A low-noise on-chip coherent microwave source

Chengyu Yan1,2, Juha Hassel3,4, Visa Vesterinen4, Jinli Zhang1,5, Joni Ikonen1, Leif Grönberg4, Jan Goetz1,3 and Mikko Möttönen1,4

The scaling up of quantum computers operating in the microwave domain requires advanced control electronics, and the use of integrated components that operate at the temperature of the quantum devices is potentially beneficial. However, such an approach requires ultralow power dissipation and high signal quality to ensure quantum-coherent operations. Here we report an on-chip device that is based on a Josephson junction coupled to a spiral resonator and is capable of coherent continuous-wave microwave emission. We show that the characteristics of the device accurately follow a theory based on the perturbative treatment of a capacitively shunted Josephson junction as the gain element. The infidelity of typical quantum gate operations due to phase noise of this cryogenic 25 pW microwave source is less than 0.1% up to 10 ms evolution time, which is below the infidelity caused by dephasing in state-of-the-art superconducting qubits. Together with future cryogenic amplitude and phase modulation techniques, our approach may lead to scalable cryogenic control systems for quantum processors.

Quantum computing has developed rapidly in the last decade using a range of different physical systems1–7. For example, semiconductor- and superconductor-based qubits with frequencies in the microwave regime have been extensively studied7,8. In such systems, the control of a large quantum processor is typically implemented by channeling a sequence of microwave pulses to the qubits operating at many different frequencies. This can be achieved using conventional room-temperature electronics. However, this approach requires a large number of broadband connections scaling linearly with the number of qubits, to transmit signals from room temperature to the temperature of the refrigerator that hosts the quantum processor (typically below 100 mK). For the scaling up of these quantum computing systems, the heavily attenuated bundles of coaxial microwave cables determine much of the cooling power and physical size of the refrigerator9. Cable lengths also lead to latencies and limitations in, for example, quantum error correction10,11.

Cryogenic-integrated control electronics can potentially overcome these challenges. Independent of how the cryogenic control electronics are realized, any viable approach will need stable microwave sources integrated in the relevant operating environment to create, for example, the carrier frequencies for the modulated pulses12. Josephson-junction-based sources have been previously studied in the field of radio astronomy as local oscillators (LOs) for receivers in the millimetre- and submillimetre-wave bands13. However, for typical solid-state quantum information processing applications operating in the sub-20GHz band, the specific requirements are stringent. In particular, an on-chip microwave source should exhibit very low power dissipation, long coherence time, and low noise. Some of these properties have been previously explored with prototype devices based on quantum dots14–16 and Josephson junctions (JJs)17,18 embedded in resonators. Similar designs can also find applications in generating non-classical radiation19,20. However, detailed design guidelines for specific applications are generally lacking, and it remains unclear whether the signal quality of these systems will be sufficient for high-fidelity qubit operations.

In this Article, we report an on-chip coherent microwave source based on a JJ strongly coupled to a spiral resonator. We develop a quantitative theoretical model for the resonator-coupled JJ that operates in a particular parameter regime and can yield a stable and coherent continuous-wave microwave output. Our theory is verified by experiments that also provide relevant system parameters, including the output power and microwave generation efficiency. We also confirm the applicability of our source for quantum-coherent operations by measuring the phase noise of the oscillator output, and provide the total phase noise spectrum up to large offset frequencies and evaluate its impact on the dephasing of an ideal qubit.

We analyse, in particular, the source-induced infidelity of the identity operation as well as the NOT gate for an ideal qubit, and conclude that they lie below 0.1% up to a 10–ms evolution. We achieve this performance metric with a total cryogenic power consumption of 200 pW—which is compatible with the millikelvin environment—and a generation efficiency of about 15%. We, thus, suggest that the source is compatible for driving qubits in schemes, in which it is combined with additional amplitude and phase modulation components. Due to the high output power, our on-chip source may also be of potential use in other applications, such as writing and retrieving quantum information encoded in spin ensembles21 and pumping a microwave-to-optical photon converter22.

Theory and design. Our oscillators are realized as capacitively shunted Josephson junctions (C-shunt JJs) coupled to a resonator. Under certain conditions described below, the phase dynamics of a C-shunt JJ locks to a radiation field that couples to it23. The phase-locked and biased junction generates power at the corresponding angular frequency $\omega_0$.

