Continuity from neutron matter to color-superconducting quark matter with $^3P_2$ superfluidity

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Abstract

I clarify how the concept of quark-hadron continuity, which was previously considered in the context of the asymptotic color-flavor locked phase with ideal three-flavor symmetry, is applied to two-flavor matter. Our observation is that neutron star matter can continuously be connected to two-flavor color-superconducting (2SC) phase with an additional condensate of $d$-quarks in the $^3P_2$ channel. I discuss here two aspects of this novel phase. First, I introduce the notion of continuity based on the patterns of symmetry breaking and the corresponding order parameters, and then explain qualitatively the physics mechanism of $d$-quark $^3P_2$ pairing in analogy to nuclear physics. Our finding serves as the theoretical underpinning for the phenomenological construction of the equation of state in the neutron star environment.

Keywords: Dense QCD matter, Quark-hadron continuity, Neutron star, Color superconductivity, Equation of state

1. Introduction

Whether there is a phase boundary between hadronic matter and quark matter at high densities has been a long-standing problem since the advent of QCD. I shall present here a possibility of continuous connection between hadronic matter and color-superconducting quark matter, based on our recent work [1]. The idea of quark-hadron continuity is ascribed to the identical symmetry breaking patterns and the low-lying excitations in both quark and hadronic phase. In three-flavor symmetric case, where color-flavor locking (CFL) occurs, continuity is established [2]. By contrast, in two-flavor case, the concept of continuity between nuclear matter and two-flavor color superconductor (2SC) is generally thought to be invalidated due to the apparent difference in the global symmetries realized in the both phases. In this contribution, I shall concentrate on the two-flavor case and discuss that the continuity from $^3P_2$ neutron superfluid to two-flavor quark matter is still possible with an additional down quark condensate in $^3P_2$ channel [1].

It is worth mentioning that quark-hadron continuity is significant in cold and dense QCD matter, especially when we consider the phase structure and the bulk properties. A Ginzburg-Landau analysis shows that matter at sufficiently low temperature allows for a critical point in the phase diagram, and goes through a smooth crossover from the hadronic to the quark phase with increasing density (see, e.g., Ref. [3]). The bulk properties of dense matter is characterized by a quantity such as the equation of state (EoS). Now quark-hadron continuity is the baseline for the phenomenological construction of EoS [4].
2. Quark-hadron continuity — aspects of symmetry and order parameter

In the conventional color superconductor, a diquark condensate in $^1S_0$ channel is formed

$$\langle \hat{q}_{\alpha A} C \gamma^5 \hat{q}_{BB} \rangle \propto \epsilon_{\alpha \beta \gamma} \epsilon_{ABC} (\hat{\Phi}^C) ,$$

(1)

where the charge conjugation matrix $C \equiv i \gamma^0 \gamma^2$ is inserted to form a Lorentz scalar. Greek ($\alpha, \beta, \ldots$) and capital ($A, B, \ldots$) indices represent color and flavor, respectively. The color-superconducting phase can be thought of as a Higgs phase of QCD. Diquarks play here the same role as the Higgs boson does in the electro-weak sector of the standard model. It has previously been shown in lattice gauge theories that the confinement phase and the Higgs phase are connected without a phase boundary [5]. Owing to this fact, we cannot distinguish between the chiral (confinement) phase and the color superconductor (Higgs) phase as long as the global symmetries are the same in both phases; it leads to the concept of quark-hadron continuity.

The symmetry of QCD comprises local gauge symmetry, chiral symmetry and baryon number symmetry: $G_{\text{QCD}} = [SU(3)_C] \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B$. Neglecting the strange ($s$) quark mass, the chiral part of $G_{\text{QCD}}$ can be ideally treated as $N_f = 3$. In reality, however, $s$ is much heavier than the up ($u$) and the down ($d$) quarks ($m_s / m_u, m_d \sim 30$), so it is more natural to consider isospin-symmetric $N_f = 2$ systems.

Here, I first review the conventional continuity in $N_f = 3$ from the viewpoint of residual symmetry in the color-superconducting phase and the corresponding order parameters. Then I will show our novel analysis in $N_f = 2$ case, which also allows for continuity with a new element of $^3P_2$ $d$-quark condensate.

2.1. $N_f = 3$ case

In this case, color-flavor locking occurs. The CFL ansatz for the diquark condensate Eq. (1) reads $\langle \hat{\Phi}^{\alpha A} \rangle = \delta^{\alpha A} \Delta_{\text{CFL}}$. The residual symmetry in the CFL phase is $G_{\text{CFL}} = SU(3)_{C-L+R}$. Thus, the pattern of symmetry breaking in the CFL phase is $G_{\text{QCD}} \rightarrow G_{\text{CFL}}$. The residual global symmetry in the CFL phase $G_{\text{CFL}}$ is the same as that of the hadronic phase, where the chiral and the baryon number symmetries are spontaneously broken. Thus, there exists quark-hadron continuity for $N_f = 3$ case.

