Kinetic energy driven superconductivity in the electron doped cobaltate 
\( \text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O} \)

Bin Liu, Ying Liang, and Shiping Feng
Department of Physics, Beijing Normal University, Beijing 100875, China
Wei Yeu Chen
Department of Physics, Tamkang University, Tamsui 25137, Taiwan

Within the charge-spin separation fermion-spin theory, we have shown that the mechanism of superconductivity in the electron doped cobaltate \( \text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O} \) is ascribed to its kinetic energy. The dressed fermions interact occurring directly through the kinetic energy by exchanging magnetic excitations. This interaction leads to a net attractive force between dressed fermions, then the electron Cooper pairs originating from the dressed fermion pairing state are due to the charge-spin recombination, and their condensation reveals the superconducting ground state. The superconducting transition temperature is identical to the dressed fermion pair transition temperature, and is suppressed to a lower temperature due to the strong magnetic frustration. The optimal superconducting transition temperature occurs in the electron doping concentration \( \delta \approx 0.29 \), and then decreases for both underdoped and overdoped regimes, in qualitative agreement with the experimental results.

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The discovery of superconductivity in the doped cobaltate \( \text{Na}_{0.35}\text{CoO}_2 \cdot 1.3\text{H}_2\text{O} \) has engendered great interest in transition metal oxides\(^1\). Although the superconducting (SC) transition temperature is much less than that in the doped cuprate superconductors, both systems share many common features. The doped cobaltate has a lamellar structure consisting of the two-dimensional (2D) \( \text{CoO}_2 \) layers separated by a thick insulating layer of \( \text{Na}^+ \) ions, where the one-half spin \( \text{Co}^{4+} \) ions are arranged on a triangular lattice, and it becomes a superconductor after hydration that significantly enhances the distance between \( \text{CoO}_2 \) layers\(^1\). This structure is similar to the cuprate superconductors in the sense that they also have a layered structure of the square lattice of the \( \text{CuO}_2 \) plane separated by insulating layers\(^2\). The undoped cuprate is a Mott insulator with the antiferromagnetic (AF) long-range-order (AFLRO), then changing the carrier concentration by ionic substitution or increasing the oxygen content turns these compounds into the SC state leaving the AF short-range correlation still intact\(^2\). The optimal doping occurs at 0.15 holes (electrons) added to the half-filled band of the parent compound\(^2\). However, \( \text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O} \) is viewed as an electron doped Mott insulator, where superconductivity appears with electron doping\(^7\). The optimal doping for superconductivity occurs at 0.3 electrons per \( \text{Co} \) above the ground-state \( \text{Na}_{0.3}\text{CoO}_2 \cdot 1.3\text{H}_2\text{O} \), which is a half-filled two-electron \( t_{2g} \) derived band\(^4\). Although the ferromagnetic correlation is present in \( \text{Na}_x\text{CoO}_2 \) for the large electron doping concentration \( (x \approx 0.7) \)\(^6\), the AF short-range spin correlation in \( \text{Na}_x\text{CoO}_2 \) and \( \text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O} \) in the low electron doping concentration \( (\delta \approx 0.35) \) has been observed from nuclear quadrupolar resonance and thermopower as well as other experimental measurements\(^1\). Moreover, the SC transition temperature displays the same kind of doping controlled behavior that is observed in the doped cuprates, i.e., the optimal SC transition temperature occurs in a narrow range of the electron doping concentration, and decreases for both underdoped and overdoped regimes\(^4\), in analogy to the phase diagram of the cuprate superconductors\(^9,10\). Therefore the superconductivity developed with the background of the AF spin correlation in the doped cobaltate suggests the importance of the strong electron correlation as in the doped cuprates\(^2\).

