The Ups and Downs of Baryon Oscillations

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ABSTRACT

Baryon acoustic oscillations, measured through the patterned distribution of galaxies or other baryon tracing objects on very large (\(\gtrsim 100\) Mpc) scales, offer a possible geometric probe of cosmological distances. Pluses and minuses in this approach’s leverage for understanding dark energy are discussed, as are systematic uncertainties requiring further investigation. Conclusions are that 1) BAO offer promise of a new avenue to distance measurements and further study is warranted, 2) the measurements will need to attain \(\sim 1\)% accuracy (requiring a 10000 square degree spectroscopic survey) for their dark energy leverage to match that from supernovae, but do give complementary information at 2% accuracy. Because of the ties to the matter dominated era, BAO is not a replacement probe of dark energy, but a valuable complement.

1. Introduction

This paper provides a pedagogical introduction to baryon acoustic oscillations (BAO), accessible to readers not necessarily familiar with details of large scale structure in the universe. In addition, it summarizes some of the current issues – plus and minus – with the use of BAO as a cosmological probe of the nature of dark energy. For more quantitative, technical discussions of these issues, see White (2005).

The same year as the detection of the cosmic microwave background, the photon bath remnant from the hot, early universe, Sakharov (1965) predicted the presence of acoustic oscillations in a coupled baryonic matter distribution. In his case, baryons were coupled to cold electrons rather than hot photons; Peebles & Yu (1970) and Sunyaev & Zel’dovich (1970) pioneered the correct, hot case. In the modern picture of these oscillations (see Meiksin, White, & Peacock (1999) for a comprehensive technical treatment, and Eisenstein (2005) for an exciting visual perspective), when the universe was hot enough for matter to be ionized, the photons and (charged) baryons were tightly coupled through electromagnetic forces. This made the mean free path for the photons short compared to \(ct\), the free streaming distance, and so the photons and baryons acted as a fluid medium, capable of supporting perturbations in the form of acoustic waves.
The largest scale of the acoustic waves was set by the sound horizon. Due to the rapidity of the decoupling process after the universe recombined, and the lack of significant interactions thereafter, these largest scale wavelengths remain in close to their primordial state today. Such oscillations take the form of patterns on this primordial sound horizon scale, and harmonics, in the spatial distribution of photons and baryons. These acoustic waves in the photon number (or temperature) distribution were detected some 33 years after the CMB discovery. The acoustic waves in the baryon spatial distribution were detected in 2005.

The pattern in the CMB photons shows up as peaks and troughs of order unity deviation, making precision measurement of the angular scale of the primordial sound horizon possible with modern wide area surveys such as the WMAP satellite. The theoretical derivation of the physical scale as a function of cosmology is straightforward due to the simple, well understood physics entering the photon-baryon coupling and decoupling, and the linear nature of the acoustic perturbations, due to fluctuation amplitudes of less than $10^{-4}$ in the number density, seeded (presumably) by early universe inflation. From the two elements of the measured angular scale and theoretical physical scale, one obtains the angular distance to the decoupling epoch.

The baryon side of the effect is similar, but with some important differences. For one thing, we do not detect baryons directly in the way we detect photons. Instead we detect light emitted from processes involving baryons or electrons, or light affected in some way by the gravitational potential of mass in a structure (galaxy or cluster, say). While electrons should trace the baryon pattern well, and can be neglected in the mass effects (since a proton outweighs an electron by some 2000 times), other important components of mass exist besides baryons. Indeed cold dark matter particles contribute six times more than baryons to the mass density of the universe. Thus, the spatial pattern of oscillations traced by massive structures has been diluted relative to the primordial baryon acoustic oscillations. Furthermore, the baryons after decoupling found a ready made pattern of gravitational potentials, from the cold dark matter, waiting to influence them. So the amplitude of the baryon acoustic oscillations is not of order one, like the photons, but rather $\lesssim 5\%$. On the plus side, these oscillations are not relative to a $< 10^{-4}$ base perturbation amplitude, but rather one of order $10^{-1}$, since the matter perturbations have been amplified by gravitational instability since the time of decoupling.

