Subthreshold photoproduction of charm

M.A.Braun and B.Vlahovic
North Carolina Central University, Durham, NC, USA

Abstract

Charm photoproduction rates off nuclei below the nucleon threshold are estimated using the phenomenologically known structure functions both for $x < 1$ and $x > 1$. The rates rapidly fall below the threshold from values $\sim 10$ pb for Pb close to the threshold (at 7.5 GeV) to $\sim 1$ pb at 6 GeV.

1 Introduction

In view of the envisaged upgrade of the CEBAF facility up to 12 GeV it becomes important to have relatively secure predictions about the production rates of charm on nuclear targets below the threshold for the nucleon target. This note aims at such predictions. From the start it has to be recalled that the dynamical picture of charm production at energies close to the threshold is much more complicated than at high energies. On the one hand, the production rates cannot be described by the standard collinear factorization expression but involve gluon distributions both in $x$ and transverse momentum, which are unknown at small transverse momenta (in the confinement region). On the other hand, in the immediate vicinity of the threshold the simple photon-gluon fusion mechanism of charm production becomes overshadowed by multiple gluon exchanges and formation of colourless bound states with lower mass, as compared to open charm. As a result, theoretical studies of the charm production near the threshold are known to be notoriously difficult. To avoid all these complications we study charm production at not too small distance from the thresholds, where all effects due to multiple gluon exchanges are hopefully small. To be more concrete, taking the charmed quark mass $m_c = 1.5$ GeV, we have the threshold of open charm photoproduction on the proton target at the incident photon energy $E^{th}_1 = 7.78$ GeV and on the deuteron target at 5.39 GeV. As we shall argue (see Appendix), our treatment is hopefully valid at incident photon energies $E$ excluding regions at distances $\sim 0.3$ GeV from these thresholds, say, $5.7 < E < 7.5$ GeV. Closer to the thresholds multigluon exchanges and bound state formation may change our predictions considerably.

In fact all estimates show that the ratio of the double to single gluon exchange contributions is determined by the parameter $\alpha_s(M^2)[\mu/(m_c\Delta x)]^2$, where $\mu$ is the light quark mass, $M$ is the mass scale for the gluon coupling to the charmed system and $\Delta x$ is the distance from the threshold for the scaling variable of the produced charmed system (1, see also Appendix). Obviously as $\Delta x \rightarrow 0$ multiple gluon exchanges begin to play the dominant role. They also bind the produced charmed quarks into colourless bound states, thus changing the
threshold value. However, with $\Delta x$ growing their contribution becomes suppressed. Taking the effective $\mu \simeq \Lambda_{QCD} = 0.3$ GeV, and $M = 2m_c$ so that $\alpha_s(M^2) \sim 0.3$ we find the suppression factor of the order $75(\Delta x)^2$. This factor determines the region where one can neglect all multiple gluon exchanges and bound state formation. In a more detail this suppression factor will be discussed in Appendix.

Neglecting multiple gluon exchanges, subthreshold production will be determined by the gluon distribution in the "cumulative" region ($x > 1$ at large energies or massless target). As mentioned, in the low energy region the charm production rate involves the gluon distribution not only in $x$ (collinear factorization) but also in both $x$ and $k^2_\perp$ ($k_\perp$ factorization). We shall use the simplest approximation about the structure of this combined distribution, assuming the dependence on $x$ and $k^2_\perp$ factorized. For the latter dependence we shall use a monopole form, prompted by the perturbation theory. As for the former one, we shall exploit the existing (scarce) data on the nuclear structure functions at $x > 1$, from which we shall extract the relevant gluon distribution, using the DGLAP evolution equation. With all these simplifications, we hope to be able to predict the rates up to factor $2 \div 3$.

2 Kinematics and cross-sections

2.1 The charm production cross-section

Consider the exclusive process

$$\gamma + A \rightarrow C \bar{C} + A^*, \quad (1)$$

where $A$ is the target nucleus of mass $m_A$ and $A^*$ is the recoil nuclear system of mass $m_A^*$. We denote the total mass of the $C \bar{C}$ system as $M$. Obviously $M \geq 2m_c$ where $m_c$ is the mass of the C-quark, which we take as 1.5 GeV. The inclusive cross-section for charm photoproduction is obtained after summing over all states of the recoil nuclear system.

