Evidence for Superfluidity in a Resonantly Interacting Fermi Gas

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(Dated: March 22, 2022)

We observe collective oscillations of a trapped, degenerate Fermi gas of $^6$Li atoms at a magnetic field just above a Feshbach resonance, where the two-body physics does not support a bound state. The gas exhibits a radial breathing mode at a frequency of $2837(05)$ Hz, in excellent agreement with the frequency of $\nu_H \equiv \sqrt{10\nu_x\nu_y}/3 = 2830(20)$ Hz predicted for a hydrodynamic Fermi gas with unitarity limited interactions. The measured damping times and frequencies are inconsistent with predictions for both the collisionless mean field regime and for collisional hydrodynamics. These observations provide the first evidence for superfluid hydrodynamics in a resonantly interacting Fermi gas.

PACS numbers: 03.75.Ss, 32.80.Pj

Strongly-interacting two-component Fermi gases provide a unique testing ground for the theories of exotic systems in nature, ranging from super-high temperature superconductors to neutron stars and nuclear matter. The feature which all of these systems have in common is a strong interaction between pairs of spin-up and spin-down particles. In atomic Fermi gases, tunable, strong interactions are produced using a Feshbach resonance [1, 2, 3]. Near the resonance, the zero-energy s-wave scattering length $a$ exceeds the interparticle spacing, and the interparticle interactions are unitarity-limited and universal [1, 2, 3]. In this region, high temperature Cooper pairing has been predicted [4, 5, 6].

In this Letter, we present measurements of the frequencies and damping times for the radial hydrodynamic breathing mode of a trapped, highly degenerate gas of $^6$Li atoms just above a Feshbach resonance [11]. These observations provide the first evidence of superfluid hydrodynamics in a resonantly interacting two-component Fermi gas.

Hydrodynamic behavior in a collisionless quantum gas at very low temperature is known to be a hallmark of superfluidity. Previously, we observed hydrodynamic, anisotropic expansion of a strongly interacting, ultracold, two-component Fermi gas [12]. However, an initially collisionless gas could have become collisionally hydrodynamic as the Fermi surface significantly deformed during the expansion [13]. Thus, the observations suggested superfluid hydrodynamics, but were not conclusive.

Superfluidity has been observed in Bose-Einstein condensates (BECs) of molecular dimers, which have been produced from a two-component strongly interacting Fermi gas. The first experiments produced the dimer BECs at a magnetic field below the Feshbach resonance, where the atoms have a nonzero binding energy [14, 15, 16, 17]. Although the two-body physics does not support a bound state at magnetic fields above the Feshbach resonance, for fermionic atoms, the many-body physics does. Observations of BECs originating from such dimers are consistent with the existence of preformed pairs [18, 19]. These experiments indirectly explore the microscopic structure, while our experiments are complementary in that they directly measure the macroscopic dynamics.

One method for distinguishing between a BEC and a superfluid Fermi gas is to examine their collective hydrodynamic modes at low temperature where the trapped gas is collisionless [20]. For a weakly repulsive BEC contained in a nearly cylindrically symmetric trap, the radial breathing mode occurs at a frequency of

$$\nu_B = 2\sqrt{\nu_x\nu_y},$$

(1)

where $\nu_i$ are the harmonic oscillation frequencies in Hz of a noninteracting gas in the $i^{th}$ direction of the trap. In contrast to a weakly repulsive BEC, a superfluid Fermi gas in a cigar-shaped trap is predicted to have a radial breathing mode at the hydrodynamic frequency

$$\nu_H = \sqrt{10/3}\nu_x\nu_y.$$

(2)

This result is obtained in the unitarity limit, where the shift from the interparticle interactions vanishes for a hydrodynamic gas [21, 22]. In general, hydrodynamics with the frequency $\nu_H$ can arise from superfluidity or from collisions in a normal fluid. However, at low temperature, Pauli blocking is expected to suppress the collision rate in a Fermi gas, increasing the damping of the collisionally hydrodynamic modes as the temperature is lowered.

