Influence of Ohmic Heating on Advection-Dominated Accretion Flows

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ABSTRACT

Advection-dominated, high-temperature, quasi-spherical accretion flow onto a compact object, recently considered by a number of authors, assume that the dissipation of turbulent energy of the flow heats the ions and that the dissipated energy is advected inward. It is suggested that the efficiency of conversion of accretion energy to radiation can be very much smaller than unity. However, it is likely that the flows have an equipartition magnetic field with the result that dissipation of magnetic energy at a rate comparable to that for the turbulence must occur by Ohmic heating. We argue that this heating occurs as a result of plasma instabilities and that the relevant instabilities are current driven in response to the strong electric fields parallel to the magnetic field. We argue further that these instabilities heat predominantly the electrons. We conclude that the efficiency of conversion of accretion energy to radiation can be much smaller than unity only for the unlikely condition that the Ohmic heating of the electrons is negligible.

Subject headings: accretion, accretion disks—galaxies: active—plasmas—magnetic fields—stars: magnetic fields—X-rays: stars
1. Introduction

Advection-dominated accretion flows have been intensely studied during the past several years (for example, Narayan and Yi 1994; Narayan and Yi 1995; Abramowicz et al. 1995; Nakamura et al. 1996; Chakrabarti 1996). The basic dynamical equations for accretion disks including the advection of entropy were first discussed by Paczyński and Bisnovatyi-Kogan (1981) and Muchotrzeb and Paczyński (1982). In contrast with the widely applied theory of thin accretion disks of Shakura (1973) and Shakura and Sunyaev (1973) where the disk material cools efficiently by local radiation of viscously generated energy, the further assumption that the viscous dissipation heats the ions and that the cooling is inefficient. [A recent paper by Esin et al. (1996) treats advection dominated accretion flows assuming that the viscous dissipation heats the ions and that a constant fraction \( f \) of this dissipated energy is advected inward and that the fraction \( 1 - f \) is locally radiated. The further assumption that the energy exchange between ions and electrons is by Coulomb scattering leads to conditions with the ion temperature \( T_i \) much larger than the electron temperature \( T_e \) so that the cooling is inefficient. [A recent paper by Esin et al. (1996) treats advection dominated accretion flows assuming \( T_i = T_e \).] The radiative efficiency, the power output in radiation divided by \( \dot{M} c^2 \) (with \( \dot{M} \) is the mass accretion rate), is found to be very small compared with unity. The advection-dominated accretion flows tend to be quasi-spherical and optically thin (except for cyclotron radiation as discussed below) with radial inflow speed \( v_r \approx -\alpha v_K \), azimuthal speed \( v_\phi \approx \text{const.} v_K \ll v_K \), ion thermal speed \( v_{si} \approx \text{const.} v_K \sim v_K \) (Narayan and Yi 1995), where \( v_K \equiv (GM/r)^{1/2} \) is the Kepler speed and \( \alpha \) is the dimensionless viscosity parameter of Shakura (1973) usually assumed to be in the range \( 10^{-3} - 1 \).

In §2 we discuss magnetized accretion flows and the importance of Ohmic dissipation in addition to the earlier considered viscous dissipation. We argue that the Ohmic heating is due to plasma instabilities which heat the electrons. In §3 we treat a model for the radial variation of electron and ion temperatures assuming that a fraction \( g \) of the dissipated energy goes into heating the electrons and a fraction \( 1 - g \) goes into heating the ions. The electrons cool by bremsstrahlung and cyclotron radiation and exchange energy with ions by Coulomb collisions. In §4 we discuss conclusions of this work.

