Hints of Cosmic String Induced Discontinuities in the COBE Data?

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I. INTRODUCTION

We apply new statistical tests on the four year 53 GHz DMR data and show that there is significant probability for the existence of coherent temperature discontinuities hidden in the CMB maps around the Galactic Poles. Comparing Monte Carlo simulations with the DMR maps we find that the probability for the existence of a coherent discontinuity with amplitude \(0.2 \times (\frac{2}{3})_{\text{rms}}\) superposed on the gaussian fluctuations is more than double than the corresponding probability when no discontinuity is present. This result is consistent with the existence of a horizon size long cosmic string with mass per unit length \(\mu\) given by \(G\mu = \gamma_s \approx 10^{-6}/\pi\) where \(\gamma_s\) is the string velocity and \(\alpha\) is the corresponding Lorentz factor.

New statistical tests have recently been proposed for the detection of coherent discontinuities (edges) hidden in CMB maps [1]. The main advantage of these statistics is that they focus on the large scale coherence properties of CMB maps and are therefore effective even in cases of low resolution maps provided that the sky area covered is large. These properties are shared by the maps produced by the DMR instrument of COBE. The goal of this paper is the application of the new statistical tests analyzed in Ref. [2] on the four year DMR maps in the regions of the Galactic Poles (approximatelly 90° \(\times\) 90° or 32 \(\times\) 32 pixel maps). The statistics calculated for these maps are the skewness, the kurtosis, the Sample Mean Difference (SMD) [4] and the Maximum Sample Difference (MSD) [4]. These results are then compared with the corresponding results obtained from a large number of gaussianized DMR maps (similar power spectrum and random phases in Fourier space) with and without a coherent discontinuity superposed. Thus for each applied statistic we find the value of the superposed coherent discontinuity amplitude \(s = \frac{\alpha}{\sqrt{32}}\) that is most consistent with the actual DMR data.

In particular we ask the following question: ‘Assume that the DMR map around the North Galactic Pole is gaussian but has small coherent temperature discontinuities superposed on it. What is the most probable value of the amplitude of the superposed discontinuity? Also, what is the ratio of the probability that a discontinuity of the given amplitude is present over the probability that no discontinuity is present?’ . Clearly, for a gaussian map we would find that the most probable value of the discontinuity amplitude is 0.

Before describing the technique followed to address the above questions we will briefly review the statistical tests we have used (the notation used here is the same as in Ref. [3]). These tests involve both conventional statistics (skewness and kurtosis) and new statistics (MSD and SMD) optimized for the detection of coherent discontinuities in 1d and 2d pixel maps. Assuming a 32 \(\times\) 32 standardized temperature pixel map \(T_{ij}\) \((i,j = 1,...,32)\), the skewness \(s\) and the kurtosis \(k\) are defined as:

\[
s = \frac{1}{T} \sum_{i,j} T_{ij}^3/32^2
\]

\[
k = \frac{1}{T} \sum_{i,j} T_{ij}^4/32^2
\]

These are conventional statistics and their values for gaussian maps with uncorrelated pixels are \(s = 0, k = 3\). To define the statistics MSD and SMD we consider a partition of the CMB in two parts separated by a random line. In this study we have considered straight lines but the analysis can be generalized to more general types of lines with no significant changes in the results. Let \(k\) denote the set of parameters that define the partition line and let \(T_u\) and \(T_l\) be the mean temperatures of the two parts of the map (the indices ‘\(u\)’ and ‘\(l\)’ stand for ‘upper’ and ‘lower’ parts). The statistical variable \(Y_k\) is defined as:

\[
Y_k = T_u - T_l
\]

The statistics Sample Mean Difference (SMD) and Maximum Sample Difference (MSD) are defined as :

\[
SMD = \frac{1}{N} \sum_k Y_k
\]

\[
MSD = \max_k (Y_k)
\]

where \(N\) is the total number of partitions. Both MSD and SMD approach asymptotic values for large \(N\).
Ref. [1] it was shown that the presence of the coherent discontinuity $k_0$ can be detected much more efficiently by the statistics SMD and MSD than by the skewness and kurtosis. A physically motivated mechanism which can lead to the production of a coherent discontinuity on CMB maps is the presence of a moving long cosmic string in our horizon [2].

