Optimal Design and Dynamic Control of an ISD Vehicle Suspension Based on an ADD Positive Real Network

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ABSTRACT As a two-terminal mass element, an inerter makes up for the lack of mass impedance in a traditional vehicle suspension system. A passive Inerter-Spring-Damper (ISD) suspension can significantly improve vibration isolation performance in a low frequency range, while the impact is not obvious in other frequency bands. To isolate a wider frequency of vibrations in a vehicle suspension system on the basis of an ISD suspension, this paper combines the concepts of using an Acceleration-Driven-Damping (ADD) approach, which can effectively suppress vibrations in medium and high frequency bands, and an ISD passive network, which is superior in low frequency bands. First, an ideal model optimization design method of an ISD vehicle suspension based on an ADD positive real network is proposed, and a second-order model is built. The parameters are optimized by means of the artificial fish swarm algorithm considering both positive real constraints and suspension performance constraints. Then, on the basis of the optimal ideal model, a radial basis function sliding mode controller is designed to control the ISD vehicle suspension system using a mechatronic inerter. Finally, simulations and experiments are carried out to verify the dynamic performance of the proposed ISD suspension. The results show that a controllable ISD suspension system based on an ADD positive real network can improve the suspension performance effectively and isolate a wide range of frequency vibrations. This research provides theoretical evidence and methodological guidance for further enriching the design and dynamic control of ISD suspension systems.

INDEX TERMS Vehicle suspension, inerter, ADD positive real network, sliding mode control, bench test.

I. INTRODUCTION

The structural reform of the automobile industry not only attracts the attention of electric vehicles and other new fields, but also drives people’s higher demand for vehicle performance including ride comfort [1]–[3]. Vehicle vibration reduction is closely related to tires, vehicle suspension and seats. As the main shock absorption system, the performance of the suspension system plays an important role in vehicle ride comfort and handling stability [4].

In 2002, Professor Smith of the University of Cambridge proposed a mass element “inerter” [5]. As a two-terminal element, an inerter makes up for the lack of mass impedance in the mechanical network of a traditional passive suspension system based on “spring-dampers”, thus providing a new solution for improving the system performance of vehicle suspensions. The application of an inerter in suspension design promoted the rapid development of “Inerter-Spring-Damper (ISD)” suspension system [6], [7]. Similar to a traditional suspension, ISD vehicle suspensions can also be categorized into passive suspension and active controllable suspension systems.

For passive ISD suspensions, Frank optimized six different combinations of ISD suspension parameters, and obtained the global optimal solution of ride comfort and tire grounding based on a quarter suspension model [8]. On the basis of a bilinear matrix inequality, Papageorgiou used an iterative method to solve the positive real controller, and took the $H_2$ and $H_\infty$ forms of the suspension performance index as the optimization objective to analyze suspension...
performance [9]. Summarizing the research on the characteristics of passive ISD suspensions, it is noted that passive ISD vehicle suspension systems employing inerters can effectively suppress body vibrations in the low frequency band, but the improvement effect in other frequency bands is not obvious [10]–[13]. Therefore, improving the vibration transmission characteristics in wider frequency bands has become a key goal in the study of vehicle ISD suspension systems.

Active suspension is a new type of suspension system with actuators and control methods. Active suspension addresses the shortcomings of passive suspension and semi-active suspension and expands the range of parameter adjustment. Active suspension adjusts suspension performance according to the actual driving situation and road feedback signals, so that a vehicle can always be in an optimal state under different working conditions [14]. In controllable ISD suspension research, Professor Chen offered six kinds of controllable suspensions with inerter devices [15]. The results showed that a controllable suspension with an inerter can improve suspension performance compared with that of traditional semi-active suspensions. Soong of the University of Malaya designed a nonlinear control method of two-stage switching inertance according to whether the suspension working space exceeded a preset threshold, effectively reducing the root-mean-square value of vehicle body acceleration under the random road input conditions [16]. Hu further proposed a new type of inerter with a continuously changeable inertance, and offered two control methods based on frequency information and phase information. A test proved that a semi-active suspension with the new inerter could effectively suppress the system vibration [17]. In addition, Hu put forward a control method with a skyhook inerter configuration with the aim of suppressing vehicle body vibrations, and three methods—an on-off control, anti-chatter on-off control, and continuous control—were designed to improve riding comfort [18]. He subsequently proposed dividing the semi-active suspension into passive suspension and semi-active suspension and used the network synthesis method to solve the optimal function of the passive suspension. Afterwards, the semi-active suspension was constructed by combining an adjustable damper with the optimal passive network [19]. Professor Chen further proposed a semi-active suspension system with adjustable inertance and damping coefficient online. The semi-active suspension, with a semi-active inerter and a semi-active damper, could track the target active control force more effectively, and all of the ride comfort, suspension deflection, and road holding performances were improved [20]. It can be concluded that the current controllable ISD suspension is mainly achieved by optimization algorithms, semi-active damping and inerter designs, and there has been no targeted research on vibration suppression from the perspective of frequency-domain performance. In addition to the control method which has been studied, another approach with unique performance has attracted our attention. Savaresi proposed a new control strategy called Acceleration-Driven-Damping (ADD) control [21]. The main advantages of ADD control are that it aims to suppress the inertial force and can effectively improve vehicle ride comfort in medium and high frequency bands [22, 23].

