Renormalization-group flows and charge transmutation in string theory

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Abstract

We analyze the behaviour of heterotic squashed-Wess–Zumino–Witten backgrounds under renormalization-group flow. The flows we consider are driven by perturbation creating extra gauge fluxes. We show how the conformal point acts as an attractor from both the target-space and world-sheet points of view. We also address the question of instabilities created by the presence of closed time-like curves in string backgrounds.

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1 Introduction

The purpose of this note is to analyze the behavior of certain string backgrounds under world-sheet renormalization-group flows. In our set-up, these flows are driven by world-sheet operators, which create, in the target space-time, extra \( U(1) \), electric or magnetic, gauge fields and push the string off criticality.

World-sheet renormalization group has been investigated both from the general conformal field theory (CFT) and from the geometrical, target space viewpoints \[1, 2\]. The motivations are diverse: study the stability of the background against off-critical excursions, search for new critical string backgrounds, eventually explore string theory off-shell, etc.

In the presence of “impurities” such as branes or orbifold fixed points in non-compact target spaces, or when background electric or magnetic fields are switched on, tachyons may in general appear \[3, 4\]. World-sheet renormalization-group techniques are then useful for investigating the relaxation process of the original unstable vacuum, towards a new, stable infrared fixed point. Such relaxation is usually a tachyon condensation \[5, 6, 7\], that can be accompanied by emission of particles (in the form \( e.g. \) of charged pairs) \[8, 9\].

The presence of closed time-like curves can also trigger decays. It was argued years ago \[10\] that gravitational solutions with such chronological pathologies might naturally
evolve towards chronologically safe backgrounds. This has been recast more recently in the framework of string vacua \cite{11, 12} with some preliminary results. It is clear that one would gain insight by studying renormalization-group flows in an appropriately chosen parameter space for families of string backgrounds.

World-sheet renormalization group can be studied directly at the level of the two-dimensional CFT. Any relevant operator can be used to leave the conformal point, and the necessary tools are in principle available for computing the beta-functions and determining the flows. This procedure is usually perturbative. It turns out that it is trustful \cite{1} in determining the new conformal point in the IR only when the operators responsible for the flow are marginally relevant (conformal dimensions $\Delta = \bar{\Delta} = 1$, but only at first order in the deformation parameter), or almost relevant (conformal dimensions $\Delta = \bar{\Delta} = 1 - \varepsilon$). In the case of deformations with irrelevant operators we return back to the conformal point towards the IR. In the framework of Wess–Zumino–Witten (wzw) models \cite{13} (which capture \textit{e.g.} the $S^3$ and AdS$_3$ spaces with NS background fluxes), such operators exist only at large level $k$. Hence, their operator product expansions involve a plethora of fields, and the actual computation of their beta function is very intricate. In order to overcome this difficulty, we will here use an alternative method, more geometric and based on target-space techniques.

This note is organized as follows. In section 2 we present a quick review of the heterotic squashed wzw models \cite{14, 15}. Then in section 3 we introduce a perturbation and study the system as the RG flow, in the corresponding two-dimensional $\sigma$-model, takes it back to the conformal point. In section 4 we show that this is consistent with the CFT results.

\section{Squashed wzw models}

One of the most appealing properties of WZW models is that they allow for both an exact CFT bidimensional description and a simple spacetime interpretation in terms of group manifolds. Current-current deformations allow to explore their moduli space, leading in general to models that keep the integrability properties but may lack a nice spacetime description. Special attention is deserved by the asymmetric deformations in which the two currents come from different sectors of the theory; in this case, in fact, together with the nice CFT properties, the spacetime geometry remains simple to describe in terms of
squashed groups.

