Interplay between periodicity and nonlinearity of indirect excitons in coupled quantum wells

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Abstract
Inspired by a recent experiment of localization–delocalization transition (LDT) of indirect excitons in lateral electrostatic lattices (Remeika et al 2009 Phys. Rev. Lett. 102 186803), we theoretically investigate the interplay between periodic potential and nonlinear interactions of indirect excitons in coupled quantum wells. It is shown that the model involving both attractive two-body and repulsive three-body interactions can lead to a natural account for the LDT of excitons across the lattice when reducing lattice amplitude or increasing particle density. In addition, the observations that the smooth component of the photoluminescent energy increases with increasing exciton density and that the exciton interaction energy is close to the lattice amplitude at the transition are also qualitatively explained. Our model provides an alternative way of understanding the underlying physics of the exciton dynamics in lattice potential wells.

(Some figures may appear in colour only in the online journal)

1. Introduction

Indirect excitons (spatially separated electron–hole pairs) in coupled quantum wells (CQWs) can have a long lifetime and a high cooling rate. With these two merits, Butov et al have successfully cooled the trapped excitons to the order of 1 K and have observed surprisingly that excitons form two, inner and external, rings and that periodic bright spots can appear in the external ring [1–3]. Although there is no clear evidence that these excitons are condensed into the Bose–Einstein condensation (BEC) state [4–8], it is fascinating enough to see some puzzling particle number distributions in various confined potential wells [9, 10]. More recently, periodic potential (lattices) due to gate voltages were created for indirect excitons of a built-in electric dipole moment. It was interesting to observe the abnormal localization–delocalization transition (LDT) for transport across the lattice with reducing lattice amplitude or increasing particle density [11]. This gives a unique opportunity to understand further the complex dynamical behaviors of indirect excitons.

In the literature [4, 12], a charge-separated transportation mechanism was proposed, which gave a satisfactory explanation for the formation of exciton rings and the dark region between the inner and the external rings. In particular, this mechanism was further confirmed by photoluminescence images of a single quantum well [12]. However the origin of periodic bright spots in the external ring is still under debate [13–15]. Alternatively a self-trapped interaction model involving an attractive two-body interaction and a repulsive three-body interaction was proposed for these systems [16, 17]. This model can give a good account of the periodic bright spots in the external ring. In addition, it also explained well the abnormal exciton distribution in an impurity potential, where the photoluminescence (PL) pattern becomes much more compact than a Gaussian with a central intensity dip, exhibiting an annular shape with a darker central region [9]. Moreover, the model also captured some experimental details;
for instance, the dip can turn into a tip at the center of the annular cloud when the sample is excited by higher power lasers.

The success of the self-trapped interaction model in understanding various phenomena has motivated us to use the same model to investigate the LDT phenomenon [11]. It will be shown that the complex LDT as well as other dynamic behaviors can also be qualitatively explained by this model.

The paper is organized as follows. Section 2 is divided into five subsections to introduce in detail the self-trapped interaction model in use. Most importantly, we justify the basis for the model. In section 3, numerical calculations and detailed discussions on the LDT of transport are given. Section 4 is devoted to a brief summary.

2. Model and method

In this section, we introduce and justify the model used in this paper. In particular, the physical origin of the proposed nonlinear terms will be discussed in detail.

2.1. Quantum degeneracy

We first recall the results of electron and hole creation, transportation, and formation of the exciton pattern shown in [2]. As pointed out in [4, 12], when electrons and holes are first excited by high-power laser, they are actually charge separated and have a small recombination rate because the drift speed of hot electrons (of smaller effective mass) is larger than that of hot holes. No true exciton is formed at this stage. They can travel a long distance from the laser spot, after which hot electrons and holes collide with the lattice and are cooled down. Due to the neutrality of the CQWs, negative charges will slow down and accumulate far away from the laser spot. Cold electrons and holes will then meet and eventually form excitons at the boundary of the opposite charges. Alternatively, in the experiment of [9], where an impurity potential is induced near the excitation spot, particles are cooled down to form the excitons in the well.

