Anomalous behavior of neutron refraction index in a perfect crystal near the Bragg reflex

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Abstract. Anomalous behavior of neutron refraction index in a perfect crystal near Bragg resonance was studied. This phenomenon is connected with the resonance behavior of potential of neutron interaction with crystal near the Bragg reflex. The amplitude of this resonance is equal to magnitude of g-harmonic of neutron interaction potential \( V_g \) and width is about the Bragg width of reflex. Recently, it was shown that for the case of noncentrosymmetric crystal this effect result in a large electric field acting on a neutron (value of the field can reach about \( 10^8 \) V/cm). This effect is planed using to search for the electric dipole moment of a neutron. If the degree of crystal imperfect is less than the Bragg reflection width (case of perfect crystal) the width of the reflex is determined by the own width of crystal reflex that is about \( 10^{-5} \) of the neutron energy. The value of g-harmonics of interaction of neutron with crystal \( V_g \) and optical potential of the interaction of neutron with crystal \( V_0 \) are usually about the same. Therefore the variation of neutron energy on a \( 10^{-5} \) of its value will change significantly a potential of neutron interaction with crystal.

1. Introduction.

Electric dipole moments (EDMs) of elementary particles belong to the most sensitive probes for CP violation beyond the Standard Model of Particle Physics. Constraining or detecting EDMs of different systems allows to gather experimental information about models for new physics that is complementary to high energy physics data.

The statistical sensitivity of any experiment to measure the nEDM is determined by the product \( E \tau \sqrt{N} \), where \( E \) is the value of the electric field, \( \tau \) the duration of the neutron’s interaction with the field and \( N \) the number of counted neutrons.

Experiments with non-centrosymmetric crystals exploit the interplanar electric field. For quartz, this field was measured to be \( E = 2 \cdot 10^8 \) V/cm [4], several orders of magnitude higher than the electric field achievable in vacuum or liquid helium.
Time of neutron stay in the field $\tau$ is determined by the neutron velocity in crystal and abnormal refraction index can be the source of a systematic effects.

2. Neutron optics near the Bragg reflection

For the periodic crystal structure it is convenient to present the potential of neutron interaction with crystal as Fourier series over the reciprocal lattice vectors $g$:

$$
V(r) = \sum_a \sum_g V_a(\vec{r} - \vec{r}_a) = \sum_g V_g \exp(i\vec{g}\vec{r}) = V_0 + \sum_g 2V_g \cos(\vec{g}\vec{r} + \varphi_g) \quad (1)
$$

where $V_a(\vec{r} - \vec{r}_a)$ is the potential of single atom, $\vec{r}_a$ is the atom position, $V_g = v_g \exp(i\varphi_g)$, $v_g$ and $\varphi_g$ are the amplitude and phase of $g$-harmonic $V_g$, $g = 2\pi / d$, $d$ is the interplanar distance. Here we take into account $V_g = V_g^*$, because we consider the real value potentials.

When the Bragg condition is not satisfied for any system of crystallographic planes the passage of neutron through the crystal is determined average potential $V_0$, otherwise all is determined by the $g$ harmonic of the potential of the interaction of the neutron with the crystal $V_g$. Energy of the neutrons moving through the crystal close to the Bragg condition can be represented in the form:

$$
\tilde{E}_k = E_k - V_0 + V_g \cdot \frac{1}{\Delta_B} \quad (2)
$$

where $\Delta_B = (E_k - E_{k_g}) / V_g = 2(E_k - E_B) / V_g$ is parameter of the deviation from the Bragg condition for system of crystallographic planes $g$. $\vec{k}_g = \vec{k} + \vec{g} - \vec{r}$ is the wavevectors of the reflected wave. $E_k = h^2k^2 / 2m$, $E_{k_g} = h^2k_g^2 / 2m$ are the energies of a neutron in the $|k\rangle$ and $|k_g\rangle$ state, respectively. $E_B = h^2g^2 / (8m\sin^2\theta_B)$ is the energy of the neutron corresponding to the exact Bragg condition.

According to Eq. (2), the neutron energy increases unlimitedly close to the exact the Bragg condition ($\Delta E \equiv E_k - E_B \rightarrow 0$). For this reason, perturbation theory is inapplicable at $\Delta E \equiv |V_g| \equiv v_g$. For the exact Bragg condition ($\Delta E = 0$) two neutron states with the moments $\vec{h}k$ and $\vec{h}k_g$ have the same energy, that corresponds to the confluent state with the energy $E_k$. The amplitudes of these states are about the same. As a result, it is necessary to solve the two level problem.

More accurate consideration of the last term in equation (2) results in:

$$
\tilde{E}_k = E_k - V_0 + V_g \cdot \frac{\Delta_B}{\Delta_B^2 + 1} \quad (3)
$$

One can see from the expression (3) that the last term quickly change on the scale of the Bragg reflex width $\Delta_B$, i.e. for the neutron energy range $E_B - V_g < E_k < E_B + V_g$ ($V_g / E_B \approx 10^{-5}$).The magnitude of $V_g$ and $V_0$ are about the same, therefore there is resonance like dependence of the potential of neutron interaction with the crystal on the neutron energy. For example, for the quartz crystal diffraction plane (110) $V_g = 4 \cdot 10^{-8} eV$, $V_0 = 10^{-7} eV$, $E_B = 3.2 \cdot 10^{-3} eV$ for the diffraction angle close to $\pi / 2$. 
3. Method.

