The gravitino abundance in supersymmetric ‘new’ inflation models

David H. Lyth

Department of Physics,
Lancaster University,
Lancaster LA1 4YB. U. K.

Abstract

We consider the abundance of gravitinos created from the vacuum fluctuation, in a class of ‘new’ inflation models for which global supersymmetry is a good approximation. Immediately after inflation, gravitinos are produced, with number density determined by equations recently presented by Kallosh et. al. (hep-th/9907124) and Giudice et. al. (hep-ph/9907510). Unless reheating intervenes, creation may continue, maintaining about the same number density, until the Hubble parameter falls below the gravitino mass. In any case, the abundance of gravitinos created from the vacuum fluctuation exceeds the abundance from thermal collisions in a significant regime of parameter space, leading to tighter cosmological constraints.

Introduction

Gravitinos are created in the early Universe with a cosmologically significant abundance. They are certainly created by thermal collisions after reheating [1], and some time ago [2] it was pointed out that they may also be created non-thermally, starting from the vacuum fluctuation that exists well before horizon exit during inflation. It was conjectured that the initial gravitino number density from this mechanism would be of order \( m_{3/2}(t)^3 \), where the effective gravitino mass \( m_{3/2}(t) \) is at most of order the Hubble parameter. In that case, gravitino creation from the vacuum is insignificant compared with creation from thermal collisions.

Recently, the mode function equations determining the gravitino abundance have been worked out [3, 4], for models in which a single real scalar field dominates the density and pressure of the Universe. On the basis of these equations, their authors have pointed out that gravitinos are created just after inflation with number density of order \( M^3 \), where \( M \) is the mass of the inflaton. In many models of inflation, this already makes gravitino creation from the vacuum much more important than creation from thermal collisions. Furthermore, creation may continue, maintaining about the same number density, until the Hubble parameter falls below the true gravitino mass \( m_{3/2} \) [4], making gravitino creation from the vacuum even more significant.
In this note, we consider a specific class of supersymmetric ‘new’ inflation models. We estimate the abundance of gravitinos for both the minimal case, where creation ends soon after inflation, and the maximal case where it continues until the Hubble parameter falls below the gravitino mass.

The model The models that we consider invoke a tree-level supergravity theory, containing no physical fields except the gravitino, and a complex scalar field φ with the minimal kinetic term. (The degrees of freedom corresponding to the spin 1/2 partner of the scalar field are eaten by the helicity 1/2 components of the gravitino field.) There is a holomorphic superpotential $W(\phi_1)$, leading to the potential

$$V = F^2 - 3M_P^2|m_{3/2}(t)|^2$$

and

$$F^2 \equiv e^{\frac{1}{2}|\phi_1|^2/M_P^2} \left| \frac{dW}{d\phi_1} + M_P^{-2} \phi_1^* W \right|^2$$

and

$$m_{3/2}(t) \equiv e^{\frac{1}{2}|\phi_1|^2/M_P^2} W / M_P^2.$$ (3)

In the vacuum, $m_{3/2}(t)$ is the gravitino mass, denoted $m_{3/2}$ without an argument.

The superpotential is of the form $W(\phi_1)$, leading to the potential

$$W = V_0^\frac{1}{p} \phi_1 \left( 1 - \frac{1}{p+1} \left( \sqrt{2} \phi_1 / v \right)^p \right),$$ (4)

with $p \geq 3$. The real parameter $v$ is taken to be small on the Planck scale, $v \ll M_P$ where $M_P = 2.4 \times 10^{18}$ GeV.

