Optimization of Concrete Made with Abakaliki Quarry Dust as Fine Aggregate Using Scheffe’s Optimization Model

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ABSTRACT

In recent years, Nigeria has witnessed rapid development especially in the area of infrastructural development like roads, bridges, buildings etc. The conventional methods used in concrete mixing have its own peculiar problems, like time wasting, material wasting and errors. These problems have been the cause of structural failures which has given rise to loss of life and properties. Hence the need to develop a method that will take care of all these anomalies witnessed in the conventional method. This work is aimed at removing these anomalies by the use of Scheffe’s optimization method. This optimization method can predict the compressive strength of a concrete given the mix ratios and also predict the mix ratios required to give a compressive strength for a particular concrete made by completely replacing river sand with quarry dust. With this method it will be easy to predict the compressive strength of concrete based on the type of structure it is to be used for, thereby eliminating the problems associated with structural collapse due to errors in concrete mixing by conventional method.

Keyword: Quarry dust; Concrete; Scheffe’s optimization model; compressive strength; simplex design

1. INTRODUCTION

Concrete is the product of the combination of water, cement, fine aggregate and coarse aggregate, when these mixes are not properly mixed together the target compressive strength and other parameter required will not be achieved. In the construction industries structural failures occur due to improper concrete mix from wrong mixes and it poses a very big challenge, especially in the third world countries like Nigeria. Hence, the need to do away with the old conventional methods and develop a method that will eliminate all these problems associated with them.

The use of statistical method which has found its way in the industries in the area of optimization of products such as gasoline, food products and detergent is a welcome development (Simon 2003). The use of statistical experimental design approach in concrete mixture proportioning helps structural engineers to evolve the best possible design in the area of cost, weight, reliability or a combination of these parameter (Rajsekaran 2005). In their
work Kalntari at el (2009) acknowledged that the selection of mix proportion is a very important process in the selection of suitable components required for concrete production and also the means of maximizing some important parameter like compressive strength, durability and smooth consistency.

Fine aggregate is an important material in the production of concrete, because it affects the strength of concrete to a large extent (Shamin at el 2006). The quarry dust used as fine aggregate in this experiment is a waste product produced from the crushing of quarry stones in the quarry industries. The quarry dust which is classified as a dust pollutant, if not properly disposed can cause so many environmental problems. So the use of quarry dust as fine aggregate in the production of concrete will go a long way in reducing its pollution effects on human and the environment (Mahzuz at el 2001). Since compressive strength is the criterion used in the determination of the quality of concrete (Troxel et al 1968), it is very clear that if a concrete is to be used to construct any structure, the compressive strength must have to be determine to make sure that the structure can carry the intending load. Apart from compressive strength other properties are also very important, they comprises of workability, durability, strength development and economy (Waziri et al 2011). In this work, Scheffe’s Optimization Model which is a statistical experiment design approach in concrete mix proportioning, is used to optimize the compressive strength of concrete produced with quarry dust as fine aggregate.

2. SIMPLEX DESIGN

The \((q,m)\) simplex lattice designs are characterized by the symmetric arrangement of points within the experimental region and a well chosen polynomial equation to represent the response surface over the entire simplex region. The polynomial has exactly as many parameters as the number of points in the associated simplex lattice design.

The \((q,m)\) simplex lattice design given by Scheffe in 1958 consist of \(q^m+1\)C\(_m\) points, where each components proportion take \((m+1)\) equally spaced value

\[
x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots, 1
\]

where

\(i = 1,2,\ldots, q\), \(q\) ranges between 0 and 1 and all possible mixture with these components proportions are used.

For \((4,2)\) simplex lattice, it can be written in the form \(4+2-1\)C\(_2\) = 5C\(_2\) = 10 points. \(x_i\) can be taken as \(m + 1 = 4\) possible values; \(x_i = 0, \frac{1}{2}, 1\) with which possible design points \((1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right)\), and can be represented as shown below in fig 1.
The properties of fresh and hardened concrete are called responses (Simeon et al, 1997). These responses can be put in a polynomial function of pseudo component of the mixture as proposed by Scheffes (1958) and Simon et al (1997) as shown below:

\[ Y = b_0 + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijk} X_i X_j X_k + \ldots + \sum b_{i_1i_2 \ldots i_n} X_{i_1} X_{i_2} \ldots X_{i_n} + e \]

where

- \( 1 \leq i \leq q \), \( 1 \leq i \leq j \leq q \), \( 1 \leq i \leq j \leq k \leq q \), and \( 1 \leq i1 \leq i2 \ldots \ldots \leq in \leq q \) respectively.
- \( b_0 \) = arbitrary constant
- \( e \) = random error
- \( Y \) = the response