In general, a passive ISD suspension offers distinctive vibration isolation performance in low frequency ranges, but the effect is not as obvious in other frequency bands. To isolate a wider frequency vibration range of an ISD vehicle suspension, this paper combines the concepts of the ADD approach which can effectively suppress vibrations in medium and high frequency bands, and an ISD passive network, which is superior in low frequency bands.

The organization of this paper is as follows. First, an ideal model of an ISD vehicle suspension based on a second-order ADD positive real network is proposed in Section II. Then, considering both positive real constraints and vehicle suspension performance constraints, the parameters of the second-order ADD model are optimized by the artificial fish swarm algorithm in Section III. In addition, a radial basis function sliding mode controller is designed using a mechatronic inerter and simulations are carried out in Section IV. Finally, the experimental results of ISD suspension performance are analyzed in Section V. Some conclusions are drawn in Section VI.

II. DYNAMIC MODEL OF ISD VEHICLE SUSPENSION

In this paper, a quarter dynamic model of a vehicle ideal suspension is established as the research object, and the model structure is shown in Figure 1:

In Figure 1, \( m_s \) is the sprung mass, \( m_u \) is the unsprung mass, \( k \) is the spring stiffness of the suspension, \( c_s \) is the semi-active damping coefficient of ADD control, \( k_t \) is the equivalent spring stiffness of the tire, \( z_s \) is the vertical displacement of the sprung mass, \( z_u \) is the vertical displacement of the unsprung mass, \( z_r \) is the vertical input displacement of road roughness, and \( T(s) \) is the impedance expression of a positive real network system. According to Newton’s second law, the Laplace transformation of its dynamic equations are as follows:
TABLE 1. Quarter vehicle model parameters.

| Parameter                                      | Value    |
|------------------------------------------------|----------|
| Sprung mass $m_s$/kg                           | 320      |
| Unsprung mass $m_u$/kg                         | 45       |
| Spring stiffness $k_s$ (N/m)                    | 22000    |
| Tire stiffness $k_t$ (N/m)                      | 190000   |
| Damping coefficient of the traditional passive suspension $c_s$ (N-s/m) | 1000     |

follows:

$$m_s z_s'' + [k_s z_s + s c_s](z_s - z_u) = 0$$
$$m_u z_u'' + k_t (z_u - z_r) - [k_s z_s + s c_s](z_s - z_u) = 0$$  (1)

In formula (1), $z_s$, $z_u$ and $z_r$ are the Laplace transformation forms of the corresponding variables, and $s$ is the Laplace variable. A second-order positive real network system is studied in this paper, and its velocity impedance expression is:

$$T(s) = \frac{A s^2 + B s + C}{D s^2 + E s + F}$$  (2)

In the formula, the values of $A$, $B$, $C$, $D$, $E$ and $F$ are greater than or equal to zero. In particular, $D$, $E$ and $F$ are not all equal to zero [24].

The damping coefficient $c_s$ of ADD control is obtained with the following formula:

$$c_s = \begin{cases} 
    c_1 & \text{if } \dot{z}_s(\dot{z}_s - \dot{z}_u) \geq 0 \\
    c_2 & \text{if } \dot{z}_s(\dot{z}_s - \dot{z}_u) < 0 
\end{cases}$$  (3)

The parameters of the quarter vehicle suspension model are shown in Table 1:

III. OPTIMIZATION OF ISD VEHICLE SUSPENSION

The artificial fish swarm algorithm is a bottom-up intelligence optimization algorithm with global convergence [25]–[27]. The algorithm seeks an optimal solution based on the basic behaviors of fish in a certain range; the main processes include fish swarm initialization, foraging behavior, clustering behavior, tail chasing behavior and random behavior.

