The two dimensional antiferromagnetic Heisenberg model in the presence of an external field

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We present numerical results on the zero temperature magnetization curve and the static structure factors of the two dimensional antiferromagnetic Heisenberg model in the presence of an external field. The impact of frustration is also studied.

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I. INTRODUCTION

The discovery of high-$T_c$ superconductors has renewed the interest in the two dimensional antiferromagnetic spin-$\frac{1}{2}$ Heisenberg Hamiltonian:

$$H = \sum_{<x,y>} \vec{S}(x) \cdot \vec{S}(y) + \alpha \sum_{|x,y|} \vec{S}(x) \cdot \vec{S}(y).$$ (1.1)

The ground state of (1.1) with nearest neighbors couplings $<x,y>$ is considered to yield a good description of the antiferromagnetic properties in the undoped copper oxide planes. Doping destroys the antiferromagnetic order and this effect has been studied intensively by adding to the nearest neighbor Hamiltonian a second term with next to nearest neighbor couplings $|x,y|$ in diagonal directions of strength $\alpha=J_2/J_1$. The behavior of the static structure factors $S_i(\vec{p}=(p_1,p_2),\alpha,N) = 4 \sum \exp(i\vec{p} \cdot \vec{x}) \langle S_i(0)S_i(\vec{x}) \rangle$ - which are just the Fourier transform of the spin-spin correlators $\langle S_i(0)S_i(\vec{x}) \rangle$, $i=1,2,3$ - changes with $\alpha$:

1.) For $0 < \alpha < 0.3$ one finds a strong divergence

$$S_i(\vec{p}=(p_1,p_2),\alpha,N \rightarrow \infty) = 4N m^+ + \alpha$$ (1.2)

at the momentum $\vec{p}=(\pi,\pi)$, indicating antiferromagnetic order with a non vanishing staggered magnetization $m^+(\alpha=0) \approx 0.308$.

2.) For $\alpha > 0.7$ such a divergence is found at $\vec{p}=(0,\pi)$ and $\vec{p}=(\pi,0)$, which is a signature for collinear antiferromagnetic order.

It is unclear, what happens in between $0.3 < \alpha < 0.7$. One can imagine, that the singularity is less divergent and starts to move along some trajectory from $\vec{p}=(\pi,\pi)$ to $\vec{p}=(\pi,0)$ and $\vec{p}=(0,\pi)$. The reconstruction of this trajectory would demand a very precise determination of the static structure factor $S_i(\vec{p},\alpha,N)$ with a high resolution in the momentum plane.

Moving singularities in static structure factors were observed and analyzed for the one dimensional antiferromagnetic Heisenberg model in the presence of an external field $B$. The relevant ground states, which enter in the computation of the static structure factors have total spin $S=\pm M(B)N$, where $M=M(B)$ is given by the magnetization curve. The following phenomena have been found in the one dimensional antiferromagnetic Heisenberg model at fixed magnetization $M$:

1) the longitudinal structure factor is almost constant for $p_3(M) = \pi(1-2M) < p \leq \pi$:

$$S_3(p,M) \approx 2(1-2M)$$ (1.3)

and develops a cusp type singularity at $p = p_3(M)$. The critical exponent $\eta_3(M)$ at this singularity changes with the external field - i.e. with $M$.

2) The transverse structure factor is almost constant for $0 < p < p_1(M) = 2\pi M$:

$$S_1(p,M) \approx 2M$$ (1.4)

and develops a ”break” type singularity at the momentum $p_1(M)$. Moreover, it diverges at $p=\pi$ with a field-dependent critical exponent $\eta_1(M)$.

The singularities in the static structure factors are accompanied with zero frequency excitations (”soft-modes”) in the dynamical structure factors. The effect of frustration on the position $p_3(M)$, $p_1(M)$ and the type $\eta_3(M)$, $\eta_1(M)$ of the singularities has been studied as well. Frustration does not change the position but drastically changes the type of the singularities.

It is the purpose of this paper to investigate the singularities in the static structure factors of the two dimensional spin-$\frac{1}{2}$ Heisenberg model in the presence of an external field.

