Numerical Solution of One-dimensional Telegraph Equation using Cubic B-spline Collocation Method

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Abstract
In this paper, a collocation approach is employed for the solution of the one-dimensional telegraph equation based on cubic B-spline. The derived method leads to a tri-diagonal linear system. Computational efficiency of the method is confirmed through numerical examples whose results are in good agreement with theory. The obtained numerical results have been compared with the results obtained by some existing methods to verify the accurate nature of our method.

Keywords: Telegraph equation; Cubic B-spline method; Collocation

1 Introduction

We consider the following second order one-dimensional linear hyperbolic equation:

\[ \frac{\partial^2 u}{\partial t^2} + 2\alpha \frac{\partial u}{\partial t} + \beta^2 u = \frac{\partial^2 u}{\partial x^2} + f(x,t), \quad a \leq x \leq b, \quad t \geq 0, \]  

(1.1)

with the initial conditions,

\[ u(x, 0) = f_0(x), \quad a \leq x \leq b, \]  

(1.2)

\[ \frac{\partial u(x, 0)}{\partial t} = f_1(x), \quad a \leq x \leq b, \]  

(1.3)

and subject to the boundary conditions,

\[ u(a, t) = g_0(t), \quad u(b, t) = g_1(t), \quad t \geq 0, \]  

(1.4)

where \(\alpha\) and \(\beta\) are known positive constant coefficients. We assume that \(f_0(x), f_1(x)\) and their derivatives are continuous functions of \(x\), and \(g_0(t), g_1(t)\) and their derivatives are continuous functions of \(t\). Equations of the form (1.1) arise in the study of propagation of electrical signals in a cable of transmission line and wave phenomena. In fact the telegraph equation is more suitable than ordinary diffusion equation in modeling reaction diffusion [1, 2]. Furthermore, we should mention that with the appropriate coefficient and forcing terms, the one-dimensional telegraph equation describes a diverse array of physical systems; for example, the propagation of voltage and current signals.

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in coaxial transmission lines of negligible leakage conductance and/or resistance [3]. For $\alpha > 0$, $\beta = 0$ Eq. (1.1) represents a damped wave equation.

During past years, much attention has been given in the literature to the development, analysis and implementation of stable methods for the numerical solution of second-order hyperbolic equation, (see [4]). Recently, Mohanty et al. [5, 6], developed new three-level implicit unconditionally stable alternating direction implicit schemes for the two and three-space dimensional linear hyperbolic equations. These schemes are second-order accurate both in space and time. Dehghan and Shokri [7] developed a numerical method to solve the one-dimensional telegraph equation using Thin Plate Splines (TPS) Radial Basis Function (RBF). A numerical method based on the interpolating scaling functions were described by Lakestani and N. Saray [8]. Evans and Hasan [9] applied an Alternating Group Explicit (AGE) method to obtain numerical solution of the telegraph equation.

In this paper for the second order one-dimensional linear telegraph equation a two-level explicit spline-difference method to obtain numerical solution of the telegraph equation.

**2 Description of method**

We consider a uniform mesh $\Delta$ with the grid points $\lambda_{j,n}$ to discretize the region $\Omega = [a, b] \times [0, T]$. Each $\lambda_{j,n}$ is the vertices of the grid points $(x_j, t_n)$, where $x_j = a + jh$, $j = 0, 1, 2, \ldots, N$ and $t_n = nk$, $n = 0, 1, 2, \ldots$ and $h$ and $k$ are mesh sizes in the space and time direction respectively.

First of all, we discretize the problem in time variable using the following finite difference approximation with uniform mesh sizes in the space and time direction respectively.

The vertices of the grid points $(x_j, t_n)$, where $x_j = a + jh$, $j = 0, 1, 2, \ldots, N$ and $t_n = nk$, $n = 0, 1, 2, \ldots$ and $h$ and $k$ are mesh sizes in the space and time direction respectively.

