Interaction between Two Inclined Cracks in Bonded Dissimilar Materials subjected to Various Stresses

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Abstract. This paper deals with the interaction between two inclined cracks in the upper part of bonded dissimilar materials subjected to various stresses which is normal stress (Mode I), shear stress (Mode II), tearing stress (Mode III) and mixed stress. This problem is formulated into hypersingular integral equations (HSIE) by using modified complex potentials (MCP) with the help of continuity conditions of the resultant force and displacement functions where the unknown is the crack opening displacement (COD) function and the tractions along the crack as the right hand terms. Then, the curve length coordinate method and appropriate quadrature formulas are used to solve numerically the obtained HSIE to compute the stress intensity factors (SIF) in order to determine the stability behavior of materials containing cracks. Numerical results showed the behavior of the nondimensional SIF at the cracks tips. It is observed that the various stresses and the elastic constants ratio are influences to the value of nondimensional SIF at the crack tips.

1. Introduction

In engineering structures, it is very important to study on stability and safety of the materials and one of the subarea is crack problems. A number of papers have been publish to investigate the behavior of SIF at the crack tips for crack problems in an infinite plane [1,2], half plane [3,4] and bonded dissimilar materials [5-7]. The nondimensional SIF for the crack problems in bonded dissimilar materials subjected to various stress was calculated by using the body force method with continuous distributions along cracks [5]. A single crack problem in the upper part of bonded dissimilar materials subjected to various stress was solved using HSIE [6]. The HSIE were used to solve the nondimensional SIF for multiple cracks problems in the upper part of bonded dissimilar materials subjected to shear stress [7].

This paper is the extended study from [6] with focus to the single crack only and [7] multiple crack subjected to shear stress only. We investigates the behavior of nondimensional SIF for interaction between two inclined cracks in the upper part of bonded dissimilar materials subjected to the various stresses such as normal stress, shear stress, tearing stress and mixed stress.
2. Problem Formulation

The stress components \( \sigma_x, \sigma_y, \sigma_{xy} \), the resultant force function \( f(X,Y) \), and the displacements \( u,v \) are the fundamental equations of complex variable function and related to the complex potential functions \( \phi(z) \) and \( \psi(z) \) as follows

\[
\sigma_y - \sigma_x + 2i\sigma_{xy} = 2\left[z\phi'(z) + \psi'(z)\right]
\]

\[
f = -Y + iX = \phi(z) + z\phi'(z) + \psi(z)
\]

\[
2G(u + iv) = \kappa\phi'(z) - z\phi''(z) - \psi'(z)
\]

where \( G \) is shear modulus of elasticity, \( \kappa = 3 - 4v \) for plane strain, \( \kappa = (3 - v)/(1 + v) \) for plane stress and \( v \) is Poisson’s ratio [8]. The derivative of resultant force function (2), yields

\[
\frac{d}{dz}(-Y + iX) = \phi'(z) + \phi'(z) + \frac{dz}{dz} \left[z\phi''(z) + \psi'(z)\right] = N + iT
\]

where the normal \( (N) \) and tangential \( (T) \) traction along the crack segment \( z, z + dz \).

By using the modified complex potentials and apply continuity conditions for the resultant force (2) and displacement functions (3), and substituting into (4), then we can obtained the HSIE for two cracks in the upper part of bonded dissimilar materials as follows

\[
\left\{ N(t_j) + iT(t_j) \right\}_j = \frac{1}{\pi} \int_{t_j} g_j(t_j) dt_j + \frac{1}{2\pi} \int_{t_j} D_j(t_j, t_j) g_j(t_j) dt_j
\]

\[
+ \frac{1}{2\pi} \int_{t_k} D_k(t_k, t_k) g_k(t_k) dt_k + \frac{1}{2\pi} \int_{t_k} D_k(t_k, t_k) g_k(t_k) dt_k
\]

where \( j, k = 1, 2 (j \neq k) \), and \( g_j(t_j) \), \( D_j(t_j, t_j) \) and \( D_k(t_k, t_k) \) are defined in [7].

The curve length coordinate method is used to solve the HSIE (5) in order to transform the COD function \( g(t) \) to the square root singularity as follows

\[
g(t) = H(x) \sqrt{a^2 - t^2}
\]

Then we apply the appropriate quadrature formulas [9].

