Scalar Gravitation and Extra Dimensions

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Abstract

Gunnar Nordström constructed the first relativistic theory of gravitation formulated in terms of interactions with a scalar field. It was an important precursor to Einstein’s general theory of relativity a couple of years later. He was also the first to introduce an extra dimension to our spacetime so that gravitation would be just an aspect of electromagnetic interactions in five dimensions. His scalar theory and generalizations thereof are here presented in a bit more modern setting. Extra dimensions are of great interest in physics today and can give scalar gravitational interactions with similar properties as in Nordström’s original theory.

1 Introduction

Modern theories of gravitation based on supergravity or superstrings contain in general one or more scalar fields which will modify at some level the original tensor theory of Einstein. What rôle they play in the real world is not at all clear. But recent developments in cosmology such as the need for a very fast inflationary phase\footnote{\textsuperscript{1}Invited talk at \textit{The Gunnar Nordström Symposium on Theoretical Physics}, Helsinki, August 27 - 30, 2003.} in the early Universe and dark energy to explain the slower acceleration much later\footnote{\textsuperscript{2}}, are now being discussed in terms of such scalar fields with gravitational interactions. Most of the underlying, fundamental theories require spacetime to have more than the four we know about today. Such extra dimensions must be microscopic or curled up in some highly non-trivial way. These ideas are so interesting that they have inspired much experimental work during the last few years\footnote{\textsuperscript{3}}.

It was the Finnish physicist Gunnar Nordström who 90 years ago was the person who laid the first seeds in this research program which today has grown to a large, international effort. He was the first to construct a consistent, relativistic theory of gravitation described by the interactions of a massless, scalar field. Even if this theoretical proposal did not turn out to be physically correct and was after a couple of years replaced by Einstein’s tensor theory\footnote{\textsuperscript{4}}, it represented an important milestone
in the investigation of gravitational phenomena. Probably more important was his
daring idea to explain unknown physics in our known spacetime by known physics
in a spacetime with extra, compactified dimensions. This was several years before
the more detailed theories of Kaluza\cite{5} and Klein\cite{6} and lies today at the heart of
the most fundamental theories of Nature ever formulated.

We will here give a short summary of the main ideas behind Nordström’s different
contributions in a bit more modern formulation. As an example of how scalar fields
appear in theories of gravitation with extra dimensions, we will consider the radion
field which codes for the size of the compactified space. In four dimensions, it
appears as a correction to Einstein’s theory and has several properties in common
with the scalar field first introduced for gravitation by Nordström.

\section{Scalar Gravitation}

The first gravitational theories were inspired by electrostatics. Since Newton’s law
for the gravitational force

\begin{equation}
F_N = -G \frac{m_a m_b}{r^2}
\end{equation}

between two masses \(m_a\) and \(m_b\) with separation \(r\) has exactly the same form as
Coulomb’s law

\begin{equation}
F_C = \frac{e_a e_b}{4\pi r^2}
\end{equation}

for the force between two charges \(e_a\) and \(e_b\) at the same separation, it has been
tempting for more than a hundred years to describe them in a similar language\cite{7}.
The only, and crucial difference, is that the gravitational force is always attractive
while the electrostatic force can be both attractive and repulsive, depending on the
relative sign of the charges. The rôle of the electromagnetic potential will then be
played by a scalar gravitational field.

\subsection{Non-relativistic interactions}

Let us see how this analogy can be formulated more closely in a Lagrangian de-
scription. In electrostatics the electric field is given in terms of the potential as
\(E = -\nabla \phi\). The static Lagrangian is therefore

\begin{equation}
\mathcal{L} = \frac{1}{2} E^2 - \rho \phi = \frac{1}{2} (\nabla \phi)^2 - \rho \phi
\end{equation}
where $\rho(r) = \sum_a e_a \delta(r - r_a)$ is the charge density. The corresponding equation of motion is just Poisson’s equation $\nabla^2 \phi = -\rho$. We now find the interaction energy $E$ from the Hamiltonian $H = \int d^3x (-L)$ which after a partial integration gives

$$E = \int d^3x \left( \frac{1}{2} \phi \nabla^2 \phi + \rho \phi \right)$$

Using now the Poisson equation in the first term and writing it in momentum space as $k^2 \phi_k = \rho_k$ where the Fourier-transformed charge density is $\rho_k = \sum_a e_a e^{i k \cdot r_a}$, we get back to the Coulomb energy

$$E = \frac{1}{2} \int d^3x \rho \phi = \frac{1}{2} \sum_k \frac{\rho_k \rho_{-k}}{k^2} = \sum_{a < b} \frac{e_a e_b}{4\pi|\mathbf{r}_a - \mathbf{r}_b|} \quad (4)$$

after throwing away the infinite self-energies.

