Application of the heavy quark expansion: $|V_{ub}|$ and spectral moments

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The use of inclusive $B$ decays to determine $|V_{ub}|$, $|V_{cb}|$, $m_b$ and heavy quark matrix elements via spectral moments is discussed.

1. $|V_{ub}|$

A precise and model independent determination of the magnitudes of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{ub}$ is important for testing the Standard Model at $B$ factories via the comparison of the angles and the sides of the unitarity triangle.

$|V_{ub}|$ is notoriously hard to measure model-independently. The first extraction of $|V_{ub}|$ from experimental data relied on a study of the lepton energy spectrum in inclusive charmless semileptonic $B$ decay [11], a region in which (as will be discussed) the rate is highly model-dependent. $|V_{ub}|$ has also been measured from exclusive semileptonic $B \to \rho \ell \bar{v}$ and $B \to \pi \ell \bar{v}$ decay [2]. These exclusive determinations also suffer from model dependence, as they rely on form factor models (such as light-cone sum rules [3]) or quenched lattice calculations at the present time (for a review of recent lattice results, see [4]).

In contrast, inclusive decays are quite simple theoretically, and if it were not for the huge background from decays to charm, it would be straightforward to determine $|V_{ub}|$ from inclusive semileptonic decays. Inclusive $B$ decay rates can be computed model independently in a series in $\Lambda_{QCD}/m_b$ and $\alpha_s(m_b)$ using the heavy quark expansion (HQE) [5 6 7 8]. At leading order, the $B$ meson decay rate is equal to the $b$ quark decay rate. The leading nonperturbative corrections of order $\Lambda_{QCD}^2/m_b^2$ are characterized by two heavy quark effective theory (HQET) matrix elements, usually called $\lambda_1$ and $\lambda_2$. These matrix elements also occur in the expansion of the $B$ and $B^*$ masses in powers of $\Lambda_{QCD}/m_b$,

$$m_{B(B^*)} = m_b + \Lambda - \frac{\lambda_1 + 3(-1)\lambda_2}{2m_b} + \ldots$$

Similar formulae hold for the $D$ and $D^*$ masses. The parameters $\Lambda$ and $\lambda_1$ are independent of the heavy $b$ quark mass, while there is a weak logarithmic scale dependence in $\lambda_2$. The measured $B^* - B$ mass splitting fixes $\lambda_2(m_b) = 0.12 \text{GeV}^2$, while $m_b$ (or, more precisely, a well-defined short-distance mass such as $m_b^{(15)}$ [9 10]) and $\lambda_1$ may be determined from other physical quantities (as will be discussed in the second part of this talk). Since the parton level decay rate is proportional to $m_b^2$, the uncertainty in $m_b$ is a dominant source of uncertainty in the relation between $B \to X_u \ell \bar{v}$ and $|V_{ub}|$; an uncertainty in $m_b$ of 50 MeV corresponds to a $\sim 5\%$ determination of $|V_{ub}|$ [2 11].

Unfortunately, the $B \to X_u \ell \bar{v}$ rate can only be measured imposing severe cuts on the phase space to eliminate the $\sim 100$ times larger $B \to X_c \ell \bar{v}$ background. Since the predictions of the OPE are only model independent for sufficiently inclusive observables, these cuts can destroy the convergence of the expansion. This is the case for two kinematic regions for which the charm background is absent and which have received much attention: the large lepton energy region, $E_\ell > (m_b^2 - m_{X_c}^2)/2m_b$, and the small hadronic invariant mass region, $m_X < m_D$ [12 13 14 15].

