Role of switching-on and -off effects in the vacuum instability

T. C. Adorno*1,2, R. Ferreira†5, S. P. Gavrilov‡1,3, and D. M. Gitman§1,4,5

1Department of Physics, Tomsk State University, Lenin Prospekt 36, 634050, Tomsk, Russia;
2College of Physical Science and Technology, Hebei University, Wusidong Road 180, 071002, Baoding, China;
3Department of General and Experimental Physics, Herzen State Pedagogical University of Russia, Moyka embankment 48, 191186, St. Petersburg, Russia;
4P. N. Lebedev Physical Institute, 53 Leninskiy prospekt, 119991, Moscow, Russia;
5Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, CEP 05508-090, São Paulo, S.P., Brazil;

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Abstract

We find exact differential mean numbers of fermions and bosons created from the vacuum due to a composite electric field of special configuration. This configuration imitates a finite switching-on and -off regime and consists of fields that switch-on exponentially from the infinitely remote past, remains constant during a certain interval $T$ and switch-off exponentially to the infinitely remote future. We show that calculations in the slowly varying field approximation are completely predictable in the framework of a locally constant field approximation. Beyond the slowly varying field approximation, we study effects of fast switching-on and -off in a number of cases when the size of the dimensionless parameter $\sqrt{eET}$ is either close or exceeds the threshold value that determines the transition from a regime sensitive to on-off parameters to the slowly varying regime for which these effects are secondary.

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Particle creation, Schwinger effect, time-dependent external field, Dirac and Klein-Gordon equations.

*tg.adorno@gmail.com, tg.adorno@mail.tsu.ru
†rafaelu@gmail.com
‡gavrilovsergeyp@yahoo.com, gavrilovsp@herzen.spb.ru
§gitman@if.usp.br
1 Introduction

The consistent consideration of quantum processes in the vacuum violating backgrounds has to be done in the framework of nonperturbative calculations for quantum field theory, in particular, QED. Different analytical and numerical methods were applied to study the effect of electron-positron pair creation from vacuum; see recent reviews [1, 2]. Among these methods, there are ones based on the existence of exact solutions of the Dirac (or Klein-Gordon) equation in the corresponding background fields; e.g., see Refs. [3, 4]. They give us exactly solvable models for QED that are useful to consider the characteristic features of theory and could be used to check approximations and numeric calculations. Recently, we present the review of particle creation effects in time-dependent uniform external electric fields that contains three most important exactly solvable cases: Sauter-like electric field, \( T \)-constant electric field, and exponentially growing and decaying electric fields [5]. These electric fields switched on and off at the initial and the final time instants, respectively. We refer to such kind of external fields as the \( t \)-electric potential steps.

Choosing parameters for the exponentially varying electric fields, one can consider both fields in the slowly varying regime and fields that exist only for a short time in a vicinity of the switching on and off time. The case of the \( T \)-constant electric field is distinct. In this case the electric field is constant within the time interval \( T \) and is zero outside of it, that is, it is switched on and off "abruptly" at definite instants. The model with the \( T \)-constant electric field is important to study the particle creation effects; see Ref. [5] for the review. Then the details of switching on and off for the \( T \)-constant electric field is of interest. To estimate the role of the switching on and off effects for the pair creation due to the \( T \)-constant electric field we consider a composite electric field that grows exponentially in the first interval \( t \in I = (-\infty, t_1) \), remains constant in the second interval \( t \in II = [t_1, t_2] \), and decreases exponentially in the last interval \( t \in III = (t_2, +\infty) \). We essentially use notation and final formulas from Ref. [5].

The article is organized as follows: In Sec. 2 we introduce the composite field and summarize details concerning the exact solutions of the Dirac equation with such a field. We find exact formulas for the differential mean number of particles created from the vacuum, the total number of particles created from the vacuum and the vacuum-to-vacuum transition probability. In Sec. 3 we consider the general properties of the differential mean numbers of pairs created. We visualize how these mean numbers are distributed over the quantum numbers, especially in cases where asymptotic approximations involved are not applicable. In Sec. 4 we compute differential and total quantities in some special field configurations of interest. We show that the results for slowly varying fields are completely predictable using recently developed version of a locally constant field approximation. We study configurations that simulate finite switching-on and -off processes within and beyond the slowly varying regime. Final comments are placed in Sec. 5.
2 IN and OUT solutions in a composite electric field

In this section we summarize general aspects on exact solutions of the Dirac equation with the field under consideration and briefly discuss the calculation of differential and total numbers of pairs creation.

The composite electric field a \( d = D + 1 \) dimensional Minkowski space-time is homogeneous, positively oriented along a single direction \( \mathbf{E} (t) = (E^i (t) = \delta^i_1 E (t), \ i = 1, \ldots, D) \) and described by a vector along the same direction, \( A^\mu = (A^0 = 0, \mathbf{A} (t)), \ \mathbf{A} (t) = (A^i (t) = \delta^i_1 A_x (t)) \), whose explicit forms are

\[
E (t) = E \begin{cases} 
  e^{k_1 (t-t_1)}, & t \in I, \\
  1, & t \in II, \\
  e^{-k_2 (t-t_2)}, & t \in III, 
\end{cases}
\]

\[
A_x (t) = A \begin{cases} 
  k_1^{-1} (-e^{k_1 (t-t_1)} + 1 - k_1 t_1), & t \in I, \\
  -t, & t \in II, \\
  k_2^{-1} (e^{-k_2 (t-t_2)} - 1 - k_2 t_2), & t \in III, 
\end{cases}
\]

where \( t_1 < 0 \) and \( t_2 > 0 \) are fixed time instants. Throughout the text, we refer to I as the switching-on interval, III as the switching-off interval and II as the constant field interval. This field configuration encompasses the \( T \)-constant field \([6]\), characterized by the absence of exponential parts, and the peak field \([7]\).

