STATISTICAL COMPLEXITY AND NONTRIVIAL COLLECTIVE BEHAVIOR IN ELECTROENCEPHALOGRAPHIC SIGNALS

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Abstract

We calculate a measure of statistical complexity from the global dynamics of electroencephalographic (EEG) signals from healthy subjects and epileptic patients, and are able to establish a criterion to characterize the collective behavior in both groups of individuals. It is found that the collective dynamics of EEG signals possess relative higher values of complexity for healthy subjects in comparison to that for epileptic patients. To interpret these results, we propose a model of a network of coupled chaotic maps where we calculate the complexity as a function of a parameter and relate this measure with the emergence of nontrivial collective behavior in the system. Our results show that the presence of nontrivial collective behavior is associated to high values of complexity; thus suggesting that similar dynamical collective process may take place in the human brain. Our findings also suggest that epilepsy is a degenerative illness related to the loss of complexity in the brain.

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The concept of complex systems has become a new paradigm for the search of mechanisms and a unified interpretation of the processes of emergence of structures, organization and functionality in a variety of natural and artificial phenomena in different contexts [Badii & Politi, 1997; Mikhailov & Calenbuhr, 2002; Kaneko & Tsuda, 2000]. One common criterion for defining complexity is the emergent behavior: collective structures, patterns and functions that are absent at the local level arise from simple interaction rules between the constitutive elements in a system. Phenomena such as the spontaneous formation of structures, organization, spatial patterns, chaos synchronization, collective oscillations, spiral waves, segregation and differentiation, formation and growth of domains, and social consensus, are examples of self-organizing processes that occur in various contexts such as physical, chemical, biological, physiological, social and economic systems.

There has been much interest in the study of the phenomenon of emergence of nontrivial collective behavior in the context of systems of interacting chaotic elements [Kaneko, 1990; Chaté & Manneville, 1992a, 1992b; Pikovsky & Kurths, 1994; Shibata & Kaneko, 1998; Cosenza, 1998; Cosenza & González, 1998; Cisneros et al., 2002; Manrubia et al., 2004]. Nontrivial collective behavior is characterized by a well defined evolution of macroscopic quantities coexisting with local chaos. Models based on coupled map networks have been widely used in the investigation of collective phenomena that appear in many complex systems [Kaneko & Tsuda, 2000]. In particular, networks of coupled chaotic maps can exhibit nontrivial collective behavior.

A paradigmatic example of a complex system is provided by the human brain. It consists of a highly interconnected network of millions of neurons. The local dynamics of a neuron in general behaves as a non-linear excitable element [Herz et al., 2006]. From the signal of a single neuron it is not possible to understand the highly structured collective behavior and functions of the brain.

In this paper we investigate the relative complexity of the human brain by considering the collective dynamics that arise from the local dynamics of groups of neurons, as manifested in electroencephalographic (EEG) signals. We calculate a measure of complexity from the global dynamics of EEG signals from healthy subjects and epileptic patients, and are able to establish a criterion to characterize the collective behavior in both groups of individuals. It is found that the collective dynamics of EEG signals possess relative higher values of complexity for healthy subjects in comparison to that for epileptic patients. Our results
support the view that epilepsy is characterized by a loss of complexity in the brain, as indicated by measurements of the dimension correlation [Babloyantz A. & Destexhe A., 1986], algorithmic complexity [Rapp et al., 1994], and anticipation of seizures [Martinerie et al., 1998].

In order to interpret our results, we propose a model of coupled chaotic maps where we calculate the measure of complexity as a function of a parameter and relate this measure with the emergence of nontrivial collective behavior in the system. Our results show that the appearance of nontrivial collective behavior is associated to high values of complexity; thus suggesting that similar dynamical collective process may take place in the human brain.

