A Straightforward Procedure for Deriving the Biaxial Interaction Diagrams of RC Sections in Fire

Duc Toan Pham 1, Hong Hai Nguyen 2, Sabine Boulvard 1

1 Centre Scientifique et Technique du Bâtiment (CSTB), 84 avenue Jean Jaurès, Champs-sur-Marne, 77447 Marne-la-Vallée Cedex 2, France
2 INFRANEO, 140 avenue Jean Lolive, 93500 Pantin, France
ductoan.pham@cstb.fr

Abstract. Based on the lower bound static approach of the yield design (or limit analysis) theory, this contribution presents a straightforward computational procedure for establishing the biaxial interaction diagrams of RC sections in fire conditions, taking into account the experimentally-based relationships linking the degradation of material strength properties to the temperature increase. In the present approach, material characteristics are introduced in two steps: (i) a preliminary heat transfer analysis for evaluating the temperature distribution on the RC section and (ii) the introduction of reduced factors as functions of temperature into both the concrete and steel strength properties. For illustrative purpose, calculations will be conducted on a typical RC section subjected to different fire exposures. Finally, the theoretical predictions will be compared to those obtained from numerical simulations using a finite element software.

1. Introduction

The biaxial interaction diagrams represent a widely used practical tool for the design of reinforced concrete (RC) sections under axial load and bending moments about their two axes. At ambient temperature, these biaxial interactions diagrams can be classically established within the framework of ultimate limit state design (ULSD) where the strain limitations of concrete and reinforcing steel are prescribed (see for example EN 1992-1-1 [1]). However, the implementation of such a calculation procedure becomes much more complex in the case of RC sections subjected to fire loading, since the strain limitations of concrete and reinforcing steel are now temperature dependent. In addition, elevated temperature leads to a degradation of constituent material properties.

Based on the lower bound static approach of the yield design (or limit analysis) theory (Salençon [2], de Buhan [3]), this contribution presents a straightforward computational procedure for calculating the biaxial interaction diagrams of RC sections subjected to fire loading, in addition to classical mechanical loading conditions. Indeed, while the classical methods are usually based on conventional limitations of strains of constituent materials, the yield design approach only requires that their stress (and not strain) limitations be prescribed, with no reference to other mechanical characteristics. In this case material characteristics may be introduced in two steps: (i) a preliminary heat transfer analysis for evaluating the temperature distribution on the RC section and (ii) the introduction of reduced factors as functions of temperature into both the concrete and steel strength properties (see Pham et al. [4] for more details). As a result, a semi-analytical solution of axial load-biaxial bending moments failure surfaces could be obtained for any prescribed temperature...
distribution. For illustrative purpose, calculations will be conducted on a typical RC section subjected to different fire exposures. Finally, the theoretical predictions will be compared to those obtained from numerical simulations using a finite element software.

2. Material strength properties
In what follows, it will be assumed that spalling of concrete does not occur and the bonding between the reinforcing bars and the surrounding concrete is perfect. In addition, the tensile strength of concrete is neglected due to the brittleness of concrete failure under tensile stresses.

Following the approach proposed by Chen [5], Nielsen and Hoang [6], Bleyer and de Buhan [7] or more recently Pham et al. [4], a modified Mohr-Coulomb failure condition is adopted for the strength criterion of concrete material. With the tensile strength neglected, the expression of the criterion may be written as follows:

\[
\phi^c(\sigma) = \sup_{i,j=1,2,3} \left\{ \sigma_i (1 + \sin \varphi) - \sigma_j (1 - \sin \varphi) - f_i (1 - \sin \varphi) \right\} \leq 0
\]  

(1)

where \( \sigma_i, i = 1,2,3 \), are the principal stress components, \( f_i \) is the uniaxial compressive strength and \( \varphi \) is the internal friction angle at ambient temperature.

Neglecting the shear and bending resistance of steel reinforcing bars, their strength properties are characterized by the following condition on the axial force \( n \):  

\[
|n| \leq n_0 = A_s f_y
\]  

(2)

where \( n_0 \) is its tensile-compressive resistance, equal to the product of the steel yield strength \( f_y \) by its cross-sectional area \( A_s \).

Figure 1. Reduction factors as decreasing functions of temperature (EN 1992-1-2 [7])

In fire conditions, the internal friction angle of concrete is assumed to remain unaffected by the temperature increase while its compressive strength \( f_c \) and the yield strength \( f_y \) of steel will be reduced through the introduction of the non-dimensional multiplicative factors (Figure 1):
where \( k_c \) and \( k_s \) are decreasing functions of the temperature \( \theta \), equal to one for \( \theta = 20^\circ \text{C} \) (ambient temperature).

