Non-cascaded Control Barrier Functions for the Safe Control of Quadrotors

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Abstract—Researchers have developed various cascaded controllers and non-cascaded controllers for the navigation and control of quadrotors in recent years. It is vital to ensure the safety of a quadrotor even if a controller tends to make the quadrotor unsafe. To this end, this paper proposes a non-cascaded Control Barrier Function (CBF) for a quadrotor controlled by either cascaded controllers or a non-cascaded controller. Incorporated with Quadratic Programming (QP), the non-cascaded CBF can simultaneously regulate the magnitude of the total thrust and the torque of the quadrotor determined by a controller, so as to ensure the safety of the quadrotor. The non-cascaded CBF establishes a non-conservative forward invariant safe region, in which the controller of a quadrotor is fully or partially effective in the navigation or the pose control of the quadrotor. The non-cascaded CBF is applied to a quadrotor performing aggressive roll maneuvers in simulations to evaluate the effectiveness of the non-cascaded CBF.

I. INTRODUCTION

Researchers have made great progress with the design, navigation, and control of quadrotors in recent years. Several challenging tasks (e.g., surveillance, delivery, and rescue [1], [2], [3], [4]) have been addressed by quadrotors. However, a navigation strategy, a control strategy, or manual manipulations may lead a quadrotor into an unsafe state (e.g., collisions), especially when developing a new navigation strategy or a new control strategy [5]. It’s crucial to ensure the safety of a quadrotor, even if a navigation strategy, a control strategy, or manual manipulations tend to make the quadrotor unsafe.

Researchers have proposed different approaches to address the safety of systems. Model Predictive Control (MPC) [6] has been applied to ensure safety for systems subject to various constraints. Wabersich et al. proposed a safe filter based on MPC to transfer a constrained system into an unconstrained safe system [7]. Then a Reinforcement Learning (RL) algorithm [8] can be applied to the system safely. Researchers also have paid attention to providing safe guarantees for systems based on Control Barrier Functions (CBFs) [9]. To handle the safety of complex systems, researchers have proposed Exponential Control Barrier Functions (ECBFs) [10] and High-order Control Barrier Functions (HOCBFs) [11], [12]. To enhance the feasibility of CBFs for systems with input constraints, Agrawal et al. proposed a subset of the safe set of a system and designed a controller to render the subset forward invariant to ensure the safety of the system [13]. Chen et al. introduced a nominal control law and a backup set to handle the safety of systems with input constraints [14].

Control Barrier Functions (CBFs) [9] have been developed to ensure safety in several applications (e.g., adaptive cruise control [15], multi-robot systems [16], carrier landing [17]). CBFs have been applied to quadrotors by some researchers as well in recent years. In [5], a position level CBF was designed for assisting human operators to teleoperate quadrotors safely. However, this study is based on the assumption that the Euler angles of a quadrotor are small and cannot be applied to quadrotors performing aggressive maneuvers as a result. Wang et al. studied the trajectory planning of a team of quadrotors based on differential flatness and CBFs. CBFs were used to modify nominal trajectories for quadrotors to avoid collisions [16]. To address the collision safety constraints suffered by a planar quadrotor, time-varying control Lyapunov functions were used to guarantee stability of the quadrotor and time-varying CBFs were developed to guarantee safety of the quadrotor in [18]. The method was extended to guarantee the safety of a three-dimensional quadrotor in [19]. Khan et al. proposed cascaded CBFs for a cascaded control architecture of quadrotors to enforce safety, allowing independent safety regulation in the altitude domain and in the lateral domain of a quadrotor [20]. Singletary et al. proposed a nominal control law and a backup set to guarantee the safety of a quadrotor with input constraints [21]. However, [18], [19], [20], [21] focused on quadrotors with cascaded controllers and developed cascaded CBFs for quadrotors. The cascaded CBFs cannot be applied to quadrotors with non-cascaded controllers (e.g., an end-to-end learning-based controller [22], [23]). Moreover, the effectiveness of the cascaded CBFs in the case that quadrotors are in abnormal state has not been verified. The abnormal states of a quadrotor refer to the states that a quadrotor usually doesn’t reach according to a conventional motion planner, such as up-side-down and tumbling.

