Model of Extended Mechanics and Non-Local Hidden Variables for Quantum Theory
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Abstract
Newtonian physics is describes macro-objects sufficiently well, however it does not describe microobjects. A model of Extended Mechanics for Quantum Theory is based on an axiomatic generalization of Newtonian classical laws to arbitrary reference frames postulating the description of body dynamics by differential equations with higher derivatives of coordinates with respect to time but not only of second order ones and follows from Mach principle. In that case the Lagrangian $L(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)}, ...)$ depends on higher derivatives of coordinates with respect to time. The kinematic state of a body is considered to be defined if n-th derivative of the body coordinate with respect to time is a constant (i.e. finite). First, kinematic state of a free body is postulated to invariable in an arbitrary reference frame. Second, if the kinematic invariant of the reference frame is the n-th order derivative of coordinate with respect to time, then the body dynamics is describes by a 2n-th order differential equation. For example, in a uniformly accelerated reference frame all free particles have the same acceleration equal to the reference frame invariant, i.e. reference frame acceleration. These bodies are described by third-order differential equation in a uniformly accelerated reference frame.

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1 Introduction
Classical Newtonian mechanics is essentially the simplest way of mechanical system description with second-order differential equations, when higher order time derivatives of coordinates can be neglected. The extended model of mechanics with higher time derivatives of coordinates is based on generalization of Newton’s classical axiomatics onto arbitrary reference frames (both inertial and non-inertial ones) with body dynamics being described with higher order differential equations. Newton’s Laws, constituting, from the mathematical viewpoint, the axiomatics of classical physics, actually postulate the assertion that the equations describing the dynamics of bodies in inertial frames are second-order differential equations. However, the actual time-space is almost without exception non-inertial, as it is almost without exception that there exist (at least
weak) fields, waves, or forces perturbing an ideal inertial frame. It corresponds to Mach’s principle [1] with general statement “Local physical laws are determined by the large-scale structure of the universe.” Non-inertial nature of the actual time-space is also supported by observations of the practical astronomy that expansion of the reality occurs with an acceleration. In other words, actually any real reference frame is a non-inertial one; and such physical reality can be described with a differential equation with time derivatives of coordinates of the order exceeding two, which play the role of additional variables. This is evidently beyond the scope of Newtonian axiomatics. Aristotle’s physics considered velocity to be proportional to the applied force, hence the body dynamics was described by first derivative differential equation. Newtonian axiomatics postulates reference frames, where a free body maintains the constant velocity of translational motion. In this case the body dynamics is described with a second order differential equation, with acceleration being proportional to force [2]. This corresponds to the Lagrangian depending on coordinates and their first derivatives (velocities) of the body, and Euler-Lagrange equation resulting from the principle of the least action. This model of the physical reality describes macrocosm fairly good, but it fails to describe micro particles. Both Newtonian axiomatics and the Second Law of Newton are invalid in microcosm. Only averaged values of observable physical quantities yield in the microcosm the approximate analog of the Second Law of Newton; this is the so-called Ehrenfest’s theorem. The Ehrenfest’s equation yields the averaged, rather than precise, ratio between the second time derivative of coordinate and the force, while to describe the scatter of quantum observables the probability theory apparatus is required. As the Newtonian dynamics is restricted to the second order derivatives, while micro-objects must be described with equations with additional variables, tending Planck’s constant to zero corresponds to neglecting these variables. Hence, offering the model of extended Newtonian dynamics, we consider classical and quantum theories with additional variables, describing the body dynamics with higher order differential equations. In our model the Lagrangian shall be considered depending not only on coordinates and their first time derivatives, but also on higher-order time derivatives of coordinates. Classical dynamics of test particle motion with higher-order time derivatives of coordinates was first described in 1850 by M. Ostrogradskii [3] and is known as Ostrogradskii’s Canonical Formalism. Being a mathematician, M. Ostrogradskii considered coordinate systems rather than reference frames. This is just the case corresponding to a real reference frame comprising both inertial and non-inertial reference frames. In a general case, the Lagrangian takes on the form \((n \to \infty)\)

\[ L = L(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)}). \] (1)
2 Theory of Extended Mechanics

Let us consider in more detail this precise description of the dynamics of body motion, taking into account of real reference frames. To describe the extended dynamics of a body in an any coordinate system (corresponding to arbitrary reference frame) let us introduce concepts of kinematic state and kinematic invariant of an arbitrary reference frame.

