Universal scaling of forest fire propagation

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In this paper we use a variant of the Watts-Strogatz small-world model to predict wildfire behavior near the critical propagation/nonpropagation threshold. We find that forest fire patterns are fractal and that critical exponents are universal, which suggests that the propagation/nonpropagation transition is a second-order transition. Universality tells us that the characteristic critical behavior of propagation in real (amorphous) forest landscapes can be extracted from the simplest network model.

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Because of heterogeneous conditions of weather, fuel and topography encountered, the spread of large fires looks fractal, as revealed by satellite maps [1]. This suggests stochastic modeling. Until now this spread has been modeled using regular networks, as well as cellular automata to include site weights [2-5]. However there is some evidence that networks with only local contacts do not mimic real fires very well [6-8] as they cannot take into account physical effects beyond the nearest neighbors of a burning site (i.e. an item of vegetation), such as radiation from flames. Moreover, wind and topography may induce anisotropy, which in turn reduces the effective dimension of the propagation to less than two. Here we propose a new model based on the social small-world network (swn) which was initially proposed by Watts and Strogatz [9] (Fig. 1a). The present model is built from a two-dimensional \( L \times L \) size lattice which includes both short-range connections, via the nearest neighbors of a burning site, and long-range connections in its radiative influence zone (Fig. 1b). The so-called long-range spotting process, whereby burning firebrands produced when shrubs and trees burn rapidly are lofted by the fire plume and transported downwind to start new fires in recipient fuel beds, is not considered here as it introduces a different kind of transition [10]. The influence zone of a burning site is characterized by two parameters, \( l_x \) and \( l_y \), which are dependent on fire and fuel conditions and expressed in an arbitrary length unit (\( \delta l \)) corresponding to the lattice parameter. The vector \( \vec{s} = (s_x, s_y) \) is the position vector of any site in the influence zone with respect to the burning site position taken as the origin. Isotropic propagation, i.e. with no wind and no slope effects, corresponds to circular patterns (\( l_x = l_y \)). A high value of the ratio \( AR = l_y/l_x \) corresponds to a strong anisotropy of the front shape induced by the terrain slope and/or wind effects in the propagation direction (here the \( y- \) direction). Fire patterns then become elliptical. As shown in Fig. 2, fire patterns exhibit a \emph{swn} behavior only locally, which explains why the present model is referred to as a local small-world network (lswn). These lswn effects increase as the proportion of combustible sites, \( p \), decreases. The model has shown an excellent agreement with known experimental data [11]. In addition to its capacity to mimic the phenomena of fire spread in nature, the present model produces super-real-time simulations of fire patterns. We then focus on the very powerful feature of universality of forest fire propagation, as has been done for the study of complex systems, such as biological systems [12].

Assuming every burning site to be a point-like source of radiation, the amount of power received by a site located within the influence zone decreases inversely with the square of the distance from the burning site

\[
\Delta P = \delta P \left( \frac{s_x^2}{l_x^2} + \frac{s_y^2}{l_y^2} \right)^{-1} \tag{1}
\]

The term \( \delta P \) is the minimum amount of power required for a combustible site to start to degrade. It can be determined from either experiments or deterministic simulations [13]. The lswn model uses a time-weighting procedure on sites in order to model fire propagation through a network of vegetation items. It is based on the knowledge of two characteristic times, namely the time required for a site to achieve complete combustion, \( t_c \), and that of thermal degradation before ignition, \( t_{TD} \). As an example, the nearest neighbour of a burning site in the propagation direction receives an amount of power equal to \( \delta P(l_y/\delta l)^2 \), whereas a site at the border of the influence zone of this burning site receives an amount of power \( \delta P \) and requires a time of \( t_{TD} \) to reach ignition. Fuel properties (e.g. load
distribution, type, or moisture content) do not affect the weighting procedure but the characteristic times should be modified. The concept of both time-weighting procedure and influence zone differentiates the $lsun$ model from the usual percolation model, although it has been shown to behave as a regular network for system sizes much larger than that of the influence zone [14].