The poor behaviour of the OPE for these quantities is slightly subtle, because in both cases there is sufficient phase space for many different resonances to be produced in the final state, so an inclusive description of the decays is still appropriate. However, in both of these regions of phase space the $B \to X_u \ell \bar{v}$ decay products are dominated by high energy, low invariant mass hadronic states,

$$E_\ell \sim m_b, \ m_X^2 \sim m_{X_c}^2 \sim \Lambda_{QCD}^2 m_b \ll m_b^2;$$

In this region the differential rate is very sensitive to the details of the wave function of the $b$ quark in the $B$ meson [16]. This can be seen simply from the kinematics. A $b$ quark in a $B$ meson has momentum

$$p_b^\mu = m_b v^\mu + k^\mu$$

where $v^\mu$ is the four-velocity of the quark, and $k^\mu$ is a small residual momentum of order $\Lambda_{QCD}$. If the momentum transfer to the final state leptons is $q$, the invariant mass of the final state hadrons is

$$m_X^2 = (m_b v + k - q)^2 = (m_b v - q)^2 + 2k \cdot (m_b v - q) + O(\Lambda_{QCD}^2).$$

Over most of phase space, the second term is suppressed relative to the first by one power of $\Lambda_{QCD}/m_b$, and so may be treated as a perturbation. This corresponds to the usual
OPE. However, in the region $E_X$ is large and $m_X$ is small, $m_b^2 - q^2 = (E_X, 0, 0, E_X) + O(\Lambda_{QCD})$ is almost light-like, and the first two terms are the same order,

$$m_X^2 = (m_b v - q)^2 + 2E_X k_+ + \ldots, \quad k_+ \equiv k_0 + k_3. \quad (5)$$

The differential rate in this “shape function region” is therefore sensitive at leading order to the wave function $f(k_+)$ which describes the distribution of the light-cone component of the residual momentum of the $b$ quark. $f(k_+)$ is a nonperturbative function and cannot be calculated analytically, so the rate in the region (2) is model-dependent even at leading order in $\Lambda_{QCD}/m_b$.

Near the endpoint, the OPE for the $E_\ell$ spectrum has the form (where $y = 2E_\ell/m_b$)

$$\frac{d\Gamma}{dy} \sim 2\theta(1-y) - \frac{\lambda_1}{3m_b^2} \delta(1-y) - \frac{\rho_1}{9m_b^2} \delta'(1-y) + \ldots$$

$$- \frac{\lambda_2}{3m_b^2} \delta(1-y) - \frac{11\lambda_2}{m_b^2} \delta(1-y) + \ldots + \ldots \quad (6)$$

For $1 - y \sim \Lambda_{QCD}/m_b$, the terms on the first line are all parametrically $O(1)$, and so must be summed to all orders, and the result may be written as a convolution of $f(k_+)$ with the parton-level rate $[16]$. The terms of the second line are less singular near $y = 1$, and so correspond to subleading effects, which will be discussed later.

The situation is illustrated in Fig. 1(a-b), where the lepton energy and hadronic invariant mass spectra are plotted in the parton model (dashed curves) and incorporating a simple one-parameter model for the distribution function (solid curves) $[17]$.

$$f(k_+) = \frac{32}{\pi^2 \Lambda^2} (1-x)^2 e^{-\frac{1}{2} (1-x)^2 / \Lambda^2} \theta(1-x) \quad (7)$$

where

$$x \equiv \frac{k_+}{\Lambda}, \quad \Lambda = 0.48 \text{ GeV}. \quad (8)$$

The differences between the curves in the regions of interest indicate the sensitivity of the spectrum to the precise form of $f(k_+)$, in both curves, the unshaded side of the vertical line denotes the region free from charm background. Because $m_b^2 \sim \Lambda_{QCD} m_B$, the integrated rate in this region is very sensitive to the form of $f(k_+)$, complicating the issue of determining $|V_{ub}|$ model-independently.

