STRUCTURAL CHANGE AND ECONOMIC DYNAMICS: RETHINKING FROM THE COMPLEXITY APPROACH

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Abstract. Economic systems have evolved through time thereby changing the structure that characterizes them. These changes respond to technological changes that transform economies into highly interconnected systems. The modifications in the norms that guide the behaviour of organizations and, therefore the functioning of the economy, are a first case of this transformation. The industrialization process, through the incorporation of increasing returns to scale in different sectors, and the introduction of service activities are other examples. Another form to represent structural change is the change of the values of the variables that characterize the state space of an economic system. This research article is an effort to put together and compare, from the complexity approach, different approaches for structural change and dynamics of economic systems. We start by briefly presenting the complexity approach in general and in economics. Then, we put forward three approaches highlighting structural change.

1. Introduction. Approaching the economy from the complexity allows understanding and explaining economic issues and phenomena that do not necessarily correspond to the neo-Walrasian paradigm. In general, this paradigm highlights the interaction and communication between agents with complete information and no distortions, the coordination of activities with no frictions or asymmetries, and the achievement of equilibria states characterized by different types of stability. In contrast, we take the complexity approach, which considers all system’s behaviors in context with its environment. We use a non-reductionist perspective to describe the system’s properties. For that purpose, we define the economic system to study—i.e. a country, a region, an organization, then we describe its internal and the environment’s properties, the relations and interactions, and the way they change in time.

In general, complex systems involved many heterogeneous components that interact dynamically and give rise to superior levels or scales structures— that exhibit common behaviors. In these systems there exists the emergence of behaviors, from

2010 Mathematics Subject Classification. 91B55, 91B66.
Key words and phrases. Estructural change, dynamics, regimes, complex systems, networks.
the interactions, that cannot be inferred simply from the individual behavior of their parts. The complex systems’ characteristics include: self-organization, emergence, feedback, criticality, path dependency, initial conditions, heterogeneity, dynamic and evolution [3].

In particular, self-organization has to do with the parts of the system affecting each other through the coordination, interaction and interdependency of individuals or units. These individuals self-organize and obtain an aggregate result using local rules of behavior. Examples include: ecosystems, markets, cities, and transport systems such as roads. Emergence refers to the relation between the properties of the parts of the system and the collective properties of the whole. The aggregate behavior is not equal to the sum of the individual behaviors.

Complex systems are dynamic by nature and have multiple equilibria. Therefore, to select one equilibrium from another we have to use dynamic considerations. Reaching an equilibrium or another, will depend on initial conditions and the trajectory taken by agents, which may vary. Finally, complex systems are adaptive. As adaptive systems, they change their behavior in response to the environment. The effect of a change in the environment in the system cannot be understood only considering the direct impact, but we also have to consider indirect effects. This adaptive characteristic means complex systems evolve.

Congruently, the economy is a complex adaptive system composed of heterogeneous agents that interact between them and that are influenced and influence other agents. In these systems the aggregate behavior cannot be deduced from the behavior of a representative agent [34]. The central idea is how the parts in the economy coordinate and how self-organization leads to phase transitions that change from one state to another. The economy evolves and may or may not converge to a stationary state or equilibrium. If it does converge, this may be efficient or not. The focus is redirected towards understanding the mechanism behind the transition from an equilibrium to another rather than focusing on achieving a stable one.

Therefore, the complexity approach allows going beyond the simplistic and reductionist view of standard economics. It also makes evident the need to apply techniques and methods, such like networks and computational models, that allow incorporating explicit structure of interactions between agents or components of the system, increasing returns, dynamics and evolution in the analysis [20, 34]. When we introduce such elements into the economic research we may obtain a more complete and informative analysis that would facilitate the design of economic policies and improve their execution.

In our work we highlight, from three perspectives, the structure and dynamics of the economic system, framed within the complexity approach, focusing on the different trajectories countries may follow. Structure refers to the components of the macroeconomic aggregates and the patterns of interactions between them [43]. Structural change refers to modifications of these patterns in a long period of time and these can be abrupt or smooth. Consequently, the functioning and performance of the system may change. A process of structural change should lead to deep changes in the profile of specialization of an economy and in the novel production processes. From this viewpoint, a structural change process would consist of an emergent property of a complex system that is the consequence of multiple feedback processes between the heterogeneous components of the system. Then, a process of structural change requires the presence of nonlinear relationships between the dynamical specification of the models, where path dependence is relevant and the
possibility of unexpected changes is latent. Structural change is possible because of the dynamic nature of the economic system. In this context, the structural change is an emergent property of a complex system. In Economics, a structural change happens when there is a change in the basic ways an economy operates, caused by deep changes in economic development, in capital and labor, in resource availability or in a political system. Structural change can be initiated by strategy decisions or deep changes in the quantity or type of resources, profound alterations in population or the society [38].

The object of study of this manuscript, the economic system and its structural change, is closely related to the multidisciplinary literature of the input-product network and the international trade network [2, 4, 18, 47, 22]. These investigations apply complex networks for the analysis of the relationships between sectors of economic activities and between countries, and the implications of these relationships in certain economic phenomena over time. But the complex approach is not the only one that considers the economy as a dynamical system. [19] shows how the anti-equilibrium approach presented by [23] also considers the economy as a dynamic system, where the macroeconomy is derived from the structure of the economy.

