Contact interactions probe effective dark-matter models at the LHC

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Abstract – Effective field theories provide a simple framework for probing possible dark-matter (DM) models by re-parametrising full interactions into a reduced number of operators with smaller dimensionality in parameter space. In many cases these models have four particle vertices, e.g., $\bar{q}q\chi\bar{\chi}$, leading to the pair production of dark-matter particles, $\chi$, at a hadron collider from initial state quarks, $q$. In this analysis we show that for many fundamental DM models with $s$-channel DM couplings to $\bar{q}q$ pairs, these effective vertices must also produce quark contact interactions (CI) of the form $\bar{q}qqq$. The respective effective couplings are related by the common underlying theory which allows one to translate the upper limits from one coupling to the other. We show that at the LHC, the experimental limits on quark contact interactions give stronger translated limits on the DM coupling than the experimental searches for dark-matter pair production.

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Introduction. – An explanation of the cosmological dark-matter puzzle is one of the most important questions facing modern physics. Currently, a compelling explanation of this phenomenon is via so-called Weakly Interacting Massive Particles (WIMPs) [1–3]. The heightened interest in WIMPs is due to the fact that a neutral particle which interacts with roughly the strength of the weak force gives the correct DM abundance.

Many experiments are currently under way in the hope of finding the DM particle. The most well known are direct detection searches that aim to see DM collisions with atomic nuclei [4]. Another possible way to see a signal could come from the annihilation of DM particles in high-density regions of the Universe, see, e.g., [5].

More recently, interest has turned to the possibility that particle colliders could produce dark matter. Under the WIMP hypothesis, dark matter is assumed to be a neutral stable particle with a mass around the electroweak symmetry breaking scale. Therefore, it is natural to ask if the signatures of these particles could be seen at TeV scale colliders. Using a model-independent or effective theory approach [6–8], it is possible to relate expected production cross-sections to the relic density [9] and/or to the direct detection cross-section [10–12]. Since the dark-matter candidates are only weakly interacting, observations require Standard Model particles to be produced in association. These are usually a photon or jet that has been radiated from the initial state and lead to a mono-photon (or -jet) topology. Many studies have now investigated these kinds of signals in a model-independent or effective theory approach [10–28]. In the case of supersymmetry the same signal can be used to search for compressed spectra [29–31].

In the effective field theory approach, higher-dimensional operators are employed as an approximation to a full theory that includes dark matter. For this approach to be valid, the full theory must contain at least one additional heavy particle that mediates the interaction between the Standard Model and dark matter. Consequently, it is possible that the presence of these new states may have observable effects at colliders and these should be explored. For example, if the interaction between dark matter and the Standard Model is via an $s$-channel process, the mediator could produce a di-jet resonance [25,26]. In this case, the search for di-jet resonances is often more powerful than the more direct mono-jet searches for the dark-matter particle. However, they are limited to models where the mediator can be produced on-shell at the collider in question. In this case, the validity of using an effective theory to describe the dark-matter production must be called in question.

Instead of using a di-jet resonance search to constrain the effective model, we propose to look for deviations from the Standard Model in $q\bar{q}$ contact interactions. In the case of an $s$-channel mediator, the operator will lead to a deviation from standard QCD interactions and thus can
be searched for. In fact ATLAS and CMS [32–35] have searches of this kind in various final states.

In this study we focus on deviations in the high-energy di-jet spectrum. We choose di-jets, since if a mediator couples to a \(qq\) initial state, it is guaranteed to also mediate the production of di-jets. We can compare to the mono-jet analyses by noting that the same interaction between the mediator and initial-state quarks must also exist there. Consequently, if we assume that the interaction between the mediator and dark-matter particles is perturbative (\(g \leq \sqrt{4\pi}\)), the mono-jet search sets a limit on the interaction between the mediator and quarks. We show that in the region of parameter space where the effective theory is valid (mediator mass \(\gtrsim 1\) TeV) and perturbativity is not violated, the contact interaction search leads to the most stringent limit.

We first show how an effective theory of interactions can be derived in the limit of a large mediator mass by giving an example fundamental model, and explain the limits on this effective theory coming from both contact interactions and mono-jet searches for dark matter. We continue with the comparison of limits on the effective couplings and show that at the LHC contact interaction bounds lead to more stringent limits. We finish by addressing the question, in which way other fundamental theories may be expected to have different bounds on the underlying couplings.

