Stability Analysis for the Expansive Soil Slope Considering the Influence of Cracks Based on the Upper Bound Method

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Abstract. Crack is an important factor in slope stability analysis. The permeability of expansive soil is small, and its rainfall infiltration is limited during rainfall. Cracks provide a great channel for rainfall infiltration, which increases the infiltration depth and porewater pressure, further affecting the deep soil and ultimately leading to expansive soil landslides. A stability analysis method that considered the crack effect for the expansive soil slope is established, which based on the upper bound theory of limit analysis. And the effects of crack depth and crack angle on the stability of the expansive soil slope are explored and discussed. The results show that: The safety factor decreases with the increase of crack depth, and is the smallest under the consideration of the influence of earthquake action and deformation force. The effect of the crack angle on the slope safety coefficient are insignificant. When the crack angle greater than 70°, the safety factor becomes smaller. Compared with the crack angle, the crack depth exerts a greater influence on the slope safety factor.

1. Introduction

Expansive soil was a type of special clay with several cracks, strong expansibility and super consolidation characteristics. The cracks strongly influenced the properties of expansive soil, such as strength, deformation and seepage. Once expansive soil was exposed to the atmosphere, soil moisture changed repeatedly, soil structure disintegrated rapidly, cracks developed continuously, water permeability increased continuously, and strength decreased rapidly until it reached zero, leading to special engineering problems [1-3].

Expansive soil could easily form cracks in the wet–dry cycle of rainfall evaporation due to its strong expansion and contraction characteristics, such as ‘big cracks on a sunny day, spitting yellow water from cracks after the rain’. So far, numerous studies have found that cracks play an important role in the stability of expansive soil slopes. Several scholars have argued that multiple cracks were the key factors that affected the instability of expansive soil slopes. Experimental studies have been conducted on the expansive soil strength indices. By exploring the role of cracks with respect to the instability of expansive soil slopes, the relationship between crack initiation and the instability characteristics of expansive soil slopes could be explained. Moreover, the slope stability analysis method, which considered the influence of cracks and the reinforcement method of limiting crack development with geomembrane has been proposed [4-5]. Shi et al. [6] also considered the crack developing depth was a key problem to slope stability of the expansive soil, and established the crack developing depth calculation model which considered the water loss rate and the cumulative damages. Hou et al. [7] through the example analysis showed that the existence of fissures in expansive soil...
slopes had remarkable influence for the rainfall infiltration and porewater pressure distribution. Morris et al. [8] tried to use approaches like the linear elastic mechanics, the linear elastic fracture mechanics and shear failure to study soil cracking length. Qi et al. [9] studied expansive soil slope stability considering the cracks and coupling effects by numerical analysis, it was proved that both the coupling behaviour and cracks had adverse effects on the slope stability of expansive soils and hence should be taken into account in conventional engineering practice. Li et al. [10] considered the Guangxi white strong expansive soil as a research object and calculated the stability of expansive soil slope infiltrated by rainwater. The results revealed that whether the crack was considered greatly influenced the stability of the expansive soil slope. Yao et al. [11] analysed the stability of expansive soil slopes regarding the effect of cracks and rainfall infiltration and compared the differences when cracks were considered or not by presenting engineering examples. The results indicated that the slope infiltration and stability analysis that considered the influence of cracks were more reasonable and practical. Generally, the existing studies have mainly focused on the existence and distribution of cracks that affect the stability of expansive soil slopes. However, the effects of the depth of expansive soil cracks and the expansion deformation on the stability of expansive soil slopes have rarely been studied.

Therefore, the present work aims to establish a calculation program of the stability of the two-stage expansive soil slope supported by an anchor frame beam along the new Yun Gui Expressway based on upper bound theory and deformation force. The stability analysis of expansive soil slopes is investigated, considering the influence of cracks on the stability of the expansive soil slope. And the effects of crack depth and inclination angle on the stability of the expansive soil slope were discussed based on the influencing factors.

2. Establishment of slope model considering the influence of cracks

Multi-crack is a typical feature of expansive soil, and the crack is an important factor in slope stability analysis. The permeability of expansive soil is small, the rainfall infiltration is limited, and the existence of cracks provides a good channel for rainwater infiltration; these characteristics further cause the landslide of expansive soil slopes. As shown in Figure 1, we consider the cracks connected with the slip surface, where the depth and hydrostatic pressure of the cracks are denoted by \( h_w \) and \( P_w \) (under rainfall), respectively.

\[
h_w = nH (0 \leq n \leq 1) \tag{1}
\]

\[
P_w = \frac{1}{2} \gamma_w h_w^2 \tag{2}
\]

From Formula (1), when \( n=0 \), the crack depth is 0; when \( n=1 \), the crack depth is the slope height.

