Two-Photon and Two-gluon Decays of
0\(^{++}\) and 2\(^{++}\) P-wave Heavy Quarkonium States

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By neglecting the relative quark momenta in the propagator term, the two-photon and two-gluon decay amplitude of heavy quarkonia states can be written as a local heavy quark field operator matrix element which could be obtained from other processes or computed with QCD sum rules technique or lattice simulation, as shown in a recent work on \(\eta_c,b\) two-photon decays. In this talk, I would like to discuss a similar calculation on P-wave \(\chi_{c,0,2}\) and \(\chi_{b,0,2}\) two-photon decays. We show that the effective Lagrangian for the two-photon decays of the P-wave \(\chi_{c,0,2}\) and \(\chi_{b,0,2}\) is given by the heavy quark energy-momentum tensor local operator and its trace, the \(\bar{Q}Q\) scalar density. A simple expression for \(\chi_{c,0}\) two-photon and two-gluon decay rate in terms of the \(f_{\chi_{c,0}}\) decay constant, similar to that of \(\eta_c\) is obtained. From the existing QCD sum rules value for \(f_{\chi_{c,0}}\), we get 5 keV for the \(\chi_{c,0}\) two-photon width, somewhat larger than measurement.

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I. INTRODUCTION

First of all, I would like to dedicate this talk to the memory of Professor Giuseppe Nardulli, who, with great kindness and generosity has initiated the long and fruitful collaboration I have with the members of the Physics Department and INFN at the University of Bari.

In the non-relativistic bound state calculation [1, 2], the two-photon and two-gluon decay rates for P-wave quarkonium states depend on the derivative of the spatial wave function at the origin which has to be extracted from potential models, unlike the two-photon decay rate of S-wave \(\eta_c\) and \(\eta_b\) quarkonia which can be predicted from the corresponding \(J/\psi\) and \(\Upsilon\) leptonic widths using heavy quark spin symmetry (HQSS) [3], there is no similar prediction for the P-wave \(\chi_c\) and \(\chi_b\) states and all the existing theoretical values for the decay rates are based on potential model

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calculations [1, 4–14].

Since the matrix element of a heavy quark local operator between the vacuum and P-wave quarkonium state is also given, in bound state description, by the derivative of the spatial wave function at the origin, one could express the P-wave quarkonium two-photon and two-gluon decay amplitudes in terms of the matrix element of a local operator with the appropriate quantum number, like the heavy quark \( \bar{Q}Q \) scalar density or axial vector current \( \bar{Q}\gamma_\mu \gamma_5 Q \). We have thus an effective Lagrangian for the two-photon and two-gluon decays of P-wave quarkonia in terms of heavy quark field operator instead of the traditional bound state description in terms of the wave function. This effective Lagrangian can be derived in a simple manner by neglecting the relative quark momentum in the heavy quark propagator as in non-relativistic bound state calculation. In this talk, I would like to report on a recent work [15] using the effective Lagrangian approach to describe the two-photon and two gluon decays of P-wave heavy quarkonia state, similar to that for S-wave quarkonia [3]. This was stimulated by the recent new CLEO measurements [16, 17] of the two-photon decay rates of the charmonium P-wave 0\(^{++}\), \( \chi_{c0} \) and 2\(^{++}\), \( \chi_{c2} \) states. We obtain an effective Lagrangian for P-wave quarkonium decays in terms of the heavy quark energy-momentum tensor and its trace and that the two-photon and two-gluon decay rates of \( \chi_{c0,2} \) and \( \chi_{b0,2} \) can be expressed in terms of the decay constants \( f_{\chi_{c0}} \) and \( f_{\chi_{b0}} \), similar to that for \( \eta_c \) and \( \eta_b \), which are given, respectively, by \( f_{\eta_c} \) and \( f_{\eta_b} \). Then a calculation of \( f_{\chi_{c0}} \) and \( f_{\chi_{b0}} \) by sum rules technique [18, 19] or lattice simulation [20, 21] would give us a prediction of the P-wave quarkonia decay rates. In fact, as shown below, \( f_{\chi_{c0}} \) obtained in [18] implies a value of 5 keV for the \( \chi_{c0} \) two-photon width, somewhat larger than measurement. In the following I will present only the main results, as more details are given in the published paper [15].

