Scalar Dark Matter Models with Significant Internal Bremsstrahlung

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There has been interest recently on particle physics models that may give rise to sharp gamma ray spectral features from dark matter annihilation. Because dark matter is supposed to be electrically neutral, it is challenging to build weakly interacting massive particle models that may accommodate both a large cross section into gamma rays at, say, the Galactic center, and the right dark matter abundance. In this work, we consider the gamma ray signatures of a class of scalar dark matter models that interact with Standard Model dominantly through heavy vector-like fermions (the vector-like portal). We focus on a real scalar singlet $S$ annihilating into lepton-antilepton pairs. Because this two-body final-state annihilation channel is d-wave suppressed in the chiral limit, $\sigma_{f\bar{f}v} \propto v^4$, we show that virtual internal bremsstrahlung emission of a gamma ray gives a large correction, both today and at the time of freeze-out. For the sake of comparison, we confront this scenario to the familiar case of a Majorana singlet annihilating into light lepton-antilepton pairs, and show that the virtual internal bremsstrahlung signal may be enhanced by a factor of (up to) two orders of magnitude. We discuss the scope and possible generalizations of the model.

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I. INTRODUCTION

The nature of Dark Matter (DM), which is supposed to account for about 80% of all mass in the universe, is one of the big mysteries of physics. It is also one of the strongest indication for possible physics beyond the Standard Model (SM) of particle physics. Indeed the dominant view is that dark matter is made of new, neutral and stable (or very long-lived) particles. Among the plethora of possible DM candidates, weakly interacting massive particles or WIMPs have many attractive features. First and foremost their abundance may be naturally explained through thermal freeze-out, a mechanism that is very robust, insensitive to unknown, higher scale physics, and points to an almost unique prediction for the annihilation cross section of DM in the early universe, \( \langle \sigma v \rangle \sim 10^{-26} \text{cm}^3 \cdot \text{s}^{-1} \). This feature also paves the way for the strategies for the identification of DM, provided that DM annihilates into –and thus interact with– SM degrees of freedom: at colliders, through direct detection, using low background detectors, or through indirect detection, which is the topic of the present work.

Indirect detection rests on the possibility that DM, which supposedly is accumulated in various parts of the universe, to begin with the central region of our own galaxy, may annihilate into SM particles or messengers, thus contributing to the cosmic flux of particles that reach the Earth or its vicinity. One important issue with this search strategy is that the bulk of the cosmic rays is expected to be of astrophysical origin and thus is somewhat uncertain. This, combined with the fact that the spectral energy distribution of messengers produced from DM annihilation is generically featureless, somewhat limits our ability to non-ambiguously identify DM – of course we may use the data to set exclusion limits and, in practice we do so, since there is no yet any clear signal of DM from the sky.

Possible exceptions to this rule of thumb is offered by so-called smoking guns, that is signatures that in principle have no counterpart of astrophysical origin. An important instance for our purpose is a gamma ray line (i.e. a monochromatic photon) from DM annihilation into \( \gamma\gamma \) \[^1\] or \( Z\gamma \) \[^3\] (see \[^4\] for a recent review). Gamma ray lines are actively being searched, most notably by the Fermi satellite \[^5\] and the HESS telescope \[^6\], and again the absence of signal has so far only permit to set exclusion limits. Since DM is neutral, its annihilation in \( \gamma\gamma \) should proceed through radiative corrections, and so is \( a \text{ priori} \) suppressed by a factor \( \propto \alpha^2 \lesssim 10^{-4} \) compared to annihilation into fermion or gauge boson pairs, \( f\bar{f} \) or \( WW \), which not only are supposed to determine the relic abundance but also lead to a large \( \gamma \) ray continuum. However this is not a no-go theorem and following the recent claim of a possible excess of gamma rays around \( E_\gamma \sim 130 \text{ GeV} \) in the Fermi telescope data \[^7\] \[^8\], much works have been dedicated to find new ways to circumvent this conclusion \[^9\] \[^18\] (see also \[^19\] \[^24\]). Despite the apparent fading of the significance of the signal \[^5\], we believe that it remains of interest to look for further alternative scenarios.

In the present work, we specifically focus on virtual internal bremsstrahlung (VIB) as a possible way of producing an enhanced, sharp gamma ray spectral feature from DM annihilation \[^25\] \[^26\]. VIB is a process by which a gamma ray in the final state is, roughly speaking, emitted by a charged virtual particle \(^1\). While being suppressed by a factor of \( \alpha \), it may actually dominate DM annihilation if the two-body process is suppressed. The canonical example is the case of two Majorana particles \( \chi \) annihilating into a pair of light

[^1]: Distinction from of soft photons through final state radiation (FSR) may be made in a gauge invariant way \[^27\].
fermions \( f \bar{f} \). Because of Pauli principle, annihilation of the Majorana particles must be either in a s-wave spin 0 state or p-wave spin 1. In the chiral limit \( m_f = 0 \), the latter case is the only possibility as the effective coupling between the pair of \( \chi \) and the \( f \bar{f} \) is of the pseudo-vector type (which implies that the final state is \( J = 1 \)). So annihilation of the Majorana pair is either chirally or p-wave suppressed, \( \sigma_{f\bar{f}v} \propto m_f^2 \) or \( \propto v^2 \) respectively [28].

While of little practical importance for freeze-out in the early universe, for which \( v^2 \sim 0.24 \), the suppression is dramatic at the galactic center, where \( v \sim 10^{-3} \). On the contrary, the emission of a photon in the final state allows for \( J = 0 \), so 3-body annihilation may proceed in the s-wave channel, which is by far the dominant process at the galactic center.

This beautiful mechanism has been first proposed within the framework of the MSSM [25], but since then has been considered and studied further in more general terms (see e.g. [27, 29–33]). In particular, a very simple scenario has been proposed in [34] in an attempt to explain the putative 130 GeV Fermi excess. The dark matter candidate is a right-handed neutrino, and the signal is annihilation into lepton-antilepton pairs together with a gamma. In this work, it has been shown that both the relic abundance and the Fermi measurement could be simultaneously explained, assuming a slight \( O(10) \) astrophysical boost of the gamma ray signal. In the present work, we consider a scalar dark matter candidate instead, with properties which are otherwise very similar to those of the heavy neutrino of [34], hence we will adopt this instance as a benchmark to which to compare our model. The basic facts we will use is that the annihilation of a real scalar DM candidate into fermion-antifermions pair is either s-wave but chirally suppressed \( \sigma_{f\bar{f}v} \propto m_f^2 \), like the Majorana case, or d-wave suppressed, \( \sigma_{f\bar{f}v} \propto v^4 \). The extra velocity suppression compared to the Majorana case may seem harmless if we consider annihilation in the early universe, but will show otherwise. In particular we will show that VIB annihilation may give to very significant annihilation into gamma rays for candidates that match the measured cosmic abundance. For the sake of the argument we will consider a very simple toy model, and limit ourselves to a leptophilic scalar DM candidate.

We begin with a discussion of the basic features of the model, including its annihilation into lepton-antilepton pairs. Next we give some details of our calculation of its 3-body annihilation in the VIB channel and compare the results with the Majorana case. In particular we show that the VIB signal is strongly enhanced in the scalar case compared to the Majorana case. In the last section we discuss the possible generality of this results, possible drawbacks, and prospects. We finish with some conclusions.

II. THE MODEL

The model we consider is very simple. It consists of a real scalar particle, \( S \), which we take to be leptophilic for the sake of our argument. By this we mean that it has Yukawa couplings only to SM leptons. More specifically, in this section we consider couplings to the right-handed ones \( (l_R) \). We will discuss other possibilities in Sec. IV D. We also introduce heavy vector-like leptons \( (\Psi) \). At this stage it does not matter whether there is one or many (like one per SM generation) heavy leptons. Their Yukawa interactions are thus of the form

\[
\mathcal{L} \supset y_l S \bar{\Psi} l_R + h.c.\ ,
\]

In this specific instance the \( \Psi \) are thus \( SU(2)_L \) singlet and obviously their hypercharges are equal \( Y_\Psi = Y_{l_R} \). We want to consider the possibility that the \( S \) is a dark matter candidate.
FIG. I: Diagrams contributing to the annihilation amplitude (t and u channels) through the vector-like portal.

