Research Article

Propagation and Interaction of Two Parallel Internal Cracks under Tensile Stress in Cuboid Glass

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The behavioral characteristics of multiple cracks are important factors leading to material failures. However, to date, the fracturing and interactions of internal cracks have received minimal research due to the challenges in creating real internal cracks in brittle solids without incurring any damages to the surfaces. Therefore, this study investigated the fracturing processes and interactions between two parallel penny-shaped internal cracks under tensile stress conditions using both experimental and numerical simulation methods. Glass was used as the experimental material in this study. The results showed that the vertical spacing \( d \) between the two cracks was an important factor which affected both the crack interactions and the failure loads. When the \( d \) was small, attraction and coalescence occurred between the two cracks. However, when the \( d \) was large, the fracturing was observed to occur in the plane where one crack was located. A crack propagation arc appeared in all of the examined specimens. The loads of the crack initiation and the strength levels of the specimens were determined to be positively related to the crack spacing \( d \). It was found that the larger the \( d \) was, the larger the crack initiation load and tensile strength values would be. In the present study, the crack paths in the numerical simulations based on the MTS criterion were determined to be consistent with the experiment results.

1. Introduction

The fracturing and growth of cracks are common problems in the field of engineering [1]. For a single crack, the stress distribution around the crack tip is not disturbed by the possible adjacent cracks, and its propagation path can be predicted artificially and accurately when the external boundary conditions are known. When multiple cracks are involved, the stress distribution around the cracks becomes extremely complex. The interaction between multiple cracks causes the coalescence and the deflection of crack propagation path. The interactions between multiple cracks are important factors leading to material failures [2–8]. For practical engineering, the interaction of multiple cracks is more common and more practical. Therefore, it is of major significance to carry out research regarding the fracturing processes and interactions of multiple cracks.

In previous studies, many researchers have performed investigations regarding the interactions and fracturing of multiple cracks. Dyskin et al. [9] studied the propagation of inclined cracks arranged in parallel in horizontal direction and in parallel in vertical direction. The results show that the cracks arranged in horizontal direction hinder each other’s propagation; that is, the wing crack propagation is even smaller than that of a single initial crack. When the initial cracks are arranged along the vertical direction, the wing cracks caused by the outer and inner tips of the initial cracks increase. Guo et al. [10] made prefabricated parallel cracks in unsaturated resin materials, studied the penetration of parallel cracks under uniaxial compression, and summarized the four stages of growth and penetration of parallel cracks. Zhou et al. [11] used 3D printing to make samples with multiple cracks and carried out experimental research. It was found that the angle of bridge between cracks had a great impact on the compressive strength and peak strain of the samples. Lin et al. [12] carried out uniaxial compression test to study the failure characteristics of layered rock mass with double cracks. The test results show that the crack angles
have a great influence on the peak strength of the sample. Kolari [13] established a finite element model with multiple cracks, carried out numerical calculation, and obtained the propagation law of multiple cracks. The numerical simulation results are in good agreement with the experimental results. Bi et al. [14] used the meshless method to carry out the numerical simulation of biaxial compression of multiple crack model and obtained the shear failure mode of the model. Fender et al. [15] proposed an influence rule of length and time under the action of double crack attraction and provided a concrete expression. Dalbe et al. [16] found that there was not only attraction but also repulsion between two cracks under uniaxial stress conditions. In another related study, Schwaab et al. [17] successfully determined the critical conditions for the transitions of double cracks from attraction to repulsion and presented a numerical explanation.

However, under normal condition, the majority of fracturing problems in engineering tend to be three-dimensional problems with internal cracks being the key factors [18, 19]. At the present time, the current research (whether theoretical, experimental, or numerical) often focuses on surface cracks and penetrating cracks. However, the research has rarely involved the examination of internal crack interactions. The challenges are that it is difficult to make real internal cracks in brittle solids without incurring any damages to the surfaces. The previous experimental studies involving the interactions of internal cracks, such as those conducted by Zhu et al. [20] and Fu [21], included uniaxial compression experiments on resin specimens with double internal cracks. However, these internal cracks were simulated by mica sheets embedded in the samples. Wang et al. [22] introduced a 3D-internal laser-engraved method (3D-ILC) to make internal cracks in intact brittle solids without incurring damages to the surfaces of the samples. The internal cracks made by 3D-ILC method are real cracks compared to the mica sheets in previous research.

