BARYON MASSES AND PION–NUCLEON
σ–TERM
TO SECOND ORDER IN THE QUARK
MASSES

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Abstract

We analyze the octet baryon masses and the pion–nucleon σ–term in the framework of heavy baryon chiral perturbation theory. In contrast to previous investigations, we include all terms up-to-and-including quadratic order in the light quark masses. The pertinent low–energy constants are fixed from resonance exchange. This leaves as the only free parameter the baryon mass in the chiral limit, $\hat{m}$. We find $\hat{m} = 749\pm125$ MeV together with $\sigma_{\pi N}(0) = 48\pm10$ MeV. We discuss various implications of these results.

1 Introduction

The analysis of the octet baryon masses in the framework of chiral perturbation theory already has a long history, see e.g. [1–12]. In this paper, we present the results of a first calculation including all terms of order $O(m_q^2)$, where $m_q$ is a generic symbol for any one of the light quark masses $m_u,d,s$. We work in the isospin limit $m_u = m_d$ and neglect the electromagnetic corrections. Previous investigations only considered mostly the so–called computable corrections of order $m_q^2$ or included some of the finite terms at this order. This, however, contradicts the spirit of chiral perturbation theory (CHPT) in that all terms
at a given order have to be retained, see e.g. [13–15]. In general, the quark mass expansion of the octet baryon masses takes the form

\[ m = \hat{m} + \sum_q B_q m_q + \sum_q C_q m_q^{3/2} + \sum_q D_q m_q^2 + \ldots \] (1)

modulo logs. Here, \( \hat{m} \) is the mass in the chiral limit of vanishing quark masses and the coefficients \( B_q, C_q, D_q \) are state-dependent. Furthermore, they include contributions proportional to some low-energy constants which appear beyond leading order in the effective Lagrangian. In this letter, we evaluate these coefficients for the octet baryons \( N, \Lambda, \Sigma \) and \( \Xi \). In addition, we also calculate the pion–nucleon \( \sigma \)-term which is intimately related to the quark mass expansion of the baryon masses [3,7,16]. For some comprehensive reviews, see e.g. [17–20].

2 Effective Lagrangian

To perform the calculations, we make use of the effective meson–baryon Lagrangian. Our notation is identical to the one used in [7] and we discuss here only the new terms. Denoting by \( \phi \) the pseudoscalar Goldstone fields (\( \pi, K, \eta \)) and by \( B \) the baryon octet, the effective Lagrangian takes the form

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}^{(1)}_{\phi B} + \mathcal{L}^{(2)}_{\phi B} + \mathcal{L}^{(4)}_{\phi B} + \mathcal{L}^{(2)}_\phi + \mathcal{L}^{(4)}_\phi \] (2)

where the chiral dimension \( (i) \) counts the number of derivatives and/or meson mass insertions. The baryons are treated in the extreme non–relativistic limit [21,22], i.e. they are characterized by a four–velocity \( v_\mu \). In this approach, there is a one–to–one correspondence between the expansion in small momenta and quark masses and the expansion in Goldstone boson loops, i.e. a consistent power counting scheme emerges. The form of the lowest order meson–baryon Lagrangian is standard, see e.g. [7], and the meson Lagrangian is given in [15]. The dimension two meson–baryon Lagrangian can be written as (we only enumerate the terms which contribute)

\[ \mathcal{L}^{(2)}_{\phi B} = \mathcal{L}^{(2,\text{stand})}_{\phi B} + \sum_{i=1}^{10} b_i O_{i}^{(2)} , \] (3)