Why then isn’t it trivial to detect baryon acoustic oscillations, if their absolute amplitude is so much greater than the photon acoustic peaks? Unfortunately, many more CMB photons are available to be detected – $10^{15}$ pass through an outstretched hand each second, while there are fewer than $10^{10}$ galaxies in the entire visible universe. Furthermore, there is much
greater confusion of the baryon signal; many photons from the faint light given off by a distant galaxy must be detected before we even know there is a galaxy there tracing the primordial baryon distribution, and even more before we precisely know its three dimensional spatial location – and that is for just one galaxy in the pattern. Still, this feat has been accomplished with the Sloan Digital Sky Survey (Eisenstein et al. (2005)).

Using the same theoretical physical horizon scale calculated for the CMB photons, and measuring the angular scale of the baryon acoustic oscillations at some redshift from a large galaxy redshift survey, their ratio provides the angular distance scale to that redshift. This is often called a standard ruler test, in analogy to the supernova (SN) standard candle test. Because the measured angle is the ratio of the detected angular scale to the physical horizon at decoupling, this distance measure is in some sense tied to the early universe rather than to the recent universe. This will be important later (see §4). Moreover, because one can measure a three dimensional pattern of galaxies (while only a two dimensional picture of the CMB sky), one can also derive a ratio corresponding to a radial distance, basically the Hubble parameter at some redshift.

Thus baryon acoustic oscillations (BAO) offers a distance probe of the universe, and the expansion history and cosmological parameters including dark energy properties that enter the distance. Because the galaxies (or other baryon sensitive objects) are used merely as markers of spatial position, BAO is a geometric test in the sense of not needing to know galaxy properties or masses (this is not absolutely true, as discussed in §3). Only the wavelength of the oscillations follows simply from the primordial coupling to the CMB, not the amplitude, so BAO does not characterize the cosmic growth history. In this sense, this probe is in the same class as Type Ia supernovae.

In the following text, we consider the positive and negative aspects of BAO as a cosmological probe, and the role it can play alone and in complementarity with other techniques. We will see that, like every probe, corrections need to be applied. We identify some systematic issues that need to be addressed for robust estimation of its power in constraining dark energy.

2. BAO are simple, SN are complicated or 
   SN are rich, BAO are meager?

We indicated above why BAO are so much harder to detect and characterize than CMB acoustic oscillations, and that after all the work of observing distant galaxies one ends with a single quantity for each galaxy – its three dimensional position. This is both a feature and
a bug. The use of galaxies as markers does not depend on the galaxy properties, other than as they influence the detection (selection effects). SN are also used as geometric markers of the expansion history, but their distance measures depend on their luminosities, as well as intervening effects on the detected flux, such as dust. Properties of the SN that influence the detected flux complicate their use as cosmological probes to the extent that we remain ignorant of such effects, i.e. how much they contribute systematic uncertainties.

Since BAO depend on the galaxy positions, they appear much simpler. But systematic uncertainties arise here too. For example, just as the SN flux must be properly translated to its emitted luminosity, the measured galaxy redshifts must be put in terms of their radial position. This seems straightforward to accomplish, but at the level of precision required for a dark energy probe, corrections from redshift space to real space are not automatically trivial. We cannot a priori assume that although BAO appear to involve simple physics that their systematic uncertainties are negligible; we must compute them through theoretical calculations and compare them to simulations, and try the method out with real world data. We discuss some of these uncertainties in the next section.

In the presence of systematics, we must take seriously the section title question. Indeed the dependence of BAO on just a single measured property offers the feature of simplicity, but is this a bug as well? Each SN observed does not provide merely a single data point on the distance-redshift diagram, but rather a rich array of information that serves for crosschecks and systematics control. A SN used in a cosmological survey has a complete flux history from shortly after explosion, through maximum light, and into the nebular phase, spanning over two months in the SN rest frame. This is obtained in multiple passbands, covering the rest-frame visible light and possibly extending into the ultraviolet and near infrared. An image of the SN relative to its host galaxy is part of the data, giving details on galaxy type and morphology, location of the SN in the core or outskirts, etc. A spectrum of the SN provides detailed information on the physics, including through line velocities, shapes, and strengths. This rich stream of data allows robust crosschecks on the use of the SN as a cosmological distance indicator. No such crosschecks exist for the BAO method.