For the corresponding gravitational interaction between several masses, we can introduce a massless, scalar field $\phi$ described by the canonical Lagrangian density $L_\phi = (1/2)(\partial_\mu \phi)^2$. In the static limit we then have for the gravitational field

$$L = -\frac{1}{8\pi G} (\nabla \phi)^2 - \rho \phi \quad (5)$$

where now $\rho(r) = \sum_a m_a \delta(r - r_a)$ is the mass density and $G$ is Newton’s gravitational constant entering his force law $(1)$. This should be compared with $(3)$ for the electrostatic case. Notice the sign difference in the kinetic part. It results in the corresponding field equation

$$\nabla^2 \phi = 4\pi G \rho \quad (6)$$

The gravitational interaction energy now follows as in the electrostatic case and becomes

$$E = \frac{1}{2} \int d^3x \rho \phi = -G \sum_{a < b} \frac{m_a m_b}{|\mathbf{r}_a - \mathbf{r}_b|} \quad (7)$$

It is attractive as it should be.

### 2.2 Nordström theory I

In order to move out of this non-relativistic or static description, such a scalar theory of gravitation must be made consistent with the special theory of relativity. This was the first problem Nordström considered. The simplest generalization of the field equation $(6)$ is just

$$\Box \phi = -4\pi G \rho \quad (8)$$
when written in terms of the d’Alembertian operator $\square = \partial_t^2 - \nabla^2$. This is just what Nordström first tried. He could then calculate the gravitational field from masses in arbitrary motion.

What was needed next, was to find the equation of motion for a particle moving in this external field. This problem required a more dramatic answer. Nordström found that the inertial mass of the particle was no longer constant and would depend exponentially on the field\[7\][9]. At that time this was not considered to be problematic and was thought to be similar to its dependence on velocity in the special theory of relativity. The four-velocity of the particle $u\mu$ then satisfies the equation of motion

$$\ddot{u}_\mu + \dot{\phi}u_\mu = \partial_\mu \phi$$

(9)

where the dot derivative is with respect to proper time. All particles will fall the same way in a gravitational field independently of their masses so the weak equivalence principle is satisfied.

One immediate problem with this theory was that it could not be derived from an action principle. But more importantly, it was stressed by Einstein that the mass density $\rho$ in (8) should be a world scalar and preferably the trace $T$ of the energy-momentum tensor $T_{\mu\nu}$ of the system. The whole question about the relationship between the inertial and gravitational mass of a particle had to be reconsidered\[10\].

### 2.3 Nordström theory II

This effort resulted shortly in his second scalar gravitational theory\[11\]. The mass density $\rho$ was now assumed to be proportional to the trace $T_m$ of the energy-momentum tensor setting up the gravitational field, i.e. $\rho = g(\phi)T_m$ where $g(\phi)$ is some function to be determined. From the requirement of proportionality between inertial and gravitational masses, he found $g(\phi) = 1/\phi$ after a constant shift of the potential\[12\]. The field equation (8) is therefore replaced by

$$\phi \Box \phi = -4\pi G T_m$$

(10)

and was now non-linear. The electromagnetic field has $T_m = 0$ and it would therefore not have any gravitational interactions. As a result, there would not be any bending of light near stellar masses in this theory.

The dynamics of a gravitational system of particles could now be derived from an action principle. In particular, the motion of a particle in an external field follows from the variational principle

$$\delta \int ds \phi(x) = 0$$

(11)
with the line element \( ds = \sqrt{g_{\mu\nu}u^\mu u^\nu}d\tau \) where \( u_\mu \) is the 4-velocity and \( \tau \) the proper time. Instead of (9), it gives the modified equation of motion

\[
\phi u_\mu + \dot{\phi} u_\mu = \partial_\mu \phi
\]

which obviously also satisfies the weak equivalence principle.

