Numerical Simulation of Stratified Values of the Vertical Gravity Gradient Generated by a Pair of Closely Spaced Isolated Inhomogeneities with Excessive Density

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Abstract. In the article is presented a mathematical model and a computational algorithm for solving the inverse problem of gravity gradiometry – a problem of localization (separability) of a pair of gravitational inhomogeneities – isolated ore bodies (oil and gas domes) with excessive positive (negative) density based on the gravitational anomalies generated by them. Anomalies are empirical data for solving the inverse problem, which is relevant for geological exploration, monitoring and geophysical support of developed fields, as well as for practical problems of engineering geology.

1. Introduction

This paper proposes a method that can be used for studying low-contrast geological objects and for solving some engineering and geoeological problems, as well as for monitoring the state of the natural and engineering objects. In particular, this method can be used for monitoring oil and gas fields in operation, as well as for planning built-up areas and designing structures with high safety requirements.

The mathematical model for similar works is the direct and inverse problems of gravity gradiometry.

The primary stage of the study is the gravimetric survey and interpretation of experimental data. Interpretation of gravity exploration data is divided into qualitative and quantitative ones.

Qualitative interpretation consists of the anomalous field features analysis. The basis of the method of qualitative interpretation is the method of analogies. The data of the gravity gradiometry is compared with data from other geophysical methods, as well as with verified data on already studied territories. For a more detailed and accurate study of the territory, a scheme for the distribution of anomalies is made based on the results of qualitative interpretation.

Quantitative interpretation consists of solving of the direct and inverse problems.

The direct problem is to calculate the characteristics of the gravitational field generated by a body of a known shape with a known density, the location of which is known. This task has the only sustainable solution.

The inverse problem of gravity gradiometry is to determine the location, depth, shape and size of the ore intrusion (dome) through the measuring of the gravitational anomalies generated by the shape, depth, and density of the underlying rocks heterogeneities.
The solution of the gravity gradiometry inverse problem is not unique, because bodies that differ in shape and density may have similar characteristics of gravity. The solution to this problem is unstable. It is an ill-posed problem [1, 2]. The theory of ill-posed problems and practical problems of mathematical geophysics are presented in the works [1, 2, 3, 4, 5]. Problems related to diagnostics of separability of heat sources and elastic vibrations are considered in publications [6, 7, 8, 9].

The solution of a specific inverse problem, including the problem of gravity gradiometry, is performed by iterative methods. Usually these are consecutive solutions to a series of direct problems, so that at each subsequent step the deviation of the calculated data from the empirical data decreases, and ideally tends to zero. The initial approximation is chosen taking into account a priori information about the geological structure of the region, the data from other geophysical exploration methods, as well as qualitative interpretation of empirical data from gravimetric surveys. This is usually a graphical representation of empirical data.

For gravimetric measurements on a certain profile, the initial empirical data for solving the inverse problem of gravity gradiometry is presented in the form of a graph, the extrema of the graph correspond to the anomalies of the gravitational field. For area shooting, the values of the function of two variables are obtained. In this case, the isolines provide a visual representation of the gravity distribution over the area.

This paper presents the results of numerical simulation of the vertical gradient gravity anomaly generated by a pair of isolated closely placed bodies, which density differs markedly from the density of the composing rocks.

The idea of the proposed method is based on the fact that when moving away from the surface, the influence of inhomogeneities on the value of the vertical gradient is smoothed out. Therefore, by comparing the results of the gravimeter measurement of the magnitude of gravity at two or more fixed levels at the points of the studied geological object, in some cases it is possible to confidently localize inhomogeneities. In practice, the use of this method involves the use of non-traditional technology: instead of surface measurements of the characteristics of the gravitational field, only vertical gradient measurements are required using a controlled aircraft, for example, a quadcopter with a modern gravimeter installed on it.

This work is a continuation of research [10, 11].