The Dirac equation\(^1\)

\[
i \partial_t \psi (x) = H (t) \psi (x), \quad H (t) = \gamma^0 (\gamma \mathbf{P} + m), \]

\[
P_x = -i \partial_x - U (t), \quad \mathbf{P}_\perp = -i \nabla_\perp, \quad U (t) = q A_x (t),
\]

\(^1\)The subscript “\( \perp \)” denotes spacial components perpendicular to the electric field (e. g. \( \mathbf{x}_\perp = \{ x^2, \ldots, x^D \} \)) and \( \psi (x) \) is a \( 2^{[d/2]} \)-component spinor (\([d/2]\) stands for the integer part of the ratio \( d/2 \)). As usual, \( m \) denotes the electron mass, \( \gamma^\mu \) are \( \gamma \)-matrices in \( d \) dimensions and \( U (t) \) denotes the potential energy of a particle with algebraic charge \( q \). We select the electron as the main particle, \( q = -e \) with \( e \) representing the absolute value of the electron charge. Hereafter we use the relativistic system of units \(( \hbar = c = 1) \), except when indicated otherwise.
The constant spinors satisfy
\[ \gamma^0 \gamma^1 v_{\chi,\sigma} = \chi v_{\chi,\sigma}, \quad v^\dagger_{\chi,\sigma} v_{\chi',\sigma'} = \delta_{\chi,\chi'} \delta_{\sigma,\sigma'}, \] (5)
where \( \chi = \pm 1 \) are eigenvalues of \( \gamma^0 \gamma^1 \) and \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{[d/2]-1}) \) represent a set of additional eigenvalues, corresponding to spin operators compatible with \( \gamma^0 \gamma^1 \). The constant spinors are subjected to additional conditions depending on the space-time dimensions, whose details can be found in Ref. [5]. After these substitutions, the Dirac spinor can be obtained through the solutions of the second-order ordinary differential equation\(^2\)
\[ \left\{ \frac{d^2}{dt^2} + [p_x - U(t)]^2 + \pi_\perp^2 - i\chi \dot{U}(t) \right\} \varphi_n(t) = 0, \quad \pi_\perp = \sqrt{\mathbf{P}_\perp^2 + m^2}. \] (6)

In the switching-on I and -off III intervals, the solutions are expressed in terms of Confluent Hypergeometric Functions (CHFs.),
\[ \varphi^j_n(t) = b^2_1 y_1^j(\eta_j) + b^1_1 y_2^j(\eta_j), \]
\[ y_1^j(\eta_j) = e^{-\eta_j/2} \eta_j^\nu_j \Phi(a_j, c_j; \eta_j), \]
\[ y_2^j(\eta_j) = e^{\eta_j/2} \eta_j^{-\nu_j} \Phi(1-a_j, 2-c_j; -\eta_j), \] (7)
while at the constant interval II, the solutions are expressed in terms of Weber Parabolic Cylinder Functions (WPCFs.),
\[ \varphi_n(z) = b^+ u_+(z) + b^- u_-(z), \]
\[ u_+(z) = D_{\beta+(\chi-1)/2}(z), \quad u_-(z) = D_{-\beta-(\chi+1)/2}(iz). \] (8)

At these equations, \( a_j, c_j, \nu_j \) and \( \beta \) are parameters
\[ a_1 = \frac{1}{2} (1 + \chi) + i \Xi_1, \quad a_2 = \frac{1}{2} (1 + \chi) + i \Xi_2^+, \]
\[ \Xi_j^\pm = \frac{\omega_j \pm \Pi_j}{k_j}, \quad c_j = 1 + 2 \nu_j, \quad \nu_j = \frac{i \omega_j}{k_j}, \quad \beta = \frac{i \lambda}{2}, \]
\[ \omega_j = \sqrt{\Pi_j^2 + \pi_\perp^2}, \quad \Pi_j = p_x - \frac{eE}{k_j} \left[ (-1)^j + k_j t_j \right], \quad \lambda = \frac{\pi_\perp^2}{eE}. \] (9)
\(^2\)For scalar particles, the exact solutions for the Klein-Gordon equation \( \phi_n(x) \) are connected with the scalar functions as \( \phi_n(x) = \exp(i \mathbf{p} r) \varphi_n(t) \). Since spinning degrees-of-freedom are absent in this case, \( n = \mathbf{p} \) and \( \chi = 0 \) in Eq. (6) as well as in all subsequent formulas.
\(z\) and \(\eta_j\) are time-dependent functions

\[
\eta_1(t) = i h_1 e^{k_1(t-t_1)}, \quad \eta_2(t) = i h_2 e^{-k_2(t-t_2)}, \quad h_j = \frac{2eE}{k_j^2}, \quad (10)
\]

\[
z(t) = (1 - i) \xi(t), \quad \xi(t) = \frac{eEt - p_x}{\sqrt{eE}}, \quad (11)
\]

and \(b^j_1, b^j_2, b^{\pm}\) are constants, fixed by initial conditions. In addition, the index \(j\) in Eqs. (7), (9) and (10) distinguish quantities associated to the switching-on (\(j = 1\)) from the switching-off (\(j = 2\)) intervals.

In virtue of asymptotic properties of the CHFs, at \(t \to \pm \infty\), the solutions given by Eq. (7) can be classified as particle/antiparticle states

\[
+\varphi_n(t) = +\mathcal{N} \exp \left( i \pi \nu_1 / 2 \right) y_1(t_1), \quad -\varphi_n(t) = -\mathcal{N} \exp \left( -i \pi \nu_1 / 2 \right) y_1(t_1), \quad t \in I,
\]

\[
+\varphi_n(t) = +\mathcal{N} \exp \left( -i \pi \nu_2 / 2 \right) y_2(t_2), \quad -\varphi_n(t) = -\mathcal{N} \exp \left( i \pi \nu_2 / 2 \right) y_2(t_2), \quad t \in III, \quad (12)
\]

since, at the infinitely remote past \(t \to -\infty\) and future \(t \to +\infty\), the set above behaves as plane-waves,

\[
\zeta \varphi_n(t) = \zeta \mathcal{N} e^{-i \zeta \omega_1 t}, \quad t \to -\infty, \quad \zeta \varphi_n(t) = \zeta \mathcal{N} e^{-i \zeta \omega_2 t}, \quad t \to +\infty, \quad (13)
\]

where \(\omega_1\) denotes the energy of initial particles at \(t \to -\infty\), \(\omega_2\) denotes the energy of final particles at \(t \to +\infty\) and \(\zeta\) labels electron (\(\zeta = +\)) and positron (\(\zeta = -\)) states. With the help of such solutions, one may construct IN \(\{ \zeta \psi(x) \}\) and OUT \(\{ \zeta \bar{\psi}(x) \}\) sets of Dirac spinors. The normalization constants \(\zeta \mathcal{N} = \zeta CV_{(d-1)}^{-1/2}\) and \(\zeta \mathcal{N} = \zeta CV_{(d-1)}^{-1/2}\) are calculated with respect to the usual inner product for Fermions and Bosons, where \(\zeta C\) and \(\zeta C\) given by

\[
\zeta C = \begin{cases} \left( \frac{2\omega_1 q_1^j}{\omega_1} \right)^{-1/2} & \text{Fermi}, \\ \left( \frac{2\omega_1}{\omega_1} \right)^{-1/2} & \text{Bose} \end{cases}, \quad \zeta C = \begin{cases} \left( \frac{2\omega_2 q_2^j}{\omega_2} \right)^{-1/2} & \text{Fermi}, \\ \left( \frac{2\omega_2}{\omega_2} \right)^{-1/2} & \text{Bose} \end{cases}, \quad q_j^c = \omega_j - \chi \zeta \Pi_j. \quad (14)
\]

For further details, e.g., see Ref. [5].

