An enhanced nonparametric EWMA sign control chart using sequential mechanism

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Abstract

Control charts play a significant role to monitor the performance of a process. Nonparametric control charts are helpful when the probability model of the process output is not known. In such cases, the sampling mechanism becomes very important for picking a suitable sample for process monitoring. This study proposes a nonparametric arcsine exponentially weighted moving average sign chart by using an efficient scheme, namely, sequential sampling scheme. The proposal intends to enhance the detection ability of the arcsine exponentially weighted moving average sign chart, particularly for the detection of small shifts. The performance of the proposal is assessed, and compared with its counterparts, by using some popular run length properties including average, median and standard deviation run lengths. The proposed chart shows efficient shift detection ability as compared to the other charts, considered in this study. A real-life application based on the smartphone accelerometer data-set, for the implementation of the proposed scheme, is also presented.

1. Introduction

Statistical process control (SPC) is a collection of tools for the monitoring of process parameters. The most valuable of these tools is control chart (cf. Montgomery [1]). Shewhart, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts (cf. [2–4]) are the commonly used control chart structures to monitor the parameters of the process. The simplicity and ease of interpretation make Shewhart charts more common in use, but they are relatively insensitive to small shifts in process parameters, whereas, CUSUM and EWMA control charts are mostly used for the detection of smaller shifts in process parameters (cf. [1]).

In parametric control charts, the parent distribution of the process production is usually known and commonly assumed to be a normal. If the distribution of the process production is unknown, the traditional control limits no longer remain effective and the detection ability of parametric control charts can be negatively affected. This leads us to the development of control charts that are not specifically designed under the assumption of normality or any other parametric distribution. In SPC literature, the nonparametric control charts are widely employed and have numerous advantages for the monitoring of real processes (cf. Chakraborti...
et al. [5]). For recent literature on nonparametric charts, the interested readers may go through the contributions by [6–14].

In SPC literature, various sampling techniques are used to improve the performance of the parametric and nonparametric control charts. Of these, simple random sampling (SRS), (cf. Montgomery [1]), double sampling (DS) (cf. Croasdale [15]), ranked set sampling (RSS) and its different forms (cf. [16–17]), repetitive sampling (RS) (cf. [18–19]) and variable sampling interval (VSI) (cf. [20–21]) are famous ones. Balamurali and Jun [22] showed that the RS scheme is more efficient than single and double sampling schemes but it is not better than the sequential sampling (SS) scheme. The SS was introduced by Wald [23] as a tool for more effective industrial quality control during second world war. The SS is a sampling plan in which an undetermined number of samples are tested one by one, accumulating the results, until a decision can be made. In SS the sample size i.e., \( n \) is not fixed in advanced. Balamurali and Jun [22] mentioned that the SS is more efficient as compared to DS procedure. The SS and RS schemes are quite similar to each other. Both sampling schemes have a similar pair of limits and decision criteria is same for both designs. The only difference exists between these two designs when a sample falls in the no-decision interval. In RS, the sampler discards the sample that falls in the no-decision interval and the resampling will continue until a decision is reached. On the other hand, in SS, the sampler doesn’t ignore the sample that falls in the no-decision interval, the sampler draws a new sample and update the information with previous sample, until sample statistic falls in either of the decisive zones.

By exploring the literature, we found that no study as of yet, utilizes the SS scheme for increasing the efficiency of the nonparametric control charts. To fill this gap, we propose a nonparametric EWMA sign chart, based on arcsine transformation, using the SS scheme, for efficient monitoring of process location. The rest of the article is as follows: the description of the existing and proposed charts is presented in Section 2. The performance comparisons are provided in Section 3. A real application of the proposed chart is given in Section 4. Finally, the summary and conclusions are provided in Section 5.

2. Description of nonparametric control charts

In this section, we provide a brief description of some useful non-parametric charts such as: the nonparametric EWMA sign (EWMA-Sign), the arcsine EWMA (AEWMA-Sign) charts, proposed by Yang et al. [24], and the nonparametric CUSUM sign (CUSUM-Sign) chart proposed by Yang and Cheng [25].