with the \( O_{i}^{(2)} \) monomials in the fields of chiral dimension two. Typical forms are \( \text{Tr}(\bar{B}[u_\mu, [u^\mu, B]]) \), \( \text{Tr}(\bar{B}(v \cdot u, [v \cdot u, B])) \) or \( \bar{B}B\text{Tr}(u_\mu u^\mu) \), with \( u_\mu = iu^\dagger \partial_\mu U u^\dagger \), \( u = \sqrt{U} \) and \( U = \exp(i\phi/F_P) \) collects the pseudoscalars. The form of \( \mathcal{L}^{(2,\text{stand})}_{\phi B} \)
is [7],
\[ L^{(2,\text{stand})}_{\phi B} = b_D \text{Tr}(\bar{B}\{\chi, B\}) + b_F \text{Tr}(\bar{B}[\chi, B]) + b_0 \text{Tr}(\bar{B}B) \text{Tr}(\chi) , \] (4)
i.e. it contains three low–energy constants and \( \chi = u^\dagger \chi u^\dagger + u \chi^\dagger u \) is proportional to the quark mass matrix \( \mathcal{M} = \text{diag}(m_u, m_d, m_s) \) since \( \chi = 2B\mathcal{M} \). Here, \( B = -\langle 0|\bar{q}q|0 \rangle / F_P^2 \) and \( F_P \) is the pseudoscalar decay constant. All low–energy constants in \( L^{(2)}_{\phi B} \) are finite. A subset of the \( b_i \) has been estimated in [23] by analyzing kaon–nucleon scattering data. The splitting of the dimension two meson–baryon Lagrangian in Eq.(3) is motivated by the fact that while the first three terms appear in tree and loop graphs, the latter ten only come in via loops. There are seven terms contributing at dimension four,
\[ L^{(4)}_{\phi B} = \sum_{i=1}^{7} d_i O_i^{(4)} , \] (5)
with typical forms of the \( O_i^{(4)} \) are \( \tilde{B}B\text{Tr}(\chi^2) \) or \( \text{Tr}(\bar{B}[\chi, [\chi, B]]) \). At this stage, we take \( m_u = m_d \neq m_s \). For \( m_u \neq m_d \), there is an additional term in \( L^{(4)}_{\phi B} \). The explicit expressions for the various terms in Eqs.(3,5) can be found in [24]. We point out that there are 20 a priori unknown constants. In addition, there are the \( F \) and \( D \) coupling constants (subject to the constraint \( F + D = g_A = 1.25 \)) from the lowest order Lagrangian \( L^{(1)}_{\phi B} \). What we have to calculate are all one–loop graphs with insertions from \( L^{(1,2)}_{\phi B} \) and tree graphs from \( L^{(2,4)}_{\phi B} \). We stress that we do not include the spin–3/2 decuplet in the effective field theory [6], but rather use these fields to estimate the pertinent low–energy constants (resonance saturation principle). We therefore strictly count in small quark masses and external momenta with no recourse to large \( N_c \) arguments.

3 Baryon masses and pion–nucleon \( \sigma \)–term

The form of the terms \( \sim m_q \) and \( \sim m_q^{3/2} \) for the baryon masses and \( \sigma_{\pi N}(0) \) is standard, we use here the same notation as Ref.[7]. The \( q^4 \) contribution to any octet baryon mass \( m_B \) takes the form
\[ m_B^{(4)} = \epsilon_{1,B}^P M_P^4 + \epsilon_{2,B}^P M_P^2 M_Q^2 \]
\[ + \epsilon_{3,B}^P M_P^4 \ln\left(\frac{M_P^2}{\lambda^2}\right) + \epsilon_{4,B}^P M_P^2 M_Q^2 \ln\left(\frac{M_P^2}{\lambda^2}\right) , \] (6)
with \( P, Q = \pi, K, \eta \) and \( \lambda \) the scale of dimensional regularization. The explicit form of the state–dependent prefactors \( \epsilon_{i,B} \) can be found in Ref.[24]. Notice
the appearance of mixed terms $\sim M_P^2 M_Q^2$ which were not considered in most existing investigations. The fourth order contribution to the baryon masses contains divergences proportional to (using dimensional regularization)

$$L = \frac{\lambda^{d-4}}{16\pi^2} \left\{ \frac{1}{d-4} - \frac{1}{2} \left[ \ln(4\pi) + 1 - \gamma_E \right] \right\}, \quad (7)$$

with $\gamma_E = 0.572215$. These are renormalized by an appropriate choice of the low–energy constants $d_i$,

