Annual modulation sensitivity in cold dark matter searches

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The sensitivity of experiments looking for annual modulated signals is discussed and analyzed. The choice of energy intervals for rate integration and enhancing the signal-to-noise ratio of the predicted WIMP signal is addressed. Special emphasis is placed on quantifying the minimum required exposure, \( MT \), under experimental conditions. The difficulty reduces to establish the proper separation between the rate due to the unmodulated part of the WIMP signal and the rate of spurious background present in any experiment. The problem is solved by placing an upper bound to the unmodulated part of the signal using the best exclusion plot. We find that the lowest backgrounds achieved result in exposures in the range \( MT \sim 5 – 100 \text{ kg yr} \) for masses \( m_\chi > 100 \text{ GeV} \) depending on the energy threshold of the detector. While our results are valid for Ge and NaI detectors, the formulae derived apply to other elements as well. Prescriptions are given to estimate exposures for higher background experiments.

I. INTRODUCTION

Weakly interacting massive particles (WIMP’s) are most likely the main constituents of the galactic halo. Constraints on the strength of the interaction and mass of the WIMP’s have been obtained by several experiments using germanium, silicon, and NaI(Tl) crystals. For a coupling constant of the order of the weak coupling constant, the mass of these WIMP’s must be, in any case, heavier than the 40 GeV lower bound already imposed by the measurement of the width of the \( Z^0 \). The same measurement has also placed bounds on some supersymmetric candidates.

Drukier et al. suggested that a distinctive signature of WIMP’s would be the annual modulation of the detection rate. The modulation originates in the orbital motion of the Earth around the Sun that produces a variation in the relative velocity between the Earth and the WIMP’s, thus altering the dark matter flux during the year. Extracting this fluctuation from data that, certainly, contain the contribution of spurious events is a task that demands care. The problem to be confronted with is the following: to extract a feeble signal from the data, we require a large signal-to-noise ratio so that the relative statistical fluctuations are small compared with the sought after signal. At the same time, efforts to reduce the background of unwanted (no dark matter) events prevent achieving large signal-to-noise ratios. There is no compromise solution possible; if a low counting rate has been attained, a long exposure is required to detect an even smaller modulation.

The main purpose of this work is to examine this issue quantitatively and to establish some guidelines upon which upcoming data, like those from the Sierra Grande experiment, can be analyzed. Although the subject of annual modulation, and its details, have been discussed by several authors, we feel that the sensitivity of modulation searches, namely the amount of exposure needed to resolve a given fluctuation embedded in a background, has not been yet addressed quantitatively in the literature. A semi-quantitative approach was advanced in Ref. where results for required exposures were presented. Our procedure to obtain minimum exposures however, relies on simpler assumptions and give lower estimates.

In the first section we briefly introduce the formalism needed for calculating the expected signal-to-noise ratios. For the sake of simplicity, we illustrate our calculations for the case of a germanium detector later on we extend the results to a sodium iodide detector. We take into account the influence of the crossing of the spectra due to their annual fluctuation and choose energy intervals of integration that maximize the expected signal-to-noise ratios of the modulated rate. The next section analyzes quite generally the sensitivity of this type of experiments, addressing its difficulties and common misuses. Estimates of the exposures for the best experiment are presented together with guidelines for experiments with higher backgrounds. Most of our results are not strongly influenced by astrophysical uncertainties.

II. PREDICTED SIGNAL

The expected total rate of events due to the recoil of nuclei elastically scattered by WIMP’s will be the product of the cross section, the WIMP’s flux, and the number of target nuclei in a detector of atomic mass number \( A \). For an incoming WIMP of mass \( m_\chi \) and velocity \( v \) the differential counting rate in the recoil-energy interval \( T \), \( T + dT \), is given by

\[
\frac{dR}{dT} = \frac{N_A}{A} \left( \frac{\rho_{halo}}{m_\chi} \right) \int_{v_{min}}^{v_{max}} f(v, \sigma_v, V_Z) v \frac{d\sigma}{dv}(v) dv,
\]

(2.1)
where \( N_A \) is Avogadro’s number, \( \rho_{\text{halo}}/m_\chi \) is the number density of WIMP’s, and \( f(v, \sigma_v, V_\odot) \) is the Maxwell-Boltzmann velocity distribution of the WIMP’s for an observer on the Earth. The velocity distribution is a function of \( V_\odot(t) \), the velocity of the Earth with respect to the galactic rest frame varying annually and \( \sigma_v \), the dispersion velocity of the WIMP’s in the galactic halo. The integration limits of (2.1) are, \( v_{\text{max}} \), the maximum velocity of the WIMP’s \( (v_{\text{max}} = V_\odot + v_{\text{esc}} , \text{where} \ v_{\text{esc}} \ \text{is the escape velocity from our galaxy}) \), and \( v_{\text{min}} \), the minimum WIMP velocity necessary to contribute to a particular energy of the recoil spectrum.

