Self-similar Solution of Hot Accretion Flow with Thermal Conduction and Anisotropic Pressure

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Abstract

We explore the effects of anisotropic thermal conduction, anisotropic pressure, and magnetic field strength on the hot accretion flows around black holes by solving the axisymmetric, steady-state magnetohydrodynamic equations. The anisotropic pressure is known as a mechanism for transporting angular momentum in weakly collisional plasmas in hot accretion flows with extremely low mass accretion rates. However, anisotropic pressure does not extensively impact the transport of the angular momentum, it leads to shrinkage of the wind region. Our results show that the strength of the magnetic field can help the Poynting energy flux overcome the kinetic energy flux. This result may be applicable to the understanding of the hot accretion flow in the Galactic Center Sgr A* and the M87 galaxy.

Unified Astronomy Thesaurus concepts: Accretion (14); Supermassive black holes (1663); Hydrodynamics (1963); Active galactic nuclei (16)

1. Introduction

According to the temperature of the accretion flow, black hole accretion disks can be divided into two broad types: cold and hot. Intensive analytical studies and simulations have been done on hot accretion flows since the pioneer study of Narayan & Yi (1994). It is well known that a variety of accreting systems in the universe with low mass accretion rates such as low-luminosity active galactic nuclei (LLAGNs) as well as black hole X-ray binaries in the quiescent and hard states can be explained by a hot accretion flow model (Yuan & Narayan 2014). Indeed, hot accretion flow is an efficient model to describe the accretion flow around a majority of nearby galaxies with extremely low-luminosity AGNs, including our Galactic Center, Sagittarius A* (Sgr A*), and M87.

Based on observations of black hole X-ray binaries and LLAGNs, an outflow (wind and jet) can be launched from a hot accretion flow (Tominesi et al. 2010, 2014; Crenshaw & Kraemer 2012; Wang et al. 2013; Cheung et al. 2016; Ma et al. 2019). A wind can push the gas around the black hole outward and effectively change the black hole accretion rate in hot accretion systems. Wind is not only a key ingredient of accretion flows that can impact their dynamics and structure, but feedback of the wind on the surrounding medium can also play a major role in the formation and evolution of galaxies (Fabian 2012; Kormendy & Ho 2013; Naab & Ostriker 2017).

Three different wind-launching mechanisms have been proposed, including thermally driven (Begelman et al. 1983; Font et al. 2004; Luketic et al. 2010; Waters & Proga 2012), magnetically driven (Blandford & Payne 1982; Lynden-Bell 1996, 2003), and radiation-driven wind (Murray et al. 1995; Proga et al. 2000; Proga & Kallman 2004; Nomura & Ohsuga 2017). In terms of hot accretion flow, the first two mechanisms play a major role in producing a wind, since the radiation is negligible (Yuan & Narayan 2014). To investigate the existence of wind in a hot accretion flow, a large number of hydrodynamical (HD) and magnetohydrodynamical (MHD) numerical simulations have been carried out so far (e.g., Igumenshchev & Abramowicz 1999; Stone et al. 1999; Hawley et al. 2001; Machida et al. 2001; Stone & Pringle 2001; Pen et al. 2003; De Villiers et al. 2003, 2005; Yuan & Bu 2010; Pang et al. 2011; McKinney et al. 2012; Narayan et al. 2012; Yuan et al. 2012a, 2012b; Li et al. 2013; Yuan et al. 2015; Bu & Gan 2018; Inayoshi et al. 2018, 2019).

Hot accretion flows with very low mass accretion rates are in fact virially hot and low-density plasmas (Narayan et al. 1998; Quataert 2003; Yuan & Narayan 2014). Since the density is very low, the plasma is collisionless, i.e., the Coulomb mean free path is many orders of magnitude larger than the system size. Therefore, the electrons should move large distances to exchange energy with other particles. In such a case, thermal conduction is important and can significantly change the dynamics of the accretion flow. That said, it is well known that the magnetorotational instability (MRI) is a mechanism to induce magnetohydrodynamic turbulence to drive the angular momentum outward in accretion flows (Balbus & Hawley 1991). Consequently, the MHD equations of hot accretion flow should include thermal conduction. Moreover, the ions’ mean free path is also quite large compare to the Larmor radius in a hot accretion flow with a low mass accretion rate. The ions move along the field lines. Therefore, the pressure should be anisotropic. We can treat the anisotropic pressure as a perturbation to the ideal MHD and include it to the MHD equations.

