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Abstract

We examine the unsteady MHD free convective flow of a chemically reacting incompressible fluid over a vertical permeable plate under the influence of thermal radiation, Dufour and heat source/sink. The dimensionless governing equations are solved analytically using the three term perturbation method. Expressions for velocity, temperature and concentration for the flow are obtained and presented graphically. The analysis shows that Casson fluid parameter (β) increases the velocity; Dufour number increases the velocity and velocity; magnetic field force decreases the velocity; Chemical reaction rate increases the temperature but decreases the velocity and concentration; Grashof numbers increase the velocity when their values are increasingly varied. Furthermore, skin fiction coefficient, Nusselt number and Sherwood number for different values of governing parameters are calculated and the results are summarized in tabular form.

Keywords: MHD, Casson fluid, Dufour effect, Free convection, Chemical reaction.

I. Introduction

Non-Newtonian fluid flow occurs in various branches of chemical and material processing engineering. There are various types of non-Newton fluids such as micropolar fluid, couple stress fluid, Viscoelastic fluid and power-law fluid and so on. Likewise, there is another non-Newtonian fluid model known as the Casson fluid model. It is a part of mechanics based mostly at the time conception that a fluid
particle could also be taken into thought as continuous in a very structure. The Casson fluid model is one of the non-Newtonian fluid models that illustrate characteristics of the yield stress. Additionally, Casson fluid acts as a solid when the shear stress is applied less than the yield stress, and it moves if the applied shear stress is greater than the yield stress. There are important applications in the processing industry and biomechanics of polymers. The following are examples of Casson fluid: jelly, tomato sauce, honey, soup, concentrated fruit juices, gypsum paste etc. Alao et al. [I] studied the effects of thermal radiation, soret and dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. Charankumaret al. [II] suggested the work on chemical reaction and soret effects on cassonmhd fluid flow over a vertical plate. Choudhary et al. [III] discussed unsteady MHD casson fluid flow through porous medium with heat source/sink and time dependent suction. Falana et al. [IV] investigated the effect of brownian motion and thermophoresis on a nonlinearly stretching permeable sheet in a nanofluid. Ferdows et al. [V] developed the scaling group transformation for MHD boundary layer free convective heat and mass transfer flow past a convectively heated nonlinear radiating stretching sheet. Ibrahim et al. [VI] examined the unsteady MHD micropolar fluid flow and heat transfer over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Idowu and Falodun [VII] studied soret–dufour effects on mhd heat and mass transfer of walter’s viscoelastic fluid over a semi-infinite vertical plate: spectral relaxation analysis. Kim et al. [VIII] studied Heat and mass transfer in MHD micropolar fluid flow over a vertical moving porous plate in a porous medium. Mahmud and Sattar [IX] developed unsteady MHD free convection and mass transfer flow in a rotating system with hall current, viscous dissipation and joule heating. Mohan et al. [X] suggested an unsteady mhd free convection flow of casson fluid past an exponentially accelerated infinite vertical plate through a porous media in the presence of thermal radiation, chemical reaction and heat source or sink. Narayana et al. [XI] studied the effects of hall current and radiation absorption on MHD micropolar fluid in a rotating system. Ojelra and Naresh Kumar [XII] developed unsteady MHD mixed convective flow of chemically reacting and radiating couple stress fluid in a porous medium between parallel plates with soret and dufour effects. Okuyade et al. [XIII] studied the unsteady MHD free convective chemically reacting fluid flow over a vertical plate with thermal radiation, Dufour, Soret and constant suction effects. Pal et al. [XIV] examined perturbation technique for unsteady MHD mixed convection periodic flow, heat and mass transfer in micropolar fluid with chemical reaction in the presence of thermal radiation. Rajakumaret al. [XV] investigated radiation, dissipation and Dufour effects on MHD free convection Casson fluid flow through a vertical oscillatory porous plate with ion-slip current. Raju et al. [XVI] developed the analytical study of MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction. Sattar and Maleque [XVII] studied unsteady MHD natural convection flow along an accelerated porous plate with Hall current and mass transfer in a rotating porous medium. Seth et al. [XVIII] proposed the Heat and Mass Transfer effects on unsteady MHD natural convection flow of a
chemically reactive and radiating fluid through a porous medium past a moving vertical plate with arbitrary ramped temperature. Seth et al. [XIX] investigated effects of hall current and rotation on unsteady MHD natural convection flow with heat and mass transfer past an impulsively moving vertical plate in the presence of radiation and chemical reaction. Sharma et al. [XX] investigated the soret and dufour effects on unsteady MHD mixed convection flow past a radiative vertical porous plate embedded in a porous medium with chemical reaction. Srinivas et al. [XXI] developed pulsating flow of casson fluid in a porous channel with thermal radiation, chemical reaction and applied magnetic field. Ullah et al. [XXII] developed soret and dufour effects on unsteady mixed convection slip flow of casson fluid over a nonlinearily stretching sheet with convective boundary condition. Vedavathi et al. [XXIII] chemical reaction and radiation and dufour effects on casson magneto hydro dynamics fluid flow over a vertical plate with heat source/sink. Venkateswarlu and Satyanarayana [XXIV] studied the effects of thermal radiation on unsteady mhd micropolar fluid past a vertical porous plate in the presence of radiation absorption. Vijayaragavan et al. [XXV] examined Heat and Mass Transfer in radiative casson fluid flow caused by a vertical plate with variable magnetic field effect. Thus the present investigation is concerned with the study of variable magnetic field effect and radiative Casson fluid in two dimensional MHD flow, heat and mass transfer of viscous incompressible fluid pass a permeable vertical plate in a porous medium. The effects of various physical parameters on the velocity, temperature and concentration profiles as well as on local skin friction co-efficient, local Nusselt number and local Sherwood number are shown graphically.

