Incremental Observer Abstraction for Opacity Verification and Synthesis

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Abstract With the proliferation of communication networks and mobile devices, the privacy and security concerns on their information flow are raised. Given a critical system that may leak confidential information, the problem consists of verifying and also enforcing opacity by designing supervisors, to conceal confidential information from unauthorized persons. To find out what the intruder sees, it is required to construct an observer of the system. In this paper, we consider incremental observer generation of modular systems, for verification and enforcement of current-state opacity and current-state anonymity. The synchronization of the subsystems generate a large state space. Moreover, the observer generation with exponential complexity adds even larger state space. To tackle the complexity problem, we prove that observer generation can be done locally before synchronizing the subsystems. The incremental local observer generation along with an abstraction method lead to a significant state space reduction compared to traditional monolithic methods. The existence of shared unobservable events is also considered in the incremental approach. Thus, we investigate the problem in two cases with two abstraction methods. Moreover, we present an illustrative example, where the results of verification and enforcement of current-state opacity are shown on a modular multiple floor/elevator building with an intruder.

Keywords Transition systems. Abstraction . Current-state opacity . Compositional verification . Modular observers . Synthesis . Supervisory control theory

1 Introduction

With the rapid growth of large communication networks and online services, and their diverse applications, ranging from modern technologies in defence and e-banking to
health care and autonomous vehicles, security and privacy concerns on their information flow are raised. This means that unauthorized people should not acquire the information flow in these services, for instance being able to track and identify the real-time location information of users. There are various notions on security and privacy for different applications based on their vulnerability to intruders.

One category of security notions, (Focardi and Gorrieri, 1996) concerns the information flow from the system to the outside observer, which is called opacity (Saboori and Hadjicostis, 2007; Jacob et al., 2016). Opacity is a general and formal security property that has been widely investigated for DESs for finite automata (Saboori and Hadjicostis, 2007; Bryans et al., 2008; Saboori and Hadjicostis, 2008, 2014), and also for Petri nets (Bryans et al., 2005; Tong et al., 2016, 2017a). A system is opaque, if for any secret behavior, there exists at least one non-secret behavior that looks indistinguishable to the intruder (Saboori and Hadjicostis, 2007; Lafortune et al., 2018). The security notion is investigated for automata (Jacob et al., 2016) using either state-based predicates (Saboori and Hadjicostis, 2007, 2011a; Jacob et al., 2016; Tong et al., 2017b), or language-based predicates (Badouel et al., 2006; Saboori and Hadjicostis, 2008; Cassez, 2009; Lin, 2011; Tong et al., 2016). Also, in Wang et al. (2018), the language-based opacity for real-time automata is investigated.

Depending on the modeling formalism of the system and the secret, there are different opacity notions. Current-state opacity (Saboori and Hadjicostis, 2014; Tong et al., 2017b) requires that the sequence of observable events seen by the intruder never allows the external observer to unambiguously determine that the current state of the system falls within a given set of secret states. A number of various examples and applications are presented in Saboori and Hadjicostis (2011a). Anonymity is also a special case of opacity (Bryans et al., 2008; Lin, 2011). In Wu et al. (2014), it is used for location privacy and is called current-state anonymity, where the servers that access the user’s location information are regarded as intruders. An intruder can be modeled as an observer of the system, meaning that it has full knowledge about the system structure, while it is only able to see the observable events of the system.

The state-based formulations enable us to use various state estimators to verify opacity. State estimation is a general technique that can be used for any problem where an observer is required. Opacity verification is related to the state estimation problem, and it requires that the set of possible states after a finite number of observations can be calculated.

Observers achieved by subset construction (Cassandras and Lafortune, 2008), are deterministic finite automata that estimate the set of possible current states for verifying properties of interest. This makes a clear connection between state-based notions of opacity and their verification with observers. There are several works that exploit observer generation for opacity verification (Saboori and Hadjicostis, 2011b, 2013; Wu and Lafortune, 2013; Wu and Lin, 2018). Verifying weak opacity can be done with polynomial time complexity, while verifying strong opacity cannot (Cassez et al., 2012). In Zhang et al. (2012), instead of constructing an observer that has exponential complexity, they build a detector with polynomial complexity to specifically verify weak opacity. A system is weakly (strongly) opaque, if some (all) strings in the secret language are confused with some strings in the non-secret language (Jacob et al., 2016). However, for strong opacity verification that is relevant to what is inves-
tigated in this work, it is required to construct an observer. The work by Tong et al. (2017a) also addresses the verification of state-based opacity for systems modeled by Petri nets. Moreover, Wu and Lafortune (2013) show that there exists a polynomial-time transformation between different notions of opacity.

Ensuring opacity and some of its variants on a system is usually performed exploiting supervisory control (Ramadge and Wonham, 1989) as in Takai and Oka (2008); Takai and Kumar (2009). Given a system that is not current-state opaque with respect to a secret, it is required to design a maximally permissive supervisor that restricts the behavior of the system to turn it into a current-state opaque system. The design of supervisors to enforce opacity is also sometimes called opacity enforcement. In Badouel et al. (2006), the language-based opacity and a set of intruders having different observations are considered. The work by Dubreil et al. (2008, 2010) is also on the language-based opacity enforcement for one intruder. Enforcing opacity using supervisory control techniques is also investigated by Saboori and Hadjicostis (2008). They propose methods for designing optimal supervisors to enforce two different opacity properties, with the assumption that the supervisor can observe all events (Saboori and Hadjicostis, 2012). In the work by Yin and Lafortune (2016); Tong et al. (2018), to enforce current-state opacity, the assumption that all controllable events should be observable is relaxed. The work by Falcone and Marchand (2014) is on runtime verification and enforcement of opacity to validate several levels of opacity on a system. In Wu and Lafortune (2014); Ji et al. (2018) authors propose a novel enforcement mechanism, based on the use of insertion functions that change the output behavior of the system, by inserting additional observable events.

To verify or synthesize a supervisor to enforce the current-state opacity of a modular system, it is required to generate the system’s observer. Given the exponential complexity of observer generation, as well as the interaction between system components, especially for large complex modular systems, it is highly probable to encounter the state space explosion problem while performing verification and synthesis. For this reason, reduction methods play an important role in making the procedure feasible. Bourouis et al. (2017) use binary decision diagram technique (Bryant, 1992) to abstract very moderate size graphs, as a method for the verification of three different opacity variants. Moreover, they prove that the opacity properties are preserved by composition, which guarantees that local verification of these properties can also be performed. In Zhang and Zamani (2017), a bisimulation-based method to verify the infinite-step opacity of nondeterministic finite transition systems is proposed. Since this abstraction is based on strong bisimulation it has a minor reduction capability compared to the abstraction recently proposed by the authors (Noori-Hosseini et al., 2018). Our work is on an alternative equivalence-based definition of bisimulation, called visible bisimulation equivalence (Lennartsson and Noori-Hosseini, 2018). In this abstraction, local events are abstracted incrementally when a set of subsystems are synchronized in the verification of current-state opacity. In this paper, this incremental technique is further developed, applying even stronger abstractions than branching bisimulation.

Existing methods have mainly focused on the basic opacity formulation and not so much on the computational complexity. Specially, the ability to improve the computational efficiency for modular structure has been explored by few works (Yin and
None of these works are on the verification of opacity notions. Pola et al. (2017) is based on critical observer calculation that is used for checking critical observability. They show that a global observer in a modular system is isomorphic to the synchronization of local observers. However, they assume automata with full observations, which is an oversimplification of the problem. In (Masopust, 2018) a counterexample for the assumption in (Pola et al., 2017) is presented which illustrates that in a system with partial observation and in the presence of shared unobservable events, the general observer is not equal to the synchronization of local observers. In Noori-Hosseini et al. (2018), we assumed partial observation, but only local unobservable events. In this paper this assumption is relaxed and shared unobservable events are also considered. Therefore, our results in this work, further generalize and improve the results of the mentioned papers. Since we incrementally combine compositional abstraction and local observer generation interchangeably, new local events appear and are removed. Hence, we illustrate and prove that in a modular system with partial observation and shared unobservable events, considering two simple rules, the monolithic observer is equal to the synchronization of local observers.

Comparing (Pola et al., 2017) and our previous work (Noori-Hosseini et al., 2018; Lennartson and Noori-Hosseini, 2018), which we exploit bisimulation abstraction incrementally, it results in more efficient approach. Moreover, in this paper we improve the previous results and go for an even more powerful reduction method as in (Flordal and Malik, 2009; Mohajerani et al., 2014). In (Masopust, 2018) they show that it is very unlikely that there is a polynomial time algorithm for deciding the critical observability in modular automata systems. Yin and Lafortune (2017) propose different modular approaches to avoid computing the monolithic model and resulting state space explosion. Saboori and Hadjicostis (2010) is on initial-state opacity of modular DESs. However, since we use incremental abstraction, our approach is more efficient. In (Petriccone et al., 2012), they provide an approach to deal with the analysis of safety criticality for complex air traffic management systems. They propose a technique for complexity reduction. Moreover, they use an ordinary bisimulation that does not consider local events, which means that our bisimulation approach in (Noori-Hosseini et al., 2018) is more efficient. Since bisimulation generates comparatively marginal reduction compared to conflict equivalence, the proposed procedure in this paper is much more efficient.

To the best of our knowledge, there is no previous work on incremental local observer generation and abstraction for current-state opacity verification and supervisor synthesis with partial observations including shared unobservable events. Compositional methods for modular systems, (Flordal and Malik, 2009) exploit abstraction to remove states and transitions that are unnecessary for the synthesis purpose, i.e., they build the synchronous composition incrementally, replacing each individual component by its simpler version that still preserves the main characteristics of the component. Conflict-preserving abstractions, (Malik et al., 2007) are the reduction methods that can be used for the synthesis purpose. In that work it is assumed that, the synthesized supervisor components cannot observe or disable previous abstracted events. This makes abstracted events unobservable. When it comes to supervisor calculation, the closed-loop behaviour is the property to be preserved after simplification.
In (Mohajerani et al., 2014, 2017), a synthesis equivalence is introduced for this purpose. (Mohajerani et al., 2014) proposes an improved version of the compositional method that produces more memory efficient supervisors. Also, (Mohajerani et al., 2017) investigates the compositional abstraction-based synthesis of least restrictive, controllable, and nonblocking supervisors for DESs that are given as a large number of finite-state machines.

In this work, a modular system is considered as a nondeterministic finite automata with an intruder that has partial observation over the system. We formulate the compositional verification of current-state opacity and current-state anonymity for modular systems. Here, we exploit incremental observer generation and abstraction for verifying current-state opacity and anonymity. Each non-safe state of the observer is considered as a forbidden state and the opacity verification problem changes to a non-blocking verification problem. To tackle the complexity problem, we prove that the observer generation can be done locally before synchronizing the subsystems, in the presence of shared unobservable events. Also, the conflict equivalence abstraction (Malik, 2004) is incrementally applied on each subsystem where local events are identified incrementally and abstracted after each synchronization. Since the reduction is applied after synchronization, a significant part of the state space in ordinary synchronization, incrementally become local and the same approach is also applied on the local unobservable events, that are avoided in each step. This compositional local observer generation along with an abstraction method results in a significant state space reduction and enables opacity verification and supervisor synthesis for much larger systems.

