Dressing of superconducting qubits by their interaction with a low frequency photon reservoir

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Abstract. Dressing of states of superconducting qubits by their interaction with low frequency photons is investigated. We show that the derivative of the self-energy function of a qubits state at its eigenenergy is equal to the total probability to find the system in the virtual states where the reservoir degrees of freedom manifest themselves explicitly. This parameter determines the strength of the qubit-reservoir interaction. The results of calculations of the self-energy function show that contributions to the dressed states of qubits can be very significant.

1. Introduction

The rapid progress in development of device design and fabrication methods has allowed to increase the coherence time of superconducting qubits by five orders of magnitude [1–7]. Nevertheless the further improvement in coherence time is still needed [8–13]. Any qubits during its evolution interacts with the surrounding environment, and the interaction gives rise to the decoherence [14, 15]. In superconducting circuits various physical processes give rise to the dephasing [16–19], such as fluctuating charged impurities, and flux noise. These fluctuations cause the $1/f$ flux [20,21]. One of the sources of dephasing in superconducting qubits is the low frequency photon noise [21,22]. In some sense this dephasing is similar to the phonon dephasing in the quantum dots. In both cases one deals with thermal massless low energy bosons. The quantum fluctuations during which the reservoir degrees of freedom manifest themselves in virtual states dressed an qubit. As a consequence, the state of an qubit becomes dressed. The vector of such a state is the sum of the bare state and the state in that determines the probability to find the system in virtual state, and summing these probabilities we get the total probability $P_{\text{virt}}$ find the system in the virtual states. In this paper we show that $P_{\text{virt}}$ can be derived from the self-energy functions characterizing quantum fluctuations in superconducting qubits. This point is demonstrated on the example of quantum fluctuations induced by low frequency photon noise.

2. Method

In our investigation of the problem we use the generalized dynamical equation (GDE), which in \cite{23, 24} has been derived as a most general dynamical equation consistent with current
concepts of quantum physics. Being equivalent to the Schrödinger equation in the case when the interaction in a quantum system is instantaneous, this equation allows one to extend quantum dynamics to the case of nonlocal-in-time interactions. GDE is formulated in terms of the operator $\tilde{S}(t_2,t_1)$, which describes the contribution to the evolution operator from the processes in which the interaction begins at time $t_1$ and ends at time $t_2$. This equation allows one to obtain $\tilde{S}(t_2,t_1)$ for any $t_1$ and $t_2$, if the contributions from the processes associated with infinitesimal duration times $t_2 - t_1$ of the interaction are known. Most of the contribution to the evolution operator in the limit $t_2 \rightarrow t_1$ comes from the processes associated with the fundamental interaction in a system. Denoting this contribution by $H_{int}(t_2,t_1)$ we get boundary condition $\tilde{S}(t_2,t_1) \rightarrow H_{int}(t_2,t_1)$ at $t_2 \rightarrow t_1$. The evolution operator in the Schrödinger picture can be represented in the form [25–29].

$$U_S(t,0) = \frac{i}{2\pi} \int_{-\infty}^{\infty} dz \exp(-izt)G(z)$$

with $G(z) = G_0(z) + G_0(z)T(z)G_0(z)$, where $G_0(z) = (z - H_0)^{-1}$, $H_0$ is the free Hamiltonian, and the operator $T(z)$ is defined as

$$T(z) = i \int_{0}^{\infty} d\tau \exp(-iz\tau) \exp(-iH_0t_2) \tilde{S}(t_2,t_1) \exp(iH_0t_1),$$

which is used with the boundary condition $T(z) \rightarrow B(z)$ at $|z| \rightarrow \infty$, where

$$B(z) = i \int_{0}^{\infty} d\tau \exp(i\tau) \exp(-iH_0t_2) M_{int}(t_2,t_1) \exp(iH_0t_1).$$

The contribution to the Green operator $G(z)$, which comes from the processes associated with the self-interaction of particles, has the same structure as the free Green operator $G_0(z)$. That is why it is natural to replace $G_0(z)$ by the propagator $G_0^{(\nu)}(z)$, which describes the evolution of particles interacting only with vacuum and hence has the structure $\langle m' | G_0^{(\nu)}(z) | m \rangle = \langle m' | m \rangle (z - E_m - C_m(z))^{-1}$, where $|m\rangle$ are eigenvectors of the free Hamiltonian $(H_0 | m \rangle = E_m | m \rangle)$. Correspondingly, the operator $T(z)$ should be replaced by the operator $M(z)$, which describes the evolution of particles interacting not only with vacuum. These operators are related as follows:

$$G(z) = G_0(z) + G_0(z)T(z)G_0(z) = G_0^{(\nu)}(z) + G_0^{(\nu)}(z)M(z)G_0^{(\nu)}(z),$$

