Reggeon exchange from AdS/CFT

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Using the AdS/CFT correspondence in a confining background and the worldline formalism of
gauge field theories, we compute scattering amplitudes with an exchange of quark and antiquark in
the $t$-channel corresponding to Reggeon exchange. It requires going beyond the eikonal approxima-
tion, which was used when studying Pomeron exchange. The wordline path integral is evaluated
through the determination of minimal surfaces and their boundaries by the saddle-point method at
large gauge coupling $g^2 N_c$. We find a Regge behaviour with linear Regge trajectories. The slope is
related to the $q\bar{q}$ static potential and is four times the Pomeron slope obtained in the same frame-
work. A contribution to the intercept, related to the Lüscher term, comes from the fluctuations
around the minimal surface.

I. INTRODUCTION

The aim of our study is to continue our investigation using the AdS/CFT correspondence\textsuperscript{[12]} for application to
the description\textsuperscript{[3–5]} of high energy scattering in the strong coupling (nonperturbative) regime of QCD. As is well
known, the high energy scattering amplitudes may be phenomenologically described by the exchange of states which correspond to singularities in the complex angular momentum in the crossed channel. These Regge singularities are
of two types. Pomeron exchange is leading in elastic scattering at high energy and corresponds to vacuum quantum
numbers while Reggeon exchanges are subleading and involves a priori various quantum number exchanges. The
high energy behaviour of amplitudes $s^{\alpha_{P,R}(t)}$ is characterized by universal Pomeron and Reggeon trajectories, whose
parameters are obtained from the analysis of various soft hadronic reactions. Indeed they differ both in intercept and
slope, as shown by typical values\textsuperscript{[3]}: $\alpha_P(t) \approx 1.08 + 0.25 t$ and $\alpha_R(t) \approx .55 + .93 t$, for the dominant trajectories. Qualitatively the difference in intercept between Pomeron and Reggeon trajectories can be related in QCD to the
effect of the exchange of wee partons\textsuperscript{[7]}, which are gluons or $q\bar{q}$ for the Pomeron and valence quarks for the Reggeons.

However a more precise theoretical determination from quantum field theories is made difficult by its nonperturbative
character.

Previous theoretical approaches to this strong coupling problem have mainly focused on the Pomeron trajectory\textsuperscript{[11]}. One technical reason seems to rely on the applicability of the eikonal approximation in non-perturbative
QCD calculations for quark-(anti-)quark scattering with vacuum quantum number exchange for which the quark
propagators are essentially mapped onto Wilson lines following the straight line classical quark trajectories.

Using the same approximation in the framework of the AdS/CFT correspondence\textsuperscript{[3–5,12]} the elastic $qq$ or $q\bar{q}$
scattering amplitude is given by the expectation value of a Wilson loop, which is related to a minimal surface problem
in an appropriate confining version of the AdS/CFT correspondence e.g. the AdS Black Hole (AdS BH) geometry\textsuperscript{[13]}. The physical amplitude is obtained through analytic continuation from Euclidean to Minkowski signature. The
classical approximation gives rise to the determination of the Pomeron slope and to a unit intercept, while fluctuations
around the minimal surface have the effect of adding a shift to the intercept above one. The values obtained in Ref.
\textsuperscript{[4,5]} agree well with the Pomeron trajectory.

In this paper we would like to extend this approach to the investigation of amplitudes which are mediated by the
exchange of a Reggeon. We therefore have to consider the exchange of quarks in the $t$ channel for which the eikonal
approximation is obviously no longer valid. This necessitates the introduction of new tools to express the scattering
amplitudes in terms of quantities computable using the AdS/CFT correspondence. Our method is to use the so-called
worldline formalism\textsuperscript{[14,15]} with fermionic spin factors\textsuperscript{[16–19]} which expresses the 4-point function corresponding to
the interaction of four $q\bar{q}$ states in terms of a path integral over quark trajectories in spacetime. We will again use AdS/CFT to compute the Wilson loop VEV along these trajectories. Then we will perform the path integral over the trajectories using semiclassical approximation. An important point is that we use the standard confining backgrounds of AdS/CFT, and thus neglect the effect of fermion loops.

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The relation between scattering amplitudes and the 4-point function involves the LSZ reduction and the knowledge of the wavefunction of the asymptotic states. For sake of simplicity we will assume in the present paper that the wavefunction is just the product of free spinors. The factors due to the propagation of the asymptotic states towards and from the interaction region (which are truncated by LSZ reduction) are not included in our calculation. A more refined analysis would have to discuss the structure of the confined $q\bar{q}$ bound states which is beyond the scope of our study.