Because $v$ is real, $\text{Im} \phi_1$ is driven to zero, leaving the canonically-normalized inflaton field $\phi = \sqrt{2} \text{Re} \phi_1$. Since $W$ has no constant term, the quadratic terms in Eq. (1) cancel. At $|\phi_1| \ll M_P$, global supersymmetry is a good approximation except near the minimum of the potential. This gives

$$V \simeq F^2 \simeq \left| \frac{dW}{d\phi_1} \right|^2 = V_0 \left( 1 - (\phi / v)^p \right)^2.$$ (5)

The field equation is

$$\ddot{\phi} + 3H \dot{\phi} + V' = 0$$

where $H$ is the Hubble parameter, related to the energy density by $3M_P^2 H^2 = \rho$. The density and pressure are

$$\rho = V + \frac{1}{2} \dot{\phi}^2$$

and

$$P = -V + \frac{1}{2} \dot{\phi}^2.$$ (8)

\footnote{For $p \geq 5$ one also needs $\phi / v \gg (v / M_P)^{p-1}$, which is assumed.}
Inflation occurs in the regime $\phi \ll v$, while $\phi$ is rolling away from the origin. The vacuum expectation value (vev) of $\phi$ is precisely $v$ in this approximation, and $V = 0$ corresponding to unbroken supersymmetry. The mass of $\phi$ in the vacuum is

$$M \sim \frac{V_0^{1/2}}{v} \sim H_\ast \left(\frac{M_P}{v}\right) \gg H_\ast,$$

where $H_\ast$ is the Hubble parameter at the end of inflation.

Before proceeding, we note that this class of inflation models is reasonably well-motivated. The form of the superpotential may be motivated by invoking a $Z_p$ symmetry ($R$-symmetry). Such a symmetry allows additional terms only of order $(\phi_1/v)^{1+n_p}$ ($n \geq 2$). The assumption of a practically minimal kinetic function (Kähler potential) $K = |\phi_1|^2$ is not completely unreasonable, since of the expected higher-order terms $\sim M_P^{2-2n}|\phi_1|^{2n}$ only the first need be suppressed [3]. The main limitation on the model is the requirement that the neglected fields all have vevs much less than $M_P$; indeed, just one field of order $M_P$, with the minimal kinetic term, will make the potential too steep for inflation [7], and there is no reason why non-minimal terms or additional fields should flatten the potential.

Near the minimum of the potential, corresponding to the vacuum of the one-field theory under consideration, we have to use the supergravity expression Eq. (1). This gives negative vacuum energy,

$$V_{\text{vac}} = F_{\text{vac}}^2 - 3M_P^2 m_{3/2}^2$$

$$3M_P^2 m_{3/2}^2 \simeq -3V_0(v/M_P)^2$$

$$F_{\text{vac}}^2 \sim V_0(v/M_P)^4.$$

In the true vacuum, the potential (practically) vanishes. This means that the true vacuum values of $F$ and/or the gravitino mass must be generated by some other sector of the Lagrangian, than the one used for the model of inflation. One hypothesis [3] is that the model gives the true gravitino mass, with the additional sector generating only the true vacuum value of $F$ which as usual we denote by $M_2^2$. For definiteness we adopt this hypothesis, which actually seems the most natural in view of the requirement that there be no Planck-scale vevs, at least during inflation. (To implement supersymmetry breaking without Planck-scale vevs, one might invoke a gauge-mediated mechanism or a Fayet-Iliopoulos term, neither of which would significantly affect the gravitino mass.)

We shall make estimates for the cases $p = 3$, $p = 4$ and $p \gg 2$. A relation between $V_0$ and $v$ is provided by the COBE measurement of the cosmic microwave background anisotropy [7],

$$X(p) = \left(\frac{M_P}{v}\right)^{p/2} \left(\frac{V_0^{1/2}}{M_P^2}\right),$$

where

$$X \equiv 5.3 \times 10^{-4} p^{-\frac{1}{p+2}} [N(p-2)]^{-\frac{p-1}{p-2}}.$$

with $\dot{\rho} = -3H(\rho + P)$.
Here $N$ is the number of $e$-folds of slow-roll inflation after cosmological scales leave the horizon. We take $N = 50$, leading to the following estimates.

\[
p = 3 : \quad \frac{v}{M_P} \simeq 10^2 \left( \frac{V_0^{1/4}}{M_P} \right)^4
\]

