Properties of Globular Clusters in Galaxy Clusters: Sensitivity from the Formation and Evolution of Globular Clusters

So-Myoung Park1, Jihye Shin1, Rory Smith2, and Kyungwon Chun1

1 Korea Astronomy and Space Science Institute, 776 Daedeok-daero, Yuseong-gu Daejeon 34055, Republic of Korea; jshin@kasi.re.kr
2 Universidad Técnica Federico Santa María, 3939 Vicuña Mackenna, San Joaquín Santiago 8940897, Chile

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Abstract

We investigate the properties of globular clusters (GCs) in a galaxy cluster, using the particle tagging method with a semianalytical approach in a cosmological context. We assume GCs form from dark matter halo mergers and their metallicity is assigned based on the stellar mass of the host dark matter halos and the formation redshift of GCs. Dynamical evolution and disruption of GCs are considered using semianalytical approaches, controlled by several free parameters. In this paper, we investigate how our results are changed by the choice of free parameters. We compare our fiducial results with representative observations, including the mass ratio between the GC system and its host galaxy, the GC occupancy, the number fraction of blue GCs, and the metallicity gradient with the GC mass. Because we can know the positions of GCs with time, comparison with additional observations is possible, e.g., the median radii of the GC system in individual galaxies, the mean projected density profiles of intracluster GCs, and the metallicity and age gradients of GCs with a clustercentric radius. We also find that the specific mass of the GC system in each galaxy is different with a clustercentric radius.

Unified Astronomy Thesaurus concepts: Galaxy clusters (584); Galaxy formation (595); Globular star clusters (656); Computational astronomy (293)

1. Introduction

Globular clusters (GCs) are found in every type of galaxy, from dwarfs to cD galaxies. Their typical mass range is $10^5 - 10^6 M_\odot$, and their present mass function (MF) shows a log-normal shape (Fall & Zhang 2001; Waters et al. 2006; Jordan et al. 2007b; Lameli-Núñez et al. 2020). Because their ages are typically nearly a Hubble time ago ($\sim 10-12$ Gyr), they are fossils that represent the extreme star formation at high redshift (e.g., Glatt et al. 2008; Chies-Santos et al. 2011; Powalka et al. 2017; Usher et al. 2019). GCs in galaxies and galaxy clusters provide valuable information about the merging history of galaxies and galaxy clusters and the environment of GC formation (Mackey et al. 2010; Olschanski & Sorce 2018; Kruíjssen et al. 2019b; Massari et al. 2019; Dolfi et al. 2021). Therefore, GCs are useful for understanding the formation and evolution of their host galaxies (Brodie & Strader 2006; Forbes et al. 2018).

GC observations have been conducted in local groups and galaxy clusters (Côté et al. 2004; Jordan et al. 2007a; Sarajedini et al. 2007; Carter et al. 2008; Ferrarese et al. 2012; Harris et al. 2013; Brodie et al. 2014; Piotto et al. 2015; Fahron et al. 2020) and they have revealed important relations between GCs and their host galaxies. Basically, most galaxies of $M_{stellar} \geq 10^9 M_\odot$ have GCs (GC occupancy; Peng et al. 2008; Georgiev et al. 2010; Sánchez-Janssen et al. 2019; Carlsten et al. 2022), where $M_{stellar}$ is the galaxy stellar mass. Although there is a nonlinear relation between the total and stellar mass of galaxies (Behroozi et al. 2013a; Hudson et al. 2015), galaxies including GCs show a constant mass fraction at $z = 0$ of $M_{GCs}/M_{halo} \sim 5 \times 10^{-5}$ (the $M_{GCs}/M_{halo}$ relation; Peng et al. 2008; Harris et al. 2013; Hudson et al. 2014; Harris et al. 2015), where $M_{GCs}$ is a GC system mass in each galaxy, and $M_{halo}$ is the total galaxy mass. Previous papers have suggested that the $M_{GCs}/M_{halo}$ relation is the result of hierarchical galaxy mergers (e.g., Choksi & Gnedin 2019a; El-Badry et al. 2019; Bastian et al. 2020).

GCs in galaxies exhibit color bimodality, which is commonly observed in almost all massive early-type galaxies in Virgo itself but is not universally observed (Larsen et al. 2001; Forbes 2005; Harris et al. 2006; Peng et al. 2006; Waters et al. 2009; Brodie et al. 2012; Tonini 2013; Harris et al. 2016; Bastian & Lardo 2018). Blue GCs are metal-poor, while red GCs are metal-rich. The color bimodality of GCs might imply that there were two star formation epochs or mechanisms in the history of galaxies (Brodie & Strader 2006; Usher et al. 2012).

Based on the color bimodality of GCs, there are three different formation scenarios of blue and red GCs (see Brodie & Strader 2006, for a review): the major merger model, the in situ scenario, and the dissipationless accretion scenario. The major merger model is for when blue GCs form in protogalactic fragments, while red GCs form from the gas-rich major merger of galaxies (Ashman & Zepf 1992). The in situ scenario is when blue GCs form simultaneously as galaxies form but the formation is restrained by the pressure from the supernova explosion that causes gas to be expelled from the star-forming galaxy. Then, red GCs form from the expelled gas, which falls back into the galaxy due to gravity (Forbes et al. 1997; Harris et al. 1999). Finally, in the dissipationless accretion scenario, blue GCs form by the dissipationless accretion of neighboring dwarf galaxies, and red GCs form in situ in massive seed galaxies (Côté et al. 1998).

On the other hand, there is an alternative GC formation scenario, based on Lambda cold dark matter cosmology: the hierarchical merging scenario (Muratov & Gnedin 2010). In this scenario, the hierarchical build-up of galaxies by mergers forms GCs and one does not need to consider the formation of...
blue and red GCs independently. Instead, the metallicity of GCs is assigned by the $M_{\text{stellar}}$ of their host galaxies and the formation redshift of GCs.

To trace the formation and evolution of GCs, various numerical simulations have been performed. Due to the wide dynamical range between GCs and their host galaxies, high-resolution cosmological simulations are required. Because of the huge advancements in computational hardware, recent high-resolution cosmological hydrodynamic simulations have been performed so they have started to directly resolve the GC formation with somewhat limited internal resolution (Boley et al. 2009; Kimm et al. 2016; Li et al. 2017; Kim et al. 2018; Lahén et al. 2020; Ma et al. 2020, and references therein). However, they still suffer from the limitations of spatial/mass resolution, simulation volume, and huge calculation times.

As an extension to the approach of the high-resolution cosmological hydrodynamic simulations, the E-MOSAICS project performs a self-consistent hydrodynamical zoom-in simulation with subgrid modeling (Pfeffer et al. 2018; Kruijssen et al. 2019a; Reina-Campos et al. 2022b, 2022c). In this hydrodynamical approach, we can infer the formation site and condition of the GC formation. The E-MOSAICS project applies the GC mass loss due to stellar evolution, tidal shock, two-body relaxation, and dynamical friction, but only dynamical friction is calculated in post-processing. It increases the calculation time of modeling so there is a limitation to investigating the properties of GCs by exploring the impact of parameter variations.

Traditionally, dark matter (DM)-only simulations with a semianalytical approach for the GC formation are implemented. DM-only simulations require a short computational time, but generally they do not make clear predictions on the GC spatial distributions (Prieto & Gnedin 2008; Li & Gnedin 2014; Choksi et al. 2018; El-Badry et al. 2019).

The particle tagging method (PTM) that tags DM or stellar particles as GCs has also been applied in cosmological DM-only or hydrodynamical simulations (Corbett Moran et al. 2014; Mistani et al. 2016; Ramos-Almendares et al. 2018, 2020; Chen & Gnedin 2022). The advantages of the PTM are that it can trace the position and velocity information of GCs and the computational time is much shorter than high-resolution cosmological simulations due to the post-processing and without running full simulations. Previous studies using this method, however, do not consider various processes of dynamical evolution of GCs. For example, Ramos-Almendares et al. (2018, 2020) neglect the mass-loss process of GCs to concentrate on the tidal stripping that causes GCs to escape their host galaxies and become intracluster globular clusters (ICGCs; Lee et al. 2010). Chen & Gnedin (2022) apply the stellar and tidal mass loss of GCs but neglect the mass loss by two-body relaxation, dynamical friction, and tidal shocks.

