Research Article

Crankshaft Optimization Based on Experimental Design and Response Surface Method

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1. Introduction

The crankshaft is one of the most significant parts in the engine and has a direct impact on engine performance. During the engine's operation, the crankshaft bears the combined action of periodic gas pressure, reciprocating inertia force, and torque [1]. The crankshaft also bears the bending and torsional load, which leads to fatigue failure or even fracture failure during the operation of the crankshaft. Once the crankshaft fails, it will affect the regular operation of the engine and easily lead to safety accidents. Through the finite element simulation analysis of the crankshaft, the position and stress value of the crankshaft which is prone to damage in the working process is obtained. Through modal analysis, it can determine the area with the large amplitude of the crankshaft to prevent resonance [2, 3]. Through harmonic response analysis, the natural frequency and weak parts of the crankshaft excited under specific load can be obtained, which provides a theoretical guidance scheme for the subsequent optimal design of the crankshaft. The response surface of the crankshaft is analyzed by using design expert software and DOE theory [4, 5]. The three-dimensional response surface is obtained, and the optimal size of the crankshaft is gained to realize the rapid optimization of the crankshaft in terms of mass and equivalent stress. The three-dimensional model of the crankshaft restored in equal proportion is imported into ANSYS Workbench to calculate the working condition strength [6]. The calculation result is close to the measured value and has high simulation accuracy.
Many researchers at home and abroad have done a lot of research on the dynamic characteristics of the engine crankshaft. Aiming at the problem of torsional vibration and noise in the working process of a crankshaft, Zhang made a modal analysis of the crankshaft. The first natural frequency (NF) is defined as the objective function, and the three-dimensional parameters of the crankshaft are defined as the input parameters. The objective function is augmented to maximize the first NF [7]. To deepen the understanding of the dynamics of the engine crankshaft, Chen established the three-dimensional model of the crankshaft by using Pro/E, and imported ANSYS to carry out two kinds of free and constrained modal analysis, the NF and vibration type, and optimized it on this basis [8]. Zhou established a multibody dynamics model of a crankshaft based on the theory of multibody dynamics. The dynamic characteristics of the crankshaft are simulated, and the dynamic and static characteristics are optimized by combining artificial neural network and genetic algorithm [9]. To reduce the vibration fracture of the crankshaft, Xu studied the engine crankshaft made of QT800 material, carried out the modal analysis and harmonic response analysis, and located the location where the vibration damage of the crankshaft is easy to occur [10]. Yu et al. combined flexible body dynamics with the finite element method to conduct a dynamic simulation of the crankshaft and had a deep understanding of the stress and life of the crankshaft [11]. Mourelatos used the two-stage dynamic substructure method based on the finite element method to predict the dynamic response of the crankshaft [12]. Deshbhratar and Suple studied the mode of the crankshaft and explored the relationship between the frequency and vibration mode of the crankshaft [13]. Sani and Noor gave the vibration mode and NF of the crankshaft and connected the DAS with the sensor on the crankshaft to complete the excitation experiment, which was verified by the finite element method [14]. The above literature mainly studies the static and dynamic characteristics of the crankshaft and obtains its maximum equivalent stress, fatigue life, and NF. However, the crankshaft structure has not been optimized.

After analyzing the mechanical and vibration characteristics of the crankshaft, optimization is an important content. The optimization of the crankshaft is also one of the hot topics of the current research. Cui verified that the crankshaft met the design requirements through dynamic and static characteristic analysis and fatigue analysis after optimizing the design of the crankshaft [15]. Wang et al. constructed the structural strength response surface model of the crankshaft through the central combination experiment and least square method and optimized the design of the crankshaft with critical dimensions as optimization parameters [16]. Song et al. used the DOE method to input the vital dimensions of the crankshaft and carried out the multiobjective optimization design of the crankshaft, which significantly reduced the crankshaft mass and maximum equivalent stress [17]. Li et al. put forward the multiscientific design method of the crankshaft, established the parametric model of piston connecting rod crankshaft, and carried out multiobjective optimization design with mass, strength, and thermal performance as constraints [18]. Garg established the crankshaft model with Pro/E and carried out the static analysis. Finally, the crankshaft was optimized with the rule of unchanged crankshaft shape, and the goal is to reduce the equivalent stress [19]. Xun et al. carried out the bending fatigue test and stress test on the crankshaft using the finite element method. Then, the static strength of the crankshaft is defined as the optimization goal, and the crankshaft is optimized [20]. Cevik carried out the topology optimization design of the crankshaft. While ensuring the stiffness and strength, the crankshaft was lightweight to reduce the crankshaft mass and moment of inertia. After optimization, the stress value increased [21]. Ketia et al. analyzed the dynamic characteristics of engine components under harsh conditions. The maximum stress of the crankshaft is simulated by bench test, and the fatigue experimental analysis of two kinds of the crankshaft is also carried out [22]. Feng et al. established the dynamic model of the V-shaped diesel engine crankshaft in ADAMS, calculated the crankshaft fatigue life under various working conditions, analyzed the life, and found the position of crankshaft stress concentration [23]. Zhang and Fenglan analyzed the fracture failure of the crankshaft of heavy-duty vehicles and expansively evaluated the crankshaft through material testing, mechanical property verification, and other procedures [24]. The above literature has optimized the crankshaft using many methods, such as lightweight design, topology optimization, and multiobjective optimization. The aim is to improve the performance of the crankshaft. However, the optimization goal is mainly aimed at the mass and maximum equivalent stress of the crankshaft without considering its NF and vibration characteristics.