![FIG. 1: Diagrams for \( \bar{Q}Q \) annihilation to two photons.](image)

II. EFFECTIVE LAGRANGIAN FOR \( \chi_{c0,2} \rightarrow \gamma\gamma \) AND \( \chi_{b0,2} \rightarrow \gamma\gamma \)

By neglecting term containing the relative quark momenta \( q \) in the quark propagator [25] (\( Q_{c,b}^2 \) being the heavy quark charge), the P-wave part of the \( c\bar{c} \rightarrow \gamma\gamma, gg \) and \( b\bar{b} \rightarrow \gamma\gamma, gg \) amplitudes
represented by diagrams in Fig. 1 are
\[ M(Q\bar{Q} \to \gamma\gamma) = -e^2 Q^2 c,b A_{\mu\nu} \bar{v}(p_2) T_{\mu\nu} u(p_1) \]
with \( A_{\mu\nu} \) the photon part of the amplitude and the heavy quark part \( T_{\mu\nu} \) given by
\[
A_{\mu\nu} = -2\epsilon_1 \cdot k_2 \epsilon_2 k_{1\nu} + 2\epsilon_1 \cdot \epsilon_2 k_{2\mu} k_{1\nu} \\
T_{\mu\nu} = (q_{1\mu} - q_{2\mu}) \gamma_\nu
\]
which can be obtained directly from the following effective Lagrangian for two-photon and two-gluon decay of \( P \)-wave heavy quarkonia states
\[
\mathcal{L}_{\text{eff}}(Q\bar{Q} \to \gamma\gamma) = -ic_1 A_{\mu\nu} \bar{Q}(\vec{\partial}_\mu - \vec{\partial}_\nu)\gamma_\nu Q
\]
\[
c_1 = -e^2 Q^2 c,b [(k_1 - k_2)^2 / 4 - m_Q^2]^{-2}
\]
With the matrix element of \( \theta_{Q\mu\nu} = \bar{Q}(\vec{\partial}_\mu - \vec{\partial}_\nu)\gamma_\nu Q \) between the vacuum and \( \chi_{c0,2} \) or \( \chi_{b0,2} \) given by \((Q^2 = M^2)\)
\[
<0|\theta_{Q\mu\nu}|\chi_0> = T_0 M^2 (-g_{\mu\nu} + Q_\mu Q_\nu / M^2), \\
<0|\theta_{Q\mu\nu}|\chi_2> = -T_2 M^2 \epsilon_{\mu\nu}.
\]
The two-photon decay amplitudes are then easily obtained:
\[
M(\chi_0 \to \gamma\gamma) = -e^2 Q^2 c,b \frac{T_0 A_0}{[M^2/4 + m_Q^2]^2}
\]
\[
M(\chi_2 \to \gamma\gamma) = -e^2 Q^2 c,b \frac{T_2 A_2}{[M^2/4 + m_Q^2]^2}
\]
with \( T_2 = \sqrt{3} T_0 \) from HQSS and
\[
A_0 = \left(\frac{3}{2}\right) M^2 (M^2 \epsilon_1 \cdot \epsilon_2 - 2\epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1)
\]
\[
A_2 = M^2 \epsilon_{\mu\nu} [M^2 \epsilon_{1\mu} \epsilon_{2\nu} - 2(\epsilon_1 \cdot k_2 \epsilon_{2\mu} k_{1\nu} + \epsilon_2 \cdot k_1 \epsilon_{1\mu} k_{2\nu} \\
+ \epsilon_1 \cdot \epsilon_2 k_{1\mu} k_{2\nu})]
\]
For QCD sum rules calculation or lattice simulation, it is simpler to compute the trace of the energy-momentum tensor \( \theta_{Q\mu\nu} \) given by \( 2m_Q \bar{Q}Q \). We have then
\[
\bar{v}(p_2) T_{\mu\nu} u(p_1) = 2m_Q \bar{v}(p_2) u(p_1)
\]
The problem of computing the two-photon or two-gluon decay amplitude of $\chi_{c0,2}$ and $\chi_{b0,2}$ states is reduced to computing the decays constants $f_{\chi_{c0}}$ and $f_{\chi_{b0}}$ defined as

$$< 0 | \bar{Q} Q | \chi_0 > = m_\chi_0 f_{\chi_0}$$

Thus $T_0$ is given directly in terms of $f_{\chi_0}$ without using the derivative of the $P$-wave spatial wave function at the origin.