With the exact solutions discussed above, one can write complete sets of solutions for the whole time interval \(t \in (-\infty, +\infty)\). Using the classification (12) and the solutions given by Eq. (8), Dirac spinors (4) (or Klein-Gordon solutions) for all time duration can be calculated from the following
set of solutions,

\[ +\varphi_n(t) = \begin{cases} \kappa g (-|^+) - \varphi_n(t) + g (+|^+) + \varphi_n(t), & t \in I \\ b_1^+ u_+(t) + b_1^- u_-(t), & t \in II \\ +\mathcal{N} \exp(-i\pi \nu_2/2) y_1^2(\eta_2), & t \in III \end{cases} \]

\[ -\varphi_n(t) = \begin{cases} -\mathcal{N} \exp(-i\pi \nu_1/2) y_1^1(\eta_1), & t \in I \\ b_2^+ u_+(t) + b_2^- u_-(t), & t \in II \\ g (+|_) + \kappa g (-|_) - \varphi_n(t), & t \in III \end{cases} \]

where \( b_{1,2}^\pm, g (\pm|^+) \), and \( g (\pm|_) \) are some coefficients, \( g (\zeta'|\zeta) = g (\zeta'|\zeta)^* \). Here \( \kappa \) is an auxiliary constant that allow us to present solutions for the Klein-Gordon \((\kappa = -1)\) or Dirac \((\kappa = +1)\) equations. For the solutions of the Dirac equation, the \( g \)-coefficients satisfy unitarity relations

\[ \sum_{\zeta} g (\zeta'|\zeta) g (\zeta'|\zeta') = \sum_{\zeta} g (\zeta'|\zeta) g (\zeta'|\zeta') = \delta_{\zeta,\zeta'} \]  

(17)

while for the solutions of the Klein-Gordon equation, the \( g \)-coefficients satisfy unitarity relations

\[ \sum_{\zeta} \zeta g (\zeta'|\zeta) g (\zeta'|\zeta') = \sum_{\zeta} \zeta g (\zeta'|\zeta) g (\zeta'|\zeta') = \zeta \delta_{\zeta,\zeta'} , \]

(18)

To obtain the \( g \)-coefficients, we conveniently consider continuity conditions at instants \( t_1, t_2 \)

\[ \pm\varphi_n(t_{1,2} - 0) = \pm\varphi_n(t_{1,2} + 0), \quad \partial_t \pm\varphi_n(t_{1,2} - 0) = \partial_t \pm\varphi_n(t_{1,2} + 0) , \]

substitute appropriate normalization constants for each case, given by Eqs. [14], and use Wronskian determinants for CHFs. and WPCFs. After these manipulations, one can readily verify that \( g (-|^+) \) and \( g (+|_) \) for the Dirac case reads

\[ g (-|^+) = \sqrt{\frac{q_1}{8eE\omega_1q_2^2\omega_2}} \exp \left[ \frac{i\pi}{2} \left( \nu_1 - \nu_2 + \beta + \frac{\lambda}{2} \right) \right] \left[ f_1^- (t_2) f_2^+ (t_1) - f_1^+ (t_2) f_2^- (t_1) \right] , \]

\[ g (+|_) = \sqrt{\frac{q_2}{8eE\omega_2q_1^2\omega_1}} \exp \left[ \frac{i\pi}{2} \left( \nu_2 - \nu_1 + \beta + \frac{\lambda}{2} \right) \right] \left[ f_1^+ (t_1) f_2^- (t_2) - f_1^- (t_1) f_2^+ (t_2) \right] , \]

\[ f_{k_1}^\pm (t_j) = \left[ (-1)^j k_j n_j \frac{dy_k^j (\eta_j)}{d\eta_j} + y_k^j (\eta_j) \partial_t \right] u_\pm (z) \bigg|_{t=t_j} , \]

(19)
while for the Klein-Gordon case have the form,

\[
g(\mid \rangle) = -\frac{1}{\sqrt{8eE\omega_1\omega_2}} \exp \left[\frac{i\pi}{2} (\nu_1 - \nu_2 + \beta) \right] \left[ f_1^+ (t_2) f_2^+ (t_1) - f_1^+ (t_2) f_2^+ (t_1) \right] \Big|_{\chi=0} ,
\]

\[
g(\rangle \mid) = \frac{1}{\sqrt{8eE\omega_1\omega_2}} \exp \left[\frac{i\pi}{2} (\nu_2 - \nu_1 + \beta) \right] \left[ f_1^+ (t_1) f_2^- (t_2) - f_1^- (t_1) f_2^+ (t_2) \right] \Big|_{\chi=0} .
\] (20)

Taking into account that the \(g\)'s coefficients establish the Bogoliubov transformations, one may compute fundamental quantities concerning vacuum instability for Fermions (the Dirac case) and Bosons (the Klein-Gordon case), for example, the differential mean number of pairs created from the vacuum \(N_{cr}^n\), the total number \(N_{cr}\) and the vacuum-to-vacuum transition probability \(P_v\) as

\[
N_{cr}^n = \left| g(\mid \rangle) \right|^2 , \quad N_{cr} = \sum_n N_{cr}^n ,
\]

\[
P_v = \exp \left[ \kappa \sum_n \ln \left( 1 - \kappa N_{cr}^n \right) \right] .
\] (21)

### 3 General properties of the differential mean numbers of pairs created

The \(g\)-coefficients (19) and (20) enjoy certain properties under time/momentum reversal that result in symmetries for differential quantities. More precisely, the simultaneous change

\[
k_1 \rightleftharpoons k_2 , \quad t_1 \rightleftharpoons -t_2 , \quad p_x \rightleftharpoons -p_x ,
\] (22)

yields to a number of identities, for instance, \(\Pi_1 \rightleftharpoons -\Pi_2 , \quad \omega_1 \rightleftharpoons \omega_2 , \quad a_1 \rightleftharpoons a_2 , \quad c_1 \rightleftharpoons c_2\) so that \(g(\mid \rangle)\) and \(g(\rangle \mid)\) are related by

\[
g(\mid \rangle) \rightleftharpoons \kappa g(\rangle \mid) ,
\] (23)

implying, in particular, that \(N_{cr}^n\) (and therefore total quantities) are even with respect to the exchanges (22). Moreover (19) and (20) are even with respect to \(p_\perp\), so that all quantum quantities in Eq. (21) are symmetric with respect to the momenta \(p\) (for Fermions, these quantities does not depend on spin polarizations as well). Such properties are helpful in computing asymptotic estimates in several regimes, some of them discussed in subsequent section.

