The flavour problem and family symmetry
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Abstract

We show how two different family symmetries can be used to address the flavour problem in $SO(10)$-like models. The first is based on a gauged $U(1)_F$, whose problems disappear in the context of a type I string model embedding. The second is based on $SU(3)_F$, the maximal family group consistent with $SO(10)$; in this case the family symmetry is more constraining, so we merely look at it in the context of a supersymmetric field theory.

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1 Introduction

The flavour problem in SUSY models falls into two parts. The first is to understand the origin of the Yukawa couplings, and with the see-saw mechanism to understand the origin of the Majorana masses. The second is to understand why flavour changing and CP violating processes induced by SUSY loops is so small. Any theory of flavour should address both problems simultaneously. Consider, $\mu \rightarrow e \gamma$ [1]:

$$BR(\mu \rightarrow e\gamma) \approx \frac{\alpha^3}{G_F^2} f_{21}(M_2, \mu, m_{\tilde{\nu}}) \left| m_{L_{21}}^2 \right|^2 \tan^2 \beta$$

(1)

The simplest way to suppress processes like these is to suppress off diagonal sfermion masses. There are two general ways of generating off diagonal sfermion masses. The first, ‘primordial’, is where there are off diagonal elements in the SCKM basis. The second is RGE generated, where either Higgs triplets in running from the GUT scale to the EW scale, or for leptons, running the MSSM with right handed neutrinos from the GUT scale to the lightest RH neutrino mass. In general both will be present; models of flavour are concerned with suppressing primordial contributions.

2 U(1) Family symmetry

We consider here the $SO(10)$-like model of flavour $SU(4)_P \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_F$. Here the family index is promoted to being a gauge index under $U(1)_F$, and the matter representations under 4, 2, 2, 1 are:

$$F^i = (4, 2, 1)_{(1, 0, 0)} ; \bar{F}^i = (\bar{1}, 1, 2)_{(4, 2, 0)}$$

$$h = (1, 2, 2)_0$$

(2)

(3)

The exotic Higgs fields are $H, \bar{H}$, and $\Phi$. The first two break $SU(4)_P \otimes SU(2)_R \rightarrow SU(3)_c \otimes U(1)_Y$. The latter breaks $U(1)_F$. Their representation and VEVs are:

$$H = (4, 1, 2)_0, \quad \langle H \rangle_M \approx \sqrt{\delta} ; \quad \bar{H} = (\bar{4}, 1, 2)_0, \quad \langle \bar{H} \rangle_M \approx \sqrt{\delta}$$

$$\Phi = (1, 1, 1)_{-1}, \quad \langle \Phi \rangle_M \approx \epsilon$$

(4)

(5)

The phenomenological values of the expansion parameters are $\delta = \epsilon = 0.22$. These are of use when we use Froggatt-Nielsen operators [2] to generate the Yukawa matrices and the Majorana matrix for the RH neutrino fields:

$${}^\text{Yukawa:} \quad F^i \bar{F}^j \left( \frac{H \bar{H}}{M^2} \right)^{n_{ij}} \left( \frac{\Phi}{M} \right)^{p_{ij}} \rightarrow F^i \bar{F}^j C_{ij} \delta^{n_{ij}} \epsilon^{p_{ij}}$$

(6)

$${}^\text{Majorana:} \quad F^i \bar{F}^j \left( \frac{H \bar{H}}{M^2} \right) \left( \frac{H \bar{H}}{M^2} \right)^{q_{ij}} \left( \frac{\Phi}{M} \right)^{q_{ij}} \rightarrow F^i \bar{F}^j M_3 \delta \epsilon^{q_{ij}}$$

(7)
With a certain operator texture \[3\], we can then write out the Majorana matrix, and the Yukawa matrices, at the GUT scale:

\[
\frac{M_{RR}(M_X)}{M_3} \sim \begin{pmatrix}
\delta \epsilon^6 & \delta \epsilon^6 & \delta \epsilon^4 \\
\delta \epsilon^6 & \delta \epsilon^4 & \delta \epsilon^2 \\
\delta \epsilon^4 & \delta \epsilon^2 & 1
\end{pmatrix}
\] (8)

\[
Y^u(M_X) \sim \begin{pmatrix}
\frac{\sqrt{2}}{5} \delta \epsilon^5 & \frac{\sqrt{2}}{5} \delta \epsilon^3 & \frac{2}{\sqrt{5}} \delta^2 \epsilon \\
0 & \frac{8}{5} \delta \epsilon^2 & 0 \\
0 & \frac{8}{5} \delta \epsilon^2 & r_t
\end{pmatrix}
\] (9)