The next point to be considered is the dynamic aspect of the $lsun$ model. At each time step, $\delta t$, taken as the time unit, a burning site contributes to the increase in the degradation level of the sites connected to it before ignition. Once ignited, these burning sites each contribute to the thermal degradation and ignition of the sites located in their own influence zone. Fire cannot propagate if a site ceases to burn before the complete thermal degradation of its nearest neighbours. The dynamics of propagation in the $y$-direction can be characterized by the ratio $R$ defined as

$$R = \left( \frac{L_y}{\delta l} \right)^2 \frac{t_c}{t_{TD}} \quad (2)$$

There exists therefore a dynamic threshold, below which fire cannot propagate even in a homogeneous vegetation. For line-ignition conditions, the dynamic threshold value was found [14] to be 1/2 whereas it is unity for point-ignition conditions. For the latter conditions, above the dynamic threshold, a ratio $t_c/t_{TD}$ smaller than unity means that only a fraction $\delta c/t_{TD}$ of the influence zone of the burning site will be ignited when this site finishes burning.

The fire propagation process is initiated here by igniting the site located at the center of the first line. The statistical averaging process is carried out by generating $N$ samples, $N$ being ranged from 500 to 1000, which ensures that fluctuations are small far from the percolation threshold.

The geometric propagation (percolation) threshold of the network corresponds to a minimum concentration of active sites, $(p_c)$, above which propagation along a path connecting the opposite sides of the network occurs. In the usual percolation theory [15] for infinite size networks, the correlation length $\xi$, defined as the average cluster size, diverges at $p_c$ as a power law $\xi \propto (p - p_c)^{-\nu}$, $\nu$ being the critical exponent of the correlation length [15]). Near $p_c$, the average burned mass is dominated by that of the largest cluster which behaves as a power law of $p_c - p$,

$$< m > \propto (p_c - p)^{-\gamma} \quad (3)$$

Below $p_c$, for system sizes $L$ smaller than $\xi$, $< m >$ scales as $(L/\delta l)^{D_f}$ where $D_f$ is the fractal dimension of the largest cluster mass and is $p$-dependent. For very large system sizes, it saturates as $(\xi/\delta l)^{D_f}$. The size at which the saturation occurs is therefore a measure of the correlation length. At $p_c$, the correlation length diverges and the average burned mass scales as

$$< m(L) > \propto L^{-\gamma/\nu} \quad (4)$$

which leads to $D_f(p_c) = \gamma/\nu$.

In the present study we first determined the percolation threshold, the fractal dimension $D_f$ and the critical exponents as functions of $l_y/\delta l$ for isotropic fire propagations (Fig. 3). Different techniques can be used to determine the percolation threshold by varying the concentration of active sites. It corresponds to the peak of the percolation probability $p$-derivative [12]. It also corresponds to the maximum propagation time as well as the maximum propagation time fluctuations. These fluctuations are largest at the percolation threshold since the largest cluster is mainly composed of critical ‘red’ bonds which stop the propagation if they are cut [13, 11]. These techniques that are systematically used to determine the percolation threshold provide very similar values. In the present case, the percolation threshold decreases as $(l_y/\delta l)^{-1.58}$ (Fig. 3a). The correlation length critical exponent $\nu$ is in agreement with that of the usual percolation theory [15], $\nu = 4/3$ (Fig. 3b), as expected from a re-normalization procedure [16] of the isotropic influence zone. Model results show that for isotropic propagation, long-range radiative effects preserve the features of the phase transition of the usual percolation theory.

The propagation of real fires is generally anisotropic due to windy and/or hilly terrain conditions, which enhances overhead flame radiation. As shown in Fig. 4a, the percolation threshold varies as a power law of the anisotropy ratio, $AR$ (defined in Fig. 1), with an exponent of $-2/3$. Such behaviour has also been predicted for anisotropy crossover in percolation [17]. At $p_c$ the average burned mass scales as $(L/\delta l)^{D_f}$ with a fractal dimension of the physical support of fire propagation lower than two (Fig. 4c), whatever the anisotropy ratio $AR > 1$. By analogy with isotropic propagation, the dimension of the propagation support can thus be estimated within the range of $[1.2 - 1.5]$. The value of the corresponding correlation length exponent is approximately $\nu = 5/3$ (Fig. 4b), which agrees well with the value extrapolated from the fit of $\nu(D)$ data obtained for Euclidian system dimensions [12] $(2 \geq D \leq 6)$ (Fig. 5). Results summarized in Table 1 confirm the universality of the critical exponents whatever the network symmetry.

