Regular phantom black holes

K.A. Bronnikov  
Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow, Russia; Institute of Gravitation and Cosmology, PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia. E-mail: kb20@yandex.ru

J.C. Fabris  
Departamento de Física, Universidade Federal do Espírito Santo, Vitória, 29060-900, Espírito Santo, Brazil. E-mail: fabris@cco.ufes.br

For self-gravitating, static, spherically symmetric, minimally coupled scalar fields with arbitrary potentials and negative kinetic energy (favored by the cosmological observations), we give a classification of possible regular solutions to the field equations with flat, de Sitter and AdS asymptotic behavior. Among the 16 presented classes of regular solutions are traversable wormholes, Kantowski-Sachs (KS) cosmologies beginning and ending with de Sitter stages, and asymptotically flat black holes (BHs). The Penrose diagram of a regular BH is Schwarzschild-like, but the singularity at \( r = 0 \) is replaced by a de Sitter infinity, which gives a hypothetic BH explorer a chance to survive. Such solutions also lead to the idea that our Universe could be created from a phantom-dominated collapse in another universe, with KS expansion and isotropization after crossing the horizon. Explicit examples of regular solutions are built and discussed. Possible generalizations include \( k \)-essence type scalar fields (with a potential) and scalar-tensor theories of gravity.

PACS numbers: 04.70.Bw 95.35.+d 98.80.-k

Observations provide more and more evidence that the modern accelerated expansion of our Universe is governed by a peculiar kind of matter, called dark energy (DE), characterized by negative values of the pressure to density ratio \( w \). Moreover, by current estimates, even \( w < -1 \) seems rather likely \([26,27,28]\), though many of such estimates are model-dependent. Thus, assuming a perfect-fluid DE with \( w = \text{const} \) implies, using various observational data (CMB, type Ia supernovae, large-scale structure), \(-1.39 < w < -0.79\) at 2\( \sigma \) level \([20]\). Considerations of a variable DE equation of state \([26,27,28]\) also allow highly negative values of \( w \). A model-independent study \([5]\) of data sets containing 172 SNIa showed a preferable range \(-1.2 < w < -1\) for the recent epoch. Similar figures follow from an analysis of the Chandra telescope observations of hot gas in 26 X-ray luminous dynamically relaxed galaxy clusters \([6]\): \( w = -1.20^{+0.24}_{-0.26} \).

Moreover, a highly negative \( w \) makes negligible the undesirable DE contribution to the total energy density in the period of structure formation. Thus, even if the cosmological constant, giving precisely \( w = -1 \), is still admitted by observations as possible DE, there is a need for a more general framework allowing \( w < -1 \).

The perfect-fluid description of DE is plagued with instability at small scales due to imaginary velocity of sound; more consistent descriptions providing \( w < -1 \) use self-interacting scalar fields with negative kinetic energy (phantom scalars) or tachyonic fields \([29,30,31]\) (see also references therein). To avoid the obvious quantum instability, a phantom scalar may perhaps be regarded as an effective field description following from an underlying theory with positive energies \([25,26]\). Curiously, in a classical setting, a massless phantom field even shows a more stable behavior than its usual counterpart \([12,13]\). A fundamental origin of phantom fields is under discussion, but they naturally appear in some models of string theory \([7]\), supergravities \([14]\) and theories in more than 11 dimensions like F-theory \([15]\).

If a phantom scalar, be it basic or effective, is part of the real field content of our Universe, it is natural to seek its manifestations not only in cosmology but also in local phenomena, in particular, in black hole (BH) physics, as, e.g., in the recent works on DE accretion onto BHs \([16,17]\) and on BH interaction with a phantom shell \([18]\).

We are trying here to find out which kinds of regular static, spherically symmetric configurations may be formed by a phantom scalar field itself. Since it does not respect the usual energy conditions, nonsingular solutions of particular physical interest for BH physics and/or cosmology could be expected. Our main finding is, in our view, the existence of regular asymptotically flat BH solutions with an expanding, asymptotically de Sitter Kantowski-Sachs (KS) cosmology beyond the event horizon. It is, to our knowledge, quite a new way of avoiding a BH central singularity, alternative to known solutions with a regular center (see, e.g., \([19,20,21]\)).

