Scalar meson masses and mixing angle in a $U(3) \times U(3)$ Linear Sigma Model.

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Abstract

Meson properties are considered within a $U(3) \times U(3)$ Linear Sigma Model (LSM). The importance of the $U_A(1)$-breaking term and the OZI rule violating term in the generation of meson masses and mixing angles is stressed. The LSM parameters are fitted to the pseudoscalar meson spectrum thus giving predictions for scalar meson properties. The model predicts a scalar meson $\bar{q}q$ nonet whose members are: $\{\sigma(\approx 400), f_0(980), \kappa(\approx 900) \text{ and } a_0(980)\}$ resonances. Scalar meson mixing angle (in the $\{|ns>,|s>\}$ basis) is predicted to be $\phi_S \approx -14^\circ$. Therefore the $f_0(980)$ is predominantly strange while the $\sigma(\approx 400)$ is mostly non-strange. The model also gives a pseudoscalar mixing angle $\phi_P \approx 35^\circ$ which corresponds to $\theta_P \approx -19^\circ$ in the singlet-octet basis. Comparison with chiral perturbation theory shows that $L_8$ is saturated by scalar mesons exchange. However, LSM predicts that $L_5$ is not saturated by scalar mesons exchange as assumed in the literature.
1. Introduction

Although we all believe in QCD as the theory describing strong interactions, its non-perturbative nature at low energies prevents its direct application at the hadron level, thus being necessary to consider models for the strong interactions which exhibit the general features of QCD and be useful in the computation of hadron properties.

Concerning the light quark sector, the most important characteristic of QCD is the chiral symmetry exhibited by the QCD Lagrangian in the limit when the light quarks become massless. This has been the starting point for most of phenomenological approach to low energy QCD. The whole chiral machinery have been summarized in the so called Chiral Perturbation Theory (CHPT)[1] formalism (and its extensions [2,3,4]) which is still a very active field.

Another general approach which over the past few years has gained renewed interest is the $1/N_c$ expansion of QCD[5]. Concerning the meson sector this approach tell us that in the large $N_c$ limit, mesons are stable particles grouped in nonets, whose masses remain finite. Meson interactions are suppressed by $(\frac{1}{N_c})^{\frac{n}{2}-1}$ where $n$ denotes the number of mesons in the interacting vertex.

From the whole hadron spectrum the less-well-understood is the scalar sector. In spite of the fact that the existence of the singlet scalar meson ($\sigma$) was proposed long time ago, today we still do not have an unambiguous identification of this particle and the corresponding nonet of scalar mesons. The lowest lying scalar mesons which have been firmly established are the $I=0$ ($f_0(980)$) and the $I=1(a_0(980))$, but their quark content is still controversial [6]. Among others, the argument against the considering of the $a_0(980)$ as the isovector $\bar{q}q$ resonance is its rather small coupling to two photons which is in disagreement with naive quark model calculations [6,7]. Although a $K\bar{K}$ molecule picture calculation [7] is closer to, it is still far from experimental results[6,8]. Moreover, in the later calculations there are large uncertainties due to theoretical assumptions [7].

In addition to the above mentioned resonances we have well-established scalar resonances above 1 GeV (e.g $K^*(1430)$) and many others whose status is rather uncertain and have been conservatively labeled as ($f_0(400-1200), f_0(1370), f_0(1500), a_0(1450)$ etc.) by the PDG[8]. The identification of the $q\bar{q}$ scalar resonances is further complicated by the possibility of the
existence of non-$q\bar{q}$ states in the same energy region and with the same quantum numbers. From the $\frac{1}{N_c}$ perspective some of these states (glue-balls) are also stable in the large $N_c$ limit.

In the standard CHPT machinery, scalar meson effects would appear at $O(p^4, p^6)$ due to the “integrating out” of these degrees of freedom. This is an acceptable procedure whenever scalar meson masses lie above the low energy region, typically above $\lambda_\chi$ (chiral symmetry breaking scale), and we are not interested in the scalar mesons phenomenology. However, it is well possible that a very broad and not so heavy scalar resonance ($\sigma$) exist [9,10]. In this case, these degrees of freedom should be explicitly included in the effective description of low energy QCD.

The most promising possibility is to consider scalar degrees of freedom within CHPT. This theory is known to be a systematic expansion in powers of (small) momentum. However, predictive power is substantially lost at $O(p^4)$ and beyond due to the appearance of a large set of low energy constants (LEC) which have to be phenomenologically fixed. This problem can be circumvented by invoking saturation by meson resonances [11]. Contributions of scalar meson resonances exchange to the CHPT LEC have been considered in the literature [11]. Here again, the problem of the scalar mesons arise as we still don’t have confident experimental information on these resonances and is not possible to make reliable estimations of the contribution of scalar resonances exchange to CHPT low energy constants. It has been assumed e.g. that scalar meson resonances exchange saturate $L_5$ and $L_8$ [11].

Extension of CHPT beyond the very low energy domain is problematic. The heavy field approach is necessary for spin 1/2 fermions (and vector mesons) which forces us to consider a double expansion and to the introduction of spin 3/2 degrees of freedom [2]. Again, introduction of spin 3/2 resonances forces us to consider a new (triple) expansion (“small scale” expansion[3]) and the theory rely upon a spin 3/2 field theory which for decades has been known to suffer from serious yet unresolved inconsistencies[1]. On the other side, it is well known that “golden plated” (i.e. low energy parameters free) predictions of CHPT at $O(p^4)$ (e.g. $\gamma\gamma \rightarrow \pi^0\pi^0$ [14]) fail in the description of data at threshold were it is supposed to work well. Perhaps this is an indication that

\[\text{it has been argued in [12] that only the leading order term in the 1/m expansion of spin } \frac{3}{2} \text{ fields could be theoretically consistent. Explicit calculations [3] and a Hamiltonian quantization of the theory (see Eq.(28) of Ref.[13]) gives support to this argument.}\]
we are already missing something which well it could be the contributions of a 
not so heavy scalar meson. From the CHPT perspective this is equivalent to 
say that $O(p^6)$ contributions to that process are saturated by scalar mesons 
exchange.

Another possibility for the description of scalar mesons phenomenology 
is the linear realization of chiral symmetry. However, although a pure-meson 
LSM is a renormalizable model, it is not clear how a systematic expansion 
can be carried out as there is no obvious small parameter. Nevertheless, it is 
symptomatic that one-loop corrections e.g. to meson masses have been found 
by Chan and Haymaker to be small[15]($\sim 10\%$). This raises the possibility 
of a weak regime in the effective LSM description of low energy QCD at 
least in the meson sector. In this concern, it is quite remarkable that in a 
$SU(2) \times SU(2)$ LSM the derivative coupling of pions (characteristic of non-
linear realizations) is recovered in the LSM after delicate (“miraculous”[16]) 
cancellations when we expand in inverse powers of the center of mass energy. 
In this sense a LSM gives the same results (at leading order) as CHPT in 
the very low energy regime[15].