To be more concrete consider a heterotic SWZW model on a group $G$ of dimension $d$ and rank $r$. The asymmetric current-current deformation is realized by adding the operator

$$\mathcal{O} = \frac{\sqrt{kk_g}}{2\pi} \int d^2 z \sum_{a,b=1}^r c_{ab} \left( J^a(z) - \frac{i}{k} f^a_{MN} \psi^M \psi^N \right) \bar{J}^b(\bar{z}),$$

(1)

where $J^a$ are currents in the Cartan torus $T \subset G$, $\psi^M$ are the fermionic superpartners and $\bar{J}^a$ are anti-holomorphic currents belonging to the gauge sector. The engineering dimension of the operator is obviously $(1, 1)$ and, as it has been shown in [16], $\mathcal{O}$ is truly marginal (i.e. at every order in deformation) for any value of the parameter matrix $c_{ab}$, since the currents commute. In other words, with the aid of $\mathcal{O}$, we reach an $r$-dimensional space of exact CFT’s.

As described in [15], the background fields corresponding to the new sigma–model can be read using a technique bearing many resemblances to a Kaluza–Klein reduction and consist in a metric, a Kalb–Ramond field and a $U(1)^r$ (chromo-)magnetic field. As announced above the description remains simple and the all-order exact expression can be given in terms of Maurer–Cartan currents $J^\mu$ on $G$ as follows:

$$\begin{cases}
\begin{aligned}
\text{d}s^2 &= \sum_{\mu \in G/T} J^\mu J^\mu + \left(1 - h^2\right) \sum_{a \in T} J^a J^a, \\
H[3] &= \frac{1}{2} f_{\mu \nu \rho} J^\mu \wedge J^\nu \wedge J^\rho & \mu \in G/T, \\
F^a &= h \sqrt{\frac{k}{k_g}} f^a_{\mu \nu} J^\mu \wedge J^\nu & \mu \in G/T, a \in T,
\end{aligned}
\end{cases}$$

(2)

where we chose $c_{ab} = h \delta_{ab}$. In particular we see that the metric is the one of a squashed group i.e. we still have the structure of a $T$ fibration over $G/T$ but the radius of the fiber changes with $h$. A special value of the deformation parameter is singled out: for $h < 1$ the metric is positive definite, while for $h > 1$ the signature changes. The apparently singular $h = 1$ value can nevertheless be reached by a limiting procedure whose geometrical interpretation is the trivialization of the fiber. We end up with an exact CFT on a $G/T$ background sustained by a (chromo-)magnetic field.

The simplest example is given by $G = SU(2)$ where we have (in Euler coordinates)

---

1It would be a genuine reduction if we had done the construction in type II or in a bosonic theory. In this case the current $J^a$ would just be the anti-holomorphic derivative of an internal coordinate $X^a$. 
the following background fields:

\[
\begin{align*}
\text{d } s^2 &= \text{d } \theta^2 + \text{d } \psi^2 + \text{d } \phi^2 + \cos \theta \text{ d } \psi \text{ d } \phi - h^2 \left( \text{d } \psi + \cos \theta \text{ d } \phi \right)^2, \\
B &= \cos \theta \text{ d } \psi \wedge \text{d } \phi, \\
A &= 2h \left( \text{d } \psi + \cos \theta \text{ d } \phi \right),
\end{align*}
\]

\hspace{1cm} (3)

corresponding, in the \( h \to 1 \) limit, to a \( S^2 \) geometry.

3 RG-flows for compact groups: geometric approach

We present here the geometric, target-space techniques for analyzing RG flows in two-dimensional theories. These techniques apply to any compact group. We will however expand on the case of \( SU(2) \) since it captures all the relevant features.

3.1 The parameter space

The model that we have presented in the previous section is conformal; for this reason we expect to find it as a fixed point in an RG flow. To verify this claim let us introduce a two-parameter family of \( \sigma \) models generalizing the exact backgrounds of Eq. (2); a possible choice consists in adding a new magnetic field, this time coming from a higher dimensional right sector. Explicitly

\[
\begin{align*}
\text{d } s^2 &= \sum_{\mu \in G/T} J^\mu J^\mu + \left( 1 - h^2 \right) \sum_{a \in T} J^a J^a, \\
H[3] &= \frac{h}{2h} f_{\mu \nu \rho} J^\mu \wedge J^\nu \wedge J^\rho \quad \mu \in G/T, \\
F^a &= \frac{h + \bar{h}}{2} \sqrt{\frac{k}{k_a}} f_{\mu \nu} J^\mu \wedge J^\nu \quad \mu \in G/T, a \in T, \\
\bar{F}^a &= \frac{h - \bar{h}}{2} \sqrt{\frac{k}{k_a}} f_{\mu \nu} J^\mu \wedge J^\nu \quad \mu \in G/T, a \in T
\end{align*}
\]