The plan is as follows. After writing the field equations, we mention some no-go theorems for both normal and phantom scalars (without proofs), making sure that they leave sufficient freedom for solutions of interest. Then follows a simple qualitative analysis which reveals 16 classes of possible nonsingular solutions and the properties of the potential necessary for their existence. Using the inverse problem method, we construct simple explicit examples of different types of solutions, which include, among others, regular KS cosmologies, asymptotically de Sitter both in the past and in the future, and regular black holes, asymptotically flat or anti-de Sitter cosmologies.
(AdS) in the static (R) region and asymptotically de Sitter in the nonstatic (T) region. Some generalizations of these results are indicated in conclusion.

We start with the action for a self-gravitating minimally coupled scalar field with an arbitrary potential $V(\phi)$

$$S = \int \sqrt{-g} d^4x [R + \varepsilon g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - 2V(\phi)], \quad (1)$$

where $R$ is the scalar curvature, $\varepsilon = +1$ corresponds to a usual scalar field with positive kinetic energy and $\varepsilon = -1$ to a phantom field. For the general static, spherically symmetric metric

$$ds^2 = A(\rho) dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho) (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2)$$

and $\phi = \phi(\rho)$, the scalar field equation and three independent combinations of the Einstein equations read

$$(A r^2 \phi')' = \varepsilon r^2 dV/d\phi, \quad (3)$$

$$(A') r^2 = -2r^2 V; \quad (4)$$

$$2r''/r = -\varepsilon \phi'^2; \quad (5)$$

$$A(r^2)'' - r^2 A' = 2, \quad (6)$$

where the prime denotes $d/d\rho$. Eq. (4) follows from (4)–(6), which, given a potential $V(\phi)$, form a determined set of equations for the unknowns $r(\rho)$, $A(\rho)$, $\phi(\rho)$. Eq. (6) can be once integrated giving

$$B' \equiv \left(\frac{A}{r^2}\right)' = \frac{2(\rho_0 - \rho)}{r^4}, \quad (7)$$

where $B(\rho) = A/r^2$ and $\rho_0$ is an integration constant.

The coordinate $\rho$ has been chosen in such a way that Killing horizons, if any, correspond to regular zeros of the function $A(\rho) = A(\rho - \rho_0)^p$ where $p \in \mathbb{N}$ is the order of the horizon limit. The metric is static where $A(\rho) > 0$ (in R regions), while where $A < 0$ (in T regions) $\rho$ is a time coordinate, and describes a homogeneous anisotropic KS cosmology.

Some general consequences of Eqs. (4)–(6) (no-go theorems) constrain the nature of possible solutions.

(A) For $\varepsilon = +1$ one cannot obtain wormholes or configurations ending with a regular 3-cylinder of finite radius $r$. This result follows from Eq. (6) (giving $r'' \geq 0$) and is valid independently of the large $r$ behaviour of the metric — flat, AdS or any other. For phantom fields Eq. (6) gives $r'' \geq 0$, and such a restriction is absent. Thus, for a free massless phantom field wormhole solutions are well known since the 70s.

(B) Particlelike (or starlike) solutions (PLS), i.e., asymptotically flat solutions with a regular center, are not excluded for both kinds of scalar fields but under certain constraints on the potential. Thus, for $\varepsilon = +1$, PLS cannot be obtained with $V(\phi) \geq 0$.

For $\varepsilon = -1$, on the contrary, no PLS can appear if $V(\phi) \leq 0$, and PLS with mass $m > 0$ require a potential of alternating sign. The (Schwarzschild) mass $m$ is defined here by assuming $r \approx \rho$ and $A \approx 1 - 2m/\rho$ as $\rho \to \infty$.

(C) A no-hair theorem, extending the one known for $\varepsilon = +1$, sounds as follows: The only asymptotically flat BH solution to Eqs. (4)–(6) is characterized by $\phi = c$ and the Schwarzschild metric in the whole domain of outer communication, if either (i) $\varepsilon = +1$ and $V(\phi) \geq 0$ or (ii) $\varepsilon = -1$, $V(\phi) \leq 0$, and $r' > 0$ at and outside the event horizon.

