Meta-Stable Brane Configurations with Multiple NS5-Branes

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Abstract

Starting from an $\mathcal{N}=1$ supersymmetric electric gauge theory with the multiple product gauge group and the bifundamentals, we apply Seiberg dual to each gauge group, obtain the $\mathcal{N}=1$ supersymmetric dual magnetic gauge theories with dual matters including the gauge singlets. Then we describe the intersecting brane configurations, where there are NS-branes and D4-branes (and anti D4-branes), of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of this gauge theory.

We also discuss the case where the orientifold 4-planes are added into the above brane configuration. Next, by adding an orientifold 6-plane, we apply to an $\mathcal{N}=1$ supersymmetric electric gauge theory with the multiple product gauge group (where a single symplectic or orthogonal gauge group is present) and the bifundamentals. Finally, we describe the other cases where the orientifold 6-plane intersects with NS-brane.
1 Introduction

It is known that the dynamical supersymmetry breaking in meta-stable vacua \( \mathcal{N} = 1 \) SQCD with massive fundamental flavors (for recent developments on supersymmetry breaking, see also the review paper [2]). Due to the extra mass term for quarks in the magnetic superpotential, not all F-term equations can be satisfied and the supersymmetry is broken. The meta-stable brane configurations of type IIA string theory corresponding to \( \mathcal{N} = 1 \) SQCD with massive fundamental flavors have been found in [3, 4, 5] (for the brane dynamics and supersymmetric gauge theory, see also the review paper [6]). There is also the meta-stable brane configuration [7] corresponding to the gauge theory where there exists an extra adjoint matter as well as fundamentals.

Giveon and Kutasov [8] have found the type IIA brane configuration consisting of three NS-branes, D4-branes and anti-D4-branes (\( \overline{D4} \)-branes). The meta-stable vacua of [1] occur in some region of parameter space when the D4-branes and \( \overline{D4} \)-branes can decay and the geometric misalignment of flavor D4-branes arises. The mass term in the magnetic superpotential corresponds to the relative displacement of two NS5’-branes along the (45) directions and the dual quarks can be represented by the bifundamentals of product gauge group.

Adding an orientifold 4-plane (O4-plane) only to the brane configuration of [8] implies that the gauge group is a product of a symplectic group and an orthogonal group and the geometric misalignment of flavor D4-branes [9] leads to the brane configuration of [10]. The \( \mathcal{N} = 1 \) product gauge group theory [11, 12] is realized by three NS-branes, two kinds of D4-branes, and two kinds of D6-branes [13]. When an orientifold 6-plane (O6-plane) is added into the brane configuration of [8], the gauge group is a product of two unitary groups with extra matters as well as bifundamentals and the type IIA brane configuration consists of five NS-branes, D4-branes, \( \overline{D4} \)-branes and O6-plane. Similarly, the geometric misalignment of flavor D4-branes [9] leads to the brane configurations of [14] or [15] depending on the O6-plane charge.

One can generalize the work of [8] further by adding more NS-branes to that brane configuration. For the \( \mathcal{N} = 1 \) triple product group gauge theory, the supersymmetric electric brane configuration consists of four NS-branes, three kinds of D4-branes, and three kinds of D6-branes [13, 16]. Then the triple product gauge group theory with bifundamentals only can be realized by four NS-branes, three kinds of D4-branes and \( \overline{D4} \)-branes [17]. The meta-stable vacua of [18] occur in some region of parameter space when the D4-branes and \( \overline{D4} \)-branes can decay and the geometric misalignment of flavor D4-branes arises.

Now adding an orientifold 4-plane only to the brane configuration of [17] leads to the
fact that the gauge group is a triple product of a symplectic group and an orthogonal group
alternatively and the geometric misalignment of flavor D4-branes [17] reduces to the brane
configuration of [19]. One can add an orientifold 6-plane into the brane configuration of
[13] [16] together with extra outer NS-branes. Then one of the type IIA brane configurations
consists of six NS-branes, D4-branes, \( D_4 \)-branes and O6-plane. The geometric misalignment
of flavor D4-branes [17] leads to the brane configurations of [18]. Different O6-plane charge
will give rise to other brane configuration. When there are seven NS-branes, D4-branes, \( D_4 \)-
branes and O6-plane, the geometric misalignment of flavor D4-branes [20] gives rise to the
brane configurations [21].

Recently, from the \( \mathcal{N} = 1 \) triple product group gauge theory with fundamentals and
bifundamentals, the meta-stable brane configurations consisting of four NS-branes, three kinds
of D4-branes, and three kinds of D6-branes are found [22] and its generalization to multiple
product gauge groups also is discussed in [22] [23].

In this paper, along the line of [8] [9] [17] [20], we present the intersecting brane configurations
of type IIA string theory corresponding to the new meta-stable nonsupersymmetric meta-
stable vacua in four dimensional \( \mathcal{N} = 1 \) multiple product gauge theory with matters. For the
\( \mathcal{N} = 1 \) multiple product gauge group theory with bifundamentals, the supersymmetric electric
brane configuration in type IIA string theory consists of \( (n + 1) \) NS-branes, and \( n \) kinds of
D4-branes (and anti D4-branes) and the gauge group is a product of \( n \) unitary groups with
bifundamentals. For a given supersymmetric electric gauge theory which does not have any
superpotential, one takes both the mass deformation for bifundamentals and its Seiberg dual.
Then we construct the meta-stable brane configurations in type IIA string theory. Adding
the O4-plane or O6-plane to these brane configurations are described.

In section 2, we describe the type IIA brane configuration corresponding to the electric
theory based on the \( \mathcal{N} = 1 \) gauge theory with gauge group \( \prod_{i=1}^{n} SU(N_{c,i}) \) and bifundamentals,
and deform this theory by adding the mass term for the bifundamentals for each gauge group.
Then we construct the Seiberg dual magnetic theories for each gauge group with corresponding
dual matters as well as additional gauge singlets. After that, the nonsupersymmetric brane
configurations are found by recombination and splitting for the flavor D4-branes.

In section 3, we consider the type IIA brane configuration corresponding to the electric
theory based on the \( \mathcal{N} = 1 \) \( \prod_{i=1}^{n-2} Sp(N_{c,i}) \times SO(2N_{c,i+1}) \times Sp(N_{c,n}) \) or \( \prod_{i=1}^{n-1} Sp(N_{c,i}) \times SO(2N_{c,i+1}) \) gauge theory and bifundamentals, and deform this theory by adding the mass
terms for the bifundamentals for each gauge group. We present the Seiberg dual magnetic
theories for each gauge group with corresponding dual matters as well as additional gauge
singlets. The nonsupersymmetric brane configurations are found. When the orientifold 4-
plane charge is reversed, it is straightforward to proceed similarly.

In section 4, we study the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1\ Sp(N_{c,1}) \times \prod_{i=2}^{n} SU(N_{c,i})$ gauge theory with bifundamentals, and deform this theory by adding the mass term for the quarks for each gauge group. Explicitly we construct the Seiberg dual magnetic theories for each gauge group factor with corresponding dual matters as well as extra gauge singlets and the nonsupersymmetric brane configurations are found.

In section 5, we explain the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1\ SO(N_{c,1}) \times \prod_{i=2}^{n} SU(N_{c,i})$ gauge theory with bifundamentals, and deform this theory by adding the mass term for the quarks for each gauge group. After that we describe the Seiberg dual magnetic theories for each gauge group factor with corresponding dual matters as well as extra gauge singlets. The nonsupersymmetric brane configurations are found from the magnetic brane configurations.

In section 6, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1\ \prod_{i=1}^{n} SU(N_{c,i})$ gauge theory with bifundamentals, a symmetric flavor and a conjugate symmetric flavor for $SU(N_{c,1})$, and deform this theory by adding the mass term for the quarks for each gauge group. Then we construct the Seiberg dual magnetic theories for each gauge group with corresponding dual matters as well as additional gauge singlets. After that, the nonsupersymmetric brane configurations are found by recombination and splitting for the flavor D4-branes.

In section 7, we consider the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1\ \prod_{i=1}^{n} SU(N_{c,i})$ gauge theory with bifundamentals, eight-fundamentals, an antisymmetric flavor and a conjugate symmetric flavor for $SU(N_{c,1})$, and deform this theory by adding the mass terms for the quarks for each gauge group. We present the Seiberg dual magnetic theories for each gauge group with corresponding dual matters as well as additional gauge singlets. The nonsupersymmetric brane configurations are found.

Finally, in section 8, we summarize what we have found in this paper.

There exist some related works \[24\]-[59] on the meta-stable vacua in different directions.

2 Meta-stable brane configurations with $(n+1)$ NS-branes

The type IIA brane configuration \[13, 16\] corresponding to $\mathcal{N} = 1$ supersymmetric electric gauge theory(see also \[22\]) with gauge group

$$SU(N_{c,1}) \times \cdots \times SU(N_{c,n})$$
and with the \((n-1)\) bifundamentals \(F_i\) charged under \((1, \ldots, 1, \Box_i, \Box_{i+1}, 1, \ldots, 1)_i\) and their complex conjugate fields \(\tilde{F}_i\) charged \((1, \ldots, 1, \Box_i, \Box_{i+1}, 1, \ldots, 1)_i\) where \(i = 1, 2, \ldots, (n-1)\) can be described by the \(NS5'_1\)-brane(012389), the \(NS5_2\)-brane(012345), \(\cdots\), the \(NS5_{n+1}\)-brane for odd number of gauge groups(or the \(NS5'_{n+1}\)-brane for even number of gauge groups), \(N_{c,1}, N_{c,2}, \ldots, N_{c,n}\)-color D4-branes(01236). See the Figure 1 for the details on the brane configuration.

Let us place the \(NS5'_1\)-brane at the origin \(x^6 = 0\) and denote the \(x^6\) coordinates for the \(NS5_2\)-brane, \(\cdots\), the \(NS5_{n+1}\)-brane for odd \(n\)(or the \(NS5'_{n+1}\)-brane for even \(n\)) are given by \(x^6 = y_1, y_1 + y_2, \cdots, \sum_{j=1}^{n-1} y_j + y_n\) respectively. The \(N_{c,1}\) D4-branes are suspended between the \(NS5'_1\)-brane and the \(NS5_2\)-brane, the \(N_{c,2}\) D4-branes are suspending between the \(NS5_2\)-brane and the \(NS5'_3\)-brane, \(\cdots\) and the \(N_{c,n}\) D4-branes are suspended between the \(NS5'_n\)-brane and the \(NS5_{n+1}\)-brane for odd \(n\)(or between the \(NS5_n\)-brane and the \(NS5'_{n+1}\)-brane for even \(n\)). The fields \(F_i\) and \(\tilde{F}_i\) correspond to 4-4 strings connecting the \(N_{c,i}\)-color D4-branes with \(N_{c,i+1}\)-color D4-branes. We draw this \(\mathcal{N} = 1\) supersymmetric electric brane configuration in Figure 1A(1B) when \(n\) is odd(even) for the vanishing mass for the fields \(F_i\) and \(\tilde{F}_i\). The gauge couplings of \(SU(N_{c,1}), SU(N_{c,2}), \cdots, SU(N_{c,n})\) are given by a string coupling constant \(g_s\), a string scale \(\ell_s\) and the \(x^6\) coordinates \(y_i\) for \(n\) NS-branes through

\[
g_1^2 = \frac{g_s \ell_s}{y_1}, \quad g_2^2 = \frac{g_s \ell_s}{y_2}, \quad \cdots, \quad g_n^2 = \frac{g_s \ell_s}{y_n}.
\]

Let us deform the theory by Figure 1A. Displacing the two \(NS5'\)-branes, \(NS5'_{i}\)-brane and \(NS5'_{i+2}\)-brane, relative each other in the

\[v \equiv x^4 + ix^5\]
direction, characterized by \((\Delta x)_{i+1}\), corresponds to turning on a quadratic mass-deformed superpotential for the fields \(F_i\) and \(\tilde{F}_i\) as follows:

\[
W_{\text{elec}} = m_{i+1} F_i \tilde{F}_i (\equiv m_{i+1} \Phi_{i+1}), \quad \text{when } i \text{ is odd}
\]  

(2.1)

where the \(i\)-th gauge group indices in \(F_i\) and \(\tilde{F}_i\) are contracted, each \((i+1)\)-th gauge group index in them is encoded in \(\Phi_{i+1}\) and the mass \(m_{i+1}\) is given by

\[
m_{i+1} = \frac{(\Delta x)_{i+1}}{\ell_s^2}.
\]

(2.2)

The gauge-singlet \(\Phi_{i+1}\) for the \(i\)-th gauge group which was in the adjoint representation for the \((i+1)\)-th gauge group can be represented by

\[
(1_1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, 1_{i+2}, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)
\]

under the gauge group where the gauge group is broken from \(SU(N_{c,i+1})\) to \(SU(N_{c,i+1} - N_{c,i+2})\). Then the \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})\) matrix. The \(NS5'_{i+2}\)-brane together with \((N_{c,i+1} - N_{c,i+2})\)-color D4-branes is moving to the \(+v\) direction for fixed other branes during this mass deformation. In other words, the \(N_{c,i+2}\) D4-branes among \(N_{c,i+1}\) D4-branes are not participating in the mass deformation. Then the \(x^5\) coordinate \((\equiv x)\) of \(NS5'_i\)-brane is equal to zero while the \(x^5\) coordinate of \(NS5'_{i+2}\)-brane is given by \((\Delta x)_{i+1}\). Giving an expectation value to the meson field \(\Phi_{i+1}\) corresponds to recombination of \(N_{c,i}\) and \(N_{c,i+1}\)-color D4-branes, which will become \(N_{c,i}\) or \(N_{c,i+1}\)-color D4-branes in Figure 1A such that they are suspended between the \(NS5'_i\)-brane and the \(NS5'_{i+2}\)-brane and pushing them into the \(w \equiv x^8 + ix^9\)

direction. We assume that the number of colors satisfies \(N_{c,i+1} \geq N_{c,i} - N_{c,i-1} \geq N_{c,i+2}\). Now we draw this brane configuration in Figure 2A for nonvanishing mass for the fields \(F_i\) and \(\tilde{F}_i\).

Let us deform the theory by Figure 1B. Displacing the two \(NS5'\)-branes, the \(NS5'_{i-1}\)-brane and the \(NS5'_{i+1}\)-brane, relative each other in the \(v\) direction, charaterized by \((\Delta x)_{i-1}\), corresponds to turning on a quadratic mass-deformed superpotential for the fields \(F_{i-1}\) and \(\tilde{F}_{i-1}\) as follows:

\[
W_{\text{elec}} = m_{i-1} F_{i-1} \tilde{F}_{i-1} (\equiv m_{i-1} \Phi_{i-1}), \quad \text{when } i \text{ is even}
\]  

(2.3)

where the \(i\)-th gauge group indices in \(F_{i-1}\) and \(\tilde{F}_{i-1}\) are contracted, each \((i-1)\)-th gauge group index in them is encoded in \(\Phi_{i-1}\) and the mass \(m_{i-1}\) is given by

\[
m_{i-1} = \frac{(\Delta x)_{i-1}}{\ell_s^2}.
\]

(2.4)
The gauge-singlet $\Phi_{i-1}$ for the $i$-th gauge group which was in the adjoint representation for the $(i-1)$-th gauge group is in the representation

$$(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1_i, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)$$

under the gauge group where the gauge group is broken from $SU(N_{c,i-1})$ to $SU(N_{c,i-1} - N_{c,i-2})$. Then the $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix. The $NS5'_{i-1}$-brane together with $(N_{c,i-1} - N_{c,i-2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation. In other words, the $N_{c,i-2}$ D4-branes among $N_{c,i-1}$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate ($\equiv x$) of $NS5'_{i-1}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i-1}$-brane is given by $(\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i-1}$ corresponds to recombination of $N_{c,i-1}$- and $N_{c,i-2}$-color D4-branes, which will become $N_{c,i-1}$- or $N_{c,i-2}$-color D4-branes in Figure 1B such that they are suspended between the $NS5'_{i-1}$-brane and the $NS5'_{i+1}$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i-1} \geq N_{c,i} - N_{c,i+1} \geq N_{c,i-2}$.

Now we draw this brane configuration in Figure 2B for nonvanishing mass for the fields $F_{i-1}$.

2.1 $\mathcal{N} = 1 \ SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i}) \times \cdots \times SU(N_{c,n})$ magnetic theory

Let us first consider the Seiberg dual for the middle gauge group factor. There are two magnetic duals depending on whether the gauge group factor occurs at odd chain or even chain.
2.1.1 When the dual gauge group occurs at odd chain

Starting from Figure 1A, moving the NS5\(_{i+2}\)-brane with \((N_{c,i+1} - N_{c,i+2})\) D4-branes to the \(+v\) direction leading to Figure 2A, and interchanging the NS5\(_{i}'\)-brane and the NS5\(_{i+1}\)-brane, one obtains the Figure 3A.

![Figure 3: The \(N = 1\) magnetic brane configuration for the gauge group containing \(SU(\tilde{N}_{c,i} = N_{c,i-1} + N_{c,i+1} - N_{c,i})\) where \(i\) is odd, corresponding to Figure 2A with D4- and \(\overline{D4}\)-branes (3A) and with a misalignment between D4-branes (3B) when the two NS5\(_i\)'-branes are close to each other. The number of tilted D4-branes in 3B can be written as \(N_{c,i} - N_{c,i-1} - N_{c,i+2} = N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i}\).]

Before arriving at the Figure 3A, there exists an intermediate step where the \((N_{c,i+1} + N_{c,i-1} - N_{c,i})\) D4-branes are connecting between the NS5\(_{i+1}\)-brane and the NS5\(_i\)'-brane, \((N_{c,i+1} - N_{c,i+2})\) D4-branes connecting between the NS5\(_i\)'-brane and NS5\(_{i+2}\)-brane, and \(N_{c,i+2}\) D4-branes between the NS5\(_i\)'-brane and the NS5\(_{i+3}\)-brane. By introducing \(-N_{c,i+2}\) D4-branes and \(-N_{c,i+2}\) anti-D4-branes between the NS5\(_{i+1}\)-brane and NS5\(_i\)'-brane, reconnecting the former with the \(N_{c,i+1}\) D4-branes connecting between NS5\(_{i+1}\)-brane and the NS5\(_i\)'-brane (therefore \(N_{c,i+1} - N_{c,i+2}\) D4-branes) and moving those combined \((N_{c,i+1} - N_{c,i+2})\) D4-branes to \(+v\)-direction, one gets the final Figure 3A where we are left with \((N_{c,i} - N_{c,i+2} - N_{c,i-1})\) anti-D4-branes between the NS5\(_{i+1}\)-brane and NS5\(_i\)'-brane.

The dual gauge group from Seiberg dual relation is given by

\[
SU(N_{c,1}) \times \cdots \times SU(N_{c,i-1}) \times SU(\tilde{N}_{c,i}) \times SU(N_{c,i+1}) \times \cdots \times SU(N_{c,n}) \tag{2.5}
\]

with \(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}\) where the matter contents are the bifundamentals \(f_i\) in \((1_1, \cdots, 1, \Box_i, \Box_{i+1}, 1, \cdots, 1_n)\), \(\Box_i\) in the representation \((1_1, \cdots, 1, \Box_i, \Box_{i+1}, 1, \cdots, 1_n)\) in addition to \((n-2)\) bifundamentals \(F_j\) and \(\tilde{F}_j\), \(j = 1, 2, \cdots, i-1, i+1, \cdots, n\) and the gauge singlet \(\Phi_{i+1}\) for the \(i\)-th dual gauge group in the adjoint representation for the \((i+1)\)-th dual gauge group, i.e., \((1_1, \cdots, 1, (N_{c,i+1} - N_{c,i+2})^2 - 1, 1, 1_{i+2}, \cdots, 1_n)\) plus a singlet under the dual gauge group \((2.4)\) where the gauge group is broken from \(SU(N_{c,i+1})\) to \(SU(N_{c,i+1} - N_{c,i+2})\).
When two NS5'-branes in Figure 3A are close to each other, then it leads to Figure 3B by realizing that the number of \((N_{c,i+1} - N_{c,i+2})\) D4-branes connecting between \(NS_{5i+1}\)-brane and \(NS_{5'i+2}\)-brane can be rewritten as \((N_{c,i} - N_{c,i+2} - N_{c,i-1})\) plus \(\tilde{N}_{c,i}\). If we ignore all the D4-branes and NS-branes located at the left hand side of \(NS_{i+1}\)-brane and at the right hand side of \(NS_{i}'\)-brane from Figure 3, then the brane configuration becomes the one in \([8]\). The Figure 2 of \([17]\) is contained in the Figure 3. In particular, the brane configuration from the \(NS_{5i+1}\)-brane to the \(NS_{5i+3}\)-brane is exactly same as the one of \([17]\).

The cubic superpotential with the mass term \((2.1)\) with \((2.2)\) in the dual theory is given by

\[
W_{dual} = \Phi_{i+1} f_i \tilde{f}_i + m_{i+1} \text{tr} \Phi_{i+1}. \tag{2.6}
\]

Here the magnetic fields \(f_i\) and \(\tilde{f}_i\) correspond to 4-4 strings connecting the \(\tilde{N}_{c,i}\)-color D4-branes (that are connecting between the \(NS_{5i+1}\)-brane and the \(NS_{5'i+2}\)-brane in Figure 3B) with \(N_{c,i+1}\)-flavor D4-branes (that are a combination of three different D4-branes in Figure 3B). Among these \(N_{c,i+1}\)-flavor D4-branes, only the strings ending on the upper \((N_{c,i+1} - N_{c,i} + N_{c,i-1})\) D4-branes and on the tilted middle \((N_{c,i} - N_{c,i+2} - N_{c,i-1})\) D4-branes in Figure 3B enter the cubic superpotential term. Although the \((N_{c,i+1} - N_{c,i+2})\) D4-branes in Figure 3A cannot move any directions for fixed other branes, the tilted \((N_{c,i} - N_{c,i+2} - N_{c,i-1})\)-flavor D4-branes can move \(w\) direction in Figure 3B. The remaining upper \(\tilde{N}_{c,i}\) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
N_{c,i+1} - N_{c,i+2} = (N_{c,i} - N_{c,i-1} - N_{c,i+2}) + \tilde{N}_{c,i}.
\]

The brane configuration for zero mass for the bifundamental \(F_1\) and \(\tilde{F}_1\), which has only a cubic superpotential from \((2.6)\), can be obtained from Figure 3A by moving the upper \(NS_{5'i+2}\)-brane together with \((N_{c,i+1} - N_{c,i+2})\) color D4-branes into the origin \(v = 0\). Then the number of dual colors for D4-branes becomes \(\tilde{N}_{c,i}\) between \(NS_{5i+1}\)-brane and \(NS_{5'i}\)-brane and \(N_{c,i+1}\) between two \(NS_{5}'\)-branes as well as \(N_{c,i-1}\) D4-branes between \(NS_{5i-1}\)-brane and \(NS_{5i+1}\)-brane. Or starting from Figure 1A and moving the \(NS_{5i+1}\)-brane to the left all the way past the \(NS_{5}'\)-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 3A is stable as long as the distance \((\Delta x)_{i+1}\) between the upper \(NS_{5}'\)-brane and the lower \(NS_{5}'\)-brane is large, as in \([8]\). If they are close to each

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1In general, there are also the extra terms in the superpotential \(\Phi' f_{i-1} f_i + \Phi'' \tilde{f}_{i-1} \tilde{f}_i + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1}\) where we define \(\Phi' = F_i F_{i-1}\) and \(\Phi'' = \tilde{F}_i \tilde{F}_{i-1}\), coming from different bifundamentals. However, the F-term conditions, \(\Phi' f_i + \Phi_{i-1} \tilde{f}_{i-1} = 0 = \Phi'' \tilde{f}_i + \Phi_{i-1} f_{i-1}\) lead to \(\Phi' = \Phi'' = f_{i-1} = \tilde{f}_{i-1} = 0\). Therefore, these extra terms do not contribute to the one loop computation up to quadratic order.
other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 3B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.5) and superpotential (2.6). The \((N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i})\) flavor D4-branes of straight brane configuration of Figure 3B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and \(NS_{5,i+1}\)-brane from the DBI action, by following the procedure of [8], as long as the distance \(y_{i+2}\) corresponding to the \(NS_{5,i+3}\)-brane goes to the infinity because the presence of an extra \(NS_{5,i+3}\)-brane does not affect the DBI action. For the finite and small \(y_{i+2}\), the careful analysis for DBI action is needed in order to obtain the bending curve connecting two \(NS_{5}'\)-branes.

When the upper \(NS_{5}'\)-brane (or \(NS_{5}'_{i+2}\)-brane) is replaced by coincident \((N_{c,i+1} - N_{c,i+2})\) D6-branes and the \(NS_{5,i+3}\) is rotated by an angle \(\frac{x}{2}\) in the \((v,w)\) plane in Figure 3B, this brane configuration reduces to the one found in [22] where the gauge group was given by \(\cdots \times SU(n_{c,i-1}) \times SU(n_{f,i} + n_{c,i+1} + n_{c,i-1} - n_{c,i}) \times SU(n_{c,i+1}) \times \cdots\) with \(n_{f,i}\) multiplets, \(\tilde{n}_{f,i}\) multiplets, bifundamentals, and gauge singlets. In particular, the Figure 3B of [22] with vanishing flavors \(Q\) and \(Q''\) is contained in this modified Figure 3B running from the \(NS_{5,i-1}\)-brane to the \(NS_{5,i+3}\)-brane. Then the present number \((N_{c,i+1} - N_{c,i+2})\) corresponds to the \(n_{f,i}\), the number \(N_{c,i}\) corresponds to \(n_{c,i}\), the number \(N_{c,i-1}\) corresponds to \(n_{c,i-1}\), and the number \(N_{c,i+2}\) corresponds to the \(n_{c,i+1}\). Note that the number of D4-branes touching \(NS_{5}'_{i+2}\)-brane in Figure 3B is equal to \((N_{c,i+1} - N_{c,i+2})\).

The quantum corrections can be understood for small \((\Delta x)_{i+1}\) by using the low energy field theory on the branes. The low energy dynamics of the magnetic brane configuration can be described by the \(N = 1\) supersymmetric gauge theory with gauge group (2.5) and the gauge couplings for the three gauge group factors are given by

\[
\begin{align*}
g_{i-1, mag}^2 &= \frac{g_s \ell_s}{(y_i + y_{i-1})}, & g_{i, mag}^2 &= \frac{g_s \ell_s}{y_i}, & g_{i+1, mag}^2 &= \frac{g_s \ell_s}{(y_i + y_{i+1})}.
\end{align*}
\]

The dual gauge theory has a gauge singlet \(\Phi_i+1\) and bifundamentals \(f_i, \tilde{f}_i, F_j\), and \(\tilde{F}_j\) under the dual gauge group (2.5) and the superpotential corresponding to Figures 3A and 3B is given by

\[
W_{dual} = h\Phi_{i+1} f_i \tilde{f}_i - h\mu_{i+1}^2 \text{tr} \Phi_{i+1}, \quad h^2 = g_{i+1, mag}^2, \quad \mu_{i+1}^2 = \frac{-(\Delta x)_{i+1}}{2\pi g_s f_i^3}.
\]

Then \(f_i \tilde{f}_i\) is a \(\tilde{N}_{c,i} \times \tilde{N}_{c,i}\) matrix where the \((i+1)\)-th gauge group indices for \(f_i\) and \(\tilde{f}_i\) are contracted with those of \(\Phi_{i+1}\) while \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})\) matrix. Although the field \(f_i\) itself is an antifundamental in the \((i+1)\)-th gauge group which is a different representation for the usual standard quark coming from D6-branes, the product
\(f_i \tilde{f}_i\) has the same representation for the product of quarks and moreover, the \((i + 1)\)-th gauge group indices for the field \(\Phi_{i+1}\) play the role of the flavor indices, as in comparison with the brane configuration in the presence of D6-branes before.

Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{i+1}\) cannot be satisfied if the \((N_{c,i+1} - N_{c,i+2})\) exceeds \(\tilde{N}_{c,i}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f_i \tilde{f}_i - \mu_{i+1}^2 \delta_{b}^2 = 0\), and \(\Phi_{i+1} f_i = 0 = \tilde{f}_i \Phi_{i+1}\). Then the solutions for these are given by

\[
\begin{align*}
< f_i > &= \begin{pmatrix} \mu_{i+1} \mathbf{1}_{\tilde{N}_{c,i}} & 0 \\ 0 & \mathbf{0} \end{pmatrix}, \\
\tilde{f_i} &= \begin{pmatrix} \mu_{i+1} \mathbf{1}_{\tilde{N}_{c,i}} & 0 \\ 0 & \mathbf{0} \end{pmatrix}, \\
< \Phi_{i+1} > &= \begin{pmatrix} 0 & 0 \\ 0 & M_{i+1} \mathbf{1}_{(N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i})} \end{pmatrix} 
\end{align*}
\]

(2.7)

where the zero of \(< f_i >\) is a \((N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i}) \times \tilde{N}_{c,i}\) matrix, the zero of \(< \tilde{f}_i >\) is a \(\tilde{N}_{c,i} \times (N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i})\) matrix and the zeros of \(< \Phi_{i+1} >\) are \(\tilde{N}_{c,i} \times \tilde{N}_{c,i}, \tilde{N}_{c,i} \times (N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i}),\) and \((N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i}) \times \tilde{N}_{c,i}\) matrices. Then one can expand these fields around on a point \(2.7\) and arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \(V_{\text{eff}}^{(1)}\) for \(M_{i+1}\) leads to the positive value for \(m^2_{M_{i+1}}\) implying that these vacua are stable.