In this research investigation, which was based on the 3D-ILC method, experiments regarding the fracturing processes of two 3D internal cracks were performed. The internal crack propagation process, crack morphology, and fracture features were analyzed. Meanwhile, the fracture paths were simulated using 3D numerical simulations based on the MTS. It was indicated in this study that the simulation results were in accordance with the experimental results.

2. Experimental Set-Up

2.1. Sample Material. In this study, BK7 glass was adopted as the sample material. This was a type of borosilicate crown glass chosen for its outstanding optical properties of transparency and photoelasticity. The glass was essentially a continuous, uniform, isotropic, linearly elastic, and brittle material at room temperature. In addition, the BK7 glass was also found to be an ideal material for photoelasticity, which provided a basis for the qualitative observations of the stress distribution patterns in this study’s experiments, as well as the tracking of the locations of the crack tips.

2.2. Sample Preparation. Two sets of samples were set up in this study, named Group A and Group B, and the vertical crack spacing d was set as 4 mm in Group A and 8 mm in Group B. The size parameters of the cubic specimen with double internal cracks were as follows: The side length of the cube specimen was a = 60 mm, the internal crack produced by 3D-ILC technology was a standard round crack, and the crack size was 2r = 10 mm. The internal crack was round in shape, and the horizontal spacing of the crack was L = 6 mm. The size proportions of the double crack cube specimen are detailed in Figure 1, and the experimental scheme is shown in Table 1.

2.3. Testing System and Tensile Device. A SUNS-650W electro-hydraulic servo loading system was used in this study’s experiments, as detailed in Figure 2. The loading rate was set as 0.05 kN/s, which was within the quasi-static loading range. A camera monitored the dynamic changes in the crack growth within the specimen. In order to avoid the problems of stress concentrations and eccentric effects, a set of tensile fixtures was designed, as shown in Figure 3. A universal hinge was set between the tensile device and the sample for the purpose of ensuring that the axis was under tension. The adopted method ensured that the stress distribution was uniform, which resulted in a high rate of stretching success. The adhesive used in this study was a steel adhesive. It was found that after bonding was completed, a maximum tensile strength of 30 MPa could be reached after being left at room temperature for 10 days.
2.4. Photoelastic Test System. It has been found that glass generally exhibits optical isotropy in a stress-free state. However, when subjected to stress, the refractive index characteristics tend to change, showing optical anisotropy. That is to say, when a beam of light passes through glass under internal stress conditions, it will produce two rays with different propagation speeds. These are ordinary light o which follows the law of refraction and extraordinary light e which does not follow the law of refraction. This phenomenon is referred to as stress birefringence, in which o and e share the same frequencies and vibration directions. There will be a fixed phase difference, and interference fringes can be generated. These effects can achieve the stress field distributions and dynamic change observations of the sample ends and areas surrounding cracks, by which the stress concentrations can be determined and the test eccentricity corrected. According to the stress-optical law, when the light is incident on a test object, due to the birefringence effects, the principal stress will have the following relationship with the corresponding refractive index:

\[ n_1 - n_2 = (C_1 - C_2) (\sigma_1 - \sigma_2), \]

where \( \sigma_1 \) and \( \sigma_2 \) represent the principal stresses of a sample under ballast, respectively; \( n_1 \) and \( n_2 \) are the refractive indices in the directions of \( \sigma_2 \) and \( \sigma_1 \); and \( C_1 \) and \( C_2 \) indicate the material stress-optical coefficients. The optical path difference \( \Delta \) produced by polarized light passing through a sample is as follows:

\[ \Delta = (n_1 - n_2)h, \]

where \( h \) represents the thickness of the medium and the correspondence between stress and optical quantity can be established by (1) and (2):

\[ (\sigma_1 - \sigma_2) = \frac{\Delta}{hC}. \]

3. Experimental Results

The crack propagation process observed in this study is shown in Figure 4. In the figure, the first row provides the results of the photo elastic effects. This study performed analysis from the following three aspects: (1) crack propagation process, (2) crack initiation and failure loads, and (3) failure morphology and fracture surface characteristics.