$$d_i = d_i^\nu(\lambda) + \Gamma_i L. \quad (8)$$

In fact, all seven $d_i$ are divergent. The appearance of these divergences is in marked contrast to the $q^3$ calculation which is completely finite (in the heavy fermion approach). In what follows, we set $\lambda = 1$ GeV. Similarly, the fourth order contribution to the pion–nucleon $\sigma$–term can be written as

$$\sigma^{(4)}_{\pi N}(0) = M^2_\pi \left[ \epsilon_{1,\sigma}^P M^2_P + \epsilon_{2,\sigma}^P M^2_P \ln \left( \frac{M^2_P}{\lambda^2} \right) + \epsilon_{3,\sigma}^Q M^2_P \ln \left( \frac{M^2_Q}{\lambda^2} \right) \right], \quad (9)$$

with the $\epsilon_{i,\sigma}$ given in [24]. Here, the renormalization is somewhat more tricky. It can most efficiently performed in a basis of a linearly independent subset of the operators $dO_i^{(4)}/dm_q$, $q = (u, d, s)$, as detailed in Ref.[24]. A good check on the rather lengthy expressions for the nucleon mass and $\sigma_{\pi N}(0)$ is given by the Feynman–Hellmann theorem, $\dot{m}(\partial m_N/\partial \dot{m}) = \sigma_{\pi N}(0)$, with $\dot{m}$ the average light quark mass, $\dot{m} = (m_u + m_d)/2$. We remark here that in contrast to the order $q^3$ calculation, the shift to the Cheng–Dashen point, $\sigma_{\pi N}(2 M^2_\pi) - \sigma_{\pi N}(0)$, is no longer finite, i.e. there appear $t$–dependent divergences. We therefore do not consider this $\sigma$–term shift in what follows. We will also not discuss in detail the two kaon–nucleon $\sigma$–terms, $\sigma_{KN}^{(1,2)}(t)$, for similar reasons in this letter.

4 Resonance saturation

Clearly, we are not able to fix all the low–energy constants appearing in $\mathcal{L}_{\phi B}^{(2,4)}$ from data, even if we would resort to large $N_c$ arguments. We will therefore use the principle of resonance saturation to estimate these constants. This works very accurately in the meson sector [25–27]. In the baryon case, one has to account for excitations of meson ($R$) and baryon ($N^*$) resonances. One writes down the effective Lagrangian with these resonances chirally coupled to the Goldstones and the baryon octet, calculates the pertinent Feynman diagrams to the process under consideration and, finally, lets the resonance masses go to infinity (with fixed ratios of coupling constants to masses). This generates
higher order terms in the effective meson–baryon Lagrangian with coefficients expressed in terms of a few known resonance parameters. Symbolically, we can write

$$\tilde{\mathcal{L}}_{\text{eff}}[U, B, R, N^*] \rightarrow \mathcal{L}_{\text{eff}}[U, B].$$  \hspace{1cm} (10)$$

Here, there are two relevant contributions. One comes from the excitation of the spin-3/2 decuplet states and the second from t–channel scalar and vector meson excitations, cf. Fig. 1. It is important to stress that for the resonance contribution to the baryon masses, one has to involve Goldstone boson loops. This is different from the normal situation like e.g. in form factors or scattering processes. Consider first the decuplet contribution. We treat these field relativistically and only at the last stage let the mass become very large. The pertinent interaction Lagrangian between the spin–3/2 fields (denoted by ∆), the baryon octet and the Goldstones reads (suppressing SU(3) indices)

$$\mathcal{L}_{\Delta B\phi} = \frac{\mathcal{C}}{\sqrt{2}F_P} \bar{\Delta}^{\nu} T \Theta_{\nu \mu}(Z) B \partial^\mu \phi + \text{h.c.}, \hspace{1cm} (11)$$

with $T$ the standard $\frac{1}{2} \rightarrow \frac{3}{2}$ isospin transition operator, $F_P = 100$ MeV the (average) pseudoscalar decay constant and $\mathcal{C} = 1.5$ determined from the decays $\Delta \rightarrow B\pi$. The Dirac matrix operator $\Theta_{\mu \nu}(Z)$ is given by

$$\Theta_{\mu \nu}(Z) = g_{\mu \nu} - \left(Z + \frac{1}{2}\right)\gamma_{\mu} \gamma_{\nu}. \hspace{1cm} (12)$$