Our devices reside in the parameter regime corresponding to relatively high Josephson coupling energies in the range of $E_J/h \gtrsim 1$ THz, where $h$ is the Planck constant. Furthermore, it is known that the required harmonic phase-locking conditions are favoured in the case24 of $\omega \gg \omega_0$, that is, the generated angular frequency $\omega$ sufficiently exceeds the plasma angular frequency of the
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\[ I = \frac{e}{2C} \left( 1 - \frac{I}{I_0} \right) \left( 1 - \left( \frac{I}{I_1} \right)^2 \right), \]

where \( h = h/(2\pi) \), \( I_0 \) is the d.c. current through the junction, \( I_1 \) is the critical amplitude of the self-sustained oscillation of the system at angular frequency \( \omega_1 \), \( I_1 = \frac{2e}{\pi \alpha} \), \( I_1 \) is the critical current of the junction and \( I_1 \) refers to the \( n \)-th order Bessel function of the first kind. As detailed in Supplementary Section 2, \( I_0 \) can be expressed as \( I_0 = I_0 - \frac{\Phi_0}{2\pi} \), where \( I_0 \) and \( R \) are the total bias current and resistance parallel to the junction at d.c. bias, respectively. The fact that \( R_1 \) is negative for positive \( I_0 \) is a manifestation of the power generation ability.

To provide an intuitive understanding of the electrodynamics of the source, we note that the power output properties are related to the real component of impedance \( R_1 \), whereas the imaginary component \( X_1 \) slightly modifies the resonator frequency. In the steady state, the photon emission rate from the JJ to the resonator equals that of the resonator decay, satisfied for \( R_1 = -R_1 \) (Fig. 1c). Formally, we can express the total losses including the Josephson effect by \( R_1 + R_1 \) such that the total effective quality factor assumes the form \( Q_{\text{tot}} = \sqrt{L/C_{\text{r}}/(R_1 + R_1)} \), where we have neglected the effect of \( X_1 \) for simplicity. Thus, the sustained oscillations corresponding to \( R_1 = -R_1 \) lead to a diverging \( Q_{\text{tot}} \). Due to the amplitude dependence of \( R_1 \), a single well-defined amplitude for the sustained oscillation is obtained at each bias point.

The total RF power generation is given by \( P_{\text{out}} = -\left( Q/Q_0 \right) R_1 I^2 / 2 = \left( Q/Q_0 \right) I^2 \omega_0 / (2e) \), where \( Q \) and \( Q_0 \) are the experimentally measurable total and external quality factors excluding the Josephson effect (Supplementary Information). The phase-locking condition implies \( I_0 < I_0 \), such that the maximum power output is obtained by optimizing the load such that \( R_1 = 0.68 Q_0 / Q_0 \) and minimizing the residual microwave losses such that the microwave efficiency \( \eta_{\text{microwave}} \equiv Q/Q_0 \approx 1 \) (Supplementary Section 2 and Extended Data Fig. 3).

To account for the imaginary component, we note that \( X_1 \) yields a small correction to the reactance of the excited mode, and results in a small shift in the output frequency. Its bias-point dependence maps into the corresponding dependence for the output frequency. This phenomenon is anticipated to affect the quality of the output signal, since a fluctuating bias point leads to fluctuations in the frequency and hence in the phase of the output, as studied for different types of Josephson oscillators in the literature. The relative effect of \( X_1 \) in the total reactance is minimized by maximizing the characteristic impedance of the resonator \( \sqrt{L/C_{\text{r}}} \). This provides a guideline for resonator optimization to achieve higher frequency and phase stability of the output signal: maximize the impedance under the constraints set by the resonator design. Although we can achieve much higher resonator impedances given our design rules, we set here the impedance to roughly 100 \( \Omega \) so that the dependence between the emission frequency and d.c. bias can be conveniently observed experimentally. This allows us to explicitly demonstrate the validity of our model. Further improvement in the phase stability and hence the linewidth of the output signal can be realized by increasing the characteristic impedance of the resonator, which may be obtained by decreasing the metallization ratio of the distributed transmission-line resonator design and by using a thin film of high-kinetic-inductance material.