Our analysis is based on a numerical computation of the ground states at fixed magnetization $M=S/N$ on the following square lattices:

$$N = 4 \times 4, \quad N = 6 \times 6$$ (1.5)

with periodic boundary conditions and

$$N = 3 \times 3 + 1, \quad N = 5 \times 5 + 1, \quad N = 7 \times 7 + 1$$ (1.6)
with helical boundary conditions. In the latter case the Hamiltonian can be considered to be one dimensional

\[
H = \sum_{x=1}^{N} \vec{S}(x) \left( \vec{S}(x+1) + \vec{S}(x+k) \right) \\
+ \alpha \left( \sum_{x=1}^{N} \vec{S}(x) \left( \vec{S}(x+k-1) + \vec{S}(x+k+1) \right) \right) 
\]

(1.7)

with four types of couplings. Square lattices are realized for:

\[
N = k^2 + 1, \quad k = 3, 5, 7 \quad (1.8)
\]

\[
N = k^2 - 1, \quad k = 3, 5, 7 \quad (1.9)
\]

as can be seen for \(k=5, N=26\) in Fig. 1.

The helical boundary conditions yield a quantization of the two dimensional momenta \(\vec{p} = (p_1, p_2)\):

\[
p_1 = \frac{2\pi}{N} n, \quad p_2 = \frac{2\pi}{N} kn, \quad n = 0, \ldots, \frac{N}{2} \quad (1.10)
\]

which differs from the usual quantization due to periodic boundary conditions.

The outline of the paper is as follows. In section 2 we present the magnetization curves \(M(\alpha, B)\) at zero temperature, as they depend on the external field \(B\) and the frustration parameter \(\alpha\). The field-dependence of the static structure factors at momentum \(\vec{p} = (\pi, \pi)\) is discussed in section 3. The momentum dependence of the static structure factors turns out to be smooth for the unfrustrated model, as is demonstrated in section 4. Therefore, we did not find any signature for field-dependent soft-modes in this case \((\alpha = 0.0)\). The situation changes if we frustrate the system sufficiently \((\alpha = 0.5)\). There we observe cusp-type singularities in the field-dependence of the static structure factors at certain momenta (section 5).

II. THE MAGNETIZATION CURVE

Let us start with the ground state energies per site \(\epsilon(\alpha, M = \frac{1}{2}, N)\) at fixed magnetization \(M = \frac{1}{2}\) and frustration \(\alpha\). The numerical results for the systems (1.5) and (1.6) scale with \(M\) except for the vicinity of \(M=0\) and \(M=\frac{1}{2}\). Near saturation \(M \to \frac{1}{2}\), the ground state energy per site

\[
\epsilon(\alpha, M = \frac{1}{2}, N) = \frac{\alpha + \frac{1}{2}}{M - \frac{1}{2}} \quad (2.1)
\]

and its first derivative

\[
\frac{\epsilon(\alpha, M = \frac{1}{2} - \frac{1}{N}, N) - \epsilon(\alpha, M = \frac{1}{2}, N)}{M - \frac{1}{2}} = 4, \quad (2.2)
\]

with \(\alpha \leq \frac{1}{2}\) are known for all system sizes \(N\). Finite size effects appear in the difference ratios:

\[
(M - \frac{1}{2})^{-1}(\epsilon(\alpha, M, N) - \epsilon(\alpha, M = \frac{1}{2}, N)) = D_1(\alpha, Z) = 4 - \frac{1}{2}D_1(\alpha)Z - \frac{1}{2}D_2(\alpha) Z^2 - \frac{1}{2}D_3(\alpha) Z^3 
\]

(2.3)

They can be absorbed in the definition of an "optimized" scaling variable:

\[
Z = (1 - 2M) - \frac{c(M)}{N} \quad (2.4)
\]

with

\[
c(M) = 2(\frac{1}{2} - 1/N)^{\kappa}. \quad (2.5)
\]

The choice for \(c(M)\) guarantees, that the expansion (2.3) satisfies the "initial" condition (2.2) for \(M = \frac{1}{2} - \frac{1}{N}\) i.e. \(Z=0\) independent of the exponent \(\kappa\). For \(\kappa \simeq 5\), we achieve the best scaling of the numerical data as can be seen from Fig. 2. A fit of the numerical data to a Taylor expansion in \(Z\) yields the coefficients \(D_n(\alpha), \ n = 1, 2, 3\) listed in Table 1. We have repeated the fit with one additional term in the Taylor expansion (2.3). Such a fit yields only slight modifications of the first two coefficient.

