Bridging Hadronic CP-Violation to Twist-Three Distributions

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Abstract

The nucleon sigma term of the isoscalar and isovector quark chromo-magnetic dipole moments are essential inputs for the determination of the P, CP-odd pion-nucleon couplings induced by quark chromo-electric dipole moments. We demonstrate that the former can be mapped to the third moment of the nucleon twist-three chiral-odd distribution functions \( \{ e^q(x) \} \) which are in principle measurable in semi-inclusive deep inelastic scattering processes. We perform a survey on existing model calculations as well as experimental data on \( e^u(x) + e^d(x) \) and derive a predicted range for the isoscalar chromo-magnetic dipole moment sigma term.

1. Introduction

The precise determination of low-energy matrix elements of strongly-interacting bound states has been a long-standing problem in nuclear and particle physics. It is interesting for many reasons: on the one hand, it helps improving our current understanding of Quantum Chromodynamics (QCD) in its confinement region; on the other hand, it provides necessary theoretical inputs for searches of the Beyond Standard Model (BSM) physics via precision experiments in nuclear and atomic systems. These calculations are however very challenging due to the non-perturbative nature of QCD at the hadronic scale. Although in some cases lattice QCD offers convincing first-principle calculations to the problem, the technical complexities of such calculations may greatly vary depending on the details of the desired matrix elements.

An alternative approach to the problem is to relate the hadronic matrix elements of interest to experimental observables. An example of this kind is the series work by Jaffe and Ji in Refs. [1, 2] which pointed out that the nucleon tensor charges \( \delta q \), defined as:

\[
\langle PS| i\sigma^{\mu\nu}\gamma_5 q |PS\rangle = -2\delta q S^\mu,
\]

where \( \vec{S} \) is the spin vector of the nucleon, could be related to the first moment of the quark’s transversity distribution function \( h_1^q(x) \) as:

\[
\delta q = \int_{-1}^{1} dx h_1^q(x) = \int_0^1 dx \left( h_1^q(x) - h_1^\bar{q}(x) \right).
\]

The distribution \( h_1^q(x) \) probes the difference in probability for a quark to be polarized parallel or anti-parallel to a transversely-polarized hadron and can be measured in experiment. This observation stimulated a number of experiments to extract the tensor charges through semi-inclusive deep inelastic scattering (SIDIS), semi-inclusive \( e^+e^- \) annihilation and \( \pi^0/\eta \)-exclusive electroproduction [3, 4, 5, 6]. For instance, Ref. [3, 7] quoted the following numerical values for the proton tensor charges:

\[
\delta u = 0.413 \pm 0.133, \quad \delta d = -0.229 \pm 0.094
\]

at the scale \( Q^2 = 2.4 \text{ GeV}^2 \). Future experiments at SoLID are expected to further reduce the uncertainty by one order of magnitude [7]. The direct experimental determination of tensor charges is important...
in many aspects. Firstly, it provides an accuracy check to the corresponding calculations on lattice as well as low-energy QCD models. Secondly, tensor charges are important QCD inputs for the probe of BSM physics through precision experiments such as electric dipole moment (EDM) searches [8, 9] and nuclear beta decay [10] so the reduction of their theoretical errors allows for a more precise transcription of experimental bounds into constraints on BSM parameters.

In this paper we apply the idea above to the study of a different class of precision observables, namely the P and CP-odd pion-nucleon couplings \( \{ \hat{g}_I \} \) induced by the dimension-5 quark chromo-electric dipole moment (cEDM) operators. These couplings are interesting because in many cases they are the main contributors to EDMs of multi-nucleon systems [9]. Therefore, experimental limits on nuclear and atomic EDMs may set direct constraints on \( \{ \hat{g}_I \} \). If one could figure out the precise functional dependence of \( \hat{g}_I \) on the Wilson coefficients of the cEDM operators, then the limits on nuclear and atomic EDMs may be used to effectively constrain the CP-violating parameters in relevant BSM scenarios.

Chiral symmetry relates \( \{ \hat{g}_I \} \) to the nucleon mass shifts induced by both the quark masses and the P, CP-even quark chromo-magnetic dipole moment (cMDM) operators \([11, 12, 13, 14, 15, 16, 17, 18]\). These mass shifts can be expressed as nucleon matrix elements of the form \( \langle N | \bar{q}q | N \rangle \) and \( \langle N | \bar{q}q \cdot Gq | N \rangle \). The first matrix element is just the ordinary QCD sigma term which is one of the standard objects in lattice study. The second, however, is much complicated as it involves explicit gluon degrees of freedom (DOFs) in the operator. The corresponding lattice techniques for its evaluation is not yet well-established.

The first matrix element is just the ordinary QCD sigma term which is one of the standard objects in lattice study. The second, however, is much complicated as it involves explicit gluon degrees of freedom (DOFs) in the operator. The corresponding lattice techniques for its evaluation is not yet well-established. Therefore, we shall offer here an alternative point of view, namely to relate \( \langle N | \bar{q}q \cdot Gq | N \rangle \) to a special class of higher-twist distribution functions which can be probed in SIDIS experiments. This is interesting because it represents yet another nice interplay between the precision frontier in BSM searches and studies of hadronic and nuclear structures. It points out another direction in the research of hadronic CP violation apart from the conventional lattice/low-energy QCD models approach and provides extra motivation for the future improvement of experimental measurement of higher-twist observables.

This paper is arranged as follows: in Section 2 we briefly review the main results in Ref. [17], namely the theoretical formulation of the P, CP-odd pion-nucleon couplings \( \{ \hat{g}_I \} \) in the presence of cEDM and cMDM operators, in particular how \( \hat{g}_I \) could be written in terms of the cMDM matrix elements; in Section 3 we introduce the chiral-odd twist-three distribution function \( e^3(x) \) and summarize its properties; Section 4 represents our central result, where we construct a mapping between the cMDM sigma terms (and hence \( \hat{g}_I \)) and the third Mellin moment of \( e^3(x) \); in Section 5 and 6 we analyze the existing model calculations and experimental results of \( e^3(x) \) and extract their implications on the isoscalar cMDM sigma term; in Section 7 we draw our final conclusions.