**Demonstration of coherent emission.** After the calibration of the gain of the amplification chain (Supplementary Fig. 1), the microwave emission spectrum is measured with a spectrum analyser (Extended Data Fig. 2 shows the setup) and shown in Fig. 2a as a function of the d.c. bias current \( I_0 \) which is converted to the d.c. bias voltage by the on-chip shunt resistor. We observe three bias regions with distinctive signatures in both current–voltage \( (I–V) \) characteristics and emission spectrum as follows: the supercurrent state for \( I_0 < 10 \mu A \) (region I), self-induced Shapiro step \( I_0 < 13 \mu A \) (region II) and the normal state with the resistance determined by the shunt resistor for \( I_0 > 19 \mu A \) (region III). There is no photon emission in region I and negligible emission in region III. In contrast, a major emission occurs at the Shapiro step in region II. The Shapiro step is a manifestation of the self-induced locking of the Josephson and resonator dynamics, leading to the measurable power emission. The central frequency of the emitted signal shifts towards a higher frequency with increasing \( I_0 \) owing to the abovementioned \( X_1 \) (Fig. 2a). This behaviour is well captured by our model.
The emitted power, as shown in Fig. 2b, is obtained by integrating the emission spectrum over frequency. The power increases almost linearly with increasing $I_b$, again well captured by the model. The output power exceeds 20 pW for $I_b > 16\,\mu A$ peaking at 28 pW ($-75.5$ dBm) with a corresponding d.c. power $P_{\text{d.c.}} = 17.7\,\mu A \times 10.7\,\mu V = 189\,\text{pW}$ ($-67.2$ dBm). This suggests a d.c.-to-RF conversion efficiency of 15% at the maximum output power. Note that this efficiency is not the fundamental limit, but bounded by the requirement of unconditional stability and single-valued bias path (Supplementary Section 2 and Extended Data Fig. 3 provide a detailed discussion). As shown in Fig. 2c, the typical emission linewidth is $4.1 \pm 0.1$ kHz, which is roughly five times as sharp as that obtained in ref. 18 ($-22$ kHz). Such a narrow linewidth suggests the potential for a noticeable improvement in phase stability over previous coherent cryogenic sources $^{16-18,28,29}$, although further advancements are needed for the source to be applicable in qubit control. Especially, relative phase drifts between sources used to drive different qubits may complicate and degrade the implementation of multi-qubit quantum logic. However, note that ultrastable sources usually involve active frequency and phase stabilization techniques rather than operating in the open-loop scenario employed above.
Interestingly, we only use a single-pole room-temperature commercial low-pass filter on the d.c. bias line in the present setup, whereas filters at multiple temperature stages have been utilized in previous studies. On one hand, our filtering scheme relaxes some of the burden required to build and test the experimental setup. On the other hand, improvements in the filtering scheme in our setup may lead to further narrowing of the spectral line.

To find evidence that the output of our source is composed of microwaves in a coherent state, we utilize the heterodyne detection technique (Extended Data Fig. 2). The output field is demodulated by a local reference tone to yield the in-phase (I) and quadrature-phase (Q) components. The frequency of the reference tone is detuned from the central emission frequency by 62.5 MHz, which is optimized for our setup. The results of 10^6 samples are summarized as a two-dimensional (2D) probability distribution (Fig. 2d). The probability distribution of the output shows a nearly Gaussian shape with respect to the intensity of the radiation field or the radius (Fig. 2d), as detailed in Extended Data Fig. 4. The Gaussian ensembles rotate at an intermediate frequency of 62.5 MHz in the IQ plane and hence resembles a ring with a finite radius and width. This coincides with the distribution of a coherent state averaged over different phase shifts; hence, our observation provides evidence of the coherent character of the emission. Note that this heterodyne setup is much more prone to noise than the setup utilizing the spectrum analyser, and consequently, we employ the spectrum analyser setup to study the quantitative performance of the source.

In Extended Data Fig. 6, we provide data on a reference device differing from that discussed here mainly in its lower critical current (roughly 1.8 µA) and having a lumped-element LC resonator. In addition, the resonator impedance is ~3.8Ω as opposed to ~75Ω for the spiral resonator. As expected from the discussion above, the frequency of the emitted signal (Extended Data Fig. 6) is much more sensitive to the bias current than for the spiral resonator sample. Thus, the emitted signal is expected to experience excessive phase noise. Nevertheless, the good agreement between the experimental data with our theory for this sample of very different parameters than those of the spiral resonator sample provides a convenient verification of our model.

To gain an additional understanding on the possible limitations of linewidth of the generated signal, we utilize the well-established injection-locking technique. Here the frequency of the injection tone f inj is fixed to that of the free emission at a given bias, whereas the injection power P inj is swept.

Figure 3a illustrates that in our experiments, the injection tone induces a very sharp peak into the emission spectrum, into which the whole emission gradually shifts with increasing injection power. For P inj = −100 dBm, we only observe a single emission peak with a linewidth of 1 Hz (Fig. 3b). Interestingly, this linewidth seems to be limited by the smallest resolution bandwidth of the spectrum analyser used in our experiment; further, our more detailed study of the measured spectrum (Extended Data Fig. 8) suggests that assuming a Lorentzian lineshape for the injection-locked source, its linewidth is of the order of 1 mHz or below. In general, it is possible to measure very small linewidths with an advanced hardware setup, for example, by carrying out the Fourier transform of the IQ traces after sufficient averaging. Yet, we note that the typical linewidths of the state-of-the-art superconducting qubits are in the kilohertz range, that is, comparable to the linewidth of our source without injection locking (Fig. 2c). We have also measured the emission spectrum with a fixed P inj but varying f inj and the results agree well with the Adler theory, as shown in Fig. 3c.d and Extended Data Fig. 5 and discussed in Supplementary Section 3.