Effective couplings from a fundamental model. –

We start with a simple formulation of an example model to describe the interaction of a new dark-matter particle \(\chi\) with Standard Model quarks \(q\). We choose \(\chi\) to be a Dirac fermion and analyze pair production \(qq \to \chi\chi\) from initial-state quarks, via a heavy vector mediator \(V\) from a \(U(1)\) gauge theory. A particle \(X\) is assumed to have mass \(M_X\). We consider the following Lagrangian for this model:

\[
\mathcal{L}_{UV} = \tilde{q}(i\slashed{D} - M_q)q + \chi(i\slashed{D} - M_\chi)\chi + \frac{1}{2} M_V^2 V_\mu V^\mu - \frac{1}{4} V^{\mu\nu}V_{\mu\nu} - g_q \tilde{q}q \gamma^\mu P_L V^\mu - g_\chi \chi \gamma^\mu P_L \chi V^\mu, \tag{1}
\]

where we have used the projection operator

\[
P_L \equiv \frac{(1 - \gamma^5)}{2}. \tag{2}
\]

The first four terms include both kinematic and mass terms for all the fields (with the standard Abelian field strength tensor \(V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu\) for the vector mediator). The last terms describe chiral interactions of the vector particle \(V^\mu\) with both fermions \(\chi\) and \(q\) via dimensionless coupling strengths \(g_\chi\) and \(g_q\). The particular choice of a chiral interaction leads to effective operators that are commonly analysed in experimental studies, e.g., [32,34]. We consider other operators at the end of this work.

The DM particle \(\chi\) is assumed to interact with the Standard Model only by exchanging the new mediator \(V\), i.e., it is unchanged under any Standard Model gauge group and neither couples to the respective gauge bosons nor the Higgs particle.

The new mediator leads to new interaction channels for the Standard Model quarks, which are shown in fig. 1. At a hadron collider, an off-shell mediator that is created by two initial-state quarks can either produce a pair of quarks, describing elastic quark scattering, or produce a pair of the new particle \(\chi\). Since both processes depend on the strength of the initial-state coupling \(g_q\), their cross-sections are related.

If we now assume that the mass of the mediator, \(M_V\), lies far beyond the accessible center-of-mass energy \(\sqrt{s}\) of the partons in any scattering process we want to analyze at a hadron collider, we can integrate out the vector field and expand the remainder of the effective Lagrangian up to leading order in \(s/M_V^2\) (see, e.g., [36]),

\[
\mathcal{L}_{\text{eff}} = \tilde{q}(i\slashed{D} - M_q)q + \chi(i\slashed{D} - M_\chi)\chi
- \frac{g_q^2}{2M_V^2} (q_L \gamma^\mu q_L q_L \gamma_\mu q_L - g_q g_L q_L \gamma^\mu q_L \bar{\chi} L \gamma_\mu \chi L
- \frac{g_\chi^2}{2M_V^2} \bar{\chi} L \gamma^\mu \chi L \bar{\chi} L \gamma_\mu \chi L, \tag{3}
\]

with the left-handed component of the quark field \(q_L \equiv P_L q\). The last term describes the scattering of the dark-matter particle \(\chi\) with itself, which is of no interest in this analysis and is therefore omitted henceforth. We combine the pre-factors of the two remaining effective vertices by defining the effective couplings \(G_q \equiv g_q^2/M_V^2\), describing a contact interaction (CI) between four Standard Model quarks, and \(G_\chi \equiv g_\chi g_L/M_V^2\), which gives the scattering strength between quarks and the DM particle \(\chi\).

To be consistent with the perturbative approach of using tree-level diagrams only, the dimensionless couplings \(g\) must not be larger than \(\sqrt{4\pi}\). Thus, in addition to the restriction \(M_V^2 \geq \sqrt{s}\) demanded for the effective approximation to be valid, only the limited parameter space \(0 < G_i < 4\pi/\sqrt{s}\) is allowed for both effective couplings \(G_i\).

Experimental limits on the effective couplings. –

The two effective couplings we derived have to be probed differently at a hadron collider. Firstly, \(G_q\) describes the elastic scattering of quarks and can be analysed by looking for deviations compared to Standard Model predictions.
for high-energy di-jet production. This analysis has been performed by both the ATLAS \cite{32} and CMS \cite{34} Collaborations at the LHC. Since there also exist Standard Model diagrams for this type of scattering, limits on $G_q$ depend on how the Standard Model terms interfere with the new contribution of the effective operator. We conservatively take the lowest limits given for destructive interference, which CMS quotes as

$$G_q \leq 4\pi(7.5\text{ TeV})^{-2} \quad (4)$$

at 95\% CL, determined with an integrated luminosity of 2.2 fb$^{-1}$ at 7 TeV center-of-mass energy.

