Vibration Suppression of an Axially Moving Web in a Multi-Span Roll-to-Roll Microcontact Printing System

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

| As Published          | https://doi.org/10.1007/s42417-018-0047-y |
|-----------------------|--------------------------------------------|
| Publisher             | Springer Singapore                         |
| Version               | Author’s final manuscript                  |
| Citable link          | https://hdl.handle.net/1721.1/131436        |
| Terms of Use          | Creative Commons Attribution-Noncommercial-Share Alike |
| Detailed Terms        | http://creativecommons.org/licenses/by-nc-sa/4.0/ |
Vibration Suppression of an Axially Moving Web in a Multi-Span Roll-to-Roll Microcontact Printing System

Sajid Ali* and Muhammad A. Hawwa
Department of Mechanical Engineering
King Fahd University of Petroleum and Minerals
Dhahran 31261, Saudi Arabia

David E. Hardt
Mechanical Engineering Department
Massachusetts Institute of Technology
Cambridge, MA 02319, USA

(*) Corresponding Author: drsajidak@gmail.com; sajidali@kfupm.edu.sa

Abstract

The focus of this work is to investigate the vibration suppression of an axially moving web traveling between multiple rolls. Web axial tension and axial speed, decisive parameters in the equation of motion, that describes system dynamics, are rigorously obtained by considering the rolls-web coupled system’s dynamics. The proposed control method is based on imposing a suitable boundary condition and applying control torques at rolls, such that the vibration energy at the end of web decays. The non-linear dynamic equation is realized by applying the Hamilton’s principle. Using a finite difference and state space approach, the partial differential equation of motion is converted into a system of coupled first-order ordinary differential equations (ODE’s) in time by eliminating the spatial variable. Effects of vibration suppression are demonstrated by numerical methods. Results from both the experiments and numerical simulations show that the proposed method can effectively suppress the vibration of the axially moving web, thereby protecting the web from excessive oscillations.

Keywords: Axially moving web; Coupled nonlinear vibration; Finite difference method; Vibration control; Microcontact printing (µCP).
1. Introduction

Because of their importance in industrial processing of thin materials (textiles, papers, polymers, metals, and composites), axially moving string models are located at the heart of web dynamics studies. Researchers started to investigate the problem of vibration of axially moving strings since the 1950s [1-3]. Since then, the subject has been under increasing investigation [4]. In light of the fact that a significant number of engineering applications involving axially moving strings occur in roll-to-roll systems, formulating the problem of an axially moving string in isolation from the roll dynamics limited the model utility. Web tension regulation in roll-to-roll (R2R) system was considered for the first time in [5] and subsequently in [6] where models of web tension were presented based on continuum mechanics principles. Dynamic models for the unwinding / rewinding rolls and free web span from first principles using Newton’s laws and conservation of mass principle were derived in [7] and [8]. The roll-to-roll dynamic model was based on the laws of physics [9]: (i) Hooke’s law web elasticity, (ii) Coulomb’s law accounting for the contact between the web and the roll, including friction, (iii) The conservation of mass law describing the coupling between the axial speed of and the tension in the web, and (iv) The roll rotational dynamics. Many researchers over the years applied the approach outlined in line [9] to develop the web tension-velocity model in a roll-to-roll system, [10-18].

A limiting factor in roll-to-roll micro contact printing that influences the quality of web processing is mechanical vibration that can naturally arise as a system’s dynamics unavoidable phenomenon. Mechanical vibrations in R2R systems can be root caused at the rolls if they have an out of round shape, are out of balance, are misaligned, accelerate, decelerate or stop. Web vibrations can also be caused by an intermittent processing style that typically characterizes material deposition, ink printing, and physical and chemical curing. It is thus desirable to suppress vibration of translating web to improve performance. Many researchers have focused on finding a solution for the problem of web vibration by means of isolation. To achieve a good understanding of what have been done so far, published studies can be classified into two categories:

(I) Studies focused on the passive vibration suppression by optimizing damping / stiffness in order to minimize the vibration amplitude [e.g., 19-24]. For an axially moving web (modeled as a string), a vibration suppression technique by utilizing a hydraulic roll was presented in [19]. It was shown in [20] that the web displacement is bounded when a bounded distributed force is applied to it transverse direction. Passive nonlinear targeted energy transfer technique was demonstrated by Viguie et al. [21] to passively suppress the vibration. Foda [22] achieved the vibration dissipation by placing some passive elements and actuators at selected positions along the web span. Nonlinear-targeted energy transfer (TET) was applied to suppress the excessive vibration of an axially moving web with transverse wind loads [23]. A nonlinear energy sink (NES) was attached to the web to absorb vibrational energy. Boundary control approach was utilized to suppress the transverse vibration that occurred during axial motion [24].

