We describe how non-minimal coupling term between the Higgs boson and gravity can lead to the chaotic inflation in the Standard Model without introduction of any additional degrees of freedom. Produced cosmological perturbations are predicted to be in accordance with observations. The tensor modes of perturbations are practically vanishing in the model.

1 Introduction

This talk is based on the recent work\textsuperscript{1}, and closely follows it. Note, that the expression for the inflationary potential presented here differs from the one presented in the original work—both expressions coincide in the region relevant for inflation, while the expression given here has a wider range of validity (down to the Standard Model regime).

The fact that our universe is almost flat, homogeneous and isotropic is often considered as a strong indication that the Standard Model (SM) of elementary particles is not complete. Indeed, these puzzles, together with the problem of generation of (almost) scale invariant spectrum of perturbations, necessary for structure formation, are most elegantly solved by inflation\textsuperscript{2,3,4,5,6,7}.

The majority of present models of inflation require an introduction of an additional scalar—the “inflaton”. Inflaton properties are constrained by the observations of fluctuations of the Cosmic Microwave Background (CMB) and the matter distribution in the universe. Though the mass and the interaction of the inflaton with matter fields are not fixed, the well known considerations prefer a heavy scalar field with a mass $\sim 10^{13}$ GeV and extremely small self-interacting quartic coupling constant $\lambda \sim 10^{-13}$ for realization of the chaotic inflationary scenario\textsuperscript{8}. This value of the mass is close to the GUT scale, which is often considered as an argument in favour of existence of new physics between the electroweak and Planck scales.

It was recently demonstrated in\textsuperscript{1} that the SM itself can give rise to inflation, provided non-minimal coupling of the Higgs field with gravity. The spectral index and the amplitude of tensor
perturbations can be predicted and be used to distinguish this possibility from other models for inflation; these parameters for the SM fall within the 1σ confidence contours of the WMAP-5 observations.

To explain our main idea, let us consider the Lagrangian of the SM non-minimally coupled to gravity,

\[ L_{\text{tot}} = L_{\text{SM}} - \frac{M^2}{2} R - \xi H^\dagger H R, \]

where \( L_{\text{SM}} \) is the SM part, \( M \) is some mass parameter, \( R \) is the scalar curvature, \( H \) is the Higgs field, and \( \xi \) is an unknown constant to be fixed later. The third term in (1) is in fact required by the renormalization properties of the scalar field in a curved space-time background, so, in principle, it should be added to the usual SM Lagrangian with some constant. Here, we will analyse the situation with large non-minimal coupling parameter \( \xi \gg 1 \), but still not too large for the non-minimal term to contribute significantly to the Plank mass in the SM regime (\( H \sim v \)), i.e. \( \sqrt{\xi} \ll 10^{17} \). Thus, we have \( M \simeq M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18} \text{ GeV} \).

It is well known that inflation has interesting properties in models of this type. However, in these works the scalar was not identified with the Higgs field of the SM. Basically, most attempts were made to identify the inflaton field with the GUT Higgs field. In this case one naturally gets into the regime of induced gravity (where, unlike this paper, \( M = 0 \) and \( M_P \) is generated from the non-minimal coupling term by the Higgs vacuum expectation value). In this case the Higgs field decouples from the other fields of the model, which is generally undesirable. Here we demonstrate, that when the SM Higgs boson is coupled non-minimally to gravity, the scales for the electroweak physics and inflation are separate, the electroweak properties are unchanged, while for much larger field values the inflation is possible.

The paper is organised as follows. We start from discussion of inflation in the model, and use the slow-roll approximation to find the perturbation spectrum parameters. Then we will argue in Section 3 that quantum corrections are unlikely to spoil the classical analysis we used in Section 2. We conclude in Section 4.

2 Inflation and CMB fluctuations

Let us consider the scalar sector of the Standard Model, coupled to gravity in a non-minimal way. We will use the unitary gauge \( H = h/\sqrt{2} \) and neglect all gauge interactions for the time being, they will be discussed later in Section 3. Then the Lagrangian has the form:

\[ S_J = \int d^4x \sqrt{-g} \left\{ - \frac{M^2 + \xi h^2}{2} R + \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\} . \]

This Lagrangian has been studied in detail in many papers on inflation, we will reproduce here the main results of. Compared to [1], we present a better approximation for the inflationary potential here. To simplify the formulae, we will consider only \( \xi \) in the region \( 1 \ll \sqrt{\xi} \ll 10^{17} \), in which \( M \simeq M_P \) with very good accuracy.