**Definition:** Kinematic state of a body is set by \( n \)-th time derivative of coordinate. The kinematic state of the body is defined provided the \( n \)-th time derivative of body coordinate is zero, the \( (n-1) \)-th time derivative of body coordinate being constant. In other words, we consider the kinematic state of the body defined if \( (n-1) \)-th time derivative of body coordinate is finite. Let us note that a reference frame performing harmonic oscillations with respect to an inertial reference frame does not possess any definite kinematic state.

Considering the dynamics of particles in arbitrary reference frames, we suggest the following two postulates.

**Postulate 1.** Kinematic state of a free body is invariable. This means that if the \( n \)-th time derivative of a free body coordinate is zero, the \( (n-1) \)-th time derivative of body coordinate is constant. That is,

\[
\frac{d^n q}{dt^n} = 0, \quad \frac{d^{n-1} q}{dt^{n-1}} = \text{const}.
\]  

(2)

In the extended model of dynamics, conversion from a reference frame to another one will be defined as:

\[
q' = q_0 + \dot{q} t + \frac{1}{2!} \ddot{q} t^2 + \ldots + \frac{1}{n!} \dot{q}^{(n)} t^n.
\]  

(3)

**Postulate 2.** If the kinematic invariant of a reference frame is \( n \)-th time derivative of coordinate, then the body dynamics is described with the differential equation of the order \( 2n \):

\[
\alpha_{2n} \dot{q}^{(2n)} + \ldots + \alpha_0 q = F(t, q, \dot{q}, \ddot{q}, \ldots, \dot{q}^{(n)}).
\]  

(4)

This means that the Lagrangian depends on \( n \)-th time derivative of coordinate, so variation when applying the least action principle will yield the order higher by a unity. Therefore, the dynamics of a free body in a reference frame with \( n \)-th order derivative being invariant shall be described with a differential equation of the order \( 2n \). To consider dynamics of a body with an observer in an arbitrary coordinate system, varying the action function for \( n \)-th order kinematic invariant, we obtain the equation of the order \( 2n \):

\[
\delta S = \delta \int L(t, \dot{q}', q') dt = \int \sum_{n=0}^{N} (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{q}^{(n)}} \delta \dot{q}^{(n)} \ dt = 0.
\]  

(5)

Expanding into Taylor’s series the function \( q = q(t) \) yields:

\[
q = q_0 + \dot{q} t + \frac{1}{2!} \ddot{q} t^2 + \ldots + \frac{1}{n!} \dot{q}^{(n)} t^n.
\]  

(6)
It is well known that the kinematic equation in inertial reference frames of Newtonian physics contains the second time derivative of coordinate, that is, acceleration:

$$q_{\text{Newton}} = q_0 + vt + \frac{1}{2}a t^2.$$  \hspace{1cm} (7)

Let us denote the additional terms with higher derivatives as

$$q_r = \frac{1}{3!}q^{(3)} t^3 + \ldots + \frac{1}{n!}q^{(n)} t^n.$$  \hspace{1cm} (8)

Then

$$q = q_{\text{newton}} + q_r.$$  \hspace{1cm} (9)

In our case, the discrepancy between descriptions of the two models is the difference between the description of test particles in the model of Extended Mechanics with Lagrangian $L(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)}, ...)$ and Newtonian dynamics in inertial reference frames with the Lagrangian $L(t, q, \dot{q})$:

$$\int [L(t, q, \dot{q}, ..., \dot{q}^{(n)}) - L(t, q, \dot{q})] dt = h,$$  \hspace{1cm} (10)

$h$ being the discrepancy (error) between descriptions by the two models. Comparing this value with the uncertainty of measurement in inertial reference frames, expressed by the Heisenberg uncertainty relation, the equation (10) can be rewritten as

$$S(t, q, \dot{q}, ..., \dot{q}^{(n)}) - S(t, q, \dot{q}) = h.$$  \hspace{1cm} (11)