\[
Y^d(M_X) \sim \begin{pmatrix}
\frac{2}{\sqrt{5}} \delta \epsilon^4 & -\frac{\sqrt{2}}{5} \delta \epsilon^3 & \frac{4}{\sqrt{5}} \delta^2 \epsilon \\
\frac{8}{5} \delta \epsilon^5 & \frac{2}{\sqrt{5}} \delta \epsilon^2 + \frac{16}{5\sqrt{5}} \delta^2 \epsilon^2 & \frac{2}{\sqrt{5}} \delta^2 \\
\end{pmatrix}
\] (10)

\[
Y^\nu(M_X) \sim \begin{pmatrix}
\frac{\sqrt{2}}{5} \delta \epsilon^5 & \frac{2}{5} \delta \epsilon^3 & 0 \\
0 & \frac{6}{5\sqrt{5}} \delta \epsilon^2 & 2 \delta \\
0 & \frac{6}{5\sqrt{5}} \delta \epsilon^2 & r_\nu
\end{pmatrix}
\] (11)

\[
Y^e(M_X) \sim \begin{pmatrix}
\frac{6}{5} \delta \epsilon^5 & 0 & 0 \\
-3 \sqrt{\frac{2}{5}} \delta \epsilon^2 + \frac{12}{5\sqrt{5}} \delta \epsilon^2 & \frac{2}{\sqrt{5}} \delta \epsilon & -3 \sqrt{\frac{2}{5}} \delta^2 \\
\sqrt{2} \delta \epsilon^2 & \frac{2}{\sqrt{5}} \delta \epsilon & 1
\end{pmatrix}
\] (12)

This model is consistent with all laboratory data; A global analysis \[3\] assuming universal gaugino and sfermion masses but non-universal Higgs masses, with \(\tan \beta = 50\) is consistent with sparticle and Higgs mass limits, fermion masses and mixing angles including the LMA MSW, muon \(g-2\), \(b \to s\gamma\) and the LFV constraints.

Furthermore, in this model it is possible that the decay \(B_s \to \mu^+ \mu^-\) could occur at a rate \[4\] close to the current limit of \(BR(B_s \to \mu \mu) < 2.0 \times 10^{-6}\) which is much larger than the Standard Model prediction of \(BR(B_s \to \mu \mu) \sim 1.5 \times 10^{-10}\).

We now turn to understanding why the primordial off-diagonal terms are small. This is entirely natural in the context of a type I string embedding. In fact, a string construction which leads to a continuum of SUSY Pati-Salam models is known \[5\] (See also \[6\]). The idea here is that everything is set up on two intersecting D5-branes. One of them, the 5\(_1\)-brane has the gauge group \(U(4)_1 \otimes U(2)_L \otimes U(2)_R\), and the other, the 5\(_2\)-brane has the gauge group \(U(4)_2\). Higgs states are present which can break \(U(4)_1 \otimes U(4)_2 \to U(4)_{PS}\). Most of the \(U(1)\)s are broken by the GS mechanism, but one remains: \(U(1)_F\). Furthermore, in this model \(R_2 \ll R_1\), (“single brane limit”), and we have approximate gauge coupling unification.

The important point from a flavour point of view is that in this model the Pati-Salam and Family breaking fields can have auxiliary fields which may contribute to symmetry breaking; despite the fact that these are much smaller.
Figure 1: $B_s \rightarrow \mu^+ \mu^-$ predictions
than the dominant contributions coming from moduli, they can contribute to the A-terms at the same order as the moduli, since [7]:

$$A \supset F_\phi \partial_\phi \ln Y = F_\phi \partial_\phi \ln n = F_\phi \frac{n}{\Phi} \propto nm_{3/2}$$  \hspace{1cm} (13)

(The last step is since $F_\phi \propto m_{3/2} \phi$). Having done this, you need a parameterisation for the auxiliary field VEVs. The usual one is that of goldstino angles. In order to get the auxiliary field VEVs for the family group Higgs, it was necessary to pick a definite brane allocation. Picking it to be an intersection state, the F-term VEVs and then the soft terms can be written down [8]. Where $\sum_\alpha X_\alpha^2 = 1$:

$$m_F^2 = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}, \quad a = 1 - \frac{3}{2} (X_S^2 + X_{T_3}^2), \quad b = 1 - 3X_{T_3}^2 \hspace{1cm} (14)$$

$$M_3 \approx M_2 \approx M_1 = \sqrt{3}m_{3/2}X_{T_3} \equiv M_{1/2} \hspace{1cm} (15)$$

$$A = \sqrt{3}m_{3/2} \begin{pmatrix} d_1 + d_H + 5d_\Phi & d_1 + d_H + 4d_\Phi & d_1 + d_H + 2d_\Phi & d_2 + d_H + d_\Phi \\ d_1 + d_H + 4d_\Phi & d_1 + d_H + 2d_\Phi & d_2 + d_H + d_\Phi \\ d_3 + d_H + 4d_\Phi & d_3 + d_H + 2d_\Phi & d_4 \end{pmatrix} \hspace{1cm} (16)$$