To summarize, the main contributions of this paper are as follows. We exploit modular nondeterministic finite state automata with partial observation, specifically considering shared unobservable events. We prove that the synchronization of local observers is isomorphic to the monolithic observer of the original synchronized system. Applying the incremental observer generation and abstraction, results in significant reduction due to the incremental local events removal. Moreover, the construction of local observers before synchronization generates less state space. The efficiency of our work is illustrated by a computer-based example, where the results are shown on current-state opacity/anonymity verification and also, synthesis of a supervisor to enforce it in a multiple floor/elevator building with an intruder. Although (Masopust, 2018) claimed that the observer generation for modular systems is not solvable, we have a good answer to that in this paper. Our work is also an improvement of (Pola et al., 2017) and Noori-Hosseini et al. (2018) papers. The comparison results between the monolithic and compositional approaches are also shown in the tables. Our opacity evaluation is not based on any specific strategy. Moreover, we transform opacity verification and enforcement to a non-blocking problem.

We investigate this problem in two cases with two abstraction methods.

The remainder of the paper is organized as follows. In Section 2, we recall preliminaries of the work. The problem statement is described in Section 3. Section 4 is on efficient generation of observers and includes a proposition on the possibility of generating local abstracted observers before synchronization. In Section 5 the current-state opacity/anonymity notions for both single and modular systems are described. Section 6 provides the discussions on the effect of existence of shared unob-
servable events in the system, and the necessity of defining rules before abstraction during the incremental observer generation procedure. Section 7 provides an example to illustrate the efficiency of the method by applying it on the current-state opacity verification of a multiple floor/elevator building. It also includes the comparison with the verification using monolithic observer generation. Moreover, Section 8 is on the compositional synthesis of a secure system. Section 9 concludes the paper.

2 Preliminaries

A transition system \( G = (X, \Sigma, T, I, AP, \lambda) \) where \( X \) is a set of states, \( \Sigma \) is a finite set of events, \( T \subseteq X \times \Sigma \times X \) is a transition relation, where \( t = (x, a, x') \in T \) includes the source state \( x \), the event label \( a \), and the target state \( x' \) of the transition \( t \). A transition \( (x, a, x') \) is also denoted \( x \xrightarrow{a} x' \). \( I \subseteq X \) is a set of possible initial states, \( AP \) is a set of atomic propositions, and \( \lambda : X \to 2^{AP} \) is a state labeling function.

A subset \( L \subseteq \Sigma^* \) is called a language. Moreover, for events set \( \Omega \subseteq \Sigma \), the projection \( P_{\Omega} : \Sigma^* \to \Omega^* \) is the operation that maps events from \( \Sigma^* \) to \( \Omega^* \), by removing strings from \( \Sigma^* \), that do not belong to \( \Omega^* \). In the composition of subsystems, see Def. 2, events that are not included in any synchronization with other subsystems are called local events. Such events are central in the generation of observers and abstractions.

Model including \( \varepsilon \) transitions. The transition system \( G \) is now extended to include transitions labeled by the empty \( \varepsilon \). In this paper, the \( \varepsilon \) event will explicitly be used for local unobservable events. The alphabet is extended to \( \Sigma \cup \{\varepsilon\} \), and a sequence of \( \varepsilon \) transitions \( x = x_0 \xrightarrow{\varepsilon} x_1 \xrightarrow{\varepsilon} \cdots \xrightarrow{\varepsilon} x_n = x' \), \( n \geq 0 \), is denoted \( x \xrightarrow{\varepsilon} x' \). A corresponding sequence, including possible \( \varepsilon \) transitions before, after and in between events in a string \( s \in \Sigma^* \), is denoted \( x \xrightarrow{\varepsilon} x' \). The epsilon closure of a state \( x \) is defined as \( R_\varepsilon(x) = \{ x' \mid x \xrightarrow{\varepsilon} x' \} \), and for a set of states \( Y \subseteq X \) we write \( R_\varepsilon(Y) = \bigcup_{x \in Y} R_\varepsilon(x) \).

A nondeterministic transition system generally includes a set of initial states, \( \varepsilon \) labeled transitions, and/or alternative transitions with the same event label. A transition function for an event \( a \in \Sigma \) in a nondeterministic transition system is defined as \( \delta(Y, a) = R_\varepsilon(\{ x' \mid (\exists x \in Y) x \xrightarrow{a} x' \in T \}) \). An extended transition function is then inductively defined, for \( s \in \Sigma^* \) and \( a \in \Sigma \), as \( \delta(I, sa) = \delta(\delta(I, s), a) \) with the base case \( \delta(I, \varepsilon) = R_\varepsilon(I) \). Furthermore, the language for a nondeterministic transition system is defined as \( \mathcal{L}(G) = \{ s \in \Sigma^* \mid (\exists x \in I) \delta(x, s) \neq \emptyset \} \).

Local transitions and hidden \( \tau \) events. To obtain efficient abstractions, a special \( \tau \) event label is used for transitions with local observable events. The lack of communication with other subsystems means that the \( \tau \) event is hidden from the rest of the environment. The closure of \( \tau \)-transitions in a finite path \( x = x_0 \xrightarrow{\tau} x_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} x_n = x' \), \( n \geq 0 \), is denoted \( x \xrightarrow{\tau} x' \).
Note the difference between $\varepsilon$ and $\tau$ events. Unobservable local events are replaced by $\varepsilon$ before an observer is generated, which removes any $\varepsilon$ transitions. Observable local events are then replaced by $\tau$ to model that they are hidden before performing any abstraction. In process algebra, the replacement of any specific event by the event $\tau$ is called hiding, cf. Milner (1989). A transition system $G$ where the events in $\Sigma^h$ are hidden and replaced by $\tau$ is denoted $G^{\Sigma^h}$.

**Partition $\Pi$ and block $\Pi(x)$** To obtain abstracted transition systems, states $x, y \in X$ that can be considered to be equivalent in some sense, denoted $x \sim y$, are merged into equivalence classes $[x] = \{y \in X \mid x \sim y\}$, also called blocks. These blocks, which are non-overlapping subsets of $X$, divide the state space into the quotient set $X/\sim$, also called a partition $\Pi$ of $X$. The block/equivalence class including state $x$ is denoted $\Pi(x) = [x]$. A partition $\Pi_1$ that is finer than a partition $\Pi_2$, denoted $\Pi_1 \preceq \Pi_2$, means that $\Pi_1(x) \subseteq \Pi_2(x)$ for all $x \in X$. The partition $\Pi_2$ is then said to be coarser than $\Pi_1$.

**Invisible, visible and stuttering transitions** For a given state partition $\Pi$, a transition $x \xrightarrow{\tau} x'$ is invisible if $\Pi(x) = \Pi(x')$, while a transition $x \xrightarrow{a} x'$ is visible if $a \neq \tau$ or $\Pi(x) \neq \Pi(x')$. A path $x \xrightarrow{\tau_1} x_1 \xrightarrow{\tau_2} \cdots \xrightarrow{\tau_n} x_n \xrightarrow{a} x'$ is called a stuttering transition, denoted $x \xrightarrow{a}^{\ast} x'$, if $\Pi(x) = \Pi(x_1) = \cdots = \Pi(x_n)$, and $a \neq \tau$ or $\Pi(x_n) \neq \Pi(x')$. This means that the first $n$ transitions are invisible, while the last one is visible. A block stuttering transition corresponding to $x \xrightarrow{a}^{\ast} x'$ is denoted $\Pi(x) \xrightarrow{a}^{\ast} \Pi(x')$.

**Visible bisimulation** Different types of bisimulations, used for abstraction, are either defined for labeled transition systems, only including event labels (often called actions) on the transitions, or for Kripke structures, only including state labels (Baier and Katoen, 2008). In this work, shared events are required for synchronization of subsystems, while state labels are used to model security properties. Recently, Lennartson and Noori-Hosseini (2018) introduced an abstraction for transition systems including both event and state labels, called visible bisimulation. It is directly defined as an equivalence relation based on block stuttering transitions, and more specifically on the set of event-target-blocks $\Gamma_{\Pi}(x) = \{ \frac{a}{n} \xrightarrow{\Pi} \Pi(x') \mid x \xrightarrow{a}^{\ast} x' \}$ that defines all possible stuttering transitions from an arbitrary state $x$.

**Definition 1 (Visible bisimulation equivalence)** Given a transition system $G = \langle X, \Sigma, T, I, AP, \lambda \rangle$ and the state label partition $\Pi_{\lambda}(x) = \{ y \in X \mid \lambda(x) = \lambda(y) \}$, a partition $\Pi$, for all $x \in X$ determined by the greatest fixpoint of the fixpoint equation

$$\Pi(x) = \{ y \in X \mid \Pi \preceq \Pi_{\lambda} \land \Gamma_{\Pi}(x) = \Gamma_{\Pi}(y) \},$$

is a visible bisimulation (VB) equivalence, and states $x, y \in \Pi(x)$ are visibly similar, denoted $x \sim y$. \qed
**Quotient transition system** Blocks are the states in abstracted transition systems, and the notion partition \( I \) is used in the computation of this model, while the resulting reduced model takes the equivalence perspective. It is therefore called *quotient transition system*, and for a given partition \( I \) it is defined as \( G/\sim = \langle X/\sim, \Sigma, T_\sim, I_\sim, AP, \lambda_\sim \rangle \), where \( X/\sim = \{ [x] | [x] = I(x) \} \) is the set of block states (equivalence classes), \( T_\sim = \{ [x] \stackrel{a}{\rightarrow} [x'] | x \stackrel{a}{\rightarrow} x'([x'] \neq [x] \lor a \neq \tau) \} \) is the set of block transitions, here specifically defined for VB, \( I_\sim = \{ [x] | x \in I \} \) is the set of initial block states, and \( \lambda_\sim([x]) = \lambda(x) \) is the block state label function, where it is assumed that \( \lambda(x) = \lambda(y) \), \( \forall y \in [x] \).

Visibly bisimilar states \( x \sim y \) in \( G \) are also visibly bisimilar to the block state \([x]\) in \( G/\sim\), i.e. \([x] \sim x\) for all \( x \in X \). Furthermore, \( G \) and \( G/\sim\) are VB equivalent, denoted \( G \sim G/\sim\). Combining hiding of a set of events \( \Sigma^h \) for a system \( G \), followed by the generation of the quotient transition system, results in the *abstracted transition system* \( G^{\Sigma^h/\sim} \equiv G^{A_{\Sigma^h}} \). This also means that \( G^{\Sigma^h/\sim} \sim G^{A_{\Sigma^h}} \).

**Synchronous composition** The definition of the synchronous composition in Hoare (1978) is adapted to \( \tau \) events, where such events in different subsystems are not synchronized, although they share the same event label. They are simply considered as local events, which is natural since the hiding mechanism where an event is replaced by the invisible \( \tau \) event is only applied to local events. This results in the following definition of the synchronous composition, including \( \tau \) event labels.

