Bilepton and exotic quark mass limits in 331 models from $Z \to b\bar{b}$ decay

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Abstract

We study the effect of new physics on the Z-decay into $b\bar{b}$ pairs in the framework of 331 models. The decay $Z \to b\bar{b}$ is computed at one loop level and, using previous results, we evaluate this branching fraction in the framework of 331 models. A wide range of the space parameter of the model is considered and possible deviations from the standard model predictions are explored. From precision measurements at the Z-pole we find the allowed region for $M_{J_3}$, $M_\chi$ at 95% CL.
I. INTRODUCTION

The Standard Model (SM) [1] has passed almost all the experimental test in the past, except for the recently discovered neutrino oscillation [2]. The SM predictions have been extensively checked, for example, in the neutral currents phenomenology, the GIM mechanism [3] and the invisible decay of the Z, where three light neutrino families can be accommodated with experimental data. One loop quantum corrections at the $Z\bar{b}b$ vertex [4] have shown the first hint of the top-quark mass, latter discovered at FERMILAB [5]. Moreover, oblique corrections show that the Higgs mass value should be at the level of the electroweak scale [6].

However, there are some fundamental questions that remain unanswered within the SM framework. The first experimental clue came from neutrino oscillations as an evidence for some physics beyond the SM, but the SM also has no answer to the fermion and neutrino mass scales and mixing angles, the number of families, the origin of electric charge quantization, among a list of important questions [7]. Some of these problems may be a hint to new physics above the electroweak scale and below the Planck scale and are addressed in extended models using different theoretical ideas. For example, the naturalness problem is solved in supersymmetric theories. Quadratic quantum corrections to the Higgs mass can be rendered stable in supersymmetric extensions of the SM, in particular the Minimal Supersymmetric SM (MSSM) where the Higgs mass can be light [8]. The stability of the Higgs mass can also be obtained in the recently proposed Little Higgs theories [9]. On the other hand, Warped Extra Dimension theories can explain the fine tuning between the electroweak scale and the Planck scale [10]. Within these frameworks one can naturally have symmetry breaking without scalar fields.

The so-called (331) models, based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group, has been recently proposed [11]. In these models the cancellation of anomalies imposes to have three fermion families. There are some models with one and three families where this number is regulated by the value of the parameter $\beta$ in the definition of the electromagnetic charge [12]. In three families models, there are two classes of them studied in the literature with $\beta = -\sqrt{3}$, $\beta = 1/\sqrt{3}$, where the first of them has bileptons in the spectrum. These models can be tested in the near future at the LHC and have an interesting phenomenology. Besides bileptons they also predict the existence of exotic quarks. There are some studies
on the mass limits for these particles \[13, 14, 15, 16, 17\]. The exchange of doubly charged bileptons in the processes $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ have been analyzed at center of mass energies from 189 GeV to 207 GeV and it is found that $(g_L/M_\chi)^2 \approx 10^{-6}$ GeV$^{-2}$ up to 95% CL \[13\]. From the contribution of bileptons to the oblique parameters S and T a bound $M_\chi > 500$ GeV has been established \[14\]; from muonium - antimuonium conversion the $M_\chi > 850$ GeV bound is found \[15\]. Fermion pair production in $e^+e^-$ annihilation allows to put the limit $M_\chi > 740$ GeV; this last bound is less stringent than the previously mentioned but it is less dependent on free parameters \[16\]. LHC production has been studied in \[17\].