2. Accretion Flows with B Field

In quasi-spherical accretion onto a compact object of mass \( M \) of Schwarzschild radius \( r_s = 2GM/c^2 \) (for a black hole) the accreting matter is likely to be permeated by a magnetic field \( B(r,t) \). Typically the accreting matter is ionized and consequently highly conducting with the result that the magnetic field is frozen into the flow. One result of this is that \( |B_r| \propto r^{-2} \). Thus the magnetic energy-density varies as \( E_{mag} = B^2/8\pi \propto r^{-4} \). On the other hand the kinetic energy-density varies as \( E_{kin} = \rho v^2/2 \propto r^{-4} \).

Thus one can expect that equipartition between magnetic and kinetic energy-densities occurs in the flow at a large distance \( r = r_{equi} \gg r_s \) (Shvartsman 1971) and that it is maintained for smaller \( r \). Further accretion for \( r < r_{equi} \) is possible only if magnetic flux is destroyed by reconnection and the magnetic energy \( E_{mag} \) is dissipated. The dissipation of magnetic energy was first taken into account by Bisnovatyi-Kogan and Ruzmaikin (1974) who showed that accretion for conditions of equipartition (\( E_{mag} \sim E_{kin} \)) is accompanied by the dissipation of magnetic energy into heat with entropy \( s \) (per unit mass) production rate \( \rho T(ds/dr) = -3B^2/(16\pi r) \). We point out that the Ohmic dissipation of the magnetic energy is an important, possibly dominant heating process in advection-dominated accretion flows with \( E_{mag} \sim E_{kin} \). In this regard note that although Narayan and Yi (1995) assume an equipartition magnetic field, they do not consider the Ohmic heating.

The basic equations for accretion flows with \( E_{mag} \sim E_{kin} \) are

\[
\frac{\partial v}{\partial t} + v \cdot \nabla \rho = -\frac{1}{\rho} \nabla p + \frac{1}{\rho c} J \times B + \nu_m \nabla^2 v
\]

(1a)

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta_m \nabla^2 B
\]

(1b)

where \( v(r,t) \) is the flow velocity, \( p(r,t) \) the pressure, \( g = -\nabla(GM/|r|) \) the gravitational acceleration, \( \nu_m \) the microscopic kinematic viscosity coefficient, and \( \eta_m \) the microscopic magnetic diffusivity.

It is well known that the microscopic classical transport coefficients \( \nu_m \) and \( \eta_m \) are much too small to directly influence on the macroscopic flow \( v \) and magnetic field \( B \) evolution. For example, for conditions pertinent to a flow onto a massive black hole, \( n \sim 10^{12} \text{ cm}^{-3} \), \( T_i \sim 10^{12} \text{ K} \), and \( B \sim 10^8 \text{ G} \) for \( r \sim r_s \), the Reynolds number for the flow \( Re_v = r|v|/\nu_m \sim 10^{24} \), where \( \nu_m \sim \tau_{gi}^2/\tau_{ii} \) is the viscosity
appropriate for a tangled magnetic field (Braginskii 1965; Paczyński 1978), and where \( r_{gi} \sim 10^2 \text{cm} \) is the ion gyro-radius and \( \tau_{ni} \sim 10^6 \text{s} \) is the ion-ion Coulomb scattering time, and \( \omega_{ci} \tau_{ni} \gg 1 \) with \( \omega_{ci} \sim 10^9 / \text{s} \) the ion cyclotron frequency. [Under some conditions it is possible that \( \nu_m > r_{gi} / \tau_{nt} \) as discussed by Subramanian, Becker, and Kafatos (1996)]. The magnetic Reynolds number \( Re_B = r|\mathbf{v}| / \eta_m \sim 10^{27} \), where \( \eta_m = c^2 / (4\pi \sigma_S) \) with \( \sigma_S \) the Spitzer conductivity.

