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Jumps and stock market variance during the COVID-19 pandemic: Evidence from international stock markets

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ABSTRACT

Based on the work of Buncic and Gisler (2017), this paper investigates whether the roles of jump components will change in forecasting the volatility of international equity markets during the COVID-19 pandemic. Interestingly, in contrast to the conclusions of Buncic and Gisler (2017), we find jump components of the international equity indices are useful to predict the international stock markets’ volatility during the COVID-19 pandemic. Our study tries to provide new evidence of jump components in stock markets.

1. Introduction

Drastic shocks, also known as jumps, can often lead to sharp fluctuations in volatility and contain contents of huge movements of asset markets (Aït-Sahalia et al., 2015; Bandi and Renò, 2016). Commonly considered for volatility forecasting, researchers find that jumps contain predictive information for volatility forecasting (Eraker et al., 2003; Becker et al., 2009; Clements and Liao, 2017; Ma et al., 2019; Maneesoonthorn et al., 2020). However, Buncic and Gisler (2017) find jump components are not effective in forecasting the realized volatility for international equity markets. We are inspired to investigate the roles of jumps in predicting the realized volatility of the stock market.

It is noteworthy that the COVID-19 pandemic has brought a fierce strike to the whole world, and this pandemic continues to provide its destructive influence. Thus, based on the work of Buncic and Gisler (2017), we investigate whether the roles of jump components will change in forecasting the volatility of international equity markets during this pandemic. In other words, are jump components helpful for forecasting the realized volatility of international equity markets? This paper contributes to the existing literature by checking the predictability of jumps for international equity markets during the COVID-19 pandemic.

2. Methodology

2.1. Realized volatility, bipower variation and jumps

The log-price \( p_t \) is assumed to follow a continuous-time diffusion process driven by Brownian motion, which can be:

\[
dp_t = \mu dt + \sigma dW_t + k dq_t,
\]

(1)

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where $u_t$ is a locally bounded drift term, $\sigma_t$ is a volatility process that is bounded away from zero, $W_t$ is standard Brownian motion, and $q_t$ is a counting process with (possibly) time-varying intensity.

The quadratic variation (QV) of the log-price process is:

$$
QV_t \rightarrow \int_0^t \sigma^2(s) \, ds + \sum_{0<s\leq t} k^2(s),
$$

where $\int_0^t \sigma^2(s) \, ds$ is the integrated variance (IV) of the process and it is the continuous part of the quadratic variation, while $\sum_{0<s\leq t} k^2(s)$ is the squared jump component between 0 and $t$ and is the discontinuous part of the quadratic variation.

Following Andersen and Bollerslev (1998), the realized variance (RV) can be:

$$
RV_t = \sum_{i=1}^M r_{t,i}^2,
$$

where $r_{t,i}$ represents the $i$th intraday return of day $t$, and $M$ is the interval. When the intraday sampling frequency increases, $\text{plim}_{M \to \infty} RV_t = QV_t$.

According to Barndorff-Nielsen and Shephard (2004), the bi-power variance (BPV) is:

$$
BPV_t = \frac{\pi}{2} \left( \frac{M}{M-2} \right) \sum_{i=1}^M |r_{t,i}||r_{t,i-1}|,
$$

where $\left( \frac{M}{M-2} \right)$ is a finite sample bias correction term.

Thus, $\text{plim}_{M \to \infty}(RV_t - BPV_t) = \sum_{0<s\leq t} k^2(s)$.

To make the jumps to be non-negative, following Barndorff-Nielsen and Shephard (2004) and Andersen et al. (2007), the jump component is:

$$
J_t = \max\{RV_t - BPV_t, 0\},
$$

The continuous component then can be:

$$
C_t = RV_t - J_t = RV_t - \max\{RV_t - BPV_t, 0\},
$$

Moreover, following Barndorff-Nielsen and Shephard (2006), the “remarkable” jumps can be:

$$
Z_t = \Delta^{-1/2} \frac{(RV_t - BPV_t)RV_t^{-1}}{\sqrt{\left( \frac{M}{M-1} \right) \left( \frac{M}{M-2} \right) \max\left( 1, \frac{10}{BPV_t} \right)}},
$$

where $TQ_t$ is the realized tri-power quarticity.