Nomenclature

| K | Permeability of the porous medium |
| M | Magnetic parameter |
| Q | Heat absorption parameter |
| R | Thermal radiation |
| T | Temperature on the plate[K] |
| T₀ | Temperature at the plate[K] |
| g | Gravitational acceleration [ms⁻²] |

| Prefix/Suffix |
| Sₑ | Schmidth number |
| Pₑ | Prandtle number |
| Kₑ | Chemical reaction parameter |
| Gₑ | Grashof number |
| Gₘ | Modified Grashof number |
| qₑ | Thermal radiation |
| Nu | Nusselt number |
| Sh | Sherwood number |

Greek Symbols

| ρ | Fluid density[kg/m] |
| μ | Viscosity |
| β | Casson fluid parameter |
| τ | Skin fiction |

II. Formulation and Solution

MHD Casson fluid of incompressible, viscous, electrically-conducting fluid over a vertical plate moving with radiation and chemical reaction in the presence of
Dufour effect is considered. The rheological equation of state for an isotropic and incompressible flow of Casson fluid is

\[
\tau_{ij} = \begin{cases} 
2 \left[ \mu_g + \frac{P_y}{\sqrt{2\pi}} \right] e_{ij}, & \pi > \pi_c \\
2 \left[ \mu_B + \frac{P_y}{\sqrt{2\pi}} \right] e_{ij}, & \pi < \pi_c 
\end{cases}
\]  

Where \( \pi \) is the product based on the non-Newtonian fluid, \( \pi_c \) is a critical value of this product, \( \mu_B \) is the plastic dynamic viscosity of the non-Newtonian fluid, \( e_{ij} \) is the \((i,j)\)th component of deformation rate.

Let the \( x - \) axis is taken along the plate in the vertically upward direction and \( y - \) axis is taken normal to the plate. At time \( t \leq 0 \) the plate and fluid are at the same temperature \( T_0 \). At time \( t > 0 \), the plate is exponentially accelerated with a velocity \( u = u_0 e^{at} \) in its own plane and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is raised to \( C_w \). It is also assumed that there is a first order chemical reaction between the fluid and the species concentration. The reaction is assumed to take place entirely in the stream which is shown in Fig.1. Then under the usual Boussinesq approximation the unsteady flow is governed by the following equations:

\[
\frac{\partial u}{\partial z} = \nu \left[ 1 + \frac{1}{\beta} \right] \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k} u + \frac{g \beta_T}{k} [T - T_0] + \frac{g \beta_C}{k} [C - C_w] 
\]

\[
\frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial z} - \frac{Q_0}{\rho c_p} [T - T_0] + \frac{D_m K_T}{C_s c_p} \frac{\partial^2 C}{\partial z^2} 
\]