So complexity, and simplicity, can be either a feature or a bug. Neither should be automatically ruled out, but rather the use of all the data and the impact of remnant systematic uncertainties must be calculated in detail. We do point out, however, that both methods have distinct advantages in being geometric methods, where the objects are markers only. This also means that cosmologically unbiased selection of the best markers is allowed to provide the tightest constraints possible on cosmological parameters.
3. Systematic Uncertainties for Baryon Acoustic Oscillations

While BAO indeed appear cleaner in physics basis and application than many other probes, this does not mean we should neglect careful scrutiny and computation to verify this. Because of the short history of measurement of BAO, the list below of areas needing examination is likely incomplete.

3.1. Bias

As mentioned above, while the primordial spatial pattern exists in baryons, we measure the light from assemblages of matter. The relation between these quantities is referred to as the bias, and can in general be scale varying. It seems likely that on the large scales of the BAO pattern, the bias will be smooth. Preliminary simulation studies have been carried out by Seo & Eisenstein (2005) agreeing with this. For a smooth “tilt” to the matter power spectrum caused by bias, one can add fitting parameters, e.g. polynomial coefficients. This can remove the effects of bias on measuring the oscillation scale, but it is not clear how much the additional parameters degrade or otherwise affect the scale estimation when trying to achieve percent level accuracy. Some simulations do appear to show an increasing shift in oscillation peak location as the bias increases, even after this correction.

Another approach has been proposed by Dolney et al. (2004), where halo model bias parameters are fit by means of measuring higher order correlations plus the matter power spectrum. So while definitive calculations need to be done, bias uncertainty is likely to be tractable and probably will not substantially interfere with the BAO method.

3.2. Nonlinear mode coupling

As pointed out in the Introduction, BAO actually have an advantage over the acoustic oscillations in the CMB in that the absolute amplitude of the matter fluctuations when measured in the recent universe is much higher than the photon density fluctuations. However, this means that the simple, linear physics treatment as in the CMB does not transfer over with the same high degree of robustness – one might say that the CMB is 99.99% linear while the BAO are 90-99% linear. This is a nonnegligible difference.

The slight degree of nonlinearity causes coupling between modes, or scales, in the baryon spatial pattern. This coupling increases as the matter fluctuations grow, becoming more significant as one approaches the more recent universe. Such scale coupling smears the baryon
acoustic oscillations, rendering them difficult to discern and possibly changing the scale. One can avoid these effects by only looking at the longest wavelengths (where unfortunately sample variance, or the size of the survey, gives an increasingly large error), but then one restricts the number of oscillations available for measuring the standard ruler scale. Commonly, one estimates that one to two peaks are visible for observations at $z \approx 0.3$, three to four peaks for $z \approx 1.5$, and six to seven peaks for $z \approx 3$.

Such a characterization of linearity, e.g. requiring the mass fluctuation amplitude to be less than 0.5, is likely overoptimistic. Figure 1 shows the results of a $1024^3$ particle, $2048^3$ grid PM simulation in a 700 $h^{-1}$Mpc box, from M. White (White 2005). The curves in the upper panel show the linear theory prediction for the mass variance per logarithmic wavemode $k$ and the points give the simulation results, showing the deviation at higher $k$ as nonlinear effects enter. The bottom panel shows the ratio of power relative to a zero baryon content universe, but the main point is the rapidity with which the simulation results (including nonlinear effects) deviate from the linear theory curve wherein the BAO are readily apparent – even though the nonlinear effects are only at the 5-10% level. At $z = 1$, one might be able to convince oneself one sees three peaks, and at $z = 3$ maybe four, but there is a huge difference between being able to detect that a peak exists and being able to characterize the wavelength at the $\leq 1\%$ level.

