The Casimir Energy Paradox of the QCD String

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It is widely thought that the early onset of the asymptotic Casimir energy with unit conformal charge signals bosonic string formation of the confining flux connecting a static quark-antiquark pair in QCD. This is observed on a scale where most of the string eigenmodes do not exist and the few stable modes above the ground state are displaced. Hints for the resolution of this paradox are suggested.

1. Puzzle in the QCD String Spectrum

A new analysis of the fine structure in the QCD string spectrum was presented at Lattice 2002 \cite{1}. Shortly afterwards, two papers were submitted for publication with focus on complementary aspects of the same problem \cite{1,2}.

Ref. \cite{1} reported a comprehensive study of the QCD string spectrum as the quark–antiquark separation $R$ was varied in the range $0.2 \text{ fm} < R < 3.0 \text{ fm}$ (Fig. 1). On the shortest length scale, the excitations were consistent with short distance physics without string imprint in the spectrum. A crossover region below 2 fm was identified with a dramatic rearrangement of the level orderings. On the largest length scale of 3 fm, the spectrum exhibited string-like excitations with asymptotic $\pi/R$ string gaps split by a fine structure. It is quite remarkable that the torelon spectrum, which is free of end effects, exhibits a similar fine structure, as reported for the first time at this conference \cite{3}.

In a complementary study \cite{2}, the Casimir energy and the related effective conformal charge, $C_{\text{eff}}(R) = -12R^3F'(R)/(\pi(D-2))$, were isolated where $F(R)$ is the force between the static color sources and $D$ is the space-time dimension of the gauge theory. With unparalleled accuracy, $C_{\text{eff}}(R)$ was determined for the gauge group SU(3) in three and four dimensions in the range $0.2 \text{ fm} < R < 1.0 \text{ fm}$ below the crossover region of the string spectrum. It was suggested that the rapid change of the effective conformal charge from what is expected in the running Coulomb law to $C_{\text{eff}}(R) \approx 1$ well below 1 fm is a signal for early bosonic string formation. The results are surprising because the scale $R$ is not large compared with the expected width of the confining flux, and, perhaps more quantitatively, the string imprint in the Casimir energy is observed in the $R$ range where the spectrum exhibits complex non-string behavior, as shown in Fig. 1.

After a long series of comprehensive studies, we report here new results in the three dimensional $Z(2)$ gauge model and a simple resonance model for a better understanding of the seemingly paradoxical situation.

2. String Formation and $C_{\text{eff}}$ in $Z(2)$ Model

References to earlier work on the three-dimensional $Z(2)$ gauge model can be found in a recent paper on the finite temperature properties of the $Z(2)$ string \cite{4}. The well-known dual transformation of the model to Ising variables facilitates very efficient simulations and analytic studies of the Casimir energy and the string spectrum in the $\Phi^4$ field theory setting. This is illustrated in Figs. 2 and 3. We applied the simple definition $C_{\text{eff}}(R) = -24R^2(E'(R) - \sigma)/\pi(D-2)$

\textsuperscript{*}Talk presented by J. Kuti.
with the string tension $\sigma$ determined in high precision separate runs from the ground state of long torelons. The simulations in Fig. 2 are compared with the predictions of the Nambu-Goto (NG) string model and the analytic first term in the $\hbar$ expansion of the equivalent $\Phi^4$ field theory setting. The NG spectrum with fixed end boundary conditions in $D$ dimensions was first calculated in Ref. [4] with the result $E_N = \sigma R(1 - \frac{D-2}{12(D-1)} + \frac{2\pi N}{aR})^2$ where $N=0$ is the string ground state. Although there exists an inconsistency in the quantization of angular momentum rotations around the $q\bar{q}$ axis at finite $R$ values unless $D = 26$, the problem asymptotically disappears in the $R \to \infty$ limit [5]. It has been ex-
pected that predictions for strings emerging from field theory and their effective bosonic string description at large $R$ will be similar to that of the NG model with corrections which are increasingly important at smaller $R$.

The field theory calculation of $C_{\text{eff}}$ includes the classical energy of the string-like soliton being formed and the sum of zero-point energies which are dominated by glueball scattering states in the bulk! The few stable and displaced “string-like” modes are also contributing. How scattering states might conspire to produce $C_{\text{eff}} \approx 1$ is further illustrated below in a simple resonance model of massless scalar field theory. Our results are closer to the leading order field theory calculation and deviate substantially from the predictions of the NG string model, particularly at smaller $R$ values. This differs from the tantalizing findings of Ref. [4] where simulations of the finite temperature $Z(2)$ string were presented to be in agreement with the 1–loop NG string model. For further illustration, the first string excitation in Fig. 3 is compared with the NG model and its loop expansion which completely breaks down below 1 fm. The leading order field theory calculation, which

Figure 1. Short distance degeneracies and crossover in the QCD string spectrum from Ref. [1] where the notation is explained. The symbol LW indicates the $R$ range of $C_{\text{eff}}(R)$ in Ref. [2].

Figure 2. The red points are from high precision Z(2) simulations. The solid black curve with NG label is the full NG prediction, $C_{\text{eff}}(R) = 1$ is the asymptotic string result (tree-level NG), the dashed blue line is the 1-loop NG approximation. The solid red line represents the analytic first term in the $\hbar$ expansion.
Figure 3. The energy gap $\Delta E$ above the ground state is plotted as $\Delta E/(N\pi/R) - 1$ to show percentage deviations from the asymptotic string level for $N=1$. Several $Z(2)$ simulations with cyan, blue, red, and green points are combined with good scaling properties. The open circles represent $D=3$ $SU(2)$ results after readjusting the ratio of the string tension $\sigma$ to the glueball mass in $Z(2)$. The black line is the full NG prediction, the dashed blue and green lines are 1-loop and two-loop NG approximations, respectively.

does not assume string formation, accounts for the shape of the spectrum quite well.

3. Simple Resonance Model

Consider a massive scalar field in one space and one time dimension interacting with an external source $J(x)$ according to the Lagrangian $L = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} M^2 \Phi^2 - J(x)\Phi^2$. The field is confined between two opaque barriers represented by repulsive delta-function potentials $J(x) = -\frac{1}{2} M^2 (\Theta(x) - \Theta(x-L)) + \lambda (x + \delta(x-L))$ where $\lambda$ is tunable and the field is rendered massless inside a square well. The field eigenmodes $E_n(L)$ and $C_{eff}(L)$ can be calculated as shown in Fig. 4. There are only four bound "string modes" for the two choices of $\lambda$. On the left side $\lambda = 10$, $C_{eff} = 0.99$ and the sharp resonance spectrum is located at the expected string positions. On the right, $\lambda = 0$ and the sharp resonances disappear from the bulk scattering state spectrum. Nevertheless, $C_{eff} = 0.85$ is only 15 percent off from the $L = \infty$ limit. A hidden mechanism on phase shifts might keep the conformal charge close to what it would be for a perfect massless string spectrum trapped inside.

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REFERENCES

1. K.J. Juge, J. Kuti, and C. Morningstar, Phys. Rev. Lett. 90, (2003) 161601.
2. M. Lüscher and P. Weisz, JHEP 0207, (2002) 049.
3. K.J. Juge, J. Kuti, F. Maresca, C. Morningstar, and M. Peardon, this Proceedings.
4. M. Caselle, M. Hasenbusch, M. Panero, JHEP 0301 (2003) 057.
5. J. F. Arvis, Phys. Lett. 127B, (1983) 106.