Phase noise and its infidelity in driving an ideal qubit.
We extract phase noise from the emission spectrum under injection locking with P inj = −100 dBm, where the injection tone contributes less than 1% of the total power. Our results presented in Fig. 4a show that L rapidly decays with increasing frequency offset f off from f inj. It reaches ~95 decibels relative to the

![Graph](image-url)
carrier per hertz (dBc Hz⁻¹) at \(f_{\text{off}} = 10\) kHz, which is about 15 dB below the corresponding value for a typical lab-grade LO operating at room temperature, but it needs further improvement to be compatible with high-precision LOs such as that used to generate the injection tone. The measured phase noise eventually saturates to \(\sim -120\) dBc Hz⁻¹ at \(\sim 5\) MHz. The saturation is mainly determined by noise added by our amplification chain (Supplementary Fig. 4). The noise floor can be possibly subtracted to a large extent from the source noise by carefully averaging the data for offset frequencies exceeding 5 MHz (Extended Data Fig. 7).

It is possible to minimize the influence of system noise using a cross-correlation technique and thus obtain the actual \(\mathcal{L}\) at large \(f_{\text{off}}\). We leave this extension for future research. Nevertheless, we note that \(\mathcal{L} = -116\) dBc Hz⁻¹ at an offset frequency of 1 MHz is well below \(-99\) dBc Hz⁻¹ obtained by the quantum-dot-based on-chip microwave source studied elsewhere.

Owing to the large output power and hence the large signal-to-noise ratio, phase noise \(\mathcal{L}\) yields the dominating noise of the device up to relatively large offset frequencies. This motivates us to examine the influence of phase noise on apparent qubit dephasing as well as gate and operation fidelity. We consider the source, augmented with a noiseless pulse and phase modulator, to drive an ideal qubit that is free of intrinsic dephasing and dissipation. Following the framework shown elsewhere, we calculate the infidelity of quantum operations, defined as \(1 - F_\text{av}(\tau)\), where the averaged operational fidelity is denoted by \(F_\text{av}(\tau)\).

We have

\[
F_\text{av}(\tau) \approx \frac{1}{2} [1 + e^{-X(\tau)}],
\]
where the evolution time is denoted by $\tau$,

$$X(\tau) = \frac{1}{2\pi} \int_0^\infty \frac{df}{df} \left( \frac{1}{10 \text{dBc} \cdot \text{Hz}^{-1}} \right) \frac{1}{Hz} \sum_{l,x,y,z} G_{x,y,z}(f, \tau),$$

and $G_{x,y,z}(f, \tau)$ is a filter function that quantifies the action of the control Hamiltonian and hence depends on the specific quantum operation (Supplementary Section 5).

Figure 4b shows the calculated infidelity for prototypical quantum operations: Ramsey, Hahn echo and NOT gate operations. The infidelity is ~0.1% for all these operations after a long evolution time of $\tau = 10$ ms. These infidelities are an order of magnitude lower than those achieved by a typical lab-grade LO12. However, in the short-$\tau$ limit, the calculated infidelity is about an order of magnitude higher than that obtained from lab-grade LO due to the overestimated phase noise $\mathcal{N}$ at large $f_{\text{offset}}$ arising from the amplification chain. The low-offset-frequency components dominate in the long-$\tau$ limit. On the other hand, both low- and high-frequency components have a noticeable contribution in the short-$\tau$ limit, as discussed in the Supplementary Information. For comparison, the infidelities of the operations are reduced by an order of magnitude if we extract them from the phase noise measured for the LO. These infidelities are also substantially affected by the noise floor set by the amplification chain. Identical fidelity analysis with the noise floor subtracted are shown in Extended Data Fig. 7.

The above-measured low noise of our microwave source suggests that the device is a promising candidate for controlling state-of-the-art superconducting qubits with coherence times currently reaching 100 $\mu$s (refs. 12–14).

Conclusions

We reported an on-chip coherent microwave source based on a $\downarrow$J$^\downarrow$ strongly coupled to a spiral resonator. The source can generate microwave signals with a narrow linewidth (<1 Hz), low noise ($<-120$ dBc Hz$^{-1}$), high output power (>25 pW) and a d.c.-to-RF power conversion efficiency of about 15%. The output power is two orders of magnitude higher than that of previous double-quantum-dot sources24–26 and aluminium-junction sources27 operated at millikelvin temperatures (Table 1). We calculated that the expected infidelity bound arising from the phase noise of our source in a typical quantum-logic operation is below 0.1% up to 10-ns evolution, ensuring that the signal quality is sufficient for the control of state-of-the-art quantum systems.

We used the injection-locking technique in an effort to study the intrinsic limitations of the oscillator. As for any oscillating source independent of technology, frequency and phase stabilization techniques need to be used for best performance. An alternative to this direct injection-locking scheme is to bring a reference tone to the cryogenic temperature at a low-frequency band, reducing the bandwidth requirements from room temperature, and to use frequency multiplication techniques25 to generate the injection tone.

Furthermore, the integration of superconducting quantum interference devices with the source would allow the emission frequency to be tuned without substantial amplitude modulation and hence could enable frequency stabilization based on phase-locked loops, which are typically used in the context of voltage-controlled oscillators at room temperature, and for which superconducting counterparts have been demonstrated11.

