Holographic energy density, dark energy sound speed, and tensions in cosmological parameters: $H_0$ and $S_8$

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Abstract. Interesting discrepancies in cosmological parameters are challenging the success of the $\Lambda$CDM model. Direct measurements of the Hubble constant $H_0$ using Cepheid variables and supernovae turn out to be higher than inferred from the Cosmic Microwave Background (CMB). Weak galaxy lensing surveys consistently report values of the strength of matter clustering $\sigma_8$ lower than values derived from the CMB in the context of $\Lambda$CDM. In this paper we address these discrepancies in cosmological parameters by considering Dark Energy (DE) as a fluid with evolving equation of state $w_{\text{de}}(z)$, constant sound speed squared $c_s^2$, and vanishing anisotropic stress $\sigma$. Our $w_{\text{de}}(z)$ is derived from the Holographic Principle and can consecutively exhibit radiation-like, matter-like, and DE-like behaviour, thus affecting the sound horizon and the comoving angular diameter distance, hence $H_0$. Here we show DE sound speed plays a part in the matter clustering behaviour through its effect on the evolution of the gravitational potential. We compute cosmological constraints using several data set combinations including primary CMB, CMB lensing, redshift-space-distortions, local distance-ladder, supernovae, and baryon acoustic oscillations. In our analysis we marginalise over $c_s^2$ and find $c_s^2 = 1$ is excluded at $\gtrsim 3\sigma$. For our baseline result including the whole data set we found $H_0$ and $\sigma_8$ in good agreement (within $\approx 2\sigma$) with low redshift probes. Our constraint for the baryon energy density $\omega_b$ is however in $\approx 3\sigma$ tension with BBN constraints. We conclude evolving DE also having non-standard clustering properties [e.g., $c_s^2(z,k)$] might be relevant for the solution of current discrepancies in cosmological parameters.

Keywords: dark energy theory, cosmological parameters from CMBR, cosmological parameters from LSS

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1 Introduction

While the concordance model provides a reasonable, good phenomenological description of most astrophysical measurements [1–4], it also becomes clear that our ignorance about the nature of Dark Matter as well as the so-called cosmological constant problem represent major drawbacks in the model. In addition, over the past years we have seen the emergence of pretty interesting discrepancies (e.g., the Hubble constant $H_0$, the strength of matter clustering $\sigma_8$) in cosmological parameters whose understanding could reveal new physics disregarded in the standard cosmological model [5–25].

Although Bayesian analyses show that the standard cosmological model \( \Lambda \)CDM performs better than its simplest alternatives [26], there exists the possibility that more elaborate models could explain the shortcomings \( \Lambda \)CDM is facing. Dynamical Dark Energy and Modified Gravity (MG) have become the two leading approaches when trying to explain the late-time accelerating universe [27–30]. There is however no conclusive evidence for new Dark Energy (DE) fields or deviations from General Relativity [31–33].

Within the wide spectrum of proposals to address the DE problem, there is a hypothesis known as the Holographic Principle (HP). Roughly speaking, the HP asserts that everything inside a region of space can be described by bits of information confined to the boundary [34–39]. This non-extensive scaling would suggest that quantum field theory ceases to be valid in a large volume. Nevertheless, it is also true that the performance of local quantum field theory at describing particle phenomenology is quite remarkable. It turns out that a relationship between ultraviolet (UV) and infrared (IR) cut-offs of an effective quantum field theory could make these regimes compatible with each other [40]. If $\rho$ is the quantum zero-point energy density associated to a UV cut-off, the total energy in a region of size $L$ should not exceed the mass of a black hole of the same size, namely,

$$L^3 \rho \leq L M_p^2,$$

(1.1)

where $M_p$ is the reduced Planck mass. The largest, allowed IR cut-off $L_{\text{IR}}$ saturates the inequality (1.1) so that the maximum energy density in the effective theory is given by

$$\rho = 3 \gamma^2 M_p^2 L_{\text{IR}}^{-2},$$

(1.2)
where $\gamma$ is an arbitrary parameter. The UV/IR relationship (1.2) is a consequence of recognising that quantum field theory overestimates states. Moreover, it offers a possible way of understanding the cosmological constant problem [41–43], one of the main shortcomings of the standard cosmological model $\Lambda\text{CDM}$.

Interestingly, the UV/IR relation (1.2) has been widely applied in cosmology as an alternative to the cosmological constant causing the late-time accelerating expansion in the concordance model. These kinds of cosmological models are now known as Holographic Dark Energy (HDE) models (see [44] for a review). In this context, the IR cut-off $L_{\text{IR}}$ has a cosmological origin and various choices are found in the literature [45–51]. Despite being appealing as an alternative to $\Lambda\text{CDM}$, the HDE models investigated here are not derived from a Lagrangian which is a disadvantage when studying the evolution of cosmological perturbations: since HDE models do not have a Lagrangian, we cannot derive equations of motion for linear order perturbations.\(^1\) Nevertheless, fairly general theories relying on scalar and vector fields (e.g., scalar-vector-tensor theories [53]) could provide background phenomenology matching HDE models while allowing the investigation of cosmological perturbations. Here we will adopt a phenomenological approach and assume the existence of a DE fluid having an evolving equation of state $w_{\text{de}}(a)$ derived from the UV/IR relation (1.2). As for the description of DE perturbations, we opt for a constant sound speed in the fluid rest-frame $\hat{c}^2_s$ and vanishing anisotropic stress $\pi = 0$.

In this work we want to determine whether or not HDE is viable given current astrophysical measurements. Although cosmological constraints have been computed for HDE models (see, for instance, [54–68]), a few details have been overlooked. Firstly, while HDE models usually feature an evolving $w_{\text{de}}(a)$ which might cross the phantom divide $w_{\text{de}} = -1$, this behaviour is not properly addressed in the literature when also considering the evolution of perturbations. Here we will take it into consideration by using the Parameterized Post-Friedmann (PPF) formalism [69]. Secondly, when modelling DE perturbations, studies exist which a priori set $\hat{c}^2_s$ to a constant value. However, this choice could bias cosmological constraints as it directly affects the clustering properties of DE. In our investigation we marginalise over $\hat{c}^2_s$ and inquire about its phenomenological signatures in the context of HDE. Thirdly, with regard to cosmological constraints of HDE models, most studies focus on the background evolution and use only low red-shift data to constrain the parameter space fully disregarding the impact on earlier stages of the Universe. Here we fill this gap in the literature by also studying the impact of HDE on linear order perturbations.

The manuscript is organised as follows. In section 2 we set our notation, discuss the particular HDE model and explain its background phenomenology as well as the behaviour of linear order perturbations. In sections 3–4 we present and discuss results for cosmological constraints. Finally, in section 5 we give our conclusions.

2 Theoretical framework and holographic dark energy model

The Einstein-Hilbert action reads

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa} + \mathcal{L}_m \right), \quad (2.1)$$

where $\mathcal{L}_m$ denotes the Lagrangian for any matter fields appearing in the theory, $g$ is the determinant of the metric $g_{\mu\nu}$, $R$ is the Ricci scalar and $\kappa \equiv 8\pi G$ is a constant with $G$ being

\(^1\)However, see ref. [52] for a relation between HDE and massive gravity theory that could provide a framework for investigating perturbations.
the bare Newton’s constant. By applying the Principle of Least Action we can derive the well known Einstein field equations

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}, \]

(2.2)

where \( R_{\mu\nu} \) is the Ricci tensor and \( T_{\mu\nu} \) is the energy-momentum tensor of matter fields. Since observations and simulations indicate that on large enough scales the Universe is statistically homogeneous and isotropic also having vanishing curvature [1, 70–72], here we will assume a flat, linearly perturbed Friedmann-Lemaître-Robertson-Walker metric (FLRW). In the conformal Newtonian gauge [73]

\[ ds^2 = a(\tau)^2 \left[ -(1 + 2\psi(\vec{x}, \tau))d\tau^2 + (1 - 2\phi(\vec{x}, \tau))d\vec{x}^2 \right], \]

(2.3)

where \( a(\tau) \) is the scale factor, and \( \psi, \phi \) denote the gravitational potentials. As usual we will consider the material content as described by a perfect fluid with energy-momentum tensor

\[ T^\mu_\nu = P_{\text{fld}} \delta^\mu_\nu + (\bar{\rho}_{\text{fld}} + P_{\text{fld}}) U^\mu, \]

(2.4)

where \( \bar{\rho}_{\text{fld}}, P_{\text{fld}}, \) and \( U^\mu \) respectively denote the energy density, pressure, and four-velocity vector of the fluid. At first order the four-velocity vector is given by

\[ U^\mu = a(\tau)^{-1}(1 - \psi, \vec{u}), \]

which satisfies \( U^\mu U_\mu = -1 \), with \( \vec{u} = \vec{x} \). Taking into account linear perturbations, the elements of the energy-momentum tensor are given by

\[ T^0_0 = -\bar{\rho}_{\text{fld}} + \delta \rho_{\text{fld}}, \]

(2.5)

\[ T^i_i = (\bar{\rho}_{\text{fld}} + P_{\text{fld}}) u_i, \]

(2.6)

\[ T^i_j = (\bar{P}_{\text{fld}} + \delta P_{\text{fld}}) \delta^i_j + \Sigma^i_j, \]

(2.7)

where \( \bar{\rho}_{\text{fld}}, \bar{P}_{\text{fld}} \) are background quantities and only depend on time. The perturbations \( \delta \rho_{\text{fld}}, \delta P_{\text{fld}}, \Sigma^i_j \) depend on \( (\vec{x}, \tau) \). The anisotropic stress tensor of the fluid is defined as

\[ \Sigma^i_j = T^i_j - \delta^i_j T^k_k/3. \]