In this paper we want to study the allowed region for the spectrum of the $J_3$ particles, all of them with fractional electric charge $\pm5/3$, and for the bilepton $\chi^{++}$ for some $(331)$ models \[11\]. This allowed region is constrained by comparing the data with radiative corrections to the decay $Z \rightarrow b\bar{b}$. A $\chi^2$ study can be made taking $\Gamma_Z$, $R_b$, $R_c$, $R_t$ and $\Gamma(Z \rightarrow b\bar{b})$ as parameters. Both the $A_b$ and $A_{FB}^b$ asymmetries are within $3\sigma$ from the SM predictions and we are not going to take these into account in our study \[18\]. On the other hand, as this model has a left-handed structure it can not resolve this last issue at the one loop level because the contribution to the value of $g_{LZ}^{bbZ}$ is very small \[19\]. The mixing of the neutral current with the additional one (i.e., the $Z$ and $Z'$ neutral currents) needs a mixing angle of the order of $\sin \theta = -0.3625$ to explain the $A_b$ and $A_{FB}^b$ deviations, and this value is not compatible with the value one can obtain from the analysis of the $Z$-pole observables and the weak charge $Q_W$, which is of the order of $\sin \theta \leq 10^{-3} - 10^{-4}$ \[20\].

In the following sections we present the model, the new physics contribution to the LEP observables and the conclusions.

II. THE 331 MODEL

The fermionic states can be taken as in Ref.\[21\]:

\[
{f^m_L} = \begin{pmatrix} \nu_m \\ l_m \\ l_m^+ \end{pmatrix}_L \sim (1,3,0) ; \quad Q_3 = \begin{pmatrix} t \\ b \\ J_3 \end{pmatrix}_L \sim (3,3,2/3) ; \quad Q^i_L = \begin{pmatrix} d_i \\ -u_i \\ J_i \end{pmatrix}_L \sim (3,\bar{3},-1/3)
\]

\[
u_{a,R} \sim (3, 1, 5/3) ; \quad d_{a,R} \sim (3, 1, -1/3)
\]
where \( a, m = 1, 2, 3 \); \( i = 1, 2 \) represent the two light families of the SM. The exotic quarks \( J_3 \) and \( J_{1,2} \) have electric charge \( \pm 5/3 \) and \( \pm 4/3 \) respectively and then, they do not mix in the mass matrix. Only the \( J_3 \) quark contributes to the one loop quantum corrections to \( Z \to b\bar{b} \). This model does not have exotic leptons. This implies that the one loop new physics contribution to \( Z \to l^-l^+ \) is suppressed with respect to the SM one. The electric charge is

\[
Q = T_{3L} - \sqrt{3}T_{8L} + Y_N
\]

where \( T_{3L}, T_{8L} \) and \( Y_N \) are the generators of the \( SU(3)_L \) and \( U(1)_X \), respectively.

The scalar fields content, responsible for the symmetry breaking and the fermion masses, is given by three Higgs triplets:

\[
\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1, 3, 1) \quad \eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (1, 3, 0) \\
\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1, 3, -1)
\]

In order to give mass to the neutrino sector one can also introduce a Higgs sextet:

\[
S = \begin{pmatrix} \sigma_1^0 \\ s_2^+ \\ s_1^- \\ s_2^- \\ s_1^{++} \\ \sigma_2^0 \end{pmatrix} \sim (1, 6, 0).
\]

The vacuum expectation values (VEV) for the scalar fields are:

\[
\langle \rho \rangle \sim \frac{u}{\sqrt{2}} , \quad \langle \eta \rangle \sim \frac{v}{\sqrt{2}} , \quad \langle \delta \rangle \sim \frac{v_\sigma}{\sqrt{2}} , \quad \langle \chi \rangle \sim \frac{w}{\sqrt{2}}
\]

where the VEV’s values are chosen to obey the following relations:

\[
w \gg u, v, v_\sigma \sim V \\
V^2 = u^2 + v^2 + v_\sigma^2.
\]
With the mass eigenstates and the would-be Goldstone bosons coming from the Higgs potential \[12, 22\] one can write the relevant couplings in order to compute the one loop correction to \( Z \rightarrow b \bar{b} \). The bosons are written as:

