Note on Seiberg Duality in Matrix Model

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Abstract: In this note, we give a method to derive the Seiberg duality by the matrix model. The key fact we used is that the effective actions given by matrix model method should be identical for both electric and magnetic theories. We demonstrate our method for SQCD with $U(N)$, $SO(N)$ and $Sp(N)$ gauge groups.

Keywords: Matrix Model, Seiberg Duality.
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### 1. Introduction and Motivation

The field theory v.s. matrix model conjecture proposed by Diikgraaf and Vafa \[1, 2, 3\] has intrigued a lot of works from various perspectives. The original idea comes from the relationship between field theory and string theory, but later the conjecture is proved by pure field theory methods in \[4, 5\] for adjoint matter and in \[6\] for massive fundamental flavors and adjoint matter (The generalization to massless flavors has been given in \[17\] based on the work of Seiberg \[6\]). With these achievements, matrix model becomes another alternative way to investigate many interesting problems in fields theories, like the new duality demonstrated in \[7\] (the generalization to other cases in \[8, 9, 10\]) and related works in \[11, 12\].

Besides these successes of matrix model, we also like to know the limit of the new method. The baryonic deformation has been addressed in \[19, 20, 21, 22\] where it has been showed that although the baryonic deformation complexes the boundary condition in matrix model, there is a way to sum up relative contributions for field theory in matrix model expansion. The multi-trace deformation was investigated in \[16, 30\] where it was pointed out \[16\] that the direct matrix model integration of multi-trace deformation does not give back correct results in field theory, but by linearization trick we can reduce the multi-trace problem to the single-trace problem. Except the adjoint and fundamental flavors, other matter contents have been considered in \[23\] where it was found that the conjecture failed with these more general matter fields. For example, the gauge theory $Sp(N)$ with antisymmetric chiral fields agrees with the matrix model up to $N/2$ loops in the perturbative theory, but discrepancy shows at $N/2 + 1$ loops. We also like to ask what is the correct matrix model description (if it exists) for chiral theories because of their role in phenomenology.

The question we like to address in the note is the Seiberg duality in matrix model. Seiberg duality of $\mathcal{N} = 1$ theories \[24, 25, 26, 27\] is a very nontrivial statement above...
two different UV theories in IR. It states that these two theories (the electric theory and the magnetic theory) will flow to same (nontrivial) conformal fixed point in IR. With the new method of matrix model, it is natural to apply to the Seiberg duality. In [14, 15], explicit calculations in matrix models have been done for both electric and magnetic theories of SQCD with mass deformations of quarks and it has been shown that the effective actions are same for both theories, thus checked the Seiberg duality. Generalizations to $SO/Sp$ groups are given in [28, 29].

However, as we emphasized in [15], these calculations serve as the check of Seiberg duality and we want to ask more profound question: could we derive the Seiberg duality from the matrix model? If we could, matrix model will be another powerful tool to study the duality in field theory.

Let us analyze this question. The first idea to derive Seiberg duality in matrix model is to try to find a proper transformation of matrix superpotential in one theory (for example the electric theory). However, it seems this naive method does not work. There are several reasons. First, familiar transformations (like the Legendre transformation) change one theory into another equivalent theory while the dual pair are total different UV theories. This can be seen from another point of view. The dual pair will contribute to same effective action in IR, while the effective action in IR is not directly related to the free energy of matrix models, but through

$$W_{\text{eff}} = N_c \frac{\partial F_{\chi=2}(S, g)}{\partial S} + F_{\chi=1}(S, g)$$  \hspace{1cm} (1.1)$$

The relationship (1.1) shows that if $W_e = W_g$, with general different $N_c$ for dual pair we will have $F_e \neq F_g$, i.e., they are two different matrix theories with total different free energies.

The second reason can be also seen from (1.1) that the matrix model does not have any memory about the rank of gauge group. We recover the information of rank only when we go from the free energy to the effective action where the rank $N_c$ appears as a multiplier. It tells us that we should not seek the transformation of Seiberg duality in matrix model at the level of free energy (or the matrix model superpotential), but at the level of effective action. More concretely, starting with two matrix models with superpotential $W_{e,\text{tree}}$ and $W_{g,\text{tree}}$, we do the independent matrix model integrations and calculate effective actions $W_{e,\text{eff}}$ and $W_{g,\text{eff}}$. These effective actions will be functions of glueball field $S$ and other fields as well as coupling constants. The idea is that if we require $W_{e,\text{eff}} \equiv W_{g,\text{eff}}$ as functions of all variables, we may derive the Seiberg duality. We will show the idea works, at least for these examples we will discuss in this note.
2. The Seiberg dual theory of $U(N_c)$ group