It is remarkable to note, that the first coefficient \(D_1(\alpha)\) decreases with \(\alpha\) and vanishes at some value \(\alpha = \alpha^*\) where \(0.49 < \alpha^* < 0.5\). This has important consequence for the magnetization curve \(M=M(B, \alpha)\) which is obtained from \(\epsilon(\alpha, M, N = \infty)\) by differentiation:

\[
\frac{\partial \epsilon}{\partial M} = B(M). \quad (2.6)
\]

For \(0 < \alpha < \alpha^*\), where \(D_1(\alpha) > 0\), we derive from (2.3) and (2.6) a linear behavior near saturation \(M \to \frac{1}{2}\)

\[
M = \frac{1}{2} - \frac{4 - B}{D_1(\alpha)} \quad (2.7)
\]

For \(\alpha = \alpha^*\) - where \(D_1(\alpha^*) = 0\) - however, we find a square root behavior:

\[
M = \frac{1}{2} - \sqrt{\frac{4 - B}{2D_2(\alpha^*)}} \quad (2.8)
\]

A similar phenomenon has been observed in the one dimensional antiferromagnetic Heisenberg model with frustration \(\alpha\). There it turns out, that the magnetization curve has a square root behavior of the type (2.8) in the spin fluid phase \(0 \leq \alpha < \alpha_0\) with \(\alpha_0 \simeq 0.241\) and a quartic root behavior in the vicinity of the transition point \(\alpha\), namely at \(\alpha=0.25\).
Let us go back to the discussion of the magnetization curves in the two dimensional model. The coefficients $D_n(\alpha)$, $n = 1, 2, 3$ in the polynomial (2.3) were fitted by the data points in the regime $Z < 2$ ($\frac{1}{4} < M < \frac{1}{4}$). The extrapolation of (2.3) to larger $Z$-values (smaller $M$ values) is close to and away from the numerical data for $\alpha=0.0$ and $\alpha=0.5$, respectively. In other words: $D(\alpha = 0.0, Z)$ seems to be a smooth function in $Z$ in contrast to $D(\alpha = 0.5, Z)$ which changes its slope substantially somewhere between $Z=0.3$ and $Z=0.4$. In order to determine the exact position $Z$, where the slope may be discontinuous, we would need numerical results on larger systems, which allow for a better resolution of the $Z$ dependence of (2.3). A discontinuity in the slope of $D(\alpha = 0.5, Z)$ at $Z = Z_0$ (i.e. $M = M_0$) means, that the magnetization curve (2.6) has a plateau at height $M = M_0$.

The existence of a plateau in the magnetization curve for $\alpha > 0.5$ has been predicted by S. Gluzman[4], who studied the equivalent Bose gas problem and found a gap. In Figs. 3a,b we present the magnetization curves for $\alpha = 0.0$ and $\alpha = 0.5$ as they follow from an analysis of finite system results with the method of Bonner and Fisher[2]. Finite size effects appear to be small for $\alpha = 0.0$, as can be seen from a comparison of the numerical data for the small system sizes ($N = 24, 26$) with those for larger systems ($N = 36, 48, 50$) shown in the insets of Fig. 3a.

For $\alpha = 0.5$, however, we observe a sensitivity of the numerical data to the size of the system and to the boundary condition. The results for $N = 24, 26$ obtained with helical boundary conditions have a shoulder around $M = \frac{1}{4}$, which might indicate the emergence of a "plateau". Unfortunately, we do not reach this shoulder with our largest systems $N = 48, 50$ shown in the upper insets of Fig. 3b. The lower inset of Fig. 3b contains the results for $N = 36$ with periodic boundary conditions. Here, we observe a shoulder around $M = 0.42$ which seems to be absent in the upper insets for the systems with $N = 48, 50$. The variation of the magnetization curve with $\alpha$ on the $4 \times 4$ system has been computed by Lozovik and Notch[4]. These authors claim to see indications for "plateaus" at various $M$-values for $\alpha \simeq 0.538$.

