Radial consolidation of prefabricated vertical drain-reinforced soft clays under cyclic loading

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\begin{abstract}
Previous studies have revealed that prefabricated vertical drains (PVDs) are effective in stabilizing the soft subgrade soils under traffic-induced cyclic loads. A consolidation model of soft clays under cyclic loading incorporating both radial and vertical drainages, and nonlinear variation of compressibility and permeability is presented in this paper. The prediction of the model matches well with the result from a large-scale cyclic triaxial test conducted on a soil sample installed with a single PVD. Parametric analysis has also been carried out to explore the impacts of cyclic degradation parameter, cyclic stress ratio, nonlinear compressibility and permeability, cyclic loading frequency, drainage condition, and number of loading cycles on the behaviors of the PVD-reinforced soil. Results indicate that radial drainage helps to decelerate the accumulation of excess pore pressures under cyclic loading due to two factors: increased rate of excess pore pressure dissipation and decreased rate of internal excess pore pressure generation. In addition, radial drainage also facilitates the rapid dissipation of excess pore pressures during rest period, leading to a denser soil which is stronger for resisting the subsequent set of cyclic loads. The findings of this study demonstrate the usefulness of PVDs in reinforcing soft subgrades under cyclic loading conditions, e.g., road pavement, railway embankment, etc.
\end{abstract}

\section*{Introduction}
Soft subgrade soils with relatively high fine fraction and water content usually have undesirable geotechnical properties such as low bearing capacity [39–41], high compressibility [42–44], and creep degradation [45–48]. The technique of installing prefabricated vertical drains (PVDs) with surcharge loading is economically attractive for dewatering and compacting the soft soil deposits before infrastructure constructions [1–3]. PVDs are usually installed in the subgrade in either a square or triangular pattern, with 1 to 2 m drain spacing. The hydraulic gradient induced by preloading generates horizontal flows of pore water into PVDs, thereby accelerating the dissipation of excess pore pressures and shortening the consolidation time [4–8,75]. To further optimize the radial consolidation process in terms of consolidation time and lateral deformation, combining PVDs with both surcharge load and vacuum pressure is recommended [9,10]. However, vacuum preloading has also been reported to be responsible for the formation of a clogging zone around the PVD caused by the overall movement of the soil toward the PVD [57,58]; in this clogging zone, permeability anisotropy is likely to be introduced due to the reduced coefficient of horizontal permeability as a result of clay particles being vertically oriented [58]. Several case studies of soil improvement with PVDs in Tianjin Port (China) [11,49,50], Port of Brisbane (Australia) [12], National Soft Soil Testing Facility (Ballina, Australia) [51,52], and Bangkok (Thailand) [53] have been published. As for the theoretical analysis, the large strain analysis has been recently incorporated into the radial consolidation theory, along with other influencing factors like nonlinear variation in compressibility and permeability [10,13], smear zone [14,15], non-Darcian flow [14], well resistance [15,16], vacuum pressure [10,17,18], and creep [15,19].

On the other hand, traffic-induced excess pore pressures can be observed in soft subgrades after the infrastructures (e.g., highways, railway embankments, airport runways and tunnels) built on/in them are open to the public [30,54,56,68,74]. The effectiveness of vertical drains in facilitating the rapid dissipation of excess pore pressures for sand-like soils susceptible to liquefaction was verified by [23], after which PVDs were demonstrated effective in preventing the excess pore pressures from accumulating to critical values for clay-like soils that are susceptible to cyclic softening, by conducting the partially-drained cyclic loading tests using a large-scale cylindrical triaxial equipment [24].

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Recently, a geocomposite-PVD system has been proved to be efficient in preventing subgrade instability and fluidisation under cyclic loading [59]; PVDs help to prevent subgrade fluidisation at shallow depths, while geocomposites allow adequate surficial drainage and effective confinement at the ballast/subgrade interface [59]. Accompanying with the dissipation of excess pore pressures, settlements continue accumulating under traffic loads and during rest periods (i.e., also known as post-cyclic recompression) [20–22,54–56,68–73], as shown in Fig. 1.