2.1 Background

From the time-time component of eq. (2.2) and using the unperturbed (i.e., \( \psi = \phi = 0 \)) FLRW metric (2.3), we obtain

\[ H^2 = \frac{\kappa}{3}(\bar{\rho}_r + \bar{\rho}_m + \bar{\rho}_{\text{de}}), \]

(2.8)

where the Hubble parameter \( H \equiv \frac{1}{a(t)} \frac{da}{dt} \), and \( \rho_{\text{de}}, \rho_r, \rho_m \) respectively denote DE, radiation, and matter energy densities. While radiation and matter will be taken into account as in the standard cosmological model \( \Lambda \)CDM, we will consider DE as a fluid with energy density given by (1.2). We choose the so-called GO cut-off [47]

\[ L^{-2}_{\text{IR}} \equiv \alpha H^2 + \beta \frac{dH}{dt} \]

(2.9)

\[ ^2 \text{Unless stated otherwise, throughout this paper we adopt the following conventions: speed of light } c = 1, \tau \text{ is the conformal time, } \vec{x} \text{ denotes conformal comoving coordinates, and the metric signature is } (- + + +). \]
where $\alpha$ and $\beta$ are dimensionless constants. Eqs. (1.2) and (2.9) allow us to define a HDE density
\[ \rho_{\text{de}} = \frac{3}{\kappa} \left( \alpha H^2 + \beta \frac{dH}{dt} \right), \tag{2.10} \]
where the constant $\gamma$ was absorbed by $\alpha$ and $\beta$. Taking into account eq. (2.10), we can rewrite the Friedmann equation (2.8) as
\[ H^2 = \Omega_{\text{r},0} H_0^2 a^{-4} + \Omega_{\text{m},0} H_0^2 a^{-3} + \left( \alpha H^2 + \beta \frac{a dH^2}{2 da} \right). \tag{2.11} \]
We define $E^2 \equiv \frac{H^2}{M_0^2}$ and find an analytical solution for the differential equation (2.11) given by
\[ E^2(a) = \Omega_{\text{r},0} a^{-4} + \Omega_{\text{m},0} a^{-3} + \Omega_{\text{de},0} a^{\frac{2(\alpha - 1)}{\beta}}, \tag{2.12} \]
where
\[ \Omega_{\text{r},0} \equiv \left( 1 + \frac{\alpha - 2\beta}{1 - \alpha + 2\beta} \right) \Omega_{\text{r},0}, \tag{2.13} \]
\[ \Omega_{\text{m},0} \equiv \left( 1 + \frac{2\alpha - 3\beta}{2 - 2\alpha + 3\beta} \right) \Omega_{\text{m},0}, \tag{2.14} \]
\[ \Omega_{\text{de},0} \equiv \left( 1 - \frac{2\Omega_{\text{m},0}}{2 - 2\alpha + 3\beta} - \frac{\Omega_{\text{r},0}}{1 - \alpha + 2\beta} \right), \tag{2.15} \]
and the effective parameter densities satisfy $\Omega_{\text{m},0} + \Omega_{\text{r},0} + \Omega_{\text{de},0} = 1$. Note that the HDE parameter density reads
\[ \Omega_{\text{de}} = \left( \frac{\alpha - 2\beta}{1 - \alpha + 2\beta} \right) \Omega_{\text{r},0} a^{-4} + \left( \frac{2\alpha - 3\beta}{2 - 2\alpha + 3\beta} \right) \Omega_{\text{m},0} a^{-3} + \Omega_{\text{de},0} a^{\frac{2(\alpha - 1)}{\beta}}. \tag{2.16} \]
Assuming a barotropic fluid with $P_{\text{de}} = w_{\text{de}} \rho_{\text{de}}$, from the condition for energy conservation
\[ \frac{d\rho_{\text{de}}}{dt} + 3H \rho_{\text{de}} (1 + w_{\text{de}}) = 0 \tag{2.17} \]
and eqs. (2.10) and (2.12), we can derive the equation of state for our DE fluid
\[ w_{\text{de}}(a) = \frac{\left( \frac{2\alpha - 3\beta - 2}{3\beta} \right) \Omega_{\text{de},0} a^{\frac{2(\alpha - 1)}{\beta}} + \left( \frac{2\beta - \alpha}{3\alpha - 6\beta - 3} \right) \Omega_{\text{r},0} a^{-4}}{\left( \frac{2\alpha - 3\beta}{2 - 2\alpha + 3\beta} \right) \Omega_{\text{m},0} a^{-3} + \left( \frac{\alpha - 2\beta}{1 - \alpha + 2\beta} \right) \Omega_{\text{r},0} a^{-4} + \Omega_{\text{de},0} a^{\frac{2(\alpha - 1)}{\beta}}}. \tag{2.18} \]

Figure 1 shows the evolution of parameter densities as well as the HDE equation of state $w_{\text{de}}(a)$ in eq. (2.18). It becomes clear that when $\alpha > 2\beta$ the HDE equation of state evolves from radiation-like [$w_{\text{de}}(a) \approx 1/3$] to pressure-less matter-like [$w_{\text{de}}(a) \approx 0$] until reaching a DE-like [$w_{\text{de}}(a) < -1/3$] behaviour at late times. Consequently, a non-vanishing HDE (2.10) can effectively add both pressure-less matter and radiation to the cosmological model [see eqs. (2.13)–(2.14)]. While for the case where $\alpha = 2\beta$ there is no radiation-like behaviour of HDE in the early universe, HDE contributes to the effective matter parameter

\[ \frac{d}{da} \approx aH \frac{d}{da}. \]

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\[ ^3 \] As it is usual, we define the density parameters $\Omega_{\text{r},0} \equiv \frac{M_0^2}{\kappa} \rho_{\text{r},0}$ and use $\frac{d}{dt} = aH \frac{d}{da}$. 

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Figure 1. Evolution of parameter densities and HDE equation of state $w_{de}$ for the HDE model. Note that $w_{de}$ can cross the phantom divide. Here we use $\Omega_{m,0} = 0.31$ and $\Omega_{r,0} = 8.5 \times 10^{-5}$. While in the lower panel ($\alpha > 2\beta$) the HDE equation of state has a radiation-like behaviour at early times, in the upper panel ($\alpha = 2\beta$) $w_{de}$ is matter-like when the universe is under radiation dominance.

density in the matter dominated epoch. Since in this work we focus on a possible explanation for the late time accelerating expansion of the Universe and its relation with the formation of structures, we constrain the HDE model to satisfy $\Omega_{r,0}^{\text{eff}} = \Omega_{r,0}$ later when computing cosmological constraints. From eq. (2.13), the latter is fulfilled for $\alpha = 2\beta$. In this way we make sure that the early universe is described as in the $\Lambda$CDM model. Later in section 4 we will expand on this constraint in relation with previous works.

With the constraint $\alpha = 2\beta$, the HDE density in eq. (2.10) becomes the well known Ricci Dark Energy (RDE) which only has a single free parameter. Then, the normalised Hubble parameter (2.12) and the HDE equation of state (2.18) are simplified

\[
E^2(a) = \Omega_{r,0} a^{-4} + \left(1 + \frac{\alpha}{4 - \alpha}\right) \Omega_{m,0} a^{-3} + \left(1 - \frac{\Omega_{m,0}}{1 - \frac{\alpha}{4}} - \Omega_{r,0}\right) a^{\frac{4-\alpha}{\alpha}}, \tag{2.19}
\]

\[
w_{de}(a) = \frac{\alpha - 4}{3\alpha \left(1 - \frac{\alpha (4 - \alpha)^2}{(4 - \alpha)(1 - \Omega_{r,0}) + 4\Omega_{m,0}}\right)} \frac{\alpha}{\alpha - 4}. \tag{2.20}
\]

From eq. (2.19) we can easily extract the HDE density

\[
\tilde{\rho}_{de}(a) = \frac{\alpha}{(4 - \alpha)} \Omega_{m,0} a^{-3} + \left(1 - \frac{4}{(4 - \alpha)} \Omega_{m,0} - \Omega_{r,0}\right) a^{\frac{4-\alpha}{\alpha}}, \tag{2.21}
\]
where \( \tilde{\rho}_{\text{de}} \equiv \frac{\dot{\rho}_{\text{de}}}{H_0^2} \). Moreover, from the conservation equation (2.17) we can derive an expression for the HDE pressure \( \tilde{P}_{\text{de}} \equiv \frac{\rho_{\text{de}}}{H_0^2} \)

\[
\tilde{P}_{\text{de}}(a) = \frac{4 - \alpha}{3\alpha} \left( 1 - \frac{4}{(4 - \alpha)} \Omega_{m,0} \right) a^{\frac{4 - \alpha}{\alpha}},
\]  

(2.22)

which in turn allows us to compute the adiabatic sound speed squared for the DE fluid

\[
c_a^2 \equiv \frac{d\tilde{P}_{\text{de}}}{d\tilde{\rho}_{\text{de}}} = w_{\text{de}} - \frac{w'_{\text{de}}}{3(1 + w_{\text{de}})} = \frac{4(\alpha - 4)}{3\alpha} \left( \frac{\Omega_{m,0}}{(\alpha - 1)(\alpha - 4)(1 - \Omega_{r,0}) + 4\Omega_{m,0}} \right) a^\frac{\alpha - 4}{\alpha}.
\]  

(2.23)

We will use the previous expressions to derive simplified, approximate solutions for the DE perturbations during matter dominance in the next section.

