Dynamics of a director reorientation and optical response of polymer films filled with a liquid crystal under strong electric field

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Abstract. The dynamics of the director reorientation in nematic liquid crystals (NLC) confined by cylindrical cavities of porous polymeric films under strong electric field $E$ with has been investigated theoretically. The main attention was paid to the specific mode of field application characterized by abrupt changes of the applied voltage’s polarity. In experiments with porous films filled with a liquid crystal 5CB such mode resulted in appearance of strong peak–like decreasing of an optical transparence of the films. Two mechanisms of such unusual response based on assumption of electrically induced motion of ions and overall motion of a liquid were considered and applied to explain experimental results.

1. Introduction
At present liquid crystals (LC), which play a key role in display industry are considered as very promising materials for non display applications, like E-paper, sensors and photonics [1, 2]. In the latter case there is a number of new requirements different from those applied for liquid crystal displays (LCD). In particular, decreasing of operating times for LC devices becomes of a great importance. Different ways of a solution of this problem, like usage of ferroelectric liquid crystals [2] were proposed. While FLC show very short operating times, their application in photonics devices meets the additional problems, like possibility of irreversible changes of a layer structure of FLC imposed by mechanical deformations.

The alternative way to decrease operation times is to use composite liquid crystal materials of different types [3, 4] where liquid crystals are confined by cavities of micro or nano sizes [5]. Among such materials, porous films filled with liquid crystals are of a special interest. These materials are characterized by well oriented pores of submicron or nanometer diameters. It provides an electric control of light propagation with operating times in microsecond range even in the case of nematic liquid crystals. For example, aluminum and silicon oxide porous films filled with LC were proposed as proper materials for fiber optics applications [6, 7]. Computer modeling of the electrically induced static changes of the initial orientational structure and optical properties was performed on the base of
a minimization of the free energy [8] taking into account the dielectric contribution, which is proportional to the squared value of electric field intensity and does not depend on the field polarity. It was found, that application of strong enough electric field to the initially deformed escaped radial (ER) structure can induce a surface anchoring breaking which results in transition to the homogeneous orientation of LC along a pore axis and essential changes of an effective refractive indexes of LC.

Recently [9] we performed electro-optical investigations of polymeric (PET) porous films, filled with a standard nematic (5CB) and found a strong peak-like optical response arising after a very fast (about 10 ns) change of the electric field polarity. In this paper we consider electro-kinetic phenomena including both the electrically induced ion’s motion and the overall motion of an anisotropic liquid as a base of mechanisms, describing peculiarities of the above mentioned optical response.

2. Experimental results

In our experiments we used PET films of thickness \( h = 23 \mu m \) and different porosity \( P \) with cylindrical and randomly allocated pores (diameters \( d = 0.2, 0.3, 0.4, 0.5 \) and \( 0.6 \mu m \)). Prepared samples were vacuumed and filled with well studied nematic 5CB characterized by a high positive value of a dielectric permittivity anisotropy (approximately \( \Delta \varepsilon = 12 \) at room temperature). Samples were thoroughly wiped to remove parasite surface layers of LC and were placed between two glass substrates with ITO electrodes. The distance between the LC cell and the photodetector (22cm) was fixed in all experiments. Measurements were fulfilled at a room temperature via standard electro-optical setup with different position of input polarization.

It was established that the optical response of the samples on the applied voltage drastically depended on the frequency range and the application schema. In the case of video-pulses with changeable polarity at low frequencies (\( f<f_c \)) the optical response arises as an impulse of light intensity which decays with the time. In case of radio-pulse of high enough frequency (\( f>f_c \)) electric field induces an overall change in the light intensity, which is typical for an electro-optical response of LC layer in essentially shorter than the electric pulse duration.

![Figure 1](image_url)

**Figure 1.** Electro-optical response of the sample with a pore diameter 0.3 \( \mu m \) (\( \delta=0, \) without analyzer) on the applied square wave signals (\( f=50 \) Hz < \( f_c \)) for two values of voltage \( U: \) (a) \( U=80V<U^* \), (b) \( U=170V>U^* \). Left inset shows the (b) response in different time scale and right inset shows typical response on application of high frequency radio pulse (\( f=5 \) kHz > \( f_c \), \( U=170V \)).