The soils with and without radial drainage might have different trends of volumetric strains (Fig. 1). For example, during the cyclic loading, the soil with combined radial and vertical drainages is expected to have a smaller value of excess pore pressure and meanwhile experience a larger volumetric strain in comparison with the soil with vertical drainage only (i.e., $\varepsilon_{\text{v,cyc}(v)} > \varepsilon_{\text{v,cyc}(rv)}$). Supposing that the rest period is sufficiently long for excess pore pressures fully dissipating, the soil with combined drainages will undergo a smaller volumetric strain (i.e., $\varepsilon_{\text{v,cyc}(rv)} < \varepsilon_{\text{v,rest}(v)}$) due to the less accumulation of excess pore pressure at the end of the cyclic loading [69,70]. It is very much likely that the total volumetric strains in these two conditions are not the same, i.e., $\varepsilon_{\text{v,cyc}(rv)} + \varepsilon_{\text{v,rest}(rv)} \neq \varepsilon_{\text{v,cyc}(v)} + \varepsilon_{\text{v,rest}(v)}$. However, the effect of drainage conditions (i.e., with and without radial drainage) on volumetric strains has yet to be investigated.

For modeling PVD-induced radial consolidation under cyclic loading, there are solutions for a PVD unit cell [60,61] and for a triaxial loading condition [25]. In [25], finite difference method is used to solve the partial differential equation that combines the internal excess pore pressure generation predicted by an undrained cyclic model with radial consolidation theory. However, some assumptions (e.g., constant permeability and compressibility) were made and only radial drainage was considered in this model. In addition, the predicted internal excess pore pressure generation due to undrained cyclic loading might be overestimated at the later loading stage. Hence in this paper, a consolidation model of soft clays under cyclic loading considering both radial and vertical drainages is presented. The effect of nonlinear variation of permeability and compressibility is considered and the internal excess pore pressure generation is calculated according to the undrained cyclic model [26]. The proposed model is first verified by the results of a cyclic triaxial test conducted previously. The accumulation of excess pore pressures during partially-drained cyclic loading under the impacts of cyclic degradation parameter, cyclic stress ratio, nonlinear compressibility and permeability, cyclic loading frequency, and number of loading cycles are investigated. Finally, how the drainage condition affects the development of volumetric strains during both cyclic loading and rest period is demonstrated. The effect of principal stress rotation [62–65] on the responses of excess pore pressures and volumetric strains under moving wheel loading is not considered within the scope of this study. Well resistance and smear effect are not incorporated in the proposed model, as an ideal drain is assumed.

Consolidation model for soft clay improved by PVDs under cyclic loading

Consolidation with internal excess pore pressure generation

The internal excess pore pressure generated by cyclic loading has been incorporated into either one-dimensional consolidation [27–29] or radial consolidation [25,30] to evaluate the accumulation of excess pore pressures under partially-drained cyclic loading condition. If there exists both radial and vertical drainages, the following equation can be given accompanied with a schematic procedure (Fig. 2):

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial r} = c_u \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + c_v \left( \frac{\partial^2 u}{\partial z^2} \right)$$

where $u$ is the excess pore pressure, $r$ is the radial distance from the centerline of the soil cylinder, $z$ is the depth from the top permeable surface, $c_u$ is the coefficient of radial consolidation and $c_v$ is the coefficient of vertical consolidation.