The rest of the paper is organized as follows. Second section presents a model of the structure of an economy based on the idea of anti-equilibrium [23] and regulation and control of the economy [24, 32]; we represented the economy using its social accounting matrix (SAM). In this context, structural change is a change of stages. In the third section we present a computational model of structural change of the productive system of a country represented by its input-output matrix that forms a complex network. Structural change is driven by technological progress through innovations. Innovations materialize as new products and new means of production thereby changing the input-output relations; these changes modify the structure of the system and its properties. The fourth section presents the concepts of regimes and regimes shifts in economics and their relation to structural change. Regimes are commonly described as areas of a state space in which the state variables exhibit characteristic behaviors and take on characteristic quantities. Finally, in the fifth section, we discuss some points and conclude. Each of the three perspectives raises the interaction between heterogeneous agents and the feedback between the different components of the economic system. In each of the sections we use a database that considers these elements, highlighting the empirical nature of the analysis. The models we proposed throughout the manuscript express the economy as a dynamic system. Moreover, the procedures and techniques that we propose to simulate the trajectories of the systems are diverse, but capture the same idea: structural change. These are difference equations, network theory and input-output models, and multivariable statistical methods to identify regions of the state space and to show their trajectories.

2. Anti-equilibrium and change of stages of an economy. In this section we address structural change as the change of an economy from a stage of functioning to another. This general question has been responded from the anti-equilibrium approach [23], and the regulation and control of the economy [24, 32] approaches. These are based on the analogy between an economic system and a cybernetic device. This device has a sphere of control (C) and a real sphere (R) controlled by units (or mechanisms) that learn and respond to the movements and the path of the involuntary (or routinary) actions of the units of the real sphere.
The economy is conceived as a complex system of organizations that has, each one, a unit of control and a real unit. In this formulation the organizations are the productive branches integrated by firms and the groups of homes. These firms and homes belong to branches and groups respectively, and interact by the emission and reception of quantity or price signals.

**The database**

The information of the exchange between productive branches or groups of homes as suppliers and as demand agents is organized in a SAM of \( n \) branches and \( m \) groups. The foundations of the models of the control and regulation approach on a SAM situate the modeling on an applied economics perspective [41]. It is a route to quantify, simulate, and calibrate the models.

This SAM is defined as follows:

\[
Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}
\]

where \( Z \) is the complete matrix of \( (n+m,n+m) \) rows, \( Z_{11} \) is the inter-sectoral submatrix of \( (n,n) \) rows, \( Z_{21} \) is the inter groups’ branches submatrix of \( (m,n) \) rows, and \( Z_{12} \) is the inter branches’ groups submatrix of \( (n,m) \) rows.

To close the relationship between the supply and the demand of the economy it inserts \( Z \) in the next table in which the sum of the first \( n+m \) rows equals that of the first \( n+m \) columns:

\[
\begin{bmatrix} z_{11} & z_{12} & v & f \\ z_{21} & 0 & 0 & w_f \\ g_1 & g_2 & 0 & G_f \end{bmatrix} l_{n+m+2} = \begin{pmatrix} X \\ W \\ G \end{pmatrix} l_{n+m+2}^t
\]

where \( v \) is the submatrix of the flows of investment of \( (n,1) \), \( f \) is the submatrix of flows of the balance of foreign trade of \( (n,1) \), \( w_f \) is the submatrix of flows of the balance of foreign income of the homes of \( (m,1) \), \( g_1 \) is the submatrix of flows of the surplus of the firms (or dividend not distributed) of \( (1,n) \), \( g_2 \) is the submatrix of flows of savings of the homes and \( G_f \) is the surplus of the rest of the world plus the variation of the international reserves; \( x \) is the submatrix of the gross value of production of \( (n,1) \), \( w \) is the submatrix of the income of the homes of \( (m,1) \), \( V \) of \( (1,1) \) is the total investment of the economy and \( F \) of \( (1,1) \) is the total balance of foreign trade; the ‘ denotes transposition of the respective matrices and \( t \) is the sum vector of the correspondent dimension.

The accounting identity of flows of the economy is:

\[
G = (g_1 g_2) l_{n+m} + G_f = V + l_n^t f + G_f
\]

that is:

\[
S = (g_1 g_2) l_{n+m} - l_n^t f = V
\]

i.e. saving identical to investment.

To complete the description of the economy it is necessary to introduce a stock-flow identity; there is a link that shows the connection between the stocks of capital of each type of machinery, equipment and buildings and the flow of investments that increase their stocks. This is the simplest link between stocks and flows; it is only a conversion of the production of flows of commodities by origin into capital stocks by types of goods. This is:
where \( k_t \) is the matrix of the capital stocks accumulated at the end of period \( t \).

Clearly a great number of the entries of the \( v \) and \( k \) matrices are zero because there are not capital goods in the classification of commodities.

To set a more general accounting framework it is necessary to introduce a transactions matrix \( K \) that shows the interchange of the capital goods by productive branch of origin into the respective productive branch of destination:

\[
Kl_n = k_t - k_{t-1} \tag{4}
\]

\[
l_n'K = k'_{dt} \tag{5}
\]

where \( K \) is the transactions matrix of the capital goods between the \( n \) productive branches of origin in the respective productive branches of destination and \( k'_{dt} \) is the capital goods matrix by destination of \((1, n)\) rows.

**The Interaction among agents and equilibrium conditions**

The interdependence of the agents is reflected in the weighted graph associated to the matrix of proportions in which each agent, productive branch or home, exchange with each other:

\[
A = Z\hat{x}^{-1} \tag{6}
\]

where \( \hat{x}^{-1} \) is the column matrix wrote as a diagonal matrix and inverted.

Now it is possible to write the balance identity as the following equilibrium conditions by quantities:

\[
\begin{pmatrix}
X \\
W
\end{pmatrix}
= \begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & 0
\end{pmatrix}
\begin{pmatrix}
X \\
W
\end{pmatrix}
+ \begin{pmatrix}
v \\
0
\end{pmatrix}
+ \begin{pmatrix}
f \\
w_f
\end{pmatrix}
= (I - A)
\quad \quad \text{and by prices:}
\[
\dot{g} = (g_1, g_2)' = (c, r)' \begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & 0
\end{pmatrix} = p' (I - A) \tag{7}
\]

where \( c \) is the submatrix of the costs of the intermediate inputs of \((1, n)\) and \( r \) is the submatrix of \((1, m)\) formed with the rates of remuneration of the primary inputs supplied by the homes.

