Verification for measurement-only blind quantum computing

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Blind quantum computing is a new secure quantum computing protocol where a client who does not have any sophisticated quantum technology can delegate her quantum computing to a server without leaking any privacy. It is known that a client who has only a measurement device can perform blind quantum computing [T. Morimae and K. Fujii, Phys. Rev. A 87, 050301(R) (2013)]. It has been an open problem whether the protocol can enjoy the verification, i.e., the ability of client to check the correctness of the computing. In this paper, we propose a protocol of verification for the measurement-only blind quantum computing.

I. INTRODUCTION

Blind quantum computing \[\text{(BQC)}\] is a secure delegated quantum computing, where a client (Alice), who does not have enough quantum technology, delegates her quantum computing to a server (Bob), who has a fully-fledged quantum computer, without leaking any information about her computation to Bob. A blind quantum computing protocol for almost classical Alice was first proposed by Broadbent, Fitzsimons, and Kashefi [1] by using the measurement-based model due to Raussendorf and Briegel [11]. In their protocol, Alice only needs a device which emits randomly rotated single-qubit states. Later it was shown that weak coherent pulses, instead of single-photon states, are sufficient for blind quantum computation [3]. Recently, it was shown that blind quantum computing can be verifiable [2, 3, 10]. Here, verifiable means that Alice can test Bob’s computation [2, 3, 10]. The verifiability is an important requirement, since Alice cannot recalculate the result of the delegated computation by herself to check the correctness (remember that she does not have any quantum computer), and therefore if there is no verification method, she might be palmed off with a wrong result by a fishy company who tries to sell a fake quantum computer [3, 10]. The verifiable blind protocol was experimentally demonstrated with a photonic qubit system [3, 10].

Recently, another type of blind quantum computing protocol was proposed in Ref. [2]. In this protocol, Alice needs only a device that can measure quantum states. One advantage of this protocol is that the security is device independent [2, 3, 12, 20, 22], and is based on the no-signaling principle [16], which is more fundamental than quantum physics. However, it has been an open problem whether the protocol can enjoy verification.

In this paper, we propose a verification protocol for the measurement-only blind quantum computing. We will propose two protocols. Interestingly, our protocols are based on the combination of two different concepts from different fields: the no-signaling principle [16] from the foundation of physics and the topological quantum error correcting code [14, 15, 17] from a practical application in quantum information. The no-signaling principle means that a shared quantum (or more general) state cannot be used to transmit information. It is one of the most central principles in physics, and known to be more fundamental than quantum physics (i.e., there is a theory which is more non-local than quantum physics but does not violate the no-signaling principle [16]). The topological quantum error correcting code is a specific type of the quantum error correcting code which cleverly uses the topological order of exotic quantum symmetry-breaking systems to globally encode logical states.

II. TOPOLOGICAL MEASUREMENT-BASED QUANTUM COMPUTATION

The Raussendorf-Harrington-Goyal state \([\text{RHG})] is the three-dimensional graph state with the elementary cell given in Fig. 1 (a). Defects in the graph state are created by Z measurements on \([\text{RHG}) as usual in the cluster measurement-based model. Topological braiding of defect tubes can implement some Clifford gates [14, 15, 17]. Non-Clifford gates, that are necessary for the universal quantum computation, are implemented by the magic state preparation and distillation [18]. A string of Z operators acting on the resource state, which has at least one open edge, is considered as an error, and its edge(s) is detected by syndrome measurements of cubicles of X operators (Fig. 1(b)). A string of Z operators on the resource states, which connects or surrounds defects (Fig. 1(c)), is not detected, and can be a logical error. Local adaptive measurements can implement quantum computation as well as syndrome error detection.

III. FIRST PROTOCOL

Let us explain our first protocol. The basic idea of our protocol is illustrated in Fig. 2. Bob prepares the resource state, and Alice performs measurements.

More precisely, our protocol runs as follows (Fig. 3). First, Bob prepares a universal resource state, and sends each qubit of it to Alice one by one (Fig. 3 (a)). Alice measures each qubit until she remotely
Fig. 1: The topological measurement-based quantum computation. (a) The elementary cell of the Raussendorf-Harrington-Goyal state. Green balls are qubits, and red bonds are CZ gates. (b) The error detection. Red strings are errors. Green boxes are syndrome operators. (c) Undetected errors or logical operations. Blue tubes are defects. Red and yellow strings are strings of operators, which surround or connect defects, respectively.

Fig. 2: Our setup. Bob first prepares a resource state. Bob next sends each particle to Alice one by one. Alice measures each particle according to her algorithm.

