Subsonic and Transonic Flow Simulation for an Oscillating T-Tail
—Effects of In-plane Motion of Horizontal Tail Plane—*

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The flutter speed of a T-tail depends strongly on the angle of attack or dihedral angle of the horizontal tail plane (HTP). The unsteady rolling moment acting on the HTP oscillating in yaw and sideslip, which is induced by the bending and torsion of the vertical tail plane, plays a critical role in this phenomenon. In this paper, a 3D Navier-Stokes code that takes into account the in-plane motion of the HTP is developed, and the unsteady flow features of transonic flow around an oscillating T-tail are clarified, especially, in relation to the mechanism generating the rolling moment.

Key Words: T-tail, Flutter, Transonic Flow, Sideslip and Yaw, CFD

1. Introduction

The T-tail flutter speed depends strongly on the angle of attack or dihedral angle of the horizontal tail plane (HTP). (Note that the flutter speed of a conventional (single) wing does not depend on the angle of attack in the case of subsonic flight unless flow separation occurs.) This phenomenon was first recognized through the investigation of an accident involving a Handley Page Victor which was lost due to T-tail flutter in 1954.1) The mechanism of the phenomenon can be explained by the fact that the rolling moment generated by the yawing and sideslip oscillation of the HTP, which is induced by the torsion and bending oscillations of the vertical tail plane (VTP), depends on the angle of attack (or steady lift) and the dihedral angle of the HTP. This mechanism has been clearly shown in the flutter analyses of several T-tail configurations by Washizu et al.,2) Jennings and Berry3) and van Zyl and Mathews.1) Washizu et al.2) and Jennings and Berry3) used quasi-steady strip aerodynamic forces to evaluate the rolling moment due to the in-plane motion of the HTP. Van Zyl and Mathews4) extended the conventional doublet lattice method to account for these aerodynamic loads using the lifting-line concept proposed by Queijo.5) Their method4) can be applied to T-tail flutter analyses in a subsonic compressible flow. The exact lifting-surface theory for a wing oscillating in yaw and sideslip was developed by Isogai and Ichikawa in 1973.5) Isogai7) also extended this theory to T-tail configuration, which takes into account the in-plane motion of the HTP. Unfortunately however, the theory can only be applied to incompressible flow, though the effect of compressibility, especially transonic flow, on T-tail flutter is important. Ruhlin and Sandford9) conducted experimental studies of transonic T-tail flutter for a wide-body transport airplane. They found that the transonic flutter boundary of the anti-symmetric flutter of this T-tail showed an unusually sharp dip between the Mach numbers 0.92 and 0.98. This phenomenon clearly shows the importance of establishing the prediction method of T-tail flutter, especially, in the transonic regime.

Recently, computational fluid dynamics (CFD) has been applied to the transonic flutter of T-tail configurations.9–11) Arizono et al.9,10) presented the numerical simulations of a T-tail configuration using an Euler code. They also compared their results with experimental data, although the results of their numerical simulation show a considerable discrepancy with those of the experiment. Additionally, in their studies, the effects of the angle of attack and the dihedral angle of the HTP were not studied in detail. Attorni et al.11) also conducted the T-tail flutter simulation of P180 aircraft using an Euler code. However, the accuracy of the method was not confirmed because no comparison with the experimental data was made. Isogai12) presented a numerical method for computing the subsonic and transonic flows for a wing oscillating in yaw and sideslip using 3D Navier-Stokes (NS) equations as the first step towards a complete T-tail configuration. He showed that, by introducing a new coordinate system oscillating in yaw and sideslip, the existing 3D NS code can easily be modified to account for in-plane motions. The calculated rolling moments showed good agreement with the existing experimental data obtained for incompressible flow, and the effect of compressibility, especially the effects of the shock wave in the transonic flow on the rolling moment, is clarified.