To the best knowledge of the authors, a CBF that can ensure the safety of quadrotors controlled by non-cascaded controllers and can address the safety issue of quadrotors in abnormal state is not available yet. To address the above-mentioned two issues, this paper proposes a novel non-cascaded CBF for quadrotors. The main contributions of this paper are as follows.

• This paper develops a non-cascaded CBF for quadrotors controlled by either cascaded controllers or non-cascaded controllers. The non-cascaded CBF can maintain a quadrotor within a safe region in Cartesian space, even if the quadrotor attains an abnormal state.

• Based on a physical engine, this paper verifies the effec-

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tiveness of the non-cascaded CBF for quadrotors in the case of aggressive roll maneuvers.

The rest of this paper is organized as follows. The preliminaries of this paper are presented in Section II. Section III illustrates a non-cascaded CBF for quadrotors. In Section IV, the non-cascaded CBF is applied to a quadrotor performing aggressive roll maneuvers based on a physical engine. Section V summarizes this paper.

II. PRELIMINARY

This section demonstrates the dynamics of a quadrotor, exponential control barrier functions, and the problem formulation of the safety of a quadrotor.

A. Dynamics of a Quadrotor

In this section the dynamics of a quadrotor is demonstrated based on a body frame $F_B$ fixed to the quadrotor and an inertial frame $F_I$, as shown in Fig. 1. The body frame is defined by axes $x_B, y_B$, and $z_B$, and the inertial frame is defined by axes $x_I, y_I$, and $z_I$. Euler angles with a Z-Y-X sequence are used to define the roll $\phi$, pitch $\theta$, and yaw $\psi$ angles between the body frame and the inertial frame.

![Fig. 1. Notations of a quadrotor](image)

The dynamics of a quadrotor can be expressed as [24]

$$
\begin{align*}
\dot{v} &= -Gz_t + \frac{1}{m} f_T z_B \\
\dot{R} &= R\Omega \\
J \dot{\omega}_B &= \tau - \omega_B \times J \omega_B + \tau_g \\
\dot{r} &= v
\end{align*}
$$

(1)

where $v = [v_x, v_y, v_z]^T \in \mathbb{R}^3$ represents the translational velocity of the quadrotor in the inertial frame, $G$ is the acceleration of gravity, $m$ is the mass of the quadrotor, $z_t$ is a unit vector in the positive direction of the $z_B$ axis of the inertial frame, $f_T$ denotes the magnitude of the total thrust of the quadrotor, $z_B$ is a unit vector in the positive direction of the $z_B$ axis of the body frame, $\Omega$ is the rotation matrix from the body frame $F_B$ to the inertial frame $F_I$ defined in [24], $\omega_B = [p, q, r]^T$ represents the angular velocity of the quadrotor in the body frame. $\omega_B$ can be expressed as

$$
\Omega = \begin{bmatrix}
0 & -r & q \\
-r & 0 & -p \\
q & p & 0
\end{bmatrix}
$$

(2)

$J = \text{diag}(J_{xx}, J_{yy}, J_{zz})$ is the inertia matrix of the quadrotor. $\tau = [\tau_x, \tau_y, \tau_z]^T$ is the torque generated by the rotors of the quadrotor in the body frame. $\tau_g$ denotes a gyroscopic torque.

$r = [r_x, r_y, r_z]^T$ is the position of the quadrotor in the inertial frame.

