Interpreting the Spin-down Evolutions of Isolated Neutron Stars with Hall Effects

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ABSTRACT

The observed long-term spin-down evolution of isolated radio pulsars cannot be explained by the standard magnetic dipole radiation with a constant braking torque. However, how and why the torque varies still remains controversial, which is a major issue in understanding neutron stars. Many pulsars have been observed with significant long-term changes of their spin-down rates modulated by quasi-periodic oscillations. Applying the phenomenological model of pulsar timing noise we developed recently to the observed precise pulsar timing data of isolated neutron stars, here we show that the observed long-term evolutions of their spin-down rates and quasi-periodic modulations can be explained by Hall effects in their crusts. Therefore the evolution of their crustal magnetic fields, rather than that in their cores, dominates the observed long term spin-down evolution of these young pulsars. Understanding of the nature of pulsar timing noise not only reveals the interior physics of neutron stars, but also allows physical modeling of pulsar spin-down and thus improves the sensitivity of gravitational wave detections with pulsars.

Subject headings: stars: neutron–pulsars: individuals (B1828-11)–magnetic fields

1. Introduction

Pulsars are very stable natural clocks with observed steady pulses. However, many pulsars exhibit significant timing irregularities, i.e., unpredicted arrival times of pulses. Hobbs et al. (2010, hereafter H2010) carried out so far the most extensive study of the long-term

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timing irregularities of 366 pulsars, and ruled out some timing noise models in terms of observational imperfections, random walks (Boynton 1972; Alpar et al. 1986; Cheng 1987a, 1987b), and planetary companions (Cordes & Shannon 2008). Lyne et al. (2010, hereafter L2010) found that timing behaviors often result from two different spin-down rates, and pulsars switch abruptly between these states, often quasi-periodically, leading to the observed spin-down patterns. The deviations of pulsars’ spin frequency $\dot{\nu}(t)$ from its long-term trend are correlated with changes in the pulse shapes (see Fig. 3 and 4 of L2010), and therefore are magnetospheric in origin. By modeling the observed precise timing data of pulsars, we will show that the long-term linear change of $\dot{\nu}$ is consistent with the timescale of Hall drift, and the oscillatory structure of $\dot{\nu}$ and the changes in pulse shapes can be produced by the Hall waves in the crust of a neutron star (NS). Consequently the mechanism of magnetic field evolution and the origin of the timing noise of pulsars are better understood, which may improve the sensitivity of gravitational wave detections with pulsars.

2. Phenomenological model and observational test

Assuming the pure magnetic dipole radiation in vacuum as the braking mechanism for a pulsar’s spin-down (e.g. Lorimer & Kramer 2004), we have

$$\dot{\nu} = -AB(t)^2\nu^3,$$

in which $A = 8\pi^2 R^6 \sin \theta^2 / 3c^3 I$ is a constant, $B(t)$ is the strength of the dipole magnetic field at the surface of the NS, $R (\sim 10^6 \text{ cm})$, $I (\sim 10^{45} \text{ g cm}^2)$, and $\theta (\sim \pi/2)$ are the radius, moment of inertia, and angle of magnetic inclination, respectively. Previously, we constructed a phenomenological model for $B(t)$ with a long-term evolution modulated by short-term oscillations (Zhang & Xie 2012a and 2012b, hereafter Paper I and Paper II, respectively):

$$B(t) = B_L(t)(1 + \sum \kappa_i \sin(\phi_i + 2\pi \frac{t}{T_i})),$$

where $t$ is the pulsar’s age, and $\kappa_i (\ll 1)$, $\phi_i$, $T_i$ are the amplitude, phase and period of the $i$-th oscillating component, respectively. $B_L(t) = B_0(t/t_0)^{-\alpha}$, in which $B_0$ is the field strength at age $t_0$, and $\alpha$ is the power law index. Then from Equation (2) and taking only the dominating oscillating component, we obtained the analytic approximation for $\ddot{\nu}$ (Paper I):