We choose a reference system in which the target nucleus with momentum $A p$ is at rest and the incoming photon with momentum $q$ is moving along the $z$-axis in the opposite direction, so that $q_+ = q_\perp = 0$. The photoproduction cross-section corresponding to (1) is then obtained via the imaginary part of the diagram in Fig. 1 as

$$\sigma_{A \rightarrow A^*} = A \int \frac{d^4k}{(2\pi)^4} \delta((Ap - k)^2 - m_A^* x) \left( \frac{\Gamma_{AA^*}(k^2)}{k^2} \right)^2 \sigma_g(M^2, k^2). \quad (2)$$

Here $\Gamma_{AA^*}(k^2)$ is the vertex for gluon emission from the target; $\sigma_g(M^2, k^2)$ is the photoproduction cross-section off the virtual gluon of momentum $k$. We have also introduced the scaling variable for the gluon as $x = k_+/p_+$. Due to $q_+ = 0$ this is also the scaling variable for the observed charm. Note that this definition, which is standard at large energies and produced masses, is not at all standard at moderate scales. In particular this $x$ does not go to unity at the threshold for the nucleon target. Rather the limits for its variation converge to a common value 0.76. For the nuclear targets with $A >> 1$ its minimal value at the nucleon threshold is well below unity ($\sim 0.64$). One should have this in mind when associating this $x$ with the gluon distribution: it follows that for a nuclear target, for energies going noticeably below the nucleon threshold, the cumulative region (prohibited for the free nucleons kinematics) includes not only values of $x$ above unity but also a part of the region $x < 1$.

\footnote{Some crude estimates were earlier reported in [3]. Their order agrees with the present calculations at energies not too close to the threshold.}
We use the $\delta$-function to integrate over $k_-$ to obtain the cross-section (2) as

$$\sigma_{A \rightarrow A^*} = A \int \frac{dx^2d^2k_1}{2(A-x)(2\pi)^3} \left( \frac{\Gamma_{AA^*}(k^2)}{k^2} \right)^2 \sigma_g(M^2, k^2).$$  \hspace{1cm} (3)

In these variables we find

$$M^2 = xs_1 + k^2, \quad s_1 = 2pq$$  \hspace{1cm} (4)

and

$$k^2 = xAm^2 - \frac{x}{A-x}m_A^*2 - \frac{A}{A-x}k_1^2,$$  \hspace{1cm} (5)

where we have put $p^2 = m^2$, the nucleon mass squared, neglecting the binding.

The limits of integration in (3) are determined by the condition $M^2 \geq 4m^2_c$, which leads to

$$x(s_1 + Am^2) - \frac{x}{A-x}m_A^*2 - \frac{A}{A-x}k_1^2 - 4m^2_c \geq 0.$$  \hspace{1cm} (6)

Since $k_1^2 \geq 0$, one gets

$$x(s_1 + Am^2) - \frac{x}{A-x}m_A^*2 - 4m^2_c \geq 0,$$  \hspace{1cm} (7)

from which one finds the limits of integration in $x$ for the transition $A \rightarrow A^*$:

$$x_{1}^{A \rightarrow A^*} \leq x \leq x_{2}^{A \rightarrow A^*}$$  \hspace{1cm} (8)

where

$$x_{1,2}^{A \rightarrow A^*} = \frac{1}{2s} \left( As - m_A^*2 + 4m^2_c \pm \sqrt{[As - (m_A^* + 2m^2_c)]^2} \right)$$  \hspace{1cm} (9)

and $s = s_1 + Am^2$. The limits of integration in $k_1$ at a given $x$ are determined by (6).

