Magneto Optical Rotation in the Doppler- broadened Spectrum of Rb atoms in Intermediate fields

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Abstract. Doppler- broadened absorption spectrum of rubidium ($^{87}$Rb and $^{85}$Rb) D$_2$ line has been studied in the presence of a magnetic field of intermediate strength i.e. up to 5 mT. Resonant non - linear magneto optical rotation is measured in the Doppler broadened spectrum of rubidium atomic vapour. Low light intensity approximation in the intermediate field region has been made to calculate the magneto optical rotation and to compare with experimental data. The enhancement of non-linear magneto optical rotation is discussed taking into account hole burning and circular birefringence in the intermediate field region.

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1. Introduction

Non-linear Magneto Optical Rotation (NMOR) is a useful tool for precision metrology. Recent developments in this area suggest the possibility of highly sensitive magnetometry, magneto- optical switching, occurrence of parity and time-reversal violation and electric dipole moment in atoms [1-5]. Faraday and Voigt effects are the most prominent magneto optical effects and there are other non - linear magneto optical effects such as the Hanle effect, level crossing etc. If the frequency of the incident radiation matches the transition frequency of the medium then Faraday Effect is called Resonant Faraday Effect. Resonant magneto-optical effects have large magnitudes compared to simple Faraday or Voigt effects in transparent media such as glasses and liquids.

When linearly polarized laser field interacts resonantly with an atom in the presence of an external magnetic field, the different polarization components of the incident field couple to different transitions between the Zeeman sublevels. As a result, the absorption and dispersion of the different polarization components may be different, giving rise to such effects as dichroism and birefringence which result in changes in the polarization ($\sigma_{+}$ and $\sigma_{-}$) of the transmitted light.

NMOR has already been studied extensively for weak fields (geophysical range) [6-8]. However, to our knowledge, no studies have been reported for the rubidium atomic vapour in fields of intermediate strength. Weis et al. have discussed the Faraday rotation in the intermediate field region for cesium atoms using low laser power approximation [9]. We have followed their method to calculate the NMOR for Rb atomic vapour. We have observed the Doppler broadened spectrum of rubidium ($^{87}$Rb and $^{85}$Rb) atomic vapour. We have studied these effects experimentally for intermediate magnetic field strengths ($g\mu_B B \approx A_J$) upto 5 mT. High field NMOR for calcium has been discussed by G. S. Agarwal et al. [10].

In this paper we present laser power and magnetic field dependent NMOR in the Doppler broadened spectrum. The Bennet structure related NMOR and coherence NMOR are distinguished with the help of power dependent measurements. NMOR in the Rb atomic...
vapour at room temperature (22°C) is due to ground state Zeeman coherences through ‘transit effect’ (uncoated cell without buffer gas) and Bennet structure effects [8]. A preliminary analysis of the experimental data has been made. The intensity dependent rotation predicts that saturation effects play a vital role in enhancing the non-linear effects.

2. Theory

87Rb (~27.85%) and 85Rb (~72.15%) are naturally occurring isotopes of rubidium. The selection rules for the D 2 line transition are $\Delta J = 0$ or $\pm 1$, $\Delta I = \pm 1$, $\Delta S = 0$; for hyperfine transition we have $\Delta F = 0$ or $\pm 1$, and $\Delta M_F = 0$ or $\pm 1$. Here, $F=I+J$ to $|I-J|$, where $I$ is the nuclear spin and $J$ is the electronic total angular momentum quantum number. The Hamiltonian which includes the hyperfine interaction in an intermediate magnetic field [11] is:

$$H_{HFS} = A_J (I.J) + \frac{B_J}{2I(2I-1)(2J-1)} [3I(I+1)J(J+1)] + g_J \mu_B I.B - g_I \mu_N I.B$$

where $A_J$ is the magnetic hyperfine structure constant and $B_J$ is the electric quadrupole interaction constant. The last two terms in the above equation represent the interaction of the electronic and nuclear magnetic moments with the external magnetic field. Here $g_J$ and $g_I$ are the electronic and nuclear g factors.