In this paper, we investigate the properties of GCs in galaxy clusters, combining the advantages of both PTM and a semianalytical approach: the position information from PTM and the speed from a semianalytical approach. Based on the DM-only cosmological simulation, we tag DM particles as GCs and apply the dynamical evolution of each GC. For the GC formation, we adopt the hierarchical merging scenario, which assumes GCs form from DM halo mergers and the metallicity of GCs is a function of $M_{\text{stellar}}$ of their host DM halos and GC formation redshift. The evolution of GCs is described by the semianalytical approach so it can be easily adjusted by varying the parameter sets that control the analytical recipes. We investigate how parameter variations can affect the properties of GCs with the advantage of the speed of our method and then we compare our results with various observations, including position information. In this study, first, we introduce our method optimized for GC formation and evolution. Second, we take advantage of the speed at which we can explore parameter space with our method. This enables us to investigate the sensitivity of our results to possible parameters, allowing us to better understand which physical processes are important. Finally, we compare our fiducial results with various observations. Note that our main purpose is not to tune our fiducial parameter set to reproduce the observations exactly.

This paper is constructed as follows. In Section 2, we introduce our PTM and show how GCs form and evolve in a cosmological context with the semianalytical approach. Section 3 shows a comparison of our results with observations and how the choice of parameter set can affect the results. In Section 4, we compare our results with additional observations. Sections 5 and 6 discuss and summarize our results.

## 2. Method for GC Formation and Evolution

In this section, we introduce our PTM with the semianalytical approach to describe the formation and evolution of GCs. For the GC formation, we adopt the hierarchical merging scenario (e.g., Choksi et al. 2018; Choksi & Gnedin 2019b; El-Badry et al. 2019; Bastian et al. 2020, and references therein). For the GC mass loss, we apply the stellar evolution and two-body relaxation. For the GC disruption, we apply tides from host galaxies and the orbital decay by dynamical friction.

### 2.1. Halo Mass Assembly History

We build the cluster mass assembly history using three sets of high-resolution DM-only cosmological simulations from $z=200$ to 0 (Taylor et al. 2019) with the $N$-body code, GADGET2 (Springel 2005). The original simulations were carried out with cosmological parameters $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, $\Omega_b = 0.04$, and $h = 0.68$ with a box of (140 Mpc $h^{-1}$)$^3$. The mass of each particle is $1.7 \times 10^2 M_{\odot} h^{-1}$ with a softening length ($\epsilon$) of 5.469 kpc $h^{-1}$. The power spectrum is calculated by the CAMB package$^5$ (Lewis et al. 2000).

From these original simulations, Virgo cluster analog DM halos$^3$ are identified at $z = 0$. We choose three cluster DM halos (Targets 1–3), which have small Lagrangian volumes, among various Virgo cluster analog DM halos in the original simulations to reduce the calculation time. Next, we resimulate Targets 1–3 with a zoom-in technique, which involves rerunning the interesting regions of the low-resolution simulations with higher resolution (Porter 1985; Katz & White 1993; Navarro & White 1994). Multiscale initial conditions with positions and velocities were generated by the MUSIC package$^4$ (Hahn & Abel 2011). The high-resolution particle mass is $3.32 \times 10^6 M_{\odot} h^{-1}$ with $\epsilon = 0.683$ kpc $h^{-1}$. Targets 1–3 have an $M_{\text{halo}}$ of $9.25 \times 10^{13}$, $1.14 \times 10^{14}$, and $1.26 \times 10^{14} M_{\odot} h^{-1}$.

$^3$ http://camb.info
$^4$ The Virgo cluster is one of the nearest galaxy clusters so various kinds of observations are implemented: the ACS Virgo cluster survey (Côté et al. 2004), the next generation Virgo cluster survey (NGVS; Ferrarese et al. 2012), and the Extended Virgo Cluster Catalog (Kim et al. 2014), and so on. Therefore, we can use various observational properties of GCs in the Virgo cluster.
$^5$ https://www.n-очка.ee/ohahn/MUSIC/
respectively. The virial ratio, $2T/|U|$, of Targets 1–3 is 1.56, 1.19, and 1.31, respectively, where $T$ is a kinetic energy and $U$ is a potential energy so Target 1 is the most unrelaxed cluster among them.

We identify halo and subhalo structures with the modified six-dimensional phase-space DM halo finder, ROCKSTAR\textsuperscript{5} (Behroozi et al. 2013b) and make merger trees using Consistent Trees (Behroozi et al. 2013c). To define the DM halo in ROCKSTAR, we set the minimum number to 20 particles so that the minimum $M_{\text{halo}}$ is $6.64 \times 10^7 \, M_\odot \, h^{-1}$. Throughout this paper, however, we use DM halos more massive than an $M_{\text{halo}}$ of $10^9 \, M_\odot \, h^{-1}$, which consists of more than $\sim$300 DM particles. The equivalent $M_{\text{stellar}}$ of $M_{\text{halo}} \approx 10^9 \, M_\odot \, h^{-1}$ would be $5.97 \times 10^5 \, M_\odot \, h^{-1}$. Here, we convert $M_{\text{halo}}$ to $M_{\text{stellar}}$ using the $M_{\text{stellar}}$ and $M_{\text{halo}}$ relation from weak lensing data (Hudson et al. 2015). This match is what was done in the observation so we can compare our results with the observations identically. The slope of stellar MF of the observation is $-0.56$ (Lan et al. 2016), and Targets 1–3 have slopes of stellar MF with $-0.54$, $-0.54$, and $-0.57$, respectively.

### 2.2. PTM for GCs

The hierarchical merging scenario is adopted to make the GCs in our model. We assume GCs form by DM halo mergers that are larger than a minimum mass ratio of $\gamma_{\text{MR}}$, which is a free parameter in our model. When GCs form, the initial $M_{\text{GC}}$ in each DM halo has a linear relation with $M_{\text{halo}}$ (Peng et al. 2008; Harris et al. 2013; Hudson et al. 2014; Harris et al. 2015):

$$M_{\text{GC}} = \eta M_{\text{halo}},$$

where $\eta$ is a constant formation efficiency and a free parameter in our model.

We assume that one DM particle represents one GC, although the mass of the GC is not equal to the mass of the DM particle. The initial $M_{\text{GC}}$, is distributed into individual GCs with a power-law initial globular cluster mass function (GCMF) (Elmegreen 2018):

$$dN/dM = M_0 M^{-2},$$

where $M_0$ is a normalization constant. We determine the minimum mass of the GC ($M_{\text{min}}$) is $10^5 \, M_\odot$ because we assume that most GCs below $10^5 \, M_\odot$ are destroyed due to two-body relaxation. If we normalize the probability of the MF, Equation (2) is

$$1 = \int_{M_{\text{min}}}^{\infty} M_0 M^{-2} \, dM,$$

which gives $M_0 = M_{\text{min}}$. Then, using the transformation method, the initial mass of each GC, $M_i$, is selected by a random number, $0 < r < 1$:

$$M_i = \frac{M_{\text{min}}}{r}.$$  \hspace{1cm} (4)

We repeat Equation (4) until the sum of the initial mass of each GC in individual DM halos reaches the required $M_{\text{GCs}}$ value at the GC formation epoch, and in this way, the number of GCs is also determined. The same number of DM particles from each DM halo is then selected to trace out the location of the GCs. Here, we pick particles according to in order of their bound energies, which assumes GCs form in the deepest location of the potential potential of DM halos. Finally, a different GC initial mass is assigned to each of the tagged DM particles, allowing us to trace their position and velocity down to $z = 0$.

