A Probe of New Physics in Top Quark Pair Production at $e^-e^+$ Colliders

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ABSTRACT

We describe how to probe new physics through examination of the form factors describing the $Z-t-\bar{t}$ couplings via the scattering process $e^-e^+ \rightarrow t\bar{t}$. We focus on experimental methods on how the top quark momentum can be determined and show how this can be applied to select polarized samples of $t\bar{t}$ pairs through the angular correlations in the final state leptons. We also study the dependence on the energy and luminosity of an $e^-e^+$ collider to probe a CP violating asymmetry at the $10^{-2}$ level.
1. **Introduction**

Large CP violating effects are required to have the cosmological baryon asymmetry produced at the weak phase transition. If such effects exist, can they be probed in hadron and electron colliders? Schmidt and Peskin studied a CP violation mechanism suggested by Weinberg and concluded that it is possible to probe a CP violating asymmetry in high–energy hadron collider experiments via the production of top quark–antiquark pairs. A detailed study on determining what observables are optimum for measuring the form factors governing the electric and magnetic dipole moments has been performed by Atwood and Soni. In this paper it is shown that to probe the same level of CP violating asymmetry at the Next Linear Collider (NLC) with $\sqrt{s} = 500$ GeV requires $3 \times 10^4 \text{fb}^{-1}$ to produce about $2 \times 10^7 \ t\bar{t}$ pairs, which is about a hundred times larger than the proposed integrated luminosity. (We use NLC to represent a generic $e^-e^+$ supercollider.)

To slightly improve the measurement on the CP violating asymmetry in $e^-e^+$ collisions, we propose a different experimental observable from the one suggested in Ref. 3 by measuring the polarization of the $t$ and $\bar{t}$ in the process $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^+\nu\bar{\nu}$. We show that the efficiency of untangling the different polarizations for the $t$ and $\bar{t}$ increases as the energy of the $e^-e^+$ collider increases at the cost of decreasing the total event rate. The net effect is that a 500 GeV machine might do a better CP violation measurement than a 1 TeV machine.

In present experimental data, the $Z - t - \bar{t}$ coupling was measured via its contribution to the decay of $Z \rightarrow b\bar{b}$ and the mass shift of the $Z$ boson at the loop level. It was argued that based on the electroweak chiral lagrangian analysis, the form factors describing the $Z - t - \bar{t}$ interaction can be different from the ones predicted by the Standard Model (SM). We propose measuring these form factors by considering the polarization of the $t$ and $\bar{t}$. We show how to identify the polarization of the $t$ and $\bar{t}$ and how to reconstruct the momenta of the $t$ and $\bar{t}$ via the decay mode $t\bar{t} \rightarrow bl^+\nu l\bar{b} \bar{\nu}$, where $l$ can be either an electron or a muon. We conclude that the form factors can be tested up to a few percent for the NLC with a luminosity of 50 fb$^{-1}$. For simplicity, in this paper we follow the argument in Ref. 7 and assume that the dominant non–standard form factors are only present in the $Z - t - \bar{t}$ coupling and treat the couplings of the photon and $W^\pm$ boson with fermions as described in the SM.

This paper is organized as follows. In Section 2 we present the interaction lagrangian and its corresponding helicity amplitudes which describe the production of $t\bar{t}$ pairs via $e^-e^+$ annihilation using general form factors. There, we also present the Born level cross sections in the Standard Model. In Section 3 we review the requirements for discussing CP violation and in Section 4 we examine how well the form factors that govern the vector and axial vector interactions can be measured. In Section 5 we discuss how to reconstruct the top quark momentum in the $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^+\nu\bar{\nu}$ and $e^-e^+ \rightarrow t\bar{t} \rightarrow l^+ \text{ jets}$ mode including the effects of initial state radiation. In Section 6 we discuss top quark polarization and how to select polarized event samples, then in Section 7 we give our conclusions.
2. The General Form Factors and Helicity Amplitudes

2.1. Conventions and Amplitudes

In a recent paper\(^8\) we studied the most general form factors for the coupling of \(t\) and \(\bar{t}\) with either of the vector bosons \(\gamma\) or \(Z\). Decomposing this interaction over the basis given by the Clifford algebra in terms of the form factors \(F_{1,2,3}^{R,L}\) yields the general lagrangian,

\[
\mathcal{L}_{\text{int}} = g \left[ Z_{\mu} \bar{t}^\mu \left( F_1^{Z(L)} P_- + F_1^{Z(R)} P_+ \right) t - \frac{1}{v} \partial_\nu Z_{\mu} \bar{t} \sigma^{\mu\nu} \left( F_2^{Z(L)} P_- + F_2^{Z(R)} P_+ \right) t \right.
\]

\[
+ \partial_\mu Z_{\mu} \bar{t} \left( F_3^{Z(L)} P_- + F_3^{Z(R)} P_+ \right) t + A_{\mu} \bar{t} \gamma^\mu \left( F_1^{\gamma(L)} P_- + F_1^{\gamma(R)} P_+ \right) t
\]

\[
- \frac{1}{v} \partial_\nu A_{\mu} \bar{t} \sigma^{\mu\nu} \left( F_2^{\gamma(L)} P_- + F_2^{\gamma(R)} P_+ \right) t + \partial_\mu A_{\mu} \bar{t} \left( F_3^{\gamma(L)} P_- + F_3^{\gamma(R)} P_+ \right) t \right],
\]

where \(P_\pm = \frac{1}{2}(1 \pm \gamma_5), i\sigma^{\mu\nu} = -\frac{1}{2} \left[ \gamma^\mu, \gamma^\nu \right]\), and \(v = (\sqrt{2}G_F)^{-1/2} \sim 246\) GeV.

Applying the Gordon decomposition, the vertex factors from the \(Z-t-\bar{t}\) and \(\gamma-t-\bar{t}\) interactions become

\[
\Gamma_\gamma^Z = \frac{g}{2} \bar{t}^\gamma \left( A_Z - B_Z \gamma^5 \right) + \frac{p(t)^\mu - p(\bar{t})^\mu}{2} \left( C_Z - D_Z \gamma^5 \right) + \frac{p(t)^\mu + p(\bar{t})^\mu}{2} (E_Z - F_Z \gamma^5)\big| t, \\
\Gamma_\gamma^\gamma = \frac{g}{2} \bar{t}^\gamma \left( A_\gamma - B_\gamma \gamma^5 \right) + \frac{p(t)^\mu - p(\bar{t})^\mu}{2} \left( C_\gamma - D_\gamma \gamma^5 \right) + \frac{p(t)^\mu + p(\bar{t})^\mu}{2} (E_\gamma - F_\gamma \gamma^5)\big| t,
\]

where

\[
A_{[\gamma,Z]} = F_1^{[\gamma,Z](L)} + F_1^{[\gamma,Z](R)} - \frac{2m_t}{2} \left( F_2^{[\gamma,Z](L)} + F_2^{[\gamma,Z](R)} \right),
\]

\[
B_{[\gamma,Z]} = F_1^{[\gamma,Z](L)} - F_1^{[\gamma,Z](R)},
\]

\[
C_{[\gamma,Z]} = \frac{2}{v} \left( F_2^{[\gamma,Z](L)} + F_2^{[\gamma,Z](R)} \right),
\]

\[
D_{[\gamma,Z]} = \frac{2}{v} \left( F_2^{[\gamma,Z](L)} - F_2^{[\gamma,Z](R)} \right),
\]

\[
E_{[\gamma,Z]} = - 2 \left( F_3^{[\gamma,Z](L)} + F_3^{[\gamma,Z](R)} \right),
\]

\[
F_{[\gamma,Z]} = 2 \left( F_3^{[\gamma,Z](L)} - F_3^{[\gamma,Z](R)} \right).
\]

(2.3)

