The Loop Corrections to the Parity-Violating Elastic Electron-Proton Scattering

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We calculate the two-boson-exchange (TBE) corrections to the parity-violating asymmetry of the elastic electron-proton scattering in a parton model using the formalism of generalized parton distributions (GPDs).

Keywords: parity-violating; electron-proton scattering; generalized parton distributions

1. Strangeness content and Parity-Violating measurement

Strangeness content is one of the most important questions in hadron structure because it sheds light on the role of the quark-anti quark sea in the ground state properties. One important tool is elastic parity-violating electron-proton scattering, which has been used to probe the charge and magnetization distributions of the strange quark within the nucleon. At the tree level, the parity-violating asymmetry \( A_{PV} \) arises from the interference of diagrams with one-photon-exchange (OPE) and \( Z \)-boson exchange shown in Fig 1(a) and (b), respectively. The interactions between photon, \( Z \)-boson and proton are described by the form factors of the proton defined as

\[
\langle p' \mid J_{\mu}^p \mid p \rangle = \pi(p') \left[ F_{1}^{\gamma p} \gamma_{\mu} + F_{2}^{\gamma p} \frac{i \sigma_{\mu \nu} q^{\nu}}{2M} \right] u(p),
\]

\[
\langle p' \mid J_{\mu}^{Z} \mid p \rangle = \pi(p') \left[ F_{1}^{Zp} \gamma_{\mu} + F_{2}^{Zp} \frac{i \sigma_{\mu \nu} q^{\nu}}{2M} + G^{Z}_{A} \gamma_{\mu} \gamma_{5} \right] u(p),
\]

where \( q=p'-p \) and \( M \) is the mass of the proton. \( F_{1,2}^{\gamma p} \) are the form factors of the proton electromagnetic current, \( F_{1,2}^{Zp} \) are the form factors of the proton neutral weak current. The \( A_{PV} \) can be expressed by the form factors defined above as

\[
A_{PV} = \frac{\sigma_{R} - \sigma_{L}}{\sigma_{R} + \sigma_{L}} = -\frac{G_{F} Q^{2}}{4\pi \alpha \sqrt{2}} \frac{A_{E} + A_{M} + A_{A}}{\epsilon(G_{E}^{p})^{2} + \tau(G_{M}^{p})^{2}},
\]

(2)
where $G_F$ is Fermi constant, $Q^2=−q^2$, $τ=Q^2/4M^2$, and $ε=(1+2(1+τ)\tan^2 \frac{θ_W}{2})^{-1}$. $A_E$, $A_M$, and $A_A$ are defined as $A_E = εG_{Zp}^pG_{E}^p$, $A_M = τG_{Zp}^pG_{M}^p$, and $A_A = (-1 + 4\sin^2 θ_W)\sqrt{τ(1+τ)(1-ε^2)}G_{Zp}^gG_{M}^p$, where $θ_W$ is the weak mixing (Weinberg) angle and $G_{Zp}^g = F_1^γZp - τF_2^γZp$, $G_{Zp}^g = F_1^γZp + F_2^γZp$. When combined with proton and neutron electromagnetic form factors and with the use of charge symmetry, one obtains the following relation:

$$G_{Zp}^{E,M} = (1 - 4\sin^2 θ_W)G_{Zp}^{E,M} - G_{Zp}^{g,n} - G_{Zp}^{n,m}. \quad (3)$$

$G_{Zp}^{E,M}$ can be extracted from the $A_{PV}$ through Eq.(2) and consequently the strange form factors $G_{Zp}^{E,M}$ can be determined through Eq.(3). Four experimental programs SAMPLE, HAPPEX, A4, and Gl0 have been designed to measure the $A_{PV}$, which is small and ranges from 0.1 to 100 ppm.

The main theoretical uncertainty is from higher-order electroweak radiative corrections which have been carefully studied in.\textsuperscript{6,7} They have been considered the interference of the one-loop diagrams shown in Figs. 1(c) and 1(d) in\textsuperscript{6} at both of quark and hadron levels under a zero momentum-transfer approximation in the low energy limit. To refine our knowledge it is important to go beyond the “zero momentum-transfer approximation”.

At high $Q^2$, the two-photon-exchange (TPE) box diagram was suggested\textsuperscript{8} to explain the discrepancy between the measurement of the proton electric to magnetic form factor ratio $R = G_E/G_M$, from Rosenbluth technique and polarization transfer technique.\textsuperscript{9} The TPE effect has been evaluated in\textsuperscript{10} in a parton model using GPD’s where the handbag approximation is used. The contribution of the interference of the TPE process of Fig. 1(c) with diagram of Figs. 1(a) and 1(b) to the $A_{PV}$, has been evaluated in\textsuperscript{11} in the same parton model used in.\textsuperscript{10} It was found that indeed the TPE correction to the $A_{PV}$ can reach several percent in certain kinematics, becoming comparable in size with existing experimental measurements of strange-quark effects in the proton neutral weak current.
On the other hand, the contribution of the interference of the TPE process of Fig. 1(c) with diagram of Figs. 1(a) and 1(b) to the $A_{PV}$, and the interference of the $\gamma$-$Z$ exchange process of Fig. 1(d) with diagram of Fig. 1(a) have both been evaluated in\textsuperscript{12} in a hadronic model where the excited intermediate states are neglected and the on-shell nucleon form factors are inserted. It has been found that these effects own strong $\epsilon$ and $Q^2$ dependencies in the parity-violating $ep$ scattering.