We assume that the metallicity of GCs is assigned by the $M_{\text{stellar}}$ of their host DM halos and redshift at the GC formation epoch (Li & Gnedin 2014; Choksi et al. 2018; Choksi & Gnedin 2019b). We adopt the stellar mass–metallicity model (Ma et al. 2016; Choksi et al. 2018):

$$[\text{Fe/H}] = \log \left\{ \frac{M_{\text{stellar}}}{10^{10.5} \, M_\odot} \right\} \left( 1 + z \right)^{-\alpha},$$  \hspace{1cm} (5)

where $\alpha_m = 0.35$ and $\alpha_z = 0.9$ (e.g., Choksi et al. 2018; Chen & Gnedin 2022). Here, we define metal-poor (blue) GCs as $[\text{Fe/H}] < -1.0$ and metal-rich (red) GCs as $[\text{Fe/H}] > -1.0$.

### 2.3. Mass Evolution and Disruption of GCs

After GCs form, they undergo mass loss and disruption by several internal and external processes: stellar evolution, two-body relaxation, tides from host galaxies, the orbital decay by dynamical friction, and tidal shocks (e.g., Spitzer 1987; Gieles et al. 2011; Shin et al. 2013; Webb et al. 2014; Madrid et al. 2017, and references therein). We include recipes for these processes in our PTM with the semianalytical approach. Note that we do not consider the GC disruption by tidal shocks because our simulations are DM-only simulations and we do not model the gas and stellar distributions in each galaxy.

#### 2.3.1. Mass Loss of GCs by Stellar Evolution and Two-body Relaxation

The GC mass evolution is approximated to the first-order differential equation (Fall & Zhang 2001):

$$\frac{dM}{dt} = -(\nu_{\text{se}} + \nu_{\text{ts}}) M,$$

where $\nu_{\text{se}}$ and $\nu_{\text{ts}}$ are time-dependent fractional mass-loss rates by the stellar evolution and two-body relaxation, respectively.

For $\nu_{\text{se}}$, we use the stellar evolution model (Hurley et al. 2000), assuming the initial mass function (IMF) of each GC is distributed by a Kroupa IMF (Kroupa 2002). We tabulate the changes in the mass of stars with different masses as a function of time, $\nu_{\text{se}}(t)$, so we can calculate the amount of mass loss of each GC after they form. We assume a constant metallicity (0.1 Z$_\odot$), which is a typical metallicity of observed GCs.7 Figure 1 shows the evolution of GCMF in the brightest cluster galaxy (BCG). The thin black line denotes the MF without a mass loss or disruption of GCs (original MF), and the green line shows the evolved MF, where we only apply the mass loss by the stellar evolution. The stellar evolution does not change the shape of the MF but it moves the original MF to the low-mass part.

For $\nu_{\text{ts}}$, we adopt the formula in Spitzer (1987):

$$\nu_{\text{ts}} = \frac{\xi_s}{t_{\text{th}}}.$$  \hspace{1cm} (7)

\footnote{For simplicity, we do not use the metallicity assigned by Equation (5).}
normal shape by two-body relaxation. The shape of the original MF is changed to a log-
applying the mass loss by the stellar evolution and two-body
for the evolution and the disruption of GCs: the stellar evolution
8 For simplicity, we assume the tidal radius is the Jacobi radius throughout the
Figure 1. The dynamical evolution of GCMF in the BCG according to recipes
for the evolution and the disruption of GCs: the stellar evolution (SE), two-
the green, blue, and red thin lines show the variation in the GCMF after applying
\( r_{h0} \) is the initial half-mass radius of GCs. We treat \( r_{h0} \)
where \( \xi_e \) of 0.033 is the normalization factor and \( t_{th} \) is the half-
mass relaxation timescale:
\[
 t_{th} = \frac{\bar{M}_1^{1/2} r_{h0}^{3/2}}{7.25 \bar{n} G^2/3 \ln \Lambda},
\]
where the \( \bar{m} \) of 0.87 \( M_\odot \) is the mean stellar mass of a Kroupa
IMF, \( \ln \Lambda \) of 12 is the Coulomb logarithm, a typical value for
GCs, and \( r_{h0} \) is the initial half-mass radius of GCs. We treat \( r_{h0} \)
as a free parameter in our model.
The blue line in Figure 1 shows the evolved MF after applying the mass loss by the stellar evolution and two-body relaxation. The shape of the original MF is changed to a log-
form by two-body relaxation.

2.3.2. Disruption of GCs by Tides and Dynamical Friction
To consider tides from host DM halos where GCs belong, we divided GCs into those in the isolated regime and those in the tidal regime, using \( \mathcal{M} = r_{J2}(R)/r_J(t) \), where \( r_{J2}(t) \) is a half-
mass radius and \( r_J(t) \) is the Jacobi radius \(^8\) with time (Gieles &
Baumgardt 2008). The Jacobi radius is the distance from the cluster center to the Lagrange points \( L_1 \) and \( L_2 \) so it is used to define the boundary where stars dynamically belong to the GC.
The variation of \( r_J \) is mainly driven by the internal dynamics of GCs, while \( r_J \) is mainly changed by the external dynamics of host galaxies.

Gieles & Baumgardt (2008) find that if \( \mathcal{M} > \mathcal{M}_c \), star clusters can be treated in the tidal regime, while for \( \mathcal{M} < \mathcal{M}_c \) they can be treated in the isolated regime, where \( \mathcal{M}_c \) is a criterion used to

divide into the tidal regime and the isolated regime. Gieles &
Baumgardt (2008) found that \( \mathcal{M} = 0.05 \), but we treat \( \mathcal{M}_c \) as a free parameter. In our model, we assume GCs staying in the tidal regime longer than the fixed number of \( t_{th} (T_{dur}) \) are totally
destroyed, where \( T_{dur} \) is the duration time when \( \mathcal{M} \) is larger than \( \mathcal{M}_c \). That is, \( T_{dur} \) determines how long GCs are affected by the tidal force from their host galaxies. We find that \( T_{dur} \) cannot change our results significantly so we use a fixed value of \( T_{dur} = 0.5 \) Gyr.
The evolution of \( r_J \) is described as follows. During the first
\( t_{th} \), the stellar evolution is the main driver making GCs expand so the \( r_{h0} \) of GCs expands \( \sim 1.3 \) times during the first \( t_{th} \) by the stellar evolution. During the pre-core-collapse stage of GCs, we assume the 1.3 \( r_{h0} \) is not changed so 1.3 \( r_{h0} \) is maintained until the first \( t_{cc} \), where \( t_{cc} \) is the core collapse timescale of \( \approx 10 \) \( t_{th} \) (e.g., Spitzer 1987; Gürkan et al. 2004). After the first \( t_{cc} \), because the binary-driven post-core-collapse expansion is

\[
 r_{th}(t) = r_{h0}(t/t_{cc})^{2(\nu+1)/3}, 
\]
where \( \nu \approx 0.1 \) (e.g., Goodman 1984).
The evolution of \( r_J \) is described as follows. If we assume GCs are mainly affected by the tidal force of their host DM halos, \( r_J(t) \) is
\[
 r_J(t) = (M/2M_g)^{1/3} R_g, 
\]
where \( M_g \) is the enclosed mass of host DM halos where GCs are located, and \( R_g \) is the galactocentric radius. Because we can trace the position of GCs in their host DM halos with time, we can calculate \( R_g \) as a function of time. To calculate \( M_g \), we use the halo concentration and the virial radius in the results by the ROCKSTAR DM halo finder, which assumes the Navarro–
Frenk–White profile of DM halos (Navarro et al. 1996; Jimenez et al. 2003; Mo et al. 2010). When we calculate \( r_J \), we assume that only DM particles contribute to the \( M_g \) so we do not consider the mass of stellar and gas components. The contribution of stellar and gas components to \( M_g \) will be included in a follow-up study (e.g., Li & Gnedin 2014; Choksi et al. 2018).
Finally, we can calculate \( r_J \) and \( r_J \) with time using Equations (9) and (10). We assume GCs that satisfy \( \mathcal{M} > \mathcal{M}_c \) longer than \( T_{dur} = 0.5 \) Gyr are totally destroyed. The thin red line in Figure 1 shows the MF after applying tides together with the stellar evolution and two-body relaxation. Compared with the blue line, the number of low-mass GCs is decreased due to the tidal force by their host DM halos.