This paper aims to improve vehicle ride comfort. The root mean square (RMS) value of vehicle body acceleration under the condition of a random road input is selected as the optimization objective. The optimal objective function expression of an ISD vehicle suspension based on a second-order positive real network is:

$$Y = \frac{X_1}{X_{1\text{pas}}}$$  (4)

where $Y$ is the objective function of the ISD vehicle suspension, $X_1$ is the RMS value of body acceleration of the ISD suspension to be optimized, and $X_{1\text{pas}}$ is the RMS value of body acceleration of a traditional passive suspension.

At first, the positive real constraints of the transfer function are considered. According to formula (2), the parameters to be optimized for biquadratic transfer function are $A$, $B$, $C$, $D$, $E$, $F$, and the damping coefficients $c_1$ and $c_2$ of the ADD control. The constraints of the parameters are:

$$\begin{cases} 
    c_1 > 0, & c_2 > 0 \\
    A, B, C, D, E, F \geq 0 
\end{cases}$$  (5)

Among them, $D$, $E$ and $F$ are not all equal to zero. The parameters also meet the following conditions [28]:

$$B E - (\sqrt{A F} - \sqrt{C D}) \geq 0$$  (6)

Then, the constraints of the suspension performance are considered:

$$X_2 \leq X_{2\text{pas}}, \quad X_3 \leq X_{3\text{pas}}$$  (7)

where $X_{2\text{pas}}$ and $X_{3\text{pas}}$ are the RMS restraint range of suspension working space and dynamic tire load respectively. $X_2$ and $X_3$ are the RMS value to be optimized.

According to the probability of normal distribution, $X_2$ is a third of the effective value of suspension working space. In addition, 8 cm is selected for the commercial vehicles studied in this paper. For dynamic tire load, $X_3$ is a third of the value of the static load, and the static load studied in this paper is 3650 N [29]. The overall flow of the artificial fish swarm algorithm is shown in Figure 2.
The road excitation for the optimization process is a random road input at the speed of 20 m/s [30]. The suspension parameters of the optimization process are shown in Table 1. After several optimizations, the values in Table 2 are selected as the optimization results. The final optimization result \( Y = 0.6899 \).

According to the optimization results in Table 2, the expression of the second-order ADD positive real network is:

\[
T(s) = \frac{5.314 \times 10^4 s^2 + 9.515 \times 10^5 s + 3.085 \times 10^6}{91 s^2 + 495.3 s + 8475} \tag{8}
\]

According to passive network synthesis, a biquadratic impedance transfer function can be realized in series and parallel with no more than 9 elements at most. In terms of formula (8) and [31], the optimized biquadratic transfer function meets the regular conditions and can be realized passively in series and parallel with 5 elements. Therefore, the optimized mechanical network structure corresponding to the second-order positive real network is shown in Figure 3.

After converting the biquadratic impedance transfer function to the corresponding mechanical components, the values of each corresponding component are shown in Table 3.

The suspension structure shown in Figure 3 and the data presented in Tables 1-3 are used to establish the frequency domain and time domain models of the ISD suspension. Figure 4 shows a comparison of body acceleration gains of the second-order ADD positive real network, a traditional passive suspension, the ADD control and the positive real network. Among them, the structure of the traditional passive suspension is a damping \( c \) and a spring \( k \) in parallel between \( m_s \) and \( m_u \), and its parameters are shown in Table 1. The structure of the ADD control is an adjustable damping \( c_s \) and a spring \( k \) in parallel between \( m_s \) and \( m_u \), and its parameters (Including \( c_1 \) and \( c_2 \)) are shown in Table 1 and 2. The structure of the positive real network is a spring \( k \) and \( T(s) \) in parallel between \( m_s \) and \( m_u \), and its parameters are shown in Table 1 and 2. The structure of the second-order ADD positive real network is shown in Figure 3, and its parameters are shown in Table 1-3.

It can be seen from Figure 4 that the body acceleration gains of the ADD control are slightly higher than those of the traditional passive suspension in the low frequency band (0.7-1.2 Hz). In the medium frequency (1.5-10 Hz) and high frequency bands (11.5-15 Hz), the body acceleration gains of the ADD control are significantly lower than those of the traditional passive suspension except for the peak value at the main frequency of wheel vibration. These results indicate that the ADD control can effectively suppress vibrations in the medium and high frequency bands. For the positive real network with an inerter, the peak value at the main frequency
of vehicle body vibration is significantly suppressed, while the peak value at the main frequency of wheel vibration is increased to a certain extent, which shows that the positive real network can effectively reduce vibrations at the main frequency of vehicle body vibration; but aggravates vibrations at the main frequency of wheel vibration. For the second-order ADD positive real network, the vibration peak value at the main frequency of vehicle body vibration is 15.4% lower than that of the traditional passive suspension, and 13.5% lower than the ADD control. In the medium and high frequency bands, the vehicle body vibrations are also effectively suppressed, and the resonance peak value at the main frequency of wheel vibration is significantly smaller than that of the positive real network. It is concluded that the ADD positive real network proposed in this paper makes effective use of the characteristics of the ISD suspension in the low frequency band and the ADD control in the medium and high frequency bands, thus achieving a vibration control effect in a wider frequency band.

