What is the gravity dual of the confinement/deconfinement transition in holographic QCD?

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Abstract.
We study the gravity dual of the four dimensional pure Yang-Mills theory through D4 branes proposed by Witten (holographic QCD). In the holographic QCD, it has been widely believed that the confinement phase in the pure Yang-Mills theory corresponds to the AdS D4 soliton in the gravity and the deconfinement phase corresponds to the black D4 brane. We inspect this conjecture carefully and show that the correspondence between the black D4 brane and the deconfinement phase is not correct. Instead, by using a slightly different set up, we find an alternative gravity solution called “localized soliton”, which would properly related to the deconfinement phase. In this case, the confinement/deconfinement transition is realized as a Gregory-Laflamme type transition. We find that our proposal naturally explains several known properties of QCD.

1. Introduction
In this study [1], we will focus on holographic QCD from D4 branes [2, 3, 4, 5, 6]. We will discuss some problems with the usual correspondence between the confinement/deconfinement transition in QCD and the Scherk-Schwarz transition between an AdS soliton and a black D4 brane in the gravity dual. Some of these problems were first discussed in [7]. Especially we will show that the black D4 brane cannot be identified with the (strong coupling continuation of the) deconfinement phase in QCD in four dimensions. As a resolution of these problems, we will propose an alternative scenario in which the confinement/deconfinement transition corresponds to a Gregory-Laflamme transition [8, 9, 10] between a uniformly distributed AdS soliton and a localized AdS soliton in the IIB frame. The scenario we propose suggests that we need to reconsider several previous results in holographic QCD including the Sakai-Sugimoto model [6].

2. Holographic construction of QCD
In this section, we will review the construction of four dimensional SU(N) pure Yang-Mills theory (YM4) from N D4 branes [2]. Let us first consider a 10 dimensional Euclidean spacetime with an $S^1$ and consider D4 branes wrapping on the $S^1$. We define the coordinate along this $S^1$ as $x_4$ and its periodicity as $L_4$. The effective theory on this brane is a 5 dimensional supersymmetric Yang-Mills theory (SYM5) on the $S^1_{L_4}$. For the fermions on the brane, the boundary condition along the circle can be AP (antiperiodic) or P (periodic); to specify the theory, we must pick
one of these two boundary conditions. Let us take the AP boundary condition. This gives rise to fermion masses proportional to the Kaluza-Klein scale $1/L_4$, leading to supersymmetry breaking (this is called the SS — Scherk-Schwarz— mechanism). This, in turn, induces masses for the adjoint scalars and for $A_4$, which are proportional to $\lambda_4/L_4$ at one-loop. Therefore, if $\lambda_4$ is sufficiently small and the dynamical scale $\Lambda_{YM}$ and temperature are much less than both the above mass scales, then the fermions, adjoint scalars and KK modes are decoupled and the 5 dimensional supersymmetric Yang-Mills theory is reduced to a four dimensional pure Yang-Mills theory. The precise conditions to obtain the YM4 from the SYM5 are

$$\lambda_4 \ll 1, \quad \beta \gg \frac{L_4}{\lambda_4}, \quad (1)$$

By taking the large $N$ limit of this system a la Maldacena [11] at low temperatures, we obtain the dual gravity description of the compactified 5 dimensional SYM theory [2], which consists, at low temperatures, of a solitonic $D4$ brane solution wrapping the $S^1_{L_4}$. This gravity solution is not always valid. E.g. in order that the stringy modes can be ignored, we should ensure that the curvature in string units must be small [11]. This condition turns out to be equivalent to

$$\lambda_4 \gg 1. \quad (2)$$

This is opposite to the condition (1). Thus, this gravity solution can describe the SYM5 but cannot directly describe the YM4. This is a common problem in the construction of holographic duals of non-supersymmetric gauge theories. Although we cannot compare the gravity and gauge theory directly, we can extrapolate the properties of the gauge theory from the gravity. Many interesting results, including the qualitative predictions in [2, 3, 4, 6], have been obtained using this prescription. Therefore we can expect that the thermodynamics of the Yang-Mills theory is also described through the gravity.

