Generalized Reconfigurable Intelligent Surfaces: From Transmitting and Reflecting Modes to Single-, Group-, and Fully-Connected Architectures

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Abstract—Reconfigurable intelligent surfaces (RISs) are envisioned as a promising technology for future wireless communications. With various hardware realizations, RISs can work under different modes (reflective/transmissive/hybrid) or have different architectures (single-/group-/fully-connected). However, most existing research focused on either reflective RISs or single-connected hybrid RISs while there is lack of a comprehensive study for RISs unifying different modes/architectures. In this paper, we solve this issue by analyzing and proposing a general RIS-aided communication model which unifies the reflective/transmissive/hybrid modes and single-/group-/fully-connected architectures. With the proposed model, we consider jointly designing the transmit precoder and RIS beamformer to maximize the sum-rate for RIS-aided systems. Leveraging fractional programming theory, the original sum-rate maximization problem is equivalently transformed into a multi-block optimization, which can be solved by block coordinate descent methods. We also provide simulation results to compare the performance of RISs with different modes/architectures. Compared with single-connected hybrid RISs, fully- and group-connected hybrid RISs can increase the sum-rate by around 75% and 37% for Rayleigh fading channels.

Index Terms—Architectures, modes, reconfigurable intelligent surface (RIS), RIS-aided communication model.

I. INTRODUCTION

Reconfigurable intelligent surfaces (RISs), which can build controllable radio environments and improve communication quality in a cost-effective manner [1]-[3], have recently been regarded as a revolutionary technology for wireless communication research. An RIS is a two-dimensional planar surface which consists of a large number of nearly passive elements with ultra-low power consumption. Each element has a multilayer structure composed of rectangle patches and tunable devices, e.g., positive intrinsic negative (PIN) diodes. Each PIN diode of each element can be independently switched between “ON” and “OFF” states, thereby generating different responses (i.e., phase shifts and amplitudes) for incident signals [4]. Thus, RISs can be flexibly deployed into various wireless communication systems to modify the propagation environment without much power consumption.

There are various tunable surface designs for RIS realizations [5]. On one hand, when each element of the RIS has different hardware realizations (i.e., the number of layers, size, thickness, the number and distributions of PIN diodes), the RIS can work under three modes: transmission, reflection, and hybrid transmission and reflection [6], [7]. As for the reflective/transmissive RIS, incident signals are either reflected from or transmitted through the RIS towards the same/opposite side as the transmitter. For the hybrid transmissive and reflective RIS, incident signals are split into two parts and can simultaneously arrive at both sides of the RIS. On the other hand, RISs with different architectures have recently been modeled and designed by using scattering parameter network analysis [8]. According to the connection topology among different RIS elements, RIS can be classified into three kinds of architectures: single-, group-, and fully-connected RIS. The single-connected RIS, in which each element is not connected with each other, has been widely considered in existing research. Furthermore, when all/part of RIS elements are connected with each other, fully-/group-connected RISs can be implemented to further enhance the RIS performance.

The single-connected reflective RIS is most commonly considered in previous research. Beamforming designs for single RIS-aided wireless communication systems were developed using different metrics, e.g., power minimization, rate maximization, max-min fairness [9]-[11]. Considering the high hardware complexity and cost of realizing RISs with infinite/high-resolution phase shifts, research on practical finite/low-resolution cases was also developed [12]-[14]. To improve the system performance, some researchers studied the deployment of multiple RISs and coordination problems [15]-[17]. In addition, RIS has also been investigated to provide performance enhancement in various scenarios, e.g., Wireless Power Transfer (WPT) [18], Simultaneous Wireless Information and Power Transfer (SWIPT) [19]. Developing accurate and efficient channel estimation strategies for RIS-aided communication systems was another important topic. For example, different channel estimation algorithms for RIS-aided systems were proposed based on either traditional communication theories [20] or machine learning approaches [21]. While the aforementioned research on reflective single-connected RIS [9]-[21] assumes an ideal RIS reflection model, practical RIS model analysis has been proposed for both narrowband [22] and wideband systems [23]-[25].

Limited work has been carried out for reflective RISs with different architectures. [8] first proposed to analyze the modeling of RIS based on the scattering parameter network and categorized the RIS into the three single-, group-, and
fully-connected RIS architectures. Interestingly, in contrast to single-connected RIS that relies on diagonal phase shift (scattering) matrix to only adjust the phases of the impinging waves, the group- and fully-connected RIS rely on non-diagonal matrix which enables them to adjust not only the phases but also the magnitudes of the impinging waves, therefore providing additional performance enhancements over single-connected RIS. Theoretical analysis and numerical discussions for an RIS-aided SISO system were also provided in [8] to compare these three architectures. Following this work, the authors in [26] further considered designs of single-/group- and fully-connected RIS with discrete values, and discussed different grouping strategies for group-connected RISs. Results suggest that while four resolution bits are needed in single-connected RIS, only a single resolution bit is sufficient in fully-connected RIS, simplifying significantly the future development of these promising RIS architectures. Motivated by those benefits of non-diagonal phase matrices, authors in [27] branched out to propose another novel RIS architecture with non-diagonal phase shift matrices, which resulted in a higher rate compared to conventional single-connected cases.

Recently, some researchers have started to focus on single-connected hybrid RISs. Xu et al. [28] introduced the concept of hybrid transmissive and reflective RIS, which was also referred to as simultaneously transmitting and reflecting RIS (STAR-RIS) or intelligent omni surface (IOS) [5], [29], generated hardware and channel models for RISs, and analyzed the achievable diversity gain. In [30], the authors proposed three practical operating protocols for hybrid RISs and considered corresponding beamforming designs in both unicast and multicast systems. Following the hardware model proposed in [28] and the practical operating protocols in [30], authors in [31]-[34] further diversified application scenarios by considering various designs, including beamforming design [31], resource allocation [32], coverage characterization [33], and secrecy design [34], for orthogonal multiple access (OMA) and/or non-orthogonal multiple access (NOMA) networks.

Nevertheless, it is worth noting that the aforementioned studies about RISs are either restricted to reflective RISs with different architectures [9]-[27] or for single-connected hybrid RISs [28]-[34]. There is a lack of a thorough analysis of RISs under different modes and with different architectures, which motivates this work. The main contributions of this paper are summarized as follows:

First, we analyze and propose a general RIS-aided communication model using microwave network theory. The proposed general model is derived by characterizing an RIS as multiple antennas connected to a multiple-port reconfigurable impedance network. Based on some assumptions and corollaries, we show that the existing STAR-RIS-aided communication model used in [28]-[34] is a special instance of the proposed general model.

Second, we propose a generalized RIS model by investigating three modes (reflective, transmissive, and hybrid (STAR)) and three architectures (single-, group-, and fully-connected), in total nine cases. This is the first paper to characterize RIS by taking nine different modes/architectures into consideration. The generalized RIS model in this paper can contain almost all of the existing RIS models (except for some practical RIS models proposed in [22]-[25] and the novel non-diagonal architecture proposed in [27]).

Third, with the proposed generalized RIS model, we consider the joint transmit precoder and RIS beamformer design to maximize the sum-rate for a RIS-aided multiuser multiple input single output (MU-MISO) system. Since the proposed model generates new and different constraints for RIS with different modes/architectures, existing algorithms for reflective RISs or single-connected hybrid RISs are not feasible any more. In this paper, we first propose a general solution which is suitable for RISs with nine modes/architectures. Then for single-connected RIS, we propose an efficient solution, which has similar performance but lower computational complexity compared to the general one. Convergence and complexity of the proposed algorithms are analyzed and compared.

Fourth, we provide simulation results to evaluate the performance of the proposed design. We compare the sum-rate performance when the RIS is under nine different modes/architectures. Results show that under Rayleigh fading conditions, fully- and group-connected hybrid RISs can achieve 75% and 37% higher sum-rate than single-connected hybrid ones. In the meanwhile, the sum-rate performance achieved by fully-connected hybrid RISs over that by fully-connected reflective/transmissive ones can be around 20% for Rician fading channels.

Organization: Section II introduces an RIS-aided communication model. Then Section III presents a generalized RIS model based on architecture/mode analysis and design. With the proposed model, Section IV formulates an optimization problem and illustrates solutions for the formulated problem. Section V evaluates the performance of the proposed design. Finally, Section VI provides conclusions.

Notations: Boldface lower-case and upper-case letters indicate column vectors and matrices, respectively. \( \mathbb{C} \) and \( \mathbb{R} \) denote the set of complex and real numbers, respectively. \( \mathbb{E}\{\cdot\} \) represents statistical expectation. \((\cdot)^*\), \((\cdot)^T\), \((\cdot)^H\), and \((\cdot)^{-1}\) denote the conjugate, transpose, conjugate-transpose operations, and inversion, respectively. \( \mathbb{R}\{\cdot\} \) denotes the real part of a complex number. \( I_L \) indicates an \( L \times L \) identity matrix. \( \mathbf{0} \) denotes an all-zero matrix. \( \|\mathbf{A}\|_F \) denotes the Frobenius norm of matrix \( \mathbf{A} \). \( |a| \) denotes the norm of variable \( a \). \( |\mathbf{A}| \) denotes the size of set \( \mathcal{A} \). \( \mathbf{diag}(\cdot) \) denotes a diagonal matrix. \( \text{blkdiag}(\cdot) \) denotes a block matrix such that the main-diagonal blocks are matrices and all off-diagonal blocks are zero matrices. \( j = \sqrt{-1} \) denotes imaginary unit. \( \text{Tr}(\cdot) \) denotes the summation of diagonal elements of a matrix. Finally, \( (\mathbf{A})_{i,:} \), \( (\mathbf{A})_{,:j} \), and \( (\mathbf{A})_{i,j} \) denote the \( i \)-th row, the \( j \)-th column, and the \((i,j)\)-th element of matrix \( \mathbf{A} \), respectively.

