Nonperturbative QCD Phenomenology and Light Quark Physics

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Recent progress in modeling QCD for hadron physics through truncated Dyson-Schwinger equations is reviewed. Special emphasis is put upon comparison of dressed quark propagators and the dressed quark-gluon vertex with lattice-QCD results.

1. Quark DSE solutions and Lattice Data

The renormalised dressed-quark propagator, \( S(p) \), is the solution of the Dyson-Schwinger equation (DSE)

\[
S^{-1}(p) = Z_1 \int_q g^2 D_{\mu\nu}(q) \frac{\lambda^i}{2} \gamma_\mu S(q) \Gamma^i(q, p).
\]

wherein the dressed-quark self-energy is

\[
\Sigma(p, \Lambda) = Z_1 \int_q \gamma^a D_{\mu\nu}(q) \frac{\lambda^i}{2} \gamma_\mu S(q) \Gamma^i(q, p).
\]

In Eq. (2), \( D_{\mu\nu}(q) \) is the renormalised dressed gluon propagator; \( k = p - q \). \( \Gamma^i(q, p) \) is the renormalised dressed quark-gluon vertex; and \( Z_1 = Z_2 + Z_4 \). \( Z_2 \) and \( Z_4 \) are, respectively, renormalisation constants for the quark-gluon vertex, quark wave function and current-quark mass. In Eq. (2), \( f^A \) is an integral with a translationally invariant ultraviolet regularization at mass-scale \( \Lambda \).

The general form of the dressed-quark propagator is

\[
S(p) = Z(p^2; \mu^2)/[i\gamma \cdot p + M(p^2)]
\]

where \( Z(p^2; \mu^2) \) and \( M(p^2) \) are the wave function renormalization function and running mass-function respectively. The \( Z_2 \) and \( Z_4 \) are determined by solving the gap equation, Eq. (1), subject to the renormalization condition

\[
S^{-1}(p)|_{\mu^2=\mu^2} = i\gamma \cdot p + M(\mu)
\]

at some large spacelike \( \mu^2 \).

The gap equation, Eq. (1), is coupled to the DSEs satisfied by the gluon 2-point function and the vertex 3-point function which in turn involve other \( n \)-point functions; a tractable model is defined by a truncation. At least one nonperturbative, chiral symmetry preserving truncation exists [12] and the first term in that scheme is the renormalization-group-improved rainbow gap equation, wherein the self-energy, Eq. (2), assumes the form

\[
\int_q \gamma^a G(k^2) D^{\text{free}}(k) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu,
\]

where \( k = p - q \) and \( D^{\text{free}}(k) \) is the Landau gauge free-gluon propagator.

In Eq. (3), \( G(k^2) \) is an effective coupling, onto which has been mapped the combined effect of dressing both the gluon propagator and quark-gluon vertex. In the ultraviolet, \( G(k^2) \) matches \( 4\pi \alpha_s^{-1-\text{loop}}(k^2) \). It has been found that a one-parameter infrared form for \( G(k^2) \) can produce an excellent description for the ground state pseudoscalar and vector mesons and their electroweak form factors and decays [34]. The advent of lattice-QCD data for the quark [4] and gluon [6] propagators provides an opportunity for an examination of the QCD content of \( G(k^2) \).

A first effort has been made to unravel the infrared contribution to \( G(k^2) \) from the dressed quark-gluon vertex [7]. Since the lattice gluon propagator is infrared-suppressed, the required vertex will be infrared-enhanced to be empirically successful. In that work, an effective kernel \( G(k^2) \) is constructed from the quenched lattice [6] Landau gauge gluon propagator \( D^{\text{latt}}(k^2) \) and an effective infrared behavior of the vertex determined by requiring that the DSE reproduce
the quenched lattice data [8] for the quark propagator. The kernel is given the form

\[ \frac{1}{k^2} G(k^2) = D^{\text{latt}}(k^2) \Gamma_1(k^2), \]