IV. SLIDING MODE CONTROL AND SIMULATIONS

A. DESIGN OF THE RBF SLIDING MODE CONTROL SYSTEM

In this paper, the suspension system with the ADD control is used as the reference model, and the optimized second-order suspension system is simplified. As the control actuator, the mechatronic inerter is used in the controllable model. It can be regarded as the combination of control force and inerter [32]. The equivalent model of the sliding mode control is shown as follows.

The dynamic equations of the equivalent controlled model of the sliding mode control in Figure 5 (a) are:

\[
\begin{align*}
    m_s \ddot{z}_s + k(z_s - z_u) + b(\dot{z}_s - \dot{z}_u) + F_1 &= 0 \\
    m_u \ddot{z}_u + k(\dot{z}_s - z_r) - k(z_s - z_u) - b(\dot{z}_s - \dot{z}_u) - F_1 &= 0
\end{align*}
\]  

(9)

In the formula, \( b \) is the inertance of the inerter, \( F_1 \) is the active output force of the inerter, \( z_r \) is the vertical displacement of the road, \( z_u \) and \( z_{sr} \) are the vertical displacements of the unsprung mass, and \( z_s \) and \( z_{sr} \) are the vertical displacements of the sprung mass.

The state variable \( X \) and output variable \( Y \) of the controlled model system are:

\[
\begin{align*}
    X &= [\dot{z}_s, \dot{z}_u, z_s, z_u, z_r]^T \\
    Y &= \left[ \ddot{z}_s - z_u, k(z_s - z_r) \right]^T
\end{align*}
\]

(10)

The state space equations of the controlled system are:

\[
\begin{align*}
    \dot{X} &= AX + BU_m + EW \\
    Y &= GX + DW
\end{align*}
\]

(11)

Among them, \( M = (m_s + b)(m_u + b) - b^2 \phi \)

Similarly, the state space equations of the reference model of the sliding mode control are:

\[
\begin{align*}
    \dot{X}_r &= A_r X_r + B_r W \\
    Y_r &= C_r X_r + D_r W
\end{align*}
\]

(12)

\[
A_r = \begin{bmatrix}
    -\frac{m_s}{m_u} & \frac{m_u}{m_u} & 0 & -\frac{m_u}{m_u} & 0 \\
    T(s) + c_s & T(s) + c_s & 0 & 0 & 0 \\
    T(s) + c_s & T(s) + c_s & 0 & 0 & 0 \\
    k & k & 0 & 0 & 0 \\
    k & k & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_r = \begin{bmatrix}
    0 & 0 & 0 & -4.44 \sqrt{G_0} \nu \\
    -\frac{m_s}{m_u} & \frac{m_u}{m_u} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(13)

\[
C_r = \begin{bmatrix}
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(14)

The switching surface function is defined as:

\[
    s = Ce = [c_1, c_2, \ldots, c_n][e_1, e_2, \ldots, e_n]^T = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n
\]

(15)

Generally, the value of \( c_n \) is 1. According to the error vector, the value of \( n \) is 3.

The sliding mode controller designed in this paper can make the controlled suspension system track the output of the
ideal reference model, thus generating a sliding mode in error dynamics. According to the dynamic model, the tracking error vector \( e \) is composed of the integration of the sprung mass displacement error, the sprung mass displacement error and the sprung mass velocity error between the controlled model and the reference model:

\[
e = [e_1, e_2, e_3]^T = \begin{bmatrix} z_s - z_{sr} \; \dot{z}_s - \dot{z}_{sr} \; \ddot{z}_s - \ddot{z}_{sr} \end{bmatrix}(14)
\]