The paper is organized as follows. In section II the different ingredients of the worldline formalism, namely quark trajectories, mass term, spin factor and Wilson loop VEV, are introduced. In section III we explain how to perform averaging over gauge fields for the Wilson loop VEV in the AdS BH background. In section IV we introduce the geometry of the minimal surface and its boundaries and compute the relevant spin factor, while in section V we evaluate the remaining path integral over the boundaries and give the final results, leading to the determination of the Reggeon trajectory. In section VI we give a summary of our results and discussion. An appendix is devoted to the mathematical derivation of a generic spin factor for trajectories embedded in a 3-dimensional subspace of the 4-dimensional spacetime.

II. SCATTERING AMPLITUDES WITH REGGEON EXCHANGE

Figure 1
Spacetime picture of a meson-meson scattering process mediated by Reggeon exchange. The impact parameter axis is perpendicular to the longitudinal $t-y$ plane.

Let us consider a meson-meson scattering process

\[(11') + (22') \rightarrow (33') + (44'),\]

where the continous lines 1-3, 2-4 correspond to spectator quark and antiquark, while the dashed lines 1'-2', 3'-4' correspond to annihilated and produced quark-antiquark pairs. The labels correspond to the initial and final spacetime position 4-vectors that we fix for our calculation.

The spacetime picture of this process is schematically illustrated in figure 1, where the impact parameter axis $x$ is perpendicular to the longitudinal $t-y$ plane. Note that the impact parameter is defined w.r.t. the spectator quark asymptotic trajectories.

The amplitude corresponding to the scattering process (1) can be schematically written as

\[\langle \text{out} | S_F(3', 4'| \mathcal{A}) S_F(1, 3| \mathcal{A}) S_F(4, 2| \mathcal{A}) S_F(2', 1'| \mathcal{A}) | \text{in} \rangle \]

where $|\text{out}\rangle$ and $|\text{in}\rangle$ are wavefunctions for the outgoing and incoming mesons (up to modifications due to LSZ reduction formulae). In formula (2), $S_F(X, Y | \mathcal{A})$ denotes the full quark propagator between spacetime points $X$ and $Y$ in a given background gauge field configuration $\mathcal{A}$, while the correlation function $\langle \ldots \rangle_{\mathcal{A}}$ stands for averaging over these configurations.

Let us first perform the calculations for the above scattering amplitude rotated into Euclidean space. Then we will rotate it back to Minkowski space using the substitutions.
\[
\begin{align*}
t & \to it \\
\theta & \to -i\chi \sim -i\log s
\end{align*}
\]  
(3)  

where \(\theta\) is the angle between the asymptotic straight lines \(1 \to 3\) and \(2 \to 4\). In impact parameter space we use the worldline expression for the (Euclidean) fermion propagator in a background gauge field \(A = A_\mu^\alpha(X^\mu)\) as a path integral over classical trajectories [13][19]:

\[
S_F(X,Y|A) = \int_0^\infty dT e^{-mT} \int D X^\mu(\tau) \delta(\dot{X}^2 - 1) I[X^\mu(\tau)] P e^i \int A_\mu(X(\tau)) \cdot X^\mu d\tau
\]  
(5)

Here the path integral is over trajectories \(X^\mu(\tau)\) joining \(X\) and \(Y\), parametrized by \(\tau \in (0,T)\). Because of the delta function, \(T\) is also the total length of the trajectory. The quark mass dependence appears in the first exponential.

The colour and gauge field dependence is encoded in the (open) Wilson line along the trajectory \(P e^i \int A_\mu(X(\tau)) \cdot X^\mu d\tau\), while the spin \(1/2\) character of the quark is responsible for the appearance of the spin factor:

\[
I[X^\mu(\tau)] = P \prod_1^\infty \left( \frac{1 + \dot{X}^\mu_T \gamma_\mu}{2} \right) \ldots \frac{1 + \dot{X}^\mu(\frac{2}{N}T) \gamma_\mu}{2} \frac{1 + \dot{X}^\mu(T) \gamma_\mu}{2}
\]  
(6)

where the second equality gives a suitably regularized definition of the infinite product along the trajectory \(X^\mu(\tau)\).

Note that each of the \(N\) factors in this expression is a projector due to the fact that \(\dot{X}^2 = 1\). This spin factor was first formulated for \(D=3\) in [16] and later for arbitrary \(D\) in [17,18]. In practice it was computed explicitly in \(D=2\) and \(D=3\), but not in general for \(D > 3\). One of our goals is to compute it for the configuration of figure 1 in \(D = 4\) spacetime. The calculation is presented in the Appendix.

