SOLUTION OF MULTI OBJECTIVE TRANSPORTATION PROBLEM UNDER FUZZY ENVIRONMENT

R Gowthami\textsuperscript{a} and K Prabakaran\textsuperscript{b} *
Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur, Chennai - 603203, INDIA

*Corresponding author: prabakak2@srmist.edu.in; gowthami1111997@gmail.com

ABSTRACT
The crisp transportation problem is one of the special case of linear programming problems. It is explained with the hypothesis that the cost parameters are indicated accurate way. But in the present world, the transportation parameters may be uncertain due to many uncontrolled factors; this gives rise to fuzzy environment. Here, we proposed a method for the solution of fully fuzzy multi objective transportation problems involving triangular fuzzy numbers. We solve the fully fuzzy multi objective transportation problems without converting it into crisp form. A numerical example is discussed to demonstrate the theory formulated in this article.

Key words: Multi-objective transportation problem, Triangular fuzzy numbers, parametric form, Fuzzy ranking, Fuzzy arithmetic.

1. INTRODUCTION
The transportation problem is particular case of linear programming problems which handle with the allocation of a product from variety of sources of supply to variety of destinations of demand in such a way that the total transportation cost is minimum. Decision making depends exclusively on a single condition is insufficient whenever the decision–making process handled with the complex organizational situation. So, we must understand the presence of numerous criteria that lead to the improvement of multi-criteria decision making. To formulate and solve the transportation problem, the judgment parameters such as supply, demand, unit transportation cost, time, etc. are known precisely.

But in real life situations, the data obtainable is of vague in nature and there is an inexactness degree of vagueness or uncertainty present in the problem under consideration. The uncertainty and inexactness can be tackle by the concept of fuzzy sets can be used as an important decision making tool. These imprecise data may be represented by fuzzy numbers. A fuzzy set introduced by Zadeh\cite{19} in 1965 is applied to tackle such uncertain environments.

Most of the authors have given their ideas to solve the different types of fuzzy transportation models to save time and money. Abd El-Wahed\cite{2} gave an interactional fuzzy goal programming approach for multi-objective transportation problems in 2006. Sudhakar and Navaneetha Kumar\cite{16} used the zero-suffix method for solving two stage multi objective fuzzy transportation problems in 2010. Amit Kumar and Anila Gupta\cite{3} projected an innovative technique for solution of linear multi objective
transportation problems involving fuzzy parameters in 2012. Khan and Das [7] worked on fuzzy multi objective optimization with mention to multi objective transportation problems in 2014. In 2016, Muruganandam and Srinivasan[12] obtained an optimal solution for a multi objective two stage fuzzy transportation problem. Sheema et.al[15] worked on multi objective fractional transportation problem in fuzzy environment in 2017. Here, we solve the multi-objective fuzzy transportation problem based on new algorithm without converting it to a classical one.

2. PRELIMINARIES

In this section we recall the basic concepts and the results of triangular fuzzy number and their arithmetic operations.

Definition 2.1

A fuzzy set \( \tilde{a} \) defined on the set of real numbers \( \mathbb{R} \) is said to be a fuzzy number if its membership function \( \tilde{a} : \mathbb{R} \rightarrow [0,1] \) has the following characteristics:

(i) \( \tilde{a} \) is convex, i.e. \( \tilde{a}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{a}(x_1),\tilde{a}(x_2)\} \), for all \( x_1, x_2 \in \mathbb{R} \) and \( \lambda \in [0,1] \)

(ii) \( \tilde{a} \) is normal i.e. there exists an \( x \in \mathbb{R} \) such that \( \tilde{a}(x) = 1 \).

(iii) \( \tilde{a} \) is Piecewise continuous.

Definition 2.2

A fuzzy number \( \tilde{a} \) on \( \mathbb{R} \) is said to be a triangular fuzzy number (TFN) if its membership function \( \tilde{a} : \mathbb{R} \rightarrow [0,1] \) has the following characteristics:

\[
\tilde{a}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3 \\
0, & \text{elsewhere}
\end{cases}
\]

We denote this triangular fuzzy number by \( \tilde{a} = (a_1, a_2, a_3) \). We use \( F(\mathbb{R}) \) to denote the set of all triangular fuzzy numbers. Also if \( m = a_s \) represents the modal value or midpoint, \( \alpha = (a_2-a_1) \) represents the left spread and \( \beta = (a_3-a_2) \) represents the right spread of the triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) \), then the triangular fuzzy number \( \tilde{a} \) can be represented by the triplet \( \tilde{a} = (\alpha, m, \beta) \), i.e. \( \tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta) \).