Additionally, following Andersen et al. (2007), we set $\alpha$ as the significance level and the critical value, which is denoted as $\Phi_\alpha$. The jump component can be denoted as follows:

$$
C_{J_t} = I( Z_t > \Phi_\alpha ) \cdot \max( RV_t - BPV_t, 0 ),
$$

where $I( \bullet )$ is an indication function.

### 2.2. HAR-RV-type models

**Model 1:** HAR-RV

$$
\log RV_{t+1} = \beta_0 + \beta_1 \log RV_t + \beta_2 \log RV W_t + \beta_3 \log RV M_t + \epsilon_{t+1},
$$

where RV$_t$, RVW$_t$, and RVM$_t$ represent daily, weekly and monthly RV, respectively. Moreover, RVW$_t = \frac{1}{5} \sum_{i=1}^5 RV_i$, RVM$_t = \frac{1}{22} \sum_{i=1}^{22} RV_i$, and $\epsilon_{t+1}$ represents the disturbance term.

Based on Buncic and Gisler (2017), jump components are not useful to predict the stock market volatility. In this paper, we try to check whether the contents of jumps are helpful for predicting stock market volatility during the COVID-19 pandemic based on the HAR-CRV-CJ model.
Table 1
Descriptive statistics.

| Equity index RV | Country      | Observations | Mean  | Std.dev | Skewness | Kurtosis | Jarque-Bera        | Q(20)                   | ADF                      |
|-----------------|--------------|--------------|-------|---------|----------|----------|--------------------|--------------------------|--------------------------|
| SPX             | United States| 5515         | 0.0001| 0.0003  | 10.9804  | 201.4261 | 9,415,270.6964*** | 25,810.5580***           | −31.5832***              |
| FTSE            | United Kingdom| 5547        | 0.0001| 0.0003  | 15.8377  | 413.769  | 39,722,802.2189*** | 12,316.8599***           | −43.4482***              |
| N225            | Japan        | 5347         | 0.0001| 0.0002  | 9.1627   | 127.6124 | 3,695,384.4501*** | 16,632.6800***           | −34.3812***              |
| GDAXI           | Germany      | 5574         | 0.0002| 0.0003  | 7.7867   | 100.0298 | 2,375,539.0198*** | 26,745.9680***           | −30.5722***              |
| AORD            | Australia    | 5556         | 0.0001| 0.0001  | 18.0814  | 491.5454 | 56,125,920.1610***| 16,746.1869***           | −36.3125***              |
| FCHI            | France       | 5612         | 0.0001| 0.0002  | 9.1087   | 129.3657 | 3,983,130.2228*** | 21,998.1593***           | −32.1119***              |
| HSI             | Hong Kong    | 5389         | 0.0001| 0.0002  | 10.7017  | 187.3056 | 7,964,309.2765*** | 21,330.0847***           | −33.9895***              |
| KS11            | South Korea  | 5413         | 0.0001| 0.0002  | 9.3285   | 159.079  | 5,774,367.8597*** | 26,917.8647***           | −29.4234***              |
| AEX             | The Netherlands| 5610        | 0.0001| 0.0002  | 7.8819   | 94.3215  | 2,133,490.8420*** | 26,138.4720***           | −28.9730***              |
| SSMI            | Switzerland  | 5512         | 0.0001| 0.0002  | 12.2359  | 220.9085 | 11,322,846.6330***| 18,506.3164***           | −33.7436***              |
| IBEX            | Spain        | 5575         | 0.0001| 0.0002  | 9.3113   | 147.7301 | 5,140,005.2531*** | 14,536.9034***           | −34.6935***              |
| NSEI            | India        | 5443         | 0.0001| 0.0004  | 25.549   | 885.0314 | 177,873,988.8841***| 3466.9611***             | −48.0011***              |
| MXX             | Mexico       | 5516         | 0.0001| 0.0002  | 13.2484  | 290.0526 | 19,458,562.2393***| 8093.5236***             | −48.9158***              |
| BVSP            | Brazil       | 5409         | 0.0002| 0.0003  | 8.878    | 111.734  | 2,878,921.1460*** | 27,475.9799***           | −29.1306***              |
| GSPTSE          | Canada       | 4916         | 0.0001| 0.0005  | 7.4784   | 2838.882 | 1,648,980.318636***| 3319.0682***             | −49.4592***              |
| TOXX50E         | Euro Area    | 5595         | 0.0002| 0.0003  | 12.3419  | 271.7286 | 17,321,101.7723***| 16,649.2033***           | −37.4944***              |
| STI             | Singapore    | 3571         | 0.0001| 0.0001  | 11.645   | 231.5952 | 8,036,586.2429*** | 5836.4154***             | −36.8374***              |
| FTMB            | Italy        | 3190         | 0.0001| 0.0002  | 6.6495   | 66.972   | 612,690.0159***   | 10,309.7749***           | −24.2406***              |