$$\ddot{\nu}_+ \simeq -2\dot{\nu}(\alpha/t_{\text{age}} \pm f),$$

where $t_{\text{age}}$ is the real age of the pulsar and $f \equiv 2\pi \kappa / T$ for the dominating oscillating component.
It has been found that Equation (3) describes adequately the statistical properties of the observed timing noises of radio pulsars (Paper I). Therefore for relatively young pulsars with $t_{\text{age}} < 3 \times 10^5$ yr, the first term in Equation (3) dominates and we should have $\ddot{\nu} > 0$ if $\alpha > 0$. Considering that the characteristic ages ($\tau_c$) of young pulsars are normally several times larger than $t_{\text{age}}$, Equation (3) thus explains naturally the observed $\ddot{\nu} > 0$ for most young pulsars with $\tau_c \lesssim 10^6$ yr. Similarly for much older pulsars, the second term in Equation (3) dominates, in agreement with the observational fact that the numbers of negative and positive $\ddot{\nu}$ are almost equal for the old pulsars in the sample of H2010. The confirmation of Equation (3) with observations naturally suggests that the observed pulsars indeed have evolutionary links, and the equation therefore provides details about the dipole magnetic field evolution of a young pulsar into an old pulsar.

However, Equation (3) has so far not been tested with the observed evolutions of individual pulsars. Following the same approach in Paper I, we obtain

$$\dot{\nu} \simeq \dot{\nu}_0 (1 + 2\Sigma \kappa_i (\sin(\phi_i + 2\pi \frac{t}{T_i} \sin \phi_i)) + \ddot{\nu}_L (t - t_0),$$

(4)

where $\dot{\nu}_0 = \dot{\nu}(t_0)$, $\ddot{\nu}_L = -2\alpha \dot{\nu}_0 / t_0$ describes the long-term monotonic variation of $\dot{\nu}(t)$. Therefore Equation (4) can be tested with the long-term monitoring observations of individual pulsars. We can obtain an estimate of $\ddot{\nu}_L$ by linearly fitting the long-term monotonic variation of reported $\dot{\nu}(t)$. Consequently we can also determine the time scale of the long-term magnetic field evolution of each pulsar (see Equation (6) in Paper I)

$$\tau_B \equiv \frac{B}{\dot{B} \nu_0} = \frac{2\dot{\nu}_0 \nu_0}{\ddot{\nu}_L \nu_0 - 3\dot{\nu}_0^2},$$

(5)

for comparisons with theoretical models of neutron star magnetic field evolution. $\tau_B < 0$ indicates magnetic field decrease and vice versa.

The sample of L2010 provides the precise histories of $\dot{\nu}$ for seventeen pulsars and thus may be applied to test Equation (4). Lyne et al. (2010) found that PSR B1828-11 clearly shows a long-term evolution trend (noticed in L2010 as a linear increase of its $\dot{\nu}$), i.e. $\ddot{\nu}_L > 0$. We show the comparison between the reported (taken from L2010) and analytically calculated $\dot{\nu}(t)$ for the pulsar in Figure 1 all the parameters are listed in Table 1. The one major difference is caused by the decrease of the oscillation periods of the reported data after $\sim 4000$ days, which indicates that the oscillations are unlikely caused by precession of the neutron star. Nevertheless, our model describes the general trend of the reported data of B1828 quite well.

By linearly fitting $\dot{\nu}$ of all other pulsars, we find that most of them also exhibit long-term monotonic evolutions modulated by short-term quasi-periodical oscillations. However, three
pulsars in the sample, namely, B1822-09, B2035+36 and J2043+2740, exhibit much more erratic short-term behaviors, which may significantly bias the fitting results; here we exclude these three pulsars from further study. We conclude that our phenomenological model can describe adequately the spin-down evolutions of fourteen pulsars in the sample of seventeen pulsars of L2010; all the observed and derived parameters for the fourteen pulsars are listed in Table 1.

