Amorphous vortex glass phase in strongly disordered superconductors

Jack Lidmar
Department of Physics, Royal Institute of Technology, AlbaNova, SE-106 91 Stockholm, Sweden
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We introduce a model describing vortices in strongly disordered three-dimensional superconductors. The model focuses on the topological defects, i.e., dislocation lines, in an elastic description of the vortex lattice. The model is studied using Monte Carlo simulations, revealing a glass phase at low temperatures, separated by a continuous phase transition to the high temperature resistive vortex liquid phase. The critical exponents $\nu \approx 1.3$ and $\eta \approx -0.4$ characterizing the transition are obtained from finite size scaling.

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Disorder has a profound impact on the phase diagram of type-II superconductors. The long range order of the Abrikosov vortex lattice in very clean samples is destabilized upon the introduction of disorder [1] and a glass state can appear instead [2, 3]. The effects of weak disorder are relatively well understood and may be treated within an elastic theory of the vortex lattice [4, 5, 6]. The resulting phase, known as the Bragg glass or the elastic vortex glass, is characterized by an algebraic decay of translational correlations and perfect orientational order, and the absence of large dislocation loops [6].

For stronger disorder dislocations start to become energetically favorable and begin to proliferate. What happens then is still a matter of controversy. One possibility is that any kind of order in the system is lost and the system becomes thermodynamically equivalent to a vortex line liquid (although perhaps a very viscous one) with a non-zero resistivity [6]. The other possibility is that the dislocations themselves remain pinned and the system freezes into a different genuine thermodynamic amorphous glass phase, full of dislocation loops, and with zero linear resistivity. Such a disorder driven transition from the Bragg glass to the vortex glass have been proposed to occur as the magnetic field is increased [6, 7].

Unfortunately, the theoretical methods based on pinning of elastic manifolds in random media, that were so successful for the Bragg glass, does not easily extend to incorporate the effects of dislocations. In this Letter we introduce and study a model that goes beyond a purely elastic description by including dislocations, and is well suited for numerical simulations.

Experimentally, the evidence for a vortex glass comes mainly from the scaling relations obeyed by the current-voltage characteristics and resistivity at a continuous phase transition [6]. The first order vortex melting line, observed in very clean samples at low magnetic fields, is replaced by a smooth continuous transition upon increasing the field or disorder. The universal critical exponents extracted from measurements in this regime provide an important characterization of the glass transition that can be compared with theoretical predictions. A large number of studies have thus been able to obtain good scaling behavior in different materials, and this fact has been taken to support the existence of an amorphous glass phase [8]. This interpretation has, however, been questioned recently [9]. In practice it may be extremely difficult to discriminate between a true thermodynamic glass and a very slowly moving non-equilibrium viscous vortex liquid (as window glass), with a small, but non-zero, resistivity due to thermally activated flux creep.

On the theoretical side a lot of interest has focused on the so called gauge glass model [10]. This approach, which is inspired by spin glasses in random magnets, combines the $U(1)$ symmetry of a superconducting order parameter with quenched disorder and frustration, which are believed necessary to create a glass phase, but has no other connection with the microscopics. The model, which consists of an XY-model in a quenched random gauge field, has a glass phase at low temperatures in three dimensions [10] (but probably not in two). Once screening of the vortex interaction, due to gauge field fluctuations, is accounted for, the glass phase disappears [11]. However, the way disorder enters the model, via quenched random fluxes instead of a random core energy coupling to the vortices, is not very realistic. More importantly, perhaps, the model lacks a length scale, the inter-vortex distance, set by the external magnetic field.

Attempts to study more realistic models have come to contradictory conclusions. Simulations of frustrated 3D XY-models with quenched disorder show evidence for a direct disorder driven transition from the Bragg glass to a vortex liquid at low disorder and fillings, and find no clear signs of any amorphous glass phase [12] (See, however, Ref [13]. On the other hand recent simulations [14] of a random pinning vortex model for much stronger disorder and very high field, corresponding to a half-filled system, do find a continuous transition, but with critical exponents quite far from experiments. None of these models included screening. Including screening in a similar model appears to destroy the glass phase [13], just as in the gauge glass. Langevin simulations of vortex lines also found no glass phase [15]. Recently, Kierfeld et al. [16] invoked a Landau theory in terms of the dislocation density, assuming the dislocation lines were directed,
and proposed a phase diagram containing an amorphous glass phase. Some support for a glass phase has also been found using the boson analogy \(^{17}\).