For four pseudo component mixture (cement, quarry dust, coarse aggregate and water) the response equation can be written as

\[ Y = b_0 + b X_1 + b X_2 + b X_3 + b X_4 + b_{11} X_1^2 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{22} X_2^2 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{33} X_3^2 + b_{34} X_3 X_4 + b_{44} X_4^2 + e \]

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The term \( e \) which is the random error can be neglected. Using the equation

\[ \sum_{i=1}^{q} X_i = 1 \]

4

where

- \( q = 4 \)
Equation 3.10 can now be written as

$$\sum_{i=1}^{4} X_i = 1$$

Since the total component in the mixture cannot be more than 1, equation 4 can be written as

$$X_1 + X_2 + X_3 + X_4 = 1$$

Multiplying equation 6 by $b_0$ yields

$$b_0 X_1 + b_0 X_2 + b_0 X_3 + b_0 X_4 = b_0$$

Multiplying equation 6 by $X_1$ yields

$$X_1^2 + X_1 X_2 + X_1 X_3 + X_1 X_4 = X_1$$

In like manner equation 6 can be multiply by $X_2$, $X_3$ and $X_4$ to give their respective values as follows

$$X_1 X_2 + X_2^2 + X_2 X_3 + X_2 X_4 = X_2$$
$$X_1 X_3 + X_2 X_3 + X_3^2 + X_3 X_4 = X_3$$
$$X_1 X_4 + X_2 X_4 + X_3 X_4 + X_4^2 = X_4$$

Making $X_i^2$ the subject of the formulas in equations 8 to 11 give respectively the following;

$$X_1^2 = X_1 - X_1 X_2 - X_1 X_3 - X_1 X_4$$
$$X_2^2 = X_2 - X_1 X_2 - X_2 X_3 - X_2 X_4$$
$$X_3^2 = X_3 - X_1 X_3 - X_2 X_3 - X_3 X_4$$
$$X_4^2 = X_4 - X_1 X_4 - X_2 X_4 - X_3 X_4$$

Substituting equation 7 and equations 12, 13, 14 and 15 into equation 3 yields

$$Y = X_1(b_o + b_1 + b_{11}) + X_2(b_o + b_2 + b_{22}) + X_3(b_o + b_3 + b_{33}) + X_4(b_o + b_4 + b_{44})$$
$$+ X_1 X_2(b_{12} - b_{11} - b_{22}) + X_1 X_3(b_{13} - b_{11} - b_{33}) + X_1 X_4(b_{14} - b_{11} - b_{44})$$
$$+ X_2 X_3(b_{23} - b_{22} - b_{33}) + X_2 X_4(b_{24} - b_{22} - b_{44})$$
$$+ X_3 X_4(b_{34} - b_{33} - b_{44})$$

The constants in parenthesis can be sum up to give other constants say $\beta$ and let
\[
\begin{align*}
\beta_1 &= b_0 + b_1 + b_{11} \\
\beta_2 &= b_0 + b_2 + b_{22} \\
\beta_3 &= b_0 + b_3 + b_{33} \\
\beta_4 &= b_0 + b_4 + b_{44} \\
\beta_{12} &= b_{12} - b_{11} - b_{22} \\
\beta_{13} &= b_{13} - b_{11} - b_{33} \\
\beta_{14} &= b_{14} - b_{11} - b_{44} \\
\beta_{23} &= b_{23} - b_{22} - b_{33} \\
\beta_{24} &= b_{24} - b_{22} - b_{44} \\
\beta_{34} &= b_{34} - b_{33} - b_{44}
\end{align*}
\]

Substituting equation 17 into equation 16 yields

\[
Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 + e
\]

Equation 19 can be written as

\[
Y = \bar{Y} + e
\]

Where \(e = \) standard error or standard deviation

\[
\bar{Y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4
\]

Equation 20 can be written in the form

\[
\bar{Y} = \sum_{i=1}^{4} \beta_i X_i + \sum_{1 \leq i < j \leq 4} \beta_{ij} X_i X_j
\]

Equation 20 has ten coefficients which is in agreement with Scheffes Simplex equation.

