A shortcut to calculate SPAM limb-darkening coefficients
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ABSTRACT
We release a new grid of stellar limb-darkening coefficients (LDCs, using the quadratic, power-2 and claret-4 laws) and intensity profiles for the Kepler, U, B, V and R passbands, based on STAGGER model atmospheres. The data can be downloaded from Zenodo https://doi.org/10.5281/zenodo.5593162. We compare the newly-released LDCs, computed by ExoTETHyS, with previously published values, based on the same atmospheric models using a so-called “SPAM” procedure. The SPAM method relies on synthetic light curves in order to compute the LDCs that best represent the photometry of exoplanetary transits. We confirm that ExoTETHyS achieves the same objective with a much simpler algorithm.

INTRODUCTION
Most codes that calculate transiting exoplanet light curves use limb darkening coefficients (LDCs) to approximate the radially-decreasing stellar intensity profile (Kreidberg 2015; Parviainen 2015; Agol et al. 2020; Morvan et al. 2021). Theoretical LDCs can be obtained by fitting a parametric law to numerical evaluations of the intensity across the stellar disk. A long list of limb darkening laws have been implemented, for which we refer to notable literature reviews (Kipping 2013; Espinoza & Jordán 2016; Agol et al. 2020). For a given law, the choice of the cost function and the sampling of the intensity profile affect the estimated LDCs. What are the best practices for determining theoretical LDCs has been the subject of a long debate (see Morello et al. 2021 and references therein).

THE SPAM METHOD
Howarth (2011) pointed out that what is best generally depends on the purpose. He produced synthetic-photometry light curves using atmosphere-model intensity profiles for a number of well-known systems, then solved for the geometric parameters and LDCs. The resulting SPAM LDCs are the most direct reference with which to compare the empirical LDCs obtained by fitting the transit light curves. Note that, by construction, the SPAM LDCs depend on the geometric parameters in addition to the stellar ones.

Morello et al. (2017) continued this experiment by exploring the behavior of various limb darkening laws as a function of stellar type and wavelengths. Additionally, they included the effect of spherical geometry to generate the synthetic light curves. It appeared that the intensity profiles obtained with SPAM LDCs, there referred to as “empirical” LDCs, provide a better match to the numerical intensities in the inner part of the stellar disk, while the discrepancy increases towards the edge. Theoretical LDCs obtained with a simple least squares fit distribute the error more evenly, forming two or more intervals in which the intensities are either overestimated or underestimated.

THE EXOTETHY S ALGORITHM
Morello et al. (2020b) compared an exhaustive list of methods for calculating LDCs from numerical intensity values, considering differently weighted and unweighted least squares fits, resampling of the intensities via interpolation, and truncation near the limb. They considered several figures of merit based on the results of synthetic light curve fits with fixed LDCs. These figures were (1) the amplitude of the light curve residuals, (2) the biases in the recovered...
transit depth and other geometric parameters, and (3) their spectral variation over the 0.25-10 μm range. All criteria converged to the same best procedure, there called “weighted-r QS”. This method consists of a weighted least squares fit, where the weight associated with a given intensity value is half the radial separation between the two neighboring points. A cutoff is applied to the spherical intensity profiles. The optimal fitting method has been confirmed by many more tests than those reported by Morello et al. (2020b), including different limb darkening laws and stellar types.

The ExoTETHyS\textsuperscript{1} package includes a stellar LDCs calculator that implements the optimal fitting method, ensuring a precision of few parts per million for transit modeling (Morello et al. 2020a). We note that ExoTETHyS should return the equivalent of SPAM LDCs, as the first figure of merit for the adopted fitting method was to provide the best match to the synthetic light curves.

