Dynamic characteristics of the annular rotor components in a rotor-bearing system under high frequency stress conditions

Chenglin Wang¹, Huating Chen¹, Weichen Sun² and H J Wang¹,⁴
¹Beijing Wuzi University, China.
²Beijing Institute of Technology, China.
³School of Mechanical and Electrical Engineering, Beijing Information Science and Technology University, China.
⁴Key Laboratory of Modern Measurement and Control Technology, Ministry of Education, Beijing Information Science and Technology University, China.

E-mail: wangchengl6688@126.com

Abstract. The dynamic characteristics of rotor components can directly affect the stability of a rotor-bearing system. Under certain conditions, such as high frequency stress, the dynamic performance of the rotor components is of particular importance. In this paper, a typical annular rotor is studied, and a testing system presented based on the base excitation resonance mass (BERM) method in order to measure key parameters, such as the radial and axial stiffness coefficients, damping coefficient, and loss factor. Based on this system, the dynamic characteristics of typical rotor components under radial and axial sine excitations below 2000 Hz were tested and analyzed. A predictive model of the characteristic parameters was then built based on the grey prediction method, and coupling between the radial and axial dimensions was evaluated. Further, the coupling mechanisms and relationships were analyzed, and the mechanism of interaction between microstructure features under high frequency stress was determined and combined with the changes to the internal microstructure of the rotor components to build a complete picture. Based on the experimental testing and theoretical results, our work suggests a strong correlation between the rotor stiffness coefficient and frequency of use, and thus provides empirical evidence and a reference for the future optimization of the dynamic characteristics of rotor components, in particular for the design of parameters for rotor components the experience high frequency stress conditions on the bearings.

Keywords: Dynamic characteristics, High frequency bearing characteristics, rotor components, grey prediction.

1. Introduction
With the rapid development of modern engineering and technology, mechanical devices are becoming more widely used and often operate under high frequency stress conditions, in particular, with the performance of rotating machinery being continuously improved by increasing rotational speeds. Rotor components are the most critical components of rotating machinery, and their dynamic characteristics directly affect the stability of the system, especially under certain loading conditions, including high frequencies and high acceleration. Moreover, dynamic performance is particularly important, as it will directly affect the potential usage of the system. Therefore, studying the mechanical properties of rotor components under high acceleration bearing conditions is of great importance [1-5].
2. Design of the testing system

2.1. System design principles

New materials are continuously being used in high frequency bearing applications. However, the quantitative relationships between the characteristic parameters, such as the stiffness coefficient, damping coefficient, loss factor, and geometry, and the operational parameters have not been fully understood so far [6-7]. There are numerous methods for testing the dynamic characteristics of rotors, for which the base excitation-resonance mass (BERM) method is a useful one. With the BERM method, the test object is placed between the excitation source and forced vibration body, and the dynamic characteristics of the test object are calculated and analyzed in order to acquire the characteristic dynamic parameters, including the stiffness coefficient and damping coefficient, by measuring the extracted feature information about the excitation source and forced vibration body, respectively. The dynamic model is based on the testing principles shown in figure 1. A shaking table is used as the source of excitation, and the test object can be simplified as the connection mechanism with stiffness coefficient and damping coefficient, for which the spring and damper are characterized by the eigenvalues (figure 1).

![Figure 1. Dynamic model of BERM method.](image)

Assuming the mass of the forced vibration body is $m$, the equation of motion can be written as

$$m\ddot{y} + c(\dot{y} - \dot{x}) + k(y - x) = 0$$  \hspace{1cm} (1)

The excitation form of the shaking table can be represented as

$$X = X_0 e^{i\omega t}$$  \hspace{1cm} (2)

The response of the forced vibration body can be described as

$$y = y_0 e^{i(\omega t - \phi)}$$  \hspace{1cm} (3)

Where, $\omega$ is the vibration frequency.

Combining the above equations, the elastic coefficient $k$ of the spring can defined as

$$k = \frac{m\omega^2(\alpha^2 - \alpha \cos \phi)}{\alpha^2 - 2\alpha \cos \phi + 1}$$  \hspace{1cm} (4)

Where, $\phi$ is the phase difference.

The damping coefficient $c$ of the tested object can be given by

$$c = \frac{m\omega \alpha \sin \phi}{\alpha^2 - 2\alpha \cos \phi + 1}$$  \hspace{1cm} (5)
Where the amplitude ratio is

$$\alpha = \frac{y_0}{x_0} = \frac{\dot{y}_0}{\dot{x}_0}$$

and the loss factor $\beta$ is

$$\beta = \frac{c\omega}{k}.$$  \hspace{1cm} (7)

Therefore, as long as the excitation amplitude $x_0$ can be measured, the amplitude ratio $x_0$ of the forced vibration body to the phase difference between them, as well as the stiffness, damping coefficient, and loss factor value of the test object can be acquired.