Definition 2.3

A triangular fuzzy number \( \tilde{a} \in F(\mathbb{R}) \) can also be represented as a pair \( \tilde{a} = (a, \bar{a}) \) of functions \( a(r) \) and \( \bar{a}(r) \) for \( 0 \leq r \leq 1 \) which satisfies the following requirements:

(i) \( a(r) \) is a bounded monotonic increasing left continuous function.

(ii) \( \bar{a}(r) \) is a bounded monotonic decreasing left continuous function.

(iii) \( a(r) \leq \bar{a}(r), \ 0 \leq r \leq 1 \)

Definition 2.4

For an arbitrary triangular fuzzy number \( \tilde{a} = (a, \bar{a}) \), the number \( a_0 = \frac{a(1)+\bar{a}(1)}{2} \) is said to be a location index number of \( \tilde{a} \). The two non-decreasing left continuous functions \( a_* = (a_0 - a) \), \( a^* = (\bar{a} - a_0) \) are called the left fuzziness index function and the right fuzziness index
function respectively. Hence every triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) \) can also be represented by \( \tilde{a} = (a_0, a_*, a^*) \).

2.5 Ranking of triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari[1] proposed a new ranking method based on the left and the right spreads at some \( \alpha \)-levels of fuzzy numbers.

For an arbitrary triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*) \) with parametric form \( \tilde{a} = (\bar{a}(r), \underline{a}(r)) \), we define the magnitude of the triangular fuzzy number \( \tilde{a} \) by

\[
\text{Mag}(\tilde{a}) = \frac{1}{2} \int_0^1 \left[ (a + \bar{a} + a_0) f(r) \, dr \right]
\]

where the function \( f(r) \) is a non-negative and increasing function on \([0,1]\) with \( f(0) = 0 \), \( f(1) = 1 \) and \( \int_0^1 f(r) \, dr = \frac{1}{2} \). The function \( f(r) \) can be considered as a weighting function. In real life applications, \( f(r) \) can be chosen by the decision maker according to the situation. In this paper, for convenience we use \( f(r) = r \).

For any two triangular fuzzy numbers \( \tilde{a} = (a_0, a_*, a^*) \) and \( \tilde{b} = (b_0, b_*, b^*) \) in \( F(R) \), we define the ranking of \( \tilde{a} \) and \( \tilde{b} \) by comparing the \( \text{Mag}(\tilde{a}) \) and \( \text{Mag}(\tilde{b}) \) on \( R \) as follows:

(i) \( \tilde{a} \succeq \tilde{b} \) if and only if \( \text{Mag}(\tilde{a}) \geq \text{Mag}(\tilde{b}) \)

(ii) \( \tilde{a} \preceq \tilde{b} \) if and only if \( \text{Mag}(\tilde{a}) \leq \text{Mag}(\tilde{b}) \)

(iii) \( \tilde{a} \approx \tilde{b} \) if and only if \( \text{Mag}(\tilde{a}) = \text{Mag}(\tilde{b}) \)

2.6 Arithmetic operation on triangular Fuzzy Numbers

Ming Ma et al.[11] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice \( L \). That is for \( a, b \in L \) we define \( a \lor b = \max \{a, b\} \) and \( a \land b = \min \{a, b\} \).