It will be seen below that there exist BH solutions where the condition $r' > 0$ is violated.

(D) Eq. (7) severely restricts the possible dispositions of Killing horizons in the resulting metric and consequently the global causal structure of space-time.

Indeed, horizons are regular zeros of $A(\rho)$ and hence $B(\rho)$. By (7), $B(\rho)$ increases at $\rho < \rho_0$, has a maximum at $\rho = \rho_0$ and decreases at $\rho > \rho_0$. It can have at most two simple zeros, bounding a range $B > 0$ (R region), or one double zero and two T regions around. It can certainly have a single simple zero or no zeros at all.

So the choice of possible types of global causal structure is precisely the same as for the general Schwarzschild-de Sitter solution with arbitrary mass and cosmological constant.

Eq. (5) does not contain $\varepsilon$, hence this result (the Global Structure Theorem) equally applies to normal and phantom fields. It holds for any sign and shape of $V(\phi)$ and under any assumption on the asymptotics. In particular, BHs with scalar hair (certainly, respecting the no-hair theorems) are not excluded. Some examples of (singular) BHs with both normal (e.g., AdS) and phantom scalar hair are known. However, asymptotically flat BHs with a regular center are ruled out.

The Hawking temperature of a horizon $\rho = h$ is determined as $T_H = \kappa/(2\pi)$ where $\kappa$ is the surface gravity at $\rho = h$. In our system,

$$\kappa = |A'(h)|/2 \quad \text{and} \quad A'(h) = (\rho_0 - h)/r^2(h). \quad (8)$$

Let us indicate the possible kinds of nonsingular solutions without restricting the shape of $V(\phi)$. Assuming no pathology at intermediate $\rho$, regularity is determined by the system behavior at the ends of the $\rho$ range. The latter may be classified as a regular infinity ($\rho \to \infty$), which may be flat, dS of AdS (other variants, like $r^2 \sim \rho$, can exist but seem to be of lesser interest), a regular center, and the intermediate case $r \to r_0 > 0$. Any kind of oscillatory behavior of $r(\rho)$ is ruled out by the constant sign of $r''$.

Suppose we have a regular infinity as $\rho = \infty$, so that $V \to V_+ = \text{const}$ while the metric becomes Minkowski (M), dS (dS) or AdS according to the sign of $V_+$. In all cases $r \approx \rho$ at large $\rho$.

For $\varepsilon = -1$, due to $r'' \leq 0$, $r$ necessarily vanishes at some $\rho = \rho_c$, which means a center, and the only possible regular solutions interpolate between a regular center and an AdS, flat or dS asymptotic; in the latter case the causal structure coincides with that of de Sitter space-time.
For $\varepsilon = -1$, there are similar solutions with a regular center, but due to $r'' \geq 0$ one may assume $\rho \in \mathbb{R}$ and obtain either $r \to r_0 = \text{const} > 0$ or $r \to \infty$ as $\rho \to \infty$. In other words, all kinds of regular behavior are possible at the other end. In particular, if $r \to r_0$, we get $A \approx -\rho^2/r_0^2$, i.e., a T region comprising a highly anisotropic KS cosmology with one scale factor ($r$) tending to a constant while the other ($A$) inflates. The scalar field tends to a constant, while $V(\phi) \to 1/r_0^2$.

Thus there are three kinds of regular asymptotics at one end, $\rho \to \infty$ (M, dS, AdS), and four at the other, $\rho \to -\infty$: the same three plus $r \to r_0$, simply $r_0$ for short. (The asymmetry has appeared we did not allow r to enter as $\rho \to \infty$. The inequality $r'' > 0$ forbids nontrivial solutions with two such $r_0$-asymptotics.) This makes nine combinations shown in Table 1. Moreover, each of the two cases labelled KS* actually comprises three types of solutions according to the properties of $A(\rho)$: there can be two simple horizons, one double horizon or no horizons between two dS asymptotics. Recalling 3 kinds of solutions with a regular center, we obtain as many as 16 qualitatively different classes of globally regular configurations of phantom scalar fields.