(4)

where the dressed vertex is represented by \( \gamma_\nu \Gamma_1(k^2) \) with

\[ \Gamma_1(k^2) = \frac{4\pi^2 \gamma_m}{Z_g} \left[ \frac{1}{d \nu} \ln(\tau + k^2/\Lambda^2_{\text{QCD}}) \right] v(k^2), \]

(5)

and \( \tau = e^2 - 1 > 1 \). The phenomenological factor \( v(k^2) \) is unity in the ultraviolet limit and the log factor combines with the leading log behavior of the gluon propagator to ensure that the entire kernel matches the 1-loop QCD behavior of the running coupling \( G(k^2) \to 4\pi^2 \gamma_m / \ln(k^2/\Lambda^2_{\text{QCD}}) \).

For the quenched, Landau-gauge study, \( N_f = 0 \), \( d_D = 13/22 \) and \( \gamma_m = 12/(33 - 2N_f) \) is the anomalous mass dimension.

Studies of corrections to the rainbow truncation show \( \Gamma_1 \) to be the dominant amplitude of the dressed vertex in respect to strength and effect on observables. It is the only amplitude that is ultraviolet divergent at one-loop level. The current-quark mass, \( m(\mu) \), was fixed by matching the DSE and lattice results [5] for \( M(p^2) \) on \( p^2 > 1 \text{GeV}^2 \) for each lattice set characterized by \( m_a \). With a DSE renormalisation scale \( \mu = 19 \text{GeV} \) and with \( m(\mu) \) in GeV, we find

\[\begin{array}{c|cccccccc}
m(\mu) & 0.018 & 0.036 & 0.072 & 0.108 & 0.144 \\
a m & 0.030 & 0.055 & 0.110 & 0.168 & 0.225 \\
\end{array}\]

(6)

In Fig. 1 we compare DSE solutions for \( M(p^2) \) with lattice results. It is apparent that the lattice-gluon and lattice-quark propagators are easily correlated via a simple infrared vertex enhancement \( v(k^2) \) depicted in Fig. 2. Such vertex enhancement has been anticipated for some time based on studies of the connection between DCSB and observables [9,10]. Note that \( v(0) \) is finite; for the \( m(\mu) \) case considered in Sec. 3 in relation to lattice data for the vertex, \( v(k^2_s) = 13.0 \), where \( k^2_s \sim 0.04 \text{GeV}^2 \) is an effective infrared scale.

Figure 1. DSE solutions with vertex dressing fit to lattice data for the quark mass function for various current quark masses.
Figure 2. Vertex factor obtained in the chiral limit (solid curve) and with the three lowest current-quark masses from Eq. (6). $\varepsilon(0)$ is finite.

With the dependence of the lattice data upon $p$ and $m(\mu)$ mapped into a DSE kernel, it is a vehicle for chiral extrapolation. It is apparent from Fig. 1 that the linear extrapolation [11] overestimates the chiral DSE mass function. We make a finer comparison in Fig. 3 which displays $M(p^2 = p^2_{\text{IR}})$ versus $m(\mu)$ where $p^2_{\text{IR}} = 0.38\text{ GeV}^2$ is the smallest $p^2$ value containing two lattice results for $M(p^2)$ at the three lowest $m(\mu)$ values. The linear trajectory suggested by the lattice results provides a poor extrapolation to $m(\mu) = 0$, giving a result 40% too large. The linearly-extrapolated lattice data produces the estimate [5]:

$$-\langle \bar{q} q \rangle_1^{0,\text{GeV}} = (0.270 \pm 0.027 \text{ GeV})^3.$$  

However, the error is purely statistical. The systematic error, to which the linear extrapolation must contribute, was not estimated. The DSE-assisted chiral limit [7] indicates that in quenched-QCD $-\langle \bar{q} q \rangle_1^{0,\text{GeV}} = (0.19 \text{ GeV})^3$, a factor of two smaller than the empirical value ($0.24 \pm 0.01 \text{ GeV})^3$. Related to this, the quenched lattice data underestimates the chiral $f^0_\pi$ by 30% [7].