2. Formalism with parionic calculation

In this Letter, we first calculate the TPE and $\gamma Z$ exchange corrections on a quark $l(k) + q(P_q) \to l(k') + q(P_{q'})$, denoted by the scattering amplitude $H$ in Fig. (2). Subsequently we embed the quarks in the proton as described through the nucleon’s GPD’s. Here only GPD’s of $u$ and $d$ quarks are included because the strange quark contribution to the box diagrams are very small.

In the standard model, the electromagnetic current and the neutral weak current of quarks are

\[
J_{em}^\mu = \sum_{f=u,d} \bar{q}_f Q_f \gamma_\mu q_f, \quad J_Z^\mu = \sum_{f=u,d} \bar{q}_f (g_f^V \gamma_\mu + g_f^A \gamma_5) q_f,
\]

where

\[
g_V^e = -1 + 4 \sin^2 \theta_W, \quad g_A^e = 1 \quad \text{and} \quad c_1^{hard} \quad \text{and} \quad c_2 \quad \text{are defined as}
\]

\[
c_1 = \frac{-1}{4\pi^2 M_Z^2} \left[ \ln \left( \frac{\lambda^2}{Q^2} \right) + \frac{\pi^2}{2} \right] + \frac{3}{16\pi^2 M_Z^2} \ln \left( \frac{\hat{t}}{s} \right),
\]

\[
c_2 = \frac{1}{16\pi^2 M_Z^2} \left[ -7 + 3 \ln \left( \frac{\hat{s}}{M_Z^2} \right) + 3 \ln \left( \frac{\hat{t}}{M_Z^2} \right) \right].
\]
\[ \hat{s} = (P_q + k)^2, \hat{u} = (P_q - k')^2 \] and \( \lambda \) is the infrared cut-off input by infinitesimal photon mass. The amplitudes are separated into the soft and hard parts, the soft part corresponds with the situation where the photon carries zero four momentum and one obtains \( c_{1}^{\text{soft}} = \frac{1}{4 \pi M_Z^2} \ln \left( \frac{\lambda^2}{Q^2} \right) + \frac{\pi^2}{2} \), therefore the hard part is \( c_{1}^{\text{hard}} = \frac{3}{16 \pi^2 M_Z^2} \ln \left( \frac{\hat{u}}{\lambda^2} \right) \). The IR divergence arising from the direct and crossed box diagrams is concealed with the bremsstrahlung interference contribution with a soft photon emitted from the electron and the proton. We limit ourselves to discuss the hard \( \gamma-Z \) exchange contribution in this article.

The next step is to calculate the \( ep \) amplitudes. These amplitudes are obtained as a convolution between an electron-quark hard scattering and a soft nucleon matrix element. The procedure is similar to the one in. In the kinematics regime where \( s = (p + k)^2, -u = -(k - p')^2 \) and \( Q^2 \) are all large enough compared to hadronic scale \( \Lambda_H \) which we choose as 1GeV. The parity-violating \( T \)-matrix can be written as

\[
T_{h,\lambda_N,\lambda_N}^{\text{PV,hard}} = -\frac{G_F}{2\sqrt{2}} \{ \bar{u}(k', h) \gamma_\mu u(k, h) \bar{u}_N(p', \lambda'_N) \\
- G_{M, h, \lambda_N, \lambda_N}^{\text{PV}} \gamma_\mu \gamma_5 u_N(p, \lambda_N) + \bar{u}(k', h) \gamma_\mu \gamma_5 u(k, h) \bar{u}_N(p', \lambda'_N) \}
\]

where

\[
\delta G_{M}^{\text{PV}} = \frac{1 + \epsilon}{2\epsilon} D - \frac{1 + \epsilon}{2\epsilon} \frac{Q^2 + 4M^2}{s - u} F,
\]

\[
\delta F_{2}^{\text{PV}} = \frac{1}{1 + \tau} \left[ \delta G_{M}^{\text{PV}} - \sqrt{\frac{1 + \epsilon}{2\epsilon} E} \right],
\]

\[
\delta G_{A}^{\text{PV}} = \frac{1 + \epsilon}{2\epsilon} F - \frac{1 + \epsilon}{2\epsilon} \frac{Q^2}{s - u} D,
\]

with

\[
D \equiv \int_{-1}^{1} dx \frac{Q^2 t_1^q + (\hat{s} - \hat{u}) t_2^q}{s - u} (H^q + E^q),
\]