2. \( \hat{g}_I \) and the cMDM sigma terms

We start by considering a two-flavor QCD Lagrangian with the inclusion of a non-zero \( \bar{q} \theta q \) term as well as the dimension-5 cMDM and cEDM operators:

\[
\mathcal{L}_{\text{QCD}} = \bar{Q} \left( i \partial - \mathcal{M} \right) Q - \frac{g_s}{2} \bar{Q} \sigma^{\mu \nu} G_{\mu \nu} (\tilde{d}_{\text{CM}} + \tilde{d}_{\text{CE}} i \gamma_5) Q - \frac{1}{4} G_{\mu \nu} G^{\mu \nu} - \frac{\bar{q} \theta q}{32 \pi^2} G_{\mu \nu} \tilde{G}_{\mu \nu},
\]

with \( Q = (u, d)^T \) the isospin doublet quark field, \( G_{\mu \nu} \) the gluon field strength tensor, \( \bar{G}_{\mu \nu} \equiv (1/2) \varepsilon_{\mu \nu \alpha \beta} G^{\alpha \beta} \) its dual tensor, \( \mathcal{M} = \text{diag}(m_u, m_d) \) the current quark mass matrix, \( \bar{q} \theta q \) the QCD-\( \theta \) term, \( \tilde{d}_{\text{CM}} = \text{diag}(\tilde{c}_u, \tilde{c}_d) \) the cMDM couplings and \( \tilde{d}_{\text{CE}} = \text{diag}(\tilde{d}_u, \tilde{d}_d) \) the cEDM couplings. Sources of explicit chiral symmetry breaking (CSB) in the Lagrangian are the quark mass matrix \( \mathcal{M} \) and the chromo-dipole coupling matrices \( \tilde{d}_{\text{CM}}, \tilde{d}_{\text{CE}} \). The first step in the standard treatment of Eq. (4) is to remove the \( \theta \)-term through an anomalous U(1)\(_A\)-rotation of the quark field. On top of that, one also performs a non-anomalous SU(2)\(_A\)-rotation to align the vacuum, i.e. to remove terms linear to the neutral pion in the low-energy effective theory that renders the vacuum energy unbounded from below [19, 12, 20, 21, 22, 18]. After such treatment one obtains:

\[
\mathcal{L}_{\text{QCD}} = \bar{Q} i \partial Q - \bar{Q} R Q - \bar{Q} L M Q L - \bar{Q} R M Q R - \frac{g_s}{2} \bar{Q} R \sigma^{\mu \nu} G_{\mu \nu} \tilde{d}_{\text{CM}} Q R - \frac{g_s}{2} \bar{Q} L \sigma^{\mu \nu} G_{\mu \nu} \tilde{d}_{\text{CM}} Q L - \frac{1}{4} G_{\mu \nu} G^{\mu \nu}. \]
The sources of explicit CSB are now encoded in the complex quark mass matrix $\mathcal{M}$ and the complex chromo-dipole coupling matrix $\tilde{d}_C$:

$$
\mathcal{M} = \mathcal{M} + \imath m_s (\bar{\theta} - \bar{\theta}_{\text{ind}}) + r \tilde{d}_{CE} \\
\tilde{d}_C = \tilde{d}_{CM} - \imath d_{CE}
$$

where

$$
m_* = \frac{\bar{m} (1 - \varepsilon^2)}{2} \\
r = \frac{1}{2} \frac{\langle 0 | \bar{Q} g_s \sigma_{\mu\nu} G^{\mu\nu} Q | 0 \rangle}{\langle 0 | Q \bar{Q} | 0 \rangle} \\
\bar{\theta}_{\text{ind}} = r \text{Tr} \left[ \mathcal{M}^{-1} \tilde{d}_{CE} \right].
$$

Here we have introduced the average quark mass $\bar{m} = (m_u + m_d) / 2$ and the relative quark mass difference $\varepsilon = (m_d - m_u) / (2 \bar{m})$. Similarly we shall also define the isoscalar and isovector dipole moment coupling constants $\tilde{x}_{0,3} \equiv (\tilde{x}_u \pm \tilde{x}_d) / 2$ (for $x = c, d$) for the convenience of discussions later. The advantage of writing the complex quark mass matrix $\mathcal{M}$ in the form of Eq. (6) is that if the Peccei-Quinn (PQ) mechanism \[23\] is at work, then $\bar{\theta}$ relaxes to $\bar{\theta}_{\text{ind}}$, simplifying the expression of $\mathcal{M}$.

In a low-energy effective theory of hadrons, the presence of the CSB sources $\mathcal{M}$ and $\tilde{d}_C$ induces shifts of hadrons masses from their values in the chiral limit as well as extra hadron-hadron interaction terms. The terms relevant to our study in the effective Lagrangian of nucleon are:

$$
\mathcal{L}_N = - \Delta m_N \bar{N} N + \frac{\delta m_N}{2} \bar{N} \tau^3 N - \frac{g_0}{2 F_\pi} \bar{N} \tau \cdot \pi N - \frac{g_1}{2 F_\pi} \bar{N} \pi_0 N - \frac{g_2}{2 F_\pi} \bar{N} \tau^3 N + \ldots,
$$

where $N = (p, n)^T$ is the nucleon isospin doublet and $F_\pi \approx 92.4$ MeV is the pion decay constant. The quantities $\Delta m_N$ and $\delta m_N$ are the average nucleon mass shift and the nucleon mass splitting respectively while $\delta m_N$ is the P, CP-odd pion-nucleon coupling with isospin $I$. Now, since the nucleon mass shifts and $\{g_I\}$ are induced by the CP-even and CP-odd components of the same CSB source (either $\mathcal{M}$ or $\tilde{d}_C$ or both), they are related to each other through chiral symmetry. This feature has been pointed out in many literatures \[11, 12, 13, 14, 15\] and their matching relations are constructed based on either the partially-conserved axial current (PCAC) relation or the leading order (LO) chiral Lagrangian. The problem is that such matching relations could be violated quite severely after the inclusion of next-to-leading order (NLO) and next-to-next-to leading order (NNLO) corrections. Ref. \[16\] shows that, when the QCD $\theta$-term is the only source of CP-violation, the matching relations derived at LO is preserved by one-loop corrections. Even though they are still broken at NNLO by counterterms, the relative corrections are expected to be small. Ref. \[18\] extended the study to include the CP-violation due to cEDM and left-right four quark operator (LR4Q) and found that, unlike the case of $\theta$-term, the matching relations induced by such operators are spoiled rather severely by one-loop corrections.