$$T_0 = \frac{f_{\chi_0}}{3}$$

Thus by comparing the expression for $\chi_{c0}$ and $\eta_c$ we could already have some estimate for the $\chi_{c0}$ two-photon and two-gluon decay rates. For $f_{\chi_{c0}}$ of $O(f_{\eta_c})$, one would expect $\Gamma_{\gamma\gamma}(\chi_{c0})$ to be in the range of a few keV.

The decay rates of $\chi_{c0,2}$, $\chi_{b0,2}$ states can now be obtained in terms of the decay constant $f_{\chi_0}$. We have:

$$\Gamma_{\gamma\gamma}(\chi_{c0}) = \frac{4\pi Q^4 \alpha^2_{em} M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}} + b)^4} \left[ 1 + B_0 (\alpha_s/\pi) \right]$$

$$\Gamma_{\gamma\gamma}(\chi_{c2}) = \left( \frac{4}{15} \right) \frac{4\pi Q^4 \alpha^2_{em} M_{\chi_{c2}}^3 f_{\chi_{c2}}^2}{(M_{\chi_{c2}} + b)^4} \left[ 1 + B_2 (\alpha_s/\pi) \right]$$

where $B_0 = \pi^2/3 - 28/9$ and $B_2 = -16/3$ are NLO QCD radiative corrections [22, 24].

This expression is similar to that for $\eta_c$:

$$\Gamma_{\gamma\gamma}(\eta_c) = \frac{4\pi Q^4 \alpha^2_{em} M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c} + b)^2} \left[ 1 - \frac{\alpha_s (20 - \pi^2)}{\pi} \right]$$

The two-gluon decay rates are:

$$\Gamma_{gg}(\chi_{c0}) = \left( \frac{2}{9} \right) \frac{4\pi \alpha_s^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}} + b)^4} \left[ 1 + C_0 (\alpha_s/\pi) \right]$$

$$\Gamma_{gg}(\chi_{c2}) = \left( \frac{4}{15} \right) \left( \frac{2}{9} \right) \frac{4\pi \alpha_s^2 M_{\chi_{c2}}^3 f_{\chi_{c2}}^2}{(M_{\chi_{c2}} + b)^4} \left[ 1 + C_2 (\alpha_s/\pi) \right]$$

where $C_0 = 8.77$ and $C_2 = -4.827$ are NLO QCD radiative corrections. As with the two-photon decay rates, the expressions for two-gluon decay rates are similar to that for $\eta_c$:

$$\Gamma_{gg}(\eta_c) = \left( \frac{2}{9} \right) \frac{4\pi \alpha_s^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c} + b)^2} \left[ 1 + 4.8 \frac{\alpha_s}{\pi} \right]$$

In a bound state calculation, using the relativistic spin projection operator [25, 26], $f_{\eta_c}$ and $f_{\chi_0}$ are given by

$$f_{\eta_c} = \sqrt{\frac{3}{32 \pi m_Q^3}} R_0(0) (4 m_Q)$$

$$f_{\chi_0} = 12 \sqrt{\frac{3}{(8 \pi m_Q^3)}} \left( \frac{R_1^2(0)}{M} \right)$$
which gives the decay amplitudes in agreement with the original calculation \[1\].

Comparing with \(f_{\eta_c}\), we have

\[
f_{\chi_c} = 6 \left( \frac{R'_1(0)}{R_0(0)M} \right) f_{\eta_c},
\]

which becomes comparable to \(f_{\eta_c}\).