It was proposed by Shakura (1973) that accretion flows are in general turbulent and that roughly equations (1) should be taken with turbulent transport coefficients \( \nu_t \) and \( \eta_t \) replacing the microscopic coefficients, and with \( \mathbf{v} \rightarrow \bar{\mathbf{v}} \) and \( \mathbf{B} \rightarrow \mathbf{B} \) interpreted as mean fields. The turbulent viscosity has a crucial role in thin Keplerian disks where it provides a mechanism for the outward transport of angular momentum. According to Shakura (1973), \( \nu_t = \alpha c_s H \), where \( \alpha \) = const. is the above-mentioned dimensionless viscosity parameter, \( c_s \) is the ion sound speed, and \( H \) is the half-thickness of the disk which is the outer scale of the turbulence. Note that for an advection-dominated flow, \( \nu_t \) and \( \eta_t \) have a crucial role in dissipating the magnetic energy in advection-dominated flows. In addition to \( \nu_t \) and \( \eta_t \), there will be a turbulent transport coefficient \( \alpha_h \) (with units of \( \text{cm/s} \)) associated with the helicity of the turbulence in a rotating accretion flow (see, for example, Ruzmaikin, Shukurov, and Sokoloff 1988).

Neglecting for the moment the possible difference between \( T_e \) and \( T_i \) and the radiative energy losses, energy conservation for the accretion flow can be expressed in terms of the mean fields as

\[
\rho \frac{d \mathbf{s}}{d \tau} = - \frac{1}{2} \rho \nu_t \left( \mathbf{v}_{i,j} + \mathbf{v}_{j,i} - \frac{2}{3} \delta_{ij} \mathbf{v}_{k,k} \right)^2 + \frac{1}{4\pi} \eta_t (\nabla \times \mathbf{B})^2 ,
\]

where \( s \) is the entropy per unit mass. The first term on the right hand side of (2) represents the viscous dissipation or heating of the plasma, and the second term the Ohmic dissipation. The two terms are of comparable magnitude for an accretion flow with \( \mathcal{E}_{mag} \sim \mathcal{E}_{kin} \) and \( \nu_t \sim \eta_t \).

However, equation (2) says nothing about the actual microscopic dissipation of energy in the plasma. Rather, it expresses the loss of energy from the outer scale (\( \sim r \) or \( H \) if \( H < r \)) of the flow \( \mathbf{v} \) and from the \( \mathbf{B} \) field by the nonlinear processes implicit in equations (1) and the presumed Kolmogorov cascade of this energy to smaller scale eddies and field structures of the flow. The turbulence may be characterized by wavenumber-frequency ensemble averaged spectra \( \langle v_{k\omega}^2 \rangle \) and \( \langle B_{k\omega}^2 \rangle \), where the wavenumber ranges from the small value corresponding to the mentioned outer scale \( k_{min} \sim r^{-1} \) to some much larger value \( k_{max} > k_{min} \). The conventional Kolmogorov description has a dissipation scale corresponding to \( k_{max} \sim (Re)^{1/4} k_{min} \) which corresponds to an unphysically small length scale using either \( Re_e \) or \( Re_B \). Thus, the actual dissipation must be due plasma instabilities.

The relevant plasma instabilities are probably current driven in response to the large mean electric field, \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} / c - \alpha_e \mathbf{B} / c + \eta_t \nabla \times \mathbf{B} / c \), which in general has a significant component parallel to \( \mathbf{B} \). It is unclear to us why current driven instabilities resulting from \( \mathbf{E}_d \) were not considered by Begelman and Chiu (1988). The typical electric field \( |\mathbf{E}| \sim 10^{6} \text{ V/cm} \) (for \( r \sim r_s \)) is much larger than the Dreicer electric field for electron runaway (Parail and Pogutse 1965), \( E_D = 4\pi e^2 (n_e / kT_e) \nu_t \Delta \sim 10^{-4} \text{ V/cm} \) for \( T_e \sim 10^9 \text{K} \), where \( n_e \) is the electron density. Thus the electrons will runaway. An electron becomes relativistic in a distance of travel of \( \sim 1 \text{ cm} \) which is comparable to the electron gyro radius. The drift speed of the electrons parallel to \( \mathbf{B} \) will be sufficient to give rise to streaming instability (Parail and Pogutse 1965). Streaming instability will occur if the electron drift velocity is larger than the ion thermal speed. In contrast with the ions, the travel distance for a proton to become relativistic is \( \sim 10^3 \text{ cm} \). However, acceleration of protons parallel to the magnetic field is strongly suppressed by scattering by magnetic fluctuations (Alfvén waves) with wavelengths of the order of the proton gyro radius which are generated by the proton streaming (Kulsrud and Pearce 1969). For these reasons we believe that most of the free energy driving the instability goes into heating the electrons. However, we also consider the case where a fraction \( g \) of the dissipated energy goes into heating the electrons (and \( 1 - g \) goes into heating the ions). We illustrate the behavior in this case with the following simple model.
3. Model

