Microlens diffusers for efficient laser speckle generation

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Abstract: Laser Speckle is the optical phenomena resulting from the random interference of coherent light. This phenomenon can be utilized to measure the Modulation Transfer Function (MTF) of detector arrays. Common devices used for speckle generation, such as integrating spheres and ground glass, suffer from low efficiencies less than 20%. Microlens diffusers are shown to be more efficient alternatives for speckle generation. An analysis of the statistical behavior of microlens diffusers is presented with emphasis on their application to MTF testing of detector arrays in the visible spectrum.

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1. Introduction

In this paper, we investigate the generation of laser speckle using a microlens diffuser. Laser speckle is used for a variety of applications such as strain and velocity measurement. The objective of our investigation is to evaluate the use of a microlens diffuser for the measurement of the Modulation Transfer Function (MTF) of a detector array. The use of laser speckle for MTF measurement has the primary advantage of enabling the measurement of the detector array without the need for intervening optics. This simplifies the MTF
measurement since the introduction of an imaging lens couples the MTF of the lens system with the measurement. The use of laser speckle for MTF measurement has been presented by the author and other research groups in the past twenty years [2-6].

A microlens diffuser is an assembly of microscopic lenses with a random distribution of feature size and depth. The average microlens diameter is typically 50 microns. The spatial distribution of the lenses is designed to minimize periodic structure that would give rise to diffraction grating like orders. An example microlens array is shown in Fig. 1.

![Microlens Array](image1)

**Fig. 1. Magnified photograph of sample microlens array. (courtesy of RPC Photonics)**

Microlens arrays can be designed to produce specifically shaped distributions such as square, circular, and annulus. These shaped distributes are achieved by controlling the radius, curvature, conic constant, and aspheric coefficients of each microlens. Standard and custom microlens arrays, called Engineered Diffusers™, are available from RPC Photonics[1]. The advantage of these diffusers over holographic optical elements is that the only loss is from Fresnel reflection from the air substrate boundary. Typical transmission efficiency is greater than 90% depending on the divergence angle. An anti-reflection coating can be used to increase this efficiency.

Lambertian microlens diffusers, called HiLAM diffusers[1], are constructed using two random microlens arrays positioned co-axially with a separation distance of 1mm. This combination of high divergence angle microlens diffusers yields a transmission efficiency of approximately 70%[1]. The angular output intensity is closely matched to the theoretical cosine distribution expected for a Lambertian source.

In this paper, we present the analysis of the statistical behavior of laser speckle generated using a coherent laser source as the input to a microlens diffuser. One advantage of the microlens diffuser over other means of generating laser speckle is the preservation of polarization. In the first analysis, we present a measurement of the degree of polarization seen at the output of the microlens diffuser. The first- and second-order statistics of the laser speckle will then be presented to demonstrate the viability of using a microlens diffuser in a laser speckle MTF measurement.

**2. Polarization preservation**

The experimental setup used to measure the degree of polarization for the microlens diffuser is shown in Fig. 2. The laser used was a linearly polarized 10mW HeNe centered at the common wavelength of 632.8 nm.
The degree of polarization of the light exiting the diffuser was measured by recording the irradiance detected by a single-point detector while rotating the linear polarizer about the optical axis of the setup. If linear polarization is maintained, then the ellipticity expressed as $\varepsilon = \sqrt{P_{\text{min}} / P_{\text{max}}}$ should be zero. The polarization orientation angle is calculated as $\theta = \tan^{-1}(P_{\text{min}} / P_{\text{max}})$. These measurements were repeated ten times and the results averaged. The measured ellipticity was 0.12 or effectively linearly polarized. The angular difference between the input angle and the angle of polarization was less than 1%. This measurement demonstrates that the HiLAM microlens diffuser preserves the input polarization. It has been demonstrated that an integrating sphere completely randomizes the polarization of the generated laser speckle [7]. This randomization requires the use of a linear polarizer positioned at the output port of the integrating sphere. The use of this polarizer decreases the efficiency of the speckle generation by 50%.

3. First-order statistics

The statistical behavior of the laser speckle produced by the microlens diffuser can be characterized by the first- and second-order statistics of the intensity of the wave field. The intensity is the primary quantity since this is what is measured by a detector array. The measured first-order statistics must be compared to the expected theoretical value to assure that the speckle is “well-behaved.” The behavior must be verified to determine if the microlens diffuser generates pure laser speckle where no unexpected artifacts are present.

