ηc production at the LHC challenges nonrelativistic-QCD factorization

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We analyze the first measurement of ηc production, performed by the LHCb Collaboration, in the nonrelativistic-QCD (NRQCD) factorization framework at next-to-leading order (NLO) in the strong-coupling constant αs and the relative velocity v of the bound quarks including the feeddown from hc mesons. Converting the long-distance matrix elements (LDMEs) extracted by various groups from J/ψ yield and polarization data to the ηc case using heavy-quark spin symmetry, we find that the resulting NLO NRQCD predictions greatly overshoot the LHCb data, while the color-singlet model provides an excellent description.

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Despite concerted experimental and theoretical efforts over the discovery of the J/ψ meson in the November revolution of 1974 (The Nobel Prize in Physics 1976), the genuine mechanism underlying the production and decay of heavy quarkonia, which are QCD bound states of heavy quark Q = c, b and its antiparticle Q̅, has remained mysterious. The effective quantum field theory of nonrelativistic QCD (NRQCD) [1] endowed with an appropriate factorization theorem [2] arguably constitutes the most probable candidate theory at the present time. This implies a separation of process-dependent short-distance coefficients (SDCs), to be calculated perturbatively as expansions in the strong-coupling constant αs, from supposedly universal long-distance matrix elements (LDMEs), to be extracted from experiment. The relative importance of the latter is subject to velocity scaling rules [3], which predict each of the LDMEs to scale with a definite power of the heavy-quark velocity v. In this way, the theoretical predictions are organized as double expansions in αs and v. A crucial feature of this formalism is that the Q\overline{Q} pair can at short distances be produced in any Fock state $n = 2S + 1L_J^z$ with definite spin S, orbital angular momentum L, total angular momentum J, and color multiplicity a = 1, 8. In this way, it complements the color-singlet (CS) model (CSM), which only includes the very 2S+1L_J^z state of the physical quarkonium, and thus cures a severe conceptual shortcoming of the latter, namely the existence of uncanceled infrared (IR) singularities beyond L = 0. However, the CSM does provide IR-finite NLO predictions for S-wave charmonia, such as the ηc and J/ψ mesons considered here.

Despite its theoretical rigor, NRQCD factorization has reached the crossroads in the J/ψ case. While a global fit to the J/ψ yields measured in hadroproduction, photoproduction, γγ scattering, and e+e− annihilation successfully pins down the leading color-octet (CO) LDMEs, (O^{J/ψ}(3S_2^0)), (O^{J/ψ}(1S_0^0)), (O^{J/ψ}(3S_1^0)), and (O^{J/ψ}(3P_0^8)), in compliance with the velocity scaling rules, the resulting predictions for J/ψ polarization in hadroproduction are in striking disagreement with measurements at the Fermilab Tevatron and the CERN LHC [3]. Vice versa, fits to data on J/ψ yield and polarization in hadroproduction work reasonably well [4,5], but hopelessly fail in comparisons to the world’s data from other than hadronic collisions [6].

Very recently, the LHCb Collaboration measured, for the first time, the prompt ηc yield, via ηc → p\bar{p} decays [10]. The data were taken at center-of-mass energies \sqrt{s} = 7 and 8 TeV in the forward rapidity range 2.0 < y < 4.5 for variable transverse momentum pt. This provides a tantalizing new opportunity to further test NRQCD factorization and, hopefully, to also shed light on the J/ψ polarization puzzle, the more so as the ηc meson is the spin-singlet partner of the J/ψ meson, which implies that the LDMEs of the two are related by heavy-quark spin symmetry (HQSS), one of the pillars of NRQCD factorization. The dominant feed-down contribution is due to the radiative decay h_c → ηcγ. The leading CS and CO Fock states of direct ηc (h_c) production are 1S_0^0, 1S_0^8, 3S_1^0, and 1P_1^0 (1P_1^8 and 1S_0^8).