\hspace{1cm} (4)

and in particular for \( SU(2) \):

\[
\begin{align*}
\text{d } s^2 &= \text{d } \theta^2 + \text{d } \psi^2 + \text{d } \phi^2 + \cos \theta \text{ d } \psi \text{ d } \phi - \bar{h}^2 \left( \text{d } \psi + \cos \theta \text{ d } \phi \right)^2, \\
B &= \frac{h}{\bar{h}} \cos \theta \text{ d } \psi \wedge \text{d } \phi, \\
A &= \left( h + \bar{h} \right) \left( \text{d } \psi + \cos \theta \text{ d } \phi \right), \\
\bar{A} &= \left( h - \bar{h} \right) \left( \text{d } \psi + \cos \theta \text{ d } \phi \right),
\end{align*}
\]

\hspace{1cm} (5)

where \( \bar{h} \) is a new parameter, describing the deviation from the conformal point. It is clear that the above background reduces to the one in Eq. (3) in the \( \bar{h} \to h \) limit. In particular we see that the metric is unchanged, the Kalb–Ramond field has a different normalization
and a new field $\tilde{A}$ appears. This configuration can be described in a different way: the geometry of a squashed sphere supports two covariantly constant magnetic fields with charge $Q = h + \bar{h}$ and $\bar{Q} = h - \bar{h}$; the RG flow will describe the evolution of these two charges from a generic $(Q, \bar{Q})$ to $(2h, 0)$, while the sum $Q + \bar{Q} = 2h$ remains constant. In this sense the phenomenon can be interpreted as a charge transmutation of $\bar{Q}$ into $Q$. The conservation of the total charge is in fact a consequence of having chosen a perturbation that keeps the metric and only changes the antisymmetric part of the background.

We can also see the background in Eq. (4) from a higher dimensional perspective where only the metric and the Kalb-Ramond field are switched on. Pictorially:

$$g = \begin{pmatrix} g_{\text{wzw}} & h J_a \\ h J_a & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \frac{h}{\bar{h}} B_{\text{wzw}} & h J_a \\ -\bar{h} J_a & 0 \end{pmatrix}$$

(6)

where $g_{\text{wzw}}$ and $B_{\text{wzw}}$ are the usual metric and Kalb–Ramond fields for the wzw model on the group $G$. More explicitly in the $SU(2)$ case:

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \cos \theta & h \\ 0 & \cos \theta & 1 & h \cos \theta \\ 0 & h & h \cos \theta & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{h}{\bar{h}} \cos \theta & \bar{h} \\ 0 & -\frac{h}{\bar{h}} \cos \theta & 0 & \bar{h} \cos \theta \\ 0 & -\bar{h} & -\bar{h} \cos \theta & 0 \end{pmatrix}$$

(7)

where the fourth entry represents the bosonized internal current. In particular this clarifies the stated right-sector origin for the new gauge field $\tilde{A}$. This higher dimensional formalism is the one we will use in the following RG analysis.

### 3.2 The renormalization group flow

The $\sigma$-model in Eq. (5) is not conformal for generic values of the parameters $h$ and $\bar{h}$; this is why it makes sense to study its behaviour under the RG flow. Following a dimensional-regularization scheme (see [17, 18, 2] and for various applications [19, 20, 21, 22]) we consider the action

$$S = \frac{1}{2\lambda} \int d^2 z (g_{\text{mn}} + B_{\text{mn}}) \partial X^m \bar{\partial} X^n,$$