For the off–shell parameter $Z$, we use $Z = -0.3$ from the determination of the $\Delta$ contribution to the $\pi N$ scattering volume $a_{33}$ [28]. The tadpole graphs shown in Fig. 1b with an intermediate vector meson only contribute at order $q^5$. This is evident in the conventional vector field formulation. In the tensor field formulation, the vertex $\text{Tr}(V_{\mu \nu} u^\mu u^\nu)$ seems to lead to a contribution at
order $q^4$. However, in a tadpole graph one needs the index contraction $\mu = \nu$ and thus has no term at order $q^4$. Matters are different for the scalars. Denoting by $S$ and $S_1$ the scalar octet and singlet with $M_S \simeq M_{S_1} \simeq 1$ GeV, respectively, the lowest order coupling to the baryon octet reads

$$L^{(0)}_{SB} = D_S \text{Tr}(\bar{B}\{S, B\}) + F_S \text{Tr}(\bar{B}[S, B]) + D_{S_1} S_1 \text{Tr}(\bar{BB})$$

(13)

where the coupling constants $D_S$, $F_S$ and $D_{S_1}$ are of the order one.\(^3\) In fact, there are no empirical data to really pin them down. From the decay pattern of the low–lying baryons one can, however, estimate these numbers to be small. We will generously vary them between zero and one. For the couplings of the scalars to the Goldstones, we use the notation of [25] and the parameters determined therein. Putting pieces together, all low–energy constants are expressed via resonance parameters and the baryon masses take the form

$$m_B = \hat{m} + m_B^{(3)} + \lambda_B (\hat{m})^{-1} + \beta D_B + \delta D_B^4 + \epsilon D_B^6 + D_B^S,$$

$$\beta = -\frac{1}{96\pi^2} \frac{m^4}{m^3}, \quad \delta = -\frac{\beta}{4\hat{m}}, \quad \epsilon = -\frac{2}{3m_\Delta}(2Z^2 + Z - 1),$$

(14)

with $m_B^{(3)}$ the contribution of $\mathcal{O}(m^{3/2})$ and $m_\Delta = 1.455$ GeV is the average decuplet mass. Notice that it is convenient to lump the $\mathcal{O}(m)$ and the $\mathcal{O}(m^2)$ corrections together as it was done in Eq.(14) (in the $D_B^2$ and $D_B^S$). We have kept explicit the baryon mass in the chiral limit. Of course, in the fourth order terms it could be substituted by the corresponding physical values. At second order, however, we would get a state–dependent shift, see e.g. [29], and we thus prefer to work with $\hat{m}$. Alternative representations for the baryon masses are given in [24]. Let us briefly explain the origin of the various terms in Eq.(14). The $\lambda$ contributions are tadpoles with $1/m$ insertions from $\mathcal{L}^{(2)}_{\phi B}$. Similarly, the $\beta$ and $\epsilon$ terms stem from tadpole graphs with insertions proportional to the low–energy constants $b_0, D, F$ and $b_i$, respectively. Note, however, that in the resonance exchange approximation not all of the ten $b_i$ are contributing. The $\delta$ terms subsume the contributions from $\mathcal{L}^{(4)}_{\phi B}$, these are proportional to the low–energy constants $d_i$. Finally, the terms of the type $D_B^S$ are the scalar meson contributions to the mass. They amount to a constant, state–dependent shift. These consist of terms of the types $\sim M_\pi^2$, $\sim M_\pi^4$ ln $M_\pi^2$ and $\sim M_\eta^2$ ln $M_\eta^2$, see [24]. To be specific, we give the coefficients $D_\beta, \delta, \epsilon$ and $\lambda$ for the nucleon,

$$\lambda_N = \frac{1}{8\pi^2 F_P^2} \left\{ -\frac{3}{32}(D + F)^2 M_\pi^4 \ln \frac{M_\pi^2}{\lambda^2} - \frac{1}{96}(D - 3F)^2 M_\eta^4 \ln \frac{M_\eta^2}{\lambda^2} \right.\right.$$

