Analysis of a nonlinear system dynamics in the Morlet wavelet basis

I M Dantsevich, M N Lyutikova, AY Novikov, SA Osmukha
Admiral F. F. Ushakov Maritime State University, Novorossiysk, Lenin Avenue 93, 353918, Russia
E-mail dantsevich.aumsu@gmail.com

Abstract. The article illustrates the methodological study basis of the multi-frequency (packet) spectrum of sea waves in the influence on the associated system "ship-outboard complex of geophysical equipment". The selection of the main harmonic of vibrations in the spectrum allows us to implement a compensator of the outboard system according to the stability conditions determined by the Morlet polynomials obtained from the decomposition of the observed dynamics of the system.

Keywords: Morlet wavelet, dynamics of outboard equipment, remotely operated underwater vehicle, outboard geo-physical system, the heave compensator, packet structure of sea waves, suspension system, statistical characteristics of unrest.

1. Introduction
Research of underwater robots Remotely operated underwater vehicle (ROV) is carried out in order to develop technologies for conducting underwater technical work. The power of modern ROVs working class is 300 kW or more. Technologically, ROVs have almost zero buoyancy, so the goal of the research is to develop control technologies that allow to compensate the dynamics of ROV when working with a set of working tools and manipulators.

The main task of the current study is to develop methods for "fast" calculation of sets of hydrodynamic coefficients based on field observations of the system dynamics for providing descent and lifting operations on carrier vessel board and ROV.

The results of studies of the observed dynamics of the system in the Morlet wavelet basis have shown that mathematical methods of decomposition, analysis and synthesis are effective with a smaller amount of calculations. The subsequent implementation of "fast" calculations in Morlet banks is performed on the basis of spectral density invariants of the decomposition power.

Frequency identification methods of control systems for marine technological complexes allow us to accurately determine quantitative indicators of forces and moments of dynamics based on the parametric description of the object's dynamics and its systems.

The paper [1] considers a systematic approach to modeling the dynamics of marine engineering systems related to nonlinear systems. The linearization of model parameters is achieved by converting forces and moments registered directly in the dynamics from the frequency domain to the time domain. The dynamics of elementary objects that together make up a more complex nonlinear system is considered.

In [1-2], the researchers Tristan Pe’rez, Thor I. Fossen and Kristin I. Pettersen described in detail mathematical models of underwater robots. The research results are implemented in many approaches to the design of navigation and ROV control systems. As follows from earlier works by Antonelli and...
Gianluca [3-4], computational approaches to hydrodynamics are widely used with the statistically averaged perturbation characteristics and identification of the control system by matrices of the attached masses of the control object.

The commonality of approaches unites the estimation of the perturbations spectrum of hydrodynamic forces and moments acting on the control object by means of Fourier analysis.

The second approach involves decomposing of the observed dynamics in the bases of orthogonal functions, which makes it easier to obtain numerical models in the course of working out the hydrodynamics of the complex.

2. Materials and Methods
The dynamics of the phase components of the wave spectrum with a set of orthogonal functions can be described by compressing and mapping the Morlet basis [5], which is named the Morlet wavelet.

Modeling of a complex two-component model of the ship's mass distribution and a tethered geophysical system on a developed sea wave is performed in order to compensate the dynamics of the geophysical system.

Let us determine that the scheme of a two-link system with a special pitching compensator allows compensation of vertical movements of the suspension of the ROV lowered over the ship's side and the cable-rope, as shown in Figure 1.

Three coordinate systems are considered when modeling the carrier-ship system and the complex of outboard equipment: geocentric oxyz; linked o'x'y'z'; movable OXYZ [5, 6].

The natural experiment studied the dynamics of the outboard system consisting of ROV and power box with power equipment – garage-digger outboard system. Usually, garage-digger contains converter electronics and a winch with a cable rope with ROV at the free end. The cable-rope transmits electrical power that provides ROV supply, as well as provides information about the control system and on-board video cameras.

![Figure 1](image)

Figure 1. Dynamics conditions of the carrier ship and the ROV system in conditions of longitudinal and transverse rolling

The center of system gravity (point B) oscillates along an elliptical trajectory (Figure 1). MP is mid-plane (diametral plane "DP"), MPV is main plane of the vessel.