### 2.2 First order perturbations

In this work we are interested in computing statistical properties of observables such as the CMB angular power spectra and the matter power spectrum as predicted by the HDE model. Here we limit ourselves to first order scalar perturbations and therefore we need to solve the differential equations governing the linearised Einstein field equations (2.2). Using the conformal Newtonian gauge (2.3) and taking into account a general fluid (2.5)–(2.7), we obtain

\[
k^2 \phi + 3 \frac{\dot{a}}{a} \left( \dot{\phi} + \frac{\dot{a}}{a} \psi \right) = 4\pi G a^2 \delta T^0_0, \tag{2.24}
\]

\[
k^2 \left( \dot{\phi} + \frac{\dot{a}}{a} \psi \right) = 4\pi G a^2 (\tilde{\rho}_{\text{fld}} + \tilde{P}_{\text{fld}}) \theta_{\text{fld}}, \tag{2.25}
\]

\[
\frac{\ddot{\phi}}{a} + \frac{\dot{a}}{a} (\dot{\psi} + 2 \dot{\phi}) + \left( \frac{2\dot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \psi + \frac{k^2}{3} (\phi - \psi) = \frac{4\pi}{3} G a^2 \delta T^i_i, \tag{2.26}
\]

\[
k^2 (\phi - \psi) = 12\pi G a^2 (\tilde{\rho}_{\text{fld}} + \tilde{P}_{\text{fld}}) \sigma_{\text{fld}}, \tag{2.27}
\]

where \( k \) is the wavenumber, the divergence of the velocity field is defined as \( \theta_{\text{fld}} \equiv ik^j u_j \), and \( \sigma_{\text{fld}} \) is the anisotropic stress. In the cases where the universe is regarded as composed by several fluids, the right-hand side in eqs. (2.24)–(2.27) is intended to be a sum over all species (e.g., radiation, matter, dark energy). From the conservation of energy-momentum \( \nabla_\mu T^{\mu\nu} = 0 \) for a single fluid we obtain

\[
\dot{\delta}_{\text{fld}} = -V_{\text{fld}} + 3(1 + w_{\text{fld}})\delta_{\text{fld}} - 3\frac{\dot{a}}{a} \left( \frac{\delta P_{\text{fld}}}{\tilde{\rho}_{\text{fld}}} - w_{\text{fld}} \delta_{\text{fld}} \right), \tag{2.28}
\]

\[
\dot{V}_{\text{fld}} = -\frac{\dot{a}}{a} (1 - 3w_{\text{fld}}) V_{\text{fld}} + \frac{\delta P_{\text{fld}}}{\tilde{\rho}_{\text{fld}}} k^2 + k^2 (1 + w_{\text{fld}}) \delta_{\text{fld}}, \tag{2.29}
\]

where we have used the scalar velocity perturbation \( V_{\text{fld}} \equiv ik_j T^j_0 / \tilde{\rho} = (1 + w_{\text{fld}}) \theta_{\text{fld}} \) and disregarded anisotropic stress.

Since we are interested in the observational signatures of the HDE cosmological model at late-times, in the remainder of this section we will discuss the behaviour of matter and DE
perturbations starting our analysis in the matter dominated epoch. We consider pressure-less matter with $w_m = 0$, $\delta P_m = 0$, $\sigma_m = 0$ and assume HDE as a fluid having DE equation of state (2.18), pressure perturbation $\delta P_{de}$, and vanishing anisotropic stress $\sigma_{de} = 0$. The latter and eq. (2.27) imply that at late-times the gravitational potentials $\phi = \psi$.

We parameterise the DE pressure perturbation as

$$\frac{\delta P_{de}}{\bar{\rho}_{de}} = c_s^2 \delta + \frac{3aH (\dot{c}_s^2 - c_a^2)}{k^2} V$$

(2.30)

where $c_s^2$ is the DE sound speed squared in the rest-frame, $\delta$ is the DE density perturbation, and $V$ is the DE velocity perturbation. Using (2.30), we rewrite eqs. (2.28)–(2.29) for the DE perturbations

$$\delta' = -\frac{V}{Ha^2} \left( 1 + 9a^2H^2 (\dot{c}_s^2 - w_{de}) + \frac{3a^3H^2w_{de}'}{k^2(1+w_{de})} \right) - \frac{3}{a} (\dot{c}_s^2 - w_{de}) \delta + 3(1+w_{de})\phi',$$

(2.31)

$$V' = -\left( 1 - 3c_s^2 - \frac{aw_{de}'}{(1+w_{de})} \right) \frac{V}{a} + \frac{k^2c_s^2}{Ha^2} \delta + (1+w_{de}) \frac{k^2}{Ha^2} \phi,$$

(2.32)

whereas for matter the perturbation equations (2.28)–(2.29) become

$$\delta'_m = -\frac{V_m}{Ha^2} + 3\phi',$$

(2.33)

$$V'_m = -\frac{V_m}{a} + \frac{k^2}{Ha^2} \phi.$$  

(2.34)

Note we can combine eqs. (2.24)–(2.25) and obtain

$$k^2\phi = -4\pi Ga^2 \sum_j \rho_j \left( \delta_j + \frac{3aH}{k^2} V_j \right).$$

(2.35)

### 2.2.1 Matter dominance

Here we will work out the solution for the system of differential equations (2.31)–(2.34) governing the evolution of matter and DE perturbations. We focus on late times starting from the epoch when matter becomes dominant so that we can safely neglect radiation in the model. Since we are interested in analytical, approximate solutions, to simplify our problem we assume that during Matter Dominance (MD) the Hubble parameter is

$$H^2 = H_0^2\Omega_{m0}^2 a^{-3}.$$  

(2.36)

For the standard cosmological model $\Lambda$CDM only matter contributes to the pressure perturbation in the right-hand side of eq. (2.26), hence the solution for the gravitational potential $\phi$ takes on a constant value under MD. The situation is different for the HDE we investigate here because the DE fluid might have not negligible contributions to the pressure perturbation. The latter is parameterised by eq. (2.30) and therefore we identify two situations where $\phi$ is constant as in $\Lambda$CDM: i) $\alpha = 2\beta$ ($c_a^2 = 0$) and $\dot{c}_s^2 = 0$ so that $\delta P_{de} = 0$; ii) the more general scenario where $\alpha$, $\beta$ are independent parameters ($c_a^2 \approx 0$) and $\dot{c}_s^2 = 0$ implying $\delta P_{de} \approx 0$. 

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We could only find analytical solutions for the perturbations when the gravitational potential takes on a constant value \( \phi_0 \). Therefore, using eq. (2.36) in eqs. (2.31)–(2.34), we find the solutions for both matter and DE perturbations in MD

\[
\delta_m = \delta_0 \left( a + 3 \frac{H_0^2 \Omega_{m,0}^{\text{eff}}}{k^2} \right),
\]

\[
V_m = -\delta_0 H_0 \sqrt{\Omega_{m,0}^{\text{eff}} a^{1/2}},
\]

\[
\delta = \delta_m,
\]

\[
V = V_m,
\]

where

\[
\delta_0 \equiv -\frac{2k^2 \phi_0}{3H_0^2 \Omega_{m,0}^{\text{eff}}}. \tag{2.41}
\]

Note that the potential \( \phi \) in eq. (2.35) also receives contributions from the DE fluid in MD: DE perturbations behave as matter perturbations [see eqs. (2.39)–(2.40)] and the background DE density is not negligible [see eq. (2.16)] under MD.

We numerically solved the system of differential equations (2.31)–(2.34) also taking into account the expression (2.35) for the gravitational potential. The latter is depicted in figure 2 for the standard cosmological model \( \Lambda \)CDM along with solutions for HDE and RDE models. It becomes clear that while models having \( \hat{c}_s^2 = 1 \) exhibit a varying gravitational potential during MD, models with a vanishing sound speed present a behaviour similar to \( \Lambda \)CDM, namely, a constant \( \phi \). Note that for the given set of cosmological parameters, HDE \( (\hat{c}_s^2 = 0) \) enters later than RDE \( (\hat{c}_s^2 = 0) \) the regime of MD; the small variation of \( \phi \) in this case is due to a non-vanishing adiabatic sound speed affecting the DE pressure perturbation, hence the gravitational potential.

In figure 3 we show the numerical solutions for the matter perturbations. We display solutions for the concordance model \( \Lambda \)CDM and for the RDE \( (\hat{c}_s^2 = 0) \) model. In the case of \( \Lambda \)CDM we also show the well-known analytical solutions in the MD epoch. Differences in the solutions of density perturbations arise mainly before horizon crossing and when DE becomes dominant. This is indeed due to the modifications introduced by the RDE via \( \Omega_{m,0}^{\text{eff}} \). With regard to the velocity perturbations, we can see that even though the dependence with the scale factor is the same in the two models, the RDE \( (\hat{c}_s^2 = 0) \) solution fully disagrees with the standard model due to \( \Omega_{m,0}^{\text{eff}} \).

In figure 4 we compare the numerical solutions for DE perturbations against the analytical, approximate solutions (2.39)–(2.40) valid in the MD regime. We carry out the comparison for HDE \( (\hat{c}_s^2 = 0) \) in the left panel and RDE \( (\hat{c}_s^2 = 0) \) in the right panel. While analytical solutions describe pretty well the behaviour of \( \delta \) and \( V \) in the RDE \( (\hat{c}_s^2 = 0) \) model during MD, we find disagreement for the HDE \( (\hat{c}_s^2 = 0) \) model. This is mainly due to the fact that in the latter full MD starts later than in RDE \( (\hat{c}_s^2 = 0) \) for the given set of cosmological parameters.

