Theory of Flux-Flow Resistivity near $H_{c2}$ for $s$-wave Type-II Superconductors

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This paper presents a microscopic calculation of the flux-flow resistivity $\rho_f$ for $s$-wave type-II superconductors with arbitrary impurity concentrations near the upper critical field $H_{c2}$. It is found that, as the mean free path $l$ becomes longer, $\rho_f$ increases gradually from the dirty-limit result of Thompson [Phys. Rev. B1, 327 (1970)] and Takayama and Ebisawa [Prog. Theor. Phys. 44, 1450 (1970)]. The limiting behaviors suggest that $\rho_f(H)$ at low temperatures may change from convex downward to upward as $l$ increases, thus deviating substantially from the linear dependence $\rho_f \propto H/H_{c2}$ predicted by the Bardeen-Stephen theory [Phys. Rev. 140, A1197 (1965)].

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Kim et al. [1] attributed finite resistivity observed in type-II superconductors to the motion of flux lines, calling it “flux-flow resistivity” $\rho_f$. Whereas the idea has been accepted widely, it still has poor quantitative understanding of the phenomenon. The early phenomenological theories based on single-flux considerations [2, 3, 4] cannot explain the steep decrease of $\rho_f$ observed near $H_{c2}$ [1]. This region, where microscopic calculations may be performed most easily, has been a subject of later theoretical works [5, 6, 7, 8]. We thereby have a quantitative theory on the flux-flow resistivity near $H_{c2}$ [5, 10], but its validity is still restricted to the dirty limit. Reasonable agreements with the theory have been reported later in a couple of experiments on microwave surface resistivity [2, 3] and flux-flow resistivity [4]. However, the latter experiment found a small discrepancy that $\rho_f$ near $T_c$ is larger than the theoretical prediction [2, 3, 4], which was attributed by Larkin and Ovchinnikov [11] to the extra phonon scattering effective at finite temperatures.

With these backgrounds, this paper provides a quantitative theory on the flux-flow resistivity near $H_{c2}$ applicable to arbitrary impurity concentrations. I thereby hope to establish the domain in which the dirty-limit theory [2, 3, 4] is valid, and look into whether the experiments [2, 3] may be explained by impurity scattering alone. Although a similar study was carried out by Ovchinnikov [12], the behaviors of $\rho_f$ at intermediate impurity concentrations have not been clarified explicitly.

I consider the s-wave pairing with an isotropic Fermi surface and the s-wave impurity scattering in an external magnetic field $H \parallel z$. I calculate the complex conductivity $\sigma(\omega)$ below microwave frequencies $\omega$ using the quasiclassical equations of superconductivity [13], and finally put $\omega \to 0$. I use the notation of Ref. [14] but leave the Hall terms out of consideration. Hence I start from the same equations as Eschrig et al. [13] in clarifying the motion of a single flux line within the linear-response regime.

The vector potential without perturbation is given by $A(r) = Br\hat{y} + A(r)$, where $B$ is the average flux density and $\hat{A}$ expresses the spatially varying part of the magnetic field satisfying $\int \nabla \times A \, dr = 0$. The corresponding retarded quasiclassical Green’s functions $f^R$ and $g^R$ can be obtained as Ref. [14] with the replacement of the Matsubara frequency $\epsilon_n$ by $-i\omega$. I adopt the units where the energy, length, magnetic field, and electric field are measured by the zero-temperature energy gap $\Delta(0)$ at $H = 0$, the coherence length $\xi_0 \equiv h\nu_f/\Delta(0)$ with $\nu_f$ the Fermi velocity, $B_0 \equiv \phi_0/2\pi\xi_0^2$ with $\phi_0 \equiv hc/2e$ the flux quantum, and $E_0 \equiv h\nu_f/e\xi_0^2$, respectively. I also put $\hbar = k_B = 1$.