Although excitons obey Bose statistics, the fermionic nature of their constituents is always important and forbids us treating excitons as nearly ideal Bose particles. However, cold excitons can be considered as a quantum degenerate gas if their thermal de Broglie wavelength is comparable to the inter-particle spacing. In the present case, the number density of the excitons is about $10^{10} \text{cm}^{-2}$ and the effective mass of the excitons is about 20% of the bare electron mass. Thus the temperature of the quantum degenerate excitons is estimated to be 3 K [18]. In fact, the lattice has been cooled down by a He refrigerator to be much lower than 1 K. Although the exciton temperature may be higher than 1 K when they are first excited, they should soon reach the order of 1 K after collision with the lattice. A direct signature of exciton BEC where the extension of the coherence is over the entire cloud was recently observed in a gas of indirect excitons confined in an electrostatic trap [7], although the experimental results are still under debate [8]. As a matter of fact, quantum degeneracy plays a crucial role in the formation of the exciton pattern. Consequently a single macroscopic wavefunction $\psi$ is physically needed to treat the degenerate exciton gas when individual matter waves synchronize and combine.

2.2. Nonequilibrium and pumped dissipation

Nonequilibrium is another crucial feature of the exciton system. Continuous replenishment is necessary to ensure the dynamical balance between pumping and dissipation. Dephasing is also a challenge in realizing the exciton condensation since phonons and impurities can lead to dephasing. At low exciton densities, $n \ll 1/\alpha_B^2$ ($\alpha_B$ is Bohr radius), relaxation due to exciton–exciton and exciton–carrier scattering can be neglected [19] and consequently the relaxation is mainly due to the so-called ‘phonon bottleneck’ effects and stimulated scattering. Since the relaxation time is dominated by the scattering of excitons with acoustic phonons, the relaxation rate decreases dramatically due to the low phonon density at low bath temperatures, $(T_b < 1 \text{ K})$ [20–22]. For the recombination process, on the other hand, because excitons in the lowest self-trapped level are quantum degenerate, they are dominated by the stimulated scattering when the occupation number is more than a critical value. Strong enhancement of the exciton scattering rate has been observed in the resonantly excited time-resolved PL experiment [18].

It is clear that even though the phonon scattering rate is higher than the radiative recombination rate, the fact is that the system does not reach thermal equilibrium. Nevertheless a stable steady state can possibly exist in the present incoherently pumped dissipative system [23, 24].

2.3. Exciton interactions

A complete description of the exciton–exciton interaction is notoriously difficult. In the case of atoms, the nucleus can be treated as infinitely heavy compared to electrons and hence the exchange effect between nuclei can be safely ignored. For excitons, in contrast, the hole has a mass comparable to that of the electron. Consequently one needs to consider the exchange effect not only between the electrons but also between the holes and, in principle, between the electrons and the holes as well. As a matter of fact, both fermionic exchange couplings and direct Coulomb forces play a crucial role between the excitons. Nevertheless one can understand the exciton–exciton interactions in a qualitative way. Firstly, excitons behave as electric dipoles as a result of the spatially separated electron and hole pairs. A dipolar interaction depends not only on the distance between two dipoles but also on the angle of their orientations. At high densities, dipoles tend to align in parallel in CQWs and Coulomb repulsion will govern. However, when two parallel-aligned excitons approach each other, the exchange interaction between two electrons (or two holes) will become more important and can contribute an attractive part. In contrast, at low densities two dipoles can change from aligning in parallel to inclined (the fluctuating dipole) and consequently the attraction between the electron
of one exciton and the hole of the other exciton will dominate instead. The van der Waals forces will result in an attraction between excitons at short range [15, 25].

Because of the existence of various attractive forces in the low-density case, it is sometimes possible for biexcitons (bound states of pairs of excitons) to form. Several theoretical works have given estimates of the exciton–exciton interaction and the biexciton binding energies with various assumptions. The dependence of the exciton and biexciton binding energies on the layer separation has been investigated by Tan et al [26]. They found that while the exciton binding energy decays slowly as the inverse of the layer separation, the biexciton binding energy decays extremely rapidly. Recent studies of the exciton–exciton interaction using a heavy-hole approximation have found that there is a critical layer separation for each electron/hole mass ratio, beyond which biexcitons become unstable with respect to dissociation into two separate excitons [27]. The quantum Monte Carlo calculations found a somewhat larger range of layer separations at which biexcitons are stable and the individual excitons still retain their identity in bound biexcitons [28].