2.2. Structural design of the testing system

Rotor components have a complex structure. In order to reflect their main dynamic characteristics, in this paper we selected widely used annular components as the test objects, and the form of component fit was selected as the most widely used shaft-hole fit, between the annular components and the excitation source and forced vibration body. To simulate the actual working environment, concentric mounting holes were set at the front and back supporting ends on the excitation source (shaking table) for installing annular test components, and the forced vibration body was machined into a shaft component form, and installed within the components.

![Mechanical drawing of the testing system](image)

**Figure 2.** Mechanical drawing of the testing system.

To improve the system generality for testing the dynamic characteristics of annular components of different thicknesses, the fit between the annular components and forced vibration body can be realized by configuring appropriate shaft sleeves. Thus, we can achieve the rapid replacement of test objects thereby reducing the number of components that need to be replaced in the testing system. Figure 3 shows the assembly relationship diagram of the test objects.
2.3. Design of testing scheme
For current applications, rotor components are mainly composed of metallic and polymeric materials. In this study, both a metallic and polymer material were selected, aluminum Al6061 and nylon, respectively. The geometric parameters of the annular test components included the outer diameter (mm), inner diameter (mm), and length (mm), and the layout of the experimental testing system is shown in figure 4.

Three-dimensional acceleration sensors were installed on the shaking table and test object, and the acceleration signals were collected using a data acquisition instrument. The data was transferred to the analysis software, which reads the amplitude and phase information from the signals, and the dynamic characteristics can then be calculated. By controlling the feature parameters such as the excitation source, stress value, and excitation amplitude, the impact of frequency on the dynamic characteristics of the rotor components can be studied. Based on the demands for our desired usage, this paper mainly focuses on measuring stiffness coefficients, but will also analyze the relationship between the stiffness coefficient and frequency of use.
The temperature during testing was 5℃-10℃, and the model number and performance parameters of the instruments used in the experiment are listed in Table 1.

**Table 1.** Model number and performance parameters of instruments used in the experiment.

| Instrument             | Performance parameters                                      | Model number                      |
|------------------------|------------------------------------------------------------|-----------------------------------|
| Shaking table          | Maximum working frequency = 2000 Hz                        | Suzhou Sushi Testing Instrument Co., Ltd. MAV-1000-4H |
|                        | Three-dimensional simultaneous vibration sinusoidal thrust = 9.8 kN |                                   |
|                        | Three-dimensional simultaneous sinusoidal thrust = 9.8 kN   |                                   |
| Acceleration sensors   | Frequency range = 1-4000 Hz                                | China orient institute of noise & vibration INV9832 |
| Data acquisition       | Supporting 8 channel signal input                          | China orient institute of noise & vibration INV3062 |
| instrument             | Supporting signal input by BNC connector                   |                                   |
| Analysis software      | Possessing basic functions including oscillographic sampling, time domain analysis, auto-regression spectrum analysis analysis, functional analysis | China orient institute of noise & vibration DASO-V10, Featured Version |

**Figure 5.** Spot map of the experimental test.
3. Analysis of experimental results

Due to the application of high-frequency stress on the rotor-bearing system, there is some level of uncertainty in the experimental results at certain frequencies, due to the influence of the dynamic characteristics of the testing system itself. These uncertainties mainly manifest in the discrete nature and relatively strong distribution inhomogeneity of the experimental data, which greatly influences the overall test results. For convenient statistical analysis, the interval frequency of adjacent test points within the testing range was set to 100 Hz, and the testing frequency values were set below 2000 Hz. In the testing process, abnormal data were eliminated, and a typical sample data set was selected for closer analysis. For the experiment, the test object was an aluminum Al6061 specimen of 40 mm external diameter and 2.5 mm thickness, and a hard nylon sample of the same measurements. The results are shown in Tables 2 and 3.

| Axial data of unidirectional testing | Frequency (Hz) | 700 | 1100 | 1400 | 1700 |
|-------------------------------------|----------------|-----|------|------|------|
|                                     | Stiffness (N/m)| 1008030.408 | 2652754.724 | 5384916.58 | 19588536.06 |
| Axial data of two-dimensional coupling testing | Frequency (Hz) | 400 | 600 | 1100 | 1700 |
|                                     | Stiffness (N/m)| 299542.3509 | 1433397.768 | 3042655.44 | 7140040.754 |
| Radial data of unidirectional testing | Frequency (Hz) | 500 | 700 | 1000 | 1500 |
|                                     | Stiffness (N/m)| 576793.1802 | 1312774.967 | 2301878.82 | 11317649.19 |
| Radial data of two-dimensional coupling testing | Frequency (Hz) | 400 | 700 | 1200 | 1600 |
|                                     | Stiffness (N/m)| 293135.2191 | 624385.4955 | 1370196.88 | 14436590.1 |