To insure its stability, we assume that the full Lagrangian is invariant under a discrete $Z_2$ symmetry,

$$S \rightarrow -S$$

and

$$\Psi \rightarrow -\Psi$$

while all the SM fields are taken to be even under $Z_2$. We will further assume that the $Z_2$ symmetry is unbroken and the lightest odd particle is the scalar, $M_\Psi > M_S$. Hence, in this model, the annihilation of DM into SM fields goes through heavy vector-like fermions exchange. Such an interaction has already been considered in [35] when studying scalar dark matter candidates in the MeV range. Following [36], we call this scenario the vector-like portal. Notice that, unlike in [36], the scalar field is taken to be a real singlet, like in other simple models of non-fermionic dark matter [37–40]. Being a scalar singlet $S$ has also renormalizable coupling to the SM scalar (aka the Higgs) portal,

$$\mathcal{L} \supset \frac{\lambda S^2}{2} |H|^2.$$  \hspace{1cm} (2)

We begin by assuming that this coupling is subdominant, but we comment on relaxing this condition in Sec. [IVD]

A. Two-body annihilation

This being laid down, we now consider the annihilation process

$$S(k_1) \ S(k_2) \rightarrow l(p_1) \ \bar{l}(p_2).$$  \hspace{1cm} (3)

For reasons that will become clear, we give a rather pedestrian derivation of a the annihilation amplitude. From Fig. [I] the t and u channels amplitudes are given by

$$M_{ff}^{(t)} = y_t^2 \bar{u}(p_1)P_L \frac{1}{\not{p}_1 - \not{k}_1 - M_\Psi} P_R v(p_2)$$

and

$$M_{ff}^{(u)} = y_t^2 \bar{u}(p_1)P_L \frac{1}{\not{p}_1 - \not{k}_2 - M_\Psi} P_R v(p_2)$$
respectively where $P_{R(L)}$ is the projector on right (left) helicity. The total amplitude reads

$$\mathcal{M}_{f\bar{f}} = \frac{1}{2} y_f^2 \bar{u}(p_1) P_L \left[ (\kappa_2 - \kappa_1)(D_{11} - D_{22}) + (\phi_2 - \phi_1)(D_{11} + D_{22}) \right] v(p_2)$$

where

$$D_{ij} = \frac{1}{(k_i - p_j)^2 - M^2_{\Psi}}.$$  

Here and in the next section we adopt the notations of [30]. Using the equations of motion for the fermion and the antifermion, this becomes

$$\mathcal{M}_{f\bar{f}} = \frac{1}{2} y_f^2 \bar{u}(p_1) \left[ P_L(\kappa_2 - \kappa_1)(D_{11} - D_{21}) - m_f(D_{11} + D_{21}) \right] v(p_2).$$  

(4)

The first term is velocity suppressed, while the second term is proportional to the fermion mass. Neglecting terms with powers of $m_f$ larger than one, using

$$D_{11} - D_{21} = D_{11}D_{21} \times 2p_1 \cdot (k_2 - k_1)$$  

(5)

and

$$D_{11} + D_{21} = D_{11}D_{21} \left( 2M^2_S - 2M^2_{\Psi} - 2p_1 \cdot p_2 \right),$$

(6)

we get

$$\mathcal{M}_{f\bar{f}} = \frac{1}{2} y_f^2 \bar{u}(p_1) \left[ P_L(\kappa_2 - \kappa_1) 2p_1 \cdot (k_1 - k_2) - m_f \left( 2M^2_S - 2M^2_{\Psi} - 2p_1 \cdot p_2 \right) \right] v(p_2) D_{11}D_{21}.$$  

(7)

The interesting point is that in the chiral limit, $m_f \to 0$, the amplitude squared becomes proportional to $(k_2 - k_1)^2 \propto v^2$, where $v$ denotes the relative velocity of the annihilating $S$ particles. Assuming that $m_f$ negligible and working in the non-relativistic limit relevant for annihilation of WIMPs, we get at leading order in $v$

$$\sigma v(SS \to l\bar{l}) = \frac{y_f^4}{60\pi} \frac{v^4}{M^2_S} \frac{1}{(1 + r^2)^4}$$

(8)

for the annihilation of $S$ into a light lepton-antilepton pair. Here $r$ refers to the ratio of masses $M_{\Psi}/M_S$.

The suppression by $v^4$ in the chiral limit is a bit unusual. For the sake of comparison, the corresponding annihilation of a pair of a gauge singlet Majorana into light leptons through a heavy charged scalar ($\Phi$), with coupling

$$\mathcal{L} \supset g_\Phi \Phi^\dagger \chi l_R + h.c.$$  

(9)

is given by

$$\sigma v(\chi \chi \to l\bar{l}) = \frac{y^4}{48\pi} \frac{v^2}{M^2_\chi} \frac{1 + r^4}{(1 + r^2)^4}$$

(10)

which shows the usual p-wave suppression $\propto v^2$ and $r = M_{\Phi}/M_{\chi}$ (see also [33] Eq.A.5). In the following, $r$ will always refer to the ratio of the mass of the next-to-lightest particle odd under the $Z_2$ (NLZP) symmetry divided by the dark matter mass and it is always larger.
than one. For identical Yukawa couplings, DM masses and ratios $r$, we have the following ratio of the averaged cross sections into lepton-antilepton pairs

$$\frac{\langle \sigma v \rangle_{\nu|\sigma_{\nu}|S}}{\langle \sigma v \rangle_{\nu|\sigma_{\nu}|\chi}} = \frac{4 \langle v^4 \rangle}{5 \langle v^2 \rangle (1 + r^4)} \lesssim 0.16$$

where the bound is obtained assuming $v^2 = 0.24$ and $r = 1$. In other words, for identical DM masses, assuming thermal freeze-out, it takes a larger Yukawa coupling for a scalar $S$ than for a Majorana $\chi$ to match the observed relic abundance of DM. This behavior will be studied in more details in Sec. IV.

### B. Chiral suppression from an effective operator perspective

The $d$-wave, $\propto v^4$, suppression of the annihilation cross section of real scalars in fermion-antifermion pairs is easy to explain. The annihilation of a pair of scalar in a $s$-wave corresponds to a CP even state. Thus, the fermion-antifermion final state must be described by a CP even scalar bilinear operator e.g. $\psi_f \bar{\psi}_f$. However the Yukawa interaction of (1) involves chiral (here right-handed) SM fermions. Hence, the $s$-wave annihilation is chirally suppressed. In other words, the amplitude of (1) derives from the dimension 5 operator:

$$O_S = m_f S^2 \bar{l} l.$$  (12)

In principle, we could have annihilation in a $p$-wave, at least based on the constraint from CP. However there is no CP-odd bilinear operator involving two identical real scalars (i.e. no current) hence the next possibility is a dimension 8 operator of the following form

$$O_T = \partial_\mu S \partial_\nu S \Theta_R^{\mu\nu}$$  (13)

where $\Theta_R^{\mu\nu}$ is the stress-energy tensor of the Dirac field $l_R$,

$$\Theta_R^{\mu\nu} = \frac{i}{2} l_R (\gamma^\mu \partial^\nu - \gamma^\nu \partial^\mu) l_R$$

Clearly the (traceless part of) $\Theta_R^{\mu\nu}$ has $J = 2$ which implies that the annihilation of a pair of real scalars is $d$-wave suppressed in the chiral limit.

The tentative conclusion of this section is that 2-body annihilation of a pair of real scalar into SM fermions is suppressed compared to the case of a Majorana particle. We will show that this will have concrete implications.

### III. VIRTUAL INTERNAL BREMSSTRAHLUNG

In this section we turn to radiative processes, and in particular to internal bremsstrahlung. This is of interest for two reasons. First, as it is well-known, the annihilation cross section in the $s$-wave is no longer suppressed, which implies that the 3-body final state process may be more important than the 2-body process, despite the suppression of the former by a factor $O(\alpha/\pi)$. This is typically the case for annihilation in light fermions (e.g. leptons) and when the relative velocity is non-relativistic, like at the galactic center ($v \sim 10^{-3}$).
FIG. II: Diagrams contributing to the amplitudes for \( SS \rightarrow \bar{u}l\gamma \) process.