Given some exogenous variables of quantities: \( \begin{pmatrix} v \\
w \end{pmatrix} \), \( \begin{pmatrix} f \\
w_f \end{pmatrix} \) and, also the surplus of the firms and homes consider exogenously, the equilibrium conditions permit to determine the quantities produced by each productive branch: \( x \), the income distribution among the homes: \( w \) and the relative prices of the commodities: \( c \) and the rates of remuneration of the primary services that deliver the homes: \( r \).

**The real and control spheres of an economy**

The R-sphere of the economy is constituted by the units of each organization that transform signals of a passive manner, i.e. without use of a voluntary decision process. The C-sphere groups the units of each organization that transforms the signals actively, i.e. following the steps of a voluntary decision process.

The C-sphere regulates the economy. The feedback process between both spheres generates different types of paths of the economy; these paths are accounted for and put forward the forms of coordination and the modes of control and regulation that
are present in a concrete economy. The use of the terms of coordination, and of control and regulation, are proper of the anti-equilibrium approach. These are rooted in the cybernetic economics tradition [45, 25, 26]. The concepts invoked by the terms connect the formal conditions of a model with the institutional arrangements that are necessary to achieve the results derived from formalization.

The states of an economy are not equilibria in the sense of equilibria in General Equilibrium Theory, but are results of a specific regulation form that depends of the modeling of each sphere and its relationships.

The next models specify distinct stages of functioning of an economy. Each one has two different structures of relationships between the R and C spheres. The distinction is based on the behavior of the agents and the information transmission between them; these characteristics are what determine the type of feedback between both spheres. Each model has two sub-models, one for each sphere.

The behavior of each agent is based on norms. A norm is the value of some variable that the agent considers as an adequate level to permit the development of her activity without unbalances between costs and benefits. The calculation of the costs and benefits of each agent depends of its learning process. This process consists in to observe the performance of the real sphere and its effects on the activity of each one.

The first model represents the real sphere as a simple determination of the flows of investment by the balance between the production of commodities and its demand to satisfy intermediate and final uses. At any time the investment is the difference between the capital stocks of two successive periods.

\[ v = x - (n + d), \quad v = \Delta k \] (8)

where \( n \) is the intermediate demand and \( d \) is the final demand composed by private and government consumption plus the balance of foreign trade \( f \). This real sphere shows how each productive branch behaves by calculating the needs of investment and modifying the capital stock in agree with this demand.

The correspondent control sphere describes the adjustment of the variations of the capital stock to its norm. When the level of stock is higher than the norm, the variation is negative and vice versa; the specification is as follows:

\[ \Delta k = \hat{\theta}(k^\ast - k) \] (9)

where \( k^\ast \) is the \((n,1)\) matrix of norms of capital stocks and \( \hat{\theta} > 0 \) is the \((n,n)\) diagonal matrix of the adjustment parameters.

This C-sphere integrates the system by means of control mechanisms that are strictly vegetative in the sense that they do not suppose nor require communication among agents. This fact characterizes the model as a system with vegetative non-communicative control realized by norms. A system with vegetative non-communicative control defines a stage of functioning of the economy; the next model integrates new mechanisms of control to arrive at other stage.

The second model represents the role of the price side in the functioning of the system. The R - sphere determines the surplus by the prices and, of these, by the movements of the flows and stocks of capital. To operate, the system requires communication by means of the fixed proportions of spending with respect to the total income of each productive branch or each group of homes.

Formally, the R-sphere is the following environment in which the control sphere is nested:
\[
\begin{pmatrix}
  x \\
  w
\end{pmatrix}
= \begin{pmatrix}
  A_{11} & A_{12} \\
  A_{21} & 0
\end{pmatrix}
\begin{pmatrix}
  x \\
  w
\end{pmatrix}
+ \begin{pmatrix}
  v \\
  0
\end{pmatrix}
+ \begin{pmatrix}
  f \\
  w_f
\end{pmatrix}
\]
\[
g' = (g_1, g_2) = (c, r)^T
\begin{pmatrix}
  A_{11} & A_{12} \\
  A_{21} & 0
\end{pmatrix}
\]

(10)

Thus the C-sphere is inserted in its environment by means of control mechanisms that relate the path of the capital stocks with the change in prices:

\[
\Delta c = -\Gamma(\hat{\theta}(k^*) - k)
\]

(11)

where the \(\Gamma > 0\) is the \((n, n)\) matrix of the adjustment of the variations to the investment flows \(\Delta k\); hence the variations to the prices follow the gaps of the capital stocks with respect to its normal levels.

This model represents a stage of functioning of the economy endowed of a new control sphere that is formed by vegetative communicative mechanisms. Here the mechanisms are adaptive as those of the previous model but there is a collection of procedures of communication though the weighted graph associated to the adjacency matrix of \(A\).

Structural change as the transformation of a stage of functioning into another

The structural change is the transformation of a stage into another. The characterization of the structural change is a parametric analysis of the paths of a stage with respect to those of the other. The types and values of a collection of parameters lead to an economy to one or another control sphere and is situated in one or another evolution stage of its functioning.