So far, only incomplete LO calculations were carried out for direct ηc production, excluding the 1S_0^8 contribution [11]. For the reasons explained above, it is an urgent matter of general interest to provide a full-fledged NRQCD analysis of prompt ηc hadroproduction, at NLO both in αs and v, and this is the very purpose of this Letter. From the J/ψ case, where such systematic investigations already exist [4,6,8], we know (i) that O(α_s) corrections may be sizable, especially in the 3P_0^8 channels, (ii) that O(α^2) corrections may be non-negligible [12,13], and (iii) that feed-down contributions to prompt production may be substantial, reaching 20–30% in the ηcJ case [7,14,15].

We work in the collinear parton model of QCD implemented in the fixed-flavor-number scheme with n_f = 3 quark flavors active in the colliding protons, which are represented by parton density functions (PDFs) evaluated at factorization scale \mu_f. At NLO in NRQCD, the
relevant partonic cross sections are given by

\[ \frac{d\sigma_{\text{prompt}}}{dE} = \sum \frac{d\sigma_{\text{SDC}}}{dE} \langle \mathcal{O} \rangle \langle \mathcal{O} \rangle \]

\[ \frac{d\sigma_{\text{qcd}}}{dE} = \sum \frac{d\sigma_{\text{qcd}}}{dE} \langle \mathcal{O} \rangle \langle \mathcal{O} \rangle \]

\[ \frac{d\sigma_{\text{pp}}}{dE} = \sum \frac{d\sigma_{\text{pp}}}{dE} \langle \mathcal{O} \rangle \langle \mathcal{O} \rangle \]

where \( \mathcal{O} \) are the Born SDCs including their \( \mathcal{O}(a_s) \) corrections, \( \mathcal{O}(a_s, a_\gamma) \) contain their \( \mathcal{O}(a_s^2) \) corrections, and the DS is \( \langle Q \rangle \) with \( Q = \mathcal{O}, P \) and \( h = h_c, h_r \) are the appropriate LDMEs. We approximate account for the mass difference between the same \( h_c \) and \( h_r \) mesons by substituting the space-time dimension in di-
Ratios \( R(n) = \frac{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}}{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}} \) measuring the \( O(\alpha_s^2) \) corrections to the SDCs as functions of \( p_T^{\eta_c} \) (left panel). Ratios \( R(n) = \frac{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}}{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}} \) measuring the \( O(\alpha_s) \) corrections to the SDCs as functions of \( p_T^{\eta_c} \) (right panel). The results for \( n = 1P_1^{[1]} \) refer to \( h_c \) production and are evaluated at \( p_T^{h_c} = p_T^{\eta_c} m_{h_c} / m_{\eta_c} \). The results for \( n = 1S_0^{[8]} \) in \( h_c \) production are not shown, but may be obtained from those in \( \eta_c \) production by rescaling as for \( n = 1P_1^{[1]} \). Red color indicates negative values.

FIG. 1: Ratios \( K(n) = \frac{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}}{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}} \) measuring the \( O(\alpha_s) \) corrections to the SDCs as functions of \( p_T^{\eta_c} \) (left panel). Ratios \( R(n) = \frac{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}}{d\sigma^{[\chi_0]} / d\sigma^{[\eta_c]} m_c^{-2}} \) measuring the \( O(\alpha_s^2) \) corrections to the SDCs as functions of \( p_T^{\eta_c} \) (right panel). The results for \( n = 1P_1^{[1]} \) refer to \( h_c \) production and are evaluated at \( p_T^{h_c} = p_T^{\eta_c} m_{h_c} / m_{\eta_c} \). The results for \( n = 1S_0^{[8]} \) in \( h_c \) production are not shown, but may be obtained from those in \( \eta_c \) production by rescaling as for \( n = 1P_1^{[1]} \). Red color indicates negative values.