In the conventional superconductors, the electrons interact by exchanging phonons. Since this interaction leads to a net attractive force between electrons, then the system can lower its potential energy by forming electron Cooper pairs\(^11\). These electron Cooper pairs condense into a coherent macroscopic quantum state, then can move freely without resistance. As a result, pairing in the conventional superconductors is always related with an increase in kinetic energy which is overcompensated by the lowering of potential energy\(^11\). On the contrary, it has been argued that the SC transition in the doped cuprates is determined by the need to reduce the frustrated kinetic energy\(^12,13\), where the driving attractive force between holes may be mediated by the exchange of spin excitations\(^14\). Since there is a remarkable resemblance in the normal- and SC-state properties between \( \text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O} \) and hole doped cuprates\(^1\), as mentioned above, and the strong electron correlation is common for both these materials, then two systems may have similar underlying SC mechanism, i.e., it is possible that superconductivity in the electron doped cobaltates is also driven by the kinetic energy as in the doped cuprates. Within the charge-spin separation fermion-spin theory\(^15\), the mechanism of superconductivity in the hole doped cuprates has been discussed\(^16\), where dressed holons interact occurring directly through the kinetic energy by exchanging the dressed spinon excitations, leading to a
net attractive force between the dressed holons, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground-state. In this case, the electron SC transition temperature is determined by the dressed holon pair transition temperature, and is a nonmonotonic doping dependence, i.e., the SC transition temperature is maximal for a particular carrier concentration and decreases both for underdoped and overdoped regimes\textsuperscript{16}, in agreement with experiments\textsuperscript{9,10}. In this paper, we study the mechanism of superconductivity in the doped cobaltates along with this line. We find that the SC transition temperature in the electron doped cobaltates is suppressed to a lower temperature due to the strong magnetic frustration.

In the electron doped cobaltates, the characteristic feature is the presence of the 2D CoO\textsubscript{2} plane\textsuperscript{1}, then the unusual behaviors are dominated by this plane. It has been argued that the essential physics of the doped CoO\textsubscript{2} plane is contained in the t-J model on a triangular lattice\textsuperscript{17},

\[
H = t \sum_{i\eta\sigma} PC_{i\sigma} C_{i+\eta\sigma} P^\dagger - \mu \sum_{i\sigma} PC_{i\sigma} C_{i\sigma} P^\dagger + J \sum_{i\eta} S_i \cdot S_{i+\eta},
\]  

(1)

where \( t < 0 \), the summation is over all sites \( i \), and for each \( i \), over its nearest-neighbor \( \eta \), \( C_{i\sigma}^\dagger (C_{i\sigma}) \) is the electron creation (annihilation) operator, \( S_i = C_{i\uparrow}^\dagger \sigma C_{i\downarrow}/2 \) is the spin operator with \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) as the Pauli matrices, \( \mu \) is the chemical potential, and the projection operator \( P \) removes zero occupancy, i.e., \( \sum_{\sigma} C_{i\sigma}^\dagger C_{i\sigma} \geq 1 \). The strong electron correlation manifests itself by the local constraint\textsuperscript{18}. For a proper treatment of the single occupancy local constraint in the doped cuprates in analytical calculations, the charge-spin separation fermion-spin theory has been developed to incorporate the constraint\textsuperscript{15}.

To apply this theory in the electron doped cobaltates, the t-J model (1) can be rewritten in terms of a particle-hole transformation \( C_{i\sigma} \rightarrow f_{i\uparrow-\downarrow}^\dagger \) as,

\[
H = -t \sum_{i\eta\sigma} f_{i\uparrow+\eta\sigma}^\dagger f_{i\downarrow+\eta\sigma} + \mu \sum_{i\sigma} f_{i\sigma}^\dagger f_{i\sigma} + J \sum_{i\eta} S_i \cdot S_{i+\eta},
\]  

(2)