### 2.2.2 Implementation in Boltzmann solver

Thus far in our treatment of perturbations, we regarded DE perturbations as governed by fluid equations. We used a prescription for the pressure perturbation given by (2.30). However, this approach has problems (e.g., divergences, instabilities) when the DE equation of
Figure 2. Evolution of the gravitational potential in $\Lambda$CDM, HDE, and RDE cosmological models. All the plots are normalised by the initial value in $\Lambda$CDM. Common cosmological parameters used to numerically solve the system of differential equations are: $\Omega_{r,0} = 0$, $\Omega_{m,0} = 0.3$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $k = 25H_0$, $\delta_0 = 1$; for RDE $\alpha = 0.91$; for HDE $\alpha = 0.88$ and $\beta = 0.39$.

state crosses the phantom divide which turns out to be the case in the HDE model we investigate here (see figure 1). Not allowing cosmological models to cross $w_{\text{de}} = -1$ could introduce unwanted bias in our modelling since the phantom divide is currently a crucial point [74]. By requiring strict energy and momentum conservation, the PPF formalism (also implemented in the Boltzmann solver CLASS) can deal with “smooth” DE crossing the phantom divide [69].

The PPF description of DE replaces the density and momentum components with a single joint dynamical variable

$$\Gamma \equiv -\frac{4\pi G a^2}{k^2 c_K} \delta \rho_{\text{de}},$$

(2.42)

thus reducing closure conditions, but requiring strict conservation of energy and momentum in its equation of motion. Here $c_K = 1 - 3K/k^2$, where $K$ is the space-time curvature that we set to $K = 0$. The evolution equation for $\Gamma$ is given by

$$\left(1 + c_t^2 k_H^2 \right) \left[ \frac{\dot{\Gamma}}{H} + \Gamma + c_t^2 k_H^2 \Gamma \right] = S,$$

(2.43)
Figure 3. Evolution of density and velocity matter perturbations in the $\Lambda$CDM and RDE ($\hat{c}_s^2 = 0$) model. We show the numerical solutions (solid and dashed curves) as well as the analytical solutions (dotted curves) in MD for the standard cosmological model. Cosmological parameters are as specified in figure 2.

where $c_T \equiv 0.4 \, \hat{c}_s$ calibrates the scale of the transition, $k_H = k^2 / a H$ and

$$S = \frac{\dot{a}}{a} \frac{4 \pi G}{H^2} \rho_{de} \left(1 + w_{de}\right) \frac{\theta_T}{k^2}$$

(2.44)

where the subscript T denotes all species except dark energy.

Figure 5 shows the output of our implementation in CLASS for a RDE model crossing the phantom divide. We use the best fit cosmological parameters in ref. [56] and depict $\Lambda$CDM along with RDE model for different values of the DE sound speed $\hat{c}_s^2$. The latter has a relevant effect in both CMB angular power spectrum and matter power spectrum (not shown in ref. [56]).

3 Data and methodology

In order to compute cosmological constraints for the cosmological model including DE as given by the DE energy density (2.10), we carried out the analysis in two parts. Firstly, we performed an analysis only taking into consideration background data. Secondly, we add
data from the CMB anisotropies and Redshift-Space-Distortions (RSD) that constrain the model further.

Data constraining the background evolution of the model include: Baryon Acoustic Oscillations (BAO) from refs. [75–77], Pantheon supernovae (SNe) data set from ref. [78], and the SH0ES local measurement of the Hubble constant (H0) from ref. [8] that we introduce as a Gaussian prior. As for data constraining linear order perturbations we take in information from CMB lensing (lensing) as well as temperature and polarization anisotropies of the CMB (TTTEEE) measured by the Planck collaboration [1], and a compilation of RSD as explained in ref. [79].

As discussed previously, we implemented the HDE cosmological model by considering a DE fluid with DE equation of state \( (2.18) \), constant sound speed \( c_s^2 \), and vanishing anisotropic stress. We carried out the implementation in the widely used Boltzmann solver CLASS. For a given set of cosmological parameters, the code computes all the relevant quantities (e.g., luminosity distances, CMB angular power spectrum) so that theoretical predictions can be compared with astrophysical measurements. We performed a Markov Chain Monte Carlo (MCMC) statistical analysis by sampling the parameter space with the code Monte
While the analysis carried out in ref. [56] seems to have fixed the DE sound speed $c_s^2 = 1$, here we marginalise over $c_s^2$. In figure 5 we show CMB angular power spectra along with

\begin{align*}
\ell(\ell+1)C_{\ell}^{TT} / 2\pi \left[ \mu K^2 \right] \\
\ell / 2\pi \left[ \mu K^2 \right] \\
\end{align*}

\begin{align*}
P_m(k) \left[ (h^{-1} \text{Mpc})^3 \right] \\
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 \\
10^1 10^2 10^3 10^4 10^5 \\
\end{align*}

**Figure 5.** CMB temperature angular power spectrum $C_{\ell}^{TT}$ and matter power spectrum $P(k, z = 0)$ for different values of $c_s^2$. We fix other cosmological parameters to the best fit values reported in table 1 of ref. [56], namely: for RDE $\omega_b = 0.0241$, $\omega_{cdm} = 0.1086$, $H_0 = 72.26 \text{km} \text{s}^{-1} \text{Mpc}^{-1}$, $n_s = 1.0871$, $\ln 10^{10} A_s = 3.122$, $\tau = 0.1382$ and $\alpha = 0.6904$ (note that authors in ref. [56] actually report $\beta = 0.3452$); for $\Lambda CDM$ $\omega_b = 0.0226$, $\omega_{cdm} = 0.1123$, $H_0 = 70.38 \text{km} \text{s}^{-1} \text{Mpc}^{-1}$, $n_s = 0.9691$, $\ln 10^{10} A_s = 3.180$, $\tau = 0.0877$.

Python [80, 81]. The latter is linked to CLASS and samples the parameter space with the default Metropolis-Hastings algorithm. In a first stage, a covariance matrix is adjusted so that the acceptance rate is $\approx 0.25$. Then, in a second stage of the analysis, the covariance matrix is fixed and the code performs $\sim 10^6$ iterations until reaching convergence which we estimate with the Gelman-Rubin statistic $R$ satisfying the condition $R - 1 \lesssim 0.01$ for all the varying parameters. We marginalise over the following cosmological parameters: baryon density today $\omega_b \equiv \Omega_b h^2$; cold dark matter density today $\omega_{cdm} \equiv \Omega_{cdm} h^2$; angular size of sound horizon at redshift $z_*$ (redshift for which the optical depth equals unity) $100\theta_* h$; log power of the primordial curvature perturbations $\ln 10^{10} A_s$; scalar spectrum power-law index $n_s$; Thomson scattering optical depth due to reionisation $\tau$; sound speed squared on the rest-frame of the fluid $\log c_s^2$; parameters determining the holographic DE density $\alpha$ and $\beta$. In our MCMC analyses we also marginalise over a few nuisance parameters whose number depends on the specific probe combination. For common cosmological parameters we use the same prior range as specified in table 1 of ref. [82].

When performing the first part of our statistical analysis (only background data), we vary the parameters $\omega_b$, $\omega_{cdm}$, $\alpha$, and $\beta$ (and $H_0$ when introduced as a Gaussian prior); the HDE parameters $\alpha$ and $\beta$ were introduced with an unbounded flat prior. In the second part of our analysis (taking into consideration background data as well as CMB anisotropies and Redshift-Space-Distortions), we set $\beta = \alpha / 2$ as argued earlier in section 2. Therefore, we vary the parameters $\omega_b$, $\omega_{cdm}$, $100\theta_*$, $\ln 10^{10} A_s$, $n_s$, $\tau$, $\log c_s^2$, and $\alpha$. In this case for the parameters describing the DE fluid we use the prior range specified in table 1.

With the analysis carried out in ref. [56] seems to have fixed the DE sound speed $c_s^2 = 1$, here we marginalise over $c_s^2$. In figure 5 we show CMB angular power spectra along with...
| Parameter  | Prior range |
|------------|-------------|
| $\alpha$  | $[0.01, 1.35]$ |
| $\log \hat{c}_s^2$ | $[-10, 0]$ |

Table 1. Flat prior bounds used in the full analysis including background and linear perturbations.

Table 2. Mean values and 68% confidence limits on cosmological parameters. Here we only use data constraining the background.

| Parameter | HDE:BAO | HDE:BAO+SNe | HDE:BAO+SNe+H0 | RDE:BAO+SNe+H0 |
|-----------|---------|-------------|----------------|---------------|
| $\omega_b$ | $0.0195_{-0.0144}^{+0.0051}$ | $0.0151_{-0.0097}^{+0.0025}$ | $0.0176_{-0.0108}^{+0.0039}$ | $0.0228_{-0.0106}^{+0.0048}$ |
| $\omega_{cdm}$ | $0.0688_{-0.0585}^{+0.0334}$ | $0.0821_{-0.0261}^{+0.0363}$ | $0.1010_{-0.0319}^{+0.0384}$ | $0.1202_{-0.0292}^{+0.0408}$ |
| $\beta$ | $0.49_{-0.13}^{+0.08}$ | $0.43_{-0.08}^{+0.04}$ | $0.40_{-0.08}^{+0.06}$ | — |
| $\alpha$ | $1.14_{-0.18}^{+0.22}$ | $0.98_{-0.13}^{+0.10}$ | $0.95_{-0.11}^{+0.10}$ | $0.92_{-0.13}^{+0.10}$ |
| $\Omega_{m,0}$ | $0.195_{-0.091}^{+0.077}$ | $0.214_{-0.054}^{+0.063}$ | $0.229_{-0.054}^{+0.062}$ | $0.268_{-0.045}^{+0.056}$ |