Now, consider the response to a spatially uniform but time-dependent perturbation $\delta A e^{-i\omega t} = \delta E e^{-i\omega t}/i\omega$ with $\delta E \perp H$. A straightforward calculation based on Eq. (71) of Ref. [14] leads to the following equation for the first-order response $\delta f^R = \delta f^R(\epsilon, k, \omega, r)$:

\[
\begin{align*}
& \left[ -2i\omega + \frac{(g^R_+) + (g^R_-)}{2\tau} + \hat{v} \cdot (\nabla - iA) \right] \delta f^R \\
& = f^R_+ f^R_- \hat{v} \cdot \delta E/\omega + (g^R_+ + g^R_-) \delta \Delta \\
& + \left( \Delta + \frac{(f^R_+)}{2\tau} \right) \delta g^R_+ + \left( \Delta + \frac{(f^R_-)}{2\tau} \right) \delta g^R_- \\
& - \frac{f^R_+ \delta g^R_+}{2\tau} + f^R_- \delta g^R_- + \frac{g^R_+ + g^R_-}{2\tau} \delta f^R \\
& \equiv \Delta + \frac{(f^R_+ \delta g^R_+)}{2\tau} + \frac{(f^R_- \delta g^R_-)}{2\tau} + \frac{g^R_+ + g^R_-}{2\tau} \delta f^R .
\end{align*}
\]

(1a)

Here $\tau$ is the relaxation time in the Born approximation, $\langle \cdots \rangle$ denotes the Fermi-surface average satisfying $\langle 1 \rangle = 1$, and $\Delta(r)$ and $\delta \Delta(r, \omega)$ are the pair potential and its first-order response, respectively. The unit vector $\hat{v} = k$ specifies a point on the spherical Fermi surface, $f^R_+ \equiv f^R(\epsilon_+, k, r)$ and $g^R_+ \equiv g^R(\epsilon_+, k, r)$ with $\epsilon_+ \equiv \epsilon_{\pm}\omega/2$, and the superscript $l$ denotes simultaneous operations of complex conjugation and $(\epsilon, k, \omega) \rightarrow (-\epsilon, -k, -\omega)$, e.g., $\delta g^R(\epsilon, k, \omega, r) = \delta g^L(-\epsilon, -k, -\omega, r)$. Finally, the normalization condition [15] enables us to write $\delta g^R$ in terms of $\delta f^R$ as

\[
\delta g^R = -\left( f^R_+ \delta f^R_+ + f^R_- \delta f^R_- \right)/(g^R_+ + g^R_-) .
\]

(1b)

Equation (1b) determines the retarded functions $\delta f^R$ and $\delta g^R$ for given $\delta E$ and $\delta \Delta$. In addition, the advanced functions are obtained directly by using Eq. (72) of Ref. [14] as $\delta f^A(\epsilon, k, \omega, r) = \delta f^R(-\epsilon, -k, \omega, r)$ and $\delta g^A(\epsilon, k, \omega, r) = -\delta g^R(\epsilon, k, -\omega, r)$. 

As for the Keldysh functions, I write them following Eschrig et al. 12 as \( \delta g^F = \delta g^F(\phi_+ - \phi_-) \) and \( \delta f^R = \delta f^R(\phi_+ - \phi_-) + \delta f^R(\phi_+ + \phi_-) \) with \( \phi_\pm = \text{tanh}(\varepsilon_\pm/2T) \). Then a simplified equation results for \( \delta g^\alpha = \delta g^\alpha(\varepsilon, \mathbf{k}, \omega, \mathbf{r}) \) as

\[
\left( -i\varepsilon + \frac{\langle R^R \rangle - \langle g^\Lambda \rangle}{2\tau} + \mathbf{v} \cdot \nabla \right) \delta g^\alpha = (\phi_+ - \phi_-) \left[ \frac{\langle g^R \rangle - \langle g^\Lambda \rangle}{\varepsilon} \mathbf{v} \cdot \mathbf{E}/\omega + f^R_+ \delta \Lambda^\dagger - f^F_+ \delta \Lambda \right]
\]

\[
+ \left( \Delta + \frac{\langle f^R_+ \rangle}{2\tau} \right) \delta f^\dagger + \left( \Delta^\dagger + \frac{\langle f^R_+ \rangle}{2\tau} \right) \delta f^\alpha
\]

\[
- \frac{f^R_+ \langle \delta f^\dagger \rangle + \langle f^R_+ \rangle \langle \delta f^\dagger \rangle + (g^R_+ - g^\Lambda)}{2\tau} \right), \quad (2a)
\]