![Fig.1. Effect of crack on slope](image)

2.1 Geometric relationship of slope

Many two-stage expansive soil (rock) slopes exist along the newly built Yun–Gui High-speed Railway, all of which are supported by an anchor frame beam. Based on the upper bound theory of limit analysis, we study the stability of the right-side two-stage slope with DK221 + 790 strengthened by bolts (cables). The logarithmic spiral of the slope failure mode is shown in Figure 2 (the vertical section is the unit length). In the two-grade high-slope section of the expansive slope, the platform width from top to bottom are \( d_0=L, d_1 \), and \( d_2=0 \); the slope from the top to the bottom slope of each
level are $\beta_1$ and $\beta_2$. The slope height at all levels from top to bottom are $\alpha_1 H$ and $\alpha_2 H$, where $\alpha_1$ and $\alpha_2$ are the height coefficients. These parameters can be determined according to the specific slope size. Assume that the failure plane AC is a logarithmic spiral plane and that the failure plane passes through the slanted leg. The sliding body ABC makes a rotational movement about a stable body below the rotational centre O and the relative helical plane AC. Therefore, the AC surface is a thin layer of velocity discontinuities. The length of chord OA is $r_0$. The inclination of chord OA and OC, respectively, are $\theta_0$ and $\theta_h$. The height of slope is $H$. The destruction mechanism of the slope is determined by three variables, namely, the slope height $H$, $\theta_0$, $\theta_h$ and the square frame beam spacing $D_L$. The homogeneous slope heights $H_1$ and $H_2$ 8 m, and the slope inclination is 33.5° (slope 1:1.5).

Fig.2. Logarithmic spiral slip surface of the right-side two-stage slope with DK221 + 790

As shown in Figure 2, the coordinate transformation reveals that the geometric relationship of the ratio of $H/r_0$ and $L/r_0$ can be expressed in terms of $\theta_0$ and $\theta_h$, as expressed as follows:

$$\frac{H}{r_0} = e^{i(\theta_0-\theta_1)} \sin \theta_0 \sin \theta_h$$

(3)

$$\frac{L}{r_0} = \cos \theta_0 - e^{i(\theta_0-\theta_1)} \cos \theta_h - \frac{1}{r_0} \sum_{i=1}^{2} \left( \frac{\alpha_i H}{\tan \beta_i} + d_i \right)$$

(4)

Where $\theta_g$ is the angle between the line OG and the horizontal line; and $\beta_3$ is the horizontal inclination of the crack OF. When the crack is vertical, $\beta_3=90^\circ$; when the crack is horizontal, $\beta_3=0^\circ$. $L_AF$ is the length of the line segment AF.

2.2 External power of slope

(1) Gravity

The slip surface cannot pass A due to the existence of cracks, and the area of sliding soil is denoted by FBCG (Figure 1). The superimposed method can be used to calculate the external power of the FBCG area. FBCG area = ABC area−AFG area, where the ABC and AFG regions are easier to calculate. Thus, the power of gravity can be expressed as

$$W_{FBCG} = W_{ABC} - W_{AFG}$$

(5)

(2) Expansion and deformation of external power caused by environmental change

Similarly, the horizontal deformation force can be expressed as

$$F_{FBCG} = F_{ABC} - F_{AFG}$$

(6)

(3) Hydrostatic pressure

The hydrostatic pressure of the force point is from the point G of the height of the $h_w/3$ position. Thus, the hydrostatic pressure is outside the power, as expressed as follows:

$$W_p = P_v \sin \phi = \gamma_w \Omega_{h_w} w_2 \sin \phi$$

(7)
(4) Internal loss rate

\[ C_{AC} = \frac{S \cdot C_{AC}}{2 \tan \phi} \left( e^{2\phi_{\theta}} \sin \theta - 1 \right) \]  

(8)

(5) Pore water pressure

\[ W_{\text{water}} = S \cdot \gamma \cdot \sin \phi \left[ \int_{r_{i}}^{r_{o}} \sin \theta - r_{i} \sin \theta \, dr_{o} \right] \]

\[ + \int_{r_{i}}^{r_{o}} \cos \theta \tan \beta - r_{i} \cos \theta \, dr_{d} \]

\[ + \int_{r_{i}}^{r_{o}} \cos \theta \tan \beta - r_{i} \cos \theta \tan \beta \, dr_{d} \]

\[ = k_{i} \cdot \gamma \cdot \sin \phi \cdot \sin \theta \]  