For arbitrary triangular fuzzy numbers \( \tilde{a} = (a_0, a_*, a^*) \) and \( \tilde{b} = (b_0, b_*, b^*) \) and \( * = \{+,-,\times,\div\} \), the arithmetic operations on the triangular fuzzy numbers are defined by \( \tilde{a} \ast \tilde{b} = (a_0 \ast b_0, a_* \lor b_*, a^* \lor b^*) \). In particular for any two triangular fuzzy numbers \( \tilde{a} = (a_0, a_*, a^*) \) and \( \tilde{b} = (b_0, b_*, b^*) \), we define

(i) Addition: \( \tilde{a} + \tilde{b} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max \{a_*, b_*\}, \max \{a^*, b^*\}) \)

(ii) Subtraction: \( \tilde{a} - \tilde{b} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \min \{a_*, b_*\}, \min \{a^*, b^*\}) \)

(iii) Multiplication: \( \tilde{a} \times \tilde{b} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max \{a_*, b_*\}, \max \{a^*, b^*\}) \)

(iv) Division: \( \tilde{a} \div \tilde{b} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \min \{a_*, b_*\}, \min \{a^*, b^*\}) \)
3 MODEL REPRESENTATION

In real life situations, usually every organizer wants to achieve multiple goals simultaneously while making transportation of goods. So MOTP developed by researchers to achieve multiple goals. Like classical transportation problem, in MOTP, quantity \( \tilde{x}_{ij} \) is to be transported from sources \( i = 1, 2, \ldots, m \) to destinations \( j = 1, 2, \ldots, n \) with cost \( \tilde{c}_{ij} \), where \( \tilde{c}_{ij} \) can be transportation cost, total delivery time, energy consumption or minimizing transportation risk etc. The \( k \) objectives \( \tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_k \) are to be associated with transportation cost, total delivery time, cost of damage and/or cost of security etc. So MOTP \( (\eta) \) can be represented mathematically as follows:

\[
\begin{align*}
\text{Min } & \tilde{z}_1 \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \\
\text{Min } & \tilde{z}_k \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}^k
\end{align*}
\]

Subject to

\[
\begin{align*}
\sum_{j=1}^{n} \tilde{x}_{ij} & \approx \tilde{\alpha}_i, \tilde{\alpha}_i \geq 0, (i=1, 2, \ldots, m) \\
\sum_{i=1}^{m} \tilde{x}_{ij} & \approx \tilde{\beta}_j, \tilde{\beta}_j \geq 0, (j=1, 2, \ldots, n) \\
\tilde{x}_{ij} & \geq 0, \text{ for all } i, j \text{ and } \sum_{i=1}^{m} \tilde{\alpha}_i \approx \sum_{j=1}^{n} \tilde{\beta}_j
\end{align*}
\]

Where \( \tilde{c}_{ij} \) co-efficient of the \( r \)th objective,

\( \tilde{\alpha}_i \) supply amount of the item for consumption at source \( i (\tilde{S}_i) \),

\( \tilde{\beta}_j \) demand of the item for consumption at destination \( j (\tilde{D}_j) \),

and \( \tilde{\alpha}_1 \geq 0, \tilde{\beta}_1 \geq 0 \).

4 NEW ALGORITHM FOR PROPOSED METHOD

**Step 1:** Formulate the given multi objective Transportation problem into the parametric form.

**Step 2:** Identify maximum row cost \( (\tilde{\alpha}) \) as \( \tilde{\alpha}_i = \max (\tilde{c}_{ij}) \), for fixed \( i, 1 \leq j \leq n \) and \( 1 \leq r \leq k \),

maximum column cost \( (\tilde{\beta}) \) as \( \tilde{\beta}_j = \max (\tilde{c}_{ij}) \), for fixed \( j, 1 \leq i \leq m \) and \( 1 \leq r \leq k \),

where

\[
\tilde{\alpha} = \{ \tilde{\alpha}_1^{1}, \ldots, \tilde{\alpha}_r^{1}, \ldots; \tilde{\alpha}_1^{m}, \ldots, \tilde{\alpha}_r^{m} \} \text{ and }
\tilde{\beta} = \{ \tilde{\beta}_1^{1}, \ldots, \tilde{\beta}_r^{1}, \ldots; \tilde{\beta}_1^{n}, \ldots, \tilde{\beta}_r^{n} \}.
\]

**Step 3:** Choose \( Q = \max_{1 \leq i \leq m, 1 \leq j \leq n} (\tilde{\alpha}_i^r, \tilde{\beta}_j^r), \forall r \).

**Step 4:** Choose the cell \( (C) \) having \( Q \) as one of its objective value. If there exist more than one cell \( (C) \) then select the one, which has maximum cost for another objectives.
Step 5: Choose the cell containing \( \min \left( \sum_{i=1}^{m} c_{ij}, \text{for fixed } j \right) \) in corresponding row or column of cell chosen in Step 4. If tie occurs, then choose the one to which highest allocation is possible.