There are several analytical investigations based on the self-similar assumption that probe the magnetic field effects in hot accretion flows (Akiyoshi & Fukue 2006; Abbassi et al. 2008; Zhang & Dai 2008; Bu et al. 2009; Mosallanezhad et al. 2014; Samadi et al. 2017; Bu & Mosallanezhad 2018; Deng & Bu 2019). In practice, analytical studies are powerful tools to understand the physical properties of inflow and the wind from accretion flow. The reasons are as follows: (1) global numerical simulations of accretion flow are expensive and time-

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consuming because of the complex physics involved such as dissipation and the magnetic field, and (2) analytical techniques are straightforward to use to check the dependency of the results on a variety of physical input parameters.

In the present study, we also adopt a radially self-similar assumption and solve the MHD equations of hot accretion flow in the vertical direction. We mainly focus on two extreme cases of the magnetic field, namely a weak and strong field. The magnetic field strength will be initialized by the usual plasma beta definition as \( \beta = \frac{p_{\text{gas}}}{B^2} \), where \( p_{\text{gas}} \) and \( p_{\text{mag}} \) represent the gas and magnetic pressure at the equatorial plane, respectively. Our main goal is to study the influence of the anisotropic pressure, magnetic pressure, magnetic field strength, and thermal conduction on the properties of the inflow and wind. More precisely, we examine how the physical input parameters such as the thermal conductivity coefficient and the initial plasma beta change the inflow and wind regions of the hot accretion flow. We also investigate the mass-flux weight properties of wind from a hot accretion flow in both weak and strong field cases.

The remainder of the manuscript is organized as follows. In Section 2, the basic equations, physical assumptions, self-similar solutions, and the boundary conditions will be introduced. The numerical results of weak and strong magnetic field cases will be presented in detail in Section 3. Finally, we will provide the summary and discussion in Section 4.

2. Numerical Method and Assumptions

2.1. Basic Equations

The basic equations of the hot accretion flow with an extremely low luminosity in the presence of viscosity, a magnetic field, anisotropic pressure, and thermal conduction can be described as

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \quad (1)
\]

\[
\rho \frac{d\mathbf{v}}{dt} = -\rho \nabla \psi - \nabla p_{\text{gas}} + \nabla \cdot \mathbf{\sigma} + \frac{1}{c} (\mathbf{J} \times \mathbf{B}) + \nabla \cdot \Pi, \quad (2)
\]

\[
\rho \frac{dp_{\text{gas}}}{dt} = \frac{\rho}{\rho} \frac{d\rho}{dt} = Q^+ + \nabla \cdot \mathbf{Q}_c - \Pi : \nabla \mathbf{v}, \quad (3)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{v} \times \mathbf{B} - \frac{4\pi}{c} \mathbf{\eta} \mathbf{j} \right), \quad (4)
\]

\[
\nabla \cdot \mathbf{B} = 0. \quad (5)
\]

In the above equations, \( \rho \) is the mass density, \( \mathbf{v} \) is the velocity, \( \psi = -GM/r \) is the Newtonian potential (where \( G \) is the gravitational constant, \( M \) is the mass of the central black hole, and \( r \) is the distance from the black hole), \( p_{\text{gas}} \) is the gas pressure, \( \mathbf{\sigma} \) is the viscous stress tensor, \( \mathbf{J} = c(\nabla \times \mathbf{B})/4\pi \) is the current density (where \( c \) is the speed of light), \( \mathbf{B} \) is the magnetic field, \( \Pi \) is anisotropic pressure, \( \eta \) is the gas internal energy, \( Q^+ \) is the total heating due to viscous and magnetic dissipation, \( Q_{\mathrm{mag}} \) is the thermal conduction, and \( \eta \) is the magnetic diffusivity. The \( d/dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla \) denotes the Lagrangian or comoving derivative. The equation of state of the ideal gas is assumed to be \( p_{\text{gas}} = (\gamma - 1)\rho e \), with \( \gamma = 5/3 \) being the adiabatic index. In fact, in accreting systems, Maxwell stress associated with MHD turbulence driven by MRI (Balbus & Hawley 1998) can transfer the angular momentum outward. Following the MHD numerical simulations, we decompose the magnetic field into a large-scale component and a turbulent component. The magnetic field, \( \mathbf{B} \), in the above equations represents the large-scale component. The effects of turbulence are described by the viscosity terms of the momentum and the energy equations. Both components can transport the angular momentum outward and produce heat. Following numerical simulations of hot accretion flow, Stone et al. (1999) and Yuan et al. (2012b), we assume the azimuthal component of the viscous stress tensor, \( \sigma_{\phi\phi} \), is the dominant component. This component is written as