Since we ignore the masses of the incoming electron and positron, the \(F_3\) form factors in Eq. (2.1) (i.e., the \(E_{[\gamma,Z]}\) and \(F_{[\gamma,Z]}\) in Eq. (2.3), whose vector coefficient is the four-momentum of either the \(\gamma\) or \(Z\) vector boson in our problem) do not contribute, however, we note that current conservation for the photon demands that

\[
B_\gamma = - \frac{(p(t) + p(\bar{t}))^2}{2m_t} \left( F_3^{\gamma(L)} - F_3^{\gamma(R)} \right).
\]

(2.4)
In the SM, at the Born level, the form factors are

\[
A_\gamma = \frac{4}{3} \sin \theta_W, \quad A_Z = \frac{1}{2 \cos \theta_W} \left( 1 - \frac{8}{3} \sin^2 \theta_W \right), \quad \theta_W
\]

\[
B_\gamma = 0, \quad B_Z = \frac{1}{2 \cos \theta_W}, \quad \gamma - e^- e^+ \text{ vertices are given by}
\]

\[
\Gamma_{Z\gamma\gamma}^\mu = ig \gamma^\mu \left( e_L^Z P_- + e_R^Z P_+ \right) \quad \text{and} \quad \Gamma_{Zee}^\mu = ig \gamma^\mu \left( e_L^e P_- + e_R^e P_+ \right),
\]

where the SM values for these couplings are

\[
e_L^Z = \frac{1}{\cos \theta_W} \left( -\frac{1}{2} + \sin^2 \theta_W \right), \quad e_R^Z = \frac{1}{\cos \theta_W} \sin^2 \theta_W, \quad \text{and} \quad e_L^e = e_R^e = -\sin \theta_W.
\]

The helicity amplitudes for \( e^- e^+ \to t\bar{t} \) are represented by \( (h_{e^-}, h_{e^+}, h_t, h_{t\bar{t}}) \), where \( h_{e^-} = -, + \) respectively indicates a left–handed and a right–handed electron. Apart from the common factor

\[
\frac{2g^2 E}{s - M_Z^2},
\]

with \( s = 4E^2 \), the nonvanishing helicity amplitudes from the diagram mediated by the \( Z \)-boson in \( e^- e^+ \to t\bar{t} \) are, in terms of these form factors \( A_Z, B_Z, C_Z \) and \( D_Z \)

\[
\begin{align*}
(- + - -)_Z &= e_L \sin \theta_t \left[ m_t A_Z - K^2 C_Z + E K D_Z \right], \\
(- + - +)_Z &= - e_L (1 + \cos \theta_t) \left[ E A_Z + K B_Z \right], \\
(- + + -)_Z &= e_L (1 - \cos \theta_t) \left[ E A_Z - K B_Z \right], \\
(- + + +)_Z &= e_L \sin \theta_t \left[ -m_t A_Z + K^2 C_Z + E K D_Z \right], \\
(+ - - -)_Z &= e_R \sin \theta_t \left[ m_t A_Z - K^2 C_Z + E K D_Z \right], \\
(+ - - +)_Z &= e_R (1 - \cos \theta_t) \left[ E A_Z + K B_Z \right], \\
(+ - + -)_Z &= - e_R (1 + \cos \theta_t) \left[ E A_Z - K B_Z \right], \\
(+ - + +)_Z &= e_R \sin \theta_t \left[ -m_t A_Z + K^2 C_Z + E K D_Z \right].
\end{align*}
\]

In the above formulas \( E = \sqrt{s}/2 \) is half the center of mass energy of the \( e^- e^+ \) annihilation and \( K = \sqrt{E^2 - m_t^2} \). The nonvanishing Born helicity amplitudes from the photon propagator diagram, \( (h_{e^-}, h_{e^+}, h_t, h_{t\bar{t}}) \), can be easily obtained from the results in Eq. (2.8) by
setting $e_L = e_R = -\sin \theta_W$ and replacing the $A, B, C, D$ form factors for the $Z$ boson with those for the photon. The common factor in this case is

$$\frac{2g^2E}{s}.$$  

The helicity amplitudes for the process $e^-e^+ \rightarrow t\bar{t}$ are therefore obtained by summing the contributions from these two diagrams:

$$(h_{e-}, h_{e+}, h_t, h_{\bar{t}}) = 2g^2E\left[\frac{(h_{e-}, h_{e+}, h_t, h_{\bar{t}})_Z}{s - M_Z^2} + \frac{(h_{e-}, h_{e+}, h_t, h_{\bar{t}})_\gamma}{s}\right].$$  

(2.9)

When calculating the cross section at the tree level, a color factor of 3 should be included for the top quark pair production. The spin average factor $\frac{1}{22}$ should also be included for an unpolarized $e^-e^+$.

**Table 2.1:** Born level production rates for $e^-e^+ \rightarrow t\bar{t}$ at $\sqrt{s} = 500$ GeV and 1 TeV using $m_t = 140$ GeV listed according to $t$ and $\bar{t}$ polarizations.

| $t\bar{t}$ Polarizations | $\sigma(\sqrt{s} = 500$ GeV) in fb | $\sigma(\sqrt{s} = 1$ TeV) in fb |
|---------------------------|-----------------------------------|----------------------------------|
| $RR = LL$                | 37.3                              | 2.69                             |
| $RL$                     | 225                               | 66.7                             |
| $LR$                     | 342                               | 104                              |
| $TT(\uparrow \downarrow)_{\text{in}} = TT(\downarrow \uparrow)_{\text{in}}$ | 93.6                              | 29.8                             |
| $TT(\downarrow \uparrow)_{\text{in}}$ | 290                               | 67.2                             |
| $TT(\uparrow \downarrow)_{\text{in}}$ | 164                               | 49.6                             |
| $TT(\uparrow \downarrow)_{\perp} = TT(\downarrow \uparrow)_{\perp}$ | 131                               | 32.5                             |
| $TT(\downarrow \uparrow)_{\perp} = TT(\uparrow \downarrow)_{\perp}$ | 190                               | 55.7                             |
| $t$ is $R$, $\bar{t}$ unpolarized | 262                               | 69.4                             |
| $t$ is $L$, $\bar{t}$ unpolarized | 379                               | 107                              |
| top only is $T(\uparrow)_{\text{in}}$ | 258                               | 79.4                             |
| top only is $T(\downarrow)_{\text{in}}$ | 383                               | 97.0                             |
| Unpolarized $t\bar{t}$ | 641                               | 176                              |
2.2. Cross Sections in the Standard Model

The production rates of polarized $t\bar{t}$ pairs at 500 GeV and 1 TeV $e^-e^+$ colliders are tabulated in Table 2.1 for a 140 GeV top quark. $LL$ (RR) denotes the production rate of $t_L\bar{t}_L$ ($t_R\bar{t}_R$) events, where $L$ labels a left-handed helicity and $R$ labels a right-handed helicity. Likewise, $RL$ ($LR$) denotes the production rate of $t_R\bar{t}_L$ ($t_L\bar{t}_R$) events. Besides the helicity cross sections, we have also included the cross sections for transverse polarizations of the $t$ and $\bar{t}$ (indicated by $T$). In the first column of Table 2.1, the subscript $\perp$ or $\parallel$ respectively indicate whether the cross sections refer to transverse polarizations of the quark pair in the $e^-e^+\rightarrow t\bar{t}$ scatter plane or perpendicular to it. If the arrows are (anti)aligned, the $t$ and $\bar{t}$ spins are (anti)aligned. An upward(downward) pointing arrow in the case of polarizations perpendicular to the scatter plane means the spin is directed in the $+\hat{y}$ ($-\hat{y}$) direction where $\hat{y} \equiv \hat{z} \times \vec{p}(t)$. (Recall the incoming electron beam is moving in the $+\hat{z}$ direction.) An upward(downward) pointing arrow in the case of polarizations in the scatter plane means the spin is directed perpendicular to the top momentum and toward the $-\hat{z}$ ($+\hat{z}$) direction.