### 2.1.2 When the dual gauge group occurs at even chain

Let us discuss the other case for the Seiberg dual of the middle gauge group factor. Starting from Figure 1B, moving the NS5\(_{c,i-1}\)-brane with \((N_{c,i-1} - N_{c,i-2})\) D4-branes to the +\(v\) direction leading to Figure 2B, and interchanging the NS5\(_c\)-brane and the NS5\(_{t+1}\)-brane, one obtains the Figure 4A.

Before arriving at the Figure 4A, there exists an intermediate step where the \((N_{c,i+1} + N_{c,i-1} - N_{c,i})\) D4-branes are connecting between the NS5\(_{t+1}\)-brane and the NS5\(_c\)-brane, \((N_{c,i-1} - N_{c,i-2})\) D4-branes connecting between the NS5\(_{t-1}\)-brane and NS5\(_{t+1}\)-brane, and \(N_{c,i-2}\) D4-branes between the NS5\(_{t-2}\)-brane and the NS5\(_{t+1}\)-brane. By introducing \(-N_{c,i-2}\) D4-branes and \(-N_{c,i-2}\) anti-D4-branes between the NS5\(_{t+1}\)-brane and NS5\(_c\)-brane, reconnecting the former with the \(N_{c,i-1}\) D4-branes connecting between NS5\(_{t+1}\)-brane and the NS5\(_c\)-brane (therefore \((N_{c,i-1} - N_{c,i-2})\) D4-branes) and moving those combined \((N_{c,i-1} - N_{c,i-2})\) D4-branes to +\(v\)-direction, one gets the final Figure 4A where we are left with \((N_{c,i} - N_{c,i-2} - N_{c,i+1})\) anti-D4-branes between the NS5\(_{t+1}\)-brane and NS5\(_c\)-brane.

The dual gauge group which is the same as previous expression is given by

\[
\cdots \times SU(N_{c,i-1}) \times SU(\tilde{N}_{c,i}) \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times SU(N_{c,i+1}) \times \cdots
\]

(2.8)
Figure 4: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,i} = N_{c,i} - N_{c,i-1} + N_{c,i+1} - N_{c,i})$ where $i$ is even, corresponding to Figure 2B with D4- and $\overline{D}4$-branes(4A) and with a misalignment between D4-branes(4B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 4B can be written as $N_{c,i} - N_{c,i+1} - N_{c,i-2} = (N_{c,i-1} - N_{c,i-2}) - \tilde{N}_{c,i}$.

where the matter contents are the bifundamentals $f_{i-1}$ in $(1_1, \cdots, 1, \square_{i-1}, \square_i, 1, \cdots, 1_n)$, $\cdots$, and $\tilde{f}_{i-1}$ in the representation $(1_1, \cdots, 1, \square_{i-1}, \square_i, 1, \cdots, 1_n)$ in addition to $(n - 2)$ bifundamentals $F_j$ and $\tilde{F}_j$ with $j = 1, 2, \cdots, i - 2, i, \cdots, (n - 1)$ and the gauge singlet $\Phi_{i-1}$ for the $i$-th dual gauge group which was in the adjoint representation for the $(i - 1)$-th dual gauge group is in the representation $(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1, \cdots, 1_n)$ plus a singlet under the dual gauge group (2.8) where the gauge group is broken from $SU(N_{c,i-1})$ to $SU(N_{c,i-1} - N_{c,i-2})$.

When two NS5'-branes in Figure 4A are close to each other, then it leads to Figure 4B by realizing that the number of $(N_{c,i-1} - N_{c,i-2})$ D4-branes connecting between NS5'$_{i-1}$-brane and NS5$_i$-brane can be rewritten as $(N_{c,i} - N_{c,i-2} - N_{c,i+1})$ plus $\tilde{N}_{c,i}$. If we ignore all the D4-branes and NS-branes located at the left hand side of NS5'$_{i-1}$-brane and at the right hand side of NS5$_i$-brane from Figure 4, then the brane configuration becomes the one in [8]. The Figure 4 of [17] is contained in the Figure 4. In particular, the brane configuration from the NS5'$_{i-1}$-brane to the NS5$_{i+2}$-brane is exactly same as the one of [17].

The cubic superpotential with the mass term (2.3) with (2.4) in the dual theory is given by

$$W_{\text{dual}} = \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} + m_{i-1} \text{tr} \Phi_{i-1}. \quad (2.9)$$

Here the magnetic fields $f_{i-1}$ and $\tilde{f}_{i-1}$ correspond to 4-4 strings connecting the $\tilde{N}_{c,i}$-color

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2In general, there are also the extra terms in the superpotential $\Phi_{i+1} f_i \tilde{f}_i + \Phi'' f_{i-1} \tilde{f}_{i-1} + \Phi' f_{i-1} \tilde{f}_i$ where we define $\Phi' = F_i F_{i-1}$ and $\Phi'' = \tilde{F}_i \tilde{F}_{i-1}$, coming from different bifundamentals. However, the F-term conditions, $\Phi_{i+1} f_i + \Phi f_{i-1} = 0 = \Phi_{i+1} \tilde{f}_i + \Phi'' \tilde{f}_{i-1}$ lead to $< \Phi' > = < \Phi'' > = < f_i > = < \tilde{f}_{i-1} > = 0$. In this case also these extra terms do not contribute to the one loop computation up to quadratic order.
D4-branes (that are connecting between the NS5\(_{i-1}'\)-brane and the NS5\(_i\)-brane in Figure 4B) with \(N_{c,i-1}\)-flavor D4-branes (that are a combination of three different D4-branes in Figure 4B). Among these \(N_{c,i-1}\)-flavor D4-branes, only the strings ending on the upper \((N_{c,i+1} - N_{c,i} + N_{c,i-1})\) D4-branes and on the tilted middle \((N_{c,i} - N_{c,i-2} - N_{c,i+1})\) D4-branes in Figure 4B enter the cubic superpotential term. Although the \((N_{c,i-1} - N_{c,i-2})\) D4-branes in Figure 4A for fixed other branes cannot move any directions, the tilted \((N_{c,i} - N_{c,i-2} - N_{c,i+1})\)-flavor D4-branes can move \(w\) direction in Figure 4B. The remaining upper \(\tilde{N}_{c,i}\) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
N_{c,i-1} - N_{c,i-2} = (N_{c,i} - N_{c,i+1} - N_{c,i-2}) + \tilde{N}_{c,i}.
\]

The brane configuration for zero mass for the bifundamental \(F_{i-1}\) and \(\tilde{F}_{i-1}\), which has only a cubic superpotential (2.9), can be obtained from Figure 4A by moving the upper NS5\(_{i-1}'\)-brane together with \((N_{c,i-1} - N_{c,i-2})\) color D4-branes into the origin \(v = 0\). Then the number of dual colors for D4-branes becomes \(\tilde{N}_{c,i}\) between NS5\(_{i+1}'\)-brane and NS5\(_i\)-brane and \(N_{c,i-1}\) between two NS5\(_i\)'-branes as well as \(N_{c,i+1}\) D4-branes between NS5\(_i\)-brane and NS5\(_{i+2}\)-brane. Or starting from Figure 1B and moving the NS5\(_{i+1}'\)-brane to the left all the way past the NS5\(_i\)-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 4A is stable as long as the distance \((\Delta x)_{i-1}\) between the upper NS5\(_i\)'-brane and the lower NS5\(_i\)'-brane is large, as in [8]. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 4B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.8) and superpotential (2.9). The \((N_{c,i} - N_{c,i+1} - N_{c,i-2})\) flavor D4-branes of straight brane configuration of Figure 4B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5\(_i\)-brane from the DBI action, by following the procedure of [8], as long as the distance \(y_{i+1}\) corresponding to the NS5\(_{i+2}\)-brane goes to \(\infty\) because the presence of an extra NS5\(_{i+2}\)-brane does not affect the DBI action. For the finite and small \(y_{i+1}\), the careful analysis for DBI action is needed in order to obtain the bending curve connecting two NS5\(_i\)'-branes.

When the upper NS5\(_i\)'-brane (or NS5\(_{i-1}'\)-brane) is replaced by coincident \((N_{c,i-1} - N_{c,i-2})\) D6-branes in Figure 4B, this brane configuration reduces to the one found in [22] where the gauge group was given by \(\cdots \times SU(n_{c,i-1}) \times SU(n_{f,i} + n_{c,i+1} + n_{c,i-1} - n_{c,i}) \times SU(n_{c,i+1}) \times \cdots\) with \(n_{f,i}\) multiplets, \(\tilde{n}_{f,i}\) multiplets, bifundamentals and gauge singlets. Then the present number \((N_{c,i-1} - N_{c,i-2})\) corresponds to the \(n_{f,i}\), the number \(N_{c,i}\) corresponds to \(n_{c,i}\), the number \(N_{c,i+1}\) corresponds to \(n_{c,i+1}\) and the number \(N_{c,i-2}\) corresponds to the \(n_{c,i-1}\). Note that the
number of D4-branes touching $N_{S}5'_{-1}$-brane in Figure 4B is equal to $(N_{c,i-1} - N_{c,i-2})$. In particular, the Figure 5B of [22] with vanishing flavors $Q'$ and $Q''$ is contained in this modified $N_{S}5'_{-1}$-brane to the $N_{S}5'_{+3}$-brane.

The quantum corrections can be understood for small $(\Delta x)_{i-1}$ by using the low energy field theory on the branes. The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group $\mathcal{G}$ and the gauge couplings for the three gauge group factors are given by

$$g^2_{i-1,mag} = \frac{g_{s}^2\ell_{s}}{(y_i + y_{i-1})}, \quad g^2_{i,mag} = \frac{g_{s}^2\ell_{s}}{y_i}, \quad g^2_{i+1,mag} = \frac{g_{s}^2\ell_{s}}{(y_i + y_{i+1})}.$$  

The dual gauge theory has a meson field $\Phi_{i-1}$ and bifundamentals $f_{i-1}, \tilde{f}_{i-1}, F_j$, and $\tilde{F}_j$ and the superpotential corresponding to Figures 4A and 4B is given by

$$W_{dual} = h\Phi_{i-1}f_{i-1}\tilde{f}_{i-1} - h\mu^2_{i-1} tr\Phi_{i-1}, \quad h^2 = g^2_{i-1,mag}, \quad \mu^2_{i-1} = -\frac{(\Delta x)_{i-1}}{2\pi g_{s}\ell_{s}^3}.$$  

Then $f_{i-1}\tilde{f}_{i-1}$ is a $\tilde{N}_{c,i} \times N_{c,i}$ matrix where the $(i-1)$-th gauge group indices for $f_{i-1}$ and $\tilde{f}_{i-1}$ are contracted with those of $\Phi_{i-1}$ while the $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix. Although the field $f_{i-1}$ itself is an antifundamental in the $i$-th gauge group which is a different representation for the usual standard quark coming from D6-branes, the product $f_{i-1}\tilde{f}_{i-1}$ has the same representation for the product of quarks and moreover, the $(i-1)$-th gauge group indices for the field $\Phi_{i-1}$ play the role of the flavor indices, as in comparison with the brane configuration in the presence of D6-branes before.

Therefore, the F-term equation, the derivative $W_{dual}$ with respect to the meson field $\Phi_{i-1}$ cannot be satisfied if the $(N_{c,i-1} - N_{c,i-2})$ exceeds $\tilde{N}_{c,i}$. So the supersymmetry is broken. That is, there exist three equations from F-term conditions: $f_{i-1}^2\tilde{f}_{i-1} = \mu^2_{i-1} \delta_{i}^{a} = 0$, and $\Phi_{i-1}f_{i-1} = 0 = \tilde{f}_{i-1}\Phi_{i-1}$. Then the solutions for these are given by

$$< f_{i-1} > = \begin{pmatrix} \mu_{i-1}^1 \tilde{N}_{c,i} \\ 0 \end{pmatrix}, \quad < \tilde{f}_{i-1} > = \begin{pmatrix} \mu_{i-1}^1 N_{c,i} \\ 0 \end{pmatrix},$$

$$< \Phi_{i-1} > = \begin{pmatrix} 0 \\ 0 \\ M_{i-1} 1_{(N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i})} \end{pmatrix}.$$  

(2.10)

where the zero of $< f_{i-1} >$ is a $(N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i}) \times N_{c,i}$ matrix, the zero of $< \tilde{f}_{i-1} >$ is a $\tilde{N}_{c,i} \times (N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i})$ matrix and the zeros of $< \Phi_{i-1} >$ are $\tilde{N}_{c,i} \times \tilde{N}_{c,i}$, $\tilde{N}_{c,i} \times (N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i})$, and $(N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i}) \times \tilde{N}_{c,i}$ matrices. Then one can expand these fields around on a point (2.10) and arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{\text{eff}}^{(1)}$ for $M_{i-1}$ leads to the positive value for $m^2_{M_{i-1}}$ implying that these vacua are stable.

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2.2 $\mathcal{N} = 1$ $SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,n})$ magnetic theory

Let us consider the Seiberg dual for the last gauge group factor. There are two magnetic duals depending on whether the gauge group factor occurs at odd chain or even chain.

2.2.1 When the dual gauge group occurs at odd chain

Let us consider other magnetic theory for the same electric theory in previous section. By applying the Seiberg dual to the $SU(N_{c,n})$ factor and interchanging the $NS5'_n$-brane and the $NS5_{n+1}$-brane, one obtains the Figure 5A. Before arriving at the Figure 5A, there exists an intermediate step where $N_{c,n-1}$ D4-branes between $NS5'_{n-1}$-brane and the $NS5_{n+1}$-brane, the $N_{c,n-2}$ D4-branes are connecting between the $NS5'_{n-2}$-brane and the $NS5'_{n-1}$-brane, and $(N_{c,n-1} - N_{c,n})$ D4-branes are connecting between the $NS5_{n+1}$-brane and $NS5'_n$-brane. By rotating $NS5_{n-1}$-brane by an angle $\frac{\pi}{2}$ which will become $NS5'_{n-1}$-brane, moving it with the $(N_{c,n-1} - N_{c,n-2})$ D4-branes to $+v$ direction where we introduce $(N_{c,n} - N_{c,n-2})$ D4-branes and $(N_{c,n} - N_{c,n-2})$ anti D4-branes between the $NS5_{n+1}$-brane and the $NS5'_n$-brane, one gets the final Figure 5A where we are left with $(N_{c,n} - N_{c,n-2})$ anti-D4-branes between the $NS5_{n+1}$-brane and the $NS5'_n$-brane.

Figure 5: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,n} = N_{c,n-1} - N_{c,n})$, where $n$ is odd, with D4- and $\overline{D4}$-branes(5A) and with a misalignment between D4-branes(5B) when the two $NS5'$-branes are close to each other. The number of tilted D4-branes in 5B can be written as $N_{c,n} - N_{c,n-2} = (N_{c,n-1} - N_{c,n-2}) - \tilde{N}_{c,n}$.

The dual gauge group is given by

$$SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})$$ (2.11)

and the matter contents are the field $f_{n-1}$ charged under $(1, \cdots, 1_{n-2}, \square_{n-1}, \square_n)$ and their conjugates $\tilde{f}_{n-1}$ $(1, \cdots, 1_{n-2}, \square_{n-1}, \square_n)$ under the dual gauge group (2.11) and the gauge-singlet $\Phi_{n-1}$ which is in the adjoint representation for the $(n-1)$-th gauge group, in other
words, \((1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n) \oplus (1_1, \cdots, 1_n)\) under the dual gauge group where the gauge group is broken from \(SU(N_{c,n-1})\) to \(SU(N_{c,n-1} - N_{c,n-2})\). Then the \(\Phi_{n-1}\) is a \((N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})\) matrix. Only \((N_{c,n-1} - N_{c,n-2})\) D4-branes can participate in the mass deformation.

When two NS5'-branes in Figure 5A are close to each other, then it leads to Figure 5B by realizing that the number of \((N_{c,n-1} - N_{c,n-2})\) D4-branes connecting between NS5'_{n-1}-brane and NS5_{n+1}-brane can be rewritten as \((N_{c,n} - N_{c,n-2})\) plus \(\tilde{N}_{c,n}\). The Figure 5 of [17] is contained in the Figure 5. In particular, the brane configuration from the NS5'_{n-2}-brane to the NS5'_{n}-brane is exactly same as the one of [17].

The cubic superpotential with the mass term is given by

\[
W_{\text{dual}} = \Phi_{n-1} f_{n-1} \tilde{F}_{n-1} + m_{n-1} \text{tr} \Phi_{n-1}
\]  

(2.12)

where we define \(\Phi_{n-1}\) as \(\Phi_{n-1} \equiv F_{n-1} \tilde{F}_{n-1}\) and the \(n\)-th gauge group indices in \(F_{n-1}\) and \(\tilde{F}_{n-1}\) are contracted, each \((n-1)\)-th gauge group index in them is encoded in \(\Phi_{n-1}\). Here the magnetic fields \(f_{n-1}\) and \(\tilde{f}_{n-1}\) correspond to 4-4 strings connecting the \(\tilde{N}_{c,n}\)-color D4-branes (that are connecting between the NS5'_{n-1}-brane and the NS5_{n+1}-brane in Figure 5B) with \(N_{c,n-1}\)-flavor D4-branes. Among these \(N_{c,n-1}\)-flavor D4-branes, only the strings ending on the upper \((N_{c,n-1} - N_{c,n})\) D4-branes and on the tilted \((N_{c,n} - N_{c,n-2})\) D4-branes in Figure 5B enter the cubic superpotential term. Although the \((N_{c,n-1} - N_{c,n-2})\) D4-branes in Figure 5A cannot move any directions for fixed other branes, the tilted \((N_{c,n} - N_{c,n-2})\)-flavor D4-branes can move \(w\) direction. The remaining upper \(\tilde{N}_{c,n}\) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
(N_{c,n-1} - N_{c,n-2}) = (N_{c,n} - N_{c,n-2}) + \tilde{N}_{c,n}.
\]

The brane configuration in Figure 5A is stable as long as the distance \((\Delta x)_{n-1}\) between the upper NS5'-brane and the lower NS5'-brane (or NS5'_{n}-brane) is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 5B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group \((2.11)\) and superpotential \((2.12)\). The \((N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})\) flavor D4-branes of straight brane configuration of Figure 5B bend since there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. As mentioned in [9], the two NS5'-branes are located at different side of NS5_{n+1}-brane in Figure 5B and the DBI action computation for this bending curve should be taken into account.

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential \((2.12)\), can be obtained from Figure 5A by moving the upper NS5'-brane
together with \((N_{c,n-1} - N_{c,n-2})\) color D4-branes into the origin \(v = 0\). Then the number of
dual colors for D4-branes becomes \(N_{c,n-1}\) between the \(NS5'_{n-1}\)-brane and the \(NS5_{n+1}\)-brane,
\(N_{c,n-2}\) between the \(NS5'_{n-2}\)-brane and the \(NS5'_{n-1}\)-brane and \(\tilde{N}_{c,n}\) between \(NS5_{n+1}\)-brane
and \(NS5'_{n}\)-brane. Or starting from Figure 1B and moving the \(NS5'_{n}\)-brane to the right all the
way past the \(NS5'_{n+1}\)-brane, one also obtains the corresponding magnetic brane configuration
for massless case.

The low energy dynamics of the magnetic brane configuration can be described by the
\(\mathcal{N} = 1\) supersymmetric gauge theory with gauge group and the gauge couplings for the three
gauge group factors are given by

\[
g_{n-2,mag}^2 = \frac{g_s \ell_s}{y_{n-2}}, \quad g_{n-1,mag}^2 = \frac{g_s \ell_s}{(y_n + y_{n-1})}, \quad g_{n,mag}^2 = \frac{g_s \ell_s}{y_n}.
\]

The dual gauge theory has a meson field \(\Phi_{n-1}\) and bifundamentals \(f_{n-1}, \tilde{f}_{n-1}, F_j\) and \(\tilde{F}_j\)
under the dual gauge group (2.11) and the superpotential (2.12) corresponding to Figures 5A and
5B is given by

\[
W_{dual} = h \Phi_{n-1} f_{n-1} \tilde{f}_{n-1} - h \mu_{n-1}^2 \text{tr} \Phi_{n-1}, \quad h^2 = g_{n-1,mag}^2, \quad \mu_{n-1}^2 = \frac{(\Delta x)_{n-1}}{2\pi g_s \ell_s^3}.
\]

Then \(f_{n-1} \tilde{f}_{n-1}\) is a \(\tilde{N}_{c,n} \times \tilde{N}_{c,n}\) matrix where the \((n-1)\)-th gauge group indices for \(f_{n-1}\) and \(\tilde{f}_{n-1}\)
are contracted with those of \(\Phi_{n-1}\) while the \(\Phi_{n-1}\) is a \((N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})\)
matrix. The product \(f_{n-1} \tilde{f}_{n-1}\) has the same representation for the product of quarks and
moreover, the \((n-1)\)-th gauge group indices for the field \(\Phi_{n-1}\) play the role of the flavor
indices.

Therefore, the F-term equation, the derivative \(W_{dual}\) with respect to the meson field \(\Phi_{n-1}\)
cannot be satisfied if the \((N_{c,n-1} - N_{c,n-2})\) exceeds \(\tilde{N}_{c,n}\). So the supersymmetry is broken.
That is, there exist three equations from F-term conditions: \(f_{n-1}^a \tilde{f}_{n-1,b} - \mu_{n-1}^2 \delta^a_b = 0\) and
\(\Phi_{n-1} f_{n-1} = 0 = \tilde{f}_{n-1} \Phi_{n-1}\). Then the solutions for these are given by

\[
< f_{n-1} > = \begin{pmatrix} \mu_{n-1} 1_{\tilde{N}_{c,n}} \\ 0 \end{pmatrix}, \quad < \tilde{f}_{n-1} > = \begin{pmatrix} \mu_{n-1} 1_{\tilde{N}_{c,n}} \\ 0 \end{pmatrix},
\]

\[
< \Phi_{n-1} > = \begin{pmatrix} 0 \\ M_{n-1} 1_{(N_{c,n-1} - N_{c,n-2}) - \tilde{N}_{c,n}} \end{pmatrix}
\]

where the zero of \(< f_{n-1} \) is a \((N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n}) \times \tilde{N}_{c,n}\) matrix, the zero of \(< \tilde{f}_{n-1} \)
is a \(\tilde{N}_{c,n} \times (N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})\) matrix and the zeros of \(< \Phi_{n-1} \) are \(\tilde{N}_{c,n} \times \tilde{N}_{c,n}\)
\(\tilde{N}_{c,n} \times (N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})\) and \((N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n}) \times \tilde{N}_{c,n}\) matrices. Then one can
expand these fields around on a point (2.13) and one arrives at the relevant superpotential
up to quadratic order in the fluctuation. At one loop, the effective potential \(V_{eff}^{(1)}\) for \(M_{n-1}\)
leads to the positive value for \(m_{M_{n-1}}^2\) implying that these vacua are stable.
2.2.2 When the dual gauge group occurs at even chain

Let us discuss the other case for the Seiberg dual of the last gauge group factor. Starting from Figure 1A, moving the $NS5'_{n-1}$-brane with $(N_{c,n-1} - N_{c,n-2})$ D4-branes to the $+v$ direction leading to Figure 2B, and interchanging the $NS5_n$-brane and the $NS5'_{n+1}$-brane, one obtains the Figure 6A.

Figure 6: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,n} = N_{c,n-1} - N_{c,n})$, where $n$ is even, with D4- and $\overline{D4}$-branes(6A) and with a misalignment between D4-branes(6B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 6B can be written as $N_{c,n} - N_{c,n-2} = (N_{c,n-1} - N_{c,n-2}) - \tilde{N}_{c,n}$.

Before arriving at the Figure 6A, there exists an intermediate step where the $(N_{c,n-1} - N_{c,n})$ D4-branes are connecting between the $NS5'_{n+1}$-brane and the $NS5_n$-brane, $(N_{c,n-1} - N_{c,n-2})$ D4-branes connecting between the $NS5'_{n-1}$-brane and $NS5'_n$-brane, and $N_{c,n-2}$ D4-branes between the $NS5_{n-2}$-brane and the $NS5'_{n+1}$-brane. By introducing $-N_{c,n-2}$ D4-branes and $-N_{c,n-2}$ anti-D4-branes between the $NS5'_{n+1}$-brane and $NS5_n$-brane, reconnecting the former with the $N_{c,n-1}$ D4-branes connecting between $NS5'_{n+1}$-brane and the $NS5_n$-brane (therefore $N_{c,n-1} - N_{c,n-2}$ D4-branes) and moving those combined $(N_{c,n-1} - N_{c,n-2})$ D4-branes to $+v$-direction, one gets the final Figure 6A where we are left with $(N_{c,n} - N_{c,n-2})$ anti-D4-branes between the $NS5'_{n+1}$-brane and $NS5_n$-brane.

The dual gauge group that is the same as before is given by

$$SU(N_{c,1}) \times \cdots \times SU(N_{c,n-2}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})$$

(2.14)

where the matter contents are the bifundamentals $f_{n-1}$ in $(1_1, \cdots, 1, \square_{n-1}, \square_n)$, and $\tilde{f}_{n-1}$ in the representation $(1_1, \cdots, 1, \square_{n-1}, \square_n)$ in addition to $(n-2)$ bifundamentals $F_j$ and $\overline{F}_j$, $j = 1, 2, \cdots, n-2$ and the gauge singlet $\Phi_{n-1}$ for the $n$-th dual gauge group in the adjoint representation for the $(n-1)$-th dual gauge group, i.e., $(1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n)$ plus a singlet under the dual gauge group (2.14) where the gauge group is broken from $SU(N_{c,n-1})$ to $SU(N_{c,n-1} - N_{c,n-2})$. 
When two NS5'-branes in Figure 6A are close to each other, then it leads to Figure 6B by realizing that the number of \( (N_{c,n-1} - N_{c,n-2}) \) D4-branes connecting between \( NS5'_{n-1} \)-brane and \( NS5_n \)-brane can be rewritten as \( (N_{c,n} - N_{c,n-2}) \) plus \( \tilde{N}_{c,n} \). The Figure 6 looks similar to the previous Figure 4. If we ignore all the D4-branes and NS-branes located at the left hand side of \( NS_{n+1}' \)-brane from Figure 6, then the brane configuration becomes the one in [8].

The cubic superpotential with the mass term in the dual theory is given by

\[
W_{dual} = \Phi_{n-1} f_{n-1} \tilde{f}_{n-1} + m_{n-1} \text{tr} \Phi_{n-1}.
\] (2.15)

Here the magnetic fields \( f_{n-1} \) and \( \tilde{f}_{n-1} \) correspond to 4-4 strings connecting the \( \tilde{N}_{c,n} \)-color D4-branes(that are connecting between the \( NS5'_{n-1} \)-brane and the \( NS5_n \)-brane in Figure 6B) with \( N_{c,n-1} \)-flavor D4-branes(that are a combination of three different D4-branes in Figure 6B). Among these \( N_{c,n-1} \)-flavor D4-branes, only the strings ending on the upper \( (N_{c,n-1} - N_{c,n}) \) D4-branes and on the tilted \( (N_{c,n} - N_{c,n-2}) \) D4-branes in Figure 6B enter the cubic superpotential term (2.15). Although the \( (N_{c,n-1} - N_{c,n-2}) \) D4-branes in Figure 6A cannot move any directions for fixed other branes, the tilted \( (N_{c,n} - N_{c,n-2}) \)-flavor D4-branes can move any direction in Figure 6B. The remaining upper \( \tilde{N}_{c,n} \) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
N_{c,n-1} - N_{c,n-2} = (N_{c,n} - N_{c,n-2}) + \tilde{N}_{c,n}.
\]

The brane configuration for zero mass for the bifundamental \( F_{n-1} \) and \( \tilde{F}_{n-1} \), which has only a cubic superpotential (2.15), can be obtained from Figure 6A by moving the upper \( NS5'_{n-1} \)-brane together with \( (N_{c,n-1} - N_{c,n-2}) \) color D4-branes into the origin \( v = 0 \). Then the number of dual colors for D4-branes becomes \( \tilde{N}_{c,n} \) between \( NS5'_{n+1} \)-brane and \( NS5_n \)-brane and \( N_{c,n-1} \) between two NS5'-branes as well as \( N_{c,n-2} \) D4-branes between \( NS5_{n-2} \)-brane and \( NS5'_{n-1} \)-brane. Or starting from Figure 1B and moving the \( NS5'_{n+1} \)-brane to the left all the way past the \( NS5_n \)-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 6A is stable as long as the distance \( (\Delta x)_{n-1} \) between the upper NS5'-brane and the lower NS5'-brane is large, as in [8]. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 6B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.14) and superpotential (2.15). The \( (N_{c,n} - N_{c,n-2}) \) flavor D4-branes of straight brane configuration of Figure 6B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and \( NS5_n \)-brane from the DBI action, by following the procedure of [8], as long as the distance \( y_{n-3} \) goes to \( \infty \).
because the presence of an extra $NS5_{n-2}$-brane does not affect the DBI action. For the finite and small $y_{n-3}$, the careful analysis for DBI action is needed in order to obtain the bending curve connecting two $NS5$-branes.