The source term $\Delta u_i$ corresponds to the internal excess pore pressure generated by undrained cyclic loading (Fig. 2). For simplicity, an assumption of $\Delta u_i = \Delta u_{\text{uc}}$ was often made previously [30], suggesting that drainage does not affect the internal excess pore pressure generation. However, $\Delta u_i$ generated under the partially-drained condition is affected by the stress history and dissipation of excess pore pressure, and therefore would differ from that obtained from the undrained cyclic loading, i.e., $\Delta u_i \neq \Delta u_{\text{uc}}$. For example (Fig. 2), one soil sample experiences undrained cyclic loading with constant volume (i.e., point $o$ to point $A$ in $u-t$ plane), while the other one experiences partially-drained cyclic loading with a reduced void ratio (i.e., point $o$ to point $E$ in $u-t$ plane). It is recognized that less internal excess pore pressure will be generated due to the reduced void ratio [71–73]. Therefore, if identical cyclic loading cycles are applied afterwards from time $t_i$ to $t_{i+1}$, the internal excess pore pressure generation under partially-drained condition is less than that under undrained condition, i.e., $\Delta u_i < \Delta u_{\text{uc}}$ [25]. Furthermore, if the soil failed at point $C$ under the undrained condition, $\Delta u_i$ would not be available afterwards (i.e., $t > t_i$) for predicting the variation of excess pore pressures under the partially-drained condition (e.g., point $H$). So, in the current study, $\Delta u_{\text{ic}}$ is evaluated based on the initial state of point $E$ rather than point $A$, by using the undrained cyclic model proposed by [26].

![Fig. 1. A schematic diagram for the trends of excess pore pressures and volumetric strains.](image1.png)

![Fig. 2. A procedure for evaluating excess pore pressures under partially-drained cyclic loading.](image2.png)
Internal excess pore pressure generation

Ni et al. [26] proposed an undrained cyclic loading model based on the framework of modified Cam-clay model. To capture the soil behavior unchanged but with size reduced by the elastic unloading:

\[
d_x = \frac{1}{\xi_1 N + \xi_2} \frac{dp}{p_y}
\]

(2)

where \(p_y\) is a hardening parameter which can be considered as a pre-consolidation pressure, \(p_x\) is a variable defined by [31]:

\[
p_x = p + \left( \frac{q}{M} \right)^{\frac{1}{\kappa}}
\]

(3)

where, \(M\) is the slope of the critical state line in \(p' - q\) space; \(p'\) and \(q\) are the effective mean stress and deviator stress defined by the major \(\sigma'_1\) and minor \(\sigma'_2\) principal stresses as \(p' = \frac{1}{2}(\sigma'_1 + 2\sigma'_2)\) and \(q = \sigma'_1 - \sigma'_2\), respectively.

\(\xi_1\) and \(\xi_2\) are cyclic degradation parameters corresponding to how much the yield surface contracts when the soil is elastically unloaded, and therefore determine the amounts of excess pore pressures and strains that are generated for each cycle. \(\xi_1 = 0\), then Eq. (2) can be simplified to that of [32,33].

The calculation of effective stress under undrained cyclic loading is demonstrated against the stress path for normally consolidated soils, as shown in Fig. 3.

**Step 1** For the loading part of the first cycle (from \(A\) to \(A^*\)).

The effective mean stress \(p'\) at any point (with deviator stress \(q\)) between \(A\) and \(A^*\) is given by:

\[
p' = p + \left( \frac{q}{M} \right)^{\frac{1}{\kappa}}
\]

(4)

where \(k\) and \(\kappa\) are the slopes of the normal compression and swelling lines in \(e-\log p'\) space, respectively, where \(e = 1 + e\) is the specific volume and \(e\) is the void ratio. At point \(A\), \(q_A\) is equal to the cyclic stress \(q_{cyc}\) and the corresponding yield stress \(p_{d,1}^\prime\) can be expressed as:

\[
p_{d,1}^\prime = p_A + (q_A/M)^{\frac{1}{\kappa}}/p_A
\]

(5)

**Step 2** For the unloading part of all the cycles (from \(A\) to \(A^*\), from \(B\) to \(B^*\), from \(C\) to \(C^*\), and from \(D\) to \(D^*\)).