Fig. 3: Our protocol. Here, $|\Psi_P\rangle \equiv P(|R\rangle \otimes |+\rangle^{\otimes N/3} \otimes |0\rangle^{\otimes N/3})$, $P$ is a $N$-qubit permutation, and $|R\rangle$ is a universal resource state.

Throughout this paper, we assume that there is no communication channel from Alice to Bob. Then, due to the no-signaling principle, Bob cannot learn anything about $P$. If Bob can learn something about $P$, Alice can transmit some message to Bob by encoding her message into $P$, which contradicts to the no-signaling principle.

Bob sends each qubit of $\sigma_q|\Psi_P\rangle$ to Alice one by one, and Alice does the measurement-based quantum computation on $\sigma_q|\Psi_P\rangle$ with correcting $\sigma_q$ (Fig. 3(c)). This means that before measuring $j$th qubit of $\sigma_q|\Psi_P\rangle$ she applies $\sigma_j^k$ on $j$th qubit, where $\sigma_j^k$ is the restriction of $\sigma_j$ on $j$th qubit. For example, $(I \otimes X \otimes Z)|_j = XZ$. Qubits belonging to $|R\rangle$ are used to implement the Alice’s desired quantum computation. States $|0\rangle$ and $|+\rangle$ are used as “traps” $\openbullet$. In other words, she measures $Z$ on $|0\rangle$ and $X$ on $|+\rangle$, and if she obtains the minus result (i.e., $|1\rangle$ or $|-\rangle$ state), she aborts the protocol. If results are plus for all traps, she accepts the result of the measurement-based quantum computation on $|R\rangle$.

IV. VERIFIABILITY

Now we show that if all measurements on traps show the correct results, the probability that a logical state of Alice’s computation is changed is exponentially small. In other words, the probability that Alice is fooled by Bob becomes small.

We define $|+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $P$ is an $N$-qubit permutation, which keeps the order of qubits in $|R\rangle$. This permutation is randomly chosen by Alice and kept secret to Bob.

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Since Bob might be dishonest, he might deviate from the above procedure. His general attack is a creation of a different state $\rho$ instead of $\sigma_q|\Psi_P\rangle$. If he is honest, $\rho = \sigma_q|\Psi_P\rangle\langle\Psi_P|\sigma_q^†$. If he is not honest, $\rho$ can be any state. However, for any $N$-qubit state $\rho$, there exists a completely-positive-trace-preserving (CPTP) map
which satisfies $\rho = \sum_j E_j \sigma_q |\Psi_P\rangle \langle \Psi_P| \sigma_j E_j^\dagger$, where $E_j \equiv \sum_\alpha C_j^\alpha \sigma_\alpha$, is a Kraus operator of the CPTP map, and $C_j^\alpha$ is a complex number (see Appendix A). Since $E_j^\dagger E_j$ is a POVM, $I = \sum_j E_j^\dagger E_j = \sum_j \sum_{\alpha,\beta} C_j^\alpha C_j^{\beta\dagger} \sigma_\beta \sigma_\alpha$, we obtain $\sum_j \sum_\alpha |C_j^\alpha|^2 = 1$.

Bob does not know $q$. Therefore, from Bob’s view point, the state is averaged over all $q$:

$$\frac{1}{4^N} \sum_q \sum_j \sum_{\alpha,\beta} C_j^\alpha C_j^{\beta\dagger} \sigma_\alpha \sigma_\beta |\Psi_P\rangle \langle \Psi_P| \sigma_q \sigma_q^+ E_j^\dagger \sigma_q$$

$$= \frac{1}{4^N} \sum_q \sum_{j,\alpha,\beta} C_q^\alpha C_j^{\beta\dagger} \sigma_\alpha \sigma_\beta |\Psi_P\rangle \langle \Psi_P| \sigma_q \sigma_q^+ \sigma_q^\dagger \sigma_q^+ E_j^\dagger \sigma_q$$

$$= \frac{1}{4^N} \sum_{j,\alpha} \sum_q |C_j^\alpha|^2 \sigma_\alpha |\Psi_P\rangle \langle \Psi_P| \sigma_q \sigma_q^+ \sigma_q^\dagger \sigma_q^+ = \sum_{j,\alpha} \tilde{C}_\alpha \sigma_\alpha |\Psi_P\rangle \langle \Psi_P| \sigma_q \sigma_q^+ \sigma_q^\dagger \sigma_q^+, \quad (1)$$

