Unusual destruction and enhancement of superfluidity of atomic Fermi gases by population imbalance in a one-dimensional optical lattice

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We study the superfluid behavior of a population imbalanced ultracold atomic Fermi gases with a short range attractive interaction in a one-dimensional (1D) optical lattice, using a pairing fluctuation theory. We show that, besides widespread pseudogap phenomena and intermediate temperature superfluidity, the superfluid phase is readily destroyed except in a limited region of the parameter space. We find a new mechanism for pair hopping, assisted by the excessive majority fermions, in the presence of continuum-lattice mixing, which leads to an unusual constant BEC asymptote for $T_c$ that is independent of pairing strength. In result, on the BEC side of unitarity, superfluidity, when it exists, may be strongly enhanced by population imbalance.

Ultracold atomic Fermi gases have been an ideal system for quantum simulation and quantum engineering, due to their multiple tunable parameters. Using a Feshbach resonance, one can vary the effective pairing strength from the weak coupling BCS limit to strong pairing BEC limit. As another widely explored parameter, population imbalance leads to Fermi surface mismatch and thus renders pairing more difficult, causing suppressed superfluid transition temperature $T_c$, or complete destruction of superfluidity at high population imbalances. Among other tunable parameters is the geometry of the system; one can put the Fermi gas in an optical lattice, such as a one-dimensional (1D) optical lattice (OL), which we shall explore here. Unlike the widely studied 3D continuum or 3D lattice (for which each lattice site contains 1 or 2 fermions), 1DOL is distinct in that it is a lattice-continuum mixed system with each lattice “site” now containing many fermions. Such a system has not been properly studied in the literature, with and without a population imbalance.

Population imbalance $p$ has been widely known to suppress or destroy superfluidity. Indeed, in a 3D homogeneous system, superfluidity is completely destroyed at $T = 0$ in the unitary and BCS regimes, leaving only possible intermediate temperature superfluids (ITSF). Nonetheless, in the BEC regime, stable superfluid exists even with very high $p$, and all minority fermions are paired up. This has been naturally understood as a consequence of vanishing Pauli blocking effect in the deep BEC regime, where the distribution of the constituent fermions in a Cooper pair spreads out over the entire momentum space and thus pairs can happily coexist with excessive fermions.

In this Letter, we show that the pairing and superfluid behavior of a Fermi gas, when subject to a short-range attractive interaction ($U < 0$) in 1DOL with $p \neq 0$, is very different due to the lattice-continuum mixing. We find that superfluidity may be readily destroyed by population imbalance, except for a very restricted parameter range (away from small $t$, large $p$, and large lattice constant $d$), where population imbalance gives rise to an extra mechanism for pair hopping. When a BEC superfluid does exist, this leads to an unusual constant BEC asymptote for $T_c$, and then can substantially increase $T_c$ on the BEC side of unitarity as compared to the balanced case, in which $T_c$ decreases with interaction strength. In addition, not all minority fermions are paired up in the BEC limit. We demonstrate that these unusual behaviors are associated with the mixing of a 2D continuum plane and a discrete lattice dimension, which leads to a constant ratio of $\Delta^2/\mu$, unlike in a 3D continuum or 3D lattice. This mixing enables enhanced pair hopping processes assisted by excessive fermions.

There have seemingly been many theoretical studies on Fermi gases in optical lattices. However, most studies have used the chemical potentials and magnetization as control variables and are thus limited to the weak and intermediate pairing strength regimes. In a 3D attractive Hubbard model with $U > 0$ and without population imbalance, superfluid in the deep BEC regime exists only at low fillings. On the other hand, 1DOL of $^4$Li has been realized experimentally with and without population imbalance. However, its phase diagram is yet to be explored. We emphasize that 1DOL is fundamentally a 3D system, albeit anisotropic. It cannot be compared with a genuine 2D or 1D lattice, which has usually no more than 2 fermions per site, and does not support true long-range order as we study here.

