On Load Shedding in Complex Event Processing

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Abstract

Complex Event Processing (CEP) is a stream processing model that focuses on detecting event patterns in continuous event streams. While the CEP model has gained popularity in the research communities and commercial technologies, the problem of gracefully degrading performance under heavy load in the presence of resource constraints, or load shedding, has been largely overlooked. CEP is similar to “classical” stream data management, but addresses a substantially different class of queries. This unfortunately renders the load shedding algorithms developed for stream data processing inapplicable. In this paper we study CEP load shedding under various resource constraints. We formalize broad classes of CEP load-shedding scenarios as different optimization problems. We demonstrate an array of complexity results that reveal the hardness of these problems and construct shedding algorithms with performance guarantees. Our results shed some light on the difficulty of developing load-shedding algorithms that maximize utility.

1 Introduction

The complex event processing or CEP model has received significant attention from the research community [6, 28, 35, 36, 49, 51], and has been adopted by a number of commercial systems including Microsoft StreamInsight [1], Sybase Aleri [3], and StreamBase [2]. A wide range of applications, including inventory management [4], behavior monitoring [46], financial trading [5], and fraud detection [8, 50], are now powered by CEP technologies.

A reader who is familiar with the extensive stream data processing literature may wonder if there is anything new here, or if CEP is just another name for stream data management. While both kinds of system evaluate queries over data streams, the important difference is the class of queries upon which each system focuses. In the traditional stream processing literature, the focus is almost exclusively on aggregate queries or binary equi-joins. By contrast, in CEP, the focus is on detecting certain patterns, which can be viewed as multi-relational non-equi-joins on the time dimension, possibly with temporal ordering constraints. The class of queries addressed by CEP systems requires different evaluation algorithms and different load-shedding algorithms than the class previously considered in the context of stream data management.

As an example of a CEP-powered system, consider the health care monitoring system, HyReminder [46], currently deployed at the University of Massachusetts Memorial Hospital. The HyReminder system tracks and monitors the hygiene compliance of health-care workers. In this hospital setting, each doctor wears an RFID badge that can be read by sensors installed throughout the hospital. As doctors walk around the hospital, the RFID badges they wear trigger “event” data, which is transmitted to a central CEP engine. The CEP engine in turn looks for patterns to check for hygiene compliance. As one example, according to the US Center for Disease Control (CDC) [12], a doctor who enters or exits a patient room (which is captured by
sensors installed in the doorway and encoded as an Enter-Patient-Room event or Exit-Patient-Room event) should cleanse her hands (encoded by a Sanitize event) within a short period of time. This hygiene regulation can be tracked and enforced using the following CEP queries.

Q1: SEQ(Sanitize, Enter-Patient-Room) within 1 min
Q2: SEQ(Exit-Patient-Room, Sanitize) within 1 min

In the HyReminder system, these two CEP queries monitor the event sequence to track sanitization behavior and ensure hygiene compliance. As another example consider CIMS [4], which is a system also powered by CEP and deployed in the same University of Massachusetts hospital. CIMS is used for inventory management and asset tracking purposes. It captures RFID events triggered by tags attached to medical equipment and expensive medicines, and uses CEP to track supply usage and reduce inventory cost [15].

While the emergence of CEP model has spawned a wide variety of applications, so far research efforts have focused almost exclusively on improving CEP query join efficiency [6, 28, 35, 36, 49, 51]. Load shedding, an important issue that has been extensively studied in traditional stream processing [9, 20, 25, 26, 42, 43, 47, 48, 52], has been largely overlooked in the new context of CEP.

Like other stream systems, CEP systems often face bursty input data. Since over-provisioning the system to the point where it can handle any such burst may be uneconomical or impossible, during peak loads a CEP system may need to “shed” portions of the load. The key technical challenge herein is to selectively shed work so as to eliminate the less important query results, thereby preserve the more useful query results as defined by some utility function.

More specifically, the problem we consider is the following. Consider a CEP system that has a number of pattern queries, each of which consists of a number of events and is associated with a utility function. During peak loads, memory and/or CPU resources may not be sufficient. A utility-maximizing load shedding scheme should then determine which events should be preserved in memory and which query results should be processed by the CPU, so that not only are resource constraints respected, but also the overall utility of the query results generated is maximized.

We note that, in addition to utility-maximization in a single CEP application, the load-shedding framework may also be relevant to cloud operators that need to optimize across multiple CEP applications. Specifically, stream processing applications are gradually shifting to the cloud [33]. In cloud scenarios, the cloud operator in general cannot afford to provision for the aggregate peak load across all tenants, which would defeat the purpose of consolidation. Consequently, when the load exceeds capacity, cloud operators are forced to shed load. They have a financial incentive to judiciously shed work from queries that are associated with a low penalty cost as specified in Service Level Agreements (SLAs), so that their profits can be maximized (similar problems have been called “profit maximization in a cloud” and have been considered in the Database-as-a-Service literature [18, 19]). Note that this problem is naturally a utility-maximizing CEP load shedding problem, where utility essentially becomes the financial rewards and penalties specified in SLAs.

While load shedding has been extensively studied in the context of general stream processing [9, 20, 25, 26, 42, 43, 47, 48, 52], the focus there is aggregate queries or two-relation equi-join queries, which are important for traditional stream joins. The emerging CEP model, however, demands multi-relational joins that predominantly use non-equi-join predicates on timestamp. As we will discuss in more detail in Section 4 the CEP load shedding problem is significantly different and considerably harder than the problems previously studied in the context of general stream load shedding.

We show in this work that variants of the utility maximizing load shedding problems can be abstracted as different optimization problems. For example, depending on which resource is constrained, we can have three problem variants, namely CPU-bound load shedding, memory-bound load shedding, and dual-bound load shedding (with both CPU- and memory-bound). In addition we can have integral load shedding, where event instances of each type are either all preserved in memory or all shedded; and fractional load shedding, where a sampling operator exists such that a fraction of event instances of each type can be sampled
Table 1: Problem variants for CEP load shedding

|                | Memory-bound | CPU-bound | Dual-bound |
|----------------|--------------|-----------|------------|
| Integral       | IMLS         | ICLS      | IDLS       |
| Fractional     | FMLS         | FCLS      | FDLS       |

According to a predetermined sampling ratio. Table I summarizes the six variants of CEP load shedding studied in this paper: IMLS (integral memory-bound load shedding), FMLS (fractional memory-bound load shedding), ICLS (integral CPU-bound load shedding), FCLS (fractional CPU-bound load shedding), IDLS (integral dual-bound load shedding), and FDLS (fractional dual-bound load shedding).

We analyze the hardness of these six variants, and study efficient algorithms with performance guarantees. We demonstrate an array of complexity results. In particular, we show that CPU-bound load shedding is the easiest to solve: FCLS is solvable in polynomial time, while ICLS admits a FPTAS approximation. For memory-bound problems, we show that IMLS is in general NP-hard and hard to approximate. We then identify two special cases in which IMLS can be efficiently approximated or even solved exactly, and describe a general rounding algorithm that achieves a bi-criteria approximation. As for the fractional FMLS, we show it is hard to solve in general, but has approximable special cases. Finally, for dual-bound load shedding IDLS and FDLS, we show that they generalize memory-bound problems, and the hardness results from memory-bound problems naturally hold. On the positive side, we describe a tri-criteria approximation algorithm and an approximable special case for the IDLS problem.

The rest of the paper is organized as follows. We first describe necessary background of CEP in Section 2, and introduce the load shedding problem in Section 3. We describe related work in Section 4. In Section 5, Section 6, and Section 7 we discuss the memory-bound, CPU-bound and dual-bound load-shedding problems, respectively. We conclude in Section 8.

2 Background: Complex Event Processing

The CEP model has been proposed and developed by a number of seminal papers (see [6] and [51] as examples). To make this paper self-contained, we briefly describe the CEP model and its query language in this section.

2.1 The data model

Let the domain of possible event types be the alphabet of fixed size \( \Sigma = \{E_i\} \), where \( E_i \) represents a type of event. The event stream is modeled as an event sequence as follows.

**Definition 1.** An event sequence is a sequence \( S = (e_1, e_2, ..., e_N) \), where each event instance \( e_j \) belongs to an event type \( E_i \in \Sigma \), and has a unique time stamp \( t_j \). The sequence \( S \) is temporally ordered, that is \( t_j < t_k, \forall j < k \).

**Example 1.** Suppose there are five event types denoted by upper-case characters \( \Sigma = \{A, B, C, D, E\} \). Each character represents a certain type of event. For example, for the hospital hygiene monitoring application HyReminder, event type \( A \) represents all event instances where doctors enter ICU, \( B \) denotes the type of events where doctors washing hands at sanitization desks, etc.

A possible event sequence is \( S = (A_1, B_2, C_3, D_4, E_5, A_6, B_7, C_8, D_9, E_{10}) \), where each character is an instance of the corresponding event type occurring at time stamp given by the subscript. So \( A_1 \) denotes that at time-stamp 1, a doctor enters ICU. \( B_2 \) shows that at time 2, the doctor sanitizes his hands. At time-stamp 6, there is another enter-ICU event \( A_6 \), so on and so forth.

Following the standard practice of the CEP literature [6, 7, 36, 51], we assume events are temporally ordered by their timestamps. Out-of-order events can be handled using techniques from [16, 35].

**Definition 2.** Given an event sequence \( S = (e_1, e_2, ..., e_N) \), a sequence \( S' = (e_{i_1}, e_{i_2}, ..., e_{i_m}) \) is a subsequence of \( S \), if \( 1 \leq i_1 < i_2 < ... < i_m \leq N \).
Note that the temporal ordering in the original sequence is preserved in subsequences, and a subsequence does not have to be a contiguous subpart of a sequence.

2.2 The query model

Unlike relational databases, where queries are typically ad-hoc and constructed by users at query time, CEP systems are more like other stream processing systems, where queries are submitted ahead of time and run for an extended period of time (thus are also known as long-standing queries). The fact that CEP queries are known a priori is a key property that allows queries to be analyzed and optimized for problems like utility maximizing load shedding.

Denote by \( Q = \{Q_i\} \) the set of CEP queries, where each query \( Q_i \) is a sequence query defined as follows.

**Definition 3.** A CEP sequence query \( Q \) is of the form \( Q = SEQ(q_1, q_2, \ldots, q_n) \), where \( q_k \in \Sigma \) are event types. Each query \( Q \) is associated with a time based sliding window of size \( T(Q) \in \mathbb{R}^+ \), over which \( Q \) will be evaluated.

We then define the skip-till-any-match query match semantics.

**Definition 4.** In skip-till-any-match, a subsequence \( S' = (e_{i_1}, e_{i_2}, \ldots, e_{i_n}) \) of \( S \) is considered a query match of \( Q \) if:

1. Pattern matches: Event \( e_{i_l} \) in \( S' \) is of type \( q_l \) for all \( l \in [1, n] \),
2. Within window: \( t_{i_n} - t_{i_1} \leq T(Q) \).

We illustrate query matches using Example 2.

**Example 2.** We continue with Example 1 where the event sequence \( S = (A_1, B_2, C_3, D_4, E_5, A_6, B_7, C_8, D_9, E_{10}) \). Suppose there are a total of three queries: \( Q_1 = SEQ(A, C) \), \( Q_2 = SEQ(C, E) \), \( Q_3 = SEQ(A, B, C, D) \), all having the same window size \( T(Q_1) = T(Q_2) = T(Q_3) = 5 \).

Both sequences \((A_1, C_3)\) and \((A_6, C_8)\) are matches for \( Q_1 \), because they match patterns specified in \( Q_1 \), and are within the time window 5. However, \((A_1, C_8)\) is not a match even though it matches the pattern in \( Q_1 \), because the time difference between \( C_8 \) and \( A_1 \) exceeds the window limit 5.

Similarly, \((C_3, E_5)\) and \((C_8, E_{10})\) are matches of \( Q_2 \); \((A_1, B_2, C_3, D_4)\) and \((A_6, B_7, C_8, D_9)\) are matches of \( Q_3 \).