This study takes the crankshaft of an automobile engine as the research object. The minor structures of the crankshaft, such as holes, chamfer, and thread lines, are appropriately simplified. The main goal of a static crankshaft analysis is to determine the mechanical characteristics of the crankshaft under load. To maintain the crankshaft's stable operation, a static analysis of the crankshaft under limited conditions is required to determine whether the crankshaft can maintain good mechanical characteristics under maximum load. The optimization technique is then established by using the design of expert software to get the crankshaft's regression equation and regression model. Finally, the crankshaft is recreated using the optimization approach.

The review arrangements are as follows: in Section 2, the finite element analysis of the crankshaft is carried out. The static analysis of the crankshaft is to identify its mechanical characteristics. The purpose of dynamic characteristic and harmonic response analysis of the crankshaft is to research its vibration characteristics. Section 3 is the optimization of the crankshaft structure. It is necessary to conduct the sensitivity analysis of the crankshaft parameters to determine the design variables. The response surface model and mathematical model are established by the RSM. The multiobjective optimization algorithm is used to solve mathematical model. Section 4 is the conclusions.
2. Finite Element Analysis of the Crankshaft

In this section, the mechanical analysis of the crankshaft is studied and the corresponding load parameters are obtained. Then the static analysis of the crankshaft is carried out to obtain its maximum equivalent stress and maximum deformation, which provides the basis for subsequent optimization. Finally, the modal analysis and harmonic response analysis of the crankshaft are conducted to study the dynamic performance of the crankshaft.

2.1. Modeling and Mechanical Analysis of the Crankshaft

The basic parameters of the crankshaft used in this study are shown in Table 1. Due to the complex structure of crankshaft, considering that some minor features have no practical significance for the actual analysis of crankshaft, it will also lead to grid singularity, long calculation time, and low calculation accuracy. Therefore, the minor features of the crankshaft, such as holes, chamfer, and thread lines, are appropriately simplified. The three-dimensional model of the crankshaft is constructed by SolidWorks software, as shown in Figure 1.

For any part, the selection of materials has a decisive impact on its performance. The crankshaft is a high-speed rotating part bearing periodic loads. It is easy to produce large inertial forces (IF). Therefore, it requires that the manufacturing materials of the crankshaft have relatively low density and high strength and stiffness. In this way, it can reduce the fatigue damage under periodic loads and reduce the rotational IF. Based on economy and usability, QT800-2 is used as the manufacturing material of the crankshaft in this study, and its main mechanical parameters are shown in Table 2.

The internal combustion engine with a four-cylinder ignition sequence of 1-2-4-3 is taken as the research goal. Combined with practice, the equivalent stress and deformation generated by the power stroke of the crankshaft are the largest, and the primary inducement of the deformation of the crankshaft is to bear the excessive bending load so that the torsional stress is ignored in the calculation. To simplify the analysis, it is set that when the piston is at the top dead center position in the cylinder, the maximum load is directly borne by the crankshaft pin. Therefore, when the piston of the third cylinder is at the end of the compression stroke, the stress of the crankshaft is analyzed [25]. The movement diagram of the crankshaft connecting rod assembly is shown in Figure 2.

The meanings of the symbols in Figure 2 are as follows: A is the connecting rod small end center. B is the connecting rod big end center. L is the center distance between the small end and the big end of the connecting rod. R is the crankshaft radius. S is the distance between the top dead center (TDC) and bottom dead center (BDC). α is the crankshaft angle. β is the connecting rod swing angle. ω is the angular speed of the crankshaft.

The meanings of the symbols in Figure 3 are as follows: m1 is assumed to be the concentrated mass point that moves back and forth in a straight line with the piston at the center of the small end of the connecting rod. m2 is assumed to be the concentrated mass point that rotates around the rotation.

\[
\beta = \sin^{-1}(\lambda \sin \alpha). \quad (1)
\]

In equation (1), \( \lambda \) is the crank connecting rod ratio.

The connecting rod angular velocity can be calculated by
\[
\omega_1 = \beta = \frac{\lambda \omega}{\sqrt{1 - \lambda^2 \sin^2 \alpha}}. \quad (2)
\]

The connecting rod angular acceleration can be calculated by
\[
\epsilon = \beta = -\omega_1^2 \lambda (1 - \lambda^2) \sin \alpha \cos \beta \quad (3)
\]

The acceleration of the piston can be calculated by
\[
a = R \omega_1^2 (\cos \alpha - \lambda \cos 2 \alpha). \quad (4)
\]

When the crankshaft rotates one turn, the maximum acceleration of the piston acceleration is 14767.4 m/s².

The equation of the combustion pressure of the combustible mixture borne by the piston is
\[
F_g = \frac{\pi D^2}{4} (P_g - P_0). \quad (5)
\]

Here, \( D \) is the piston diameter. \( P_g \) is the maximum engine burst pressure. \( P_0 \) is the absolute pressure in the crankcase; usually \( P_0 \) is 0.1 MPa.