2.1. EWMA-Sign chart

Let \( X \) be the variable of interest with mean value \( \theta \) and \( T = X - \theta \) defines the respective deviations from its mean value. Let \( p \) denote the proportion of positive deviations i.e., \( p = P(T > 0) \). For in-control process, \( p = 0.5 \) and for out-of-control process, \( p = p_1 \neq 0.5 \). The sign test statistic is written as:

\[
T^+ = \sum_{j=1}^{n} I(X_j - \theta > 0),
\]

where \( I(.) \) is given as:

\[
I(X_j - \theta > 0) = \begin{cases} 
1, & \text{if } T^+ = (X_j - \theta > 0) \\
0, & \text{otherwise}
\end{cases}.
\]

where \( j = 1, 2, \ldots, n \).
Koti and Babu [26] showed that $T^*$ follows the binomial distribution with parameters $n$ and $p$. Moreover, $E(T^*) = n/2$ and $Var(T^*) = n/4$, respectively. The EWMA statistic based on (1) is written as:

$$EWMA_{T_i} = \lambda T_i + (1 - \lambda)EWMA_{T_{i-1}}$$

(2)

where $\lambda$ is the smoothing parameter ranging from 0 to 1.

Yang et al. [24] proposed the EWMA-Sign chart to monitor the process target. The mean and variance of the EWMA statistic in (2) are respectively given as (Abbasi [27] and Yang et al. [24]):

$$E(\text{EWMA}_{T_i}) = n/2 \text{ and } Var(\text{EWMA}_{T_i}) = \frac{\lambda}{2 - \lambda} \left( \frac{n}{4} \right).$$

The asymptotic control limits of Yang et al. [24] chart are

$$UCL_{\text{EWMA}_{T_i}} = \frac{n}{2} + L\sqrt{\frac{\lambda}{2 - \lambda} \left( \frac{n}{4} \right)},$$

$$CL_{\text{EWMA}_{T_i}} = \frac{n}{2},$$

$$LCL_{\text{EWMA}_{T_i}} = \frac{n}{2} - L\sqrt{\frac{\lambda}{2 - \lambda} \left( \frac{n}{4} \right)},$$

where $L$ is the width of the control limits.

### 2.2. AEWMA-Sign chart

Yang et al. [24] observed that due to the asymmetric behavior of the binomial distribution for small to moderate sample size $n$, the in-control average run length ($\text{ARL}_0$) values of the EWMA sign chart are not equal to the usually known value of 370 when $p = 0.5$. So to overcome this deficiency, Yang et al. [24] applied the arcsine transformation i.e., $T = \sin^{-1}(\sqrt{p})$. The distribution of $T$ under the arcsine transformation follows the normal distribution with mean $\sin^{-1}(\sqrt{p})$ and variance $\left( \frac{1}{4n} \right)$. The EWMA statistic based on the arcsine transformation is defined as:

$$EWMA_{T_i} = \lambda T_i + (1 - \lambda)EWMA_{T_{i-1}}$$

(4)

The starting value of $EWMA_{T_i}$ is set as the mean value of $T$ as $EWMA_{T_0} = \sin^{-1}(\sqrt{0.5})$. The mean and variance of the $EWMA_{T_i}$ are $E(\text{EWMA}_{T_i}) = \sin^{-1}(\sqrt{0.5})$ and $Var(\text{EWMA}_{T_i}) = \frac{\lambda}{2 - \lambda} \left( \frac{1}{4n} \right)$, respectively (cf. Yang et al. [24]).

So, the control limits of the arcsine EWMA sign chart are:

$$UCL_{\text{EWMA}_{T_i}} = \sin^{-1}(\sqrt{0.5}) + L\sqrt{\frac{\lambda}{2 - \lambda} \left( \frac{1}{4n} \right)},$$

$$CL_{\text{EWMA}_{T_i}} = \sin^{-1}(\sqrt{0.5}),$$

$$LCL_{\text{EWMA}_{T_i}} = \sin^{-1}(\sqrt{0.5}) - L\sqrt{\frac{\lambda}{2 - \lambda} \left( \frac{1}{4n} \right)},$$

(5)
where \( p = 0.5 \) represents the in-control state of the process. If any \( \text{EWMA}_T \geq \text{UCL}_{\text{EWMA}} \) or \( \text{EWMA}_T \leq \text{LCL}_{\text{EWMA}} \), the process is considered to be out-of-control. The AEWMA-Sign chart shows slightly better shift detection ability as compared to the EWMA-Sign chart (cf. Yang et al. [24]).

