On phase transition signal in VHM inelastic collisions

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Abstract

The primary intent is to show that the signal of first order phase transition may be observed at experiment if and only if the multiplicity is sufficiently large. We discuss corresponding phenomenology from the point of view of experiment.

I. Introduction: the aim

The thermodynamical approach becomes more and more popular for description of heavy ion inelastic interactions [1]. Generally it based on the assumption that there is equilibrium in a final state and the Plank distribution is useable. This is a formal basis of the approach in frame of which the thermodynamical parameters are introduced [2] and corresponding results are impressive.

Here we will apply the field-theoretical approach which seems more general and, to all appearance, it allows to introduce model-free thermodynamic description. Namely, we will try to derive thermodynamics from usual S-matrix theory [3]. It is evident that there is no direct connection between equilibrium thermodynamics and S-matrix theory and our primary aim is to fined the constrains in frame of which this connection exist.

We will try to describe final state by thermodynamical parameters. The particle distribution, as it is usual in S-matrix, reflects the dynamics of interacting fields. In result we construct the S-matrix interpretation of thermodynamics, or shortly the "S-matrix thermodynamics". This approach is the natural development of Wigner-functions formalism [4] given in interpretation of Carruthers and Zachariasen [5]. Some ideas of Schwinger-Keldysh [6] and Niemy-Semenoff [7] was used deriving S-matrix equilibrium thermodynamics.

For sake of definiteness, we will consider following quantity [3]:

\[ \mu(n, s) \simeq - \frac{T(n, s)}{n} \ln \sigma_n(s) \]  

(1)

to illustrate our approach. It was interpreted as the work which is necessary for a particle production. If this is so then \( \mu(n, s) \) must be sensitive to the first order phase transition. Indeed, if the state is unstable against particle production then \( \mu(n, s) \) must decrease with number of particles. We will discuss this idea latter.

We will consider

— the physical meaning of \( \mu(n, s) \)

and

— how it can be measured.

It must be noted that r.h.s. of definition (1) contains \( \sim (1/n) \) corrections. Therefore only the very high multiplicity (VHM) events will be considered.
It must be noted that in VHM region the dynamical models (Regge, LLA QCD) cannot be used [3] and the thermodynamical description is mostly useful.

It is important that $\mu(n, s)$ is defined by the directly measurable quantities:

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- $n$ is the multiplicity;
- $T(n, s)$ is the mean energy of produced particles;
- $\sigma_n(s)$ is the multiplicity distribution normalized on unite.

Despite the fact that all this quantities are simple and directly measurable we must discuss them in more details. The point is that above definition of $\mu$ have physical, experimental, meaning in the frame of some restrictions.

**II. Multiplicity**

By definition $n$ is the measured multiplicity. This means that it may include:

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- only the observed in given experiment charged particles;
- particles in restricted range of rapidity.

This feature makes the task of measurement of $\mu(n, s)$ real.

But there is difficulty: multiplicity must be sufficiently large. I will explain this restriction somewhat later.

**III. Temperature**

The temperature is first important parameter. In $S$-matrix approach it is introduced as the Lagrange multiplier of energy conservation law. From experimental point of view this means that $T(n, s)$ is the mean energy of produced particles. But considering event-by-event measurement the mean energy is the fluctuating quantity. Therefore, $T$ would have physical meaning if and only if fluctuations of $T$ are Gaussian. It must be underlined that this is the necessary condition since formally the expansion of cross section near $T(n, s)$ have zero convergence radii. In result $T(n, s)$ have physical meaning if and only if the inequalities [3]:

$$\frac{\langle \prod_{i=1}^{l} (\epsilon_i - \langle \epsilon \rangle) \rangle_{n, s}}{\langle \prod_{i=1}^{2l} (\epsilon_i - \langle \epsilon \rangle) \rangle_{n, s}} << 1, \ l = 3, 4, ..., (2)$$

are satisfied. Here $\epsilon_i$ is the energy of $i$-th particle and averaging is performed over all events at given multiplicity $n$ and energy $s$. This condition will be used for definition of VHM region.

The ratio of correlators (1) at $l = 3$ was investigated recently using CDF data and it was shown that it falls down with $n$ for $T \sim 1GeV$. This result will be published soon.

One can say that if (2) is hold than the system is in thermal equilibrium state. This means that particle energy spectra is Boltzmann-like and is defined by one parameter, $T(n, s)$. Such state is necessary to observe phase transition. But it is not clear is this condition sufficient to use the equilibrium thermodynamics formalism? We will see that there is also additional condition.

