Duality and Cosmology

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Abstract

In some recent theories including Quantum SuperString theory we encounter duality - it arises due to a non commutative geometry which in effect adds an extra term to the Heiserberg Uncertainity Principle. The result is that the micro world and the macro universe seem to be linked. We show why this is so in the context of a recent cosmological model and a physical picture emerges in the context of the Feynman-Wheeler formulation of interactions.

1 Introduction

Nearly a century ago several Physicists including Lorentz, Poincare and Abraham amongst others tackled unsuccessfully the problem of the extended electron[1, 2]. An extended electron appeared to contradict Special Relativity, while on the other hand, the limit of a point particle lead to inexplicable infinities. Dirac finally formulated an equation in which the physically relevant or ”renormalized” mass was finite and consisted of the bare mass and the electromagnetic mass which become infinite in the limit of point particles, no doubt, but the infinities cancel one another. This approach lead to non-causal effects, which were circumvented by a formalism of Feynman and Wheeler, in which the interaction of a charge with the rest of the universe was considered, and also not just the point charge, but its neighbourhood had to be taken into account.

These infinities persisted for many decades. Infact the Heisenberg Uncertainity Principle straightaway leads to infinities in the limit of spacetime points. It was only through the artifice of renormalization that ’t Hooft could finally
circumvent this vexing problem, in the 1970s. Nevertheless it has been realized that the concept of spacetime points is only approximate\[3, 4, 5, 6, 7\]. We are beginning to realize that it may be more meaningful to speak in terms of spacetime foam, strings, branes, non commutative geometry, fuzzy spacetime and so on\[8\]. This is what we will now discuss.

2 Duality

We consider the well known theory of Quantum SuperStrings and also an approach in which an electron is considered to be a Kerr-Newman Black Hole, with the additional input of fuzzy spacetime.

As is well known, String Theory originated from phenomenological considerations in the late sixties through the pioneering work of Veneziano, Nambu and others to explain features like the s-t channel dual resonance scattering and Regge trajectories\[9\]. Originally strings were conceived as one dimensional objects with an extension of the order of the Compton wavelength, which would fudge the point vertices of the s-t channel scattering graphs, so that both would effectively correspond to one another (Cf.ref.\[3\]). The above strings are really Bosonic strings. Raimond\[10\], Scherk\[11\] and others laid the foundation for the theory of Fermionic strings. Essentially the relativistic Quantized String is given a rotation, when we get back the equation for Regge trajectories,

\[ J \leq (2\pi T)^{-1}M^2 + a_0 \hbar \]  

with \(a_0 = +1(+2)\) for the open (closed) string (1)

When \(a_0 = 1\) in (1) we have gauge Bosons while \(a_0 = 2\) describes the gravitons. In the full theory of Quantum Super Strings, or QSS, we are essentially dealing with extended objects rotating with the velocity of light, rather like spinning black holes. The spatial extention is at the Planck scale while features like extra space time dimensions which are curled up in the Kaluza Klein sense and, as we will see, non commutative geometry appear\[12, 13\].

We next observe that it is well known that the Kerr Newman of charged spinning Black Hole itself mimics the electron remarkably well including the purely Quantum Mechanical anomalous \(g = 2\) factor\[14\]. The problem is that there would be a naked singularity, that is the radius would become
complex,
\[ r_+ = \frac{GM}{c^2} + ib, b \equiv \left( \frac{G^2Q^2}{c^8} + a^2 - \frac{G^2M^2}{c^4} \right)^{1/2} \] (2)

where \( a \) is the angular momentum per unit mass.

This problem has been studied in detail by the author in recent years [15, 16]. Indeed it is quite remarkable that the position coordinate of an electron in the Dirac theory is non Hermitian and mimics equation (2), being given by
\[ x = (c^2p_1H^{-1}t + a_1) + \frac{i}{2}\hbar(\alpha_1 - c p_1 H^{-1})H^{-1}, \] (3)

where the imaginary parts of (2) and (3) are both of the order of the Compton wavelength.

The key to understanding the unacceptable imaginary part was given by Dirac himself [17], in terms of zitterbewegung. The point is that according to the Heisenberg Uncertainty Principle, space time points themselves are not meaningful- only space time intervals have meaning, and we are really speaking of averages over such intervals, which are at least of the order of the Compton scale. Once this is kept in mind, the imaginary term disappears on averaging over the Compton scale.

In this formulation, the mass and charge of the electron arises due to zitterbewegung effects at the Compton scale [14]. These masses and charges are renormalized in the sense of the Dirac mass in the classical theory, alluded to in section 1.

