Development of Equations Relates the Factors Affecting Riverbank Stability Using Dimensional Analysis

Asad H. Aldefae1,*, Rusul A. Alkhafaji1
1 Department of Civil Engineering, University of Wasit, Iraq, Wasit, Kut.

* Corresponding author: asadaldefae@uowasit.edu.iq

Abstract. Many previous studies had been investigated the riverbank stability since of its great significance in prohibition flood risk that may command many economic and environmental problems. Investigating the riverbank stability due to development of cracks that represents the main causes of failure is very important, to prepossess recurring landslides by characterizing the riskiest areas and remediating them using a suitable method. In this paper, Dimensional analysis (DA) was carried out to formulate reasonable hypotheses about the complex physical conditions related to riverbank stability investigation. Two empirical equations were derived; the first one for the rate of settlement in riverbank soil, and the other for the failure angle of the slope. Depending on the experimental results that were applied in the software of IBM SPSS Statistics, v24, both equations gave good compatibility where the correlations coefficients were 0.9877 and 0.9826 respectively.

Keywords: Dimensional analysis, Riverbank stability, Soil settlement, Angle of failure.

1. Introduction

Riverbank stability study is considered as one of the most important and complicated studies due to the interplay between the influences of each of the geometric, hydraulic, and morphologic factors. The importance of this issue is highlighted by the role of the riverbank in preventing the dangers of floods and the resulting environmental and economic damage such as land loss, impact on aquatic life, changes in river levels, and accumulations of sediments in estuaries. The investigation of riverbank stability depends on the influence of many factors, among them the soil erosion of the riverbank, the angle of slope failure, and the amount of settlement of the riverbank soil.

Many studies can be found in the literature to predict the behavior of the riverbank by deriving empirical equations. Because of the strong relationship between the angle of slope failure $\beta$ (see Figure 1 below) with the depth and location of tension cracks [1], thus in critical conditions, the angle of slope failure corresponds to the angle of cohesion, and based on Taylor’s theory [2] concluded the following equation to calculate the failure angle using a simple geometrical shape of the riverbank.

$$\beta = (\alpha + \phi)/2$$

Where $\beta$ is the angle of failure, $\alpha$ is the angle before failure, $\phi$ is the angle of internal friction. An equation to calculate the angle of failure in case of neglecting the pore water pressure and the hydrostatic confining pressure in the tension cracks is developed by [3]:

$$\beta = 0.5\tan^{-1}((H/H')^2(1 - K_r^2)\tan \alpha + \phi)$$
Where $H$, the height of the bank is, $H'$ is the height above the vertical part of the bank (see figure 1) while $k_r$ is a topographical factor. Another equation was developed by [4] to calculate the failure angle of the slope depending on the depth of the tension cracks before and after the collapse occurs.

$$\beta = \tan^{-1}\left(\frac{H_2}{B_w+H_3/\tan \alpha}\right)$$  \hspace{1cm} (3)

Where $B_w$ is the width of the failure block. Figure 1 illustrates the parameters used for calculating the angle of failure plane.

A set of field and experimental data to derive a relationship to estimate the depth of tension crack to analyze bank stability is used by [5], through which the angle of the slope failure can be assessed. The relative errors between the observed and calculated values for the model were 4%, 23%, and 27% for the angle of failure, the depth of tension crack and the bank respectively. An analysis was developed to predict the stability of riverbank by [6]. This analysis applies to a steep and inconsistent riverbank (i.e. no typical shape of section along the river), where the effect of hydrostatic pressure and pore water pressure was included in this analysis. A theoretical analysis about the effect of hydrodynamic conditions on the stability of non-cohesive riverbank in meandering and straight rivers, where a relationship was derived to predict the critical angle of the slope as a function of the dimensionless variables is provided [7]. It was found that it is directly proportional to the square of flow velocity and inversely with the median diameter of the grain of bank materials, and it is higher in meandering rivers than in straight rivers. A theoretical study was conducted by [8] to predict the erosion rate of the riverbank by using dimensional analysis, where the effect of physical properties of both fluvial erosion and mass failure and their interaction with each other was taken into consideration, two empirical equations were derived to predict the rate of riverbank erosion and with a prediction accuracy of 60%.