In section 2, we present a finite-difference approximation to discretize the Eq. (1.1) in time variable. In section 3, we apply cubic B-spline collocation method to solve the problem in space direction. In section 4, numerical experiments are conducted to demonstrate the efficiency of the proposed method computationally.

This paper is organized as follows. In section 2, we present a finite-difference approximation to discretize the Eq. (1.1) in time variable. In section 3, we apply cubic B-spline collocation method to solve the problem in space direction. In section 4, numerical experiments are conducted to demonstrate the efficiency of the proposed method computationally.

Note that we have computed the numerical results by Matlab programming.

In this paper, we discretize the Eq. (1.1) in space variable. In this paper, we discretize the Eq. (1.1) in space variable.

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B-spline functions have some attractive properties. Due to the being piecewise polynomial, they can be integrated and differentiated easily. Since they have compact support, numerical methods in which B-spline functions are used as a basic function lead to matrix system including band matrices [10]. Such systems can be handled and solved with low computational cost. Therefore, spline solutions of partial differential equations are suggested in many studies [11, 12].

In this paper, we discretize the problem in space variable using the following finite difference approximation with uniform mesh sizes in the space and time direction respectively.

First of all, we discretize the problem in time variable using the following finite difference approximation with uniform step size $k$.

\[ u^n_j \approx \frac{u^{n+1}_j - 2u^n_j + u^{n-1}_j}{k^2}, \tag{2.5} \]

\[ u^n_j \approx \frac{u^{n+1}_j - u^{n-1}_j}{2k}, \tag{2.6} \]

and

\[ u^n_{x x} \approx \frac{u^{n+1}_{x x} + u^{n-1}_{x x}}{2}. \tag{2.7} \]

Substituting the above approximations into discretized form of Eq. (1.1), we have,

\[ \frac{u^{n+1}_j - 2u^n_j + u^{n-1}_j}{k^2} + 2\alpha \frac{u^{n+1}_j - u^{n-1}_j}{2k} + \beta^2 u^n_j = \frac{(u_{x x})^{n+1}_j + (u_{x x})^{n-1}_j}{2} + f(x_j, t_n), \tag{2.8} \]

thus, we obtain,

\[ (1 + \alpha k)u^{n+1}_j - \frac{k^2}{2} (u_{x x})^{n+1}_j = \frac{k^2}{2} (u_{x x})^{n-1}_j + \frac{k^2}{2} f(x_j, t_n) - (\beta^2 k^2 - 2)u^n_j - (1 - \alpha k)u^{n-1}_j, \tag{2.9} \]

after some simplifications, the above equation can be written in the following form,

\[ (1 + \alpha k)u^* - \frac{k^2}{2} (u_{x x})^* = r(x), \tag{2.10} \]
where,
\[
r(x) = \frac{k^2}{2} u''_{xx} + k^2 f(x, t_n) - (\beta^2 k^2 - 2)u'' - (1 - \alpha k)u''_{xx},
\]
with the boundary conditions,
\[
u'(a) = g_0(t_n), \quad u'(b) = g_1(t_n).
\]
In order to start any computations using the above formula we need the values of \(u\) at the nodal points at the zero and first level times.

| Table 1. Values of \(B_j(x)\) and its derivatives at the nodal points. |
|-----------------|-------|-------|-------|-------|-------|
| \(x\)           | \(x_{j-2}\) | \(x_{j-1}\) | \(x_j\) | \(x_{j+1}\) | \(x_{j+2}\) |
| \(B_j(x)\)      | 0     | 1     | 4     | 1     | 0     |
| \(B'_j(x)\)     | 0     | 3/h   | 0     | -3/h  | 0     |
| \(B''_j(x)\)    | 0     | 6/h^2 | -12/h^2 | 6/h^2 | 0     |