### Table I: Regular solutions with $\rho \in \mathbb{R}$ for $\varepsilon = -1$

| $A$S | $M$ | dS | $r_0$ |
|------|-----|----|-------|
| AdS | wormhole | wormhole | black hole | black hole |
| M   | wormhole | sym | black hole | black hole |
| dS  | sym | sym | KS* | KS* |

Examples of each behavior may be found in an algorithmic manner by properly choosing the function $r(\rho)$ and invoking the inverse problem method: $B(\rho)$ and $A(\rho)$ are then obtained from Eq. (7) (and $B(\rho)$ always behaves as described above), after that $\phi(\rho)$ is yielded by Eq. (5) and $V(\phi)$ by Eq. (4). A critical requirement is that $r(\rho)$ must satisfy the inequality $r'' \leq 0$ for $\varepsilon = 1$ and $r'' \geq 0$ for $\varepsilon = -1$. The function $V(\phi)$ is restored from known $V(\rho)$ and $\phi(\rho)$ provided the latter is monotonic, which is the case if到处 everywhere $r'' \neq 0$.

The potential $V$ tends to a constant and, moreover, $dV/d\phi \to 0$ at each end of the $r$ range. Therefore any model from the above classes requires a potential with at least two zero-slope points (not necessarily extreme) at different values of $\phi$. Suitable potentials are, e.g., $V = V_0 \cos^2(\phi/\phi_0)$ and the Mexican hat potential $V = (\lambda/4)(\phi^2 - \eta^2)^2$ where $V_0, \phi_0, \lambda, \eta$ are constants. A flat infinity certainly requires $V_+ = 0$, while a de Sitter asymptotic can correspond to a maximum of $V$ since phantom fields tend to climbing up the slope of the potential rather than rolling down, as is evident from Eq. (6). Accordingly, Faraoni [4], considering spatially flat isotropic phantom cosmologies, has shown that if $V(\phi)$ is bounded above by $V_0 = \text{const} > 0$, the de Sitter solution is a global attractor. Very probably this conclusion extends to KS cosmologies after isotropization.

We will now give a transparent analytic example, leaving for the future more elaborated models with better motivated potentials. So we put $\varepsilon = -1$,

$$r = (\rho^2 + b^2)^{1/2}, \quad b = \text{const} > 0.$$  

and use the inverse problem scheme. Eq. (7) gives

$$B(\rho) = A(\rho)/r^2(\rho)$$

$$= \frac{c}{b^2} + \frac{1}{b^2 + \rho^2} + \frac{\rho_0}{b^3} \left( \frac{b \rho}{b^2 + \rho^2} + \tan^{-1} \frac{\rho}{b} \right),$$

where $c = \text{const}$. Eqs. (5) and (4) then lead to expressions for $\phi(\rho)$ and $V(\phi)$:

$$\phi = \pm \sqrt{2} \tan^{-1}(\rho/b) + \phi_0,$$

$$V = -\frac{c}{b^2} \left[ 1 + \frac{2\rho^2}{r^2} \right] - \frac{\rho_0}{b^3} \left[ 3b \rho + \left( 1 + \frac{2\rho^2}{r^2} \right) \tan^{-1} \frac{\rho}{b} \right].$$

with $r = r(\rho)$ given by (11). In particular,

$$B(\pm \infty) = -\frac{1}{3} V(\pm \infty) = \frac{2bc \pm \pi \rho_0}{2b^3}.$$  

Choosing in (11), without loss of generality, the plus sign and $\phi_0 = 0$, we obtain for $V(\phi)$ ($\psi := \phi/\sqrt{2}$):

$$V(\phi) = -\frac{c}{b^2} \left[ 3 - 2 \cos^2 \psi \right] - \frac{\rho_0}{b^3} \left[ 3 \sin \psi \cos \psi + \psi(3 - 2 \cos^2 \psi) \right].$$

The solution behavior is controlled by two integration constants: $c$ that moves $B(\rho)$ up and down, and $\rho_0$ which fixes the maximum of $B(\rho)$. Both $r(\rho)$ and $B(\rho)$ are even functions if $\rho_0 = 0$, otherwise $B(\rho)$ loses this symmetry.