Careful work needs to be carried out to determine how much residual uncertainty is caused by nonlinear mode coupling, and by lack of complete understanding of the true nonlinear mass power spectrum.

### 3.3. Redshift space distortions

Baryon acoustic oscillations show themselves in the three dimensional spatial pattern of the baryon distribution on large scales. Besides the difficulties due to not detecting baryons directly (§3.1), complications arise due to not detecting the radial distance directly. This is the well known problem of translating from a redshift space distribution to a real space distribution. In particular, massive structures show three effects: a stretching along the line of sight (finger of god) effect due to internal velocity dispersion in the gravitational potential, a squashing due to large scale infall, and a distortion due to the spacetime geometry shear effect discussed by Alcock & Paczyński (1979) (basically arising because points at the same distance emit light signals at the same value of the cosmic expansion, while points with radial separation emit at different values).

The formalism for dealing with these distortions has been developed in various limits,
e.g. by Kaiser (1987); Ballinger, Peacock, & Heavens (1996); Matsubara & Szalay (2002). These distortions, needing to be fit, add some level of uncertainty to determining the baryon acoustic oscillation scale. One can employ a polynomial fit to the slope induced by this systematic in the power spectrum to reduce its effect. With such a procedure, the redshift distortion clearly does not prevent detection of the peaks – however we do not have quantification of how it affects the precision characterization of the peaks, i.e. is the scale still accurately determined at the $\leq 1\%$ level? Recall that we are interested in the $k$-space scale of the oscillations, so even if the residual amplitude of the systematics after correction is at a few percent this does not guarantee recovery of the length scale to $\leq 1\%$.

Moreover, if we rely on the theory of redshift distortions as mentioned above, we must take into account the results by Scoccimarro (2004) showing the failure of its elements outside their limiting validity, in particular the influence of nongaussianities and nonlinearities. It seems more robust to attempt to make such corrections through simulations. This can certainly be done but will require a comprehensive suite of various cosmological models.

### 3.4. Other Systematics

Other systematics that enter at a low, but not yet defined, level include selection function effects. As for SN, the fact that galaxies serve merely as markers for the BAO method means that we can pick and choose a sparse sample of objects. This helps reduce the time and cost of a survey, but the sample cannot be too sparse or shot noise effects will enter once the survey has been divided into redshift bins or subclasses (for crosschecks). Care must be taken that the selection is homogeneous in any quantity that contains cosmology dependence. In determining precision redshifts, one must avoid line confusion or blending, and be wary of the effect of intragalactic structure variation over a spectrograph slit, such as from star forming regions.

BAO indeed have strong advantages in avoiding flux and color calibration issues (of course, if such calibration is accomplished for a supernova program, it will have widespread benefits for most fields of astronomy, but that is not a strictly dark energy issue). Similarly, dust extinction should not be a problem, unless Milky Way extinction somehow apodizes power on large scales to confuse oscillations (which seems farfetched). Similarly, as long as a BAO survey is properly designed so flux threshold limits do not interfere with redshift completeness, gravitational lensing magnification should not be an issue. It could enter through distortion of the standard ruler – remember lensing magnifies flux by magnifying scales, i.e. so-called convergence lensing or scale shrinking Linder (1988a) – but this should be negligible on the large scales involved in BAO.
Some theory systematics exist as well. One example already mentioned is the details of the nonlinear power spectrum: this probably does not have features harmonically related to the BAO scale, but incomplete correction for even “broadband” tilts can degrade determination of the scale. On the standard ruler side of the calculation, the major uncertainty is the value of the physical matter density $\Omega_m h^2$; the Planck CMB survey should determine this to 0.9%, which is precise but not perfectly so. Glazebrook & Blake (2005) show that the residual uncertainty on the matter density translates into an up to 40% degradation of the constraints on dark energy parameters. Eisenstein & White (2004) showed that effects of curvature and neutrinos will not adversely affect the standard ruler use. Computation of the CMB power spectrum, for fitting to the photon acoustic oscillations, is also not perfect: Corasaniti et al. (2004) showed that 10-15% errors in CMBfast can occur for dynamical dark energy models, at both the lowest multipoles and in the acoustic peaks region.