2.1 Faraday rotation in the case of Doppler broadened spectrum

Linearly polarized light incident on the sample can be resolved into left circularly ($\sigma_-$) and right circularly ($\sigma_+$) polarized components. The magnetic sublevels are degenerate in the absence of a magnetic field and that results in a coincidence of the optical resonance frequencies for $\sigma_-$ and $\sigma_+$. In the presence of an external magnetic field, the Zeeman shifts lead to a difference between the resonance frequencies for the two circular polarizations. The plane of polarization is rotated by an angle

$$\varphi = \pi (n_x - n_y) \frac{l}{\lambda}$$

where $n$ is the refractive index and $l$ is the length of the sample and $\lambda$ is the wavelength of laser.

Low power (laser intensity) approximation [9] is applicable only when the medium is optically thin and there is no variation in the output intensity with respect to the input field. A theoretical description for NMOR in the limit of low laser intensity and for intermediate magnetic fields has been developed by Weis et al. The refractive index for the circularly polarized EM plane wave propagating in a Doppler broadened medium along a magnetic field $B$ can be written as

$$n_x (\omega, FM, F'M \pm 1) = 1 + \frac{(C / \Gamma_D)}{\left( F'M \pm 1 | \tilde{F'M} \right)^2} \left[ x = (\tilde{\omega}_{FM \pm 1} - \tilde{\omega}_{FM}) / \Gamma_D, y = \gamma / 2 \Gamma_D \right]$$

where $\gamma$ is the width of the transition, and the tildes denote wave functions and energies perturbed by the magnetic field and $C = N (2\pi / \hbar) (2J + 1) (2I + 1)$, $N$ is the number density of atoms in the ground state. $\Gamma_D = \omega (2k_B T / mc^2)^{1/2}$ is the Doppler width and the plasma dispersion function $Z(x, y) = \sqrt{\pi} \int_{-\infty}^{\infty} \frac{exp(-\xi^2)}{\xi(x+iy)} d\xi$
For the ground state (5S\(_{1/2}\)), the electric quadrupole coupling constant \(B_J\) is zero and the magnetic dipole term \(A_J\) is quite large (1011.9 MHz for \(^{85}\text{Rb}\) and 3417.3 MHz for \(^{87}\text{Rb}\)). This shows that the low field approximation \((g_J\mu_BB \ll A_J)\) is applicable in the ground state and in this case \(F\) is a good quantum number. Our earlier work \([12]\) indicates that the non-linear nature of Zeeman sublevels is more pronounced in the excited state (5P\(_{3/2}\)) and it is due to the coupling between different \(F'\) levels with the same \(M_{F'}\) value. Here \(F'\) is not a good quantum number and the energies \(\hbar\omega_{F',M'}\) have to be calculated by diagonalizing the complete Hamiltonian. The energy eigenvalues are shown as functions of the applied magnetic field in figure 1.

The perturbed wavefunctions are given by:

\[
\left| F', M' \right\rangle = \sum_{F} \alpha_{F,M',F,M'} \left| F'', M' \right\rangle,
\]

where \(\alpha\) is the matrix which diagonalizes \(H_{HFS}\). Using the above expression in equation (3) gives

\[
n_{\pm}(\omega, FM, F'M \pm 1) - 1
\]

\[
= (C/\Gamma_D) \left| J' \right| D \left| J \right|^2 Z(x, y) \sum_{F'} (-1)^{F+F'-M} \alpha_{F',M,F,M'} [(2F + 1)(2F'' + 1)]^{1/2} \left( \begin{array}{c} F'' J' I \\ J F' 1 \end{array} \right) \left( \begin{array}{c} F'' 1 \\ -M \mp 1 \pm 1 M \end{array} \right)^2
\]

Here in our calculation we assume \(\alpha_{F',M,F,M'}\) as unity.