The main mechanism by which strings can produce CMB fluctuations on angular scales larger than $1^\circ - 2^\circ$ has been well studied both analytically [3] and simulated numerically [4] and is known as the Kaiser-Stebbins effect [3]. According to this effect, moving long strings present between the time of recombination $t_{rec}$ and the present time $t_0$, produce step-like temperature discontinuities between photons that reach the observer through opposite sides of the string. These discontinuities are due to the peculiar nature of the spacetime around a long string which even though is locally flat, globally has the geometry of a cone with deficit angle $8\pi G\mu$ ($G$ is Newton’s constant, $\mu$ is the mass per unit length of the string and we have used units with $c = 1$). The magnitude of the discontinuity is proportional to the deficit angle, to the string velocity $v_s$ and depends on the relative orientation between the unit vector along the string $\hat{s}$ and the unit photon wave-vector $\hat{k}$. It is given by [3]:

$$\frac{\delta T}{T} = \pm 4\pi G\mu v_s \gamma_s \hat{k} \cdot (\hat{v}_s \times \hat{s})$$

(6)

where $\gamma_s$ is the relativistic Lorentz factor and the sign changes when the string is crossed. The angular scale over which this discontinuity persists is given by the radius of curvature of the string which according to simulations [3] is approximately equal to the horizon scale. The growth of the horizon from $t_{rec}$ to $t_0$ results in a superposition of a large number of step-like temperature seeds of all sizes starting from about $2^\circ$ (the angular size of the horizon at $t_{rec}$) to about $180^\circ$ (the present horizon scale). By the Central Limit Theorem (CLT) this large number of superposed seeds results in a pattern of fluctuations that obeys gaussian statistics. Thus the probability distribution for the temperature of each pixel of a CMB map with resolution larger than about $1^\circ - 2^\circ$ is a gaussian [3].

However, on large angular scales, despite the large number of superposed seeds there is also coherence of induced fluctuations. The fluctuations on these scales may be viewed as a superposition of a gaussian scale invariant background coming mainly from small scale seeds plus a small number of step-like discontinuities which are coherent and persist on angular scales larger than $100^\circ$. The presence of coherent discontinuities due to late long strings, superposed on a gaussian background appears to be a generic feature of cosmic string models. In what follows we will explore the consequences on non-Gaussianity of this generic feature of the cosmic string models.

II. STATISTICAL TESTS

Most of the statistical tests for the detection of non-Gaussianity focus on small scale properties (e.g. peaks) of CMB maps, because it is usually thought that the CLT tends to hide non-Gaussianity on large scales due to the large number of superposed seeds on these scales. This expectation, however, is incorrect for the type of non-Gaussianity induced by moving late long strings. The basic feature of this type of non-Gaussian fluctuations is the large scale coherence and can thus be best detected by a specially designed statistical test. Such are the tests SMD and MSD analysed both analytically and numerically in Ref. [1].

Here, we apply these statistics along with the more conventional skewness and kurtosis on the four year 53 GHz DMR data. To find the probability that the DMR maps (around the Galactic Poles) are gaussian with a superposed discontinuity of a given amplitude $\frac{\delta T}{T} = \alpha \times (\frac{\delta T}{rms})$, we apply the following technique:

1. We consider two $32 \times 32$ pixel maps of the 53 GHz (A+B)/2 DMR data (monopole and dipole removed, optimum Galactic cut: 3881 surviving pixels) around the Galactic Poles [3] (about $90^\circ \times 90^\circ$ on the sky). We calculate the statistics: skewness, kurtosis, SMD and MSD.

2. We construct a large number (1200) of gaussianized maps obtained from the DMR map by randomizing the phases in Fourier space using a gaussian spectrum with 0 mean and variance equal to the DMR measured spectrum.

3. For each one of the constructed maps we calculate the four statistics: skewness, kurtosis, SMD and MSD.

4. We superpose on the constructed maps a random coherent discontinuity of amplitude $\alpha \times (\frac{\delta T}{rms})$, obtained by a straight line partition and recalculate the skewness, kurtosis, SMD, and MSD statistics.

5. Using the results from steps 3 and 4 we construct the probability distribution for the skewness, kurtosis, SMD and MSD for various values of $\alpha$ (e.g. Figs. 1, 2).