Some GCs are gradually moving into the center of DM halos because of dynamical friction, described as orbital decay. We assume GCs that are destroyed by orbital decay can contribute to the formation of nuclear star clusters (NSCs) (e.g., Antonini et al. 2012; Gnedin et al. 2014; Sánchez-Janssen et al. 2019, and references therein). If GCs are in the circular orbit and only the DM contributes to the velocity dispersion of GCs, the dynamical friction timescale (Binney & Tremaine 2008; Gnedin et al. 2014) is
\[
 t_{df} = 0.65 \text{ Gyr} \left( \frac{R_g}{\text{kpc}} \right)^2 \left( \frac{V_c(R_g)}{\text{km s}^{-1}} \right) \left( \frac{M(t)}{10^5 M_\odot} \right)^{-1} f_r, 
\]
where \( V_c(R) \) is a circular velocity of the DM halo at \( R_g \), and \( f_r = 0.5 \) (Gnedin et al. 2014). Here, we assume \( V_c \sim \sigma \), where \( \sigma \)
is the velocity dispersion of the host DM halo. We calculate the new \( f_{\text{MM}} \) whenever the host DM halos of GCs are changed due to DM halo mergers. If \( f_{\text{MM}} \) is shorter than the timescale when GCs are staying in their host DM halos, we assume GCs are destroyed and treated as NSCs.

Finally, the thick black line in Figure 1 shows the final GCMF at \( z = 0 \). The orbital decay makes the MF slightly lower than the one without it (the thin red line), maintaining a log-normal shape with a peak mass at \( \sim 5.5 \times 10^3 \, M_\odot \). To compare our final GCMF with the observation, we overplot the present GCMF of M49, whose \( M_{\text{halo}} \approx 10^{14} \, M_\odot \) is similar to that of Targets 1–3 (the average \( M_{\text{halo}} \approx 10^{14} \, M_\odot \)). The GCMF of M49 is converted from the GC luminosity function with the constant mass-to-light ratio of 2.69 in the \( g \) band (Jordán et al. 2007b; Willmer 2018). We find that the peak mass and the log-normal shape match well between our model and the observation. Note that, if tidal shocks that we do not consider for the GC disruption are applied, the GCMF can move to the low-mass part while leaving the log-normal shape nearly invariant (e.g., Fall & Zhang 2001; Prieto & Gnedin 2008; Shin et al. 2008; Pfeffer et al. 2018; Li & Gnedin 2019; Reina-Campos et al. 2022a).

3. Comparison of Model Results with Observations

In this section, we compare our models with three representative observations at \( z = 0 \), the \( M_{\text{GCMF}}-M_{\text{halo}} \) relation, the GC occupancy, and the number fraction of blue GCs (\( f_{\text{blue}} \)) in each galaxy. Hereafter, we use the term galaxy to refer to the structure, which contains \( M_{\text{halo}} \) in both our simulation and the observation. To measure the similarity between our results and the observations quantitatively, we use the Kolmogorov–Smirnov (K-S) probability (\( P_{\text{K-S}} \)) and the \( \chi^2 \) value

\[
\chi^2 = \frac{\sum (x_o - x_m)^2}{N},
\]

where \( x_o \) is the expected numbers, \( x_m \) is the observed numbers, and \( N \) is the total number of data.

To compare our results with the observation, we use two GC catalogs in this section: the Hubble Space Telescope/Advanced Camera for Surveys (HST/ACS) Virgo cluster survey (Côté et al. 2004), and the next generation Virgo cluster survey (NGVS; Ferrarese et al. 2012). The HST/ACS Virgo cluster survey observes the 100 early-type galaxies in the Virgo cluster using the Advanced Camera for Surveys on board the Hubble Space Telescope in the F475W and F850LP bandpasses (\( \approx \text{Sloan } g \) and \( z \)). The wide field channel of the ACS has a field of view of 202″ × 202″, which translates into a 16.2 × 16.2 kpc field of view at the distance of the Virgo cluster (16.5 Mpc). Deep images in F475W and F850LP provide the brightest 90% of the GC luminosity function in 100 early-type galaxies with a sample of 13,000 GCs. The NGVS covers the Virgo cluster from its core to its virial radius (a total area of 104 deg\(^2\)) in the \( ugriz \) bandpasses, using the 1 deg\(^2\) MegaCam instrument on board the Canada–France–Hawaii Telescope (CFHT). It reaches a point-source depth of \( g \approx 25.9 \) mag (10\( \sigma \)) and a surface brightness limit of \( \mu_g \approx 29 \) mag arcsec\(^{-2}\).

3.1. Fiducial Model

Our fiducial parameter set is determined to reproduce the overall feature of the observations, the \( M_{\text{GCMF}}-M_{\text{halo}} \) relation, GC occupancy, and \( f_{\text{blue}} \), but is not tuned to match the observations exactly. Our fiducial values for free parameters are as below. GCs are formed until \( z = 1 \), which corresponds to the lookback time of 8 Gyr, with an estimated minimum age of GCs in the Virgo cluster (e.g., Ko et al. 2022). We assume all GCs form in DM halo mergers of \( \gamma_{\text{MB}} = 0.1 \) with the initial log \( \eta = -3.3 \). We assume that \( n_{\text{io}} \) is 3 pc, which is the median half-light radii of typical GCs in the Virgo cluster (e.g., Jordán et al. 2005) and GCs with \( \Psi_{\gamma} > 0.05 \) for \( t_{\text{dur}} = 0.5 \) Gyr are totally destroyed. The values of the fiducial parameter set are shown in the first row of Table 1.

Figure 2 shows the \( M_{\text{GCMF}}-M_{\text{halo}} \) relation of our fiducial results and the observation of the ACS Virgo cluster survey (Peng et al. 2008). We convert \( M_{\text{stellar}} \) of observed galaxies to \( M_{\text{halo}} \), using the \( M_{\text{halo}}-M_{\text{stellar}} \) relation (Hudson et al. 2015). The observation and our fiducial results have a \( \chi^2 \) of 0.025 and 0.057, respectively, but the overall trend is very similar to each other. The mean \( M_{\text{GCMF}}/M_{\text{halo}} \) of our fiducial results is \( 3.5 \times 10^{-5} \), which is similar to the observation of \( M_{\text{GCMF}}/M_{\text{halo}} \approx 10^{-5} \) (Harris et al. 2013). The gray lines are fitting lines of the observation (dashed) and our fiducial results (solid). The slopes of fitting lines of our fiducial results and the observation are 1.04 and 1.14, respectively.

Figure 3 shows the GC occupancy (the number ratio of galaxies that contain GCs among entire galaxies) of our fiducial results compared to the observation of NGVS (Sánchez-Janssen et al. 2019, see their Table 4). In both our fiducial results and the observations, most galaxies more massive than \( M_{\text{stellar}} \approx 10^9 \, M_\odot \) contain GCs so the overall match is good (\( P_{\text{K-S}} \sim 0.96 \)). The number of low-mass galaxies that contain GCs is smaller than massive galaxies because GCs that form in low-mass galaxies are easily destroyed due to their low mass and GCs in low-mass galaxies can move to massive galaxies by galaxy mergers.

The left panel in Figure 4 shows the number fraction of blue GCs in host galaxies (\( f_{\text{blue}} \)). We define blue GCs as \( \text{[Fe/H]} < -1.0 \) and red GCs as \( \text{[Fe/H]} > -1.0 \). For comparison, we overplot the observation of the ACS Virgo cluster survey (Peng et al. 2008). The gray-dashed line is the fitting line of the observed data points of the Virgo cluster (see Harris et al. 2015, their Equation (3)). Values of \( \chi^2 \) of our fiducial results and the observation for \( f_{\text{blue}} \) are 0.041 and 0.022, respectively. Our fiducial results show higher \( \chi^2 \) than the observation because there are many low-mass halos that contain only blue GCs (e.g., Peng et al. 2008; Harris et al. 2015). Red GCs can form in halos more massive than \( M_{\text{stellar}} \approx 2.6 \times 10^8 \, M_\odot \) at \( z = 1 \) by Equation (5). In our simulations, the fraction of massive halos at \( z = 1 \) is only \( \sim 0.01 \) so red GCs can form relatively less than blue GCs. However, the overall behavior is similar to what is observed (increasingly red GCs as the halo mass increases), even if the quantitative match is not very good.