The working length of the communication cable-rope (CR) ship's carrier and the ROV garage-digger (GD) binary system is up to 6000 meters. The operating voltage transmitted to the plug-in converter is usually 2500 volts, which means that the cable weight can be reduced.

Figure 2 shows experimental measurements of system dynamics with the following parameters: the point of descent of the cable-rope (CR) from the last roller of the ship's fodder hoisting device (FHD)
and the underwater vehicle. The coordinates are: \( X = -18.6 \) m (aft of the ship from amidships), \( Y = -0.5 \) m (down from the MVP), \( Z = 0 \) (from DP).

![Figure 2. Results of registration during the natural experiment of vertical vibrations of the equipment complex with the inactive compensator.](image)

The natural experiment of vertical vibrations of the equipment complex with the inactive compensator is carried out with the indexes: cable length 6000 m; fading sea waves 5 points; course angle of movement (CAM) = 90 degrees; 1 is vertical movement \( Y_{sp} \) of the point B (red); 2 is vertical movement \( Y_{gd} \) of the complex (blue).

When calculating the behavior of ROV while the ship is pitching, we used a spatial mathematic model of the system as the device itself and the power supply unit. The dynamics of CR was described by a system of nonlinear partial differential equations [6].

For a related coordinate system, the position of the center of gravity of the suspension point is defined as:

\[
m \frac{d^2 r}{dt^2} = (m + \lambda_0) g + F_i,
\]

where \( r \) is the position vector of the mass center; \( m + \lambda_0 \) is the mass system with added masses of water; \( F_i \) is the reaction force of the suspension and Archimedes force outboard system.

Equations of moments of forces of the connected coordinate system are:

\[
\begin{align*}
J_x \frac{d^2 \phi}{dt^2} & = \sum_i F_i \times r_i, \\
J_y \frac{d^2 \psi}{dt^2} & = \sum_i F_i \times r_i, \\
J_z \frac{d^2 \zeta}{dt^2} & = \sum_i F_i \times r_i,
\end{align*}
\]

where \( r \) is the position vector of the gravity center of the suspension system masses; \( F_i \) is the reaction forces acting on the suspension system (Archimedean force and the suspension reaction force) in the system of connected coordinates \( x', y', z' \) and angles \( \phi, \psi, \zeta \) [6].

Equations (1-2) for a complex of outboard equipment consisting of a garage-deepener and an underwater vehicle, it is necessary to take into account the attached water masses. The complex represents shapes of a complex streamlined shape.
Two cases of modeling the dynamics of the outboard equipment complex can be determined: at depth and on the surface at the time of docking of the complex and onboard equipment on the carrier vessel deck.

Figure 2 shows the results of calculating the vertical movement of the ROV when the GD winch is not working. The results refer to a 5-point attenuating wave, CAM = 900, and the length of the CR of GD is 500 m. The length of the CR of the ROV connecting the ROV and GD is 8 m.

It should be noted that due to the irregularity of GD oscillations, the relative speed of the ROV and GD at their junction is also a random value.

To estimate the possible values of the relative vertical velocity GD and the ROV and jerks in CR during docking we performed calculations of the GD and the ROV movement with regular oscillations of GD at an average frequency and an amplitude of 3% exceedance for the state intensity of 4 and 5 points, CAM = 90° and CR = 500 m and GD equal to 6000 m.

To reduce the relative velocities of GD, ROV and the intensity of jerks in the CR, it is necessary to take measures to reduce the fluctuations of GD.

Such measures include holding the vessel at the junction of the ROV and GD in the position of CAM = 1800, reducing the intensity of the wave at which the connection is made, and reducing the depth of immersion at which the GD is located at the junction. However, the most effective way to reduce the intensity of GD vibrations is to include a vertical GD vibration compensator in the FHD.

The compensator can be either a passive type in which movement of the piston is proportional to change of the cable wire tension or active type, the moving piston of which is controlled by a dedicated system.

The work [7] shows that for effective operation, a passive compensator must have a very low stiffness, which is practically impossible for implementation. The feasibility of using an active compensator requires additional evaluation.