(II) Papers concerned with the active vibration techniques for precise control of time varying conditions at high moving speeds [e.g., 25-37]. Sensors and actuators were applied along the length of an axially travelling string with the objective to avoid excessive transverse vibrations by Yang and Mote [25]. Lin and Hu [26] presented a vibration...
suppression technique based on the wave absorption or impedance matching, which was achieved by actuators and sensors placed on the axially moving web span. In an attempt to minimize the amplitude of transverse vibration in an axially moving web, actuators were installed at the web upstream and web downstream as controlling force generators by Tau and Ying [27]. Li et al. [28] introduced an active pivoting roll that adaptively decouples adjacent spans, thereby isolating a controlled span from bounded disturbances in its adjacent uncontrolled span. The proposed method in [28] was further verified experimentally. Web speed tracking and transverse vibration control was achieved by installing force actuators in the web span through a mechanical guide and regulating the applied torques at the web boundaries by Nagrakatti et al. [29]. Yang et al. [30] proposed a control scheme for an axially moving web under varying applied axial tension using a hydraulic actuator to divide the web into two domains, a controlled one and an uncontrolled one. Kim et al. [31] presented a transverse vibration control of an axially moving pre-tensioned string by introducing an electro-hydraulic actuator. Chen and Zhang [32] used a tensioner as an actuator to design an adaptive controller for the vibration of an axially moving web. Lin et al. [33] developed an adaptive boundary control for vibration suppression of an axially moving accelerated/decelerated belt system. By utilizing Lyapunov-based back stepping method, a boundary control was proposed for vibration suppression of the belt system. Zhao et al. [34] developed a vibration suppression strategy to asymptotically stabilize the web. A disturbance observer was proposed to attenuate the effects of unknown boundary disturbances of an axially moving belt system by Lin et al. [35].

The literature review reveals that there were many active as well as passive techniques available to suppress the non-linear vibration in axially moving web. It has been shown that boundary control is an efficient transverse vibration control method in axially moving webs. However, for a process such as integrated roll-to-roll microcontact printing, actuators at the boundaries can generate large enough forces to influence the quality of the product. Similarly, implementing the distributed control technique required the installation of actuators and sensors along the length of the axially moving web, which makes this method not feasible in a process already having so many constraints. Therefore, developing a proper control method to suppress the transverse vibrations of the axially moving webs in a roll-to-roll microcontact printing process represents a formidable challenge. In this paper, our goal is to come up with a cost effective vibration control methodology of axially moving webs. When a web travels at high speeds, active vibration control can be costly due to the employment of sophisticated hardware for sensing and actuation. Understanding the coupling between R2R system’s dynamics and the transverse web vibration is utilized in this work to extract boundary condition at one roll that can lead to mitigating vibration. Therefore, it is intended to rigorously consider the coupling between the roll-to-roll system’s dynamics and the transverse vibration of the web between the rolls to propose a control method by introducing a suitable boundary condition such that the vibration energy at the end of the web decays. In addition, numerical simulations and experiments are performed to demonstrate the effectiveness of the proposed control methodology.

2. Dynamic Modelling (R2R μCP)

While the basic physics of printing with molecular and liquid inks has been demonstrated at the micron and submicron level, the printing scale up of large areas and high production rates has not yet been demonstrated. A
unique lab-scale microcontact-printing machine with high-speed web handling capabilities has been developed under the collaboration held between KFUPM and MIT [2008-2016]. The printing machine (shown in Figure 1) combines robust controls for the basic machine movement along with the ability to use simple external interfaces (via Ethernet) for related experimental hardware. In particular, the design facilitates well-controlled programmable web tension while allowing for external devices such as precision print heads and optical contact sensors. Although the web handling system uses off-the shelf components, the printing zone employs a large radius impression roll that comprises a polished glass cylinder rotating on segmented air bearing. This allows for very low web drag while permitting direct optical access to the printing zone.

![Image](image.png)

**Fig. 1.** (a) The MIT Lab Scale R2R Micro Contact Printing Machine, (b) System’s Critical Component Layout.

### 2.1 Roll-to-Roll Dynamic Model

The roll-to-roll microcontact-printing machine is schematically shown in Fig. 2.
Following the mathematical modeling of R2R systems presented by Brandenburg [44] and Koç et al. [10], by assuming small width of the web in comparison to its length and considering small bending stiffness with no slip condition at the boundaries (rolls), the web tension-velocity relations for each web span are given by

\[ \frac{dT_u}{dt} \approx \left( \frac{E A R_u}{L_1} \right) \Omega_u + \left( \frac{E A R_{id1}}{L_1} + \frac{T_2}{L_1} R_{id1} \right) \Omega_{id1} \]  

\[ \frac{dT_r}{dt} \approx \left( \frac{E A R_b}{L_2} - \frac{T_3}{L_2} R_b \right) \Omega_b + \left( \frac{T_2}{L_2} R_{id1} - \frac{E A R_{id1}}{L_2} \right) \Omega_{id1} \]  

\[ \frac{dT_{id1}}{dt} \approx \left( \frac{T_3 R_b}{L_3} - \frac{E A R_b}{L_3} \right) \Omega_b - \left( \frac{T_3}{L_3} R_{id2} - \frac{E A R_{id2}}{L_3} \right) \Omega_{id2} \]  

\[ \frac{dT_{id2}}{dt} \approx \left( \frac{T_5 R_r}{L_4} - \frac{E A R_r}{L_4} \right) \Omega_r + \left( \frac{T_4 R_{id2}}{L_4} - \frac{E A R_{id2}}{L_4} \right) \Omega_{id2} \]

where \( u, r, id, \) and \( b \) stand for unwinding roll, rewinding roll, idler roll, and backing roll, respectively. \( T_i \) is the tension transmitted in the \( i \)th web span. \( E \) is the web modulus of elasticity. \( A \) is the web cross section, \( L_i \) is the web span length. \( R_j \) is the radius of designated roll \( j \). \( \Omega_j \) is the angular speed of the designates roll.