When the engine is running, the piston connecting rod assembly moves in a high-speed straight line, which will produce a great IF. So the IF must be considered in the stress analysis of the crankshaft. In the crank and connecting rod mechanism, the plane compound motion of the connecting rod is composed of various movements, and the force analysis process will be very complex when calculating the IF. Therefore, this study uses the mass substitution method to simplify the treatment. That is, the mass of the connecting rod is concentrated at one point, and a part of the mass of the connecting rod is concentrated at the small end of the connecting rod and moves back and forth with the piston. The other part of the mass is concentrated at the big end of the connecting rod and rotates with the crankshaft. The mass substitution diagram of the simplified crankshaft connecting rod is shown in Figure 3.

| Parameter name | Parameter value (mm) |
|----------------|---------------------|
| Length of the crankshaft | 430.0 |
| Main journal diameter | 55.0 |
| Crankpin diameter | 48.0 |
| Length of the main journal | 25.0 |
| Length of the crankpin | 25.0 |
| Fillet radius of the crankpin | 3.50 |
| Fillet radius of the main journal | 4.00 |

Table 1: Main parameters of the crankshaft.
center of the crankshaft at the center of the big end of the connecting rod. \( L \) is the center distance between the small end and the big end of the connecting rod. \( I_1 \) is the distance from the centroid of the connecting rod to the center of the small end of the connecting rod. \( H \) is the centroid position of the connecting rod.

Thus, the reciprocating inertia force of the equivalent concentrated mass of the piston assembly and the small end of the connecting rod is obtained as

\[
F_j = (m_p + m_1)a. \tag{6}
\]

Here, \( m_p \) is equivalent concentrated mass of the piston assembly. When the piston moves to the TDC, it produces the maximum IF. The value is \( F_{j_{\text{max}}} = 37036.64 \) N.

The centrifugal inertia force generated by the rotation of mass \( m_2 \) around the crankshaft is shown as follows:

\[
F_r = m_2 R \omega^2. \tag{7}
\]

The value is \( F_r = 14567.54 \) N. Since the bending of the connecting rod is not considered in this review, when the piston moves to the TDC of the compression stroke, the connecting rod bears the maximum tensile load \( F_1 \). The equation of the maximum tensile force \( F_1 \) is calculated as follows:

\[
F_1 = F_{j_{\text{max}}} + F_r. \tag{8}
\]

The value is \( F_1 = 51604.18 \) N.

At the same time, the maximum compressive force \( F_2 \) can be calculated as follows:

\[
F_2 = (F_g - F_r) \cos \beta - F_r \cos (\alpha + \beta). \tag{9}
\]

The value is \( F_2 = 28715.01 \) N.
2.2. Static Analysis of the Crankshaft. The primary purpose of static analysis of the crankshaft is to identify the mechanical characteristics under mechanical load. To ensure the stable operation of the crankshaft, it is necessary to conduct static analysis of the crankshaft under the limit conditions to identify whether the crankshaft can ensure good mechanical properties under the maximum load. The equivalent stress cloud chart and deformation cloud chart under the limited load of the crankshaft are analyzed to determine the position of stress concentration and maximum deformation, and then the design is optimized. Static analysis is divided into three steps: preprocessing, solution, and postprocessing. The first two steps mainly include the establishment of 3D model, the definition of material properties, and the generation of finite element mesh model. The 3D model of crankshaft has been established. Then, the crankshaft model needs to have meshed. The mesh generation methods provided by ANSYS include automatic mesh generation, mapped mesh generation, face swept mesh generation, and hybrid mesh generation. Because the 3D model of crankshaft is complex, it may not be able to generate a complete hexahedral mesh [26]. Therefore, the tetrahedral free mesh generation method is selected to establish mesh model. This can not only ensure the accuracy of the analysis but also control the problem of the amount of calculation, so as to avoid the excessive amount of calculation, resulting in too long analysis time and waste of time and cost. The obtained crankshaft finite element model has 229,115 elements and 336,377 nodes, as shown in Figure 4.

The finite element analysis of the crankshaft usually includes setting boundary conditions, adding load, and solution sets. It is necessary to apply the load to the meshed model according to the actual working conditions.

In ANSYS Workbench software, the load types include pressure load, concentrated load, and bearing load. In the working procedure, the crankshaft bears the combined load of the connecting rod piston assembly. Therefore, the load borne by the crankshaft is applied to the crankshaft pin.

The ignition sequence of the four-cylinder engine used in this review is 1-2-4-3. Combined with practice, the equivalent stress and deformation of the crankshaft during the power stroke are the largest. The deformation of the crankshaft is mainly caused by the bending load so that the torsional stress is ignored in the calculation. To simplify the analysis, the crankshaft pin bears the maximum load when the piston is at the TDC. Therefore, when the piston of the third cylinder is at the end of the compression stroke, the stress of the crankshaft is analyzed. Follow the principle that upward is positive and downward is negative, and carry out the static analysis under the condition of ignition of the third cylinder.

Add a force on the crankpin, select the definition basis as the component, and create a new coordinate system with the midpoint of the crankpin as the coordinate origin. Then, this coordinate system is used to set the Y-axis component of each force to $-51604.18\text{N}$ (pressure condition) or $28715.01\text{N}$ (tension condition) according to the working condition of each cylinder to comprehensive the load application.

After the load is applied, it is necessary to restrict the displacement of the crankshaft to prevent the occurrence of rigid displacement in the solution process, resulting in the failure of the solution.

Because the crankshaft has adopted the thrust device, the main journal can only move axially. Therefore, the Y and Z degrees of freedom of the main journal is restricted, so it can only move with X degrees of freedom. The cylindrical support is used to restrict the main journal of the crankshaft. After the solution is completed, the results of equivalent stress and total deformation are obtained. The results are shown in Figures 5(a) and 5(b).