Take for example, the first unloading part from \(A\) to \(A^*\). The effective mean stress remains constant while the deviator stress decreases. At point \(A^*\), the yield stress after unloading or the yield stress for the second cycle can be calculated:

\[
p_{d,1}^\prime = \frac{p_{d,1}^\prime (p_{d,1}^\prime / p_{d,1})^{\frac{1}{\kappa}}}{p_{d,1}^\prime}
\]

(6)

where \(p_{d,1}^\prime\) is the loading parameter corresponding to point \(A^*\).

**Step 3** For the reloading part of the following cycles (from \(A^*\) through \(B^*\) to \(B\), from \(B^*\) through \(C^*\) to \(C\), from \(C^*\) through \(D^*\) to \(D\)).

Take for example, the reloading part from \(A^*\) through \(B^*\) to \(B\). when the stress path moves from point \(A^*\) to \(B\), the soil behaves elastically for \(q < q_{cyc}\). The deviator stress \(q_{yielding}\) causing the re-yielding of the soil can be given by:

\[
q_{yielding} = \sqrt{(p_{d,1}^\prime - p_{d,1}^\prime)M^2p_{d,1}^\prime}
\]

(7)

Afterwards, the stress path moves from point \(B^*\) to \(B\) for \(q_{yielding} < q < q_{cyc}\) and the effective mean stress can be calculated as described in step 1. Once \(p\) is known for each load increment in the above-mentioned steps, \(\Delta u_0\) can be obtained by using the principle of effective stress.

**Computational procedure**

The procedure of obtaining \(\Delta u_0\) is the same as that of obtaining \(\Delta u_0\), except that \(\Delta u_0\) is predicted based on the initial state of point \(E\) rather than point \(A\) (Fig. 2). During the time interval \(dt = \epsilon_1 + \epsilon_0\), by assuming that the process of the internal excess pore pressure generation due to undrained cyclic loading and excess pore pressure dissipation traces along point \(E\) → point \(F\) → point \(G\) (Fig. 2), the accumulation of excess pore pressures during partially-drained cyclic loading can be calculated according to Fig. 4. Due to the excess pore pressure dissipation, the stress path illustrated in Fig. 3 changes accordingly, as shown in Fig. 5.

During the partially-drained cyclic loading, the pore water flows out of the soil and the pore volume decreases. To consider the effect of decreasing pore volume on soil permeability, the following empirical equations are used to link the permeability to the void ratio:

\[
e = e_0 - C_{kh} \log \left( \frac{k_{hv}}{k_{h}} \right)
\]

(8)

\[
e = e_0 - C_{kv} \log \left( \frac{k_{h}}{k_{v}} \right)
\]

(9)

where \(e_0\) is the initial void ratio, \(k_{hv}\) and \(k_{hv}\) are the initial coefficients of horizontal permeability and vertical permeability respectively, and \(C_{kh}\) and \(C_{kv}\) are the change indexes of horizontal permeability and vertical permeability respectively.

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![Fig. 3. Stress path in undrained cyclic loading (modified after [26]).](image-url)
The compressibility of the soil also changes during the consolidation process. Previous studies on post-cyclic recompression of clays suggested using Eq. (10) to correlate the volumetric strain $\varepsilon_v$ due to post-cyclic recompression with the magnitude of excess pore pressure $u_r$, which accumulates during the previous cyclic loading [30,66,67,69,70]:

$$\varepsilon_v = \frac{\alpha_c C_s}{1 + \varepsilon_0} \log \left( \frac{p'_c}{p'_c - u_r} \right)$$

(10)

where $\alpha_c$ is an experimental parameter and $C_s$ is the swelling index. According to [30,69,70], the value of $\alpha_c$ is between 1 and the ratio of $C_c$ to $C_s$.

