Why are light nuclei so important?

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2012 J. Phys.: Conf. Ser. 403 012027
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Why are light nuclei so important?

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Abstract. This contribution explores the structure of light nuclei and how the harmonic oscillator may be used as a simple basis for their understanding. There is a particular focus on the nuclei $^{12}\text{C}$ and $^9\text{Be}$.

1. Context

The quest to understand the structure of nuclei from stable to the drip-lines and from light to heavy, has driven the subject for the last several decades. Over that period nuclear theory has evolved considerably with the appearance of density functional approaches which have an ambition to be universal, the Greens Function Monte Carlo (GFMC) with an $ab\text{ initio}$ motivated nucleon-nucleon interaction with the inclusion of three-body forces, antisymmetrised molecular dynamics and fermionic molecular dynamics approaches which use an effective nucleon-nucleon interaction with nucleons simulated by Gaussian wave-packets and finally the no-core shell model (which featured strongly in the present conference). Many of these approaches are limited to the domain of light nuclei and hence it is here that their overlap is maximal. From an experimental perspective, the documentation of the structural properties of the light nuclei is most complete and hence provides the most stringent test of nuclear theory.

In many cases the structure of nuclei is driven by the behavior of, and correlations among, the valence nucleons. Deformation, pairing, evolution of shell structure and collective modes are all induced by the valence particles. In heavy nuclei the number of valence nucleons is relatively small (relative to the core nucleons), but this is not the case in light systems. Here the effect of the correlations becomes amplified and in certain systems distinct cluster-like structures appear. These cluster structures display particular spatial symmetries which are reminiscent of close packing of spheres. Remarkably, many of these symmetries can be found in the deformed harmonic oscillator, which in principle simulates the independent particle motion of the nucleons within the mean-field. In fact, the symmetries displayed by the deformed harmonic oscillator also reveal the emergence of cluster like structures in neutron-rich nuclei, such as $^9\text{Be}$.

Even though the spectroscopy of the light nuclei has been subjected to the greatest scrutiny, it remains clear that there are substantial gaps in our knowledge – witness the missing first excited $1/2^+$ state in $^9\text{B}$ (the analogue is known in $^9\text{Be}$) and the lack of clarity regarding the structure of the excitation energy spectrum of $^{12}\text{C}$. Pinning down some of these open experimental questions and constraining the experimental situation is critical as this provides the foundation for further refining models of nuclei.

This contribution explores both the simple underlying ideas connected with the evolution of cluster-like structures and some of the developments in determining the structure of light nuclei, especially $^{12}\text{C}$. Section 2 describes the symmetries of the deformed harmonic oscillator, Section 3 the
symmetries and structure of $^{12}$C, Section 4 recent experimental measurements of the structure of $^{12}$C and finally Section 5 explores the nature of clustering in neutron-rich systems, e.g. $^7$Be.

2. The deformed harmonic oscillator and symmetries in symmetric and asymmetric nuclear matter

It is well-known that the solutions of the Schrödinger equation for the deformed harmonic oscillator (DHO) potential provides a reasonable approximation to the more realistic Woods-Saxon potential. With the addition of the spin-orbit interaction a reasonable description of the shapes and single-particle structure of light stable nuclei can be found. The solutions of the deformed harmonic oscillator are shown in Fig. 1.

![Figure 1](image_url)  
**Figure 1.** Energy levels of the deformed harmonic oscillator (left). The numbers at the level crossings at deformations of 1:1, 2:1, 3:1 etc, correspond to the degeneracy. The repeating of the spherical pattern twice at 2:1 indicates that two degenerate harmonic potentials co-exist. The right hand plots show the densities of the simplest 2:1 deformed configuration, which would correspond to $^8$Be. The density is formed from the $[n_x, n_y, n_z] = [0,0,0]$ and $[0,0,1]$ harmonic oscillator levels (b), which may be decomposed into two alpha-particles (c). (a) shows the density calculated in the $x$-$z$ plane.