### 1.1 Optimized Cuts

One solution to the problem of sensitivity to nonperturbative effects is to find a set of cuts which eliminate the charm background but do not destroy the convergence of the OPE, so that the distribution function $f(k_+)$ is not required. In Ref. [18] it was pointed out that this is the situation for a cut on the dilepton invariant mass. Decays with

$$q^2 > (m_B - m_D)^2 \quad (9)$$

must arise from $b \rightarrow u$ transition. Such a cut forbids the hadronic final state from moving fast in the $B$ rest frame, and simultaneously imposes $m_X < m_D$ and $E_X < m_D$. Thus, the light-cone expansion which gives rise to the shape function is not relevant in this region of phase space $[14, 19]$. The effect of convoluting the $q^2$ spectrum with the model distribution function in Eq. (7) is illustrated in Fig. 1(c).

The region selected by a $q^2$ cut is entirely contained within the $m_X^2$ cut, but because the dangerous region of high energy, low invariant mass final states is not included, the OPE does not break down. The price to be paid is that the relative size of the unknown $\Lambda_{QCD}/m_b$ terms in the OPE grows as the $q^2$ cut is raised. Equivalently, as was stressed in $[20]$, the effective expansion parameter for integrated rate inside the region (9) is $\Lambda_{QCD}/m_b$, not $\Lambda_{QCD}/m_b$. In addition, the integrated cut rate is very sensitive to $m_b$, with a $\pm 80$ MeV error in $m_b$ corresponding to a $\sim \pm 10\%$ uncertainty in $|V_{ub}|$ $[20, 21]$.

A further important source of uncertainty arises from weak annihilation (WA) graphs $[22]$. WA arises at $O(\Lambda_{QCD}^3/m_b^3)$ in the OPE, but is enhanced by a factor of $\sim 16\pi^2$ because there are only two particles in the final state compared with $b \rightarrow u\nu\ell$. Because WA contributes only at the endpoint of the $q^2$ spectrum, it is independent of $q^2_{cut}$ and $m_{cut}$:

$$\frac{d\Gamma_{WA}}{dq^2} \sim (B_2 - B_1) \delta(q^2 - m_b^2) \quad (10)$$

$B_1$ and $B_2$ are matrix elements which are equal for both charged and neutral $B$’s under the factorization hypothesis, and so the size of the WA effect depends on the size of factorization violation. Assuming factorization is violated at the $10\%$ level gives a corresponding uncertainty in $|V_{ub}|$ from a pure $q^2$ cut of $\sim 10\%$ $[22]$, however, this estimate is highly uncertain, being proportional to $16\pi^2 \times$ (factorization violation). In addition, since the contribution is fixed at maximal $q^2$, the corresponding uncertainty grows as the cuts are tightened, reducing the integrated rate.

These uncertainties may be reduced by considering more complicated kinematic cuts: in $[21]$ it was proposed that by combining cuts on both the leptonic and hadronic invariant masses the theoretical uncertainty on $|V_{ub}|$ could be minimized. For a fixed cut on $m_X$, lowering the bound on
Introducing a small dependence on sources is given for a variety of cuts in Table 1. Since the illustrated in Fig. 2. The estimated uncertainty from other processes are negligible. The structure function, some other process \[16,23\]. The best way to measure the dependence at leading order \[16,24\]. At tree level, the relation is, and subleading twist corrections, this spectrum is directly proportional to the structure function,

\[
\frac{d\Gamma}{dE_{\gamma}} = \frac{G_F^2|V_{ub}V_{us}^\ast|^2|C_{7}^{\text{eff}}|^2m_b^5f(E_{\gamma})}{32\pi^4}.
\]