### 2.3. CUSUM-Sign chart

Using the statistic given in (1), Yang and Cheng [25] developed the two plotting statistic i.e., \( C^+_t \) and \( C^-_t \) of the CUSUM sign chart as follows:

\[
C^+_t = \max(0, C^+_{t-1} + T^+_t - (np_0 + k)) \\
C^-_t = \min(0, C^-_{t-1} - (np_0 - k) + T^-_t)
\]

where \( t = 1, 2, \ldots \) and initially, \( C^+_1 = 0 \) and \( C^-_1 = 0 \). The statistics given in (6) are plotted against their control limits \( h \) and \( -h \), respectively. The process is considered to be out-of-control if \( C^+_t \geq h \) or \( C^-_t \leq -h \), else, it is in-control. For \( k = 0.5, h = 10.65 \) and \( n = 10 \), the \( ARL_0 \) of the CUSUM–Sign chart is 370.

### 2.4. Proposed arcsine EWMA sign chart

In this section, we combine the idea of SS scheme with the non-parametric arcsine EWMA sign chart, namely the SAEWMA-Sign chart. The SS scheme is more economical and time-saving in comparison to the RS and DS schemes. In SS scheme undetermined number of samples are tested one by one, adding the results until a decision can be made. The construction of the SAEWMA-Sign chart is based on the following two steps:

**Step I:** A sample of size \( n \) is selected for the computation of the EWMA statistics, using the expression given in (4).

**Step II:** The SAEWMA-Sign chart has two pairs of control limits which consist of two upper control limits i.e., \( \text{UCL}_1 \) and \( \text{UCL}_2 \) and two lower control limits i.e., \( \text{LCL}_1 \) and \( \text{LCL}_2 \). The four control limits of the proposed chart, based on SS scheme are given as follows (cf. Aslam et al. [19]):

\[
\begin{align*}
\text{UCL}_1 &= \sin^{-1}(\sqrt{0.5}) + L_1 \sqrt{\frac{\hat{\lambda}}{2 - \hat{\lambda}} \left( \frac{1}{4n} \right)} \\
\text{LCL}_1 &= \sin^{-1}(\sqrt{0.5}) - L_1 \sqrt{\frac{\hat{\lambda}}{2 - \hat{\lambda}} \left( \frac{1}{4n} \right)} \\
\text{UCL}_2 &= \sin^{-1}(\sqrt{0.5}) + L_2 \sqrt{\frac{\hat{\lambda}}{2 - \hat{\lambda}} \left( \frac{1}{4n} \right)} \\
\text{UCL}_2 &= \sin^{-1}(\sqrt{0.5}) - L_2 \sqrt{\frac{\hat{\lambda}}{2 - \hat{\lambda}} \left( \frac{1}{4n} \right)}
\end{align*}
\]

In (7), \( L_1 \) and \( L_2 \) (\( L_1 \geq L_2 \)) are the two control limits coefficients to be determined. The decision criteria of SAEWMA-Sign chart is outlined as:

i. the process is stated as out-of-control if \( \text{EWMA}_{\hat{\lambda}_i} \geq \text{UCL}_i \) or \( \text{EWMA}_{\hat{\lambda}_i} \leq \text{LCL}_i \);  

ii. if \( \text{LCL}_2 \leq \text{EWMA}_{\hat{\lambda}_i} \leq \text{UCL}_2 \) the process is declared to be in-control;
iii. if \( LCL_1 \leq EWMA_T i \leq LCL_2 \) or \( UCL_2 \leq EWMA_T i \leq UCL_1 \) then continue sampling and go to step I (cf. Fig 1).

**Special Case**: If \( L_1 = L_2 \), then the proposed scheme is similar to the AEWMA-Sign chart under the SRS scheme. So, the proposed chart is a special case of the chart proposed by Yang et al. [24].

### 3. Performance assessment

There are a variety of measures that can be used to evaluate the performance of control charts. Some of the important measures, used in this study are:

**Average run length (ARL)** is broadly used by the researchers to assess the performance of control charts. The in-control and out-of-control ARLs are denoted by \( ARL_0 \) and \( ARL_1 \), respectively. Some researchers recommend the use of standard deviation run length (SDRL) and median run length (MDRL), due to the skewed behavior of the run length (RL) distribution.

The ARL, MDRL and SDRL are defined as:

\[
ARL = \frac{\sum_m (RL)_m}{m},
\]

\[
MDRL = \text{Median}(RL),
\]

\[
SDRL = \sqrt{E(\text{RL})^2 - (E(\text{RL}))^2}.
\]
We have adopted Monte Carlo (MC) simulations based on $5 \times 10^4$ iterations to find the results. The advantages of MC simulation over the other methods can be seen in Dyer [28].