This theory was immediately seized upon by Einstein who in the same year 1913, reformulated it in an elegant way and presented it as the first consistent, relativistic theory of gravitation[13]. It is described by a massless, scalar field \( \phi(x) \) with the standard energy-momentum tensor

\[
T_{\mu\nu} = \frac{1}{4\pi G} \left[ \partial^\mu \phi \partial^\nu \phi - \frac{1}{2} \eta^{\mu\nu} (\partial_\lambda \phi)^2 \right]
\]

where \( \eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1) \) is the Minkowski metric. In addition, matter of density \( \rho \) and 4-velocity \( u^\mu \) contributes

\[
T_{m\mu\nu} = \rho u^\mu u^\nu
\]

so that the trace \( T_m = \rho \phi \). The total energy-momentum tensor \( T_{\mu\nu} + T_{m\mu\nu} \) is now conserved. This follows from the divergence \( \partial_\mu T_{\mu\nu} = \Box \phi (\partial^\nu \phi)/4\pi G = -\rho (\partial^\nu \phi) \) when we use the field equation of motion (8). On the other hand, \( \partial_\mu T_{m\mu\nu} = \rho u^\mu \partial_\mu (\phi u^\nu) \) when we impose matter conservation \( \partial_\mu (\rho u^\mu) = 0 \). Since \( u^\mu \partial_\mu = d/d\tau \), this becomes \( \partial_\mu T_{m\mu\nu} = \rho (\partial^\nu \phi) \) from the particle equation of motion (12). The total divergence therefore adds up to zero and we have conservation of energy and momentum for the whole, interacting system.

### 2.4 Einstein-Fokker geometric reformulation

At the same time as Nordström developed this scalar field theory, Einstein was struggling with the development of his own, geometric formulation of a relativistic theory of gravitation. It was therefore clear to him that the variational principle (11) for the particle motion gives a geodesic equation in a curved spacetime with metric \( g_{\mu\nu}(x) = \phi^2(x)\eta_{\mu\nu} \). This represents a conformal transformation by a factor \( \phi^2 \) and the resulting spacetime will have a non-zero Ricci curvature tensor. In particular, the Ricci scalar is

\[
R = -\frac{6}{\phi^3} \Box \phi
\]

as found by Einstein and Fokker[14]. Under the conformal transformation the trace of the energy-momentum tensor will be transformed into \( T \rightarrow T/\phi^4 \). This can now be used to rewrite the Nordström gravitational equation (10) as

\[
R = 24\pi GT
\]
where on the right-hand side now enters the contributions from both matter and other possible sources. Here we have for the first time a purely geometric description of gravitational interactions. One can only speculate how much this equation was on Einstein’s mind when he the following year formulated his general theory of relativity where the gravitational tensor field equation has exactly the same structure.

2.5 Scalar theories of gravitation

One can obviously wonder if the Nordström gravitational theory is unique. In order to investigate that, let us consider a non-relativistic particle moving in a gravitational potential \( \phi \). It has the Lagrangian \( L_p = (1/2)m v^2 - m\phi \). Neglecting relativistic terms of the order of \( \phi v^2 \), this follows from the Lorentz-invariant action

\[
S_p = -m \int ds (1 + \phi)
\]

where \( ds \) is the relativistic line element introduced in (11). This can be generalized to

\[
S_p = -m \int ds A(\phi)
\]

where the function \( A(\phi) \to \phi + \text{const} \) in the weak-field limit \( \phi \to 0 \). It results in the equation of motion

\[
\frac{d}{d\tau} (A u_{\mu}) = A'(\partial_{\mu}\phi)
\]

where \( A' \) is the derivative of \( A \) with respect to \( \phi \). If \( A = e^\phi \), we recover Nordström’s first particle equation of motion (9) while \( A = \phi \) gives his second equation (12).