### III. THE FIELD-DEPENDENCE OF THE STATIC STRUCTURE FACTORS AT $\vec{p}=(\pi, \pi)$

In the one dimensional spin-$\frac{1}{4}$ antiferromagnetic Heisenberg model, the field-dependence of the longitudinal and transverse structure factors at $p=\pi$ looks very different[1]. Finite size effects are small in the longitudinal case, the data points nicely scale with $M$ (for $M > 0$) and develop a logarithmic singularity for $M \to 0$. Finite size effects are very strong in the transverse case, the data points do not scale at all with $M$ and indicate that the singularity persists at $p=\pi$ if the external field is switched on. The same behavior can be seen in the two dimensional case. In Fig. 4 we present the longitudinal structure factors $S_3(p = (\pi, \pi), M, \alpha, N)$ with $\alpha = 0.0$ and $\alpha = 0.5$. All the data points follow one unique curve. Finite size effects turn out to be very small for larger values of $M$. For decreasing values of $M$ however, we find increasing finite-size effects. This is a signature for the emergence of a singularity in the limit $M \to 0$. Note also, that the longitudinal structure factor decreases with the frustration parameter $\alpha$.

The transverse structure factors $S_1(p = (\pi, \pi), M, \alpha, N)$ are shown in Fig. 5a and Fig. 5b for $\alpha=0.0$ and $\alpha=0.5$, respectively. The strong finite-size dependence for $\alpha=0.0$ indicates, that a weak magnetic field strengthens the singularity at $p = (\pi, \pi)$ and thereby the antiferromagnetic order in the transverse structure factor. Note also the drastic change when we pass from the unfrustrated case (Fig. 5a) to the frustrated case (Fig. 5b). In the latter case the finite-size dependence is less pronounced. Moreover, there appears a dip at $M = \frac{1}{2}$ in the data for $N = 24, 26$ and at $M = 0.42$ for $N = 36$ (inset Fig. 5b). In both cases, this is just the position, where we found a shoulder in the corresponding magnetization curves. The dip at $M = \frac{1}{2}$ has a good chance to survive in the thermodynamical limit; it is at least consistent with the numerical results on $N = 24, 26, 36, 48, 50$. The dip at $M = 0.42$ in the data points seems to be a mere finite-size and/or boundary value effect.

### IV. THE MOMENTUM DEPENDENCE OF THE STATIC STRUCTURE FACTOR IN THE UNFRUSTRATED MODEL ($\alpha = 0$)

It was pointed out in the introduction that the static structure factors of the one dimensional spin-$\frac{1}{4}$ antiferromagnetic Heisenberg model develop singularities at field-dependent momenta $(p_1(M) = 2\pi M, p_2(M) = \pi(1 - 2M))$. We therefore address here the question, whether such singularities can be found in the momentum dependence of the static structure factors for the two dimensional model as well. It will turn out, that the appearance of these singularities crucially depends on the frustration parameter $\alpha$. In this section we first treat the unfrustrated case $\alpha=0.0$.

The magnetization $M = \frac{\vec{p}}{2\pi}$ is realized on three systems (1.5) (1.6) with $N=24, 36, 48$. The numerical values for the longitudinal structure factor $S_3(p = (p_1, p_2), M = \frac{2\pi}{\alpha}, \alpha = 0, N), N=24, 36, 48$ in the first Brillouin zone can be read off Fig. 6. These values appear to be constant along lines of constant:

$$x = \cos p_1 + \cos p_2$$

represented by the dashed curves in Fig. 6. The behavior can be seen directly along the line $p_1 + p_2 = \pi$, $x = 0$. Therefore, we expect the longitudinal structure factor to
depend on the variable $x$ only. In Fig. 7 we have plotted the longitudinal structure factor at $M=\frac{1}{2} \pi$ ($N=24, 36$), $M=\frac{3}{4} \pi$ ($N=24, 48$) and $M=\frac{5}{6} \pi$ ($N=24, 36, 48$) versus $x$. We observe a constant behavior for $-2 < x < x_3(M)$, and afterwards a decrease for $x_3(M) < x < 2$. $x_3(M)$ increases with $M$.