Let us then focus on the forces acting on each roll: (i) the inertial torque, (ii) the torque caused by the web tension, (iii) the motor torque, and (iv) the friction torque. Assuming slowly varying roll inertia and radii, the dynamic equilibrium equation of each roll can then be written as

\[ \frac{d\Omega_u}{dt} = \frac{1}{\frac{1}{2} \pi W [R_{ur}^4 (\rho_r - \rho_w) + \rho_w R_u^4]} \left( R_u T_2 + B_u \Omega_u - C_u \right) 
+ \Omega_u \left( \rho_w * W * h \left( (R_u(t))^3 * \Omega_u \right) \right) \]
The system of equations (1-9) consists of nonlinear coupled equations that describe the R2R system’s dynamics. In light of the fact that the unwinding roll radius decreases and the rewinding roll radius increases by the same amount at the same time, equations (5) and (9) are supplemented with the following equations that describe radii variation:

\[
\frac{d\Omega_{id1}}{dt} = \left[ \frac{T_3}{J_{id1}} - \frac{T_2}{J_{id1}} \right] R_{id1} - \frac{1}{J_{id1}} C_{id1}
\]

\[
\frac{d\Omega_{b}}{dt} = \left[ \frac{T_4}{J_b} - \frac{T_1}{J_b} \right] R_b - \frac{1}{J_b} C_b
\]

\[
\frac{d\Omega_{id2}}{dt} = \left[ \frac{T_5}{J_{id2}} - \frac{T_4}{J_{id2}} \right] R_{id2} - \frac{1}{J_{id2}} C_{id2}
\]

\[
\frac{d\Omega_r}{dt} = \frac{1}{2 \pi W} \left[ R_{rr}^4 (\rho_r - \rho_w) + \rho_w R_r^4 \right] 
+ \Omega_r \left( \rho_w W h \left( (R_r(t))^3 \right) \right)
\]

The system of equations (1-9) consists of nonlinear coupled equations that describe the R2R system’s dynamics. In light of the fact that the unwinding roll radius decreases and the rewinding roll radius increases by the same amount at the same time, equations (5) and (9) are supplemented with the following equations that describe radii variation:

\[
R_u(t) = R_{0u} - \frac{\theta_u}{2\pi} h
\]

\[
R_r(t) = R_{0r} + \frac{\theta_r}{2\pi} h
\]

where \(R_{0u}\) and \(R_{0r}\) are the initial radii of unwinding and rewinding rolls, respectively, \(h\) is the web thickness, and \(\theta_u\) and \(\theta_r\) are the angular displacements of unwinding and rewinding rolls, respectively. Note that in every turn, the radius of unwinding roll decreases by an amount that is equal to the thickness of the web and in the same time the radius of the rewinding roll increases by the same amount. Due to changing the radii of rolls with time, moments of inertia also change with time as

\[
J_u(t) = \frac{1}{2} \pi W \left[ R_{rr}^4 (\rho_r - \rho_w) + \rho_w [R_u(t)]^4 \right]
\]

\[
J_r(t) = \frac{1}{2} \pi W \left[ R_{rr}^4 (\rho_r - \rho_w) + \rho_w [R_r(t)]^4 \right]
\]

where \(\rho_{rr}\) is the density of roll material, \(R_{rr}\) is the radius and \(W\) is the width of each roll.

### 2.2 Web Vibration Model

Let us consider an axially moving string in a roll-to-roll system with axial tension \(T\), axial speed \(v\) and transverse deflection \(w(x,t)\). Mechanical energy of the web is given by

\[
E(t) = T + U
\]

where \(T\) is the kinetic energy and \(U\) is the potential energy for the axially moving web are given by
\[ \tau = \int_0^1 \rho A \left[ \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial x} \right] dx \quad (15) \]

\[ U = \frac{1}{2} \int_0^1 T \left( \frac{\partial w}{\partial x} \right)^2 dx \quad (16) \]

where \( \rho \) is the density and \( A \) is the area of the web. Using equations (15) and (16), the total mechanical energy \( E(t) \) of the system is given by

\[ E(t) = \frac{1}{2} \int_0^1 \left[ \left( \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial x} \right)^2 \right] dx + \frac{1}{2} \int_0^1 T \left( \frac{\partial w}{\partial x} \right)^2 dx \quad (17) \]

Normalizing equation (17) by utilizing \( \sqrt{\rho A t} \) as a temporal parameter, and \( \sqrt{\frac{T A}{\rho}} \) as a reference velocity transforms the equation of total mechanical energy into the following dimensionless one:

\[ E(t_*) = \frac{1}{2} \int_0^1 \left[ \left( \frac{\partial w_*}{\partial t_*} + v_* \frac{\partial w_*}{\partial x_*} \right)^2 \right] dx_* + \frac{1}{2} \int_0^1 \left( \frac{\partial w_*}{\partial x_*} \right)^2 dx_* \quad (18) \]

Note that variables with stars are dimensionless, and \( v_* \) is a dimensionless transport speed that represents the ratio of the physical velocity to the wave velocity in the web material. Applying a material derivative on the mechanical energy and utilizing the equation of motion

\[ \dot{E}(t_*) = \int_0^1 \left[ \left( \frac{\partial w_*}{\partial t_*} + v_* \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right] dx_* + \int_0^1 \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial t_* \partial x_*} \right) dx_* + \int_0^1 \left( \frac{\partial w_*}{\partial t_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) dx_* \quad (19) \]

which can be written as

\[ \dot{E}(t_*) = 2 \cdot v_* \int_0^1 \left[ \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right] dx_* + \int_0^1 \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial t_* \partial x_*} \right) dx_* + \int_0^1 \left( \frac{\partial w_*}{\partial t_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) dx_* \quad (20) \]

