Renormalizable $SO(10)$ GUT with Suppressed Dimension-5 Proton Decays

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Abstract

We study a renormalizable SUSY $SO(10)$ GUT model where the Yukawa couplings of single $10$, single $126$ and single $120$ fields, $Y_{10}, Y_{126}, Y_{120}$, account for the quark and lepton Yukawa couplings and the neutrino mass. We pursue the possibility that $Y_{10}, Y_{126}, Y_{120}$ reproduce the correct quark and lepton masses, CKM and PMNS matrices and neutrino mass differences, and at the same time suppress dimension-5 proton decays (proton decays via colored Higgsino exchange) through their texture, so that the soft SUSY breaking scale can be reduced as much as possible without conflicting the current experimental bound on proton decays. We perform a numerical search for such a texture, and investigate implications of that texture on unknown neutrino parameters, the Dirac CP phase of PMNS matrix, the lightest neutrino mass and the $(1, 1)$-component of the neutrino mass matrix in the charged lepton basis. Here we concentrate on the case when the active neutrino mass is generated mostly by the Type-2 seesaw mechanism, in which case the correlation between the suppression of dimension-5 proton decays and the neutrino parameters is expected to be most direct.
1 Introduction

The $SO(10)$ grand unified theory (GUT) [1,2] is a well-motivated scenario beyond the Standard Model (SM), since it unifies the SM gauge groups into an anomaly-free group, it unifies the SM matter fields and the right-handed neutrino of each generation into one 16 representation, and it accommodates the seesaw mechanism for the tiny neutrino mass [3, 4, 5, 6, 7]. Renormalizable $SO(10)$ GUT models [8]-[29], where the electroweak-symmetry-breaking Higgs field originates from $10$, $126$, $120$ fields (or some of them) and the SM Yukawa couplings stem from renormalizable terms $\tilde{Y}_{10}^{16}1016 + \tilde{Y}_{126}^{126}16 + \tilde{Y}_{120}^{120}16$ (or part of them), are particularly interesting, because the SM Yukawa couplings and the active neutrino mass are described in a unified manner with fundamental Yukawa couplings $\tilde{Y}_{10}$, $\tilde{Y}_{126}$, $\tilde{Y}_{120}$. Specifically, the up-type quark, down-type quark, charged lepton and neutrino Dirac Yukawa matrices are derived as

\[
Y_u = Y_{10} + r_2 Y_{126} + r_3 Y_{120}, \quad Y_d = r_1 (Y_{10} + Y_{126} + Y_{120}), \quad Y_e = r_1 (Y_{10} - 3Y_{126} + r_e Y_{120}),
\]

\[
Y_D = Y_{10} - 3r_2 Y_{126} + r_\nu Y_{120}, \quad \text{with} \quad Y_{10} \propto Y_{10}, \quad Y_{126} \propto Y_{126}, \quad Y_{120} \propto Y_{120}, \quad \text{and} \quad r_1, r_2, r_3, r_e, r_\nu \text{ being numbers.}
\]

The Majorana mass for right-handed neutrinos and the Type-2 seesaw contribution to the active neutrino mass are both proportional to $Y_{126}$.

Supersymmetric (SUSY) GUT models are currently severely constrained by the non-observation of proton decay through dimension-5 operators from colored Higgsino exchange [33, 34], the most stringent bound being on the $p \to K^+\nu$ mode [35]. This constraint is imminent in SUSY renormalizable $SO(10)$ GUT models, because natural unification of the top and bottom quark Yukawa couplings requires $\tan \beta \sim 50$. For such large $\tan \beta$, right-handed dimension-5 operators $E^cU^cU^cD^c$ give a significant contribution to the $p \to K^+\bar{\nu}_\tau$ decay [36], and it is hard to realize a cancellation in the $E^cU^cU^cD^c$ operators’ contribution and that of left-handed dimension-5 operators $QQQL$ to the $p \to K^+\bar{\nu}_\tau$ decay and a cancellation in the $QQQL$ operators’ contributions to the $p \to K^+\bar{\nu}_\mu$ decay. Besides, it is impossible to enhance the colored Higgsino mass well above $2 \times 10^{16}$ GeV (by some adjustment of the mass spectrum of GUT-scale particles that modifies the unification conditions) because the $SO(10)$ gauge coupling becomes non-perturbative immediately above the thresholds of the components of rank-5 $126 + \overline{126}$ fields. Although one can increase the soft SUSY breaking scale to suppress dimension-5 proton decays, the higher the SUSY particle masses, the more the naturalness of the electroweak scale is lost. In this situation, it is worth recalling that it is the fundamental Yukawa couplings $\tilde{Y}_{10}, \tilde{Y}_{126}, \tilde{Y}_{120}$ that determine the coefficients of the dimension-5 operators. There may be a texture of the fundamental Yukawa couplings that suppresses dimension-5 proton decays and at the same time reproduces the correct quark and lepton Yukawa couplings and neutrino mass matrix. Specifically, as the up quark Yukawa coupling is a specially small Yukawa coupling in the minimal SUSY Standard Model (MSSM) with $\tan \beta \sim 50$, if those components of the
Yukawa matrices $\tilde{Y}_{10}, \tilde{Y}_{126}, \tilde{Y}_{120}$ responsible for dimension-5 proton decays are related to the up quark Yukawa coupling, then dimension-5 proton decays are maximally suppressed. The above idea has been sought for in Refs. [37, 38] based on the model that includes single $10$, single $\mathbf{126}$ and single $120$ fields [17, 18, 19].

In this paper, we perform a numerical search for such a texture in the model that includes single $10$, single $\mathbf{126}$ and single $120$ fields, by the following steps. First, we spot those components of the Yukawa matrices $Y_{10}, Y_{126}, Y_{120}$ (proportional to $\tilde{Y}_{10}, \tilde{Y}_{126}, \tilde{Y}_{120}$) which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. Next, we numerically fit the experimental data on the quark and lepton masses, CKM and PMNS mixing matrices and neutrino mass differences in terms of $Y_{10}, Y_{126}, Y_{120}$, and meanwhile we minimize the components of $Y_{10}, Y_{126}, Y_{120}$ spotted above. In this way, we numerically discover a texture of the fundamental Yukawa couplings that suppresses dimension-5 proton decays and reproduces the correct fermion data. We further discuss implications of the texture on unknown neutrino parameters, in particular the Dirac CP phase of PMNS matrix, $\delta_{\text{pmns}}$, the lightest neutrino mass, $m_1$, and the $(1,1)$-component of the neutrino mass matrix in the charged lepton basis, $m_{ee}$, that regulates the neutrinoless double beta decay.

The present paper focuses on the case when the active neutrino mass is dominated by the Type-2 seesaw contribution coming from the tiny vacuum expectation value (VEV) of $\mathbf{126}$ field, whereas the Type-1 seesaw contribution resulting from integrating out right-handed neutrinos is assumed subdominant. In this case, the neutrino mass matrix is directly proportional to $Y_{126}$ and a close connection between the suppression of dimension-5 proton decays and the neutrino parameters is expected.

This paper is organized as follows: In Section 2 we review the renormalizable SUSY $SO(10)$ GUT model where the electroweak-symmetry-breaking Higgs field originates from single $10$, single $\mathbf{126}$ and single $120$ fields. We also re-derive the dimension-5 proton decay partial widths, and clarify the relation between the dimension-5 proton decays and the Yukawa couplings $Y_{10}, Y_{126}, Y_{120}$. In Section 3 we spot those components of the Yukawa matrices $Y_{10}, Y_{126}, Y_{120}$ which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. In Section 4 we perform a numerical search for a texture of $Y_{10}, Y_{126}, Y_{120}$ that suppresses dimension-5 proton decays and at the same time reproduces the correct fermion data, and discuss a connection between the suppression of dimension-5 proton decays and the neutrino parameters. Section 5 summarizes the paper.
2 Renormalizable SUSY $SO(10)$ GUT

We consider a SUSY $SO(10)$ GUT model that contains fields in $10$, $126$, $\overline{126}$, $120$ representations, denoted by $H$, $\Delta$, $\overline{\Delta}$, $\Sigma$, and three matter fields in $16$ representation, denoted by $\Psi_i$ ($i = 1, 2, 3$ is the flavor index). The model also contains fields in $210$, $45$, $54$ representations, denoted by $\Phi$, $A$, $E$, which are responsible for breaking $SU(5)$ subgroup of $SO(10)$. The most general renormalizable Yukawa couplings are given by

$$W_{\text{Yukawa}} = (\tilde{Y}_{10})_{ij} \Psi_i H \Psi_j + (\tilde{Y}_{126})_{ij} \Psi_i \overline{\Delta} \Psi_j + (\tilde{Y}_{120})_{ij} \Psi_i \Sigma \Psi_j$$

where $\tilde{Y}_{10}$ and $\tilde{Y}_{126}$ are $3 \times 3$ complex symmetric matrices and $\tilde{Y}_{120}$ is a $3 \times 3$ complex antisymmetric matrix. The electroweak-breaking-Higgs fields of the Minimal SUSY Standard Model (MSSM), $H_u, H_d$, are linear combinations of $(1, 2, \pm \frac{1}{2})$ components of $H$, $\Delta$, $\overline{\Delta}$, $\Sigma$, $\Phi$. Accordingly, the Yukawa coupling for up-type quarks, $Y_u$, that for down-type quarks, $Y_d$, and that for charged leptons, $Y_e$, and the Dirac Yukawa coupling for neutrinos, $Y_D$, are derived as

$$W_{\text{Yukawa}} \supset (Y_u)_{ij} Q_i H_u U_c^i + (Y_d)_{ij} Q_i H_d D_c^i + (Y_e)_{ij} L_i H_d E_c^i + (Y_D)_{ij} L_i H_u N_c^i$$

where $Y_u$, $Y_d$, $Y_e$, $Y_D$ are given by

$$Y_u = Y_{10} + r_2 Y_{126} + r_3 Y_{120},$$

$$Y_d = r_1 (Y_{10} + Y_{126} + Y_{120}),$$

$$Y_e = r_1 (Y_{10} - 3 Y_{126} + r_e Y_{120}),$$

$$Y_D = Y_{10} - 3r_2 Y_{126} + r_D Y_{120}$$

at a $SO(10)$ breaking scale. Here $Y_{10} \propto \bar{Y}_{10}, Y_{126} \propto \bar{Y}_{126}, Y_{120} \propto \bar{Y}_{120}$ and $r_1, r_2, r_3, r_e, r_D$ are numbers. By a phase redefinition, we take $r_1$ to be real positive.