Using (5) we may pass from the integration variable $k_1^2$ to $|k^2|$. Summing over all states of the recoiling nucleus $A^*$ we get

$$\sigma_{A} = \int_{x_{1}^{(A)}}^{x_{2}^{(A)}} xdx \int_{|k^2|_{\text{min}}}^{x_{s1} - 4m^2_c} d|k^2| \sigma_g(xs_1 - |k^2|, k^2)\rho(x, |k^2|),$$  \hspace{1cm} (10)

where

$$\rho(x, |k^2|) = \frac{\pi}{2(2\pi)^3} \sum_{A^*} \left( \frac{\Gamma_{AA^*}(k^2)}{k^2} \right)^2,$$  \hspace{1cm} (11)

$$|k^2|_{\text{min}} = \frac{A}{A-x}x^2m^2$$  \hspace{1cm} (12)

and $x_{1,2}^{(A)}$ is determined by (9) with $m_A^*$ put to its minimal value $m_A^* = m_A = Am$.

The threshold energy corresponds to $x_{1}^{(A)} = x_{2}^{(A)}$ or $As = (Am + 2m^2_c)^2$. In terms of the photon energy $E$ we have $s_1 = 2mE$ and the threshold energy is found to be

$$E_{A}^{\text{th}} = 2m_c \left( 1 + \frac{1}{A} \frac{m_c}{m} \right).$$  \hspace{1cm} (13)

It steadily falls with $A$ from the nucleon target threshold. With $m_c = 1.5$ GeV we find (in GeV):

$$E_{1}^{\text{th}} = 7.79, \quad E_{2}^{\text{th}} = 5.39, \quad E_{3}^{\text{th}} = 4.60, \quad E_{12}^{\text{th}} = 3.40, \quad E_{207}^{\text{th}} = 3.02$$  \hspace{1cm} (14)

down to the value $2m_c = 3$ GeV for infinitely heavy nucleus.
2.2 High-energy limit

To interpret $\rho$ in Eq. (10) it is instructive to study its high-energy limit, which corresponds to taking $s_1 >> m_c^2$ and both quantities much greater than the nucleon mass. Assuming that the effective values of the gluon virtuality are limited (and small) one then gets for the nucleon target ($A = 1$)

$$\sigma_1 = \int_{4m_c^2/s_1}^{1} xdxg(xs_1)\int_{0}^{xs_1}d|k^2|\rho(x,|k^2|).$$

(15)

Here we also neglect the off-mass-shellness of the cross-section off the gluon, considering $|k^2| << 4m_c^2$. The obtained formula is precisely the standard collinear factorization formula with the identification

$$xg(x, M^2) = \int_{0}^{M^2}d|k^2|x\rho(x,|k^2|).$$

(16)

Thus the quantity $\rho(x,|k^2|)$ obviously has a meaning of the double distribution of gluons in $x$ and $|k^2|$.

3 The gluon distribution $\rho(x,|k^2|)$

To find the double distribution of gluons in $x$ and $|k^2|$ one may be tempted to use (16) and simply differentiate $xg(x, M^2)$ in $M^2 = |k^2|$. However (16) is only true for $x << 1$. At finite $x$ the derivative $dg(x, m^2)/dM^2$ is not positive and cannot be interpreted as the double gluonic distribution.

To avoid this problem, we choose a different, somewhat simplified approach. We assume a simple factorizable form for the double density $\rho(x,|k^2|)$ and choose the $|k^2|$ dependence in accordance with the perturbation theory, with an infrared cutoff in the infrared region:

$$\rho(x,|k^2|) = \frac{a(x)}{|k^2| + \Lambda^2}.\quad (17)$$

Function $a(x)$ can be obtained matching (17) with the observed $xg(x, M^2)$ at a particular point $M_0^2$. Since we are interested in the threshold region, we take $M_0 = 2m_c$ to finally obtain

$$\rho(x,|k^2|) = \frac{g(x, 4m_c^2)}{\ln(4m_c^2/\Lambda^2 + 1)} \frac{1}{|k^2| + \Lambda^2}.\quad (18)$$

The recipe (18) amounts to taking in (16) $\rho$ dependent also on $M^2$, with the latter dependence factorized. Our calculations show that the results are rather weakly dependent of the infrared cutoff chosen in the interval $0.4 ÷ 0.7$ GeV.