3.1. Predictive analysis of the stiffness coefficient based on the grey prediction method

Considering the discrete nature and distribution inhomogeneity of the experimental data, the sample data must be processed to acquire characteristic parameter values under loading at different frequencies. Combined with the data distribution characteristics, the grey prediction method can be used to analyze the stiffness data [8].
Table 3. Stiffness test results of hard nylon component.

|                                | Frequency (Hz) | 1300  | 1400  | 1700  | 2000  |
|--------------------------------|----------------|-------|-------|-------|-------|
| Axial data of unidirectional   |                | 9716  | 1042  | 1213  | 3551  |
| testing                        |                | 14.70 | 9.05  | 4.05  | 48.47 |
|                                |                |       |       |       |       |
| Axial data of two-dimensional   |                | 700   | 1100  | 1800  | 1900  |
| coupling testing               |                |       |       |       |       |
|                                |                | 4547  | 2934  | 4108  | 4328  |
|                                |                | 0.495 | 2.573 | 4.248 | 7.048 |
|                                |                |       |       |       |       |
| Radial data of unidirectional   |                | 500   | 700   | 1000  | 1700  |
| testing                        |                |       |       |       |       |
|                                |                | 5691  | 1393  | 2260  | 6341  |
|                                |                | 18.35 | 91.87 | 343.3 | 1113.142 |
|                                |                |       |       |       |       |
| Radial data of two-dimensional   |                | 400   | 900   | 1500  | 1800  |
| coupling testing               |                |       |       |       |       |
|                                |                | 2871  | 6668  | 8860  | 4310  |
|                                |                | 27.42 | 1.756 | 0.839 | 5.261 |

3.2. Establishment of the grey prediction model

In order to reduce the influence of the high growth sequence on the fitting accuracy, the original sequence data are subjected to a logarithmic transformation. Here, the original sequence data are represented by \( X^{(00)}(K_i) \), and the transformed logarithmic sequence data is represented by \( X^{(01)}(K_i) = \ln(X^{(00)}(K_i)) \).

Assuming \( X^{(0)}(K_i) = \{X^{(0)}(K_1), X^{(0)}(K_2), ..., X^{(0)}(K_n)\} \), if the internal \( \Delta K_i = K_i - K_{i-1}, i=2, 3..., n \), is not a constant, then \( X^{(0)}(K_i) \) is a non-equidistant sequence.

Assuming \( X^{(1)}(K_i) = \{X^{(1)}(K_1), X^{(1)}(K_2), ..., X^{(1)}(K_n)\} \), if \( X^{(1)}(K_i) = \sum_{j=1}^{i} X^{(01)}(K_j) \Delta K_j, \ i=1, 2, ..., n \), then \( X^{(1)}(K_i) \) is the first-order accumulated generating (1-AGO) sequence of the non-equidistant sequence \( X^{(0)}(K_i) \).

For the 1-AGO sequence, the albinism differential equation can be built as follows:

\[
\frac{dx_1^{(1)}(t)}{dt} + ax_1^{(1)}(t) = u, \ i=1,2,...,n
\]

If we stipulate \( x_1^{(1)}(K_1) = X^{(0)}(K_1) \) when \( t=K_1 \), then the response function is

\[
x_1^{(1)}(K_i) = \left[ X^{(0)}(K_i) - \frac{u}{a} \right] e^{-a(K_i-0)} + \sum_{i=1}^{n} \frac{u}{a} i=1,2,...,n
\]
After reduction, the expression of the model is

$$\bar{x}_i^{(0)}(K_{i+1}) = \frac{1}{\Delta K_{i+1}} (1 - e^{a\Delta K_{i+1}}) \left[ X^{(0)}(K_i) - \frac{u}{a} \right] e^{-a(K_{i+1} - K_i)}$$  \hspace{1cm} (10)

The summary parameters of the above model, $a$ and $u$, can be obtained through $[a, u]^T = (B^T B)^{-1} B^T Y_N$, where

$$B = \begin{bmatrix} -Z^{(1)}(K_2)1 \\ -Z^{(1)}(K_3)1 \\ \vdots \\ -Z^{(1)}(K_n)1 \end{bmatrix}$$

