Localization under the effect of randomly distributed decoherence

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Abstract. Electron transport through disordered quasi one-dimensional quantum systems is studied. Decoherence is taken into account by a spatial distribution of virtual reservoirs, which represent local interactions of the conduction electrons with their environment. We show that the decoherence distribution has observable effects on the transport. If the decoherence reservoirs are distributed randomly without spatial correlations, a minimal degree of decoherence is necessary to obtain Ohmic conduction. Below this threshold the system is localized and thus, a decoherence driven metal-insulator transition is found. In contrast, for homogenously distributed decoherence, any finite degree of decoherence is sufficient to destroy localization. Thus, the presence or absence of localization in a disordered one-dimensional system may give important insight about how the electron phase is randomized.

1 Introduction

The electron transport through nanosystems takes place in an intermediate regime between classical and quantum transport [1–4]. Thus, a quantum description of the system must take the effect of decoherence into account. This raises then the fundamental, but up to now only partially answered question, whether decoherence enhances or reduces transport. In this respect tight-binding chains [5–10], molecular wires [11–13] and aggregates [14–17] were studied. Surprisingly it was found that the transport can be enhanced by decoherence, a phenomenon referred to as decoherence-assisted transport. Recently, it has also been discussed, if Anderson localization can be observed in the presence of many-body interactions [18–21]. As many-body interactions are a source of decoherence, this corresponds to the question, if localization is possible under the effects of decoherence.

In this paper, we address this question using a statistical model for the effects of decoherence [7–9, 11]. We use virtual decoherence reservoirs [22, 23], where the electrons are absorbed and reinjected after randomization of phase and momentum. These decoherence reservoirs represent local interactions of the conduction electrons with their environment. Their spatial distribution is governed by the underlying microscopic decoherence processes. For example, when the decoherence is caused by random, uncorrelated scattering, also the decoherence reservoirs are distributed in this way. Afterwards, the transport property of interest (e.g. resistance or conductance) is ensemble averaged over the spatial distributions of decoherence reservoirs. In contrast, when a continuous loss of the electron phase is considered, a homogenous distribution of decoherence reservoirs is used. Continuous phase randomization was considered by Pastawski et al. [12, 13, 24]. It is also used in other virtual reservoir approaches [6, 25–30] and other phenomenological models [10, 14–17, 21, 31]. However, it is not clear if continuous decoherence is justified in every system.

\[ p = 1/l_{\phi} \]

Fig. 1. (Color online) Resistivity $\rho$ of an infinitely long disordered tight-binding chain as a function of the degree of decoherence $p$ (inverse phase coherence length $1/l_{\phi}$). If the decoherence is distributed homogeneously (dashed curves), $\rho$ is Ohmic for any $p > 0$. If the decoherence distribution is random and uncorrelated (solid curves), a minimal degree of decoherence $p^*$ is necessary to obtain Ohmic conduction, whereas below this threshold the system is localized ($\rho \to \infty$). $p^*$ as a function of the disorder $\sigma$ is shown in the inset.
Studying disordered tight-binding chains and ribbons, here we explain why the distribution of the decoherence reservoirs has a significant effect on the transport. When the decoherence reservoirs are distributed randomly without spatial correlations, we find length-independent Ohmic resistivity only if the degree of decoherence exceeds a certain threshold, or in other words, if the phase coherence length is sufficiently short. Otherwise the system is localized, which is indicated by a divergence of the resistivity at a critical degree of decoherence, see the solid curves in figure 1. When the decoherence reservoirs are distributed homogenously, the transport is Ohmic for any finite degree of decoherence, see the dashed curves in figure 1. Our results may help to learn from resistance measurements how the decoherence is distributed and how the electron phase information is lost in the system.

2 Quantum transport in presence of phase randomizing reservoirs

In general, we consider electron transport through a quantum system, described by a single-particle Hamiltonian $H$. The system is in contact with two real source and drain reservoirs as well as several virtual decoherence reservoirs.