\[
\begin{align*}
\rho &= \left( \begin{array}{c}
G^+_W \\
\frac{iG^+_Z + V}{\sqrt{2}} \\
0
\end{array} \right) ;
\eta = \left( \begin{array}{c}
-G^+_W \\
\frac{-iG^+_Z + V}{\sqrt{2}} \\
0
\end{array} \right) ;
\chi = \left( \begin{array}{c}
G^-_Y \\
G^-_X \\
\frac{w + iG^+_Z}{\sqrt{2}}
\end{array} \right)
\end{align*}
\]

where \( G^\pm_W, G_Z, G^\pm_Y \) and \( G^\pm_X \) are the would-be Goldstone bosons for the fields \( W^\pm, Z, Y^\pm \) and \( \chi^\pm \) respectively. The \( \rho \) and \( \eta \) components give origin to the charged Higgs \( H^+ \), odd-\( A^0 \), even-\( H^0 \) (all with masses of the order of \( W, Z, Y \)) coming from the electroweak scale. The Higgs fields \( \rho^+, \eta^+_2 \) and \( Re(\chi^0) \) have a mass proportional to the scale \( w \). The other fields of \( \chi \) give origin to the would-be Goldstone bosons of \( \chi^{++, Y-} \) and \( Z' \). All the scalar fields, except for \( h^0 \), have masses of the order of the first symmetry breaking \( M_\chi \).

The covariant derivative can be written in terms of the mass eigenstates in the following way:

\[
D_\mu = \partial_\mu + i e Q A_\mu + i \frac{g}{c_W} (T_{3L} - s^2_W Q) Z_\mu + i \frac{g}{\sqrt{1 - 3 t^2_W}} \left( \sqrt{3 t^2_W} (Q - T_3) + T_8 \right) Z'_\mu
\]

\[
+ \frac{i g}{2} \left[ W^+_\mu (T_1 - i T_2) + Y^-_\mu (T_4 - i T_5) + \chi^+_\mu (T_6 - i T_7) + h.c. \right]
\]

where the \( T_i \) are the \( SU(3)_L \) generators. The gauge neutral fields are related in the following way:

\[
\begin{pmatrix}
W^3_\mu \\
W^8_\mu \\
B_\mu
\end{pmatrix} =
\begin{pmatrix}
s_W & c_W & 0 \\
-\sqrt{3} s_W & \sqrt{3} s_W t_W & \sqrt{1 - 3 t^2_W} \\
c_W \sqrt{1 - 3 t^2_W} & -s_W \sqrt{1 - 3 t^2_W} & \sqrt{3 t^2_W}
\end{pmatrix}
\begin{pmatrix}
A_\mu \\
Z_\mu \\
Z'_\mu
\end{pmatrix}
\]

where \( W^3, W^8 \) and \( B_\mu \) are the gauge fields for the groups \( SU(3)_L \) and \( U(1)_X \) respectively. The \( Z - Z' \) mixing is defined as

\[
\begin{pmatrix}
Z_\mu \\
Z'_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
Z_{1\mu} \\
Z_{2\mu}
\end{pmatrix}
\]

Finally, the Yukawa lagrangian is:

\[
-L_Y = \lambda_3 \bar{Q}_{3L} J_{3R} \chi + \lambda_{ij} \bar{Q}_{iL} J_{jR} \chi^* + \lambda_{3a} \bar{Q}_{3L} d_{aR} \rho + \lambda_{ia} \bar{Q}_{iL} u_{aR} \rho^* + \lambda_{3a} \bar{Q}_{3L} u_{aR} \eta + \lambda_{ia} \bar{Q}_{iL} d_{aR} \eta^* + h.m.n.f_m.lj_m.l f^c_n.l \eta^* + h.c.
\]
where the Yukawa constants $\lambda_{33}$ and $\lambda_3$ are given by:

$$
\lambda_{33} \sim \frac{g}{\sqrt{2}} \frac{m_b}{M_W}, \quad \lambda_3 \sim \sqrt{2} g \frac{m_{J_3}}{M_X}.
$$

