Damping of electromagnetic waves in low-collision electron-ion plasmas

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Abstract

Using previously developed method [1, 2] of two-dimensional Laplace transform we obtain the characteristic equations \( k(\omega) \) for electromagnetic waves in low-collision fully ionized plasma of a plane geometry. We apply here a new, different from the one used in [1, 2], iteration procedure of taking into account the Coulomb collisions. The kinematical waves are collisionally damping in the same extent as electromagnetic waves. Despite the different from [2] appearance of the dispersion (poles) equation, the obtained decrements for fast and slow wave modes coincide with results obtained in [2], if one neglects the terms of higher orders in \( v_x^2/c^2 \), (\( v_x \) and \( c \) are electron and light velocities). We point out how one can determine mutually dependent boundary conditions allowing to eliminate simultaneously both the backward and kinematical waves for transversal as well as for longitudinal oscillations.

PACS numbers: 52.25 Dg; 52.35 Fp.
Key words: plasma oscillations; plasma waves; Landau damping; Coulomb collisions; collision damping; Vlasov equations; kinematical waves; plasma echo.

1 Introduction

Propagation of electromagnetic waves in low-collision fully ionized plasma is described by asymptotic solution of the coupled kinetic and Maxwell equations. The trivial fact is known: an exponential solution \( \exp(ikx - \omega t) \), proposed by L. Landau in 1946 (in the simplest case of a plane geometry) with complex \( \omega \), is not a solution of either the Vlasov equations (for longitudinal plasma waves), nor the equations for transversal waves. Nevertheless in the available literature one usually admits that namely Landau solution is true but the above mentioned equations must be correspondingly corrected by additional terms according to Landau rules of passing around poles in calculation of logarithmically divergent integrals appearing at substitution of the solution into the primary equations.

The proposed method of two-dimensional Laplace transformation combined with Vlasov prescription of calculating divergent integrals in the sense of principal value allows one to obtain very simply asymptotical solutions of the original equations.

In this work we briefly describe both the techniques and the results of the proposed in [2] new iteration procedure. Following to this method one replaces Laplace image \( Q_{p_1p_2}(\vec{v}_e) \) of the Coulomb collision term \( Q(\vec{v}_e, x, t) \) by the term

\[
f_{p_1p_2}^{(1)}(\vec{v}_e) \left[ \frac{Q^0_{p_1p_2}(\vec{v}_e)}{f_{p_1p_2}^0(\vec{v}_e)} \right],
\]

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where \( f_{p_1p_2}(\vec{v}_r) \) is Laplace image of the perturbation of electron distribution function, \( Q^o_{p_1p_2}(\vec{v}_r) \) is Laplace image of the collision term, calculated in the null-iteration approximation using Laplace image \( f_{p_1p_2}^{(e)} \) of the collisionless approximation of \( f_{p_1p_2}^{(e)} \)

\[
f_{p_1p_2}^{(e)}(\vec{v}_r) = \frac{|e|}{m_e} \frac{\partial f_0^{(e)}(\vec{v})}{\partial v_z} \cdot \frac{E_{p_1p_2}}{p_1 + v_x p_2} + \frac{v_x f_{p_1}^{(e)}(\vec{v})}{p_1 + v_x p_2}.
\]

In our boundary problem we can suppose \( f_{p_2}^{(e)} = 0 \).

The method of subtraction of unphysical backward field waves suggested in [2] at non-zero \( f_{p_1}^{(e)} \) and \( f_{p_2}^{(e)} \) does not yet define single-valued dependence of \( f_{p_1}^{(e)} \) and \( f_{p_2}^{(e)} \) on \( \vec{v}_r \). As we said before, the boundary conditions are not independent and the given boundary electrical field \( E(0, t) \) defines the boundary function \( f_1^{(e)}(\vec{v}, 0, t) \). Such an interrelation of these quantities can be determined through the natural condition of absence of kinematical waves. In the case \( E(0, t) = E_0 \cos(\omega t) \), \( p_1 = \pm i \omega \), the general expression for \( f_{p_1}^{(e)}(\vec{v}) \) is

\[
f_{p_1}^{(e)}(\vec{v}) = \frac{a(\vec{v})}{p_1 + i \omega} + \frac{a^*(\vec{v})}{p_1 - i \omega},
\]

where symbol * means complex conjugation.