The current from the $j$th to the $i$th reservoir is calculated by means of the Landauer formula [1–3]

$$I_{ij} = \frac{e}{h} \int dE T_{ij} (f_j - f_i), \quad (1)$$

where $T_{ij}$ is the coherent transmission between the reservoirs and $f_j$ are the energy distribution functions of the reservoirs. The energy-distribution functions of the source and drain reservoir are assumed to be Fermi functions $f_{S/D}$. The energy-distribution functions of the virtual reservoirs are determined by the constraint that the total (energy-resolved) current at a virtual reservoir has to vanish. At an infinitesimal bias voltage and at zero temperature, the total resistance of such a system, measured in units of $h/e^2$, is given by (see e.g. [24])

$$R = \frac{1}{T_{DS} + \sum_{i,j} T_{ij} R_{ij} T_{jS}}, \quad (2)$$

where

$$R_{ij}^{-1} = \begin{cases} -T_{ij} & i \neq j, \\ \sum_{k \neq i} T_{ik} & i = j. \end{cases} \quad (3)$$

The sum in (2) is over the virtual reservoirs, whereas the sum in (3) is over all reservoirs including source and drain.

When virtual decoherence reservoirs are introduced within a one-dimensional quantum system and when complete phase randomization is assumed, the coherent transmission $T_{ij}$ is limited to nearest neighbors and thus, the system can be subdivided into smaller coherent subsystems. Simplifying (2) under these conditions, the resistance of the system is given by the sum of the subsystem resistances [7]

$$R = \sum_i \frac{1}{T_{i+1,i}}, \quad (4)$$

The above formulae for the resistance reflect Kirchhoff’s law for networks of resistances $1/T_{ij}$, which have to be calculated by quantum mechanics. Applying the nonequilibrium Green’s function approach [1–3], the transmission from the $j$th to the $i$th reservoir is given by

$$T_{ij} = 4 \text{Tr} \left( \text{Im} \left( \Gamma_i \right) \text{Im} \left( \Gamma_j \right) G^+ \right), \quad (5)$$

where the Green’s function is defined as

$$G = \left[ E - H - \sum_k \Gamma_k \right]^{-1}. \quad (6)$$

The influence of each of the reservoirs is taken into account by a self-energy $\Gamma_k$, which is in general a complex-valued function and thus, connects the isolated quantum system to the environment.

Here, the decoherence is considered as a statistical process due to (dynamical) scattering. Thus, after calculating the resistance by means of (2) or (4) for a given spatial distribution of decoherence reservoirs, the transport property of interest (e.g. resistance or conductance) is ensemble averaged over the decoherence distributions.

3 Results

3.1 Disordered tight-binding chains

We begin with tight-binding chains of length $N$, which are described by the Hamiltonian

$$H = \sum_{i=1}^N \varepsilon_i \langle \hat{i} | i \rangle + \sum_{i=1}^{N-1} t \langle \hat{i} | i+1 \rangle \langle i+1 | \hat{i} \rangle. \quad (7)$$

The coupling $t$ between neighboring sites is assumed to be homogenous and is used as the energy unit $t = 1$. The onsite energies $\varepsilon_i$ are distributed independently according to a probability distribution $w(\varepsilon)$ with mean 0 and variance $\sigma^2$. In order to keep the discussion clear and simple, we consider here only the band-center $E = 0$ and wide-band contacts $\Gamma_k = -i \langle k | \langle k \rangle$, where $k$ denotes the sites connected to the $k$th reservoir (i.e. an end of the chain). However, we stress that the main results of this paper are still valid outside the band-center and for arbitrary self-energies.