3.1. Propagation of the Internal Cracks

3.1.1. Attraction between Two Internal Cracks. As shown in Figure 5, “petal-shaped” stress fringes had appeared at the crack tips, which indicated that the stress was concentrated in those areas. When the crack spacing \( d \) was small (Group A), the inner tip of the prefabricated crack had begun to propagate first, while the tip near the edge of the specimen had only slightly expanded. Then, after expanding a certain

| Group no. | Vertical spacing D/mm | Stress fringes | Crack growth | Physical experiment | Fracture surface | SIF distribution | Crack path |
|-----------|------------------------|----------------|--------------|---------------------|-----------------|-----------------|------------|
| Group A   | 4                      | √              | √            | √                  | √               | √               | √          |
| Group B   | 8                      | √              | √            | √                  | √               | √               | √          |

Figure 2: Loading system.

Figure 3: Stretching fixture.
Figure 4: Process of growth of the cracks. (a) Group A. (b) Group B.

Figure 5: Crack interactions observed in the experiments. (a) Attraction of two cracks in 3D in our experiment. (b) Attraction of two cracks in 2D in Ref. [15–16].
distance, the crack tips began to attract and become closer to each other, displaying a “hook” shape. At that time, the stress fringes at the crack tips were not “bracketed.” This was found to be similar to the 2D double crack law published in the Physical Review Letters [15, 16], as shown in Figure 5. However, 3D crack propagation morphology could clearly be observed in this study’s experiments. For example, when a “hook-shaped” crack was formed, the inside tip of the prefabricated crack had expanded at a significantly slower rate. Meanwhile, the rate at the outer tip had increased significantly. The stress fringes of the outside tip of the prefabricated crack showed a sharp color change, and the crack eventually penetrated the sample.

3.1.2. Dominant Crack Failure. As detailed in Figure 5, when the crack spacing $d = 8$ mm (Group B), the stress fringe range of the inner tip of the prefabricated crack was larger than that of the outer tip. However, the difference between the two was significantly smaller than those of Group A. It was observed that during the initial stage of the crack propagation, both the inside and outside tips of the prefabricated cracks had slowly expanded, but the propagation rate of the

| Crack morphology | Fracture surface |
|------------------|------------------|
| "Crescent-shaped" step | Propagation arc |
| Fractured surface | Group A |
| Interaction "funnel" | Propagation arc |
| Fractured surface | Group B |

Table 2: Failure patterns of the samples in the experiments.
inside tip was greater than that of the outside tip. Then, after expanding a certain distance, the inside tips attracted and became closer. As a result, the dominant crack accelerated, broke through the sample, and finally caused failure of the material to occur. It was considered to be worth noting in this study that the failure surface was also attracted by another prefabricated crack, and the failure surface was observed to be significantly three-dimensionally bent toward the other precrack.

3.2. Crack Initiation and Failure Loads. The crack initiation load of the sample was determined to be positively related to the crack pitch $d$. It was found that the larger the internal crack pitch was, the larger the initiation load would be. The ratios of the crack initiation load to the tensile strength were 5.77% and 7.82%, respectively. In addition, it was observed that the failure loads were positively related to the crack pitch $d$. In this study, the ratios of the failure load to the tensile strength of the intact specimen were 43.36% and 52.95%, respectively.