Let us first make two comments about the calculation of (2):

i) Since the initial and final mesons are colour singlets, the four Wilson lines close to form a single Wilson loop, and the gauge averaging factorizes out of the expression:

\[
\left\langle \text{tr} P e^i \int_C A \right\rangle_A
\]  
(7)

where the contour \(C\) follows the quark trajectories \(1 \to 3' \to 4' \to 2' \to 1'\) (following the contours sketched on figure 1).

ii) The spin factor matrices

\[
I[1 \to 3]_{\alpha_1 \alpha_3} I[4 \to 2]_{\alpha_4 \alpha_2} I[2' \to 1']_{\alpha_{2'} \alpha_1} I[3' \to 4']_{\alpha_{3'} \alpha_{4'}}
\]  
(8)

are contracted with the initial and final spinor wavefunctions like \(u_{\alpha_1}(p_1) \bar{u}_{\alpha_1'}(p_1)\), corresponding to a simple approximation for the wave-functions of the external mesons as mentioned in the introduction.

### III. AVERAGING OVER THE GAUGE FIELDS — THE ADS BLACK HOLE BACKGROUND

According to the AdS/CFT correspondence the expectation value of a Wilson loop is given by the string partition function with the condition that the AdS string ends on the curve \(C\) which is placed on the boundary of the AdS geometry, which represents the physical spacetime [21]. Using the saddle point approximation (at large \(g^2N\) gauge coupling) gives the formula:

\[
\langle W(C) \rangle \approx \text{Fluctuations}(C) \cdot e^{-\frac{\text{Area}(C)}{2\pi \alpha'}}
\]  
(9)

where \(\text{Area}(C)\) is the minimal area of a string worldsheet evaluated in the \textit{curved} geometry of the background. This background has to be chosen dual to the appropriate gauge field theory. \(\alpha'\) is the AdS string tension. The prefactor \(\text{Fluctuations}(C)\) is the contribution of quadratic fluctuations around the minimal surface.

In this paper we will use the confining Black Hole (BH) background proposed in [13]. Let us recall the main features of the evaluation of the expectation values of Wilson loops in this geometry.

When the Wilson loop is large enough (in comparison to the horizon radius, which sets the confinement scale of the theory), the minimal surface is concentrated near the horizon and we can use the flat space approximation. Then [1] takes the form.
\( \langle W(C) \rangle = \text{Fluctuations}(C) \cdot e^{-\frac{1}{2\alpha'_{\text{eff}}} \text{Area}_{\text{FLAT}}(C)} \) 

where now the minimal surface area is evaluated in the flat metric and the parameter \( \alpha'_{\text{eff}} \) involves a scale related to the BH metric near the horizon. The version of AdS background that we are using does not allow to determine \( \alpha'_{\text{eff}} \) from first principles. It can be directly related, however, to the static quark-antiquark potential obtained from a rectangular Wilson loop of size \( T \times L \):

\[ \langle W(T \times L) \rangle \equiv e^{TV(L)} = e^{T \left[-\frac{4}{2\alpha'_{\text{eff}}} + n_{\perp} \frac{\pi^2}{24} L + \cdots \right]} , \]

where \( T, L \) and \( T/L \gg 1 \).

The first term in the potential comes from the area of the rectangle, while the second one is due to fluctuations. Note that, when calculating the fluctuations around a minimal surface near the horizon in the BH backgrounds there could be \( n_{\perp} = 7, 8 \) massless bosonic modes. In general we will keep \( n_{\perp} \) as a free parameter in order to accommodate other geometric realizations of a confining gauge theory. It is at this stage that the string picture from AdS/CFT may deviate from the old ‘effective string’ picture of QCD. The \( n_{\perp} \) need not be necessarily equal to \( 4 - 2 = 2 \). The ambient number of effective dimensions in the dual string theory may be indeed higher. The question of its interpretation in terms of gauge field theory collective degrees of freedom remains an interesting open question.

Within our approximation we will just use the two parameters \( \alpha'_{\text{eff}} \) and \( n_{\perp} \) which parametrize the behaviour of string theory in the confining geometry. It is interesting to note that even in the absence of an \textit{ab initio} theoretical determination of these parameters, they can be obtained from e.g. lattice QCD calculations and used to predict the behaviour of scattering amplitudes at high energy.

### IV. EVALUATION OF THE SCATTERING AMPLITUDES

We will now proceed to evaluate the scattering amplitude. In this section we will just concentrate on the classical configurations leaving the evaluation of the contribution of quadratic fluctuations to the following section.