When the upper $NS5'$-brane (or $NS5'_{n-1}$-brane) is replaced by coincident $(N_{c,n-1} - N_{c,n-2})$ D6-branes and the $NS5_{n-2}$ is rotated by an angle $\frac{\pi}{2}$ in the $(v, w)$ plane in Figure 6B, this brane configuration reduces to the one found in [22] where the gauge group was given by $SU(n_c, 1) \times \cdots \times SU(n_{c,n-2}) \times SU(n_{f,n} + n_{c,n-1} - n_{c,n})$ with $n_{f,n}$ multiplets, $\tilde{\eta}_{f,n}$ multiplets, bifundamentals and gauge singlets. Then the present number $(N_{c,n-1} - N_{c,n-2})$ corresponds to the $n_{f,n}$, the number $N_{c,n}$ corresponds to $n_{c,n}$ and the number $N_{c,n-2}$ corresponds to the $n_{c,n-1}$. Note that the number of D4-branes touching $NS5'_{n-1}$-brane in Figure 6B is equal to $(N_{c,n-1} - N_{c,n-2})$. In particular, the Figure 2B of [22] with vanishing flavors $Q$ and $Q'$ is contained in this modified Figure 6B running from the $NS5_{n-2}$-brane to the $NS5_{n}$-brane.

The quantum corrections can be understood for small $(\Delta x)_{n-1}$ by using the low energy field theory on the branes. The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (2.14) and the gauge couplings for the three gauge group factors are given by

$$g_{n-2, mag}^2 = \frac{g_s \ell_s}{y_{n-2}}, \quad g_{n-1, mag}^2 = \frac{g_s \ell_s}{(y_n + y_{n-1})}, \quad g_{n, mag}^2 = \frac{g_s \ell_s}{y_n}.$$  

The dual gauge theory has a gauge singlet $\Phi_{n-1}$ and bifundamentals $f_{n-1}, \tilde{f}_{n-1}, F_j$, and $\tilde{F}_j$ and the superpotential (2.15) corresponding to Figures 6A and 6B is given by

$$W_{dual} = h\Phi_{n-1} f_{n-1} \tilde{f}_{n-1} - \hbar \mu_{n-1}^2 \text{tr} \Phi_{n-1}, \quad h^2 = g_{n-1, mag}^2, \quad \mu_{n-1}^2 = -\frac{(\Delta x)_{n-1}}{2\pi g_s \ell_s^3}.$$  

Then $f_{n-1} \tilde{f}_{n-1}$ is a $\tilde{N}_{c,n} \times \tilde{N}_{c,n}$ matrix where the $(n-1)$-th gauge group indices for $f_{n-1}$ and $\tilde{f}_{n-1}$ are contracted with those of $\Phi_{n-1}$ while $\Phi_{n-1}$ is a $(N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})$ matrix. Although the field $f_{n-1}$ itself is an antifundamental in the $n$-th gauge group which is a different representation for the usual standard quark coming from D6-branes, the product $f_{n-1} \tilde{f}_{n-1}$ has the same representation for the product of quarks and moreover, the $(n-1)$-th gauge group indices for the field $\Phi_{n-1}$ play the role of the flavor indices, as in comparison with the brane configuration in the presence of D6-branes before.

Therefore, the F-term equation, the derivative $W_{dual}$ with respect to the meson field $\Phi_{n-1}$ cannot be satisfied if the $(N_{c,n-1} - N_{c,n-2})$ exceeds $\tilde{N}_{c,n}$. So the supersymmetry is broken. That is, there exist three equations from F-term conditions: $f_{n-1}^a \tilde{f}_{n-1,b} - \mu_{n-1}^2 \delta^a_b = 0$, and
\( \Phi_{n-1} f_{n-1} = 0 = \tilde{f}_{n-1} \Phi_{n-1} \). Then the solutions for these are given by

\[
< f_{n-1} > = \left( \mu_{n-1} 1_{\tilde{N}_{c,n}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right), \quad < \tilde{f}_{n-1} > = \left( \mu_{n-1} 1_{\tilde{N}_{c,n}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right),
\]

\[
< \Phi_{n-1} > = \begin{pmatrix} 0 \\ 0 \end{pmatrix} M_{n-1} 1_{(N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})}
\]

where the zero of \(< f_{n-1} >\) is a \((N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n}) \times \tilde{N}_{c,n}\) matrix, the zero of \(< \tilde{f}_{n-1} >\) is a \(\tilde{N}_{c,n} \times (N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})\) matrix and the zeros of \(< \Phi_{n-1} >\) are \(\tilde{N}_{c,n} \times \tilde{N}_{c,n}\), \(\tilde{N}_{c,n} \times (N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})\), and \((N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n}) \times \tilde{N}_{c,n}\) matrices. Then one can expand these fields around on a point \((2.16)\) and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \(V^{(1)}_{\text{eff}}\) for \(M_{n-1}\) leads to the positive value for \(m_{M_{n-1}}^2\) implying that these vacua are stable.

2.3 \( \mathcal{N} = 1 \ SU(\tilde{N}_{c,1}) \times \cdots \times SU(N_{c,n}) \) magnetic theory

Now we turn to the last case. Let us consider the Seiberg dual for the first gauge group factor. Starting from Figure 1A, moving the \(NS5'_3\)-brane with \((N_{c,2} - N_{c,3})\) D4-branes to the \(+v\) direction leading to Figure 2A, and interchanging the \(NS5'_1\)-brane and the \(NS5_2\)-brane, one obtains the Figure 7A.

![Figure 7](image_url)

Figure 7: The \( \mathcal{N} = 1 \) magnetic brane configuration for the gauge group containing \( SU(\tilde{N}_{c,1} = N_{c,2} - N_{c,1}) \) with D4- and \( \overline{D4} \)-branes(7A) and with a misalignment between D4-branes(7B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 7B can be written as \(N_{c,1} - N_{c,3} = (N_{c,2} - N_{c,3}) - \tilde{N}_{c,1} \).

Before arriving at the Figure 7A, there exists an intermediate step where the \((N_{c,2} - N_{c,1})\) D4-branes are connecting between the \(NS5_2\)-brane and the \(NS5'_1\)-brane, \((N_{c,2} - N_{c,3})\) D4-branes connecting between the \(NS5'_1\)-brane and \(NS5'_3\)-brane, and \(N_{c,3}\) D4-branes between the \(NS5'_1\)-brane and the \(NS5_4\)-brane. By introducing \(-N_{c,3}\) D4-branes and \(-N_{c,3}\) anti-D4-branes between the \(NS5_2\)-brane and \(NS5'_1\)-brane, reconnecting the former with the \(N_{c,2}\) D4-branes...
connecting between \(NS5_2\)-brane and the \(NS5'_1\)-brane (therefore \((N_{c,2} - N_{c,3})\) D4-branes) and moving those combined \((N_{c,2} - N_{c,3})\) D4-branes to \(+v\)-direction, one gets the final Figure 7A where we are left with \((N_{c,1} - N_{c,3})\) anti-D4-branes between the \(NS5_2\)-brane and \(NS5'_1\)-brane.

The dual gauge group is given by

\[
SU(\widetilde{N}_{c,1} \equiv N_{c,2} - N_{c,1}) \times SU(N_{c,2}) \times \cdots \times SU(N_{c,n})
\] (2.17)

where the matter contents are the bifundamentals \(f_1\) in \((\Box_1, \Box_2, \cdots, 1_n)\), and \(\tilde{f}_1\) in the representation \((\Box_1, \Box_2, 1, \cdots, 1_n)\) in addition to \((n - 2)\) bifundamentals \(F_j\) and \(\tilde{F}_j\), \(j = 2, 3, \cdots, n\) and the gauge singlet \(\Phi_2\) for the first dual gauge group in the adjoint representation for the second dual gauge group, i.e., \((1_1, (N_{c,2} - N_{c,3})^2 - 1, 1_3, \cdots, 1_n)\) plus a singlet under the dual gauge group where the gauge group is broken from \(SU(N_{c,2})\) to \(SU(N_{c,2} - N_{c,3})\).

When two \(NS5'\)-branes in Figure 7A are close to each other, then it leads to Figure 7B by realizing that the number of \((N_{c,2} - N_{c,3})\) D4-branes connecting between \(NS5_2\)-brane and \(NS5'_3\)-brane can be rewritten as \((N_{c,1} - N_{c,3})\) plus \(\widetilde{N}_{c,1}\). The Figure 7 looks similar to Figure 3. If we ignore all the D4-branes and NS-branes at the right hand side of \(NS'_1\)-brane from Figure 7, then the brane configuration becomes the one in [8].

The cubic superpotential with the mass term \([2.1]\) in the dual theory is given by

\[
W_{\text{dual}} = \Phi_2 f_1 \tilde{f}_1 + m_2 \text{tr} \Phi_2.
\] (2.18)

Here the magnetic fields \(f_1\) and \(\tilde{f}_1\) correspond to 4-4 strings connecting the \(\widetilde{N}_{c,1}\)-color D4-branes (that are connecting between the \(NS5_2\)-brane and the \(NS5'_3\)-brane in Figure 7B) with \(N_{c,2}\)-flavor D4-branes (that are a combination of three different D4-branes in Figure 7B). Among these \(N_{c,2}\)-flavor D4-branes, only the strings ending on the upper \((N_{c,2} - N_{c,1})\) D4-branes and on the tilted \((N_{c,1} - N_{c,3})\) D4-branes in Figure 7B enter the cubic superpotential term \([2.18]\). Although the \((N_{c,2} - N_{c,3})\) D4-branes for fixed other branes in Figure 7A cannot move any directions, the tilted \((N_{c,1} - N_{c,3})\)-flavor D4-branes can move \(w\) direction in Figure 7B. The remaining upper \(\widetilde{N}_{c,1}\) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
N_{c,2} - N_{c,3} = (N_{c,1} - N_{c,3}) + \widetilde{N}_{c,1}.
\]

The brane configuration for zero mass for the bifundamental \(F_1\) and \(\tilde{F}_1\), which has only a cubic superpotential \([2.17]\), can be obtained from Figure 7A by moving the upper \(NS5'_3\)-brane together with \((N_{c,2} - N_{c,3})\) color D4-branes into the origin \(v = 0\). Then the number of dual colors for D4-branes becomes \(\widetilde{N}_{c,1}\) between \(NS5_2\)-brane and \(NS5'_1\)-brane and \(N_{c,2}\) between two \(NS5'\)-branes as well as \(N_{c,3}\) D4-branes between \(NS5'_3\)-brane and \(NS5_4\)-brane. Or starting
from Figure 1A and moving the NS5_2-brane to the left all the way past the NS5_1'-brane, one
also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 7A is stable as long as the distance ($\Delta x$)_2 between the
upper NS5'-brane and the lower NS5'-brane is large, as in [8]. If they are close to each other,
then this brane configuration is unstable to decay and leads to the brane configuration in
Figure 3B. One can regard these brane configurations as particular states in the magnetic
brane theory with the gauge group [2.17] and superpotential [2.18]. The ($N_{c,2} - N_{c,3} - \tilde{N}_{c,1}$)
flavor D4-branes of straight brane configuration of Figure 7B bend due to the fact that there
exists an attractive gravitational interaction between those flavor D4-branes and NS5_2-brane
from the DBI action, by following the procedure of [8], as long as the distance $y_3$ goes to $\infty$
because the presence of an extra NS5_4-brane does not affect the DBI action. For the finite
and small $y_3$, the careful analysis for DBI action is needed in order to obtain the bending
curve connecting two NS5'-branes.

When the upper NS5'-brane(or NS5'_3-brane) is replaced by coincident ($N_{c,2} - N_{c,3}$) D6-
branes and the NS5_4 is rotated by an angle $\frac{\pi}{2}$ in the $(v, w)$ plane in Figure 7B, this brane
configuration reduces to the one found in [22] where the gauge group was given by
$SU(n_{f,1} + n_{c,2} - n_{c,1}) \times SU(n_{c,2}) \times \cdots$ with $n_{f,i}$ multiplets, $\tilde{n}_{f,i}$ multiplets, bifundamentals and gauge
singlets. Then the present number ($N_{c,2} - N_{c,3}$) corresponds to the $n_{f,1}$, the number $N_{c,1}$
corresponds to $n_{c,1}$, the number $N_{c,3}$ corresponds to $n_{c,2}$. Note that the number of D4-branes
touching NS5'_3-brane in Figure 7B is equal to ($N_{c,2} - N_{c,3}$). In particular, the Figure 2B of
[22] with vanishing flavors $Q$ and $Q'$ is contained in this modified Figure 7B running from the
NS5_2-brane to the NS5_4-brane.

The quantum corrections can be understood for small ($\Delta x$)_2 by using the low energy field
theory on the branes. The low energy dynamics of the magnetic brane configuration can
be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group and the gauge
couplings for the three gauge group factors are given by

$$g_{1, mag}^2 = \frac{g_s \ell_s}{y_1}, \quad g_{2, mag}^2 = \frac{g_s \ell_s}{(y_2 + y_1)}, \quad g_{3, mag}^2 = \frac{g_s \ell_s}{y_3}.$$  

The dual gauge theory has a gauge singlet $\Phi_2$ and bifundamentals $f_1, \tilde{f}_1, F_j$, and $\tilde{F}_j$ under the
dual gauge group and the superpotential corresponding to Figures 7A and 7B is given by

$$W_{\text{dual}} = h \Phi_2 f_1 \tilde{f}_1 - h \mu_2^2 \text{tr} \Phi_2, \quad h^2 = g_{2, mag}^2, \quad \mu_2^2 = \frac{(\Delta x)_2}{2 \pi g_s \ell_s^2}.$$  

Then $f_1 \tilde{f}_1$ is a $\tilde{N}_{c,1} \times \tilde{N}_{c,1}$ matrix where the second gauge group indices for $f_1$ and $\tilde{f}_1$ are
contracted with those of $\Phi_2$ while $\Phi_2$ is a $(N_{c,2} - N_{c,3}) \times (N_{c,2} - N_{c,3})$ matrix. Although the
field \( f_1 \) itself is an antifundamental in the second gauge group which is a different representation for the usual standard quark coming from D6-branes, the product \( f_1 \tilde{f}_1 \) has the same representation for the product of quarks and moreover, the second gauge group indices for the field \( \Phi_2 \) play the role of the flavor indices, as in comparison with the brane configuration in the presence of D6-branes before.

Therefore, the F-term equation, the derivative \( W_{\text{dual}} \) with respect to the meson field \( \Phi_2 \) cannot be satisfied if the \((N_{c,2} - N_{c,3}) \) exceeds \( \bar{N}_{c,1} \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( f_1^a \tilde{f}_{1,b} - \mu_1^2 \delta_{ab} = 0 \), and \( \Phi_2 f_1 = 0 = \tilde{f}_1 \Phi_2 \). Then the solutions for these are given by

\[
< f_1 > = \left( \begin{array}{c} \mu_2 1_{\bar{N}_{c,1}} \\ 0 \end{array} \right), \quad < \tilde{f}_1 > = \left( \begin{array}{c} \mu_2 1_{\bar{N}_{c,1}} \\ 0 \end{array} \right), \quad < \Phi_2 > = \left( \begin{array}{cc} 0 & 0 \\ 0 & M_2 1_{(N_{c,2} - N_{c,3} - \bar{N}_{c,1})} \end{array} \right)
\]

(2.19)

where the zero of \(< f_1 > \) is a \((N_{c,2} - N_{c,3} - \bar{N}_{c,1}) \times \bar{N}_{c,1} \) matrix, the zero of \(< \tilde{f}_1 > \) is a \(\bar{N}_{c,1} \times (N_{c,2} - N_{c,3} - \bar{N}_{c,1}) \) matrix and the zeros of \(< \Phi_2 > \) are \(\bar{N}_{c,1} \times \bar{N}_{c,1}, \bar{N}_{c,1} \times (N_{c,2} - N_{c,3} - \bar{N}_{c,1}), \) and \((N_{c,2} - N_{c,3} - \bar{N}_{c,1}) \times \bar{N}_{c,1} \) matrices. Then one can expand these fields around on a point (2.19) and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \( V_{\text{eff}}^{(1)} \) for \( M_2 \) leads to the positive value for \( m_{M_2}^2 \) implying that these vacua are stable.

3 Meta-stable brane configurations with \((n+1)\) NS-branes plus O4-plane

The type IIA brane configuration \([60]\) corresponding to \( \mathcal{N} = 1 \) supersymmetric electric gauge theory (see also \([22]\)) with gauge group

\[
\begin{align*}
\text{Sp}(N_{c,1}) \times \text{SO}(2N_{c,2}) \times \cdots \times \text{SO}(2N_{c,n-1}) \times \text{Sp}(N_{c,n}) & \quad \text{for odd } n, \\
\text{Sp}(N_{c,1}) \times \text{SO}(2N_{c,2}) \times \cdots \times \text{Sp}(N_{c,n-1}) \times \text{SO}(2N_{c,n}) & \quad \text{for even } n,
\end{align*}
\]

and with the \((n - 1)\) bifundamentals \( F_i \) charged under \((1, \cdots, 1, \square_i, \square_{i+1}, 1, \cdots, 1_n) \) where \( i = 1, 2, \cdots, (n - 1) \) can be described by the \( \text{NS}5'_1\)-brane, the \( \text{NS}5_2\)-brane, \cdots, the \( \text{NS}5_{n+1}\)-brane for odd number of gauge groups (or the \( \text{NS}5'_{n+1}\)-brane for even number of gauge groups), \(2N_{c,1}-, 2N_{c,2}-, \cdots, \) and \(2N_{c,n}\)-color D4-branes. There exists the \( O4^{\pm}\)-planes \((01236)\) which act as \((x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9) \) and they have RR charge \( \pm 1 \). See the Figure 8 for the details on the brane configuration.

Let us place the \( \text{NS}5'_1\)-brane at the origin \( x^0 = 0 \) and denote the \( x^0 \) coordinates for the \( \text{NS}5_2\)-brane, \cdots, the \( \text{NS}5_{n+1}\)-brane for odd \( n \) (or the \( \text{NS}5'_{n+1}\)-brane for even \( n \)) are given by
Figure 8: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $\prod_{i=1}^{n-2} Sp(N_{c,i}) \times SO(2N_{c,i+1})$ for 8A and $\prod_{i=1}^{n-1} Sp(N_{c,i}) \times SO(2N_{c,i+1})$ for 8B and bifundamentals $F_i$ with vanishing mass for the bifundamental.

$x^6 = y_1, y_1 + y_2, \ldots, \sum_{j=1}^{n-1} y_j + y_n$ respectively. The $2N_{c,1}$ D4-branes are suspended between the $NS5'_1$-brane and the $NS5_2$-brane, the $2N_{c,2}$ D4-branes are suspending between the $NS5_2$-brane and the $NS5'_3$-brane, $\cdots$ and the $2N_{c,n}$ D4-branes are suspended between the $NS5'_n$-brane and the $NS5_{n+1}$-brane for odd $n$ (or between the $NS5_n$-brane and the $NS5'_{n+1}$-brane for even $n$). The fields $F_i$ correspond to 4-4 strings connecting the $2N_{c,i}$-color D4-branes with $2N_{c,i+1}$-color D4-branes. We draw this $\mathcal{N} = 1$ supersymmetric electric brane configuration in Figure 8A(8B) when $n$ is odd(even) for the vanishing mass for the fields $F_i$.

Let us deform the theory by Figure 8A. Displacing the two $NS5'$-branes, $NS5'_r$-brane and $NS5'_{i+2}$-brane, relative each other in the $v$ direction, characterized by $(\Delta x)_{i+1}$, corresponds to turning on a quadratic mass-deformed superpotential for the fields $F_i$ as follows:

$$W_{\text{elec}} = m_{i+1} F_i F_i (\equiv m_{i+1} \Phi_{i+1}), \quad \text{when } i \text{ is odd}$$

(3.1)

where the $i$-th gauge group indices in $F_i$ and $F_i$ are contracted, each $(i + 1)$-th gauge group index in them is encoded in $\Phi_{i+1}$ and the mass $m_{i+1}$ is given by

$$m_{i+1} = \frac{(\Delta x)_{i+1}}{\ell_s^2}. \quad \text{(3.2)}$$

The gauge-singlet $\Phi_{i+1}$ for the $i$-th gauge group is in the adjoint representation for the $(i + 1)$-th gauge group, i.e.,

$$(1_1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})(2N_{c,i+1} - 2N_{c,i+2} - 1), 1_{i+2}, \cdots, 1_n)$$

under the gauge group where the gauge group is broken from $SO(2N_{c,i+1})$ to $SO(2N_{c,i+1} - 2N_{c,i+2})$. Then the $\Phi_{i+1}$ is a $2(N_{c,i+1} - N_{c,i+2}) \times 2(N_{c,i+1} - N_{c,i+2})$ matrix. The $NS5'_{i+2}$-brane together with $2(N_{c,i+1} - N_{c,i+2})$-color D4-branes is moving to the $\pm v$ direction for fixed other
branes during this mass deformation. In other words, the $2N_{c,i+2}$ D4-branes among $2N_{c,i+1}$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate (≡ $x$) of NS5'$_i$-brane is equal to zero while the $x^5$ coordinate of NS5'$_{i+2}$-brane is given by $±(Δx)_{i+1}$. Giving an expectation value to the meson field $Φ_{i+1}$ corresponds to recombination of $2N_{c,i}$- and $2N_{c,i+1}$-color D4-branes, which will become $2N_{c,i}$- or $2N_{c,i+1}$-color D4-branes in Figure 8A such that they are suspended between the NS5'$_i$-brane and the NS5'$_{i+2}$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i+1} ≥ N_{c,i} − N_{c,i−1} + 2 ≥ N_{c,i+2}$ where $i$ is odd. Now we draw this brane configuration in Figure 9A for nonvanishing mass for the fields $F_i$.

![Figure 9: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $[\prod_{i=1}^{n−2} Sp(N_{c,i}) × SO(2N_{c,i+1})] × Sp(N_{c,n})$ for 9A and $\prod_{i=1}^{n−1} Sp(N_{c,i}) × SO(2N_{c,i+1})$ for 9B with nonvanishing mass for the bifundamental. The $2N_{c,i+1}$ D4-branes in 8A are decomposed into $2(N_{c,i+1} − N_{c,i+2})$ D4-branes which are moving to $±v$ direction in $\mathbb{Z}_2$ symmetric way in 9A and $2N_{c,i+2}$ D4-branes which are recombined with those D4-branes connecting between NS5'$_{i+2}$-brane and NS5'$_{i+3}$-brane in 9A. The $2N_{c,i−1}$ D4-branes in 8B are decomposed into $2(N_{c,i−1} − N_{c,i−2})$ D4-branes which are moving to $±v$ direction in $\mathbb{Z}_2$ symmetric way in 9B and $2N_{c,i−2}$ D4-branes which are recombined with those D4-branes connecting between NS5'$_{i−2}$-brane and NS5'$_{i−1}$-brane in 9B.

Let us deform the theory by Figure 8B. Displacing the two NS5' -branes, the $NS5'_i$-brane and the $NS5'_{i+1}$-brane, relative each other in the $±v$ direction, characterized by $Δx_{i−1}$ corresponds to turning on a quadratic mass-deformed superpotential for the fields $F_{i−1}$ as follows:

$$W = m_{i−1}F_{i−1}F_{i−1}(≡ m_{i−1}Φ_{i−1}), \quad \text{when } i \text{ is even}$$

(3.3)

where the $i$-th gauge group indices in $F_{i−1}$ are contracted, each $(i−1)$-th gauge group index in them is encoded in $Φ_{i−1}$ and the mass $m_{i−1}$ is given by

$$m_{i−1} = \frac{(Δx)_{i−1}}{ℓ^2_s}. \quad \text{(3.4)}$$

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The gauge-singlet $\Phi_{i-1}$ for the $i$-th gauge group is in the adjoint representation for the $(i-1)$-th gauge group, i.e.,

$$(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})(2N_{c,i-1} - 2N_{c,i-2} + 1), 1_i, \cdots, 1_n)$$

under the gauge group where the gauge group is broken from $Sp(N_{c,i-1})$ to $Sp(N_{c,i-1} - N_{c,i-2})$. Then the $\Phi_i$ is a $2(N_{c,i-1} - N_{c,i-2}) \times 2(N_{c,i-1} - N_{c,i-2})$ matrix. The $NS5'_i$-brane together with $2(N_{c,i-1} - N_{c,i-2})$-color D4-branes is moving to the $\pm v$ direction for fixed other branes during this mass deformation. In other words, the $2N_{c,i-2}$ D4-branes among $2N_{c,i-1}$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate (idented by $x$) of $NS5'_i$-brane is equal to zero while the $x^5$ coordinate of $NS5'_i$-brane is given by $\pm (\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i-1}$ corresponds to recombination of $2N_{c,i-1}$ and $2N_{c,i}$-color D4-branes, which will become $2N_{c,i}$- or $2N_{c,i}$-color D4-branes in Figure 8B such that they are suspended between the $NS5'_i$-brane and the $NS5'_i$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i-1} \geq N_{c,i} - N_{c,i+1} - 2 \geq N_{c,i-2}$ where $i$ is even. Now we draw this brane configuration in Figure 9B for nonvanishing mass for the fields $F_{i-1}$.

Basically the brane configurations are the same as the ones in previous section if one includes the mirrors but the careful treatment for the appropriate O4-plane charge is needed in order to obtain the correct number of D4-branes.

### 3.1 $N = 1$ $Sp(N_{c,1}) \times \cdots \times SO(2\tilde{N}_{c,i})[Sp(\tilde{N}_{c,i})] \times \cdots$ magnetic theory

#### 3.1.1 When the dual gauge group occurs at odd chain

Starting from Figure 9A and interchanging the $NS5'_i$-brane and the $NS5'_i$-brane, one obtains the Figure 10A. By introducing $-2N_{c,i+2}$ D4-branes and $-2N_{c,i+2}$ anti-D4-branes between the $NS5'_i$-brane and $NS5'_i$-brane, reconnecting the former with the $N_{c,i+1}$ D4-branes connecting between $NS5'_i$-brane and the $NS5'_i$-brane (therefore $2(N_{c,i+1} - N_{c,i+2})$ D4-branes) and moving those combined $(N_{c,i+1} - N_{c,i+2})$ D4-branes to $+v$-direction (and their mirrors to $-v$ direction), one gets the final Figure 10A where we are left with $2(N_{c,i} - N_{c,i+2} - N_{c,i-1} + 2)$ anti-D4-branes between the $NS5'_i$-brane and $NS5'_i$-brane.

The dual gauge group is

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i}) \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

(3.5)

3 When the O4-plane charges are reversed, then the gauge group will be either $\prod_{i=1}^{n-2} SO(2N_{c,i}) \times Sp(2N_{c,i+1}) \times SO(2N_{c,n})$ for odd $n$ or $\prod_{i=1}^{n-1} SO(N_{c,i}) \times Sp(2N_{c,i+1})$ for even $n$. One can analyze these cases also by realizing that the adjoint of symplectic gauge group is symmetric matrix while the adjoint of orthogonal gauge group is antisymmetric.
The \( \mathcal{N} = 1 \) magnetic brane configuration for the gauge group containing \( Sp(\tilde{N}_{c,i} = N_{c,i-1} + N_{c,i+1} - N_{c,i} - 2) \) with D4- and \( \overline{D4} \)-branes(10A) and with a misalignment between D4-branes(10B) when the two NS5’-branes are close to each other. The number of tilted D4-branes in 10B can be written as
\[
N_{c,i} - N_{c,i-1} - N_{c,i+2} + 2 = N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i}.
\]
The \( x \)-coordinate of NS5\(_i'+2\)-brane is given by \( \pm (\Delta x)_{i+1} \).

The matter contents are the field \( f_i \) charged under \((1, \cdots, 1, i-1, 2\tilde{N}_{c,i}, 2N_{c,i+1}, 1, \cdots, 1_n)\) under the dual gauge group and the gauge-singlet \( \Phi_{i+1} \) that is in the adjoint representation for the \((i+1)\)-th dual gauge group, i.e.,
\[
(1, \cdots, 1, (N_{c,i+1} - N_{c,i+2})(2N_{c,i+1} - 2N_{c,i+2} - 1), i_{i+2}, \cdots, 1_n)
\]
under the dual gauge group where the gauge group is broken from \( SO(2N_{c,i+1}) \) to \( SO(2N_{c,i+1} - 2N_{c,i+2}) \). That is, the \( \Phi_{i+1} \) is an \( 2(N_{c,i+1} - N_{c,i+2}) \times 2(N_{c,i+1} - N_{c,i+2}) \) antisymmetric matrix.