From the $\frac{1}{N_c}$ perspective, a LSM description of low energy $(\bar{q}q)$ meson-
meson interactions can be accurate whenever other $\frac{1}{N_c} \to 0$ surviving non-$\bar{q}q$
states be much more massive than the LSM degrees of freedom. From lattice 
calculations and theoretical estimates [17] the lowest lying scalar glue-ball 
has a mass of $\simeq 1.5GeV$ or above, hence a LSM can be naively expected to 
be a good approximation below $1GeV$.

Recently, there has been renewed interest in the linear realization of chiral 
symmetry [9,18,19,20]. Some linkage between a $SU(2) \times SU(2)$ quark-
level LSM and non-perturbative QCD have been established [18] and double-
counting ambiguities have been addressed [19]. On the other hand, signals 
of scalar ($\sigma(650)$ and $\kappa(900)$) resonances in reanalysis of $\pi\pi$-$KK$ scattering 
have been argued[20] to be reproduced within a pure-meson LSM in its 
$U(3) \times U(3)$ version[21,22]. 
The pure-meson $U(3) \times U(3)$ LSM has also been used by Tornqvist as a 
field theoretical framework where to interpret the results arising from his 
unitarized quark model [9]. However in the Ishida reanalysis of data[20] the 
$I = 1/2$ scalar resonance turns out to have a mass around $900MeV$ whereas 
Tornqvist identifies this resonance with the $K^*(1430)$ and some signal below 
the $K^-\pi$ threshold [9].
Although the $U(3) \times U(3)$ LSM model has previously been studied in the literature [15,21,22], we consider that a complete analysis of its phenomenological implications is still missing. This is particularly important for scalar mesons since available experimental information on these particles is unclear as can be seen from the last three versions of the RPP. Furthermore, the theory (when written in terms of the appropriate fields) allows us to transparently identify the corresponding quark content of mesons which is particularly useful in the identification of $\bar{q}q$ scalar mesons.

In this work we explore predictions for scalar mesons properties arising from a $U(3) \times U(3)$ LSM with $U_A(1)$ breaking. This model has already been considered by t’Hooft [23], in its $U(2) \times U(2)$ version, in connection with the explanation of the $U_A(1)$-problem by instanton contributions. We begin by carrying out a detailed analysis of the $U(3) \times U(3)$ LSM predictions for meson masses and mixing angles. We shall be concerned with the meson sector only putting fermions aside for a while as they are not relevant for the physical quantities studied here. We consider a $U(3) \times U(3)$ LSM in order to phenomenologically incorporate the effects of the $U_A(1)$ symmetry breaking. These effects are well known in the pseudoscalar sector but, as we shall see below, they give the scalar sector unexpected properties. The $U_A(1)$ breaking effects in the scalar sector are explicitly exhibited and turn out to be crucial in the understanding of scalar mesons.

In order to make contact with the quark content of scalar mesons we start working in the usual singlet-octet basis but we switch later to the strange-non-strange basis[24]. Whenever possible, we use pseudoscalar mesons experimental information in order to fix the parameters of the model in such a way that we can make predictions for scalar mesons. It turns out that one of the parameters ($\lambda'$) cannot be fixed from pseudoscalar spectrum as only the scalar mixed sector depends directly on it. Hence, we choose to analyze the mixed scalar meson masses and mixing angle dependence on $\lambda'$. This parameter has been suggested by Tornqvist to be small ($\lambda'$ used in this work corresponds to $4\lambda'_T$ in Ref.[9]), and bosonization of a Nambu-Jona-Lasinio (NJL) model extended with a t’Hooft like interaction [25] predicts $\lambda' \approx O(1/N_{\chi}^3)$. As the author of [25] points out, this result is consistent with but not necessarily required by QCD. We include this term here as it is compatible with chiral symmetry and renormalizability in the usual sense. In spite of some sensitivity of scalar meson masses to the $SU(3)$ symmetry breaking, the model indicates that is likely to identify the scalar nonet with
the \( \{\sigma(\approx 400), f_0(980), \kappa(900) \text{ and } a_0(980)\} \) resonances which according to the above discussion leads to a scenario where the (broad) sigma meson would give dominant contributions in the very low energy domain of QCD. We obtain a scalar mixing angle \( \phi_s \approx -14^\circ \). The pseudoscalar mesons mixing angle predicted by this model is \( \phi_P \approx 35^\circ \) which corresponds to \( \theta \approx -19^\circ \) in the usual singlet-octet basis.

We also study scalar mesons exchange contributions to the \( \mathcal{O}(p^4) \) CHPT low energy constants. Expectations for the saturation of \( L_8 \) are corroborated but LSM predicts that \( L_5 \) is not saturated by meson resonances exchange.

2. \( U(3) \times U(3) \) Linear Sigma Model.

The \( U(3) \times U(3) \) LSM was originally proposed by Levy[19] in a quark-meson version. The meson sector of this theory was also considered by Gasiorowicz and Geffen [22].

The \( U(3) \times U(3) \) symmetric Lagrangian is written in terms of a nonet of scalar (\( \sigma_i \)) and a nonet of pseudo-scalar (\( P_i \)) fields. A nonet is represented by the matrix:

\[
F \equiv \frac{1}{\sqrt{2}} \lambda_i f_i \quad i = 0, \ldots 8.
\]

where \( \lambda_i(i = 1, \ldots, 8) \) are the Gell-Mann matrices, \( \lambda_0 \equiv \sqrt{\frac{2}{3}} 1 \) and \( f_i \) denotes a generic field. We will use \( \langle \rangle \) throughout this paper to denote trace over the \( U(3) \) structure.

The Lagrangian is given by

\[
\mathcal{L} = \mathcal{L}_{sym} + \mathcal{L}_{SB}
\]  

(1)

where \( \mathcal{L}_{sym} \) denotes the \( U(3) \times U(3) \) symmetric Lagrangian:

\[
\mathcal{L}_{sym} = \frac{1}{2} (\partial_\mu M)(\partial^\mu M^\dagger) - \frac{\mu^2}{2} X(\sigma, P) - \frac{\lambda}{4} Y(\sigma, P) - \frac{\lambda'}{4} X^2(\sigma, P)
\]

with \( M = \sigma + iP \), and \( X, Y \) stand for the \( U(3) \times U(3) \) chirally symmetric terms:

\[
X(\sigma, P) = \langle MM^\dagger \rangle
\]

\[
Y(\sigma, P) = \langle (MM^\dagger)^2 \rangle = \langle (\sigma^2 + P^2)^2 \rangle = 2(\sigma^2 P^2 - (\sigma P)^2) + 2(\sigma^2 P^2 - (\sigma P)^2).
\]
The chiral symmetry breaking terms are given by

\[ \mathcal{L}_{SB} = \langle c\sigma \rangle - \beta Z(\sigma, P) \]  

where

\[ Z(\sigma, P) = \{\det(M) + \det(M^\dagger)\} \]

\[ = \frac{1}{3} [\langle \sigma \rangle (\langle \sigma \rangle^2 - 3\langle P \rangle^2) - 3\langle \sigma \rangle \langle \sigma^2 - P^2 \rangle + 6\langle P \rangle \langle \sigma P \rangle \]

\[ + 2\langle \sigma (\sigma^2 - 3P^2) \rangle] \]

and \( c \equiv \frac{1}{\sqrt{2}} \lambda_i c_i \), with \( c_i \) constant. The most general form of \( c \) which preserves isospin and gives PCAC is such that the only non-vanishing coefficients are \( c_0 \) and \( c_8 \). The former gives, by hand, the pseudo-scalar nonet a common mass, while the latter breaks the \( SU(3) \) symmetry down to isospin. These parameters can be related to quark masses in QCD. The \( Z \) term in Eq。(2) is \( U(3)_V \times SU(3)_A \) symmetric and is reminiscent of the quantum effects in QCD breaking the \( U_A(1) \) symmetry.

