Optimal Pilot Symbols Ratio in terms of Spectrum and Energy Efficiency in Uplink CoMP Networks

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Abstract—In wireless networks, Spectrum Efficiency (SE) and Energy Efficiency (EE) can be affected by the channel estimation that needs to be well designed in practice. In this paper, considering channel estimation error and non-ideal backhaul links, we optimize the pilot symbols ratio in terms of SE and EE in uplink Coordinated Multi-point (CoMP) networks. Modeling the channel estimation error, we formulate the SE and EE maximization problems by analyzing the system capacity with imperfect channel estimation. The maximal system capacity in SE optimization and the minimal transmit power in EE optimization, which both have the closed-form expressions, are derived by some reasonable approximations to reduce the complexity of solving complicated equations. Simulations are carried out to validate the superiority of our scheme, verify the accuracy of our approximation, and show the effect of pilot symbols ratio.

Index Terms—Spectrum efficiency, Energy efficiency, Coordinated multi-point, Channel estimation error, Pilot symbols ratio.

I. INTRODUCTION

In Coordinated Multi-point (CoMP) networks, Spectrum Efficiency (SE) and Energy Efficiency (EE), the important indexes in wireless networks [1], [2], can be both improved by the provided cooperative diversity gains [3], [4]. For uplink transmission, Joint Reception CoMP (JR-CoMP), the efficient uplink CoMP scheme, is characterized by simultaneous reception at multiple cooperative nodes with fully data and control information exchange [5]. It is known that the closed-form approximation for EE-SE trade-off is derived with idealistic and realistic accuracy demonstration on the assumption that the backhaul links are prefect [3]. However, ideal backhaul links and perfect Channel State Information (CSI) are limited and impossible in practical networks [6]. Therefore, we mainly study the SE and EE maximization based on non-ideal backhaul links and imperfect CSI.

In practice, only imperfect and non-real-time (long-term) CSI can be obtained and exchanged in the networks due to the channel estimation error and the capacity limited backhaul links. These limitations need to be well considered and have already caught much research attentions. In [7], considering the backhaul link constraint, different theoretical uplink CoMP concepts are analyzed. And the framework incorporating these concepts is provided with practical CoMP algorithms. In [8], the optimal clustering and rate allocation scheme for JR-CoMP networks with delayed CSI feedback is proposed by taking a stochastic decision approach. However, these works only assume non-ideal backhaul links and imperfect CSI sharing but without the consideration of channel estimation error.

It is known that the channel estimation is usually carried out by periodic pilot or training symbols transmission, where the estimation error occurs inevitably [9]–[11]. It is straightforward that too much and too less pilot symbols in channel estimation will both affect the system performance, so its suitable ratio needs to be determined rationally. In terms of SE and EE, decreasing transmit power and reducing pilot symbols will overcome system throughput deterioration, which unfortunately leads to the degradation of Bit Error Probability (BEP) performance [9]. In order to find the optimal trade-off, the power and spacing of the pilot symbols are optimized to maximize SE in adaptive modulation OFDM networks with the imperfect CSI [10]. And in [11], closed-form BEP expression is derived for Filter Bank Multicarrier (FBM) networks, based on which the optimal power allocation between pilot and data symbols is proposed to minimize BEP. In uplink CoMP networks with perfect backhaul links, the channel estimation error is modeled and the system-level computer simulation is conducted to investigate system and user throughput [12]. However, to the best of our knowledge, the pilot symbols assignment for optimal SE and EE in JR-CoMP networks with non-ideal backhaul links has never been discussed so far.

In this paper, we investigate optimal pilot symbols ratio for channel estimation to maximize SE and EE in uplink CoMP networks with non-ideal and limited backhaul links. Modeling the channel estimation error, we first formulate the SE and EE maximization problems by deriving the system capacity. Utilizing the optimal derivative conditions in SE maximization and the Lagrange multiplier method in SE maximization, the optimal pilot symbols ratio at every cooperative node are derived, based on which the maximal system capacity in SE optimization and the minimal transmit power in EE optimization are obtained consequently. To avoid solving complicated equations, we introduce some reasonable approximate relationships to get the closed-form expressions of these results. Finally, numerical simulations validate the superiority of our scheme and the accuracy of our approximation, and reveal that with the increase of pilot symbols ratio, SE and EE first rise rapidly in channel estimation limited region, and then decrease slowly in useful information limited region.