It has been pointed out that the effective interaction between excitons will be attractive when the separation between two excitons is about 3–6 exciton radii [29]. In the current experiment, the exciton density is about 10^{10} cm^{-2}. For this density, the average distance between the excitons is about 100 nm. As the exciton Bohr radius \( a_B \) is about 10–50 nm [9], the average distance between excitons is about 2–10 exciton radii. Thus it is reasonable to assume that the two-body interaction is in the attractive regime. In fact, the attractive interaction between the exciton has been considered as a possible candidate to describe the pattern formation observed by experiments [29, 30].

The direct clues of exciton interaction come from the complex exciton distribution revealed by the experiments. It is clear that the interaction between excitons is neither purely attractive, nor purely repulsive. If it is purely repulsive, it could drive the system towards a homogeneous distribution. In contrast, if it is purely attractive, the system will become unstable and eventually collapse when the exciton density is higher than a critical value. Thus it is reasonable to speculate that at low densities the interaction is dominated by an attraction, while at high densities a repulsion must exist to prevent the system against collapse [16].

In the dilute limit, we thus model the indirect excitons in CQWs as governed by a two-body attraction \((-g_1)\) and a three-body repulsion \((g_2)\) (see details section 2.4). While this ad hoc model has no microscopic origin, it is believed to capture the important physics for the low-density exciton system. In this model, self-trapped interactions are referred to the exciton patterns, which are mainly determined by the interaction terms, \(-g_1 n + g_2 n^2\). A reliable many-body calculation is obviously not feasible at the moment, in particular for extremely high densities. The observation that the exciton cloud first contracts and then expands on increasing the excitation power gives a strong support to our model [9]. Our model can also give a natural and self-consistent explanation of the particle density dependence of the FWHM broadening and PL energy shift found in experiments [31].

At low densities, the interaction between excitons is dominated by the attractive one. With increasing density (by increasing the laser power), the low-temperature exciton cloud will contract. This means that the attractive interaction hampers the exciton motion. At the mean-field level, this corresponds to an increase of the self-trapped potential. The larger the particle density is, the larger the attractive interaction is, and the exciton cloud will contract further. At low temperatures, the uncertainty principle governs the exciton motion and consequently it results in a higher mean exciton energy and larger energy dispersion. A higher mean exciton energy indicates a blue-shift of the PL peak and a larger energy dispersion means that the PL peak is broadened (its FWHM becomes large).

When the particle density is further increased, three-body repulsion becomes more important and the self-trapped interaction becomes weak. The PL peaks will have a red-shift and become sharp again. In fact, with an increase of the laser power in the low particle density regime, the phenomenon that PL spectra broaden first and then become sharper has been observed (see figure 3(c) of [31]). This, in addition to the support given by the experimental data in [9], provides further evidence that the interaction between excitons is likely the combination of a two-body attraction and a three-body repulsion. With increasing laser power, a blue-shift of the PL peaks has been reported in [25].

2.4. Model Hamiltonian

Based on the discussions above, it is physically reasonable to model the highly degenerate nonequilibrium exciton gas by the following nonlinear Schrödinger equation [16, 17]

\[
-\frac{\hbar^2}{2m^*} \nabla^2 \psi_j + (V_{\text{ex}} - g_1 n + g_2 n^2) \psi_j = E_j \psi_j,
\]

(1)

where \( V_{\text{ex}} \) denotes the external potential created by a laterally modulated gate voltage, \( n \) is the local probability density, to be specified later, and \( m^* \) is the effective mass of exciton.

Equation (1) reduces to the famous (attractive) Gross–Pitaevskii (GP) equation when \( g_2 = 0 \). Unlike the GP equation, which is usually used to determine the ground states of a low-temperature bosonic gas with a short- or zero-range two-body interaction, equation (1) considers an additional three-body interaction. More importantly, taking into account the effect of complex interactions and the nonequilibrium condition, equation (1) is to be solved not only for the ground state but also for excited states. \( \psi_j(r) \) and \( E_j \) in (1) are thus the \( j \)th eigenfunction and eigenvalue respectively. That is to say, when a macroscopically ordered state forms in the ring or in the impurity (disorder) potential well, excitons are actually distributed over the discrete energy levels, although usually only the lowest few levels are involved. Particles with energy greater than the self-trapped potential energy cannot be bounded and are ruled out in the calculations. Considering the
special property of the system, the local probability density of excitons is taken to be
\[ n(r) = \sum_{j=1}^{N} \eta(E_j)|\psi_j(r)|^2, \tag{2} \]
where \( N \) denotes the total number of bound states involved and \( \eta(E_j) \) is the probability (or distribution) function for energy level \( E_j \) that satisfies the normalization condition \( \sum_{j=1}^{N} \eta(E_j) = 1 \). More details concerning the distribution function \( \eta(E_j) \) are given in section 2.5. The eigenfunction is normalized under \( \int_{d\Omega} |\psi_j(r)|^2 d\Omega = 1 \).