The above-mentioned issue is partially alleviated for the case of cMDM/cEDM following the work from Ref. \[17\] that suggests the following form of matching (PQ symmetry is assumed for simplicity):

$$
g_0 = \tilde{d}_0 \left( \sigma_C^3 + \frac{r \sigma^3}{\bar{m}} \right), \\
g_1 = -2 \tilde{d}_3 \left( \sigma_C^0 - \frac{r \sigma^0}{\bar{m}} \right),
$$

($\tilde{g}_2$ is always of higher order and is neglected) where we have defined the isoscalar and isovector QCD
sigma terms $\sigma^{0,3}$ and cDM sigma terms $\sigma^{0,3}_C$ of a proton state $|P\rangle$ (with momentum $P^\mu$) as follows$^1$

$$
\begin{align*}
\sigma^0 &= \left. \frac{\partial \Delta m_N}{\partial \bar{m}} \right|_{\bar{m}} = \frac{\bar{m}}{2m_N} \langle P | \bar{Q} | P \rangle \\
\sigma^3 &= \left. \frac{\partial \delta m_N}{\partial \bar{m}} \right|_{\bar{m}} = \frac{\bar{m}}{m_N} \langle P | \bar{Q} \tau^3 | P \rangle \\
\sigma^0_C &= \left. \frac{\partial \Delta m_N}{\partial c_0} \right|_{c_0=0} = \frac{1}{4m_N} \langle P | \bar{Q} g_\sigma \sigma^{\mu\nu} G^{\mu\nu} \tau^3 | P \rangle \\
\sigma^3_C &= \left. \frac{\partial \delta m_N}{\partial c_3} \right|_{c_3=0} = -\frac{1}{2m_N} \langle P | \bar{Q} g_\sigma \sigma^{\mu\nu} G^{\mu\nu} \tau^3 | P \rangle
\end{align*}
$$

where $m_N$ is the nucleon mass. The good thing about the relations in Eq. (9) is that they are preserved exactly by one-loop corrections. The matchings are violated by $O(p^4)$ counterterms but the amount of violation is in general not larger than 10%.

It is instructive to write Eq. (9) as

$$
\tilde{g}_I = \tilde{g}_I |_{\text{dir}} + \tilde{g}_I |_{\text{vac}},
$$

i.e. to split $\tilde{g}_I$ into the sum of “direct” and “vacuum alignment” contribution, which correspond to the first and second term at the right side of Eq. (9) respectively; the direct contribution depends on $\sigma^{0,3}$ while the vacuum alignment contribution depends on $\sigma^{0,3}$ as well as the vacuum condensate ratio $r$ and the current quark masses. The advantage of such splitting is that it separates the unknown quantities from the (relatively) known ones. Parameters in $\tilde{g}_I |_{\text{vac}}$ can in general be obtained from current results of lattice studies as well as from QCD sum rule analysis. Techniques to calculate QCD sigma terms on lattice are quite well-established. Simulations with $N_f = 2$ provide numerical estimations for the isoscalar and isovector quark mass parameters and sigma terms: $\bar{m} \approx 3.6$ MeV, $\varepsilon \approx 0.33$, $\sigma^0 \approx 37$ MeV and $\sigma^3 \approx 2.9$ MeV $^{21,25,26}$ which are also confirmed by more recent lattice calculations $^{27}$. The isoscalar sigma term $\sigma^0$ may also be extracted from low-energy $\pi N$-scattering, but the outcomes are in general much larger than the lattice result (see $^{28}$ and references therein). Meanwhile a rough estimate of the vacuum condensate ratio $r$ may be obtained from QCD sum rule: $r \approx 0.4$ GeV$^2$ $^{29}$. It however drops out when we construct the ratio between the vacuum alignment contribution to $\tilde{g}_0$ and $\tilde{g}_1$:

$$
\frac{\tilde{g}_0}{\tilde{g}_1} |_{\text{vac}} = \frac{\sigma^3}{2\varepsilon^0} \frac{\tilde{d}_0}{\tilde{d}_3} \approx 0.12 \frac{\tilde{d}_0}{\tilde{d}_3}.
$$

That is, if we neglect the direct contribution and assume that $\tilde{d}_0 \sim \tilde{d}_3$, then one may conclude that $\tilde{g}_1 \gg \tilde{g}_0$, which is now a common wisdom in the low-energy community.

With all of these said, we still cannot claim to have a complete understanding of $\tilde{g}_I$ without knowing the precise values of the cDM sigma terms $\sigma^{0,3}_C$ that show up in the direct contribution terms. Unlike the QCD sigma terms, the lattice study of cDM sigma terms is still at its beginning stage, but could be carried out in a similar way with some technical complications (see $^{17}$ for discussions). Here, however, we shall provide an alternative direction in which $\sigma^{0,3}_C$ can be obtained through the measurement of the chiral-odd twist-three distribution function $e^q(x)$ (which will be introduced in the next section) in SIDIS experiments, analogous to the determination of nucleon tensor charges from the transversity distribution function $h_1^T(x)$.

3. $e^q(x)$ and its moments

This section serves as a brief review of the basic properties of the chiral-odd twist-three distribution function $e^q(x)$ which details can be found in $^{30}$ and references therein. First, for a proton state $|P\rangle$,

---

$^1$We choose the normalization of the state as $\langle P|P'\rangle = (2\pi)^4 2E_P \delta^4(\vec{P} - \vec{P'})$. Here we also point out two types in Eq. (34) of Ref. $^{17}$: there should be a factor of 2 and -2 multiplied to $\langle p|\bar{q} q |q\rangle |p\rangle$ and $\langle p|q g_\sigma g_\sigma G^{\mu\nu} \tau^3 |q\rangle |p\rangle$ respectively.

$^2$The quark mass parameters are evaluated at $\mu = 2$ GeV whereas the sigma terms are scale-invariant.
\( e^q(x) \) can be written as the matrix element of a quark bilinear with light-cone separation \([12]\):

\[
e^q(x) = \frac{1}{2m_N} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{q}(0)[0,\lambda n]q(\lambda n)|P\rangle.
\]  

(13)

where \( n^\mu \) is a basis vector on the light cone and \([0,\lambda n]\) is the gauge link operator that restores explicit gauge invariance of the distribution function. This function is non zero at \(-1 \leq x \leq 1\) and satisfies \( e^q(-x) = e^q(x) \). The scalar bilinear operator could be decomposed by the mean of operator identity \([31, 32, 33, 34]\):

\[
\bar{q}(0)[0,\lambda n]q(\lambda n) = \bar{q}(0)q(0) + \frac{1}{2} \int_0^1 du \int_0^u dv \bar{q}(0)\sigma_{\alpha\beta}z\delta(0,uz)g_s G_{\alpha\nu}(vz)v\nu[vz,uz]q(uz)
\]

\[-i m_q \int_0^1 du \bar{q}(0)\bar{q}(0)[0,uz](i\partial - m_q)q(uz)\]

\[-\frac{i}{2} \int_0^1 du \left[ \bar{q}(0)\bar{q}(0)[0,uz](i\partial - m_q)q(uz) + \bar{q}(0)(i\partial - m_q)\bar{q}(0)[0,uz]q(uz) \right],
\]  

(14)

with \( m_q \) the current quark mass. These are the “singular”, “pure twist-three”, “quark mass” and “equation of motion” (EOM) term respectively \([30]\). The EOM term does not contribute to physical matrix elements, so \( e^q(x) \) can be decomposed into:

\[
e^q(x) = e^q_{\text{sing}}(x) + e^q_{\text{tw}3}(x) + e^q_{\text{mass}}(x)
\]  