$$\frac{\varepsilon}{u_b} = 6.304 \times 10^{-8} \left(\frac{U}{u_b}\right)^{1.198} (\beta)^{-0.413} (S°)^{-0.719}$$  \hspace{1cm} (4)

$$\frac{\varepsilon}{u_b} = 6.142 \times 10^{-8} \left(\frac{U}{u_b}\right)^{1.198} \left(\frac{h_b}{d_{50}}\right)^{0.005} (\beta)^{-0.416} (S°)^{-0.719}$$  \hspace{1cm} (5)

Where $\varepsilon$ is the erosion rate, $u_b$ is the average velocity within the total depth, $U$ is the shear velocity, $\beta$ is the bank angle, $S°$ is the channel slope, $h_b$ is the bank height, and $d_{50}$ is the mean particle diameter (corresponding to 50% finer than this diameter).

Knowing the factors that contribute to affecting riverbank stability is a precondition for developing future forecasts by simulation using any method of modeling of riverbank stability, in addition to the need for field and experimental data. The basic principle in the modeling process requires identifying the relevant variables and then linking these variables using the physical laws. As there is scarcity in
literature studies related the settlement of riverbanks and its impact on stability, this paper presents one of the most important methods of modeling, which is the dimensional analysis where two empirical equations were derived. The first equation was for the settlement rate, and the second one for the failure angle of the slope.

2. Dimensional Analysis
Dimensional analysis is widely used to study the dimensions of various physical quantities of almost any engineering problem. This procedure is used to obtain information about large complex systems and as a means of checking mathematical and physical equations. Since the conversion of units from one dimensional unit to another is often somewhat complicated, thus, dimensional analysis is used to produce relationships between different physical quantities by identified their fundamental dimensions such as mass, M, length, L, and time, T. The dimensional analysis is used to verify the reasonableness of derivative equations and is used to formulate reasonable hypotheses about the complex physical conditions that can be tested by experimentation [9]. In this paper, the dimensional analysis was carried out twice, one for the riverbank settlement and the other for the angle of failure.

2.1 Buckingham’s Pi Theorem
For describing the dimensionless groups of the selected variables of the problem, Buckingham’s π theorem has been performed. The theorem assumes that “If the equation \( F(q_1, q_2, q_3, \ldots) = 0 \) is complete, the solution of this equation will be in the form of \( F(\pi_1, \pi_2, \pi_3, \ldots \pi_n - k) = 0 \), where the \( \pi \) terms are independent vectors of the parameters \( q_1, q_2, q_3, \ldots \), and they are dimensionless in the unit as a fundamental dimension” [10]. In mathematical meaning, a complete dimensional homogenous equation, concerning \( n \) physical quantities that are acting in terms of \( m \) fundamental quantities will be succinct to a mathematical relationship between \( n-m \) as dimensionless groups. For example, if there are nine \( (9) \) physical quantities involved in the relationship of the physical problem with three \( (3) \) fundamental physical quantities, six \( (6) \) sets of dimensionless groups will be formed.

2.2 Dimensional Variables
The first step in the dimensional analysis method is to determine the variables that affect riverbank stability based on the factors that govern the stability of the riverbank, which include hydraulic, morphological, and geometric factors in addition to the effect of the flow duration. Hydraulic parameters include variables related to the flow characteristics, which are fluid density \((\rho)\), dynamic viscosity \((\mu)\), gravitational acceleration \((g)\), flow depth \((y)\), and flow velocity \((v)\). Morphological parameters include variables related to soil properties; soil density \((\rho_s)\), soil cohesion \((c)\), angle of friction between soil particles \((\phi)\), and soil settlement \((S)\). Geometric factors include parameters related to the shape and dimensions of the channel section, which are, the length of the channel \((L)\), width of the channel bed \((B)\), bed slope \((S_v)\), side slope \((S_z)\), angle of failure \((\beta)\), angle before failure \((\alpha)\), the width of failure block \((B_{wo})\), and the depth of tension crack \((Y)\).

