Cosmological constraints on Chaplygin gas dark energy from galaxy clusters X-ray and supernova data

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(Dated: September 6, 2018)

The recent observational evidences for the present accelerated stage of the Universe have stimulated renewed interest for alternative cosmologies. In general, such models contain an unknown negative-pressure dark component that explains the supernova results and reconciles the inflationary flatness prediction ($\Omega_F = 1$) and the cosmic microwave background measurements with the dynamical estimates of the quantity of matter in the Universe ($\Omega_m \simeq 0.3 \pm 0.1$). In this paper we study some observational consequences of a dark energy candidate, the so-called generalized Chaplygin gas which is characterized by an equation of state $p_C = -A/\rho_C^\alpha$, where $A$ and $\alpha$ are positive constants. We investigate the prospects for constraining the equation of state of this dark energy component by combining Chandra observations of the X-ray luminosity of galaxy clusters, independent measurements of the baryonic matter density, the latest measurements of the Hubble parameter as given by the HST Key Project and data of the Supernova Cosmology Project. We show that very stringent constraints on the model parameters can be obtained from this combination of observational data.

PACS numbers: 98.80.Es; 95.35.+d; 98.62.Sb

I. INTRODUCTION

One of the most important goals of current cosmological studies is to unveil the nature of the so-called dark energy or quintessence, the exotic negative-pressure component responsible for the accelerating expansion of our Universe. Over the last years, a number of candidates for this dark energy have been proposed in the literature, with the vacuum energy density (or cosmological constant) and a dynamical scalar field apparently constituting the most plausible explanations. From the observational viewpoint, these two class of models are currently considered our best description of the observed Universe whereas from the theoretical viewpoint they usually face fine-tuning problems, notably the so-called cosmological constant problem as well as the cosmic coincidence problem, i.e., the question of explaining why the vacuum energy or the scalar field dominates the Universe only very recently. The later problem happens even for tracker versions of scalar field models in which the evolution of the dark energy density is fairly independent of initial conditions.

Among the many dark energy candidates, a recent and very interesting proposal has been suggested by Kamenshchik et al. and developed by Bilić et al. and Bento et al.. It refers to the so-called Chaplygin gas (C), an exotic fluid whose equation of state is given by

$$p_C = -A/\rho_C^\alpha,$$

with $\alpha = 1$ and $A$ a positive constant. Actually, the above equation for $\alpha \neq 1$ generalizes the original Chaplygin equation of state proposed in Ref. whereas for $\alpha = 0$, the model behaves like scenarios with cold dark matter plus a cosmological constant ($\Lambda$CDM).

In the context of the Friedman-Robertson-Walker (FRW) cosmologies, whether one inserts Eq. (1) into the energy conservation law ($u_\mu T^{\mu\nu}_{;\nu} = 0$), the following expression for the energy density is immediately obtained

$$\rho_C = A + B \left(\frac{R_o}{R}\right)^{3(1+\alpha)} \equiv \rho_C^o,$$

or, equivalently,

$$\rho_C = \rho_{C.o} \left[ A_s + (1 - A_s) \left(\frac{R_o}{R}\right)^{3(1+\alpha)} \right] \equiv \rho_{C.s},$$

where $\rho_{C.o}$ is the current energy density (from now on a subscript $o$ means present day quantities, and $C$ denotes either the Chaplygin gas or its generalized version). The function $R(t)$ is the cosmic scale factor, $B = \rho_{C.o}^{1+\alpha} - A$ is a constant and $A_s = A/\rho_{C.s}^{1+\alpha}$ is a quantity related to the present day Chaplygin adiabatic sound speed ($v_s^2 = \alpha A/\rho_{C.s}^{1+\alpha}$). As can be seen from the above equations, the C-gas interpolates between non-relativistic matter ($\rho_C(R \to 0) \simeq \sqrt{B/R^3}$) and negative-pressure dark component regimes ($\rho_C(R \to \infty) \simeq \sqrt{A}$). This particular behavior of the Chaplygin gas inspired some authors to propose a unified scheme for the cosmological “dark sector”, an interesting idea which has also been considered in many different contexts (see, however, [11]).
In the theoretical front, a connection between the Chaplygin equation of state and string theory had long been identified by Bordemann and Hoppe [12] and Hoppe [13] (see also [14] for a detailed review). As explained in such references, a Chaplygin gas-type equation of state is associated with a parametric description of the invariant Nambu-Goto d-brane action in a \(d + 2\) spacetime. In the light-cone parameterization, such an action reduces itself to the action of a Newtonian fluid which obeys Eq. (1) with \(\alpha = 1\), with the C-gas corresponding effectively to a d-branes gas in a \((d+2)\)-dimensional spacetime.