For normal stress, the condition and the normal and tangential component with an angle of the crack \( \alpha \) is defined as follows

\[
\frac{1}{E_1} \sigma_{y_1} = \frac{1}{E_2} \sigma_{y_2}, \quad N + iT = -p \cos^2 \alpha + i p \sin \alpha \cos \alpha
\]

for shear stress is defined as follows

\[
\frac{1}{E_1} \sigma_{y_1} = \frac{1}{E_2} \sigma_{y_2}, \quad N + iT = -p \sin^2 \alpha - i p \sin \alpha \cos \alpha
\]

for tearing stress is defined as follows

\[
\frac{1 + 2v_1}{E_1} \sigma_{y_{11}} = \frac{1 + 2v_2}{E_2} \sigma_{y_{22}}, \quad N + iT = 2p \sin \alpha \cos \alpha + i p \left( \cos^2 \alpha - \sin^2 \alpha \right)
\]
whereas, for mixed stress is defined as follows
\[
\frac{1-v_1}{E_1} \sigma_{n1} = \frac{1-v_2}{E_2} \sigma_{n2}, \quad N + iT = -p + io \tag{10}
\]
where \( E_1 = 2G_1(1+v_1) \) and \( E_2 = 2G_2(1+v_2) \) are Young’s modulus of elasticity for upper and lower parts of bonded dissimilar materials, respectively.

3. Results and Discussion

SIF at the crack tips is defined as
\[
K_{A_j} = (K_1 - iK_2)_{A_j} = \sqrt{2\pi} \lim_{t\rightarrow 0^+} \left[ t^{-1/2} \right] g'(t) = \sqrt{a\tau F_{A_j}} \tag{11}
\]
where \( j = 1,2 \) and \( F_{A_j} = F_{1A_j} + iF_{2A_j} \) is the nondimensional SIF at cracks tips \( A_j \).

Consider two parallel inclined cracks in the upper part of bonded dissimilar materials subjected to various stresses as defined in Figure 1(b). Figure 1 and figure 2 show the nondimensional SIF for \( R/h = 0.9 \) and \( \alpha \) varies.

Figure 1 shows the nondimensional SIF versus \( \alpha \) subjected to normal stress (Mode I) with red line and shear stress (Mode II) with black line at all cracks tips. It is observed that, for normal stress as \( \alpha \) and \( G_2/G_1 \) increase the nondimensional SIF \( F_1 \) decreases at all cracks tips. Whereas the nondimensional SIF \( F_2 \) increases for \( \alpha < 45^\circ \) and decreases as \( G_2/G_1 \) increases at all cracks tips. For shear stress, as \( \alpha \) increases \( F_1 \) increases and as \( G_2/G_1 \) increases \( F_1 \) decreases at all cracks tips. Whereas \( F_2 \) decreases for \( \alpha < 45^\circ \) and increases as \( G_2/G_1 \) at all cracks tips.

Figure 2 shows the nondimensional SIF versus \( \alpha \) subjected to tearing stress (Mode III) with black line and mixed stress with red line at all cracks tips. It is found that, for tearing stress as \( \alpha \) increases \( F_2 \) decreases and \( F_1 \) decreases at all cracks tips. Whereas as \( G_2/G_1 \) increases \( F_1 \) increases at all cracks tips but \( F_2 \) increases at crack tip \( A_1 \) for \( \alpha > 50^\circ \), at crack tip \( A_2 \) for \( \alpha > 20^\circ \), at crack tip \( B_1 \) for \( \alpha > 60^\circ \) and at crack tip \( B_2 \) for \( \alpha > 30^\circ \). For mixed stress, as \( \alpha \) increases \( F_1 \) increases and as \( G_2/G_1 \) increases \( F_2 \) decreases at all cracks tips. Whereas \( F_2 \) does not show any significant differences as \( \alpha \) increases at all cracks tips. As \( G_2/G_1 \) increases \( F_1 \) decreases at crack tips \( A_1 \) and \( B_2 \), but increases at tip \( A_2 \) and does not show any significant differences at tip \( B_1 \).
Figure 1. Nondimensional SIF versus $\alpha$ subject to normal stress (red line) and shear stress (black line).
4. Conclusions

This paper deals with the interaction between two inclined cracks in the upper part of bonded dissimilar materials subjected to various stresses which is normal stress (Mode I), shear stress (Mode II), tearing stress (Mode III) and mixed stress. From the numerical results we conclude that the behavior of nondimensional SIF depends on the various stresses and the elastic constants ratio. This observation indicate that the strength of materials decreases as increases for shear and mixed stresses. However the strength of materials increases as increases for all various stresses.

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