Thus, combining data on \(B \to X_{s}\gamma\) with data from \(B \to X_{u}f\bar{\nu}\), one can eliminate the dependence on the structure function and therefore determine \(|V_{ub}|\) with no model dependence at leading order \[16,24\]. At tree level, the relation is

\[
\frac{|V_{ub}|}{V_{ub}\Gamma_{s}} = \left( \frac{3\alpha}{\pi} |C_{7}^{\text{eff}}|^2 \frac{\Gamma_{s}(E_c)}{\Gamma_{s}(E_c)(1 + \delta(E_c))} \right)\]

where

\[
\Gamma_{u}(E_c) \equiv \int_{E_c}^{m_b/2} dE_{\gamma} \frac{d\Gamma_{u}}{dE_{\gamma}}
\]

\[
\Gamma_{s}(E_c) \equiv \frac{2}{m_b} \int_{E_c}^{m_b/2} dE_{\gamma}(E_{\gamma} - E_c) \frac{d\Gamma_{s}}{dE_{\gamma}}
\]

and \(\delta(E_c)\) contains terms suppressed by \(O(\Lambda_{QCD}/m_b)\). An analogous relation holds for the hadronic invariant mass spectrum \[13,14\]. In addition to higher twist effects, there are perturbative corrections to \[13\]. Most important of these are the parametrically large Sudakov logarithms, which have been summed to subleading order \[24\]. In addition, contributions from additional operators which contribute to \(B \to X_{s}\gamma\) have been calculated \[25\]. The CLEO collaboration \[26\] recently used a variation of this approach to determine \(|V_{ub}|\) from their measurements of the \(B \to X_{s}\gamma\) photon spectrum and the charged lepton spectrum in \(B \to X_{u}f\bar{\nu}\).

The subleading corrections to \[13\] contained in \(\delta(E_c)\) have only recently been studied \[27,28,29,30\]. These are analogous to higher twist effects in DIS, and there are two separate effects, each of which is large.

First of all, the \(O(1/m_b)\) corrections happen to have a large numerical prefactor. This is easiest to see by looking at the

1.2 \(f(k_\gamma)\) and subleading corrections

Alternatively, one can reduce the theoretical uncertainty in \(|V_{ub}|\) by measuring the universal structure function \(f(k_\gamma)\) in other processes \[16,23\]. The best way to measure the structure function \(f(k_\gamma)\) is from the photon energy spectrum of the inclusive decay \(B \to X_{s}\gamma\). Up to perturbative
OPE for the lepton spectrum in semileptonic $b \to u$ decay, Eqn. (6). The terms in the first line are universal, and so are the same for $B \to X_{s}\gamma$ and $B \to X_{c}\ell\bar{\nu}_{\ell}$ decays. The subleading terms on the second line are not universal, and sum to subleading distribution functions [27]. Note that the subleading $\Delta_{s}$ term has a coefficient of 11, whereas the corresponding coefficient in the $B \to X_{s}\gamma$ is 3, giving an $O(A_{QCD}/m_{b})$ correction which is enhanced by a factor of 8 over the naive dimensional estimate. Since this is just the first term of an infinite series, one cannot immediately determine the size of the subleading correction, but for a simple model the corresponding shift of $|V_{ub}|$ is plotted in Fig. 5. For a charged lepton cut of 2.3 GeV, this corresponds to a $\sim 15\%$ shift in the extracted value of $|V_{ub}|$. While this is a substantial shift, it was argued in [30] that the magnitude of this shift is quite insensitive to the model chosen for the subleading distribution function, and so the corresponding uncertainty in $|V_{ub}|$ is much smaller than the overall shift of $\sim 15\%$.

A second source of uncertainty arises because of the WA graphs discussed in the previous section. In the region near $y = 1$, the WA graph is the first term of an infinite series which resums into a sub-subleading (relative order $(1/m_{b}^{2})$) distribution function [29]. As before, the size of the WA contribution is difficult to determine reliably; the authors of [29] estimate the corresponding uncertainty in $|V_{ub}|$ to be at the $\sim 10\%$ level (with unknown sign) for a cut $E_{\ell} > 2.3$ GeV. For both subleading effects, the fractional uncertainty in $|V_{ub}|$ is reduced considerably as the cut on $E_{\ell}$ is lowered below 2.3 GeV.