(8)

where $g$ and $B$ are the fields in Eq. (6). The beta-equations at two-loop order in the expansion in powers of the overall coupling constant $\lambda$ and the field redefinitions for the
internal coordinates $X^i$ turn out to be:

\[
\begin{aligned}
\beta_{\lambda^*} &= \frac{d\lambda^*}{dt} = -\frac{\lambda^*}{2\pi} \left(1 - \frac{k^2}{\hbar^2}\right) \left(1 + \frac{\lambda^*}{8\pi} \left(1 - 3\frac{k^2}{\hbar^2}\right)\right), \\
\beta_h &= \frac{dh}{dt} = \frac{\lambda^*}{8\pi} \left(1 - h^2\right) \left(1 + \frac{\lambda^*}{8\pi} \left(1 - 3\frac{k^2}{\hbar^2}\right)\right), \\
\beta_{\bar{h}} &= \frac{d\bar{h}}{dt} = -\frac{\lambda^*}{8\pi} \left(1 + h^2\right) \left(1 - \frac{k^2}{\hbar^2}\right), \\
X^i &= X^i - \frac{\lambda^*}{16} \left(1 - h^2\right) \left(1 - 4\frac{k^2}{\hbar^2} + 3\frac{k^4}{\hbar^4}\right),
\end{aligned}
\]

where $\lambda^* = \lambda g^*$, $g^*$ being the dual Coxeter number, is the effective coupling constant ($\lambda^* = N\lambda$ for $G = SU(N)$). The contributions at one- and two-loop order are clearly separated. In the following we will concentrate on the one-loop part and we will comment on the two-loop result later. Let us then consider the system:

\[
\begin{aligned}
\beta_{\lambda^*} &= \frac{d\lambda^*}{dt} = -\frac{\lambda^*}{4\pi} \left(1 - \frac{k^2}{\hbar^2}\right), \\
\beta_h &= \frac{dh}{dt} = \frac{\lambda^*}{8\pi} \left(1 - h^2\right) \left(1 - \frac{k^2}{\hbar^2}\right), \\
\beta_{\bar{h}} &= \frac{d\bar{h}}{dt} = -\frac{\lambda^*}{8\pi} \left(1 + h^2\right) \left(1 - \frac{k^2}{\hbar^2}\right).
\end{aligned}
\]

This can be integrated by introducing the parameter $z = \bar{h}/h$ which makes one of the equations redundant. The other two become:

\[
\begin{aligned}
\dot{\lambda}^* &= -\frac{\lambda^*}{4\pi} (1 - z^2), \\
\dot{z} &= -\frac{\lambda^*}{4\pi} (1 - z^2).
\end{aligned}
\]

By inspection one easily sees that $\dot{\lambda}/\lambda = \dot{z}/z$, implying $\lambda(t) = Cz(t)$, where $C$ is a constant. This was to be expected since $C$ is proportional to the normalization of the topological WZ term. Since we are dealing with a compact group it turns out that $C$ is, as in [13], quantized with:

\[
C_k = \frac{2\pi}{k}, \quad k \in \mathbb{N}.
\]

Now it’s immediate to separate the system and find that $z(t)$ is defined as the solution to the implicit equation:

\[
-\frac{t}{2k} = \frac{1}{z_0} - \frac{1}{z(t)} + \log \frac{(z(t) + 1)(z_0 - 1)}{(z(t) - 1)(z_0 + 1)}
\]

with the initial condition $z(0) = z_0$. A similar expression was found in [23, 13]. The reason for this is, as pointed out previously [24], that the conformal model ($\bar{h} = h$) in its higher-dimensional representation (the one in Eq. (6)) coincides with a $G \times H$ WZW model after a suitable local field redefinition.

As it is usually the case in the study of non-linear dynamics, a better understanding of the solution is obtained by drawing the RG flow. In a $(z, \lambda^*)$ plane, the trajectories are
The flow diagram for this system in the \((h, \bar{h})\) plane, Fig. 1(b), shows how the system relaxes to equilibrium after a perturbation. In particular we can see how increasing \(\bar{h}\) leads to a new fixed point corresponding to a value of \(h\) closer to 1.

