Observations and three-dimensional photoionization modelling of the Wolf–Rayet planetary nebula Abell 48

A. Danehkar,1 † H. Todt,2 B. Ercolano3,4 and A. Y. Kniazev5,6,7
1Department of Physics and Astronomy, Macquarie University, Sydney, NSW 2109, Australia
2Institut für Physik und Astronomie, Universität Potsdam, Karl-Liebknecht-Str. 24/25, D-14476 Potsdam, Germany
3Universitäts-Sternwarte München, Ludwig-Maximilians Universität München, Scheinerstr. 1, D-81679 München, Germany
4Exzellenzcluster Universe, Technische Universität München, Boltzmannstr. 2, D-85748 Garching, Germany
5South African Astronomical Observatory, PO Box 9, 7935 Observatory, Cape Town, South Africa
6Southern African Large Telescope Foundation, PO Box 9, 7935 Observatory, Cape Town, South Africa
7Sternberg Astronomical Institute, Lomonosov Moscow State University, Moscow 119992, Russia

Accepted 2014 January 28. Received 2014 January 28; in original form 2013 September 10

ABSTRACT
Recent observations reveal that the central star of the planetary nebula Abell 48 exhibits spectral features similar to massive nitrogen-sequence Wolf–Rayet stars. This raises a pertinent question, whether it is still a planetary nebula or rather a ring nebula of a massive star. In this study, we have constructed a three-dimensional photoionization model of Abell 48, constrained by our new optical integral field spectroscopy. An analysis of the spatially resolved velocity distributions allowed us to constrain the geometry of Abell 48. We used the collisionally excited lines to obtain the nebular physical conditions and ionic abundances of nitrogen, oxygen, neon, sulphur and argon, relative to hydrogen. We also determined helium temperatures and ionic abundances of helium and carbon from the optical recombination lines. We obtained a good fit to the observations for most of the emission-line fluxes in our photoionization model. The ionic abundances deduced from our model are in decent agreement with those derived by the empirical analysis. However, we notice obvious discrepancies between helium temperatures derived from the model and the empirical analysis, as overestimated by our model. This could be due to the presence of a small fraction of cold metal-rich structures, which were not included in our model. It is found that the observed nebular line fluxes were best reproduced by using a hydrogen-deficient expanding model atmosphere as the ionizing source with an effective temperature of \( T_{\text{eff}} = 70 \, \text{kK} \) and a stellar luminosity of \( L_\star = 5500 \, \text{L}_\odot \), which corresponds to a relatively low-mass progenitor star \((\sim 3 \, \text{M}_\odot)\) rather than a massive Pop I star.

Key words: stars: Wolf–Rayet – ISM: abundances – planetary nebulae: individual: Abell 48.

1 INTRODUCTION
The highly reddened planetary nebula Abell 48 (PN G029.0+00.4) and its central star (CS) have been the subject of recent spectroscopic studies (Wachter et al. 2010, Depew et al. 2011, Todt et al. 2013, Frew et al. 2013). The CS of Abell 48 has been classified as Wolf–Rayet [WN5] (Todt et al. 2013), where the square brackets distinguish it from the massive WN stars. Abell 48 was first identified as a planetary nebula (PN) by Abell (1955). However, its nature remains a source of controversy whether it is a massive ring nebula or a PN as previously identified. Recently, Wachter et al. (2010) described it as a spectral type of WN6 with a surrounding ring nebula. But, Todt et al. (2013) concluded from spectral analysis of the CS and the surrounding nebula that Abell 48 is rather a PN with a low-mass CS than a massive (Pop I) WN star. Previously, Todt et al. (2010) also associated the CS of PB 8 with [WN/C] class. Furthermore, IC 4663 is another PN found to possess a [WN] star (Miszalski et al. 2012).

A narrow-band H\(_\alpha\)+[N\(_\mathrm{II}\)] image of Abell 48 obtained by Jewitt et al. (1986) first showed its faint double-ring morphology. Zuckerman & Aller (1986) identified it as a member of the elliptical morphological class. The H\(_\alpha\) image obtained from the SuperCOSMOS Sky H\(_\alpha\) Survey (Parker et al. 2008) shows that the angular dimensions of the shell are about 46\(\arcsec\) \times 38\(\arcsec\), and are used throughout this paper. The first integral field spectroscopy of Abell 48 shows the same structure in the H\(_\alpha\) emission-line profile.

* Based on observations made with the Australian National University (ANU) Telescope at the Siding Spring Observatory, and the Southern African Large Telescope (SALT) under programs 2010-3-RSA,OTH-002.
† E-mail: ashkibiz.danehkar@students.mq.edu.au

© 2014 RAS
suitable sky window has been selected from the science data for the sky subtraction purpose.

The positions observed on the PN are shown in Fig. 1(a). The centre of the IFU was placed in two different positions in 2010 and 2012. The exposure time of 20 min yields a signal-to-noise ratio of \( S/N \gtrsim 10 \) for the \([\text{O\ II}]\) emission line. Multiple spectroscopic standard stars were observed for the flux calibration purposes, notably Feige 110 and EG 274. As usual, series of bias, flat-field frames, arc lamp exposures, and wire frames were acquired for data reduction, flat-fielding, wavelength calibration and spatial calibration.

Data reductions were carried out using the IRAF pipeline WIFES (version 2.0; 2011 Nov 21). The reduction involves three main tasks: WFTABLE, WFCAL and WFREDUCE. The IRAF task WFTABLE converts the raw data files with the single-extension Flexible Image Transport System (FITS) file format to the Multi-Extension FITS file format, edits FITS file key headers, and makes file lists for reduction purposes. The IRAF task WFCAL extracts calibration solutions, namely the master bias, the master flat-field calibration solutions, namely the master bias, the master flat-field, and the wire frames. The IRAF task WFREDUCE applies the calibration solutions to science data, subtracts sky spectra, corrects for differential atmospheric refraction, and applies the flux calibration using observations of spectrophotometric standard stars.

A complete list of observed emission lines and their flux

---

**Table 1.** Journal of the IFU observations with the ANU 2.3-m Telescope.

| PN   | Date (UT) | \( \lambda \) range (\( \AA \)) | \( R \) | Exp.(s) |
|------|-----------|----------------------------------|------|--------|
| Abell 48 | 2010/04/22 | 4415–5589                      | 7000 | 1200 |
|       |           | 5222–7070                       | 7000 | 1200 |
|       | 2012/08/23 | 3295–5906                      | 3000 | 1200 |
|       |           | 5462–9326                      | 3000 | 1200 |

**Table 2.** Observed and dereddened relative line fluxes of the PN Abell 48, on a scale where \( H_\beta = 100 \). Uncertain and very uncertain values are followed by ‘*’ and ‘**’, respectively. The symbol ‘+’ denotes blended emission lines.