(15)

\[
p = 4 : \quad \frac{v}{M_P} \simeq 10^3 \left( \frac{V_0^{1/4}}{M_P} \right)
\]

(16)

\[
p \gg 2 : \quad \frac{v}{M_P} \simeq 10^5 p \left( \frac{V_0^{1/4}}{M_P} \right)^2
\]

(17)

This determines the potential in terms of $m_{3/2}$, as follows.

\[
p = 3 : \quad V_0^{1/4} \sim \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{3/8} \times 10^{11} \text{ GeV}
\]

(18)

\[
p = 4 : \quad V_0^{1/4} \sim \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{1/3} \times 10^{12} \text{ GeV}
\]

(19)

\[
p \gg 2 : \quad V_0^{1/4} \sim \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{1/4} p^{-1/4} \times 10^{13} \text{ GeV}.
\]

(20)

If $m_{3/2}$ is the true gravitino mass, it presumably lies roughly in the range 1 keV to 100 GeV.

**The helicity 1/2 gravitino mass** The model contains only a single chiral superfield, with minimal kinetic term, which can be taken to be real. The gravitino field therefore obeys the Rarita-Schwinger equation, with a time-dependent mass given by Eq. (3), and constraints to eliminate unphysical degrees of freedom. The equation and the constraints have to be evaluated in the curved spacetime corresponding to the expanding Universe. This gives separate mode function equations for the helicity 1/2 and helicity 3/2 states, as seen by a comoving observer.

The helicity 3/2 mode function satisfies the same equation as a spin 1/2 particle with mass $m_{3/2}(t)$ \[3, 4, 9\]. There is practically no creation of helicity 3/2 gravitinos from the vacuum in the present model, corresponding to the conformal invariance of the massless Dirac equation.

The helicity 1/2 mode function, satisfies the same equation as a spin 1/2 particle with mass \[3, 4, 9\]

\[
\tilde{m}(t) = m_{3/2}(t) - \frac{3}{2} m(1 + A_1) - \frac{3}{2} H A_2 + \mu ,
\]

(21)

\(^2\)This is the expression given in \[3\]. The result of \[3\] leads to an identical expression \[10\], except that the last term of Eq. (21) is $-\mu$. This discrepancy does not affect order of magnitude calculations.
where

\[ A_1 \equiv \frac{P - 3M_\rho^2 m_{3/2}(t)}{\rho + 3M_\rho^2 m_{3/2}(t)} \equiv \cos \chi \]  

(22)

\[ A_2 \equiv \frac{2 - 3M_\rho \dot{m}_{3/2}(t)}{3 \rho + 3M_\rho^2 m_{3/2}(t)} \equiv \sin \chi \]  

(23)

\[ A \equiv A_1 + iA_2 = e^{i\chi} \]  

(24)

\[ \mu \equiv -\frac{1}{2} \dot{\chi} . \]  

(25)

Using Eqs. (1), (7) and (8), we can write Eq. (22) as

\[ A_1 = \frac{1}{2} \dot{\phi}^2 - F^2 \]  

\[ \frac{1}{2} \dot{\phi}^2 + F^2 . \]  

(26)

As already noted, the helicity 1/2 components of the gravitino field have, in this model, the same dynamics as a spin 1/2 field with effective mass \( \tilde{m} \). They are produced with momentum \( k/a \) if there is appreciable violation of a weak adiabaticity condition \[ |(a\dot{\tilde{m}})| \ll \omega^2 \equiv k^2 + (a\tilde{m})^2 , \]  

(27)

where the prime denotes differentiation with respect to conformal time, \( d/d\eta = ad/dt \), and the average is over a conformal time interval \( \omega^{-1} \). (As usual, \( a \) is the scale factor of the Universe, \( H = \dot{a}/a \).) In practice \( k_{\text{max}} \), the biggest \( k \) for which significant creation occurs, is simply the biggest value achieved by \( a\tilde{m} \), within the regime where \( \tilde{m} \) varies non-adiabatically \( (|\dot{\tilde{m}}| \sim \tilde{m}^2) \).