The rest of the paper is organized as follows. System model is described in Section II SE and EE optimizations...
are resolved in Section III and Section V. Simulations are provided in Section VI, followed by conclusion in Section VII.

II. SYSTEM MODEL

In uplink JR-CoMP networks, a common single-antenna User Equipment (UE) transmits information to $M$ coordinated Base Stations (BSs), which can process the received signal cooperatively. These $M$ BSs are connected through non-ideal and limited backhaul links (e.g., wireless links), which cannot afford too much traffic loads. Therefore, with the consideration of backhaul exchange delay and in order to reduce backhaul overhead, only long-term CSI can be exchanged during long time interval among the $M$ coordinated BSs. Due to the limitation, only the non-coherent JR-CoMP can work in this situation. Moreover, the CSI is detected and estimated at each BS with errors, i.e., only imperfect CSI can be obtained. The flat block fading is assumed to characterize the complex channel gain between BS $m$ and the common UE, denoted by $h_m$. $W$ is the system bandwidth and $n$ is the Additive White Gaussian Noise (AWGN) with power $N = W \times N_0$, where $N_0$ is the Power Spectral Density (PSD) of the noise.

A. Signal Model

In each time frame, the desired information $x$ is transmitted to the cluster of $M$ cooperative BSs by the UE with transmit power $P$. The received signal at BS $m$ can be expressed as

$$y_m = \sqrt{P} h_m x + i_m + n,$$  \hspace{1cm} (1)

where $E\{ |x|^2 \} = 1$. $i_m$, with the power $I_m$, is the overall interference at BS $m$ caused by other UEs, using the same resource, outside the $M$ coordinated BSs.

Therefore, with perfect synchronization among the cluster and considering the imperfect channel estimation, the non-coherent combined received signal can be expressed as

$$y = \sum_{m=1}^M \sqrt{P} h_m x + \sum_{m=1}^M \sqrt{P} \tilde{h}_m x + \sum_{m=1}^M (i_m + n),$$ \hspace{1cm} (2)

where $\tilde{h}_m$ is the result of imperfect channel estimation, $\tilde{h}_m$ is the corresponding error, and $\tilde{h}_m = h_m + \hat{h}_m$.

B. Channel Estimation Error

It is known that the channel estimation is based on pilot symbols detection followed by data demodulation in wireless networks [9]. The periodicity of pilot symbols depends on the channel coherence over time, frequency and space etc. It is assumed that pilot symbols are transmitted during the channel coherence interval, i.e., $L$ symbols frame in this paper.

We denote the ratio of the pilot symbols for channel estimation by $\alpha_m$ ($0 \leq \alpha_m \leq 1$) at BS $m$, therefore, $(1-\alpha_m)$ represents the other part for useful information. Under block fading channel, the Minimum Mean Square Error (MMSE) of the estimation at BS $m$ can be expressed [9], as given by

$$e_m = \frac{1}{1 + \alpha_m \cdot L \cdot SNR_m},$$ \hspace{1cm} (3)

where $SNR_m$ is SNR of received signal at BS $m$, given by

$$SNR_m = \frac{P|h_m|^2}{I_m + N}.$$ \hspace{1cm} (4)

Hence, $\bar{h}_m$ and $\hat{h}_m$ then can be formulated in terms of $e_m$, as written by

$$|\bar{h}_m|^2 = |h_m|^2 \cdot e_m, \quad |\hat{h}_m|^2 = |h_m|^2 \cdot (1 - e_m).$$ \hspace{1cm} (5)