Majorana mass for the right-handed neutrinos is obtained as $(Y_{126})_{ij} \overline{\overline{\Delta}} N_i^c N_j^c$ where $\overline{\overline{\Delta}}$ denotes $\overline{\overline{\Delta}}$'s VEV. Integrating out $N_i^c$ yields an effective operator $L_i H_u L_j H_u$, which we call the Type-1 seesaw contribution. Additionally, the $(1, 3, 1)$ component of $\overline{\overline{\Delta}}$ mixes with that of $E$ after $SO(10)$ breaking. Integrating out the $(1, 3, 1)$ components yields an effective operator $L_i H_u L_j H_u$, which we call the Type-2 seesaw contribution. This paper centers on the case where the Type-2 seesaw contribution dominates over the Type-1 one, in which case the Wilson coefficient of the Weinberg operator $(C_\nu)_{ij} L_i H_u L_j H_u$ satisfies

$$(C_\nu)_{ij} \propto (Y_{126})_{ij}$$

at a $SO(10)$ breaking scale.
$H, \Delta, \tilde{\Delta}, \Sigma, \Phi$ contain pairs of $(3, 1, -\frac{1}{2})$ and $(\bar{3}, 1, \frac{1}{2})$ components, which we call ‘colored Higgs fields’ and denote by $H_C^A, \overline{H}_C^B$ ($A, B$ are labels), respectively. Exchange of $H_C^A, \overline{H}_C^B$ gives rise to dimension-5 operators inducing a proton decay. Those couplings of $H_C^A, \overline{H}_C^B$ which contribute to such operators are

$$W_{\text{Yukawa}} \supset \sum_A \left[ \frac{1}{2} (Y_L^A)_{ij} Q_i H_C^A Q_j + (Y_L^A)_{ij} Q_i \overline{H}_C^A L_j + (Y_R^A)_{ij} E_i^c H_C^A U_j^c + (Y_R^A)_{ij} U_i^c \overline{H}_C^A D_j^c \right] .$$ (8)

where $Y_L^A, Y_R^A, \overline{Y}_R^A$ are proportional to $Y_{10}, Y_{126}$ or $Y_{120}$, and $Y_L^A$ are proportional to $Y_{10}$ or $Y_{126}$. After integrating out $H_C^A, \overline{H}_C^B$, we get effective dimension-5 operators contributing to proton decay,

$$-W_5 = \frac{1}{2} C^{ijkl}_{5L} (Q_k Q_l) (Q_i L_j) + C^{ijkl}_{5R} E_k^c U_i^c U_i^c D_j^c$$ (9)

(in the first term, isospin indices are summed in each bracket) where

$$C^{ijkl}_{5L} (\mu = \mu_{H_C}) = \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_L^A)_{kl} (\overline{Y}_L^B)_{ij} - \frac{1}{2} (Y_L^A)_{il} (\overline{Y}_L^B)_{kj} - \frac{1}{2} (Y_L^A)_{ik} (\overline{Y}_L^B)_{lj} \right\} \bigg|_{\mu = \mu_{H_C}} .$$ (10)

$$C^{ijkl}_{5R} (\mu = \mu_{H_C}) = \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{kl} (\overline{Y}_R^B)_{ij} - (Y_R^A)_{il} (\overline{Y}_R^B)_{kj} \right\} \bigg|_{\mu = \mu_{H_C}} .$$ (11)

and $\mathcal{M}_{H_C}$ denotes the mass matrix for $H_C^A, \overline{H}_C^B$ and $\mu_{H_C}$ is taken around the eigenvalues of $\mathcal{M}_{H_C}$.

We concentrate on the $(Q_k Q_l) (Q_i L_j)$ operators’ contributions to the $p \to K^+ \bar{\nu}_\alpha (\alpha = e, \mu, \tau)$ and $p \to K^0 e^+_\beta (e_\beta = e, \mu)$ decays and the $E_k^c U_i^c U_i^c D_j^c$ operators’ contribution to the $p \to K^+ \bar{\nu}_\tau$ decay. For other decay modes, the $(Q_k Q_l) (Q_i L_j)$ operators’ contributions to the $N \to \pi e^+_\beta$ and $p \to \eta e^+_\beta$ decays are suppressed in the same texture that suppresses the above contributions as we comment in Section 3. The rest of the decay modes are bounded only weakly [39] and so we do not discuss them in this paper.

The contribution of the $C^{ijkl}_{5L} (Q_k Q_l) (Q_i L_j)$ term to the $p \to K^+ \bar{\nu}_\alpha (\alpha = e, \mu, \tau)$ decays is given by

$$\Gamma(p \to K^+ \bar{\nu}_\alpha) \big|_{\text{from } c_{5L}} = C \left| \beta_H (\mu_{\text{had}}) \frac{G_F}{\sqrt{\pi}} \left\{ \left( 1 + \frac{D}{3} + F \right) C_{LL}^{\alpha \mu} (\mu_{\text{had}}) + \frac{2D}{3} C_{LL}^{\mu \alpha} (\mu_{\text{had}}) \right\} \right|^2 .$$ (12)

Here $\alpha_H, \beta_H$ denote hadronic matrix elements, $D, F$ are parameters of the baryon chiral Lagrangian, and $C_{LL}$ are Wilson coefficients of the effective Lagrangian, $-\mathcal{L}_6 \supset C_{LL}^{ijkl} (\psi_{u k} \psi_{d l}) (\psi_{d l} \psi_{u k})$.
where $\psi$ denotes a SM Weyl spinor and spinor index is summed in each bracket. The Wilson coefficients $C_{LL}^{\mu}$ satisfy

$$C_{LL}^{\text{sq}ud}(\mu_{\text{had}}) = A_{LL}^\alpha(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_W}{m_{\tilde{q}}^2} F g_2 \left( C_{5L}^{\text{sq}ud} - C_{5L}^{\text{uads}} \right) |_{\mu = \mu_{\text{SUSY}}},$$

$$C_{LL}^{\text{dus}}(\mu_{\text{had}}) = A_{LL}^\alpha(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_W}{m_{\tilde{q}}^2} F g_2 \left( C_{5L}^{\text{dus}} - C_{5L}^{\text{uads}} \right) |_{\mu = \mu_{\text{SUSY}}},$$

Here $F$ is a loop function factor depending on the SUSY particle mass spectrum. $A_{LL}^\alpha(\mu_{\text{had}}, \mu_{\text{SUSY}})$ accounts for renormalization group (RG) corrections in the evolution from soft SUSY breaking scale $\mu_{\text{SUSY}}$ to a hadronic scale where the values of $\alpha_H, \beta_H$ are reported. Hereafter, we neglect RG corrections involving $u, d, s$ quark Yukawa couplings, and accordingly, quark flavor mixings along the RG evolution are neglected. The Wilson coefficients $C_{5L}$ are related to the colored Higgs Yukawa couplings as

$$C_{5L}^{\text{sq}ud}(\mu_{\text{SUSY}}) - C_{5L}^{\text{uads}}(\mu_{\text{SUSY}}) = A_{L}^\alpha(\mu_{\text{SUSY}}, \mu_{\text{HC}}) \sum_{A,B} (M_{H_C})^{-1}_{AB} \frac{3}{2} \left\{ (Y_A^L)^{ud} (Y_L^B)_{s\alpha} - (Y_A^L)_{ds} (Y_L^B)^{u\alpha} \right\} |_{\mu = \mu_{\text{HC}}},$$

$$C_{5L}^{\text{dus}}(\mu_{\text{SUSY}}) - C_{5L}^{\text{uads}}(\mu_{\text{SUSY}}) = A_{L}^\alpha(\mu_{\text{SUSY}}, \mu_{\text{HC}}) \sum_{A,B} (M_{H_C})^{-1}_{AB} \frac{3}{2} \left\{ (Y_A^L)^{us} (Y_L^B)_{d\alpha} - (Y_A^L)_{ds} (Y_L^B)^{u\alpha} \right\} |_{\mu = \mu_{\text{HC}}},$$

where $A_{L}^\alpha(\mu_{\text{SUSY}}, \mu_{\text{HC}})$ accounts for RG corrections in the evolution from $\mu_{\text{HC}}$ to $\mu_{\text{SUSY}}$.

