Introduction

The grouting has been a widely used method to strengthen weak soil in geotechnical engineering, in terms of fracturing, permeation, filling and compaction [1-6]. The essence of grouting process is that the cement grout is injected into soil under high grouting pressure to densify it, and then the cement grout will become a whole with soil after hardening [7-10]. In this way, the mechanical properties of soil could be enhanced.

Related to the evaluation of reinforcement effect in engineering applications, one of the core issues in soil grouting applications is to obtain the strength of soil after grouting [11-12]. However, the strength of hardened grout-soil consolidation involves not only the
cement grout but also the soil, as shown in Fig. 1. Given that there are no mature quantitative methods at present, the strength of hardened grout-soil consolidation in engineering cases is usually estimated by experience [13-14], which is easy to result in large errors, and then affect the accurate evaluation of engineering quality. Therefore, the issue of strength calculation for hardened grout-soil consolidation should be received considerable attention.

Numerous studies have tested the strength of the hardened grout-soil consolidation by conducting simulation tests. Specifically, Hu et al. and Han et al. performed simulated grouting test in fractured rock mass and then obtained the uniaxial compressive strength of the rock mass after grouting [15-16], respectively. Cheng et al. carried out grouting field tests in the sedimentary layer to reveal the grouting reinforcement mechanism, as well as to obtain the strength of soil after grouting [17]. Besides, Yang et al. developed grouting tests in gravel to study the relationship between the compressive strength of consolidation body and the injection parameters, as well as the grout properties [18]. Furthermore, Zhang et al. performed fracturing grouting tests in fully weathered fault, and then the conclusions can be drawn that the uniaxial compressive strength of fault soil, and the cohesion of grout-soil interface after grouting can be increased by 181%~2535% and 93%~274%, respectively [19]. These studies contribute to better understand the grouting reinforcement mechanism, however, the experiments are developed under specific conditions, and the conclusions tend to be qualitative descriptions, which has limitations in application.

To deal with the aforementioned problems, a few studies have focused on the quantitative strength calculation method of hardened grout-soil consolidation. Based on the skeleton support of hardened grout for soft clay, Qian et al. developed a composite model consisting of soft clay and hardened grout, which can be used to calculate the compression modulus of the consolidation body [20]. Furthermore, on basis of area equivalence principle, homogenization method and deformation harmonization principle, Guo et al. obtained the expression of the equivalent elastic parameters of the consolidation body after grouting [21-22]. These studies have initially realized quantitative strength calculation of the hardened grout-soil consolidation, however, they only considered the properties of soil and hardened grout, ignoring the characterization of the physical process of grouting. As mentioned above, the grouting strengthens the soil by injection and compaction under high pressure, then, the relationship between key grouting parameters (e.g. grouting pressure, injection rate and grout viscosity) and strength of the hardened grout-soil consolidation should be studied.

Based on the influence of key grouting parameters on the size of hardened grout, a quantitative strength calculation method of hardened grout-soil consolidation is developed in this study, which can reflect the influence of grouting pressure, injection rate, grout viscosity and other parameters on the strength of consolidation body. Therefore, comparison of theoretical values and experimental values are conducted to verify the suitability of this strength calculation method.

**Methods**

**Assumptions**

The following assumptions are made to simplify the calculation. According to the previously developed analytical model for grout propagation in soil [23], the width of hardened grout decreases along the diffusion direction. It is assumed that the width of hardened grout remains constant along the diffusion direction, which can be calculated by means of the integral mean value theorem. Besides, the viscosity of grout remains constant during the grouting process, without considering the change of grout viscosity with time. The soil and hardened grout are assumed to be isotropic, and the hardened grout-soil consolidation is assumed to be transversely isotropic. Furthermore, the hardened grout-soil consolidation follows the assumption that the strain is uniform and the stress is distributed according to the stiffness, without considering the change of the elastic parameters of soil due to the compaction of the injected grout.

**Formula Derivation**

A parameter that can characterize the equivalent injection ratio of grout in the soil, $m$, is defined in this study, which is equal to the ratio of the average width of hardened grout ($\bar{b}$) to the length of soil unit ($L$), and it can be written as

$$m = \frac{\bar{b}}{L}$$

(1)

On basis of the previous study on the grout diffusion process in soil, the grout pressure, $p$, is axially applied to soil at the grout-soil interface within the range of $D$
(grouting reinforcement range), which causes the soil to compress and deform, therefore, the stress-deformation relationship follows the hyperbolic model. At any diffusion distance, the width of hardened grout \( b \) is equal to the compressive deformation of soil \( (\Delta \varepsilon) \), can be expressed as

\[
b = (\varepsilon - \varepsilon_0)D = \left( \frac{p}{A + Bp} - \frac{\sigma_3}{A + B\sigma_3} \right)D
\]

where \( \varepsilon \) = volumetric strains of soil; \( \varepsilon_0 \) = initial volumetric strains of soil; \( A, B \) = coefficient; \( \sigma_3 \) = minor principal stress of soil.