As proved in the Appendix A (including the general case of arbitrary energies $E$ and self-energies $\Gamma$), the disorder averaged resistance of the coherent chain of $N$ sites is given by the compact analytical formula

$$\left\langle \frac{1}{T_N} \right\rangle = \int \frac{1}{T_N} \prod_{i=1}^N w(\varepsilon_i) d\varepsilon_i = \frac{1}{2} \left( 1 + \alpha_+ e^{N/\xi} + \alpha_- e^{-N/\xi} (-1)^N \right), \quad (8)$$

where $\langle \cdot \rangle$ denotes disorder averaging and

$$2 \alpha_\pm = 1 \pm \text{sech}(\xi^{-1}), \quad (9)$$
The coherence length and is a function of the disorder strength $\sigma$. Any decoherence distribution, for which the number of coherent subsystems (8) exceeds their exponentially decreasing frequency of occurrence (12). Any decoherence distribution, where the transition between Ohmic and localized behavior appears. It can be related by (11) to a critical phase transition:

$$\rho_{\xi<\ell} \propto e^{(1/\xi-1)/\ell} N.$$  

The root of the exponent $\xi^{-1} - \ell^{-1} = 0$ determines the critical degree of decoherence

$$\rho_c = 1 - e^{-1/\ell},$$  

where the transition between Ohmic and localized behavior appears. It can be related by (11) to a critical phase coherence length and is a function of the disorder strength $\sigma$, see (10). In a completely different way, we obtained (17) already in [8]. Here, its derivation by the new analytical formula (8) and (12) allows to understand the statistical origin of the decoherence-induced insulator-metal transition:

Localization is found to survive decoherence, when the exponentially increasing resistance of the long coherent subsystems (8) exceeds their exponentially decreasing frequency of occurrence (12). Any decoherence distribution, for which the number $u_j$ of coherent subsystems decreases with their length $j$ faster than exponentially will show only Ohmic behavior. A simple example is to distribute the decoherence reservoirs randomly under the constraint that at least after $j_{\text{max}}$ normal bonds a decoherence reservoir has to be introduced. The corresponding $u_j$ has then a cut-off $u_j > j_{\text{max}} = 0$ and thus, decreases faster than exponentially. In this case the system is Ohmic for any finite degree of decoherence. Also for a homogenous decoherence distribution, where all subsystems have the same size $\ell_\phi$, the resistivity

$$\rho_{\text{hom}} = \frac{1}{\ell_\phi} \left\langle \frac{1}{T_{\ell_\phi}} \right\rangle$$  

is Ohmic for any finite degree of decoherence.

If however, $u_j$ decreases with $j$ asymptotically more slowly than exponentially, the system will always be localized, in spite of decoherence. This behavior appears for example, if the probability $p_j$ of having coherent subsystems of length $j$ (i.e. $j-1$ succeeding normal bonds) decreases as $p_j \propto j^{-\gamma}$ with an arbitrary constant $\gamma > 0$. [9] we have shown that in a modified model pure dephasing can also be obtained, i.e. complete phase randomization but conservation of the momentum at a virtual reservoir. However, this would change the resistivity only by an additive constant and does not affect the main results of this paper.
This strong influence of the decoherence distribution on the transport is the main result of this paper and is summarized in figure 1, where the resistivity of the infinite long chain is shown as a function of the degree of decoherence. The dashed curves for homogenous decoherence clearly show decoherence-assisted transport, and agree qualitatively well with other studies assuming homogenous decoherence [5, 6, 10, 12–17]. The solid curves for random uncorrelated decoherence exhibit divergencies at the critical degree of decoherence \( p^* \), which is shown as a function of the disorder strength \( \sigma \) in the inset of figure 1. Thus, in contrast to homogenous decoherence, where the transport is Ohmic for any \( p > 0 \), we find for random uncorrelated decoherence a metal-insulator transition at \( p^* \).