The description of the dynamic electro-optical response, made above, is based on a concept of change in the effective refractive index under the electric field, and does not include a mechanism of
its influence on orientational structure of LC inside pores. In particular, the backward relaxation of \( \Omega(r) \) to the initial state (\( \Omega(r) = \pi/2 \)) with the times essentially smaller than the pulse duration, cannot be explained by a traditional dielectric mechanism, describing reorientation of LC director inside pores.

This mechanism is based on an existence of dielectric torque produced by an electric field, with the value proportional to the square of the electric field strength. So, it is not changed at the change of polarity, except for the very fast (in order of nanoseconds) time intervals of the polarity inversion. The typical times of orientational relaxation of LC are essentially longer, and orientational structure has not to be changed during the change of polarity.

The well-known backflow effect, arising on a short time with fast switching on/off the electric field, could cause short peaks in optical response close to those shown above. Nevertheless in our case the polarity inversion time is much shorter (nanoseconds) than that, needed to form back flows.

In this paper we considered two mechanisms of such unusual optical response. The first one is based on a change of LC orientational structure caused by the integral shear flow (electro-osmotic flow). The second mechanism is connected with electrically induced motion of ions inside pores, filled with LC, which causes to the polarization of the system and adds a contribution to the total electric field.

3. Electro-osmotic flow

Electro-osmotic flows can arise due to electro-kinetic phenomena, observed in capillaries of different sizes and filled with polar isotropic liquids, such as a water and water solutions [10]. It is well known that the physical background of the electro-kinetic phenomena is connected with the absorption of polar molecules by the inner surface of the capillary, which leads to polarization of the surface and arising of the electrical double layer with thickness \( l_0 \) (of order some nanometers for typical water solutions). It produces non-homogeneous radial distribution of ions inside a capillary in the near surface diffuse layer with thickness \( l_0 > l_0 \). In turn, the application of DC (or low frequency) axial electric field \( E_z \) to the liquid induces a motion of ion’s system which also results in arising of overall steady (or oscillating) electro-osmotic flow through a capillary. The flow direction depends on the field direction, so the flow oscillates with the same frequency as the AC electric field frequency. It is worth noting that the profile of a steady flow appears for a relatively short time \( \tau_v \), which can be determined as \( \tau_v \approx \rho R^2 / \eta \) [11], where \( \rho \) and \( \eta \) – are density and viscosity of a liquid. Estimations made for the water inside capillary with radius \( R=1 \mu m \) gives \( \tau_v \approx 10^{-6} \)s. Increase of the viscosity to the typical for LC values \( 10^{-2} \ldots 10^{-1} \text{Pa.s} \) and decrease of \( R \) should lead to the corresponding decrease of time \( \tau_v \), which is essentially smaller than the typical values of a period of \( E_z \) oscillations in the experiments mentioned above. In these experiments we used the samples of porous films with removed (after filling) parasitic layers of LC from the outer surfaces. It has to form the two meniscuses, which confine LC inside a cylindrical pore inside the pore (figure 2).

![Figure 2. Schematic sectional view of a pore with two meniscuses on each side.](image)

In this case electro-osmotic flow should lead to the deformation of meniscuses and arising of quasi elastic restore force \( F^\alpha \), proportional to the surface tension coefficient \( \sigma \), which can induce slowing of the electro-osmotic flow and it can even stop when the maximum value of the pressure gradient \( P^\alpha \)
exceeds the value of the electro-osmotic pressure gradient $P^E_z$. Expression for $P^E_z$ can be obtained from the equation for the volumetric flow velocity $Q$ induced by the pressure gradient $P^E_z$ and the electric field $E_z$ [14]. In particular, for the “plug” flow ($kR>>1$) simplified expressions for $Q$ can be written as:

$$Q = \frac{PR^2S}{8\eta} \left[ \frac{\varepsilon \varepsilon' \zeta}{\eta} \right] SE_z$$  \hspace{1cm} (1)

where $S=\pi R^2$ – cross section area of the pore, $k = \frac{l}{D}$ -1, $\varepsilon$ – dielectric permittivity of a liquid, $\zeta$ – so-called zeta potential.