The cubic superpotential with the mass term (3.1) and (3.2) in the dual theory is given by
\[
W_{dual} = \Phi_{i+1} f_i f_i + m_{i+1} \text{tr} \Phi_{i+1}.
\]

Here the magnetic field \( f_i \) corresponds to 4-4 strings connecting the \( 2\tilde{N}_{c,i} \)-color D4-branes (that are connecting between the NS5\(_i+1\)-brane and the NS5\(_i'+2\)-brane including the mirrors) with \( 2N_{c,i+1} \)-flavor D4-branes (that are a combination of three different D4-branes including the mirrors in Figure 10B). Among these \( 2N_{c,i+1} \)-flavor D4-branes, only the strings ending on the upper and lower \( 2(N_{c,i+1} - N_{c,i} + N_{c,i-1} - 2) \) D4-branes and on the tilted \( 2(N_{c,i} - N_{c,i+2} - N_{c,i-1} + 2) \) D4-branes including the mirrors in Figure 10B enter the cubic superpotential term (3.6).

When the upper and lower half NS5\(_i'+2\)-branes are replaced by coincident \((N_{c,i+1} - N_{c,i+2})\) D6-branes and the NS5\(_i+3\) is rotated by an angle \( \frac{\pi}{2} \) in the \((v, w)\) plane in Figure 10B, this

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\( ^4 \)There are also the extra terms in the superpotential \( \Phi' f_{i-1} f_i + \Phi_{i-1} f_i f_{i-1} \) where we define \( \Phi' \equiv F_i F_{i-1} \), coming from different bifundamentals. However, the F- term condition, \( \Phi' f_i + 2\Phi_{i-1} f_{i-1} = 0 \) leads to \( < \Phi' > = < f_{i-1} > = 0 \). Therefore, these extra terms do not contribute to the one loop computation up to quadratic order.
brane configuration reduces to the one found in \cite{22} where the gauge group was given by 
\[ \cdots \times SO(2n_c,i-1) \times Sp(n_{f,i} + n_{c,i+1} + n_{c,i-1} - n_{c,i} - 2) \times SO(2n_c,i+1) \times \cdots \] with \( n_{f,i} \) multiplets, bifundamentals and gauge singlets. Then the present \( (N_{c,i+1} - N_{c,i+2}) \) corresponds to the \( n_{f,i} \), the number \( N_{c,i-1} \) corresponds to \( n_{c,i-1} \), the number \( N_{c,i+2} \) corresponds to \( n_{c,i+1} \) and \( N_{c,i} \) corresponds to the \( n_{c,i} \). The dual gauge theory has a meson field \( \Phi_{i+1} \) and bifundamentals \( f_i \) under the dual gauge group \((3.3)\) and the superpotential \((3.6)\) corresponding to Figures 10A and 10B is given by
\[
W_{\text{dual}} = h\Phi_{i+1}f_if_i - h\mu_{i+1}^2 \text{tr} \Phi_{i+1}, \quad h^2 = g_{i+1,\text{mag}}, \quad \mu_{i+1}^2 = -\frac{(\Delta x)_{i+1}}{2\pi g_s \ell_s^3}.
\]
Then \( f_i f_i \) is a \( 2\tilde{N}_{c,i} \times 2\tilde{N}_{c,i} \) matrix where the \((i+1)\)-th gauge group indices for \( f_i \) are contracted with those of \( \Phi_{i+1} \) while the \( \Phi_{i+1} \) is a \( 2(N_{c,i+1} - N_{c,i+2}) \times 2(N_{c,i+1} - N_{c,i+2}) \) matrix. Therefore, the F-term equation, the derivative \( W_{\text{dual}} \) with respect to the meson field \( \Phi_{i+1} \) cannot be satisfied if the \( 2(N_{c,i+1} - N_{c,i+2}) \) exceeds \( 2\tilde{N}_{c,i} \). So the supersymmetry is broken. That is, there exist two equations from F-term conditions: \( f_i f_i - \mu_{i+1}^2 \delta_{a,b} = 0 \) and \( \Phi_{i+1} f_i = 0 \). Then the solutions for these are given by
\[
<f_i> = \begin{pmatrix}
\mu_{i+1} + 1 & 2\tilde{N}_{c,i} \\
0 & 0
\end{pmatrix}, \quad <\Phi_{i+1}> = \begin{pmatrix}
0 & 0 \\
0 & M_{i+1} 1_{(N_{c,i+1} - N_{c,i+2} - \tilde{N}_{c,i}) \otimes \sigma_2}
\end{pmatrix}.
\]

### 3.1.2 When the dual gauge group occurs at even chain

Let us discuss the other case for the Seiberg dual of the middle gauge group factor. Let us consider other magnetic theory for the same electric theory in previous subsection. Starting from Figure 9B and interchanging the \( NS5_i \)-brane and the \( NS5_i'_{i+1} \)-brane, one obtains the Figure 11A.

By introducing \(-2N_{c,i-2} \) D4-branes and \(-2N_{c,i-2} \) anti-D4-branes between the \( NS5_i'_{i+1} \)-brane and \( NS5_i \)-brane, reconnecting the former with the \( 2N_{c,i-1} \) D4-branes connecting between \( NS5_i'_{i+1} \)-brane and the \( NS5_i \)-brane (therefore \( 2(N_{c,i-1} - N_{c,i-2}) \) D4-branes) and moving those combined \( (N_{c,i-1} - N_{c,i-2}) \) D4-branes to the \(+v\)-direction (and their mirrors to the \(-v\) direction), one gets the final Figure 11A where we are left with \( 2(N_{c,i} - N_{c,i+1} - N_{c,i-2} - 2) \) anti-D4-branes between the \( NS5_i'_{i+1} \)-brane and \( NS5_i \)-brane. The dual gauge group is
\[
\cdots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \cdots \quad (3.7)
\]
The matter contents are the field \( f_{i-1} \) charged under \((1, \cdots, 1_{i-2}, 2N_{c,i-1}, 2\tilde{N}_{c,i}, 1_{i+1}, \cdots, 1_n)\) under the dual gauge group and the gauge-singlet \( \Phi_{i-1} \) for the \( i \)-th dual gauge group in the adjoint representation for the \((i - 1)\)-th dual gauge group, i.e.,
\[
(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})(2N_{c,i-1} - 2N_{c,i-2} + 1), 1_i, \cdots, 1_n)
\]
Figure 11: The \( \mathcal{N} = 1 \) magnetic brane configuration for the gauge group containing \( SO(2N_{c,i} = 2(N_{c,i-1} + N_{c,i+1} - N_{c,i} + 2)) \) with D4- and \( \overline{D}4 \)-branes (11A) and with a misalignment between D4-branes (11B) when the two NS5' -branes are close to each other. The number of tilted D4-branes in 11B can be written as \( N_{c,i} - N_{c,i-1} - N_{c,i-2} - 2 = N_{c,i-1} - N_{c,i-2} - N_{c,i} \). The \( x \) coordinate of NS5'\(_i\)-brane is given by \( \pm (\Delta x)_i \).

under the dual gauge group where the gauge group is broken from \( Sp(N_{c,i-1}) \) to \( Sp(N_{c,i-1} - N_{c,i-2}) \). Then the \( \Phi_i \) is a \( 2(N_{c,i-1} - N_{c,i-2}) \times 2(N_{c,i-1} - N_{c,i-2}) \) matrix.

The cubic superpotential with the mass term (3.3) and (3.4) in the dual theory is given by

\[
W_{\text{dual}} = \Phi_i f_i f_i + m_i \text{tr} \Phi_i
\]

where we define \( \Phi_i \) as \( F_i F_i \) and the \( i \)-th gauge group indices in \( F_i \) are contracted, each \( (i - 1) \)-th gauge group index in them is encoded in \( \Phi_i \). Here the magnetic field \( f_i \) corresponds to 4-4 strings connecting the 2\( \tilde{N}_{c,i} \)-color D4-branes (that are connecting between the NS5'\(_i\)-brane and the NS5'\(_{i-1}\)-brane in Figure 11B) with 2\( N_{c,i-1} \)-flavor D4-branes including the mirrors (which are realized as corresponding D4-branes in Figure 11A).

When the NS5'-brane (or NS5'\(_{i-1}\)-brane) is replaced by coincident \( (N_{c,i-1} - N_{c,i-2}) \) D6-branes and the NS5\(_i\)-2 is rotated by an angle \( \frac{\pi}{2} \) in the \((v, w)\) plane in Figure 11B, this brane configuration leads to the one found in [22] where the gauge group was given by \( \cdots \times Sp(n_{c,i-1}) \times SO(2n_{f,i} + 2n_{c,i+1} + 2n_{c,i-1} - 2n_{c,i} + 4) \times Sp(n_{c,i+1}) \times \cdots \) with \( 2n_{f,i} \) multiplets, bifundamentals and gauge singlets. Then the present \( (N_{c,i-1} - N_{c,i-2}) \) corresponds to \( n_{f,i} \), the number \( N_{c,i} \) corresponds to \( n_{c,i} \), the number \( N_{c,i+1} \) corresponds to \( n_{c,i+1} \) and the number \( N_{c,i-2} \) corresponds to \( n_{c,i-2} \). The dual gauge theory has a meson field \( \Phi_i \) and bifundamentals \( f_i \) under the dual gauge group (3.7) and the superpotential (3.8) corresponding to Figures 5.

\[
W_{\text{dual}} = \Phi_i f_i f_i + m_i \text{tr} \Phi_i
\]

\[<\Phi'_i> = <f_i> = 0.\]

In this case also these extra terms do not contribute to the one loop computation up to quadratic order.

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5There are also the extra terms in the superpotential \( \Phi_i f_i f_i + \Phi'_i f_i f_i \) where we define \( \Phi' \equiv F_i F_i \), coming from different bifundamentals. However, the F- term condition, \( 2\Phi_i f_i + \Phi'_i f_i = 0 \) leads to \( <\Phi' > = <f_i> = 0.\)
Therefore, the F-term equation, the derivative $W_i\not\equiv 0$ cannot be satisfied if the $2(\mathbf{N}_{c,i} - \mathbf{N}_{c,i-2})$ matrix. Therefore, the F-term equation, the derivative $W_{dual}$ with respect to the meson field $\Phi_{i-1}$ cannot be satisfied if the $2(\mathbf{N}_{c,i} - \mathbf{N}_{c,i-2})$ exceeds $2\tilde{N}_{c,i}$. So the supersymmetry is broken. That is, there exist two equations from F-term conditions: $f^a_{i-1}f^b_{i-1} - \mu_{i-1}^2 \delta^{a,b} = 0$ and $\Phi_{i-1}f_{i-1} = 0$. Then the solutions for these are given by

$$< f_{i-1} > = \left( \begin{array}{c} \mu_{i-1} 1_{2\tilde{N}_{c,i}} \\ 0 \end{array} \right), \quad < \Phi_{i-1} > = \left( \begin{array}{c} 0 \\ 0 \end{array} \right).$$

### 3.2 $\mathcal{N} = 1$ Sp($\mathbf{N}_{c,1}$) $\cdots \times$ Sp($\tilde{\mathbf{N}}_{c,n}$)[SO($2\tilde{\mathbf{N}}_{c,n}$)] magnetic theory

#### 3.2.1 When the dual gauge group occurs at odd chain

Let us consider other magnetic theory for the same electric theory. One can think of the following dual gauge group

$$\text{Sp}(\mathbf{N}_c) \times \cdots \times \text{SO}(2\mathbf{N}_{c,n-1}) \times \text{Sp}(\tilde{\mathbf{N}}_{c,n} = \mathbf{N}_{c,n-1} - \mathbf{N}_{c,n} - 2)$$

by performing the magnetic dual for the last gauge group. By applying the Seiberg dual to the $\text{Sp}(\mathbf{N}_{c,n})$ factor from Figure 9A and interchanging the $\text{NS5}'$-brane and the $\text{NS5}_{n+1}$-brane, one obtains the Figure 12A.

By rotating $\text{NS5}_{n-1}$-brane by an angle $\frac{\pi}{2}$, moving it with the $\mathbf{N}_{c,n-1} - \mathbf{N}_{c,n-2}$ D4-branes to $+v$ direction where we introduce $2(\mathbf{N}_{c,n} - \mathbf{N}_{c,n-2})$ D4-branes and $2(\mathbf{N}_{c,n} - \mathbf{N}_{c,n-2})$ anti D4-branes between the $\text{NS5}_{n+1}$-brane and the $\text{NS5}'_n$-brane, one gets the final Figure 12A where we are left with $2(\mathbf{N}_{c,n} - \mathbf{N}_{c,n-2} + 2)$ anti-D4-branes between the $\text{NS5}_n$-brane and the $\text{NS5}'_n$-brane. When two NS5'-branes in Figure 12A are close to each other, then it leads to Figure 12B by realizing that the number of $(\mathbf{N}_{c,n-1} - \mathbf{N}_{c,n-2})$ D4-branes connecting between the $\text{NS5}'_n$-brane and the $\text{NS5}_{n+1}$-brane can be rewritten as $(\mathbf{N}_{c,n} - \mathbf{N}_{c,n-2} + 2) + \tilde{N}_{c,n}$. The Figure 10 of [17] is contained in the Figure 12. In particular, the brane configuration from the $\text{NS5}'_n$-brane to the $\text{NS5}_n$-brane is exactly same as the one of [17].

The matter contents are the field $f_{n-1}$ charged under $(\mathbf{1}_1, \cdots, \mathbf{1}_{n-2}, 2\mathbf{N}_{c,n-1}, 2\tilde{\mathbf{N}}_{c,n})$ under the dual gauge group and the gauge-singlet $\Phi_{n-1}$ that is in the adjoint representation for the $(n-1)$-th dual gauge group, $(\mathbf{1}_1, \cdots, \mathbf{1}_{n-2}, (\mathbf{N}_{c,n-1} - \mathbf{N}_{c,n-2})(2\mathbf{N}_{c,n-1} - 2\mathbf{N}_{c,n-2} - 1), \mathbf{1}_n)$ under the dual gauge group (3.9) where the gauge group is broken from $\text{SO}(2\mathbf{N}_{c,n-1})$ to
Figure 12: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $Sp(\tilde{N}_{c,n} = N_{c,n-1} - N_{c,n} - 2)$ with D4- and $\overline{D4}$-branes (12A) and with a misalignment between D4-branes (12B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 12B can be written as $N_{c,n-2} = N_{c,n} - 1 - N_{c,n} - 2\right) + 2 = (N_{c,n-1} - N_{c,n-2}) - \tilde{N}_{c,n}$. The $x$ coordinate of NS5$n-1$-brane is given by $\pm (\Delta x)_{n-1}$.

$SO(2N_{c,n-1} - 2N_{c,n-2})$. That is, the $\Phi_{n-1}$ is an $2(N_{c,n-1} - N_{c,n-2}) \times 2(N_{c,n-1} - N_{c,n-2})$ antisymmetric matrix.

The cubic superpotential with the mass term in the dual theory is given by

$$W_{dual} = \Phi_{n-1} f_{n-1} f_{n-1} + m_{n-1} \text{tr} \Phi_{n-1}$$

(3.10)

where we define $\Phi_{n-1}$ as $\Phi_{n-1} \equiv F_{n-1} F_{n-1}$ and the $n$-th gauge group indices in $F_{n-1}$ are contracted, each $(n-1)$-th gauge group index in $F_{n-1}$ is encoded in $\Phi_{n-1}$. Here the magnetic field $f_{n-1}$ correspond to 4-4 strings connecting the $2\tilde{N}_{c,n}$-color D4-branes including the mirrors (that are connecting between the NS5$n-1$-brane and the NS5$n+1$-brane in Figure 12B) with $2N_{c,n-1}$-flavor D4-branes. Among these $2N_{c,n-1}$-flavor D4-branes, only the strings ending on the $2(N_{c,n-1} - N_{c,n} - 2)$ D4-branes and on the tilted $2(N_{c,n} - N_{c,n-2} + 2)$ D4-branes in Figure 12B enter the cubic superpotential term (3.10).

The dual gauge theory has a meson field $\Phi_{n-1}$ and bifundamentals $f_{n-1}$ under the dual gauge group (3.9) and the superpotential (3.10) corresponding to Figures 12A and 12B is given by

$$W_{dual} = h\Phi_{n-1} f_{n-1} f_{n-1} - h\mu_{n-1}^2 \text{tr} \Phi_{n-1}, \quad h^2 = g_{n-1,mag}^2, \quad \mu_{n-1}^2 = -\frac{(\Delta x)_{n-1}}{2\pi g_s \ell_s^3}.$$  

Then $f_{n-1} f_{n-1}$ is a $2\tilde{N}_{c,n} \times 2\tilde{N}_{c,n}$ matrix where the $(n-1)$-th gauge group indices for $f_{n-1}$ are contracted with those of $\Phi_{n-1}$ while $\Phi_{n-1}$ is a $2(N_{c,n-1} - N_{c,n-2}) \times 2(N_{c,n-1} - N_{c,n-2})$ matrix. Therefore, the F-term equation, the derivative $W_{dual}$ with respect to the meson field $\Phi_{n-1}$ cannot be satisfied if the $2(N_{c,n-1} - N_{c,n-2})$ exceeds $2\tilde{N}_{c,n}$. So the supersymmetry is
broken. That is, there exist two equations from F-term conditions: 
\[ f^a_{n-1}f^b_{n-1} - \mu^2_{n-1}\delta^{ab} = 0 \]
and \( \Phi_{n-1}f_{n-1} = 0 \). Then the solutions for these are given by
\[
< f_{n-1} > = \left( \frac{\mu_{n-1}1_{2\tilde{N}_{c,n}}}{0} \right), \quad < \Phi_{n-1} > = \left( \begin{array}{cc} 0 & \begin{array}{cc} 0 & M_{n-1}1_{(N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n}) \otimes i\sigma_2} \end{array} \end{array} \right)
\]

3.2.2 When the dual gauge group occurs at even chain

One can think of the following dual gauge group
\[ Sp(N_{c,1}) \times \cdots \times Sp(N_{c,n-1}) \times SO(2\tilde{N}_{c,n} \equiv 2(N_{c,n-1} - N_{c,n} + 2)) \]  
(3.11)

by performing the magnetic dual for the last gauge group. By applying the Seiberg dual to the
\( SO(2N_{c,n}) \) factor from Figure 9B and interchanging the \( NS5_n \)-brane and the \( NS5'_{n+1} \)-brane, one obtains the Figure 13A.

Figure 13: The \( \mathcal{N} = 1 \) magnetic brane configuration for the gauge group containing
\( SO(2\tilde{N}_{c,n} = 2(N_{c,n-1} - N_{c,n} + 2)) \) with D4- and \( \overline{D}4 \)-branes(13A) and with a misalignment between D4-branes(13B) when the two \( NS5_n \)-branes are close to each other. The number of tilted D4-branes in 13B can be written as \( N_{c,n} - N_{c,n-2} - 2 = (N_{c,n-1} - N_{c,n-2}) - \tilde{N}_{c,n} \). The \( x \) coordinate of \( NS5'_{n-1} \)-brane is given by \( \pm(\Delta x)_{n-1} \).

By moving the \( NS5'_{n-1} \)-brane with the \( (N_{c,n-1} - N_{c,n-2}) \) D4-branes to \(+v\) direction where we introduce \(-2N_{c,n-2}\) D4-branes and \(-2N_{c,n-2}\) anti D4-branes between the \( NS5'_{n+1} \)-brane and the \( NS5_n \)-brane, one gets the final Figure 13A where we are left with \( 2(N_{c,n} - N_{c,n-2} - 2) \) anti-D4-branes between the \( NS5_{n+1} \)-brane and the \( NS5_n \)-brane. When two \( NS5' \)-branes in Figure 13A are close to each other, then it leads to Figure 13B by realizing that the number of \( (N_{c,n-1} - N_{c,n-2}) \) D4-branes connecting between the \( NS5'_{n-1} \)-brane and the \( NS5_n \)-brane can be rewritten as \( (N_{c,n} - N_{c,n-2} - 2) \) plus \( \tilde{N}_{c,n} \).

The matter contents are the field \( f_{n-1} \) charged under \( (1_1, \cdots, 1_{n-2}, 2N_{c,n-1}, 2\tilde{N}_{c,n}) \) under the dual gauge group (3.11) and the gauge-singlet \( \Phi_{n-1} \) that is in the adjoint representation for the \((n-1)\)-th dual gauge group, \( (1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})(2N_{c,n-1} - 2N_{c,n-2} + 1), 1_n) \)
under the dual gauge group where the gauge group is broken from \( Sp(N_{c,n-1}) \) to \( Sp(N_{c,n-1} - N_{c,n-2}) \). That is, the \( \Phi_{n-1} \) is an \( 2(N_{c,n-1} - N_{c,n-2}) \times 2(N_{c,n-1} - N_{c,n-2}) \) symmetric matrix. The cubic superpotential with the mass term in the dual theory is given by

\[
W_{\text{dual}} = \Phi_{n-1} f_{n-1} f_{n-1} + m_{n-1} \text{tr} \Phi_{n-1} \quad (3.12)
\]

where we define \( \Phi_{n-1} \) as \( \Phi_{n-1} \equiv F_{n-1} F_{n-1} \) and the \( n \)-th gauge group indices in \( F_{n-1} \) are contracted, each \( (n-1) \)-th gauge group index in \( F_{n-1} \) is encoded in \( \Phi_{n-1} \). Here the magnetic field \( f_{n-1} \) correspond to 4-4 strings connecting the \( 2\tilde{N}_{c,n} \)-color D4-branes including the mirrors (that are connecting between the \( NS5'_{n-1} \)-brane and the \( NS5_n \)-brane in Figure 13B) with \( 2N_{c,n-1} \)-flavor D4-branes. Among these \( 2N_{c,n-1} \)-flavor D4-branes, only the strings ending on the \( 2(N_{c,n-1} - N_{c,n} + 2) \) D4-branes and on the tilted \( 2(N_{c,n} - N_{c,n-2} - 2) \) D4-branes in Figure 13B enter the cubic superpotential term (3.12).

When the upper NS5'-brane (or \( NS5'_{n-1} \)-brane) is replaced by coincident \( (N_{c,n-1} - N_{c,n-2}) \) D6-branes and the \( NS5_{n-2} \)-brane is rotated by an angle \( \frac{\pi}{2} \) in the \((v,w)\) plane in Figure 13B, this brane configuration reduces to the one found in [22] where the gauge group was given by \( \times \cdots \times SO(2n_{c,n-2}) \times Sp(n_{c,n-1}) \times SO(2n_{f,n} + 2n_{c,n-1} - 2n_{c,n} + 4) \) with \( n_{f,n} \) multiplets, bifundamentals and gauge singlets. Then the present number \( (N_{c,n-1} - N_{c,n-2}) \) corresponds to the \( n_{f,n} \), the number \( N_{c,n} \) corresponds to \( n_{c,n} \) and the number \( N_{c,n-2} \) corresponds to the \( n_{c,n-1} \). The dual gauge theory has a meson field \( \Phi_{n-1} \) and bifundamentals \( f_{n-1} \) under the dual gauge group (3.11) and the superpotential (3.12) corresponding to Figures 13A and 13B is given by

\[
W_{\text{dual}} = h \Phi_{n-1} f_{n-1} f_{n-1} - h_{n-1}^2 \text{tr} \Phi_{n-1}, \quad h^2 = g_{n-1,\text{mag}}, \quad \mu_{n-1}^2 = -\frac{(\Delta x)_{n-1}}{2\pi g_s \ell_s^3}.
\]

Then \( f_{n-1} f_{n-1} \) is a \( 2\tilde{N}_{c,n} \times 2\tilde{N}_{c,n} \) matrix where the \( (n-1) \)-th gauge group indices for \( f_{n-1} \) are contracted with those of \( \Phi_{n-1} \) while \( \Phi_{n-1} \) is a \( 2(N_{c,n-1} - N_{c,n-2}) \times 2(N_{c,n-1} - N_{c,n-2}) \) matrix. Therefore, the F-term equation, the derivative \( W_{\text{dual}} \) with respect to the meson field \( \Phi_{n-1} \) cannot be satisfied if the \( 2(N_{c,n-1} - N_{c,n-2}) \) exceeds \( 2\tilde{N}_{c,n} \). So the supersymmetry is broken. That is, there exist two equations from F-term conditions: \( f_{n-1}^a f_{n-1}^b - \mu_{n-1}^2 \delta^{a,b} = 0 \) and \( \Phi_{n-1} f_{n-1} = 0 \). Then the solutions for these are given by

\[
< f_{n-1} > = \begin{pmatrix} \mu_{n-1} & 1_{2\tilde{N}_{c,n}} \\ 0 & 0 \end{pmatrix}, \quad < \Phi_{n-1} > = \begin{pmatrix} 0 & 0 \\ M_{n-1} 1_{2(N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})} \end{pmatrix}.
\]

### 3.3 \( \mathcal{N} = 1 \) \( Sp(\tilde{N}_{c,1}) \times \cdots \times Sp(N_{c,n})[SO(2N_{c,n})] \) magnetic theory

Let us consider the Seiberg dual for the first gauge group factor. Starting from Figure 8A, moving the \( NS5'_{n} \)-brane with \( (N_{c,2} - N_{c,3}) \) D4-branes to the \(+v\) direction leading to Figure

\[
\text{Figure 13B}
\]

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9A, and interchanging the $NS5'_1$-brane and the $NS5_2$-brane, one obtains the Figure 14A.

Figure 14: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $Sp(\tilde{N}_{c,1} = N_{c,2} - N_{c,1} - 2)$ with D4- and $D4'$-branes (14A) and with a misalignment between D4-branes (14B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 14B can be written as $N_{c,1} - N_{c,3} + 2 = (N_{c,2} - N_{c,3}) - \tilde{N}_{c,1}$. The $x$ coordinate of $NS5'_3$-brane is given by $\pm (\Delta x)_2$.

By introducing $-2N_{c,3}$ D4-branes and $-2N_{c,3}$ anti-D4-branes between the $NS5_2$-brane and $NS5'_1$-brane, reconnecting the former with the $2N_{c,2}$ D4-branes connecting between $NS5_2$-brane and the $NS5'_1$-brane (therefore $2(N_{c,2} - N_{c,3})$ D4-branes) and moving those combined $2(N_{c,2} - N_{c,3})$ D4-branes to $\pm v$-direction, one gets the final Figure 14A where we are left with $2(N_{c,1} - N_{c,3} + 2)$ anti-D4-branes between the $NS5_2$-brane and $NS5'_1$-brane. The dual gauge group is given by

$$Sp(\tilde{N}_{c,1} \equiv N_{c,2} - N_{c,1} - 2) \times SO(2N_{c,2}) \times \cdots \times Sp(N_{c,n})[SO(2N_{c,n})]$$

(3.13)

where the matter contents are the bifundamentals $f_1$ in $(\Box_1, \Box_2, \cdots, 1_n)$, in addition to $(n-2)$ bifundamentals $F_j$, $j = 2, 3, \cdots, n$ and the gauge singlet $\Phi_2$ for the first dual gauge group in the adjoint representation, i.e., $(1_1, (N_{c,2} - N_{c,3})(2N_{c,2} - 2N_{c,3} - 1), 1_3, \cdots, 1_n)$ under the dual gauge group where the gauge group is broken from $SO(2N_{c,2})$ to $SO(2N_{c,2} - 2N_{c,3})$.