Performing the customary calculations we obtain for the divergences of the vector and axial-vector currents:

\[ \partial_\mu A_\mu^j = (c_0 d_{ojk} + c_8 d_{8jk})P_k \]

\[ \partial_\mu V_\mu^j = c_8 f_{8jk}\sigma_k. \]

The choice (2) for the breaking term gives PCAC in the usual way

\[ \partial_\mu A_\mu^\pi = (\sqrt{\frac{2}{3}}c_0 + \sqrt{\frac{1}{3}}c_8)\pi \equiv m_\pi^2 f_\pi \pi \]

\[ \partial_\mu A_\mu^K = (\sqrt{\frac{2}{3}}c_0 - \frac{1}{2} \sqrt{\frac{1}{3}} c_8)K \equiv m_K^2 f_K K \]

The linear \( \sigma \) term in Eq. (2) induces \( \sigma \)-vacuum transitions which give to \( \sigma \) fields a non-zero vacuum expectation value (hereafter denoted by \{ \}). Linear terms can be eliminated from the theory by performing a shift to a new scalar field \( S = \sigma - V \) such that \( \{S\} = 0 \), where \( V \equiv \{\sigma\} \). This shift generates new three-meson interactions and mass terms.

For the sake of simplicity lets write \( V = Diag(a, a, b) \) where \( a, b \) are related to \( \{\sigma\} \) through
\begin{align*}
a &= \frac{1}{\sqrt{3}}\{\sigma_0\} + \frac{1}{\sqrt{6}}\{\sigma_8\} \\
b &= \frac{1}{\sqrt{3}}\{\sigma_0\} - \frac{2}{\sqrt{6}}\{\sigma_8\}
\end{align*}

It is convenient to write the shifted meson Lagrangian as:

\[ L_{MM}(\sigma, P) = \sum_{n=0}^{4} L_n \]

where terms containing products of \( n \) fields are collected in \( L_n \). Explicitly

\[ \begin{align*}
L_0 &= -\frac{\mu^2}{2}(2a^2 + b^2) - \frac{\lambda'}{4}(2a^2 + b^2)^2 - \frac{\lambda}{4}(2a^4 + b^4) - 2\beta a^2 b + \langle CV \rangle \\
L_1 &= -(\mu^2 + \lambda'(2a^2 + b^2))\langle SV \rangle - \lambda((a^2 + ab + b^2)\langle SV \rangle - ab(a + b)\langle S \rangle) \\
&\quad - 2\beta(a(a + b)\langle S \rangle - a\langle SV \rangle) + \langle CS \rangle \\
L_2 &= -\frac{\mu^2}{2}X(S) - \lambda'(\langle SV \rangle^2 + \frac{1}{2}(2a^2 + b^2)(S^2 + P^2)) \\
&\quad - \lambda(\langle (a + b)V - ab \rangle\langle S^2 + P^2 \rangle + \frac{1}{2}(VS)^2 - \frac{1}{2}(VP)^2) \\
&\quad - \beta[(2a + b)(\langle S \rangle^2 - \langle P \rangle^2 - \langle S^2 - P^2 \rangle) - 2\langle S \rangle\langle VS \rangle + 2\langle P \rangle\langle VP \rangle + 2\langle V(S^2 - P^2) \rangle] \\
L_3 &= -\beta Z(S) - \lambda'(\langle SV \rangle\langle S^2 + P^2 \rangle - \lambda(V(S^3 + P^2S + SP^2 - PSP))) \\
L_4 &= -\frac{\lambda}{4}Y(S, P) - \frac{\lambda'}{4}X^2(S, P)
\end{align*} \] (8)

The condition \( L_1 = 0 \) gives:

\[ \sqrt{\frac{2}{3}}c_0 + \sqrt{\frac{1}{3}}c_8 = \sqrt{2}a(\xi + 2\beta b + \lambda a^2) \]

\[ \sqrt{\frac{2}{3}}c_0 - \frac{1}{2}\sqrt{\frac{1}{3}}c_8 = \frac{1}{\sqrt{2}}(a + b)\left(\xi + 2\beta a + \lambda(a^2 - ab + b^2)\right) \]

where, for convenience, we defined \( \xi \equiv \mu^2 + \lambda'(2a^2 + b^2) \).
3. Meson masses and mixing angles.

Meson masses are given by $L_2$. A straightforward calculation reveals the following masses for the non-mixed sectors.

\[ m_\pi^2 = \xi + 2\beta b + \lambda a^2 \]
\[ m_K^2 = \xi + 2\beta a + \lambda (a^2 - ab + b^2) \]
\[ m_\sigma^2 = \xi - 2\beta b + 3\lambda a^2 \]
\[ m_\kappa^2 = \xi - 2\beta a + \lambda (a^2 + ab + b^2), \] (10)

where we have denoted $a$ and $\kappa$ the scalar mesons analogous to $\pi$ and $K$.

For the mixed sectors we obtain the following Lagrangian.

\[ L_2^P = -\frac{1}{2}(m_{0P}^2 P_0^2 + m_{8P}^2 P_8^2 + 2m_{08P}^2 P_0 P_8) \] (11)
\[ L_2^S = -\frac{1}{2}(m_{0S}^2 S_0^2 + m_{8S}^2 S_8^2 + 2m_{08S}^2 S_0 S_8) \]

where

\[ m_{0P}^2 = \xi + \frac{1}{3}[\lambda(2a^2 + b^2) - 4\beta(2a + b)] \]
\[ m_{8P}^2 = \xi + \frac{1}{3}[\lambda(a^2 + 2b^2) + 2\beta(4a - b)] \] (12)
\[ m_{08P}^2 = \frac{\sqrt{2}}{3}(a - b)\left(\lambda(a + b) + 2\beta\right) \]
\[ m_{0S}^2 = \xi + \frac{1}{3}[4\beta(2a + b) + 3\lambda(2a^2 + b^2) + 2\lambda'(2a + b)^2] \]
\[ m_{8S}^2 = \xi + \frac{1}{3}[-2\beta(4a - b) + 3\lambda(a^2 + 2b^2) + 4\lambda'(a - b)^2] \]
\[ m_{08S}^2 = \frac{\sqrt{2}}{3}(a - b)[-2\beta + 3\lambda(a + b) + 2\lambda'(2a + b)] \]