In our experiments with a trapped Fermi gas, Pauli blocking of collisions is expected to be effective at the lowest temperatures achieved. Nevertheless, a weakly damped radial mode at precisely the hydrodynamic frequency of $\nu_H$ is observed, and the damping rate decreases strongly as the temperature is lowered below $\approx 30\%$ of the Fermi temperature for a noninteracting gas. These observations provide the first evidence for superfluid hydrodynamics in a resonantly interacting Fermi gas.

We measure the frequencies and damping times of the radial breathing mode of the $^6$Li Fermi gas near the Fes-
hibach resonance at 822 (3) G \cite{19}. The frequencies measured at several temperatures and magnetic fields are compared to the predictions (Eqs. 1 and 2) based on the trap oscillation frequencies measured by parametric resonance. A cross check of the measurement method is provided by measuring the breathing mode of the noninteracting gas.

We begin by preparing a degenerate, 50-50 mixture of the lowest spin-up and spin-down states of $^6$Li atoms in an ultrastable CO$_2$ laser trap as described previously \cite{12}. Forced evaporation at a chosen magnetic field in the range 770-910 G produces a highly degenerate, unitarity-limited sample. The trap depth is lowered by a factor of $\simeq 580$ over 4 s, then recompressed to 4.6% of the full trap depth in 1 s and held for 1 s to assure equilibrium. This produces a degenerate sample of $N/2 \simeq 1.5 \times 10^5$ atoms per spin-state at a temperature of $\simeq 0.15 T_F$. The corresponding Fermi temperature for a noninteracting gas is then $T_F = \hbar \bar{\omega}(3N)^{1/3} = 2.5 \mu K$ ($\bar{\omega} = 2\pi \times (\mu_L T_F \mu_S)^{1/3}$), small compared to the final trap depth of 35 $\mu K$.

From the images of the released cloud, the number of atoms is determined by integrating the column density. We have measured the resolution of the entire imaging system to be 5.5 $\mu m$. For low temperatures, $T/T_F \leq 0.4$, the ratio $T/T_F$ is determined by fitting a Thomas-Fermi profile for a noninteracting Fermi gas to the transverse $(x)$ distribution obtained by integrating the column density in the axial $(z)$ direction. Very good fits are obtained for the noninteracting gas at 526 G. At 910 G, this procedure yields temperatures as low as $T/T_F = 0.06$ and excellent fits. However, at fields closer to resonance, slightly higher temperatures are obtained, and the shape may not be precisely Thomas-Fermi due to many-body effects. For measurements near resonance at higher temperatures $T/T_F = 0.5 - 1.2$, where the gas is not perfectly hydrodynamic, temperature estimation is less precise. Here, we simply assume hydrodynamic expansion of a Maxwell-Boltzmann spatial distribution for a noninteracting gas. The measured temperatures therefore indicate the trend but not necessarily the absolute temperature. We find consistency between the measured number of trapped atoms, the temperature, and the initial cloud size obtained by hydrodynamic scaling \cite{12}. For the strongly interacting gas, we include a reduction of the cloud radius arising from the mean field \cite{2}. By correcting selected images for our estimated saturation $I/I_{sat} = 0.2$, we estimate that the true temperatures are lower by 0.03 $T_F$ and the true atom numbers are increased by a factor near 1.15 compared to the values given in Table I.

Trap oscillation frequencies at 4.6% of the full well depth are measured by parametric resonance in a weakly-interacting sample. The gas is cooled by forced evaporation over 25 seconds to temperatures of 0.3 $T_F$ at a field of 300 G and the trap depth is then modulated by 0.5% for 1 s. During this period, the low collision rate produces little damping, but permits the gas to thermalize.