Here we use a previously developed pairing fluctuation theory. It goes beyond the BCS mean-field treatment by self-consistently including finite momentum pairing in the self energy, which thus contains two parts, $\Sigma(K) = \Sigma_{sc}(K) + \Sigma_{pg}(K)$, where $\Sigma_{sc}(K) = -\Delta c^\dagger_0(\bar{K})$ and $\Sigma_{pg}(K) = \sum_q t_{pq}(Q)\tilde{G}_0(Q-K)$, corresponding to the contributions of the Cooper pair condensate and finite momentum...
pairs, respectively. We shall follow the notations of Ref. [25], such that \( h = k_B = 1 \), and four momenta \( K \equiv (\omega_n, k) \), \( Q \equiv (\Omega, q) \), \( \Sigma_a = T \sum_q \), etc. Here \( G_0(K) \) is the non-interacting Green’s function, \( t_{pg}(Q) \) the Matrix, \( \Delta_{sc} \) the order parameter, and \( \omega_n (\Omega) \) the odd (even) Matsubara frequency. The finite momentum pairing directly leads to the presence of a pseudogap when it becomes strong. This theory has been applied to 3D homogeneous and trapped Fermi gases [11][26][27], as well as on a 3D or quasi-2D lattice [12][19], and has been used by other groups [21][23][30].

Now we adapt this theory for 1DOL by modifying the noninteracting atomic dispersion into \( \epsilon_{k\sigma} = \epsilon_k - \mu - \mu_{\sigma} \equiv k_{\parallel}^2/2m + 2t[1 - \cos(k_2d)] - \mu_{\sigma} \), where \( k_{\parallel} = (k_x, k_y) \) is the in-plane momentum, and \( \mu_{\sigma} \) the chemical potential for spin \( \sigma = \uparrow, \downarrow \). This one-band lattice dispersion is justified when the band gap in the \( z \) direction is tuned to be much greater than the Fermi energy in the \( xy \) plane. The derivation of our self-consistent equations is otherwise the same, so that we shall present the result directly, with an emphasis on the unusual new findings caused by population imbalance and the lattice-contiuum mixing.

In the superfluid phase, we define the pseudogap via \( \Delta_{pg}^2 = - \sum_Q t_{pg}(Q) \), so that the total gap \( \Delta \) is given by \( \Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2 \), which leads to the self energy \( \Sigma_{\sigma}(K) \approx -\Delta^2 G_0,\sigma(K) \), and the full Green’s function

\[
G_{\sigma}(K) = \frac{\nu_{\sigma}^2}{i\nu_{\sigma} - E_{k\sigma}} + \frac{\nu_{\sigma}^2}{i\nu_{\sigma} + E_{k\sigma}},
\]

where \( \nu_{\sigma}^2 = (1 + \xi_k/E_k)/2 \), \( \nu_{\sigma} = (1 - \xi_k/E_k)/2 \), \( E_{k\uparrow} = E_k - h, E_{k\downarrow} = E_k + h \), and \( E_k = \sqrt{\xi_k^2 + \Delta^2} \), \( \xi_k = \epsilon_k - \mu, \mu = (\mu_{\uparrow} + \mu_{\downarrow})/2, h = (\mu_{\uparrow} - \mu_{\downarrow})/2 \). Then we have the number equations,

\[
n = \sum_k \left[ \frac{1}{2} + \frac{f(E_{k\uparrow}) - f(E_{k\downarrow})}{2} \right],
\]

\[
p_m = \sum_k \left[ f(E_{k\uparrow}) - f(E_{k\downarrow}) \right],
\]

where \( p = (n_\uparrow - n_\downarrow)/n, \xi(x) = [f(x + h) + f(x - h)]/2, \) and \( f(x) = 1/(e^{x/T} + 1) \). We have the following gap equation with pair chemical potential \( \mu_p = 0 \) in the superfluid phase,

\[
m = 4\pi a \sum_k \left[ \frac{1}{2\epsilon_k} - \frac{1}{2} \frac{\nu_{\uparrow}^2}{E_k} \right] + a_0\mu_p,
\]

where the interaction \( U \) has been replaced by the \( s \)-wave scattering length \( a \) via \( U^{-1} = m/4\pi a - \sum_k 1/2\epsilon_k \). Here a finite \( \mu_p \) extends this equation into the non-superfluid phase. We caution that the parameter \( a \) does not necessarily yield the experimentally measured scattering length, which is better reflected by an effective scattering length \( a_{\text{eff}} \) such that \( 1/a_{\text{eff}} = \sqrt{2md}/a \). (See Supplementary Secs. I, II and Fig. S1). The coefficient \( a_0 \) is determined via Taylor expanding \( t_{pg}^{-1}(Q) \) on the real frequency axis, \( t_{pg}^{-1}(\Omega, q) \approx a_0\Omega^2 + a_0(\Omega - \Omega_q + \mu_p), \) with \( \Omega_q = B_q q_0^2 + 2t_B[1 - \cos(q_x d)] \). Here \( B_r = 1/2M_r \), with \( M_r \) being the effective pair mass in the \( xy \)-plane, and \( t_B \) is the effective pair hopping integral. Then we have the pseudogap equation