In the case of a $SU(3)$ symmetric vacuum ($a = b$ or equivalently $\{\sigma_S\} = 0$), the pseudoscalar octet has a common squared mass which differs from the pseudoscalar singlet squared mass by a term proportional to $\beta$ (the $U_A(1)$-breaking term). Keeping $a \neq b$ amounts to simultaneously consider $U_A(1)$-breaking and $SU(3)$ breaking contributions to the $\eta - \eta'$ splitting. Similar results are obtained for the scalar nonet but also $\lambda'$ contributes to the octet-singlet squared mass difference in this case. In order to exhibit the mass...
pattern in the general case of chiral symmetry broken down to isospin, and in order to assess the relevant aspects of the theory it is instructive to shift to new fields in the mixed sector. This change of basis explicitly shows the role played by the $U_A(1)$-breaking term in the generation of scalar and pseudoscalar meson masses. Also, quark content is transparent and the role played by the different terms in the generation of masses, mixing, and violation to OZI rule, is obvious. Switching to the new fields ($\{ s >, |ns > \}$ basis [24]) defined by:

$$P_{ns} = \sqrt{\frac{1}{3}} P_8 + \sqrt{\frac{2}{3}} P_0$$  \hspace{1cm} (13)

$$P_s = -\sqrt{\frac{2}{3}} P_8 + \sqrt{\frac{1}{3}} P_0$$

and similar relations for the scalar mixed fields, we obtain an analogous Lagrangian to Eq. (11) where

$$m_{P^s}^2 = \xi - 2\beta b + \lambda a^2$$

$$m_{P^s}^2 = \xi + \lambda b^2$$

$$m_{P_8-ns}^2 = -2\sqrt{2}\beta a$$

$$m_{S^s}^2 = \xi + 2\beta b + 3\lambda a^2 + 4\lambda' a^2$$

$$m_{S^s}^2 = \xi + 3\lambda b^2 + 2\lambda' b^2$$

$$m_{S_8-ns}^2 = 2\sqrt{2}(\beta + \lambda') a$$  \hspace{1cm} (14)

The vacuum expectation values of the $s - ns$ scalar fields are simply related to the parameters $a, b$ as: $\{ \sigma_{ns} \} = \sqrt{2} a, \{ \sigma_s \} = b$.

From the above relations is clear that in the $\beta = 0$ case, the pseudoscalar sector gets diagonalized and even in the general case when chiral symmetry is broken down to isospin ($a \neq b$), the $U_A(1)$-breaking term $Z$ is required in order to break the $\eta_{ns} - \pi$ degeneracy [22]. In the scalar sector, however, we still have a non-zero mixing when $\beta = 0$ due to the $\lambda'$ term.

It is interesting to analyze, step by step, the results so far obtained from a $1/N_c$ perspective. According to $\frac{1}{N_c}$ counting rules [5], OZI rule allowed three(four)-meson interactions should be $\frac{1}{\sqrt{N_c} (\frac{1}{N_c})}$ suppressed. OZI rule violating terms, like $\beta$ and $\lambda'$ in Eq.(1), are further suppressed by higher powers of $\frac{1}{N_c}$. As shown in [25], $\beta \approx \mathcal{O}(\frac{1}{N_c^{1/2}})$ while $\lambda'$ is even more suppressed.
Thus, based on $\frac{1}{N_c}$ counting rules, we would expect $\beta$ and $\lambda'$ contributions to physical quantities to be small as compared with $\lambda$ contributions.

At leading order in the $\frac{1}{N_c}$ expansion, chiral symmetry is exact and mesons are stable and grouped in degenerated chiral multiplets. At $O(\frac{1}{N_c})$ only the $\lambda$ term must be included. This term generates (under the adequate “conditions”) vacuum instabilities in the LSM potential leading to the spontaneous breaking of chiral symmetry. New three-meson interactions and mass terms are generated. Higher order terms ($\beta$ and $\lambda'$) allowed by chiral symmetry can then be incorporated. This procedure would give new mass terms coming from the $\lambda$ term only. The $U_A(1)$ breaking term would give just $(1/N_c^{3/2}$ suppressed) three-meson interactions. As we know that even before the chiral transition, both the $U_A(1)$ symmetry is broken, and OZI rule violating transitions, though suppressed, are allowed transitions, we should include these terms before spontaneous symmetry breaking occur. Proceeding in this way, these terms also generates new mass terms and three-meson interactions which are formally subleading as compared with contributions coming from the $\lambda$ term. As the vacuum expectation values of scalar fields are $O(\sqrt{N_c})$, the new mass terms and three meson interactions generated by the $U_A(1)$ breaking term are $O(\frac{1}{N_c})$ and $O(\frac{1}{N_c^{3/2}})$ respectively. If we adopt the $\frac{1}{N_c}$ counting rule for $\lambda'$ as found in [25], meson masses contributions coming from the $\lambda'$ term are $O(\frac{1}{N_c^{2}})$ while three-meson interactions generated by this term are $O(\frac{1}{N_c^{2}})$. This is, however, not a clear issue as discussed in [25].

In the limit when $c_0, c_8$ (or equivalently the corresponding $c_s, c_{ns}$) → 0 we have spontaneous breaking of symmetry. In this case, and considering a $SU(3)$ symmetric vacuum, pseudoscalar mesons are massless (GB) except for the $\eta'$ meson which acquire a non-zero mass due to the combined effects of the $U_A(1)$-breaking and the spontaneous breaking of symmetry. Notice however that this mass (related to t’Hooft mass in Ref. [25]) is formally subleading ($O(\frac{1}{N_c})$) in the $\frac{1}{N_c}$ expansion. In this case, the $\{|s>, |ns>, \}$ pseudoscalar fields are mixed by the effects of the $U_A(1)$-breaking term. The scalar meson octet acquire a common mass and the scalar singlet is split due to the effects of both, the $U_A(1)$-breaking term and the OZI rule forbidden (but $U(3) \times U(3)$ chirally symmetric) term $\lambda'$.

Including the explicit chiral symmetry breaking terms $c_0, c_8$ (or equivalently $c_{ns}, c_s$) breaks the $SU(3)$ symmetry down to isospin, splits the isospin multiplets in both sectors and mixes the $\{|1>, |8>, \}$ fields. Pseudoscalar
strange-non-strange fields remain mixed by the effects of the $U_A(1)$-breaking term only, while scalar strange-non-strange fields remain mixed by both, the $U_A(1)$-breaking term and the $\lambda'$ term.

Let’s now turn to the mixing analysis. For the final purposes of this work (scalar mesons quark content and phenomenology) it is convenient to take the $\{|ns>, |s>\}$ basis in Eq.(13) as our starting point. The physical fields are linear combinations of $\{P_s, P_{ns}\}$ which diagonalize $L_P^2$:

$$\eta = P_{ns} \cos(\phi_P) - P_s \sin(\phi_P)$$  
$$\eta' = P_{ns} \sin(\phi_P) + P_s \cos(\phi_P).$$  

The relative angle between the $\{|1>, |8>\}$ basis and the $\{|S>, |NS>\}$ basis in Eq (13) corresponds to $\vartheta = -54.73^\circ$.

