Large-Scale Test Facility for Modeling Bubble Behavior and Liquid Metal Two-Phase Flows in a Steel Ladle

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A new experimental facility has been designed and constructed which represents a 1:5.25 model of an industrial 185 t steel ladle. This setup is intended for systematic investigations of complex liquid metal multiphase flows created by gas blowing from the bottom. Two tons of a Sn-40 wt pct Bi alloy are employed as working fluid, its thermophysical properties are very similar to those of liquid steel. The relatively low operating temperatures ($T \sim 200$ °C) compared to the real industrial process allow the use of powerful measuring techniques for characterizing the behavior of the gas phase and resulting flow regimes. Argon gas is injected through diverse plug systems into a cylindrical fluid vessel which is equipped with a pressure tight lid to achieve low-pressure conditions for vacuum processing. This paper presents first measurements of the gas distribution close to the free liquid metal surface for various gas flow rates, plug positions and types. Moreover, the pressure in the vessel has been varied between 1 mbar and ambient pressure. The experiments provide a copious data base about flow regimes, void fraction, liquid and bubble velocities, and bubble properties, which can be used to provide so far unknown boundary conditions for numerical simulations of various metallurgical reactors such as steelmaking converters or steelmaking ladles.

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I. INTRODUCTION

In metallurgy, gas–liquid two-phase flows play an essential role in the mixing, degassing and refining of molten metals. In particular, ladle treatment of liquid metals by gas injection is highlighted as the crucial processing stage mainly responsible for inclusion cleanliness in steel, special steels or aluminum alloys. The reliable prediction of hydrodynamic processes in gas stirred ladles is of utmost importance for optimization and safe process control. Although numerical tools are currently developing at a breathtaking pace, experimental data are indispensable for code validation in the foreseeable future. The actual state of knowledge about the rising behavior of gas bubbles in liquid metals is insufficient for an adequate numerical modeling of related industrial processes. This includes relevant multiphase flows in metallurgical vessels, such as electric arc furnaces, converters, ladles, tundishes or casting molds.

Researchers have been working intensively on this topic for decades, and a recently published review article provides an insightful overview.\(^1\) Numerous water modeling and numerical simulations have been performed for studying gas stirring in bottom blown ladles.\(^2-19\) In order to obtain estimates with respect to the processing efficiency of a steelmaking ladle the majority of studies focus on investigations of the mixing time as a function of various process parameters. Measurements are carried out for determining the

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bubble induced global fluid flow, the size and shape of the liquid–gas interface or the position and dynamics of the slag eye.

Using powerful optical measurement techniques, water models provide direct and easy access to essential measurement quantities, enable the acquisition of high-resolution data and may even provide a visualization of the plume dynamics. Experiments in liquid metals are much more difficult to perform than in water, but due to the significant differences in material properties measurements in water do not adequately represent the physics of the real process. Heavy liquid metals such as steel differ from many transparent fluids, among other things, due to the distinct higher values of density \( \rho_L \) and surface tension \( \sigma \), which leads to significant discrepancies in the Morton number \( \text{Mo} = \frac{\nu_L \rho_L \Delta \rho g}{\sigma} \) (\( \nu_L \) denotes the kinematic viscosity and \( \Delta \rho \) stands for the density difference between liquid and gas and \( g \) is the gravitational acceleration) by more than two orders of magnitude (\( \text{Mo}_{\text{steel}} \approx 10^{-13}, \text{Mo}_{\text{water}} \approx 10^{-11} \)). It must be expected that gas bubbles in liquid metals behave fundamentally different from those in water, especially with regard to bubble formation, dispersion, coalescence or breaking of bubbles. Furthermore, the gas bubble behavior in a heavy liquid metal is characterized by a significant turbulence level at high Reynolds numbers \( \text{Re} = \frac{u_T d_B}{\nu_L} \). Assuming values of 5 mm for the bubble diameter \( d_B \) and a terminal velocity \( u_T \) of 0.3 m/s in liquid steel results in a Re number of about 2000 (see Table I in Section II–A for the material properties of steel). By contrast, the comparison of the Eötvös number \( \text{Eo} = \frac{\Delta \rho g d_B^2}{\sigma} \) or the Weber number \( \text{We} = \frac{\rho_L u_T^2 d_B}{\sigma} \), which are determined by the ratio \( \rho_L / \sigma \), does not show distinct differences. Thus, it can be assumed that the behavior of single bubbles does not differ that much. However, the processes in the steel ladle are dominated by bubble plumes at high gas contents, the dynamics of which are essentially determined by the turbulence on different scales. There is also a significantly higher rate of bubble–bubble interactions, such as coalescence or bubble breakup, in which surface tension plays a decisive role. Coalescence and bubble breakup play an important role with regard to the resulting bubble size distribution and the interfacial area within the melt. Coalescence reduces the number of the bubbles and decreases the gas–liquid interfacial area. First experimental studies concerning the bubble coalescence and breakup in liquid metals have been reported recently.20, 21 Another essential aspect is the formation of the bubbles, e.g., at a porous plug. While these gas injectors are definitely wetted in water, this issue is not fully clarified for molten steel. Studies have shown that gas injectors are usually inadequately wetted in liquid metals. Poorly wetted injectors cause the formation of significantly larger gas bubbles and lower the frequency of bubble detachment.22

So far, there are very few studies that have dealt with liquid metal physical modeling of the processes in the steel ladle. Xie and Oeters performed measurements of the flow velocity in a ladle-shaped vessel filled with Wood’s metal at centric gas blowing.23 The measured data obtained by means of permanent magnet probes reveal that a circulating flow field with an upwardly directed liquid flow in the bubble plume zone is established in the vessel. A continuing study also considered eccentric gas injection and measured bubble velocities using a double-contact resistance probe.24 The mean bubble rising velocities were found to be almost constant along the radius and the radial distribution of the void fraction shows a Gaussian distribution. Tokunaga et al. reconstructed skirted He bubbles in Wood’s metal after detachment from the gas injector while wobbling bubbles were found under comparable conditions in water.25, 26 The skirted bubbles disintegrate into smaller bubbles with growing distance from the gas injection point approaching a Gaussian distribution of the void fraction. Thunman et al. used the eutectic alloy GaInSn covered by a MgCl₂-glycerol solution to simulate the slag layer for studying the slag entrainment around the open eye.27