After modulation, imaging at 526 G is used to measure the release energy versus drive frequency. Well-resolved resonances are obtained at $2\omega_x = 2\pi \times 3200(20)$ Hz and $2\omega_y = 2\pi \times 3000(20)$ Hz. Because of the low frequency, the axial resonance is measured at full trap depth. We obtain $2\omega_x = 2\pi \times 600(20)$ Hz, yielding $2\omega_z = 2\pi \times 140(5)$ Hz at 4.6% of full trap depth, including a quadratically combined magnetic field curvature contribution of 21 Hz. From these measurements, $\mu_L \equiv \sqrt{\nu F_{\perp}} = 1550(20)$ Hz.

To excite the transverse breathing mode, the trap is turned off abruptly ($\leq 1 \mu s$) and turned back on after a delay of $t_0 = 50 \mu s$. To show that our excitation is a weak perturbation, we estimate the energy increase, which arises principally from the change in potential energy in the transverse directions. Assuming approximately ballistic expansion, $\Delta E_{\perp} = E_{\perp}(\omega_{\perp} t_0)^2/2 = 0.1 E_{\perp}$. For initial temperatures of 0.1-0.15 $T_F$, the corresponding temperature change is $\Delta T/T_F \simeq 0.05$ when the gas thermalizes, consistent with our measurements. To measure the frequency and damping time, the breathing mode is excited and the sample is held for a variable time $t_{hold}$, after which the trap is extinguished suddenly, releasing the gas. The cloud is allowed to expand for 1 ms and then imaged using a camera beam probe pulse of 5 $\mu s$ duration arranged to produce a two-level system as described previously \cite{12}.

To study a noninteracting sample, breathing modes are excited at 526 G, where the scattering length is nearly zero \cite{23, 24}, after cooling at 300 G as described above. Fig. \ref{fig:1} shows $\sqrt{(x^2)}$ for the expanded gas $(\langle x \rangle \equiv 0)$, plotted as a function of the hold time in the trap, $t_{hold}$. Fitting an exponentially damped sinusoid $x_{rms} + A \exp(-t/t_{damp}) \sin (2\pi vt + \varphi)$ to the data, we obtain the frequency $v = 3212(30)$ Hz and the damping time $t_{damp} = 2.04$ ms, where the errors are from the fit. The damping time is consistent with a small anharmonicity of the gaussian profile of the trap potential.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Excitation of the breathing mode in a noninteracting Fermi gas of $^6$Li at 526 G. Error bars denote one-$\sigma$ uncertainty.}
\end{figure}

The strongly interacting gas exhibits longer damping times at low temperature. Fig. 2(a-c) shows the results at 870 G as the temperature is lowered from $T/T_F = 0.50$ to 0.17. The damping time increases from 1.4(0.1) ms to 3.85 (0.4) ms. At the lowest temperature, the measured
The measured oscillation frequency for the transverse breathing mode can be compared to those expected for a noninteracting gas, for a weakly repulsive Bose gas, and for a hydrodynamic Fermi gas. For a weakly repulsive Bose gas, using eqs. 1 and 2 we predict $\nu_H = 2\nu_\perp = 3100$ Hz and for a hydrodynamic gas, $\nu_H = \sqrt{10/3}\nu_\perp = 2830(20)$ Hz.

The measured oscillation frequency of 3212(30) Hz for the breathing mode of the noninteracting gas is in excellent agreement with the frequency 3200(20) Hz measured for a hydrodynamic Fermi gas, and it differs significantly from that of the noninteracting gas and the weakly interacting Bose gas. Similar results are obtained at 860 G and 880 G and 910 G at slightly lower temperatures. For the axial direction, the measured amplitude of the oscillation is consistent with zero.

On the molecular side, just below resonance at 815 G at $T/T_F = 0.14$, we obtain $\nu = 2931(19)$, which is just above the $\nu_H$, consistent with predictions that the response near resonance is fermionic [21, 22]. At a much lower field of 770 G, we find $\nu = 3000(150)$ closer to the predicted Bose frequency of 3100 (20) Hz. However, the data is not of as high quality as that shown in Fig. 2.

