Curved BPS domain wall solutions in four-dimensional $\mathcal{N} = 2$ supergravity

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Abstract

We construct four-dimensional domain wall solutions of $\mathcal{N} = 2$ gauged supergravity coupled to vector and to hypermultiplets. The gauged supergravity theories that we consider are obtained by performing two types of Abelian gauging. In both cases we find that the behaviour of the scalar fields belonging to the vector multiplets is governed by the so-called attractor equations known from the study of BPS black hole solutions in ungauged $\mathcal{N} = 2$ supergravity theories. The scalar fields belonging to the hypermultiplets, on the other hand, are either constant or exhibit a run-away behaviour. These domain wall solutions preserve $1/2$ of supersymmetry and they are, in general, curved. We briefly comment on the amount of supersymmetry preserved by domain wall solutions in gauged supergravity theories obtained by more general gaugings.

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1 Introduction

Recently there has been a lot of interest in constructing domain wall solutions in gauged supergravity. As consequence of the AdS/CFT correspondence [1, 2, 3], domain wall solutions can provide the supergravity duals of field theories near a fixed point and may, in addition, give a holographic picture of the renormalization group (RG) flow. In order to understand this in more detail it is important to have explicit solutions on the supergravity side.

The BPS domain wall solutions of five-dimensional gauged supergravity theories describe the RG flow between superconformal field theories in four dimensions (for an example see [4]). Here, the gravitational $AdS_5$ backgrounds arise as the near horizon geometry of type IIB D3-branes located at certain transversal six-dimensional spaces. In the same way, four-dimensional $AdS_4$ supergravities emerge, for example, from M-theory membranes located at certain transversal eight-dimensional spaces. These are then holographically related to three-dimensional conformal field theories [5], which are however not very well understood at the present time. In this paper we will focus on BPS domain wall solutions of gauged supergravity theories with eight supercharges ($gauged \mathcal{N} = 2$ supergravity) in four dimensions. Domain wall solutions in theories with less supersymmetries were previously considered in [6, 7].

The four-dimensional $\mathcal{N} = 2$ gauged supergravity theories we will consider are obtained by performing a gauging either of a $U(1)$ subgroup of the $SU(2)$ R-symmetry or of a particular Abelian Killing symmetry of the universal hypermultiplet moduli space. The former results in models with a potential term for the vector scalars only, whereas the latter results in a potential term for the vector scalars and for the dilaton field entering the universal hypermultiplet. In the context of string theory, the latter models can be obtained by Calabi-Yau threefold compactifications of type II string theory in the presence of internal H-fluxes [1, 11]-[15]. The vacuum structure of these type II Calabi-Yau threefold compactifications was discussed in some detail in [11]-[15]. It was shown that $\mathcal{N} = 2$ supersymmetric ground states with flat four-dimensional Minkowski spacetime are only possible in certain degeneration limits of the underlying Calabi-Yau spaces (otherwise $\mathcal{N} = 2$ supersymmetry gets completely broken at generic points in the Calabi-Yau moduli space). Thus, in general, non-vanishing H-fluxes do allow for supersymmetric anti-de-Sitter groundstates with non-vanishing cosmological constant $\Lambda_4$. This implies that the associated four-dimensional field equations possess solutions describing non-flat gravitational backgrounds, such as domain walls.

In this paper we will construct four-dimensional domain wall solutions of the gauged supergravity theories described above. We will show that (as already indicated in [15, 16]) the behaviour of the scalar fields belonging to the vector multiplets is governed by a set of equations already known from the study of BPS black hole solutions in

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It is conceivable [8, 9] that string Calabi-Yau compactifications with internal H-fluxes in the field theory limit are T-dual to D3-branes in the presence of transversal RR-fluxes [10].
ungauged $\mathcal{N} = 2$ supergravity theories, namely the so-called attractor equations [17]-[26]. We will also show that in general, the four-dimensional BPS equations are solved by curved walls, i.e. there is in general also a three-dimensional cosmological constant $\Lambda_3$ on the domain wall. In fact, the curvature of the wall must cancel contributions coming from an expression closely related to the $U(1)$-Kähler connection (as we will see, this effect already occurs in the context of pure $\mathcal{N} = 2$ supergravity). This is in contrast with what happens in five dimensions in analogous models. Since such a connection is not present in five dimensions (and whenever the pullback of the $SU(2)$ connection is trivial), the five-dimensional BPS equations only allow for flat domain wall solutions [27, 28].

We can then give the following dictionary between the quantities which play a role in black hole solutions of ungauged $\mathcal{N} = 2$ supergravity and those of the corresponding domain wall solutions of gauged $\mathcal{N} = 2$ supergravity:

- In gauged $\mathcal{N} = 2$ supergravity we will use the following symplectic invariant superpotential:

$$W = e^{s\phi} \left( \alpha_I L^I - \beta^I M_I \right), \quad s = 0, 2,$$  

where the symplectic vector $(L^I, M_I)$ $(I = 0, \ldots, N_V)$ denotes the sections which depend on the vector scalar fields, and $\phi$ is the dilaton field. In the case of the gauging of a particular Abelian isometry of the universal hypermultiplet moduli space, the entries of the constant symplectic vector $(\alpha_I, \beta^I)$ correspond to the electric/magnetic $U(1)^{N_V+1}$ charges of the universal hypermultiplet and emerge in type IIA/B string compactifications as internal H-fluxes on Calabi-Yau threefolds with $h^{1,1}(h^{2,1}) = N_V$ for type IIA (type IIB) [12, 13]. The dependence of this superpotential on the vector scalars is, on the other hand, the direct analogue of the central charge $Z$ of the $\mathcal{N} = 2$ supersymmetry algebra that plays a role in the context of BPS black holes in ungauged $\mathcal{N} = 2$ supergravity. Here, the $(\alpha_I, \beta^I)$ are just the electric/magnetic $U(1)^{N_V+1}$ charges of the black holes.