The mixing angle can be obtained from the following relation

$$\sin(2\phi_P) = \frac{2m_{P_{ns}}^2}{m_{\eta'}^2 - m_{\eta}^2}. \quad (16)$$

The diagonal masses are

$$m_{\eta}^2 = \frac{1}{2}(m_{P_{ns}}^2 + m_{P_s}^2) - \frac{1}{2}R_P$$
$$m_{\eta'}^2 = \frac{1}{2}(m_{P_{ns}}^2 + m_{P_s}^2) + \frac{1}{2}R_P \quad (17)$$

where

$$R_P \equiv [(m_{P_{ns}}^2 - m_{P_s}^2)^2 + 4m_{P_{s,ns}}^4]^{1/2} \quad (18)$$

From the above relations trace invariance of the mass matrix is obvious

$$m_{\eta'}^2 + m_{\eta}^2 = m_{P_s}^2 + m_{P_{ns}}^2. \quad (19)$$

Similar relations to Eqs. (16,17,18,19) hold for the corresponding scalar quantities with the $P \to S$ replacement. Using Eqs. (14,16) we obtain

$$\sin(2\phi_P) = \frac{-4 \sqrt{2} \beta a}{m_{\eta'}^2 - m_{\eta}^2} \quad (20)$$
$$\sin(2\phi_S) = \frac{4 \sqrt{2}(\beta + \lambda'b)a}{m_{\eta'}^2 - m_{\eta}^2} \quad (21)$$
where we have denoted $f, f'$ the physical scalar mesons analogous to the $\eta, \eta'$ mesons.

From Eqs.(10,12) the Gell-Mann-Okubo (GMO) relations read

$$3m_{8P}^2 - 4m_K^2 + m_\pi^2 = -2\lambda(a - b)^2$$  \hspace{1cm} (22)
$$3m_{8S}^2 - 4m_\kappa^2 + m_a^2 = 2(\lambda + 2\lambda')(a - b)^2. \hspace{1cm} (23)$$

These equations reflect the well known fact that violations to GMO relations are second order in the $SU(3)$ breaking parameter. Notice that corrections to GMO relations have different sign in the scalar and pseudoscalar sectors.

On the other hand, Eqs.(17,18) yield

$$(m_{\eta'}^2 - m_\eta^2)^2 = [\lambda(a^2 - b^2) - 2\beta b]^2 + 32\beta^2 a^2. \hspace{1cm} (24)$$

In addition to the $SU(3)$ breaking, the well known role of the $U_A(1)$ breaking in the $\eta, \eta'$ mass splitting is explicitly exhibited in this last equation.

4. Predictions

Before presenting the phenomenological extraction for the parameters entering in the model, we would like to stress that there are some predictions which are independent of some of the particular values for the parameters in the interacting Lagrangian. These predictions are relations between meson masses, three-meson couplings and the scalar meson vacuum expectation values which are independent of the specific values for the remaining parameters entering in the model. Some of these relations which at present are interesting for phenomenological applications and can be straightforwardly deduced from $\mathcal{L}_2, \mathcal{L}_3$ are:

$$m_\kappa^2 = \frac{(a + b)m_K^2 - 2am_\pi^2}{b - a}$$
$$g_{a_0K^+\kappa^-} = \frac{m_\kappa^2 - m_\pi^2}{\sqrt{2}(a + b)}$$
$$g_{a_0K^-\kappa^+} = \frac{m_a^2 - m_\kappa^2}{\sqrt{2}(b - a)} \hspace{1cm} (25)$$
$$g_{\pi_0\kappa^+\kappa^-} = \frac{m_\kappa^2 - m_\pi^2}{\sqrt{2}(a + b)} = \frac{m_K^2 - m_\pi^2}{\sqrt{2}(b - a)}$$
The above relations concerns the non-mixed sectors. In these sectors, the OZI rule forbidden term $\lambda'$ gives similar contributions to all masses (encoded in $\xi = \mu^2 + \lambda'(2a^2 + b^2)$). This is not true for the mixed sectors where the effects of the $U_A(1)$ breaking and OZI rule forbidden term $\lambda'$ are important. A more complete (although not exhaustive) list of such relations arising solely from chiral symmetry and the way it is broken can be found in the general treatment of Schechter and Ueda[15]. As shown in that work, in a general treatment of the subject nothing can be said about the $I = 0$ and $I = 1$ scalar meson masses in the general case of chiral symmetry broken down to isospin.

There are two interesting relations more arising from LSM. The first one, concerns scalar and pseudoscalar mixing angles. In fact, from Eqs.(21,22) we obtain

$$ (m_{\eta'}^2 - m_{\eta}^2)\sin(2\phi_P) = -(m_{f'}^2 - m_{f}^2)\sin(2\phi_S) + 4\sqrt{2}\lambda'ab. \quad (26) $$

The second, is a relation between scalar and pseudoscalar meson masses. From Eqs.(10,14,19) we get:

$$ m_{\eta'}^2 + m_{\eta}^2 - 2m_{K}^2 = -(m_{f'}^2 + m_{f}^2 - 2m_{\pi}^2 - 2\lambda'(2a^2 + b^2)) \quad (27) $$

The $\lambda'$ term in these equations is expected to be $\frac{1}{N_c}$ suppressed as compared with the remaining terms [25] (the l.h.s in Eq. (27) $= 2\beta(2a+b) + \lambda(a-b) = \mathcal{O}(\frac{1}{N_c})$), thus, disregarding this term LSM gives striking predictions for scalar meson masses and mixing angle $\phi_S$ which are valid modulo $\frac{1}{N_c}$ corrections. In the case $\lambda' = 0$, Eq.(27) reproduces Dmitrasinovic’s sum rule [25]. This sum rule was derived in the context of a Nambu-Jona-Lasinio model extended with a t’Hooft interaction which when bosonized reduces to a LSM like the one considered in this work, with definite predictions for the parameters in Eq.(1). In particular, this approach predicts $\lambda' = 0$ [25]. We wish to emphasize, however, that the $\lambda'$ term in Eq.(27) turns out to be crucial in the identification of scalar mesons. Without this term there seems to be no place for a light $\sigma$ as a member of the scalar meson nonet.

We now turn to the fixing of the parameters entering in the LSM. The model has 6 free parameters $(a, b, \mu, \lambda', \lambda, \beta)$ which can be fixed from phenomenology. As can be seen form Eqs (10,12,14), all the meson masses, but the mixed scalar meson ones, depend upon $(a, b, \lambda, \beta)$ and the combination
\[ \xi = \mu^2 + \lambda'(2a^2 + b^2). \] A phenomenological extraction of the value of \( \lambda' \) necessarily requires the use of information on the mixed scalar meson properties. However, this information is still controversial and the spirit of this work is to provide further understanding on this sector. Instead of fixing the value of \( \lambda' \) we analyze scalar meson masses and mixing angle as a function of this parameter.