(15)

where

\[
e^q_{\text{sing}}(x) = \frac{\delta(x)}{2m_N} \langle P|\bar{q}(0)q(0)|P\rangle
\]

\[
e^q_{\text{mass}}(x) = -\frac{i m_q}{2m_N} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \lambda \int_0^1 du \langle P|\bar{q}(0)\gamma[0,uz]q(uz)|P\rangle
\]

\[
e^q_{\text{tw}3}(x) = \frac{1}{4m_N} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \lambda^2 \int_0^1 du \int_0^u dv \langle P|\bar{q}(0)\sigma_{\alpha\beta}n[0,uz]g_s G_{\alpha\nu}(vz)v\nu[vz,uz]q(uz)|P\rangle.
\]  

(16)

In particular, the quark mass term satisfies the following relation:

\[
x e^q_{\text{mass}}(x) = \frac{m_q}{m_N} f^q(x)
\]  

(17)

where \( f^q(x) \) is the twist-two unpolarized quark distribution function:

\[
f^q(x) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{q}(0)\gamma[0,\lambda n]q(\lambda n)|P\rangle
\]  

(18)

that satisfies \( f^q(-x) = -f^q(x) \). The Mellin moments of \( f^q(x) \) can be defined as

\[
f^n = \int_{-1}^1 dx x^n f^q(x)
\]  

(19)

and similarly for \( e^q(x) \). We shall adopt the convention that \( f^n \) is called the \((n+1)\)-th moment of \( f^q(x) \). Notice that the integral over \( x \) ranges from \(-1\) to \(1\) so it includes simultaneously the effect of parton and anti-parton. Eq. (17) then implies the following relation between the moments of \( e^q_{\text{mass}} \) and \( f^q \):

\[
e^n_{\text{mass}} = \frac{m_q}{m_N} f^n \quad \forall n \geq 0.
\]  

(20)

\( ^3 \)Our convention for covariant derivative is \( D_\mu = \partial_\mu - ig_s A_\mu \).
There are well-known sum rules for the first and second moment of $e^q(x)$ respectively:

$$
e_0^q = \frac{1}{2m_N} \langle P|\bar{q}(0)q(0)|P \rangle$$

$$
e_1^q = \frac{m_q}{m_N} N_q$$

(21)

where $N_q$ is the valence quark number of the quark flavor $q$ in the proton. At first sight, the first moment gives us the QCD sigma term given in Eq. (10), however it is of minor practical use because $e_1^q$ is saturated by the $\delta$-function which is experimentally inaccessible. Meanwhile, the second moment of $e^q$ comes merely from the quark mass term and vanishes in the chiral limit.

We are particularly interested in the third moment of $e^q(x)$ [35, 34, 36, 37]:

$$
e_2^q = e_{2,\text{mass}}^q + e_{2,\text{tw3}}^q$$

$$= \frac{m_q}{m_N} f_1^q + \frac{1}{4m_N(P^+)^2} \sum_{i=1}^2 \langle P|\bar{q}(0)\sigma^{+i} g_s G^{+i}(0)q(0)|P \rangle.$$  

(22)

where the light-cone components of a four-vector $a^\mu$ are defined as $a^\pm = (1/\sqrt{2})(a^0 \pm a^3)$. We shall now argue that the third moment of $e^q(x)$ is contributed mostly by $e_{2,\text{tw3}}^q(x)$. First of all, $e_{2,\text{mass}}^q$ receives a current quark mass suppression: $m/m_N \sim 4 \times 10^{-3}$; at the same time, both experiments and lattice simulations suggest $f_1^q \sim 10^{-1}$ at the hadron scale [38]. These together give $e_{2,\text{mass}}^q \sim 10^{-3} - 10^{-4}$. On the other hand, since $e_2^q$ is boost-independent in the $z$-direction, we may work in the proton rest frame where $P^+ = m_N/\sqrt{2}$. The order of magnitude of $e_{2,\text{tw3}}^q$ can be roughly estimated using naive dimensional analysis (NDA) [39, 40]:

$$
e_{2,\text{tw3}}^q \sim \frac{1}{2m_N^3} \times \frac{\alpha_s}{4\pi} \chi^3$$

(23)

where $\alpha_s = g_0^2/4\pi$ and $\chi \sim 1$ GeV is the so-called CSB scale. To get a feeling, we take $\alpha_s \approx 0.5$ at $\mu = 1$ GeV in MS-scheme [41]: that gives $e_{2,\text{tw3}}^q \sim 10^{-2}$ which is at least an order of magnitude larger than $e_{2,\text{mass}}^q$. One may also worry that there could be accidental cancellation between different regions of $x$ in the $x^2 e_{2,\text{tw3}}^q(x)$ integral that makes $e_{2,\text{tw3}}^q$ unexpectedly small; but in the following we shall argue that this is very unlikely. First, realize that $e_{2,\text{tw3}}^q(x)$ is a regular function at $-1 \leq x \leq 1$ (unlike $e_{\text{mass}}^q(x)$ that diverges at small $x$) and its first moment vanishes. This implies that the sizes of $e_{2,\text{tw3}}^q(x)$ at intermediate and large values of $x$ should be comparable to those at small $x$ such that cancellation can occur in the $x$-integration. Now, since the integral for its third moment is weighted more at regions of large $x$ due to the $x^2$ factor, mutual cancellation is unlikely to occur and the outcome should be significantly non-zero. Therefore, it is reasonable to assume that $e_2^q$ is dominated by the pure twist-three contribution:

$$
e_2^q \approx e_{2,\text{tw3}}^q.$$  

(24)

Finally, we shall quote some important results of the renormalization group (RG) evolution of $e^q(x)$ that was studied in several papers [33, 35, 36]. Ref. [34] shows that $e^q(x)$ satisfies a DGLAP-type evolution equation in large-$N_c$ limit upon neglecting quark mass. The evolution of its moments is given by:

$$
e_n^q(\mu) \bigg|_{N_c \to \infty} = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_n^+/b} e_n^q(\mu_0)$$

(25)

where $b = (11N_c - 2N_f)/3$, $\gamma_n^+ = 2N_c \left\{ \psi(n+1) + \gamma_E - \frac{1}{2} - \frac{1}{2(n+1)} \right\}$, $\psi(z)$ is the digamma function and $\gamma_E$ is the Euler constant. For the third moment which is particularly important for us, we shall adopt an improved evolution formula including the $1/N_c^2$ corrections [36] (also assuming $m_q = 0$):

$$
e_2^q(\mu) = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{6.11/b} e_2^q(\mu_0).$$