To compute \(u^1\) we may use the initial conditions \(u(x, t_0) = f_0(x)\) and \(u_t(x, t_0) = f_1(x)\). Using Taylor series for \(u\) at \(t = t_0 + k\) following [13] we have,
\[
u^1 = u^0 + ku^0_t + \frac{k^2}{2} u''^0 + O(k^3),
\]
u^0 and \(u_t^0\) are known from initial conditions exactly thus we need to compute term \(u''^0\). By using Eq. (1.1) we obtain,
\[
u''^0 = [u_{xx} + f(x, t) - 2\alpha u_t - \beta^2 u]\big|_{t=0}.
\]
Now substituting (2.14) and initial conditions into (2.13) we can obtain an approximation for \(u\) at \(t = t_0 + k\),
\[
u^1 = f_0(x) + kf_1(x) + \frac{k^2}{2} [u_{xx} + f(x, t) - 2\alpha u_t - \beta^2 u]\big|_{t=0} + O(k^3).
\]

3 B-spline collocation method
In this section we use the B-spline collocation method to solve Eq. (1.1) with the boundary conditions (2.12). Let \(\Delta^* = \{a = x_0 < x_1 < \ldots < x_N = b\}\) be the partition in \([a, b]\). We define the cubic B-spline for \(j = -1, 0, \ldots, N + 1\) by the following relation in [14] as,
\[
B_{3, j} = \frac{1}{h^3} \begin{cases} 
(x - x_{j-2})^3, & x \in [x_{j-2}, x_{j-1}], \\
3h(x - x_{j-1}) + 3h(x - x_{j-1})^2 - 3(x - x_{j-1})^3, & x \in [x_{j-1}, x_j], \\
3h(x_{j+1} - x) + 3h(x_{j+1} - x^2) - 3(x_{j+1} - x)^3, & x \in [x_j, x_{j+1}], \\
(x_{j+2} - x)^3, & \text{otherwise}.
\end{cases}
\]

Our numerical treatment for solving Eq. (1.1) using the collocation method with cubic B-splines is to find an approximate solution \(\hat{S}(x)\) to exact solution \(u(x, t)\) in the form,
\[
\hat{S}(x) = \sum_{j=-1}^{N+1} \hat{c}_j(t) B_{3, j}(x),
\]
where \(\hat{c}_j(t)\) are unknown time dependent parameters to be determined from the boundary conditions and collocation of the differential equation. The values of \(B_{3, j}(x)\) and its two derivatives may be tabulated as in Table 1.

Using approximate function (3.16) and cubic B-spline (3.15), the approximate values at the knots of \(\hat{S}(x)\) and its derivatives are determined in terms of the time dependent parameters \(\hat{c}_j(t)\) as,
\[
\hat{S}(x) = \hat{c}_{j-1} + 4\hat{c}_j + \hat{c}_{j+1},
\]
where \(\hat{c}_j(t)\) are unknown time dependent parameters to be determined from the boundary conditions and collocation of the differential equation. The values of \(B_{3, j}(x)\) and its two derivatives may be tabulated as in Table 1.
\[
\dot{S}(x) = 3(\dot{c}_{j+1} - \dot{c}_{j-1}), \quad (3.18)
\]
\[
\ddot{S}(x) = 6(\dot{c}_{j-1} - 2\dot{c}_j + \dot{c}_{j+1}). \quad (3.19)
\]