Since $LR$ and $RL$ only depend on the form factors $A$ and $B$, it is possible to untangle the contributions from $A$ and $B$ by studying the polarized states $t_L\bar{t}_R$ and $t_R\bar{t}_L$. One important issue is how well the NLC can bound these form factors for a given integrated luminosity.

If one can separate the polarization states of $LL$ and $RR$ from the others, then one can measure the asymmetry

$$A_1 = \frac{LL - RR}{LL + RR}$$

(2.10)

or

$$A_2 = \frac{LL - RR}{LL + RR + LR + RL}.$$  

(2.11)

These asymmetries are important for studying CP violation.

3. CP Violation

3.1. Form Factors and CP Violation

If the form factor $D$ in Eq. (2.2) is not zero, the theory is CP violating. \[ Specifically, from Eq. (2.8) it is apparent that the difference between the $RR$ and $LL$ cross sections is linearly dependent on $D$ through

$$(RR - LL) \propto Re\left[\left(\frac{A_Z}{s-m^2_Z} + \frac{A_\gamma}{s}\right)D_Z^*\right],$$

(3.1)

where the “*” indicates complex conjugation and in this equation the photon is assumed to preserve its SM behavior, namely, $D_\gamma = 0$. From the helicity amplitudes listed in Eq. (2.8), one can construct a quantity sensitive to CP violation, $A_1 = \frac{LL-RR}{LL+RR}$, to measure the real part of $D$. The imaginary part of $D$ can be examined by studying the transverse polarization of the top quark pairs perpendicular to the scatter plane. The advantage of being able to measure the polarization states of the top quark pair is that one can measure $A_1$ instead of $A_2 = \frac{LL-RR}{LL+RR+LR+RL}$ which may lose some of its effectiveness due
Figure 3.1: This plot displays the variation of the asymmetry $A_1$ in Eq. (2.10) with the mass of the Higgs boson for Weinberg’s model using $\sqrt{s} = 500, 1000$ GeV and $m_t = 140$ GeV.

to the dilution acquired by having a larger denominator from the $RL$ and $LR$ events when studying the CP violation effects.

In Weinberg’s model there is a dependence on the mass of the Higgs boson that enters in the form factor $D$, 

$$
\text{Re}\{D_{[\gamma,Z]}\} = 2\text{Im}\{Z_2\} A_{[\gamma,Z]}^{\text{Born}} \left( \frac{m_t^4}{4\pi v^3 s\beta} \right) \left[ 1 - \frac{m_H^2}{s\beta^2} \ln \left( 1 + \frac{s\beta^2}{m_H^2} \right) \right],
$$

where $\beta = \sqrt{1 - 4m_t^2/s}$ and $A_{[\gamma,Z]}^{\text{Born}}$ refers to the SM values at the Born level for $A_{\gamma}$ and $A_Z$ given in Eq. (2.5). For models with two or more Higgs doublets Weinberg showed that $2\text{Im}\{Z_2\} \leq \sqrt{2}$ with a reasonable choice of Higgs vacuum expectation values. When we compute $A_1$ for a 500 GeV machine with $m_t = 140$ GeV (see Fig. 3.1), we find $A_1 = -1.57\%$ for $m_H = 100$ GeV and $A_1 = -0.43\%$ for $m_H = 1$ TeV. The percentage decrease in the magnitude of the CP violating asymmetry is less for a heavier top quark, but the general trend is a decrease in $|A_1|$ as the Higgs boson mass gets larger.

Using Weinberg’s model, it is apparent that there is also a variation of the CP violating asymmetry with the beam energy of the collider. In Fig. 3.2 we plot $A_1$ versus the center of mass energy for Higgs boson masses of 100, 400 and 1000 GeV using $m_t = 140$ GeV. Comparing a 1 TeV machine with a 500 GeV machine we see that the $A_1$ asymmetry value is slightly larger in magnitude for the higher energy machine. It is important to note, however, that the $RR$ and $LL$ event rates, which are the ones used for studying CP violation, are smaller by an order of magnitude for the 1 TeV machine when compared with the 500 GeV machine (see Table 2.1). Because of this reduction in statistics, the 500 GeV machine is preferred for testing CP violation.

The question is, “What luminosity is needed to measure the CP violating asymmetry $A_1$ at the $10^{-2}$ level at the NLC?”
3.2. Requirements for studying CP Violation

To test CP violation, we use the dilepton mode,

\[ e^- e^+ \rightarrow t \bar{t} \rightarrow b \bar{b} l^+ l^- \nu \bar{\nu}, \tag{3.3} \]

which has a branching ratio of \( \frac{2.11}{219} = \frac{2}{81} \) for \( l = e, \mu \). At the Born Level in the Standard Model the unpolarized cross section for \( e^- e^+ \rightarrow t \bar{t} \) with \( m_t = 140 \) GeV is (see Table 2.1)

\[ \sigma (e^- e^+ \rightarrow t \bar{t}) = \begin{cases} 641 \text{ fb} & \text{for } \sqrt{s} = 500 \text{ GeV}, \\ 176 \text{ fb} & \text{for } \sqrt{s} = 1 \text{ TeV}, \end{cases} \tag{3.4} \]

while the cross section for the production of \( t \) and \( \bar{t} \) with identical helicities is

\[ \sigma (e^- e^+ \rightarrow t_L \bar{t}_L) = \sigma (e^- e^+ \rightarrow t_R \bar{t}_R) = \begin{cases} 37 \text{ fb} & \text{for } \sqrt{s} = 500 \text{ GeV}, \\ 2.7 \text{ fb} & \text{for } \sqrt{s} = 1 \text{ TeV}. \end{cases} \tag{3.5} \]

To measure an asymmetry \( A_1 \sim 10^{-2} \) requires that the number of events, \( N \), be large enough so that the statistical error in the measurement, \( \frac{1}{\sqrt{N}} \), is less than \( 10^{-2} \). For the optimistic value of \( \frac{1}{\sqrt{N}} \approx 10^{-2} \) we can estimate the luminosity required to observe this CP violating asymmetry,

\[ L = \frac{81N}{2\epsilon [\sigma (e^- e^+ \rightarrow t_R \bar{t}_R) + \sigma (e^- e^+ \rightarrow t_L \bar{t}_L)]}, \tag{3.6} \]

where \( \epsilon \) is the efficiency of finding \( LL \) or \( RR \) events. Assuming we are able to solve for the kinematics of the dilepton mode in 70% of the events (see Section 5) and the acceptance of the kinematic cuts in selecting \( LL \) or \( RR \) events is \( \frac{1}{4} \) (see Section 6), then \( \epsilon \approx 0.18 \). This implies the required luminosity to observe CP violation through \( A_1 \sim 10^{-2} \) in top quarks of \( m_t = 140 \) GeV is \( L \approx 3 \times 10^4 \text{ fb}^{-1} \) at \( \sqrt{s} = 500 \) GeV (i.e., about \( 2 \times 10^7 t \bar{t} \) pairs are required).

4. How Well Can Form Factors be Measured at the NLC?

The bounds determined from the experimental data, which limit the size of the form factors governing the top quark interaction with the vector boson, do not eliminate the possibility of nonuniversal couplings, as discussed by Peccei and Zhang \[ \text{[7]} \] making it important to test the couplings as completely as we can. When determining form factors for an s-channel process where the intermediate state is a vector particle, one benefits by the information conveyed by the variation in the angular distribution of the final state.\[ \text{[10]} \] From Eq. (2.8) it is clear the \( t \bar{t} \) helicity states are produced with a polar angle distribution of either \( \sin^2 \theta \) or \( (1 \pm \cos \theta)^2 \), so noting that the form factors carry no angular dependences, it is possible to use the methods described above to solve for the direction of motion for the top quark and then use the differential cross section as described through the parameters \( c_0, c_+, c_- \) by

\[ \frac{d\sigma}{d\cos \theta} = \frac{\sqrt{E^2 - m_t^2}}{32\pi s E} \left[ c_0 \sin^2 \theta + c_+ (1 + \cos \theta)^2 + c_- (1 - \cos \theta)^2 \right]. \tag{4.1} \]
Figure 4.1: In $d\sigma/d \cos\theta$ we see the dependence of the production rates for polarized $t\bar{t}$ states on the top quark polar angle. This angle $\theta$ is measured with respect to the incoming $e^-$ beam which is taken to move along the $+\hat{z}$ direction.