4 Results and discussion

MCMC results for the first part of our analysis constraining the background evolution are summarised in figure 6 and table 2. Mean values for the HDE parameters $\alpha$ and $\beta$ are in good agreement with previous works using different data sets [57, 67, 83]. Although we do not put any hard bound for $\alpha$ and $\beta$ in our MCMC analysis, in figure 6 we can clearly see that there are no samples in the region satisfying $\alpha < 2\beta$. This is due to the fact that in this region the HDE density becomes negative and we have required the condition $\rho_{de} \geq 0$ to hold. Figure 7 shows the behaviour of parameter densities and $w_{de}$ for a model having $\alpha < 2\beta$ (upper panel) as well as the best fit for the case HDE:BAO+SNe+H0 in table 2 (lower panel). While in the upper panel we clearly see that $w_{de}$ has a singularity when $\Omega_{de}$ changes sign, the lower panel shows a non-negligible amount of HDE during both radiation dominated epoch and DM domination. This behaviour can be understood if we note that, whatever probe combination in table 2, samples for $\alpha$ and $\beta$ satisfy $\alpha > 2\beta$ and therefore HDE effectively contributes to matter and radiation [see eqs. (2.13)–(2.14)]. Consequently, we observe in figure 6 a degeneracy between the matter parameter density $\Omega_{m,0}$ and the HDE parameter $\alpha$. The degeneracy is even more evident for the green contours showing results for the RDE model.

Next we explain the second part of our analysis where data constraining linear order perturbations are also considered and we set $\beta = \alpha/2$. Statistical information derived from our MCMC results is presented in table 3, while figure 8 contains confidence contours and marginalised 1D posteriors. Several comments can be made. Firstly, by comparing figures 6 and 8 we note that while background data sets seem compatible with each other, also taking Planck data into consideration introduces a discordance in the determination of some cosmo-
Figure 6. 1D marginalised likelihoods as well as confidence contours (i.e., 68% and 95%) for the HDE (i.e., gray, red, and blue) cosmological model. Green contours and curves show results for the RDE model with BAO+SNe+H0. Note that here we plot the matter density parameter $\Omega_{m,0} \equiv (\omega_b + \omega_{cdm})/h^2$ which is a derived parameter in our analysis. Dashed, vertical line indicates the result obtained by the Planck collaboration using the standard cosmological model (see table 2, column TTTEEE+lowE+lensing in ref. [1]). Vertical bands indicate DES results (68% and 95% confidence intervals) for $\Lambda$CDM reported in ref. [13]. Logical parameters (see dark, blue and light, green contours in figure 8), namely, $H_0$, $\omega_{cdm}$, $\sigma_8$, $n_s$, and $\alpha$. Secondly, with regard to the RDE parameter $\alpha$, we can see that in this case it is well constrained and its mean value is significantly shifted towards lower values than reported in table 2. Although we use different data sets, constraints for $\alpha$ agree at the 2$\sigma$ level with results in ref. [56]; there are however noticeable differences in other cosmological parameters such as $\omega_b$, $\omega_{cdm}$, $n_s$, $\tau$, and $\Omega_{m,0}$. Thirdly, even though the sound speed squared $\log \hat{c}_s^2$ hits the lower bound in the prior and we can only set an upper limit, it becomes clear from our results that $\hat{c}_s^2 = 1$ is excluded by more than 3$\sigma$. This result calls in question the...
Figure 7. Evolution of parameter densities and HDE equation of state $w_{de}$ for the HDE model. In the lower panel we use the best fit values for HDE:BAO+SNe+H0 ($\alpha = 0.906$, $\beta = 0.349$, $H_0 = 72.90\text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega_{m,0} = 0.224$). The magenta, dotted curve shows $w_{de}$ for the best fit of RDE:BAO+SNe+H0 ($\alpha = 0.893$, $H_0 = 73.12\text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega_{m,0} = 0.281$). We set $\Omega_r,0 = 8.5 \times 10^{-5}$, and for the upper panel we use $\Omega_m,0 = 0.2713$. Assumption of fixing $\tilde{c}_s^2$ in ref. [56] (presumably to $\tilde{c}_s^2 = 1$). A comparison between figures 5 and 9 clearly confirms that the DE sound speed plays a role in the analysis of RDE. A lower DE sound speed along with changes in $\omega_b$, $\omega_{cdm}$, $n_s$, and $\tau$ show a much better fit than previous results fixing $\tilde{c}_s^2 = 1$. Fourthly, regardless of the probe combination, the only parameter which shows relatively good agreement with the baseline result reported by the Planck collaboration for the $\Lambda$CDM model is the Thomson scattering optical depth due to reionisation $\tau$. All other parameters in common with the standard cosmological model appear relatively discrepant. Fifthly, our analysis discloses a slight degeneracy between $\alpha$ and the parameters $H_0$, $\omega_b$, $\omega_{cdm}$, and $\sigma_8$ (see figure 8). Sixthly, while for the analysis in table 2 only including background data the DE equation of state $w_{de}(a = 1) \approx -1$ (see lower panel in figure 7), the analysis in table 3 also including CMB and RSD data yields $w_{de}(a = 1) < -1$ (see figure 10). Then, despite having a present DE budget similar to the $\Lambda$CDM ($\Omega_{de}(a = 1) \approx 0.7$), we conclude the RDE model struggles to simultaneously fit low and high redshift data. In figure 9 we show the CMB TT angular power spectrum and the matter power spectrum corresponding to the best fits of analyses in table 3 and figure 8. For the sake of comparison we also depicted the Plack baseline result for the $\Lambda$CDM model. Main differences in the CMB angular power spectrum appear on very large angular scales where cosmic
variance dominates the error budget and the Sachs-Wolfe effect becomes important: the enhancement of power at small \( \ell \) is due to lower values for the spectral index than in \( \Lambda \)CDM as well as the evolution of gravitational potentials (affected by \( \alpha \) and \( \epsilon^2_\delta \)) when DE dominates the energy budget (see also left panel of figure 11). Concerning the matter power spectrum, we can see that it is heavily modified with respect to the \( \Lambda \)CDM solution depending on the data set used in the analysis. Except for the combination TTTEEE+lensing+SNe, data favour more power on large scales than in the \( \Lambda \)CDM model. Except for the combination TTTEEE+lensing+BAO+H0, data favour less power on small scales than in the standard model. Our baseline result TTTEEE+lensing+BAO+RSD+SNe+H0 keeps the angular acoustic scale in good agreement with the Planck collaboration baseline result. Right panel of figure 11 shows the effect of changing \( \alpha \) and \( \epsilon^2_\delta \) that we explain below.

In what follows we focus on our baseline result TTTEEE+lensing+BAO+RSD+SNe+H0 which seems to bring in relative good agreement CMB and Large Scale Structure data. First, note that although we do not use data from the Dark Energy Survey (DES), our derived constraints for \( S_8 \) and \( \Omega_{m,0} \) (see table 3) are compatible with DES measurements are \( S_8 = 0.776 \pm 0.017 \) and \( \Omega_{m,0} = 0.339^{+0.032}_{-0.031} \) within \( \approx 2\sigma \). Second, we can see in the right panel of figure 9 than on large scales (wavenumber \( k \lesssim 10^{-2} \) h Mpc\(^{-1}\)) beyond the reach of current galaxy surveys, the RDE model predicts more power than the standard model. This difference is due to a few changes with respect to the concordance model: i) a smaller spectral index (\( n_s = 0.9649 \) in \( \Lambda \)CDM); ii) a slight shift in the pivot scale; iii) differences in the evolution of matter perturbations in the RDE model (see figure 3). Third, we constrain the redshift dependence in the linear matter power spectrum by using the parameter \( f\sigma_8 \) through the likelihood RSD which relies on linear perturbation theory.\(^4\) Our constraint for the strength of matter clustering \( \sigma_8 \) turns out to be lower than in the \( \Lambda \)CDM model Planck baseline. Since matter velocity perturbations might be greater in RDE than in \( \Lambda \)CDM (see figure 3), we expect less matter clustering in RDE. Moreover, since the r.m.s. linear theory mass fluctuation in a

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\(^4\)We also computed non-linear corrections for the RDE using HALOFIT [84]. However, since HALOFIT is optimised for the standard model, we do not use it in this work. Even when including these non-linear corrections we observe less power on small scales in the RDE model than predicted by the \( \Lambda \)CDM model. Non-linear evolution of DE perturbations could also be investigated along the lines explained in refs. [85, 86].
Figure 8. 1D marginalised likelihoods as well as confidence contours (i.e., 68% and 95%) for the RDE cosmological model. Dashed, vertical and dotted, horizontal black lines indicate the results obtained by the Planck collaboration using the standard cosmological model (see table 2, column TTTEEE+lowE+lensing in ref. [1]). Note that here we plot the Hubble constant $H_0$ and the strength of matter clustering $\sigma_8$ which are derived parameters in our analysis. Vertical, dashed, magenta line indicates SH0ES value $73.04 \, \text{km} \, s^{-1} \, \text{Mpc}^{-1}$ [8]. Dashed, vertical and dotted, horizontal green lines indicate DES value $\sigma_8 = 0.733$ for the analysis of large scale structure combining three two-point correlation functions [13].

sphere of radius $R = 8 \, \text{Mpc} \, h^{-1}$ at $z = 0$

$$(\sigma_8)^2 = \frac{1}{2\pi^2} \int d \log k W^2(kR) k^3 P_m(k),$$

where $W(kR)$ is a spherical top-hat filter, is predominantly determined by contributions on small scales and the matter power spectrum $P_m$ predicts less power in RDE than in $\Lambda$CDM, we can expect $\sigma^{\text{RDE}}_8 < \sigma^{\Lambda\text{CDM}}_8$ as we indeed found.
Figure 9. Left: CMB temperature angular power spectrum $C_{\ell}^{TT}$. Right: linear theory matter power spectrum $P_{m}(k, z = 0)$. We plot the baseline result reported by the Planck collaboration ($\Lambda$CDM) along with best fits of analyses in figure 8 for the RDE model. Our baseline result (TTTEEE+lensing+BAO+RSD+SNe+H0) has the following best fit values for cosmological parameters: $\omega_b = 0.02351$, $\omega_{cdm} = 0.1214$, $100\theta_s = 1.04087$, $ln10^{10}A_s = 3.014$, $n_s = 0.9440$, $\tau = 0.0551$, $\alpha = 0.651$, log $\hat{c}_s^2 = -6$, $\sigma_8 = 0.736$, $H_0 = 71.44$ km s$^{-1}$ Mpc$^{-1}$.