- The attractor equations (see eq.(3.29)) determine the running of the vector scalar fields from the domain wall to the supersymmetric extrema, $DW = 0$, reached at spatial infinity. In fact, the vector scalar fields are stabilized in the sense that their values at spatial infinity only depend on the constants $(\alpha_I, \beta^I)$. Therefore the attractor equations are relevant for the RG-flow in the corresponding field theories. In the context of $\mathcal{N} = 2$ black holes the same stabilization equations determine the evolution of the vector scalars from spatial infinity towards the horizon, where the central charge is extremized, $DZ = 0$, and the scalars are entirely expressed in terms of the $(\alpha_I, \beta^I)$.

- For $\mathcal{N} = 2$ domain walls, the extremum of the potential with respect to the vector scalar fields, $V_{\text{extr.}} \sim |W|^2$, is again a function only of the $(\alpha_I, \beta^I)$ and corresponds to a four-dimensional cosmological constant $\Lambda_4$ at spatial infinity.
The analogous quantity, $|Z|^2_{\text{extr.}}$, is just the entropy $S$ of the $\mathcal{N} = 2$ black holes, i.e. the area of the horizon.

- We will show that the quantity (see section 3)
  \[ A^Y = \frac{1}{2} e^{-U(Y_I - \bar{Y}^I)} \, d \, (F_I - \bar{F}_I) \]  
  provides the three-dimensional cosmological constant $\Lambda_3$ on the domain wall. This means that a non-vanishing one-form $A^Y$ leads to curved anti-de-Sitter like domain wall solutions. On the black hole side, the same object corresponds to the angular momentum of stationary but in general non-static solutions [23, 25, 26].

- Finally, closely related brane configurations play a role in the comparison of $\mathcal{N} = 2$ domain wall and black hole solutions. In type IIA superstring compactified on a Calabi-Yau threefold $M$, D4-branes, being $\beta^A$-times wrapped around 4-cycles of $M$, together with $\alpha_0$ D0-branes, lead to black holes with magnetic charges $p^A$ and one electric charge $\alpha_0$. The corresponding entropy is determined by the triple intersection form $C_{ABC}$. Increasing the dimensionality of the branes by two, i.e. considering boundstates of wrapped D6-branes together with $\alpha_0$ D2-branes, leads to membranes in four dimensions, which represent the source for the supergravity domain wall solutions in four uncompactified dimensions. Similarly, in type IIB theory compactified on a Calabi-Yau threefold, D3-branes wrapped around 3-cycles provide charged black hole solutions, whereas the same configurations of wrapped D5-branes correspond to domain wall solutions.

The paper is organized as follows. In the next section we will recall certain aspects of gauged $\mathcal{N} = 2$ supergravity in four dimensions. In section 3 we will solve the BPS equations for domain wall solutions. Section 4 contains a discussion of our solutions as well as various concluding remarks.

## 2 $\mathcal{N} = 2$ gauged supergravity

Domain walls are codimension one solutions that separate the spacetime into regions corresponding to different vacua. In the simplest case, a domain wall is supported by a gauge potential that couples to its world volume. The field strength of this gauge potential is dual to a cosmological constant. In a more general setting with non-trivial couplings to scalar fields, this cosmological constant appears as an extremum of the potential term in the Lagrangian. The resulting solution then describes a flow towards an extremum, and if the potential possesses several extrema, the solution may interpolate between them. In the following we will be interested in constructing domain wall solutions in $\mathcal{N} = 2$ gauged supergravity theories in four dimensions.

The $\mathcal{N} = 2$ gauged supergravity theories that we consider are based on abelian vector multiplets (labelled by an index $I = 0, \ldots, N_V$) and hypermultiplets coupled to the
\( \mathcal{N} = 2 \) supergravity fields. Potentials that are allowed by \( \mathcal{N} = 2 \) supersymmetry are then obtained by performing a gauging of (some of) the various global symmetries. There are two different types of gaugings, namely (i) one can either gauge some of the isometries of the moduli space of ungauged \( \mathcal{N} = 2 \) supergravity or (ii) one can gauge (part of) the \( SU(2) \) R-symmetry, which only acts on the fermions. In \( \mathcal{N} = 2 \) supergravity, at the two-derivative level, the moduli space is a direct product \( \mathcal{M} = \mathcal{M}_V \times \mathcal{M}_H \), where \( \mathcal{M}_V \) and \( \mathcal{M}_H \) are parameterized by the scalars belonging to the vector multiplets and to the hypermultiplets, respectively. In the following, we will only consider Abelian gaugings, either of (some of) the isometries of the hyper scalar manifold \( \mathcal{M}_H \) or of the \( SU(2) \) R-symmetry of \( \mathcal{N} = 2 \) supergravity. We refer to [29, 30, 31] for a detailed description of the gauging.