Another interesting connection is related to recent attempts of describing the dark energy component through the original Chaplygin gas or its generalized version. Such a possibility has provoked a growing interest for exploring the observational consequences of this fluid in the cosmological context. For example, Fabris et al. [17] analyzed some consequences of such scenario using type Ia supernovae data (SNe Ia). Their results indicate that a Universe completely dominated by the Chaplygin gas is favored when compared with ACDM models. Recently, Avelino et al. [16] used a larger sample of SNe Ia and the shape of the matter power spectrum to show that such data restrict the model to a behaviour that closely matches that of an ACDM models while Bento et al. [18, 19] showed that the location of the CMB peaks imposes tight constraints on the free parameters of the model. More recently, Dev, Alcaniz & Jain [15] and Alcaniz, Jain & Dev [1] investigated the constraints on the C-gas equation of state from strong lensing statistics and high-\(z\) age estimates, respectively, while Silva & Bertočz [28, 29] who analyzed the X-ray luminosity of galaxy clusters derived. We also examine the limits from a statistical combination between X-ray data and recent SNe Ia observations. Finally, in Sec. IV, we finish the paper by summarizing the main results and comparing our constraints with others derived from independent analyses.

### II. THE CHAPLYGIN GAS MODEL

The FRW equation for a spatially flat, homogeneous, and isotropic scenarios driven by nonrelativistic matter and a separately conserved C-gas component reads

\[
\left(\frac{\dot{R}}{R}\right)^2 = H_o^2 \Omega_m \left(\frac{R_o}{R}\right)^3 + (1 - \Omega_m) [A_s + (1 - A_s) \left(\frac{R_o}{R}\right)^{3(\alpha + 1)}],
\]

where an overdot denotes time derivative, \(H_o = 100 h \text{Km.s}^{-1}\text{Mpc}^{-1}\) is the present value of the Hubble parameter, \(\Omega_m\) is the matter density parameter, and the dependence of the C-gas energy density with the scale factor comes from Eq. (3).

The comoving distance \(r_1(z)\) to a light source located at \(r = r_1\) and \(t = t_1\) and observed at \(r = 0\) and \(t = t_o\) is given by

\[
r_1(z) = \frac{1}{R_o H_o} \int_{x'}^1 \frac{dx}{x^2 F(x, \Omega_m, A_s, \alpha)^{1/2}}.
\]

where \(x' = R(t)/R_o = (1 + z)^{-1}\) is a convenient integration variable and the dimensionless function \(F(x, \Omega_m, A_s, \alpha)\) is given by

\[
F = \left[\Omega_m x^{-3} + (1 - \Omega_m) \left(A_s + \frac{(1 - A_s)}{x^{3(\alpha + 1)}}\right)\right]^{1/2}.
\]

Now, in order to derive the constraints from X-ray gas mass fraction on the C-gas, let us consider the concept of angular diameter distance, \(D_A(z)\). Such a quantity is defined as the ratio of the source distance to its angular diameter, i.e.,

\[
D_A = \frac{\ell}{\theta} = R(t_1) r_1 = (1 + z)^{-1} R_o r_1(z),
\]

which provides, when combined with Eq. (5),

\[
D_A^c = \frac{H_o^{-1}}{(1 + z)} \int_{x'}^1 \frac{dx}{x^2 F(x, \Omega_m, A_s, \alpha)}.\]

As one may check, for \(A_s = 0\) and \(\alpha = 1\) the above expressions reduce to the standard cold dark matter model (SCDM). In this case, the angular diameter distance can be written as

\[
D_A^{SCDM} = \frac{2 H_o^{-1}}{(1 + z)^{3/2}} \left((1 + z)^{1/2} - 1\right).
\]
III. LIMITS FROM X-RAY GAS MASS FRACTION

Following Allen et al. [28, 29] and Lima et al. [31], we consider the Chandra data consisting of six clusters distributed over the redshift interval $0.1 < z < 0.5$. The data are constituted of regular, relatively relaxed systems for which independent confirmation of the matter density parameter results is available from gravitational lensing studies. The X-ray gas mass fraction ($f_{\text{gas}}$) values were determined for a canonical radius $r_{2500}$, which is defined as the radius within which the mean mass density is $2500$ times the critical density of the Universe at the redshift of the cluster. In order to generate the data set the SCDM model with $H_0 = 50\text{Km.s}^{-1}\text{Mpc}^{-1}$ was used as the default cosmology (see [28] for details).