Let \(\dot{S}(x)\) satisfies the equation (2.10) plus the boundary conditions, thus we have,
\[
L \dot{S}(x_j) = r(x_j), \quad 0 \leq j \leq N, \quad (3.20)
\]
\[
\dot{S}(x_0) = g_0(t_n), \quad \dot{S}(x_N) = g_1(t_n),
\]
where \(Lu = (1 + \alpha k)u - \frac{k^2}{2}(u_x)_x\). Substituting (3.16) into (3.20) and using (3.17) and (3.19), we have,
\[
(1 + \alpha k)(\dot{c}_{j-1} + 4\dot{c}_j + \dot{c}_{j+1}) - \frac{k^2}{h^2}(\dot{c}_{j-1} - 2\dot{c}_j + \dot{c}_{j+1}) = 0, \quad 1 \leq j \leq N - 1,
\]
simplifying the above equation leads to the following system of linear equations,
\[
((1 + \alpha k)h^2 - 3k^2)\dot{c}_{j-1} + (4(1 + \alpha k)h^2 + 6k^2)\dot{c}_j + ((1 + \alpha k)h^2 - 3k^2)\dot{c}_{j+1} = h^2r_j, \quad 1 \leq j \leq N - 1.
\]

To obtain a unique solution for \(\dot{C} = (\dot{c}_{-1}, \dot{c}_0, ..., \dot{c}_{N+1})\) we need to use the boundary conditions. Using the first boundary condition we have,
\[
u(a, t_n) = \dot{S}(a) = g_0(t_n) = \dot{c}_{-1} + 4\dot{c}_0 + \dot{c}_1, \quad (3.23)
\]
by eliminating \(\dot{c}_{-1}\) from the above equation and the equation (3.22) for \(j = 0\) we have,
\[
18k^2\dot{c}_0 = h^2r_0 + (3k^2 - h^2(1 + \alpha k))g_0(t_n).
\]

Similarly, using the boundary condition,
\[
u(b, t_n) = \dot{S}(b) = g_1(t_n) = \dot{c}_{N-1} + 4\dot{c}_N + \dot{c}_{N+1},
\]
and eliminating \(\dot{c}_{N+1}\) from the above equation and equation (3.22) for \(j = N\) we have,
\[
18k^2\dot{c}_N = h^2r_N + (3k^2 - h^2(1 + \alpha k))g_1(t_n).
\]

Associating (3.24) and (3.26) with (3.22), we obtain a linear \((N+1) \times (N+1)\) system of equations which can be written in the matrix form as,
\[
A\dot{c} = \dot{b}, \quad (3.27)
\]
where,
\[
\begin{bmatrix}
18k^2 & 0 & 0 \\
(1 + \alpha k)h^2 - 3k^2 & 4(1 + \alpha k)h^2 + 6k^2 & (1 + \alpha k)h^2 - 3k^2 \\
& \ddots & \ddots \\
(1 + \alpha k)h^2 - 3k^2 & 4(1 + \alpha k)h^2 + 6k^2 & 0 \\
0 & 0 & (1 + \alpha k)h^2 - 3k^2
\end{bmatrix}
\]
\[
A = \begin{bmatrix}
(1 + \alpha k)h^2 - 3k^2 & 4(1 + \alpha k)h^2 + 6k^2 & (1 + \alpha k)h^2 - 3k^2 \\
& \ddots & \ddots \\
(1 + \alpha k)h^2 - 3k^2 & 4(1 + \alpha k)h^2 + 6k^2 & 0 \\
0 & 0 & (1 + \alpha k)h^2 - 3k^2
\end{bmatrix},
\]
\[
(3.28)
\]
\[
\hat{\beta} = \begin{pmatrix}
\frac{h^2 r_0 + (3k^2 - h^2(1 + \alpha k))g_0(t_n)}{h^2 r_1} \\
\vdots \\
\frac{h^2 r_{N-1} + (3k^2 - h^2(1 + \alpha k))g_1(t_n)}{h^2 r_N}
\end{pmatrix},
\hat{\epsilon} = \begin{pmatrix}
\hat{\epsilon}_0 \\
\hat{\epsilon}_1 \\
\vdots \\
\hat{\epsilon}_N
\end{pmatrix}.
\]

(3.29)

Since \(\alpha > 0\), it is easily seen that \(A\) is strictly diagonally dominant, hence nonsingular by Gershgorin’s theorem (see [14] and the references therein). Since \(A\) is nonsingular, we can solve (3.27) for \((\hat{\epsilon}_0, \hat{\epsilon}_1, \ldots, \hat{\epsilon}_N)\) and substitute into the boundary equations (3.23) and (3.25) to obtain \(\hat{\epsilon}_{-1}\) and \(\hat{\epsilon}_{N+1}\). Hence the method of collocation applied to Eq. (1.1) using a basis of cubic B-splines has a unique solution given by (3.16).