Note: Descriptive statistics of RVs of equity indices are exhibited. In line with Jarque and Bera (1987), we set the null hypothesis of a normal distribution for each variable. Ljung and Box (1978) propose the Ljung-Box statistic called Q(n); in our study, the 20th order serial correlation is tested. The Augmented Dickey-Fuller test is used to test whether the time series is stationary. Asterisks ***, ** and * denote rejections of null hypothesis at 1%, 5% and 10% levels.
Table 2
Results of the out-of-sample $R^2$ test.

| Forecasting models | $T_{os}$ | $R^2_{os}$ (%) | MSPE-Adj. | p-value |
|-------------------|---------|----------------|-----------|---------|
| HAR-RV-JSPX       | 451     | 0.6312         | 1.0163    | 0.1547  |
| HAR-RV-JPTSE      | 456     | **16.6876**    | 2.1613    | 0.0153  |
| HAR-RV-JN225      | 437     | 1.3794*        | 1.4031    | 0.0803  |
| HAR-RV-JGDAI      | 455     | **6.1398**     | 1.2846    | 0.0995  |
| HAR-RV-JAORD      | 459     | **1.7383**     | 2.3304    | 0.0099  |
| HAR-RV-JFCXI      | 465     | 2.1905*        | 1.5148    | 0.0649  |
| HAR-RV-JHISI      | 443     | -0.6181        | -0.4218   | 0.6634  |
| HAR-RV-JKS11      | 447     | **0.9491**     | 1.8856    | 0.0297  |
| HAR-RV-JAEX       | 464     | -14.5774       | -0.3758   | 0.6465  |
| HAR-RV-JSSMI      | 456     | -102.4253      | 1.5731    | 0.0578  |
| HAR-RV-JBEX       | 463     | **4.4091**     | 1.6832    | 0.0462  |
| HAR-RV-JNSEI      | 442     | -1.9548        | -0.6991   | 0.7577  |
| HAR-RV-JMX        | 452     | **4.6145**     | 2.9519    | 0.0016  |
| HAR-RV-JBVSP      | 436     | **4.9235**     | 1.8756    | 0.0304  |
| HAR-RV-JGSPTSE    | 449     | **5.6046**     | 2.3765    | 0.0087  |
| HAR-RV-JSTOXX50E  | 449     | **4.8872**     | 1.783     | 0.0373  |
| HAR-RV-JSTI       | 451     | -0.9095        | 0.4801    | 0.3156  |
| HAR-RV-JFTMIB     | 456     | -0.4061        | 0.1548    | 0.4385  |

Notes: Columns display forecasting models, the effective number of out-of-sample observations $T_{os}$, $R^2_{os}$ (%), MSPE-adjusted statistic, p-value, respectively. If the $R^2_{os}$ (%) is larger than zero, implying that forecasting model outperform the benchmark model. Asterisk ***, ** and * denote rejections of null hypothesis at 1%, 5% and 10% level.