The motivation for this study originates from discussions and evaluations made within the former VDEh Technical Committee for Fluid Mechanics.28 A new large-scale test facility for the systematic investigation of swarms of gas bubbles in a tin-bismuth alloy (Sn-40 wt pct Bi, liquidus temperature \( T_L \approx 170 ^\circ \text{C} \)) was designed and built at HZDR. The thermophysical properties of Sn-40 wt pct Bi (e.g., density, surface tension, viscosity, etc.) are quite similar to those of liquid steel. Moreover, the very low process temperature compared to steel allows the use of various measurement techniques to obtain reliable and high-resolution data on the flow processes. This experiment provides reliable and high-resolution data regarding two-phase flow regime, velocity distribution, bubble properties and local gas

| Table I. Material Parameters for Sn-40 wt pct Bi and Ar in Comparison to Liquid Iron and Two Selected Steel Grades |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|             | Temperature     | Density, \( \rho \) (kg/m³) | Kin. Viscosity, \( \nu \) (10⁻⁹ m²/s) | \( \sigma_{\text{el}} \) (10⁵ A/(V m)) | Surface Tension, \( \sigma \) (N/m) |
| Sn-40 wt pct Bi | 205             | 7912.0          | 300.0           | 1.410           | 0.456           |
| Liquid Iron   | 1590            | 6960.2          | 760.8           | 0.682           | 1.847           |
| Stainless Steel X5CrNi18-10 | 1510         | 6833.0          | 807.6           | 0.682           | 1.847           |
| Stainless Steel X2CrNi-Mo18-14-3 | 1510       | 6873.3          | 802.9           | 0.721           | 1.377           |
| Argon Gas     | 20              | 1.66*           | 13414.7         | 0.436           | 0.718           |

*Solution of the ideal gas equation.
distribution. This extensive database is currently used to provide previously unknown boundary conditions for CFD (computational fluid dynamics) simulations of various metallurgical reactors, such as steel converters or ladles. This paper provides a detailed description of the experimental setup and the measurement concept. Furthermore, the capabilities of this test facility are demonstrated here by presenting and discussing first measurements carried out under various gas blowing and pressure conditions.

II. EXPERIMENTAL SETUP

A. Industrial Reference Case and Similarity Considerations

The experimental setup on laboratory scale is equivalent to a 1:5.25 model of a 185 tons ladle from Dillinger which is chosen as industrial reference case, as this has already been the subject of several investigations into flow behavior of liquid steel by gas injection both under atmospheric pressure and under vacuum.[29–34] The operating temperature in the real process is about 1600 °C. Figure 1 shows a schematic of the industrial facility with the position of the gas injection plugs. The ladle is slightly conical in shape and has an inner diameter of 3.15 m at the bottom. The bath level of the liquid steel is typically at a height of 3.2 m. Thus, the height-to-diameter ratio of the bath domain is about unity. Slit plugs installed in the bottom of the ladle are used for inert gas injection (Ar). As depicted in Figure 1, the gas can be injected at different positions at the bottom of the vessel. Typically, the argon gas volume flow rate is in a range between 200 and 600 NL/min (normal liters per minute).

In order to determine the gas volume flow rate in the model, the modified Froude number[12]

\[ \text{Fr}^* = \frac{\rho_G Q_G}{\rho_L - \rho_G g L^2} \]  

is considered here, where \( \rho_G \) and \( Q_G \) are the density and the volume flow rate of injected gas, while \( L \) is the typical length scale. (Indices G, L, M, and R stand for gas, liquid, model and reference case.)

Assuming Froude similarity, the gas volume flow rate for the model \( Q_{G,M} \) is determined by

\[ Q_{G,M} = \sqrt{\frac{\rho_{G,R} \rho_{L,M}}{\rho_{G,M} \rho_{L,R}}} \left( \frac{L_M}{L_R} \right)^{\frac{s}{2}} Q_{G,R}, \]  

where the density of the gas at 20 °C in the industrial reference case \( \rho_{G,R} \) and in the model \( \rho_{G,M} \) are assumed to be equal. The density of liquid Sn-40 wt pct Bi at 205 °C \( \rho_{L,R} \) is 7912 kg/m\(^3\) and the density of the liquid steel \( \rho_{L,M} \) is 6873 kg/m\(^3\) for steel grade X2CrNiMo18-14-3 material properties and the relevant references are given in Table I.

The ratio of the characteristic length of the model \( L_M \) to the reference case \( L_R \) represents the scaling factor of the experiment. Inserting these numerical values in Eq. [2] gives the following result:

\[ Q_{G,M} = \sqrt{\frac{7912}{6873} \left( \frac{1}{5.25} \right)^{\frac{5}{2}}} Q_{G,R} \approx 0.017 Q_{G,R} \]  

A true-to-scale representation of the gas volume flows of \( Q_{G,R} = 200 \) to 600 NL/min commonly used in the industrial process results in an application of volume flow rates between 3.4 and 10.2 NL/min in the experimental model.

B. Fluid Vessel and Gas Injection

The model of the industrial ladle is a cylindrical vessel made of stainless steel with an inner diameter of 600 mm. The bottom of the vessel is not straight but slightly concave as shown in Figure 2. The side wall is equipped with electric heaters and thermal isolation to control the temperature during the experiments. The setup is integrated in the LIMMCAST test facility, which is used at HZDR for the physical modeling of flow processes in continuous steel casting.[33] LIMMCAST is operated with the binary alloy Sn-40 wt pct Bi whose relevant material properties are compared with those of liquid steel in Table I for the operating temperature of 205 °C. A steel plate is positioned above the bottom of the vessel. Aside from the fact that this flat shape of the bottom better reflects the situation in the real ladle, this plate contains the gas injectors. To fill the vessel before starting the experiments, the melt is pushed by pressurized argon into the vessel from a storage tank through a feed line in the bottom. The filling process is finished when a level of 600 mm above the steel plate is reached. Then the height-to-diameter ratio of the fluid volume corresponds to the typical value in the industrial process (aspect ratio unity).