2. Beyond Ladder-rainbow

Since the pseudoscalar states are dominated by chiral symmetry and its pattern of dynamical and explicit breakdown, most work on extensions of the BSE kernel beyond the rainbow-ladder level have sought to preserve this symmetry. Particular extensions of the self-energy beyond rainbow must be accompanied by related extensions of the ladder kernel so that the axial vector Ward-Takahashi is preserved. In the presence of an empirical amount of DCSB, this identity ensures the Goldstone nature of the chiral limit pseudoscalars, and the physical mass patterns, independently of model details [13]. This may be implemented constructively if a model of the dressed quark-gluon vertex is defined in terms of Feynman diagrams. Several studies [1,2,14] have employed the Abelian-like series

$$Z_1 F^{\lambda a b} = \frac{\lambda^a}{2} \Gamma_{\nu}(k, p) = \frac{\lambda^a}{2} \gamma_{\nu} - \int_q D_{\rho \lambda}(q) \frac{\lambda^b}{2} \gamma_{\rho}$$

$$\times S(k + q) \frac{\lambda^a}{2} \gamma_{\nu} S(p + q) \frac{\lambda^b}{2} \gamma_{\lambda} + \cdots ,$$  \hspace{1cm} (7)

which has the ladder-rainbow truncation of Eq. (2) as the first term.

The corresponding Bethe–Salpeter kernel in this scheme is generated as the sum of terms produced by cutting a quark line in the resulting diagrammatic representation of the quark self-energy. Due to the complexity of a BSE calculation with a multi-loop kernel, most numerical implementations [1,2,14] have exploited the algebraic structure that follows from use of the in-
Figure 4. $1^{++} (a_1/f_1)$ axialvector meson BSE eigenvalue from the ladder kernel ($G = 0$) and from the dressed vertex model ($G > 0$).

However, in the $1^{++} (a_1/f_1)$ channel, we find a repulsive effect of 290 MeV above the ladder kernel result, yielding a value close to $m_{\text{expt}}^{a_1}$. We are not able to compare these findings with previous work on vertex dressing since such P-wave states do not have solutions in the models considered previously for that purpose [12,14]. Other studies of $a_1$ and $b_1$ based on the ladder-rainbow truncation have used a separable approximation where the quark propagators are the phenomenological instruments [19,20,21], these studies find more acceptable masses for both states in the vicinity of 1.3 GeV.

3. Nonperturbative Model of the Quark-gluon Vertex

We denote the dressed-quark-gluon vertex for gluon momentum $k$ and quark momentum $p$ by $ig t^c \Gamma_\sigma(p + k, p)$, where $t^c = \lambda^c/2$ and $\lambda^c$ is an SU(3) color matrix. Through $O(g^2)$, i.e., to 1-loop, the amplitude $\Gamma_\sigma$ is given, in terms of Fig. 6, by $\Gamma_\sigma(p + k, p) = Z_{1F} \gamma_\sigma + \Gamma^A_\sigma + \Gamma^{NA}_\sigma + \ldots$.