All of the hydraulic, morphological, and geometric parameters that affect both of soil settlement and angle of failure are summarized in Table 1 whereas the parameters that related to the appearance of tension cracks (i.e., angle of failure, angle before failure, the width of failure block, and depth of tension cracks) only affect the angle of failure.
Table 1. The classification of parameters affecting riverbank stability and angle of failure

| 1. Parameters Characterizing the Flow | Symbol | Unit | Dimension |
|--------------------------------------|--------|------|-----------|
| Density of Fluid                     | \( \rho \) | Kg/m\(^3\) | ML\(^{-3}\) |
| Dynamic Viscosity of Fluid           | \( \mu \) | Kg/m.s | ML\(^{-1}\)T\(^{-1}\) |
| Gravitational Acceleration           | \( g \) | m/s\(^2\) | LT\(^{-2}\) |
| Approach Flow Depth                  | \( y \) | m | L |
| Approach Flow Velocity               | \( v \) | m/s | LT\(^{-1}\) |
| 2. Parameters Characterizing Soil Properties | Symbol | Unit | Dimension |
| Density of Soil                      | \( \rho_s \) | Kg/m\(^3\) | ML\(^{-3}\) |
| Cohesion of Soil                     | \( c \) | Kg/m\(^2\) | ML\(^{-2}\) |
| Angle of Friction                    | \( \phi \) | - | - |
| Soil Settlement                      | \( S \) | m | L |
| 3. Parameters Characterizing Channel Geometry | Symbol | Unit | Dimension |
| Length of the Channel                | \( L \) | m | L |
| Width of the Channel Bed             | \( B \) | m | L |
| Bed Slope                            | \( S_o \) | - | - |
| Side Slope                           | \( S_s \) | - | - |
| Angle of Failure                     | \( \beta \) | - | - |
| Angle before Failure                 | \( \alpha \) | - | - |
| Width of Failure Block               | \( B_w \) | m | L |
| Depth of Tension Crack               | \( Y \) | m | L |
| 4. Time                              | Symbol | Unit | Dimension |
| Duration of Flow                     | \( t \) | hour | T |

2.3 Modeling Procedures

2.3.1 Settlement of the Riverbank Soil. The settlement of the riverbank soil can be expressed as a function of the relationship between the above variables using Buckingham \( \pi \)-theorem. The similarity or \( \pi \)-group theorem means the prediction of the prototype performance from the model test observations by the application of \( \pi \)-group such as Reynolds number or Froude number, the prediction of flow condition using river models assist in the design of flood control structure as well as the analysis of sediment transport in the river.

\[
S = f(\rho, \mu, g, y, v, \rho_s, C, \phi, L, B, S_o, S_s, t)
\]  

(6)

\[
f(S, \rho, \mu, g, y, v, \rho_s, C, \phi, L, B, S_o, S_s, t) = 0
\]  

(7)

Since there are fourteen variables \((n = 14)\), and the repeated variables are \((m = 3)\) selected as \((\rho, v, \text{and } B)\), according to \( \pi \)-theorem the dimensionless group acquired are \((n - m = 11)\), each group has to implicate \((m + 1 = 4)\) variables, as shown below:

\[
f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}) = 0
\]  

(8)

\[
\pi_1 = \rho^{a_1} \cdot v^{b_1} \cdot B^{c_1} \cdot S
\]  

(9)

\[
\pi_2 = \rho^{a_2} \cdot v^{b_2} \cdot B^{c_2} \cdot \mu
\]  

(10)

\[
\pi_3 = \rho^{a_3} \cdot v^{b_3} \cdot B^{c_3} \cdot g
\]  

(11)

\[
\pi_4 = \rho^{a_4} \cdot v^{b_4} \cdot B^{c_4} \cdot y
\]  

(12)

\[
\pi_5 = \rho^{a_5} \cdot v^{b_5} \cdot B^{c_5} \cdot \rho_s
\]  