A formal solution of the indirect exciton wavefunction should also allow for scattering states apart from the bound ones. This may be of particular interest when considering lowering the potential depth or increasing exciton density. While including the scattering states is beyond the scope of the current paper, it is estimated that as the particle energy is greater than the self-trapped potential energy, this part will likely only contribute to the smooth background.

The two- and three-body coupling constants, \( g_1 \) and \( g_2 \), are defined positive, which by dimension counting will be proportional to \( N \) and \( N^2 \), with \( N \) the number of excitons. Note that exciton number density is proportional to the excitation laser power \( P \), thus \( g_1 \) is also proportional to \( P \).

When length and energy are scaled in units of \( \sigma_{\text{PL}} \) and \( \epsilon_0 = h^2/m^*\sigma_{\text{PL}}^2 \), with \( \sigma_{\text{PL}} \) the root-mean-square radius of the exciton cloud observed via PL [17], equation (1) reduces to
\[ -\frac{1}{2} \nabla^2 \psi_j(r) + [V_{\text{ex}}(r) - \alpha j_n + \alpha z n^2]\psi_j(r) = E_j \psi_j(r), \tag{3} \]
where \( \alpha_1 = m^* N g_1/h^2 \) and \( \alpha_2 = m^* N^2 g_2/h^2\sigma_{\text{PL}}^2 \). It is useful to estimate the values of \( g_1 \) and \( g_2 \) with respect to the real system. Taking figure 1(b) as an example, it is estimated that \( n \approx 3.0 \times 10^{10} \text{ cm}^{-2} \). As the experiment reveals \( \sigma_{\text{PL}} = 10 \mu m \), the trapped exciton number \( N = \pi \sigma_{\text{PL}}^2 n \approx 9.4 \times 10^4 \). With \( \alpha_1 = 20 \) and \( \alpha_2 = 0.001 \sigma_{\text{PL}}^2 = 0.4 \) used in figure 1(b), we obtain \( g_1 \approx 4.93 \times 10^{-20} \text{ meV} \) and \( g_2 \approx 6.54 \times 10^{-36} \text{ meV} \).

2.5. Distribution function

The clues for determining the distribution function \( \eta(E_j) \) in (2) come from the temperature-dependent PL peaks. Four sharp peaks in the PL spectra corresponding to the emission of indirect exciton states have been observed recently in a so-called ‘elevated trap’ [31]. The individual localized states are ascribed to the confinement from both the elevated trap potential and the self-trapped potential. For a high-temperature PL spectra such as at 10 K, the height of the PL peak corresponding to the high excited state is higher than that in the ground state (see figure 2(c) of [31]). This indicates that the excitons have a large probability distribution in the high excited state. With the reduction of the temperature, say to 5.1 K, the height of the PL peak corresponding to the high excited state becomes low while the height of the three PL peaks corresponding to the two ground states becomes high. It seems that the excitons of high excited states relax to the low exciting state with the reduction of temperature. At low temperature, four peaks of almost the same height are shown in the PL spectra. The excitons are distributed in the four discrete levels with almost the same probability. In particular, at very low temperatures such as \( T = 2.7 \text{ K} \), the low-energy peak becomes very low.

Determination of the distribution function \( \eta \) requires consideration of both the complex energy relaxation and recombination processes. Two factors are important in determining the actual form of \( \eta \). Firstly, excitons are Bose quasiparticles and, at temperatures that are not too low, the distribution can be approximated by the (classical) Boltzmann function, \( \exp[-E_j/T] \), where \( T \) is considered as an effective temperature related to the lattice temperature. The other important factor is the energy dependence of the exciton luminescence efficiency. Low-energy excitons have a high luminescence efficiency. We then use \( \exp(E_j/E_0) \) to describe this effect, where \( E_0 \) is an effective energy scale related to the exciton lifetime. As a consequence, a temperature and energy dependent exciton distribution is the outcome of the competition between the above two effects. The weight of \( \eta(E_j) \rightarrow \eta(E_j, T) \) can thus be taken to be
\[ \eta(E_j, T) = C_0 \exp \left[ -\frac{(E_j - \mu)}{T} \right] \exp \left[ \frac{(E_j - \mu)}{E_0} \right] = C \exp \left[ \frac{\alpha(E_j - \mu)}{E_0} \right] \tag{4} \]
where \( C \) is the normalization factor, \( \mu \) is the chemical potential, and \( \alpha = 1/E_0 - 1/T \) can be viewed as an ‘effective temperature’. The above distribution function has been used to study the spatial and temperature dependence of exciton energy in coupled quantum wells [17].