The integrated control of quantum systems are currently being pursued in various approaches, including techniques based on cryogenic semiconductors and techniques based on optical-to-microwave transducers28. Semiconductor-based oscillators with an output power of about 0.2 $\mu$W at 1.5 K (ref. 30) and even full semiconductor-based cryogenic control systems operating at temperatures of a few kelvins and with power consumption of the order of 100 mW have been demonstrated33. In addition, cryogenic semiconductor electronics for megahertz-level voltage pulses operating at a temperature of 100 mK and at a power consumption of 18 nW per cell per megahertz of operation frequency have been demonstrated34. Although less mature, all-superconducting control electronic concepts are probably superior in terms of power efficiency.

Ideally, the power dissipated at the base temperature is efficiently converted into the control signals of a quantum system. We have, thus, provided a design framework based on the perturbation approach to optimize the efficiency against the boundary conditions stemming from the requirements of stable operation. The framework provides a convenient tool to design the oscillators for the desired power level, useful in customizing for a given application. Experimentally, the generated signal corresponds to the order of $10^2$–$10^3$ photons within a gate length of 10–100 ns, typical for superconducting qubits. Enhancements in qubit quality increase the needed driving power, thus rendering a relatively high power output necessary, as shown in Extended Data Fig. 9 (Supplementary Section 7 provides a detailed discussion). Even a higher output power level is technologically feasible by adjusting the junction parameters.

A continuous-wave source with satisfactory power and signal quality alone is not sufficient for the control of quantum systems. A feasible scheme for millikelvin-operated full waveform control may be a combination of our source with cryogenic microwave phase shifters29,30 or flux-tunable resonators31 and quantum-circuit refrigerators32,33. Moreover, cryogenic mixing or parametric frequency conversion techniques are an alternative option for full waveform generation. We also anticipate that a stable microwave source is useful in other contexts than direct qubit control such as those based on single-flux quantum logic33. Compared with semiconductor counterparts, single-flux quantum logic is orders of magnitude more power efficient33. In general, the field of integrated control systems in cryogenic solid-state quantum technology is still at its infancy and it is expected that a power-efficient low-phase-noise reference oscillator will be a necessary component, acting as the primary source of microwave power and as a reference master clock.

In the future, we aim to study the properties of cascaded cryogenic sources, where one injection-locked source works as the locking tone for other sources. In such a scenario, the total

| Device | Operation temperature | Output power | Linewidth | Phase noise |
|--------|-----------------------|--------------|-----------|-------------|
| Nb junction device (this work) | 10 mK | 28 pW | 4 kHz | ~116 dBc Hz$^{-1}$ at 1 MHz (lock) |
| Al junction device28 | 10 mK | 0.255 pW | 22 kHz | N/A |
| Double quantum dot35 | 10 mK | 0.2 pW | 5.6 kHz | ~99 dBc Hz$^{-1}$ at 1.3 MHz (lock) |
| SiGe HBT oscillator36 | 4 K | 0.2 μW | 200 kHz | ~112 dBc Hz$^{-1}$ at 1 MHz |
| Crysogenic HEMT oscillator36 | 1.4 K | 0.2 μW | N/A | ~112 dBc Hz$^{-1}$ at 1 MHz |

Key parameters of cryogenic sources gathered from the indicated references, including those for high-electron-mobility transistors (HEMTs) and heterojunction bipolar transistors (HBTs). Here ‘N/A’ refers to a case where the corresponding data were not found. The ‘lock’ in the phase noise column indicates that the noise was measured under injection locking. The figures for the linewidth are given without injection locking.
injection power delivered from room temperature, and hence the number of microwave cables required, is independent of the size of the cryogenic control system such as the number of qubits in a large-scale quantum computer. We also aim to study the thermalization of the output impedance of the source. Although the measured phase noise also includes the effect of finite temperature, thermal photons leaking into different parts of a quantum computer may lead to additional undesired dephasing. However, it should be possible to thermalize the shunt resistor well below 100 mK, even at high output powers. In fact, a cryogenic source may be able to use the exponential thermal suppression of noise photons at the signal frequency, in contrast to signals generated at room temperature, after which the suppression of noise photons by cryogenic attenuation or filtering leads to an equal relative suppression of the signal power.

Methods

Theoretical model. Let us develop an analytical model for the Josephson oscillators based on a C-shunt JJs coupled in parallel with a resonator circuit. The concept is to first derive an analytical model for the junction as a gain element. Then, the gain properties are analysed for the resonator-coupled junction. The mathematical tools used here include (1) well-known conditions of junction phase locking to an RF drive, (2) a perturbative harmonic analysis of the junction to provide the gain properties under the RF drive and phase-locking conditions and (3) using a power-balance criterion to show that the gain and phase-locking conditions satisfy a stable sustained oscillating mode by the self-generated RF drive. For (2) above, the junction properties are described as an effective impedance, the negative real part of which is the manifestation of the gain. The model provides a convenient engineering tool for obtaining the basic properties of the oscillator. Namely, it provides simple design criteria for stable oscillator operation that can be used to predict the output power, d.c.-to-RF conversion efficiency, and curiously, the operation-point-dependent output frequency.