**Fig. 1:** The Physical Model of the problem

The governing equations are

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\[
\frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} - K_r(C - C_\infty) \tag{4}
\]

The initial and boundary condition are

\[
u = 0, T = T_\infty, C = C_\infty, \quad t \leq 0, \quad \forall z \tag{5}
\]

\[
u = u_0 e^{at}, T = T_\infty, C = C_\infty, \quad t > 0, \quad \text{at} \ z = 0 \tag{6}
\]

\[
u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as} \ z \rightarrow \infty \tag{7}
\]

The Radiative heat flux \(q_r\) can be expressed as the form

\[
\frac{\partial q_r}{\partial z} = -4a^* \sigma \left[ T_\infty^4 - T^4 \right] \tag{8}
\]

Where \(\sigma\) is Stefan Boltzmann constant N and \(k\) is mean absorption coefficient.

It is assumed that the temperature differences within the flow are small and \(T^4\) may be expressed as a linear function of the temperature. This is obtained by expanding \(T^4\) in Taylor’s series about and neglecting the higher order terms, thus we get

\[
T^4 \approx 4T_\infty^3T - 3T_\infty^4 \tag{9}
\]

Substituting (8) and (9) in (3), we get

\[
\frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial z^2} - \frac{16a^* \sigma T_\infty^3}{\rho c_p} \left[ T - T_\infty \right] + \frac{D_n K_T}{C_S c_p} \frac{\partial^2 C}{\partial z^2} \tag{10}
\]

We introduced the following non-dimensional variables

\[
u^* = \frac{\nu}{U_0}, \quad \nu^* = \frac{\nu}{w_0}, \quad z^* = \frac{w_0 z}{\nu}, \quad U_* = \frac{U_*}{U_0}, \quad U^*_p = \frac{U^*_p}{U_0}, \quad t^* = \frac{tw_0^2}{\nu}, \quad \theta = \frac{T - T_\infty}{T_\infty - T_\infty}, \tag{11}
\]

\[K = \frac{k w_0^2}{\nu^3}, \quad \text{Pr} = \frac{\nu \rho c_p}{\alpha}, \quad \text{Gr} = \frac{g \beta \nu (C_w - C_\infty)}{u_0 w_0^2}, \quad \text{Gm} = \frac{g \beta \nu (T_w - T_\infty)}{u_0 w_0^2}, \tag{12}
\]

\[K_r = \frac{K \nu}{w_0^2}, \quad S_r = \frac{\nu}{D}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad M^2 = \frac{\sigma B_\infty^2 \nu}{\rho u_0 w_0^2} \tag{13}
\]

By using non-dimensional variables, the governing equations can be reduces to

\[
\frac{\partial u}{\partial z} = \left[ 1 + \frac{1}{M^2} \right] \frac{\partial^2 u}{\partial z^2} - \left[ M + \frac{1}{k} \right] u + \text{Gr} \theta + \text{Gm} \phi \tag{14}
\]
\[ \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} - \left[ \frac{R}{Pr} + Q \right] \theta + Du \frac{\partial^2 \phi}{\partial z^2} \]  

(12)

\[ \frac{\partial \phi}{\partial z} = \frac{1}{Sr} \frac{\partial^2 \phi}{\partial z^2} - K, \phi \]  

(13)

The initial and boundary conditions reduces to

\[ u = 0, \quad \theta = 0, \quad \phi = 0, \quad t \leq 0, \quad \forall z \]  

(14)

\[ u = e^{\eta}, \quad \theta = 1, \quad \phi = 1, \quad t > 0, \quad \text{at } z = 0 \]  

(15)

\[ u \to 0, \quad \theta \to \theta_\infty, \quad \phi \to \phi_\infty \quad \text{as } z \to \infty \]  

(16)

In order to solve the coupled non-linear system of equations (11) to (13) with the boundary conditions (14) to (16), the following simple perturbation technique is used. The governing equations (11) to (13) are expanded in the powers of Eckert number (\( \varepsilon \ll 1 \)).

\[ u = u_0(z) + \varepsilon e^{\eta} u_1(z) + \varepsilon^2 e^{2\eta} u_2(z) + O(\varepsilon^3) \]  

(17)

\[ \theta = \theta_0(z) + \varepsilon e^{\eta} \theta_1(z) + \varepsilon^2 e^{2\eta} \theta_2(z) + O(\varepsilon^3) \]  