Fig. 1—Schematic view of the industrial reference case.
The bottom plate is held in position by four stainless steel tubes. These are connected to two rings in the upper part of the vessel to give the entire installation the necessary stability. In addition, the pipes serve as gas supply to guide argon gas to the connections in the bottom plate. The fluid container is equipped with a lid that is self-sealing under low-pressure conditions. The vessel is connected to a vacuum pump, which allows the realization of various pressure conditions above the liquid level during the experiments, starting at atmospheric pressure (1 bar) and going down to low-pressure values to 1 mbar. Such a situation corresponds to the typical conditions for vacuum treatment like VOD applications (vacuum oxygen decarburization). The white arrows in Figure 2(b) indicate the position of the gas injection pipes for two independently operated plugs. The entire setup comprising the vessel with lid and experimental equipment is shown in Figure 2(c).

Figures 3(a) and (b) show two different plug types used for gas injection. Four different positions are prepared for gas injection in the bottom steel plate as shown in Figure 3(c): Diverse plug types with an outer
diameter of 29 mm can be installed at the center and three off-centered locations, the specification of which is based on the positioning of the gas supply in the real ladle. In this paper, we present measurements with only the center plug position \((x = 0 \text{ cm})\) and an off-center plug position \((x = 160 \text{ cm})\) near the sidewall in operation. Figure 3(d) gives the exact positions of the plugs indicated by the blue circles. Each plug is connected to a gas flow controller (MKS, model GE50A) providing a maximum volume flow rate of 6.85 NL/min. Unused locations are covered by blind flanges.

C. Measuring Concept

Instrumentation of liquid metal flows in general and the determination of flow variables such as velocity, pressure, void fraction, properties of a dispersed phase, etc., in particular is considered very challenging due to the opacity, high temperatures or chemical aggressiveness of the liquid. Whichever diagnostic method is chosen, some serious restrictions always exist to apply the different sensors, for instance, the material properties of the liquid metal, the amount of impurities, the velocity range, the accuracy of the method, or the presence of electromagnetic fields. As a consequence, there is a very constrained choice of commercially available techniques to measure the velocity structure of opaque fluid flows at high temperatures. All sensors and algorithms for data processing employed in this study are in-house developments made by HZDR.

The focus of the present study is primarily on the properties of the gas phase, in particular the distribution of the gaseous phase in the fluid (void fraction) as well as the size and velocity of the gas bubbles. Furthermore, the velocity in the liquid phase is monitored at selected positions.

1. Visual inspection of the free surface

Recordings of the free liquid metal surface are carried out using the full-frame mirror less interchangeable-lens camera Sony α7 III model ILCE-7M3 with the lens Sony SEL FE 28-70 mm F3.5-5.6 OSS. Standard videos are recorded in Full HD (1920 × 1080 pixel) applying a frame rate of 50 fps. Furthermore, twice as fast frame rates of 100 fps are achieved in the slow motion mode. The higher frame rates are specifically required to capture the high dynamics of gas bubbles at the melt surface under low-pressure conditions. In the experiments at ambient pressure, the camera is positioned centrally above the liquid metal surface at a distance of
one meter. From this position the entire liquid metal surface can be completely captured. In the case of measurements at low-pressure conditions, the lid must remain closed. Here, images are acquired through a laterally mounted glass observation window in the lid with a viewing angle of 25 deg in respect of the vertical axis. With this arrangement, about 80 pct of the free metal surface can be detected, except for an edge region below the observation window. Following features can be observed at the free liquid metal surface: movement of the level, location and dynamics of the bubble plume.

2. Electrical resistance probes for measuring the void fraction and bubble characteristics

Local resistance probes are a rather classical approach for measuring the void fraction in multiphase flows.[45–47] The use of more sophisticated double-contact probes also enable to determine the bubble velocity and bubble size.[48–50] Both types of probes are shown in Figure 4. The sensitive part of the resistance probes is an electrically conducting tip (x5CrNi18-10 steel wire, diameter 0.3 mm) in direct contact with the liquid metal. The probe is supplied with an alternating voltage (1 to 10 kHz), which causes an electric current to flow from the probe tip through the liquid metal to a counter electrode. The latter can be the vessel wall or the cladding tube of the probe. The gas contact at the probe wire is detected by an interruption of the current provided that the bubble completely encloses the sensitive tip of the sensor. Due to the huge difference in the electrical conductivity between the gas and the liquid metal, we obtain very sharp signals easy to evaluate by a threshold method. The electrical insulation of the wire made of the polymer polyetheretherketon (PEEK) allows a maximum permanent operating temperature of 230 °C.

A double-contact sensor contains two electrical contacts, which have a specified distance in axial direction. Figure 4(b) contains a photograph of a double-contact probe. Figure 5 shows a typical signal for a sensor with two tips. When the gas bubble hits and envelops the tip of the probe (contact 1), the current is interrupted in the first electrical loop. After a short time the bubble arrives at the second contact (contact 2) and causes to interrupt the electric current in the second electrical loop. The contact time of the bubble \( t_B = t_3 - t_1 \) at probe 1 is the time interval, in which the current is zero. The local void fraction at the sensor position is determined as the ratio between the sum of the bubble contact times and the entire measuring time.

By taking into account the time difference \( \Delta t_u = t_2 - t_1 \) between the start times of the bubble events at both contacts and the distance \( D \) between the contacts, the vertical velocity of the bubble interface can be calculated as

\[
u_B = \frac{\Delta t_u}{D}
\]  

[4]

If the velocity is known, then the length of the vertical secant of the bubble can be obtained from the gas contact time at the first probe:

\[
s_B = u_B(t_3 - t_1)
\]

[5]

Our measuring system involves various arrays of up to 64 single probes allowing for the reconstruction of the gas distribution in the horizontal cross-section. Figure 6(a) displays the 8 × 8 sensor arrangement of single-contact probes, where the lateral distance between the individual sensors is 10 mm in each case. In the sketch the sensor block is positioned in the center of the horizontal cross-section. It can be moved toward the
side walls using two rails. Another version is the $21 \times 3$ array as shown in Figure 6(b). In both versions the sensor tips are immersed 60 mm into the melt. The possibility to move these measurement arrangements also in vertical direction is foreseen, but was not yet implemented at the time of the measurements presented here.