Thus by comparing the expression for \(\chi_{c0}\) and \(\eta_c\) we could already have some estimate for the \(\chi_{c0}\) two-photon and two-gluon decay rates. For \(f_{\chi_{c0}}\) of \(O(f_{\eta_c})\), one would expect \(\Gamma_{\gamma\gamma}(\chi_{c0})\) to be in the range of a few keV. As shown in Table 1, the predicted two-photon width of \(\chi_{c0}\) from the sum rules value \(f_{\chi_{c0}} = 357\,\text{MeV} \rangle_{18}\) is however almost twice the CLEO value, but possibly with large theoretical uncertainties in sum rules calculation as to be expected, while a recent calculation \[27\] implies a larger decay rates for \(\chi_{c0}\). The measured ratio \(\Gamma_{\gamma\gamma}(\chi_{c2})/\Gamma_{\gamma\gamma}(\chi_{c0})\) is then \(\approx 0.24 \pm 0.09\), somewhat bigger than the predicted value of about 0.14 as shown in Table 1 together with the CLEO measurement of the decay rates \[10\] which gives \((2.53 \pm 0.37 \pm 0.26)\,\text{keV}\) and \((0.60 \pm 0.06 \pm 0.06)\,\text{keV}\) for \(\chi_{c0}\) and \(\chi_{c2}\) respectively.

| Reference     | \(\Gamma_{\gamma\gamma}(\chi_{c0})(\text{keV})\) | \(\Gamma_{\gamma\gamma}(\chi_{c2})(\text{keV})\) | \(R = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})}\) |
|---------------|----------------------------------------|----------------------------------------|----------------------------------------|
| Barbieri[1]   | 3.5                                     | 0.93                                   | 0.27                                   |
| Godfrey[4]    | 1.29                                    | 0.46                                   | 0.36                                   |
| Barnes[5]     | 1.56                                    | 0.56                                   | 0.36                                   |
| Gupta[7]      | 6.38                                    | 0.57                                   | 0.09                                   |
| Münz[8]       | 1.39 \pm 0.16                           | 0.44 \pm 0.14                          | 0.32^{+0.16}_{-0.12}                  |
| Huang[9]      | 3.72 \pm 1.10                           | 0.49 \pm 0.16                          | 0.13^{+0.11}_{-0.06}                  |
| Ebert[10]     | 2.90                                    | 0.50                                   | 0.17                                   |
| Schuler[11]   | 2.50                                    | 0.28                                   | 0.11                                   |
| Crater[12]    | 3.34 – 3.96                             | 0.43 – 0.74                            | 0.13 – 0.19                            |
| Wang[13]      | 3.78                                    | –                                      | –                                      |
| Laverty[14]   | 1.99 – 2.10                             | 0.30 – 0.73                            | 0.14 – 0.37                            |
| This work     | 5.00                                    | 0.70                                   | 0.14                                   |
| Exp(CLEO)[16] | 2.53 \pm 0.37 \pm 0.26                  | 0.60 \pm 0.06 \pm 0.06                 | 0.24 \pm 0.04 \pm 0.03                 |
| Exp(Average)[16] | 2.31 \pm 0.10 \pm 0.12                | 0.51 \pm 0.02 \pm 0.02                | 0.20 \pm 0.01 \pm 0.02                |

TABLE 1: Potential model predictions for \(\chi_{c0,2}\) two-photon widths compared with this work.

The two-photon \(\chi_{c0,2}, \chi_{c0,2}'\) branching ratios are independent of \(f_{\chi_{c0}}\)

\[
\mathcal{B}(\chi_{c0}, \chi_{c0} \rightarrow \gamma\gamma) = \frac{9}{2} Q_c^4 \frac{\alpha_{em}^2}{\alpha^2} \left( 1 + (B_0 - C_0) \frac{\alpha_s}{\pi} \right),
\]

\[
\mathcal{B}(\chi_{c2}, \chi_{c2} \rightarrow \gamma\gamma) = \frac{6}{5} Q_c^4 \frac{\alpha_{em}^2}{\alpha^2} \left( 1 + (B_2 - C_2) \frac{\alpha_s}{\pi} \right)
\]
with $B_0 = \pi^2/3 - 28/9, B_2 = -16/3, C_0 = 8.77, C_2 = -4.827$. Apart from QCD radiative correction factors, the expressions for branching ratios are very similar to that for $\eta_c$ and $\eta_c'$:

$$B(\eta_c, \eta_c' \to \gamma \gamma) = \frac{9}{2} Q_c^4 \frac{\alpha_m^2}{\alpha_s^2} \left(1 - 8.2 \frac{\alpha_s}{\pi}\right)$$

with $\alpha_s$ evaluated at the appropriate scale.

