Formation, stability, and dynamics of vector bright solitons in a trapless Bose–Einstein condensate with spin–orbit coupling

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Abstract

We consider a binary self-attractive Bose–Einstein condensate with Rashba spin–orbit coupling (SOC) in two-dimensional (2D) free space. The formation, stability, and dynamics of vector bright solitons are elucidated through numerical analysis and variational approximation. It is found that dynamically stabilized vector bright solitons can be formed in 2D free space with appropriate parameters and ramp schemes, and its motional trajectory and stability show nontrivial behavior and strong dependence on the direction of the force generated by the SOC. Finally, the case of the periodic modulation of SOC is discussed, and an experimental protocol is also given.

1. Introduction

Soliton—solitary waves that maintain their shape as they propagate—is ubiquitous in nature, and appear in contexts as diverse as nonlinear optical media, superfluids and superconductors, and recently in Bose–Einstein condensate (BEC) [1–7]. Given their versatility, ultracold atom systems offer us additional layers of tunability, leading to the arbitrarily parametric variations. Because of the well-known collapse property in high-dimensional scenarios, great effort has been devoted to search for systems with stable solitary waves, where different physical mechanisms have been elaborated. Typically, a spatial modulation of the Kerr nonlinearity of the optical material is designed in the field of nonlinear optics [8, 9], while an external trapping potential [10–12], a rapid oscillation of the strength of contact interaction [13–16], or nonlocal nonlinearity [17, 18] has been proposed for ultracold atomic systems.

Recently, the successfully experimental realization of artificial spin–orbit coupling (SOC) in BEC has broadened the frontier of ultracold atomic gases used for quantum simulations [19–27]. This degree of freedom opens up a new avenue to study the fundamental properties of various topologic defects due to the close relationship between the spin and motional degrees of freedom [28–40]. In the presence of SOC, a variety new types of solitons, such as half-vortex gap solitons [41], discrete and continuum composite solitons [42], and many others, have been predicted. More specifically, the two-dimensional (2D) composite solitons [43] and stable 3D solitons without the ground state [44] in free space have been reported in a binary BEC under the effect of SOC, in which the stability mechanism comes from the balance between attractive nonlinearity and the modification of the dispersion induced by the SOC.

So far, the formation, stability, and dynamics of vector bright solitons in a trapless BEC with SOC have not been discussed in detail, which is what we attempt to do in this paper. Our results show that dynamically stabilized vector bright solitons can be formed in 2D free space with appropriate parameters and ramp schemes. More importantly, the dynamics and stability of such solitons show nontrivial behavior and strong dependence on the direction of the force generated by the SOC.
This paper is organized as follows. In section 2, we introduce the theoretical model describing a binary spin–orbit-coupled (SO-coupled) BEC, and briefly introduce the numerical method. The effects of SOC on the formation, stability, and dynamics of vector bright solitons are investigated by using the mean-field theory in section 3. Finally, the main results are summarized in section 4.

2. The theoretical model

We consider a binary SO-coupled BEC with attractive contact interactions and confined in a quasi-2D axisymmetric trap, which can be realized by adding a very strong harmonic confinement along the axial direction. The second-quantized Hamiltonian of the model under consideration is given by \( \hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \), where

\[
\hat{H}_0 = \int \mathrm{d} \mathbf{r} \hat{\psi}^\dagger (\mathbf{r}) \left( -\hbar^2 \nabla^2 / 2m + V_{\text{ext}}(\mathbf{r}) + V_{\text{so}} \right) \hat{\psi}(\mathbf{r}),
\]
\[
\hat{H}_{\text{int}} = \int \mathrm{d} \mathbf{r} \left( \frac{g_{\uparrow\uparrow}}{2} \hat{n}_\uparrow^2 + \frac{g_{\downarrow\uparrow}}{2} \hat{n}_\downarrow^2 + g_{\uparrow\downarrow} \hat{n}_\uparrow \hat{n}_\downarrow \right),
\]