To estimate \( k_{\text{max}} \) in this model, we need to follow the evolution of \( \tilde{m} \). During inflation, it is slowly varying, with \( \tilde{m} \approx m_{3/2}(t) \ll H_* \). After inflation, \( \phi \) oscillates about its vev. Except during the first few Hubble times, the amplitude \( \phi_0 \) is much less than \( v \), and \( \phi - v \approx \phi_0 \sin Mt \). As long as global supersymmetry is a good approximation, this gives

\[ F^2 \simeq \frac{1}{2}M^2 \phi_0^2 \sin^2 M \]  

(28)

\[ \frac{1}{2} \dot{\phi}^2 \simeq \frac{1}{2}M^2 \phi_0^2 \cos^2 M \]  

(29)

From Eq. (26), this gives \( A_1 = \cos 2Mt \) and therefore

\[ \tilde{m}(t) \simeq \mu(t) \simeq -M . \]  

(30)

(The overall sign of \( \tilde{m} \) is determined by Eqs. (3) and (23), but it is not physically significant.) The mass \( M \) of the inflaton is also the mass of the inflatino. In the model under consideration the latter, in turn, becomes the goldstino in the limit of global
supersymmetry. This is the physical reason why $\tilde{m}$ rises sharply in magnitude just after inflation. The sharp rise creates gravitinos, with $k_{\text{max}} \sim a_* M$, where a star denotes the end of inflation.

Taking this model literally, the negative vacuum energy means that the Hubble parameter falls to zero in the early Universe,

$$3M_P^2 H^2 = \rho(t) \simeq M^2 \phi_0^2(t) + V_{\text{vac}} \quad (31)$$
$$\simeq M^2 \phi_0^2(t) - 3M_P^2 m_{3/2}^2. \quad (32)$$

When $H$ vanishes, the Universe attains its maximum size. After that, it recollapses. Global supersymmetry is a good approximation for $F$ except near $F = 0$, because its amplitude $M\phi_0$ never falls below $\sqrt{3}M_P m_{3/2} \gg F_{\text{vac}}$. As a result, $\tilde{m}$ maintains the constant value $M$, and gravitino production occurs only at the end of inflation. Taking the model literally, we therefore have

$$k_{\text{max}} \sim a_* M. \quad (33)$$

where the star denotes the end of inflation.

In reality, the energy density is not given by Eq. (32), but by

$$3M_P^2 H^2 = \rho(t) \simeq M^2 \phi_0^2(t), \quad (34)$$

with the negative inflaton contribution to the vacuum energy canceled by whatever field is responsible for supersymmetry breaking in the true vacuum. It is argued elsewhere that as a result, gravitino creation will continue unless reheating intervenes, with every-increasing $k_{\text{max}} \sim a M$, until the Hubble parameter falls below the true gravitino mass. In the present model this epoch corresponds to

$$\left(\frac{a_*}{a}\right)^{\frac{7}{4}} \sim \frac{\rho^{\frac{3}{4}}}{V_0^{\frac{1}{4}}} \sim \left(\frac{m_{3/2}}{H_*}\right)^{\frac{1}{4}} \quad (35)$$
$$\sim \left(\frac{v}{M_P}\right)^{\frac{1}{2}}, \quad (36)$$

leading to

$$k_{\text{max}} \sim a_* M (M_P/v)^{\frac{3}{2}}. \quad (37)$$

The abundance of gravitinos created from the vacuum Since we are dealing with fermions, the occupation number of each helicity state is $\leq 1$. It is expected to be

---

3The goldstino is the fermionic component of the chiral superfield responsible for spontaneous global supersymmetry breaking. In the early Universe, when the time-derivative of the scalar component, not just the $F$ component, contributes to supersymmetry breaking, the goldstino mass does not vanish (see also [11]).
of order 1 at \( k = k_{\text{max}} \), giving number density \( n \)

\[
    n \simeq \frac{2}{4\pi^2} a^{-3} \int_0^{k_{\text{max}}} k^2 dk
\]

\[
    \simeq 10^{-2} a^{-3} k_{\text{max}}^3,
\]

\[ (38) \]