4.1. Out-of-sample analysis

3. Data

Following the Buncic and Gisler (2017), we apply daily realized measures data from the Oxford-Man Institute’s Quantitative Finance Realized Library. The 18 international stock indices are the S&P 500 (SPX, United States), the FTSE 100 (FTSE United Kingdom), the Nikkei 225 (N225, Japan), the DAX 30 (GDAXI, Germany), the All Ordinaries (AORD, Australia), the CAC 40 (FCHI, France), the Hang Seng (HSI, Hong Kong), the KOSPI (KS11, South Korea), the AEX (AEX, The Netherlands), the Swiss Market Index (SSMI, Switzerland), the IBEX 35 (IBEX, Spain), the S&P CNX Nifty (NSEI, India), the IPC Mexico (MXX, Mexico), the Bovespa (BVSP, Brazil), the S&P TSX (GSPTSE, Canada), the Euro STOXX 50 (STOXX50E, Euro area), the FT Straits Times (STI, Singapore), and the FTSE MIB (FTMIB, Italy). The sample period ranges from January 1, 2000 to December 30, 2021 except for the S&P 500 and the FTSE MIB indices, which start from May 2, 2002 and June 1, 2009, respectively. Table 1 shows the descriptive statistics. From Table 1, we find that all the international stock indices’ RVs have right skew and high kurtosis. The Jarque-Bera (JB) test shows that no Gaussian distributions exist in all the RV at the 1% significance level. The Ljung-Box test shows that all the RVs have serial auto-correlations up to the 20th order at the 1% significance level. The Augmented Dickey-Fuller (ADF) test shows that all the RVs have no unit root at the 1% significance level, further showing that the data series are stationary.

4. Empirical results

4.1. Out-of-sample analysis

In line with Paye (2012) and Liang et al. (2020), the out-of-sample $R^2(R^2_{os})$ method is efficient in capturing the distinction among the predictability models. The out-of-sample $R^2$ statistic is:

$$R^2_{os} = 1 - \frac{\sum_{j=1}^{M} (RV_j - RV_j^o)^2}{\sum_{j=1}^{M} (RV_j - \bar{RV})^2}, j = \text{Model}(2),$$

where $RV_j$ is the actual realized volatility, $RV_j^o$ is the prediction from model $j$, where $j \in \text{Model}(2)$, and $RV_j^o$ is the volatility forecasting from the benchmark model. A positive $R^2_{os}$ of a model shows that this model is superior to the benchmark model. Following Clark and West (2007), the MSPE metric is applied to check the difference among the models for oil futures market volatility.
Table 2 shows the results of the out-of-sample $R^2$ test for 18 equity markets during the COVID-19 pandemic. For all the international equity indices, the first out-of-sample observation starts on March 11, 2020. We can observe some remarkable findings from Table 2. During the COVID-19 pandemic, the values of $R^2_{oos}$ are significantly positive for 11 of 18 equity markets, including FTSE, N225, GDAXI, AORD, FCHI, KS11, IBEX, MXX, BVSP, GSPTSE, and STOXX50E, implying that jump components of the international equity indices are useful to improve forecasting performance for most of the observed indices during the COVID-19 pandemic. However, Buncic and Gisler (2017) find that jumps are not useful for predicting the volatility of international equity indices.

Why do the roles of jumps change during the COVID-19 pandemic? A possible reason can be that jumps are able to predict stock market volatility based on the channel of investor sentiment. Jumps often contain valuable information that is connected to extreme conditions (Ma et al., 2019). More specifically, people are more sensitive to sharp fluctuations (jumps) in the stock market during the COVID-19 pandemic or the crisis because of the increase of investor fear increase (Smales and Kininmonth, 2016; Ergun and Durukan, 2017; Goldstein et al., 2017; Chang et al., 2020; Ftiti et al., 2021). In addition, existing studies find that models tend to have better performance during recessions (Rapach et al., 2010; Neely et al., 2014).