Several measures of complexity have been proposed in the literature. Here we employ the concept of statistical complexity, introduced by Lopez-Ruiz et al. [1995]. This quantity is based on the statistical description of a system at a given scale, and it has been shown to be capable of discerning among different macroscopic structures emerging in complex systems [Sánchez & Lopez-Ruiz, 2005]. The amount of complexity $C$ is obtained by computing the product between the entropy $H$, and a sort of distance to the equipartition state in the system, named the disequilibrium $D$. Thus, the statistical complexity is defined as [Lopez-Ruiz et al., 1995]

$$C = H \cdot D = -K \sum_{s=1}^{R} p_s \log p_s \cdot \sum_{s=1}^{R} \left( p_s - \frac{1}{R} \right)^2,$$

where $H$ and $D$ are, respectively, the entropy and the disequilibrium; $p_s$ represents the probability associated to the state $s$; $R$ is the number of states, and $K$ is a positive normalization constant. Note that $p_s$ may vary for different levels of observation, reflected in $R$. The quantity $C$ can quantify relative values of complexity in a specific system at a given level of description.

The EEG data base used in this study consists of records from 40 individuals in an age range between 22 and 48 years old. These individuals are classified into four groups: (I) a group of 10 healthy subjects; (II) a group of 10 epileptic patients receiving treatment with Phenobarbital for at least 18 months; (III) a group consisting of 18 epileptic patients that have not yet received medical treatment; and (IV) a group of 2 epileptic patients who experienced spontaneous seizures during the EEG recording. All the epileptic patients in the data base were diagnosed generalized epilepsy manifested through tonic-clonic seizures.

The record of the EEG signal from each individual was carried out over 19 channels
connected to scalp electrodes according to the international 10−20 system [Jasper, 1958]. The potentials were measured with respect to a reference level consisting of both ears short-circuited. The signal was digitalized at a sampling frequency of 256 Hz and A/D conversion of 12 bits, and filtered to bandwidth between 0.5 and 30 Hz. All the EEG signals were recorded with the individuals at rest and with eyes closed, during a one-hour period, between 8 a.m. and 10 a.m. We consider continuous segments of the EEG signals containing between 15000 to 30000 points, with no significant artifacts.

A channel in the EEG signal represents an average of a set of neurons in an specific part of the brain. The 19 channels in each EEG can be considered as simultaneous time series coming from different parts of an interacting dynamical network. The collective behavior of such a dynamical network at a given time \( t \) can be described by, the instantaneous mean field \( h_t \), defined as

\[
h_t = \frac{1}{M} \sum_{j=1}^{M} e_{jt},
\]

where \( e_{jt} \) is the real value registered by electrode \( j \) at discrete time \( t \), \( j = 1, \ldots, M \); and \( M = 19 \) is the number of electrodes.

The probability distribution of the mean field values corresponding to a given EEG signal is constructed from the time series of \( h_t \) calculated for that signal. We define the number of states \( \mathcal{R} \) as a partition consisting of \( \mathcal{R} \) equal size segments on the range of values of \( h_t \). Next, the probability \( p_s \) associated to the \( s \) state is calculated. Here we set \( \mathcal{R} = 10^3 \) for all the EEG signals. Then, Eq. 1 is used to calculate the statistical complexity \( C \) associated to the mean field for each EEG.

Figure 1 shows the statistical complexity of the mean field of the EEG signal for all the individuals in each group from the data base. Figure 1 indicates that the complexity measure for the global dynamics of the EEG signal is higher in healthy subjects (group I) in relation to that of epileptic patients (groups II, III, and IV). The lowest levels of complexity correspond to the patients undergoing epileptic seizures (group IV). This measure of complexity allows us to discern between healthy subjects and epileptic patients. Among EEG signals from epileptic patients, those signals from patients under treatment seem to possess slightly greater complexity than those from patients without treatment. However, the quantity \( C \) is not very efficient for distinguishing between the different groups of epileptic patients. The spread of the values of the complexity found in group I indicates a greater
variability in the statistical properties of EEG signals from the healthy subjects.

FIG. 1: Statistical complexity of healthy subjects (group I), epileptic patients subject to treatment (group II), epileptic patients without treatment (group III), and epileptic patients undergoing seizures (group IV). Vertical lines indicate the limits for each group. The order of the individuals in each group does not have any meaning.

It is important to say that other choices for the partition of states of the EEG signal are possible, and hence different values for the $H$ and $D$ measures can be obtained, without affecting the overall behavior of the statistical quantity $C$. For instance, Rosso et al. [2003] divided the time axis of EEG signals into non-overlapping temporal windows on which the wavelet energy at different resolution levels can be calculated.