A query with the skip-till-any-match semantics essentially looks for the conjunction of occurrences of event types in a specified order within a time window. Observe that in skip-till-any-match a subsequence does not have to be contiguous in the original sequence to be considered as a match (thus the word *skip* in its name). Such queries are widely studied \([2, 6, 28, 35, 36, 49, 51]\) and used in CEP systems.

We note that there are three additional join semantics defined in \([6]\), namely, skip-till-next-match, partition-contiguity and contiguity. In the interest of space and also to better focus on the topic of load shedding, the details of these join semantics are described in Appendix A of the load shedding problem formulated in this work is agnostic of the join semantics used.

We also observe that although there are CEP language extensions like negation \([28]\) and Kleene closure \([23]\), in this work we only focus on the core language constructs that use conjunctive positive event occurrences. We leave such query extensions for load shedding as future work.

3 CEP Load Shedding

It is well known that continuously arriving stream data is often bursty \([20, 42, 48]\). During times of peak loads not all data items can be processed in a timely manner under resource constraints. As a result, part of the input load may have to be discarded (shedded), resulting in the system retaining only a subset of data items. The question that naturally arises is which queries should be preserved while others shedded? To answer this question, we introduce the notion of utility to quantify the “usefulness” of different queries.
3.1 A definition of utility

In CEP systems, different queries naturally have different real-world importance, where some query output may be more important than others. For example, in the inventory management application \[4, 15\], it is naturally more important to produce real-time query results that track expensive medicine/equipment than less expensive ones. Similarly, in the hospital hygiene compliance application \[46\], it is more important to produce real-time query results reporting serious hygiene violations with grave health consequences than the ones merely reporting routine compliance.

We define *utility weight* for each query to measure its importance.

**Definition 5.** Let \( Q = \{Q_i\} \) be the set of queries. Define the *utility weight* of query \( Q_i \), denoted by \( w_i \in \mathbb{R}^+ \), as the perceived usefulness of reporting one instance of match of \( Q_i \).

A user or an administrator familiar with the application can typically determine utility weights. Alternatively, in a cloud environment, operators of multi-tenancy clouds may resort to service-level-agreements (SLAs) to determine utility weights. In this work we simply treat utility weights as known constants. We note that the notion of query-level weights has been used in other parts of data management literature (e.g., query scheduling \[38\]).

The total utility of a system is then defined as follows.

**Definition 6.** Let \( C(Q_i, S) \) be the number of distinct matches for query \( Q_i \) in \( S \). The *utility* generated for query \( Q_i \) is

\[
U(Q_i, S) = w_i \cdot C(Q_i, S)
\]

(1)

The sum of the utility generated over \( Q = \{Q_i\} \) is

\[
U(Q, S) = \sum_{Q_i \in Q} U(Q_i, S).
\]

(2)

Our definition of utility generalizes previous metrics like the *max-subset* \[20\] used in traditional stream load shedding literature. *Max-subset* aims to maximize the number of output tuples, and thus can be viewed as a special case of our definition in which each query has unit-weight.

We also note that although it is natural to define utility as a linear function of query matches for many CEP applications (e.g., \[4, 46\]), there may exist applications where utility can be best defined differently (e.g., a submodular function to model diminishing returns). Considering alternative utility functions for CEP load shedding is an area for future work.

**Example 3.** We continue with Example 2 using the event sequence \( S \) and queries \( Q_1, Q_2 \) and \( Q_3 \) to illustrate utility.

Suppose the utility weight \( w_1 \) for \( Q_1 \) is 1, \( w_2 \) is 2, and \( w_3 \) is 3. Since there are 2 matches of \( Q_1 \), \( Q_2 \) and \( Q_3 \), respectively, in \( S \), the total utility is \( 2 \times 1 + 2 \times 2 + 2 \times 3 = 12 \).

3.2 Types of resource constraints

In this work, we study constraints on two common types of computing resources: CPU and memory.

**Memory-bound load shedding.** In this first scenario, memory is the limiting resource. Normally, arriving event data are kept in main memory for windowed joins until time-stamp expiry (i.e., when they are out of active windows). During peak loads, however, event arrival rates may be so high that the amount of memory needed to store all event data might exceed the available capacity. In such a case not every arriving event can be held in memory for join processing and some events may have to be discarded.
Example 4. In our running example the event sequence \( S = (A_1, B_2, C_3, D_4, E_5, A_6, B_7, C_8, D_9, E_{10} \ldots) \),
with queries \( Q_1 = \text{SEQ}(A, C) \), \( Q_2 = \text{SEQ}(C, E) \) and \( Q_3 = \text{SEQ}(A, B, C, D) \).
Because the sliding window of each query is 5, we know each event needs to be kept in memory for 5 units of time.
Given that one event arrives in each time unit, a total of 5 events need to be simultaneously kept in memory.

Suppose a memory-constrained system only has memory capacity for 3 events. In this case “shedding” all events of type B and D will sacrifice the results of \( Q_3 \) but preserves A, C and E and meets the memory constraint. In addition, results for \( Q_1 \) and \( Q_2 \) can be produced using available events in memory, which amounts to a total utility of \( 2 \times 1 + 2 \times 2 = 6 \). This maximizes utility, for shedding any other two event types yields lower utility.

CPU-bound load shedding. In the second scenario, memory may be abundant, but CPU becomes the bottleneck.
As a result, again only a subset of query results can be processed.

Example 5. We revisit Example 4 but now suppose we have a CPU constrained system. Assume for simplicity that producing each query match costs 1 unit of CPU. Suppose there are 2 unit of CPU available per 5 units of time, so only 2 matches can be produced every 5 time units.

In this setup, producing results for \( Q_2 \) and \( Q_3 \) while shedding others yields a utility of \( 2 \times 2 + 2 \times 3 = 10 \) given the events in \( S \). This is utility maximizing because \( Q_2 \) and \( Q_3 \) have the highest utility weights.

Dual-bound load shedding. Suppose now the system is both CPU bound and memory bound (dual-bound).

Example 6. We continue with Example 5. Suppose now due to memory constraints 3 events can be kept in memory per 5 time units, and in addition 2 query matches can be produced every 5 time units due to CPU constraints.

As can be verified, the optimal decision is to keep events A, C and E while producing results for \( Q_1 \) and \( Q_2 \), which yields a utility of \( 2 \times 1 + 2 \times 2 = 6 \) given the events in \( S \). Note that results for \( Q_3 \) cannot be produced because it needs four events in memory while only three can fit in memory simultaneously.

3.3 Types of shedding mechanisms

In this paper, we consider two types of shedding mechanisms, an integral load shedding, in which certain types of events or query matches are discarded altogether; and a fractional load shedding, in which a uniform sampling is used, such that a portion of event types or query matches is randomly selected and preserved.

Note that both the above mentioned load-shedding mechanisms are relevant in an online setting. That is, settings in which a shedding decision is made for the current event before the next arriving event is processed. This is in contrast to offline load shedding, where decisions are made after the whole event sequence has arrived. The reason we only focus on online load shedding is practicality – most stream applications demand real-time responsiveness; an offline algorithm that works after the entire event sequence has arrived is unlikely to be practically useful.

Performance of online algorithms is oftentimes measured against their offline counterparts to develop quality guarantees like competitive ratios [12]. However, we show in the following that meaningful competitive ratios cannot be derived for any online CEP load shedding algorithms.

Proposition 1. No online CEP load shedding algorithm, deterministic or randomized, can have competitive ratio better than \( \Omega(n) \), where \( n \) is the length of the event sequence.

The full proof of this proposition can be found in Appendix B. Intuitively, to see why it is hard to bound the competitive ratio, consider the following adversarial scenario. Suppose we have a universe of \( 3m \) event types, \( \Sigma = \{E_i\} \cup \{E''_i\} \cup \{E''''_i\} \), \( i \in [m] \).
Let there be 2m queries \( \text{SEQ}(E_i, E''_i) \) and \( \text{SEQ}(E''_i, E''''_i) \), \( \forall i \in [m] \), each with unit utility weight.
The stream is known to be \( (e_1, e_2, \ldots, e_m, X) \), where \( e_i \) is either of type \( E_i \) or \( E''_i \) with equal probability.
In addition, \( X \) is drawn from the uniform distribution on \( \{E''''_i : i \in [m]\} \).
Lastly, suppose the system only has enough memory to hold two events. The optimal offline algorithm can look at the type of event \( X \), denoted by \( E_k \), and keep the corresponding event \( e_k \) (of type \( E_k \) or \( E'_k \)) that arrived previously, to produce a match (either \( (E_k, E'_k) \) or \( (E'_k, E''_k) \), as the case may be) of utility of 1. In comparison, an online algorithm needs to select one event into memory before the event type of \( X \) is revealed. (Note that the offline algorithm cannot just output results based on the last event \( X \) given the form of the input, because \( e_k \) could be either \( E_k \) or \( E'_k \).) Thus, the probability of producing a match is \( \frac{1}{m} \), and the expected utility is also \( \frac{1}{m} \).

This result essentially suggests that we cannot hope to devise online algorithms with good competitive ratios. In light of this result, in what follows, we will characterize the arriving event stream, and focus on optimizing the expected utility of online algorithms without further discussing competitive ratio bounds.

### 3.4 Modeling CEP systems

At a high level, the decision of which event or query to shed should depend on a number of factors, including utility weights, memory/CPU costs, and event arrival rates. Intuitively, the more important a query is, the more desirable it is to keep constituent events in memory and produce results of this query. Similarly the higher the cost is to keep an event or to produce a query match, the less desirable it is to keep that event or produce that result. The rate at which events arrive is also important, as it determines CPU/memory costs of a query as well as utility it can produce.

In order to study these trade-offs in a principled way, we consider the following factors in a CEP system. First, we assume that the utility weight, \( w_i \), which measures the importance of query \( Q_i \) installed in a CEP system, is provided as a constant. We also assume that the CPU cost of producing each result of \( Q_i \) is a known constant \( c_i \), and the memory cost of storing each event instance of type \( E_j \) is also a fixed constant \( m_j \). Note that we do not assume uniform memory/CPU costs across different events/queries, because in practice event tuples can be of different sizes. Furthermore, the arrival rate of each event type \( E_j \), denoted by \( \lambda_j \), is assumed to be known. This is typically obtained by sampling the arriving stream \([17, 40]\). Note...
that characteristics of the underlying stream may change periodically, so the sampling procedure may be invoked at regular intervals to obtain an up-to-date estimate of event arrival rates.

Furthermore, we assume that the “expected” number of matches of \( Q_i \) over a unit time, denoted by \( n_i \), can also be estimated. A simple but inefficient way to estimate \( n_i \) is to sample the arriving event stream and count the number of matches of \( Q_i \) in a fixed time period. Less expensive alternatives also exist. For example, in Appendix C we discuss an analytical way to estimate \( n_i \), assuming an independent Poisson arrival process, which is a standard assumption in the performance modeling literature [34]. In this work we will simply treat \( n_i \) as known constants without further studying the orthogonal issue of estimating \( n_i \).

Lastly, note that since \( n_i \) here is the expected number of query matches, the utility we maximize is also optimized in an expected sense. In particular, it is not optimized under arbitrary arrival event strings (e.g., an adversarial input). While considering load shedding in such settings is interesting, Proposition 1 already shows that we cannot hope to get any meaningful bounds against certain adversarial inputs.

The symbols used in this paper are summarized in Table 2, and our main approximation results are listed in Table 3.