II. RIS-AIDED COMMUNICATION MODEL

Consider an RIS-aided wireless communication system, where an \( N \)-antenna transmitter indexed by \( \mathcal{N} = \{1, \ldots, N\} \) serves \( K \) multi-antenna users with the assistance of an RIS. Each user has \( N_k \) antennas, \( \forall k \in \mathcal{K} = \{1, \ldots, K\} \). In general,
we consider an $M$-cell RIS indexed by $\mathcal{M} = \{1, \ldots, M\}$ where the $m$th cell consists of antenna $m$ and antenna $m+M$. Therefore, in this $M$-cell RIS, there are $2M$ antennas connected to a $2M$-port reconfigurable impedance network, as shown in Fig. 1. According to [8], assuming that the multiple antennas at the transmitter/receiver/RIS are perfectly matched with no mutual coupling, the overall channel matrix between the transmitter and each user, $H_{\text{all},k} \in \mathbb{C}^{N_k \times N_k}$, $k \in \mathcal{K}$, can be expressed as

$$H_{\text{all},k} = H_{d,k} + \mathbf{H}_k \Phi \mathbf{G}, \forall k \in \mathcal{K},$$

where $H_{d,k} \in \mathbb{C}^{N_k \times N_k}$, $\mathbf{H}_k \in \mathbb{C}^{N_k \times 2M}$, $\forall k \in \mathcal{K}$, and $\mathbf{G} \in \mathbb{C}^{2M \times N_k}$ denote channel matrices from the transmitter to user $k$, from the RIS to user $k$, and from the transmitter to the RIS, respectively. $\Phi \in \mathbb{C}^{2M \times 2M}$ denotes the scattering matrix of the $2M$-port reconfigurable impedance network [8]. According to microwave network theory [37], [8], $\Phi$ should satisfy $\Phi^H \Phi \preceq I_{2M}$. Particularly, when the $2M$-port reconfigurable impedance network is lossless, we have the condition that $\Phi^H \Phi = I_{2M}^2$.

To show that existing RIS-aided communication model [28]-[34] is a special case of our proposed model, we make the following assumptions:

A1: Each antenna has a uni-directional radiation pattern.

A2: In each cell, the two antennas are back to back placed so that each antenna covers half space as shown in Fig. 2.

With these assumptions, we can divide the $M$-cell RIS into 2 sections, where the first section consists of antennas $1$-$M$ and the other section consists of antennas $M+1$-$2M$, and thus the $M$-cell RIS partitions the whole space into two sides, which are respectively covered by the two sections. Accordingly, we can partition the overall channel matrix (1) as

$$H_{\text{all},k} = H_{d,k} + \mathbf{H}_k \Phi \mathbf{G}, \forall k \in \mathcal{K},$$

where $H_{d,k} \in \mathbb{C}^{N_k \times N_k}$, $\mathbf{H}_k \in \mathbb{C}^{N_k \times 2M}$, $\forall k \in \mathcal{K}$, and $\mathbf{G} \in \mathbb{C}^{2M \times N_k}$ denote channel matrices from the transmitter to user $k$, from the RIS to user $k$, and from the transmitter to the RIS, respectively. $\Phi \in \mathbb{C}^{2M \times 2M}$ denotes the scattering matrix of the $2M$-port reconfigurable impedance network [8]. According to microwave network theory [37], [8], $\Phi$ should satisfy $\Phi^H \Phi \preceq I_{2M}$. Particularly, when the $2M$-port reconfigurable impedance network is lossless, we have the condition that $\Phi^H \Phi = I_{2M}^2$.

Without loss of generality, we assume that the transmitter and $K$ users, indexed by $\mathcal{K} = \{1, \ldots, K\}$, are located at one side of the RIS (covered by section 1), and $K_2 = K - K_1$ users, indexed by $\mathcal{K}_2 = \{K_1 + 1, \ldots, K\}$, are located at the other side of the RIS (covered by section 2). To facilitate understanding, we illustrate the locations of the transmitter, RIS, and users from a top view in Fig. 3. Following assumptions A1 and A2, we can deduce two corollaries:

C1: The channel from the transmitter to section 2 of the RIS is zero, that is $\mathbf{G}_2 = \mathbf{0}$, as the transmitter is not covered by the uni-directional radiation pattern of section 2.

C2: The channel from section $i$ of the RIS to the user belongs to $\mathcal{K}_j$ is zero $\forall i \neq j$, that is $\mathbf{H}_{r,i} = \mathbf{0}$, $\forall k \in \mathcal{K}_j, \forall i \neq j$. 

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1Here we use “cell” instead of “element” since in existing works each RIS element contains one antenna, while in this work each cell has two antennas. The “cell” is illustrated in Fig. 1.

2Here we should explain that matrix $\Phi$ is a scattering matrix, which provides an integrated description of a network as seen at its $2M$ ports. The scattering matrix associates the voltage of incident waves with reflected waves from the ports. When the multi-port network is lossless, no power can be delivered to the network so that the incident and reflected power are equal to each other. In this case, it can be derived that the scattering matrix satisfies a unitary constraint [37].

3In practice the uni-directional radiation pattern can be achieved by using microstrip patch antennas or by adding a reflecting ground plane.
as the user belongs to \( K_j \) is not covered by the unidirectional radiation pattern of section \( i \).

Leveraging corollaries C1 and C2, we can simplify the channel matrix (2) as

\[
 H_{all,k} = \begin{cases} 
 H_{d,k} + \prod_{k,1} \Phi_{1,k}, G, & k \in K_1, \\
 H_{d,k} + \prod_{k,2} \Phi_{2,k}, G, & k \in K_2.
\end{cases}
\]  

(3)

We denote users within the coverage of section 1 of the RIS as reflective users, indexed by \( K_r = K_1, K_r = K_1 \). Similarly, we denote users within the coverage of section 2 as transmissive users, indexed by \( K_t = K_2, K_t = K_2 \). Then, using auxiliary notations, \( H_k = H_{d,k}, \forall k \in K_i, \forall i \in \{1, 2\}, \Phi_r = \Phi_{1,1}, \Phi_t = \Phi_{2,1}, \) and \( G = G_1 \), we can rewrite the simplified channel (3) as

\[
 H_{all,k} = \begin{cases} 
 H_{d,k} + H_r \Phi_1 G, & k \in K_1, \\
 H_{d,k} + H_t \Phi_2 G, & k \in K_2.
\end{cases}
\]  

(4)

Given that \( \Phi_r^{H} \Phi_r = I_M \) in the above discussion, \( \Phi_1 \) and \( \Phi_r \) should satisfy the following constraint:

\[
 \Phi_r^{H} \Phi_r + \Phi_t^{H} \Phi_t = I_M.
\]  

(5)

Based on the above analysis, we have verified that the communication model utilized in existing STAR-RIS works [28]-[34] is a special instance of our models (4) and (5). In the following section, we will propose a generalized RIS model by investigating different cases of the constraint (5) and will show that the model utilized in STAR-RIS works [28]-[34] is a special case of our proposed model.

III. ARCHITECTURE/MODE ANALYSIS AND DESIGN

A. Architecture Analysis and Design

In [8], three kinds of RIS architectures which have different RIS element connection topologies have been proposed. In this subsection, we generalize this concept by analyzing and designing the \( M \)-cell RIS architecture with different RIS connection topologies as detailed below.

1) Single-Connected Architecture: In this architecture, RIS cells are not connected to each other. Thus, matrices \( \Phi_r, \Phi_t \) are all diagonal, which are given by

\[
 \Phi_r = \text{diag}(\phi_{1,1}, \ldots, \phi_{r,M}),
\]  

(6a)

\[
 \Phi_t = \text{diag}(\phi_{1,1}, \ldots, \phi_{t,M}),
\]  

(6b)

where the entries \( \phi_{i,m}, \forall m \in M \), satisfy the constraints

\[
 |\phi_{i,m}|^2 + |\phi_{i,m}|^2 = 1, \forall m \in M.
\]  

(7)

2) Fully-Connected Architecture: We propose and design a generalized RIS architecture referred to as fully-connected architecture where all RIS cells are connected to each other through reconfigurable impedance components. Thus, \( \Phi_r, \Phi_t \) are all full matrices satisfying the constraint

\[
 \Phi_r^{H} \Phi_r + \Phi_t^{H} \Phi_t = I_M.
\]  

(8)

The fully-connected architecture is able to provide the best RIS performance due to the most general constraint (8).

