Minimum density of irrigation of the rotor of a film centrifugal heat and mass-exchange apparatus

N H Zinnatullin¹, G N Zinnatullina¹, A I Haibullina², L S Sabitov³, N.F. Kashapov³

¹Kazan National Research Technological University, 68 Karla Marks, street, Kazan, 420015, Russian Federation
²Kazan State Power Engineering University, 51 Krasnoselskaya street, Kazan, 420066, Russian Federation
³Kazan Federal University, 18 Kremlyovskaya street, Kazan, 420008, Russian Federation

haybullina.87@mail.ru

Abstract. For anomalous-viscous centrifugal film based on the energy and moment approaches, equations for the critical fracture radius and minimum irrigation density are obtained. Experimental results are more consistent with data from the moment approach.

1. Introduction

Centrifugal devices, in which the film flow of a liquid is realized, are widely used in chemical technology for carrying out hydromechanical, heat and mass transfer processes [1, 2]. The basis of these devices is the rotor, which is a shaft with nozzles in the form of a flat disk, cone, sphere or other curvilinear bowl. These devices process materials that characterize different rheological properties: viscous liquids, abnormally viscous liquids, viscous-plastic media, etc. [3, 4].

Effective operation of centrifugal film apparatus is possible only if a stable film of liquid is formed. Possible ruptures of the liquid film and excessive irrigation reduce the efficiency of the technological processes, since this either reduces the working surface, or because of the increase in the film thickness, its resistance increases. Therefore, the study of the behavior of a centrifugal liquid film remains relevant from the point of view of determining the minimum irrigation density.

The phenomenon of rupture of a liquid film must also be investigated from the standpoint of dispersion, since it is accompanied by the formation of surface jets and their disintegration into droplets on the edge of the nozzle [5].

Known work on the study of the rupture of a liquid film with an isothermal flow in a gravitational field [6-8]. When studying the rupture of a liquid film, two approaches are used: energetically and forcefully. With the force approach, an equilibrium of the acting forces is formed at the critical point of rupture. In the energy approach, it is assumed that the liquid-gas-solid system chooses such a
situation when the energy minimum of the system is possible. If a bursting flow is energetically more favorable for such a system, a discontinuity of flow will be observed, and vice versa.

The force method did not find application: the calculated critical fracture radius is much smaller than the experimental one; energy analysis of the breakdown of a liquid film for a gravitational field gives a satisfactory agreement between the calculated and experimental data.

To calculate minimum irrigation density \( \Gamma_{\text{min}} \) Hobler offers a formula:

\[
\Gamma_{\text{min}} = \frac{\mu}{4} \left[ \frac{2 \sigma_3 (1 - \cos \Theta_0)}{\rho g} \right]^{2/3} \left[ \left( \frac{\rho^2 g}{\mu^2} \right) \right]^{3/5}
\]  

(1)

where \( \rho, \mu \) — density and dynamic viscosity of a fluid, respectively [kg/m\(^3\)], [Pa·sec]; \( \sigma_3 \) — surface tension, respectively, at the interface of the solid-liquid, solid-gas, liquid-gas [N/m]; \( \Theta_0 \) — wetting angle [rad.].

Hobler’s ideas were developed in Mikilavinan’s works for a horizontal film of liquid moving under the action of a tangential shear stress on a free surface.

Known work on the study of the rupture of a liquid film - the appearance of a ”dry spot” - with non-isothermal flow of the medium [9, 10].

It should be noted that in all these works only rheologically simple media were studied, i.e. only Newtonian fluids. In industrial vehicles, more complex rheological media are mainly processed.

The rupture of a liquid film is obviously a consequence of hydrodynamic instability. However, the theory of hydrodynamic instability does not contain information about the conditions for the occurrence of “dry spots”, streams. It does not take into account a very significant parameter - wetting angle \( \Theta_0 \).

Consider the steady-state isothermal flow of anomalous-viscous fluid flowing in the form of a thin laminar film along a rotating flat nozzle (Fig. 1).