**Expected results and Reflections**

The main idea behind this formulation is to extract the characteristics, in terms of the complexity perspective, presented in the argument developed above. The aspects that are necessary to consider include the following points:

1. Processes: the economic activity is the outcome of mutual actions of its participants that influence the variations, perturbations and alterations of the interchanges and the interdependence historically established.
2. Interaction: the agents integrated to the processes interconnect their actions generating outputs, outcomes and consequences that characterize and express the joint economic performance of the system.
3. Emergency: the individual behaviours of the organizations composed by different types of agents follow diverse paths; the functioning of the economic system depend of the conducts of these entities and its connections, however the global path of the system is not the weighted sum (or aggregation) of the individual paths of these unitary takers of decision.
4. Transition: the processes pass by distinct phases that exhibit a set of patterns configuring economic structures and reveal the modes of change.

Consider the next characteristics of the specified model:

1. The different processes are composed by the actions and reactions of the organizations that have a unit guided by behaviours based on routines and one other that takes informed decisions. The levels of behaviour from a ground of involuntary routines to a sophisticated decision taking show that the participants are complicated dual organizations. The initial information synthetized in the SAM give the historical panorama of the diversity and the variety of the organizations and his interaction rules.
2. The interaction is seen as the interconnected and weighted graph associated to the adjacent matrix of the transactional processes. The interdependence is a constitutive fact of the economic system.

3. The paths of the system, in each stage, are a result of the interaction of decision makers that act by vegetative, or vegetative communicative, control mechanisms. The movement of the system across a trajectory surges from the behaviours of the organizations and its interdependence. The set of possible trajectories has structural attributes like patterns of the trajectory set, stability or viability of the system. These properties are unexplainable by the joint behaviour of the participants. Hence, the presence of supervenient properties beyond of the behaviours and the paths of each one and all of the participants characterize, in the system, the emergency pattern that conforms also its evolution.

4. The transit from one stage to another is not only a change of phase, it is much more because it represents a transformation of the structure of system from a vegetative to a vegetative communicative control sphere, and so on in line with the other stages of the approach of the control and regulation of the economies.

The anti-equilibrium approach is a response to the dissatisfaction with some versions of the Theory of General Equilibrium at the last third of the Twentieth Century. It advanced some of the issues of the conjunction of the behavioural, institutional, and evolutionary theories of the first decade of the Twenty First Century that are important contributors to the complexity perspective.

The models in this section used the SAM, price, and capital data to model structural change as the transition from one stage to another. In a new stage there will be a new control sphere. The models use many of the elements found in complex systems such as heterogeneous interacting agents that adapt. The structure of the system is a result of these interactions. In the next section we present a model of structural change based on technological progress that focuses on the productive structure of the economy and its evolution. The economy is also viewed as a complex system composed of heterogeneous interacting agents, where the unit of analysis is a sector. The structure of the economy emerges from the interactions between sectors.

3. A computational model of structural change. In this section we propose a model structural change in a long period of time that simulates structural change (regime shifts) in the economic systems based on its structural properties. For such a purpose we take the productive structure of the economy as determined by the technological relationships connecting the sectors together in the production process through the supply and demand of inputs. The purpose of the analysis is to study the underlying mechanism generating the productive structure of the economic system through time driven by technological progress. Structural change is the change in the pattern of interactions between sectors in the economy, therefore is captured as changes in the structural properties of the system and in the measures that describe the related network.

The data base

The Input-output (I-O) model assumes the technical relationships connecting the sectors in the production process fixed ruling out substitution between inputs. But, if we have several states of an economy through time, we can observe changes in the technological relationships as differences in the I-O matrices. Structural
decomposition in I-O analysis studies the nature of these differences as coming from changes in the components of final demand and technology [35, 16, 43]. These changes represent new economic transactions and new ways of combining inputs to produce more efficiently. However, structural decomposition does not explain the transition from one state of the economy to the other or how the changes in the technological structure emerged. We go beyond the I-O model and incorporate substitution of inputs as one of the elements of the dynamic model.

In the model the driver of the dynamics of the I-O structure is technological progress. Technological change is realized by innovations, which are the result of positive feedbacks of R&D efforts. Through technological progress the structure of the economy evolves incorporating past innovations, now materialized into new products and new procedures to produce, which result in new technological links between sectors and, thus, change the I-O structure of the economy [17].

We introduce the dynamics in the model borrowing some ideas from network formation and network dynamics. In particular, the generic mechanism behind the dynamics of the structure of the economy is a preferential-attachment-like mechanism, where I-O connections change based on the centrality of the sectors. The identification of this mechanism provides insights for the policy making process because it allows us to understand how the productive structure changes according to its properties. To undertake the investigation we represent the economy as a weighted and directed network using I-O matrices. The advantages of this analysis are being able to propose a computational model of the evolution of the productive structure of the economy through technological progress using empirical data.

**The interactions among economic activities**

To represent the productive structure of an economy as a complex network we use the intermediate demand table to compute the direct coefficients matrix according to the I-O model [27]. The intermediate demand table contains the flow of resources between sectors, where the sectors in the rows represent the sellers and the sectors in the columns represent the buyers. In the I-O model, total output of a sector $x_i$ is expressed as a function of the demand for the different commodities produced in the economy. Production is defined as follow:

$$x = (I - A)^{-1}d = Ld$$  \hspace{1cm} (12)

where $A = [a_{ij}]$ is the $(n, n)$ matrix of direct coefficients defined as $a_{ij} = z_{ij}/x_j$, $x$ is the $(n, 1)$ column vector of output, in which $z_{ij}$ represent inter-sectoral sales by sector $i$ to sector $j$, and $d$ the $(n, 1)$ column vector of final demand, $L = (I - A)^{-1} = [l_{ij}]$ is an $(n, n)$ matrix known as the Leontief inverse or the total requirements matrix. The elements of the Leontief inverse give information of the direct and indirect requirements of inputs. We compute total production as the sum of the elements of the $(n, 1)$ column vector of output.