Simplifying each term in equation (20) as follows:

1st Term

\[ \int_0^1 \left[ \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right] dx_* = \left. \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right|_0^1 - \int_0^1 \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) dx_* \quad (21a) \]

\[ \int_0^1 \left[ \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right] dx_* = \left. \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right|_0^1 - \int_0^1 \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) dx_* \quad (21b) \]

\[ \int_0^1 \left[ \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right] dx_* + \int_0^1 \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) dx_* = \left. \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right|_0^1 \quad (21c) \]

\[ 2 \cdot \int_0^1 \left[ \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right] dx_* = \left. \left( \frac{\partial w_*}{\partial x_*} \right) \left( \frac{\partial^2 w_*}{\partial x_*^2} \right) \right|_0^1 \quad (21d) \]
which lead to

\[ \int_0^1 \left[ \frac{\partial w_x}{\partial x} \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right] dx_s = \frac{1}{2} \left( \left. \frac{\partial w_x}{\partial x} \right|_0^{\frac{1}{2}} \right) \]  \hspace{1cm} (21)

2nd Term

\[ \int_0^1 \left( \frac{\partial w_x}{\partial x} \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right) dx_s = \int_0^1 \left[ \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right] dx_s + \int_0^1 \left[ \left( \frac{\partial w_x}{\partial t_x} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right] dx_s \]  \hspace{1cm} (22a)

\[ \int_0^1 \left( \frac{\partial w_x}{\partial x} \left( \frac{\partial^2 w_x}{\partial t \cdot \partial x} \right) + \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right) dx_s

= \left[ \left( \frac{\partial w_x}{\partial x} \right) \left( \frac{\partial w_x}{\partial t} \right) \right]_0^1 - \int_0^1 \left[ \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right] dx_s + \int_0^1 \left[ \left( \frac{\partial^2 w_x}{\partial t \cdot \partial x} \right) \left( \frac{\partial w_x}{\partial x} \right) \right] dx_s \]  \hspace{1cm} (22b)

\[ \int_0^1 \left( \frac{\partial w_x}{\partial x} \left( \frac{\partial^2 w_x}{\partial t \cdot \partial x} \right) + \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right) dx_s

= \left[ \left( \frac{\partial w_x}{\partial x} \right) \left( \frac{\partial w_x}{\partial t} \right) \right]_0^1 - \int_0^1 \left[ \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right] dx_s + \int_0^1 \left[ \left( \frac{\partial^2 w_x}{\partial t \cdot \partial x} \right) \left( \frac{\partial w_x}{\partial x} \right) \right] dx_s \]  \hspace{1cm} (22c)

\[ \int_0^1 \left( \frac{\partial w_x}{\partial x} \left( \frac{\partial^2 w_x}{\partial t \cdot \partial x} \right) + \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right) dx_s

= \left[ \left( \frac{\partial w_x}{\partial x} \right) \left( \frac{\partial w_x}{\partial t} \right) \right]_0^1 - \int_0^1 \left[ \left( \frac{\partial w_x}{\partial t} \right) \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right] dx_s + \int_0^1 \left[ \left( \frac{\partial^2 w_x}{\partial t \cdot \partial x} \right) \left( \frac{\partial w_x}{\partial x} \right) \right] dx_s \]  \hspace{1cm} (22d)

which lead to

\[ \int_0^1 \left( \frac{\partial w_x}{\partial x} \left( \frac{\partial^2 w_x}{\partial x^2} \right) \right) dx_s = \left( \left. \frac{\partial w_x}{\partial x} \right|_0 \left( \frac{\partial w_x}{\partial t} \right) \right) \]  \hspace{1cm} (22)

Substituting equations (21) and (22) into equation (20) and differentiating with respect to time give

\[ \dot{E}(t_s) = v_s \left( \left. \frac{\partial w_x}{\partial x} \right|_0 \right) + \left( \left. \frac{\partial w_x}{\partial t} \right|_0 \right) \]  \hspace{1cm} (23)

Applying the limits of integration,
\[ \dot{E}(t_c) = v_c \left( \left( \frac{\partial w_c(1,t)}{\partial x_c} \right)^2 - \left( \frac{\partial w_c(0,t)}{\partial x_c} \right)^2 \right) + \left( \frac{\partial w_c(1,t)}{\partial x_c} \right) \frac{\partial w_c(1,t)}{\partial t_c} \] 

\[ - \left( \frac{\partial w_c(0,t)}{\partial x_c} \right) \frac{\partial w_c(0,t)}{\partial t_c} \]  

(24)

or,

\[ \dot{E}(t_c) = -v_c \left( \frac{\partial w_c(0,t)}{\partial x_c} \right)^2 + \left( \frac{\partial w_c(1,t)}{\partial x_c} \right) \left[ \frac{\partial w_c(1,t)}{\partial t_c} + v_c \left( \frac{\partial w_c(1,t)}{\partial x_c} \right) \right] \]  

(25)