The theory we want to discuss is the $U(N_c)$ gauge group with $N_f$ flavors $Q_i, \bar{Q}_i^\alpha$ and arbitrary deformation $W_{tree} = V(M)$ of meson fields $M_i^j = Q_i^\alpha \bar{Q}_\alpha^j$ where $\alpha$ is color index. The matrix model integration of the prototype has been given in [13] by the insertion of delta-function $\delta(M_i^j - Q_i^\alpha \bar{Q}_\alpha^j)$ with results as

$$W_{eff}(S, M) = (N_c - N_f)S[1 - \log \frac{S}{A^3}] - S \log \left(\frac{\det(M)}{A^{2N_f}}\right) + V_{tree}(M) \quad (2.1)$$

This is a pretty neat result because usually we can not do the matrix model integration exactly\(^2\). For this simple example with arbitrary deformation of $V(M)$, (2.1) is exact. As a simple exercise we can take $V(M) = m_i^j Q_i^\alpha \bar{Q}_\alpha^j = \text{tr}(mM)$ which has been done explicitly in [14]. Equation (2.1) gives

$$W = (N_c - N_f)S[1 - \log \frac{S}{A^3}] - S \log \left(\frac{\det(M)}{A^{2N_f}}\right) + \text{tr}(mM)$$

Integrated out $M$ by

$$\frac{\partial W}{\partial M} = 0 = -SM^{-1} + m$$

we get

$$W = N_cS[1 - \log \frac{S}{A^3}] - S \log \left(\frac{\Lambda^{N_f}}{\det(m)}\right) \quad (2.2)$$

which matches the result in [14].

Now we will apply above general result given by Demasure and Janik to our Seiberg dual pair. Given the electric theory as above with arbitrary deformation $V(M)$, we try to find the proper magnetic theory $U(\tilde{N}_c)$ with $N_f$ flavors $q_i, \tilde{q}_i^\alpha$, singlets $M$ and proper superpotential $V(q, \tilde{q}, M)$. The first step we need to do is to integrate the magnetic matrix model. Here we have fields $q_i, \tilde{q}_i^\alpha$ and gauge singlets $M$. Should we integrate them all in matrix model? The answer is no. We need only integrate fields $q_i, \tilde{q}_i^\alpha$ in matrix model while keeping $M$ as parameters. It is because fields $M$ are gauge singlets. So according to the field theory analysis in [1, 2, 3], we should leave $M$ untouched at the level of free energy and add them back to the effective action directly by the prescription (1.1). This point has also been emphasized in [16, 17]. Using this new understanding, we redo the integration of magnetic matrix model in [14, 15] at Appendix to show the consistence.

\(^1\)Various results in the SQCD like $\mathcal{N} = 1$ theory with $U(N)$ gauge group in matrix model can be found in [33].

\(^2\)The matrix model integration of delta-function requires that the rank $M$ of matrix is larger than the number $N_f$ of flavors. Since we have kept $N_f$ fixed while taking the large $M$ limit in the matrix model integration, the condition is satisfied.
Since we do not need to integrate fields $M$, the matrix model integration of magnetic theory is same prototype as discussed by Demasure and Janik and we can write down the effective superpotential directly as

$$W_{g,\text{eff}}(S, M, \tilde{M}) = (\tilde{N}_c - N_f)\tilde{S}[1 - \log \left(\frac{\tilde{S}}{\Lambda^3}\right)] - \tilde{S} \log \left(\frac{\det(\tilde{M})}{\Lambda^{2N_f}}\right) + V(M, \tilde{M}) \quad (2.3)$$

where to distinguish the magnetic theory from the electric theory, we use tilde for our notations in magnetic theory and $\tilde{M}^i_j$ is the magnetic meson given by $q_i \cdot \tilde{q}^j$. To compare with the electric theory (2.1) we need to integrate out the magnetic meson $\tilde{M}$.