\[
a_0\Delta_{pg}^2 = \sum_q \frac{b(\Omega_q)}{\sqrt{1 + 4\frac{a_0}{a_1}(\Omega_q - \mu_p)}},
\]

with \( b(x) = 1/(e^{x/T} - 1) \) and pair dispersion \( \Omega_q = (\sqrt{a_0^2[1 + 4\mu(\Omega_q - \mu_p)/a_0] - a_0})/2a_1 \), which reduces to \( \Omega_q = \Omega_q - \mu_p \) when \( a_1/a_0 \ll 1 \), e.g., in the BEC regime.

Equations (2) - (5) form a closed set of self-consistent equations, which will be solved for \( (\mu_{\uparrow}, \mu_{\downarrow}, \Delta_{pg}, T_c) \) with \( \Delta_{sc} = 0 \), and for \( (\mu_{\uparrow}, \mu_{\downarrow}, \Delta, \Delta_{pg}) \) in the superfluid phase. For our numerics, we consider \( p > 0 \), and define Fermi momentum \( k_F = (3\pi^2n)^{1/3} \) and Fermi energy \( E_F \equiv k_BT_F = \hbar^2k_F^2/2m \).

The asymptotic solution in the BEC limit, \( \mu \to -\infty \), can be obtained analytically fully for \( p = 0 \) or partially for \( p > 0 \). For \( p = 0, \mu_{\uparrow} = \mu \). However, for \( p > 0 \), we have \( \mu_{\uparrow} > 0 \) at \( T_c \) throughout the BCS-BEC crossover, and \( \mu_{\downarrow} = 2\mu - \mu_{\uparrow} \). This is self-consistently justified by the solution that \( T_c \ll \Delta \ll |\mu| \) in the BEC regime. Therefore, in the BEC limit, \( f(E_{k\uparrow}^\dagger) = f(\xi_k^\dagger) = 0 \) for all \( p \), but \( f(E_{k\downarrow}^\dagger) = f(\xi_k^\dagger) = 0 \) only for \( p = 0 \). From the number equations, we obtain

\[
\mu = -te^{d/a} + 2t + \frac{2\pi d n_{\pi}}{m},
\]

dominated by the leading two-body term. The exponential dependence of \( \mu \) on \( d/a \) results from the quasi-two-dimensionality since \( |k_z| \ll \pi/d \). In the \( t \to 0 \) 2D limit, one finds \( d/a \approx \ln |\mu|/\pi \), which diverges logarithmically. To leading order corrections in powers of \( 1/\mu \), we have

\[
(1 - p)n = -m\Delta^2 \frac{4\pi d}{4\pi d} - \frac{np\Delta^2}{2\mu}, \text{ or }
\]

\[
\Delta = \sqrt{4\pi |\mu|d(1 - p)n/m} \frac{1 - \pi d n_{\pi}}{m}. \n\]

At \( T_c, \Delta_{pg} = \Delta \). Note that \( \Delta^2/\mu \) approaches a constant in the BEC limit, in contrast to its counterpart in a 3D homogeneous case, where \( \Delta \sim |\mu|^{1/4} \) so that \( \Delta^2/\mu \to 0 \).

For \( p = 0 \), one can easily obtain

\[
B_{\parallel} = \frac{1}{4m}, \text{ and } t_B = \frac{t^2}{2} \approx \frac{t}{2}e^{-d/a},
\]

which yields

\[
T_c \approx \frac{\pi a n}{2m} = \frac{k_F a}{3\pi} T_F \n\]

in the BEC regime via the pseudogap equation [13].