3. Phase structure of the dual gravity
In this section we investigate the thermodynamical phase structure of the gravity and compare the result with the phase structure of the pure Yang-Mills theory. To discuss holographic QCD at finite temperatures, we begin by compactifying Euclidean time in the boundary theory on a circle with periodicity $\beta = 1/T$. In order to determine the gravitational theory, we need to fix the periodicity of fermions in the gauge theory along the time cycle. Let us recall that the gauge theory of interest here is pure Yang Mills theory in four dimensions, which does not have fermions. Fermions reappear when the validity condition of the gravity description (2) is enforced and $\Lambda_{YM}$ goes above the KK scales $1/L_4$. In this sense, fermions are an artifact of the holographic method and in the region of validity of the pure YM theory, the periodicity of the fermion should not affect the gauge theory results.

Indeed we can obtain the thermal partition function of the YM4 from the SYM5 on $S^1_\beta \times S^1_{L_4}$ with either the (AP,AP) or (P,AP) boundary condition $^1$:

$$Z_{\text{SYM5}}^{\text{AP,AP}} = \text{Tr} e^{-\beta H_{\text{SYM5}}} \to \text{Tr} e^{-\beta H_{\text{YM4}}}, \quad (\lambda_4 \to 0, L_4/\lambda_4 \beta \to 0),$$

$$Z_{\text{SYM5}}^{\text{P,AP}} = \text{Tr} (-1)^F e^{-\beta H_{\text{SYM5}}} \to \text{Tr} e^{-\beta H_{\text{YM4}}}, \quad (\lambda_4 \to 0, L_4/\lambda_4 \beta \to 0). \quad (3)$$

As a consequence, it is pertinent to study these two boundary conditions.

The gravity solutions appearing in the (AP,AP) case are summarized as [12]

$$\begin{array}{|c|c|c|}
\hline
\text{Low temperature} & \text{Solitonic D4 solution} & W_0 = 0, W_4 \neq 0 \\
\text{High temperature} & \text{Black D4 solution} & W_0 \neq 0, W_4 = 0 \\
\hline
\end{array} \quad (5)$$

$^1$ Here and elsewhere the boundary conditions will always refer to those of the boundary theory along $S^1_\beta \times S^1_{L_4}$, respectively.
Between these two solutions, a phase transition called Scherk-Schwarz transition happens at \( \beta = L_4 \). Here \( W_0 \) and \( W_4 \) are the Polyakov loop operators:

\[
W_0 = \frac{1}{N} \text{Tr}Pe^{i \int_0^1 A_0 dx^0}, \quad W_4 = \frac{1}{N} \text{Tr}Pe^{i \int_0^{L_4} A_4 dx^0}.
\]

(6)

Vacuum expectation values of these operators characterize the phases of the gauge theory.

The gravity solutions appearing in the (P,AP) case are

| Low temperature | Solitonic D4/uniformly smeared solitonic D3 |
|-----------------|---------------------------------------------|
| High temperature| Localized solitonic D3 solution              |

\[
W_0 = 0, W_4 \neq 0, \quad W_0 = 0, W_4 = 0
\]

(7)

Note that we need to take a T-dual along the temporal \( S^1 \) circle and go to the IIB frame to see the high temperature region, since the masses of the winding modes of the IIA string wrapping along the temporal circle become light when \( T > \sqrt{\lambda_4}/L_4 \). This T-dual maps the solitonic D4 solution in the IIA to solitonic D3 branes uniformly smeared along the dual temporal circle in the IIB. Above a critical temperature \( \sim L_4/\lambda_4 \), this uniform configuration becomes meta-stable and the solitonic D3 branes are localized on the dual temporal circle. This phase transition is a Gregory-Laflamme (GL) type transition \([8, 9, 10]\).

The phases in the 4 dimensional pure Yang-Mills theory are

| Low temperature | Confinement phase |
|-----------------|-------------------|
| High temperature| Deconfinement phase|

Here the value of \( W_4 \) is non-zero, since the 4 dimensional Yang-Mills theory appears in a weak coupling and small \( L_4 \) regime in the SYM5.