| \( \lambda_{\text{lab}} (\AA) \) | ID  | Mult | \( F(\lambda) \) | \( I(\lambda) \) | Err(%) |
|-------------------------------|-----|------|------------------|------------------|-------|
| 3726.03                       | [O\ II] | F1  | 20.72            | 128.96           | 25.7  |
| 3728.82                       | [O\ III] | F1  | *               | *                | *     |
| 3808.75                       | [Ne\ II] | F1  | 7.52             | 38.96            | 9.4   |
| 4340.47                       | H\alpha  | H5  | 21.97            | 54.28            | 17.4  |
| 4471.50                       | He\beta   | V14 | 3.76             | 7.42             | 12.0  |
| 4861.33                       | H\beta-2  | H4  | 100.00           | 100.00           | 6.2   |
| 4958.91                       | [O\ III] | F1  | 117.78           | 99.28            | 5.3   |
| 5006.84                       | [O\ III] | F1  | 411.98           | 319.35           | 5.2   |
| 5754.60                       | [N\ II]  | F3  | 1.73             | 0.43             | 40.8  |
| 5875.66                       | He\beta   | V11 | 87.70            | 18.97            | 5.3   |
| 6312.10                       | [S\ III] | F3  | 4.47             | 0.60             | 46.9  |
| 6461.95                       | C\II     | V17.04 | 3.36            | 0.38             | 26.2  |
| 6548.10                       | [N\ II]  | F1  | 252.25           | 26.09            | 5.2   |
| 6562.77                       | H\alpha  | H3  | 2806.94          | 280.00           | 5.1   |
| 6583.50                       | [N\ II]  | F1  | 874.83           | 87.28            | 5.3   |
| 6678.16                       | He\beta   | V46 | 55.90            | 5.07             | 5.3   |
| 6716.44                       | [S\ II]  | F2  | 85.16            | 7.44             | 5.1   |
| 6730.82                       | [S\ II]  | F2  | 92.67            | 7.99             | 5.5   |
| 7135.80                       | [Ar\ III] | F1  | 183.86           | 10.88            | 5.2   |
| 7236.42                       | C\II     | V3  | 29.96            | 1.63             | 20.7  |
| 7281.35                       | He\beta   | V45 | 11.08            | 0.58             | 41.3  |
| 7751.43                       | [Ar\ III] | F1  | 111.83           | 4.00             | 34.5  |
| 9068.60                       | [S\ III] | F1  | 1236.22          | 19.08            | 5.3   |

| \( c(H_\beta) \) | \( \text{Err(\%)} \) |
|-----------------|-----------------|
| 3.10 ± 0.04    | 1.076 ± 0.067  |
| 1354.6 ± 154.2 |

---

2 OBSERVATIONS AND DATA REDUCTION

Integral field spectra listed in Table 1 were obtained in 2010 and 2012 with the 2.3-m ANU telescope using the Wide Field Spectrograph (WiFeS; Dopita et al. 2007, 2010). The observations were done with the spectral resolution of \( R \sim 7000 \) in the 441.5–707.0 nm range in 2010 and \( R \sim 3000 \) in the 329.5–932.6 nm range in 2012. The WiFeS has a field-of-view of \( 25'' \times 38'' \) and each spatial resolution element of \( 1'' \times 0.5'' \) (or \( 1'' \times 1'' \)). The spectral resolution of \( R(= \lambda/\Delta \lambda) \sim 3000 \) and \( R \sim 7000 \) corresponds to a full width at half-maximum (FWHM) of \( \sim 100 \) and 45 km s\(^{-1}\), respectively. We used the classical data accumulation mode, so a
intensities are given in Table 2 on a scale where $H\beta = 100$. All fluxes were corrected for reddening using $I(\lambda)_{\text{corr}} = F(\lambda)_{\text{obs}} 10^{(\lambda(H\beta) - 1 + f(\lambda))}$. The logarithmic $c(H\beta)$ value of the interstellar extinction for the case B recombination ($T_e = 10000$ K and $N_e = 10000$ cm$^{-3}$; Storey & Hummer 1995) has been obtained from the H$\alpha$ and H$\beta$ Balmer fluxes. We used the Galactic extinction law $f(\lambda)$ of Howarth (1983) for $R_V = A(V)/E(B-V) = 3.1$, and normalized such that $f(H\beta) = 0$. We obtained an extinction of $c(H\beta) = 3.1$ for the total fluxes (see Table 2). Our derived nebular extinction is in excellent agreement with the value derived by Todt et al. (2013) from the stellar spectral energy (SED). The same method was applied to create $c(H\beta)$ maps using the flux ratio H$\alpha$/H$\beta$, as shown in Fig. 2(b). Assuming that the foreground interstellar extinction is uniformly distributed over the nebula, an inhomogeneous extinction map may be related to some internal dust contributions. As seen, the extinction map of Abell 48 depicts that the shell is brighter than other regions, and it may contain the asymptotic giant branch (AGB) dust remnants.

3 KINEMATICS

Fig 2 shows the spatial distribution maps of the flux intensity, continuum, radial velocity and velocity dispersion of H$\alpha$ $\lambda6563$ and [N II] $\lambda6584$ for Abell 48. The white contour lines in the figures depict the distribution of the emission of H$\alpha$ obtained from the SHS (Parker et al. 2005), which can aid us in distinguishing the nebular borders from the outside or the inside. The observed velocity $v_{\text{obs}}$ was transferred to the local standard of rest (LSR) radial velocity $v_{\text{LSR}}$ by correcting for the radial velocities induced by the motions of the Earth and Sun at the time of our observation. The transformation from the measured velocity dispersion $\sigma_{\text{obs}}$ to the true line-of-sight velocity dispersion $\sigma_{\text{true}}$ was done by $\sigma_{\text{true}} = \sqrt{\sigma_{\text{obs}}^2 - \sigma_{\text{ins}}^2}$, i.e. correcting for the instrumental width (typically $\sigma_{\text{ins}} \approx 42$ km/s for $R \sim 3000$ and $\sigma_{\text{ins}} \approx 18$ km/s for $R \sim 7000$) and the thermal broadening ($\sigma_{\text{th}}^2 = 8.3 T_e [\text{kK}] / Z$, where $Z$ is the atomic weight of the atom or ion).

We have used the three-dimensional morpho-kinematic modeling program SHAPE (version 4.5) to study the kinematic structure. The program described in detail by Steffen & Lopez (2006) and Steffen et al. (2011), uses interactively moulded geometrical polygon meshes to generate the 3D structure of objects. The modeling procedure consists of defining the geometry, emissivity distribution and velocity law as a function of position. The program produces several outputs that can be directly compared with long slit or IFU observations, namely the position–velocity (P–V) diagram, the 2-D line-of-sight velocity map on the sky and the projected 3-D emissivity on the plane of the sky. The 2-D line-of-sight velocity map on the sky can be used to interpret the IFU velocity maps. For best comparison with the IFU maps, the inclination (i), the position angle ‘PA’ in the plane of the sky, and the model parameters are modified in an iterative process until the qualitatively fitting 3D emission and velocity information are produced. We adopted a model, and then modified the geometry and inclination to conform to the observed H$\alpha$ and [N II] intensity and radial
velocity maps. For this paper, the three-dimensional structure has then been transferred to a regular cell grid, together with the physical emission properties, including the velocity that, in our case, has been defined as radially outwards from the nebular centre with a linear function of magnitude, commonly known as a Hubble-type flow (see e.g. Steffen et al. 2009).

The morpho-kinematic model of Abell 48 is shown in Fig. 3(a), which consists of a modified torus, the nebular shell, surrounded by a modified hollow cylinder and the faint outer halo. The shell has an inner radius of 10″ and an outer radius of 23″ and a height of 23″. We found an expansion velocity of $v_{\text{exp}} = 35 \pm 5$ km s$^{-1}$ and a LSR systemic velocity of $v_{\text{sys}} = 65 \pm 5$ km s$^{-1}$. Our value of the LSR systemic velocity is in good agreement with the heliocentric systemic velocity of $197 \pm 5$ km s$^{-1}$. Based on the systemic velocity, Abell 48 must be located at a distance of less than 2 kpc, as it is located at the dusty Galactic disc.