supplemented by the local constraint \( \sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} \leq 1 \) to remove double occupancy, where \( f_{i\sigma}^\dagger (f_{i\sigma}) \) is the hole creation (annihilation) operator, and \( S_i = f_{i\uparrow}^\dagger f_{i\downarrow}/2 \) is the spin operator in the hole representation. Then the hole operators can be expressed as, \( f_{i\uparrow} = a_{i\uparrow}^\dagger S_i^- \) and \( f_{i\downarrow} = a_{i\downarrow}^\dagger S_i^+ \), in the charge-spin separation fermion-spin theory\textsuperscript{15}, where the spinful fermion operator \( a_{i\sigma} = e^{-i\Phi_{i\sigma}} a_i \) describes the charge degree of freedom together with some effects of the spin configuration rearrangements due to the presence of the doped electron itself (dressed fermion), while the spin operator \( S_i \) describes the spin degree of freedom (dressed spinon), then the single occupancy local constraint, \( \sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} = S_i^+ a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger S_i^- + S_i^- a_{i\uparrow} a_{i\downarrow}^\dagger + S_i^+ a_{i\downarrow} a_{i\uparrow}^\dagger = a_i^\dagger (S_i^+ S_i^- + S_i^- S_i^+) = 1 - a_i^\dagger a_i \leq 1 \), is satisfied in analytical calculations, and the double dressed fermion occupancy, \( a_{i\sigma}^\dagger a_{i-\sigma}^\dagger = e^{i\Phi_{i\sigma}} a_i^\dagger e^{i\Phi_{i-\sigma}} = 0 \) and \( a_{i\sigma} a_{i-\sigma} = e^{-i\Phi_{i\sigma}} a_i e^{-i\Phi_{i-\sigma}} = 0 \), are ruled out automatically. These dressed fermion and spinon have been shown to be gauge invariant, and in this sense, they are real and can be interpreted as the physical excitations\textsuperscript{15,19}. It has been emphasized that this dressed fermion \( a_{i\sigma} \) is a spinless fermion \( a_i \) incorporated a spin cloud \( e^{-i\Phi_{i\sigma}} \) (magnetic flux), and is a magnetic dressing. In other words, the gauge invariant dressed fermion carries some spin messages, i.e., it shares its nontrivial spinon environment\textsuperscript{20}. Although in common sense \( a_{i\sigma} \) is not a real spinful fermion, it behaves like a spinful fermion. In this charge-spin separation fermion-spin representation, the low-energy behavior of the t-J model (2) can be expressed as\textsuperscript{15},

\[
H = -t \sum_{i\eta\sigma} (a_{i\uparrow} S_i^- a_{i+\eta\uparrow}^\dagger S_{i+\eta}^- + a_{i\downarrow} S_i^+ a_{i+\eta\downarrow}^\dagger S_{i+\eta}^+) - \mu \sum_{i\sigma} a_{i\sigma}^\dagger a_{i\sigma} + J_{\text{eff}} \sum_{i\eta} S_i \cdot S_{i+\eta},
\]  

(3)

with \( J_{\text{eff}} = (1 - \delta)^2 J \), and \( \delta = \langle a_{i\sigma} a_{i\sigma} \rangle = \langle a_i^\dagger a_i \rangle \) is the electron doping concentration. In this case, the magnetic energy \( (J) \) term in the t-J model is only to form an adequate dressed spinon configuration\textsuperscript{12}, while the kinetic energy \( (t) \) term has been transferred as the dressed fermion-spinon interaction, which dominates the essential physics. This dressed fermion-spinon interaction is quite strong, and can induce the doped fermion pairing state (then the electron pairing state and superconductivity) by exchanging dressed spinon excitations in a higher power of the electron doping concentration \( \delta \) as in the doped cuprates\textsuperscript{16}. As in the conventional superconductors, the SC state in the doped cobaltates also is characterized by electron Cooper pairs, forming SC quasiparticles, then the order parameter for the electron Cooper pair can be expressed as,

\[
\Delta = \langle f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{i\uparrow}^\dagger \rangle = \langle a_{i\uparrow} a_{i\downarrow} S_i^+ S_i^- - a_{i\downarrow} a_{i\uparrow} S_i^- S_i^+ \rangle.
\]  

(4)