(9)

(6) External power of earthquake load

\[ W_{k} = W_{h} + W_{sv} \]  

(10)

3. Slope safety factor analysis

3.1 Safety factor

(1) In view of the influence of cracks, the external power is equal to the internal energy dissipation power, formulas shown as follows:

\[ W_{FBKG} = C_{AC} \]  

(12)

(2) In view of the influence and hydrostatic pressure of cracks, the external power and internal energy dissipation power are equal, formulas expressed as follows:

\[ W_{FBKG} + W_{p} = C_{AC} \]  

(13)

(3) In view of the influence of cracks, water pressure (crack and pore) and deformation force, the external power is equal to the internal energy dissipation power, that is,

\[ W_{FBKG} + F_{FBKG} = C_{AC} \]  

(14)

(4) In view of the influence of cracks and earthquakes, the external power is equal to the internal energy dissipation power, which can be obtained as follows:

\[ W_{FBKG} + W_{k} = C_{AC} \]  

(15)

(5) In view of the influence of cracks and earthquakes, the external power is equal to the internal energy dissipation power, which can be expressed as follows:

\[ W_{FBKG} + F_{FBKG} + W_{k} = C_{AC} \]  

(16)

(6) In view of the influence of cracks, water pressure (cracks and pores) and deformation forces and earthquakes, the external power is equal to the internal energy dissipation power, which can be obtained as follows:

\[ W_{FBKG} + W_{p} + W_{\text{water}} + F_{FBKG} + W_{k} = C_{AC} \]  

(17)

(7) Calculation results analysis

A crack angle of \( \beta_{c}=90^\circ \), \( n_{i}=0.1 \), is selected in this study. The other relevant parameters are listed in Table 1. The safety factor calculation results of the slope considering the effects of pore water pressure are shown in Table 2. The software automatically outputs the slope slip surface graph under the three conditions, as shown in Figure 3.

| Table 1. The Parameters of two grade expansive soil slope |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \( d_{i}/m \) | \( \alpha_{i} \) | \( \alpha_{i} \) | \( \beta_{i}/^\circ \) | \( \beta_{i}/^\circ \) | \( H/m \) | \( \gamma/(kN/m^3) \) | \( c/kPa \) | \( \varphi/^\circ \) | \( \beta_{i} (kN/m^3) \) | \( k_{h} \) | \( k_{v} \)
| 3 | 1/2 | 1/2 | 33.7 | 33.7 | 16 | 18.85 | 10 | 15 | 1.22 | 0.1 | 0.2/3 |

As shown in Table 2, the safety factor of the slope is 0.903 when the effect of self-weight is considered. The safety factor of the slope is 0.828 when the self-weight and cracks are considered. The
safety factor is reduced by 8.31% because of the crack; and the slope safety factor is 0.825 when the self-weight, crack and hydrostatic pressure of cracks are considered, indicating that the hydrostatic pressure of the crack slightly influences the safety factor of the slope. The safety factor of the slope is 0.772 when the self-weight, deformation force and crack are considered. The safety factor of the slope is 0.688 when the effects of dead weight, seismic force and crack are considered. The safety factor of the slope is 0.558 when the self-weight, deformation force and seismic force are considered; the safety factor of the slope is 0.492 when the self-weight, deformation force, seismic force and crack are considered. The coefficient of decrease is low, and the maximum percentage of decrease is 11.83%, indicating that the crack is unfavourable to slope stability. In addition, the effect of the hydrostatic pressure of the crack on the safety coefficient is minimal, and the influence of the crack on the slope safety is less than those of the earthquake (down to 17.65%) and the pore water pressure (down to 16.17%) greater than that of the deformation force (down to 7.8%).

Table 2. Comparison of safety factors

| Condition                        | Safety factor | Reduction percentage |
|----------------------------------|---------------|---------------------|
|                                 | Cracks are not considered | Cracks are considered |
| Self-respect                     | 0.903         | 0.828               | 8.31%                |
| Weight + crack hydrostatic pressure | 0.895         | 0.825               | 7.82%                |
| Weight + deformation force       | 0.833         | 0.757               | 9.12%                |
| Weight + earthquake              | 0.742         | 0.696               | 6.20%                |
| Weight + deformation force + earthquake | 0.686         | 0.664               | 3.21%                |
| Weight + deformation force + earthquake + water pressure | 0.558         | 0.492               | 11.83%               |

As shown in Figure 3, when the effect of cracks is considered, the length of the sliding surface of the slope is shortened and communicates with the cracks. The three operating conditions considered for the initial radius of the logarithmic helical sliding surface are as follows: self-weight + largest crack; self-weight + deformation force + crack; and self-weight + earthquake + minimum crack.