### Table 3: Summary of approximation results

| Approximation Ratio | Formula | Theorem |
|---------------------|---------|---------|
| IMLS               | \( p/(1 - f) \) | 4       |
| IMLS\(^m\) (loss minimization) | \((\frac{1}{\tau}, \frac{1 - \tau}{1 - \tau})\) bi-criteria approximation, for any \( \tau \in (0, 1) \) | 5       |
| ICLS               | \( 1 + \epsilon \), for \( \epsilon > 0 \) | 10      |
| IDLS               | \( p/(1 - f) \) | 12      |
| IDLS\(^m\) (loss minimization) | \((\frac{1}{\tau}, \frac{1 - \tau}{1 - \tau}, \frac{1}{1 - \tau})\) tri-criteria approximation, for any \( \tau \in (0, 1) \) | 11      |

Relative Approximation Ratio (see Definition 8)

| FMLS (under some assumptions) | \( 1 - O \left( \frac{|\Sigma|}{\sqrt{\Sigma}} \right) \left( t^2 + 1 \right)^{-\frac{1}{2}} \) where \( t = \min \left\{ \min_{E_j} \left\{ \frac{\lambda_j}{M_j} \right\}, \frac{1}{\sqrt{|\Sigma|}} \right\} \) | 7       |
| Absolute Approximation Ratio (see Definition 7) | \( O \left( 1 - \frac{k!}{(k-d)!k^d} \right) \) where \( k > d \) | 8       |

### 4 Related work

Load shedding has been recognized as an important problem, and a large body of work in the stream processing literature (e.g., [10, 9, 20, 26, 32, 42, 43, 47, 48]) has been devoted to this problem. However, existing work in the context of traditional stream processing predominantly considers the equi-join of two streaming relations. This is not directly applicable to CEP joins, where each join operator typically involves multi-relational non-equi-join (on time-stamps). For example, the authors in [32] are among the first to study load shedding for equi-joins operators. They proposed strategies to allocate memory and CPU resources to two joining relations based on arrival rates, so that the number of output tuples produced can be maximized.
Similarly, the work [20] also studies the problem of load shedding while maximizing the number of output tuples. It utilizes value distribution of the join columns from the two relations to produce optimized shedding decisions for tuples with different join attribute values.

However, the canonical two-relation equi-join studied in traditional stream systems is only a special case of the multi-relational, non-equijoin that dominates CEP systems. In particular, if we view all tuples from $R$ (resp. $S$) that have the same join-attribute value $v_i$ as a virtual CEP event type $R_i$ (resp. $S_i$), then the traditional stream load shedding problem is captured as a very special case of CEP load shedding we consider, where each “query” has exactly two “event types” ($R_i$ and $S_i$), and there are no overlapping “event types” between “queries”. Because of this equi-join nature, shedding one event has limited ramifications and is intuitively easy to solve (in fact, it is shown to be solvable in [20]). In CEP queries, however, each event type can join with an arbitrary number of other events, and different queries use overlapping events. This significantly complicates the optimization problem and makes CEP load shedding hard.

In [9], sampling mechanisms are proposed to implement load shedding for aggregate stream queries (e.g., SUM), where the key technical challenge is to determine, in a given operator tree, where to place sampling operators and what sampling rates to use, so that query accuracy can be maximized. The work [43] studies the similar problem of strategically placing drop operator in the operator tree to optimize utility as defined by QoS graphs. The authors in [42] also consider load shedding by random sampling, and propose techniques to allocate memory among multiple operators.

The works described above study load shedding in traditional stream systems. The growing popularity of the new CEP model that focuses on multi-relational non-equijoin calls for another careful look at the load-shedding problem in the new context of CEP.

5 Memory-bound load shedding

Recall that in the memory-bound load shedding, we are given a fixed memory budget $M$, which may be insufficient to hold all data items in memory. The problem is to select a subset of events to keep in memory, such that the overall utility can be maximized.

5.1 The integral variant (IMLS)

In the integral variant of the memory-bound load shedding problem, a binary decision, denoted by $x_j$, is made for each event type $E_j$, such that event instances of type $E_j$ are either all selected and kept in memory ($x_j = 1$), or all discarded ($x_j = 0$). The event selection decisions in turn determine whether query $Q_i$ can be selected (denoted by $y_i$), because output of $Q_i$ can be produced only if all constituent event types are selected in memory. We formulate the resulting problem as an optimization problem as follows.

\[
\text{(IMLS)} \quad \max \sum_{Q_i \in \mathcal{Q}} n_i w_i y_i \quad (3) \\
\text{s.t.} \quad \sum_{E_j \in \Sigma} \lambda_j m_j x_j \leq M \quad (4) \\
y_i = \prod_{E_j \in Q_i} x_j \quad (5) \\
y_i, x_j \in \{0, 1\} \quad (6)
\]

The objective function in Equation (3) says that if each query $Q_i$ is selected ($y_i = 1$), then it yields an expected utility of $n_i w_i$ (recall that as discussed in Section 3, $n_i$ models the expected number of query matches of $Q_i$ in a unit time, while $w_i$ is the utility weight of each query match). Equation (4) specifies the memory constraint. Since selecting event type $E_j$ into memory consumes $\lambda_j m_j$ memory, where $\lambda_j$ is the
arrival rate of $E_j$ and $m_j$ is the memory cost of each event instance of $E_j$, Equation (4) guarantees that total memory consumption does not exceed the memory budget $M$. Equation (5) ensures that $Q_i$ can be produced if and only if all participating events $E_j$ are selected and preserved in memory ($x_j = 1$, for all $E_j \in Q_i$).

5.1.1 A general complexity analysis

We first give a general complexity analysis. We show that this shedding problem is NP-hard and hard to approximate by a reduction from the Densest k-Sub-Hypergraph (DKSH).

Theorem 1. The problem of utility maximizing integral memory-bound load shedding (IMLS) is NP-hard.

A proof of the theorem can be found in Appendix D. We show in the following that IMLS is also hard to approximate.

Theorem 2. The problem of IMLS with $n$ event types cannot be approximated within a factor of $2^{(\log n)^{\delta}}$, for some $\delta > 0$, unless $3SAT \in DTIME(2^{n^{3/4+\epsilon}})$.

This result is obtained by observing that the reduction from DKSH is approximation-preserving. Utilizing an inapproximability result in [29], we obtain the theorem above (a proof is in Appendix E).

It is worth noting that DKSH and related problems are conjectured to be very hard problems with even stronger inapproximability than what was proved in [29]. For example, authors in [24] conjectured that Maximum Balanced Complete Bipartite Subgraph (BCBS) is $n^\epsilon$ hard to approximate. If this is the case, utilizing a reduction from BCBS to DKSH [29], DKSH would be at least $n^\epsilon$ hard to approximate, which in turn renders IMLS $n^\epsilon$ hard to approximate given our reduction from DKSH.

While it is hard to solve or approximate IMLS efficiently in general, in the following sections we look at constraints that may apply to real-world CEP systems, and investigate special cases that enable us to approximate or even solve IMLS efficiently.

5.1.2 A general bi-criteria approximation

We reformulate the integral memory-bound problem into an alternative optimization problem (IMLS$^l$) with linear constraints as follows.

(IMLS$^l$) \[ \max \sum_{Q_i \in Q} n_i w_i y_i \] \text{s.t.} \[ \sum_{E_j \in \Sigma} \lambda_j m_j x_j \leq M \] \[ y_i \leq x_j, \forall E_j \in Q_i \] \[ y_i, x_j \in \{0, 1\} \] (7)

Observing that Equation (5) in IMLS is essentially morphed into an equivalent Equation (8). These two constraints are equivalent because $y_i, x_j$ are all binary variables, $y_i$ will be forced to 0 if there exists $x_j = 0$ with $E_j \in Q_i$.

Instead of maximizing utility, we consider the alternative objective of minimizing utility loss as follows. Set $\hat{y}_i = 1 - y_i$ be the complement of $y_i$, which indicates whether query $Q_i$ is un-selected. We can change the utility gain maximization IMLS$^l$ into a utility loss minimization problem IMLS$^m$. Note that utility gain is maximized if and only if utility loss is minimized.

(IMLS$^m$) \[ \min \sum_{Q_i \in Q} n_i w_i \hat{y}_i \] \text{s.t.} \[ \sum_{E_j \in \Sigma} \lambda_j m_j x_j \leq M \] \[ \hat{y}_i \geq 1 - x_j, \forall E_j \in Q_i \] \[ \hat{y}_i, x_j \in \{0, 1\} \] (9) (10) (11)
In IMLS\textsuperscript{m} Equation (10) is obtained by using $\hat{y}_i = 1 - y_i$ and Equation (8). Using this new minimization problem with linear structure, we prove a bi-criteria approximation result. Let $OPT$ be the optimal loss with budget $M$ in a loss minimization problem, then an $(\alpha, \beta)$-bi-criteria approximation guarantees that its solution has at most $\alpha \cdot OPT$ loss, while uses no more than $\beta \cdot M$ budget. Bicriteria approximations have been extensively used in the context of resource augmentation (e.g., see [41] and references therein), where the algorithm is augmented with extra resources and the benchmark is an optimal solution without augmentation.

**Theorem 3.** The problem of IMLS\textsuperscript{m} admits a $(\frac{1}{1-\tau}, \frac{1}{1-\tau})$ bi-criteria-approximation, for any $\tau \in [0, 1]$.

For concreteness, suppose we set $\tau = \frac{1}{2}$. Then this result states that we can efficiently find a strategy that incurs at most 2 times the optimal utility loss with budget $M$, while using no more than $2M$ memory budget.

**Proof.** Given a parameter $\frac{1}{1-\tau}$, we construct an event selection strategy as follows. First we drop the integrality constraint of IMLS\textsuperscript{m} to obtain its LP-relaxation. We solve the relaxed problem to get an optimal fractional solutions $x^*_j$ and $\hat{y}^*_i$. We can then divide queries $Q$ into two sets, $Q^a = \{Q_i \in Q | \hat{y}^*_i \leq \tau\}$ and $Q^r = \{Q_i \in Q | \hat{y}^*_i > \tau\}$. Since $\hat{y}_i$ denotes whether query $Q_i$ is un-selected, intuitively a smaller value means the query is more likely to be accepted. We can accordingly view $Q^a$ as the set of “promising” queries, and $Q^r$ as “unpromising” queries.

The algorithm works as follows. It preserves every query with $\hat{y}^*_i \leq \tau$, by selecting constituent events of $Q_i$ into memory. So the set of query in $Q^a$ is all accepted, while $Q^r$ is all rejected.

We first show that the memory consumption is no more than $\frac{1}{1-\tau} M$. From Equation (10), we know the fractional solutions must satisfy

$$x^*_j \geq 1 - \hat{y}^*_i, \forall E_j \in Q_i$$  \hspace{1cm} (12)

In addition, we have $\hat{y}^*_i \leq \tau, \forall Q_i \in Q^a$. So we conclude

$$x^*_j \geq 1 - \tau, \forall Q_i \in Q^a, E_j \in Q_i$$  \hspace{1cm} (13)

Since we know $x^*_j$ are fractional solutions to IMLS\textsuperscript{m}, we have

$$\sum_{E_j \in Q^a} m_j \lambda_j x^*_j \leq \sum_{E_j \in Q^a \cup Q^r} m_j \lambda_j x^*_j = M$$  \hspace{1cm} (14)

Here we slightly abuse the notation and use $E_j \in Q^a$ to denote that there exists a query $Q \in Q^a$ such that $E_j \in Q$. Combining (13) and (14), we have

$$\sum_{E_j \in Q^a} m_j \lambda_j (1 - \tau) \leq M$$

Notice that $\sum_{E_j \in Q^a} m_j \lambda_j$ is the total memory consumption of our rounding algorithm, we have

$$\sum_{E_j \in Q^a} m_j \lambda_j \leq \frac{M}{1-\tau}$$

Thus total memory consumption cannot exceed $\frac{M}{1-\tau}$.