In the previously known nonperturbative calculations of scattering amplitudes using Wilson line/loop formalism, it was crucial to consider the eikonal approximation for the quark trajectories. This is very well justified for elastic amplitudes (Pomeron trajectories), where all quarks are effectively spectators i.e. their spacetime trajectories are supposed not to be deflected by the gluon field. On the contrary, the Reggeon exchange amplitude implies that two quark lines (i.e. \( 2' \rightarrow 1' \) and \( 3' \rightarrow 4' \) in figure 1) are exchanged in the \( t \) channel and thus the corresponding propagators cannot be described within the eikonal approximation.

#### Spectator quarks

The eikonal approximation is expected still to be valid for the spectator quarks (i.e. \( 1 \rightarrow 3 \) and \( 2 \rightarrow 4 \) in figure 1). It will be convenient to assume that the spectator quarks are heavy, while the exchanged quarks are light. Hence the spectators just follow straight line trajectories, while the path integrals of noneikonal type have to be done w.r.t the trajectories \( 2' \rightarrow 1' \) and \( 3' \rightarrow 4' \).

The spin factors for the spectator quarks are simple. Indeed, all the terms in the product \( \bar{u}_3 \alpha_3(p_1) u_1(p_1) \) are identical as \( \bar{X} \) is constant along the trajectory. Since they are projectors, the product reduces to a single term. Therefore the spin factor is just

\[ \frac{1 + \hat{X}\mu\gamma\mu}{2} \equiv \frac{1 + \hat{p}_{\mu}}{2} \quad i = 1, 2 . \]

When we act with this projector on the spinor \( u(p_i) \), we have \( \hat{p}_{\mu} u(p_i) = |p_i| u(p_i) \equiv m u(p_i) \), so that e.g. \( J[1 \rightarrow 3] u(p_1) = u(p_1) \). Contracting this with the outgoing spinor will give the prefactor (in the forward direction)

\[ \bar{u}_{\alpha_3}(p_1) u_{\alpha_1}(p_1) = \delta_{\alpha_3\alpha_1} . \]
Exchanged quarks

Let us consider the path integral expression for \(2\) after inserting the four quark propagators \(3\). Integrating first over the gauge field configurations (for fixed trajectories), the amplitude can be be schematically rewritten as

\[
\int DX^\mu(\tau) \left\{ \delta(\dot{X}^2 - 1) \cdot (\text{Spin factors}) \cdot \langle W(1 \to 3' \to 4' \to 2' \to 1') \rangle_{A} \cdot e^{-m(\text{Length}[2'\to 1'] + \text{Length}[3'\to 4'])} \right\},
\]

where the contributions of the lengths of the spectator trajectories are included in the implicit normalization. Since we assumed that the spectator quarks are heavy and follow straight line trajectories, the Wilson loop is formed out of two straight lines and the trajectories \(2' \to 1'\) and \(3' \to 4'\).

The dominant contribution in \(14\) will come from evaluating the Wilson loop expectation value (saddle point in \(\alpha'_f\) c.f. \(3\)) through the minimal area in the AdS BH bulk having this loop as the boundary.

Let us now assume that the exchanged quarks are nearly massless. Our main assumption is that their trajectories will be constrained to remain on the minimal surface spanned between the two (infinite) spectator trajectories. This is a kind of softness assumption which is sensible in the classical approximation that we consider.

The relevant minimal surface is the helicoid \(4\) parametrized by

\[
t = \tau \cos p\sigma \\
y = \tau \sin p\sigma \\
x = \sigma
\]

where

\[
p = \theta/L,
\]

\[
\sigma = -L/2 \ldots L/2.
\]

and \(L\) is the impact parameter distance. The nearly massless exchanged quarks follow trajectories on the helicoid, which are defined by specifying the functions \(\tau(\sigma)\). These trajectories form the upper and lower edges of the Wilson loop and will be obtained through a subsequent minimization.

Averaging over the gauge fields involves, according to \(14\), the calculation of the area of the piece of helicoid between the boundaries \(\pm \tau(\sigma)\). Using the results \(4\) we obtain

\[
\text{Area}[\tau(\sigma)] = \int_{-L/2}^{L/2} d\sigma \int_{-\tau(\sigma)}^{\tau(\sigma)} d\tau \sqrt{1+p^2\tau^2} = \int_{-L/2}^{L/2} d\sigma \left\{ \tau(\sigma)\sqrt{1+p^2\tau^2(\sigma)} + \frac{1}{p} \log \left(p\tau(\sigma) + \sqrt{1+p^2\tau^2(\sigma)} \right) \right\}
\]

while the length of the edges is given by

\[
\text{Length}[\tau(\sigma)] = \int_{-L/2}^{L/2} d\sigma \sqrt{1+p^2\tau^2(\sigma) + \left(\frac{d\tau(\sigma)}{d\sigma}\right)^2}.
\]