The computational algorithms for the computation of different run length measures is described below:

i. Generate a random sample of size $n$ from the binomial distribution, having parameters $n$ and $p = p_0 = 0.5$, call it $T_i$.

ii. Compute the $EWMA_{T_i}$ statistics using the expression given in (4).

iii. For a fixed level of $\lambda$, select values for $L_1$ and $L_2$ for the computation of control limits in (6), for a pre-specified $ARL_0$.

iv. The sample number at which the plotting statistic falls outside the $UCL_1$ or $LCL_1$ is called a run length. If $LCL_1 \leq EWMA_{T_i} \leq LCL_2$ or $UCL_2 \leq EWMA_{T_i} \leq UCL_1$, we continue resampling and repeat steps (i)–(iii) unless the plotting statistics falls in either of the decisive zones.

v. Repeat steps (i)-(v) $5 \times 10^4$ times to compute the in-control $ARL$ as the mean of these run lengths.

For the out-of-control $ARL$, shifts are introduced by generating random observations from Binomial distribution using parameters $n$ and $p = p_1 \neq 0.5$. To evaluate the performance of the proposed chart, we chose various combination of $L_1$, $L_2$, $n$ and $\lambda$, to achieve a pre-specified $ARL_0$. It is to be mentioned that the design parameter $L_2$ is obtained by using the formula $L_2 = L - \varphi \cdot L$ where $L$ is defined earlier in Section 2 and $\varphi$ helps in defining the non-decisive zone.

For our study purpose, we used $\lambda = 0.05$ and 0.25 for the proposed chart and found the control chart multipliers $L_1$ and $L_2$ for fixing $ARL_0 = 370$. Moreover, we have used $\varphi = 0.02(0.02)0.1$ in this study.

For these design parameters, we have obtained the run length properties of the proposed chart such as $ARL$, $MDRL$ and $SDRL$. These results are provided in Table 1. From Table 1, we advocate the following interesting points:

i. The $ARL_0$ values are close to the desired value of 370 when the value of $p = 0.5$ (for example for $\lambda = 0.05, L_1 = 2.665, L_2 = 2.619, \varphi = 0.02, ARL_0 = 369$ and for $\lambda = 0.25, L_1 = 3.271, L_2 = 3.155, \varphi = 0.02, ARL_0 = 370$).

ii. It is noted that the efficiency of the proposed chart to detect small shifts in the process location, increases as the value of $\lambda$ decreases (for example for $\lambda = 0.25, L_1 = 3.362, L_2 = 3.09, \varphi = 0.04, p_1 = 0.51, ARL_1 = 303$ and for $\lambda = 0.05, L_1 = 2.667, L_2 = 2.565, \varphi = 0.04, p_1 = 0.51, ARL_1 = 257$).

iii. It is observed that the shift detection ability of the proposed scheme increases as the value of $\varphi$ increase (for example for $\lambda = 0.05, L_1 = 2.665, L_2 = 2.619, p_1 = 0.51, \varphi = 0.02, ARL_1 = 272$ and for $\lambda = 0.05, L_1 = 2.665, L_2 = 2.619, p_1 = 0.51, \varphi = 0.08, ARL_1 = 228$).

iv. The values of $MDRL$ and $SDRL$ decreases as the value of $\varphi$ increases (for example for $\lambda = 0.05, p_1 = 0.52, \varphi = 0.02, MDRL = 120, SDRL = 150$, and for $\lambda = 0.05, p_1 = 0.52, \varphi = 0.1, MDRL = 93, SDRL = 112$).

v. The $MDRL$ and $SDRL$ also decreases with an increase in the level of $p_1$, considering fixed $\lambda$ and $\varphi$. 
To get more insight of the run length distribution for the proposed chart, we also computed the run length properties at varying levels of \( n \) and \( \lambda \). As the value of the \( n \) increases, the detection ability of the proposed chart increases. For example, for \( n = 10, p_1 = 0.55, ARL_1 = 42 \) and \( n = 15, p_1 = 0.55, ARL_1 = 31 \) (cf. Table 2 and Fig 2). On the other hand, as the value of \( \lambda \) increases the shift detection ability of the proposed chart decreases. For example, for \( \lambda = 0.05, p_1 = 0.6, ARL_1 = 17 \) and \( \lambda = 0.5, p_1 = 0.6, ARL_1 = 35 \) (cf. Table 3 and Fig 3).