Aside these properties, it is useful to visualize how the differential mean numbers \(N_{cr}^n\) are distributed over the quantum numbers (for instance \(p_x\)) to outline some preliminary remarks concerning pair creation, especially in cases where asymptotic approximations of the WPCFs. and CHFs. involved in the \(g\)-coefficients are not applicable.\(^3\) To this end, we present below some plots

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\(^3\)For example, when the argument of WPCFs. \(z_j\) or of CHFs. \(\eta_j\) are finite quantities. Also when the parameters \(a_j, c_j\) are also finite.
of the mean number of particles created from the vacuum $N_{cr}^{n}$ as a function of $p_x$ for different values of $k_1$, $k_2$ and $T$ (Figs. 1, 2 for Fermions and 3, 4 for Bosons) for a fixed amplitude $E$ of the composite field. For the sake of simplicity, we set $p_\perp = 0$ and select a convenient system of units, in which besides $\hbar = c = 1$ the electron mass is also set equal to the unity, $m = 1$. In this system, the Compton wavelength corresponds to one unit of length $\lambda_e = \hbar mc = 1 \approx 3.8614 \times 10^{-14}$ m, the Compton time corresponds to one unit of time $\lambda_e/c = 1 \approx 1.3 \times 10^{-21}$ s and the electron rest energy corresponds to one unit of energy $mc^2 = 1 \approx 0.511$ MeV. In all plots below, the longitudinal momentum $p_x$, time duration $T$ and phases $k_j$ are relative to electron’s mass $m$, corresponding to dimensionless quantities, i. e., $p_x/m$, $mT$ and $k_j/m$, respectively.

Figure 1: (color online) Differential mean number of Fermions created from the vacuum $N_{cr}^{n}$ (solid lines) by a symmetrical composite field, with $k_1 = k_2 = k$ and amplitude $E = E_c = m^2/e = 1$ fixed. Graph (A) shows distributions with $k/m = 1$ fixed, while Graph (B) shows $mT$ is fixed, $mT = 5$. In (A), the solid lines labeled with (i), (ii) and (iii) refers to $mT = 5$, $mT = 10$ and $mT = 50$, respectively. In (B), (i), (ii) and (iii) refers to $k/m = 0.1$, $k/m = 0.05$ and $k/m = 0.01$, respectively. The horizontal dashed line corresponds to the uniform distribution $e^{-\pi\lambda}$ which, in this system of units and $p_\perp = 0$, is $e^{-\pi}$.

The results displayed in all pictures above, reveal wider distributions corresponding to composite electric fields with larger $T$ (red/dark blue lines for Fermions/Bosons in graphs (a), Figs. 1, 3) or smaller $k_j$ (red/dark blue lines for Fermions/Bosons in graphs (b), Figs. 1, 3) and thinner distributions corresponding to opposite configurations, associated with smaller $T$ (orange/purple lines for Fermions/Bosons in graphs (a), Figs. 1, 3) or larger $k_j$ (orange/purple lines for Fermions/Bosons in graphs (b), 1, 3). Once the time duration is designated by $T$ and $k_j^{-1}$ ($k_j^{-1}$ represent a scale of time duration for increasing and decreasing phases), these results are consistent with the fact that the larger the duration of an electric field, the longer it has to accelerate pairs. Therefore larger values to $p_x/m$ are expected to occur in cases corresponding to electric fields with larger time duration. Moreover, it should be noted that the distributions above tend to the uniform distribution $N_{cr}^{n} = e^{-\pi\lambda}$ (horizontal dashed lines) for $T$ and $k_j^{-1}$ sufficiently large. This is not unexpected since
the composite field tends to a constant field as soon as $T$ and $k_j^{-1}$ increase, becoming infinitely constant in the limit $T \to \infty$ and $k_j^{-1} \to \infty$. At last, but not least, observing Figs. 2, 4 we find that asymmetrical configurations ($k_1 \neq k_2$) yields to asymmetrical distributions. This is associated with the fact that different phases $k_1, k_2$ implies in different times to accelerate pairs during the switching-on and -off processes in general. An interpretation of these results follows from the semiclassical analysis: Electrons created from the vacuum have quantum numbers $p_x$ within the range $-eE(T/2 + k_1^{-1}) \leq p_x \leq eE(T/2 + k_2^{-1})$, corresponding to longitudinal kinetic momenta $\Pi_x(t) = p_x + eA_x(t)$ which, at $t \to +\infty$, varies according to $-eE(T + k_1^{-1} + k_2^{-1}) \leq \Pi_x(+\infty) \leq 0$. Assuming that pairs are materialized from the vacuum with zero longitudinal momentum $\Pi_x(t) = 0$, it follows from the classical equations of motion that the kinetic longitudinal momentum at $t \to +\infty$ has the form $\Pi_x(+\infty) = -e \int_t^{+\infty} dt' E(t')$, where $t$ is the time of creation. Thus, if an electron is created at $t \to -\infty$, its longitudinal kinetic momentum at $t \to +\infty$ is maximal (in absolute value) $\Pi_x(+\infty) = -eE(T + k_1^{-1} + k_2^{-1})$. At the same time, its longitudinal kinetic momentum is expressed in terms of $p_x$ as $\Pi(+\infty) = p_x - eE(T/2 + k_2^{-1})$, which means that such a electron is found to have the a minimal value to $p_x$, namely $p_x \to p_x^{\text{min}} = -eE(T/2 + k_1^{-1})$. On the other hand, if the electron is created at $t \to +\infty$, then its longitudinal kinetic momentum tends to zero, $\Pi_x(+\infty) \to 0$, which means that the corresponding quantum number $p_x$ tends to its maximum, $p_x \to p_x^{\text{max}} = eE(T/2 + k_2^{-1})$. According to this interpretation, asymmetric configurations result in asymmetric distributions. This explains asymmetric distributions in graphs (C) and (D), Figs. 2, 4 for instance.