Spin factor

It remains to evaluate the remaining piece of the amplitude \(14\), the spin factor associated to the trajectories \(2' \to 1'\) and \(3' \to 4'\). Since the quark trajectories are contained in the helicoid, they lie in a 3 dimensional hypersurface of the 4D physical spacetime. For these types of trajectories the spin factor simplifies substantially. The result of the calculation presented in the Appendix boils down to the following expression

\[
I[\dot{X}] = \frac{1 + \dot{X}^\mu(T)\gamma^\mu}{2} \cdot \frac{1 + \dot{X}^\mu(0)\gamma^\mu}{2} \cdot \left(\frac{1 + \dot{X}(T) \cdot \dot{X}(0)}{2}\right)^{-1}
\]

which depends only on the initial and final direction vectors \(\dot{X}^\mu(0), \dot{X}^\mu(T)\), and not on the specific form of the whole trajectory, provided it is smooth.

If we contract the projectors with the appropriate spinors \(u(p_1), \bar{v}(p_2)\) the projectors act like the identity after using the free Dirac equation (recall the discussion after formula \(2\)). The scalar products \(\dot{X}(T) \cdot \dot{X}(0)\) are equal to \(-\cos \theta \to -s\), therefore in the Minkowskian high energy limit we obtain after contraction the factor

\[
\mathcal{I} = \bar{v}_{\alpha_3'}(p_1)u_{\alpha_2'}(p_2) \cdot \bar{u}_{\alpha_3}(p_2)v_{\alpha_3'}(p_1) \cdot \frac{1}{s^2} = -\frac{1}{s} \cdot \sigma^3_{\alpha_3', \alpha_3'} \cdot \sigma^3_{\alpha_2', \alpha_2'}.
\]
The final result is obtained through the computation of the path integral over the trajectories of the exchanged quarks parametrized by the function $\tau(\sigma)$. Using the results of the previous section, formula (14) can be rewritten as

$$I \cdot \int D\tau(\sigma) \text{Fluctuations}[\tau(\sigma)] e^{-\frac{1}{2\pi\alpha'_\text{eff}} \text{Area}[\tau(\sigma)] - 2m \text{Length}[\tau(\sigma)]}.$$  \hspace{1cm} (23)

The $\text{Area}[\tau(\sigma)]$ is given by formula (19), $\text{Fluctuations}[\tau(\sigma)]$ is the contribution of quadratic fluctuations of the string around the given minimal surface defined by the trajectory $\tau(\sigma)$, see (10). $\text{Length}[\tau(\sigma)]$ appears in formula (20).

Let us perform this integral by saddle point in the limit of $\alpha'_\text{eff}$ small. As a consequence we may consider both the $m$ dependence and the fluctuation contribution as prefactors not entering the saddle point equations. This should be true for small enough masses $m$. We will discuss this point later on.

The Euler-Lagrange equations are

$$\frac{\partial \text{Area}[\tau(\sigma)]}{\partial \tau} = \sqrt{1 + p^2 \tau(\sigma)^2} = 0.$$  \hspace{1cm} (24)

The solution is a constant complex $\tau(\sigma) = \pm i/p$. Here the complex value has to be understood in the sense of applying the steepest descent method to the path integral (23), and deforming the integration contours into the complex plane.

Physically this indicates an instability of the minimal surface problem associated with the scattering process. This is reminiscent of a similar phenomenon encountered in the semiclassical study of high energy scattering of asymptotic open string states in flat spacetime c.f. Ref. [28].

Substitution of the classical solution $p\tau(\sigma) = -i$ into (23) gives a non vanishing contribution from the logarithm:

$$e^{-\frac{1}{2\pi\alpha'_\text{eff}} \text{Area}[-i/p]} = e^{-\frac{4\pi^2}{4\pi}\theta} e^{-\frac{L^2}{4\pi\alpha'_\text{eff}}}$$  \hspace{1cm} (25)

after analytical continuation to Minkowski space.

Fluctuations

Let us evaluate the contribution of quadratic fluctuations of the string worldsheet around the minimal surface defined by the saddle point trajectories (24).

Since the classical trajectory is constant (albeit complex), we will calculate the result for fixed $\tau(\sigma) = T$ for real $T$, and then analytically continue to $T = -i/p$.

