Total Reaction Cross Section in an Isospin-Dependent Quantum Molecular Dynamics (IDQMD) Model

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The isospin-dependent quantum molecular dynamics (IDQMD) model is used to study the total reaction cross section $\sigma_R$. The energy-dependent Pauli volumes of neutrons and protons have been discussed and introduced into the IDQMD calculation to replace the widely used energy-independent Pauli volumes. The modified IDQMD calculation can reproduce the experimental $\sigma_R$ well for both stable and exotic nuclei induced reactions. Comparisons of the calculated $\sigma_R$ induced by $^{11}$Li with different initial density distributions have been performed. It is shown that the calculation by using the experimentally deduced density distribution with a long tail can fit the experimental excitation function better than that by using the Skyrme-Hartree-Fock calculated density without long tails.

It is also found that $\sigma_R$ at high energy is sensitive to the long tail of density distribution.

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The total reaction cross section $\sigma_R$ has been extensively studied theoretically and experimentally. [1-10] There are two kinds of theoretical models to calculate $\sigma_R$. The first is the low energy theory based on the interaction potential. Such models are not successful for the reaction beyond 10-15 MeV/nucleon above the Coulomb barrier. The second is the high energy microscopic Glauber theory based on the individual nucleon-nucleon collisions in the overlap volume of the projectile and target. However, the roles of mean field and medium effect are difficult to be discussed in the Glauber-type model. In recent years, Ma et al. developed a new method to study $\sigma_R$ with helps of transport theory and Glauber model. [5] Originally, the BUU model [11] was taken as a tool to investigate $\sigma_R$. [5] Later, the quantum molecular dynamics (QMD) model was applied to study $\sigma_R$ [13] in the same spirit as Ref. [5]. The reaction dynamics in transport theory at intermediate energy is mainly governed by the mean field, two-body collisions, and Pauli blocking. To investigate the isospin effects, the above three dynamical ingredients should include properly isospin degrees of freedom to obtain an isospin-dependent quantum molecular dynamics (IDQMD). It is also important that the samples of neutrons and protons in the phase space should be treated separately in the initialization of projectile and target nuclei. In this Letter, the IDQMD model is introduced to calculate $\sigma_R$. This model incorporates the isospin-dependence of mean-field, nucleon-nucleon cross section, and Pauli blocking. It has been widely used to study the multi-fragmentation and the collective flow. In the IDQMD model, neutrons and protons are distinguishable in the initialization.

The density distributions for the initial projectile and target nuclei are determined from the Skyrme-Hartree-Fock (SHF) calculation [17] with parameter set SKM. The stability of the propagation of the initialized nuclei has been checked in details and can last at least 200 fm/c according to the evolutions of the average binding energies and the root mean square radii of the initialized nuclei. In the IDQMD model, once the distance between the two nucleons is less than $r_{nn}$, the collision may occur, where $r_{nn}$ is defined as

$$r_{nn} = \sqrt{\sigma_{nn}(\sqrt{s})/\pi}$$

with $\sigma_{nn}(\sqrt{s})$ representing the nucleon-nucleon reaction cross section and $\sqrt{s}$ representing the nucleon energy in the center of mass system. Whenever a collision occurs, the final momenta of the scattering nucleons can be easily obtained as a consequence of momentum conservation and the coordinates are updated in terms of newtonian equation (classical trajectory). The six-dimensional phase space radius of one nucleon is the product of its $\Delta P$ and $\Delta R$. Then we calculate and check the phase spaces around the final states of the scattering nucleons. Thus it is easy to determine the probabilities ($P_1$ and $P_2$) for each of the two scattering nucleons that their final phase spaces are already occupied by other nucleons by comparing with the defined Pauli volume, i.e. $4(\Delta P \Delta R)^3/3$, in six-dimensional phase space, where $\Delta R$ is the minimum radius which is allowed to be occupied by itself in coordinate space and the $\Delta P$ is the same but in momentum space. The collision is then blocked with a probability

$$P_{\text{block}} = 1 - [1 - \text{min}(P_1, 1)][1 - \text{min}(P_2, 1)].$$

Correspondingly, the collision is allowed with probability $(1 - P_{\text{block}})$. Whenever a collision is blocked, the momenta of the scattering nucleons will be replaced by the values they are prior to scattering. More details could be found in Refs. [12,15].
The following formula of in-medium reaction cross section is used in the present IDQMD calculation,
\[
\sigma_{nn} = (13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{-4}) \nonumber \\
\frac{1.0 + 7.772E_{lab}^{0.006}\rho^{1.48}}{1.0 + 18.0\rho^{1.46}} 
\sigma_{np} = (-70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta) 
\frac{1.0 + 20.88E_{lab}^{0.04}\rho^{2.02}}{1.0 + 35.86\rho^{1.90}} 

\beta = \sqrt{1 - \frac{1}{\gamma^2}}, \gamma = \frac{E_{lab}}{931.5} + 1 \quad (1) 
\]
This formula includes both effects of incident energy Elab and nucleon matter density \( \rho \). It shows that the medium effect is important at intermediate energies and becomes smaller at higher energies but does not vanish.[18, 19].