(12)

III. 331 MODELS AND $Z \rightarrow b\bar{b}$

Some of the Feynman rules that we need for the computation of $Z \rightarrow b\bar{b}$ are shown in Figure 1. The new physics diagrams that contribute to one loop order are shown in Figure 2. We can write the one loop contribution to the width $\Gamma(Z \rightarrow b\bar{b})$ as

$$
\Delta \Gamma = i \frac{e}{4s_W c_W} \gamma_\mu (1 - \gamma_5) Re(\delta_{b}^{SM} + \delta_{b}^{NP})
$$

(13)

with

$$
\delta_{b}^{NP} = -\frac{\alpha}{4\pi s_W} F_b
$$

(14)

and

$$
F_b = \sum_j (I_j(x, y) - I_j(0, y)) = I(x, y) - I(x, 0).
$$

(15)

The variables $x$ and $y$ take the value $x = (m_f/M_G)^2$ and $y = (M_Z/M_G)^2$, $m_f$ is the mass of the fermion circulating into the loop and $M_G$ is the mass of the gauge field. The new physics contribution comes with $m_{J_3}$ and $M_X$. The 331 model has a left-handed structure; this implies that the one loop contribution of these fields has, for each vertex, exactly the same tensorial structure as the SM contribution. Using Ref. [4], and substituting $m_t \rightarrow m_{J_3}$, $M_W \rightarrow M_X$ and taking into account the Feynman rules shown in Figure 1 we can compute the $I_j(x, y)$ one loop functions:

$$
I(x, y) = -\frac{5}{3} s_W^2 \left[ \frac{3}{2} + \frac{2}{3} x + \left( 1 - \frac{4}{3} x \right) f_{0a} + \left( \frac{1}{3} + y + \frac{1}{6} x(-4 + x) \right) f_{2a} - \right.
\left. \left( y + x + \frac{x^2}{2} \right) f_{1a} + s_W^2 \left[ \frac{-3}{4} c_W - 3x + 2 \left( \frac{4}{3} c_W + x \right) f_{0b} + (x(1 - x) + \right.ight.
\left. \left. \frac{c_W}{2} \left( y + \frac{1}{2}(1 - x) \right) \right) f_{2b} + x(c_W - 1 + 2x) f_{1b} \right] - \frac{3}{8} c_W + \frac{3}{2} x + \right.
\left. \left( \frac{3}{4} c_W - x \right) f_{0b} + \left( -\frac{1}{2} x(1 - x) + \frac{c_W}{4} (y + \frac{1}{2}(1 - x)) \right) f_{2b} + \right.
\left. \left( \frac{1}{2} c_W - x \right) x f_{1b} + \left( 1 - \frac{2}{3} s_W^2 \right) x \left( -1 + \frac{x}{1 - x} + \frac{x^2 \ln x}{(1 - x)^2} \right) + \right.
\left. \left( \frac{1}{2} + 2 s_W^2 \right) \left( \frac{x}{1 - x} + \frac{x^2 \ln x}{(1 - x)^2} \right) \right)
$$

(16)
where $x = (m_{J_3}/M_χ)^2$ and $y = (M_Z/M_χ)^2$. The $f_{ia(b)}$ functions can be computed using Ref.\[4\] and are shown in the Appendix.

The exotic quarks $J_1, J_2$ do not couple to the bottom quark $b$. As we have already said, they do not mix with $J_3$ because they have different electric charge. Besides, the gauge fields $Z', Y$ do not contribute up to one loop. Scalar field contributions can be summarized in the following way. All $χ$ components are would-be Goldstone bosons, except for the third (neutral) one that does not couples to the $b$-quark. The first two components of $η$ and $ρ$ act exactly as two Higgs doublets. The contributions due to $h^0, H^0, A^0$ are strongly suppressed because the $b$-quark would be into the loop and, moreover, the couplings would be proportional to $m_b$.

