Analytic calculations of the spectra of ultra high energy cosmic ray nuclei. II. The general case of background radiation.

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June 15, 2010

Abstract

We discuss the problem of ultra high energy nuclei propagation in extragalactic background radiations. The present paper is the continuation of the accompanying paper I where we have presented three new analytic methods to calculate the fluxes and spectra of Ultra High Energy Cosmic Ray (UHECR) nuclei, both primary and secondary, and secondary protons. The computation scheme in this paper is based on the analytic solution of coupled kinetic equations, which takes into account the continuous energy losses due to the expansion of the universe and pair-production, together with photo-disintegration of the nuclei. This method includes in the most natural way the production of secondary nuclei in the process of photo-disintegration of the primary nuclei during their propagation through extragalactic background radiations. In paper I, in order to present the suggested analytical schemes of calculations, we have considered only the case of the Cosmic Microwave Background (CMB) radiation, in the present paper we generalize this computation to all relevant background radiations, including infra-red (IR) and visible/ultra-violet radiations, collectively referred to as Extragalactic Background Light (EBL). The analytic solutions allow transparent physical interpretation of the obtained spectra. EBL plays an important role at intermediate energies of UHECR nuclei. The most noticeable effect of the EBL is the low-energy tail in the spectrum of secondary nuclei.
1 Introduction

In the accompanying paper [1], hereafter paper I, we have discussed three different analytic methods to study the propagation of Ultra High Energy Cosmic Ray (UHECR) nuclei through background radiations. In order to give a clear explanation of the analytic procedures, we have only discussed propagation in the Cosmic Microwave Background (CMB) neglecting all other relevant backgrounds. In the present paper we will extend our method including all relevant background radiations, obtaining more complete results for the expected UHE nuclei spectra. Apart from the CMB we include the interaction with Infrared (IR) and optical photons, to which we refer collectively as Extragalactic Background Light (EBL).

The importance to study the UHECR nuclei as primary radiation have been already discussed in the Introduction of paper I. It is enough to remind here again that according to recent Auger data [2], the primary UHECR at energy higher than \((3 - 4) \times 10^{18}\) eV are dominated by heavy nuclei. Propagation of nuclei through extragalactic background radiation results in a distortion of their energy spectra, different from that for UHE protons. There are many papers, cited in paper I, where propagation of UHE nuclei through CMB and EBL have been studied by Monte Carlo simulations. One of the most detailed study has been performed in [3]. In the present paper we study this propagation analytically using the Coupled Kinetic Equations (CKE method, see paper I), with both CMB and EBL radiations being included. We compute the fluxes and spectra of UHECR nuclei (primary and secondary) and secondary protons using, as in paper I, the hypothesis of a power-law generation spectrum, and assuming that source composition is given by nuclei with fixed atomic mass number \(A_0\). We focus in this paper on the influence of the EBL on the propagation of UHE nuclei, discussing the effects of this background on the predicted spectra in comparison with the CMB.

The EBL radiation is emitted by astrophysical objects at present and past cosmological epochs and subsequently is modified by red-shift and dilution due to the expansion of the Universe. The EBL energy spectrum is dominated by two peaks one at the optical and the other at IR energies, produced respectively by direct emission from stars, and by thermal radiation from dust. At lower energies the background spectrum is completely dominated by the CMB.

Measurements of the EBL using direct observations is very difficult because of the foreground emissions, mainly from our own galaxy, interplanetary dust radiation and reflected zodiacal light from the Sun [4]. The optical EBL flux was evaluated by the measurements of the wide-field planetary camera on board of the Hubble space telescope. The procedure consists in measuring the total background in three different bands and subtracting the zodiacal light and the galactic foregrounds [5]. The near-IR flux was measured by the DIRBE instrument onboard the Cosmic Background Explorer (COBE) satellite [6], also such observations are affected by source subtraction techniques and modeling of the zodiacal light. In the far-IR regime the EBL can be directly observed with less pollution from foregrounds, these observations have been carried out by DIRBE [7] and FIRAS [8] instruments both onboard COBE.

Indirect observations of EBL are also used [4]. One indirect method is based on the integration of galaxy counts that helps in setting the reliable lower limits to the expected background and also in determining the spectral energy distribution of the EBL, mainly at frequencies for which no COBE data are available. Another indirect way of evaluating the EBL density is based on the observations of TeV \(\gamma\)-rays [4], using the pair-production absorption features of \(\gamma\gamma_{EBL} \rightarrow e^+e^-\). Using it one can deduce the intensity of EBL.
Using TeV \( \gamma \)-rays observations from blazars, the upper limits on the expected EBL have been obtained (see [4] and references therein).