The hyper scalar manifold \( \mathcal{M}_H \) is a quaternionic space, and hence it possesses three complex structures \( J^x \) as well as a triplet of Kähler two-forms \( K^x \) (here \( x = 1, 2, 3 \) denotes an \( SU(2) \) index). The holonomy group is \( SU(2) \times Sp(n_H) \) and the Kähler forms have to be covariantly constant with respect to the \( SU(2) \) connection. The isometries of \( \mathcal{M}_H \) are generated by a set of Killing vectors \( k_I^x \),

\[
q^u \to q^u + k_I^x \epsilon^I ,
\]

and therefore the gauging of (some of) the Abelian isometries results in the introduction of gauge covariant derivatives via the replacement \( dq^u \to dq^u + k_I^x A^I \). In order to maintain supersymmetry, the gauging of the isometries has to preserve the quaternionic structure, which implies that the Killing vectors have to be tri-holomorphic. This is the case whenever it is possible to express the Killing vectors in terms of a triplet of real Killing prepotentials \( P_I^x \), as follows:

\[
K^x_{uv} k_I^v = -\nabla_u P_I^x \equiv -\partial_u P^x_I - \epsilon^{xyz} \omega^y_u P^z_I . \tag{2.2}
\]

Here \( \omega^y_u \) are the \( SU(2) \) connections, which are related to the Kähler forms by \( K^x_{uv} = -\nabla_{[u} \omega^x_v] \). By using the Pauli matrices \( \sigma^x \) one can also revert to matrix notation and write

\[
(P_I)_{ij} = \sum_{x=1}^3 P_I^x (\sigma^x)_i^k \varepsilon_{jk} , \tag{2.3}
\]

where \( \varepsilon_{jk} \) denotes the two-dimensional antisymmetric \( \varepsilon \)-symbol.

In this paper we will, for concreteness, only consider the quaternionic space associated with the so-called universal hypermultiplet. Classically, it is given by the coset space \( SU(2,1)/U(2) \). Its Kähler potential is, in a certain parameterization, given by

\[
K = -\log[ S + \bar{S} - 2(C + \bar{C})^2] . \tag{2.4}
\]

This coset space has two Abelian isometries which are generated by the Killing vectors associated to shifts in the imaginary parts of \( S \) and \( C \). The gauging of these two Abelian isometries has been discussed in [12] (we refer to [27, 32, 33, 34] for a discussion of the
gauging of (some of) the other isometries of the universal hypermultiplet). In this paper, however, we will only consider the case when the shift in the imaginary part of $S$ is gauged. The associated Killing vector is given by

$$k_I = \frac{\partial}{\partial q^a} = -\alpha_I i(\partial_S - \partial_{\bar{S}}),$$  \hspace{1cm} (2.5)$$

where the $\alpha_I$ are constant and real. This is the four-dimensional analogue of the gauging of five-dimensional supergravity discussed in \[27\]. The case where one independently gauges both the Abelian isometries results in a potential which makes the construction of explicit domain wall solutions rather difficult, and will not be discussed here.

The Killing prepotential associated with the Killing vector (2.5) is, in matrix notation (2.3), given by

$$P_{ij} = e^{2\phi} \alpha_I \sigma^{1}_{ij}, \quad \sigma^{1}_{ij} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{ij},$$  \hspace{1cm} (2.6)$$

where the dilaton is given by $e^{-2\phi} = S + \bar{S} - 2(C + \bar{C})^2$.

It can be shown \[29\] that the gauging of a $U(1)$ subgroup of the $SU(2)$ R-symmetry results in a Killing prepotential of the form $P_I = \alpha_I \sigma^1$, where the $\alpha_I$ are again constant and real. Thus, the Abelian gaugings we will consider in this paper are characterised by a Killing prepotential of the form

$$P_{ij} = e^{s\phi} \alpha_I \sigma^{1}_{ij}, \quad s = 0, 2.$$  \hspace{1cm} (2.7)$$

The class of spacetime metrics that we will consider in the following is given by

$$ds^2 = e^{2U(z)} \hat{g}_{mn} dx^m dx^n + e^{-2pU(z)} dz^2$$  \hspace{1cm} (2.8)$$

with $\hat{g}_{mn} = \hat{g}_{mn}(x^m)$. Here we denote spacetime indices by $x^\mu = (x^m, z)$, and the corresponding tangent space indices by $a = (0, 1, 2, 3)$. The constant $p$ is introduced for later convenience. We assume Lorentz invariance in the three-dimensional subspace $a = (0, 1, 2)$. The metric $\hat{g}_{mn}$ is thus a three-dimensional constant curvature metric. We take the solutions to be uncharged, that is we set the gauge fields to zero. In general, however, as a consequence of the gauging of (some of) the isometries of $\mathcal{M}_H$, some of the hyper scalar fields are charged and therefore they act as sources for the gauge fields. The associated currents are given by $h_{\mu u} k_u^I (dq^\nu + k_\nu^J A^J)$. Then, in order for a vanishing gauge field to be a consistent solution of its equation of motion, either the associated current has to vanish or the charged hyper scalars have to satisfy the equation $h_{\mu u} k_\nu^I dq^\nu = 0$. This implies that the hyper scalar fields that are not constant have to be neutral. This will indeed turn out to be the case for the gauging (2.5), where the only non-trivial hyper scalar field is the neutral dilaton field $\phi$.

In the absence of gauge fields, the supersymmetry transformation laws for the gravitini $\psi_{\mu i}$, the gaugini $\lambda^A_i \ (A = 1, \ldots, N_V)$ and the hyperini $\zeta_\alpha$ read \[33\] \[29\]

$$\delta \psi_{\mu i} = D_\mu \epsilon_i - \frac{1}{2} P_{ij} \bar{L}^I \gamma_\mu \epsilon^j,$$
\[ \delta \lambda_i^A = \gamma^\mu \partial_\mu z^A \epsilon_i + \mathcal{P}_{ij} g^{AB} D_B L_i \bar{\epsilon}^j, \]
\[ \delta \zeta_\alpha = V_{u\alpha} \gamma^\mu \partial_\mu q^u \varepsilon_{ij} \bar{\epsilon}^j - 2V_{u\alpha} k^u L_i \epsilon_i. \quad (2.9) \]

Here we have denoted the scalar fields residing in the abelian vector multiplets by \( L^I \).