We generalize equation (2) by taking into account (a) that \( T_i \) and \( T_e \) may differ with energy exchange between ions and electrons by Coulomb collisions, (b) that the Ohmic plus viscous dissipation heats electrons and ions as discussed below, and (c) that the main energy loss is from optically thin bremsstrahlung and optically thick cyclotron emission. Note that the thickness of the flow \( H/r \) is not restricted. Note also that in contrast with Narayan and Yi (1995), no assumption is made that a constant fraction \( f \) of the dissipated energy is advected inward. Hence

\[
\frac{3}{2} \frac{dT_i}{dt} - \frac{T_i}{\rho} \frac{d\rho}{dt} = (1-g) \mathcal{H} - \nu_{te}(T_i - T_e), \tag{3a}
\]

\[
\frac{3}{2} \frac{dT_e}{dt} - \frac{T_e}{\rho} \frac{d\rho}{dt} = g \mathcal{H} - C_{brem} - C_{cyc} + \nu_{te}(T_i - T_e), \tag{3b}
\]

where \( g \leq 1 \) is the fraction of the Ohmic plus viscous dissipation which goes into heating the electrons. We assume \( g = \text{const} \), which we view as more physically plausible than the assumption that \( f = \text{const} \), of Narayan and Yi. For simplicity of the formulae we assume \( T_i < m_i c^2 \) and \( T_e < m_e c^2 \), where \( T_i \) and \( T_e \) are measured in ergs. Here,

\[
\nu_{te} \approx \frac{4(2\pi)^2 ne^4}{m_im_e} \left( \frac{T_e}{m_e} + \frac{T_i}{m_i} \right)^{-\frac{1}{2}} \ell n \Lambda
\]

is the ion-electron energy exchange rate with \( \ell n \Lambda = \mathcal{O}(20) \) the Coulomb logarithm (Spitzer 1940); \( \mathcal{H} \approx (9/4)m_i \alpha (c_i/v_K)^2 v_K^2 \mathcal{J} / r \) is the heating rate per ion with \( \mathcal{J} = 1 - (r_S/r)^2 \); \( C_{brem} \approx n \sigma_T \alpha_f m_e c^3 (T_e/m_e c^2)^{1/2} \) is the bremsstrahlung cooling rate per electron with \( n \) the electron or ion density, \( \sigma_T \) the Thomson cross section, and \( \alpha_f \) the fine structure constant; and \( C_{cyc} \approx T_e \omega_p^2 \mathcal{M}_e^3 / (8\pi^3 n c^2 r) \) is the self-absorbed cyclotron radiation cooling rate per electron with \( \mathcal{M}_e \gg 1 \) the cut-off harmonic number of the cyclotron radiation below which the radiation is self-absorbed (Trubnikov 1958). For \( \mathcal{M}_e \gg (2/9)\mu \gg 1 \), with \( \mu \equiv m_e c^2/T_e \), Trubnikov’s analysis gives \( \mathcal{M}_e \approx (2\mu/9)(1 + \ell n(D)/\mu)^{1/3} \), where \( D \approx \omega_p^2 r / (\omega_{ce} \mathcal{M}_e) \), with \( \omega_p \) and \( \omega_{ce} \) the electron plasma and cyclotron frequencies respectively. Trubnikov’s expression for \( C_{cyc} \) is similar to that of Narayan and Yi (1995).