The cross section \( \sigma_R \) can be written as [5]
\[
\sigma_R = 2\pi \int b[1 - T(b)]db = 2\pi \int b[1 - \exp(-N)]db \quad (2) 
\]
where the transport function \( T(b) \) can be obtained from the average n-n collision number \( N \) as a function of the impact parameter \( b \). More details could be found in Ref. [5].

In the previous IDQMD model, the volume occupied by nucleon in the projectile and target (Pauli volume) was a constant \( (h^3/2) \). This constant could be deduced from the lowest limit of the uncertain relationship between the momentum and the coordinate.[14] Recently it was found that the Pauli volume is sensitive to the incident energy and should not be a constant qualitatively.[20] Calculation of the average collision number \( N \) of the \( ^{12}\text{C} + ^{12}\text{C} \) system at high incident energy shows that the average collision number still has an uptrend after the evolution time of 200fm/c. However, other studies indicated that the average collision number has been saturated after 50fm/c at high incident energy.[13] This indicates that the invariable Pauli volume used in the previous IDQMD model may not be suitable and the Pauli volume may be energy-dependent. From formula (2) we know that \( \sigma_R \) can be decided by the average collision number \( N \). It is also clear that \( N \) is sensitive to the probability of the Pauli blocking in the collisions. Thus, the energy dependence of the Pauli volume becomes very important in the calculation of \( \sigma_R \). Up to now, there has not yet been parameterized formula to describe the energy-dependent Pauli volume.

We calculate the cross section \( \sigma_R \) of the \( ^{12}\text{C} + ^{12}\text{C} \) reaction system in an IDQMD framework in a wide energy range as shown in Fig. 1(a). Here soft EOS and in-medium \( \sigma_{nn} \) are used. The initialized density distribution of \( ^{12}\text{C} \) comes from the SHF calculation. The solid line with open circles shows the calculated results with the constant Pauli volume. It is obvious that they can not reproduce the experimental data. With adjusted

![Figure 1](image-url)
FIG. 2: Experimental density distribution (solid) of $^{11}$Li ($\rho_1$) [21] and the calculated one (dashed) with the Skyrme-Hartree-Fock density distribution $\rho_0$.

The experimental ones at high incident energy for about a few percent since the SHF calculation of $^{11}$Li does not give the long tail as experimental one, which is expected to play an important role in the calculation of $\sigma_R$. This indicates that the density distribution calculated with the SHF density distribution is not appropriate to those halo nuclei [21] in the IDQMD calculation. It is obvious that the calculated $\sigma_R$ at several hundreds of MeV/u is more sensitive to the long-tail density distribution. Since the central densities are also different between them, more conclusions about the density effect on $\sigma_R$ can be obtained by further studies. The research along this line is in progress.

Figure 3(b) shows the calculated results of excitation function of $\sigma_R$ for the $^{12}$C + $^{27}$Al reaction system. In Fig. 3, all the filled circles represent the experimental data.[4,22] The solid line with open squares shows the calculated results with the IDQMD, where the initialized density distribution of $^{27}$Al and $^{12}$C are calculated by using the SHF model. Figure 3 shows that the IDQMD calculation with the energy-dependent Pauli volume and the density distribution of the SHF model gives a good trend of excitation function of $\sigma_R$. For reaction induced by stable nuclei, the calculated results can fit the experimental values well in a wide energy range. With the experimental density distribution the IDQMD model can also give the fine results of $\sigma_R$ induced by halo nuclei.

Figure 4 gives the calculation of $\sigma_R$ of Li isotopes on $^{12}$C target at 790MeV/u. The density distribution of $^{11}$Li is from the experiment and the others are calculated with the SHF model. It shows that $\sigma_R$ varies smoothly with mass number up to 9Li. There is a sudden increase of $\sigma_R$ between 9Li and $^{11}$Li which is corresponding to the halo structure in $^{11}$Li. Our calculation results reproduce both the experimental values and the sudden change between 9Li and $^{11}$Li fairly well.

In conclusion, the IDQMD model has been introduced to study the total reaction cross section $\sigma_R$ by using the energy-dependent Pauli volume which is deduced from the excitation function of $\sigma_R$ of $^{12}$C+$^{12}$C. The calculated results can reproduce the experimental $\sigma_R$ quite well. It is interesting to investigate further the energy and isospin effects of Pauli volume in collision. For halo nuclei, the calculated results by using experimental density distribution are better than that by using the SHF density distribution. It is suggested that the long tail of the density distribution plays an important role for the halo nuclei and $\sigma_R$ at high energy is sensitive to the long tail distribution of halo nuclei. Since the IDQMD model incorporates the isospin-dependences of mean-field, nucleon-nucleon cross section, and Pauli blocking, it is also interesting to study each isospin-effect on $\sigma_R$ and other physical quantities. In the IDQMD calculation, the clusters can be
judged by the relative momenta and coordinates with an isospin-dependent modified coalescence model and the momentum distribution of the projectile fragments can be obtained easily. Thus, the IDQMD model can be used to study the total reaction cross section, fragmentation cross sections and momentum distribution of fragment of halo nuclei simultaneously. It can be used to give a more comprehensive and reliable criterion of halo structure, which is of great significance. These works are in progress.

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