For later convenience we now also introduce a non-holomorphic section which we write as \( V^T = (L^I, M_I) \). It is subject to the symplectic constraint \( i(L^I M_I - L^I M_I) = 1 \). The assignment of chiral weights \( c \) is as follows. The non-holomorphic section \( V \) has \( c = -1 \), whereas the supersymmetry parameters \( \epsilon_i \) and \( \bar{\epsilon}^i \) have \( c = \frac{1}{2} \) and \( c = -\frac{1}{2} \), respectively.

Inspection of (2.9) shows that the following quantity will play a central role when solving the equations resulting from the vanishing of the supersymmetry transformation laws (2.9):
\[ W = e^{s\phi} \alpha_i L^I, \quad s = 0, 2. \quad (2.10) \]

Supersymmetric domain wall solutions will, in general, only preserve part of \( \mathcal{N} = 2 \) supersymmetry. A condition on the supersymmetry parameters that is consistent with local Lorentz invariance in the subspace \( a = (0,1,2) \) (and, hence, with (2.8)) is the following,
\[ \epsilon_i = A_{ij} \gamma_3 \bar{\epsilon}^j, \quad (2.11) \]
where \( A_{ij} \) denotes an \( SU(2) \) matrix. Since the spacetime metric (2.8) only has a non-trivial dependence on the coordinate \( z \), we have \( A_{ij} = A_{ij}(z) \). Taking the complex conjugate of (2.11) and recalling that \( (\epsilon_i)^* = \epsilon^i \) (as well as using a convention where \( \gamma_3 \) is real) then yields that \( (A^* A)^i_j \bar{\epsilon}^j = \epsilon^i \). Let us now assume that (2.11) is the only restriction on the supersymmetry parameters, so that \( (A^* A)^i_j = \delta^i_j \). On the other hand, inserting (2.11) into the gravitino variation \( \delta \psi_{mi} \) and demanding its vanishing yields \( A_{ij} \sim \mathcal{P}_{ij} L^I \). Now, since for general gaugings the condition \( (A^* A)^i_j = \delta^i_j \) is in general not satisfied, we conclude that for general gaugings (2.11) cannot be the only restriction on the supersymmetry parameters, but that there are further constraints leading to an additional (or possibly complete) breaking of supersymmetry. For the Abelian gaugings (2.7) considered in this paper, however, \( (A^* A)^i_j = \delta^i_j \) is satisfied, so that (2.11) is the only condition on the supersymmetry parameters, thus leading to domain wall solutions which preserve 1/2 of \( \mathcal{N} = 2 \) supersymmetry.

3 Solving the BPS equations

In this section we will construct supersymmetric domain wall solutions (2.8) for the Abelian gaugings specified by (2.5) and (2.7).

The symplectic extension of (2.10) is given by
\[ W = e^{s\phi} \alpha_i L^I - \beta^I M_I, \quad s = 0, 2. \quad (3.1) \]
Here the $\beta^I$ denote real constants associated with the dual Killing vector $k^I$ and the dual Killing prepotential $\mathcal{P}_{ij}^I$. The quantity (3.1) is symplectically invariant, provided that $(\alpha_I, \beta^I)$ transforms as a vector under symplectic transformations. In the context of string theory, a constant vector $(\alpha_I, \beta^I)$ does indeed arise when turning on $H$-fluxes on Calabi-Yau threefolds [11, 12]. Moreover, in the presence of these fluxes, it is then possible to engineer supersymmetric domain wall solutions by wrapping D-branes around appropriate supersymmetric cycles. We take this to be an indication that, in general, supersymmetric domain wall solutions do depend on both $\alpha_I$ and $\beta^I$.

We will therefore consider the following symplectic extension of the supersymmetry transformation rules (2.9) (which are valid in the absence of gauge fields),

\[
\begin{align*}
\delta \psi_{\mu i} &= D_\mu \epsilon_i - \frac{1}{2} W \sigma_{ij}^I \gamma_\mu \epsilon^j , \\
\delta \lambda_i^A &= \gamma_\mu \partial_\mu z^A \epsilon_i + g^{AB} \bar{D}_B W \sigma_{ij}^I \epsilon^j , \\
\delta \zeta_\alpha &= V^i u \left( \gamma_\mu \partial_\mu q^u \epsilon_i - 2(k_u^I L^I - k^I u M^I) \epsilon_i \right) ,
\end{align*}
\]  
(3.2)

with $W$ given by (3.1). The BPS solutions we will construct in this section are obtained by demanding the vanishing of the supersymmetry variations (3.2), subject to a condition of the form (2.11). It should be pointed out, however, that we are not aware of any construction of gauged supergravity theories giving rise to supersymmetry transformation rules of the type (3.2) involving not only $\mathcal{P}_I$ and $k_u^I$ but also their duals. Our justification for starting with (3.1) and (3.2) is twofold. On the one hand we will be able to solve the system of equations resulting from the vanishing of (3.2). On the other hand, as stated above, there are examples of supersymmetric domain wall solutions in string theory which are characterised by both the parameters $\alpha_I$ and $\beta^I$ appearing in (3.1), and we would like to be able to reproduce them.

We now impose the following condition on the supersymmetry parameters, in accordance with the discussion given below (2.11),

\[
\epsilon_i = \bar{h} \sigma_{ij}^I \gamma^3 \epsilon^j ,
\]  
(3.3)

where $h = h(z)$ denotes a phase factor with chiral weight $c = -1$.