We take the direct coefficients matrix $A$ as the weighted adjacency matrix of the productive network. This is a weighted directed graph with self-loops [5, 6, 2]. A node represents a sector and weighted directed edges represent technological relationships between sectors. Self-loops capture the idea of a sector using its own product as input. The weighted adjacency matrix of the productive network has entries $w_{ij} = a_{ij} > 0$ for $i \neq j$ if sector $i$ has a technological relationship by supplying inputs to sector $j$. $a_{ij}$ is the $ij - th$ element of the direct coefficients matrix. Since the network is directed we have $w_{ij} \neq w_{ji}$.
Using the direct coefficients as weights of the links in the network represents an advantage because it is capturing heterogeneity in the intensity or strength of the connections between nodes and not only the existence of a link. When the density of the network is very high, the distribution of weights provides additional and useful information.

Changes in the input-output structure

To start studying structural change, first we identify changes between I-O matrices. We have a series of input-output matrices representing states of the economy in different periods of time. We, then, compare two by two to observe changes in the structure of connections.

\[ A_{m,n}(t) = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix} \]

where both matrices are symmetric and of the same size. To see if there was a change in the structure we measure: \( A(t) - A(t + 1) \). Then, for every entry of the matrix, we evaluate if:

\[ a_{ij}(t) - a_{ij}(t + 1) \begin{cases} = 0 & \text{if no change} \\ < 0 & \text{if connection was strengthen} \\ > 0 & \text{if connection was weaken} \end{cases} \]

To observe the set of these changes together we can make a scatterplot between the elements of \( A_i \) and \( A_{i+1} \). If the points corresponding to the connections between \( i \) and \( j \) in both matrices are above the 45-degree line then there was a strengthen of the connection. Similarly, if the points corresponding to the connections are below the 45-degree line, then there were changes that weaken the connections. We can identify the sectors that show the largest changes of edge weights as those that created new connections or those that changed the weight of existing connections and are, therefore, key for the structural change of an economy during a specific period. During the process of structural change there will be winners and losers. Those sectors that lose connections or that see the weights of their connections weaken will be the losers because sectors have substituted their products for others.

Changes in the structural properties

A first property of the network is the number of connections or interactions between nodes. The fraction of actual connections with respect to the potential number of connections that could exist if all nodes were connected is the density of the input-output network. Density can be compared from year to year to see how the structure has changed. Another measure is the degree, which is a local measure of centrality that counts the number of adjacent connections that a sector has. According to the direction of the connections, a sector has an in-degree and an out-degree. In the I-O context, in-degree measures the number of transactions that a sector undertakes to buy inputs. Out-degree is a measure of the number of transactions taking place when a sector is supplying inputs to other sectors. They are computed as the row and column sum of the (non-weighted) adjacency matrix \( A \), which is equal to 1 if there exists a link pointing from \( j \) to \( i \) and zero otherwise.
To evaluate the number and magnitude of the connections between nodes network analysis provides two measures called weighted degree or strengths. According to the direction of the connections a sector may have an in-strength and an out-strength. These are measured as the row and column sum of the weighted adjacency matrix, $A$, which is the direct coefficients matrix. A more informative measure is one that considers direct and indirect connections and the fact that not all connections are of the same quality, where a node with a smaller number of high-quality links may outrank one with a larger number of mediocre links. [15] proposed the eigenvector method to measure linkages considering that connections should be weighted according to their importance, where inputs from a sector with high linkages receive a larger weight in the process than inputs from a sector with lower linkages. Linkages computed with this method as the right and left principal eigenvectors of the direct input coefficient matrix $A$ and the output matrix $x$ respectively, are related to the eigenvector centrality in network analysis. This centrality satisfies:

$$Ax = \lambda x,$$

where $A$ is the adjacency matrix which is the direct input coefficient matrix and $\lambda$ is the largest eigenvalue.\(^1\)

However, eigenvector centrality for directed networks, which have an asymmetric adjacency matrix, has problems for computing the centrality of nodes outside strongly connected components, which receive scores of zero [36]. To get around this problem [21] proposed to compute authority and hub scores for directed networks. Authority and hub scores are a generalization of eigenvector centrality for directed networks. These scores are global centralities and have a mutually reinforcing relationship where good authorities are pointed by good hubs and good hubs point to good authorities [21]. The authority centrality of a node is proportional to the sum of the hub centralities of the nodes that point to it, and the hub centrality of a node is proportional to the authority centrality of the nodes it points to [36].\(^2\) Authority scores are characterized by equation 15 and hub scores are defined by equation 16 below:

$$Ax = \lambda x$$  \hspace{1cm} (13)

$$a = (I - \lambda A^T A)^{-1} - 1) \lambda$$  \hspace{1cm} (14)

$$h = (I - \lambda AA^T)^{-1} - 1) \lambda$$  \hspace{1cm} (15)

Therefore, the authority and hub scores are given by the eigenvector of $AA^T$ and $A^T A$ with the same eigenvalue.

\textbf{A Dynamic Model of Structural Change}

To model technological progress in the productive network we use tools from network theory. Essentially, we use a version of a network dynamic mechanism called preferential attachment. There are different models of network formation and network dynamics. These generative network models model the mechanisms by which networks are created. The idea is to explore the generative mechanisms to see

\(^1\)Eigenvector centrality was first proposed in [9] as a power measure in social networks. One interpretation of eigenvector centrality is given in [10], where it counts the number of walks of all lengths, weighted inversely by length, which emanate from a node, so it can be interpreted as an accessibility index.

\(^2\)sectors that are highly independent from other sectors’ inputs would have a centrality of zero even if these sectors sell their own product to other sectors as input.
what structures they produce and if those structures are close to what we observe in reality. Examples of this type of generative mechanisms for the growth of networks include preferential attachment, vertex copying models, and network optimization [36]. The preferential attachment mechanism was first called cumulative advantage by [40]. The original idea is that the probability that nodes create new links is proportional to the number of links they already have.