Now for \( p > 1 \), one has to solve for \( \mu \) and \( T_c \) numerically, since \( \mu_{\uparrow} > 0 \). We have \( E_{k\uparrow}^\dagger \approx \xi_k^\dagger + \frac{4\pi d n_{\pi}}{m} \), then Eq. (3) becomes

\[
p_m = \sum_k f(\xi_k^\dagger + \frac{4\pi d n_{\pi}}{m}).
\]
In the BEC regime, \( T_c \) is controlled by the inverse \( T \)-matrix expansion. The coefficients \( a_0 \) (and pair density \( n_p \)) and \( a_1 \) are given by

\[
n_p = a_0 \Delta^2 = n_\downarrow - \sum_k f(\xi_k^\downarrow) - f(\xi_k^\uparrow + 4\pi n_p m) \Delta^2, \quad (12)
\]

\[
a_1 \Delta^2 = A + \frac{n_\downarrow}{4|\mu|}(1 + B). \quad (13)
\]

For the inverse pair mass, we have

\[
B_\parallel = \frac{1}{4m} + \delta B_\parallel, \quad t_B = \frac{t^2}{n_p} \left[ C + \frac{n_\downarrow}{2|\mu|}(1 - D) \right]. \quad (14)
\]

Here \( A, B, C, D \) and \( \delta B_\parallel \) depend on \( \mu \) and \( T \) only. They can be readily obtained via the inverse \( T \)-matrix expansion (see Supplementary Sec. III for concrete expressions). For \( p = 0 \), \( \mu \rightarrow -\infty \) so that \( f(\xi_k^\downarrow) \), as well as \( A, B, C, D \) and \( \delta B_\parallel \), all vanish. Then we recover \( n_p = n_\downarrow = n/2, a_1 \Delta^2 = -n/8\mu \) and Eq. (9).

Of paramount importance is that population imbalance leads to these extra terms in \( a_0, a_1, B_\parallel \) and \( t_B \), which are associated with the excessive majority fermions via the Fermi functions. Equation (14) suggests that the pair motion in the \( z \) direction for \( p \neq 0 \) is now strongly enhanced by these fermions as an extra pair hopping mechanism; a minority fermion may hop to the next site by exchanging its majority partner to one that is already there, which is guaranteed by the existence of a transverse continuum dimension, since the “site” is actually a 2D plane. Obviously, this extra mechanism will be dominant over the usual virtual pair unbinding-rebinding process \( [2] \) in the BEC regime. Indeed, the pair hopping integral \( t_B \) and thus \( T_c \) approach constant BEC asymptotes rather than decreasing with \( 1/k_F a \). Furthermore, Eq. (12) indicates that \( n_p < n_\downarrow \) for \( p \neq 0 \), namely, not all minority fermions are paired in the BEC limit.

In the deep BEC regime, Eq. (6) determines \( \mu \), and Eq. (8) yields the gap \( \Delta \), for given \( 1/k_F a \). Then \( \mu \uparrow \) and \( T_c \) can be obtained via solving Eq. (5) (with \( \Delta_{pg} = \Delta \)) along with Eq. (11), with the help of Eqs. (12) and (13). Finally, \( \mu \downarrow = 2\mu - \mu \uparrow \).

Shown in Fig. 1 the zero \( T \) phase diagram for \( (t/E_F, k_F d) = (0.2, 0.5) \). These parameters allow a relatively large polarized superfluid (pSF) phase (yellow shaded region) in the BEC regime. A normal phase lies to the left of the black solid line of \( T_{MF} = 0 \). The red line is given by the instability condition \( \partial^2 \Omega/\partial \Delta^2 = 0 \) against phase separation, following Refs. [5, 10], where \( \Omega \) is the thermodynamic potential. The blue \( T_c = 0 \) curve is determined by \( t_B = 0 \). A possible pair density wave (PDW) state emerges in the dotted region where \( t_B \) becomes negative \([3] \). The rest space has an unstable mean-field solution of pSF at \( T = 0 \). The grey and brown shaded regions allow ITSF; the former has a lower \( T_c \), whereas the latter does not but is unstable at low \( T \), as shown in the inset for \( 1/k_F a = 2 \). To compare with the 3D continuum case \([10] \), we label the top axis with \( 1/k_F a_{\text{eff}} \). Apparently, the position of the Normal/Unstable boundary is roughly the same, but stable pSF solution no longer exists here for high \( p \gtrsim 0.5 \). Instead, a PDW phase emerges.