At the zero doping, the t-J model is reduced to the Heisenberg model. Many authors\textsuperscript{21} have shown unambiguously that as in a square lattice, there is indeed AFLRO in the ground state of the AF Heisenberg model on a triangular lattice. However, this AFLRO is destroyed more rapidly with increasing doping than that on a square lattice due to the strong geometry frustration. Thus there is no AFLRO away from the zero doping, i.e., \( \langle S_i^z \rangle = 0 \), where the dressed spinons form the disordered spin liquid state, then the dressed spinon correlation function \( \langle S_i^+ S_j^- \rangle = \langle S_i^+ S_j^+ \rangle \), and the order parameter for the electron Cooper pair in Eq. (4) can be written as \( \Delta_a = \langle S_i^+ S_j^- \rangle \Delta_a \), with the dressed fermion pairing order parameter \( \Delta_a = \langle a_{i\uparrow} a_{i\downarrow} - a_{i\downarrow} a_{i\uparrow} \rangle \).
This shows that the dressed fermion pairs move freely in the background of the disordered spin liquid state, and then the physical properties of the SC state are essentially determined by the dressed fermion pairing state. In this case, the mechanism of superconductivity in the doped cuprates has been discussed based on the Eliashberg’s strong coupling theory. Following the discussions in Ref.16, we obtain the self-consistent equations in terms of the equation of motion method that is satisfied by the full dressed fermion diagonal and off-diagonal Green’s functions \( g(i-j, t - t') = \langle \langle a_{i\sigma}(t); a_{j\sigma}'(t') \rangle \rangle \) and \( \Im \langle i-j, t - t' \rangle = \langle \langle a_{i\sigma}'(t); a_{j\sigma}(t') \rangle \rangle \) as

\[
g(i\omega_n, k) = g^{(0)}(i\omega_n, k) + g^{(0)}(i\omega_n, k)\Sigma_1^{(a)}(i\omega_n, k)
\times g(i\omega_n, k) - \Sigma_2^{(a)}(-i\omega_n, -k)\Im \Sigma_1^{(a)}(i\omega_n, k), \tag{5a}
\]

\[
\Im \langle i-j, t - t' \rangle = \langle \langle a_{i\sigma}'(t); a_{j\sigma}(t') \rangle \rangle , \tag{5b}
\]

respectively, where the mean-field (MF) dressed fermion Green’s function \( g^{(0)}(i\omega_n, k) = i\omega_n - \xi_k \) with the MF dressed fermion excitation spectrum \( \xi_k = Zt\gamma_k - \mu \), \( Z \) is the number of the nearest neighbor sites, \( \gamma_k = (1/Z) \sum_{\eta} e^{i\eta \cdot \vec{k}} = [\cos(k_x + 2\cos(k_y/2)\cos(\sqrt{3}k_y)/2)]/3 \), the dressed spinon correlation function \( \chi = \langle S^+_i S^-_{i+\hat{y}} \rangle \), while the self-energies have been obtained as,

\[
\Sigma_1^{(a)}(i\omega_n, k) = (Zt)^2 \frac{1}{N^2} \sum_{p,p',\pm} \gamma^2_{p+\pm\hat{k}} \sum_{ip_m} g(ip_m + i\omega_n, p + k)
\times \frac{1}{\beta} \sum_{ip_m'} D^{(0)}(ip_m', p') \times D^{(0)}(ip_m' + ip_m, p' + p), \tag{6a}
\]

\[
\Sigma_2^{(a)}(i\omega_n, k) = (Zt)^2 \frac{1}{N^2} \sum_{p,p',\pm} \gamma^2_{p+\pm\hat{k}} \sum_{ip_m} \Im \langle -ip_m' - i\omega_n, -p - k \rangle
\times \frac{1}{\beta} \sum_{ip_m'} D^{(0)}(ip_m', p') \times D^{(0)}(ip_m' + ip_m, p' + p), \tag{6b}
\]