**Definition 2 (Synchronous composition including \( \tau \) events)** Consider two transition systems \( G_i = \langle X_i, \Sigma_i, T_i, I_i, AP_i, \lambda_i \rangle \), \( i = 1, 2 \). The synchronous composition of \( G_1 \) and \( G_2 \) is defined as

\[
G_1 \parallel G_2 = \langle X_1 \times X_2, \Sigma_1 \cup \Sigma_2, T, I_1 \times I_2, AP_1 \cup AP_2, \lambda \rangle
\]

where

\[
(x_1, x_2) \stackrel{a}{\rightarrow} (x_1', x_2') \in T : a \in (\Sigma_1 \cap \Sigma_2) \setminus \{\tau\}, \quad x_1 \stackrel{a}{\rightarrow} x_1' \in T_1, \quad x_2 \stackrel{a}{\rightarrow} x_2' \in T_2,
\]

\[
(x_1, x_2) \stackrel{\tau}{\rightarrow} (x_1', x_2) \in T : a \in (\Sigma_1 \setminus \Sigma_2) \cup \{\tau\}, \quad x_1 \stackrel{\tau}{\rightarrow} x_1' \in T_1,
\]

\[
(x_1, x_2) \stackrel{\tau}{\rightarrow} (x_1, x_2') \in T : a \in (\Sigma_2 \setminus \Sigma_1) \cup \{\tau\}, \quad x_2 \stackrel{\tau}{\rightarrow} x_2' \in T_2.
\]

and \( \lambda : X_1 \times X_2 \rightarrow 2^{AP_1 \cup AP_2} \).

Any \( \varepsilon \) transitions are handled in the same as \( \tau \) transitions, since they are also assumed to be local. On the other hand, before subsystems are synchronized, local observers will in this work be generated. This means that any \( \varepsilon \) transitions will be removed before synchronization.

**Nonblocking and controllable supervisor** In order to determine whether a system satisfies a given specification or not, the system has to be *verified*, and if it fails, the system is restricted by *synthesizing a supervisor*. This means that states from which it is not possible to reach a desired marked state, called *blocking states*, are removed. Furthermore, any uncontrollable events that can be executed by the plant are not allowed to be disabled by the supervisor (Ramadge and Wonham, 1989; Wonham et al.,
Thus, a supervisor is synthesized to avoid blocking states and disabling uncontrollable events. Such a nonblocking and controllable supervisor is also maximally permissive, meaning that it restricts the system as little as possible.

3 Problem statement

The focus of this paper is to generate reduced observers that still preserve relevant properties, to be able to verify different security notions. It is also shown how supervisors can be generated, avoiding states that do not satisfy desired properties. This section presents the main problem statements of the paper, the incremental generation of reduced observers, with and without presence of shared unobservable events, and some security notions that will be analyzed by such reduced observers.

3.1 Incremental abstraction for modular systems

A transition system, including a number of subsystems $G_i, i \in \mathbb{N}^+$ that are interacting by synchronous composition, is defined as

$$G = \parallel_{i \in \mathbb{N}^+} G_i = G_1 \parallel G_2 \parallel \cdots \parallel G_n.$$  

A straightforward approach to analyze such a modular system is to compute the explicit monolithic transition system $G$. However, there are limitations on memory and computation time in the generation and analysis of such monolithic systems. An alternative approach is to avoid building the explicit monolithic system, by analyzing each individual subsystem first. In this case, local events of each subsystem are hidden and abstracted based on the desired property to be preserved. Moreover, after every synchronization of subsystems more local events may appear and thus, additional abstraction is possible. This step by step combined hiding, abstraction and synchronization is here called incremental abstraction. In (Flordal and Malik, 2009), this approach is proposed for verification, and is then called compositional verification.

3.2 Incremental observer generation

The focus of this paper is on verification of security properties, while a simple extension towards synthesis is shown in the end of the paper. The security properties are analyzed by constructing an observer, where only observable events are involved. The generated observer is deterministic and computed by subset construction (Hopcroft et al., 2001).

Since the observer generation as well as the synchronization of the subsystems have exponential complexity, the incremental abstraction mentioned above is of interest. This approach can be applied if the observer generation is divided into local observers that are synchronized. When all unobservable events are local, i.e. no shared unobservable events are involved, it is shown in Section 4 that an observer of
the monolithic system $G$, denoted $\mathcal{O}(G)$, also can be computed by the synchronous composition of the local observers of its subsystems. Thus,

$$\mathcal{O}(G) = \bigparallel_{i \in \mathbb{N}^+} \mathcal{O}(G_i).$$

(2)

The security properties considered in this work result in observer states that are either safe or non-safe. Introducing the state label $N$ for the non-safe states, visible bisimulation can be used in an incremental abstraction, still preserving the separation between the two types of states.

For two synchronized subsystems, $G_1 \parallel G_2$, the sets of local events in $G_1$ and $G_2$ are $\Sigma^h_1$ and $\Sigma^h_2$, respectively, and the events in $\Sigma^h_{12}$ are the shared events between the two subsystems that become local after the synchronization, see also Example 1. Thus, the set $\Sigma^h = \Sigma^h_1 \cup \Sigma^h_2 \cup \Sigma^h_{12}$ includes all events that can be hidden after the synchronization. Using the notations $G^{\Sigma^h}$ for hiding the events in $\Sigma^h$, $G^{A\Sigma^h}$ for abstraction including hiding, and the equivalence $G^{\Sigma^h} \sim G^{A\Sigma^h}$, it is also shown in Section 4 that an abstraction of $\mathcal{O}(G_1 \parallel G_2)^{\Sigma^h}$, including the local observer generation in (2), can be incrementally generated as

$$\mathcal{O}(G_1 \parallel G_2)^{\Sigma^h} \sim (\mathcal{O}(G_1)^{A\Sigma^h_1} \parallel \mathcal{O}(G_2)^{A\Sigma^h_2})^{A\Sigma^h_{12}}.$$

(3)

Repeating this incremental abstraction procedure when more subsystems are included still implies that only observers of individual subsystems $\mathcal{O}(G_i)$ are required. Furthermore, the repeated abstraction means that often systems with a moderate state space are synchronized, especially when a number of local events are obtained after each synchronization.

Since (3) only includes one type of state label ($N$), it can also be expressed in terms of marked and non-marked states. Therefore, the problem can also be identified as a non-blocking problem, and more efficient abstractions (coarser state partitioning) than visible bisimulation can be used. This is further described in Section 4.

When no explicit set of hidden events is included in the abstraction operator $A$, the default set of events to be hidden is assumed to be all local observable events. Assuming that this set is $\Sigma^h$ for transition system $G$, it means that $\mathcal{O}(G)^{A\Sigma^h}$ is often simplified to $\mathcal{O}(G)^A$, where we also note that the observer is generated before the abstraction is performed.

3.3 Incremental observer generation with shared unobservable events

For systems also including shared unobservable events, such events cannot be replaced by $\varepsilon$ due to the synchronization with other subsystems. This means that a complete observer can not be computed by composing local observers as in (2) before subsystems have been synchronized such that no shared unobservable events remain. On the other hand it is shown in Section 6 that observers can also be computed incrementally, such that shared unobservable events have to be retained, while transitions with local unobservable events can be removed in a partial observer generation.

To clarify this partial observer generation, the more detailed observer operator $\mathcal{O}_{\Sigma^\varepsilon}(G)$ is introduced, where the subscript $\Sigma^\varepsilon$ includes the set of local unobservable
events that are replaced by $\varepsilon$ before the observer generation. Similar to the sets of hidden events in (3), the sets of local unobservable events in $G_1$ and $G_2$ are $\Sigma_1^a$ and $\Sigma_2^a$, respectively, and the events in the set $\Sigma_{12}^c$ are the shared unobservable events in $G_1$ and $G_2$ that become local after the synchronization $G_1 \parallel G_2$, see also Example 1. Thus, the set $\Sigma^c = \Sigma_1^c \cup \Sigma_2^c \cup \Sigma_{12}^c$ includes all unobservable events that can be replaced by $\varepsilon$ when the observer is generated after the synchronization. In Section 6 it is shown that an observer alternatively can be generated incrementally as

$$O_{\Sigma^c}(G_1 \parallel G_2) = O_{\Sigma_{12}^c} \left( O_{\Sigma_1^c}(G_1) \parallel O_{\Sigma_2^c}(G_2) \right),$$

(4)

where the shared unobservable events in $\Sigma_{12}^c$ are preserved until they become local. Also observe the special case with no shared unobservable event ($\Sigma_{12}^c = \emptyset$), where (4) simplifies to (2). Furthermore, the observer generation, combined with the incremental abstraction, results in

$$O_{\Sigma^c}(G_1 \parallel G_2)^{\Sigma_h} \sim O_{\Sigma_{12}^c} \left( O_{\Sigma_1^c}(G_1)^{\Delta^A_{\Sigma_{12}^c}} \parallel O_{\Sigma_2^c}(G_2)^{\Delta^A_{\Sigma_{12}^c}} \right).$$

(5)

Note that the observer generation is always performed before corresponding abstraction. Observable and unobservable events are here incrementally replaced by $\tau$ and $\varepsilon$, respectively, when they become local. The mix between step-wise abstraction and partial observer generation means that some events are replaced by $\varepsilon$ first after one or more abstractions. To be able to construct correct partial observers, this implies that some restrictions must be included in the incremental abstractions. This is solved in Section 6 by introducing additional temporary state labels (other than labels for non-safe states).

When no explicit set of unobservable events is included in the observer operator $O$, the default set is assumed to be all local unobservable events. Assuming that this set is $\Sigma^c$ for transition system $G$, it implies that $O_{\Sigma^c}(G)$ is often simplified to $O(G)$, where the observer is generated after the events in $\Sigma^c$ have been replaced by $\varepsilon$.

**Example 1** This example illustrates the incremental replacement of local events by $\varepsilon$ or $\tau$ in (5). The events $a$, $b$, $c$ and $d$ are observable, while the events $u$ and $v$ are unobservable. Fig. 1 shows that the events $a$, $d$ and $v$ are shared. To generate the local observers $O(G_i)$, $i = 1, \ldots, 3$, local unobservable events are replaced by $\varepsilon$, and $\Sigma_1^u = \{u\}$, $\Sigma_2^u = \emptyset$, and $\Sigma_3^u = \emptyset$. Although event $v$ is unobservable, it is shared between $G_1$ and $G_2$ and is not replaced by $\varepsilon$ at this level. However, it becomes local after the synchronization $G_1 \parallel G_2$, which means $\Sigma_{12}^v = \{v\}$. Moreover, $\Sigma_{12}^u = \emptyset$ and $\Sigma_{23}^u = \emptyset$. In the hiding process of local observable events before abstraction, the sets of hidden events are $\Sigma_1^b = \{b\}$, $\Sigma_2^b = \{c\}$, $\Sigma_3^b = \emptyset$, $\Sigma_{12}^b = \{a\}$, $\Sigma_{13}^b = \emptyset$, and $\Sigma_{23}^b = \{d\}$. \hfill $\square$

### 3.4 Security properties

The two security properties that are studied in this work are current-state opacity (CSO) and current-state anonymity (CSA), a minor modification of CSO. It is assumed that an intruder knows the model of the system and has access to the observ-
able events. Thus, an intruder can generate an observer of the system, and security violation can be formulated as the existence of non-safe states in this observer.