\[
E \equiv \int_{-1}^{1} dx \frac{Q^2 t_1^q + (\hat{s} - \hat{u}) t_2^q}{s - u} (H^q - \tau E^q),
\]

\[
F \equiv \int_{-1}^{1} dx \frac{Q^2 t_2^q + (\hat{s} - \hat{u}) t_1^q}{s - u} \sigma_{gm}(x) \tilde{H}^q,
\]

here \( H^q, E^q \) and \( \tilde{H}^q \) are the GPD’s for a quark in the nucleon. To estimate the amplitudes of Eq. one needs to specify a model for the GPD’s. Again we follow to adopt an unfactorized model of GPD’s in terms of a forward parton distributions and a gaussian factor in \( x \) and \( Q^2 \).
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Fig. 3. Direct and crossed box diagrams to describe the two-photon exchange and $\gamma$-$Z$ contribution to the lepton-quark scattering processes.

Fig. 4. TPE and $\gamma$-$Z$ exchange corrections to the $A_{PV}$ as functions of $\epsilon$. The left of upper panel for $Q^2=1.0$ GeV$^2$, the right for $Q^2=2.0$ GeV$^2$. The left of lower panel for $Q^2=5.0$ GeV$^2$, the right for $Q^2=9.0$ GeV$^2$. 1$\gamma \times 2\gamma$ (dot line) denotes to the correction only coming from the interference of FIG.1(a) and FIG.1(c), $Z \times 2\gamma$ (dash line) denotes to the correction only coming from interference of FIG.1(b) and FIG.1(c), $2\gamma$ (dash dot line) denotes to $1\gamma \times 2\gamma + Z \times 2\gamma$, Total (solid line) denotes to the full TPE and $\gamma$-$Z$ exchange corrections including the interference of FIG.1(a) and FIG.1(d).

3. Results

In Fig. 4 we show the TPE and $\gamma Z$-exchange corrections to the $A_{PV}$ by plotting $\delta$, defined by

$$A_{PV}(1\gamma + Z + 2\gamma + \gamma Z) = A_{PV}(1\gamma + Z)(1 + \delta), \quad (10)$$
at four different values of $Q^2 = 1.0, 2.0, 5.0$ and $9.0$ GeV$^2$. $A_{PV}(1\gamma + Z)$ denotes the $A_{PV}$ arising from the interference between OPE and $Z$-boson-exchange, i.e., Figs. 1(a) and 1(b) while $A_{PV}(1\gamma + Z + 2\gamma + \gamma Z)$ includes the effects of TPE and $\gamma Z$-exchange. The full results are represented by solid curves. We also show in Fig. 2 the interferences between OPE and TPE ($1\gamma \times 2\gamma$), by dotted lines, as well as that between $Z$-exchange and TPE ($Z \times 2\gamma$) in dashed lines, with their sum ($2\gamma$) denoted by dot-dashed lines.

We first reproduce the result in,\textsuperscript{11} which demonstrates that $1\gamma \times 2\gamma$ contribution partially cancels that of $Z \times 2\gamma$ and their sum are always positive. The similar feature also appears in the result of hadronic model calculation.\textsuperscript{12} However, in\textsuperscript{12} those curves all decrease when $\epsilon$ increase and vanish when $\epsilon=1$ but our GPD’s calculation results show different feature for that and with much higher value. It needs further study to understand the reason of this difference. Furthermore, the solid lines and the dot-dashed lines are found to be almost parallel. It means that their difference, namely the $\gamma$-$Z$ effects, essentially own very mild $\epsilon$ dependencies.

There are still some box diagrams which are not included in the hadronic model calculation. Those are the diagrams with two-$Z$-boson exchange (TZE) diagrams and two-$W$-boson exchange (TWE) exchange. We also calculate these diagrams by using same parton model. In the hadronic model their contributions are expected to be very small, because of the main contribution of loop integral is from low loop momentum due to the inserted form factors functioning as regulators. However, in the partonic model, the main contribution is from high loop momentum and it is straightforward to calculate it in this letter. The sum of TZE and TWE effect in our model is demonstrated at Fig.\textsuperscript{5} It is clear the those effect varies with $Q^2$ in the similar way as other effects, namely it increases when $Q^2$ decrease. Besides, we
find that effect becomes sensitive to $\epsilon$ in high $\epsilon$ region.

4. Conclusions

We conclude that the TBE exchange effects contribute to $A_{PV}$ at a level of one or more per cent. Most of them are very sensitive to $Q^2$ and their magnitudes increase when $Q^2$ decrease, except TPE exchange effects. Hence, the claim made by$^{12}$ that $\gamma$-$Z$ exchange effect is overestimated in the zero momentum transfer approximation is also supported by partonic calculation. Besides, we find that two-$Z$/W-boson-effect is not negligible in the partonic calculation and it is about same magnitude as TPE and $\gamma$-$Z$ exchange effect. These TBE effects are very important when extracting the s-content of nucleon, but the impact of these diagrams to extract strangeness is in the further study.

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