By comparing Eqs. (5) with (9) and (10) we can see that the pion and kaon decay constants are related to the scalar vacuum expectation values in the following way:

\[
f_\pi = \sqrt{2a} \quad f_K = \frac{1}{\sqrt{2}}(a + b). \tag{28}
\]

As the \( \beta \) parameter is a direct measure of the \( \eta_{NS} - \pi \) splitting we will use \((m_{\eta}, m_{\eta'}, m_\pi)\) in order to fix the values of \((\beta, \lambda, \xi)\). From Eqs(10,14,17,23) we obtain that \( \beta \) is determined by the following equation:

\[
8(a^2 + 2b^2)\beta^2 - 4b(2m_\pi^2 - m_{\eta'}^2 - m_\eta^2)\beta + (m_{\eta'}^2 - m_\eta^2)(m_\eta^2 - m_\pi^2) = 0 \tag{29}
\]

and \( \lambda \) and \( \xi \) are obtained from:

\[
\lambda = \frac{2m_\pi^2 - m_{\eta'}^2 - m_\eta^2 - 6b\beta}{a^2 - b^2}
\]

\[
\xi = m_\pi^2 - 2b\beta - \lambda a^2
\]

The solutions to these equations depend upon the scalar fields vacuum expectation values. We choose to fix \( a \) only from Eq. (28) and analyze everything as a function of \( x = \frac{b-a}{2a} \) which is a measure of the \( SU(3) \) breaking. The solutions to Eq.(29) are stable under changes in \( x \). This stability of the \( \beta \) parameter is in contrast with the remaining parameters which are sensitive to the chosen value for \( x \). From the two solutions \((\beta \approx -750\,MeV, -1550\,MeV)\) to equation (29), the first one is ruled out by phenomenology. This can straightforwardly seen from the \( K \) mass which is plotted in Fig. 1 as a function of \( x \) for both solutions. The isospin averaged \( K \) mass corresponds to \( x \approx 0.39 \). However, as shown in Fig.1 the \( K \) mass is not very sensitive to \( x \) and allows for a wide range of values (roughly \( x \in [0.3, 0.5] \)) for this parameter.

Concerning the sign of \( \beta \), it differ from the sign of the corresponding quantity obtained in Ref. [25]. The sign of \( \beta \) can be reversed by redefining
chiral transformations with a global sign. As a consequence, relations in Eq.(28) and solutions to Eq. (29) gets modified by a sign, i.e. this change is equivalent to the transformations \( \beta \to -\beta \) and \( a, b \to -a, -b \). Meson masses and mixing angles are invariant under these transformations as can be easily seen from Eqs.(10,12,14). All three-meson couplings are changed by a global sign and four-meson couplings remain invariant. This relative sign between three and four-meson couplings should have no physical consequences. This is so at least for meson decays and meson-meson scattering. In other words, the \( U_A(1) \) breaking term should not distinguish between otherwise equivalent degenerate vacuum states.

Meson masses for the non-mixed scalar sector \((a, \kappa)\) do not depend on the particular value for \( \lambda' \) depending only on the \( \xi \) combination of \( \lambda' \) and \( \mu \). The mixed scalar meson \((f, f')\) masses do depend on the particular value for \( \lambda' \).

In Fig.2 the dependences of \( m_a, m_\kappa \) are shown as a function of \( x \). The value \( x = 0.39 \) corresponds to \( m_\kappa \approx 900 \text{MeV}, m_a \approx 910 \text{MeV} \). A value of 980 MeV for the \( a_0 \) meson requires \( x \approx 0.27 \) which is reasonably close to the set of values allowed by the fit to the \( K \) mass.

Results for the mixed scalar meson masses are shown in Fig. 3 as a function of \( \lambda' \). Clearly the \( f' \) scalar meson (the scalar analogous to \( \eta' \) ) can be identified with the \( f_0(980) \) meson and the \( f \) meson (scalar analogous to \( \eta \) ) has a mass around 400 MeV. Hereafter we call these mesons \( f_0 \) and \( \sigma \) respectively. A 980MeV mass for the \( f_0 \) meson corresponds to \( \lambda' \approx 4 \). This value gives \( m_\sigma \approx 375 \text{MeV} \).

Small values for \( \lambda' \) as required by the identification of the \( f_0(980) \) as the scalar singlet suggested by Fig. 3 is roughly consistent with NJL model expectations [25] and Tornqvist conclusions (recall \( \lambda' = 4\lambda'_T \))[9]. However, definitively LSM predicts a non-zero value for this parameter. On the other hand, in Tornqvist picture the \( \kappa \) meson is identified with the \( K^*(1430) \) meson. A \( \kappa \) meson mass of 1430 MeV would require \( x \approx 0.14 \) which is unlikely in the light of the the results for the \( K \) mass shown in Fig. 1. Furthermore, a calculation of the \( a_0 \to \gamma\gamma \) decay within a SU(3) Linear Chiral Model gives support to the identification of both the \( a_0(980) \) and the \( \kappa(900) \) as members of the scalar \( \bar{q}q \) nonet [26]. A complete reanalysis of scalar meson interactions within LSM is under investigation.

As to the mixing angles, the pseudoscalar mixing angle (which is independent of \( \lambda' \) ), can be extracted from Eq.(20). We obtain \( 33.2^\circ \leq \phi_P \leq 38.5^\circ \) for
0.27 \leq x \leq 0.39. This range of values correspond to $-16.2^\circ \leq \theta_P \leq -21.5^\circ$ for the pseudoscalar mixing angle in the usual singlet-octet basis. The central value ($x = 0.335$) $\phi_P = 35.5^\circ$ is in agreement with the experimental result as extracted from $\eta, \eta'$ radiative decays ($\theta_P \simeq -19^\circ$)[8,27]. This result for the pseudoscalar mixing angle can be considered as a consistency check for the model.

The scalar mixing angle can be extracted from Eq.(21). This angle depends on $\lambda'$ and for say, $\lambda' = 4$ and $x = 0.39$ we obtain $\phi_S \approx -14^\circ$. This value is very stable under changes in $x$. It roughly lies in the range $-5^\circ \leq \phi_S \leq -19^\circ$ for $10 \geq \lambda' \geq 0$.

Concerning the kaon decay constant in Eq.(28), the predicted value is $f_K = (1 + x)f_\pi$ with $x \in [0.27, 0.39]$. This value is not too far from its experimental result [8] $f_K^{exp} = 1.22 f_\pi^{exp}$. Furthermore, one-loop corrections[15] to $x$ have been shown to be $\simeq 10\%$ ($x$ corresponds to $\simeq 2$ times the expansion parameter $\frac{\langle \sigma_8 \rangle}{\sqrt{2} \sigma_0}$ used by Chan and Haymaker in reference [15]) and in the right direction to bring down the predicted value for $f_K$ even closer to experimental results. A complete reanalysis of the results of the model at one one-loop level would be desirable to confirm the gross results emerging at tree level.