The right panel in Figure 4 shows the mean metallicity of blue and red GCs with each GC mass (\( M_{\text{GCMF}} \)) of our fiducial results and the observation of the ACS Virgo cluster survey (Peng et al. 2006; Choksi et al. 2018). In observations, the mean metallicity for blue GCs increases as \( M_{\text{GCMF}} \) increases (see the blue-dashed line), which is known as the blue tilt. Similar to the observation, our fiducial result (the solid line) also shows the blue tilt (e.g., Choksi et al. 2018; Usher et al. 2018), but the mean metallicity of blue and red GCs tends to have offsets to slightly lower and higher values of 0.1–0.3 dex.
3.2. Testing Sensitivity to Parameters

In this section, we investigate how our model results are affected by the variation of free parameters. To try to understand the importance and impact of free parameters on our results, we systematically vary each free parameter from the fiducial values (see the first row in Table 1) while keeping the others fixed (bold numbers in Table 1 show the changed values). Free parameters that we can change in our model are the redshift cut \( z_c \), the merger ratio \( \gamma_{MR} \), the initial mass ratio of \( M_{GC} \) and \( M_{halo} \) \( (\log \eta) \), the initial half-mass radius of GCs \( \rho_0 \), and the ratio of \( \rho_0 \) and \( r_J \) \( (R) \). Table 1 summarizes the free parameters and the altered values we consider for them. Note that the fiducial value is chosen to reproduce the overall observations, but is not finely tuned to match all observations exactly (see Section 3.1).

For comparison with our fiducial results, we use the \( M_{GC} - M_{halo} \) relation, GC occupancy, and \( f_{blue} \). Figures 5–7 show the results of each parameter set, compared to our fiducial parameter results.

Redshift cut. First, we investigate how GC properties are changed with the redshift cut \( z_c \), which limits the minimum GC formation epoch. The fiducial value of \( z_c \) is 1. However, young GCs exist in extragalactic systems (Glatt et al. 2008; Ko et al. 2018; Usher et al. 2019) and the mass and radii of young massive star clusters (YMCs) are similar to GCs in the local universe (Portegies Zwart et al. 2010; Longmore et al. 2014; Forbes et al. 2018). Thus, some simulation papers continue to make GCs until \( z_c = 0 \), assuming that YMCs could be one of

### Table 1

| Parameter Setting | \( z_c \) | \( \gamma_{MR} \) | Initial log \( \eta \) | \( \rho_0 \) pc | \( \rho_J \) pc | \( M_{GC} - M_{halo} \) | GC Occupancy | \( f_{blue} \) |
|-------------------|--------|----------------|----------------|-----------|-----------|----------------|--------------|-----------|
| Fiducial          | 1.0    | 0.1            | -3.3           | 3.0       | 0.05      | 0.057         | 0.96         | 0.041     |
|                   | 0.0    | 0.1            | -3.3           | 3.0       | 0.05      | 0.053         | 1.00         | 0.043     |
|                   | 4.0    | 0.1            | -3.3           | 3.0       | 0.05      | 0.212         | 0.00         | 0.051     |
|                   | 1.0    | 0.01           | -3.3           | 3.0       | 0.05      | 0.132         | 0.81         | 0.033     |
|                   | 1.0    | 0.3            | -3.3           | 3.0       | 0.05      | 0.103         | 0.63         | 0.033     |
|                   | 1.0    | 0.1            | -3.0           | 3.0       | 0.05      | 0.045         | 0.94         | 0.037     |
|                   | 1.0    | 0.1            | -4.0           | 3.0       | 0.05      | 0.204         | 0.04         | 0.065     |
|                   | 1.0    | 0.1            | -3.3           | 2.0       | 0.05      | 0.115         | 0.10         | 0.051     |
|                   | 1.0    | 0.1            | -3.3           | 10.0      | 0.05     | 0.102         | 0.02         | 0.056     |
|                   | 1.0    | 0.1            | -3.3           | 3.0       | 0.05     | 0.053         | 0.98         | 0.035     |
|                   | 1.0    | 0.1            | -3.3           | 3.0       | 0.005    | 0.355         | 0.00         | 0.254     |

Note. Column (1): the lowest redshift for the GC formation. Column (2): the minimum DM halo merger mass ratio. Column (3): the initial mass fraction of the GC system and their host galaxies. Column (4): the initial half-mass radius. Column (5): the criteria of the ratio between the half-mass radius and the Jacobi radius. Column (6): the \( \chi^2 \) value for the \( M_{GC} - M_{halo} \) relation. Column (7): the K-S probability for GC occupancy. Column (8): the \( \chi^2 \) value for the number fraction of blue GCs. Bold numbers show the changed values.

Figure 2. The \( M_{GC} - M_{halo} \) relation at \( z = 0 \). Black open circles represent observations of the Virgo cluster. Orange solid dots represent our results with the fiducial parameter set. Linear fitting lines of the observations (Harris et al. 2013) and our fiducial results are denoted by the gray-dashed line and the dashed line, respectively.

Figure 3. The GC occupancy. The black-dashed line denotes the observations of the Virgo cluster (Peng et al. 2008). The orange line denotes our results with the fiducial parameter set.
the precursors of future GCs (e.g., Prieto & Gnedin 2008; Li & Gnedin 2014; Choksi et al. 2018; El-Badry et al. 2019; Chen & Gnedin 2022). Applying this assumption in our model, $z_c = 1$ is changed to $z_c = 0$. On the other hand, the average age of observed GCs in the Milky Way is old (10–13 Gyr) so we also test using $z_c = 4$.

The results are shown in the first and second panels in the first rows of Figures 5–7. In Figure 5, the height of $M_{\text{GCs}}$ with $z_c = 0$ is 0.30 dex higher than that of the $z_c = 1$ case. The slope of the $z_c = 0$ case is 1.13 similar to that of the $z_c = 1$ case of 1.04. In Figure 6, the GC occupancy of the $z_c = 0$ case is similar to that of the $z_c = 1$ case ($P_{\text{hS}} = 1.0$). Because both the $z_c = 0$ and $z_c = 1$ cases experience the epoch of the maximum number of galaxy merger frequencies at $z = 3$, this results in the similar trend of both the $z_c = 0$ and $z_c = 1$ cases. In Figure 7, the $z_c = 0$ case can make more red GCs than the $z_c = 1$ case because many red GCs form at low redshift. Thus, the $z_c = 0$ case has more galaxies that have lower $f_{\text{blue}}$ than the $z_c = 0$ case.

The $z_c = 4$ case differs more strongly from the $z_c = 1$ case because galaxies stop making GCs before the epoch of the peak galaxy merger frequency ($z = 3$). It means insufficient GCs are made overall (see Figures 5 and 6) and the early formation of GCs makes galaxies have mostly blue GCs (see Figure 7 and Equation (5)). Therefore, the $z_c = 4$ case shows a worse match than the $z_c = 0$ case.

When we stop making GCs at $z = 1$, it matches the overall observed properties of GCs in the Virgo cluster. It implies that the GC formation in galaxy clusters after $z = 1$ might be rare. One possible explanation is that the cosmic star formation rate (SFR) has a peak at between $z = 2$ and 1 and the SFR is rapidly decreasing after $z = 1$ in the range of $10^{11} M_\odot < M_{\text{halo}} < 10^{15} M_\odot$ (Behroozi et al. 2019). The other is that after galaxies fall into the galaxy cluster, they lose their gas by harassment and ram pressure stripping (Moore et al. 1996; Hester 2006; Boselli et al. 2019). In this case, although the galaxy mergers continue after falling into the galaxy cluster, GCs cannot form.

**Merger ratio.** We consider the minimum galaxy merger ratio to produce the GC system ($\gamma_{\text{MR}}$) is 0.1 as a fiducial value so both major mergers and minor mergers may make GCs. We use 0.01 rather than 0.1 to include the extreme minor mergers and use 0.3 for major mergers only.