(18)

\[ \phi = \phi_0(z) + \varepsilon e^{\eta} \phi_1(z) + \varepsilon^2 e^{2\eta} \phi_2(z) + O(\varepsilon^3) \]  

(19)

Substituting equations (17),(18) and (19) into the equations (11), (12) and (13) and equating harmonic and non-harmonic terms and neglecting the higher order terms and obtain the following equations.

\[ B \frac{d^2 u_0}{dz^2} - \left( M + \frac{1}{K} \right) u_0 = -Gr\theta_0 - G\phi_0 \]  

(20)

\[ B \frac{d^2 u_1}{dz^2} - \left( M + \frac{1}{K} + n \right) u_1 = -Gr\theta_1 - G\phi_1 \]  

(21)

\[ B \frac{d^2 u_2}{dz^2} - \left( M + \frac{1}{K} + 2n \right) u_2 = -Gr\theta_2 - G\phi_2 \]  

(22)

\[ \frac{1}{Pr} \frac{d^2 \theta_0}{dz^2} - \left( \frac{R}{Pr} + Q \right) \theta_0 = -Du \frac{d^2 \phi_0}{dz^2} \]  

(23)

\[ \frac{1}{Pr} \frac{d^2 \theta_1}{dz^2} - \left( \frac{R}{Pr} + Q + n \right) \theta_1 = -Du \frac{d^2 \phi_1}{dz^2} \]  

(24)
The analytical solutions of the equations (20)-(28) are given by

\[
\begin{align*}
\frac{1}{P_r} \frac{d^2 \theta_2}{dz^2} & - \left[ \left( \frac{R}{P_r} + Q \right) + 2n \right] \theta_2 = - Du \frac{d^2 \phi_2}{dz^2} \\
\frac{d^2 \phi_0}{dz^2} - K_r S_e \phi_0 & = 0 \\
\frac{d^2 \phi_1}{dz^2} - \left[ K_r + n \right] S_e \phi_1 & = 0 \\
\frac{d^2 \phi_2}{dz^2} - \left[ K_r + 2n \right] S_e \phi_2 & = 0
\end{align*}
\]

(25)

(26)

(27)

(28)

The skin friction co-efficient, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. These parameters can be defined and determined as follows.

\[
\begin{align*}
\tau & = \left( \frac{\partial u}{\partial z} \right)_{z=0} = a_{21} + \epsilon e^{\eta} \left( a_{22} \right) + \epsilon^2 e^{2\eta} \left( a_{23} \right) \\
Nu & = \left( \frac{\partial \theta}{\partial z} \right)_{z=0} = a_{24} + \epsilon e^{\eta} \left( a_{25} \right) + \epsilon^2 e^{2\eta} \left( a_{26} \right) \\
Sh & = \left( \frac{\partial \phi}{\partial z} \right)_{z=0} = a_{27} + \epsilon e^{\eta} \left( a_{28} \right) + \epsilon^2 e^{2\eta} \left( a_{29} \right)
\end{align*}
\]

(32)

(33)

(34)

III. Results and Discussion

In the present study, we have to select \( \epsilon = 0.01, \quad t = 0.1, \quad n = 1 \) while \( \beta, Du, Gm, Gr, Kr, M, R, Scare \) varied over a range, which are listed in the figures. Fig.2
shows that the variation in velocity profiles of z for various values in Casson fluid parameter (β). Here we illustrate that as the values of Casson fluid parameter (β) increases it leads to increases in velocity. Fig.3 displays the velocity profile increases on increasing the dufour number (Du) in the region away from the plate. This implies that Dufour number (Du) tends to raises the fluid velocity in the region away from the plate. The variation in velocity profile with z for different values in Modified Grashof number (Gm) is shown in Fig.4. The modified Grashof number (Gm) defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases as modified Grashof number (Gm) increases. Fig.5 depicts the various values of z for various values in Grashof numbers (Gr). This figure reflects that with increase in Grashof numbers (Gr) there is increase in fluid velocity due to improvement of buoyancy force. Fig.6 witnesses the effect of chemical reaction parameter (Kr) on the profile of velocity distribution and it is indicated that as (Kr) increases the velocity profile decreases with the boundary layer thickness. Fig.7 demonstrates the velocity profiles against for various magnetic field parameters (M) and here we observed that the velocity decreases as the existence of magnetic field becomes stronger. This conclusion agrees with the fact that the magnetic field exerts retarding force on the free-convection law. From Fig.8 the effect of radiation parameter R on the velocity profiles increases with an increase in R. Fig.9 depicts the value of Schmidt number (Sc) increase in the peak velocity is seen near the plate. Whereas for higher values of Schimth number (Sc) the peak velocity shifts closer to the plate. Fig.10 represents the effect of Dufour number (Du) on the concentration profile. As the Dufour number (Du) increases, the concentration profiles of the fluid increases in the boundary layer region. Fig.11 represent the variations in temperature profile for different value of chemical reaction parameter (Kr), here we observe that the temperature decreases with a growing of chemical reaction parameter (Kr). Fig.12 shows that the temperature profile for different values of Prandtl number (Pr), here we observe that the temperature decreases with a rising of Prandtl number (Pr). From figure 13, we observe that Schmidt number increases as the increasing of the various temperature profiles.