It is possible to get rid of the non-minimal coupling to gravity by making the conformal transformation from the Jordan frame to the Einstein frame

\[ \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega(h)^2 = 1 + \frac{\xi h^2}{M_P^2}. \]

This transformation leads to a non-minimal kinetic term for the Higgs field. So, it is convenient
to make the change to the new scalar field $\chi$ with

$$
\frac{d\chi}{dh} = \sqrt{\Omega^2 + \frac{2M_P^2}{\sqrt{\Omega^2}} \left( \frac{d(\Omega^2)}{dh} \right)^2} = \frac{\sqrt{1 + (\xi + 6\xi^2)\frac{h^2}{M_P^2}}}{1 + \xi \frac{h^2}{M_P^2}}.
$$

(4)

Finally, the action in the Einstein frame is

$$
S_E = \int d^4x \sqrt{-\hat g} \left\{ -\frac{M_P^2}{2} \hat R + \frac{\partial \mu \chi \partial^\mu \chi}{2} - U(\chi) \right\},
$$

(5)

where $\hat R$ is calculated using the metric $\hat g_{\mu\nu}$ and the potential is

$$
U(\chi) = \frac{1}{\Omega(h(\chi))^2} \frac{\lambda}{4} (h(\chi)^2 - v^2)^2.
$$

(6)

For small field values $h, \chi < M_P/\xi$ the change of variables is trivial, $h \simeq \xi$ and $\Omega^2 \simeq 1$, so the potential for the field $\chi$ is the same as that for the initial Higgs field and we get into the SM regime. For $h, \chi \gg M_P/\xi$ the situation changes a lot. In this limit the variable change (4) is

$$
\Omega(h)^2 \simeq \exp \left( \frac{2\chi}{\sqrt{6}M_P} \right).
$$

(7)

The potential for the Higgs field is exponentially flat for large $\xi$ and has the form

$$
U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left( 1 - \exp \left( -\frac{2\chi}{\sqrt{6}M_P} \right) \right)^2.
$$

(8)

The full effective potential in the Einstein frame is presented in Fig. 1. It is the flatness of the potential at $\chi \gtrsim M_P$ which makes the successful (chaotic) inflation possible.

Basically, there are two distinct scales—for low field values $h, \chi \ll M_P/\xi$ we have the SM, for high field values $h \gg M_P/\sqrt{\xi}$ ($\chi > M_P$) we have inflation with exponentially flat potential (8) and the Higgs field is decoupled from all other SM fields (because $\Omega \propto h$, see Section 3). In the intermediate region $M_P/\xi \ll h \ll M_P/\sqrt{\xi}$ ($M_P/\xi \ll \chi < M_P$) the coupling with other particles is not suppressed ($\Omega \sim 1$), while the potential and change of variables are still given by (8) and (7).

Analysis of the inflation in the Einstein frame\(^b\) can be performed in the standard way using the slow-roll approximation. The slow roll parameters (in notations of\(^b\)) can be expressed analytically as functions of the field $h(\chi)$ using (4) and (6) (we give here the expressions for the case $h^2 \gtrsim M_P^2/\xi \gg v^2, \xi \gg 1$, exact expressions can be found in 16),

$$
\epsilon = \frac{M_P^2}{2} \left( \frac{dU/d\chi}{U} \right)^2 \simeq \frac{4M_P^4}{3\xi^2 h^4},
$$

(9)

$$
\eta = M_P^2 \frac{d^2U/d\chi^2}{U} \simeq \frac{4M_P^4}{3\xi^2 h^4} \left( 1 - \frac{\xi h^2}{M_P^2} \right),
$$

(10)

$$
\zeta^2 = M_P^2 \frac{d^3U/d\chi^3 dU/d\chi}{U^2} \simeq \frac{16M_P^6}{9\xi^3 h^6} \left( \frac{\xi h^2}{M_P^2} - 3 \right).
$$

(11)

\(^a\)The following two formulae have wider validity range than those in\(^b\), which are valid only for $h \gg M_P/\sqrt{\xi}$.

\(^b\)The same results can be obtained in the Jordan frame\(^b\)\(^b\).

\(^c\)These formulas are valid up to the end of the slow roll regime $h_{end}$, while the formulas (10) and (11) in\(^b\) are applicable only for the earlier inflationary stages, $h^2 \gg M_P^2/\xi$, which is sufficient to calculate primordial spectrum parameters $n_s$ and $r$. 
Slow roll ends when $\epsilon \simeq 1$, so the field value at the end of inflation is $h_{\text{end}} \simeq (4/3)^{1/4} M_P/\sqrt{\xi} \simeq 1.07 M_P/\sqrt{\xi}$. The number of e-foldings for the change of the field $h$ from $h_0$ to $h_{\text{end}}$ is given by

$$N = \int_{h_{\text{end}}}^{h_0} \frac{1}{M_P^2} U \left(\frac{d\chi}{dh}\right)^2 \, dh \simeq \frac{3}{4} \frac{h_0^2 - h_{\text{end}}^2}{M_P^2/\xi}. \quad (12)$$

We see that for all values of $\sqrt{\xi} \ll 10^{17}$ the scale of the Standard Model $v$ does not enter in the formulae, so the inflationary physics is independent on it.

After end of the slow roll the $\chi$ field enters oscillatory stage with diminishing amplitude. After the oscillation amplitude falls below $M_P/\sqrt{\xi}$, the situation returns to the SM one, so at this moment the reheating temperature $T_{\text{reh}} \gtrsim \left(\frac{15\lambda}{8\pi g^*}\right)^{1/4} M_P/\xi \simeq 1.5 \times 10^{13}$ GeV, where $g^* = 106.75$ is the number of degrees of freedom of the SM. Careful analysis may give a larger temperature generated during the decay of the oscillating $\chi$ field, but definitely below the energy scale at the end of the inflation $T_{\text{reh}} < (\frac{15\lambda}{8\pi g^*})^{1/4} M_P/\sqrt{\xi} \simeq 2 \times 10^{15}$ GeV.