3. SCHEFFES OPTIMIZATION EQUATION

The concrete is made of quarry dust, coarse aggregate, cement and water, so the coefficients of the polynomial is of 4,2 polynomial and is as shown in Fig. 1.

At the vertex A the value of \(X_1 = 1, \) and \(X_2 = X_3 = X_4 = 0\) and similarly

At B \(X_2 = 1, \) and \(X_1 = X_3 = X_4 = 0\)

At C \(X_3 = 1, \) and \(X_1 = X_2 = X_4 = 0\)

At D \(X_4 = 1, \) and \(X_1 = X_2 = X_3 = 0\)

While at the midpoint between vortex A and B, \(X_1 X_2 X_3 \) and \(X_4 \) is \(1/2, 1/2, 0, 0\) respectively. Similarly at midpoints between A and C, A and D, B and C, B and D, C and D, give respectively
A and C $X_1, X_2, X_3$ and $X_4$ is $1/2, 0, 1/2, 0$
A and D $X_1, X_2, X_3$ and $X_4$ is $1/2, 0, 1/2$
B and C $X_1, X_2, X_3$ and $X_4$ is $0, 1/2, 1/2, 0$
B and D $X_1, X_2, X_3$ and $X_4$ is $0, 1/2, 0, 1/2$
C and D $X_1, X_2, X_3$ and $X_4$ is $0, 0, 1/2, 1/2$

Now let’s designate $Y_i$ as $n_i$ and $Y_{ij}$ as $n_{ij}$, where $n_i$ is the response to pure components and $n_{ij}$ is the response to mixture components $i$ and $j$. Remember, if $X_i = 1$ and $X_j = 0$, since $j \neq i$ then

$$n_i = \beta_i$$  \hspace{1cm} 22

Equation 1 implies that the coefficient $\beta_i$ and $n_i$ are the responses to the pure components which means that equation 22 can be written as

$$\sum_{i=1}^{4} \beta_i X_i = \sum_{i=1}^{4} n_i X_i$$  \hspace{1cm} 23

Now substituting the response values into equation 21 give respectively

$$\begin{cases}
n_1 = \beta_1 \\
n_2 = \beta_2 \\
n_3 = \beta_3 \\
n_4 = \beta_4
\end{cases}$$  \hspace{1cm} 24

In general equation 24 can be summary and written as

$$n_i = \beta_1$$  \hspace{1cm} 25

In a similar manner, the values of the midpoints between $X_1, X_2, X_3$ and $X_4$ can be substitute into equation 25 to give respectively

$$\begin{cases}
n_{12} = \frac{1}{2} \beta_1 + \frac{1}{2} \beta_2 + \frac{1}{4} \beta_{12} \\
n_{13} = \frac{1}{2} \beta_1 + \frac{1}{2} \beta_3 + \frac{1}{4} \beta_{13} \\
n_{14} = \frac{1}{2} \beta_1 + \frac{1}{2} \beta_4 + \frac{1}{4} \beta_{14} \\
n_{23} = \frac{1}{2} \beta_2 + \frac{1}{2} \beta_3 + \frac{1}{4} \beta_{23} \\
n_{24} = \frac{1}{2} \beta_2 + \frac{1}{2} \beta_4 + \frac{1}{4} \beta_{24} \\
n_{34} = \frac{1}{2} \beta_3 + \frac{1}{2} \beta_4 + \frac{1}{4} \beta_{34}
\end{cases}$$  \hspace{1cm} 26

Equation 26 can be summarized and be written as
\[ n_{ij} = \frac{1}{2} \beta_i + \frac{1}{2} \beta_j + \frac{1}{4} \beta_{ij} \]  

27

Rearranging equation 25 and equation 27 give

\[ \beta_i = n_i \]  

28

\[ \beta_{ij} = 4n_{ij} - 2\beta_i - 2\beta_j \]  

29

Equation 29 can further be written as

\[ \beta_{ij} = 4n_{ij} - 2n_i - 2n_j \]  