where \( m \) is the atom mass and assumed to be equal for the two components. \( \hat{\psi} = [\hat{\psi}_\uparrow(\mathbf{r}), \hat{\psi}_\downarrow(\mathbf{r})]^T \) denotes collectively the spinor Bose field operators with \( \uparrow, \downarrow \) referring to two pseudospin components, \( \mathbf{r} = (x, y) \), \( \hat{n}_\uparrow = \hat{\psi}_\uparrow \hat{\psi}_\uparrow^\dagger \), \( \hat{n}_\downarrow = \hat{\psi}_\downarrow \hat{\psi}_\downarrow^\dagger \), and the energy is expressed in units of oscillator energy \( \hbar^2 a_{\text{so}}^2 / m \). The effective intra- and inter-component contact interactions parameters can be written as \( g_{\uparrow\uparrow} = \sqrt{8\pi \hbar^2 a_{\text{so}}^2 / m a_\text{so}} \) and \( g_{\downarrow\uparrow} = \sqrt{8\pi \hbar^2 a_{\text{so}}^2 / m a_\text{int}} \) being the corresponding s-wave scattering lengths and \( a_\text{so} = \sqrt{\hbar/(M\omega_z)} \). To highlight the effect of SOC, in the present work we focus on the case with \( g_{\uparrow\downarrow} = g_{\downarrow\uparrow} = g < 0 \) (attractive contact interactions), which is similar to three dimensional free space case [44].

We implement the mean-field approximation, where the collisions between the condensate atoms and the thermal cloud are neglected. The field-operators \( \hat{\psi}_\uparrow \) in equation (1) thus can be replaced by the macroscopic wave-functions \( \psi_\uparrow \). The dynamics of such a system can be well described by the following dimensionless Gross–Pitaevskii (GP) equations

\[
\frac{i}{\hbar} \frac{\partial \psi_\uparrow}{\partial t} = \left( -\frac{1}{2} \nabla^2 + V_{\text{so}} + V_{\text{ext}}(t) + g (|\psi_\uparrow|^2 + |\psi_\downarrow|^2) \right) \psi_\uparrow,
\]
\[
\frac{i}{\hbar} \frac{\partial \psi_\downarrow}{\partial t} = \left( -\frac{1}{2} \nabla^2 + V_{\text{so}} + V_{\text{ext}}(t) + g (|\psi_\uparrow|^2 + |\psi_\downarrow|^2) \right) \psi_\downarrow,
\]

here we work in dimensionless units by introducing the scales characterizing the trapping potential: the length is expressed in units of oscillator length \( \sqrt{\hbar/(m\omega_z)} \), the time is expressed in units of \( 1/\omega_{\perp} \), and the energy is expressed in units of oscillator energy \( \hbar\omega_{\perp} \). To obtain the periodic modulation of SOC, we make the strength of SOC oscillate at frequency \( \Omega \). In addition, to avoid nonadiabatic disturbances, we gradually switch on the periodically varying SOC and simultaneously turn off the radial confinement potential as [13, 45],

\[
\kappa(t) = f(t)(\kappa_0 + \kappa_\text{so} \sin \Omega t),
\]
\[
\omega_{\perp}^2(t) = 1 - f(t),
\]

where \( f(t) \) is a ramp function given by

\[
f(t) = \begin{cases} t/T & (0 \leq t \leq T) \\ 1 & (t > T). \end{cases}
\]

We here want to note that the ramp scheme \( f(t) \) is very important for the formation and the stability of the vector bright solitons for both fixed and periodically varying SOC. Without this scheme, i.e. \( f(t) = 1 \), no stable soliton can be formed. In addition, it is found that if the nonadiabaticity of ramp function is not negligible, the lowest exciting modes will be excited. The longer the ramp time \( T \), the better the adiabaticity. In the absence of SOC and external potential, equation (2) is nothing but a natural extension of Manakov model to the 2D setting [46]. In the presence of external potential and SOC, it is hard to obtain the exactly analytic solutions of equation (2). In what follows we numerically solve the time-dependent version of GP equations (2) with parameters described by equations (3)–(5) by using the time-splitting pseudospectral method [47], and systematically perform numerical simulations for a variety of the parameters to study the formation, stability and dynamics of vector bright solitons.