The symmetries are revealed in the degeneracies of the energy levels as the potential is prolate deformed ($\varepsilon_x>0$). At deformations of 2:1 and 3:1 the spherical degeneracies (2, 6, 12, 20,…) are repeated two and three times, respectively. These symmetries extend to the behaviour of the densities
calculated for systems of equal, and even, numbers of protons and neutrons. For example, in the formation of $^8$Be within this framework, the density would correspond to the sum of the densities associated with the $[n_x, n_y, n_z] = [0,0,0]$ and $[0,0,1]$ orbitals. These are shown in Fig. 1 (right hand side). The two centred symmetry is evident. These densities can be decomposed into two centres:

$$\phi_{\alpha}(\pm) = \frac{1}{\sqrt{2}} \left[ [0,0,0] \pm [0,0,1] \right]. \quad (1)$$

The resulting densities which may be associated with alpha-particles are also shown in Fig. 1 (part c). These ideas were explored at length in Ref. [1]. The case of $^8$Be is rather trivial, consisting of only two centres and two alpha-particles. The $^{24}$Mg nucleus is much more complex, being composed of 6 alpha-particles. This point is worth emphasising...... even though the clusters themselves do not exist explicitly as quasi-free alpha-particles, the symmetries reflect the underlying geometric arrangements

![Diagram](image_url)

**Figure 2.** Comparison between the Nilsson-Strutinsky calculations of the potential energy surface of $^{24}$Mg [2] (center) and the Alpha Cluster Model structures, ACM, (density plots around the outside) [3], harmonic oscillator, HO, and Hartree-Fock, HF [4].
of clusters. Experimentally, the nucleus $^8$Be has a prolate deformation (two touching alpha-particles), $^{12}$C is oblate (triangular arrangement of 3 alpha-particles), $^{16}$O spherical (tetrahedral arrangement of 4 alpha-particles), $^{20}$Ne is prolate with an octupole deformation ($^{16}$O+alpha structure)…… Again, this does not imply the existence of free alpha-particles but that the symmetry is retained in the ground-state. For the clustering to appear explicitly an excitation which is equivalent to the associated $N$-alpha decay threshold is required – the Ikeda rule [6]. For these reasons the alpha-cluster structure in the $^8$Be ground-state is explicitly developed – it is unbound to two alpha-decay by 92 keV.

Fig. 2 illustrates a comparison between four different types of calculation. In the centre is the potential energy surface associated with a Nilsson-Strutinsky (NS) type calculation [2]. Here the local minima in the potential energy surface are associated with shell-structure in the deformed energy level scheme, which echo the shell structure in the DHO. Around the outside, and with a one-to-one correspondence, are calculations from an alpha cluster model (ACM) [3] and from the mean-field Hartree Fock (HF) approach [4]. In each case it is possible to identify the different shapes in the ACM and HF calculations uniquely with the minima in the potential energy surface. Moreover, from knowledge of the underlying configuration associated with the potential energy minima of the NS potential energy surface, it is possible to deduce also the harmonic oscillator structure. Correspondingly, the harmonic oscillator (HO) densities are shown next to those of the ACM and HF. Remarkably, the symmetries that are found in the HO are repeated in the ACM and HF. This underlines the fact that these symmetries are a necessary component of models of light nuclei and are a reflection of the underlying cluster structure.

3. $^{12}$C and dynamical symmetries

In the case of $^{12}$C, the system can be constructed from a variety of geometric arrangements of three alpha-particles. It might be expected that the compact equilateral-triangle arrangement is the lowest energy configuration. Such an arrangement possesses a $D_{3h}$ point group symmetry. The corresponding rotational and vibrational spectrum is described by a form [6]

$$E = E_0 + Av_1 + Bv_2 + C(l + 1) + D(K \pm 2l)^2,$$  

(2)

where $v_1, v_2$ are vibrational quantum numbers, and $v_2$ is doubly degenerate; $l = v_2, v_2 - 2, \ldots 1$ or 0, $L$ is angular momentum, $M$ would be its projection on a laboratory fixed axis and $K$ a body-fixed axis [6]. $A, B, C$ and $D$ are adjustable parameters. The spectrum of states predicted by the choice $A = 7.0, B = 9.0, C = 0.8$ and $D = 0.0$ MeV is shown in Fig. 3 [6].