30

Substituting equation 28 and equation 30 into equation 18 yields

\[ Y = n_1X_1(1 - 2X_2 - 2X_3 - 2X_4) + n_2X_2(1 - 2X_1 - 2X_3 - 2X_4) + n_3X_3(1 - 2X_1 - 2X_2 - 2X_4) + n_4X_4(1 - 2X_1 - 2X_2 - 2X_3) + 4X_1X_2n_{12} + 4X_1X_3n_{13} + 4X_1X_4n_{14} + 4X_2X_3n_{23} + 4X_2X_4n_{24} + 4X_3X_4n_{34} \]  

31

Recalling equation 6 gives

\[ X_1 + X_2 + X_3 + X_4 = 1 \]  

6

Multiplying equation 6 by 2 gives

\[ 2X_1 + 2X_2 + 2X_3 + 2X_4 = 2 \]  

32

Subtracting 1 from equation 32 (both RHS and LHS) gives

\[ 2X_1 + 2X_2 + 2X_3 + 2X_4 - 1 = 1 \]  

33

Rearranging equation 33 gives

\[ 2X_1 - 1 = 1 - 2X_2 - 2X_3 - 2X_4 \]  

34

Similarly

\[
\begin{align*}
2X_2 - 1 &= 1 - 2X_1 - 2X_3 - 2X_4 \\
2X_3 - 1 &= 1 - 2X_1 - 2X_2 - 2X_4 \\
2X_4 - 1 &= 1 - 2X_1 - 2X_2 - 2X_3
\end{align*}
\]  

35

Substituting equation 34 and 35 into equation 31 yields

\[ Y = n_1X_1(2X_1 - 1) + n_2X_2(2X_2 - 1) + n_3X_3(2X_3 - 1) + n_4X_4(2X_4 - 1) + 4X_1X_2n_{12} + 4X_1X_3n_{13} + 4X_1X_4n_{14} + 4X_2X_3n_{23} + 4X_2X_4n_{24} + 4X_3X_4n_{34} \]  

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Equation 36 is the final Scheffes optimization equation for the mixture.
Table 1. Mixture Proportions for Actual and Pseudo Components.

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 1.1. N | 1.2. X₁ | 1.3. X₂ | 1.4. X₃ | 1.5. X₄ | 1.6. RESPONSE | 1.7. Z₁ | 1.8. Z₂ | 1.9. Z₃ | 1.10. Z₄ |
| 111. N₁ | 1.12. 1 | 1.13. 0 | 1.14. 0 | 1.15. 0 | 1.16. n₁ | 1.17. 0.45 | 1.18. 1 | 1.19. 1 | 1.20. 2.5 |
| 112. N₂ | 1.22. 0 | 1.23. 1 | 1.24. 0 | 1.25. 0 | 1.26. n₂ | 1.27. 0.5 | 1.28. 1 | 1.29. 1.5 | 1.30. 3 |
| 113. N₃ | 1.32. 0 | 1.33. 0 | 1.34. 1 | 1.35. 0 | 1.36. n₃ | 1.37. 0.55 | 1.38. 1 | 1.39. 2 | 1.40. 4 |
| 114. N₄ | 1.42. 0 | 1.43. 0 | 1.44. 0 | 1.45. 1 | 1.46. n₄ | 1.47. 0.6 | 1.48. 1 | 1.49. 3 | 1.50. 6 |
| 115. N₅ | 1.52. 0.5 | 1.53. 0.5 | 1.54. 0 | 1.55. 0 | 1.56. n₅ | 1.57. 0.475 | 1.58. 1 | 1.59. 1.25 | 1.60. 2.75 |
| 116. N₆ | 1.62. 0.5 | 1.63. 0 | 1.64. 0.5 | 1.65. 0 | 1.66. n₆ | 1.67. 0.5 | 1.68. 1 | 1.69. 1.5 | 1.70. 3.25 |
| 117. N₇ | 1.72. 0.5 | 1.73. 0 | 1.74. 0 | 1.75. 0.5 | 1.76. n₇ | 1.77. 0.525 | 1.78. 1 | 1.79. 2 | 1.80. 4.25 |
| 118. N₈ | 1.82. 0 | 1.83. 0.5 | 1.84. 0.5 | 1.85. 0 | 1.86. n₈ | 1.87. 0.525 | 1.88. 1 | 1.89. 1.75 | 1.90. 3.5 |
| 119. N₉ | 1.92. 0 | 1.93. 0.5 | 1.94. 0 | 1.95. 0.5 | 1.96. n₉ | 1.97. 0.55 | 1.98. 1 | 1.99. 2.25 | 1.100. 4.5 |
| 1101. N₃₄ | 1.102. 0 | 1.103. 0 | 1.104. 0.5 | 1.105. 0.5 | 1.106. n₃₄ | 1.107. 0.575 | 1.108. 1 | 1.109. 2.5 | 1.110. 5.0 |
Table 2. Mixture Proportions at the Control points showing the Actual and Pseudo Components.