Figure 4.2: These plots present the angular correlation between the charged leptons produce from the polarized $t$ and $\bar{t}$ decays in $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^-l^+\nu\bar{\nu}$. Boosting to the individual center of mass frame for the decays of the $t$ and $\bar{t}$, we plot the cosine of the angle between the momentum of each charge lepton and the boost axis as defined by the top momentum. Note how each combination of $t,\bar{t}$ helicities (RR,LL,RL,LR) occupy separate quadrants.

From Eq. (2.8) and Eq. (4.1) we obtain

$$c_0 = c_{0+} + c_{0-},$$

$$c_{0\pm} = 6 \left( g^2 E \right)^2 \left\{ \left| e_Z^Z m_t A_Z - K^2 C_Z \pm EK D_Z \frac{s - m_Z^2}{s} \right|^2 + \left| e_L^\gamma m_t A_\gamma - K^2 C_\gamma \pm EK D_\gamma \right|^2 \right\},$$

$$c_{\pm} = 3 \left( g^2 E \right)^2 \left\{ \left| e_L^Z E A_Z \pm KB Z \frac{s - m_Z^2}{s} \right|^2 \right. + \left. \left| e_L^\gamma E A_\gamma \pm KB_\gamma \frac{s - m_Z^2}{s} \right|^2 \right\}.$$ (4.2)

Untangling the form factors from such distributions is not trivial for various reasons. For instance, it is possible for there to be cancellations among the form factors making deviations from the SM difficult to observe. One result of these complications is that given a certain polar angle distribution described by a finite number of data points carrying some doubt in precision, there are different sets of values for the form factors that can satisfy the constraints such a distribution gives. With this in mind, nevertheless, we can ask how well experiments at the NLC might be able to constrain the form factors.

We now proceed to ask how well the fit to the angular distribution can do to get the form factors. We show in Fig. 4.1 the lowest order SM production rates for polarized $t\bar{t}$ pairs as a function of the scattered polar angle, $\theta_t$, measured in the center–of–mass frame of the $t\bar{t}$ pair at the NLC. In Table 4.1 and Table 4.2 we present bounds that represent a 68% and a 90% confidence level on the range for determining the form factors for the $Z-t-\bar{t}$ interaction. These bounds were obtained using MINUIT to fit the polar angle distribution of the top quark as generated for the NLC ($\sqrt{s}$=500 GeV, with 50fb$^{-1}$).
**Figure 4.3:** The distribution for the production of polarized $t\bar{t}$ pairs, $d\sigma/d[(1-r)/(1+r)]$ vs. $(1-r)/(1+r)$ for $r \equiv E(l^+)/E(l^-)$ when angular cuts are applied to the final state $l^+$ and $l^-$ in the respective decay frames of the $t$ and $\bar{t}$.

**Table 4.1:** The upper and lower bounds indicate the statistical range within which we can determine the form factors from the angular distribution of the top quark in $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b} l^+ l^- \nu \bar{\nu}$ given a $50 \text{ fb}^{-1}$ luminosity at the NLC within a 68% confidence level.

| Form Factor | SM VALUE | Upper | Lower |
|-------------|----------|-------|-------|
| $F_{1Z}^{(L)}$ | 0.395 | 0.419 | 0.370 |
| $F_{1Z}^{(R)}$ | -0.175 | -0.153 | -0.197 |
| $F_{2Z}^{(L)}$ | 0.0 | 0.013 | -0.009 |
| $F_{2Z}^{(R)}$ | 0.0 | 0.013 | -0.009 |

**Table 4.2:** The upper and lower bounds indicate the statistical range within which we can determine the form factors from the angular distribution of the top quark in $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b} l^+ l^- \nu \bar{\nu}$ given a $50 \text{ fb}^{-1}$ luminosity at the NLC within a 90% confidence level.

| Form Factor | SM VALUE | Upper | Lower |
|-------------|----------|-------|-------|
| $F_{1Z}^{(L)}$ | 0.395 | 0.473 | 0.311 |
| $F_{1Z}^{(R)}$ | -0.175 | -0.107 | -0.254 |
| $F_{2Z}^{(L)}$ | 0.0 | 0.084 | -0.024 |
| $F_{2Z}^{(R)}$ | 0.0 | 0.084 | -0.024 |

integrated luminosity) with no constraints on the kinematics. We expect 30,000 $t\bar{t}$ events, but if we focus on the mode where the two $W$'s decay leptonically to $\mu^+$ or $e^+$, there is a branching ratio reduction to 1,500 events. These 1,500 events were collected into twenty bins for the fit. Allowing only one form factor to vary at a time, the results presented in Table 4.1 (Table 4.2) indicate that within the 68% (90%) confidence limit, it should be
Figure 4.4: Same as [Fig. 4.1] except for a purely left–handed $e^-$ beam.

possible to find $F_1^L$ to within about 6% (20%), while $F_1^R$ can only be known to within roughly 11% (40%). The $F_2$ form factors are zero in the SM, and the fit indicates that their values can be known to within about $0.0 \pm 0.01$ ($0.0^{+0.08}_{-0.02}$) at a confidence level of 68% (90%).

By taking our knowledge of the polarization behavior to untangle some of the form factors before comparing them with data, it may be possible to improve these bounds. In particular from Eq. (2.8) it is apparent that the $t\bar{t}$ states of opposite helicity only depend on $A$ and $B$. By making cuts (to be discussed in Section 6), we can isolate $RL$ and $LR$ contributions and investigate $A$ and $B$ from this data subset with a minimum contamination from the $LL$ and $RR$ states. Afterwards, we can proceed to the $RR$ and $LL$ contributions to study $C$ and $D$. Performing the same procedures as we did with the unpolarized cross section, we take 750 events from the unconstrained $LR$ sample and use MINUIT to fit for $F_1^L(Z)$ and find $0.395 \pm 0.014$ at a confidence level of 68%, which is roughly two percent better than we did with the unpolarized distribution. Further improvement in the form factor determination can come from the increased statistics (by about a factor of 7) obtained by including the events from $e^-e^+ \rightarrow t\bar{t} \rightarrow l^\pm + \text{jets}$ in the analysis. For the $l^\pm + \text{jets}$ mode, where the branching ratios take 9,000 unpolarized events from the original 30,000, the results indicate that within the 68% (90%) confidence limit, it should be possible to find $F_1^{(L)}$ to within about 3% (8%), while $F_1^{(R)}$ can be known to within roughly 5% (18%). In this case the $F_2$ form factors can be known to within about $0.0 \pm 0.004$ ($0.0 \pm 0.02$) at a confidence level of 68% (90%).

More information can be gained by focusing on polarization states for the top quarks besides pure helicity states. For example, the interference terms that result between the product of the imaginary parts of the amplitudes with the real parts do not exist in the total rate expressed by Eq. (4.1) and Eq. (4.2), but they do appear in the rate for top quark spins perpendicular to the scatter plane. To see this, note that the polarization asymmetry comparing top spins pointing in opposite directions perpendicular to the scatter plane is proportional to

$$\sum _{h_{e^-}, h_{e^+}, h_\bar{t}} \text{Im} \left[ (h_{e^-}, h_{e^+}, h_\bar{t} = +, h_\bar{t}) (h_{e^-}, h_{e^+}, h_\bar{t} = -, h_\bar{t}) \right].$$

Thus, angular correlations and decay plane correlations will further improve the determination of the form factors $F_1$ and $F_2$.

