Modeling the Cryogenic Texture Formation Process in the Soil Around the Pipeline

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Abstract. This article is concerned with the study and analysis of heat exchange processes, moisture transfer and cryogenic texture formation in fine dispersed soils when laying and operating underground pipelines. The segregation frost heaving around cold big-diameter pipelines buried in the soil has been investigated. The method of modeling the secondary frost heaving is presented. The conditions under which this model is applicable as a result of assumptions made during its construction are described. The model is presented in a stringent mathematical formulation. The checking numerical case has been calculated. The process of ice layers formation graph is given. Under certain conditions one layer can be formed and under the other conditions two layers can be formed. Mostly it depends on the pipe temperature. For this case the guidelines for soil frost heaving prevention are given. The guidelines are general and applicable to other cases.

1. Introduction

Frost heaving of fine-dispersed soil is due to the moisture migration to the freezing front or inside the frozen layer under the temperature gradient influence. In this case the cryogenic texture of the soil is formed: the layers of frozen soil are interlaid with bands of almost pure ice. When the soil is frozen, the moisture in the melt zone migrates relatively quickly, and the growth of the ice band occurs over time measured by days and weeks. In the frozen zone free moisture freezes at the phase transition front, and the inherent moisture freezes in a wide range of sub-freezing temperatures. The moisture conductivity coefficient of the inherent moisture is small, but when the temperature field has a constant gradient over a period of years, the frost heaving still occurs, and ice bands are formed. This happens, for example, near the cold pipe of the gas pipeline.

According to the terminology [1], we call such type of heaving secondary. Our work is devoted to this type of heaving. This article shows results of the secondary heaving research.

The relevance of the work is due to the fact that the secondary frost heaving adversely affects the pipelines stability, operational reliability and cost-effectiveness.
The scientific novelty lies in the fact that the problem solved in [2-4] for the case of plane-parallel symmetry is solved here in cylindrical coordinates. The method of transition from plane to radial problems described in [5] is applied. This allowed us to construct a model of secondary heaving based on the results of [2-7].

2. Research objective

Experimental and theoretical studies of heat exchange processes in frozen soils are summarized in a number of substantial academic works: [8-21].

This article considers the segregation mechanism of the secondary heaving around the pipe under the following assumptions:

1. The temperature field is close to stationary.
2. The soil is fine-dispersed, homogeneous.
3. The pores are completely filled with moisture, the pores are air-free.
4. External mechanical loads are absent.

A cylindrically shaped specimen of fine-dispersed soil of the initial radius \( r_2 \) with a pipe of radius \( r_1 \) inside is considered. Thus around the pipe there is soil ring with an initial thickness \( l = r_2 - r_1 \) (Figure 1). Moisture fills all the pores of the soil, the soil is air-free.

![Figure 1. Cold pipe surrounded by soil](image)

The temperature on the surface of the pipe is constant \( T_1 < 0 \), and on the side surface of the soil specimen the temperature is \( T_2 > T_1 \), here the inflow of moisture is provided due to constant humidity. There are no external loads.

According to [2], the unfrozen moisture migrates towards the temperature lowering, it penetrates through the lateral surface and moves towards the pipe. Soil swells and its thickness \( l \) increases over time: \( l = l(t) \).

Due to moisture transfer the soil particles move along the coordinate \( r \) at a speed of \( \nu(r, t) \).

Moisture content \( W \) and ice content \( L \), temperature \( T \), dry unit weight \( \rho_{ck} \) are defined as function of \( (r, t) \). As in [4], humidity and iciness are relative values and, accordingly, equal to

\[
W = \frac{m_w}{\rho_{ck}} \quad L = \frac{m_i}{\rho_{ck}}
\]

(1)

where \( m_w \) and \( m_i \) is the water mass and ice mass. The dry unit weight is defined as follows:
\begin{equation}
\rho_{ck} = \begin{cases} 
\rho_{ck}0 W_c < W_m \\
\frac{\gamma_w}{W_c + b} W_c \geq W_m 
\end{cases}
\end{equation}

where $W_c, W_m$ are the total moisture content and the total soil moisture capacity respectively, and

\begin{equation}
W_c = W + L
\end{equation}

$\gamma_w$ is moisture density, and coefficient $b = \frac{\gamma_w}{\rho_{ck}0} - W_m$

When $T > 0$ total moisture content is $W_c = W$, as $L = 0$

When is $T \leq 0$ the so-called unfrozen moisture curve $W_{uw}(T)$ is experimentally testable physical specification of this type of soil, at subfreezing temperature $W(T) = W_{uw}(T)$, if $W_c \geq W_{uw}(T)$.