We begin our analysis from an RF-driven C-shunt JJ, as shown in Fig. 1. In the first step, we consider a bare C-shunt JJ. Assume that the junction is subjected to an RF current $I_{RF} = I_0 \sin(\omega t)$ such that $\omega \gg \nu_c$, where $\nu_c = \frac{1}{\sqrt{2 \pi C_C}}$ is the junction plasma frequency. $I_0$ is the Josephson inductance and $C_C$ is the capacitance parallel to the junction. Capacitance $C_C$ is dominated by the shunt capacitance since the intrinsic capacitance of the junction is negligible. Under the above assumptions, the RF drive couples predominantly capacitively through the parallel connections of the junction and shunt capacitance. Thus, the unperturbed voltage, neglecting the Josephson effect, across the tunnel junction is simply expressed as

$$V(t) = \frac{1}{C_C} \int I_0 \sin(\omega t) dt = -\frac{I_0}{C_C} \cos(\omega t) + \langle U_t \rangle,$$  

where the constant of integration is the d.c. voltage across the junction.

We employ the a.c. Josephson relation to obtain the phase across the junction as

$$\phi = \frac{2 \pi}{\Phi_0} \int V(t) dt = -\frac{2 \pi I_0}{\Phi_0 C_C} \sin(\omega t) \left(1 - \phi_i \right) + \frac{2 \pi}{\Phi_0} (U) t - \phi_i,$$  

where we have defined the constant of integration as $-\phi_i$. Next, we utilize the Josephson current–phase relation $I = \sin(\phi)$, which provides the current through the Josephson tunnel element as

$$I_i(t_0, I_0, \phi_i) = \sum_{i=-\infty}^{\infty} I_m(i) \sin(\omega t - \phi_i).$$  

Let us first consider the direct current ($I_D$) through the tunnel element. In equation (6), this follows from the term $m = 1$ as

$$\langle I_i \rangle = I_i(1) \sin(-\phi_i).$$  

In fact, $\langle I_i \rangle = I_0 - \frac{2 \pi}{\Phi_0} \Phi_0$ is related to the external direct bias current (Supplementary Fig. 3). Solving for $\phi_i$ yields

$$\phi_i = \arcsin \left( -\frac{\langle I_i \rangle}{I_i(1)} \right).$$  

To address the RF properties, we consider the fundamental-frequency component of $I_i(t)$, which follows from equation (6), picking terms $m = 0$ and $m = 2$ as

$$I_c(t) = I_c(0) \sin(\omega t - \phi_0) + I_c(2) \sin(-\omega t - \phi_2),$$  

Using a Bessel recurrence formula and equation (8) leads to

$$I_c(t) = I_c(0) (I(1) - I(2)) \left(1 - \frac{\langle I_i \rangle}{I_i(1)} \right)^2 \sin(\omega t) + 2 \frac{\langle I_i \rangle}{I_i(1)} I_c(2) \cos(\omega t).$$  

In the capacitively shunted junction case, the RF current $I_c(t)$ couples back to the shunt capacitance since we assume that it is the dominant impedance at frequency $\omega$. As this happens, a voltage emerges on top of $V_c$ of equation (3). Marking this voltage perturbation as $V_c(t)$, we obtain

$$V_c(t) = \frac{1}{\alpha C_C} \left[ I_c(1) - I_c(2) \right] \left(1 - \frac{\langle I_i \rangle}{I_i(1)} \right)^2 \sin(\omega t) - 2 \frac{\langle I_i \rangle}{I_i(1)} I_c(2) \sin(\omega t).$$  

Let us convert equations (3) and (11) into the frequency domain by identifying the in-phase $\sin(\omega t)$ and quadrature $\cos(\omega t)$ components to write $V_{i\omega}(\omega) = V_{i\omega}(\omega) + V_{q\omega}(\omega)$. Furthermore, it is practical to write the output in the form of impedance $Z_{i\omega}(\omega) = V_{i\omega}(\omega)/I_{i\omega}(\omega)$. From equations (3) and (11) and by arranging the quadratures, it follows

$$Z_{i\omega}(\omega) = -\frac{2 \langle I_i \rangle}{I_i(1)} \frac{1}{\omega C_C} + \frac{1}{\omega \tau_1} \left[ I_c(1) - I_c(2) \right] \left(1 - \frac{\langle I_i \rangle}{I_i(1)} \right)^2.$$  

The insertion of $I_i = \frac{\omega P_{in}}{\Phi_0}$ here yields

$$Z_{i\omega}(\omega) = -\frac{\omega P_{in}}{\Phi_0} I_i(1) + \frac{1}{\omega \tau_1} \left[ I_c(1) - I_c(2) \right] \left(1 - \frac{\langle I_i \rangle}{I_i(1)} \right)^2.$$  

We stress that the real part of the junction impedance is negative if $|I_i| > 0$ and hence represents gain. Note that here the direction of positive current is fixed by the choice that we operate at the first positive-voltage Shapiro step. The imaginary part of the junction impedance equals that of the shunt capacitor modified by a non-trivial perturbative term due to the Josephson effect. Equation (13) yields equations (3) and (2) for the real and imaginary components of junction impedance, respectively.

