Inhomogeneous color superconductivity and the cooling of compact stars

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Abstract. In this talk I discuss the inhomogeneous (LOFF) color superconductive phases of Quantum Chromodynamics (QCD). In particular, I show the effect of a core of LOFF phase on the cooling of a compact star.

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Study of the phase diagram of Quantum Chromodynamics (QCD) in extreme condition of density and/or temperature has attracted a lot of interest in recent years. In particular, high density and low temperature conditions make room for a new state of deconfined quark matter known as Color Superconductor [1] (see [2] for reviews). Understanding this phase is an important challenge both for the purely theoretical aspects and for the phenomenological implications. As a matter of fact, the study of Color Superconductivity (CSC) allows for a deeper knowledge of the phase diagram of QCD; moreover, one is expected to find high baryon densities and low temperatures in the core of the compact stellar objects: as a consequence, it is interesting to understand the way CSC modifies the properties of such stars (equations of state, transport coefficient, cooling properties), in order to get a more accurate knowledge of these intriguing stellar objects.

In three flavor QCD and at asymptotically high density the ground state of CSC is known to be the Color-Flavor-Locked (CFL) state [3]. In this state of matter the color and the flavor degrees of freedom are linked together and the ground state is invariant under transformations in the diagonal group $SU(3)_c + V$. At moderate densities, as can be found in the core of compact stars, one has to keep into account electrical and color neutrality conditions and finite mass effects of the quarks [4]. As a consequence, the Fermi spheres of the pairing quarks are likely to be mismatched and the CFL state can be disfavored. In this case more exotic patterns of condensation can occur, and the ground state of QCD in these conditions is still a matter of debate (see for example [5] and references therein).

Among the various candidates I discuss here the crystalline color superconductor, known in literature as the LOFF phase [6]; the LOFF state is characterized by a non vanishing total momentum of the pair. In particular for the three flavor case I consider
here the simplest one-plane wave structure defined by

\[ \langle \psi_{\alpha i}(x)C\gamma_5\psi_{\beta j}(x) \rangle \propto \sum_{I=1}^{3} \Delta_I e^{2i\mathbf{q}_I \cdot \mathbf{r}}\epsilon_{\alpha\beta I} \epsilon_{ijI} \]  

(1)

\[ (i, j = 1, 2, 3 \text{ flavor indices, } \alpha, \beta = 1, 2, 3 \text{ color indices}); \] it has been considered for the first time in the three flavor QCD contest in Ref. [7] and it was found energetically favored with respect to other phases of QCD in a certain range of values of the strange quark mass \( M_s \). In Eq. (1), \( 2\mathbf{q}_I \) represents the momentum of the Cooper pair and the gap parameters \( \Delta_1, \Delta_2, \Delta_3 \) describe respectively \( d-s, u-s \) and \( u-d \) pairing. For sufficiently large \( \mu \) the energetically favored phase is characterized by \( \Delta_1 = 0, \Delta_2 = \Delta_3 \) and \( \mathbf{q}_2 = \mathbf{q}_3 \). This phase turns out to be also chromomagnetically stable [8]. In [9] more sophisticated ansatz have been considered, and the window of \( M_s \) where the LOFF phase exists has been enlarged.

If LOFF matter is present in the core of a compact star then it affects the neutrino emissivity, and consequently the cooling process of the star itself. In the following I discuss the role of the LOFF phase on the cooling of neutron stars.

Neutrino emissivity is defined as the energy loss by \( \beta \)-decay per volume unit per time unit [10]. In [11] a simplified approach based on the study of three different toy models of stars has been used. The first model (denoted as I) is a star consisting of noninteracting nuclear matter (neutrons, protons and electrons) with mass \( M = 1.4 M_\odot \), radius \( R = 12 \text{ km} \) and uniform density \( n = 1.5 n_0 \), where \( n_0 = 0.16 \text{ fm}^{-3} \) is the nuclear equilibrium density. The nuclear matter is assumed to be electrically neutral and in beta equilibrium. The second model (II) is a star containing a core of radius \( R_1 = 5 \text{ km} \) of neutral unpaired quark matter at \( \mu = 500 \text{ MeV} \), with a mantle of noninteracting nuclear matter with uniform density \( n \). Solution of the Tolman-Oppenheimer-Volkov equations gives a mass-radius relation so that a mass \( M = 1.4 M_\odot \) corresponds to a star radius \( R_2 = 10 \text{ km} \). The model III is represented by a compact star containing a core of electric and color neutral three flavors quark matter in the LOFF phase, with \( \mu = 500 \text{ MeV} \) and \( M_s^2/\mu = 140 \text{ MeV} \).

The main processes of cooling are dominated by neutrino emission in the early stage of the lifetime of the pulsar and by photon emission at later ages. The cooling rate is governed by the following differential equation:

\[ \frac{dT}{dt} = - \frac{L_\nu + L_\gamma}{V_{nm} c_{\nu}^{nm} + V_{qm} c_{\nu}^{qm}} = - \frac{V_{nm} \epsilon_{V}^{nm} + V_{qm} \epsilon_{V}^{qm} + L_\gamma}{V_{nm} c_{\nu}^{nm} + V_{qm} c_{\nu}^{qm}}. \]  

(2)

Here \( T \) is the inner temperature at time \( t \); \( L_\nu \) and \( L_\gamma \) are neutrino and photon luminosities, i.e. emissivity by the corresponding volume. The superscripts \( nm \) and \( qm \) refer, respectively, to nuclear matter and quark matter including the superconductive phase; \( c_{\nu}^{nm} \) and \( c_{\nu}^{qm} \) denote specific heats of the two forms of hadronic matter. Eq. (2) is solved imposing a given temperature \( T_0 \) at a fixed early time \( t_0 \) (we use \( T_0 \rightarrow \infty \) for \( t_0 \rightarrow 0 \)). To compute the neutrino emissivity of nuclear and unpaired quark matter the standard textbook results are used [10, 12]; for the LOFF phase I refer to [11].