The function $C_m(z)$ describes self-interaction of particles in the state $|m\rangle$, and the condition $z - E_m - C_m(z) = 0$ determines the physical masses of the particles. Taking into account (5), one can rewrite (3) in terms of $M(z)$ and $C_m(z)$, satisfying the boundary conditions $M(z) \rightarrow B_c(z)$, $C_m(z) \rightarrow \langle m | B_\delta(z) | m \rangle$. Here $B_c(z)$ is the part of the interaction operator describing the proper interaction between particles, and $B_\delta(z)$ describes their self-interaction: $B(z) = B_c(z) + B_\delta(z)$, where "singular" term $B_\delta(z)$ has the structure $\langle m' | B_\delta(z) | m \rangle = \langle m' | m \rangle B_\delta^{(m)}(z)$. On the example of superconducting qubits we show that the self-energy function carries information
about the strength of the qubit-reservoir interaction and the probabilities to find the qubit in the bare state and in the virtual states. In terms of $M(z)$ equation for the self-energy function takes form:

$$
\frac{d \langle n | C(z) | n \rangle}{dz} = -\sum_{m} \frac{\langle n | M(z) | m \rangle \langle m | M(z) | n \rangle}{(z - E_{m} - C_{m}(z))^{2}}.
$$

(6)

3. Quantum fluctuations in superconducting qubits in a low frequency photons reservoir

The system under consideration is a Cooper pair box [7, 21, 30]. Under appropriate conditions (charging energy $E_{C}$ much larger than the Josephson coupling $E_{J}$ and temperatures $k_{B}T \ll E_{J}$) only two charge states are important, and the Hamiltonian of the qubit $H_{Q}$ reads

$$
H_{Q} = \frac{\delta E_{C}}{2} \sigma_{z} - \frac{E_{J}}{2} \sigma_{x},
$$

(7)

where the charge basis $\{ |0\rangle, |1\rangle \}$ is expressed using the Pauli matrices, and the bias $\delta E_{C} \equiv E_{C} (1 - \frac{eV_{x}}{h})$ can be tuned by varying the applied gate voltage $V_{x}$. The Hamiltonian (7) describes only qubits as quantum objects. This treatment is only reasonable in the limit of large number of photons. The processes in which superconducting qubits interact with single microwave photons are described by circuit QED. The ground and excited states of a qubit are described by the vectors $|g, n\rangle$ and $|e, n\rangle$. When the microwave cavity is close to resonance with a qubit, its eigenstates determined by the Jaynes-Cummings are of the form:

$$
|+ , n\rangle = \cos \theta_{n} |e, n\rangle + \sin \theta_{n} |g, n+1\rangle,
$$

(8)

$$
|- , n\rangle = - \sin \theta_{n} |e, n\rangle + \cos \theta_{n} |g, n+1\rangle,
$$

(9)

where $\tan (2\theta_{n}) = -\Omega_{n} / \Delta$ with $n$, $\Delta = \omega_{L} - \omega_{R}$ and $\Omega_{n}$ being the number of photons in the mode $\omega_{L}$, the detuning ($\omega_{R} = \omega_{e} - \omega_{g}$) and the Rabi frequency, respectively. The energies of the dressed states (8) and (9) are given by (here and below, we use the natural system of units in which $\hbar = c = 1$)

$$
E_{\pm,n} = \left( n + \frac{1}{2} \right) \omega_{L} + \omega_{R} / 2 \pm \Omega_{R}^{2} / 2,
$$

(10)

where $\Omega_{R}^{2}$ is the renormalized Rabi frequency. The splitting $\Omega$ between the levels $|+, n\rangle$ and $|-, n\rangle$ is, in general, small compared with $k_{B}T$, and, as a consequence, the dephasing charge noise and the low frequency photon noise in the microwave cavity can induce transitions between dressed states of the same manifold. Let us now consider the interaction of a charge qubit coupled to a stripline cavity with a reservoir of low frequency photons in the cavity. The ground and excited states be the states $|0\rangle$ and $|1\rangle$, respectively. This choice of the qubit is motivated by the fact that the ground state is almost the pure state $|0\rangle$, and only in the states where the charge differs from zero the qubit interacts with the environment.

Let us consider the qubit dressed only by the thermal low frequency photons in a stripline cavity. Let $|\mu\rangle$ be the eigenstate of the reservoir of low frequency photons with the energy $E_{\mu}$. The eigenstates $|\mu\rangle$ with the energy $E_{\mu}$ describe the reservoir environment as a system with almost infinite number degrees of freedom, and hence the change in energy of the states $|\mu\rangle$ caused by the interaction of a charge qubit coupled to a stripline cavity with a reservoir of low frequency photons is extremely small compared to the energy of these states. For this reason, it is convenient to set $E_{\mu}$ to be the zero energy point. The zero point for $z$ is changed correspondingly $z' = z - E_{\mu} \rightarrow z$. The reservoir-qubit interaction creates and destroys particles in the reservoir. Creating a particle raises the energy, whereas the particle destruction lowers the energy. In general, the interaction consists of a number of creation and destruction processes.
The eigenstates for which the contributions from these processes to the energy is positive will be referred to as $| N; q^+, \mu \rangle$. Replacing all the creation (destruction) operators of additional particles in the state $| q^+, \mu \rangle$ with the energy $E_q$ by the corresponding destruction (creation) operators of the same modes we arrive at the state $| N; q^-, \mu \rangle$ with the energy $-E_q$. The equation for the self-energy function of a charge qubit coupled to the low frequency photon reservoir takes the form:

$$\frac{dC_{N\mu}(z)}{dz} = -\sum_{j, q^\pm} \langle N; \mu | M(z) | j, \mu, q^\pm \rangle \langle j; \mu, q^\pm | M(z) | N; \mu \rangle \left( z - E_N \mp E_q - C_{N\mu}(z) \mp E_q \right)^2. \quad (11)$$