Setup allows us to change velocity of the crystal relatively to the laboratory coordinate system, that allows us to change it time the deviation from Bragg condition

$$\Delta_b(t) = \Delta_{b_0} + 4 \frac{E_B}{V_g} \nu(t)$$

(4)

where \( \nu \) is velocity of the neutron. The initial deviation from Bragg condition \( \Delta_{b_0} \) was determined by the difference of the crystal interplanar distance, which was regulated by the temperature difference \( T_{13} = T_1 - T_3 \) of the crystals K1 and K3. The parameter deviation is defined as:

$$\Delta_{b_0} = 4 \frac{E_B}{V_g} \chi T_{13}$$

(5)

here \( \chi \) is coefficient of thermal expansion of the crystal along the reciprocal lattice vector.

Acceleration effect of the neutron is determined by the changing of potential of neutron interaction with the crystal during the time of neutron stay in crystal , and can be written as:

$$\Delta E(t_0) = \tilde{E}(t_2) - \tilde{E}(t_1) = \frac{1 - \Delta_{b_0}^2}{(1 + \Delta_{b_0}^2)^2} \frac{4 E_B}{\nu} \Delta \nu(t_0)$$

(6)

here \( \Delta \nu(t_0) \) - is the change of crystal velocity, \( t_0 \) is the time of neutron entrance the crystal.

4. Experiment.

For the experimental measurements of the refractive index of a neutron in a perfect crystal was made at the WWR-M reactor (PNPI, Gatchina).

**Figure 1.** The experimental setup

Main crystal K3 was moved by the piezomotor with the oscillation frequency \( \nu_c = 4.5 \text{ kHz} \) \( (t_c = 222 \text{ ms}) \), with an amplitude equal to 0.3 mm or 0.15 mm. The length of the crystal in the direction of the neutron velocity was \( L = 5 \text{ cm} \), the neutron wavelength \( \lambda_n = 4.9 \text{Å} \). Thus, the neutron time of flight through the crystal was approximately equal to a quarter of the period of oscillation of the crystal \( t_n = 62 \text{ ms} \approx t_c / 4 \).

Crystals K1 and K2 as the double-crystal spectrometer. Where K1 is the monochromator and K2 is analyzer.: Neutron energy analysis after passage through the main crystal K3 was
fulfilled by changing the temperature of crystal K2 and therefore the interpalnar distance and reflected neutron wavelength due to the temperature expansion

\[ \lambda = 2d \sin(\theta_B) \] (7)

where \( d \) is the interpalnar distance.

5. Results.
The value of neutron acceleration in crystal is show in Figure 2. The two crystal temperature difference \( T_{13} \) is directly connected with the deviation from Bragg condition, see (5), the calculated Bragg width of the reflex (110) of quartz crystal in a unit of \( T_{13} \) is about 1K.

![Figure 2](image-url)

**Figure 2.** The change of energy of the neutron after the passage of the accelerated crystal.

One can see that effect of neutron accelerating reach about (20-30)neV, while the zero harmonics of neutron interaction with the crystal \( V_0 \) is about 100neV.

The value of neutron interaction with the crystal can be calculated from the effect of neutron accelerating (6) and (3). The result is shown in Figure 3. One can see that varying the incident neutron energy on a relative value a few \( 10^{-5} \) change the neutron interaction with crystal on about 40neV, that is about the zero harmonic \( V_0 \).
6. Conclusion:
We have studied the refraction index of a neutron in crystal near the Bragg reflection. It is shown that the dependence of neutron interaction potential with crystal on the deviation from Bragg condition is characterized by the resonance shape with the width of an order of the width of Bragg reflection (for thermal and cold neutrons $\Delta E/E \sim 10^{-5}$).

This fact opens up new prospects for the development of ultrahigh-resolution neutron optics experiment and new devices for managing the neutron beam.

A new phenomenon, the acceleration of the neutron near the Bragg resonance, was observed for the case of neutron passage through the crystal moving with variable speed.

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7. References
[1] Fedorov V V, Voronin V V New possibility of a search for neutron EDM by polarization method in neutron diffraction by a crystal without center of symmetry, in Physics of Atomic Nuclei and Elementary Particles 1996 Proc. of the XXX PNPI Winter School 123
[2] Fedorov V V, Lapin E G, Semenikhin S Y and Voronin V V The effect of cold neutron spin rotation at passage through a noncentrosymmetric crystal 2002 Appl. Phys. A74 s91- s93
[3] Fedorov V V, Jentschel M, Kuznetsov I A, Lapin E G, Lelièvre-Berna E, Nesvizhevsky V, Petoukhov A, Yu Semenikhin S, Soldner T, Voronin V V and Braginetz Yu P Measurement of the neutron electric dipole moment via spin rotation in a non-centrosymmetric crystal 2010 Physics Letters B 694 22-25
[4] Alexeev V L, Fedorov V V, Lapin E G and et al. 1989 Nucl. Instr. and Meth. A 284 181 JETP 69 (1989) 1083.