By equating amplitudes of the kinematical waves to zero one obtains linear integral equations for determination of \( a(\vec{v}) \) and \( a^*(\vec{v}) \):

\[
\left[ \frac{|e|}{m_e} \frac{\partial f_0^{(e)}}{\partial v_z} E_{p_1p_2} \right] p_1 = \pm i \omega
\]

\[
+ v_x f_{p_1}^{(e)}(\vec{v}) = 0
\]

where \( E_{p_1p_2} \) (see Eq.(37) in [2]) contains integrals of the type

\[
\int \frac{f_{p_1}^{(e)} u_x u_z d^2 \vec{u}}{p_1 + u_x p_2}.
\]

These equations define uniquely the dependence \( a(\vec{v}) \) on \( \vec{v} \). But determined in this way \( f_{p_1}^{(e)}(\vec{v}) \) can not be used to eliminate unphysical backward field waves in \( E(x, t) \), as was supposed in [2]. To this end one must use the boundary condition for \( F_{p_1} \) (that is Laplace transform of \( \partial E(x, t)/\partial x |_{x=0} \)).

In this way function \( f_{p_1}^{(e)} \) has the form

\[
f_{p_1}^{(e)}(\vec{v}_r) \sim \frac{\partial f_0^{(e)}}{\partial v_z} \eta(\vec{v}),
\]

where \( \eta(\vec{v}) \) is some complicated function of \( \vec{v} \). Assuming that factor \( \partial f_0^{(e)}/\partial v_z \) is the main in the dependence of \( f_{p_1}^{(e)}(\vec{v}_r) \) on \( \vec{v} \) we can use Eq.(5) with replacement \( \eta(\vec{v}) \rightarrow \eta(\vec{v}) \) for rough estimates. Then, in the expression for collision term, \( f_{p_1}^{(e)} \) in Eq.(2) can be approximately omitted (both terms in Eq.(2) have the same structure in \( \vec{v} \)).

Analogous considerations in the case of longitudinal waves lead to the determination of \( f_{p_1}^{(e)} \), but there are no other free boundary conditions to eliminate the backward waves in \( E(x, t) \). This fact leads to the inevitable conclusion that the normal boundary component of electrical field \( E(0, t) = E_0 \cos(\omega t) \) is broken at the plasma boundary due to the surface charge. This plasma boundary field can be found with the proportional changing of \( f_{p_1}^{(e)}(\vec{v}) \) and \( E_0 \rightarrow E_0', E_{p_1} \rightarrow E_{p_1}' \) in linear equation of the type (4) without changing field amplitude \( E_0 \) in \( E_{p_1} \) in equation for \( E_{p_1p_2} \) (see Eq.(37) in [2]).
2 Collisional damping of electromagnetic waves

Characteristic equation which is an equation for double poles \( p_1, p_2 \) of Laplace images of electrical field \( E(x,t) \) and distribution function \( f_1^{(e)}(\vec{v}_e,x,t) \) has been obtained in [2]. For \( E(0,t) \sim E_0 \exp(\omega t) \) it has the following form

\[
G(p_1, p_2) = (p_1 - \omega) \left[ p_2^2 - \frac{p_1^2}{c^2} + \frac{\omega^2}{c^2} \int_{-\infty}^{\infty} \frac{v_x d^3\vec{v}}{p_1 + v_x p_2} \left( \frac{\partial f_0^{(e)}}{\partial v_x} - \frac{m_e Q_{p_1 p_2}^0(\vec{v})}{|e| E_{p_1 p_2}} \right) \right] = 0, \tag{7}
\]

where \( Q_{p_1 p_2}^0(\vec{v}) \sim E_{p_1 p_2} \) (see [2], Eqs.(27)-(29)).

The integrals are defined according to Vlasov principal value prescription. The residue of Eq.(7) at pole \( p_1 = \omega \) defines poles in \( p_2 \).