The cubic superpotential with the mass term in the dual theory is given by

$$W_{\text{dual}} = \Phi_2 f_1 f_1 + m_2 \text{tr } \Phi_2.$$ 

(3.14)

Here the magnetic fields $f_1$ correspond to 4-4 strings connecting the $2\tilde{N}_{c,1}$-color D4-branes (that are connecting between the $NS5_2$-brane and the $NS5'_3$-brane in Figure 14B) with $2N_{c,2}$-flavor D4-branes (that are a combination of three different D4-branes in Figure 14B). Among these $2N_{c,2}$-flavor D4-branes, only the strings ending on the $2(N_{c,2} - N_{c,1} - 2)$ D4-branes and on the tilted middle $2(N_{c,1} - N_{c,3} + 2)$ D4-branes in Figure 14B enter the cubic superpotential term (3.14).
When the upper NS5’-brane (or NS5’-brane) is replaced by coincident \((N_{c,2} - N_{c,3})\) D6-branes and the NS5’-brane is rotated by an angle \(\frac{\pi}{2}\) in the \((v, w)\) plane in Figure 14B, this brane configuration reduces to the one found in \([22]\) where the gauge group was given by \(Sp(n_{f,1} + n_{c,2} - n_{c,1} - 2) \times SO(2n_{c,2}) \times \cdots\) with \(n_{f,i}\) multiplets, multiplets, bifundamentals and gauge singlets. Then the present number \((N_{c,2} - N_{c,3})\) corresponds to the \(n_{f,1}\), the number \(N_{c,1}\) corresponds to \(n_{c,1}\), and the number \(N_{c,3}\) corresponds to \(n_{c,2}\). The dual gauge theory has a meson field \(\Phi_2\) and bifundamentals \(f_1\) and \(F_j\) under the dual gauge \([3.13]\) group and the superpotential \([3.14]\) corresponding to Figures 14A and 14B is given by

\[
W_{\text{dual}} = h \Phi_2 f_1 f_1 - h \mu_2^2 \text{tr} \Phi_2, \quad h^2 = g_{2,\text{mag}}^2, \quad \mu_2^2 = \frac{(\Delta x)_2^2}{2\pi g_s f_s^3}. \]

Then \(f_1 f_1\) is a \(2\tilde{N}_{c,1} \times 2\tilde{N}_{c,1}\) matrix where the second gauge group indices for \(f_1\) are contracted with those of \(\Phi_2\) while \(\Phi_2\) is a \(2(N_{c,2} - N_{c,3}) \times 2(N_{c,2} - N_{c,3})\) matrix. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_2\) cannot be satisfied if the \(2(N_{c,2} - N_{c,3})\) exceeds \(2\tilde{N}_{c,1}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f_1^i \tilde{f}_1^j - \mu_2^2 \delta^{ab} = 0,\) and \(\Phi_2 f_1 = 0.\) Then the solutions for these are given by

\[
< f_1 > = \left( \begin{array}{c} \mu_2^2 \text{I}_{2\tilde{N}_{c,1}} \\ 0 \end{array} \right), \quad < \Phi_2 > = \left( \begin{array}{cc} 0 & 0 \\ 0 & M_2 \text{I}_{(N_{c,2} - N_{c,3} - \tilde{N}_{c,1}) \otimes i\sigma_2} \end{array} \right). \]

As in footnote \([3]\) when the O4-plane charges are reversed, then the gauge group will be either \([\prod_{i=1}^{n-2} SO(2N_{c,i}) \times Sp(2N_{c,i+1})] \times SO(2N_{c,n})\) for odd \(n\) or \([\prod_{i=1}^{n-1} SO(N_{c,i}) \times Sp(2N_{c,i+1})]\) for even \(n\). By following what we done so far, one can analyze the magnetic duals for these cases also by realizing that the adjoint of symplectic gauge group is symmetric matrix while the adjoint of orthogonal gauge group is antisymmetric.

4 Meta-stable brane configurations with 2n NS-branes plus O6−-plane

The type IIA brane configuration, by generalizing the brane configurations \([61, 62, 18]\) to the case where there are more NS-branes, corresponding to \(\mathcal{N} = 1\) supersymmetric electric gauge theory (see also \([23]\)) with gauge group

\[Sp(N_{c,1}) \times SU(N_{c,2}) \cdots \times SU(N_{c,n})\]

and with the \((n-1)\) bifundamentals \(F_i\) charged under \((1_1, \cdots, 1, i, i+1, 1, \cdots, 1_n)\) and their complex conjugate fields \(\tilde{F}_i\) charged \((1_1, \cdots, 1, i, i+1, 1, \cdots, 1_n)\) where \(i = 1, 2, \cdots, (n-1)\).
can be described by the $NS_{5}'$-brane, the $NS_{5}$-brane, $\cdots$, the $NS_{5}'_{n}$-brane for odd number of gauge groups (or the $NS_{5}_{n}$-brane for even number of gauge groups), $2N_{c,1}$, $N_{c,2}$, $\cdots$, and $N_{c,n}$-color D4-branes. See the Figure 15 for the details on the brane configuration. The $O6^{−}$-plane acts as $(x^{4}, x^{5}, x^{6}) \rightarrow (-x^{4}, -x^{5}, -x^{6})$ and has RR charge $-4$.

Let us place an $O6$-plane at the origin $x^{6} = 0$ and denote the $x^{6}$ coordinates for the $NS_{5}'_{1}$-brane, $\cdots$, the $NS_{5}'_{n}$-brane for odd $n$ (or the $NS_{5}_{n}$-brane for even $n$) are given by $x^{6} = y_{1} + y_{2}, \cdots, \sum_{j=1}^{n-1} y_{j} + y_{n}$ respectively. The $2N_{c,1}$ D4-branes are suspended between the $NS_{5}'_{1}$-brane and its mirror, the $N_{c,2}$ D4-branes are suspending between the $NS_{5}'_{1}$-brane and the $NS_{5}_{2}$-brane, $\cdots$ and the $N_{c,n}$ D4-branes are suspended between the $NS_{5}_{n-1}$-brane and the $NS_{5}'_{n}$-brane for odd $n$ (or between the $NS_{5}'_{n-1}$-brane and the $NS_{5}_{n}$-brane for even $n$). The fields $F_{i}$ and $\tilde{F}_{i}$ correspond to 4-4 strings connecting the $N_{c,i}$-color D4-branes with $N_{c,i+1}$-color D4-branes. We draw this $\mathcal{N} = 1$ supersymmetric electric brane configuration in Figure 15A (15B) when $n$ is odd (even) for the vanishing mass for the fields $F_{i}$ and $\tilde{F}_{i}$.

There is no superpotential in Figure 15A. Let us deform this theory. Displacing the two $NS_{5}'$-branes relative each other in the $+v$ direction, characterized by $(\Delta x)_{i-1}$, corresponds to turning on a quadratic mass-deformed superpotential for the field $F_{i-1}$ and $\tilde{F}_{i-1}$ as follows:

$$W_{elec} = m_{i-1} F_{i-1} \tilde{F}_{i-1} (\equiv m_{i-1} \Phi_{i-1}), \quad \text{when } i \text{ is odd} \quad (4.1)$$

where the $i$-th gauge group indices in $F_{i-1}$ and $\tilde{F}_{i-1}$ are contracted and the mass $m_{i-1}$ is given by

$$m_{i-1} = \frac{(\Delta x)_{i-1}}{\ell_{s}^{2}}. \quad (4.2)$$

The gauge-singlet $\Phi_{i-1}$ for the $i$-th gauge group is in the adjoint representation for the $(i-1)$-th gauge group, i.e., $\left(N_{c,i-1} - N_{c,i-2}\right)^{2} - 1, \cdots, 1_{n} \oplus (1_{1}, \cdots, 1_{n})$ under the
gauge group where the gauge group is broken from $SU(N_{c,i-1})$ to $SU(N_{c,i-1} - N_{c,i-2})$. The $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix. The $NS5'_{i-2}$-brane together with $(N_{c,i-1} - N_{c,i-2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation (and their mirrors to $-v$ direction). Then the $x^5$ coordinate of $NS5'_{i-2}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i-2}$-branes is given by $(\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i-1}$ corresponds to recombination of $N_{c,i-1}$- and $N_{c,i-2}$-color D4-branes, which will become $N_{c,i-1}$ or $N_{c,i}$-color D4-branes in Figure 15A such that they are suspended between the $NS5'_{i-2}$-brane and the $NS5'_{i}$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i-1} \geq N_{c,i} - N_{c,i+1} \geq N_{c,i-2}$.

Now we draw this brane configuration in Figure 16A for nonvanishing mass for the fields $F_{i-1}$ and $\tilde{F}_{i-1}$.

![Figure 16](image-url)

Figure 16: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_{c,1}) \times \prod_{i=2}^{n} SU(N_{c,i})$ and bifundamentals $F_{i}$ and $\tilde{F}_{i}$ with nonvanishing mass for the bifundamentals when the number of gauge groups factor $n$ is odd (16A) and even (16B). The $N_{c,i-1}$ D4-branes in 15A are decomposed into $(N_{c,i-1} - N_{c,i-2})$ D4-branes which are moving to $+v$ direction in 16A and $N_{c,i-2}$ D4-branes which are recombined with those D4-branes connecting between $NS5'_{i-3}$-brane and $NS5'_{i-2}$-brane in 16A. The $N_{c,i+1}$ D4-branes in 16B are decomposed into $(N_{c,i+1} - N_{c,i+2})$ D4-branes which are moving to $+v$ direction in 16B and $N_{c,i+2}$ D4-branes which are recombined with those D4-branes connecting between $NS5'_{i+1}$-brane and $NS5'_{i+2}$-brane in 16B.

Let us deform the theory by Figure 15B. Displacing the two $NS5'$-branes, the $NS5'_{i-1}$-brane and the $NS5'_{i+1}$-brane, relative each other in the $v$ direction, characterized by $(\Delta x)_{i+1}$, corresponds to turning on a quadratic mass-deformed superpotential for the fields $F_{i}$ and $\tilde{F}_{i}$ as follows:

$$W = m_{i+1} F_{i} \tilde{F}_{i} (\equiv m_{i+1} \Phi_{i+1}), \quad \text{when } i \text{ is even} \quad (4.3)$$

where the $i$-th gauge group indices in $F_{i}$ and $\tilde{F}_{i}$ are contracted, each $(i + 1)$-th gauge group...
index in them is encoded in $\Phi_{i+1}$ and the mass $m_{i+1}$ is given by

$$m_{i-1} = \frac{(\Delta x)_{i+1}}{l_s^2}. \quad (4.4)$$

The gauge-singlet $\Phi_{i+1}$ for the $i$-th gauge group is in the adjoint representation for the $(i + 1)$-th gauge group, i.e.,

$$(1_1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, 1_{i+2}, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)$$

under the gauge group where the gauge group is broken from $SU(N_{c,i+1})$ to $SU(N_{c,i+1} - N_{c,i+2})$. Then the $\Phi_{i+1}$ is a $(N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})$ matrix. The $NS5'_{i+1}$-brane together with $(N_{c,i+1} - N_{c,i+2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation. In other words, the $N_{c,i+2}$ D4-branes among $N_{c,i+1}$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate ($\equiv x$) of $NS5'_{i-1}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i+1}$-brane is given by $(\Delta x)_{i+1}$. Giving an expectation value to the meson field $\Phi_{i+1}$ corresponds to recombination of $N_{c,i}$ and $N_{c,i+1}$-color D4-branes, which will become $N_{c,i}$- or $N_{c,i+1}$-color D4-branes in Figure 15B such that they are suspended between the $NS5'_{i-1}$-brane and the $NS5'_{i+1}$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i+1} \geq N_{c,i} - N_{c,i-1} \geq N_{c,i+2}$. Now we draw this brane configuration in Figure 16B for nonvanishing mass for the fields $F_i$ and $\tilde{F}_i$.

### 4.1 $\mathcal{N} = 1$ $Sp(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i}) \times \cdots \times SU(N_{c,n})$ magnetic theory

#### 4.1.1 When the dual gauge group occurs at odd chain

Starting from Figure 16A and interchanging the $NS5_{i-1}$-brane and the $NS5'_i$-brane (and their mirrors), one obtains the Figure 17A.

By introducing $-N_{c,i-2}$ D4-branes and $-N_{c,i-2}$ anti-D4-branes between the $NS5'_i$-brane and $NS5_{i-1}$-brane, reconnecting the former with the $N_{c,i-1}$ D4-branes connecting between $NS5'_i$-brane and the $NS5_{i-1}$-brane (therefore $(N_{c,i-1} - N_{c,i-2})$ D4-branes) and moving those combined $(N_{c,i-1} - N_{c,i-2})$ D4-branes to $+v$-direction, one gets the final Figure 17A where we are left with $(N_{c,i} - N_{c,i+1} - N_{c,i-2})$ anti-D4-branes between the $NS5'_i$-brane and $NS5_{i-1}$-brane.

The dual gauge group is given by

$$Sp(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n}). \quad (4.5)$$

The matter contents are the field $f_{i-1}$ charged under $(1_1, \cdots, 1_{i-2}, N_{c,i-1}, \tilde{N}_{c,i}, \cdots, 1_n)$, and its conjugate field $\tilde{f}_{i-1}$ charged under $(1_1, \cdots, 1_{i-2}, \tilde{N}_{c,i-1}, \tilde{N}_{c,i}, \cdots, 1_n)$ under the dual gauge
Figure 17: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,i} = N_{c,i-1} + N_{c,i+1} - N_{c,i})$ with D4- and $D4$-branes(17A) and with a misalignment between D4-branes(18B) when the two NS5’-branes are close to each other. The number of tilted D4-branes in 18B can be written as $N_{c,i} - N_{c,i+1} - N_{c,i-2} = N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i}$. The cubic superpotential with the mass term (4.1) and (4.2) is given by

$$W_{\text{dual}} = \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} + m_{i-1} \text{tr} \Phi_{i-1}.$$ (4.6)

Here the magnetic fields $f_{i-1}$ and $\tilde{f}_{i-1}$ correspond to 4-4 strings connecting the $\tilde{N}_{c,i}$-color D4-branes(are that connecting between the $NS5’_{i-2}$-brane and the $NS5_{i-1}$-brane in Figure 17B) with $N_{c,i-1}$-flavor D4-branes(are that realized as corresponding D4-branes in Figure 17A).

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (4.5) and the gauge couplings for the three gauge group factors are given by similarly. The dual gauge theory has a meson field $\Phi_{i-1}$ and bifundamentals $f_{i-1}$, and $\tilde{f}_{i-1}$ under the dual gauge group (4.5) and the superpotential (4.6) corresponding to Figures 17A and 17B is given by

$$W_{\text{dual}} = h \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} - h \mu_{i-1}^2 \text{tr} \Phi_{i-1}, \quad h^2 = g_{i-1, \text{mag}}, \quad \mu_{i-1}^2 = -\frac{(\Delta x)_{i-1}}{2\pi g_s f_s^3}.$$ 

Then $f_{i-1} \tilde{f}_{i-1}$ is a $\tilde{N}_{c,i} \times \tilde{N}_{c,i}$ matrix where the $(i - 1)$-th gauge group indices for $f_{i-1}$ and $\tilde{f}_{i-1}$ are contracted with those of $\Phi_{i-1}$ while $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix.

---

6There exist also the extra terms in the superpotential $\Phi_{i+1} f_i \tilde{f}_i + \Phi' f_{i-1} \tilde{f}_i + \Phi' f_{i-1} f_i$ where we define $\Phi' \equiv F_i F_{i-1}$ and $\Phi'' \equiv F_i F_{i-1}$, coming from different bifundamentals. However, the $F$- term conditions, $\Phi_{i+1} \tilde{f}_i + \Phi' f_{i-1} = 0 = \Phi_{i+1} f_i + \Phi'' \tilde{f}_{i-1}$ lead to $< \Phi' >= < \Phi'' > = < f_i > = < \tilde{f}_i > = 0$. Also these extra terms do not contribute to the one loop computation up to quadratic order.
When the upper NS5'-brane(or NS5' -brane) is replaced by coincident \((N_{c,i-1} - N_{c,i-2})\) D6-branes and the NS5_{i-3} is rotated by \(1/2\) in Figure 17B, this brane configuration looks similar to the one found in [24] where the gauge group was given by \(Sp(n_{c,1}) \times \cdots \times SU(n_{f,i} + n_{c,i+1} + n_{c,i-1} - n_{c,i}) \times SU(n_{c,i+1}) \times \cdots \) with \(n_{f,i}\) multiplets, \(\tilde{n}_{f,i}\) multiplets, bifundamentals and singlets. Then the present \((N_{c,i-1} - N_{c,i-2})\) corresponds to the \(n_{f,i}\), \(N_{c,i}\) corresponds to \(n_{c,i}\), \(N_{c,i+1}\) corresponds to \(n_{c,i+1}\) and \(N_{c,i-2}\) corresponds to the \(n_{c,i-1}\). Therefore, the F-term equation, the derivative \(W_{dual}\) with respect to the meson field \(\Phi_{i-1}\) cannot be satisfied if the \((N_{c,i-1} - N_{c,i-2})\) exceeds \(\tilde{N}_{c,i}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f_{i-1}^a \tilde{f}_{i-1,b} - \mu_{i-1}^a \delta_b^a = 0\) and \(\Phi_{i-1} f_{i-1} = 0 = \tilde{f}_{i-1} \Phi_{i-1}\). Then the solutions for these are given by

\[
<f_{i-1}> = \begin{pmatrix} \mu_{i-1} & 1 \\ 0 & 0 \end{pmatrix} \tilde{N}_{c,i}, \quad <\tilde{f}_{i-1}> = \begin{pmatrix} \mu_{i-1} & 1 \\ 0 & 0 \end{pmatrix} \tilde{N}_{c,i}, \quad <\Phi_{i-1}> = \begin{pmatrix} 0 \\ 0 \end{pmatrix} M_{i-1} (N_{c,i-1} - N_{c,i-2} - \tilde{N}_{c,i}).
\]

4.1.2 When the dual gauge group occurs at even chain

Let us consider other magnetic theory for the same electric theory. Starting from Figure 16B and interchanging the NS5'_{i-1}-brane and the NS5_{i}-brane(and their mirrors), one obtains the Figure 18A.

![Figure 18: The \(\mathcal{N} = 1\) magnetic brane configuration for the gauge group containing \(SU(\tilde{N}_{c,i} = N_{c,i-1} + N_{c,i+1} - N_{c,i})\) with D4- and \(D4\)-branes(18A) and with a misalignment between D4-branes(18B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 18B can be written as \(N_{c,i} - N_{c,i-1} - N_{c,i+2} = (N_{c,i+1} - N_{c,i+2}) - \tilde{N}_{c,i}\). By reconnecting the \((N_{c,i+1} - N_{c,i+2})\) D4-branes connecting between the NS5_{i}-brane and the NS5'_{i-1}-brane with those D4-branes connecting between NS5'_{i-1}-brane and the NS5'_{i+1}-brane where we introduce \(-N_{c,i+2}\) D4-branes and \(-N_{c,i+2}\) anti D4-branes and moving those combined D4-branes to \(+v\)-direction(and their mirrors to \(-v\) direction), one gets the final
Figure 18A where we are left with \((N_{c,i} - N_{c,i+2} - N_{c,i-1})\) anti-D4-branes between the \(NS5_i\)-brane and the \(NS5_{i-1}'\)-brane. We assume that the number of colors satisfies \(N_{c,i+1} \geq N_{c,i} - N_{c,i-1} \geq N_{c,i+2}\).

The dual gauge group is given by

\[
Sp(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n}).
\] (4.7)

The matter contents are the field \(f_i\) charged under \((1, \cdots, \Box_i, \Box_{i+1}, \cdots, 1_n)\), and their conjugates \(\tilde{f}_i\) charged \((1_1, \cdots, \Box_i, \Box_{i+1}, \cdots, 1_n)\) under the dual gauge group (4.7) and the gauge-singlet \(\Phi_{i+1}\) which is in the adjoint representation for the \(i\)-th dual gauge group, in other words, \((1_1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)\) under the dual gauge group (4.7) where the gauge group is broken from \(SU(N_{c,i+1})\) to \(SU(N_{c,i+1} - N_{c,i+2})\). Then the \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})\) matrix. Only \((N_{c,i+1} - N_{c,i+2})\) D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term (1.3) and (1.4) is given by

\[
W_{\text{dual}} = \Phi_{i+1} f_i \tilde{f}_i + m_{i+1} \text{tr} \Phi_{i+1}
\] (4.8)

where we define \(\Phi_{i+1}\) as \(\Phi_{i+1} = \tilde{F}_i \tilde{F}_i\) and the \(i\)-th gauge group indices in \(F_i\) and \(\tilde{F}_i\) are contracted, each \((i+1)\)-th gauge group index in them is encoded in \(\Phi_{i+1}\). Here the magnetic fields \(f_i\) and \(\tilde{f}_i\) correspond to 4-4 strings connecting the \(\tilde{N}_{c,i}\)-color D4-branes (that are connecting between the \(NS5_i\)-brane and the \(NS5_{i-1}'\)-brane in Figure 18B) with \(N_{c,i+1}\)-color D4-branes. Among these \(N_{c,i+1}\)-color D4-branes, only the strings ending on the upper \((N_{c,i-1} + N_{c,i+1} - N_{c,i})\) D4-branes and on the tilted \((N_{c,i} - N_{c,i+2} - N_{c,i-1})\) D4-branes in Figure 18B enter the cubic superpotential term (4.8).

When the upper \(NS5\)-brane (or \(NS5_{i-1}'\)-brane) is replaced by coincident \((N_{c,i+1} - N_{c,i+2})\) D6-branes with a rotation of \(NS5_{i+2}\)-brane in Figure 18B, this brane configuration looks similar to the one found in [23] where the gauge group was given by \(Sp(n_{c,1}) \times \cdots \times SU(n_{f,i} + n_{c,i+1} + n_{c,i-1} - n_{c,i}) \times SU(n_{c,i+1}) \times \cdots\) with \(n_{f,i}\) multiplets, \(\tilde{n}_{f,i}\) multiplets, bifundamentals and gauge singlets. Then the present \((N_{c,i+1} - N_{c,i+2})\) corresponds to the \(n_{f,i}\), \(N_{c,i}\) corresponds to \(n_{c,i}\), \(N_{c,i-1}\) corresponds to \(n_{c,i-1}\) and \(N_{c,i+2}\) corresponds to \(n_{c,i+1}\). The low energy dynamics of the magnetic brane configuration can be described by the \(\mathcal{N} = 1\) supersymmetric gauge theory with gauge group (4.7) and the gauge couplings for the three gauge group factors are given by similarly. The dual gauge theory has a meson field \(\Phi_{i+1}\) and bifundamentals \(f_i, \tilde{f}_i\)

\[\text{Of course, there are also the extra terms in the superpotential } \Phi' f_{i-1} f_i + \Phi'' \tilde{f}_{i-1} \tilde{f}_i + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} \text{ where we define } \Phi' \equiv F_i F_{i-1} \text{ and } \Phi'' \equiv \tilde{F}_i \tilde{F}_{i-1}, \text{ coming from different bifundamentals. However, the } F\text{-term conditions, } \Phi' f_i + \Phi_{i-1} \tilde{f}_{i-1} = 0 = \Phi'' \tilde{f}_i + \Phi_{i-1} f_{i-1} \text{ lead to } <\Phi'> = <\Phi''> = <f_{i-1}> = <\tilde{f}_{i-1}> = 0. \text{ Then, these extra terms do not contribute to the one loop computation up to quadratic order.}\]
under the dual gauge group \((4.7)\) and the superpotential \((4.8)\) corresponding to Figures 18A and 18B is given by

\[
W_{\text{dual}} = h\Phi_{i+1}f_i\tilde{f}_i - h\mu_{i+1}^2 \text{tr} \Phi_{i+1}, \quad h^2 = g_{i+1, \text{mag}}, \quad \mu_{i+1}^2 = \frac{(\Delta x)_{i+1}}{2\pi g_s l_s^3}.
\]

Then \(f_i\tilde{f}_i\) is a \(\widetilde{N}_{c,i} \times \widetilde{N}_{c,i}\) matrix where the \((i+1)\)-th gauge group indices for \(f_{i+1}\) and \(\tilde{f}_{i+1}\) are contracted with those of \(\Phi_{i+1}\) while \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2})\) matrix.

Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{i+1}\) cannot be satisfied if the \((N_{c,i+1} - N_{c,i+2})\) exceeds \(\tilde{N}_{c,i}\). So the supersymmetry is broken.

That is, there exist three equations from F-term conditions: \(f^a_i\tilde{f}_{i,b} - \mu_{i+1}^2 \delta^a_b = 0\) and \(\Phi_{i+1} f_i = 0 = \tilde{f}_i \Phi_{i+1}\). Then the solutions for these are given by

\[
< f_i > = \left( \begin{array}{c} \mu_{i+1}^1 \widetilde{N}_{c,i} \\ 0 \end{array} \right), \quad < \tilde{f}_i > = \left( \begin{array}{c} \mu_{i+1}^1 \widetilde{N}_{c,i} \\ 0 \end{array} \right),
\]

\[
< \Phi_{i+1} > = \left( \begin{array}{cc} 0 & 0 \\ 0 & M_{i+1}(N_{c,i+1} - N_{c,i+2}) - \widetilde{N}_{c,i} \end{array} \right).
\]

4.2  \(\mathcal{N} = 1\) Sp\((N_{c,1}) \times \cdots \times SU(\widetilde{N}_{c,n})\) magnetic theory

4.2.1 When the dual gauge group occurs at odd chain

Starting from Figure 15A, moving the \(NS5'_n\) -brane with \((N_{c,n-1} - N_{c,n-2})\) D4-branes to the \(+v\) direction leading to Figure 16A, and interchanging the \(NS5_{n-1}\)-brane and the \(NS5'_{n}\)-brane, one obtains the Figure 19A.

Figure 19: The \(\mathcal{N} = 1\) magnetic brane configuration for the gauge group containing \(SU(\widetilde{N}_{c,n} = N_{c,n-1} - N_{c,n})\) with D4- and \(\overline{D4}\)-branes(19A) and with a misalignment between D4-branes(19B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 19B can be written as \(N_{c,n} - N_{c,n-2} = (N_{c,n-1} - N_{c,n-2}) - \widetilde{N}_{c,n}\).

By introducing \(-N_{c,n-2}\) D4-branes and \(-N_{c,n-2}\) anti-D4-branes between the \(NS5'_n\)-brane and \(NS5_{n-1}\)-brane, reconnecting the former with the \(N_{c,n-1}\) D4-branes connecting between
NS5\textsubscript{\textprime} N-brane and the NS5\textsubscript{n-1} N-brane (therefore \((N_{c,n-1}-N_{c,n-2})\) D4-branes) and moving those combined \((N_{c,n-1}-N_{c,n-2})\) D4-branes to +\(v\)-direction, one gets the final Figure 19A where we are left with \((N_{c,n}-N_{c,n-2})\) anti-D4-branes between the NS5\textsubscript{\textprime} N-brane and NS5\textsubscript{n-1} N-brane.

The dual gauge group is given by

\[ Sp(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n}) \]  

(4.9)

and the matter contents are the bifundamentals \(f_{n-1}\) in \((\mathbf{1}, \cdots, \mathbf{1}_{n-2}, \mathbf{N}_{n-1}, \mathbf{n})\), and \(\tilde{f}_{n-1}\) in the representation \((\mathbf{1}, \cdots, \mathbf{1}_{n-2}, \mathbf{N}_{n-1}, \mathbf{n})\) in addition to \((n-2)\) bifundamentals \(F_{j}\) and \(\tilde{F}_{j}\), \(j = 1, 2, \cdots, (n-2)\) and the gauge singlet \(\Phi_{n-1}\) for the \(n\)-th dual gauge group is in the representation \((\mathbf{1}, \cdots, \mathbf{1}_{n-2}, (N_{c,n-1}-N_{c,n-2})^2-1, \mathbf{1}_n)\) plus a singlet under the dual gauge group (4.9) where the gauge group is broken from \(SU(N_{c,n-1})\) to \(SU(N_{c,n-1} - N_{c,n-2})\).

The cubic superpotential with the mass term in the dual theory is given by

\[ W_{\text{dual}} = \Phi_{n-1} f_{n-1} \tilde{f}_{n-1} + m_{n-1} \text{tr} \Phi_{n-1} \]  

(4.10)

Here the magnetic fields \(f_{n-1}\) and \(\tilde{f}_{n-1}\) correspond to 4-4 strings connecting the \(\tilde{N}_{c,n}\)-color D4-branes(that are connecting between the NS5\textsubscript{\textprime} \(n\)-2-brane and the NS5\textsubscript{n-1} \(n\)-brane in Figure 19B) with \(N_{c,n-1}\)-flavor D4-branes(that are a combination of three different D4-branes in Figure 19B). Among these \(N_{c,n-1}\)-flavor D4-branes, only the strings ending on the upper \((N_{c,n-1}-N_{c,n})\) D4-branes and on the tilted \((N_{c,n} - N_{c,n-2})\) D4-branes in Figure 19B enter the cubic superpotential term (4.10).