The corresponding $x$-dependence of the transverse structure factor is shown in Fig. 8. For $x \to -2 (\vec{p} \to (\pi, \pi))$ we see here the emergence of a singularity in accord with the strong finite-size dependence of the transverse structure factor $S_1(\vec{p} = (\pi, \pi), M, \alpha = 0, N) - 2M$ discussed in section 3. Moreover, this quantity is almost zero for $x > x_1(M)$, where $x_1(M)$ decreases with $M$. At $M=0$ the longitudinal and transverse structure factor coincide. Their dependence on the variable (4.1) is shown in Fig. 9 for $N = 24, 26, 36$.

In the one dimensional case the longitudinal and transverse structure factors turned out to be constant for $p > p_3(\pi)=\pi(1-2M)$ and $p < p_1(\pi)=\pi 2M$, respectively. However, there one finds cusp-type singularities at the "soft mode" momenta $p=p_3(\pi)$ and $p=p_1(\pi)$. The smooth momentum dependence of the static structure factors for the unfrustrated two dimensional Heisenberg model seems to exclude the existence of field-dependent soft-modes.

V. HUNTING FOR SOFT-MODES

In one and two dimensions, the antiferromagnetic Heisenberg models with nearest neighbor couplings differ substantially under various aspects:

1. Fluctuations are stronger in one than in two dimensions i.e. the antiferromagnetic order increases with the dimension.

2. There are field-dependent soft-modes in one dimension; they seem to be absent in two dimensions.

3. Near saturation ($B \to 4, M \to \frac{1}{2} \pi$) the magnetization curves show a linear behavior in two dimensions but a square root behavior in one dimension. As was pointed out in section 2, this behavior changes, if the frustration parameter exceeds a critical value $\alpha^*$.

If these features are correlated, we might hope to find soft-modes in the two dimensional antiferromagnetic Heisenberg model as well, provided we weaken the antiferromagnetic order by frustrating the system. In order to test this hypothesis, we need a reliable criteria for the existence or nonexistence of soft-modes.

In the one dimensional case, we looked for singularities (breaks, cusps etc) in the momentum distribution of the structure factors at fixed magnetization $M$. As is demonstrated in Figs. 10a,b for the one dimensional case, pronounced structures are produced as well by the soft-modes in the $M$-dependence. The position of these singularities $M_1(p) = \frac{\pi}{2} p$ and $M_3(p) = \frac{\pi}{2} p$ define the soft-mode trajectories, which travel from $p = 0$ (ferromagnetic order) to $p = \pi$ (antiferromagnetic order) and vice versa. Keeping fixed the momentum, we have to meet the soft-mode at an appropriate value of the magnetization. In this respect it is much easier to find the soft-modes in the one dimensional case.

In case of the frustrated two dimensional Heisenberg model, the points $\vec{p} = (\pi, \pi)$ and $\vec{p} = (0, 0)$ play a special role. Singularities in the structure factors at these points indicate antiferromagnetic and collinear antiferromagnetic order, respectively. Therefore, it is rather plausible to assume, that the soft-mode trajectories connect these points.