(26)
4. Relations between cMDM sigma terms and $e_2^q$

Now we shall demonstrate that the third moment of $e^q(x)$ is related to the cMDM sigma terms defined in Eq. (28). This can be seen by considering the following parameterization of the (spin-averaged) $q\bar{q}\cdot Gq$ matrix element:

$$\langle P|\bar{q}(0)g_\sigma G^{\alpha\nu}(0)\sigma_\alpha^\nu q(0)|P\rangle = A^q m_N (m_N^2 g^{\alpha\nu} - P^\mu P^\nu) + B^q m_N P^\mu P^\nu,$$

where $A^q$ and $B^q$ are dimensionless, scale-dependent invariant matrix elements. The cMDM sigma term obviously depends on both $A^q$ and $B^q$:

$$\langle P|\bar{q}(0)g_\sigma G^{\alpha\nu}(0)\sigma_\alpha^\nu q(0)|P\rangle = 3A^q m_N^3 + B^q m_N^3.$$  

On the other hand, $e_2^q_{tw3}$ depends on another combination of $A^q$ and $B^q$:

$$e_2^q_{tw3} = \frac{A^q - B^q}{4}.$$  

Combining with the approximation in Eq. (24) we thus obtain:

$$\sigma_0^c \approx m_N^2 \left(3(e_2^u + e_2^d) + B^u + B^d\right)$$

$$\sigma_3^c \approx -2m_N^2 \left(3(e_2^u - e_2^d) + B^u - B^d\right).$$  

This is the central result of the paper: the cMDM sigma terms $\sigma_0^c, \sigma_3^c$ are matched to $e_2^u, e_2^d$, barring the two unknown constants $B^u, B^d$. Furthermore, we observe that upon a non-relativistic (NR) reduction of the quark fields, these constants have to vanish. This can be easily seen by working in the nucleon’s rest frame (i.e. $P^\mu = (m_N, 0)$) and realizing that in the NR limit the quark bilinear $\bar{q}\sigma_\alpha^\nu q$ is non-zero only when $\alpha, \nu \neq 0$, so the only solution is that $B^q = 0$. It is well-known that the symmetry relations obtained from NR quark models are in most cases identical to those implied by spin-flavor symmetry, which is a direct consequence of the large $N_c$-expansion [22]. Therefore, in this work we shall simply assume the $B^q$ term to be subdominant and neglect it in the numerical analysis. In-depth theoretical investigations in the future are of course necessary to better understand the relative size between $A^q$ and $B^q$.

Eq. (30) allows us to perform a more detailed separation of the unknown hadronic inputs from the known ones in the matching relations of $\tilde{g}_I$:

$$\tilde{g}_0 = \frac{\sigma_0^0}{m_N} - 6m_N^2 (e_2^u - e_2^d) - 2m_N^2 (B^u - B^d)$$

$$\tilde{g}_1 = -2\tilde{d}_3 \left\{ -\frac{\sigma_0^0}{m_N} + 3m_N^2 (e_2^u + e_2^d) + m_N^2 (B^u + B^d) \right\}.$$  

Within the curly bracket at the right side in both lines of Eq. (31), the first term is relatively well-known from current lattice study and QCD sum rule, the second term is directly related to the chiral-odd twist-three distribution function which can be experimentally probed while the third term is so-far unknown but it should be no larger than the second term based on general argument. Therefore, determination of $e^q(x)$ from experiments allows us to pin down the second term as well as to constrain the size of the third term. By doing so one may achieve a better understanding of the theoretical uncertainty in the matching relation of $\tilde{g}_I$.

5. Model predictions

We now turn to the numerical analysis of the cMDM sigma terms. It is instructive to first collect some useful information from various low-energy models of QCD. Most of these models do not have gluons as explicit DOFs so a direct calculation of cMDM sigma terms is impossible. However, they are capable in calculating $e^q(x)$ and we may then extract their implications on the $\sigma_0^c, \sigma_3^c$ using Eq. (30)
upon neglecting the \(B^q\) terms. Here we shall analyze several model-based studies of \(e^q(x)\) of the proton, including bag model \[2, 33\], spectator model \[44\], chiral quark-soliton model (\(\chi\) QSM) \[45, 46, 47\] and light-front constituent quark model (LFCQM) \[48\].

First, to see how well these models reproduce QCD predictions, we quote their prediction for first and second moments of \(e^u(x) + e^d(x)\) which, by QCD, should satisfy the sum rules given in Eq. (21). For the first moment, dispersion relation and chiral perturbation theory suggest that it should be quite large: \(e_0^u + e_0^d \approx (6 - 10)\) at a typical DIS scale, say \(Q^2 = 1.5\) GeV\(^2\) \[39\]. From the results in Table 1 we find that the bag model and spectator model fail to satisfy the first sum rule while \(\chi\) QSM and LFCQM are consistent with QCD predictions. Looking back at Eq. (19) where one finds that the first moment comes entirely from the \(\delta\)-function, we conclude that the bag model and spectator model are unable to reproduce the effect of the \(\delta\)-function. The \(\chi\) QSM does have an explicit \(\delta\)-function although its “regular” piece also contributes to a slight fraction of the first moment; the LFCQM result does not have an explicit \(\delta\)-function but its effect is somehow redistributed to \(e_{tw3}\) and \(e_{mass}\). For the second moment, Eq. (21) suggests \(e_1^u + e_1^d = 3m_q/m_N \sim 10^{-2}\) and this is clearly not satisfied by any of these models. This is, however, understandable because the sum rule of \(e_1^q\) in Eq. (21) is a consequence of QCD EOM while each low-energy model satisfies their own EOM that usually involves constituent-like quark masses, making their values of second moment too large.

Finally, based on the shape of \(e^q(x)\) plotted for each model we are able to deduce the third moment of the isosinglet combination \(e^u(x) + e^d(x)\); see Table 1 for the summary. Note that the outcomes are consistent with the NDA analysis in Eq. (23). With these results, we deduce \(\sigma_0^e\) using Eq. (30) at the respective model scale and evolve it to \(Q^2 = 1\) GeV\(^2\) for mutual comparison. This provides us a range of model-predicted values of \(\sigma_0^e\) as \((0.085 - 0.24)\) GeV\(^2\). We may use this to compare the relative importance between the “direct” and “vacuum alignment” contribution to \(\tilde{g}_1\):

\[
\frac{\tilde{g}_1|_{dir}}{\tilde{g}_1|_{vac}} = -\frac{\bar{m}}{r} \frac{\sigma_0^e}{\sigma_0^u} \approx -0.21 \left(3(e_2^u + e_2^d) + B^u + B^d\right),
\]

where we have taken the lattice value for \(\bar{m}, \sigma_0^u\) and the sum-rule estimation of \(r\). Substituting the model predictions for \(e_2^u + e_2^d\) given in Table 1 we find its contribution to the ratio above to take the following range:

\[
\left(\frac{\tilde{g}_1|_{dir}}{\tilde{g}_1|_{vac}}\right)_{e_2} = -(0.02 - 0.06),
\]

upon neglecting \(B^{u,d}\). This implies that the “vacuum alignment” contribution to \(\tilde{g}_1\) dominates over the direct contribution, thus we in fact already have a fair estimation of eEDM-induced \(\tilde{g}_1\) based on the currently-available data. We shall check this conclusion by studying the implications from the existing experimental results of \(e^q(x)\) in the next section.