Assuming $Q=0$ (closed channel) one can easily get expression for the electrically induced pressure gradient:

$$P^E_z = 8 \left( \frac{\varepsilon \varepsilon' \zeta}{R^2} \right) E_z$$  \hspace{1cm} (2)

The estimated value of $\zeta = -12$ mV for PET-5CB interface at the “plug” flow approximation was obtained from the direct measurements of volumetric velocities of the LC flow through porous films [12].

Maximum value (170V) of the voltage, applied to samples in the electro-optical experiments, mentioned above, corresponds to the value $E_z \approx 7$V/µm. For linear dependence $P^E_z$ on $E_z$, predicted by eq. (2) it results in the maximal values of the electrically induced pressure gradient $P^E_z$ and pressure difference $\Delta P^m = P^E_z$ equal, correspondently to $2.6 \cdot 10^8$Pa/m and $6.10^4$Pa for $R = 150$nm. The latter value can be compared with the pressure difference in capillary $\Delta P^m$ arising due to deformation of two menisci induced by the electro-osmotic flow by using well-known expression for a capillary pressure $P^m$:

$$P^m = \frac{2\sigma \cos \theta}{R}$$  \hspace{1cm} (3)

where $\theta$ – the contact angle which depends on the wetting conditions. For the simplicity, we will consider the case of menisci with the initially flat surface ($\theta = \pi / 2$ weak wetting), deforming to the ideal wetting state ($\theta = 0$). In accordance with (3) it leads to the next expression for the maximum value of the capillary pressure difference $\Delta P^m$, acting in the direction, opposite to those for $\Delta P^E$:

$$\Delta P^m = 4\sigma / R$$

Assuming the value of $\sigma = 3.3 \cdot 10^2$Pa.m for 5CB [14] one can get $\Delta P^m = 8.8 \cdot 10^5$Pa, which is 15 times higher than previously estimated value of $\Delta P^E$. So, the presented above estimations confirm the possibility of the significant slowing of an electro-osmotic flow slowing and stopping due to deformation of the menisci.

The equation, which describes the dynamics of such process, can be obtained by analysis of the forces, acting on the sample of a liquid confined inside a pore. For this aim, it is convenient to express the contact angle $\theta$ in terms of height $H$ of the spherical segment corresponding to each meniscus $\cos \theta = 2(\pi/2)^{2} \Delta H + (H/R)^2)^{1/2}$.

At the initial state ($E_z=0$, $H = H_0$) capillary forces are compensated, so there is no pressure gradient or flow. Under the influence of the electrically induced flow, both menisci are deformed, which causes the change of $H$ from $H_0$ to $H(t)$. The expression describing a connection between $\Delta H(t) = H(t) - H_0$ and $\Delta V(t) = V(t) - V_0$ (change of the spherical segment volume) in the case of weak deformations ($\Delta H<<R$) can be written as:

$$\Delta V(t) = \left( \frac{\pi R^3}{2} \right) \left[ 1 + \left( \frac{H_0}{R} \right)^2 \right] \Delta H(t)$$  \hspace{1cm} (4)
Thus volumetric velocity of the flow $Q(t)$ is expressed as:

$$Q(t) = \frac{d}{dt} \left[ \Delta V(t) \right] = \left( \frac{\pi R^2}{2} \right) \left[ 1 + \left( \frac{H_0}{R} \right)^2 \right] \frac{d}{dt} \left[ \Delta H(t) \right] \quad (5)$$

The deformation of meniscuses results in increasing of the total area $\Delta S$ of the spherical segments’ surface, defined for the case of small deformations ($\Delta H << R$) as $\Delta S = \Delta S_1 + \Delta S_2 = 2\pi(\Delta H)^2$ which, in turn, leads to the rise of the surface energy $\Delta W = \sigma \Delta S = 2\pi \sigma (\Delta H)^2$ and the appearance of the quasi elastic force $F^\sigma = -(dW/d)(\Delta H) = -4\pi \sigma (\Delta H)$, which tries to return a system to the initial state.