3. Results and discussion

This section presents the major results of the paper. It is aimed at giving a theoretical picture for the LDT of a degenerate
are calculated self-consistently using equations (1)–(4). The spot. Spatial distributions of low-energy exciton density considered as a background within the range of the laser

The nondegenerate high-energy (hot) excitons are simply
highly degenerate low-energy (or cool) excitons, based
figure 3 of [11].

number density or the lattice potential amplitude, as shown in
(FWHM) of the exciton cloud will change with the particle
increase. As a matter of fact, the full width at half maximum
distance between the two separated degenerate exciton clouds
increase. When the exciton number is increased with higher
excitation power $P$, degenerate excitons can delocalize and
spread beyond the excitation spot (see figures 1(b), (c), (e),
and (f)). In these cases, the effect of attraction becomes
more important, which results in more energy eigenstates
becoming involved in the self-trapping. The first-excited states
are a nearly two-fold degenerate p-wave with a node at the
center. Superposition of the ground-state s-wave and the two
first-excited-state p-wave wavefunctions leads to
an annular distribution with a hole at the center in the
case without lattices (see figure 1(f)). With lattices, the
two-fold degeneracy is lifted and, consequently, ring-shaped
delocalized excitons shift to two separate clouds at high
excitation power (see figure 1(c)).

3.2. Spatially dependent PL energy

A PL image of indirect excitons in the delocalized regime,
potted as energy versus space, is presented in figure 2(a)
of [11]. The image was measured at the center $y = 0$ of
the exciton cloud and integrated over $\Delta y = 0.5 \mu m$. As a matter
of fact, both the integrated PL intensity $I(x)$ and the average
PL energy $\bar{h}0(x)$ show a small modulation at the lattice period
superimposed on a smoothly varying profile. In particular,
the average PL energy shows a domed curve shape.

In fact in CQWs, the spatially dependent energy of
indirect excitons, which is directly related to the PL energy
$\bar{h}0$, involves four major contributions. The first part is the
intrinsic energy, which includes contributions due to the band
gap, the Coulomb interaction between electrons and holes,
and the electric potential used to form indirect excitons. The
intrinsic energy of exciton mainly contributes to the smooth
part of the PL energy $\bar{h}0$ since it does not change with position
when the experimental conditions remain unchanged.

The second part is the kinetic energy $E_k$. Near the laser
spot, excitons are hot, with a large kinetic energy. It is believed
that this is the main reason why a high-energy distribution is
located in the center regime. The third part is the interaction
energy $E_I$. It also contributes to the smooth part of the PL
energy $\bar{h}0$. Within our model, when the exciton interaction
is in the repulsion-dominant regime near the laser spot, $E_I$
increases with the exciton density $n$ and thus smooth part
to the competition between the complex interactions and the
nonequilibrium energy distribution. Apart from the periodic
lattice potential, any additional weak external potential will
only have a minor effect on the results. In our calculations,
the wavefunction is chosen to be a Gaussian initially. All
trapped energy eigenstates $\psi_j$ are solved consistently and will
contribute to the spatial distribution.

As mentioned above, at low excitation powers $P$, the
low-energy exciton density profile essentially coincides with
the excitation laser spot. In our model, this arises due to the
dominant attraction between excitons in the dilute limit and
at the same time only the ground state contributes to the
self-trapping. The ground-state wavefunction is s-wave with a
maximum at the center (see figures 1(a) and (d)). Degenerate
excitons are localized and do not travel beyond the excitation
spot.