Step 6: Make maximum possible allocation to the cell selected in Step 5 and cross out the row or column for which supply / demand is satisfied.

Step 7: Repeat the procedure of Step 3 to Step 6 for remaining sources and destinations until whole supply or demand requirements are not met.

5 NUMERICAL EXAMPLE

Example 5.1: Consider a fully multi-objective fuzzy transportation problem the data for the fuzzy cost and time together with the supply and demand are measured to be triangular fuzzy numbers and is given below

|      | D1       | D2       | D3       | D4       | Supply  |
|------|----------|----------|----------|----------|---------|
| S1   | (1,1.5,2)| (1,2.3)  | (5,7.9)  | (4,6,8)  | (7,8.9) |
|      | (3,4,5)  | (2,4.6)  | (2,3,4)  | (1,3.5)  |         |
| S2   | (1,1.5,2)| (7,8.5,10)| (2,4,6)  | (3,4,5)  | (17,19,21)|
|      | (4,5,6)  | (7,8.9)  | (7,8.5,10)| (9,10,11)|         |
| S3   | (7,8.9)  | (7,9,11) | (3,4,5)  | (5,6,7)  | (16,17,18)|
|      | (4,6,8)  | (1,2,3)  | (3,4,5,6) | (1,1,5,2)|         |
| Demand| (10,11,12)| (2,3,4)  | (13,14,15)| (15,16,17)|         |

| Sources | D1              | D2 | D3             | D4             | Supply  |
|---------|-----------------|----|----------------|----------------|---------|
| S1      | (1.5,0,5-0.5r,0.5-0.5r) | (2,1-r,1-r) | (7,2-2r,2-2r) | (6,2-2r,2-2r) | (8,1-r,1-r) |
|         | (4,1-r,1-r)     |    | (3,1-r,1-r)    | (3,2-2r,2-2r)  |         |
| S2      | (1.5,0,5-0.5r,0.5-0.5r) | (8,5,1,5-1.5r,1.5-1.5r) | (4,2-2r,2-2r) | (4,1-r,1-r) | (19,2-2r,2-2r) |
|         | (5,1-r,1-r)     |    | (8.5,1.5-1.5r,1.5-1.5r) | (10,1-r,1-r)  |         |
| S3      | (8,1-r,1-r)     | (9,2-2r,2-2r) | (4,1-r,1-r) | (6,1-r,1-r) | (17,1-r,1-r) |
|         | (6,2-2r,2-2r)   |    | (4,1,5,1.5-1.5r) | 1.5-0.5-      |         |
Table 5.3: Fuzzy transportation problem after giving the allocation

| Sources | D1 | D2 | D3 | D4 | Supply |
|---------|----|----|----|----|--------|
| S1      | (1.5,0.5-0.5r,0.5-0.5r) | (2.1-r,1-r) | (7,2-2r,2-2r) | (6,2-2r,2-2r) | (8,1-r,1-r) |
|         | (4.1-r,1-r) | (4.2-2r,2-2r) | (3,1-r,1-r) | (3,2-2r,2-2r) | (5,1-r,1-r) |
| S2      | (1.5,0.5-0.5r,0.5-0.5r) | (8.5,1.5-1.5r,1.5-1.5r) | (4.2-2r,2-2r) | (4,1-r,1-r) | (10,1-r,1-r) |
|         | (5,1-r,1-r) | (8,1-r,1-r) | (8.5,1.5-1.5r,1.5-1.5r) | (8,1-r,1-r) | (19,2-2r,2-2r) |
| S3      | (8,1-r,1-r) | (9,2-2r,2-2r) | (4,1-r,1-r) | (6,1-r,1-r) | (17,1-r,1-r) |
|         | (6,2-2r,2-2r) | (2,1-r,1-r) | (4,5,1.5-1.5r,1.5-1.5r) | (1,1-r,1-r) | (1.5,0.5-0.5r,0.5-0.5r) |
|         | | | | | (16,1-r,1-r) |
| Demand  | (11,1-r,1-r) | (3,1-r,1-r) | (14,1-r,1-r) | (16,1-r,1-r) |