The expression for the viscous friction force $F_v$, which acts on the LC confined inside a pore, can be simplified by considering sliding of a quasi solid cylinder with the length $L$, separated from the inner surface of a pore by the layer of liquid of thickness, equal to the Debye length $l_D$ with a linear velocity profile in the gap between the surfaces of the pore and the cylinder. Such approximation correlates with the “plug” type of the flow mentioned above for a liquid moving with the average velocity $v_0 = Q(t)/\pi R^2$ everywhere except for the thin diffuse layer, characterized by high viscosity losses due to large values of the velocity gradient $u = dv(r)/dr$ which considered to be constant value, defined as $u = v_0/l_D$ in accordance with the simplified model.

Taking into account (5), one can get the next expression for the viscous friction force:

$$F^v = -\eta 2\pi RL_u = -\eta \pi L \left( \frac{R}{l_D} \right) \left[ 1 + \left( \frac{H_0}{R} \right)^2 \right] \frac{d}{dt} \left[ \Delta H(t) \right] \quad (6)$$

The expression for the electro-osmotic force $F^E$, which induces a motion of the cylinder, can be easily obtained from (2):

$$F^E = P^E \pi R^2 L = 8\pi \left( \varepsilon \varepsilon_0 \right) E_z L \quad (7)$$

The cylinder motion, under the action of the forces described above, is described by the Newton’s second law:

$$m \left( \frac{d^2 x}{dt^2} \right) = F^E + F^\sigma + F^v \quad (8)$$

where $x = \Delta H$ and $m = \rho \pi R^2 L$ is the cylinder’s mass. Equation (8) can be converted to canonical form, which describes vibrations of the pendulum in the presence of frictional forces:

$$\frac{d^2 x}{dt^2} + 2 \beta \left( \frac{dx}{dt} \right) + \omega_0^2 x = A^E(t) \quad (9)$$

where:

$$\beta = \left[ \frac{\eta}{2\rho R l_D} \right] \left[ 1 + \left( \frac{H_0}{R} \right)^2 \right] \quad \omega_0^2 = \frac{4\sigma}{\rho LR^2} \quad A^E(t) = 8 \left[ \varepsilon \varepsilon_0 \varepsilon_0^* \right] E_z(t)$$

The solution of the equation (9) depends on the ratio:

$$\beta^2 \omega_0^2 = \eta^2 L \left[ 1 + \left( \frac{H_0}{R} \right)^2 \right] \frac{1}{16 \sigma \rho l_D^2} \quad (10)$$
With the typical values for 5CB $\eta \approx 0.05$ Pa.s, $\sigma \approx 3.3 \times 10^{-2}$ Pa.m, $\rho \approx 10^3$ kg/m$^3$ and considering $L = h = 23 \mu$m, $l_0 \approx 15 \mu$m, $H_0 = 0$ it results in the next estimate $\beta^2/\omega_0^2 \approx 2.10^5 >> 1$ which is in accordance with the motion of an over damped pendulum, with the characteristic time $\tau^0$ written as:

$$\tau^0 = \frac{2\beta}{\omega^2} = \left( \frac{\eta L}{4\sigma} \right) \left( \frac{R}{l_0} \right) \left[ 1 + \left( \frac{H_0}{R} \right)^2 \right]$$  \hspace{1cm} (11)

For typical values $R=200$nm, $\sigma = 3.3 \times 10^{-2}$ N/m, $l_0=10$nm and $\eta = 10^{-1}$Pa.s, the estimated value of $\tau^0 \approx 350 \mu$s which is essentially smaller than the typical values of $t_{\text{dec}}$ shown in figure 3. Such deviation can be partly explained by the lag of a director relaxation, induced by the shear flow, relatively to the slowdown process by elastic energy.