The ice bands appear only in frozen soils with high initial moisture content, so let us assume that:

\begin{equation}
W_c \geq W_m \quad W_c \geq W_{uw}
\end{equation}

This implies:

\begin{equation}
\rho_{ck} = \frac{\gamma_w}{W_c + b}
\end{equation}

In addition, when

\begin{equation}
T \leq 0 \quad W_c = W + L = W_{uw}(T) + L
\end{equation}

Let us assume that when $t = 0$ the soil has a constant temperature $T = T_2$ and moisture content $W_c = W_0$, and the soil thickness is $l = l(0)$.

3. Theory section of the research

The energy and mass transfer are described by the following equations:

\begin{equation}
\frac{\partial H}{\partial t} + \text{div } (H \nu ) = -\text{div } (Q_T) ;
\end{equation}

\begin{equation}
\frac{\partial (m)}{\partial t} + \text{div } (m \nu ) = -\text{div } (q_w)
\end{equation}

where $H = \int_0^T C_{ef} dT$ is the enthalpy of the environment,

\begin{equation}
C_{ef} = \begin{cases} 
C, & T > 0 \\
C + \kappa (dW/dT), & T \leq 0 
\end{cases}
\end{equation}

$C$ is volumetric heat capacity; $Q_T$ is heat flux density; $q_w$ is the moisture flux density; $\kappa$ is heat of moisture freezing, $m$ is the total moisture mass ($m_w + m_i$) contained per unit volume of the considered specimen. Taking into account the equalities (1) and (3), we have $m = W_c \rho_{ck}$. According to [4], the heat flux and moisture flux densities are defined from the following equations:

\begin{equation}
q_w = -k_0 K(\rho_{ck},T)\text{grad}W
\end{equation}

\begin{equation}
Q_T = -\lambda \text{grad}T
\end{equation}

where $k_0$ is a constant, $K(\rho_{ck},T)$ is the moisture conductivity coefficient, $\lambda$ is the thermal conductivity coefficient.
Now the system of equations (7)-(8) can be rewritten as follows:

$$\begin{align*}
\frac{\partial}{\partial t} (W_c \rho_{ck} ) + \text{div} (W_c \rho_{ck} \nu) &= k_0 \text{div} (K(\rho_{ck}, T) \cdot \text{grad} W) \quad \text{(9)} \\
\frac{\partial H}{\partial t} + \text{div}(HT) &= \text{div}(\lambda \cdot \text{grad}(T)) \quad \text{(10)}
\end{align*}$$

Let us define initial condition on the soil specimen boundaries for these equations:

$$\begin{align*}
T( r,0 ) &= T_2; \\
T( r_1, t ) &= T_1; r_1 < r < l(t), \tau > 0 \\
T( l, t ) &= T_2; \\
W( r,0 ) &= W( r_2, t ) = W_0;
\end{align*}$$

(11)

for $W_c < W_*$ when $r = r_1$ we have $\frac{\partial W}{\partial r} = 0$ ;

for $W_c \geq W_*$ when $r = r_2$ we have $W( r_1, t ) = W_{\text{in}}( t )$

Let us turn our attention to the boundary conditions when $r = r_1$. When the total moisture content reaches the level $W_*$, its further increase is impossible due to the disruption of water films surrounding the soil particles. It is in these films that the unfrozen moisture moves. The soil particles velocity is defined by the formula (12) obtained by the authors in [4]

$$v = \frac{q_w}{\gamma_w}$$

(12)

The system (9) - (11) describes the process of moisture transfer in the soil in the initial period of time, while the front velocity $r = \xi$ is significant and the temperature field is nonstationary. We believe that when $t > t_1$, the temperature is a known function $T = T(r, t)$, and does not depend on $\rho_{ck}$. Due to the deformation of the soil specimen the temperature field changes in a quasi-stationary way. Following the method [3], taking into account quasi-stationarity, we transform the system (9) - (11) to one equation with one unknown variable $\rho_{ck}$, which will be valid only in the frozen zone:

$$\begin{align*}
\frac{\partial \rho_{ck}}{\partial t} + F_1( \rho_{ck}, r, t ) \frac{\partial \rho_{ck}}{\partial r} &= -F_2( \rho_{ck}, r, t ), \\
r_1 < r < \xi
\end{align*}$$