The Faraday rotation

\[
\Phi_F = \frac{\epsilon_0 I}{2c} \sum_{F',M} \text{Re}[n_+ (\omega, FM, F'M + 1) - n_- (\omega, FM, F'M - 1)]
\]

In the strong laser field limit, the motion of atoms in the transverse direction is neglected as discussed by Novikova \([13]\). It gives a rotation of the plane of polarization \(\varphi\) and absorption \(A\) as

\[
\varphi = x \log(1 + \frac{1}{x^2})
\]

\[
A \sim \frac{1}{2c} \sum_{F',M} \text{Re}[n_+ (\omega, FM, F'M + 1) - n_- (\omega, FM, F'M - 1)]
\]

\[
= \frac{\epsilon_0 I}{2c} \sum_{F',M} \text{Re}[n_+ (\omega, FM, F'M + 1) - n_- (\omega, FM, F'M - 1)]
\]
\[ A \sim x \arctan \frac{1}{x} \]  

(6)

where \( x = \frac{S\gamma}{|\Omega|} \), \( \Omega \) is the Rabi frequency and \( S \) is the Zeeman level shift.

3. Experimental Setup

A block diagram of our experimental setup is shown in figure 2.

![Figure 2. A Schematic diagram of an Experimental setup](image)

ECDL-External Cavity Diode Laser, ND filter- Neutral Density filter, P-Polarizer, PBS-Polarized beam splitter, PD1, PD2- Photo diodes, H-Helmholtz coil.

The Rb vapour cell is kept inside a Helmholtz coil. For the NMOR measurements in the Faraday geometry, the applied field should be parallel to the direction of the laser beam. A commercial Sacher Lasertechnik 780 nm wavelength external cavity diode laser in Littrow configuration with a large coarse tuning range \( 10 \text{ nm} < \Delta \lambda < 90 \text{ nm} \) and a moderate piezoelectric fine tuning range \( 0.3 \text{ nm} < \Delta \lambda < 0.5 \text{ nm} \) was used. The pyrex 25 x 75 mm rubidium vapour cell without buffer gas supplied by Sacher Lasertechnik contains the natural isotope mixture.

A pump beam alone was used for the NMOR measurements. A polarimetric method was used to measure the NMOR (Faraday rotation). This pump beam can simultaneously act as a control and probe field for the NMOR.

3.1. Detection Scheme for NMOR

The detection scheme employs a ‘‘balanced polarimeter’’ in which a sample is placed between a polarizer and a polarizing beam splitter PBS (analyzer) whose transmission axes are oriented at 45° to one another. PD1 and PD2 detect the signal for resultant polarization (\( \sigma_+ \), \( \sigma_- \)) components.

Rotation angle \( \varphi = \frac{1}{2} \arcsin \left( \frac{I_1 - I_2}{I_1 + I_2} \right) \)

Transmission power = \( I_1 + I_2 \)

\( I_1 \) and \( I_2 \) are the light intensities detected in the two output channels (PD1 and PD2) of the analyzer. In our experiment, a balanced receiver was used to initialize the photo diodes (PD1 and PD2) output in the earth magnetic field (~ 0.04 mT). We have nullified the resultant output of both the photo diodes with respect to the earth magnetic field. Therefore our measurements in intermediate fields are not affected by the earth’s field.
4. Results and Discussion

The $D_2$ line structure of rubidium is complex because of the excited state energy levels which are closely spaced and consists of 6 allowed transitions: $F = 1$ to $F' = 0, 1, 2$; $F = 2$ to $F' = 1, 2, 3$ for $^{87}\text{Rb}$, and $F = 2$ to $F' = 1, 2, 3$; $F = 3$ to $F' = 2, 3, 4$ for $^{85}\text{Rb}$. The first excited state magnetic sub level of $^{85}\text{Rb}$ is shown in figure 1. The non linearity of these Zeeman levels is large compared to that of the ground state. Furthermore, the non linearity is less pronounced for $^{87}\text{Rb}$ compared to $^{85}\text{Rb}$. We have considered the Doppler broadened absorption spectrum and the line spacings of the excited state Zeeman sub levels are less than the Doppler width (~550 MHz). It results in an overlap of all transitions within the same Doppler profile.

Intermediate magnetic field strength NMOR, upto 5 mT, for the atomic transitions, $F=1$ to $F'=0, 1, 2$; $F=2$ to $F'=1, 2, 3$ for $^{87}\text{Rb}$ and $F=2$ to $F'= 1, 2, 3$; $F=3$ to $F'= 2, 3, 4$ for $^{85}\text{Rb}$ is shown in figure 3. These measurements have been carried out for the saturation power (~2 mW) and for the low power regimes (~0.4 mW and ~0.15 mW).

