Analysis of Data Clusters Obtained by Self-Organizing Methods

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Abstract

The self-organizing methods were used for the investigation of financial market. As an example we consider data time-series of Dow Jones index for the years 2002-2003 (R. Mantegna, cond-mat/9802256). In order to reveal new structures in stock market behavior of the companies drawing up Dow Jones index we apply SOM (Self-Organizing Maps) and GMDH (Group Method of Data Handling) algorithms. Using SOM techniques we obtain SOM-maps that establish a new relationship in market structure. Analysis of the obtained clusters was made by GMDH.

keywords: self-organizing map, group method of data handling.

1 Introduction

Complex systems we meet in economics and finances are usually described by huge amounts of data, parameters and often have a non-predictable and chaotic behavior. To reveal interrelations among these parameters and data new methods of computer analysis have to be developed. These problems are of exceptional interest now when computer technologies provide enormous possibilities for collecting, storing and processing of information obtained by tracing system behavior. It is believed that this information is particularly important for the prediction of the system behavior. In this case researchers dealing with such types of systems often try to apply statistical [1, 2] or neurons methods [3, 4] for the discovery of new knowledge about the system. In framework of this approach self-adjustable methods known as self-organization methods [3, 4, 5, 6, 7] are especially interesting because they could be applied in autonomous regime without external setting for every particular case. In this article we use two types of self-organization methods for investigation of financial market behavior by analyzing data series of Dow Jones index. The study of the financial markets behavior is a very important and actual question because financial markets are described by inadequate prior information; they include many variables and are

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characterized by fuzzy elements. Therefore, market structures are extremely difficult for analytical description, and their dynamics is hard to predict.

2 Processing data by self-organizing methods

We investigate data time-series of Dow Jones index for the years 2002-2003\(^1\). These data are presented in a table, each column of which contains information about corresponding stock price during each working day for the years 2002 and 2003 separately. Each line contains information about every day state of all indices. We use time-series of the prices for 27 companies drawing up Dow Jones index, which are calculated at the end of each working day.\(^2\)

|    | AA | AXP | BA | BS | CAT | CVX | DD | DIS | EK | GE | GM | GT | IBM | IP | JPM | KO | MCD | MMM | MO | MRK | PG | S   | T   | UN  | UTX | XOM |
|----|----|-----|----|----|-----|-----|----|-----|----|----|----|----|-----|----|-----|----|-----|-----|----|-----|----|-----|-----|----|-----|
| 1  |    |     |    |    |     |     |    |     |    |    |    |    |     |    |     |    |     |     |    |     |    |    |     |    |     |    |
| ... |    |     |    |    |     |     |    |     |    |    |    |    |     |    |     |    |     |     |    |     |    |    |     |    |     |    |
| 245|    |     |    |    |     |     |    |     |    |    |    |    |     |    |     |    |     |     |    |     |    |    |     |    |     |    |

Table 1: Input data

For finding relationships in collective dynamics of prices we have normalized the input data

\[
x_{ji} = \frac{x'_{ji} - x_{jmin}^i}{x_{jmax}^i - x_{jmin}^i}, \quad j = 1, 245, \quad i = 1, 27,
\]  

where \(x'_{ji}\) is non-normalized elements of input Table 1, \(x_{jmax}^i\) and \(x_{jmin}^i\) are maximal and minimal non-normalized elements of \(i\)-column, respectively. In order to reveal some structures in stock market behavior for the companies under consideration we apply SOM (Self-Organizing Maps) \(^3\) and GMDH (Group Method of Data Handling) algorithms \(^6\)\(^7\) and compare these results.

The SOM is an unsupervised-learning neural-network method that provides a similarity graph of input data. It represents obtained results in a very convenient way. A typical simplified version of the SOM algorithm consists of two steps iterated for every sample \(^5\): a) finding the best matching units; b) adaptation of the weights. Initially all neuron weights are initialized by uniform distribution on the interval \([0, 1]\).

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\(^1\)AA-Alcoa, ALD-Allied Capital, AXP-American Express Co, BA-Boeing Co, BS-Bethlehem Steel, CAT-Caterpillar Inc., CVX-Chevron Corp. & Texaco Inc., DD-Du Pont, DIS-Walt Disney Co., EK-Eastman Kodak Co., GE-General Electric, GM-General Motors, GT-Goodyear Tire, IBM-IBM Corp., IP-International Paper, JPM-Morgan JP, KO-Coca Cola Co., MCD-McDonalds Corp., MMM-Minnesota Mining, MO-Philips Morris, MRK-Merck & Co Inc., PG-Procter & Gamble, S-Sears Roebuck, T-AT & T, UN-Unilever, UTX-United Tech, XOM-Exxon Corp