Perhaps most worrisome is that the matter power spectrum depends on the standard scenario in the dark matter and dark energy physics. If we lose the gamble and the dark energy is not simple – e.g. sound speed not equal to the speed of light, anisotropic stress nonzero, or coupling nonzero – then these properties confuse the BAO technique. For example they can change the turnover scale in the pure CDM power spectrum used to calibrate the oscillations and can even add new oscillations. In this sense BAO are not a geometric probe, not following directly from the metric as SN distances do.

3.5. Summary of Influence of Systematics

The method of BAO clearly has several major pluses in the area of systematics control, such as substantial freedom from flux calibration issues (though it does still have spectral, or redshift, calibration issues). While the linearity of the physics is a strong plus, it is not total, and indeed the effects of nonlinearities might be the most worrying of those mentioned above. Still, this should be less severe than for the weak lensing method. All the issues brought up in this section (with the possible exception of the “non-geometric” one) are likely to be tractable with sufficient effort. But that effort has not yet been put in, and so we cannot say for sure that all the systematic uncertainties will not contribute at the 1% level.

In the next section we will consider BAO as being purely statistically limited, without systematic errors, and see that the question of whether 1% accuracy can be achieved is crucial.
4. BAO are more precise distance indicators than SN, or SN are more precise dark energy indicators than BAO?

If our goal is understanding dark energy, then we must keep this in mind when discussing the precision of measurements carried out for different techniques. For example, the CMB power spectrum, and the distance to the last scattering surface, can be measured quite precisely, but they contain relatively little leverage on determining dark energy properties.

One issue, related to the richness/meagerness argument above, is the density of distance measurements. That is, if we are using distances to map the expansion history of the universe, how fine in detail is the map? BAO are fundamentally limited in that millions of galaxies must be binned together to provide an accurate measurement of the oscillation wavelength, and crucially one cannot subdivide the redshift bins below this scale. Since the scale corresponds to a comoving size of $100 \, h^{-1} \, \text{Mpc}$, the Nyquist frequency of the oscillations imposes a requirement that $\Delta z \geq 0.2$. SN, by contrast, can map the expansion arbitrarily finely, subject only to observational constraints not any fundamental limitation. Note that such a difference in ability is not included in the following analysis in terms of a smooth, slowly varying equation of state.

It is important to remember that any estimations of dark energy constraints must use a well behaved description of dark energy. Equation of state parametrizations that blow up quickly to unphysical values will give hypersensitive, inaccurate constraints. For example the parametrization $w(z) = w_0 + w_1 z$ can overstate the dark energy constraints by a factor 3 (Linder & Huterer (2005)).

Calculations, e.g. Seo & Eisenstein (2003, 2005); Blake & Glazebrook (2003); Glazebrook & Blake (2005), show that 10000 square degree spectroscopic redshift surveys can achieve percent, or possibly subpercent, statistical precision on distances. If the systematic uncertainties do not degrade this, it is comparable to or better than SN distance measurements, which are at the 1% level. Moreover, the radial modes of BAO provide the Hubble parameter $H(z)$, rather than the angular distance that is the integral of this quantity. Since dark energy enters the distances through $H(z)$, a determination of the bare quantity seems advantageous. These properties of BAO appear promising.

However, we must then propagate the measured distances through to the dark energy constraints, and here a subtlety arises. As stated in the Introduction, one actually measures relative distances, in both the BAO and SN cases. The SN distances can be viewed as either luminosity distances to some redshift $z$ convolved with an additional parameter $M$ involving the absolute luminosity and Hubble constant, or equivalently the luminosity distance to some redshift $z$ relative to some low redshift value (formally $d(10 \, \text{pc})$). The BAO distances involve
the sound horizon scale, found through CMB angular scale measurements to the decoupling epoch (formally the last scattering surface). So they can be thought of as distances to some redshift $z$ relative to the value at a high redshift, $z = 1089$.