Fig.1. Fluid flow pattern

Orthogonal Coordinate System \( \varphi, \phi, z \) rotates together with the disk, the axis of fluid flow is symmetrical.

The rheological equation of state is represented as a power relation (“power law”), which for a one-dimensional flow has the form:

\[
\tau_{\varphi r} = k \left( \frac{dW_r}{dz} \right)^n
\]

(2)

where \( \tau_{\varphi r} \) — shear stress; \( k \) and \( n \) — rheological fluid constants; \( W_r \) — adial velocity For anomalous-viscous pseudoplastic fluid \( n < 1 \).
In the axial region (shock region), a three-dimensional flow of a liquid occurs, starting with $r_0$, a two-dimensional flow occurs due to centrifugal forces; therefore, the computational region begins with $r > r_0$.

Works [11, 12] are devoted to determining the length of the initial section $r_0$.

Consider the total energy of a liquid film element having a surface area $S_0 = 2\pi r \Delta r$. If we allow the rupture of this element, we will have (Fig. 2):

$$S_0 = S_1 + S_2$$

(3)

where $S_1$ and $S_2$ - respectively wetted and dry areas.

The full energy $E_0$ of element $S_0$ could be written

$$E_0 = E_\kappa + E_\sigma$$

(4)

where $E_\kappa$ and $E_\sigma$ - respectively, the kinematic and surface energy of the liquid film.

![Fig.2. Liquid film rupture pattern](image)

As shown by preliminary calculations for cases where a rupture of a centrifugal liquid film is possible, the flow can be considered one-dimensional [13, 14]. Then, assuming the similarity of the velocity profile in the film continuous and discontinuous flow, the kinematic energy can be represented as:

$$E_\kappa = \int_0^{\delta_0} \frac{W_r^2}{2} \rho \, dr = \int_0^{\delta_0} S_1 \rho \frac{W_r^2}{2} \, d\delta$$

(5)

Taking into account the velocity profile obtained in [14], we obtain:

$$E_\kappa = \frac{\rho S_1 n^2}{(2n+1)(3n+2)} \left( \frac{\rho W_r^2}{K} \right)^{2n/3n+2} \delta_0^n$$

(6)

To simplify the mathematical calculations we introduce dimensionless complexes:

$$\tilde{\delta}_0 = \frac{\delta_0}{R}; \quad \tilde{r} = \frac{r}{R}; \quad \tilde{S}_1 = \frac{S_1}{S_0}; \quad \tilde{S}_2 = 1 - \frac{S_2}{S_0}; \quad \tilde{J} = \frac{J}{K}; \quad \tilde{W}_r = \frac{\rho W_r^2 R^3}{\sigma^3}$$
It is easy to show that in the presence of a film break, the irrigation density will be expressed as:
\[
\Gamma = \rho \cdot \frac{\Delta r}{S_1} \text{ or } \Gamma = \frac{\Gamma_F}{X}, \quad \Gamma_F = \frac{\rho r}{2\pi R}
\]
Hereinafter, the index F will mark the parameters defined without taking into account the rupture of the liquid film. Liquid film thickness $\delta_0$ taking into account the introduced dimensionless complexes will have the form:
\[
\bar{\delta}_0 = \left(\frac{2n+1}{n}\right)^{2n+1} \left(\frac{Z_F}{X}\right)^{n} \left(\frac{\phi}{W_{f, y}}\right)^{n+1}
\]  
(7)

We present equation (6) to a dimensionless form:
\[
\bar{E} = \frac{E_K}{\sigma_3 \cdot S_0} = \left(\frac{n}{3n+2}\right)^n \left(\frac{2n+1}{n}\right)^{2n+1} \cdot W_{f, y}^{-n} \cdot \left(\frac{\phi}{\rho r}\right)^{3n+1} \cdot Z_F^{3n+2} \cdot X^{-\frac{n+1}{2n+1}} + X (1 - \cos \Theta_0 + \frac{\sigma_2}{\sigma_3})
\]  
(8)