Due to the loss of grout pressure, the width of hardened grout decreases gradually along the diffusion direction, therefore, the average width of hardened grout, \( \bar{b} \), can be obtained by taking the integral mean value theorem of Eq. (2) from the injection hole to the maximum diffusion distance, and it is evaluated by

\[
\bar{b} = \frac{1}{p} \int_0^p \left( \frac{p}{A + Bp} - C \right)dp
\]

Where \( C = \sigma_3/(A+B\sigma_3) \). Besides, \( p \) is equal to the injection pressure at the injection hole, and then it decreases to 0 at the maximum diffusion distance.

Integrating Eq. (3) leads to

\[
\bar{b} = \frac{1}{p} \int_0^p \left( \frac{p}{A + Bp} - C \right)dp = \frac{p}{B} \log(A + Bp)
\]

Then, the average width of hardened grout \( \bar{b} \) can be written as

\[
\bar{b} = \frac{AD \log(A + Bp)}{B^2} + \frac{D(1-BC)}{B}
\]

Introducing Eq. (5) into Eq. (1) leads to

\[
m = \frac{AD \log(A + Bp)}{B^2L} + \frac{D(1-BC)}{BL}
\]

The coefficient, \( k \), can be expressed as

\[
k = n(1 - m)/m
\]

Where \( n \) is the ratio of elastic modulus of soil to hardened grout.

Therefore, the strength calculation method developed by Guo et al. [20] can be improved as

\[
E = \frac{E_0}{r_{max} n L} \int_0^{r_{max}} \frac{f(r, \mu, q)}{d(r, \mu, q)} dr - \frac{AD \log(A + Bp)}{B^2} + \frac{D(1-BC)}{BL}
\]

Where \( E = \) elasticity modulus of the hardened grout-soil consolidation; \( E_c = \) compression modulus of soil; \( \nu = \) Poisson's ratio of the soil; \( \nu_g = \) Poisson's ratio of the hardened grout.

The above calculation method is based on the average width of hardened grout, and finally establishes the relationship between the grout pressure and the strength of the hardened grout-soil consolidation. Therefore, the following analysis concentrates on the influence of other key grouting parameters on the strength of the hardened grout-soil consolidation.

As obtained in previous studies [24], the relationship between grout diffusion distance \( (r) \) and width of hardened grout \( (b) \) can be expressed as

\[
r = r_{max} e^{\frac{\pi b}{4h + ACD}}
\]

Where \( r_{max} \) = the maximum grout diffusion distance; \( \mu = \) viscosity of the grout; \( q = \) injection rate of the grout.

Then, the relationship between the width of hardened grout and other key grouting parameters, e.g. viscosity and injection rate of the grout, is established by Eq. (9), which can be expressed by

\[
b = f(r, \mu, q)
\]

By taking the integral mean value theorem of Eq. (10) from the injection hole to the maximum diffusion distance, \( b \) can be evaluated by

\[
\bar{b} = \frac{1}{r_{max}} \int_0^{r_{max}} f(r, \mu, q) dr
\]

Therefore, \( m \) can be written as

\[
m = \frac{\bar{b}}{L} = \frac{1}{r_{max} L} \int_0^{r_{max}} f(r, \mu, q) dr
\]

Considering viscosity and injection rate of the grout, the formula of elastic parameters of the hardened grout-soil consolidation can be changed to

\[
E = \frac{E_0}{r_{max} n L} \int_0^{r_{max}} f(r, \mu, q) dr - \frac{AD \log(A + Bp)}{B^2} + \frac{D(1-BC)}{BL}
\]

\[
\nu = \frac{\nu_g (1-n^2) + \nu (1-n^2)}{(1-n^2) + k(1-n^2)}
\]

(13)
To sum up, the relationship between key grouting parameters and strength of the hardened grout-soil consolidation is established by Eq. (8) and Eq. (13).

**Table 1. Elastic parameters of soil and hardened grout.**

| Material         | Elasticity modulus (MPa) | Poisson’s ratio |
|------------------|--------------------------|-----------------|
| Soil             | 3                        | 0.42            |
| Hardened grout   | 420 (7d)                 | 0.18            |
|                  | 500 (14d)                | 0.16            |

Fig. 2. Change in elastic modulus of the hardened grout-soil consolidation with grout pressure.