The error dynamics equation is:

\[
\dot{e} = A_e e + B_e U + H_e X_r
\]

\[
A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -m_s k & 0 & 0 \end{bmatrix} \\
B_e = \begin{bmatrix} 0 \\ 0 \\ -m_s \frac{k}{M} \end{bmatrix}^T \\
H_e = \begin{bmatrix} \frac{1}{m_s} & -\frac{1}{m_s} & 0 \\ 0 & \frac{1}{m_s} & \frac{k}{m_s} \\ \frac{1}{m_s} & \frac{1}{m_s} & \frac{k}{m_s} \end{bmatrix}
\]

Expression (13) can be written in the form of block matrix:

\[
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -c_s & -c_k & 0 \\ \frac{m_s k}{M} & \frac{m_s k}{M} & \frac{1}{M} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \end{bmatrix} s
\]

(16)

When \( s = c_1 e_1 + c_2 e_2 + e_3 = 0, \dot{s} = c_1 \dot{e}_2 + c_2 \dot{e}_3 + \dot{e}_3 = 0 \), equation (16) can be simplified as:

\[
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -c_s & -c_k \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}
\]

(18)

In conclusion, the sliding mode control law is as follows:

\[
\dot{u}^* = \frac{F_d - [CB_e]^{-1} C (A_e e + H_e X_r)}{m_u} 
\]

(21)

In this paper, the pole assignment method is used and the expected closed-loop pole group needs to be determined. The characteristic polynomial of equation (19) is \( D(\lambda) = \lambda^2 + c_2 \lambda + c_1 \), and the standard form of the second-order system transfer function is:

\[
\Phi(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}
\]

(19)

Therefore, \( c_1 = \frac{w_n^2}{2}, \; c_2 = 2\xi w_n \).

To stabilize the system and ensure the dynamic characteristics of the sliding mode, the two characteristic roots should be conjugate complex roots, and located in the left half plane of the complex plane. In this paper, the number of expected poles is 3; one pair of dominant poles are \( s_1 \) and \( s_2 \), and the other is a far pole. The performance of the system is mainly determined by the dominant pole, while the far pole has little influence. Therefore, the characteristic roots of the sliding mode equation are distributed on the left plane of the complex plane. By setting the system indexes of the overshoot \( \sigma \% \leq 12\% \) and the peak time \( t_p \leq 0.7 \), the coefficient vector \( C = [31.413, 6.266, 1] \) of the switching function is determined.

Equation (13) is derived and made to be equal to zero:

\[
\frac{ds(x)}{dt} = C \dot{e} = C (A_e e + B_e U + H_e X_r) = 0
\]

(20)

The equivalent control force \( F_d \) in the sliding mode region is obtained:

\[
F_d = -[CB_e]^{-1} C (A_e e + H_e X_r)
\]

(21)

The law of isokinetic approach is:

\[
\frac{ds}{dt} = -\varepsilon \text{sgn}(s) \quad \varepsilon > 0
\]

(22)

According to the Lyapunov stability criterion, if \( V(x) \) is positive definite and the derivative of \( V(x) \) is negative definite, the system is asymptotically stable. In this paper, the Lyapunov function is \( V(x) = s^T \cdot s / 2 \), and the derivative of \( V(x) \) is less than zero. Therefore, the system is stable.

Then, the sliding mode control of the system is:

\[
u^* = F_d + [CB_e]^{-1} s = F_d - \frac{M}{m_u} \varepsilon \text{sgn}(s)
\]

(23)

In conclusion, the sliding mode control law is as follows:

\[
u = \begin{cases} u^*, & u^*(\dot{z}_s - \dot{z}_u) \geq 0 \\ 0, & u^*(\dot{z}_s - \dot{z}_u) < 0 \end{cases}
\]

(24)

Furthermore, the radial basis function (RBF) neural network is used to optimize the switch of sliding mode control and improve the smoothness of the output signal. Specifically, the input of the RBF neural network is the switching function \( s \) and its derivative of sliding mode control. In addition, there are fifteen neuroses selected as the hidden layer and the Gaussian function is taken as the radial basis function. Finally, the output of the RBF neural network is provided by the switching term of sliding mode controller. Figure 6 shows a schematic of the suspension control system.

The inputs of RBF neural network are:

\[
\begin{cases} x_1 = s \\ x_2 = \dot{s} \end{cases}
\]

(25)

The radial basis vector of the hidden layer is:

\[
H = [h_1, h_2, \cdots, h_{15}]
\]

(26)

\[
h_j = w_j \exp \left( -\frac{\|x_i - c_j\|^2}{b_j} \right), \quad j = 1, 2, \cdots, 15
\]

(27)
where $c_j$ is the Gaussian function center value of node $j$, $b_j$ is the network baseband parameter of node $j$, and $w_j$ is the weight of node $j$ between the hidden layer and output layer.