Using the infrared dust maps of Schlegel et al. (1998), we found a mean reddening value of $E(B-V) = 11.39 \pm 0.64$ for an aperture of 10′′ in diameter in the Galactic latitudes and longitude of $(l, b) = (29, 0, 0.4)$, which is within a line-of-sight depth of $\leq 20$ kpc of the Galaxy. Therefore, Abell 48 with $E(B-V) \simeq 2.14$ must have a distance of less than 3.3 kpc. Considering the fact that the Galactic bulge absorbs photons overall 1.9 times more than the Galactic disc (Driver et al. 2007), the distance of Abell 48 should be around 2 kpc, as it is located at the dusty Galactic disc.

4 NEBULAR EMPIRICAL ANALYSIS

4.1 Plasma diagnostics

The derived electron temperatures ($T_e$) and densities ($N_e$) are listed in Table 3 together with the ionization potential required to create the emitting ions. We obtained $T_e$ and $N_e$ from temperature-sensitive and density-sensitive emission lines by solving the equilibrium equations of level populations for a multilevel atomic model using EQUID code (Howarth & Adams 1981). The atomic data sets used for our plasma diagnostics from collisionally excited lines (CELs), as well as for abundances derived from CELs, are given in Table 4. The diagnostics procedure to determine temperatures and densities from CELs is as follows: we assume a representative initial electron temperature of 10,000 K in order to derive $N_e$ from [S ii] line ratio; then $T_e$ is derived from [N ii] line ratio in conjunction with the mean density derived from the previous step. The calculations are iterated to give self-consistent results for $N_e$ and $T_e$. The correct choice of electron density and temperature is important for the abundance determination.

We see that the PN Abell 48 has a mean temperature of $T_e([\text{N ii}]) = 6980 \pm 930$ K, and a mean electron density of $N_e([\text{S ii}]) = 750 \pm 200$ cm$^{-3}$, which are in reasonable agreement with $T_e([\text{N ii}]) = 7200 \pm 750$ K and $N_e([\text{S ii}]) = 1000 \pm 130$ cm$^{-3}$ found by Todt et al. (2013). The uncertainty on $T_e([\text{N ii}])$ is order of 40 percent or more, due to the weak flux intensity of [N ii] $\lambda 5755$, the recombination contribution, and high interstellar extinction. Therefore, we adopted the mean electron temperature from our photoionization model for our CEL abundance analysis.

Table 3 also lists the derived He i temperatures, which are lower than the CEL temperatures, known as the ORL-CEL temperature discrepancy problem in PNe (see e.g. Liu et al. 2000, 2004b).

| Parameter | Value |
|-----------|-------|
| $r_{\text{out}}$ (arcsec) | 23 ± 4 |
| $\delta r$ (arcsec) | 13 ± 2 |
| $h$ (arcsec) | 23 ± 4 |
| $i$ | $-35^\circ \pm 2^\circ$ |
| PA | $135^\circ \pm 2^\circ$ |
| GPA | $197^\circ 48' \pm 2^\circ$ |
| $v_{\text{sys}}$(km/s) | 65 ± 5 |
| $v_{\text{exp}}$(km/s) | 35 ± 5 |

Using the infrared dust maps of Schlegel et al. (1998), we found a mean reddening value of $E(B-V) = 11.39 \pm 0.64$ for an aperture of 10′′ in diameter in the Galactic latitudes and longitude of $(l, b) = (29, 0, 0.4)$, which is within a line-of-sight depth of $\leq 20$ kpc of the Galaxy. Therefore, Abell 48 with $E(B-V) \simeq 2.14$ must have a distance of less than 3.3 kpc. Considering the fact that the Galactic bulge absorbs photons overall 1.9 times more than the Galactic disc (Driver et al. 2007), the distance of Abell 48 should be around 2 kpc, as it is located at the dusty Galactic disc.

4 NEBULAR EMPIRICAL ANALYSIS

4.1 Plasma diagnostics

The derived electron temperatures ($T_e$) and densities ($N_e$) are listed in Table 3 together with the ionization potential required to create the emitting ions. We obtained $T_e$ and $N_e$ from temperature-sensitive and density-sensitive emission lines by solving the equilibrium equations of level populations for a multilevel atomic model using EQUID code (Howarth & Adams 1981). The atomic data sets used for our plasma diagnostics from collisionally excited lines (CELs), as well as for abundances derived from CELs, are given in Table 4. The diagnostics procedure to determine temperatures and densities from CELs is as follows: we assume a representative initial electron temperature of 10,000 K in order to derive $N_e$ from [S ii] line ratio; then $T_e$ is derived from [N ii] line ratio in conjunction with the mean density derived from the previous step. The calculations are iterated to give self-consistent results for $N_e$ and $T_e$. The correct choice of electron density and temperature is important for the abundance determination.

We see that the PN Abell 48 has a mean temperature of $T_e([\text{N ii}]) = 6980 \pm 930$ K, and a mean electron density of $N_e([\text{S ii}]) = 750 \pm 200$ cm$^{-3}$, which are in reasonable agreement with $T_e([\text{N ii}]) = 7200 \pm 750$ K and $N_e([\text{S ii}]) = 1000 \pm 130$ cm$^{-3}$ found by Todt et al. (2013). The uncertainty on $T_e([\text{N ii}])$ is order of 40 percent or more, due to the weak flux intensity of [N ii] $\lambda 5755$, the recombination contribution, and high interstellar extinction. Therefore, we adopted the mean electron temperature from our photoionization model for our CEL abundance analysis.

Table 3 also lists the derived He i temperatures, which are lower than the CEL temperatures, known as the ORL-CEL temperature discrepancy problem in PNe (see e.g. Liu et al. 2000, 2004b).

| Parameter | Value |
|-----------|-------|
| $r_{\text{out}}$ (arcsec) | 23 ± 4 |
| $\delta r$ (arcsec) | 13 ± 2 |
| $h$ (arcsec) | 23 ± 4 |
| $i$ | $-35^\circ \pm 2^\circ$ |
| PA | $135^\circ \pm 2^\circ$ |
| GPA | $197^\circ 48' \pm 2^\circ$ |
| $v_{\text{sys}}$(km/s) | 65 ± 5 |
| $v_{\text{exp}}$(km/s) | 35 ± 5 |
Table 4. References for atomic data.

| Ion   | Transition probabilities | Collision strengths |
|-------|--------------------------|---------------------|
| N^+  | Bell et al. (1995)       | Stafford et al. (1994) |
| O^+  | Zeippen (1987)           | Pradhan et al. (2006) |
| O^2+ | Storey & Zeippen (2000)  | Lennon & Burke (1994) |
| Ne^2+| Landi & Bhatia (2005)    | McLaughlin & Bell (2000) |
| S^+  | Mendoza & Zeippen (1982) | Ramsbottom et al. (1996) |
| S^2+ | Mendoza & Zeippen (1982) | Taval & Gupta (1999) |
| Ar^2+| Béjont & Hansen (1986)   | Galavis et al. (1995) |

To determine the electron temperature from the He I λλ7281/6678, respectively. Similarly, we got T_e(He I) = 6960 K for He I λλ7281/6768 and T_e(He I) = 7510 K for λλ7281/6678 from the measured nebular spectrum by Todt et al. (2013).