In CSO verification, the states of the observer that exclusively include secret states are called non-safe states. By definition, a system is current-state opaque, if there is no non-safe state in the observer. On the other hand, a system is current-state anonymous, if there is no singleton state in the observer. The singleton states are considered as non-safe states in CSA verification. In Section 5, both opacity and anonymity notions for modular systems are described.

Moreover, for the synthesis of current-state opaque/anonymous systems, that is limited to systems including only local unobservable events, uncontrollable events are introduced such that an efficient supervision equivalence abstraction can be used to find the supervisor.

4 Efficient generation of observers

Since the computation of an observer has exponential complexity (Cassandras and Lafortune, 2008), it is shown in this section how the incremental abstraction in (3) can be used to significantly lower the computational complexity. All unobservable events are in this section assumed to be local and can therefore immediately be replaced by \( \varepsilon \). Based on this assumption, it is shown how local observers can be directly generated before the incremental abstraction is applied.

4.1 Incremental observer abstraction for modular systems

For a nondeterministic transition system \( G \), a deterministic transition system with the same language as \( L(G) \), called an observer \( O(G) \), is generated by subset construction (Hopcroft et al., 2001), where \( O(G) = \langle \hat{X}, \Sigma, \hat{T}, \hat{I}, AP, \hat{\lambda} \rangle \), and \( \hat{X} = \{ Y \in 2^X \mid (\exists s \in L(G)) Y = \delta(I, s) \} \), \( \hat{T} = \{ Y \overset{a}{\rightarrow} Y' \mid Y' = \delta(Y, a) \} \), and \( \hat{I} = R_{\varepsilon}(I) \). The relation between \( \hat{\lambda}(Y) \) and \( \lambda(x) \) is application dependent, see Section 5.

Introduce the transition function \( \hat{\delta}(Y, a) \overset{def}{=} \hat{\delta}(Y, a) \) and the extended transition function, inductively defined as \( \hat{\delta}(\hat{I}, sa) = \hat{\delta}(\hat{I}, s), a \) with the base case \( \hat{\delta}(\hat{I}, \varepsilon) = \hat{I} \). It is then easily shown that \( \hat{\delta}(\hat{I}, s) = \delta(I, s) \), see (Hopcroft et al., 2001). This means that \( L(O(G)) = L(G) \).

![Fig. 1](image-url) Three subsystems with local and shared, observable and unobservable events.
For a modular system (1) with partial observation, the monolithic observer can be computed by first generating local observers for each subsystem before they are synchronized. This is possible, since the following lemma shows that the same monolithic observer is obtained when synchronization is made before and after observer generation. Compared to a recent result in (Pola et al., 2017), this lemma is a generalization including unobservable events, and the proof is more straightforward. It was first presented in Noori-Hosseini et al. (2018).

**Lemma 1 (Modular observers)**

Let \( G_1 \) and \( G_2 \) be two nondeterministic transition systems with no shared unobservable events. Then,

\[
O(G_1 \parallel G_2) = O(G_1) \parallel O(G_2).
\]

Proof: Consider the language of the synchronized system \( L(G_1 \parallel G_2) \) and the projection \( P_i : (\Sigma_1 \cup \Sigma_2)^* \rightarrow \Sigma_i^* \) for \( i = 1, 2 \), where the alphabet \( \Sigma_i \) only includes observable events. After a string \( s \in L(G_1 \parallel G_2) \) has been executed, the set of reachable states can be expressed as \( Y_1 \times Y_2 \), where

\[
Y_i = \{ x | (\exists x_0 \in I_i) x_0 \xrightarrow{P_i(s)} x \}, \ i = 1, 2.
\]

Assume that there are transitions \( x_i \xrightarrow{a} x_i' \) in \( G_i \) for \( i = 1, 2 \), where \( a \in \Sigma_1 \cap \Sigma_2 \), \( x_i \in Y_i \), and \( x_i' \in Y_i' \). Then there is a corresponding transition \( (x_1, x_2) \xrightarrow{a} (x_1', x_2') \) in \( G_1 \parallel G_2 \). Thus, subset construction of \( G_1 \parallel G_2 \) generates the transition \( Y_1 \times Y_2 \xrightarrow{a} Y_1' \times Y_2' \). Since \( Y_i \) and \( Y_i' \) are also states in \( O(G_i) \), the corresponding transition in \( O(G_1) \parallel O(G_2) \) is \( (Y_1, Y_2) \xrightarrow{a} (Y_1', Y_2') \). With similar arguments for \( a \in \Sigma_1 \setminus \Sigma_2 \) and \( a \in \Sigma_2 \setminus \Sigma_1 \), we find that for a given string \( s \in L(O(G_1 \parallel G_2)) = L(O(G_1) \parallel O(G_2)) \), the reachable states included in the block states of \( O(G_1 \parallel G_2) \) and \( O(G_1) \parallel O(G_2) \) are the same. Indeed, the bijective function

\[
f : 2^{X_1 \times X_2} \rightarrow 2^{X_1} \times 2^{X_2}, \quad \text{where} \quad f(Y_1 \times Y_2) = (Y_1, Y_2)
\]

for \( Y_i \in 2^{X_i}, \ i = 1, 2 \), shows that the two transition systems are isomorphic and therefore structurally equal (denoted by the equal sign \( = \)). □

**Online estimation** This lemma shows that estimation of a modular system can be implemented by running local observers combined with online synchronization. This is a dramatic simplification compared to estimation based on traditional monolithic observers.

In the following proposition, abstraction is added to the result of Lemma 1. The proposition is valid for any abstraction that is congruent with respect to (wrt) synchronization and hiding.

**Proposition 1 (Incremental abstraction of modular observers)** Let \( G_1 \) and \( G_2 \) be two nondeterministic transition systems with no shared unobservable events but hidden observable events in the set \( \Sigma^h \equiv \Sigma_1^h \cup \Sigma_2^h \cup \Sigma^h_{12} \), where \( \Sigma^h \) includes local events in \( G_i \), \( i = 1, 2 \), and \( \Sigma^h_{12} \) includes shared events in \( G_1 \) and \( G_2 \). For an arbitrary
abstraction equivalence $G^{\Sigma_h} \sim G^{A^{\Sigma_h}}$ that is congruent wrt synchronization and hiding, the abstraction of the following observer can be incrementally generated as

$$O(G_1 \parallel G_2)^{\Sigma_h} \sim (O(G_1)^{A^{\Sigma_h}} \parallel O(G_2)^{A^{\Sigma_2}})^{A^{\Sigma_{h2}}}.$$ 

Proof: Combining Lemma 1 with hiding of the local events in $G_1$ and $G_2$, we find that

$$O(G_1 \parallel G_2)^{\Sigma_h} \sim O(G_1)^{A^{\Sigma_h}} \parallel O(G_2)^{A^{\Sigma_2}}.$$ 

For an arbitrary equivalence $G \sim H$, congruence wrt synchronization means that $G \parallel R \sim H \parallel R$. Thus,

$$O(G_1 \parallel G_2)^{\Sigma_h} \sim O(G_1)^{A^{\Sigma_h}} \parallel O(G_2)^{A^{\Sigma_2}}.$$ 

Now, also hiding the shared events in $\Sigma_{h2}$, combined with congruence wrt hiding ($G \sim H$ implies $G^{\Sigma_h} \sim H^{\Sigma_h}$) and one more abstraction, we finally obtain

$$O(G_1 \parallel G_2)^{\Sigma_h} \sim (O(G_1)^{A^{\Sigma_h}} \parallel O(G_2)^{A^{\Sigma_2}})^{A}\Sigma_{h2}.$$ 

\[\square\]

### 4.2 Incremental observer algorithm and transformation to nonblocking verification

An incremental observer generation including abstraction is presented for modular systems in Algorithm 1.

| Algorithm 1 Incremental observer generation including abstraction |
|---------------------------------------------------------------|
| **input** $G_1, \ldots, G_n$  
| **output** $O(G)^{A}$  
| 1: for $i \in N_n$ do  
| 2: $G_i := O(G_i)$  
| 3: end for  
| 4: $\pi_{\Omega} := \{\{1\}, \{2\}, \ldots, \{n\}\}$  
| 5: repeat  
| 6: Choose $\Omega_1, \Omega_2 \in \pi_{\Omega}$ according to some heuristics  
| 7: $\Omega := \Omega_1 \cup \Omega_2$  
| 8: $G_{\Omega} := G^A_{\Omega_1} \parallel G^A_{\Omega_2}$  
| 9: Replace $G^A_{\Omega_1}$ and $G^A_{\Omega_2}$ by $\Omega$ in $\pi_{\Omega}$  
| 10: until $\Omega = N_n$  
| 11: $O(G)^{A} := G^A_{\Omega}$ |

Fig. 2 Observer generation and incremental abstraction of a modular transition system $G = \bigparallel_{i \in N_n} G_i$. 

Heuristics In the selection of the sets $\Omega_1$ and $\Omega_2$ and corresponding transition systems $G_{\Omega_1}$ and $G_{\Omega_2}$, to be abstracted in Algorithm 1, a natural approach is to first select a group of transition systems with few transitions. Among them, the two systems with the highest proportion of local events are chosen to be abstracted. In this way, a significant reduction of states and transitions is achieved by the abstractions, and the intermediate system after the synchronization $G_\Omega := G_{\Omega_1}^A \parallel G_{\Omega_2}^A$ also becomes smaller.

Algorithm 1, including this heuristics, is an adaption of a method suggested by Flordal and Malik (2009) for incremental verification. They call it compositional verification, and the focus is on nonblocking and controllability properties, while the formulation here is adapted to incremental observer generation and specific observer properties.

Transformation to nonblocking verification by extended local observers

The security related verification and synthesis problems considered in this paper are all related to identification of specific non-safe observer state properties, see Section 3.4. In CSO, states that exclusively include secret states are non-safe, and in CSA, singleton states are considered as non-safe states. Non-safe states in an observer may formally be considered as forbidden states, and the verification as a forbidden state problem. This verification problem can be solved by introducing the state label $N$ for the non-safe states, and then use visible bisimulation as abstraction in Algorithm 1.

Since the problem only includes two types of states, safe and non-safe, an alternative to generic state labels and visible bisimulation is to transform the forbidden state problem to a nonblocking problem. All forbidden (non-safe) states in each individual observer $\mathcal{O}(G_i)$ are then augmented with a self-loop labeled by $w_i$, $i = 1, \ldots, n$. The resulting local observers are called $\mathcal{O}_{w_i}(G_i)$. For each such observer, a two-state automaton $G_{NS_i}$, shown in Fig. 3, is then introduced. It includes a marked state with a self-loop on the set of observable events $\Sigma_i^o$ in $G_i$ and a transition via the event $w_i$ to a non-marked state. The extended local observer

$$\mathcal{O}_e(G_i) = \mathcal{O}_{w_i}(G_i) \parallel G_{NS_i}$$

then obtains non-marked blocking states added to every occurrence of a $w_i$ self-loop in $\mathcal{O}_{w_i}(G_i)$. Thus, every forbidden state in $\mathcal{O}(G_i)$ results in a direct transition to a blocking state in the extended local observer, while all original states in $\mathcal{O}_{w_i}(G_i)$ become marked in $\mathcal{O}_e(G_i)$. The reason is that no state in $\mathcal{O}_{w_i}(G_i)$ is explicitly marked,
meaning that every state is implicitly considered to be marked in the synchronization. If any blocking states remain in the total extended observer
\[ O_e(G) = O_e(G_1) \parallel O_e(G_2) \parallel \cdots \parallel O_e(G_n), \]
this observer is not nonblocking, and the observer \( O(G) \) includes one or more non-safe states.