We gradually increase the strength of SOC and switch off the trap according to the liner ramp (5) with \( T = 10 \). For the initial states, it has been previously shown that the Townes soliton, which is fundamental to
understanding the self-similar collapse of solutions to the 2D nonlinear Schrödinger equation, can be stabilized by an adequate modulation of the nonlinearity [14, 48, 49]. This fact motivates us to build the explicitly initial states as the superposition of stabilized Townes solitons, which can be written as \( \psi_{\text{f1}} = \Phi_1 \equiv \alpha_{\text{f1}} \Phi_2 \) with the superposition coefficients being real constants and satisfying the normalization condition \( \alpha_{\text{f1}}^2 + \alpha_{\text{f2}}^2 = 1 \).

Without loss of generality, we set \( \alpha_{\text{f1}} = \alpha_{\text{f2}} = 1/\sqrt{2} \) throughout this work, and find similar results for different choices of the superposition coefficients. Here we want to note that the vortical components in the mixed-modes (MMs) of [43] (which are built as superposition of states with topological charges \((0, -1)\) and \((0, +1)\) in the two components) vanish in the limit of \( N \rightarrow N_c = 2N_c/(1 + \gamma) \), where \( N_c \) being the well-known collapse threshold for fundamental (Townes) soliton in the free 2D space and \( \gamma \) being the strength of the inter-component (XPM) interactions. The MM degenerates into the two-component Townes soliton, similarly to our initial states.

### 3. Dynamically stabilized vector bright solitons

For a SO-coupled system, metastable solitons can exist in 3D free space in the context of binary atomic condensates by combing contact self-attractive and SOC [44]. In the following section, the effects of both fixed and periodically varying SOC on the dynamics of the system are investigated. It is found that dynamically stabilized vector bright solitons can exist in a trapless 2D BEC by gradually turning on the SOC and simultaneously switching off the external potential, and its motional trajectory and stability show nontrivial behavior and strong dependence on the direction of the force generated by the SOC.

We first consider the system with fixed SOC, i.e. \( \kappa_{\text{f1}} = 0 \) and \( \kappa(t) = \kappa_{\text{f2}}(t) \). It is well known that the solitary wave in equation (2) without SOC is unstable against attractive interactions once the trap is removed. For the non-interacting system \((g = 0)\) with SOC, we gradually switch on the SOC and simultaneously switch off the external potential according to the ramp function given by equation (5).

It is observed that the two matter-wave packages are always spatially separated, showing phase separation along \( y \)-direction, which is similar to the SO-coupled dipolar BEC [50]. With time going on, the initially solitary-like wave packets are unstable and spread out gradually, accompanied by weak excitations, as shown in figure 1 for \( \kappa(t) = 0.5f(t) \). After completely switching off the external potential at \( t = 10 \), the SOC keeps constantly and its net effect is repulsive, leading to an unstable system. Hence we conclude that the system cannot be stabilized without the help of the attractive contact interactions.

In the presence of attractive contact interactions, such as \( g = -5 \) shown in figure 2(a), the wave packets diffuse to some extent but keeps the basic configuration unchanged. To give a clear description of the generic scenarios, the corresponding time evolutions of peak densities \( \max_r |\psi_{\text{f1}}(r, t)|^2 \) and monopole moments \( \langle r \rangle_{\text{f1}} = \int r |\psi_{\text{f1}}(r, t)|^2 \text{d}r \) are shown in figures 3(a) and 4(a), respectively. It is easy to see that within the period of ramp function, the peak densities of both components decrease with time, accompanied by rapid oscillations. After completely switching off the external potential, the peak densities exhibit slow oscillations and almost keep constants, which gives rise to the stability mechanism through the competition between the attractive contact interactions and SOC. The monopole moments of both components increase due to the slight spreadings of densities during the period of ramp function, as presented in figure 4(a). Here we want to note that the time evolutions of both the peak densities and monopole moments are isotropic throughout the whole process.