The mechanism that we use in this chapter to model the dynamics of an economy’s structure is of the type of preferential attachment in the sense that sectors in the productive network make new connections and change the strength of existing ones according to their centrality. We maintain the number of sectors fixed given the sectors classification we have through time. Nevertheless, we can still think of new nodes been added to a much more disaggregated network, which increases the size of the sectors we actually observe at a higher level of aggregation and create a new link between two existing sectors.

The dynamic model we propose is driven by technological progress in the productive network based on the changes in the structural properties of the network. Technological progress generates innovations which can change the structure of the network through two effects: 1) A new link because there is a new sector in a highly disaggregated economy, or a new relationship between two existing sectors; 2) A change in the technological coefficients or weights of the links due to substitution of inputs to produce.

Innovations are a function of R&D intensity, which is, at the same time, a function of production. More production means more money to invest in R&D and the more R&D intensive the higher the probability of creating an innovation. New innovations have a probability of realizing and are sector specific. When an innovation realizes it changes the way a sector produces by introducing a new connection or changing the intensity of the existing connections between sectors according to the sectors’ centrality. The sector to receive the new connection is chosen randomly from a set of sectors with low centrality.

Initial conditions are an early stage of development of the structure when the density of the network was lower, thus there were fewer connections between sectors. We can compare different generative mechanisms depending on which centrality measure we use. Then, we can observe which of these mechanisms was able to generate the closest approximation to the evolution of the structure of the economy. The general idea is the following. Sectors with high in-centrality are very directly dependent on other sectors’ output to produce their own during a given period. Instead, sectors with low in-centrality are sectors that directly require only a few inputs from other sectors to produce, so are more independent. Conversely, sectors with high out-centrality are common suppliers to most of the sectors in the economy and, at the same time, they rely strongly on their sells. Finally, sectors with low out-centrality supply inputs to only a few sectors. Then, a sector with a high in-centrality is more likely to change the intensity of the existing connections rather than forming new links. This process is assumed to represent substitution between inputs, were the increase in the intensity of one connection implies the proportional reduction on the others. Comparably, a sector with low out-centrality is assumed to be more likely to form a new link due to an innovation. Within this sector with low out-centrality an innovation is transformed into a new variety, for example computer parts in the electronics sector in the 1970s. This will create new links to other sectors with positive weights. In the simulation exercises we defined low
centralities as values below the average of the centralities and high centralities are those values above. We assumed that the magnitude of this new link will have a random component drawn out from a distribution determined as the best fit of the actual distribution of weights. The intensity or weight of a new connection between sector i and sector j will take the following value:

$$w_{ij}(t + 1) = w^*$$

(17)

where $w^*$ is a random number drawn from the best fit distribution with mean and standard deviation equal to the mean and standard deviation values of the actual weights distribution. The change in the weights between two existing sectors is performed as follows:

$$w_{ij}(t + 1) = w_{ij}(t) + w^*$$

(18)

Since the change in weights represents substitution between inputs, when $w_{ij}$ increases, $w_{ik}$ decreases for all $k \in N_i$ and $k \neq i$ proportionally in the following way:

$$w_{ik}(t + 1) = w_{ik}(t) - \frac{w^*}{N_i}$$

(19)

where $N_i$ is the number of input suppliers of $i$.

After a new link is created and after a weight has changed, the system updates to these new conditions. First the direct coefficients matrix gets updated incorporating the new links and the changes in the weights of the existing links, $A(t+1)$ according to equation 17, equation 18 and equation 19 Second, production and $R&D$ will update according to:

$$x(t + 1) = (I - A(t + 1))^{-1}d(t + 1) = L(t + 1)d(t + 1)$$

(20)

and

$$R&D_i = (0.01)x_i(t)$$

(21)

where $x(t + 1)$ is the new production $(n, 1)$ vector, $I$ is the identity matrix, $A(t+1)$ is the new $(n, n)$ technological coefficients matrix, and $d(t + 1)$ is the $(n, 1)$ final demand vector which is taking its actual value each year.

Each round, if $R&D$ increased, then the probability of a sector of having an innovation increases through a multiplicative positive feedback parameter $f > 0$. After each round the network structure updates. At time $t$, the direct coefficients matrix is indexed $t$, $A(t)$, and so to the other variables at hand. We suggest performing 1000 Monte Carlo experiments of the simulation model. The value of the variables at each $t$ will be the mean of those 1000 experiments.

**Expected Results and Reflections**

The results that we expect to obtain from the simulation of the computational model are:

- The trajectory of the simulated density of the network that represents structural change at the aggregate level.
- The trajectory of the production of each sector and of the economy that represents how an aggregate property of the system changes.
- The evaluation of congruent structural changes in the simulated economy:
- Changes in the number of connections and their weights between sectors.
- Changes in the structural properties of the network: centrality of sectors.
• The evaluation of the generative mechanism should answer: did the dynamic mechanism generated a trajectory that approximated the one observed for the economy under investigation?

If the approximation was close, then we found a mechanism that generates the structural change of the economy and that could potentially foresee where it is heading. The generative mechanism that closely approximated the trajectory of the economy under investigation has not to be the only one. This mechanism may be valid for one economic system but not for another one. For example, it may approximate the trajectory of some European countries but may not approximate the dynamics of Latin American countries. The generative mechanism found may be valid for a specific period of time but not for others.

To be able to build a dynamic model of structural change such as the one we propose in this paper, we need a long time series of input-output matrices built under the same methodology. Once we found a generative model of structural change for an economic system, we can draw some lessons for industrial policy to be implemented.