(13)

where

$$\begin{align*}
F_1 &= \frac{k_0}{\gamma_w} \left( K \cdot \varphi + \rho_{ck} \cdot \varphi \cdot \frac{\partial K}{\partial \rho_{ck}} \right); \\
F_2 &= \frac{k_0}{\gamma_w} \cdot K \cdot \rho_{ck} \cdot \left( \frac{\partial \varphi}{\partial r} + \varphi \cdot \frac{1}{r} \right);
\end{align*}$$

the initial conditions are:

$$\begin{align*}
W_c( r,0 ) &= W_0; \\
\rho_{ck}( r,0 ) &= \rho_n = \frac{\gamma_w}{W_0 + b},
\end{align*}$$

(14)

where $\xi$ is the phase transition boundary (Figure 1). When calculating the ice bands formation process, melt zone is not taken into account, and on the melt zone boundary we can set the following conditions
\[
W_c(\xi, t) = W_0; \rho_{ck}(\xi, t) = \frac{\gamma_w}{W_0 + b}.
\]

Calculations and experiments [16] show that the soil specimen may have areas where \( \rho_{ck} = 0 \), i.e., pure ice bands. In the mode of unilateral freezing one of these bands is formed on the inner surface of the specimen \( r = r_1 \). Therefore, the boundary condition here is set for \( r = r_1^* \) (Figure 1):

\[
\rho_{ck}(r_1^*, t) = \frac{\gamma_w}{W_{uw}(T(r_1^*, t)) + b}
\]

The value of \( r_1^* \) is established in the course of the solution.

Problem \((13 - 14)\) is solved by the method of characteristics. Where characteristics overlap, the soil bands overlap. The density increases dramatically, the moisture conductivity coefficient \( K \) increases dramatically and the ice band is formed. Real soils cannot have a density higher than \( \rho_{ck0} \).

4. The numerical simulation results

Due to the limited length of the article, we omit intermediate conversions. They are similar to those given in [3], but in cylindrical coordinates.

The dependence of the unfrozen moisture amount on the temperature is given by [3]

\[
W_{uw}(T) = \begin{cases} 
W_0, & T \geq 0 \\
W_0e^{-\beta T^2/2}, & T < 0
\end{cases}
\]

where \( \beta \) is the coefficient which determines the curve shape \( W_{uw} \).

According to theoretical and experimental studies [1], [16], pure ice bands are formed at a sub-freezing temperature close to the stationary temperature, so all calculations are made only in the frozen zone. The coefficient of the homogeneous soil moisture conductivity found experimentally [16] is a function of the soil density and \( K(\rho_{ck}) = (\rho_{ck})^{2.3} \).

The initial data for calculations:

\[
T_1 = -5^\circ C; T_2 = 2^\circ C; r_1 = 0.1m; r_2 = 0.25m; k_0 = 1.4 \cdot 10^{-7} m^2/s; \\
W_0 = 0.42; \rho_{ck0} = 1.25 \cdot 10^3 kg/m^3; \rho_* = 0.5kg/m^3; \beta = 1K^{-1}.
\]

In addition, it was accepted that the moisture exchange stops, when \( \rho_{ck}(W_c^*) = \rho_* \), i.e \( \rho_{ck} < \rho_* \).

Figure 2 shows the graph of the distribution of the soil dry units density \( \rho_{ck} \) when \( t=580 \). Here the ice band formation area is well visible. The results are qualitatively consistent with the Yershov’s experiments [16].

![Figure 2. The dependence of the soil dry units density on the radius at t=580 h](image-url)
If the cold pipe is operated at a constant temperature \( T_p \leq -2^\circ C \) for a long time, then an ice ring is formed on its surface. The second ice band is formed where the unfrozen moisture curve has a bending point.

The distance between the bands is much greater than in a flat specimen [3]. This is explained by the qualitative difference in heat exchange in the plane-parallel and cylindrical areas. When \( T_p > -2^\circ C \) only one ice band is formed on the pipe surface.

5. **Guidelines for the pipeline protection from the frost heaving**

Calculations must be carried out according to the above method and one of two variants of protection should be chosen:

1) the trench into which the pipe is laid is to be wider than the second ice ring and is to be filled with nonfrost-heaving soil;
2) the pipe is additionally heat-insulated to prevent the second ice band occurrence, in that case the removed soil volume is much smaller.

The choice of the variant should be technically and economically justified.

6. **Theoretical inference:**

1) the ice bands are formed in the frozen zone, but not on the phase transition boundary;
2) cryogenic texture is continuously formed throughout the frozen zone for a long time after its freezing;
3) ice bands are formed under quasi-stationary temperature conditions.

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