4.2 Ionic and total abundances from ORLs

Using the effective recombination coefficients (given in Table 4), we determine ionic abundances, X_i^+,/H^+. From the measured intensities of optical recombination lines (ORLs) as follows:

\[ \frac{N(X^+)}{N(H^+)} = \frac{I(\lambda)}{I(H^\beta)} \frac{\alpha_{\text{eff}}(H^\beta)}{4861} \alpha_{\text{eff}}(\lambda), \]

(1)

where I(\lambda) is the intrinsic line flux of the emission line \lambda emitted by ion X^+. I(H^\beta) is the intrinsic line flux of H^\beta, \alpha_{\text{eff}}(H^\beta) the effective recombination coefficient of H^\beta, and \alpha_{\text{eff}}(\lambda) the effective recombination coefficient for the emission line \lambda.

Abundances of helium and carbon from ORLs are given in Table 6. We derived the ionic and total abundances from He I λ4471, λ5876 and λ6678 lines. We assumed the Case B recombination for the He I lines (Porter et al. 2012, 2013). We adopted an electron temperature of T_e = 5000 K from He I lines, and an electron density of N_e = 1000 cm^{-3}. We averaged the He^+/H^+ ionic abundances from the He I λ4471, λ5876 and λ6678 lines with weights of 1.3, roughly the intrinsic intensity ratios of these three lines. The total He/H abundance ratio is obtained by simply taking the sum of He^+/H^+ and He^{2+}/H^+. However, He^{2+}/H^+ is equal to zero, since He II λ4686 is not present. The C^{2+} ionic abundance is obtained from C II λλ6462 and λ7236 lines.

4.3 Ionic and total abundances from CELs

We determined abundances for ionic species of N, O, Ne, S and Ar from CELs. To deduce ionic abundances, we solve the statistical equilibrium equations for each ion using EQUIB code, giving level population and line sensitivities for specified N_e = 1000 cm^{-3} and T_e = 10000 K adopted according to our photoionization modelling. Once the equations for the population numbers are solved, the ionic abundances, X_i^+/H^+, can be derived from the observed line intensities of CELs as follows:

\[ \frac{N(X^+)}{N(H^+)} = \frac{I(\lambda_{ij}) \alpha_{ij}(\lambda) \alpha_{\text{eff}}(H^\beta) N_e}{I(H^\beta) 4861 A_{ij} n_i}, \]

(2)

where I(\lambda_{ij}) is the dereddened flux of the emission line \lambda_{ij} emitted by ion X^{i+} following the transition from the upper level i to the lower level j, I(H^\beta) the dereddened flux of H^\beta, \alpha_{\text{eff}}(H^\beta) the effective recombination coefficient of H^\beta, A_{ij} the Einstein spontaneous transition probability of the transition, n_i the fractional population of the upper level i, and N_e is the electron density.

Total elemental and ionic abundances of nitrogen, oxygen, neon, sulphur and argon from CELs are presented in Table 7. Total elemental abundances are derived from ionic abundances using the ionization correction factors (icf) formulas given by Kingsburgh & Barlow (1994). The total O/H abundance ratio is obtained by simply taking the sum of the O^+/H^+ derived from [O II] λλ3726,3729 doublet, and the O^{2+}/H^+ derived from [O III] λλ4959,5007 doublet, since He II λ4686 is not present, so O^{3+}/H^+ is negligible. The total N/H abundance ratio was calculated from the N^{+}/H^+ ratio derived from the [N II] λλ6548,6584 doublet, correcting for the unseen N^{2+}/H^+ using,

\[ \frac{N}{H} = \left( \frac{N^{+}}{H^+} \right) \left( \frac{O}{O^+} \right). \]

(3)

The Ne^{2+}/H^+ is derived from [Ne III] λ4866 line. Similarly, the
Table 7. Empirical ionic abundances derived from CELs.

| Ion    | λ(Å) | Mult | Value* |
|--------|------|------|--------|
| N$^+$  | 6548.10 | F1 | 1.356\(-5\) |
|        | 6583.50 | F1 | 1.486\(-5\) |
| Mean   |        |     | 1.421\(-5\) |
| icf(N) |       |     | 3.026   |
| N/H    |       |     | 4.299\(-5\) |
| O$^+$  | 3727.43 | F1 | 5.251\(-5\) |
| O$^{2+}$ | 4958.91 | F1 | 1.024\(-4\) |
|        | 5006.84 | F1 | 1.104\(-4\) |
| Average|       |     | 1.064\(-4\) |
| icf(O) |       |     | 1.0     |
| O/H    |       |     | 1.589\(-4\) |
| Ne$^{2+}$ | 3868.75 | F1 | 4.256\(-5\) |
| Ne/H   |       |     | 6.358\(-5\) |
| S$^+$  | 6716.44 | F2 | 4.058\(-7\) |
|        | 6730.82 | F2 | 3.896\(-7\) |
| Average|       |     | 3.977\(-7\) |
| S$^{2+}$ | 9068.60 | F1 | 5.579\(-6\) |
| S/H    |       |     | 1.126   |
| Ar$^{2+}$ | 7135.80 | F1 | 9.874\(-7\) |
| Ar/H   |       |     | 1.494   |

* Assuming $T_e = 10\,000$ K and $N_e = 1000$ cm$^{-3}$.

unseen Ne$^+/H^+$ is corrected for, using

$$\frac{\text{Ne}}{\text{H}} = \left( \frac{\text{Ne}^2+/\text{H}^+}{\text{O}^{2+}/\text{O}} \right).$$  (4)

For sulphur, we have S$^+/H^+$ from the [S ii] $\text{λ}6716,6731$ doublet and S$^{2+}/H^+$ from the [S iii] $\lambda9069$ line. The total sulphur abundance is corrected for the unseen stages of ionization using

$$\frac{\text{S}}{\text{H}} = \left( \frac{\text{S}^+/\text{H}^+ + \text{S}^{2+}/\text{H}^+}{1 - \left( \frac{\text{O}^{2+}/\text{O}}{\text{S}^{2+}/\text{H}^+} \right)^3} \right)^{1/3}.$$  (5)

The [Ar iii] $\lambda7136$ line is only detected, so we have only Ar$^{2+}/H^+$. The total argon abundance is obtained by assuming Ar$^+/Ar = N^{+}/N$:

$$\frac{\text{Ar}}{\text{H}} = \left( \frac{\text{Ar}^{2+}/\text{H}^+}{1 - N^{+}/N} \right)^{-1}.$$  (6)

As it does not include the unseen Ar$^{2+}$, so the derived elemental argon may be underestimated.

The 3-D photoionization model was obtained through an iterative process, involving the comparison of the predicted H$\beta$ luminosity $L_{\text{H}\beta}$ (erg s$^{-1}$), the flux intensities of some important lines, relative to H$\beta$ (such as [O iii] $\lambda5007$ and [N ii] $\lambda6584$), with those measured from the observations. The free parameters included distance and nebular parameters. We initially used the stellar luminosity ($L_*=6000L_\odot$) and effective temperature ($T_{\text{eff}}=70kK$) found by Todt et al. (2013). However, we slightly adjusted the stellar luminosity to match the observed line flux of [O iii] emission line. Moreover, we adopted the nebular density and abundances derived from empirical analysis in Section 4, but they have been gradually adjusted until the observed nebular emission-line spectrum was reproduced by the model. The best-fitting $L_{\text{H}\beta}$ depends upon the distance and nebula density. The plasma diagnostics yields $N_e = 750$–1000 cm$^{-3}$, which can be an indicator of the density range. Based on the kinematic analysis, the distance must be less than 2 kpc, but more than 1.5 kpc due to the large interstellar extinction. We matched the predicted H$\beta$ luminosity $L_{\text{H}\beta}$ with the value derived from the observation by adjusting the distance and nebular density. Then, we adjusted abundances to get the best emission-line spectrum.