The contribution of the $C_{5L}^{ijkl}(Q_k Q_l)(Q_i L_j)$ term to the $p \to K^0 c^+ \beta^-$ ($e^+ = e, \mu$) decays is given by

$$\Gamma(p \to K^0 c^+ \beta^-) = C \left| \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} (1 - D + F) C_{LL}^{\text{dus}}(\mu_{\text{had}}) \right|^2.$$  (17)

Here $C_{LL}$ are Wilson coefficients of the effective Lagrangian, $-\mathcal{L}_6 \supset C_{LL}^{ijkl}(\psi_{uLk}^\dagger \psi_{dLi})(\psi_{uLi}^\dagger \psi_{dLk})$, which satisfy

$$C_{LL}^{\text{dus}}(\mu_{\text{had}}) = A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_W}{m_{\tilde{q}}^2} F g_2 \left( -C_{5L}^{\text{dus}} + C_{5L}^{\text{uads}} \right) |_{\mu = \mu_{\text{SUSY}}},$$

1 When writing $C_{5L}^{\text{sq}ud}$, we mean that $Q_i$ is in the flavor basis where the down-type quark Yukawa coupling is diagonal and that the down-type quark component of $Q_i$ is exactly $s$ quark (the up-type quark component of $Q_i$ is a mixture of $u, c, t$). Likewise, $Q_k$ is in the flavor basis where the down-type quark Yukawa coupling is diagonal and its down-type component is exactly $d$ quark, and $Q_l$ is in the flavor basis where the up-type quark Yukawa coupling is diagonal and its up-type quark component is exactly $u$ quark. The same rule applies to $C_{5L}^{\text{dus}}$ and others.
where $A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}})$ accounts for RG corrections. The Wilson coefficients $C_{5L}$ are related to the colored Higgs Yukawa couplings as

$$C_{5L}^{u\beta u} (\mu_{\text{SUSY}}) - C_{5L}^{s\beta u} (\mu_{\text{SUSY}}) = A_L(\mu_{\text{SUSY}}, \mu_H) \sum_{A,B} (M^{-1}_{H_C})_{AB} \left\{ \frac{3}{2} \left( (Y^A_L)_{us}(Y^B_L)_{u\beta} - (Y^A_L)_{uu}(Y^B_L)_{s\beta} \right) \right\}_{\mu=\mu_H},$$

where $A_L(\mu_{\text{SUSY}}, \mu_H)$ accounts for RG corrections.

The contribution of the $C_{5R}^{ijkl} E_k^r U_i^c U_j^c D_j^c$ term to the $p \rightarrow K^+ \bar{\nu}_\tau$ decay is given by

$$\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)|_{\text{from } C_{5R}} = C \left| \alpha_H(\mu_{\text{had}}) \frac{1}{f_\pi} \left\{ \left(1 + \frac{D}{3} + F\right) C_{5R}^{u\beta u} (\mu_{\text{had}}) + \frac{2D}{3} C_{5R}^{s\beta u} (\mu_{\text{had}}) \right\}^2. $$

Here $C_{5R}$ are Wilson coefficients of the effective Lagrangian, $-\mathcal{L}_6 \supset C_{5R}^{ijkl}(\bar{\psi}_{\nu_{lR}} \psi_{d_{Ri}})(\bar{\psi}_{u_{cR}} \psi_{c_{Rj}})$, which satisfy

$$C_{5R}^{u\beta u}(\mu_{\text{had}}) = A_{5R}^{u\beta u}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{m_{\tau_R}}{m_{\tau_R}} F^r (V_{\text{ckm}})_{r\gamma} y_{\tau_R} C_{5R}^{u\beta u} (\mu_{\text{had}}) \Big|_{\mu=\mu_{\text{SUSY}}};$$

$$C_{5R}^{s\beta u}(\mu_{\text{had}}) = A_{5R}^{s\beta u}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{m_{\tau_R}}{m_{\tau_R}} F^r (V_{\text{ckm}})_{r\gamma} y_{\tau_R} C_{5R}^{s\beta u} (\mu_{\text{had}}) \Big|_{\mu=\mu_{\text{SUSY}}};$$

where $V_{ij}^{\text{ckm}}$ denotes $(i,j)$-component of CKM matrix. $F^r$ is another loop function factor depending on the SUSY particle mass spectrum. $A_{5R}^{r}(\mu_{\text{had}}, \mu_{\text{SUSY}})$ accounts for RG corrections.

The Wilson coefficients $C_{5R}$ are related to the colored Higgs Yukawa couplings as

$$C_{5R}^{u\beta u}(\mu_{\text{SUSY}}) = A_{5R}^{u\beta u}(\mu_{\text{SUSY}}, \mu_H) \sum_{A,B} (M^{-1}_{H_C})_{AB} \left\{ (Y^A_R)_{\tau t} (Y^B_R)_{u\beta} - (Y^A_R)_{uu} (Y^B_R)_{s\beta} \right\}_{\mu=\mu_H},$$

$$C_{5R}^{s\beta u}(\mu_{\text{SUSY}}) = A_{5R}^{s\beta u}(\mu_{\text{SUSY}}, \mu_H) \sum_{A,B} (M^{-1}_{H_C})_{AB} \left\{ (Y^A_R)_{\tau t} (Y^B_R)_{us} - (Y^A_R)_{uu} (Y^B_R)_{ls} \right\}_{\mu=\mu_H},$$

where $A_{5R}^{r}(\mu_{\text{SUSY}}, \mu_H)$ accounts for RG corrections.

The flavor-dependent terms in Eqs. (15), (16), (19), (23), (24) are related to the fundamental Yukawa couplings $Y_{10}, Y_{120}, Y_{120}$ as follows. Since $Y_{10}^A$ is defined as $W_{\text{Yukawa}} \supset (Y_{10}^A)_{ij} Q_i H_C^A Q_j$, its flavor indices are symmetric and thus $Y_{10}^A$ is not proportional to $Y_{120}$. Therefore, we can

\footnote[2]{\textit{g}_t, \textit{y}_\tau$ in Eqs. (21), (22) are Yukawa couplings of MSSM and so already include the factors of $1/\sin\beta$ and $1/\cos\beta$, respectively.}
write without loss of generality ($\alpha = e, \mu, \tau$ and $\beta = e, \mu$)

\[
\sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y^A_L)_{ud}(\mathcal{W}^B_L)_{s\alpha} - (Y^A_L)_{ds}(\mathcal{W}^B_L)_{u\alpha} \right\}
\]

\[
= \frac{1}{M_{H_C}} \left[ a \left\{ (Y_{10})_{uLd_L}(Y_{10})_{s_L\alpha_L} - (Y_{10})_{d_Ls_L}(Y_{10})_{u_L\alpha_L} \right\} + b \left\{ (Y_{10})_{uLd_L}(Y_{126})_{s_L\alpha_L} - (Y_{10})_{d_Ls_L}(Y_{126})_{u_L\alpha_L} \right\} \\
+ c \left\{ (Y_{10})_{uLd_L}(Y_{120})_{s_L\alpha_L} - (Y_{10})_{d_Ls_L}(Y_{120})_{u_L\alpha_L} \right\} + d \left\{ (Y_{126})_{uLd_L}(Y_{10})_{s_L\alpha_L} - (Y_{126})_{d_Ls_L}(Y_{10})_{u_L\alpha_L} \right\} \\
+ e \left\{ (Y_{126})_{uLd_L}(Y_{120})_{s_L\alpha_L} - (Y_{126})_{d_Ls_L}(Y_{120})_{u_L\alpha_L} \right\} \right] ,
\]  
(25)

\[
\sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y^A_L)_{us}(\mathcal{W}^B_L)_{d\alpha} - (Y^A_L)_{ds}(\mathcal{W}^B_L)_{u\alpha} \right\} = (\text{Above expression with exchange } d_L \leftrightarrow s_L),
\]

\[
\sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y^A_L)_{us}(\mathcal{W}^B_L)_{u\beta} - (Y^A_L)_{us}(\mathcal{W}^B_L)_{s\beta} \right\}
\]

\[
= \frac{1}{M_{H_C}} \left[ a \left\{ (Y_{10})_{uLs_L}(Y_{10})_{u_L\beta_L} - (Y_{10})_{u_Lu_L}(Y_{10})_{s_L\beta_L} \right\} + b \left\{ (Y_{10})_{uLs_L}(Y_{126})_{u_L\beta_L} - (Y_{10})_{u_Lu_L}(Y_{126})_{s_L\beta_L} \right\} \\
+ c \left\{ (Y_{10})_{uLs_L}(Y_{120})_{u_L\beta_L} - (Y_{10})_{u_Lu_L}(Y_{120})_{s_L\beta_L} \right\} + d \left\{ (Y_{126})_{uLs_L}(Y_{10})_{u_L\beta_L} - (Y_{126})_{u_Lu_L}(Y_{10})_{s_L\beta_L} \right\} \\
+ e \left\{ (Y_{126})_{uLs_L}(Y_{120})_{u_L\beta_L} - (Y_{126})_{u_Lu_L}(Y_{120})_{s_L\beta_L} \right\} \right] ,
\]  
(27)

