Numerical analysis of the ultimate compressibility of concrete with indirect reinforcement for plotting a stress-strain diagram

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Abstract. Indirect reinforcement is one of the effective ways to increase the strength and deformation characteristics, which is achieved by creating a volumetric stress state in concrete. For the design of reinforced concrete structures with indirect reinforcement, the calculation method based on a nonlinear deformation model is well suited. For such calculations, it is required to construct the stress-strain diagram, which will most accurately describe the relationship $\sigma$-$\varepsilon$. The article presents a numerical analysis of increasing the bearing capacity and deformations of concrete of various classes with indirect reinforcement in the form of welded meshes. Selected formulas to determine strength and ultimate compressibility. The problems arising in determining the compressibility limit of concrete with indirect reinforcement for engineering calculations are noted. Recommendations for calculating the parametric points of the stress-strain diagram are given.

1. Introduction
For a long time concrete is one of the basic building materials in the construction of civil and industrial buildings. In this regard, the task of improving design solutions and calculation methods at the stage of operation and during emergency impacts is urgent [1-7]. Indirect reinforcement is one of the effective ways to increase the strength and deformation characteristics, which is achieved by creating a volumetric stress state in concrete. Researchers offer various design solutions for indirect reinforcement, which have shown their effectiveness depending on the operating characteristics and the stress-strain state of structures: the spiral reinforcement [8-11], tube-concrete elements with various cross-sectional shapes [12-16], welded meshes [17-19], etc.

For the design of reinforced concrete structures with indirect reinforcement, the calculation method based on a nonlinear deformation model is well suited [20]. It can be used for various cross-sectional configurations and reinforcement distribution. In this case, one of the most important stages is the choice of the stress-strain diagram [21,22], which will most accurately describe the relationship $\sigma$-$\varepsilon$. To plot a stress-strain diagram, it is necessary to determine the parametric points characterizing stresses $\sigma$ and relative deformations $\varepsilon$ at various stages of work. The study [23] notes that when determining the characteristic values of compression deformations within the framework of the theory of limit states, difficulties arise associated with the different influence of indirect reinforcement on the ultimate compressibility of concrete of different strength.

The purpose of this work is a numerical analysis of the effect of indirect reinforcement in the form of transverse welded meshes on the growth of the bearing capacity and deformability of concrete of
various strengths, as well as the development of practical recommendations for plotting of concrete deformation diagrams.

2. Materials and Methods
For the numerical analysis, a simplified three-line state diagram was selected (Figure 1) [20]. Subscript “3” is added for concrete under triaxial compression created by indirect reinforcement.

The values of the stresses in the upper section of the diagram $R_{c3}$ are proposed to be determined based on formula (1) presented in the article [24].

$$R_{c3} = \left[ \frac{1 - \rho_{xy}}{2} + \sqrt{\frac{(1 - \rho_{xy})^2}{2} + 9 \rho_{xy}} \right] R_c;$$

where $R_c$ is a prismatic concrete compressive strength; $\psi_c = 0.375$ is a lateral compression unevenness coefficient; $\mu_{xy}$ is an indirect reinforcement ratio; $R_{s,xy}$ is a tensile strength of indirect reinforcement rods.

Indirect reinforcement coefficient can be determined by the formula:

$$\mu_{xy} = \frac{A_{sx} L_x + A_{sy} L_y}{L_x L_y S},$$

where $A_{sx}, A_{sy}$ – the total area of the reinforcing bars in the direction of the X and Y axes; $L_x, L_y, S$ are shown in Figure 2.

It is proposed to determine the relative deformations $\varepsilon_{c03}$ and $\varepsilon_{c3u}$ on the basis of formula (3), proposed in [25] on the basis of processing a large sample of experimental data for concrete samples of various strengths. In this article, formula (3) is presented with the conversion to the prism strength $R_p$.

$$\varepsilon_{b03} = e^n \varepsilon_{b0}; \varepsilon_{b3u} = e^n \varepsilon_{bu}$$

$$n = \left( 2.9224 - 0.00408 R_c \right) \left( 0.3124 + 0.0022 R_b \right),$$

where $\sigma_{c,xy}$ is a lateral compression stress.
Figure 2. Compressed element with indirect reinforcement meshes.