| N  | X_1 | X_2 | X_3 | X_4 | RESPONSE | Z_1 | Z_2 | Z_3 | Z_4 |
|----|-----|-----|-----|-----|----------|-----|-----|-----|-----|
| 1.121 | 1.122. 0.25 | 1.123. 0.25 | 1.124. 0.25 | 1.125. 0.25 | 1.126. n_1 | 1.127. 0.525 | 1.128. 1 | 1.129. 1.875 | 1.130. 3.875 |
| 1.131 | 1.132. 0.4 | 1.133. 0.4 | 1.134. 0.2 | 1.135. 0 | 1.136. n_2 | 1.137. 0.49 | 1.138. 1 | 1.139. 1.4 | 1.140. 3 |
| 1.141 | 1.142. 0.4 | 1.143. 0 | 1.144. 0.2 | 1.145. 0.4 | 1.146. n_3 | 1.147. 0.53 | 1.148. 1 | 1.149. 2 | 1.150. 4.2 |
| 1.151 | 1.152. 0 | 1.153. 0.4 | 1.154. 0.4 | 1.155. 0.2 | 1.156. n_4 | 1.157. 0.54 | 1.158. 1 | 1.159. 2 | 1.160. 4 |
| 1.161 | 1.162. 0.2 | 1.163. 0.4 | 1.164. 0 | 1.165. 0.4 | 1.166. n_{12} | 1.167. 0.53 | 1.168. 1 | 1.169. 2 | 1.170. 4.1 |
| 1.171 | 1.172. 0.3 | 1.173. 0.3 | 1.174. 0.2 | 1.175. 0.2 | 1.176. n_{13} | 1.177. 0.515 | 1.178. 1 | 1.179. 1.75 | 1.180. 3.65 |
| 1.181 | 1.182. 0.2 | 1.183. 0.2 | 1.184. 0.3 | 1.185. 0.3 | 1.186. n_{14} | 1.187. 0.535 | 1.188. 1 | 1.189. 2 | 1.190. 4.1 |
| 1.191 | 1.192. 0.2 | 1.193. 0.3 | 1.194. 0.2 | 1.195. 0.3 | 1.196. n_{23} | 1.197. 0.53 | 1.198. 1 | 1.199. 1.95 | 1.200. 4 |
| 1.201 | 1.202. 0.3 | 1.203. 0.2 | 1.204. 0.2 | 1.205. 0.3 | 1.206. n_{24} | 1.207. 0.525 | 1.208. 1 | 1.209. 1.9 | 1.210. 3.95 |
| 1.211 | 1.212. 0.25 | 1.213. 0.25 | 1.214. 0.5 | 1.215. 0 | 1.216. n_{34} | 1.217. 0.5125 | 1.218. 1 | 1.219. 1.625 | 1.220. 3.375 |
### Table 3. Compressive strength at 28 days.