For the proton at $x < 1$ the gluon distribution $g(x, 4m_c^2)$ can be taken from numerous existing fits to the experimental data. In our calculations we have used GRV95 LO \cite{4}. For the nucleus in the non-cumulative region

$$x_1^{(1)} < x < x_2^{(1)}\quad (19)$$

we use the simplest assumption $g_A(x, Q^2) = A g_1(x, Q^2)$ neglecting the EMC effect in the first approximation. Obviously this not a very satisfactory approximation at $x$ quite close to the threshold for the proton target. However our estimates are in any case not justified in this region, since, as mentioned in the introduction, multiple gluon exchanges and bound states formation begin to play a leading role in the immediate vicinity of the threshold.

For the nuclei in the cumulative region (outside region (19)) the gluon distribution may be estimated using, first, the existing data for the nuclear structure functions in this region
and, second, the popular hypothesis that at sufficiently low $Q^2 = Q_0^2$ the sea and gluon distributions vanish and hadrons become constructed exclusively of valence quarks. Then one can find the gluon distribution at a given $Q^2$ from the standard DGLAP evolution equation, with the quark distributions determined from the experimental data on the structure functions at $x > 1$ and evolved back to $Q^2 = Q_0^2$. In practice we took the initial valence distributions in carbon at $Q_2 = Q_0^2$ in the form

$$u(x, Q_0^2) = d(x, Q_0^2) = a e^{-bx} \quad (20)$$

and the rest of the distributions equal to zero. Then we calculated the carbon structure function at $x > 1$ and $Q_2$ in correspondence with the data of [5] and chose the parameters $a$ and $b$ to fit the data. With thus chosen $a$ and $b$ we finally calculated the gluon distribution in carbon at the scale $4m_c^2$. Our obtained gluon distributions in carbon for $Q_0 = 0.4$ and 0.7 GeV/c are shown in Fig. 2 for $1 < x < 2$. As one observes, the dependence on the choice of $Q_0$ is very weak in this interval. The slopes result equal to 11.4 ($Q_0 = 0.4$ GeV/c) and 11.2 ($Q_0 = 0.7$).

The distribution for other nuclei was taken from the A-dependence, chosen in accordance with the experimental data for hadron production at $x > 1$ as $\propto A^{1+0.3x} \quad [6]$.

4 Numerical results

The cross-section (10) involves the photon-gluon fusion cross-section $\sigma_g$ off mass shell. The integration over the gluon virtuality starts from $|k^2| \sim m^2$. If one assumes $m/M \to 0$ then the bulk of the contribution will come from the region of small $|k^2|$ (with a logarithmic precision). In reality $m/M$ is not so small. However, to simplify our calculations, as a first approximation, we have taken the photon-gluon cross-section on the mass shell, where it is known to be [5]

$$\sigma_g(M^2) = \pi \alpha_e \alpha_s e^2 c \frac{1}{M^4} \int_{t_1}^{t_2} dt \left[\frac{t}{u} + \frac{u}{t} + \frac{4m_c^2 M^2}{tu} \left(1 - \frac{m_c^2 M^2}{tu}\right)\right] \quad (21)$$

Here $e_c$ is the quark charge in units $e$, $u = -M^2 - t$ and the limits $t_{1,2}$ are given by

$$t_{1,2} = -\frac{M}{2} [M \pm \sqrt{M^2 - 4m_c^2}] \quad (22)$$

We take the strong coupling constant $\alpha_s = 0.3$.

Our gluon distribution depends on two parameters: the infrared cutoff $\Lambda$ in (19) and the value of $Q_0$, at which the sea and gluon distributions die out. The order of both is well determined, but still one can vary them to some degree. In our calculations we took both $\Lambda$ and $Q_0$ equal to 0.4 or 0.7 GeV/c.

With these values for the parameters we obtain the cross-sections for charm photoproduction on Pb shown in Fig. 3. As one observes, the dependence on both $\Lambda$ and $Q_0$ is relatively weak: in the whole range of their variation the cross-sections change by less than 30%. Fig. 4 illustrates the A-dependence of the cross-sections (with $\Lambda = Q_0 = 0.4$ GeV/c). To have the idea of the number of nucleons which have to interact together to produce charm at fixed energy below threshold we show the limits of integration $x_1$ and $x_2$ in Fig. 5.