Now it comes to the key point. Since we require $W_{e,\text{eff}} = W_{g,\text{eff}}$ for arbitrary deformation $V(M)$, it is conceivable that we should have $V(M, \tilde{M}) = V(M) + f(M, \tilde{M})$ where $f(M, \tilde{M})$, which describes the interaction of $M$ and $q_i \cdot \tilde{q}^j$, does not depend on the deformation $V(M)$. Because $M$ is gauge singlet and adjoint under the flavor symmetry $U(N_f)$, the interaction of $M$ and $q_i \cdot \tilde{q}^j$ should be like $\sum \text{tr}(M^p \tilde{M}^{q} M^{p} \tilde{M}^{q} \ldots)$. Integrating out the magnetic meson $\tilde{M}$, we have equation

$$\frac{\partial W_g}{\partial \tilde{M}} = 0 = -\tilde{S} \tilde{M}^{-1} + \frac{\partial f(M, \tilde{M})}{\partial \tilde{M}} \quad (2.4)$$

From (2.4) we suppose to solve $\tilde{M}$, put it back to $W_{g,\text{eff}}$ and compare with $W_{e,\text{eff}}$. Especially we should have term $S \log(\det(M))$ by putting $\tilde{M}$ back to term $\tilde{S} \log(\det(\tilde{M}))$. It is hard to imagine we can have this result unless the solution is $\tilde{M}^{-1} \sim M^n$. In another word,

$$f(M, \tilde{M}) = \text{tr}(\tilde{M} \frac{M^n}{\mu^{2n-1}}) \quad (2.5)$$

where $\mu$ is a scale constant. Under this assumption, we have

$$\tilde{M}^{-1} = \frac{M^n}{\tilde{S} \mu^{2n-1}} \quad (2.6)$$

Putting it back to $W_{g,\text{eff}}$ and simplifying, we get

$$W_{g,\text{eff}} = n\tilde{S} \det(M) + \tilde{N}_c \tilde{S} - \tilde{N}_c \tilde{S} \log \tilde{S} + \tilde{S} \log \left(\frac{\Lambda^{3N_c-N_f}}{\mu^{2n-1}N_f}\right) \quad (2.7)$$

where we have neglected the term $V(M)$ in $W_{g,\text{eff}}$ (we will neglect the same term in $W_{e,\text{eff}}$). The result should be compared with the effective action of electric theory

$$W_{e,\text{eff}} = -S \det(M) + (N_c - N_f)S - (N_c - N_f)S \log S + S \log \Lambda^{3N_c-N_f} \quad (2.8)$$

which is just regrouped of equation (2.1). Comparing the first term of (2.7) and (2.8) we get the first condition

$$-S = n\tilde{S} \quad (2.9)$$
Using (2.9) to second and third terms we get
\[ \widetilde{N}_c = n(N_f - N_c) \]
(2.10)
From this we see that \( n \) must be positive integer. Comparing the last term we get
\[ \Lambda^{3N_c - N_f} (\widetilde{\Lambda}^{3\widetilde{N}_c - \widetilde{N}_f})^{\frac{1}{n}} = (-n)\frac{\widetilde{N}_c}{n} (\mu^{2n-1})^{\frac{N_f}{n}} \]
(2.11)
Now it is clear that when \( n = 1 \), equations (2.9), (2.10) and (2.11) are exactly the dual relations of Seiberg dual pair. Notice that just by requiring the matching of \( W_{e,eff} \) and \( W_{g,eff} \) we cannot exclude the possibility \( n \neq 1 \). However, from (2.9) we see that when \( n \neq 1 \), \(|S| \neq |\widetilde{S}|\), so it is very natural to choose \( n = 1 \). In fact by the symmetry of dual pair and the dual theory of the dual theory will go back to original theory, we should choose \( n = 1 \). To see this, notice that
\[ S \rightarrow [\widetilde{S} = -\frac{S}{n}] \rightarrow [\widetilde{\widetilde{S}} = -\frac{S}{n^2}] . \]