\[
\sigma_{\phi\phi} = \rho \nu_1 \frac{\partial}{\partial r} \left( \frac{v_0}{r} \right), \quad (6)
\]

where \( \nu_1 \) is the kinematic viscosity coefficient, which is evaluated as

\[
\nu_1 = \frac{\alpha_1}{\rho \Omega_k} (p_{\text{gas}} + p_{\text{mag}}), \quad (7)
\]

where \( \Omega_k \equiv GM/r^3 \) is the Keplerian velocity, \( \alpha_1 \) is the viscosity parameter, and \( p_{\text{mag}} = |\mathbf{B}|^2/(8\pi) \) is the magnetic pressure. The heating rate \( Q^+ \) in Equation (3) is decomposed into two components, i.e., the viscous heating and the magnetic field dissipation heating, as

\[
Q^+ = Q_{\mathrm{vis}} + Q_{\mathrm{res}}, \quad (8)
\]

with

\[
Q_{\mathrm{vis}} = \nabla \cdot \mathbf{v}, \quad (9)
\]

\[
Q_{\mathrm{res}} = \frac{4\pi}{c^2} \mathbf{j} \cdot \mathbf{j}. \quad (10)
\]

Here, the magnetic diffusivity can be set as \( \eta = \eta_0/(\rho \Omega_k) (p_{\text{gas}} + p_{\text{mag}}) \) to satisfy the self-similar solutions in the radial direction. Following the numerical simulations (e.g., Braginskii 1965; Balbus 2004; Chandra et al. 2015), we model the anisotropic pressure \( \Pi \) with an anisotropic viscosity,

\[
\Pi = -3 \rho \nu_2 \left[ \frac{b^2}{b} : \nabla \mathbf{v} - \frac{\nabla \cdot \mathbf{v}}{3} \right] \left[ \frac{b^2}{b} - \frac{1}{3} \right], \quad (11)
\]

where \( b = B/|B| \) is a unit vector in the direction of magnetic field, and \( I \) is the unit tensor. The anisotropic viscosity coefficient \( \nu_2 \) is written as

\[
\nu_2 = \frac{\alpha_2}{\rho \Omega_k} (p_{\text{gas}} + p_{\text{mag}}). \quad (12)
\]

The hot accretion flow is taken to be axisymmetric and steady state \((\partial / \partial \phi = \partial / \partial t = 0)\). We adopt spherical polar coordinates \((r, \theta, \phi)\) to solve Equations (1)–(5). To avoid complexity due to considering all components of the magnetic field, in our current analytical study, we assume only the toroidal component of the field as

\[
\mathbf{B} = B_\phi(r, \theta) \mathbf{e}_\phi, \quad (13)
\]

which automatically satisfies \( \nabla \cdot \mathbf{B} = 0 \).\(^8\) With the help of the above assumptions, the three components of the current

\(^8\)In a future study, we relax this assumption and consider all components of the magnetic field to show the role of a nonzero vertical magnetic flux.
density, $J$, can be read as

$$J = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (B_0 \sin \theta),$$

(14)

$$J_b = -\frac{1}{r} \frac{\partial}{\partial r} (r B_0),$$

(15)

$$J_\phi = 0.$$  

(16)

By considering the toroidal component of the magnetic field, the anisotropic tensor can be reduced to

$$\Pi = \begin{bmatrix} \Pi_{rr} & 0 & 0 \\ 0 & \Pi_{\theta\theta} & 0 \\ 0 & 0 & \Pi_{\phi\phi} \end{bmatrix},$$

(17)

with

$$\Pi_{rr} = \Pi_{\theta\theta} = - \frac{1}{2} \Pi_{\phi\phi} = \rho \nu \left[ \frac{v_r}{r} + \frac{v_\theta}{r} \cot \theta - \frac{1}{3} \frac{\partial}{\partial r} (r^2 v_r) - \frac{1}{3} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) \right].$$

(18)