(13)

\[
\pi_6 = \rho^{a_6} \cdot v^{b_6} \cdot B^{c_6} \cdot C
\]  

(14)
\[ \pi_7 = \rho a^7 * v^{b_7} * B^{c_7} * \emptyset \]  (15)
\[ \pi_8 = \rho a^8 * v^{b_8} * B^{c_8} * L \]  (16)
\[ \pi_9 = \rho a^9 * v^{b_9} * B^{c_9} * S_o \]  (17)
\[ \pi_{10} = \rho a^{10} * v^{b_{10}} * B^{c_{10}} * S_S \]  (18)
\[ \pi_{11} = \rho a^{11} * v^{b_{11}} * B^{c_{11}} * t \]  (19)

Taking the first group and evaluating \((a_1, b_1, and c_1)\) by expression the variables in term of \((mass \ M, length \ L, time \ T)\) as shown below:

\[ \pi_1 = \rho a_1 * v^{b_1} * B^{c_1} * S \]
\[ M^0 L^0 T^0 = (ML^{-3}) a_1 * (LT^{-1}) b_1 * (L) c_1 * L \]

For M: \(a_3 = 0\)

For L: \(-3a_1 + b_1 + c_1 + 1 = 0\)

\[ b_1 + c_1 = -1 \]

For T: \(-b_1 = 0\)

So \(c_1 = -1\)

And \(\pi_1 = \frac{S}{B}\)

By the same way:

\[ \pi_2 = \frac{\mu}{\rho v B} \]
\[ \pi_3 = \frac{g B}{v^2} \]
\[ \pi_4 = \frac{y}{B} \]
\[ \pi_5 = \frac{\rho s}{\rho} \]
\[ \pi_6 = \frac{C}{\rho B} \]
\[ \pi_7 = \emptyset \]
\[ \pi_8 = \frac{L}{B} \]
\[ \pi_9 = S_o \]
\[ \pi_{10} = S_s \]
\[ \pi_{11} = \frac{\rho v t}{C} \]

\[ f \left( \frac{S}{B}, \frac{\mu}{\rho v B}, \frac{g B}{v^2}, \frac{y}{B}, \frac{\rho s}{\rho}, \frac{C}{\rho B}, \emptyset, \frac{L}{B}, S_o, S_s, \frac{\rho v t}{C} \right) = 0 \]  (20)
\[ \frac{S}{B} = f \left( \frac{\mu}{\rho v B}, \frac{g B}{v^2}, \frac{y}{B}, \frac{\rho s}{\rho}, \frac{C}{\rho B}, \emptyset, \frac{L}{B}, S_o, S_s, \frac{\rho v t}{C} \right) \]  (21)

The general form of the analysis can be simplified by some suppositions and according to the circumstances in this study; the inverse of the term \(\frac{\mu}{\rho v B}\) multiplied by the term \(\frac{L}{B}\), can be represented the Reynold number \(\frac{\rho v t}{\mu}\), which it is usually an ineffective parameter (as the flow is turbulent that was modeled using an empirical approach) and because of the complexity of turbulent flow nature which prevents from establishing a general mathematical solution, so, it is neglected, the term \(\frac{g B}{v^2}\) can be multiplied by the term \(\frac{y}{B}\), then taking the square root of the term inverse, which would represent the Froude number \(Fr = \frac{v}{\sqrt{g y}}\) water density and soil density are constant in all models so the term \(\frac{\rho s}{\rho}\) is dropped, and as the bed slope and the side slope are constant in all models, the terms \(S_o\) and \(S_s\) are neglected, by multiplying the term \(\frac{\rho v t}{C}\) by the inverse of the term \(\frac{C}{\rho B}\) resulting in the term \(\frac{\rho v t}{C}\).