Further analysis of obtained results requires additional consideration of the orientational relaxation processes under the action of the flow and the electric field. In turn, it depends on the type of the initial orientational structure of LC, realized at strong confinement inside pores. It NMR experiments [13] fulfilled with the similar system (polycarbonate porous membrane filled with LC), indicated on the possibility for LC to form two different types of an orientational structure – planar polar (PP) and escaped radial with point defects (ERPD) dependently on the radius $R$ and anchoring strength $W$. The results of our dielectric investigations of PET-5CB system, which will be published elsewhere, correspond to PP structure, characterized by a director distribution in the plane of the film. For this case, one can expect electrically induced Fredericksz transition with the threshold intensity $E_z^F$ and voltage $U_F$ roughly defined for strong surface anchoring as $E_z^F = (\pi/d)[K_{33}/(\varepsilon_0 K_{13})]^{1/2}$, $U_F = E_z^F h = (\pi h/d)[K_{13}/(\varepsilon_0 K_{33})]^{1/2}$, where $K_{33}$ – the Frank’s module. The dependence of $U_F$ on $d$ obtained by using data for 5CB is presented in table 1.

| $d$, $\mu$m | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  |
|--------------|------|------|------|------|------|
| $U_F$, V     | 102  | 68   | 51   | 41   | 34   |

Table 1. Calculated values of $U_F$ for samples with different pore sizes.

Figure 3. Characteristic times ($t_{\text{inc}}$ and $t_{\text{dec}}$) of negative peak as functions of a pore diameter and corresponding dependence of threshold voltages $U^*$ and $U_F$.

The calculated values of $U_F$ are slightly lower, than the experimentally determined threshold voltages, presented in figure 3, which correspond to the strong negative peak in the electro-optical response (figure 1). Moreover, the account of weak anchoring effects results in decreasing of $U_F$ more pronounced for small values of $d$ [14]. Nevertheless, the electro-osmotic flow can play the essential
role in destabilization of the initial PP structure. Indeed, while the torques acting on a director from electric field and boundaries are equal to zero at $E_z < E_z^F$, the electro-osmotic flow produces the non-zero torque, which tends to rotate a director towards the pore’s axis. So, it can act as a trigger mechanism creating the escape of a director from the film plane with further enforcing due to arising of a dielectric torque. The initial PP structure can be restored by the elastic torques on a stage of decay of the shear flow due to capillary forces. The relatively weak positive changes in the optical response, shown in figure 1 correspond to such scenario.

In more details, the influence of electric field on the orientational structure, induced by the electro-osmotic flow and the dielectric torque, can be obtained from the general equations of nematodynamics [14]. For the aim of simplicity, we will use the equation of a director motion for the escaped radial configuration, written in terms of angle $\Psi (r,t) = \pi/2 - \Omega (r,t)$, where $\Omega$ is the polar angle, which can be considered as a balance torque equation, written as:

$$M_K + M_d + M_\beta = M_v$$  (12)

where

$$M_K = K \left\{ \frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \sin \Psi \cos \Psi \right\}$$  (13)

the Frank’s type torque due to elastic curvature of a director field, where $K$ – the effective Frank’s constant,

$$M_E = \frac{[\epsilon_\perp \epsilon_{II} \Delta \epsilon \sin \Psi \cos \Psi E_z^2]}{(\epsilon_{II} + \Delta \epsilon \cos^2 \Psi)^2}$$  (14)

the dielectric torque, which is proportional to $E_z^2$, where $\epsilon_{\perp} = \epsilon (\Psi=0), \epsilon_{II} = \epsilon (\Psi=0), \Delta \epsilon = \epsilon (\Psi=\pi/2) - \epsilon (\Psi=0)$ - the principal values and anisotropy of a dielectric permittivity,

$$M_\beta = -\frac{1}{2} \frac{\partial v}{\partial r} (\gamma_2 \cos 2\Psi - \gamma_1)$$  (15)

the hydrodynamic torque, proportional to the velocity gradient, where $\gamma_1 = (\alpha_3 - \alpha_2) > 0$ – the rotational viscosity coefficient, $\gamma_2 = (\alpha_1 + \alpha_2) < 0$ – the reactive parameter arising due to connection between the director and the velocity field.

