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Calculation method of reliability on combine harvester transmission belt by considering dynamic stress

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Abstract. Transmission belt is one of the most likely to fail parts of combine harvester, which affecting the machine reliability seriously. Dynamic strength occurs along with vibration during the operation and must be taken into account when calculating reliability, especially in harsh working environment like harvesting. However, the existing calculation method of reliability on combine harvester transmission belt didn’t take the dynamic strength into account. In this research, a reliability calculation method was proposed based on the dynamic analysis of transmission belt. The nonlinear dynamic equation was built using string and beam model. Through the equation, relationship between belt speed and dynamic stress was deduced. Considering dynamic stress and regarding uncertain parameters as random uncertain parameters, reliability calculation model was built. Finally, an example was presented and the above mentioned dynamic reliability calculation method was simulated to verify the theoretical analysis in this paper and tested by the Monte-Carlo method.

1. Introduction

In recent decades, agricultural equipment keep developing rapidly. Reliability has become a new trend of combine harvester, especially in Asia. Combine harvester have a lot of working parts and a complex structure, any problem would cause machine failure and delay the harvesting time [1]. V belt is widely used in combine harvesters, however, the heavy load and strong vibration make it prone to failure. The reliability calculation in use didn’t take dynamic stress into account and it is difficult to achieve the desired results. In fact, compared with other parts, transmission belt is most likely to fail during harvesting operations in the field [2]. So it is necessary to consider the influence of dynamic stress when calculating the reliability of belt.

Most of the mechanical models of V-belt are based on the flat-belt mechanics model, including creep theory, shear theory and CMM model [3]. Circumferential and lateral vibration would occurs when V-belt is in operation [4]. Some reliability design methods had been developed based on the basic criteria of V-belt which is having sufficient strength and no slippage occur during power transmission [5]. Qin [6] deduced parameters sensitivity of V-belt reliability and discussed the reliability design method in the case of incomplete probability information. Based on reliability design, Gong [7] and Qing [8] discussed the optimization design algorithm of the belt. Dynamic stress and its effect on failure of belt were analyzed by Wen [9], but the reliability calculation was not involved.
This paper focuses on the reliability calculation method of V-belt by considering dynamic stress. Nonlinear dynamic equation was built and the relationship between belt speed and dynamic stress was deduced. Considering dynamic stress and uncertain parameters, reliability calculation model was built and tested by Monte-Carlo method.

2. Nonlinear dynamic equation of belt
Since the bending stiffness of belt can be neglected during movement process, the dynamic model of belt could be simplified as an axially moving string with viscoelastic properties [10]. Since bending stiffness could be ignored, the belt can be simplified as axial moving string and modelled as an Euler-Bernoulli beam [11]. Selecting a section of belt that does not contact the pulley as subject (shown in Figure.1). Let the horizontal direction be the $X$ axis and the vertical direction be the $Y$ axis to establish absolute coordinate system $XOY$. The relative coordinate system was established on the belt before deformation occurs. Take the fixed point on the belt as the coordinate origin. Let $u$ and $w$ stand for the displacement of the point $(x, y)$ in the $X$, $Y$ direction and they were function of position and time.

\[ u = u(x, y, t) \quad w = w(x, y, t) \]  

Relative coordinates can be written as

\[ \begin{align*}
\ddot{x} & = v + \frac{\partial u}{\partial x} \frac{\partial v}{\partial t} + \frac{\partial u}{\partial t} \\
\ddot{y} & = v + \frac{\partial w}{\partial x} \frac{\partial v}{\partial t} + \frac{\partial w}{\partial t}
\end{align*} \]  

The acceleration relation:

\[ \begin{align*}
\frac{d^2x}{dt^2} & = v^2 \frac{\partial^2 u}{\partial x^2} + 2v \frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} \\
\frac{d^2y}{dt^2} & = v^2 \frac{\partial^2 w}{\partial x^2} + 2v \frac{\partial^2 w}{\partial x \partial t} + \frac{\partial^2 w}{\partial t^2}
\end{align*} \]  

The quality of micro element $dx$ was $\rho Adx$ (shown in Figure.2). Ignoring the moment of inertia and shear deformation, $dx$ could be regarded as Euler-Bernoulli beam.
Points AB move to A' B'

$$\overline{AA'} = \vec{u}i + w\vec{j}$$  $$\overline{BB'} = (u + \frac{\partial u}{\partial x}i + w\frac{\partial w}{\partial x}j)$$  \hspace{1cm} (4)

It can be concluded from the position relation in Figure.2:

$$\overline{AA'} + \overline{A'B'} = \overline{AB} + \overline{BB'} = d\vec{x}i + \overline{BB'}$$  \hspace{1cm} (5)