Fig. 3. Change in Poisson’s ratio of the hardened grout-soil consolidation with grout pressure.

**Results and Discussion**

**Influence of Grout Pressure on Strength**

The main aim of this paper is to study the relationship between key grouting parameters and strength of the hardened grout-soil consolidation. In engineering practice, the grouting reinforcement effect is usually controlled by adjusting these three parameters i.e. grouting pressure, injection rate and grout viscosity. Therefore, they are selected to study the effect of grouting parameters on strength.

By conducting laboratory tests, the elastic parameters of soil and hardened grout are tabulated in Table 1 [25], respectively.

Based on the calculation method developed above, the theoretical values of elastic parameters of the hardened grout-soil consolidation under different grout pressure have been obtained, and Fig. 2 shows the change in elastic modulus of the consolidation body with grout pressure.

It can be concluded from Fig. 2 that higher grout pressure is beneficial to increase the elastic modulus of the consolidation body, e.g. when the grout pressure is selected as 100 kPa, the elastic modulus of the consolidation body after 7 days and 14 days are 6.77 MPa and 7.50 MPa, respectively, which are increased by 126% and 249% compared to before grouting. This is mainly because within a certain range, the higher grout pressure results in greater soil compaction, which makes the hardened grout larger in size and plays a stronger role of skeleton support. Furthermore, the overall relationship between the grout pressure and the elastic modulus of the hardened grout-soil consolidation is nonlinear. With the gradual increase of the grout pressure, the increment of the elastic modulus of the consolidation body caused by the same grout pressure becomes smaller and smaller, which is because as the strength of soil gradually increases, the difficulty of compacting the soil with grout pressure continues to increase.

Fig. 3 shows that higher grout pressure tends to result in smaller Poisson's ratio of the hardened grout-soil consolidation. Specifically, with the grout pressure selected to be 100 kPa, the Poisson's ratio of the consolidation body after 7 days and 14 days are 0.29 and 0.27, and they are decreased by 29.3% and 34.4% compared with that without grouting, respectively. Besides, the grout pressure and the Poisson's ratio of the hardened grout-soil consolidation also show a nonlinear relationship. The decrease of the Poisson's ratio of the consolidation body caused by the same grout pressure gradually reduces with the increase of grout pressure, which is also due to the continuous increase in soil strength.

**Influence of Grout Viscosity on Strength**

The relationship between grout viscosity and strength of the hardened grout-soil consolidation has also been studied, and the values of related parameters are tabulated in Table 2.

As shown in Fig. 4a), within a certain range, the greater the grout viscosity, the higher the elastic
The calculations show that the elastic modulus of consolidation body increases with the increase of injection rate. For example, when the injection rate of grout is increased from 1e-3 m³/s to 5e-3 m³/s, the elastic modulus of the consolidation body after 7 days is increased from 3.46 MPa to 3.88 MPa, which is 15.3% and 29.3% higher than that before grouting, and 14 days later it can even increase by 18.3% and 35.0%, respectively. Besides, Fig. 5b) shows that the increase of injection rate will lead to the decrease of Poisson's ratio of the consolidation body. Specifically, as the injection rate increases from 1e-3 m³/s to 5e-3 m³/s, after 7 days and 14 days, the Poisson's ratio decreases from 0.393 to 0.373, and from 0.389 to 0.366, respectively. This is mainly because the high injection rate of grout is conducive to the formation of the large-size hardened grout to strengthen the soil more effectively.

Comparison

Experiment Design

The experiment setup is made of high-strength welded steel, and it is sealed by screws and bolts to...
realize grout injection under high pressure [12], which can reach up to 3 MPa. The thickness, inner diameter and height of the experiment setup are 25 mm, 184 mm and 400 mm, respectively. Besides, the grout injection also requires matching pipelines and grouting pump, and the sensors are placed inside the experiment setup to obtain pressure data during grouting.

The above calculation method assumes that there is only one major fracture filled with hardened grout to strengthen the soil, and other small branch fractures are not considered in this study, therefore, this condition should be simulated in the experiment. The major fracture filled with hardened grout, with the inclination $\alpha$ of 0°, 45° and 90°, are induced in the soil, as shown in Fig. 7, and the reason for setting the inclination is to simulate the major fractures with different directions.

The properties of the soil used in this study are tabulated in Table 3.

Fig. 5. Change in elastic parameters of the hardened grout-soil consolidation with injection rate. a) Elasticity modulus, b) Poisson’s ratio.

