The $\omega\sigma\gamma$-vertex in light cone QCD sum rules

A. Gokalp ∗ and O. Yilmaz †

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

(November 23, 2018)

We investigate the $\omega\sigma\gamma$-vertex and estimate the coupling constant $g_{\omega\sigma\gamma}$ in the framework of the light cone QCD sum rules. We compare our result with the values of this coupling constant deduced from a phenomenological analysis of $\omega \to \pi\pi\gamma$ decays.

PACS numbers: 12.38.Lg;13.40.Hq;14.40.Aq

The determination of the hadronic observables from experimental data, such as coupling constants and form factors, requires physics information at large distances. However, such information cannot be obtained directly from the fundamental QCD Lagrangian in this nonperturbative domain. Among the various nonperturbative methods, traditional QCD sum rules [1] have proved very useful in studying the properties of low-lying hadrons. Further progress has been achieved by an alternative method known as the QCD sum rules on the light cone [2–5] based on the operator product expansion on the light cone, which is an expansion over the twist of the operators rather than dimensions as in the traditional QCD sum rules. In the present work, we study the $\omega\sigma\gamma$-vertex and we estimate the coupling constant $g_{\omega\sigma\gamma}$. The traditional QCD sum rules approach requires for its applicability that in all the channels of the process under consideration the virtuality should be large [1]. However, for the $\omega\sigma\gamma$-vertex, the virtuality in gamma channel is not large. For this reason, in the present calculation we employ the light cone QCD sum rules method.

The low-mass scalar mesons have fundamental importance in the phenomenology of low energy QCD and from the point of view of hadron spectroscopy. Over the years, experimental evidence has accumulated for their existence [6] and different proposals about their nature and about their quark substructure have been put forward. At present, whether they are conventional $q\bar{q}$ states [6], $\pi\pi$, $KK$ molecules [6], or multiquark $q^2\bar{q}^2$ states [6] is still a subject of debate. On the other hand, they are relevant hadronic degrees of freedom, and therefore the role they play in hadronic processes should also be studied besides the questions of their nature.

The $\omega\sigma\gamma$-vertex has importance in different areas of hadron physics. In the calculation of the electromagnetic form factors of the deuteron, $\omega\sigma\gamma$-vertex plays a special role [11]. The $\omega\sigma\gamma$-exchange current compensates the large effect of the $\rho\sigma\gamma$-exchange current contribution which is commonly included in calculations of elastic electron-deuteron scattering, and therefore the knowledge of the coupling constant $g_{\omega\sigma\gamma}$ is essential in such studies. Furthermore, at low energies near threshold in the electromagnetic production reactions of vector mesons on nucleon targets, scalar and pseudoscalar meson exchange mechanisms become important [12], and in particular the coupling constant $g_{\omega\sigma\gamma}$ may be needed as a physical

∗Electronic address: agokalp@metu.edu.tr

†Electronic address: oyilmaz@metu.edu.tr
input for the studies of photoproduction of $\omega$-mesons on nucleons near threshold.

In order to study the $\omega\sigma\gamma$-vertex and to estimate the coupling constant $g_{\omega\sigma\gamma}$, we consider the two point correlation function with photon

$$T_\mu(p,q) = i \int d^4x e^{ip\cdot x} < \gamma(q) | T \{ j_\mu(x) j_\sigma(0) \} | 0 >$$

where $j_\mu = \frac{1}{6}(\pi^\mu \gamma_\mu u^a + \bar{d}^\mu \gamma_\mu d^a)$ and $j_\sigma = \frac{1}{2}(\pi^\sigma u^b + \bar{d}^\sigma d^a)$ are the interpolating currents for $\omega$ and $\sigma$ mesons with $u$ and $d$ denoting up and down quark fields, respectively, and $a$, $b$ are the color indices. The overlap amplitudes of these interpolating currents with the meson states are defined as

$$< 0 | j_\omega^\mu | \omega > = \lambda_\omega u_\mu$$
$$< 0 | j_\sigma | \sigma > = \lambda_\sigma$$

where $u_\mu$ is the polarization vector of $\omega$ meson. The coupling constant $g_{\omega\sigma\gamma}$ is defined through the effective Lagrangian

$$\mathcal{L} = \frac{e}{m_\omega} g_{\omega\sigma\gamma} \partial^\alpha \omega^\beta (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \sigma$$