It is useful to rewrite equations (3) in dimensionless form. Note that \( d/d\hat{r} = \nu_r (d/dr) \) with \( \nu_r = -(3/2)\alpha T_i v_K \), and that \( H/r = \hat{T}_i^{\frac{3}{2}} \), number density of electrons or ions \( n = \hat{M}/(6\pi G m_i r^2 \hat{T}_i^{\frac{3}{2}} v_K) \), mass density \( \rho = n m_i \), magnetic field \( B = [2\hat{M} v_K/(3\pi r^2 \hat{T}_i^{\frac{3}{2}})]^{1/2} \), where \( \hat{T}_i \equiv T_i/\tau_v \) with \( \tau_v = GM m_i / r \) the virial temperature. We also normalize the electron temperature with the same \( T_v \), \( \hat{T}_e \equiv T_e/T_v \). Equations (3) become

\[
\frac{d\hat{T}_i}{d\hat{r}} = -(1-g)\hat{\mathcal{H}} + \hat{\mathcal{A}}(\hat{T}_i - \hat{T}_e), \tag{4a}
\]

\[
\frac{d\hat{T}_e}{d\hat{r}} = -(2 + \hat{\zeta})(g - \hat{\zeta})\hat{\mathcal{H}} + \hat{\mathcal{C}}_{brem} + \hat{\mathcal{C}}_{cyc} - (2 + \hat{\zeta})\hat{\mathcal{A}}(\hat{T}_i - \hat{T}_e), \tag{4b}
\]

where \( \hat{\mathcal{A}} \approx \hat{\mathcal{J}} / 2\hat{r} \),

\[
\hat{\mathcal{C}}_{brem} \approx \frac{2\hat{\zeta} \alpha f}{27\alpha_f^2} \left( \frac{m_e}{m_i} \right) \left( \frac{\hat{M}_c^2}{L_E} \right) \left( \frac{\hat{T}_i^{\frac{3}{2}}}{\hat{T}_e^{\frac{1}{2}}} \right), \tag{4c}
\]

\[
\hat{\mathcal{C}}_{cyc} \approx \frac{1}{9\pi^2} \frac{\alpha^2}{2\alpha_f^2} \left( \frac{m_i}{m_e} \right)^3 \left( \frac{r_e}{r_S} \right) \left( \frac{\hat{M}_c^2}{L_E} \right) \left( \frac{\hat{T}_i^{\frac{3}{2}}}{\hat{T}_e^{\frac{1}{2}}} \right), \tag{4f}
\]

where \( L_E \equiv 4\pi GM m_i c / \sigma_T \) is the Eddington luminosity, and \( r_e \equiv c^2 / (m_e c^2) \) is the classical radius of the electron. The terms \( d\hat{T}_i / d\hat{r} \) and \( d\hat{T}_e / d\hat{r} \) in equations (4) describe the advection of energy by the flow. Apart from the cyclotron cooling the different terms depend only on \( \alpha \) and \( \hat{M}_c^2 / L_E \). The cyclotron cooling is relatively more important for accretion onto a stellar mass object than for accretion onto a massive black hole. The assumed condition for optically thin bremsstrahlung radiation requires \( (\hat{M}_c^2 / \alpha L_E) \hat{r}^{-\frac{1}{2}} < 1 \) for \( \hat{T}_i = \mathcal{O}(1) \).