The purpose of the present paper is to extend the method proposed for a single wing in the study by Isogai13) to a T-tail configuration, for which the effect of in-plane motion of the HTP is taken into account. Additionally, the effects of compressibility on the flow around the oscillating T-tail are evaluated; especially, the effects of the shock wave and shock-induced flow separation on the rolling moment around the HTP.
2. Numerical Method

As the numerical method to take into account the effect of the in-plane motion of the HTP, we utilize the same method proposed by Isogai, that is, we introduce new coordinate systems that move with the in-plane motion of the HTP. Figure 1 shows the definition of the coordinate system. The $x'y'z'$ coordinate system is fixed to the free-stream and the $x'z'$ coordinate system is the coordinate system that moves with the in-plane motion of the HTP. (In Fig. 1, $h_t$ and $h_f$ are the displacements of the HTP and VTP normal to their time mean surfaces, respectively.) The relation between the two coordinate systems can be given by

$$x = x' + \Psi z', \quad y = y', \quad z = z' + H - \Psi(x' + a) \tag{1}$$

where $H$ and $\Psi$ are the sideslip and yawing displacements of the HTP, respectively, and where the higher-order terms of $\Psi$ are neglected. (See Appendix I for the derivations of Eq. (1), (2) and (3).) In Eq. (1) $a$ is the position of the axis of yaw (positive toward the leading-edge). (Note that length is made non-dimensional by the semichord ($b$) at the junction.) The conventional 3D NS code can be easily modified by changing the metric terms\textsuperscript{13) such as $\xi_k, \xi_k, \xi_k, \xi_k,$ etc., which transform the 3D NS equations in the $x'y'z'$ coordinate system into the computational space ($\xi, \eta, \zeta$), namely,

$$\xi_t = J(y'(\Psi x' + z') - y'(-\Psi z' + z_0))$$
$$\xi_z = J(z'x' - x'z')$$
$$\xi_y = J(y'x' + z') - y'(-\Psi x' + z_0))$$
$$\eta_t = J(x'z' - x'z')$$
$$\eta_z = J(y'x' + z') - y'(-\Psi x' + z_0))$$
$$\zeta_t = J(y'z' - z_0)$$
$$\zeta_z = J(y'z' - z_0)$$
$$\zeta_y = J(y'z' - z_0)$$

$$\xi$ = $-((d\Psi/dt)e_{\xi} - z_{\xi}) - (dH/dt - d\Psi/dt(x' + a))\zeta$$
$$\eta = -((d\Psi/dt)e_{\eta} - z_{\eta}) - (dH/dt - d\Psi/dt(x' + a))\zeta$$
$$\zeta = -((d\Psi/dt)e_{\zeta} - z_{\zeta}) - (dH/dt - d\Psi/dt(x' + a))\zeta \tag{2}$$

where $t$ is the non-dimensional time ($t = T(U/b)$; $T$: time; $U$: free-stream velocity) and where $J$ is the Jacobian of the transformation metrics and is given by

$$J^{-1} = x'_{\xi}y'z'_{\eta} + x'_{\eta}y'z'_{\xi} + x'_{\xi}y'z'_{\xi}$$
$$-x'_{\xi}y'z'_{\eta} - x'_{\eta}y'z'_{\xi} - x'_{\xi}y'z'_{\xi} \tag{3}$$

Note that no special treatment of the far field boundary condition and the surface boundary condition (no slip condition) on the HTP is necessary in the new $x'y'z'$ coordinate system once the conventional metrics are replaced with those given by Eq. (2). However, some caution is necessary as to the displacement of the VTP surface. When we denote the displacement of the VTP normal to its time mean surface as $h_f(x', y', t)$, the $z'$ coordinate of the displaced surface can be expressed from Eq. (1) as

$$z' = h_f(x', y', t) - H + \Psi(x' + a) \tag{4}$$

This relation should be noted in the grid generation process.