6. Implications from experiments

We find two existing literatures that attempted the direct extraction of \(e^q(x)\) from experiment: Refs. 49, 50 among which only the former is published in a peer-reviewed journal, and therefore we shall use it to obtain an estimate of \(e_2^u + e_2^d\). In Ref. 49, the combination \(e(x) = e^u(x) + (1/4)e^d(x)\) at the scale \(Q^2 = 1.5\) GeV\(^2\) was extracted from the azimuthal asymmetry \(A_{LU}\) in the SIDIS process \(ep \rightarrow e\pi^+X\) which was measured by the CLAS Collaboration 51. Such extraction required the knowledge of the Collins fragmentation function \(H_C^T\) which was deduced from the HERMES data 52, 53, 54.

It is non-trivial to translate the outcome in Ref. 49 into our desired third moment of \(e^u(x) + e^d(x)\) due to two reasons: (1) the measured combination \(e(x)\) is not isoscalar and (2) there are altogether only four data points available which lie within \(0.18 < x < 0.37\). Therefore, extra assumptions are needed in order to extract the most information out of it. First, we will assume \(e^q(x) \approx e^u(x)\) which holds in the large \(N_c\) limit 30 and is shown to be a fair approximation within a \(\chi\) QSM even for \(N_c = 3\) 16; this
assumption and $\sigma(-x) = \sigma(x)$ together give $\sigma(x) = \sigma^u(x) + (1/4)\sigma^d(-x)$. Second, given the limited amount of data points, we shall assume a simple Gaussian-like parameterization of $\sigma^u(x)$:

$$\sigma^u(x) = A \exp \left\{ -\frac{(x-x_0)^2}{2\sigma^2} \right\}$$

which qualitatively describes most of the model predictions. The parameters $\{A, x_0, \sigma\}$ are to be fitted to experimental data.

Our fitting proceeds as follows. First, we fit the four data points using the parameterization in Eq. (34) which returns a best-fitted value of $x_0 \approx 0.15$, in rough agreement with the phenomenological model predictions. Next we fix $x_0 = 0.15$ and repeat the fit to obtain the two remaining parameters: $A = 2.08 \pm 0.54$ and $\sigma = 0.15 \pm 0.03$; the fitted curve is shown in Fig. 1. With these we may compute the third moment of $\sigma^u(x) + \sigma^d(x)$ by varying $A$ and $\sigma$ within their respective allowed region, which then gives:

$$0.03 < e_3^u + e_3^d < 0.13, \quad Q^2 = 1.5 \text{ GeV}^2. \tag{35}$$

The order of magnitude of the outcome is the same as that from the models, again indicating that the isovector $P$, CP-odd coupling $\bar{g}_1$ is dominated by the “vacuum alignment” contribution $\bar{g}_1|_{\text{vac}}$.

The fitting above is very crude due to the limited data, but it can be greatly improved with the future accumulation of more data points in a wider range of $x$. At the same time, different SIDIS processes (such that the $e p \to e \pi^+\pi^- X$ process analyzed in Ref. [50]) should be considered simultaneously in order to disentangle the isoscalar and isovector combinations of $\{e_3^d\}$. Even though their role in the $P$, CP-odd pion-nucleon couplings seems to be subdominant as indicated in this paper, these experiments still provide invaluable information about the cMDM matrix elements which could be contrasted to future lattice calculations and thus serve as a direct test of the lattice precision.

7. Conclusions

Searches of BSM physics through precision tests of fundamental symmetries in low-energy hadronic systems often require precise understanding of QCD matrix elements that are difficult to compute from first principle, and a way out is to map them to experimentally measurable quantities: examples are the nucleon tensor charges and the isoscalar sigma term. In this work we focus on the cMDM sigma terms $\sigma_{C}^{0,3}$ which are essential in the study of the $P$, CP-odd pion-nucleon coupling $\bar{g}_1$ induced by dimension-5 cEDM operators. In the absence of lattice calculations, we investigate the possibility to extract information of such matrix elements through observables in DIS processes. We find that, barring some unknown but suppressed additive constants, $\sigma_{C}^{0.3}$ could be related to the third moment of the chiral-odd twist three distribution function $e^\sigma(x)$ that could be measured in various SIDIS experiments. Surveying on existing

| Model                  | $e_0^u + e_0^d$ | $e_1^u + e_1^d$ | $e_2^u + e_2^d$ | $\sigma_C^0 \cdot \text{GeV}^{-2}$ |
|------------------------|-----------------|-----------------|-----------------|----------------------------------|
| Bag Model [2]          | 3.6             | 0.19            | 0.032           | 0.085                            |
| Spectator Model [44]   | 1.7             | 0.20            | 0.042           | 0.11                             |
| $\chi$QSM [47]        | 9.3             | 0.19            | 0.063           | 0.17                             |
| LFCQM [48]             | 7.5             | 0.48            | 0.092           | 0.24                             |
| Lattice [24, 25] + Sum Rule | 8.0         | 0.012           | N/A             | N/A                              |

Table 1: Model predictions for the first, second, third moment of $e^u(x) + e^d(x)$ in a proton evolved to $\mu = 1$ GeV using Eq. (25) and (26), and the implied value for $\sigma_C^0$, at the same scale using Eq. (30) with the subdominant terms $B_{u,d}$ neglected. The last row is obtained by substituting the lattice results of the quark masses and sigma term into the sum rule of the first and second moment in Eq. (21). The starting points of the QCD evolution are: 400 MeV (bag model), 500 MeV (spectator model), 600 MeV ($\chi$QSM), 720 MeV (LFCQM) and 2 GeV (lattice) respectively.
model studies and limited experimental data of $e^q(x)$, we arrive at $\sigma_0^1 \sim 10^{-1} \text{ GeV}^2$ which implies a subdominant contribution to $g_1$ comparing to the “vacuum alignment” term. Nevertheless, improvement of the $e^q(x)$ measurements is still worthwhile in terms of providing a gauge for any future attempts in the first-principle calculation of the cMDM sigma terms which will directly test our understanding of non-perturbative QCD.