**Conflict equivalence abstraction** Conflict equivalence, introduced by Malik et al. (2004), preserves the nonblocking property of an automaton. This means that an automaton is nonblocking if and only if its conflict equivalence abstraction is also nonblocking. This abstraction, here denoted \( A_c \), generally generates more efficient reductions compared to the visible bisimulation abstraction, here denoted \( A_v \). The reason is that only the nonblocking property is preserved, while visible bisimulation, including divergence sensitivity, preserves temporal logics similar to CTL* (Lennartsson and Noori-Hosseini, 2018).

By introducing the extended observer \( O_e \) as observer operator in Algorithm 1, an incremental observer based on the abstraction \( A_c \) is efficiently computed. This is possible, since conflict equivalence is congruent wrt hiding and synchronization (Malik et al., 2004). The DES software tool Supremica (Åkesson et al., 2006) includes an incremental conflict equivalence implementation based on (Flordal and Malik, 2009).

As an alternative, Algorithm 1 can also be implemented based on the visible bisimulation abstraction \( A_v \) and the original local observers including the non-safe state label \( N \). Note that this abstraction is also congruent wrt hiding and synchronization.

The following example illustrates the transformation to a nonblocking problem. Furthermore, the efficiency of the conflict equivalence and the visible bisimulation abstractions is demonstrated.

**Example 2** Consider the subsystem \( G_i, i \in \mathbb{N}_n^+ \) in Fig. 4, where \( v_i \) is a local unobservable event and therefore replaced by \( \varepsilon \) before observer generation. The events \( a_i \) and \( c_i \) are local observable and the events \( b_i \) and \( b_{i+1} \) are observable but shared between neighbor subsystems, except the local events \( b_1 \) and \( b_{n+1} \). The local observer \( O(G_i) \) is also shown in Fig. 4.

The transition system \( G_i \) is assumed to have one secret state, state 2. Thus, the observer state 2 is non-safe from both CSO and CSA point of view. This non-safe state with state label \( N \) in \( O(G_i) \) is a forbidden state to which a \( w_i \) self-loop is added in \( O_w(G_i) \) in Fig. 4. Including the 2-state model \( G_{NS_i} \) as depicted in Fig. 5, gives the extended local observer \( O_e(G_i) = O_w(G_i) \parallel G_{NS_i} \), where the \( w_i \) self-loop is replaced by a \( w_i \) transition to a blocking state, also shown in Fig. 5.

In Table 1 the complexity of the incremental extended observer \( O_e(G)^{A_c} \), including abstraction based on conflict equivalence, is compared with the extended observer without abstraction \( O_e(G) = \bigparallel_{i \in \mathbb{N}_n^+} O_e(G_i) \) for different number of subsystems \( n \). The result shows the strength of including the incremental abstraction, where the number of states \( |X| \) and transitions \( |T| \) including abstraction is constant independent of \( n \), due to the specific structure of the problem.

Somewhat surprisingly, the incremental visible bisimulation abstraction \( A_v \) gives an even larger reduction down to only 2 states and 2 transitions, independent of \( n \).
This is shown in (Noori-Hosseini et al., 2018). The reason why the conflict equivalence abstraction $\mathcal{A}_c$ does not achieve such an extreme reduction in this example is that the extended local observer $\mathcal{O}_e(G_i)$ includes an additional blocking state as a marker for the non-safe state. Thus, it is clear that for systems with special structures as the one in this example, visible bisimulation can be even more efficient than conflict equivalence. 

\[\square\]

5 Opacity and anonymity for modular systems

So far we have shown how a specific type of states in an observer called non-safe states can be identified in an efficient way for modular systems. In this section a more
Table 1 Comparison of the number of states, transitions and the elapsed time after calculation of the abstracted extended observer $O_e(G)^{Ac}$ using incremental abstraction based on conflict equivalence, and the modular extended observer $O_e(G)$ without abstraction.

| $n$ | $|\hat{X}|$ | $|\hat{T}|$ | $t_e$ (ms) | $|\hat{X}|$ | $|\hat{T}|$ | $t_e$ (s) |
|-----|------------|------------|-----------|------------|------------|----------|
| 3   | 16         | 57         | 5         | 64         | 328        | 0.007    |
| 5   | 16         | 57         | 5         | 1,024      | 8,448      | 0.021    |
| 8   | 16         | 57         | 5         | 65,536     | 847,872    | 2.62     |
| 9   | 16         | 57         | 5         | 262,144    | 3,801,088  | 11.19    |
| 12  | 16         | 57         | 6         | $\approx 9.5 \cdot 10^6$ | - | - |

detailed definition of these non-safe states is given. This is done for the two security problems current state opacity and anonymity, focusing on modular structures.

A centralized architecture is considered, including one single intruder of the system. It is assumed that the intruder has full knowledge of the system structure. However, it can only observe a subset of all system events, included in the set of observable events. Based on its observations, the intruder is assumed to be able to construct an observer of the system, where only observable events are included as transition labels.

5.1 Current state opacity and anonymity

In current-state opacity (CSO) (Saboori and Hadjicostis, 2014; Jacob et al., 2016), the goal is to evaluate if it is possible to estimate any secret states in a system based on its observable events. For a transition system $G$, let $X_S \subseteq X$ be the set of secret states. This system is then said to be opaque if for every string of observable events $s \in \mathcal{L}(G)$, each corresponding state set $Y = \delta(I, s)$ that includes secret states also includes at least one non-secret state from the set $X \setminus X_S$.

**Definition 3 (Current state opacity)** Consider a nondeterministic transition system $G$, where any unobservable events are replaced by $\epsilon$ and $X_S \subseteq X$ is the set of secret states. For a string of observable events $s \in \mathcal{L}(G)$, the block state $Y = \delta(I, s)$ is safe if

$$Y \cap X_S \neq \emptyset \quad \Rightarrow \quad Y \nsubseteq X_S$$

Furthermore, $G$ is current state opaque if for all strings $s \in \mathcal{L}(G)$, all corresponding block states $Y = \delta(I, s)$ are safe.

Since the block states $Y = \delta(I, s)$ in this CSO definition are states in the corresponding observer $O(G)$, the following proposition follows immediately.

**Proposition 2 (Current state opacity and safe/non-safe observer states)** A transition system $G$ is current-state opaque if and only if all states in the observer $O(G)$ are safe. Furthermore, a state $Y$ in $O(G)$ is non-safe if it only includes secret states from $G$, i.e. $Y \subseteq X_S$. 

\[
\]
According to this proposition, a transition system $G$ is current-state non-opaque if and only if at least one state in the observer $O(G)$ only includes secret states from $G$, and is therefore non-safe.

*Current state anonymity* With the increasing popularity of location-based services for mobile devices, the privacy concerns about the unwanted revelation of user’s current location is raised. For this reason the notion of CSO is adapted, and a new related notion called *current state anonymity* (CSA) is introduced (Wu et al., 2014). CSA captures the observer’s inability to know for sure the current locations of moving patterns.

**Definition 4 (Current state anonymity)** Consider a nondeterministic transition system $G$, where any unobservable events are replaced by $\varepsilon$. For a string of observable events $s \in \mathcal{L}(G)$, the block state $Y = \delta(I, s)$ is safe if this state set is not a singleton ($|Y| > 1$). Furthermore, $G$ is current state anonymous if for all strings $s \in \mathcal{L}(G)$, all corresponding block states $Y = \delta(I, s)$ are safe. 

In the same way as for CSO, the block states $Y = \delta(I, s)$ in this CSA definition are states in the corresponding observer $O(G)$, which directly implies the following proposition.

**Proposition 3 (Current state anonymity and safe/non-safe observer states)** A transition system $G$ is current-state anonymous if and only if all block states in the observer $O(G)$ are safe. Furthermore, a block state $Y$ in $O(G)$ is non-safe if it is a singleton ($|Y| = 1$).

According to this proposition, a transition system $G$ is current-state non-anonymous, if and only if at least one block state $Y$ in the observer $O(G)$ is a singleton and is therefore non-safe. Obviously, anonymity is evaluated by verifying that no observer block state is a singleton state. This is natural, since more than one system state in each observer block state implies an uncertainty in determining the exact location of a moving pattern.

The following example shows the observers for CSO and CSA, including their different $N$ (non-safe) state label interpretations.

**Example 3** Consider the transition system $G$ in Fig. 6, where the observable events have been hidden by $\tau$, the unobservable event has been replaced by $\varepsilon$, and the secret states are labeled by $N$. In the observer generation, the source and target states of the $\varepsilon$ transition are merged. Although the observers are structurally equal, depending on the verification problem, the interpretation of the non-safe states and thus, the state labeling differs.

The block state $\{1, 2\}$ in the observer $O_{\text{csO}}(G)$ has label $N$, because both states 1 and 2 are secret states. On the other hand, the corresponding state in $O_{\text{csA}}(G)$ does not have state label $N$, as it is not a singleton state. In the abstraction $O_{\text{csO}}(G)^A$, the $N$-labeled states with a $\tau$ transition in between are merged, while no reduction is achieved for $O_{\text{csA}}(G)^A$. It is observed that the state label $N$ is preserved during the abstraction.
To summarize this subsection, a block state $Y$ in the observer $O(G)$ is non-safe and is augmented with state label $N$, in CSO verification when $Y \subseteq X_{\infty}$, and in CSA verification when $|Y| = 1$. These results will now be generalized to modular systems.

5.2 Current state opacity and anonymity for modular systems

For a modular system $G = \|_{i \in \mathbb{N}_+} G_i$, current state opacity and anonymity are now expressed in terms of safety of the local block states $Y_i$ in $G_i$.

**Definition 5 (Current state opacity for modular systems)** Consider a modular system $G = \|_{i \in \mathbb{N}_+} G_i$. For a string of observable events $s \in L(G)$, the block state $Y = (Y_1, Y_2, \ldots, Y_n) = \delta(I, s)$ is safe if all local block states $Y_i$, $i \in \mathbb{N}_+$ are safe. Furthermore, $G$ is current state opaque if for all strings $s \in L(G)$, all corresponding block states $Y = \delta(I, s)$ are safe. \( \Box \)

According to this definition, it is enough that one local block state $Y_i$ is non-safe, for the global block state $Y = (Y_1, Y_2, \ldots, Y_n)$ to become non-safe. The synchronous composition in Def. 2 also generates this result, by taking the union of the actual non-safe state labels $N$ from the individual subsystems. This semantics is motivated by the fact that the intruder then is able to recognize that a specific subsystem $G_i$ is in a secret state. From a security point of view this is enough for the whole system $G$ to be non-opaque.

If for instance each subsystem models one moving pattern, say a person, and subsystem $G_i$ is in a non-safe block state $Y_i$. Then, the person modeled by $G_i$ is obviously in a secret state. Another scenario, only including one person, means that the person is in a secret state in $G_i$, while local states of other subsystems may represent the absence of the single person. This modeling scenario is applied in the multiple floor/elevator building in Sect. 7.