By assuming that the baryonic mass fraction in galaxy clusters provides a fair sample of the distribution of baryons at large scale (see, for instance, [32]) and that $f_{\text{gas}} \propto D_A^{3/2}$ [20], the model function is defined as [28]

$$f_{\text{gas}}^{\text{mod}}(z_i) = \frac{b\Omega_b}{(1 + 0.19h^{3/2})\Omega_m} \left[ 2h \frac{D_A^{\text{SCDM}}(z_i)}{D_A^{\text{CDM}}(z_i)} \right]^{1.5},$$ \hspace{1cm} (10)

where the bias factor $b \simeq 0.93$ [33] is a parameter motivated by gas dynamical simulations that takes into account the fact that the baryon fraction in clusters is slightly depressed with respect to the Universe as a whole [33]. The term $(2h)^{3/2}$ represents the change in the Hubble parameter between the default cosmology and quintessence scenarios and the ratio $D_A^{\text{SCDM}}(z_i)/D_A^{\text{CDM}}(z_i)$ accounts for deviations in the geometry of the universe from the default cosmology (SCDM model). In Fig. 1 we show the behavior of $f_{\text{gas}}^{\text{mod}}$ as a function of the redshift for some selected values of $A_s$ and $\alpha$ and fixed values of $\Omega_m = 0.3$, $\Omega_b h^2 = 0.0205$ and $h = 0.72$.

![FIG. 1: The model function $f_{\text{gas}}^{\text{mod}}$ as a function of the redshift for selected values of $A_s$ and $\alpha$ and fixed values of $\Omega_m = 0.3$, $\Omega_b h^2 = 0.0205$ and $h = 0.72$.](image)

FIG. 2: a) The $\Delta\chi^2$ contours for the $\alpha-A_s$ plane according to the X-ray data discussed in the text. The contours correspond to 68%, 95% and 99% confidence levels. The value of the matter density parameter has been fixed at $\Omega_m = 0.3$. b) $\Omega_m - A_s$ plane for the original C-gas model ($\alpha = 1$). At 95.4% we find $\Omega_m = 0.3 \pm 0.02$ while the entire range of $A_s$ is allowed. Note that the X-ray data constrain tightly the matter density parameter.

For the SCDM data. The 68%, 95% and 99% confidence levels are defined by the conventional two-parameters $\chi^2$ levels 2.30 and 6.17, respectively.

In Fig. 2a we show contours of constant likelihood for the $\Omega_m - A_s$ plane, using a $\chi^2$ minimization with the priors $\Omega_b h^2 = 0.0205 \pm 0.0018$ [36] and $h = 0.72 \pm 0.08$ [37] for the range of $A_s$ and $\alpha$ spanning the interval $[0,1]$ in steps of 0.02,

$$\chi^2 = \sum_{i=1}^{6} \left[ \frac{[f_{\text{gas}}^{\text{mod}}(z_i, \Omega_m, A_s, \alpha) - f_{\text{gas}, i}]}{\sigma_{f_{\text{gas}, i}}} \right]^2 + \left[ \frac{\Omega_b h^2 - 0.0205}{0.0018} \right]^2 + \left[ h - 0.72 \right]^2,$$ \hspace{1cm} (11)

where $\sigma_{f_{\text{gas}, i}}$ are the symmetric root-mean-square errors for the SCDM data. The 68.3% and 95.4% confidence levels are defined by the conventional two-parameters $\chi^2$ levels 2.30 and 6.17, respectively.
the X-ray data discussed earlier. From the above equation we find that the best fit model occurs for \( A_s = 1 \) which, according to Eq. (4), is independent of the index \( \alpha \) and equivalent to a \( \Lambda \)CDM universe. Such model corresponds to a accelerating scenario with the deceleration parameter \( q_0 = -0.55 \). From this figure, we also see that both \( A_s \) and \( \alpha \) are quite insensitive to these data and that, at 95.4\% c.l., one can limit the parameter \( A_s \) to be > 0.52. Figure 2b shows the plane \( \Omega_m - \Omega_s \) for the conventional C-gas \((\alpha = 1)\). As one should expect from different analyses [28, 31], the matter density parameter is very well constrained by this data set while the parameter \( A_s \) keeps quite insensitive to it. The best fit occurs for models lying in the interval \( A_s = [0, 1] \) and \( \Omega_m = 0.3 \). At 95.4\% c.l., we find \( 0.268 \leq \Omega_m \leq 0.379 \). For a X-ray analysis where the Cg plays the role of a unified model for dark matter/energy, see [38].