On the system with $N = 5 \times 5 - 1 = 24$ sites, the momentum closest to $\vec{p} = (\pi, \pi)$ and $\vec{p} = (0, 0)$ are $\vec{p} = (\frac{\pi}{2}, \frac{\pi}{2})$ and $\vec{p} = (\frac{\pi}{6}, \frac{\pi}{6})$, respectively. Here, we have the best chance to observe soft-mode effects in the $M$-distributions of the static structure factors. The sensitivity of the $M$-distributions to a change of the frustration parameter can be seen in Figs. 11a,b and 12. In the unfrustrated case $\alpha = 0.0$ (open symbols), the $M$-distributions are smooth. Switching on the frustration parameter $\alpha$ (solid symbols), we observe the emergence of a peak at $M = \frac{1}{3}$ in $S_1(\vec{p} = (\frac{\pi}{4}, \frac{\pi}{4}), M, \alpha)$ (Fig. 11a) and at $M = \frac{1}{5}$ in $S_3(\vec{p} = (\frac{\pi}{6}, \frac{\pi}{6}), M, \alpha)$ (Fig. 11b), respectively. The $M$-distribution of the transverse structure factor at $\vec{p}=\left(\frac{\pi}{3}, \frac{\pi}{3}\right)\pi$ is shown in Fig. 12. The data for $\alpha = 0.5$ show a break at $M=0.42$, which reminds us to the structure observed in the one dimensional case shown in Fig. 10b.

VI. DISCUSSION AND CONCLUSION

In this note, we have studied the zero temperature properties of the two dimensional spin-$\frac{1}{2}$ antiferromagnetic Heisenberg model in the presence of an external field. Our results for the unfrustrated and the frustrated model can be summarized as follows.

1. The unfrustrated model: Owing to small finite-size effects and a weak dependence on the boundary conditions, the magnetization curve and the momentum and field-dependence of the static structure factors can be determined with fairly good accuracy from the rather small systems $(1.5),(1.6)$. This statement holds away from the "critical values" $\vec{p}=(\pi, \pi), M=0$. At the momentum $\vec{p}=(\pi, \pi)$ the longitudinal structure factor is finite for $M > 0$, whereas the transverse structure factor is divergent if the system size $N$ goes to infinite. At fixed magnetization $M$, the momentum dependence of the static structure factor is smooth and well described.
We have restricted our considerations to a limited range field-dependent momentum sufficiently. They might originate from a soft-mode with $\vec{p}$ and $\vec{p}$ dependence of the trajectory are much too small, in order to determine the field-size and the boundary conditions. We do not know, whether these phenomena appear - varies with the system size and the boundary conditions. We do not know, whether these phenomena are finite-size effects or will survive in the thermodynamical limit.

We looked for singularities in the field-dependence of the static structure factors at fixed momenta. Indeed maxima emerge for momenta close to $\vec{p}=(\pi, \pi)$ and $\vec{p}=(0, \pi), \vec{p}=(\pi, 0)$ if the system is frustrated sufficiently. They might originate from a soft-mode with field-dependent momentum $\vec{p}(M)$ connecting $\vec{p}=(\pi, \pi)$ and $\vec{p}=(0, \pi), \vec{p}=(\pi, 0)$. However, our systems (1.5) (1.6) are much too small, in order to determine the field-dependence of the trajectory $\vec{p}(M)$.

We have restricted our considerations to a limited range $0 \leq \alpha \leq \frac{1}{2}$ in the frustration parameter $\alpha$. Going beyond $\alpha=\frac{1}{2}$ we meet level crossings. E.g. the ground states in the sectors with $S_z=\frac{N}{2}-1$ (called ”one magnon states”) have different momenta for $\alpha < \frac{1}{2}$ and $\alpha > \frac{1}{2}$, respectively. Level crossings are ”felt” by the Lanczos algorithm, which meets increasing problems to find the “true” ground state among two competing states. The problem can be solved, if these two states differ in their quantum numbers. One of the two competing states is filtered out, if the starting vector has the appropriate quantum numbers. Therefore, an extension of our analysis to the strong frustration regime $\alpha > \frac{1}{2}$ demands for a careful analysis of the ground state quantum numbers in the sectors with a given total spin. We expect, that level crossings for $\alpha > \frac{1}{2}$ will generate interesting phenomena. E.g. they might be responsible for the plateau in the magnetization curve predicted by S. Gluzman.

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FIG. 1. The square lattice $5 \times 5 + 1 = 26$ with helical boundary condition.

FIG. 2. Scaling behavior of the difference ratio (2.3) in the strong field limit for the unfrustrated ($\alpha = 0.0$ open symbols) and the frustrated case ($\alpha = 0.5$ solid symbols). The curves represent polynomial fits in the scaling variable (2.4).