The differential cross section in Eq. (2.1) is given, in general, by

\[
\frac{d\sigma}{dT} = \frac{\sigma_o(m_\chi)}{T_{\text{max}}} F^2(T),
\]

(2.2)

where \( \sigma_o(m_\chi) \) is the cross section at zero momentum transfer (of a heavy Dirac neutrino, for example), \( T_{\text{max}} = 2\mu^2 v^2 / m_N \) (\( \mu \) is the WIMP-nucleus reduced mass and \( m_N \) the nucleus mass), and \( F^2(T) \) is a standard Bessel nuclear form factor, \( 1 \) that depends on the momentum transfer and which takes into account the loss of coherence of the interaction. The calculated recoil spectrum must also be convoluted with a relative efficiency function for germanium, \( 1 \) accounting for the efficiency of the recoiling process in generating an ionization signal. We refer therefore to the deposited energy \( E \), instead of \( T \), in the following formulae. Table I lists the values of \( \rho_{\text{halo}}, v_{\text{esc}}, \sigma_v \), and \( V_\odot \), the velocity of the Sun around the galactic center, used in this work.

### Crossing energies

It is known that, the differential energy spectra of June and December cross at specific energies that depend, among other factors, on the WIMP mass. The reason for the crossing of the spectra is that in June the velocity distribution increases by 15 km s\(^{-1}\) enabling the WIMP’s to deposit a higher fraction of their kinetic energy. This crossing can significantly alter the amplitude of the modulation depending on the energy interval chosen to integrate the differential rate.

The calculated crossing energies, \( E_c \), (dotted and dashed lines) are shown in figure 1, for the case of a heavy Dirac neutrino, as a function of the WIMP mass.

| TABLE I. List of parameters used. |
|-----------------------------------|
| \( \rho_{\text{halo}} = 0.3 \ \text{GeV cm}^{-3} \) |
| \( V_\odot = 230 \ \text{km s}^{-1} \) |
| \( v_{\text{esc}} = 570 \ \text{km s}^{-1} \) |
| \( \sigma_v = 270 \ \text{km s}^{-1} \) |

The figure explores also the dependence of the crossing energy on different halo parameters. The halo density for example, has no effect on the crossing energies since it affects only the absolute rates. The crossing is neither affected by the assumed escape velocity, \( v_{\text{esc}} \), or the value of \( V_\odot \) (which in turn modifies \( V_\odot \)) but, it is sensitive to the dispersion velocity, \( \sigma_v \), which for WIMP masses higher than 500 GeV changes by 2 keV, when \( \sigma_v \) is increased from 270 km s\(^{-1}\) to 300 km s\(^{-1}\). The dispersion velocity of WIMP’s in the galactic halo is a rather uncertain parameter, ranging from 246 km s\(^{-1}\) to 323 km s\(^{-1}\) [4], so here we adopt a standard value of 270 km s\(^{-1}\).

### Theoretical signal-to-noise ratio

To extract a signal from the data it is necessary to establish the maximum of the theoretical signal-to-noise ratio. The total signal as a function of time can be expressed in the form,

\[
S(t) = \int \frac{dR}{dE'} [V_\odot(t)]dE' = S_o + S_m \cos(\omega t) + O(S_m^2),
\]

(2.3)

where we define \( S_o \) as the unmodulated part, \( S_m \) as the amplitude of the modulation, \( \omega = 2\pi/\tau \ (\tau = 365 \ \text{d}) \), and \( t \) is measured from June \( 2^{\text{nd}} \). From the differential rates evaluated at June, \( S_J = S(t = 0) \), and December, \( S_D \equiv S(t = \tau/2) \) expressions for \( S_m \) and \( S_o \) can be obtained,

\[
S_m = \frac{1}{2} [S_J - S_D] \quad S_o = \frac{1}{2} [S_J + S_D]. \quad (2.4)
\]

Assuming that \( S_m \ll S_o \), the theoretical signal-to-noise ratio is defined to be,

\[
(s/n)_{\text{th}} = \frac{S_m}{\sqrt{S_o}} \sqrt{MT} \quad (S_m \ll S_o),
\]

(2.5)
where the product $MT$ is the exposure of the detector, namely its mass times the overall exposition time to the WIMP flux. This is the function we need to maximize. Notice that in (2.3) it is assumed that no spurious background is present. A more realistic case will be considered in the next section.