3. Physical implications

3.1. Inclination evolution?

We first investigate the alternative possibility that the long-term linear change of $\dot{\nu}(t)$ is caused by the change of the inclination angle of a NS, rather than the change of its magnetic field strength. Observationally, the inclination angle change would also produce a long-term change on the width of pulses. For a circular beam, the change of its width can be given by the geometrical relation

$$
\Delta W = W_0 - 4 \arcsin \left[ \frac{\left( \sin^2 \frac{\gamma}{2} - \sin^2 \frac{\beta + \Delta \theta}{2} \right)}{\sin(\theta - \Delta \theta) \sin(\theta + \beta)} \right]^{1/2},
$$

where $\Delta \theta$ is the magnitude of the inclination angle change that can be obtained from $\sin^2 \Delta \theta / \sin^2 \theta \approx \dot{\nu}_L \Delta t / \dot{\nu}_0 \approx 0.0017$ for PSR B1828−11 ($\Delta t$ is the time span of the observations), $\beta$ is the impact angle, $\gamma$ is the half angular width of the radiating cone (Gil et al. 1984),

$$
\gamma = 2 \arcsin \left[ \frac{\sin^2 \frac{W_0}{4} \sin \theta \sin(\theta + \beta)}{\sin^2 \frac{\beta}{2}} \right]^{1/2},
$$

$W_0 \approx 0.04P$ is the initial pulse profile width of B1828−11 (L2010). Thus, $\Delta W$ only depends on $\theta$ and $\beta$. There are two observational constraints for $\theta$ and $\beta$ for B1828−11: (a) $\Delta W \approx -0.28 \pm 0.51$ ms (L2010); and (b) its position angle (P.A.) gives $\sin \theta / \sin \beta \sim 5.35$ (Gould & Lyne 1998).

We show the comparison of the two constraints on $\theta$ and $\beta$ in Figure 2. One can see that condition (a) requires $\beta \lesssim 1.2$ deg, which is much smaller than that from condition (b); thus the possibility of inclination angle change is ruled out for B1828−11. It is therefore reasonable to assume that the long-term linear changes of $\dot{\nu}(t)$ for other pulsars in the sample are also not caused by the changes of their inclination angles, because the patterns of their spin-down evolutions are very similar to PSR B1828−11.
3.2. Long-term magnetic field evolution

Goldreich & Reisenegger (1992) studied several avenues for magnetic field decay in isolated NSs: ohmic decay, ambipolar diffusion, and Hall drift. Depending on the strength of the magnetic fields, each of these processes may dominate the evolution, and the ambipolar diffusion is only important for magnetars. Ohmic decay occurs in both the fluid core and solid crust of a NS (Sang & Chanmugam 1987; Urpin et al. 1994; Page et al. 2000). It is inversely proportional to the electric conductivity and independent of the strength of magnetic fields. If an electric current flows vertically in magnetic fields, the fields exerts a transverse force on the moving charge carriers. The resulting Hall drift of the carriers can transport magnetic fields from the inner crust to the outer crust (Jones 1988). The Hall effect is non-dissipative and thus cannot be a direct cause of magnetic field decay. However, it can enhance the rate of ohmic dissipation, since only electrons are mobile in the solid crust, and their Hall angle is large. Consequently the evolution of magnetic fields resembles that of vorticity, and then the fields undergo a turbulent cascade terminated by ohmic dissipation at small scales (Goldreich & Reisenegger 1992; Muslimov 1994; Biskamp & Müller 1999; Urpin & Shalybkov 1999; Rheinhardt & Geppert 2002; Geppert & Rheinhardt 2002; Pons & Geppert 2010; Geppert et al. 2013).