For partially-drained cyclic loading, the following Eq. (11) may be given by replacing $u_r$ in Eq. (10) with $u$ in a partially-drained condition [30]:

$$e = \varepsilon_0 - \alpha_c C_s \log \left( \frac{p'_c}{p'_c - u} \right)$$

(11)

The coefficient of volumetric compressibility can be derived from Eq. (11):

$$m_v = 0.434 \alpha_c C_s \left( 1 + \varepsilon_0 \right) \left( \frac{p'_c}{p'_c - u} \right)$$

(12)

Eq. (12) can be used to obtain the value of $m_v$ for predicting the volumetric strain either during cyclic loading or rest period. The coefficients of radial and vertical consolidation can be given by the definition $c_h = k_h / (m_v \gamma_w)$ and $c_v = k_v / (m_v \gamma_w)$, respectively.

Model verification

The proposed model is verified by the result obtained from a partially-drained cyclic triaxial test [34]. The large-scale cylindrical dynamic triaxial equipment which was designed and built at the University of Wollongong was used. This apparatus is capable of accommodating 300 mm diameter by 600 mm high soil samples and utilizes a hydraulic type dynamic actuator to apply cyclic loads. A single PVD (cross-sectional dimension: 20 mm $\times$ 5 mm) was driven into the clay sample along the centerline using a rectangular mandrel. Care was taken during the installation of the PVD and the removal of the mandrel, to avoid any excessive disturbance of the soil. Water was then poured through the PVD to prevent drain unsaturation. Only radial drainage was available during the test. An excess pore pressure transducer was also installed in the soil sample at the depth of 150 mm from the top surface and 60 mm from the centerline. In the test, the soil sample was subjected to the cyclic loading with $N = 15,000$ cycles, $f = 1$ Hz, and CSR = 0.6 (i.e., CSR is defined as the ratio of cyclic deviator stress $q_{cyc}$ to the static undrained shear strength), followed by a rest period of 48 h. Fig. 6 presents the variation of excess pore pressure recorded by transducer T2 during the test. The result of an undrained cyclic loading test with the same CSR and $f$ is also given in Fig. 6 for comparison. Without
Table 1

| Parameter | Set 1 (Fig. 6) | Set 2 (Figs. 7 and 8) | Set 3 (Fig. 9) | Set 4 (Figs. 10 and 11) |
|-----------|----------------|-----------------------|----------------|------------------------|
| \(\lambda(C_v)\) | 0.17 (0.39) | 0.25 (0.59) | 0.25 (0.59) | 0.25 (0.59) |
| \(\alpha(C_v)\) | 0.04 (0.09) | 0.05 (0.12) | 0.05 (0.12) | 0.05 (0.12) |
| \(\eta\) | 1.87 | 1.2 | 1.2 | 1.2 |
| \(\eta_0\) | 30 | 30 | 30 | 30 |
| \(\eta_f\) | 30 | 30 | 30 | 30 |
| \(\xi_1\) | 1.32 | 1.0 | 1.0 | 1.0 |
| \(\xi_2\) | 280 | – | 40 | 150 |
| \(\eta_0\) | 0.15 | 0.15 | 0.15 | 0.15 |
| \(\eta_1\) | 0.01 | 0.01 | 0.01 | 0.01 |
| \(\eta_2\) | 0.6 | 0.6 | 0.6 | 0.6 |
| \(k_{00}\) (m/s) | \(3.15 \times 10^{-8}\) | \(3 \times 10^{-8}\) | \(3 \times 10^{-8}\) | \(3 \times 10^{-8}\) |
| \(k_{01}\) (m/s) | \(-\) | \(1 \times 10^{-7}\) | \(1 \times 10^{-7}\) | \(1 \times 10^{-7}\) |
| \(\varepsilon_0\) | 0.57 | 0.75 | 0.75 | 0.75 |
| \(\varepsilon_1\) | 0.75 | 0.75 | 0.75 | 0.75 |
| \(\alpha\) | 2.2 | 1.25 | 1.25, 2.5, 3.75, 5, 6.25 |

The parameters used for the model prediction.