For the local probes immersed in the liquid, it should be noted that the accuracy of the measurement results is also influenced by the bubble-sensor interaction. In order to estimate the influence of the invasive sensors on the bubble behavior, a preliminary study was carried out using X-ray radiography to visualize the interaction of the bubbles with an arrangement of three sensors in the ternary alloy GaInSn. It was found that the inertial forces during bubble rise are sufficiently large so that the bubbles trajectories are not affected by the presence of the sensor. The eutectic composition of GaInSn has a comparably high density as Sn-40 wt pct Bi, thus the X-ray tests are representative for the situation in the experimental facility. Moreover, the bubbles are only slightly deformed when they hit the sensor tip.

3. Ultrasound-Doppler-velocimetry (UDV) for measuring velocity

The ultrasound-doppler-velocimetry (UDV) is applied to measure the velocity field of the Sn-40 wt pct Bi alloy. This method is based on a pulse-echo technique, where ultrasonic bursts of a few cycles are emitted from an ultrasonic transducer and propagate along the measuring line perpendicular to the transducer surface. The pulses are scattered by micro particles suspended in the fluid, their echo signals are received by the same transducer. The movement of the scattering particles, which are always present in liquid metals at a technical level of purity, results in a small time shift of the signal structure between two successive bursts from which the velocity can be calculated. Knowing the speed of sound in Sn-40 wt pct Bi (which is $c_s \approx 2240 \text{ m/s}$) allows us to determine the particle position along the ultrasound propagation from the detected time delay between the burst emission and the echo reception. Combining both information enables the reconstruction of the velocity profile along the measuring line. For more detailed descriptions of the measuring principle and respective applications in liquid metal flows, we refer to the relevant literature. The instrument DOP 3010 from Signal Processing Lausanne is used for performing the measurements. The operating temperature of 205 °C of the melt to be measured requires the use of special sensors with integrated acoustic wave guides in order to protect the piezoelectric element from destruction by overheating. Figure 7 shows both the radial and the azimuthal arrangement of up to 9 wave guide probes with a length of 300 mm that are used simultaneously during the measurements. The spatial resolution is about 1 mm in axial direction and distance between the sensors is 29 mm. Profiles of the vertical velocity component are obtained over a measuring depth of about 100 mm. A temporal resolution of 10 Hz is achieved, when all 9 sensors were operated. This value can be increased to almost 90 Hz, if only one probe is used.

The measured velocity is composed of signals from both the melt and the rising bubbles. The influence of the bubbles was estimated in a special high-speed measurement configuration with an acquisition rate of about 80 Hz using a single ultrasonic sensor. The gas bubbles are detected as peaks in the echo signal due to the reflection of the ultrasonic beam from the bubble surface. When a bubble is identified, the corresponding sections in the velocity signal are removed by an algorithm before temporal averaging. Corresponding analysis shows that cutting out the sections of the signal containing the bubble information results in a minor reduction in velocity of less than five percent. Therefore, the results presented in this paper are not corrected.

III. RESULTS AND DISCUSSION

We present the first measurements carried out at the facility to demonstrate the capabilities of the setup and the instrumentation. The design is very flexible and offers various options for varying the configuration or process parameters, such as the argon gas volume flow rate, number and type of the plugs for gas injection, gas injection positions and the pressure conditions in the vessel. Gas volume flow rates up to 5 NL/min are applied in the experiments. Table II gives an overview of the used plug positions (see Figure 3(d)), plug types, gas volume flow rates and pressure conditions at the free surface.

If the gas volume flow rate has to be adjusted, the start of the respective measurement is usually delayed for at least 5 minutes to allow the new flow regime to develop. Since the movement of gas bubbles in bubble swarms has a stochastic character, certain minimal measurement times are necessary to obtain statistically reliable values. In our experiments, the recordings last between 10 and 20 minutes, with the measurement duration chosen so that the statistical error remains less than 10 pct. The choice of the measuring duration is based on some estimates of the statistical variation of the individual measured variables, which were probed in a few prior test measurements.
A. Gas Injection at the Center Plug (Case A)

The measurement program was started with experiments using a single plug as gas injector in the center of the bottom plate. The slit plug and the porous plug shown in Figures 3(a) and (b), respectively, are used. A comparison of both plug types by measurements of the gas distribution, the number of bubbles detected and bubble properties using the resistance probes shows only marginal differences, which are significantly smaller than the statistical measurement error. The sensor positions are quite close to the free surface of the melt (immersion depth 60 mm). This means that the type of gas injector obviously has no measurable influence on the behavior of the bubbles at the liquid surface. Due to the differences in effective gas injection ports, it could be assumed that bubbles of different numbers and sizes should form immediately above the injection point. If this is the case, many small gas bubbles result from the porous plug injection and the increased interactions between the bubbles during ascent will lead to coalescence of bubbles. This could explain why the expected

Fig. 6—Schematics and photographs of the sensor arrays of electrical resistance probes, (a) $8 \times 8$ array, (b) $21 \times 3$ array.
Fig. 7—Schematic sketch and photograph of radial (a) and azimuthal arrangement (b) of the ultrasound sensors.

### Table II. Overview of the Different Measurement Configurations Used in the Study

| Case   | Center Plug | Wall Plug | Plug Type    | Gas Volume Flow Rates (NL/min) | Pressure      |
|--------|-------------|-----------|--------------|---------------------------------|---------------|
| Case A | ●           |           | porous/slit  | 0.5, 2, 3.4, 5                  | ambient       |
| Case B | ●           |           | porous/slit  | 0.5, 2, 3.4, 5                  | ambient       |
| Case C | ●           | ●         | slit         | 0.5, 2, 3.4, 5                  | ambient       |
| Case D | ●           |           | porous       | 0.5                            | 10 to 1000 mbar |
differences in bubble formation are not detectable in measurements near the surface. A verification requires measurements on different vertical positions. However, since this is not yet possible in the current setup, we will postpone this question until later and will not discuss a comparison of slit plug and porous plug further in this paper.