The limits of integration of equation (2.3) are arbitrary and therefore the theoretical signal-to-noise ratio (2.3) is effectively energy dependent,

$$\frac{(s/n)_{th}(E)}{\sqrt{S_o(E)}} = \frac{S_m(E)}{\sqrt{S_o(E)}} \sqrt{MT}.$$  

$$\text{(2.6)}$$

In the case of germanium detectors the crossing energies, $E_c$, usually lie between the energy threshold of the detector, $E_i$, and an upper limit, $E_f$, representative of the highest energy depositions produced by WIMP’s. The maximum signal-to-noise ratio, for low-mass WIMP’s, are obtained when the differential rates of June and December are integrated between $E > E_c$ and $E_f$, since the spectra there differ over a larger fraction of the total energy window, $(E_i, E_f)$. For heavier WIMP’s, however, with larger values of $E_c$, the maximum signal-to-noise ratios will be attained when the differential rates is integrated in the low energy region, from $E_i$ to $E < E_c$.

The theoretical signal-to-noise ratio for 1 kg yr exposure as a function of the WIMP mass is shown in Fig. 2; only integrals from $E$ to $E_f$ are plotted. For each integration interval the corresponding ratio is calculated and plotted at $E$, while $E$ is varied between 0 and $E_f$. From the figure one can extract, for each WIMP mass, the value of $E$ that maximizes the signal-to-noise ratio, $E_{s/n}$.

In a similar fashion, we define $E'_{s/n}$ as the value of $E$ that maximizes $(s/n)_{th}$, when $E$ varies from $E_i$ up to $E_f$. In Fig. 3 the values of $E_{s/n}$ and $E'_{s/n}$ are plotted as a function of the WIMP mass.

\begin{figure}[h]
\centering
\includegraphics[width=0.9\textwidth]{fig2.png}
\caption{Theoretical signal-to-noise ratios in Ge for 1 kg yr for several WIMP masses. The ratios were calculated for all energy intervals running from $E$ to $E_f = 50$ keV and plotted at $E$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.9\textwidth]{fig3.png}
\caption{Values of $E_{s/n}$ ($E'_{s/n}$) in Ge maximizing the theoretical signal-to-noise ratios when the differential rate is integrated between $E$ and $E_f$ ($E_i$ and $E$).}
\end{figure}

For the next section a quantitative measure of the annual effect is needed. We calculate therefore, the percentage ratio of the modulated over the unmodulated signal, $\alpha = \frac{S_m}{S_o}$, and plot it as a function of the mass of the WIMP in Fig. 4. The solid-line is obtained using the energy intervals that maximize $(s/n)_{th}(E)$ with relative intensities between 2 – 8% (solid line). In the same figure a similar ratio but integrated between an arbitrary minimum energy, $E_i = 2.5$ keV, and $E_f = 50$ keV (dashed line) is also shown to illustrate the importance of the choice of the energy interval. This second curve shows a dramatic decrease in the modulation percentage for the mass that symmetrizes, with respect to $E_c$, the interval.

\begin{figure}[h]
\centering
\includegraphics[width=0.9\textwidth]{fig4.png}
\caption{Percentage of predicted annual modulation in Ge in two illustrative cases. An optimized case (solid line) using the energy intervals that maximize $(s/n)_{th}(E)$ and a non-optimized one (dash line) where $E_i$ has been arbitrarily selected to be 2.5 keV.}
\end{figure}

*An upper limit of 50 keV seems adequate since the energy deposited in a germanium detector by a 10 TeV WIMP with a mean velocity of 340 km $s^{-1}$ is of that order.
of integration (≈ 300 GeV). Table II lists the values of $E_{s/n}$ and $E'_{s/n}$, the second with three different threshold energies, and the corresponding values of $S_m/S_o$.