We would like to pause for a moment and put the above results in perspective. The target-space of the sigma-model under consideration is a squashed three-sphere with two different magnetic fields. Along the flow, a transmutation of the two magnetic charges occurs: the system is driven to a point where one of the magnetic charges vanishes. This fixed point is an ordinary squashed-wzw (of the type studied in Sec. 2), that supports a single magnetic charge.

As we pointed out in Sec. 2 in the squashed-wzw, the magnetic field is bounded by a critical value, \(h = 1\). As long as \(h \leq 1\), the geometry is a genuine squashed three-sphere.
For $h > 1$, the signature becomes Lorentzian and the geometry exhibits closed time-like curves. Although of limited physical interest, such a background can be used as a laboratory for investigating the fate of chronological pathologies along the lines described above. In particular we see that under the perturbation we are considering the model shows a symmetry between the $h > 1$ and $h < 1$ regions. In fact the presence of closed time-like curves doesn’t seem to make any difference, but for the fact that regions with different signatures are disconnected, i.e. the signature of the metric is preserved under the RG flow. It is clear that these results are preliminary. To get a more reliable picture for closed time-like curves, one should repeat the above analysis in a wider parameter space, where other RG motions might appear and deliver a more refined stability landscape.

4 RG-flows for compact groups: CFT approach

In order to make contact with genuine CFT techniques, we must identify the relevant operators which are responsible for the $(h, \bar{h})$ deformation of the $G \times H$ original WZW model $(H = U(1)^{\text{rank} G})$. At lowest approximation, all we need is their conformal dimensions in the unperturbed theory.

Following [1], let $\mathcal{L}_0$ be the unperturbed (conformal) action and $\mathcal{O}_i$ the operators of conformal dimension $\Delta_i$. Consider the perturbed model, with Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + g^i \mathcal{O}_i .$$

The tree-level beta-functions read:

$$\beta^i(g) = (\Delta_i - 1)g^i ,$$

where $g^i$ is supposed to be small, for the perturbative expansion of $\beta^i$ to hold.

The $G \times H$ primary operator we need can be written as follows:

$$\mathcal{O} = \sum_{\mathbb{a}, b} \langle t^{\mathbb{a} \mathbb{b}} g t^{\bar{\mathbb{b}}} g^{-1} \rangle \langle t^{\mathbb{a} \partial} g g^{-1} \rangle \langle t^{\bar{\mathbb{b}} g^{-1}} \bar{\partial} g \rangle = \sum_{\mathbb{a}, b} \Phi^{\mathbb{a} \mathbb{b}} J^{\mathbb{a} \bar{\mathbb{b}}} \bar{J}^{\bar{\mathbb{b}}} ,$$

where $\Phi^{\mathbb{a} \mathbb{b}}$ is a primary field transforming in the adjoint representation of the left and right groups $G$. As such, the total conformal dimensions are

$$\Delta = \bar{\Delta} = 1 + \frac{g^*}{g^* + k} ,$$

One should be very careful in the choice of signs in these formulae. In [1] the time variable, in fact, describes the evolution of the system towards the infrared and as such it is opposite with respect to the $t = \log \mu$ convention that we used in the previous section (as in [13]).
where \( g^* \) is the dual Coxeter number and as such the operator is irrelevant (in the infrared).

Specializing this general construction to our case we find that the action for the fields in Eq. (6) is:

\[
L = \frac{k}{4\pi} \left\{ L_0 + \left( \frac{h}{\bar{h}} - 1 \right) \sum_{\beta\alpha, b} \Phi^{\beta\alpha} J^{\beta\alpha} \bar{J}^{\beta\alpha} + \frac{\bar{h}}{h} (h + \bar{h}) \sum_i J^{\alpha_i} \bar{J}^{\alpha_i} + \frac{h}{\bar{h}} (h - \bar{h}) \sum_{i, \beta\alpha} J^i \Phi^{\alpha_i, \beta\alpha} \bar{J}^{\beta\alpha} \right\},
\]