Second, the emission from (essentially) the virtual intermediate particle, depending on the ratio of its mass and that of DM, may show a sharp spectral feature, which may even mimic a monochromatic gamma ray line. As mentioned in the introduction, the literature on this topic is vast. In this section we specifically follow the detailed and pedagogical approach of [30] to derive the annihilation cross section of the scalar \( S \) to \( \mathcal{O}(\alpha) \). We warn the reader that 1/ the result is disappointingly simple – the spectrum of gamma rays and leptons/antileptons is the same as in the case of a Majorana particle– and 2/ this fact was already discussed in the literature [32] (see also [41] in the case of light dark matter). Our excuse to re-iterate is that we believe that it is of some interest of being a bit more detailed (and compare to the calculation of the Majorana case of, say, [30]) and also that the precise normalization is of prime importance for indirect searches – so that it is perhaps worth providing an independent check.

A. Derivation of 3-body cross-section

The relevant \( t \)-channel diagrams are shown in Fig. II There are also the three exchange diagrams (\( u \)-channel), which add up in the scalar case (while they subtract in the Majorana case). As before, \( k_1 \) and \( k_2 \) denote the momenta of the annihilating dark matter particles, \( p_1 \) and \( p_2 \) denote the momenta of the outgoing leptons and we write \( k \) the momentum of the photon,

\[
S(k_1) S(k_2) \rightarrow \bar{I}_R(p_1) I_R(p_2) \gamma(k)
\]

and write

\[
\mathcal{M} \cdot \epsilon^* = e g^2 L \left[ \mathcal{M}^{(t)}_a + \mathcal{M}^{(u)}_a + \mathcal{M}^{(t)}_b + \mathcal{M}^{(u)}_b + \mathcal{M}^{(t)}_c + \mathcal{M}^{(u)}_c \right]
\]

where the \( t,u \) superscript refers to the \( t,u \)-channel diagram. Having in mind s-wave annihilation, we may set \( k_1 = k_2 = K \), so that in this limit and after some obvious manipulations, the first two amplitudes, which are associated to inner emission, may be written as

\[
\mathcal{M}^{(t)}_a = \bar{u}(p_1) P_L \left[ (M_F^2 + M_S^2) \epsilon^* - 2 \epsilon^* \cdot KK \right] v(p_2) D_1 D_2 \equiv \mathcal{M}^{(u)}_a
\]

where

\[
D_i = \frac{1}{(p_i - K)^2 - M_W^2}.
\]

Similarly the final state radiation amplitudes may be written as

\[
\mathcal{M}^{(t)}_b = \bar{u}(p_1) P_L \left[ 2 \epsilon^* \cdot p_1 + \epsilon^* \cdot \bar{K} \right] \bar{K} v(p_2) D_2 D(p_1 + k) \equiv \mathcal{M}^{(u)}_b
\]
where the final state fermion propagator
\[ D(p_1 + k) = \frac{1}{(p_1 + k)^2 - m_f^2} = \frac{1}{2p_1 \cdot k} \]
displays the usual infrared divergent behaviour (here we work in the chiral limit \( m_f = 0 \)), and
\[ \mathcal{M}_c^{(t)} = \bar{u}(p_1) P_L \bar{K} [2p_2 \cdot \epsilon^* + \epsilon |k|] v(p_2) D_1 D(p_2 + k) \equiv \mathcal{M}_c^{(u)}. \]
Now it is easy to see that the potentially IR divergent pieces cancel from the FSR (Eqs. (15) and (16)) amplitudes. Using 2\( K = p_1 + p_2 + k \) and the equation of motion \( \bar{u}(p_1) \gamma_\mu = 0 \), \( \bar{p}_2 v(p_2) \) and \( k^2 = 0 \), the \( \mathcal{M}_b \) amplitude reduces to
\[ \mathcal{M}_b = \bar{u}(p_1) P_L [2\epsilon^* \cdot p_1 \bar{k} + \epsilon^* \bar{\epsilon} \bar{p}] v(p_2) D_2 D(p_1 + k) = \bar{u}(p_1) P_L \epsilon^* v(p_2) D_2. \]
Similarly the \( \mathcal{M}_c \) amplitude is simply given by
\[ \mathcal{M}_c = \bar{u}(p_1) P_L \epsilon^* v(p_2) D_1. \]
Using
\[ D_1 + D_2 = -(2M_\Psi^2 - 2M_\Sigma^2 + 2K \cdot (p_1 + p_2)) D_1 D_2, \]
we may combine the six amplitudes to get
\[ \mathcal{M}_{\text{tot}} = \bar{u}(p_1) P_L (k \cdot (p_1 + p_2) \epsilon^* - \epsilon^* \cdot (p_1 + p_2) \bar{k}) v(p_2) D_1 D_2. \]
The total amplitude is manifestly gauge-invariant (\( \mathcal{M}_{\text{tot}} = 0 \) for \( k \rightarrow \epsilon^* \) as it should be), but the derivation makes clear that both the internal and FSR processes are necessary for this to occur. It is also IR divergence free, as expected on general grounds (see [27]). Indeed, IR divergences in FSR, which are \( \mathcal{O}(\alpha) \) are expected to cancel with similar contributions from the interference of a tree level amplitude and its \( \mathcal{O}(\alpha) \) one-loop radiative corrections (see e.g. [42]). In the case at hand, just like in the case of the annihilation of Majorana particles, the tree level amplitude in an s-wave initial state vanishes in the chiral limit.

B. Spectrum and cross section dependence in \( r \) for VIB

The amplitude (18) bears little resemblance to the one of the Majorana case (see e.g. Eq.(4.12) in [30]). Yet it gives rise to precisely the same gamma ray spectrum. Defining the 3-body annihilation cross section as
\[ \sigma_{2 \rightarrow 3} = \frac{|\mathcal{M}|^2}{128 \pi^3} dx dy \]
where \( v = \sqrt{k_1 \cdot k_2 - m_1 m_2} / E_1 E_2 \) refers to the relative velocity of the \( S \) particles and \( x, y \) are the reduced energy parameters \( x = 2E_\gamma / \sqrt{s} \) and \( y = 2E_f / \sqrt{s} \), with \( s \) the Mandelstam variable corresponding to the center-of-mass energy squared, we obtain the following amplitude squared for the 3-body annihilation with emission of a photon
\[ |\mathcal{M}_{\gamma}|^2 = \frac{32 \pi \alpha y f^4}{M_\Sigma^2} \frac{4(1 - y)(2 + 2x^2 + 2x(y - 2) - 2y + y^2)}{(1 - r^2 - 2x)^2(3 + r^2 - 2x - 2y)^2} \]
with \( r = M_\Phi / M_\chi \), in agreement with [32].

As above, it is of interest to compare this expression to the one obtained in the same limit in the Majorana case (see [32] and also Eq.A.1 in [33])

\[
\frac{1}{4} \sum_{\text{spin}} |M_\chi|^2 = \frac{4 \pi \alpha g_l^4}{M_\chi^2} \frac{4(1 - y)(2 + 2x^2 + 2x(y - 2) - 2y + y^2)}{(1 - r^2 - 2x)^2(3 + r^2 - 2x - 2y)^2}
\]  

(21)

where \( r = M_\Phi / M_\chi \). Clearly the dependence on \( x \) and \( y \) are precisely the same. It is interesting to notice that, all other things being kept constant, the cross section is larger by a factor of 8 in the scalar case compared to the Majorana case. A factor of 4 comes clearly from the spin average. The extra factor of 2 is related to the projection of the Majorana pair into a spin zero initial state (i.e. there is a factor of \( 1/\sqrt{2} \) in the amplitude). Let us emphasize that the relevant normalization scalar versus Majorana is not important for our argumentation, what matters is 2-body versus 3-body.

![Figure III](image)

**FIG. III:** Spectra \( dN_\gamma/\gamma dx = d\log[\sigma_\gamma]/dx \) as a function of \( x = E_\gamma / M_{dm} \) for several values of \( r \).