where $M_{H_C}$ denotes a typical value of the eigenvalues of $\mathcal{M}_{H_C}$, and $a, b, c, d, e, f, g, h, j$ are numbers determined from the colored Higgs mass matrix $^{10-15}$. Here $(Y_{10})_{uLd_L}$ denotes the $(1, 1)$-component of $Y_{10}$ in the term $(Y_{10})_{ij} \Psi_i H \Psi_j$ in the flavor basis where the left-handed up-type quark component of $\Psi_i$ has the diagonalized up-type quark Yukawa coupling, and the left-handed down-type quark component of $\Psi_j$ has the diagonalized down-type quark Yukawa coupling. $(Y_{10})_{d_Ls_L}, (Y_{126})_{u_Ld_L}$ and others are defined analogously. Since each of $Y^A_R, \overline{Y}^A_R$ is
proportional to $Y_{10}$, $Y_{126}$ or $Y_{120}$, we can write

$$\sum_{A,B} (\mathcal{M}_{HC}^{-1})_{AB} \left\{ (Y^A_R \tau_t (\overline{Y}^B_R)_{ud} - (Y^A_R \tau_u (\overline{Y}^B_R)_{td}) \right\}$$

$$= \frac{1}{M_{HC}} \left[ a \{(Y_{10})_{\tau t R} (Y_{10})_{u R d R} - (Y_{10})_{\tau R u R} (Y_{10})_{t R d R}\} + b \{(Y_{10})_{\tau R t R} (Y_{126})_{u R d R} - (Y_{10})_{\tau R u R} (Y_{120})_{t R d R}\} 
+ c \{(Y_{10})_{\tau R t R} (Y_{120})_{u R d R} - (Y_{10})_{\tau R u R} (Y_{120})_{t R d R}\} 
+ d \{(Y_{126})_{\tau R t R} (Y_{10})_{u R d R} - (Y_{126})_{\tau R u R} (Y_{10})_{t R d R}\} + e \{(Y_{126})_{\tau R t R} (Y_{126})_{u R d R} - (Y_{126})_{\tau R u R} (Y_{126})_{t R d R}\} 
+ f \{(Y_{126})_{\tau R t R} (Y_{120})_{u R d R} - (Y_{126})_{\tau R u R} (Y_{120})_{t R d R}\} 
+ g \{(Y_{120})_{\tau R t R} (Y_{120})_{u R d R} - (Y_{120})_{\tau R u R} (Y_{120})_{t R d R}\} \right],$$

(28)

$$\sum_{A,B} (\mathcal{M}_{HC}^{-1})_{AB} \left\{ (Y^A_R \tau t (\overline{Y}^B_R)_{us} - (Y^A_R \tau u (\overline{Y}^B_R)_{ts}) \right\} = (\text{Above expression with replacement } d_R \rightarrow s_R),$$

(29)

where $a, b, c, d, e, f, g, h, j$ are the same numbers as those in Eqs. (25)-(27).

3 Components of the Yukawa matrices that can be reduced

We spot those components of the Yukawa matrices $Y_{10}, Y_{126}, Y_{120}$ which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. Specifically, we attempt to reduce the pair-products of the components of $Y_{10}, Y_{126}, Y_{120}$ that appear in Eqs. (25)-(27) (e.g. $(Y_{10})_{\tau R t R} (Y_{10})_{u R d R}$) to the order of the up quark Yukawa coupling times the top quark Yukawa coupling $O(y_u y_t)$. As a matter of fact, some pair-products cannot simultaneously be reduced because of the requirement that $Y_{10}, Y_{126}, Y_{120}$ reproduce the correct quark and lepton Yukawa couplings. In this circumstance, we tune the colored Higgs mass matrix such that coefficients $a, b, c, d, e, f, g, h, j$ in Eqs. (25)-(29) realize cancellations among the problematic pair-products. Finally, we present “those components of the Yukawa matrices $Y_{10}, Y_{126}, Y_{120}$ that can be reduced” as well as an example of the colored Higgs mass matrix that gives coefficients $a, b, c, d, e, f, g, h, j$ that realize the above-mentioned cancellations.

- Focus on Eq. (28). We have $(Y_{10})_{\tau R t R} + \tau_2 (Y_{126})_{\tau R t R} + \tau_3 (Y_{120})_{\tau R t R} = y_t \times (\text{mixing angle between } t_L \text{ and } \tau_R \text{ components})$, and since $t_L$ and $\tau_R$ are both 3rd generation components,
As a matter of fact, it is impossible to simultaneously reduce \( (Y_{10})_{\tau_R t_R} \), \( (Y_{26})_{\tau_R t_R} \) because \( Y_{120} \) is an antisymmetric matrix. Consequently, one or both of \( (Y_{10})_{\tau_R t_R} \) and \( (Y_{26})_{\tau_R t_R} \) are always on the order of the top quark Yukawa coupling \( y_t \). Hence, in order to reduce the Yukawa coupling pair-products in Eq. (28), it is necessary to reduce

\[
(Y_{10})_{u_R d_R}, \quad (Y_{26})_{u_R d_R}, \quad \text{and} \quad (Y_{120})_{u_R d_R}.
\]  

(30)

Eq. (28) also contains terms of the form \( (Y_A)_{\tau_R u_R} (Y_B)_{t_R d_R} \) (\( A, B = 10, 126, 120 \)). They can be estimated to be \( \sin^2 \theta_{13}^\text{ckm} y_t^2 \) (\( \theta_{i j}^\text{ckm} \) denotes the \((i, j)\)-mixing angle of CKM matrix), which is numerically close to \( y_u y_t \). Hence, we do not need to reduce \( (Y_A)_{\tau_R u_R} \) or \( (Y_A)_{t_R d_R} \) further.

- Focus on Eq. (29). For the same reason as above, we have to reduce

\[
(Y_{10})_{u_R s_R}, \quad (Y_{26})_{u_R s_R}, \quad \text{and} \quad (Y_{120})_{u_R s_R}.
\]  

(31)

Eq. (29) also contains terms of the form \( (Y_A)_{\tau_R u_R} (Y_B)_{t_R s_R} \) (\( A, B = 10, 126, 120 \)), which are estimated to be \( \sin \theta_{13}^\text{ckm} \sin \theta_{23}^\text{ckm} y_t^2 \). They contribute to the \( p \to K^+ \nu_\tau \) decay amplitude by a similar amount to the terms \( (Y_A)_{\tau_R u_R} (Y_B)_{t_R d_R} \) in Eq. (28), because these terms enter the decay amplitude in the form \( V_{ts}^\text{ckm} (Y_A)_{\tau_R u_R} (Y_B)_{t_R d_R} + V_{td}^\text{ckm} (Y_A)_{\tau_R u_R} (Y_B)_{t_R s_R} \). CKM matrix satisfies \( |V_{ts}^\text{ckm}| \lesssim \sin \theta_{23}^\text{ckm} \) and \( |V_{td}^\text{ckm}| \sim \sin \theta_{13}^\text{ckm} \). Therefore, we tolerate the terms \( (Y_A)_{\tau_R u_R} (Y_B)_{t_R s_R} \) and do not reduce \( (Y_A)_{\tau_R u_R} \) or \( (Y_A)_{t_R s_R} \) further.

- As a matter of fact, it is impossible to simultaneously reduce \( (Y_{10})_{u_R s_R} \), \( (Y_{26})_{u_R s_R} \) and \( (Y_{120})_{u_R s_R} \) to \( O(y_u) \). This is because Eq. (31) gives

\[
(Y_{10})_{u_R s_R} + (Y_{26})_{u_R s_R} + (Y_{120})_{u_R s_R} = \frac{1}{r_1} (Y_d)_{u_R s_R} \\
\simeq \frac{y_t}{y_b} y_s \times (\text{mixing angle between } s_L \text{ and } u_R),
\]  

(32)

where \( r_1 \) is estimated to be \( y_b/y_t \) so that the top and bottom quark Yukawa couplings are reproduced. The mixing angle between \( s_L \) and \( u_R \) is estimated to be the Cabibbo angle \( \lambda \lesssim 0.22 \) and thus we get \( (Y_{10})_{u_R s_R} + (Y_{26})_{u_R s_R} + (Y_{120})_{u_R s_R} \approx 0.22 \times \frac{y_t}{y_b} y_s \), which is much greater than the up quark Yukawa coupling \( y_u \).

A way out is to adjust the colored Higgs mass matrix such that coefficients \( c, f \) in Eqs. (25)-(29) are zero,

\[
c = f = 0.
\]  

(33)
Then, we are exempted from reducing \((Y_{120})_{u_R s_R}\), because \((Y_{120})_{u_R s_R}\) appears only in the term \((Y_{120})_{\tau_R \tau_R} (Y_{120})_{u_R s_R}\) and the component \((Y_{120})_{\tau_R \tau_R}\) is suppressed because \(Y_{120}\) is an antisymmetric matrix. As a bonus, it is no longer necessary to reduce \((Y_{120})_{u_R d_R}\).

- Focus on Eqs. (25), (26). Since \(\alpha\) ranges in the whole three flavors, it is difficult to reduce \((Y_A)_{s_L} s_L\), \((Y_A)_{d_L} s_L\) and \((Y_A)_{u_L} s_L\) \((A = 10, 126, 120)\) for all \(\alpha\). Hence, we leave these Yukawa couplings untouched and instead reduce \((Y_B)_{u_L} d_L\), \((Y_B)_{u_L} s_L\) and \((Y_B)_{d_L} s_L\) \((B = 10, 126)\) (one side of the Yukawa coupling pair-products).