### III. DETECTION SENSITIVITY

Generalizing the concepts introduced in the previous section the total rate under experimental conditions can be expressed as,

$$S_{tot} = B + S_o + S_m \cos(\omega t),$$

(3.1)

where $B$ is the rate of spurious background. The experimental signal-to-noise ratio ($S_m \ll S_o$) is then,

$$(s/n)_{exp} = \frac{S_m}{\sqrt{S_{tot}}} \sqrt{MT} \approx \frac{\alpha S_o}{\sqrt{B + S_o}} \sqrt{MT},$$

(3.2)

where $\alpha = S_m/S_o$ is as in Table I. Relation (3.2) is always valid; however, determining the real magnitude of integration (≈ 300 GeV). Table II lists the values of $E_{s/n}$ and $E'_{s/n}$, the second with three different threshold energies, and the corresponding values of $S_m/S_o$.

### TABLE II. Energies that maximize the signal-to-noise ratio in Ge. Depending on the WIMP mass the signal-to-noise ratio was obtained integrating the differential rates of June and December, $dR$, in two cases: when $\int_{E}^{E'} dR$ was considered, $E_{s/n}$ corresponds to the value of $E$ maximizing the expected signal-to-noise ratio, and when $\int_{E}^{E'} dR$ was computed, $E'_{s/n}$ corresponds to the value of $E$ maximizing the signal-to-noise. $E_f = 50$ keV and $E_i = 0, 2, 3$ keV were used.

| $m_x$ (GeV) | $E_{s/n}$ (keV) | $E'_{s/n}$ (keV) | $\alpha \equiv S_m/S_o$ (%) |
|------------|----------------|----------------|--------------------------|
| 30         | 3.5            | 8.5            | 7.80                     |
| 40         | 5.3            | 9.4            | 7.40                     |
| 50         | 6.9            | 10.0           | 6.95                     |
| 60         | 8.3            | 10.4           | 6.53                     |
| 70         | 9.5            | 10.6           | 6.14                     |
| 80         | 10.7           | 10.8           | 5.85                     |
| 90         | 11.7           | 11.0           | 5.56                     |
| 100        | 12.5           | 11.2           | 5.28                     |
| 200        | 17.7           | 11.6           | 3.83                     |
| 300        | 20.1           | 11.2           | 3.24                     |
| 400        | 21.5           | 12.0           | 2.93                     |
| 500        | 9.1            | 10.0           | 2.67                     |
| 600        | 9.5            | 10.4           | 2.48                     |
| 700        | 9.9            | 10.6           | 2.38                     |
| 800        | 10.1           | 11.0           | 2.39                     |
| 900        | 10.3           | 11.0           | 2.39                     |
| 1000       | 10.3           | 11.2           | 2.41                     |
| 2000       | 11.1           | 12.0           | 2.42                     |
| 3000       | 11.3           | 12.2           | 2.43                     |
| 4000       | 11.5           | 12.4           | 2.42                     |
| 5000       | 11.5           | 12.4           | 2.43                     |
| 6000       | 11.7           | 12.6           | 2.43                     |
| 7000       | 11.7           | 12.6           | 2.43                     |
| 8000       | 11.7           | 12.6           | 2.43                     |
| 9000       | 11.7           | 12.6           | 2.43                     |

The best scenario, just mentioned, is that where $\gamma \equiv S_{tot}/S_L \approx 1, B \approx 0$, and the minimum required exposure is therefore,

$$MT_{min} = \left( \frac{s/n}{\alpha} \right)^2 \frac{1}{S_L}.$$  

(3.3)

Conversely, when $S_{tot}/S_L \gg 1$, longer exposures are required to achieve a similar sensitivity. In this case the necessary exposure is,

$$MT = \gamma MT_{min} \approx \frac{B}{S_L} MT_{min}, \quad (\gamma \gg 1).$$

(3.4)

To obtain $S_L$ we resort to the exclusion plot, $\sigma(m_x)$, using the relation,

$$S_L \approx \int_{E_{s/n}}^{E_f} \frac{dR[\sigma(m_x)]}{dE} dE.$$  

(3.5)

1 Of course, we do not know for sure if $S_L$ is purely a dark-matter signal or still has some spurious background hidden in it. The best we can do is to assume that it is pure dark matter and estimate the exposure required to detect a modulated signal of value $\alpha$. If no modulation is found, it is either because there is none, or because $S_L / S_o$, and consequently our original assumption of a negligible background was wrong. The recipe then calls for a further reduction of the background and an increase in the exposure.
The differential rate, $dR(\sigma)/dE$, on the left-hand side is calculated as in section II but using the experimental exclusion, $\sigma(m_\chi)$, instead of the theoretical zero-momentum cross section,

$$\frac{d\sigma}{dT} = \frac{\sigma(m_\chi)}{T_{max}} F^2(k). \quad (3.7)$$

To maintain self-consistency we integrate the differential rate using the energy limits and the value of $\alpha$ derived in the previous section for each WIMP mass. Thus, the predicted signal is renormalized to make it compatible with the values of $\sigma(m_\chi)$ obtained from the exclusion plot.