It also follows from the normalization condition that \( \delta f^\alpha = \delta f^\alpha(\varepsilon, \mathbf{k}, \omega, \mathbf{r}) \) is given in terms of \( \delta g^\alpha \) by

\[
\delta f^\alpha = \left( f^R_+ \delta g^\dagger + f^F_+ \delta g^\alpha \right) \left( g^R_+ - g^\Lambda \right). \quad (2b)
\]

Finally, the pair potential \( \Delta \) is determined by

\[
\delta \Delta = \frac{N(0)V_0}{4i} \int_{-\varepsilon_c}^{\varepsilon_c} d\varepsilon \left\langle \delta f^R(\phi_+ - \phi_-) \right. \right. 
\]

\[
\left. \left. + \delta f^\alpha \right\rangle , \quad (3)
\]

where \( N(0) \) is the density of states per spin at the Fermi level, \( V_0 > 0 \) is the s-wave pairing interaction, and \( \varepsilon_c \) is the cut-off energy. Once the solution to Eqs. (1)-(3) is obtained self-consistently, the transport current \( \delta j = \delta j(\omega, \mathbf{r}) \) is calculated by

\[
\delta j = -\frac{eN(0)V_0}{2} \int_{-\infty}^{\infty} d\varepsilon \left\langle \mathbf{v} \left( \delta g^R(\phi_+ - \delta g^\Lambda(\phi_+ + \delta g^\alpha) \right) \right. \right. 
\]

\[
\left. \left. \right\rangle . \quad (4)
\]

The corrections caused by \( \delta j \) to the magnetic field and the charge density may be neglected safely for the relevant weak-coupling superconductors. Equations (1)-(4) are still exact within the linear-response regime and form a basic starting point to calculate complex conductivity of type-II superconductors at arbitrary magnetic fields.

I now concentrate on the region near \( H_c2 \) and solve Eqs. (1)-(4) by expanding every quantity up to second order in \( \Delta(\mathbf{r}) \) as \( f^R = f^R(1) + f^R(2), f^F = f^F(1) + f^F(2), \mathbf{A} = \tilde{\mathbf{A}}(2), \delta \Delta = \delta \Delta(1) + f^R(2), \delta \Lambda = \delta \Lambda(1) + f^F(2), \) and \( \delta g(2) = \delta g(0) + \delta g(2) \), with \( g^R(0) = 0 \) as seen from Eq. (1). The superscript (1) in \( \delta f^\alpha \) and \( \delta \Lambda(\mathbf{r}) \) will be dropped as it causes no confusions.

The zeroth-order quantity \( \delta g^{(0)} \) is obtained easily from Eq. (2a) as

\[
\delta g^{(0)} = \frac{2\tau}{1 - i\omega\tau} \frac{\phi_+ - \phi_-}{\omega} \mathbf{v} \cdot \mathbf{E}. \quad (5)
\]

When put into Eq. (1), this expression yields the normal-state Drude conductivity \( \sigma_n \), as it should.

Next, Eq. (1) is simplified for the first-order \( \delta f^R \) into

\[
\left[ -i\varepsilon + \frac{1}{2\tau} + \sqrt{B} \sin \frac{\theta}{2} \left( e^{-i\varphi_\alpha} - e^{i\varphi_\alpha} \right) \right] \delta f^R
\]

\[
= \frac{f^R_+ + f^F_+}{2\tau} \frac{\mathbf{v} \cdot \mathbf{E}}{\omega} + \delta \Delta + \frac{\langle \delta f^R \rangle}{2\tau}, \quad (6)
\]

where \( \alpha^\dagger \) and \( \alpha \) are creation and annihilation operators satisfying \( [\alpha, \alpha^\dagger] = 1 \) and \( \langle \theta, \varphi \rangle \) are the polar angles of \( \mathbf{v} \). Equation (6) can be solved with the Landau-level expansion (LLE) method 17 by expanding \( \delta \Delta \) and \( \delta f^R \) in periodic basis functions of the vortex lattice as