The parameters needed for the following analysis are given in Table 1 (i.e., parameter sets 2, 3, and 4). The values of excess pore pressures and volumetric strains shown in this section are corresponding to zero deviator stress (i.e., the lowest \(u/p_{0-0}\) in each cycle).

Cyclic degradation parameters

Cyclic degradation parameters affect the reduction in size of the yield surface after each elastic unloading and therefore determine the internal excess pore pressure generation during the following reloading stage. Larger values of \(\xi_1\) and \(\xi_2\) usually mean a smaller contraction of the yield surface during unloading, leading to a smaller expansion of the yield surface caused by reloading. Fig. 7 (a) shows the effect of \(\xi_2\) on the normalized excess pore pressure \(u/p_{0-0}\). As \(\xi_2\) increases, \(u/p_{0-0}\) decreases accordingly. For \(\xi_2 = 30\) and 35, the value of \(u/p_{0-0}\) reaches a critical value around 0.75, which triggers the failure of the soil. For \(\xi_2 = 45\) and 50, peak values of \(u/p_{0-0}\) are observed around 400 and 600 cycles respectively, indicating that the dissipation rate of excess pore pressure gradually exceeds the generation rate of internal excess pore pressure with the increasing number of loading cycles. Fig. 7 (b) presents the effect of \(\xi_1\) on \(u/p_{0-0}\). It is indicated that \(u/p_{0-0}\) is very sensitive to the value of \(\xi_1\). This is because the importance of the product of \(\xi_1\) and \(N\) increases as \(N\) increases based on Eq. (2). For \(\xi_1 = 0.12\) and 0.15, the failure of the soil can be avoided.

Cyclic stress ratio and cyclic loading frequency

Fig. 8 presents the effect of cyclic loading frequency on the accumulation of excess pore pressures during partially-drained cyclic loading. Three CSR values (i.e., 0.5, 0.55, and 0.8) are used for the prediction as they cover the range of critical CSR from 0.5 to 0.8 [35–38]. For CSR = 0.8 (Fig. 8 (a)), there is no or marginal influence of cyclic loading frequency on \(u/p_{0-0}\) for \(f\) ranging from 0.05 to 1 Hz. This phenomenon implies that the generation rate of internal excess pore pressure is much larger than the dissipation rate of excess pore pressure, resulting in a rapid failure of the soil within 5 cycles. For CSR = 0.55 (Fig. 8 (b)), the curves corresponding to different cyclic loading frequencies start to diverge. The soil loaded at a lower \(f\) has more time for excess pore pressure dissipation within each cycle and therefore has a lower \(u/p_{0-0}\) for a given number of loading cycles. Although failure occurs for all the cyclic loading frequencies, the smaller the \(f\), the more loading cycles the soil is able to sustain. As CSR decreases to 0.5 (Fig. 8 (c)), the divergence between different curves becomes more marked. At this CSR, the internal excess pore pressure generation within each cycle might be comparable to the excess pore pressure dissipation and therefore lowering the loading frequency can even prevent the failure of the soil (i.e., \(f = 0.05\) Hz).

Permeability and compressibility

Fig. 9 presents the effect of nonlinear variation of permeability and compressibility on the prediction of the proposed model with large-scale cyclic triaxial test result for comparison.

radial drainage, the excess pore pressure increased rapidly to \(u/p_{0-0} = 0.45\) at the end of 34,466 cycles. By contrast, with radial drainage, the excess pore pressure at T2 slowly increased to \(u/p_{0-0} = 0.27\) after 15,000 cycles, which was reduced by approximately 30 % compared with the undrained condition at the same number of loading cycles. The proposed model is used for prediction with the soil parameters given in Table 1 (i.e., parameter sets 2, 3, and 4). The values of excess pore pressures shown in this section are corresponding to zero deviator stress (i.e., the lowest \(u/p_{0-0}\) in each cycle).