In the case of procedure (1) the pole equation differs in form from the pole equation in [2]:

\[
G(p_1, p_2) = (p_1 - \omega) \left[ p_2^2 - \frac{p_1^2}{c^2} + \frac{\omega^2}{c^2} \int \frac{\partial f_0^{(e)}}{\partial v_x} \frac{v_x d^3\vec{v}}{p_1 + v_x p_2 - \Phi_{p_1 p_2}^0(\vec{v})} \right] = 0, \tag{8}
\]

where

\[
\Phi_{p_1 p_2}^0(\vec{v}) \equiv \frac{Q_{p_1 p_2}^0(\vec{v})}{f_1^0(\vec{v}, p_1, p_2)} \tag{9}
\]
does not contain the value \( E_{p_1 p_2} \); \( f_1^0(\vec{v}, p_1, p_2) \) is defined by Eq.(3).

In analogy with [2], one uses approximation

\[
\int_{-\infty}^{\infty} F(v_x) f_1^0(v_x) dv_x = \int_0^\infty \left[ F(v_x) + F(-v_x) \right] f_1^0(v_x) dv_x \simeq \frac{F(v_{0x}) + F(-v_{0x})}{2}, \tag{10}
\]

where coefficient \( 1/2 \) appears owing to difference in normalization of distribution functions taken in intervals \((-\infty, \infty)\) and \((0, \infty)\) and

\[
v_{0x} \equiv \sqrt{v_x^2}. \tag{11}
\]

Then one obtains

\[
\int \frac{\partial f_0^{(e)}}{\partial v_x} \frac{v_x d^3\vec{v}}{p_1 + v_x p_2 - \Phi_{p_1 p_2}^0(\vec{v})} \simeq \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} \Theta(v_{0x}, v_y, v_z) \frac{\partial f_0^{(e)}}{\partial v_z} \frac{v_z d^3\vec{v}}{v_x} \tag{12}
\]

where

\[
\Theta(v_{0x}, v_y, v_z) \equiv \left[ \frac{p_1 - \frac{1}{2} \Phi_{p_1 p_2}^+(v_{0x}, v_y, v_z)}{p_1^2 - v_{0x}^2 p_2^2 - p_1 \Phi_{p_1 p_2}^+(v_{0x}, v_y, v_z) + v_{0x} p_2 \Phi_{p_1 p_2}^-(v_{0x}, v_y, v_z)} \right]
\]

and

\[
\Phi_{p_1 p_2}^+(v_{0x}, v_y, v_z) \equiv \Phi_{p_1 p_2}^0(v_{0x}, v_y, v_z) + \Phi_{p_1 p_2}^0(-v_{0x}, v_y, v_z).
\]

The replacement \( \Phi_{p_1 p_2}^0(v_{0x}, v_y, v_z) \to \Phi_{p_1 p_2}(v_{0x}, v_y, v_z), \ v_x^2 \to v_{0x}^2 \simeq kT/m_e \) is made after taking derivatives in \( v_x, v_y, \) and \( v_z \) in the differential operator \( Q_{p_1 p_2}^0(\vec{v}); \) integrals in \( dv_y \) and \( dv_z \) can be approximately estimated by their mean values:

\[
\int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} \Theta(v_{0x}, v_y, v_z) \frac{\partial f_0^{(e)}}{\partial v_z} \frac{v_z d^3\vec{v}}{v_x} \simeq N_y N_z \Theta(v_{0x}, v_{0y}, v_{0z}) \tag{13}
\]

with evident normalization constants \( N_y \) and \( N_z \).

After elementary transformations one obtains dispersion (poles) equation in the form

\[
\left[ \left( p_2^2 - \frac{p_1^2}{c^2} \right) \left( p_1^2 - v_{0x} p_2^2 - p_1 a + v_{0x} p_2 b \right) - \frac{\omega^2}{c^2} p_1 \left( p_1 - \frac{a}{2} \right) \right] = 0, \tag{14}
\]
where

\[ a \equiv \Phi^+_{p_1p_2}(v_{0x}, v_{0y}, v_{0z}) = \frac{4\lambda}{3v_{0x}^2} \frac{p_1^4 - 2p_1^2v_{0x}^2p_2^2 + 5v_{0x}^4p_2^4}{(p_1^2 - v_{0x}^2p_2^2)^2} \]  