It was found in [23,26] that formulas (1) and (3) provide a better agreement between the calculation results and experimental data than formulas SP 63.13330.2018 [20]. In this regard, they are accepted as the main ones for further calculations.

Due to the difficulties in the analytical description of the triaxial stress state for concrete [25-28], most of the engineering formulas for determining the strength and ultimate compressibility of concrete with indirect reinforcement were obtained on the basis of empirical data on the average strength of the tested samples. The same can be said about expressions (1) and (3).

It should be noted that the effect of indirect reinforcement on the strength and deformability of concrete is that the lower the strength, the greater the increase in the compressive strength and ultimate compressibility at the same indirect reinforcement ratio. But to perform engineering calculations, it is necessary to use the values of strength characteristics that provide the required reliability of building structures. In this regard, characteristic compressive strength $R_{ck}$ and the design value of strength $R_{cd}$ are used, which are less than the experimental mean values of strength $R_{cm}$. This leads to the fact that, when plotting a stress-strain diagram and substituting reduced values of strength characteristics in the formulas, the compressibility limit can be significantly overestimated for the selected class of concrete.

For concretes of various classes, strength and deformation calculations were performed, required to plot a three-line stress-strain diagram. Calculations were performed with various ratios of indirect reinforcement $\mu_x=0.005$; $\mu_y=0.02$; $\mu_z=0.035$; $\mu_y=0.05$. When carrying out calculations, various values of concrete strength for a given class of strength were used to substitute in formulas (1) and (3): $R_{cd}$, $R_{cm}$, $R_{cn}$.

Table 1 shows the strength values for concrete of the considered concrete classes. When determining the strength $R_{cm}$; the coefficient of variation $v$ is taken equal to 0.135. For clarity, Figure 3 shows three-line stress-strain diagrams (for concrete of a compressive strength class 30 with an indirect reinforcement coefficient $\mu_y=0.005$) based on the design value of strength $R_{cd}$, the characteristic compressive strength $R_{ck}$ and the mean value of strength $R_{cm}$. 
Figure 3. Stress-strain diagrams for concrete of strength class 30.
- unconfined concrete;
- $R_{cm}$, $\mu_{xy}=0.005$;
- $R_{cn}$, $\mu_{xy}=0.005$;
- $R_{cd}$, $\mu_{xy}=0.005$;

Table 1. Compressive strength

| Strength classes | $R_{cd}$, MPa | $R_{cb}$, MPa | $R_{cm}$, MPa |
|------------------|---------------|---------------|---------------|
| 20               | 11.5          | 15.0          | 19.2          |
| 30               | 17.0          | 22.0          | 28.4          |
| 40               | 22.0          | 29.0          | 36.7          |
| 50               | 27.5          | 36.0          | 45.9          |
| 60               | 33.0          | 43.0          | 55.1          |
| 70               | 37.0          | 50.0          | 61.8          |
| 80               | 41.0          | 57.0          | 68.5          |
| 90               | 44.0          | 64.0          | 73.5          |
| 100              | 44.7          | 71.0          | 79.3          |

3. Results and discussion

Table 2 shows some of the calculation results that most clearly demonstrate the problem under consideration. Figure 4 graphically presents the calculation results depending on the design value of strength $R_{cd}$. For the calculations performed, the indirect reinforcement was taken with a yield strength $R_{y}=500$ MPa.

Based on the calculation results, an increase in the bearing capacity for the cases considered was obtained, which ranged from 1.14 to 2.8 times, depending on the reinforcement coefficient $\mu_{xy}$ and the design value of strength $R_{cd}$. At the same time, the increase in the compressibility limit turned out to be significantly larger and ranged from 1.62 to 6.98 times depending on the reinforcement coefficient $\mu_{xy}$ and the design value of strength $R_{cd}$.