3) Group-Connected Architecture: It is obvious that when the number of RIS cells \( M \) gets larger, the circuit topology of the fully-connected RIS will become extremely complicated. To achieve a good trade-off between the RIS performance and circuit complexity, we propose and design a generalized RIS architecture referred to as group-connected architecture. In the group-connected RIS, the \( M \) cells are divided into \( G \) groups indexed by \( G = \{1, \ldots, G\} \) with each group of RIS cells utilizing the fully-connected architecture. For simplicity, here we assume that all groups have the same size \( M/G \) and we define \( G_g = \{(g-1)M + 1, \ldots, gM\} \) as the set of RIS cell indexes for group \( g \). It should be noted that there are various grouping strategies while exploring different kinds of grouping strategies is left as future work. Thus, \( \Phi_r, \Phi_t \) are block diagonal matrices, which are given by

\[
 \Phi_r = \text{blkdiag}(\Phi_{r,1}, \ldots, \Phi_{r,G}),
\]  

(9a)

\[
 \Phi_t = \text{blkdiag}(\Phi_{t,1}, \ldots, \Phi_{t,G}),
\]  

(9b)

where \( \Phi_{r,g} \in C^{M \times M}, \forall g \in G \) satisfies the constraint

\[
 \Phi_r^{H} \Phi_{r,g} + \Phi_t^{H} \Phi_{t,g} = I_M, \forall g \in G.
\]  

(10)

B. Mode Analysis

According to the proportion of energy split for reflection and transmission, the RIS is able to realize the following three different modes [11]:

1) Reflection: The RIS only reflects signals towards the same side as the transmitter, i.e., \( \Phi_r = 0, \Phi_t^{H} \Phi_t = I_M \).

2) Transmission: Incident signals can only penetrate the RIS, i.e., \( \Phi_r = 0, \Phi_t = 0, \Phi_t^{H} \Phi_t = I_M \).

3) Hybrid Reflection and Transmission: Incident signals can both reflect from and transmit through the RIS, which leads to a dual function of reflection and transmission, i.e., \( \Phi_r \neq 0, \Phi_t 
eq 0, \Phi_r^{H} \Phi_r + \Phi_t^{H} \Phi_t = I_M \).

C. Unified Architecture and Mode

Combining the three different RIS architectures and the three different RIS modes, there are in total nine cases. For clarity, the RIS models for the nine cases are summarized in Table I.

| Modes       | Architectures                  | Single-connected | Group-connected | Fully-connected |
|-------------|--------------------------------|------------------|-----------------|----------------|
| Reflective  | \[\phi_{r,m}\] = 1, \forall m  | \(\Phi_r^{H} \Phi_r = I_M, \forall g \in G\) | \(\Phi_r^{H} \Phi_r = I_M\) | \(\Phi_r^{H} \Phi_r = I_M\) |
| Transmissive| \[\phi_{r,m}\] = 1, \forall m  | \(\Phi_t^{H} \Phi_t = I_M, \forall g \in G\) | \(\Phi_t^{H} \Phi_t = I_M\) | \(\Phi_t^{H} \Phi_t = I_M\) |
| Hybrid      | \[\phi_{r,m}\]^2 + \[\phi_{t,m}\]^2 = 1, \forall m \in M | \(\Phi_r^{H} \Phi_r + \Phi_t^{H} \Phi_t = I_M\) | \(\Phi_r^{H} \Phi_r + \Phi_t^{H} \Phi_t = I_M\) | \(\Phi_r^{H} \Phi_r + \Phi_t^{H} \Phi_t = I_M\) |
Remark 1. In [8], an M-element RIS is modeled as M antennas connected to an M-port reconfigurable impedance network. In this work, we further generalize the RIS model by modeling an M-cell RIS as 2M antennas connected to a 2M-port network. Namely, each RIS “element” in [8] contains a single antenna, while each RIS “cell” in this work contains two back to back placed uni-directional antennas, which are able to support the reflective and transmissive modes. Particularly, when the RIS works in the reflective mode, our proposed model boils down to the model in [8], as shown in the first row (starting from “Reflective”) of Table I.

Remark 2. Most of the prior works [9]-[21] focus on single-connected reflective RISs, i.e., \( \phi_{t,m} = 0, \phi_{r,m} = 1 \forall m \), which is a subset of the model proposed in [8] and a special case as summarized in [8] and a special case in the first row of Table I. The recently introduced concept, STAR-RIS/IOS [28]-[34], which is essentially the single-connected hybrid RIS as shown in Table I, is also a particular instance of the proposed model. To conclude, reflective RISs and single-connected hybrid RISs have been researched, but group/fully-connected hybrid RISs have never been researched.

In the following section, we will focus on the beamforming design for an RIS-aided wireless communication system so as to investigate and compare the nine cases of RIS.

IV. JOINT TRANSMIT PRECODER AND RIS BEAMFORMER DESIGN

A. Problem Formulation

We consider an RIS-aided MU-MISO system with an \( N \)-antenna base station (BS), an \( M \)-cell RIS, and \( K \) single-antenna users (including \( K_e \) reflective users indexed by \( K_e \) and \( K_t \) transmissive ones indexed by \( K_t, K_e \cup K_t = K \)) based on Section II. In this section, we have following assumptions: i) We assume direct links between the BS and users are blocked so that there are only BS-RIS-user links. ii) We assume exact and instantaneous channel state information (CSI) is available at the BS, and our proposed design provides a lower bound on the performance of practical systems.

Let \( s = [s_1, \ldots, s_K]^T \in \mathbb{C}^K \) be the transmit symbol vector with \( \mathbb{E}\{ss^H\} = \mathbf{I}_K \). Transmit symbols are first precoded at the BS by a precoder matrix \( \mathbf{W} \in [w_1, \ldots, w_K] \in \mathbb{C}^{N \times K} \), where \( w_k \in \mathbb{C}^N \) is the precoding vector for user \( k, \forall k \in K \). Then they are up-converted to the RF domain via \( N \) RF chains. After propagating through the RIS-aided channels, the signals are corrupted by additive Gaussian white noise (AGWN). Thus, the received signal for each user is given by

\[
y_k = h_k^H \Phi_t G w_k s_k + n_k,
\]

\[
= h_k^H \Phi_t G w_k s_k + h_k^H \Phi_t L \sum_{p \in K, p \neq k} w_p s_p + n_k, \quad \forall i \in \{t, r\}, \forall k \in K_i,
\]

where \( h_k \in \mathbb{C}^M \) denotes the channel vector between the RIS and user \( k \), and \( n_k \sim \mathcal{CN}(0, \sigma_k^2) \) denotes the AGWN, \( \forall k \in K \).

Define \( \tilde{h}_k = (h_k^H \Phi_t G)^H, \forall i \in \{t, r\}, \forall k \in K_i \). Then the signal-to-interference-plus-noise ratio (SINR) for each user can be calculated as

\[
\gamma_k = \frac{|h_k^H \Phi_t G w_k|^2}{\sum_{p \in K, p \neq k} |h_k^H \Phi_t G w_p|^2 + \sigma_k^2}, \forall i \in \{t, r\}, \forall k \in K_i.
\]

(12)

Our goal is to jointly design the transmit precoder and RIS beamformer to maximize the sum-rate for the MU-MISO system, subject to power constraint and the RIS constraint shown in Table I. Therefore, the joint transmit precoder and RIS beamformer design problem can be formulated as

\[
\begin{align}
\max_{\mathbf{w}, \Phi_t, \Phi_r} f_o(\mathbf{W}, \Phi_t, \Phi_r) &= \sum_{k \in K} \log_2(1 + \gamma_k) \tag{13a} \\
\text{s.t.} \quad \Phi_t \text{ and } \Phi_r \text{ satisfy Table I}, \tag{13b} \\
\|W\|_F^2 &\leq P, \quad \tag{13c}
\end{align}
\]

where \( P \) is the total transmit power at the BS. Problem (13) is difficult to solve due to the complex form of the objective and the non-convex constraints of the RIS. To tackle this difficulty, in the following subsections, we attempt to first transform problem (13) into a more tractable multi-variable/block optimization based on fractional programming theory and then iteratively cope with each block.