3.1 Theoretical calculation

The first-order statistics of polarized laser speckle intensity are known to be equivalent to a negative exponential probability density function (PDF) [8]. This function changes when a two-dimensional detector array is used to sample the laser speckle field. The use of a detector array with finite size results in an integration or averaging of the speckle intensity over the active area of each pixel. This produces a new PDF for the integrated speckle intensity. The theoretical PDF of an integrated speckle pattern can be approximated analytically [9]

$$p_I(I) \equiv \left( \frac{\mathcal{H}}{\langle I \rangle} \right)^{\mathcal{H}-1} I^{-\mathcal{H}-1} \exp \left( -\frac{\mathcal{H}}{\langle I \rangle} \right) \frac{\Gamma(\mathcal{H})}{\Gamma(\mathcal{H}+1)} \text{ for } I \geq 0$$ (1)

The correlation parameter, $\mathcal{H}$, can be calculated from the aperture area, $S_m$, and the speckle correlation area, $S_c$. For a uniform square scattering spot of size, $L$, the correlation parameter, $\mathcal{H}$, can be calculated as,

$$\mathcal{H} = \left[ \frac{2}{\sqrt{S_m}} \int_0^{S_c} \left( 1 - \frac{\Delta x}{\sqrt{S_m}} \right) \text{sinc}^2 \left( \frac{\Delta x}{\sqrt{S_c}} \right) d\Delta x \right]^{-2}. \quad (2)$$

Fig. 2. Experimental setup for polarization preservation measurement.
For a square detector aperture, w, the value of $S_m = w^2$ the correlation area, $S_c$, is,

$$S_c = 2 \int_{0}^{\infty} \int_{0}^{\infty} \text{sinc}^2 \left( \frac{L \Delta x}{\lambda z} \right) \text{sinc}^2 \left( \frac{L \Delta y}{\lambda z} \right) d\Delta x d\Delta y,$$  \hspace{1cm} (3)

3.2 Measurement

The PDF of the speckle intensity was performed using a CCD detector array with 640 horizontal elements and 480 vertical elements. Each detector element is square with dimensions 5.6um by 5.6um. The experimental setup used for this measurement is shown in Fig. 3.

![Experimental setup for measuring PDF of laser speckle intensity.](image)

For our experimental setup $\lambda=632.8\text{nm}$, $z=2.9\text{cm}$, $w=5.6\text{um}$ (detector element width), and the scattering aperture is $L=2.54\text{mm}$. Therefore, the values of $S_m$ and $S_c$ are $3.14\times10^{-11}\text{m}^2$ and $2.5\times10^{-11}\text{m}^2$, respectively, and through numerical integration of Eq. (2) $M$ is found to be 2.7. The measured PDF for the speckle intensity was produced from a histogram of the intensity each static speckle pattern. An average of 10 histograms was used to improve accuracy. Uncorrelated speckle patterns were achieved for each image using small movements (approximately 100um) of the diffuser. The A comparison of the theoretical PDF, calculated using Eq. (1), and the measured PDF are given in Fig. 4.

![Comparison of measured to theoretical probability density functions for the intensity of the laser speckle generated with the micro lens diffuser.](image)

This comparison demonstrates that the first-order statistics of the speckle intensity are a close match to the theoretical curve. The deviation between the measurement and the theoretical curves is in part due to the approximation used in the calculation of Eq. (1) [9].
Another contribution to the observed deviation is that the degree of polarization is 1%. The resulting speckle intensity is then the sum of two statistically independent intensities. The overall effect is the addition of noise in the measurement. This type of noise in the intensity values of the speckle pattern would result in a deviation of the intensity histogram.

4. Second-order statistics

The second-order statistics of the laser speckle irradiance pattern is calculated from the estimate of the power spectral density (PSD). This was first calculated by Goldfischer to be a delta function component at zero frequency plus an extended component having a form equivalent to the normalized autocorrelation of the speckle generating aperture [10].

4.1 Theoretical calculation

In this analysis, a square aperture was used to measure the modulation transfer function (MTF) of a CCD as a means of verifying that the second-order statistics are closely matched to what is expected theoretically. This method was used in Boreman [7] using an integrating sphere as a means of generating laser speckle. The PSD of the laser speckle irradiance resulting from a square aperture is expressed using Gaskill’s special functions [11]:

$$PSD_{input, theo}(\xi, \eta) = \langle I \rangle^2 \left[ \delta(\xi, \eta) + \left( \frac{\lambda z}{L} \right)^2 \Lambda \left( \frac{\lambda z}{L} \xi \right) \Lambda \left( \frac{\lambda z}{L} \eta \right) \right]$$

(4)

Where $\Lambda(x) = 1 - |x|$ for $|x| \leq 1$, zero otherwise. The fixed variables are: distance, $z$, from the aperture to the measurement plane, the wavelength, $\lambda$, and dimensions of the square aperture, $L$. This theoretical $PSD_{input, theo}$ will be used to determine the MTF of the array following the method employed by Boreman [7].