Another interesting aspect of our analysis concerns the constraint for the Hubble constant $H_0$. Figure 8 indicates that the RDE model can simultaneously relax the current tension in $H_0$ and $\sigma_8$ (see case TTTEEE+lensing+BAO+SNe). The angular acoustic scale

$$\theta_s = \frac{r_s(z_*)}{D_A(z_*)},$$

is pretty well constrained by CMB observations. In eq. (4.2), $r_s$ and $D_A$ respectively denote the sound horizon

$$r_s = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz,$$

and the comoving angular diameter distance

$$D_A = \int_0^{z_*} \frac{dz}{H(z)}.$$

While for the $\Lambda$CDM Planck baseline

$$100\theta_s = 1.04110, \quad r_s(z_*) = 144.531055 \text{ Mpc}, \quad d_A(z_*) = 12.738778 \text{ Mpc},$$

our baseline result (the best fit)

$$100\theta_s = 1.04087, \quad r_s(z_*) = 136.800000 \text{ Mpc}, \quad d_A(z_*) = 12.053113 \text{ Mpc},$$

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Figure 10. Evolution of parameter densities and DE equation of state $w_{\text{de}}$ for the RDE model. Here we use the best fit values for our baseline result (TTTEEE+lensing+BAO+RSD+SNe+H0) analysis in figure 8. For sake of comparison we also plot the $\Lambda$CDM baseline result by the Planck collaboration.

where the comoving angular diameter distance is related to the angular diameter distance $d_A$ via $D_A(z) = (1 + z)d_A$. The RDE model decreases both the sound horizon and the comoving angular diameter distance while keeping the angular acoustic scale in good agreement with the $\Lambda$CDM solution. These changes can be understood from figure 12. On the one hand, in the RDE model the expansion rate is enhanced with respect to the standard model in two stages ($10^0 \lesssim z \lesssim 10^5$ and at late times $z \lesssim 10^{-1}$). On the other hand, for a relatively short period of time the universe expands faster in $\Lambda$CDM than in RDE for $10^{-1} \lesssim z \lesssim 10^0$.

While the Early Dark Energy model of ref. [87] mainly changes the sound horizon through the enhancement of $H(z)$ prior to recombination, the RDE model introduces changes in the expansion rate prior and post recombination.

Although our RDE baseline result brings into $\approx 2\sigma$ agreement $H_0$ and $\sigma_8$, other parameters get shifted in order to maintain the fit to primary CMB and RSD data. Besides the shift of $n_s$ towards lower values than allowed in the $\Lambda$CDM analysis of Planck, we also obtain a value of baryon matter density $\omega_b$ higher than in the Planck $\Lambda$CDM baseline result. This

\[ \text{In the EDE scenario alleviating the $H_0$ tension, a similar situation occurs. However, the EDE exacerbates the $\sigma_8$ tension while including large scale structure data in the analysis [88].} \]
value actually exacerbates the existing $\approx 2\sigma$ discrepancy in the $\Lambda$CDM model with values inferred from Big Bang Nucleosynthesis (BBN) \cite{89}.

We finalize our discussion by comparing our findings with previous results. First, whereas in ref. \cite{56} authors do not seem to have properly included radiation in their analysis when solving for the background evolution (see their eq. (5) for the Hubble parameter), here we have derived the full expression in eq. (2.19) (or the more general eq. (2.12)). Second, since the RDE model can cross the phantom divide $w_{de} = -1$ we have used the PPF formalism so that perturbations behave properly. Authors in ref. \cite{56} split the whole region of $w_{de}$ into three regions ending up with a much more involved implementation. Third, while in ref. \cite{56} values $n_s > 1$ are preferred, our analysis favours values $n_s < 1$. This is due to the much better constraining power added by Planck data on small scales in comparison to WMAP data.

5 Conclusions

Holographic Dark Energy (HDE) seems to offer a plausible alternative to the simple, but troublesome cosmological constant. While the holographic approach does not require modifications to General Relativity, neither does it require new dynamical Dark Energy (DE) fields, it does need an arbitrary choice. In these kinds of DE models the length scale fixing

\footnote{Recent analyses using an improved rate of deuterium burning estimate a $\omega_b = 0.02233 \pm 0.00036$ in excellent agreement with the Planck baseline result \cite{4}. This value is discrepant with our baseline RDE result at the $3\sigma$ level.}
the maximum energy density in the effective field theory becomes important. In the literature there exist a few ultraviolet/infrared relationships that avoid the problem of causality as well as the coincidence issue. In this work we studied two infrared (IR) cut-offs: the Ricci scalar curvature (also dubbed RDE) and its more general version the Granda-Oliveros (GO) IR cut-off.

Thus far most works studying phenomenological aspects of HDE models have focused on the background. Here we scrutinised HDE models derived from both GO and RDE cut-offs also considering linear order perturbations. Our investigation refined upon previous works as we summarise below.

Concerning the background evolution. We noted that in previous works radiation is not properly taken into account when solving for the Hubble parameter. We showed this might not be a good approximation: depending on the cut-off choice, new terms might appear playing a part in early stages of the universe expansion. As a result, the holographic energy density using the GO cut-off can be radiation-like, matter-like, or DE-like depending on the component that dominates the energy budget. For the RDE IR cut-off the holographic energy density can only be matter-like or DE-like, since the constraint $\alpha = 2\beta$ excludes a radiation-like behaviour (see figure 1). Interestingly, the peculiar behaviour in the holographic energy density changes the expansion rate prior and post-recombination (see figure 12).
Consequently, the RDE cosmological model can decrease both sound horizon and comoving angular diameter distance while keeping good agreement with CMB measurements of the angular acoustic scale. This feature might help to relax the current discrepancy in the Hubble constant between low and high redshift probes.

Considering a DE fluid having a background evolution matching the RDE model, we proceeded to the investigation of linear order perturbations and assumed our DE as having a constant sound speed $\hat{c}_s^2$ as well as vanishing anisotropic stress. We managed to find analytical, approximate solutions for matter and DE perturbations in the regime of matter dominance and when the gravitational potential can be regarded as constant. Our findings show that matter perturbations behave slightly different with respect to the standard cosmological model $\Lambda$CDM (see figure 3). Due to the matter-like behaviour of the holographic density in the RDE model, DE clusters in the same way as matter perturbations when its sound speed vanishes.

Differences in the matter clustering properties of the RDE model with respect to $\Lambda$CDM become apparent in the matter power spectrum. We implemented the RDE model in the popular Boltzmann solver CLASS so that predictions for the statistical properties of CMB and matter fluctuations could be computed. Whereas previous works artificially considered the possibility of $w_{de}$ crossing the phantom divide, here we used the PPF formalism fully granting energy-momentum conservation. Another important difference with respect to previous works concerns the treatment we gave to $\hat{c}_s^2$. In the literature, a RDE model with fixed $\hat{c}_s^2 = 1$ has been investigated. Nevertheless, we showed this choice might not be appropriate as it heavily affects matter clustering through changes in the gravitational potential during matter dominance (see figure 2).

We computed cosmological constraints for the RDE model marginalising over $\hat{c}_s^2$. In our investigation we regarded CMB data (i.e., temperature, polarisation, lensing), baryon acoustic oscillations, supernovae, redshift space distortions, and the local measurement of the Hubble constant. For our baseline result using all data sets we found that $\hat{c}_s^2 = 1$ is excluded at $\gtrsim 3\sigma$. We also obtained a value of the strength of matter clustering $\sigma_8$ lower than in the $\Lambda$CDM model and in good agreement ($\approx 2\sigma$) with recent DES results. Our constraint on the Hubble constant $H_0$ value is also in good concordance ($\approx 2\sigma$) with local, model independent measurements using Cepheid variables. This can be explained by the presence of a turning point in the Hubble parameter $H(z)$ for $z < 0.1$ which is still consistent with SNe observations [90]. While the scalar spectral index $n_s$ takes on lower values in the RDE model than in the $\Lambda$CDM analysis, the constraint on the baryon density $\omega_b$ in our RDE baseline result appears to be in $\approx 3\sigma$ tension with BBN measurements (see figure 8 and table 3).

