Abstract. We analyze the prospects at a muon collider for measuring chargino masses in the $\mu^+\mu^- \rightarrow \tilde{\chi}^+\tilde{\chi}^-$ processes in the threshold region. We find that a measurement of the lightest chargino mass to better than 200 MeV is possible with 100 fb$^{-1}$ luminosity. The muon sneutrino mass can also be simultaneously measured to a few GeV.

INTRODUCTION

Particle masses can be measured quite accurately by producing them near threshold. This has been demonstrated recently at LEP II where $W$ pairs were produced at $\sqrt{s} = 161$ GeV, and a precise measurement of the $W$ mass has been obtained. We have recently shown that future high-luminosity $\mu^+\mu^-$ colliders can measure the $W$ boson, top quark and Higgs boson masses with high precision in the processes $\ell^+\ell^- \rightarrow WW, t\bar{t}, ZH$ [1–3]. Threshold production of chargino pairs at a muon collider offers a possible way of measuring the chargino mass and also the muon sneutrino mass that is involved in the production process [4].

Muon colliders [5–9] could be especially useful tools in precision measurements of particle masses, widths, and couplings. Initial state radiation from muons is reduced compared to electrons, and muon colliders have negligible beamstrahlung. The threshold regions for particle production depend on the particle widths. In fact, one can in principle measure the widths of the $W$ boson, the top quark, and the Higgs boson width by performing the appropriate measurements of the production cross sections near threshold. When the lightest chargino is dominantly gaugino, its width is usually negligibly small and the threshold cross section is controlled only by angular momentum considerations and the characteristics of the colliding beam. The measurement of the chargino mass via the threshold cross section has been considered previously for electron-positron machines in Ref. [10,11]. We consider...
the measurement at a muon collider with high luminosity, carefully taking into account the beam effects and reoptimizing cuts to eliminate the background in the threshold region. We assume here that the muon collider has a relatively modest beam energy spread of \( R = 0.1\% \), where \( R \) is the rms spread of the energy of a muon beam\(^2\). We assume that 100 fb\(^{-1}\) integrated luminosity is available; high luminosity is necessary if the threshold measurement is to prove interesting.

A precision measurement of the chargino mass will be highly desirable to test patterns of supersymmetry breaking. For example the relationship between the lightest neutralino and the lightest chargino masses can be used to test the existence of a universal soft SUSY-breking parameter. The chargino pair production process has been investigated beyond the tree-level recently [14]. A precision measurement of the cross section can test radiative corrections coming from heavy squarks.

**SIGNAL AND BACKGROUND**

A simultaneous measurement of the chargino and sneutrino masses requires a sampling of the cross section at at least two points. As in other threshold measurements, the statistical precision on the chargino mass is maximized just above \( 2m_{\tilde{\chi}^\pm} \). However as is evident from Fig. 1, a change in the cross section at \( \sqrt{s} = 2m_{\tilde{\chi}^\pm} + 1 \) GeV can be due to a variation in the sneutrino mass, so a second measurement of the cross section must be taken at a higher \( \sqrt{s} \) where the dependence of the cross section on the chargino mass and the slepton mass is different. It turns out to be advantageous for the chargino mass measurement to choose this higher energy measurement at a point where the chargino cross section is not flat.

The precision that can be obtained in the chargino mass depends substantially on the chargino mass itself: the heavier the chargino the smaller the production cross section. The cross section also depends on the mass of the sneutrino which appears in the \( t \)-channel. The contribution from the sneutrino graph interferes destructively with the \( s \)-channel graphs. If the lightest chargino is gaugino-dominated, then changing the parameters of the chargino mass matrix essentially changes the mass but not the chargino couplings significantly. Therefore one can envision a measurement of the cross section that depends on just two parameters: the chargino mass \( m_{\tilde{\chi}^\pm} \) and the sneutrino mass \( m_{\tilde{\nu}} \).

The chargino decay mode is \( \tilde{\chi}^\pm \rightarrow \tilde{\chi}^0 f \bar{f} \). If \( m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0} > M_W \) then \( W \) exchange dominates and the final state is comprised of 49\% purely hadronic events, 42\% mixed hadronic-leptonic events, and 9\% purely leptonic events (these ratios are determined by the \( W \) branching fractions). The width of the chargino has a negligible impact on the threshold cross section even the two body decay is possible \( (m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0} > M_W) \) if the lightest chargino is gaugino-dominated. There are several

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\(^2\) The most recent TESLA design envisions a beam energy spread of \( R = 0.2\% \) [12] while the NLC design expects a beam energy spread of \( R = 1.0\% \). A high energy \( e^+e^- \) collider in the large VLHC tunnel would have a beam spread of \( \sigma_E = 0.26 \) GeV [13] which should give numbers precisions comparable to those considered here.
backgrounds to the chargino pair signal, the largest being $\mu^+\mu^- \rightarrow W^+W^-$. The cross section is reduced near threshold, so the cuts to reduce this background need to be reoptimized.