### 1.3 Summary for $|V_{ub}|$

We are left in the fortunate situation of having a number of theoretically clean methods of determining $|V_{ub}|$ from inclusive decays, each of which has advantages and disadvantages which are summarized in Table 2. Because the different techniques have different sources of uncertainty, agreement between these methods (combined with future unquenched lattice predictions for $B \to \pi\ell\bar{\nu}$ decays) will give evidence that the different sources of uncertainty have been correctly estimated.

In the future, experimental measurements can help reduce the theoretical errors in a number of ways:

- better determinations of $m_{b}$ (through, for example, moments of $B$ decay distributions) can reduce the largest single source of uncertainty for determinations using optimized cuts,
- the size of WA effects may be tested by comparing $D^{0}$ and $D_{s}$ semileptonic decays, or by extracting $|V_{ub}|$ from $B^{\pm}$ and $B^{0}$ decays separately,
- improved measurements of the $B \to X_{s}\gamma$ spectrum will give an improved determination of $f(k_{s})$, and
- studying the dependence of the extracted value of $|V_{ub}|$ as a function of the lepton cut $E_{\ell}$ can test the size of the subleading twist terms in (13).
Table 2. Comparison of different kinematic cuts for the determination of $|V_{ub}|$ from inclusive decays.

| Cut | % of rate | Good | Bad |
|-----|-----------|------|-----|
| $E_{\ell} > \frac{m_{D}^{2} - m_{S}^{2}}{2m_{b}}$ | ~ 10% | simplest to measure | depends on $f(k^{+})$ (and subleading corrections) |
| $m_{X} < m_{D}$ | ~ 70% | lots of rate | reduced phase space - duality issues? |
| $q^{2} > (m_{B} - m_{D})^{2}$ | ~ 20% | insensitive to $f(k^{+})$ | depends on $f(k^{+})$ (and subleading corrections) |
| “Optimized cuts” | up to ~ 45% | insensitive to $f(k^{+})$ | very sensitive to $m_{b}$ |
| | | lots of rate | WA corrections may be substantial |
| | | can move cuts away from kinematic limits and still have small uncertainties | effective expansion parameter is $\Lambda_{QCD}/m_{c}$ |
| | | | - less rate than pure $m_{X}$ cut, and more complicated to measure |

2 Spectral Moments

Since differential rates may be computed in the HQE as a power series in $\alpha_{s}(m_{b})$ and $\Lambda_{QCD}/m_{b}$, an unlimited number of spectral moments may be computed. Different moments have different dependence on the nonperturbative parameters of the HQE, so a simultaneous fit to multiple moments allows these parameters to be determined experimentally. In addition, consistency between different observables provides a powerful check of the validity of the HQE to inclusive decays. Moments which have received particular attention are moments of the charged lepton energy spectrum \([41]\) and hadronic invariant mass spectrum \([32]\) in $B \rightarrow X_{\ell}\ell\nu$ decays, and moments of the photon energy spectrum in $B \rightarrow X_{\gamma}$ decays \([33]\).

By comparing the first moment of the photon spectrum in $B \rightarrow X_{\gamma}$ with the first moment of the hadronic invariant mass spectrum, the CLEO collaboration \([34]\) determined $\Lambda = 0.35 \pm 0.07 \pm 0.10$ GeV and $\lambda_{1} = -0.236 \pm 0.071 \pm 0.078$ GeV\(^2\) (where the first error is experimental and the second theoretical). More recently, additional moments have been measured by CLEO\([35]\), BABAR \([36]\) and DELPHI \([37]\), providing enough constraints to perform a global fit to the HQE including terms of order $1/m_{b}^{4}$. Two such global fits have been recently performed. Ref. \([38]\) found