C. System Capacity

With imperfect channel estimation, $S_m = P|\tilde{h}_m|^2$ turns out to be the useful signal strength at BS $m$, while $Z_m = P|h_m|^2$ degrades into the interference, which is treated as a noise for average performance due to the randomness of channel estimation each time in practice. Among the received power $S_m$, $\alpha_m \cdot S_m$ is devoted to pilot symbols demodulation, thus only $(1-\alpha_m) \cdot S_m$ is available for desired information. Therefore, using (2) and (5), the SNR for desired information can be obtained under non-coherent combination, as given by

$$SNR = \frac{\sum_{m=1}^M (1 - \alpha_m) P|h_m|^2 (1 - e_m)}{\sum_{m=1}^M P|h_m|^2 e_m + \sum_{m=1}^M I_m + M \cdot N}.$$ \hspace{1cm} (6)

For simplicity and without loss of generality, it is assumed that the overall interference power $I_m$ is identical at each BS for average performance. Then substituting (4) and (6) into (5), the SNR is then transformed, as expressed by

$$SNR = \frac{\sum_{m=1}^M (1 - \alpha_m) SNR_m}{\sum_{m=1}^M \frac{SNR_m}{1 + \alpha_m \cdot L \cdot SNR_m} + M}.$$ \hspace{1cm} (7)

According to Shannon capacity formula, the achievable uplink data rate $C$ can be attained, as given by

$$C = W \cdot \log_2 (1 + SNR).$$ \hspace{1cm} (8)

III. SE OPTIMIZATION

The SE is defined as the radio between the achievable uplink data rate and the system bandwidth, as given by

$$\eta_S = \frac{C}{W} = \log_2 (1 + SNR).$$ \hspace{1cm} (9)

Due to monotonicity of Log function, maximizing $\eta_S$ is equivalent to maximizing $SNR$. Thus, given fixed transmit power $P$, the SE optimization problem can be formulated as

$$\max_{\{\alpha_m\}} SNR, \quad (P1)$$

s.t. $0 \leq \alpha_m \leq 1, \ m \in \{1, 2, \ldots, M\}$.

It can be proved that (P1) is concave since its Hessian matrix is negative definite and the feasible region is linear, which guarantee the existence of exclusive maximum. Moreover, using this concave property, the optimal $\alpha_m$, denoted by $\alpha'_m$, must satisfies $0 < \alpha'_m < 1$ because $SNR > 0$ when $0 < \alpha_m < 1$ and $SNR = 0$ when $\alpha_m = 0$ or $\alpha_m = 1$. 


Proposition 1: With fixed transmit power $P$ at the common UE, the approximate maximal system capacity $C^*$ in non-coherent JR-CoMP networks can be presented by

$$C^* = 2 \cdot W \cdot \log_2 \left( \frac{-b_{SE} + \sqrt{b_{SE}^2 + 4 \cdot M \cdot c_{SE}}}{2 \cdot M} \right),$$

where

$$b_{SE} = \frac{2}{M} \cdot \sum_{m=1}^{M} \frac{2 \cdot SNR_m}{\sqrt{L} \cdot SNR_m},$$

$$c_{SE} = \frac{M}{\sum_{m=1}^{M} \left( \frac{L \cdot SNR_m + 2}{L} + 1 \right)}.$$

Proof: The first derivative of $SNR$ can be expressed as

$$\frac{\partial SNR}{\partial \alpha_m} = -\frac{L \cdot SNR_m^2 \left( \alpha_m^2 \cdot L \cdot SNR_m + 2 \cdot \alpha_m - 1 \right)}{\left( \sum_{m=1}^{M} \frac{SNR_m}{\alpha_m + L \cdot SNR_m} + M \right) \left( 1 + \alpha_m \cdot L \cdot SNR_m \right)^2}.$$

Resolving (16) and considering $\sqrt{SNR + 1} > 0$, $SNR^*$ can be derived, as given by

$$SNR^* = \left( \frac{-b_{SE} + \sqrt{b_{SE}^2 + 4 \cdot M \cdot c_{SE}}}{2 \cdot M} \right)^2 - 1.$$

Therefore, by substituting (17) into (8), the maximal system capacity $C^*$ can be obtained and formulated as (10).