| Points of observation | Replica 1 (KN) | Replica 2 (KN) | Replica 3 (KN) | Mean Cube strength (KN) | Mean cube strength (N/mm²) | Predicted cube strength (N/mm²) |
|-----------------------|----------------|----------------|----------------|-------------------------|-----------------------------|--------------------------------|
| n₁                    | 1.229. 470.77  | 1.230. 483.50  | 1.231. 463.36  | 1.232. 472.54           | 1.233. 21.00               | 1.234. 21.00                   |
| n₂                    | 1.236. 287.80  | 1.237. 246.75  | 1.238. 313.47  | 1.239. 282.68           | 1.240. 12.56               | 1.241. 12.56                   |
| n₃                    | 1.243. 265.90  | 1.244. 245.53  | 1.245. 258.33  | 1.246. 256.59           | 1.247. 11.04               | 1.248. 11.04                   |
| n₄                    | 1.250. 201.70  | 1.251. 198.53  | 1.252. 167.83  | 1.253. 189.35           | 1.254. 8.42                | 1.255. 8.42                    |
| n₁₂                   | 1.257. 397.69  | 1.258. 368.91  | 1.259. 345.60  | 1.260. 370.73           | 1.261. 16.48               | 1.262. 16.48                   |
| n₁₃                   | 1.264. 200.80  | 1.265. 190.91  | 1.266. 167.48  | 1.267. 186.28           | 1.268. 8.28                | 1.269. 8.28                    |
| n₁₄                   | 1.271. 185.23  | 1.272. 176.26  | 1.273. 182.87  | 1.274. 181.45           | 1.275. 8.06                | 1.276. 8.06                    |
| n₂₃                   | 1.278. 249.69  | 1.279. 190.26  | 1.280. 231.39  | 1.281. 244.61           | 1.282. 10.87               | 1.283. 10.87                   |
| n₂₄                   | 1.285. 250.37  | 1.286. 248.61  | 1.287. 265.55  | 1.288. 254.84           | 1.289. 11.33               | 1.290. 11.33                   |
| n₃₄                   | 1.292. 245.77  | 1.293. 236.79  | 1.294. 230.70  | 1.295. 238.75           | 1.296. 10.61               | 1.297. 10.61                   |
| C₁                    | 1.299. 360.67  | 1.300. 392.64  | 1.301. 350.64  | 1.302. 367.98           | 1.303. 10.36               | 1.304. 9.78                    |
| C₂                    | 1.306. 214.32  | 1.307. 204.71  | 1.308. 193.22  | 1.309. 204.08           | 1.310. 9.07                | 1.311. 12.666                  |
| C₃                    | 1.313. 273.89  | 1.314. 225.04  | 1.315. 262.73  | 1.316. 255.59           | 1.317. 11.36               | 1.318. 7.525                   |
| C₄                    | 1.320. 243.89  | 1.321. 221.04  | 1.322. 237.87  | 1.323. 234.27           | 1.324. 10.41               | 1.325. 11.079                  |
| C₁₂                   | 1.327. 316.16  | 1.328. 377.64  | 1.329. 329.64  | 1.330. 341.15           | 1.331. 11.16               | 1.332. 10.906                  |
| C₁₃                   | 1.334. 273.67  | 1.335. 287.41  | 1.336. 322.64  | 1.337. 294.57           | 1.338. 13.09               | 1.339. 10.518                  |
| C₁₄                   | 1.341. 273.43  | 1.342. 327.98  | 1.343. 258.58  | 1.344. 286.66           | 1.345. 12.74               | 1.346. 9.344                   |
| C₂₃                   | 1.348. 249.23  | 1.349. 359.35  | 1.350. 290.92  | 1.351. 299.83           | 1.352. 13.33               | 1.353. 10.086                  |
| C₂₄                   | 1.355. 354.35  | 1.356. 204.28  | 1.357. 388.15  | 1.358. 315.59           | 1.359. 12.03               | 1.360. 9.486                   |
| C₃₄                   | 1.362. 199.17  | 1.363. 165.48  | 1.364. 183.46  | 1.365. 182.70           | 1.366. 10.12               | 1.367. 9.5                     |

Note: the cube strength in N/mm² is derived from dividing the force by $150 \times 150$ mm².
4. MATERIALS AND METHODS

The quarry dust used here was collected from quarry industry in Umuoghara in Abakaliki, Ebonyi state in Nigeria. The fine aggregate was washed thoroughly to remove unwanted debris and later dried; the quarry dust was later graded in accordance with BS 812 part 1:1995.

The coarse aggregate was crushed granite chippings of nominal size 20mm produce from Umuoghara quarry industrial site. The ordinary Portland cement used was Dangote cement which conformed to BS 12. The cement was well protected from dampness to avoid lumps. The water used was portable tap water from Imo State Water Board fit for domestic consumption. The quarry dust used was passed through sieve (2mm) and retained in sieve (150μm) and the coarse aggregate used was passed through sieve (25mm) and retained in sieve (20mm).

The batching of the concrete was carried out by weighing the different constitutes materials based on ten different mix ratios.