Over the range of magnetic fields studied, our measured oscillation frequencies $\nu(B)$ at the lowest temperatures show the same magnetic field dependence as those of Rudi Grimm’s group [25]. However, our data show a much smaller frequency shift with respect to the expected hydrodynamic frequency $\nu_H$.

We estimate the frequency shifts $\Delta \nu \equiv \nu(\text{meas}) - \nu(\text{actual})$ arising from anharmonicity in the trapping potential [26]. For the hydrodynamic frequency, $\Delta \nu_H = -(32/25)\sqrt{10/3}\nu_\perp M\omega_\perp^2 x_{\text{rms}}^2/(b_{H}^2 U)$, where $U$ is the trap depth, $M$ is the $^6$Li mass, and $\omega_\perp = 2\nu_\perp$. Here, $b_H$ is the hydrodynamic expansion factor, 11.3 after 1 ms [27]. The shift in the geometric mean of the transverse frequencies $\Delta \nu_\perp = -(6/5)\nu_\perp M \omega_\perp^2 x_{\text{rms}}^2/(b_T^2 U)$, where $b_T = 10.3$ is the ballistic expansion factor at 1 ms. Using Table I these results yield a net $\Delta \nu \approx \Delta \nu_H - \sqrt{10/3} \Delta \nu_\perp = +24$ Hz at 870 G and $T/T_F = 0.17$. For the three higher temperatures at 870 G, we find $\Delta \nu \approx -35$ Hz at $T/T_F = 0.33$ and 0.5, and -16 Hz at $T/T_F = 1.15$, consistent with the measured $\sim -60$ Hz shift below the lowest temperature data.

We have also investigated the effect of decreasing the

| B(G) | $T/T_F$ | $N(10^5)$ | $x_{\text{rms}}(\mu\text{m})$ | $\nu(\text{Hz})$ | $\tau_{\text{damp}}(\text{ms})$ |
|------|---------|------------|----------------|-----------------|-----------------|
| 526* | 0.30(0.02) | 288(18) | 35.2 | 3212(30) | 2.04(0.4) |
| 770 | 0.13(0.03) | 138(20) | 29.3 | 3000(150) | 2.00(1.1) |
| 815 | 0.14(0.04) | 198(24) | 24.0 | 2931(19) | 3.60(1.5) |
| 860 | 0.14(0.04) | 294(26) | 28.6 | 2857(16) | 3.67(1.1) |
| 870* | 0.17(0.06) | 288(30) | 32.0 | 2837(05) | 3.85(0.4) |
| 870† | 0.15(0.03) | 225(36) | 33.5 | 2838(06) | 6.01(1.4) |
| 870‡ | 0.18(0.04) | 207(28) | 41.8 | 5938(18) | 1.44(0.2) |
| 870* | 0.33(0.02) | 370(24) | 46.1 | 2754(14) | 2.01(0.3) |
| 870* | 0.50(0.06) | 290(32) | 45.1 | 2775(08) | 1.39(0.1) |
| 870* | 1.15(0.10) | 244(10) | 41.7 | 2779(50) | 1.08(0.4) |
| 880 | 0.12(0.04) | 258(30) | 30.0 | 2836(16) | 3.95(1.5) |
| 910 | 0.11(0.06) | 268(17) | 27.8 | 2798(15) | 3.30(1.1) |

TABLE I: Breathing mode frequencies $\nu$, and damping times $\tau_{\text{damp}}$. B is the applied magnetic field, $T/T_F$ is the initial temperature in units of the Fermi temperature and N is the total number of atoms, uncorrected for saturation. $x_{\text{rms}}$ is the time-averaged root-mean-square size of the oscillating cloud. 