In Fig. III we show the dependence of the photon spectrum

\[
\frac{dN_\gamma}{dx} = \frac{M_{dm}}{\sigma_{\gamma ll}} \frac{d\sigma_{\gamma ll}}{dE_\gamma}
\]

(22)

in the parameter \( r \). The spectra have been obtained integrating the amplitude [20] over \((1 - x) \leq y = E_\gamma / M_{dm} \leq 1\). We see that for larger values of \( r \) the amplitude of the spectra is lower but the spread is larger. The sharp feature in the spectrum also moves to smaller \( E_\gamma < M_{dm} \) with increasing \( r \).

Let us emphasize that \( dN_\gamma/\gamma dx \) is independent of the scalar/Majorana nature of the dark matter candidate and this concordance has already been elucidated in [32]. It is related to
the fact that the scalar and Majorana initial states, while both \( L = S = 0 \), differ only by their parity, the scalar case being clearly CP even. We have nothing new to add here, but merely repeat their argument, which stems from the fact that the amplitude \( \text{(18)} \) may be derived from the effective operator

\[
\mathcal{O}_S = \left( \partial_\mu \bar{l}_R \gamma_\nu l_R + \bar{l}_R \gamma_\nu \partial_\mu l_R \right) F^{\mu\nu}
\]

while in the Majorana case it is related to

\[
\mathcal{O}_\chi = \left( \partial_\mu \bar{l}_R \gamma_\nu l_R + \bar{l}_R \gamma_\nu \partial_\mu l_R \right) \tilde{F}^{\mu\nu},
\]

where \( \tilde{F}^{\mu\nu} \) is the dual of \( F^{\mu\nu} \), which only amounts to exchanging the role of the \( \vec{E} \) and the \( \vec{B} \) of the photon and thus gives rise to the same spectrum.

Although the 3-body cross section \( \sigma_{\gamma ll} \) differ for Majorana and scalar dark matter by numerical factors, we know that the \( r \) dependence is universal and, in the appendix, we give a formulation of \( \langle \sigma v \rangle_{\gamma ll} \) that emphasize this fact. Notice that a useful approximate expression of \( \langle \sigma v \rangle_{\gamma ll} \) has also been derived previously in [27].

C. Annihilation into \( \gamma \gamma \) versus VIB

For the sake of completeness, and because of their close relation to internal Bremsstrahlung, we discuss in this section the annihilation of DM in gamma ray pairs. As is well-know, the s-wave annihilation of a pair of Majorana particles in monochromatic gamma rays may be derived from a chiral anomaly argument [2, 25]. Concretely, in the chiral limit and for \( r = M_\phi / M_\chi \gg 1 \), the amplitude for annihilation may be obtained by simply replacing, in the box diagram corresponding \( \chi \chi \rightarrow \gamma \gamma \), the scalar propagator by an effective contact interaction between the Majorana particles and the light fermions. Since this effective coupling is of axial-vector type, the resulting triangular diagram, which only involves light fermions, is precisely the one that arises in the derivation of the chiral anomaly. This implies in particular that the amplitude is non-vanishing even for massless fermions. For \( r > \sim 1 \) and in the chiral limit, the annihilation cross section is simply given by [2]

\[
\sigma v(\chi \chi \rightarrow \gamma \gamma) = \frac{\alpha^2}{64\pi^3} \frac{g^4}{M_\chi^2} \frac{1}{r^4}.
\]

The general expression, still in the chiral limit but valid for all \( r \) (including \( r = 1 \)) as obtained from the calculation of the box diagrams [25], is given for reference in the appendix.

Naively we would expect a similar result to hold for the annihilation of scalar particles, with the proviso that the initial state is CP even in this case, while it is CP odd for Majorana particles, thus by replacing the chiral anomaly with the trace anomaly. Concretely, in the limit \( r = M_\psi / M_S \gg 1 \), we would replace the heavy fermion propagator in the box diagrams by the effective contact interaction of \( \text{(13)} \) and then use the trace anomaly, \( \Theta_\mu^\alpha \propto F_\alpha^\beta F^{\alpha\beta} \) to estimate the annihilation into gamma ray lines. The most immediate consequence of this argument is that the annihilation amplitude should be finite in the chiral limit, just like in the Majorana case.\(^2\) While this turns out to be the case, the argument seems to be incorrect

\(^2\) As a way of comparison, notice that this is very different from the contribution of light fermions to the decay of the Higgs in two photons, which vanishes in the chiral limit, see e.g. [13].
or, at the very least, it does not give the dominant contribution to the process. Indeed
the trace anomaly would lead to a cross section that scales like $M^{-8}_\psi$ (the effective operator of \[13\] is dimension 8), while a calculation of the full box diagram gives a results which actually scales like $M^{-4}_\psi$, again as in the Majorana case. More precisely, following the result quoted in \[17\], which is based on a calculation made in \[44\], we get in the chiral limit and for $r \gg 1$,

$$\sigma_v(SS \rightarrow \gamma\gamma) = \frac{\alpha^2}{36\pi^3} \frac{y_l^4}{M^2_S} \frac{1}{r^4},$$

which, all other things being kept constant, differs from the corresponding expression in the Majorana case simply by a factor of $16/9$. We are currently re-doing the calculations of the box diagram made in \[44\], for its own sake of and for the possible implications for direct detection (see next section). For the time being, we tentatively conclude that there is a subtle distinction between the Majorana and scalar cases. This is further illustrated by the fact that the exact dependence of $\sigma_v(SS \rightarrow \gamma\gamma)$ on $r$, which is given for reference in the appendix, diverges at $r = 1$ in the chiral limit, as illustrated in Fig. IV.

Using these results, we show as an illustration the differential photon spectrum associated to the $\gamma\gamma$ (blue lines), $\gamma Z$ (green lines) and $\gamma\ell\bar{\ell}$ (red lines) channels as well as their sum (grey lines) in Fig. V. The spectra have been normalized by the sum of the annihilation cross-sections into the three final states, i.e.

$$\frac{dN_i}{dx} = \frac{1}{\langle \sigma v \rangle_\gamma} \frac{d\langle \sigma v \rangle_i}{dx} \quad \text{for} \quad i = \gamma\gamma, \gamma Z, \gamma\ell\bar{\ell}$$

(23)

where $x = E_\gamma/M_{dm}$ and $\langle \sigma v \rangle_\gamma = \sum_i \langle \sigma v \rangle_i$. We have chosen $M_{dm} = 100$ GeV and we have considered annihilation into one single lepton specie, with the Yukawa coupling set to one. For these parameters, we have obtained the cross sections $\langle \sigma v \rangle_{\gamma\gamma}$ and $\langle \sigma v \rangle_{\gamma\ell\bar{\ell}}$ listed in Table I.

FIG. IV: $\langle \sigma v \rangle_{\gamma\gamma}$ for Majorana and real scalar pairs as a function of $r$ in the chiral limit $m_f = 0$ (the cross sections are given for $y_l = g_l = 1$ and $M_{dm} = 100$ GeV).
FIG. V: Normalized spectra \( \frac{dN}{dx} \) of Eq. (23) as a function of \( x = E_\gamma / M_{dm} \) for two values of \( r = 1.1 \) (left) and 2.0 (right) for scalar (continuous lines) and Majorana (dotted lines) dark matter with \( M_{dm} = 100 \text{ GeV} \). The spectra have been convoluted with a gaussian kernel assuming a relative energy resolution \( \Delta E/E = 0.1 \).

| Majorana | Scalar |
|----------|--------|
| \( r = 1.1 \) | \( r = 2 \) |
| \( \langle \sigma v \rangle_{\gamma\ell} \) | \( 1.2 \times 10^{-27} \) | \( 1.3 \times 10^{-29} \) |
| \( \langle \sigma v \rangle_{\gamma\gamma} \) | \( 4.1 \times 10^{-29} \) | \( 3.0 \times 10^{-30} \) |

TABLE I: Cross-sections in units of \( \text{cm}^3/\text{s} \) for \( M_{dm} = 100 \text{ GeV} \) and \( y_\ell = g_\ell = 1 \)

We have estimated the \( \langle \sigma v \rangle_{\gamma Z} \) cross-sections using

\[
\langle \sigma v \rangle_{\gamma Z} \simeq 2 \tan[\theta_W]^2 \left( 1 - \left( \frac{M_Z}{2M_{dm}} \right)^2 \right)^3 \langle \sigma v \rangle_{\gamma\gamma},
\]

with \( \theta_W \) is the Weinberg angle, as in the case of Higgs decay into two photons or \( \gamma Z \) (see e.g. [43]). From these numbers, we see once more that, for a fixed set of model parameters \( M_{dm}, y_\ell = g_\ell \) and \( r \), the radiative processes are always larger for scalar particles, especially for Bremsstrahlung.