The state of a quadrotor can be represented by $\xi = [x, y, z, v_x, v_y, v_z, \phi, \theta, \psi, p, q, r]^T$. The control input of the quadrotor is $u = [f_T, \tau_x, \tau_y, \tau_z]^T$. The rotor velocities of the quadrotor can be derived based on the control input according to [25]

$$
\begin{bmatrix}
\dot{f}_T \\
\dot{\tau}_x \\
\dot{\tau}_y \\
\dot{\tau}_z
\end{bmatrix} =
\begin{bmatrix}
k_F & k_F & k_F & k_F \\
0 & k_F L & 0 & -k_F L \\
-k_F L & 0 & k_F L & 0 \\
k_M & -k_M & k_M & -k_M
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_T^2 \\
\dot{\omega}_T^2 \\
\dot{\omega}_T^2 \\
\dot{\omega}_T^2
\end{bmatrix}
$$

where $L$ represents the distance between a rotor and the center of the quadrotor. $k_F$ and $k_M$ are positive constants determined by air density and the shape of propellers. $\omega_t (i = 1, 2, 3, 4)$ denotes the rotational velocity of the $i$th rotor of the quadrotor.

For a quadrotor, one also has [25]

$$
\begin{bmatrix}
\dot{m} r \\
\dot{m} \dot{r} \\
\dot{m} \dot{r}^\prime
\end{bmatrix} =
\begin{bmatrix}
-m g z_t + f_T z_B \\
f_T \omega_t \times z_B + 2 f_T \omega_t \times z_B + f_T (\omega_t \times z_B + \omega_t \times \omega_t \times z_B)
\end{bmatrix}
$$

(4)

where $\omega_t = R \omega_B$ is the angular velocity of the quadrotor in the inertial frame. In practice, the rotor dynamics are relatively fast for the pose control of a quadrotor, so one can assume that the rotor thrusts as well as the control input of a quadrotor $u = [f_T, \tau_x, \tau_y, \tau_z]^T$ are instantaneously achieved [25]. Since quadrotors are usually equipped with digital controllers in practice, given a certain control input, the quadrotor will achieve the control input instantaneously and maintain the control input for a period of time. Thus, one can assume that the derivative of the magnitude of the total thrust $f_T$ is zero. Then, (4) can be simplified as

$$
\begin{bmatrix}
\dot{m} r \\
\dot{m} \dot{r} \\
\dot{m} \dot{r}^\prime
\end{bmatrix} =
\begin{bmatrix}
-m g z_t + f_T z_B \\
f_T \omega_t \times z_B \\
f_T (\omega_t \times z_B + \omega_t \times \omega_t \times z_B)
\end{bmatrix}
$$

(5)

B. Exponential Control Barrier Function

Exponential Control Barrier Function (ECBF) is introduced to address the high relative degree safety constraints of a system [10]. The ECBF has been used to design cascaded CBFs for quadrotors in [20]. Without loss of generality, one can assume a nonlinear affine system

$$
\dot{x} = f(x) + g(x)u
$$

(6)

where $f$ and $g$ are locally Lipschitz. $x \in D \subset \mathbb{R}^n$ is the state and $u \in U \subset \mathbb{R}^m$ is the control input of the system. The safety of the system can be guaranteed by enforcing the invariance of a safe set [9]. In particular, one can consider a set $C$ defined as the superlevel of a continuously differentiable function $h(x): \mathbb{R}^n \to \mathbb{R}$, yielding

$$
C = \{x \in \mathbb{R}^n : h(x) \geq 0\}
$$

$$
\partial C = \{x \in \mathbb{R}^n : h(x) = 0\}
$$

$$
\text{Int}(C) = \{x \in \mathbb{R}^n : h(x) > 0\}
$$

(7)

The set $C$ is referred to as a safe set.
Definition 1. [9] A set \( C \) is forward invariant if for every \( x(t_0) \in C, x(t) \in C \) for all \( t \in [t_0, t_1) \), where \( t_0, t_1 \in \mathbb{R} \) and \( t_1 > t_0 \geq 0 \). The system (6) is safe with respect to the set \( C \) if the set \( C \) is forward invariant.