Figure 8 presents distributions of the void fraction measured by means of the $21 \times 3$ array of electrical resistance probes. The gas is injected through a porous plug installed at the center in the bottom plate. The gas volume flow rate is varied between 0.5 and 5.0 NL/min. The void fraction and the extent of the bubble plume increase as the gas volume flow rate increases. The radial distribution of the void fraction resembles the shape of a Gaussian distribution (solid lines in Figure 8(e)) with the maximum at a position nearly above the injection position which is in agreement with the findings in References 24 and 26

The double-contact resistance probe was installed at the central position of the sensor array directly above the injection site. This allows the determination of the vertical bubble velocity and the secant length of the gas bubbles at this location. The results of the measurements are depicted in Figure 9. The symbols mark the measured values, while the solid lines have been interpolated using rational polynomials for better illustration. It is obvious that an increase of the gas volume flow will also rise the number of bubbles (number of counts) detected at the probe.

It can also be seen in Figure 9(b) that the number of bubbles increases for all bubble sizes. At higher gas volume flow rates, there is also a considerable number of large bubbles with secant lengths larger 10 to 15 mm that are not found at the small gas volume flow rate of 0.5 NL/min. Similar observations apply to the bubble velocity as shown in Figure 9(a). For larger $Q_G$, the maximum of detected bubbles shifts toward larger bubble velocities. While the majority of bubbles rise with velocities in the range between 0.2 and 0.4 m/s near the surface, velocities as high as about 0.8 m/s are reached at the highest gas volume flow rate of $Q_G = 5 \text{ NL/min}$.

The rising gas bubbles drive a global motion in the vessel. In the immediate vicinity of the bubble plume, the liquid metal is transported upwards with the gas phase and descends within the peripheral areas near the side walls. Figure 10 presents corresponding time-averaged values of the liquid velocity obtained by UDV confirming the existence of this large-scale flow structure. In the region around the axis ($r = 0$) there is a clear upward flow (see Figure 10(a)), which increases with growing gas volume flow rate, while Figure 10(b) reveals a downward flow along a radius of $r = 260 \text{ mm}$. The velocities in this region do not show a clear dependence on $Q_G$, which is certainly due to the fact that the backflow extends over a much larger area than the spatially confined bubble plume.

B. Gas Injection at the Plug Position Near the Side Wall (Case B)

Corresponding measurements were carried out for the gas injection position of $x = 160 \text{ mm}$. Figures 11(a) through (d) contain two-dimensional distributions of the local void fraction for various gas volume flow rates measured by the $8 \times 8$ sensor array. In contrast to the central gas injection, the center of the measured gas concentration is no longer located above the injection position here, but is shifted toward the side wall. The distance to the side wall decreases as the gas flow increases. This is also reflected in the linear profiles of gas distribution in Figure 11(e). As we have already seen in the last section, the rising gas bubbles create a

![Fig. 8—Case A: Gas injection through a porous plug at the central position (marked by the blue arrow in (e)): (a to d) Two-dimensional distribution of the local void fraction measured by the $21 \times 3$ sensor array; (a) 0.5 NL/min, (b) 2.0 NL/min, (c) 3.4 NL/min, (d) 5.0 NL/min, (e) radial profiles of the void fraction obtained from the center line ($y = 0 \text{ mm}$) of the data presented in (a to d). The solid lines represent Gaussian fits.](image-url)
localized region of upward moving fluid. For reasons of continuity, this jet continuously entrains fluid from the surrounding volume.

If such a jet occurs near the wall, this inflow will be disturbed on the wall side, since there is a much smaller ambient volume. This creates an additional pressure gradient that pulls the jet toward the wall. The shape of the cross-section of the plume changes from a circular shape to an elliptical one that adapts to the curved wall of the vessel as shown in Figure 11(d). The modification of the gas injection position also changes the global recirculation flow in the vessel as shown in Figure 12. The upward flow caused by the bubbles is detected near the injection site. An increase in gas volume flow causes higher fluid velocities and a shift of the jet toward the side wall.

C. Simultaneous Gas Injection Through Two Plugs (Case C)

Figure 13 illustrates the gas distribution near the free surface, if two plugs are operated in parallel with the same gas volume flow rate. In this experiment the $8 \times 8$ sensor array is used where the position of the array was varied by moving it on two rails. Both bubble plumes are separately recognizable, but it becomes obvious that
both influence each other with the interaction becoming stronger as the gas volume flow rate increases. While at the lowest gas volume flow rate of 0.5 NL/min the centers of the bubble plumes roughly coincide with the injector positions, an attraction of both plumes is observed at \( Q_G = 2 \) NL/min. The outer plume moves inward and for the plume released from the center plug even a bistable state exists, \( i.e.\), it fluctuates between two distinguished positions.

The mutual attraction of the plumes is due to the same mechanism responsible for the approach of the lateral plume to the wall in the previous section. The alternating emergence of the central plume at two different positions is maintained also at \( Q_G = 3.4 \) NL/min.
However, the lateral plume is clearly oriented toward the wall. With a further increase in $Q_G$ (see Figure 13(d)), the gas distribution shifts even more clearly to the side wall. The plumes are now found either directly on the wall or near the location of the lateral injector.

In another experiment, the procedure of gas injection was modified in such a way that a volume flow rate of 3.4 NL/min is continuously injected via the plug in the middle position, while $Q_G$ is gradually increased from 0.5 NL/min to 5 NL/min at the outer position.

The corresponding results are presented in Figure 14. The phenomenon of mutual attraction of both plumes can also be observed here, which is evidently more pronounced the higher the respective gas volume flows are chosen. First, the smaller plume moves from the outside position to the stronger plume in the middle (see Figures 14(a) and (b)). This tendency reverses as soon as the outer plume features the same or some higher gas flow rate (see Figures 14(c) and (d)).

D. Gas Injection Under Low-Pressure Conditions (Case D)

In the following we consider the effect of gas stirring under the conditions of lowered pressure in the vessel. For this purpose, the vessel was closed with the lid and the gas volume above the liquid metal was evacuated by means of a vacuum pump. In this way, the pressure in the vessel can be reduced to values of $< 10$ mbar. This obviously has a significant effect on the behavior of the gas bubbles.