The efficiency of an active compensator depends on its design parameters, mainly on the size of the piston stroke and the speed of its movement, as well as on the characteristics of the piston movement control system. It is obvious that the movement of the piston leads to a proportional change in the released length of the CR on which the ROV is lowered (and raised).

3. Results

3.1. Mathematical model of a low-stiffness compensator

Studies of pitching compensators are carried mostly to investigations of vertical fluctuations of the outboard system. However, fluctuations in the system center of gravity (suspension points) in reality frequently perform complex oscillations along an elliptical trajectory. The slope of the trajectory corresponds to the period of transverse pitching of the vessel, as shown in Figure 1.

In addition to the geocentric and mobile coordinate systems (Figure 1), we will implement a third coordinate system "linked". The suspension point (point B) moves on the wave relatively to the mass center of the vessel. For these reasons, it is necessary to add the calculated values in the form of coordinate systems to the vertical coordinates in the form of increments to the mobile coordinate system associated with the center of mass gravity of the ship.

Modeling is feasible for these extreme cases with recalculation of the experiment data from reference data.

Hydrodynamic characteristics are calculated in a related coordinate system, the origin of which is taken at the point of the center of gravity of the joint garage-digger and ROV system.

The axis $OY'$ coincides with the axis of the supporting frame and is directed upward along the cable-rope of the system's attachment, the axis $OX'$ forms the right three vectors with the axis $OY'$ at a perpendicular vector $OZ'$.

In practice, it is assumed that the attached mass of structural elements is approximately 20% of the total mass of the drum, plates and base. The center of the garage-digger is located in the center of the drum with the cable rope.
3.2. Computational models of the outboard system dynamics

It is quite problematic to solve the equation describing the dynamics of the outboard system. Due to errors in the calculated values of the attached water masses, flow, and the influence of waves, this problem can only be evaluated from the perspective of probability theory.

The study of workflow models allows us to obtain good results for the right part of expression (2). As follows from the results of the experiment, the dynamics of the outboard equipment is described by harmonic functions.

Spectral sets of registered realizations within the observed dynamics \([-\varphi, +\varphi]\) are defined as [8]:

$$\int_{-\varphi}^{\varphi} d\sin(\varphi, t) \frac{d\varphi}{dt} dt = 0. \quad (3)$$

Let us define a wavelet as:

$$\psi \left( \frac{\varphi - t}{a} \right) \exp \left( j\sin \left( \frac{\varphi - t}{a} \right) \right). \quad (4)$$

The transformation of the \(\sin(\varphi, t)\) function takes place over the phase parameters so that:

$$\vartheta(\varphi, t) = \arctan \left\{ \frac{A_1 \psi_i(\varphi - t/a) \sin(\varphi_i(\varphi - t/a)) + A_2 \psi_i(\varphi - t/a) \cos(\varphi_i(\varphi - t/a))}{A_1 \psi_i(\varphi - t/a) \cos(\varphi_i(\varphi - t/a)) + A_2 \psi_i(\varphi - t/a) \cos(\varphi_i(\varphi - t/a))} \right\}, \quad (5)$$

for each set \(\{\varphi - , \varphi + \}\): \(\vartheta(\varphi - , t_1) = \vartheta(\varphi + , t_1)\) if \(\vartheta(\varphi - , t_2) = \vartheta(\varphi + , t_2)\).

The initial values of the shift and scale transformation functions are equated [9]:

$$\psi_i \left( \frac{t_2 - t_1}{\varphi - t} \right) \sin \left( \varphi_i \left( \frac{t_2 - t_1}{\varphi - t} \right) \right) = \psi_i \left( \frac{t_2 - t_1}{\varphi + t} \right) \sin \left( \varphi_i \left( \frac{t_2 - t_1}{\varphi + t} \right) \right) \quad (6)$$

$$\psi_i \left( \frac{t_2 - t_1}{\varphi - t} \right) \sin \left( \varphi_i \left( \frac{t_2 - t_1}{\varphi - t} \right) \right) = \psi_i \left( \frac{t_2 - t_1}{\varphi + t} \right) \sin \left( \varphi_i \left( \frac{t_2 - t_1}{\varphi + t} \right) \right). \quad (7)$$