B. Overview of the Joint Design Framework

We start by taking the fractional terms \( \gamma_k, \forall k \in K \) out of the \( \log(\cdot) \) function in the original objective \( f_o(\mathbf{W}, \Phi_t, \Phi_r) \). Based on the Lagrangian Dual Transform [35], [36], \( f_o(\mathbf{W}, \Phi_t, \Phi_r) \) can be equivalently transformed into the following form:

\[
\begin{align}
f_o(\mathbf{W}, \Phi_t, \Phi_r, \mathbf{\nu}) &= \sum_{k \in K} \left( \log_2(1 + \tau_k) - \nu_k \right. \\
&\quad+ \left. \frac{1 + \tau_k}{1 + \nu_k} |\tilde{h}_k^H \Phi_t G w_k|^2 \right) \\
&\quad+ \sum_{p \in K} \left( |\tilde{h}_k^H \Phi_t G w_p|^2 + \sigma_k^2 \right), \tag{14}
\end{align}
\]

where \( \mathbf{\nu} = [\nu_1, \ldots, \nu_K]^T \in \mathbb{R}^K \) is an auxiliary vector. It can be seen from (14) that we remove the original fractional terms in the \( \log(\cdot) \) function of \( f_o(\mathbf{W}, \Phi_t, \Phi_r) \), but introduce a summation of new fractional terms. The main difference is that these new fractional terms are independent from the \( \log(\cdot) \) function and thus more tractable. Then we can apply Quadratic Transform [35], [36] to transform these fractional parts into integral expressions and reformulate the objective function \( f_o(\mathbf{W}, \Phi_t, \Phi_r, \mathbf{\nu}) \) as:

\[
\begin{align}
f_r(\mathbf{W}, \Phi_t, \Phi_r, \mathbf{\nu}) &= \sum_{k \in K} \left( \log_2(1 + \tau_k) - \nu_k \right. \\
&\quad+ \left. 2\sqrt{1 + \nu_k} \mathbb{R}\{\sigma_k^H \tilde{h}_k^H w_k\} - |\tau_k|^2 \sum_{p \in K} \left( |\tilde{h}_k^H \Phi_t G w_p|^2 + 2 \sigma_k^2 \right) \right), \tag{15}
\end{align}
\]

where \( \mathbf{\tau} = [\tau_1, \ldots, \tau_K]^T \in \mathbb{C}^K \) denotes another auxiliary vector. Now with newly introduced two auxiliary vectors \( \mathbf{\nu} \) and \( \mathbf{\tau} \), the original problem (13) can be transformed into:

\[
\max_{\mathbf{w}, \Phi_t, \Phi_r, \mathbf{\nu}, \mathbf{\tau}} f_r(\mathbf{W}, \Phi_t, \Phi_r, \mathbf{\nu}, \mathbf{\tau}) \tag{16a}
\]

\[
\text{s.t.} \quad (13b), (13c). \tag{16b} \]
Algorithm 1 Joint Transmit Precoder and RIS Design

Input: \( h_{i,k}, \forall i \in \{t,r\}, \forall k \in \mathcal{K} \), \( G, P \).

Output: \( \Phi_\ell^*, \Phi_\tau^*, W^* \).

1: Initialize \( \Phi_\ell, \Phi_\tau, W, D_1, \ldots, D_G \).
2: while no convergence of objective (13a) do
   3: Update \( \ell^* \) by (17) in Section IV-C.
   4: Update \( \tau^* \) by (18) in Section IV-C.
   5: Update \( W^* \) by (20) in Section IV-D.
   6: Update \( \Phi_{\ell/\tau}^* \) by solving problem (21) in Section IV-E.
3: end while
8: Return \( \Phi_{\ell/\tau}^*, W^* \).

Problem (16) is a typical multi-variable block problem. A well-known direction is to solve it based on block coordinate descent (BCD) iterative algorithms [38]. Given appropriate initial values of \( W \) and \( \Phi_{\ell/\tau}^* \), we iteratively update the above blocks until convergence. The proposed joint design framework is summarized in Algorithm 1.

In the following subsections, we will decompose problem (16) into several sub-problems in an iterative manner and discuss the solution for each block in detail. Specifically, solutions to two auxiliary vectors, i.e., blocks \( \ell \) and \( \tau \), will be presented in Section IV-C. Then, the solution to the transmit precoder, i.e., block \( W \), will be provided in Section IV-D. Finally, the solution to the RIS beamformer, i.e., block \( \{ \Phi_\ell, \Phi_\tau \} \), will be discussed in Section IV-E.

C. Auxiliary Vectors: Blocks \( \ell \) and \( \tau \)

When \( W, \Phi_\ell, \Phi_\tau, \) and \( \tau \) (or \( \ell \)) are fixed, the sub-problem with respect to \( \ell \) (or \( \tau \)) is an unconstrained convex optimization, whose solution can be easily derived by setting \( \partial_{\ell} \{ \ell \Phi_\ell \} = 0 \) (or \( \partial_{\tau} \{ \tau \Phi_\tau \} = 0 \)). Then we can obtain the following optimal solutions for each auxiliary variables \( \ell_k \) and \( \tau_k \) as:

\[
\ell_k^* = \frac{\gamma_k}{\sum_{p \in \mathcal{K}} |\mathbf{h}_k^H \mathbf{w}_p|^2 + \sigma_k}, \forall k \in \mathcal{K},
\]

\[
\tau_k = \frac{\sqrt{1 + \lambda_k \mathbf{h}_k^H \mathbf{w}_k}}{\sum_{p \in \mathcal{K}} |\mathbf{h}_k^H \mathbf{w}_p|^2 + \sigma_k}, \forall k \in \mathcal{K}.
\]

D. Transmit Precoder: Block \( W \)

With given \( \Phi_\ell, \Phi_\tau, \) and \( \ell \), the sub-problem with respect to \( W \) can be written as the following form:

\[
\begin{align*}
\max_{W} & \quad \sum_{k \in \mathcal{K}} \left( 2\sqrt{1 + \lambda_k} \mathbb{R} \{ \ell_k^* \mathbf{h}_k^H \mathbf{w}_k \} - |\tau_k|^2 \sum_{p \in \mathcal{K}} |\mathbf{h}_k^H \mathbf{w}_p|^2 \right) \\
\text{s.t.} & \quad ||W||_F^2 \leq P, \\
\end{align*}
\]

where \( \mathbf{h}_k \triangleq \tau_k \mathbf{h}_k, \forall k \in \mathcal{K} \). Since both the objective function and the constraint of problem (19) are convex, we can use some classical optimization methods to find its optimal solution. Here we adopt the Lagrange multiplier method and introduce a multiplier \( \lambda \geq 0 \) for power constraint (19b). Then by checking the first-order optimality condition, we can get the optimal solution of \( W \):

\[
\mathbf{w}_k^* = \left( \sum_{p \in \mathcal{K}} \mathbf{h}_p \mathbf{h}_p^H + \lambda^* \mathbf{I}_N \right)^{-1} \sqrt{1 + \lambda_k \mathbf{h}_k^H \mathbf{w}_k}, \forall k \in \mathcal{K},
\]

where \( \lambda^* \) can be obtained by a simple bisection search.

E. RIS Beamformer: Block \( \{ \Phi_\ell, \Phi_\tau \} \)

When \( W, \tau, \) and \( \ell \) are determined, the sub-problem with respect to \( \Phi_\ell \) and \( \Phi_\tau \) can be written as

\[
\begin{align*}
\max_{\Phi_\ell, \Phi_\tau} & \quad \sum_{i \in \{t,r\}} \sum_{k \in \mathcal{K}_i} \left( 2\mathbb{R} \{ \tau_k^* \mathbf{h}_k^H \Phi_\ell \mathbf{g}_k \} - |\tau_k|^2 \sum_{p \in \mathcal{K}} |\mathbf{h}_k^H \mathbf{g}_p|^2 \right) \\
\text{s.t.} & \quad \Phi_\ell \text{ and } \Phi_\tau \text{ satisfy Table I}, \\
\end{align*}
\]

where \( \tau_k = \sqrt{1 + \lambda_k \mathbf{h}_k^H \mathbf{g}_k} \). It can be easily observed from Table I that both single- and fully-connected architectures are special cases of the group-connected architecture. Therefore, in the following section, we will first provide a general solution for the group-connected case, which can easily boil down to other cases, and then propose an efficient algorithm for the single-connected case to further reduce the complexity.

1) A General Solution for Group-Connected RIS: Problem (21) can be written as the following form

\[
\begin{align*}
\max_{\Phi_\ell, \Phi_\tau} & \quad \sum_{i \in \{t,r\}} \left( 2\mathbb{R} \{ \text{Tr}(\Phi_i X_i) \} - \text{Tr}(\Phi_i Y_i \Phi_i^H Z_i) \right) \\
\text{s.t.} & \quad \Phi_i^{g,g} + \Phi_i^{r,g} = I_{M_i}, \forall g \in \mathcal{G}, \\
& \quad \Phi_i = \text{blkdiag}(\Phi_{i,1}, \ldots, \Phi_{i,G}), \forall i \in \{t,r\}, \\
\end{align*}
\]

where \( X_i \in \mathbb{C}^{M \times M}, Y_i \in \mathbb{C}^{M \times M}, \) and \( Z_i \in \mathbb{C}^{M \times M} \) are respectively defined as

\[
\begin{align*}
X_i & \triangleq \sum_{k \in \mathcal{K}_i} \tau_k^* \mathbf{g}_k \mathbf{h}_k^H, \quad Y_i \triangleq \sum_{p \in \mathcal{K}} \mathbf{g}_p \mathbf{g}_p^H, \\
Z_i & \triangleq \sum_{k \in \mathcal{K}_i} |\tau_k|^2 \mathbf{h}_k^H \mathbf{h}_k^H, \forall i \in \{t,r\}. \\
\end{align*}
\]