The relative abundance at nucleosynthesis is \( n_s \)

\[
    n_s \simeq 10^{-2} \left( \frac{k_{\text{max}}}{a_*} \right)^3 \frac{\gamma T_R}{V_0},
\]

\[ (40) \]

where \( s \) is the entropy density at nucleosynthesis, and \( \gamma^{-1} \) is the increase in entropy per comoving volume (if any), between reheating at temperature \( T_R < V_0^{1/4} \) and nucleosynthesis.

In the present model, a minimal estimate for the gravitino abundance is provided by Eq. (33), whereas a maximal one is provided by Eq. (37). Let us suppose first that the maximal estimate Eq. (37) is correct. Depending on the value of \( p \), we find

\[
    p = 3 : \quad n/s \sim 10^{-10} \left( \frac{M_P}{V_0^{1/4}} \right)^4 \frac{\gamma T_R}{M_P}
\]

\[ (41) \]

\[
    p = 4 : \quad n/s \sim 10^{-15} \left( \frac{M_P}{V_0^{1/4}} \right)^3 \frac{\gamma T_R}{M_P}
\]

\[ (42) \]

\[
    p \gg 2 : \quad n/s \sim 10^{-27} p^{-5} \left( \frac{M_P}{V_0^{1/4}} \right)^8 \frac{\gamma T_R}{M_P}.
\]

\[ (43) \]

The cosmological significance \( \gamma^{1} \) of the gravitino depends on its true mass \( m_{3/2} \). A gravitino with mass more than a few TeV has no effect because it decays well before nucleosynthesis, but such a big mass is regarded as unlikely.

A gravitino with mass in the range \( 100 \text{ MeV} \lesssim m_{3/2} \lesssim 1 \text{ TeV} \) decays around or after nucleosynthesis, but before the present. This range includes the value \( m_{3/2} \sim 100 \text{ GeV} \) to 1 TeV, expected in gravity-mediated models of supersymmetry breaking. Observation then requires \( \gamma^{1} \)

\[
    n/s \lesssim 10^{-13}.
\]

\[ (44) \]

(To be precise, the upper bound depends on the mass and is in the range \( 10^{-12} \) to \( 10^{-15} \).) The abundance of gravitinos from thermal collisions is \( n/s \sim 10^{-13}(\gamma T_R/10^9 \text{ GeV}) \), leading to the bound \( \gamma T_R \gtrsim 10^9 \text{ GeV} \). Using instead Eqs. (18)–(20) and Eqs. (41)–(43), we find the following results.

\[
    p = 3 : \quad \gamma T_R \lesssim 10^6 \text{ GeV}
\]

\[ (45) \]

\[
    p = 4 : \quad \gamma T_R \lesssim 100 \text{ GeV}
\]

\[ (46) \]

\[
    p \gg 2 : \quad \gamma T_R \lesssim 10^{-25} \text{ GeV}.
\]

\[ (47) \]
A gravitino with mass $m_{3/2} \lesssim 100\text{ MeV}$ survives to the present, and is a dark matter candidate. This includes the range predicted by gauge-mediated models of supersymmetry breaking, which is $1\text{ keV} \lesssim m_{3/2} \lesssim 100\text{ GeV}$ with the upper decades disfavoured. The present density is

$$\Omega_{3/2} \simeq 10^5 \left( \frac{m_{3/2}}{100\text{ keV}} \right) \frac{n}{s}$$

If creation from thermal collisions dominates, then very roughly

$$\Omega_{3/2} \sim \left( \frac{100\text{ keV}}{m_{3/2}} \right) \left( \frac{T_R}{10^4\text{ GeV}} \right).$$

Using instead Eqs. (18)–(20) and Eqs. (41)–(43), we find the following results for $m_{3/2} = 100\text{ keV}$;

$$p = 3 : \quad \gamma T_R \lesssim 10^{11}\text{ GeV}$$

$$p = 4 : \quad \gamma T_R \lesssim 10^4\text{ GeV}$$

$$p \gg 2 : \quad \gamma T_R \lesssim 10^{-17}\text{ GeV}.$$

These results are quite dramatic. The bound on the reheat temperature is generally stronger than the one coming from thermal collisions, indicating that gravitino production from the vacuum is dominant. For $p \gg 2$, the abundance is so big that an epoch of thermal inflation [13] would be needed.