According to the above suppositions, the final form of equation (16) would be as illustrates in the following equation:
\[ \frac{S}{B} = f(Fr, \emptyset, \frac{\rho v t}{C}) \]  

(22)

2.3.2 Failure Angle of the Slope. The parameters that affect the angle of failure are the same parameters affect the settlement of riverbank in addition to the parameters related to the appearance of tension cracks; all of them are illustrated in Table 1. The failure angle of the riverbank can be expressed as a function of the above variables using Buckingham \( \pi \)-theorem as shown below:

\[ \beta = f(\rho, \mu, g, y, v, \rho_s, C, \emptyset, S, L, B, S_o, S_s, \alpha, B_w, Y, t) \]  

(23)

\[ f(\rho, \mu, g, y, v, \rho_s, C, \emptyset, S, L, B, S_o, S_s, \alpha, \beta, B_w, Y, t) = 0 \]  

(24)

The number of variables \( (n = 18) \), and repeated variables \( (m = 3) \) selected as \( (\rho, v, and Y) \), so the dimensionless group acquired are \( (n - m = 15) \) according to \( \pi \)-theorem, and as shown below:

\[ f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{15}) = 0 \]  

(25)

\[ \pi_1 = \rho^{a_1} \cdot v^{b_1} \cdot Y^{c_1} \cdot \mu \]  

(26)

\[ \pi_2 = \rho^{a_2} \cdot v^{b_2} \cdot Y^{c_2} \cdot g \]  

(27)

\[ \pi_3 = \rho^{a_3} \cdot v^{b_3} \cdot Y^{c_3} \cdot y \]  

(28)

\[ \pi_4 = \rho^{a_4} \cdot v^{b_4} \cdot Y^{c_4} \cdot \rho_s \]  

(29)

\[ \pi_5 = \rho^{a_5} \cdot v^{b_5} \cdot Y^{c_5} \cdot C \]  

(30)

\[ \pi_6 = \rho^{a_6} \cdot v^{b_6} \cdot Y^{c_6} \cdot \emptyset \]  

(31)

\[ \pi_7 = \rho^{a_7} \cdot v^{b_7} \cdot Y^{c_7} \cdot S \]  

(32)

\[ \pi_8 = \rho^{a_8} \cdot v^{b_8} \cdot Y^{c_8} \cdot L \]  

(33)

\[ \pi_9 = \rho^{a_9} \cdot v^{b_9} \cdot Y^{c_9} \cdot B \]  

(34)

\[ \pi_{10} = \rho^{a_{10}} \cdot v^{b_{10}} \cdot Y^{c_{10}} \cdot S_o \]  

(35)

\[ \pi_{11} = \rho^{a_{11}} \cdot v^{b_{11}} \cdot Y^{c_{11}} \cdot S_s \]  

(36)

\[ \pi_{12} = \rho^{a_{12}} \cdot v^{b_{12}} \cdot Y^{c_{12}} \cdot \beta \]  

(37)

\[ \pi_{13} = \rho^{a_{13}} \cdot v^{b_{13}} \cdot Y^{c_{13}} \cdot \alpha \]  

(38)

\[ \pi_{14} = \rho^{a_{14}} \cdot v^{b_{14}} \cdot Y^{c_{14}} \cdot B_w \]  

(39)

\[ \pi_{15} = \rho^{a_{15}} \cdot v^{b_{15}} \cdot Y^{c_{15}} \cdot t \]  

(40)

For the first group, \( (a_1, b_1, and c_1) \) would be evaluated by expression the variables in term of \( (MLT) \), as shown below:

\[ \pi_1 = \rho^{a_1} \cdot v^{b_1} \cdot Y^{c_1} \cdot \mu \]

\[ M^0L^0T^0 = (ML^{-3})^{a_1} \cdot (LT^{-1})^{b_1} \cdot (L)^{c_1} \cdot (ML^{-1}T^{-1}) \]

For M: \( a_1 + 1 = 0 \)

\[ a_1 = -1 \]
For L: $-3a_1 + b_1 + c_1 - 1 = 0$

\[
b_1 + c_1 = -2
\]

For T: $-b_1 - 1 = 0$

\[
b_1 = -1
\]