In Fig. 1 the star surface temperature as a function of time is shown (see [13] for similar results obtained in other models). Solid line (black online) is for model I; dashed
FIGURE 1. (Color online) Surface temperature $T_s$, in Kelvin, as a function of time, in years, for the three toy models of pulsars described. Solid black curve refers to a neutron star formed by nuclear matter with uniform density $n = 0.24$ fm$^{-3}$ and radius $R = 12$ Km (model I); dashed line (red online) refers to a star with $R_2 = 10$ km, having a mantle of nuclear matter and a core of radius $R_1 = 5$ Km of unpaired quark matter, interacting via gluon exchange (model II); dotted curve (blue online) refers to a star like model II, but in the core there is quark matter in the LOFF state; see [11] for more details. All stars have $M = 1.4 M_\odot$.

curve (red online) refers to model II; the dotted line (blue online) is for model III and it is obtained for the following values of the parameters: $\mu = 500$ MeV, $M_s^2/\mu = 140$ MeV, $\Delta_1 = 0$, $\Delta_2 = \Delta_3 \simeq 6$ MeV. For unpaired quark matter $\alpha_s \simeq 1$, accordingly to the one loop beta function of QCD, corresponding to $\mu = 500$ MeV and $\Lambda_{\text{QCD}} = 250$ MeV. The use of perturbative QCD at such small momentum scales is however questionable. Therefore the results for model II should be considered with some caution and the curve is plotted only to allow a comparison with the other models. In any case it is important to remark that the apparent similarity between the LOFF curve and the unpaired quark curve depends on the fact that the LOFF phase is gapless. This yields a parametric dependence on temperature analogous to that of the unpaired quark matter: $c_V \sim T$ and $\epsilon_\nu \sim T^6$. However the similarity between the curves of models II and III should be considered accidental because emissivity of unpaired quark matter depends on the value we assumed for the strong coupling constant.

A final remark: the improvement of the pairing condensation ansatz and, more important, of the model of the stars would allow a direct comparison with the observational data. Nevertheless we expect our results capture the essential physics: indeed from our knowledge of the two flavor LOFF phase [14] we may argue that fermion gapless excitations are peculiar of the crystalline color superconductivity; since these gapless excitations are responsible for the rapid cooling, a neutron star with a LOFF core should cool faster than the cooling of a star made only of nuclear matter. If a careful comparison with the observational data (see for example [15]) could allow to rule out slow cooling for star masses in the range we have considered, this would favor either the presence of condensed mesons [16] or quark matter in a gapless state in the core (since gapped quarks emit neutrinos very slowly).
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REFERENCES

1. M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B 422, 247 (1998) [arXiv:hep-ph/9711395]; R. Rapp, T. Schäfer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998) [arXiv:hep-ph/9711396]; D. T. Son, Phys. Rev. D 59, 094019 (1999) [arXiv:hep-ph/9812287]; R. D. Pisarski and D. H. Rischke, Phys. Rev. D 61, 074017 (2000) [arXiv:nucl-th/9910056].

2. K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333; M. G. Alford, Ann. Rev. Nucl. Part. Sci. 51, 131 (2001) [arXiv:hep-ph/0012047]; G. Nardulli, Riv. Nuovo Cim. 25N3, 1 (2002) [arXiv:nucl-th/0203047]; S. Reddy, Acta Phys. Polon. B 33, 4101 (2002) [arXiv:nucl-th/0211045]; T. Schäfer, arXiv:hep-ph/0304281; D. H. Rischke, Prog. Part. Nucl. Phys. 52, 197 (2004) [arXiv:nucl-th/0305039]; M. Alford, Prog. Theor. Phys. Suppl. 153, 1 (2004) [arXiv:nucl-th/0312007]; M. Buballa, Phys. Rept. 407, 205 (2005) [arXiv:hep-ph/0402234]; H. c. Ren, arXiv:hep-ph/0404074; I. Shovkovy, arXiv:nucl-th/0410091; T. Schäfer, arXiv:hep-ph/0509068.

3. M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537, 443 (1999) [arXiv:hep-ph/9804403].

4. A. W. Steiner, S. Reddy and M. Prakash, Phys. Rev. D 66, 094007 (2002) [arXiv:hep-ph/0202037]; S. Reddy, Acta Phys. Polon. B 33, 4101 (2002) [arXiv:nucl-th/0211045]; T. Schäfer, arXiv:hep-ph/0304281; D. H. Rischke, Prog. Part. Nucl. Phys. 52, 197 (2004) [arXiv:nucl-th/0305039]; M. Alford, Prog. Theor. Phys. Suppl. 153, 1 (2004) [arXiv:nucl-th/0312007]; M. Buballa, Phys. Rept. 407, 205 (2005) [arXiv:hep-ph/0402234]; H. c. Ren, arXiv:hep-ph/0404074; I. Shovkovy, arXiv:nucl-th/0410091; T. Schäfer, arXiv:hep-ph/0509068.

5. A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47, 1136 (1964); P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964); M. G. Alford, J. A. Bowers and K. Rajagopal, Phys. Rev. D 63, 074016 (2001); R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. 76, 263 (2004).

6. R. Casalbuoni, R. Gatto, N. Ippolito, G. Nardulli and M. Ruggieri, Phys. Lett. B 575, 181 (2003) [Erratum-ibid. B 582, 279 (2004)] [arXiv:hep-ph/0307325].

7. S. L. Shapiro and S. A. Teukolsky, Black Holes, White Dwarfs and Neutron Stars, (New York: Wiley, 1983).