown that the reactive power losses are greatly reduced compared to the initial case (Bouchekara, 2013) from −4.6066 to −25.076 using ALO technique.

5. Robustness test

In order to check the robustness of the ant-lion optimizer for solving continues optimal power flow problems, 10 times trials with various search agents for cases 1–5. Tables 2–11 presents the statistical results achieved by the PSO, FA and ALO algorithms for OPF problems for various cases. From these tables, it is clear that the optimum objective function values obtained by ant-lion optimizer are near to every trial and minimum compare to PSO and FA algorithms. Its proofs the robustness of ant-lion optimizer (ALO) to solve OPF problem.

6. Conclusion

In this work, ant-lion optimizer, firefly algorithm, and particle swarm optimization algorithm are successfully applied to standard 30-bus IEEE systems to solve the optimal power flow problem for the various types of cases: fuel cost, active power loss, reactive power loss, voltage deviation and voltage stability index. The obtained results give the optimum sets of control variables with ALO, PSO and FA Algorithms which demonstrate the effectiveness of the different techniques. The solutions obtained from the ALO approach has good convergence characteristics and gives the better optimum results compared to FA and PSO techniques which confirm the strength of recommended algorithm. Further, we can improve the algorithms efficiency using different types of penalty handling approaches: adaptive, Deb, MQM, static methods etc. and different randomization techniques: adaptive and levy flight approaches for better exploration and exploitation.

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References

Abou El Ela, A. A., & Abido, M. A. (2010). Optimal power flow using differential evolution algorithm. Electric Power Systems Research, 80, 878–885. http://dx.doi.org/10.1016/j.epsr.2009.12.018
Al-Rashidi, M. R., & El-Hawary, M. E. (2009). Applications of computational intelligence techniques for solving the revived optimal power flow problem. Electric Power Systems Research, 79, 694–702. http://dx.doi.org/10.1016/j.epsr.2008.10.004
Bakirtzis, A. G., Biskas, P. N., Zoumas, C. E., & Petridis, V. (2002). Optimal power flow by enhanced genetic algorithm. IEEE Transactions on Power Systems, 17, 229–236. http://dx.doi.org/10.1109/TPWRS.2002.1007886
Belhodj, C. A., & Abido, M. A. (1999). An optimized fast voltage stability indicator. In Electric Power International Conference on Engineering (pp. 79–83). Budapest: PowerTech.

Bouchekara, H. R. E. H. (2013). Optimal power flow using black-hole-based optimization approach. Applied Soft Computing, 24, 879–888.

Bouchekara, H. R. E. H., Abido, M. A., & Boucherma, M. (2014). Optimal power flow using teaching-learning-based optimization technique. Electric Power Systems Research, 114, 49–59.

Bouchekara, H. R. E. H., Abido, M. A., Chaib, A. E., & Mehasni, R. (2014). Optimal power flow using the league championship algorithm: A case study of the Algerian power system. Energy Conversion and Management, 87, 58–70. http://dx.doi.org/10.1016/j.enconman.2014.06.088

Bouktir, T., Llabdar, R., & Slimani, L. (2005). Optimal power flow of the Algerian electrical network using ant colony optimization method. Leonardo Journal of Sciences, 6, 43–57.

Carpentier, J. (1962). Contribution à l’étude du Dispatching Economique [Contribution to the economic dispatch problem]. Bulletin de la Société Française des Électriciens, 3, 431–447.

Duman, S., Güvenç, U., Sönmez, Y., & Yörükeren, N. (2012). Optimal power flow using gravitational search algorithm. Energy Conversion and Management, 59, 86–95. http://dx.doi.org/10.1016/j.enconman.2012.02.024

Frank, S., Steponavice, I., & Rebennack, S. (2012a). Optimal power flow: A bibliographic survey I. Energy Systems, 3, 221–258. http://dx.doi.org/10.1007/s12667-012-0056-y

Frank, S., Steponavice, I., & Rebennack, S. (2012b). Optimal power flow: A bibliographic survey II. Energy Systems, 3, 259–289. http://dx.doi.org/10.1007/s12667-012-0057-y

Kessel, P., & Glavitsch, H. (1986). Estimating the voltage stability of a power system. IEEE Transactions on Power Delivery, 1, 346–354. http://dx.doi.org/10.1109/TPWRD.1986.4308013

Lee, K., Park, Y., & Ortiz, J. (1985). A united approach to optimal real and reactive power dispatch. IEEE Transactions on Power Apparatus and Systems, 104, 1147–1153. http://dx.doi.org/10.1109/TPAS.1985.323466

Mirjalili, S. (2015). The ant lion optimizer. Advances in Engineering Software, 83, 80–98. http://dx.doi.org/10.1016/j.advengsoft.2015.01.010

Niknam, T., Norimani, M. R., Jabbari, M., & Malekpour, A. R. (2011). A modified shuffle frog leaping algorithm for multi-objective optimal power flow. Energy, 36, 6420–6432. http://dx.doi.org/10.1016/j.energy.2011.09.027

Ongsakul, W., & Tantimaporn, T. (2006). Optimal power flow by improved evolutionary programming. Electric Power Components and Systems, 34, 79–95. http://dx.doi.org/10.1080/15325000691001458

Soliman, S. A. H., & Mantawy, A. H. (2012). Modern optimization techniques with applications in electric power systems, energy systems. New York, NY: Springer. http://dx.doi.org/10.1007/978-1-4614-1752-1

Yildiz, A. R. (2012). A comparative study of population-based optimization algorithms for turning operations. Information Sciences, 210, 81–88. http://dx.doi.org/10.1016/j.ins.2012.03.005