The materials were mixed thoroughly before adding the prescribed quantity of water and then mixed further to produce fresh concrete. The fresh concrete produced was filled into a cone and the slump obtained. The freshly mixed concrete was remixed and then filled into moulds in approximately 50 mm layers with each layer given 25 strokes of the tempering rod. The concrete was towelled off level with the top of the mould and the specimen stored under damp sack for 24 hours in the laboratory before de-moulding and cured for 28 days. A total of 120 cubes were produced.

The testing of the hardened cubes was carried out after 28 days, using a compressive testing machine. Load on the cube were applied at the rate of 15 N/mm² per minute in accordance with BS 1881 part 116,(1993).

5. RESULTS AND ANALYSIS

The compressive strength at 28 days obtained from the laboratory is as shown in Table 3

5.1 Scheffe’s Mix Model for the actual value

Substituting the values of the ten responses \(n_1, n_2, n_3, n_4, n_{12}, n_{13}, n_{14}, n_{23}, n_{24}, \) and \(n_{34}\) into equation 37 gives

\[
Y = 21X_1(2X_1 - 1) + 12.56X_2(2X_2 - 1) + 11.04X_3(2X_3 - 1) + 8.42X_4(2X_4 - 1) \\
+ 65.92X_1X_2 + 33.12X_1X_3 + 32.24X_1X_4 + 43.48X_2X_3 + 45.32X_2X_4 \\
+ 42.44X_3X_4 + e
\]
| 1.368. Response | 1.369. $Y_e$ | 1.370. $Y_m$ | 1.371. $D_i = Y_m - Y_e$ | 1.372. $D_A - D_i$ | 1.373. $(D_A - D_i)^2$ |
|-----------------|--------------|--------------|-----------------|----------------|------------------|
| 1.374. C1       | 1.375. 10.36 | 1.376. 9.78  | 1.377. -0.58    | 1.378. -0.698  | 1.379. 0.487204  |
| 1.380. C2       | 1.381. 9.07  | 1.382. 12.666| 1.383. 3.596    | 1.384. -4.874  | 1.385. 23.75588 |
| 1.386. C3       | 1.387. 11.36 | 1.388. 7.525 | 1.389. -3.835   | 1.390. 2.557   | 1.391. 6.538249  |
| 1.392. C4       | 1.393. 10.41 | 1.394. 11.079| 1.395. 0.669    | 1.396. -1.947  | 1.397. 3.790809  |
| 1.398. C5       | 1.399. 11.16 | 1.400. 10.906| 1.401. -0.254   | 1.402. -1.024  | 1.403. 1.048576  |
| 1.404. C6       | 1.405. 13.09 | 1.406. 10.518| 1.407. -2.572   | 1.408. 1.294   | 1.409. 1.674436  |
| 1.410. C7       | 1.411. 12.74 | 1.412. 9.344 | 1.413. -3.396   | 1.414. 2.118   | 1.415. 4.485924  |
| 1.416. C8       | 1.417. 13.33 | 1.418. 10.086| 1.419. -3.244   | 1.420. 1.966   | 1.421. 3.865156  |
| 1.422. C9       | 1.423. 12.03 | 1.424. 9.486 | 1.425. -2.544   | 1.426. 1.266   | 1.427. 1.602756  |
| 1.428. C10      | 1.429. 10.12 | 1.430. 9.5   | 1.431. -0.62    | 1.432. -0.658  | 1.433. 0.432964  |

1.434. $\sum D_i$  
1.435. $\sum D_i$  
1.436. $\sum (D_A - D_i)$  
1.437. $(D_A - D_i)^2$  
1.438. $47.68195$  
1.439. $D_A = \sum D_i / N$  
1.440. $-1.278$  
1.441. $S^2 = \sum (D_A - D_i)^2 / (N - 1)$  
1.442. $5.297994$  
1.443. $S = \sqrt{S^2}$  
1.444. $2.301737$  
1.445. $T = D_A \times N^{0.5} / S$  
1.446. $1.7558$  

Table 4. Student-Statistical T-Test Sheffe’s Model for Quarry Dust (Two-Tailed T-Test).

t from the table (Appendix E) is given as $t_\alpha (V) = t_{0.05(9)} = 1.833$, and calculated $t = 1.756$. Therefore, $t$ from the table is higher than $t$ calculated; so the difference between the lab result and the model result is insignificant.
Table 5. F-Statistical Test for Scheffe’s Model for Quarry Dust.