(19)

where \( \beta\alpha \) runs over all currents, \( i \) over the internal currents in \( H \) and \( J^{\alpha_i} \) is the wzw current of the Cartan subalgebra of \( G \) coupled to the internal \( \bar{J}^{\alpha_i} \). The extra terms can be interpreted as combinations of operators in the \( G \times H \) model. The beta-functions are thus computed following Eq. (16) with the coupling \( g = h/\bar{h} - 1 \). We obtain

\[
\frac{d}{dt} \left( \frac{h}{\bar{h}} - 1 \right) \bigg|_{\bar{h}=h} = \frac{g^*}{g^* + k} \left( \frac{h}{\bar{h}} - 1 \right) + \cdots = \left( \frac{g^* - g^{*2}}{k} \right) \left( \frac{h}{\bar{h}} - 1 \right) + \cdots,
\]

(20)

where the dots after the first equality denote higher order terms in the \( (h/\bar{h} - 1) \)-expansion and after the second equality, in addition to that, higher order terms in the \( 1/k \)-expansion. This result is the same as the one in [25] since, as we have mentioned, there is the a local field redefinition that maps this model at the conformal point to the \( G \times H \) wzw model. The above result is to be compared with the results following from Eq. (20) when they are expanded around \( h = \bar{h} \). We obtain:

\[
\frac{d}{dt} \left( \frac{h}{\bar{h}} - 1 \right) \bigg|_{\bar{h}=h} = \left( \frac{g^* - g^{*2}}{k} \right) \left( \frac{h}{\bar{h}} - 1 \right) + \frac{1}{2} \left( -\frac{g^*}{k} + \frac{g^{*2}}{k^2} \right) \left( \frac{h}{\bar{h}} - 1 \right)^2 + \cdots.
\]

(21)

We see that these results agree to first order in the coupling \( h/\bar{h} - 1 \).

The extra information that we obtain from this calculation is about the interpretation for the two-loop beta-function we described in the previous section. The one-loop corrections to (16) are of the form \( C_{ijk} g^i g^j \), where \( C_{ijk} \) are related to the three-point function of the unperturbed theory [1]. This coefficient is a measure of the dimension of the operator \( O_i \) in the theory perturbed by the set of all operators. Such a computation goes beyond the scope of the present note. Nevertheless, (21) predicts the coefficient of the term \( (h/\bar{h} - 1)^2 \) to second order in the \( 1/k \)-expansion and it seems that such a computation is feasible from the CFT viewpoint at least as a series expansion in \( 1/k \).
5 Conclusions

In this work, we have analyzed the phase space of squashed WZW models, away from the original conformal point. Our analysis is given in detail for the compact group $SU(2)$ and can be generalized to any compact group. We have restricted ourselves to deviations from the conformal point, generated by switching on simultaneously two distinct magnetic (or electric) fields. The corresponding backgrounds may have interesting interpretation in terms of NS5-branes. We have investigated the phase diagram using geometrical, target-space techniques, as well as standard CFT renormalization-group methods. Our results can be summarized as follows: the squashed-WZW models are found, as expected, as IR fixed points in the RG flow, and this result is confirmed from both a target space and world-sheet point of view. The field theory interpretation of this flow consists in what we have called charge transmutation. One $U(1)$ charge transforms into another $U(1)$ while the total charge is conserved. For large values of the parameter $h$ the backgrounds under consideration contain closed time-like curves. These do not seem to change the behaviour of the flow and the model remains stable, at least under the deformation we consider.

This charge transmutation enters the class of phenomena that are expected to take place when a metastable string background jumps to a stable one through a non-critical path. These include tachyon condensation, particle production and other interesting physical phenomena: the RG flow around the conformal point is a tool to get information on the dynamics of the relaxation. Our geometrical tools are well-fitted to describe the latter provided we allow for more parameters in the phase space. A generalization of our approach may also allow to address more thoroughly the issue of instabilities triggered by the presence of closed time-like curves. Of course this is all very preliminary and in particular much still remains to be done in clarifying the link between energy minimization and time evolution in non-compact and time-dependent backgrounds. A first step in this direction consists in investigating non-compact groups, like $SL(2,\mathbb{R})$, for which some aspects (e.g. related to Zamolodchikov’s C-theorem) of the underlying theory remain obscure.
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