The approach taken in this Letter starts from an elastic theory of the vortex lattice, but modified to include dislocations. The basic degrees of freedom in our model is the two-dimensional displacement vector \(\mathbf{u}_i\) and a tensor field \(\mathbf{B}\) that accounts for the dislocations. Since the clean Abrikosov vortex lattice usually has hexagonal symmetry we define the model on a stacked triangular lattice with \(N = L^3\) sites and unit lattice constant, and with the displacements \(\mathbf{u}_i\) living on the vertices and the field \(\mathbf{B}_{ij}\) living on the links connecting nearest neighbors \(i\) and \(j\). The Hamiltonian reads

\[
\mathcal{H} = \frac{1}{2} \sum_{(ij)} (\mathbf{w}_{ij} - \mathbf{F}_{ij})^2, \tag{1}
\]

where \((ij)\) denote all nearest neighbor pairs on the stacked triangular lattice, \(\mathbf{w}_{ij} = \mathbf{u}_i - \mathbf{u}_j + \mathbf{B}_{ij}\), and \(\mathbf{u}_i \in \mathbb{R}^2\). The \(\mathbf{B}_{ij}\) are integer multiples of the elementary triangular lattice vectors \(\mathbf{a}_1 = (1,0), \mathbf{a}_2 = (\frac{1}{2}, \frac{\sqrt{3}}{2})\) in the plane, i.e., \(\mathbf{B}_{ij} = m \mathbf{a}_1 + n \mathbf{a}_2, m, n \in \mathbb{Z}\), and contain all the information about the dislocations. More precisely the dislocation density is given by the lattice curl of \(\mathbf{B}\), implying that dislocations form lines with conserved Burgers vector as they should. A dual representation in terms of interacting dislocation lines can be obtained by integrating out \(\mathbf{u}_i\), but Eq. \((1)\) turns out to be more convenient for our purposes. The coupling to disorder occurs via the random variable \(\mathbf{F}_{ij} \sim \mathcal{N}(0,\sigma^2)\). For simplicity we take the maximum possible disorder strength, with \(\mathbf{F}_{ij}\) independently and uniformly distributed in the planar unit cell of the triangular lattice.

In the absence of dislocations all \(\mathbf{B}_{ij}\) are zero and the model reduces to an ordinary elastic model of the vortex lattice with compression modulus \(c_{11} = 3/2\), tilt modulus \(c_{44} = 1\), and shear modulus \(\sigma_{eq} = 3/2\) \(^{22}\). The generalization to other values of these (bare) elastic constants is straightforward via the addition of a term proportional to \(\sum_{(ij)} (\mathbf{w}_{ij} \cdot \mathbf{e}_{ij})^2 [\mathbf{e}_{ij}\) being the unit vector connecting nearest neighbor sites \(i\) and \(j\)] to Eq. \((1)\) and/or by changing the coupling in the \(z\)-direction to make it anisotropic. In this work we will, however, restrict to the isotropic model defined above.

The presence of thermally excited dislocations will renormalize the elastic constants and eventually cause a transition at which they drop to zero. The renormalized elastic constants can be obtained by taking the second derivative of the free energy with respect to a deformation, \(\mathbf{w}_{ij} \rightarrow \mathbf{w}_{ij} + \varepsilon_{\mu\nu}(\mathbf{e}_{ij} \cdot \mathbf{e}_\nu)\mathbf{e}_\mu\). This gives

\[
C_{\mu\nu|\rho\sigma} = \frac{\partial^2 f}{\partial \varepsilon_{\mu\rho} \partial \varepsilon_{\nu\sigma}} = \frac{1}{2} \delta_{\mu\rho} \delta_{\nu\sigma} (3 - \delta_{\mu,\nu})
\]

\[
- \frac{N}{T} \langle (\sigma_{\mu\nu} \sigma_{\rho\sigma}) - (\sigma_{\mu\nu}) (\sigma_{\rho\sigma}) \rangle, \tag{2}
\]

where \(\langle \cdots \rangle\) denotes a thermal average, and the \(3 \times 2\) stress tensor \(\sigma\) is given by

\[
\sigma_{\mu\nu} = \frac{1}{N} \sum_{(ij)} (\mathbf{e}_{ij} \cdot \mathbf{e}_\mu)(\mathbf{e}_\nu \cdot [\mathbf{B}_{ij} - \mathbf{F}_{ij}]). \tag{3}
\]