We then need to show that the utility loss is bounded by a factor of $\frac{1}{1-\tau}$. Denote the optimal loss of IMLS\textsuperscript{m} as $l^*$, and the optimal loss with LP-relaxation as $\tilde{l}^*$. We then have $\tilde{l}^* \leq l^*$ because any feasible solution to IMLS\textsuperscript{m} is also feasible to the LP-relaxation of IMLS\textsuperscript{m}. In addition, we know
\[
\sum_{Q_i \in Q^r} n_i w_i \hat{y}^*_i \leq \sum_{Q_i \in Q^r \cup Q'} n_i w_i \hat{y}^*_i = l^* \leq l^*
\]

So we can obtain

\[
\sum_{Q_i \in Q^r} n_i w_i \hat{y}^*_i \leq l^*
\] (15)

Based on the way queries are selected, we know for every rejected query

\[
\hat{y}^*_i \geq \tau, \forall Q_i \in Q^r
\] (16)

Combining (15) and (16), we get

\[
\sum_{Q_i \in Q^r} n_i w_i \tau \leq l^*
\]

Observing that \(\sum_{Q_i \in Q^r} n_i w_i\) is the utility loss of the algorithm, we conclude that

\[
\sum_{Q_i \in Q^r} n_i w_i \leq \frac{l^*}{\tau}
\]

This bounds the utility loss from optimal \(l^*\) by a factor of \(\frac{1}{\tau}\), thus completing the proof. \(\Box\)

Note that since our proof is constructive, this gives an LP-relaxation based algorithm to achieve \((\frac{1}{\tau}, \frac{1}{1-\tau})\) bi-criteria-approximation of utility loss.

5.1.3 An approximable special case

Given that memory is typically reasonably abundant in today’s hardware setup, in this section we will assume that the available memory capacity is large enough such that it can hold at least a few number of queries. If we set \(f = \frac{\max_i \sum_{E_j \in Q_i} \lambda_j m_j}{M}\) to be the ratio between the memory requirement of the largest query and available memory \(M\). We know if \(M\) is large enough, then each query uses no more than \(fM\) memory, for some \(f < 1\).

In addition, denote by \(p = \max_{E_j} |\{Q_i | E_j \in Q_i, Q_i \in Q\}|\) as the maximum number of queries that one event type participates in. We note that in practice there are problems in which each event participates in a limited number of queries. In such cases \(p\) will be limited to a small constant.

Assuming both \(p\) and \(f\) are some fixed constants, we obtain the following approximation result.

**Theorem 4.** Let \(p\) be the maximum number of queries that one event can participate in, and \(f\) be the ratio between the size of the largest query and the memory budget defined above, IMLS admits a \(\frac{p}{1-\tau}\)-approximation.

The idea here is to leverage the fact that the maximum query-participation \(p\) is a constant to simplify the memory consumption constraint, so that a knapsack heuristic yields utility guarantees. In the interest of space we present the full proof of this theorem in Appendix F.

5.1.4 A pseudo-polynomial-time solvable special case

We further consider the multi-tenant case where multiple CEP applications are consolidated into one single server or into one cloud infrastructure where the same set of underlying computing resources is shared across applications.

In this multi-tenancy scenario, since different CEP applications are interested in different aspects of real-world event occurrences, there typically is no or very limited sharing of events across different applications (the hospital hygiene system HyReminder and hospital inventory management system CIMS as mentioned in the Introduction, for example, have no event types in common. So do a network intrusion detection...
application and a financial application co-located in the same cloud). Using a hyper-graph model, a multi-tenant CEP system can be represented as a hyper-graph $H$, where each event type is represented as a vertex and each query as a hyper-edge. If there is no sharing of event types among CEP applications, then each connected component of $H$ corresponds to one CEP application. Let $k$ be the size of the largest connected component of $H$, then $k$ is essentially the maximum number of event types used in any one CEP application, which may be limited to a small constant (the total number of event types across multiple CEP applications is not limited). Assuming this is the case, we have the following special case that is pseudo-polynomial time solvable.

**Theorem 5.** In a multi-tenant CEP system where each CEP tenant uses a disjoint set of event types, if each CEP tenant uses no more than $k$ event types, the problem of IMLS can be solved in time $O(|\Sigma| |Q| M^{2k^2})$.

Our proof utilizes a dynamic programming approach developed for Set-union Knapsack problem [27]. The full proof of this theorem can be found in Appendix G.

We note that we do not assume the total number of event types across multiple CEP tenants to be limited. In fact, the running time grows linearly with the total number of event types and queries across all CEP tenants.

Lastly, we observe that the requirement that events in each CEP tenant are disjoint can be relaxed. As long as the sharing of event types between different CEP tenants are limited, such that the size of the largest component of $H$ mentioned above is bounded by $k$, the result in Theorem 5 holds.

### 5.2 The fractional variant (FMLS)

In this section, we consider the fractional variant of the memory-bound load shedding problem. In this variant, instead of taking an all-or-nothing approach to either include or exclude all event instances of certain types in memory, we use a random sampling operator [31], which samples some arriving events uniformly at random into the memory. Denote by $\bar{x}_j \in [0,1]$ the sampling probability for each event type $E_j$. The fractional variant memory-bound load shedding (FMLS) can be written as follows.

\[
(FMLS) \quad \max \sum_{Q_i \in Q} n_i w_i \bar{y}_i \tag{17}
\]

s.t. \[
\sum_{E_j \in \Sigma} \lambda_j m_j \bar{x}_j \leq M \tag{18}
\]

\[
\bar{y}_i = \prod_{E_j \in Q_i} \bar{x}_j \tag{19}
\]

\[
0 \leq \bar{x}_j \leq 1 \tag{20}
\]

The integrality constraints are essentially dropped from the integral version of the problem, and are replaced by constraints in (20). We use fractional sampling variables $\bar{x}_j$ and $\bar{y}_i$ to differentiate from binary variables $x_j, y_i$. Note that Equation (19) states that the probability that a query result is produced, $\bar{y}_i$, is the cross-product of sampling rates of constituent events since each event is sampled randomly and independently of each other.

#### 5.2.1 A general complexity analysis

In the FMLS formulation, if we fold Equation (19) into the objective function in (17), we obtain

\[
\max \sum_{Q_i \in Q} n_i w_i \prod_{E_j \in Q_i} \bar{x}_j \tag{21}
\]

This makes FMLS a polynomial optimization problem subject to a knapsack constraint (18).
Since we are maximizing the objective function in Equation (21), it is well known that if the function is concave, then convex optimization techniques [14] can be used to solve such problems optimally. However, we show that except the trivial case where each query has exactly one event (i.e., (21) becomes linear), in general Equation (21) is not concave.

**Lemma 1.** If the objective function in Equation (21) is non-trivial (that is, at least one query has more than one event), then (21) is non-concave.

We show the full proof of Lemma 1 in Appendix H.

Given this non-concavity result, it is unlikely that we can hope to exploit special structures of the Hessian matrix to solve FMLS. In particular, convex optimization techniques like KKT conditions or gradient descent [14] can only provide local optimal solutions, which may be far away from the global optimal.

On the other hand, while the general polynomial program is known to be hard to solve [11, 44], FMLS is a special case where all coefficients are positive, and the constraint is a simple knapsack constraint. Thus the hardness results in [11, 44] do not apply to FMLS. We show the hardness of FMLS by using the Motzkin-Straus theorem [37] and a reduction from the Clique problem.

**Theorem 6.** The problem of fractional memory-bound load shedding (FMLS) is NP-hard. FMLS remains NP-hard even if each query has exactly two events.

The full proof of this theorem can be found in Appendix I.

So despite the continuous relaxation of the decision variables of IMLS, FMLS is still hard to solve. However, in the following we show that FMLS has special structure that allows it to be solved approximately under fairly general assumptions.

### 5.2.2 Definitions of approximation

We will first describe two definitions of approximation commonly used in the numerical optimization literature.

The first definition is similar to the approximation ratio used in combinatorial optimization.

**Definition 7.** [30] Given a maximization problem \( P \) that has maximum value \( v_{\text{max}}(P) > 0 \). We say a polynomial time algorithm has an absolute approximation ratio \( \epsilon \in [0, 1] \), if the value found by the algorithm, \( v(P) \), satisfies \( v_{\text{max}}(P) - v(P) \leq \epsilon v_{\text{max}}(P) \).

The second notion of relative approximation ratio is widely used in the optimization literature [22, 30, 39, 44].

**Definition 8.** [30] Given a maximization problem \( P \) that has maximum value \( v_{\text{max}}(P) \) and minimum value \( v_{\text{min}}(P) \). We say a polynomial time algorithm has a relative approximation ratio \( \epsilon \in [0, 1] \), if the value found by the algorithm, \( v(P) \), satisfies \( v_{\text{max}}(P) - v(P) \leq \epsilon (v_{\text{max}}(P) - v_{\text{min}}(P)) \).

Relative approximation ratio is used to bound the quality of solutions relative to the possible value range of the function. We refer to this as \( \epsilon \)-relative-approximation to differentiate from \( \epsilon \)-approximation used Definition 7.

Note that in both cases, \( \epsilon \) indicates the size of the gap between an approximate solution and the optimal value. So smaller \( \epsilon \) values are desirable in both definitions.

### 5.2.3 Results for relative approximation bound

In general, the feasible region specified in FMLS is the intersection of a unit hyper-cube, and the region below a scaled simplex. We first normalize (18) in FMLS using a change of variables. Let \( \bar{x}_j' = \frac{\lambda_j x_j v^c}{M} \) be the scaled variables. We can obtain the following formulation FMLS'.

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\begin{align*}
&\text{(FMLS')} \quad \max \sum_{Q_i \in Q} n_i w_i \prod_{E_j \in Q_i} \frac{M}{\lambda_j m_j} \bar{x}'_j \quad (22) \\
&\text{s.t.} \quad \sum_{E_j \in \Sigma} \bar{x}'_j = 1 \quad (23) \\
&0 \leq \bar{x}'_j \leq \frac{\lambda_j m_j}{M} \quad (24)
\end{align*}

Note that the inequality constraint \((18)\) in FMLS is now replaced by an equality constraint \((23)\) in FMLS'. This will not change the optimal value of FMLS as long as \(\sum_{E_j} \frac{\lambda_j m_j}{M} \geq 1\) (otherwise, although \(\sum_{E_j \in \Sigma} \bar{x}'_j = 1\) is unattainable, the memory budget becomes sufficient and the optimal solution is trivial). This is because all coefficients in \((17)\) are positive, pushing any \(x_i\) to a larger value will not hurt the objective value. Since the constraint \((18)\) is active for at least one global optimal point in FMLS, changing the inequality in the knapsack constraint to an equality in \((23)\) will not change the optimal value.

Denote by \(d = \max \{|Q_i|\}\) the maximum number of event types in a query. We will assume in this section that \(d\) is a fixed constant. Note that this is a fairly realistic assumption, as \(d\) tends to be very small in practice (in HyReminder \([46]\), for example, the longest query has 6 event types, so \(d = 6\)). Observe that \(d\) essentially corresponds to the degree of the polynomial in the objective function \((22)\).

**An approximation using co-centric balls.** Using a randomized procedure from the optimization literature \([30]\), we show that FMLS' can be approximated by bounding the feasible region using two co-centric balls to obtain a (loose) relative-approximation ratio as follows.

**Theorem 7.** The problem FMLS' admits a relative approximation ratio \(\epsilon\), where \(\epsilon = 1 - O \left( |\Sigma|^{-\frac{d}{2}} \frac{t^2 + 1}{t^2} \right)^{\frac{d}{2}}\) and \(t = \min \left( \min_{E_j} \left( \frac{\lambda_j m_j}{M} \right), \frac{1}{\sqrt{|\Sigma|}} \right)\).

A proof of this result can be found in Appendix I. Note that this is a general result that only provides a loose relative approximation bound, which is a function of the degree of the polynomial \(d\), the number of event types \(|\Sigma|\), and \(\frac{\lambda_j m_j}{M}\), and cannot be adjusted to a desirable level of accuracy.

**An approximation using simplex.** We observe that the feasible region defined in FMLS' has special structure. It is a subset of a simplex, which is the intersection between a standard simplex (Equation \((23)\)) and box constraints (Equation \((24)\)).

There exists techniques that produces relative approximations for polynomials defined over simplex. For example, \([22]\) uses a grid-based approximation technique, where the idea is to fit a uniform grid into the simplex, so that values on all nodes of the grid are computed and the best value is picked as an approximate solution. Let \(\Delta_n\) be an \(n\) dimensional simplex, then \((x \in \Delta_n | xx \in \mathbb{Z}_+^n\) is defined as a \(k\)-grid over the simplex. The quality of approximation is determined by the granularity the uniform grid: the finer the grid, the tighter the approximation bound is.