When the upper NS5\textsubscript{\textprime} \(n\)-brane(or NS5\textsubscript{\textprime} \(n\)-2-brane) is replaced by coincident \((N_{c,n-1}-N_{c,n-2})\) D6-branes and the NS5\textsubscript{n-3} is rotated by \(\frac{\pi}{2}\) in Figure 19B, this brane configuration looks similar to the one found in [23] where the gauge group was given by \(Sp(n_{c,1}) \times \cdots \times SU(n_{c,n-1}) \times SU(n_{f,n}+n_{c,n-1}-n_{c,n})\) with \(n_{f,n}\) multiplets, \(\tilde{n}_{f,n}\) multiplets, bifundamentals and singlets. Then the present \((N_{c,n-1}-N_{c,n-2})\) corresponds to the \(n_{f,n}\), \(N_{c,n}\) corresponds to \(n_{c,n}\), and \(N_{c,n-2}\) corresponds to the \(n_{c,n-1}\). The dual gauge theory has a meson field \(\Phi_{n-1}\) and bifundamentals \(f_{n-1}, \tilde{f}_{n-1}, F_{j}\), and \(\tilde{F}_{j}\) and the superpotential corresponding to Figures 19A and 19B is given by

\[ W_{\text{dual}} = h \Phi_{n-1} f_{n-1} \tilde{f}_{n-1} - h \mu_{n-1}^2 \text{tr} \Phi_{n-1}, \quad \mu_{n-1}^2 = g_{n-1, \text{mag}}^2, \quad h^2 = g_{n-1, \text{mag}}^2 \frac{(\Delta x)^{n-1}}{2\pi g_s^3}. \]

Then \(f_{n-1} \tilde{f}_{n-1}\) is a \(\tilde{N}_{c,n} \times \tilde{N}_{c,n}\) matrix where the \((n-1)\)-th gauge group indices for \(f_{n-1}\) and \(\tilde{f}_{n-1}\) are contracted with those of \(\Phi_{n-1}\) while \(\Phi_{n-1}\) is a \((N_{c,n-1}-N_{c,n-2}) \times (N_{c,n-1}-N_{c,n-2})\) matrix. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{n-1}\) cannot be satisfied if the \((N_{c,n-1}-N_{c,n-2})\) exceeds \(\tilde{N}_{c,n}\). So the supersymmetry is broken.
That is, there exist three equations from F-term conditions: \( f_{n-1}^a \tilde{f}_{n-1,b} - \mu_{n-1}^2 \delta_b^a = 0 \), and \( \Phi_{n-1} f_{n-1} = 0 = \tilde{f}_{n-1} \Phi_{n-1} \). Then the solutions for these are given by

\[
< f_{n-1} > = \begin{pmatrix} \mu_{n-1} 1_{N_{c,n}} \\ 0 \end{pmatrix}, \quad < \tilde{f}_{n-1} > = \begin{pmatrix} \mu_{n-1} 1_{N_{c,n}} \\ 0 \end{pmatrix}, \quad < \Phi_{n-1} > = \begin{pmatrix} 0 & 0 \\ 0 & M_{n-1} 1_{(N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n})} \end{pmatrix}.
\]

### 4.2.2 When the dual gauge group occurs at even chain

Let us consider other magnetic theory for the same electric theory. By applying the Seiberg dual to the \( SU(N_{c,n}) \) factor and interchanging the \( NS5'_{n-1} \)-brane and the \( NS5_n \)-brane, one obtains the Figure 20A.

![Figure 20: The \( N = 1 \) magnetic brane configuration for the gauge group containing \( SU(N_{c,n}) \) with D4- and \( \overline{D4} \)-branes(20A) and with a misalignment between D4-branes(20B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 20B can be written as \( N_{c,n} - N_{c,n-2} = (N_{c,n-1} - N_{c,n-2} - \tilde{N}_{c,n}) \). By rotating \( NS5_{n-2} \)-brane by an angle \( \frac{\pi}{2} \) which will become \( NS5'_{n-2} \)-brane, moving it with the \( (N_{c,n-1} - N_{c,n-2}) \) D4-branes to \( +y \) direction where we introduce \( (N_{c,n} - N_{c,n-2}) \) D4-branes and \( (N_{c,n} - N_{c,n-2}) \) anti D4-branes between the \( NS5_n \)-brane and the \( NS5'_{n-1} \)-brane, one gets the final Figure 20A where we are left with \( (N_{c,n} - N_{c,n-2}) \) anti-D4-branes between the \( NS5_n \)-brane and the \( NS5'_{n-1} \)-brane.

The gauge group is given by

\[
SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(N_{c,n}) \equiv N_{c,n-1} - N_{c,n}
\] (4.11)

and the matter contents are the field \( f_{n-1} \) charged under \( (1_1, \cdots, 1_{n-2}, \Box_{n-1}, \Box_n) \) and their conjugates \( \tilde{f}_{n-1} \) \( (1_1, \cdots, 1_{n-2}, \Box_{n-1}, \Box_n) \) under the dual gauge group and the gauge-singlet \( \Phi_{n-1} \) which is in the adjoint representation for the \( (n - 1) \)-th gauge group, in other
words, \((1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n) \oplus (1_1, \cdots, 1_n)\) under the dual gauge group \(\mathbb{Z}_2\) where the gauge group is broken from \(SU(N_{c,n-1})\) to \(SU(N_{c,n-1} - N_{c,n-2})\). Then the \(\Phi_{n-1}\) is a \((N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})\) matrix. Only \((N_{c,n-1} - N_{c,n-2})\) D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term is given by

\[
W_{\text{dual}} = \Phi_{n-1} f_{n-1} \widetilde{f}_{n-1} + m_{n-1} \text{tr} \Phi_{n-1}
\]

(4.12)

where we define \(\Phi_{n-1}\) as \(\Phi_{n-1} = F_{n-1} \widetilde{F}_{n-1}\) and the \(n\)-th gauge group indices in \(F_{n-1}\) and \(\widetilde{F}_{n-1}\) are contracted, each \((n-1)\)-th gauge group index in them is encoded in \(\Phi_{n-1}\). Here the magnetic fields \(f_{n-1}\) and \(\widetilde{f}_{n-1}\) correspond to 4-4 strings connecting the \(\widetilde{N}_{c,n}\)-color D4-branes (that are connecting between the NS5\(_{n-2}'\)-brane and the NS5\(_n\)-brane in Figure 20B) with \(N_{c,n-1}\)-flavor D4-branes. Among these \(N_{c,n-1}\)-flavor D4-branes, only the strings ending on the upper \((N_{c,n-1} - N_{c,n})\) D4-branes and on the tilted \((N_{c,n} - N_{c,n-2})\) D4-branes in Figure 20B enter the cubic superpotential term \((4.12)\).

The low energy dynamics of the magnetic brane configuration can be described by the \(\mathcal{N} = 1\) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by similarly. The dual gauge theory has a gauge singlet \(\Phi_{n-1}\) and bifundamentals \(f_{n-1}, \widetilde{f}_{n-1}, F_j\) and \(\widetilde{F}_j\) under the dual gauge group and the superpotential corresponding to Figures 20A and 20B is given by

\[
W_{\text{dual}} = h \Phi_{n-1} f_{n-1} \widetilde{f}_{n-1} - h \mu_{n-1}^2 \text{tr} \Phi_{n-1}, \quad h^2 = g_{n-1, \text{mag}}, \quad \mu_{n-1}^2 = -\frac{(\Delta x)_{n-1}}{2\pi g_s s^3}.
\]

Then \(f_{n-1} \widetilde{f}_{n-1}\) is a \(\widetilde{N}_{c,n} \times \widetilde{N}_{c,n}\) matrix where the \((n-1)\)-th gauge group indices for \(f_{n-1}\) and \(\widetilde{f}_{n-1}\) are contracted with those of \(\Phi_{n-1}\) while the \(\Phi_{n-1}\) is a \((N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})\) matrix. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{n-1}\) cannot be satisfied if the \((N_{c,n-1} - N_{c,n-2})\) exceeds \(\widetilde{N}_{c,n}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f_{n-1}^a \tilde{\Phi}_{n-1,b} - \mu_{n-1}^2 \delta^a_b = 0\) and \(\Phi_{n-1} f_{n-1} = 0 = \tilde{\Phi}_{n-1} \Phi_{n-1}\). Then the solutions for these are given by

\[
< f_{n-1} > = \left( \begin{array}{c} \mu_{n-1}^{-1} \mathbf{1}_{\widetilde{N}_{c,n}} \\ 0 \end{array} \right), \quad < \widetilde{f}_{n-1} > = \left( \begin{array}{c} \mu_{n-1} \mathbf{1}_{\widetilde{N}_{c,n}} \\ 0 \end{array} \right),
\]

\[
< \Phi_{n-1} > = \left( \begin{array}{cc} 0 & 0 \\ M_{n-1} \mathbf{1}_{(N_{c,n-1} - N_{c,n-2}) - \widetilde{N}_{c,n}} & 0 \end{array} \right).
\]

4.3 \(\mathcal{N} = 1\) \(Sp(N_{c,1}) \times SU(\widetilde{N}_{c,2}) \times \cdots \times SU(N_{c,n})\) magnetic theory

Let us consider the Seiberg dual for the second gauge group factor. Starting from Figure 15A, moving the \(NS5'_{3}\)-brane with \((N_{c,3} - N_{c,4})\) D4-branes to the \(+v\) direction leading to Figure
16A, and interchanging the $NS5'_1$-brane and the $NS5_2$-brane, one obtains the Figure 21A.

![Figure 21](image)

**Figure 21:** The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,2} = 2N_{c,1} + N_{c,3} - N_{c,2})$ with D4- and $\tilde{D}$4-branes(21A) and with a misalignment between D4-branes(21B) when the two $NS5'_1$-branes are close to each other. The number of tilted D4-branes in 21B can be written as $N_{c,2} - N_{c,4} - 2N_{c,1} = (N_{c,3} - N_{c,4}) - \tilde{N}_{c,2}$.

By introducing $-N_{c,4}$ D4-branes and $-N_{c,4}$ anti-D4-branes between the $NS5_2$-brane and $NS5'_1$-brane, reconnecting the former with the $N_{c,3}$ D4-branes connecting between $NS5_2$-brane and the $NS5'_1$-brane (therefore $(N_{c,3} - N_{c,4})$ D4-branes) and moving those combined $(N_{c,2} - N_{c,4} - 2N_{c,1})$ anti-D4-branes between the $NS5_2$-brane and $NS5'_1$-brane.

The dual gauge group is given by

$$Sp(N_{c,1}) \times SU(\tilde{N}_{c,2} = 2N_{c,1} + N_{c,3} - N_{c,2}) \times SU(N_{c,3}) \times \cdots \times SU(N_{c,n})$$

where the matter contents are the bifundamentals $f_2$ in $(1_1, \square_2, \square_3, 1, \cdots, 1_n)$, and $\tilde{f}_2$ in the representation $(1_1, \square_2, \square_3, 1, \cdots, 1_n)$ in addition to $(n-2)$ bifundamentals $F_j$ and $\tilde{F}_j$, $j = 1, 3, \cdots, (n-1)$ and the gauge singlet $\Phi_3$ for the second dual gauge group in the adjoint representation for the third dual gauge group, i.e., $(1_1, 1_2, (N_{c,3} - N_{c,4})^2 - 1, 1, \cdots, 1_n)$ plus a singlet under the dual gauge group where the gauge group is broken from $SU(N_{c,3})$ to $SU(N_{c,3} - N_{c,4})$.

The cubic superpotential with the mass term in the dual theory is given by

$$W_{\text{dual}} = \Phi_3 f_2 \tilde{f}_2 + m_3 \text{tr} \Phi_3.$$  

Here the magnetic fields $f_2$ and $\tilde{f}_2$ correspond to 4-4 strings connecting the $\tilde{N}_{c,2}$-color D4-branes (that are connecting between the $NS5_2$-brane and the $NS5'_2$-brane in Figure 21B) with $N_{c,3}$-flavor D4-branes (that are a combination of three different D4-branes in Figure 21B). Among these $N_{c,3}$-flavor D4-branes, only the strings ending on the upper $(2N_{c,1} + N_{c,3} - N_{c,2})$
D4-branes and on the tilted \((N_{c,2} - N_{c,4} - 2N_{c,1})\) D4-branes in Figure 21B enter the cubic superpotential term \((4.14)\).

When the upper NS5'-brane(or NS5'_r-brane) is replaced by coincident \((N_{c,3} - N_{c,4})\) D6-branes and the NS5_q is rotated by an angle \(\frac{\pi}{2}\) in the \((v, w)\) plane in Figure 21B, this brane configuration reduces to the one found in \([23]\) where the gauge group was given by \(Sp(n_{c,1}) \times SU(n_{f,2} + n_{c,3} + 2n_{c,1} - n_{c,2}) \times SU(n_{c,3}) \times \cdots\) with \(n_{f,i}\) multiplets, \(n_{f,i}\) multiplets, bifundamentals and gauge singlets. Then the present number \((N_{c,3} - N_{c,4})\) corresponds to the \(n_{f,2}\), the number \(N_{c,2}\) corresponds to \(n_{c,2}\), the number \(N_{c,4}\) corresponds to \(n_{c,3}\), and the number \(N_{c,1}\) corresponds to the \(n_{c,1}\). The dual gauge theory has a meson field \(\Phi_3\) and bifundamentals \(f_2, \tilde{f}_2, F_j\), and \(\tilde{F}_j\) under the dual gauge group \((4.13)\) and the superpotential \((4.14)\) corresponding to Figures 21A and 21B is given by

\[
W_{\text{dual}} = h\Phi_3 f_2 \tilde{f}_2 - h\mu_3^2 \text{tr} \Phi_3, \quad h^2 = g_{3, \text{mag}}, \quad \mu_3^2 = \frac{(\Delta x)_3}{2\pi g_s f_s^3}.
\]

Then \(f_2 \tilde{f}_2\) is a \(\tilde{N}_{c,2} \times \tilde{N}_{c,2}\) matrix where the third gauge group indices for \(f_2\) and \(\tilde{f}_2\) are contracted with those of \(\Phi_3\) while the \(\Phi_3\) is a \((N_{c,3} - N_{c,4}) \times (N_{c,3} - N_{c,4})\) matrix. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_3\) cannot be satisfied if the \((N_{c,3} - N_{c,4})\) exceeds \(\tilde{N}_{c,2}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f_2^i \tilde{f}_2^b - \mu_3^2 \delta^i_b = 0\), and \(\Phi_3 f_2 = 0 = \tilde{f}_2 \Phi_3\). Then the solutions for these are given by

\[
<f_2 > = \begin{pmatrix} \mu_3 1_{\tilde{N}_{c,2}} \\ 0 \end{pmatrix}, \quad <\tilde{f}_2 > = \begin{pmatrix} \mu_3 1_{\tilde{N}_{c,2}} \\ 0 \end{pmatrix},
\]

\[
<\Phi_3 > = \begin{pmatrix} 0 \\ 0 \\ M_{31}(N_{c,3} - N_{c,4} - \tilde{N}_{c,2}) \end{pmatrix}.
\]

5 Meta-stable brane configurations with 2\(n\) NS-branes plus \(O6^+\)-plane

The type IIA brane configuration, by generalizing the brane configurations \([62, 18]\) to the case where there are more NS-branes, corresponding to \(\mathcal{N} = 1\) supersymmetric electric gauge theory(see also \([23]\)) with gauge group

\[
SO(N_{c,1}) \times SU(N_{c,2}) \cdots \times SU(N_{c,n})
\]

and with the \((n - 1)\) bifundamentals \(F_i\) charged under \((1_1, \cdots, 1, \square_i, \square_{i+1}, 1, \cdots, 1_n)\) and their complex conjugate fields \(\tilde{F}_i\) charged \((1_1, \cdots, 1, \bar{\square}_i, \bar{\square}_{i+1}, 1, \cdots, 1_n)\) where \(i = 1, 2, \cdots, (n - 1)\)
can be described by the $NS5_1$-brane, the $NS5'_2$-brane, $\cdots$, the $NS5_n$-brane for odd number of gauge groups (or the $NS5'_n$-brane for even number of gauge groups), $Nc,1$, $Nc,2$, $\cdots$, and $Nc,n$-color D4-branes. See the Figure 22 for the details on the brane configuration. The $O6^+$-plane acts as $(x^4,x^5,x^6) \rightarrow (-x^4,-x^5,-x^6)$ and has RR charge $+4$.

Figure 22: The $\mathcal{N}=1$ supersymmetric electric brane configuration for the gauge group $SO(Nc,1) \times \prod_{i=2}^{n} SU(Nc,i)$ and bifundamentals $F_i$ and $\tilde{F}_i$ with vanishing mass for the bifundamentals when the number of gauge groups factor $n$ is odd(22A) and even(22B). We do not draw the mirrors of the branes appearing in the left hand side of $O6^+$-plane.

Let us place an $O6$-plane at the origin $x^6 = 0$ and denote the $x^6$ coordinates for the $NS5_1$-brane, $\cdots$, the $NS5_n$-brane for odd $n$ (or the $NS5'_n$-brane for even $n$) are given by $x^6 = y_1, y_1 + y_2, \cdots, \sum_{j=1}^{n-1} y_j + y_n$ respectively. The $Nc,1$ D4-branes are suspended between the $NS5_1$-brane and its mirror, the $Nc,2$ D4-branes are suspending between the $NS5_1$-brane and the $NS5'_2$-brane, $\cdots$ and the $Nc,n$ D4-branes are suspended between the $NS5'_n-1$-brane and the $NS5_n$-brane for odd $n$ (or between the $NS5_{n-1}$-brane and the $NS5'_n$-brane for even $n$). The fields $F_i$ and $\tilde{F}_i$ correspond to 4-4 strings connecting the $Nc,i$-color D4-branes with $Nc,i+1$-color D4-branes. We draw this $\mathcal{N}=1$ supersymmetric electric brane configuration in Figure 22A(22B) when $n$ is odd(even) for the vanishing mass for the fields $F_i$ and $\tilde{F}_i$.

There is no superpotential in Figure 22A. Let us deform this theory. Displacing the two NS5'-branes relative each other in the $+v$ direction, characterized by $(\Delta x)_{i+1}$, corresponds to turning on a quadratic mass-deformed superpotential for the field $F_i$ and $\tilde{F}_i$ as follows:

$$W = m_{i+1}F_i\tilde{F}_i (\equiv m_{i+1}\Phi_{i+1})$$

where the $i$-th gauge group indices in $F_i$ and $\tilde{F}_i$ are contracted and

$$m_{i+1} = \frac{(\Delta x)_{i+1}}{\ell_s^2}.$$  

The gauge-singlet $\Phi_{i+1}$ for the $i$-th gauge group is in the adjoint representation for the $(i+1)$-th gauge group, i.e., \((1_1, \cdots, 1_i, (Nc,i+1 - Nc,i+2)^2 - 1, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)\) under the gauge
group where the gauge group is broken from $SU(N_{c,i+1})$ to $SU(N_{c,i+1} - N_{c,i+2})$. The $\Phi_{i+1}$

is a $(N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})$ matrix. The $NS5'_{i+1}$-brane together with $(N_{c,i+1} - N_{c,i+2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation (and their mirrors to $-v$ direction). Then the $x^5$ coordinate of $NS5'_{i-1}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i-1}$-branes is given by $(\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i+1}$ corresponds to recombination of $N_{c,i}$- and $N_{c,i+1}$-color D4-branes, which will become $N_{c,i}$ or $N_{c,i+1}$-color D4-branes in Figure 22A such that they are suspended between the $NS5'_{i-1}$-brane and the $NS5'_{i+1}$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i+1} \geq N_{c,i} - N_{c,i-1} \geq N_{c,i+2}$.

Now we draw this brane configuration in Figure 23A for nonvanishing mass for the fields $F_i$ and $\tilde{F}_i$.

Let us deform the theory by Figure 22B. Displacing the two NS5'-branes, the $NS5'_{i-2}$-brane and the $NS5'_{i-2}$-brane, relative each other in the $v$ direction, characterized by $(\Delta x)_{i-1}$, corresponds to turning on a quadratic mass-deformed superpotential for the fields $F_{i-1}$ and $\tilde{F}_{i-1}$ as follows:

$$W = m_{i-1} F_{i-1} \tilde{F}_{i-1} (\equiv m_{i-1} \Phi_{i-1})$$

where the $i$-th gauge group indices in $F_{i-1}$ and $\tilde{F}_{i-1}$ are contracted, each $(i-1)$-th gauge

![Figure 23: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SO(N_{c,1}) \times \prod_{i=2}^{n} SU(N_{c,i})$ and bifundamentals $F_{i}$ and $\tilde{F}_{i}$ with nonvanishing mass for the bifundamentals when the number of gauge groups factor $n$ is odd(23A) and even(23B). The $N_{c,i+1}$ D4-branes in 23A are decomposed into $(N_{c,i+1} - N_{c,i+2})$ D4-branes which are moving to $+v$ direction in 23A and $N_{c,i+2}$ D4-branes which are recombined with those D4-branes connecting between $NS5'_{i+1}$-brane and $NS5_{i+2}$-brane in 23A. The $N_{c,i-1}$ D4-branes in 23B are decomposed into $(N_{c,i-1} - N_{c,i-2})$ D4-branes which are moving to $+v$ direction in 23B and $N_{c,i-2}$ D4-branes which are recombined with those D4-branes connecting between $NS5_{i-3}$-brane and $NS5'_{i-2}$-brane in 23B.](image-url)
group index in them is encoded in $\Phi_{i-1}$ and the mass $m_{i-1}$ is given by
\[ m_{i-1} = \frac{(\Delta x)_{i-1}}{\ell_s^2}. \]

The gauge-singlet $\Phi_{i-1}$ for the $i$-th gauge group is in the adjoint representation for the $(i-1)$-th gauge group, i.e.,
\[
(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1_i, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)
\]
under the gauge group where the gauge group is broken from $SU(N_{c,i-1})$ to $SU(N_{c,i-1} - N_{c,i-2})$. Then the $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix. The $NS5'_{i-2}$-brane together with $(N_{c,i-1} - N_{c,i-2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation. In other words, the $N_{c,i-2}$ D4-branes among $N_{c,i-1}$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate (\(\equiv x\)) of $NS5'_{i}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i-2}$-brane is given by $(\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i-1}$ corresponds to recombination of $N_{c,i-1}$- and $N_{c,i}$-color D4-branes, which will become $N_{c,i-1}$- or $N_{c,i}$-color D4-branes in Figure 22B such that they are suspended between the $NS5'_{i-2}$-brane and the $NS5'_{i}$-brane and pushing them into the $w$ direction. Now we draw this brane configuration in Figure 23B for nonvanishing mass for the fields $F_{i-1}$ and $\tilde{F}_{i-1}$.

5.1 $\mathcal{N} = 1 \ SO(N_{c,1}) \times SU(\tilde{N}_{c,2}) \times \cdots \times SU(N_{c,n})$ magnetic theory

By applying the Seiberg dual to the $SU(N_{c,2})$ factor and interchanging the $NS5_1$-brane and the $NS5'_2$-brane (and their mirrors), one obtains the Figure 24A.

Figure 24: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,2} = N_{c,1} + N_{c,3} - N_{c,2})$ with D4- and $\overline{D}4$-branes (24A) and with a misalignment between D4-branes (24B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 24B can be written as $N_{c,2} - N_{c,4} - N_{c,1} = (N_{c,3} - N_{c,4}) - \tilde{N}_{c,2}$. 

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By rotating $NS5'$-brane by an angle $\frac{\pi}{2}$, moving it with $(N_{c,3} - N_{c,4})$ D4-branes to $\pm v$-direction (and their mirrors to $-v$ direction), one gets the final Figure 24A where we are left with $(N_{c,2} - N_{c,1} - N_{c,4})$ anti-D4-branes between the $NS5'_2$-brane and $NS5_1$-brane.

The dual gauge group is given by

$$SO(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{c,1} + N_{c,3} - N_{c,2}) \times SU(N_{c,3}) \times \cdots \times SU(N_{c,n}).$$

The matter contents are the field $f_2$ charged under $(1_1, \tilde{N}_{c,2}, \tilde{N}_{c,3}, \cdots, 1_n)$ and their conjugates $\tilde{f}_2$ charged under $(1_1, \tilde{N}_{c,2}, N_{c,3}, \cdots, 1_n)$ under the dual gauge group and the gauge-singlet $\Phi_3$ for the second dual gauge group in the adjoint representation for the third dual gauge group, i.e., $(1_1, 1_2, (N_{c,3} - N_{c,4})^2 - 1, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)$ under the dual gauge group where the gauge group is broken from $SU(N_{c,3})$ to $SU(N_{c,3} - N_{c,4})$. Then the $\Phi_3$ is a $(N_{c,3} - N_{c,4}) \times (N_{c,3} - N_{c,4})$ matrix.

The cubic superpotential with the mass term is given by (4.14) where we define $\Phi_3$ as $\Phi_3 \equiv F_2 \tilde{F}_2$ and the second gauge group indices in $F_2$ and $\tilde{F}_2$ are contracted, each third gauge group index in them is encoded in $\Phi_3$. Here the magnetic fields $f_2$ and $\tilde{f}_2$ correspond to 4-4 strings connecting the $\tilde{N}_{c,2}$-color D4-branes (that are connecting between the $NS5_1$-brane and the $NS5'_3$-brane in Figure 24B) with $N_{c,3}$-flavor D4-branes (which are realized as corresponding D4-branes in Figure 24A).

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by the expressions in subsection 4.3. The dual gauge theory has a meson field $\Phi_3$ and bifundamentals $f_2$ and $\tilde{f}_2$ under the dual gauge group and the superpotential corresponding to Figures 24A and 24B is given by the expressions in subsection 4.3. Then $f_2 \tilde{f}_2$ is a $\tilde{N}_{c,2} \times \tilde{N}_{c,2}$ matrix where the third gauge group indices for $f_2$ and $\tilde{f}_2$ are contracted with those of $\Phi_3$ while $\Phi_3$ is a $(N_{c,3} - N_{c,4}) \times (N_{c,3} - N_{c,4})$ matrix. The product $f_2 \tilde{f}_2$ has the same representation for the product of quarks and moreover, the third gauge group indices for the field $\Phi_3$ play the role of the flavor indices.

When the upper $NS5'$-brane (or $NS5'_3$-brane) is replaced by coincident $(N_{c,3} - N_{c,4})$ D6-branes and the $NS5'_1$-brane is rotated by $\frac{\pi}{2}$ in the $(v, w)$ plane in Figure 24B, this brane configuration looks similar to the one found in [23] where the gauge group was given by $SO(n_{c,1}) \times SU(n_{f,2} + n_{c,1} + n_{c,3} - n_{c,2}) \times SU(n_{c,3}) \times \cdots$ with $n_{f,i}$ multiplets, $\tilde{n}_{f,i}$ multiplets, bifundamentals and gauge singlets. Then the present $n_{c,4}$ corresponds to the $n_{c,3}$, $n_{c,2}$ corresponds to $n_{c,2}$, $N_{c,1}$ corresponds to $n_{c,1}$, and $(N_{c,3} - N_{c,4})$ corresponds to the $n_{f,2}$. Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi_3$ cannot be satisfied if the $(N_{c,3} - N_{c,4})$ exceeds $\tilde{N}_{c,2}$. So the supersymmetry is broken. That is, there exist
three equations from F-term conditions: \( f_a^2 \tilde{f}_{2,b} - \mu_3 \delta_b^a = 0 \) and \( \Phi_3 f_2 = 0 = \tilde{f}_2 \Phi_3 \). Then the solutions for these are given by the expressions in subsection 4.3. Then one can expand these fields around on a point and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \( V^{(1)}_{\text{eff}} \) for \( M_3 \) leads to the positive value for \( m_{M_3}^2 \) implying that these vacua are stable.

5.2 \( \mathcal{N} = 1 \) \( SO(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i}) \times \cdots \times SU(N_{c,n}) \) magnetic theory

5.2.1 When the dual gauge group occurs at odd chain

Let us consider other magnetic theory for the same electric theory. Starting from Figure 23A and interchanging the NS5\(_i\)-brane and the NS5\(_i\)-brane (and their mirrors), one obtains the magnetic brane configuration which is exactly the same as the Figure 18A, in subsection 4.1.2., with a replacement of \( O_6^+\)-plane instead of \( O_6^-\)-plane. Let us call this as a “modified” Figure 18A.

The dual gauge group is given by

\[
SO(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n}).
\]

The matter contents are the field \( f_{i-1} \) charged under \( (1, \cdots, \square_{i-1}, \square_i, \cdots, 1_n) \), and their conjugates \( \tilde{f}_{i-1} \) charged \( (1, \cdots, \square_{i-1}, \square_i, \cdots, 1_n) \) under the dual gauge group and the gauge-singlet \( \Phi_{i+1} \) which is in the adjoint representation for the \( i\)-th dual gauge group, in other words, \( (1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, \cdots, 1_n) \oplus (1, \cdots, 1_n) \) under the dual gauge group where the gauge group is broken from \( SU(N_{c,i+1}) \) to \( SU(N_{c,i+1} - N_{c,i+2}) \). Then the \( \Phi_{i+1} \) is a \( (N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2}) \) matrix. Only \( (N_{c,i+1} - N_{c,i+2}) \) D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term is given by (4.8) in subsection 4.1.2 and we define \( \Phi_{i+1} \) as \( \Phi_{i+1} \equiv F_i \tilde{F}_i \) and the \( i\)-th gauge group indices in \( F_i \) and \( \tilde{F}_i \) are contracted, each \( (i+1)\)-th gauge group index in them is encoded in \( \Phi_{i+1} \). The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential (4.8), can be obtained from modified Figure 18A by moving the upper NS5\(_i\)-brane together with \( (N_{c,i+1} - N_{c,i+2}) \) color D4-branes into the origin \( v = 0 \) (and their mirrors).