As expected, the cross-sections rapidly fall for energies below threshold. Their energy dependence cannot be fit with a simple exponential (in fact they fall faster than the exponential). As to the absolute values, for Pb the cross-section fall from $\sim 10$ pb immediately below the threshold down to $\sim 1$ pb at $E = 6$ GeV. The $A$ dependence is close to linear.
5 Discussion

We have estimated charm photoproduction rates for nuclear targets below the nucleon target threshold. The estimates require knowledge of the gluon distribution in both $x$ and $k^2_{\perp}$ in a wide region of the momenta including the confinement region.

Our estimates were based on a simple factorization assumption and introduction of an infrared cutoff. Another approximation has been to take the photon-gluon fusion cross-section on its mass-shell. We are of the opinion that this second approximation is not very serious, as compared to the first one. In any case it can easily be dropped for the price of considerable complication of the calculation.

Our predictions are infrared cutoff dependent. However the cutoff dependence results weak for variations of the cutoffs in a reasonable interval.

In our study we assumed the standard mechanism of charm production via gluon-photon fusion (a single gluon exchange between light and heavy quarks). It can be shown that this mechanism dominates, provided one is not too close to the threshold (see Appendix).

6 Acknowledgements

M.A.B. is thankful to the Faculty of Science of the NCCU for hospitality.

7 Appendix. Multiple gluon exchange

7.1 Kinematics and phase volume

To study the relative weight of multiple gluon exchange we consider a simplified picture with a mesonic target composed of a light quark and antiquark of mass $\mu$. We neglect the binding, so that the meson mass is just $2\mu$. We shall compare contributions to heavy flavour production of the three amplitudes Fig. 6a – c. Amplitude $a$ corresponds to a single gluon exchange between light and heavy quarks, amplitudes $b$ and $c$ to double gluon exchange. We use the light-cone variables and denote $k_{i+} = z_ip_+, \quad p_{i+} = x_ip_+, \quad i = 1, 2$.

The phase volume for the reaction is given by

$$dV = \frac{1}{16(2\pi)^6} \frac{d^3k_1d^3p_1d^3p_2}{z_1x_1x_2} \delta(R_e - R), \quad (23)$$

where $d^3k_1 = dz_1d^2k_{1\perp}$ etc and the $\delta$-function refers to conservation of the light-cone energy (the "-" component of the momentum). Its argument contains the external energy $R_e = 2pq + 4\mu^2 = 2\mu E + 4\mu^2$, and the energy of the produced particles $R = \sum_{i=1}^2(m^2_{c,i\perp}/z_i + \mu^2_{i\perp}/x_i)$

The minimal value of $R$ determining the production threshold occurs at

$$z_i = z_0 = \frac{m_c}{m_c + \mu}, \quad x_i = x_0 = \frac{\mu}{m_c + \mu}, \quad k_{i\perp} = p_{i\perp} = 0, \quad i = 1, 2, \quad (24)$$

and is equal to $\min R = R_0 = 2(m_c + \mu)^2$.

We shall study our amplitudes near the threshold, so that $x_i$ will be small and $z_i$ will be close to unity. We put $z_i = z_0 + \zeta_i, \quad x_i = x_0 + \xi_i, \quad i = 1, 2$ and develop $R$ near the threshold keeping terms of the second order in $\zeta$’s, $\xi$’s and transverse momenta. We present the difference $R_e - R$ in the form $R_e - R_0 = \Delta R_0$, where dimensionless $\Delta$ measures the distance from the threshold and is supposed to be small. Finally we rescale our variables as follows

$$\zeta_i = \tilde{\zeta}_i \sqrt{\Delta z_0^2 R_0/M^2}, \quad \xi_i = \tilde{\xi}_i \sqrt{\Delta x_0^2 R_0/m^2}, \quad k_{i\perp} = \tilde{k}_{i\perp} \sqrt{\Delta z_0 R_0}, \quad p_{i\perp} = \tilde{p}_{i\perp} \sqrt{\Delta x_0 R_0}. \quad (25)$$
We use the Coulomb gauge for the interaction between quarks and neglect the contribution from the transverse momenta in it. Then the interaction depends only on the scaling variables, and for the transition between, say, light quarks from the transverse momenta in it. Then the interaction depends only on the scaling variables, and for the transition between, say, light quarks $p_1 + p_2 \rightarrow p_1' + p_2'$ is given by

$$V(p_1, p_2|p_1', p_2') = 4\pi\alpha_s \frac{(x_1 + x_1')(x_2 + x_2')}{(x_1 - x_1')^2}. \quad (27)$$

We shall assume that the initial light quarks have their momenta equal to $p$, so that their scaling variable is equal to unity. We omit the the common factor due to their binding into the initial target meson. Finally we consider photoproduction, so that $q^2 = 0$ and choose a system in which $q_+ = q_\perp = p = 0$.