\[
\delta \Delta(\omega, \mathbf{r}) = \sqrt{V} \sum_{N=0}^{\infty} \delta \Delta_N(\omega) \psi_{N\mathbf{q}}(\mathbf{r}), \quad (7a)
\]

\[
\delta f^R(\varepsilon, \mathbf{k}, \omega, \mathbf{r}) = \sqrt{V} \sum_{m=-\infty}^{\infty} \sum_{N=0}^{\infty} \delta f^R_{mN}(\varepsilon, \theta, \omega) e^{im\varphi} \psi_{N\mathbf{q}}(\mathbf{r}), \quad (7b)
\]

where \( N \) denotes the Landau level, \( \mathbf{q} \) is an arbitrary chosen magnetic Bloch vector, \( V \) is the volume of the system, and \( \psi_{N\mathbf{q}}(\mathbf{r}) \) satisfies \( a\psi_{N\mathbf{q}} = \sqrt{V} \psi_{N-1\mathbf{q}} \) and \( a^\dagger \psi_{N\mathbf{q}} = \sqrt{V} + 1 \psi_{N+1\mathbf{q}} \). The quantities without perturbations are expanded similarly. Near \( H_c2, \Delta(\mathbf{r}) \) may well be approximated using only the \( N = 0 \) level as \( \Delta(\mathbf{r}) = \sqrt{V} \Delta_0(\varepsilon_0\mathbf{q}) \). The coefficients \( \Delta_0 \) and \( f^R_{mN} \) have already been obtained in Ref. 16 satisfying \( f^R_{mN} = \delta mN\Delta_0 f^R_{mN} \) and \( \Delta(\mathbf{r}) = \sqrt{V} \Delta_0(\varepsilon_0\mathbf{q}) \). It then follows from Eqs. (6) and (8) that \( \delta f^R_{mN} \) and \( \delta \Delta_N \) may be written as

\[
\delta \Delta_N = \delta \Delta_0 \delta \Delta_1, \quad (8a)
\]

\[
\delta f^R_{mN} = \Delta_0 \left( \delta f^R_{mN-1} + \delta f^R_{mN+1} \right), \quad (8b)
\]

Equation (8b) is thereby transformed into \( (\mu = \pm) \)

\[
\sum_{N'} \mathcal{M}_{NN'} \delta f^R_{N'\mu} = \left( f^R_{N+} + f^R_{N-} \right) \delta E_\mu \sin \theta/4\omega
\]

\[
+ \delta_{N0} \delta_{\mu+} \left( \delta \Delta_0 + \langle \delta f^R_{N+} \rangle/2\tau \right), \quad (9)
\]

where \( E_\pm \equiv E_x \pm iE_y, f^R_{N\pm} \equiv f^R_{N}(\varepsilon_0\mathbf{q}) \), and

\[
\mathcal{M}_{NN'} \equiv -i\varepsilon \delta \Delta_{N'} + \beta \sqrt{N+1} \delta \Delta_{N'} - \beta \sqrt{N} \delta \Delta_{N-1}, \quad (10a)
\]

with \( \varepsilon \equiv \varepsilon + i/2\tau \) and \( \beta = \sqrt{B} \sin \theta/2\sqrt{2} \). To solve Eq. (9), let us write the inverse of the matrix \( \mathcal{M} \) as

\[
K^\prime_{NN'} \equiv (\mathcal{M}^{-1})_{NN'}. \quad (10b)
\]

Then \( \delta f^R_{N\mu} \) is obtained formally as

\[
\delta f^R_{N\mu} = K^{R\mu}_{NN'} \delta E_\mu /\omega + \delta_{\mu+} K^{R}_{N} \left( \delta \Delta_0 + \langle \delta f^R_{N+} \rangle/2\tau \right), \quad (10c)
\]

where \( K^{R}_{N} \) is defined by

\[
K^{R}_{N} \equiv \frac{\sin \theta}{4} \sum_{N'} \mathcal{M}_{NN'} \left( f^R_{N+} + f^R_{N-} \right), \quad (10d)
\]

with \( f^R_{N} \) already obtained as

\[
f^R_{N} = K^{R}_{N}/(1 - \langle K^{R}_{0} \rangle/2\tau). \quad (10e)
\]
Taking the average angle of Eq. (10), we obtain
\[ \langle \delta f^{R+} \rangle = \frac{\langle K_1^R \rangle}{1 - (K_1^R)^2} \delta E_+ + \frac{\langle K_1^I \rangle}{1 - (K_1^I)^2} \delta \Delta_1 \] (10f)
Equation (10) determines \( \delta f^{R+}_{N\mu} \) efficiently and fixes the retarded response \( \delta f^{R-} \).