From the tails of a bimodal distribution. *Shown in the figures. †For $\nu = 2931(19)$, which is just above the $\nu_H$, consistent with predictions that the response near resonance is fermionic [21, 22]. At a much lower field of 770 G, we find $\nu = 3000(150)$ closer to the predicted Bose frequency of 3100 (20) Hz. However, the data is not of as high quality as that shown in Fig. 2.

Over the range of magnetic fields studied, our measured oscillation frequencies $\nu(B)$ at the lowest temperatures show the same magnetic field dependence as those of Rudi Grimm’s group [25]. However, our data show a much smaller frequency shift with respect to the expected hydrodynamic frequency $\nu_H$. The measured oscillation frequencies for the transverse breathing mode can be compared to those expected for a noninteracting gas, for a weakly repulsive Bose gas, and for a hydrodynamic Fermi gas. For a weakly repulsive Bose gas, using eqs. 1 and 2 we predict $\nu_H = 2\nu_\perp = 3100$ Hz and for a hydrodynamic gas, $\nu_H = \sqrt{10/3}\nu_\perp = 2830(20)$ Hz.

The measured oscillation frequency of 3212(30) Hz for the breathing mode of the noninteracting gas is in excellent agreement with the frequency 3200(20) Hz measured by the parametric resonance method for the x-direction which is imaged in the experiments. Hence, the parametric resonance method and the breathing mode excitation method yield identical frequencies to better than 1%.

For the strongly interacting gas at 870 G and $T/T_F = 0.17$, the measured radial breathing mode frequency 2837(05) Hz, is in excellent agreement with the prediction $\nu_H = 2830(20)$ for a hydrodynamic Fermi gas, and it differs significantly from that of the noninteracting gas and the weakly interacting Bose gas. Similar results are obtained at 860 G and 880 G and 910 G at slightly lower temperatures. For the axial direction, the measured amplitude of the oscillation is consistent with zero.

Results for different temperatures and magnetic fields are given in Table I.
harmonic contribution to temperature dependence of our damping rates: The an-
tent with a damping rate which scales linearly with trap
damping time of the

time is large over a broad temperature range, inconsistent
ation or collisional hydrodynamics. It is therefore diffi-
cult to see how the observations can be explained without

oscillation amplitude to less than 10% by reducing $t_0$ to
25$\mu$s at 870 G and $T/T_F = 0.15$. For small amplitudes,
the aspect ratio of the cloud changes very little. Hence,
we expect that the deformation of the Fermi surface is
very small, so that collisional behavior is not induced.
We obtain $\nu = 2838(16)$ Hz, in precise agreement with
the results for $t_0 = 50\mu$s. The damping time is somewhat
increased to 6.00(1.4) ms, presumably due to the smaller
energy input, rather than reduced anharmonicity (since
the frequency is unchanged).

The measured damping time shows a rapid increase
with decreasing temperature, Fig. 3. The open circle
shows the result at a trap depth increased by a factor
of 4.1. The open circle shows the result when the trap depth is increased by a
factor of 4.1. The measured hydrodynamic frequency scales
as $\sqrt{4 T}$ within 3% and the product $\nu_{\text{meas}} \tau_{\text{damp}}$ is consis-
tent with those at lower trap depth. This is consistent with a damping rate which scales linearly with trap
frequency, as expected for unitarity-limited interactions,
where the rate scales with the Fermi energy. Note that
anharmonicity cannot make a major contribution to the
temperature dependence of our damping rates: The an-
harmonic contribution to $1/\tau_{\text{damp}}$ would be proportional
to the shift and therefore independent of trap depth for
fixed $T/T_F$ and $N$. Then, $\nu \tau_{\text{damp}} \propto \nu$, and the vertical
position of the open circle would increase by a factor of 2.
Further, the anharmonic contribution is identical for the
three highest temperature points where the mean cloud
sizes are nearly the same.