Cumming et al. (2004, hereafter C2004) found that, in isolated NSs with relatively pure crusts, the Hall effect dominates over ohmic decay after a time \( t_{\text{switch}} \simeq 10^4 B_{12}^{-3} \) yr, where \( B_{12} \) is in units of \( 10^{12} \) G. This is consistent with the fact that \( \tau_B \) is much shorter than the ohmic timescale \( \tau_{\text{ohm}} \gtrsim 10^6 \) yr. At lower densities of the crust, the degenerated electrons contribute to the pressure, and the Hall timescale (\( \tau_{\text{Hall}, \text{Outer}} = 5.7 \times 10^4 \frac{\text{yr}}{B_{12}} \rho_{12}^{5/3} \left( \frac{Y_e}{0.25} \right)^{11/3} \left( \frac{g_{14}}{2.45} \right)^{-2} \)) (8) in which \( \rho_{12} = \rho/10^{12} \) g cm\(^{-3} \), \( Y_e \) is the number fraction of electrons, and \( g_{14} \) is the local gravity, assumed constant. At densities greater than neutron drip, the neutrons dominate the pressure (C2004),

\[
\tau_{\text{Hall, Inner}} = \frac{1.2 \times 10^7 \text{ yr}}{B_{12}} \rho_{14}^{7/3} Y_n^{10/3} \left( \frac{Y_e}{0.05} \right) \left( \frac{f_n}{0.5} \right)^2 \left( \frac{g_{14}}{2.45} \right)^{-2},
\]

(9)

where \( Y_n \) is the number fraction of neutrons, and \( f_n \) is the factor that accounts for the interactions between neutrons.

In the top panel of Figure 3, it is shown that there is no significant correlation between \( \tau_c \) and \( |\tau_B| \). In the second and third panels, we show that \( \tau_{\text{Hall, Outer}} \) is consistent with \( |\tau_B| \)
for eight pulsars and \( \tau_{\text{Hall, Inner}} \) is for the other six pulsars. Here, the outer and inner crusts are defined by \( 0.25 < \rho_{12} < 4 \) and \( 0.25 < \rho_{14} < 4 \), respectively, yielding the upper and lower limits for \( \tau_{\text{Hall}} \) plotted in the second and third panels of Figure 3. This implies that the Hall drift is responsible for their observed long-term evolutions in \( B \). In other words, the evolution of their crustal magnetic fields, rather than that in their cores, dominates the observed long term spin-down evolution of these young pulsars. Assuming \( \tau_{\text{Hall}} = |\tau_B| \), we obtain an effective density \( \rho_B \), which may be useful to indicate the location of the majority of the magnetic field lines in the NS crusts. The values of \( \rho_B \) are shown in the bottom panel of Figure 3.

The Hall drift can also pump energy from an internal strong toroidal field to the dipolar poloidal component on a timescale of \( \tau_{\text{Hall}} \) (Pons et al. 2012; Gourgouliatos & Cumming 2014), resulting in increased \( B \) for a pulsar. Therefore the Hall drift can cause both long term decrease and increase of \( B \) in a pulsar. Figure 3 shows that all the fourteen pulsars are quite young (with \( 10^4 < \tau_c < 3 \times 10^6 \) yr) and only three of the four pulsars are observed with long term increase of \( B \), i.e., \( \tau_B < 0 \). This is consistent with our previous result that the long-term decrease of magnetic fields are more frequently observed for young pulsars (Paper I), suggesting that the Hall drift tends to generate more magnetic field decay than increase.

Very recently, Geppert et al. (2013) found that the Hall drift can produce small-scale strong surface magnetic field anomalies (spots) due to the interaction of the “initially” dipolar field with the strong toroidal crustal component, which is fundamental for generating observable radio emission. Besides, there is a large scale dipolar poloidal component which has impact on the braking behavior, and its evolution due to Hall drift may be responsible for the long-term monotonic evolutions of \( \dot{\nu} \).