It is worth noting that there are quadratic terms in problem (22), which makes different groups of \( \Phi_{i/g,g} \) mingle with each other. To efficiently solve this problem, we attempt to split out one pair \( \Phi_{i,g} \) and \( \Phi_{r,g} \) for group \( g \) from the objective function with fixed other pairs, and focus on the design of each pair. To this end, we first split objective (22a) group-by-group as:

\[
\begin{align*}
\sum_{i \in \{t,r\}} & \quad \left( 2\mathbb{R} \{ \text{Tr}(\Phi_i X_i) \} - \text{Tr}(\Phi_i Y_i \Phi_i^H Z_i) \right) \\
= & \quad \sum_{i \in \{t,r\}} \left( 2 \sum_{g=1}^{G} \mathbb{R} \{ \text{Tr}(\Phi_{i,g} X_{i,g}) \} \\
& - \text{Tr} \left( \sum_{p=1}^{G} \Phi_{i,p} \sum_{g=1}^{G} Y_{p,g} \Phi_{i,g,p}^H Z_{i,g,p} \right) \right),
\end{align*}
\]

where \( X_{i,g} = [X_i]_{(g-1)M+1:gM}, Y_{p,g} = [Y]_{(g-1)M+1:pM,(g-1)M+1:gM} \in \mathbb{C}^{M \times M}, \) and
Objective function decreases fastest \cite{39}. The tangent space of a manifold is referred to as a tangent vector. All these tangent vectors at a point form the tangent space. Each tangent space can be formulated as follows:

$$\min_{\Phi_g} \tilde{f}_g(\Phi_g) = \text{Tr}(\Phi_g Y_{g,g} H^T Z_{g,g} - 2R\{ \Phi_g \tilde{X}_g \})$$  \hspace{1cm} \text{(26a)}$$

$$\text{s.t. } \Phi_g H^T \Phi_g = \mathbf{I}_M, \forall g \in \mathcal{G}. \hspace{1cm} \text{(26b)}$$

The main challenge of solving problem (26) is the unitary constraint (26b). One possible solution is to do some relaxations and solve the relaxed problem. However, it is inevitable that there will be performance loss since the result is not based on the original problem. Moreover, when the value of $\Phi_g$ is not that "good", the convergence of the overall BCD procedure might not be guaranteed. In order to get a “good” solution of problem (26), we adopt a manifold algorithm \cite{39}, whose main idea is to construct all available solutions of the problem as a manifold, and then transform the original constrained problem on the Euclidean space into an unconstrained one on the manifold space. Then many algorithms on the Euclidean space, e.g., Trust-Region (TR), Conjugate-Gradient (CG), and Broyden–Fletcher–Goldfarb–Shanno (BFGS) methods \cite{40}, can be easily extended to the manifold space with some necessary projections. In the following we will introduce the procedure of the manifold algorithm by applying the CG method.

**Manifold Construction:** Constraint (26b) forms a $2M^2$-dimensional complex Stiefel manifold \cite{40}, i.e.,

$$M_g = \{ \Phi_g \in \mathbb{C}^{2M \times M} : \Phi_g H^T \Phi_g = \mathbf{I}_M \}, \forall g \in \mathcal{G}, \hspace{1cm} \text{(27)}$$

and then problem (26) becomes an unconstrained optimization on the Stiefel manifold, i.e.,

$$\Phi^*_g = \arg \min_{\Phi_g \in M_g} \tilde{f}_g(\Phi_g), \forall g \in \mathcal{G}. \hspace{1cm} \text{(28)}$$

Here we should explain that a manifold is a topological space which “locally” resembles the Euclidean space. To be specific, the moving direction of a point on the manifold is referred to as a tangent vector. All these tangent vectors at this point, which include all possible directions this point can move to, form the tangent space. Each tangent space can be regarded as a Euclidean space which has one tangent vector, namely, Riemannian gradient, pointing to the direction that the objective function decreases fastest \cite{39}. The tangent space of the manifold (27) at point $\Phi_g$ is given as

$$T_{\Phi_g} M_g = \{ T_g \in \mathbb{C}^{2M \times M} : R\{ \Phi_g H T_g \} = 0_M \}, \forall g \in \mathcal{G}. \hspace{1cm} \text{(29)}$$

**Riemannian Gradient:** In the CG method, the Euclidean gradient is required to calculate the Riemannian gradient. Therefore, we first calculate the Euclidean gradient of the objective $\tilde{f}_g(\Phi_g)$ as

$$\nabla \tilde{f}_g(\Phi_g) = 2Z_g \Phi_g Y_g - 2X_g^H, \forall g \in \mathcal{G}. \hspace{1cm} \text{(30)}$$

Then the Riemannian gradient can be calculated by projecting the Euclidean gradient onto the tangent space \cite{39}:

$$\nabla \tilde{M}_g \tilde{f}_g(\Phi_g) = \text{Pr}_{\Phi_g} (\nabla \tilde{f}_g(\Phi_g)) = \nabla \tilde{f}_g(\Phi_g) - \Phi_g \text{chdiag} (\nabla \tilde{f}_g(\Phi_g)), \forall g \in \mathcal{G}, \hspace{1cm} \text{(31)}$$

where $\text{Pr}_{\Phi_g} (\cdot)$ denotes the projection function, and $\text{chdiag}(\cdot)$ chooses all diagonal elements of a matrix to construct a diagonal matrix. Now we can apply the CG method. At the $v$-th iteration of the CG method, we successively do the following steps:

**S1:** Find the descent direction:

$$\Xi_g^v = -\nabla \tilde{M}_g \tilde{f}_g(\Phi_g^v) + \mu^v_g \text{Pr}_{\Phi_g} (\Xi_g^{v-1}), \forall g \in \mathcal{G}, \hspace{1cm} \text{(32)}$$

where $\mu^v_g$ denotes the CG update parameter. There are many choices for updating this parameter, e.g., Fletcher-Reeves formula, Polak-Ribiére formula, and Hestenes-Stiefel formula \cite{41}. Here we adopt a Riemannian version of the Polak-Ribiére formula, which is given by

$$\mu^v_g = \frac{\text{Tr} \left( [\nabla \tilde{M}_g \tilde{f}_g(\Phi_g^v)]^H \nabla \tilde{M}_g \tilde{f}_g(\Phi_g^v) \right)}{\text{Tr} \left( [\nabla \tilde{M}_g \tilde{f}_g(\Phi_g^{v-1})]^H \nabla \tilde{M}_g \tilde{f}_g(\Phi_g^{v-1}) \right)}, \forall g \in \mathcal{G}. \hspace{1cm} \text{(33)}$$

**S2:** Perform a retraction \cite{39}:

$$\Phi_g^{v+1} = \text{ReTr}_{\Phi_g} (\delta_g \Xi_g^v) = (\Phi_g^v + \delta_g \Xi_g^v) (\mathbf{I}_M + (\delta_g^2 \Xi_g^v)^H \Xi_g^v)^{-1/2}, \forall g \in \mathcal{G}, \hspace{1cm} \text{(34)}$$
Algorithm 3 Efficient Solution for Single-Connected RIS

Input: $\mathbf{h}_{i,k}, \forall i \in \{t,r\}, \forall k \in \mathcal{K}$, $\mathbf{G}$, $\mathbf{t}$, $\mathbf{r}$, $\mathbf{W}$, $\Phi_t$, $\Phi_r$.
Output: $\Phi_t^*, \Phi_r^*$.
1: Calculate $\mathbf{y}_{e/i}$, $\tilde{\varphi}_{i/r}$ by (38).
2: while no convergence of $\Phi_t$, $\Phi_r$ do
3: for $m = 1 : M$ do
4: Update $\theta_{i/r,m}$ by (41).
5: Update $\alpha_{i,m}^*$ by golden-section search.
6: Calculate $\phi_{i/r,m}^*$ by (43).
7: end for
8: end while
9: Return $\Phi_{i/r}^* = \text{diag}(\phi_{i/r,M}^*, \ldots, \phi_{i/r,1}^*)$.

with definitions

$$
\mathbf{V}_i \triangleq \sum_{k \in \mathcal{K}} |\mathbf{r}_k|^2 \sum_{p \in \mathcal{K}} \mathbf{v}_{k,p}^H \mathbf{V}_{k,p}, \quad (38a)
$$

$$
\mathbf{\bar{v}}_i \triangleq \sum_{k \in \mathcal{K}} \sqrt{1 + i_k \mathbf{V}_{k,k}} \mathbf{r}_k, \forall i \in \{t, r\}. \quad (38b)
$$

Similar to the proposed general solution for group-connected hybrid RIS, here we also split the objective and focus on the design for one pair, i.e., $\phi_{i,m}, \phi_{r,m}$, while fixing the others. Therefore, the sub-objective for $\phi_{t,m}$ and $\phi_{r,m}$ is given by

$$
z_m(\phi_{t,m}, \phi_{r,m}) = \sum_{i \in \{t,r\}} \left( |\mathbf{V}_{i,m}| |\phi_{i,m}|^2 + 2\Re\left\{ \sum_{n \neq m} |\mathbf{V}_{i,m,n}| \phi_{i,m} - |\tilde{\varphi}_{i,m}| \phi_{i,m}^* \right\} \right) \quad (39)
$$

where $\delta_g$ is the step size and can be searched by backtracking algorithms [39]. $\text{re}(\cdot)$ denotes the retraction function which maps a point in the tangent space into the manifold.