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References

[1] R. L. Jaffe, X.-D. Ji, Chiral odd parton distributions and polarized Drell-Yan, Phys. Rev. Lett. 67 (1991) 552–555. doi:10.1103/PhysRevLett.67.552

[2] R. L. Jaffe, X.-D. Ji, Chiral odd parton distributions and Drell-Yan processes, Nucl. Phys. B375 (1992) 527–560. doi:10.1016/0550-3213(92)90110-W

[3] Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, Extraction of Quark Transversity Distribution and Collins Fragmentation Functions with QCD Evolution, Phys. Rev. D93 (1) (2016) 014009. arXiv:1505.05589, doi:10.1103/PhysRevD.93.014009

[4] M. Radici, A. Courtoy, A. Bacchetta, M. Guagnelli, Improved extraction of valence transversity distributions from inclusive dihadron production, JHEP 05 (2015) 123. arXiv:1503.03495, doi:10.1007/JHEP05(2015)123

[5] G. R. Goldstein, J. O. Gonzalez Hernandez, S. Liuti, Flavor dependence of chiral odd generalized parton distributions and the tensor charge from the analysis of combined $\pi^0$ and $\eta$ exclusive electro-production data. arXiv:1401.0438

[6] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia, A. Prokudin, Simultaneous extraction of transversity and Collins functions from new SIDIS and e+e- data, Phys. Rev. D87 (2013) 094019. arXiv:1303.3822, doi:10.1103/PhysRevD.87.094019
[7] Z. Ye, N. Sato, K. Allada, T. Liu, J.-P. Chen, H. Gao, Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, Unveiling the nucleon tensor charge at Jefferson Lab: A study of the SoLID case, Phys. Lett. B767 (2017) 91–98. arXiv:1609.02449 doi:10.1016/j.physletb.2017.01.046

[8] H. Gao, T. Liu, Z. Zhao, Nucleon tensor charge and electric dipole moment arXiv:1704.00113.

[9] J. Engel, M. J. Ramsey-Musolf, U. van Kolck, Electric Dipole Moments of Nucleons, Nuclei, and Atoms: The Standard Model and Beyond, Prog.Part.Nucl.Phys. 71 (2013) 21–74. arXiv:1303.2371 doi:10.1016/j.ppnp.2013.03.003.

[10] V. Cirigliano, S. Gardner, B. Holstein, Beta Decays and Non-Standard Interactions in the LHC Era, Prog. Part. Nucl. Phys. 71 (2013) 93–118. arXiv:1303.6953 doi:10.1016/j.ppnp.2013.03.005.

[11] M. Pospelov, Best values for the CP odd meson nucleon couplings from supersymmetry, Phys.Lett. B530 (2002) 123–128. arXiv:hep-ph/0109044 doi:10.1016/S0370-2693(02)01263-7.

[12] J. Bsaisou, U.-G. Meißner, A. Nogga, A. Wirzba, P- and T-Violating Lagrangians in Chiral Effective Field Theory and Nuclear Electric Dipole Moments, Annals Phys. 359 (2015) 317–370. arXiv:1412.5471 doi:10.1016/j.aop.2015.04.031.

[13] J. de Vries, U.-G. Meißner, Violations of discrete spacetime symmetries in chiral effective field theory, Int. J. Mod. Phys. E25 (05) (2016) 1641008. arXiv:1509.07331 doi:10.1142/S0218301316410081.

[14] J. Bsaisou, C. Hanhart, S. Liebig, U.-G. Meißner, A. Nogga, et al., The electric dipole moment of the deuteron from the QCD \( \theta \)-term, Eur.Phys.J. A49 (2013) 31. arXiv:1209.6306 doi:10.1140/epja/i2013-13031-x.

[15] E. Mereghetti, W. Hockings, U. van Kolck, The Effective Chiral Lagrangian From the Theta Term, Annals Phys. 325 (2010) 2363–2409. arXiv:1002.2391 doi:10.1016/j.aop.2010.03.005.

[16] J. de Vries, E. Mereghetti, A. Walker-Loud, Baryon mass splittings and strong CP violation in SU(3) Chiral Perturbation Theory, Phys. Rev. C92 (4) (2015) 045201. arXiv:1506.06247 doi:10.1103/PhysRevC.92.045201.

[17] J. de Vries, E. Mereghetti, C.-Y. Seng, A. Walker-Loud, Lattice QCD spectroscopy for hadronic CP violation, Phys. Lett. B766 (2017) 254–262. arXiv:1612.01567 doi:10.1016/j.physletb.2017.01.017.

[18] C.-Y. Seng, M. Ramsey-Musolf, Parity-violating and time-reversal-violating pion-nucleon couplings: Higher order chiral matching relations, Phys. Rev. C96 (6) (2017) 065204. arXiv:1611.08063 doi:10.1103/PhysRevC.96.065204.

[19] J. de Vries, E. Mereghetti, R. Timmermans, U. van Kolck, The Effective Chiral Lagrangian From Dimension-Six Parity and Time-Reversal Violation, Annals Phys. 338 (2013) 50–96. arXiv:1212.0990 doi:10.1016/j.aop.2013.05.022.

[20] R. F. Dashen, Some features of chiral symmetry breaking, Phys. Rev. D3 (1971) 1879–1889. doi:10.1103/PhysRevD.3.1879.

[21] V. Baluni, CP Violating Effects in QCD, Phys.Rev. D19 (1979) 2227–2230. doi:10.1103/PhysRevD.19.2227.

[22] T. Bhattacharya, V. Cirigliano, R. Gupta, E. Mereghetti, B. Yoon, Dimension-5 CP-odd operators: QCD mixing and renormalization, Phys. Rev. D92 (11) (2015) 114026. arXiv:1502.07325 doi:10.1103/PhysRevD.92.114026.

[23] R. Peccei, H. R. Quinn, CP Conservation in the Presence of Instantons, Phys.Rev.Lett. 38 (1977) 1440–1443. doi:10.1103/PhysRevLett.38.1440.
[24] G. Bali, P. Bruns, S. Collins, M. Deka, B. Glasle, et al., Nucleon mass and sigma term from lattice QCD with two light fermion flavors, Nucl. Phys. B866 (2013) 1–25. arXiv:1206.7034 doi:10.1016/j.nuclphysb.2012.08.009

[25] G. de Divitiis, et al., Leading isospin breaking effects on the lattice, Phys. Rev. D87 (11) (2013) 114505. arXiv:1303.4896 doi:10.1103/PhysRevD.87.114505

[26] S. Aoki, et al., Review of lattice results concerning low-energy particle physics. arXiv:1607.00299

[27] G. S. Bali, S. Collins, D. Richtmann, A. Schfer, W. Skdner, A. Sternbeck, Direct determinations of the nucleon and pion $\sigma$ terms at nearly physical quark masses, Phys. Rev. D93 (9) (2016) 094504. arXiv:1603.00827 doi:10.1103/PhysRevD.93.094504

[28] J. Ruiz de Elvira, M. Hoferichter, B. Kubis, U.-G. Meißner, Extracting the $\sigma$-term from low-energy pion-nucleon scattering, J. Phys. G45 (2) (2018) 024001. arXiv:1706.01465 doi:10.1088/1361-6471/aa9422

[29] V. M. Belyaev, B. L. Ioffe, Determination of the baryon mass and baryon resonances from the quantum-chromodynamics sum rule. Strange baryons, Sov. Phys. JETP 57 (1983) 716–721, [Zh. Eksp. Teor. Fiz.84,1236(1983)].