The Lagrangian for the scalar, gravitational field coupled to particles with a scalar mass density \( \rho \), will now be the covariant generalization of the non-relativistic Lagrangian (5) we started out with,

\[
\mathcal{L} = \frac{1}{8\pi G} \eta^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - \rho A(\phi)
\]

It gives the modified field equation \( \Box \phi = -4\pi G \rho A' \). There is no change in the energy-momentum tensor for the field, while for the matter part we now have instead \( T_{\mu\nu}^m = \rho A u^\mu u^\nu \). The total energy-momentum for the system is conserved as before. Eliminating the mass density from the field equation using the trace \( T_m = \rho A \), we find a generalized Nordström field equation

\[
A(\phi) \Box \phi = -4\pi G A'(\phi) T_m
\]
In the weak-field limit it is equivalent to the previous equations.

As already stated, the choice $A = \phi$ reproduces the second Nordström theory. Taking instead $A = 1 + \phi$, one obtains a theory investigated by Bergmann[15]. The choice $A = e^\phi$ is inspired by his first theory and is given as a home-work problem in the textbook by Misner, Thorne and Wheeler[16]. It has more recently been used by Shapiro and Teukolsky as a laboratory for numerical, relativistic gravitation[17]. None of these theories give neither any deflection of light nor the right perihelion shift for Mercury.

3 Extra Dimensions

There are no experimental indications that our spacetime has any extra dimensions. Experiments in high energy physics demonstrate that the strong and electroweak particle forces are confined to only four dimensions down to distances of the order of $10^{-18}$ m. On the other hand, our understanding of gravitational interactions has only been verified down to separations of the order of $10^{-4}$ m[3]. Thus it is conceivable that gravity can extend out into extra dimensions of size smaller than this, but still large compared with the range of the electroweak force[19].

The possibility that spacetime has more than four dimensions, was first contemplated by Nordström[18]. This was in order to unify his scalar theory of gravitation with Maxwell’s theory of electromagnetism. Now it turned out that this particular gravitational theory got a very short lifetime. But just the idea that spacetime has extra dimensions lies today at the heart of the most fundamental theories of quantum gravity and superstrings. Even if these new dimensions are so microscopically small that they never can be directly detected, they can still have very profound, indirect consequences in that a complicated and asymmetrical description of the basic interactions in our four-dimensional spacetime are just the reflections of a highly compact and symmetrical description seen from a higher-dimensional spacetime.

3.1 Five-dimensional Maxwell theory

In addition to constructing a relativistic theory of gravitation, Nordström wanted also to unify it with electromagnetic theory[18]. For this he envisaged our four-dimensional spacetime to be endowed with an extra, fifth dimension. Both of these interactions should then be part of a Maxwell theory in five dimensions,

$$\mathcal{L} = -\frac{1}{4} F_{ab} F^{ab} - J_a A^a$$  \hspace{1cm} (22)
where Latin indices now take on five values. The generalized field strengths are \( F_{ab} = \partial_a A_b - \partial_b A_a \) where the electromagnetic vector-potential has the components \( A^a = (A^\mu, \phi/\sqrt{4\pi G}) \), i.e. incorporating the gravitational potential \( \phi \). The corresponding 5-vector for the current densities is \( J^a = (J^\mu, \rho \sqrt{4\pi G}) \) where \( \rho \) is the mass density.

From here follows the 5-dimensional wave equation

\[
\partial^2 A_a - \partial_a (\partial^b A_b) = -J_a
\]  

(23)

We now see that ordinary current conservation \( \partial_\mu J^\mu = 0 \) in our spacetime implies that \( \rho \) is independent of the coordinate in the fifth direction, \( \partial_5 \rho = 0 \). Nordström then observed that when the vector-potential is also made independent of this new coordinate, the 5-dimensional wave equation separates (23) into the standard Maxwell wave equation \( \partial^2 A_\mu - \partial_\mu (\partial^\nu A_\nu) = -J_\mu \) plus his wave equation (8) for the gravitational field.

This daring and beautiful proposal was generally overlooked and generated little interest. One reason can be that it was published at the same time as WW I broke out and not much later Einstein had the correct theory of gravitation[4].