The surface energy in the framework of the considered approximation (no adsorption) at the phase interface can be represented as [15]
\[
E_\sigma = S_1 \sigma_3 + S_1 \sigma_1 + S_2 \sigma_2
\]  
(9)

According to Young's law [16], which includes only the thermodynamic properties of the interface, one can write:
\[
\cos \Theta_0 = (\sigma_2 - \sigma_1) / \sigma_3
\]

Then, taking dimensioning into account, we get:
\[
\bar{E}_\sigma = \frac{E_\sigma}{\sigma_3 S_0} = X (1 - \cos \Theta_0) + \frac{\sigma_2}{\sigma_3}
\]  
(10)

Total energy of the element $S_0$ in dimensionless parameters it looks like:
\[
\bar{E}_0 = \left(\frac{n}{3n+2}\right)^n \left(\frac{2n+1}{n}\right)^{2n+1} \cdot W_{f, y}^{-n} \cdot \left(\frac{\phi}{\rho r}\right)^{3n+1} \cdot Z_F^{3n+2} \cdot X^{-\frac{n+1}{2n+1}} + X (1 - \cos \Theta_0) + \frac{\sigma_2}{\sigma_3}
\]  
(11)

Since the condition $\frac{dE_0}{dX} = 0$ for $\frac{d^2E_0}{dX^2} > 0$ corresponds to the minimum energy of the system, then from (11) we have:
\[
-A_1 W_{f, y} \left(\frac{n}{2n+1}\right)^n \left(\frac{\phi}{\rho r}\right)^{3n+1} \cdot Z_F^{3n+2} \cdot X^{\frac{3n+2}{2n+1}} + (1 - \cos \Theta_0) = 0
\]  
(12)

где $A_1 = \left(\frac{n+1}{2n+1}\right)^{n+1} \left(\frac{n}{3n+2}\right) \left(\frac{2n+1}{n}\right)^{2n+1}$. From equation (12) we obtain the value of X:
If $X$ varies from zero to unity, then the flow over the disk is characterized by ruptures of the liquid film, i.e. dry areas $S_2$. Given that the case $X = 1$ corresponds to the beginning of the gap, from equation (13) we obtain the critical radius:

$$r_K = \left[ \frac{1}{2\pi A_2} \right]^{3n+2} \frac{1}{(2n+1)^{3n+2}}$$

where $A_2 = \left[ \frac{2n+1}{n} \right]^{n/2n+1} \left[ \frac{3n+2}{n+1} \right]^{3n+2}$

From the physics of the phenomenon, it follows that the case $X = 1$ with $r = 1$ corresponds to the minimum irrigation density of the rotor apparatus. Then from equation (14) we get

$$F_{\min} = \frac{2\pi}{wR} A_2 \sigma_3 \left( \phi W_{I_y} \right)^{n} \frac{1}{3n+2}$$

Consider another approach for determining the rupture of a centrifugal liquid film. As shown by numerous experiments, usually the initial point of rupture of the film is placed along the nozzle radius to the periphery. At some point, an equilibrium is established and, at the same time, a roller of liquid forms on the border of the dry and wetted areas. This equilibrium state is characterized by equality of not forces, but moments of forces, made up for a roller [Fig. 3].

![Fig. 3. Liquid film rupture pattern](image-url)

The equation of moments for the equilibrium state of the roller, taking into account the surface, centrifugal forces and the force of the velocity head, we obtain:
\[ M_\sigma + M_\omega + M_w = 0 \quad (16) \]

\[ M_\sigma = \sigma_3 \delta_0 \]
\[ M_\omega = - \int_0^{h_0} \rho \delta_0 \omega^2 r dh \]
\[ M_w = - \int_0^{h_0} \rho \frac{W_r^2}{2} d\delta \]

We believe that the roller has a cylindrical surface with a radius \( r_b \), with \( r_b \ll R \) and

\[ h_0 = r_b (1 - \cos \Theta_0) \]