[30] A. V. Efremov, P. Schweitzer, The Chirally odd twist 3 distribution $e(a)(x)$, JHEP 08 (2003) 006. arXiv:hep-ph/0212044 doi:10.1088/1126-6708/2003/08/006

[31] I. I. Balitsky, V. M. Braun, Evolution Equations for QCD String Operators, Nucl. Phys. B311 (1989) 541–584. doi:10.1016/0550-3213(89)90168-5

[32] V. M. Braun, I. E. Filyanov, Conformal Invariance and Pion Wave Functions of Nonleading Twist, Z. Phys. C48 (1990) 239–248, [Yad. Fiz.52,199(1990)]. doi:10.1007/BF01554472

[33] A. V. Belitsky, D. Mueller, Scale dependence of the chiral odd twist - three distributions $h(L)(x)$ and $e(x)$, Nucl. Phys. B503 (1997) 279–308. arXiv:hep-ph/9702354 doi:10.1016/S0550-3213(97)00432-X

[34] J. Kodaira, K. Tanaka, Polarized structure functions in QCD, Prog. Theor. Phys. 101 (1999) 191–242. arXiv:hep-ph/9812449 doi:10.1143/PTP.101.191

[35] I. I. Balitsky, V. M. Braun, Y. Koike, K. Tanaka, Q**2 evolution of chiral odd twist - three distributions $h(L)(x)$, $Q**2$ and $e(x)$, $Q**2$ in the large N(c) limit, Phys. Rev. Lett. 77 (1996) 3078–3081. arXiv:hep-ph/9605439 doi:10.1103/PhysRevLett.77.3078

[36] Y. Koike, N. Nishiyama, Q**2 evolution of chiral odd twist - three distribution $e(x)$, $Q**2$, Phys. Rev. D55 (1997) 3068–3076. arXiv:hep-ph/9609207 doi:10.1103/PhysRevD.55.3068

[37] M. Burkardt, Transverse force on quarks in deep-inelastic scattering, Phys. Rev. D88 (2013) 114502. arXiv:0810.3589 doi:10.1103/PhysRevD.88.114502

[38] M. Deka, T. Streuner, T. Doi, S. J. Dong, T. Draper, K. F. Liu, N. Mathur, A. W. Thomas, Moments of Nucleon’s Parton Distribution for the Sea and Valence Quarks from Lattice QCD, Phys. Rev. D79 (2009) 094502. arXiv:0811.1779 doi:10.1103/PhysRevD.79.094502

[39] S. Weinberg, Larger Higgs Exchange Terms in the Neutron Electric Dipole Moment, Phys. Rev. Lett. 63 (1989) 2333. doi:10.1103/PhysRevLett.63.2333

[40] A. Manohar, H. Georgi, Chiral Quarks and the Nonrelativistic Quark Model, Nucl. Phys. B234 (1984) 189–212. doi:10.1016/0550-3213(84)90231-1
A. Deur, S. J. Brodsky, G. F. de Teramond, The QCD Running Coupling, Prog. Part. Nucl. Phys. 90 (2016) 1–74. arXiv:1604.08082 doi:10.1016/j.ppnp.2016.04.003

R. F. Dashen, E. E. Jenkins, A. V. Manohar, Spin flavor structure of large N(c) baryons, Phys. Rev. D51 (1995) 3697–3727. arXiv:hep-ph/9411234 doi:10.1103/PhysRevD.51.3697

A. I. Signal, Calculations of higher twist distribution functions in the MIT bag model, Nucl. Phys. B497 (1997) 415–434. arXiv:hep-ph/9610480 doi:10.1016/S0550-3213(97)00231-9

R. Jakob, P. J. Mulders, J. Rodrigues, Modeling quark distribution and fragmentation functions, Nucl. Phys. A626 (1997) 937–965. arXiv:hep-ph/9704335 doi:10.1016/S0375-9474(97)00586-5

P. Schweitzer, The Chirally odd twist three distribution function e**alpha(x) in the chiral quark soliton model, Phys. Rev. D67 (2003) 114010. arXiv:hep-ph/0303011 doi:10.1103/PhysRevD.67.114010

Y. Ohnishi, M. Wakamatsu, pi N sigma term and chiral odd twist three distribution function e(x) of the nucleon in the chiral quark soliton model, Phys. Rev. D69 (2004) 114002. arXiv:hep-ph/0312044 doi:10.1103/PhysRevD.69.114002

C. Cebulla, J. Ossmann, P. Schweitzer, D. Urbano, The Twist-3 parton distribution function e**a(x) in large-N(c) chiral theory, Acta Phys. Polon. B39 (2008) 609–640. arXiv:0710.3103

C. Lorcé, B. Pasquini, P. Schweitzer, Transverse pion structure beyond leading twist in constituent models, Eur. Phys. J. C76 (7) (2016) 415. arXiv:1605.00815 doi:10.1140/epjc/s10052-016-4257-8

A. V. Efremov, K. Goeke, P. Schweitzer, Azimuthal asymmetries at CLAS: extraction of e**a(x) and prediction of A(UL), Phys. Rev. D67 (2003) 114014. arXiv:hep-ph/0208124 doi:10.1103/PhysRevD.67.114014

A. Courtoy, Insights into the higher-twist distribution e(x) at CLAS, arXiv:1405.7659

H. Avakian, et al., Measurement of beam-spin asymmetries for pi + electroproduction above the baryon resonance region, Phys. Rev. D69 (2004) 112004. arXiv:hep-ex/0301005 doi:10.1103/PhysRevD.69.112004

A. V. Efremov, K. Goeke, P. Schweitzer, Azimuthal asymmetry in electroproduction of neutral pions in semiinclusive DIS, Phys. Lett. B522 (2001) 37–48, [Erratum: Phys. Lett. B544,389(2002)]. arXiv:hep-ph/0108213 doi:10.1016/S0370-2693(01)01258-8, 10.1016/S0370-2693(02)02518-2

A. Airapetian, et al., Observation of a single spin azimuthal asymmetry in semiinclusive pion electro production, Phys. Rev. Lett. 84 (2000) 4047–4051. arXiv:hep-ex/9910062 doi:10.1103/PhysRevLett.84.4047

A. Airapetian, et al., Single spin azimuthal asymmetries in electroproduction of neutral pions in semiinclusive deep inelastic scattering, Phys. Rev. D64 (2001) 097101. arXiv:hep-ex/0104005 doi:10.1103/PhysRevD.64.097101