FIG. 2: The LHCb [10] measurements of \( d\sigma / dp_T \) for prompt \( \eta_c \) hadroproduction at \( \sqrt{s} = 7 \text{ TeV} \) (upper panel) and 8 TeV (lower panel) are compared with the default predictions of NRQCD (solid lines) and the CSM (dot-dashed lines) at NLO, but without relativistic corrections, evaluated with the four LDME sets in Table I. The theoretical errors as explained in the text are indicated by the yellow and blue bands, respectively. For comparison, also the default contributions due to the individual Fock states are shown. Red color indicates negative values.
NRQCD and CSM and default predictions including $\mathcal{O}(\alpha_s)$ but excluding $\mathcal{O}(v^2)$ corrections, evaluated with the four LDME sets in Table I. In Table II, the same information is presented for the LHCb $p_T$ bins, together with three theoretical errors. The first one is due to the lack of knowledge of the values of $\langle P^h(n) \rangle$ and the $\mathcal{O}(v^2)$ corrections to the HQSS relations (3). Both effects are estimated by evaluating Eq. (1) with $\langle P^h(n) \rangle = \xi m_0^2 \langle O^h(n) \rangle$ and varying $\xi$ in the range $-0.5 < \xi < 0.5$, so that $\xi$ is of order $v^2 \approx 0.23$ as obtained from potential model calculations [22]. The second theoretical error is due to unknown corrections beyond $\mathcal{O}(\alpha_s)$, which are estimated by varying $\mu_\alpha$, $\mu_r$, and $\mu_f$ by a factor of two up and down relative to their default values. The third one is due to the fit errors in the LDMEs specified in Table I. The error bands shown in Fig. 2 are obtained by adding these three errors in quadrature.

In Fig. 2, the default NRQCD predictions are also broken down to the individual Fock state contributions. Evidently, the $h_c$ feeddown contribution is negligible owing to the small $J^{P=1}$ and $S_0^{(8)}$ SDCs, a feature that could not be anticipated without explicit calculation, the more so as the $\chi_{cJ}$ feeddown contribution to prompt $J/\psi$ production is quite significant. The most striking feature is, however, that the CSM, which is basically made up just by the $I^{P=1}_S$ contribution, yields an almost perfect description of the LHCb data, leaving practically no room for CO contributions. While the $I^{S=0}_S$ and $P^{J=1}_1$ contributions comply with this condition for all four $J/\psi$ LDME sets considered, the latter dictate a very sizable $S_0^{(8)}$ contribution, which overshoots the LHCb data by up to about one order of magnitude. Even the LDME set that describes the LHCb data best, namely the one of Ref. [4], yields an unacceptable $\chi^2/d.o.f.$ value of 257/7 with respect to the default NRQCD predictions. If we take the lower borders of the respective error bands in Fig. 2 as a reference, then $\chi^2/d.o.f.$ comes down to 36.7/7, which is still very poor.

In our second approach, we determine the $\eta_c$ and $h_c$ LDMEs without recourse to the $J/\psi$ and $\chi_{cJ}$ LDMEs, by directly fitting the LHCb data under certain simplifying assumptions. First, we neglect the $h_c$ feeddown contributions by appealing to their dramatic suppression in Fig. 2. Second, we neglect the $S_0^{(8)}$ and $P^{J=1}_1$ contributions to direct $\eta_c$ production because of the $\mathcal{O}(v^4)$ suppression of their LDMEs relative to the $I^{S=0}_S$ one, which is not compensated by an inverse hierarchy in the respective SDCs. In fact, the $I^{S=0}_S$ SDCs are only of the same order as the $I^{S=0}_S$ ones, while the $P^{J=1}_1$ ones are even smaller. We are then left with the $I^{S=0}_S$ and $S_0^{(8)}$ contributions to direct $\eta_c$ production. As in Table II, we include $\mathcal{O}(\alpha_s)$ corrections, but neglect $\mathcal{O}(v^2)$ corrections. Our fitting procedure is as follows. We first determine $\langle O^{nc}(I^{S=0}_S) \rangle$ from the $\eta_c \to \gamma\gamma$ partial decay width [23].

