Softening of Roton and Phonon Modes in a Bose-Einstein Condensate with Spin-Orbit Coupling

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Roton-type excitations usually emerge from strong correlations or long-range interactions, as in superfluid helium or dipolar ultracold atoms. However, in weakly short-range interacting quantum gases, the recently synthesized spin-orbit (SO) coupling can lead to various unconventional phases of superfluidity, and give rise to an excitation spectrum of roton-maxon character. Using Bragg spectroscopy we study a SO coupled Bose-Einstein condensate of $^{87}$Rb atoms, and show that the excitation spectrum in a “magnetized” phase clearly possesses a two-branch and roton-maxon structure. As Raman coupling strength $\Omega$ is decreased, a roton-mode softening is observed, as a precursor of the phase transition to a stripe phase that spontaneously breaks spatially translational symmetry. The measured roton gaps agree well with theoretical calculations. Further, we determine sound velocities both in the magnetized and the non-magnetized phase, and a phonon-mode softening is observed around the phase transition in between. The validity of the $f$-sum rule is examined.

Roton and phonon are two typical excitation modes of superfluids. They were first introduced by Landau in his phenomenological explanation on superfluidity of liquid helium [1], and an experimental observation was realized about two decades later [2]. The emergence of roton mode in superfluid helium originates from strong density correlations. In weakly interacting ultracold gases, long-range dipole-dipole interactions can induce a roton-maxon dispersion [3, 4], which were recently observed in a Bose-Einstein condensate (BEC) with cavity-mediated long-range interactions [5]. Across the phase transition from a superfluid to a supersolid phase, a softening of roton mode was further demonstrated [5]. An important question naturally arises: can an excitation spectrum of roton-maxon character be observed in a quantum gas with weak and short-range interactions?

Recently, artificial one-dimensional SO coupling has been synthesized in ultracold bosonic [6] and fermionic [8, 9] atoms by two counter-propagating Raman lasers that couple the momentum of an atom to its internal states of the F = 1 manifold, generated by a bias magnetic field. In addition, the BEC is illustrated by two Bragg beams with approximately parallel polarization, which are symmetric about the $y$ axis.

By a quadratic Zeeman shift $\varepsilon = 4.53k_r$ with recoil momentum $k_r$, the $|m_F = 1\rangle$ state is effectively suppressed, and the system can be regarded as a spin-1/2 system. The single-particle Hamiltonian along the SO coupling direction (the $x$-direction) is given by ($\hbar = 1$)

$$H_0 = \frac{(k_x - k_r\sigma_z)^2}{2m} + \frac{\delta}{2}\sigma_z + \frac{\Omega}{2}\sigma_x, \quad (1)$$

where $m$ is the atom mass, $\Omega$ is the Raman coupling strength, and $\delta$ is the two-photon detuning, which is fine tuned to be $\delta = 0$ in experiment. Symbols $\sigma_z$ and $\sigma_x$ represent the Pauli matrices, with $|m_F = -1\rangle$ for spin $|\uparrow\rangle$ and $|m_F = 0\rangle$ for spin $|\downarrow\rangle$. For each given $k_x$, Eq. (1) has two eigenstates with energy $E_+(k_x) > E_-(k_x)$ for the upper (+) and the lower (−) branch of single-particle dispersion [10, 11], respectively. The lower branch has two degenerate minima for $\Omega < 4E_r$ ($E_r = k_r^2/2m$), denoted by $\pm k_{\text{min}}$, and has a single minima at $k_x = 0$ for
Ω > 4Eₐ. With interatomic interactions of ⁸⁷Rb atoms being taken into account, it has been shown [⁶, ¹²] that for Ω < 0.2Eₐ, atoms condense in a superposition of two components with opposite momenta ±kₘᵟᵣₚ, exhibiting the stripe order. For 0.2Eₐ < Ω < 4Eₐ, this system maintains the magnetized phase, where atoms condense at kₘᵟᵣₚ or −kₘᵟᵣₚ. When Ω > 4Eₐ, the single-particle dispersion has only one single minimum at zero momentum, and the Bose gas hence exhibits no magnetization, i.e., the non-magnetized phase.