It follows that the bare state $| \Psi^0_N \rangle \equiv | N \rangle$ with $N$ Cooper pairs ($N \to 1$) is described by the corresponding "free" Green operator

$$\langle \Psi^0_N | \tilde{G}_0(z) | \Psi^0_N \rangle = \frac{1}{z - E_N(0) - C_N(z)} = Z \frac{1}{z - E_N - C_N(z) - i \frac{\Gamma_N}{2}}, \quad (12)$$

where the energy $\Delta E_N = \text{Re} C_N(z = E_N)$ is included into the energy $E_N$, and

$$\tilde{C}_N(z) = \frac{1}{Z} (C_N(z) - C_N(E_N) - \eta) \quad (13)$$

$$\frac{\Gamma_N}{2} = \text{Im} \frac{C_N(E_N)}{1 - \eta}. \quad (14)$$

Here $\eta = \frac{dC_N(z)}{dz} \bigg|_{z=E_N}$ and $Z = (1 - \eta)^{-1}$. Substituting (5) into (1) we get

$$A_{N\mu}(t) = \langle \Psi^0_N | U(t, 0) | \Psi^0_N \rangle = Z^2 \exp \left( -\frac{\Gamma_N}{2} t \right). \quad (15)$$

Thus, the probability $P_N(t) = |A_{N\mu}(t)|^2$ for the qubit to remain in the same bare state $| \Psi^0_N \rangle$ after a time $t$ is not only damped with the rate $\Gamma_N$, but also is less than unit even at time $t = 0$. To explain this fact, note that due to quantum fluctuations the qubit spends not the whole time in its bare state, a part of time it is in a virtual state where the reservoir degrees of freedom manifest themselves. As it follows from the definition of the operator $M(z)$, the probability amplitude $a_{N\nu\mu}$ to find the system in the virtual state $| \nu, \mu \rangle$ is

$$a_{N\nu\mu} = \frac{\langle N; \nu, \mu | M \left( z = E_{N\mu} \right) | \Psi^0_{N\mu} \rangle}{-E_{\nu} - C_{N\mu}(E_{N\mu})}. \quad (16)$$

Thus, for the total probability to find the system in virtual states we get

$$P_{\text{virt}} = \sum_{\nu} |a_{N\nu\mu}|^2. \quad (17)$$

The above means that the state vector of the qubit dressed by the reservoir takes the form

$$| \Psi_N \rangle = | \Psi^0_N \rangle + \sum_{\nu} a_{N\nu\mu} | \Psi^0_{N\nu} \rangle \quad (18)$$

Let us make the approximation in which only one photon is created or annihilated, and the operator $M(z)$ is reduced to the Hamiltonian

$$H_I = \sum_N \sum_q g(a_q + a_q^\dagger) |N\rangle \langle N|, \quad (19)$$
where the operators $a_q^\dagger$ and $a_q$ describe the creation and annihilation of the photon with momentum $q$ and frequency $\omega_q$, $|x\rangle$ is the state of the qubit. We also neglect the operator $C(z)$ in the denominator.

Averaging (11) in this approximation yields

$$
\frac{dC_N(z)}{dz} = -\frac{L}{2\pi} \int \frac{\omega^2 e^{-\omega^2}}{\omega_c^2 e^{-\omega^2}} \left\{ \frac{g^2 [1 + n(\omega)]}{(z - E_N - C_N (z - \omega) - \omega)^2} + \frac{g^2 n(\omega)}{(z - E_N - C_N (z + \omega) + \omega)^2} \right\} d\omega,
$$

(20)

where $L$ is the resonator length, $n(\omega) = \frac{1}{e^{\frac{\omega}{T} - 1}}$ and coupling constant $g = \pi \Omega$ is defined by the shift with Rabi frequency $\Omega$.

![Graph](image)

**Figure 1.** The derivative of the self-energy function $\eta = \frac{dC(z=E_N)}{dz}$.

4. Conclusion

We have shown that the derivative of the self-energy function at $z = E_N$ is equal to the total probability $P_{\text{virt}}$ to find the system in virtual states in which the reservoir degrees of freedom manifest themselves explicitly. We have also derived the form of qubit states dressed by its interaction with a reservoir. The results of calculations of the probability $P_{\text{virt}}$ to find the qubit in the state (18) can be very high, and this can have a significant effect on dephasing processes.

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