- Unfortunately, at least one of \((Y_{10})_{u_L} s_L\), \((Y_{12})_{u_L} s_L\), \((Y_{12})_{d_L} s_L\) is on the order of \(V_{\text{ckm}} \frac{y_t}{y_b} y_s\), and consequently, some of the Yukawa coupling pair-products in Eqs. (25), (26) cannot be suppressed to \(O(y_u y_t)\) for all \(\alpha\). This is seen from two equalities,

\[
(Y_{10})_{s_L c_L} + (Y_{12})_{s_L c_L} + (Y_{12})_{s_L c_L} \approx \frac{y_t}{y_b} y_s \times (\text{mixing angle between } c_L \text{ and } s_R),
\]

and

\[
(Y_{10})_{d_L s_L} + (Y_{12})_{d_L s_L} - V_{\text{ckm}} \left\{ (Y_{10})_{u_L s_L} + (Y_{12})_{u_L s_L} \right\} = V_{\text{ckm}} \left\{ (Y_{10})_{c_L s_L} + (Y_{12})_{c_L s_L} \right\} + V_{\text{ckm}} \left\{ (Y_{10})_{t_L s_L} + (Y_{12})_{t_L s_L} \right\}.
\]

Since \(c_L\) and \(s_R\) are both 2nd generation components, the mixing in Eq. (34) is nearly maximal. Also, \((Y_{120})_{s_L c_L}\) is suppressed compared to \((Y_{10})_{c_L s_L}, (Y_{12})_{c_L s_L}\) because \(Y_{120}\) is an antisymmetric matrix. Hence, we have \((Y_{10})_{c_L s_L} + (Y_{12})_{c_L s_L} \approx \frac{y_t}{y_b} y_s (\mu_{HC})\), and from Eq. (35) we conclude that at least one of \((Y_{10})_{u_L s_L}, (Y_{12})_{u_L s_L}, (Y_{10})_{d_L s_L}\) and \((Y_{12})_{d_L s_L}\) is on the order of \(V_{\text{ckm}} \frac{y_t}{y_b} y_s\).

A natural way out is to reduce \((Y_{10})_{u_L} s_L\) and \((Y_{12})_{u_L} s_L\), while tuning coefficients \(a, b, d, e\) such that \(a (Y_{10})_{d_L s_L} + d (Y_{12})_{d_L s_L} = 0\) and \(b (Y_{10})_{d_L s_L} + e (Y_{12})_{d_L s_L} = 0\) hold. This choice is because \((Y_{10})_{u_L} s_L\) and \((Y_{12})_{u_L} s_L\) can more easily be related to the tiny up quark Yukawa coupling.

- Finally, focus on Eq. (27). Since we leave \((Y_A)_{s_L} c_L\) and \((Y_A)_{s_L} u_L\) untouched, we have to reduce \((Y_{10})_{u_L} u_L\) and \((Y_{12})_{u_L} u_L\).

To sum up, in order to suppress dimension-5 proton decays, we have to reduce the following Yukawa couplings:

\[
(Y_{10})_{u_R d_R}, (Y_{12})_{u_R d_R}, (Y_{10})_{u_R s_R}, (Y_{12})_{u_R s_R},
(Y_{10})_{u_L d_L}, (Y_{12})_{u_L d_L}, (Y_{10})_{u_L u_L}, (Y_{12})_{u_L u_L}, (Y_{10})_{u_L s_L}, (Y_{12})_{u_L s_L}
\]

---

3 One might hope that the term \(V_{\text{ckm}}^{\mu} \{ (Y_{10})_{t_L s_L} + (Y_{12})_{t_L s_L} \}\) cancels the term \(V_{\text{ckm}}^{\mu} \{ (Y_{10})_{c_L s_L} + (Y_{12})_{c_L s_L} \}\), but this is not compatible with the correct quark Yukawa couplings.
Meanwhile, we have to adjust the colored Higgs mass matrix such that $c = f = 0$, $a(Y_{10})_{d_{L}d_{L}} + d(Y_{126})_{d_{L}d_{L}} = 0$ and $b(Y_{10})_{d_{L}d_{L}} + e(Y_{126})_{d_{L}d_{L}} = 0$ hold.

We comment on the $N \rightarrow \pi e^+_\beta$ and $p \rightarrow \eta e^+_\beta$ decays. Their decay amplitudes contain terms obtained by replacing $s$ with $d$ in Eq. (27). Therefore, by reducing $(Y_{10})_{u_{L}d_{L}}$, $(Y_{126})_{u_{L}d_{L}}$, $(Y_{10})_{u_{L}u_{L}}$ and $(Y_{126})_{u_{L}u_{L}}$, these decay modes are also suppressed.

We present an example of the colored Higgs mass matrix that realizes $c = f = 0$ and $a/d = b/e$. The latter is a necessity condition for $a(Y_{10})_{d_{L}s_{L}} + d(Y_{126})_{d_{L}s_{L}} = 0$ and $b(Y_{10})_{d_{L}s_{L}} + e(Y_{126})_{d_{L}s_{L}} = 0$.

To study the colored Higgs mass matrix, we have to write the superpotential for $H$, $\Delta$, $\Delta$, $\Sigma$, $\Phi$, $A$, $E$ fields, introduce $SO(10)$-breaking VEVs, and specify the colored Higgs components of the fields. To this end, we use the result of Ref. [42]. The notation for fields is common for our paper and Ref. [42] except that $\Phi^{120}$ field is written as $D$ in Ref. [42]. We define the couplings, coupling constants and masses for the fields according to Eqs. (2),(3) of Ref. [42] (our definition of the coupling constants and masses is reviewed in Appendix). We employ the same notation for the VEVs of $\Delta$, $\Delta$, $\Phi$, $A$, $E$ as Ref. [42], and write the $(3, 1, -\frac{1}{3})$ and $(\bar{3}, 1, \frac{1}{3})$ components as Table 3 of Ref. [42].

Now we present the example of the colored Higgs mass matrix. It satisfies

$$\lambda_{18} = 0, \quad \lambda_{20} = 0, \quad \frac{\lambda_{21}}{\lambda_{19}} = \frac{3}{\lambda_{17}},$$

$$i A_1 = -\frac{1}{6} \frac{\lambda_{21}}{\lambda_{19}} \Phi_3, \quad i A_2 = -\frac{\sqrt{3}}{6} \frac{\lambda_{21}}{\lambda_{19}} \Phi_2, \quad E = 0. \quad (37)$$

The VEV configuration in the second line of Eq. (37) satisfies the $F$-flatness conditions (displayed in Eq. (28) of Ref. [42]) for any values of $\frac{\lambda_{21}}{\lambda_{19}}$, $\Phi_1$, $\Phi_2$, $\Phi_3$, $\tau_R v_R$ if one tunes the parameters $m_1, m_2, m_4, \lambda_1, \lambda_2, \lambda_6, \lambda_9, \lambda_{10}$ appropriately. Given Eq. (37), the colored Higgs mass matrix is

---

4 We have confirmed that the mass matrix of the $(1, 2, \pm \frac{1}{2})$ fields given in Eq. (68) of Ref. [42] is correct. However, we argue that in the colored Higgs mass matrix given in Eq. (69) of Ref. [42], the sign of the term $\frac{2}{5} \sqrt{3} \lambda_7 A_2$ in $m^{(1,1,-\frac{1}{2})}$ should be minus. Otherwise, we have confirmed that Eq. (69) of Ref. [42] is correct. We argue that in the superpotential of the VEVs given in Eq. (27) of Ref. [42], the sign of the term $\lambda_6[A_1(-\frac{1}{2}) + A_2(-\frac{3}{5}v_6)]$ should be flipped. Otherwise, we have confirmed that Eq. (27) of Ref. [42] is correct.
given by

\[ W \supset \left( H^{(3,1,\frac{1}{2})} \Delta^{(3,1,\frac{1}{2})} (6,1,1) \Delta^{(3,1,\frac{1}{2})} (10,1,3) \phi^{(3,1,\frac{1}{2})} \Sigma^{(3,1,\frac{1}{2})} (6,1,3) \Sigma^{(3,1,\frac{1}{2})} (10,1,1) \right) \mathcal{M}_H \]

where

\[ \mathcal{M}_H = \begin{pmatrix}
    m_3 & \frac{m_2}{\sqrt{30}} & -\frac{m_2}{\sqrt{10}} & -\frac{m_2}{\sqrt{30}} & -\sqrt{\frac{2}{15}} \lambda_4 \Phi_3 & \frac{\lambda_5 \Phi_5}{\sqrt{6}} & 0 & 0 \\
    \frac{\lambda_1 \Phi_3}{\sqrt{30}} & m_2 + i \lambda_21 \frac{\lambda_4 \Phi_3}{\lambda_19 \sqrt{30} \sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
    m_2 - i \lambda_21 \frac{\lambda_4 \Phi_3}{\lambda_19 \sqrt{30} \sqrt{2}} & -\frac{m_2}{\sqrt{15}} \lambda_4 \Phi_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
    -2 \frac{\lambda_1 \Phi_3}{\sqrt{6}} & 0 & \frac{\lambda_21 \Phi_3}{\lambda_19 \sqrt{2}} & -\lambda_17 \Phi_2 & 0 & 0 & 0 & 0 \\
    -\lambda_1 \Phi_3 & 0 & \lambda_21 \frac{\Phi_3}{\lambda_19 \sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\
    -\sqrt{\frac{2}{3}} \lambda_1 \Phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    \end{pmatrix}
\]