5.1 The ionizing spectrum

The hydrogen-deficient synthetic spectra of Abell 48 was modelled using stellar model atmospheres produced by the Potsdam Wolf-Rayet (PoWR) models for expanding atmospheres (Gräfener et al. 2002; Hamann & Gräfener 2004). It solves the non-local thermodynamic equilibrium (non-LTE) radiative transfer equation in the comoving frame, iteratively with the equations of statistical equilibrium and radiative equilibrium, for an expanding atmosphere under the assumptions of spherical symmetry, stationarity and homogeneity. The result of our model atmosphere is shown in Fig.5. The model atmosphere calculated with the PoWR code is for the stellar surface abundances H:He:C:N:O = 10:85:0.3:5:0.6 by mass, the stellar temperature $T_{\text{eff}} = 70kK$, the transformed radius $R_\star = 0.54R_\odot$ and the wind terminal velocity $v_\infty = 1000$ km s$^{-1}$. The best photoionization model was obtained with an effective temperature of 70 kK (the same as PoWR model used by Todt et al. 2013) and a stellar luminosity of $L_*/L_\odot = 5500$, which is close to $L_*/L_\odot = 6000$ adopted by Todt et al. (2013). This stellar lu-
minosity was found to be consistent with the observed H\(\beta\) luminosity and the flux ratio of [O iii]/H\(\beta\). A stellar luminosity higher than 5500 L\(_\odot\) produces inconsistent results for the nebular photoionization modelling. The emission-line spectrum produced by our adopted stellar parameters was found to be consistent with the observations.

### 5.2 The density distribution

We initially used a three-dimensional uniform density distribution, which was developed from our kinematic analysis. However, the interacting stellar winds (ISW) model developed by [Kwok et al. (1978)] demonstrated that a slow dense superwind from the AGB phase is swept up by a fast tenuous wind during the PN phase, creating a compressed dense shell, which is similar to what we see in Fig. 6. Additionally, [Kahn & West (1985)] extended the ISW theory for bipolar PNe (see e.g. Mellema 1994, 1997). As shown in Fig. 6, we adopted a density structure with a toroidal wind mass-loss geometry, similar to the ISW model. In our model, we defined a density distribution in the cylindrical coordinate system, which has the form

\[ N_\text{H}(\tau) = N_0[1 + (\tau/\tau_{\text{in}})^{-\alpha}] \]

where \(\tau\) is the radial distance from the centre, \(\alpha\) the radial density dependence, \(N_0\) the characteristic density, \(\tau_{\text{in}} = r_{\text{out}} - \delta r\) the inner radius, \(r_{\text{out}}\) the outer radius and \(\delta r\) the thickness.

The density distribution is usually a complicated input parameter to constrain. However, the values found from our plasma diagnostics (\(N_\text{e} = 750–1000\) cm\(^{-3}\)) allowed us to constrain our density model. The outer radius and the height of the cylinder are equal to \(r_{\text{out}} = 23'\) and the thickness is \(\delta r = 13'\). The density model and distance (size) were adjusted in order to reproduce \(I(H\beta) = 1.355 \times 10^{-10}\) erg s\(^{-1}\) cm\(^{-2}\), dereddened using c(H\(\beta\))=3.1 (see Section 2). We tested distances, with values ranging from 1.5 to 2.0 kpc. We finally adopted the characteristic density of \(N_0 = 600\) cm\(^{-3}\) and the radial density dependence of \(\alpha = 1\). The value of 1.90 kpc found here, was chosen, because of the best predicted H\(\beta\) luminosity, and it is in excellent agreement with the distance constrained by the synthetic spectral energy distribution (SED) from the PoWR models. Once the density distribution and distance were identified, the variation of the nebular ionic abundances were explored.

![Figure 4](image)

**Figure 4.** Ionic abundance maps of Abell 48. From left to right: spatial distribution maps of singly ionized Helium abundance ratio He\(^+\)/H\(^+\) from He I ORLs (4472, 5877, 6678); ionic nitrogen abundance ratio N\(^+\)/H\(^+\) (\(\times 10^{-5}\)) from [N ii] CELs (5755, 6548, 6584); ionic oxygen abundance ratio O\(^{2+}\)/H\(^+\) (\(\times 10^{-4}\)) from [O iii] CELs (4959, 5007); and ionic sulphur abundance ratio S\(^{2+}\)/H\(^+\) (\(\times 10^{-4}\)) from [S ii] CELs (6716, 6731). North is up and east is towards the left-hand side. The white contour lines show the distribution of the narrow-band emission of H\(\alpha\) in arbitrary unit obtained from the SHS.

![Figure 5](image)

**Figure 5.** Non-LTE model atmosphere flux (solid line) calculated with the PoWR models for the surface abundances H:He:C:N:O = 10:85:0.3:5:0.6 by mass and the stellar temperature \(T_{\text{eff}} = 70\) kK, compared with a blackbody (dashed line) at the same temperature.

### Table 8. Input parameters for the MOCASSIN photoionization model.

| Stellar and Nebular Parameters | Nebulos Abundances | Model | Obs. |
|-------------------------------|-------------------|-------|------|
| \(T_{\text{eff}}\) (kK)        | 70                | He/H  | 0.120| 0.124|
| \(L_\star\) (L\(_\odot\))     | 5500              | C/H   | \(\times 10^3\) | 3.00 | – |
| \(N_\odot\) (cm\(^{-3}\))     | 800-1200          | N/H   | \(\times 10^5\) | 6.50 | 4.30 |
| \(D\) (kpc)                   | 1.9               | O/H   | \(\times 10^5\) | 1.40 | 1.59 |
| \(r_{\text{out}}\) (arcsec)  | 23                | Ne/H  | \(\times 10^5\) | 6.00 | 6.36 |
| \(\delta r\) (arcsec)        | 13                | S/H   | \(\times 10^6\) | 6.00 | 6.73 |
| \(b\) (arcsec)               | 23                | Ar/H  | \(\times 10^6\) | 1.20 | 1.48 |
Table 9. Dereddened observed and predicted emission-line fluxes for Abell 48. References: D13 – this work; T13 – Todt et al. (2013). Uncertain and very uncertain values are followed by ‘*’ and ‘**’, respectively. The symbol ‘+’ denotes blended emission lines.