3. Yield design auxiliary problem for the determination of the biaxial interaction diagram

Referring to an orthonormal frame \( Oxyz \) (Fig. 2), the structure under consideration is a beam of thickness \( h \) along the \( Oz \)-axis, width \( b \) along the \( Oy \)-axis and length \( L \) along the \( Ox \)-axis. It is subjected to mechanical loading conditions as follows:

- Body forces (self-weight) are neglected.
- The section \( x = 0 \) is in smooth contact with a fixed vertical plane.
- The section \( x = L \) is in smooth contact with a plane in rotation about \( \Delta = \{(y, z)|\delta - z\alpha_y - y\alpha_z = 0\} \)
- Lateral sides are stress free.

Figure 2. RC beam considered for the determination of the biaxial interaction diagram

In these conditions \( \delta \), \( \alpha_y \) and \( \alpha_z \) may be interpreted as the generalized kinematic variables associated by duality to the generalized stress variables \( (N, M_y, M_z) \) (see Bleyer and de Buhan [8] for more details).

Considering any kinematically admissible (K.A.) velocity field \( U \) (which complies with the velocity boundary conditions), the work of the external forces in any such field may be written as follows:

\[
W_e(U) = N\dot{\delta} + M_y\dot{\alpha}_y + M_z\dot{\alpha}_z
\]

where \( N, M_y \) and \( M_z \) may be interpreted as the axial force along \( Ox \) and bending moments about \( Oy \) and \( Oz \) exerted on the section \( x = L \) of the beam, which is thus subjected to a three-parameter loading mode.

According to the yield design theory (Salençon [2], de Buhan [3]), the so-called domain \( K \) of potentially safe loads \( (N, M_y, M_z) \) is the set of loads which can be equilibrated by a stress distribution
in the beam (stress tensor fields in the concrete, tensile force distributions along the reinforcements), verifying the respective strength conditions (1) and (2) on any point of the beam. The boundary of this domain is called the biaxial interaction diagram of the RC beam subjected to combined axial load and biaxial bending moments.

4. **Lower bound static approach**

Given any position of Δ, the two uniaxial stress distributions shown in figure 3 are considered. In configuration shown in figure 3(a) both concrete and reinforcing bars reach their positive tensile (resp. negative compressive) strengths when located below (resp. above) Δ. The opposite applies in configuration shown in figure 3(b).

![Figure 3](image)

**Figure 3.** Stress distributions in the RC beam used in the lower bound static approach of yield design

The corresponding values of the three loading parameters \( N, M_y, \) and \( M_z \) in equilibrium with such stress distributions may be calculated, leading to the determination of the biaxial interaction diagram of the RC beam under the combined actions of axial load and biaxial bending moments.

5. **Illustrative examples and preliminary comparison with numerical simulations using a commercial software**

For illustrative purpose, the following set of data has been selected (Figure 4):

- Rectangular section \( b \times h = 300 \times 500 \) mm²,
- Normal weight concrete with siliceous aggregates, \( f_c = 30 \) MPa,
- Six hot rolled reinforcing steel bars of diameter 16 mm, with 36 mm of concrete cover, \( f_y = 500 \) MPa,
- Reduction factors \( k_c \) and \( k_s \) are considered temperature dependent referred to EN 1992-1-2 [7] (Figure 1).
- Four faces exposed to ISO 834 fire (EN 1991-1-2 [9]).

![Diagram of RC cross-section](image)

**Figure 4.** Example of a RC cross-section used for illustrative purpose

The temperature distribution of the RC cross-section may be obtained by either the use of any thermal software or available experimental results. Figure 5 shows, for example, the temperature distributions predicted for instance by SAFIR program [10], while figure 6 displays the corresponding \((M_x, M_y)\) interaction curves for different axial compressive loads \(N\). As expected, it can be immediately seen that the temperature increase leads to a significant decrease of the interaction curves.

![Temperature distributions](image)

**Figure 5.** Temperature distributions on the beam cross-section for 1 h and 2 h of fire exposures
Figure 6. Examples of \((M_y, M_z)\) interaction curve for different axial loads \(N\) (at different fire exposures)

For comparative purpose, numerical simulations of a RC beam subjected to fire have also been carried out using SAFIR software [10]. It should be noted that the 2D heat transfer analysis (Figure 5) is independently performed on the beam cross-section, without taking into account the mechanical boundary conditions. The beam is then modelled as an assemblage of independent “fibers”. The stress-strain relationships of concrete and steel at elevated temperature are those proposed in EN 1992-1-2 [7]. Figure 7 shows that the points resulting from such numerical simulations are close to those predicted by the proposed method.

Figure 7. Comparison between the numerical results with those derived from the proposed method

6. Conclusions
In this contribution, the lower bound static approach of the yield design has been applied to the determination of the biaxial interaction diagrams of RC sections subjected to fire loading, in addition to classical mechanical loading conditions. The main feature of the proposed approach is the introduction of reduced factors as functions of temperature into both the concrete and steel strength properties. A semi-analytical solution of such failure loads may be obtained for any prescribed temperature distribution, making it possible to study the influence of fire-induced temperature increase on the strength reduction of RC structural members. Due to its simplicity, the proposed method allows performing parametric studies in a rather quick way. In addition, it has been shown that the theoretical
predictions are favourably compared with those obtained from numerical simulations using a finite element software.

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