3.3. Short-term magnetic field oscillations

The above discussed theoretical expectation that the Hall drift can produce both increase and decrease of \( B \) suggests that an oscillatory mode of \( B \) in a NS crust may also be produced; however, the Hall timescales (either \( \tau_{\text{Hall, Outer}} \) or \( \tau_{\text{Hall, Inner}} \)) are too long for the observed periodicities in pulsar timing noises (with \( 0.4 < T < 4.3 \) yr as listed in Table 1). On the other hand, the diffusive motion of the magnetic fields can perturb the background magnetic fields (here the dipole magnetic fields) at the base of the NS crust. Such perturbations propagate as circularly polarized “Hall waves” along the background field lines upward into the lower density regions in the crusts (Thompson & Duncan 1996). The Hall waves can strain the
crust with a wave period \( C2004 \),
\[
P_{\text{Hall}} \approx \frac{\tau_{\text{Hall},b}}{n^2} \approx \frac{10^7 \gamma \tau}{B_{12} n^2},
\]
where \( \tau_{\text{Hall},b} \) is the Hall timescale at the base of the crust, and \( n \) is the number of nodes over the crust. The elastic response of the crust to the Hall wave can induce angular displacement
\[
\theta_s = \frac{d\xi}{dz} = -\frac{B_s}{4\pi \mu} \delta B,
\]
where \( \xi \) is the fluid displacement, \( z \) is the depth of the crust, and \( \mu \) is the shear modulus. The maximum strain occurs at or above the turning point. Using the WKB scaling, it is found that at the turning point \( C2004 \),
\[
\theta_{s,\text{turn}} = 3 \times 10^{-7} B_{12}^2 n^{13/9} \frac{\delta B_b}{B},
\]
in which \( \delta B_b \) is the amplitude of the mode at the base of the crust. The density at the turning point can then be calculated from \( C2004 \)
\[
\rho_{\text{turn}} \approx \frac{2 \times 10^{12}}{Y_e n^{4/3}} \text{ g cm}^{-3}.
\]

Since most of the dipole field lines are collected at the polar regions of NSs, the strains at the regions are especially strong. The elastic responses of the crusts to the waves induce changes of the areas of polar regions, producing both pulse shape changes and oscillations in \( \dot{\nu} \). The quasi-periodical oscillations in \( \dot{\nu} \) and time residuals can be caused by the Hall waves, i.e., \( P_{\text{Hall}} \sim T \). The calculated parameters from the above equations are also listed in Table 1. The values of \( \rho_{\text{turn}} \approx 10^8 \text{ g/cm}^3 \) are moderate for the theoretically predicted density at the turning point. Above the turning point, the perturbation on magnetic fields \( \delta B \) decreases to match the boundary condition \( \delta B = 0 \). The variation in \( \dot{\nu} \) roughly correlates with \( \theta_{s,\text{turn}} \) as
\[
\sin^2 \theta_{s,\text{turn}}/\sin^2 \theta \approx \Delta \dot{\nu}_0/\dot{\nu}_0,
\]
where \( \Delta \dot{\nu}_0 \) is the amplitude of the oscillations in \( \nu_0 \).

From Equation \( \text{(4)} \), we have \( \Delta \dot{\nu}_0/\dot{\nu}_0 = 4\kappa \). For B1828–11, \( \Delta \dot{\nu}_0/\dot{\nu}_0 = 4 \times 10^{-3} \), which is roughly the same as 0.5\( \Delta \dot{\nu}/\dot{\nu} \) = 0.36 (listed in Table 1); here \( \Delta \dot{\nu} \) is defined as the peak-to-peak amplitude of the oscillations. The small difference between \( \Delta \dot{\nu}_0/\dot{\nu}_0 \) and 0.5\( \Delta \dot{\nu}/\dot{\nu} \) is due to the partial cancellation of the two harmonics with the same amplitude, but different phases. For the other thirteen pulsars, \( \kappa \) is not available and we thus assume \( \Delta \dot{\nu}_0/\dot{\nu}_0 = 0.5 \Delta \dot{\nu}/\dot{\nu} \), which is reasonable because only one significant periodicity has been observed for each of them. Assuming \( \theta = \pi/4 \), \( \theta_{s,\text{turn}} \) can be calculated, as listed in Table 1. Finally we can constrain \( \delta B_b/B \) (the last column in the two tables), which is consistent with \( \delta B_b/B \sim 1 \) usually assumed in theoretical calculations (e.g. Thompson & Duncan 1996).
4. Summary, conclusion, discussion and future perspectives

In this work, we applied the phenomenological model of pulsar timing noise we developed recently to the observed precise timing data of young radio pulsars, and analyzed the influence of the Hall effects in NS crusts on the magnetic field evolutions. This is the first time that the two aspects of the Hall effect, i.e., Hall drift and waves, are correlated with observational data for individual pulsars. We obtained the following conclusions:

