A Joint Transform Correlator Encryption System Based On Complex Amplitude Modulation for Grayscale Images

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Abstract. Aiming at the problem that the joint transform correlator (JTC) encryption system is difficult to perform experiments on encrypted grayscale images, a JTC encryption system based on complex amplitude modulation for grayscale images system is proposed. In this system, the complex amplitude modulation is implemented by means of single pixel hologram coding, so that the encryption of the grayscale image can be completed by using a single spatial light modulator with pure phase in the JTC encryption experiment.

1. Introduction
The double random phase encoding (DRPE) technique proposed by Refregier and Javidi in 1995 pioneered the use of optical information technology to solve encryption problems [1]. Since the introduction of this method, optical image encryption has attracted the attention of many researchers, and has gradually become a research hotspot. A variety of new optical image encryption methods have been proposed, such as optical encryption based on wavelet transform [2], Optical encryption based on QR code [3, 4], optical encryption based on optical holography [5], optical encryption based on joint transform correlator (JTC) [6-8], etc. The JTC optical encryption system has (1) does not require a complex-conjugate key, (2) the key mask does not need precise alignment, (3) the encrypted image is a grayscale image for easy recording and transmission, has been widely used in the field of optical encryption.

Since the input plane of the JTC encryption system is a light field with complex amplitude distributed, in optical experiments, a phase spatial light modulator (SLM) and an amplitude SLM are generally required to cooperate with each other. The input plane cannot be directly utilized using a single phase SLM or an amplitude SLM. The dual SLM not only increases the overhead, but also has complex problems such as matching alignment. At present, there are two main methods for implementing JTC encryption using a single pure phase SLM. 1. When encrypting a binary image, the binary image is modulated by the phase information of the complex amplitude to include phase information [9]; When encrypting a grayscale image, the grayscale image is encoded into a binary image, and then the phase information is used to modulate the binary image, such as QR code [10], binary code [11]. However, grayscale image coding increases bandwidth usage and reduces encryption efficiency.

In order to solve these problem, this paper uses pure phase SLM single-pixel hologram coding to achieve complex amplitude modulation. The method has a simple structure, and only a single pure
phase spatial light modulator and a 4f filtering system can realize complex amplitude modulation of the light field, thereby realizing an encrypted gray image.

2. Complex amplitude modulation method

\( E(x, y) \) Denotes the complex amplitude information of the target light field. The Euler formula can be used to decompose the target light field into two parts. The specific process is as follows

\[
E(x, y) = A(x, y)e^{i\phi(x, y)} = \frac{A_{\text{max}}}{2} e^{i\alpha(x, y)} + \frac{A_{\text{max}}}{2} e^{i\beta(x, y)}
\]

\[
\alpha(x, y) = \varphi(x, y) + \cos^{-1}\left[\frac{A(x, y)}{A_{\text{max}}}\right]
\]

\[
\beta(x, y) = \varphi(x, y) - \cos^{-1}\left[\frac{A(x, y)}{A_{\text{max}}}\right]
\]

In the above formula, \( A(x, y) \) is the amplitude of the \( E(x, y) \) , \( \varphi(x, y) \) is the phase of the \( E(x, y) \) . \( A_{\text{max}} \) Is the maximum of the \( A(x, y) \) .

It can be seen from Formula (1) that the amplitude distribution of the two parts of the light field obtained by the decomposition is uniform. It is only necessary to use the phase mask to generate these two parts of the light field, and to make them coherent to recover the target light field. The phase mask of the generated two-part light field is divided into two areas of "a" and "b" in a checkerboard pattern. The "an" area of one phase mask and the "b" area of the other phase mask are recombined into a new one as shown in Fig.1. The plane wave illuminates the combined phase mask and can be modulated into a complex amplitude distribution of the target light field.

Fig 1. Combination of phase mask (a) phase mask 1, (b) phase mask 2, (c) combined phase mask.

With this method, complex amplitude modulation can be achieved using only a phase SLM, but a phase jump occurs at the junction of the two phase mask, resulting in high frequency noise when using the combined phase mask. Therefore, this method needs to be used in conjunction with a low-pass 4f filtering system.