Even more useful, however, may be the purity of the $t\bar{t}$ polarization which is obtained by polarizing the electron beam. The majority of the cross section is comprised of left–handed top quarks from the start. By selecting a left–handed polarization for the electron beam, helicity conservation suppresses the production of right–handed top quarks.
that move along the initial electron direction of motion. This can be seen in Fig. 4.4 where for each top quark helicity combination we plot $d\sigma/d\cos\theta$ vs. $\cos\theta$. What this provides is a cleaner measure of the form factor dependence $EA + KB$, as apparent from the amplitudes shown in Eq. (2.8). Similarly, a right–handed polarization for the electron beam will provide a cleaner sample of right–handed top quarks where the form factor probed is $EA - KB$.

5. How to reconstruct the Kinematics of the Top Quarks

In reality, the initial state electron or positron at high energies will radiate photons along the beam axis either due to initial state radiation (ISR) during the $e^+e^-$ interaction or due to the interaction with the classical electromagnetic field formed by having a dense bunch of charged particles in the interaction region (beamstrahlung). Such radiation tends to move along the beam direction such that neither the center–of–mass energy nor the boost of the $t\bar{t}$ pair is known, complicating the analysis discussed in the previous section. Despite this, the effects of initial state radiation are such that most of the time the center–of–mass energy of the $t\bar{t}$ pair is close to the beam energy. Furthermore, it is possible to design experiments such that the beamstrahlung is minimized, so we only consider the bremsstrahlung effect. Nevertheless, it is still possible to determine the momentum of the top quark.

We direct our attention to solving for the top quark momentum for two cases: The first case will be the mode where the top quark decays leptonically while the top antiquark decays hadronically, $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{l}^+\nu bqq';$ and the second case will be the mode where both top quarks decay leptonically, $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{l}^+\nu b\bar{l}^-\bar{\nu}$. For the initial state radiation we use the distribution for getting an $e^-$ (or $e^+$) with a fraction $z$ of its original beam energy, given by Kuraev and Fadin to order $\lambda$,

$$D(z, s) = \frac{\lambda}{2} (1 - z) \frac{\lambda}{8\lambda} - 1 - \frac{1}{4} \lambda (1 + z),$$

where $\lambda \equiv \frac{2\alpha}{\pi} (\ln \frac{s}{m_e^2} - 1)$ for original center of mass energies of $\sqrt{s}$ and electron mass $m_e$.

In Section 6.2 we show that there was little difficulty in associating the proper bottom flavored quark with the proper top flavored quark, so we will drop the concern about this ambiguity for this part of the discussion.

5.1. $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{l}^+\nu bqq'$

When one top quark decays hadronically and the other decays leptonically, the only information we are missing to describe the momenta and energies of the event completely is the component of the neutrino momentum that moves along the beam axis. The transverse momentum of the neutrino is known by applying conservation of momentum. Next, we apply a familiar formula which provides a doubly degenerate solution for the longitudinal momentum of the neutrino,

$$p_z(\nu) = a \pm \sqrt{a^2 - (1 - b^2) \left( a^2 - b^2p_T^2(\nu) \right)} \quad (5.1)$$
Figure 5.1: The center–of–mass energy of the $e^-e^+$ annihilation when ISR is taken into account.

In the above formula,

$$a = -\frac{(\mathbf{p}_T(bl^+) \cdot \mathbf{p}_T(\nu) + \frac{1}{2} (m_t^2 - M_{bl^+}^2))}{p_z(bl^+)}$$

$$b = \frac{E(bl^+)}{p_z(bl^+)}$$

(5.2)

where $\mathbf{p}(\nu) = (\mathbf{p}_T(\nu), p_z(\nu))$ is the momentum of the neutrino separated into directions respectively transverse and along the Z–axis, while $p(bl^+) = (E(bl^+), \mathbf{p}(bl^+))$ describes the four–momentum of the two–particle system of mass $p(bl^+) \cdot p(bl^+) = M_{bl^+}^2$ composed of the bottom quark and the charged lepton. With the idea that polarization studies are a means of investigating top quark properties after its discovery, the top quark mass is assumed to be known and its decay width narrow. In our Monte Carlo study, the widths of the $W$ bosons have been included for event simulation, so this computation was performed by scanning a range of values for $m(W^-)$ and $m(W^+)$ about the peak value of $m_W$ and accepting the first solution that was found. Attempting to account for some detector inefficiency, this procedure also included a gaussian smearing of the momentum for the $b$ and $\bar{b}$ with $\Delta E(b)/E(b) = 0.5/\sqrt{E(b)}$ and for the visible leptons with $\Delta E(l^+)/E(l^+) = 0.15/\sqrt{E(l^+)}$. (More detailed discussion is given in Section 6.2.) Random chance yields a 50% probability for selecting the correct solution for $p_z(\nu)$, but we need to do better than that. What needs to be resolved then, is how to remove this degeneracy in the $z$–momentum of the neutrino.

Table 5.1: The efficiency in solving for the top quark momentum using initial state radiation, smearing, and a Breit–Wigner width on the $W$–bosons is described here by three values: the percentage of solveable configurations; the cosine of the angle between the true top quark direction and the direction given by solving the kinematics ($\zeta \equiv \cos(t_{true}, t_{solved})$); the root mean squared of $\zeta$.

| SOLUTIO EFFICIENCY (unknown $m_W$, smearing and ISR included) | %Solved | $<\zeta>$ | $<\zeta^2>$ – $<\zeta>^2$ |
|---------------------------------------------------------------|---------|----------|-----------------|
| $l^+ + jets$                                                  | 84      | 0.978    | 0.090           |
| Best $m_W$                                                    | 72      | 0.981    | 0.073           |
| $l^+ l^- bb\nu\bar{\nu}$                                    | 68      | 0.957    | 0.142           |

We have two straightforward means at our disposal for selecting the best neutrino solution. In one case we take advantage of the fact that the width of the $W$–boson is
narrow with respect to its mass and select the \( p_z(\nu) \) that gives a value for \((p(\nu) + p(e^\pm))^2\) that is closest to \(m_W^2\). In the second case we take advantage of the feature that the initial state radiation is strongly peaked for soft emissions, as shown in Fig. 5.1, and select the \( p_z(\nu) \) that gives a value for the center–of–mass energy that is closest to the original beam energy. To measure how these two methods compare for determining the top quark momentum, we tabulate three values in Table 5.1. First, as a matter of statistics, we give the percentage of the time this method was able to find a solution. Selecting the best \( m_W \) provided the largest number of solutions where only 16% of the cross section was lost while choosing the smallest loss of energy through radiation failed to find a solution 28% of the time. What may be more important in polarization studies than losing an extra 12% of the events, however, is getting a more precise measure for the direction of the top quark. Averaging the cosine of the angular separation between the true top quark direction and the direction obtained by selecting the solution to the quadratic formula for \( p_z(\nu) \), which we label \( \zeta \equiv \cos(\theta_{\text{true}}, \theta_{\text{solved}}) \), it was found that the minimum radiation method had a value for \( \zeta \) slightly closer to unity. Not only is the average of this angle important, but also the spread of this distribution, \(<\zeta^2> - <\zeta>^2\), where it was better to accept those solutions which provided the value of \( M(t\bar{t}) \) closest to the original beam energy.

With either of these two methods, there is little loss in statistics and very little change in the polar angle distribution of the top quarks, making it possible to study the form factors through the polar angle distribution of the top quark.

### 5.2. \( e^-e^+ \to t\bar{t} \to b\bar{b}l^+\nu l^-\bar{\nu} \)

When the top quark and antiquark decay leptonically and both the boost and the mass of the \( t\bar{t} \) pair are unknown, there is a great deal of information that must be extracted out of the measurement of the four visible particles \((b, \bar{b}, l^+, l^- \text{ for } l = e, \mu)\). Detailed discussion on such kinematics can be found in Ref. 16. The four–momenta of both the \( \nu \) and \( \bar{\nu} \) must be determined using the center of mass energy as an unknown. This means we have eight unknowns. Generalizing the procedures that have been discussed here earlier, however, we find that we can solve this system exactly, albeit within an eight–fold degeneracy at most.