1 As North and South Galactic Pole regions we have used faces 0 and 5 of the COBE quadrilateraled spherical cube projection (CSC) in galactic pixelization, according to the plots and pixel numbering schemes found at:

http://www.gsfc.nasa.gov/astro/cobe/skymap_info.html
6. We use the results from steps 1 and 5 to find the probability for obtaining the already calculated, from step 1, COBE DMR values of skewness, kurtosis, SMD and MSD in the gaussianized maps as a function of $\alpha$. Thus we have four probability distributions, one for each statistic: $P_s(\alpha), P_k(\alpha), P_{SMD}(\alpha), P_{MSD}(\alpha)$ (Figs. 3, 4).

For convenience we will hereafter use normalized probability distributions defined as:

$$Q(\alpha) \equiv \frac{P(\alpha)}{P(\alpha = 0)}$$

($7$)

$Q(\alpha)$ helps us answer the question: 'How probable is the existence of a coherent discontinuity within the studied maps?'. Fig. 1 shows the probability distribution of the skewness, for various values of $\alpha$. Clearly, this statistic is insensitive to the detection of a steplike coherent discontinuity in the data. The situation is similar for the kurtosis.

Thus, these two conventional statistics cannot differentiate between a gaussian map and a map with a small superposed coherent discontinuity with $\alpha \leq 0.8$. This is clearly seen in Figs. 3, 4 where $Q_s(\alpha), Q_k(\alpha)$ are shown to be quite flat functions of $\alpha$.

The situation is quite different for the MSD and SMD statistics. The probability distribution for the MSD, for example, is clearly very sensitive to the value of $\alpha$ and its maximum occurs for $\alpha = 0.2$, thus favouring the existence of a discontinuity (Fig. 2).

Clearly, the distributions $Q_{MSD}(\alpha)$ and $Q_{SMD}(\alpha)$, as it is shown in Figs. 3, 4, demonstrate a conspicuous peak at $\alpha \approx 0.2$ with $Q_{MSD}(\alpha=0.2) \approx 2.4$ indicating that the presence of a discontinuity with $\alpha \approx 0.2$ is more than twice as probable compared to a purely gaussian map. We therefore conclude that there are clear hints for the existence of a temperature discontinuity in the 53 GHz (A+B)/2 DMR data in the region of the North Galactic Pole superposed on the gaussian fluctuations (Fig. 3).

We have repeated the analysis for the data in the region of the South Galactic Pole (Fig. 4) with very
similar results, indicating the presence of a similar type of discontinuity.

The presence of such discontinuities on top of the anticipated gaussian fluctuations could be due to the presence of a long string in our present horizon. According to the results from the application of the MSD statistic, such a string would go approximately through the galactic coordinates (230°.98, 71°.68) and (330°.25, 48°.98) in the North Galactic Pole region (or pixels 612 and 29 according to the quadrilateralized spherical cube). The corresponding discontinuity in the South Galactic Pole goes through the coordinates (315°.00, −73°.57) and (122°.02, −73°.12) or pixels 5733 and 5553. It is straightforward to check that the position of this discontinuity relative to the dipole anisotropy excludes the possibility that the discontinuity is an artifact of improper removal of the dipole anisotropy.

III. CONCLUSIONS

We have shown that the 53 GHz (A+B)/2 DMR data in the regions around the Galactic Poles show clear hints for the existence of hidden temperature discontinuities coherent on large angular scales. These hints have been revealed by the application of the new MSD and SMD statistics which were especially designed for the detection of coherent discontinuities hidden in the fluctuation patterns. We have also shown that the conventional statistics of skewness and kurtosis are unable to reveal these features. The 53 GHz DMR maps in the regions around the Galactic Poles are consistent with the presence of no discontinuities at the 1σ level, but the probability for the existence of a coherent discontinuity with amplitude $\frac{\delta T}{T} = \alpha \times (\frac{\delta T}{T})_{rms} \simeq 4 \times 10^{-6}$ is more than double compared to the probability that no discontinuity is present. This result could be interpreted as an indication for the existence of a moving long string in our present horizon with $G \mu \nu / \gamma_s \simeq 10^{-6}/\pi$.

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