By increasing the amplitude of the fixed SOC from 0.5 to 0.6, it is interesting to observe that the wave packets basically retain their initial shapes, and dynamically stabilized vector bright solitons are formed after the external potential is switched off. This can be easily seen from the total density distribution of the system shown in the fifth column of figure 2(b), where the total density distribution of the system is almost unchanged during the

![Figure 1. The time evolution of the non-interacting system \((g = 0)\) for fixed SOC. The strength of SOC \( \kappa(t) = 0.5f(t) \) is gradually switched on and simultaneously the external potential is switched off from \( t = 0 \) to \( t = 10 \) according to the ramp function given by equation (5). The units of length is \( a_s = \sqrt{\hbar/(Ma_e)} \) and the scale of each plot is \( 6 \times 6 \) in units of \( a_e \).](image)
whole process. Compared with the former case, the oscillation frequencies of both the peak densities and the monopole moments exhibit relatively rapid oscillations, showing strong dependence on the strength of SOC, as shown in figures 3(b) and 4(b), respectively. In addition, it is found that the monopole moments decrease owing to the less spreading of densities. In this case, the repulsive SOC accurately balances the attractive contact interactions, resulting in the stability of the initial states after the external potential is removed. To further prove the stability of the vector solitons, the time evolution of $\langle y \rangle_{t=1} = \int |\psi_{1,1}(r,t)|^2 dr$ is exhibited in figure 5, from which we can see that it trends to a constant after the oscillations are averaged out. We want to note that only the average value of $y$ is shown here because $\langle x \rangle_{t=1} = \int x |\psi_{1,1}|^2 dr$ and $\langle r \rangle_{t=1} = \int r |\psi_{1,1}|^2 dr$ increase with time owing to the drift motion of the vector solitons, which will be discussed below.
Figure 4. The time evolution of monopole moment \( \langle r \rangle_{\pm 1} = \int |\psi_{\pm 1}|^2 \, |r| \, dr \) corresponding to figure 2, where the left column is for component \( \psi_{\uparrow} \), the right is for component \( \psi_{\downarrow} \), and the inset in (c) is the local expansion of \( \langle r \rangle_{2} \). The other parameters in (a)–(c) are the same as those in figures 2(a)–(c).

Figure 5. The time evolution of \( \langle y \rangle_{\pm 1} = \int y |\psi_{\pm 1}|^2 \, |y| \, dy \) corresponding to figure 2. The parameters in (a)–(c) are the same as those in figures 2(a)–(c).
A typical phenomenon in figure 2 is the whole system is moving spatially and deviating from the origin along the $-x$-direction. This can be understood by the fact that the SOC with ramp scheme not only balance the attractive contact interactions, but also generates the force that drives the soliton motion. To explicitly analyzing the mechanism of moving, we transform the Hamiltonian into the form neglecting the external potential and contact interactions

$$\mathcal{H} = \frac{p_x^2}{2} + \frac{p_y^2}{2} + \kappa(t)(p_x\sigma_x + p_y\sigma_y)$$

$$= \frac{(p_x + \kappa(t)\sigma_x)^2}{2} + \frac{(p_y + \kappa(t)\sigma_y)^2}{2} - \frac{\kappa^2(t)}{2},$$

where $p_x$ and $p_y$ are the momentum along $x$- and $y$-direction, respectively, and the vector potential can be written as

$$\vec{A} = x\kappa(t)\sigma_x + y\kappa(t)\sigma_y.$$  

(7)

The force induced by the SOC can be written as \[ F_A = \frac{\partial \vec{A}}{\partial t} = x\frac{\partial \kappa(t)}{\partial t}\sigma_x + y\frac{\partial \kappa(t)}{\partial t}\sigma_y. \]

(8)

We calculate the average value of $\langle \sigma_{x,y,z} \rangle = \psi^\dagger \sigma_{x,y,z} \psi$, and obtain

$$\sum \langle \sigma_x \rangle \gg \sum \langle \sigma_y \rangle \simeq 0,$$

$$\sum \langle \sigma_y \rangle \gg \sum \langle \sigma_z \rangle \simeq 0.$$  

(9)

It is easy to see that compared with $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$, the force acting on the system mainly comes from $\langle \sigma_z \rangle$. To give an analytic analysis, we further employ the Gaussian variational method, where the variational wave functions are assumed to be

$$\psi_i = \frac{1}{\sqrt{2\pi R}} e^{-\frac{e^2(x_0+\delta i)^2}{2R^2}} e^{ikx},$$

$$\psi_i = \frac{1}{\sqrt{2\pi R}} e^{-\frac{e^2(x_0+\delta i)^2}{2R^2}} e^{ikx},$$