The $H^+$ contribution is suppressed with respect to the new physics contributions coming with $m_{J_3}$ and $M_χ$ because of the suppression factors $(m_t/m_{H^+})^2$ and $(m_{J_3}/M_χ)^2$ respectively, where the $H^+$-mass is of the order of $M_χ$ but $m_{J_3} \gg m_t$ in these models. The Yukawa term $λ_{33} J_3 b ρ^{++}$ gives origin to the $ρ^{++}$ contribution, and it is suppressed because $λ_{33} \sim m_b$. The $η_2^+$ does not couples to particles in the loop.

From the $Z - Z'$ mixing we obtain the relation

$$\delta g_R^{Zbb} = -\frac{t_W s_W}{\sqrt{3 + 9 t_W^2}} \sin \theta$$

This formula implies that $\sin \theta \simeq -0.3625$ is the value that can explain the discrepancy for the asymmetry $\delta g_R^{Zbb}$, as already mentioned in the introduction.

We will consider the following observables: $Γ(Z → b\bar{b})$, $R_b$, $R_c$, $R_l$ and $Γ_Z$. The new physics contribution to them can be written as:

$$Γ(Z → b\bar{b}) = Γ(Z → b\bar{b})^{SM}(1 + δ^{NP}_b),$$

$$Γ_Z = Γ_Z^{SM}[1 + Br(Z → b\bar{b})^{SM}δ^{NP}_b],$$

$$R_b = R_b^{SM}[1 + δ^{NP}_b(1 - R_b^{SM})],$$

$$R_c = R_c^{SM}[1 - R_b^{SM}δ^{NP}_b],$$

$$R_l = R_l^{SM}[1 + R_b^{SM}δ^{NP}_b].$$

The theoretical and experimental values are given in Table 1 \[23\]. In order to determine the allowed region in the space parameter for $m_{J_3}$ and $M_χ$ we compute a $χ^2$ fit with the parameters of Table 1 at 95% confidence level. The two $A^{FB}_b$ and $A_b$ asymmetries are not
\[ \Gamma(Z) \rightarrow b\bar{b} \text{ [MeV]} = 0.3775 \pm 0.0004 \]

\[ R_b = 0.21638 \pm 0.00066 \]

\[ R_c = 0.1720 \pm 0.0030 \]

\[ R_l = 20.784 \pm 0.043 \]

| Experiment | Theory |
|------------|--------|
| 2.4952 ± 0.0023 | 2.4972 ± 0.0012 |
| 0.3775 ± 0.0004 | 0.3758 ± 0.0001 |
| 0.21638 ± 0.00066 | 0.21544 ± 0.00014 |
| 0.1720 ± 0.0030 | 0.17233 ± 0.00005 |
| 20.784 ± 0.043 | 20.763 ± 0.014 |

**TABLE I: LEP observables directly sensitive to the Zb\bar{b} vertex**

taken into account in our analysis because of the \( 3\sigma \) discrepancy between the theoretical SM prediction and the measured values.

In Figures 3 and 4 we show the allowed 95\% CL regions delimited by the two curves in the space parameter of \( M_{\chi} \) and \( M_{J_3} \).

**IV. CONCLUSIONS**

The sensitivity of precision observables at the Z-pole to the structure of the \( Z \rightarrow b\bar{b} \) vertex allows to study new physics effects. These can originate deviations from the standard model prediction, and, comparing with the experimental errors one can deduce some limits in the masses of bilepton and exotic quarks in 331 models. New limits on these particles are found in this way. We performed a naive analysis of the three triplets scalar sector contribution of our model to the \( Zb\bar{b} \)-vertex and we found that this contribution is small relative to the one coming from diagrams with \( \chi^{++} \) and \( J_3 \) in the loop. For the 331 model we have an important equation for the bilepton \( \chi^{++} \) and the exotic quark \( J_3 \) masses from the Z-pole. This implies that in the case of a bilepton with a mass of the order of 700 GeV were found in future colliders then an exotic \( J_3 \) quark charged \( \pm 5/3 \) and with a mass in the range 1500 – 4000 GeV should also be found.