Definition 2. [9] For system (6), given a set \( C \subseteq D \in \mathbb{R}^n \) defined as the superlevel set of a \( r \)-times continuously differentiable function \( h(x): D \subseteq \mathbb{R}^n \rightarrow \mathbb{R} \), then \( h(x) \) is an ECFB if there exits a row vector \( \mathbf{K}_a \in \mathbb{R}^n \) such that

\[
\begin{align*}
\sup_{u \in U} [L^p h(x) + L^p h^{-1} h(x) u] & \geq -\mathbf{K}_a \mathbf{h}(x) \\
h(x(t)) & \geq \mathbf{C}_b A^d \mathbf{h}(x(t)) \geq 0, \quad h(x(t_0)) \geq 0
\end{align*}
\]

where

\[
\mathbf{h}(x) = \begin{bmatrix} h(x) \\ h(x) \\ \vdots \\ h^{n-1}(x) \end{bmatrix}, \quad A^d = \begin{bmatrix} A^d \end{bmatrix}, \quad \mathbf{C}_b = [1 \ 0 \ \cdots \ 0]
\]

and the matrix \( A^d \) is dependent on the choice of \( \mathbf{K}_a \).

One can define a family of functions \( v_i : D \rightarrow \mathbb{R} \) and corresponding superlevel sets \( C_i, i = 0, \ldots, r_b \) as follows [9]

\[
v_0 = h(x), \quad C_0 = \{ x : h(x) \geq 0 \},
\]

\[
v_i = v_{i-1} + p_i v_0(x), \quad C_i = \{ x : v_i(x) \geq 0 \},
\]

\[
v_r = v_{r-1} + p_r v_0(x), \quad C_r = \{ x : h_r(x) \geq 0 \},
\]

where \( p_i \) is an adjustable parameter and one has \( p_i > 0 \) (i = 1, 2, …, \( r_b \)).

Theorem 1. [9] If \( C_b \) is forward-invariant and \( x(t_0) \in \bigcap_{i=0}^{r_b} C_i \), then \( C_0 \) is forward-invariant.

C. Problem Formulation

This paper addresses the safety of a quadrotor in the form of maintaining the quadrotor within a safe region in Cartesian space. A safe region is defined as

\[
\mathcal{S} = \{ \mathbf{r} \in \mathbb{R}^3 | r_{\text{min}} \leq \mathbf{r} \leq r_{\text{max}} \}
\]

where \( \leq \) represents element-wise inequality. \( r_{\text{min}} \) and \( r_{\text{max}} \) are the lower bound and the upper bound of \( \mathbf{r} \), respectively. For a quadrotor with the initial position \( \mathbf{r}(t_0) \in \mathcal{S} \) and a nominal control input in terms of the total thrust and the torque generated by the rotors, one needs to ensure that the position of the quadrotor satisfies \( \mathbf{r}(t) \in \mathcal{S} \) where \( t \geq t_0 \), by regulating the nominal control input.

III. SAFE CONTROL OF QUADROTORS BASED ON NON-CASCADED CONTROL BARRIER FUNCTIONS

To ensure the safety of a quadrotor controlled by cascaded controllers or a non-cascaded controller, this section proposes a non-cascaded CBF for the quadrotor.

A. Non-cascaded Control Barrier Functions

This paper regards a quadrotor as a system with a relative degree of four. Thus, ECFB [10] is used to design non-cascaded CBFs for the quadrotor in this section. CBFs are designed for every single \( r_x, r_y, \) or \( r_z \) component of the position of the quadrotor, and then the CBFs are integrated to achieve a CBF that can maintain the quadrotor within a given safe region.