When the exciton number is increased with higher
excitation power $P$, degenerate excitons can delocalize and
spread beyond the excitation spot (see figures 1(b), (c), (e),
and (f)). In these cases, the effect of attraction becomes
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J. Phys.: Condens. Matter 24 (2012) 455301 T F Xu et al
Figure 2. (a) Exciton normalized probability density distribution \(n(x, y_0)\) as a function of \(x\) (\(y_0\) is denoted in figure 1(c)). (b) \(x\) dependence of lattice potential energy density \(E_p(x, y_0) = V_{el}(x)n(x, y_0)\). (c) \(x\) dependence of interaction energy density \(E_i(x, y_0)\) and kinetic energy density \(E_k(x, y_0)\). (d) separates the contributions of \(E_{sx}(x, y_0)\) and \(E_{sy}(x, y_0)\) due to the attraction and the repulsion for \(E_i(x, y_0)\) (see text). The parameters used are the same as those in figure 1(c), except a larger \(v_0 = 120\) is used to account for the experiment. The vertical line shows that maxima in \(n(x, y_0)\) correspond to minima in \(E_p(x, y_0)\). The results are intended to be compared to those in figure 2 of [11]. All energy densities are in units of \(\epsilon_0 \equiv \hbar^2/m^*\sigma_{pl}^2\).

of \(\hbar v_0\) will increase with increasing excitation power \(P\), as observed in experiments. Moreover, the reduction of the cloud size corresponds to the increase of the exciton density when the excitation power \(P\) remains unchanged. Therefore the smooth part of \(\hbar v_0\) will increase with a reduction of the cloud size. The last part is the lattice potential energy \(E_p\). Within the Thomas–Fermi approximation, it is easy to verify that \(E_p\) dominates over \(E_i\) and \(E_k\) in such a dilute system (see later). This is why maxima in the exciton number density (or equivalently the PL intensity) correspond to minima in the periodic potential energy density \(E_p\) (see figure 2). PL intensity profiles \((I\) versus \(x)\) show two broadened peaks in figure 2(c) of [11]. Since the PL intensity is proportional to the exciton number density, we ascribe it to the exciton inner ring.

Apart from the smooth part mainly due to the intrinsic energy, within our model a spatially dependent kinetic energy density of degenerate excitons can be calculated by [17]

\[
E_k(r) = \sum_{j=1}^{N} \eta(E_j) \left( \frac{\partial \psi_j(r)}{\partial x} \right)^2 + \left( \frac{\partial \psi_j(r)}{\partial y} \right)^2 \right). \tag{5}
\]

In addition, the interaction (mean field) energy density can be given by \(E_i(r) \equiv E_i + E_r\), with \(E_i(r) = -g n(r)^2\) and \(E_r(r) = g_2 n(r)^3\) corresponding to the contributions due to attractive and repulsive part respectively. Moreover, the lattice potential energy density is given by \(E_p(r) = \sum_j \eta_j(E_j) V_{el}(r) \left| \psi_j(r) \right|^2\). Figure 2(a) shows the calculated spatial probability density distribution as a function of position \(x\) of highly degenerate excitons, \(n(x, y_0)\). We focus on a fixed \(y\) position \((y_0)\) corresponding to the center of the upper cloud (see figure 1(c)). The spatial lattice potential energy density \(E_p(x, y_0)\), kinetic energy density \(E_k(x, y_0)\), and interaction energy density \(E_i(x, y_0)\) are presented in figures 2(b) and (c). The results in figures 2(a)–(c) are intended to be compared to the experimental results reported in figure 2 of [11]. As mentioned already, near the center excitons are hot (nondegenerate) and are not considered in the current model.

In view of figure 2, \(E_k\) and \(E_l\) are about one order of magnitude smaller than \(E_p\). This means that the Thomas–Fermi approximation is still valid in such a dilute exciton system. Consequently the exciton number distribution is mainly determined by the periodic potential energy \(E_p\). The higher \(E_p\) is, the larger the local PL energy is. Conversely, the lower \(E_p\) is, the higher the local exciton number density is. As the PL intensity is proportional to the local exciton number density, minima in energy correspond to maxima in intensity. Similarly the largest exciton number density (and hence the PL intensity) is located in the bottom of the periodic potential energy. Similar experimental results were found at the center of the exciton pattern (\(y_0 = 0\)). It is thus speculated that the exciton temperature near the excitation spot is still very low.

Figure 2(d) separates the contributions of \(E_{sx}(x, y_0)\) and \(E_{sy}(x, y_0)\) to \(E_i(x, y_0)\), which are due to attraction and repulsion respectively. In view of figure 2(d), it is seen near the center, \(\left| E_{sx}(x, y_0) \right| > E_{sy}(x, y_0)\), which is how the term of self-trapping is defined.