2. A physical model

The vector field of gravity acceleration can be expressed in terms of a scalar function \( V(x, y, z) \) by using the gradient operator:

\[
\vec{g} = \nabla V = \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k}.
\]  

The function \( V(x, y, z) \) is called the gravitational potential. According to the physical sense, the gravitational potential is a measure of the energy that must be expended to transfer a body with a unit mass from a certain position to infinity in the field of gravity.

The first derivatives of the gravity potential \( V \) (1) are projections of the acceleration of gravity on the corresponding coordinate axes. In the global theory of gravimetry, the Oz axis is usually directed deep into the Earth, the Ox axis to the North, and the Oy axis to the East. In practical work of gravity gradiometry tasks modeling, the Oz axis is directed deep into the Earth. Accordingly, the Ox and Oy lay on the leveled surface and are perpendicular to the Oz axis. Their directions are determined by specific conditions and features of the geological structure of the object under study. The gravitational force vector is directed along the Oz axis. Its module is the measured value of gravity (also known designation is \( g_z \)). Gal (G) is taken as the unit of gravity.1 \( \text{G} = 1 \frac{\text{cm}}{\text{s}^2} \). In practice of gravimetry and gravity prospecting milligals are of use: 1 mg = 0.001 G; 1 \( \mu \text{g} = 0.001 \text{mg} \). It is clear that \( \frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0 \).

In the practice of gravimetry and gravity gradiometry, second derivatives are also used. The Hessian matrix of the function \( V \), (the tensor of the gravitational field strength)
gives an idea about the geometry of the leveled surface and the variations of the gravity magnitude.

Gravity gradients are understood as the second derivatives of $V$, which characterize the rate of change in the vertical component of gravity along the corresponding axes:

$$
\frac{\partial^2 g}{\partial z^2} = \frac{\partial^2 V}{\partial z^2}, \quad \frac{\partial^2 g}{\partial x \partial z} = \frac{\partial^2 V}{\partial z \partial x}, \quad \frac{\partial^2 g}{\partial y \partial z} = \frac{\partial^2 V}{\partial z \partial y}, \quad \frac{\partial^2 g}{\partial x^2} = \frac{\partial^2 V}{\partial x^2}, \quad \frac{\partial^2 g}{\partial y^2} = \frac{\partial^2 V}{\partial y^2}.
$$

Gradients $V_{xz}$ and $V_{zy}$ are called horizontal gradients, and $V_{zz}$ - vertical gradient.

The unit for measuring gradients is the eotvos (E), named after the Hungarian geophysicist L. Eotvos. One eotvos corresponds to a change in gravity of 0.1 mGal per 1 km. $1E=10^{-9}s^{-2}$. Gradients can be determined numerically if the values of gravity are known, or measured by using devices, such as variometers and gradientometers. The second derivatives $V_{zz}, V_{yy},$ and $V_{xx}$, allow us to calculate the curvature of the level surface.

In this work, we will be interested in vertical and horizontal gradients. In reality, the vertical gradient is 2-3 orders of magnitude higher than the horizontal ones. When the normal direction of the Oz axis is inland, the Ox axis is North, and the Oy axis is East, $V_{zz} = 3086$ E, the gradient $V_{xz}$ has a maximum at latitude 45°, equal to 8.16 E ($7.76$ E at the latitude of Novosibirsk), and the gradient $V_{zy}$ is zero.

3. **Mathematical model**

We will consider a simplified statement – an interpretation of measurements by profile. In the simplest statement, this is the problem of separability of two closely spaced bodies.

When measuring the value of the vertical gradient along the profile with increasing height above the surface, the deviation of the vertical gradient anomalies from the plane parallel to the leveled surface at the origin decreases, and the graph is smoothed.

The origin of coordinates is determined by the level surface between the bodies. The assumption of a symmetrical arrangement of heterogeneities is made to simplify the computational scheme and algorithm. The axis Ox (Figure 1) lies in the plane leveled at the point O of the surface and is directed to the protrusion of the profile, the points O, A, and B are the marks of the sign positions. The axis Oz is directed deep into the Earth, the half-Axis Oh is directed upwards. The line at the distance $h_0$ from the axis Ox is its parallel.