(15)

\[ b \equiv \Phi^-_{p_1p_2}(v_{0x}, v_{0y}, v_{0z}) = \frac{8\lambda}{3v_{0x}^2} \frac{p_1^2v_{0x}p_2 - 3p_1v_{0x}^3p_2^2}{(p_1^2 - v_{0x}^2p_2^2)^2} \]  

(16)

\[ \lambda \equiv \frac{2\pi e^4 L n_i}{m_e^2} ; \quad p_1 = i\omega ; \quad v_{0x} \approx v_{0y} \approx v_{0z} \approx \frac{kT_e}{m_e}, \]  

(17)

and further as

\[ \left[ \left( c^2 p_2^2 - p_1^2 \right) \left( p_1^2 - v_{0x}^2p_2^2 \right)^3 - \omega^2 p_1 \left( p_1^2 - v_{0x}^2p_2^2 \right)^2 \right] \]

\[ - \frac{4\lambda p_1}{(3v_{0x}^2)^{3/2}} \frac{\left( c^2 p_2^2 - p_1^2 \right) \left[ p_1^4 - 4p_1^2v_{0x}^2p_2^2 + 11v_{0x}^4p_2^4 \right]}{\left[ p_1^4 - 2p_1^2v_{0x}^2p_2^2 + 5v_{0x}^4p_2^4 \right]} \]

\[ + \frac{2\lambda p_1}{(3v_{0x}^2)^{3/2}} \omega^2 \left[ p_1^4 - 2p_1^2v_{0x}^2p_2^2 + 5v_{0x}^4p_2^4 \right] = 0. \]  

(18)

This equation is an analogue of the characteristic equation (30) in [2].

For both electron modes

\[ p_2^{(1)} = \pm \frac{\omega}{c} \sqrt{1 - \frac{\omega^2}{\omega_L^2}} + \delta^{(1)} ; \]  

(19)

\[ p_2^{(2)} = \pm \frac{i\omega}{v_{0x}} \left( 1 + \frac{v_{0x}^2\omega_L^2}{2c^2\omega^2} \right) + \delta^{(2)} ; \]  

(20)

we obtain from Eq.(18) at \( |\delta^{(1,2)}| \ll 1 \) and neglecting terms with higher orders in \( v_{0x}^2/c^2 \) in Eqs.(15), (16):

\[ \delta^{(1)} = \pm \frac{2\pi e^4 n_i L \omega_L^2}{3 \sqrt{3} v_{0x} m_e kT_e c^2 \omega \sqrt{1 - \omega_L^2/\omega^2}} ; \]  

(21)

\[ \delta^{(2)} = \pm \left( \frac{\pi e^4 n_i L \omega_L^2}{3 \sqrt{3} v_{0x} m_e kT_e} \right)^{1/3} . \]  

(22)

that coincides with corresponding expressions for \( \delta^{(1,2)} \) in [4], in spite of differences of characteristic equations.

Let us emphasize here the sharp increase of the dissipative collisional absorption of electromagnetic waves proportional to \( 1/\sqrt{1 - \omega_L^2/\omega^2} \) at \( \omega \to \omega_L + 0 \) with the dominating collisionless non-dissipative reflective evanescence of waves at \( \omega < \omega_L \).

3 Conclusions

Coincidence of collisional damping decrements for the two variants of iteration process is an evidence of the proposed calculation method correctness. The approximation (1) is more preferable than one used in [1, 2] since it has more evident physical sense. The difference of both iteration procedures appears at large values of \( v_{0x}^2/c^2 \), however in this case there is also growing contribution of relativistic corrections to the original equations.

The requirements of absence of both unphysical backward (divergent at \( x \to \infty \)) field waves and kinematical waves smearing electron distribution function \( f_1^{(e)}(\vec{v}, x, t) \) in \( v_x \) lead at the given boundary field \( E(0, t) \) to the determination of the boundary distribution function \( f_1^{(e)}(\vec{v}, 0, t) \) and the solution \( f_1^{(e)}(\vec{v}, x, t) \) with single-valued dependence on \( \vec{v}, x, \) and \( t \).
Acknowledgements The author is thankful to Dr. A. P. Bakulev for his criticism and assistance in preparing the paper in \LaTeX\ style.

References

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