4 Numerical examples

To illustrate the efficiency and applicability of our present method computationally, we consider three examples of linear telegraph equations in [2], which their exact solutions are known. We applied our method to these examples with different values of step lengths and computed solutions are compared with method in [2]. The \(L_{\infty}\), \(L_2\) and RMS errors in the solutions are tabulated in Table 2-5. These tables verify the accurate values of our present method.

**Example 4.1.** We consider the hyperbolic telegraph Eq. (1.1) with \(\alpha = \frac{1}{2}\) and \(\beta = 1\) in the interval \(0 \leq x \leq 1\). Subject to the initial conditions,

\[
\begin{align*}
&u(x, 0) = 0, \quad 0 \leq x \leq 1, \\
&u_t(x, 0) = 0, \quad 0 \leq x \leq 1,
\end{align*}
\]

and with boundary conditions,

\[u(0, t) = u(1, t) = 0.\]

(4.30)

The right hand side function is \(f(x, t) = (2 - 2t + t^2)(x - x^2)\exp(-t) + 2r^2\exp(-t)\). The exact solution is \(u(x, t) = (x - x^2)t^2\exp(-t)\).

This example has been solved by our method with the step sizes of \(k = 0.01\) and \(h = 0.005\) for various time \(t = 1, 2, \ldots, 5\). The computed solutions are compared with the exact solution and \(L_{\infty}\), \(L_2\)-errors and Root-Mean-Square (RMS) of errors in the solutions are tabulated in Table 2. These results verify that our method is considerable accurate than the method in [2].

| methods          | Time parameter | \(L_2\)-error  | \(L_{\infty}\)-error | RMS     |
|------------------|----------------|----------------|----------------------|---------|
| present method   | \(t = 1\)      | \(k = 0.01\), \(h = 0.005\) | 3.7306e−005          | 3.4031e−006 | 2.6314e−006 |
|                  | \(t = 2\)      | \(k = 0.01\), \(h = 0.005\) | 8.1795e−005          | 7.7810e−006 | 5.7694e−006 |
|                  | \(t = 3\)      | \(k = 0.01\), \(h = 0.005\) | 1.8810e−005          | 1.7217e−006 | 1.3267e−006 |
|                  | \(t = 4\)      | \(k = 0.01\), \(h = 0.005\) | 6.6650e−006          | 5.7489e−007 | 4.7011e−007 |
|                  | \(t = 5\)      | \(k = 0.01\), \(h = 0.005\) | 1.0521e−005          | 9.6949e−007 | 7.4210e−007 |
| Dosti and Nazemi[2] | \(t = 1\)      | \(k = 0.01\), \(h = 0.005\) | 1.4532e−004          | 1.9175e−004 | —         |
|                  | \(t = 2\)      | \(k = 0.01\), \(h = 0.005\) | 8.1135e−005          | 1.1387e−004 | —         |
|                  | \(t = 3\)      | \(k = 0.01\), \(h = 0.005\) | 1.2513e−004          | 1.7053e−004 | —         |
|                  | \(t = 4\)      | \(k = 0.01\), \(h = 0.005\) | 1.4742e−004          | 2.0271e−004 | —         |
|                  | \(t = 5\)      | \(k = 0.01\), \(h = 0.005\) | 7.2603e−005          | 9.8405e−005 | —         |
Example 4.2. We consider the hyperbolic telegraph Eq. (1.1) in the interval \(0 \leq x \leq \pi\). Subject to the initial conditions,

\[
\begin{align*}
  u(x, 0) &= \sin(x), \quad 0 \leq x \leq \pi, \\
  u_t(x, 0) &= -\sin(x), \quad 0 \leq x \leq \pi,
\end{align*}
\]

and the boundary conditions,

\[
 u(0, t) = u(\pi, t) = 0.
\]