The computational model presented in this section modeled structural change along a period of time for an economic system as a result of technological progress. Technological progress generated innovations, which modified the connections between sectors and, consequently, changed the structure of the network representation of the economy. These changes in the productive network exemplify structural change. The methods used considered that the economy is a complex system that is composed of heterogeneous sectors that interact and is dynamic by nature. In the next section we present the concepts of regimes and regime shifts, as structural changes, in the economic literature.

4. Regime and dynamics of regimes. In this section we review the basic notions and definitions of economic regime and regime switching. Our study draws on complexity theory, and describes how these notions appear implicitly or explicitly in different areas of the economic literature. The concepts of regime shifts are then used to represent structural changes in different economic concepts and then modelled. Any economy has a performance, a functioning and a structure that are characterized by processes resulting from the actions of the agents in the economic activity, interactions between the units that act, and emergence from individual behaviours, joint operations and transitions between phases through the respective processes. In this context, structural change refers to a change in the performance, functioning, and structure of an economy, which can be described by, and is in connection with, processes, interactions, emergence, and transitions, which constitute the complexity of the economy. In this section we introduce the concepts of economic regime and regime switching. These concepts are then used to represent structural changes in different economic concepts.

The term regime has a long history in economics. It has been implicitly or explicitly extensively used in a variety of fields, with reference not only to methodological aspects but also to analytical and economic policy, and even political issues. Still, the term is generally neither uniquely nor well defined so that it stands for different things to the various authors who have been using it. A dynamic regime is not fixed, but rather fluctuates over time and space as external forces and internal processes influence the system. A regime is a dynamic model with its own associated multidimensional domain, in which state variables exhibit characteristic behaviors or structures. Those structures can be defined either by inherent dynamic behavior
(e.g. a basin of attraction) or by the observable manifestation of them (e.g. an oligotrophic lake vs. a eutrophic one). The state space of a system can encompass multiple regimes of a variety of basin sizes and attraction strength, which in some disciplines is referred to as resilience [31, 13]. Identifying and understanding the boundaries that separate regimes is essential to understanding the regimes themselves. However, boundaries may be poorly defined, vary with exogenous and endogenous parameters, and evolve in time in response to changes in these parameters. It is easier to define regime indirectly, by defining regime switch.

Regime shifts are large, abrupt, systemic changes in the structure and function of a system, where a regime is a characteristic behavior of a system, which is maintained by mutually reinforced processes or feedback loops. The shift of regime typically occurs when a continuous smooth change in an internal process or an external variable triggers a completely different system behavior with irreversible consequences. A regime shift occurs when a system moves across regime boundaries, which are influenced by a variety of exogenous and endogenous mechanisms. These are qualitatively different from phase transitions, which are driven solely by changes in external conditions. Exogenously generated regime shifts can take several forms, including: continuous changes in parameters or in the functional form of the dynamics; or randomly distributed shocks which change the values of the state variables, the parameters and/or the very rule of motion (i.e., stochastic or random motion that is layered on top of a deterministic system). Endogenously generated shifts depend fundamentally on mechanisms internal to the system, inbuilt in its architecture or relational wiring. Shifts often occur when the system reaches and overshoots some frontier values in its state space and/or in the parameter space.

From the mathematical viewpoint, a regime switch refers to a situation where there is a change in the nature of a system of equations, i.e., a qualitative change in the functional forms of a given system taken as a model. The system behavior implied by any given model always implies the definition of a regime, in general more than one, when the mathematical model is so set up. In an economic perspective, an economic regime is a given set of rules and/or institutions, which are said to govern the economy as a system, and therefore it accounts for its qualitative (static or dynamic) behaviours. Regime, therefore, stands for such a set of rules/institutions, but sometimes, instead, it stands for the resulting qualitative behaviour, and often it is confusedly used to indicate both of them. The implied hypothesis is that they are uniquely corresponding to one another. According to [11, 8] a regime is a class of (dynamic) behaviours which are sufficiently similar from a qualitative point of view, that they can be considered (as being generated) by variants of the same basic model. Regime switches are associated with qualitative changes in the dynamics generated by changes in the model to which an economy is obeying. These changes are fundamentally discontinuous jumps or switches. Very often, they can be modelled as the result of the economy’s reaching and overshooting certain pre-determined critical or threshold values in the key state values and/or in its parameters. In this case, delay result and accumulation phenomena reflecting path dependence and the like play a crucial role.

Various mechanisms and their interactions can produce behaviour that is a compounding of smooth evolution within a given regime, and sudden qualitative discontinuities in behaviour across regimes, regime shifts (also catastrophes, [44]). Although the dynamics of the system may be linear or otherwise easily predictable
within a regime, cross-regime dynamics shifts can be reversible or irreversible. However, the evidence that such a shift has occurred or is about to occur may be subtle, complicating predictions of when a system is in or about to enter a regime change, and complicating the decision processes for controlling regime shifts. On the other hand, regime switching can be reversible or irreversible, but only reversible changes are relevant to understand the relationship between economic fluctuations and regime switching phenomena. Stringing together different regimes in variable histories that can follow in its own steps, reversibility is what makes cross-regimes dynamics one of fluctuation (e.g. a growth cycle or nearly so) or very irregular up to chaotic. Regime changes result from a change in the dominant forces. Any complex system contains different feedback systems that can evolve and combine in only a limited number of ways. Over time, a particular combination of feedbacks will tend to become dominant, leading the system to self-organize into a particular structure and function or “regime”. However, if at some point a critical threshold is passed where a different set of feedbacks become dominant, and the system experiences a change in structure and function or a “regime shift”.