The excitation spectrum of the magnetized phase is measured through Bragg spectroscopy [¹⁷, ²⁰]. We first prepare the BEC at the spin state |mₓ = −1⟩, and adiabatically ramp up Raman coupling strength Ω to the desired value. By this way, the condensate is at the minimum −kₘᵟᵣₚ, where most of atoms are in the spin state |mₓ = −1⟩. Then, we quickly switch on two Bragg lasers and hold them for 1 ~ 2ms. The Bragg beams have wavelength about λₐ = 780.24nm and are detuned 6.8GHz away from the resonance. The angle θ between the two lasers (Fig. 1) determines the momentum transfer qₓ = 2Eₐ sin(θ/2) (kₘᵟᵣₚ = 2π/λₐ), while the frequency difference ω is tuned to produce an excitation. The Bragg pulse kicks a small percent of atoms out of the condensate cloud. The intensity of Bragg lasers is adjusted to excite at most 20% atoms, such that the linear response theory applies [²¹]. Finally, with the Stern-Gerlach technique, we take spin-resolved time-of-flight (TOF) images after 24 ms of free expansion. Three examples for Ω = 2Eₐ are shown in Fig. 1b-d, which have qₓ/kₘᵟᵣₚ = −1.77, 1.77, and 1.77, respectively. The latter two have the same momentum transfer, but different frequency difference. It is worth noting that the spins of atoms in Fig. 1c flip when being kicked out from the condensate by the Bragg pulse. This is due to the lock of spin and momentum.

For each TOF image, the atom numbers, Nₐ and Nₑ, of the Bragg and the remaining condensate, are counted, and the ratio P(qₓ, ω) ≡ Nₑ/(Nₐ + Nₑ) is calculated. According to the linear response theory [²¹], the excitation spectrum can be determined by the dynamic structure factor S(qₓ, ω). An evaluation based on Fermi’s golden rule yields P(qₓ, ω) ∝ Ωₐ being fixed at an appropriate value. For Ω = 2Eₐ, Fig 1e shows the plot of the excitation efficiency P(qₓ, ω) versus frequency difference ω, for qₓ/kₘᵟᵣₚ = −1.77 (red triangles) and 1.77 (purple circles and gray diamonds). For simplicity and comparison purpose, Ωₐ is taken as the Bragg-laser intensity Ωₛ being fixed at an appropriate value. We define excitation efficiency as P(qₓ, ω) = P(qₓ, ω)/(Ωₛ/Ωₛ₀)², where Ωₛ₀ is chosen such that P(qₓ, ω) is a dimensionless quantity. The excitation efficiency equals to the dynamic structure S(qₓ, ω), apart from an unknown constant C: For a given momentum transfer qₓ, a broad range of frequency difference ω is scanned with Bragg laser intensity Ωₛ being fixed at an appropriate value. For Ω = 2Eₐ, Fig 1e shows the plot of the excitation efficiency P(qₓ, ω) versus frequency difference ω, for qₓ/kₘᵟᵣₚ = −1.77 (red triangles) and 1.77 (purple circles and gray diamonds). For simplicity and comparison purpose, Ωₛ₀ is taken as the Bragg-laser intensity Ωₛ being fixed at an appropriate value. We define excitation efficiency as P(qₓ, ω) = P(qₓ, ω)/(Ωₛ/Ωₛ₀)², where Ωₛ₀ is chosen such that P(qₓ, ω) is a dimensionless quantity. The excitation efficiency equals to the dynamic structure factor S(qₓ, ω). An evaluation based on Fermi’s golden rule yields P(qₓ, ω) ∝ Ωₛ being fixed at an appropriate value. For Ω = 2Eₐ, Fig 1e shows the plot of the excitation efficiency P(qₓ, ω) versus frequency difference ω, for qₓ/kₘᵟᵣₚ = −1.77 (red triangles) and 1.77 (purple circles and gray diamonds). For simplicity and comparison purpose, Ωₛ₀ is taken as the Bragg-laser intensity Ωₛ being fixed at an appropriate value.

The excitation spectrum in the magnetized phase clearly shows a roton-type minimum at finite momentum around qₓ = 2kₘᵟᵣₚ (see the inset of Fig. 2). We measure the roton gap Δ, defined as the excitation en-
energy at the roton minimum, and find it soften as Raman coupling strength decreases (Fig. 2b). We calculate the roton gap based on a modified Bogoliubov theory, shown as the red solid curve in Fig. 2b. The experimental data agree well with theoretical calculations. As mentioned above, for $^{87}\text{Rb}$ atoms, there is a phase transition near $\Omega_1 \approx 0.2E_r$ between the magnetized and the stripe phase, and accordingly, the roton gap is expected to vanish at $\Omega_1$. Unfortunately, our experimental data are not sufficiently accurate to figure out the precise location of $\Omega_1$. On the other hand, we do find that the roton-maxon structure disappears when $\Omega$ is tuned above a large enough value (about $3.4E_r$ in our experiment), suggesting that the roton mode is a precursor of the stripe phase with periodic fringes.