5. CHPT low energy constants and LSM

As well known, $\mathcal{O}(p^4)$ low energy constants can be fixed by invoking saturation by meson resonance exchange. In particular, the lowest order lagrangian involving scalar meson resonances is [11]:

$$L_2[S(0^{++})] = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi \rangle + \tilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \tilde{c}_m S_1 \langle \chi \rangle$$

(31)

where

$$u_\mu = i u^\dagger D_\mu U u^\dagger = u^\dagger_\mu, \quad U = u^2 = exp(-i \sqrt{2} \Phi / F), \quad \Phi = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i P_i$$

(32)

with

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi u, \quad \chi = 2B_0 \mathcal{M}.$$  

(33)

Clearly, Eq.(31) do not consider scalar meson mixing which arise at higher order in the chiral expansion. As discussed in the first of Refs. [11], scalar meson resonance exchange gives the following contributions to the $\mathcal{O}(p^4)$ low energy constants:
Octet contributions:

\[
L_1^S = -\frac{c_d^2}{6M_S^2}, \quad L_3^S = \frac{c_d^2}{2M_S^2}, \quad L_4^S = -\frac{cdcm}{3M_S^2}, \quad L_5^S = \frac{cdcm}{M_S^2},
(34)
\]

\[
L_6^S = -\frac{c_m^2}{6M_S^2}, \quad L_8^S = \frac{c_m^2}{2M_S^2}, \quad H_2^S = \frac{c_m^2}{M_S^2}.
(35)
\]

Singlet contributions:

\[
L_1^{S_1} = \frac{\tilde{c}_d^2}{2M_{S_1}^2}, \quad L_4^{S_1} = \frac{\tilde{c}_d c_m}{M_{S_1}^2}, \quad L_6^{S_1} = \frac{c_m^2}{2M_{S_1}^2}
(36)
\]

Expanding Eq. (31) we obtain

\[
\mathcal{L}_2[S(0^{++})] = \frac{2cd}{F^2} \langle S \partial_\mu \Phi \partial^\mu \Phi \rangle + \frac{2\tilde{c}_d}{F^2} S \langle \partial_\mu \Phi \partial^\mu \Phi \rangle +
\]

\[
4Bo c_m [\langle S\mathcal{M} \rangle - \frac{1}{4F^2} \langle S(\Phi^2 \mathcal{M} + \mathcal{M} \Phi^2 + 2\Phi \mathcal{M} \Phi) \rangle] +
\]

\[
4Bo \tilde{c}_m S_1 [\langle \mathcal{M} \rangle - \frac{1}{4F^2} \langle \Phi^2 \mathcal{M} + \mathcal{M} \Phi^2 + 2\Phi \mathcal{M} \Phi \rangle] + \mathcal{O}(\Phi^4)
\]

where \( F \) denotes the pseudoscalar decay constant in the chiral limit.

We can now make contact with LSM results in the SU(3) symmetric limit \( a = b \). In this limit a comparison of LSM with Eq.(37) gives:

\[
c_d = -2c_m = -\frac{F}{\sqrt{2}}, \quad \tilde{c}_d = -2\tilde{c}_m = -\frac{F}{\sqrt{6}}. \quad (38)
\]

As expected [11] the couplings \( c_d, \ c_m, \ \tilde{c}_d, \ \tilde{c}_m \) are \( \mathcal{O}(N_c^4) \) and satisfy the large \( N_c \) relations

\[
\tilde{c}_d = \frac{\varepsilon}{\sqrt{3}} c_d, \quad \tilde{c}_m = \frac{\varepsilon}{\sqrt{3}} c_m, \quad \varepsilon = 1.
(39)
\]

These results gives the following contributions of scalar resonances exchange to the \( \mathcal{O}(p^4) \) low energy constants

\[
L_1^{SC} = \frac{F^2}{12} \left( \frac{1}{M_{S_1}^2} - \frac{1}{M_S^2} \right), \quad L_3^{SC} = \frac{F^2}{4M_S^2},
(40)
\]

18
\[
L_{4}^{SC} = -\frac{F^2}{12} \left( \frac{1}{M_{S1}^2} - \frac{1}{M_{S}^2} \right), \quad L_{5}^{SC} = -\frac{F^2}{4M_{S}^2}, \\
L_{6}^{SC} = \frac{F^2}{48} \left( \frac{1}{M_{S1}^2} - \frac{1}{M_{S}^2} \right), \quad L_{8}^{SC} = \frac{F^2}{16M_{S}^2}, \quad H_{2}^{SC} = \frac{F^2}{8M_{S}^2}.
\]

There is a worth remarking point about Eq.(40). From Eqs. (14), in the \(SU(3)\) symmetric limit we obtain:

\[
\begin{align*}
  m_{0P}^2 - m_{8P}^2 &= -6\beta a \\
  m_{0S}^2 - m_{8S}^2 &= 6(\beta + \lambda' a)a.
\end{align*}
\]

From the above relations is clear that the \(U_A(1)\) breaking has an inverted effect in the mixed scalar sector as compared with its effect in the pseudoscalar sector. In the pseudoscalar sector the anomaly pushes the singlet up and the octet down while in the scalar sector it does push the singlet down and the octet up. This effect changes the signs of \(L_1, L_4, L_6\) from the naively expected ones as \(m_{0S}^2\) turns out to be smaller than \(m_{8S}^2\) due to the \(U_A(1)\) breaking effect in the scalar sector.

Scalar mesons exchange contributions to the LEC quoted in Eq. (40) have the right large \(N_c\) properties. In fact, from Eqs. (40,41) is clear that the contribution of scalar mesons exchange to \(L_1^{SC}\) is formally \(O(1)\) as it must be since, although \(L_1\) is \(O(N)\), \(2L_1 - L_2\) is \(O(1)\) and there are no scalar contributions to \(L_2\). Likewise, the remaining LEC in Eq. (40) can be easily seen to have the right large \(N_c\) behavior.

The scalar couplings in Eq.(31) can be estimated from (38,39). Using \(F = f_\pi = 93\) MeV we obtain:

\[
\begin{align*}
  c_d &= -6.56 \times 10^{-2} \text{ GeV}, \quad c_m = 3.28 \times 10^{-2} \text{ GeV} \\
  \tilde{c}_d &= -3.79 \times 10^{-2} \text{ GeV}, \quad \tilde{c}_m = 1.89 \times 10^{-2} \text{ GeV}
\end{align*}
\]

Notice that LSM predicts \(c_d c_m < 0\), thus the expected saturation of \(L_5\) by scalar meson exchange [11] is not satisfied. This can be explicitly seen by numerically evaluating the scalar contributions to CHPT LEC in Eq.(40). A first estimate can be obtained by using the large \(N_c\) limit where \(M_S = M_{S1}\).
In this limit, the only non-zero LEC are

\[ L_{SC}^3 = 2.24 \times 10^{-3}, \quad L_{SC}^5 = -2.24 \times 10^{-3}, \quad L_{SC}^8 = 0.56 \times 10^{-3}. \] (43)

We have used \( M_S = M_{a_0} = 980\text{ MeV} \) in obtaining these values. The value of \( L_{SC}^8 \) above is consistent with the saturation by scalar mesons exchange invoked in Ref.[11]. A calculation of \( L_{SC}^8 \) using the LSM estimate for the averaged octet mass \( (M_{LSM}^S \approx 912\text{ MeV}) \) gives \( L_{SC}^8 = 0.64 \times 10^{-3} \) to be compared with the experimental value \( L_{exp}^8(m_\rho) = 0.9 \pm 0.3 \times 10^{-3} \) [11]. These results are consistent with the expected saturation of \( L_8 \) by scalar mesons exchange. However, \( L_{SC}^5 \) has the wrong sign and LSM predicts that this LEC is not saturated by scalar mesons exchange. As to \( L_{SC}^3 \) it has the same sign as that found in [11] but LSM predicts a stronger effect than the one quoted there, thus reducing the total value of the resonance exchange contribution to this LEC quoted in Table 3 of Ref. [11].