![Figure 3](image)

**Figure 3.** NMOR for $^{87}\text{Rb}$ and $^{85}\text{Rb}$ at different laser intensities.

We have computed the NMOR for both the Rb isotopes using equation 4. In figure 3a, the agreement between the experimental data and the calculated results is quite good for a laser intensity of 0.15 mW and it is because we have used the low power approximation to compute NMOR. In the case of $^{87}\text{Rb}$, F=1 to F’=0, 1, 2 and F=2 to F’= 1, 2, 3, the experimental data for 0.15 mW is comparable with the calculated results only above a field of 3 mT (figures 3b and 3c). It clearly shows that the intermediate field approximation is not
valid in the weak field region. The ‘hole burning’ effect, i.e., removal of atoms from the Doppler broadened atomic distribution, can change the direction of rotation [7] and this is observed in our measurements for 2 mW laser power (figure 3d). These power dependent measurements differentiate the Bennet structure related NMOR and coherence NMOR.

The enhancement of non-linearity in the intermediate magnetic field strength is due to the non-linear Zeeman sublevel structure. In figure 3d, for $^{85}$Rb, F=2 to F’=1, 2, 3, it is noted that the structure of NMOR is different from that of the other transitions (figures 3a, 3b and 3c). It is due to the Zeeman level shift $S$ ($\sim$100 to 200 MHz for fields upto 5mT), which is comparatively smaller than that of other Zeeman level shifts ($\sim$230 to 325MHz for $^{87}$Rb, F= 1 to F’=0,1,2; $\sim$ 430 to 500 MHz for $^{87}$Rb, F=2 to F’= 1, 2, 3; $\sim$180 to 260 MHz for $^{85}$Rb, F=3 to F’= 2, 3, 4). The non-linear behaviour is dominant in the saturation power of 2 mW. Rotation varies drastically with respect to the detuning frequency (of the order of GHz) for 5 mT at the saturation power (2 mW). This is shown in figure 4.

![Figure 4. NMOR in Rb D$_2$ line with respect to detuning frequency](image)

The PD1 and PD2 signals for 2 mW are shown in figure 5. Saturation effects and overlap of transitions in the relatively high field regime (3 to 5mT) result in an enhancement of non-linear effects. Transitions F=2 to F’= 1, 2, 3 in $^{85}$Rb and F=3 to F’=2, 3, 4 in $^{85}$Rb overlap over the field of 3 to 5mT and it is shown in figure 5b. This also plays a role in NMOR and complicates the structure in the intermediate field regime. So it is necessary to have a single rubidium isotope in the vapour cell for studies of high field NMOR. In figure 5a, the structure of $^{87}$Rb, F=1 to F’=0, 1, 2 and $^{85}$Rb, F=2 to F’=1, 2, 3 absorption changes its sign over a field of 3 to 5 mT. This shows that saturation effects are important in the intermediate field region.

![Figure 5. Absorption spectra at different fields with the saturation power 2.5 mW.](image)
The low laser field (0.15 mW) NMOR in the Faraday geometry is due to circular birefringence. The high laser field (2 mW) NMOR involves circular birefringence and saturation effects. In addition to that there are non-linear effects because of intermediate magnetic field strength (upto 5mT). While the calculated values are of the same order as the measured values, the lack of a good quantitative agreement between the experimental data and calculated results may be due to inhomogeneous distribution of the laser intensity within the laser beam. Further the theory assumes that the excited state population is negligible and very low power NMOR (< 10 µW) may agree with these calculated results. The saturation and optical pumping effect are also involved in the high laser power NMOR. An appropriate approximation should be made to study these effects in the Rb D₂ line.

5. Conclusions

NMOR has been measured at intermediate fields under moderate laser intensity in Doppler broadened spectrum of ^85Rb and ^87Rb D₂ line. The data has been compared with theoretical calculation and the agreement is satisfactory in some cases. Saturation effects and the non-linear nature of Zeeman sublevels (level shift) are involved in the enhancement of NMOR. More detailed studies are in progress.

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