Figure 2: (color online) Differential mean number of Fermions created from the vacuum $N_{n}^{\text{cr}}$ (solid lines) by asymmetrical composite fields, with $k_1 \neq k_2$ and amplitude $E = E_c = m^2/e = 1$ fixed. In both graphs, $mT$ is fixed, where in Graph (C) shows $mT = 10$ and $k_1/m = 0.5$ while in Graph (D) shows $mT = 5$. In (C), the solid lines labeled with (i), (ii) and (iii) refers to $k_2/m = 1$, $k_2/m = 5$ and $k_2/m = 10$, respectively. In (D), (i) denotes $k_1/m = 0.5$, $k_2/m = 0.3$, (ii) denotes $k_1/m = 0.1$, $k_2/m = 0.07$ and (iii) denotes $k_1/m = 0.01$, $k_2/m = 0.008$. The horizontal dashed line corresponds to the uniform distribution $e^{-\pi \lambda}$ which, in this system of units and $p_\perp = 0$, is $e^{-\pi}$.
Figure 3: (color online) Differential mean number of Bosons created from the vacuum $N_{cr}^n$ (solid lines) by a symmetrical composite field, with $k_1 = k_2 = k$ and amplitude $E = E_c = m^2/e = 1$ fixed. Graph (A) shows distributions with $k/m = 1$ fixed, while Graph (B) shows $mT$ is fixed, $mT = 5$. In (A), the solid lines labeled with (i), (ii) and (iii) refers to $mT = 5$, $mT = 10$ and $mT = 50$, respectively. In (B), (i), (ii) and (iii) refers to $k/m = 0.1$, $k/m = 0.05$ and $k/m = 0.01$, respectively. The horizontal dashed line corresponds to the uniform distribution $e^{-\pi\lambda}$ which, in this system of units and $p_\perp = 0$, is $e^{-\pi}$.

4 Differential and total quantities in some special configurations

Irrespective of the $t$-electric potential step under consideration, it is known that the most favorable conditions for pair creation from the vacuum are associated with strong fields acting over a sufficiently large period of time, in which differential and total quantities are significant. For the composite electric field (I), the time duration is encoded in two sets, namely, $(k_1^{-1}, k_2^{-1})$ and $(t_1, t_2)$. The former represent scales of time duration for the increasing and decreasing phases of the electric field, defined at intervals I and III, while the latter corresponds to the time duration in which the field is constant, defined at interval II.

If the period $T$ is a relatively short (see, e.g., the cases with $mT = 5$ and $k/m < 0.5$ on the right side of Figs. [1]-[4]), the effects of pair creation tend to ones obtained for the peak field [5]. The latter field correspond to a limit of the composite field when the intermediate interval $T$ is absent. From the results above, we observe that the existence of a finite interval $T$, between "slow" switching-on and -off processes, has no significant influence on the distribution of the differential mean numbers $N_{cr}^n$ over the quantum numbers (see appropriate asymptotic formulae in Ref. [5]). The influence of the $T$-constant interval appears only in the next-to-leading order.

A composite electric field of large duration corresponds to small values for the switching-on
Figure 4: (color online) Differential mean number of Bosons created from the vacuum \( N_{cr} \) (solid lines) by asymmetrical composite fields, with \( k_1 \neq k_2 \) and amplitude \( E = E_c = m^2/e = 1 \) fixed. In both graphs, \( mT \) is fixed, where Graph (C) shows \( mT = 10 \) and \( k_1/m = 0.5 \) while in Graph (D) shows \( mT = 5 \). In (C), the solid lines labeled with (i), (ii) and (iii) refers to \( k_2/m = 1, k_2/m = 5 \) and \( k_2/m = 10 \), respectively. In (D), (i) denotes \( k_1/m = 0.5, k_2/m = 0.3 \), (ii) denotes \( k_1/m = 0.1, k_2/m = 0.07 \) and (iii) denotes \( k_1/m = 0.01, k_2/m = 0.008 \). The horizontal dashed line corresponds to the uniform distribution \( e^{-\pi\lambda} \), which, in this system of units and \( p_\perp = 0 \) is \( e^{-\pi} \).

phase \( k_1 \), switching-off phase \( k_2 \) and large \( T = t_2 - t_1 \)\(^4\) satisfying the following condition

\[
\min \left( \sqrt{eET}, eE k_1^{-2}, eE k_2^{-2} \right) \gg \max \left( 1, \frac{m^2}{eE} \right) .
\]

(24)

The condition (24) defines a configuration in which the field takes a sufficiently large time to reach the constant regime (slow switching-on process, \( k_1^{-1} \) large), remains constant over a sufficiently large interval \( T \) and finally takes another sufficiently large time to switch-off completely (slow switching-on process, \( k_2^{-1} \) large).

The most important objects in vacuum instability by external fields are the total number of particles created from the vacuum \( N_{cr} \) and the vacuum-to-vacuum transition probability \( P_v \), both given by Eq. (21). The first quantity corresponds to the summation of the differential mean numbers \( N_{cr}^n \) over the momenta \( p \), and spin degrees-of-freedom

\[
N_{cr}^n = V_{(d-1)} n_{cr}^n, \quad n_{cr}^n = \frac{J(d)}{(2\pi)^{d-1}} \int d\mathbf{p} N_{cr}^n,
\]

(25)

which, in fact, is reduced to the calculation of the density of pairs created from the vacuum \( n_{cr}^n \). Here the summation over \( \mathbf{p} \) was transformed into an integral and \( J(d) = 2^{(d/2) - 1} \) denotes the total number spin projections in a \( d \)-dimensional space. These are factored out since the numbers \( N_{cr}^n \) are independent of spin polarization. The dominant contribution of the densities \( n_{cr}^n \) in the slowly

\(^4\)Without loss of generality, we select from now on a symmetrical interval II, in which \( t_1 = -T/2 = -t_2 \).
varying regime are proportional to the total increment of the longitudinal kinetic momentum, 
\[ \Delta U = |\Pi_2 - \Pi_1| = e |A_x(\infty) - A_x(-\infty)|, \]
which is the largest parameter in the problem [8]. Hence it is meaningful to approximate the total density \( n^{cr} \) by its dominant contribution \( \tilde{n}^{cr} \), corresponding to an integral over a specific domain \( \Omega \)

\[ \Omega : n^{cr} \approx \tilde{n}^{cr} = \frac{J(d)}{(2\pi)^{d-1}} \int_{p \in \Omega} dp N^{cr}_n, \]

whose result is proportional to \( \Delta U \). As it is general for \( t \)-electric potential steps, such domain \( \Omega \) is defined by a specific range of values to the longitudinal momentum \( p_x \) and restricted values to the perpendicular momentum \( p_{\perp} \) which, under the condition (24), is

\[ \Omega : \left\{ \frac{|p_x|}{\sqrt{eE}} \leq \sqrt{eE} \frac{T}{2} + \frac{3}{2} \sqrt{h_1/2}, \sqrt{\lambda} < K_{\perp}, K_{\perp}^2 \gg \max \left( 1, \frac{m^2}{eE} \right) \right\}. \]