Taking many-body interactions explicitly into account, such a transition is found at a critical temperature \[18, 19\], which is proportional to the degree of decoherence in the system. Experiments on various nanosystems have been performed, see e.g. [32–36], where a transition from Ohmic to exponential behavior is observed. It is found [32] that this transition occurs when the phase coherence length approaches the localization length\(^2\), which agrees with our condition (17). However, to our knowledge these experiments have been done only for systems with a fixed length. In order to learn from an experiment how the decoherence is distributed, we propose to study the resistivity of linear nanosystems as a function of their length. When the decoherence is randomly distributed, we expect that below a critical temperature the resistivity increases exponentially with the chain length. Above the critical temperature the resistivity should be constant (Ohm’s law). In contrast, for homogenous decoherence we expect for any non-zero temperature Ohmic behavior, where the length of the system is increased.

### 3.2 Tight-binding ribbons and other model variations

In this Section we show that our results are not model specific, but appear more generally. We show that the transition also appears, (I) when the phase is randomized only partially at a virtual reservoir, (II) when tight-binding ribbons instead of chains are studied and, (III) when the conductivity is ensemble averaged instead of the resistivity.

When the assumption of complete phase randomization is abandoned by attaching the virtual reservoirs only to the chain, see the inset of figure 3, the coherent transmission between all reservoirs has to be taken into account and the ensemble average can be calculated only numerically by means of (2) and (3). However, as shown in figure 3, the decoherence induced transition appears for attached reservoirs (squares) as well as for completely phase randomizing reservoirs (circles). Contributions from

\( ^2 \) Note that our parameter \( \xi^{-1} \) is not the inverse localization length, which is generally defined as \( \lim_{N \to \infty} \frac{1}{N} \log \langle T_N \rangle \), but the second-order generalized Lyapunov exponent, defined as \( \lim_{N \to \infty} \frac{1}{N} \log \langle T_N \rangle \). See [8]. Anyway, both quantities are a measure for the localization in the system.

the next-nearest neighbors can be observed only for higher degrees of decoherence, as the coherent transmission between two reservoirs is exponentially suppressed with their distance. The numerical averages agree well with the analytical result (14) for completely phase randomizing reservoirs, see the solid curves. Combining this result with (18), we can conclude that attaching a homogenous distribution of decoherence reservoirs to the chain leads to Ohmic transport for any finite degree of decoherence.

The decoherence induced transition from localized to Ohmic behavior is not restricted to one-dimensional chains but is found also in two-dimensional tight-binding ribbons, see figure 4. In these ribbons decoherence is introduced by replacing randomly selected bonds with virtual reservoirs, see the marked bonds in the inset. However, a subdivi-
The averaging process, see the curve for \( p = 0.60 \) to the dashed curve, which gives the resistivity due to (15). However, the influence of the averaging process in the transitional regime is not clear, see the curve for \( p = 0.55 \), which should converge to the corresponding dashed horizontal line, and the curve for \( p = 0.50 \), which should diverge.

Finally we study, if the transition occurs also, when the decoherence average is not performed over the resistance \( \langle R \rangle \) but over the conductance \( \langle 1/R \rangle \), which corresponds to an experiment, where the bias voltage is fixed and the current through the system is measured. In this case, analytical calculations are demanded but the numerical average over random uncorrelated decoherence configurations clearly confirms that a minimal degree of decoherence is necessary for Ohmic transport, see figure 5. Below this threshold a power-law divergence is found in contrast to the exponential increase (16) in the case of averaging the resistance. Figure 5 also shows that deep in the Ohmic regime, the resistivity is independent of the averaging approach, Stone et al. [37] arrived at the same result. Using a recursive scattering approach, Stone et al. [37] arrived at the same result.