| Line          | Observed  | Predicted |
|---------------|-----------|-----------|
|               | D13       | T13       |
| \(I(H\beta)/10^{-10} \, \text{erg} \, \text{cm}^{-2} \, \text{s}^{-1}\) | 1.355 | 1.371 |
| H\beta 4861   | 100.00    | 100.00    |
| H\alpha 6563  | 286.00    | 285.32    |
| H\gamma 4340  | 54.28; 45.10 | 46.88 |
| H\delta 4102  | –        | 25.94     |
| H\epsilon 4472| 7.42;     | 6.34      |
| H\iota 5876   | 18.97     | 17.48     |
| H\epsilon 6678| 5.07      | 4.91      |
| H\iota 7281   | 0.58;: 0.70 | 0.97      |
| H\iota 4686   | –        | 0.00      |
| C\iota 6462   | 0.38      | 0.27      |
| C\iota 7236   | 1.63      | 1.90      |
| [N\iota] 5755 | 0.43;: 0.40 | 1.20      |
| [N\iota] 6548 | 26.09     | 26.00     |
| [N\iota] 6584 | 87.28     | 81.25     |
| [O\iota] 3726 | 128.96;   | 59.96     |
| [O\iota] 3729 | *         | 43.54     |
| [O\iota] 7320 | – 0.70    | 2.16      |
| [O\iota] 7330 | – 0.60    | 1.76      |
| [O\iota] 4363 | 3.40      | 2.30      |
| [O\iota] 4959 | 99.28     | 111.82    |
| [O\iota] 5007 | 319.35    | 333.66    |
| [Ne\iota] 3869| 38.96     | 39.60     |
| [Ne\iota] 3967| –         | 11.93     |
| [S\iota] 4069 | –         | 1.52      |
| [S\iota] 4076 | – 0.52    |          |
| [S\iota] 6717 | 7.44      | 10.30     |
| [S\iota] 6731 | 7.99      | 10.57     |
| [S\iota] 6312 | 0.60;:     | 2.22      |
| [S\iota] 9069 | 19.08     | 16.37     |
| [Ar\iota] 7136| 10.88     | 12.75     |
| [Ar\iota] 7751| 4.00;:     | 3.05      |
| [Ar\iv] 4712 | –         | 0.61      |
| [Ar\iv] 4741 | –         | 0.51      |

Figure 6. The density distribution based on the ISW models adopted for photoionization modelling of Abell 48. The cylinder has outer radius of 23" and thickness of 13". Axis units are arcsec, where 1 arcsec is equal to 9.30 \times 10^{-3} \, \text{pc} based on the distance determined by our photoionization model.

6.3 The nebular elemental abundances

Table 9 lists the nebular elemental abundances (with respect to H) used for the photoionization model. We used a homogeneous abundance distribution, since we do not have any direct observational evidence for the presence of chemical inhomogeneities. Initially, we used the abundances from empirical analysis as initial values for our modelling (see Section 4). They were successively modified to fit the optical emission-line spectrum through an iterative process. We obtained a C/O ratio of 2.1 for Abell 48, indicating that it is predominantly C-rich. Furthermore, we find a helium abundance of 0.12. This can be an indicator of a large amount of mixing processing in the He-rich layers during the He-shell flash leading to an increased carbon abundance. The nebulae around H-deficient CSs typically have larger carbon abundances than those with H-rich CSs (see review by De Marco & Barlow 2001). The O/H we derive for Abell 48 is lower than the solar value (O/H = 4.57 \times 10^{-4}; Asplund et al. 2009). This may be due to that the progenitor has a sub-solar metallicity. The enrichment of carbon can be produced in a very intense mixing process in the He-shell flash (Herwig et al. 1997). Other elements seem to be also decreased compared to the solar values, such as sulphur and argon. Sulphur could be depleted on to dust grains (Sofia et al. 1994), but argon cannot have any strong depletion by dust formation (Sofia & Jenkins 1998). We notice that the N/H ratio is about the solar value given by Asplund et al. (2009), but it can be produced by secondary conversion of initial carbon if we assume a sub-solar metallicity progenitor. The combined (C+N+O)/H ratio is by a factor of 3.9 larger than the solar value, which can be produced by multiple dredge-up episodes occurring in the AGB phase.

6.3 Comparison of the emission-line fluxes

Table 9 compares the flux intensities predicted by the best-fitting model with those from the observations. Columns 2 and 3 present the dereddened fluxes of our observations and those from Todt et al. (2013). The predicted emission-line fluxes are given in Column 4, relative to the intrinsic dereddened H\beta flux, on a scale where \(I(H\beta) = 100\). The most emission-line fluxes presented are in reasonable agreement with the observations. However, we notice that the [O\iota] \lambda 7319 and \lambda 7330 doublets are overestimated by a factor of 3, which can be due to the recombination contribution. Our photoionization code incorporates the recombination term to the statistical equilibrium equations. However, the recombination contribution are less than 30 per cent for the values of \(T_e\) and \(N_e\) found from the plasma diagnostics. Therefore, the discrepancy between our model and observed intensities of these lines can be due to inhomogeneous condensations such as clumps and/or colder small-scale structures embedded in the global structure. It can also be due to the measurement errors of these weak lines. The [O\iota] \lambda\lambda 7372,3729 doublet predicted by the model is around 25 per cent lower, which can be explained by either the recombination contribution or the flux calibration error. There is a notable discrepancy in the predicted [N\iota] \lambda 5775 auroral line, being higher by a factor of \(3\). It can be due to the errors in the flux measurement of the [N\iota] \lambda 5755 line. The predicted [Ar\iota] \lambda 7751 line is also 30 per cent lower, while [Ar\iota] \lambda 7136 is about 20 per cent higher. The [Ar\iota] \lambda 7751 line usually is blended with the telluric line, so the

© 2014 RAS, MNRAS 000 1–12
observed intensity of these line can be overestimated. It is the same for [S III] λ9069, which is typically affected by the atmospheric absorption band.

### 6.2 Ionization and thermal structure

The volume-averaged fractional ionic abundances are listed in Table [10](#). We note that hydrogen and helium are singly-ionized. We see that the O$^+$/O ratio is higher than the N$^+$/N ratio by a factor of 1.34, which is dissimilar to what is generally assumed in the $icf$ method. However, the O$^{2+}$/O ratio is nearly a factor of 1.16 larger than the Ne$^{2+}$/Ne ratio, in agreement with the general assumption for $icf$(Ne). We see that only 19 per cent of the total nitrogen in the nebula is in the form of N$^+$. However, the total oxygen largely exists as O$^{2+}$ with 70 per cent and then O$^+$ with 26 per cent.

The elemental abundances we used for the photoionization model returns ionic abundances listed in Table [11](#) are comparable to those from the empirical analysis derived in Section [4](#). The ionic abundances derived from the observations do not show major discrepancies in He$^+$/H$^+$, C$^{2+}$/H$^+$, N$^+$/H$^+$, O$^{2+}$/H$^+$, Ne$^{2+}$/H$^+$ and Ar$^{2+}$/H$^+$; differences remain below 18 per cent. However, the predicted and empirical values of O$^+$/H$^+$, S$^+$/H$^+$ and S$^{2+}$/H$^+$ have discrepancies of about 45, 31 and 33 per cent, respectively.

Fig. [7](#) (bottom) shows plots of the ionization structure of helium, carbon, oxygen, argon (left-hand panel), nitrogen, neon and sulphur (right-hand panel) as a function of radius along the equatorial direction. As seen, ionization layers have a clear ionization
sequence from the highly ionized inner parts to the outer regions. Helium is 97 percent singly ionized over the shell, while oxygen is 26 percent singly ionized and 70 percent doubly ionized. Carbon and nitrogen are about ~20 percent singly ionized ~80 percent doubly ionized. The distribution of N\(^+\) is in full agreement with the IFU map, given in Fig 4. Comparison between the He\(^+\), O\(^+\), and S\(^+\) ionic abundance maps obtained from our IFU observations and the ionic fractions predicted by our photoionization model also show excellent agreement.