In the classical mechanics, in inertial reference frames, the Lagrangian depends only on the coordinates and their first time derivatives. In the Extended Mechanics, in real reference frames, the Lagrangian depends not only on the coordinates and their first time derivatives, but also on their higher derivatives. Applying the least action principle [4], we obtain Euler-Lagrange equation for the Extended Mechanics:

$$\sum_{n=0}^{N} (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{q}^{(n)}} = 0,$$  \hspace{1cm} (12)

or

$$\frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \dot{q}} - \ldots + (-1)^N \frac{d^N}{dt^N} \frac{\partial L}{\partial \dot{q}^{(N)}} = 0.$$  \hspace{1cm} (13)

The Lagrangian will be expressed through quadratic functions of variables:

$$L = k q^2 - k_1 \dot{q}^2 + k_2 \ddot{q}^2 - \ldots + (-1)^\alpha k_{\alpha} \dot{q}^{(\alpha)} = \sum_{\alpha=0}^{\infty} (-1)^\alpha k_{\alpha} \dot{q}^{(\alpha)}.$$  \hspace{1cm} (14)

For our case, the action function will be:

$$S = \dot{q} \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \ldots + (-1)^\alpha \dot{q}^{(\alpha)} \frac{\partial L^{(\alpha)}}{\partial \dot{q}^{(\alpha)}} + \ldots = \sum_{\alpha=0}^{\infty} (-1)^\alpha \dot{q}^{(\alpha)} \frac{d^\alpha}{dt^\alpha} \frac{\partial L}{\partial \dot{q}^{(\alpha)}}.$$  \hspace{1cm} (15)
Or

\[ S = 2kq^2 - 2k_1\dot{q}^2 + 2k_2\ddot{q}^2 + \ldots + 2k_\alpha\dot{q}^{(\alpha)2} = 2 \sum_{\alpha=0}^{\infty} (-1)^\alpha k_\alpha \dot{q}^{(\alpha)2}. \]  

(16)

Introducing the notation

\[ F = \frac{\partial L}{\partial \dot{q}}, \quad p = \frac{\partial L}{\partial \dot{q}} \]  

(17)

\[ F^2 = \frac{\partial L}{\partial \ddot{q}}, \quad \dot{p}^3 = \frac{\partial L}{\partial \dot{q}(3)} \]  

(18)

\[ F^4 = \frac{\partial L}{\partial \dot{q}(4)}, \quad \ddot{p}^5 = \frac{\partial L}{\partial \dot{q}(5)} \]  

(19)

\[ F^{2n} = \frac{\partial L}{\partial \dot{q}(2n)}, \quad \dot{p}^{2n+1} = \frac{\partial L}{\partial \dot{q}(2n+1)} \]  

(20)

we obtain the description of inertial forces for Extended Mechanics. The value of the resulting force accounting for inertial forces can be expressed through momentums and their derivatives, expressing the Second Law of Newton for the extended Newtonian dynamics model:

\[ F - \frac{dp}{dt} + \frac{d^2}{dt^2}(F^2 - \frac{dp^3}{dt}) + \frac{d^4}{dt^4}(F^4 - \frac{dp^5}{dt}) + \ldots \frac{d^n}{dt^n}(F^n - \frac{dp^{2n+1}}{dt}) = 0. \]  

(21)

In other words, (21) can be written as

\[ \sum_{n=0}^{\infty} \frac{d^{2n}}{dt^{2n}}(F^{2n} - \frac{d^{2n+1}p^{2n+1}}{dt^{2n}}) = 0. \]  

(22)

The action function takes on the form

\[ S = \sum_{n=0}^{\infty} (-1)^n \dot{q}^{(n)}p^{n+1} = \sum_{n=0}^{N} (-1)^n \dot{q}^{(n)} \frac{\partial L}{\partial \dot{q}^{(n+1)}}. \]  

(23)

For this case, energy can be expressed as

\[ E = \alpha_0q^2 + \alpha_1\dot{q}^2 + \alpha_2\ddot{q}^2 + \ldots + \alpha_n\dot{q}^{(n)2} + \ldots \]  

(24)

Denoting the Appel's energy of acceleration [5] as \( Q, \) \( \alpha_n \) being constant factors, we obtain for kinetic energy and potential energy, respectively,

\[ E = V + W + Q \]  