In this paper, we have shown that the spatial distribution of decoherence, caused by local interactions of the conduction electrons with their environment, has a significant influence on the transport through a disordered quantum system. When the decoherence is homogeneously distributed, Ohmic conduction is found for any finite degree of decoherence. In contrast, for random uncorrelated decoherence, a minimal degree of decoherence is necessary, whereas below the system is localized. This transition is caused by the interplay of the exponentially increasing coherent resistance (8) and its exponentially decreasing importance (12). The other characteristics of our model (one-dimensional chains, complete phase randomization, decoherence average of the resistance) are not important for this transition, but help to gain much insight into the transport problem by analytical calculability. To summarize in other words, we have shown that an Anderson insulator can be stable against decoherence effects, if these are randomly distributed. This result could help to gain information from resistance measurements on the distribution of the decoherence in a nanosystem.

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A Disorder averaged resistance of coherent tight-binding chains

In this Appendix we calculate analytically the disorder averaged resistance of coherent tight-binding chains by means of generating functions. Using a recursive scattering approach, Stone et al. [37] arrived at the same result.

We consider a chain of length \( N \) described by the Hamiltonian (7), which is connected to two reservoirs by the self-energy

\[
\Gamma = \nu + i \eta
\]  

acting on the first and last site of the chain. Because of the tridiagonal structure of \( G^{-1} \) the resistance, defined as the inverse of the transmission (5), can be calculated recursively in the same way as in [8, 9]

\[
\frac{1}{T_N} = \left| \frac{r_N - \Gamma r_{N-1} - \Gamma s_N + \Gamma^2 s_{N-1}}{4 \eta^2} \right|^2
\]
with the polynomials
\[ r_i = (E - \varepsilon_i) r_{i-1} - r_{i-2}, \quad s_i = (E - \varepsilon_i) s_{i-1} - s_{i-2}, \]
\[ r_0 = 1, \quad s_1 = 1, \quad s_0 = 0. \]  
(21)

Using these recursion relations, the disorder averaged resistance can also be calculated recursively

\[
\left\langle \frac{1}{T_N} \right\rangle = \int \frac{1}{T_N} \prod_{i=1}^{N} w(\varepsilon_i) d\varepsilon_i,
\]
\[
= \frac{1}{4\eta^2} \left[ R_N + 2|\Gamma|^2 R_{N-1} + |\Gamma|^4 R_{N-2} - 4\nu S_N - 4\nu|\Gamma|^2 S_{N-1} + 4\nu^2 U_N + 2|\Gamma|^2 \right],
\]  
(22)

with

\[ R_N = \langle r_N^2 \rangle = \int r_N^2 \prod_{i=1}^{N} w(\varepsilon_i) d\varepsilon_i \]
\[ = (E^2 + \sigma^2) R_{N-1} - 2ES_{N-1} + R_{N-2}, \]
(23)

\[ S_N = \langle r_N r_{N-1} \rangle = \int r_N r_{N-1} \prod_{i=1}^{N} w(\varepsilon_i) d\varepsilon_i \]
\[ = ER_{N-1} - S_{N-1}, \]
(24)

\[ U_N = \langle r_N s_{N-1} \rangle = \int r_N s_{N-1} \prod_{i=1}^{N} w(\varepsilon_i) d\varepsilon_i \]
\[ = ES_{N-1} - U_{N-1} - 1 \]
(25)

and the initial conditions \( R_0 = 1, R_1 = S_0 = U_1 = 0. \)

In order to solve this recursion, we calculate the generating functions \( F_P(z) = \sum_{N=1}^{\infty} z^N P \) of the polynomials \( P \in \{ R, S, U \} \) and with these the generating function

\[ F_{\langle 1/T_N \rangle}(z) = \sum_{N=1}^{\infty} \left\langle \frac{1}{T_N} \right\rangle z^{N-1} \]
\[ = \frac{1}{\eta^2} \left[ F_R(z) + (2|\Gamma|^2 + \sigma^2)|\Gamma|^4 \right] \left( 1 + zF_R(z) \right) \]
\[ - 4\nu(1 + z|\Gamma|^2) F_S(z) + 4\nu^2 F_U(z) + \frac{2|\Gamma|^2}{1 - z}, \]
\[ = \frac{1}{2(1 - z)} - \frac{A(z)}{4\eta^2 N_1(z)}, \]
(26)