Figure 2. Behavior of \( T_c \) versus \( 1/k_F a \) for varying imbalance \( p \), as labeled, for a physically accessible case with \( t/E_F = 0.2 \) and \( k_F d = 0.5 \). The orange dashed \( \mu = 0 \) line separates the fermionic and bosonic regimes. Here “Normal” denotes normal Fermi gases. Besides the pSF (yellow shaded) and PDW (dotted region), and normal phases, the mean-field superfluid solution between the black (solid) and red (solid and dashed) curves is unstable at \( T = 0 \). The grey and brown areas admit ITSF. An example in the brown area is shown in the inset for \( 1/k_F a = 2 \). The green dashed and blue dotted lines show less restrictive instability conditions, with \( n_\uparrow/m \) being superfluid density. The top axis is labeled with \( 1/k_F a_{\text{eff}} \).
homogeneous case \cite{5,10}. Below the lower $T_c$, phase separation, FFLO and/or PDW states may occur. The $T_c$ solution in the yellow shaded region is unstable, corresponding to ITSF in the brown shaded area in Fig. 1. Besides the unpaired normal phase to the left, a pseudogap phase exists above (the upper) $T_c$. As $1/k_{F,a}$ increases, $T_c$ approaches a constant BEC asymptote. This should be contrasted with the dashed $p = 0$ curve, for which $T_c$ decreases with $1/k_{F,a}$ following Eq. (10). Therefore, relative to the $p = 0$ case, imbalance may substantially raise $T_c$ on the BEC side of unitarity. The plot of $T_c$ versus $p$ in the inset for $1/k_{F,a} = 2$ and 10 shows an enhancement for $p \lesssim 0.1$ and $p \lesssim 0.3$, respectively.

Our calculations reveal that as $d$ increases, the $T = 0$ pSF phase shrinks quickly. For $k_{F,d} = 1$, the upper pSF phase boundary (blue curve) in Fig. 1 moves down to $p \sim 0.11$. And for $d = 2$, the pSF phase disappears completely. The grey shaded ITSF phase extends to $1/k_{F,a} = +\infty$ at low $p$, with both an upper and lower $T_c$. Similar reduction of the superfluid phase can be achieved by decreasing $t$.

Shown in Fig. 3 is a plot of $T_c$ similar to Fig. 2 except now with increased lattice constant, $k_{F,d} = 2$. The superfluid phase exists only for relatively low $p$, exhibiting typical ITSF, with the lower $T_c$ extending to $k_{F,a} = +\infty$ for $p \lesssim 0.0085$; both the upper and lower $T_c$’s approach a constant BEC asymptote for these low $p$. For larger $p \gtrsim 0.009$, there is no superfluid in the deep BEC regime. The superfluid phase shrinks to zero as $p$ increases beyond about 0.135 at $1/k_{F,a} = -0.7$.

Finally, we examine in Fig. 4 the asymptotic behavior of various quantities versus $1/k_{F,a}$ at $T_c$ in the BEC regime for $1/E_F = 0.25$, $k_{F,d} = 2$ and $p = 0.01$. The solid and dashed lines represent the fully numerical and the BEC asymptotic solutions, respectively. Figure 4(a) demonstrates that the asymptotic solutions for $\mu$ and (pseudo)gap $\Delta$ given by Eqs. 6 and 9 work very well for $1/k_{F,a} > 2$. Figure 4(b)-(d) presents $\mu$, $\Delta$, $\mu_\|$, $\mu_z$, and $\Delta$ as a function of $1/k_{F,a}$ for $p = 0.01, 1/E_F = 0.25$ and $k_{F,d} = 2$. For comparison, also plotted are the BEC asymptotes (dashed lines), as well as $n_p$. The energy unit is $E_F$, and $2m = 1$.

We have studied various situations for a big range of $(t, p, d)$ and found that the superfluidity can be easily destroyed by large $d$ and small $t$. Reducing $t$ in Figs. 2 and 3 may shrink the pSF phase quickly, as shown in Supplementary Fig. S3. Overall, in the multidimensional $(t, d, p, T)$ phase space, especially in the BEC limit, the superfluid phase exists only for small and intermediate $d$, small $p$, relatively large $t$ and intermediate (and low) $T$.

To understand the destruction of superfluidity at large $d$ and small $t$, we note that when $d$ is large, more fermions will occupy the high $k_\|$ states. In addition, a small $t$ may further force the lattice band fully occupied, so that the Fermi surface becomes nearly dispersionless as a function of $k_z$. This makes it extremely hard to accommodate the excessive majority fermions, which will necessarily have to occupy high $k_\|$ states at a high energy cost. Furthermore, since $|k_z| \lesssim \pi/d$, the Pauli blocking effect can no longer be eliminated in the $z$ direction in the BEC regime, so that $t_B$ may be quickly suppressed to zero as $d$ and $p$ increase and/or $t$ decreases at low $T$.