![Figure 3.](image)

Spectrum of the energy levels of an equilateral triangle configuration. The bands are labeled by $(v_1; v_2)$. See Ref. [6] for further details.
The ground state band, \((v_1;v_2) = (0,0 0 )\), contains no vibrational modes and coincides well with the observed experimental spectrum. Here the states correspond to different values of \(K\) \((K = 3n, n = 0,1,2...)\) and \(L\). For \(K=0\), \(L=0, 2, 4\) etc., which is a rotation of the plane of the triangle about a line of symmetry, whereas for \(K ≠ 0\) \(L = K, K +1, K +2\)..... In the present case, \(K=0\) or 3 is plotted with the parity being given by \((-1)^K\). The \(K=0\) states coincide well with the well-known \(0^+\) (ground-state), \(2^+\) (4.4 MeV) and \(4^+\) (14.1 MeV) states. The \(K=3\) states correspond to a rotation about an axis which passes through the centre of the triangle, the first of which has spin and parity \(3^+\) and coincides with the 9.6 MeV, \(3^+\) excited state. This state may be thought of in terms of three alpha-particles rotating around the centre-of-mass each with one unit of angular momentum. Correspondingly, the next such state would be \(K=6\), \(J^P=6^+\). A prediction of this model is that there should be a \(4^+\) state almost degenerate with the \(4^+\) state. A recent measurement involving studies of the alpha-decay correlations indicated that the 13.35 MeV unnatural-parity state possessed \(J^P=4^-\) [7]. The close degeneracy with the 14.1 MeV \(4^+\) state would appear to confirm the \(D_{3h}\) symmetry. In this picture the \(0^+\) state at 7.65 MeV would correspond to a vibrational mode \((v_1=1)\). The coupling of rotational modes would then produce a corresponding \(2^+\) state at 4.4 MeV above 7.65 MeV, i.e. 12.05 MeV; there is no known \(2^+\) state at this energy, pointing to the more complex structure of this state.

4. \(^{12}\)C recent experimental developments
If the 7.65 MeV state in \(^{12}\)C has a structure similar to that of the ground-state then a \(2^+\) state close to 12 MeV is expected. The closest state which has been tabulated with these characteristics is at 11.16 MeV [8]. This state was observed in the \(^{11}\)B(\(^{3}\)He,\(d\))\(^{12}\)C reaction, but has not been observed in measurements subsequently.

![Figure 4. \(^{12}\)C excitation energy spectrum from the \(^{11}\)B(\(^{3}\)He,\(d\)) reaction. The measurement clearly demonstrates the absence of the 11.16 MeV state in \(^{12}\)C [9].](image)

A recent re-measurement of this reaction using the K600 spectrometer at iThemba (the previous measurements were made using photographic plates) demonstrates that the earlier observation of a state at 11.16 MeV was an experimental artifact and no such state exists [9].

Recent studies of the \(^{12}\)C(\(α,α'\)) [10] and \(^{12}\)C(\(p,p'\)) [11,12] reactions indicate the presence of a \(2^+\) state close to 9.6-9.7 MeV with a width of 0.5 to 1 MeV. The state is only weakly populated in these reactions, presumably due to its underlying cluster structure, and is broad. Consequently, its distinction from other broad-states and dominant collective excitations (e.g. the 9.6 MeV, \(3^+\)) makes its unambiguous identification challenging. Further evidence for such an excitation comes from measurements of the \(^{12}\)C(\(γ,3α\)) reaction performed at the HIGS facility, TUNL [13]. Here a measurable cross section for this process was observed in the region of 9-10 MeV which cannot be attributed to known states in this region. Furthermore, the angular distributions of the alpha-particles are consistent with an \(L=2\) pattern, indicating a dominant \(2^+\) component. Based on a rather simple
description of this state in terms of three alpha-particles with radii given by the experimental charge
radius, it is possible to use the 2 MeV separation between the Hoyle-state and the proposed 2’
excitation to draw some conclusions as to the arrangements of the clusters. This would indicate that
rather than a linear arrangement of the three clusters that a more appropriate description would be a
loose arrangement of the alpha-particles in something approaching a triangular structure.