$$- \frac{1}{16}(D^2 - 2DF + 3F^2) M_K^4 \ln \frac{M_K^2}{\lambda^2} \right\},$$

\(^3\) In what follows, we neglect the singlet field.
\begin{align*}
D_N^\beta &= D_N^{\beta,(2)} + D_N^{\beta,(4)} = \frac{C^2}{16\pi^2 F_P^3} \left\{ (-4\pi^2 F_P^2)(M_K^2 + 4M_F^2) \\
&+ \frac{27}{16} M_K^2 \ln \frac{M_K^2}{\lambda^2} + \frac{33}{24} M_F^4 \ln \frac{M_F^4}{\lambda^2} + \frac{7}{12} M_\eta^4 \ln \frac{M_\eta^4}{\lambda^2} \\
&+ \left[ -\frac{11}{18} M_K^2 + \frac{43}{144} M_F^2 + \frac{1}{8}(D - 3F)^2 \left( \frac{M_K^2}{4} + M_\eta^2 \right) \right] M_K^2 \ln \frac{M_K^2}{\lambda^2} \\
&+ \left[ \frac{9}{8}(D + F)^2 \left( \frac{M_K^2}{4} + M_F^2 \right) \right] M_F^2 \ln \frac{M_F^2}{\lambda^2} \\
&+ \left[ -\frac{3}{4}(D - F)^2 M_K^2 + \frac{1}{16}(223D^2 - 318DF + 351F^2)M_K^2 \right] M_K^2 \ln \frac{M_K^2}{\lambda^2} \right\} , \\
D_N^S &= \frac{C^2}{2F_P^2} \left\{ -\frac{415}{288} M_K^4 + \frac{83}{72} M_F^2 M_K^2 - \frac{31}{72} M_K^4 \right\} , \\
D_N^S &= \frac{C^2}{8\pi^2 F_P^2} \left\{ -\frac{1}{2} M_\eta^4 \ln \frac{M_\eta^4}{\lambda^2} - \frac{1}{8} M_F^4 \ln \frac{M_F^4}{\lambda^2} \right\} . \tag{15}
\end{align*}

The corresponding coefficients for the Λ, Σ and Ξ and also the \( D_B^S \) can be found in [24]. The tree contribution from \( \mathcal{L}_{\phi B}^{(2,\text{stand})} \) is subsumed in the \( D_B^\beta \).

The numerical values of the \( \lambda_B \), \( D_B^\beta \), \( D_B^\beta \), \( D_B^S \) and \( D_B^S \) are given in Table 1 (using \( F = 0.5 \), \( D = 0.75 \) and \( c_{d,m} \) from [25]). We see that the dominant terms at \( \mathcal{O}(m_\eta^2) \) are indeed the tadpole graphs with an insertion from \( \mathcal{L}_{\phi B}^{(2,\text{stand})} \) (this holds for the masses but not for \( \sigma_{\pi N}(0) \)). It is also instructive to compare the values we find from resonance exchange with the ones previously determined from KN scattering data. We have transformed the results of Ref.[23] into our notation. As can be seen from Table 2, most (but not all) coefficients agree in sign and magnitude. Note, however, that this is only a subset of the coefficients considered in this work. The full list will be given in [24]. We remark that the procedure used in [23] involves the summation of arbitrary high orders via a Lippmann–Schwinger equation and is thus afflicted with some uncertainty not controlled within CHPT.

| B | \( \lambda_B \) [GeV^2] | \( D_B^\beta \) [GeV^2] | \( D_B^\beta \) [GeV^2] | \( D_B^\beta \) [GeV^2] | \( D_B^S \) [GeV] |
|---|---|---|---|---|---|
| N | 0.0049 | −39.875 | −2.3617 | 0.0321 | −0.017 \( D_S + 0.061 F_S \) |
| Λ | 0.0179 | −60.044 | 1.0831 | 0.0617 | −0.047 \( D_S - 0.010 F_S \) |
| Σ | 0.0144 | −132.90 | −11.273 | 0.1422 | +0.051 \( D_S + 0.010 F_S \) |
| Ξ | 0.0216 | −126.46 | −5.4799 | 0.1324 | −0.037 \( D_S - 0.082 F_S \) |