In order to give an interpretation to the above results, we propose a dynamical model. We consider a system of $N$ interacting nonlinear, heterogeneous elements forming a network, where the state of element $i$ ($i = 1, 2, \ldots, N$) at time $t$ is denoted by $x^i_t$. The evolution of the state of each element is assumed to depend on its own local dynamics and on its interaction with the network, whose intensity is described by the coupling parameter $\epsilon$. Then, we consider a network of maps subjected to a global interaction as follows [Cisneros et al., 2002]

\[ x^i_{t+1} = (1 - \epsilon) f_i(x^i_t) + \frac{\epsilon}{N} \sum_{j=1}^{N} f_j(x^j_t), \]

where the function $f_i(x^i_t)$ describes the local dynamics of map $i$. The usual homogeneous globally coupled map system [Kaneko, 1990] corresponds to having the same local function
for all the elements, i.e., $f_i(x_i^t) = f(x_i^t)$. As local chaotic dynamics we choose the logarithmic map $f(x) = b + \ln |x|$, $x \in (-\infty, \infty)$, where $b$ is a real parameter. This map does not belong to the standard class of universality of unimodal or bounded maps. Robust chaos occurs in the parameter interval $b \in [-1, 1]$, with no periodic windows and no separated chaotic bands on this interval [Kawabe & Kondo, 1991]. Heterogeneity in the maps is introduced by considering $f_i(x_i^t) = b_i + \ln |x_i^t|$, where the values $b_i$ are uniformly distributed at random in the interval $[-1, 1]$.

As the macroscopic variable for this system, we consider the instantaneous mean field defined as

$$h_t = \frac{1}{N} \sum_{j=1}^{N} f_j(x_j^t). \quad (4)$$

Figure 2(a) shows the bifurcation diagram of the mean field $h_t$ of the globally coupled heterogeneous map network, Eq. (3), with $b_i \in [-1, 1]$ and $N = 10^4$, as a function of the coupling strength $\epsilon$. For each value of $\epsilon$, the mean field was calculated at each time step during a run of $10^2$ iterates starting from random initial conditions on the local maps, uniformly distributed on the interval $x_0^i \in [-8, 8]$, after discarding $10^3$ transients. The local maps are chaotic and desynchronized (see Fig. 2(b)). However, the mean field in Fig. 2(a) reveals the existence of global periodic attractors for some intervals of the coupling parameter. This is the phenomenon of nontrivial collective behavior where macroscopic order coexists with local disorder in a system of interacting dynamical elements. Different collective states emerge as a function of the coupling $\epsilon$. In this representation, collective periodic states at a given value of the coupling appear as sets of vertical segments which correspond to intrinsic fluctuations of the periodic orbits of the mean field. Increasing the system size $N$ does not decrease the amplitude of the collective periodic orbits. Moreover, when $N$ is increased the widths of the segments that make a periodic orbit in the bifurcation diagrams such as in Fig. 2(a) shrink, indicating that the global periodic attractors become better defined in the large system limit.

Figure 2(c) shows the complexity $C$ of the mean field as a function of $\epsilon$. Here, the observation level was set at $R = 15$. When the value of $\epsilon$ is small, the bifurcation diagram of $h_t$ shows a period-one collective attractor, which implies that for this parameter range the system follows the standard statistical behavior of uncorrelated disordered variables that yield a single period for its mean field. At the chosen level of resolution, the complexity
FIG. 2: (a) Bifurcation diagram of $h_t$ as a function of $\epsilon$ for a coupled heterogeneous map network, Eq. (4), with $b_i \in [-1, 1]$ and $N = 10^4$. (b) Bifurcation diagram of a local map $x^i_t$ versus $\epsilon$, exposing the underlying chaotic dynamics. (c) Complexity $C$ as a function of $\epsilon$.