As previously proven in Ref. [6], local gauge symmetry cannot be broken spontaneously. It means that the Higgs mechanism alone, which is fictitious breaking of gauge symmetry, cannot be captured by any gauge-invariant order parameter. Here, however, global symmetry breaking occurs simultaneously in $G_{\text{QCD}} \rightarrow G_{\text{CFL}}$, thus this global sector can still be captured by gauge-invariant order parameters. We can construct such order parameters by saturating gauge indices of diquark operator $\langle \hat{\Phi}^{\alpha A} \rangle$ in two ways, each of which captures the breaking of the chiral and superfluid sector of the QCD symmetry $G_{\text{QCD}}$ [7].

\[
\text{Chiral: } \quad \mathcal{M} = \delta^{\alpha A} \delta^{\beta B} \epsilon_{\alpha \beta \gamma} \epsilon_{ABC} (\hat{q}^\gamma q_\delta, \hat{q}^\delta q_\gamma) \propto (\hat{q}^\gamma q_\delta, \hat{q}^\delta q_\gamma),
\]

(2)

\[
\text{Superfluid: } \quad \Upsilon = \epsilon_{\alpha \beta \gamma} \epsilon_{\alpha \beta \gamma} \epsilon_{\gamma \delta \epsilon} \epsilon_{\delta \epsilon \mu} (\hat{q}_\delta q_\mu, \hat{q}_\epsilon q_\gamma) \propto (\epsilon_{\alpha \beta \gamma} \epsilon_{\gamma \delta \epsilon} \epsilon_{\delta \epsilon \mu} (\hat{q}_\delta q_\mu, \hat{q}_\epsilon q_\gamma)).
\]

(3)

Superfluid order parameter $\Upsilon$ takes the similar expectation value in both the quark and the hadronic phases because of quark-hadron continuity. In the hadronic phase, as it is obvious from the RHS of Eq. (3) above, it can be interpreted as $\Upsilon \propto \Lambda \Lambda$. This is the order parameter for $\Lambda$-superfluidity.

2.2. $N_f = 2$ case

The 2SC ansatz for the diquark condensate (1) reads $\langle \hat{\Phi}^{\alpha A} \rangle = \delta^{\alpha A} \delta^{\beta B} \Delta_{\text{2SC}} \equiv \Phi^{\alpha A}_{\text{2SC}}$, where we assumed the unitary gauge-fixing. The residual symmetry in the 2SC phase is $G_{\text{2SC}} = [SU(2)_C] \times SU(2)_L \times SU(2)_R \times U(1)_B$, which is apparently different from the hadronic phase. However, apart from the 2SC condensate $\Phi^{\alpha A}_{\text{2SC}}$, it is still possible to consider an additional two-flavor diquark condensate in order that quark-hadron continuity is maintained. To find a new element, here I turn the three-flavor argument the other way round, focusing on the superfluid aspect. We describe neutrons by a quark-diquark structure
n = e^{i\theta}(u_3^\alpha C\gamma^5 d_\beta) d_\gamma. Assuming quark-hadron continuity, which will be justified \textit{a posteriori}, we start out with the order parameter for $^3P_2$ neutron superfluidity:

$$\Psi_{nn} \equiv n^\dagger C\gamma^5 n \approx e^{i\theta(\vec{u}_3^\alpha C\gamma^5 d_\beta)}(u_3^\alpha C\gamma_5 d_\beta)(d_\gamma^\dagger C\gamma^5 d_\gamma).$$

The first two terms in parentheses are identical to the 2SC condensate $\Phi^\dagger_{2SC}$ in Eq. (4), while the last term $d_\gamma^\dagger C\gamma^5 d_\gamma$ is the novel element here. If we take the expectation value of $\Psi_{nn}$ in the mean-field approximation, $\Psi_{nn}$ takes the value

$$\langle \Psi_{nn} \rangle \approx \Phi^\dagger_{2SC} \Phi_{2SC} (d_\gamma^\dagger C\gamma^5 d_\gamma),$$

where we have neglected the crystalline condensation. We call this phase 2SC+$\langle dd \rangle$ phase. The residual symmetry in the 2SC+$\langle dd \rangle$ phase is the same as neutron matter, hence the continuity holds. At first glance, $d_\gamma^\dagger C\gamma^5 d_\gamma$ condensation seems not to occur because $d$-quarks are in color 6 channel, which is known to be repulsive in one gluon exchange (OGE) potential. Even though the short-range interaction via OGE is repulsive, $^3P_2$ pairing is still possible through the spin-orbit interaction as I will explain shortly.