In the simplest case $\rho_0 = c = 0$ we obtain the so-called Ellis wormhole [22]: $V = 0$ and $A = 1$.

Solutions with $\rho_0 = 0$ but $c \neq 0$ describe symmetric structures: wormholes with two AdS asymptotics if $c > 0$ and solutions with two dS asymptotics if $c < 0$. If $0 > c > -1$, there is an R region in the middle, bounded by two simple horizons, at $c = -1$ they merge into a double horizon, and $c < -1$ leads to a pure KS cosmology.

With $\rho_0 \neq 0$, the two asymptotics look differently. Let us dwell upon solutions which are flat at $\rho = \infty$. Then the constants obey the condition $2bc = -\pi \rho_0$, while the Schwarzschild mass is $m = \rho_0/3$. According to [13], for $\rho_0 < 0$ ($c > 0$) we obtain a wormhole with $m < 0$ and an AdS metric at the far end, corresponding to the cosmological constant $V_\perp < 0$. For $\rho_0 > 0$, when $V_\perp > 0$, there is a regular BH with $m > 0$ and a dS asymptotic far beyond the horizon. As any asymptotically flat BH with...
a simple horizon, it has a Schwarzschild-like causal structure, but the singularity $r=0$ in the Carter-Penrose diagram is replaced by $r=\infty$.

The horizon radius depends on both parameters $m$ and $b = \min r(\rho)$ and cannot be smaller than $b$, which also plays the role of a scalar charge: $\psi \approx \pi/2 - b/\rho$ at large $\rho$. Since $A(0) = 1 + c$, the throat $r = \rho$ is located in the R region if $c > -1$, i.e., if $3m < 2b$, at the horizon if $3m = 2b$ and in the T region beyond it if $3m > 2b$.

Such regular BHs combine the properties of BHs, whose main feature is a horizon, and wormholes, whose main feature is a throat, $r = r_{\text{min}} > 0$. The above relations between $m$ and $b$ show (and it is probably genetically true) that if the BH mass dominates over the scalar charge, the throat is invisible to a distant observer, and the BH looks almost as usual in general relativity. However, a possible BH explorer now gets a chance to survive for a new life in an expanding KS universe.

One may also speculate that our Universe could appear from collapse to a phantom BH in another, “mother” universe and undergo isotropization (e.g., due to particle creation) soon after crossing the horizon. The KS structure of our Universe is not excluded observationally if its isotropization had happened early enough, before the last scattering epoch (at redshifts $z \gtrsim 1000$). The same idea of a Null Bang instead of a Big Bang (cosmological expansion starting from a horizon rather than a singularity) was discussed in [21] for a system with a de Sitter vacuum core and a regular center in the R region.

Let us note in conclusion that the present analysis, which has revealed a wealth of regular solutions including BHs, is easily extended to more sophisticated phantom models, e.g., to those of k-essence type. Indeed, for the scalar field Lagrangian $L = P(X) - 2V(\phi)$ where $X = g^{\mu\nu}\phi_\mu\phi_\nu$ and $P$ is an arbitrary function, Eqs. (6) and (7) remain unchanged while the crucial inequality $r'' \geq 0$ holds if the theory satisfies the “phantom condition” $dP/dX < 0$. k-essence type theories, among other merits, are known to avoid inadmissible sound velocities and the stabilization problem [34].

Other obvious generalizations are scalar-tensor theories of gravity and, as their subclass, nonminimally coupled scalar fields with Lagrangians including constant $R\phi^2$. Such theories are conformally related to [1], and the conformal factors, if well-behaved, do not change the causal and asymptotic properties of the solutions.

Acknowledgments. We are thank Nelson Pinto-Neto and Jérôme Martin for helpful discussions. KB thanks the colleagues from DF-UFES for hospitality. The work was supported by CNPq (Brazil); KB was also supported by ISTC Project 1655.