\[
\begin{align*}
\frac{u}{p_{0-0}} & \approx 0.45 \\
\frac{u}{p_{0-0}} & \approx 0.27
\end{align*}
\]
compressibility on \( \frac{u}{p'}_{c0} \) at three cyclic loading frequencies (i.e., \( f = 0.05, 0.1, \) and 1 Hz). By assuming \( C_s = 0.12, C_k = 0.75, \) and \( \alpha_c = 1.25, \) five values of \( \alpha_c C_s/C_k \) ranging from 0.2 to 1.0 at an interval of 0.2 are used for the prediction. For \( f = 1 \) Hz (Fig. 9 (a)), the excess pore pressures for soils with different values of \( \alpha_c C_s/C_k \) are almost identical until the critical \( \frac{u}{p'}_{c0} \) value of 0.75 is quickly reached. As \( f \) decreases to 0.1 Hz (Fig. 9 (b)), the influence of \( \alpha_c C_s/C_k \) on \( \frac{u}{p'}_{c0} \) emerges from 400 cycles. A higher value of \( \alpha_c C_s/C_k \) results in a smaller coefficient of consolidation and hence a higher accumulation rate of excess pore pressure, which may bring the soil to failure (e.g., \( \alpha_c C_s/C_k = 1.0). \) When \( f \) further decreases to 0.05 Hz (Fig. 9 (c)), curves of \( \frac{u}{p'}_{c0} \) against \( N \) for different \( \alpha_c C_s/C_k \) values begin to diverge from an even earlier stage, i.e., 200 cycles. Similarly, a higher value of \( \alpha_c C_s/C_k \) leads to a higher \( \frac{u}{p'}_{c0} \). In addition, the peak values of \( \frac{u}{p'}_{c0} \) and following descending trends of \( \frac{u}{p'}_{c0} \) can be observed due to the prolonged loading period for each cycle which allows sufficient time for excess pore pressure dissipation.

**Drainage condition**

To investigate the effectiveness of radial drainage in improving soil performances under cyclic loads and during rest period, the excess pore pressures and volumetric strains of the soils with and without PVD reinforcement are compared. Fig. 10 shows the comparison under the impact of different CSR values. At the cyclic loading stage, the soils with combined radial and vertical drainages have lower \( \frac{u}{p'}_{c0} \) values than those with vertical drainage only at five CSR values (Fig. 10 (a)). As CSR increases, the difference in \( \frac{u}{p'}_{c0} \) between the soils with different drainage conditions increases. During the rest period, the radial drainage greatly accelerates the dissipation of excess pore pressures that accumulate during the previous cyclic loading. Fig. 10 (b) shows the comparison of volumetric strains occurring during cyclic loading and rest period. During the cyclic loading, the soils with radial drainage have slightly larger volumetric strains compared with those without radial drainage. During the rest period, due to the radial drainage, the time needed for completing the post-cyclic recompression is remarkably reduced. Before the soils with combined drainages finish volumetric change, they always have larger volumetric strains in comparison with those having vertical drainage only at a given time (e.g., \( \varepsilon_v-t \) curves in the grey rectangle). In case the excess pore pressures of the soils with vertical drainage only are allowed to fully dissipate, the corresponding volumetric strains will exceed those of the soils with combined drainages. As the volumetric strains occur accompanying the dissipation of

Fig. 8. The effect of \( f \) on \( \frac{u}{p'}_{c0} \): (a) CSR = 0.8; (b) CSR = 0.55; (c) CSR = 0.5.

Fig. 9. The effect of \( \alpha_c C_s/C_k \) on \( \frac{u}{p'}_{c0} \): (a) \( f = 1 \) Hz; (b) \( f = 0.1 \) Hz; (c) \( f = 0.05 \) Hz.