In order to make sure that the rate of change of mechanical energy is negative, the following condition is proposed

\[ \left( \frac{\partial w_c(1,t)}{\partial t_c} \right) = -v_c \left( \frac{\partial w_c(1,t)}{\partial x_c} \right) \]  

(26)

Regulating the axial speed of the web to a desired set point leads to defining the error in the speed of the set point as

\[ e = v - v_d \]  

(27)

where, \( v \) is the actual web speed and \( v_d \) is the desired web speed. Using the roll-to-roll dynamics, input torques into the unwinding and rewinding rolls are calculated from

\[ \tau_u(t) = \frac{d}{dt} (J_u \ast \Omega_u) - R_u \ast T + C_u \]  

(28)

\[ \tau_r(t) = \frac{d}{dt} (J_r \ast \Omega_r) + R_r \ast T + C_r \]  

(29)

Using \( v_u = R_u \Omega_u \), \( v_r = R_r \Omega_r \), and substituting them in equations (28) and (29) result in

\[ \tau_u(t) = \frac{d}{dt} \left( J_u \ast \frac{v_u}{R_u} \right) - R_u \ast T + C_u \]  

(30)

\[ \tau_r(t) = \frac{d}{dt} \left( J_r \ast \frac{v_r}{R_r} \right) + R_r \ast T + C_r \]  

(31)

Substituting \( v_u = v_r = v \) in equations (30) and (31) become

\[ \tau_u(t) = \frac{d}{dt} \left( J_u \ast \frac{v}{R_u} \right) - R_u \ast T + C_u \]  

(32)

\[ \tau_r(t) = \frac{d}{dt} \left( J_r \ast \frac{v}{R_r} \right) + R_r \ast T + C_r \]  

(33)

Substituting equation (27) in equations (32) and (33), the input control torques at the unwinding and rewinding rolls become

\[ \tau_u(t) = \frac{d}{dt} \left( J_u \ast \frac{v_d}{R_u} \right) + \frac{d}{dt} \left( J_u \ast \frac{e}{R_u} \right) - R_u \ast T + C_u \]  

(34)

\[ \tau_r(t) = \frac{d}{dt} \left( J_r \ast \frac{v_d}{R_r} \right) + \frac{d}{dt} \left( J_r \ast \frac{e}{R_r} \right) + R_r \ast T + C_r \]  

(35)
To make sure that the error is exponentially decaying, using \( \frac{d}{dt} \left( J \ast \frac{e}{R_u} \right) = -k_v \ast e \) in equations (34) and (35), the input control torques become

\[
\tau_u(t) = \frac{d}{dt} \left( J_u \ast \frac{v_u}{R_u} \right) - k_v \ast e - R_u \ast T + C_u
\]  
\[\text{(36)}\]

\[
\tau_r(t) = \frac{d}{dt} \left( J_r \ast \frac{v_r}{R_r} \right) - k_v \ast e + R_r \ast T + C_r
\]  
\[\text{(37)}\]

where \( k_v \) is a positive control gain. From the assumption used in the equation (36) and (37), the speed tracking error will converge to zero exponentially given as

\[
e(t) = \exp \left( \frac{-k_v \ast R}{J} \right)
\]  
\[\text{(38)}\]

Thus, providing stable controlled input torques to the winding and unwinding rolls.

3. Finite Difference Scheme

A numerical approach based on a finite difference scheme is used for the discretization of spatial coordinates. Applying the backward finite difference scheme to discretize equation (26) and applying the central difference scheme to discretize the equations of motion result in the following \( n \) second-order ODEs, where \( n \) is the total number of spatial points and \( dx \) is the step size.

\[
\begin{align*}
\frac{d^2 w_{1}}{dt^2} + \frac{v_s}{dx} \frac{dw_{1}}{dt} + \frac{(v_s^2 - 1)}{dx^2} (w_{1} - w_{2}) + \frac{v_s}{2 \ast dx} \ast w_{2} &= 0 \\
\frac{d^2 w_{2}}{dt^2} + \frac{v_s}{dx} \left( -\frac{dw_{1}}{dt} + \frac{dw_{3}}{dt} \right) + \frac{(v_s^2 - 1)}{dx^2} (w_{1} - 2 \ast w_{2} + w_{3}) + \frac{v_s}{2 \ast dx} \ast (w_{2} - w_{3}) &= 0 \\
&\vdots \\
\frac{d^2 w_{n-1}}{dt^2} + \frac{v_s}{dx} \frac{dw_{n-1}}{dt} + \frac{(v_s^2 - 1)}{dx^2} (w_{n-2} - 2 \ast w_{n-1}) + \frac{v_s}{2 \ast dx} \ast w_{n-2} &= 0 \\
\frac{d^2 w_{n}}{dt^2} &= -\frac{v_s}{2 \ast dx} \left( \frac{dw_{n}}{dt} - \frac{dw_{n-1}}{dt} \right)
\end{align*}
\]  
\[\text{(39)}\]

To solve the \( n \) second-order ODEs, state space representation is used to convert them into \( 2n \) first-order ODEs, which are given as
Numerical integrations for the two cases of uncoupled (R2R dynamics independent of transverse vibration) and coupled (R2R dynamics dependent of transverse vibration) are carried out to see the effectiveness of the proposed methodology on the vibration suppression in each case. Results are presented in Figs. 3-5 showing the transverse displacement responses monitored at the three representative locations at the web span ($x_c = 0.25, 0.50$ and $0.75$).

