Supplementary Information for “Harvesting Vibrational Energy Using Material Work Functions”

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1 Output power of ideal cycle

Here the average output power of the ideal cycle is calculated in the voltage-constrained and charge-constrained modes. The motion of the work-function energy harvester is assumed to be periodic with a period of $\Delta t$. During the cycle of operation the total capacitance of the WFEH is varied from the minimum value $C_{\text{min}}$ to the maximum value $C_{\text{max}}$. In the thermodynamical equilibrium the absolute voltage across the WFEH gap is given by the built-in voltage $V_{\text{bi}}$. When the capacitance of the WFEH is at the minimum, the electric energy stored in the WFEH is given by

$$E_{\text{Cmin}} = \frac{1}{2} (C_{\text{min}} - C_{\text{par}}) V_{\text{bi}}^2,$$

where $C_{\text{par}}$ is the parasitic capacitance. Similarly, the electric energy stored in the WFEH with the capacitance at maximum is given by

$$E_{\text{Cmax}} = \frac{1}{2} (C_{\text{max}} - C_{\text{par}}) V_{\text{bi}}^2.$$

In the voltage constrained mode any excess electric charge is immediately extracted. The voltage constrained mode yields the average output power as

$$P_{\text{idealV}} = \frac{2}{\Delta t} (E_{\text{Cmax}} - E_{\text{Cmin}}) = \frac{C_{\text{max}} V_{\text{bi}}^2}{\Delta t} \left(1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right).$$

In the charge constrained mode the WFEH capacitor is charged or discharged only when the stored electric energy reaches a maximum or minimum, respectively. These extremes are reached, when the WFEH capacitance is at maximum and minimum. In this derivation the charging and discharging is controlled with an electrical switch.

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The charge-constrained ideal cycle begins with phase 1 (see Fig. 1d): Capacitance $C$ is at the maximum and the electric charges in the WFEH are equilibrated via the connected load circuit. In phase 2 the switch connecting the WFEH to the load circuit is opened and $C$ is decreased to $C_{\text{min}}$. The stored electric energy and voltage $u$ increase rapidly as capacitance decreases. In phase 3 the switch is closed and the excess charge is extracted by the load circuit. The stored electric energy and voltage $u$ during the charge-constrained ideal cycle can be solved from equations $V_{\text{gap}} = u - V_{\text{bi}}$ and $= C_{\text{gap}} V_{\text{gap}} + C_{\text{par}} u$. Just before discharging in the phase 3 the output voltage of the WFEH is given by

$$u_{\text{ph3}} = -\left(\frac{C_{\text{max}}}{C_{\text{min}}} - 1\right) V_{\text{bi}}.$$  

(S3)

The corresponding electric energy stored in the WFEH is

$$E_{\text{ph3}} = \frac{1}{2} (C_{\text{min}} - C_{\text{par}}) \left(\frac{C_{\text{max}}}{C_{\text{min}}}\right)^2 V_{\text{bi}}^2.$$  

(S4)

The electric energy transferred to the load circuit in phase 3 is simply $E_{\text{ph3}} - E_{\text{Cmin}}$. After discharging the switch is opened and $C$ is increased to $C_{\text{max}}$ in phase 4. In phase 4 the electric energy decreases as capacitance increases. Therefore, little or no mechanical energy is needed to alter the capacitance in this phase. In phase 5 the switch is closed and the WFEH is recharged by the equilibration process. Just before charging in the phase 5 the output voltage of the WFEH is given by

$$u_{\text{ph5}} = \left(1 - \frac{C_{\text{min}}}{C_{\text{max}}}\right) V_{\text{bi}}.$$  

(S5)

The corresponding electric energy in the WFEH is

$$E_{\text{ph5}} = \frac{1}{2} (C_{\text{max}} - C_{\text{par}}) \left(\frac{C_{\text{min}}}{C_{\text{max}}}\right)^2 V_{\text{bi}}^2.$$  

(S6)