where $\tilde{C}_\alpha \equiv \sum_j |C_j^\alpha|^2$ and $\sum_\alpha \tilde{C}_\alpha = \sum_\alpha \sum_j |C_j^\alpha|^2 = 1$. Here, we have used the following equations

$$\sum_q \sigma_q \sigma_q \rho q \sigma_q^+ \sigma_q^\dagger \sigma_q^+ \sigma_q^\dagger = 0 \quad (2)$$

$$\frac{1}{4^N} \sum_q \sum_j \sigma_q^+ \sigma_q \rho q \sigma_q^+ \sigma_q^\dagger \sigma_q^\dagger = \sigma_\alpha \rho \sigma_\alpha$$

for any $\rho$ and $\alpha \neq \beta$. The second equation is easy to show. For a proof of Eq. (2), see Appendix A. Equation 1 shows that we can assume that Bob’s attack is the “random Pauli” attack, i.e., Bob randomly applies Pauli operators on each qubit.

Bob’s attacks after creating $\rho$ can also be included in the preparation of $\rho$. This is understood as follows. Let us assume that, after creating $\rho$, Bob sends a subsystem $S_1$ of $\rho$ to Alice, and then Alice measures all particles of $S_1$. After this Alice’s measurement, Bob might apply an operation on another subsystem $S_2$ of $\rho$ which has not been sent to Alice. However, Bob cannot know Alice’s measurement angles and results on $S_1$ due to the no-signaling principle, and therefore Bob’s operation on $S_2$ is independent of Alice’s measurements on $S_1$. Furthermore, Bob’s operation on $S_2$ commutes with Alice’s measurements on $S_1$. Hence we can consider as if Bob applied such an operation on $S_2$ immediately after he preparing $\rho$.

In short, we can assume that Bob’s attack is a random Pauli attack on the correct state $|\Psi_P\rangle$ as is shown in Eq. (1). Hence let us focus on $\sigma_\alpha |\Psi_P\rangle$. For many quantum error correcting code (such as the topological one [14, 15]), if the weight $|\alpha|$ of $\sigma_\alpha$ is less than a certain integer $d$ (the code distance), then such an error is detected or does not change logical states [2, 14, 15, 17]. For example, in the topological code, $d$ is determined by the defect thickness and distance between defects [14, 15].

Here, the weight $|\alpha|$ of $\sigma_\alpha$ means the number of non-trivial operators in $\sigma_\alpha$. (For example, the weight of $I \otimes XZ \otimes Z \otimes I \otimes X$ is 3.) Therefore, in order for $\sigma_\alpha$ to change a logical state of the computation, $|\alpha|$ must be larger than $d$. (To understand it, let us consider a simple example. If we encode the logical 0 as $|0_L\rangle \equiv |000\rangle$ and the logical 1 as $|1_L\rangle \equiv |111\rangle$, we must flip more than two qubits to change the logical state. A single bit flip is detected and corrected when the majority vote is done.)

Alice randomly chooses a permutation $P$. In this case, the probability of $P^t \sigma_\alpha P$ not changing any trap is at most $(\frac{2}{3})^{|\alpha|/3}$. (For a calculation, see Appendix C). Therefore, the probability that the logical state is changed and no trap is flipped is at most $\sum_{|\alpha| \geq d} \tilde{C}_\alpha (\frac{2}{3})^{|\alpha|/3} \sum_{|\alpha| \geq d} \tilde{C}_\alpha \leq (\frac{2}{3})^d/3$, where we have used the fact $\tilde{C}_\alpha \geq 0$ and $\sum_\alpha \tilde{C}_\alpha = 1$. Here, we have said “at most”, since the above sum includes the contribution from $\sigma_\alpha$ which has a weight larger than $d$ but does not contain any logical error. In this way, we have shown that the probability that Alice is fooled by Bob is exponentially small ($d$ can be sufficiently large by concatenating the code). As we have seen, no communication from Alice to Bob is required for the verification. Therefore, whatever Alice’s measurement device does, Bob cannot learn Alice’s computational information because of the no-signaling principle. In other words, the security of the protocol is device-independent.

V. SECOND PROTOCOL

Let us explain our second protocol, which uses the property of the topological code, and does not use any trap. Alice randomly chooses $k \equiv (h_1, \ldots, h_N, t_1, \ldots, t_N) \in \{0, 1\}^{2N}$, and defines the $N$-qubit operator $K_k = \bigotimes_{j=1}^N T_{j}^{h_j} H_j^{t_j}$, where $T \equiv |0\rangle \langle 0| + |i\rangle \langle i|$, and $H$ is the Hadamard operator. Note that $T^4 = XZ = T$, and $T^4 = XZ = -iX$. Next, Alice defines the $N$-qubit state $|\Psi_k\rangle = K_k |RHG^N\rangle$, where $|RHG^N\rangle$ is the $N$-qubit Raussendorf-Harrington-Goyal state [14, 15] with sufficient number of magic states being already distilled [14, 15].

