Comments on “Rates of processes with coherent production of different particles and the GSI time anomaly” by C. Giunti, Phys. Lett. B 665, 92 (2008), arXiv: 0805.0431 [hep–ph]

A. N. Ivanov a,b, E. L. Kryshenε, M. Pitschmanna, P. Kienleb,ε,ε

aAtominstitut der Österreichischen Universitäten, Technische Universität Wien, Wiedner Hauptstraße 8-10, A-1040 Wien, Österreich,
bStefan Meyer Institut für subatomare Physik, Österreichische Akademie der Wissenschaften, Boltzmanngasse 3, A-1090, Wien, Österreich,
cPetersburg Nuclear Physics Institute, 188300 Gatchina, Orlova roscha 1, Russian Federation,
dPhysik Department, Technische Universität München, D–85748 Garching, Germany

We give comments on the recent paper by Giunti (Phys. Lett. B 665, 92 (2008), arXiv: 0805.0431 [hep–ph]) with a critique of our explanation of the experimentally observed periodic time–dependence of the interference term in the rate of the K–shell electron capture decay of the H–like ions 140Pr 58+ and 142Pm 60+, as a two–neutrino–flavour mixing. We show also that this phenomenon cannot be explained by a coherent mixing of two states of a mother ion as proposed by Giunti.
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Recently Litvinov et al. [1] have observed that the K–shell electron capture (EC) decay rates of the H–like ions 140Ce 58+ or 142Nd 60+

\[ 140\text{Pr}^{58+} \rightarrow 140\text{Ce}^{58+} + \nu, \]
\[ 142\text{Pm}^{60+} \rightarrow 142\text{Nd}^{60+} + \nu, \] (1)

have unexpected oscillatory structure. According to the experimental data [1], the rates of the number \( N_{d}^{EC} \) of daughter ions 140Ce 58+ or 142Nd 60+

\[ \frac{dN_{d}^{EC}(t)}{dt} = \lambda_{EC}^{(H)}(t) N_{m}(t), \] (2)

where \( N_{m}(t) \) is the number of mother H–like ions 140Pr 58+ or 142Pm 60+ [1] and \( \lambda_{EC}^{(H)}(t) \) is the EC–decay rate, are periodic functions, caused by a periodic time–dependence of the EC–decay rates

\[ \lambda_{EC}^{(H)}(t) = \lambda_{EC}^{(H)} \left( 1 + a_{EC} \cos \left( \frac{2\pi t}{T_{d}} + \phi \right) \right) \] (3)

with a period \( T_{d} \approx 7 \text{ sec} \) and an amplitude \( a_{EC} \approx 0.20 \).

We have proposed in [2] an explanation of the periodic time–dependence of the EC–decay rates as the interference of two massive neutrinos \( \nu_{1} \) and \( \nu_{2} \) with masses \( m_{1} \) and \( m_{2} \), respectively. The period \( T_{d} \) of the time–dependence has been related to the difference \( \Delta m_{21}^{2} = m_{2}^{2} - m_{1}^{2} \) of the squared neutrino masses \( m_{2} \) and \( m_{1} \)

\[ \frac{2\pi}{T_{d}} = \frac{\Delta m_{21}^{2}}{2\gamma M_{m}}, \] (4)

where \( M_{m} \) is the mass of the mother ion and \( \gamma = 1.43 \) is a Lorentz factor [1].

For the calculation of the EC–decay rate we have used the standard weak interaction Hamilton operator

\[ H_{W}(t) = \int d^{3}x \sum_{j} U_{ej} \mathcal{H}_{W}^{(j)}(x), \] (5)

where \( U_{ej} \) are the matrix elements of the mixing matrix of massive neutrinos and \( \mathcal{H}_{W}^{(j)}(x) \) is defined by

\[ \mathcal{H}_{W}^{(j)}(x) = \frac{G_{F}}{\sqrt{2}} V_{ud}[\bar{\nu}_{\nu}(x)\gamma_{\mu}(1 - g_{A}\gamma_{5})\psi_{d}(x)] \times [\bar{\psi}_{\nu}(x)\gamma_{\mu}(1 - \gamma_{5})\psi_{e}(x)] \] (6)

\[ \epsilon = \text{arXiv:0807.2750v2 [nucl-th] 18 Jul 2008} \]

aE-mail: ivanov@kph.tuwien.ac.at
bE-mail: E.Kryshen@gsi.de
cE-mail: pitschmann@kph.tuwien.ac.at
dE-mail: Paul.Kienle@ph.tum.de
with standard notations [2].