The Reynolds Averaged Navier-Stokes (RANS) code originally developed by Isogai\textsuperscript{14) was modified to treat the T-tail configuration following the procedure just described. The planforms of the T-tail HTP and VTP used in the present study are both rectangular, for which the theoretical\textsuperscript{7) and experimental data\textsuperscript{15) are available for incompressible flow cases. The planforms of the T-tail HTP and VTP used in the present study are both rectangular, for which the theoretical\textsuperscript{7) and experimental data\textsuperscript{15) are available.}\textsuperscript{26} The span and chord length of the HTP are 0.60 m and 0.20 m, respectively, and the aspect ratio is 3.0. The span and chord length of the VTP are 0.30 m and 0.20 m, respectively, and the aspect ratio is 1.5. The airfoil sections of the HTP and VTP are NACA65A010. There is no fairing at the junction of the HTP and VTP. In the experiment of Ichikawa et al.,\textsuperscript{15) the wind tunnel model was so rigid that it could be assumed to be rigid. The VTP was oscillated in pitch around the mid-chord axis, and the rolling moment around the HTP was measured in a 2 m × 2 m low-speed wind tunnel of the National Aerospace Laboratory of Japan. The experimental data of the rolling moments was obtained for $k = 0$\textendash 0.25 (k: reduced frequency defined by $k = boo/U$; $\omega$: circular frequency) for the two cases of the mean angle of attack ($\alpha_{II}$) of the HTP; namely, $\alpha_{II} = 0$ deg and $\alpha_{II} = 5.48$ deg, respectively. The dihedral angle ($\Gamma$) of the HTP for both cases was 0 deg, and the Reynolds number was $3.2 \times 10^5$.

As the first step of the present study, the flow simulations at $M_{\infty} = 0.30$ are conducted to examine the accuracy and the reliability of the present code. Additionally, the rolling moments around the HTP are compared with the experimental data of Ichikawa et al.\textsuperscript{15) and the theoretical results of the lifting-surface theory of Isogai.\textsuperscript{7) As the next step, the transonic flow simulations for the same T-tail configuration are conducted to evaluate the effects of Mach number, reduced
frequency and angle of attack of the HTP on the rolling moment. Figure 2 shows the distribution of the surface grid generated around the present T-tail configuration. The grid is a H-H type structured grid. The number of grid point around the HTP is 120 in the \(x_0\) direction (61 points on the upper/lower surfaces, respectively), 66 points in the \(y_0\) direction (31/35 points normal to the upper/lower surfaces, respectively) and 78 points in the \(z_0\) direction (58 points on the wing surface). The number of grid points around the VTP is 120 in the \(x_0\) direction (61 points on the upper/lower surfaces, respectively), 35 points in the \(y_0\) direction and 78 points in the \(z_0\) direction (39 points normal to the upper/lower surfaces, respectively). (The effect of number of grid points and time accuracy of the code are discussed in Appendix II.) The code utilizes the Total Variation Diminishing (TVD) scheme\(^{16}\) and the Baldwin and Lomax algebraic turbulence model\(^{17}\) (Reynolds number is set to be \(3.20 \times 10^5\), which is equal to that of the experiment\(^{15}\)).

3. Results and Discussion

Before we present the results of the computation, it might be worthwhile to mention that the rolling moment around the HTP is composed of the following three components, whose generation mechanisms are different from each other.

1) The rolling moment due to the rolling motion of the HTP: This rolling moment is generated by the asymmetry of the effective angle of attack between the right and left wings, which is induced by the rolling motion of the HTP. (Note that this rolling moment is not generated in the present case since the motion of the HTP is pure yawing oscillation.) It should also be noted that this component does not depend on the angle of attack (or steady lift) or the dihedral angle of the HTP.

2) The rolling moment due to the end-plate effect: A pressure difference appears between the upper and the lower surfaces of the VTP when it oscillates in pitch and heave. This pressure difference induces the asymmetric pressure difference between the right and left wings of the HTP, which is attached on top of the VTP, generating rolling moment around the HTP. This rolling moment does not depend on the angle of attack (or steady lift) or the dihedral angle of the HTP.

3) The rolling moment due to in-plane motion: As already discussed, this rolling moment is generated by the sideslip and the yawing motion of the HTP, and it is the cause of the phenomenon that T-tail flutter speed depends on the angle of attack (or steady lift) and the dihedral angle of the HTP.