Taking the initial value of the phase equal to zero:

$$\sin \left( \varphi_i \left( \frac{t_2 - t_1}{\varphi - t} \right) \right) = 0; \quad \sin \left( \varphi_i \left( \frac{t_2 - t_1}{\varphi + t} \right) \right) = 0; \quad (8)$$

which requires the condition \(\varphi + = \varphi -\), we get:

$$\varphi_i \left( \frac{t_2 - t_1}{\varphi - k} \right) = k\pi. \quad (9)$$

If the base indicator is the Morlet wavelet [2]:

$$\varphi_k = \frac{\omega_0(t_1 - t_2)}{k\pi}, \quad (10)$$

where the lowest iteration order is the value:

$$\varphi_0 = \frac{\omega_0(t_1 - t_2)}{\pi}. \quad (11)$$

Thus, having the construction of the phase transformation of the observed dynamics and the scale coefficients, we can say that the Morlet wavelet decomposition provides the frequency set shown in Figures 3-4, which is a compression and mapping of the Morlet basis [10, 11].
Figure 3. Decomposition of functions in the basis of Morlet wavelets, dynamics of the suspension point on the ship of outboard equipment

Figure 4. Expansion of the garage-digger dynamics driven by a suspension point in the basis of Morlet wavelets

For any system described by the algorithm (7-11), it is necessary to determine a set of coefficients of the Morlet basis expansion, as shown in table 1. We estimate the statistical averaged characteristics of the harmonics of the Morlaix decomposition and calculate the contribution to the overall dynamics of the process, which allows us to achieve compensation in descent-lifting devices (FHD).
Figure 5. Morlet Wavelet "32-order", fragment, "Dynamics of decomposition coefficients in Morlet bursts"

Table 1. Calculated values of the Morlet wavelet fragment of the array (32x65)

| Coefficients of polynomial expansion in the Morlet basis max "65» | 1 | 2 | 3 | 4 | 5 | 64 | 65 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.04083408 | 4.16333634 | ... | 1.56125112 | 1.04083408 |
| 0 | 0 | 0 | 558608e-17 | 234434e-17 | 837913e-16 | 0.07071067 |
| 0 | 0 | 1.80277751 | 1.47196168 | 0.07071067 | 1.06066017 |
| 0 | 2.32737576 | 2.08166817 | 0.05784779 | 0.28906980 | 0.92539550 | 1.44365758 |
| 0 | 866932e-17 | 117217e-17 | 06434138 | 7630392 | 4707943 | 228519 |
| ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... |
| 2.753788 | 2.54951241 | 0.04476503 | 0.05000000 | 0.30000000 | 0.55000000 | 0.05000000 |
| 14654010e-17 | 658222e-17 | 27526815 | 00000001 | 00000000 | 00000000 | 00000000 |
| ... | ... | ... | ... | ... | 0.71995562 | 734 |
| ... | ... | ... | ... | 1595 | 4693 | ... |
| ... | ... | ... | ... | ... | 0.44422494 | 9651574 |
| ... | ... | ... | ... | ... | 0.07071067 | 0.88464381 |
| ... | ... | ... | ... | ... | 81186558 | 0383246 |
| ... | ... | ... | ... | ... | 0.86620580 | 6953519 |

4. Discussion
The obtained results allow us to draw several important conclusions. Decomposition of the observed dynamics of hydrodynamic forces and moments in Morlet wavelets allows us to store information about the phase of the initial sequences of signals from the dynamics sensors.

Applying the tresholding technology to the spectral power of invariants calculated in the Morlet basis allows limiting the amount of calculations in the filters of the "fast" Morlet transform. Limiting the sets of decomposition coefficients that most affect the dynamics of the descent-ascent-ROV system is a crucial rule for identifying the control system.

5. Conclusion
Decomposition of the observed sequence in the Morlet wavelet basis gives a set of harmonic components of the packet wave spectrum. The estimation of the spectral power of the decomposition in the Morlet wavelet basis defines the main vector of motion as an average characteristic of the system vibrations.
caused by a packet sea wave. The stability of the system polynomials Morlet is determined by the harmonic component of the fundamental harmonic decomposition.

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