3.3. Failure Patterns and Fracture Surfaces. The failure patterns of both samples are shown in Table 2. The precrack pitch $d$ determined the failure patterns. The main cracks were determined to be crack propagation arcs, interaction interfaces, ruby-shaped steps, crescent-shaped steps, and interaction “funnels.”

3.3.1. Crack Propagation Arcs. Crack propagation arcs appeared in all of the samples. It was observed that the smaller the crack pitch $d$ was, the more obvious the crack propagation arc would be, as illustrated in Figure 6. For example, in regard to the Group A fracture arc, the crack propagation arc displayed discontinuity in the interaction zone between the double cracks, which indicated that the prefabricated crack always expanded along a circular arc under tensile stress conditions without external interference. This could be explained by the following theory:

By assuming that the crack tip was an ellipse with a long radius $a$ and a short radius $c$ and with the vertical stress being $\sigma_A$, the following could be applied:

$$ K_I = \phi(\frac{a}{c}, \beta) = \pi^{0.5} \left[ \cos^2 \beta + \left( \frac{c}{a} \right)^2 \sin^2 \beta \right]^{0.5} / E \left( \frac{a}{c} \right), $$

(5)

$$ E \left( \frac{a}{c} \right) = \int_0^{\pi/2} \left[ 1 - \left( 1 - \left( \frac{c}{a} \right)^2 \sin^2 \varphi \right)^{0.5} \right] d\varphi, $$

(6)

where $K_I$ is a mode-I stress intensity factor, $c$ denotes the crack length, $\varphi$ is the geometric term, and $E (a/c)$ represents the elliptic integral. Therefore, in the formula, $\beta$ is a dummy variable. In other words, when $a/c \neq 1$, the following is always true:

$$ K_I \left( \beta = \frac{\pi}{2} / K_I (\beta = 0) = \left( \frac{c}{a} \right)^{0.5} \leq 1. $$

(7)

Furthermore, without external interference, the crack propagation will always tend to achieve $a/c = 1$ to form a rounded tip.

3.3.2. “Crescent-Shaped” Steps. A “crescent-shaped” step appeared in Group A, as shown in Figure 7, displaying the shape of a “crescent” curve. This “crescent-shape” step was located between the inside tips of the double cracks. It was the intersection of the inside tips of the cracks caused by the interactions between the double cracks.

3.3.3. Interaction “Funnel.” “Funnel” were observed in the specimens with large pitches between the prefabricated cracks (Group B). As can be seen in Figure 8, the fractured surface of an internal crack propagation was bent as the result of the “attraction” of another internal crack. It was found that as the crack pitch $d$ increased, the attraction between the internal cracks became increasingly smaller as the scope of a “funnel” does. Finally, the fractured surfaces returned to a plane which was perpendicular to the stretching direction.

4. Numerical Simulations

4.1. Numerical Models. This study’s numerical models for Groups A and B are shown in Figure 9. The sizes of samples and the cracks were the same as the parameters used in this study’s experiments. The sizes of the numerical model were set as $6 \times 6 \times 6$ cm. The elasticity modulus was 213 GPa,
and the Poisson’s Ratio was 0.2. The boundary conditions of the models were the same as those used in the experimental tests. Tension was applied on the upward and downward surfaces of the models in a vertical direction.

4.2. Crack Propagation Criteria. In this research investigation, the maximum tensile stress criterion (MTS) was used as the crack propagation disc, and the cracks propagated along the direction of the maximum circumferential stress \( \sigma_{\theta} \). Meanwhile, the hoop stress was related to the mode-I stress intensity factor, which could be expressed as follows:

\[
K_I(\theta) = \sigma_{\theta} \sqrt{2\pi r} = \cos \theta \left[ K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right].
\]

Therefore, if \( (\partial K_I(\theta)/\partial \theta) = 0, (\partial K_I(\theta)/\partial \theta^2) \leq 0 \), then the cracking angle \( \theta_0 \) could be obtained as follows:

\[
\theta_0 = \arccos \left( \frac{3K_{II}^2 + \sqrt{K_I^4 + 8K_I^2K_{II}^2}}{K_I^2 + 9K_{II}^2} \right). \tag{9}
\]

4.3. Distribution Patterns of the SIFs around the Pre-cracks. The values of \( d \) in the numerical model were set as 4 mm and 8 mm, respectively, which were the same as the values of \( d \) in the experiments.