For calculation of UHE nuclei spectra, the knowledge of EBL at early cosmological epochs is important and thus EBL cosmological evolution is needed. As discussed in [10] there were proposed three different methods to determine the EBL cosmological evolution: (i) evolution inferred from observations at different red-shifts, (ii) forward evolution, which begins with cosmological initial conditions and evolves them forward in time matching the present day observations [11] and (iii) backward evolution, which starts form the present day observations and evolves data backward in time [12]. At present there are a few works with calculations of the EBL with cosmological evolution included, most notably [12] and [13]. In the present paper we mainly use the EBL as presented in [12], which is a refinement of previous calculations [14], based on the data from the Spitzer infrared observatory and the Hubble Space Telescope deep survey. In [12] the EBL photon density is found from 0.03 eV up to the Lyman limit 13.6 eV for different values of the red-shift up to \( z = 6 \).

The paper is organized as follows: in section 2 we discuss the energy losses of nuclei, focusing mainly on the effects of the EBL, in section 3 we briefly review the CKE method already discussed in paper I, in section 4 we present our results on the expected fluxes of primary and secondary nuclei and secondary nucleons, and finally in section 5 the results are discussed.

We use throughout the paper the following abbreviations: UHECR for Ultra High Energy Cosmic Rays, CMB for Cosmic Microwave Background Radiation, EBL for Extragalactic Background Light in the range from Infrared to Ultraviolet, and CKE for Coupled Kinetic Equation method.

## 2 Nuclei energy losses and role of the EBL

As discussed in the accompanying paper I, the propagation of UHE nuclei through background radiations is affected by three kinds of energy losses: (i) adiabatic losses due to the expansion of the Universe, (ii) losses due to \( e^+e^- \)-pair production on the background photons (this interaction conserves the nuclei specie, i.e. \( A \) and \( Z \)) and (iii) photo-disintegration of UHE nuclei (this process changes the nuclei specie giving rise to the production of secondary nuclei and nucleons).

Using the same approach as in paper I, we will consider here two basic quantities that characterize the propagating nucleus, namely its atomic mass number \( A \) and the Lorentz factor \( \Gamma \). The use of the Lorentz factor instead of energy is more suitable because in the process of photo-disintegration, e.g. \( (A + 1) \rightarrow A + N \), the Lorentz-factors of all three particles are approximately the same, since the kinetic recoil energy of a secondary nucleus and nucleon is much smaller than the rest-mass of this particle. Therefore, during propagation the nucleus Lorentz factor changes only due to expansion of the universe and pair-production, remaining unchanged during photo-disintegration.

In paper I we have introduced three different analytic schemes to compute the fluxes of UHECR nuclei and their secondaries. In the present paper we will use only the coupled kinetic equation (CKE) scheme. As discussed in section 4 of paper I, the photo-disintegration process in the CKE method is interpreted as a decaying process, e.g. \( A \rightarrow (A - 1) + N \), which results in the disappearance of the nucleus \( A \). In this sense only the pair-production process and the Universe expansion change the Lorentz factor.
Γ, and thus energy \( E = \Gamma m_N \) until the disappearance of the nucleus \( A \). The photodisintegration of \( A \) nuclei only depletes the \( A \)-nuclei flux.

The energy spectra of UHECR nuclei have been calculated by the CKE method in paper I. The calculations in this work differ by the presence of EBL radiation, which affects only the low energy part of the spectrum. Since energies of EBL photons are higher than that of CMB, the calculated spectra are characterised by photo-disintegration which occurs at lower energies. One may also immediately understand that EBL radiation makes negligible contribution to pair-production at all energies. To explain it let us start with low Lorentz-factors \( \Gamma \), when the pair production occurs on optical and UV photons from EBL spectrum. The threshold of pair production is given by \( \Gamma_{\text{th}} \epsilon \sim 2m_e \), where \( \epsilon \) is an energy of optical/UV photon. For \( \epsilon \sim 1 \text{ eV}, \ \Gamma_{\text{th}} \sim 10^6 \), and for known density of optical/UV photons the energy losses due pair production are considerably lower than that for adiabatic energy loss \( H_0 \). At higher Lorentz factors the pair-production on CMB photons strongly dominates because of much larger density of CMB photons in comparison with the EBL. The numerical calculations confirm this conclusion.

Let us now come over to numerical calculations.

The rate of the Lorentz factor loss due to pair-production, i.e. energy loss for fixed \( A \), can be written for all Lorentz factors \( \Gamma \) taking into account only CMB radiation. It easily can be written in terms of pair-production process for protons (see also section 2 of paper I) as

\[
\left( \frac{1}{\Gamma} \frac{d\Gamma}{dt} \right)^A_{\text{pair}} \equiv \beta^A_{\text{pair}}(\Gamma, t) = \frac{Z^2}{A} \beta^p_{\text{pair}}(\Gamma, t)
\]

where \( Z \) and \( A \) are, respectively, the electric-charge number and atomic-mass number of the nucleus, and \( \beta^p_{\text{pair}}(\Gamma, t) \) is the Lorentz factor decrease rate for the proton on the CMB.

