A study on network analysis by navigation of robots using graphical measures to solve the complexity

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Abstract. In this paper, we present a graph theoretical analysis of a procedure for maintaining communication for tasks such as monitoring, identification, completion and productivity, where connectivity between the components involved in the task is required for conditional alertness. We present graphical measures and fuzzy theoretical tools to analyse the communication between the components in a network undergoing different tasks and accumulate a suitable information regarding the activities of the components by observing through sensor monitoring system. Also, we consider the efficiency of the robots in the monitoring system to control the position and orientation relative to other components to sustain communication links.

1. Introduction
Nowadays, the communication network has developed from being just a medium of information transmission to different devices, where properties like connectivity, efficiency of the medium transmitted, mechanical advantage of the components in the network are used to maintain the quality of the medium. But, it is difficult to foresee the transmission properties due to the efficiency of various factors including in the transmission channel and intrusion from other sources. So, observing the characteristics of the components involved in the network will increase the connectivity and emission of information through proper channels. We place properly designed navigation agents called ‘robots’ uniquely at different landmarks to monitor the activities in the network. These robots can sense the shortest distance between any two components in the network. Every network can be converted into a finite connected graph by representing vertices as its components and edges as the connection between these components.

The goal of such work is to develop the transmission in networks by monitoring the actions of the factors involved in the transmission by specially designed robots. So, metric dimension gives a measure to anticipate the information needs of a network by repositioning and organizing themselves to best acquire and deliver that information.

The notion of metric dimension was introduced by P J Slater in [1] and developed by Harary and Melter in [2]. Some of the applications of this navigation of robots in networks are discussed in [3] and in other scientific fields and application to problems of pattern recognition and image processing, some of which include the use of classified arrangements are given in [4]. On the other hand,
Chartrand [3] describe the graph with metric dimension 1, \( n - 1 \) and \( n - 2 \). Also, application including weighing problems and combinatorial examine and optimization [4].

2. Basic notions

We consider simple connected graph \( G = (V, E) \) for representing networks under consideration and follow the basic definition and notation in [6].

2.1 Connectivity and eccentricity

Let \( G = (V, E) \) be a connected graph and \( V(G) = \{v_1, v_2, ..., v_n\} \) be the vertex set. Suppose \( S \) is the smallest subset of \( V \) such that \( G \) will be disconnected by the removal of the vertices in \( S \). Then the cardinality of \( S \) is called the connectivity of \( G \).

2.2 Eigen vector centrality

Eigenvector centrality (eigen centrality) is a measure of the effect of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes.

We can use adjacency matrix of the graph for calculating eigen vector centrality. Let \( A = [a_{ij}] \) be the adjacency matrix of \( G \) defined by \( a_{ij} = 1 \) if there is an edge from the vertex \( v_i \) to \( v_j \) otherwise \( a_{ij} = 0 \), then the eigen vector centrality of a vertex \( v_i \) is defines as

\[
C_{vi} = \frac{1}{2} \sum_{v_k \in N(v_i)} C_{vk} = \frac{1}{2} \sum_{v_k \in EV(G)} a_{i,k} C_{vk}
\] (1)

where \( N(v_i) \) is the set of all neighbours of \( v_i \) and \( \lambda \) is the dominating or largest eigen value in magnitude of \( A \).

2.3 Metric Dimension of graph

Let \( W = \{u_1, u_2, ..., u_m\} \) be an ordered set of vertices of \( G \) and let \( v \) be a vertex of \( G \) and \( d(v, u_i), i = 1, 2, ..., m \) denote the shortest distance from \( v \) to \( u_i \). Co-ordinate of a vertex \( v \) is an ordered pair \( (d(v, u_1), d(v, u_2), ..., d(v, u_m)) \). If distinct vertices of \( G \) have distinct co-ordinates with respect to \( W \), then \( W \) is called a resolving set or location set for \( G \). A resolving set of minimum cardinality is called a basis for \( G \) and this cardinality is called the metric dimension or location number of \( G \) and is denoted by \( dim(G) \) or \( \beta(G) \).

![Figure 1. A graph with metric dimension 2 and metric basis \( W = \{v_1, v_2\} \)](image)

2.4 Efficiency of robots

Mechanical efficiency of a robot is defined by the rate at which the energy and power (input) are transferred into force and movement (output) and is calculated as

\[
Efficiency = \frac{Mechanical Advantage}{Velocity} \times 100
\] (2)

3 Known results

In this section, we consider important results that is relevant for the subsequent sections.
3.1 Metric dimension of some standard graphs
- The metric dimension of graph $G$ is 1 if and only if $G$ is a path.
- If $C_n$ is a cycle of length $n > 2$, then $\beta(C_n) = 2$.
- If $K_n$ is the complete graph with $n > 1$ then $\beta(K_n) = n - 1$.
- For the complete bipartite graph with $n + 1$ vertices, $\beta(K_{1,n}) = n - 1, n \geq 2$.