Unfortunately, both of these vanish upon averaging over the disorder realizations, \([\langle \sigma \rangle]_{av} = [C]_{av} = 0\), since \([f]_{av}\) becomes independent of \(\varepsilon_{\mu\nu}\) for the fully disordered model, where the averaging includes all possible deformations. A non-zero \(\langle \sigma \rangle\) is still indicative of a phase with rigidity, and hence we use

\[
\sigma_{\mu\nu}^2 = \left[ \sum_{\mu\nu} \langle \sigma_{\mu\nu} \rangle^2 \right]_{av} = \left[ \sum_{\mu\nu} \langle \sigma_{\mu\nu} \rangle \langle \sigma_{\mu\nu} \rangle \right]_{av} \tag{4}
\]

instead, where the second form, which involves two different replicates \(\alpha, \beta\) to avoid any bias in the expectation value, is the one used in the actual simulations. Alternatively, \(\sigma_{\mu\nu}^2\) can be calculated from Eq. \((2)\) using that \([C]_{av} = 0\),

\[
\sigma_{\mu\nu}^2 = \left[ \sum_{\mu\nu} \langle \sigma_{\mu\nu} \rangle^2 \right]_{av} = \frac{8T}{N}. \tag{5}
\]

In the solid low temperature phase (if it exists) \(\sigma_{\mu\nu}^2\) will acquire a non-zero expectation value, while in the liquid it approaches zero. Close to a continuous phase transition it obeys the finite size scaling form,

\[
\sigma_{\mu\nu}^2 = L^{1-d} \tilde{\sigma}(tL^{1/\nu}), \tag{6}
\]

where \(t = (T - T_c)/T_c\) is the reduced temperature, \(\nu\) the correlation length exponent, \(d = 3\) the dimensionality, and \(\tilde{\sigma}\) is a scaling function \(^{27}\).

Lattice order may be probed by the structure function \(S_{\mathbf{K}} = \frac{1}{N} \langle \sum_{\mathbf{i}} e^{i\mathbf{K} \cdot \mathbf{u}_i} \rangle\) at a reciprocal lattice vector \(\mathbf{K}\) of the triangular lattice, but for strong disorder, when the system might freeze into a highly disordered glassy state, this is not useful. To quantify order in a glass one may instead consider an overlap order parameter,

\[
\langle \mathbf{q}_{\mathbf{K}} \rangle = \frac{1}{N} \sum_{\mathbf{i}} e^{i\mathbf{K} \cdot (\mathbf{u}_i^0 - \mathbf{u}_i^*)}, \tag{7}
\]

between two replicates \(\alpha, \beta\). The order parameter susceptibility, given by

\[
\chi_{\mathbf{K}} = N \left[ \langle \mathbf{q}_{\mathbf{K}}^2 \rangle \right]_{av}, \tag{8}
\]

obeys the finite size scaling relation

\[
\chi_{\mathbf{K}} = L^{2-\eta \tilde{\chi}}(tL^{1/\nu}), \tag{9}
\]

which defines the critical exponent \(\eta\).

We study this model via Monte Carlo (MC) simulations. One MC sweep consists of \(N\) randomly chosen
FIG. 2: Test of equilibration in the glass. As seen, $\sigma^2_{\text{rms}}$ from Eq. (4) and (5) converge to a common value from below and above, which provides a very useful check on the equilibration.

We begin with a brief discussion of the clean model (all $F_{ij} = 0$). Figure 1 shows a sharp drop in the structure function $S_K$ and in the renormalized shear modulus $c_{66}$, and a discontinuous jump in the average energy, with a corresponding approximate delta-function peak in the specific heat $C_V$. The very sharp variation in these quantities at $T_c \approx 0.076$ signifies a discontinuous first order transition in accordance with experiments [19] as well as numerical simulations of frustrated XY-models [20] and vortex line models [21, 22]. This is also confirmed by looking at histograms for the energy or the structure function, which show an increasingly growing double-peak structure developing for the largest system sizes.