One must worry about the level of the backgrounds, and the systematic error that the residual background presents for the cross section measurement. Figure 1 indicates that with the amount of integrated luminosity we are assuming will be available for the measurement, the cross section is being measured to the few percent level, so an understanding of the background to at least this level is necessary. The backgrounds to chargino pair-production have been investigated in Refs. [15,16] where the signal efficiencies have been obtained for the various final states when the center-of-mass energy is $\sqrt{s} = 500$ GeV. The primary background is $W$ pair production which is very large, but can be effectively eliminated because the $W$’s are produced in the very-forward direction. However, if the energy is reduced so that the collider is operating in the chargino threshold region, then the effectiveness of these cuts might be reduced (the signal events might be expected to be more spherical as well). Therefore the efficiencies should be re-investigated for the threshold measurement. The overall signal efficiency of our cuts in Ref. [4] is about 10\% for the fully hadronic decays.

We have assumed here that the chargino is lighter than the muon sneutrino. If that is not the case, the chargino has a new decay mode: $\tilde{\chi}^\pm \rightarrow \ell^\pm \tilde{\nu}$. The efficiency of the cuts against background would need to be reconsidered if this mode is kinematically allowed.

A further advantage of the threshold measurement is that the chargino mass measurement is somewhat isolated from its subsequent decays. Distributions in the final state observables, say e.g. $E_{jj}$ from the decay $\tilde{\chi}^\pm \rightarrow \tilde{\chi}^0 jj$ [15], depend on the neutralino mass. The cross section for chargino pair production, on the other hand, is independent of the final state particles, and only the branching fractions and detector efficiencies for the various final states impact this measurement (as indicated above, if $m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0} > M_W$ the branching fractions of chargino decay is given essentially in terms of the $W$ branching fractions).

The chargino production cross section decreases with increasing chargino mass. Therefore the precision with which the mass can be measured is better at smaller values of the mass. Figure 2 shows the expect precision with 100 fb$^{-1}$ integrated luminosity for sneutrino masses of 300 and 500 GeV. For a lighter sneutrino, for which the destructive interference between the $s$-channel and $t$-channel graphs is more severe, the precision obtained is reduced. Furthermore inspection of Fig. 1 demonstrates the variability of the cross section is reduced for heavier sneutrino masses leading to a reduced precision measurement. The sneutrino mass can be measured to about 6 GeV accuracy for $m_{\tilde{\nu}} = 300$ GeV and to about 20 GeV accuracy for $m_{\tilde{\nu}} = 500$ GeV. This provides an indirect method of measuring the sneutrino mass (the sneutrino might be too heavy to produce directly).

The result of a fit to the chargino cross section is shown in Fig. 3, taking $M_2 = 120$ GeV and $\tan \beta = 4$ and assuming an integrated luminosity of 100 fb$^{-1}$. The cross section is measured just above the threshold $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 1$ GeV, and at a
FIGURE 1. The threshold region of $\mu^+\mu^- \to \tilde{\chi}^+\tilde{\chi}^-$ for various sneutrino masses, taking $M_2 = 100$ GeV and $\tan \beta = 4$. The rapid rise of the cross section is due to the pair production of spin-1/2 particles with small decay widths. The muon collider is assumed to have a beam energy spread of $R = 0.1\%$.

point well above the threshold, $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 20$ GeV.

POLARIZATION

It is expected that the both beams of a muon collider can be partially polarized, although with some loss of luminosity [9]. This could prove a useful tool for measuring the gaugino and Higgsino components of the chargino. When the chargino is gaugino-dominated, it couples to the left-handed $\mu^-$ because the chargino is then dominantly the partner to the $W$. Since the $WW$ background can be reduced by having substantial right-handed polarization, some improvement can be expected in the chargino mass precision, especially in the case where the chargino is higgsinos-dominated. Only the gaugino component of the chargino couples to the $t$-channel sneutrino exchange, and this can be turned off by operating with polarized $\mu$ beams.

For the gaugino-dominated chargino considered here, both the signal and background are approximately proportional to $(1 - P)^2$ where $P$ is the polarization of the two muon beams. So for fully polarized $\mu^+$ and $\mu^-$ beams one can improve the mass determination by a factor of two.
FIGURE 2. The $1\sigma$ precision obtainable in the chargino mass taking $m_{\tilde{\nu}} = 300$ and 500 GeV. The precision is better for larger sneutrino mass because the contribution from the $t$-channel sneutrino exchange diagram destructively interferes with the $s$-channel diagrams.

CONCLUSIONS

We have shown that a measurement of the lightest chargino mass to better than 200 MeV is possible by measuring the pair production cross section near threshold at a muon collider with 100 fb$^{-1}$ luminosity. This is much better than other techniques. The muon sneutrino mass can also be simultaneously measured to a few GeV.

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FIGURE 3. The $\Delta \chi^2 = 1$ contours in the chargino mass - sneutrino mass plane, taking $M_2 = 120$ GeV, $\tan \beta = 4$, and $m_{\tilde{\nu}} = 300$ and 500 GeV. This gives a chargino mass $m_{\tilde{\chi}^\pm} \approx 123$ GeV. The curves assume 50 fb$^{-1}$ of integrated luminosity is devoted to $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 1$ GeV, and 50 fb$^{-1}$ is applied at $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 20$ GeV. The chargino mass determination is better for higher sneutrino mass since the sneutrino exchange diagrams interferes destructively with the $s$-channel diagrams.

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