Total cost = \((2,1-r,1-r)(3,1-r,1-r) + (7,2-2r,2-2r)(5,1-r,1-r) + (1.5,0.5-0.5r,0.5-0.5r)(11,2-2r,2-2r) + (4.2-2r,2-2r)(8,1-r,1-r) + (4,1-r,1-r)(1.1-r,1-r) + (6,1-r,1-r)(16,1-r,1-r)\)

=\((189.5,2-2r,2-2r)\), \(\forall r \in [0,1]\)

If \(r=0\) then total cost = \((187.5,189.5,191.5)\)

Total time = \((4.2-2r,2-2r)(3,1-r,1-r) + (3,1-r,1-r)(5,1-r,1-r) + (5,1-r,1-r)(11,2-2r,2-2r) + (8.5,1.51.5r,1.5-1.5r)(8,1-r,1-r) + (4,1-r,1-r)(1,1-r,1-r) + (1.5,0.5-0.5r,0.5-0.5r)(16,1-r,1-r)\)

=\((178.5,2-2r,2-2r)\), \(\forall r \in [0,1]\)

If \(r=0\) then total time = \((176.5,178.5,180.5)\)
Example 5.2:

Consider an example of unbalanced fully multi-objective fuzzy transportation problem the data for the fuzzy cost and time together with the supply and demand are measured to be triangular fuzzy numbers and is given below

Table 5.4: Multi-objective fuzzy Transportation problem

| Source | D1        | D2        | D3        | D4        | Supply        |
|--------|-----------|-----------|-----------|-----------|---------------|
| S1     | (0.6,0.8,0.9) | (2.4,3.3,5) | (1.8,2.1,2.3) | (1.5,1.8,2) | (17.2,18,19.2) |
|        | (0.1,0.3,0.9) | (1.1,5.2)  | (1.1,4.1.6) | (0.7,0.9,1)  |               |
| S2     | (1.1,1.3,1.4) | (3.1,3.6,4.1) | (1.4,1.6,1.7) | (2.2,2.5,2.7) | (22.6,24,25.8) |
|        | (0.4,0.6,0.7) | (1.5,1.8,2.1) | (0.8,0.9,1.1) | (1.1,4.1.8)  |               |
| S3     | (1.5,1.8,2)  | (3.1,3.5,3.9) | (2.1,2.4,2.7) | (0.8,1.1.1)  | (12.6,13,13.6) |
|        | (0.6,0.9,1.1) | (1.4,1.8,2.2) | (1.3,1.5,1.7) | (0.2,0.3,0.4) |               |
| Demand | (11.2,12,12,6) | (5.4,6,6,4)  | (14.8,16,16.8) | (19,20,20.8) |               |

Table 5.5: Fuzzy transportation problem in parametric form

| Source | D1        | D2        | D3        | D4        | Supply        |
|--------|-----------|-----------|-----------|-----------|---------------|
| S1     | (0.8,0.2-0.2r,0.1-0.1r) | (3.0,6-0.6r,0.5-0.5r) | (2.1,0.3-0.3r,0.2-0.2r) | (1.8,0.3-0.3r,0.2-0.2r) | (18,0.8-0.8r,1.2-1.2r) |
|        | (0.3,0.2-0.2r,0.6-0.6r) | (1.5,0.5-0.5r,0.5-0.5r) | (1.4,0.4-0.4r,0.2-0.2r) | (0.9,0.2-0.2r,0.1-0.1r) |               |
| S2     | (1.3,0.2-0.2r,0.1-0.1r) | (3.6,0.5-0.5r,0.5-0.5r) | (1.6,0.2-0.2r,0.1-0.1r) | (2.5,0.3-0.3r,0.2-0.2r) | (24,1.4-1.4r,1.8-1.8r) |
|        | (0.6,0.2-0.2r,0.1-0.1r) | (1.8,0.3-0.3r,0.3-0.3r) | (0.9,0.1-0.1r,0.2-0.2r) | (1.4,0.4-0.4r,0.4-0.4r) |               |
| S3     | (1.8,0.3-0.3r,0.2-0.2r) | (3.5,0.4-0.4r,0.4-0.4r) | (2.4,0.3-0.3r,0.3-0.3r) | (1,0.2-0.2r,0.1-0.1r)  | (13,0.4-0.4r,0.6-0.6r) |
|        | (0.9,0.3-0.3r,0.2-0.2r) | (1.8,0.4-0.4r,0.4-0.4r) | (1.5,0.2-0.2r,0.2-0.2r) | (0.3,0.1-0.1r,0.1-0.1r) |               |
| Demand | (12,0.8-0.8r,0.6-0.6r) | (6,0.6-0.6r,0.4-0.4r)  | (16,1.2-1.2r,0.8-0.8r) | (20,1-r,0.8-0.8r)      |               |
### Table 5.6: Balanced Fuzzy transportation problem in parametric form