Determining the neutrinos’ momenta is accomplished by utilizing the fact that this process contains the decay of four particles whose masses either are or will be known and whose decay widths are narrow. From the start, we have the conservation of transverse momentum,

\[
\mathbf{V}_T + \mathbf{p}_T(\nu) + \mathbf{p}_T(\bar{\nu}) = 0, \tag{5.3}
\]

where we define the four-vector for the sum of the energies and momenta of the visible particles to be

\[
V \equiv \left( V(0), \mathbf{V} \right) = p(b) + p(\bar{b}) + p(e^+) + p(e^-). \tag{5.4}
\]

The conservation of transverse momentum provides two equations in pursuit of solving for the \( p(\nu) \) and \( p(\bar{\nu}) \). Next, from the the decays of the \( W \)’s and \( t \)'s and the masses of the \( \nu \)
Figure 5.2: The boost, $d\sigma/d\beta$ vs. $\beta = \frac{\vec{p}(t)+\vec{p}(\bar{t})}{E(t)+E(\bar{t})}$, as generated by ISR.

Figure 5.3: The angular distribution $d\sigma/d\cos\theta$ vs. $\cos\theta$ as dictated by the top momentum provided by the Monte Carlo compared against the top momentum found when solving the kinematics including the effects of ISR.

and $\bar{\nu}$ we have six more relations, two of which are nonlinear:

- Top quark mass: $m(t)^2 = (p(b) + p(\nu) + p(e^+))^2$,
- Top antiquark mass: $m(\bar{t})^2 = (p(\bar{b}) + p(\bar{\nu}) + p(e^-))^2$,
- $W^+$ mass: $m(W^+)^2 = (p(e^+) + p(\nu))^2$,
- $W^-$ mass: $m(W^-)^2 = (p(e^-) + p(\bar{\nu}))^2$,
- Neutrino mass: $0 = E(\nu)^2 - \vec{p}(\nu) \cdot \vec{p}(\nu)$,
- Antineutrino mass: $0 = E(\bar{\nu})^2 - \vec{p}(\bar{\nu}) \cdot \vec{p}(\bar{\nu})$.

In this study we have used the narrow width approximation for the top quark and top antiquark.

These relations provide two coupled quadratic realtions in $E(\bar{\nu})$ and $E(\nu)$. Combining these two quadratic equations provides a quartic equation in $E(\nu)$ which can be solved exactly, yielding a four–fold degeneracy at most for the neutrino energy. Substituting each solution for the neutrino energy, one at a time, into either of the two quadratic relations in $E(\bar{\nu})$ and $E(\nu)$ provides two solutions for $E(\bar{\nu})$ for each $E(\nu)$. The result is at most eight pairs of solutions for $(E(\bar{\nu}), E(\nu))$. Since the remaining equations are linear, the solutions for the momentum components are unique for each given $(E(\bar{\nu}), E(\nu))$ pair. What remains is to decide which solution is the correct one.

Since the initial state radiation is peaked for soft emissions, as shown in Fig. 5.1, the method we use for choosing among the degenerate solutions in this top decay mode is to select the set of $(E(\bar{\nu}), E(\nu))$ that provides a value of $M(t\bar{t})$ closest to the original beam energy. In Table 5.1, we see that we were able to obtain solutions for 68% of the cross section with $\langle \zeta \rangle = 0.957$ and $\langle \zeta^2 \rangle - \langle \zeta \rangle^2 = 0.142$. 
Figure 6.1: Helicity arguments control the preferred moving direction for the $e^+e^- \rightarrow t\bar{t}$ when both final state top quarks have identical helicities.

Figure 6.2: $E(l^+)$ and $E(l^-)$ distributions for $e^-e^+ \rightarrow t\bar{t}$ when both final state top quarks have identical helicities.

Figure 6.3: The distribution for the production of polarized $t\bar{t}$ pairs, $d\sigma/d[(1-r)/(1+r)]$ vs. $(1-r)/(1+r)$ for $r \equiv E(l^+)/E(l^-)$.

5.3. Initial State Radiation

The effects of ISR on the angular distribution of the top quarks is minimal. To demonstrate this, we show in Fig. 5.1 and Fig. 5.2 that the initial state radiation is soft, keeping the energy of the process quite close to the beam energy for a majority of the events. The $t\bar{t}$ system is hardly boosted as illustrated in Fig. 5.2, where $\beta = \frac{p(t)+\bar{p}(\bar{t})}{E(t)+E(\bar{t})}$. When we take events generated by our Monte Carlo and compare the angular distributions between the kinematics obtained by solving the eight simultaneous equations and the actual value generated by the program, we find only a slight difference, as shown in Fig. 5.3.

With the $t$ and $\bar{t}$ momenta determined, it becomes feasible to select a sample of $t\bar{t}$ pairs that is strongly biased to a particular polarization state.

6. Determining the Polarizations of the $t$ and the $\bar{t}$

6.1. Polarized $t\bar{t}$ Production

Here we consider the dilepton mode. We showed in Ref. 8 that the polarization of the $t$ (and the $\bar{t}$) can be self–analyzed from the decay $t \rightarrow bW^+ \rightarrow bl^+\nu_l$ ($\bar{t} \rightarrow b\bar{W}^- \rightarrow \bar{b}l^-\bar{\nu}_l$). For a heavy top quark, the preferred moving direction of the $l^+$ and the $l^-$ in the center–of–mass frame of the $t\bar{t}$ pair are shown in Fig. 6.1. Because of the correlations, it is possible to distinguish different polarization states of the $t\bar{t}$ pairs by the energy distribution, $d\sigma/dE(l)$, of the leptons $l^+$ and $l^-$. Because the asymmetry in $E(l^+)$ and $E(l^-)$ is magnified when the $t$ and the $\bar{t}$ are boosted, we anticipate that it becomes easier to distinguish different polarization states of the $t\bar{t}$ pair when the energy of the $e^-e^+$ collider increases, but it must be kept in mind that the production rate of $t\bar{t}$ decreases with higher beam energies. At the NLC, the energy distributions of the $l^+$ and $l^-$ are shown in Fig. 6.2.

As discussed in the previous section, $A_1$ is cleaner than $A_2$ because it does not contain the relatively large contributions from $LR$ and $RL$ in its denominators. Obviously, this can be useful only if one can separate the various polarization states for the $t\bar{t}$ pairs. One method to do this which applies the physics contained in the energy distributions of Fig. 6.2 is to make a selection of events using the energy ratio $r = E(l^+)/E(l^-)$. The events with $r \gg 1$ are mainly $RR$ events, $r \ll 1$ are mainly $LL$, and $r \sim 1$ characterizes both $RL$ and
LR events. In order not to induce the CP asymmetry while applying the kinematic cut on this variable $r$, it is necessary to make a symmetric cut on $r$. For instance, one can select the RR events by requiring $r > 3$ and the LL events by requiring $r < 1/3$. In Fig. 6.3, we show the distribution of $(1 - r)/(1 + r)$ for various polarization states of the $t\bar{t}$ pair.