Then, the output of the RBF neural network is:

$$f^* = \sum_{j=1}^{15} w_j \exp \left( -\frac{\|s - c_j\|^2}{b_j} \right), j = 1, 2, \cdots, 15$$

(28)

In this paper, the control objective of the RBF neural network is $s \dot{s} \rightarrow 0$, so the weight adjustment index is:

$$E = s \dot{s}$$

(29)

Then:

$$dw_j = -\eta \frac{\partial (s \dot{s})}{\partial w_j(t)} = -\eta \frac{\partial (s \dot{s})}{\partial u(t)} \cdot \frac{\partial u(t)}{\partial w_j(t)}$$

(30)

Let $\frac{\partial u(t)}{\partial u(t)} = \gamma$; the learning algorithm of the RBF neural network weight is:

$$dw_j = \gamma \cdot s \dot{s} \cdot h_j$$

(31)

where $w_j$ is 1.2, $c_j$ is an integer of set $[-7, 7]$, and $b_j$ is 10.

In summation, the inputs of the RBF sliding mode controller are the sprung mass displacements of the controlled and reference model, and the output is:

$$F_a = F_d + f^* = -[CB_e]^{-1} C (A_e e + H_e X_r) + f^*$$

(32)

**B. SIMULATION OF SINUSOIDAL ROAD INPUT**

In this paper, when the sinusoidal road is taken as the input, the amplitude is 10 mm and the frequency range is [0.1, 15] Hz. Figure 7 shows the performance comparisons.

It can be seen in Figure 7 that, under sinusoidal road input conditions, the controllable ISD vehicle suspension based on the ADD positive real network (controllable ISD suspension for short) studied in this paper is compared with a traditional passive suspension and a passive ISD suspension. The RMS value of body acceleration of the controllable ISD suspension is significantly reduced in the whole frequency domain especially in the high frequency domain. The RMS value of the suspension working space of the controllable ISD suspension is improved mainly in the low frequency band, and the resonance peak value of the high frequency band is slightly higher than that of the traditional passive suspension. The RMS value of the dynamic tire load of the controllable ISD suspension has been improved in the low and medium frequency bands, while its RMS value is slightly higher than that of the traditional passive suspension in the high frequency band.

In general, under the sinusoidal inputs, the ride comfort of the controllable ISD suspension is significantly improved compared with those of the traditional passive suspension and passive ISD suspension, and the vehicle body vibration is effectively suppressed over a wider frequency range.

**FIGURE 7.** Performance comparison diagrams of the sinusoidal excitation simulations.

**C. SIMULATION OF RANDOM ROAD INPUT**

Taking the traditional passive suspension, the passive ISD suspension and the controllable ISD suspension as objects, the three performance indexes of the body acceleration, suspension working space and dynamic tire load at speeds of 10 m/s, 20 m/s and 30 m/s are compared and studied, and the comparison of their RMS values is shown in Table 4. The simulations at the speed of 20 m/s are carried out under random road input conditions [33], as shown in Figure 8.
TABLE 4. Performance indexes under the sinusoidal road inputs.

| Suspension type      | Speed (m/s) | RMS of body acceleration (m/s²) | RMS of suspension working space (mm) | RMS of dynamic tire load (kN) |
|----------------------|-------------|---------------------------------|-------------------------------------|-----------------------------|
| Traditional passive  | 10          | 0.9381                          | 9.4146                              | 0.6388                      |
| ISD suspension       | 20          | 1.3096                          | 13.0047                             | 0.9004                      |
|                      | 30          | 1.5742                          | 15.4227                             | 1.0977                      |
| Passive ISD          | 10          | 0.7997                          | 9.3074                              | 0.6904                      |
| suspension           | 20          | 1.1109                          | 12.6941                             | 0.9746                      |
|                      | 30          | 1.3381                          | 15.1388                             | 1.1908                      |
| Controllable ISD     | 10          | 0.7145                          | 9.0896                              | 0.6778                      |
| suspension           | 20          | 0.9604                          | 12.4912                             | 0.9487                      |
|                      | 30          | 1.1902                          | 14.9016                             | 1.1664                      |

It can be seen from Table 4 and Figure 8 that under the conditions of a random road input and when the vehicle speeds are 10 m/s, 20 m/s and 30 m/s, compared with the traditional passive suspension, the RMS value of body acceleration of the controllable ISD suspension has been decreased by 26.7% at most and the RMS value of suspension working space of the controllable ISD suspension has been decreased by 3.8% at most. Compared with the passive ISD suspension, the RMS value of body acceleration of the controllable ISD suspension has been decreased by 13.5% at most and the RMS value of suspension working space has been decreased by 2.2% at most. The results show that the vehicle ride comfort of the controllable ISD suspension is improved obviously, while the dynamic tire load is increased slightly.