3.1. Comparative analysis

In this section, we present a comparison of the proposed scheme with the EWMA-Sign chart, the AEWMA-Sign chart and the CUSUM-Sign chart. To make valid comparisons with existing counterparts, \( ARL_0 \) of all selected charts is fixed at a pre-specified level i.e., \( ARL_0 = 370 \).
3.1.1. Proposed Vs EWMA-Sign. The ARL values of the EWMA-Sign chart and proposed SAEWMA-Sign chart are presented in Table 4, under different shift levels. The comparison reveals that SAEWMA-Sign chart performs efficiently at different shift levels (for example, with \( n = 10 \), \( p_1 = 0.51,0.53,0.55 \) and 0.7, the ARL values of the proposed SAEWMA-Sign chart \( ARL_1 = 214,82,42,7 \) whereas the corresponding \( ARL_1 = 288,106,52,8 \) for EWMA-Sign chart (cf. Table 4)). From Table 4, it is revealed that for all the choices of \( n \) and \( p_1 \) the proposed SAEWMA-Sign chart performs more efficiently relative to the EWMA-Sign chart. These results show that the proposed SAEWMA-Sign chart is far better than EWMA-Sign chart in terms of detecting all levels of shifts.

3.1.2. Proposed Vs AEWMA-Sign. The ARL values of the AEWMA-Sign chart and proposed chart are compared in Table 4 at various kinds of shifts. Based on Table 4, we observed that the proposed SAEWMA-Sign chart has significantly better performance as compared to the AEWMA-Sign chart, (for example, with \( n = 15 \), \( p_1 = 0.51,0.53,0.55 \) and 0.7 the ARL of proposed SAEWMA-Sign chart are \( ARL_1 = 188,63,31,6 \) against \( ARL_1 = 255,81,38,6 \) for the

| \( p_1 \) | \( n \) | \( L_1 \) | \( L_2 \) | \( L_3 \) | \( L_4 \) |
|-------|-------|-------|-------|-------|-------|
| 0.5   | 10    | 2.740 | 2.678 | 2.652 | 2.633 |
|       | 12    | 2.405 | 2.369 | 2.328 | 2.300 |
| 0.51  | 0.5   | 369.9 | 369   | 370   | 370   |
|       | 0.53  | 214   | 205   | 187   | 169   |
|       | 0.54  | 126   | 114   | 100   | 86    |
| 0.55  | 0.5   | 112   | 99    | 86    | 72    |
|       | 0.53  | 81    | 72    | 62    | 51    |
|       | 0.54  | 61    | 55    | 48    | 40    |
| 0.6   | 0.5   | 88    | 72    | 62    | 51    |
| 0.7   | 0.5   | 68    | 60    | 50    | 39    |
| 0.85  | 0.5   | 44    | 38    | 31    | 24    |
| 0.95  | 0.5   | 44    | 38    | 31    | 24    |
AEWMA-Sign chart). From these results, we noticed a significantly better performance of the SAEWMA-Sign chart compared to AEWMA-Sign chart.

3.1.3. Proposed Vs CUSUM-Sign. The ARL values of the CUSUM-Sign chart are reported in Table 4. From Table 4, it is revealed that the ARL of the proposed SAEWMA-Sign chart is lower than CUSUM-Sign, under all shifts in the process location (for example, with \( n = 15 \), \( p_1 = 0.51, 0.53, 0.55 \) and 0.7, of the ARLs of the proposed SAEWMA-Sign chart are \( ARL_1 = 188, 63, 31 \), against ARL values of CUSUM-Sign chart are, which are \( ARL_1 = 303, 117, 49 \)). The above-mentioned results clearly indicate that the superiority of the proposed SAEWMA-Sign chart against the CUSUM-Sign chart for all levels of shifts.

3.2. A comparative analysis among single, double, repetitive and sequential sampling based charting schemes

In this section, we present a comparison of various charting schemes based on single, double, repetitive and sequential sampling mechanisms. We have evaluated the performance of all of these schemes in terms of ARL, for varying shifts in location, by considering different widths of indeterminate zones for some useful combinations of \( \phi \) and \( p_1 \) (cf. Table 5). In order to strengthen the findings of our comparative analysis, we have also computed the results of the average number of samples used (for single, double, repetitive and sequential schemes) for in-control and out-of-control processes. These results are reported in Table 6 for the same combinations of \( \phi \) and \( p_1 \), as used for Table 5.