3. The Seiberg dual theory of \( SO(N) \) and \( Sp(N) \) groups

The checking of Seiberg duality in matrix model for \( SO(N) \) gauge group with \( N_f \) flavors \( Q_i^j \) under the non-degenerated mass deformations has been done in [28]. The procedure to derive the Seiberg duality will be parallel to \( U(N) \) case. Using the delta-function technique, the general effective superpotential under arbitrary meson deformation \( V(M) \) with \( M = Q^i \cdot Q^j \) is given by [28]
\[ W_{e,eff} = \frac{1}{2} (N_c - 2 - N_f) S [1 - \log \frac{S}{\Lambda^3} - \frac{S}{2} \log \frac{\text{det}(M)}{\Lambda^{2N_f}} + V(M) \]
(3.1)
To see this, choosing \( V(M) = \frac{1}{2} \text{tr}(mM) \) and minimizing \( W_{e,eff} \) in (3.1) regarding to \( M \) we get
\[ \frac{\partial W_{e,eff}}{\partial M} = -\frac{S}{2} M^{-1} + \frac{m}{2} = 0 \]
Putting it back to \( W_{e,eff} \) and simplifying we get
\[ W_{e,eff} = \frac{S}{2} (N_c - 2) [1 - \log \frac{S}{\Lambda^{3(N_c - 2) - N_f} \text{det}(m) \Lambda^{N_c - 2}}] \]
which is the result got in [28]. Using similar arguments (i.e., (1) \( M \) should not be integrated in matrix model; (2) the matching for arbitrary deformation \( V(M) \) and the term \( S \log \text{det}(M) \)) for the magnetic theory we will have
\[ W_{g,eff} = \frac{1}{2} (\widetilde{N}_c - 2 - N_f) \widetilde{S} [1 - \log \frac{\widetilde{S}}{\widetilde{\Lambda}^3} - \frac{\widetilde{S}}{2} \log \frac{\text{det}(\widetilde{M})}{\widetilde{\Lambda}^{2N_f}} + V(M) + \frac{1}{2 \mu^{2n-1}} \text{tr}(M^n \widetilde{M}) \]
(3.2)
3Other works of \( SO/Sp \) groups in matrix model can be found also in [34].
Integrating out meson field $\tilde{M}$ we have
\[
\frac{\partial W_{g,\text{eff}}}{\partial \tilde{M}} = \frac{\tilde{S}}{2} \tilde{M}^{-1} + \frac{M^n}{2\mu^{2n-1}} = 0
\] (3.3)
Solving $\tilde{M}$ and putting it back we simplify the effective action as (notice that we neglected the term $V(M)$)
\[
W_{g,\text{eff}} = \frac{n\tilde{S}}{2} \log \det(M) + \frac{\tilde{S}}{2}(\tilde{N}_c - 2)(1 - \log \tilde{S}) + \frac{\tilde{S}}{2} \log \frac{\Lambda^{3(\tilde{N}_c-2)-N_f}}{(\mu^{2n-1})^{N_f}}
\] (3.4)
which should be compared with
\[
W_{e,\text{eff}} = -\frac{S}{2} \log \det(M) + \frac{S}{2}(N_c - N_f - 2)(1 - \log S) + \frac{S}{2} \log \Lambda^{3(\tilde{N}_c-2)-N_f}
\] (3.5)
From the first three terms we get
\[-S = n\tilde{S}, \quad \tilde{N}_c - 2 = n(N_f - (N_c - 2))\] (3.6)
and from the last term we get
\[\Lambda^{3(\tilde{N}_c-2)-N_f}(\Lambda^{3(\tilde{N}_c-2)-N_f})^\frac{1}{n} = (-n)^{-\frac{\tilde{N}_c-2}{n}}(\mu^{2n-1})^{\frac{N_f}{n}}\] (3.7)
Similar reason as in $U(N_c)$ case tells us to choose $n = 1$. In this case, equations (3.6) and (3.7) are exactly the dual relations of Seiberg dual pair with $SO(N)$ gauge group. Notice that to compare (3.7) with the result in field theory \cite{25}, we need to set
\[\Lambda_{\text{matrix}}^{3(\tilde{N}_c-2)-N_f} = 16\Lambda_{\text{field}}^{3(\tilde{N}_c-2)-N_f}\] (3.8)
as noticed in \cite{28}.
Comparing above calculation of $SO(N_c)$ with the one of $U(N_c)$, we see they are same if we make the following replacement $N_c \rightarrow N_c - 2$. When we discuss the gauge group $Sp(N)$ we just need to use the replacement $N_c \rightarrow N_c + 2$. With this replacement we will simply write down results. Unlike the $SO(N)$ case where the meson fields $M = Q^i \cdot Q^j$ are symmetric, for $Sp(N)$ (the rank $r$ of $Sp(N)$ is $N/2$) the meson fields $M = Q^i_a Q^b_J J^{ab}$ is antisymmetric \cite{29} where $J^{ab} = i\sigma_2 \otimes 1_{r \times r}$. The effective superpotential under general meson deformation is
\[
W_{e,\text{eff}} = \frac{1}{2}(N_c + 2 - N_f)S[1 - \log \frac{S}{\Lambda^3}] - \frac{S}{2} \log \frac{\det(M)}{\Lambda^{2N_f}} + V(M)
\] (3.9)
Similar reason constraints the effective superpotential for the dual magnetic theory to be
\[
W_{g,\text{eff}} = \frac{1}{2}(\tilde{N}_c + 2 - N_f)\tilde{S}[1 - \log \frac{\tilde{S}}{\Lambda^3}] - \frac{\tilde{S}}{2} \log \frac{\det(\tilde{M})}{\Lambda^{2N_f}} + V(M) + \frac{1}{2\mu^{2n-1}} \text{tr}(M^n)\tilde{M}
\] (3.10)
Integrated out $\tilde{S}$ from (3.10) and comparing with (3.9), we get following dual relations from matrix model for $Sp(N)$ gauge group