The fluctuation determinant for the case of a helicoid bounded by two helices with $\tau = \pm T$ has already been calculated [5]. Let us briefly recall the basic steps. First one replaces the variable $\tau$ in (15)-(17) by

$$\rho = \frac{1}{p} \log(p\tau + \sqrt{1 + p^2\tau^2}).$$  \hspace{1cm} (26)

In the variables $\rho, \sigma$ the induced metric on the helicoid is conformally flat i.e.

$$g_{ab} = (\cosh^2 p\rho) \delta_{ab}.$$  \hspace{1cm} (27)

Therefore, since string theory in the AdS background is critical, we may perform the calculation for the conformally equivalent flat metric $g_{ab} = \delta_{ab}$. This reduces to a calculation of the fluctuation determinant for a rectangle of size $a \times b$ where

$$a = L$$

$$b = \frac{2L}{\theta} \log \left(pT + \sqrt{1 + p^2T^2} \right).$$  \hspace{1cm} (28)\hspace{1cm} (29)

We assume furthermore that the quadratic bosonic fluctuations are governed by the Polyakov action, as is indeed the case for string theories on AdS backgrounds. The result is just equivalent to the Lüscher term computation (c.f. [11]), and for high energies (after continuation to Minkowski space $a/b = \mathcal{O}(\log s) \gg 1$) we obtain
Analytically continuing this expression to the saddle point $T = -i/p$ gives

$$Fluctuations(\tau(\sigma) \equiv T) = \exp \left( n_{\perp} \cdot \frac{\pi}{24} \cdot \frac{\theta}{2 \log \left( pT + \sqrt{1 + p^2T^2} \right)} \right).$$

(30)

Putting together the different components contributing to (23), namely the spin factor (22), the classical minimal area contribution (25) and the fluctuation determinant (31), we get for the amplitude in impact parameter representation

$$s^{-\frac{n_{\perp}}{24}} e^{-\frac{k^2}{4\alpha_{eff} \log s}} \cdot \delta_{\alpha_1 \alpha_3} \delta_{\alpha_2 \alpha_4} \delta^3_{\alpha_1 \alpha_2 \alpha_3} \cdot \epsilon^3_{\alpha_4 \alpha_5 \alpha_6}.$$

(32)

A Fourier transform to momentum space gives the following result for the scattering amplitude:

$$s^{\frac{n_{\perp}}{24} + \alpha_{eff} t} \cdot \delta_{\alpha_1 \alpha_3} \delta_{\alpha_2 \alpha_4} \delta^3_{\alpha_1 \alpha_2 \alpha_3} \cdot \epsilon^3_{\alpha_4 \alpha_5 \alpha_6}.$$

(33)

i.e. a linear Regge trajectory

$$\alpha(t) = \frac{n_{\perp}}{24} + \alpha_{eff} t.$$

(34)

Note that we neglected in (33) possible logarithmic prefactors which are not under control at this stage of our approach.

Relation with Pomeron exchange

Let us compare our resulting amplitude with the one obtained for Pomeron exchange for which we obtained the trajectory (44)

$$\alpha_P(t) = 1 + \frac{n_{\perp}}{96} + \frac{\alpha_{eff} t}{4}.$$

(35)

The first observation is that in both cases, the slope is determined by minimal surface solutions through the logarithmic contribution in the helicoid area. The factor 4 difference in the slope comes from the specific saddle point path integral over the exchanged quark trajectories (for Reggeon exchange). It is interesting to note that this theoretical feature is in agreement with the phenomenology of soft scattering. Indeed once we fix the $\alpha_{eff}$ from the phenomenological value of the static $q\bar{q}$ potential ($\alpha'_{eff} \sim 0.9 \text{GeV}^{-2}$) we get for the slopes $\alpha_R = \alpha'_{eff} \sim 0.9 \text{GeV}^{-2}$ and $\alpha_P = \alpha'_{eff}/4 \sim 0.23 \text{GeV}^{-2}$ in good agreement with the observed slopes, c.f. section I.

The second feature is the relation between the Pomeron and Reggeon intercepts. At the classical level of our approach these are respectively 1 and 0. Note that this classical piece is in agreement with what is obtained from simple exchanges of two gluons and quark-antiquark pair, respectively, in the $t$ channel. The fluctuation (quantum) contributions to the Reggeon and Pomeron are related by a factor of 4. Adding both classical and fluctuation contributions gives an estimate which is in qualitative agreement with the observed intercepts. For $n_{\perp} = 7.8$ one gets $1.073 - 1.083$ for the Pomeron and $0.3 - 0.33$ for the Reggeon. This result is below the intercepts of around 0.5 observed for the dominant Reggeon trajectories. We will discuss some possible sources of this discrepancy in the summary.