As this model has a left-handed structure it can not resolve the \( A_b \) and \( A_{FB}^{b} \) asymmetries at the one loop level because the contribution to the value of \( g_{L}^{bZ} \) is very small. On the other hand, the mixing of the two neutral currents needs a mixing angle of the order of \( \sin \theta = -0.3625 \) to explain the asymmetry deviations, and this value is not compatible with the value one can obtain from the analysis of the Z-pole observables and the weak charge...
$Q_W$, which is of the order of $\sin \theta \leq 10^{-3} - 10^{-4}$.

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V. APPENDIX

The functions $f_{ia(b)}(x, y)$ we used to evaluate the corrections, in the limit when $x = (m_f/M_G)^2 \to 0$ are:

\[
\begin{align*}
    f_{0a} &= -2 + \ln(y) - i\pi \\
    f_{1a} &= \frac{\ln^2(1+y)}{y} + \frac{2}{y} \text{Li}_2\left(\frac{y}{1+y}\right) - \frac{2}{y} \ln(y) \ln(1+y) + i\frac{2\pi}{y} \ln(1+y) \\
    f_{2a} &= -\frac{4}{y} \left[-1 + \ln(y) + \frac{1}{y} \left(\frac{1}{2} \ln^2(1+y) \right) + \frac{2}{y} \ln^2\left(\frac{1+y}{1+y}\right) - \ln(y) \ln(1+y)\right] + \\
    &\quad i\frac{4\pi}{y} (1 - \frac{1}{y} \ln(1+y)) \\
    f_{0b} &= -2 + \sqrt{1 - 4/y} \ln\left(\frac{1 + \sqrt{1 - 4/y}}{1 - \sqrt{1 - 4/y}}\right) - i\pi \sqrt{1 - 4/y} \\
    f_{1b} &= \frac{4}{y} \left[\text{Li}_2\left(\frac{-2}{-1 + \sqrt{1 - 4/y}}\right) + \text{Li}_2\left(\frac{2}{1 + \sqrt{1 - 4/y}}\right)\right] \\
    f_{2b} &= \frac{4}{y^2} \left[y + y \sqrt{1 - 4/y} \ln\left(\frac{1 - \sqrt{1 - 4/y}}{1 + \sqrt{1 - 4/y}}\right) + 2\text{Li}_2\left(\frac{-2}{-1 + \sqrt{1 - 4/y}}\right) \right. \\
    &\quad \left. + 2\text{Li}_2\left(\frac{2}{1 + \sqrt{1 - 4/y}}\right)\right] + 4\pi \sqrt{1 - 4/y} \frac{1}{y}
\end{align*}
\]
\[ \frac{e + 3swt}{2} t_{\lambda \nu \mu}(k_1 \lambda, k_2 \nu, k_3 \mu) \]

\[ \frac{igM_X}{2eW} \left( \frac{1}{2} + 2s_W^2 \right) g_{\mu \nu} \]

\[ -ig \sqrt{2} \frac{M_{L3}}{M_X} P_L \]

\[ \frac{ig \sqrt{2}}{eW} s_W^2 \gamma_\mu \]

FIG. 1: Feynman rules for NP contributions to $Z \to b\bar{b}$.
FIG. 2: One loop NP diagrams that contribute to $Z \rightarrow b\bar{b}$. 
FIG. 3: Allowed region at 95% CL for $M_X, M_{J_3}$. 

\( M_X \) (GeV) 

\( M_{J3} \) (GeV)
FIG. 4: Allowed region at 95% CL for a wider region in the space parameters $M_X, M_{J3}$ plane.