Where, $Z^{(1)}(K_i)$ is the background value of $X^{(1)}(t)$ in the interval $[K_{i-1}, K_i]$. The background value can be calculated by the two-point smoothing expression of the traditional GM (1, 1) model as follows:

$$Z^{(1)}(K_{i+1}) = \frac{x^{(1)}(K_{i+1}) + x^{(1)}(K_i)}{2}$$  \hspace{1cm} (11)

$$Y_N = \begin{bmatrix} X^{(0)}(K_2), X^{(0)}(K_3), \ldots, X^{(0)}(K_n) \end{bmatrix}^T$$  \hspace{1cm} (12)

3.3. Numerical analysis of the stiffness coefficients based on the grey prediction method

Based on the established grey prediction model, the predicted stiffness coefficients can be calculated in MATLAB, and the results are shown in Tables 4 and 5.

### Table 4. Prediction values of stiffness coefficients of Al6061.

|                      | Frequency (Hz) | 700 | 1100 | 1400 | 1700 |
|----------------------|----------------|-----|------|------|------|
| Prediction values of |                |     |      |      |      |
| unidirectional axial | Stiffness (N/m) | 1008030.408 | 2355132.003 | 6681198.181 | 17347219.72 |
| stiffness            |                |     |      |      |      |
| Prediction values of |                | 400 | 600  | 1100 | 1700 |
| coupling axial       | Stiffness (N/m) | 299542.3503 | 1520750.969 | 2750937.377 | 7327906.453 |
| stiffness            |                |     |      |      |      |
| Prediction values of |                | 500 | 700  | 1000 | 1500 |
| unidirectional radial| Stiffness (N/m) | 576793.1794 | 1190456.029 | 2653104.184 | 10561344.75 |
Table 5. Prediction values of stiffness coefficients of hard nylon.

| Prediction values of unidirectional radial stiffness | Frequency (Hz) | 400 | 700 | 1200 | 1600 |
|-----------------------------------------------------|----------------|-----|-----|------|------|
| Stiffness (N/m)                                     |                | 293135.219 | 489507.4992 | 2041140.825 | 12115988.25 |

The precision of the algorithm was tested using the parameters: model precision $p$ and posterior error ratio $C$. From the grey prediction model of Al6061, $p = 99.2\%$ and $C = 0.25$ for axial data with unidirectional testing. According to the experimental results, the model precision is at the first level, wherein $p = 99.7\%$ and $C = 0.049$ for axial data with two-dimensional coupling. In the grey prediction model of hard nylon, $p = 99.9\%$ and $C = 0.006$ for radial data with unidirectional testing, and the model precision is of the first level according to the experimental results was $p = 98.3\%$ and $C = 0.301$ for radial data using two-dimensional coupling testing. Again, the model precision was first level.

3.4. Analysis of stiffness variation under two-dimensional coupling vibration.

Under different conditions of high frequency stress loading, the dynamic characteristics of the toroidal rotor components, especially the stiffness coefficient will change. The key factors include the size of the stress value, the direction and frequency of stress, etc., especially after high-frequency fatigue loading, the grain structure of the rotor components will change. This article focused comparing the grey prediction results for unidirectional vibration with those of two-dimensional coupling vibrations, the influence of two-dimensional coupling vibration on stiffness can be determined.
The grey prediction model was fitted by the cftool toolbox in Matlab. By analyzing the stiffness formula, an exponential relationship was found between stiffness and frequency. Hence, \( y = ax^b + c \) was selected as the fitting equation. Thus, the fitting results for the aluminum alloy annular components are based on the radial uncoupling stiffness equation \( k = 1.26 \times 10^{-5}w^{3.747} + 4.928 \times 10^5 \), radial coupling stiffness equation \( k = 3.145 \times 10^{-13}w^{6.113} \), axial uncoupling stiffness equation \( k = 1.156 \times 10^{-8}w^{4.694} + 3.424 \times 10^5 \), and axial coupling stiffness equation \( k = 0.2525w^{2.302} + 4 \times 10^5 \).

![Figure 6. Fitted stiffness curves of the aluminum alloy annular components.](image)

After fitting the stiffness curves, the stiffness of Al6061 can be found, and shows a trend of increasing stiffness with increasing frequency for both unidirectional vibrations and two-dimensional coupling vibrations. Under low frequency vibration conditions, the deformation period of metal grains is much longer than the metal straining delay time, such that the internal stress within the metal grains has plenty of time to respond. Increasing the frequency of vibration, it was found that the deformation period of the metal grains gradually approaches, and may even be shorter than, the metal straining delay time. Therefore, the internal stress of the metal grains cannot be fully released.