The photon raw spectra for the 3-body final state has been obtained in Sec. III B while in the \( \gamma\gamma \) case the spectra is just a delta function at \( E_\gamma = M_{dm} \), multiplied by two to account for photon multiplicity. For the \( \gamma Z \) final state, one expects a broader feature, due to the \( Z \) width, around \( M_{dm}(1 - M_Z^2/4M_{dm}^2) \). We follow [21, 44] that described the resulting photon spectrum with an intrinsic width that depends on the final state massive boson. All raw spectra are then convoluted with a gaussian kernel in order to account for the finite energy resolution of the detector. In Fig. V we consider a relative energy resolution of \( \Delta E/E = 0.1 \).

From the normalized spectra presented in Fig. V the differences between Majorana and scalar dark matter are not obvious. The general form of the spectra is actually very similar for both scalar (continuous curves) and Majorana (dotted curves) particles especially for \( r \approx 1 \) in which case the Bremsstrahlung drives the main features of the full spectrum. For
In the Majorana case a double line structure becomes more distinguishable (in the Majorana case, we have obtained results which are consistent with those of [45]). Notice that, for a more complete description of the photon spectrum, one should also include the continuous gamma spectrum from the $Z\gamma$ line as well as from the production of the final state leptons, in particular in the case of tau leptons. This however affects the spectra for small energies only, and this only slightly, see for instance [7].

IV. POSSIBLE PHENOMENOLOGICAL IMPLICATIONS

In this section, we first discuss the implication of d-wave suppression of the annihilation cross section on the dark matter relic abundance. This will lead us to the conclusion that larger Yukawas are needed in the scalar dark matter case than in the Majorana one in order to account for the total amount of dark matter. This in turn will imply that the cross section for radiative 3-body processes can become as important as the 2-body process in the scalar case. Finally in order to illustrate our findings we present a numerical analysis comparing the viable parameter space for the simple scalar and Majorana dark matter models defined in Sec. II.

A. Relic abundance for d-wave suppressed annihilation

To begin with, we consider the simplest scenario and assume that the relic abundance of pair of scalar in the early universe is determined by the d-wave suppressed 2-body process discussed in Sec. II. The thermally averaged cross section is given by

$$\langle \sigma v \rangle = \sum_f \frac{y_f^4 \langle v^4 \rangle}{60\pi M_S^2} \frac{1}{(1 + r^2)^4}.$$ 

Following [46] (or [47]) and using

$$\langle \sigma v \rangle = \frac{1}{60\pi} \frac{d^3k_1 d^3k_2 e^{-(E_1+E_2)/T} \sigma v}{d^3k_1 d^3k_2 e^{-(E_1+E_2)/T}} = \sqrt{\frac{x^3}{4\pi}} \int_0^\infty dv e^{-x(v^2)/4} \langle \sigma v \rangle$$

with $x = M_{dm}/T$, we get

$$\langle v^2 \rangle = \frac{6}{x_f} \quad \text{and} \quad \langle v^4 \rangle = \frac{60}{x_f^2}.$$ 

Using $x_f = 25$ for the relative temperature at freeze-out one gets $\langle v^2 \rangle = 0.24$ and $\langle v^4 \rangle = 0.1$. One has also to take into account the fact that the velocity has a slight numerical impact on the $x_f$ dependence of the relic abundance. If the thermal averaged cross section scale like $x^{-n}$, with $n = 0, 1, ..$ for s-wave, p-wave,... dominated cross section, taking into account that the Boltzmann equation for the relic abundance for $x > x_f$ takes the form

$$\frac{dY_{dm}}{dx} = -\frac{\lambda}{x^{2+n}} Y_{dm}^2$$

where $\lambda$ is a constant, and $Y_{dm}$ denote the comoving dark matter number density then [48]

$$Y_{dm} \approx \frac{(n+1)x_f^{n+1}}{\lambda}.$$
Hence, to reach the same relic abundance $\Omega_{\text{dm}} h^2$, the averaged annihilation cross section $\langle \sigma v \rangle$ in for instance a pure d-wave channel must be larger by a factor of $n + 1 = 3$ wrt the s-wave case i.e. $\langle \sigma v \rangle_{d-\text{wave}} \approx 9 \cdot 10^{-26} \text{ cm}^2\text{s}^{-1}$. Considering the velocity expansion of the annihilation cross section to the next order compared to e.g. \cite{48, 49}, writing $\sigma v = a + bv^2 + cv^4$, we have

$$\Omega_{\text{dm}} h^2 \simeq \frac{1.07 \times 10^9 x_f}{M_{pl}/\text{GeV} \sqrt{g_*} (a + 3b/x_f + 20c/x_f^2)}$$  \hspace{1cm} (25)$$

where $M_{pl} = 1.22 \times 10^9$ GeV is the Planck mass and $g_*$ is the number of relativistic degrees of freedom at the time of freeze-out.

Lest the reader think that we are splitting hairs, consider again the ratio of \cite{11} but now expressed in terms of the relic abundances,

$$\frac{\Omega_{\text{dm}} \big|_S}{\Omega_{\text{dm}} \big|_\chi} = \frac{\sum_l g_l^4 (5 \times 3) (1 + r^4) x_f^2}{\sum_l y_l^4 (4 \times 20) x_f} = \left( \frac{\sum_l g_l^4}{\sum_l y_l^4} \right) \frac{3}{16} x_f (1 + r^4).$$

The factor that multiplies the couplings is typically $O(10)$, hence larger Yukawa couplings are required to reach same abundance for the scalar than for the Majorana. It is pretty clear that this implies larger Bremsstrahlung emission in the case of scalar DM.

**B. Enhanced three-body processes for scalar dark matter**

In the Majorana case, the 3-body annihilation cross section is always small compared to the 2-body one, which is relevant for the abundance in the early universe. The ratios
\[ \frac{\langle \sigma v \rangle_{\gamma l}/\langle \sigma v \rangle_u}{\langle \sigma v \rangle_u} \] are shown in the left hand side of Fig. [VI] for Majorana (blue dashed line) and scalar (black continuous line) dark matter. The limiting values for \( r \to 1 \) are

\[ \left. \frac{\langle \sigma v \rangle_{\gamma l}}{\langle \sigma v \rangle_u} \right|_x \approx 0.015 \quad \text{and} \quad \left. \frac{\langle \sigma v \rangle_{\gamma l}}{\langle \sigma v \rangle_u} \right|_S \approx 0.76 \] (26)

taking \( x_f = 25 \). It is by itself remarkable that, for the scalar candidate, the radiative process may be almost as important as the 2-body one. We will come back to this in the next section. In the meantime we may define the “boost factor” (BF)

\[ BF = \frac{\langle \sigma v \rangle_{\gamma l}}{\langle \sigma v \rangle_u} \bigg|_S \times \frac{\langle \sigma v \rangle_{\gamma l}}{\langle \sigma v \rangle_u} \bigg|_x , \] (27)

that is equal to 50 for \( r \to 1 \) and gives the relative enhancement of the Bremsstrahlung signal of the scalar DM candidate compared to the Majorana one. It should be clear from the behaviour of the cross section that \( BF = 50 \) is actually a minimum and this is further illustrated in the right hand side of Fig. [VI].

The enhancement of the Bremsstrahlung signal from a scalar WIMP is our main result, but as such it is of no immediate use, as other processes may determine the relic abundance. In particular one has to take into account co-annihilation processes, which are important in the case of nearly degenerate particles, \( r \gtrsim 1.1 \) [19]. Also a singlet scalar candidate may have renormalizable coupling to the SM scalar (SMS). To study these effects, we have implemented the scalar and Majorana models in Micromegas [50] with the help of Feynrules [51]. Our analysis is detailed in the next section.