Device fabrication. The devices are fabricated in a multilayer process for superconducting circuits, the key element of which is a side-wall passivated spacer technique for the Nb–Al/AIO–Nb JJs. Figure 1d and Extended Data Fig. 1 summarize the structure of the device. The fabrication begins with a hydrofluoric acid dip of a 150-mm high-resistivity silicon wafer to remove oxides from the surface. The trilayer stack for the junctions with thicknesses of 100/100/100 nm is then deposited, with a target critical-current density of 100 A cm$^{-2}$. We deposit the subsequent layers as follows: a second 120-nm thick Nb layer, an atomic-layer-deposited 50-nm AIO$_x$ insulator for a parallel-plate capacitance density of roughly 1.5 fF $\mu$m$^{-2}$, a third Nb layer of 120-nm thickness, and a normal-metal layer with a thickness of about 100–130 nm for a sheet resistance of approximately 44 $\Omega$ $\mu$m$^{-1}$. The patterning of the layers is enabled by ultraviolet photolithography. The Nb layers are etched with plasma, whereas the insulators and resistors are wet etched. The Josephson oscillators are fabricated in the same batch of wafers as the travelling wave parametric amplifiers shown elsewhere.

Data availability

Data supporting the findings of this study are available at https://zenodo.org/record/5571377#.YWfLENrByUK.

Code availability

The algorithms used for the findings of this study are available within the paper and files in the Supplementary Information.

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Author contributions
C.Y. conducted the experiments and data analysis with inputs from the other authors. I.H. developed the analytical model and contributed to the data analysis. I.H. and V.V. designed the device with input from L.G. who fabricated the sample. J.Z. and I.J. assisted in the heterodyne detection. C.Y., I.H., J.G. and M.M. conceived the experimental idea. The manuscript was written by C.Y., I.H. and M.M with comments from all the authors. M.M. acted as the main supervisor of the work.

Competing interests
The authors declare the following: M.M. is a co-founder and Chief Scientist of IQM. Some of the authors are inventors in the granted patent titled ‘Vector signal generator operating on microwave frequencies, and method for generating time-controlled vector signals on microwave frequencies’ (FI128904B) applied by Aalto University Foundation Sr. and VTT Technical Research Centre of Finland and invented by M.M., J.I.H., T. Ollikainen, and J.G. Patent nos. CN11169726A, WO2020183060A1 and TW202107837A belonging to the same patent family have been applied by Aalto University. These patents relate to the work reported in this manuscript in a way that the invention involves, but is not limited to, a voltage-biased Josephson junction for microwave generation.

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Correspondence and requests for materials should be addressed to Chengyu Yan or Mikko Mottonen.

