Measurement and Visualization System of Eigenmode Shapes Considering Orthogonal Modes on Circular Membrane

Eri Zempo¹, Naoto Wakatsuki¹,², Koichi Mizutani¹,², and Yuka Maeda¹,²

¹ Graduate School of Systems and Information Engineering, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki, 305-8573 Japan
² Faculty of Engineering, Information and Systems, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki, 305-8573 Japan

E-mail: wakatuki@iit.tsukuba.ac.jp

Abstract. This work proposes a visualization system of eigenmode shapes of a tapped circular membrane of membranophones. Under tapped membrane vibration, it is not enough to divide modes in frequency domain, because of the existence of eigenmodes whose frequencies are close to each other including orthogonal modes. Therefore, we structured this visualization system of eigenmode shapes, which could divide the orthogonal modes appearing at slightly different frequencies. This system firstly measures the sound pressures emitted from the vibration on tapped circular membrane of membranophone with a circular microphone array placed in proximity to the membrane. Secondly, it extracts the circumferential shape of each eigenfrequency. In this system, dominant frequencies of estimated power spectrum density obtained by Yule-Walker method are considered as approximate eigenfrequencies of this vibration. Multiple eigenmodes whose azimuthal mode order is different, are divided by Fourier series expansion of the circumferential shapes of each eigenfrequency. In addition, orthogonal modes, which have the same mode order and slightly different frequencies are divided by considering the orthogonality of mode shapes, sine and cosine functions. Lastly, the system estimates the eigenmode shape of the whole membrane using the Bessel function of the first kind correspond to each mode order as radial shape, and shows them to users. It is expected that user could recognize the unbalance of tension distribution intuitively using this system. It is also expected the system to be applied to the tuning assistive system of the tuning of circular membranophone.

1. Introduction

Membranophones are musical instruments, which emit sounds by the vibration of membranes. Most of them have circular membranes, called “head”, whose tension distribution has an effect on the sound of the instruments, as the eigenmode frequencies and these compositions in the vibration are changed. “Tuning” is the process to adjust of the tension distribution. Players follow this process mainly before their performances.

Schematic views of snare drum are shown in Fig. 1 as an example of membranophones. The edges of heads are caught between a hoop and a cylindrical drum shell. Heads are more stretched over the edge of the shell, when you tighten the “tuning rods”, which are kinds of screws, and...
Figure 1. Schematic views of snare drum: Tightening tension rods make the head membrane stretched.

the hoop is pulled toward “tuning lug”. A membrane, usually has from six to ten tuning rods. The tension distribution of the head is determined by controlling the tensions of the rods in the tuning process. In the common practical method, the player adjusts each rod little by little, and guess the tension distribution of the head by hearing the pitches of the sounds emitted by tapping several points[1]. However, it is not easy for beginners to master these processes as there are many difficult points in them.

On the other hand, it is known that non-uniform tension makes the eigenmode shapes non-symmetric[2, 3]. This means visualizing the mode shapes might help the players to guess the tension distribution. To assist tuning or training that processes, we proposed the system, which immediately visualizes the eigenmode shape of the vibration on tapped circular membranes of membranophones[4]. In the system, the membrane vibration is measured by means of sound pressures acquired by a circular microphone array located adjacent to the membrane, and the peaks of the estimated power spectral density (PSD) of the sound are regarded as the eigenfrequencies. Each component of eigenfrequency is extracted by synchronous detection, and shown in a few tenth seconds to users.

However, it is also known that orthogonal modes on the circular membrane under non-uniform tension may cause beat in the sound; such states are likely to occur in the tuning process[2, 3]. Such a beat was observed as is in the system mentioned above, however, multiple modes including orthogonal modes should be separated for further assistance of tuning. Thus, in this paper, we applied the Fourier analysis not only to the temporal waveform, but also to the spatial distribution in azimuthal coordinate, with the aim to extract the eigenmodes considering adjacent orthogonal modes, and show their mode shapes.