Using equations (S1), (S2), (S4), and (S6) and the electric energy stored in the parasitic capacitor $C_{\text{par}} u^2/2$ the average output power of a WFEH operating in the charge constrained mode can be written as

$$P_{\text{ideal}} = \frac{C_{\text{max}} V_{\text{bi}}^2}{2\Delta t} \left[1 - \frac{C_{\text{min}}}{C_{\text{max}}}\right] \left[C_{\text{max}} + C_{\text{min}} + 2 - \frac{2 C_{\text{par}}}{C_{\text{min}}} \left[1 + \left(\frac{C_{\text{min}}}{C_{\text{max}}}\right)^2\right]\right].$$  

(4)

2 Supplementary data from work-function energy harvester prototype

2.1 Harvesting test measurements

The time dependencies of the measured and simulated voltage across different load resistors connected to the experimental Cu/Al parallel plate work-function energy harvester with different minimum plate separations $d_0$ are shown in Supplementary Fig. 1 and Supplementary Fig. 2.
Supplementary Fig. 1. Time dependencies of measured and simulated voltage across a 1.00 GΩ load resistor connected to a Cu/Al parallel plate work-function energy harvester with the minimum distance between the plates $d_0$ of (a, b) 170 µm, (c, d) 350 µm, and (e, f) 580 µm. Time is normalized with the operating period $\Delta t$. Data from low (a, c, e) and high frequencies (b, d, f) are shown.
Supplementary Fig. 2. Time dependencies of measured and simulated voltage across a 100 MΩ load resistor connected to a Cu/Al parallel plate work-function energy harvester with the minimum distance between the plates $d_0$ of 350 µm. Time is normalized with the operating period $\Delta t$. Data from low (a) and high frequencies (b) are shown.

Brass/Al and Al/Al plate material combinations were also tested in the experimental setup. The brass plate was fixed and the Al plate was moving. The value of 0.92±0.08 V was measured for the built-in voltage of the brass/Al work-function harvester (see Supplementary Fig. 3a). This value is close to the built-in voltage of the Cu/Al device (1.03±0.08 V). The measured output voltages of the Cu/Al, brass/Al, and Al/Al parallel plate work-function energy harvesters are compared in Supplementary Fig. 3b. Differences in the properties of the aluminium oxides covering the plate surfaces cause the built-in voltage to be non-zero in the Al/Al case.

Supplementary Fig. 3. (a) Determination of the built-in voltage $V_{bi}$ of the brass/Al parallel plate work-function energy harvester from the current–voltage data. The value $V_{bi} = 0.92±0.08$ V is determined from the intersection of the lines fitted to the data. (b) Measured output voltages of Cu/Al, brass/Al, and Al/Al parallel plate work-function energy harvesters connected to a 1 GΩ load resistor and operating at 0.9 Hz. The minimum distance between the plates $d_0$ is 600 µm for Cu/Al, 500 µm for brass/Al, and 300 µm for Al/Al.
2.2 Demonstration of switching operation for increased power output

The experimental data demonstrating the increase in the power output of the WFEH prototype is shown in Supplementary Fig. 4. For easier manual switching the time constant of the WFEH, $RC_{\text{max}}$, was increased by using a 4 GΩ load resistor. In order to preserve the shape of the voltage signal, digital filtering of interfering signals was not used in the switching operation. Supplementary Fig. 4 shows that although the switch is closed for over 10% of the cycle time, the average output power of the switched operation is 2.1 times that of the switchless operation.

Supplementary Fig. 4. Measured voltage across a 4.00 GΩ load resistor connected to a Cu/Al parallel plate work-function energy harvester (a) without a switch and (b) with a switch. The switch was closed when distance between the plates was near its minimum (0.6 – 0.9 s). The operation frequency of the device is 0.5 Hz and the minimum distance between the plates was 170 µm. The output power of the switched case is 2.1 times the output power of the switchless case.