(3)

describing the $\omega\sigma\gamma$-vertex [12]. The electronic decay width of $\omega$ meson, neglecting the electron mass is given as $\Gamma(\omega \rightarrow e^+ e^-) = (4\pi\alpha^2/3m_\omega^2)\lambda_\omega^2$, using the experimental value of the decay width $\Gamma(\omega \rightarrow e^+ e^-) = (0.60 \pm 0.02)$ KeV of $\omega$ meson [3], we determine the overlap amplitude $\lambda_\omega$ of $\omega$ meson as $\lambda_\omega = (0.036 \pm 0.001)$ GeV$^2$. In a previous work, we estimated the overlap amplitude $\lambda_\sigma$ employing a QCD sum rule analysis of the scalar current by considering the two-point scalar current correlation function as $\lambda_\sigma = (0.12 \pm 0.03)$ GeV$^2$ [13] since this amplitude is not available experimentally. The $< \sigma \gamma | \omega >$ matrix element, using Eq. 3, can be written as $< \sigma(p')\gamma(q)|\omega(p)> = i\frac{e}{m_\omega} g_{\omega\sigma\gamma} K(q^2)(p \cdot q u \cdot \epsilon - u \cdot q p \cdot \epsilon)$ where $q = p - p'$, $\epsilon_\mu$ is the polarization vector of the photon, and $K(q^2)$ is a form factor with $K(0) = 1$.

The theoretical part of the sum rule for the coupling constant $g_{\omega\sigma\gamma}$ is obtained in terms of QCD degrees of freedom by calculating the two point correlator in the deep Euclidean region where $p'^2$ and $(p' + q)^2$ are large and negative. In this calculation the full light propagator with both perturbative and nonperturbative contributions is used, and it is given as [14]

$$iS(x,0) = <0 | T \{ \bar{q}(x)q(0) \} | 0 >$$

$$= i\frac{\beta}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192 m_0^2} \langle \bar{q}q \rangle$$

$$- i g_s \frac{1}{16 \pi^2} \int_0^1 du \left\{ \frac{\beta}{x^2} \sigma_{\mu\nu} G^{\mu\nu}(ux) - 4i u \frac{x_\mu}{x^2} G^{\mu\nu}(ux) \gamma_{\nu} \right\} + ...$$

(4)

where terms proportional to light quark mass $m_u$ or $m_d$ are neglected. After a straightforward computation we obtain

$$T_\mu(p,q) = 2i \int d^4xe^{ip\cdot x} A(x_\sigma g_{\mu\tau} - x_\tau g_{\mu\sigma}) < \gamma(q) | \bar{q}(x)\sigma^{\tau\sigma}q(0) | 0 >$$

(5)

where $A = i/(2\pi^2 x^4)$, and higher twist corrections are neglected since they are known to make a small contribution [3]. In order to evaluate the two point correlation function further,
we need the matrix elements $<\gamma(q)|\overline{q}\sigma_{\alpha\beta}q|0>$. These matrix elements are defined in terms of the photon wave functions $[15-17]$

$$<\gamma(q)|\overline{q}\sigma_{\alpha\beta}q|0> = i\epsilon_q <\gamma_q> \int_0^1 du e^{iuqx} \{(\epsilon_\alpha q_\beta - \epsilon_\beta q_\alpha) \left[\chi(\phi(u) + x^2[g_1(u) - g_2(u)]\right]
+ [q \cdot x(\epsilon_\alpha x_\beta - \epsilon_\beta x_\alpha) + \epsilon \cdot x(x_\alpha q_\beta - x_\beta q_\alpha)] g_2(u)\} \quad , \quad (6)$$

where the parameter $\chi$ is the magnetic susceptibility of the quark condensate and $e_q$ is the quark charge, $\phi(u)$ stand for the leading twist-2 photon wave function, while $g_1(u)$ and $g_2(u)$ are the two-particle photon wave functions of twist-4. In the further analysis the path ordered gauge factor is omitted since we work in the fixed point gauge $[18]$.

The two point function $T_{\mu}(p, q)$ satisfies a dispersion relation and we saturate this dispersion relation by inserting a complete set of one hadron states into the correlation function. This way we construct the phenomenological part of the two point correlation function as

$$T_{\mu}(p, q) = \frac{<\sigma\gamma|\omega><\omega|j_{\mu}|0><0|j_{\sigma}|\sigma>}{(p^2 - m_\omega^2)(p'^2 - m_\sigma^2)} + ... \quad \text{(7)}$$

where the contributions from the higher states and the continuum starting from some threshold $s_0$ are denoted by dots. In order to take these contributions into account we invoke the quark-hadron duality prescription and replace the hadron spectral density with the spectral density calculated in QCD. In accordance with the QCD sum rules method strategy, we then equate the two representations of the two point correlation function, theoretical and phenomenological, and construct the corresponding sum rule for the coupling constant $g_{\omega\sigma\gamma}$.