The enhancement and destruction of superfluidity are easily testable in future experiments. The enhancement also suggests that a small imbalance is beneficial for achieving superfluidity experimentally.

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Q. J. Chen, J. Stajic, S. N. Tan, and K. Levin, BCS-BEC crossover: From high temperature superconductors to ultracold superfluids, Phys. Rep. 412, 1 (2005).

I. Bloch, J. Dalibard, and W. Zwerger, Many-body physics with ultracold gases, Rev. Mod. Phys. 80, 885 (2008).

M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, Fermionic superfluidity with imbalanced spin populations, Science 311, 492 (2006).

G. B. Partridge, W. Li, R. I. Kamar, Y. A. Liao, and R. G. Hulet, Resonant Fermi gases, Rep. Prog. Phys. 73, 014521 (2010).

Y. Yu and Q. J. Chen, Superfluidity in atomic Fermi gases, Phys. Rev. A 75, 063603 (2007).

Q. J. Chen, Y. He, C.-C. Chien, and K. Levin, Stability conditions and phase diagrams for two-component Fermi gases with population imbalance, Phys. Rev. A 74, 063603 (2006).

L. Radzihovsky and D. H. Sheehy, Imbalanced Feshbach-resonant Fermi gases, Rep. Prog. Phys. 73, 076501 (2010).

W. Yi and L. M. Duan, Trapped fermions across a Feshbach resonance with population imbalance, Phys. Rev. A 73, 031604(R) (2006).

C.-H. Pao, S.-T. Wu, and S.-K. Yip, Superfluid stability in the BEC-BCS crossover, Phys. Rev. B 73, 132506 (2006).

M. M. Forbes, E. Gubankova, W. V. Liu, and F. Wilczek, Stability criteria for breached-pair superfluidity, Phys. Rev. Lett. 94, 017001 (2005).

C.-C. Chien, Q. J. Chen, Y. He, and K. Levin, Intermediate temperature superfluidity in a Fermi gas with population imbalance, Phys. Rev. Lett. 97, 090402 (2006).

Q. J. Chen, Y. He, C.-C. Chien, and K. Levin, Theory of superfluids with population imbalance: Finite-temperature and BCS-BEC crossover effects, Phys. Rev. B 75, 014521 (2007).

Q. J. Chen, I. Kosztin, B. Jankó, and K. Levin, Superconducting transitions from the pseudogap state: d-wave symmetry, lattice, and low-dimensional effects, Phys. Rev. B 59, 7083 (1999).

W. Hofstetter, J. I. Cirac, P. Zoller, E. Demler, and M. D. Lukin, High-temperature superfluidity of fermionic atoms in optical lattices, Phys. Rev. Lett. 89, 220407 (2002).

I. Bloch, Ultracold quantum gases in optical lattices, Nat. Phys. 1, 23 (2005).

M. Köhl, H. Moritz, T. Stöferle, K. Günter, and T. Esslinger, Fermionic atoms in a three dimensional optical lattice: Observing Fermi surfaces, dynamics, and interactions, Phys. Rev. Lett. 94, 080403 (2005).

M. A. Cazalilla, A. F. Ho, and T. Giamarchi, Two-component Fermi gas on internal-state-dependent optical lattices, Phys. Rev. Lett. 95, 226402 (2005).

G. Orso, L. P. Pitaevskii, S. Stringari, and M. Wouters, Formation of molecules near a Feshbach resonance in a 1D optical lattice, Phys. Rev. Lett. 95, 060402 (2005).

T. Koponen, J. Kinnunen, J.-P. Martikainen, L. M. Jensen, and P. Törmä, Fermion pairing with spin-density imbalance in an optical lattice, New J. Phys. 8, 179 (2006).

C.-C. Chien, Y. He, Q. J. Chen, and K. Levin, Superfluid-insulator transitions at noninteger filling in optical lattices of fermionic atoms, Phys. Rev. A 77, 011601 (2008).

S. Giorgini, L. P. Pitaevskii, and S. Stringari, Theory of ultracold atomic Fermi gases, Rev. Mod. Phys. 80, 1215 (2008).

A. Cichy and R. Micnas, The spin-imbalanced attractive Hubbard model in d=3: Phase diagrams and BCS-BEC crossover at low filling, Ann. Phys. 347, 207 (2014).

W. Ong, C. Cheng, I. Arakelyan, and J. E. Thomas, Spin-imbalanced quasi-two-dimensional Fermi gases, Phys. Rev. Lett. 114, 110403 (2015).