In this case using asymptotic formulas given by Ref. [5] one can see that the differential mean numbers are practically uniform over a wide range of values to the kinetic momenta of the domain \( \Omega \) while decreases exponentially beyond these ranges. In leading-order approximation, the mean numbers are

\[ N^{cr}_n \sim \begin{cases} 
\exp \left( -2\pi \Xi^{-1} \right), & \text{for } p_x/\sqrt{eE} < -\sqrt{eET}/2, \\
\exp \left( -2\pi \Xi^{-1} \right), & \text{for } |p_x|/\sqrt{eE} \leq \sqrt{eET}/2, \\
\exp \left( -2\pi \Xi^+ \right), & \text{for } p_x/\sqrt{eE} > +\sqrt{eET}/2.
\end{cases} \]

It is clear that the asymptotic forms (28) specified in each range above, coincides with asymptotic forms of the \( T \)-constant and exponential electric fields; see, e. g., Ref. [5]. Thus, we see that in each domain of \( \Omega \) with a particular type of field, principal terms in the distribution \( N^{cr}_n \) do not depend on the type of fields in neighboring regions, only terms of following orders acquire such a dependence. It follows that the dominant contribution for the density of pairs created by the composite field is expressed as a sum of the dominant contribution for the \( T \)-constant and exponential electric fields,

\[ \tilde{n}^{cr} \approx \sum_j \tilde{n}^{cr}_j, \quad \tilde{n}^{cr}_j = \frac{J(d)}{(2\pi)^{d-1}} \int_{t \in D_j} dt \left[ eE_j(t) \right]^{d/2} \exp \left[ -\pi \frac{m^2}{eE_j(t)} \right], \]

where the index \( j = 1, 2, 3 \) denotes each interval of the composite field, \( D_{1,2,3} = I, II, III \). It is known [5] that

\[ \tilde{n}^{cr}_{1,3} = \frac{J(d)}{(2\pi)^{d-1}} \frac{(eE)^{d/2}}{k_{1,2}} e^{-\pi m^2/eE} G \left( \frac{d}{2}, \frac{\pi m^2}{eE} \right), \]

\[ \tilde{n}^{cr}_2 = \frac{J(d)}{(2\pi)^{d-1}} \frac{(eE)^{d/2} T}{k_{1,2}} e^{-\pi m^2/eE} \exp \left[ -\pi \frac{m^2}{eE} \right], \]

\[ \tilde{n}^{cr}_1 = \]
where \( G(\alpha, z) \) is expressed in terms of the incomplete gamma function \( \Gamma(\alpha, z) \) [10] as

\[
G(\alpha, z) = \int_{1}^{\infty} \frac{ds}{s^{\alpha+1}} e^{-z(s-1)} = e^{z} z^{\alpha} \Gamma(-\alpha, z). \tag{31}
\]

Calculating the vacuum-to-vacuum probability of the composite field, we obtain that it is product of the partial \( P_{v}^{j} \) for the \( T \)-constant and exponential electric fields, respectively, \( \ln P_{v} = \sum_{j} \ln P_{v}^{j} \); see the Ref. [5]. It is important to point out that the result above may be reproduced from the universal form for the total density of pairs created by \( t \)-electric potential steps in the slowly varying regime [8]. Such a form does not demand knowledge on the exact solutions of the Dirac/Klein-Gordon equations. This is a consequence of the fact that in the approximation by leading terms, the distribution \( N_{cr}^{n} \) in each region of \( \Omega \) is formed independently of neighboring regions.

While the results for slowly varying fields are completely predictable, configurations in which fields act over a relatively short time to reach the constant regime (fast switching-on process, \( k_{1}^{-1} \) small), remain constant over a sufficiently large interval \( T \) and takes a short interval to switch-off completely (fast switching-off process, \( k_{2}^{-1} \) small) have to be studied in more details. These configurations simulate finite switching-on and -off processes, whose considerations are discussed below.

To study such configurations, one has to compare parameters involving momenta with ones involving time scales, such as \( \sqrt{eET}, eEk_{1}^{-2} \) and \( eEk_{2}^{-2} \). Regarding the dependence on the perpendicular momenta \( \mathbf{p}_{\perp} \) for instance, it is well known that a \( t \)-electric potential step of large time duration does not create a significant number of pairs with large \( \mathbf{p}_{\perp} \). This is meaningful as long as charged pairs are accelerated along the direction of the electric field, having thereby a wider range of values of \( p_{x} \) instead \( \mathbf{p}_{\perp} \). By virtue of that, one may simplify the calculation of differential quantities and consider restricted values to \( \mathbf{p}_{\perp} \), ranging from zero till a finite number so that the inequality

\[
\sqrt{\tilde{\lambda}} < K_{\perp}, \quad K_{\perp}^{2} \gg \max \left( 1, \frac{m^{2}}{eE} \right), \tag{32}
\]

is fulfilled. Here \( K_{\perp} \) is a moderately large number that sets an upper bound to the perpendicular momenta of pairs created. Thus, taking into account the inequality above, we assume that

\[
\sqrt{eET} \gg K_{\perp}^{2}, \quad \max \left( eEk_{1}^{-2}, eEk_{2}^{-2} \right) \leq \max \left( 1, \frac{m^{2}}{eE} \right), \tag{33}
\]

As a consequence, the field satisfies the following inequalities

\[
\sqrt{eET}/2 \gg \max \left( \sqrt{eEk_{1}^{-1}}, \sqrt{eEk_{2}^{-1}} \right) \leftrightarrow \max (k_{1}T/2, k_{2}T/2) \gg 1. \tag{34}
\]

To study differential quantities in this case we select a definite sign for \( p_{x} \) which, for convenience, the negative is chosen \(-\infty < p_{x} \leq 0\). Next we use the properties of symmetry discussed in Eqs.
(22) and (23) to generalize results for $p_x$ positive. Here $\xi_1$ varies from large negative to large positive values while $\xi_2$ is always large and positive; $\Pi_1/\sqrt{eE}$ changes from large positive to large negative values while $\Pi_2/\sqrt{eE}$ that is always large and negative. However once $h_1, h_2$ are finite, we find that the asymptotic behavior of $N_n^{cr}$ is classified according to three main ranges

\[(a) \quad -\sqrt{eET} \leq \xi_1 \leq -\tilde{K}_1 \leftrightarrow \sqrt{eET} + \frac{h_1}{2} > \frac{\Pi_1}{\sqrt{eE}} \geq \tilde{K}_1 + \sqrt{\frac{h_1}{2}},\]

\[(b) \quad -\tilde{K}_1 < \xi_1 < \tilde{K}_1 \leftrightarrow \tilde{K}_1 + \sqrt{\frac{h_1}{2}} > \frac{\Pi_1}{\sqrt{eE}} > -\tilde{K}_1 + \sqrt{\frac{h_1}{2}},\]

\[(c) \quad \xi_1 \geq \tilde{K}_1 \leftrightarrow \frac{\Pi_1}{\sqrt{eE}} \leq -\tilde{K}_1 + \sqrt{\frac{h_1}{2}},\] (35)

where $\tilde{K}_1$ is a sufficiently large number satisfying $\sqrt{eET} \gg K_2$. Moreover, as long as $\xi_2$ is large and positive, $c_2$ is also large so that one case use the asymptotic approximation (9.246.1) in Ref. [9] for the WPCfs. $u_{\pm}(z_2)$ and Eq. (13.8.2) in Ref. [10] for the CHF $y_1^2(\eta_2)$, throughout all ranges above.