Another method we propose is based on the fact that the moving direction of the $l^+$ and $l^-$ are strongly correlated in the center-of-mass frame of the $t\bar{t}$ pair. To illustrate this point, we show in Fig. 4.2 the strong correlation between $\cos \theta_{t^+}$ and $\cos \theta_{t^-}$, where $\theta_{t^{\pm}}$ is the angle between the boost axis of the $t$ and the momentum of the $l^+ (l^-)$ in the rest frame of the $t (\bar{t})$. The four populated corners correspond to the four different helicity states of the $t\bar{t}$ pair. Therefore, one can select individual polarization states of $t\bar{t}$ by making cuts on this plot. However, it is important to make sure that the kinematic cuts won’t induce an artificial CP asymmetry. For example, one can select the RR and LL data sample by making a square cut of equal area on the $\cos \theta_{t^+} > 0$, $\cos \theta_{t^-} > 0$ and $\cos \theta_{t^+} < 0$, $\cos \theta_{t^-} < 0$ corners without inducing an artificial CP asymmetry when studying $A_1$. The $\cos \theta_{t^+} > 0$, $\cos \theta_{t^-} < 0$ and $\cos \theta_{t^+} < 0$, $\cos \theta_{t^-} > 0$ corners are suitable to measure the form factors $A$ and $B$ from the $RL$ and $LR$ data sample. Similarly, information on the form factors $C$ and $D$ are contained in the LL and RR samples. The key element of this approach is to be able to determine the moving direction of the $t$ via the decay mode $t\bar{t} \rightarrow b l^+ \nu_l \bar{b} l^- \bar{\nu}_l$. This method is therefore useful even when some kinematic cuts are being imposed on the final observable partons as usually done in reality for detection and triggering. In Fig. 4.3, we show the effect of selecting various corners in the configuration space of Fig. 4.2 on the distribution in $(1 - r)/(1 + r)$. The acceptance of the kinematic cuts in selecting LL or RR events is about $\frac{1}{4}$. Different polarization states of the $t\bar{t}$ dominate in different regions of $(1 - r)/(1 + r)$. Therefore, it is possible to enhance the probability of a particular polarization of the $t\bar{t}$ pair by making cuts on $(1 - r)/(1 + r)$ as to be performed in the following analysis. We also note the relative enhancement of the RR and LL rates displayed in Fig. 4.3 as a result of these selections.

6.2. The Bottom Quark Ambiguity

If the $b$ is not distinguished from the $\bar{b}$, there are two solutions provided by this procedure because it becomes possible to assign mistakenly the $\bar{b}$ with the $t$. Nevertheless, this causes only minor difficulties. To break the degeneracy, one first solves the kinematics given one combination of jet momenta assignments and then repeats the procedure for the other combination, then one compares the two results.

We performed a simplified computation without ISR where it was not necessary to apply both of the constraints provided by $m(W^+)$ and $m(W^-)$, since then we know the boost of the system. Furthermore, with the center-of-mass energy known, we did not need to invoke the nonlinear relations restricting the neutrinos to be massless. This means that given one $W$ boson mass, we were able to solve for the other. The tendency is that the combination where the $b$ quark was misidentified as the $\bar{b}$ quark generates unreasonable solutions for the $W$ boson mass that was left as a free parameter, allowing a proper identification of the $b$ quark jets. In particular, it was chosen to drop any solutions that gave a negative $W$ boson mass (this condition may be tightened). If both solutions yielded positive masses for the gauge bosons, they were subjected to a further selection criteria regarding the neutrino masses.
One effect of using a value for the mass of the $W^-$ different than $\sqrt{(p(e^-) + p(\bar{\nu}))^2}$ in the above procedure is that the solution produces magnitudes for the neutrino momenta which are no longer equivalent to their own energies. To remove some of the inefficiency caused by the finite width, $\Gamma_W$, two solutions were determined. The first solution was obtained by fixing the mass of the $W^-$ boson to $m_W$ and solving the kinematics. For the second solution we left the mass of the $W^-$ boson free and fixed the mass of the $W^+$ boson to $m_W$. An artifact of this procedure is that the worst solution of the two tends to provide neutrinos with larger values of $p(\nu)^2$ and $p(\bar{\nu})^2$. Since the neutrino is massless, selecting the solution that gives the neutrino masses closest to zero ameliorates the problems introduced by $\Gamma_W$ and the remaining problem introduced by the inability to distinguish the $b$ quark from the $\bar{b}$ antiquark.

We have used a Monte Carlo program to test the success rate of this algorithm and found a 99% efficiency in reconstructing the moving direction of the $t$ by assuming a hundred percent detection efficiency of the energy of the $b$ and $\bar{b}$. About 0.5% of the solutions misidentified $b$ quarks as $\bar{b}$ antiquarks and the other 0.5% consisted of kinematic configurations which could not provide a solution given our criteria. To determine whether one can measure CP violation effects better than a percent from this method requires further study to ensure that no artificial CP violation is induced by removing from consideration those events for which no solution was found. Attempting to account for some detector inefficiency, this procedure was also studied with the inclusion of a gaussian smearing of the momentum for the $b$ and $\bar{b}$ with $\Delta E(b)/E(b) = 0.5/\sqrt{E(b)}$ and for the visible leptons with $\Delta E(l\pm)/E(l\pm) = 0.15/\sqrt{E(l\pm)}$. The results in this case gave a 96% efficiency in assigning the bottom momenta to the proper top quarks where 3.3% of the time this assignment was incorrect and 0.7% of the events provided no solution given our criteria. It is expected that the efficiency in removing the bottom quark ambiguity should improve slightly by selecting the best values of $p(\nu)^2$ and $p(\bar{\nu})^2$ among a range of choices where we do not restrict ourselves to fixing the $W$ boson masses to $m_W$, but rather allow ourselves to check a variety of solutions for $W$ boson masses within, e.g., $m_W \pm 2\Gamma_W$. Note that in distinguishing the $b$ from the $\bar{b}$ through their kinematics as we have described, we have not taken advantage of other information which can be made available such as the charge of the leading particle in the bottom quark jet.

Summarizing, we described how the top quark analyzes its own polarization through its decay $t \rightarrow bW^+ \rightarrow b l^+\nu$ such that a moving top (anti)quark with right–handed (left–handed) helicity tends to produce the $l^+$ ($l^-$) with its momentum aligned with the top (anti)quark direction of motion. We have demonstrated that the top quark momentum can be determined well including smearing and width effects with very little ambiguity in deciding which bottom flavored quark goes with which top flavored quark. If the CP violation effects are very small, however, the error introduced by the ambiguity in distinguishing the $b$ from the $\bar{b}$ may interfere with the measurement.

A 1 TeV machine can do better than a 500 GeV machine regarding the determination of the top quark momentum, polarization, and which bottom quark should be associated with the $t$, all due to the larger boost the top quark receives.
7. Conclusion

We have shown how to determine the top quark momentum in $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^+\nu\bar{\nu}$ and $e^-e^+ \rightarrow t\bar{t} \rightarrow l^\pm + \text{jets}$ by solving for the momenta of the neutrinos, including the effects from initial state radiation. Assuming the mass of the top quark is known, an accurate determination of the top quark moving direction was obtained, even when including detector effects by smearing the particles’ momenta and taking into account the finite width effects from the $W$ boson. The efficiency of this algorithm was affected very little by the degeneracy created by treating the $b$ and $\bar{b}$ as indistinguishable, and it is possible to optimize the efficiency to account for width effects in the mass of the $W$ boson. With the top quark direction known, the door is open to performing studies using the polarization of the top quark.

Through angular correlations between the lepton and top quark directions, we have shown how to separate the different polarization states for the $t\bar{t}$ pair. Applying these tools, we have examined how CP violation effects and form factor magnitudes may be studied in $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^+\nu\bar{\nu}$ and concluded that large luminosities ($3 \times 10^4 \text{fb}^{-1}$) are required to measure a CP violation asymmetry $A_1$ at the order of $10^{-2}$ at the NLC. On the other hand, the form factors can be measured to some accuracy. For the $l^\pm + \text{jets}$ mode at a 500 GeV machine, the results indicate that within the 68% (90%) confidence limit, it should be possible to find $F_1^L$ to within about 3% (8%), while $F_1^R$ can be known to within roughly 5% (18%). In this case the $F_2$ form factors can be known to within about $0.0 \pm 0.004$ ($0.0 \pm 0.02$) at a confidence level of 68% (90%). Isolating the $RL$ and $LR$ contributions can improve the precision of the measurements of these form factors. For instance, we saw that in the dilepton mode that $F_1^L$ can be measured to about 6% at the 68% confidence level using unpolarized beams, yet by selecting the $RL$ contribution using the methods discussed in this paper, it appears possible to improve this bound to 4% in this mode.