Since dark energy is more dominant in the recent universe than in the high redshift universe, distances to $z = 1.7$, say, measured relative to low redshifts are much more sensitive to dark energy properties than those measured relative to high redshifts. Indeed, canonical models of dark energy have not merely subdominant energy densities at high redshift but basically negligible energy densities. For example, a cosmological constant model has $\Omega_A(z = 1.7) = 0.1$ and $\Omega_A(z = 1089) = 10^{-9}$.

So even if BAO could achieve somewhat more precise distance measurements than SN, plus the measurement of $H(z)$, SN could still achieve denser and more leveraging measurements. This was calculated explicitly in Linder (2003) and is examined in more detail in calculations for this article. We find that high accuracy, relatively low redshift ($z \approx 0.5$), BAO measurements are important for dark energy leverage (note that here nonlinearities will be most severe). As expected, this is not as crucial for dynamical models that retain more dark energy at $z > 1$.

### 4.1. Dark Energy Equation of State Constraints

As a baseline model, we consider 1% measurements of both the radial and tangential BAO scale at redshifts $z = 0.5, 1, 2.75, 3.25$. We find that lack of data in the intermediate range $z \approx 1.3 - 2.5$ (more difficult for ground based observations) does not change the broad characteristics. Results are tested for a fiducial cosmological constant model and a SUGRA ($w_0 = -0.82, w_a = 0.58$) model, both with $\Omega_m = 0.28$. BAO is compared or added to other data sets such as the distance to CMB last scattering of Planck quality, SN distance measurements of SNAP quality (including systematics), or weak lensing (WL) shear power spectrum of SNAP 1000 square degrees quality.

We illustrate some results in Figure 2, including the effect of diluting the BAO measurement accuracy to 2%. Parameters not shown, such as $\Omega_m$ or $\mathcal{M}$, are marginalized over. Note that since nonlinear effects are most troublesome at the $z = 0.5$ bin, this point, which carries substantial dark energy leverage, may be the hardest place to attain 1% accuracy. BAO+CMB yield 0.18, 0.47 one sigma uncertainty on $w_0, w_a$, while SN+CMB give 0.09, 0.37, and all three together provide 0.06, 0.24. The complementarity is significant. Note that only all three probes together begin to approach the “revelatory” requirement of estimating $w' = w_a/2$ to 0.1. If BAO is weakened to 2%, then BAO+CMB gives a poor 0.35, 0.90,
and the trio provides 0.08, 0.33 – minor improvement over the case without BAO. Thus, the accuracy attainable with the BAO method is important to know rigorously.

We have seen from the above results that SN data are a key element for the dark energy constraints (e.g. improving estimation of $w_0$ by a factor of 2-4 alone, and a factor 3-4 added to BAO). But given the complementarity, if BAO can provide 1% measurements then the SN data set may not need to be as stringent. Reducing the survey depth to, say, $z = 0.8$ but somehow keeping the SN systematics at the same low level as for the space based SNAP survey, degrades the BAO+CMB+SN constraints only to 0.06, 0.25. However if the BAO precision slips to 2% this becomes 0.09, 0.42. Careful study is required. We will also see later that the SN depth is an important element in several other respects.

Note that just because BAO effectively involves a distance ratio of, say, $d(z = 3)$ to $d(z = 1089)$, this does not mean that BAO can simply separate out the conditions of the universe between $z = 3$ and $z = 1089$. That is, one does not isolate the effects of “everything but” dark energy or the effects of unexpected early dark energy. Both distances entering the ratio are still integral quantities, and while the conditions at $z > 3$ are involved, they are not given separately. Still, BAO does offer the possibility of putting some constraints on $z > 3$ dark energy (though one expects the mass growth factor to be more sensitive to this property).