where the MF dressed spinon Green’s function \( D^{(0)}(i-j, t - t') = \langle \langle S^+_i(t); S^-_{j}(t') \rangle \rangle \), and has been obtained as \( D^{(0)}(ip_m, p) = [(ip_m^2 - \omega^2_p)/B_p] \), with \( B_p = \lambda[2\chi^2(\epsilon_p - 1) + \chi(\gamma_p - \epsilon)] \), the MF dressed fermion excitation spectrum \( \omega^2_p = A_1 \gamma^2_p + A_2 \gamma_p + A_3, A_1 = \alpha\lambda(\chi^2 + \chi^2/2), A_2 = -\alpha\lambda(\chi^2 + \chi^2/2) + (\alpha C^2 + (1 - \alpha)/(4Z) - \alpha \chi^2/(2Z) + (\alpha C + (1 - \alpha)/(2Z) - \alpha \chi^2/2), A_3 = \lambda^2(\alpha C^2 + (1 - \alpha)/(4Z) - \alpha \chi^2/(2Z) + (\alpha C + (1 - \alpha)/(2Z) - \alpha \chi^2/2), \lambda = 2ZJ_{eff}, \epsilon = 1 + 2t/\lambda, \phi \rangle \), the dressed fermion’s particle-hole parameters \( \phi = \langle \langle a_{i\sigma}^\dagger_{\pm\hat{k}} a_{i\sigma} \rangle \rangle \), and the dressed spinon correlation functions \( \chi^2 = \langle S^+_i S^-_{i+\hat{y}} \rangle, C = (1/Z^2) \sum_{\eta, \eta'} \langle S^+_i S^-_{i+\hat{y}} S^+_i S^-_{i+\hat{y}} \rangle, \) and \( C^2 = (1/Z^2) \sum_{\eta, \eta'} \langle S^+_i S^-_{i+\hat{y}} S^+_i S^-_{i+\hat{y}} \rangle. \) In order not to violate the sum rule of the correlation function \( \langle S^+_i S^-_{i} \rangle = 1/2 \) in the case without AFLRO, the important decoupling parameter \( \alpha \) has been introduced in the MF calculation, which can be regarded as the vertex correction.

Since the pairing force and dressed fermion gap function have been incorporated into the self-energy \( \Sigma_1^{(a)}(k) \), then it is an effective dressed fermion gap function. In particular, this effective dressed fermion gap function \( \Sigma_2^{(a)}(k) \) is an even function of \( i\omega_n \), while the other self-energy \( \Sigma_1^{(a)}(k) \) is not. It has been shown that the self-energy \( \Sigma_1^{(a)}(k) \) renormalizes the MF dressed fermion spectrum, and therefore it dominates the charge transport of the systems. As a qualitative discussion as in the doped cuprates, we neglect \( \Sigma_1^{(a)}(k) \), and only study the static limit of the effective dressed fermion gap function, i.e., \( \Sigma_2^{(a)}(k) = \Delta_a(k) \), then the dressed fermion diagonal and off-diagonal Green’s functions are obtained from Eq. (5) as,

\[
g(i\omega_n, k) = \frac{i\omega_n + \xi_k}{(i\omega_n)^2 - E_k^2}
\times \frac{1}{\nu = 1, 2} \left( 1 + \frac{\xi_k}{E_{\nu}(k)} \right) \frac{1}{i\omega_n - E_{\nu}(k)}, \tag{7a}
\]

\[
\Im \langle i-j, t - t' \rangle = \frac{-\Delta_a^*(k)}{(i\omega_n)^2 - E_k^2}
\times \frac{1}{\nu = 1, 2} \Delta_a^*(k) \frac{1}{i\omega_n - E_{\nu}(k)}, \tag{7b}
\]

with \( E_1(k) = E_k, E_2(k) = -E_k \), and the dressed fermion quasiparticle spectrum \( E_k = \sqrt{\xi_k^2 + \Delta_a(k)^2} \). Although the situation of the pairing symmetry of superconductivity in the doped cobaltate Na\(_2\)CoO\(_2\)·\(_y\)H\(_2\)O is far from reaching a consensus, many experimental data suggest non-s-wave superconductivity without a full gap. In particular, it has been argued according to the irreducible representations of the triangular lattice that there are three possible basis functions of even parity, i.e., one s-like function \( s_k = \cos(k_x + 2\cos(k_y/2)\cos(\sqrt{3}k_y)/2) + \cos(k_x + \sqrt{3}k_y)/2 \), and two d-like functions, \( d_{1k} = 2\cos(k_x + 2\cos(k_y/2)\cos(\sqrt{3}k_y)/2) - \cos(k_x + \sqrt{3}k_y)/2 \) and \( d_{2k} = 3\cos[k_x + \sqrt{3}k_y]/2 - \sqrt{3}\cos[k_x - \sqrt{3}k_y]/2 \). However, with the different linear combinations of these basis functions, it has been found in terms of the Gutzwiller approximation scheme and variational Monte Carlo simulation that the lowest energy state is the d-wave \((d_1 + id_2)\) state with the gap function \( \Delta^{(d)}(k) \propto \Delta^{(d)}(d_{1k} + id_{2k}) \). Recently, many authors have argued that it is possible that this d-wave state also is the lowest state around the electron doped regime where superconductivity appears. Therefore in the following discussions, we only consider this d-wave case \( \Delta^{(d)}(k) = \sum_{\eta, \eta'} \langle S^+_i S^-_{i+\hat{y}} S^+_i S^-_{i+\hat{y}} \rangle \).
\[ \Delta_k^{(d)} = \Delta_k^{(d)}(d_{1k} + id_{2k}) \]. In this case, we find from Eq. (6b) that the effective dressed fermion gap parameter satisfies the equation,