3 Further studies of WFEH connected to a resistor load

The analysis of the electrical properties of WFEHs can be simplified by assuming that the motion of the WFEH is decoupled from the electrical domain (i.e. the effect of the electrostatic force is neglected). The case of variable-distance parallel plate capacitor with ideal sinusoidal motion can be described by equations

\[ C(t) = \frac{\varepsilon_0 A}{d(t)} + C_{\text{par}}, \]  
\[ d(t) = \Delta d \left[ 1 - \cos \left( 2\pi \frac{t}{\Delta t} \right) \right] + d_0, \]

where $\Delta d$ is the amplitude of the motion of the plates and $d_0$ is the minimum distance between the plates. The solution of equation (2) in the case of this kind of parallel plate capacitor requires numerical methods, but analytical approximations can be calculated in few special cases. In the absence of the parasitic capacitance ($C_{\text{par}} = 0$) at high frequencies the output voltage is given approximately by
Interestingly, equation (S7) does not depend on the resistor at all. At low frequencies the output voltage is given approximately by

\[ u(t) = -2\pi V_{bi} \cdot \frac{\Delta d}{\Delta t} \cdot \frac{R}{d_0} \cdot \frac{C(t) - C_{par}}{C_{min} - C_{par}} \cdot \sin \left( 2\pi \frac{t}{\Delta t} \right). \]  

(S8)

The case of triangularly varying capacitance (i.e. triangle wave as capacitance signal) is mathematically interesting as equation (2) can be solved analytically in this case, but it can be difficult to realize a device with such a temporal capacitance variation in reality. The time dependence of triangularly varying capacitance can be written as

\[ C(t) = \begin{cases} 
(C_{max} - C_{min}) \left( 1 - \frac{2t}{\Delta t} \right) + C_{min}, & \text{when } 0 \leq t \leq \Delta t/2 \\
(C_{max} - C_{min}) \left( \frac{2t}{\Delta t} - 1 \right) + C_{min}, & \text{when } \Delta t/2 \leq t \leq \Delta t.
\end{cases} \]  

(S9)

The solution of equation (2) with equation (S9) is given below. A slightly more realistic capacitance function is the sinusoidal time dependence,

\[ C(t) = \frac{1}{2} (C_{max} - C_{min}) \left[ 1 + \cos \left( 2\pi \frac{t}{\Delta t} \right) \right] + C_{min}, \]  

(S10)

which can be used to model the case where the effective area of the parallel plate capacitor is varied sinusoidally (i.e. the plates moving along an axis parallel to both plates).

3.1 Output of circuit with resistor load and triangularly varying capacitance

With the triangular time dependence of the total capacitance given by equation (S9), the solution of equation (2) can be written as

\[ u(t) = V_{bi} \cdot \left\{ \begin{array}{l}
\frac{1}{1 - \gamma} + \left( \frac{u_{C_{max}}}{V_{bi}} - \frac{1}{1 - \gamma} \right) \frac{C(t)}{C_{max}} \left( \frac{C(t)}{C_{min}} \right)^{\gamma - 1}, & \text{when } 0 \leq t \leq \Delta t/2 \\
\frac{1}{1 + \gamma} + \left( \frac{u_{C_{min}}}{V_{bi}} - \frac{1}{1 + \gamma} \right) \frac{C(t)}{C_{min}} \left( \frac{C(t)}{C_{max}} \right)^{\gamma - 1}, & \text{when } \Delta t/2 \leq t \leq \Delta t,
\end{array} \right. \]  

(S11)

where

\[ u_{C_{min}} = u \left( t = \frac{\Delta t}{2} \right) = V_{bi} \frac{1}{1 - \gamma} \left\{ 1 + \left( 1 - \gamma \right) \frac{u_{C_{max}}}{V_{bi}} - 1 \left( \frac{C_{min}}{C_{max}} \right)^{\gamma - 1} \right\}, \]  