Figures 3-5 demonstrates the effectiveness of the proposed boundary control condition in reducing the transverse vibration at different locations of the web for the representative dimensionless speed of 0.5. Solid and dashed lines represent the transverse displacement of the web without boundary control and with boundary control conditions, respectively. With the application of boundary control, a significant reduction in the amplitude of web transverse vibration is achieved. Therefore, it can be stated that the proposed condition effectively reduces the vibrational energy and prevents the web from an excessive web vibration.
Fig. 3. Transverse displacement (with & without control) at $x_0 = 0.25$ & $v_0 = 0.5$.

Fig. 4. Transverse displacement (with & without control) at $x_0 = 0.50$ & $v_0 = 0.5$.

Fig. 5. Transverse displacement (with & without control) at $x_0 = 0.75$ & $v_0 = 0.5$. 

12
According to the control condition, energy reduction at the right boundary is directly proportional to axial speed. The higher the speed, the faster energy decay in the string. To see the effect of the axial speed of the web on the vibration reduction, the controlled transverse responses at various dimensionless speeds are plotted at various locations of the web as shown in Figs. 6–8. Comparison is made at three locations $(x_0 = 0.25, 0.50$ and $0.75)$ and at the dimensionless axial speeds of $v_0 = 0.1, 0.3, 0.4$ and $0.6$.

Fig. 6. Transverse displacement at $x_0 = 0.25$ for webs having different speeds.

Fig. 7. Transverse displacement at $x_0 = 0.50$ for webs having different speeds.

Fig. 8. Transverse displacement at $x_0 = 0.75$ for webs having different speeds.
Comparison shows a significant decrease in the amplitude of vibration at all web speeds. Amplitude reduction is, however, more prominent as the dimensionless speed is increased. Consequently, it can be concluded that at higher dimensionless speeds, with the use of controlled boundary condition, a lower amplitude of the transverse response can be achieved at the same location at the string.

Let us now examine the effect of the proposed combined control consisting of (i) regulating torques at the rolls, while (ii) imposing boundary control at the right roll. Equations (36), (37) and (40) are solved together with the system of equations (1) - (9), as a coupled system that describes the transverse vibration of the controlled axially moving web between the unwinding and the rewinding rolls.

The effect of the proposed combined control methodology on web axial speed is presented in Fig. 9. It is noted that the axial web speed remains almost constant for the most of the time of transferring the web material from one roll to another.

![Fig. 9. Axial speed comparison.](image)

Figure 9 shows a comparison of the web axial speed and tension for different values of the control gain ($k_v$).

![Fig. 10. (a) Axial speed and (b) axial tension comparison.](image)

Figure 10 shows a comparison of the web axial speed and tension for different values of the control gain ($k_v$).

Note that increasing the control gain results in decreasing the variation in axial speed, as shown in Fig. 10(a). While increasing the control gain causes a significant increase in the magnitude of the axial tension in the transient phase,
as shown in Fig. 10(b). Therefore, a better choice to avoid high levels of the transient amplitudes while keeping the speed variation small enough is to choose a positive gain between 0.3 and 0.7.

A comparison of the R2R dynamic-coupled web vibration is made for three cases (i) R2R system without boundary control keeping constant torques at the rolls, (ii) R2R system with boundary control without applying control torques at the rolls, and (iii) R2R system with boundary control and control torques. Three web locations ($x_i = 0.25, 0.50$ and 0.75) have been selected for comparative study as shown in Figs. 11-13.

**Fig. 11.** Transverse displacement at $x_i = 0.25$ for webs having variable speed.

**Fig. 12.** Transverse displacement at $x_i = 0.50$ for webs having variable speed.

**Fig. 13.** Transverse displacement at $x_i = 0.75$ for webs having variable speed.
Comparison shows that boundary control (represented by the red dashed curves) causes a significant decrease in the vibration amplitude. A further reduction in the vibration amplitude and settling time is achieved when the R2R system has both a boundary control and applied control torques at the rolls (represented by the black dashed curves).

5. Experimental Setup & Results

To experimentally verify the effectiveness of the proposed control methodology, tests are performed in the Laboratory of Manufacturing and Productivity at the Massachusetts Institute of Technology (MIT), where the microcontact printing machine is hosted. The roll-to-roll printing machine, utilized as a test bed, is a multiplane machine with two active rolls and two idler rolls. Web is fed from the unwinding roll, passes through the two idler rolls, in between the microcontact printing is performed on the moving web before it is rewound by the rewinding roll. To measure as well as control the web tension, FMS CMGZ 309 Tension Controller is installed on the system alongside with a force measuring roll. A microprocessor based PID-controller design is implemented for precise closed-loop tension control of a running web. Fiber optic micro-displacement measurement systems (μDMS) is used to measure the transverse displacement of the axially moving web. All μDMS sensors have an internal sampling rate of 10 KHz. The average filter controls how many readings the sensor will average together before sending the results to the serial port. Higher averages will slow down the sensor response and increase the resolution. The actual sample rate (readings/second) is displayed below in the live chart. At the slowest speed (4096), the sample rate is approximately 0.4 seconds per data point. At the fastest speed (2), the sample rate is 200 microseconds per point.

To implement the boundary control, the original setup is modified, shown in Fig. 14, where a foam roll is installed at a slightly offset position at the right boundary (roll).