For determining \( M_w \) the velocity profile is used for the case of one-dimensional fluid flow from [14]. As a result, the equation of moments for the roller can be represented as:

\[ \sigma_3 \delta_0 - A_3 \delta_0^2 W_r^2 \left( \frac{w}{4} \right)^2 - A_4 r_b^3 \rho w^2 r = 0 \quad (17) \]

where \( A_3 = \frac{2n^3 + 16n^2 + 5n + 2}{4(3n + 1)(3n + 2)(n + 1)^2} \), \( A_4 = (\sin \Theta_0 - \Theta_0 \cos \Theta_0 - \frac{1}{3} \sin^3 \Theta_0) \)

To determine the roller radius \( r_b \) using the Laplace equation [17], according to which the relationship between the average overpressure under the roller and the curvature of the roller can be represented as:

\[ p_{ac} = \frac{\sigma_3}{r_b} \quad (18) \]

Determine the excess pressure \( p_{ac} \) as the sum of pressures from velocity head and centrifugal force:

\[ p_{ac} = p_{we} + p_{\omega \delta} = \frac{1}{2} \int_0^{h_0} \rho w_r^2 d\delta + \frac{1}{h_0} \int_0^{h_0} \frac{1}{h_0} \int_0^{h_0} \rho b w^2 r db dh \quad (19) \]

Integration of equation (19) with regard to the velocity profile and the roller configuration allows determining the radius of the roller \( r_b \) from (18) in the form:

\[ r_b = A_5 \left( \frac{\sigma_3}{\rho w^2 r} \right)^{1/2} \quad (20) \]

where \( A_5 = \left[ \frac{2(1 - \cos \Theta_0)}{(\Theta_0 - \sin \Theta_0 \cos \Theta_0)} \right]^{1/2} \)

The joint solution of equations (17), (20) and (7) allows to determine the radius of rupture of a liquid film:
\[-r_K = \left[ A_6 \left( \frac{\rho w r}{\sigma_3} \right)^n \left( \frac{w^n K R}{\sigma_3} \right) \frac{1}{W_{ly}^{0.5}} \right]^2 \]  

(21)

In the dimensionless parameters we get:

\[-r_K = A_6^2 (2n)^n J^{2n} \frac{1}{W_{ly}} \]

where \( A_6 = \left( \frac{2n+1}{2\pi n} \right)^n \frac{1}{A_4^{2n+1}} \left( \frac{(\Theta_0 - \sin \Theta_0 \cos \Theta_0)}{2(1-\cos \Theta_0)} \right)^{\frac{3}{2(2n+1)}} \)

Given that the case \( r_K = 1 \) corresponds to the minimum irrigation density from equation (21) we get:

\[ H_{\text{min}} = A_{7} \sigma_3 \frac{1}{\rho^{2n}} \left( \frac{w^2 R}{2n} K^n \right) ^{\frac{1}{2(n+1)}} \]

(22)

where \( A_7 = \left( \frac{2}{2n+1} \right)^2 \cdot A_4 \]

\[ \frac{2n+1}{2n+1} \left( \frac{(\Theta_0 - \sin \Theta_0 \cos \Theta_0)}{2(1-\cos \Theta_0)} \right)^{\frac{3}{2(n+1)}} \]

Let us compare the equations obtained for the discontinuity radius based on the energy and moment methods. The ratio of the right parts of equations (14) and (21) gives:

\[ \frac{r_M}{r_K} = A_8 \left[ J^{3n-2} \frac{1}{W_{ly}} \right]^{\frac{2n+1}{3n+1}} \]

(23)

where \( A_8 = A_7^2 (2\pi)^{2n} \left( \frac{2n+2}{3n+1} \right) \)

Calculations show that \( \frac{r_M}{r_K} \) may be either more or less than one. In experimental conditions, in the range of change of technological parameters in the operating conditions of these devices, the condition \( \frac{r_M}{r_K} < 1 \) hard to reach. Therefore, to analyze the operation of centrifugal film apparatus, we propose the moment method. In this case, the energy method should be looked at as an analysis of the causes of the gap.