Fig. 10. The effect of drainage condition on \( \frac{u}{p'}_{c0} \) and \( \varepsilon_v \) at different values of CSR: (a) \( \frac{u}{p'}_{c0} \); (b) \( \varepsilon_v \).

Fig. 10. The effect of drainage condition on \( \frac{u}{p'}_{c0} \) and \( \varepsilon_v \) at different values of CSR: (a) \( \frac{u}{p'}_{c0} \); (b) \( \varepsilon_v \).
induced internal excess pore pressure generation and therefore reflect the amount of internal excess pore pressure generation, it is implied that the total cyclic loading-induced internal excess pore pressure generation $\sum \Delta u_p$ can be reduced by the introduction of radial drainage.

**Number of loading cycles**

To investigate how the number of loading cycles influences the soil performances, the normalized excess pore pressures $u/p_o$ and volumetric strains $\varepsilon_v$ with loading cycles $N=200, 400, 600, 800, \text{and } 1000$, respectively are given in Fig. 11. At the cyclic loading stage, the excess pore pressures of the soils without radial drainage increase rapidly as the number of loading cycles increases (Fig. 11 (a)). By contrast, the excess pore pressures of the soils with combined drainages increase at a lower rate. Fig. 11 (b) again confirms that radial drainage has the potential of reducing the total cyclic loading-induced internal excess pore pressure generation. The differences in $u/p_o$ and $\varepsilon_v$ between the soils with different drainage conditions increase as the number of loading cycles increases, indicating that the effect of radial drainage on stabilizing soils becomes gradually evident with time.

**Discussion**

The level of cyclic stress encountered in the subgrade under railway embankment is normally low to medium. With the involvement of radial drainage, the excess pore pressures can be prevented from accumulating to the critical value, thereby saving the soil from failure (Fig. 6). It is also noted that the build-up of excess pore pressures is affected by cyclic degradation parameter, cyclic stress ratio, nonlinear compressibility and permeability, cyclic loading frequency, and number of loading cycles (Figs. 7 to 11).

The excess pore pressure accumulation during partially-drained cyclic loading is in essence a problem of consolidation with the internal excess pore pressure generated by cyclic loading (Fig. 2), both of which can be affected by the drainage condition. It is self-evident that radial drainage is able to accelerate the consolidation process, i.e., radial drainage path can be ten times shorter than the vertical drainage path in a clay stratum. On the other hand, the accelerated dissipation of excess pore pressures (i.e., $(\Delta u_p + \Delta u)$ in Fig. 2) results in more strength gain of the soil, and then reduced shear strain and therefore reduced internal excess pore pressure increment generated by the subsequent cyclic loads, i.e., $\Delta u_p$ in each sub-divided loading period decreases due to radial drainage. This effect can be reflected in the volumetric strains that accompany the dissipation of excess pore pressures; the volumetric strains caused by excess pore pressures fully dissipation as shown in Figs. 10 and 11 suggest that the total cyclic loading-induced generation of excess pore pressure $\sum \Delta u_p$ can be reduced by the involvement of radial drainage. As a result, the accumulation of excess pore pressures during cyclic loading, due to the increased rate of dissipation and decreased rate of generation, is therefore reduced for the soil with combined radial and vertical drainages (Figs. 10 and 11).

It should be noted that in case cyclic loading-generated excess pore pressures fully dissipate and more volumetric strains occur in the rest period, the soils will become stronger for resisting the subsequent set of cyclic loads (e.g., the soils with vertical drainage only in Figs. 10 and 11). However, for intermittent traffic loads, the rest period between the two sets of cyclic loads is normally not sufficient for completing the process of excess pore pressure dissipation. Take one-day traffic for example, it can be assumed that one day consists of a continuous cyclic loading period and a following rest period for simplicity [30], as shown in Fig. 10 (b). It can be seen that the rapid dissipation of excess pore pressures promoted by radial drainage produces denser soils which are expected to have more desirable mechanical performances under the subsequent set of cyclic loads.
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