![Fig. 14. Modified Lab Scale R2R Micro Contact Printing Machine.](image)
Experiments are performed to measure the transverse displacement at different locations for the web with and without control systems. The transverse displacement of the web is measured by the fiber optic micro displacement measurement system (µDMS) at the three representative locations ($x = 0.25$, 0.50 and 0.75). With and without control condition responses are compared at the web axial speed “$v = 4$ in/sec” and the web transmitted tension “$T=30$ N” as shown in Figs. 15 – 17.

Fig. 15. Transverse Response Comparison at $x = 0.25$.

Fig. 16. Transverse Response Comparison at $x = 0.50$.

Fig. 17. Transverse Response Comparison at $x = 0.75$. 
Comparison of the transverse vibration response at different locations clearly shows that applying a boundary control condition at the right boundary results in smaller web vibrations. The rate of decay can be clearly increased by properly selecting the slope of the web at the controlled boundary.

The paper shows a simple, though powerful, tactic for transverse vibration reduction of axially moving webs. This proposed boundary control method can be included in a feedback / feedforward control system.

6. Conclusions

To achieve consistent patterning process at the micron and sub-micron scale of a continuous microcontact printing, it is necessary to control the web vibration during printing. In this paper, a proposed control method was introduced by a suitable adjustment of the web slope at a boundary condition in the printing zone, while applying control torques at the unwinding and rewinding rolls of a microcontact printing machine. The motivation of the proposed control methodology was decaying of the vibration energy at the end of web. The non-linear dynamic model was realized by applying the Hamilton’s principle. By applying a finite difference and state space approach, the non-linear partial differential equation were converted into a system of coupled first-order ordinary differential equations (ODE’s) in time by eliminating the spatial variable. The influence of the proposed vibration suppression methodology was demonstrated numerically. It was shown through numerical results that by imposing a boundary condition, the vibrational energy decayed, preventing the web excessive vibrations. The effect of dimensionless speed showed a significant decrease in the amplitude and the transverse displacement reduction was more prominent as the dimensionless speed was increased. Numerical simulations are also backed by experiments, which showed a significant web oscillation reduction.

Acknowledgement

Dr. Scott Nill, Ms. Larissa Nietner, and Mr. Adam Libert have had a significant contribution in the design and development of the microcontact-printing machine. Discussions with Dr. Hassen Ouakad, Dr. Hussain Al-Qahtani. And Dr. Du Xian have been quite helpful. Research support offered by KFUPM under Projects MIT13101/2 is acknowledged. The authors would like to acknowledge the support provided by Center for Clean Water and Clean Energy at MIT and KFUPM.
References:

[1] R.A. Sack, Transverse oscillations in traveling strings, British Journal of Applied Physics. 5 (1954) 224-226.

[2] S. Mahalingam, Transverse vibrations of power transmission chains, British Journal of Applied Physics. 8 (1957) 145-148.

[3] F.R. Archibald and A.G. Emslie, The vibrations of a string having a uniform motion along its length, Journal of Applied Mechanics, ASME. 25 (1958) 347–8.

[4] A.G. Ulsoy, C.D. Mote Jr. and R. Syzmani, Principal developments in band saw vibration and stability research, Holz als Roh und Werkstoff. 36 (1978) 273-280.

[5] K. Grenfell, “Tension control on paper-making and converting machinery”, Proceedings of IEEE Annual Conference on Electrical Engineering in the Pulp and Paper Industry. 9 (1963).

[6] G. Brandenburg, “The dynamics of elastic webs threading a system of rollers”, Newspaper Techniques. (1972) 12-25.

[7] D. Whitworth, “Tension variations in pliable material in production machinery”, PhD. dissertation, Loughborough University of Technology. (1979).

[8] D. Whitworth and M. Harrison, “Tension variations in pliable material in production machinery”, Journal of Applied Mathematical Modeling. 7 (1983) 189–196.

[9] H. Koç, D. Knittel, M. Mathelin and G. Abba, “Modeling and robust control of winding systems for elastic webs”, IEEE Transactions on control systems technology. 10 (2) (2002).

[10] H. Koç, D. Knittel, M. Mathelin and G. Abba, Modeling and control of an industrial accumulator in a web transport system, in Proc. Europ. Contr. Conf. ECC. (1999).

[11] H. Koç, D. Knittel, M. Mathelin and G. Abba, “Web tension control in an industrial accumulator”, in Proc. Int. Conf. Web Handling IWEB5. (1999).

[12] S. Yun, C. Han and J. Chung, “A study on the robust control algorithm for an axially moving film”, KSME International Journal. 15 (2001) 1207-1216.

[13] A. Valenzuela, J. Bentley and R. Lorenz, “Sensor less tension control in paper machines”, IEEE Transactions on industry applications. 39 (2) (2003).
[14] C. Chen, K. Chang and C. Chang, “Modeling and control of a web-fed machine”, Applied Mathematical Modelling. 28 (2004) 863–876.

[15] E. Laroche and D. Knittel, “An improved linear fractional model for robustness analysis of a winding system”, Control Engineering Practice. 13 (2005) 659–666.

[16] P. Pagilla, N. Siraskar, and R. Dwivedula, “Decentralized control of web processing lines”, IEEE Transactions on Control Systems Technology. 15 (2007) 106–117.

[17] X. Dou and W. Wang, “Robust control of multistage printing systems”, Control Engineering Practice. 18 (2010) 219-229.