The effect of the EBL is relevant only in the process of photo-disintegration of nuclei. The physics of this process enters the kinetic equation through the nucleus life-time \( \tau_A \) (see next section), that can be identified as the mean time needed for the nucleus \( A \) to lose one nucleon in the interaction with background photons. Therefore, see also section 2 of paper I, one can write

\[
\frac{1}{\tau_A} = \frac{c}{2T^2} \int_{\epsilon_0(A)}^{\infty} d\epsilon_r \sigma(\epsilon_r, A) \nu(\epsilon_r) \epsilon_r \int_{\epsilon_r/(2\Gamma)}^{\infty} d\epsilon \frac{n_{\text{begr}}(\epsilon)}{\epsilon^2}
\]

where \( \epsilon \) and \( \epsilon_r \) are the energies of background photons in the laboratory system and in the rest system of the nucleus, respectively, \( n_{\text{begr}} = (n_{\text{CMB}} + n_{\text{EBL}}) \) is the photon density of CMB and EBL background radiations, \( \sigma \) and \( \nu \) are, respectively, the photo-disintegration cross-section and the multiplicity (mean number) of ejected nucleons; as in paper I we use the cross section parameterization from [17].

In the case of CMB alone, discussed in paper I, the evolution of \( \tau_A \) with red-shift (hereafter we will refer to red-shift instead of cosmological time) was simply fixed by the CMB evolution: the number of CMB photons increases by a factor \((1+z)^3\) and their energy by a factor \((1+z)\). As discussed in the Introduction the EBL evolution with red-shift is not reliably known and several models have been put forward to describe such evolution. In the present paper we will use the evolution model of Stecker et al. [12]. In this model the dependence of the EBL spectral distribution on the red-shift is determined through a backward evolution in time of the observed spectral distribution at \( z = 0 \). This method is an empirically based calculation of the spectral energy distribution of EBL using: (i) the
luminosity dependent spectral energy distribution of galaxies based on the observations of normal galaxies, (ii) observationally based luminosity functions as discussed in \[18\] and (iii) the red-shift dependent luminosity evolution functions, empirically derived curves giving the universal star formation rate \[19\] or luminosity density \[20\]. The calculations of the work \[12\] are based on two different scenarios for the luminosity evolution: the base-line scenario and the fast evolution scenario.

In the base-line model galactic luminosities at 60 $\mu$m evolve as $(1 + z)^{3.1}$ up to $z = 1.4$, at higher red-shifts the luminosity is assumed constant with negligible emission at red-shift $z > 6$. In particular, this last assumption of the base-line scenario is supported by the observations of the Hubble space telescope, which indicate that the star formation rate drops off significantly at red-shift around $z = 6$ \[21\], similar decrease is also reported by the Subaru deep field observations of the Ly$\alpha$ emitting objects at red-shift $z = 6.5$ \[22\].

In the fast evolution scenario the galaxies luminosity is evolved as $(1 + z)^4$ in the red-shift range $0 < z < 0.8$ and as $(1 + z)^2$ in the range $0.8 < z < 1.5$. At higher red-shifts all luminosities are assumed constant with no evolution and, as in the baseline scenario, the luminosity is assumed zero at $z > 6$. This kind of evolution is based on the mid-IR luminosity functions determined from that at $z = 2$ in \[23\].

The fast evolution model corresponds to somewhat like upper limit for the EBL density, with the larger contribution at high red-shift. As a lower limit to the EBL contribution
we have also considered a third possibility that corresponds to the minimum possible EBL density at \( z > 0 \). This density can be found from the following general statement, which we have proved for any diffuse background radiation: In the case of the generation rate of background radiation \( Q(\epsilon, z) = K \epsilon^{-\alpha} (1 + z)^m \), valid up to \( z_{\text{max}} \), with arbitrary \( \alpha, m \geq 0 \) and assuming that the background photons are not absorbed, the density of diffuse background radiation at epoch \( z \) is always larger than \( n_z(\epsilon) = (1 + z)^{-3/2} n_0(\epsilon) \), where \( n_0(\epsilon) \) is the measured density at \( z = 0 \).

Once the dependence of the EBL photon density on the red-shift is determined, one can write explicitly the photo-disintegration “life-time” for a nucleus \( A \) with Lorentz factor \( \Gamma \) at any red-shift \( z \). Separating the contributions from the two backgrounds, CMB and EBL, in equation (2), one has:

\[
\frac{1}{\tau_A(\Gamma, z)} = \frac{1}{\tau_{A,CMB}(\Gamma, z)} + \frac{1}{\tau_{A,EBL}(\Gamma, z)} =
\]

\[
= \frac{cT(1 + z)}{2\pi^2 \Gamma^2} \int_{\epsilon_0(A)}^{\infty} d\epsilon_r \sigma(\epsilon_r, A) \nu(\epsilon_r) \epsilon_r \left[ -\ln \left( 1 - e^{\min(\epsilon_r/\Gamma, \epsilon_r')/(1+z)} \right) \right] +
\]

\[
+ \frac{c}{2\Gamma^2} \int_{\epsilon_0(A)}^{\infty} d\epsilon_r \sigma(\epsilon_r, A) \nu(\epsilon_r) \epsilon_r \int_{\epsilon_r/(2\Gamma)}^{\infty} \frac{d n_{\text{EBL}}(\epsilon', z)}{\epsilon'^2}
\]

where \( \tau_{A,CMB}(\Gamma, z) \) is the CMB contribution calculated in paper I and \( \tau_{A,EBL}(\Gamma, z) \) is the EBL contribution, with \( n_{\text{EBL}}(\epsilon, z) \) computed within the three evolutionary models: the baseline and fast evolution models of [12], and minimum EBL model, as described in this section.