3.1. Results of \(M_\infty = 0.30\)

Figure 3 shows the variation of the rolling moment (the magnitude and phase angle with respect to the yawing displacement of the HTP) around the HTP with respect to \(k\) for \(\alpha_\text{H} = 5.48\) deg at \(M_\infty = 0.30\). As described previously, the VTP is oscillated in pitch with the amplitude of 6 deg around the mid-chord line. Therefore, the HTP is oscillated in yaw around the mid-chord point (\(a = 0\)) of the junction. The yawing motion of the HTP can be given by

\[
\Psi = \Psi_\text{\textcircled{e}} \sin kt
\]

where \(\Psi_\text{\textcircled{e}}\) is the amplitude of the yawing oscillation, which is equal to the amplitude of the pitching oscillation of the VTP in the present case. The vertical axis of Fig. 3 is the non-dimensional form of the rolling moment defined by

\[
C_r = M_r/(4\rho U^2 b_l^2 \Psi_\text{\textcircled{e}})
\]

where \(M_r\) is the rolling moment (positive right wing down), \(\rho\) is the free-stream density and \(l\) is the semi-span of the HTP.

In the present numerical simulation, periodic solutions were obtained after two or three cycles of oscillation. In Fig. 3, the results obtained by the present numerical simulation are compared with those of the experiment\(^{15}\) and the lifting-surface...
Numerical simulations were conducted for the same T-tail and the values at the center show the mean value of the sixth station (the maximum value of the Mach number of the rel-

The error bars in the numerical results show satisfactory agreement with those of the experiment and the lifting-surface theory. Figure 4 shows the flow pattern (iso-vorticity contour) around the right-wing section of the HTP near the junction at $k = 0.30$. As seen in the figure, a small-scale flow separation on the lower surface is observed. Although the existence of a similar flow separation is confirmed during the whole cycle of oscillation, its effect on the rolling moment seems to be very small because it is confined in the area close to the junction.

### 3.2. Results of transonic flow

Numerical simulations were conducted for the same T-tail pitch oscillation around the mid-chord line of the VTP in transonic flow, and the effects of $k$ and $\alpha_{tt}$ were evaluated in detail.

#### 3.2.1. Results of $M_\infty = 0.75$ and $\alpha_{tt} = 5.48$ deg

Figure 5 shows the variation of the rolling moment with respect to $k$ that is obtained for $M_\infty = 0.75$, $\alpha_{tt} = 5.48$ deg, $\Gamma = 0$ deg, $\Psi_{\alpha} = 6$ deg and $Re = 3.20 \times 10^7$. In Fig. 5, the results obtained for $M_\infty = 0.30$ are also shown for the purpose of comparison. As seen in the figure, there is some scattering of the results for $k = 0.2$--0.5 because it was difficult to obtain a periodic solution due to the existence of large-scale flow separation around the wing sections near the junction. The error bars in the figure show the scattering of the data and the values at the center show the mean value of the sixth through eighth cycles of oscillation. From a comparison of the results of $M_\infty = 0.75$ and $M_\infty = 0.30$, a certain trend in the effect of compressibility on the behavior of rolling moment is identified. Even with a highly reduced frequency of $k = 0.50$, the difference in the magnitude between $M_\infty = 0.75$ and $M_\infty = 0.30$ is small, while there is a large difference in phase angle; that is, the phase angle at $M_\infty = 0.75$ is delayed about 30 deg behind that at $M_\infty = 0.30$. On the other hand, at the low reduced frequency of $k = 0.10$, the magnitude of $M_\infty = 0.75$ is about 45% larger than that of $M_\infty = 0.30$, while the difference in the phase angle is very small between the two. These are confirmed further in Figs. 6 and 7, where the variations of the rolling moment with respect to time during one cycle of oscillation are plotted together with that of the yawing displacement for $k = 0.10$ and $k = 0.50$, respectively. These differences of the behavior of the rolling moment seem to come from the difference of the flow patterns observed for $M_\infty = 0.75$ at $k = 0.10$ and at $k = 0.50$, respectively. Figure 8 shows the flow pattern at the typical phase of oscillation around the wing section of the right wing of the HTP near the junction at $k = 0.10$ and $k = 0.50$, respectively. The large-scale flow separations on the upper and the lower surfaces are observed for both $k = 0.10$ and $k = 0.50$, while the shock wave is observed on the upper surface for $k = 0.50$ at the phase of $kt = \pi/6$. Figure 9 shows the flow pattern (iso-density contour) around the wing section of the 53% semi-span of the right wing of the HTP at $k = 0.10$ and at $k = 0.50$, respectively. As seen in the figures, there is a remarkable difference in the shock patterns between $k = 0.10$ and $k = 0.50$. For $k = 0.10$, there appears to be a weak shock wave near the leading-edge region, while a relatively strong shock wave appears at the 20--50% chord position for $k = 0.50$. This difference in the shock patterns between the two cases clearly comes from the difference in in-plane moving velocities of the wing at the 53% semi-span station (the maximum value of the Mach number of the relative flow at 53% semi-span at $k = 0.50$ is 0.81, while that at $k = 0.10$ is 0.77). These differences in flow pattern between