(38)

\[ m_{66} = m_2 + \lambda_2 \left( \frac{\Phi_1}{10 \sqrt{6}} + \frac{\Phi_2}{30 \sqrt{2}} \right) - i \frac{\lambda_21 \lambda_6 \Phi_2}{\lambda_19 \sqrt{30} \sqrt{2}} \quad (40) \]

\[ m_{77} = m_1 + \lambda_1 \left( \frac{\Phi_1}{\sqrt{6}} + \frac{\Phi_2}{3 \sqrt{2}} + \frac{2 \Phi_3}{3} \right) - i \frac{\lambda_21 \lambda_7 \sqrt{2} \Phi_2}{\lambda_19 \sqrt{15}} \quad (41) \]

\[ m_{22} = m_6 + \frac{1}{3} \sqrt{\frac{2}{3}} \lambda_1 \Phi_1 \quad (42) \]

\[ m_{33} = m_6 + \frac{\sqrt{2}}{9} \lambda_1 \Phi_2 \quad (43) \]

The Wilson coefficients of the terms \( C_{5L}^{ijkl} (Q_i Q_l) (Q_i L_j) \), \( C_{5R}^{ijkl} E_k^e U_i^e U_i^c D_c^e \), which appear after integrating out the colored Higgs fields, are given by

\[ C_{5L}^{ijkl} (\mu = \mu_H) = C_{5R}^{ijkl} (\mu = \mu_H) \]

\[ = \begin{pmatrix}
    (\bar{Y}_{10})_{ij} \\
    (\bar{Y}_{126})_{ij} \\
    0 \\
    0 \\
    0 \\
    (\bar{Y}_{120})_{ij} \\
    (\bar{Y}_{120})_{ij} \\
\end{pmatrix} \mathcal{M}_H^{-1} \]

(44)
First, since the upper-right $5 \times 2$ part of $\mathcal{M}_{HC}$ is zero, the upper-right $5 \times 2$ part of the inverse matrix $\mathcal{M}_{HC}^{-1}$ is also zero. It follows that the terms $(\tilde{Y}_{10})_{kl}(\tilde{Y}_{120})_{ij}$ and $(\tilde{Y}_{126})_{kl}(\tilde{Y}_{120})_{ij}$ do not appear in the Wilson coefficients $C_{5L}^{ijkl}, C_{5R}^{ijkl}$, and hence $c = f = 0$ in Eqs. (25)-(29). Second, the upper-left $5 \times 5$ part of $\mathcal{M}_{HC}^{-1}$ is exactly the inverse matrix of the same part of $\mathcal{M}_{HC}$. It is possible to mathematically prove that the components of $\mathcal{M}_{HC}^{-1}$ satisfy a relation $(\mathcal{M}_{HC}^{-1})_{11} : (\mathcal{M}_{HC}^{-1})_{31} : (\mathcal{M}_{HC}^{-1})_{41} = (\mathcal{M}_{HC}^{-1})_{12} : (\mathcal{M}_{HC}^{-1})_{32} : (\mathcal{M}_{HC}^{-1})_{42}$ when the $(3,2), (4,2)$ and $(5,2)$-components of $\mathcal{M}_{HC}$ are zero as in Eq. (39). Then, since the numbers $a, d$ in Eqs. (28)-(27) are determined by $(\mathcal{M}_{HC}^{-1})_{11}, (\mathcal{M}_{HC}^{-1})_{31}, (\mathcal{M}_{HC}^{-1})_{41}$ and the numbers $b, e$ are determined by $(\mathcal{M}_{HC}^{-1})_{12}, (\mathcal{M}_{HC}^{-1})_{32}, (\mathcal{M}_{HC}^{-1})_{42}$, we get $a/d = b/e$. We comment that if $E \neq 0$, the above relation would be lost and we would in general get $a/d \neq b/e$.

We are yet to prove that Eq. (37) is compatible with the situation that all the fields have \textit{GUT-scale masses} except for one pair of $(1, 2, \pm \frac{1}{2})$ fields that give the MSSM Higgs fields and a $(1, 3, 1)$ field that has mass slightly below the GUT scale to realize the Type-2 seesaw mechanism. Also, Eq. (37) must be consistent with the right value of $a/d$ that realizes $a (Y_{10})_{dLsL} + d (Y_{126})_{dLsL} = 0$, and with the right values of $r_1, r_2, r_3, r_e$ that reproduce the correct fermion data. (Note that common coupling constants enter the colored Higgs mass matrix and the mass matrix of the $(1, 2, \pm \frac{1}{2})$ fields.) We have numerically checked that under the restriction of Eq. (37) and the condition that the mass matrix of the $(1, 2, \pm \frac{1}{2})$ fields have one zero eigenvalue, the ratio of the masses of various fields (other than the pair of $(1, 2, \pm \frac{1}{2})$ fields) and the values of $a/d$, $r_1, r_2, r_3, r_e$ vary in a wide range and there is no correlation among them. It is thus quite likely that the gauge coupling unification is achieved with a help of GUT-scale threshold corrections and the right values of $a/d$ and $r_1, r_2, r_3, r_e$ are obtained even with Eq. (37).

### 4 Numerical search for the texture of $Y_{10}, Y_{126}, Y_{120}$

We search for the texture of the Yukawa couplings $Y_{10}, Y_{126}, Y_{120}$ discussed in Section 3 i.e., the texture which reproduces the correct quark and lepton Yukawa couplings and neutrino mass matrix according to Eqs. (31)-(5), (7) and in which the components of Yukawa couplings $(Y_{10})_{uRdR}, (Y_{126})_{uRdR}, (Y_{10})_{uRsR}, (Y_{126})_{uRsR}, (Y_{10})_{uLdL}, (Y_{126})_{uLdL}, (Y_{10})_{uLuL}, (Y_{126})_{uLuL}, (Y_{10})_{uLsL}, (Y_{126})_{uLsL}$ are reduced.
4.1 Procedures

First, we numerically calculate the MSSM Yukawa coupling matrices $Y_u, Y_d, Y_e$ at scale $\mu = 2 \cdot 10^{16}$ GeV in $\overline{DR}$ scheme, and the flavor-dependent RG correction to the coefficient of the Weinberg operator in the evolution from $\mu = 2 \cdot 10^{16}$ GeV to $\mu = M_Z$, written as $R_{ij}$ and defined as $(C_\nu)_{ij}|_{\mu=M_Z} = \sum_{k,l} R_{ik} R_{jl} (C_\nu)_{kl}|_{\mu=2 \cdot 10^{16}}$ GeV. In the calculation of the RG equations, we assume the following SUSY particle mass spectrum for concreteness:

$$m_\tilde{q} = m_\tilde{\ell} = m_{H^0} = m_{H^\pm} = m_A = 20 \text{ TeV}, \quad M_{\tilde{g}} = M_{\tilde{W}} = \mu_H = 2 \text{ TeV}, \quad \tan \beta = 50. \quad (45)$$

However, we caution that the values of $Y_u, Y_d, Y_e$ at $\mu = 2 \cdot 10^{16}$ GeV and $R_{ij}$ only logarithmically depend on the SUSY particle mass spectrum and so the texture of $Y_{10}, Y_{126}, Y_{120}$ we search is not sensitive to the spectrum; for example, multiplying the spectrum with factor 10 does not change our results. We adopt the following input values for quark masses and CKM matrix parameters:

The isospin-averaged quark mass and strange quark mass in $\overline{MS}$ scheme are obtained from lattice calculations in Refs. [46, 47, 48, 49, 50, 51] as $\frac{1}{2}(m_u + m_d)(2 \text{ GeV}) = 3.373 \pm 0.080$ MeV and $m_s(2 \text{ GeV}) = 92.0 \pm 0.2$ MeV. The up and down quark mass ratio is obtained from an estimate in Ref. [52] as $m_u/m_d = 0.46(3)$. The $\overline{MS}$ charm and bottom quark masses are obtained from QCD sum rule calculations in Ref. [53] as $m_c(3 \text{ GeV}) = 0.986 - 9(\alpha_s^{(5)}(M_Z) - 0.1189)/0.002 \pm 0.010$ GeV and $m_b(m_b) = 4.163 + 7(\alpha_s^{(5)}(M_Z) - 0.1189)/0.002 \pm 0.014$ GeV. The top quark pole mass is obtained from $t\bar{t}$+jet events measured by ATLAS [54] as $M_t = 171.1 \pm 1.2$ GeV. The CKM mixing angles and CP phase are calculated from the Wolfenstein parameters in the latest CKM fitter result [55]. For the QCD and QED gauge couplings, we use $\alpha_s^{(5)}(M_Z) = 0.1181$ and $\alpha^{(5)}(M_Z) = 1/127.95$. For the lepton and W, Z, Higgs pole masses, we use the values in Particle Data Group [39].