Figure 15 shows exemplary snapshots of the free surface for a gas volume flow rate of 0.5 NL/min injected through the center plug. Corresponding movies revealing the bubble dynamics at the free surface are available as Supplementary Material (see Supplementary Movies S1 and S2). In the case of the low-pressure conditions at 10 mbar (see Figure 15(b)), the surface is very strongly disturbed and a multitude of splashes and droplets are emitted. During the bubble release the liquid metal forms a large dome and a balloon with a very thin liquid metal film is created. This balloon explodes and a bunch of small droplets of liquid metal fall back. At the same time, distinct surface waves occur which are generated by the burst of the bubbles. These waves travel toward the wall, where they are reflected. At specific volume flow rates resonant waves can appear. Such behavior is typically observed for pressure values lower than 50 mbar.

In contrast, under ambient pressure conditions as shown in Figure 15(a), we observe that the bubbles do not actually burst. The liquid metal film of the bubble, which forms the dome, appears to be thicker and heavier. The bubble collapses and the gas escapes in a rather unspectacular manner. Formation of a “balloon” as observed at low-pressure exploding in a bundle of metal droplets cannot be detected.

As in the experiments under ambient pressure conditions single-contact probes are installed in the $21 \times 3$ arrangement with an immersion depth of 60 mm. Figures 16(a) through (g) show results for the local void fraction at the sensor tips at a gas volume flow rate of 0.5 NL/min obtained at different pressures. The pressure reduction leads to a significant increase in the.
plume size and local void fraction at the center of the plume from 1 to 7 pct. The time-averaged profiles of the local void fraction along the center line of the vessel are presented in Figure 16(h).

Corresponding abundance distributions of the vertical bubble velocity and the vertical secant length of the bubble are shown in Figure 17. The signals of all bubbles detected at the double-contact probe during the 20 minutes measurement period are included in these plots. The data indicate that the sizes as well as the number of bubbles are increasing with decreasing pressure. It is particularly obvious in Figure 17(b) that the lower the pressure becomes, the more larger bubbles are detected by the probe. These large bubbles apparently reach the free surface at higher velocities as shown in Figure 17(a). The lowering of the pressure apparently contributes significantly to the increase in the total volume of the gas phase.

The bursting of the large bubbles at the free surface leads to a significant deflection of the latter and a significant increase of the interface due to the formation of films and droplets (see also Figure 15). This situation...
Fig. 16—Case D: gas injection through the porous plug at the central position (marked by the blue arrow in (h)) with a gas volume flow rate of 0.5 NL/min under varying pressure conditions in the vessel: (a to g) Two-dimensional distribution of the local void fraction measured by the 21 x 3 sensor array; (a) 1000 mbar, (b) 500 mbar, (c) 250 mbar, (d) 100 mbar, (e) 50 mbar, (f) 20 mbar, (g) 10 mbar, (h) radial profiles of the void fraction obtained from the center line (y = 0 mm) of the data presented in (a to g). The solid lines represent Gaussian fits.

Fig. 17—Case D: abundance distributions of (a) the vertical bubble velocity and (b) the vertical bubble secant length at the point where the bubble was hit by the sensor. Data are obtained by a double-contact probe in measurements with varying pressure in the vessel. The solid lines result from fitting with a rational polynomial.
is advantageous if chemical reactions in the free surface area are to be intensified. However, such disturbances of the surface also entail the risk that slag is entrained and smaller bubbles and inclusions are carried from the surface into the interior of the fluid.[57]

IV. SUMMARY AND CONCLUSIONS

A new large-scale test facility for the experimental investigations of complex multiphase flows in a tin-bismuth alloy (liquidus temperature \( \approx 170 \, ^\circ\text{C} \)) was successfully commissioned at HZDR. The test facility and the corresponding measuring systems serve as a powerful and flexible experimental platform for modeling complex multiphase flows in metallurgy and casting. Multiple series of measurement campaigns are performed to demonstrate the functionality of the facility and the instrumentation under different operating conditions and parameters.

Due to the similarity of the material properties of Sn-40 wt pct Bi and steel the physical processes in real metallurgical reactors, such as steel converters and steel ladles, can be simulated quite realistically in the experimental model. Moreover, the distinctly lower operating temperature compared to steel allows the use of appropriate measurement technologies to obtain reliable and high-resolution data on the properties of the complex multiphase flows. The main purpose of such low-temperature model experiments consists in the acquisition of an extensive database for the verification of previously unknown boundary conditions for CFD simulations and the validation of numerical models.

The experiments are carried out in a cylindrical stainless steel vessel, which resembles a 1:5.25 model of an industrial 185 t ladle from Dillinger. The setup also includes a pressure tight lid to realize low-pressure conditions (approx. 10 mbar) in the volume above the liquid metal level for vacuum processing. Argon gas is injected 600 mm below the level through various types of gas injectors, in particular through slit designs and porous injectors provided by SMS group and RHI Magnesita. Both, the position and the number of gas injectors can be varied. The gas volume flow rate is adjusted so that the Froude similarity to the production plant is fulfilled. The measurements are performed with a rectangular grid of up to 64 electrical resistance probes and an array of up to 9 ultrasonic sensors. In addition, the behavior of the gas bubbles on the free surface is recorded by video technology.

Measurements were carried out in the vessel both at ambient pressure and under low-pressure conditions. The multiphase flows occurring in the experiment are highly turbulent and the measured values of the respective flow parameters show correspondingly strong fluctuations. The pressure conditions are continuously adjustable from normal pressure to a minimum pressure of about 1 mbar which depends on the applied gas volume flow rate. Besides the pressure, the gas volume flow rate as well as the type, number and positions of the gas injectors were varied during these first experiments. Parameters such as the distribution of bubble number and void fraction, bubble velocity and bubble size, the shape and the dynamics of the bubble plume as well as velocities in the molten metal have been measured. The measurements of the resistance probes and the UDV velocity measurements complement each other very well. The results regarding the expansion and the dynamics of the bubble plume are in very good agreement. Initial tests have shown that the UDV probes can, in principle, also be used to detect the bubble positions. This approach can be further developed as complementary method to determine bubble velocities. In this sense, the disturbing influence of rising gas bubbles can also be corrected from the velocity measurements of the fluid. A suitably selected arrangement of several UDV sensors allow for a monitoring of the sloshing behavior in the vessel.