When the third cylinder is in the working condition, the stress is mainly concentrated at the transition fillet of the main journal and crankpins. Among them, the maximum equivalent stress of the transitional fillet of the crankshaft pin is as high as 76.63 MPa. There are diverse degrees of stress concentration in the transitional fillet of other main journals and crankpins, but the value is small. The stress concentration in other parts is not apparent. At the same time, it is also obvious from the figure that the journal of the third cylinder is more vulnerable to damage.

The deformation cloud diagram of the crankshaft model when the third cylinder is in the working condition is shown in Figure 5(b). It can be seen from the figure that the maximum deformation of the third cylinder in the operational state occurs on the crankpin, the deformation is 10.06 $\mu$m, and the deformation of the main journal is not apparent. The other three cylinder crankpins also have large deformation. The overall deformation is minor.

Based on the static analysis, the maximum equivalent stress and deformation of the crankshaft are relatively small. However, this analysis can only describe the mechanical characteristics of the crankshaft at rest or the stress condition of the crankshaft at a particular time during operation. For the crankshaft in real work, it is not only affected by mechanical stress but also the superposition of loads such as thermal stress caused by high-temperature gas leaked from the cylinder and the IF of high-speed rotation. Because the gas in the cylinder is periodically ignited, the crankshaft will inevitably be subjected to an alternating load. Therefore, even if the maximum equivalent stress is less than the strength limit, the fatigue failure may occur. The engine is a precision component, and the crankshaft rotates at high speed. A slight deformation of the crankshaft will lead to considerable centrifugal shift, resulting in significant vibration of the crankshaft. Thus, the matching relationship between the piston, connecting rod, and the crankshaft is destroyed, resulting in a damage to the engine. Therefore, it is necessary to decrease the maximum equivalent stress and deformation of the crankshaft to improve operational stability.

2.3. Modal Analysis of the Crankshaft. The periodic ignition of each engine cylinder will inevitably cause mechanical vibration. The crankshaft is a part with a certain mass, and vibration will also occur in the process of its rotation. Therefore, the vibration characteristics of the crankshaft are essential for the research of the crankshaft. Only by understanding the vibration characteristics of the crankshaft can we avoid the lousy vibration phenomenon in the crankshaft design. For the study of dynamic characteristics, modal analysis is the most basic and intuitive means.
The aim of modal analysis is to research the dynamic characteristics of the structure, such as the NF, damping, and vibration mode. They are important parameters in structural design under dynamic load, and they are also the basis of other dynamic analysis problems [27].

During the modal analysis of the crankshaft, constraints are added to the crankshaft in order to make the analysis results closer to the actual working conditions, so the cylindrical support is applied to the crankshaft main journal. At the same time, the default damping method is used to analyze and solve the first to sixth natural frequencies and modal vibrations of the crankshaft. The NF and modal vibration description is shown in Table 3, and the results of the modal analysis are shown in Figure 6.

The vibration of the crankshaft is mainly caused by the low-order frequency when the engine is working. Therefore, when studying the crankshaft vibration mode, only the low-order modal characteristics of the crankshaft need to be analyzed. It
Table 3: NF and modal vibration description.

| Order | NF (Hz) | Modal vibration description |
|-------|---------|----------------------------|
| 1     | 3941.6  | The balance weight of the first cylinder swings along the X-axis. |
| 2     | 3942.6  | The balance weight of the fourth cylinder swings along the X-axis. |
| 3     | 4012.0  | The balance weights of the second and third cylinders swing in the same direction along the X-axis. |
| 4     | 4036.3  | The balance weights of the second and third cylinders swing in the same direction along the X-axis. |
| 5     | 5744.9  | The balance weight of the fourth cylinder twists along the Y-axis. |
| 6     | 5745.4  | The balance weight of the first cylinder twists along the Y-axis. |

Figure 6: Continued.
can be known from Figure 6 that the maximum deformation of the crankshaft in the constrained state occurs in the second-order vibration. The crankshaft speed is 4500 revolutions per minute, and the fundamental frequency is 75 Hz, while the first NF of the crankshaft is 3941.60 Hz. The fundamental frequency is quite different from the first NF, so the crankshaft can avoid resonance when the engine is working. Therefore, the design of crankshaft can meet the initial dynamic requirements.

2.4. Harmonic Response Analysis of the Crankshaft. To further understand the dynamic characteristics of the crankshaft, the harmonic response analysis of the crankshaft is carried out based on the modal analysis. Harmonic response analysis is to determine the structural response of the model under the excitation of sinusoidal simple harmonic load with determined frequency. Therefore, the harmonic response of the crankshaft is analyzed to ensure that the crankshaft can overcome the harmful effects of forced vibration such as resonance and fatigue at different speeds. For harmonic response analysis, the frequency displacement response curve and frequency stress response curve are mainly derived.

The motion differential equation of forced vibration of the crankshaft under positive harmonic load is established as
\[ [M][\ddot{x}] + [C][\dot{x}] + [K][x] = [F]\sin(\theta t). \]  

(10)

The node displacement response is shown as follows:

\[ \{x\} = \{A\}\sin(\theta t + \phi). \]  

(11)

Here, \([M]\) is the mass matrix; \([C]\) is the damping matrix; \([K]\) is the stiffness matrix or coefficient matrix; \([x]\) is the displacement vector; \([\dot{x}]\) is the velocity vector; \([\ddot{x}]\) is the acceleration vector; \([F(t)]\) is the force vector; and \(\theta\) is the phase angle of displacement response under hysteretic excitation load.