An interesting feature of the formulae for the area (13) of the helicoid and fluctuations (30) around it, is the key role of the logarithmic term. This gives rise to the possibility of additional contributions from passing onto a different Riemann sheet ($\log \rightarrow \log + 2\pi ik$) in the course of performing analytical continuation from Euclidean to Minkowski space (44). The amplitude in impact parameter space (22) will then pick up new multiplicative factors:

$$e^{-\frac{k^2}{4\alpha_{eff} \log s}} \cdot e^{-\frac{k^2}{4\alpha_{eff} \log s}}.$$

(36)

This can be interpreted (for $k > 0$) as $k$-Pomeron exchange corrections to a single Reggeon exchange. Indeed the slope of the trajectory obtained from Fourier transform of formula (36) is just the one expected from such contributions. These are well known to describe absorptive corrections in Regge phenomenology. Note however that the intercept decreases with $k$ like $\frac{n_{\perp}}{24(1+\text{Re}k)}$ instead of growing as expected from usual absorptive correction models.
VI. SUMMARY AND DISCUSSION

In the present paper we proposed a method to study Reggeon exchange amplitudes starting from basics of nonperturbative gauge theory. Since the process involves the exchange of quarks between the incoming states the eikonal approximation, which is useful for Pomeron exchange, is no more valid and new tools have to be used. We consider then the worldline formalism which relates the scattering amplitude to a path integral over quark trajectories involving spin factors and a Wilson loop, both depending on the spacetime quark trajectories, which enter in the definition of the functional integral.

We give a semiclassical evaluation of the Wilson loop VEV using the AdS/CFT correspondence for a confining theory (AdS Black Hole). The appropriate minimal surface in the bulk turns out to be a portion of a helicoid whose boundaries, determined by minimization, are the exchanged quark trajectories, analytically continued in the complex plane. We prove that for such case, where the trajectories are contained in a 3-dimensional subspace of 4-dimensional spacetime, the spin factor depends only on the initial and final momenta and thus factor out of the functional integral. In this case this leads to a $1/s$ factor corresponding the Regge intercept 0, which is what is expected from two spin $1/2$ exchanges.

In this framework we find a Regge behaviour with linear trajectory with slope $\alpha'_{eff}$, directly related to the static $q\bar{q}$ potential. Interestingly enough from a phenomenological point of view, this is exactly four times the slope of the Pomeron trajectory obtained in [4,5] within the same approach.

The contribution of worldsheet fluctuations around the minimal surface for fixed boundaries gives rise, after analytical continuation to Minkowski space, to an intercept increment equal to $n_\perp/24$ where $n_\perp \sim 7,8$ is the number of transverse massless fluctuation modes of the string in the 10 dimensional bulk. This value is directly related to the Lüscher term coefficient in the static potential. It is four times the increment of the Pomeron intercept above 1 found in the same approach [5].

Let us add a few comments. The minimization procedure leading to (24) can be extended by including the mass term in the saddle point determination also leading to a constant trajectory solution $\tau(\sigma) = constant$. We checked that this constant goes to the value obtained in section 5 when $m \rightarrow 0$. However if $m$ is not small enough we do not expect that the minimal surface would remain helicoidal, since it could be deformed by mass terms involved in the minimization.

In addition, apart from the fluctuations around the minimal surface with fixed (saddle point) boundaries, one expects also a contribution from fluctuation of the boundary trajectories, both inside the helicoid surface, as well as outside. This is a more involved problem which deserves to be studied in the future.

We did not study in detail the helicity structures of the amplitudes. It is important since in the real world they play a rôle in distinguishing different Reggeon trajectories like $\rho$ and $\pi$ exchanges differing in the intercepts. In any case we expect no difference in the Regge slopes which are obtained independently of the spin factors. Our result, using only simplifying assumptions on the asymptotic wave functions could correspond to a combination of different Regge trajectories.

Finally, an interesting feature of AdS/CFT is the possibility of exploring the transition to shorter distances, where the curved background geometry will start to play an important role modifying the minimal surfaces and thus the behaviour of scattering amplitudes.

Note added: While completing this paper, a study of high energy scattering in the AdS/CFT framework [29], investigating different processes and using different tools, has appeared.

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Appendix — Evaluation of the spin factor

In this appendix we will evaluate the spin factor in 4D for a trajectory which is contained in the 3D hypersurface \((t, x, y)\). We will use the standard basis of gamma matrices, rotated by \(i\) to Euclidean space:

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & i\sigma^k \\ -i\sigma^k & 0 \end{pmatrix}.
\]

Following [17], we evaluate the product of projectors

\[
I[\dot{X}] = \prod_{i=0}^{N} \frac{1 + \dot{X}^\mu (\tau_i) \gamma^\mu}{2}; \quad \tau_i = i \frac{T}{N}.
\]