Table 1
Numerical values of the state–dependent coefficients in Eq.(14). The \( D_B^\beta \) are dimensionless. The \( D_B^S \) are for \( c_d, c_m \) from [25] and \( M_S = 1 \) GeV.
Table 2
Some low–energy constants from $\mathcal{L}^{(2)}_{\phi B}$ in GeV$^{-1}$. In the first row, only the decuplet contribution is given. In the second row, scalar meson exchange is added.

|                | $b_1$  | $b_2$  | $b_3$  | $b_8$  | $b_0$  | $b_F$  | $b_D$  |
|----------------|--------|--------|--------|--------|--------|--------|--------|
| Reso. ($D_S = F_S = 0$) | 0.084  | -0.144 | 0.108  | -0.216 | -0.738 | -0.264 | 0.317  |
| Reso. ($D_S = F_S = 1$) | 0.100  | -0.111 | 0.125  | -0.239 | -0.733 | -0.208 | 0.345  |
| Ref.[23]        | 0.044  | -0.145 | -0.054 | -0.165 | -0.493 | -0.213 | 0.066  |

5 Results and discussion

The only free parameter in the formula for the baryon masses, Eq.(14), is the mass of the baryons in the chiral limit, $\hat{m}$, since all low–energy constants are fixed in terms of resonance parameters. In particular, in contrast to Ref.[7], the parameters $b_0$, $b_D$ and $b_F$ are no longer free. Also, at quadratic order in the quark masses the ambiguity between $\hat{m}$ and $b_0$ is resolved, it is not necessary to involve any one of the $\sigma$–terms in the fitting procedure [5,7]. In fact, one can not find one single value of $\hat{m}$ to fit all four octet masses, $m_N = 0.9389$ GeV, $m_A = 1.1156$ GeV, $m_\Sigma = 1.1931$ GeV and $m_\Xi = 1.3181$ GeV exactly. We therefore fit these masses individually and average the corresponding values for $\hat{m}$. The contribution from scalar meson exchange only enters the uncertainties of the numbers given. This is justified since the numerical values for the couplings $F_S$ and $D_S$ are supposedly small. A more thorough discussion on this point can be found in [24]. We have $\hat{m}_N = 711$ MeV, $\hat{m}_A = 679$ MeV, $\hat{m}_\Sigma = 877$ MeV, $\hat{m}_\Xi = 728$ MeV, with the following average

$$\hat{m} = 749 \pm 125 \text{ MeV}.$$  \hspace{1cm} (16)

This number is compatible with the one found in the analysis of the pion–nucleon $\sigma$–term, where approximately 130 MeV to the nucleon mass were attributed to the strange matrix element $m_s \langle p | \bar{s}s | p \rangle$ (with a sizeable uncertainty) [30]. The spread of the various values is a good measure of the uncertainties related to this complete $q^4$ calculation. Let us take a closer look at the quark mass expansion of the nucleon mass, in the notation of Eq.(1),

$$m_N = 711 + 202 - 272 + 298 \text{ MeV} = 939 \text{ MeV}.$$  \hspace{1cm} (17)