The amplitude \( A(m \to d + \nu) \) of the EC–decay \( m \to d + \nu \), where \( m \), \( d \) and \( \nu \) are the mother ion, the daughter ion and a neutrino, has been defined as follows

\[
A(m \to d + \nu) = \sum_j U_{ej} A(m \to d + \nu_j),
\]

where the coefficients \( U_{ej} \) testify that the electron couples to the electron neutrino. In turn, the amplitude \( A(m \to d + \nu_j) \) is equal to

\[
A(m \to d + \nu_j) = -\int d^4 x \langle \nu_j d | \mathcal{H}^{(j)}_W(x) | m \rangle = \]

\[
\begin{align*}
&= -(2\pi)^4 \delta^{(4)}(k_d + k_j - k_m) \langle \nu_j d | \mathcal{H}^{(j)}_W(0) | m \rangle =
\end{align*}
\]

\[
= (2\pi)^4 \delta^{(4)}(k_d + k_j - k_m) \mathcal{M}(m \to d + \nu_j)
\]

(8)

with \( \mathcal{M}(m \to d + \nu_j) = -\langle \nu_j d | \mathcal{H}^{(j)}_W(0) | m \rangle \).

Recently Giunti has criticised this explanation [3]. According to Giunti [3], the correct neutrino wave function in the final state of the EC–decays of the H–like ions \( m \to d + \nu \) should be taken in the form

\[
| \nu_e(t) \rangle = \frac{\sum_k A_k(t) | \nu_k \rangle}{\sqrt{\sum_j |A_j(t)|^2}}
\]

(9)

with \( A_k(t) \), defined by

\[
A_k(t) = -i \int_0^t dt' \langle \nu_k d | H_W(t') | m \rangle,
\]

(10)

where \( H_W(t) \) is the weak interaction Hamilton operator Eq. [5].

As has been shown in [4], such a wave function contradicts the principles both of standard time–dependent perturbation theory [6]–[9] and of quantum field theory [10][11].

In the recent paper [5] Giunti has undertaken a new attempt to refute the explanation, proposed in [2], of the experimental data by GSI [1]. Below we comment on Giunti’s analysis of the EC–decay rate.

**The EC–decay rate**

According to Giunti [5], the decay probability \( P_{m \to d + \nu} \), defined by

\[
P_{m \to d + \nu} = |A(m \to d + \nu)|^2
\]

(11)

with \( A(m \to d + \nu) \) given by Eq. (7), is not equal to

\[
P_{m \to d + \nu} ≠ \sum_j |U_{ej}|^2 |A(m \to d + \nu_j)|^2
\]

(12)

In addition Giunti claims that the decay probability \( P_{m \to d + \nu} \), Eq. (11) has an incorrect massless limit \( m_j \to 0 \), namely

\[
P_{m \to d + \nu} = \lim_{m_j \to 0} |A(m \to d + \nu)|^2 =
\]

\[
= |A(m \to d + \nu)|^2_{\text{SM}} \sum_j |U_{ej}|^2,
\]

(13)

whereas the correct limit is

\[
P_{m \to d + \nu} = \lim_{m_j \to 0} |A(m \to d + \nu)|^2 =
\]

\[
= |A(m \to d + \nu)|^2_{\text{SM}},
\]

(14)

where \( |A(m \to d + \nu)|^2_{\text{SM}} \), calculated in the Standard Model of electroweak interactions of heavy ions, is equal to

\[
|A(m \to d + \nu)|^2_{\text{SM}} = \lim_{m_j \to 0} |A(m \to d + \nu_j)|^2.
\]

(15)

The incorrectness of these assertions is clearly seen if one takes into account correctly the contribution of the \( \delta \)–functions \( (k_d + k_j - k_m) \), describing the conservation of energy and 3–momentum in the EC–decays.