We now turn to the disordered model. In the maximally disordered model the dynamics becomes very slow at low temperatures, and it is important to check the convergence of the MC simulation. To do so we monitor several quantities averaged over increasingly longer simulation times, after discarding an equally long time for equilibration. In Fig. 2 we plot, as an example, $\sigma^2_{\text{rms}}$ calculated from Eq. (4) and (5), and $\chi_K$ as a function of MC time for the size $L = 5$ and the lowest $T$ used in the exchange MC for this size. As seen the curves saturate and become time independent which indicates that the simulation has converged at around 100000 sweeps. Similar checks are done for all sizes. Typically, 4000–7000 disorder realizations are used to form disorder averages.

In the inset of Fig. 3 we show MC data for $L^4\sigma^2_{\text{rms}}$ according to Eq. (6). Corrections to scaling are clearly visible for the smallest size, $L = 3$, which was therefore omitted from the fitting procedure. The crossing point in the inset gives an estimate of $T_c$.

FIG. 3: Finite size scaling collapse of MC data for $\sigma^2_{\text{rms}}$ according to Eq. (6). Corrections to scaling are clearly visible for the smallest size, $L = 3$, which was therefore omitted from the fitting procedure. The crossing point in the inset gives an estimate of $T_c$.

In order to overcome the slow glassy dynamics in the fully disordered system an exchange (or parallel tempering) MC algorithm [18] is used, where several systems with identical disorder configurations but different temperatures are simulated in parallel and sometimes exchanged. The temperatures were chosen in a range around $T_c$, such that the acceptance rate of temperature exchanges were no less than 15%.

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finite size scaling collapse using Eq. (9) of \( \chi^2 \) of \( \chi_K \) (averaged over the smallest reciprocal lattice vectors). As a measure of the quality of the data collapses we use the error in a \( \chi^2 \)-fit of the scaled data to a polynomial (indicated by the dotted lines in the figures). The critical parameters are estimated by adjusting them to minimize this error, and the errorbars are estimated by varying them until the data no longer scales. The final results for the critical parameters are summarized in Table I.

In summary, we have introduced a model of vortex lattices subjected to disorder. The model goes beyond an elastic description by including dislocations. Such a description should be valid as soon as the screening length is larger than the inter-vortex distance, \( \lambda \gtrsim a_v \). The model is studied via MC simulations and finite size scaling. Without disorder the model shows a first order melting transition. For strong disorder, the model has a low-temperature glass phase where the dislocation loops are pinned by the disorder. The glass melts via a continuous transition characterized by the critical exponents in Table I. We propose that the model gives an effective description of the vortex glass transition in type-II superconductors. Experiments usually find a correlation length exponent in the range \( 1 < \nu < 2 \), which compares well with the present value \( \nu = 1.3 \pm 0.2 \). Very recent simulations on disordered 3D \( XY \)-models find evidence for a glass transition with \( \nu = 1.5 \pm 0.12 \) or \( \nu = 1.1 \pm 0.2 \), \( \eta = -0.5 \pm 0.1 \) [24], consistent with the present study. It would be interesting to extend the study to include dynamic quantities, and to map out the phase diagram for intermediate disorder strengths and other values of the bare elastic constants.

* Electronic address: jlidmar@kth.se

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[25] This corresponds to a random stress. Other ways of incorporating disorder would also be interesting to consider.
[26] To avoid factors of \( \sqrt{3}/2 \) extensive quantities are normalized using \( N \) instead of the volume \( \Omega = N \sqrt{3}/2 \).
[27] The absence of any nontrivial anomalous dimension in Eq. (6) is a result of the underlying periodicity in the model. At a fixed point with such periodicity the scaling dimension of \( \mathbf{u} \) must vanish, and hence \( w \) and \( B \) scales as \( L^{-1} \) and \( \sigma \) as \( L^{-d} \).

**TABLE I**: Estimates of the critical properties of the model.

| \( T^*_c \)     | \( \nu \)     | \( \eta \)     |
|---------------|---------------|---------------|
| 0.018 ± 0.002 | 1.3 ± 0.2     | -0.4 ± 0.1    |

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FIG. 4: Finite size scaling collapse of MC data for \( \chi_K \) using Eq. (9).