The amplitude corresponding to Fig. 6a is given by

$$A^{(a)} = \frac{V((p, p)|2p - p_2, p_2)V(q - k_1, 2p - p_2|k_2, p_1)}{(\mu^2 - (2p - p_2)^2)(m_c^2 - (q - k_1)^2)}. \quad (28)$$

The two interactions near the threshold turn into $12\pi\alpha_s$ and $2\pi\alpha_s(z_2 - z_1)$ where we used the fact that $x_1, x_2 << 1$ and $z_1, z_2 \simeq 1$. For the same reason we find the two denominators as $\mu^2 - (2p - p_2)^2 \simeq 2\mu_\perp^2/x_2$ and $m_c^2 - (q - k_1)^2 \simeq 2pq$. In our dimensionless variables we obtain

$$A^{(a)} = -2c_1\sqrt{2z_0}\Delta \frac{\tilde{\zeta}_1(x_0 + \tilde{\zeta}_1\sqrt{2z_0}\Delta)}{\mu^2 + 2\mu(m_c + \mu)\Delta \tilde{p}_\perp^2}, \quad c_1 = \frac{6\pi^2\alpha_s^2}{pq}. \quad (29)$$

The amplitude corresponding to Fig. 6b is

$$A^{(b)} = \frac{V((q - k_1, p|q - k_1 + p - p_1, p)V(q - k_1 + p - p_1, p|k_2, p_2)}{(m_c^2 - (q - k_1)^2)(m_c^2 - (q - k_1 + p - p_1)^2)}. \quad (30)$$

Near the threshold the interactions become $4\pi\alpha_s$ and $4\pi\alpha_s$. The new denominator is

$$m_c^2 - (q - k_1 + p - p_1)^2 \simeq m_c^2 + 2(m_c + \mu)\Delta(\sqrt{m_ck_1} + \sqrt{\mu p_1})^2. \quad (31)$$

At $\Delta << 1$ we can drop the second term. So we get

$$A^{(b)} = -c_2\frac{1}{m_c^2}, \quad c_2 = \frac{8\pi^2\alpha_s^2}{pq}. \quad (32)$$

Finally we consider the amplitude of Fig. 6c:

$$A^{(c)} = \frac{V((k_1 - p + p_1, p|k_1, p_1)V(k_2 - p + p_2, p|k_2, p_2)}{(m_c^2 - (k_1 - p + p_1)^2)(m_c^2 - (k_2 - p + p_2)^2)}. \quad (33)$$

Near the threshold both interactions become approximately equal to $4\pi\alpha_s$ and both denominators to $m_c^2$. So the amplitude becomes

$$A^{(c)} = 2c_2\frac{pq}{m_c^2} \simeq 2c_2\frac{1}{m_c^2}, \quad (34)$$

where we have used that near the threshold $pq \simeq m_c^2$. So the amplitudes $b$ and $c$ have the same order of magnitude.
### 7.3 Cross-sections

Now we can pass to our main goal: comparison of contributions of the three amplitudes to the total cross-section for heavy flavour production. The first thing to note is that near the threshold amplitude \(a\) does not interfere with \(b\) and \(c\), since since \(A^{(a)}\) is odd in \(\xi_1\) and \(A^{(b,c)}\) do not depend on \(\xi_1\) at all. Second, since \(A^{(b)}\) and \(A^{(c)}\) are of the same order and structure it is sufficient to compare the contributions of \(A^{(a)}\) and \(A^{(b)}\). Finally, due to the fact that \(x_0\) is small, the magnitude of contributions depends on the relation between \(\Delta\) and \(x_0\). We shall study two limiting cases: \(\Delta < x_0\) (region A) and \(\Delta >> x_0\) (region B).