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**Fig. 4.** The flow pattern (iso-vorticity contour) around wing section of right HTP near junction ($M_\infty = 0.30$, $\alpha_{tt} = 5.48$ deg, $\Gamma = 0$ deg, $\Psi_{\alpha} = 6$ deg).

**Fig. 5.** Variation of rolling moment with respect to reduced frequency ($M_\infty = 0.75$, $\alpha_{tt} = 5.48$ deg, $\Gamma = 0$ deg, $\Psi_{\alpha} = 6$ deg).
$k = 0.10$ and $0.50$ seem to be the cause of the different trend in rolling moment behavior, which was pointed out previously.

### 3.2.2. Results of $M_\infty = 0.75$ and $\alpha_H = 0$ deg

As explained previously, the rolling moment generated for $\alpha_H = 5.48$ deg is composed of two components; namely, the endplate effect and that due to the in-plane motion of the HTP. The rolling moment generated for $\alpha_H = 0$ deg is composed only of the end-plate effect. Therefore, the effect of HTP in-plane motion on rolling moment can be evaluated by comparing the results of $\alpha_H = 5.48$ deg and those of $\alpha_H = 0$ deg. Figure 10 shows the variation in rolling moment (magnitude and phase angle) with respect to $k$ that is obtained for $\alpha_H = 0$ deg. In the same figure, the results obtained for $\alpha_H = 5.48$ deg are also shown for the purpose of comparison. We can see some clear trend as to the effect of the HTP angle of attack. For the highly reduced frequency of $k = 0.50$, the difference in rolling moment magnitude between $\alpha_H = 0$ deg and $\alpha_H = 5.48$ deg is very small, while the phase angle obtained for $\alpha_H = 0$ deg shows a delay of approximately 50 deg from that obtained for $\alpha_H = 5.48$ deg. On the other hand, the magnitude obtained for $\alpha_H = 5.48$ deg at the low reduced frequency of $k = 0.10$ shows a value of approximately 50% larger than that obtained for $\alpha_H = 0$ deg, while the difference in the phase angle between the two cases is very small. In order to see these trends in detail, the variations in rolling moment with respect to time during one cycle of oscillation are shown for $\alpha_H = 0$ deg and $\alpha_H = 5.48$ deg together with that of the yawing displacement for $k = 0.10$ in Fig. 11 and for $k = 0.50$ in Fig. 12, respectively. As seen in Fig. 11 ($k = 0.10$), the amplitude of the rolling moment of $\alpha_H = 5.48$ deg is about 50% larger than that of $\alpha_H = 0$ deg, while the phase angle with respect to the yawing displacement is very small between the two cases. On the other hand, as seen in Fig. 12 ($k = 0.50$), the variation in the moment of $\alpha_H = 0$ deg is delayed about 50 deg from that of $\alpha_H = 5.48$ deg, while their amplitudes are of the same order. The reason for this phenomenon seems to come from the fact that the rolling moment for $\alpha_H = 0$ deg does not contain the component due to the in-plane motion of the HTP, which is proportional to the in-plane moving velocity. (Note that the phase angle of the velocity advances 90 deg ahead of the yawing displacement.) In Fig. 13, the flow patterns (iso-density contours) around the wing sections...
of the HTP near the junction for $\alpha_H = 0\,\text{deg}$ and $\alpha_H = 5.48\,\text{deg}$ at $k = 0.10$ are shown at a typical phase of oscillation. No flow separation is observed on the upper surface for $\alpha_H = 0\,\text{deg}$, although large-scale flow separation is observed on the lower surface. On the other hand, large-scale flow separations are observed on both the upper and lower surfaces for $\alpha_H = 5.48\,\text{deg}$ (see Fig. 8 also). In Fig. 14, the flow patterns (iso-density contours) around the right wing sections at the 53% semi-span of the HTP for $\alpha_H = 0\,\text{deg}$ and $\alpha_H = 5.48\,\text{deg}$ are shown at a typical phase of oscillation, $k = 0.50$. There appear to be relatively strong shock waves on the upper surface for $\alpha_H = 5.48\,\text{deg}$ (see Fig. 9 also) although no shock-induced flow separation is observed for both cases.