Fig. 6. Experiment setup.

Fig. 7. Schematic diagram of induced major fracture in soil.
Different grout viscosities and grout pressures are designed in the experiments to verify their influence on the strength of hardened grout-soil consolidation. Specifically, the change of grout viscosity is realized by adjusting the grout ratio, and three water-cement ratios, i.e. 0.8, 1 and 1.5, are selected in this study. Furthermore, the grout pressure can be changed by adjusting the injection pressure, which are selected as 1 MPa, 1.5 MPa and 2 MPa. It should be emphasized that the grout pressure refers to the pressure that can drive the propagation of the fracture, and its value is equal to the residual value after that the injection pressure overcomes the minor principal stress of soil, pipeline pressure loss and grout viscous resistance, which can be measured by the sensors.

The manual grouting pump used in the experiment can control the grout injection rate by controlling the compression frequency, however, due to the small size of the experiment setup, the volume of grout that can be injected in a single experiment is limited. Furthermore, the experiment needs to inject a part of grout in advance to discharge the pipeline air, which makes it difficult to accurately control the grout injection rate. Therefore, this study does not consider the influence of grout injection rate.

Experimental Results and Analysis

The uniaxial compression tests were carried out on the grout-soil consolidation samples [20], e.g. the stress-strain curve of sample 1 and sample 2 are shown in Fig. 8, therefore, the strength of the grout-soil consolidation can be obtained.

The grout pressure that can drive the propagation of fracture is measured as 37.2–74.5 kPa by use of sensors in this experiment. When the grout pressure is changed within this range, the experimental results show that the strength of grout-soil consolidation after grouting can be increased by 80.7%–173%. Besides, the change of grout viscosity is realized by adjusting the water-cement ratio in the range of 0.8 to 1.5, and then the grout viscosity decreases from 0.023 to 0.006. It can be concluded from experimental results that the strength of consolidation were increased by 53.8%–192% with the increase of grout viscosity. Because it is difficult to accurately control the grouting parameters in the experiment, there are some differences between the parameters selected in the experiment and theoretical calculation. However, the experimental values still verifies the suitability of the strength calculation method developed in this study to a certain extent.

Conclusions

This study mainly focuses on the quantitative strength calculation method of hardened grout-soil consolidation, and the following conclusions can be drawn.

1) Based on that the width of hardened grout is equal to the compressive deformation of soil at any diffusion distance, a quantitative strength calculation method of hardened grout-soil consolidation is presented, which establishes the relationship between the key grouting parameters and the strength of the hardened grout-soil consolidation.

![Fig. 8. Stress-strain curves of grout-soil consolidation samples: a) Sample 1 (α = 0°, W/C ratio = 0.8, p = 1 MPa); b) Sample 1 (α = 45°, W/C ratio = 0.8, p = 1.5 MPa).](image-url)
(2) Higher grouting pressure, injection rate and grout viscosity tend to result in higher strength of the consolidation body, e.g. when the grout pressure is selected to be 100 kPa, the elastic modulus of the consolidation body after 7 days and 14 days are increased by 126% and 249% compared to that before grouting.

(3) Different grout ratios (0.8, 1 and 1.5) and grout pressures (1 MPa, 1.5 MPa and 2 MPa) are designed in the experiments to verify the suitability of the strength calculation method. When the grout pressure is changed within this range, the experimental results show that the strength of grout-soil consolidation after grouting can be increased by 80.7%–173%. The strength of consolidation were increased by 53.8%–192% with the increase of grout viscosity from 0.006 to 0.023.

Acknowledgments

This project was gratefully supported by the National Natural Science Foundation of China (Grant No. 52279115, 51909250), and Science and technology project of Shandong High-speed Group (HSB2021-86). Great appreciation is also extended to the editorial board and the reviewers of this paper.

Conflict of Interest

There is no conflict of interest among all authors.