The comparative analysis reveals the following:

i. The SS scheme detects the shifts more efficiently as compared to the single, DS and RS schemes. For instance, at \( p_1 = 0.51 \) and \( \phi = 0.1 \) the ARL values are 289, 221, 298 and 214 for single, DS, RS and SS schemes, respectively, as may be seen in Table 5.

ii. The average sample size for DS, RS and SS schemes (cf. Table 6) reveals that the SS scheme gains an edge over other schemes (for almost all shifts levels) with a marginal increase in the average number of samples used (cf. Table 5). For instance, if \( p_1 = 0.52 \) and \( \phi = 0.1 \), the average number of samples used are 10.822, 11.167 and 10.875 for DS, RS and SS schemes respectively (cf. Table 6).
iii. The performance of the DS and SS schemes are in close competition, especially when \( p_1 > 0.54 \) (cf. Table 5). The performance of the SS scheme is relatively better than the single, DS and RS schemes when \( 0.51 \leq p_1 \leq 0.54 \) (cf. Table 5).

iv. With an increase in the width of indecisive zone (i.e. \( \phi \)), the SS scheme gets an advantage over others, followed by the superiority of DS. It is to be noted that RS scheme behaves in a reverse manner, the reason being ignoring the sample falling in indecisive zone. In real applications, having a wider indecisive zone may not be very practical, and hence we have chosen the indecisive regions of practical worth.

Therefore, we can say that the proposed chart based on SS scheme offers an efficient charting structure that is relatively better in detecting all levels of shifts, as compared to the existing control charts, considered in this study.

### Table 3. Run length properties of the proposed chart for different levels of \( \lambda \) when \( n = 10 \) and \( \phi = 0.1 \).

| \( p_1 \) | \( \lambda \) | 0.05 | 0.25 | 0.5 | 0.75 |
|---|---|---|---|---|---|
| \( L_1 \) | | 2.740 | 7.514 | 4.732 | 10 |
| \( L_2 \) | | 2.405 | 2.897 | 3.098 | 2.979 |
| 0.5 | ARL | 370 | 367 | 371 | 245 |
| | MDRL | 259 | 249 | 257 | 169 |
| | SDRL | 365 | 369 | 372 | 244 |
| 0.51 | ARL | 214 | 246 | 293 | 197 |
| | MDRL | 153 | 172 | 203 | 137 |
| | SDRL | 201 | 245 | 293 | 199 |
| 0.52 | ARL | 126 | 169 | 227 | 159 |
| | MDRL | 93 | 117 | 156 | 111 |
| | SDRL | 112 | 169 | 230 | 160 |
| 0.53 | ARL | 82 | 120 | 177 | 129 |
| | MDRL | 61 | 86 | 123 | 90 |
| | SDRL | 68 | 116 | 179 | 129 |
| 0.54 | ARL | 56 | 88 | 137 | 106 |
| | MDRL | 44 | 64 | 95 | 74 |
| | SDRL | 44 | 83 | 137 | 105 |
| 0.55 | ARL | 42 | 65 | 107 | 86 |
| | MDRL | 33 | 47 | 75 | 59 |
| | SDRL | 30 | 60 | 105 | 86 |
| 0.6 | ARL | 17 | 20 | 35 | 34 |
| | MDRL | 15 | 15 | 25 | 24 |
| | SDRL | 9 | 17 | 33 | 33 |
| 0.7 | ARL | 7 | 6 | 8 | 8 |
| | MDRL | 7 | 5 | 6 | 6 |
| | SDRL | 2 | 3 | 6 | 7 |
| 0.85 | ARL | 2 | 1 | 1 | 1 |
| | MDRL | 2 | 1 | 1 | 1 |
| | SDRL | 1 | 1 | 1 | 1 |
| 0.95 | ARL | 2 | 1 | 1 | 1 |
| | MDRL | 2 | 1 | 1 | 1 |
| | SDRL | 1 | 1 | 1 | 1 |
Fig 3. ARL comparison of the proposed chart for different levels of $\lambda$ when $n = 10$ and $\phi = 0.1$.

Table 4. ARL values of the proposed and existing control charts when $\lambda = 0.05$ for different levels of $n$.