[1] A. Upadhye, M. Ishak and P.J. Steinhardt, Phys. Rev. D 72, 063501 (2005).
[2] Y. Wang and M. Tegmark, Phys. Rev. Lett. 92, 241302 (2004).
[3] U. Seljak et al., Phys. Rev. D 71, 103515 (2005).
[4] S. Hannestad and E. Mortsell, JCAP 0409, 001 (2004).
[5] U. Alam, V. Sahni, T.D. Saini and A.A. Starobinsky, Mon. Not. R. Astron. Soc. 354, 275 (2004).
[6] S.W. Allen et al., Mon. Not. R. Astron. Soc. 353, 457 (2004).
[7] A. Sen, JHEP 0204, 048 (2002); 0207, 065 (2002).
[8] V. Gorini, A. Yu. Kamenshik, U. Moschella and V. Pasquier, Phys. Rev. D 69, 123512 (2004).
[9] V. Faraoni, Class. Quantum Grav. 22, 3235 (2005).
[10] S. Nojiri and S.D. Odintsov, Phys. Lett. 562B, 147 (2003).
[11] S.M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003).
[12] K.A. Bronnikov, G. Clément, C.P. Constantinidis and J.C. Fabris, Phys. Lett. A243, 121 (1998), gr-qc/9801050.
[13] C. Armendariz-Picon, Phys. Rev. D 65, 104010 (2002).
[14] H.P. Nilles, Phys. Rep. 110, 1 (1984).
[15] N. Khviengia, Z. Khviengia, H. Lü and C.N. Pope, Class. Quantum Grav. 15, 759 (1998).
[16] E.O. Babichev, V.I. Dokuchaev and Yu.N. Eroshenko, Phys. Rev. Lett. 93, 021102 (2004).
[17] A. Frolov, “Accretion of ghost condensate by black holes”, hep-th/0404216.
[18] V. Berezin, V. Dokuchaev, Yu. Eroshenko and A. Smirnov, Class. Quantum Grav. 22, 4443 (2005).
[19] I. Dymnikova, Gen. Rel. Grav. 24, 235 (1992).
[20] K.A. Bronnikov, Phys. Rev. D 63, 044005 (2001).
[21] K.A. Bronnikov, A. Dobosz and I.G. Dymnikova, Class. Quantum Grav. 20, 3797 (2003).
[22] H.G. Ellis, J. Math. Phys. 14, 104 (1973).
[23] K.A. Bronnikov, Phys. Rev. D 64, 064013 (2001).
[24] K.A. Bronnikov, Acta Phys. Pol. B4, 251 (1973).
[25] K.A. Bronnikov, S.B. Fadeev and A.V. Michtchenko, Gen. Rel. Grav. 35, 505 (2003), gr-qc/0212065.
[26] J.D. Bekenstein, Phys. Rev. D 5, 1239 (1972); ibid., 2403; “Black holes: classical properties, thermodynamics, and heuristic quantization”, in: “Cosmology and Gravitation”, ed. M. Novello, Atlantisciences, France, 2000, pp. 1-85, gr-qc/9808028 (review).
[27] S. Adler and R.B. Pearson, Phys. Rev. D 18, 2798 (1978).
[28] K.A. Bronnikov and G.N. Shikin, Grav. & Cosmol. 8, 107 (2002), gr-qc/0109027.
[29] C.K.C. Chan, J.H. Horne and R.B. Mann, Nucl. Phys. B 447, 441 (1995).
[30] O. Bechmann and O. Lechtenfeld, Class. Quantum Grav. 12, 1473 (1995).
[31] H. Dewitt and O. Lechtenfeld, Int. J. Mod. Phys. A 13, 741 (1998), gr-qc/9612062.
[32] R. Wald, “General Relativity”, Univ. of Chicago Press, Chicago, 1984.
[33] P. Aguiar and P. Crawford, “Dust-filled axially symmetric universes with cosmological constant”, gr-qc/0009056.
[34] R.J. Scherrer, Phys. Rev. Lett. 93, 011103 (2004).
[35] J.M. Aguirregabiria, L. P. Chimento and R. Lazkoz, Phys. Rev. D 70, 023509 (2004).