We have attempted to model the data at 870 G without
invoking superfluidity. A first scenario is that the gas
is nearly collisionless at the lowest temperature, and the
long damping time and measured frequency are the result
of collisionless mean field evolution. A second scenario is
collisional hydrodynamics.

The collisionless mean field scenario requires a large
negative mean field shift to explain the difference
between the frequencies of 3212(30) Hz and 2837(20) Hz
measured for the noninteracting and strongly interacting
samples, respectively. However, for a unitarity limited
interparticle interaction with a negative $\beta = -0.55$ 28, a
Vlasov equation model 29 yields a $+90$ Hz shift relative
to 3200 Hz, while we observe a $-400$ Hz shift. Also, for our trap, the same model shows that the coupling
of the collisionless transverse modes by the interaction
would produce a noticeable beat at 370 Hz with an am-
plitude minimum at 1 ms, which is not observed. Hence,
the data are inconsistent with the collisionless scenario.

To investigate the second scenario, we considered a col-
sisional hydrodynamic model, including two-body Pauli
blocking 20, 30. A small negative shift at the higher
temperatures might arise from the mean field in the hy-
drodynamical limit 21, 22, or from the anharmonic shift
described above. Neglecting these shifts, a relaxation
approximation model 31 can be used to determine both
the breathing mode frequency and the damping time in
the hydrodynamic limit 21, 22, or from the anharmonic shift
described above. Neglecting these shifts, a relaxation
approximation model 31 can be used to determine both
the breathing mode frequency and the damping time in
terms of the measured trap oscillation frequencies, for an
arbitrary momentum relaxation rate. We find that a very
large momentum relaxation rate is needed to fit the 4 ms
damping time of the $T/T_F = 0.17$ data in a collision-
ally hydrodynamic regime. Then, the predicted damping
time is large over a broad temperature range, inconsistent
with the observed rapid decrease in damping time with
temperature. Lowering the maximum relaxation rate,
we can fit the damping times at the two highest temper-
atures. In this case, however, obtaining a 4 ms damping
time requires a temperature below $T/T_F = 0.1$, i.e.,
a nearly collisionless regime, inconsistent with observations
as described above.

In conclusion, at our lowest temperatures, we observe a
breathing mode at precisely the hydrodynamic frequency
as well as highly anisotropic hydrodynamic expansion, as
in our previous experiments 12, 29. The damping time
increases rapidly as the temperature is lowered 32, consis-
tent with a transition from collisional to superfluid hy-
drodynamics at a temperature between 0.2 and 0.3 $T_F$.
On the basis of the above arguments, we believe the data
are not consistent with either collisionless mean field evolu-
tion or collisional hydrodynamics. It is therefore diffi-
cult to see how the observations can be explained without
invoking superfluidity.

Recent theory describes the BCS-BEC crossover
regime in terms of very large fermionic pairs, com-
parable in size to the interparticle spacing 32, 33, 34, 35.
Falco and Stoof 34 predict BEC-like or BCS-like behav-
ior above the Feshbach resonance, depending on whether
$\epsilon_b \equiv \hbar^2 / (a^2 m_{\text{atom}})$ is $\leq 2 k_B T_F$, or $\geq 2 k_B T_F$ 34. Near
resonance, where $\epsilon_b << 2 k_B T_F$, the majority of fermionic
pairs (either Bose molecules or Cooper pairs) are very
large. Hence, one expects that the response of the sys-
tem to compression is fermionic, and scales with density
as $n^{2/3}$ 37. This idea is consistent with our measure-
ments and with predictions 21, 22.

This research is supported by the Chemical Sciences,
Geosciences and Biosciences Division of the Office of Ba-
sic Energy Sciences, Office of Science, U. S. Department of Energy, the Physics Divisions of the Army Research Office and the National Science Foundation, and the Fundamental Physics in Microgravity Research program of the National Aeronautics and Space Administration.

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