Wing flutter usually occurs around the reduced frequency of $k = 0.1–0.2$. The results obtained in this section clearly
show the importance of the effect of compressibility, especially the transonic flow, on the behavior of rolling moment, which plays a critical role in T-tail flutter. The effect of the HTP in-plane motion in the transonic flutter of a T-tail is now under investigation using the present 3D NS code.

4. Concluding Remarks

In the present study, as a first step to investigate the phenomenon that the flutter speed of a T-tail depends on the angle of attack and the dihedral angle of the HTP, especially in transonic flow, a numerical method to take into account the in-plane motion of the HTP was proposed, and a 3D NS code based on the method was developed. The subsonic and transonic flow simulations for a simple T-tail of a rectangular planform were conducted using the 3D NS code. The behavior of rolling moment around the HTP with respect to the reduced frequency, which plays a critical role in the phenomenon that flutter velocity depends on the angle of attack and dihedral angle of the HTP, was studied in detail. As the result, the computed rolling moment obtained for \( M_\infty = 0.30 \) shows satisfactory agreement with that of the experiment and the lifting-surface theory. As to the transonic flow, the effects of the reduced frequency and angle of attack on the behavior of rolling moment and its relation to the flow pattern, especially the shock pattern and shock-induced flow separation, were clarified.

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Equation (A.2) can be expressed in the \((x', y', z')\) coordinate system by applying the chain rule as follows:

\[
y_{\eta} = y_{\eta}' x'_{\eta} + y_{\eta}' y'_{\eta} + y_{\eta}' z'_{\eta}
\]

Since \(y_{\epsilon} = 0, y_{\zeta} = 0\) and \(y_{\eta} = 1.0\) from Eq. (1) of the main text, we obtain

\[
y_{\eta} = y_{\eta}'
\]

Similarly \(z_{\zeta}, y_{\zeta}\) and \(z_{\eta}\) in the right-hand side of Eq. (A.2) become

\[
\begin{align*}
z_{\zeta} &= -\psi z_{\xi} + z_{\eta}' \\
y_{\zeta} &= y_{\zeta}' \\
z_{\eta} &= -\psi x_{\eta}' + z_{\eta}'
\end{align*}
\]

respectively. By substituting Eq. (A.3)–(A.6) into Eq. (A.2), we obtain

\[
\xi = J(y_{\eta}'(-\psi x_{\eta}' + z_{\eta}') - y_{\zeta}'(-\psi x_{\zeta}' + z_{\zeta}'))
\]

Using similar treatment, we obtain Eq. (2) and (3) of the main text, in which the higher-order terms of \(\psi\) are also neglected.

**Appendix II: Effect of Number of Grid Points and the Time Accuracy of the Code**

The number of grid points used in the main text is \(120 \times 66 \times 78\) in the \(x', y'\) and \(z'\) directions, respectively. We call this as “normal grid.” To examine the accuracy of the normal grid, computations using a “fine grid,” namely, \(130 \times 86 \times 90\) grid points in \(x', y'\) and \(z'\) directions, respectively, were also conducted for \(M_{\infty} = 0.30\) at \(k = 0.10, 0.30\) and 0.50. The results are shown in Fig. 3 of the main text. As seen in the figure, the results using the normal grid show close agreement with those of the fine grid.

The non-dimensional time step used throughout the present computations is \(\Delta t = 0.0004\). It was also confirmed that almost identical results were obtained using \(\Delta t = 0.0008\).

S. Fu
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