Parametrizing the velocity by

\[
\dot{X}^\mu = (\dot{t}, \dot{x}, \dot{y}, \dot{z}) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta, 0),
\]

the projector has the following form

\[
\frac{1 + \dot{X}^\mu \gamma^\mu}{2} = \frac{1}{2} \begin{pmatrix}
1 + \sin \theta \cos \phi & 0 & 0 & \cos \theta + i \sin \theta \sin \phi \\
0 & 1 + \sin \theta \cos \phi & 0 & -\cos \theta + i \sin \theta \sin \phi \\
0 & 0 & -\cos \theta - i \sin \theta \sin \phi & 1 - \sin \theta \cos \phi \\
\cos \theta - i \sin \theta \sin \phi & 0 & 0 & 1 - \sin \theta \cos \phi
\end{pmatrix}.
\]

We see that this matrix is a direct sum of two \(2 \times 2\) matrices which are by themselves projectors. We may write it as

\[
\frac{1 + \dot{X}^\mu \gamma^\mu}{2} = |\theta, \phi, 1\rangle |\theta, \phi, 2\rangle \langle \theta, \phi, 1| \langle \theta, \phi, 2| \cdot \Omega
\]

where the 2-dimensional vector \(|\theta, \phi, 1\rangle\) (respectively \(|\theta, \phi, 2\rangle\)) is embedded in the 1–4 (resp. 2–3) 2-dimensional subspace of the 4-dimensional spinorial representation. Explicitly, these vectors are given by:

\[
|\theta, \phi, 1\rangle = \frac{1}{\sqrt{2(1 - \cos \phi \sin \theta)}} \begin{pmatrix}
\cos \theta + i \sin \theta \sin \phi \\
\sin \theta \cos \phi
\end{pmatrix},
\]

and

\[
|\theta, \phi, 2\rangle = \frac{1}{\sqrt{2(1 - \cos \phi \sin \theta)}} \begin{pmatrix}
-\cos \theta + i \sin \theta \sin \phi \\
\sin \theta \cos \phi
\end{pmatrix}.
\]

Thanks to the decomposition [11] the spin factor can now be easily rewritten as

\[
|\theta_N, \phi_N, 1\rangle |\theta_N, \phi_N, 2\rangle \langle \theta_0, \phi_0, 1| \langle \theta_0, \phi_0, 2| \cdot \Omega
\]

where the scalar \(\Omega\) is given by

\[
\Omega = \prod_{i=0}^{N-1} \langle \theta_{i+1}, \phi_{i+1}, 1| \theta_i, \phi_i, 1\rangle \langle \theta_{i+1}, \phi_{i+1}, 2| \theta_i, \phi_i, 2\rangle.
\]

For smooth trajectories \(\Omega\) is equal to 1. Indeed we have

\[
\langle \theta + d\theta, \phi + d\phi, 1| \theta, \phi, 1\rangle = 1 - i \frac{\sin \phi \, d\theta + \cos \phi \cos \theta \, d\theta \, d\phi}{2 - 2 \cos \phi \sin \theta}
\]

\[
\langle \theta + d\theta, \phi + d\phi, 2| \theta, \phi, 2\rangle = 1 + i \frac{\sin \phi \, d\theta + \cos \phi \cos \theta \, d\theta \, d\phi}{2 - 2 \cos \phi \sin \theta}
\]

where we neglected higher order terms in \(d\theta, d\phi\). We see therefore that the spinors get rotated in opposite directions in the two 2-dimensional subspaces of the 4-dimensional spinorial representation. Putting the two contributions together we obtain:
\[ \langle \theta + d\theta, \phi + d\phi, 1 | \theta, \phi, 1 \rangle \cdot \langle \theta + d\theta, \phi + d\phi, 2 | \theta, \phi, 2 \rangle \sim 1 + \mathcal{O}(d\theta^2, d\phi^2, d\theta d\phi) \].

Thus \( \Omega \) can acquire nontrivial contributions only from cusps.

Finally let us rewrite (44) as
\[
I[\dot{X}] = \frac{1 + \dot{X}(T)\gamma^\mu 1 + \dot{X}(0)\gamma^\mu}{2} \cdot \left( \frac{1 + \dot{X}(T)\dot{X}(0)}{2} \right)^{-1} \cdot \Omega_{\text{cusps}}
\]

in which we used the relation
\[
\langle \dot{X}(T), 1 | \dot{X}(0), 1 \rangle \langle \dot{X}(T), 2 | \dot{X}(0), 2 \rangle = \frac{1 + \dot{X}(T)\dot{X}(0)}{2}
\]
where \( \Omega_{\text{cusps}} \) gets contributions only from cusps. For the case at hand, the trajectory found from solving the Euler-Lagrange equations (24) is smooth apart from two cusps where the curves of \( \tau = \text{const} \) on the helicoid meet the straight line trajectories of spectator quarks. Since they meet at right angles, the contribution of each cusp is just 1/2 as can be derived from (50).

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