Figure 14-15 depicts the variation of species concentration C versus Z under the influence of Chemical reaction parameter (Kr) and Schmidt number (Sc). These figures indicates that the concentration level of the fluid falls due to increasing values of Chemical reaction parameter (Kr) and Schmidt number (Sc).
Fig. 2: Casson fluid (β) effect on velocity

\[
\begin{align*}
\tau &= 0.01; \, Sc = 0.22; \, Kr = 1; \, Pr = 0.71; \\
Du &= 0.60; \, M = 1.5; \, Gr = 10;
\end{align*}
\]

\( \beta = 0.5, 1, 2, 3 \)

Fig. 3: Dufour (Du) effect on velocity

\[
\begin{align*}
Sc &= 0.22; \, Kr = 1; \, Q = 0.1; \, Pr = 0.71; \\
K &= 1; \, Gr = 10;
\end{align*}
\]

\( Du = 0.6, 1, 2, 3 \)

Fig. 4: Modified Grashof number (Gm) Effect on Velocity

\[
\begin{align*}
Sc &= 0.22; \, Kr = 1; \, Q = 0.1; \, Pr = 0.71; \\
K &= 1; \, Du = 0.60;
\end{align*}
\]

\( Gm = 10, 15, 20, 25 \)
Fig. 5: Grashof Number (Gr) effect on velocity

Fig. 6: chemical reaction parameter (Kr) effect on velocity

Fig. 7: Magnetic field (M) effect on velocity
Fig. 8: Radiation (R) effect on velocity

Fig. 9: Schmidth number (Sc) effect on velocity

Fig. 10: Dufour (Du) effect on Temperature
Fig.11: chemical reaction parameter ($Kr$) effect on Temperature

$$Sc = 0.22; Kr = 1; Q = 0.1; Pr = 0.71; Du = 0.60;$$

Fig.12: Prandtl number ($Pr$) effect on Temperature

$$Sc = 0.22; Kr = 1; Q = 0.1; Pr = 0.71; Du = 0.60;$$

Fig.13: Schmidt number ($Sc$) effect on Temperature

$$Sc = 0.22, 0.3, 0.6, 0.78$$
Fig. 14: chemical reaction parameter (Kr) effect on Concentration

Fig. 15: Schmidt number (Sc) effect on Concentration

Fig. 16: Schmidt Number (Sc) effect on velocity

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Fig. 17: Schmidt Number (Sc) effect on temperature

For comparison purpose, the method proposed by Mohan et al. [X] is deliberated. The profiles of fluid velocity and concentration versus boundary layer coordinate is shown in the Fig. 16 and Fig. 17 for a numerous estimations of Sc by taking Du=0. Our results are in good agreement with the performance of Mohan et al. [X] without Dufour effect.