The RDE model has an evolving DE equation of state which according to our baseline result $w_{de}(z = 0) < -1$ (see figure 10). However, when only considering background data we found $w_{de}(z = 0) \approx -1$ (see figure 7), thus showing RDE model struggles to simultaneously fit background and perturbations data. It remains to be seen whether or not a more general IR cut-off such as GO having $\alpha > 2\beta$ (RDE has $\alpha = 2\beta$) might be able to provide a better fit to the background data than RDE while also relaxing tensions in cosmological parameters (e.g., $H_0$, $\sigma_8$, $\omega_b$). Our work shows that dynamical DE also having non standard clustering properties may play a part in the solution of discrepancies in cosmological parameters (other possibilities relying on new physics discussed, for instance, in refs. [18–22, 25, 91]). Scalar-Vector-Tensor theories provide a fairly general framework where $w_{de}(z)$, $\hat{c}_s^2(z,k)$, and $\sigma(z,k)$ worthwhile an investigation in light of current and upcoming experiments [92, 93].
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Numerical codes. Modified CLASS code reproducing results in this work can be found in the GitHub branch HDE of the repository EFCLASS.\footnote{https://github.com/wilmarcardonac/EFCLASS.git}

References

[1] PLANCK collaboration, Planck 2018 results. Part VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6 [Erratum ibid. 652 (2021) C4] [arXiv:1807.06209] [inSPIRE].

[2] DES collaboration, Cosmological Constraints from Multiple Probes in the Dark Energy Survey, Phys. Rev. Lett. 122 (2019) 171301 [arXiv:1811.02375] [inSPIRE].

[3] DES collaboration, Dark Energy Survey Year 1 Results: Cosmological Constraints from Cluster Abundances, Weak Lensing, and Galaxy Correlations, Phys. Rev. Lett. 126 (2021) 141301 [arXiv:2010.01138] [inSPIRE].

[4] V. Mossa et al., The baryon density of the Universe from an improved rate of deuterium burning, Nature 587 (2020) 210 [inSPIRE].

[5] E. Abdalla et al., Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies, J. High Energy Astrophys. 34 (2022) 49 [arXiv:2203.06142] [inSPIRE].

[6] A.G. Riess, S. Casertano, W. Yuan, L.M. Macri and D. Scolnic, Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics beyond ΛCDM, Astrophys. J. 876 (2019) 85 [arXiv:1903.07603] [inSPIRE].

[7] W.L. Freedman, Measurements of the Hubble Constant: Tensions in Perspective, Astrophys. J. 919 (2021) 16 [arXiv:2106.15656] [inSPIRE].

[8] A.G. Riess et al., A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s⁻¹ Mpc⁻¹ Uncertainty from the Hubble Space Telescope and the SH0ES Team, Astrophys. J. Lett. 934 (2022) L7 [arXiv:2112.04510] [inSPIRE].

[9] S. Joudaki et al., CFHTLenS revisited: assessing concordance with Planck including astrophysical systematics, Mon. Not. Roy. Astron. Soc. 465 (2017) 2033 [arXiv:1601.05786] [inSPIRE].

[10] C. Heymans et al., KiDS-1000 Cosmology: Multi-probe weak gravitational lensing and spectroscopic galaxy clustering constraints, Astron. Astrophys. 646 (2021) A140 [arXiv:2007.15632] [inSPIRE].

[11] O.H.E. Philcox and M.M. Ivanov, BOSS DR12 full-shape cosmology: ΛCDM constraints from the large-scale galaxy power spectrum and bispectrum monopole, Phys. Rev. D 105 (2022) 043517 [arXiv:2112.04515] [inSPIRE].
[12] A. Blanchard and S. Ilić, *Closing up the cluster tension?, Astron. Astrophys.* 656 (2021) A75 [arXiv:2104.00756] [SPIRE].

[13] DES collaboration, *Dark Energy Survey Year 3 results: Cosmological constraints from galaxy clustering and weak lensing*, Phys. Rev. D 105 (2022) 023520 [arXiv:2105.13549] [SPIRE].

[14] R.A. Battye, T. Charnock and A. Moss, *Tension between the power spectrum of density perturbations measured on large and small scales*, Phys. Rev. D 91 (2015) 103508 [arXiv:1409.2769] [SPIRE].

[15] E. Macaulay, I.K. Wehus and H.K. Eriksen, *Consistency tests of ΛCDM from the early integrated Sachs-Wolfe effect: Implications for early-time new physics and the Hubble tension*, Phys. Rev. D 104 (2021) 063524 [arXiv:2105.10425] [SPIRE].

[16] S. Vagnozzi, *Consistency tests of ΛCDM from the early integrated Sachs-Wolfe effect: Implications for early-time new physics and the Hubble tension*, Phys. Rev. D 104 (2021) 063524 [arXiv:2105.10425] [SPIRE].

[17] N. Schöneberg et al., *The H0 Olympics: A fair ranking of proposed models*, Phys. Rept. 984 (2022) 1 [arXiv:2107.10291] [SPIRE].

[18] A. Gómez-Valent et al., *Early dark energy in the pre- and post-recombination epochs*, Phys. Rev. D 104 (2021) 083536 [arXiv:2107.11065] [SPIRE].

[19] L.A. Anchordoqui, E. Di Valentino, S. Pan and W. Yang, *Dissecting the H0 and S8 tensions with Planck + BAO + supernova type Ia in multi-parameter cosmologies*, J. High Energy Astrophys. 32 (2021) 28 [arXiv:2107.13932] [SPIRE].

[20] M. Rashkovetskyi, J.B. Muñoz, D.J. Eisenstein and C. Dvorkin, *Small-scale clumping at recombination and the Hubble tension*, Phys. Rev. D 104 (2021) 103517 [arXiv:2108.02747] [SPIRE].

[21] S. Bansal, J.H. Kim, C. Kolda, M. Low and Y. Tsai, *Mirror twin Higgs cosmology: constraints and a possible resolution to the H0 and S8 tensions*, JHEP 05 (2022) 050 [arXiv:2110.04317] [SPIRE].

[22] L. Heisenberg, H. Villarrubia-Rojo and J. Zosso, *Can late-time extensions solve the H0 and σ8 tensions?, Phys. Rev. D 106* (2022) 043503 [arXiv:2202.01260] [SPIRE].

[23] M.G. Dainotti, B. De Simone, T. Schiavone, G. Montani, E. Rinaldi and G. Lambiase, *On the Hubble constant tension in the SNe Ia Pantheon sample*, Astrophys. J. 912 (2021) 150 [arXiv:2107.02117] [SPIRE].

[24] M.G. Dainotti et al., *On the Evolution of the Hubble Constant with the SNe Ia Pantheon Sample and Baryon Acoustic Oscillations: A Feasibility Study for GRB-Cosmology in 2030*, Galaxies 10 (2022) 24 [arXiv:2201.09848] [SPIRE].

[25] R. Murgia, S. Gariazzo and N. Fornengo, *Constraints on the Coupling between Dark Energy and Dark Matter from CMB data*, JCAP 04 (2016) 014 [arXiv:1602.01765] [SPIRE].

[26] A. Heavens et al., *No evidence for extensions to the standard cosmological model*, Phys. Rev. Lett. 119 (2017) 101301 [arXiv:1704.03467] [SPIRE].

[27] E.J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of dark energy*, Int. J. Mod. Phys. D 15 (2006) 1753 [hep-th/0603057] [SPIRE].

[28] T. Clifton, P.G. Ferreira, A. Padilla and C. Skordis, *Modified Gravity and Cosmology*, Phys. Rept. 513 (2012) 1 [arXiv:1106.2476] [SPIRE].

[29] M. Li, X.-D. Li, S. Wang and Y. Wang, *Dark Energy*, Commun. Theor. Phys. 56 (2011) 525 [arXiv:1103.5870] [SPIRE].
[30] K. Bamba, S. Capozziello, S. Nojiri and S.D. Odintsov, Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests, *Astrophys. Space Sci.* **342** (2012) 155 [arXiv:1205.3421] [inSPIRE].

[31] LIGO Scientific and Virgo collaborations, Tests of general relativity with GW150914, *Phys. Rev. Lett.* **116** (2016) 221101 [Erratum ibid. **121** (2018) 129902] [arXiv:1602.03841] [inSPIRE].

[32] T.E. Collett et al., A precise extragalactic test of General Relativity, *Science* **360** (2018) 1342 [arXiv:1806.08300] [inSPIRE].

[33] Planck collaboration, Planck 2015 results. Part XIV. Dark energy and modified gravity, *Astron. Astrophys.* **594** (2016) A14 [arXiv:1502.01590] [inSPIRE].

[34] G. ’t Hooft, Dimensional reduction in quantum gravity, *Conf. Proc. C* **930308** (1993) 284 [gr-qc/9310026] [inSPIRE].

[35] L. Susskind, The World as a hologram, *J. Math. Phys.* **36** (1995) 6377 [hep-th/9409089] [inSPIRE].

[36] R. Bousso, The Holographic principle, *Rev. Mod. Phys.* **74** (2002) 825 [hep-th/0203101] [inSPIRE].

[37] R. Bousso, A Covariant entropy conjecture, *JHEP* **07** (1999) 004 [hep-th/9905177] [inSPIRE].

[38] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Effective field theory, black holes, and the cosmological constant, *Phys. Rev. Lett.* **82** (1999) 4971 [hep-th/9803132] [inSPIRE].

[39] S. Weinberg, The Cosmological Constant Problem, *Rev. Mod. Phys.* **61** (1989) 1 [inSPIRE].

[40] V. Sahni and A.A. Starobinsky, The Case for a positive cosmological $\Lambda$-term, *Int. J. Mod. Phys. D* **9** (2000) 373 [astro-ph/9904398] [inSPIRE].

[41] S. Nojiri and S.D. Odintsov, Unifying phantom inflation with late-time acceleration: Scalar phantom-non-phantom transition model and generalized holographic dark energy, *Gen. Rel. Grav.* **38** (2006) 1285 [hep-th/0506212] [inSPIRE].