Figure 9 shows comparisons of the body acceleration, suspension working space and dynamic tire load in the frequency domain at a vehicle speed of 20 m/s.

It can be seen from Figure 9 that for the power spectral density (PSD) of body acceleration, the curve of the controllable ISD suspension is significantly lower than those of the traditional passive suspension and passive ISD suspension in the full frequency band, and the ability of the controllable ISD suspension to achieve wide-band vibration suppression is more prominent. For the PSD of the suspension working space and dynamic tire load, the curve of the controllable ISD suspension is significantly lower at the low frequency than that of the traditional passive suspension and passive ISD suspension. The resonance peak is effectively suppressed, but the effect in the high frequency is not significant. On the basis of the dynamic tire load of the controllable ISD suspension essentially keeping pace with that of the traditional passive suspension, the results demonstrate that the ride comfort of the controllable ISD suspension has been effectively improved.

V. TEST RESULTS AND DISCUSSION

The controllable ISD suspension system studied in this paper mainly includes a suspension spring, a tire equivalent spring, a ball screw mechatronic inerter device, a controller and an unsprung mass [34]. A single-channel hydraulic servo excitation platform is used as the platform for a bench test. The specific layout is shown in Figure 10 and its parameters are shown in Table 5.

In the construction of the test bench, the spring is used to simulate the equivalent stiffness of the tire, and the iron disk is used to simulate the unsprung mass. The spring element is
placed between the upper and lower fixture. As the control actuator, the mechatronic inerter is designed by coupling the ball screw inerter and the rotating motor, and it can be actively controlled by changing the current input of the motor [7]. The upper end of the mechatronic inerter is consolidated with the upper fixture, its lower end is connected with the lower fixture, and then the parallel structure of the mechatronic inerter device and the spring element is built.

In this paper, the ISD suspension control strategy based on ADD positive real network is downloaded by the dSPACE prototype development system. During the test, the inputs of the system is the road excitation and vertical displacement of the sprung mass of the controlled model collected by the signal acquisition instrument. This signal is transmitted to the dSPACE and synthesized with the vertical displacement of the sprung mass of the reference model under the same road input. After calculation, the error matrix of the system can be obtained. According to the output of the dSPACE and the characteristics of the rotating motor, the motor controller can control the motor current to produce the corresponding control force.

In addition, the body acceleration and suspension working space signals are collected by a signal acquisition instrument, and the dynamic tire load signals are collected by the excitation table during the test, which are analyzed in the following after processing.

A schematic diagram of the test is shown in Figure 11.

### A. TEST OF SINUSOIDAL ROAD INPUT

The sinusoidal road inputs are generated from the excitation of different frequency points, in which the input amplitude is
set as 10 mm and the frequency range is set as [0.1, 15] Hz. The comparisons of the RMS values of body acceleration, suspension working space and dynamic tire load under the sinusoidal input conditions are shown in Figure 12.

It can be seen from Figure 12 that the controllable ISD suspension studied in this paper significantly improved the RMS value of body acceleration in the full frequency range compared with those of the traditional passive suspension and passive ISD suspension. The RMS value of the suspension working space of the controllable ISD suspension is lower than that of the other two in the low frequency band (0.7-1.2 Hz). Compared with the traditional passive suspension, the RMS value of the suspension working space of the controllable ISD suspension and the passive ISD suspension both slightly increase in the high frequency band (11.5-15 Hz), but the controllable ISD suspension has a better performance in the medium frequency band (1.5-10 Hz). The RMS value of dynamic tire load of the controllable ISD suspension is lower than that of the traditional passive suspension in the low frequency band, while it is increased in the high frequency band.

In summation, in this test under the sinusoidal inputs, the body acceleration of the controllable ISD suspension is significantly improved in a full frequency band compared with those of the traditional passive suspension and passive ISD suspension. However, the improvement effect of the suspension working space and dynamic tire load are not obvious, which is consistent with the previous simulation results. In general, the controllable ISD suspension system based on an ADD positive real network achieves the vibration suppression effect in a wider frequency band, and its performance has been improved significantly compared with those of a traditional passive suspension and passive ISD suspension.