In Figs. 1 and 2 we plot \( \tau_A^{-1} \) and \( \beta_{\text{pair}}^A \) at \( z = 0 \) as function of \( \Gamma \) for various nuclei species as labelled. We have plotted \( \tau_A^{-1} \) for two different models of EBL evolution [12].
baseline (red continuous) and fast evolution (blue dotted), compared to the case of CMB alone (magenta dotted). The effect of EBL is clearly seen at intermediate energies with a tiny difference among the two choices of baseline and fast evolution.

To illustrate the effect of different evolution regimes we have plotted in Figs. 3 and 4 the photo-disintegration life-time $\tau_A$ as function of the red-shift for two fixed values of the nucleus Lorentz factors $\Gamma = 1 \times 10^8$ and $\Gamma = 1 \times 10^9$. This choice is motivated by Figs. 1 and 2 which show that the EBL effect is dominant in the range $10^8 < \Gamma < 2 \times 10^9$. Figures 3 and 4 show the variation of $\tau_A$ with $z$ for the three regimes of EBL evolution discussed above. The red continuous curve corresponds to the baseline model, the black continuous curve to the fast evolution scenario and the green dashed line to the minimum EBL, normalized at $z = 0$ to the baseline density. The effect of the EBL reveals itself with a longer life-time in the case of minimum EBL and shorter in the case of fast evolution. This result can be easily understood by taking into account that when the EBL photon density increases, the photo-disintegration process becomes more efficient and the corresponding nucleus life-time decreases. The plots of figures 3 and 4 show also that the effect of the EBL is efficient only at low red-shifts. The evolution of the CMB is strong: the density of photons increases as $(1 + z)^3$ and energies as $1 + z$. Therefore, at large red-shifts the CMB dominates. Figs. 3 and 4 show that $\tau_A$ is determined by the CMB already at $z \geq 2$. This evidence reduces the impact of the EBL evolution on our calculations: at $z < 2$ the evolutionary models do not differ much and at $z > 2$ the CMB dominates.
Figure 4: The same as in figure 3 for heavy nuclei.

3 Coupled Kinetic Equations

In this section we review the CKE method, putting together all the details discussed in paper I. The modification consists in the inclusion of the EBL photon density $n_{EBL}(\epsilon, z)$, which modifies the photo-disintegration life-time $\tau_A(\Gamma, t)$ of a nucleus in cosmological epoch $t$.

The basic kinetic equation for space density of $A$-nuclei $n_A(\Gamma, t)$ under assumption of homogeneous distribution of sources has a form:

$$\frac{\partial n_A(\Gamma, t)}{\partial t} - \frac{\partial}{\partial \Gamma} [n_A(\Gamma, t) b_A(\Gamma, t)] + \frac{n_A(\Gamma, t)}{\tau_A(\Gamma, t)} = Q_A(\Gamma, t)$$

(4)

where $t$ is the cosmological time, $b_A = -d\Gamma/dt$ is the rate of the Lorentz-factor loss, and $Q_A$ is the rate of $A$-nuclei production. Eq. (4) is valid for $A_0$, $A_0 - 1$ etc.

In Eq. (4) the rate $b_A$ of Lorentz-factor decrease includes the terms due to the expansion of the universe and pair-production on CMB, as discussed in the previous section, and using Eq. (1) for $\beta_{pair}$ this rate can be explicitly written as

$$b_A(\Gamma, z) = \Gamma \frac{Z^2}{A} \beta_{pair}^p(\Gamma, z) + \Gamma H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}.$$  

(5)

Like in paper I we consider here and hereafter the standard cosmology with $H_0 \approx 72$ km/(s Mpc), $\Omega_m \approx 0.27$ and $\Omega_\Lambda \approx 0.73$. We will often use the Hubble parameter $H(z)$ at epoch $z$ and $dt/dz$, given by

$$H(z) = H_0\sqrt{(1+z)^3\Omega_m + \Omega_\Lambda} \quad \text{and} \quad \frac{dt}{dz} = -\frac{1}{(1+z)H(z)}.$$  

(6)
Let us discuss now the generation function \( Q_A(\Gamma, t) \) in the rhs of Eq. (1). For the primary nuclei \( A_0 \) this term describes the injection rate of nuclei accelerated in the sources. Like in paper I we assume that only a single specie \( A_0 \) is accelerated (in future applications one can sum up over different \( A_0 \)). As a particular example, in this paper we will consider as primary the Iron nuclei \( A_0 = 56 \). Assuming an homogeneous distribution of sources, the injection rate for these nuclei is given by

\[
Q_{A_0}(\Gamma, z) = L_0 \left( \gamma_g - 2 \right) m_N A_0 \Gamma^{-\gamma_g}
\]  

(7)

where \( \gamma_g > 2 \) is the generation index, \( m_N \) is the nucleon mass and \( L_0 \) is the emissivity, i.e. the energy injected per unit of comoving volume and per unit time.