The requirement \( \partial_5 A^a = 0 \), that the field should have no variation in the new dimension, was introduced again several years later by Kaluza[5] in his 5-dimensional unified theory of gravitation and electromagnetism built upon Einstein’s theory. It is now called the cylinder condition and was explained by Klein[6] as a consequence of quantum mechanics when the extension in the extra dimension is so small that it is unobservable.

### 3.2 Einstein gravitation with extra dimensions

In order to illustrate some of the physics resulting from having more than four dimensions, let us with Kaluza and Klein consider Einstein gravity in a spacetime with \( D = 4 + n \). If the extra dimensions have the coordinates \( y^\alpha \), the fundamental Einstein-Hilbert action will be

\[
S = -\frac{1}{2} M_D^{2+n} \int d^4x \int d^n y \sqrt{-\bar{g}} \bar{R} \]  

(24)

where \( M_D \) is the Planck mass in this spacetime. It has the metric \( \bar{g}_{ab} \) with determinant \( \bar{g} \) and \( \bar{R} \) is the corresponding Ricci curvature scalar. Assuming that the extra dimensions are compactified and described by the internal metric \( h_{\alpha\beta} \), we assume the form

\[
d\bar{s}^2 = g_{\mu\nu}(x)dx^\mu dx^\nu - b^2(x)h_{\alpha\beta}(y)dy^\alpha dy^\beta \]  

(25)
for the full spacetime metric in its groundstate. The unknown function $b(x)$ gives the general size of the compactified volume $V_n = \int d^n y \sqrt{-h}$. With the full metric on this form, one can calculate the scalar curvature $\bar{R}$. Since the determinant $\bar{g} = gh$ now separates into a product, the full action can be written as

$$S = -\frac{1}{2} M_{D+1}^2 V_n \int d^4 x \sqrt{-g} b^n \left[ R - \frac{1}{b^2} R_n + n(n-1) g^{\mu \nu} \partial_\mu b \partial_\nu b \right]$$

(26)

after a few partial integrations and assuming the the internal curvature scalar $R_n$ is a constant[20]. The coefficient in front of the four-dimensional Ricci scalar $R$ defines the usual Planck constant

$$M_P^2 = V_n M_D^{2+n}$$

(27)

in agreement with more general considerations[19]. With two extra dimensions and the requirement that the gravitational range in these directions must not be larger than $10^{-4}$ m, one finds a value for the mass $M_{D=6}$ which can be as small as 1 TeV. In this case one would have the possibility to see the effects of quantum gravity or string theory at LHC and other future high-energy particle accelerators.

The result [26] for the action describes a scalar field $b(x)$ with gravitational interactions. It can be rewritten on a more canonical form by defining a new field $\Phi = b^n$ which then gives

$$S = -\frac{1}{2} M_P^2 \int d^4 x \sqrt{-g} \left[ \Phi R + (1 - 1/n) \frac{1}{\Phi} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - R_n \Phi^{1-2/n} \right]$$

(28)

This extended tensor theory is now recognized to be of the Brans-Dicke type[21] where the last term is a potential $V(\Phi)$ for the scalar field resulting from the curvature of the internal space. Without such a potential, it gives corrections to standard Einstein gravity which depends on the parameter $\omega = -(1 - 1/n)$ in front of the scalar kinetic energy. When $n > 1$ we will have $\omega$ to be close to one in magnitude and negative. On the other hand, solar system gravitational precision measurements require $\omega > 3500[22]$. This doesn’t mean that such scalar-tensor theories are ruled out by observations. Damour and Nordtvedt have shown that if $\omega$ gets to be dependent on $\Phi$ which easily happens when higher order corrections are included, it will be driven towards very high values today because of the cosmological evolution of the Universe[23].