Indeed, from a classical point of view also, in the extreme relativistic case, as is well known there is an extension of the order of the Compton wavelength, within which we encounter meaningless negative energies [18]. With this proviso, it has been shown that we could think of an electron as a spinning Kerr Newman Black Hole. This has received independent support from the work of Nottale [19].

We are thus lead to the picture where there is a cut off in space time intervals.

In the above two scenarios, the cut off is at the Compton scale \((l, \tau)\) the Planck scale being a special case for the Planck mass. Such discrete space time models compatible with Special Relativity have been studied for a long time by Snyder and several other scholars [20, 21, 22]. In this case it is well known that we have the following non commutative geometry
\[ [x, y] = (iu^2/h)L_z, [t, x] = (iu^2/hc)M_x, \]
\[ [y, z] = (\frac{ia^2}{\hbar})L_x, [t, y] = (\frac{ia^2}{\hbar c})M_y, \]
\[ [z, x] = (\frac{ia^2}{\hbar})L_y, [t, z] = (\frac{ia^2}{\hbar c})M_z, \]

where \( a \) is the minimum natural unit and \( L_x, M_x \) etc. have their usual significance.

Moreover in this case there is also a correction to the usual Quantum Mechanical commutation relations, which are now given by

\[ [x, p_x] = i\hbar [1 + (a/\hbar)^2 p_x^2]; \]
\[ [t, p_t] = i\hbar [1 - (a/\hbar c)^2 p_t^2]; \]
\[ [x, p_y] = [y, p_x] = i\hbar (a/\hbar)^2 p_x p_y; \]
\[ [x, p_t] = c^2 [p_x, t] = i\hbar (a/\hbar)^2 p_x p_t; \text{etc.} \]

where \( p^\mu \) denotes the four momentum.

In the Kerr Newman model for the electron alluded to above (or generally for a spinning sphere of spin \( \sim \hbar \) and of radius \( l \)), \( L_x \) etc. reduce to the spin \( \hbar^2 \) of a Fermion and the commutation relations (4) and (5) reduce to

\[ [x, y] \approx 0(l^2), [x, p_x] = i\hbar [1 + \beta l^2], [t, E] = i\hbar [1 + \tau^2] \]

where \( \beta = (p_x/\hbar)^2 \) and similar equations.

Interestingly the non commutative geometry given in (3) can be shown to lead to the representation of Dirac matrices and the Dirac equation [23]. From here we can get the Klein Gordon equation, as is well known [24, 25], or alternatively we deduce the massless string equation.

This is also the case with superstrings where Dirac spinors are introduced, as indicated above. Infact in QSS also we have equations mathematically identical to the relations (3) containing momenta (Cf.ref. [13]). This, which implies (4), can now be seen to be the origin of non-commutativity.

The non commutative geometry and fuzzyness is contained in (3). Infact fuzzy spaces have been investigated in detail by Madore and other [26, 27], and we are lead back to the equation (5). The fuzzyness which is closely tied up with the non commutative feature is symptomatic of the breakdown of the concept of the spacetime points and point particles at small scales or high energies. As has been noted by Snyder, Witten, and several other scholars, the divergences encountered in Quantum Field Theory are symptomatic of precisely such an extrapolation to spacetime points and which
necessitates devices like renormalization. As Witten points out\cite{28}, "in developing relativity, Einstein assumed that the space time coordinates were Bosonic; Fermions had not yet been discovered!... The structure of space time is enriched by Fermionic as well as Bosonic coordinates."

A related concept, which one encounters also in String Theory is Duality. Infact the relation (6) leads to,

\[ \Delta x \sim \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar} \] (7)

where \( \alpha' = l^2 \), which in Quantum SuperStrings Theory \( \sim 10^{-66} \). Witten has wondered about the basis of (7), but as we have seen, it is a consequence of (6).

This is an expression of the duality relation,

\[ R \rightarrow \alpha' / R \]

This is symptomatic of the fact that we cannot go down to arbitrarily small spacetime intervals, below the Planck scale.

There is an interesting meaning to the duality relation arising from (7) in the context of the Kerr-Newman Black Hole formulation. While it appears that the ultra small is a gateway to the macro cosmos, we could look at it in the following manner. The first term of the relation (7) which is the usual Heisenberg Uncertainity relation is supplemented by the second term which refers to the macro cosmos.