Let us first consider the variation $\delta \psi_{\mu i} = 0$. For a spacetime metric of the form (2.8), we find that

\[
\omega_m^{ab} = \hat{\omega}_m^{ab} + e^{(\nu+1)U} \partial_z U \left( \eta_3^a \eta_3^b - (a \leftrightarrow b) \right) ,
\]  
(3.4)

where $\hat{\omega}_m^{ab}$ denotes the spin connection associated to the three-dimensional metric $\hat{g}_{mn} = \tilde{e}_m^a \tilde{e}_n^b \eta_{ab}$, where $a = 0, 1, 2$. The metric $\hat{g}_{mn}$ is a constant curvature metric. In the anti-de-Sitter case, we may write the constant curvature condition as

\[
\hat{R}_{mn}^{ab} = \frac{4}{l^2} \left( \tilde{e}_m^a \tilde{e}_n^b - (m \leftrightarrow n) \right) ,
\]  
(3.5)
where $l$ denotes a real constant, which is related to the three-dimensional cosmological constant $\Lambda_3$ by $\Lambda_3 = l^{-2}$. The associated curvature scalar is then $\hat{R} = 24\Lambda_3$. The condition (3.5) can be viewed as the integrability condition associated with

\[ \hat{D}_m (h^{1/2} \epsilon_i) = \frac{i}{4} \hat{e}_m \gamma_\alpha \gamma_3 (h^{1/2} \epsilon_i). \]  

The case of zero curvature is obtained from the above by sending $l \to \infty$, whereas the de-Sitter case is obtained by replacing $l \to il$. The latter does not lead to supersymmetric domain wall solutions. We also note that the imposition of (3.6) does not lead to a further restriction of the residual supersymmetry preserved by (2.8).

Using (3.3) as well as (3.6) we then obtain from $\delta \psi_{mi} = 0$,

\[ h\bar{W} = e^{pU} \partial_z U + \frac{2i}{l} e^{-U}. \]  

Next, let us consider the variation $\delta \psi_{zi} = 0$. It yields

\[ \hat{D}_z \epsilon_i = \frac{1}{2} e^{-pU} h\bar{W} \sigma_{ij}^1 \gamma^3 h \epsilon^j. \]  

Inserting (3.3) on the r.h.s. of (3.8) yields

\[ \hat{D}_z \epsilon_i = \frac{1}{2} e^{pU} h\bar{W} \epsilon_i. \]  

Hermitian conjugation then gives

\[ \hat{D}_z \epsilon^i = \frac{1}{2} e^{pU} \bar{h}\bar{W} \epsilon^i. \]  

On the other hand, inserting (3.3) on the l.h.s. of (3.8) yields

\[ \hat{D}_z \epsilon^i = \frac{1}{2} e^{-pU} h\bar{W} \epsilon^i - (h\hat{D}_z \bar{h}) \epsilon^i. \]  

Comparison of (3.10) with (3.11) then gives

\[ h\hat{D}_z \bar{h} = \frac{1}{2} e^{-pU} \left( h\bar{W} - \bar{h}W \right) = \frac{2i}{l} e^{-(1+p)U}. \]  

Also, using $\omega_z^{ab} = 0$ and that the pullback of the SU(2) connection is trivial (as we will see later), we find that (3.9) is solved by

\[ h^{1/2} \epsilon_i = \chi_i e^{\frac{1}{2}U}, \]  

where $\chi_i$ denotes a spinor satisfying (3.6).

Next, let us consider the vanishing of the gaugini variation, $\delta \lambda^A_i = 0$. Using the special geometry relations

\[ g_{AB} = -2D_AL^I \text{Im}N_{IJ} \bar{D}_B L^J, \quad D_AM_I = \bar{N}_{IJ} D_AL^J, \quad \bar{D}_AV = 0 \]  

9
as well as (3.3) we obtain from $\delta \lambda^A = 0$,

$$g_{AB} \partial_z z^A = i \left[ D_z L^I D_B M_I - D_z M_I D_B L^I \right] = -e^{-pU} h \partial_z \bar{\Omega} W$$

(3.15)

and, hence,

$$\left( \hbar D_z L^I + i \beta^I e^{\phi - pU} \right) \bar{D}_B \bar{M}_I = \left( \hbar D_z M_I + i \alpha_I e^{\phi - pU} \right) \bar{D}_B \bar{L}^I .$$

(3.16)

Using $\bar{D}_B (\bar{L}^I M_I - L^I \bar{M}_I) = 0$ we may rewrite this as

$$\left( \partial_z (\bar{h} L^I) + i \beta^I e^{\phi - pU} \right) \bar{D}_B \bar{M}_I = \left( \partial_z (\bar{h} M_I) + i \alpha_I e^{\phi - pU} \right) \bar{D}_B \bar{L}^I ,$$

(3.17)

or equivalently as $\Pi^T \Omega \bar{U}_B = 0$, where $\Omega$ denotes the sympletic metric and where $\Pi$ and $\bar{U}_B$ denote two symplectic vectors, as follows,

$$\Pi = \left( \begin{array}{c} \partial_z (\bar{h} L^I) + i \beta^I e^{\phi - pU} \\ \partial_z (\bar{h} M_I) + i \alpha_I e^{\phi - pU} \end{array} \right), \quad \bar{U}_B = \left( \begin{array}{c} \bar{D}_B \bar{L}^I \\ \bar{D}_B \bar{M}_I \end{array} \right), \quad \Omega = \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right) .$$

(3.18)

We note that $\Pi$ has chiral weight $c = 0$.