Generally, the modular system model in Def. 5 is very flexible. Thus, it must be adjusted based on the opacity problem that is of interest, such that it is enough
with one local non-safe block state to make the total modular system current-state non-opaque.

In the following definition of current state anonymity for modular systems, all subsystems must be in a singleton state for the whole system to break the location privacy for moving patterns.

**Definition 6 (Current state anonymity for modular systems)** Consider a modular system $G = \|_{i \in \mathbb{N}_+} G_i$. For a string of observable events $s \in L(G)$, the global block state $Y = (Y_1, Y_2, \ldots, Y_n) = \delta(I, s)$ is non-safe if all local block states $Y_i, i \in \mathbb{N}_+$ are singletons and therefore non-safe. Furthermore, $G$ is current state anonymous if for all strings $s \in L(G)$, all corresponding global block states $Y = \delta(I, s)$ are safe, i.e. no such states are singleton states.

According to this definition, all local block states $Y_i$ must be singleton states and therefore non-safe, for the global block state $Y = (Y_1, Y_2, \ldots, Y_n)$ to be non-safe. The motivation for this interpretation is that the synchronized subsystems together are assumed to model a map. The location privacy is then broken if it is possible to get a specific location on such a map. This corresponds to a global singleton state, involving singleton states for all subsystems.

Unfortunately, this interpretation does not coincide with the ordinary definition of synchronous composition in Def. 2. Here it is necessary to take the intersection of the actual non-safe state labels $N$ from the individual subsystems, to generate a correct global state according to Def. 6. Thus, in current state anonymity for modular systems, the union of non-safe state labels $N$ in the synchronous composition in Def. 2 is replaced by the intersection of these state labels from the individual subsystems.

For a modular system, according to Defs. 5 and 6, the composed block state $Y = (Y_1, Y_2, \ldots, Y_n)$ is a state in the corresponding observer $O(G)$. No shared unobservable events means that this observer can also be computed by the synchronous composition of the corresponding local observers, cf. (2). This is now illustrated in the following example for the modular versions of CSO and CSA.

**Example 4** Consider the observers $O(G_1)$ and $O(G_2)$ in Fig. 7, where the event $a$ represents leaving/entering an elevator ($G_1$) and entering/leaving a corridor ($G_2$), while event $b$ represents entering/leaving a secret room. Each observer state is a block state, where state 2 in $O(G_2)$ is non-safe, labeled with $N$. For opacity analysis it implies that this block state only includes secret system states, and the union of the individual observer state labels results in the state label $N$ for the block state $(1, 2)$ in the composed observer $O(G_1) \parallel O(G_2)$.

In anonymity analysis the block state 2 in $O(G_2)$ is non-safe and therefore a singleton state. Taking the intersection of the individual observer state labels gives no state label $N$ for the block state $(1, 2)$ in the composed observer $O(G_1) \parallel O(G_2)$, since state 1 in $O(G_1)$ is not labeled with $N$. Thus, the composed system $G_1 \parallel G_2$ is anonymous but not opaque.

To summarize this subsection, for a modular system $G = \|_{i \in \mathbb{N}_+} G_i$, a global block state $Y = (Y_1, Y_2, \ldots, Y_n)$ of the observer $O(G)$ is called non-safe and is augmented with state label $N$, if
CSO: $O(G_1) \parallel O(G_2)$

CSA: $O(G_1) \parallel O(G_2)$

\[
\begin{align*}
&0 \xrightarrow{a} 1 \\
&0 \xrightarrow{a} 1 \xrightarrow{b} N \\
&0 \xrightarrow{a} 1 \xrightarrow{b} (1, 2)
\end{align*}
\]

5.3 Other types of opacity

Opacity can also be defined based on languages, see Dubreil et al. (2008), Badouel et al. (2006), and Lin (2011). For a system $G$ with a set of initial states $I$ and a language $L(G, I)$, two sublanguages are introduced, a secret language $L_S \subseteq L(G, I)$ and a non-secret language $L_{NS} \subseteq L(G, I)$, where $L_S \cap L_{NS} = \emptyset$. A projection $P$ from all events to the observable events is also introduced. The system $G$ is then language-based opaque if $L_S \subseteq P^{-1}[P(L_{NS})]$.

To verify language-based opacity (LBO), this formulation can be transformed to CSO as in (Wu and Lafortune, 2013), and then verified based on the techniques proposed in this paper. This includes a modular formulation of the transformation from LBO to CSO. Furthermore, two notions of initial state opacity (ISO) and initial/final state opacity (IFO), as presented in (Wu and Lafortune, 2013), can also be transformed to LBO and then to a CSO problem.

6 Observer abstraction for systems with shared unobservable events

For modular systems with partial observation, and in the presence of shared unobservable events, the final observer can be computed by first generating local observer of each subsystem, and then perform the synchronization. In the process of incremental local observer generation, shared unobservable events become local and are replaced by $\varepsilon$. In the work by Masopust (2018), it is shown that the equation $O(G_1 \parallel G_2) = O(G_1) \parallel O(G_2)$, is not valid anymore, when there are shared unobservable events in the system. The reason is that, in the monolithic approach $O(G_1 \parallel G_2)$, first, the synchronization is performed. Thus, there is no shared unobservable event left and all unobservable events are replaced by $\varepsilon$, when the observer is generated. While, in $O(G_1) \parallel O(G_2)$, since the synchronization is performed after the local observer generation, the shared unobservable events between subsystems still remain. However,
these events incrementally become local after step by step synchronization of local observers. Example 5 illustrates the inequality $O(G_1 \parallel G_2) \neq O(G_1) \parallel O(G_2)$ in the presence of shared unobservable events.

Example 5 Consider two transition systems $G_i$ and their observers $O(G_i)$, $i = 1, 2$, as illustrated in Fig. 8, where $u \in \Sigma_1 \cap \Sigma_2$ and $v \in \Sigma_1$ are unobservable, and $a \in \Sigma_2$ is observable. The local unobservable event $v$ is replaced by $\epsilon$ before observer generation. The synchronization of local observers is $O(G_1) \parallel O(G_2)$, and the monolithic observer of the composed system is $O(G_1 \parallel G_2)$. Synchronization means the cross product of state sets of two subsystems. However, as it is shown in $G_1 \parallel G_2$ in Fig. 8, for simplicity, it is changed to the pairs of states belonging to $G_1$ and $G_2$. It is also illustrated that $O(G_1 \parallel G_2) \neq O(G_1) \parallel O(G_2)$. The reason is the shared unobservable event $u$, that becomes local after the synchronization in $O(G_1) \parallel O(G_2)$. 

It is possible to incrementally perform abstraction before observer generation. Therefore, we evaluate its correctness, and compare it with the case that we perform observer generation before abstraction, as in the below example.

Example 6 A nondeterministic system $G$ is illustrated in Fig. 9, where events $u$ and $v$ are unobservable and the rest of events are observable. Non-safe states are labeled with $N$. In $O\{u,v\}(G)\mathcal{A}^{(a,b,c,d)}$, first, events $\{u,v\}$ are reduced during observer generation and then, events $\{a,b,c,d\}$ are abstracted from the observer. While in $O\{u,v\}(G \mathcal{A}^{(a,b,c,d)})$, first $G$ is abstracted and then, the observer is generated. As it is seen in both mentioned systems, not only they are not equal, but also their opacity result is different. This is due to that, states are merged in wrong blocks, when abstraction is performed before observer generation as in $O\{u,v\}(G \mathcal{A}^{(a,b,c,d)})$. Thus, we define two simple rules to preserve the non-determinism during abstraction and also, keep the unobservable events.

During the incremental hiding and abstraction, some sequences of transitions may appear in the system that have equal string of events, and also share a common source state, which may result in a non-deterministic behavior. This condition is called potentially nondeterministic choices (PNC)s. The CSO/CSA properties are verified based on the existence of $N$-labeled states. Thus, merging states incorrectly,
will result in wrong interpretation about the security properties of the system. To avoid these situations and make sure that all such PNCs are correctly handled, two conservative rules are defined. The rules may be applied only few times during the incremental process, as it is recommended to not use multiple identical events when modeling each subsystem. However, they make the abstraction of the current subsystem less efficient by avoiding hiding on some local events. The rules are as follows.

1. If there are transitions \( t \in T \) that have multiple identical local events \( \sigma \in \Sigma \), repeated \( n \) times, then, do not hide \( \sigma \), and add temporary state labels \( M^i_\sigma \), to the \( i \)-th target state, for \( i = 1, \ldots, n \).

2. For all transitions \( t \in T \) that have unobservable event \( \sigma_u \), add temporary state labels \( M^s_\sigma \) and \( M^t_\sigma \), to the source and target state of \( t \), respectively.

Rule 1 is applied to distinguish the PNCs from each other and thus, preventing abstraction from destroying the interpretation of the security properties. This is a conservative but, simple and easy-to-apply rule. However, it is a topic for future work to find a more efficient rule. In Fig. 10, events \( a, b \) and \( c \) are observable, and \( u \) is unobservable. Although there are multiple identical events in the system, there is no need to add temporary labels on their target states, because there is no PNCs in the system. However, applying rule 1, states are augmented with temporary labels, and thus, no abstraction happens.

Note that, these rules are merely applied on the system including shared unobservable events, otherwise there is no need to apply them. Also, rules are independent of whether a state is already augmented with label \( N \), and \( N \) labels are considered in addition to the temporary state labels. Moreover, all added temporary state labels are remained, until the reason for keeping them is resolved, i.e. when observable events become single and local, and unobservable events become local.

In the incremental procedure, the augmented state labels may evolve during observer generation. When a new block state is built during observer generation, tempo-
rary and $N$ state labels are handled separately. The new temporary label is the union of temporary state labels of all members of the block. Also, depending on whether the problem is CSO or CSA, the new $N$ label is created based on the rules mentioned in Section 5 for observer generation. For CSO, the intersection of labels of all states of the block state is considered, and for CSA, the block state should be a singleton, in order to get label $N$. In the following an examples on CSO is presented, which better illustrates the necessity of the mentioned rules.

**Example 7** Here, the same system as in Example 6 is considered, where the mentioned rules are applied on $G$, as depicted in Fig. 11. The $u$ and $v$-transitions get two temporary state labels on their source and target states. Moreover, the nondeterministic transitions get temporary state labels on the target states. All temporary state labels are added to the previously created $N$-labels.

The label of the block state $\{3, 6, 7\}$ in $O_{\{u,v\}}(G^{A^{\{a,b,c,d\}}})$ is calculated as follows. The temporary state label is the union of all temporary state labels of states 3, 6 and 7, which is $\{M_1^3, M_2^3, M_3^3, M_4^3\}$. Moreover, since the observer is generated for CSO, and in the block state $\{3, 6, 7\}$, state 6 does not have label $N$, then, the block state is not non-safe anymore. Comparing systems $O_{\{u,v\}}(G^{A^{\{a,b,c,d\}}})$ and $O_{\{u,v\}}(G^{A^{\{a,b,c,d\}}})$, one can see that both convey similar CSO interpretation. Also, after reducing event $b$, $O_{\{u,v\}}(G^{A^{\{a,b,c,d\}}})A^{(b)} = O_{\{u,v\}}(G^{A^{\{a,b,c,d\}}})$. □

The above example is on the consecutive abstractions in the incremental procedure of observer generation and abstraction, and the necessity of having rules on reaching to equal systems despite of changing the order of the two operators. On the other hand, Lemma 2 proves the correctness of the consecutive generation of observers in reaching to identical block states.