For $\alpha_s = 0.26$, $B(\eta_c \to \gamma \gamma) = 3.6 \times 10^{-4}$ to be compared with the measured value of $(2.8 \pm 0.9) \times 10^{-4}$, but this prediction is rather sensitive to $\alpha_s$, for example, with $\alpha_s = 0.28$, one would get $B(\eta_c \to \gamma \gamma) = 2.95 \times 10^{-4}$, in better agreement with the measured value of $(2.4_{-0.9}^{+1.1}) \times 10^{-4}$ and for $\chi_{c0,2}$, the predicted two-photon branching ratios would be $3.45 \times 10^{-4}$ and $4.45 \times 10^{-4}$ compared with the measured values of $(2.35 \pm 0.23) \times 10^{-4}$ and $(2.43 \pm 0.18) \times 10^{-4}$, for $\chi_{c0}$ and $\chi_{c2}$ respectively. The predicted branching ratio for $\chi_{c2}$ is rather large and one would need $\alpha_s = 0.36$ to bring the predicted value closer to experiment.

Recently, the $Z(3930)$ state above $D \bar{D}$ threshold found by Belle \cite{28} with mass $(3928 \pm 5 \pm 2)$ MeV and width $(29 \pm 10{\text{(stat)}} \pm 2{\text{(sys)}})$ MeV, consistent with $\chi_{c2}'$, seems to be confirmed by the observation of a similar state by BaBar \cite{29}, with mass $(3926.7 \pm 2.7 \pm 1.1)$ MeV and width $(21.3 \pm 6.8 \pm 3.6)$ MeV. Belle \cite{28} gives $\Gamma_{\gamma \gamma}(\chi_{c2}') \times B(D \bar{D}) = (0.18 \pm 0.05 \pm 0.03)$ keV while BaBar \cite{29} gives $\Gamma_{\gamma \gamma}(\chi_{c2}') \times B(D \bar{D}) = (0.24 \pm 0.05 \pm 0.04)$ keV for this state. If taken to be the $2^P$ excited state $\chi_{c2}'$ and assuming $B(D \bar{D}) \approx 0.70 \pm 0.30 \pm 0.32$, one would get $\Gamma_{\gamma \gamma}(\chi_{c2}') = (0.18 - 0.24 \pm 0.05 \pm 0.03)$ keV. This implies $f_{\chi_{c0}} \approx (195 - 225)$ MeV and $\Gamma_{gg}(\chi_{c0}')$ in the range $(5 - 10)$ MeV.

For $\chi_{b0,2}$ potential model calculations similar to that for $\chi_{c0,2}$, gives the two-photon width about $1/10$ of that for $\eta_b$, which implies $f_{\chi_{b0}} = f_{\eta_b}/3$, smaller than Cornell potential \cite{33} value $f_{\chi_{b0}} = 0.46 f_{\eta_b}$.

### III. Remark on the $\eta_c'$ Two-Photon Decays

Since the predicted two-photon branching ratios for $\chi_{c0,2}, \chi_{c0,2}'$ and for $\eta_c, \eta_c'$ are similar and independent of the decay constants, apart from QCD radiative corrections, as seen in Eq. \cite{23, 24} and Eq. \cite{25}, one expects a large two-photon decay rates for $\eta_c'$, it would be relevant here to mention the problem of the $\eta_c' \to \gamma \gamma$ decay rate \cite{3, 35}. The small value of $\Gamma_{\gamma \gamma}(\eta_c') = (1.3 \pm 0.6)$ keV given previously by CLEO \cite{34} is obtained by assuming $B(\eta_c' \to K_S K \pi) \approx 0.096^{+0.020}_{-0.019}$ (stat) $\pm 0.025$ (syst)

$$R(\eta_c(2S)K^+/\eta_c K^+) = \frac{B(\eta_c(2S)K^+) \times B(\eta_c(2S) \to K \bar{K} \pi)}{B(B^+ \to \eta_c K^+) \times B(\eta_c \to K \bar{K} \pi)} = 0.96^{+0.020}_{-0.019} \text{(stat)} \pm 0.025 \text{(syst)}$$