\[ 1 = -(Zt)^2 \frac{1}{N^3} \sum_{k,q,p} \sum_{\nu,\nu'} \gamma_k^{(d)} \sum_{\nu,\nu'} \frac{1}{2E_{\nu'}(k)} \]

\[ \times \frac{B_p}{\omega_{\nu'}(p)} \frac{F_{\nu',\nu'}(k, q, p)}{\omega_{\nu'}(\nu p) - \omega_{\nu}(\nu q) - E_{\nu'}(k)}. \] 

(8)

where

\[ \gamma_k^{(d)} = d_{1k} + id_{2k}, \]

\[ F_{\nu',\nu'}(k, q, p) = n_F\{E_{\nu'}(k)\}[n_B[\omega_{\nu'}(p)] - n_B[\omega_{\nu'}(q)]) + n_B[\omega_{\nu'}(p)]\}. \] 

As in the case of the doped cuprates, this gap equation must be solved simultaneously with other seven self-consistent equations, therefore all the order parameters, decoupling parameter \( \alpha \), and chemical potential \( \mu \) are determined by the self-consistent calculation. In this case, we can obtain the dressed fermion gap parameter in terms of the off-diagonal Green’s function (7b) as,

\[ \Delta_k^{(d)} = \frac{2}{N} \sum_k |\gamma_k^{(d)}|^2 \frac{\Delta_k^{(d)}}{E_k} \text{th}\{\frac{1}{2}\beta E_k\}. \] 

(9)

The dressed fermion pairing state originating from the kinetic energy term by exchanging dressed spinon excitations will also lead to form the electron Cooper pairing state as mentioned in Eq. (4). For a discussion of the physical properties of the SC state, we now need to calculate the electron off-diagonal Green’s function \( \Gamma^{(i - j, t - t')} = \langle \{f_{i\nu}(t); f_{j\nu}(t')\} \rangle \). In the framework of the charge-spin separation fermion-spin theory, it is a convolution of the dressed spinon Green’s function \( D(i\omega_m, p) \) and off-diagonal dressed fermion Green’s function \( \Sigma(i\omega_n, k) \), and can be expressed as,

\[ \Gamma^{(i\omega_n, k)} = \frac{1}{N} \sum_p \beta \sum_{i\omega_m} D^{(0)}(i\omega_m, p) \]

\[ \times \Sigma(i\omega_m - i\omega_n, p - k), \] 

(10)

in the present approximation. This convolution of the dressed spinon Green’s function and off-diagonal dressed fermion Green’s function reflects the charge-spin recombination, and can be evaluated in terms of the MF dressed spinon Green’s function and off-diagonal dressed fermion Green’s function as,

\[ \Gamma^{(i\omega_n, k)} = \frac{1}{N} \sum_{p,\nu,\nu'} \Delta_k^{(d)}(p - k) \]

\[ \times \frac{B_p}{2\omega_{\nu'}(p)} \frac{n_B[\omega_{\nu'}(p)] + n_F[E_{\nu'}(p - k)]}{\omega_{\nu'}(\nu p) - \omega_{\nu}(\nu q) - E_{\nu'}(k)}. \] 

(11)

In this case, the SC gap function is obtained in terms of this electron off-diagonal Green’s function as,

\[ \Delta_k^{(d)}(k) = \frac{1}{N} \sum_p \Delta_k^{(d)}(p - k) \text{th}\{\frac{1}{2}\beta E_{p - k}\} \frac{B_p}{2\omega_p} \]

\[ \times \text{coth}\{\frac{1}{2}\beta \omega_p\}. \] 