where

\[ A(z) = \left[ 1 - 2\nu^2 + |\Gamma|^4 \right] z^2 + \left[ 1 + (2\nu^2 - 1)(E^2 - \sigma^2) \right] \]
\[ + 2|\Gamma|^2 (1 - 2\nu E) |\Gamma|^4 z \]
\[ + E^2 + \sigma^2 + 2|\Gamma|^2 - 4\nu E + 2\nu^2, \]
(27)

\[ N_1(z) = z^3 - (E^2 - \sigma^2 - 1)z^2 + (E^2 + \sigma^2 - 1)z - 1, \]
(28)

generalizing [8] beyond the wide-band approximation. We perform a partial fraction decomposition of \( F_{\langle 1/T_N \rangle}(z) \), or for simplicity rather of

\[ \frac{A(z)}{N_1(z)} = -\frac{4\eta^2}{z - z_k}, \]
(29)

where the \( z_k \) are the roots of the polynomial \( N_1(z) \) for which Vieta’s formulas hold

\[ z_1 + z_2 + z_3 = E^2 - \sigma^2 - 1, \]
\[ z_1 z_2 + z_1 z_3 + z_2 z_3 = E^2 + \sigma^2 - 1, \]
\[ z_1 z_2 z_3 = 1. \]
(30)

In the same way, the \( \alpha_k \) are determined as

\[ \alpha_k = A(z_k) = \frac{3z_k^2 - 2}\frac{3z_k}{z_k} (E^2 - \sigma^2 - 1) + E^2 + \sigma^2 - 1 \]
\[ = \frac{3z_k^2 - 2}{z_k} \frac{3z_k}{z_k} \frac{A(z_k)}{N_1(z_k)}. \]
(31)

Using the formal power-series

\[ \frac{\alpha_k}{z - z_k} = -\frac{\alpha_k}{z_k} \sum_{N=1}^{\infty} \left( \frac{z}{z_k} \right)^{N-1} \]
(32)

in (29), we get finally the analytical formula

\[ \left\langle \frac{1}{T_N} \right\rangle = \frac{1}{2} + \frac{1}{4\eta^2} \sum_{k=1}^{3} \frac{\alpha_k}{z_k} e^{-N \log(z_k)} \]
(33)

This is the main result of this Appendix. It gives, together with \( z_k \) from (30) and \( \alpha_k \) from (31), the disorder averaged resistance of the coherent tight-binding chain of length \( N \), which is connected at its ends to reservoirs by arbitrary self-energies. To our knowledge such a compact analytical formula, namely a constant plus a sum of three exponential functions, has never been reported before in the literature.

In the case \( E = 0 \), which is mainly discussed in this paper, the roots are given by

\[ z_{1,2} = -\frac{\sigma^2}{2} \pm \sqrt{\frac{\sigma^4}{4} + 1}, \quad z_3 = -1, \]
(34)

and (33) simplifies to (8).

In the following, we discuss possible values of the roots \( z_k \) of the polynomial \( N_1(z) \), which determine the behavior of the exponential functions in (33) and thus, the behavior
of the resistance. At first, we note that \( N_1(z) \) is independent of the reservoir’s self-energy \( \Gamma \) and thus, also its roots are independent of the modeling of the reservoirs \(^8\). From \( N_1(z = 0) = -1 \) and \( N_1(z = 1) = 2\sigma^2 > 0 \), we learn that \( N_1(z) \) has at least one single real root in the interval \([0, 1]\), which is denoted by \( z_1 \) and leads to the exponential increase of the resistance. More information on the \( z_k \) can be gained by the discriminant

\[
\Delta = \sigma^8 - 2\sigma^4 (4 + 10E^2 - 2) + E^2 (E^2 - 4)^3. \tag{35}
\]