The results are shown in the third and fourth panels in the first rows of Figures 5–7. In Figure 5, the $\gamma_{\text{MR}} = 0.01$ case makes the height of $M_{\text{GCs}}$ increase 0.75 dex entirely because most galaxies undergo more frequent mergers than the $\gamma_{\text{MR}} = 0.1$ case. The slopes of the $\gamma_{\text{MR}} = 0.01$ and $\gamma_{\text{MR}} = 0.1$ cases are 1.04 and 0.97, respectively, so the slope stays quite similar. More frequent galaxy mergers of the $\gamma_{\text{MR}} = 0.01$ case make the GC occupancy higher than the $\gamma_{\text{MR}} = 0.1$ case (see Figure 6). When $\gamma_{\text{MR}} = 0.3$, the height of $M_{\text{GCs}}$ and the GC occupancy slightly decrease in each galaxy (see Figures 5 and 6). This is because fewer GCs form in individual galaxies due to a smaller major merger frequency than the minor merger frequency. On the other hand, in Figure 7, we can see that $f_{\text{blue}}$ is not significantly affected by $\gamma_{\text{MR}}$ because the total number of GCs per galaxy is just determined by $\gamma_{\text{MR}}$ while the number ratio of blue and red GCs is mainly determined by $z_c$.

The initial mass ratio between the GC system and the host galaxy. The initial mass ratio of the GC system and its host galaxies (log $\eta$) is set to be $-3.3$ at the GC formation epoch as our fiducial value. We test two more initial log $\eta$ of $-4.0$ and $-3.0$ (e.g., Choksi & Gnedin 2019a). These values assume a linear relation between log $M_{\text{GCs}}$ and log $M_{\text{halo}}$, but only the height is changed on the plot.

The panels in the second rows of Figures 5–7 show the results. In Figure 5, if the initial log $\eta$ is high, more GCs can form in each galaxy so the height of $M_{\text{GCs}}$ of the log $\eta = -3$ case is 0.03 dex higher than the log $\eta = -3.3$ case with slopes of 1.07 and 1.04, respectively. If the initial log $\eta$ is low, insufficient GCs form so the height of $M_{\text{GCs}}$ of the log $\eta = -4$ case is 0.51 dex lower than the one of the log $\eta = -3.3$ case. The GC occupancy is also affected by the initial log $\eta$ because the log $\eta = -3$ and log $\eta = -4$ cases can make more and less GCs than the log $\eta = -3.3$ case (see Figure 6). In Figure 7, the
initial log $\eta$ does not affect $f_{\text{blue}}$ because the number ratio of blue and red GCs is mainly driven by $z_c$.

The initial half-mass radius. Next, we change the initial half-mass radius ($r_{h0}$). Our fiducial value of $r_{h0}$ is 3 pc. We use 2 and 10 pc rather than 3 pc to see the effect of $r_{h0}$.

The first and second panels in the third rows in Figures 5–7 show the results. In Figure 5, the slopes of the $r_{h0} = 2$ pc and the $r_{h0} = 10$ pc cases are 1.11 and 0.98, similar to the fiducial results of 1.04, but the heights of both cases are 0.22 and 0.41 dex lower than the $r_{h0} = 2$ pc case. Both the $r_{h0} = 2$ pc and $r_{h0} = 10$ pc cases produce lower GC occupancy than the $r_{h0} = 3$ pc case entirely (see Figure 6). This means too many GCs are being destroyed, they are not changing the overall $M_{\text{GCs}}$–$M_{\text{halo}}$ trend. Instead, data points are simply being removed from the trend so the occupancy falls.

In Figure 7, if $r_{h0}$ is 2 pc, there are more red GCs than in the $r_{h0} = 10$ pc case. This is because if $r_{h0}$ is 2 pc, blue GCs that form at high redshift are destroyed by a short $t_{\text{rlx}}$. In the case of red GCs, they have formed relatively recently compared to blue GCs, so red GCs can survive until $z = 0$ in spite of a short $t_{\text{rlx}}$. When $r_{h0}$ is 10 pc, most blue and red GCs have larger $R$ than $R_c$, so they are quickly destroyed by tides from host galaxies. Although GCs have $r_{h0} = 10$ pc, some blue and red GCs that can have low $R$ might be isolated from the tides. Therefore, it makes isolated GCs have a long $t_{\text{rlx}}$ so they can survive until $z = 0$. These results are also seen in N-body simulations of individual star clusters (Webb et al. 2014; Zonoozi et al. 2016; Park et al. 2018).

The ratio of the half-mass radius to the tidal radius. We use an $R_c$ of 0.05 (e.g., Gieles & Baumgardt 2008) to divide the GCs into the tidal regime ($R > R_c$) and the isolated regime.
so GCs in the tidal regime are destroyed by the tidal force from host galaxies. To investigate how $R_c$ affects the disruption of GCs, we change $R_c$ from 0.05 to 0.5 and 0.005. The results are shown in the third and fourth panels in the third rows in Figures 5–7. The trend of $R_c$ of each GC is increasing due to Equation (9), although there is a fluctuation of $R_c$ by $r_f$. That is, GCs that have $R_c = 0.5$ have already experienced the $R_c = 0.05$ regime. This results in similar trends of the $M_{GC} - M_{halo}$ relation and the GC occupancy between the $R_c = 0.5$ and $R_c = 0.05$ cases (see Figures 5 and 6).

In Figure 7, many galaxies have lower $f_{blue}$ in the $R_c = 0.5$ case than the $R_c = 0.05$ case. Because the tides are weaker in the $R_c = 0.5$ case compared to the $R_c = 0.05$ case, more red GCs can survive from their host galaxies. As a result, none of the results with $R_c = 0.005$ are similar to the $R_c = 0.05$ case. Therefore, increasing $R_c$ does not do much but decreasing this value is very significant for our model.

4. Additional Comparison with the Observations

4.1. Comparing GC Positions with Observations

Because we use the PTM, we can trace the positions of tagged particles as GCs with time. In this section, we focus on additional observations for comparison, using GC position information. To compare our results with the observation, we add two more GC catalogs in this section: the point-source catalog in the Sloan Digital Sky Survey (SDSS) Sixth Data Release (Adelman-McCarthy et al. 2008) and the Canada–France–Hawaii Telescope Legacy Survey (CFHTLS; Hudelot et al. 2012). Lee et al. (2010) select the brightest GC candidates in a circular field with a radius of 9°, using the photometry of the point sources in the SDSS Sixth Data Release. They use the criteria for color and magnitude, $0.6 < (g - i_0) < 1.3$ and $19.5 < i_0 < 21.7$ mag, with reddening correction, and the...
The magnitude limit is $i_0 < 21.7$ mag. CHFTLS covers 155 deg$^2$ across four patches that comprise several fields with five filters: $u^*$, $g^*$, $r^*$, $i^*$, and $z^*$. For a point source, the limiting magnitude in the $i$ band is $\sim 24.7$.

First, we investigate the size of the GC system in each galaxy. Figure 8 shows the median radii of the GC system ($R_{\text{log}0.5}$) with the $M_{\text{stellar}}$ of their host galaxies. The observed effective radii of the GC system are taken from nearby galaxy groups (Hudson & Robison 2018). To measure the median radii of the GC system, we use galaxies that contain more than 10 GCs inside 0.1$R_{\text{vir}}$ (orange solid dots) or 1.0$R_{\text{vir}}$ (brown open circles). Adopting galaxies that have more than 10 GCs can improve the statistics because using 10 GCs results in higher confidence in calculating $R_{\text{log}0.5}$. When we increase the radial cut from 0.1$R_{\text{vir}}$ to 1.0$R_{\text{vir}}$, $R_{\text{log}0.5}$ slightly increases but the overall trend is almost the same. The trend of $R_{0.5}$ is found to increase with $M_{\text{stellar}}$ and it is similar to the observations of galaxies with $M_{\text{stellar}} > 10^{10} M_\odot$, although there are large vertical scatters in the observation. From $M_{\text{stellar}} = 10^8 M_\odot$ to $M_{\text{stellar}} = 10^{10} M_\odot$, the trend of $\log R_{0.5}$ is almost constant. Thus, the trend of $\log R_{0.5}$ shows a broken-power law with a broken point at $\sim 5 \times 10^{10} M_\odot$.