According to the above-mentioned idea, this paper designs two control barrier function candidates for the \( r_x \) component of the position of a quadrotor

\[
\begin{align*}
\tilde{h}_x(r_x) &= r_{\text{max}} - r_x \\
\tilde{h}_x(r_x) &= r_x - r_{\text{min}}
\end{align*}
\]

where \( r_{\text{min}} \) and \( r_{\text{max}} \) are the lower bound and upper bound of a safe \( r_x \), respectively. A safe set of the \( r_x \) component of the position of a quadrotor is

\[
C_x = \{ r_x \in \mathbb{R} | \tilde{h}_x(r_x) \geq 0, \tilde{h}_x(r_x) \geq 0 \}
\]

For \( \tilde{h}_x(r_x) \), a family of functions can be achieved according to [9]

\[
\begin{align*}
\tilde{v}_0 &= \tilde{h}_x \\
\tilde{v}_1 &= \tilde{v}_0 + p_{v_0} \tilde{v}_0 \\
\tilde{v}_2 &= \tilde{v}_1 + p_{v_1} \tilde{v}_1 \\
\tilde{v}_3 &= \tilde{v}_2 + p_{v_2} \tilde{v}_2 \\
\tilde{v}_4 &= \tilde{v}_3 + p_{v_3} \tilde{v}_3 \\
\tilde{v}_r &= \tilde{v}_{r-1} + p_{v_r} \tilde{v}_{r-1}
\end{align*}
\]

where \( p_v \) is an adjustable parameter and one has \( p_v > 0 \) (i = 1, 2, 3, 4). According to (5) and (13), \( v_x, v_y, \) and \( v_z \) depend on the control input of a quadrotor \( \mathbf{u} \). If the control input of a quadrotor \( \mathbf{u} \) can enforce the forward-invariance of \( C_x, C_y, \) and \( C_z \), the forward-invariance of \( C_b \) defined by \( \tilde{h}_x \) is guaranteed according to Theorem 1. One can ensure that \( r_x \) is kept a safe distance from \( r_{\text{max}} \) then.

Based on (13) and (15), to enforce the forward-invariance of \( C_x, C_y, \) and \( C_z \), one has

\[
\begin{align*}
\mathbf{r} \leq \mathbf{Q} \cdot r_{\text{max}}
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{r} &= \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \sum_{i=1}^{2} p_{r_i} \\ \sum_{i=1}^{3} p_{r_i} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}
\end{align*}
\]

Similar to (16), to enforce the forward-invariance of a safe set defined by \( \tilde{h}_x(r_x) \), one should make

\[
\begin{align*}
\mathbf{Q} \cdot r_{\text{min}} \leq \mathbf{x}
\end{align*}
\]

According to (11), (14), and (16), one can enforce the forward-invariance of the safe set \( C_x \), if

\[
\begin{align*}
\mathbf{Q} \cdot r_{\text{min}} \leq \mathbf{r} \leq \mathbf{Q} \cdot r_{\text{max}}
\end{align*}
\]
The safety of the \( r \) component of the position of a quadrotor is ensured then.

One can design CBFs that are similar to (13) for the \( r \) and \( \bar{r} \) components of the position of a quadrotor, according to the approach demonstrated above. By integrating the CBFs for the \( r \), \( \bar{r} \), and \( \bar{\bar{r}} \) components of the position of the quadrotor, one can obtain a non-cascaded CBF that can maintain the quadrotor within a safe region. The non-cascaded CBF for the quadrotor can be expressed as

\[
\begin{cases}
\bar{h} = r_{\text{max}} - r \\
\bar{\bar{h}} = \bar{r} - r_{\text{min}}
\end{cases}
\]

(20)

A safe set can be formulated as

\[ C = \{ r \in \mathbb{R}^3 \mid \bar{h} \geq 0, \bar{\bar{h}} \geq 0 \} \]

(21)

To enforce the forward-invariance of the safe set \( C \) and ensure the safety of the quadrotor, one should make

\[
Qr_{\text{min}} \preceq \Gamma A \preceq Qr_{\text{max}}
\]