1. The $\dot{\nu}$ evolutions for most of the pulsars in L2010 sample can be described adequately with our phenomenological model consisting a long-term monotonic change and short-term oscillations in $\dot{\nu}(t)$;

2. The observed long-term monotonic changes in $\dot{\nu}(t)$ can be interpreted with the Hall drifts in NS crusts;

3. The observed short-term oscillations in $\dot{\nu}(t)$ and in pulse shapes can be produced by the Hall waves in NS crusts.

In a unified view, the long-term evolutions in $\dot{\nu}$ can be understood as that they are caused by the Hall waves with very low harmonics ($n \lesssim 10$). Actually, Goldreich & Reisenegger (1992) conjectured a turbulent Hall cascade, transferring energy from large to small scales, with an energy spectrum $E_k \propto k^{-2}$ (where $k$ is the wavenumber). However, it is still unclear that why the high harmonics (e.g. $n \sim 1000$) dominate in all pulsars we studied here. It is probably due to some selection biases that the observational time spans for all pulsars are only a few decades. Indeed, Hobbs et al. (2010) found that the structures seen in the timing noise vary with data span; as more data are collected, more quasi-periodic features are observed. To ultimately address these questions, the power spectrum and the physical origin of the Hall waves need to be investigated in detail in future works.

As we have discussed above and shown previously, the long-term evolution of $B$ of young pulsars are quite similar to that of the (young) pulsars studied here. In addition, the huge spans of the reported braking indices of all pulsars can be naturally explained by short-term oscillations of their $B$ with characteristics also similar to that of these pulsars. Therefore it is reasonable to assume that the Hall effects we studied here are also responsible for the observed spin evolutions of essentially all pulsars, including millisecond radio pulsars. Physically both a long-lived core based dipolar field and a crustal field should coexist and be superimposed for a young pulsar. However, it is likely that the “observed” features, with e.g. $10^4 < \tau_B < 10^7$ yr, are caused by the crustal field, whose evolution proceeds faster than that of the core field, due to Hall effect and finite conductivity in its crust. As the pulsar ages,
its crustal field is gradually dissipated and the core field begins to dominate its spin-down evolution. In this scenario we would expect much longer \( \tau_B \) for millisecond pulsars.

Unfortunately, currently the data on the evolutions of \( \dot{\nu} \) of millisecond pulsars are not available with comparable details as for those normal radio pulsars studied here (L2010). This is because that millisecond pulsars have much weaker dipole magnetic fields, implying much smaller \( \theta_{s,\text{turn}} \) and \( \Delta \dot{\nu}/\dot{\nu}_0 \), which is actually the basis for using them to detect gravitational waves. Availability of higher quality timing data of millisecond pulsars from the current on-going pulsar timing array observations (Haasteren et al. 2011; Demorest et al. 2013; Manchester et al. 2013) may allow both the long-term monotonic evolutions and short oscillations to be identified for individual millisecond pulsars, which will improve the current gravitational-wave limits (Shannon et al. 2013).

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Fig. 1.— *Upper Panel*: $\dot{\nu}(t)$ for PSR B1828–11 during the past 20 years. The reported data (taken from L2010) are represented by red stars; and the solid black line is calculated from Equation (4) with the following parameters: $T_1 = 2T_2 = 500$ days, $\kappa_1 = \kappa_2 = 10^{-3}$, $\phi_1 = 5.6$, $\phi_1 = 0.5$. The dotted, solid, dot-dashed and dashed lines are linear fits for the major peak values of reported data and analytical calculation, the minor peak values of reported data and analytical calculation, respectively. *Lower Panel*: Time differences between the peak positions of reported data and analytical calculation. The differences for major peak positions and minor peak positions are represented by triangles and diamonds, respectively.