Substituting Eq. (5) into Eq. (11) we get

\[
P_{m \to d + \nu} = |A(m \to d + \nu)|^2 =
\]

\[
= \sum_j |U_{ej}|^2 [(2\pi)^4 \delta^{(4)}(k_d + k_j - k_m)]^2
\]

\[
× |\mathcal{M}(m \to d + \nu_j)|^2 + 2 \sum_{i>j} \text{Re}[U_{ej}^* U_{ei}]
\]

\[
× |\mathcal{M}^*(m \to d + \nu_i)\mathcal{M}(m \to d + \nu_j)|
\]

\[
× [(2\pi)^4 \delta^{(4)}(k_d + k_i - k_m)]
\]

\[
× [(2\pi)^4 \delta^{(4)}(k_d + k_j - k_m)].
\]

(16)

For subsequent calculations one has to use the relations [10]

\[
[(2\pi)^4 \delta^{(4)}(k_d + k_j - k_m)]^2 =
\]

\[
= VT (2\pi)^4 \delta^{(4)}(k_d + k_j - k_m).
\]

(17)
Giunti’s wave function of neutrino with lepton flavour $\ell$

Now let us make comments on Giunti’s wave function of neutrino in the final state of the $EC$-decay. In addition to the critique, expounded in [1], we would like to emphasize that Giunti’s wave function of the neutrino $\nu_\ell$ with the lepton flavour $\ell$ depends on the initial and the final states of the reaction $I_i \rightarrow I_f + \nu_\ell$ in which the neutrino $\nu_\ell$ is produced, where $I_i$ and $I_f$ are not necessary one-particle states. In order to accentuate this point we propose to rewrite the wave function Eq. (9) specifying the initial and final states as

$$|\nu_\ell(t)\rangle_{I_i I_f} = \frac{\sum_k A_k(t)_{I_i I_f} |\nu_k\rangle}{\sqrt{\sum_j |A_j(t)_{I_i I_f}|^2}}$$

with $A_k(t)_{I_i I_f}$, given by

$$A_k(t) = -i \int_0^t d\tau \langle\nu_k I_f | H_W(\tau) | I_i \rangle.$$

Hence, the neutrinos $(\nu_\ell)_{I_i I_f}$ and $(\nu_\ell')_{I_i' I_f'}$, produced in two different reactions $I_i \rightarrow I_f + \nu_\ell$ and $I_i' \rightarrow I_f' + \nu_\ell'$, are two different particles. They are not stable and the probability of the transition $(\nu_\ell)_{I_i I_f} \leftrightarrow (\nu_\ell')_{I_i' I_f'}$ is equal to

$$P(\nu_\ell \leftrightarrow \nu_\ell') = |\langle \nu_\ell' | (\nu_\ell(t)|\nu_\ell(t)\rangle_{I_f I_f'}|^2 =$$

$$= \sum_k |A_k(t)_{I_f I_f'}|^2 \sum_j |A_j(t)_{I_i I_i'}|^2,$$

where $A_k(t)_{I_f I_f'} \neq A_k(t)_{I_i I_i'}$ by definition due to different initial and final states of the reactions $I_i \rightarrow I_f + (\nu_\ell)_{I_i I_f}$ and $I_i' \rightarrow I_f' + (\nu_\ell')_{I_i' I_f'}$, respectively. Since the number of initial and final states $(I_i I_f)$ of the reactions producing neutrinos $(\nu_\ell)_{I_i I_f}$ with a lepton flavour $\ell$ is infinite, so, according to Giunti [3], there is an infinite set of neutrinos $(\nu_\ell)_{I_i I_f}$ with a leptonic flavour $\ell$.