After evaluating the Fourier transform and then performing the double Borel transformation with respect to the variables $Q_1^2 = -p'^2$ and $Q_2^2 = -(p' + q)^2$, we finally obtain the following sum rule for the coupling constant $g_{\omega\sigma\gamma}$

$$g_{\omega\sigma\gamma} = \frac{1}{6} \frac{m_\omega(e_u + e_d)}{m_\omega \lambda_\omega \lambda_\sigma} \frac{<\pi u>}{e^{m_\pi^2/M_1^2} e^{m_\pi^2/M_2^2}} \left\{ -M^2 \chi(\phi(u_0)f_0(s_0/M^2) + 4g_1(u_0) \right\} \quad (8)$$

where the function $f_0(s_0/M^2) = 1 - e^{-s_0/M^2}$ is the factor used to subtract the continuum, $s_0$ being the continuum threshold, and

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2} \quad , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \quad \text{(9)}$$

with $M_1^2$ and $M_2^2$ are the Borel parameters.

The various parameters we adopt for the numerical evaluation of the sum rule for the coupling constant $g_{\omega\sigma\gamma}$ are $<\pi u> = \pm 0.014 \pm 0.002 \text{ GeV}^3$ $[19]$ for the vacuum condensate, and $\chi = -4.4 \text{ GeV}^{-2}$ $[16,20]$ for the magnetic susceptibility of the quark condensate, $\lambda_\omega = (0.036 \pm 0.001) \text{ GeV}^2$ as determined from the experimental electronic decay width of $\omega$ meson as discussed above, $\lambda_\sigma = (0.12 \pm 0.03) \text{ GeV}^2$ which was determined using QCD sum rules method $[13]$, and $m_\omega = 0.782 \text{ GeV}$, $m_\sigma = 0.5 \text{ GeV}$. The leading twist-2 photon wave function is given as $\phi(u) = 6u(1-u)$ and the two-particle photon wave function of twist-4 is given by the expression $g_1(u) = -(1/8)(1-u)(3-u)$ $[16]$. We then study the dependence of the sum rule for the coupling constant $g_{\omega\sigma\gamma}$ on the continuum threshold $s_0$ and on the Borel parameters $M_1^2$ and $M_2^2$ by considering independent variations of these parameters. We find
that the sum rule is quite stable for $M_1^2 = 1.2$ GeV$^2$ and for $1.0$ GeV$^2 < M_2^2 < 1.4$ GeV$^2$ where these limits on $M_2^2$ are within the allowed interval for the vector channel [21]. The variation of the coupling constant as a function of the Borel parameter $M_2^2$ for the values of $s_0 = 1.1, 1.2, 1.3$ GeV$^2$ with $M_1^2 = 1.2$ GeV$^2$ is shown in Fig. 1. The sources contributing to the uncertainty in the estimated value of the coupling constant are those due to variations in the Borel parameters $M_1^2$ and $M_2^2$, in the threshold parameter $s_0$, in the estimated values of the vacuum condensate and the magnetic susceptibility of the quark condensate, and in the values of the overlap amplitudes $\lambda_\sigma$ amd $\lambda_\omega$. We take these uncertainties by a conservative estimate into account, we obtain the coupling constant $g_{\omega\sigma\gamma}$ as $|g_{\omega\sigma\gamma}| = (0.72 \pm 0.08)$.

---

**FIG. 1.** The coupling constant $g_{\omega\sigma\gamma}$ as a function of the Borel parameter $M_2^2$ for different values of the threshold parameter $s_0$ with $M_1^2=1.2$ GeV$^2$.

