Abstract

Two-quark correlations (diquarks) may play an important role in hadronic physics, particularly near the deconfinement point. This opens the possibility of a net energy gain by means of a (non-perturbative) quark pairing effect, perhaps up to stabilize diquark droplets. We address in the present work the possibility of a self-bound, stable state of bulk diquark matter.
1. Introduction

Considerable interest has been devoted to the physics of dense hadronic matter in the last years. In addition to the astrophysical and cosmological environments where a key role by QCD physics is expected, there is an obvious motivation coming from the significative improvements in accelerator facilities which may provide direct evidence of the state of matter above the saturation density $\rho_o$. Of course we have not been able to solve the dynamics of quarks and gluons in the non-perturbative regime and thus our knowledge of very important issues such as the ground state energy and the onset of phase transitions (chiral, deconfinement) remain uncertain. Lacking of reliable computations in this regime, several phenomenological models have been devised to address these points, the most popular one being the so-called M.I.T. bag [1], which is considerably successful in reproducing most features of the low-energy hadrons.

A particularly interesting suggestion related to these topics was the idea that a high-strangeness variant of the quark-gluon plasma may be absolutely stable [2] and thus the true ground state of hadronic matter. Witten’s strange matter has been the subject of activity concerning its various properties and production/detection mechanisms, as well as the corresponding astrophysical and cosmological consequences [3] (see also Ref.[4] for a discussion of ”strange baryon matter” catalyzed by kaon condensation). Amusingly, there have been also suggestions of metastable or stable exotic particles made out of a few quarks. Examples of this class where symmetry properties are crucial for the lifetime of the state are the $H$ dihyperon [5] and the $Q_\alpha$ boson [6].

Recently, another work [7] raised the possibility of substantial quark-quark ”hyperfine” interactions surviving above the deconfinement point [8]. Therefore, in this diquark picture the quarks remain correlated and lower their energy in this regime, being an intermediate state before the asymptotic freedom region. We shall discuss in this work the possibility of an absolutely stable diquark phase (quite analogous to the strange matter argument), restricting ourselves to the region of high density and low temperatures, and point out some phenomenological consequences to be analyzed elsewhere.
2. Diquark physics

The diquark suggestion is based mainly on the assumption that the most relevant interaction between quarks is of the form

$$H_I = -A \sum_{i \neq j} b_i^\dagger \sigma^a \lambda^B b_i \sigma^a \lambda^B b_j,$$  \hspace{1cm} (1)

where $i, j$ label the quarks, $b, b^\dagger$ are their annihilation and creation operators, $\sigma^a$ are the usual Pauli matrices and $\lambda^B$ the color $SU(3)$ ones. The coupling $A$ is univocally related to the strong interactions coupling constant $\alpha_S$, but has been instead fixed by fitting the $N - \Delta$ mass difference. In terms of the color-spin wavefunctions [9] it can be checked that the most attractive channel is $| \overline{3}, 0 \rangle$ for which

$$< \overline{3}, 0 | H_I | \overline{3}, 0 > = -16A$$  \hspace{1cm} (2)

Because of this fact, we shall refer to the color triplet, spin-zero, isospin zero combination as the diquark [7,8].

A mass of the diquark of $m_D \approx 575$ MeV has been derived by assuming that the $N - \Delta$ mass difference of about 300 MeV is determined by $H_I$. Hence, the expectation is that in the deconfined phase the third quark of a nucleon should pair up with another free one to maximize the attractive energy. As energies grow higher an increasing fraction of "ionized" diquarks would be found, but the former may be totally dominant in a certain density range in the case of self-gravitating matter [10,11]. The question to be answered is if such a diquark matter can exist as a self-bound state, that is to say if the energy gain is enough to stabilize a bit of that matter, or is preferred only under external pressure.

3. The stability of diquark matter

We shall address in this section the stability of a diquark collection. For the same reasons than in the strange matter case, we do not expect that the region of high-$T$, low-$\rho$ can be relevant to this issue because the entropic contribution $-TS$ in the free energy generally desestabilizes non-topological solitons of this kind with respect to an ordinary
nucleon gas [12]. We are thus led to explore the opposite case of low-\(T\) and high-\(\rho\), where diquarks have been modelled by an effective Lagrangian for a color-triplet field \(\phi\)

\[
L = \frac{1}{2} (\partial_\mu \phi^\dagger \partial^\mu \phi - m_D \phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2,
\]  