LEGEND: \( \bar{Y}_P = \sum Y_P / N \), \( \bar{Y}_M = \sum Y_M / N \), where \( N = 10 \)

| 1.447. Response | 1.448. \( Y_P \) | 1.449. \( Y_M \) | 1.450. \( Y_P - \bar{Y}_P \) | 1.451. \( Y_M - \bar{Y}_M \) | 1.452. \( (Y_P - \bar{Y}_P)^2 \) | 1.453. \( (Y_M - \bar{Y}_M)^2 \) |
|-----------------|----------------|----------------|---------------------|---------------------|----------------|----------------|
| 1.454. C1       | 1.455. 10.36   | 1.456. 9.78    | 1.457. -1.007       | 1.458. -0.309       | 1.459. 1.014049| 1.460. 0.095481|
| 1.461. C2       | 1.462. 9.07    | 1.463. 12.666  | 1.464. -2.297       | 1.465. 2.577        | 1.466. 5.276209| 1.467. 6.640929|
| 1.468. C3       | 1.469. 11.36   | 1.470. 7.525   | 1.471. -0.007       | 1.472. -2.564       | 1.473. 4.9E-05 | 1.474. 6.574096|
| 1.475. C4       | 1.476. 10.41   | 1.477. 11.079  | 1.478. -0.957       | 1.479. 0.99         | 1.480. 0.915849| 1.481. 0.9801 |
| 1.482. C5       | 1.483. 11.16   | 1.484. 10.906  | 1.485. -0.207       | 1.486. 0.817        | 1.487. 0.042849| 1.488. 0.667489|
| 1.489. C6       | 1.490. 13.09   | 1.491. 10.518  | 1.492. 1.723        | 1.493. 0.429        | 1.494. 2.968729| 1.495. 0.184041|
| 1.496. C7       | 1.497. 12.74   | 1.498. 9.344   | 1.499. 1.373        | 1.500. -0.745       | 1.501. 1.885129| 1.502. 0.555025|
| 1.503. C8       | 1.504. 13.33   | 1.505. 10.086  | 1.506. 1.963        | 1.507. -0.003       | 1.508. 3.853369| 1.509. 9E-06 |
| 1.510. C9       | 1.511. 12.03   | 1.512. 9.486   | 1.513. 0.663        | 1.514. -0.603       | 1.515. 0.439569| 1.516. 0.363609|
| 1.517. C10      | 1.518. 10.12   | 1.519. 9.5     | 1.520. -1.247       | 1.521. -0.589       | 1.522. 1.555009| 1.523. 0.346921|
| 1.524. Total    | 1.526. 113.67  | 1.527. 100.89  |                     | 1.528.              |                | 1.529. 17.95081| 1.530. 16.4077|
| 1.525.          |                |                |                     |                    |                |                |
| 1.531. Mean     | 1.532. 11.367  | 1.533. 10.089  |                     |                    |                |                |

\[
S_P^2 = \frac{\sum (Y_P - \bar{Y}_P)^2}{N - 1} = \frac{17.95081}{9} = 1.99
\]

\[
S_M^2 = \frac{\sum (Y_M - \bar{Y}_M)^2}{N - 1} = \frac{16.4077}{9} = 1.82
\]

\( S_2^2 \) is the greater of \( S_P^2 \) and \( S_M^2 \), and \( S_1^2 \) is the smaller of the two values. So \( S_P^2 = 1.99 \) and \( S_M^2 = 1.82 \). The \( F_{calculated} = S_1^2/S_2^2 = 1.99/1.82 = 1.09 \).
From statistical tables \((V_1, V_2) = t_\alpha(9,9)\), from Appendix F, \(F_{0.05(9,9)} = 3.18\). \(1/F_{table} = 1/3.18 = 0.314\). The Null Hypothesis will be accepted if \(1/F_{table} < F_{calculated} < F_{table}\); \(0.314 < 1.82 < 3.18\). The Null Hypothesis is accepted and the model is adequate for use.

6. CONCLUSION

The Scheffes optimization model for the prediction of compressive strength of concrete made with quarry dust was developed. With the model equation, it will be possible to predict for a given compressive strength the mix ratios associated with it, and for a given mix ratios the compressive strength associated with them. The predictions for the model were tested at 95% accuracy level using Student-statistical-T test and F-statistical Test and they were found to be adequate.

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