(10)

where $R$, $\delta$, $K$ are variational parameters. Substituting equation (10) into the Gross–Pitaevskii energy functional, we obtain the total variational energy as

$$E = \frac{1}{2} \left( \frac{1}{R^2} + K^2 \right) + \kappa e^{-\frac{\delta^2}{R^2}} \left( -\frac{\delta}{R^2} + K \right) + \frac{g}{8\pi R^2} \left( 1 + e^{-2\delta^2/R^2} \right).$$

(11)

Taking the parameters in figure 2(b) as an example, we find that the energy is minimized by $R \approx 1.603$, $\delta \approx 0.513$, and $K \approx -0.542$ for $\kappa = 0.6$ and $g = -5$, which are in reasonable agreement with our numerical results.

Furthermore, we consider the Heisenberg equation of motion

$$\frac{dr}{dt} = -i[H, \psi] = p + \kappa \sigma \equiv P.$$  

(12)

We note that $p$ is conversed but $P$ is not in the uniform system. The expectation of value of $P$ with respect to the variational function is given by

$$\langle P_x \rangle = K + \kappa e^{-\frac{\delta^2}{R^2}}. \quad \langle P_y \rangle = 0.$$  

(13)

For $R = 1.603$ and $\delta = 0.513$, we find $\kappa \langle \sigma_z \rangle = \kappa e^{-\delta^2/R^2} = 0.542$, which leads $\langle P_x \rangle = 0$. Thus, in the soliton at rest, the momentum $\langle P_x \rangle = K < 0$ balances with the SOC momentum $\kappa \langle \sigma_z \rangle > 0$.

Then we consider the Hamiltonian in the moving frame,

$$\mathcal{H}_v = \mathcal{H} - v p_x,$$  

(14)

where $v$ is the velocity in the $x$-direction. For example, we set $v = \pm 0.3$. By minimizing $\langle \mathcal{H}_v \rangle$, we find $\langle \mathcal{H}_v \rangle = -0.09$ for $v = 0.3$ and $\langle \mathcal{H}_v \rangle = -0.413$ for $v = -0.3$. Thus the soliton moving leftward has smaller energy than the soliton moving rightward. By the transformation $e^{-i\pi (Lx + \sigma_z)}$, the moving direction can be rotated by $\pi$ without changing the energy, and therefore, there also exists a soliton moving at $v = 0.3$ with $\langle \mathcal{H}_v \rangle = -0.413$. Thus, the above soliton with $v = 0.3$ and $\langle \mathcal{H}_v \rangle = -0.09$ is energetically unstable, which implies that acceleration in the $+x$-direction makes the soliton unstable.

When the weak force $f$ is exerted in the $\pm x$-direction, the Heisenberg equation becomes $\frac{dp_x}{dt} = f$, and hence $K$ increases as $K = K(0) + ft$. For example, we consider the moment at $ft = \pm 0.5$. By minimizing $\langle \mathcal{H} \rangle$ with
\( K = K(0) + 0.5 \) and \( K = K(0) - 0.5 \) being fixed, we obtain \( R = 1.572, \delta = 0.852 \) and \( R = 2.170, \delta = 0.396 \), respectively. These values give \( \langle P_x \rangle = K + \kappa e^{-x^2/R^2} = 0.406 \) and \(-0.461\), which are slower than 0.5 expected for the case without SOC. The force in the \(+x\)-direction more accelerates the soliton than in the \(-x\)-direction, resulting in the motions of vector bright solitons toward the \(-x\)-direction. See appendix for the movies of the dynamics of the vector bright solitons.

To give a clear description of the motion of vector bright solitons, the trajectories of the center of mass of solitons are presented in figure 6, where axisymmetric motion of vector bright solitons is observed. It is interesting to observe that motion of solitons is tend to be quasi-periodic after the external potential is switched off, and the period strongly depend on the strength of SOC.

Finally, we move to the case with periodic modulation of SOC since our initial motivation of this work is to demonstrate that whether the matter-wave bright solitons can be stabilized in 2D free space by causing the strength of SOC to oscillate rapidly. Figure 2(c) shows the time evolutions of the system with periodically varying SOC \( \kappa = f(t)(0.6 + 0.2 \sin(30t)) \). Compared with the case without the oscillating term \( 0.2 \sin(30t) \) shown in figure 2(b), there is no distinct difference in the whole dynamic evolution process and we again find a similar moving behavior of the stabilized vector bright solitons.