Giunti’s explanation of “Darmstadt oscillations”

According to Giunti [5], the interference term in the $EC$-decay rate $m \rightarrow d + \nu_c$ comes from the
mixing of the different mass–states of the mother ion \( m \). For the wave function of the initial state of the mother ion, which is not an eigenstate of the mass–operator, Giunti has proposed the following expression

\[
|m\rangle = \cos \theta |m'\rangle + \sin \theta |m''\rangle,
\]

where \( |m'\rangle \) and \( |m''\rangle \) are two states of the mother ion with masses \( M_{m'} \) and \( M_{m''} \), respectively, and \( \theta \) is a mixing angle. This means that the initial state of the mother ion is a coherent state of two eigenstates of the mass–operator \( |m'\rangle \) and \( |m''\rangle \), respectively. The final state of the EC–decay is defined by the wave function \( |\tilde{d}, \nu_e\rangle \), where \( \nu_e \) is a massless electron neutrino. Since Giunti’s calculation has no relation to the real calculation of the \( EC \)–decay rate, which is needed for the comparison with the experimental data on the rate of the number of daughter ions \([2]\), below we give a calculation of the \( EC \)–decay rate within Giunti’s approach in detail.

According to standard time–dependent perturbation theory \([6]–[9]\), the amplitude of the \( m \rightarrow d + \nu_e \) decay is defined by (see also Eq. \((10)\) and \((15)\)),

\[
A(m \rightarrow d + \nu_e)(t) = \langle d, \nu_e | H_W(\tau) | m \rangle,
\]

where \( |m\rangle \) is the wave function Eq. \((26)\) and the weak interaction Hamilton operator \( H_W(t) \) takes the form \([12]\)

\[
H_W(t) = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3x [\bar{\psi}_n(x) \gamma^\mu (1 - g_A \gamma^5) \psi_p(x)] \times [\bar{\psi}_{\nu_e}(x) \gamma_\mu (1 - \gamma^5) \psi_{\nu_e} - (x)].
\]

Suppose that the wave function of the neutrino in the \( EC \)–decay is a plane wave. In this case the amplitude of the \( EC \)–decay is equal to \([6]–[9]12\)

\[
A(m \rightarrow d + \nu_e)(t) = -i \int_0^t dt \langle d, \nu_e | H_W(\tau) | m \rangle,
\]

where \( \Delta E' = E_d(q') + E_{\nu_e}(\tilde{k}) - M_{m'} \) and \( \Delta E'' = E_d(q') + E_{\nu_e}(\tilde{k}) - M_{m''}, \) \( q' \) and \( \tilde{k} \) are 3–momenta of the daughter nucleus and the neutrino, \( E_d(q') \) and \( E_{\nu_e}(\tilde{k}) \) are the energies of the daughter ion and the neutrino, respectively, \( M_{GT} \) is the nuclear matrix element of the Gamow–Teller transition and \( \langle \psi_1^{Z_2} \rangle \) is the wave function of the bound electron in the \( H \)–like mother ion, averaged over the nuclear density \([12]\).

The rate of the neutrino spectrum is defined by \([2]\)

\[
\frac{dN_{\nu_e}(t)}{dt} = \frac{1}{2M_{m'}} \int \frac{d^3q}{(2\pi)^3 2E_d(q')} \times \frac{1}{2F + 1} \sum_{M_{\nu} = -\frac{1}{2}}^{\frac{1}{2}} \frac{d}{dt} |A(m \rightarrow d + \nu_e)(t)|^2 = \frac{3}{2F + 1} V E_{\nu_e} |M_{GT}|^2 \langle \psi_1^{Z_2} \rangle^2 
\]

\[
\times \left\{ 2 \cos^2 \theta \frac{\sin(\Delta E' t)}{(\Delta E')} + 2 \sin^2 \theta \frac{\sin(\Delta E'' t)}{(\Delta E'')} + \sin 2\theta \left[ \frac{\sin \left( \frac{\Delta E'}{2} \right)}{\frac{\Delta E'}{2}} \cos \left( \frac{\Delta E' - \Delta E''}{2} \right) t \right] \right\}
\]