INDEPENDENT CONFIRMATION OF EXOTETHYS

Maxted (2018) compiled a table of stellar LDCs based on synthetic stellar spectra\textsuperscript{2} (Chiavassa et al. 2018) computed for the stellar convection simulations of the STAGGER-grid (Magic et al. 2013). They adopted the power-2 limb darkening law (Hestroffer 1997),

\[
\frac{I(\mu)}{I(1)} = 1 - c (1 - \mu^\alpha),
\]  

(1)

that outperforms other two-coefficient laws, especially for the M-dwarf models (Morello et al. 2017). Here \( \mu = \cos \theta = \sqrt{1 - r^2} \), where \( \theta \) is the angle between the surface normal and the line of sight, and \( r \) is the radial distance from center of the stellar disk in units of its radius. In addition to the coefficients \( c \) and \( \alpha \), Maxted (2018) has tabulated the transformed coefficients,

\[
h_1 = I \left( \frac{1}{2} \right) = 1 - c \left( 1 - 2^{-\alpha} \right) \text{ and }
\]

(2)

\[
h_2 = I \left( \frac{1}{2} \right) - I(0) = c 2^{-\alpha},
\]

(3)

which were found to be less strongly correlated. The grid of LDCs published by Maxted (2018) is the only one that, according to our knowledge, is based on a SPAM-like procedure. The LDCs were derived for a standard system configuration, neglecting their dependence on the geometric parameters.

The ExoTETHyS package includes a precalculated grid of stellar intensity spectra obtained from a subset\textsuperscript{3} of the same STAGGER model atmospheres (Magic et al. 2015; Chiavassa et al. 2018). We performed grid calculations to compare the LDCs returned by ExoTETHyS with the SPAM-like ones tabulated by Maxted (2018). To appreciate the quality of the agreement between the LDCs obtained with both methods, we calculated analogous sets of LDCs using an unweighted least squares fit. Figure 1 shows the results obtained for the Kepler passband. The median absolute discrepancies between the ExoTETHyS and SPAM-like LDCs are 7–10\% for \( c \), 1.8–10\% for \( \alpha \), 1–2 \% for \( h_1 \) and \( h_2 \), respectively. The results for other passbands are similar within a factor 0.5–2. The discrepancies between the unweighted least squares and SPAM-like LDCs are typically larger by a factor 2–4 for \( c \) and \( h_2 \) and 1–2 for \( h_1 \). We provide data files (beyond those shown in the figure) and scripts to ensure the reproducibility of our results by any user; these reproducibility materials have been deposited on Zenodo: https://doi.org/doi:10.5281/zenodo.5593162.

Finally, we investigate whether the difference between the LDCs obtained in this work and those previously tabulated are solely due to the fitting method. We note that the normalized intensity profiles returned by ExoTETHyS are similar, but not identical, to those published online. Despite both calculations rely on the same stellar atmosphere models, numerical differences must have propagated through the process to get passband-integrated normalized intensity profiles (e.g., using different spectral synthesis tools, interpolation and/or passband reference files). For the Kepler passband, the median absolute discrepancy between the intensities at the limb is 0.68\%, but it can be up to 6\% for some stellar models. The small differences between ExoTETHyS and tabulated LDCs could be dominated by the differences between the underlying intensity profiles, hence suggesting an even better agreement between the ExoTETHyS and SPAM methods.

\footnotesize{\textsuperscript{1} https://github.com/ucl-exoplanets/ExoTETHyS}

\footnotesize{\textsuperscript{2} Publicly available on Pollux database: http://npollux.lupm.univ-montp2.fr}

\footnotesize{\textsuperscript{3} The new larger database will be incorporated by the end of 2021.}
CONCLUSIONS

We computed a grid of LDCs and intensity profiles for Kepler, U, B, V and R passbands by using ExoTETHyS and STAGGER atmospheric models. We compared our new LDCs with those reported by Maxted (2018) using a SPAM-like procedure that is designed to match exoplanetary transit observations. Our new LDCs are in excellent agreement with the tabulated values. These results confirm previous claims that the ExoTETHyS algorithm is essentially equivalent to the SPAM procedure, but without the need to compute synthetic transit light curves, hence requiring a much shorter computing time.

Figure 1. LDCs obtained by ExoTETHyS (red triangles) and unweighted least squares fit (blue circles) vs. LDCs tabulated by Maxted (2018) for the Kepler passband.
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