[18] T. Tran and K. Choi, “A back stepping-based control algorithm for multi-span roll-to-roll web system”, The International Journal of Advanced Manufacturing Technology. 70 (2014) 45-61.

[19] Y. Wang, X. Liu and L. Huang, “Stability analyses for axially moving strings in nonlinear free and aerodynamically excited vibrations”, Chaos, Solitons and Fractals. 38 (2008) 421–429.

[20] A. Kelleche and N. Tatar, “Control of an axially moving viscoelastic Kirchhoff string”, Applicable Analysis. (2017) 1–18.

[21] R. Viguie, G. Kerschena, J. C. Golinval, D. M. McFarland, L. A. Bergman, A.F. Vakakis and N. V. Wouw, “Using passive nonlinear targeted energy transfer to stabilize drill-string systems”, Mechanical Systems and Signal Processing. 23 (2009) 148–169.

[22] M. A. Foda, “Vibration control and suppression of an axially moving string”, Journal of Vibration and Control. 18 (2011) 58–75.

[23] Y. Zhang, J. Zang, T.Z. Yang, B. Fang, and X. Wen, “Vibration suppression of an axially moving string with transverse wind loadings by a nonlinear energy sink”, Mathematical Problems in Engineering. (2013) 1–7.

[24] A. Kelleche, N. Tatar and A. Khemmoudj, “Uniform Stabilization of an Axially Moving Kirchhoff String by a Boundary Control of Memory Type”, Journal of Dynamics and Control Systems. 23 (2017) 237–247.

[25] B. Yang and C. D. Mote, “Active vibration control of axially moving continua”, Intelligent Structural Systems. (1992) 359-402.

[26] J. Lin and J. Hu, “Vibration attenuation of an axially moving string via active in-domain control methods”, Asian Journal of Control. 4 (1999) 270-282.
C. A. Tan and S. Ying, “Active wave control of the axially moving string: Theory and Experiment”, Journal of Sound and Vibration. 236 (2000) 861–880.

Y. Li, D. Aron and C. D. Rahn, “Adaptive vibration isolation for axially moving strings: Theory and Experiment”, Automatica. 38 (2002) 379–390.

S. P. Nagarkatti, F. Zhang, B. T. Costic and D. M. Dawson, “Speed tracking and transverse vibration control of an axially accelerating Web”, Mechanical Systems and Signal Processing. 16 (2002) 337–356.

K. J. Yang, K. S. Hong and F. Matsuno, “Robust adaptive boundary control of an axially moving string under a spatiotemporally varying tension”, Journal of Sound and Vibration. 273 (2004) 1007–1029.

C. W. Kim, K. S. Hong and H. Park, “Boundary control of an axially moving string” Actuator Dynamics Included”, Journal of Mechanical Science and Technology. 19 (2005) 40–50.

L. Q. Chen and W. Zhang, “Adaptive vibration reduction of an axially moving string via a tensioner”, Journal of Sound and Vibration. 48 (2006) 1409–1415.

Y. Liu, Z. Zhao and W. He, “Stabilization of an axially moving accelerated/decelerated system via an adaptive boundary control”, ISA Transactions. 64 (2016) 394–404.

Z. Zhao, Y. Liu, F. Guo and F. Fu, “Vibration control and boundary tension constraint of an axially moving string system”, Nonlinear Dynamics. 89 (2017) 2431–2440.

Y. Liu, Z. Zhao and W. He, “Boundary control of an axially moving system with high acceleration/deceleration and disturbance observer”, Journal of the Franklin Institute. 354 (2017) 2905–2923.

Y. Liu, Z. Zhao and W. He, “Boundary control of an axially moving accelerated/decelerated belt system”, International Journal of Robust and Nonlinear Control. 26 (2016) 3849–3866.

Y. Liu, F. Guo, X. He and Q. Hui, “Boundary control for an axially moving system with input restriction based on disturbance observers”, IEEE Transactions on Systems, Man and Cybernetics: Systems. (2018), DOI: 10.1109/TSMC.2018.2843523.

W. He, S. S. Ge and D. Huang, “Modeling and vibration control for a nonlinear moving string with output constraint”, IEEE Transactions on Mechatronics. 20 (2015), 1886-1897.

W. He, Y. Ouyang and J. Hong, “Vibration control of a flexible robotic manipulator in the presence of input deadzone”, IEEE Transactions on Mechatronics. 13 (2017), 48-59.
[40] W. He and S. S. Ge, “Cooperative control of a nonuniform gantry crane with constrained tension”, Automatica. 66 (2016) 146–154.

[41] W. He and S. Zhang, “Control design for nonlinear flexible wings of a robotic aircraft”, IEEE Transactions on Control Systems Technology. 25 (2017), 351-357.

[42] W. He, X. He, M. Zou and H. Li, “PDE model-based boundary control design for a flexible robotic manipulator with input backlash”, IEEE Transactions on Control Systems Technology. (2018), DOI: 10.1109/TCST.2017.2780055.

[43] W. He, T. Meng, S. Zhang, Q. Ge and C. Sun, “Trajectory tracking control for flexible wings of a micro aerial vehicle”, IEEE Transactions on Systems, Man and Cybernetics: Systems. (2018), 10.1109/TSMC.2017.2779854.

[44] G. Brandenburg, “A mathematical model of a continuous elastic web in a system of driven, looped, rollers”, Control Engineering and Data Processing. 3 (1973) 62–69.