6. Conclusions.

We work out \( U(3) \times U(3) \) Linear Sigma Model predictions for meson masses and mixing angles at tree level. We clearly exhibit the effects of the \( U_A(1) \)-breaking term in the generation of meson masses and in the mixing of the scalar and pseudoscalar mesons. The parameters entering in the model are fitted to the pseudoscalar spectrum. The model predicts that the members of the scalar meson nonet are: \{\( a_0(980), \kappa(900), \sigma(\approx 400), f_0(980) \}\} resonances. These mesons are the analogous to \{\( \pi, K, \eta, \eta' \)\} pseudoscalar mesons. The scalar mixing angle in the {\( |ns>, |s> \)} basis is found to be \( \phi_S \approx -14^\circ \), thus the \( f_0(980) \) meson is mostly strange and the \( \sigma(\approx 400) \) is predominantly non-strange. The pseudoscalar mixing angle in this basis is \( \phi_P \approx 35^\circ \) which corresponds to \( \theta_P \approx -19^\circ \) in the singlet-octet basis. The fit of the parameters of the model gives \( \lambda' \approx 4 \). This result is along the line of Tornqvist’s conclusion in the sense that the \( \lambda_T' \) parameter (\( \lambda_T' \) in this work) is small [9]. However, we obtain a \( \kappa \) meson mass \( \approx 900\text{ MeV} \) in accordance to Ishida [20] reanalysis of \( K\bar{K} - \pi\pi \) scattering data, thus being unlikely to identify this particle with \( K^*(1430) \) meson in this model. We also study the contributions of scalar mesons exchange to the \( \mathcal{O}(p^4) \) low energy constants of CHPT. The model predicts saturation of \( L_8 \) by scalar resonances exchange as expected [11]. However, \( L_5 \) is not saturated by these resonances.

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References.

1.- S. Weinberg Physica A96 327 (1979); J. Gasser and L. Leutwyler Nucl. Phys. B250, 465 (1985); J. Gasser, M.E. Sainio and V. Svarc Nucl. Phys. B307, 779 (1988). For a review see e.g. “Dynamics of the Standard Model” J.F. Donoghue, E. Golowich and B.R. Holstein Cambridge Univ. Press (1992).

2.- E. Jenkins and A. V. Manohar Phys. Lett. B255, 558 (1991); B259, 363 (1991); E. Jenkins, A.V. Manohar and M. Wise Phys. Rev. Lett. 75, 2272 (1995).

3.- T. R. Hemmert, B. R. Holstein and J. Kambor hep-ph/9712496 (unpublished)

4.- H. Leutwyler Phys. Lett B374; P. Herrera-Siklody et. al hep-ph/9610549

5.- G. t’Hooft, Nucl. Phys B72, 461 (1974); E. Witten, Nucl. Phys B160, 57 (1979); R. Dashen, E. Jenkins and A. Manohar Phys Rev D49, 4713 (1994); D51, 3697 (1995).

6.- see e.g. H. Marksiske et.al. The Crystal Ball Coll. Phys. Rev. D41, 3324 (1990); T. Oest et. al. JADE Coll. Z. Phys. C47, 343 (1990) and references therein.

7.- T. Barnes Phys. Lett. B165, 434 (1985);

8.- Particle Data Group, R. M. Barnett et. al. Phys. Rev. D54, 1 (1996).

9.- N.A. Tornqvist hep-ph/9711483, hep-ph/9712479, unpublished; N.A. Tornqvist and M. Ross, Phys. Rev. Lett. 76 (1996) 1575.

10.- F. Sannino and J. Schechter, Phys. Rev. D52:96 (1995); M. Harada F. Sannino and J. Schechter, Phys. Rev. D54, 1991 (1996); Phys. Rev. Lett. 78, 1603 (1997).

11.- G. Ecker et.al. Nucl. Phys. B321, 311, (1989); J. F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. D39, 1947 (1989).
12.- M. Napsuciale and J. L. Lucio Phys. Lett. B384, 227 (1996); Nucl. Phys. B494, 260 (1997).

13.- V. Pascalutsa [hep-ph/9802282] (unpublished).

14.- J. Binjnens and F. Cornet, Nucl. Phys. B296, 557 (1988); J. F. Donoghue, B.R.Holstein and Y.C.Lin, Phys. Rev. D37, 2423 (1988); S. Bellucci, J. Gasser and M.E. Sainio, Nucl. Phys. B423, 80 (1994).

15.- J. Schechter and Y. Ueda Phys. Rev. D3 2874 (1971); L.H. Chan and R.W. Haymaker Phys. Rev. D7 402 (1973); Phys. Rev. D7 415 (1973).

16.- V. De Alfaro et.al. “Currents in hadron Physics”, North Holland Publ.(1973) Amsterdam, Chap. 5.

17.- C. J. Morningstar and M. Peardon Phys. Rev. D56, 4043 (1997); M. Teper [hep-ph/9712504], unpublished; V. Anisovich, [hep-ph/9712504], unpublished.

18.- L.R. Baboukhadia, V. Elias and M.D. Scadron [hep-ph/9708431], unpublished.

19.- A. Bramon, R. Riazuddin and M.D. Scadron [hep-ph/9709274], unpublished.

20.- S. Ishida et.al. [hep-ph/9712230]; M.Y. Ishida and S.Ishida [hep-ph/9712231] to be published in Proc. of Int. Conf. Hadron’97.

21.- M.Levy Nuov. Cim. LIIA 7247 (1967).

22.- S. Gasiorowicz and D.A. Geffen Rev. Mod. Phys. 41 531 (1969).

23.- G. ’tHooft, Phys. Rep. 142, 357, (1986).

24.- M. D. Scadron Phys. Rev. D26 239 (1982).

25.- V. Dmitrasinovic. Phys. Rev C53, 1383, (1996).

26.- M. Napsuciale, J.L.Lucio and M.D. Scadron in preparation.

27.- E.P.Venugopal and B.R.Holstein [hep-ph/9710382], unpublished.
Figure caption

Fig.1.- $K$ meson mass (in units of $MeV$) as a function of $x = \frac{b-a}{2a}$ for the two solutions to Eq.(29) $\beta \approx -700MeV$ (continuous line) and $\beta \approx -1550MeV$ (dashed line).

Fig.2.- $a_0$ (dashed line) and $\kappa$ (continuous line) meson masses (in units of $MeV$) as a function of $x = \frac{b-a}{2a}$.

Fig.3.- $\sigma$ (continuous line) and $\sigma'$ (dashed line) masses (in units of $MeV$) as a function of $\lambda'$ for $x = 0.39$. 