(note that the coupling \(\lambda\) differs by a factor of 4 from the usual field-theoretical value). By employing a variant of the \(P\)-matrix formalism of Jaffe and Low [13], Donoghue and Sateesh [7] obtained the value \(\lambda = 27.8\) which is indicative of the strength of the repulsion that avoids a Bose condensate at finite density. However, there is still nothing in this description that tells of the fact that diquarks are colored objects and therefore can not escape from the region where the vacuum is able to support them. These confining interactions should probably come from the neglected higher-order terms \(\sum_{n>2} \lambda_n (\phi^\dagger \phi)^n\) in the Lagrangian of eq.(3) which are untractable at present for our problem (note that as a matter of fact the mass is also renormalized by high-order interactions and should be properly denoted as \(m_D^*\) as in nuclear matter calculations, see below). We shall therefore proceed to introduce a vacuum energy density term \(\varepsilon_V\) analogous to the M.I.T. bag constant \(B\) to simulate confinement of diquarks in a finite volume (of course any another confinement model may be also adopted). It is known that in the latter case a good fit to the hadronic spectroscopy can be obtained with a value \(B^{1/4} \approx 145\text{MeV}\) [1]. In the diquark approach, this needs not to be true and we should recalculate everything to extract a sensible value for \(\varepsilon_V\); whose value does not in principle affect the determination of \(\lambda\), but is entangled with the derivation of \(m_D\). We shall treat it as a free parameter in the following of this work.

An approximate equation of state valid for self-interacting bosons as described by the Lagrangian of eq.(3) has been derived in Refs.[7,10] and will be adopted here. That description consists of assuming a Gaussian distribution function \(f(k)\) for the diquarks

\[
f(k) = \frac{N}{2\pi \sigma^{3/2}} e^{\exp(-k^2/2\sigma^2)}
\]

and minimize the energy derived from eq.(3) with respect to the width \(\sigma\). In the limits of high and low diquark densities \(n_D\) the minimization can be carried out analytically.
Furthermore, it can be checked (see for example the Fig.1 of Ref.[10]) that the low-density limit applies whenever \( n_D \leq 10n_o \) and \( f(k) \to \delta(k) \). This is safely within the range we are interested in and justifies the adoption of

\[
P_D = \frac{\lambda}{2m_D^2} n_D^2 \quad (5a)
\]

\[
\rho_D = m_D n_D \quad (5b)
\]

(where \( P_D, \rho_D \) are the diquark pressure and energy density respectively) for the equation of state.

We are now in position to investigate the possibility of a bound state of diquarks. Depending on the flavor content of the mixture, namely isoscalar matter or charge-zero matter, two cases are possible and they will be discussed separately.

\textit{a) Isoscalar matter}

Isoscalar matter having equal numbers of \( u \) and \( d \) quarks would form \( (n_u + n_d)/2 \) diquarks below a density \( \rho_s \approx \rho_o \) [10] due to the lower value of \( m_D \) compared to twice the value of the constituent quark mass. Since the densities relevant for diquark matter stability never exceed \( \rho_o \) our assumption of the absence of free quarks in this case (full pairing) will be justified \textit{a posteriori}.

Bulk matter must be electrically neutral and thus (relativistic) electrons need to be present to satisfy this condition. The electron thermodynamic quantities can be derived from the grand canonical potential \( \Omega = -\mu_e^4/12\pi^2 \) where \( \mu_e \) is the electron chemical potential. In addition, the vacuum effect \( \epsilon_V \) must be added to the energy and subtracted from the pressure as usually done. The program to compute the existence of stable states is quite simple: first we impose the necessary condition of a zero-pressure point

\[
P = P_D + P_e - \epsilon_V = 0 \quad (5)
\]

which, together with the electrical neutrality condition

\[
\frac{1}{3}n_D - n_e = 0 \quad (6)
\]
is used to determine the maximum value of $\varepsilon_V^{1/4}$ which still renders a baryochemical potential $\mu_B = (\sum \rho_i + P_i)/n_B$ lying below the nucleon mass $m_n$ for a given value of $m_D$ (strictly speaking, we should demand $\mu_B$ to be less than the energy per baryon of a $^{56}\text{Fe}$ crystal, which is $\approx 8$ MeV below $m_n$ but we have not taken into account this refinement). Because we expect that the diquark mass $m_D$ gets non-negligible corrections from higher-order contributions, the maximum $\varepsilon_V$ has been determined for a range starting at $m_D^* = 0.8 m_D$ (which is a typical effective value encountered for $m_n^*$ in many-body calculations) and going to the highest value of $m_D$ for which stability is conceivable. The resulting region is given in Fig.1.

b) Charge-zero matter

In this case the mixture already contains as many $d$ quarks as diquarks (i.e. it can be thought as being made of partially "broken" neutrons) and no electrons are needed to neutralize the bulk matter. We have assumed non-relativistic free constituent quarks with mass $m_d = 360$ MeV. Thus, eqs.(5) and (6) are replaced by

$$P = P_D + P_d - \varepsilon_V = 0$$

$$n_D - n_d = 0$$

respectively. The procedure is analogous to the isoscalar case, namely to search for the values of $\varepsilon_V^{1/4}$ which make $\mu_B \leq m_n$ as a function of $m_D$. The results are displayed in Fig.2.