![Figure 1. The shape and location of the bodies.](image)

If the gravitational survey is processed and interpreted according to the profile and, therefore, only gravitational anomalies along the stretch in this direction are taken into account, then the total force of attraction for the points of the Ox axis is calculated using the formula (2):
The vertical component of the gravitational acceleration at the point $(x, 0)$ is given by

$$V_z = V_z(x, 0) = \gamma \iiint_\Omega \rho(x', z') \frac{z' dx' dz'}{\sqrt{(x-x')^2 + z'^2}}.$$  \hspace{1cm} (2)

Formula (3) gives the values of the vertical component at the point $(x, h)$:

$$V_z(h) = V_z(x, h) = \gamma \iiint_\Omega \rho(x', z') \frac{z' dx' dz'}{\sqrt{(x-x')^2 + (h+z')^2}}.$$  \hspace{1cm} (3)

4. The discretization of the model and the computational scheme

In formulas (2), (3), the point $M(x', z')$ belongs to the domain $\Omega$, which is refined when calculating integrals (2), (3) by the numerical method.

The space sampling algorithm and the scheme of the computational algorithm are shown at figure 2.

![Figure 2](image-url)

**Figure 2.** Discretization of the model and computational scheme.

The three-dimensional space is divided into voxels of cubic shape to perform the space-sampling. When processing measurements on the profile voxels – squares, we identify them with the elements $dx' dz'$, assuming for calculations that $dx'=dz'=1$. We will take for voxels the marking $M(m, n)$ by identifying them with points as centers of the respective elements. The first index (vertically, in the quadrature formula $m$) is equal to zero at the origin, has a plus sign in the lower half – plane, a minus sign in the upper one, and grows modulo "from the center". The second index is also equal to zero at the origin, growing to the right, it is assigned with the index $n$. In the area of excess density, we put $\rho(m, n) = 1$, in the rest of the lower half-plane $\rho(m, n) = 0.5$. In the upper half-plane $\rho(m, n) = 0$.

For $m > 0$, $n > 0$, the element of the vertical gradient between the voxels $M(0, n)$ and $M(m, n)$ is equal to

$$dg_z(m, n) = \rho(m, n) \frac{m}{\sqrt{m^2 + (n-n')^2}}.$$  \hspace{1cm} (4)

The amount

$$g_z(0, n') = \sum_m dg_z(m, n) = \sum_m \rho(m, n) \frac{m}{\sqrt{m^2 + (n-n')^2}}.$$  \hspace{1cm} (5)

gives the total contribution of the column with the number $n'$ to the value of the vertical gradient of gravity at the point $(0, n)$.

The total value of the vertical gravity gradient at the point $(0, n)$ is calculated by the sum
\[ g_z(0, n) = \sum_m g_z(m, n'). \]  

(6)

Summation (6) in each column is carried out to the number \( m \), at which the value of the element (5) becomes less than \( 10^{-4} \).

Similarly, to calculate the value of the vertical gradient of gravity at height \( h_0 \) (we’ll assume \( h_0 = -m_0 \)), the formulas (4), (5) and (6) take the form:

\[ g_z(h_0, n) = \rho(-m_0, n) \frac{n-n'}{\sqrt{(m+m_0)^2+(n-n')^2}}, \]  

(7)

\[ g_z(h_0, n') = \sum_m g_z(h, n) = \sum_m \rho(-m_0, n) \frac{n-n'}{\sqrt{(m+m_0)^2+(n-n')^2}}, \]  

(8)

\[ g_z(h_0, n) = \sum_n g_z(h, n'). \]  

(9)

At each point of the Ox axis (for each \( n \) on the voxel M (0, \( n \))), we calculate the analog of the radiation patterns as follows:

Taking (6) and (7) into account, in the direction between the voxels M(0,\( n \)) and M(\( -m_0, n \)), we postpone the length segment

\[ k \cdot [g_z(0, n) - g_z(h_0, n)] = |\vec{v}(n, n)|, \]  

(10)

The vector \( |\vec{v}(n, n)| \) interprets the amount of variation in the vertical gradient between the points M (0,\( n \)) and M (\( -m_0, n \)).