We next consider \( \delta f^a \) and substitute Eqs. (3), (7b), and (8b) into Eq. (2b) with \( g^{R} = -g^{I} = 1 \). Expanding \( \delta f^a \) as Eq. (7b), we find that the coefficient of \( \delta f^a_{mN} \) can also be written as Eq. (8b). The corresponding \( \delta f^a_{N\mu} \) (\( \mu = \pm \)) is obtained easily as
\[ \delta f^a_{N\mu} = K_N^a \frac{\phi_+ - \phi_-}{\omega} \delta E_\mu, \] (11a)
with
\[ K_N^a = -\frac{\tau \sin \theta}{2(1 - i\omega \tau)} \left[ \frac{\langle f^{R+} \rangle}{1} + (-1)^N \frac{\langle f^{R-} \rangle}{1} \right]. \] (11b)

Now we are ready to calculate the pair potential \( \delta \Delta_1 \). Let us substitute the above results for \( \delta f^{R+} \) and \( \delta f^a \) into Eq. (3). Then a straightforward calculation yields the expression for \( \delta \Delta_1 \) defined by Eqs. (7a) and (8a) as
\[ \delta \Delta_1(\omega) = D(\omega) \frac{\delta E_+}{\omega}, \] (12a)
with
\[ D(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \frac{\langle K_1^R(\epsilon, \omega) \rangle}{1 - (K_1^R(\epsilon, \omega))^2} - \frac{\langle K_1^I(\epsilon, \omega) \rangle}{1 - (K_1^I(\epsilon, \omega))^2} \right] \phi(\epsilon - \omega) d\epsilon. \] (12b)

In deriving the formula, use has been made of the symmetries: \( K_N^R(\epsilon) = K_N^R(-\epsilon) \), \( K_N^I(\epsilon) = K_N^I(-\epsilon, \omega) = K_N^I(\epsilon, -\omega) = K_N^I(\epsilon, -\omega) \), and \( K_N^I(\epsilon, \omega) = (-1)^N K_N^I(\epsilon, -\omega) \). I also substituted the result \( 1/N(0)\delta_0 = \frac{\tau}{2} \) at \( H = 0 \) and put \( \epsilon \to \infty \), noting \( K_1^I(\epsilon) \to i/\epsilon \) as \( \epsilon \to \infty \). One can see easily that Eq. (12b) satisfies \( D(\omega) = D^*(\omega) \).

The second-order quantities \( \delta g^{R,a}(2) \) can be calculated similarly by expanding them as
\[ \delta g^{R,a}(2) = \sum_m \sum_K \delta g^a_{mK}(\epsilon, \theta, \omega) e^{im\varphi + iK \cdot r}, \] (13)
where \( K \) is a reciprocal lattice vector of the magnetic Brillouin zone 17. Since \( \delta \mathbf{E} \) is spatially uniform, we only need the \( K = 0 \) component within the linear-response regime. It also follows from Eq. (4) that \( m \neq \pm 1 \) are irrelevant for the current density. Let us substitute the expansions for \( f^{R} \) and \( \delta f^{R+} \) into Eq. (13) with \( g^{R} = 1 \), multiplies it by \( e^{i\varphi}/2\pi V \), and perform integrations over \( (r, \varphi) \). We thereby obtain \( g^a_{\pm} \equiv g^a_{m=\pm 1, K=0} \) as
\[ \delta g^{R}_{\pm} = \Delta_0^2 \frac{\delta E^+_{\pm}}{\omega}, \] (14a)
This algorithm can be put into a more convenient form in terms of $\mathcal{R}_N \equiv D_{N+1}/D_N$ and $\tilde{\mathcal{R}}_N \equiv \mathcal{D}_{N-1}/\mathcal{D}_N$ as follows. Let us calculate $\mathcal{R}_N$ and $\tilde{\mathcal{R}}_N$ by