In the range (a), $\xi_1$ is large and negative and $c_1$ is large as well. Then using the asymptotic expansions (9.246.2), (9.246.3) in Ref. [9] for the WPCfs. $u_{\pm}(z_1)$ and Eq. (13.8.2) in [10] for the CHF $y_1^2(\eta_1)$, one finds that the mean number of particles created, in the leading order approximation, admit the following form

\[N_n^{cr} \sim \exp \left[ -\pi \left( \lambda + \Xi_1^- - \Xi_2^+ \right) \right] \times \left\{ \begin{array}{ll}
\sinh (\pi \Xi_2^-) \sinh (\pi \Xi_1^+) , & \text{Fermi} \\
\cosh (\pi \Xi_2^-) \cosh (\pi \Xi_1^+) , & \text{Bose}
\end{array} \right.,\] (36)

as $T \to \infty$. The combination of hyperbolic functions above tends to the unity since, in this range, the frequencies $\omega_1, \omega_2$ and the parameters $\Xi_1^+, \Xi_2^-$ are large quantities, namely $\omega_1 \simeq \sqrt{eE} |\xi_1|$, $\omega_2 \simeq \sqrt{eE} \xi_2$, $\Xi_1^+ \sim \sqrt{2h_1} |\xi_1|$, $\Xi_2^- \sim \sqrt{2h_2} \xi_2$. In virtue of that, the dominant contribution of Eq. (36) has the form

\[N_n^{cr} \sim \exp \left[ -\pi \left( \lambda + 2\Xi_1^- \right) \right],\] (37)

as $T \to \infty$, being valid both for Fermions and Bosons. In this last result, the parameter $\Xi_1^-$ is a small quantity, $\Xi_1^- \sim \sqrt{h_1/2} (\lambda/2 |\xi_1|)$, so that its contribution to $N_n^{cr}$ are negligible in comparison to $\lambda$. As a result, the differential mean numbers are practically uniform over the range (a), $N_n^{cr} \sim e^{-\pi \lambda}$.

In the range (c), $\xi_1$ is large and positive and $c_1$ is also large. Hence one may use the asymptotic expansions (9.246.1) in Ref. [9] for the WPCfs. $u_{\pm}(z_1)$ and Kummer transformations for the CHF.
prove that the mean number of particles created is significantly small

\[ N_n^{cr} \sim \mathcal{F}_1 \left[ O(\xi_1^{-6}) + O(\xi_2^{-6}) + O(\xi_1^{-3}\xi_2^{-3}) \right], \tag{38} \]
as \( T \to \infty \), in which \( \mathcal{F}_1 \) is a combination of hyperbolic functions similar to Eq. (36),

\[
\mathcal{F}_1 = \frac{\exp \left[ \pi \left( \Xi_2^+ + \Xi_1^+ \right) \right]}{\sinh (2\pi \omega_2/k_2) \sinh (2\pi \omega_1/k_1)} \times \begin{cases}
\sinh (\pi \Xi_2^-) \sinh (\pi \Xi_1^-), & \text{Fermi}, \\
cosh (\pi \Xi_2^-) \cosh (\pi \Xi_1^-), & \text{Bose}.
\end{cases}
\tag{39}
\]

In this range, the frequencies \( \omega_j \) and the parameters \( \Xi_j^- \) are large quantities \( \omega_j \approx \sqrt{\epsilon E_j}, \Xi_j^- \sim \sqrt{2h_j \xi_j} \) so that, as in the range (a), \( \mathcal{F}_1 \) can be approximated to \( \mathcal{F}_1 \approx 1 \). Therefore the differential mean numbers are significantly small in this range.

In the range (b), \( \xi_1 \) varies from large negative to large positive values while \( c_1 \) varies from large to finite values. By this reason, it is not possible to use any asymptotic approximations for the special functions \( u_\pm(z_1) \) and \( y_2^1(\eta_1) \), although one can still consider the same approximations (9.246.1) in Ref. [9] and (13.8.2) in Ref. [10] for the WPCfs. \( u_\pm(z_2) \) and CHF \( y_2^1(\eta_2) \), respectively. The resulting expression shall depend explicitly on the exact forms of \( u_\pm(z_1) \) and \( y_2^1(\eta_1) \).

The most significant contribution for the differential mean numbers for positive, \( 0 \leq p_x < +\infty \), can be obtained by a similar analysis, taking into account the properties of symmetry (22) and (23). We finally find the domain of dominant contribution to the mean number of particles created. In this domain in the leading order approximation, it has a form

\[ N_n^{cr} \sim e^{-\pi \lambda} \times \begin{cases}
\exp (-2\pi \Xi_1^-), & \text{for } -\sqrt{\epsilon ET}/2 + \tilde{K}_1 < p_x/\sqrt{\epsilon E} \leq 0, \\
\exp (-2\pi \Xi_2^+), & \text{for } 0 < p_x/\sqrt{\epsilon E} \leq \sqrt{\epsilon ET}/2 - \tilde{K}_2,
\end{cases} \tag{40} \]
as \( T \to \infty \), valid for Fermions and Bosons. This approximation is almost uniform over this wide range of values to the longitudinal momentum since the parameters \( \Xi_1^- \) and \( \Xi_2^+ \) are negligible in comparison to \( \lambda \). In virtue of that, the switching-on and -off effects on the differential mean numbers, in the present configuration, manifest themselves as next-to-leading corrections to the uniform distribution \( e^{-\pi \lambda} \). This means that the influence of the switching-on and -off processes on differential quantities are negligible for \( T \) sufficiently large. From these results, the present configuration can be referred as a “fast” switching-on and -off configuration, in virtue of Eq. (34) and from the fact that the mean number of particles created are mainly characterized by the uniform distribution \( e^{-\pi \lambda} \). In this case, the leading contribution to the number density \( \tilde{n}^{cr} \), given by Eq. (26), is proportional to the total increment of the longitudinal kinetic momentum, \( \Delta U = eET \), and then the time duration \( T \). We see that both the \( T \)-constant field itself and the composite field under condition (34) can be considered as regularizations of a constant field. The present discussion encompasses the \( T \)-constant limit, characterized by the absence of exponential
parts and defined by the limit \( k \to \infty \).