The low energy dynamics of the magnetic brane configuration can be described by the \( \mathcal{N} = 1 \) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by those in subsection 4.1.2 and the superpotential corresponding to modified Figures 18A and 18B is given by the one in subsection 4.1.2. Therefore, the F-term equation, the derivative \( W_{\text{dual}} \) with respect to the meson field \( \Phi_{i+1} \) cannot be satisfied if the
(N_{c,i+1} - N_{c,i+2}) exceeds \( \tilde{N}_{c,i} \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( f_i^a \tilde{f}_{i,b} - \mu_{i+1}^2 \delta_b^a = 0 \) and \( \Phi_{i+1} f_i = 0 = \tilde{f}_i \Phi_{i+1} \). Then the solutions for these are given by previous results.

5.2.2 When the dual gauge group occurs at even chain

Starting from Figure 15B and interchanging the \( NS5_{i-1} \)-brane and the \( NS5'_i \)-brane (and their mirrors), one obtains the Figure 17A, in subsection 4.1.1, with a replacement of \( O6^+ \)-plane instead of \( O6^- \)-plane. Let us call this as a “modified” Figure 17A.

The dual gauge group is given by

\[
SO(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n})
\]

and the matter contents are the field \( f_{i-1} \) charged under \((1_1, \cdots, 1_{i-2}, N_{c,i-1}, \tilde{N}_{c,i}, \cdots, 1_n)\), and its conjugate field \( \tilde{f}_{i-1} \) charged under \((1_1, \cdots, 1_{i-2}, N_{c,i-1}, \tilde{N}_{c,i}, \cdots, 1_n)\) under the dual gauge group and the gauge-singlet \( \Phi_{i-1} \) for the \( i \)-th dual gauge group in the adjoint representation for the \((i-1)\)-th dual gauge group, i.e., \((1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1, \cdots, 1_n)\) plus a singlet under the dual gauge group. Then the \( \Phi_{i-1} \) is a \((N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})\) matrix.

The cubic superpotential with the mass term is given by \((4.6)\) in subsection 4.1.1 and the brane configuration for zero mass for the bifundamental, which has only a cubic superpotential \((4.6)\), can be obtained from modified Figure 17A by moving the upper \( NS5' \)-brane (or \( NS5'_{i-2} \)-brane) together with \((N_{c,i-1} - N_{c,i-2})\) color D4-branes into the origin \( v = 0 \)(and their mirrors). The low energy dynamics of the magnetic brane configuration can be described by the \( N = 1 \) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by the ones in subsection 4.1.1. The dual gauge theory has a meson \( \Phi_{i-1} \) and bifundamentals \( f_{i-1} \), and \( \tilde{f}_{i-1} \) under the dual gauge group and the superpotential corresponding to modified Figures 17A and 17B is the same as the one in subsection 4.1.1. Therefore, the F-term equation, the derivative \( W_{dual} \) with respect to the meson field \( \Phi_{i-1} \) cannot be satisfied if the \((N_{c,i-1} - N_{c,i-2})\) exceeds \( \tilde{N}_{c,i} \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( f_{i-1}^a \tilde{f}_{i-1,b} - \mu_{i+1}^2 \delta_b^a = 0 \) and \( \Phi_{i-1} f_{i-1} = 0 = \tilde{f}_{i-1} \Phi_{i-1} \). Then the solutions for these can be obtained. At one loop, the effective potential \( V^{(1)}_{eff} \) for \( M_{i-1} \) leads to the positive value for \( m_{M_{i-1}}^2 \) implying that these vacua are stable.
5.3 \( \mathcal{N} = 1 \) \( SO(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,n}) \) magnetic theory

5.3.1 When the dual gauge group occurs at odd chain

Let us consider other magnetic theory for the same electric theory. By applying the Seiberg dual to the \( SU(N_{c,n}) \) factor and interchanging the \( NS5'_{N_{c,n}-1} \)-brane and the \( NS5_{N_{c,n}} \)-brane, one obtains the Figure 20A which appears in subsection 4.2.2, with a replacement of \( O6^+ \)-plane instead of \( O6^- \)-plane. Let us call this a “modified” Figure 20A.

The gauge group is given by

\[
SO(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})
\]

and the matter contents are the field \( f_{n-1} \) charged under \((1_1, \cdots, 1_{n-2}, \square_{n-1}, \square_n)\) and their conjugates \( \tilde{f}_{n-1} \) \((1_1, \cdots, 1_{n-2}, \square_{n-1}, \square_n)\) under the dual gauge group and the gauge-singlet \( \Phi_{n-1} \) which is in the adjoint representation for the \((n-1)\)-th gauge group, in other words, \((1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n) \oplus (1_1, \cdots, 1_n)\) under the dual gauge group where the gauge group is broken from \( SU(N_{c,n-1}) \) to \( SU(N_{c,n-1} - N_{c,n-2}) \). Then the \( \Phi_{n-1} \) is a \((N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})\) matrix. Only \((N_{c,n-1} - N_{c,n-2})\) D4-branes can participate in the mass deformation.

The low energy dynamics of the magnetic brane configuration can be described by the \( \mathcal{N} = 1 \) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by similarly and the dual gauge theory has a meson field \( \Phi_{n-1} \) and bifundamentals \( f_{n-1}, \tilde{f}_{n-1}, F_j \) and \( \tilde{F}_j \) under the dual gauge group and the superpotential corresponding to modified Figures 20A and 20B is given by the one in subsection 4.2.2. Therefore, the F-term equation, the derivative \( W_{\text{dual}} \) with respect to the meson field \( \Phi_{n-1} \) cannot be satisfied if the \((N_{c,n-1} - N_{c,n-2})\) exceeds \( \tilde{N}_{c,n} \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( f_{n-1}^a \tilde{f}_{n-1, b} - \mu_{n-1}^2 \delta^a_b = 0 \) and \( \Phi_{n-1} f_{n-1} = 0 = \tilde{f}_{n-1} \Phi_{n-1} \). Then the solutions for these are given by similarly.

5.3.2 When the dual gauge group occurs at even chain

Starting from Figure 23B, moving the \( NS5'_{N_{c,n}-2} \)-brane with \((N_{c,n-1} - N_{c,n-2})\) D4-branes to the \(+v\) direction leading to Figure 23B, and interchanging the \( NS5_{N_{c,n}-1} \)-brane and the \( NS5'_{N_{c,n}} \)-brane, one obtains the Figure 19A in subsection 4.2.1, with a replacement of \( O6^+ \)-plane instead of \( O6^- \)-plane. Let us call this as a “modified” Figure 19A.

The dual gauge group is given by

\[
SO(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})
\]
the matter contents are the bifundamentals $f_{n-1}$ in $(1_1, \cdots, 1, \Box_{n-1}, \Box_n)$, and $\tilde{f}_{n-1}$ in the representation $(1_1, \cdots, 1, \Box_{n-1}, \Box_n)$ in addition to $(n - 2)$ bifundamentals $F_j$ and $\tilde{F}_j$, $j = 1, 2, \cdots, (n-2)$ and the gauge singlet $\Phi_{n-1}$ for the $n$-th dual gauge group in the adjoint representation for the $(n-1)$-th dual gauge group, i.e., $(1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n)$ plus a singlet under the dual gauge group where the gauge group is broken from $SU(N_{c,n-1})$ to $SU(N_{c,n-1} - N_{c,n-2})$.

The dual gauge theory has a meson field $\Phi_{n-1}$ and bifundamentals $f_{n-1}, \tilde{f}_{n-1}, F_j,$ and $\tilde{F}_j$ and the superpotential corresponding to modified Figures 19A and 19B is given by previous results. Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi_{n-1}$ cannot be satisfied if the $(N_{c,n-1} - N_{c,n-2})$ exceeds $\tilde{N}_{c,n}$. So the supersymmetry is broken.

6 Meta-stable brane configurations with $(2n + 1)$ NS-branes plus $O6^+$-plane

The type IIA brane configuration, by generalizing the brane configurations [63, 64] to the case where there are more NS-branes, corresponding to $N = 1$ supersymmetric gauge theory (see also [23]) with gauge group

$$SU(N_{c,1}) \times SU(N_{c,2}) \times \cdots \times SU(N_{c,n})$$

and with a symmetric tensor field $S$ charged under $(\frac{1}{2}N_{c,1}(N_{c,1} + 1), 1_2, \cdots, 1_n)$, the $(n-1)$ bifundametals $F_i$ charged under $(1_1, \cdots, 1, \Box_i, \Box_{i+1}, 1, \cdots, 1_n)$ and their complex conjugate fields $\tilde{F}_i$ charged $(1_1, \cdots, 1, \Box_i, \Box_{i+1}, 1, \cdots, 1_n)$ where $i = 1, 2, \cdots, (n-1)$ as well as a conjugate symmetric field $\tilde{S}$ charged under $(\frac{1}{2}N_{c,1}(N_{c,1} + 1), 1_2, \cdots, 1_n)$ can be described by the NS5-brane located at the origin, the $NS5_1$-brane, the $NS5_2$-brane, ..., the $NS5'_{n'}$-brane for odd number of gauge groups (or the $NS5_{n'}$-brane for even number of gauge groups), $N_{c,1}$, $N_{c,2}$, ..., and $N_{c,n}$-color D4-branes. See the Figure 25 for the details on the brane configuration. The $O6^+$-plane acts as $(x^4, x^5, x^6) \to (-x^4, -x^5, -x^6)$ and has RR charge +4.

Let us place an O6-plane at the origin $x^6 = 0$ and denote the $x^6$ coordinates for the $NS5'_{1}$-brane, ..., the $NS5'_{n'}$-brane for odd $n$ (or the $NS5_{n'}$-brane for even $n$) are given by $x^6 = y_1, y_1 + y_2, \cdots, \sum_{j=1}^{n-1} y_j + y_n$ respectively. The $N_{c,1}$ D4-branes are suspended between the $NS5'_{1}$-brane and its mirror, the $N_{c,2}$ D4-branes are suspended between the $NS5'_{1}$-brane and the $NS5_2$-brane, ..., and the $N_{c,n}$ D4-branes are suspended between the $NS5_{n-1}$-brane and the $NS5'_n$-brane for odd $n$ (or between the $NS5'_{n-1}$-brane and the $NS5_n$-brane for even $n$). The fields $F_i$ and $\tilde{F}_i$ correspond to 4-4 strings connecting the $N_{c,i}$-color D4-branes with $N_{c,i+1}$-color D4-branes. We draw this $N = 1$ supersymmetric electric brane configuration in
Figure 25: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $\prod_{i=1}^{n} SU(N_{c,i})$ and bifundamentals $F_{i}$ and $\tilde{F}_{i}$ with vanishing mass for the bifundamentals when the number of gauge groups factor $n$ is odd(25A) and even(25B). We do not draw the mirrors of the branes appearing in the left hand side of $O6^+$-plane.

Figure 25A(25B) when $n$ is odd(even) for the vanishing mass for the fields $F_{i}$ and $\tilde{F}_{i}$. The fields $S$ and $\tilde{S}$ correspond to 4-4 strings connecting the $N_{c,1}$-color D4-branes with $x^6 < 0$ with $N_{c,1}$-color D4-branes with $x^6 > 0$.

There is no superpotential in Figure 25A. Let us deform this theory. Displacing the two NS5’-branes relative each other in the $+v$ direction, characterized by $(\Delta x)_{i-1}$, corresponds to turning on a quadratic mass-deformed superpotential for the field $F_{i-1}$ and $\tilde{F}_{i-1}$ as follows:

$$W = m_{i-1} F_{i-1} \tilde{F}_{i-1} (\equiv m_{i-1} \Phi_{i-1})$$

where the $i$-th gauge group indices in $F_{i-1}$ and $\tilde{F}_{i-1}$ are contracted.

$$m_{i-1} = \frac{(\Delta x)_{i-1}}{\ell_s^2}.$$ 

The gauge-singlet $\Phi_{i-1}$ for the $i$-th gauge group is in the adjoint representation for the $(i-1)$-th gauge group, i.e., $(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)$ under the gauge group where the gauge group is broken from $SU(N_{c,i-1})$ to $SU(N_{c,i-1} - N_{c,i-2})$. The $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix. The $NS5'_{i-2}$-brane together with $(N_{c,i-1} - N_{c,i-2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation(and their mirrors to $-v$ direction). Then the $x^5$ coordinate of $NS5'_{i}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i-2}$-branes is given by $(\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i-1}$ corresponds to recombination of $N_{c,i-1}$ and $N_{c,i}$-color D4-branes, which will become $N_{c,i-1}$ or $N_{c,i}$-color D4-branes in Figure 25A such that they are suspended between the $NS5'_{i-2}$-brane and the $NS5'_{i}$-brane and pushing them into the $w$ direction.

Now we obtain this brane configuration which is the same as the Figure 16A except the middle NS5-brane for nonvanishing mass for the fields $F_{i}$ and $\tilde{F}_{i}$. 

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Let us deform the theory by Figure 25B. Displacing the two NS5'-branes, the NS5'\(_{-1}\)-brane and the NS5'\(_{i+1}\)-brane, relative each other in the \(v\) direction, characterized by \((\Delta x)_{i+1}\), corresponds to turning on a quadratic mass-deformed superpotential for the fields \(F_i\) and \(\tilde{F}_i\) as follows:

\[
W = m_{i+1} F_i \tilde{F}_i (\equiv m_{i+1} \Phi_{i+1})
\]

where the \(i\)-th gauge group indices in \(F_i\) and \(\tilde{F}_i\) are contracted, each \((i+1)\)-th gauge group index in them is encoded in \(\Phi_{i+1}\) and the mass \(m_{i+1}\) is given by

\[
m_{i+1} = \frac{(\Delta x)_{i+1}}{\ell_s^2}.
\]

The gauge-singlet \(\Phi_{i+1}\) for the \(i\)-th gauge group is in the adjoint representation for the \((i+1)\)-th gauge group, i.e.,

\[
(1_1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, 1_i, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)
\]

under the gauge group where the gauge group is broken from \(SU(N_{c,i+1})\) to \(SU(N_{c,i+1} - N_{c,i+2})\). Then the \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})\) matrix. The NS5'\(_{i+1}\)-brane together with \((N_{c,i+1} - N_{c,i+2})\)-color D4-branes is moving to the \(+v\) direction for fixed other branes during this mass deformation. In other words, the \(N_{c,i+2}\) D4-branes among \(N_{c,i+1}\) D4-branes are not participating in the mass deformation. Then the \(x^5\) coordinate(\(\equiv x\)) of NS5'\(_{i-1}\)-brane is equal to zero while the \(x^5\) coordinate of NS5'\(_{i+1}\)-brane is given by \((\Delta x)_{i+1}\). Giving an expectation value to the meson field \(\Phi_{i+1}\) corresponds to recombination of \(N_{c,i}\) and \(N_{c,i+1}\)-color D4-branes, which will become \(N_{c,i}\)- or \(N_{c,i+1}\)-color D4-branes in Figure 25B such that they are suspended between the NS5'\(_{i-1}\)-brane and the NS5'\(_{i+1}\)-brane and pushing them into the \(w\) direction. We assume that the number of colors satisfies \(N_{c,i+1} \geq N_{c,i} - N_{c,i-1} \geq N_{c,i+2}\). Now we obtain this brane configuration that is the same as the Figure 16B except the middle NS5-brane for nonvanishing mass for the fields \(F_i\) and \(\tilde{F}_i\).

6.1 \(\mathcal{N} = 1\) \(SU(N_{c,1}) \times \cdots \times SU(\bar{N}_{c,i}) \times \cdots \times SU(N_{c,n})\) magnetic theory

6.1.1 When the dual gauge group occurs at odd chain

Starting from Figure 25A and interchanging the NS5\(_{i-1}\)-brane and the NS5\(_{i}\)-brane (and their mirrors), one obtains the Figure 17A, in subsection 4.1.1, with a replacement of a combination of \(O6^+\)-plane and a middle NS5-brane, instead of \(O6^-\)-plane. Let us denote this as the “deformed” Figure 17A.
The dual gauge group is given by

\[ SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n}) \]

The matter contents are the field \( f_i \) charged under \( (1, \cdots, \text{i-1,} \tilde{N}_{c,i}, \text{i,} \text{i+1,} \cdots, \text{n}) \), and its conjugate field \( \tilde{f}_i \) charged under \( (1, \cdots, \text{i-1,} \tilde{N}_{c,i}, \text{i,} \text{i+1,} \cdots, \text{n}) \) under the dual gauge group and the gauge-singlet \( \Phi_{i-1} \) for the \( i \)-th dual gauge group in the adjoint representation for the \((i-1)\)-th dual gauge group, i.e., \((1, \cdots, 1, 2, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1, \cdots, 1, n)\) plus a singlet under the dual gauge group where the gauge group is broken from \( SU(N_{c,i-1}) \) to \( SU(N_{c,i-1} - N_{c,i-2}) \). Then the \( \Phi_{i-1} \) is a \((N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})\) matrix.

The cubic superpotential with the mass term is given by the one in subsection 4.1.1 and the brane configuration for zero mass for the bifundamental, which has only a cubic superpotential \( W_{bifund} \), can be obtained from deformed Figure 17A by moving the upper NS5'-brane(or NS5'_{i-2}-brane) together with \((N_{c,i-1} - N_{c,i-2})\) color D4-branes into the origin \( v = 0 \)(and their mirrors).

The low energy dynamics of the magnetic brane configuration can be described by the \( N = 1 \) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by the ones in subsection 4.1.1. The dual gauge theory has a meson \( \Phi_{i-1} \) and bifundamentals \( f_{i-1} \), and \( \tilde{f}_{i-1} \) under the dual gauge group and the superpotential corresponding to deformed Figures 17A and 17B is the same as the one in subsection 4.1.1. Therefore, the F-term equation, the derivative \( W_{dual} \) with respect to the meson field \( \Phi_{i-1} \) cannot be satisfied if the \((N_{c,i-1} - N_{c,i-2})\) exceeds \( \tilde{N}_{c,i} \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( f_{i-1}^a \tilde{f}_{i-1,b} - \mu_{i-1}^2 \delta_a^b = 0 \) and \( \Phi_{i-1} f_{i-1} = 0 = \tilde{f}_{i-1} \Phi_{i-1} \).

### 6.1.2 When the dual gauge group occurs at even chain

Let us consider other magnetic theory for the same electric theory. Starting from Figure 25B and interchanging the \( NS5'_{i-1}-\)brane and the \( NS5_{i-1}-\)brane(and their mirrors), one obtains the magnetic brane configuration which is exactly the same as the Figure 18A, in subsection 4.1.2., with a replacement of a combination of \( O6^+\)-plane and a middle NS5-brane, instead of \( O6^-\)-plane. Let us denote this as the "deformed" Figure 18A.

The dual gauge group is given by

\[ SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n}) \]

The matter contents are the field \( f_i \) charged under \( (1, \cdots, \Box, \Box_{i+1}, \cdots, 1, n) \), and their conjugates \( \tilde{f}_i \) charged \((1, \cdots, \Box, \Box_{i+1}, \cdots, 1, n)\) under the dual gauge group and the gauge-singlet \( \Phi_{i+1} \) which is in the adjoint representation for the \( i \)-th dual gauge group, in other
words, \((1_1, \cdots , 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, \cdots , 1_n)\) under the dual gauge group where the gauge group is broken from \(SU(N_{c,i+1})\) to \(SU(N_{c,i+1} - N_{c,i+2})\). Then the \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})\) matrix. Only \((N_{c,i+1} - N_{c,i+2})\) D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term is given by in subsection 4.1.2 and we define \(\Phi_{i+1}\) as \(\Phi_{i+1} = F_i \tilde{F}_i\) and the \(i\)-th gauge group indices in \(F_i\) and \(\tilde{F}_i\) are contracted, each \((i + 1)\)-th gauge group index in them is encoded in \(\Phi_{i+1}\).

The low energy dynamics of the magnetic brane configuration can be described by the \(\mathcal{N} = 1\) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by those in subsection 4.1.2 and the superpotential corresponding to deformed Figures 18A and 18B is given by the one in subsection 4.1.2. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{i+1}\) cannot be satisfied if the \((N_{c,i+1} - N_{c,i+2})\) exceeds \(\tilde{N}_{c,i}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f^a_i \tilde{f}_{i,b} - \mu_{i+1}^2 \delta^a_b = 0\) and \(\Phi_{i+1} f_i = 0 = \tilde{f}_i \Phi_{i+1}\).

### 6.2 \(\mathcal{N} = 1\) \(SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,n})\) magnetic theory

#### 6.2.1 When the dual gauge group occurs at odd chain

Starting from Figure 25A, moving the \(NS5'_{n-2}\)-brane with \((N_{c,n-1} - N_{c,n-2})\) D4-branes to the \(+v\) direction leading to Figure 23B, and interchanging the \(NS5_{n-1}\)-brane and the \(NS5'_{n}\)-brane, one obtains the Figure 19A in subsection 4.2.1, with a replacement of a combination of \(O6^+\)-plane and a middle NS5-brane, instead of \(O6^-\)-plane. Let us denote this as the “deformed” Figure 19A.

The dual gauge group is given by

\[SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})\]

the matter contents are the bifundamentals \(f_{n-1}\) in \((1_1, \cdots , 1, \Box_{n-1}, \Box_n)\), and \(\tilde{f}_{n-1}\) in the representation \((1_1, \cdots , 1, \Box_{n-1}, \Box_n)\) in addition to \((n - 2)\) bifundamentals \(F_j\) and \(\tilde{F}_j\), \(j = 1, 2, \cdots , (n - 2)\) and the gauge singlet \(\Phi_{n-1}\) for the \(n\)-th dual gauge group in the adjoint representation for the \((n - 1)\)-th dual gauge group, i.e., \((1_1, \cdots , 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n)\) plus a singlet under the dual gauge group where the gauge group is broken from \(SU(N_{c,n-1})\) to \(SU(N_{c,n-1} - N_{c,n-2})\).

The dual gauge theory has a \(\Phi_{n-1}\) and bifundamentals \(f_{n-1}, \tilde{f}_{n-1}, F_j,\) and \(\tilde{F}_j\) and the superpotential corresponding to deformed Figures 19A and 19B is given by previous results. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{n-1}\) cannot be satisfied if the \((N_{c,n-1} - N_{c,n-2})\) exceeds \(\tilde{N}_{c,n}\). So the supersymmetry is broken.
6.2.2 When the dual gauge group occurs at even chain

Let us consider other magnetic theory for the same electric theory. By applying the Seiberg dual to the $SU(N_{c,n})$ factor and interchanging the $NS5'_{n-1}$-brane and the $NS5_n$-brane, one obtains the Figure 20A which appears in subsection 4.2.2, with a replacement of a combination of $O6^+$-plane and a middle NS5-brane, instead of $O6^-$-plane. Let us denote this as the “deformed” Figure 20A.

The gauge group is given by

$$SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})$$

and the matter contents are the field $f_{n-1}$ charged under $(1_1, \cdots, 1_{n-2}, \square_{n-1}, \square_n)$ and their conjugates $\tilde{f}_{n-1}$ $(1_1, \cdots, 1_{n-2}, \square_{n-1}, \square_n)$ under the dual gauge group and the gauge-singlet $\Phi_{n-1}$ which is in the adjoint representation for the $(n-1)$-th gauge group, in other words, $(1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n) \oplus (1_1, \cdots, 1_n)$ under the dual gauge group where the gauge group is broken from $SU(N_{c,n-1})$ to $SU(N_{c,n-1} - N_{c,n-2})$. Then the $\Phi_{n-1}$ is a $(N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2})$ matrix. Only $(N_{c,n-1} - N_{c,n-2})$ D4-branes can participate in the mass deformation.

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by similarly and the dual gauge theory has a meson $\Phi_{n-1}$ and bifundamentals $f_{n-1}, \tilde{f}_{n-1}, F_j$ and $\tilde{F}_j$ under the dual gauge group and the superpotential corresponding to deformed Figures 20A and 20B is given by the one in subsection 4.2.2. Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi_{n-1}$ cannot be satisfied if the $(N_{c,n-1} - N_{c,n-2})$ exceeds $\tilde{N}_{c,n}$. So the supersymmetry is broken.

That is, there exist three equations from F-term conditions: $f^n_{n-1} f^b_{n-1} - \mu^2 \delta^n_b = 0$ and $\Phi_{n-1} f_{n-1} = 0 = \tilde{f}_{n-1} \Phi_{n-1}$.

6.3 $\mathcal{N} = 1$ $SU(\tilde{N}_{c,1}) \times \cdots \times SU(N_{c,n})$ magnetic theory

Let us consider the Seiberg dual for the first gauge group and take the mass deformation by moving the $(N_{c,2} - N_{c,3})$ D4-branes between the middle NS5-brane and the $NS5'_2$-brane to $+v$ direction. Starting from the Figure 25A, we apply the Seiberg dual to the first gauge group $SU(N_{c,1})$ factor and the $NS5'_1$-brane and its mirror are interchanged each other. Then the brane configuration is the same as Figure 25A except that the number of color $\tilde{N}_{c,1}$ is given by $\tilde{N}_{c,1} = 2N_{c,2} - N_{c,1}$ from [21] [14]. By rotating the $NS5_2$-brane by $\pi/2$ (leading to $NS5'_2$-brane) and moving it together with $(N_{c,2} - N_{c,3})$ D4-branes to $+v$ direction, then the $(N_{c,2} - N_{c,3})$
D4-branes are connecting between the $NS_{5}$-brane and the $NS_{5}'$-brane and $\tilde{N}_{c,1}$ D4-branes connecting between the middle NS5-brane and $NS_{5}'$-brane as well as $N_{c,3}$ D4-branes between the $NS_{5}'$-brane and the $NS_{5}''$-brane (and their mirrors).

Figure 26: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,1} = 2N_{c,2} - N_{c,1})$ with D4- and $\overline{D}4$-branes (26A) and with a misalignment between D4-branes (26B) when the two NS5'-branes are close to each other. The number of tilted D4-branes in 26B can be written as $N_{c,1} - N_{c,3} - N_{c,2} = (N_{c,2} - N_{c,3}) - \tilde{N}_{c,1}$.

By introducing $(N_{c,2} - N_{c,3})$ D4-branes and $(N_{c,2} - N_{c,3})$ anti-D4-branes between the middle NS5-brane and $NS_{5}'$-brane, reconnecting the former with the $(N_{c,2} - N_{c,3})$ D4-branes connecting between the $NS_{5}'$-brane and the $NS_{5}''$-brane and moving those combined D4-branes to $+v$-direction (and their mirrors to $-v$ direction), one gets the final Figure 26A.

Now we draw this $\mathcal{N} = 1$ supersymmetric magnetic brane configuration in Figure 26. We assume, as before, that the number of colors satisfies $2N_{c,2} \geq N_{c,1} \geq N_{c,2} + N_{c,3}$.

The dual gauge group is given by

$$SU(\tilde{N}_{c,1} \equiv 2N_{c,2} - N_{c,1}) \times SU(N_{c,2}) \times \cdots \times SU(N_{c,n})$$

where the number of dual color can be obtained from the linking number counting, as done in [21, 14]. The matter contents are the flavor singlet $f_{1}$ in the bifundamental representation $(\tilde{N}_{c,1}, \overline{N}_{c,2}, 1_{3}, \cdots, 1_{n})$ and its complex conjugate field $\overline{f}_{1}$ in the bifundamental representation $(\overline{N}_{c,1}, N_{c,2}, 1_{2}, \cdots, 1_{n})$, under the dual gauge group and the gauge singlet $\Phi_{2}$ in the representation for $(1_{1}, (N_{c,2} - N_{c,3})^{2} - 1, \cdots, 1_{n}) \oplus (1_{1}, \cdots, 1_{n})$ under the dual gauge group where the gauge group is broken from $SU(N_{c,2})$ to $SU(N_{c,2} - N_{c,3})$. There are also the symmetric flavor $s$ for $SU(\tilde{N}_{c,1})$ and the conjugate symmetric flavor $\overline{s}$ for $SU(\overline{N}_{c,1})$ as well as $F_{j}$ and $\overline{F}_{j}$. Then the $\Phi_{2}$ is a $(N_{c,2} - N_{c,3}) \times (N_{c,2} - N_{c,3})$ matrix. Only $(N_{c,2} - N_{c,3})$ D4-branes among $N_{c,2}$
D4-branes can participate in the mass deformation. A cubic superpotential is an interaction between dual “quarks” and a meson.

Then the dual magnetic superpotential, by adding the mass term for the bifundamental \( F_1 \) which can be interpreted as a linear term in the meson \( \Phi_2 \) to this cubic superpotential, is given by

\[
W_{\text{dual}} = \Phi_2 f_1 \tilde{f}_1 + m_2 \text{tr} \Phi_2,
\]

where \( \Phi_2 \) was defined as \( \Phi_2 \equiv F_1 \tilde{F}_1 \) and the first gauge group indices in \( F_1 \) and \( \tilde{F}_1 \) are contracted and each second gauge group index in them is encoded in \( \Phi_2 \).