The comparison of the measurements with slit plug and porous plug shows only minor differences in void fraction distribution and bubble sizes. This means that the type of gas injector has no measurable influence on the behavior of the bubbles on the fluid surface. However, these measurements were all taken directly below the surface, which is why only limited conclusions can be drawn about the conditions at the injection point and in the center of the vessel. Future investigations will therefore include measurements at different heights, in particular also in the immediate vicinity of the gas injector. For this purpose, the respective components and mounts are currently modified accordingly to allow a continuous adjustment of the height of the sensor array.

Measurements at low pressure show clearly a different behavior of the bubbles, especially on the free surface. Here, the bubbles show a strong increase in size, which obviously accelerates the closer the bubble approaches the free surface. The shape of these large bubbles can no longer be described as spherical or ellipsoidal. It is more likely to assume the occurrence of umbrella-shaped bubbles, which is why a derivation of the bubble expansion exclusively from the data of the two-wire probe may underestimate its size considerably. In order to better detect these bubbles, the sensor arrangement is further being optimized here.

The experimental program is being continued. Further systematic measurements are in preparation. In particular, an extension of the measurement setup should also allow variations of the sensor positions at different heights above the bottom of the vessel in the future. This will serve to investigate the turbulent dispersion and dynamics of the bubble plume at different heights and to clarify to what extent bubble-bubble interactions (coalescence and breakup) influence velocity and size distribution of the bubbles during their rise.

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CONFLICT OF INTEREST
The authors declare that they have no conflict of interest.

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SUPPLEMENTARY INFORMATION
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ABBREVIATIONS

| Symbol | Description |
|--------|-------------|
| $\alpha$ | Azimuthal position of ultrasound probe |
| $\rho$ | Density |
| $\rho_L$ | Density of liquid metal |
| $\rho_{LM}$ | Density of liquid metal in the model |
| $\rho_{LR}$ | Density of liquid metal in reality |
| $\rho_G$ | Density of gas |
| $\rho_{GM}$ | Density of gas in the model |
| $\rho_{GR}$ | Density of gas in reality |
| $\Delta \rho$ | Density difference between liquid and gas ($\rho_L - \rho_G$) |
| $\nu$ | Kinematic viscosity |
| $\nu_L$ | Kinematic viscosity of liquid metal |
| $\sigma$ | Surface tension |
| $\sigma_{el}$ | Electrical conductivity |
| $c_s$ | Speed of ultrasound in SnBi |
| $D$ | Distance between the two contacts of a double-contact probe |
| $d_B$ | Diameter of bubble |
| $g$ | Gravitational acceleration |
| $L$ | Typical length parameter |
| $Q_G$ | Volume flow rate of injected gas |
| $Q_{GR}$ | Volume flow rate of injected gas in reality |
| $r$ | Radius |
| $s_B$ | Vertical secant of bubble |
| $T$ | Temperature |
| $T_L$ | Liquidus temperature |
| $t_1$, $t_2$, $t_3$, $t_4$ | Instances in time |
| $t_B$ | Contact time of the bubble |
| $\Delta t_B$ | Time difference between start times of bubble events at both contacts of a double-contact probe |
| $u_B$ | Velocity of the interface of the bubble |
| $u_T$ | Terminal velocity of bubble |
| $x, y, z$ | Coordinates |