C. Numerical analysis

We have considered the scalar and Majorana dark matter models which interaction with the SM fermions is dictated by Eqs. (1) and (9), neglecting extra interactions through the SM scalar portal from Eq. (2), i.e. setting \( \lambda_S = 0 \). In addition, we have assumed that the dark matter couples to one single lepton specie, the electron. The result of a random scan over the parameter space is shown in Figs [VII] and [VIII] see also the appendix for more details. We also give the 3-body annihilation cross section (Yukawa coupling) versus the mass of the DM in Fig. [VII] (resp. Fig. [VIII]) both for the scalar (left) and Majorana (right) candidates. All the points match the observed cosmological relic abundance. Notice that we have taken into account the bremsstrahlung contribution to the effective annihilation cross section relevant for the computation of the relic abundance. In the case of the scalar dark matter such process can modify the dark matter abundance up to a 15% for the largest values of the Yukawa couplings while in the case of the Majorana dark matter it may be safely neglected (it is always < 0.5%).

The color code corresponds to different values of \( r - 1 \), which is the relative mass difference between the DM particle and the heavy charged particle. The blueish points (roughly the lower right points in the plots) correspond to dark matter candidates which mass is nearly degenerate, roughly \( r \sim 1 \), with the mass of the electrically charged heavy particle (\( \Phi \) and \( \Psi \)). In the latter case, the annihilation of the charged particles

\[ \bar{\Psi} \Psi \] or \( \Phi^\dagger \Phi \rightarrow \bar{q}q, \bar{l}l, \gamma \gamma \] (28)
FIG. VII: Annihilation cross section into three-body final states $\langle \sigma v \rangle_{\gamma\ell\ell}$ for WIMP candidates (left: scalar, right: Majorana fermion) coupling to a single family of massless leptons. The lines appearing in the upper part of the plots give an upper bounds on $\langle \sigma v \rangle_{\gamma\ell\ell}$ for $r = 1.1$ associated to Fermi LAT, HESS experiments as well as future constraints from CTA and GAMMA-400 [45]. The filled diamond correspond to the benchmark models of Tab. II and the filled rectangle corresponds to the benchmark studied in [34]. The color gradient scale is associated to the values of $r - 1$.

and the co-annihilation processes

$$\bar{\Psi}S \quad \text{or} \quad \Phi^\dagger \chi \rightarrow \bar{\ell}\gamma$$

are important for the determination of the relic abundance. The greenish points (roughly the top regions) correspond to candidates for which the dark matter annihilation into lepton-antilepton pairs,

$$SS \quad \text{or} \quad \chi\chi \rightarrow \bar{\ell}\ell$$

become progressively more important and so does Bremsstrahlung. Increasing further the ratio of masses $r$, the relative weight of the 3-body process diminishes, particularly for the Majorana case, see Sec. III B. The maximal value of $r$ is reached for $M_S \approx 400$ GeV and $M_\chi \approx 1$ TeV given our assumption on the Yukawas $y_l, g_l < \pi$, see Fig. VIII. In our numerical analysis as well as writing the co-annihilation annihilation processes as in (29) and (30), we have assumed that the coupling of Eqs. (1) and (9) is made for $l \equiv e$. For dark matter coupling to three light flavours, the results of Fig. VIII should be rescaled by a factor of $3^{-1/4} \approx 0.76$. From Fig. VIII, it is clear that significant Bremsstrahlung requires rather large Yukawa couplings $y_l, g_l \sim 1$, especially for scalar candidates. This simply reflects the fact that the annihilation is d-wave for scalars and p-wave for Majorana, as discussed in Sec. II.

For a fixed value of $r$ and $r \approx 1$, one clearly distinguish two different regimes in the Yukawa-$M_{dm}$ plane. For the lowest values of the Yukawas, the dark matter models accounting for dark matter abundance are clearly independent of $y_l$ or $g_l$. At some point the Yukawa function of $M_{dm}$ begins to bend and finally reaches a regime with $\log(y_l, g_l) \propto \log(M_{dm})$. This behaviour can be understood by comparing the dependence of the processes (28)-(30) in $y_l$ or $g_l$. For the smallest values of the coupling and $r - 1$, it is the annihilation processes of $\Psi$ and $\chi$ through $\gamma$ and $Z$ boson exchanges in (28) that dominate over all other processes and play...
the major role in fixing the dark matter abundance. The corresponding cross section is $\propto g^4$, with $g$ the weak coupling, and it is independent of $y_L, g_L$ (see Appendix). Thus no Yukawa or $M_{\text{dm}}$ dependence is to be expected in this regime. For larger values of the Yukawas, the coannihilation cross section of $\propto g^2 y_L^2$ or $g^2 g_L^2$ begin to compete with charged particle annihilation and the abundance begins to depend on the Yukawa coupling. The relative importance of both of those regimes is weighted by Boltzmann factors, $\exp[-(r-1)x_f]$ and $\exp[-(r-1)x_f]^2$ for (29) and (28) respectively [49], so that the dependence in $y_L, g_L$ becomes more pronounced for larger values of $r$. For the largest values of the Yukawas, the processes which are $\propto y_L^4$ or $g^4$ fix the dark matter density. One should be aware that the standard treatment of freeze-out mechanism and coannihilation processes [49], as implemented in numerical code like Micromegas [50], rests on the assumption that the dark matter and heavy charged particles are in chemical equilibrium. Here, as in [45], we have simply checked under which conditions the processes $\chi l \leftrightarrow \Phi \gamma$ and $S l \leftrightarrow \Psi \gamma$ are in equilibrium at the epoch of thermal freeze-out: this should be the case provided $y_L, g_L > 10^{-3}$.

Taken the Bremsstrahlung spectral features seriously, one may wonder if these candidates are compatible with constraints from the current gamma ray experiments, in particular Fermi-LAT and HESS. To this end, we also report in Fig VII with a series of upper bounds on $\sigma_{\gamma l} + 2\sigma_{\gamma \gamma}$ for $r = 1.1$ that were derived in [45] using Fermi-LAT and HESS data as well as the future constraints from the GAMMA-400 satellite mission and the Cerenkov Telescope Array (CTA). For simplicity, we report the limits in the $M_{\text{dm}} - \langle \sigma v \rangle_{\gamma l}$ plane without adding the contribution from $\sigma_{\gamma \gamma}$, which are anyway negligible for $r \lesssim 2$, see Sec. III C and also [45]. As in [7], we find that the largest possible value of the 3-body cross section for a Majorana dark matter giving rise to right dark matter abundances is always substantially smaller than current limits (so that an astrophysical boost would be required to match any excess, say the possible feature around 130). In contrast, in the scalar dark matter case, within the assumptions made so far regarding e.g. $\lambda_S = 0$, one can easily cross those limits, typically
giving rise to a more important gamma-ray flux.

| Benchmarks | $y_i$ | $r$ | $\langle \sigma v \rangle_{\gamma ll}$ | $\langle \sigma v \rangle_{\gamma \gamma}$ | $\Omega_{dm}h^2$ | $R_{3bdy}$ | $R_{ann}$ | $R_{co}$ |
|------------|------|-----|----------------|----------------|----------------|-----------|---------|---------|
| Scalar     | $y_l = 1.17$ | 1.16 | $5.4 \times 10^{-27}$ | $1.3 \times 10^{-28}$ | $0.11$ | $0.06$ | $0.28$ | $0.41$ |
| Majorana   | $g_l = 0.9$ | 1.17 | $2.2 \times 10^{-28}$ | $8.9 \times 10^{-30}$ | $0.10$ | $0.002$ | $0.95$ | $0.047$ |

TABLE II: Benchmark models for dark matter candidates with $M_{dm} = 150$ GeV which VIB signal could be associated to a gamma ray excess around 130 GeV. Cross sections are given in units of cm$^3$/s, $y_i$ refers to the Yukawa couplings $y_l$ and $g_l$ and $R_{3bdy}$, $R_{ann}$ and $R_{co}$ give the relative contribution of 3-body, annihilation and coannihilation processes, respectively, effectively contributing to the relic abundance.

