Effect of the Minor Principal Stress on Crack Initiation Stress Threshold

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Abstract. The rapid expansion of urban and mining infrastructures worldwide over the last two decades has seen an escalation of development in hard rock, including urban tunnels, deep open pits, and underground mines. However, despite many success stories, the pervasiveness of rock burst and spalling remains yet a major unresolved ground control problem in many deep mines and tunnels. Spalling is characterised as a sudden explosion-like rock failure that occurs spontaneously and could affect both the short-term and long-term viability of mining operations. Previous studies strongly show that the spalling strength in low-porosity rocks can be associated with the initiation of internal cracks in the sample. However, very limited studies are available that looks at the effect of all the principal stresses on rock crack initiation stress threshold. This paper investigates the previously ignored effect of the minor principal stress on the crack initiation stress level of sandstone cubes tested under true triaxial loading conditions. The results reveal that the minor principal stress has a profound effect on the crack initiation point. As a result of this study, a new crack mode-changing stress (CMCS) concept is also introduced which is defined as the corresponding minimum principal stress required to change rock fracturing from splitting to a sliding failure mode.

1. Introduction
In the hunt for more ores and due to tremendous improvements in excavation techniques and instrumentation methods, mining depths have increased significantly from less than 200 m in the early 1900s to above 2000 m and even further in late 2000s [1] (see Figure 1). More recently, mining depths have even exceeded 4 km, for example, in the Mponeng gold mine in South Africa which is one of the deepest mines on earth [2]. As we go deeper, many associated problems have arisen, such as high-stress conditions, high temperature, and low porosity which leads to explosion-like fractures known as stress spalling, slabbing, or rock bursting. Such failure types emerged in the early 1900s and have gained more attention since [3]. Spalling failure has been observed in different mine locations around the globe: Nickel mines in the Sudbury Basin in Canada, the Neyriz Marble Mine in Iran, metal mines in North America, Champion Reef Mine in the Kolar Gold Fields in India, the Hongtoushan copper mine in China, the Onkalo Tunnel in Finland, the Steg Lateral Adit of the Lötschberg Base Tunnel in Switzerland, and in the Garpenberg zinc mine in Sweden at a depth of 880 m [3, 4, 5, 6, 7, 8, 9, 10]. In Australia, the mining industry makes a great contribution to the economy with over 350 mines in operation [11]. In the last century, spalling failure has been observed in numerous locations around Australia: Hawkesbury Sandstone in the crown of the M5 East Motorway in Sydney, the Northside Storage tunnels and Elgas Cavern in Sydney, the tunnels constructed within Bringley Shale and the Newport Formation in the Sydney Basin, the Mount Isa Mines in Queensland where the depth of the failure in the side walls exceeded one metre, the Kalgoorlie mines in Western Australia, and the NorthConnex project in New South Wales [12, 13, 14, 15, 16]. Therefore, solving the challenges facing
this sector has attracted global attention in the last decade and millions of dollars were allocated to associated research.

Figure 1. Mining depth variation for different ores between 1920 and 2020

Given that spalling failure is associated with the initiation of internal cracks, the crack initiation (CI) concept which is defined as the onset of stress-induced damage that initiates following the closure of pre-existing cracks was used as a predictor of spalling failure. The initiation and accumulation of the induced localised tensile cracks is due to the inhomogeneous nature of rocks, mineral composition, particle size, and structural type [17, 18]. The crack initiates within the elasticity zone of the rock and, therefore, many crack initiation models were developed based on the stress-strain response of underground rock. These include the volumetric strain method [19], the lateral strain method [20], the extensional strain method [21], the crack volumetric strain method [22], the Poisson’s ratio method [23] and the lateral strain response (LSR) method [24]. Before the CI stress threshold, the rock experiences no new induced stress-damage and its strength remains under quasi-static loading [25]. Based on controlled in-situ experiments under uniaxial loading conditions, the CI point is typically in the range of 30% to 50% of the UCS value and, therefore, it has been indicated as an intrinsic property of brittle rocks regardless of the loading or environmental conditions [26, 27]. From this perspective, a crack initiation ratio (CIR) concept (the ratio of the CI stress threshold to the peak strength) has been introduced as a dimensionless indicator of spalling strength to better characterise the mechanism of catastrophic failure of underground brittle rock. While the CI point represents the onset of stress-induced cracks at a micro level, the crack damage CD stage represents the coalescence of cracks at a macro level and dilation of deformation.

The general consensus revealed that the material elasticity, porosity, tensile strength, and brittleness ratio have an impact on the crack initiation, coalescence, and propagation. Whereas the confining pressure significantly increases the CI stress threshold, it also shapes the CI mode. At low confinement, the CI is developed under a splitting mode where the pre-existing cracks remain open during the failure process which promotes higher localised tensile stress around the perimeter, while at higher confinement the predominant crack mode is a sliding mode due to the closure of pre-existing cracks that reduces the tensile stress region around the perimeter [17]. Moreover, it is proven based on experimental testing and theoretical analysis that the intermediate principal stress has a significant influence on crack propagation and hence spalling around the periphery of underground tunnels [28, 29]. However, the effect of the minor principal stress on the CI is yet not addressed well in the literature. Therefore, this paper aims to investigate the effect of the minor principal stress on the crack initiation of sandstone cubes tested under true triaxial loading conditions. The results reveal that, like the intermediate principal stress, the minor principal stress has a profound effect on the crack initiation point and the cracking mode.