Now with proper initial values, the (at least local) optimal $\Phi_t^*, \forall g \in \mathcal{G}$ can be obtained by iteratively updating $\delta_g^{-1}, \Phi_t^*, \mu_g^*$, and $\Xi_g^*$ until convergence. After solving problem (26) for all groups, the optimal reflective and transmissive matrices for each group can be split from $\Phi_t^*$, i.e.,

$$
\Phi_{t,g}^* = [\Phi_{t1}^*, \ldots, \Phi_{tM}^*], \quad \Phi_{r,g}^* = [\Phi_{r1}^*, \ldots, \Phi_{rM+1:2M}]^T, \forall g \in \mathcal{G}. \quad (35)
$$

The procedure of the above RIS beamforming design is summarized in Algorithm 2.

Remark 3. The proposed general solution is also suitable for both fully- and single-connected cases:

C1: For the fully-connected case ($G = 1$), problem (21) can be rewritten as the following form:

$$
\begin{align}
\min_{\Phi} & \quad \text{Tr}(\mathbf{Y} \Phi \mathbf{H}^T \mathbf{Z}) - 2\Re\{\text{Tr}(\mathbf{X})\} \quad (36a) \\
\text{s.t.} & \quad \mathbf{H}^T \Phi = \mathbf{I}_M, \quad (36b)
\end{align}
$$

where $\mathbf{F} \triangleq [\mathbf{F}_{t1}^H, \mathbf{F}_{r1}^H]^H \in \mathbb{C}^{2M \times M}$, $\mathbf{X} \triangleq [\mathbf{X}_{t1}, \mathbf{X}_{r1}] \in \mathbb{C}^{M \times 2M}$, and $\mathbf{Z} \triangleq \text{blkdiag}(\mathbf{Z}_{t1}, \mathbf{Z}_{r1}) \in \mathbb{C}^{2M \times 2M}$. This problem can be solved by steps 6-15 in Algorithm 2.

C2: Single-connected RIS beamformer can be directly obtained by Alg. 2 with $G = M$. However, single-connected RIS has a friendly characteristic, i.e., $\Phi_t$ and $\Phi_r$ are all diagonal matrices, which can facilitate the RIS beamformer design. Therefore, we propose a more efficient algorithm for the single-connected case, which achieves a similar performance but has lower computational complexity compared to Algorithm 2.

2) An Efficient Solution for Single-Connected RIS: When the RIS has a single-connected architecture, each non-zero element of $\Phi_{i/r}$ can be modeled as $\phi_{i/r,m} = \sqrt{\alpha_{i,m}} e^{j\theta_{i/r,m}}$, $\theta_{i/r,m} \in [0, 2\pi)$, $\alpha_{i,m} + \alpha_{r,m} = 1$, $\forall m \in \mathcal{M}$ [5]. Define $\phi_t \triangleq [\phi_{t1}, \ldots, \phi_{tM}]^T$, and $\mathbf{v}_{k,p} \triangleq (h_{k,p}^H \text{diag}(g_k))^H$, $\forall k \in \mathcal{K}$, $\forall k \in \mathcal{K}$, $\forall i \in \{t, r\}$. Problem (21) can be rewritten as

$$
\begin{align}
\max_{\phi_t, \phi_r} & \quad \sum_{i \in \{t,r\}} \left( 2\Re\{\mathbf{v}_i^H \phi_t - \phi_t^H \mathbf{V}_i \phi_t \} \right), \quad (37a) \\
\text{s.t.} & \quad |\phi_{t,m}|^2 + |\phi_{r,m}|^2 = 1, \forall m \in \mathcal{M}, \quad (37b)
\end{align}
$$

Phase Shift: From problem (40) we can observe that the phase shifts $\theta_{i/r,m}$ and amplitudes $\alpha_{i/r,m}$ can be separately determined. Therefore, we first directly let the term $\cos(\chi_{i,m} - \theta_{i,m})$ becomes a real-value optimization and problem (40) can be formulated as a function of $\alpha_{t,m}$:

$$
\begin{align}
\min_{\alpha_{t,m}, \chi_{i,m}, \theta_{i,m}} & \quad \chi_{i,m} + \pi, \quad \chi_{i,m} \in [0, \pi), \quad (40a) \\
& \quad \chi_{i,m} - \pi, \quad \chi_{i,m} \in [\pi, 2\pi), \quad \forall i \in \{t,r\}, \forall m \in \mathcal{M}. \quad (40b)
\end{align}
$$

Amplitude: With optimal phase shifts, objective $\bar{z}_m(\alpha_{t,m}, \alpha_{r,m}, \theta_{t,m}, \theta_{r,m})$ becomes a real-value optimization problem (40) can be reformulated as a function of $\alpha_{t,m}$:

$$
\begin{align}
\min_{\alpha_{t,m}, \chi_{i,m}, \theta_{i,m}} & \quad \chi_{i,m} + \pi, \quad \chi_{i,m} \in [0, \pi), \quad (41a) \\
& \quad \chi_{i,m} - \pi, \quad \chi_{i,m} \in [\pi, 2\pi), \quad \forall i \in \{t,r\}, \forall m \in \mathcal{M}. \quad (41b)
\end{align}
$$

where $\chi_m = [\mathbf{V}_{i,m}]_m - [\mathbf{V}_{i,m}]_m, \forall m \in \mathcal{M}$. It can be proved that objective (42a) is a convex function with $\alpha_{t,m} \in (0, 1)$ (the proof of the convexity of objective (42a) is given in the appendix). Therefore, the minimum of problem (42) is achieved at the minimum point. Although we cannot solve close-form solution due to the complicated form of objective (42a), we can apply efficient one-dimensional search methods, e.g., golden-section search, to find the minimum point.

After solving problem (42) and getting the optimal amplitude $\alpha_{t,m}^*$, we can obtain $\phi_{i/r,m}^*$ as:

$$
\phi_{i/r,m}^* = \sqrt{\alpha_{t,m}^*} e^{j\theta_{i/r,m}^*}, \forall m \in \mathcal{M}. \quad (43)
$$

The details of the single-connected RIS beamforming design are summarized in Algorithm 3.
TABLE II
THE NUMBER OF NON-ZERO RIS ELEMENTS AND OVERALL OPTIMIZATION COMPLEXITY FOR RIS WITH DIFFERENT ARCHITECTURES

| No. of Groups     | Group Dimension | No. of Non-zero Elements | Optimization Complexity                                      |
|-------------------|-----------------|--------------------------|-------------------------------------------------------------|
| Single-connected  | M               | 1                        | $O(1, K^2 M^2 + I_{bs} K N^3 + I_{gc} G M^3)$ by Algorithm 2 |
| Group-connected   | G               | M                        | $O(1, K^2 M^2 + I_{bs} K N^3 + I_{gc} G M^3)$ by Algorithm 3 |
| Fully-connected   | 1               | M                        | $O(1, K^2 M^2 + I_{bs} K N^3 + I_{gc} G M^3)$                |

(a) Rayleigh fading  (b) Rician fading

Fig. 4. Sum-rate versus the number of iterations ($P = 5$ dBm, $G = 8$, $N = K = 6$, $K_t = K_r = 3$, $M = 64$).

F. Convergence Analysis

The convergence of the proposed Algorithm 1 cannot be strictly proved, since the update of block $\{\Phi_t, \Phi_r\}$ has no guarantee of global optimum. Fortunately, since the update of the remaining blocks (steps 3-5 in Algorithm 1) is monotonous, the loss induced by updating block $\{\Phi_t, \Phi_r\}$ is negligible. Although we cannot provide a rigorous theoretical proof for the convergence of Algorithm 1, we evaluate the convergence performance by some simulations. Simulation results for the proposed algorithms are shown in Fig. 4. In Fig. 4, schemes “Full”, “Group”, and “Single, Alg. 2” are plotted based on Algorithms 2 and 1, while “Single, Alg. 3” is plotted based on Algorithms 3 and 1. From Fig. 4 we can observe that our proposed solutions always converge within limited iterations under different channel realizations, e.g., Rayleigh fading and Rician fading channels. These simulation results demonstrate the robustness of our proposed algorithms. Moreover, our proposed efficient solution for single-connected RIS, i.e., “Single, Alg. 3”, has a similar performance to “Single, Alg. 2”, but with a lower complexity (which will be discussed in the following subsection).

G. Complexity Analysis

In this subsection, we provide a broad complexity analysis for algorithms proposed in the previous section. As shown in Algorithm 1, four blocks are iteratively updated to find a convergent solution. In each iteration, updating blocks $\eta$ and $\tau$ require $O(K^2 M^2)$ operations; updating block $W$ has a complexity of approximately $O(K(M^2 + I_{bs} N^3))$, where $I_{bs}$ is the number of iterations for bisection search. The complexity of the proposed two algorithms for designing block $\Phi_t, \Phi_r$ will be discussed as follows.