B. TEST OF RANDOM ROAD INPUT

Under the conditions of random road input, the vehicle is driven on a class C road at speeds of 10 m/s, 20 m/s and 30 m/s. The time duration is set as 10 s, and the sampling interval is set as 0.002 s. After the collected signals of body acceleration, the suspension working space and dynamic tire load are processed, and the RMS value comparisons is shown in Table 6 and Figure 13.
TABLE 6. Performance indexes under the random road inputs.

| Suspension type | Speed  | RMS of body acceleration (m/s²) | RMS of suspension working space (mm) | RMS of dynamic tire load (kN) |
|-----------------|--------|----------------------------------|--------------------------------------|-------------------------------|
| Traditional passive suspension | 10     | 0.9578                           | 5.3675                               | 0.6759                        |
|                  | 20     | 1.4864                           | 8.5647                               | 0.9748                        |
|                  | 30     | 2.1578                           | 12.3542                              | 1.4257                        |
| Passive ISD suspension | 10     | 0.8460                           | 5.3246                               | 0.7363                        |
|                  | 20     | 1.2854                           | 8.4003                               | 1.0631                        |
|                  | 30     | 1.8818                           | 12.1590                              | 1.5594                        |
| Controllable ISD suspension | 10     | 0.7471                           | 5.2074                               | 0.7202                        |
|                  | 20     | 1.1217                           | 8.2677                               | 1.0104                        |
|                  | 30     | 1.6726                           | 11.9544                              | 1.4799                        |

From Table 6 and Figure 13, it can be seen that the comprehensive vibration isolation performance of the controllable ISD suspension studied in this paper has been significantly improved compared with those of the traditional passive suspension and passive ISD suspension under the random road input conditions. Compared with the traditional passive suspension, the RMS value of body acceleration of the controllable ISD suspension has been reduced by 24.5% at 20 m/s. Under the same conditions, the RMS value of suspension working space of the controllable ISD suspension has been decreased by 3.5%, and the RMS value of dynamic tire load has been increased by 3.7%. In addition, the RMS value of body acceleration of the controllable ISD suspension has also been improved compared with the passive ISD suspension, with the reduction of 12.7% at 20 m/s. Under the same conditions, the RMS value of suspension working space of the controllable ISD suspension has been reduced by 1.6%, and the RMS value of dynamic tire load has been reduced by 5.0%. The results show that the ride comfort of the controllable ISD suspension has been improved, and the vehicle body vibrations have also been effectively suppressed.

Figure 14 shows the comparisons of the controllable ISD suspension with the traditional passive suspension and passive ISD suspension under the random road input conditions when the speed is 20 m/s in the frequency domain.

It can be seen from Figure 14 that the PSD value of body acceleration of the controllable ISD suspension is significantly improved improvement compared with those of the traditional passive suspension and passive ISD suspension, and the vibration peak values have also been decreased. The PSD values of the suspension working space and dynamic tire load of the controllable ISD suspension are significantly lower than those of the traditional passive suspension in the low frequency band. Comparably, the effect is not significant in the medium and high frequency bands, but it is better than that of the passive ISD suspension. On the whole, under the conditions of random road input, the body acceleration index of the controllable ISD suspension system studied in this paper is improved most significantly, and the improvement effect is better than that of the passive ISD suspension, which effectively realized the effect of vehicle wide-band vibration suppression.
VI. CONCLUSION

This paper proposed an ADD positive network synthesis methodology for vehicle suspension design by using the advantages of ADD control, which can effectively suppress medium-high frequency vibrations of a vehicle body, combined with an ISD suspension passive network synthesis method, which can effectively block low frequency vibrations. In this way, the problem of ISD vehicle suspension vibration suppression in a wider frequency domain is transformed into a problem of positive real optimization and control based on the ADD network synthesis.

A model of the ISD vehicle suspension based on a second-order ADD positive real network is built, and the parameters of the ADD positive real network are obtained by an artificial fish swarm algorithm optimization. Taking the second-order ADD positive real network system as the ideal reference model, an RBF sliding mode control method is proposed to construct the ISD suspension active control system. The test results show that under sinusoidal inputs, the RMS value of body acceleration of the controllable ISD suspension is significantly improved over a wide frequency range. Compared with the traditional passive suspension, the RMS value of body acceleration of the controllable ISD suspension has been reduced by 24.5% under random road input conditions when the speed is 20 m/s. Compared with the passive ISD suspension, the RMS value of body acceleration of the controllable ISD suspension has been reduced by 12.7% under the same conditions. The results demonstrate that the controllable ISD suspension system based on the ADD positive real network can enhance the vibration suppression of medium-high frequency bands on the basis of the ISD suspension and thus comprehensively improve suspension performance.

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