Let us consider the second term in (7). We write \( \Delta p = \Delta N mc \), where \( \Delta N \) is the Uncertainity in the number of particles, \( N \), in the universe. Also \( \Delta x = R \), the radius of the universe where

\[ R \sim \sqrt{N l}, \] (8)

the famous Eddington relationship. It should be stressed that the otherwise empirical Eddington formula, arises quite naturally in a Brownian characterisation of the universe as has been pointed out earlier (Cf. for example ref.\cite{2}). Put simply (8) in the Random Walk equation

We now get, using (2),

\[ \Delta N = \sqrt{N} \]
Substituting this in the time analogue of the second term of (7), we immediately get, \( T \) being the age of the universe,

\[
T = \sqrt{N} \tau
\]  

(9)

In the above analysis, including the Eddington formula (8), \( l \) and \( \tau \) are the Compton wavelength and Compton time of a typical elementary particle, namely the pion. The equation for the age of the universe is also correctly given above. Infact in the closely related model of fluctuational cosmology (Cf. for example ref.[29]) all of the Dirac large number coincidences as also the mysterious Weinberg formula relating the mass of the pion to the Hubble constant, follow as a consequence, and are not empirical. In this formulation, in a nutshell, \( \sqrt{N} \) particles are fluctuationally created within the time \( \tau \), so that,

\[
\frac{dN}{dt} = \frac{\sqrt{N}}{\tau}
\]  

which leads to (9) (and (8)).

Next use of the well known formula, \( (M = Nm, M \) being the mass of the universe, and \( m \) the pion mass)

\[
R \approx GM/c^2,
\]

gives on differentiation and use of (10) the Hubble law, with

\[
H = c/l\sqrt{N} \approx \frac{Gm^3c}{\hbar} \text{ or } m = \left(\frac{\hbar H}{Gc}\right)^{1/3}
\]  

(11)

(11) gives the supposedly mysterious and empirical Weinberg formula connecting the pion mass to the Hubble constant.

Using (11) we can deduce that there can be a cosmological constant \( \Lambda \) such that,

\[
\Lambda \leq 0(H^2)
\]

Recent observations confirm this ever expanding and possibly accelerating feature of the universe[30]. All these relations relating large scale parameters to microphysical constants were shown to be symptomatic of what has been called, stochastic holism (Cf. also ref.[31]), that is a micro-macro connection with a Brownian or stochastic underpinning. Duality, or equivalently, relation (11) is really an expression of this micro-macro link.
3 The Dirac and Feynman - Wheeler Formulations

To appreciate this concept of holism in a more physical sense, we return to the classical description of the electron alluded to right at the beginning. We will discuss very briefly the contributions of Dirac, Feynman and Wheeler. This was built upon the earlier work of Lorentz, Abraham, Fokker and others. Our starting point is the so called Lorentz-Dirac equation[2]:

\[ ma^\mu = F^\mu_{in} + F^\mu_{ext} + \Gamma^\mu \]  

where

\[ F^\mu_{in} = \frac{e}{c} F^\mu_{in} v_v \]

and \( \Gamma^\mu \) is the Abraham radiation reaction four vector related to the self force and, given by

\[ \Gamma^\mu = \frac{2}{3} \frac{e^2}{c^3} (\dot{a}^\mu - \frac{1}{c^2} a^\lambda a^\lambda v^\mu) \]  

Equation (12) is the relativistic generalisation for a point electron of an earlier equation proposed by Lorentz, while equation (13) is the relativistic generalisation of the original radiation reaction term due to energy loss by radiation. It must be mentioned that the mass \( m \) in equation (12) consists of a neutral mass and the original electromagnetic mass of Lorentz, which latter does tend to infinity as the electron shrinks to a point, but, this is absorbed into the neutral mass. Thus we have the forerunner of renormalisation in quantum theory.

There are three unsatisfactory features of the Lorentz-Dirac equation (12). Firstly the third derivative of the position coordinate in (12) through \( \Gamma^\mu \) gives a whole family of solutions. Except one, the rest of the solutions are run away - that is the velocity of the electron increases with time to the velocity of light, even in the absence of any forces. This energy can be thought to come from the infinite self energy we get when the size of the electron shrinks to zero. If we assume adhoc an asymptotically vanishing acceleration then we get a physically meaningful solution, though this leads to a second difficulty, that of violation of causality of even the physically meaningful solutions. Let us see this briefly.
For this, we notice that equation (12) can be written in the form

\[ ma^\mu(\tau) = \int_0^\infty K^\mu(\tau + \alpha \tau_0)e^{-\alpha}d\alpha \]  

(14)

where

\[ K^\mu(\tau) = F_{in}^\mu + F_{ext}^\mu - \frac{1}{c^2}Rv^\mu, \]

\[ \tau_0 = \frac{2}{3}e^2/mc^3 \]  

(15)

and

\[ \alpha = \frac{\tau' - \tau}{\tau_0}, \]

where \( \tau \) denotes the time and \( R \) is the total radiation rate.