REFERENCES

1. Y. Liu, M. Ersson, H. Liu, P.G. Jönsson, and Y. Gan: Metall. Mater. Trans. B., 2019, vol. 50B, pp. 555–77.
2. D. Mazumdar and R.I.L. Guthrie: ISIJ Int., 1995, vol. 35, pp. 1–20.
3. S.T. Johansen, D.G.C. Robertson, K. Woje and T.A. Enghii: Metall. Trans. B, 1988, vol. 19B, pp. 745–54 and pp. 755–64.
4. D. Mazumdar and R.I.L. Guthrie: Metall. Trans. B., 1986, vol. 17B, pp. 725–33.
5. S. Kim and R.J. Fruehan: Metall. Trans. B., 1987, vol. 18B, pp. 381–90.
6. Y.Y. Sheng and G.A. Irons: Metall. Trans. B., 1992, vol. 23B, pp. 779–88.
7. M. Iguchi, S. Hosohara, T. Kondoh, Y. Itoh, and Z. Morita: ISIJ Int., 1994, vol. 34, pp. 330–37.
8. Y.Y. Sheng and G.A. Irons: Metall. Mater. Trans. B., 1995, vol. 26B, pp. 625–34.
9. M. Iguchi, R. Tsujino, K. Nakamura, and M. Sano: Metall. Mater. Trans. B., 1999, vol. 30B, pp. 631–37.
10. Y. Kishimoto, Y. Sheng, G.A. Irons, and J. Chang: ISIJ Int., 1999, vol. 39, pp. 113–22.
11. M. Iguchi and N. Kasa: Metall. Mater. Trans. B., 2000, vol. 31B, pp. 453–60.
12. K. Krishnapisharody and G.A. Irons: Metall. Mater. Trans. B., 2013, vol. 44B, pp. 1486–98.
13. J.-P. Bellot, V. De Felice, B. Dussousb, A. Jardy, and S. Hans: Metall. Mater. Trans. B., 2014, vol. 49B, pp. 13–21.
14. W. Liu, H. Tang, S. Yang, M. Wang, J. Li, Q. Liu, and J. Liu: Metall. Mater. Trans. B., 2018, vol. 45B, pp. 2681–91.
15. A.N. Conejo, R. Mishra, and D. Mazumdar: Metall. Mater. Trans. B., 2019, vol. 50B, pp. 1490–1502.
16. H. Duan, Y. Ren, and L. Zhang: Metall. Mater. Trans. B., 2019, vol. 50B, pp. 1476–89.
17. R.D. Morales, F.A. Calderon-Hurtado, K. Chattopadhyay, and S.J. Guarneros: Metall. Mater. Trans. B., 2020, vol. 51B, pp. 628–48.
18. J. Villena-Aguilar, J.A. Ramos-Banderas, C.A. Hernandez-Bocanegra, A. Uriostegui-Hernandez, and G. Solorio-Diaz: ISIJ Int., 2020, vol. 60, pp. 1172–78.
19. T. Haas, C. Schubert, M. Eickhoff, and H. Pfeifer: Metall. Mater. Trans. B., 2021, vol. 50B, pp. 903–21.
20. O. Keplinger, N. Shevchenko, and S. Eckert: Int. J. Multiphase Flow., 2018, vol. 105, pp. 159–69.
21. O. Keplinger, N. Shevchenko, and S. Eckert: Int. J. Multiphase Flow., 2019, vol. 116, pp. 39–50.
22. N. Shevchenko, S. Boden, S. Eckert, D. Borin, M. Heinze, and S. Odenbach: Eur. Phys. J. Special Topics, 2013, vol. 220, pp. 63–77.
23. Y. Xie and F. Oeters: Steel Res. Int., 1992, vol. 63, pp. 93–104.
24. Y. Xie, S. Orsten, and F. Oeters: ISIJ Int., 1992, vol. 32, pp. 66–75.
25. H. Tokunaga, M. Iguchi, and H. Tatemichi: Metall. Mater. Trans. B, 1999, vol. 30B, pp. 61–66.
26. M. Iguchi and H. Tokunaga: Metall. Mater. Trans. B, 2002, vol. 33B, pp. 695–702.
27. M. Thunman, S. Eckert, O. Hennig, J. Björkvall, and D. Sichen: Steel Res. Int., 2007, vol. 78, pp. 849–56.
28. N. Vogl and H.-J. Odenthal: Multiphase flow simulation of a steelmaking ladle—CFD benchmark of the working group Fluid Mechanics of the German Steel Institute, Proceedings of the 13th Multiphase Flow Conference and Short Course: Simulation, Experiment & Application, Dresden, 2015, pp. 1–17.
29. M. Neifer, S. Rödl, N. Bannenberg and H. Lachmund: Proceedings Scaninject VII, 1995, Lulea, Sweden, Vol. 2, pp. 283–310.
30. M. Neifer, S. Rödl, N. Bannenberg, and H. Lachmund: Stahl Eisen, 1999, vol. 117, pp. 55–63.
31. H. Lachmund, B. Prothmann, D. Huin, H. Saint-Raymond and H. Gaye: La Revue de Métallurgie–CIT, 1998, vol. 95, pp. 487–99.
32. H. Lachmund, N. Bannenberg and T. Scherrmann: La Revue de Métallurgie–CIT, 1998, vol. 95, pp. 755–64.
33. N. Bannenberg, H. Lachmund, F. Oeters, and L. Zhang: Stahl Eisen, 1999, vol. 119, pp. 37–44.
34. K. Marx, S. Rödl, H. Lachmund, and Y. Xie: Stahl Eisen, 2006, vol. 126, pp. 97–106.
35. K. Timmel, S. Eckert, G. Gerbeth, F. Stefani, and T. Wondrak: ISIJ Int., 2010, vol. 50, pp. 1134–41.
36. Y. Plevachuk, Y. Sklyarchuk, G. Gerbeth, and S. Eckert: Int. J. Mater. Res., 2010, vol. 101, pp. 839–44.
37. R.S. Hixson, M.A. Winkler, and M.L. Hodgdon: Phys. Rev. B., 1990, vol. 24, pp. 6485–91.
38. M.J. Assael, K. Kakosimos, R.M. Banish, J. Brillo, I. Egry, R.F. Brooks, P.N. Quested, K.C. Mills, A. Nagashima, Y. Sato, and W.A. Wakeham: J. Phys. Chem. Ref. Data., 2006, vol. 35, pp. 285–300.
39. K.C. Mills and R.F. Brooks: Mater. Sci. Eng. A., 1994, vol. 178, pp. 77–81.
40. T. Dubberstein, H.P. Heller, J. Klostermann, R. Schwarze, and J. Brillo: J. Mater. Sci., 2015, vol. 50, pp. 7227–35.
41. C.S. Kim: Thermophysical properties of stainless steel, Technical Report Argonne National Lab, 1975, ANL-75-55.
42. P. Pichler, B.J. Simonds, J.W. Sowards, and G. Potlacher: J. Mater. Sci., 2019, vol. 55, pp. 4081–93.
43. B. Wilthan, H. Reschab, R. Tanzer, W. Schützenhofer, and G. Potlacher: Int. J. Thermophys., 2008, vol. 29, pp. 434–44.
44. VDI Wärmeatlas, Springer, Berlin, Heidelberg, 11. Edition, 2013, pp. 357 and 425.
45. O.C. Jones and J.M. Delhaye: Int. J. Multiphase Flow., 1976, vol. 3, pp. 89–116.
46. R. van der Welle: Int. J. Multiphase Flow., 1985, vol. 11, pp. 317–45.
47. H. Turkoglu and B. Farouk: Int. J. Heat Mass Transfer., 1996, vol. 39, pp. 3401–15.
48. M. Iguchi, T. Nakatani, and H. Kawabata: Metall. Mater. Trans. B., 1997, vol. 28B, pp. 409–16.
49. Q. Wu and M. Ishii: Int. J. Multiphase Flow., 1999, vol. 25, pp. 155–73.
50. S. Eckert, G. Gerbeth, and O. Lielausis: Int. J. Multiphase Flow., 2000, vol. 26, pp. 67–82.
51. H. Kamioka and Y. Sumino: J. Phys. Soc. Japan., 1984, vol. 53, pp. 3036–41.
52. Y. Takeda: Nucl. Technol., 1987, vol. 79, pp. 120–24.
53. Y. Takeda: Nucl. Eng. Design., 1991, vol. 126, pp. 277–84.
54. S. Eckert and G. Gerbeth: Exp. Fluids., 2002, vol. 32, pp. 542–46.
55. A. Cramer, C. Zhang, and S. Eckert: Flow Meas. Instrum., 2004, vol. 15, pp. 145–53.
56. S. Eckert, G. Gerbeth, and V.I. Melnikov: Exp. Fluids., 2003, vol. 35, pp. 381–88.
57. T. Vogt, S. Boden, A. Andruszkiewicz, K. Eckert, S. Eckert, and G. Gerbeth: Nucl. Eng. Design., 2015, vol. 294, pp. 16–23.