\(^2\)For more details see http://finance.yahoo.com

\(^3\)AA-Alcoa, ALD-Allied Capital, AXP-American Express Co, BA-Boeing Co, BS-Bethlehem Steel, CAT-Caterpillar Inc., CVX-Chevron Corp. & Texaco Inc., DD-Du Pont, DIS-Walt Disney Co., EK-Eastman Kodak Co., GE-General Electric, GM-General Motors, GT-Goodyear Tire, IBM-IBM Corp., IP-International Paper, JPM-Morgan JP, KO-Coca Cola Co., MCD-McDonalds Corp., MMM-Minnesota Mining, MO-Philips Morris, MRK-Merck & Co Inc., PG-Procter & Gamble, S-Sears Roebuck, T-AT & T, UN-Unilever, UTX-United Tech, XOM-Exxon Corp
The distance between two neurons of the two-dimensional grid is found as

\[ d_j = \| x - w_j \| = \sqrt{\sum_{i=0}^{N-1} (x_i - w_{ij})^2}, \]  

(2)

where \( j \) is the index of neuron in the net, \( i \) is the dummy index of vector components, \( w_{ij} \) is the weight of synapse, which matches \( i \)-component of input vector with output neuron \( j \). Thus, we find for each vector \( x \) such a neuron \( c \), for which the distance between \( x \) and \( c \) is the smallest

\[ c \equiv \| x - w_c \| = \min_{j} d_j. \]  

(3)

Vectors of weights \( w_j \) are adapted using the following rule

\[ w_j(t+1) = \begin{cases} 
  w_j(t) + \alpha(t) h_{c_j}(t) \cdot |x(t) - w_j(t)|, & j \in N_c, \\
  w_j(t), & j \notin N_c
\end{cases} \]

(4)

where \( N_c \) describes the neighborhood of "neuron-winner" \( c \), \( \alpha(t) \) is the learning-rate factor, which decreases monotonously with the regression steps, \( h_{c_j}(t) \) is a scalar multiplier called the neighborhood function. Thus, training process leads to reduction of the distance between input signal and position of "neuron-winner" as well as to the reduction of the Euclidean distance between input vector and any vector \( w_j, j \in N_c \).

The SOM-map is obtained as the result of this mapping of vector \( x \) on neurons plane. Similar input vectors are placed closely to each other on the SOM-map. Such procedure makes it possible to single out the input information and locate the input vectors in the vicinity of similar vectors on the map without preliminary training, using only internal properties of input data. Next processing of the input data by chosen rule (4) leads to the neuron-grids training and formation of corresponding clusters. In this case the mapping of the information by the system of neurons is characterized by the adaptation of system to the information. Since the similar data are placed in the same clusters the introduction of a new vector that is similar to existing ones does not change clusters distribution significantly. But input of data vector, which have not been presented on the map until yet, leads to the rearranging of the mapping structure and formation of new clusters. Optimization of clusters and formation of SOM-grid are carried out in the process of training and are based on the procedure of minimization of errors squared sum in existing clusters. Applying the above algorithm to the input data from Table 1 for the year 2002 (27 input vectors of \( N \)-dimension, \( N = 245 \)) we obtain SOM-map and SOM-grid presented on Fig.1 and Fig. 2.

Let us consider economic interpretation of the SOM-map and SOM-grid obtained for data presented in the Table 1. Elements of the first cluster I are companies associated with transport branch (BA, GM, CAT - machine building, GT - technical branch, CVX, GM, XOM - chemical industry associated with production of oil and fuel). At the same time EK and PG, also engaged in chemical industry, are placed in the separate cluster far from other chemical companies. This fact shows their difference from all other companies. EK is basically engaged in chemical technologies
Figure 1: SOM-map 20x20. Data taken for the year 2002.

Figure 2: SOM-grid. Data taken for the year 2002.
associated with film-production, polymeric materials etc., that makes it dependent on mining companies (MRK, MMM). The second company (PG) is a world-famous manufacturer of goods for self- and household care.

Analyzing the second cluster II we can see that it consists of companies involved in health- and life-care manufacturing. DD develops new technologies of nutrition production and MRK is a global research-driven pharmaceutical company. The last company of this cluster (AXP) is engaged in financial and international banking services worldwide. Considering KO and DIS companies one can notice that in spite of the fact that they are located in separate clusters their proximity shows similar dynamics of stock prices.

The elements of the third cluster III (UN, PG, MMM) compose a subsystem, which provide goods for household care and personal products. One can understand the relationship between PG (mentioned above) and UN, which is also engaged in production of goods for self- and household care. So, these companies are presented in health care and safety markets.

It should be noted that SOM-map and SOM-grid are not changed when they are smoothed over input data by the method of the nearest neighborhood and by Savitsky-Golay method up to the range $M=10 \ [8]$. Thus SOM-grids preserve their topology on such time intervals. It indicates that the stock market also preserves its collective properties on time intervals which are shorter then 10 days. This fact can be used for short term forecasts during share price oscillation in the market.