The result is given in terms of the singular values of $Y_u, Y_d, Y_e$ and the CKM mixing angles and CP phase at $\mu = 2 \cdot 10^{16}$ GeV, as well as $R_{ij}$ in the flavor basis where $Y_e$ is diagonal ($R_{ij}$ is also diagonal in this basis), tabulated in Table 1. For each singular value of $Y_u, Y_d$, we present $1\sigma$ errors that have propagated from experimental error of the corresponding input quark mass. For the CKM mixing angles and CP phase, we present $1\sigma$ errors that have propagated from experimental errors of the input Wolfenstein parameters.
Table 1: The singular values of MSSM Yukawa couplings $Y_u$, $Y_d$, $Y_e$, and the mixing angles and CP phase of CKM matrix, at $\mu = 2 \cdot 10^{16}$ GeV in $\overline{\text{DR}}$ scheme. Also shown is the flavor-dependent RG correction $R_{ij}$ to the coefficient of the Weinberg operator in the evolution from $\mu = 2 \cdot 10^{16}$ GeV to $\mu = M_Z$, in the flavor basis where $Y_e$ is diagonal ($R_{ij}$ is also diagonal in this basis). For each singular value of the quark Yukawa matrices, we present 1σ error that has propagated from experimental error of the corresponding input quark mass, and for the CKM parameters, we present 1σ errors that have propagated from experimental errors of the input Wolfenstein parameters.

| $Y_u$     | Value with Eq. (45) |
|-----------|---------------------|
| $y_u$     | 2.69(14)×10^{-6}    |
| $y_c$     | 0.001384(14)        |
| $y_t$     | 0.478(98)           |
| $y_d$     | 0.0002908(92)       |
| $y_s$     | 0.00579(13)         |
| $y_b$     | 0.3552(23)          |
| $y_e$     | 0.00012202          |
| $y_\mu$   | 0.025766            |
| $y_\tau$  | 0.50441             |

\[
\text{cos } \theta_{13}^{\text{km}} \quad \text{sin } \theta_{12}^{\text{km}} \\
\text{cos } \theta_{13}^{\text{km}} \quad \text{sin } \theta_{23}^{\text{km}} \\
\delta_{\text{km}} \quad \text{(rad)} \\
\]

Next, we fit the MSSM Yukawa couplings $Y_u, Y_d, Y_e$ and the neutrino mixing angles and mass differences with the fundamental Yukawa couplings $Y_{10}, Y_{126}, Y_{120}$ and the numbers $r_1, r_2, r_3, r_\ell$ according to Eqs. (3)-(5),(7). Meanwhile, we minimize the following quantity:

\[
\sum_{A=10,126} \{ |(Y_A)_{uRdR}|^2 + |(Y_A)_{uRSR}|^2 + |(Y_A)_{uLdL}|^2 + |(Y_A)_{uLsL}|^2 + |(Y_A)_{uLsL}|^2 \} 
\]

To facilitate the analysis, we concentrate on the parameter region where $r_3 = 0$ (which is compatible with any values of $r_1, r_2, r_\ell$ and Eq. (37)). Then, we obtain $(Y_{10})_{uRj} + r_2(Y_{126})_{uRj} \leq y_u$ and $(Y_{10})_{uLj} + r_2(Y_{126})_{uLj} \leq y_u$ for any flavor index $j$, and it becomes easier to reduce Eq. (40) to the order of the tiny up quark Yukawa coupling $y_u$. Given $r_3 = 0$, Eqs. (3)-5 can be rearranged as follows: We fix the flavor basis such that the left-handed down-type quark components in $\Psi_i$ have diagonal $Y_d$ Yukawa coupling with real positive diagonal components.
$Y_u$, which is still symmetric, is then written as

$$Y_u = V_{CKM}^T \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c e^{2i d_2} & 0 \\ 0 & 0 & y_t e^{2i d_3} \end{pmatrix} V_{CKM}$$

(47)

where $d_2, d_3$ are unknown phases. In the same flavor basis, $Y_d$ becomes

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} V_{dR}$$

(48)

where $V_{dR}$ is an unknown unitary matrix. From Eqs. (47), (48) and the fact that $Y_{10}, Y_{126}$ are symmetric and $Y_{120}$ is antisymmetric, we get

$$Y_{126} = \frac{1}{1 - r_2} \left\{ \frac{1}{r_1} \left( Y_d + Y_d^T \right) - Y_u \right\},$$

(49)

$$\frac{1}{r_1} Y_e = Y_u - (3 + r_2) Y_{126} + r_e \frac{1}{r_1} \left( Y_d - Y_d^T \right).$$

(50)

We perform the singular value decomposition of $Y_e$ as

$$Y_e = U_{eL} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} U_{eR}^\dagger,$$

(51)

and calculate the active neutrino mass matrix in the charged-lepton-diagonal basis as

$$(M_\nu)_{\ell\ell'} \propto R_{\ell\ell'} \left( U_{eL}^T Y_{126} U_{eL} \right)_{\ell\ell'} R_{\ell'\ell'}, \quad \ell, \ell' = e, \mu, \tau,$$

(52)

where $\ell, \ell'$ denote flavor indices for the left-handed charged leptons. Utilizing Eqs. (47)-(52), we perform the fitting as follows. We fix $y_u, y_c, y_t$ and CKM matrix by the values in Table 1, while we vary $y_d/r_1, y_s/r_1, y_b/r_1$, unknown phases $d_2, d_3$, unknown unitary matrix $V_{dR}$ and complex numbers $r_2, r_e$. Here we eliminate $r_1$ by requiring that the central value of the electron Yukawa coupling $y_e$ be reproduced. In this way, we try to reproduce the correct values of $y_d, y_s, y_\mu, y_\tau, \theta_{12}^{\text{pmns}}, \theta_{13}^{\text{pmns}}, \theta_{23}^{\text{pmns}}$ and neutrino mass difference ratio $\Delta m_{21}^2/\Delta m_{32}^2$. Specifically, we require $y_d, y_s$ to fit within their respective $3\sigma$ ranges, while we do not constrain $y_b$ because $y_b$ may be subject to sizable GUT-scale threshold corrections. We impose stringent restrictions on the values of neutrino mixing angles and mass differences, because we are primarily interested in the correlation between the neutrino Dirac CP phase and the suppression of dimension-5 proton decays, and so it is essential to suppress variation of the other neutrino parameters. In particular, we require $\sin^2 \theta_{12}^{\text{pmns}}, \sin^2 \theta_{13}^{\text{pmns}}, \Delta m_{21}^2/\Delta m_{32}^2$ to fit within their respective $1\sigma$ ranges.

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Note that $Y_u$ in Eq. (2) is the complex conjugate of $Y_u$ in SM defined as $-\mathcal{L} = \bar{q}_L Y_u u_R H$. 

17
reported by NuFIT 4.1\cite{56,57}. We assume two narrow benchmark ranges of \(\sin^2 \theta_{23}^{\text{pmns}}\), since
the current experimental error of \(\sin^2 \theta_{23}^{\text{pmns}}\) is too large. Only the normal hierarchy of the
neutrino mass is considered because no good fitting is obtained with the inverted hierarchy. Finally, since the experimental errors of \(y_\mu, y_\tau\) are tiny, we only require their reproduced values
to fit within \(\pm 0.1\%\) ranges of their central values. The constraints are summarized in Table 2.

| Allowed range          |
|------------------------|
| \(y_u\)                |
| \(y_c\)                |
| \(y_t\)                |
| \(y_d\)                |
| \(y_s\)                |
| \(y_b\)                |
| \(y_e\)                |
| \(y_\mu\)              |
| \(y_\tau\)             |
| \(\cos \theta_{13}^{\text{ckm}} \sin \theta_{12}^{\text{ckm}}\) |
| \(\cos \theta_{13}^{\text{km}} \sin \theta_{23}^{\text{km}}\) |
| \(\sin \theta_{13}^{\text{km}}\) |
| \(\delta_{\text{km}}\)  |
| \(\sin^2 \theta_{12}^{\text{pmns}}\) |
| \(\sin^2 \theta_{13}^{\text{pmns}}\) |
| \(\sin^2 \theta_{23}^{\text{pmns}}\) |
| \(\Delta m_{21}^2 / \Delta m_{32}^2\) |
| \(\delta_{\text{pmns}}, \alpha_2, \alpha_3, m_1\) |
| \(d_2, d_3, V_{dR}\)   |
| \(r_1\)                |
| \(r_3\)                |
| \(r_2, r_e\)           |

Within the constraints of Table 2, we minimize the quantity Eq. (46) repeatedly starting
from different random values of \(y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, r_e\). Each fitting and
minimization result is plotted on the planes of the neutrino Dirac CP phase \(\delta_{\text{pmns}}\), the lightest
neutrino mass \(m_1\), and the absolute value of the (1,1)-component of the neutrino mass matrix
in the charged-lepton-diagonal basis \(|m_{ee}|\), versus the ”maximal proton decay amplitude”
deﬁned in the next subsection.
4.2 Results

We present the plots of fitting and minimization results obtained by the procedures of Section 4.1 on the planes of $\delta_{\text{pmns}}$, $m_3$, $|m_{ee}|$ versus the "maximal proton decay amplitude". Here "maximal proton decay amplitude" of each decay mode is defined as

$$\tilde{A}(p \to K^+\nu) = \left\{ \tilde{A}(p \to K^+\bar{\nu}_\tau)_{\text{from } C_{5R}} + \tilde{A}(p \to K^+\bar{\nu}_\tau)_{\text{from } C_{5L}} \right\}^2$$

$$+ \tilde{A}(p \to K^+\bar{\nu}_\mu)_{\text{from } C_{5L}} + \tilde{A}(p \to K^+\bar{\nu}_e)_{\text{from } C_{5L}},$$

with sum over $A, B = (10, 10), (10, 126), (126, 10), (126, 126), (120, 10), (120, 126), (120, 120),$ (53)

$$\tilde{A}(p \to K^+\bar{\nu}_\alpha)_{\text{from } C_{5L}} = g_2^2 \sum_{A, B = 10, 126} \left| 1 + \frac{D}{3} + F \right| V_{ts}^{\text{ckm}} \left\{ (Y_A)_{\tau R t R} (Y_B)_{u R d R} - (Y_A)_{\tau R u R} (Y_B)_{t R d R} \right\}$$

$$+ \frac{2D}{3} V_{td}^{\text{ckm}} \left\{ (Y_A)_{\tau R t R} (Y_B)_{u R s R} - (Y_A)_{\tau R u R} (Y_B)_{t R s R} \right\}$$