And then the optimal pilot symbols ratio $\alpha_m^*$ can be calculated by substituting (17) into (15).

IV. EE OPTIMIZATION

In this section, we will discuss the optimal ratio of pilot symbols to maximize EE with required uplink data rate, denoted by $R_{ul}$. It is known that the total power consumption of the networks contains transmit power and circuit power. The circuit power can be further decomposed into static component (drive hardware) and dynamic component (process signal), which are denoted by $P_{base}$ and $P_c = \varepsilon \cdot R_{ul}$, where $\varepsilon$ is the power consumption for transmitting a data bit. The EE is defined as the radio between the average data rate and the average total power consumption at all nodes $\Pi$, as given by

$$\eta_E = \frac{R_{ul}}{P_{total}} = \frac{R_{ul}}{P + \varepsilon \cdot R_{ul} + P_{base}},$$

which indicates that given the required uplink data rate, i.e., $R_{ul}$, maximizing $\eta_E$ is equivalent to minimizing the total transmit power $P$. Therefore, the EE optimization problem can be formulated, as given by

$$\min_{\{\alpha_m\}} P,$$

s.t. $SNR \geq 2^\frac{R_{ul}}{\alpha_{min}} - 1,$

$$0 \leq \alpha_m \leq 1, \quad m \in \{1, 2, \ldots, M\}.$$

It can be easily proved that when the optimal solution is obtained, $SNR = 2^\frac{R_{ul}}{\alpha_{min}} - 1$ must be satisfied just by reducing transmit power $P$ to make it hold.
**Proposition 2:** With the required uplink data rate $R_{ul}$ in non-coherent J-R-CoMP networks, the approximate minimal transmit power $P^*$ of the common UE can be presented by

$$P^* = \left( \frac{b_{EE} + \sqrt{b_{EE}^2 + 4 \cdot c_{EE} \cdot \sum_{m=1}^{M} \frac{|h_m|^2}{\sigma^2}}}{2 \cdot \sum_{m=1}^{M} \frac{|h_m|^2}{\sigma^2}} \right)^2,$$

where

$$b_{EE} = \sum_{m=1}^{M} 2 \cdot \sqrt{\frac{|h_m|^2}{\sigma^2}} \cdot \frac{2^{\frac{R_m}{W}}}{\sqrt{L}},$$

$$c_{EE} = M \left( 2^{\frac{R_m}{W}} - 1 \right) - 2 \cdot M \cdot \frac{R_m}{W}.$$

**Proof:** Lagrange multiplier method is adopted to prove this proposition, where the Lagrange multiplier is given by

$$L = P - \lambda_{EE} \cdot \left( SNR - 2^{\frac{R_m}{W}} + 1 \right),$$

where $\lambda_{EE}$ is the Lagrange coefficient. By letting $\frac{\partial L}{\partial \alpha_m} = 0$ and utilizing (11), the necessary condition for optimal $\alpha_m$ is

$$\alpha_m^2 \cdot L \cdot SNR_m + 2 \cdot \alpha_m - 1 = \frac{R_m}{W}.$$

By solving (21) and considering $\alpha_m > 0$, the optimal $\alpha_m$ can be expressed, as given by

$$\alpha_m = \sqrt{1 + L \cdot SNR_m \cdot \frac{2^{\frac{R_m}{W}}}{W} - 1}.$$

By substituting (22) into (7) and considering $SNR = 2^{\frac{R_m}{W}} - 1$, the following equation can be established as

$$\sum_{m=1}^{M} \left( 1 - \frac{\sqrt{\theta_m} - 1}{L \cdot SNR_m} \right) SNR_m \frac{1}{\sqrt{\theta_m} - 1} = 2^{\frac{R_m}{W}} - 1,$$

where

$$\theta_m = 1 + L \cdot SNR_m \cdot 2^{\frac{R_m}{W}}.$$

Similarly, the optimal $P$, denoted by $P^*$, can be calculated by solving (23), based on which $\alpha_m^*$ can be obtained according to (22). However, only the numerical results are acquired in this way due to the complex structures of (23).