\[
\begin{aligned}
\mathcal{R}_{N-1} &= (-i\bar{\varepsilon} + \beta^2 N\mathcal{R}_N)^{-1}, \quad \mathcal{R}_{N,\text{cut}} = i/\bar{\varepsilon}, \\
\tilde{\mathcal{R}}_{N+1} &= (-i\bar{\varepsilon} + \beta^2 N\tilde{\mathcal{R}}_N)^{-1}, \quad \tilde{\mathcal{R}}_1 = i/\bar{\varepsilon},
\end{aligned}
\tag{17a}
\]

for an appropriately chosen large $N_{\text{cut}}$. Then $K_N^N$ for $N' \leq N$ is obtained by

\[
\begin{aligned}
K_0^0 &= \mathcal{R}_0, \quad K_N^{N'} = (\mathcal{R}_N'/\tilde{\mathcal{R}}_N')K_{N-1}^{N'-1}, \\
K_N^{N+1} &= \beta\sqrt{N+1}\mathcal{R}_{N+1}K_N^{N'},
\end{aligned}
\tag{17b}
\]

One can check the convergence by increasing $N_{\text{cut}}$. It turns out that $N_{\text{cut}} = 2$ is sufficient both near $T_c$ and in the dirty limit where an analytic calculation is also possible. One can thereby reproduce the formula by Thompson [5] and Takayama and Ebisawa [8] which satisfies $\text{Im} \sigma_f^0(\omega \to 0) = 0$. In contrast, $N_{\text{cut}} \gtrsim 1000$ is required in the clean limit at low temperatures.

Using Eq. (16b) and taking the limit $\omega \to 0$, I have calculated the initial slope of the flux-flow resistivity:

\[
\alpha = \frac{H}{\rho_n} \frac{\partial \rho_f}{\partial H} \bigg|_{H=H_{c2}},
\tag{18}
\]

for various impurity concentrations and various values of the Ginzburg-Landau parameter $\kappa_{\text{GL}}$. The main $H$ dependence in Eq. (10b) lies in $\Delta_0^2 \propto H_{c2}-H$, which can be obtained accurately following Ref. [10].

Figure 1 displays the slope $\alpha$ for several values of $\xi_E/l \equiv 1/2\pi T_c \tau$ with $\kappa_{\text{GL}} = 50$. The curve TTE denotes the prediction of the dirty-limit theory by Thompson [5] and Takayama and Ebisawa [8]. Marked mean-free-path dependence is clearly seen. In fact, the slope at $T = T_c$ ($T = 0$) decreases from 4.99 (1.72) in the dirty limit to 0.89 (0.73) at $\xi_E/l = 4.0$. Thus, nonlocal effects in clean systems tend to increase the resistivity substantially over the prediction of the dirty-limit theory. This result near $H_{c2}$ also suggests that $\rho_f(H)$ at low temperatures may change from convex downward to upward as $l$ increases and may not be fit quantitatively by the Bardeen-Stephen theory $\rho_f \propto H/H_{c2}$ [2]. The slope has also been found to become steeper as $\kappa_{\text{GL}}$ approaches $1/\sqrt{2}$ due to the increase of the coefficient of $\Delta_0^2 \propto H_{c2}-H$, reaching $\alpha = 9.57$ (2.59) at $T = T_c$ ($T = 0$) for $\kappa_{\text{GL}} = 1$ and $\xi_E/l = 50$. This fact indicates the necessity of correctly identifying the material parameters, such as $\kappa_{\text{GL}}$, $\xi$, and $l$, for any detailed comparisons between the theory and experiments on the flux-flow resistivity. Besides, a careful experiment will be required, especially near $T_c$, to determine the slope $\alpha$ which may change appreciably near $H_{c2}$ [2].

In summary, this paper has developed a reliable and efficient method to calculate the flux-flow resistivity near $H_{c2}$ over all impurity concentrations and clarified large dependence of $\rho_f$ on both $l$ and $\kappa_{\text{GL}}$.

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\[\begin{align}
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