The approximate solution of equation (12) can be easily found in the case when the flow induced torque $M_\beta$ acting on a director is essentially larger, than the dielectric moment $M_E$, and elastic curvature moment $M_K$. It may occur at the intensive shear flow on the initial stage of a director relaxation (small values of the angle $\Psi$ describing the deviation of a director field from the initial state $\Psi (r,0) = 0$). Indeed, in the latter case both $M_E$ and $M_K$ decrease to zero at $\Psi (r,t) \to 0$, whereas the moment $M_\beta$ reaches its maximal absolute value

$$M_\beta^{max} = \frac{\alpha_3}{2} \frac{\partial v}{\partial r}$$  (16)

This moment tends to induce an overall orientation of LC in the direction of a flow (along a capillary axis).

So for the mentioned above approximation the torques balance equation $M_\beta = M_v$ includes only the flow induced torque, defined by eq. (15) and viscous torque:

$$M_v = \gamma_1 \frac{d\Psi}{dt}$$  (17)
Eq. (17) can be easily solved for a steady flow, when a velocity gradient $u = \partial \mathbf{v} / \partial r$ does not depend on time. The corresponding analytic solution in this case is written as:

$$t_g \Psi = \frac{t_g \Psi}{\beta} \left[ 1 - \exp{-\frac{t}{\tau}} \right]$$

$$t_g \Psi = \frac{t_g \Psi}{\beta} \left[ 1 + \exp{-\frac{t}{\tau}} \right]$$ (18)

where the flow alignment angle $\Psi_f$ and the relaxation time $\tau_f$ are defined as:

$$t_g \Psi_f = \left( \frac{\alpha_2}{\alpha_3} \right)^{1/2}$$ (19)

$$t_g \tau_f = \left[ \frac{\alpha_2 - \alpha_3}{(\alpha_2 \alpha_3)^{1/2}} \right] \left( -\frac{\partial \mathbf{v}}{\partial r} \right)^{-1}$$ (20)

By using the values of Leslie coefficients for 5 CB [14] $\alpha_2 = -0.0812$ Pa.s, $\alpha_3 = -0.0036$ Pa.s one can get $t_f \approx 45 \ u^{-1}$, where $u = (-\partial \mathbf{v} / \partial r) > 0$

For the “plug” type flow, considered above, the maximal value of the velocity gradient $u_{\text{max}}$ takes place in the boundary layer. In general, this value depends on the Debye length $l_d$ [14]. For rough estimations we used the value $l_d \approx 15$ nm, which corresponds to variations of a parameter $kR$ from 6.7 (for $R = 100$ nm) up to 20 (for $R = 300$ nm). Calculations, made with the parameters mentioned above, show, that in the near surface layer ($0.95 < kR < 1$) the mean value of $u_{\text{max}}$ varies from $2.0 \cdot 10^4$ s$^{-1}$ to $1.75 \cdot 10^4$ s$^{-1}$ at variation of $R$ from 100 nm to 300 nm. So the corresponding value of $\tau_f$ only slightly varies with a radius (from 220 $\mu$s to 250 $\mu$s). The obtained values of $\tau_f$ are in a rough agreement with the results for $t_{\text{inc}}$ shown in figure 3 and confirms that flow induced changes of orientation may play the main role in an electro-optical response arising on the first stage, which takes place after very fast (for time of about $\tau_v$) initial increasing of the flow velocity rate up to the maximal value $Q_{\text{max}}$.

Nevertheless the estimates made above are valid only for very small values of $\Psi$ at least for liquid crystals with large value of $\Delta \varepsilon$ (as 5CB). Dielectric moment $M_d$ is proportional to $E_2$ and essentially exceeds the flow induced moment $M_{\text{fl}}$ even at relatively small (some degrees) values of $\Psi$. So the alternative mechanism describing the intermediate and final stages of electro-optical response has to be proposed. It will be considered in the next paragraph.

4. Ion motion

To calculate the field $E(t)$ let’s exchange the set of charged sectors of LC, formed on the surface of the chaotically allocated pores, by the homogeneous distribution of the surface charges with an average density $\rho_{\text{f}}$, localized on the meniscus’s surfaces in pores.

In consideration of the lower and upper surfaces impact on the value $E(t)$, which takes place in case of the equal ion mobility, the expression can be written:

$$E(t) = \frac{\rho_{\text{f}}(t)}{\varepsilon_0} = \frac{q(t)n}{\varepsilon_0}$$ (21)

where $q(t)$ – the surface charge value in a pore, equal for both positive and negative ions.