As far as the reheating mechanism and the universe evolution after the end of the inflation is fixed in the model, the number of e-foldings for the the COBE scale entering the horizon can be calculated (see [23]). Here we estimate it as $N_{\text{COBE}} \simeq 62$ (exact value depends on the detailed analysis of reheating, which will be done elsewhere). The corresponding field value is $h_{\text{COBE}} \simeq 9.4 M_P/\sqrt{\xi}$. Inserting (12) into the COBE normalization $U/\epsilon = (0.027 M_P)^4$ we find the required value for $\xi$

$$\xi \simeq \sqrt{\frac{\lambda}{3}} \frac{N_{\text{COBE}}}{0.027^2} \simeq 49000 \sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2} v}. \quad (13)$$

Note, that if one could deduce $\xi$ from some fundamental theory this relation would provide a connection between the Higgs mass and the amplitude of primordial perturbations.

The spectral index $n_s = 1 - 6\epsilon + 2\eta$ calculated for $N = 60$ (corresponding to the scale $k = 0.002$ Mpc) is $n_s \simeq 1 - 8(4N + 9)/(4N + 3)^2 \simeq 0.97$. The tensor to scalar perturbation ratio $r$ is $r = 16\epsilon \simeq 192/(4N + 3)^2 \simeq 0.0033$. The predicted values are well within one sigma of the current WMAP measurements [9], see Fig. [2].
3 Radiative corrections

An essential point for inflation is the flatness of the scalar potential in the region of the field values $h \sim 10 M_P / \sqrt{\xi}$ ($\chi \sim 6 M_P$). It is important that radiative corrections do not spoil this property. Of course, any discussion of quantum corrections is flawed by the non-renormalizable character of gravity, so the arguments we present below are not rigorous.

There are two qualitatively different type of corrections one can think about. The first one is related to the quantum gravity contribution. It is conceivable to think that these terms are proportional to the energy density of the field $\chi$ rather than its value and are of the order of magnitude $U(\chi)/M_P^4 \sim \lambda/\xi^2$. They are small at large $\xi$ required by observations. Moreover, adding non-renormalizable operators $h^{2n}/M_P^{2n}$ to the Lagrangian also does not change the flatness of the potential in the inflationary region.

Other type of corrections is induced by the fields of the Standard Model coupled to the Higgs field. In one loop approximation these contributions have the structure

$$\Delta U \sim \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2},$$  

(14)

where $m(\chi)$ is the mass of the particle (vector boson, fermion, or the Higgs field itself) in the background of field $\chi$, and $\mu$ is the normalization point. Note that the terms of the type $m^2(\chi)M_P^2$ (related to quadratic divergences) do not appear in scale-invariant subtraction schemes that are based, for example, on dimensional regularisation (see a relevant discussion in 25, 26, 27, 28). The masses of the SM fields can be readily computed and have the form

$$m_{\psi, A}(\chi) = \frac{m(v) h(\chi)}{v \Omega(\chi)}, \quad m_H^2(\chi) = \frac{d^2U}{d\chi^2}$$  

(15)

for fermions, vector bosons and the Higgs (inflaton) field. It is crucial that for large $\chi$ these masses approach different constants (i.e. the one-loop contribution is as flat as the tree potential) and that (14) is suppressed by the gauge or Yukawa couplings in comparison with the tree term. In other words, one-loop radiative corrections do not spoil the flatness of the potential as well. This argument is identical to the one given in 13.

4 Conclusions

Non-minimal coupling of the Higgs field to gravity leads to the possibility of chaotic inflation in SM. Specific predictions for the primordial perturbation spectrum are obtained. Specifically, very small amount of tensor perturbations is expected, which means that future CMB experiments measuring the B-mode of the CMB polarization (PLANCK) can distinguish between the described scenario from other models (based, e.g. on inflaton with quadratic potential).

At the same time, we expect that the Higgs potential does not enter into the string coupling regime, nor generates another vacuum up to the scale of at least $M_P/\xi \sim 10^{14}$ GeV, so we expect the Higgs mass to be in the window $130\text{GeV} < M_H < 190\text{GeV}$ (see, eg. 29), otherwise the inflation would be impossible.

The inflation mechanism we discussed has in fact a general character and can be used in many extensions of the SM. Thus, the $\nu$MSM of (SM plus three light fermionic singlets) can explain simultaneously neutrino masses, dark matter, baryon asymmetry of the universe and inflation without introducing any additional particles (the $\nu$MSM with the inflaton was considered in 25). This provides an extra argument in favour of absence of a new energy scale between the electroweak and Planck scales, advocated in 27.

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\[a\] Actually, in the Jordan frame, we expect that higher-dimensional operators are suppressed by the effective Planck scale $M_P^2 + \xi h^2$.
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