(54)

$$\tilde{A}(p \to K^0\bar{e}_\beta^+) = g_2^2 \sum_{A, B = 10, 126} \left( 1 - D + F \right) \left| (Y_A)_{u L s L} (Y_B)_{u L \beta L} - (Y_A)_{u L u L} (Y_B)_{s L \beta L} \right|,$$

(55)

where $\alpha = e, \mu, \tau$ and $\beta = e, \mu$, and $y_t, y_\tau, g_2$ here are the top and tau Yukawa couplings and the weak gauge coupling at scale $\mu = 20$ TeV. From the definition above, one sees that the "maximal proton decay amplitude" is proportional to the decay amplitude when coefficients $a, b, d, e, g, h, j$ have the same absolute value and the terms with these coefficients interfere maximally constructively, under the condition of $a (Y_{10})_{d L s L} + d (Y_{126})_{d L s L} = 0$ and $b (Y_{10})_{d L s L} + e (Y_{126})_{d L s L} = 0$. (Remember that we are setting $c = f = 0$.) The square of $\tilde{A}(p \to K^+\nu)$ is proportional to the $p \to K^+\nu$ partial width when the sfermion masses are degenerate, $\mu$-term and the Wino mass are degenerate in magnitude, and $\tilde{A}(p \to K^+\bar{\nu}_\tau)_{\text{from } C_{5R}}$ and $\tilde{A}(p \to K^+\bar{\nu}_\tau)_{\text{from } C_{5L}}$ interfere maximally constructively, provided small difference in RG corrections to the right-handed and left-handed dimension-5 operators is neglected. We consider the "maximal proton decay amplitude" to be a good measure for the suppression of the dimension-5 proton decays. As a matter of fact, we have found that $\tilde{A}(p \to K^0\bar{e}_\beta^+) (\beta = e, \mu)$ are smaller than $\tilde{A}(p \to K^+\nu)$ in all the fitting and minimization results. Considering that the current experimental bound is more severe for the $p \to K^+\nu$ decay than for the $p \to K^0\bar{e}_\beta^+$ decays, it is phenomenologically more important to study the suppression of $\tilde{A}(p \to K^+\nu)$ than that of $\tilde{A}(p \to K^0\bar{e}_\beta^+)$. Therefore, we
present the plots of $\delta_{pmns}, m_1, |m_{ee}|$ versus $\tilde{A}(p \to K^+\nu)$ only, and solely discuss the suppression of $\tilde{A}(p \to K^+\nu)$.

Fig. 1 displays the results with the higher-octant benchmark where $\sin^2 \theta_{pmns}^{23} = 0.55 \pm 0.01$, and Fig. 2 displays those with the lower-octant benchmark where $\sin^2 \theta_{pmns}^{23} = 0.45 \pm 0.01$. In the plots, each dot corresponds to the result of one fitting and minimization analysis starting from a different random set of values of $y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, r_e$. 

Figure 1: Results of the fitting and minimization analysis in Section 4.1, where the quantity Eq. (46) is minimized within the constraints of Table 2. Here we choose the higher-octant benchmark where $\sin^2 \theta_{23}^{\text{pmns}} = 0.55 \pm 0.01$ in Table 2. Each dot corresponds the result of one analysis starting from a different set of random values of $y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, r_e$. From the upper to the lower panel, the horizontal line indicates the neutrino Dirac CP phase $\delta_{\text{pmns}}$, the lightest neutrino mass $m_1$, and the absolute value of the $(1,1)$-component of the neutrino mass matrix in the charged-lepton-diagonal basis $|m_{ee}|$. The vertical line shows the "maximal proton decay amplitude" $\tilde{A}(p \to K^+ \nu)$ defined in Eq. (53).
Figure 2: The same as Fig. 1 except that we choose the lower-octant benchmark where $\sin^2 \theta_{23}^{\text{pmns}} = 0.45 \pm 0.01$ in Table 2.

From the upper panels of Figs. 1-2, we observe that the dimension-5 proton decays are most suppressed for the neutrino Dirac CP phase satisfying $\pi/2 \gtrsim \delta_{\text{pmns}} \gtrsim -\pi/2$. From the middle panels, we find that the dimension-5 proton decays are most suppressed for the lightest neutrino
mass around \( m_1 \simeq 0.003 \) eV. From the lower panels, we see that the dimension-5 proton decays are most suppressed when the (1,1)-component of the neutrino mass matrix in the charged lepton basis satisfies \( |m_{ee}| \lesssim 0.0002 \) eV. The distributions of the fitting and minimization results are qualitatively the same for the higher-octant benchmark with \( \sin^2 \theta_{23}^{\text{pmns}} = 0.55 \pm 0.01 \) and the lower-octant benchmark with \( \sin^2 \theta_{23}^{\text{pmns}} = 0.45 \pm 0.01 \), which suggests that the results do not depend on the precise value of \( \sin^2 \theta_{23}^{\text{pmns}} \). The predicted range of the Dirac CP phase \( \pi/2 \gtrsim \delta_{\text{pmns}} \gtrsim -\pi/2 \) will be confirmed or falsified in long baseline neutrino oscillation experiments. At present, NuFit 5.0 \[58\] reports a slight contradiction between the results of the T2K and the NOvA long baseline experiments on the Dirac CP phase in the normal mass hierarchy case. Therefore, we cannot currently state that the above predicted range is experimentally favored or disfavored. The predicted values of \( m_1 \) and \( |m_{ee}| \), with the normal neutrino mass hierarchy, are beyond the reach of on-going and future cosmological and low-energy experiments.

5 Summary

In the renormalizable SUSY SO(10) GUT model which includes single 10, single 126 and single 120 fields and where the renormalizable terms \( \tilde{Y}_{10} 16 10 16 + \tilde{Y}_{126} 16 126 16 + \tilde{Y}_{120} 16 120 16 \) account for the quark and lepton Yukawa couplings and neutrino mass matrix, we have pursued the possibility that a texture of the fundamental Yukawa couplings \( \tilde{Y}_{10}, \tilde{Y}_{126}, \tilde{Y}_{120} \) suppresses dimension-5 proton decays while reproducing the correct fermion data. Here we have assumed that the active neutrino mass comes mostly from the Type-2 seesaw mechanism. First, we have spotted those components of the Yukawa matrices \( Y_{10}(\propto \tilde{Y}_{10}), Y_{126}(\propto \tilde{Y}_{126}), Y_{120}(\propto \tilde{Y}_{120}) \) which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. Next, we have performed a numerical search for the texture of \( Y_{10}, Y_{126}, Y_{120} \) by fitting the data on the quark and lepton masses, CKM and PMNS matrices and neutrino mass differences and at the same time minimizing the above-spotted components of the Yukawa matrices. We have investigated implications of the texture on unknown neutrino parameters and found that the "maximal proton decay amplitude", which quantifies how much dimension-5 proton decays are suppressed by the Yukawa couplings, is minimized in the region where the neutrino Dirac CP phase satisfies \( \pi/2 \gtrsim \delta_{\text{pmns}} \gtrsim -\pi/2 \), the lightest neutrino mass is around \( m_1 \simeq 0.003 \) eV, and the (1,1)-component of the neutrino mass matrix in the charged lepton basis satisfies \( |m_{ee}| \lesssim 0.0002 \) eV. The above results do not depend on the precise value of \( \theta_{23} \) neutrino mixing angle.

23
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Appendix

We review our definition of the coupling constants and masses for $H, \Delta, \overline{\Delta}, \Sigma, \Phi, A, E$ fields in $10, 126, \overline{126}, 120, 210, 45, 54$ representations, which follows Eq. (2) of Ref. [42]. The couplings are defined in the same way as Eq. (3) of Ref. [42]. Note that $120$ representation field is written as $D$ in Ref. [42], while we write it as $\Sigma$. The coupling constants are defined as

\begin{equation}
W = \frac{1}{2} m_1 \Phi^2 + m_2 \overline{\Delta} \Delta + \frac{1}{2} m_3 H^2 \\
+ \frac{1}{2} m_4 A^2 + \frac{1}{2} m_5 E^2 + \frac{1}{2} m_6 \Sigma^2 \\
+ \lambda_1 \Phi^3 + \lambda_2 \Phi \overline{\Delta} \Delta + (\lambda_3 \Delta + \lambda_4 \overline{\Delta}) H \Phi \\
+ \lambda_5 A^2 \Phi - i \lambda_6 A \overline{\Delta} \Delta + \frac{\lambda_{12}}{120} \varepsilon A \Phi^2 \\
+ E (\lambda_8 E^2 + \lambda_9 A^2 + \lambda_{10} \Phi^2 + \lambda_{11} \Delta^2 + \lambda_{12} \overline{\Delta}^2 + \lambda_{13} H^2) \\
+ \Sigma^2 (\lambda_{14} E + \lambda_{15} \Phi) \\
+ \Sigma \{ \lambda_{16} H A + \lambda_{17} H \Phi + (\lambda_{18} \Delta + \lambda_{19} \overline{\Delta}) A + (\lambda_{20} \Delta + \lambda_{21} \overline{\Delta}) \Phi \} \tag{57}
\end{equation}

where $\varepsilon$ denotes the antisymmetric tensor in $SO(10)$ space.

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