Where, the $d$ parameters are:

$$d_1 = X_S - X_{T_1} - X_{T_2}, \quad d_2 = \frac{1}{2}X_S - X_{T_1} - \frac{1}{2}X_{T_3} \hspace{1cm} (17)$$

$$d_3 = \frac{1}{2}X_S - X_{T_1} - X_{T_2} + \frac{1}{2}X_{T_3}, \quad d_4 = -X_{T_1} \hspace{1cm} (18)$$

$$d_H = (S + S^*)^{1/2}X_H + (T_3 + T_3^*)^{1/2}X_{TH} \hspace{1cm} (19)$$

$$d_\Phi = (S + S^*)^{1/4}(T_3 + T_3^*)^{1/4}X_\Phi \hspace{1cm} (20)$$

| Point | $X_S$ | $X_{T_1}$ | $X_{T_2}$ | $X_H$ | $X_{TH}$ |
|-------|-------|-----------|-----------|-------|-----------|
| A     | 0.50  | 0.50      | 0.00      | 0.00  | 0.00      |
| B     | 0.54  | 0.49      | 0.00      | 0.00  | 0.00      |
| C     | 0.27  | 0.27      | 0.84      | 0.00  | 0.00      |
| D     | 0.27  | 0.27      | 0.00      | 0.60  | 0.60      |

Table 1: Four benchmark points for the $U(1)_F$ model

Having done this, we consider for benchmark points, A-D (given in tab. 1), which represent the various independent sources of flavour violation. A represents no primordial FV, B represents primordial FV from the moduli alone, C represents primordial FV from the Froggatt-Nielsen fields alone, and D represents primordial FV from the Pati-Salam Higgs fields alone. Fig. 2 shows two lines for each benchmark point, one with the see-saw contribution removed by removing the RH neutrino field, and one with a seesaw contribution. We see
that we are able to stay within the experimental limits at point C, and that
the see-saw source can reduce the amount of FV coming from the primordial
sector (see fig. point D).

We have assumed that the D-terms coming from $U(1)_F$ are small to keep
d-term contributions to LFV small; however with a similar setup, it is possible
to suppress LFV by having the D-terms very large.

![Figure 2: $\mu \rightarrow e\gamma$ in the $U(1)_F$ model. The dashed line represents a model with no RH neutrino field, removing the see-saw contribution to the process. The solid line represents 'physics', with FV coming in general from both primordial and see-saw sources. The horizontal line is the 2002 experimental limit.](image)

### 3 $SU(3)$ family symmetry

We now look at a grand unified, non-abelian theory of flavour, $SO(10) \otimes SU(3)_F$. We also impose a $U(1) \otimes Z_2 \otimes \mathbb{R}$ global symmetry, which will only allow certain Yukawa and $L_{\text{soft}}$ operators. Then we break $SO(10)$ first to the Pati-Salam group and then to the MSSM group by two instances of Wilson line breaking.
Having done this, we break $SU(3)_F \rightarrow SU(2)_F$ with the field $\phi_3$, and then we break $SU(2)_F \rightarrow 0$ with the field $\phi_{23}$:

$$\langle \phi_3 \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \phi_{23} \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ (21)

The charges under $U(1)_X$ restrict the form of the operators that form the Yukawa elements and soft terms. Once you choose a set to match with the Yukawa sector, then the operators which form the soft terms are completely fixed. For example, one set of field assignments [11] allows two (amongst many) operators:

$$\frac{1}{M^2} \psi_3 \psi_3 h \rightarrow \begin{pmatrix} 0 \\ \tau \end{pmatrix}, \quad \Sigma \frac{1}{M^2} \psi_3 \psi_3 h \rightarrow \begin{pmatrix} 0 \\ y e^2 \tau \end{pmatrix}$$ (22)