1) The General Solution for Group-Connected RIS: As summarized in Algorithm 2, the optimization of group-connected RIS is based on an iterative design. In each iteration, the objective is divided into $G$ sub-problems, each of which is solved by a manifold version of the CG method (steps 7-13 in Algorithm 2) with complexity $O(I_{gc} G M^3)$, where $I_{gc}$ denotes the number of iterations for the CG method. Therefore, calculating group-connected RIS requires $O(I_{gc} G M^3)$ operations, where $I_{gc}$ denotes the number of iterations for Algorithm 2. The overall complexity is $O(I(K^2 M^2 + I_{bs} K N^3 + I_{gc} G M^3))$, where $I$ denotes the number of iterations in Algorithm 1. Recall that fully- and single-connected RISs are special cases of group-connected ones, we can easily derive the corresponding complexity as follows:

C1: For the fully-connected case, i.e., $G = 1$, there is no need to do iterations for RIS design so that the complexity of updating RIS beamformer is $O(I_{gc} G M^3)$. The overall complexity is thus $O(I(K^2 M^2 + I_{bs} K N^3 + I_{gc} G M^3))$.

C2: For the single-connected case, i.e., $G = M$, optimizing RIS requires $O(I_{gc} G M)$. And the overall complexity is $O(I(K^2 M^2 + I_{bs} K N^3 + I_{gc} G M))$.

2) The Efficient Solution for Single-Connected RIS:Updating $\{\Phi_t, \Phi_r\}$ by Algorithm 3 requires $O(I_{sc} M)$ operations, where $I_{sc}$ denotes the number of iterations. Thus, the complexity for joint transmit beamformer and single-connected RIS beamformer design is $O(I(K^2 M^2 + I_{bs} K N^3 + I_{sc} M))$.

To summarize, when RIS has different architectures, $\Phi_t$ and $\Phi_r$ will have different numbers of non-zero elements related to the number of RIS cells $M$ and groups $G$. Therefore, the complexity for beamforming design will be different. To provide a clear comparison, we summarize the number of non-zero RIS elements and optimization complexity for different architectures in Table II.

V. PERFORMANCE EVALUATION

In this section, we present simulation results to demonstrate the performance of the RIS-aided MU-MISO system when the RIS has nine different modes/architectures. We adopt both Rayleigh fading and Rician fading channels (with a fixed Rician factor $\kappa = 3$ dB) for BS-RIS and RIS-user links. The
signal attenuation is set as $\zeta_0 = -30$ dB at a reference distance 1 m for all channels. The path loss exponent of BS-RIS and RIS-user channels are all set as $\varepsilon = 2.2$. The noise power at each user is set as $\sigma_{\mathrm{N}}^2 = -80$ dBm, $\forall i \in \{t, r\}$, $\forall k \in \mathcal{K}_i$. The relative position among the BS, RIS, and users is shown in Fig. 5, where the distance between the BS and the RIS is set as $d_{\mathrm{BI}} = 50$ m, and $K$ users are randomly located close to the RIS with the same distance $d_{\mathrm{IU}} = 2.5$ m.

Now we examine the sum-rate performance of the RIS-aided MU-MISO system when RIS has different modes and architectures. Fig. 6 shows the sum-rate performance versus the transmit power. From Fig. 6 we can obtain the following conclusions: i) With the same architecture, the “hybrid” scheme can outperform other modes. For example, for Rician fading channels, the “hybrid, full” scheme can achieve around 20% higher sum-rate than “Transmissive/Reflective, Full” schemes. This is because the hybrid RIS can fully utilize the multiuser diversity, while part of users are blocked under other modes. Therefore, the “hybrid” scheme can realize a full-dimensional service coverage, which indicates that the scope of application for hybrid RISs can be much more flexible than “transmissive/reflective” ones. ii) Under the same mode, the “full” scheme always achieves the best sum-rate performance. For example, for Rayleigh fading channels, the sum-rate of “hybrid, full” and “hybrid, group” schemes is around 75% and 37% higher than that of the “hybrid, single” scheme. iii) With the same architecture, the performance gap between the hybrid RIS and transmissive/reflective RIS for Rician fading channels is larger than that for Rayleigh fading channels, which demonstrates that hybrid RISs are more suitable in Rician fading environments. iv) When RISs are under the same mode, the performance gap between fully-/group-connected RISs and single-connected ones for Rayleigh fading channels is larger than that for Rician fading channels. This fact shows that the advantage of fully-/group-connected RIS is much more prominent in Rayleigh fading propagations, therefore confirming results in [8].

Then in Fig. 7, we plot sum-rate as a function of the number of RIS elements $M$. It can be observed from Fig. 7 that the sum-rate for all schemes grows with the increase
of $M$. More importantly, the slope of the sum-rate-versus-$M$ curve for fully-/group-connected RIS is larger than that for single-connected RIS. This phenomenon can be explained by Table II, which demonstrates that the number of non-zero elements grows quadratically with increasing $M$ for fully-/group-connected RIS, but grows linearly for single-connected case: The more the number of non-zero elements, the higher the beam control flexibility.

Finally in Figs. 8(a), 8(c) and Figs. 8(b), 8(d), we plot sum-rate versus the number of transmissive and reflective users under different channel fadings. From Figs. 8(a)-8(d) we have the following observations: i) With the same architecture and same numbers of transmissive/reflective users, the “hybrid” scheme can always outperform its competitors. ii) With the growth of the number of transmissive/reflective users, the performance gap between the “hybrid” scheme and the “transmissive/reflective” one becomes smaller. iii) When the number of total users the BS can simultaneously serve is fixed, e.g., circled points in Figs. 8(a)-8(d), the “hybrid” scheme still achieves better performance than “reflective/transmissive” schemes. This fact demonstrates that the “hybrid” scheme provides an additional power splitting in RIS to serve users, while users share the same RIS beamformer in “transmissive/reflective” schemes.

VI. CONCLUSION

In this paper, we analyze and propose a general RIS-aided communication model unifying three modes, namely, transmissive, reflective, and hybrid, and three architectures, namely, single-, group-, fully-connected architectures, totally nine cases. Particularly, the existing STAR-RIS-aided communication model [28]-[34] is a special case of our proposed model.

With the proposed model, we consider the joint transmit precoder and RIS beamformer design to maximize the sum-rate for an RIS-aided MU-MISO system. We first transform the original problem into a multi-block optimization based on fractional programming theory. For the design of RIS block, we propose a general algorithm, which is suitable for different architectures, and an efficient algorithm specifically for single-connected RIS. Convergence and complexity analysis for the proposed algorithms are also provided.

Finally, we provide a comprehensive comparison for RIS with nine different modes/architectures. Simulation results show that fully- and group-connected hybrid RISs can achieve around 75% and 37% higher sum-rate performance than single-connected hybrid RISs. Meanwhile, using fully-connected hybrid RISs can improve the sum-rate performance by around 20% compared with using fully-connected transmissive/reflective ones. Based on the general and unified RIS model, there are many issues worth being studied for future research, which include, but are not limited to, fast channel estimation, the extension to wideband systems, as well as the deployment of RIS in different scenarios, e.g., rate-splitting multiple access (RSMA), integrated sensing and communication (ISAC), etc.

APPENDIX

PROOF OF CONVEXITY OF OBJECTIVE (42A)

We first abstract objective as a real-value function $\bar{z}(x) = ax^2 + bx + c\sqrt{1-x^2}$ with one-to-one correlations, i.e., $x = \sqrt{a_{x,m}} \in (0, 1)$, $a = v_m \in \mathbb{R}$, $b = -2|x_{t,m}| < 0$, $c = -|x_{t,m}| < 0$. Then the proof is given as follows:

**Proof.** Calculate first- and second-order derivatives of $\bar{z}(x)$:

$$\nabla \bar{z}(x) = 2ax + b - \frac{cx}{\sqrt{1-x^2}}, \quad \nabla^2 \bar{z}(x) = 2a - \frac{c}{(1-x^2)^{\frac{3}{2}}}.$$

(44)

Then we discuss the characteristic of $\nabla \bar{z}(x)$ under the following two conditions:

**C1** When $a \geq 0$, it is obvious that $\nabla^2 \bar{z}(x) > 0$ so that $\nabla \bar{z}(x)$ is a monotonically increasing function with $x \in (0, 1)$.

Based on the above discussion, we turn to judge the range of $\nabla \bar{z}(x)$ by examining the following two limitations:

$$\lim_{x \to 0} \nabla \bar{z}(x) = b < 0, \quad \lim_{x \to 1} \nabla \bar{z}(x) = +\infty > 0.$$

(45)

In this case, $\bar{z}(x)$ is first monotonically decreasing and then monotonically increasing within the range $x \in (0, 1)$.

**C2** When $a < 0$, calculate the third-order derivative of $\bar{z}(x)$:

$$\nabla^3 \bar{z}(x) = -3cx(1-x^2)^{-\frac{5}{2}},$$

(46)

which is always larger than zero when $x \in (0, 1)$. Thus, $\nabla^2 \bar{z}(x)$ is monotonically increasing with $x \in (0, 1)$. Then we calculate

$$\lim_{x \to 0} \nabla^2 \bar{z}(x) = 2a - c,$$

(47)

which has the following two conditions:

**C2.1** When $2a - c \geq 0$, $\nabla^2 \bar{z}(x) > 0$ for $x \in (0, 1)$. Thus, $\nabla \bar{z}(x)$ is also monotonically increasing within the
range $x \in (0, 1)$ and the same conclusion as C1 can be obtained.