It is impossible for the strain to reach the corresponding value at the same time of the stress. The deformation cannot establish the equilibrium state before the elastic wave is transmitted from the force application region and does not return from the object edge. Therefore, within the elastic limit loading strain behind the stress, but also need to overcome the static force between the grains, so the strain extremum of viscoelastic damping material under high-frequency simple harmonic load is smaller than the same amplitude of low-frequency excitation force, resulting in an increase of stiffness coefficient. At the same time, since the relative movements between the grain boundaries in the metal are affected by shear stress \( \tau = G \frac{dv}{dt} \), the rate of variation of the vibration velocity \( v \) also increases with increasing vibration frequency. Hence, the shear stress must be overcome in order to also increase. As a result, stiffness of the metal is improved under high frequency vibrations.

After coupling the excitation vibrations in two directions, the value of the stiffness coefficient of the metal shows a significant decline within the high frequency range. This can be explained by the change in relative movement between the grain boundaries in the crystalline metal due to the coupling vibration. Under the effects of coupling vibrations, the vibration of the crystal in one direction can promote relative movement of the grain boundary in the same direction to cause sliding of the grain boundary. Moreover, the interaction forces between grain boundaries in another direction are weakened, thus reducing the required minimum force to initiate mutual movement between the grain boundaries. Under the condition of mutual coupling in multiple directions, it becomes easier to produce relative sliding between crystals, thus leading to a decrease in the stiffness of the metal material under the vibration coupling condition.
It should be noted, the internal microstructure of the component varies constantly under the action of high frequency loading \([9]\). Photos of the metallographic microstructure of the Al6061 test object used in the experiment are shown in figure 7.

![Figure 7](image_url)

**Figure 7.** Microscopic images of the metallographic structure the Al6061 test object (40 mm external diameter; 2.5 mm thickness) used in the experiment. (a) Before experiment (b) After experiment.

From the comparative analysis of microstructures (Figure 7), it can be seen that prior to testing, the grains presented a clearly zonal arrangement with an average grain diameter of approximately 10.72 µm, in which black dots correspond to the strengthening phase of Mg\(_2\)Si (with a diameter of 5.14 µm), as shown in Figure 7 (a). After testing, the resultant microstructure is composed of equiaxed crystals, and grains have grown to approximately 19.31 µm due to thermal effects, and the grain diameter of the strengthening phase of Mg\(_2\)Si is 5.26 µm, as shown in Figure 7 (b). During the vibration process, induced obvious grain changes and grain boundary migration of the continuous shearing force.

Furthermore, in the experimental process some friction effects can be observed, as well as convective heat transfer between the shaft sleeve and test components, which further promote grain changes and grain boundary migration activities. Moreover, free grains can rotate by themselves, which leads to a constantly changing direction of thermal diffusion. Thus, under the comprehensive effects of various factors, the direction of grain growth varies constantly, and grains are gradually transformed into equiaxed grains. To verify the above conclusions, the thickness of the test object was increased to 4.5 mm, and the experimental results were found to be similar to those obtained for a thickness of 2.5 mm.

![Figure 8](image_url)

**Figure 8.** Microscopic images of the metallographic structure the Al6061 test object (40 mm external diameter; 4.5 mm thickness) used in the experiment. (a) Before experiment (b) After experiment.
Finally, if we change the material of the test object to hard nylon with a 40 mm external diameter and 4.5 mm thickness, changes in microstructure before and after experiment present a similar trend.

![Microscopic images of the metallographic structure the hard nylon test object (40 mm external diameter; 4.5 mm thickness) used in the experiment.](image)

**Figure 9.** Microscopic images of the metallographic structure the hard nylon test object (40 mm external diameter; 4.5 mm thickness) used in the experiment.

### 4. Conclusions
In this paper, a system for testing the dynamic characteristics of rotor components was designed based on the BERM method, and the relationships between the different parameters (stiffness value, damping coefficient, and loss factor) of various damping materials and frequency was investigated. The stiffness coefficient of typical rotor parts was found to increase up to a frequency of 2000 Hz, and demonstrated typical nonlinear characteristics. If the rotor components suffered synchronal action from simultaneous axial and radial loading in two directions, the stiffness coefficient values were affected, and presented a decreasing trend. The main reasons can be attributed to the influence of the operational mode and microstructure of the internal grains. Based on the above conclusions, optimization can be achieved in the design of rotor components, and reasonable operating parameters can be set, thus enhancing the reliability of the system as a whole.

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