Propagating through background radiations the Iron nuclei are photo-disintegrated producing, through a photo-disintegration chain, secondary nuclei with \( A < A_0 \) and secondary nucleons (neutrons decay very fast to protons). Therefore, we have only three types of propagating particles: primary nuclei \( A_0 \), secondary nuclei with \( A < A_0 \) and secondary protons.

The dominant process of photo-disintegration is the one nucleon (\( N \)) emission, namely the process \((A+1) + \gamma \rightarrow A + N\). All three nuclear particles have the same Lorentz factor \( \Gamma \). Then the production rate of secondary \( A \)-nucleus and \( A \)-associate proton is given by

\[
Q_A(\Gamma, z) = Q_{A_0}(\Gamma, z) = \frac{n_{A+1}(\Gamma, z)}{\tau_{A+1}(\Gamma, z)} \tag{8}
\]

where \( \tau_{A+1} \) is the photo-disintegration life-time of \((A+1)\)-nucleus.

Equation (8) is the basis of the CKE method. The generation rate for \( A \)-nuclei in kinetic equation (4) is given by \( n_{A+1}(\Gamma, z) \) found as a solution of the preceding equation for \( A+1 \) nuclei, i.e. Eq. (8) provides coupling of these two equations (see section 4 of paper I for more details).

The first in the chain of CKE is the equation for primaries \( A_0 \) with the generation term given by Eq. (7):

\[
\frac{\partial n_{A_0}(\Gamma, t)}{\partial t} - \frac{\partial}{\partial \Gamma} \left[ n_{A_0}(\Gamma, t)b_{A_0}(\Gamma, t) \right] + \frac{n_{A_0}(\Gamma, t)}{\tau_{A_0}(\Gamma, t)} = Q_{A_0}(\Gamma, t). \tag{9}
\]

Its solution reads

\[
n_{A_0}(\Gamma, z) = \int_z^{z_{\text{max}}} \frac{dz'}{(1 + z')H(z')} Q_{A_0}(\Gamma', z') \frac{d\Gamma'}{d\Gamma} e^{-\eta_{A_0}(\Gamma', z')}. \tag{10}\]

The second is the equation (4) for \( A_0 - 1 \) nuclei with generation rate \( n_{A_0}(\Gamma', z')/\tau_{A_0}(\Gamma', z') \). For an arbitrary secondary nuclei \( A \) the generation term is provided by \( n_{A+1}(\Gamma, z) \) found from preceding equation, and thus \( n_A(\Gamma, z) \) is given by

\[
n_A(\Gamma, z) = \int_z^{z_{\text{max}}} \frac{dz'}{(1 + z')H(z')} \frac{n_{A+1}(\Gamma', z') d\Gamma'}{\tau_{A+1}(\Gamma', z')} e^{-\eta_A(\Gamma', z')} \tag{11}\]

The exponential term in Eq. (11) is given by

\[
e^{-\eta_A(\Gamma', z')} = \exp \left[ -\int_z^{z'} \frac{dz''}{(1 + z'')H(z'')} \frac{1}{\tau_A(\Gamma'', z'')} \right], \tag{12}\]
The physical meaning of the factor \( \exp(-\eta) \) becomes clear if Eq. (12) is re-written in terms of the cosmological time \( t \) as
\[
e^{-\eta A(\Gamma',t')} = \exp \left[ - \int_{t}^{t'} \frac{dt''}{\tau_A(\Gamma'',t'')} \right],
\]
in which one easily recognizes the survival probability during the propagation time \( t' - t \) for a nucleus with fixed \( A \). Therefore, \( \exp(-\eta) \) provides the suppression of large \( z' \) in the integral in Eq. (11).

As we already emphasized, in each of the coupled kinetic equations \( A = const \), though at any \( z' \) a probability of \( A = const \) is suppressed by \( \exp(-\eta) \). Two consequences follow from such description.

First, the maximum red-shift \( z_{\text{max}} \) in Eqs. (10) and (11) formally corresponds to the maximum acceleration Lorentz factor \( \Gamma_{\text{max}} \) in evolution of the Lorentz factor at fixed \( A \). As a matter of fact the suppression factor \( \exp(-\eta) \) controls automatically the maximum attainable Lorentz factor. Therefore, one avoids using \( z_{\text{min}} \) and \( z_{\text{max}} \) from trajectory calculations. In fact \( z_{\text{min}} = z \), while effective \( z_{\text{max}} \) is controlled by survival probability \( \exp(-\eta) \).

