Abstract: Controlling the quadrature measured by a homodyne detector is a universal task in continuous-variable quantum optics. However, deriving an error signal that is linear across the entire range of quadrature angles remains an open experimental problem. Here we propose a scheme to produce such an error signal through the use of a universally tunable modulator.

1. Introduction

Homodyne detection is a measurement technique that involves combining a signal with a phase-coherent reference (usually derived from the same source) — the local oscillator (LO) — at a beam-splitter, and measuring the difference photocurrents of the two outputs. This is a powerful technique used in quantum optics to analyze field quadratures and reconstruct quantum states of light [1, 2]. In particular, it is deployed in the measurement of gravitational waves, characterizing entanglement-based quantum key distribution systems, and other several other quantum information tasks [3–6].

However, optical homodyne detection requires precise control of the relative phase between the signal and LO — i.e. the “homodyne angle” — that determines the detected quadrature. This problem is compounded if there is a need to deterministically vary the homodyne angle across its entire range of $[0, \pi/2]$ — such as in the characterization of ponderomotive squeezed light sources [REF] — with minimal and systematic-free changes in the optical setup, while employing a linear error signal. Traditionally, modulation of the signal and/or LO, in purely amplitude or purely phase has been used to derive a linear error signal around a pre-determined quadrature angle [5, 6]. Producing a linear error signal for an arbitrary homodyne angle requires modulation of the LO field at arbitrary angles in the amplitude-phase plane [7].

Here we present a scheme to produce an error signal for controlling the relative phase of the local oscillator and signal in a homodyne detection scheme utilizing a universally tunable modulator (UTM) — a modified electro-optic amplitude modulator [8]. The UTM allows the modulation of light at arbitrary angles in the amplitude-phase plane which can be used to produce a linear error signal at all homodyne angles.

2. Laboratory Setup

2.1. Balanced Homodyne Detection

Balanced homodyne detection (BHD) is a subclass of homodyne detection that uses a 50/50 beamsplitter for combining the signal and LO. By adding two photodetectors to the outputs of the beamsplitter, the detected currents can be subtracted to yield a final measurement signal which produces information about the quantum state of our system. A full treatment of BHD’s benefits can be found in Kumar [9].

BHD can be used to measure an electromagnetic mode at different quadratures (phase/amplitude, for example). Discounting the effects of the UTM and the signal-producing device under
Fig. 1. The experimental setup consists of a Mach-Zehnder Interferometer arranged for balanced homodyne detection. The lower path contains the amplitude-phase transducer under consideration (i.e. a Fabry-Perot cavity) and is referred to as the "signal path". The upper path is the LO pickoff and includes the UTM for production of an error signal and piezo mirror to actuate the path length.

consideration (SIG), after the first beamsplitter (\(BS_{LO}\)) we have the fields \(E_s = \frac{E_0 e^{i\omega t + \phi}}{2}\) and \(E_{LO} = \frac{E_0 e^{i\omega t}}{2}\) where \(E_0\) is the field amplitude before \(BS_{LO}\), \(\omega\) is the carrier frequency, and we define \(E_s\) to have a \(\pi\) phase shift after reflection off of the beamsplitter. 

\(E_s\) and \(E_{LO}\) acquire a relative phase dependent on the difference in path lengths which we represent as the angle \(\theta\) in the LO term, leaving us with \(E_{LO} = \frac{E_0 e^{i\omega t + \phi}}{2}\). The two beams then interfere at the second beamsplitter (\(BS_{HD}\)) and produce the following outputs:

\[
E_1 = \frac{1}{2} E_0 e^{i\omega t} [1 + e^{i\theta}], \quad E_2 = \frac{1}{2} E_0 e^{i\omega t} [e^{i\theta} - 1]
\]  

(1)

The beams are impingent upon the two photodetectors, and the difference in photocurrent yields a \(\theta\) dependent signal which determines the measurement quadrature and information we gain from the system.

2.2. UTM

In Fig. 1 the UTM modulates the LO path. For a beam modulated at \(\omega_m\), the total field can be approximated as:

\[
\vec{E} = E_0 + E_+ e^{i\omega_m t} + E_- e^{-i\omega_m t}
\]  

(2)

where \(E_+ = \frac{\tilde{A} e^{i\phi_1}}{2}\) and \(E_- = \frac{\tilde{A} e^{i\phi_2}}{2}\). \(\tilde{A} = A e^{i\phi_A}\) and \(\tilde{P} = P e^{i\phi_P}\) represent AM/PM respectively [10]. One controls the UTM by tuning the two crystal drives \(\tilde{\delta}_1 = \delta_1 e^{i\sigma_1}\) and \(\tilde{\delta}_2 = \delta_2 e^{i\sigma_2}\) to set \(\tilde{A}\) and \(\tilde{P}\):

\[
\tilde{A} = \frac{1}{2} \tan(\frac{\sigma}{2})(\delta_1 - \delta_2), \quad \tilde{P} = \frac{1}{2} (\delta_1 + \delta_2)
\]  

(3)

See Yam et al. for derivations. Here, \(\sigma\) represents the phase difference between the electric field components along the UTM’s left and right diagonals.