| Source | D1               | D2               | D3               | D4               | D5               | Supply         |
|--------|------------------|------------------|------------------|------------------|------------------|----------------|
| S1     | (0.8,0.2-0.2r,0.1-0.1r) | (3.0,6-0.6r,0.5-0.5r) | (2.1,0.3-0.3r,0.2-0.2r) | (1.8,0.3-0.3r,0.2-0.2r) | (0.0-r,0-r)    | (18.0,8-0.8r,1.2-1.2r) |
|        | (0.3,0.2-0.2r,0.6-0.6r) | (1.5,0.5-0.5r,0.5-0.5r) | (1.4,0.4-0.4r,0.2-0.2r) | (0.9,0.2-0.2r,0.1-0.1r) | (0.0-r,0-r)    |                             |
| S2     | (1.3,0.2-0.2r,0.1-0.1r) | (3.6,0.5-0.5r,0.5-0.5r) | (1.6,0.2-0.2r,0.1-0.1r) | (2.5,0.3-0.3r,0.2-0.2r) | (0.0-r,0-r)    | (24.1,4-1.4r,1.8-1.8r)    |
|        | (0.6,0.2-0.2r,0.1-0.1r) | (1.8,0.3-0.3r,0.3-0.3r) | (0.9,0.1-0.1r,0.2-0.2r) | (1.4,0.4-0.4r,0.4-0.4r) | (0.0-r,0-r)    |                             |
| S3     | (1.8,0.3-0.3r,0.2-0.2r) | (3.5,0.4-0.4r,0.4-0.4r) | (2.4,0.3-0.3r,0.3-0.3r) | (1.0,2-0.2r,0.1-0.1r) | (0.0-r,0-r)    | (13.0,4-0.4r,0.6-0.6r)    |
|        | (0.9,0.3-0.3r,0.2-0.2r) | (1.8,0.4-0.4r,0.4-0.4r) | (1.5,0.2-0.2r,0.2-0.2r) | (0.3,0.1-0.1r,0.1-0.1r) | (0.0-r,0-r)    |                             |
| Demand | (12,0.8-0.8r,0.6-0.6r) | (6.0,6-0.6r,0.4-0.4r) | (16,1.2-1.2r,0.8-0.8r) | (20,1-r,0.8-0.8r) | (1,0-r,0-r)     |                             |