Second, ratios \( d\Gamma/d\Gamma' \) in Eqs. (10) and (11) are characterised by \( A = const \), and therefore for their calculation the formula (68) from appendix B of paper I is valid:
\[
\frac{d\Gamma'}{d\Gamma} = \frac{1+z'}{1+z} \exp \left[ \frac{Z^2}{A} \int_{z}^{z'} dz'' (1+z'')^2 \frac{d\bar{b}^p_0(\bar{\Gamma})}{d\bar{\Gamma}} \frac{d\Gamma'}{d\Gamma} \right],
\]
where \( \bar{b}^p_0(\bar{\Gamma}) = -d\Gamma/dt \) is the Lorentz-factor loss per unit time for protons at \( z = 0 \) due to pair production process.

Finally, we address the calculation of the secondary protons associated to the production of secondary \( A \) nuclei in the process \( (A+1) + \gamma \rightarrow A + N \). For the space density of \( A \)-associate protons the notation \( n^A_p \) will be used.

The kinetic equation for proton propagation reads
\[
\frac{\partial n^A_p(\Gamma,t)}{\partial t} - \frac{\partial}{\partial \Gamma} [b_p(\Gamma,t)n^A_p(\Gamma,t)] = Q_p(\Gamma,t)
\]
where \( Q_p \) is the rate of proton production given Eq. (8) and \( b_p(\Gamma,t) = -d\Gamma/dt \) is the Lorentz-factor decrease rate due to expansion of the universe (adiabatic energy losses), pair-production and photo-pion production both on the CMB radiation field [16].

The solution of Eq. (15) is given by
\[
n^A_p(\Gamma,z) = \int_{z}^{z_{\text{max}}} \frac{dz'}{(1+z)H(z)} Q^A_p(\Gamma',z') \left( \frac{d\Gamma'}{d\Gamma} \right)_p
\]
where \( d\Gamma/d\Gamma' \) for protons is given in [15] and paper I. It can be calculated from Eq. (14) putting there \( Z^2 = A \), and including in \( b^p_0(\Gamma) \) the photo-pion production energy losses.

### 4 Spectra

In this section we will discuss the spectra of different species of primaries, as well as the spectra of secondary nuclei and protons, produced by photo-disintegration of the primaries during propagation. The main emphasis is given to the impact of the EBL on the spectra.
We will consider the three models for the EBL cosmological evolution discussed in section 2: the baseline and the fast evolution models of [12] and the minimum EBL as presented in section 2. In order to isolate the impact of the background radiation on the spectra, we will assume that only one nucleus species $A_0$ is accelerated and injected into space according to Eq. (7). The general case of mixed injection composition can be obtained by summing the fluxes with different $A_0$. Sources are assumed homogeneously distributed in the universe.

The injection spectrum is taken in the power-law form with the index and maximum energy at the source fixed throughout the paper as $\gamma_g = 2.3$ and $E_{\text{max}} = Z_0 \times 10^{21}$ eV. The emissivity $L_0$, i.e. the energy injected in unit comoving volume per unit time, is not specified and the calculated diffuse spectra are given in arbitrary units.

4.1 Primary nuclei

The spectra of primary nuclei are the simplest for calculation and understanding. The spectrum of primary $A_0$ is given by Eq. (10) with the generation term $Q_{A_0}$ and survival probability $\exp(-\eta_0)$ described by Eq. (7) and Eq. (12), respectively. Because of the large space density of the CMB photons, the spectrum of primary nuclei is formed first due to interaction with the CMB photons, and then is distorted (much more slowly) by the EBL. As explained above, as far as the EBL is concerned, only photo-disintegration is important and should be taken into account. The primary nuclei are photo-disintegrated by the EBL photons only at low Lorentz factors (at $\Gamma \leq 2 \times 10^9$, see section 2) i.e. at
these Lorentz factors the primary spectrum is depleted by the EBL. Numerically this depletion is described by diminishing of $\tau_A$ and thus by decreasing of survival probability $\exp(-\eta_{A0})$, which suppresses the flux (10). A side effect of this interaction is the production of secondary nuclei and protons with the same Lorentz-factors $\Gamma \leq 2 \times 10^9$. Fig. 5 confirms these expectations. The spectra calculated for the CMB alone are shown by dotted magenta curves. The EBL radiation always suppresses these spectra. All three evolution models (baseline, evolution and minimum EBL) show almost identical suppression, because they differ essentially only at large red-shift where the CMB dominates (see section 2). For heavy nuclei suppression is stronger.

4.2 Secondary nuclei

The spectrum of the secondary nuclei is given by Eq. (11) with the generation term and survival probability presented by Eqs. (8) and (12), respectively. In presence of the EBL these two factors works in opposite directions: the survival probability suppresses the flux just like in the case of the primary nuclei, while the generation term, inversely proportional to $\tau_A$, increases the flux. We can give a qualitative argument, based on the trajectory calculations, that generation term dominates and interaction with the EBL radiation always increases the secondary nuclei flux in comparison with the CMB.