In the MHD case, thermal conductivity is mostly anisotropic, being suppressed in the direction transverse to the magnetic field. The form of the thermal conduction can be written as

$$Q_T = \kappa \nabla T,$$

(19)

where $T$ is the gas temperature and $\kappa$ is the thermal conduction coefficient. Although the saturated form of the thermal conductivity is appropriate for hot accretion flows we adopt a standard form of thermal conduction in which the heat flux depends linearly on the local temperature gradient. Consequently, the radial dependency of the thermal conductivity coefficient $\kappa$ is considered to be a power-law function of radius to preserve radial self-similarity, i.e., $\kappa(r) = \kappa_0 r^{1/2 - n}$, where $n$ is the density index (see Tanaka & Menou 2006 and Khajenabi & Shadmehri 2013 for more details). By substituting all of the above assumptions and definitions into Equations (1)–(5), we obtain partial differential equations (PDEs) presented in Appendix A.

2.2. Self-similar Solutions

Based upon the numerical HD and MHD simulations of hot accretion flow, time-averaged inflow and outflow mass accretion rates decrease with radius due to the existence of wind (Stone et al. 1999; Yuan et al. 2012a, 2012b). Consequently, their results show that the density profile becomes flatter than in the case with a constant mass accretion rate, in the range of $10r_s < r < 100r_s$ (far away from the strong gravity of the central black hole and also the outer boundary conditions), where $r_s$ is the Schwarzschild radius. In this study, following the numerical simulations, we consider a radial power-law form for the physical variables including density to remove the radial dependency of the variables. To do so, we adopt the following self-similar solutions in the radial direction by adopting a fiducial radius $r_0$. The solutions can be written as

$$\rho(r, \theta) = \rho_0 \left( \frac{r}{r_0} \right)^{-n} \rho(\theta),$$

(20)

$$v_r(r, \theta) = v_0 \left( \frac{r}{r_0} \right)^{-1/2} v_r(\theta),$$

(21)

$$v_\theta(r, \theta) = v_0 \left( \frac{r}{r_0} \right)^{-1/2} v_\theta(\theta),$$

(22)

$$v_\phi(r, \theta) = v_0 \left( \frac{r}{r_0} \right)^{-1/2} v_\phi(\theta),$$

(23)

$$c_s(r, \theta) = c_s(\theta),$$

(24)

$$B_0(r, \theta) = B_0 \left( \frac{r}{r_0} \right)^{-\left(\alpha/2\right)\left(-1/2\right)} b_0(\theta),$$

(25)

where $r_s$, $\rho_0$, $v_0$, $c_s$, and $B_0$ are the units of length, density, velocity, and magnetic field, respectively. The ordinary differential equations (ODEs) can be derived by substituting the above self-similar solutions into the PDEs (A1)–(A6). The coupled system of ODEs is presented in Appendix B. The ODEs (B1)–(B6) consist of six physical variables: $v_r(\theta)$, $v_\theta(\theta)$, $v_\phi(\theta)$, $\rho(\theta)$, $c_s(\theta)$, and $b_0(\theta)$ as well as their derivatives.

2.3. Boundary Conditions

All physical variables are assumed to be even symmetric, continuous, and differentiable at the equatorial plane. Hence, $v_r(\theta) = v_r(\pi - \theta)$, $v_\theta(\theta) = -v_\theta(\pi - \theta)$, $v_\phi(\theta) = v_\phi(\pi - \theta)$, $c_s(\theta) = c_s(\pi - \theta)$, and $\rho(\theta) = \rho(\pi - \theta)$. Since we include the latitudinal component of the velocity, $v_\theta$, due to the even symmetric assumption, its value will be zero at this boundary. Thus, the following boundary conditions will be imposed at $\theta = \pi/2$:

$$\frac{d\rho}{d\theta} = \frac{dc_2}{d\theta} = \frac{dv_r}{d\theta} = \frac{dv_\theta}{d\theta} = \frac{dv_\phi}{d\theta} = \frac{db_0}{d\theta} = v_\theta = 0.$$  

(26)

We define the plasma beta, which is the ratio of the gas to the magnetic pressure. To study the models with weak and strong magnetic fields, we then set its value at the equatorial plane as

$$\beta_0 = \frac{\rho_{\text{gas}}}{\rho_{\text{mag}}} = \frac{2\rho c_s^2}{b_0^2}.$$  

(27)