Here the magnetic fields \( f_1 \) and \( \tilde{f}_1 \) correspond to 4-4 strings connecting the \( \tilde{N}_{c,1} \)-color D4-branes (that are connecting between the middle NS5-brane and the NS5'_{2}-brane in Figure 26B) with \( N_{c,2} \)-flavor D4-branes. Among these \( N_{c,2} \)-flavor D4-branes, only the strings ending on the upper \( (2N_{c,2} - N_{c,1}) \) D4-branes and on the tilted \( (N_{c,1} - N_{c,2} - N_{c,3}) \) D4-branes in Figure 26B enter the cubic superpotential term. Note that the summation of these D4-branes is equal to \( (N_{c,2} - N_{c,3}) \).

When the upper NS5'-brane (or NS5'_{2}-brane) is replaced by coincident \( (N_{c,2} - N_{c,3}) \) D6-branes and the NS5'_{3} is rotated by an angle \( \frac{\pi}{2} \) in the \( (v, w) \) plane in Figure 26B, this brane configuration reduces to the one found in [23] where the gauge group was given by

\[
\text{SU}(2n_{f,1} + 2n_{c,2} - n_{c,1}) \times SU(n_{c,1}) \times \text{SU}(n_{c,2}) \times n_{f,1} \text{ multiplets}, \tilde{n}_{f,1} \text{ multiplets}, \text{ bifundamentals}, \text{ a symmetric flavor}, \text{ a conjugate symmetric flavor}, \text{ and various gauge singlets}.
\]

Then the present number \( (N_{c,2} - N_{c,3}) \) corresponds to the \( n_{f,1} \), the number \( N_{c,1} \) corresponds to \( n_{c,1} \) and the number \( N_{c,3} \) corresponds to the \( n_{c,2} \).

The low energy dynamics of the magnetic brane configuration can be described by the \( \mathcal{N} = 1 \) supersymmetric gauge theory with gauge group and the gauge couplings for the gauge group factors are given by similarly. The dual gauge theory has a meson field \( \Phi_2 \) and bifundamental \( f_1 \) in the representation \( (\tilde{N}_{c,1}, \overline{N}_{c,2}, \cdots, 1_n) \) under the dual gauge group and the superpotential corresponding to the Figures 26A and 26B is given by

\[
W_{\text{dual}} = h \Phi_2 f_1 \tilde{f}_1 - h \mu_2 \text{tr} \Phi_2, \quad h = g_{2,\text{mag}}, \quad \mu_2 = -\frac{(\Delta x)^2}{2\pi g_s f_3^2}.
\]

Then \( f_1 \tilde{f}_1 \) is a \( \tilde{N}_{c,1} \times \tilde{N}_{c,1} \) matrix where the second gauge group indices for \( f_1 \) and \( \tilde{f}_1 \) are contracted with those of \( \Phi_2 \) while \( \Phi_2 \) is a \( (N_{c,2} - N_{c,3}) \times (N_{c,2} - N_{c,3}) \) matrix. Therefore, the F-term equation, the derivative of \( W_{\text{dual}} \) with respect to the meson field \( \Phi_2 \) cannot be satisfied if the \( (N_{c,2} - N_{c,3}) \) exceeds \( \tilde{N}_{c,1} \). So the supersymmetry is broken. That is, there are three equations from F-term conditions: \( f_1^a \tilde{f}_{1,b} - \mu_2 \delta_1^a = 0 \), \( \Phi_2 f_1 = 0 \), and \( \tilde{f}_1 \Phi_2 = 0 \). Then the
solutions for these are given by

\[
< f_1 >= \left( \frac{\mu_2}{\tilde{N}_{c,1}} \right), \quad < \tilde{f}_1 >= \left( \frac{\mu_2}{\tilde{N}_{c,1}} 0 \right), \quad < \Phi_2 >= \left( 0 \begin{array}{c} M_2 1_{(N_{c,2} - N_{c,3} - \tilde{N}_{c,1})} \end{array} \right).
\]

7 Meta-stable brane configurations with \((2n + 1)\) NS-branes, \(O6^\pm\)-planes, and D6-branes

The type IIA brane configuration, by generalizing the brane configurations [65] [66] [67] to the case where there are more NS-branes, corresponding to \(\mathcal{N} = 1\) supersymmetric electric gauge theory (see also [23]) with gauge group

\[
SU(N_{c,1}) \times SU(N_{c,2}) \cdots \times SU(N_{c,n})
\]

and with an antisymmetric tensor field \(A\) charged under \((\frac{1}{2}N_{c,1}(N_{c,1} - 1), 1, \cdots, 1_n)\), a conjugate symmetric tensor field \(\tilde{S}\) charged under \((\frac{1}{2}N_{c,1}(N_{c,1} + 1), 1, \cdots, 1_n)\), an eight fundamentals \(\hat{Q}\) charged under \((N_{c,1}, 1, \cdots, 1_n)\), the \((n - 1)\) bifundamentals \(F_i\) charged under \((1, \cdots, 1, \Box_i, \Box_{i+1}, 1, \cdots, 1_n)\) and \(\tilde{F}_i\) charged \((1, \cdots, 1, \Box_i, \Box_{i+1}, 1, \cdots, 1_n)\) where \(i = 1, 2, \cdots, (n - 1)\) can be described by the \(NS5_1\)-brane, the \(NS5_2\)-brane, \cdots, the \(NS5_n\)-brane for odd number of gauge groups (or the \(NS5'_n\)-brane for even number of gauge groups), \(N_{c,1}\)-, \(N_{c,2}\)-, \cdots, and \(N_{c,n}\)-color D4-branes. See the Figure 27 for the details on the brane configuration. The \(O6^\pm\)-planes act as \((x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)\) and has RR charge \(\pm 4\).

![Figure 27: The \(\mathcal{N} = 1\) supersymmetric electric brane configuration for the gauge group \(\prod_{i=1}^n SU(N_{c,i})\) and bifundamentals \(F_i\) and \(\tilde{F}_i\) with vanishing mass for the bifundamentals when the number of gauge groups factor \(n\) is odd(27A) and even(27B). We do not draw the mirrors of the branes appearing in the left hand side of O6-plane. We denote \(NS/O6/D6\) as a combination of \(NS5^{'-}\)-brane, \(O6^\pm\)-planes and D6-branes.](image)

Let us place an O6-plane at the origin \(x^6 = 0\) and denote the \(x^6\) coordinates for the \(NS5_1\)-brane, \cdots, the \(NS5_n\)-brane for odd \(n\) (or the \(NS5'_n\)-brane for even \(n\)) are given by \(x^6 = y_1, y_1 + y_2, \cdots, \sum_{j=1}^{n-1} y_j + y_n\) respectively. The \(N_{c,1}\) D4-branes are suspended between
the NS5′-brane and its mirror, the $N_{c,2}$ D4-branes are suspending between the NS5′-brane and the NS5′-brane, \cdots and the $N_{c,n}$ D4-branes are suspended between the NS5′-brane and the NS5′-brane for odd $n$(or between the NS5′-brane and the NS5′-brane for even $n$). The fields $F_1$ and $\tilde{F}_i$ correspond to $4\times 4$ strings connecting the $N_{c,i}$-color D4-branes with $N_{c,i+1}$-color D4-branes. The fields $A$ and $\tilde{S}$ correspond to $4\times 4$ strings connecting the $N_{c,1}$-color D4-branes with $x^6 < 0$ with $N_{c,1}$-color D4-branes with $x^6 > 0$. We draw this $N = 1$ supersymmetric electric brane configuration in Figure 27A(27B) when $n$ is odd(even) for the vanishing mass for the fields $F_i$ and $\tilde{F}_i$.

There is no superpotential in Figure 27A. Let us deform this theory. Displacing the two NS5′-branes relative each other in the $+v$ direction, characterized by $(\Delta x)_{i+1}$, corresponds to turning on a quadratic mass-deformed superpotential for the field $F_i$ and $\tilde{F}_i$ as follows:

$$W = m_{i+1}F_i\tilde{F}_i(\equiv m_{i+1}\Phi_{i+1})$$

where the $i$-th gauge group indices in $F_i$ and $\tilde{F}_i$ are contracted and the mass $m_{i+1}$ is given by

$$m_{i+1} = \frac{(\Delta x)_{i+1}}{\ell_s^2}.$$ 

The gauge-singlet $\Phi_{i+1}$ for the $i$-th gauge group is in the adjoint representation for the $(i+1)$-th gauge group, i.e., $(\mathbf{1}_1, \cdots, \mathbf{1}_i, (N_{c,i+1} - N_{c,i+2})^2 - \mathbf{1}, \cdots, \mathbf{1}_n) \oplus (\mathbf{1}_1, \cdots, \mathbf{1}_n)$ under the gauge group where the gauge group is broken from $SU(N_{c,i+1})$ to $SU(N_{c,i+1} - N_{c,i+2})$. The $\Phi_{i+1}$ is a $(N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})$ matrix. The NS5′-brane together with $(N_{c,i+1} - N_{c,i+2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation(and their mirrors to $-v$ direction). Then the $x^5$ coordinate of NS5′-brane is equal to zero while the $x^5$ coordinate of NS5′-branes is given by $(\Delta x)_{i+1}$. Giving an expectation value to the meson field $\Phi_{i+1}$ corresponds to recombination of $N_{c,i}$ and $N_{c,i+2}$-color D4-branes, which will become $N_{c,i}$ or $N_{c,i+1}$-color D4-branes in Figure 27A such that they are suspended between the NS5′-brane and the NS5′-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i+1} \geq N_{c,i} - N_{c,i-1} \geq N_{c,i+2}$.

Now we obtain this brane configuration and this is the same as the Figure 23A except that there are NS5′-brane, $O6^-$-plane and eight D6-branes at the origin $x^6 = 0$ for nonvanishing mass for the fields $F_i$ and $\tilde{F}_i$.

Let us deform the theory by Figure 27B. Displacing the two NS5′-branes, the NS′-brane and the NS′-brane, relative each other in the $v$ direction, characterized by $(\Delta x)_{i-1}$, corresponds to turning on a quadratic mass-deformed superpotential for the fields $F_{i-1}$ and $\tilde{F}_{i-1}$ as follows:

$$W = m_{i-1}F_{i-1}\tilde{F}_{i-1}(\equiv m_{i-1}\Phi_{i-1})$$
where the $i$-th gauge group indices in $F_{i-1}$ and $\tilde{F}_{i-1}$ are contracted, each $(i-1)$-th gauge group index in them is encoded in $\Phi_{i-1}$ and the mass $m_{i-1}$ is given by

$$m_{i-1} = \frac{(\Delta x)_{i-1}}{\ell_s^2}.$$

The gauge-singlet $\Phi_{i-1}$ for the $i$-th gauge group is in the adjoint representation for the $(i-1)$-th gauge group, i.e.,

$$(1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1_i, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)$$

under the gauge group where the gauge group is broken from $SU(N_{c,i-1})$ to $SU(N_{c,i-1} - N_{c,i-2})$. Then the $\Phi_{i-1}$ is a $(N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})$ matrix. The $NS5'_{i-2}$-brane together with $(N_{c,i-1} - N_{c,i-2})$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation. In other words, the $N_{c,i-2}$ D4-branes among $N_{c,i-1}$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate (≡ $x$) of $NS5'_{i}$-brane is equal to zero while the $x^5$ coordinate of $NS5'_{i-2}$-brane is given by $(\Delta x)_{i-1}$. Giving an expectation value to the meson field $\Phi_{i-1}$ corresponds to recombination of $N_{c,i-1}$ and $N_{c,i}$-color D4-branes, which will become $N_{c,i-1}$ or $N_{c,i}$-color D4-branes in Figure 27B such that they are suspended between the $NS5'_{i-2}$-brane and the $NS5'_{i}$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies $N_{c,i-1} \geq N_{c,i} - N_{c,i+1} \geq N_{c,i-2}$. Now we obtain this brane configuration and this is the same as the Figure 23B except that there are $NS5'$-brane, $O6^-$-plane and eight D6-branes at the origin $x^6 = 0$ for nonvanishing mass for the fields $F_{i-1}$ and $\tilde{F}_{i-1}$.

### 7.1 $N = 1$ $SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i}) \times \cdots \times SU(N_{c,n})$ magnetic theory

#### 7.1.1 When the dual gauge group occurs at odd chain

Let us consider other magnetic theory for the same electric theory. Starting from Figure 25B and interchanging the $NS5'_{i-1}$-brane and the $NS5'_{i}$-brane (and their mirrors), one obtains the magnetic brane configuration which is exactly the same as the Figure 18A, in subsection 4.1.2., with a replacement of $NS/O6/D6$ instead of $O6^-$-plane. Let us call this as “modified” Figure 18A.

The dual gauge group is given by

$$SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} \equiv N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n}).$$

The matter contents are the field $f_i$ charged under $(1, \cdots, \Box_i, \Box_{i+1}, \cdots, 1_n)$, and their conjugates $\tilde{f}_i$ charged $(1, \cdots, \Box_i, \Box_{i+1}, \cdots, 1_n)$ under the dual gauge group and the gauge-singlet $\Phi_{i+1}$ which is in the adjoint representation for the $i$-th dual gauge group, in other
words, \((1_1, \cdots, 1_i, (N_{c,i+1} - N_{c,i+2})^2 - 1, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)\) under the dual gauge group where the gauge group is broken from \(SU(N_{c,i+1})\) to \(SU(N_{c,i+1} - N_{c,i+2})\). Then the \(\Phi_{i+1}\) is a \((N_{c,i+1} - N_{c,i+2}) \times (N_{c,i+1} - N_{c,i+2})\) matrix. Only \((N_{c,i+1} - N_{c,i+2})\) D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term is given by \((4.8)\) in subsection 4.1.2 and we define \(\Phi_{i+1} = F_i \tilde{F}_i\) and the \(i\)-th gauge group indices in \(F_i\) and \(\tilde{F}_i\) are contracted, each \((i + 1)\)-th gauge group index in them is encoded in \(\Phi_{i+1}\). The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential \((4.8)\), can be obtained from modified Figure 17A by moving the upper NS5’-brane together with \((N_{c,i+1} - N_{c,i+2})\) color D4-branes into the origin \(v = 0\) (and their mirrors).

The low energy dynamics of the magnetic brane configuration can be described by the \(\mathcal{N} = 1\) supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by those in subsection 4.1.2 and the superpotential corresponding to modified Figures 18A and 18B is given by the one in subsection 4.1.2. Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi_{i+1}\) cannot be satisfied if the \((N_{c,i+1} - N_{c,i+2})\) exceeds \(\tilde{N}_{c,i}\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(f_{a}^i \tilde{f}_{i,b} - \mu_{i+1}^2 \delta_{i}^a = 0\) and \(\Phi_{i+1} f_i = 0 = \tilde{f}_i \Phi_{i+1}\).

### 7.1.2 When the dual gauge group occurs at even chain

Starting from Figure 25A and interchanging the NS5\(_{i-1}\)-brane and the NS5’\(_i\)-brane (and their mirrors), one obtains the Figure 17A, in subsection 4.1.1, with a replacement of NS/O6/D6 instead of O6\(^*\)-plane. Let us call this as “modified” Figure 17A.

The dual gauge group is given by

\[
SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,i} = N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \cdots \times SU(N_{c,n})
\]

The matter contents are the field \(f_{i-1}\) charged under \((1_1, \cdots, 1_{i-1}, N_{c,i-1}, \overline{N}_{c,i}, \cdots, 1_n)\), and its conjugate field \(\tilde{f}_{i-1}\) charged under \((1_1, \cdots, 1_{i-2}, \overline{N}_{c,i-1}, \overline{N}_{c,i}, \cdots, 1_n)\) under the dual gauge group and the gauge-singlet \(\Phi_{i-1}\) for the \(i\)-th dual gauge group in the adjoint representation for the \((i - 1)\)-th dual gauge group, i.e., \((1_1, \cdots, 1_{i-2}, (N_{c,i-1} - N_{c,i-2})^2 - 1, 1, \cdots, 1_n)\) plus a singlet under the dual gauge group where the gauge group is broken from \(SU(N_{c,i-1})\) to \(SU(N_{c,i-1} - N_{c,i-2})\). Then the \(\Phi_{i-1}\) is a \((N_{c,i-1} - N_{c,i-2}) \times (N_{c,i-1} - N_{c,i-2})\) matrix.

The cubic superpotential with the mass term is given by the one \((4.6)\) in subsection 4.1.1 and the brane configuration for zero mass for the bifundamental, which has only a cubic superpotential \((4.6)\), can be obtained from modified Figure 17A by moving the upper NS5’-brane (or NS5’\(_{i-2}\)-brane) together with \((N_{c,i-1} - N_{c,i-2})\) color D4-branes into the origin.
\[ v = 0 \text{(and their mirrors). The low energy dynamics of the magnetic brane configuration can be described by the } \mathcal{N} = 1 \text{ supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by the ones in subsection 4.1.1. The dual gauge theory has a meson } \Phi_{i-1} \text{ and bifundamentals } f_{i-1}, \text{ and } \tilde{f}_{i-1} \text{ under the dual gauge group and the superpotential corresponding to modified Figures 17A and 17B is the same as the one in subsection 4.1.1. Therefore, the F-term equation, the derivative } W_{\text{dual}} \text{ with respect to the meson field } \Phi_{i-1} \text{ cannot be satisfied if the } (\mathcal{N}_{c,i-1} - \mathcal{N}_{c,i-2}) \text{ exceeds } \tilde{\mathcal{N}}_{c,i}. \text{ So the supersymmetry is broken. That is, there exist three equations from F-term conditions: } f_{i-1}^a \tilde{f}_{i-1,b} - \mu_{i-1}^2 \delta_b^a = 0 \text{ and } \Phi_{i-1} f_{i-1} = 0 = \tilde{f}_{i-1} \Phi_{i-1}.
\]

\[ \text{7.2 } \mathcal{N} = 1 \text{ } SU(N_{c,1}) \times \cdots \times SU(\tilde{N}_{c,n}) \text{ magnetic theory} \]

\[ \text{7.2.1 When the dual gauge group occurs at odd chain} \]

Let us consider other magnetic theory for the same electric theory. By applying the Seiberg dual to the } SU(N_{c,n}) \text{ factor and interchanging the } NS5'_{n-1}-\text{brane and the } NS5_n-\text{brane, one obtains the Figure 20A which appears in subsection 4.2.2, with a replacement of } NS/O6/D6 \text{ instead of } O6^-\text{-plane. Let us call this as “modified” Figure 20A.}

The gauge group is given by

\[ SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} = N_{c,n-1} - N_{c,n}) \]

and the matter contents are the field } f_{n-1} \text{ charged under } (1_1, \cdots, 1_{n-2}, \square_{n-1}, \square_n) \text{ and their conjugates } \tilde{f}_{n-1} (1_1, \cdots, 1_{n-2}, \square_{n-1}, \square_n) \text{ under the dual gauge group and the gauge-singlet } \Phi_{n-1} \text{ which is in the adjoint representation for the } (n-1)\text{-th gauge group, in other words, } (1_1, \cdots, 1_{n-2}, (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n) \oplus (1_1, \cdots, 1_n) \text{ under the dual gauge group where the gauge group is broken from } SU(N_{c,n-1}) \text{ to } SU(N_{c,n-1} - N_{c,n-2}). \text{ Then the } \Phi_{n-1} \text{ is a } (N_{c,n-1} - N_{c,n-2}) \times (N_{c,n-1} - N_{c,n-2}) \text{ matrix. Only } (N_{c,n-1} - N_{c,n-2}) \text{ D4-branes can participate in the mass deformation.}

The low energy dynamics of the magnetic brane configuration can be described by the } \mathcal{N} = 1 \text{ supersymmetric gauge theory with gauge group and the gauge couplings for the three gauge group factors are given by similarly and the dual gauge theory has a meson } \Phi_{n-1} \text{ and bifundamentals } f_{n-1}, \tilde{f}_{n-1}, F_j \text{ and } \tilde{F}_j \text{ under the dual gauge group and the superpotential corresponding to modified Figures 20A and 20B is given by the one in subsection 4.2.2. Therefore, the F-term equation, the derivative } W_{\text{dual}} \text{ with respect to the meson field } \Phi_{n-1} \text{ cannot be satisfied if the } (N_{c,n-1} - N_{c,n-2}) \text{ exceeds } \tilde{\mathcal{N}}_{c,n}. \text{ So the supersymmetry is broken. That is, there exist three equations from F-term conditions: } f_{n-1}^a \tilde{f}_{n-1,b} - \mu_{n-1}^2 \delta_b^a = 0 \text{ and } \Phi_{n-1} f_{n-1} = 0 = \tilde{f}_{n-1} \Phi_{n-1}.\]

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7.2.2 When the dual gauge group occurs at even chain

Starting from Figure 25A, moving the $NS5'_{n-2}$-brane with $(N_{c,n-1} - N_{c,n-2})$ D4-branes to the $+v$ direction leading to Figure 23B, and interchanging the $NS5_{n-1}$-brane and the $NS5'_{n}$-brane, one obtains the Figure 19A, with a replacement of $NS/O6/D6$ instead of $O6^*$-plane. Let us call this as “modified” Figure 19A.

The dual gauge group is given by

$$SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{c,n-1} - N_{c,n})$$

the matter contents are the bifundamentals $f_{n-1}$ in $(1_1, \cdots, 1, \Box_{n-1}, \Box_n)$, and $\tilde{f}_{n-1}$ in the representation $(1_1, \cdots, 1, \Box_{n-1}, \Box_n)$ in addition to $(n-2)$ bifundamentals $F_j$ and $\tilde{F}_j$, $j = 1, 2, \cdots, (n-2)$ and the gauge singlet $\Phi_{n-1}$ for the $n$-th dual gauge group in the adjoint representation for the $(n-1)$-th dual gauge group, i.e., $(1_1, \cdots, 1_{n-2}; (N_{c,n-1} - N_{c,n-2})^2 - 1, 1_n)$ plus a singlet under the dual gauge group where the gauge group is broken from $SU(N_{c,n-1})$ to $SU(N_{c,n-1} - N_{c,n-2})$.

The dual gauge group has a $\Phi_{n-1}$ and bifundamentals $f_{n-1}, \tilde{f}_{n-1}, F_j, \tilde{F}_j$ and the superpotential corresponding to modified Figures 19A and 19B is given by previous results. Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi_{n-1}$ cannot be satisfied if the $(N_{c,n-1} - N_{c,n-2})$ exceeds $\tilde{N}_{c,n}$. So the supersymmetry is broken.

7.3 $\mathcal{N} = 1 \ SU(\tilde{N}_{c,1}) \times \cdots \times SU(N_{c,n})$ magnetic theory

Let us apply the Seiberg dual to the first gauge group $SU(N_{c,1})$ factor. Starting from Figure 27A and moving the $NS5_1$-brane to the left all the way past the middle $NS5'$-brane (and the mirror of $NS5_1$-brane to the right of the middle $NS5'$-brane), one obtains the Figure 28A. By introducing $(N_{c,2} - N_{c,3})$ D4-branes and $(N_{c,2} - N_{c,3})$ anti-D4-branes between $NS5_1$-brane and its mirror, one gets the final Figure 28A where we are left with $(N_{c,2} - N_{c,3} - \tilde{N}_{c,1})$ anti-D4-branes between the $NS5_1$-brane and its mirror.

The gauge group is given by

$$SU(\tilde{N}_{c,1} \equiv 2N_{c,2} - N_{c,1} + 4) \times SU(N_{c,2}) \times \cdots \times SU(N_{c,n})$$

where the number of dual color can be obtained from the linking number counting, as done in [21]. The matter contents are the flavor singlet $f_1$ in the bifundamental representation $(\tilde{N}_{c,1}, \overline{N}_{c,2}, \cdots, 1_n)$ and its complex conjugate field $\tilde{f}_1$ in the bifundamental representation $(\overline{N}_{c,1}, N_{c,2}, \cdots, 1_n)$, and the gauge singlet $\Phi_2 \equiv F_1 F_1$ in the representation for $(1_1, (N_{c,2} - N_{c,3})^2 - 1, 1_3, \cdots, 1_n) \oplus (1_1, \cdots, 1_n)$, under the dual gauge group where the
Figure 28: The $N = 1$ magnetic brane configuration for the gauge group containing $SU(\tilde{N}_{c,1}) = 2N_{c,2} - N_{c,1} + 4)$ with D4- and $\overline{D4}$-branes (28A) and with a misalignment between D4-branes (28B) when the two NS5' branes are close to each other.

The gauge group is broken from $SU(N_{c,2})$ to $SU(N_{c,2} - N_{c,3})$. There are also the antisymmetric flavor $a$, the conjugate symmetric flavor $\tilde{s}$ and eight fundamentals $\hat{q}$ for $SU(\tilde{N}_{c,1})$ as well as $F_j$ and $\tilde{F}_j$.

Then the dual magnetic superpotential, by adding the mass term for the bifundamental, is given by

$$W_{dual} = \Phi_2 f_1 \tilde{f}_1 + m_2 \text{tr} \Phi_2 + \hat{q} \tilde{s} \hat{q}.$$  

Here the magnetic fields $f_1$ and $\tilde{f}_1$ correspond to 4-4 strings connecting the $\tilde{N}_{c,1}$-color D4-branes (that are connecting between the NS5$_1$-brane and the NS5'$_2$-brane in Figure 28B) with $N_{c,2}$-flavor D4-branes (which are realized as corresponding D4-branes in Figure 28A). Among these $N_{c,2}$-flavor D4-branes, only the strings ending on the upper $\tilde{N}_{c,1}$ D4-branes and on the tilted $(N_{c,2} - N_{c,3} - \tilde{N}_{c,1})$ D4-branes in Figure 28B enter the above cubic superpotential term. Note that the summation of these D4-branes is equal to $(N_{c,2} - N_{c,3})$.

When the NS5'$_2$-brane which is connected by $\tilde{N}_{c,1}$ D4-branes is replaced by $(N_{c,2} - N_{c,3})$ D6-branes and the NS5$_3$-brane is rotated by $\frac{\pi}{2}$ in Figure 28B, the brane configuration reduces to the one in [23] where the gauge group is given by $SU(2n_{f,1} + 2n_{c,2} - n_{c,1} + 4) \times SU(n_{c,2}) \times SU(n_{c,3}) \times \cdots$ with $n_{f,1}$ fundamentals, bifundamentals, an antisymmetric flavor, a conjugate symmetric flavor, eight fundamentals and various gauge singlets. Then the our $N_{c,1}$ corresponds to the $n_{c,1}$, the number $(N_{c,2} - N_{c,3})$ corresponds to $n_{f,1}$, and our $N_{c,3}$ corresponds to the $n_{c,2}$.

The gauge couplings for the gauge group factors are given by similarly and the superpotential corresponding to Figures 28A and 28B is given by

$$W_{dual} = h\Phi_2 f_1 \tilde{f}_1 - h\mu_2^2 \text{tr} \Phi_2 + \hat{q} \tilde{s} \hat{q}, \quad h^2 = g_{2,\text{mag}}^2, \quad \mu_2^2 = \frac{(\Delta x)^2}{2\pi g_s \ell_s^2}.$$
Then the product $f_1 \tilde{f}_1$ is a $\tilde{N}_{c,1} \times \tilde{N}_{c,1}$ matrix where the second gauge group indices for $f_1$ and $\tilde{f}_1$ are contracted with those of $\Phi_2$ while $\mu_2^2$ is a $(N_{c,2} - N_{c,3}) \times (N_{c,2} - N_{c,3})$ matrix. Therefore, the F-term equation, the derivative of $W_{\text{dual}}$ with respect to the meson field $\Phi_2$ cannot be satisfied if the $(N_{c,2} - N_{c,3})$ exceeds $\tilde{N}_{c,1}$. So the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. That is, there are five equations from F-term conditions: $f_1^a \tilde{f}_{1,b} - \mu_2^2 \delta_b^a = 0$, $\Phi_2 f_1 = 0$, $\tilde{f}_1 \Phi_2 = 0$, $\hat{q} \tilde{s} = 0$, and $\hat{q} \hat{q} = 0$. Then the solutions for these are given by

$$\langle f_1 \rangle = \left( \begin{array}{c} \mu_2 1_{\tilde{N}_{c,1}} \\ 0 \end{array} \right), \quad \langle \tilde{f}_1 \rangle = \left( \begin{array}{c} \mu_2 1_{\tilde{N}_{c,1}} \\ 0 \end{array} \right), \quad \langle \Phi_2 \rangle = \left( \begin{array}{c} 0 \\ 0 \\ M_2 1_{(N_{c,2} - N_{c,3} - \tilde{N}_{c,1})} \end{array} \right),$$

$$\langle \hat{q} \rangle = 0, \quad \langle \tilde{s} \rangle = 0.$$

8 Conclusions

The meta-stable brane configurations we have found are summarized by Figures 3-7 for the theory described in section 2, by Figures 10-14 for the theory given in section 3, by Figures 17-21 for the theory in section 4. Moreover, those are described by Figure 24 and “modified” Figures 17-20 for the theory described in section 5, by Figure 26 and “deformed” Figures 17-20 for the theory described in section 6, and by Figure 28 and “modified” Figures 17-20 for the last theory described in section 7. If we replace the upper NS5'-brane in these Figures with the coincident D6-branes, the corresponding brane configurations become nonsupersymmetric minimal energy brane configurations found in [22, 23] previously.

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