The result in \([22]\), however, does not apply here because the feasible region of FMLS' represents a subset of the standard simplex. We note that there exist a special case where if \(\frac{\lambda_j m_j}{M} \geq 1, \forall j\) (that is, if the memory requirement of a single event type exceeds the memory capacity), the feasible region degenerates to a simplex, such that we can use grid-based technique for approximation.

**Theorem 8.** In FMLS', if for all \(j\) we have \(\frac{\lambda_j m_j}{M} \geq 1\) then the problem admits a relative approximation ratio of \(\epsilon\), where
\[
\epsilon = O \left( 1 - \frac{k!}{(k - d)!k^d} \right)
\]
for any \(k \in \mathbb{Z}^+\) such that \(k > d\). Here \(k\) represents the number of grid points along one dimension.
Note that as \( k \to \infty \), approximation ratio \( \varepsilon \to 0 \). The full proof of this result can be found in Appendix K. This result can provide increasingly accurate relative approximation bound for a larger \( k \). It can be shown that for a given \( k \), a total of \( \binom{|\Sigma|+k-1}{|\Sigma|-1} \) number of evaluations of the objective function is needed.

### 5.2.4 Results for absolute approximation bound

Results obtained so far use the notion of relative approximation (Definition 8). In this section we discuss a special case in which FMLS’ can be approximated relative to the optimal value (Definition 7).

We consider a special case in which queries are regular with no repeated events. That is, in each query, no events of the same type occur more than once (e.g., query SEQ(A,B,C) is a query without repeated events while SEQ(A,B,A) is not because event A appears twice). This may be a fairly general assumption, as queries typically detect events of different types. HyReminder [46] queries, for instance, use no repeated events in the same query (two such example Q1 and Q2 are given in the Introduction). In addition, we require that each query has the same length as measured by the number of events.

With the assumption above, the objective function Equation (22) becomes a homogeneous multi-linear polynomial, while the feasible region is defined over a sub-simplex that is the intersection of a cube and a standard simplex. We extend a random-walk based argument in [39] from standard simplex to the sub-simplex, and show an (absolute) approximation bound.

**Theorem 9.** In FMLS’, if \( \frac{\lambda_j m_j}{M} \) are fixed constants, in addition if every query has no repeated event types and is of same query length, \( d \), then a constant-factor approximation can be obtained for FMLS’ in polynomial time. In particular, we can achieve a \( (1 - \beta \left( \frac{k!}{(k-d)!k^d} \right)) \)-approximation, by evaluating Equation (21) at most \( \binom{|\Sigma|+k-1}{|\Sigma|-1} \) times, where \( \beta = \min \left( \min_j \left( \frac{\lambda_j m_j}{M} , 1 \right) \right) d \), and \( k > d \) is a parameter that controls approximation accuracy.

We use a scaling method to extend the random-walk argument in [39] to the sub-simplex in order to get the desirable constant factor approximation. A proof of Theorem 9 can be found in Appendix L. Note that, by selecting \( k = O(d^2) \) we can get \( \frac{k!}{(k-d)!k^d} \) close to 1. Also note that if \( \frac{\lambda_j m_j}{M} \geq 1 \) for all \( j \), \( \beta = 1 \) and we can get an approximation arbitrarily close to the optimal value by using large \( k \).

### 6 CPU-bound load shedding

In this section we consider the scenario where memory is abundant, while CPU becomes the limiting resource that needs to be budgeted appropriately. CPU-bound problems turn out to be easy to solve.

#### 6.1 The integral variant (ICLS)

In the integral variant CPU load shedding, we again use binary variables \( y_i \) to denote whether results of \( Q_i \) can be generated. For each query \( Q_i \), at most \( n_i \) number of query matches can be produced. Assuming the utility weight of each result is \( w_i \), and the CPU cost of producing each result is \( c_i \), when \( Q_i \) is selected \( (y_i = 1) \) a total of \( n_i w_i \) utility can be produced, at the same time a total of \( n_i c_i \) CPU resources are consumed. That yields the following ICLS problem.

\[
\text{(ICLS)} \quad \max \sum_{Q_i \in \mathcal{Q}} n_i w_i y_i \quad \text{(25)}
\]

\[
\text{s.t.} \quad \sum_{Q_i \in \mathcal{Q}} n_i c_i y_i \leq C \quad \text{(26)}
\]

\[
y_i \in \{0, 1\} \quad \text{(27)}
\]
ICLS is exactly the standard 0-1 knapsack problem, which has been studied extensively. We simply cite a result from [45, Chapter 5] for completeness.

**Theorem 10.** [45] The integral CPU-bound load shedding (ICLS) is NP-complete. It admits a fully polynomial approximation scheme (FPTAS).

ICLS is thus an easy variant among the load shedding problems studied in this work.

### 6.2 The fractional variant (FCLS)

Similar to the memory-bound load shedding problems, we also investigate the fractional variant where a sampling operator is used to select a fixed fraction of query results. Instead of using binary variables $y_i$, we denote by $\overline{y}_i$ the percentage of queries that are sampled and processed by CPU for output. The integrality constraints in ICLS are again dropped, and we can have the following FCLS formulation.

$$\text{(FCLS)} \quad \max \sum_{Q_i \in \mathcal{Q}} n_i w_i \overline{y}_i$$

s.t. $$\sum_{Q_i \in \mathcal{Q}} n_i c_i \overline{y}_i \leq C$$

$$0 \leq \overline{y}_i \leq 1, \text{ for all } i$$

Since $n_i, w_i, c_i$ are all constants, this is a simple linear program problem that can be solved in polynomial time. We conclude that FCLS can be efficiently solved.

### 7 Dual-bound load shedding

Lastly, we study the dual-bound load shedding problem, where both CPU and memory resources can be limited.

#### 7.1 The integral variant (IDLS)

The integral dual-bound load shedding (IDLS) can be formulated by combining CPU and memory constraints.

$$\text{(IDLS)} \quad \max \sum_{Q_i \in \mathcal{Q}} n_i w_i y_i$$

s.t. $$\sum_{E_j \in \Sigma} \lambda_j m_j x_j \leq M$$

$$\sum_{Q_i \in \mathcal{Q}} n_i c_i y_i \leq C$$

$$y_i \leq \prod_{E_j \in Q_i} x_j$$

$$x_j, y_i \in \{0, 1\}$$

Binary variables $x_j, y_i$ again denote event selection and query selection, respectively. Note that in order to respect the CPU constraint, not all queries whose constituent events are available in memory can be produced. This is modeled by an inequality in Equation (31).

We show that the loss minimization version of IDLS admits a tri-criteria approximation.
Theorem 11. Denote by $M$ the given memory budget $C$ the given CPU budget, and $l^*$ the optimal utility loss with that budget. IDLS admits $(\frac{1}{\tau}, \frac{1}{1-\tau}, \frac{1}{1-\tau})$ tri-criteria-approximation for any $\tau \in [0, 1]$. That is, for any $\tau \in [0, 1]$, we can compute a strategy that uses no more than $\frac{1}{\tau} M$ memory $\frac{1}{1-\tau} C$ CPU, and incur no more than $\frac{1}{1-\tau} l^*$ utility loss.

The idea of the proof is to use LP-relaxation and round the resulting fractional solution, which is similar to Theorem 3. Detailed proof of Theorem 11 can be found in Appendix M. In addition, we show that the approximable special case of IMLS (Theorem 4) also holds for IDLS. Details of the proof can be found in Appendix N.

Theorem 12. Let $p$ be the maximum number of queries that one event can participate in, and $f = \frac{\max_{i} \sum_{j \in Q_i} m_j \lambda_j}{M}$ be the ratio between the size of the largest query and the memory budget. IDLS is $\frac{p}{1-\tau}$-approximable in pseudo polynomial time.

7.2 The fractional variant (FDLS)

The fractional dual-bound problem once again relaxes the integrality constraints in IDLS to obtain the following FDLS problem.

\[
\text{(FDLS)} \quad \max \sum_{Q_i \in Q} n_i u_i y_i \\
\text{s.t.} \quad \sum_{E_j \in \Sigma} \lambda_j m_j \overline{\tau}_j \leq M \\
\quad \sum_{Q_i \in Q} n_i c_i y_i \leq C \quad (33) \\
\quad \overline{y}_i \leq \prod_{E_j \in Q_i} \overline{\tau}_j \quad (34) \\
\quad \overline{x}_j, \overline{y}_i \in [0, 1]
\]

First of all, we note that the hardness result in Theorem 6 still holds for FDLS, because FMLS is simply a special case of FDLS without constraint (33).

The approximation results established for FMLS, however, do not carry over easily. If we fold (34) into Equation (32), and look at the equivalent minimization problem by taking the inverse of the objective function, then by an argument similar to Lemma 1, we can show that objective function is non-convex in general. In addition, we can show that except in the trivial linear case, the constraint in Equation (33) is also non-convex using a similar argument. So we are dealing with a non-convex optimization subject to non-convex constraints.

It is known that solving or even approximating non-convex problems with non-convex constraints to global optimality is hard [14, 21]. Techniques dealing with such problems are relatively scarce in the optimization literature. Exploiting special structure of FDLS to optimize utility is an interesting area for future work.

8 Conclusions and future work

In this work we study the problem of load shedding in the context of the emerging Complex Event Processing model. We investigate six variants of CEP load shedding under various resource constraints, and demonstrate an array of complexity results. Our results shed some light on the hardness of load shedding CEP queries, and provide some guidance for developing CEP shedding algorithms in practice.
CEP load shedding is a rich problem that so far has received little attention from the research community. We hope that our work will serve as a springboard for future research in this important aspect of the increasingly popular CEP model.

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A Additional CEP join semantics

In order to define additional CEP join semantics, we define two additional types of subsequences.

**Definition 9.** Given an event sequence \( S = (e_1, e_2, ..., e_N) \) and a subsequence \( S' = (e_{i_1}, e_{i_2}, ..., e_{i_m}) \). The subsequence \( S' \) is called a contiguous subsequence of \( S \), if \( i_j + 1 = i_{j+1} \), for all \( 1 \leq j \leq m - 1 \).

The subsequence \( S' \) is called a type-contiguous subsequence of \( S \), if for all events \( e_{i_j} \) of type \( E_{i_j} \), where \( j \in [1, m] \), there does not exist another event \( e_k \) in \( S \), where \( i_{j-1} < k < i_j \), such that \( e_k \) is also of the type \( E_{i_j} \).

We illustrate these definitions using the following example.

**Example 7.** Suppose there are four event types \( \Sigma = \{A, B, C, D\} \). Let the stream of arriving events be \( S = (A_1, B_2, C_3, D_4, A_5, B_6, B_7, C_8, D_9) \), where the subscript denotes the time-stamp of the event occurrence.

In this example, \( (A_1, B_2, C_3) \) is a contiguous subsequence, because intuitively there is no intervening events between \( A_1 \) and \( B_2 \), or \( B_2 \) and \( C_3 \). However, \( (A_5, B_6, C_8) \) is not a contiguous subsequence, because there is a \( B_7 \) between \( B_6 \) and \( C_8 \), violating the definition of contiguity.

Although \( (A_5, B_6, C_8) \) is not a contiguous subsequence, it is a type-contiguous subsequence, because there is no event of type \( B \) between \( A_5 \) and \( B_6 \), and no event of type \( C \) between \( B_6 \) and \( C_8 \). The subsequence \( (A_5, B_7, C_8) \), on the other hand, is not type-contiguous subsequence because there is an event of type \( B \), \( B_6 \), between \( A_5 \) and \( B_7 \).

We are now ready to define skip-till-next-match, and contiguity\(^1\) as follows.

**Definition 10.** Join semantics skip-till-any-match, skip-till-next-match, and contiguity determine what event sequence constitutes a query match. Specifically, a subsequence \( S' = (e_{i_1}, e_{i_2}, ..., e_{i_m}) \) of \( S \) is considered a query match of \( Q = SEQ(q_1, q_2, ..., q_n) \) if:

- Event \( e_{i_l} \) in \( S' \) is of type \( q_l \), for all \( l \in [1, n] \) (pattern matches), and \( t_{i_m} - t_{i_1} \leq T(Q) \) (within window).