Here, the dimensionless magnetic pressure is defined as $p_{\text{mag}} = b_0^2/2$. Since the maximum density is located at the equatorial plane, we adopt $\rho(\pi/2) = 1$ for all sets of input parameters. By substituting the above boundary conditions into Equations (B1)–(B6), we can get the following equations at the equatorial plane

$$\frac{dv_\theta}{d\theta} = \left( n - \frac{3}{2} \right) v_r,$$

(28)

$$-\frac{1}{2} v_r^2 - v_\theta^2 = -1 + \left( n + 1 + \frac{n - 1}{\beta} \right) c_s^2,$$

(29)

$$-\alpha_2 (n - 2) \left( 1 + \frac{1}{\beta} \right) \frac{1}{3 \frac{d\theta}{d\theta}} c_s^2,$$

(30)
It is worth noting that without thermal conduction, i.e., $\kappa_0=0$, the above set of equations simply gives us the values of $v_r$, $v_\phi$, and $c_\phi$ at the equatorial plane. In the presence of thermal conduction, we have another unknown parameter, i.e., \((d^2c^2_s/\text{d}\theta^2)\), at the equatorial plane. To have a self-consistent solution, we follow the Khajenabi & Shadmehri (2013) approach and guess the radial velocity at the equatorial plane. We can then simply obtain the value of other physical variables such as $c_\phi$, $v_\phi$, and $b_\phi$ from Equations (28)–(30) and (27). We also impose the following physical boundary conditions at the angle where the radial velocity becomes null, i.e., $v_r(\theta_b)=0$,

$$c^2_s(\theta_b) = \frac{\pi}{2} - \theta_b$$  \hspace{1cm} (32)

The above dimensionless constraint at $\theta_b$ represents the fact that the scale height of the accretion disk, $H \equiv c_s/\Omega_k$, should be equal to the disk height, i.e., $c_s/\Omega_k \cong (\pi/2 - \theta_b) r$. More precisely, the height of the disk is proportional to the wind temperature. By integrating Equations (B1)–(B6), we find the angle at which the radial velocity of the flow becomes null. At this angle we check whether the wind condition (Equation (32)) at $\theta_b$ is satisfied. If not, we choose another value for the radial velocity at the equatorial plane until Equation (32) is satisfied. We then integrate the system of ODEs from the equatorial plane to a certain critical angle denoted as $\theta_b$ where we encounter numerical errors. More precisely, the mass density or the gas pressure will get very close to zero at this inclination. We consider this angle as the upper boundary of the flow structure. We think that with a simple radial self-similar solution we cannot describe the flow structure near the rotation axis. The reason is as follows: if the solution does not end at an upper boundary, $\theta_b$, the net mass accretion rate $\dot{M}_{\text{net}}$, at a specific radius $r$ is calculated as

$$\dot{M}_{\text{net}} = \sqrt{GM} \left[ 4\pi \int_{\pi/2}^{\pi} \rho(\theta) v_r(\theta) \sin \theta \text{d}\theta \right] r^{3/2-n} = 4\pi \sqrt{GM} \left[ \int_{\pi/2}^{\theta_b} \rho(\theta) v_r(\theta) \sin \theta \text{d}\theta \right] r^{3/2-n} + \int_{\theta_b}^{\pi} \rho(\theta) v_r(\theta) \sin \theta \text{d}\theta = \dot{M}_{\text{in}} + \dot{M}_{\text{out}}.$$  \hspace{1cm} (33)

The mass conservation implies that a net mass accretion rate should be a constant for a steady accretion flow. The constant mass accretion rate can only happen in the following two cases. (1) When $n = 3/2$, $\dot{M}_{\text{net}}$ is forced to not change with radius. This case is discussed in Narayan & Yi (1995), resulting in a solution in which the flow is radial (with rotation), i.e., $v_\phi = 0$. Therefore, no outflow has been obtained in their solution. (2) When $n \neq 3/2$, the integration term in the above equation must be zero, resulting in the case $\dot{M}_{\text{net}} = 0$. This requires that the outflow exactly equals the inflow at a certain radius, which in fact is not physical (no accretion process happens). Since we should have a nonzero net accretion rate, the inflow rate must be larger than the outflow rate at a certain radius, and the solution must be truncated at a certain angle near the rotational axis, namely $\theta_b$. Consequently, the solution with outflow ($n < 3/2$) cannot satisfy self-similar approximations near the rotation axis.