Here we have used the relation

\[
[(2\pi)^3 \delta^{(3)}(\vec{q} + \vec{k})]^2 = V (2\pi)^3 \delta^{(3)}(\vec{q} + \vec{k}),
\]

where \( (2\pi)^3 \delta^{(3)}(\vec{0}) = V \) is the normalisation volume \([10]\). For sufficiently long time we get \([6]–[11]\)

\[
\frac{dN_{\nu_e}(t)}{dt} = \frac{3}{2F + 1} V E_{\nu_e} |M_{GT}|^2 \langle \psi_1^{Z_2} \rangle^2 
\]

\[
\times \left\{ \cos^2 \theta (2\pi) \delta(\Delta E') + \sin^2 \theta (2\pi) \delta(\Delta E'') \right\} \times \cos(\Delta M_m t),
\]

where \( \Delta M_m = M_{m'} - M_{m''}, \Delta E' = E_d(\tilde{k}) + E_{\nu_e}(\tilde{k}) - M_{m'}, \) and \( \Delta E'' = E_d(\tilde{k}) + E_{\nu_e}(\tilde{k}) - M_{m''}. \) The \( EC \)–decay rate \( \lambda_{EC}^{(m)}(t) \) from the coherent
state $|m\rangle$ is defined by
\[ \lambda_{EC}^{(m)}(t) = \int \frac{d^3k}{(2\pi)^32E_\nu} \frac{1}{V} \frac{dN_\nu(t)}{dt} \] (33)
Substituting Eq. (32) into Eq. (33) and integrating over the neutrino phase volume we get
\[ \lambda_{EC}^{(m)}(t) = \lambda_{EC}\left\{ 1 + 2\sin2\theta\cos(\Delta M_m t) \right\} , \] (34)
where $\lambda_{EC}$ is the $EC$–decay constant, calculated in [12].

Thus, one can show that for the initial state of the mother ion, given by Eq. (26), the $EC$–decay rate has a periodic interference term with a period $T_d$, defined by the mass difference $\Delta M_m = M_{m'} - M_{m''}$, which is equal to
\[ \Delta M_m = \frac{2\pi\gamma\hbar}{T_d c^2} = 8.45 \times 10^{-16} \text{eV}/c^2 , \] (35)
where $\gamma = 1.43$ is the Lorentz factor [1]. A reduction of the $EC$–decay rate Eq. (34) to the experimental shape [1]
\[ \lambda_{EC}(t) = \lambda_{EC}\left\{ 1 + a_{EC}\cos\left(\frac{2\pi t}{T_d} + \phi \right) \right\} \] (36)
with $a_{EC} \simeq 0.20$ [1] can be carried out by changing $|m'\rangle \rightarrow e^{i\phi'}|m'\rangle$ and $|m''\rangle \rightarrow e^{i\phi''}|m''\rangle$, giving $\phi = \phi' - \phi''$, and setting $\theta \simeq 2.8\pi^0$.

The problem of such an explanation of the "Darmstadt oscillations" is as follows. If there exist two states of the mother ion $|m'\rangle$ and $|m''\rangle$ with a mass–difference $\Delta M_m = 8.45 \times 10^{-16} \text{eV}/c^2$, giving the contribution to the $EC$–decay through the coherent state $|m\rangle$, given by Eq. (28), the contribution to the $EC$–decay should be also from the coherent state $|\tilde{m}\rangle$
\[ |\tilde{m}\rangle = - \sin\theta|m'\rangle + \cos\theta|m''\rangle , \] (37)
which is not also an eigenstate of the mass–operator and orthogonal to the state $|m\rangle$. The coherent state $|\tilde{m}\rangle$, given by Eq. (37), can be produced in the system of mother ions on the same footing as the coherent state $|m\rangle$, given by Eq. (28). Indeed, the mother ions, injected into the Storage Ring, are produced by means of a fast projectile fragmentation with a statistical population of the states $|m'\rangle$ and $|m''\rangle$, which are eigenstates of the mass–operator. Hence, the probabilities $P_m$ and $P_{\tilde{m}}$ of the appearance of the coherent states $|m\rangle$ and $|\tilde{m}\rangle$, related by $P_m + P_{\tilde{m}} = 1$, should be equal $P_m = P_{\tilde{m}} = \frac{1}{2}$ due to a principle indistinguishability of these states.