J. Kangara, C. Cheng, S. Pegahian, I. Arakelyan, and J. E. Thomas, Atom pairing in optical superlattices, Phys. Rev. Lett. 120, 083203 (2018).

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[1] Q. J. Chen, J. Stajic, S. N. Tan, and K. Levin, BCS-BEC crossover: From high temperature superconductors to ultracold superfluids, Phys. Rep. 412, 1 (2005).

[2] I. Bloch, J. Dalibard, and W. Zwerger, Many-body physics with ultracold gases, Rev. Mod. Phys. 80, 885 (2008).

[3] M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, Fermionic superfluidity with imbalanced spin populations, Science 311, 492 (2006).

[4] G. B. Partridge, W. Li, R. I. Kamar, Y. A. Liao, and R. G. Hulet, Resonant Fermi gases, Rep. Prog. Phys. 73, 014521 (2010).

[5] W. Yi and L. M. Duan, Trapped fermions across a Feshbach resonance with population imbalance, Phys. Rev. A 73, 031604(R) (2006).

[6] C.-H. Pao, S.-T. Wu, and S.-K. Yip, Superfluid stability in the BEC-BCS crossover, Phys. Rev. B 73, 132506 (2006).

[7] M. M. Forbes, E. Gubankova, W. V. Liu, and F. Wilczek, Stability criteria for breached-pair superfluidity, Phys. Rev. Lett. 94, 017001 (2005).

[8] C.-C. Chien, Q. J. Chen, Y. He, and K. Levin, Intermediate temperature superfluidity in a Fermi gas with population imbalance, Phys. Rev. Lett. 97, 090402 (2006).

[9] Q. J. Chen, Y. He, C.-C. Chien, and K. Levin, Theory of superfluids with population imbalance: Finite-temperature and BCS-BEC crossover effects, Phys. Rev. B 75, 014521 (2007).

[10] Q. J. Chen, I. Kosztin, B. Jankó, and K. Levin, Superconducting transitions from the pseudogap state: d-wave symmetry, lattice, and low-dimensional effects, Phys. Rev. B 59, 7083 (1999).

[11] W. Hofstetter, J. I. Cirac, P. Zoller, E. Demler, and M. D. Lukin, High-temperature superfluidity of fermionic atoms in optical lattices, Phys. Rev. Lett. 89, 220407 (2002).

[12] I. Bloch, Ultracold quantum gases in optical lattices, Nat. Phys. 1, 23 (2005).

[13] M. Köhl, H. Moritz, T. Stöferle, K. Günter, and T. Esslinger, Fermionic atoms in a three dimensional optical lattice: Observing Fermi surfaces, dynamics, and interactions, Phys. Rev. Lett. 94, 080403 (2005).

[14] M. A. Cazalilla, A. F. Ho, and T. Giamarchi, Two-component Fermi gas on internal-state-dependent optical lattices, Phys. Rev. Lett. 95, 226402 (2005).

[15] G. Orso, L. P. Pitaevskii, S. Stringari, and M. Wouters, Formation of molecules near a Feshbach resonance in a 1D optical lattice, Phys. Rev. Lett. 95, 060402 (2005).

[16] T. Koponen, J. Kinnunen, J.-P. Martikainen, L. M. Jensen, and P. Törmä, Fermion pairing with spin-density imbalance in an optical lattice, New J. Phys. 8, 179 (2006).

[17] C.-C. Chien, Y. He, Q. J. Chen, and K. Levin, Superfluid-insulator transitions at noninteger filling in optical lattices of fermionic atoms, Phys. Rev. A 77, 011601 (2008).

[18] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Theory of ultracold atomic Fermi gases, Rev. Mod. Phys. 80, 1215 (2008).

[19] A. Cichy and R. Micnas, The spin-imbalanced attractive Hubbard model in d=3: Phase diagrams and BCS-BEC crossover at low filling, Ann. Phys. 347, 207 (2014).

[20] W. Ong, C. Cheng, I. Arakelyan, and J. E. Thomas, Spin-imbalanced quasi-two-dimensional Fermi gases, Phys. Rev. Lett. 114, 110403 (2015).

[21] J. Kangara, C. Cheng, S. Pegahian, I. Arakelyan, and J. E. Thomas, Atom pairing in optical superlattices, Phys. Rev. Lett. 120, 083203 (2018).