The analytical solution is given as \(u(x, t) = \exp(-t)\sin(x)\). For this example, we solved the equation in two cases \(\alpha = 4, \beta = 2\) and \(\alpha = 6, \beta = 2\). For each of these two cases we have,

\[
f(x, t) = (2 - 2\alpha + \beta^2)\exp(-t)\sin(x).
\]

The \(L_\infty\), \(L_2\)-errors and Root-Mean-Square (RMS) of errors are tabulated in Table 3 and 4 for \(t=0.4, 0.8, 1.2, 1.6\) and 2, with the step sizes of \(k = 0.0001\) and \(h = 0.02\). Errors in the solution are compared with the results in [2].

| methods          | Time | parameter | \(L_2\)-error | \(L_\infty\)-error | RMS   |
|------------------|------|-----------|----------------|---------------------|-------|
| present method   | \(t = 0.4\) | \(k = 0.0001, h = 0.02\) | 8.7477e-006 | 1.0101e-006 | 7.1188e-007 | |
|                  | \(t = 0.8\) | \(k = 0.0001, h = 0.02\) | 1.5778e-005 | 1.8219e-006 | 1.2840e-006 | |
|                  | \(t = 1.2\) | \(k = 0.0001, h = 0.02\) | 1.8294e-005 | 2.1124e-006 | 1.4888e-006 | |
|                  | \(t = 1.6\) | \(k = 0.0001, h = 0.02\) | 1.8144e-005 | 2.0951e-006 | 1.4766e-006 | |
|                  | \(t = 2\)   | \(k = 0.0001, h = 0.02\) | 1.6638e-005 | 1.9211e-006 | 1.3539e-006 | |
| Dosti and Nazemi[2] | \(t = 0.4\) | \(k = 0.0001, h = 0.02\) | 2.4226e-003 | 2.9000e-003 | — | |
|                  | \(t = 0.8\) | \(k = 0.0001, h = 0.02\) | 3.1924e-003 | 3.2000e-003 | — | |
|                  | \(t = 1.2\) | \(k = 0.0001, h = 0.02\) | 3.0592e-003 | 2.8000e-003 | — | |
|                  | \(t = 1.6\) | \(k = 0.0001, h = 0.02\) | 2.6269e-003 | 2.3000e-003 | — | |
|                  | \(t = 2\)   | \(k = 0.0001, h = 0.02\) | 2.1396e-003 | 1.8000e-003 | — | |

Also we applied our method to this example for the case2 : \(\alpha = 6, \beta = 2\) with step lengths \(k = 0.0001, h = 0.02\) and \(t=0.4, 0.8, 1.2, 1.6\) and 2. The errors in the solution are tabulated in table 4.

| methods          | Time | parameter | \(L_2\)-error | \(L_\infty\)-error | RMS   |
|------------------|------|-----------|----------------|---------------------|-------|
| present method   | \(t = 0.4\) | \(k = 0.0001, h = 0.02\) | 8.1943e-008 | 1.9225e-008 | 1.1474e-008 | |
|                  | \(t = 0.8\) | \(k = 0.0001, h = 0.02\) | 2.7784e-007 | 6.3658e-008 | 3.8905e-008 | |
|                  | \(t = 1.2\) | \(k = 0.0001, h = 0.02\) | 5.3419e-007 | 1.2066e-007 | 7.4802e-008 | |
|                  | \(t = 1.6\) | \(k = 0.0001, h = 0.02\) | 8.1866e-007 | 1.8272e-007 | 1.1463e-007 | |
|                  | \(t = 2\)   | \(k = 0.0001, h = 0.02\) | 1.1118e-006 | 2.4609e-007 | 1.5569e-007 | |