### Table 5.7: Fuzzy transportation problem after giving the allocation

| Source | D1               | D2               | D3               | D4               | D5               | Supply         |
|--------|------------------|------------------|------------------|------------------|------------------|----------------|
| S1     | (0.8,0.2-0.2r,0.1-0.1r) | (3.0,6-0.6r,0.5-0.5r) | (2.1,0.3-0.3r,0.2-0.2r) | (1.8,0.3-0.3r,0.2-0.2r) | (0.0-r,0-r)    | (18.0,8-0.8r,1.2-1.2r) |
|        | (0.3,0.2-0.2r,0.6-0.6r) | (1.5,0.5-0.5r,0.5-0.5r) | (1.4,0.4-0.4r,0.2-0.2r) | (0.9,0.2-0.2r,0.1-0.1r) | (0.0-r,0-r)    |                             |
|        | (5.0,8-0.8r,1.2-1.2r) | (6.0,6-0.6r,1.4-1.4r) | (7.1-r,0.8-0.8r) | | | |
|      | Demand | Total cost | S2       | S3       | Total time |      |
|------|--------|------------|----------|----------|------------|------|
|      | (12,0.8-0.8r,0.6-0.6r) | (6,0.6-0.6r,0.4-0.4r) | (16,1.2-1.2r,0.8-0.8r) | (20,1-r,0.8-0.8r) | (1,0-r,0-r) |      |
| S2   | (1.3,0.2-0.2r,0.1-0.1r) | (3,6,0.5-0.5r,0.5-0.5r) | (1.6,0.2-0.2r,0.1-0.1r) | (2.5,0.3-0.3r,0.2-0.2r) | (0,0-r,0-r) | (24,1.4-1.4r,1.8-1.8r) |
|      | (0.6,0.2-0.2r,0.1-0.1r) | (1.8,0.3-0.3r,0.3-0.3r) | (0.9,0.1-0.1r,0.2-0.2r) | (1.4,0.4-0.4r,0.4-0.4r) | (0,0-r,0-r) | (13,0.4-0.4r,0.6-0.6r) |
|      | (7,0.8-0.8r,0.6-0.6r)  | (16,1.2-1.2r,0.8-0.8r) | (16,1.2-1.2r,0.8-0.8r) | (24,1.4-1.4r,1.8-1.8r) | (0,0-r,0-r) | (13,0.4-0.4r,0.6-0.6r) |
| S3   | (1.8,0.3-0.3r,0.2-0.2r) | (3.5,0.4-0.4r,0.4-0.4r) | (2.4,0.3-0.3r,0.3-0.3r) | (1.0,2-0.2r,0.1-0.1r) | (0,0-r,0-r) | (13,0.4-0.4r,0.6-0.6r) |
|      | (0.9,0.3-0.3r,0.2-0.2r) | (1.8,0.4-0.4r,0.4-0.4r) | (1.5,0.2-0.2r,0.2-0.2r) | (0.3,0.1-0.1r,0.1-0.1r) | (0,0-r,0-r) | (13,0.4-0.4r,0.6-0.6r) |
| Demand | (12,0.8-0.8r,0.6-0.6r) | (6,0.6-0.6r,0.4-0.4r) | (16,1.2-1.2r,0.8-0.8r) | (20,1-r,0.8-0.8r) | (1,0-r,0-r) |      |

**Total cost** = (0.8,0.2-0.2r,0.1-0.1r) (5,0.8-0.8r,1.2-1.2r) + (3,0.6-0.6r,0.5-0.5r) (6,0.6-0.6r,0.4-0.4r) + (1.8,0.3-0.3r,0.2-0.2r) (7,1-r,0.8-0.8r) + (1.3,0.2-0.2r,0.1-0.1r) (7,0.8-0.8r,0.6-0.6r) + (1.6,0.2-0.2r,0.1-0.1r) (16,1.2-1.2r,0.8-0.8r) + (0.0-r,0-r) (1,1.4-1.4r,1.8-1.8r) + (1.0,2-0.2r,0.1-0.1r) (13,0.4-0.4r,0.6-0.6r)

= (82.3,1.4-1.4r,1.8-1.8r), \( \forall r \in [0,1] \)

If r=0 then total cost = (80.9, 82.3, 84.1)

**Total time** = (0,3,0.2-0.2r,0.6-0.6r) (5,0.8-0.8r,1.2-1.2r) + (1.5,0.5-0.5r,0.5-0.5r) (6,0.6-0.6r,1.4-1.4r) + (0.9,0.2-0.2r,0.1-0.1r) (7,1-r,0.8-0.8r) + (0.6,0.2-0.2r,0.1-0.1r) (7,0.8-0.8r,0.6-0.6r) + (0.9,0.1-0.1r,0.2-0.2r) (16,1.2-1.2r,0.8-0.8r) + (0.0-r,0-r) (1,1.4-1.4r,1.8-1.8r) + (0.3,0.1-0.1r,0.1-0.1r) (13,0.4-0.4r,0.6-0.6r)

= (39,3.1.4-1.4r,1.8-1.8r) , \( \forall r \in [0,1] \)

If r=0 then total time = (37.9, 39.3, 41.1)
6. CONCLUSION

In the above model, we developed a technique for the solution of multi objective transportation problem. Using the proposed algorithm we discussed numerical examples without changing them into equivalent crisp transportation problem. Here we represented the triangular fuzzy numbers in terms of location index and fuzziness index functions and a solution is obtained in its parametric form. Additionally the projected technique is more flexible for the Decision maker to come to a decision $r \in [0,1]$ regarding upon the condition.