Harmonic response analysis provides two common analysis methods in ANSYS Workbench: the modal superposition method and the complete method. The modal superposition method can balance the dual economy of calculation accuracy and calculation time. Furthermore, it is suitable for less precise solutions. The complete method can effectively improve the calculation accuracy, but it consumes too much time and memory. Therefore, the complete method is selected as the harmonic response analysis method. The load and displacement boundary conditions are set, and the load conditions and displacement constraints of the above static analysis are used. At the same time, it can be seen from Table 3 that the NF range of the crankshaft is 3941.6–5745.4 Hz. Since the excitation frequency range should be greater than the NF range of the crankshaft, it is necessary to expand the excitation frequency range and set it to 1000–8000 Hz. To obtain a better response curve, the solution interval is 400, which is solved based on the first to the sixth natural frequencies in modal analysis.

The displacement-frequency response curves of the crankshaft in the X, Y, and Z directions is obtained by harmonic response analysis, as shown in Figure 7.

![Figure 7: Displacement-frequency response curve. (a) X direction. (b) Y direction. (c) Z direction.](image-url)
3. Optimization of the Crankshaft

This section mainly studies the optimization of the crankshaft structure. First, the sensitivity analysis of the crankshaft parameters is carried out to determine the variables to be optimized. Then, the response surface model is analyzed by the RSM, and the design variables and objective functions of the crankshaft are established. Finally, the multiobjective optimization algorithm is used to solve mathematical model, and the optimized results are discussed.

3.1. Sensitivity Analysis of the Crankshaft. To optimize the crankshaft structure, it is necessary to determine the design variables, namely, the dimensional parameters, and carry out the sensitivity analysis to determine the dimensional parameters that have a significant impact on the output parameters. Second, based on the DOE, the RSM is used to fit the mathematical function of design variables and output parameters. The variance analysis of the fitted mathematical function is accomplished to determine that the fitted function can accurately reflect the relationship between design variables and output parameters. At the same time, the response surface analysis is carried out to clarify the interaction between the design variables and the output parameters. Finally, the fitted mathematical function is solved by the NSGA-II method, and the Pareto solution is obtained.

The purpose of finite element analysis of the crankshaft is to optimize its structure and make it more reasonable. The sensitivity analysis can pave the way for the optimal design of the crankshaft. Over the sensitivity analysis, the size parameters that are most sensitive to the target function in the optimization development are resolute [28]. Therefore, the key size parameters can be selected as the variable according to the sensitivity analysis consequences. The main journal diameter, fillet radius of the main journal, crankpin diameter, and fillet radius of crankpin are selected as the size parameters. The position diagram of each size parameters in the crankshaft is shown in Figure 8.

The description of the size parameters, symbol, initial value, and variation range of each size parameter are shown in Table 4.

In the mathematical sense, sensitivity reflects the change gradient of the objective function to the design variable [29]. In the numerical calculation, the initial dimension parameter is set as \( d \), and the eigenvector \( \{f\} \) is defined as an implicit function. When the parameter \( d \) changes slightly, the first-order Taylor expansion of the \( u \)-order eigenvector \( \{f^{(u)}\} \) is shown as follows:

\[
\{f^{(u)}(d + \Delta d)\} = \{f^{(u)}(d)\} + \sum_{i=1}^{n} \frac{\partial \{f^{(u)}\}}{\partial (d_i)} \Delta d_i. \tag{12}
\]

Then, equation (12) is rewritten as

\[
[S][\Delta d] = [\Delta f]. \tag{13}
\]

where \([\Delta f]\) is the residual vector as shown in the following equation:

\[
[\Delta f] = \{f^{(u)}(d + \Delta d)\} - \{f^{(u)}(d)\}. \tag{14}
\]

\([\Delta d] = [\Delta d_1, \Delta d_2, \ldots, \Delta d_n] \) is the modification of design parameters. \([S]\) is the sensitivity matrix, expressed as

Table 4: Size parameters setting.

| No. | Description of the size parameters | Symbol | Initial value (mm) | Variation range (mm) |
|-----|-----------------------------------|--------|-------------------|---------------------|
| 1   | Main journal diameter             | \( x_1 \) | 55.0              | 49.5~60.5           |
| 2   | Fillet radius of the crankpin      | \( x_2 \) | 3.50              | 3.15~3.85           |
| 3   | Crankpin diameter                 | \( x_3 \) | 48.0              | 43.2~52.8           |
| 4   | Fillet radius of the main journal  | \( x_4 \) | 4.00              | 3.6~4.4             |

![Figure 8: Diagram of initial size parameters.](image)
Figure 9: Results of sensitivity analysis.

\[
[S] = \begin{bmatrix}
\frac{\partial f_1}{\partial d_1} & \frac{\partial f_1}{\partial d_2} & \cdots & \frac{\partial f_1}{\partial d_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_m}{\partial d_1} & \frac{\partial f_m}{\partial d_2} & \cdots & \frac{\partial f_m}{\partial d_n}
\end{bmatrix}
\]

where \( n \) is the number of design parameters to be modified and \( m \) is the eigenvector number. \( f \) is the objective function, such as \( NF \) and equivalent stress.