Solving from equations (3.4) and (3.6), the minimum exposure can be expressed as

$$MT_{min} = \left(\frac{s/n}{\alpha}\right)^2 \left(\int_{E_{s/n}}^{E_f} \frac{dR[\sigma(m_\chi)]}{dE} dE\right)^{-1}. \quad (3.8)$$

FIG. 5. Predicted minimum exposures in Ge as a function of WIMP mass for three energy thresholds, $E_i = 0, 2,$ and 3 keV. The curves were derived using the exclusion plot of Ref. 3.

In Fig. 5 and Table II we show the minimum exposures (in kg yr) for the lowest rate experiment as a function of WIMP mass based on the modulation percentages calculated in Table I. As a reasonable criterion to distinguish the signal, we have used that $s$ has to be at least $2\sigma$ larger than the statistical uncertainty or, what is the same, that $s/n = 2$. The figure and table were obtained using for $\sigma(m_\chi)$ the data from Ref. 3, properly rescaled to germanium nuclei. As mentioned in the previous section, for low-mass WIMP’s, the energy intervals maximizing the signal-to-noise ratios tend to be located above the crossing energies. For these masses the minimum exposures are obtained integrating relation (3.8) between $E_{s/n}$ and $E_f$. Heavier WIMP’s maximize their $s/n$ at energy intervals below their crossings. Therefore, the three curves corresponding to different energy thresholds were obtained integrating (3.8) between $E_i = 0, 2,$ and 3 keV and $E_{s/n}$. Notice that the sensitivity decreases roughly as $\alpha^{-2}$.

For an arbitrary experiment with a total rate $S_{tot}$, the required exposure can be estimated from equation (3.3), in units of the exposure of the lowest rate experiment. The difference between the two can be attributed to a spurious background of magnitude $B = (\gamma - 1)S_L$.

An analogous calculation can be done for a NaI detector using the measured relative efficiencies (quenching factors, $q(I) = 0.09, q(Na) = 0.30$) and generalizing the formulae of section II to the case of a two-nuclei target. Similar results to those of previous sections are found with the significant distinction that because of the rather low quenching factor of iodine, the values of $E_c$, $E_{s/n}$, and $E_{s/n}'$ occur at low energies (< 10 keV). For NaI a lower value of $E_f$ was used (30 keV). Furthermore, since the iodine nucleus cross section is larger than that of sodium, iodine is the main contributor to the total rate at low energies (at high energies its nuclear form factor strongly diminishes the interaction rate).

The minimum exposure for NaI as a function of WIMP mass is shown in Fig. 6. Due to the large cross section of iodine smaller exposures than in the case of germanium are expected. Nevertheless, as can be appreciated from the figure, $MT_{min}$ is now more dependent on the threshold energy of the detector.

FIG. 6. Predicted minimum exposures in NaI as a function of WIMP mass for three energy thresholds, $E_i = 0, 1,$ and 2 keV according to the exclusion plot of Ref. 3.

IV. CONCLUSIONS

We have performed a quantitative prediction of annual modulation rates for Ge and NaI detectors. To this aim, we found first the adequate energy intervals where to integrate the rate so that the theoretical signal-to-noise ratio is at its maximum. Using these optimized theoretical rates and assuming that the lowest experimental rate achieved so far contains no spurious events we then estimated the minimum exposure required for the predicted modulated rates as a function of WIMP mass. From the
results obtained we can draw a few conclusions. To avoid artificially large signal-to-noise ratios, an arbitrary experiment with a total rate, \( S_{\text{tot}} \), has to be compared with the best experiment, \( S_L \), and should attribute the difference in counting rate to the presence of a background of the order of \( B = (\gamma - 1)S_L \). \( \gamma = S_{\text{tot}}/S_L \). This arbitrary experiment requires an exposure of \( MT = \gamma MT_{\text{min}} \) to reach the same level of sensitivity as the best one. Current exclusion plots correspond to exposures in the range \( MT \sim 10 - 60 \text{ kg yr} \) for masses \( m_\chi > 100 \text{ GeV} \) (Ge) and between \( MT \sim 5 - 90 \text{ kg yr} \) for masses \( m_\chi > 300 \text{ GeV} \) (NaI). A strong dependence in NaI of the minimum exposure with threshold energy is also predicted. In Ge, the sensitivity is independent from the threshold energy for low WIMP masses.