We know that a possibility of describing particle creation by the \( T \)-constant field in the slowly varying approximation depends on the value of dimensionless parameter \( \sqrt{eET} > 1 \). According to condition (33) a magnitude of the lower boundary \( \vartheta = \min \sqrt{eET} \) is proportional to \( m^2/eE \) if \( m^2/eE > 1 \). Accordingly, a contribution of switching-on and -off processes to the particle creation effect becomes more pronounced for not too strong fields. It is useful to compare switching-on and -off effects for the \( T \)-constant field and for the composite field in the case when the parameter \( \sqrt{eET} \) approaches the above mentioned threshold values. From the plots on the left side of Figs. 1 - 4 one can see that \( \sqrt{eET} = 10 \) is near threshold value. To this end we compute exact plots of the mean differential number of Fermions (19) and Bosons (20) created as a function of \( p_x/m \) for two typical cases of critical field and very strong field, respectively. In the case of the \( T \)-constant field, we calculate the \( p_x/m \) dependence using exact Eqs. (4.9) and (4.11) given in Ref. [5]. Results of these computations are presented on Figs. 5 and 6. We see that differential mean numbers of pairs created by the composite electric field (solid lines) and the \( T \)-constant field (dashed lines) oscillate around the uniform distribution \( e^{-\pi \lambda} \). It can be seen that for fields with a critical magnitude, \( E = E_c \) and \( \sqrt{eET} = 10 \) (plots (a)), the oscillations around the uniform distribution are greater than for fields with overcritical magnitude, \( E = 10E_c \) and \( \sqrt{eET} = 10\sqrt{10} \) (plots (b)), both for the composite field and the \( T \)-constant field.

![Figure 5](image-url)

**Figure 5:** (color online) Differential mean number of electron/positron pairs created from the vacuum by a symmetric composite field (solid red lines, labeled with (i)) with \( k_1/m = k_2/m = 1 \) and by a \( T \)-constant field (dashed light red lines, labeled with (ii)). In panel (A), \( E = E_c \) while in panel (B), \( E = 10E_c \). In both cases, \( mT = 10 \) and \( \mathbf{p}_\perp = 0 \). The horizontal dashed black line denotes the uniform distributions, being \( e^{-\pi} \) in (A) and \( e^{-\pi/10} \) in (B).

We see that distributions \( N^{cr}_n \) for the \( T \)-constant field always oscillate greater around the uniform distribution than for the composite field. And in the case of bosons, these deviations from the uniform distribution are more significant. On the other hand, the plot of \( N^{cr}_n \) for the \( T \)-constant field is more “rectangular” than for the composite field (for overcritical magnitudes). Such
Figure 6: (color online) Differential mean number of scalar particles created from the vacuum by a symmetric composite field (solid blue lines, labeled with (i)) with $k_1/m = k_2/m = 1$ and by a $T$-constant field (dashed light blue lines, labeled with (ii)). In panel (A), $E = E_c$ while in panel (B), $E = 10E_c$. In both cases, $mT = 10$ and $p_{\perp} = 0$. The horizontal dashed black line denotes the uniform distributions, $e^{-\pi}$ in (A) and $e^{-\pi/10}$ in (B).

Wide distributions arose because of contributions of exponential tails, $|p_x|/m < \left(\frac{eE}{m^2}\right)\left(\frac{mT^2}{2} + \frac{m}{k}\right)$. Note also that for $|p_x|/m > \left(\frac{eE}{m^2}\right)\left(\frac{mT^2}{2} + \frac{m}{k}\right)$, mean numbers for the composite field for both magnitudes are negligible, whereas for the $T$-constant field this is not always true: in fact, for critical magnitudes, the mean numbers for $|p_x|/m$ slightly larger than $\left(\frac{eE}{m^2}\right)\left(\frac{mT^2}{2}\right)$ are not negligible, although they are small. The characteristic behavior in the case of the slowly varying regime, when $\tilde{n}^{cr} \sim T$, is quite noticeable in the case of fermions already for the value of the dimensionless parameter $\sqrt{eET} = 10$ and is pronounced for large values of this parameter. It can be concluded that for fermions the quantity $\sqrt{eET} = 10$ is close to the threshold value. However, for bosons at $\sqrt{eET} = 10$, the approximation of the slowly varying regime does not work yet. To be applicable this approximation requires larger values of the parameter $\sqrt{eET}$. The slowly varying regime is working for both fermions and bosons at $\sqrt{eET} = 10\sqrt{10}$. Comparing these two cases, we see that the regularization by switching on and off exponential fields is less disturbing than by the $T$-constant field, which entails considerable oscillations in the distributions, and can even lead to sharp bursts $N^{cr}_n$ in narrow regions of $p_x$. The latter circumstance, however, is not essential for estimating of dominant contributions for the density of created pairs due to the very strong $T$-constant field. However, the above calculation method which is using the composite field is more realistic and preferable for the analysis of next-to-leading terms.

5 Concluding remarks

We find exact formulas for differential mean numbers of fermions and bosons created from the vacuum due to the composite electric field of special configuration that simulate finite switching-
on and -off processes within and beyond the slowly varying regime. We show that the results for slowly varying fields are completely predictable using recently developed version of a locally constant field approximation. Using exact results beyond the slowly varying regime, we find that the leading contribution to the number density of created pairs is independent of fast switching-on and -off if the time duration $T$ of a slowly varying field is sufficiently large. It means that composite fields of such configurations can be used as regularizations of a slowly varying field, in particular, of a constant field. We have studied effects of fast switching-on and -off in a number of cases, when the value of the total increment of the longitudinal kinetic momentum, characterized by the dimensionless parameter $\sqrt{\varepsilon ET} > 1$, approaches the threshold that determines the transition from a regime that is sensitive to parameters of on-off switching to the slowly varying regime. It is shown that for bosons this threshold value is much higher. We see that the regularization by faster switching on and off is more disturbing, which entails considerable oscillations in distributions, and can even lead to sharp bursts $N_{cr}^{n}$ in narrow regions of $p_x$. The latter circumstance, however, is not essential for estimating of dominant contributions to the density of created pairs due to the very strong field. However, the above calculation method which is using the composite field is more realistic and preferable for the analysis of next-to-leading terms. Thus, details of switching-on and -off may be important for a more complete description of the vacuum instability in some physical situations, for example, in physics of low dimensional systems, such as graphene and similar nanostructures, whose transport properties may be interpreted as pair creation effects under low energy approximations.

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