(22)

where

\[
\Gamma = \begin{bmatrix}
\prod_{i=1}^{2} P_i & \prod_{i=1}^{2} P_i & I_{3 \times 3} & O_{3 \times 3} & O_{3 \times 3} \\
\prod_{i=1}^{3} P_i & \sum_{i=1}^{3} P_i & P_i & I_{3 \times 3} & O_{3 \times 3} \\
\prod_{i=1}^{4} P_i & \sum_{i=1}^{4} P_i & P_i & P_i & I_{3 \times 3}
\end{bmatrix}
\]

\[
Q = \begin{bmatrix}
\prod_{i=1}^{2} P_i & \prod_{i=1}^{2} P_i & \prod_{i=1}^{2} P_i & \prod_{i=1}^{2} P_i
\end{bmatrix}^T
\]

\[
\Lambda = \begin{bmatrix}
\bar{r}^T, \bar{r}^T, \bar{\bar{r}}^T, \bar{\bar{r}}^T
\end{bmatrix}^T
\]

\[
P_i = \text{diag}(p_{i1}, p_{i2}, p_{i3})
\]

(23)

where \( \bar{r} \), \( \bar{\bar{r}} \), \( \bar{\bar{\bar{r}}} \) can be adjusted by the control input of the quadrotor according to (5).

B. Safe Control of a Quadrotor

According to [9], the safe control of a quadrotor can be achieved based on a Quadratic Program (QP) that incorporates CBFs. As shown in Fig. 2, for a nominal control input \( u_0 \) in this paper, a safe control input can be determined by

\[
\begin{align*}
&\text{argmin}_{u \in U} ||u(t) - u_0(t)|| \\
&\text{s.t.} \\
&Qr_{\text{min}} \preceq \Gamma A(\xi, u) \preceq Qr_{\text{max}} \\
&u_{\text{min}} \preceq u \preceq u_{\text{max}}
\end{align*}
\]

(24)

where \( u_{\text{min}} \) and \( u_{\text{max}} \) are the lower bound and upper bound of the control input \( u \), respectively.

![Flow diagram of the safe control of a quadrotor based on a non-cascaded CBF](image)

In this paper, non-cascaded CBFs are designed for quadrotors with a nominal control input consisting of the magnitude of the total thrust and the torque generated by rotors. However, it should be pointed out that the non-cascaded CBFs can be applied to quadrotors with a nominal control input consisting of rotor thrusts or rotor velocities (e.g., [22]) based on (3).

IV. Simulations

To evaluate the effectiveness of the developed non-cascaded CBF in ensuring the safety of quadrotors, the non-cascaded CBF is applied to a quadrotor performing aggressive roll maneuvers in simulations. The simulations are conducted in a Gazebo simulator [26], a physical engine that has been applied to several studies of quadrotors [27], [28]. An IF750A quadrotor is used in simulations, as shown in Fig. 3. The parameters of the IF750A quadrotor are listed in Table I. \( f_{\text{max}} \) and \( f_{\text{min}} \) are the maximum and the minimum feasible magnitude of the total thrust of the quadrotor, respectively. \( \tau_{\text{max}}, \tau_{\text{min}}, \tau_{\text{max}}, \tau_{\text{min}}, \tau_{\text{max}}, \text{and} \tau_{\text{min}} \) denote the feasible range of the torque generated by rotors.

![IF750A quadrotor used in simulations](image)

**Table I** Parameters of the IF750A Quadrotor

| Parameter | Value       | Parameter | Value       |
|-----------|-------------|-----------|-------------|
| \( m \) (kg) | 1.50        | \( J_{gy} \) (N.m²) | 0.039       |
| \( J_{gy} \) (N.m²) | 0.051       | \( f_{\text{max}} \) (N) | 39.000      |
| \( f_{\text{min}} \) (N) | 0.000       | \( \tau_{\text{max}} \) (N.m) | 5.130       |
| \( \tau_{\text{min}} \) (N.m) | 0.024       | \( \tau_{\text{max}} \) (N.m) | -0.024      |
| \( \tau_{\text{min}} \) (N.m) | -5.130      | \( \tau_{\text{max}} \) (N.m) | -10.000     |