By adopting this method, the outflow region extends to $\theta_b < \theta < \theta_c$. We think our solution in the region of $\theta_b < \theta < \pi/2$ is physical due to the fact that the solution satisfies the equations and boundary conditions at both ends as well as at $\theta = \theta_b$.

### 3. Numerical Results

To avoid a shock being caused by a supersonic inflow such as a jet near the central region, which is a source of deviation from the radial self-similar assumptions, we neglect the region within $10r_g$. We consider a radial power-law form for the physical variables to reduce the radial dependency of variables and solve the system of equations only in the theta direction. Therefore, we believe our radial self-similar solutions are unique and stable. By this methodology, the PDEs are reduced to ODEs (Equations (B1)–(B6)), which we solved numerically. The equations were integrated from the equatorial plane, $\theta = \pi/2$, toward a critical angle, $\theta = \theta_b$. The solutions are in the following parameter space: $\alpha_1 = 0.1$, $\eta_1 = 0.1$, $\gamma = 5/3$, and $n = 0.5$. To explore how much the anisotropic pressure can affect the physical variables of the accretion flow, we plotted them in Figure 1 for three different values of $\alpha_2 = [0.01, 0.1, 0.2]$ in the vertical direction, shown by dotted, dashed, and solid lines, respectively. For this figure, $\beta_s = 5000$, which represents the weak magnetic field case. From left to right and top to bottom, Figure 1 shows the radial velocity, $v_r$, the latitudinal velocity, $v_\phi$, the rotational velocity, $v_\theta$, the poloidal velocity, $v_p$, the density, $\rho$, and the sound speed squared, $c^2_s$, respectively. All velocities are plotted in the units of Keplerian values and the density is in the units of maximum density at the equatorial plane at that radius. It is clear that the anisotropic pressure does not considerably affect the physical quantities; however, it can decrease the amount of the wind. For instance, in the top left panel, the wind region extends from the dotted-dashed line that is the $v_r = 0$ line by around $\theta \sim 35^\circ$ for $\alpha_2 = 0.01$, while for $\alpha_2 = 0.2$ this region shrinks and goes until $\theta \sim 43^\circ$. As was expected, the radial velocity of the wind is much higher than that of the inflow. In the top right panel, there is an increase in the absolute value of the latitudinal velocity, with the maximum amount occurring between $\theta \sim 45^\circ$–$50^\circ$. Then, we can see that the absolute value of this quantity decreases toward the polar region. Therefore, the poloidal velocity of the flow shows an increasing trend from the equatorial plane toward the polar region. In the middle left panel, it is shown that $v_\phi$ increases with $\theta$. The bottom left panel illustrates the density drops from the equatorial plane toward the polar region. In addition, the anisotropic pressure can increase the density especially in the wind region. This implies that as the amount of the wind is enhanced with decreasing $\alpha_2$, the density decreases due to the sweeping of more gas away by the wind. As you can see in the bottom right panel of Figure 1, the sound speed increases from the disk toward the polar region.

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4. We also examine anisotropic pressure effects for the strong magnetic field case and the results were almost identical to Figure 1.
We have not only examined the effects of the anisotropic pressure but we have also investigated the influence of the thermal conductivity and the magnetic field strength on the flow properties. In the left and right panels of Figure 2, the radial velocity is plotted for the weak and strong magnetic field with $\beta = [5000, 50]$, respectively. We also consider four different values for the conductivity coefficient as $\kappa_\perp = [0.04, 0.06, 0.08, 0.1]$, illustrated by dotted, dashed, dotted-dashed, and solid lines, respectively. This figure clearly shows that in the weak magnetic field, thermal conduction does...
not affect the inflow radial velocity. However, in the case of the strong magnetic field, thermal conduction can increase the inflow radial velocity. In addition, thermal conduction can decrease the radial wind velocity, disregarding the magnetic field strength.

Figure 3 shows the dependency of $q_0$ on $k_0$ with $\alpha_1 = 0.1$, $\alpha_2 = 0.05$, $n = 0.5$, $\gamma = 5/3$, and $\eta_0 = 0.1$. The two models with weak, $\beta = 5000$, and strong, $\beta = 500$, magnetic fields are marked by triangles and squares, respectively. The inverse behavior of these two parameters can be clearly seen from this figure: $q_0$ decreases as $k_0$ increases. As $q_0$ is the declination angle between the inflow and wind areas, increasing $k_0$ leads to a gradual shrinkage of the wind region. For the weak magnetic field, the wind region is expanded respect to the strong magnetic field. For instance, for $\kappa = 0.06$ the wind region in the weak magnetic field starts from $\theta \sim 61^\circ$, while in strong magnetic field it starts from $\theta \sim 59^\circ$.