2. True Triaxial Testing
Early attempts to model a truly in-situ condition of underground rock were carried out by Mogi in the early 1970s who developed a true triaxial testing system capable of independently applying orthogonal
loading on rock specimens [30]. Afterwards, the fabrication and design of true triaxial testing systems prospered. Nowadays, over 100 true triaxial testing apparatuses are utilised in soil, concrete, and rock testing [31, 32]. The Geotechnical Engineering Centre (GEC) of the University of Queensland owns a multifunctional true triaxial testing system with the capability of performing true triaxial loading, permeability, hydraulic fracking, and thermo-mechanical coupled modelling with a temperature of up to 100 degrees, and it is designed to accommodate acoustic emission and ultrasonic monitoring instrumentations to assess the progressive damage [33, 34, 35].

Sandstone bulk samples were extracted from a quarry in the state of Queensland and transported to the GEC where cutting and grinding took place. After grinding, the samples were oven-dried at 50°C for 24 hours before testing (Figure 2). To study the failure characteristics of the sandstone cubes, several generalised triaxial compression stress state tests ($\sigma_1 > \sigma_2 = \sigma_3$), true triaxial tests ($\sigma_1 > \sigma_2 > \sigma_3$), and generalised triaxial tensile stress state tests ($\sigma_1 = \sigma_2 > \sigma_3$) were carried on 50 mm cubes as per the designated loading path in Table 1.

![Figure 2. Test setup: (a) 50mm sandstone cubes for the triaxial and true triaxial testing, and (b) sample positioned inside the true triaxial machine](image)

Before testing, a seating load of 5 kN was maintained in the direction of the principal stresses ($\sigma_1, \sigma_2, \sigma_3$) to avoid sample eccentricity while loading. The testing was carried under a loading rate of 15 kN/min to satisfy the ISRM recommendations. The major principal stresses ($\sigma_1$) were increased from the seating load to sample failure where intermediate ($\sigma_2$) and minor ($\sigma_3$) principal stresses were increased simultaneously from the seating load to the designated load as shown in Table 1 and Figure 3.

It can be deduced from Figure 3 that the peak strength of the sandstone increases from the generalised triaxial compression stress state ($\sigma_1 > \sigma_2 = \sigma_3$) to the generalised triaxial tensile stress state ($\sigma_1 = \sigma_2 > \sigma_3$) at different $\sigma_2$ values for all constant values of $\sigma_3 = 5, 10, 20$ and 40 MPa. That is, the peak strength of the sandstone first increased and then decreased with the increase of $\sigma_2$ at constant $\sigma_3$ values but is still in general greater than that of the generalised triaxial compression stress state which is in line with the reported observations in the literature [34, 36].

| 3D Stress State | \( \sigma_3 \) (MPa) | Loading Condition | 3D Stress State | \( \sigma_3 \) (MPa) | Loading Condition |
|-----------------|----------------------|-------------------|-----------------|----------------------|-------------------|
| \( \sigma_1 > \sigma_2 = \sigma_3 \) | 5 | Triaxial compression stress | \( \sigma_1 = \sigma_2 > \sigma_3 \) | 20 | Triaxial tensile stress |
|                | 10 |                             |                | 40 |                             |
|                | 20 |                             |                | 5  |                             |
|                | 40 |                             |                | 10 |                             |
| \( \sigma_1 = \sigma_2 > \sigma_3 \) | 5  | Triaxial tensile stress    |                | 20 | True triaxial               |
|                | 10 |                             |                | 40 |                             |
3. Crack Initiation Results

The crack initiation in this study was determined based on four models, as summarised in Table 2 and Figure 4. Several sets of $\sigma_1 > \sigma_2 \geq \sigma_3$ at $\sigma_2=40$ and 60 MPa at different values of $\sigma_3=5, 10, 20$ and 40 MPa were deliberately selected to better understand the effect of $\sigma_3$ on the CI stress (represented by a red rectangle in Figure 3). The variation of $\sigma_3$ with $\sigma_1$ at same $\sigma_2$ values is shown in Figure 5. The CI values of the selected stresses were determined as summarised in Table 3 along with the standard deviation (SD) and coefficient of variation (CoV) and were then plotted in Figure 6.