In both considered cases we have found that the maximum allowed values of the vacuum energy density $\varepsilon_V^{1/4}$ are lower than the "canonical" $B^{1/4} \approx 145$ MeV obtained for the M.I.T. bag. In fact, this is not very surprising since the naive sum of diquark + constituent quark energies inside an hadron is already quite close to $m_n$ and therefore such a model would not require a large value of $\varepsilon_V$ to fit its mass. We conclude from these results that a self-bound diquark matter is in principle possible.

The zero-pressure, self-bound diquark state is found to be less dense than strange matter having a density $\rho \approx 4B \approx 4 \times 10^{14} \text{g cm}^{-3}$. The density of the hypothetical diquark
chunks is mainly determined by the value of $\rho_D$ and may lie between $\simeq 10^{13} \, g \, cm^{-3}$ to $1.9 \times 10^{14} \, g \, cm^{-3}$, which are the extreme limits derived from the windows in Figs. 1 and 2.

It is important to remind that even if the vacuum energy density happens to fall inside the stability window it is clear that the latter should be in any case a bulk effect. This is to avoid the spontaneous conversion of nuclei into a diquark soup. In the strange matter picture a minimum (threshold) baryon number $A_{\text{min}} \simeq 10 - 100$ is postulated to exist to preclude the conversion, because if so a simultaneous decay of $\sim A/3$ quarks producing a net strangeness must occur to achieve an energy gain. In the diquark case a similar situation holds, although strange quark production is not needed in principle. The suppression is related to the low probability for an isolated quark of a given nucleon to overlap with another one of a neighbour nucleon (with the right symmetry numbers) inside a nucleus. A few-diquark configuration should be desestabilized by their interactions and to achieve a net energy gain we should reach the bulk limit. In other words confinement acts as a barrier against the decay and stabilization would only be possible once matter is compressed beyond $\rho_o$ and the quarks are free to recombine as they wish (this has been previously noted in Ref.[7]).

On the other hand, it should be noted that we have not demonstrated that interacting diquarks are stable under decays of the type $\phi \to q\bar{q}$. If they are not, the diquark matter would have a lifetime of a fraction of second [14] which is nevertheless very long for particle physics standards and would lead to observable consequences.

4. Conclusions and discussion

We have entertained the possibility of a stable phase of matter composed of diquarks, the latter being quark pairs in the maximally attractive state of the hyperfine interaction Hamiltonian (eq.(1)). This would require a negative pressure produced by the QCD vacuum properties lower than the M.I.T. value $B^{1/4} \simeq 145\text{MeV}$, but as far as we know, not excluded by any experimental fact. The stability window has been found to be slightly larger for the isoscalar than for the charge-zero case.

On phenomenological grounds diquark matter strongly resembles the extensively
addressed strange matter case. Note, however, that there is an important difference connected with the bulk electric charge. Strange matter needs to be neutralized by electrons because the strange quark is massive and these effects lead to a low $Z/A$ value for a given nugget. On the other hand, it has been found that diquark matter can be found in either a isoscalar or charge-zero content, depending on the parent nuclear matter. This means that electrons are not always necessary for ensuring the electrical neutrality of the chunk. A study analogous to the one on strangelet formation [15] would then be interesting to address the probability of a ”diquarklet” production in the central-rapidity region of heavy ion colliders, which are near the isoscalar case. On the other hand it is possible that pure diquark stars may exist, having a maximum mass [16] of $M_{MAX} \simeq 3\left(\frac{\lambda}{27.8}\right)^{1/2}\left(\frac{575\text{MeV}}{m_D}\right)^2 M_\odot$, perhaps covered with a normal matter crust [17]. Note that this contrasts with the diquark stars of Refs.[10,11] where diquark matter was assumed to appear only under pressure. Several previous works on the combustion of nuclear matter [18], supernovae [19], etc. made for the strange matter case can be carried over for the diquark case as well and will be the subject of future publications.

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Figure captions

**Figure 1.** The stability window of isoscalar diquark matter (hatched) as a function of the vacuum energy density $\varepsilon_V$ and the (effective) diquark mass $m_D$ (see the text for details). Both axis are measured in MeV.

**Figure 2.** The same as in Fig.1 for charge-zero diquark matter.