Hysteresis is the dependence of the state of a system on its history and it has to do with the concept of path dependency in complex systems. Hysteresis emphasizes the role of history and time in a system, demonstrating that the system has memory and that its dynamics depends on past events. Hysteretic systems have two important properties. First, the reversal of discontinuous change requires that a system change back past the conditions at which the change first occurred. This occurs because systemic change alters feedback processes that maintain a system in a particular regime. Second, hysteresis greatly enhances the role of history in a system, and demonstrates that the system has memory in that its dynamics are shaped by past events.

Conditions at which a system shifts its dynamics from one regime to another are often called thresholds. Thresholds can depend on different parameters of the model, and can change with time and space. Economists have used the theory to identify both the internal mechanisms, which can increase the resilience of a particular regime (see [14]), and the thresholds at which external pressures can overwhelm these internal stabilizing mechanisms and cause a regime shift (see [30]).

A regime shift initially represents a loss of resilience -resilience is the capacity of a system to re-organize and respond to a perturbation by recovering quickly. Crossing a single threshold between alternative regimes often leads to a cascading effect in which multiple thresholds across scales of space, time, and organization may be broken. The regime that this cascading effect ultimately produces has a tendency to be highly resilient and resistant, for instance, to management strategies that might seek to restore the earlier regime. The amount of time a system spends in one particular regime depends fundamentally on the self-organization and resilience of the system; if the system is not highly resilient to external disturbances or perturbations, it may move into a different regime.

Although the dynamics of the system may be linear or otherwise easily predictable within a regime, shifts or transitions between regimes are most often due to nonlinear responses and relationships, and occur very abruptly. Nonlinearity makes predicting when systems are in or about to enter a transition, as well as whether systems can return to the original regime, very difficult. Compared to transitions that are simply nonlinear, the presence of hysteresis makes prediction of a regime shift extremely difficult, and recovery following such a shift requires
changing critical parameters beyond a minimum threshold amount. As the shift or transition phase occurs quickly, few to no data are typically collected in this phase. The theory of regime shifts is in fact included in the framework of nonlinear dynamics, state spaces and dynamic attractors. Nonlinear systems can present more than one stable basin of attraction, that we would call a regime, which is stable due to a number of negative feedback loops that hold it within that state. At each perturbation, the system changes the attractor, as it moves away it moves towards a critical phase transition area far from its equilibrium, an unstable regime governed by positive feedback where some small event can get amplified rapidly driving the system through the phase transition into another basin of attraction. The system has two or more basins of attraction and can flip between them.

To analyze the basic properties of multiple regime phenomena, it is useful to capture the basic properties in a mathematical model. Usually a dynamical model is represented by a system of difference or differential equations and then mathematical research on dynamic regimes starts with one such system of equations. Several tasks have to be solved:

- The identification of alternative dynamic regimes and the boundaries that separate them. Dynamic regimes can be identified with steady states or attractors of the system. If this is the case, to describe regimes we have to determinate the basins of attraction. When two, or more, attractors are present in a system, each attractor has a basin of initial conditions, which lead asymptotically to that attractor. The basin boundaries are the sets, which separate different basins and they commonly have very complicated fractal structure [37, 39]. In the presence of one dimensional systems and related to generating partitions, one can identify regimes with increasing-decreasing intervals [33]. The analogous situation in greater dimension can be given by piecewise defined systems [42].

- To describe the basic properties of the dynamic regimes. Once we have identified the alternative regimes, we can study its stability and reversibility, we can compute the dimension of the regime, the presence of hysteresis and the elasticity and amplitude of resilience. [29, 46] It is also possible to study the dependence of regime boundaries and other characteristics on the parameters of the system.

- What are the mechanisms of regime change? Clearly, if the system has only stable states, then to shift from one regime to another requires a sudden impulse or an external perturbation or we have to change the parameters of the model. This situation can be modeled, for example, with Markov chains. But it is also possible to have endogenous shifts [11].

- How can we describe dynamics across regimes? Multiple regime models present a twofold dynamics: across regimes and within each regime. While dynamics within regimes is represented in the traditional form of differential-difference equations, dynamics across regimes can be represented via symbolic [48] and coded dynamics [12]. Another tool to represent multi-regime models are hybrid systems (see [28]), where there is an interacting mechanism between discrete (representing regime shifts) and continuous dynamics (representing dynamics within each regime).

5. Conclusions. The three perspectives summarized in the previous sections depart from empiric evidence of the economic activity towards the construction of models. Therefore, the database is the origin of the analysis. The framework used
along the three sections is the complexity approach; we consider the economy as a complex system composed of heterogeneous agents. For this reason we center the analysis on the interactions between these agents. The exposition of the three perspectives was made considering the chronological order of their introduction into the economic debate. We started with the lower level and moved to the major level of generality of the problem. As we advance from the first to the third section, the modeling incorporated a wider range of methods and techniques including matrix algebra, graphs, network analysis, computational modeling, and mathematical models.

The models proposed in each of the three perspectives are examples of the essential relation between how structural change can be conceptualized specific of stages of the economy, variations in the levels of complexity viewed as changes in the productive structure as a result of technological progress, and regime shifts and the methods and techniques used to simulate the specific transition of stages, levels, or regimes. Structural change, independently of how it is conceptualized, is placed at the center of the analysis. We redirect the focus away from the modeling methods and techniques, towards the change and dynamics of the systems.

Heterogeneity, interactions, the emergence of structures, and the feedback between the components of the systems-real and control spheres in the anti-equilibrium approach, network of connections of input-output and its evolution, and regimes shifts-are subsidiary aspects of the structural economic change and its occurrence. Structural change is the emergent propriety of the systems in each perspective presented because all their properties and simultaneous relationships inside the system are necessary for the occurrence of structural change. Emergence is the consequence of applying the complexity approach and it can only be simulated by dynamical models built with elements that go beyond the use of deductive and inductive methods.

**Acknowledgments.** We would like to thank the comments and suggestions made by two referees.

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Received December 2018; revised March 2019.

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