3.2 Capacity $l$ - index of a robot
The *Capacity $l$ - index* of a basis element $u_i$ for $i = 1, 2, \ldots, m$ is denoted by $Cal(u_i)$ and is defined as the number of vertices of $G$ identified by $u_i$ with respect to $d(u_i, u_j) = l$ for $j = 1, 2, \ldots, n$.

![Figure 2. $W = \{R_1, R_2\}$. Here $l = 1, 2, 3$. For $l = 1, Cal(R_1) = 2, Cal(R_2) = 2$. For $l = 2, Cal(R_1) = 2, Cal(R_2) = 2$. For $l = 3, Cal(R_1) = 1, Cal(R_2) = 1$](image)

4 Network monitoring by graphical measures
Network monitoring is an analytic process where all networking components like client, internet, firewalls, servers, sensor node, mobile node and base station are monitored and estimated continuously to preserve and enhance their availability. Network monitoring should be dynamic. Monitoring the issues continuously helps in finding problems at the initial stage and thereby we can prevent network failures.

4.1 Network monitoring using efficiency of robots
Eigenvector centrality identify a node's importance while giving attention to the significance of its neighbours and also tells us which nodes are prominent based on the topological structure of the network. The following figures represents topological structure of a network and its graphical representation.

![Figure 3. Network device system](image)
Let $G = (V, E)$, with $V = \{v_1, v_2, ..., v_5, R_1, R_2, R_3, R_4\}$ and basis $W = \{R_1, R_2, R_3, R_4\}$. Following table gives the capacity $l$-index of the basis elements (robots) with respect to $l = 1, 2, 3, 4, 5$.

### Table 1. capacity $l$-index of basis elements.

| Robots | $l$ | $\text{Cal}(R_i)$ | Max $\text{Cal}(R_i)$ |
|--------|-----|-------------------|---------------------|
| $R_1$  | 1   | 1                 | 1                   |
| $R_2$  | 1   | 1                 |                     |
| $R_3$  | 1   | 1                 |                     |
| $R_4$  | 1   | 1                 |                     |
|        |     |                   |                     |
| $R_1$  | 2   | 2                 | 3                   |
| $R_2$  | 3   | 3                 |                     |
| $R_3$  | 2   | 3                 |                     |
| $R_4$  | 2   | 2                 |                     |
|        |     |                   |                     |
| $R_1$  | 2   | 2                 |                     |
| $R_2$  | 1   | 1                 |                     |
| $R_3$  | 3   | 3                 |                     |
| $R_4$  | 1   | 1                 |                     |
|        |     |                   |                     |
| $R_1$  | 3   | 3                 |                     |
| $R_2$  | 2   | 3                 |                     |
| $R_3$  | 4   | 2                 |                     |
| $R_4$  | 3   | 3                 |                     |
|        |     |                   |                     |
| $R_1$  | 1   | 1                 |                     |
| $R_2$  | 1   | 1                 |                     |
| $R_3$  | 5   | 1                 |                     |
| $R_4$  | 0   | 0                 |                     |

For $l = 1$, all basis elements identifies the same number of vertices and has equal importance. For $l = 2$, $\text{MaxCal}(R_i)$ occurred for $R_2, R_3$, for $l = 3$, $R_1$ identifies more vertices, for $l = 4$, $R_1, R_4$ identifies more vertices and for $l = 5$, $R_1, R_2, R_3$ identifies same number of vertices. So, based on the data positions (landmarks) $R_1, R_2, R_3$ plays an important role in monitoring the activities of the components in the network device system.
4.2 Network monitoring using eigen vector centrality

The following table represents the eigen vector centrality of each vertex in the network to show the most prominent nodes and level of influence of a node within a network. Each node within the network will be given a value, the higher the score the greater the level of influence within the network.

Let the adjacency matrix of the given network be

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

We use power method iteration process to find the normalized value and the eigen vector centrality of each vertex. After finite number of iterations, the sequence of normalized value and the corresponding eigen vector components will converge. Let \(X_{9\times1}\) be the required eigen vector component vector and we start the iteration process with initial approximation vector \(X_0 = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]^T\).

**Iteration: 1**

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
1 \\
1 \\
1 \\
1 \\
1
\end{bmatrix}
= \begin{bmatrix}
0.1623 \\
0.3246 \\
0.4870 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493
\end{bmatrix}
\]

The normalized value of the vector = \(\sqrt{38} = 6.16\).