Therefore, based on the maximum stress intensity factor \( K_I \) and \( K_{II} \) of the samples, \( K_{in}/|K_{max}| \) was defined as the normalized \( K \), where \( i = I, II, \) and \( n = 1 \sim 2 \) were the sample numbers. The normalized \( K_I \) and \( K_{II} \) were drawn and distributed around the crack tips, as shown in Figure 10. In the figure, A and B are the location identifiers of the front edges of the double inner crack, and the laws were as follows:

1. Under the same tensile conditions, the larger the crack pitch \( d \) was, the smaller the normalized \( K_I \) and \( K_{II} \) would be, and the more difficult the crack would be to initiate. This was consistent with the

\[\text{Figure 8: Attraction between two cracks in Group B.}\]

\[\text{Figure 9: Numerical models.}\]
law in which the larger the prefabricated pitch \( d \) was, the greater the crack initiation of the specimen would be, as described in Section 3.3.

(2) For the normalized \( K_1 \), the value on the inside of the prefabricated crack was greater than that on the outside, which indicated that there were interactions between the inside tips of the double crack. This resulted in partial propagation occurring on the inside before occurring on the outside, which was consistent with the test results.

(3) The normalized \( K_{II} \) gradually decreased from the inside of the crack to both sides, and the \( K_{II} \) of the outside tip of the crack was almost zero. Therefore, it was assumed that the interactions between the inner sides of the double cracks had caused the \( K_{II} \) distribution to occur. Next, the crack propagation direction had changed, which resulted in the phenomena of “double attraction” of the double cracks occurring. This was found to be consistent with the experimental test results.

4.4. Simulations of the Crack Propagation Paths

4.4.1. Group A. As shown in Figure 11, for Group A, the prefabricated internal cracks had attracted each other at the inner tips of the cracks after expanding a certain distance. Then, the calculations of the inner tips of the cracks were suspended due to the overlapping of the cracks, which was consistent with the failure process observed in this study’s tests.

4.4.2. Group B. As shown in Figure 12, for Group B, after the precracks had spread out to a certain distance, the inner tips of the cracks first displayed a tendency of attracting each other. Then, after a certain degree of “hook-shaped” spread had been reached, the tips had gradually returned to the plane perpendicular to the tensile direction under the conditions of tensile stress. Subsequently, the cracks had eventually broken through the specimen and led to failure, which was consistent with the fractured surfaces of the specimens observed in this study’s experimental tests.

5. Conclusions

(1) Due to the fact that the internal crack pitch \( d \) had varied, the specimens were divided into a crack “intersection” failure group (\( d = 4 \) mm) and a dominant crack failure group. The main crack forms included crack propagation arcs, interaction interfaces, “rugby-shaped” steps, “crescent-shaped” steps, and interaction “funnels.” Crack propagation arcs...
Figure 11: Numerical simulation results of Group A.

Figure 12: Numerical simulation results of Group B.
had appeared in all of the specimens. The interaction interfaces and the “rugby-shaped” steps appeared in the Group A \((d = 4 \text{ mm})\), while the “crescent-shaped” steps and interaction “funnels” appeared in Group B \((d = 8 \text{ mm})\).

(2) The crack initiation loads of the specimens were found to be positively related to the crack pitch \(d\). It was observed that the larger the internal crack pitch was, the larger the crack initiation load would be. In addition, the failure loads were determined to also be positively related to the crack pitch \(d\).

(3) The crack paths in the numerical simulations based on the MTS criterion were found to be consistent with the results of this study’s experimental tests, which could be explained by the \(K\) distributions at the crack tips.

**Data Availability**

All the data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

**Conflicts of Interest**

The authors declare no conflict of interests.

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