This looks similar for the other octet baryons. We conclude that the quark mass corrections of order $m_q$, $m_q^{3/2}$ and $m_q^2$ are all of the same size.\footnote{Note that for certain values of the scalar couplings, the fourth order term $m_N^{(4)}$ can be significantly smaller. However, the nucleon mass is much more sensitive to...}
it was argued that only the leading non–analytic corrections (LNAC) $\sim m_q^{3/2}$ are large and that further terms like the ones $\sim m_q^2$ are modestly small, of the order of 100 MeV. This would amount to an expansion in $M_K^2/(4\pi F_P)^2 \sim 1/6$ with a large leading term. This expectation is not borne out by our results, the next corrections are as large as the LNACs. These findings agree with the meson cloud model calculation of Gasser [3]. A last remark about the baryon masses concerns the deviation from the Gell-Mann-Okubo relation, $\Delta_{\text{GMO}} = (3m_\Lambda + m_\Sigma - 2m_N - 2m_\Xi)/4$ which empirically is about 6.5 MeV. We find $\Delta_{\text{GMO}} = 31$ MeV, which is larger in magnitude than the value found in [5,6]. We remark that in our case the decuplet contribution is contained in the $m_q^2$ contributions and not in the $m_q^{3/2}$ as in [5]. Therefore, in our case, $\Delta_{\text{GMO}}$ is dominated by the $m_q^2$ piece. The sizeable uncertainty in the chiral limit masses, Eq.(16), does not allow for a very accurate statement about this very small quantity. It is also very sensitive to the scalar couplings. To get a better handle on this issue, one either has to be able to fix all pertinent low–energy constants at order $m_q^2$ from data or improve upon the resonance saturation estimate used here by including e.g. the mass splitting within the decuplet and the SU(3) breaking of the decuplet-octet-meson couplings. A better understanding of this topic is, of course, at the heart of the determination of the quark mass ratio $R = (m_d - m_u)/(m_s - \hat{m})$ from the baryon masses (once the electromagnetic corrections have been included).

The pion–nucleon $\sigma$–term is completely fixed. Using for $\hat{m}$ its average, we find (no scalar resonance contribution)

$$\sigma_{\pi N}(0) = 48 \text{ MeV},$$

with an uncertainty of about $\pm 10$ MeV due to the spread in $\hat{m}$. This number compares favourably with the one found in Ref.[30], $\sigma_{\pi N}(0) = 44 \pm 9$ MeV. We stress that this result reflects a very non–trivial consistency for the complete calculation to quadratic order in the quark masses using the resonance saturation principle in the scalar sector. The additional contribution from the scalar meson exchange is accounted for in the $\pm 10$ MeV uncertainty. To be specific, we have $\sigma_{\pi N}(0) = (48 + D_S \cdot 0.5 - F_S \cdot 2.0)$ MeV. It is furthermore instructive to disentangle the various contributions to $\sigma_{\pi N}(0)$ of order $q^2$, $q^3$ and $q^4$, respectively,

$$\sigma_{\pi N}(0) = 54 - 33 + 27 \text{ MeV} = 48 \text{ MeV},$$

which shows a moderate convergence, i.e. the terms of increasing order become successively smaller. Still, the $q^4$ contribution is important. Also, at this order no $\pi\pi$ rescattering effects are included. We notice that using the values for the scalar contribution than the other octet masses.
$b_{0,D,F}$ and $b_i$ as determined in Ref.[23] leads to a much increased fourth order contribution.

6 Summary and outlook

In this paper, we have used heavy baryon chiral perturbation theory to calculate the octet baryon masses to quadratic order in the quark masses, including 20 local operators with unknown coefficients. These low–energy constants were fixed by resonance exchange. The dominant contribution comes indeed from the excitation of the spin–3/2 decuplet fields. Tadpole graphs with scalar meson exchange only lead to small corrections. This left us with one free parameter, the baryon mass in the chiral limit, which could be determined within 18% accuracy, and is compatible with the value inferred for the strange matrix–element $m_s \langle p | \bar{s}s | p \rangle$ in Ref.[30]. Furthermore, the pion–nucleon $\sigma$–term comes out surprisingly close to its empirical value, $\sigma_{\pi N}(0) = 48$ MeV, with an uncertainty of about $\pm 10$ MeV. This first exploratory $q^4$ study of the three flavor scalar sector of baryon CHPT points towards a significant improvement compared to previous investigations which were mostly confined to so–called “computable” corrections and/or fitted a few of the pertinent low–energy constants. However, the calculation is not yet accurate enough to determine the quark mass ratio $R$ reliably from the octet masses. Furthermore, we did not address the kaon–nucleon $\sigma$–terms, the corresponding shifts to the pertinent Cheng-Dashen points together with the strangeness content of the proton here. We will come back to these topics in Ref.[24].

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