(12)

which shows that the symmetry of the electron Cooper pair is the same as the symmetry of the dressed fermion pair, i.e., the SC gap function can be written as \( \Delta_k^{(d)}(k) = \Delta_k^{(d)}(d_{1k} + id_{2k}) \). With the help of Eqs. (12) and (9), the SC gap parameter can be obtained as \( \Delta_k^{(d)} = -\chi \Delta_k^{(d)} \), as in the doped cuprates. Our present theory also indicates that there is a coexistence of the electron Cooper pair and short-range AF correlation in the doped cobaltates, and therefore the short-range AF fluctuation can persist into superconductivity. This is because that the AF fluctuation is dominated by the scattering of dressed spinons, which has been incorporated into the electron off-diagonal Green’s function and hence the electron Cooper pair in terms of the dressed spinon Green’s function. This result is consistent with some experiments. We have performed a numerical calculation for the gap parameters, and the results of the dressed fermion (a) and SC (b) gap parameters in the d-wave symmetry as a function of the electron doping concentration \( \delta \) at \( T = 0.001J \) and \( t/J = -2.5 \) are shown in Fig. 1, where both values of the dressed fermion and

\[ \Delta_k^{(d)} \]
SC gap parameters increase with increasing doping in the underdoped regime, and reach the maximal values for a particular doping concentration, then decrease in the overdoped regime. Although there is a coexistence of the electron Cooper pair and short-range AF correlation, the value of the SC gap parameter is still suppressed by this AF fluctuation. The SC gap function in Eq. (12) also indicates that the SC transition temperature $T_c$ occurring in the case of $\Delta_0^{(d)} = 0$ is identical to the dressed fermion pair transition temperature occurring in the case of $\Delta_0^{(d)} = 0$. This SC transition temperature $T_c$ as a function of electron doping concentration $\delta$ in the d-wave symmetry for $t/J = -2.5$ is plotted in Fig. 2 in comparison with the experimental data taken from $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$ (inset). It is shown that the maximal SC transition temperature $T_c$ occurs around the optimal electron doping concentration $\delta \approx 0.29$, and then decreases for both underdoped and overdoped regimes. Using a reasonable estimation value of $J \sim 10$mev to 20mev in $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$, the SC transition temperature in the optimal doping is $T_{\text{optimal}} \approx 0.02J \approx 3K \sim 6K$. Our these results are in qualitative agreement with the experimental data.

Since $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$ is the electron doped Mott insulator on a triangular lattice, therefore the system has strong geometrical spin frustration. This magnetic frustration also induces the strong charged carrier’s quantum fluctuation. In comparison with the case in the doped cuprates, we find that the SC transition temperature in the doped cobaltates is suppressed heavily to a lower temperature due to both strong magnetic frustration and dressed fermion’s quantum fluctuation. In Ref.16, the dressed holon’s quantum fluctuation is not considered, and therefore leads to an obvious weakness that the SC transition temperature in the doped cuprates is too high, and not suppressed in the overdoped regime. Recently, this weakness has been cured by considering the dressed holon’s quantum fluctuation28, and the results show that the SC transition temperature in the doped cuprates in the optimal doping is suppressed to an reasonable value, and then decreases for both underdoped and overdoped regimes, which also confirm that both strong magnetic frustration and charged carrier’s quantum fluctuation suppress the SC transition temperature.

In summary, we have discussed the mechanism of superconductivity in the electron doped cobaltate $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$ within the charge-spin separation fermion-spin theory. It is shown that the mechanism of superconductivity is ascribed to the kinetic energy. The dressed fermions interact occurring directly through the kinetic energy by exchanging magnetic excitations. This interaction leads to a net attractive force between dressed fermions, then the electron Cooper pairs originating from the dressed fermion pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground state. The electron SC transition temperature is identical to the dressed fermion pair transition temperature, and is suppressed to a lower temperature due to the strong magnetic frustration. The optimal SC transition temperature occurs in the electron doping concentration $\delta \approx 0.29$, and then decreases for both underdoped and overdoped regimes, in qualitative agreement with the experimental observations.

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FIG. 2. The superconducting transition temperature as a function of the electron doping concentration in the d-wave symmetry for $t/J = -2.5$. Inset: the experimental result on $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$ taken from Ref. 4

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