Consider first the case of the CMB only and a secondary nuclei $A$ at $z_0 = 0$. Let us study like in section 2.2 of paper I the backward evolution trajectories $A(z) = A(A, \Gamma, z)$ and $\Gamma(z) = \Gamma(A, \Gamma, z)$, where $A$ and $\Gamma$ are the values at $z_0 = 0$. At the generation red-shift
$z = z_g$, by definition $A(z_g) = A_0$, and $\Gamma(z_g) = \Gamma_g$. Let us now switch on the EBL. $A(z)$ starts to increase earlier and reaches $A_0$ earlier, i.e. at smaller $z_g$. Hence $\Gamma_g = \Gamma(z_g)$ decreases, and the number of generated primary nuclei $Q_{A_0} \propto \Gamma_g^{-\gamma}$ increases.

We may put this effect in other words: the EBL accelerates the evolution of $A(z)$, and $z_g$, where $A(z)$ reaches $A_0$, becomes smaller, $\Gamma_g$ becomes smaller, too, and the generated flux becomes larger.

The fluxes of secondary nuclei are shown in Fig. 6 for heavy nuclei and in Fig. 7 for light nuclei. The fluxes with the CMB only are shown by magenta dotted lines. As anticipated these fluxes are always lower than ones with the EBL taken into account (three upper curves). The fluxes corresponding to different EBL also obey the above hierarchy: the weaker EBL, the smaller secondary nuclei flux. In Figs. 6 and 7 one can see that minimum EBL (black dotted curves) corresponds to the lowest flux among the three EBL versions. The only exclusion is given by $A = 55$, because of anomalous vicinity to primary $A = 56$.

### 4.3 Secondary protons

The flux of secondary protons $p$ born simultaneously with brother-nuclei $A$ in the process $(A+1) + \gamma \rightarrow A + N$ is given by Eq. (16), with the same generation rate (8) as for nuclei $A$. We call these protons $A$-associate and denote their space density $n_p^A(\Gamma, z)$. We calculate the total proton flux summing up $n_p^A(\Gamma, z)$ over all $A < A_0$.

As one can see from Eq. (16) the flux of secondary protons is affected by the EBL only through the injection term $Q_{p}^{A}$, i.e. through the photo-disintegration life-time of the parent nucleus $A+1$ (see Eq. 8). Increasing the EBL flux one decreases the life-time $\tau_{A+1}$, making more efficient the production of secondary-protons. One can see this hierarchy of the proton fluxes in Figs. 8 and 9: the proton fluxes increase with the EBL flux. This
Figure 8: Flux of secondary protons accompanying production of secondary $A$-nuclei (as labelled). Three different versions of the EBL are plotted: baseline model (red continuous), fast evolution (blue dotted) and minimum EBL (black dotted). The case of CMB alone (see paper I) is shown by dotted magenta curve. One can observe the hierarchy of proton fluxes: the larger proton flux corresponds to larger EBL (see text).

effect is restricted to the Lorentz factor range $1 \times 10^8 < \Gamma < 2 \times 10^9$, where the effect of the EBL plays a relevant role (see figures 1 and 2). Caused by the Lorentz-factor equality $\Gamma_{A+1} = \Gamma_p$, it corresponds to dominant proton production on the EBL with energies $E_p \leq 2 \times 10^{18}$ eV. For larger energies $E_p$ and larger red-shifts the production on the CMB dominates (see Figs. 8 and 9). From this discussion it follows that the proton production on the EBL occurs in the energy range of minor importance for UHECR study.

To conclude this section we demonstrate in Fig. 10 the comparison of the observed fluxes of primary nuclei, taken as pure Iron, the secondary nuclei and secondary protons, with the injection parameters as fixed in section 3. The flux of nuclei are grouped summing over different nuclei species, as shown in the figure, while the flux of secondary protons is given as the total flux summed over all associated nuclei with $A \leq A_0$ (see section 3). Four groups of nuclei are presented in Fig. 10: primary Iron, heavy secondaries $40 < A < 56$, intermediate secondaries $26 < A < 39$ and light secondaries $2 < A < 25$. One can observe that primary Iron and heavy secondaries dominate over secondary protons and light secondary nuclei.

In the left panel of Fig. 10 we give the fluxes in the case of baseline model, while in the right panel in the case of fast evolution. Comparison of the secondary-nuclei fluxes in these two figures follow the general rule of an increase of the flux at intermediate Lorentz-factors $\Gamma < 2 \times 10^9$ in the models with larger EBL, i.e. in the fast evolution model with a larger EBL density at high red-shifts.
Figure 9: The same as in figure for lighter secondaries (as labelled).

Figure 10: Total flux of nuclei and protons produced by the injection of Iron at the source in the case of EBL evolution given by baseline model (left panel) and fast evolution model (right panel). The fluxes are summed over the nuclei species as labelled, the total flux of secondary protons is shown by black dotted curve.