We also overplot the initial log $R_{0.5}$ with black dots in Figure 8 to show how the GC system size has changed from their initial size. In our model, it is difficult to define the initial log $R_{0.5}$ in each galaxy because GCs are added whenever galaxies experience mergers. Thus, we just overplot the initial log $R_{0.5}$ at the first merger in each galaxy for simplicity. We find that the initial log $R_{0.5}$ is an extension of the current log $R_{0.5}$.

Next, we investigate the distribution of ICGCs in galaxy clusters. Various galaxy cluster surveys have detected ICGCs and found that the number density profile of blue ICGCs is more extended than that of red ICGCs (Lee et al. 2010; Peng et al. 2011; Durrell et al. 2014; Madrid et al. 2018; Harris et al. 2020). Figure 9 shows the projected density profiles of the blue...
blue GCs can escape their host galaxies by accretion or merger. It makes blue ICGCs have a higher mass density profile than red ICGCs.

In Figure 10, we investigate the mean metallicity (left panel) and the mean age (right panel) of GCs as a function of a clustercentric radius. For comparison, we overplot the NGVS observation as open circles (Ko et al. 2022). In the left panel of Figure 10, both our fiducial results and the observation show a decreasing mean metallicity of blue and red GCs with the clustercentric radius. Especially, the gradient and the height of the mean metallicity of blue GCs match the observation well. However, in the case of red GCs, there is an offset of the mean metallicity between our fiducial results and the observation. We revisit this issue in the discussion (Section 5). In the right panel of Figure 10, the mean age of blue GCs is decreasing with the clustercentric radius in both our fiducial results and the observation but there is no overlap. However, there is a possibility that the age of GCs is estimated as younger (1–2 Gyr) because integrated stellar populations from the observations can be affected by changing the morphology of the horizontal branch (Ko et al. 2022). In this case, there might be an overlap of blue GCs between our fiducial results and the observation but the age of red GCs in our fiducial model is still younger than the observation.

4.2. Properties of the GC System with the Clustercentric Radius

To understand how properties of the GC system are changed by the clustercentric radius, we investigate the $M_{\text{GC}}-M_{\text{halo}}$ relation and the specific mass ($S_{M}=100M_{\text{GC}}/M_{\text{stellar}}$, Peng et al. 2008). We divide galaxies into inner ($r<1$ Mpc) and outer galaxies ($r>1$ Mpc).

The left panel in Figure 11 shows the $M_{\text{GC}}-M_{\text{halo}}$ relation of our fiducial results. The slopes of the inner and outer galaxies are 1.06 and 0.94, respectively, and the height of the inner galaxies is 1 dex higher than the outer galaxies. The right panel in Figure 11 shows the $S_{M}$ of our fiducial results and the observation of the ACS Virgo cluster survey (Peng et al. 2008). The inner galaxies have higher $S_{M}$ than the outer galaxies in both our fiducial results and the observation (e.g., Peng et al. 2008; Harris et al. 2013). There are insufficient high-mass and low-mass galaxies in both our simulation and the observation, respectively, so it results in a significant difference between our fiducial model and the observation at high-mass and low-mass regions of galaxies (e.g., Moustakas et al. 2016; Carlsten et al. 2022). However, we can still see a U-shape of the $S_{M}$ of the inner galaxies in both our fiducial results and the observation.

Overall, Figure 11 shows that the inner galaxies have more GCs than the outer galaxies. This results from the inner galaxies generally experiencing more frequent mergers than the outer galaxies due to an early infall to galaxy clusters.

5. Discussion

In this paper, the most important issue we try to tackle is how the various free parameters impact on the GC properties at $z=0$. In Section 3.2, we investigate how the GC properties can be changed by the various values of free parameters: the redshift cut ($z_c$), the merger ratio ($\gamma_{\text{MR}}$), the initial mass ratio between the GC system and the host galaxy ($\log \eta$), the initial half-mass radius ($r_{h0}$), and the ratio of $r_{h0}$ to $r_{J}$ ($\Omega$). These parameters change the $M_{\text{GC}}-M_{\text{halo}}$ relation, GC occupancy,
and \( f_{\text{blue}} \) significantly. Among them, \( z_c, \gamma, \text{ initial} \log \eta \), and \( \gamma \) are sensitive free parameters because they affect the \( M_{\text{GC}}-M_{\text{halo}} \) relation, GC occupancy, and \( f_{\text{blue}} \) simultaneously. This means the environment of the GC formation and the tide are important to build up the current properties of GCs in galaxy clusters. However, because we examine the effect of parameter variations by changing only one parameter value and leaving the rest of the values fixed, we do not know whether there are interdependencies or degeneracy among the parameters. In addition, one parameter can affect some of the observations significantly but cannot affect the rest of the observations. For example, the \( M_{\text{GC}}-M_{\text{halo}} \) relation is less affected by \( n_{\text{gas}} \) but GC occupancy and \( f_{\text{blue}} \) are significantly altered by \( \gamma_{\text{MBR}} \). Various parameter combinations are needed to derive the formation conditions for galaxies and GCs. In the future, we propose using the Markov chain Monte Carlo (MCMC) method to improve our understanding of how various parameter combinations can affect the final results and the match to observations.

Throughout the paper, we assume that the initial \( \log \eta \) is a constant. However, stars form from cold gas so previous semianalytical models have assumed a constant fraction between \( M_{\text{GC}} \) and the cold gas mass (\( M_{\text{gas}} \)), which is a function of \( M_{\text{stellar}} \) and \( z \) (e.g., Li \\& Gnedin 2014; Choksi et al. 2018; Choksi \\& Gnedin 2019a, 2019b). Instead of using a constant initial fraction between \( M_{\text{GC}} \) and \( M_{\text{halo}} \), we also tried to adopt a constant fraction between \( M_{\text{GC}} \) and \( M_{\text{gas}} \), using the best model parameters in Choksi et al. (2018). The \( M_{\text{GC}}-M_{\text{halo}} \) relation at \( z = 0 \) of our fiducial results matches the observation, while the \( M_{\text{GC}}-M_{\text{halo}} \) relation at \( z = 0 \) with a constant fraction between \( M_{\text{GC}} \) and \( M_{\text{gas}} \) is an order lower than the observation of the Virgo cluster. We infer that the different method for GC disruption by tide and the additional GC disruption, dynamical friction, which Choksi et al. (2018) did not consider, might demand a higher initial \( \log \eta \). We will investigate how additional GC mass-loss and disruption processes can affect the initial mass fraction of the GC system and its host galaxy.

Our fiducial results can reproduce the overall GC properties in the galaxy cluster, the \( M_{\text{GC}}-M_{\text{halo}} \) relation, GC occupancy, and the decreasing \( f_{\text{blue}} \) with \( M_{\text{halo}} \). However, our fiducial results have some limitations: the abundance of low-mass galaxies that have only blue GCs (the left panel of Figure 4), and the offsets of the mean metallicity of blue and red GCs between our model and the observation (the right panel of Figures 4 and 10). Despite the variations in the free parameters that we make, there is generally a much higher number of low-mass galaxies that only have only blue GCs than the observation (Figure 7).

To improve the match, we change \( \alpha_m = 0.35 \) to \( \alpha_m = 0.1 \) and \( \alpha_c = 0.9 \) to \( \alpha_c = 2.0 \) in the metallicity model (Equation (5)), while keeping the free parameters as the fiducial values (see Table 1). The results are shown in Figure 12. It is interesting that despite changing metallicity parameters to match the height of our model with the observation (see the bottom left panel), \( f_{\text{blue}} \) is not altered significantly (see the top left panel). We still can see a similar trend of blue GCs between the changed model and the observation. In the upper right panel, the mean metallicity of red GCs in the observation is 0.3 dex higher than our model. In the case of blue GCs, the trend of the changed model is the same with the observation up to \( M_{\text{GC}} \sim 10^6 \, M_\odot \). In the case of massive blue GCs (\( M_{\text{GC}} > 10^6 \, M_\odot \)), there is an offset between the changed model and the observation but we can still see a blue tilt. In the lower right panel, we do not see a clear age gradient in the blue GCs and there is no overlap between our model and the observation.