(26)
and the Belle measurement \[37\]
\[
\mathcal{B}(B^+ \to \eta_c K^+) \times \mathcal{B}(\eta_c \to K^0 K\pi) = (6.88 \pm 0.77^{+0.55}_{-0.66}) \times 10^{-5}
\] (27)

BABAR obtains \[36\]
\[
\mathcal{B}(\eta_c' \to K_S K \pi) = (1.9 \pm 0.4\text{(stat)} \pm 1.1\text{(syst)})\%.
\] (28)
as quoted by CLEO \[38\]. This new BABAR value for \(\mathcal{B}(\eta_c' \to K_S K \pi)\) is considerably smaller than the corresponding value \(\mathcal{B}(\eta_c \to K_S K \pi) = (7.0 \pm 1.2)\%\) \[17\] for \(\eta_c\).

Thus with the BaBar result for \(\mathcal{B}(\eta_c' \to K_S K \pi)\) and the CLEO measurement \[34\]
\[
R(\eta_c'/\eta_c) = \frac{\Gamma(\gamma \gamma)(\eta_c') \times \mathcal{B}(\eta_c' \to K_S K \pi)}{\Gamma(\gamma \gamma)(\eta_c) \times \mathcal{B}(\eta_c \to K_S K \pi)} = 0.18 \pm 0.05 \pm 0.02
\] (29)
one would get \[38\]
\[
\Gamma(\eta_c' \to \gamma \gamma) = (4.8 \pm 3.7)\text{ keV}
\] (30)
in agreement with the predicted value
\[
\Gamma(\eta_c' \to \gamma \gamma) = (4.1 \pm 2.3)\text{ keV}
\] (31)
while the assumption of near equality of the \(K_S K\pi\) branching ratios for \(\eta_c\) and \(\eta_c'\)
\[
\mathcal{B}(\eta_c' \to K_S K \pi) \approx \mathcal{B}(\eta_c \to K_S K \pi)
\] (32)
and the Belle ratio \[39\]
\[
R(\eta_c'/\eta_c K) = \frac{\mathcal{B}(B \to K \eta_c(2S) \times \mathcal{B}(\eta_c(2S) \to K_S K^- \pi^+)}{\mathcal{B}(B^0 \to K \eta_c) \times \mathcal{B}(\eta_c \to K_S K^- \pi^+)} = 0.38 \pm 0.12 \pm 0.05
\] (33)
would lead to \[38\]
\[
\Gamma(\gamma \gamma)(\eta_c') = (1.3 \pm 0.6)\text{ keV}
\] (34)
which is rather small compared with the predicted value given in Eq. \([31]\) above.
IV. CONCLUSION

In conclusion, we have derived an effective Lagrangian for $\chi_{c0,2}$ and $\chi_{b0,2}$ two-photon and two-gluon in terms of the decay constants $f_{\chi_{c,b0}}$, similar to that for $\eta_{c,b}$ in terms of $f_{\eta_{c,b}}$.

Existing sum rules calculation, however produces a two-photon width about 5 keV, somewhat bigger than the CLEO measured value. It remains to be seen whether a better determination of $f_{\chi_{c,b}}$ from lattice simulation or QCD sum rules calculation could bring the $\chi_{c0,2}$ two-photon decay rates closer to experiments or higher order QCD radiative corrections and large relativistic corrections are needed to explain the data.

The problem of two-photon width of $\eta'_c$ would go away if more data could confirm the small BaBar value for $B(\eta'_c \to K_SK\pi)$ compared with $B(\eta_c \to K_SK\pi)$.

As relativistic corrections should be small for $P$-wave bottomia $\chi_{b0,2}$ states, two-photon and two-gluon decays could provide a test of QCD and a determination of $\alpha_s$ at the the $m_b$ mass scale.