The impact of the thermal conduction on other physical variables of the accretion flow in weak and strong magnetic field models, $\beta = [5000, 50]$, is examined in Figures 4 and 5, respectively. Latitudinal velocity in the units of the Keplerian value is plotted in the top left panel of Figure 4. Although the thermal conduction does not show any effects on $v_\theta$ around the equatorial plane, it causes the absolute value of the wind latitudinal velocity to increase. In the case of the strong magnetic field in Figure 5, the absolute value of the latitudinal velocity drastically increases toward the polar region. In the top right panel of Figure 4 it is seen that when the magnetic field is weak, thermal conductivity shows a different effect on the rotational velocity of the inflow and wind. In the inflow, the thermal conductivity reduces the rotational velocity; however, it can increase the wind velocity. We cannot see this inverse behavior in the inflow and wind region when the magnetic field magnifies. In the case of the strong magnetic field, thermal conduction causes a decrease in the rotational velocity of both the inflow and wind (top right panel of Figure 5). In both cases of the magnetic field strength, the density increases with thermal conduction, and this implies that thermal conduction can make the disk thicker. In addition, in the weak magnetic field the density drops more smoothly, rather than in the case of the strong magnetic field (bottom left panels of Figures 4 and 5). The sound speed squared in the units of the Keplerian velocity is plotted in the bottom right panel of both Figures 4 and 5. A comparison between these two panels shows that thermal conduction can reduce the sound speed and thus the temperature of the flow. Further, it can be seen that for the higher values of the thermal conduction, the sound speed increases smoothly from the equatorial plane toward the polar region. Moreover, when the magnetic field strength increases, the sound speed decreases in the wind region.

The Bernoulli parameter, considering only the toroidal magnetic field, is defined as

$$\text{Be} = \frac{1}{2}V^2 + h + \psi + \frac{B_\phi^2}{4\pi\rho}$$  \hspace{1cm} (34)
where $h = \gamma p / \left( \rho (\gamma - 1) \right)$ is enthalpy. The top row panels of Figure 6 show the Bernoulli parameter for four values of the conductivity coefficients, $\kappa = [0.04, 0.06, 0.08, 0.1]$, and also for the weak and strong magnetic fields, $\beta = [5000, 50]$, respectively. As you can see, enthalpy is higher in the weak magnetic field compared to the strong magnetic field. However, we cannot see significant changes in the Bernoulli parameter when the strength of the magnetic field is amplified.

We also calculated the wind power, which includes kinetic, thermal, and Poynting energy fluxes as

$$
\begin{align*}
P_k(r) &= 2\pi r^2 \int_{0}^{90^\circ} \rho \max(v_r^3, 0) \sin \theta d\theta, \\
P_{\text{th}}(r) &= 4\pi r^2 \int_{0}^{90^\circ} \rho e \max(v_r, 0) \sin \theta d\theta, \\
P_B(r) &= 4\pi r^2 \int_{0}^{90^\circ} S_r \max(v_r / |v_r|, 0) \sin \theta d\theta,
\end{align*}
$$

where $S_r$ is the radial component of Poynting flux and is defined as

$$
S_r = v_r \frac{B^2}{4\pi}.
$$

The results are plotted in the bottom row of Figure 6 with respect to the conductivity coefficient, where the left and right panels are for weak and strong magnetic fields, respectively. All components of the energy flux decrease with the conductivity coefficient for both magnetic field cases except for the Poynting energy flux in the weak magnetic field, which increases with this parameter. Clearly, the thermal energy flux dominates two other fluxes in both weak and strong magnetic field cases. Moreover, when the magnetic field is weak the kinetic energy flux is larger than the Poynting energy flux. As the magnetic field magnifies, the Poynting energy flux prevails over the kinetic energy.