| $n$ | Charts | Profiles | $p_1$ | $p_1$ | $p_1$ | $p_1$ | $p_1$ | $p_1$ | $p_1$ | $p_1$ |
|-----|--------|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | Proposed | ARL      | 370   | 214   | 126   | 82    | 56    | 42    | 17    | 7     | 4     | 2     |
|     |         | MDRL     | 259   | 153   | 93    | 62    | 44    | 33    | 15    | 7     | 4     | 2     |
|     |         | SDRL     | 365   | 201   | 112   | 69    | 44    | 30    | 9     | 2     | 1     | 1     |
|     | AEWMA–Sign | ARL  | 369   | 288   | 174   | 106   | 72    | 52    | 19    | 8     | 4     | 3     |
|     |         | MDRL     | 253   | 204   | 126   | 80    | 55    | 41    | 17    | 8     | 4     | 3     |
|     |         | SDRL     | 357   | 272   | 159   | 90    | 56    | 38    | 10    | 3     | 1     | 1     |
|     | EWMA–Sign | ARL      | 371   | 292   | 171   | 108   | 70    | 51    | 19    | 8     | 4     | 3     |
|     |         | MDRL     | 261   | 205   | 126   | 79    | 54    | 41    | 17    | 8     | 4     | 3     |
|     |         | SDRL     | 362   | 277   | 154   | 92    | 54    | 37    | 9     | 2     | 1     | 1     |
|     | CUSUM–Sign | ARL      | 376   | 317   | 216   | 138   | 90    | 63    | 20    | 8     | 4     | 3     |
|     |         | MDRL     | 260   | 221   | 155   | 99    | 67    | 48    | 18    | 8     | 4     | 3     |
|     |         | SDRL     | 372   | 304   | 203   | 125   | 77    | 52    | 11    | 3     | 1     | 0     |
| 15  | Proposed | ARL      | 371   | 188   | 100   | 63    | 43    | 31    | 13    | 6     | 3     | 2     |
|     |         | MDRL     | 266   | 132   | 74    | 48    | 34    | 26    | 12    | 5     | 3     | 2     |
|     |         | SDRL     | 357   | 175   | 86    | 50    | 33    | 21    | 6     | 2     | 1     | 0     |
|     | AEWMA–Sign | ARL      | 369   | 255   | 138   | 81    | 53    | 38    | 15    | 6     | 3     | 2     |
|     |         | MDRL     | 261   | 186   | 101   | 62    | 43    | 31    | 13    | 6     | 3     | 2     |
|     |         | SDRL     | 354   | 250   | 122   | 67    | 39    | 25    | 6     | 2     | 1     | 0     |
|     | EWMA–Sign | ARL      | 369   | 256   | 140   | 82    | 53    | 38    | 15    | 6     | 4     | 3     |
|     |         | MDRL     | 258   | 183   | 102   | 63    | 43    | 32    | 13    | 6     | 4     | 3     |
|     |         | SDRL     | 350   | 239   | 124   | 66    | 38    | 25    | 6     | 2     | 1     | 0     |
|     | CUSUM–Sign | ARL      | 368   | 303   | 193   | 117   | 73    | 49    | 15    | 6     | 3     | 2     |
|     |         | MDRL     | 270   | 215   | 137   | 85    | 54    | 37    | 13    | 5     | 3     | 2     |
|     |         | SDRL     | 384   | 293   | 182   | 107   | 65    | 40    | 8     | 2     | 1     | 0     |

(Continued)
4. A real-life application on smartphone accelerometer data

In this section, we provide a real-life application of an accelerometer data-set for the proposed and the other schemes, considered in this study. An accelerometer is a device which has extensive variety of applications in various fields, such as to measure vibration on machines, cars, air blast pressure, earthquake and aftershocks etc. In this study, we have selected the smartphone accelerometer data-set for the monitoring purpose. This application presents the enactment of control charts for accelerometer data monitoring. We have selected total 50 subgroups of size 10 for this study (cf. Riaz et al. [29]). For the construction of the proposed and the AEWMA-Sign schemes, we used the following parameters, $\lambda = 0.25$, \(L_1 = 3.492\), \(L_2 = 3.026\), \(P = 0.06\), \(L = 3.291\) and \(ARL_0 \approx 370\).

The monitoring statistics given in (2) and (6) of the proposed SAEWMA-sign and the AEWMA-sign charts are thus constructed using the control limits given in (3) and (6), respectively. By observing the charts in Fig 4, following observations can be made for the smartphone accelerometer data-set:

Table 4. (Continued)