A 1 TeV machine can do better than a 500 GeV machine in determining $F_1^{L,R}$ because the relative sizes of the $RR$ and $LL$ production rates become small and furthermore, the top quark is boosted more in a 1 TeV machine thereby allowing a better determination of its direction. A 1 TeV machine makes the CP asymmetry measurement more difficult, however, because of the smaller rates in $RR$ and $LL$.

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References

1. N. Turok and J. Zadrozny, *Phys. Rev.* **65** (1990) 2331; M. Dine, P. Huet, R. Singleton and L. Susskind, *Phys. Lett.* **257B** (1991) 351; L. McLerran, M. Shaposhnikov, N. Turok and M. Voloshin, *Phys. Lett.* **256B** (1991) 451; A. Cohen, D. Kaplan and A. Nelson, *Phys. Lett.* **263B** (1991) 86.

2. C.J.C. Im, G.L. Kane and P.J. Malde, University of Michigan at Ann Arbor preprint UM-TH-92-27.

3. C.R. Schmidt and M.E. Peskin, *Phys. Rev. Lett.* **69** (1992) 410.

4. S. Weinberg, *Phys. Rev. Lett.* **63** (1989) 2333; *Phys. Rev.* **D42** (1990) 860.

5. D. Atwood and A. Soni in *Phys. Rev.* **D45** (1992) 2405.

6. see, e.g., P. Chen, *Phys. Rev.* **D46** (1992) 1186; P. Chen, SLAC preprint SLAC-PUB-5914, presented at the 9th International Workshop on Photon-Photon Collisions (PHOTON-PHOTON '92), San Diego, CA, 22-26 Mar 1992; Ronald Settles in Les Rencontres de Physique de la Vallee D’Aoste: Results and Perspectives in Particle Physics, La Thuile, Italy, Mar 8-14, 1992 and at 27th Rencontre de Moriond: Electroweak Interactions and Unified Theories, Les Arcs, France, Mar 15-22, 1992, and at 4th San Miniato Topical Seminar on the Standard Model and Just Beyond, San Miniato, Italy, Jun 1-5, 1992.

7. R.D. Peccei and X. Zhang, *Nucl.Phys.* **B337** (1990) 269; R. Peccei, S. Peris and X. Zhang, *Nucl.Phys.* **B349** (1991) 305.

8. G.L. Kane, G.A. Ladinsky and C.-P. Yuan, *Phys. Rev.* **D45** (1992) 124.

9. C. Jarlskog, *Phys. Rev.* **D35** (1987) 1685.

10. J.C. Collins and G.A. Ladinsky in *Proceedings of the Polarized Collider Workshop*, The Pennsylvania State University, 1990, ed. J. Collins, S. Heppelmann and R. Robinett (AIP Conference Proceedings No. 223, American Institute of Physics, New York, 1991).

11. MINUIT, Application Software Group, Computing and Networks Division, CERN, Geneva, Switzerland.

12. Talk presented by M. Peskin at the Workshop on Physics and Experiments with Linear $e^+e^-$ Colliders, Waikoloa, Hawaii, 26-30 April 1993.

13. E.A. Kuraev and V.S. Fadin, *Yad. Fiz.* **41** (1985) 733 ( *Sov. J. Nucl. Phys.* **41** (1985) 466).

14. D. Chang, W.-Y. Keung and I. Phillips, CERN preprint CERN-TH.6658/92.

15. G.L. Kane, published in the *Proceedings of the 12th SLAC Summer Institute on Particle Physics*, 23 July–3 August 1984, Stanford, California.

16. R.H. Dalitz and G.R. Goldstein, *Phys. Lett.* **B287** (1992) 225; preprint OUTP-93-16P.

17. L.L. Frankfurt, et al., *Phys. Lett.* **B230** (1989) 141.
Table Captions

Table 2.1 Born level production rates for $e^- e^+ \rightarrow t\bar{t}$ at $\sqrt{s} = 500$ GeV and 1 TeV using $m_t = 140$ GeV listed according to $t$ and $\bar{t}$ polarizations.

Table 4.1 The upper and lower bounds indicate the statistical range within which we can determine the form factors from the angular distribution of the top quark in $e^- e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^-l^+\nu\bar{\nu}$ given a 50 fb$^{-1}$ luminosity at the NLC within a 68% confidence level.

Table 4.2 The upper and lower bounds indicate the statistical range within which we can determine the form factors from the angular distribution of the top quark in $e^- e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l^-l^+\nu\bar{\nu}$ given a 50 fb$^{-1}$ luminosity at the NLC within a 90% confidence level.

Table 5.1 The efficiency in solving for the top quark momentum using initial state radiation, smearing, and a Breit–Wigner width on the $W$–bosons is described here by three values: the percentage of solvable configurations; the cosine of the angle between the true top quark direction and the direction given by solving the kinematics ($\zeta \equiv \cos(t_{true}, t_{solved})$); the root mean squared of $\zeta$. 
Figure Captions

Figure 3.1 This plot displays the variation of the asymmetry $A_1$ in Eq. (2.10) with the mass of the Higgs boson for Weinberg’s model using $\sqrt{s} = 500, 1000$ GeV and $m_t = 140$ GeV.

Figure 3.2 This plot displays the variation of the asymmetry $A_1$ in Eq. (2.10) with $\sqrt{s}$ for Weinberg’s model using $m_t = 140$ GeV and $m_H = 0.1, 0.4, 1.0$ TeV.

Figure 4.1 In $d\sigma/d\cos \theta$ we see the dependence of the production rates for polarized $t\bar{t}$ states on the top quark polar angle. This angle $\theta$ is measured with respect to the incoming $e^-$ beam which is taken to move along the $+\hat{z}$ direction.

Figure 4.2 Same as Fig. 4.1 except for a purely left–handed $e^-$ beam.

Figure 5.1 The center–of–mass energy of the $e^-e^+$ annihilation when ISR is taken into account.

Figure 5.2 The boost, $d\sigma/d\beta$ vs. $\beta = \frac{E(t)+E(\bar{t})}{E(t)+E(\bar{t})}$, as generated by ISR.

Figure 5.3 The angular distribution $d\sigma/d\cos \theta$ vs. $\cos \theta$ as dictated by the top momentum provided by the Monte Carlo compared against the top momentum found when solving the kinematics inculding the effects of ISR.

Figure 6.1 Helicity arguments control the preferred moving direction for the $e^+, e^-$ as they are produced in the respective rest frames of the $t, \bar{t}$ decays ($t \rightarrow b e^+ \nu, \bar{t} \rightarrow \bar{b} e^- \bar{\nu}$). The subscript $R (L)$ indicates right–handed (left–handed) quark helicity.

Figure 6.2 $E(l^+)$ and $E(l^-)$ distributions for $e^-e^+ \rightarrow t\bar{t}$ when both final state top quarks have identical helicities.

Figure 6.3 The distribution for the production of polarized $t\bar{t}$ pairs, $d\sigma/d[(1 - r)/(1 + r)]$ vs. $(1 - r)/(1 + r)$ for $r \equiv E(l^+)/E(l^-)$.

Figure 6.4 These plots present the angular correlation between the charged leptons produce from the polarized $t$ and $\bar{t}$ decays in $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b} l^+ l^- \nu\bar{\nu}$. Boosting to the individual center of mass frame for the decays of the $t$ and $\bar{t}$, we plot the cosine of the angle between the momentum of each charge lepton and the boost axis as defined by the top momentum. Note how each combination of $t, \bar{t}$ helicities (RR,LL,RL,LR) occupy separate quadrants.

Figure 6.5 The distribution for the production of polarized $t\bar{t}$ pairs, $d\sigma/d[(1 - r)/(1 + r)]$ vs. $(1 - r)/(1 + r)$ for $r \equiv E(l^+)/E(l^-)$ when angular cuts are applied to the final state $l^+$ and $l^-$ in the respective decay frames of the $t$ and $\bar{t}$. 