4. Influencing factors analysis

4.1 Crack depth on safety factor
The crack angle is 90°, and the crack depths are 1, 2, 3 and 4 m. Three different conditions are selected, and the other parameters are left unchanged. The calculation results are shown in Table 3 and Figure 4. As shown in Figure 4, the safety factor decreases with the increase of crack depth. As shown in Table 3, when the crack depth is 4 m, the safety factor is 0.71. The safety factor is 0.651 when the influence of the deformation force is considered. The safety factor is reduced to 0.569 when earthquake and deformation force are considered.

| Crack depth/m | 0   | 1   | 2   | 3   | 4   |
|---------------|-----|-----|-----|-----|-----|
| Safety factor | 0.903 | 0.868 | 0.800 | 0.754 | 0.710 |
|               | 0.833 | 0.781 | 0.739 | 0.702 | 0.651 |
|               | 0.688 | 0.662 | 0.638 | 0.621 | 0.569 |

| Pull T/kN |
|-----------|
| 0         |
| 50        |
| 100       |
| 150       |

4.2 Crack angle on safety factor

The crack depth is 1.6 m, the crack inclinations are 110°, 100°, 90°, 80° and 70° and the values of the tension of the anchor rod are 0, 50, 100 and 150kN. The other parameters are left unchanged. The calculation results are shown in Table 4 and Figure 5. As shown in Figure 5, the change of the crack dip slightly affects the safety factor. Within a certain range, the change of the crack dip slightly influences the pull of the rock bolt. Table 4 shows that when the crack dip angle is 70°, the safety factor is 0.649 when the slope is not supported by rock bolts and 0.809 when the anchor tension is 50 kN. The safety factor is 1.223 when the anchor tension is 100 kN, and the slope is safe and stable.

| Crack angle/° | 110 | 100 | 90 | 80 | 70 |
|---------------|-----|-----|----|----|----|
| Safety factor | 0.665 | 0.664 | 0.664 | 0.648 | 0.649 | 0 |
|               | 0.825 | 0.824 | 0.823 | 0.807 | 0.809 | 50 |
|               | 1.24 | 1.239 | 1.238 | 1.222 | 1.223 | 100 |
|               | 1.655 | 1.654 | 1.653 | 1.637 | 1.639 | 150 |
Fig. 5. Relationship between crack angle and safety factor

Fig. 6. Slope sliding surface under different crack dip angles

Fig. 7. Slip surface of slope under different crack depths

Figures 6 and 7 show the landslide profiles of different cracks with different crack depths. As shown in Figure 6, the logarithmic helical slip planes $\theta_0$, $\theta_h$, $r_0$, and $L$ don’t change under the five types of crack inclinations, and the slope inclination only has a slight effect on the upper part of the slope. Therefore, the slit angle slightly affects the slope safety factor. As shown in Figure 7, the logarithmic helical slip planes $\theta_0$, $r_0$ and $L$ change to vary degrees under various crack depths of 1, 2, 3 and 4 m. Therefore, the crack depth exerts a greater influence than the crack dip angle on the slope safety factor.
5. Conclusions
(1) The safety factor decreases with the increase of the crack depth, and the safety coefficient considering the influence of earthquake and deformation force is the smallest. When the crack depth is 4 m, the safety factor is 0.71. When the deformation force is considered, the safety factor is 0.651. When the effects of earthquake and deformation are considered, the safety factor is reduced to 0.569.

(2) The change of the crack angle slightly affects the safety factor. When the crack angle is 70°, the safety coefficient is small. When the slope has no bolt support, the safety factor is 0.649. When the anchor tension is 50 kN, the safety factor is 0.809; when the anchor tension is 100 kN, the safety factor is 1.223, and the slope is safe and stable.

(3) As the crack depths are 1, 2, 3 and 4 m, the crack depth exerts a greater influence on the slope safety factor than the crack dip angle. And the logarithmic spiral slip planes L, θ₀, and r₀ have varying degrees of difference under various crack depths.

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