### 3.3 Radions and quintessence

In this extra-dimensional approach the Brans-Dicke scalar field $\Phi(x)$ stems from the radial size $b(x)$ of the compactified space. For this reason it is called a radion and
is generic in this kind of theories. It plays an important rôle in today’s cosmological theories\cite{24}, and is used both for inflation\cite{25} and quintessence in the Universe\cite{26}. These physical phenomena are usually discussed in the Einstein frame which arises from a conformal transformation $g_{\mu\nu} \rightarrow A^2(\Phi)g_{\mu\nu}$ of the metric in the Brans-Dicke action\cite{28} such that the coefficient of the Einstein-Hilbert term $R$ is $\Phi$-independent and with the value one. This comes about since this term will transform as $R \rightarrow A^{-2}R - 6A^{-3}\Box A$ where now the $\Box$ operator is the d’Alembertian in curved space with metric $g_{\mu\nu}$. This is the same conformal transformation as used by Einstein and Fokker in their reformulation of the Nordström scalar theory, but then from flat Minkowski space. One obtains the desired result with the choice $A = \Phi^{-1/2}$ which now will appear as the new scalar field in the Lagrangian.

In order to get the kinetic term on canonical form, we let $A = A(\phi)$ satisfy

$$\frac{1}{A} \frac{\partial A}{\partial \phi} = -\frac{\alpha}{M_P}$$

where the parameter $\alpha$ is related to the Brans-Dicke parameter $\omega$ by

$$\alpha^2 = \frac{1}{4\omega + 6}$$

Since this is constant, we thus simply have $A = e^{-\alpha\phi/M_P}$. The resulting action in the Einstein frame is therefore

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2}M_P^2 R + \frac{1}{2}g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \tilde{V}(\phi) \right]$$

where $\tilde{V}$ is the transformed potential,

$$\tilde{V}(\phi) = \frac{1}{2}M_P^2 A^4(\phi)V(\Phi)$$

If the original potential $V$ is just a constant, as would be the case if the compact space was flat and we instead had included a cosmological constant in the $D$-dimensional action\cite{24}, the resulting potential in the Einstein frame is seen to be an exponential. Such a potential has many attractive features which today is used in realistic quintessence models\cite{27}.

Specific models depend on the number $n$ of extra dimensions and what kind of fields are included in the fundamental action. A more detailed investigation of such a model with $n = 2$ and an additional scalar field in the extra-dimensional spacetime has been performed by Albrecht, Burgess, Ravndal and Skordis\cite{28}. Due to quantum effects, the Brans-Dicke parameter will be field dependent and thus evolve into a sufficiently large value today so to be consistent with solar system gravity tests.
In addition, the same quantum effects give a radiative correction to the purely exponential potential which then takes the more realistic form previously proposed by Albrecht and Skordis\[29\].

The radion field also couples to matter. In the original frame this would have been described by an action $S_m = S_m[g_{\mu \nu}, \psi]$ where $\psi$ stands for a generic set of matter fields. Under the conformal transformation it changes into $S_m \rightarrow S_m[A^2(\phi)g_{\mu \nu}, \psi]$ which means that the radion field will modify the fundamental coupling constants in the matter Lagrangian. Since $\phi$ varies with the evolution of the Universe, this means that the fundamental constants also will vary with time. There are some indications that this might actually be the case for the electromagnetic fine-structure constant\[30\].

From the above it is seen that the coupling of the radion to matter is only through the factor $A^2$ in the metric. This coupling will then manifest itself in the equation of motion for the radion which now becomes\[20\]

$$\Box \phi + \ddot{V}'(\phi) = \frac{\alpha}{M_P} T_m$$ (33)

The Brans-Dicke parameter $\alpha$ gives the deviation of the coupling strength from that of the graviton. We notice that this wave equation is of the same form as the Nordström scalar field equation when the potential is zero.

### 4 Conclusion

Scalar fields are abundant in modern theories of gravitation and other fundamental interactions. It was Nordström who first constructed such a consistent, relativistic theory based on a single field. Although his scalar theory was shortly afterwards replaced by Einstein’s tensor theory, scalar fields in this connection are today again being investigated and with greater vigor than ever before. There are many experimental indications from physics and cosmology that they are present in our physical world, but it requires much more work to find out how they fit in with the other fundamental fields we know.

The intellectual leap he made suggesting that our spacetime has extra dimensions in order to unify the different interactions we see around us, has today resulted in a large number of very promising theoretical proposals and experimental efforts. It will probably never be possible to rule out such an extension of our conventional spacetime. One can instead only hope that their presence in some way soon will be demonstrated and bring order and beauty to our understanding of the physical world.
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