So $c_1 = -1$

And $\pi_1 = \frac{\mu}{\rho v_Y}$

By the same way:

\[
\begin{align*}
\pi_2 &= \frac{g_Y}{v^2}, \\
\pi_3 &= \frac{y}{Y}, \\
\pi_4 &= \frac{\rho_S}{\rho}, \\
\pi_5 &= \frac{C}{\rho Y}, \\
\pi_6 &= \emptyset, \\
\pi_7 &= \frac{S}{Y}, \\
\pi_8 &= \frac{L}{Y}, \\
\pi_9 &= \frac{B}{Y}, \\
\pi_{10} &= S_s, \\
\pi_{11} &= S_o
\end{align*}
\]

$\pi_{12} = \beta$, $\pi_{13} = \alpha$, $\pi_{14} = \frac{B_w}{Y}$, $\pi_{15} = \frac{v t}{Y}$

\[
f \left( \frac{\mu}{\rho v_Y}, \frac{g_Y}{v^2}, \frac{y}{Y}, \frac{\rho_S}{\rho}, \frac{C}{\rho Y}, \emptyset, \frac{S}{Y}, \frac{L}{Y}, \frac{B}{Y}, S_s, S_o, \beta, \alpha, \frac{B_w}{Y}, \frac{v t}{Y} \right) = \frac{v}{\sqrt{g_Y}}
\]

The same suppositions that mentioned in the previous paragraph for the analysis of the bank settlement are applied here, according to which, the general form of the analysis can be simplified by neglecting the terms $(\frac{\mu}{\rho v_Y}, \frac{L}{Y}, \frac{\rho_S}{\rho}, S_s, S_o)$, multiplying of the term $\frac{g_Y}{v^2}$ by the term $\frac{y}{Y}$, then by taking the square root of the term inverse, resulting in the Froude number $Fr = \frac{v}{\sqrt{g_Y}}$, neglecting the term $\frac{B}{Y}$, as the width of the channel bed is constant, the term of the slope angle before failure $\alpha$ is neglected, as it is constant, the term $\frac{v t}{Y}$ can be multiplied by the inverse of the term $\frac{C}{\rho Y}$, which results in the term $\frac{B_w}{Y}$.

According to the above suppositions, the final form of equation (37) would be as illustrates in the following equation:

\[
\beta = f \left( Fr, \emptyset, \frac{B_w}{Y}, \frac{v t}{C} \right)
\]

3. IBM SPSS Statistics, v24

3.1 Curve Estimation Analysis

The Curve Estimation procedure produces the curve estimation regression statistics, and most of the related plots for 11 different curve estimation regression models can be produced using the curve estimation procedure. For each dependent variable, a separate model is produced, and also all the predicted values could be saved, whereas the rest, subsequently, prediction intervals as new variables. For each model: regression coefficients, multiple $R$, $R^2$, adjusted $R^2$, standard error of the estimate, analysis-of-variance table, predicted values, residuals, and prediction intervals. Models: linear, logarithmic, inverse, quadratic, cubic, power, compound, S-curve, logistic, growth, and exponential.

3.2 Non-linear Regression Analysis

Nonlinear regression is a method of finding a nonlinear model of the relationship between the dependent variable and a set of independent variables. Unlike traditional linear regression, which is restricted to estimating linear models, nonlinear regression can estimate models with arbitrary relationships between independent and dependent variables.
4. Results

After the dependent variable was related with the independent variables by a function, for each one of the settlement of the bank and the angle of failure by performing the dimensional analysis which was described in the previous section, each of these relationships were applied in the software (IBM SPSS Statistics, v24) to create an empirical formula by using regression analysis.