[42] S. Nojiri and S.D. Odintsov, Covariant Generalized Holographic Dark Energy and Accelerating Universe, *Eur. Phys. J. C* **77** (2017) 528 [arXiv:1703.06372] [inSPIRE].

[43] L.N. Granda and A. Oliveros, Infrared cut-off proposal for the Holographic density, *Phys. Lett. B* **669** (2008) 275 [arXiv:0810.3149] [inSPIRE].

[44] C. Gao, F. Wu, X. Chen and Y.-G. Shen, A Holographic Dark Energy Model from Ricci Scalar Curvature, *Phys. Rev. D* **79** (2009) 043511 [arXiv:0712.1394] [inSPIRE].

[45] S. Nojiri, S.D. Odintsov and T. Paul, Different Faces of Generalized Holographic Dark Energy, *Symmetry* **13** (2021) 928 [arXiv:2105.08438] [inSPIRE].

[46] S. Nojiri, S.D. Odintsov and T. Paul, Barrow entropic dark energy: A member of generalized holographic dark energy family, *Phys. Lett. B* **825** (2022) 136844 [arXiv:2112.10159] [inSPIRE].

[47] S. Nojiri, S.D. Odintsov and T. Paul, Early and late universe holographic cosmology from a new generalized entropy, *Phys. Lett. B* **831** (2022) 137189 [arXiv:2205.08876] [inSPIRE].
C. Lin, An effective field theory of holographic dark energy, *JCAP* **07** (2021) 003 [arXiv:2101.08092] [nSPIRE].

L. Heisenberg, Scalar-Vector-Tensor Gravity Theories, *JCAP* **10** (2018) 054 [arXiv:1801.01523] [nSPIRE].

X. Zhang, Holographic Ricci dark energy: Current observational constraints, quintom feature, and the reconstruction of scalar-field dark energy, *Phys. Rev. D* **79** (2009) 103509 [arXiv:0901.2262] [nSPIRE].

L. Xu and Y. Wang, Observational Constraints to Ricci Dark Energy Model by Using: SN, BAO, OHD, fgas Data Sets, *JCAP* **06** (2010) 002 [arXiv:1006.0296] [nSPIRE].

Y. Wang, L. Xu and Y. Gui, Probing Ricci dark energy model with perturbations by using WMAP seven-year cosmic microwave background measurements, BAO and Type Ia supernovae, *Phys. Rev. D* **84** (2011) 063513 [arXiv:1110.4401] [nSPIRE].

I.A. Akhlaghi, M. Malekjani, S. Basilakos and H. Haghi, Model selection and constraints from Holographic dark energy scenarios, *Mon. Not. Roy. Astron. Soc.* **477** (2018) 3659 [arXiv:1804.02989] [nSPIRE].

H. Hossienkhani, N. Azimi and H. Yousefi, Constraints on the Ricci dark energy cosmologies in Bianchi type I model, *Int. J. Geom. Meth. Mod. Phys.* **18** (2021) 2150095 [nSPIRE].

M. Malekjani, M. Rezaei and I.A. Akhlaghi, Can Holographic dark energy models fit the observational data?, *Phys. Rev. D* **98** (2018) 063533 [arXiv:1809.08792] [nSPIRE].

Z.-P. Huang and Y.-L. Wu, Cosmological Constraint and Analysis on Holographic Dark Energy Model Characterized by the Conformal-age-like Length, *Int. J. Mod. Phys. A* **27** (2012) 1250130 [arXiv:1202.5163] [nSPIRE].

T.-F. Fu, J.-F. Zhang, J.-Q. Chen and X. Zhang, Holographic Ricci dark energy: Interacting model and cosmological constraints, *Eur. Phys. J. C* **72** (2012) 1932 [arXiv:1112.2380] [nSPIRE].

Y. Wang and L. Xu, Current Observational Constraints to Holographic Dark Energy Model with New Infrared cut-off via Markov Chain Monte Carlo Method, *Phys. Rev. D* **81** (2010) 083523 [arXiv:1004.3340] [nSPIRE].

W. Fang, W. Hu and A. Lewis, Crossing the Phantom Divide with Parameterized Post-Friedmann Dark Energy, *Phys. Rev. D* **78** (2008) 087303 [arXiv:0808.3125] [nSPIRE].
[70] D.W. Hogg et al., Cosmic homogeneity demonstrated with luminous red galaxies, *Astrophys. J.* **624** (2005) 54 [astro-ph/0411197] [SPIRE].

[71] C. Marinoni, J. Bel and A. Buzzi, The Scale of Cosmic Isotropy, *JCAP* **10** (2012) 036 [arXiv:1205.3309] [SPIRE].

[72] PLANCK collaboration, Planck 2015 results. Part XVI. Isotropy and statistics of the CMB, *Astron. Phys. Astrophys.* **594** (2016) A16 [arXiv:1506.07135] [SPIRE].

[73] C.-P. Ma and E. Bertschinger, Cosmological perturbation theory in the synchronous and conformal Newtonian gauges, *Astrophys. J.* **455** (1995) 7 [astro-ph/9506072] [SPIRE].

[74] M. Kunz and D. Sapone, Crossing the Phantom Divide, *Phys. Rev. D* **74** (2006) 123503 [astro-ph/0609040] [SPIRE].

[75] BOSS collaboration, The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample, *Mon. Not. Roy. Astron. Soc.* **470** (2017) 2617 [arXiv:1607.03155] [SPIRE].

[76] F. Beutler et al., The 6dF Galaxy Survey: Baryon Acoustic Oscillations and the Local Hubble Constant, *Mon. Not. Roy. Astron. Soc.* **416** (2011) 3017 [arXiv:1106.3366] [SPIRE].

[77] A.J. Ross, L. Samushia, C. Howlett, W.J. Percival, A. Burden and M. Manera, The clustering of the SDSS DR7 main Galaxy sample. Part I. A 4 per cent distance measure at $z = 0.15$, *Mon. Not. Roy. Astron. Soc.* **449** (2015) 835 [arXiv:1409.3242] [SPIRE].

[78] PAN-STARRS1 collaboration, The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, *Astrophys. J.* **859** (2018) 101 [arXiv:1710.00845] [SPIRE].

[79] R. Arjona, J. García-Bellido and S. Nesseris, Cosmological constraints on nonadiabatic dark energy perturbations, *Phys. Rev. D* **102** (2020) 103526 [arXiv:2006.01762] [SPIRE].

[80] B. Audren, J. Lesgourgues, K. Benabed and S. Prunet, Conservative Constraints on Early Cosmology: an illustration of the Monte Python cosmological parameter inference code, *JCAP* **02** (2013) 001 [arXiv:1210.7183] [SPIRE].

[81] T. Brinckmann and J. Lesgourgues, MontePython 3: boosted MCMC sampler and other features, *Phys. Dark Univ.* **24** (2019) 100260 [arXiv:1804.07261] [SPIRE].

[82] PLANCK collaboration, Planck 2013 results. Part XVI. Cosmological parameters, *Astron. Astrophys.* **571** (2014) A16 [arXiv:1303.5076] [SPIRE].

[83] A. Oliveros, M.A. Sabogal and M.A. Acero, Barrow holographic dark energy with Granda-Oliveros cutoff, *Eur. Phys. J. Plus* **137** (2022) 783 [arXiv:2203.14464] [SPIRE].

[84] VIRGO CONSORTIUM, Stable clustering, the halo model and nonlinear cosmological power spectra, *Mon. Not. Roy. Astron. Soc.* **341** (2003) 1311 [astro-ph/0207664] [SPIRE].

[85] L.R. Abramo, R.C. Batista, L. Liberato and R. Rosenfeld, Structure formation in the presence of dark energy perturbations, *JCAP* **11** (2007) 012 [arXiv:0707.2882] [SPIRE].

[86] L.R. Abramo, R.C. Batista, L. Liberato and R. Rosenfeld, Physical approximations for the nonlinear evolution of perturbations in inhomogeneous dark energy scenarios, *Phys. Rev. D* **79** (2009) 023516 [arXiv:0806.3461] [SPIRE].

[87] V. Poulin, T.L. Smith, T. Karwal and M. Kamionkowski, Early Dark Energy Can Resolve The Hubble Tension, *Phys. Rev. Lett.* **122** (2019) 221301 [arXiv:1811.04083] [SPIRE].

[88] J.C. Hill, E. McDonough, M.W. Toomey and S. Alexander, Early dark energy does not restore cosmological concordance, *Phys. Rev. D* **102** (2020) 043507 [arXiv:2003.07355] [SPIRE].

[89] R.J. Cooke, M. Pettini and C.C. Steidel, One Percent Determination of the Primordial Deuterium Abundance, *Astrophys. J.* **855** (2018) 102 [arXiv:1710.11129] [SPIRE].
[90] E.Ó. Colgáin and M.M. Sheikh-Jabbari, A critique of holographic dark energy, Class. Quant. Grav. 38 (2021) 177001 [arXiv:2102.09816] [inSPIRE].

[91] S.J. Clark, K. Vattis, J. Fan and S.M. Koushiappas, The $H_0$ and $S_8$ tensions necessitate early and late time changes to $\Lambda$CDM, arXiv:2110.09562 [inSPIRE].

[92] V.I. Sabla and R.R. Caldwell, Microphysics of early dark energy, Phys. Rev. D 106 (2022) 063526 [arXiv:2202.08291] [inSPIRE].

[93] W. Cardona, J.B. Orjuela-Quintana and C.A. Valenzuela-Toledo, An effective fluid description of scalar-vector-tensor theories under the sub-horizon and quasi-static approximations, JCAP 08 (2022) 059 [arXiv:2206.02895] [inSPIRE].