Table 2. Calculation results.

| Strength classes | $\mu_{xy}=0.005$ | $\mu_{xy}=0.02$ |
|------------------|-------------------|-----------------|
|                  | $R_{cd}=f(R_{cd})$ | $R_{cd}=f(R_{cd})$ | $R_{cd}=f(R_{cd})$ | $R_{cd}=f(R_{cd})$ | $R_{cd}=f(R_{cd})$ | $R_{cd}=f(R_{cd})$ |
|                  | $R_{cd}$, MPa     | $R_{cd}$, MPa   | $R_{cd}$, MPa   | $R_{cd}$, MPa   | $R_{cd}$, MPa   | $R_{cd}$, MPa   |
| 20               | 16.5 1.43 2.83 2.57 2.34 | 24.0 2.08 4.67 4.24 3.81 |
| 30               | 22.5 1.32 2.45 2.34 2.01 | 31.5 1.85 4.02 3.57 3.16 |
| Strength classes | $\mu_{xy}=0.005$ | $\mu_{xy}=0.05$ |
|------------------|------------------|------------------|
| 20               | 28.6 2.49 5.95 5.43 4.91 | 32.2 2.80 6.98 6.43 5.87 |
| 30               | 37.3 2.20 5.16 4.62 4.09 | 41.9 2.46 6.14 5.55 4.97 |
| 40               | 44.6 2.03 4.63 4.07 3.55 | 49.9 2.27 5.56 4.95 4.34 |
| 50               | 52.1 1.89 4.16 3.61 3.10 | 58.1 2.11 5.04 4.41 3.80 |
| 60               | 59.3 1.80 3.77 3.23 2.75 | 65.9 2.00 4.60 3.96 3.36 |
| 70               | 64.4 1.74 3.53 3.01 2.54 | 71.4 1.93 4.32 3.68 3.09 |
| 80               | 69.4 1.69 3.32 2.81 2.36 | 76.7 1.87 4.07 3.44 2.86 |
| 90               | 73.0 1.66 3.18 2.68 2.24 | 80.7 1.83 3.90 3.27 2.71 |
| 100              | 77.3 1.63 3.03 2.54 2.12 | 85.2 1.79 3.71 3.09 2.55 |
But, since formulas (1) and (3) were originally derived for experimental mean strengths, the results for deformations are obviously overestimated. And if we take the mean values of concrete strength when calculating the parameters of the compression diagram of concrete with indirect reinforcement, then the calculated increase in the compressibility limit will be significantly smaller and will be from 1.32 to 5.87 times, depending on the reinforcement coefficient $\mu_{xy}$ and the mean value of strength $R_{cm}$.

The need to use the mean values of concrete strength in calculations corresponds to the physical meaning of formulas (1) and (3) and is confirmed by previous studies [23]. It should be noted that earlier calculations using the formulas proposed in SP 63.13330.2018 lead to similar conclusions, but the agreement with the experimental data turns out to be worse even when calculating using the average values of concrete strength [23].

Thus, when plotting stress-strain diagrams of concrete with indirect reinforcement, the calculation of deformations based on the design value of strength $R_{cd}$ leads to a significant overestimation of the relative strain values $\varepsilon_{c03}$ and $\varepsilon_{c3u}$ (18-45%).

4. Conclusions
Indirect reinforcement with transverse welded meshes significantly increases the strength of concrete due to the creation of a volume stress state: for the cases considered, the increase was from 1.14 to 2.8 times. The effect on the increase in the compressibility limit is even more significant: for the cases considered, the increase was from 1.32 to 5.87 times.

For the correct plot of stress-strain diagrams of concrete with indirect reinforcement SP 63.13330.2018 should be supplemented with instructions for determining the average strength values for concretes of various classes. Without these information, the calculated ultimate compressibility of concrete with indirect reinforcement is overestimated (up to 45% excess).

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