Fig. 2.— Comparison between the allowed area in the $\theta - \beta$ plane from $\Delta W$ and that from the position angle constraint for B1828–11.
Fig. 3.— $\tau_c$, $\tau_{\text{Hall, outer}}$, $\tau_{\text{Hall, inner}}$ and $\rho_B$ versus $|\tau_B|$. In all panels: the crosses and circles represent $\tau_B > 0$ and $\tau_B < 0$, respectively. In the second and third panels: the crosses, circles, solid line and dotted line represent $\tau_{\text{Hall, outer}}(\rho_{12} = 1)$, $\tau_{\text{Hall, inner}}(\rho_{14} = 1)$, $\tau_{\text{Hall, outer}} = \tau_B$ and $\tau_{\text{Hall, inner}} = \tau_B$, respectively. In bottom panel: The outer crust and inner crust mean the density values that are derived from the equations $\tau_{\text{Hall, outer}} = \tau_B$, and $\tau_{\text{Hall, inner}} = \tau_B$, respectively.
Table 1: Observed and derived parameters for the pulsars with long-term monotonic changes and short-term quasi-periodical oscillations in $\dot{\nu}$; the former are taken from L2010.

| Pulsar    | $\nu_0$ | $-\dot{\nu}_0$ | $\dot{\nu}_L$ | B    | $\tau_B$ | $T$ | $\Delta\dot{\nu}/\dot{\nu}$ | n    | $\theta_{s,\text{turn}}$ | $\frac{\Delta M_p}{B}$ | unity |
|-----------|---------|----------------|----------------|------|----------|-----|-----------------------------|------|--------------------------|----------------------|-------|
| B0740−28  | 5.99    | 604.4          | 5.68           | 5.36 | 2.2      | 0.4 | 0.66                        | 2.28 | 4.06                     | 0.07                 |
| B0919+06  | 2.32    | 7.34           | 1.23           | 2.45 | -4.0     | 1.6 | 0.68                        | 1.6  | 4.1                      | 0.54                 |
| B0950+08  | 3.95    | 3.59           | -0.15          | 0.77 | 14.2     | 1.2 | 0.84                        | 3.33 | 4.58                     | 2.10                 |
| B1540−06  | 1.41    | 1.75           | 0.04           | 2.53 | -33.1    | 4.3 | 1.71                        | 0.96 | 6.54                     | 1.68                 |
| B1642−03  | 2.58    | 11.84          | 0.10           | 2.66 | 119      | 4.3 | 2.52                        | 0.94 | 7.96                     | 1.91                 |
| B1714−34  | 1.52    | 22.75          | -0.13          | 8.12 | 12.6     | 4.3 | 0.79                        | 0.54 | 4.45                     | 0.26                 |
| B1818−04  | 1.67    | 17.70          | 0.20           | 6.23 | 31.0     | 1.2 | 0.85                        | 1.17 | 4.61                     | 0.15                 |
| B1826−17  | 3.26    | 58.85          | 0.16           | 4.18 | 12.3     | 2.2 | 0.68                        | 1.06 | 4.12                     | 0.34                 |
| B1828−11  | 2.47    | 36.7           | 8.72           | 4.99 | -3.3     | 1.4 | 0.71                        | 1.2  | 4.2                      | 0.20                 |
| B1839+09  | 2.62    | 7.50           | -0.63          | 2.07 | 6.85     | 1.1 | 2.00                        | 2.12 | 7.07                     | 0.86                 |
| B1903+07  | 1.54    | 11.76          | -0.15          | 5.73 | 17.8     | 1.4 | 6.80                        | 1.10 | 13.1                     | 0.53                 |
| B1907+00  | 0.98    | 5.33           | 0.08           | 7.59 | 504      | 1.4 | 0.75                        | 0.96 | 4.33                     | 0.12                 |
| B1929+20  | 3.74    | 5.86           | -1.57          | 1.08 | 2.3      | 1.7 | 0.31                        | 2.3  | 2.8                      | 1.10                 |
| B2148+63  | 2.63    | 1.18           | -0.07          | 0.82 | 10.5     | 2.6 | 1.69                        | 2.18 | 6.50                     | 4.90                 |