Table 2. Crack initiation models used in this study [25]

| Model                        | Comment                                                                 |
|------------------------------|-------------------------------------------------------------------------|
| Volumetric Strain Method [19]| The volumetric strain curve is used in this model to determine the onset of dilatancy. Therefore, the CI stress is the point where the curve deviates from its linearity. |
| Lateral Strain Method [20]   | The axial strain in this model remains linear after the closure of pre-existing cracks until the onset of an unstable crack growth stage. Hence, the CI is the point where the lateral strain deviates from the linearity. |
| Extensional Strain Method [21]| The end of the linear portion of the curve representing lateral strain versus axial strain is considered to be the CI stress. |
| Crack Volumetric Strain Method [22] | This model was proposed to determine the CI based on crack volumetric strain versus axial strain, especially with specimens that have a high crack density. |

Table 3. Crack initiation determination

| $\sigma_1$ (MPa) | $\sigma_2$ (MPa) | $\sigma_3$ (MPa) | CI Volumetric Strain [MPa] | CI Lateral Strain [MPa] | CI Extensional Strain [MPa] | CI Crack Volumetric Strain [MPa] | SD  | CoV (%) | Average |
|------------------|------------------|------------------|---------------------------|-------------------------|-----------------------------|-----------------------------------|-----|---------|---------|
| 91               | 40               | 5                | 27                        | 27                      | 28                          | 29                                | 0.9 | 3.5     | 27.75   |
| 105              | 40               | 10               | 40                        | 38                      | 39                          | 38                                | 0.9 | 2.5     | 38.75   |
| 124              | 40               | 20               | 68                        | 66                      | 68                          | 68                                | 1.0 | 1.5     | 67.50   |
| 174              | 40               | 40               | 89                        | 90                      | 88                          | 88                                | 0.9 | 1.1     | 88.75   |
| 101              | 60               | 5                | 28                        | 28                      | 34                          | 28                                | 3.0 | 10.1    | 29.50   |
| 117              | 60               | 10               | 43                        | 40                      | 40                          | 40                                | 1.5 | 3.7     | 40.75   |
| 133              | 60               | 20               | 72                        | 71                      | 68                          | 68                                | 2.1 | 2.9     | 69.75   |
| 188              | 60               | 40               | 92                        | 92                      | 89                          | 91                                | 1.4 | 1.6     | 91.00   |
Figure 4. CI determination models: a) volumetric strain method b) lateral strain method c) extensional strain method d) crack volumetric strain method [24, 37]

Figure 5. Variation in major and minor principal stresses at same intermediate principal stresses

In Figures 5 and 6, the variation in the major principal stresses and CI stresses with the minor principal stresses was investigated under the constant values of intermediate principal stresses of $\sigma_2 = 40$ and $60 \text{ MPa}$. It is obvious from Figure 5 that the major principal stress increases linearly with the increase in the minor principal stress which is unlike the non-linear increase of $\sigma_1$ with $\sigma_2$ at constant $\sigma_3$ values as shown in Figure 3. On the other hand, at constant values of intermediate principal stresses, the CI stress first increases linearly at low values of the minor principal stresses and then increases in a non-linear fashion at higher minor principal stresses which means a change in the cracking pattern took place, as shown in Figure 6. That is, there is a critical value where the CI stress deviates from its linearity and this could be attributed to the change in cracking mode from a splitting mode (at low confinement) to a sliding mode (at higher confinement), as discussed above and highlighted in the literature [17, 38]. Therefore, the minor principal stress where the cracking mode changed from a splitting mode to a
sliding mode can be defined as a crack mode-changing stress (CMCS) which is the point where the CI stress deviates from its linearity on the CI stress versus $\sigma_3$ curve. Therefore, it can be concluded that the minor principal stress has a significant effect on the CI and has a profound effect in controlling the cracking mode. In other words, the effect of the minor principal stress on the crack initiation is not trivial nor should be neglected.

![Graph showing variation of CI stress with minor principal stress at same intermediate principal stresses](image)

**Figure 6.** Variation of CI stress with the minor principal stress at same intermediate principal stresses

4. **Conclusion**

The spalling phenomenon is explosion-like fractures attributed to high-stress conditions, high temperature, and low porosity in shallow to deep mines. The crack initiation concept has been introduced as a predictor of spalling failure. In line with the fact that the crack initiates within the rock elasticity zone, many crack initiation models were developed based on the stress-strain response of underground rock. While the intermediate principal stress has a significant effect on the crack initiation and propagation, limited studies are available in the literature on the effect of the minor principal stress on crack initiation. Therefore, this study aims to investigate the effect of the minor principal stress on crack initiation.

At the same intermediate principal stress, the CI stress increases linearly at low values of minor principal stress and then increases non-linearly at higher minor principal stress which is indicated the change in the cracking pattern at a micro-level. Therefore, a crack mode-changing stress (CMCS) concept is introduced and defined as the point where the CI stress deviates from its linearity on the CI versus $\sigma_3$ curve where the cracking mode is changing from a splitting mode (at low confinement) to a sliding mode (at higher confinement). In other words, a CMCS is the minor principal stress required to change the cracking mode from a splitting to a sliding mode. However, further investigation on the CMCS on wide ranges of intermediate principal stresses is an endeavour that is underway and reserved for future publications. In conclusion, the minor principal stress has a significant effect on the crack initiation and plays a vital role in controlling the cracking mode.

5. **References**

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