Finally, suppose that the minimal estimate Eq. (33) is correct. Then we have

$$p = 3 : \quad \frac{n}{s} \sim 10^{-8} \frac{\gamma T_R}{M_P}$$

$$p = 4 : \quad \frac{n}{s} \sim 10^{-11} \left( \frac{M_P}{V_0^{1/2}} \right) \frac{\gamma T_R}{M_P}$$

$$p \gg 2 : \quad \frac{n}{s} \sim 10^{-15} p^{-3} \left( \frac{M_P}{V_0^{1/2}} \right)^4 \frac{\gamma T_R}{M_P}.$$

In this minimal case, there is no regime of parameter space where gravitinos created from the vacuum are dominant, in the simplest case that the model is responsible for the true gravitino mass. In the more general case, they are again dominant in a significant region of parameter space.

**Conclusion** In a specific class of inflation models, giving a potential $V \simeq V_0(1-\phi/v)^p$, we have made minimal and maximal estimates, for the abundance of gravitinos created from the vacuum fluctuation. In the maximal case, gravitinos from this mechanism may be so abundant that an era of thermal inflation is needed to dilute them. In the minimal case, gravitinos from the vacuum fluctuation have no significant effect, if the gravitino mass given by the model is the true mass.
Acknowledgements

I am indebted to Toni Riotto and Andrei Linde for useful discussions.

References

[1] T. Moroi, hep-ph/9503210; S. Sarkar, Rep. on Prog. in Phys. 59 (1996) 1493.
[2] D. H. Lyth and D. Roberts, hep-ph/9609441, unpublished; D. H. Lyth, D. Roberts and M. Smith, Phys. Rev. D 57 (1998) 7120.
[3] R. Kallosh, L. Kofman, A. Linde and A. Van Proeyen, hep-th/9907124.
[4] G. F. Giudice, I. Tkachev and A. Riotto, JHEP 9908:009 (1999).
[5] D. H. Lyth, hep-ph/9912313.
[6] K. Kumekawa, T. Moroi, and T. Yanagida, Prog. Theor. Phys. 92 (1994) 437; K. I. Izawa and T. Yanagida, Phys. Lett. B393 (1997) 331; K. I. Izawa, M. Kawasaki and T. Yanagida, Phys. Lett. B411 (1997) 249.
[7] D. H. Lyth and A. Riotto, Phys. Rep. 314 (1999) 146.
[8] A. L. Maroto and A. Mazumdar, hep-ph/9904206.
[9] M. Lemoine, hep-ph/9908333.
[10] D. H. Lyth, hep-ph/9909387 (v3), to appear in Phys. Lett. B.
[11] G. F. Giudice, A. Riotto and I. Tkachev, hep-ph/9911302.
[12] A. L. Maroto and J. R. Pelaez, hep-ph/9912212.
[13] P. Binétruy and M. K. Gaillard, Phys. Rev. D 34 (1986) 3069; G. Lazarides and Q. Shafi, Nucl. Phys. B392 (1993) 61; D. H. Lyth and E. D. Stewart, Phys. Rev. D 53 (1996) 1784; T. Barreiro, E. J. Copeland, D. H. Lyth and T. Prokopec, Phys. Rev. D 57 (1998) 7345; T. Asaka, J. Hashiba, M. Kawasaki and T. Yanagida, Phys. Rev. D 58 (1998) 083509; T. Asaki and M. Kawasaki, hep-ph/9905467.