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{32\pi\alpha^2}{81m_0^2} \left[ 1 - (20 - \pi^2) \frac{\alpha_s}{3\pi} \right] \langle O^{nc}(I^{S=0}_S) \rangle,$$

and then use it as input to fit the $\langle O^{nc}(S_0^{(8)}) \rangle$ to the LHCb data. We are entitled to do so, since the difference between the CS LDMEs for production and decay are of $\mathcal{O}(v^4)$ [2]. In our determination of $\langle O^{nc}(I^{S=0}_S) \rangle$, we set $\alpha = 1/137$ and $\alpha_s(2m_\pi) = 0.26$, and adopt the values $\Gamma_{\eta_c} = (32.3 \pm 1.0)$ MeV and $\text{Br}(\eta_c \to \gamma\gamma) = (1.57 \pm 0.12) \times 10^{-4}$ from Ref. [21]. We then obtain $\langle O^{nc}(S_0^{(8)}) \rangle = (0.24 \pm 0.02)$ GeV$^3$, in reasonable agreement with the values of its HQSS counterpart $\langle O^{1/4}(S_0^{(8)}) \rangle$ in Table Iii and $\langle O^{nc}(S_1^{(8)}) \rangle = (3.3 \pm 2.3) \times 10^{-3}$ GeV$^3$, yielding an excellent description of the LHCb data, with $\chi^2/d.o.f. = 1.4/6$, as may be seen from Fig. 3. By HQSS, this provides an independent determination of $\langle O^{1/4}(S_0^{(8)}) \rangle = \langle O^{nc}(S_0^{(8)}) \rangle$. Observing that this value falls short of the lowest value in Table Iii namely the one from Ref. [4], by 6.47 standard deviations, we recover the striking disagreement encountered in our first approach. Such a low value of $\langle O^{1/4}(S_0^{(8)}) \rangle$ is in conflict with the ideas behind the high-$p_T$ fits in Refs. [4, 5], which suggest a large $\langle O^{1/4}(S_0^{(8)}) \rangle$ value to render the $S_0^{(8)}$ contributions dominating in high-$p_T$ $J/\psi$ hadroproduction and to explain both the $J/\psi$ yield and polarization observed experimentally. However, unlike the $J/\psi$ case, the theoretical prediction of direct $\eta_c$ hadroproduction is well under control. In fact, there are no large NLO corrections in neither the CS or CO channels, and the $h_c$ feeddown contributions are also small.

To summarize, we calculated, for the first time, the $\mathcal{O}(\alpha_s)$ corrections to the $I^{S=0}_S$ and $P^{J=1}_1$ SDCs as well as...
the $\mathcal{O}(v^2)$ corrections to the $^{1}S_{0}^{[1]}$, $^{1}P_{1}^{[1]}$, and $^{1}P_{0}^{[8]}$ SDCs. Using $\eta_c$ LDMEs derived via HQSS from up-to-date $J/\psi$ LDMEs [4, 6–8], we demonstrated that the CS contribution alone can nicely describe the new LHCb data on prompt $\eta_c$ hadroproduction [10], while the full NLO NRQCD predictions yield unacceptably large $\chi^2$/d.o.f. values, of 5.24 and above. On the other hand, the CO contribution is almost exclusively exhausted by the $^{3}S_{1}^{[1]}$ channel, and the $h_c$ feeddown contribution is negligibly small. This allowed us to directly fit $\langle\mathcal{O}^{u}(S_{0}^{[8]})\rangle$ to the LHCb data after determining $\langle\mathcal{O}^{u}(S_{0}^{[8]})\rangle$ from $\Gamma(h_c \to \gamma\gamma)$, both in NRQCD through $\mathcal{O}(\alpha_s)$. Conversion to $\langle\mathcal{O}^{J/\psi}(S_{0}^{[8]})\rangle$ via HQSS yielded a value that undershoots the expectation from the velocity scaling rules by about one order of magnitude and the respective results from the NLO NRQCD fits to $J/\psi$ production data currently on the market [4, 6–8] by at least 6.47 standard deviations. Taking for granted that the LHCb results [10] and the HQSS relations [3] can be trusted and observing that the kinematic region probed falls into mid-$p_T$ range, where neither large logarithms $\ln(p_T^2/m^2)$ nor factorization breaking terms are expected, we are led to conclude that either the universality of the LDMEs is in question or that another important ingredient to current NLO NRQCD analyses has so far been overlooked.

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