2. Method of measurement and visualization eigenmode shapes

2.1. Ideal membrane theory

It is well known that the harmonic vibration $z(t)$ on ideal circular membrane under uniform tension is given by:

$$z(r, \phi, t) = R(r) \Phi(\phi)e^{j\omega t},$$

where,

$$\Phi(\phi) = Ae^{\pm jm\phi},$$

$$R(r) = J_m(k_{mn}r),$$

for polar coordinates $(r, \phi)$, where $J_m$ are Bessel functions of first kind, of order $m$, and $m$ are the mode order of the radial coordinate for the vibration[5]. $k_{mn}$ are the solution for the equation $J_m(k_{mn}r) = 0$, correlated, from lowest to highest, to azimuthal mode order $n$ for the vibration.
Figure 2. Mode shape and frequency-rate to (0, 1) fundamental mode on ideal membrane under uniform tension

Figure 3. An example of the estimated eigenmode shapes: to calculate the estimated eigenmode shape for each frequency, \( f_d \), the measurement values, \( \varphi_d(i) \), and the Bessel functions which have appropriate orders, are used as azimuthal and radial shapes, respectively.

Figure 2 shows the first ten eigenmode shapes on the ideal membranes. Diameters and concentric lines in the figure represent nodal lines for each mode. \((m, n)\) indicates mode order, \( f_r \) is the frequency relative to the (0, 1) mode. The vibration on a circular membrane is the superposition of these eigenmodes. Every \((m, n)\) mode except for \( m = 0 \), consists of a pair of orthogonal modes. These have the same eigenfrequency on the ideal membrane under uniform tension, which is called “degenerated”, while the frequencies of these orthogonal modes are split into different frequencies and they cause beat sounds under non-uniform tension. In our system, eigenmode shapes are shown by using and reflecting these knowledges.

2.2. Measurement of vibration and extraction of eigenmodes

Let \( z_i(l) \) as the offset-eliminated sound pressures of the vibration of the head, acquired by a circular microphone array located adjacent to the membrane, where \( i \) and \( l \) indicate the channel index of microphones and the sample index, respectively. The dominant frequencies of the estimated PSD, obtained by Yule-Walker method, are regarded as the approximate eigenfrequencies, \( f_d \), and let them from lower to higher, as \( f_1, f_2, \ldots \). Figure 3 shows a typical eigenmode shapes estimated by the process written above.

Next, the azimuthal eigenmode shapes on each eigenfrequencies, \( f_d \), are extracted. These shapes are calculated by the internal product of sine function, whose frequency is \( f_d \), and the
temporal waveform of each channel as,

$$\varphi_d(i) = \sum_l z_i(l)e^{2\pi f_d l}. \quad (4)$$

The real part of the left member in eq. 4, $\text{Re}[\varphi_d(i)]$, denotes the estimated azimuthal eigenmode shapes.

In addition, multiple modes including orthogonal modes would be observed by Fourier analysis to azimuthal coordinate. This method is based on that the measurements are periodic to azimuthal coordinate, as the microphone array located in a circle, and especially that the azimuthal eigenmode shape is represented as sine function[6].

By applying Fourier series expansion to the estimated azimuthal eigenmode shapes extracted in the former process, $\varphi_d(i)$, multiple modes, which have different azimuthal mode order $m$, can be separated as,

$$\tilde{\varphi}_{dm} = \sum_i \varphi_d(i)e^{2\pi mi}. \quad (5)$$

If multiple of $\tilde{\varphi}_{dm}$ which have enough value for each $\tilde{d}$ exist, it is thought as the multiple eigenmodes exist around at the frequency $f_d$, and these modes might be observed in $\varphi_d(i)$ extracted above.