Processing data time-series of Dow Jones index for the year 2003 with SOM-algorithm, we obtain the results, which are a little bit different from those for 2002 (Fig.3). In fact one can notice that EK and MRK are separated from other indices.

Nevertheless, SOM - maps preserve their topological similarity as a whole. This evidences close internal relationships between the companies that determine the economy of the country. SOM - grid for 2002 - 2003 ($N= 490$ working days) also proves this fact (Fig 4). Figures 1, 3 and 4 represent the dynamics of stock prices of the considered companies for the years 2002, 2003 and 2002-2003, respectively. All the figures show that EK and MRK companies maintain their position relative to the others and this proves the hypothesis about their close relation (EK develops chemical technologies in the area of photo, video films and polymeric materials while MRK is a mining company). One can assume that MRK is connected with production of raw materials for EK. Comparing the figures it is easy to see that high technological companies involved in developing the defensive technologies (for example, BA, CAT, UTX) fall in the same cluster. Probably these cooperative behavior is influenced by the events in Iraq in 2003. In this case the stock prices behavior of these companies is very similar. It should be noted that ALD company working in the area of financial services and insurance is close enough to the companies described above (BA, CAT, UTX). This fact can be explained by the increased interest of people and businesses in real estate and safety insurance.
Figure 3: SOM-grid. Data taken for the year 2003.

Figure 4: SOM-grid. Data taken for the years 2002-2003.
3 The GMDH analysis

Dynamics of collective stock market behavior can be analyzed by methods of multiple regression. But these methods take into account the whole set of input data and overload the final model (equation of regression). Self-organizing algorithms do not have this disadvantage. Group Method of Data Handling (GMDH) creates the model that includes only the most influential variables. The GMDH algorithms are based on a sorting-out procedure of model simulation and provide the best model according to the criterion given by the researcher. This model describes relations between their elements and the state of the whole system. Most of GMDH algorithms use polynomial referenced functions. General connections between input and output variables can be shown by Volterra functional series. A discrete analogue of Volterra series is Kolmogorov-Gabor polynomial

\[ y = a_0 + \sum_{i=1}^{N} a_i x_i + \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_i x_j + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} a_{ijk} x_i x_j x_k + ..., \]  

where \( y \) - output variable vector, \((x_1, x_2, ..., x_N)\) - input data, \((a_1, ..., a_N, ..., a_{ij}, ..., a_{ijk}, ...)\) - vector of coefficients or weights. Input data might consist of independent variables, functional expressions or finite residues. The key feature of GMDH algorithms is a partition of input data into two subsets. The first one is used to compute coefficients of the polynomial using the list square technique and to evaluate internal error by some criterion. The second one is used to calculate external error using information, which is not applied for the coefficients computations. Principles of self-organization manifest themselves in rationalization of optimal polynomial search. Internal criterion monotonously decreases when complexity of polynomials increases, simultaneously external criterion passes its minimum. Then it is possible to choose polynomial of optimal complexity, which is unique for this criterion. In other words, we provide sorting-out procedure for partial polynomials to find polynomial of optimal complexity (optimal model). It shows the dependence of the output variable on the most influential variables, which are chosen from all input variables. External criterion reaches its minimum on optimal model. Interpretation of the results is similar to multiple regression logic: the bigger is the coefficient - the more influential is the variable near it.

The combinatorial algorithm was used for providing complete search between all possible polynomials of the first order. The peculiarity of this algorithm comprises gradual change of candidate polynomials. In this case the number of variables gradually increases from 1 to \( N \). For our investigation we use unbiased criterion and variation criterion.

The complete set of polynomials includes \( k = 2^{27} - 1 \) models and the computations require a lot of time. Thus, we take into account the elements of the certain cluster and its neighbors created by SOM algorithm (Fig.1). Therefore, it makes it possible to analyze bigger set of elements relationships and obtain certain hierarchy, to find the most influential variables in each cluster.

We estimated that for each cluster fewer a 15 vectors might play a role in stock behavior. As a rule, the obtained polynomials of different complexity correspond
to the results presented by SOM. The number of the dependent variables in the models varies from 3 to 8. In some cases GMDH models describe relationships among elements of one cluster. For example, relationships among companies S, MCD and MO are not evident beforehand. Analytical expression for such relationships is

\[ Y_{MCD} = -0.03 + 0.69X_S + 0.34X_{MO}. \] (6)

Considering relationships between elements of the cluster I, which was described above, one obtains the following model

\[ Y_{CVX} = -0.029 + 0.7X_{GM} + 0.43X_{BA}. \] (7)

It is easy to see that the model includes only three elements of the cluster. It was not evident that relationships between these three companies don’t depend on the stock behavior of the other companies drawing up Dow Jones index. In the most cases the relationships we obtain have bigger numbers of elements, some of which have large coefficients.