Although we change \( \alpha_m \) and \( \alpha_c \) substantially in the metallicity model to reproduce the observation shown in the lower left panel in Figure 12, our model still has a limitation in reproducing the GC metallicity in detail: the abundance of low-mass galaxies that have only blue GCs, the height of the mean metallicity of red GCs, and the age trend of blue and red GCs with a cluster-centric radius. We expect that this issue could be improved by changing GC formation scenarios.

In addition, we adopt the metallicity model in which the metallicity is a function of \( M_{\text{stellar}} \) and \( z \), which means the metallicity bimodality represents the age bimodality. However, some galaxies that have metallicity bimodality do not reflect an
age difference (Beasley et al. 2000; Hempel et al. 2007). Instead, the age–metallicity distribution of GCs at $z = 0$ is a useful tool to infer galaxy assembly (Forbes et al. 2010). We revisit assembly histories of the Virgo cluster using the age–metallicity space in our next paper.

Our model naturally produces the GC color bimodality due to the hierarchical merging scenario and the metallicity model (Equation (5)) so our model is not proper for investigating some galaxies that do not have the GC color bimodality (e.g., M31 and many ellipticals; Larsen et al. 2001). However, our fiducial results can reproduce the overall observed GC properties in the galaxy cluster, not only the $M_{\text{GC}}-M_{\text{halo}}$ relation, the GC occupancy, and the decreasing $f_{\text{blue}}$ with $M_{\text{halo}}$, but also a discrete mean metallicity between blue and red GCs. Therefore, we suggest that our model is still useful to investigate the GC properties like the GC metallicity not only the halo–galaxy relation but also GC occupancy, the halo–halo relation of our results with the $M_{\text{GC}}-M_{\text{halo}}$ relation and the NGVS (Lee et al. 2010; Schuberth et al. 2010; Strader et al. 2011; Durrell et al. 2014; Ko et al. 2020; Chaturvedi et al. 2022).

In Section 3, we compared our results with three representative observations. The GC occupancy is an important observation because sometimes the GC occupancy is wrong even if the $M_{\text{GC}}-M_{\text{halo}}$ relation looks fine. However, the sensitivity of $f_{\text{blue}}$ does not seem to be higher than the other observational results because it is not changed significantly by the variation of free parameters. As we mentioned, if we solve the limitations of the GC metallicity using other GC formation scenarios, $f_{\text{blue}}$ might be an important observation, which can constrain our model to match the observation. In addition, if we use the velocity information, it might be an important observation to constrain the best parameter set, which provides the formation environment of GCs exactly.

Due to the tremendous recent developments in GC observations, the positions, velocities, and other properties of (IC)GCs are now estimated in multiple other galaxy clusters besides the Virgo cluster, e.g., Coma, Abell, Perseus, Fornax clusters in cluster surveys: the HST/ACS Virgo cluster survey (Côté et al. 2004), the NGVS (Ferrarese et al. 2012), the HST/ACS Coma cluster survey (Carter et al. 2008), the HST/ASC Fornax cluster survey (Jordán et al. 2007b), and the next generation Fornax survey (Eigenthaler et al. 2018). Using our PTM with the semianalytical approach, we are in an excellent position to compare our models with various state-of-the-art observations at low redshifts. As a result, we can study variations in environmental properties or conditions that can help reproduce the current properties of GCs and their host galaxies.

6. Summary

We investigate the properties of GCs and host galaxies in galaxy clusters, using cosmological zoom-in simulations for the Virgo cluster. We use a PTM with the semianalytical approach, assuming the hierarchical merging scenario: GCs form from galaxy mergers and their metallicity is assigned based on the stellar mass of host galaxies and formation redshift of GCs. We apply the internal and external mechanisms to the evolution of each GC: stellar evolution, two-body relaxation, tides, and dynamical friction. Using the semianalytical approach, the formation and evolution of GCs are controlled by free parameters. The main goal of the paper is not to reproduce the observations quite well but to test the sensitivity to physical processes for GC formation. Our results are summarized below.

1. Our fiducial parameter set can reproduce not only the $M_{\text{GC}}-M_{\text{halo}}$ relation but also GC occupancy, the decreasing $f_{\text{blue}}$ with $M_{\text{halo}}$, and the blue tilt. However, our fiducial parameter set has a limitation in reproducing the observed GC metallicity in detail: the abundance of low-mass galaxies that have only blue GCs (the left panel

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**Figure 11.** Left: the $M_{\text{GC}}-M_{\text{halo}}$ relation of our results with the fiducial parameter set. Green and magenta colors represent the inner and outer galaxies, respectively. The thick lines represent linear fitted lines. Right: the $S_{\text{0}}$ of GCs with $M_{\text{stellar}}$. The thick solid and dashed lines denote our results with the fiducial parameter set and the observation of the Virgo cluster, respectively. Solid dots denote our fiducial results and open circles denote the observation.
of Figure 4), the mean metallicity (the right panel of Figure 4 and the left panel of Figure 10), and the age trend of the blue and red GCs (the right panel of Figure 10).

2. Among free parameters, $z_{c}$, $\gamma_{MR}$, initial $\log \eta$, and $R$ are important parameters because they affect the $M_{GC}-M_{halo}$ relation, GC occupancy, and $f_{blue}$, simultaneously. These parameters affect the formation and evolution of GCs so we will investigate the environment of GC formation and evolution using the MCMC method.

3. The position information, traced by the PTM, makes it possible for us to compare with additional observations: the GC system size in each galaxy ($\log r_{0.5}$), the projected density profiles of blue and red ICGCs, the metallicity and age gradient with the clustercentric radius, and the radial dependence of the specific frequency ($S_{M}$).

However, there are offsets of the mean metallicity and age of red GCs between our fiducial model and the observation of the Virgo cluster (Figure 10).

In recent times, large samples of (IC)GCs in various galaxy clusters have been observed; therefore, the demand for modeling that attempts to simultaneously reproduce multiple aspects of their properties has never been higher. In the future, we plan to use our model to investigate the GCs in various galaxy clusters, to understand how their properties depend on their formation environment such as cluster mass, cluster dynamical state, and cluster merger history.

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Appendix A
Schechter Initial GCMF

Our model adopts the power-law initial GCMF. To see the effect of the initial GCMF, we test our model with the Schechter initial GCMF (Schechter 1976; Gieles 2009; Adamo et al. 2020). Figure 13 shows the $M_{GC} - M_{halo}$ relation, GC occupancy, and $f_{\text{blue}}$. We find that there is no large difference between the power-law and Schechter initial GCMFs.

Appendix B
Effect of Changing the Minimum Mass of GCs

Our model adopts the minimum mass of GCs ($M_{\text{min}}$) as $10^5 M_\odot$ because we assume that the low-mass GCs are quickly destroyed by two-body relaxation. To investigate the effect of $M_{\text{min}}$, we change $10^5 M_\odot$ to $10^4 M_\odot$. Figure 14 shows the $M_{GC} - M_{halo}$ relation, GC occupancy, and $f_{\text{blue}}$. We find that the results are not significantly different with $M_{\text{min}} = 10^5 M_\odot$. In addition, we use GCs whose mass is higher than $10^4 M_\odot$ at $z = 0$ to analyze our results because the observed mass range of GCs is $10^4 - 10^6 M_\odot$. Thus, we suggest that $M_{\text{min}}$ does not affect the results significantly.

ORCID iDs
So-Myoung Park https://orcid.org/0000-0003-1889-325X
Jihye Shin https://orcid.org/0000-0001-5135-1693
Rory Smith https://orcid.org/0000-0001-5303-6830
Kyungwon Chun https://orcid.org/0000-0001-9544-7021

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