A recent claim by the DAMA collaboration \cite{15} deserves a comment. The best exclusion plot used throughout this work corresponds to an experimental mean rate of at least, \( \langle S_L \rangle \approx 2 \text{ c/kg d} \) (energy region 2 - 20 keV), obtained by the same group after rejecting events by pulse shape analysis. For the annual modulation analysis, however, no rejection is allowed and therefore the corresponding experimental rate is higher, \( S_{\text{tot}} \approx 1 \text{ c/keV kg d} \times 18 \text{ keV} \approx 18 \text{ c/kg d} \). This implies the presence of a spurious background that increases the necessary exposure required to detect a modulation by a factor \( \gamma = S_{\text{tot}}/S_L \approx 9 \). According to our calculations then, the required exposure (\( \approx 65 \text{ kg yr} \) for a 60 GeV WIMP) was not achieved by the DAMA group since, at the moment of their announcement, they had analyzed a statistics of only 12.5 kg yr. It is interesting to note however, that our results predict that the DAMA collaboration with its 115.5 kg detector should be sensitive to low (\( \lesssim 90 \text{ GeV} \)) as well as high-mass (\( \gtrsim 1 \text{ TeV} \)) WIMPS in approximately three years of data acquisition.

Future improvements in background reduction will certainly push further down the limits of dark matter signal and at the same time will make the small annual fluctuations harder to distinguish. Nevertheless, in view of Figs. 5, 6, and equation (3.5), higher-background experiments with large masses will still be suitable to explore the low and high mass regions.

| \( m_\chi \) (GeV) | \( \int_{E_i}^{E_f} dR(\sigma) \) (c/kg d) | \( \int_{E_i}^{E_i/\gamma} dR(\sigma) \) (c/kg d) | \( MT_{\text{min}} \) (c/kg y) | \( MT_{\text{min}} \) (c/kg y) |
|------------------|------------------|------------------|------------------|------------------|
| 30               | 2.035            | 0.94             | 2.01             | 3.52             |
| 40               | 1.057            | 5.06             | 6.83             | 8.43             |
| 50               | 0.681            | 10.29            | 11.91            | 14.63            |
| 60               | 0.534            | 20.68            | 23.44            | 26.17            |
| 70               | 0.447            | 23.76            | 26.48            | 29.20            |
| 80               | 0.398            | 26.88            | 29.60            | 32.32            |
| 90               | 0.360            | 30.00            | 32.72            | 35.44            |
| 100              | 0.344            | 33.22            | 36.06            | 38.78            |
| 200              | 0.253            | 62.50            | 65.24            | 67.98            |
| 300              | 0.223            | 15.96            | 18.68            | 21.40            |
| 400              | 0.208            | 19.08            | 21.80            | 24.52            |
| 500              | 0.193            | 22.20            | 25.02            | 27.84            |
| 600              | 0.180            | 25.42            | 28.24            | 31.06            |
| 700              | 0.168            | 28.64            | 31.46            | 34.28            |
| 800              | 0.157            | 31.86            | 34.68            | 37.50            |
| 900              | 0.148            | 35.10            | 38.32            | 41.74            |
| 1000             | 0.140            | 38.32            | 41.54            | 44.96            |
| 2000             | 0.122            | 46.24            | 49.76            | 53.28            |
| 3000             | 0.116            | 54.16            | 57.68            | 61.20            |
| 4000             | 0.110            | 62.08            | 65.60            | 69.12            |
| 5000             | 0.106            | 70.00            | 73.62            | 77.54            |
| 6000             | 0.102            | 77.92            | 81.54            | 85.46            |
| 7000             | 0.099            | 85.84            | 89.16            | 93.08            |
| 8000             | 0.095            | 93.76            | 97.48            | 101.40           |
| 9000             | 0.092            | 101.68           | 105.20           | 109.12           |

TABLE III. Rates normalized to be compatible with the best exclusion plot of Bernabei et al. \cite{3} and corresponding exposures for several WIMP masses in Ge for the intervals of integration of Table II.
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