(25)

\[ V = \alpha_0q^2 \]  

(26)

\[ W = \alpha_1\dot{q}^2 \]  

(27)

\[ Q = \alpha_2\ddot{q}^2 + \ldots + \alpha_n\dot{q}^{(n)2} + \ldots \]  

(28)
The Hamilton-Jacobi equation for the action function will take on the form

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q,$$

(29)

The first addend in (28) is the so-called Appel's energy of acceleration [5]. Let us compare $Q$ with the Bohm’s quantum potential [6] and complement the equation (29) with the continuity equation. If $Q \approx \alpha_2 \frac{\nabla^2 S}{m^2}$ (here, the value of the constant is chosen $\alpha_2 = \frac{\hbar}{2m}$). Hence, in the first approximation we obtain for the function $\psi = e^{i\hat{S}}$, the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + V\psi.$$

(30)

3 Conclusions

Our case corresponds to Lagrangian $L(t, q, \dot{q}, ..., \dot{q}^{(n)}, ...)$, depending on coordinates, velocities and higher time derivatives, which we call additional variables, extra addends, or hidden variables. In arbitrary reference frames (including non-inertial ones) additional variables (addends) appear in the form of higher time derivatives of coordinates, which complement both classical and quantum physics. It should be noted that these hidden (addition) variables can be used to complement the quantum description without violating von Neumann theorem, as this theorem is not applied for non-linear reference frames, while the extended mechanics assumes employing any reference frames, including non-linear ones. For example, if we consider a spaceship with two observers in different cabins, one can see that this system is non-ideal, the inertial forces (or pseudo-forces) could constitute additional variables here. In this case, superposition of the two distributions obtained by the observers could yield a non-zero correlation factor, though each of the two observations has a seemingly random nature. If the fact that the reference frame is non-inertial and hence there exist additional variables in the form of inertial effects is ignored, then non-local correlation of seemingly independent observations would seem surprising. This example could visualize not only the interference of corpuscle particles, but also the non-local character of quantum correlations when considering the effects of entanglement. Newtonian mechanics (i.e. without additional derivatives) work so well in applications is valid in the framework of its applicability with a certain accuracy. The Extended Mechanics has a wider field of applicability, with Newtonian mechanics being its particular case. When Newtonian mechanics is invalid the Extended Mechanics acquires additions variables in the form of higher derivatives. Averaging we obtain Erenfest’s theorem. Introducing higher derivatives (i.e. hidden variables) means transition to Quantum Mechanics. Thus quantum mechanical experiments confirm the Extended Mechanics as well. Analogies Extended Mechanics to Bohm’s mechanics confirm this viewpoint. The next question appears: is this equation linear or not? We consider that the contribution of non-linear composed is small and in the first approximation gives
the Schroedinger equation. The model admits non-linear generalization. The present model of extended Newtonian dynamics is generalize but not alternative to Newtonian Dynamics because its extended mechanics to arbitrary reference frames. It is physics of arbitrary reference frames. Extended Mechanics describes the dynamics of mechanical systems for arbitrary reference frames and not only for inertial reference frames as Newtonian Dynamics. Newtonian Dynamics can describe non-inertial reference frames as well introducing fiction forces. In Extended Mechanics (Dynamics) we have fiction forces naturally and automatically from new axiomatic and we needn’t have inertial reference frame. Model of Extended Dynamics is differs from Newtonian Dynamics in the case of micro-objects description.

References

[1] Mach Ernst Die Mechanik in ihrer Entwickelung : historisch-kritisch dargestellt, 1883.

[2] Newton I. Philosophiae naturalis principia mathematica. London, 1687. 220 p.

[3] M. V. Ostrogradskii Memoire sur les equations differentielles relatives aux problemes des isoperim'etres // Memoires de l’Academie Imperiale des Sciences de Saint-Peterbourg, v. 6, 1850. P. 385.

[4] Lagrange J.I. Mecanique analitique. Paris, De Saint, 1788. 131 p.

[5] Appel P., Traite’ de Me’caique Rationelle, Paris, Ganthier-Villars e’diteur, 1953.

[6] Bohm D. A suggested interpretation of the quantum theory in terms of ”hidden” variables I //Physical Review. 1952. v. 85, P. 166.