C2.2: When $2\alpha - c < 0$, further examine

$$\lim_{x \to 1} \nabla^2 \tilde{z}(x) = +\infty > 0,$$

which proves that $\nabla \tilde{z}(x)$ is first monotonically decreasing and then increasing within the range $x \in (0, 1)$. Combining the above two limitations (45), we can derive that $\tilde{z}(x)$ has the same trend as $\nabla \tilde{z}(x)$ for $x \in (0, 1)$.

To summarize, $\tilde{z}(x)$ is a convex function with only one minimum point for $x \in (0, 1)$, which completes the proof.

REFERENCES

[1] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. de Rosny, and S. Tremayne, “Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and road ahead,” IEEE J. Sel. Areas Commun., vol. 38, no. 11, pp. 2540-2525, Nov. 2020.

[2] S. Gong, X. Lu, D. T. Hoang, D. Niyato, L. Shu, D. I. Kim, and Y.-C. Liang, “Toward smart wireless communications via intelligent reflecting surfaces: A contemporary survey,” IEEE Commun. Surveys & Tutorials, vol. 22, no. 4, pp. 2283-2314, Fourth Quarter 2020.

[3] K.-K. Wong, K.-F. Tong, Z. Chu, and Y. Zhang, “A vision to smart radio environment: Surface wave communication superhighways,” IEEE Wireless Commun., vol. 28, no. 1, pp. 112-119, Feb. 2021.

[4] Q. Wu and R. Zhang, “Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network,” IEEE Commun. Mag., vol. 58, no. 1, pp. 106-112, Jan. 2020.

[5] J. Xu, Y. Liu, X. Mu, J. T. Zhou, L. Song, H. V. Poor, and L. Hanzo, “Simultaneously transmit and reflecting (STAR) intelligent omni-surfaces, their modeling and implementation,” Sept. 2021. [Online]. Available: https://arxiv.org/abs/2108.06233

[6] Z. Feng, B. Clerckx, and Y. Zhao, “Waveform and beamforming design for intelligent reflecting surface aided wireless power transfer: Single-user and multi-user solutions,” IEEE Trans. Wireless Commun., to appear.

[7] Z. Zhao, B. Clerckx, and Z. Feng, “IRS-aided SWIPT: Joint waveform, active, and passive beamforming design under nonlinear harvester model,” IEEE Trans. Commun., to appear.

[8] B. Zheng, C. You, and R. Zhang, “Intelligent reflecting surface assisted multi-user OFDMA: Channel estimation and training design,” IEEE Trans. Wireless Commun., vol. 19, no. 12, pp. 8315-8329, Dec. 2020.

[9] Y. Han, W. Tang, S. Jin, C. Wen, and X. Ma, “Large intelligent surface-aided MISO uplink communication network: Feasibility and power minimization for perfect and imperfect CSI,” IEEE Trans. Wireless Commun., vol. 20, no. 7, pp. 4513-4526, July 2021.

[10] J. Xu, Y. Liu, X. Mu, and O. A. Dobre, “Deep residual learning for coordinated passive beamforming for distributed intelligent reflecting surfaces network,” in Proc. IEEE Veh. Technol. Conf. (VTC), Virtual Conference, May 2020.

[11] B. Zheng, C. You, and R. Zhang, “Double-IRS assisted multi-user MIMO: Cooperative passive beamforming design,” IEEE Trans. Wireless Commun., vol. 20, no. 7, pp. 4513-4526, July 2021.

[12] W. Mei and R. Zhang, “Intelligent reflecting surface for multi-path beam routing with active/passive beam splitting and combining,” IEEE Commun. Lett., to appear.

[13] S. Shen, B. Clerckx, and R. Murch, “Modeling and architecture design of reconfigurable intelligent surfaces using scattering parameter network,” IEEE Trans. Commun., vol. 68, no. 9, pp. 5849-5863, Sept. 2020.

[14] W. Cai, H. Li, M. Li, and Q. Liu, “Practical modeling and beamforming for intelligent reflecting surface aided wideband systems,” IEEE Trans. Commun., vol. 24, no. 7, pp. 1568-1571, July 2020.

[15] H. Li, W. Cai, Y. Liu, M. Li, Q. Liu, and Q. Wu, “Intelligent reflecting surface enhanced wideband MIMO-OFDM communications: From practical model to reflection optimization,” IEEE Trans. Wireless Commun., vol. 69, no. 7, pp. 4807-4820, July 2021.

[16] M. Nerini and B. Clerckx, “Reconfigurable intelligent surfaces based on single, group, and fully connected discrete-value impedance networks,” Sept. 2021. [Online]. Available: https://arxiv.org/abs/2110.00077

[17] S. Shen, B. Clerckx, and R. Murch, “Modeling and architecture design of reconfigurable intelligent surfaces using scattering parameter network analysis,” IEEE Trans. Veh. Technol., to appear.

[18] J. Xu, Y. Liu, X. Mu, and O. A. Dobre, “STAR-RIS: Simultaneously transmitting and reflecting reconfigurable intelligent surfaces,” IEEE Commun. Lett., vol. 29, no. 9, pp. 3134-3138, Sept. 2021.

[19] H. Zhang, S. Zeng, B. Di, Y. Tan, M. D. Renzo, M. Debbah, Z. Han, H. V. Poor, and L. Song, “Intelligent omnisurfaces for full-dimensional wireless communications: Principles, technology, and implementation,” IEEE Commun. Mag., vol. 60, no. 2, pp. 39-45, Feb. 2022.

[20] X. Mu, Y. Liu, L. Guo, J. Lin, and R. Scholz, “Simultaneously transmitting and reflecting (STAR) RIS aided wireless communications,” IEEE Trans. Veh. Technol., to appear.

[21] C. Liu, X. Liu, D. W. K. Ng, and J. Yuan, “Deep residual learning for coordinated passive beamforming for distributed intelligent reflecting surfaces network,” in Proc. IEEE Veh. Technol. Conf. (VTC), Virtual Conference, May 2020.

[22] B. Zheng, C. You, and R. Zhang, “Double-IRS assisted multi-user MIMO: Cooperative passive beamforming design,” IEEE Trans. Wireless Commun., vol. 20, no. 7, pp. 4513-4526, July 2021.

[23] K. Shen and W. Yu, “Fractional programming for communication systems–Part I: Power control and beamforming,” IEEE Trans. Signal Process., vol. 66, no. 10, pp. 5036-5049, Sept. 2019.

[24] Z. Feng, B. Clerckx, and Y. Zhao, “WAVEFORM and beamforming design for intelligent reflecting surface aided wireless power transfer: Single-user and multi-user solutions,” IEEE Trans. Wireless Commun., to appear.

[25] W. Cai, H. Li, M. Li, and Q. Liu, “Practical modeling and beamforming for intelligent reflecting surface aided wideband systems,” IEEE Trans. Commun., vol. 24, no. 7, pp. 1568-1571, July 2020.

[26] H. Li, W. Cai, Y. Liu, M. Li, Q. Liu, and Q. Wu, “Intelligent reflecting surface enhanced wideband MIMO-OFDM communications: From practical model to reflection optimization,” IEEE Trans. Wireless Commun., vol. 69, no. 7, pp. 4807-4820, July 2021.

[27] M. Nerini and B. Clerckx, “Reconfigurable intelligent surfaces based on single, group, and fully connected discrete-value impedance networks,” Sept. 2021. [Online]. Available: https://arxiv.org/abs/2110.00077

[28] S. Shen, B. Clerckx, and R. Murch, “Modeling and architecture design of reconfigurable intelligent surfaces using scattering parameter network analysis,” IEEE Trans. Veh. Technol., to appear.

[29] J. Xu, Y. Liu, X. Mu, and O. A. Dobre, “STAR-RIS: Simultaneously transmitting and reflecting reconfigurable intelligent surfaces,” IEEE Commun. Lett., vol. 29, no. 9, pp. 3134-3138, Sept. 2021.

[30] H. Zhang, S. Zeng, B. Di, Y. Tan, M. D. Renzo, M. Debbah, Z. Han, H. V. Poor, and L. Song, “Intelligent omnisurfaces for full-dimensional wireless communications: Principles, technology, and implementation,” IEEE Commun. Mag., vol. 60, no. 2, pp. 39-45, Feb. 2022.

[31] X. Mu, Y. Liu, L. Guo, J. Lin, and R. Scholz, “Simultaneously transmitting and reflecting (STAR) RIS aided wireless communications,” IEEE Trans. Veh. Technol., to appear.

[32] C. Liu, X. Liu, D. W. K. Ng, and J. Yuan, “Deep residual learning for coordinated passive beamforming for distributed intelligent reflecting surfaces network,” in Proc. IEEE Veh. Technol. Conf. (VTC), Virtual Conference, May 2020.

[33] B. Zheng, C. You, and R. Zhang, “Double-IRS assisted multi-user MIMO: Cooperative passive beamforming design,” IEEE Trans. Wireless Commun., vol. 20, no. 7, pp. 4513-4526, July 2021.