The deformation of AB section is $ds$:

$$ds = |\overline{A'B'} - \overline{AB}| = \sqrt{(1 + \frac{\partial u}{\partial x})^2 + \left(\frac{\partial w}{\partial x}\right)^2} \, dx$$  \hspace{1cm} (6)

The longitudinal dynamic strain of the belt:

$$\varepsilon_d = \frac{ds - dx}{dx} = \sqrt{(1 + \frac{\partial u}{\partial x})^2 + \left(\frac{\partial w}{\partial x}\right)^2} - 1$$ \hspace{1cm} (7)

Taylor expansion of equation (7) and omit high-order terms, the relationship between dynamic strain and displacement:

$$\varepsilon_d = \frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2$$ \hspace{1cm} (8)

According to the stress-strain relationship in Kelvin model viscoelastic theory, dynamic stress:

$$\sigma_{id} = E\varepsilon_d + \eta(\partial \varepsilon_d / \partial t) = E^\ast \varepsilon_d(x,t)$$ \hspace{1cm} (9)

Where $E$ is elasticity modulus, $\eta$ is dynamic viscous resistance coefficient, $E^\ast$ is equivalent elastic modulus.

Longitudinal dynamic strain $\varepsilon_d$ is only related to $\partial u / \partial x$ and $\partial w / \partial x$, which means the dynamic strain and dynamic stress of are related to the speed of belt. The total tension force of belt:

$$F = F_0 + \sigma_{id}A = F_0 + E^\ast \varepsilon_d(x,t)A$$ \hspace{1cm} (10)
Where $F_o, A$ are, respectively, initial tension of the belt, cross sectional area of belt.

Linear damping force is proportional to the absolute velocity, the damping force on the $dx$:

$$F_{cs} = cA dx \frac{dx}{dt} \quad F_{cy} = cA dx \frac{dy}{dt} \quad (11)$$

The force of micro element $dx$ is shown in Figure 3. Dynamic equation of micro segment $dx$:

$$\left(F + \frac{\partial F}{\partial s} ds\right) \cos(\theta + \frac{\partial \theta}{\partial s} ds) - F \cos \theta - cA dx \frac{dx}{dt} = \rho A dx \frac{d^2 x}{dt^2} \quad (12)$$

$$\left(F + \frac{\partial F}{\partial s} ds\right) \sin(\theta + \frac{\partial \theta}{\partial s} ds) - F \sin \theta - cA dx \frac{dy}{dt} = \rho A dx \frac{d^2 y}{dt^2} \quad (13)$$

When $\theta$ is small, the following approximate relation exists: $\sin \theta \approx \theta \quad \cos \theta \approx 1 - \frac{1}{2} \theta^2 \quad ds = dx/\cos \theta \quad \dot{\omega}/\dot{x} = \theta$. Equation (13) (14) can be changed as:

$$\rho A dx \frac{d^2 x}{dt^2} = \frac{\partial \sigma}{\partial t} dx - F \left[ \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} dx + \frac{1}{2} F \left( \frac{\partial^2 w}{\partial x^2} dx \right)^2 \right] - cA dx \frac{dx}{dt} \quad (14)$$

$$\rho A dx \frac{d^2 y}{dt^2} = \frac{\partial \sigma}{\partial t} \left( \frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) dx - F \frac{\partial^2 w}{\partial x^2} dx - cA dx \frac{dy}{dt} \quad (15)$$

The first and the second terms of Equation (14) (15) stand for the viscoelasticity of belt; the third term of (14) (15) stand for the linear damping of belt.

The nonlinear dynamical equation of the belt which is only related to displacement can be obtained according to the Eq. (2) (3) (10) (14) (15). The dynamic stress $\sigma_{d}$ can be calculated by bringing in data.

3. Belt Stress Analysis

The stress of belt mainly include tensile stress $\sigma_{t}$, bending stress $\sigma_{b} = E h/d$, and centrifugal stress $\sigma_{c} = qv^2/A$. Tensile stress of belt $\sigma_{t}$ includes static tensile stress $\sigma_{s}$ and dynamic tensile stress $\sigma_{id}$.

$$\sigma_{s} = F_{i}/A \quad (16)$$

$$F_{i} = F \frac{e^{\alpha}}{e^{\alpha} - 1} \quad (17)$$
Where $F_t$, $F_e$, $\alpha$, $f$ and $F_0$ are, respectively, tensile stress of tight side, maximum effective pulling force, wrap angle, friction coefficient and initial tension of the belt.