REFERENCES

[1] Abbasbandy S and Hajjari T 2009 A new approach for ranking of trapezoidal fuzzy numbers, *Computers and Mathematics with Applications*. 57 pp 413–419.
[2] Abd W. F. and El-Wahed 2006 Interactive fuzzy goal programming for multi-objective transportation problems, *Omega*, 34 pp 158-166.
[3] Anila Gupta and Amit Kumar 2012 A new method for solving linear multi-objective transportation problems with fuzzy parameters, *Applied Mathematical Modelling* 36 pp 1421-1430.
[4] Bellman R E and Zadeh L A 1970 Decision Making in a fuzzy environment, *Management Science*, 17 pp 141-164.
[5] Chanas Sand Kulcha D 1966 A concept of optimal Solution of the transportation with fuzzy cost co-efficient, *Fuzzy Sets and systems*, 82 (9) pp 299 – 305.
[6] Kaufmann A and Gupta M.M 1985 Introduction to Fuzzy Arithmetic’s: Theory and Applications, Van Nostrand Reinhold, New York.
[7] Khan A J and Das D K 2014, Fuzzy multi objective optimization: With reference to multi objective transportation problem, Recent Research in Science and Technology, 6(1) pp 274-282.
[8] Lakhveer Kaur, MadhuchandaRakshit, & Sandeep singh 2018 A New Approach to solve Multi-objective Transportation Problem, Application *Applied Mathematics: An International Journal*, 13 pp 141-164.
[9] Maxler R T and Arora J S, Survey of multi-objective optimization methods for engineering 2009*Struct Multidisc Optim.,* 26 pp 369 - 395.
[10] Melita E Vinoliah and Ganesan K 2018 Solution to a Multi-objective Fuzzy Transportation Problem-A New Approach, *International Journal of Pure and Applied Mathematics*, 119 pp 385-393.
[11] Ming Ma, Menahem Friedman, Abraham kandel 1999 A new fuzzy arithmetic, *Fuzzy sets and systems*, 108 pp 83-109.
[12] Muruganandam S and Srinivasan R 2016, Optimal Solution for Multi-Objective Two Stage Fuzzy Transportation Problem, *Asian Journal of Research in Social Sciences and Humanities*, 6(5) pp 744-752.
[13] Nagoor Gani and Razak K A 2006 Two stage fuzzy transportation problem, *Journal of Physical Sciences*, 10 pp 63 – 69.
[14] Sharif Uddin M, Sushanta K. Roy, and Mesbahuddin Ahmed M 2018 An approach to solve multi-objective transportation problem using fuzzy goal programming and genetic algorithm, *AIP Conference Proceedings* 1978, 470095.
[15] Sheema Sadia, Neha Gupta and Qazi M. Ali 2017 Multiobjective Fractional Transportation Problem in Fuzzy Environment, *International Journal of Mathematical Archive*, 8(12) pp197-209.
[16] Sudhakar V J 2010 Solving the Multi-Objective Two Stage Fuzzy Transportation Problem by Zero Suffix Method, *Journal of Mathematics Research*, 4 pp 135-140.
[17] Thorani Y L P and Ravishankar N 2014 Fuzzy multi-objective transportation model based on new ranking index on generalized LR fuzzy numbers, *Applied Math. Sciences*, **8**(138)pp 6849-6879.

[18] Waiel, F. Abd and El-Wahed 2001 A multi-objective transportation problem under fuzziness, *Fuzzy sets and systems*, **117** pp 27 – 33.

[19] Zadeh L A 1965 Fuzzy sets, *information and control*, **8** pp 338-353.