Table 12 lists mean temperatures weighted by the ionic abundances. Both [N\(^{III}\)] and [O\(^{III}\)] doublets, as well as He\(^{I}\) lines arise from the same ionization zones, so they should have roughly similar values. The ionic temperatures increasing towards higher ionization stages could also have some implications for the mean temperatures averaged over the entire nebula. However, there is a large discrepancy by a factor of 2 between our model and ORL empirical value of \(T_e\) (He\(^{I}\)). This could be due to some temperature fluctuations in the nebula (Peimbert 1967, 1971). The temperature fluctuations lead to overestimating the electron temperature deduced from CELs. This can lead to the discrepancies in abundances determined from CELs and ORLs (see e.g. Liu et al. 2000). Nevertheless, the temperature discrepancy can also be produced by bi-abundance models (Liu 2003, Liu et al. 2004a), containing some cold hydrogen-deficient material, highly enriched in helium and heavy elements, embedded in the diffuse warm nebular gas of normal abundances. The existence and origin of such inclusions are still unknown. It is unclear whether there is any link between the assumed H-poor inclusions in PNe and the H-deficient CSs.

### 7 CONCLUSION

We have constructed a photoionization model for the nebula of Abell 48. This consists of a dense hollow cylinder, assuming homogeneous abundances. The three-dimensional density distribution was interpreted using the morpho-kinematic model determined from spatially resolved kinematic maps and the ISW model. Our aim was to construct a model that can reproduce the nebular emission-line spectra, temperatures and ionization structure determined from the observations. We have used the non-LTE model atmosphere from Todt et al. 2013 as the ionizing source. Using the empirical analysis methods, we have determined the temperatures and the elemental abundances from CELs and ORLs. We notice a discrepancy between temperatures estimated from [O\(^{III}\)] CELs and those from the observed He\(^{I}\) ORLs. In particular, the abundance ratios derived from empirical analysis could also be susceptible to inaccurate values of electron temperature and density. However, we see that the predicted ionic abundances are in decent agreement with those deduced from the empirical analysis. The emission-line fluxes obtained from the model were in fair agreement with the observations.

We notice large discrepancies between He\(^{I}\) electron temperatures derived from the model and the empirical analysis. The existence of clumps and low-ionization structures could solve the problems (Liu et al. 2000). Temperature fluctuations have been also proposed to be responsible for the discrepancies in temperatures determined from CELs and ORLs (Peimbert 1967, 1971). Previously, we also saw large ORL–CEL abundance discrepancies in other PNe with hydrogen-deficient CSs, for example Abell 30 (Ercolano et al. 2003a) and NGC 1501 (Ercolano et al. 2004). A fraction of H-deficient inclusions might produce those discrepancies, which could be ejected from the stellar surface during a very late thermal pulse (VLTP) phase or born-again event (Iben & Renzini 1983). However, the VLTP event is expected to produce a carbon-rich stellar surface abundance (Herwig 2001), whereas in the case of Abell 48 negligible carbon was found at the stellar surface (C/He = \(3.5 \times 10^{-3}\) by mass; Todt et al. 2013).

The stellar evolution of Abell 48 still remains unclear and needs to be investigated further. But, its extreme helium-rich atmosphere (85 percent by mass) is more likely connected to a merging process of two white dwarfs as evidenced for R Cor Bor stars of similar chemical surface composition by observations (Clayton et al. 2007, García-Hernández et al. 2009) and hydrodynamic simulations (Staff et al. 2012, Zhang & Jeffery 2013, Menon et al. 2013).

We derived a nebula ionized mass of \(\sim 0.8 \, M_\odot\). The high C/O ratio indicates that it is a predominantly C-rich nebula. The C/H ratio is largely over-abundant compared to the solar value of \(\sim 0.02\) (Asplund et al. 2009), while oxygen, sulphur and argon are under-abundant. Moreover, nitrogen and neon are roughly similar to the solar values. Assuming a sub-solar metallicity progenitor, a 3rd dredge-up must have enriched carbon and nitrogen in the AGB-phase. However, extremely high carbon must be produced through mixing processing in the He-rich layers during the He-shell flash. The low N/O ratio implies that the progenitor star never went through the hot bottom burning phase, which occurs in AGB stars with initial masses more than \(5M_\odot\) (Karakas & Lattanzio 2007, Karakas et al. 2009). Comparing the stellar parameters found by the model, \(T_{\text{eff}} = 70\, \text{kK}\) and \(L/L_\odot = 5500\), with VLTP evolutionary tracks from Blocker (1995), we get a current mass of \(\sim 0.62M_\odot\), which originated from a progenitor star with an initial mass of \(\sim 3M_\odot\). However, the VLTP evolutionary tracks by Miller Bertolami & Althaus (2006) yield a current mass of \(\sim 0.52M_\odot\) and a progenitor mass of \(\sim 1M_\odot\), which is not consistent with the derived nebula ionized mass. Furthermore, time-scales for VLTP evolutionary track (Blocker 1995) imply that the CS has a post-AGB age of about \(\sim 9\,000\, \text{yr}\), in agreement with the nebula’s
age determined from the kinematic analysis. We therefore conclude that Abell 48 originated from an $\sim 3 \, M_\odot$ progenitor, which is consistent with the nebula’s features.

**ACKNOWLEDGMENTS**

AD warmly acknowledges the award of an international Macquarie University Research Excellence Scholarship (iMQRES). BE is supported by the German Research Foundation (DFG) Cluster of Excellence “Origin and Structure of the Universe”. AYK acknowledges the support from the National Research Foundation (NRF) of South Africa. We would like to thank Prof. Wolf-Rainer Hamann, Prof. Quentin A. Parker and Dr. David J. Frew for helping in the observing proposal writing stage, and the staff at the ANU Siding Spring Observatory for their support. We would also like to thank Prof. Simon Jeffery and Dr. Amanda Karakas for illuminating discussions and helpful comments. We would also like to thank Dr. Kyle DePew for carrying out the 2010 ANU 2.3 m observing run. AD thanks Dr. Milorad Stupar for assisting with the 2012 ANU 2.3 m observing run and his guidance on the IRAF pipeline WIFES, Prof. Quentin A. Parker and Dr. David J. Frew for helping in the observing proposal writing stage, and the staff at the ANU Siding Spring Observatory for their support. We would also like to thank the anonymous referee for helpful suggestions.