Example 4.3. We consider Eq. (1.1) with \(\alpha = 6, \beta = 2\) and the initial conditions,

\[
\begin{align*}
  u(x, 0) &= \sin(x), \quad 0 \leq x \leq 1, \\
  u_t(x, 0) &= 0, \quad 0 \leq x \leq 1,
\end{align*}
\]
and the boundary conditions,

\[
\begin{align*}
  u(0,t) &= 0, \quad 0 \leq x \leq 1, \\
  u(1,t) &= \cos(t)\sin(1), \quad 0 \leq x \leq 1,
\end{align*}
\] (4.36)

The right hand side function is \( f(x,t) = -2\alpha \sin(t)\sin(x) + \beta^2 \cos(t)\sin(x) \). The analytical solution is \( u(x,t) = \cos(t)\sin(x) \). The \( L_\infty \), \( L_2 \)-errors and Root-Mean-Square (RMS) of errors are obtained in Table 5 for \( t = 0.2, 0.4, 0.6, 0.8 \) and \( 1 \), with the step sizes of \( k = 0.001 \), \( h = 0.005 \). Errors in the solution are compared with results in [2].

Table 5. Errors in the solution of Example 4.3

| methods                      | Time parameter | \( L_2 \)-error \( h = 0.005 \) | \( L_\infty \)-error \( h = 0.005 \) | RMS                |
|------------------------------|----------------|-------------------------------|-----------------------------------|--------------------|
| present method               |                |                               |                                   |                    |
| \( t = 0.2 \)                | \( k = 0.001 \), \( h = 0.005 \) | 1.9084e−009                   | 2.2011e−010                       | 1.3461e−010        |
| \( t = 0.4 \)                | \( k = 0.001 \), \( h = 0.005 \) | 7.7279e−009                   | 8.8253e−010                       | 5.4508e−010        |
| \( t = 0.6 \)                | \( k = 0.001 \), \( h = 0.005 \) | 1.6591e−008                   | 1.8853e−009                       | 1.1703e−009        |
| \( t = 0.8 \)                | \( k = 0.001 \), \( h = 0.005 \) | 2.7801e−008                   | 3.1478e−009                       | 1.9609e−009        |
| \( t = 1 \)                  | \( k = 0.001 \), \( h = 0.005 \) | 4.0810e−008                   | 4.6056e−009                       | 2.8185e−009        |
| Dosti and Nazemi[2]          | \( t = 0.2 \)   | \( k = 0.001 \), \( h = 0.005 \) | 1.3037e−005                       | 2.4279e−005        | —                  |
| \( t = 0.4 \)                | \( k = 0.001 \), \( h = 0.005 \) | 5.1678e−005                   | 7.9315e−005                       | 7.9315e−005        | —                  |
| \( t = 0.6 \)                | \( k = 0.001 \), \( h = 0.005 \) | 8.2370e−005                   | 1.2097e−004                       | 1.2097e−004        | —                  |
| \( t = 0.8 \)                | \( k = 0.001 \), \( h = 0.005 \) | 1.0356e−004                   | 1.4883e−004                       | 1.4883e−004        | —                  |
| \( t = 1 \)                  | \( k = 0.001 \), \( h = 0.005 \) | 1.1592e−004                   | 1.6462e−004                       | 1.6462e−004        | —                  |

5 Conclusion

In this work, we develop a two-level explicit difference scheme based on application of cubic B-spline collocation method. Due to the nature of B-spline, the complexity of proposed method is less and the arising linear system is a tri-diagonal which can be solved easily. Implementation of our method is easy and accurate, this has been verified by test examples. Errors in the solution are measured by \( L_\infty \), \( L_2 \)-errors and Root-Mean-Square (RMS) of errors.

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