The observed softening of roton gap can find its origin in Raman-dressed interaction. In the presence of SO coupling, interatomic interaction becomes anisotropic, and this anisotropy can be modified by tuning $\Omega$.

This can be revealed by calculating the interaction energy for a condensate of different components. As shown in Ref. [6], for $^{87}\text{Rb}$ atoms, we have interaction energy $E_{\text{i}} \approx 1/2 \int d^3r \left[ (c_0 + c_2/2)|n_{\uparrow}\uparrow(r)|^2 + c_2/2|n_{\downarrow}\downarrow(r)|^2 - n_{\uparrow}\downarrow(r)^2 \right] + (c_2 + c_0\Omega^2/8E_r^3)n_{\uparrow\downarrow}(r)\langle n_{\uparrow\downarrow}(r) \rangle$, where the spin-independent interaction $c_0 = 7.79 \times 10^{-12}\text{Hz cm}^3$, the spin-dependent interaction $c_2 = -3.61 \times 10^{-14}\text{Hz cm}^3$, and $n_{\uparrow\downarrow}(r)$ and $n_{\downarrow\uparrow}(r)$ respectively represent the spatial density of the components at $-k_{\text{min}}$ and $k_{\text{min}}$. This means that the interaction energy for a condensate of two dressed components $\pm k_{\text{min}}$ has additional energy terms compared to the energy for a single-component condensate at $k_{\text{min}}$ or $-k_{\text{min}}$. Accordingly, one can give an estimation of the roton gap as $\Delta \approx c_0 n\Omega - \Omega_1^2/16E_r^2$ for $\Omega > \Omega_1$ with $n$ for the condensate density and $\Omega_1 \approx 0.2E_r$ marking the phase transition point between the stripe and the magnetized phase.

We also measure sound velocities both in the magnetized and the non-magnetized phase, and find a softening of the phonon mode near the phase transition between these two phases. In the magnetized phase, the excitation spectrum exhibits linear dispersions in the long wavelength limit, i.e. $E_{\text{B}}(q_x) = -c_1q_x$ for $q_x < 0$ and $E_{\text{B}}(q_x) = c_2q_x$ for $q_x > 0$; see the inset of Fig. 2b. Here...
Ref. [14]. In Fig. 4a, we plot the static structure factor $S(q_x)$ for $\Omega = 2E_r$. (a) Static structure factor $S(q_x)$ (red circles) and the contribution $S_f(q_x)$ (blue circles) from the lower branch of excitation spectrum. The theoretical calculations based on local density approximations are shown as red and blue solid curves. The relative contribution $S_f(q_x)/S(q_x)$ is shown in the inset. (b) Energy-weighted moment $M_1(q_x)$. The good agreement between the experimental data (blue circles) demonstrates the validity of the $f$-sum rule.

where constant $C$ is chosen such that among the measured data, the maximum of $S(q_x)$ is equal to unity. The blue circles in Fig. 4a represent the contribution $S_f(q_x)$ from the lower branch of excitation spectrum. The solid lines in Fig. 4a are obtained from theoretical calculations based on local density approximation. They agree with experimental data, except for those three points with very small momentum transfer $q_x$. The relative contribution $S_f(q_x)/S(q_x)$ is shown in the inset of Fig. 4a, which rapidly decreases as the momentum transfer becomes larger. The measured 1st moment $M_1(q_x)$ for $\Omega = 2E_r$ is plotted versus $q_x$ in Fig. 4b. These experimental data can be well described by a quadratic curve, demonstrating the validity of the $f$-sum rule at least in the magnetized phase.

We have shown that despite interatomic interactions are weak and short-ranged, the SO coupled $^{87}$Rb condensate has an excitation spectrum of roton-maxon character in the magnetized phase, which softens near the phase transition to the stripe phase. The sound velocities are also measured and a phonon-mode softening is observed. We mention that in condensed-matter physics and ultracold atomic physics, measurement of excitation spectrum is in itself of important role in revealing the properties of low-temperature phases [24]. The observed linear dispersion near $q_x = 0$ is an important feature of superfluidity. Further, the measured roton-maxon structure of excitation spectrum, its disappearance for large $\Omega$, and the softening of the roton gap, strongly support the predicted ground-state phase diagram for of the SO coupled Bose gas of $^{87}$Rb atoms.

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