3. JTC Encryption System based on Complex Amplitude Modulation for Grayscale Images

The schematic diagram of the JTC encryption system is shown in Fig.2 (a). The \( x, y \) Cartesian coordinate system is established with the center of the input plane as the origin. The complex amplitude obtained by multiplying the gray image \( c(x, y) \) by the random phase mask \( r(x, y) \) is decomposed according to Formula (1), and the phase mask is combined according to the method of II, and placed in the object window, the center coordinate is \((-a,0)\); the other random phase mask is placed as a key in the key window with its center coordinates \((-\alpha,0)\). Under the illumination of a monochromatic plane wave, the joint power spectrum is recorded by the CCD on the spectrum plane by optical Fourier transform, that is, the encrypted image is as follows.
\[ E(u,v) = \sum (c(x+a,y) \cdot r(x+a,y) + k(x-a,y))^2 \]
\[ = [C(u,v) \ast R(u,v)]^2 + |K(u,v)|^2 \]
\[ + [C(u,v) \ast R(u,v)] K(u,v) \exp[-j2\pi(2a)u] \]
\[ + [C(u,v) \ast R(u,v)] K(u,v) \ast \exp[-j2\pi(-2a)u] \]  \hspace{1cm} (4)

In the above formula, \((u,v)\) and \((x',y')\) are the spatial frequency coordinates on the spectrum plane respectively, and \(u = \frac{x'}{\lambda f}, v = \frac{y'}{\lambda f} : C(u,v) \cdot R(u,v)\) And \(K(u,v)\) are individual Fourier-transform results for \(c(x,y)\), \(r(x,y)\) and \(k(x,y)\). The \(\mathcal{Z}\{\cdot\}\) stands for Fourier transformation, the symbol \(*\) represents convolution, and the notation \([\cdot]^*\) denotes a complex conjugate.

Fig. 1 (b) sketches a JTC decryption system. A key, centered at point \((a,0)\), is put on the input plane. A cryptograph is laid over the spectrum plane, such that their centers coincide. When illuminated by monochromatic plane waves, the key experiences optical Fourier transformation and creates decrypting beams on the spectrum plane. Those beams pass through the cryptograph, undergo reverse optical Fourier transformation, and generate a light field on the output plane with a complex amplitude of \([6]\)

\[ g(x',y') = \mathcal{Z}^{-1}[\mathcal{Z}[k(x-a,y)P(u,v)]] \]
\[ = k(x',y') \ast [r(x',y')c(x',y')] \ast [r(x',y')c(x',y')] \ast \delta(x' - a, y') \]
\[ + k(x',y') \ast \delta(x' - a, y') \]
\[ + [r(x',y')c(x',y')] \ast [r(x',y')c(x',y')] \ast \delta(x' - 3a, y') \]
\[ + [r(x',y')c(x',y')] \ast \delta(x' + a, y') \]  \hspace{1cm} (5)

In the above formula, \(\otimes\) stands for the correlation operator, and \((x'',y'')\) are the rectangular coordinates on the output plane.

The intensity distribution of the light field represented by the fourth term is plaintext. When the center distances of the light fields shown in the formula are sufficiently large, they are spatially separated from each other, and a decrypted image can be obtained at the position \((-a,0)\).

Figure 2. Diagrams of JTC encryption and decryption (a) JTC encryption system, (b) JTC decryption system.
4. Simulation
To verify the validity of the JTC encryption system based on complex amplitude modulation for grayscale images, the encryption and decryption process was simulated. The original grayscale image used in the simulation is a 200×200-pixel grayscale image, as shown in Fig.3. The laser wavelength is 632.8nm and the Fourier lens focal length is 400mm.

![Figure 3. Original grayscale image.](image)

The 200×200-pixel random phase mask and key mask of the pixels used in the simulation are shown in Fig.4 (a) and (b), respectively. The complex amplitude obtained by multiplying the grayscale image shown in Fig.3 and the random phase mask by the Formula(1) is decomposed, and the phase template combined by the method of II is as shown in Fig. 4(c). When encrypting, place the combined phase mask in the object window and place the key mask in the key window. Both windows on the input plane are 200×200 pixels. The object window’s center is at (−300,0), while the key window’s center is at (300,0). Calculate the joint power spectrum according to Formula (4), and eliminate the noise term according to the noise cancellation method in [7] and divide by the key power spectrum to obtain the cryptograph as shown in Fig. 4(d).

When decrypting, the key is placed on the input plane of the decryption system, and its center coordinate is (300,0). The cryptograph after the noise reduction processing is placed on the spectrum plane, and its center coincides with the center of the spectrum plane. The original decrypted image on the output plane is calculated by equation (5) as shown in Fig. 4(e).

![Figure 4. The results of simulation (a) random phase mask, (b) key, (c) Combined phase mask, (d) encrypted image, (e) decrypted image](image)

5. Conclusion
In this paper, the complex amplitude modulation of a single SLM with pure phase is realized by the method of checkerboard hologram coding. A joint transform correlator encryption system based on complex amplitude modulation for grayscale images is proposed. The system is simple in structure and easy to implement. The system is more efficient than other binary encoding methods for grayscale images. The correctness of the system is verified by the simulation of encryption and decryption.
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