It should be noted that the relationships between companies obtained by this method are not reciprocal. Being bounded by the equation some companies more sufficiently influence the stock price dynamics of others than those of their own. For example, we consider cluster III, which includes UN, MMM and PG (Fig.2). The obtained models are

\[ Y_{PG} = 0.29 + 0.37X_{MMM} + 0.27X_{UN} - 0.12X_{AXP} + 0.1X_T \] (8)

and

\[ Y_{MMM} = 0.07 + 0.64X_{UN} + 0.4X_{PG} - 0.13X_{KO} - 0.05X_{MO}. \] (9)

These equations show that relationships between elements of cluster III are essential. In particular, the dynamics of \( Y_{PG} \) depends on the dynamics of \( X_{MMM} \) and vice versa. In the same time \( Y_{PG} \) also depends on the dynamics of \( X_{AXP} \) and \( X_T \), but \( Y_{MMM} \) depends on the dynamics of other indices, such as \( X_{KO}, X_{MO} \).

We found that the relationships between prices dynamics of the companies are very robust. Let us consider an example describing dynamics of cluster I.

\[ Y_{AA} = 0.02 + 0.57X_{GT} + 0.48X_{CAT} - 0.01X_{GM}. \] (10)

Easy to see that \( X_{GT} \) is the most influential element for \( Y_{AA} \). The price \( X_{CAT} \) has less weight than other prices. Eliminating from consideration \( X_{GT} \) and leaving other elements we obtain a new model, which has a new set of elements. But \( X_{CAT} \) belongs to both models and has the biggest weight in a new expression.

\[ Y_{AA} = -0.004 + 0.46X_{CAT} + 0.35X_{CVX} + 0.28X_{GE}. \] (11)

The same procedure could be applied to the data for the year 2003 and makes it possible to find analytical expressions for the stock market behavior of any company drawing up Dow Jones index. We do not display these results in order to restrict the volume of the paper. In addition, this investigation has a particular interest for people dealing with specific finance investigation.

8
4 Conclusions

The results obtained by SOM algorithm are sufficiently demonstrative and make it possible to understand deep relationships inherent to such a complex system like stock market. The GMDH algorithm, in its turn, makes it possible to establish analytical dependence of the stock prices of the companies, which have correlated behavior. The introduced self-organizing methods complement each other. The obtained results are self-consistent and allow to find an optimal non-overloaded model, which is easy for economic interpretation. Using the combination of such methods promises to be very perspective.

References

[1] John Y. Campbell, Andrew W. Lo, Archie Craig MacKinlay, John W. Campbell, Andrew Y. Lo: The Econometrics of Financial Markets, Princeton University Press, Princeton (1997)

[2] Cox D. R., Hinkley D. V. and Barndorff-Nielsen : Time series models in econometrics, finance and other fields, Chapman & Hall (1996)

[3] A.-P. N. Refenes, A. N. Burgess, Y. Bentz: Neural Networks in Financial Engineering: A Study in Methodology (1997) IEEE Transactions on Neural Networks vol 8, n 6, November 1997

[4] G. J. Deboeck and T. Kohonen (eds): Visual Explorations in Finance with Self Organizing Maps, Springer Finance, New York (1998)

[5] T. Kohonen, Self-Organizing Map, 2nd ed., Springer-Verlag, Berlin, 1995.

[6] J.A. Muller, A.G. Ivakhmenko and F. Lemke, GMDH algorithms for complex system modelling, Mathematical and Computer Modelling of Dynamical Systems, vol.4. no.4. pp. 275-316, 1998.

[7] L. Anastasakis and N. Mort, The development of self-organization techniques in modelling: a review of the group method of data handling (GMDH), ACSE Research Report No 813, University of Sheffield, UK, 2001 or www.shef.ac.uk/acse/research/students/l.anastasakis/813.pdf

[8] E. Ferster, B. Rents, Methods of correlation and regression analysis: Transl. from german - Moskow: Finance and statistics, 1983. (In russian)

[9] T. Kohonen, S. Kasaki, K. Lagus, et.al. Self-Organization of a massive document collection. IEEE transaction of Neural Networks 2000, V.11, 13, pp.574-585.

[10] F. Lillo and R. Mantegna, Variety and volatility in financial markets, Physical Review E, vol.62. no.5. pp.6126-6134, 2000.

[11] H.R. Madala and A.G. Ivakhnenko, Inductive Learning Algorithms for Complex Systems Modeling, Boca Raton: CRC Inc., 1994.
[12] R. Mantegna, Hierarchical Structure in Financial Markets, e-print cond-mat/9802256 v.1, 1998.