Similarly, we define vectors \( \vec{v}(n, n - 1) \) and \( \vec{v}(n, n + 1) \), \( \vec{v}(n, n - 2) \) and \( \vec{v}(n, n + 2) \) so on.

Calculations using formula (10) and similar ones are performed up to the number \( k \) at which the element value \( \vec{v}(n, n \pm k) \) becomes less than \( 10^{-4} \).

The sum of the vectors is calculated, analogous to the resultant on the entire Ox axis:

\[ \vec{R}(n) = \sum_{k=-N}^{N} \vec{v}(n, k). \]  

(11)

The coefficient \( k \) in formula (10) is selected empirically during a computational experiment.

5. The results of the numerical experiment

Figure 3 shows radiation patterns at significant points (see figure 1) with direction gradation through 5 voxels.

In cases where the heterogeneities are separable (in practice, this corresponds to the fact that the ore bodies lie close to the surface and are located at a considerable distance), the diagram vectors have an asymmetric distribution (Figure 2), and their sum will have a non-zero component along the Ox axis.

The set of vectors \( \vec{R}(n) \) will have different deviations from the vertical direction.

![Figure 3. Radiation patterns.](image-url)
In the case of positive excess density, the computational experiment showed a declination in the direction of the attracting mass. For clarity, on figures 4, 5 and 6, the resultants are shown without a distinction for the module. They show the view of the graph and the direction field of the variation of the vertical gradient of gravity at different locations of ore bodies along the protrusion of the profile.

Numerical calculations were performed using the above algorithm with varying input conditions. The depth of the columns (value $z_0$), the distance between the columns (value $d$), and other parameters of the model scheme (Figure 1) were changed, except for the densities of the composing rocks and heterogeneities. Linear dimensions in the computational algorithm are presented in the amount of voxels. The value of $d$ is varied from 20 to 160 voxels, $z_0$ - from 0 to 30, $h_0$ - from 10 to 30 voxels. The step of the computational grid is 6 voxels.

![Figure 4](image4.png)

**Figure 4.** Heterogeneities are clearly separable by the graph and separable by the radiation patterns.

![Figure 5](image5.png)

**Figure 5.** Heterogeneities are inseparable from the graph and inseparable from the radiation patterns.

![Figure 6](image6.png)

**Figure 6.** Heterogeneities are indistinguishable from the graph, but are distinguishable from the radiation patterns.

At figure 4, the deviation of the resultants is shown with gradation 40 voxels, in figures 5 and 6 – 30 voxels. The application of the stratified measurement method involves the usage of modern technical sources and allows you to reduce the volume of drilling operations and labor costs for conducting ground measurements.
6. Conclusions

Numerical experiments show that the proposed algorithm allows to expand the range of diagnosed heterogeneities with the use of modern technologies and technical sources.

The method based on the application of this algorithm can be used to determine karst formations and the accumulation of ground water. As well as for localization of fragments of underground engineering structures, during preliminary survey of the territory allocated for the construction of the object and other tasks of engineering geology.

The method does not give an exact answer for sufficiently deep occurrence of masses with excessive density and such heterogeneities, the characteristic dimensions of which exceed the distance between them. The capabilities of the method can be expanded by improving the accuracy and reliability of measurements, as well as by improving the computational algorithm.

Technologies that correspond to this method allow not to take into account the Bouguer corrections for the initial preparation of gravitational measurement data, since their contribution is included in the measurement of the vertical gradient at each level. When performing the operational measurement operations using an aircraft, you can ignore corrections for the influence of astronomical factors. Gravimetric measurements on moving vehicle will require adjustment for the Eötvös effect. But, accounting of the velocity vector component contribution in the latitudinal direction is as reliable as accounting of the geographical latitude.

7. References

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