Table 1: The influence of different parameters on Nusselt Number (Nu) with fixed values of other parameters.

| Kr | Pr | Du  | Sc  | R   | Nu number (Nu) |
|----|----|-----|-----|-----|----------------|
| 1  | 0.71 | 0.60 | 0.22 | 1   | -0.9865        |
| 2  | 0.9380 |
| 3  | 0.8958 |
| 1  | 1    | 0.9767 |
| 2  | 0.9443 |
| 3  | 0.9141 |
| 0.2 | -1.0287 |
| 0.4 | -1.0046 |
| 0.5 | -0.9970 |
| 0.3 | -0.9677 |
| 0.4 | -0.9459 |
| 0.5 | -0.9255 |
| 2  | 1.4077 |
| 3  | 1.7312 |
| 4  | 2.0037 |

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Table 2: The influence of different parameters on the Skin friction coefficient ($\tau$) with fixed values of other parameters.

| M  | K_r | G_r | G_m | S_c | P_r | R  | $\beta$ | Du | Skin friction ($\tau$) |
|----|-----|-----|-----|-----|-----|----|---------|----|-----------------------|
| 5  | 1   | 10  | 10  | 0.22| 0.71| 1  | 0.5     | 0.6 | 0.5583                |
| 2.5|     |     |     |     |     |    |         |     | 0.3959                |
| 3.5|     |     |     |     |     |    |         |     | 0.2442                |
| 1.5| 2   | 3   | 4   | 8   | 9   | 8  | 0.3     |     | 0.4742                |
|    |     |     |     |     |     | 9  | 0.4     |     | 0.4197                |
|    |     |     |     |     |     |    | 0.5     |     | 0.3790                |
|    |     |     |     |     |     |    |         |     | 0.3276                |
|    |     |     |     |     |     |    |         |     | 0.4430                |
|    |     |     |     |     |     |    |         |     | 0.2809                |
|    |     |     |     |     |     |    |         |     | 0.4196                |
|    |     |     |     |     |     |    |         |     | 0.5221                |
|    |     |     |     |     |     |    |         |     | 0.4861                |
|    |     |     |     |     |     |    |         |     | 0.4569                |
|    |     |     |     |     |     |    |         |     | 0.5641                |
|    |     |     |     |     |     |    |         |     | 0.5826                |
|    |     |     |     |     |     |    |         |     | 0.5996                |
|    |     |     |     |     |     |    |         |     | 0.4160                |
|    |     |     |     |     |     |    |         |     | 0.3281                |
|    |     |     |     |     |     |    |         |     | 0.2650                |
|    |     |     |     |     |     |    |         |     | 0.8366                |
|    |     |     |     |     |     |    |         |     | 1.0804                |
|    |     |     |     |     |     |    |         |     | 1.1929                |
|    |     |     |     |     |     |    |         |     | 0.5390                |
|    |     |     |     |     |     |    |         |     | 0.5487                |
|    |     |     |     |     |     |    |         |     | 0.5535                |
Table 3: The influence of different parameters on Sherwood Number (Sh) with fixed values of other parameters.

| $K_r$ | $S_c$ | $\varepsilon$ | $n$ | Sherwood Number (Sh) |
|-------|-------|----------------|-----|----------------------|
| 1     | 0.22  | 0.01           | 1   | -0.4765              |
| 2     |       |                |     | -0.6724              |
| 3     | 0.3   |                |     | -0.8229              |
|       | 0.4   |                |     | -0.5564              |
|       | 0.5   |                |     | -0.6425              |
|       | 0.02  |                |     | -0.7183              |
|       | 0.03  |                |     | -0.4841              |
|       | 0.04  |                |     | -0.4919              |
|       |       |                | 2   | -0.5000              |
|       |       |                | 3   | -0.4791              |
|       |       |                | 4   | -0.4819              |
|       |       |                |     | -0.4850              |

IV. Conclusion

In this study, we examined the Dufour effects on unsteady MHD free convection flow fast a vertical plate through porous medium in the slip regime with heat source/sink. The governing equations are solved analytically by three term harmonic and non-harmonic perturbation method. Presently, we illustrate the flow characteristics for velocity, temperature and concentration and show the how the flow fields are influenced by the parameters.

1. When the values of Casson fluid parameter ($\beta$), Modified Grashof number ($G_m$), Grashof number ($G_r$), Dufour number ($D_u$) are increases, the velocity fields increases while an increase in the chemical reaction parameter ($K_r$), magnetic field ($M$), $R$ and $S_c$, the velocity decreases.

2. An increase in the chemical reaction parameter ($K_r$), Dufour number ($D_u$), Prandtl number ($P_r$), the temperature increases while an increasing the values of $R$ decreases the temperature.

3. An increase in Schmidt number ($S_c$) and chemical reaction parameter ($K_r$) decreases the concentration fields.
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