$$m_{b}^{S} = 4.75 \pm 0.10 \text{ GeV}$$

$$V_{cb} = (40.8 \pm 0.9) \times 10^{-3}$$

while Ref. \([39]\) found, from just the DELPHI moments,

$$m_{b}(1 \text{ GeV}) = 4.59 \pm 0.08 \pm 0.01 \text{ GeV}$$

$$m_{c}(1 \text{ GeV}) = 1.13 \pm 0.13 \pm 0.03 \text{ GeV}$$

$$V_{cb} = (41.1 \pm 1.1) \times 10^{-3}$$

(corresponding to $m_{b}^{S} = 4.69$ GeV.) Ref. \([39]\) also performed a fit using the pole mass scheme. Matrix elements arising up to $O(1/m_{b}^{3})$ in the HQE were also determined from each of the fits.

In the approach of Ref. \([38]\), the charm quark mass is determined by the heavy quark relation

$$m_{b} - m_{c} = \bar{m}_{b} - \bar{m}_{D} - \Lambda_{1} \left( \frac{1}{2m_{c}} - \frac{1}{2m_{b}} \right) + (\rho_{1} - \tau_{1} - \tau_{3}) \left( \frac{1}{4m_{c}^{2}} - \frac{1}{4m_{b}^{2}} \right) + O \left( \frac{1}{m_{c}^{3}} \right)$$

(17)

(where $\bar{m}_{B(D)} \equiv (m_{B(D)} + 3m_{B(D)}^{c})/4$ is the spin-averaged meson mass, and $\rho_{1}$ and the $\tau_{i}$'s are matrix elements of order $\Lambda_{QCD}^{3}$). In the approach of Ref. \([39]\), the charm quark is not treated as heavy, and its mass is taken to be a free parameter, to be fit from the moments. The second approach has the advantage that it does not require an expansion in $\Lambda_{QCD}/m_{c}$. It however has the disadvantage that if the $c$ quark is not treated as heavy, a systematic determination of the parameters $\lambda_{2}$ and $\rho_{2}$ (arising at $O(1/m^{2}_{d})$ and $O(1/m^{3}_{d})$, respectively) from the $D - D^{\ast}$ and $B - B^{\ast}$ mass splittings can no longer be performed.

It is amusing to note that setting experimental errors in the global fit of Ref. \([38]\) to zero, one obtains the uncertainties $\delta|V_{cb}| = \pm 0.35 \times 10^{-3}$, $\delta(m_{b}) = \pm 35$ MeV, which give an indication of the limiting theoretical uncertainty in the analysis.

One can also use this fit to make precise predictions of other moments as a crosscheck. Bauer and Trott \([40]\) examined fractional moments of the lepton spectrum with a variety of cuts, and found certain moments that were very insensitive to nonperturbative effects. These moments may therefore be predicted with small uncertainties using the
values of $\tilde{\Lambda}$ and $\lambda_1$ determined from other moments, providing a stringent test of the HQE for inclusive decays. Later, these moments were measured by CLEO\[35\] and were found to agree beautifully with the theoretical predictions:

$$D_3 = \frac{\int_{1.6 \text{ GeV}} \frac{d^3 m}{dE_\ell} dE_\ell}{\int_{1.5 \text{ GeV}} \frac{d^3 m}{dE_\ell} dE_\ell} = \left\{ \begin{array}{l} 0.5190 \pm 0.0007 \quad \text{(T)} \\
0.5193 \pm 0.0008 \quad \text{(E)} \end{array} \right.$$  

$$D_4 = \frac{\int_{1.6 \text{ GeV}} \frac{d^3 m}{dE_\ell} dE_\ell}{\int_{1.5 \text{ GeV}} \frac{d^3 m}{dE_\ell} dE_\ell} = \left\{ \begin{array}{l} 0.6034 \pm 0.0008 \quad \text{(T)} \\
0.6036 \pm 0.0006 \quad \text{(E)} \end{array} \right.$$  

(18)

(where “T” and “E” denote theory and experiment, respectively).