$$-S = n\tilde{S}, \quad \tilde{N}_c + 2 = n(N_f - (N_c + 2))$$  \hspace{1cm} (3.11)

$$\Lambda^{3(N_c+2)-N_f}(\Lambda^{3(\tilde{N}_c+2)-N_f})^{-\frac{1}{n}} = (-n)^{-\frac{N_c+2}{n}}(\mu^{2n-1})^{\frac{N_f}{n}}$$  \hspace{1cm} (3.12)

The requirement of two time dualities going back to original theory picks up $n = 1$ solution.

These examples we discussed in this paper are simple and standard. It will be interesting to generalize above method to other dual theories found in field theory, for example, the one discussed by Kutasov and Schwimmer in [32, 31]. Unlike these did in this paper for which general effective actions are known by matrix model, we do not know results for generalized Seiberg dual theories at this moment. But if we manage to do it by matrix model, it should be possible to derive the dual theory by the matrix model method.

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A. Matrix integration in magnetic theory

For the simplest magnetic theory with mass deformation

$$W_g = \text{tr}(mM) + \frac{1}{\mu}q_i\tilde{M}^i_j\tilde{q}_j$$  \hspace{1cm} (A.1)

the matrix integration has been done in [14, 15], where we integrated all fields $q, \tilde{q}$ as well as the gauge singlet fields $M$. However, from the field theory analysis in [4, 5, 6] as well as emphasized in [16, 17], we should only integrate fields $q, \tilde{q}$ in matrix model and leave terms which are gauge invariant to the effective superpotential. This method has been used to generalize the work of Seiberg [1] with massive flavors to the case of massless flavors in [17] where as a by-product, the original proposal of insertion of delta-function with fundamental flavors [13] has been explained (see also [18] from another point of view about the delta-function). With these new understanding, we should redo the matrix model integration for above magnetic superpotential (A.1). It is similar to the example given in [17], but we include following calculations for completeness which can also be considered as another example for the justification of the delta-function.

Now let us do the calculation. The matrix model integration for $q, \tilde{q}$ can be found in [14] where meson fields $\frac{M^i_j}{\mu}$ have been treated as mass parameters. The result is

$$W_{g,\text{eff}} = \tilde{N}_c(\tilde{\Lambda}^{3\tilde{N}_c-N_f} \text{det}(\frac{M}{\mu}))^{-\frac{1}{N_c}} + \text{tr}(mM)$$  \hspace{1cm} (A.2)
where the first term comes after integrating out the glueball field $\tilde{S}$ and the second term, from the original tree level superpotential without matrix model integration. Next step is to minimize meson fields $M$. From (A.2) we have

$$\frac{\partial W_{g,\text{eff}}}{\partial M} = 0 = \left(\tilde{\Lambda}^{2N_c-N_f} \det\left(\frac{M}{\mu}\right)\right)^\frac{1}{N_c} M^{-1} + m$$

which gives us

$$\det(M)^\frac{N_c-N_f}{N_c} = (-)^{N_f} \left(\frac{\tilde{\Lambda}^{2N_c-N_f}}{\mu^{N_f}}\right)^\frac{N_f}{N_c} \left(\det(m)\right)^{-1}$$

(A.3)

(A.4)

Putting them back we get

$$W_{g,\text{eff}} = \left(\tilde{N}_c - N_f\right)\left(\frac{\tilde{\Lambda}^{2N_c-N_f}}{\mu^{N_f}}\right)^\frac{1}{N_c} \det(M)^\frac{1}{N_c-N_f}$$

which is exactly the correct effective superpotential of magnetic theory.

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