For the sake of illustration we consider two benchmark models (see Table II) that could be relevant for the possible excess of gamma rays around $E_{\gamma} = 130$ GeV in the Fermi-LAT data [7, 8]. Both candidates have a relic abundance $\Omega_{dm}h^2 \sim 0.1$. The relative contributions of the various processes to their annihilation cross section at freeze-out [49] are given by $R_{3bdy}$ for dark matter annihilation into the 3-body channels, $R_{ann}$ for annihilation into 2-body and $R_{co}$ for coannihilation processes, while the annihilation of the heavy charged particles contribute for $1 - R_{3bdy} - R_{ann} - R_{co}$. The dark matter mass is taken to be 150 GeV, following [7] that pointed out that the best-fit 3-body cross section is $\langle \sigma v \rangle_{3bdy}^{best} \sim 6.2 \times 10^{-27}$ cm$^3$/s with $M_{dm} \sim 150$ GeV — in the case of a gamma ray line, the best fit is for about $\langle \sigma v \rangle_{\gamma \gamma} \approx 1.27 \times 10^{-27}$ cm$^3$/s with a mass of $\sim 130$ GeV. From Fig. VII, it is clear that $\langle \sigma v \rangle_{3bdy}^{best}$ can be reached for a scalar dark matter candidate, while an extra, possibly astrophysical boost factor would be required for a Majorana candidate.

In Table II, the Majorana benchmark candidate is chosen so as to maximize the 3-body annihilation cross section. In order for this candidate to saturate the data, a boost factor of about 10 would be needed. The scalar benchmark has also $M_{dm} \sim 150$ GeV but this time the Yukawa coupling is chosen so that the candidate could account for a gamma ray at $E_{\gamma} \sim 130$ GeV, see the resulting spectra in Fig. IX. Unfortunately no distinction can be made between the shapes of scalar and Majorana benchmark spectra, even for optimistic resolutions such as $\Delta E / E \sim 0.02$, we thus only show the scalar dark matter one. The points associated to our benchmark models correspond to the filled black diamonds in Figs. VII and VIII. For reference, we also report the $M_{dm} \sim 130$ GeV candidate put forward in [34], which is assumed to couple to the three families of leptons with universal Yukawa coupling $g_l = 0.52$ for $l = e, \mu, \tau$ and to a heavy scalar ($\Phi$) with mass such that $r = M_{\Phi}/M_\chi = 1.1$.

D. Prospects and possible extensions

So far we have considered a specific scenario, assuming the coupling of a real scalar DM candidate to a heavy vector-like leptonic particle which we take to be singlet under $SU(2)$. This we have done for the sake of simplicity and illustration of the possible enhancement of the VIB in the case of a scalar compared to a Majorana DM candidate. A systematic analysis

3 Notice that the value of $\langle \sigma v \rangle_{\gamma ll} = 1.21 \times 10^{-28}$ obtained here is two times larger than the number reported in [34]. We expect that this is due to a factor of two rescaling of $\langle \sigma v \rangle_{\gamma ll}$ in [34] which is relevant when compared to gamma ray flux constraints on $\langle \sigma v \rangle_{\gamma \gamma}$.
FIG. IX: Spectra $\langle \sigma v \rangle_\gamma \frac{dN_i}{dx}$, with $\langle \sigma v \rangle_\gamma$ defined as in Eq. (23), as a function of $E_\gamma$ for the scalar dark matter benchmark model of Table II ($M_{dm} = 150$ GeV). The spectra have been convoluted with a gaussian kernel assuming a relative energy resolution of $\Delta E/E = 0.1$.

of the possible extensions of this scenario is beyond the scope of the present work, and will be presented elsewhere. In this section we briefly summarize some possible forthcoming results.

One possible improvement would be to consider the continuous gamma ray emission associated to weak gauge bosons (which we have neglected here) especially generalizing the interaction (1) to the case of Yukawa couplings to $SU(2)$ doublet vector-like leptons. In the latter case, new contribution $SS \rightarrow W\bar{l}\nu$ to 3-body processes should be taken into account.\textsuperscript{4} Beside the obvious boost of the signal that we may expect in the scalar case compared to the Majorana case (which would couple to new a scalar doublet), the results will however match the existing analysis of [33], since the scalar and Majorana dark matter spectra are expected to be precisely identical.

Possibly more interesting would be to consider Yukawa couplings to heavy vector-like quarks. In this case, there are potentially two aspects that are worth being studied in more details. To begin with, we have the possibility of the VIB of a gluon. As emphasized in the previous section, in the case of scalar particles annihilation, the VIB of a photon is substantial, possibly similar in magnitude with the 2-body process, at least for annihilation in the early universe. The ratio of the VIB of a gluon to that of a photon being given by

$$\frac{\langle \sigma v \rangle_{gqq}}{\langle \sigma v \rangle_{\gamma qq}} = \frac{N_c^2 - 1}{2N_c} \frac{\alpha_s}{Q^2 \alpha}$$

which is about $\sim 40$ for up-like quarks, and $\sim 150$ for down-like quarks. These features

\textsuperscript{4} Notice that in our numerical analysis we took into account the annihilation into $Z\bar{l}l$ for the calculation of the relic abundance which typically give rise to $\langle \sigma v \rangle_{Zll} \sim \langle \sigma v \rangle_{\gamma ll}/3$
imply that the VBI of a gluon will be potentially more relevant in determining the relic abundance than the corresponding two body process, at least for light quarks. This in turn would imply that the annihilation rate into gamma rays is fixed by \(31\) a possibility very much in the spirit of the scenario considered in \([14]\). We will address such processes with gluons and quarks in final states in a work in progress, as well as the constraints from the measurements of the antiproton flux in cosmic rays, see e.g. \([11, 14]\).

A second interesting aspect in scalar dark matter models coupled to heavy vector-like quarks is direct detection. A priori the analysis should be similar to the Majorana case discussed in \([14]\). In particular, for the scalar effective coupling to quarks, the relevant interaction should be related to the effective operator \([12]\), with the obvious substitution of leptons by quarks. We expect though some subtle differences for the contribution of the so-called twist-2 operator, which is formally related to the operator \([13]\). As is well known, in the Majorana case the proper determination of direct detection collision cross section requires the evaluation of the box diagram in kinematic regimes in which the direct use of the trace anomaly may give wrong results \([52]\). The equivalent process for scalar DM through a vector-like portal has not yet been studied and will be addressed in a future work.

In our work, we have assumed that the quartic coupling of the real scalar to the Standard Model scalar may be neglected. An obvious generalization of the results presented here would be to relax such an assumption. For the time being, we just emphasize the trivial fact that this coupling may help decreasing the VIB cross section. Indeed, as shown in Fig. \([VII]\) there are many WIMP scalar candidates with a too large flux into gamma rays, and this all the way up to \(M_{\text{dm}} \sim 900\) GeV. If another annihilation channel is opened, like through the SM scalar, the Yukawa coupling may be smaller, and these candidates may become viable. This is not quite the same for the Majorana candidates, for which the signal is systematically below the current constraints, and thus which would require some astrophysical boost to saturate the observations. As an example, let us take from our scans a scalar candidate with \(M_{\text{dm}} = 150\) GeV, \(r = 1.32\) and \(y_l = 1.8\) which relic abundance \(\Omega_{\text{dm}} h^2 = 0.1\) but, \(\langle \sigma v \rangle_{\gamma ll} = 1.110^{-26}\) cm\(^3\)/s is excluded by Fermi-LAT constraint. By allowing for \(\lambda_S = -0.06\) and decreasing \(y_l\) to 1.5, we can still account for \(\Omega_{\text{dm}} h^2 = 0.1\) while getting a \(\langle \sigma v \rangle_{\gamma ll} = 5.310^{-27}\) cm\(^2\)/s below the limits and near \(\langle \sigma v \rangle_{\gamma ll}^{\text{best}}\). Notice though that in the latter case, new channels \(SS \to WW, ZZ, HH\) open and give rise to a continuous gamma ray component with a 2-body annihilation cross section that is no more d-wave suppressed. In addition, direct detection searches can also test such a scenario given that the scattering cross-section on a proton amounts to \(8.4510^{-46}\) cm\(^2\) and is nearly excluded by the Xenon100 limits \([53]\). See also \([54]\) for recent study of latest and future constraints on singlet scalar dark matter and SMS portal.