In a previous work [22], we studied the $\omega \to \pi\pi\gamma$ decays by adding the amplitude of $\sigma$-meson intermediate state to the amplitude calculated within the framework of chiral perturbation theory and vector meson dominance. We used the experimental value for the decay rate $\Gamma(\omega \to \pi^0\pi^0\gamma)$ and we calculated the coupling constant $g_{\omega\sigma\gamma}$ as a function of the $\sigma$ meson parameters $m_\sigma$ and $\Gamma_\sigma$. If we use the values for these parameters the values $m_\sigma = 478$ MeV and $\Gamma_\sigma = 324$ MeV as suggested by the Fermilab E791 Collaboration [7], we then obtain the coupling constant $g_{\omega\sigma\gamma}$ in the framework of this phenomenological analysis as $g_{\omega\sigma\gamma} = 0.13$ and $g_{\omega\sigma\gamma} = -0.27$, since the theoretical calculation of the decay rate results in a quadratic expression for the coupling constant $g_{\omega\sigma\gamma}$. We note that these values are consistent with the interval of values for this coupling constant that we deduced from the experimental upper limit of the $\Gamma(\omega \to \pi^+\pi^-\gamma)$ decay rate which is $1.20 > g_{\omega\gamma} > -1.34$ for the above values of the $\sigma$ meson parameters [22]. The result of our present calculation utilizing QCD sum rules method is in reasonable agreement with the results obtained from
the phenomenological analysis of the $\omega \rightarrow \pi\pi\gamma$ decays. However, we note that in the present work we consider the $\sigma$ meson in the narrow resonance limit and do not take its finite width into account. Since $\sigma$ meson has a large width \cite{7}, the use of simple pole spectrum is somewhat doubtful. The effect of the width of the resonance can be taken into account within the framework of traditional QCD Laplace sum rules method, but in the light cone QCD sum rules which forms the appropriate approach for the present problem it seems that the Laplace transform method cannot be used. Therefore, in our analysis the error that is induced by the variation of threshold, which is included in the final quoted error above, may be considered as an estimation of the error resulting from the narrow width approximation.

\begin{thebibliography}{99}
\bibitem{1} M. A. Shifman, A. I. Vainstein, V. I. Zakharov, Nucl. Phys. \textbf{B147}, 385, 448, 519 (1979).
\bibitem{2} V. L. Chernyak, A. R. Zhitnitsky, Phys. Rep. \textbf{112}, 173 (1984).
\bibitem{3} I. I. Balitskii, V. M. Braun, A. V. Kolesnichenko, Nucl. Phys. \textbf{B312}, 509 (1998).
\bibitem{4} P. Ball, V. M. Braun, Phys. Rev. \textbf{D58}, 094016 (1998).
\bibitem{5} V. M. Braun, Planetary talk given at the IV th International Workshop on Progress in heavy Quark Physics, Rostock, Germany, 20-22 September 1997, hep-ph/9801222.
\bibitem{6} Particle Data Group, D. E. Groom et al., Eur. Phys. J. \textbf{C15}, 1 (2000).
\bibitem{7} E. M. Aitala et al. (E791 collaboration), Phys. Rev. Lett. \textbf{86}, 770 (2001).
\bibitem{8} E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, J. E. Ribeiro, Z. Phys. \textbf{C30}, 615 (1986); N. A. Törnqvist and M. Roos, Phys. Rev. Lett. \textbf{76}, 1575 (1996).
\bibitem{9} J. Weinstein and N. Isgur, Phys. Rev. \textbf{D41}, 2236 (1990).
\bibitem{10} R. J. Jaffe, Phys. Rev. \textbf{D15}, 267 (1977); \textbf{D17}, 1444 (1978).
\bibitem{11} M. Garcon, J. W. Van Orden, nucl-th/0102049.
\bibitem{12} B. Friman and M. Soyeur, Nucl. Phys. \textbf{A600}, 477 (1996).
\bibitem{13} A. Gokalp, O. Yilmaz, Phys. Rev. \textbf{D64}, 034012 (2001).
\bibitem{14} V. M. Belyaev, V. M. Braun, A. Khodjamirian, R. Ruckl, Phys. Rev \textbf{D51}, 6177 (1995).
\bibitem{15} G. Eilam, I. Halperin, R. R. Mendel, Phys. Lett.\textbf{B361}, 137 (1995).
\bibitem{16} A. Ali, V. M. Braun, Phys. Lett.\textbf{B359}, 223 (1995).
\bibitem{17} A. Khodjamirian, G. Stoll, D. Wyler, Phys. Lett.\textbf{B358}, 129 (1995).
\bibitem{18} A. V. Smilga, Sov. J. Nucl. Phys. \textbf{35}, 271 (1982).
\bibitem{19} P. Colangelo, A. Khodjamirian, in \textit{Boris Ioffe Festschrift ”At the Frontier of Particle Physics, Handbook of QCD"}, edited by M. Shifman, World Scientific, Singapore (2001) 1495, (hep-ph/0010175).
\bibitem{20} V. M. Belyaev, Ya. I. Kogan, Yad. Fiz. \textbf{40},1035 (1984) [Sov. J. Nucl. Phys. \textbf{40}, 659 (1984)].
\bibitem{21} V. L. Eletsky, B. L. Ioffe, Ya. I. Kogan, Phys. Lett.\textbf{B 122}, 423 (1983).
\bibitem{22} A. Gokalp, O. Yilmaz, Phys. Lett. \textbf{B 494}, 69 (2000).
\end{thebibliography}