According to the Eq. (17) (18), static tensile stress $\sigma_{ts}$ can be expressed as

$$\sigma_{ts} = 2F_0 \frac{1 - \frac{1}{e^{\alpha}}}{1 + \frac{1}{e^{\alpha}}} e^{\alpha} - 1$$

According to the Eq. (9) (19), tensile stress of belt $\sigma_i$ can be expressed as

$$\sigma_i = \sigma_{ts} + \sigma_{id} = 2F_0 \frac{1 - \frac{1}{e^{\alpha}}}{1 + \frac{1}{e^{\alpha}}} e^{\alpha} + E f e e (x, t)$$

The fatigue strength condition of the belt can be described as

$$\sigma_{\max} = \sigma_t + \sigma_k + \sigma_i \leq \sigma$$

Where $\sigma$ is allowable stress of belt.

4. Reliability model

Belt speed $v$, equivalent friction coefficient $f$, wrap angle $\alpha$, center distance $a$ and initial tension $F_0$ are random variables in the belt. Considering these random variables are independent and obeying normal distribution. The set of these random variables are

$$x = [x_1, x_2, \cdots, x_n]$$

Means and variances are

$$\mu_x = [\mu_{x_1}, \mu_{x_2}, \cdots, \mu_{x_n}]$$

$$\sigma_x = [\sigma_{x_1}, \sigma_{x_2}, \cdots, \sigma_{x_n}]^T$$

According to the basic design criteria of belt, the maximum stress must be less than the allowable stress. The dangerous point of the belt is the position that about to leave pulley. We can assume that the belt wouldn’t be failure as long as the critical point satisfies the stress condition. The strength reliability function of the drive belt can be expressed as

$$g = [\sigma_t] - \sigma_i = g(x_1, x_2, \cdots, x_n)$$

The strength failure probability can be expressed as

$$P = P(g \leq 0)$$

The reliability index of strength reliability function is
The failure probability of strength reliability function is

\[ P = \Phi(-\beta) \quad (27) \]

The Reliability of belt can be expressed as

\[ R = 1 - P \quad (28) \]

Equation (28) is the reliability model of belt by considering dynamic stress. The reliability of the belt can be calculated according to the equations above.

5. Numerical example and discussion
Take some of the belts used in combine harvesters as example. The parameters were shown in Table.1 and the statistical property of random parameters were shown in Table.2.

| Table.1 Parameters of belt |
|----------------------------|
| parameters                 | value |
| diameter of the small pulley | \( d_i = 90mm \) |
| diameter of the large pulley | \( d_s = 315mm \) |
| datum length                | \( L_s = 1600mm \) |
| depth of section            | \( h = 0.008m \) |
| mass per unit length        | \( q = 0.1kg/m \) |
| elasticity modulus of belt  | \( E = 364MPa \) |
| cross sectional area of belt| \( A = 100mm^2 \) |

| Table.2 Statistical property of random parameters |
|-----------------------------------------------|
| parameters                  | mean  | variance |
| linear velocity of belt \( n / r \cdot min^{-1} \) | 1440   | 20       |
| centre distance \( a / mm \)                  | 470    | 6.2      |
| wrap angle \( \alpha / rad \)                 | 2.739  | 0.03     |
| friction coefficient \( f \)                  | 0.47   | 0.002    |
| transmission power \( P / kw \)               | 4.4    | 0.4      |

The reliability of the belt at different speeds were calculated using the proposed method and Monte Carlo calculation results were compared as shown in Figure.4.
As can be seen from Figure 4, when belt speed is less than 5.6 m/s, the reliability is close to 1. Because the stress of belt is far less than the allowable stress when belt speed is low.

The reliability of the belt is significantly reduced in the vicinity of 6 m/s. It is a dangerous speed range that must be avoid. Dynamic stress increasing significantly and reliability decreasing sharply when the belt speed is close to the critical speed. This dangerous area would not be found if dynamic stress was not taken into account when calculating reliability. Therefore, the reliability calculation method of belt mentioned in this paper could reflect the actual situation more accurately, especially working in vibration environment.

With the increasing of speed and leaving from the dangerous speed range, dynamic stress decreases and reliability improves again. Finally, with the continuously increase of speed, reliability decrease gradually. That is because centrifugal force increase with speed.

The figure shows that the results of the two methods are similar, which means the reliability calculation method proposed in the paper is effective and accurate.

6. Conclusion
The nonlinear dynamic equation was built and the relationship between belt speed and dynamic stress was deduced. Considering dynamic stress and regarding uncertain parameters as random uncertain parameters, reliability calculation model was built. Finally, an example was presented and the dynamic reliability calculation method was simulated to verify the theoretical analysis in this paper and tested by the Monte-Carlo method. Dangerous speed range can be found through reliability calculation. So as to provide some theoretical basis for the design and reliability calculation of transmission belt of combine harvester.

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