(S12)

\[ u_{C_{max}} = u(t = 0) = u(t = \Delta t) = V_{bi} \frac{1 + \gamma}{1 + \gamma} \frac{1 + \gamma}{1 - \gamma} \left\{ 1 - \left( \frac{C_{min}}{C_{max}} \right)^{\gamma - 1} \right\} - 1 \left( \frac{C_{min}}{C_{max}} \right)^{\gamma + 1}. \]  

(S13)
and

\[
y = \frac{\Delta t}{2RC_{\text{max}}} \cdot \frac{1}{1 - \frac{C_{\text{min}}}{C_{\text{max}}}}
\]  

(S14)

Equation (S11) allows the average output power of the WFEH to be written as

\[
p_{\text{ave}} = \frac{2C_{\text{max}}V_{bi}^2}{\Delta t} \cdot \frac{1}{1 - \frac{C_{\text{min}}}{C_{\text{max}}}} \left\{ 1 - \frac{2}{\gamma} \left[ 1 - \frac{1}{1 - \frac{1}{1 - \frac{u_{\text{Cmax}}}{V_{bi}}} - 1 \right] \cdot \frac{C_{\text{max}}}{C_{\text{min}}} - 1 \right\}
\]

\[
+ \frac{1}{1 - 2\gamma} \left[ \frac{1}{1 + \gamma} \right] \cdot \frac{C_{\text{max}}}{C_{\text{min}}} - 1 \right\}
\]

\[
+ \frac{1}{1 + 2\gamma} \left[ \frac{1}{1 + \gamma} \right] \cdot \frac{C_{\text{max}}}{C_{\text{min}}} - 1 \right\}
\]

\[
= \frac{C_{\text{max}}}{C_{\text{min}}},
\]

(S15)

3.2 Normalized equation

A normalized equation of the general case can be obtained by using the normalized time \( t' = t/\Delta t \) as a new variable. This and divisions by \( C_{\text{max}} \) and \( V_{bi} \) allows equation (2) to be written as

\[
\frac{C_{\text{max}}}{C_{\text{max}}} \cdot \frac{d}{dt'} \left( \frac{u}{V_{bi}} \right) + \frac{u}{V_{bi}} \left[ \frac{d}{dt'} \left( \frac{C}{C_{\text{max}}} \right) + \frac{\Delta t}{RC_{\text{max}}} \right] = \frac{d}{dt'} \left( \frac{C}{C_{\text{max}}} \right),
\]  

(S16)

where \( RC_{\text{max}}/\Delta t \) is the normalized time constant, which can also be considered as the normalized resistance. The normalized average electric power dissipated in the load resistor, \( P_{\text{norm}} = \frac{\Delta t}{RC_{\text{max}}} \left( \frac{u}{V_{bi}} \right)^2 \), is related to the average output power of the WFEH, \( P_{\text{ave}} \), by \( P_{\text{norm}} = P_{\text{ave}} \cdot \frac{\Delta t}{C_{\text{max}}V_{bi}^2} \).

3.3 Effect of capacitance variation

Further analysis of the WFEH connected to a load resistor is straightforward with the normalised equation (S16) as it contains only a small number of independent parameters. The average output powers obtained
with three different cases of capacitance variation (equations (5), (6), (S9), and (S10)) are compared in Supplementary Fig. 5.

Supplementary Fig. 5. (a) Comparison of the time-evolutions of the total capacitance $C$ in the cases of triangular and sinusoidal waves and the parallel plate capacitor with variable plate distance (equations (9), (10), (S9), and (S10)) with $C_{\text{max}}/C_{\text{min}} = 5$. (b-d) Ratios of the average output power $P_{\text{ave}}$ and the charge-constrained ideal power $P_{\text{idealQ}}$ of work-function energy harvester with a resistor load as functions of $C_{\text{max}}/C_{\text{min}}$ in the case of (b) triangular and (c) sinusoidal capacitance variation and (d) parallel plate capacitor with variable plate distance. The curves in (b)-(d) correspond to the various values of the normalized time constant $RC_{\text{max}}/\Delta t$. The data was calculated numerically using equations (5), (6), (S9), (S10), and (S16) with $C_{\text{par}} = 0$.