The general solution to the equation $\Pi^T \Omega \bar{U}_B = 0$ constructed out of the symplectic vectors $V, \bar{V}, U_A$ and $\bar{U}_A$ is given by

$$\Pi = C \bar{h} \bar{V} + D \bar{h} \bar{V} + E \bar{h} (\partial_z \bar{z} \bar{z}) \bar{U}_A ,$$

(3.19)

with parameters $C, D$ and $E$. This is so because $V^T \Omega \bar{U}_B = 0$ as well as $\bar{V}^T \Omega \bar{U}_B = 0$, whereas $U_A^T \Omega \bar{U}_B = i g_{AB} \neq 0$. We now determine $C$ and $D$ by considering the sympletic product of (3.19) with $\bar{h} \bar{V}$ and $\bar{h} \bar{V}$, i.e. $\bar{h} \bar{V}^T \Omega \Pi$ and $\bar{h} \bar{V}^T \Omega \Pi$. We then find that

$$C = -\partial_z U , \quad D = \partial_z U - h \partial_z \bar{h}$$

(3.20)

by virtue of (3.7) and (3.12).

The parameter $E$, on the other hand, can be determined by contracting $\Pi$ with $\bar{h} U_A$, i.e. from $U_A^T \Omega \bar{U}_B = i g_{AB}$ as well as (3.13) then yields

$$E = 1 .$$

(3.21)

Thus, it follows from (3.19) that

$$\left( \partial_z (\bar{h} L^I) + \partial_z U \bar{h} L^I \right) = - i e^{\phi - pU} \left( \begin{array}{c} \beta^I \\ \alpha_I \end{array} \right) + h \partial_z \bar{z} \bar{z} \left( \begin{array}{c} \bar{D}_B \bar{L}^I \\ \bar{D}_B \bar{M}_I \end{array} \right) + \partial_z U \left( \begin{array}{c} \bar{h} L^I \\ \bar{h} M_I \end{array} \right) - h \partial_z \bar{h} \left( \begin{array}{c} \bar{h} L^I \\ \bar{h} M_I \end{array} \right) .$$

(3.22)
This we rewrite as

\[
\left( \frac{\partial_z Y^I}{\partial_z F_I} \right) = -i e^{s\phi+(1-p)U} \left( \frac{\beta^I}{\alpha_I} \right) + e^U h \partial_z z^B \left( \frac{D_B L^I}{D_B M_I} \right) + \partial_z U \left( \frac{\bar{Y}^I}{\bar{F}_I} \right) - h \partial_z h \left( \frac{\bar{Y}^I}{\bar{F}_I} \right),
\]  

(3.23)

where we introduced the chiral invariant variables

\[
Y^I = e^U \bar{h} L^I, \quad F_I(Y) = e^U \bar{h} M_I.
\]  

(3.24)

Inserting (3.24) into \( i(\bar{L}^I M_I - L^I \bar{M}_I) = 1 \) yields

\[
e^{2U} = i \left[ \bar{Y}^I F_I(Y) - Y^I \bar{F}_I(Y) \right].
\]  

(3.25)

Using

\[
e^U h \partial_z z^B \left( \frac{\bar{D}_B L^I}{\bar{D}_B M_I} \right) = e^U h \partial_z \left( \frac{\bar{L}^I}{\bar{M}_I} \right) = \partial_z \left( \frac{\bar{Y}^I}{\bar{F}_I} \right) - \partial_z U \left( \frac{\bar{Y}^I}{\bar{F}_I} \right) - h \partial_z h \left( \frac{\bar{Y}^I}{\bar{F}_I} \right),
\]  

(3.26)

it follows from (3.23) that

\[
\partial_z \left( \frac{Y^I - \bar{Y}^I}{F_I - \bar{F}_I} \right) = -i e^{s\phi+(1-p)U} \left( \frac{\beta^I}{\alpha_I} \right).
\]  

(3.27)

If we now set

\[
e^{s\phi} = e^{(p-1)U},
\]  

(3.28)

then we may integrate (3.27),

\[
\left( \frac{Y^I - \bar{Y}^I}{F_I - \bar{F}_I} \right) = i \left( \frac{h^I - \beta^I z}{h_I - \alpha_I z} \right) = i \left( \frac{H^I}{H_I} \right),
\]  

(3.29)

where the \((H^I, H_I)\) denote harmonic functions. These equations, which determine the behaviour of the physical vector scalar fields \(z^A = Y^A/Y^0\), are also known from the study of BPS black hole solutions in ungauged \( \mathcal{N} = 2 \) supergravity theories, where they are called attractor equations [17, 18].

Let us now return to (3.28). For the case \( s = 0 \) (corresponding to an Abelian gauging of the \( SU(2) \) R-symmetry), (3.28) holds for the convenient choice \( p = 1 \). For the case \( s = 2 \) (corresponding to the Abelian gauging of a particular isometry of the universal hyper moduli space) we will show below that (3.28) solves the equation resulting from the vanishing of the hyperino variation, provided that \( p = -3 \).

Next, consider inserting (3.29) into (3.12). This yields

\[
l^{-1} = -\frac{1}{4} e^{s\phi} \left( \alpha_I h^I - \beta^I h_I \right).
\]  

(3.30)
Since \( l \) has to be constant, we conclude that it is not possible to have a non-vanishing three-dimensional cosmological constant \( \Lambda_3 \) for the case \( s = 2 \). That is, for \( s = 2 \) the domain wall solutions have to be flat \((\alpha_I h^I - \beta^I h_I = 0)\), whereas for \( s = 0 \) they may be curved. This implies that in the latter case there is no restriction on the integration constants \((h^I, h_I)\), whereas in the former case there are such restrictions.