There are operators which contribute to the Majorana matrix as well. In the end, setting $O(1)$ coefficients empirically, we get:

$$Y_u \sim \begin{pmatrix} 0 & 1.2 \epsilon^3 & 0.9 \epsilon^3 \\ -1.2 \epsilon^3 & -\frac{2}{3} \epsilon^2 & -\frac{2}{3} \epsilon^2 \\ 0.9 \epsilon^3 & -\frac{4}{3} \epsilon^2 & 1 \end{pmatrix} \tau, \quad Y_d \sim \begin{pmatrix} 0 & 1.6 \epsilon^3 & 0.7 \epsilon^3 \\ -1.6 \epsilon^3 & -\epsilon^2 & -\epsilon^2 + \epsilon^3 \\ 0.7 \epsilon^3 & -\epsilon^2 & 1 \end{pmatrix} \tau$$ (23)

$$Y_\nu \sim \begin{pmatrix} 0 & 1.6 \epsilon^3 \\ -1.6 \epsilon^3 & -\alpha \epsilon^2 & -\alpha \epsilon + \epsilon^3 \nu \tau \end{pmatrix} \tau, \quad Y_\epsilon \sim \begin{pmatrix} 0 & 1.6 \epsilon^3 & 0.7 \epsilon^3 \\ -1.6 \epsilon^3 & 3 \epsilon^2 & 3 \epsilon^2 \\ -\epsilon^3 & 3 \epsilon^2 & 1 \end{pmatrix} \tau$$ (24)

$$\frac{1}{M_{RR}} \sim \begin{pmatrix} \epsilon^6 \epsilon^3 \\ e^{6 \epsilon^3} \\ 1 \end{pmatrix}$$ (25)

The first RH neutrino dominates, and we predict $m_2/m_3 \sim \tau$, $\tan \theta_{23} \sim 1.3$, $\tan \theta_{12} \sim 0.66$ and $\theta_{13} \sim \tau$.

We also constrain the order of terms allowed in $L_{\text{soft}}$. The scalar matrices come from $D$-terms, and the trilinears come from the $F$-terms [12]:

$$L_{\text{soft}} \supset \left\{ \frac{S^\dagger S \psi^\dagger \psi}{M^2} + \frac{S^\dagger S \psi^\dagger \phi_3 \psi \phi}{M^2} + \cdots \right\}_D + \left\{ \cdots \right\}_F$$ (26)

So, then the scalar matrices get terms like

$$m_\psi^2 \supset \begin{pmatrix} m_0^2 \\ m_0^2 \\ m_0^2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \rho \cdot n_0^2 + \cdots$$ (27)

This leads to a characteristic pattern of SUSY masses, with FCNCs suppressed by high powers of $\epsilon$ and $\tau$.
4 Conclusions

A $U(1)$ family symmetry allows an understanding of fermion masses and mixings, but does not address the SUSY flavour problem without additional theoretical input. Such models are motivated by string theories where $U(1)$s are abundant, and SUSY flavour changing may be controlled in a type I string embedding, even if the theory controls sparticle masses, dangerous new primordial flavour changing arise from Yukawa operators which lead to large off-diagonal soft trilinears.

A $SU(3)$ family symmetry allows (anti)symmetric Yukawa matrices, with SUSY flavour changing controlled by the family symmetries. However, it is hard to get non-abelian family symmetries from string theory.

References

[1] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53 (1996) 2442 arXiv:hep-ph/9510309.

[2] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[3] T. Blazek, S. F. King and J. K. Parry, JHEP 0305, 016 (2003) arXiv:hep-ph/0303192.

[4] T. Blazek, S. F. King and J. K. Parry, arXiv:hep-ph/0308068.

[5] L. L. Everett, G. L. Kane, S. F. King, S. Rigolin and L. T. Wang, Phys. Lett. B 531 (2002) 263 arXiv:hep-ph/0202100.

[6] G. Shiu and S. H. Tye, Phys. Rev. D 58 (1998) 106007 arXiv:hep-th/9805157. S. F. King and D. A. Rayner, Nucl. Phys. B 607 (2001) 77 arXiv:hep-ph/0012070.

[7] S. A. Abel and G. Servant, Nucl. Phys. B 597 (2001) 3 arXiv:hep-th/0009098. S. A. Abel and G. Servant, Nucl. Phys. B 611 (2001) 43 arXiv:hep-ph/0105262. S. Abel, S. Khalil and O. Lebedev, Phys. Rev. Lett. 89 (2002) 121601 arXiv:hep-ph/0112260. G. G. Ross and O. Vives, Phys. Rev. D 67 (2003) 095013 arXiv:hep-ph/0211279.

[8] S. F. King and I. N. Peddie, arXiv:hep-ph/0307091.

[9] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66 (2002) 010001.

[10] S. Pokorski, These proceedings

[11] S. F. King and G. G. Ross, arXiv:hep-ph/0307190. S. F. King and G. G. Ross, Phys. Lett. B 520 (2001) 243 arXiv:hep-ph/0108112.
[12] M. R. Ramage and G. G. Ross, arXiv:hep-ph/0307389
G. G. Ross, L. Velasco-Sevilla, O. Vives, in preparation