**Iteration: 2**

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0.1623 \\
0.3246 \\
0.4870 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493
\end{bmatrix}
= \begin{bmatrix}
0.3246 \\
0.6493 \\
0.8115 \\
1.1360 \\
0.8115 \\
0.6493 \\
0.6493 \\
0.6493 \\
0.6493
\end{bmatrix}
\]

The normalized value of the vector = \(\sqrt{4.6358} = 2.1530\).
Iteration: 3

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0.1507 \\
0.3015 \\
0.3769 \\
0.5276 \\
0.3769 \\
0.3015 \\
0.3015 \\
0.3015 \\
0.2261 \\
\end{bmatrix}
= 
\begin{bmatrix}
0.3015 \\
0.5276 \\
1.0552 \\
0.7538 \\
1.4321 \\
0.3679 \\
0.3679 \\
0.3679 \\
0.3679 \\
0.2261 \\
\end{bmatrix}
\]

The normalized value of the vector = \(\sqrt{4.6697} = 2.1609\).

The following gives five more iteration results on the normalized value and the corresponding eigen vector:

Iteration: 4 - normalized value of the vector = \(\sqrt{4.6831} = 2.1640\);

\[
[0.1128 \ 0.2901 \ 0.3545 \ 0.5318 \ 0.4029 \ 0.3062 \ 0.3062 \ 0.3062 \ 0.2256]^T
\]

Iteration: 5 - normalized value of the vector = \(\sqrt{4.6896} = 2.1655\);

\[
[0.1339 \ 0.2157 \ 0.4837 \ 0.3497 \ 0.6697 \ 0.1860 \ 0.1860 \ 0.1860 \ 0.1637]^T
\]

Iteration: 6 - normalized value of the vector = \(\sqrt{4.4521} = 2.11\)

\[
[0.1022 \ 0.2927 \ 0.3455 \ 0.5466 \ 0.4301 \ 0.3173 \ 0.3173 \ 0.3173 \ 0.2292]^T
\]

Iteration: 7 - normalized value of the vector = \(\sqrt{4.6941} = 2.1655\)

\[
[0.1315 \ 0.2012 \ 0.4803 \ 0.3486 \ 0.6736 \ 0.1933 \ 0.1933 \ 0.1933 \ 0.1533]^T
\]

Iteration: 8 - normalized value of the vector = \(\sqrt{4.6982} = 2.1675\)

\[
[0.1302 \ 0.1925 \ 0.4782 \ 0.3474 \ 0.6760 \ 0.1970 \ 0.1970 \ 0.1970 \ 0.1497]^T
\]

Next iteration will have the same values as that of the previous iterative values.

Therefore, the eigen vector corresponding to the eigen vector centrality of the nine vertices are given by:

| Table 3. Eigen vector centrality values |
|----------------------------------------|
| Vertices | Eigen vector centrality \((C_v)\) |
|----------|-------------------------------|
| \(v_1\)  | 0.1302                        |
| \(v_2\)  | 0.1925                        |
| \(v_3\)  | 0.4782                        |
| \(v_4\)  | 0.3474                        |
| \(v_5\)  | 0.6760                        |
| \(R_1\)  | 0.1970                        |
| \(R_2\)  | 0.1970                        |
| \(R_3\)  | 0.1970                        |
| \(R_4\)  | 0.1497                        |
Obviously, the basis elements $R_1, R_2, R_3$ has high priority with respect to its eigen centrality values when compared with the $C_{R_4}$. So, the most noticeable positions to place the robots to monitor the actions of the other components in the network is $R_1, R_2, R_3$.

4.3 Network monitoring using centrality and eccentricity of vertices

The eccentricity of a node $v$ in a network is the maximum distance from $v$ to any other node. The reciprocal of eccentricity is used as a measure of the importance of a node within a network. The associated centrality index measure then estimates the degree to which a network is dominated by a particular node.

Table 3. Eccentricity as a measure of centrality index to locate most important node in the network.

| Vertices | $e(v)$ | Centrality($C_e(v)$) |
|----------|--------|----------------------|
| $v_1$    | 5      | 0.2                  |
| $v_2$    | 4      | 0.25                 |
| $v_3$    | 3      | 0.33                 |
| $v_4$    | 3      | 0.33                 |
| $v_5$    | 4      | 0.25                 |
| $R_1$    | 5      | 0.2                  |
| $R_2$    | 5      | 0.2                  |
| $R_3$    | 5      | 0.2                  |
| $R_4$    | 4      | 0.25                 |

Evidently, the basis elements $R_1, R_2, R_3$ has identical values for eccentricity and centrality measures. So, the landmarks to be place the robots in the network is suitable to monitor the activities of the components in the network.

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