Before turning to the hyperino variation \( \delta \zeta_\alpha = 0 \), let us perform two consistency checks on the solution given above. Let us first return to (3.7). Using (3.27) as well as (3.28) the real part of (3.7), \( hW + \bar{h}W = 2e^{3U} \partial_z U \), may be rewritten as

\[
\partial_z e^{2U} = \left( \alpha_I (\bar{Y}^I + Y^I) - \beta^I (F_I + \bar{F}_I) \right) = i \partial_z \left( \bar{Y}^I F_I - Y^I F_I \right),
\]

which is in perfect agreement with (3.25). Next, let us consider \( h D_z \bar{h} = h \partial_z \bar{h} - i A_z \), where \( A_z = A_z^Y - i \partial_z \log \frac{h}{n} \), \( A_z^Y = \frac{1}{2} e^{-2U} (Y^I - \bar{Y}^I) \partial_z (F_I - \bar{F}_I) \). Using (3.24) it follows that \( h D_z \bar{h} = -i A_z^Y = -\frac{1}{2} e^{-2U} (\alpha_I h^I - \beta^I h_I) \) which, upon using (3.30), precisely yields (3.12).

Finally, let us turn to the variation \( \delta \zeta_\alpha = 0 \). For the case \( s = 0 \) we have \( k_I^u = 0, k^I u = 0 \), so that \( \delta \zeta_\alpha = 0 \) is solved by constant hyper scalars, \( q^u = \text{const} \). On the other hand, for \( s = 2 \) we have \( k_I^{s-s} = -2 \alpha^I \) and \( k^I s-s = -2 \beta^I \). We solve the hyperino variation by setting \( S - \bar{S} = \text{const} \) and \( C = \text{const} \), so that also in this case the pullback of the \( SU(2) \) connection is trivial. Then, using \( \partial_\mu (S + \bar{S}) = \partial_\mu e^{-2\phi} \), it follows that

\[
\delta \zeta_\alpha = V_{s+s,\alpha}^i \gamma^0 \partial_\mu e^{-2\phi} \epsilon_{ij} \ell^j + 4 V_{s-s,\alpha}^i (\alpha_I L^I - \beta^I M_I) \epsilon_i.
\]

Using

\[
V_{s+s,\alpha}^{i\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & e^{2\phi} \\ e^{-2\phi} & 0 \end{pmatrix}^i \quad \text{and} \quad V_{s-s,\alpha}^{i\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -e^{2\phi} \\ e^{2\phi} & 0 \end{pmatrix}^i
\]

as well as (3.3) and \( \epsilon_{12} = 1 \), we obtain

\[
e^{3U} \partial_z e^{-2\phi} = 4 \frac{h}{n} (\alpha_I L^I - \beta^I M_I) = 4 e^{-2\phi} hW.
\]

It follows that \( hW = h\bar{W} \), and hence we obtain from (3.7) that \( t^{-1} = \infty \). Using (3.7) it follows that (3.34) is solved by

\[
e^{-2(\phi - \phi_0)} = e^{4U}.
\]

This is consistent with (3.28) provided that \( p = -3 \).

Let us briefly summarize some of the properties of the domain wall solutions we have constructed. For both cases \((s = 0, 2)\) we find that the behaviour of the vector scalar fields is determined by a set of attractor equations given in (3.29). For the case \( s = 0 \), the hyper scalars are all constant and the domain wall solutions are, in general, curved, with the three-dimensional cosmological constant \( \Lambda_3 = l^{-2} \) determined by (3.30). The associated spacetime metric is given by (2.8) with \( p = 1 \). For the case \( s = 2 \), on the other hand, we find that the dilaton field exhibits the run-away behaviour (3.33). The domain wall solutions must be flat, and the associated spacetime metric is given by (2.8) with \( p = -3 \).
4 Discussion

The domain wall solutions specified by equations (2.8), (3.25), (3.29) and (3.30) solve the Killing spinor equations for any prepotential function describing the couplings of abelian vector multiplets to supergravity. In the case that only the R-symmetry is gauged ($s = 0$), the hyper scalars are constant and the solutions possess an asymptotic ($z \to \infty$) AdS vacuum. In the case $s = 2$ (corresponding to the gauging of an axionic shift symmetry), on the other hand, the dilaton is not constant but instead given by (3.35). Thus, it is not stabilized by the potential. In either case, the behaviour of the scalars in the vector multiplets is determined by the same set of attractor equations (3.29) known from the study of BPS black holes in ungauged supergravity. Moreover, the domain wall may possess constant AdS-like curvature according to (3.30). This feature also has a counterpart in single-center black hole solutions, namely in the angular momentum carried by these stationary solutions. There is the following one-to-one correspondence between quantities determining black hole solutions and domain wall solutions with non-trivial vector scalars:

| black hole | domain wall |
|------------|-------------|
| central charge $Z$ | superpotential $W$ |
| entropy $S$ | cosmological constant $V_{extr.}$ |
| angular momentum $J$ | constant wall curvature $\Lambda_3$ |

This analogy is related to the fact that both types of solutions can be reduced to equivalent one-dimensional systems \[23, 16\].

The fact that all the regular critical points are reached at the boundary of the AdS space ($e^{2U} \to \infty$) excludes the possibility of a regular RG-flow and/or of trapping of gravity near the wall \[37, 38\]. From the RG-flow point of view the AdS vacuum corresponds to an UV fixed point, and when moving towards negative $z$ the warp factor $e^{2U}$ decreases monotonically (in accordance with a $c$-theorem). If we do not add a source, then $e^{2U}$ will have to pass through a zero at some finite value of $z$, which describes a singular end-point of the RG flow. As in any other cases with vector multiplets only, the absence of an IR fixed point forces the solution to run into a singularity.