The color factors reveal two important considerations. The color factor of the (Abelian-like) term $\Gamma^A_\sigma$ would be given by $t^a t^b = C_F = (N_c^2 - 1)/2N_c$ for the strong dressing of the photon-quark vertex, i.e., in the color singlet channel. The octet $\Gamma^{NA}_\sigma$ is of opposite sign and is suppressed by a factor $1/(N_c^2 - 1)$: single gluon
exchange between a quark and antiquark has relatively weak repulsion in the color-octet channel, compared to strong attraction in the color-singlet channel. Net attraction for the gluon vertex (at least to this order) is provided by the non-Abelian term $\Gamma^\text{NA}$. The nonperturbative model [22] for the dressed quark-gluon vertex as described in Sec. 1 required. The non-Abelian term $\Gamma^\text{NA}$ dominates to a greater extent two diagrams $\Gamma^\text{A} + \Gamma^\text{NA}$ into dressed versions determined solely from an existing ladder-rainbow model DSE kernel that has 1-loop QCD renormalization group improvement. Two DSE models are employed. The first (DSE-Lat) [7] is the mapping of quenched lattice data for the gluon propagator into a continuum ladder-rainbow kernel with effective gluon vertex as described in Sec. 1. In this sense, it represents quenched dynamics. The second (DSE-MT) [8] provides a good one-parameter fit to a wide variety of light quark meson physics; in this sense it represents unquenched dynamics. Both are implemented through the substitution $g^2 D_0(q^2) \rightarrow G(q^2)/q^2$ with bare quark propagators replaced by rainbow DSE solutions. Note that this procedure makes a corresponding nonperturbative extension of $\Gamma^{\mu \nu \sigma}_3$. Our justification is one of simplicity; no new parameters are introduced.

The general nonperturbative vertex at $k = 0$ has a representation in terms of three invariant amplitudes; $\Gamma^\mu_\sigma(p, p) = \gamma_\mu \lambda_1(p^2) - 4p_\mu \gamma \cdot p \lambda_2(p^2) - i 2p_\mu \lambda_3(p^2)$: the lattice-QCD data [8] is provided in terms of the $\lambda_i(p^2)$. A useful comparison is the Abelian Ward identity $\Gamma^\text{WI} \mu \nu \sigma(p, p) = -i \partial S^{-1}(p)/\partial p_\sigma$. In Fig. 7 we display the DSE-Lat model results in a dimensionless form for comparison with the (quenched) lattice data. The renormalization scale of the lattice data is $\mu = 2 \text{ GeV}$ where $\lambda_1(\mu) = 1, A(\mu) = 1$. We compare to the lattice data set for which $m(\mu) = 60 \text{ MeV}$. The same renormalization scale and conditions have been implemented in the DSE calculations. Without parameter adjustment, the model reproduces the lattice data for $\lambda_1$ and $\lambda_3$ quite well. The Abelian Ansatz (Ward Identity), while clearly inadequate for $\lambda_1$ below 1.5 GeV, reproduces $\lambda_3$. The DSE model $\lambda_2$ is evidently too weak, although the lattice data has large errors. The evident infrared enhancement at $k^2_2 \sim 0.04 \text{ GeV}^2$ is $\lambda_1(k^2_0) \sim 2.3$, a factor of 6 smaller than what the analysis of the quark propagator in Sec. 1 required. The non-Abelian term $\Gamma^\text{NA}$ dominates to a greater extent

$\Gamma^\text{NA}$. Our nonperturbative model [22] for the dressed quark-gluon vertex is defined by extensions of the

\[ \Gamma^\mu_\sigma(p, p) = \gamma_\mu \lambda_1(p^2) - 4p_\mu \gamma \cdot p \lambda_2(p^2) - i 2p_\mu \lambda_3(p^2) \]

\[ (\lambda_1(p^2) = 1, \lambda_3(p^2) = 3) \]
than what the ratio of color factors \((-9)\) would suggest; it also distributes its infrared strength to favor \(\lambda_1\) more so than does \(\Gamma_A^\lambda\). The present model results could be summarized quite effectively by ignoring \(\Gamma_A^\sigma\). Due to the definition of the two DSE models, their comparison \([22]\) provides an estimate of the effects of the quenched approximation. The effects are comparable to present uncertainties in the lattice data.

Recent work on a model of the quark-gluon vertex using a bare \(\Gamma_{\mu\sigma}^{3g}\) vertex with DSE solutions for the gluon propagator and an Ansatz for dressing both internal quark-gluon vertices has produced results similar to the present work, except that the \(m(\mu) = 115\) MeV case is considered \([25]\). Evidently the detailed infrared structure of \(\Gamma_{\mu\sigma}^{3g}\) is not crucial to present considerations.

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