4.2. Complementarity with Cosmic Growth Probes

None of the probes considered above have dependence on mass growth, and so are incapable of comparing the expansion history vs. the growth history to test the theoretical framework (see, e.g., Linder (2005a)). Whether the dark energy arises from a new physical component, e.g. a high energy physics scalar field, or a modification of the theory of gravity is a crucial question. Answering this is a key requirement for understanding the physics of acceleration. Thus, just as a purely SN experiment would not be sufficiently revelatory about dark energy, a purely BAO experiment is not acceptable. We therefore consider measurement of the weak lensing shear power spectrum, as estimated for the SNAP satellite, as another probe of dark energy to be taken in complementarity. Note that this, like BAO but unlike SN, is here treated with purely statistical errors.

Figure 3 shows that WL adds appreciable information enabling tighter dark energy constraints. Recall, however, that we also want each probe, or at least expansion history and growth history separately, to stand on their own to allow for crosschecks and test of the theoretical framework (e.g. modifications of Einstein gravity). With this kept firmly in
mind, we examine the constraints upon combining WL data with the other probes.

First, note that WL+CMB without either BAO or SN gives constraints on \( w_0, w_a \) of 0.13, 0.49, so complementarity with other probes is desired. (One could also consider extension of the area of the space quality survey, or addition of other weak lensing techniques such as higher order correlations or cross-correlation cosmography. This would help the WL probe stand as more comparable to the expansion history constraints.)

For SN+CMB+WL, the constraints are 0.066, 0.23; BAO+CMB+WL attain 0.034 (0.055), 0.17 (0.27) for BAO at the 1% (2%) level. Again we see that it is important to find whether the systematic uncertainties allow BAO determination at the 1% level – and to realize that SN are the only pure geometric probe, immune to microphysics in the dark energy sector. All probes combined give dark energy bounds of 0.031, 0.14. We find that BAO does add value to SN and WL methods, and SN adds value to BAO and WL methods, especially when the BAO accuracy is 2%. Reducing the SN survey depth to \( z = 0.8 \) is more harmful now, with degradations in the case of all probes combined up to the 22%, 39% level on \( w_0, w_a \).

Note that synergy exists between surveys carrying out WL and BAO measurements. A large scale photometric survey for WL would basically supply a large scale photometric BAO survey and be an important selector for a large scale spectroscopic BAO survey, while a spectroscopic BAO survey can help calibrate photometric redshifts for the WL survey. Increasing the area of the WL survey to 4000 square degrees provides 20% improvement in the dark energy parameter constraints using all the probes. Keeping WL at 1000 square degrees but adopting the dynamical SUGRA fiducial model gives complete combination constraints of \( \sigma(w_0) = 0.020, \sigma(w' = w_a/2) = 0.029 \), with a minimum variance equation of state uncertainty \( \sigma(w_{\text{min}} = w(z = 0.43)) = 0.0095! \)

This combination of probes seems quite exciting in their prospects for revealing the nature of dark energy – if the systematic uncertainties can be kept below the statistical levels employed here. Also remember that we should require tight constraints not only from all probes jointly, but from each individually. Only thus will we have confidence in the new physics discovered.

### 4.3. Other Cosmological Probe Aspects of BAO

One can consider other uses of BAO measurements, such as constraints on spatial curvature in conjunction with the cross-correlation cosmography technique of weak lensing (Bernstein (2005)). Note that this aims at a measurement of curvature, not a test of the constancy
of the spatial curvature over cosmic time. The latter is sometimes mistakenly attributed to CMB measurements, saying that the CMB measures curvature at the epoch of last scattering and this can be compared to measurements today. This statement is untrue, since the CMB measures the integral of the curvature effects (insofar as they can be separated from all the other component energy densities) from $z = 0 - 1089$.

While measuring the spatial curvature is a worthy goal, there are some aspects to consider. It is not directly telling us anything about the nature of dark energy, though it does help through breaking degeneracies. Furthermore, the proposed measurement method relies on two techniques, neither of which has been matured through a history of implementation and systematics studies. So one should not use this to drive “optimization” of other probes around these (if that were even possible). On the other hand, SN+WL also measures curvature to 1-2%. Finally, measurements at moderately high redshifts, e.g. around $z \approx 3$ that is one of the two main ranges for applying BAO, are actually extraordinarily insensitive to spatial curvature: the dependence of cosmological distances on curvature goes through a null at $z \approx 3$ (see Linder (2005b) for further examination of curvature).