Now we compare this phase structure in the Yang-Mills theory and the above gravity results. In the low temperature regime, the properties of the Polyakov loop operators in the solitonic D4 brane agree with the confinement phase. Thus these two phases may be identified. In the high temperature regime, the localized D3 soliton has the same properties of the Polyakov loops in the deconfinement phase whereas the black D4 brane solution does not. Thus the localized D3 solution may correspond to the deconfinement phase but the black D4 brane does not. Indeed the expected phase structures of the SYM5 with the (AP,AP) and (P,AP) boundary condition are shown in Figure 1. In the (AP,AP) case, at least one phase transition occurs between the black D4 brane solution and the deconfinement phase. (This is also expected through the \( Z_2 \) symmetry: \( S_3^1 \leftrightarrow S_{L_4}^1 \)[7]. See also [13].) Therefore the previous conjecture that the black D4 brane corresponds to the deconfinement phase of the Yang-Mills theory is not correct.

Alternatively, in the (P,AP) case, the gravity solutions in the strong coupling regime may smoothly continue to the phases of the weakly coupled 4 dimensional Yang Mills theory. Thus we propose that we should look at these gravity solutions in the (P,AP) case to investigate the dynamics of the Yang-Mills theory. Especially the GL transition between the smeared D3 soliton and the localized D3 soliton would correspond to the confinement/deconfinement transition in the Yang-Mills theory\(^2\).

4. Gregory-Laflamme transition as a Hagedorn transition

Our proposal opens up the interesting possibility of a relation between the GL transition and the Hagedorn transition. It is known that the GL instability is an instability of the KK modes of the graviton along the compact circle \([8]\). In our case, the KK modes along the dual temporal

\( ^2 \) The lattice super Yang Mills study in [14] has analyzed the phase structure in the context of D1 branes compactified on (P,AP) circles, and obtained the similar result to Figure 1. This supports our proposal in the D4 branes.
Figure 1. The phase structures of the five dimensional SYM on $S^3_β \times S^1_{L_4}$ with the (AP,AP) and (P,AP) boundary condition. The gravity analysis is valid in the strong coupling region (the blue region). The 4 dimensional YM description is valid in the upper green regime. The lower green regime in the (AP,AP) case is the mirror of the upper one via the $Z_2$ symmetry $β \leftrightarrow L_4$. The solid black lines correspond to a minimal extrapolation of the phase boundaries through the intermediate region. The dotted line denotes another possible phase transition which is allowed by the $Z_2$ symmetry.

circle, which cause the GL instability in the IIB description, are mapped to winding modes around the temporal circle through the T-duality [15, 16]. This indicate that the GL transition is associated with the excitation of the winding modes of the IIA string. This is similar to the Hagedorn transition in string theory [17], where the temporal winding modes cause the instability. Thus the GL transition in the IIB description might correspond to the Hagedorn transition in the IIA description.

Note that on the large $N$ gauge theory side also, the confinement/deconfinement transition has been shown to be related to the Hagedorn transition [18, 19]. This makes it plausible that the Hagedorn transition in the Yang-Mills theory continues to the Hagedorn transition in the IIA string, which, as we argued above, is possibly the dual of the GL transition in the IIB supergravity.

5. Conclusion
In this study, we showed that the identification between the deconfinement phase and the black D4 brane solution in the (AP,AP) case is not correct and proposed a resolution by using the (P,AP) case. This result suggests that we need to reconsider several previous results, in which the black D4 brane was employed, in holographic QCD including the Sakai-Sugimoto model [6]. One important ingredient in the Sakai-Sugimoto model is the mechanism of chiral symmetry restoration at high temperatures [7]. In [1], we proposed a new mechanism for chiral symmetry restoration in our framework. Another important issue is to explore what geometry corresponds to the deconfinement phase in the real time formalism. Especially the investigation of the viscosity ratio is interesting.

The problems similar to the above had also been encountered in the study of two dimensional bosonic gauge theory in [20]. This indicates that the issues addressed in this paper are rather general in the discussion of holography for non-supersymmetric gauge theories at finite temperatures.
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