We also examined the behaviour of the rate of spread, $ros$, defined as the time derivative of the distance covered by the head fire front at steady state. For homogeneous systems, the $ros$ increases first with the square of the impact length (Eq.1), whereas during steady propagation it is found to increase as $(l_y/\delta l)^{2.6}$ due to the collective contribution of the neighboring burning sites. For inhomogeneous systems, we propose a new exponent, $\kappa$, of the scaling law of the $ros$ above the percolation threshold. To the best of our knowledge, there is no equivalent exponent in the usual percolation theory. Below $p_c$ fire cannot reach the opposite side and the steady state is never attained. Above $p_c$ fire spreads at the same rate...
within large clusters independently of the system size $L$. For an isotropic propagation, the critical exponent of the ros seems to be independent of the impact parameter (Table 1).

A special emphasis is put on the problem of diffusion in the lsvn model. The statistical fluctuations of the distance covered by the front vary with time as $t^\alpha$ where the exponent $\alpha$ is $p$-dependent. This exponent is around $3/4$ for a density $p$ close to $p_c$, which indicates a super-diffusive propagation (it may be remembered that $\alpha = 1/2$ for a diffusive propagation). It becomes unity above the percolation threshold leading to a ballistic transport, as a result of radiation beyond the nearest neighbours.

Three results emerge from the present study. First, the propagation / non-propagation transition is a second-order phase transition, the critical exponents being universal. This means that model results obtained on simple lattices near critical points remain valid for amorphous networks representative of wildland landscapes. It is found that the range of variation of the lsvn percolation threshold is reduced to about $\pm10\%$ as the network symmetry varies. Second, at the percolation threshold, fire propagation is super-diffusive. Above $p_c$ it is ballistic. Third, the dimension of the physical support of anisotropic propagation is fractal, whereas that of isotropic one is Euclidian. The fractal dimension of the support allowed us to estimate the critical exponent $\nu$ for fractal dimensions. We are at present examining the role of spotting in the propagation of wildland fires.

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TABLE I: critical exponents of fire propagation. $\nu_{th}$ is deduced from $\kappa = \nu_{th} - 1/\nu_{th}$.

| $l_x/l$ | $l_y/l$ | Lattice | $p_c$ | $\gamma$ | $D_f$ | $\nu$ |
|--------|--------|---------|-------|---------|-------|-------|
| 3      | 3      | Square  | 0.225 | 2.26 (±0.02) | 1.88 (±0.06) | 1.24 (±0.05) |
|        |        | Triangular | 0.178 | 2.19 (±0.04) | 1.63 (±0.04) | 1.34 (±0.06) |
|        |        | Amorphous | 0.200 | 2.09 (±0.04) | 1.67 (±0.04) | 1.25 (±0.06) |
| 2      | 5      | Square  | 0.225 | 2.06 (±0.05) | 1.26 (±0.03) | 1.65 (±0.08) |
|        |        | Triangular | 0.195 | 2.17 (±0.04) | 1.34 (±0.03) | 1.63 (±0.07) |
|        |        | Amorphous | 0.210 | 1.84 (±0.03) | 1.12 (±0.02) | 1.64 (±0.06) |
| 2      | 8      | Square  | 0.165 | 2.53 (±0.04) | 1.50 (±0.02) | 1.69 (±0.05) |
|        |        | Triangular | 0.135 | 2.12 (±0.03) | 1.23 (±0.04) | 1.72 (±0.08) |
|        |        | Amorphous | 0.146 | 2.06 (±0.02) | 1.27 (±0.02) | 1.62 (±0.05) |
| 2      | 2      | Square  | 0.405 | 0.45 (±0.07) | 1.25 (±0.04) | 1.22 (±0.06) |
| 3      | 3      | Square  | 0.225 | 0.54 (±0.01) | 1.31 (±0.01) | 1.24 (±0.05) |
| 5      | 5      | Square  | 0.085 | 0.54 (±0.03) | 1.31 (±0.02) | 1.33 (±0.06) |
(a) Long-range connections

- Burning site
- Empty site
- "Healthy" site
- Site affected by the burning site

(b) Influence zone

- $l_x$
- $l_y$
- $\delta l$
Propagation

Percolation threshold

Critical & scaling exponents

| 2 | 3 | 4 | 5 |
|---|---|---|---|
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| 2 | 3 | 4 | 5 |
| 1.5 | 2.0 | 2.5 | 3.0 |

(a) Non propagation

(b)
Percolation Threshold (a)

Critical & scaling exponents (b)
Average burned mass

System size \( \frac{L}{\delta_l} \)

AR=1.5
AR=2.5
AR=4
Present data

Exponent $\nu$ vs. dimension $D$