And in addition,

1. For contiguity:
   - The subsequence \( S' \) has to be a contiguous subsequence.
2. For skip-till-next-match:
   - The subsequence \( S' \) has to be a type-contiguous subsequence.
3. For skip-till-any-match:
   - The subsequence \( S' \) can be any subsequence.

We illustrate query matches using Example\(^8\)

**Example 8.** We revisit the event sequence used in Example\(^7\) Suppose there are three queries, \( Q_1 = \) skip-till-any-match\((A, B, C)\), \( Q'_1 = \) skip-till-next-match\((A, B, C)\), \( Q''_1 = \) contiguity\((A, B, C)\), where \( T(Q_1) = T(Q'_1) = T(Q''_1) = 5 \).

For \( Q_1 \), \( (A_1, B_2, C_3) \), \( (A_5, B_6, C_8) \) and \( (A_5, B_7, C_8) \) are all query matches, because they are valid subsequences, the pattern matches \( \{A, B, C\} \) specified in the query, and events are within the time window \( S \). However, \( (A_1, B_7, C_8) \) is not a match even though pattern matches, because the difference in time stamp between \( C_8 \) and \( A_1 \) is over the window limit 5.

For \( Q'_1 \), \( (A_1, B_2, C_3) \) and \( (A_5, B_6, C_8) \) are query matches. Subsequence \( (A_5, B_7, C_8) \) is not a match because it is not a type-contiguous subsequence.

\(^1\)An additional semantics, partitioned-contiguity, was discussed in \[6\]. Since partitioned-contiguity can be simulated using contiguity by sub-dividing event types using partition criteria, for simplicity it is not described here.
For \( Q_i \), only subsequence \( (A_1, B_2, C_3) \) is a valid match, because \( (A_5, B_6, C_8) \) and \( (A_5, B_7, C_8) \) are not contiguous subsequences.

B Proof of Proposition \(^1\)

Proof. First, we show that no deterministic online shedding algorithm can achieve a competitive ratio that is independent of the length of the event sequence. In order to establish this, it is sufficient to specify a distribution over event sequences for which the competitive ratio—defined to be the ratio of the expected utility obtained by the algorithm and the (offline) optimal utility—achieved by any deterministic algorithm must depend on the length of the sequence.

We construct such a distribution as follows. Suppose we have a universe of \( 3m \) event types, \( \Sigma = \{E_i\} \cup \{E'_i\} \cup \{E''_i\}, i \in [m] \). Assume that there are \( 2m \) queries \( SEQ(E_i, E''_i) \) and \( SEQ(E'_i, E''_i) \), \( \forall i \in [m] \), each with unit utility weight. The event sequence is of the form \( (e_1, e_2, ..., e_m, X) \), where for all \( i \in [m] \), \( e_i \) is set to be either \( E_i \) with probability \( \frac{1}{2} \) or \( E'_i \) with probability \( \frac{1}{2} \). In addition, \( X \) is drawn from the uniform distribution over \( \{E''_i : i \in [m]\} \).

Next, we show that no randomized algorithm can do any better. Using Yao’s principle \(^{[53]}\), we know that the expected utility of a randomized algorithm against the worst case input stream, is no more than the expected utility of the best deterministic algorithm against the input distribution. Since we know any deterministic algorithm against the input distribution constructed above is \( \frac{1}{m} \), it follows that the expected utility of a randomized algorithm against the worst case input stream from this input distribution is at most \( \frac{1}{m} \), thus completing the proof.

C Query match estimations

In this section we describe a way in which the expected query matches of \( Q_i \) over a unit time, \( n_i \), can be computed. It is not hard to imagine that \( n_i \) can be estimated, because a brute force approach is to simply sample the arriving event stream to count the number of query matches, \( \hat{N}_i \) over time-window \( T(Q_i) \), so that the number of query matches over unit time, \( n_i \), can be simply computed as \( n_i = \frac{\hat{N}_i}{T(Q_i)} \). In this section

\(^2\)Note that the algorithm can only report queries that have a match, i.e., it cannot report false positives. Hence, if \( X \) is \( E''_i \), then the algorithm cannot simply guess and declare a match, say \( (E_k, E''_k) \), without any knowledge of \( e_k \). Specifically, \( e_k \) could instead be \( E'_k \) and in such a case the declared match would be incorrect.
we will present an analytical approach to estimate \( n_i \) by using event arrival rates \( \lambda_j \) under the skip-till-any-match semantics. Note that our discussion is only to show that there exist ways to estimate \( n_i \), so that we can treat \( n_i \) as constants in our load shedding problem formulation.

Assuming that events arrive in Possion process with arrival rate \( \lambda_i \) (a typical assumption in the performance modeling literature [34]), our estimation is produced as follows. First, for each event type \( E_j \in Q_i \), the expected number of \( E_j \) occurrences in \( T(Q_i) \), denoted as \( l_j \), can be computed as \( l_j = \lambda_j T(Q_i) \). Set \( \sigma(Q_i) \) as the set of event types that are part of \( Q_i \), \( |Q_i| \) as the total number of events in \( Q_i \), and \( m(E_j) \) be the number of occurrences of event type \( e_i \) in \( Q_i \) (as an example, a query \( Q = (A, A, B) \) would have \( \sigma(Q) = \{A, B\} \), \( m(A) = 2 \), \( m(B) = 1 \), and \( |Q| = 3 \). Let \( L = \sum_{e_i \in \sigma(Q_i)} l_j \) be the total number of occurrences of events relevant to \( Q_i \) in time window \( T(Q_i) \), then the expected number of query matches of \( Q_i \) in \( T(Q_i) \) can be estimated as

\[
\left( \frac{L}{|Q_i|} \right) \frac{\prod_{e_i \in \sigma(Q_i)} \prod_{k=0}^{m(e_i)} (l_i - k)}{\prod_{r=0}^{|Q_i|} (L - r)} \tag{35}
\]

The formula of above is obtained as follows. Given a total of \( L \) event occurrences in \( T(Q_i) \), we only pick a total \( |Q_i| \) events to form a query match. Let us pick the first \( |Q_i| \) events to form random permutations and compute the probability that the first \( |Q_i| \) events produces a match. Let the first position of \( Q_i \) be an event of type \( E_{p1} \). The probability of actually seeing the event that type is \( \frac{l_{p1}}{L} \). For the second event in \( Q_i \), if it is also of type \( E_{p1} \), the probability of seeing that event that type is \( \frac{l_{p1} - 1}{L - 1} \), otherwise it is \( \frac{l_{p1}}{L} \), so on and so forth. In the end we have \( \frac{\prod_{e_i \in \sigma(Q_i)} \prod_{k=0}^{m(e_i)} (l_i - k)}{\prod_{r=0}^{|Q_i|} (L - r)} \). Given that there are a total of \( \left( \frac{L}{|Q_i|} \right) \) such possible positions out of \( L \) event occurrences and each of which is symmetric, the expected count of query matches can be expressed as the product of the two, thus the Equation (35). Given that over time \( T(Q_i) \) this many number of event matches can be produced, we can conclude that over unit time the expected number of query matches \( n_i \), is thus

\[
n_i = \frac{1}{T(Q_i)} \left( \frac{L}{|Q_i|} \right) \frac{\prod_{e_i \in \sigma(Q_i)} \prod_{k=0}^{m(e_i)} (l_i - k)}{\prod_{r=0}^{|Q_i|} (L - r)}
\]

### D Proof of Theorem

**Proof.** We obtain the hardness result by a reduction from Densest \( k \)-sub-hypergraph (DKSH). Recall that given a hypergraph \( G = (V, H) \), the decision version of DKSH is to determine if there exists an induced subgraph \( G' = (V', H') \), such that \( |V'| \leq k \), and \( |H'| \geq B \) for some given constant \( B \).

Given any instance of the DKSH problem, we construct an IMLS problem as follows. We build a bijection \( \phi : V \to \Sigma \), so that each vertex \( v_j \in V \) corresponds to one event type \( E_j \in \Sigma \). For each hyperedge \( h_i \in H \), using vertices \( v_k \) that are endpoints of \( h_i \) to construct a corresponding query \( Q_i \), such that \( \forall v_k \in h_i, \phi(v_k) \in Q_i \). Set all \( E_j \) to have unit cost \( (m_j \lambda_j = 1) \) and all \( Q_i \) have unit weight \( (w_i n_i = 1) \), and lastly set the memory budget to \( k \).

We first show the forward direction, that is if there exists a solution to DKSH, i.e., a \( k \)-sub-hypergraph with at least \( B \) hyper-edges, then there exists a solution to IMLS with no more than \( k \) memory cost but achieves \( B \) utility. Suppose the subgraph \( G' = (V', H') \) is the solution to DKSH. In the corresponding IMLS problem, if we keep all event types \( \phi(v') \), \( v' \in V' \), our solution has a memory cost of \( |V'| \), which is no more than \( k \) since \( G' \) is a \( k \)-sub-hypergraph. This ensures the constructed solution is feasible. Furthermore, the utility of the IMLS solution is exactly \( |H'| \) by our unit weight construction. Since we know \( |H'| \geq B \), this completes the proof in this direction.
In the other direction, let the set of events selected in IMLS be $S \subseteq \Sigma$ while the set of dropped events be $D = \Sigma \setminus S$. The set of vertices $V_s$ corresponding to $S$ induces a subgraph $G_s = (V_s, H_s)$. $S$ respects memory-bounds in IMLS implies $|V_s| \leq k$, so $G_s$ is a valid $k$-sub-hyper-graph. In addition, $S$ produces utility $B$ in IMLS ensures that $|H_s| \geq B$, thus completing the hardness proof.

### E Proof of Theorem 2

**Proof.** We note that the reduction from Densest $k$-sub-hypergraph (DKSH) discussed above is approximation preserving. Utilizing a hardness result of DKSH [29], which establishes that DKSH cannot be approximated within a factor of $2^{(\log n)^{\delta}}$ for some $\delta > 0$, we obtain the inapproximability result of IMLS.

### F Proof of Theorem 4

**Proof.** We modify the IMLS problem in Equation (3)-(6) to a relaxed version IMLS’ as follows.

\[
(\text{IMLS’}) \quad \max \sum_{Q_i \in \Omega} n_i w_i y_i \\
\text{s.t.} \quad \sum_{Q_i} (y_i \sum_{E_j \in Q_i} \lambda_j m_j) \leq M \\
y_i = \prod_{E_j \in Q_i} x_j \\
x_j, y_i \in \{0, 1\}
\]  

We essentially replace Equation (4) and Equation (5) in IMLS with Equation (36). Note that the left side of Equation (36) is an overestimate of the memory consumption of a particular event/query selection strategy. First, set $x_j^*, y_i^*$ as an optimal solution to IMLS, where each event selected participates at least one selected query. That is, if we set $X = \{x_j^* | x_j^* = 1, \exists Q_i \ni E_j, y_i^* = 1\}$, we have $X = \emptyset$. There always exists one such optimal solution, because otherwise we can force $x_j^* = 0, \forall x_j^* \in X$ without changing the optimal value $\text{opt}^*$ (by definition no events $E_j$ corresponding to $x_j^* \in X$ participates in queries $Q_i$ with $y_i^* = 1$), thus obtaining one optimal solution with $X = \emptyset$. With this $x_j^*, y_i^*$ we can show the following inequality

\[
\sum_{E_j \in \Sigma} \lambda_j m_j x_j^* \leq \sum_{Q_i} (y_i^* \sum_{E_j \in Q_i} \lambda_j m_j)
\]  

This is the case because we know $X = \emptyset$, so $\forall x_j^* = 1, \exists Q_i \ni E_j$ where $y_i^* = 1$. It ensures that for each term $\lambda_j m_j x_j^*$ where $x_j^* = 1$ from the left side of Equation (37), there exists one matching term $y_i^* \lambda_j m_j$ on the right side, where $E_j \in Q_i, y_i^* = 1$, thus providing the inequality.