3.3. Interplay between periodic potential and nonlinear interaction

In view of the results shown in figures 3(a) and (b) of [11], it is surprising to see that the FWHM of the exciton cloud along the x direction (across the lattice) decreases with increasing lattice amplitude \(v_0\). Conversely, the excitation power needed for the transition from localized to delocalized regimes increases with increasing \(v_0\) (see figure 3(c) of [11]). This is likely due to the reduction of the particle tunneling rate across the lattice. Therefore a larger laser power is needed to obtain a delocalized distribution of excitons when increasing \(v_0\). However, the exciton transport along the y direction is only weakly affected by the lattice. The LDT for this direction shifts slightly to lower excitation powers with increasing \(v_0\).

When \(v_0\) is increased, the self-trapped interaction energy becomes relatively weaker compared to the periodic potential energy. It causes the excitons to be strongly localized in every single well. The particle tunneling rate across the lattice will be largely suppressed. In this case, the exciton distribution will be strongly bound to the periodic potential. Consequently the critical number density for LDT across the lattice to occur will increase. This explains why a blue-shift of exciton critical density is observed when LDT occurs across the lattice.

Figure 3 studies the effects of exciton number density and lattice amplitude on LDT of degenerate excitons. As we focus on the dynamics of the outer cloud (pattern) only, the FWHM is defined as the distance (the extension) between the centers of the two clouds along the y direction (see figure 1(c)). In figures 3(a) and (b), we show the FWHM as a function of
and hence the exciton number \( n \) and lattice potential amplitude respectively. When the excitation power is low, both attractive and repulsive interaction energies are small. All particles are thus sitting in the ground state and self-trapped in the shallow mean-field potential well. In this case, the FWHM of the exciton cloud is equal to zero and remains unchanged in the lower excitation power regime (see figures 3(a) and (b) with \( g_1 = 10 \)). When \( P \) (or \( n \)) is increased to above a certain critical point, LDT occurs. In this case, the attractive interaction dominates and in addition to the ground state, excitons can also sit in the two-fold degenerate first-excited states. When the one-dimensional lattice potential is present, the two-fold degeneracy of the excited states is lifted and consequently two separated delocalized exciton clouds form.

In the present model, LDT of degenerate excitons is completely determined by the nonlinearity (self-trapped interactions). To study this effect further, in figure 3(c) we show the critical number density \( n \) for LDT as a function of \( v_0 \) along the lattice direction. Since the periodicity has little effect along the lattice direction, the critical \( n \) for LDT remains roughly unchanged with \( v_0 \) along the lattice direction.

When the excitation power \( P \) is extremely low, only intrinsic energy contributes to the exciton PL energy as the interaction energy \( E_I \) is immaterial. When \( P \) is increased, \( E_I \) becomes more and more important and plays a more important role in the PL energy. As a matter of the fact, \( h\Delta \omega \) (defined as the difference between \( h\omega \) and that associated with the lowest \( P \) for LDT) will be dominated by \( E_I \). Moreover, with the application of the lattice potential, the potential energy \( E_P \) will also contribute to the PL energy. Consequently \( h\Delta \omega \approx E_I + E_P \). Figure 3(d) shows the calculated interaction energy density \( E_I (0, y_0) \) at the LDT point as a function of lattice amplitude \( v_0 \). Here \((x, y) = (0, y_0)\) corresponds to the center of the outer cloud.

It is interesting to see experimentally that \( h\Delta \omega \) is finite at \( v_0 = 0 \) (no lattice potential) at the center of the excitation spot. When \( v_0 \) is large, a remarkable relation \( h\Delta \omega \sim v_0 \) is observed. These dynamic behaviors are accounted for well by the current model focusing on the degenerate excitons. It also gives strong support to the scenario that the kinetic energy of hot excitons is actually low.

4. Summary

In summary, we have investigated the interplay between the periodic potential and nonlinear interaction of indirect excitons in coupled quantum wells, with and without a lattice potential. The model takes into account the competition between a two-body attraction and a three-body repulsion along with a reasonable nonequilibrium energy distribution. It gives an alternative qualitatively good account of the localization–delocalization transitions of excitons across the lattice when the particle density is increased or the lattice amplitude is reduced.

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