**IV. Cross section**

Last quantity is the multiple production cross section $\sigma_n(s)$. Introducing temperature, having equilibrium condition one may consider $\sigma_n$ as the $n$-particle partition function. This assumption means that exist the set of condition in frame of which we can set $\sigma_n$ equal to partition function of equilibrium thermodynamics.

It must stressed that only the equilibrium would be considered since this case is mostly simple and in the time may be excluded from the field-theoretical formalism
following to ergodic hypothesis. That requirement leads to the following condition:

\[ T(n, s) \ll m, \]  

(3)

where the produced hadron mass, \( m \), is a natural scale parameter. It must be noted that (3) is the pure theoretical condition since in this case one may apply the low-temperature expansion. In its frame \( \sigma_n \) is defined by the partition function

\[ \rho = Tr[e^{-\beta H(\phi) + \beta j \phi}], \quad \beta = 1/T, \quad H = \int d^3 x h \]  

(4)

where \( h \) is the Hamiltonian density and and \( j \) is the source of external particles. Notice that \( \phi = \phi(x) \) is the time independent field. Low-temperature expansion is considered in [9].

Inequality (3) impose definite limitations for experiment:

— the intermediate incident energy experiments are mostly useful;
— the multiplicity must be sufficiently high;
— the central rapidity region, where the momentum of produced particles is small, must be considered.

We will assume that this requirements can be hold. At the end let us calculate \( \mu(n, s) \) defined in (1).

V. Example: phase space integral

Let us assume that the particle production dynamics is restricted only by the energy-momentum conservation law. It must be noted that if \( n \gg 1 \) then the ration (2) is small in considered model, \( \sim n^{(2-l)/2} \rightarrow 0, \ l = 3, 4, \ldots \) Therefore \( T(n, s) \) have physical meaning.

A. In the case of non-relativistic particles production the phase space is occupied densely and the temperature decease with \( n \):

\[ T(n, s) \simeq \frac{2m(n_{\text{max}} - n)}{3n} \ll O(1/n), \quad n_{\text{max}} = \sqrt{s}/m, \]  

(5)

but chemical potential

\[ \mu(n, s) \simeq \frac{m}{n_{\text{max}} - n} \ln \frac{1}{n_{\text{max}} - n} \]  

(6)

increase unrestrictedly when \( (n_{\text{max}} - n) \rightarrow 0 \). This is consequence of dense distribution of particle in the phase space.

B. Unfortunately I can not give closed expression of the case when momentum of particle \( |k| \sim m \). In the case of \( |k| \gg m \) one can find:

\[ T(n, s) \simeq \sqrt{s}/n \]  

(7)

and

\[ \mu(n, s) \simeq \frac{3}{4} \frac{m}{n} \frac{n_{\text{max}}}{n} \ln \frac{n_{\text{max}}}{2n}, \quad 1 \ll n \ll n_{\text{max}} \]  

(8)

decrease with \( n \). This is a consequence of empty phase space.

Resulting picture given on Fig.1.

VI. Example: Ising model
The interaction can be simply included if the lowest order of low-temperature expansion is considered. Having (4) one may consider various model Hamiltonian in the frame of Ising model [10], when

\[ h = \frac{\pi^2}{2} + (\nabla \varphi)^2/2 + m(T - T_c)\varphi^2/2 + \lambda \varphi^4/4, \lambda > 0, \]  

see for details [9]. It is important that in VHM region one may use the semiclassical approximation.

**A. Stable ground state, \( T > T_c \), Fig.2.**

In that case the temperatures decrease with multiplicity:

\[ T(n, s)/m \sim n^{2/3} \]  

but chemical potential increase with \( n \):

\[ \mu(n, s)/m \sim n^2. \]  

Therefore, the stable vacuum leads to repulsion and production of one additional particle needs additional work. This is natural explanation of (11), Fig.3.

**B. Unstable ground state, \( T < T_c \), Fig.4.**

In this case the temperature tends to its critical value:

\[ T(n, s) \sim T_c(1 - \gamma/n^4) \]  

and chemical potential decrease with \( n \), Fig.5:

\[ \mu(n, s) \sim T_c(1 - \gamma/n^4)/n^5. \]  

This result has evident explanation: the boiling state is unstable against particles evaporation.

Therefore, interaction may drastically change \( n \) dependence of thermodynamical parameters.

**VII. Conclusions**

One may conclude:

— \( \mu(n, s) \) is sensitive to the interaction character: it decrease if the ground state is unstable.

— considering the very high multiplicity events one my hope to investigate collective phenomena in hadron system.

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Figure 1: Pure phase space.

Figure 2: Stable ground state disturbed by $j$.

Figure 3: Stable ground state.
Figure 4: Solid line: undisturbed by $j$ potential and dotted line includes $j$.

Figure 5: Unstable ground state.