More insights can be obtained if we look at the time evolutions of the peak density and monopole moment, and the trajectories of the center of mass of the vector bright solitons, which are shown in figures 3(c), 4(c), and 6(c) with local expansion plots, respectively. As shown in such figures, although the global dynamics of the system is similar to the cases with fixed SOC, there exists locally rapid oscillating in the time evolutions of both peak density and monopole moment. Actually, the dynamics of such a system can be separated into two parts: a rapidly oscillating part with small amplitude and a slow, smoothly varying part, which is in a sense reminiscent of the cases in [13, 14]. Consequently, we conclude that the periodical varying SOC has no effect on the stability of the system essentially, but it cause the locally rapid oscillations in dynamics.

In real experiments, the contact interactions can be changed by modifying atomic collisions, which are experimentally feasible due to the flexible and precise control of the scattering lengths achievable by
magnetically tuning the Feshbach resonances. The strength of SOC can be precisely controlled by optics means, and the external trapping potential can be changed independently by using an optical trap. However, in current experiments, it is actually difficult to realized time-oscillating SOC by using the Raman coupling. In recent years, many groups have proposed to realize the SOC by using magnetic field gradient or the optical Raman lattice. With the progress of new experimental scheme, we hope that experimental physicists will be interested in the realization of time-oscillating SOC. Finally, we note that all the nonlinear contact parameters are set equal to each other in this work, but most of our results with unequal contact interactions will be qualitatively the same.

Last but not least, we would like to emphasize that in this work, only a simple 2-component spinor BEC is considered. The stable and metastable vortex-bright solitons were investigated in detailed in a two- and three-dimensional SO-coupled three-component hyperfine spin-1 BEC\[52–54\], and the generation, phase separation, and collision dynamics of overlapping quasi-1D vector solitons are investigated in \[55\]. Our work can be extended to a spinor three-component system, where more free parameters can be included and the interplay between short-range contact interactions and SOC can lead to more abundant dynamic behaviors.

4. Conclusions

To summarize, we have investigated the formation, stability and dynamics of a trapless binary self-attractive BEC in the presence of both fixed and periodically varying SOC. We found that dynamically stabilized vector bright solitons can be formed in 2D free space if we gradually turning on the SOC and simultaneously switching off the external potential. In the presence of periodically varying SOC, the system shows locally rapid oscillations in dynamics. Moreover, by using variational approximation and numerical simulations, we found that the SOC not only balance the attractive contact interactions but also generates the force that drives the solitons motion. The dynamics of such solitons show nontrivial behavior and its stability show strong dependence on the direction of the force generated by the SOC.

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Appendix. The dynamics and stability of vector bright solitons under forces in different directions

We investigate the dynamics and stability of vector bright solitons under forces in different directions, as shown in the supplemental material\[5\], where the density difference $|\psi_1|^2 - |\psi|^2$ are plotted in all the animations. We first prepare the stationary solitons in free space using the imaginary-time method, and then the forces in different directions are exerted on such solitons.

Animation 1 shows the dynamics and stability of the solitons under the force in $-x$-direction. The solitons are accelerated in the $-x$-direction as expected, which are similar to the states of figure 9 in [43]. In Animation 2, the force is exerted in the $+x$-direction. It is easy to see that the solitons disappear immediately. This indicates that the stability of solitons strongly depends on the direction of the force, which agree well with our numerical results shown in figure 2.

Next, we prepare the stationary solitons in the rotating frame with a weak harmonic potential, since there is no stable solution without harmonic potential. Animation 3 shows the dynamics in the laboratory frame. We see that the solitons rotate in the weak harmonic potential. Animation 4 shows the case of a different initial state, where the solitons rotates faster than the above case in the same harmonic potential.

Consequently, we conclude that the dynamics and stability of the vector bright solitons shows nontrivial behavior and strong dependence on the direction of the force generated by the SOC.

See supplemental material is available online at stacks.iop.org/NJP/22/033006/mmedia for movies of the dynamics and stability of vector bright solitons under forces in different directions.
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