4. Summary and Discussion

In this paper, we solve the equations of the hot accretion flow by considering a magnetic field, anisotropic pressure, and...
We investigate the properties of the inflow and wind of the collisionless plasmas in hot accretion flows with very low mass accretion rates. As an increase or decrease in the magnetic field strength makes a pressure anisotropy, we compare the physical quantities of the accretion flow in two cases with a weak and a strong magnetic field. We decompose the magnetic field into a large-scale component and a turbulent component. We only consider the toroidal component of magnetic field. The effects of turbulence are described by the viscosity. Both components can transfer the angular momentum outward and produce heat. We modeled the anisotropic pressure with an anisotropic viscosity. By considering the self-similar solutions, we solved the ODEs numerically. We examined the effect of the anisotropic pressure on the properties of the inflow and wind. We found that the impact of the thermal conductivity on the flow to some extent depends on the magnetic field strength. In the strong magnetic field, thermal conduction tends to decrease the rotational velocity of the wind. Thermal conductivity also tends to shrink the wind region. Our results showed that neither thermal conduction nor the magnetic field strength can impressively impact the Bernoulli parameter. We explored the energy flux of the wind in detail to know how thermal conduction and the magnetic field strength can affect the thermal, kinetic, and Poynting energy fluxes. We found that, except the Poynting energy flux, thermal conduction can decrease all components of the energy flux in both weak and strong magnetic fields. What is more, the strength of the magnetic field can help the Poynting energy flux overcome the kinetic energy flux. In terms of black hole mass, the main application of our hot accretion results is to dim black hole sources, including the supermassive black hole in our Galactic Center, to low-luminosity active galactic nuclei (AGNs), and to quiescent and hard states of black hole X-ray binaries. Additional physics, namely radiative cooling and self-gravity, can play a significant role in the construction of realistic physical models. These additional physics may not necessarily allow a self-similar solution. But in some cases,
they can help to determine some of the parameters of the self-similar model.

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Appendix A
Partial Differential Equations in Spherical Coordinates

To simplify the set of Equations (1)–(5), we consider the flow to be in a steady state and axisymmetric ($\partial / \partial t = \partial / \partial \phi = 0$). The spherical polar coordinate system ($r$, $\theta$, $\phi$) is adopted. We assume all components of the velocity field, $v = v_r \hat{r} + v_\theta \hat{\theta} + v_\phi \hat{\phi}$. We further assume only the toroidal component of magnetic field is presented. By substituting all assumptions and definitions introduced in Section 2.1 into Equations (1)–(5), we obtain the following partial differential equations (PDEs). Hence, the continuity equation is reduced to the following form

$$
\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho v_\theta \sin \theta) = 0.
$$

(A1)
The three components of the momentum equation are:

$$\rho \left[ v_r \frac{\partial v_r}{\partial r} + \frac{v_r}{r} \left( \frac{\partial v_r}{\partial \theta} - v_\theta \right) - \frac{v_\theta^2}{r} \right] = -\frac{GM\rho}{r^2} - \frac{\partial p_{gas}}{\partial r} + \frac{J_0 B_0}{4\pi} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \Pi_{\theta} \right) - \frac{1}{r} \left( \Pi_{\theta \theta} + \Pi_{r \theta} \right), \quad (A2)$$

where

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_r}{r} \left( \frac{\partial v_r}{\partial \theta} + v_r \right) - \frac{v_\theta^2}{r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r^3 \sigma_{\theta} \right). \quad (A4)$$

Finally, the induction equation will be written as follows

$$\frac{\partial}{\partial r} \left( rv_r B_\phi \right) + \frac{\partial}{\partial \theta} \left( v_\theta B_\phi \right) - \frac{1}{\partial \theta} \left( \eta \mathcal{L} \right) \right) + \frac{\partial}{\partial \theta} \left( r \eta \mathcal{L} \right) = 0. \quad (A6)$$

**Appendix B**

*Ordinary Differential Equations*

The ODEs can be derived just by substituting self-similar solutions presented in Section 2.2 into the partial differential Equations (A1)–(A6) as

$$\rho \left[ \frac{3}{2} - n \right] v_r + \frac{dv_r}{d\theta} + v_\theta \cot \theta \right] + v_\theta \frac{d\rho}{d\theta} = 0, \quad (B1)$$

$$\rho \left[ -\frac{1}{2} v_r^2 + \frac{dv_r}{d\theta} - v_\theta^2 - v_\phi^2 \right] = -\rho + (n + 1) p_{gas} + j_\theta b_\phi - \alpha_2 p_{tot} (n - 2) \times \left[ \frac{1}{2} v_r + \frac{2}{3} v_\theta \cot \theta - \frac{1}{3} \frac{dv_\theta}{d\theta} \right], \quad (B2)$$

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