It should be noted, however, that there is currently poor resolution to note that the BABAR measurement is not completely model independent, but depends on the assumed spectrum of excited $D$ resonances. In the model used, there is no contribution to the $B$ semileptonic width from excited $D$ states with masses below $\sim 2.4$ GeV. This is an assumption which is in conflict with the results of the HQE, as noted a number of years ago by Greffm and Kaudin [41].

Figure 4. Comparison of the BABAR measurements of the hadron invariant mass spectrum vs. the lepton energy cut (black squares), and the HQE prediction not including BABAR hadronic mass data (red triangles), from Ref. [38].

Denoting the lowest excited state as $D^{**}$, a lower bound on the first moment of the hadronic invariant mass spectrum may be obtained [32] by assuming that all rate which does not go to the $D$ or $D^*$ goes to the $D^{**}$. This gives the inequality

$$\langle s_H - \bar{m}_D^2 \rangle = \bar{m}_D^2 \left( 0.051 \frac{\alpha_s}{\pi} + 0.23 \frac{\tilde{\Lambda}}{m_B} + \ldots \right)$$  

$$\geq \Gamma_{D^*}(m_{D^*}^2 - \bar{m}_D^2) + \Gamma_{D^\prime}(m_{D^\prime}^2 - \bar{m}_D^2) + \Gamma_D(m_D^2 - \bar{m}_D^2)$$

where $\Gamma_X \equiv \Gamma(B \rightarrow X\nu)/\Gamma_{S,L}$ is the fraction of the semileptonic width which goes to the final state $X$, and the first line gives the first two terms in the HQE. Combining the HQE prediction with the measured ratio of the semileptonic $B \rightarrow D$ and $B \rightarrow D^*$ widths [42]

$$\Gamma_D = 0.31(1 - \Gamma_{D^*}), \quad \Gamma_{D^*} = 0.69(1 - \Gamma_{D^\prime})$$

and taking $m_{D^*} = 2.45$ GeV leads to the upper bound

$$\Gamma_{D^\prime} < 0.22$$

which is in conflict with the experimental semileptonic branching fraction to excited states of $\sim 0.35$. Thus, if the prediction for the first moment of the hadronic invariant mass spectrum is valid, a non-negligible fraction of the semileptonic $B$ width must be to excited $D$ states with $m_X < 2.45$ GeV, which would also bring the BABAR results in better agreement with theory [43]. It will be interesting to see how this situation evolves.

3 Conclusions

The heavy quark expansion (HQE) has proven very successful at giving a model-independent description of inclusive $B$ decays. Theory and experiment are now at the stage where a precision determination of $|V_{ub}|$ is possible from inclusive decays. The challenge is to incorporate kinematic cuts that exclude $b \rightarrow c$ decays without introducing large uncertainties (theoretical or experimental). Cutting on the lepton invariant mass $q^2$ or an optimized combination of $q^2$ and the hadronic invariant mass $m_X$ gives a result that is insensitive to the nonperturbative light-cone distribution function $f(k_\perp)$, at the expense of a difficult experimental measurement. Cutting on $m_X$ or the energy of the charged lepton $E_\ell$ is easier experimentally, but introduces dependence on the nonperturbative parton distribution function $f(k_\perp)$. At leading order in $1/m_B$, $f(k_\perp)$ may be determined from $B \rightarrow X\gamma$ decays, but there are potentially large sub-leading corrections to this relation (at least for the cut on $E_\ell$) which may limit the ultimate precision of this method.

Spectral moments from semileptonic $b \rightarrow c$ and radiative $b \rightarrow s$ decays are now being used to study the HQE at the $O(1/m_B^3)$ level, and to determine the values of the hadronic matrix elements which arise in the expansion. Global fits to a variety of moments now allow $V_{ub}$ to be determined with...
an uncertainty at the 2% level, and a short-distance $m_b$ with
an uncertainty at the 100 MeV level. These determinations
are not yet limited by theory, and so these determinations
are likely to improve in the future.

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