We have solved equations (4) starting from different given ‘initial’ values of \( \hat{T}_i \) and \( \hat{T}_e \) at large \( \hat{r} = 10^3 \), different accretion rates \( \hat{M}_c^2 = (0.01 - 1) L_E \), values of \( \alpha \), and values of \( g = 0 - 1 \), and integrating inward. For the accretion rates where advection-dominated
flows are suggested to occur (Narayan and Yi 1995), $M_c^2 \leq 0.1 M_E$ for $\alpha = 0.1$, we find that the the scaled ion temperature $T_i$ remains almost constant, whereas the scaled electron temperature $T_e$ decreases rapidly as $\dot{r}$ decreases from $10^3$. In this limit, the Coulomb energy exchange between ions and electrons is negligible. The advection terms on the left-hand-side of equation (3b) are also negligible. Consequently, the Ohmic heating of the electrons $g \mathcal{H}$ goes into radiation, mainly cyclotron radiation; that is, $g \mathcal{H} \approx \mathcal{C}_{\text{cy}}$. The total radiation is the volume integral of $g \mathcal{H} n$ which gives $g G M M / (2 r_i)$, where $r_i$ is the inner radius of the flow. Thus, the radiative efficiency is reduced by a factor of $g$ from that of a thin disk with $T_i = T_e \ll 1$ which is the volume integral of $\mathcal{H} n$. This efficiency can be very small compared with unity only if $g$ is very small compared with unity.

4. Conclusions

This work considers magnetized advection-dominated accretion flows where the magnetic field is in equipartition with the turbulent motions of the flow (Shvartsman 1971). The magnetic energy density of the flow must be dissipated by Ohmic heating with a rate comparable to that of the viscous dissipation (Bisnovatyi-Kogan and Ruzmaikin 1974). We argue that the Ohmic and viscous dissipation must occur as a result of plasma instabilities. Further, we argue that the instabilities are likely to be current driven in response to the electric field (associated with the turbulent motion) which has a significant component parallel to the magnetic field. These instabilities are likely to heat mainly the electrons. We have analysed a model for the radial variation of the electron and ion temperatures assuming that a constant fraction $g$ of the viscous plus Ohmic heating goes into heating the electrons and a fraction $(1 - g)$ goes into heating the ions. In contrast with Narayan and Yi (1995), we do it not assume that a constant fraction $f$ of the dissipated energy is advected inward by the flow. The electrons cool by bremsstrahlung and cyclotron radiation and exchange energy with the ions by Coulomb collisions. At large accretion rates $\dot{M}$, Coulomb collisions act to give $T_i \approx T_e$, high radiative efficiency, and geometrically-thin, optically-thick disk accretion. For small accretion rates, where advection-dominated accretion flows are suggested to occur, and only Coulomb energy exchange between ions and electrons, a regime of optically thin accretion flows with a large difference between ion and electron temperatures ($T_e \ll T_i$) exists (Shapiro, Lightman, & Eardley 1976). Here, we emphasize that the accretion flow properties depend critically on the Ohmic heating of the electrons. For small accretion rates where the electron temperature is much less than the ion temperature, we show that the Ohmic heating of the electrons gives a radiative efficiency which is reduced by a factor of $g$ from that for a thin disk. Thus, the tiny radiative efficiencies ($< 10^{-3}$) found by Narayan and Yi (1995) correspond to tiny values of $g$ which are unlikely for the reasons discussed in §2.

Plasma instabilities due to electron-ion streaming (for electron drift velocity larger than the ion thermal speed) may greatly enhance the energy exchange between ions and electrons. In this case the two-temperature regime disappears, the ion and electron temperatures collapse to small values, $T_{i,e} \ll 1$, and the disk is geometrically thin. That is, advection-dominated accretion flows do not occur (Fabian & Rees 1995).

We thank Drs. M.M. Romanova and H.H. Fleishmann for valuable discussions. This work was supported by NSF grant AST-9320068 and a grant from the CRDF Foundation. The work of GBK was also supported by Russian Fundamental Research Foundation grant No. 96-02-16553. The work of RVEL was also supported by NASA grant NAGW 2293.

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