| \(n\) | Charts | Profiles | \(P_s\) |
|-------|--------|----------|--------|
|       |        | 0.5 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 0.6 | 0.7 | 0.85 | 0.95 |
| 20    | Proposed | ARL  | 370 | 169 | 86 | 51 | 35 | 26 | 11 | 5 | 3 | 2 |
|       |         | MDRL | 262 | 120 | 65 | 40 | 28 | 22 | 10 | 5 | 3 | 2 |
|       |         | SDRL | 352 | 156 | 73 | 39 | 24 | 16 | 4 | 1 | 1 | 0 |
|       | AEWMA–Sign | ARL  | 368 | 234 | 115 | 66 | 43 | 31 | 12 | 5 | 3 | 2 |
|       |         | MDRL | 259 | 168 | 86 | 51 | 35 | 26 | 11 | 5 | 3 | 2 |
|       |         | SDRL | 357 | 221 | 101 | 51 | 29 | 19 | 5 | 1 | 1 | 0 |
|       | EWMA–Sign | ARL  | 370 | 235 | 116 | 66 | 43 | 31 | 12 | 6 | 3 | 3 |
|       |         | MDRL | 262 | 167 | 86 | 51 | 35 | 26 | 11 | 5 | 3 | 3 |
|       |         | SDRL | 355 | 221 | 100 | 52 | 29 | 19 | 5 | 1 | 0 | 0 |
|       | CUSUM–Sign | ARL  | 358 | 286 | 173 | 98 | 61 | 40 | 12 | 5 | 2 | 2 |
|       |         | MDRL | 254 | 202 | 122 | 71 | 45 | 30 | 10 | 4 | 2 | 2 |
|       |         | SDRL | 345 | 276 | 165 | 89 | 53 | 33 | 6 | 1 | 1 | 0 |

Table 5. ARL values of single, DS, RS and SS schemes when \(\lambda = 0.05\) and \(n = 10\).

| \(\varphi\) | Sampling Schemes | \(P_s\) |
|-----------|------------------|--------|
|           | 0.5 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 0.6 | 0.7 | 0.85 | 0.95 |
| 0.02      | Single | 369 | 289 | 178 | 110 | 73 | 52 | 19 | 8 | 4 | 3 |
|           | DS | 369 | 272 | 164 | 104 | 69 | 50 | 19 | 8 | 4 | 3 |
|           | RS | 369 | 292 | 181 | 112 | 74 | 53 | 19 | 8 | 4 | 3 |
|           | SS | 369 | 272 | 166 | 103 | 69 | 50 | 19 | 8 | 4 | 3 |
| 0.06      | Single | 369 | 289 | 178 | 110 | 73 | 52 | 19 | 8 | 4 | 3 |
|           | DS | 371 | 245 | 147 | 92 | 64 | 46 | 18 | 7 | 4 | 3 |
|           | RS | 369 | 292 | 182 | 112 | 75 | 54 | 19 | 8 | 4 | 3 |
|           | SS | 369 | 243 | 145 | 91 | 63 | 46 | 18 | 7 | 4 | 3 |
| 0.1       | Single | 369 | 289 | 178 | 110 | 73 | 52 | 19 | 8 | 4 | 3 |
|           | DS | 374 | 221 | 128 | 80 | 56 | 42 | 17 | 7 | 4 | 2 |
|           | RS | 370 | 298 | 186 | 115 | 77 | 55 | 19 | 8 | 4 | 3 |
|           | SS | 370 | 214 | 126 | 79 | 56 | 42 | 17 | 7 | 4 | 2 |

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The AEWMA-Sign scheme proposed by Yang et al. [24] shows an out-of-control signal at sample # 30 (cf. Fig 4).

The proposed SAEWMA chart based on the SS scheme offers three out-of-control signals at sample points 29, 30 and 31 (cf. Fig 4), which indicates the quick and better shift detection ability of the proposed scheme as compared to the AEWMA-Sign scheme.

We may conclude that the proposed scheme outshines the AEWMA-Sign scheme for detecting shifts in process location of the smartphone accelerometer data. The real life application also supported the finding in Section 3.

### 5. Summary, conclusions and recommendations

An efficient sampling strategy can be very effective in reducing the amount of waste produced by a process. Sequential sampling is one such mechanism. In this study, we have introduced a...
nonparametric arcsine EWMA sign chart, namely the SAEWMA-sign chart, based on the sequential sampling, in order to increase the detection ability of the arcsine EWMA sign chart. This performance analysis revealed that the proposed chart is an efficient chart that offers higher sensitivity to different types of changes in process parameters. It is also revealed that the proposed chart has quicker shift detection ability under all the design parameters as compared to the competing charts including EWMA-Sign, the AEWMA-Sign and the CUSUM-Sign charts. A real-life data set based on smartphone accelerometer is presented for the implementation of the proposed chart. The said application favors the new chart as a more beneficial statistical tool to detect abnormalities in process location.

The scope of this study may be extended to various other directions for future research such as memory charts for attributes and variables, single and multivariate quality characteristics of the process, under sequential sampling mechanism. Moreover, the proposed chart can be further investigated for parent skewed process distributions.

Supporting information
S1 Dataset.

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