References

1. ZHANG Q.S., ZHANG L.Z., LI P., FENG X. New progress in grouting reinforcement theory of water-rich soft stratum in underground engineering. Hazard Control in Tunnelling and Underground Engineering, 1 (1), 47, 2019.
2. SONG R.X., ZHAO Y.H., MI W.J., HAN K., JIANG Y.H. Application of curtain grouting in watery and large-span loess tunnel [J]. Hazard Control in Tunnelling and Underground Engineering, 3 (2), 43, 2021.
3. DU X.M., FANG H.Y., WANG S.Y., XUE B.H., WANG F.M. Experimental and practical investigation of the sealing efficiency of cement grouting in tortuous fractures with flowing water. Tunnelling and Underground Space Technology, 108, 103693, 2021.
4. SHI J.Q., XIAO Y., HU J., WU H.R., LIU H.L., HAEHEMAN W. Small-strain shear modulus of calcareous sand under anisotropic consolidation. Canadian Geotechnical Journal, DOI: 10.1139/cgj-2021-0329, 2021.
5. HAN C.H., WEI J.C., ZHANG W.J., YANG F., XIE D.L., XIE C. Quantitative permeation grouting in sand layer with consideration of grout properties and medium characteristics. Construction & Building Materials, 327, 126947, 2022.
6. BEZUIJEN A., TE GROTHAUS R., VAN TOL A.F., BOSCH J.W., HAASEMAM J.K. Analytical model for fracture grouting in sand. Journal of Geotechnical and Geoenvironmental Engineering, 137 (6), 611, 2001.
7. LI L.P., SHANG C.S., CHU K.W. Large-scale geomechanical model tests for stability assessment of super-large cross-section tunnel. Tunnelling and Underground Space Technology, 109, 103756, 2021.
8. YUN J.W., PARK J.J., KWON Y.S. Cement-based fracture grouting phenomenon of weathered granite soil. KSCE Journal of Civil Engineering, 21 (1), L, 2016.
9. LI M.T., ZHANG X., KUANG W., ZHOU Z. Grouting reinforcement of shallow and small clearance tunnel. Polish Journal of Environmental Studies, 30 (1), 752, 2021.
10. HEIDARI M., TONON F. Ground reaction curve for tunnels with jet grouting umbrellas considering jet grouting hardening. International Journal of Rock Mechanics & Mining Sciences, 76, 200, 2015.
11. ZHANG Q.S., LI P., WANG G. Parameters optimization of curtain grouting reinforcement cycle in Yonglian tunnel and its application. Mathematical Problems in Engineering, 12, 2, 2015.
12. LI P., ZHANG Q.S., ZHANG X. Analysis of fracture grouting mechanism based on model test. Rock and Soil Mechanics, 35 (11), 3221, 2014.
13. LI P., ZHANG Q.S. Grouting diffusion characteristics in faults considering the interaction of multiple grouting. International Journal of Geomechanics, 17 (5), 1, 2016.
14. LI P., XIONG J.C., LIU Y. A self-control and visual grouting model test system and measurement method. Measurement, 154, 107481, 2020.
15. HU W., SUI W.H., WANG D.L., MA L.X. Experimental investigation on mechanical properties and representative elementary volume for chemically grouted fractured rock masses. China Science Paper, 8 (5), 408, 2013.
16. HAN L.J., ZONG Y.J., HAN G.L. Study of shear properties of rock structural plane by grouting reinforcement. Rock and Soil Mechanics, 32 (9), 2570, 2011.
17. CHENG P., ZOU J.F., LI L. Experiment of fracture grouting in alluvium with physical model. Earth Science-Journal of China University of Geosciences, 38 (3), 649, 2013.
18. YANG S.C., YANG X., CHENG X. Experimental analysis of grouting consolidation in stratum of soft rheology-plastic sludge and powder clay. Journal of Southeast University ( Natural Science Edition), 41 (6), 1283, 2011.
19. ZHANG Q.S., LI P., ZHANG X. Model test of grouting strengthening mechanism for fault gouge of tunnel. Chinese Journal of Rock Mechanics & Engineering, 34 (5), 924, 2015.
20. GUO Y.W. PhD thesis, Study of mechanical model of reinforced soils by pre grouting and its application in tunneling practices. Beijing Jiaotong University, China, 2016.
21. GUO Y.W., HE S.H., GUAN X.M. Theoretical study of plane equivalent elastic model of composite soils with fracturing grouting. Rock and Soil Mechanics, 36 (8), 2193, 2015.
22. GUO Y.W., HE S.H., ZHANG A.K. A Three-dimensional equivalent elastic model of composite soils with fracturing grouting. Rock and Soil Mechanics, 37 (7), 1877, 2016.
23. LI P., ZHANG Q.S., LI S.C., ZHANG X. Time-dependent empirical model for fracture propagation in soil grouting. Tunnelling and Underground Space Technology, 94, 103, 2019.
24. LI P. PhD thesis, Mechanical mechanism and control method of fracturing grouting for argillaceous fault, Shandong University, China, 2017.
25. LI P., ZHANG Q.S., ZHANG X. Comparison research on reinforcement characteristics of cement slurry and C-S slurry for inhomogeneous fault medium. Journal of Basic Science and Engineering, 24 (4), 840, 2016.