4.1 Settlement of the Riverbank Soil

The data obtained from the experimental work conducted by [11] were substituted in equation (22) that obtained from the dimensional analysis to determine the riverbank settlement. The correlation ratio and the type of relationship between the dependent variable (settlement) and each one of the independent variables were individually tested using curve estimation analysis. It was found that the independent variable represented by the term \( \left( \frac{\rho v t}{c} \right) \) is the most closely associated with the dependent variable, since the correlation coefficient \( R = 0.997, R^2 = 0.995 \), the standard error of the estimate \( SEE = 0.002\% \), and that the type of function was cubic. The non-linear regression analysis was then performed to obtain the predicted values of settlement, which was graphically represented by a best-fit line with the measured values, as shown in figure 2.

\[
S = 0.003 + 5.815 \times 10^{-5} \left( \frac{\rho v t}{c} \right) - 1.479 \times 10^{-8} \left( \frac{\rho v t}{c} \right)^2 + 1.35 \times 10^{-12} \left( \frac{\rho v t}{c} \right)^3
\]

(44)

Where \( S \) is the soil settlement of the riverbank, \( \rho \) is the water density, \( v \) is the flow velocity, \( t \) is the flow duration, \( C \) is the soil cohesion, and \( B \) is the bed width.

It can be seen that the angle of internal friction is not mentioned in the equation above because the effect of \( \emptyset \) is much less than the effect of cohesion force, in addition to that when testing the relationship of \( \emptyset \) with the settlement in SPSS program, the correlation coefficient was very weak.

4.2 Failure Angle of the Slope

In the regression analysis for the angle of failure, the equation (43) with the data obtained from the experimental work conducted by [11] was applied in the SPSS software after the relationship between the dependent variable (angle of failure) with each one of the independent variables was tested using the curve estimation analysis, where the independent variable that most closely associated with the angle of failure was represented by the term \( \left( \frac{R_m}{Y} \right) \) with a correlation coefficient \( R = 0.991, R^2 = 0.983 \), the standard error of the estimate \( SEE = 2.9\% \), and the function type was cubic. The measured
values of the angle of failure which was calculated by [11] using an equation developed by [4] were plotted with the predicted values obtained from the regression analysis by a best-fit line, where $R^2 = 0.9826$, as shown in figure 3.

\[ \beta = 0.205 + 99.622 \left( \frac{B_w}{Y} \right) - 62.004 \left( \frac{B_w}{Y} \right)^2 + 10.554 \left( \frac{B_w}{Y} \right)^3 \]  

(45)

Where, $\beta$ is the angle of failure, $B_w$ is the width of failure block, and $Y$ is the depth of tension crack.

It could be seen also that $\Phi$ is not mentioned here because the correlation coefficient between $\Phi$ and the failure angle is very small. So, very slightly effect and is neglected.

5. Model Performance

The purpose of conducting the regression analysis is to find a statistically significant relationship that links the dependent variable with the independent variables, relying on experimental results obtained from a study conducted by [11]. Other used a physical model by modifying new flume apparatus that is a locally manufactured by certain standards [12]. The results gave a good agreement between the measured and predicted values, as it was explained in detail in the previous paragraph. The empirical equations that conducted using SPSS software, also provide additional confidence in the interpretation of the experimental results where it showed that the settlement of the bank is affected by the flow duration and the soil cohesion as shown in figure (4), as the amount of soil settlement in the model with long-term flow duration (24 hr.) was 19.5 mm at the end of the test, while in the model with short-term flow duration (5 hr.) it reached only 10.3 mm at the end of the test.

![Figure 4](image-url)
While the failure angle of the slope is affected by the depth of tension crack and the width of failure block, [11], found that the failure angle of the slope is affected by the decrease of the bank height more than the bank width, where the bank’s angle was decreased by 74.9%. In comparison, the decrease in the width and height of the bank were 85% and 76.5%, respectively, figure (5) shows the development of tension crack appearance in the experimental model.

**Figure 5.** Development of tension cracks appear in the experimental test, [11].

### 6. Conclusions

In this paper, two empirical equations were presented to predict riverbank stability, the first equation for calculating the amount of settlement in riverbank soil as a function of flow velocity, flow duration, fluid density, soil cohesion force, and width of the channel bed. The results gave a good agreement between the observed and calculated values by a correlation coefficient of (0.9877). The second equation for calculating the failure angle of the slope as a function of both the width of failure block and the depth of tension crack, which also gave good agreement between the observed and calculated values by a correlation coefficient of (0.9826). These equations were tested using data from a laboratory study conducted using a flume device to investigate the stability of the riverbank.

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