This inequality guarantees that any feasible solution of IMLS’ will be feasible for IMLS. This is because given Equation (36) in IMLS’ and Equation (37), we know Equation (4) in IMLS holds, so the solution to IMLS’ will produce the same value in IMLS but still respect the constraints in IMLS.

In addition, we show the following inequality is true

\[
\frac{1}{p} \sum_{Q_i} (y_i^* \sum_{E_j \in Q_i} \lambda_j m_j) \leq \sum_{E_j \in \Sigma} \lambda_j m_j x_j^*
\]  

25
Recall that we know each event $E_j$ participates at most $p$ number of queries. If each event participates exactly one query, we know $\sum_{Q_i} (y_i^* \sum_{E_j \in Q_i} \lambda_j m_j) = \sum_{E_j \in \Sigma} \lambda_j m_j x_j^*$. Now if each event appears in no more than $p$ queries, then for all $j$, the coefficients of each term $\lambda_j m_j$ on the right hand side is no more than $p$, thus the inequality

With this inequality we further define the program IMLS$''$. 

\[
(\text{IMLS$''$}) \quad \max \sum_{Q_i \in Q} n_i w_i y_i \\
\text{s.t. } \frac{1}{p} \sum_{Q_i} (y_i^* \sum_{E_j \in Q_i} \lambda_j m_j) \leq M
\] 

(39)

\[
y_i = \prod_{E_j \in Q_i} x_j \\
x_j, y_i \in \{0, 1\}
\]

Solution feasible to IMLS will be feasible to IMLS$''$, because we only replace Equation (41) in IMLS with Equation (39), and we know Equation (38) holds. Let the optimal value to IMLS$''$ be $\text{opt}''^*$, the optimal value of IMLS $\text{opt}^*$, then

\[
\text{opt}''^* \geq \text{opt}^*
\]

(40)

Furthermore, observe that both IMLS$'$ and IMLS$''$ are now simple one dimensional knapsack problems, where IMLS$''$ has $p$ times more knapsack capacity than IMLS$'$ (because we can rewrite Equation (39) as $\sum_{Q_i} (y_i^* \sum_{E_j \in Q_i} \lambda_j m_j) \leq pM$). Suppose we solve IMLS$'$ using a simple knapsack heuristics, by picking queries (setting $y_i = 1$) in the descending order of $n_i w_i \lambda_j m_j$ until the knapsack $M$ is filled, and denote the value so produced by $\text{val}'$. We can show $\text{val}'$ is good relative to $\text{opt}''^*$ in the following sense.

\[
\text{val}' \times \frac{p}{1 - f} \geq \text{opt}''^*
\]

(41)

To see why this is the case, suppose in the knapsack in IMLS$'$, we cheat by allowing each query to be divided into fractional parts. The optimal value in this modified version, $\hat{\text{opt}}'$, can be produced by simply packing queries in the order of $n_i w_i \lambda_j m_j$. The difference between $\hat{\text{opt}}'$ and the heuristics $\text{val}'$ is the largest when the last query cannot fit in its entirety using IMLS$'$ heuristics. Since we know each query is relatively small compared to the budget $M$ (bounded by $fM$), we know

\[
\frac{\text{val}'}{\text{opt}'} \geq \frac{1 - f}{1} = 1 - f
\]

(42)

Next, we know if we cheat again by allowing fractional queries in IMLS$''$, the optimal value $\hat{\text{opt}}''$ is no more than $p$ times $\hat{\text{opt}}'$,

\[
\hat{\text{opt}}'' \leq p \times \hat{\text{opt}}'
\]

(43)

because knapsack size has grown $p$ times but we fill knapsacks strictly based on their $n_i w_i \lambda_j m_j$ values.

Lastly, it is easy to see that

\[
\hat{\text{opt}}'' \geq \text{opt}''^*
\]

(44)

Combining Equation (42) (43) (44) we can derive Equation (41). Since we also know Equation (40), we get

\[
\text{val}' \times \frac{p}{1 - f} \geq \text{opt}^*
\]

(45)

thus completing the proof. 

\[\square\]
G Proof of Theorem 5

Proof Sketch. The IMLS problem studied in this work can be formulated as a Set-union Knapsack problem [27]. A dynamic programming algorithm is proposed in [27] for Set-union Knapsack, which however runs in exponential time. If we define an adjacency graph $G$ by representing all events as graph vertices, and each pair of vertices are connected if the corresponding events co-occur in the same query. The exponent of the running time is shown to be no more than the cut-width of the induced adjacency graph $G$, $cw(G)$. Recall that cut-width of a graph $G$ is defined as the smallest integer $k$ such that the vertices of $G$ can be arranged in a linear layout $[v_1, ..., v_n]$ such that for every $i \in [1, n-1]$, there are at most $k$ edges with one endpoint in $\{v_1, ..., v_i\}$ and another endpoint in $\{v_{i+1}, ..., v_n\}$.

In the context of a multi-tenant CEP system, assuming each non-overlapping CEP tenant uses at most $k$ event types, then the size of the largest component of the adjacency graph $G$ is at most $k$. This, combines with the fact that the degree of each vertex in each component is at most $k$, ensures that $cw(G) \leq k^2$. The running time of the dynamic programming approach in [27] can then be bounded by $O(|\Sigma||Q| M^{k^2})$. Note that this result is pseudo-polynomial, because the running time depends on the value of memory budget $M$ instead of the number of bits it needs to represent it.

H Proof of Lemma 1

Proof. Denote $H$ as the Hessian matrix of (21) representing its second order partial derivatives. In the trivial case where (21) is just a linear function (each query has exactly one event type), $H$ is an all-zero matrix with all-zero eigenvalues, which is trivially concave.

In general, (21) is a nonlinear polynomial (i.e., at least one query has more than one event). Since (21) is a polynomial with positive coefficients and positive exponents, and $\theta_j \geq 0, \forall j$, we know all non-zero second order partial derivatives of (21) are positive, and thus all non-zero entries of $H$ are positive. Denote by $h_{ij}$ the $i$th row, $j$th column entry of $H$, given that $h_{ij} \geq 0, \forall i, j$, we know the trace of the Hessian matrix $tr(H) = h_{11} + h_{22} + ... + h_{|\Sigma||\Sigma|} \geq 0$. From linear algebra, we know that $tr(H)$ equals the sum of the eigenvalues of $H$.

Since $H$ is a Hessian matrix, we know it is symmetric and its eigenvalues are all real. In addition, at least one eigenvalue is non-zero because $H$ is not an all-zero matrix.

We show by contradiction that $H$ must have at least one positive eigenvalue. Suppose this is not the case. Since $H$ has at least one non-zero eigenvalue, all its non-zero eigenvalues have to be negative, which implies that the sum of all eigenvalues are negative, thus we have $tr(H) < 0$. This contradicts with the fact that $tr(H) \geq 0$. Therefore $H$ must have at least one positive eigenvalue, which ensures that $H$ is non-concave.

I Proof of Theorem 6

Proof. We show the hardness of this problem by a reduction from the Clique problem. Given a graph $G = (V, E)$, the decision version of the Clique problem is to determine if there exists a clique of size $k$ in $G$.

From any instance of the Clique problem, we construct an instance of the FMLS problem as follows. We build a bijective function $\phi : V \rightarrow \Sigma$ to map each vertex $v_j \in V$ to an event $\phi(v_j) \in \Sigma$. We set a unit memory consumption for each event ($\lambda_j m_j = 1$), and a unit knapsack capacity. So we get $\sum_{E_j \in \Sigma} \theta_j \leq 1$. Furthermore, given an edge $e_i = (v_i, v_k) \in E$, we build a query $(\phi(v_i), \phi(v_k))$ with unit utility weight.
(n_i w_i = 1). We then essentially have a unit-weight, unit-cost, length-two-query FMLS that corresponds to the graph G. This gives rise to the following bilinear optimization problem subject to a knapsack constraint.

$$\max \sum_{Q_i:(E_l,E_k) \in Q} \pi_l \pi_k$$

s.t. $\sum_{E_j \in \Sigma} \pi_j \leq 1$

$$0 \leq \pi_j \leq 1$$

(46)

Since we are dealing with a maximization problem, and the coefficients of the objective are all non-negative, increasing $\pi_j$ values will not “hurt” the objective. Thus, the knapsack constraint in (46) can be changed into a standard simplex constraint $\sum_{E_j \in \Sigma} \pi_j = 1$ without changing the optimal value of the problem.

Given the FMLS defined above, the decision version of FMLS is to decide if there exists a fractional strategy such that the total utility value is at least $a$. We first show that if there exists a clique of size $k$ in $G$, then the value of the FMLS we construct is at least $\frac{k-1}{2k}$. To show this connection, we use the Motzkin-Straus theorem [37], which states that global maximum over the standard simplex is attained when values are distributed evenly among the largest clique from the graph. So if there is a clique of size $k$ in graph $G$, then the $k$-clique has a total of $\binom{k}{2}$ number of edges. Since each edge produces a value of $\frac{1}{k}^2$, the the optimal value of FMLS is at least $\binom{k}{2} \frac{1}{k}^2 = \frac{k-1}{2k}$.

We now show the other direction, that is if the optimal value FMLS problem we construct is no less than $\frac{k-1}{2k}$, then the graph $G$ has a clique of size no less than $k$. We show this by contradiction. Suppose the size of the maximum clique of $G$ is $c$, $c < k$. Then by the Motzkin-Straus theorem [37] the global maximum of the FMLS we construct is at most $\frac{c-1}{2c}$, which is less than $\frac{k-1}{2k}$ because $c < k$. This contradicts with the fact that the optimal value is no less than $\frac{k-1}{2k}$, therefore, the graph must have a clique of size at least $k$.

We note that by using a reduction from Clique, we have shown that even in the restricted case where the objective function in Equation (21) is a bi-linear function (a summation of quadratic square-free monomials), or in other words each query has exactly two events, FMLS remains NP-hard.

\[\square\]

J Proof of Theorem 7

Proof. Let $B(t) = \{x \in \mathcal{R}^n, \|x\|_2 \leq t\}$ be a ball constraint. We show that the feasible region of FMLS’, denoted by $S$, satisfies $B(t) \subseteq S \subseteq B(1)$, where $t = \min(\min_{E_j}(\frac{\lambda_j}{M}), \frac{1}{\sqrt{\|\Sigma\|}})$.

First, we know that any feasible solution satisfies $\|x\|_1 \leq 1$. It can be shown that $\forall x, \|x\|_2 \leq \|x\|_1$. Thus we know $\|x\|_2 \leq 1$, and $S \subseteq B(1)$.

On the other hand, the ball inside $S$ is limited by the shortest edge of the hyper-rectangle, $\min_{E_j}(\frac{\lambda_j}{M})$, and the largest possible ball inside standard simplex, which has a radius of $\frac{1}{\sqrt{\|\Sigma\|}}$. So we have

$$t = \min(\min_{E_j}(\frac{\lambda_j}{M}), \frac{1}{\sqrt{\|\Sigma\|}})$$

The largest ball inside $S$ is thus $B(t)$.

Authors in [30] show that if a convex feasible region contains a ball constraint, and in addition is bounded by another ball constraint, the polynomial program can be approximated with a relative approximation ratio that is a function of the degree of the polynomial, and the radius of the ball constraints, namely the ratio is $1 - \frac{d+1}{2} \cdot (d+1)^{-\frac{4d}{2d+1}}$. Given that $d$ is assumed to be a fixed constant, thus our result in the theorem.

\[\square\]
Furthermore, we use and a set of box constraints. homogeneous polynomial with non-negative coefficients, defined over the intersection of a standard simplex on the grid. points, hence the algorithm performs at most over the uniform grid defined as $0 \leq x \leq 1$. Since we know that the CEP queries are regular, that is, there is no repeated events in the same query, and each query has the same number of events, we can conclude that the objective function (22) is a multi-linear, homogeneous function.

In addition, since we know that the CEP queries are regular, that is, there is no repeated events in the same query, and each query has the same number of events, we can conclude that the objective function (22) is a multi-linear, homogeneous function.