In a broader perspective, one may consider a much more extended parity odd sector. For definiteness, consider the so-called Inert Doublet Model (IDM) \([55, 57]\). In its simplest incarnation it consists of the addition of a single scalar doublet, \(H_2\), odd under a \(Z_2\) parity, with no expectation value. A simple and very interesting extension consists in the addition of an odd right-handed neutrino field \([56]\) with a Majorana mass term, a model in which the mass of the SM neutrinos is generated radiatively at one loop. In the same spirit, we may consider the possibility of introducing heavy vector-like doublets, be them lepton or quark-like, all odds under \(Z_2\). Of course this scenario implies the introduction of a large number of new parameters, to begin with the Yukawa couplings, but it also broadens the
range of possibilities for the IDM, in particular regarding not only gamma ray features,\textsuperscript{5} but also potentially antimatter in cosmic rays, new direct detection channels, and particle physics signatures with flavour changing processes such as $\mu \rightarrow e + \gamma$. \textsuperscript{6}

V. CONCLUSIONS

In this work we have discussed a simple dark matter model that may lead to a significant gamma ray spectral feature. It consists of real scalar DM particle that interacts with SM leptons through heavy vector-like charged fermions (the so-called vector-like portal). The most striking feature of this model is the possibility of an enhanced annihilation of dark matter in a process with virtual internal bremsstrahlung (VIB). This rests on the fact that, 1/ in the chiral limit the annihilation cross section in lepton antilepton pairs is d-wave suppressed, i.e. $\propto v^4$ and, taking all other things constants (dark matter mass, mass of the intermediate particle and the Yukawa couplings) 2/ the bremsstrahlung cross section is relatively larger than in the case of annihilation of Majorana particles. These two features taken together imply that the VIB feature is much more enhanced for real scalar dark matter compared to Majorana candidates.

For the sake of illustration, we have studied in more details a concrete, albeit simplistic case of a real scalar with a Yukawa coupling to the SM right-handed electron (or equivalently for all practical purpose with universal coupling to all lepton). Our main result may be read from Fig. VII that shows the annihilation cross section into 3–body final state $\langle \sigma v \rangle_{\gamma ll}$, and thus the associated gamma ray flux, is comparatively much stronger for scalar than for Majorana dark matter, provided that these candidates account for all the dark matter cosmological abundance. For completeness, we have also compared the annihilation rates in monochromatic gamma rays of real scalar and Majorana candidates. In the near future, we intend to generalize this framework to the case of heavy vector-like quarks.

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Note added : while our work was being completed, the article \cite{61} was published on the ArXiv, finding similar results for real scalar dark matter.

\textsuperscript{5} Notice that the vanilla IDM already presents VBI gamma ray features \cite{58}.

\textsuperscript{6} Notice that such processes are potentially present in the singlet scalar scenario discussed here but are tuned to be irrelevant by taking the matrix Yukawa couplings, to be diagonal in lepton flavour space. See for instance \cite{59} for an analysis in Majorana dark matter case and \cite{60} for a recent study of flavour changing process in the vector-like portal scenario.
Appendix

In this appendix, we provide some more expressions of annihilation cross-sections involved in the determination of the relic abundance or the gamma ray signal. Notice though that our numerical results of Sec. IV C were obtained using Micromegas code which automatically compute all the two body processes at tree level so that their expressions given below in the non-relativistic limit are only given for the sake of completeness. We implemented the dark matter 3-body annihilation cross-sections into $\gamma \bar{\nu}_l l$ and $Z \bar{\nu}_l l$ in Micromegas. Also, notice that annihilation into two gammas were neglected in the computation of the relic abundance.

In Sec. IV, we neglected s- and p-wave contributions to the scalar dark matter annihilation cross-section. Here we give the s- to d-wave contributions to the 2-body annihilation cross section $SS \rightarrow \bar{l} l$, to leading order in $m_f$ and $v$,

$$\sigma v(\SS \rightarrow \bar{l} l) = \frac{y_i^4}{4\pi M_i^2} \frac{1}{(1 + r^2)^2} \left[ \frac{m_i^2}{M_i^5} - \frac{4}{6} \frac{m_f^2 v^2 (1 + 2r^2)}{M_i^5 (1 + r^2)^2} + \frac{1}{15} \frac{v^4}{(1 + r^2)^2} \right].$$  \hspace{1cm} (32)

Concerning the 3-body cross section $\sigma_{\gamma ll}$, we emphasize that the $r$ dependence is universal rewriting $\langle \sigma v \rangle_{\gamma ll}$ as

$$\langle \sigma v \rangle_{\gamma ll} = \frac{y_i^4}{32\pi^2} \frac{K_i}{M_i^2} F(r)$$  \hspace{1cm} (33)

with $y_i = g_i, y_l$ and $K_i = 1/8, 1$ for $i = \chi, S$. Equation (33) was obtained integrating the amplitude (20) over the domain $[0 \leq x = \frac{E_\gamma}{M_i} \leq 1]$ and $(1 - x) \leq y = \frac{E_f}{M_i} \leq 1$.

The function $F(r)$ is identical for the Majorana and scalar cases and it is shown in Fig. X. It behaves like $r^{-8}$, as expected, which is also the behaviour of the 2-body cross section in the scalar case. On the contrary, in the Majorana case the 2-body cross section behaves like $r^{-4}$. Notice that a useful approximate expression of $\langle \sigma v \rangle_{\gamma ll}$ has also been derived previously in [27] in the case of scalar dark matter. The latter reads

$$\langle \sigma v \rangle_{\gamma ll} \approx \frac{\alpha K_i y_i^4}{8\pi^2 M_i^2} \left( r^2 + 1 \right) \left( \frac{\pi^2}{6} - \log \left[ \frac{r^2 + 1}{2r^2} \right] \right) - 2Li_2 \left[ \frac{r^2 + 1}{2r^2} \right] + \frac{4r^2 + 3}{r^2 + 1} + \frac{4r^4 - 3r^2 - 1}{2r^2} \log \left[ \frac{r^2 - 1}{r^2 + 1} \right],$$  \hspace{1cm} (34)

and agrees with our findings in the case of scalar dark matter using $g_i = g_l, y_l$ and $K_i = 1/8, 1$ for $i = \chi, S$ as in our Eq. (33).

For the annihilation into $\gamma \gamma$, we follow the compact formulation of [17] inspired by [25] in the Majorana case and [44] in the scalar case. In the notations used here, in the chiral limit, we thus made use of

$$\langle \sigma v \rangle_{\gamma \gamma} = \frac{\alpha^2 g_i^4}{256\pi^3 M_i^2} \mathcal{I}(r)^2$$

with $\mathcal{I}(r) = \int_0^1 \frac{dx}{x} \log \left( \frac{-x^2 + (1 - r^2)x + r^2}{x^2 + (-1 - r^2)x + r^2} \right)$ \hspace{1cm} (35)
FIG. X: Function of $F(r)$ of Eq. (33) with $r = M_\Phi/M_S$ (scalar DM) or $r = M_\Phi/M_\chi$ (Majorana DM).

for Majorana dark matter and

$$
\langle \sigma v \rangle_{\gamma \gamma} = \frac{\alpha^2 y_f^4}{64\pi^3 M_S^2} |A(r)|^2
$$

with $A(r) = 2 - 2 \log \left[ 1 - \frac{1}{r^2} \right] - 2r^2 \arcsin^2 \left[ \frac{1}{r} \right]$  

(36)

for scalar dark matter. Notice though that $\langle \sigma v \rangle_{\gamma \gamma}$ diverges in the limit $r \to 1$ in the scalar case while it is under control in the Majorana case [25]. In [17], it is mentioned that the approximation for $\langle \sigma v \rangle_{\gamma \gamma}$ for scalar dark matter is actually expected to break down in such a limit when $m_f \to 0$ is considered. We leave for future work a more detailed analysis of $\langle \sigma v \rangle_{\gamma \gamma}$ in this framework.

Finally, let us mention that we have obtained the results presented in Figs. VII and VIII performing random scans over the parameter space:

$$
1 < r < 5
$$

$$
10^{-3} < y_t, g_t < \pi
$$

$$
45 \text{ GeV} < M_{dm} < 10^4 \text{ GeV}
$$

(37)

setting the coupling of the scalar dark matter to zero and imposing that the dark matter relic abundance is $0.09 < \Omega_{dm} h^2 < 0.13$. Imposing that $y_f < \pi$ allow us to obtain viable dark matter candidates for $r < 3(4)$ in the scalar (Majorana) dark matter case.

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