**Lemma 2 (Incremental observer generation)** Let $G = (X, \Sigma, T, I, AP, \lambda)$, be a nondeterministic transition system including shared unobservable events. $\Sigma^e_i$, $i = 1, 2$, are sets of local unobservable events that are replaced by $\varepsilon$ before the first and the second observer generation. The choice of $\Sigma^e_1$ and $\Sigma^e_2$ are arbitrary. Consider $\Sigma^e_0 = \Sigma^e_1 \cup \Sigma^e_2$, then, $O_{\Sigma^e_0}(G) = O_{\Sigma^e_1}(O_{\Sigma^e_2}(G))$.

Proof: Consider the language of the system $L(G)$ and the projections $P_1 : \Sigma^e \rightarrow (\Sigma \setminus \Sigma_1^e)^*$, $P_2 : \Sigma^e \rightarrow (\Sigma \setminus \Sigma_2^e)^*$ and $P : \Sigma^e \rightarrow (\Sigma \setminus \Sigma^e)^*$. Moreover, $G_1 \triangleq O_{\Sigma_1^e}(G) = (Y_1, \Sigma \setminus \Sigma_1^e, T_1, I_1, AP_1, \lambda_1)$ and $G_2 \triangleq O_{\Sigma_2^e}(G) = (Y_2, \Sigma \setminus \Sigma_2^e, T_2, I_2, AP_2, \lambda_2)$.

After a string $s \in L(G)$ has been executed, the set of reachable states in $O(G)$ can be expressed as $Y_1 = \{x | (\exists x_0 \in I) x_0 \xrightarrow{P_1(s)} x\}$. Moreover, after a string $t \in L(G)$ has been executed, the set of reachable states in $O(G)$ can be expressed as $Y_2 = \{x | (\exists x_0 \in I) x_0 \xrightarrow{P_2(t)} x\}$.
\( \mathcal{L}(O_{\Sigma_1}(G)) \) has been executed in \( O_{\Sigma_1}(G) \), where \( t = P_1(s) \), the set of reachable states in \( O_{\Sigma_1}(O_{\Sigma_1}(G)) \) is \( Y_2 = \{ Y_1 \mid \exists Y_1^0 \in I \} \). On the other hand, consider the isomorphic function \( f \) that is a set of sets, and outputs the union of sets as \( f : \{ Y_1, \ldots, Y_n \} \rightarrow \bigcup_i Y_i \) for \( i \in \mathbb{N}_n^+ \). Thus, \( Y = f(Y_2) \).

\[
Y = \{ x \mid (\exists x_0 \in I) \ x_0 \xrightarrow{P(s)} x \} = \{ x \mid (\exists x_0 \in I) \ x_0 \xrightarrow{P_2(P_1(s))} x \}.
\]

To go further we need to calculate the projection \( P_2(P_1(s)) : \Sigma^* \rightarrow ((\Sigma \setminus \Sigma_f^e) \setminus \Sigma_1^e)^* = (\Sigma \setminus \Sigma_f^e)^* \). On the other hand \( P : \Sigma^* \rightarrow (\Sigma \setminus \Sigma_f^e)^* \), which indicates that \( \forall s \in P_2(\mathcal{L}(G)) \), \( P_2(P_1(s)) = P(s) \), and the final set of reachable states \( Y_2 = Y \).

Lemma 3 proves the equivalence of systems after the change in the order of observer generation and abstraction.

**Lemma 3** (Visible bisimulation equivalence of \( O_{\Sigma^*}(G) \) and \( O_{\Sigma^*}(G^A) \)) Let \( G = (X, \Sigma, T, I, AP, \lambda) \) be a transition system. Then, \( O_{\Sigma^*}(G^A) \simeq \ (O_{\Sigma^*}(G^{\Sigma^K}))^A \).

**Proof:** We have \( O_{\Sigma^*}(G) \sim O_{\Sigma^*}(G^{\Sigma^K}) \). Then, for an arbitrary abstraction equivalent, \( O_{\Sigma^*}(G^{\Sigma^K}) \sim O_{\Sigma^*}(G^{\Sigma^K}) \). We then write \( O_{\Sigma^*}(G^{\Sigma^K}) \sim O_{\Sigma^*}(G^{\Sigma^K}) \), and prove it in the following.

Since in the application of this lemma \( G \) is an observer, it is deterministic. However, the non-determinism may happen later, during the incremental procedure. Another assumption is that there are assigned temporary state labels based on the mentioned rules, on both sides of the equivalence.
The state space is defined as $X \equiv X_r \cup X_\tau$, where $X_r$ is the set of states that are connected with $\epsilon$ and are merged during observer generation, and $X_\tau$ is the set of states that are connected with $\tau$ transitions and are merged during abstraction. Moreover, the partitions of the state space of an ordinary $G$ is defined as $\Pi_G \equiv \Pi_\tau X \cup \Pi_\epsilon X$, where the number of elements in each block is one, i.e. $X = \{x_1, x_2, \ldots, x_n\}$ and $\Pi_G = \{\{x_1\}, \{x_2\}, \ldots, \{x_n\}\}$. Considering this partitioning with singleton sets, makes it possible to proceed and show the effect of abstraction and observer generation on the state space. The proof can be based on the relation between these blocks. The elements in $X_\tau$ are singletons. The observer operator never acts on $\tau$, thus, we split the partition into the parts that are changed after observer generation and the rest.

The new partition is defined as $\Pi_{G,A} \equiv \Pi_\tau^A \cup \Pi_\epsilon^A = \Pi_\tau X^A \cup \Pi_\epsilon X^A$. Finally, $\Pi_{G,A} = \Pi_\tau^{O(G^A)} \cup \Pi_\epsilon^{O(G^A)} = \Pi_\tau^O \cup \Pi_\epsilon^O$. □

The below theorem, proves the general formulation of the incremental observer generation and abstraction in the presence of shared observable events, considering the two mentioned rules.

**Theorem 1 (Incremental local S-observer generation and abstraction)**

In the presence of shared unobservable events, initially, $G_1$ is entered as $O(G_1)$. For the $\Sigma^n$ there are some restrictions on the event abstraction, which are presented in the following. The general equation is as follows.

$$O_{\Sigma^1}(G_1 \parallel G_2)^{A_{\Sigma^1}} \simeq_{v} O_{\Sigma^2}(O_{\Sigma^1}(G_1)^{A_{\Sigma^1}} \parallel O_{\Sigma^2}(G_2)^{A_{\Sigma^2}})^{A_{\Sigma^2}} \quad (6)$$

Starting from the left hand side of the equivalence, $[O_{\Sigma^1}(G_1 \parallel G_2)]^{A_{\Sigma^1}}$, based on Lemma 1 and Lemma 2, we have $[O_{\Sigma^1}(O_{\Sigma^1}(G_1) \parallel O_{\Sigma^2}(G_2))]^{A_{\Sigma^1}}$. Then, following the result of Lemma 3, $[O_{\Sigma^2}([O_{\Sigma^1}(G_1)]^{A_{\Sigma^1}} \parallel [O_{\Sigma^2}(G_2)]^{A_{\Sigma^2}})]^{A_{\Sigma^2}}$, where the abstraction operator $A$, can be applied locally on each observer. Then, the specific events set of each observer are replaced by $\tau$, as in $[O_{\Sigma^2}([O_{\Sigma^2}(G_2)]^{A_{\Sigma^2}} \parallel [O_{\Sigma^1}(G_1)]^{A_{\Sigma^1}})]^{A_{\Sigma^1}}$. □

The benefit of following the right hand side of the Equation (6) is that we can generate the local observers with less complexity and also get more local events, incrementally, and reduce them through observer generation and abstraction.

The algorithm for the incremental observer generation and abstraction in the presence of shared unobservable events, is the same as Algorithm 2, except that in the for-loop, after the first observer generation, the two rules are applied on all subsystems and, temporary state labels are added and, kept until the reason for keeping them is resolved. Moreover, the equation in line 8 of Algorithm 2 is replaced with $G_2 := O_T((G_2)_I)^A \parallel ((G_2)_2)^A$. The $O_T$ operator, also removes the temporary state labels that are not required anymore, in addition to generating the observer.
7 Security of a multiple floor/elevator building

In order to demonstrate the practical use of the modular and incremental verification procedure, a security problem is formulated based on an $n$-story building with $m$ elevators on each floor. First, for better understanding of the problem, an analytical example is given for $n = 2$ and $m = 2$.

**Example 8** Transition systems for a two-story building with floor models $F^i$ and $F^2$ and elevator models $E^1$ and $E^2$ are shown in Fig. 12. Each floor consists of two corridors and two elevators. Elevator entrances are located in states 2 and 4 in $F^i$ and $E^j$ for $i, j = 1, 2$. There are card readers in the corridors and elevators, which are represented by the events $c^i_j$ and $e^i_j$, respectively. The subscript $j$ indicates the corridor and elevator and the superscript $i$ corresponding floor. The events $u^j$ and $d^j$ indicate the upward and downward movement of the $j$-th elevator, respectively. The observer of the $i$-th floor $O(F^i)$ is also shown in Fig. 12, where no specific set of initial states is assumed ($X_{F^i}^0 = X^F_i$), and the opacity depends on the choice of secret states.

Assume that there are only secret states in the second floor model $F^2$, which means that there can only be non-safe states in the observer $O(F^2)$. The opacity of the total system $F_1 \parallel F_2 \parallel E_1 \parallel E_2 \parallel$ is therefore determined by the state labels of $O(F^2)$. If the secret state is $X_{F^2}^S = \{1\}$ or $X_{F^2}^S = \{3\}$, the system is opaque, since all block states in $O(F^2)$ then include non-secret states, meaning that all states are safe. On the other hand, $X_{F^2}^S = \{1, 3\}$ results in a non-opaque system, since the state $\{1, 3\}$ in $O(F^2)$ then becomes non-safe. \(\blacksquare\)
Building with n floors and m elevators Consider an n-story building with m elevators on each floor. The transition system models of floors and elevators are depicted in Fig. 13. There are $n$ floor models $F^i$, $i \in \mathbb{N}^+_n$, and $m$ elevator models $E^j$, $j \in \mathbb{N}^+_m$. Each floor consists of corridors, rooms and elevators. Elevator entrances are located in states $2, 4, \ldots, 2m - 2, 2m$ in $F^i$ and states $2, 4, \ldots, 2n - 2, 2n$ in $E^j$. Corridors are connected through doors that open using card readers. The card readers are installed at the entrances of the elevators. Passing through the entrances of corridors and elevators are shown by events $c$ and $e$, respectively. Note that $c^j_i$ ($e^j_i$) indicates the $j$-th corridor (elevator) on the $i$-th floor. The events $u_j$ and $d_j$ represent the upward and downward movement of the $j$-th elevator, respectively. The staff moving patterns can be tracked by observing the records of their ID cards that are read by the card readers. All floors have similar structure, but have different secret states which are places in the building that have a storage for secret documents.