3. Justifying the 2SC+$\langle dd \rangle$ phase

3.1. Dynamical mechanism favoring $d$-quark pairing in $^3P_2$ channel

A prototype example of the $^3P_2$ superfluid is realized for neutrons inside neutron stars. Neutrons undergo pairing in a $^1S_0$ state at $n_B < 0.5 \, n_0$ (with $n_0 = 0.16 \, \text{fm}^{-3}$, the normal nuclear density) and in the $^3P_2$ state at $n_B > n_0$. One can find $^1S_0$ and $^3P_2$ neutron superfluid in the inner crust and the outer core of neutron stars, respectively. This realization of $^3P_2$ superfluidity is based on the observed pattern of nucleon-nucleon (NN) scattering phase shifts [8, 9]. The phase shift of the $^1S_0$ partial wave changes sign from positive to negative with increasing energy of the two nucleons, indicating that the pairing interaction turns from attractive to repulsive with increasing Fermi energy. Consequently, pairing in the $^1S_0$ channel is disfavored at high densities and taken over by pairing in the $^3P_2$ channel. This property is attributed to the short-repulsive core in the singlet $S$-wave and the significant attraction selectively generated by the spin-orbit interaction in the triplet $P$-wave with total angular momentum $J = 2$. The selective attraction for $J = 2$ channel is generated through the factor $-\hat{L} \cdot \hat{S} = -[J(J + 1) - L(L + 1) - S(S + 1)]/2$ in the spin-orbit potential, which are +2, +1 and −1 in $^3P_0$, $^3P_1$ and $^3P_2$ states, respectively. With an extra minus sign in the spin-orbit potential, there is attraction only in $^3P_2$ channel. All other $S$- and $P$-wave NN phase shifts in isospin $I = 1$ channel are smaller or repulsive in matter dominated by neutrons.

The same mechanism applies in quark matter favoring $^3P_2$ pairing between $d$-quarks as specifically described in Ref. [1]. Short-range repulsion arises between $d$-quarks due to the repulsive nature of OGE in color 6 channel. Also, near the confinement region in the continuity picture, short-distance repulsive interaction between $d$-quarks can be thought of as emerging from quark-gluon exchange in a non-relativistic quark model picture. Spin-orbit force in a long range also arises between quarks. One can derive it employing Nambu–Jona-Lasinio (NJL) model with vector coupling with a sizable effect. This effect could also be understood as naturally emerging from the Fermi-Breit reduction of the OGE potential. Therefore, pairing between $d$-quarks in the $^1S_0$ channel is also disfavored and taken over by pairing in the $^3P_2$ channel, justifying our picture with 2SC+$\langle dd \rangle$ phase.

3.2. Relation with the dense matter EoS and neutron star phenomenology

In the context of previous mean-field calculations of color-superconducting quark matter in an NJL-type model, a four-fermion coupling term in the $^3P_2$ channel has so far been missing. It would then be instructive to see how the interaction in this channel could be enlarged through the coupling to the energy-momentum tensor. Considering the four-fermion coupling term in the $^3P_2$ diquark channel, i.e.,

$$\hat{I}_P = (\bar{\psi}_L \gamma^\dagger \gamma C\phi^\dagger)(\psi_L \gamma \gamma \phi).$$

Fierz rearranging this $\hat{I}_P$ term, one finds a term with a direct correspondence to the energy-momentum tensor in the fermionic sector, $T_{\mu\nu} = \bar{\psi} i\gamma^\mu \partial^\nu \psi$. For matter in equilibrium, $T_{\mu\nu} = \text{diag}[e, P, P, P]$, with the
energy density $\varepsilon$ and the pressure $P$ of fermionic matter. Extracting only the contribution relevant for the energy-momentum tensor, the tree-level expectation value of $\hat{I}_P$ reads

$$\langle \hat{I}_P \rangle \approx \frac{3}{4} P^2.$$  \hspace{1cm} (8)

It is evident from this expression that the $^3P_2$ diquark interaction couples to the pressure which is a macroscopic quantity. Even if the direct mixing between the quark-antiquark (hole) and the diquark sectors may not be large, the superfluid energy gap can be enhanced by the macroscopic expectation value of the energy-momentum tensor as given in Eq. (8).

If we regard $P$ here as a function of the energy density $\varepsilon$, it is nothing but the EoS $P = P(\varepsilon)$. Quark-hadron continuity scenario is now also relevant in the context of neutron stars, particularly for the phenenomenological construction of the dense matter EoS [4] as is evident from the direct correspondence in the expression of Eq. (8). The EoS is constructed within three-window modeling, where the boundary conditions are set by the nuclear EoS at $n_0 < 2n_B$ and the quark EoS at $n_B > 5n_0$. The intermediate region is interpolated with smooth function, whose physics background is quark-hadron continuity (see, e.g., Fig. 4 of Ref. [4] for an illustration of the three-window modeling). Our two-flavor scenario here gives more convincing baseline for the three-window modeling than the three-flavor scenario, as most of the nuclear EoS at $n_B \leq 2n_0$ is calculated for nucleon degrees of freedom. Conversely, recent attempts to extract the neutron star EoS directly from astrophysical observations, such as that utilizing deep neural network [10, 11], may provide a basis for judging the continuity hypothesis in the future.

4. Summary

In this work, we showed that neutron matter realized inside neutron stars are continuously connected to two-flavor color superconductor. We found that the quark phase counterpart of the neutron $^3P_2$ superfluidity is what we call $2SC+(dd)$ phase, which is a $2SC$ phase with $(dd)$ pairing in $^3P_2$ channel. The mechanism favoring this phase is two-fold: One is that the short-range repulsive force emerging from quark-gluon exchange, which can also be interpreted with OGE potential. The other is that the long-range attraction arising from the spin-orbit configuration favoring $J = 2$ in particular.

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