The sensitivity analysis results are shown in Figure 9. The parameters \( x_1 \) and \( x_3 \) have a significant impact on the mass and the deformation of the crankshaft. The sensitivity values of \( x_1 \) and \( x_3 \) are negatively correlated with the deformation of the crankshaft, that is, the increase of the two parameters will reduce the deformation of the crankshaft. The sensitivity value of \( x_2 \) and \( x_4 \) is positively correlated with the deformation of the crankshaft, that is, the increase of the two parameters will increase the deformation of the crankshaft. The sensitivity value of \( x_3 \) is negatively correlated with the equivalent stress, and it is the most sensitive parameter to the equivalent stress among the four parameters. In conclusion, the optimization of the crankshaft is mainly realized by changing the main journal diameter \( x_1 \) and crank pin diameter \( x_3 \).

3.2. Design of Experiment and Response Surface Analysis.

In the field of design, the experiment involves real physical experiments and computational simulation experiments. The DOE is a technique based on probability theory and mathematical statistics to define the design variable samples with less input to obtain as many output responses as possible. Using this method can decrease the workload of the experiment and save time. The normally used DOE methods include orthogonal experiment design, central composite design, Box–Behnken method, Latin hypercube sampling design, and Taguchi method.

In this review, the Latin hypercube sampling design method is used for the design of the experiment. It is a small sampling method, and the number of experiments can be flexibly selected. The numbers of experiments required are less than that of the central composite design. It can evaluate the nonlinear influence of design variables and is applicable to the code test when all design variables are measured values.

The scheme of each experiment is determined according to the Latin hypercube sampling design, and then the experiment is carried out to obtain the experiment results. The sample points are 25. The experiment process is to use the finite element method to establish the model of 25 groups of sample points, calculate each group of sample points by finite element method, and obtain the experiment results. The sample points and results of the design experiment are shown in Table 5.

Based on the results of the experiment design, the RSM is used to fit the mathematical equation. The RSM replaces the implicit or actual function that needs to spend a lot of time to determine with an easy to handle function or surface. It is a proxy model method based on interpolation or approximation [30]. After the DOE is completed, the functional relationship between the input parameters and output parameters can be attained by the RSM. This functional relationship simplifies the complex input-output response, such as complex finite element analysis and calculation, into a simple mathematical equation significantly decreasing the calculation time.

The expression of the response surface model is as follows:

\[
y = \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_i x_j
\]

where \( n \) is the number of design variables, \( x_i \) and \( x_j \) is the design variable, \( y \) is the dependent variable, and \( \alpha_0, \alpha_i, \alpha_{ij} \) is the response surface factor. The response surface coefficient vector is estimated by the least square method as

\[
\alpha = (\Phi^T \Phi)^{-1} (\Phi^T y).
\]

The response surface sample vector is shown as follows:

\[
\Phi = \begin{bmatrix}
1 & x_1 & 1 & x_1 & 2 & \cdots & x_1 & N \\
1 & x_2 & 1 & x_2 & 2 & \cdots & x_2 & N \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
1 & x_m & 1 & x_m & 2 & \cdots & x_m & N
\end{bmatrix}
\]

Here \( N \) is the number of response surface model basis functions.

The results in Table 5 are imported into the design expert software to select the quadratic polynomial regression analysis method, take the main journal diameter \( (x_1) \), crankpin fillet \( (x_2) \), crankpin diameter \( (x_3) \), and main journal fillet \( (x_4) \) as the design variable and the mass \( (y_1) \), equivalent stress \( (y_2) \), and deformation \( (y_3) \) as the objective function, respectively. The regression equation between the design variable and the objective function is obtained.
| Run | Main journal diameter (mm) | Fillet radius of the crankpin (mm) | Crankpin diameter (mm) | Fillet radius of the main journal (mm) | Mass (kg) | Maximum equivalent stress (MPa) | Maximum deformation (μm) |
|-----|---------------------------|-----------------------------------|------------------------|---------------------------------------|-----------|-------------------------------|--------------------------|
| 1   | 55.00                     | 3.44                              | 43.78                  | 3.97                                  | 12.04     | 88.29                         | 11.07                    |
| 2   | 56.76                     | 3.64                              | 49.54                  | 3.94                                  | 12.49     | 73.10                         | 9.60                     |
| 3   | 49.72                     | 3.50                              | 51.07                  | 3.74                                  | 12.04     | 70.30                         | 9.96                     |
| 4   | 55.88                     | 3.81                              | 48.38                  | 3.90                                  | 12.36     | 75.09                         | 9.93                     |
| 5   | 50.60                     | 3.28                              | 45.70                  | 4.22                                  | 11.81     | 82.95                         | 11.03                    |
| 6   | 59.84                     | 3.58                              | 45.31                  | 4.29                                  | 12.53     | 83.34                         | 10.35                    |
| 7   | 55.44                     | 3.67                              | 49.92                  | 4.26                                  | 12.42     | 73.36                         | 9.70                     |
| 8   | 59.40                     | 3.33                              | 48.77                  | 4.00                                  | 12.67     | 75.69                         | 9.51                     |
| 9   | 52.80                     | 3.70                              | 46.08                  | 4.16                                  | 11.99     | 81.62                         | 10.76                    |
| 10  | 51.92                     | 3.30                              | 52.61                  | 4.38                                  | 12.30     | 68.54                         | 9.55                     |
| 11  | 53.68                     | 3.22                              | 43.39                  | 3.65                                  | 11.91     | 88.88                         | 11.21                    |
| 12  | 51.48                     | 3.78                              | 47.62                  | 4.03                                  | 11.98     | 76.75                         | 10.54                    |
| 13  | 54.56                     | 3.75                              | 46.46                  | 3.84                                  | 12.15     | 81.46                         | 10.45                    |
| 14  | 57.20                     | 3.42                              | 51.46                  | 3.68                                  | 12.63     | 69.84                         | 9.14                     |
| 15  | 53.24                     | 3.84                              | 44.93                  | 4.32                                  | 11.97     | 83.09                         | 11.06                    |
| 16  | 58.52                     | 3.39                              | 50.30                  | 3.87                                  | 12.68     | 72.44                         | 9.27                     |
| 17  | 58.08                     | 3.36                              | 47.23                  | 4.35                                  | 12.48     | 79.34                         | 10.00                    |
| 18  | 56.32                     | 3.53                              | 51.84                  | 3.71                                  | 12.59     | 69.84                         | 9.17                     |
| 19  | 51.04                     | 3.19                              | 46.85                  | 3.78                                  | 11.89     | 80.69                         | 10.61                    |
| 20  | 50.16                     | 3.61                              | 52.22                  | 3.81                                  | 12.14     | 69.16                         | 9.75                     |
| 21  | 54.12                     | 3.56                              | 49.15                  | 4.19                                  | 12.26     | 74.27                         | 9.93                     |
| 22  | 58.96                     | 3.72                              | 44.16                  | 3.62                                  | 12.38     | 86.16                         | 10.65                    |
| 23  | 60.28                     | 3.47                              | 50.69                  | 4.06                                  | 12.86     | 71.98                         | 9.10                     |
| 24  | 52.36                     | 3.25                              | 44.54                  | 4.13                                  | 11.87     | 85.08                         | 11.11                    |
| 25  | 57.64                     | 3.16                              | 48.00                  | 4.10                                  | 12.48     | 77.82                         | 9.79                     |