### 4.2. Robustness check

To ensure that our results are robust, we consider a different out-of-sample period in this subsection. This period ranges from 23 January 2020 to the end of the sample period, which begins with the lockdown of the city of Wuhan in China because of the outbreak of COVID-19. This may affect China’s stock market volatility and have linkage effects on international equity markets. The empirical results are shown in Table 3. We find that out-of-sample $R^2$ are significantly positive for 10 of 18 equity markets in this period. The results are consistent with the previous conclusion except for the AORD index. These results are consistent with the out-of-sample results.

### 5. Conclusion

Extending the work of Buncic and Gisler (2017), this paper checks the roles of jump components for predicting the volatility of international equity markets during the COVID-19 pandemic. We find that jump components of the international equity indices are useful to predict the international stock markets’ volatility during the COVID-19 pandemic, which is inconsistent with the results of Buncic and Gisler (2017). Our study emphasizes the importance of jump components in stock market volatility during the COVID-19 pandemic. As the COVID-19 pandemic continuously affects the world economy, understanding the information of jumps is essential for market participants and policy makers.

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*The world Health Organization announced the outbreak of COVID-19 pandemic on March 11, 2020.*

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Table 3
Results of the out-of-sample $R^2$ test based on city sealing of Wuhan.

| Forecasting models | $T_{os}$ | $R^2_{oos}$ (%) | MSPE-Adj. | $p$-value |
|--------------------|----------|----------------|-----------|-----------|
| HAR-RV-JSPX        | 484      | 1.8465**       | 1.2446    | 0.1066    |
| HAR-RV-JFTSE       | 490      | 5.9328**       | 2.2895    | 0.011     |
| HAR-RV-JN225       | 469      | 2.3115*        | 1.9275    | 0.027     |
| HAR-RV-JGDAXI      | 489      | 6.2267*        | 1.4449    | 0.0742    |
| HAR-RV-JAORD       | 492      | 1.9765         | 0.359     | 0.6402    |
| HAR-RV-JFCHI       | 499      | 1.8996**       | 1.453     | 0.0731    |
| HAR-RV-JHSI        | 475      | 0.5310         | 0.3124    | 0.6226    |
| HAR-RV-JKS11       | 479      | 1.0285**       | 2.0025    | 0.0226    |
| HAR-RV-JAES        | 498      | 14.6608        | 0.3932    | 0.6529    |
| HAR-RV-JISSM       | 490      | 0.7356         | 1.6969    | 0.0449    |
| HAR-RV-JIBEX       | 497      | 6.5273*        | 1.8138    | 0.0349    |
| HAR-RV-JNSEI       | 475      | 2.0407         | 0.63      | 0.7356    |
| HAR-RV-JMXX        | 485      | 4.2967**       | 3.0599    | 0.0011    |
| HAR-RV-JBVSP       | 468      | 4.5394**       | 1.8931    | 0.0292    |
| HAR-RV-JGSPTSE     | 482      | 6.0653***      | 2.6071    | 0.0046    |
| HAR-RV-JSTOXX50E   | 482      | 3.0928*        | 1.5087    | 0.0657    |
| HAR-RV-JSTI        | 484      | 0.5158         | 0.5265    | 0.2993    |
| HAR-RV-JFTMIB      | 489      | 0.2879         | 0.1818    | 0.4281    |

Notes: Columns display forecasting models, the effective number of out-of-sample observations $T_{os}$, the $R^2_{oos}$ (%), MSPE-adjusted statistic, $p$-value, respectively. If the $R^2_{oos}$ (%) is larger than zero, implying that forecasting model outperform the benchmark model. Asterisk ***, ** and * denote rejections of null hypothesis at 1%, 5% and 10% level.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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