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Extended Data Fig. 1 | Details of the spiral-resonator sample. Schematic layered structure of the device in the vicinity of the Josephson junction. The AlOx layer deposited after the second niobium layer is not shown for clarity, but instead, the positions of the contact holes for the galvanic connections between the second and the third niobium layer are highlighted by the red rectangles. The Josephson junction is formed at the topmost region where the first and second niobium layers cross by selective etching and side-wall passivation of the first layer before the deposition of the second layer. This schematic figure corresponds to the area enclosed by the red rectangle in Fig. 1d.
Extended Data Fig. 2 | Schematic experimental setup. The output of the source is channeled either to the heterodyne setup (left position of the switch) or to the spectrum analyzer (right position of the switch). The millikelvin bias tee is used to combine the direct bias current with the possible injection tone. When the switch is at the left position, we do not use the isolators in between the source and the first amplifier, but a 3-dB attenuator instead. Some components of minor importance are not shown for the sake of clarity.
Extended Data Fig. 3 | Stability diagram of a Josephson oscillator and trade-off between stability and power efficiency. a, Total microwave load $R_1$ for the oscillator to exhibit stable sustained oscillation as a function of the direct current through the junction $\langle I \rangle$. The load values are given relative to the optimum load corresponding to $R_1 \approx 0.68 \frac{\Phi_0}{2eC_2S\omega_3}$, noting that $1/Q_t \propto R_1$. The bias point $\langle I \rangle/I_c \approx 0.58$ provides the maximum power for the optimal load but is unstable against any variations of the load. For lower bias points, the range of allowed relative loads is finite and lies between the shown minimum and maximum. b, Computed total efficiency of the microwave generation $\eta_{\text{tot}}$ as a function of $\langle I \rangle$ with the assumption of unconditional stability and single-valued bias path (see Supplementary Section 2). In this case, the total efficiency is a product of the direct-current (d.c.) efficiency $\eta_{\text{dc}}$ and the microwave efficiency $\eta_{\text{mw}}$. If the output load of the device can be considered fixed, we may lift the requirement of unconditional stability and the microwave efficiency can be engineered to unity at relevant values of $\langle I \rangle$. If the Shapiro step can be found by other means than the single-valued bias path, the d.c. efficiency can be taken to unity.
Extended Data Fig. 4 | Measured probability distribution of the spiral-resonator source without injection locking. a, Probability distribution of the output signal in the in-phase–quadrature (IQ) plane after digital down-conversion and filtering (see Supplementary Section 3) of the data for Fig. 2d. b, Probability distribution of the normalized photon number for the data in a. The blue colour shows the experimental data and the solid red line is a corresponding Gaussian fitting with a full width at half maximum of 0.037. c, Extracted phase from the IQ measurement of a real-time trace after down-conversion and filtering. Occasionally, we observe enhanced noise which we attribute to the heterodyne measurement setup. Thus, these data are not used for the evaluation of the phase noise of the source.
Extended Data Fig. 5 | Emission spectrum of the spiral-resonator source under injection locking. a–d, Measured power spectral density with respect to the output of the source as a function of the spectral frequency \( f_{sa} \) and of the injection frequency \( f_{inj} \) at a fixed injection power as indicated. In a, we provide a three-dimensional image to highlight the sharpness of the peaks. The dashed lines provide the predictions of the peak positions from the Adler theory. In b, \( \Delta f \) is defined as the width of the frequency range where the emission signal is phase locked to the injection tone.
Extended Data Fig. 6 | Illustration of the sample and measurement results for the microwave source based on a lumped-element resonator.

a, False-colour scanning-electron-microscope image of a lumped-element device nominally identical to that measured. The ground plane is denoted by red colour. The bias line and the lumped-element resonator, consisting of a straight inductive strip $L_1$ and an AlO$_x$ parallel-plate capacitor $C_1$, is highlighted in yellow. The shunt capacitor $C_s$ appears in cyan colour. The coupling capacitance $C_2$ is shown in magenta. The scratches and black dots on the surface are not present on the measured device. The layered structure of the Josephson junction is identical to that in the spiral-resonator device, apart from the junction area of roughly $3 \mu m^2$. The scale bar is $60 \mu m$.
b, Power spectral density emitted from the source as a function of the bias current and emission frequency. The red dashed trace indicates the position of the emission peak predicted by equation (S1).
c, Voltage measured across the junction as a function of bias current.
d, Output power as a function of bias current. The measured data (red dots) are in good agreement with equation (S11) (blue solid trace).
e, Measured probability distribution of the source output in the in-phase–quadrature (IQ) plane at $I_b = 3.2 \mu A$ obtained using the heterodyne measurement setup.
Extended Data Fig. 7  | Effect of the subtraction of the noise floor on the phase noise and calculated operation fidelity of the injection-locked spiral-resonator source. a, Measured phase noise of the source before (red) and after (black) the subtraction of the noise floor. The noise floor is measured after turning off the local oscillator and the bias current of the source. To obtain the phase noise with the noise floor subtracted, a careful averaging is carried out within the frequency range indicated by the two-headed arrow. We average both the original signal and the noise floor in order to mitigate the fluctuations in the phase noise. b, Calculated bound for the operation infidelity corresponding to the data in a before (solid lines) and after (dashed lines) the subtraction of the noise floor for Ramsey (blue colour), Hahn echo (red colour), and NOT gate (green colour) operations.
Extended Data Fig. 8 | Detailed analysis of the linewidth of the output signal under 100-dBm injection locking. Measured (markers) power spectral density of the spiral-resonator output signal as a function of frequency offset from the injection tone. Note the logarithmic scale on the vertical axis as opposed to the linear scale in Fig. 3b for these data. The resolution of the spectrum analyzer is set to 1 Hz, leading to a least-square Gaussian fit with a full width at half maximum (FWHM) of 0.97 Hz (red line). We also show Voigt fits corresponding to 1-Hz FWHM of the Gaussian component and FWHM of 1 mHz (magenta line) and 10 mHz (yellow line) for the Lorentzian components. The four data points at the lowest and highest frequency offsets (grey colour) are significantly affected by the non-ideality of the Gaussian filter in the used spectrum analyzer, and consequently cannot be interpreted to reflect the line broadening of the source itself. See Supplementary Section 6 for the discussion.
Extended Data Fig. 9 | Output power requirement in qubit gate drive. The number of qubits drivable with an oscillator power of 25 pW, as a function of the Purcell decay rate for different indicated gate lengths. See Supplementary Section 7 for the discussion.