Bob prepares a universal resource state, and sends each qubit of it to Alice one by one. Alice does measurements and creates $\sigma_\alpha |\Psi_k\rangle$ in Bob’s laboratory, where $\sigma_\alpha$ is the byproduct of the measurement-based quantum computation. Due to the no-signaling principle, Bob cannot learn $k$. Bob sends each qubit of $\sigma_\alpha |\Psi_k\rangle$ to Alice one by one, and Alice does her topological measurement-based quantum computation with correcting $\sigma_\alpha K_k$. If Alice detects any error, she aborts the protocol.

Again, because of Eq. (1), we can assume that Bob’s attack is a random Pauli attack. Therefore let us focus on $\sigma_\alpha |\Psi_k\rangle$. In order for $\sigma_\alpha$ to change a logical state without being detected by syndrome measurements, $\sigma_\alpha$ must contain at least one string $s_\alpha$ of operators which connects or surrounds defects (Fig. (1) (c)) [14, 15, 17].
Since Alice randomly chooses $k$, the probability that all operators in $K_i^j s_a K_k$ become $Z$ or $XZ$ operators is at most $(\frac{3}{4})^{s_a} |s_a|$, where $s_a$ is the weight of $s_a$. Note that $|s_a| \geq d$ because it connects or surrounds defects.

Hence, the probability that the logical state is changed and Alice does not detect any error is at most $\sum_{|\alpha| \geq d} C_{\alpha} (\frac{3}{4})^{s_{\alpha}} \leq (\frac{3}{4})^{d}$. In short, our second protocol is also verifiable. Again, the device-independent security is guaranteed by the no-signaling principle.

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Appendix A: Existence of a CPTP map

Let $\{ |\phi_k \rangle\}_{k=1}^{2^N}$ be any orthonormal basis of the $N$-qubit Hilbert space, $\langle \phi_k | \phi_j \rangle = \delta_{kj}$. We diagonalize the $N$-qubit state $\rho$ as $\rho = \sum_{\alpha=2}^{2^N} \lambda_j |\lambda_j \rangle \langle \lambda_j |$. Let us take $E_{jk} = \sqrt{\lambda_j} |\phi_k \rangle \langle \phi_k |$. Then for any $N$-qubit state $\eta = \sum_{\alpha, \beta} \eta_{\alpha \beta} |\phi_\alpha \rangle \langle \phi_\beta |$, the probability that all $\delta_{kj}$ indices $\eta$ can detect is $\sum_{\alpha, \beta} \eta_{\alpha \beta} \sqrt{\lambda_j} \langle \phi_k | \phi_\alpha \rangle \langle \phi_\beta | \phi_k | = 1$. Furthermore, $\sum_{j,k} E_{jk}^\dagger E_{jk} = \sum_{j,k} \sqrt{\lambda_j} \sqrt{\lambda_j} |\phi_k \rangle \langle \phi_k | = 1$.

Appendix B: Proof of Eq. (2)

For the convenience of readers, we here give the proof of Eq. (2). Since $\alpha \neq \beta$, there exists an index $j$ such that $\sigma_\alpha | j \rangle \neq \sigma_\beta | j \rangle$. For any such $\sigma_\alpha | j \rangle$ and $\sigma_\beta | j \rangle$, we can always take $S \in \{ X, Z \}$ such that $S$ anticommutes only one of $\sigma_\alpha | j \rangle$ and $\sigma_\beta^\dagger | j \rangle$. Let us define $Q \equiv I^{|j|} \otimes S \otimes I^{N-|j|}$. Then, $\sum_q \sigma_\alpha^j \sigma_\alpha q \sigma_\beta^\dagger q \sigma_q = \sum_q (Q q_\alpha^j) \sigma_\alpha (Q q) \rho (Q q_\alpha) \sigma_\beta^\dagger (Q q) = \sum_q (Q q^\dagger Q) \sigma_\alpha (Q q) \rho (Q q^\dagger Q) \sigma_\beta^\dagger (Q q) = - \sum_q \sigma_\alpha^j \sigma_\alpha q \sigma_q^\dagger \sigma_\beta q$. We can obtain the same result for $\max(a, b, c) = b$. For $\max(a, b, c) = c$, we have only to replace $\frac{a}{3}$ with $\frac{1}{3}$.

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