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[1] R. Barbieri, R. Gatto and R. Kogerler, Phys. Lett. B 60, 183 (1976).
[2] For a review, see N. Brambilla et al., Heavy quarkonium physics, CERN Yellow Report, CERN-2005-005, 2005.
[3] J. P. Lansberg and T. N. Pham, Phys. Rev. D 74, 034001 (2006); ibid D 75, 017501 (2007).
[4] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[5] T. Barnes, Proceedings of the IX International Workshop on Photon-Photon Collisions, edited by D. O. Caldwell and H. P. Paar(World Scientific, Singapore, 1992) p. 263.
[6] G. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 46, R1914 (1992).
[7] S. N. Gupta, J. M. Johnson, and W. W. Repko, Phys. Rev. D 54, 2075 (1996);
[8] C. R. Münz, Nucl. Phys. A 609, 364 (1996).
[9] H.-W. Huang and K.-Ta Chao, Phys. Rev. D 54, 6850 (1996); errata Phys. Rev. D 56 1821 (1996).
[10] D. Ebert, R. N. Faustov and V. O. Galkin, Mod. Phys. Lett. A 18, (2003).
[11] G. A. Schuler, F. A. Berends, and R. van Gulik, Nucl. Phys. B 523, 423 (1998).
[12] H. W. Crater, C. Y. Wong and P. Van Alstine, Phys. Rev. D 74, 054028 (2006).
[13] G.-Li Wang, Phys. Lett. B 653, 206 (2007).
[14] J. T. Laverty, S. F. Radford, and W. W. Repko, arXiv:0901.3917 [hep-ph].
[15] J. P. Lansberg and T. N. Pham, Phys. Rev. D 79, 094016 (2009).
[16] K. M. Ecklund et al (CLEO Collaboration), Phys. Rev. D 78, 091501 (2008) and other recent results quoted therein.
[17] C. Amsler, et al, Particle Data Group, Review of Particle Physics, Phys. Lett. B 667, 1 (2008).
[18] V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Phys. Rept. 41, 1 (1978).
[19] L. J. Reinders, H. R. Rubinstein and S. Yazaki, Phys. Lett. B 113, 411 (1982).
[20] J. J. Dudek, R. G. Edwards and D. G. Richards, Phys. Rev. D 73, 074507 (2006).
[21] T. W. Chiu, T. H. Hsieh, and Ogawa [TWQCD Collaboration], Phys. Lett. B 651, 171 (2007).
[22] R. Barbieri, M. Caffo, R. Gatto and E. Remiddi, Phys. Lett. B 95, 93 (1980);
    Nucl. Phys. B 192, 61 (1981).
[23] W. Kwong, P. B. Mackenzie, R. Rosenfeld and J. L. Rosner, Phys. Rev. D 37, 3210 (1988).
[24] M. Mangano and A. Petrelli, Phys. Lett. B 352, 445 (1995).
[25] J. H. Kühn, J. Kaplan and E. G. O. Safiani, Nucl. Phys. B 157, 125 (1979).
[26] B. Guberina, J. H. Kühn, R. D. Peccei, and R. Ruckl, Nucl. Phys. B 174, 317 (1980).
[27] P. Colangelo, F. De Fazio, and T.N. Pham, Phys. Lett. B 542, 71 (2002).
[28] S. Uehara et al. (BELLE collaboration), Phys. Rev. D 96, 082003 (2006).
[29] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 81, 092003 (2010).
[30] P. Colangelo, Private Communication.
[31] O. Lakhina and E. S. Swanson, Phys. Rev. D 74, 014012 (2006).
[32] B-Q. Li and K. T. Chao, arXiv:0903.5506 [hep-ph].
[33] E. Eichten and C. Quigg, Phys. Rev. D 52, 1726 (1995).
[34] D. M. Asner et al. (CLEO Collaboration)], Phys. Rev. Lett. 92, 142001 (2004).
[35] T. N. Pham, Proceedings of the International Workshop on Quantum Chromodynamics Theory and Experiment, Martina Franca, Valle d’Itria, Italy, 16-20 June 2007, AIP Conf. Proc. 964, 124, (2007);
    J. P. Lansberg and T. N. Pham, Proceedings of the Joint Meeting Heidelberg-Li`ege-Paris-Wroclaw Hadronic Physics HLPW 2008, Spa Belgium, 6-8 March 2008, AIP Conf. Proc. 1038, 259 (2008).
[36] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 78, 012006 (2008).
[37] F. Fang et al. (BELLE collaboration), Phys. Rev. Lett. 90, 071801 (2003).
[38] D. Cronin-Hennessy et al. (CLEO Collaboration), Phys. Rev. D 81, 052002 (2010).
[39] S. K. Choi et al. (BELLE collaboration), Phys. Rev. Lett. 89, 102001 (2002).