Supplementary Fig. 5 shows that the triangular capacitance variation (equation (S9)) yields the highest power output: A maximum of 24% of the output power of an ideal WFEH operating in the charge-constrained mode, $P_{\text{idealQ}}$, and 87% of the output power of the voltage-constrained mode, $P_{\text{idealV}}$, can be extracted with the values $RC_{\text{max}}/\Delta t = 0.33$ and $C_{\text{max}}/C_{\text{min}} = 4.9$. In the design of the WFEH $R$ should be chosen in such a way that the optimal value of $RC_{\text{max}}/\Delta t$ can be reached. The case of sinusoidally varying capacitance (equation (S10)) is similar to the triangular case, but the power ratio $P_{\text{ave}}/P_{\text{idealQ}}$ decreases slightly faster with increasing $C_{\text{max}}/C_{\text{min}}$ than in the triangular case. In the sinusoidal case, a maximum of 17% of $P_{\text{idealQ}}$ and 58% of $P_{\text{idealV}}$ can be extracted with the values $RC_{\text{max}}/\Delta t = 0.32$ and $C_{\text{max}}/C_{\text{min}} = 4.5$. The case of parallel plate capacitor with varying distance of the plates (equations (5) and (6)) produces the smallest power: A maximum of 13% of $P_{\text{idealQ}}$ and 37% of $P_{\text{idealV}}$ with the values $RC_{\text{max}}/\Delta t = 0.24$ and
$C_{\text{max}}/C_{\text{min}} = 3.2$. In addition, the power ratio $P_{\text{ave}}/P_{\text{idealQ}}$ decreases rapidly with increasing $C_{\text{max}}/C_{\text{min}}$. Although the power ratio $P_{\text{ave}}/P_{\text{idealQ}}$ decreases with increasing $C_{\text{max}}/C_{\text{min}}$, the absolute power increases slightly with increasing $C_{\text{max}}/C_{\text{min}}$ in the case triangular and sinusoidal cases with high values of $RC_{\text{max}}/\Delta t$. In the case of the variable-distance parallel plate capacitor the absolute power increases slightly at very low values of $RC_{\text{max}}/\Delta t$ and decreases otherwise.

4 Output power of work-function charge pump connected to storage capacitor

Here the average charging power of the circuit shown in Fig. 5c is calculated. The analysis applies also to the circuit of Fig. 5a. The voltage of the storage capacitor after one cycle of operation is given by

$$u_{\text{sto}} = \frac{C_{\text{sto}}}{C_{\text{sto}} + C_{\text{min}}} \cdot u_{\text{sto0}} - \frac{C_{\text{max}} - C_{\text{min}}}{C_{\text{sto}} + C_{\text{min}}} \cdot V_{\text{bi}},$$

(S17)

where $u_{\text{sto0}}$ is the initial voltage of the storage capacitor. Using equation (S17) and the formula for geometric series the time evolution of the voltage can be written as

$$u_{\text{sto}}(N) = u_{\text{sat}} \left[ 1 - \left( \frac{C_{\text{sto}}}{C_{\text{sto}} + C_{\text{min}}} \right)^N \right],$$

(S18)

where $N = t/\Delta t$ is the number of cycles and $u_{\text{sat}} = -\left(C_{\text{max}}/C_{\text{min}} - 1\right)V_{\text{bi}}$ is the saturation voltage, i.e. the maximum achievable voltage, which is also the voltage of the WFEH just before connection to $C_{\text{sto}}$. This analysis does not take the finite resistance of the wires into account. Equation (S18) allows calculation of the key parameters of the system in the case the storage capacitor is initially empty. The electric energy in the storage capacitor is given by

$$E_{\text{sto}} = \frac{1}{2} C_{\text{sto}} u_{\text{sat}}^2 \left[ 1 - \left( \frac{C_{\text{sto}}}{C_{\text{sto}} + C_{\text{min}}} \right)^N \right]^2. \quad \text{(S19)}$$