Region A refers to the production immediately above the threshold. In this case we can take \(x_{1,2} \simeq x_0\). Then we find the contribution of amplitude \(a\) to the cross-section as

\[
\sigma^{(a)} = V_0 \Delta^7/2 \frac{1}{x_0^3} [2c_1 \sqrt{2\Delta x_0^2} \mu^2] I^{(a)},
\]

where \(I^{(a)}\) is a certain integral of the order unity. The contribution of the amplitude \(b\) to the cross-section will be

\[
\sigma^{(b)} = V_0 \Delta^7/2 \frac{1}{x_0^3} [c_2 \frac{1}{m_c^2}]^2 I^{(b)},
\]

where \(I^{(b)}\) is another integral of the order unity. The ratio of these two cross-sections will have the order

\[
\frac{\sigma^{(b)}}{\sigma^{(a)}} \sim \frac{\mu^2(m_c + \mu)^2}{m_\perp^4} \sim \frac{\mu^2}{m_\perp^2} \sim \frac{x_0^2}{\Delta}.
\]

Thus immediately above the threshold the contribution from amplitudes \(b\) and \(c\) dominate. However with the growth of \(\Delta\), in the region \(x_0^2 << \Delta\) (and \(<< x_0\) to still remain in region A) the contribution of amplitudes of \(b\) and \(c\) become suppressed by factor \(\mu/m_c\).

In region B we can approximate \(x_{1,2} \simeq \tilde{\xi}_{1,2} \sqrt{2x_0}\). To avoid logarithmic divergence in \(\tilde{\xi}_{1,2}\) we cut off the integration region from below at values of the order \(\sqrt{x_0}/\Delta\). We also note that the integral over \(\tilde{p}_{1\perp}^2\) appearing in the contribution of amplitude \(a\) is well convergent at values of \(\tilde{p}_{1\perp}^2 \sim x_0/\Delta\) so that we may neglect \(\tilde{p}_{1\perp}^2\) in the argument of the \(\delta\)-function and separate the integration over \(\tilde{p}_{1\perp}\) as a factor

\[
\int \frac{d^2\tilde{p}_{1\perp}}{[\mu^2 + 2\mu(m_c + \mu)\Delta]\tilde{p}_{2\perp}^2} = \frac{\pi}{2\mu^3(m_c + \mu)\Delta}.
\]

We find the cross-section from \(A^{(a)}\) as

\[
\sigma^{(a)} = V_0 \Delta^{5/2} \frac{1}{2x_0} [2c_1 2\Delta \sqrt{\Delta/x_0}]^2 \frac{\pi}{2\mu^3(m_c + \mu)\Delta} J^{(a)},
\]

where \(J^{(a)}\) is an integral of the order \(\ln(x_0/\Delta)\). The cross-section from \(A^{(b)}\) is found to be

\[
\sigma^{(b)} = V_0 \Delta^{5/2} \frac{1}{2x_0} [c_2 \frac{1}{m_c^2}]^2 J^{(b)},
\]

with \(J^{(b)}\) an integral of the order \(\ln^2(x_0/\Delta)\).

The ratio of the two cross-section turns out to be of the same order up to a logarithmic factor

\[
\frac{\sigma^{(b)}}{\sigma^{(a)}} \sim \frac{\mu^2(m_c + \mu)^2}{m_\perp^4} \frac{\ln \Delta}{x_0} \sim \frac{\mu^2}{m_\perp^2} \frac{\ln m_c \Delta}{\mu} \sim \frac{x_0^2}{\Delta} \frac{\ln \Delta}{x_0}
\]

and so the contribution of amplitude \(a\) clearly dominates in region B, where \(\Delta >> x_0\). The suppression factor for the contribution of the amplitudes \(b\) and \(c\) with double gluon exchange...
is found to be $m_c^2/(\mu^2\Delta)$. With $m_c/\mu \sim 5$ it is of the order $25\Delta$. Taking into account that double gluon exchange involves a coupling constant at the heavy flavour mass scale will add a factor $\sim 3$ more. So in the end we find a suppression factor of the order $75\Delta$, which implies that at a distance of $0.3$ GeV from the threshold the contribution of the double gluon exchange drops by a factor $\sim 3$.