To ensure the safety of the quadrotor in simulations, the parameters of a non-cascaded CBF shown in (23) are set to \( p_{i1} = p_{i2} = 1.000, p_{i3} = p_{i4} = 4.000, p_{i5} = p_{i6} = 5.000, p_{i7} = p_{i8} = 5.000, p_{i9} = 1.000, p_{i10} = 5.000, p_{i11} = 10.000, p_{i12} = 10.000.\)

A. Safe Control of a Quadrotor Performing Aggressive Roll Maneuvers

A quadrotor may attain an abnormal state in practice, intentionally or unintentionally. To further evaluate the effectiveness of the non-cascaded CBFs in the case that quadrotors are in abnormal state, this paper applies the non-cascaded CBFs to a quadrotor that tends to perform “Barrel Roll”, an aggressive roll maneuver that can move the quadrotor into abnormal state [29]. A safe region is defined as

\[
\mathcal{S} = \{ r \in \mathbb{R}^3 \mid \begin{bmatrix} -4.000, & -4.000, & 2.000 \end{bmatrix}^T \preceq r \preceq \begin{bmatrix} 4.000, & 4.000, & 13.000 \end{bmatrix}^T \} \text{ (unit : m)}
\]

(25)

Assume that a controller gives a nominal control input \( u_0 = [19.670 \text{ N}, 0.000 \text{ N.m}, -5.130 \text{ N.m}, 0.000 \text{ N.m}]^T \) to make the quadrotor to perform the "Barrel Roll" by rolling aggressively. The initial state of the quadrotor is \( \xi = \begin{bmatrix} \end{bmatrix} \)
\[0.000, 0.000, 9.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000\]^T (units: m, rad, m/s, and rad/s).

The position of the quadrotor performing aggressive roll maneuvers is presented in Fig. 4. The non-cascaded CBF can maintain the quadrotor within the safe region indeed, even if the quadrotor tends to roll aggressively. The orientation and the behaviors of the quadrotor performing aggressive roll maneuvers are shown in Fig. 5 and Fig. 6, respectively. In Fig. 6, yellow arrows mark the orientation of the \(z_B\) axis of the body frame \(F_B\). According to 5 and Fig. 6, one can see that the quadrotor has rolled several times and attains an abnormal state several times. The non-cascaded CBF can maintain the quadrotor within the safe region, even if the quadrotor is in abnormal state.

![Initial position of the quadrotor](image)

Fig. 4. Position of the quadrotor performing aggressive roll maneuvers

To further study the performance of the non-cascaded CBF, this paper investigates the nominal control input and the safe control input of the quadrotor performing aggressive roll maneuvers, as shown in Fig. 7. The non-cascaded CBF regulates the nominal control input of the quadrotor during the whole simulation to maintain the quadrotor within the safe region. Although the non-cascaded CBF enforces the quadrotor to relax aggressive roll maneuvers to uphold safety, the quadrotor has finished the "Barrel Roll" according to Fig 6. This suggests that the non-cascaded CBF regulates rather than takes over the control input. Thus, the non-cascaded CBF enables the quadrotor to perform maneuvers and tasks, to some extent, while enforcing the safety of the quadrotor.

![Control inputs of the quadrotor](image)

Fig. 7. Control inputs of the quadrotor performing aggressive roll maneuvers

V. CONCLUSIONS

This paper developed a non-cascaded CBF for quadrotors that may attain abnormal states controlled by cascaded controllers or non-cascaded controllers. Based on the dynamics of a quadrotor, ECBF was used to design a non-cascaded CBF for quadrotors with a nominal control consisting of the magnitude of the total thrust and the torque generated by rotors. The non-cascaded CBF has been applied to a quadrotor performing aggressive roll maneuvers to ensure the safety of the quadrotors in simulations. The results of the simulations verify the effectiveness of the non-cascaded CBF, even if a quadrotor is in abnormal state.