measure considers the macroscopical variable as laying in a single state, thus giving $C = 0$. The complexity $C$ remains zero up to a critical value of the coupling $\epsilon_c \simeq 0.04$. The onset of the complexity at the value $\epsilon_c$ resembles a first order phase transition. As the periodicity of the collective orbit increases, more states are occupied by the probability distribution of the mean field $h_t$. The probability distribution of $h_t$ corresponding to a periodic collective state is not uniform and consists of a set of distinct “humps”. A nonuniform probability distribution and few occupied states lead to larger values of the complexity $C$, as observed for the period-two collective orbit. When the system enters chaotic collective motion, more states are occupied by the probability distribution of the mean field and therefore this probability becomes more uniform. As a consequence, the complexity decreases.
The emergence of ordered collective behavior in the coupled map network, Eq. \(3\), cannot be attributed to the existence of windows of periodicity nor to chaotic band splitting in the local dynamics. Figure 2 shows that higher values of complexity are associated to the occurrence of nontrivial collective behavior in a network of interacting dynamical elements. We have obtained similar results for different network topologies and different local map dynamics. This result adds support to the concept of complexity as an emergent behavior; in this case the ordered collective behavior is not present at the local level. Furthermore, these results suggest that similar dynamical collective processes may take place in the human brain. From the dynamics of a single neuron as an excitable element it is not possible in general to characterize the collective behavior of the brain, as the collective behavior of the coupled map network cannot be inferred from the knowledge of the dynamics of a single map. Thus, the lower values of complexity found in the global dynamics of the EEG signals from epileptic patients may be associated to a decrease in the ability to generate collective organization and functions in the brain affected by such physiological condition.

As we have mentioned, the measure of the statistical complexity depends on the level of observation (number of states \(R\)) considered for the computations. Figure 3(a) shows the statistical complexity as a function of the number of states for the mean field value from an EEG signal of a healthy subject. In Fig. 3(b) a similar plot of \(h_i\) is shown for the value of the coupling parameter \(\epsilon = 0.2\) in the coupled heterogeneous map network, Eq. \(3\).

![Graph](image)

**FIG. 3:** (a) Complexity \(C\) as a function of \(R\) for the mean field from EEG signals for a healthy subject. (b) \(C\) as a function of \(R\) for the coupled heterogeneous map network, Eq. \(3\), \(\epsilon = 0.2\).

Figure 3 shows that the measures of the complexity in both systems tend to constant,
asymptotic values when the number of states $R$ is increased. For the coupled map network, the number of states used in the computations was $R = 15$, while for the EEG signals we employed $R = 1000$; these values lie in the corresponding asymptotic regime for each system.

In the heterogeneous coupled maps network, the size of the system $N$ can be related to the minimum number of states or level of description $R_c$ that should be used in the calculation of $C$. The width of a vertical segment in the bifurcation diagram of $h_t$ at the level of description $R_c$ is given by $R_c^{-1}$. In addition, we have observed that this width is proportional to the statistical dispersion of the points inside the segment, which in turn decreases with the size of the system following the law of large numbers as $N^{1/2}$. Then,

$$\frac{1}{R_c} \sim \frac{1}{\sqrt{N}} \Rightarrow N \sim R_c^2,$$  

(5)

thus, for a given system size $N$, it is possible to estimate the minimum number of states that should be considered for the computation of the statistical complexity in this system. Correspondingly, for the resolution level $R = 15$ that we employed in Fig. 2, the minimum system size that could have been used is $N \sim 225$.

In summary, we have shown that the measure of statistical complexity Eq. (11) used in this work is useful in the analysis of theoretical and real systems. There exists a morphological difference between the global dynamics in EEG signals from healthy subjects and those from epileptic patients that can be revealed by employing this tool. The calculations show that epilepsy is associated to a low level of statistical complexity.

We have also shown that the mean field of a network of heterogeneous chaotic maps contains relevant information about the complexity of the system which cannot be derived from the knowledge of the behavior of the local maps. A high degree of complexity is associated to the emergence of nontrivial collective behavior. These results can be related to those of Cisneros et al. [2002] who showed that the prediction error used to measure the mutual information transfer between a local and a global variable in a similar network decreased when nontrivial collective behavior arises in the system. Thus, a high level of complexity can also be associated to an increase in the mutual information transfer between macroscopic and microscopic variables in a system.

Finally, our intention is not to describe the human as network of interacting chaotic elements, but to show that the collective properties related to the concept of complexity are
qualitatively similar, and that a rise in complexity can be associated with the emergency of collective ordered behavior in different systems.

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