The average charging power depends on the voltage $u_{\text{sto}}$ as

$$P_{\text{sto}} = \frac{C_{\text{sto}}}{\Delta t} \ln \left( \frac{C_{\text{sto}}}{C_{\text{sto}} + C_{\text{min}}} \right) \left( u_{\text{sto}} - u_{\text{sat}} \right) u_{\text{sto}},$$

(S20)

$$= \frac{C_{\text{min}} u_{\text{sat}}^2}{\Delta t} \cdot \frac{C_{\text{sto}}}{C_{\text{min}} \ln \left( \frac{C_{\text{sto}}}{C_{\text{sto}} + C_{\text{min}}} \right)} \left( \frac{u_{\text{sto}}}{u_{\text{sat}}} - 1 \right) \frac{u_{\text{sto}}}{u_{\text{sat}}}. \quad \text{(S21)}$$

After charging $N_{\text{mpp}} = \ln(2)/\ln(1+C_{\text{min}}/C_{\text{sto}})$ cycles the charging power reaches a maximum of

$$P_{\text{sto}}^{\text{max}} = \frac{C_{\text{sto}} u_{\text{sat}}^2}{4 \Delta t} \ln \left( 1 + \frac{C_{\text{min}}}{C_{\text{sto}}} \right). \quad \text{(S21)}$$

The voltage corresponding to the maximum power point is $u_{\text{sto}}^{\text{mpp}} = u_{\text{sat}}/2$. This voltage corresponds to 25% of the maximum electric energy ($E_{\text{sto}}^{\text{mpp}} = \frac{1}{2} C_{\text{sto}} u_{\text{sat}}^2$) which can be stored in the storage capacitor. Using $N_{\text{mpp}}$ equation (S19) can be written as

$$E_{\text{sto}} = \frac{1}{2} C_{\text{sto}} u_{\text{sat}}^2 \left[ 1 - \left( \frac{1}{2} \right)^{N/N_{\text{mpp}}} \right]^2. \quad \text{(S22)}$$
5 Comparison to electrostatic energy harvesters

The average output power of an ideal electrostatic harvester operating in the voltage constrained mode is given by

\[ P_{\text{ideal}_{\text{ESV}}} = \frac{V_{\text{in}}^2}{2\Delta t} \left( 1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right) \]  \hspace{1cm} (S23)

where \( V_{\text{in}} \) is the charging voltage supplied by the external power source and the parasitic capacitances are included in \( C_{\text{min}} \) and \( C_{\text{max}} \). In ideal charge-constrained mode the average output power of an electrostatic energy harvester is given by [Roundy2002]

\[ P_{\text{ideal}_{\text{ESQ}}} = \frac{C_{\text{max}} V_{\text{in}}^2}{2\Delta t} \left( 1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right) \cdot \frac{C_{\text{max}}}{C_{\text{min}}} \]  \hspace{1cm} (S24)

The ratios of ideal powers of work function and electrostatic energy harvesters in the case \( V_{\text{bi}} = V_{\text{in}} \) are compared in Supplementary Fig. 6. In the voltage constrained mode the ideal power of an electrostatic energy harvester is exactly half of the ideal power of the corresponding WFEH. This is due to the fact that the work-function energy harvesters can harvest energy also from charging, whereas in the electrostatic energy harvesters the electric energy corresponding to the initial charge is merely borrowed from the local energy storage.

Supplementary Fig. 6. Ratio of ideal powers of work function and electrostatic energy harvester as functions of \( C_{\text{max}}/C_{\text{min}} \) in the voltage constrained mode, \( P_{\text{ideal}_{\text{ESV}}}/P_{\text{ideal}_{\text{WFEH}}} \), and in the charge constrained mode, \( P_{\text{ideal}_{\text{ESQ}}}/P_{\text{ideal}_{\text{WFEH}}} \). Calculated with equations (3), (4), (S23), and (S24) by assuming equal \( V_{\text{bi}} \) and \( V_{\text{in}} \).