Two scenarios One of the staff wants to place a secret document in one of the secret locations in the building. There is an intruder that knows the structure of the system and have access to the records of card readers. The question is then if the intruder can have the knowledge that the secret document is in that specific location. In this case, an opaque system means that even a very careless staff with no specific strategy can place the secret document in any of the secret places, without being concerned that the secret being revealed. Most opacity examples have only one strategy for reaching to an opaque solution, while in this example, no specific strategy is needed as long as the system is opaque.
As an alternative scenario, consider the case when some card readers do not work due to power failure. Since corresponding doors in that case cannot be opened, related transitions are then removed.

**Results** The CSO verification results for the building in Fig. 13, with different number of floors and elevators, are presented in Table 2 for non-opaque systems. Results for the alternative scenario, where some doors can not be opened due to power failure, are shown in Table 3. Restrictions are then introduced such that all systems become opaque.

In the first column of both tables, the pair \((n, m)\) shows the number of floors and elevators. The second column in Table 2 includes the set of secret states on each floor. The set of secret states in Table 3 are the same as in Table 2, but omitted due to shortage of space. The column in both tables that is indicated with a \(\star\) sign, shows the floors with local non-safe states, and in Table 3 also the corridors where card readers do not work.

The number of states \(|\hat{X}|\) and transitions \(|\hat{T}|\), as well as the elapsed time for the verification \(t_e\), are then presented for observers with abstraction \(O(G)^A\) and without abstraction \(O(G)\). The verification is based on the transformation to a nonblocking problem, including conflict equivalence abstraction, as presented in Sect. 4.2.

The results in Tables 2 and 3 clearly demonstrate the strength of including the incremental abstraction. The number of states in Table 3 for three floors and four elevators is more than 1000 times larger when abstraction is not included, and no solution is obtained without abstraction for the larger systems. We also observe that for the non-opaque case in Table 2 the computation does not continue when one of the non-safe states has been found, which makes it much faster than the verification of opaque systems, where the whole reachable state space (although abstracted) needs to be evaluated.

### 8 Compositional Synthesis of A Secure System

So far, the task was compositional nonblocking verification, where the controllability of the events was irrelevant. In this section, the task is to enforce current-state opacity to modular non-opaque systems. Since the local observer \(O(G_i)\) of each individual component is considered, all subsystems are deterministic and all events are observable. Most synthesis algorithms produce the least restrictive supervisor, which restricts the system as little as possible while still being controllable and nonblocking (Ramadge and Wonham, 1989). Compositional methods (Flordal and Malik, 2009) use abstraction to remove states and transitions that are unnecessary for the synthesis purpose. Conflict-preserving abstractions (Malik et al., 2007) are the reduction methods that can be used for the synthesis purpose. In the aforementioned work it is assumed that the synthesized supervisor components cannot observe or disable previous abstracted events. Therefore, this makes abstracted events unobservable.

When it comes to supervisor calculation, the closed-loop behaviour is the property to be preserved after simplification. In (Mohajerani et al., 2014, 2017), a synthesis equivalence is introduced for this purpose. (Mohajerani et al., 2014) proposes
Table 2  Results for non-opaque systems, where number of states and transitions plus execution time are given for observers with abstraction $O(G)^A$ and without abstraction $O(G)$. The $\star$ column shows the floors with local non-safe states.

| $F^i/E^j$-model | $O(G)^A = O(G_i)^A$ | $O(G) = O(G_i)$ |
|------------------|---------------------|-----------------|
| $(n, m)$         | $X^i_S$             | $\hat{X}$ | $\hat{T}$ | $t_e$ (ms) | $\hat{X}$ | $\hat{T}$ | $t_e$ (ms) |
| (1,1)            | $X^1_S = \{1\}$    | $F^1$ | 9 | 13 | 2 | 7 | 11 | 8 |
| (1,3)            | $X^3_S = \{1,2,3\}$ | $F^1$ | 26 | 50 | 4 | 19 | 39 | 9 |
| (2,2)            | $X^2_S = \{3\}$    | $X^3_S = \{1,5\}$ | $F^2$ | 49 | 127 | 5 | 775 | 2,913 | 15 |
| (2,3)            | $X^2_S = \{3\}$    | $X^3_S = \{1,5\}$ | $F^2$ | 25 | 52 | 5 | 3,823 | 14,567 | 44 |
| (3,3)            | $X^3_S = \{1\}$    | $X^2_S = \{1,3\}$ | $X^3_S = \{1,5\}$ | $F^3$ | 32 | 89 | 6 | 347,445 | 1,732,836 | 10,918 |
| (3,4)            | $X^3_S = \{1\}$    | $X^2_S = \{1,3\}$ | $X^3_S = \{5,7\}$ | $F^3$ | 42 | 118 | 7 | $\approx 2.6 \cdot 10^6$ | o.m. | – |
| (4,3)            | $X^3_S = \{1\}$    | $X^2_S = \{1,3\}$ | $X^3_S = \{1,5\}$ | $X^4_S = \{1,2,3\}$ | $F^4$ | 32 | 105 | 4 | $\approx 16 \cdot 10^6$ | o.m. | – |
| (5,3)            | $X^3_S = \{1,3\}$  | $X^2_S = \{1,5\}$ | $X^3_S = \{1,5\}$ | $X^4_S = \{1,2,3\}$ | $X^5_S = \{3,4,5\}$ | $F^5$ | 32 | 121 | 9 | $\approx 22.6 \cdot 10^6$ | o.m. | – |

an improved version of the compositional method that produces more memory efficient supervisors. Also, (Mohajerani et al., 2017) investigates the compositional abstraction-based synthesis of least restrictive, controllable, and nonblocking supervisors for DESs that are given as a large number of finite-state machines. Compositional methods build the synchronous composition incrementally, replacing each individual component $O(G_i)$ by its simpler version $O(G_i)^A$ that still preserves the main characteristics of the components. Thus, here the method in (Mohajerani et al., 2017) is used and the task is to remove the states that violate the specification. Since, the blocking states are undesired states and we would like to avoid them, the $w$-events are considered as uncontrollable.
Table 3  Results for opaque systems, where number of states and transitions plus execution time are given for observers with abstraction $O(G)^A$ and without abstraction $O(G)$. The $\star$ sign column shows the corridors where card readers do not work.

| $F^i/E^j$-model | $O(G)^A =\parallel O(G_i)^A$ | $O(G) =\parallel O(G_i)$ |
|------------------|-----------------------------|-----------------------------|
| $(n, m)$         | $|X|$ | $|\mathcal{F}|$ | $t_e$ (ms) | $|X|$ | $|\mathcal{F}|$ | $t_e$ (ms) |
| $(1,1)$          | $\star$ | 2 | 1 | 1 | 2 | 2 | 7 |
| $(1,3)$          | $(c_1^1)$ | 12 | 19 | 3 | 12 | 24 | 8 |
| $(2,2)$          | $(c_1^2)$ | 13 | 30 | 7 | 487 | 2,014 | 12 |
| $(2,3)$          | $(c_2^2)$ | 44 | 112 | 10 | 1,699 | 6,450 | 24 |
| $(3,3)$          | $(c_1^3, c_3^3)$ | 266 | 1,048 | 45 | 75,520 | 380,508 | 1,673 |
| $(3,4)$          | $(c_2^3, c_4^3)$ | 413 | 1,667 | 106 | 591,867 | 3,042,099 | 18,058 |
| $(4,3)$          | $(c_1^4, c_3^4, c_1^4)$ | 3,043 | 17,807 | 10,530 | $\approx 2 \cdot 10^6$ | o.m. | – |
| $(5,3)$          | $(c_1^5, c_3^5, c_1^5, c_2^5)$ | 19,556 | 142,087 | 24', 39" | $\approx 23 \cdot 10^6$ | o.m. | – |

The current-state opacity notion is verified by checking the existence of non-safe states in the system. For this reason, non-safe states are transformed to blocking states and the current-state opacity verification problem becomes a non-blocking problem. In order to design a non-blocking supervisor, the undesired states should be avoided and therefore, the transitions to these blocking states are considered as uncontrollable transitions. Thus, we can synthesise a current-state opaque transition system if and only all uncontrollable transitions to blocking states are avoided.

**Proposition 4** A monolithic supervisor is current-state opaque if and only all uncontrollable transitions to blocking states are avoided.\hfill $\square$

Table 4 shows $|X|$, $|T|$ and the elapsed time ($t_e^S$) for generating both compositional and monolithic supervisors, for the non-opaque systems explained in Section 7. The model has the same structure with similar set of secret states $X_S$ in each floor.

Table 4 Number of states and transitions of the supervisor for the compositional and monolithic synthesis for a non-opaque system (compositional supervisor and monolithic supervisor).

| $F^i/E^j$-model | compositional $S$ | monolithic $S$ |
|------------------|------------------|------------------|
| $(n, m)$         | $|X|$ | $|T|$ | $t_e^S$ (s) | $|X|$ | $|T|$ | $t_e^S$ (s) |
| $(1,1)$          | 2 | 3 | 0.008 | 2 | 3 | 0.01 |
| $(1,3)$          | 1 | 0 | 0.02 | 1 | 0 | 0.02 |
| $(2,2)$          | 2 | 5 | 0.05 | 2 | 5 | 0.13 |
| $(2,3)$          | 3 | 10 | 0.09 | 3 | 6 | 0.31 |
| $(3,3)$          | $(3,7)$ | 10,66 | 70 | 61 | 700 | 441 |
| $(3,4)$          | $(3,6)$ | 12,60 | 54 | – | – | – |
Example 9  For better understanding of the synthesis, again a 2-story building with two elevators and the floor observer depicted in Fig. 12(a) and (b) are considered. The set of secret states for both floors are $X^S_1 = \{3\}$ and $X^S_2 = \{1, 5\}$. As it can be seen in the Fig. 12(c), The observer of the floors has no state exclusively including 3, while there is a state in the second floor exclusively includes 5. This state is an non-safe state, which makes the second floor to be a non-opaque transition system. To make the whole system opaque, we synthesise a non-blocking and controllable supervisor that restrict the second floor transitions from entering the non-safe state. The supervisor is depicted in Fig. 14(a).

9 Conclusions

To tackle the exponential complexity of the computation of a monolithic observer for the current-state opacity verification and enforcement, we apply the compositional reduction approach. To be able to do that, we show that the observer generation can be performed locally before synchronization of the subsystems. This paper proves the possibility of local observer generation in modular systems, before any synchronization, and shows that the same monolithic observer is achieved in the end. This results in an efficient approach for system’s security verification and enforcement. The significant reduction that is achieved using this method, is shown through a computer-based example for verifying and designing a secure system. The example investigates the current-state opacity in a multiple floor/elevator building with an intruder. However, the transformation of other security and privacy notions for modular systems is also shown. A procedure is also developed for modular verification of current-state opacity based on local observers. The results show a significant state space reduction...
to solve these problems for complex systems. An interesting next step is to build a detector with polynomial complexity instead of an observer, to be able to verify and enforce opacity for larger modular systems.

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