Like SE maximization in the former section, some approximate relationships are introduced, as expressed by

$$\gamma_m \gg 1, \quad \vartheta_m \approx \gamma_m - 1,$$

$$\frac{1}{L \sqrt{L \cdot \frac{|h_m|^2}{\sigma^2} \cdot 2^{\frac{R_m}{W}}}} \approx 0,$$

based on which, (23) can be transformed, as given by

$$P \sum_{m=1}^{M} \frac{|h_m|^2}{\sigma^2} - b_{EE} \sqrt{P} - c_{EE} = 0.$$

By solving (24) and considering $\sqrt{P} > 0$, the optimal $P$ can be formulated, as given by (19).

And then the optimal pilot symbols ratio $\alpha_m^*$ can be obtained by substituting (19) into (22).

**V. Simulation results**

In this section, the typical scenario, involving three macro BSs, is considered in simulations to validate our optimal pilots assignment, where the parameters are specified in Table I. Besides our schemes, some other schemes are contained in simulations for comparison, as described by

- **Precise Optimization Scheme (POS):** Obtain $\alpha_m^*$ and calculate $SNR^*$, $P^*$ by solving the equation precisely.
- **Approximate Optimization Scheme (AOS):** Obtain $\alpha_m^*$, $SNR^*$, and $P^*$ by approximate expressions.
- **Genetic Algorithm Scheme (GAS):** Obtain $\alpha_m^*$ by complicated genetic algorithm in Matlab.
- **Traditional Scheme (TA):** Distribute the same $\alpha$ for each BS without optimization.

**TABLE I**

| Parameters                          | Values          |
|-------------------------------------|-----------------|
| System bandwidth (W)                | 10MHz           |
| Noise power spectral density ($N_0$)| -174 dBm/Hz     |
| Cooperative BSs number ($M$)        | 3               |
| Distance between UE and BSs ($d_1$, $d_2$, $d_3$) | 200,250,300 m |
| Average path loss (PL)              | 30 + 40 log_{10} d dB |
| Maximum transmit power ($P_{max}$)  | 46 dBm          |
| Static circuit power consumption ($P_{base}$) | 50 mW        |
| Dynamic circuit factor ($e$)         | 2 mW/Mbps       |
| The channel coherence interval ($L$) | 10^{-3}         |

In Fig. 1, it is observed clearly that POS can attain the highest achievable data rate for all transmit powers, indicating the validity and superiority of our scheme in terms of SE. The achievable data rates for all schemes will rise with the increment of transmit power $P$, but the growth rates all become lower because of the logarithmic relationship between the data rate and the SNR. It can be seen that AOS, much more simple than POS, can obtain the same SE performance, which will be more practical in reality. Therefore, the approximation is reasonable and quite precise. The randomness of GAS also

![Fig. 1. The achievable data rate region for different transmit powers.](image-url)
the channel estimation limited region, a little growth of $\alpha$ will improve the channel estimation considerably and obtain a performance boost due to the property of MMSE (inverse function w.r.t $\alpha$). On the contrary, when $\alpha$ is large enough, the channel estimation improves very slow due to the inverse function in MMSE. If we continue to increase $\alpha$, less symbols will be left and the performance will deteriorate. This region is called useful information limited region.

VI. CONCLUSION

In this paper, with the consideration of channel estimation error and non-ideal backhaul links, the optimal pilot symbols ratio is derived to maximize SE and EE in uplink JR-CoMP networks. The maximal system capacity in SE optimization and the minimal transmit power in EE optimization are also obtained by solving complicated equations. In order to reduce the complexity, some reasonable approximations are introduced to obtain the closed-form expressions of these results. Simulation reveals the superiority of our scheme, the accuracy of our approximations, and the effect of pilot symbols ratio.

ACKNOWLEDGEMENT

The work was supported by National Nature Science Foundation of China Project (Grant No.61471058), Hong Kong, Macao and Taiwan Science and Technology Cooperation Projects (2014DFT10320, 2016YFE0122900), the 111 Project of China (B16006) and Beijing Training Project for The Leading Talents in S&T (No. Z141101001514026).

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