To parse the expressions above, it useful to consider what operations on the drives \(\delta_1\) and \(\delta_2\) meaningfully change \(\tilde{A}\) and \(\tilde{P}\). It turns out that the phase difference between the drives is the most important parameter for setting the AM/PM ratio. If the drives are in phase, then \(\tilde{A} = 0\) and \(\tilde{P} = 1\). If the drives are out of phase, the opposite occurs and only amplitude modulation takes place. Between these two extremes, numerous AM-PM admixtures can be generated.
3. Error Signal Calculations

In order to compute the error signal of the BHD setup with the UTM and signal beams included, we must first consider expressions for $E_s$ and $E_{LO}$:

$$E_s = ?, E_{LO} = (E_0 + E_e^{j\omega t} + E_e^{-j\omega t})e^{j\theta}$$  

(4)

The "?" in $E_s$ signifies the black box, which can be composed of real and complex parts. The expression for $E_{LO}$ is the same as the expression for a beam modulated at $\omega$ in equation (4), except multiplied by $e^{j\theta}$ to account for the phase shift of the homodyne angle.

Combining these two beams at the beamsplitter yields $E_1$ and $E_2$:

$$E_1 = \frac{1}{2}E_s + \frac{1}{2}E_{LO}, E_2 = \frac{1}{2}E_s e^{j\pi} + \frac{1}{2}E_{LO}$$  

(5)

The only difference is the $\pi$ phase shift in $E_2$, a result of the total reflections off the beamsplitters.

We can get the power at each photodetector by squaring the electric field.

$$P_1 = \frac{1}{4}(E_s e^{-j\omega t} E_s^{*} e^{-j\omega t} + E_s e^{-j\omega t} E_e^{*} e^{j\omega t} + E_s^{*} e^{j\omega t} E_e e^{j\omega t} + E_s^{*} e^{j\omega t} E_e^{*} e^{-j\omega t})$$

$$+ \frac{1}{4}(E_0 E_s e^{j\omega t} + E_0 E_e^{*} e^{j\omega t} + E_0^{*} e^{-j\omega t} + E_0 E_e e^{-j\omega t})$$  

(6)

$$P_2 = -\frac{1}{4}(E_s e^{-j\omega t} E_s^{*} e^{-j\omega t} + E_s e^{-j\omega t} E_e^{*} e^{j\omega t} + E_s^{*} e^{j\omega t} E_e e^{j\omega t} + E_s^{*} e^{j\omega t} E_e^{*} e^{-j\omega t})$$

$$+ \frac{1}{4}(E_0^{*} E_s e^{j\omega t} + E_0 E_e^{*} e^{j\omega t} + E_0 E_s e^{-j\omega t} + E_0^{*} E_e e^{-j\omega t})$$  

(7)

Notice the only difference is the negative sign in the first term of $P_2$. Subtracting the two for BHD yields cancellation and the form:

$$P_f = \frac{1}{2}(E_s e^{-j\omega t} E_s^{*} e^{-j\omega t} + E_s e^{-j\omega t} E_e^{*} e^{j\omega t} + E_s^{*} e^{j\omega t} E_e e^{j\omega t} + E_s^{*} e^{j\omega t} E_e^{*} e^{-j\omega t})$$  

(8)

We can simplify by splitting into sin and cos and by defining $\epsilon(\theta)$:

$$P_{sqz} = Re[\epsilon(\theta)] \cos(\omega t) + Im[\epsilon(\theta)] \sin(\omega t)$$  

(9)

where,

$$\epsilon(\theta) = Re[E_s E_0^{*} e^{-j\theta}] \tilde{A} + Im[E_s E_0^{*} e^{-j\theta}] \tilde{P}$$  

(10)

The $\cos(\omega t)$ term is defined as the in-phase error signal (I phase), while the $\sin(\omega t)$ term denotes the in-quadrature phase (Q phase). Solving for when $\epsilon(\theta) = 0$, we attain the following condition:

$$- \frac{\tilde{A}}{\tilde{P}} = \frac{Im[E_s E_0^{*} e^{-j\theta}]}{Re[E_s E_0^{*} e^{j\theta}]}$$  

(11)

The error signal can thus be made to cross zero at any desirable $\theta$ by setting the modulation phases such that the condition above is met.
4. **Graph and Sensitivity**

Useful error signals have the property of being proportional to small changes in of the locking parameter (and therefore must change sign). By graphing the I phase of the error signal in equation (13), we see that this is indeed the case for $P_f$. See Fig. 2, where $\epsilon$ was plotted against $\theta$ for different phase differences of the UTM input drives, effectively adjusting the ratio of $\tilde{A}$ and $\tilde{P}$. We can determine the susceptibility of our error signal to noise by calculating the phase sensitivity at the shot noise limit for some given power. It is easiest to calculate if $P_f$ is not zero at zero. A UTM phase difference of $\pi/2$ accomplishes this. Then, at a power of 2 W, we get a shot noise limited phase sensitivity of 0.76 nrad.

![Homodyne Angle vs. Error Signal](image)

Fig. 2. The I phase cavity error signal as a function of homodyne angle. The different curves represent different phase differences between the UTM drives of $-\pi$ (purple), $-\pi/2$ (red), 0 (blue), $\pi/2$ (orange), $\pi$ (green).

5. **Conclusion**

We have shown here that one can use the variable AM/PM control provided by a UTM to lock the measurement quadrature of a homodyne measurement. By varying the phase difference between the UTM drives, locking at arbitrary $\theta$ is possible with a linear error signal. The work builds off of previous applications of the UTM [10], providing an efficient technique for continuous control of a generalized homodyne angle.

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