For \( \Delta < 0 \), we have the real root \( z_1 \) and two complex conjugate roots \( z_3 = \pm z_2^* \). From the third Vieta formula we learn that \( z_2z_3 = |z_2|^2 = 1/z_1 > 1 \). Therefore, the complex roots cause by their phase an oscillation, which is exponentially suppressed with the chain length. For \( \Delta \geq 0 \) all three roots are real. Again, we learn from the third Vieta formula \( z_2z_3 = 1/z_1 > 1 \). If \( z_2, z_3 > 0 \), only one of them can be less than 1. However, two roots in the interval \([0, 1]\) contradict to \( N_1(0) = -1 \) and \( N_1(1) = 2\sigma^2 > 0 \), which allows only an odd number of roots in this interval. Therefore both, \( z_2 \) and \( z_3 \) are larger 1. If \( z_2, z_3 < 0 \), only one of them can be in the interval \([-1, 0] \), which contradicts to \( N_1(0) = -1 \) and \( N_1(-1) = -2E^2 < 0 \) allowing even an even number of roots in this interval. Therefore both, \( z_2 \) and \( z_3 \) are less than \(-1 \). In both cases their contributions to the resistance are exponentially suppressed.

To summarize, we have only a single real root \( z_1 \) in the interval \([0, 1]\), which dominates the resistance for \( N \to \infty \)

\[
\langle \frac{1}{N} \rangle = \frac{\alpha_1}{4\xi} e^{N\log(z_1)}. \tag{36}
\]

This equation also clarifies that the decoherence induced transition appears also in the case of arbitrary energies \( E \) and self-energies \( \Gamma \).

### B Resistivity of disordered tight-binding chains under the effect of decoherence

In this Appendix we calculate analytically the resistivity (14) of infinitely long (\( N \to \infty \)), disordered tight-binding chains under the effect of decoherence. Using (8) and (12) we obtain

\[
\rho = \sum_{j=1}^{N \to \infty} \frac{u_j}{N} \langle \frac{1}{T_j} \rangle
\]

\[
\to \frac{\rho^2}{2} \sum_{j=1}^{\infty} e^{-(j-1)/\ell} \left[ 1 + \alpha e^{j/\xi} + \alpha_\pm e^{-j/\xi} \langle -1 \rangle \right]
\]

\[
= \frac{\rho^2}{2} \left( \sum_{j=0}^{\infty} e^{-(j-1)/\ell} + \alpha_+ e^{j/\xi} \sum_{j=0}^{\infty} e^{j/\xi} \langle -1 \rangle \right)
\]

\[
- \alpha_\pm e^{-j/\xi} \sum_{j=0}^{\infty} e^{-j/\xi} \langle -1 \rangle \right). \tag{37}
\]

While the first and third geometric series converge for any finite degree of decoherence \( \ell > 0 \), the second geometric series converges only if \( \xi > \ell \). In this case by performing the sums we get

\[
\rho = \frac{\rho^2}{2} \left[ \frac{1 - e^{-1/\ell}}{1 - e^{-1/\ell}} + \frac{\alpha_+ e^{1/\xi}}{1 - e^{1/\xi - 1/\ell}} - \frac{\alpha e^{-1/\xi}}{1 - e^{-1/\xi - 1/\ell}} \right]. \tag{38}
\]

Substituting (10) in (9), we can express \( \alpha_\pm \) as a function of the disorder \( \sigma \)

\[
2\alpha_\pm = 1 \pm \frac{1}{\sqrt{\sigma^2 + 1}}. \tag{39}
\]

Using \( e^{-1/\ell} = 1 - p \) and \( e^{1/\xi} = \pm \frac{\sigma^2}{\sigma^2 + 1} \), we obtain after straightforward algebra (15).

If \( \xi < \ell \), from the second series in (37) follows directly (16).

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