FIG. 3. The magnetization curves computed on finite systems with $N = 24, 26, 36, 48$ and $50$ with the method of Bonner and Fisher [10] a) the unfrustrated case $\alpha = 0.0$ b) the frustrated case with $\alpha = 0.5$.

FIG. 4. The field-dependence of the longitudinal structure factor $S_3(\mathbf{p} = (\pi, \pi), M, \alpha, N)$ for $\alpha = 0.0$ (open symbols) and $\alpha = 0.5$ (solid symbols). The curves are shown to guide the eye.

FIG. 5. The field-dependence of the transverse structure factor $S_1(\mathbf{p} = (\pi, \pi), M, \alpha, N)$ for a) $\alpha = 0.0$, b) $\alpha = 0.5$.

FIG. 6. The momentum dependence of the longitudinal structure factor $S_3(\mathbf{p} = (p_1, p_2), M = M_0, \alpha = 0.0, N)$ at fixed magnetization.

FIG. 7. Scaling of the longitudinal structure factor $S_3(\mathbf{p} = (p_1, p_2), M, \alpha = 0.0, N)$ in the variable $x = \cos p_1 + \cos p_2$ at fixed $M = \frac{M_0}{2}$ and $M = \frac{5}{12}$.

FIG. 8. Same as Fig. 7 for the transverse structure factor. The curves are shown to guide the eye.

FIG. 9. Same as Fig. 7 for $M = 0.0$.

FIG. 10. Field dependence of the one dimensional structure factors at fixed momenta $p = \frac{4}{3} \pi, \frac{2}{3} \pi, \frac{1}{3} \pi, \frac{1}{4} \pi$ a) the longitudinal case b) the transverse case.

FIG. 11. Field dependence of the two dimensional longitudinal structure factors at fixed momenta a) $\mathbf{p} = \pi(\frac{2}{3}, \frac{1}{3})$ b) $\mathbf{p} = \pi(\frac{4}{3}, \frac{1}{3})$ open symbols $\alpha = 0.0$ and solid symbols $\alpha = 0.5$.

FIG. 12. Field dependence of the two dimensional transverse structure factor at fixed momentum $\mathbf{p} = \pi(\frac{4}{3}, \frac{1}{3})$.
FIG. 3a
$S_3(p=\pi, M, \alpha, N)$

FIG. 4
\[ S_1(p=(\pi, \pi), M, \alpha=0.0, N) - 2M \]
$S_1(\bar{p}=(\pi,\pi), M, \alpha=0.5, N) - 2M$
$S_3(\bar{p}=(p_1,p_2), M=5/12, \alpha=0.0, N)$

FIG. 6
$S_3(p=(p_1,p_2), M, \alpha = 0.0, N)$

$M = 1/3$

$M = 3/8$

$M = 5/12$

FIG. 7
\[ S_1(\bar{p}=(p_1, p_2), M, \alpha=0.0, N) - 2M \]

- \( M = \frac{5}{12} \)
- \( M = \frac{1}{3} \)
- \( M = \frac{3}{8} \)

FIG. 8
\[ S_3(\vec{p}=(p_1, p_2), \alpha=0, \alpha=0, N) \]

\[ \cos(p_1) + \cos(p_2) \]

FIG. 9
$S_1(p,M,\alpha=0.0,N) - 2M$

FIG. 10b
$S_3(\bar{\rho}=\frac{2}{3}, \frac{2}{3}, \pi, M, \alpha, N)$

FIG. 11a
$S_3(\bar{\rho}=(1/6,5/6)\pi,M,\alpha,N)$
$S_1(\bar{p}=(2/3,2/3), M, \alpha, N) - 2M$

FIG. 12

- △ $N=48, \alpha=0.0$
- ▲ $N=48, \alpha=0.5$
- ○ $N=24, \alpha=0.0$
- ● $N=24, \alpha=0.5$
- ◊ $N=36, \alpha=0.0$
- ♦ $N=36, \alpha=0.5$