We carried out experimental studies on a flat disk for a 90% aqueous solution of glycerol (Fig.4) and for a 2.5% aqueous solution of CMC (Fig.5).
Fig. 4. The dependence of the dimensionless critical radius of the dimensionless complex J: with \( n = 1, K = 1.8 \times 10^{-1} \text{ He}^n \text{ m}^2, \rho = 1.23 \times 10^3 \text{ kg/m}^3, R = 12.5 \times 10^{-2} \text{ m}, \sigma = 48.7 \times 10^{-3} \text{ N/m}, \Theta_0 = 1.223 \text{ rad}, \alpha = \frac{\pi}{2} \text{ rad}, q = (0.55 \div 1.7) \times 10^{-6} \text{ m}^3/\text{sec}, \omega = 30 \div 120 \text{ rad \cdot sec}

Fig. 5. The dependence of the dimensionless critical radius of the flow rate and angular velocity \( \omega \): with \( n = 0.77, K = 1.57 \times 10^{-1} \text{ Nsec}^n \text{ m}^2, \rho = 1.017 \times 10^3 \text{ kg/m}^3, R = 12.5 \times 10^{-2} \text{ m,} \Theta_0 = 1.024 \text{ rad,} \sigma = 53.3 \times 10^{-3} \text{ N/m,} \alpha = \frac{\pi}{2} \text{ rad,} 1 - r_K = f(\omega) \text{ with } q = 1 \times 10^{-6} \text{ m}^3/\text{sec,} 2 - r_K = f(q) \text{ with } \omega = 60 \text{ rad \cdot sec}

The rheological constants \( K \) and \( n \) were determined by the method of two capillaries of different lengths (excluding the initial section) on a constant pressure viscometer, \( \rho \) – densimeter, \( \sigma \) – by the method of ring break, \( \Theta_0 \) - largest lifting fluid when wetting the sample. Experiments were carried out on the setup described in [5]. In each experiment, a significant zone of deviations of the rupture radius was observed (~ 15%). For the experimental adopted their average value.

As can be seen from the graphs, there is a satisfactory agreement between the experimental and calculated data.

The study was carried out by a grant from the Russian Science Foundation (Project No. 18-79-10136).
References

[1] Razinov A And, Klinov A B, Dyakonov G S 2017 Kazan 860 p
[2] Zinnatullin NH, Musin DT, Gimranov FM, Sabirova NK 2004 Kazan 116 p
[3] Davydov AV, Bronkaya VV, Zinnatullin NX 2012 Herald of Kazan Technological University 15 23 pp 145-47
[4] Zinnatullin NH, Antonov VV, Nafikov IM, Bulatov AA 1996 Engineering Physics Journal 69 1 pp 112-17
[5] Zinnatullin N X, Nafikov I M 1984 News of higher educational institutions Ser. Chemistry and Chemical Technology 27 6 pp 722-26
[6] Hartley D E, Murgatroya W 1964 Jnt. J. Heat Mass Transfer 7 9 pp 1003-15
[7] Hobler T 1968 Chemia Stosowana 38 pp 265-74
[8] Mikielewicz J, Moszynski J R 1976 Jnt. J. Heat Mass Transfer 19 7 771-76
[9] Conlson H 1973 Chem. Jug. Tech. 45 6 pp 362-68
[10] Ganiev BG, Bokov AE 1980 Physics Engineering Magazine 39 4 pp 581-91
[11] Charwat A F, Kelly R E, Gazley C 1972 J. Fluid Mech. 53 2 pp 227-58
[12] Zinnatullin NH, Bulatov A.A., Nikolaev.SG 1994 Interuniversity collection: Mass transfer processes and apparatuses of chemical technology pp 97-101
[13] Zinnatullin NH, Vachagin KD, Tyabin NV 1965 Proceedings of the Kazan Institute of Chemical Technology 35 pp 146-53
[14] Zinnatullin NH, Vachagin KD, Tyabin NV 1968 Engineering Physics Journal 15 2 234-240
[15] Summ BA, Goryunov Yu V 1976 Moscow 231 p
[16] Zimon AD 1974 Moscow 195 p
[17] Levin VG 1959 Moscow 699 p