![Diagram of silicon strip detectors and excitation energy spectra](image)

**Figure 5.** [Top] Arrangement of the 4 silicon strip detectors used in the measurements in Ref. [11].
The beam passes from bottom to top in this picture. [Bottom] Carbon-12 excitation energy spectra. a)
The blue line shows the measurement at 22 MeV. The spectrum corresponding to measurements at 26
MeV is shown by the dots. The backgrounds obtained by gating above the 8Be peak (bold dashed line)
and both above and below the Q-value peak (dot-dashed line) are both illustrated. b) Fit to the 26 MeV
data is given by the blue solid line. The polynomial background (red-line) and line-shape for the new
peak (shaded area) is shown. c) Excitation energy spectrum for events not proceeding via the decay to
the 8Be ground-state. The proposed new state is indicated by “??”.

A natural extension of such a conclusion is that there should also be a collective 4+ state. Using the
simple \( j(j+1) \) scaling, a 4+ excitation close to \( E_x(12\text{C}) = 14 \text{ MeV} \) would be expected. Recent
measurements of the two reactions \(^{9}\text{Be}(\alpha,3\alpha)n\) and \(^{12}\text{C}(\alpha,3\alpha)^4\text{He} \) have been performed [14]. In these
measurements three alpha-particles were detected in an array of four silicon strip detectors (shown in
Fig 5). The analysis required that two of the three alpha-particles came from the decay of the ground-state of $^8$Be. For the decay of $^{12}$C to $^9$Be+$\alpha$ this ensures that the decay process can proceed through only natural parity states (i.e. 0+, 1-, 2+ ...). This restricts the complexity of the excitation energy spectrum. The measurements for both the $^9$Be and $^{12}$C targets reveal the known 3-, 1- and 4+ states at 9.64, 10.84 and 14.08 MeV, respectively. However, there is an additional component to the spectrum close to 13.3 MeV (marked by ‘??’) with a width estimated to be 1.7 MeV (Fig. 5). It is believed that this is not a contaminant and is observed with similar properties in all spectra. Angular correlation measurements made using the $^{12}$C target are not definitive, but indicate a 4+ assignment.

5. Neutron-rich clusters

It has long been documented by von Oertzen and others [15] that when neutrons are added to systems in which there is a well established alpha-cluster structure, the neutrons inhabit a multicentre system and are in fact exchanged between the centres. In this way the orbits of the neutrons are analogues of those occupied by electrons in atomic molecules, with the one exception that the Pauli Exclusion Principle must be observed with respect to the cores. In the case of $^9$Be, molecule is formed from two alpha-particles and a neutron. At each centre the neutron should occupy a $p$-orbital and as a consequence the molecular orbital is a linear combination of $p$-orbitals. These linear combinations give rise to sigma and pi-type molecular orbitals which are characterized by the projection of the angular momentum onto the axis of symmetry. The pi-configuration ($\ell=1$) has been linked to the $^9$Be ground-state (3/2-) and the sigma-orbit ($\ell=0$) the 1/2+ first excited state.

Figure 6 shows the deformed harmonic oscillator and the wave-functions.

These configurations can be linked to the deformed harmonic oscillator and hence to other nuclear models, e.g. the Nilsson model. Fig. 6 shows the deformed harmonic oscillator and the wave-functions.
associated with each level. Fig. 7 shows the types of molecular wave-functions that can be formed from the superposition of two p-type orbitals. There is an obvious connection between the molecular and harmonic oscillator orbits. In the formation of the nucleus \(^{9}\text{Be}\) in the deformed harmonic oscillator basis, the valence neutron would occupy the series of levels associated with the degeneracy ‘6’ at a deformation of 2:1. These orbits have a structure which overlaps strongly with the pi (Fig. 7b) and sigma (Fig. 7e) molecular orbitals. Within the Nilsson scheme these levels have angular momenta projections 3/2 and 1/2 respectively – precisely as observed experimentally.

6. Summary
The structure of light nuclei is sensitive to nucleon-nucleon correlations which can manifest themselves as clusterisation. In the ground states of nuclei the explicit clusterisation is suppressed but the symmetries are retained. These are seen in the fingerprint of the excited states which can be described by dynamical symmetries which reflect those of the underlying structure. The spectrum of \(^{12}\text{C}\) is a good example of this. The exchange of neutrons between alpha-particle cores can be described in terms of molecular orbitals. Both cluster symmetries and molecular orbitals may be understood within the framework of the deformed harmonic oscillator.

Acknowledgements
I would like to acknowledge the very significant contributions the work described here from collaborators at iThemba Labs South Africa, Notre Dame USA and the University of Birmingham UK.

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