The $EC$–decay rate $\lambda_{EC}^{(\tilde{m})}(t)$ of the $EC$–decay $\tilde{m} \rightarrow d + \nu_e$ from the coherent state $|\tilde{m}\rangle$ is equal to
\[ \lambda_{EC}^{(\tilde{m})}(t) = \lambda_{EC}\left\{ 1 - 2\sin2\theta\cos(\Delta M_{\tilde{m}} t) \right\} . \] (38)
The total $EC$–decay rate, caused by the $EC$–decays of the H–like heavy ions from the states $|m\rangle$ and $|\tilde{m}\rangle$, is defined by
\[ \lambda_{EC}(t) = P_m\lambda_{EC}^{(m)}(t) + P_{\tilde{m}}\lambda_{EC}^{(\tilde{m})}(t) = \lambda_{EC}\left\{ 1 + 2\sin2\theta(P_m - P_{\tilde{m}})\cos(\Delta M_{\tilde{m}} t) \right\} . \] (39)
Since there is no physical reason for $P_m \neq P_{\tilde{m}}$, setting $P_m = P_{\tilde{m}}$ one gets no interference terms in the $EC$–decay rate of the H–like heavy ion in the approach proposed by Giunti [5].

Summary

We have shown that Giunti’s critique of our approach is based, technically, on the missing of the $\delta$–functions, responsible for the conservation of energy and 3–momentum in the $EC$–decay and, globally, on the misunderstanding of the standard procedure for the calculation of the decay rates.

Giunti’s wave functions for neutrinos in the final state of the $EC$–decay of the H–like heavy ions or generally in any weak interaction producing or absorbing neutrinos make no sense, since they require an infinite number of neutrinos with a lepton flavour $\ell$. These neutrinos are not stable and oscillate with a finite probability.

As regards Giunti’s explanation of the “Darmstadt oscillations” we assert the following. Apart from the existence of a superweak interaction, leading to the mass–splitting of the H–like heavy ions of order $O(10^{-15} \text{eV}/c^2)$, which is hardly possible in reality, Giunti’s approach the total $EC$–decay rate of the H–like ions should be defined by $EC$–decays from two coherent states $|m\rangle$ and $|\tilde{m}\rangle$, given by Eqs. (28) and (37), respectively. These coherent states can appear in the system of mother ions with probabilities $P_m$ and $P_{\tilde{m}}$, respectively, and related by $P_m + P_{\tilde{m}} = 1$. The equality $P_m = P_{\tilde{m}} = \frac{1}{2}$, caused by a statistical equivalence of the coherent states $|m\rangle$ and $|\tilde{m}\rangle$ in
the system of mother ions, injected into the Storage Ring, shows the absence of the interference term in the EC–decay rate

$$\lambda_{EC}(t) = P_m \lambda^{(m)}_{EC}(t) + P_{\tilde{m}} \lambda^{(\tilde{m})}_{EC}(t) = \lambda_{EC}. \quad (40)$$

Thus, Giunti’s explanation of the “Darmstadt oscillations” has no physical ground and it is erroneous by definition.

Our analysis of Giunti’s explanation of the “Darmstadt oscillations” can be formulated more generally as “a non–existence of periodic time dependent interference terms in the EC–decay rates of the H–like heavy ions for the coherence in the initial state of the mother ions”. Indeed, the states $|m\rangle$ as well as the orthogonal state $|\tilde{m}\rangle$ can be treated as coherent states of $|m'\rangle$ and $|m''\rangle$, produced in the statistical system of mother ions injected into the Storing Ring. Due to a statistical equivalence of these states, the probabilities $P_m$ and $P_{\tilde{m}}$ of the appearance of the coherent states $|m\rangle$ and $|\tilde{m}\rangle$ in the system of mother ions should be equal $P_m = P_{\tilde{m}}$. This prohibits the appearance of the interference term in the EC–decay rate (see Eq. (39) and Eq. (40)).

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