One can also consider BAO as a means of testing the reciprocity, or thermodynamic, relation. This is phrased either as a redshift scaling between angular distances and luminosity distances or “third party” angular distances between points not including the observer. Most commonly the relation is phrased as $d_l = (1 + z)^2 d_a$. This has a long history in cosmology, dating from the 1930s with Tolman, Ruse, Etherington, through Weinberg (1972), and a general proof in terms of the Raychaudhuri equation and the second law of thermodynamics by Linder (1988b). Due to this thermodynamic origin (basically, if two identical blackbodies sent photons to each other over cosmic distances then work would be done unless the relation held), it is of very general applicability. As long as the propagating photons obey Liouville’s Theorem, then the relation must work. In this sense it is an equivalent problem to measuring the evolution of the CMB temperature: if $T(z) \neq T_0(1 + z)$, we would far more likely blame the measurement than believe a breakdown of physics. So using BAO (measuring $d_a$) and SN (measuring $d_l$) to test for violation of the reciprocity relation is not likely to be actually useful (apart from the slim chance of there really being a violation such as from photon-axion mixing, say). It is an interesting idea, but one would not advocate having $T(z)$ measurements drive CMB observations either.

5. Conclusion

In summary, baryon acoustic oscillations offer another promising cosmological probe. Astrophysicists should certainly pursue its development, theoretically, algorithmically, and
observationally, to learn how to practically carry out large spectroscopic galaxy redshift surveys and extract the information, and to obtain realistic estimation of its accuracy. Ground based observations serve as the starting point, and probably dominant source, of data. A 10000 square degree spectroscopic survey, free from systematics, is a challenging endeavor. Space observations may have a role to play in covering the redshift range \( z \approx 1 - 2 \), though this does not appear to be crucial, but the reduced noise from space could play a useful role.

It is important to keep in mind, that BAO is not a panacea nor is it effortless. Corrections do need to be applied, and the residual systematics, while promising, require hard work to quantify at the 1% level. Recall that BAO is intrinsically limited in the fineness of the expansion history mapping possible, and less sensitive to dark energy for the same precision due to its tie to the matter dominated era. Finally, a “nonstandard” dark matter or dark energy sector could throw BAO awry since they are not a purely geometric probe, while SN remain clean.

The presence of such ups and downs holds for any cosmological probe. Given the few ways we have of robustly understanding the new physics behind the accelerating universe, the baryon acoustic oscillations method is a welcome addition. Pressing forward with further study, and actual observational application, BAO may offer important complementarity to supernovae, and weak lensing, probes for understanding dark energy.

This work has been supported in part by the Director, Office of Science, Department of Energy under grant DE-AC02-05CH11231. I thank Chris Blake, Daniel Eisenstein, Gary Bernstein, Karl Glazebrook, and Martin White for useful conversations.

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Fig. 1.— Baryon acoustic oscillations mostly preserve their origin from primordial linear perturbations of the photon-baryon fluid. But as matter perturbations grow, nonlinear effects gradually dissolve the standard ruler scale. The top panel shows the mass power per logarithmic $k$ mode at three redshifts from an N-body simulation, compared to the linear results. The bottom panel shows the ratio relative to the “no baryon” linear case, with the curve including baryons. The simulation points deviate from the clear oscillatory structure even when the nonlinearities are quite modest. Figure courtesy of M. White (White 2005).
Fig. 2. — Dark energy equation of state parameter estimates for various combinations of cosmological probes using next generation data. Even 1% BAO do not match the leverage of SN, but BAO can provide useful complementarity and a crosscheck.
Fig. 3.— Dark energy equation of state parameter estimates for various combinations of cosmological probes using next generation data. Weak lensing complements both BAO and SN, and all probes together provide strong constraints.