If the orthogonal modes have slightly different frequencies to each other, two eigenfrequencies, $f_{d,a}$ and $f_{d,b}$, can be found as,

$$f_{d,a,b} = \arg_{f_d \text{ max}} \left[ \sum_i z_i(l)e^{2\pi m(i+f_{si})/360} \right], \quad (6)$$

where, $f_{si}$ and $\theta_0$ denote the number of the microphones and the initial angle of the sine function, respectively. Through these processes, the pair of orthogonal mode can be separated by discrete Fourier transform even if their eigenfrequency is very close to each other. This is because their mode shape is orthogonal, that is, the inner product of their mode shape is equal to zero.

3. Experimental data on 13 inch Tom-Tom

3.1. Settings on the experiment

We measured the vibration of a head on 13 inch Tom-Tom as an example of membranophones. The diameter of this Tom-Tom is 318 mm, and the depth is 279 mm. A head is stretched over
Figure 5. Estimated PSD by Yule-Walker method, on #0.

Figure 6. The estimated azimuthal eigenmode shapes extracted by Fourier analysis to the temporal waveform

Figure 7. Components of each mode order m, for each frequencies extracted by Fourier analysis to the azimuthal

on only one side, to observe the vibration of a single membrane. The microphone #0 – 12 are arranged circularly, inside of the shell, and located adjacent to the membrane with the sampling angular frequency, \( f_{si} = 13 \), which is equal to the number of the microphones. These microphone position is shown in Fig. 4. Sound pressures by each microphones are measured in 5 sec with the sampling frequency \( f_{sl} = 30 \) kHz.

3.2. Results of the experiment

Figure 5 shows the estimated PSD obtained by Yule-Walker method. The estimated dominant frequency, \( f_{d} \{1, 2, ..., 10\} = \{118, 250, 348, 449, 496, 547, 573, 603, 639, 708\} \) (Hz) were extracted as \( f_d \) from the lowest.
The azimuthal mode shapes correspond to each peak in Fig. 5, are shown in Fig. 6. Some figures such as (b), (c), (e), have curves like sine functions, while other such as (g),(i), draw intricate curves, relatively. In addition, Fig. 7 shows the modes such as (g), (i) have multiple modes at close frequencies.

Figure 8 shows the extraction orthogonal modes on (1,1). (1,1) mode on this vibration has two orthogonal modes; one has a nodal line on $\phi = \theta_0 = 16^\circ$ with eigenfrequency 250.6 Hz, the other has a nodal line on $\phi = \theta_0 = 106^\circ$ with eigenfrequency 249 Hz.

4. Conclusion
In this work, with the aim to extract and visualise eigenmode shapes of the vibration on tapped circular membranes, we proposed the system to extract the multiple modes including orthogonal mode appearing in close frequencies. by applying the Fourier analysis not only to the temporal waveform, but also to the spatial distribution in azimuthal coordinate. Besides, we measured the vibration of the membrane on 13 inch Tom-Tom, and showed the multiple modes could be extracted. The system for visualizing eigenmode shapes utilizing this method is expected to help users to tune the tension of the membrane of membranophones, and also to learn the tuning process.

References
[1] R. Toulson, C. C. Crigny, P. Robinson and P. Richardson (2009). The perception and importance of drum tuning in live performance and music production. Journal on the Art of Record Production 4 1.
[2] R. Worland (2008). Drum tuning: an experimental analysis of membrane modes under non-uniform tension. Proceedings of Meeting on Acoustics 5.
[3] R. Worland (2010). Nomal modes of a musical drumhead under non-uniform tension. The Journal of the Acoustical Society of America 127 (1): 525-533.
[4] E. Zempo, N. Wakatsuki, K. Mizutani, and Y. Maeda (2017). Measurement and Visualization System of Vibration Mode Shape on Tapped Membrane. Proceedings of the Technical Committee on Musical Acoustics of the Acoustical Society of Japan, 36 (3): 1-6 (in Japanese).
[5] N. Fletcher and T Rossing (1998). "The Physics of Musical Instruments", Springer.
[6] T. Yudasaka, N. Wakatsuki, and K. Mizutani (2014). Acoustic model of cymbal considering geometrical nonlinearity. The Journal of the Acoustical Society of Japan, 70 (7): 362-370 (in Japanese).