\[
y_1 = 8.77 - 6.56 \times 10^{-4} x_1 - 0.01 x_2 - 3.88 \times 10^{-4} x_3 - 0.02 x_4 + 1.06 \times 10^{-6} x_1 x_2 - 5.12 \times 10^{-7} x_1 x_3 + 3.51 \times 10^{-4} x_1 x_4 + 2.79 \times 10^{-4} x_2 x_3 - 3.50 \times 10^{-5} x_2 x_4 - 4.33 \times 10^{-6} x_3 x_4 + 7.17 \times 10^{-4} x_1^2 + 1.98 \times 10^{-3} x_2^2 + 5.73 \times 10^{-4} x_3^2 + 2.74 \times 10^{-3} x_4^2,
\]
\[
y_2 = 314.07 - 1.56 x_1 + 70.52 x_2 - 9.66 x_3 - 13.59 x_4 + 0.04 x_1 x_2 + 0.02 x_1 x_3 + 0.53 x_1 x_4 - 0.51 x_2 x_3 - 4.54 x_2 x_4 + 0.23 x_3 x_4 - 0.02 x_1^2 - 4.60 x_2^2 + 0.07 x_3^2 - 1.37 x_4^2,
\]
\[
y_3 = 44.41 - 0.28 x_1 + 0.04 x_2 - 0.84 x_3 + 0.28 x_4 - 1.79 \times 10^{-3} x_1 x_2 - 5.87 \times 10^{-4} x_1 x_3 - 7.17 \times 10^{-3} x_1 x_4 + 2.42 \times 10^{-3} x_2 x_3 + 2.06 \times 10^{-2} x_2 x_4 + 2.69 \times 10^{-4} x_3 x_4 + 2.31 \times 10^{-3} x_1^2 + 4.19 \times 10^{-3} x_2^2 + 6.75 \times 10^{-3} x_3^2 + 2.35 \times 10^{-2} x_4^2.
\]

The response surface fitting effect diagram is established to judge the accuracy of the approximate model. Under the same conditions, the predicted value obtained by the approximate model is set as the ordinate, and the actual value is set as the abscissa. The certainty coefficient $R^2$ and root mean square error (RMSE) can also reflect the overall accuracy of the approximate model, and its calculation formula is as follows:

\[
R^2 = \frac{\sum_{i=1}^{n} (Y_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2},
\]

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y}_i)^2},
\]

where $n$ is the number of experiments, $y_i$ is the value calculated by the finite element method, such as the deformation and equivalent stress. $Y_i$ is the predicted value calculated by the response surface mathematical model. $\bar{y}_i$ is the mean value of calculated by the response surface mathematical model.

According to the above analysis, the fitting effect of the RSM approximate model is obtained, as shown in Figure 10. The analysis shows that the predicted value is consistent with the actual value. The fitting effect of the mass, the equivalent stress, and the deformation in the approximate model of the RSM is good. The accuracy is high, and the certainty coefficient is greater than 0.9. It can replace the finite element simulation for subsequent optimization design.

Based on the analysis of variance and the regression equation, the response surface is established, as shown in Figures 11–13. Considering four design variables, only the response surface between some design variables and the objective function is given.
It can be seen from Figure 11 that with the increase of the crankpin diameter, the crankshaft mass increases slowly. With the increase of the main journal diameter, the crankshaft mass increases significantly, so the whole response surface is steep. By comparing the two design variables, the crankshaft mass is mainly affected by the main journal diameter. In the subsequent optimization of the crankshaft, the main journal diameter should be appropriately controlled to avoid an increase in the mass.

It can be seen from Figure 11 that with the increase of the crankpin diameter, the crankshaft mass increases slowly. With the increase of the main journal diameter, the crankshaft mass increases significantly, so the whole response surface is steep. By comparing the two design variables, the crankshaft mass is mainly affected by the main journal diameter. In the subsequent optimization of the crankshaft, the main journal diameter should be appropriately controlled to avoid an increase in the mass.