In [39], in order to obtain an approximation on the simplex, $\Delta_{|\Sigma|}$, an exhaustive search is performed over the uniform grid defined as $\Delta_{|\Sigma|}(k) := \{ x \in \Delta_{|\Sigma|} | kx \in \mathbb{Z}^+ \}$. This uniform grid contains $|\Sigma|^{1+k-1}$ points, hence the algorithm performs at most $|\Sigma|^{1+k-1}$ function evaluations to determine an optimal point on the grid.

In the following we extend the result in [39], by showing that a similar algorithm exists for multi-linear, homogeneous polynomial with non-negative coefficients, defined over the intersection of a standard simplex and a set of box constraints.

Denote by $d$ the degree of the objective function (or, equivalently, the number of events in each query). Furthermore, we use $\tau_j$ to denote $\frac{\lambda_jm_j}{M}$ in (24) for succinctness. Note here, $\tau_j \leq 1$, $\forall j$, because otherwise given the simplex constraint (23) and the fact that all $x_j$ are positive, we can replace $\tau_j$ with $\tau_j > 1$ in $0 \leq x_j \leq \tau_j$ as $0 \leq x_j \leq 1$.

Similar to [39], first we define $p \in \Delta_n$ be a vector in the standard simplex. Denote by $\zeta(p)$ a discrete random variable distributed as $Pr\{\zeta(p) = i\} = p(i)$.

As in [39], we define a random process $x$ as $x_0(p) = 0 \in R^n$.
In addition, we define a “scaled-down” version $x'$ of $x$ to reflect the box constraint (24).

$$x^{n(i)}_k(p) = x^{(i)}_k(p) \tau_i$$

Note that for all $k$, any realization $x^{n(i)}_k(p)$ lies on the uniform grid $\Delta_{|\Sigma|}(k)$. Since $x'$ is a linearly scaled version of $x$, we can use results in Equation (2.5) and (2.6) of [39], to obtain mean, variance, and covariance of $x'$ as follows.

$$\mu^{n(i)}_k(p) = E[x^{n(i)}_k(p)] = kp^{(i)} \tau_i$$

$$E \left[ (x^{n(i)}_k(p) - \mu^{n(i)}_k) ^2 \right] = kp^{(i)} (1 - p^{(i)}) \tau_i^2$$

$$E \left[ (x^{n(i)}_k(p) - \mu^{n(i)}_k) \cdot (x^{n(j)}_k(p) - \mu^{n(j)}_k) \right] = -kp^{(i)} p^{(j)} \tau_i \tau_j$$

Similarly, we can reuse results of $E[x_k^n(p)]$ in Lemma 2 of [39]. In particular, we have

$$E_k^n(p) = E[x_k^n(p)] = E[x_k^0(p)] \tau^n$$

(48)

Now we need to show that the approximation result in Lemma 3 of [39] still holds. To be consistent with the minimization problem studied in [39], we get the inverse of (22), and consider the corresponding minimization version of the IMLS'.

Let the objective function under consideration be $f(x) := \sum_{\alpha \in A} f_\alpha x^\alpha$, where $\alpha$ is a multi-index in $\mathbb{Z}_+^{|\Sigma|}$. Also, say $f^*_{k}$ is the minimum value attained among all grid points as defined by the random walk $x'(p)$, where $p$ is set to the optimal vector in our feasible region (23) and (24) that minimizes $f$. We know $f^*_{k}$ is no more than the expectation of the function value over the random walk.

$$f^*_{k} \leq E[f\left(\frac{1}{k}x'_k(p)\right)]$$

(49)

Since $f$ is degree $d$ homogeneous, we have

$$E[f\left(\frac{1}{k}x'_k(p)\right)] = \sum_{\alpha \in A} f_\alpha E[\frac{1}{k^d}x_k^\alpha(p)]$$

(50)

Utilizing a result from Lemma 2 of [39] and Equation (48), we get

$$\sum_{\alpha \in A} f_\alpha E[\frac{1}{k^d}x_k^\alpha(p)] = \frac{k!}{(k-d)!k^2} \sum_{\alpha \in A} f_\alpha p^\alpha \tau^\alpha$$

(51)

Furthermore, let $f'_*$ be the minimum value attained in the feasible region (that is, at $p$), we know

$$f'_* = \sum_{\alpha \in A} f_\alpha p^\alpha$$

(52)

By assumption, $\tau_j$s are constant hence there exists a constant $\beta$ such that $\tau^\alpha \geq \beta$ for all $\alpha$. Combining (49), (50), (51), (52), we conclude that

$$f^*_{k} \leq \beta \cdot \frac{k!}{(k-d)!k^2} f'_*$$

Therefore, we get a constant factor approximation as

$$f^*_{k} - f'_* \leq \left(1 - \beta \frac{k!}{(k-d)!k^2}\right) (-f'_*)$$

30
Note that when scaling down random walk from $x(p)$ to $x'(p)$, we essentially move grid points beneath the simplex (that is, into the region between the simplex, and axis planes). Technically, these points are not in feasible region. However, we notice that in IMLS’, all coefficients in the objective function are non-negative. Thus, in the minimization problem corresponding to IMLS’, all coefficients are non-positive. We can then essentially “move up” grid points beneath the simplex onto the simplex plane by increasing values. Since all coefficients are non-positive, this will not hurt our objective function, we will get at least as good a value as $f^*_k$.

Thus, we will be able to find feasible points in the simplex using the minimum value among all grid points, and obtain a constant approximation factor.

**M Proof of Theorem 11**

**Proof.** We first obtain the following formulation IDLS$^m$ equivalent to IDLS. Again we use $\hat{y}_i = 1 - y_i$ be the complement of $y_i$, which indicates whether query $Q_i$ is un-selected.

$$(IDLS^m) \begin{array}{ll} \min & \sum_{Q_i \in Q} n_i w_i \hat{y}_i \\ \text{s.t.} & \sum_{E_j \in \Sigma} \lambda_j m_j x_j \leq M \\ & \sum_{Q_i \in Q} n_i c_i (1 - \hat{y}_i) \leq C \\ & \hat{y}_i \geq 1 - x_j, \forall E_j \in Q_i \\ & \hat{y}_i, x_j \in \{0, 1\} \end{array} (53)$$

Since we have shown in Theorem 3 that given a parameter $\frac{1}{\tau}$, solving the LP-relaxed version of IDLS$^m$ and then make selection decision based on the rounding of fractional solutions $x_j^*$ and $\hat{y}_i^*$ ensures that memory consumption does not exceed $\frac{M}{1 - \tau}$ and utility does not exceed $\frac{l^*}{1 - \tau}$, what left to be shown is that CPU consumption does not exceed $\frac{C}{1 - \tau}$.

We again divide queries $Q$ into the promising set of queries $Q^a = \{Q_i \in Q | \hat{y}_i^* \leq \tau \}$ and the unpromising set $Q^r = \{Q_i \in Q | \hat{y}_i^* > \tau \}$.

For the set of selected queries, $Q^a$, from (53) we know

$$\sum_{Q_i \in Q^a} n_i c_i (1 - \hat{y}_i) \leq \sum_{Q_i \in Q} n_i c_i (1 - \hat{y}_i) \leq C \quad (54)$$

Since $\forall Q_i \in Q^a$, $\hat{y}_i \leq \tau$,

$$\sum_{Q_i \in Q^a} n_i c_i (1 - \hat{y}_i) \leq \sum_{Q_i \in Q^a} n_i c_i (1 - \hat{y}_i)$$

Combining with (54) we get

$$\sum_{Q_i \in Q^a} n_i c_i (1 - \tau) \leq C$$

Because $\sum_{Q_i \in Q^a} n_i c_i$ is the total CPU consumption, we can obtain that

$$\sum_{Q_i \in Q^a} n_i c_i \leq \frac{C}{1 - \tau}$$

This bounds the CPU consumption and completes our proof.

□
N Proof of Theorem 12

Proof. We define the following problem variant IDLS$^p$.

\[(\text{IDLS}^p) \max \sum_{Q_i \in Q} n_i w_i y_i \]
\[
\text{s.t.} \sum_{Q_i} y_i \left( \sum_{E_j \in Q_i} \lambda_j m_j \right) \leq p M \\
\sum_{Q_i} n_i c_i y_i \leq C \\
y_i \leq \prod_{E_j \in Q_i} x_j \\
x_j, y_i \in \{0, 1\}
\]

(55)

Because we know each event is shared by at most $p$ number of queries, so for any solution $y_i, x_j$ to IDLS we have

\[
\sum_{Q_i} y_i \left( \sum_{E_j \in Q_i} \lambda_j m_j \right) \leq p \sum_{E_j \in \Sigma} \lambda_j m_j x_j = p M
\]

(56)

This implies that a solution feasible to IDLS must be feasible to IDLS$^p$, because from ILDS to IDLS$^p$ only (55) is modified and we have ensured that any solution $y_i, x_j$ to IDLS must also satisfy (55). Let $OPT$ and $OPT^p$ be the optimal value of IDLS and IDLS$^p$, we can guarantee

\[
OPT^p \geq OPT
\]

(57)

Further introduce the problem IDLS$^{ns}$

\[(\text{IDLS}^{ns}) \max \sum_{Q_i \in Q} n_i w_i y_i \]
\[
\text{s.t.} \sum_{Q_i} y_i \left( \sum_{E_j \in Q_i} \lambda_j m_j \right) \leq M \\
\sum_{Q_i} n_i c_i y_i \leq C \\
y_i \leq \prod_{E_j \in Q_i} x_j \\
x_j, y_i \in \{0, 1\}
\]

(58)

We show that there exists a solution to IDLS$^{ns}$ that has a value of $\frac{1}{p} OPT^p$. We construct this as follows. First initialize the solution to IDLS$^{ns}$ by setting $y_i^{ns} = 0, \forall Q_i$. Given the optimal solution $y_i^{ps}, x_j^{ps}$ to IDLS$^p$, we order non-zero $y_i^{ps}$s by $\frac{n_i w_i}{\sum_{E_j \in Q_i} \lambda_j m_j}$ descendingly. We can then iterate through non-zero $y_i^{ps}$s using that order, and set the corresponding $y_i^{ns}$ to 1, and stop when picking the next item violates the budget requirement, so that $y_i^{ns}$ respects constraint (58).
It apparently also satisfies other constraints of IDLS\textsuperscript{ns}s because they are not changed from IDLS\textsuperscript{p} to IDLS\textsuperscript{ns}s.

Let this solution to IDLS\textsuperscript{ns}s has value \( v_{\text{ns}} \). We know

\[
v_{\text{ns}} \frac{d}{1 - \frac{1}{f}} \geq \text{OPT}^p
\]

This is the case because first, given that \( f = \frac{M}{\max_i \sum_{E_j \in Q_i} \lambda_j m_j} \), using our construction, the slack of \( M - \sum_i y_{\text{ns}}^i (\sum_{E_j \in Q_i} \lambda_j m_j) \) is at most \( \frac{M}{f} \), or we have used at least \( M - \frac{M}{f} \) memory. In addition, since we pick \( y_{\text{ns}}^i \) by their value to weight ratio \( \frac{n_i w_i}{\sum_{E_j \in Q_i} \lambda_j m_j} \), increasing memory budget from \( y_{i}^\text{ns} \) to \( y_{i}^p \) by a factor of \( p \) increases value by no more than a factor of \( p(\frac{1}{1 - \frac{1}{f}}) \).

Given \( \text{OPT}^\text{ns} \geq v_{\text{ns}} \), we have

\[
\text{OPT}^\text{ns} \geq \frac{1 - \frac{1}{f}}{p} \text{OPT}^p
\]

Combining this with (57) we know

\[
\text{OPT}^\text{ns} \geq \frac{1 - \frac{1}{f}}{p} \text{OPT}
\]

Notice that ILDS\textsuperscript{ns}s is the well-studied multi-dimensional knapsack problem with dimensionality equals 2. Such problems can be solved in pseudo-polynomial-time to obtain the optimal value \( \text{OPT}^\text{ns} \). Since the solution so produced is guaranteed to be within \( \frac{1 - \frac{1}{f}}{p} \text{OPT} \), our proof is thus complete. \( \square \)