The MTF of the detector array is measured by capturing the laser speckle irradiance pattern directly without the use of imaging optics. An estimate of the PSD of the irradiance pattern is calculated using the expression:

$$PSD_{output, meas}(\xi) = \frac{1}{M} \sum_{i=0}^{M-1} \left| \mathbb{S}\{image(0 : N - 1, i)\} \right|^2$$

(5)

The values $N$ and $M$ correspond to the number of discrete pixels in the horizontal and vertical dimensions of the array, respectively. The symbol, $\mathbb{S}$, represents the one-dimensional Fourier Transform. The expression shown in Eq. (5) is equivalent to an ensemble average of the one-dimensional Fourier transforms of the horizontal rows in the image. The MTF is then expressed as,

$$MTF_{meas}(\xi) = \sqrt{\frac{PSD_{output, meas}(\xi)}{PSD_{input, theo}(\xi)}}$$

(6)

4.2 Measurement

The MTF of the CCD detector array was measured to verify the second-order statistics of the laser speckle generated using the HiLAM diffuser. The experimental setup used for this measurement was identical to the one used in Fig. 3 with the only change being the distance, $z=4.5cm$. The distance was adjusted so that the cut-off frequency of the input PSD, $L/\lambda z$, matched the Nyquist value of the array 89.29 cycles/mm to avoid aliasing.
The measured output PSD is plotted with the theoretical input PSD in Fig. 5.

![Graph of measured output PSD vs. theoretical input PSD](image)

**Fig. 5.** Measured output PSD shown as solid line and theoretical input PSD shown as dotted line.

The values from zero to 5 cycles/mm for the “Output PSD” are shown to be greater than the “Input PSD” curve. This is consistent with the delta function, $\delta(\xi,0)$, component inherent in the theoretical expression given in Eq. 4. These values were discarded since they do not reflect a performance characteristic of the array measured. The “Output PSD” has been normalized to the next value after the last delta component value.

The theoretical MTF of the CCD is calculated as the product of the sampling and detector-footprint MTFs expressed as,

$$MTF_{theo, CCD}(\xi) = \text{sinc}^2(\frac{\xi}{w})$$

(7)

where $w$ in this case represents the detector width and equivalent sampling spacing of 5.6 um.

![Graph of measured MTF vs. theoretical MTF](image)

**Fig. 6.** Measured MTF of CCD detector array shown as solid line and theoretical MTF is shown as dotted line.

The calculated $MTF_{meas}$ for the CCD array using Eq. 6 is shown in Fig. 6 as “Measured MTF.” The “Measured MTF” is the result of dividing the “Output PSD” by the “Input PSD”
from Fig. 5. The “Measured MTF” ends prematurely at 85 cycles/mm rather than extending to 90 cycles/mm. This is a result of a processing artifact where the division of the output by the input PSDs yields values greater than the expected “Theoretical MTF.” These values have been discarded from the “Measured MTF” shown in Fig. 6.

The measured MTF is closely matched to the theoretical MTF of the CCD. This is presented as verification that the second-order statistics of the laser speckle generated with the HiLAM diffuser are accurate.

Another method of verifying the second-order statistics would be to calculate backwards to determine the actual PSD input. This would be evaluated as;

$$PSD_{input,actual} = \frac{PSD_{output,meas}}{MTF_{Theo,CCD}^2}$$  \hspace{1cm} (8)

Unfortunately, the $$PSD_{input,actual}$$ will be influenced by the MTF of the detector array since a difference between the $$MTF_{meas}$$ and $$MTF_{Theo,CCD}$$ is expected. For completeness, this comparison is provided in Fig. 7.

![Fig. 7. Measured PSD input is shown as a solid-line and theoretical PSD input is shown as a dashed-line.](image)

For our analysis, we assumed that a close match between the $$MTF_{Theo,CCD}$$ and $$MTF_{meas}$$ using the theoretical PSD input would be sufficient (as shown in Fig. 6). Ultimately, an independent measurement of the MTF of the detector array could be used. This information was not available for this analysis.

5. Summary

The first- and second-order statistics of the laser speckle generated using the HiLAM microlens diffuser has been presented. The measurements of the statistics presented show that microlens diffusers are a viable method for generating laser speckle with known PSD and they could be used for MTF testing of detector arrays as a valid alternative to ground glass or integrating spheres.