It can be seen from Figure 12 that the main journal diameter and the fillet radius of the crankpin have a significant impact on the maximum equivalent stress. The fillet radius of the crankpin has the most significant impact, and its interaction with the main journal diameter is also significant. With the increase of the main journal diameter, the maximum equivalent stress increases slowly, and then decreases rapidly. So the whole shape is a parabola. With the increase of the fillet radius of the crankpin, the maximum equivalent stress of the crankshaft gradually increases, and then decreases slowly. Therefore, the interaction effect of the
two design variables is noticeable and presents a quadratic convex surface, which further proves the rationality of the quadratic regression equation. Within the value range of the selected design variable, the interaction should be considered comprehensively.

It can be seen from Figure 13 that the crankshaft deformation decreases dramatically with the crankpin diameter increase. At the same time, as the main journal diameter increases, the deformation reduces, resulting in a steep response surface. By comparing the two design variables, the change in the crankpin diameter has the great impact on the crankshaft deformation. The crankpin diameter can be appropriately adjusted in the crankshaft optimization to reduce deformation.

To sum up, the mechanical properties of the crankshaft studied in this review are mainly determined by the main journal diameter and the crankpin diameter, and other design variables have little influence on it. The results of response surface analysis show that the main journal diameter and crankpin diameter have a significant impact on the output parameters, indicating that the results of response surface analysis are consistent with the sensitivity analysis. In the practice of crankshaft structure design, it is necessary to improve the mechanical properties of the crankshaft. The first choice is to reasonably design the main journal diameter and crankpin diameter.

3.3. Results of Optimization and Discussion. The Multi-objective Genetic Algorithm (MOGA) is an adaptive probabilistic search algorithm that simulates the process of natural evolution. It has inherent parallelism and better global optimization ability. It can automatically obtain and guide the optimized search space and adjust the search direction without determining the rules. The Nondominated Sorted Genetic Algorithm-II (NSGA-II) is an improved genetic algorithm of NSGA. It puts forward the comparison operator of crowding degree and crowding degree and introduces the elite strategy to improve the race level. While ensuring no significant increase in the crankshaft mass, reduce the maximum equivalent stress and maximum deformation. According to the response surface model of the objective function, a multiobjective optimization model and model constraints are established as

\[
\begin{align*}
\text{Find} & \quad X = [x_1, x_2, x_3, x_4]^T, \\
\text{min} & \quad \begin{bmatrix} y_1, y_2, y_3 \end{bmatrix}, \\
\text{Subject to,} & \quad x_i \leq x_i \leq X_i (i = 1, 2, 3, 4). 
\end{align*}
\]

\[x_i, X = [x_1, x_2, x_3, x_4]^T\] is the design variable as shown in Table 4. \(\min \begin{bmatrix} y_1, y_2, y_3 \end{bmatrix}\) represents to find the minimum value of the three objective functions. The importance of objective function is higher, lower, and lower, respectively. \(x_i \leq x_i \leq X_i\) represents the range of design variables.

In order to improve the mechanical properties of the crankshaft without increasing the crankshaft mass, equation (22) is optimized. The optimized main journal diameter is 49.65 mm, the fillet radius of the crankpin is 32.3 mm, the crankpin diameter is 52.54 mm, and the fillet radius of the main journal is 3.70 mm. The three-dimensional model of the crankshaft is established again according to the optimized size, and then the static analysis and dynamic analysis of the reconstructed model are carried out to obtain the optimization results in Table 6.

It can be seen from Table 6 that the equivalent stress, deformation, and mass of the crankshaft are optimized, in which the maximum equivalent stress is reduced by 9.43%, the maximum deformation is reduced by 3.68%, and the mass is reduced by 1.30%. At the same time, the first three natural frequencies are reduced by 10.28, 10.28, and 10.34%. After optimization, the stiffness and strength of the crankshaft are improved. The purpose of optimal design is achieved.

4. Conclusion

Due to the complexity of the crankshaft motion analysis, this research divides the finite element analysis of the crankshaft into two working conditions: compression and tension. The stress distribution and the riskiest area of the crankshaft are found through static examination. The NF range of the crankshaft is determined through modal analysis, and then the frequency that should be avoided is found through harmonic response analysis, which ensures the service life of the crankshaft. Based on the DOE, the sensitivity analysis is used to determine the design variables. The crankshaft structure is optimized by ANSYS Workbench according to the Latin hypercube sampling design response surface design method, and the optimized parameters and optimal candidate points of the crankshaft are obtained. Finally, according to the optimization scheme, the crankshaft is reconstructed. The maximum equivalent stress of the optimized crankshaft model is reduced by 9.43%, the maximum deformation is reduced by 3.68%, and the mass is reduced by 1.30%. At the same time, the first, the second, and the third natural frequencies are reduced by 10.28, 10.28, and 10.34%, respectively.

| Table 6: Comparison before and after optimization. |
|-----------------------------------|
| Before optimization | After optimization | Decrease (%) |
| Mass (kg) | Equivalent stress (MPa) | Deformation (μm) | First order NF (Hz) | Second order NF (Hz) | Third order NF (Hz) |
| 12.27 | 76.63 | 10.06 | 3941.6 | 3942.6 | 4012.0 |
| 12.11 | 69.40 | 9.69 | 3536.6 | 3537.4 | 3597.2 |
| 1.30 | 9.43 | 3.68 | 10.28 | 10.28 | 10.34 |
Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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