On the other hand, in the context of string theory, it is possible to engineer domain wall solutions with a non-trivial dilaton field in terms of D-branes wrapped around supersymmetric cycles. It is thus reasonable to put appropriate sources at some place where the warp factor is still positive, say at $z = 0$. These source terms will then appear on the right-hand side of the harmonic equations

$$
\partial^2 H_I \sim \alpha_I \delta(z) \quad , \quad \partial^2 H^I \sim \beta^I \delta(z) \quad ,
$$

(4.1)
where \((\alpha_I, \beta^I)\) are the charges in a basis of the wrapped cycles. For dilatonic walls that are flat (which correspond to the case \(s = 2\) considered in this paper), there are two ways in which one can solve (4.1). The first possibility consists in continuing the solution through the source in a symmetric way, which implies the replacement \(H_I \rightarrow h_I + \alpha_I |z|, H^I \rightarrow h^I + \beta^I |z|\), and which is equivalent to a sign change in the flux vector \((\alpha_I, \beta^I)\) while passing through the source at \(z = 0\). In this case the scalar fields and the warp factor \(e^{2U}\), which are determined from (3.29), are \(Z_2\)-symmetric.

One may, however, also consider the case where the flux jumps from zero to a finite value, i.e. on the side behind the source we can set \(\alpha_I = 0, \beta^I = 0\), which is equivalent to performing the replacement \(H_I \rightarrow h_I + \alpha_I \frac{1}{2}(z + |z|), H^I \rightarrow h^I + \beta^I \frac{1}{2}(z + |z|)\), so that for negative \(z\) the \(H_I\) and \(H^I\) are constant and the spacetime is flat. Thus, by adding sources, we cut off the (singular) part of the spacetime and instead glue on either an identical piece (yielding a \(Z_2\) symmetric solution) or flat spacetime (corresponding to vanishing flux on one side of the wall).

Let us discuss a few concrete examples of dilatonic domain walls. Consider the case where the only non-vanishing components of \((\alpha_I, \beta^I)\) are given by \(\alpha_0\) and \(\beta^A (A = 1, 2, 3)\). In type IIA theory, the corresponding domain wall solution can then be obtained by a torus compactification of the D-brane intersection \(2 \times 6 \times 6 \times 6\), where the 6-branes wrap a 4-cycle in the internal space and where the common 2-brane represents the domain wall in the four non-compact directions. The dual configuration with non-vanishing \(\alpha_A (A = 1, 2, 3)\) and \(\beta^0\) is described by the intersection \(4 \times 4 \times 4 \times 8\), where the 4-branes wrap a 2-cycle and where the 8-brane wraps the whole internal space. These solutions can of course also be mapped onto the type IIB side, where they are given by the brane intersections \(5 \times 5 \times 5 \times 5\) and \(3 \times 5 \times 5 \times 7\), respectively. This picture in terms of brane intersections is the one appropriate for torus compactifications. For generic Calabi-Yau threefold compactifications, a domain wall solution with non-vanishing \(\alpha_0\) and \(\beta^A\) is obtained on the type IIA side by wrapping a single 2- and a single 6-brane around a holomorphic 0- and a holomorphic 4-cycle of the Calabi-Yau manifold. Similarly, on the type IIB side, a domain wall solution is obtained by wrapping a single 5-brane around a supersymmetric 3-cycle. In either case the symplectic flux vector \((\alpha_I, \beta^I)\) is the decomposition of the corresponding brane charges in a basis of 2-, 3- or 4-forms.

Let us describe the type IIA solution with non-vanishing \(\alpha_0\) and \(\beta^A\) in some more detail. In the limit where the volume of the Calabi-Yau threefold is taken to be large, the associated homogenous function \(F(Y)\) is given by

\[
F(Y) = \frac{D_{ABC} Y^A Y^B Y^C}{Y_0}, \quad D_{ABC} = -\frac{1}{6} C_{ABC},
\]

where the coefficients \(C_{ABC}\) denote the intersection numbers of the 4-cycles of the threefold \((A = 1, \ldots, N_V)\). Solving the attractor equations (3.29) in terms of the
harmonic functions $H_0$ and $H^A$, and using (3.25), yields

$$Y^0 = Y^0, \quad (Y^0)^2 = \frac{D_{ABC}H^A H^B H^C}{4H_0}, \quad Y^A = \frac{i}{2} H^A, \quad e^{2U} = -4Y^0H_0. \quad (4.3)$$

Since the curvature on the wall (3.30) is zero, it follows from (3.7) that $\bar{h}\bar{W} = \bar{h}W$. Using (3.24) as well as (3.35), we then obtain $\bar{h}W = \pm |W| = e^{-5U}(\alpha_0 Y^0 - \beta A F_A(Y))$, and hence

$$|W|^2 = \frac{1}{16} e^{-10U} |Y^0|^2 D_{ABC}(\frac{\alpha_0}{H_0} H^A H^B H^C + 3\beta A H^B H^C)|^2. \quad (4.4)$$

There are some issues that we didn’t address in this paper and which may be worth studying in the future. First, when evaluating the pullback of the $SU(2)$ connection of the quaternionic space for the $s = 2$ solution we find that it is trivial. In general, however, we expect that it will also contribute to the curvature of the domain wall. This effect, by the way, may also happen for BPS walls in five-dimensional gauged supergravity. Second, the geometry of the curved wall is $AdS_3$ and it would be interesting to understand whether a global identification can give rise to a BTZ black hole. Third, in the case of dilatonic walls, the curvature on the wall had to be zero $(\alpha_I h^I - \beta^I h_I = 0)$. It may, however, happen that this restriction on the curvature is circumvented by allowing the dilaton also to depend on the worldvolume coordinates. And finally, it would be interesting to investigate more general potentials for the dilaton field. These can be obtained either by gauging some of the non-compact isometries of the universal hypermultiplet moduli space or by turning on some of the NS 3-form fluxes in type IIB Calabi-Yau compactifications. In particular, $SL(2, Z)$ covariant superpotentials may eliminate the run-away behavior of the dilaton field and hence provide a vacuum which is stable with respect to the dilaton field.

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