A. Joint analysis with SNe Ia

By combining the X-ray and SNe Ia data sets, more stringent constraints on the cosmological parameters \( \Omega_m \) and \( A_s \) are obtained. As it was shown elsewhere, the parameter \( \alpha \) is highly insensitive to the SNe Ia data. To perform such analysis, we follow the conventional magnitude-redshift test (see, for example, [31]) and use the SNe Ia data set that corresponds to the primary fit C of Perlmutter et al. [40] together with the highest supernova observed so far, i.e, the 1997ff at \( z = 1.755 \) and effective magnitude \( m_{eff} = 26.02 \pm 0.34 \) [41] and two newly discovered SNe Ia, namely, SN 2002dc at \( z = 0.475 \) and \( m_{eff} = 22.73 \pm 0.23 \) and SN 2002dd at \( z = 0.95 \) and \( m_{eff} = 24.68 \pm 0.2 \) [42]. Figures 3a, 3b and 4 show the results of our analysis. In Fig. 3a we display contours of the combined likelihood analysis for the parameter space \( A_s - \alpha \). In comparison with Fig. 2a we see that the available parameter space is reasonably modified with the value of \( A_s \) constrained to be > 0.73 at 95.4\% c.l. and the entire interval of \( \alpha = [0, 1] \) allowed. The best fit model occurs for values of \( A_s = 0.98 \) and \( \alpha = 0.93 \) with \( \chi^2_{min} = 61.38 \) and \( \nu = 61 \) degrees of freedom \((\chi^2_{min}/\nu \approx 1.0)\). The most restrictive limits from this joint analysis are obtained for the original version of C-gas \((\alpha = 1)\). In this case, the plane \( \Omega_m - A_s \) (Fig. 3b) is tightly constrained with the best fit values located at \( A_s = 0.98 \) and \( \Omega_m = 0.3 \) with \( \chi^2_{min}/\nu \approx 1.0 \). At 95.4\% this analysis also provides \( A_s \geq 0.84 \) and \( 0.273 \leq \Omega_m \leq 0.329 \). Note that the contours \( \alpha - A_s \) (Fig. 3a) and \( \Omega_m - A_s \) (Fig. 2b) are almost orthogonal, thereby explaining the shape of the \( \Omega_m - A_s \) plane appearing in Fig. 3b. We also observe that by extending the \( \alpha - A_s \) plane to the interval \([0, 2]\) the new best fit values \((A_s = 1.02 \text{ and } \alpha = 0.45)\), although completely modified in comparison with the previous ones, are still in agreement with the causality \((A_s \leq 1/\alpha)\) imposed by the fact that the adiabatic sound speed \( v_s^2 = dp/d\rho \) in the medium must be less than or equal to the light velocity (see Eq. 1). Some basic results of the above analysis are displayed in Fig. 4.
IV. DISCUSSION AND CONCLUSIONS

Alternative cosmologies with a quintessence component (dark energy) may provide an explanation for the present accelerated stage of the universe as suggested by the SNe Ia results. In this work we have focused our attention on a possible dark energy candidate, the so-called Chaplygin gas. The equation of state of this dark energy component has been constrained by combining Chandra observations of the X-ray luminosity of galaxy clusters and independent measurements of the Hubble parameter and of the baryonic matter density as well as from a statistical combination between X-ray data and recent SNe Ia observations. We have shown that stringent constraints on the free parameters of the model, namely \( A_s \), \( \alpha \) and \( \Omega_m \), can be obtained from this combination of observational data.

It is also interesting to compare the results derived here with another independent analyses. For example, using only SNe Ia data, Fabris et al. [15] found \( A_s = 0.93^{+0.07}_{-0.20} \) for the original C-gas model (\( \alpha = 1 \)) with the matter density parameter constrained by the interval \( 0 \leq \Omega_m \leq 0.35 \). The same analysis for \( \Omega_m = \Omega_0 = 0.04 \) (in which the C-gas plays the role of both dark matter and dark energy) provides \( A_s = 0.87^{+0.13}_{-0.13} \). These values agree at some level with the ones obtained from statistics of gravitational lensing (SGL), i.e., \( A_s \geq 0.87 \) and age estimates of high-z galaxies (OHRG’s), \( A_s \geq 0.88 \). Both results have constraints on the parameters \( \Omega_m \) and \( A_s \) readily obtained. From the above analyses we also note that the \( \alpha \) parameter is more strongly restricted if causality requirements \( (\Omega_\alpha^2 \leq 1) \) are imposed (see Figure 4). However, it seems that a even better method to place limits on such a parameter is through the physics of the perturbations, i.e., CMB and LSS data (see, e.g., [23, 24]).

Acknowledgments

The authors are supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brasil) and CNPq (62.0053/01-1-PADCT III/Milenio).
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[46] Note that for $A_s = 1$, Eq. (4) does not depend on the parameter $\alpha$. Therefore, the smoothness of the curves at these points is a consequence of the step used for the parameters in the code.