An upper surface charge is proportional to the number of ions $M(t)$ reached that surface for the time $t$ and can be written as:
\[ q_1(t) = q_0^e \mu^* \pi R^2 \int_0^t [E_0 - E_i(t')] \, dt' \]  

Equations (21) and (22) lead to the next differential equation for \( E_i(t) \):

\[ \frac{dE_i(t)}{dt} + \alpha E_i(t) = \alpha E_0 \]  

where

\[ \alpha = \frac{\pi R^2 q_0^e n c \mu^*}{\varepsilon_0} = \frac{P q_0^e c \mu^*}{\varepsilon_0} \]  

The solution to the equation (23) corresponds to the simple law:

\[ E_i(t) = E_0 [1 - e^{-\alpha t}] \]  

with the time of relaxation

\[ \tau_q = \alpha^{-1} = \frac{\varepsilon_0}{P q_0^e c \mu^*} = \frac{\varepsilon_0}{P \sigma} \]  

where \( \sigma \) - specific conductance of LC. Estimations made in accordance with (26) give us \( \tau_q=4.4\,\text{ms} \) for typical values of the specific conductance of LC \((2.10^8\,\text{Om}\,\text{m}^{-1})\) and the film porosity \((0.1)\), which corresponds to the typical values of experimentally measured decay times.

Obtained expressions enable us to explain qualitatively the dependence of the decay time for electro-optical response on the pore radius. It is known that the electric field intensity has to be increased with reducing the pore radius, to have a notable impact on the refractive index of LC. It corresponds to the low values of the shielding field \( E_i \) in comparison with the field \( E_0 \) and the small values of the decay times to the initial state, in the framework of the model mentioned above. At the same time, the rise of the pore sizes leads to the increasing of the intensity range where strong changes of orientation take place. It results in the rise of the ratio \( E_i/E_0 \), and increasing (accordingly to (26)) of a decay time, needed for relaxation of the system to the initial state.

The quantitative comparison of the theory with the experiment can be done by using the dependence of the threshold voltages \( U^* \) on a pore diameter, which results in an appearance of the pike-shaped response (figure 1).

The data from this graph can be used to calculate the field intensity \( E_c = U^*/h \), corresponding to the appearance (or disappearance) of a strong electro-optical response, to the initial intensity \( E_0 = U_0/h \) which is constant for the data shown in figure 1. Taking into account the above-mentioned remarks one can get from equation (25) the next expressions:

\[ E = E_0 - E_i(t) = E_0 e^{-\alpha t} \]  

\[ t_{\text{dec}}(d) = \tau_q \ln\left( \frac{U_0}{U^*(d)} \right) \]

Due to the variety of the ion diffusion mechanisms, it is reasonable to consider the specific conductance of LC as a fitting parameter.

According to the data (figure 3) the dependence \( U^*(d) \) can be described by the law \( U^*(d) = A/d \)

It results in the next expression for a decay time:

\[ t_{\text{dec}}(d) = \tau_q \ln\left( \frac{U_0}{A} \right) d \]
The obtained dependence (29) is in an agreement with the results of experiment, as it shown in figure 3.

5. Conclusion
In this paper two physical mechanisms, which can be responsible for the specific electro-optical response of a liquid crystal, infiltrated into pores of a porous polymer film are considered. Both mechanisms are connected with electro-kinetic phenomena, namely motion of ions and overall electro-osmotic flow, arising when an axial electric field is applied to the liquid crystal confined by a pore’s surface. The first mechanism describes a deformation of menisci, induced by electro-osmotic flow, which results in arising of capillary forces and slowing of the flow. Such mechanism can act as a trigger which stimulates orientational changes in the vicinity of the Freedericksz transition at the abrupt change of the field polarity. The second mechanism describes time variations of the effective value of the electric field’s strength arising due to the periodical changes of direction of ion’s motion at a periodical changes of the field polarity. This mechanism can be responsible for a breaking and restoring of surface anchoring leading to a sharp negative peak in the electro-optical response.

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