7.4 Hadrons with more quarks

Generalization to hadrons with more quarks is straightforward, although quite cumbersome due to rapid proliferation of diagrams. However the basic findings remain unchanged. Indeed all the difference between amplitudes with a single and double gluon exchange between light and heavy quarks comes from the fact that a soft propagator of the order $x/\mu^2$ in the amplitude with a single gluon exchange is substituted by a hard propagator of the order $1/(pq)$ in the amplitude with two gluon exchanges. This difference persists for any number of quarks in the hadron, although the total number of soft propagator grows. So although the overall dependence on $\Delta$ will change (in accordance with the quark counting rules), the relation between cross-sections with a single and double gluon exchange will remain the same.

7.5 Bound states

One may wonder if the production cross-section is dominated by the formation of final bound states, via diagrams as Fig. 7a, which looks as quark rearrangement without any gluon exchange [2]. However one has to recall that in the bound state of a light and a heavy quark (D-meson) the typical configuration requires $p_{i+}/l_{i+} = \mu/m_D$ and $k_{i+}/l_{i+} = m_c/m_D$, where we neglect the binding taking $m_D = m_c + \mu$. The initial light quarks have however $p_{i+} = p_+$. So for their binding into D-mesons, they have to diminish their longitudinal momenta by at least two hard gluon exchanges, as shown in Fig. 7b. But the process in Fig. 7b contributes actually a part of the cross-section generated by the amplitude $A^{(c)}$ studied in the preceding subsection, which corresponds to the immediate binding of the open charm into D-mesons. Above the threshold of the open charm production its contribution can only be smaller than the total rate of open charm production. True, immediately below this threshold, at distances of the order of the binding energy, this mechanism is obviously the only one that contributes, in agreement with the estimates above for very small $\Delta$'s. However, as we have seen, with the growth of $\Delta$ the strength of multiple interactions between light and heavy quarks necessary to produce them in a state appropriate for their binding rapidly goes down. With them goes down also the corresponding part of the cross-section due to immediate binding.

References

[1] S.J.Brodsky, E.Chudakov, P.Hoyer and J.M.Laget, Phys. Lett. B 498 (2001) 23.
[2] B.Kopeliovich, private communication
[3] M.A.Braun and B.Vlahovic, hep-ph/0209261
[4] M.Glueck, E.Reya and A.Vogt, Z.Phys. C 67 (1995) 433.
[5] BCDMS collab., Z.Phys. C 63 (1994) 29.
[6] Y.D.Bayukov et al., Phys. Rev C 20 (1979) 764; N.A.Nikiforov et al., Phys. Rev. C 22 (1980) 700.
[7] J.Smith and W.L. van Neerven, Nucl. Phys. B 374 (1992) 36.
8 Figure captions

Fig. 1. The forward scattering amplitude corresponding to reaction (1). Heavy quarks are shown by double lines.

Fig. 2. The cumulative \((x > 1)\) gluon distributions in carbon at \(Q^2 = 4m_c^2\). The upper (lower) curve corresponds to \(Q_0 = 0.4(0.7)\) GeV/c.

Fig. 3. The charm photoproduction subthreshold cross-sections off Pb for different choice of parameters \(\Lambda\) and \(Q_0\). Curves from top to bottom correspond to \((\Lambda, Q_0) = (0.7, 0.4), (0.4, 0.4), (0.7, 0.7)\) and \((0.4, 0.7)\) GeV/c.

Fig. 4. The charm photoproduction subthreshold cross-sections for different targets. Curves from bottom to top correspond to \(A = 12, 64\) and 207.

Fig. 5. The limits of \(x\)-integration for different photon energies and nuclear targets. Curves from bottom to top correspond to \(A = 12, 64\) and 207.

Fig. 6. Amplitudes for charm photoproduction off a meson with a single \((a)\) and double \((b, c)\) gluon exchanges between light and heavy quarks. The latter are shown with double lines. Vertical lines correspond to gluonic exchanges.

Fig. 7. Amplitudes for the \(D\bar{D}\) photoproduction off a meson. Diagram \(a\) is equivalent to diagram \(b\), which shows how the produced quarks acquire their momenta appropriate for the binding. Notations are as in Fig. 6.