It can be seen that equation (14) differs from the usual equation of Newtonian Mechanics, in that it is non local in time. That is, the acceleration \( a^\mu(\tau) \) depends on the force not only at time \( \tau \), but at subsequent times also. Let us now try to characterise this non locality in time. We observe that \( \tau_0 \) given by equation (15) is the Compton time \( \sim 10^{-23} \) secs. So equation (14) can be approximated by

\[ ma^\mu(\tau) \approx K^\mu(\tau + \xi \tau_0) \approx K^\mu(\tau) \]  

(16)

Thus as can be seen from (16), the Lorentz-Dirac equation differs from the usual local theory by a term of the order of

\[ \frac{2}{3}e^2/3c^3 \hat{a}_{\mu} \]  

(17)

the so called Schott term.

So, the non locality in time is within intervals \( \sim \tau_0 \), the Compton time exactly what we encountered in section 2.

It must also be reiterated that the Lorentz-Dirac equation must be supplemented by the asymptotic condition of vanishing acceleration in order to be meaningful. That is, we have to invoke not just the point electron, but also distant regions into the future as boundary conditions.

Finally it must be borne in mind that the four vector \( \Gamma^\mu \) given in (13) can also be written as

\[ \Gamma^\mu = \frac{e}{2c}(F^\mu_{ret} - F^\mu_{adv})v_\nu \]  

(18)
In (18) we can see the presence of the advanced or acausal field which has been considered unsatisfactory. In fact this term, as is well known directly leads to the Schott term (17). Let us examine this non local feature. As is known, considering the time component of the Schott term (17) we get (cf.ref.[2])

\[ \frac{dE}{dt} \approx R \approx \frac{2 e^2 c}{3 r^2} \left( \frac{E}{mc^2} \right)^4, \]

where \( E \) is the energy of the particle.

whence integrating over the period of non locality \( \sim \tau_0 \) we can immediately deduce that \( r \), the dimension of spatial non locality is given by

\[ r \sim c \tau_0, \]

that is of the order of the Compton wavelength. This follows in any case in a relativistic theory, given the above Compton time. This term represents the effects within the neighbourhood of the charge.

What we have done is that we have quantified the space-time interval of non locality - it is of the order of the Compton wavelength and time. This contains the renormalization effect and gives the correct physical mass.

We now come to the Feynman-Wheeler action at a distance theory[32, 33]. They showed that the apparent acausality of the theory would disappear if the interaction of a charge with all other charges in the universe, such that the remaining charges would absorb all local electromagnetic influences was considered. The rationale behind this was that in an action at a distance context, the motion of a charge would instantaneously affect other charges, whose motion in turn would instantaneously affect the original charge. Thus considering a small interval in the neighbourhood of the point charge, they deduced,

\[ F_{ret}^{\mu} = \frac{1}{2} \{ F_{ret}^{\mu} + F_{adv}^{\mu} \} + \frac{1}{2} \{ F_{ret}^{\mu} - F_{adv}^{\mu} \} \] \hspace{1cm} (19)

The left side of (19) is the usual causal field, while the right side has two terms. The first of these is the time symmetric field while the second can easily be identified with the Dirac field above and represents the sum of the responses of the remaining charges calculated in the vicinity of the said charge.

From this point of view, the self force or in the earlier Kerr-Newman formulation, effects within the Compton scale, turns out to be the combined
reaction of the rest of the charges, or in the earlier duality and cosmological considerations, the holistic effect.

## 4 Duality and Scale

In a previous communication\textsuperscript{[34]} it was shown that we could consider a scaled Planck constant

\[ h_1 \approx N^{3/2} \hbar \]

such that we would have

\[ R = \frac{h_1}{Mc} \]

It is interesting to note that these relations are essentially the same as the second or extra term in (7) viz.,

\[ l^2 \frac{\Delta p}{\hbar} \sim \Delta x \]

with \( \Delta p = \sqrt{Nm} \) and \( \Delta x = R \) as before. This can be easily verified. In other words the two terms of the modified Heisenberg Uncertainty relation (7) represents two scales. The first term represents the micro scale with the Planck constant, while the second term represents the macro scale with the scaled Planck constant \( h_1 \), both being linked, as noted earlier.

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