**REFERENCES**

Abell G. O., 1955, PASP, 67, 258
Asplund M., Grevesse N., Sauval A. J., Scott P., 2009, A&A, 47, 481
Bell K. L., Hibbert A., Stafford R. P., 1995, Phys. Scr, 52, 240
Biémont E., Hansen J. E., 1986, Phys. Scr, 34, 116
Blöcker T., 1995, A&A, 299, 755
Clayton G. C., Geballe T. R., Herwig F., Fryer C., Asplund M., 2009, ARA&A, 281, 2812
Danehkar A., Parker Q. A., Ercolano B., 2013, MNRAS, 434, 1513
Davey A. R., Storey P. J., Kisielius R., 2000, A&AS, 142, 85
De Marco O., Barlow M. J., 2001, Ap&SS, 275, 53
Depew K., Parker Q. A., Miszalski B., De Marco O., Frew D. J., Acker A., Kovacevic A. V., Sharp R. G., 2011, MNRAS, 414, 2812
Dopita M., Hart J., McGregor P., Oates P., Bloxham G., Jones D., 2007, Ap&SS, 310, 255
Dopita M. et al., 2010, Ap&SS, 327, 245
Dopita M. A. et al., 1996, ApJ, 460, 320
Driver S. P., Popescu C. C., Tuffs R. J., Liske J., Graham A. W., Allen P. D., de Propris R., 2007, MNRAS, 379, 1022
Ercolano B., Barlow M. J., Storey P. J., 2005, MNRAS, 362, 1038
Ercolano B., Barlow M. J., Storey P. J., Liu X.-W., Rauch T., Werner K., 2003a, MNRAS, 344, 1145
Ercolano B., Morisset C., Barlow M. J., Storey P. J., Liu X.-W., 2003b, MNRAS, 340, 1153
Ercolano B., Storey P. J., 2006, MNRAS, 372, 1875
Ercolano B., Wesson R., Zhang Y., Barlow M. J., De Marco O., Rauch T., Liu X.-W., 2004, MNRAS, 354, 558
Ercolano B., Young P. R., Drake J. J., Raymond J. C., 2008, ApJS, 175, 534
Frew D. J. et al., 2013, preprint (arXiv:e-prints:1301.3994)
Galavis M. E., Mendoza C., Zeippen C. J., 1995, A&AS, 111, 347
Garcia-Hernández D. A., Hinkle K. H., Lambert D. L., Eriksson K., 2009, ApJ, 696, 1733
Goñi T. J., Finkbeiner D. P., Davis M., 2006, MNRAS, 365, 1039
Gräfener G., Koesterke L., Hamann W.-R., 2002, A&A, 387, 244
Hamann W.-R., Gräfener G., 2004, A&A, 427, 697
Herwig F., 2001, Ap&SS, 275, 15
Herwig F., Blöcker T., Schönberger D., El Eid M., 1997, A&A, 324, L81
Howarth I. D., 1983, MNRAS, 203, 301
Howarth I. D., Adams S., 1981, Program EQUIB. University College London, (Wesson R., 2009, Converted to FORTRAN 90)
Huang K.-N., 1985, Atomic Data and Nuclear Data Tables, 32, 503
Iben, Jr. I., Renzini A., 1983, ARA&A, 21, 271
Jewitt D. C., Danielson G. E., Kupferman P. N., 1986, ApJ, 302, 727
Kahn F. D., West K. A., 1985, MNRAS, 212, 837
Karakas A., Lattanzio J. C., 2007, PASA, 24, 103
Karakas A. I., van Raai M. A., Lugaro M., Sterling N. C., Dinerstein H. L., 2009, ApJ, 690, 1130
Kingsbury R. L., Barlow M. J., 1994, MNRAS, 271, 257
Kwok S., Purton C. R., Fitzgerald P. M., 1978, ApJ, 219, L125
Landi E., Bhatia A. K., 2005, Atomic Data and Nuclear Data Tables, 89, 195
Landi E., Del Zanna G., Young P. R., Dere K. P., Mason H. E., Landini M., 2006, ApJS, 162, 261
Lennon D. J., Burke V. M., 1994, A&AS, 103, 273
Liu X.-W., 2003, in IAU Symposium, Vol. 209, Planetary Nebulae: Their Evolution and Role in the Universe, Kwok S., Dopita M., Sutherland R., eds., p. 339
Liu X.-W., Storey P. J., Barlow M. J., Danziger I. J., Cohen M., Bryce M., 2000, MNRAS, 312, 585
Liu Y., Liu X.-W., Barlow M. J., Luo S.-G., 2004a, MNRAS, 353, 1251
Liu Y., Liu X.-W., Luo S.-G., Barlow M. J., 2004b, MNRAS, 353, 1231
Maccì W. J., Dutra C. M., 1992, A&A, 262, 271
McLaughlin B. M., Bell K. L., 2000, Journal of Physics B Atomic Molecular Physics, 33, 597
Mellema G., 1996, Ap&SS, 245, 239
Mellema G., 1997, A&A, 321, L29
Mendoza C., Zeippen C. J., 1982, MNRAS, 198, 127
Menon A., Herwig F., Denissenkov P. A., Clayton G. C., Staff J., Pignatari M., Paxton B., 2013, ApJ, 772, 59
Miller Bertolami M. M., Althauser L. G., 2006, A&A, 454, 845
Miszalski B., Crowther P. A., De Marco O., Koppelin J., Moffat A. F. J., Acker A., Hillwig T. C., 2012, MNRAS, 423, 934
Porter R. A., Phillipps S., Pierce M., et al., 2005, MNRAS, 362, 689
Peimbert M., 1967, ApJ, 150, 825
Peimbert M., 1971, Boletin de los Observatorios Tonantzintla y Tacubaya, 6, 29
Porter R. L., Ferland G. J., Hummer D. G., 2001, MNRAS, 328, L81
Porter R. L., Ferland G. J., Storey P. J., Detisch M. J., 2012, MNRAS, 425, L28
Porter R. L., Ferland G. J., Storey P. J., Detisch M. J., 2013, MNRAS, 433, L89
Pradhan A. K., Montenegro M., Nahar S. N., Eissner W., 2006, MNRAS, 366, L6
Ramsbottom C. A., Bell K. L., Staff M., 1996, Atomic Data and Nuclear Data Tables, 63, 57
Schlegel D. J., Finkbeiner D. P., Davis M., 1998, ApJ, 500, 525
Smits D. P., 1996, MNRAS, 278, 683
Sofia U. J., Cardelli J. A., Savage B. D., 1994, ApJ, 430, 650

© 2014 RAS, MNRAS 000, 000
Sofia U. J., Jenkins E. B., 1998, ApJ, 499, 951
Staff J. E. et al., 2012, ApJ, 757, 76
Stafford R. P., Bell K. L., Hibbert A., Wijesundera W. P., 1994, MNRAS, 268, 816
Steffen W., García-Segura G., Koning N., 2009, ApJ, 691, 696
Steffen W., Koning N., Wenger S., Morisset C., Magnor M., 2011, IEEE Transactions on Visualization and Computer Graphics, 17, 454
Steffen W., López J. A., 2006, RMxAA, 42, 99
Storey P. J., Hummer D. G., 1995, MNRAS, 272, 41
Storey P. J., Zeippen C. J., 2000, MNRAS, 312, 813
Tayal S. S., Gupta G. P., 1999, ApJ, 526, 544
Todt H. et al., 2013, MNRAS, 430, 2302
Todt H., Peña M., Hamann W.-R., Gräfener G., 2010, A&A, 515, A83
Verner D. A., Yakovlev D. G., 1995, A&AS, 109, 125
Verner D. A., Yakovlev D. G., Band I. M., Trzhaskovskaya M. B., 1993, Atomic Data and Nuclear Data Tables, 55, 233
Wachter S., Mauerhan J. C., Van Dyk S. D., Hoard D. W., Kafka S., Morris P. W., 2010, AJ, 139, 2330
Wright N. J., Barlow M. J., Ercolano B., Rauch T., 2011, MNRAS, 418, 370
Zeippen C. J., 1987, A&A, 173, 410
Zhang X., Jeffery C. S., 2012, MNRAS, 419, 452
Zuckerman B., Aller L. H., 1986, ApJ, 301, 772