5 Discussion and Conclusions

The present paper is the continuation of the accompanying paper I, where different methods for the analytic calculations of UHE nuclei spectra have been studied, including CMB as the only background radiation. In the present work we have performed more realistic calculations, including additionally the EBL as background radiation and focusing on its role. We used the Coupled Kinetic Equations method as the most transparent and precise. The most important element of this method is the generation rate com-
mon for the production of secondary nuclei $A$ and secondary nucleons $N$ in the process $(A + 1) + \gamma_{\text{begr}} \rightarrow A + N$. It has a form $n_{A+1}(\Gamma)/\tau_{A+1}(\Gamma)$ and is given explicitly by Eq. (5). It couples two successive kinetic equations for $n_A(\Gamma)$ and $n_{A+1}(\Gamma)$ and has a key importance for the calculation of $n_A(\Gamma)$ and $n_p(\Gamma)$.

In this section we describe the impact of the EBL on the calculated spectra in comparison with CMB. The physical understanding of these effects is a great advantage of the analytic methods.

The role of the EBL is limited by photo-disintegration of the low-energy nuclei with Lorentz-factors in the range $1 \times 10^8 < \Gamma < 2 \times 10^9$, when the energies of CMB photons become too low for photo-disintegration. At the same time the continuous $e^+ e^-$-energy loss occurs in this energy range due to the interaction with CMB photons as much more numerous. At $\Gamma > 2 \times 10^9$ both photo-disintegration and pair production are dominated by CMB and the spectra calculated in paper I become fully applicable.

Let us discuss first the impact of the EBL on the primary nuclei $A_0$ accelerated in the sources. Their spectrum is formed first due to the interaction with CMB, because the number of CMB photons is much larger than that of EBL. The role of the EBL consists in a flux suppression in the range $1 \times 10^8 < \Gamma < 2 \times 10^9$ due to photo-disintegration. The calculated spectra of primary nuclei are exposed in Figs. 5. The EBL suppression of the spectra is clearly seen. The steepening of spectra is much different from GZK cutoff.

Consider next the secondary nuclei $n_A(\Gamma)$ in the same Lorentz factor interval as above. Apart from the described suppression of the flux, there is a regeneration of the flux due to the photo-disintegration of $A + 1$ nuclei on the EBL. As it is explained in section 4.2 this regeneration always dominates and, as a result, the low-energy tail of the $A$-nuclei spectrum develops in the CMB-produced spectrum (see Figs. 6 and 7). This effect of low-energy extension of the secondary-nuclei spectrum is most remarkable influence of the EBL photo-dissociation on the CMB-produced spectrum. It is clearly seen in Figs. 6 and 7.

Finally, we discuss secondary protons. They are produced in $(A + 1) + \gamma_{\text{begr}} \rightarrow A + N$ photo-dissociation with the same Lorentz-factor as the accompanying $A$-nucleus and with the same generation rate. The total density of the secondary protons is found by summing $n_p^A(\Gamma)$ over all $A < A_0$. The protons are produced in the same Lorentz-factor range as the secondary nuclei, i.e. with energies $1 \times 10^{17} - 2 \times 10^{18}$ eV. In contrast to the secondary nuclei these protons are not photo-disintegrated and undergo small energy losses. In unrealistic models when only heavy nuclei, e.g. Iron, are accelerated in the sources, photo-disintegration can be the only mechanism of proton production at $E_p \sim 10^{18}$ eV and below. They can comprise about 10% of the total flux (see Fig. 10). It gives the minimum proton flux compatible with hypothesis of only heavy nuclei acceleration. In all realistic models a primary proton component must be present and the protons fraction must be much higher.

One can see from all calculated spectra that the differences in the cosmological evolution regimes for the EBL do not change strongly the spectra. It occurs because all evolution regimes are normalized by the same EBL flux at $z = 0$, while CMB radiation becomes more essential at large $z$ due to a more rapid evolution. An exceptional case is given by the spectra of secondary nuclei, see Figs. 6 and 7, because all the effect on the low-energy tail is caused by the EBL.

In Fig. 10 the fluxes of secondary nuclei and protons produced by primary (accelerated) Iron nuclei are exposed. The secondary nuclei are presented as three groups: heavy secondaries (summed over $A$ from 40 to 56), the intermediate group ($A$ from 26 to 39)
and the light group ($A$ from 2 to 25). The proton flux is summed over all $A$. One can see that heavy secondary nuclei dominate. The spectrum 'cutoff' caused by photodisintegration is less steep and starts earlier than the GZK cutoff. Flux of secondary protons is subdominant.

As the aim of these two papers we consider a study of fundamental properties of propagation of UHE nuclei through CMB and EBL in an analytic approach. In a forthcoming paper of this series we will build more realistic models for a direct comparison with the observational data.

Acknowledgements

We thank Pasquale Blasi, Yurii Eroshenko and Askhat Gazizov for valuable discussions. This work is partially funded by the contract ASI-INAF I/088/06/0 for theoretical studies in High Energy Astrophysics and by the Gran Sasso Center for Astroparticle Physics (CFA) funded by European Union and Regione Abruzzo under the contract P.O. FSE Abruzzo 2007-2013, Ob. CRO. The work of SG is additionally funded by the grant of President of RF SS-3517.2010.2, VB and SG - by FASI grant under state contract 02.740.11.5092.

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