On the other hand, $G_\chi$ describes dark-matter pair production. These particles are usually invisible at the LHC, since they do not interact significantly with the detector at the LHC due to their small coupling to the Standard Model. Mono-photons \cite{37,38} or mono-jets \cite{39,40} (radiated from the initial state) are characteristic for this kind of interaction and have been probed by both experiments\footnote{To be precise, the limits have been determined for the vertices $q\gamma^\mu q\chi^n\chi$ and $q\gamma^\mu q\chi^n\gamma^\nu\chi$ individually. However, for a large mass range the bounds are similar. Therefore, we assume the same limits on the coupling $q_L\gamma^\mu q_L\chi_L\gamma^\nu\chi_L$.}. The currently strongest upper bound on $G_\chi$ is given by a mono-jet analysis of CMS \cite{39} for an integrated luminosity of 5.0 fb$^{-1}$ at 7 TeV,

$$G_\chi \leq (765 \text{ GeV})^{-2}, \quad (5)$$

which holds for $M_\chi = 10 \text{ GeV}$ at 90\% CL. Different (larger or smaller) values for $M_\chi$ lead to weaker bounds.

**Comparing experimental limits.** – The quoted limits on $G_q$ and $G_\chi$ differ significantly, due to the very different techniques involved in the respective analyses. However, the two effective couplings have common ingredients which implicitly relate them. Consequently, we may reasonably translate the limit from $G_q$ into an upper bound on $G_\chi$ and see how this bound compares to the experimental limit given in eq. (5).

Since $G_\chi$ depends on $g_\chi$ whereas $G_q$ does not, there is no 1:1 correspondence between the two effective couplings and they are a priori independent. However, we only have restricted parameter values for the coupling constants $g_\chi$ and $g_q$ to be in agreement with the perturbative picture. Taking the definition for $G_\chi$ and restricting $g_\chi, g_q \leq \sqrt{4\pi}$ by perturbation theory, it follows that

$$G_\chi \leq \frac{4\pi}{M_V^2}. \quad (6)$$

Furthermore we may relate $G_\chi$ to $G_q$ in order to apply the experimental limit known for $G_q$. According to the definitions of the two effective couplings, it follows that

$$G_\chi = \frac{g_\chi}{M_V}\sqrt{G_q}. \quad (7)$$

Fig. 2: (Color online) Exclusion limits on the effective coupling constant $G_\chi$ for a given mediator mass $M_V$ according to experimental limits from mono-jet searches (red), experimental limits on the contact interaction $G_q$ (green), the perturbative restriction on the fundamental coupling constants $g_i$ (blue) and the validity condition for the effective field theoretical approach (purple). For the latter, the limit $M_V \geq 1 \text{ TeV}$ should be understood as a rough estimate given by the typical parton energies at the LHC. The limit $G_\chi \leq (765 \text{ GeV})^{-2} \approx 1.7 \text{ TeV}^{-2}$, eq. (5), from mono-jet searches assumes a dark-matter mass $M_\chi \approx 10 \text{ GeV}$, which is the most optimistic scenario and leads to the strongest bound. The upper bound from contact interactions assumes destructive interference which gives the most conservative limit. For mediator masses consistent with the effective approach, we see that the bound from contact interactions is most stringent.

With the experimental limits on $G_q$, given in eq. (4) and the perturbative restriction $g_\chi \leq \sqrt{4\pi}$, we find

$$G_\chi \leq \frac{1}{M_V}\frac{4\pi}{7.5 \text{ TeV}}. \quad (8)$$

In fig. 2 we compare the excluded parameter regions in the $G_\chi$–$M_V$ plane according to the different restrictions in eqs. (6)–(8) and the effective approach. It can be seen that in the mediator mass range from 1 TeV up to 7 TeV, the translated experimental limit on the quark contact interaction $G_q$ gives the strongest restrictions on the parameter space of the effective theory for the dark-matter particle $\chi$. In particular, the limits are stronger than the experimental constraints on $G_\chi$ from mono-jet searches.

For larger mediator masses, demanding that the theory is perturbative gives stronger upper limits than both of the experimental searches. However, the perturbative upper bound is static whereas the experimental sensitivity will gradually improve over time as more data is collected.

In the small $M_V$ limit below 1 TeV, experimental limits on $G_q$ can only give weak statements on the allowed parameter space and the mono-jet searches give the strongest exclusion limit. Unfortunately, the typical energies involved in general scattering processes at the LHC are likely to be at or above the TeV scale. According to the requirements of an effective theory to be valid, mediator masses below 1 TeV cannot be analysed reasonably in

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that framework and experimental limits cannot be trusted anymore. However, we note that the searches for di-jet resonances can lead to bounds in these mass ranges and in many cases, these are more stringent than those coming from mono-jet searches [25,26].

We thus conclude that for mediator masses that allow the application of an effective approximation, experimental limits from contact interaction searches or perturbativity bounds put stronger restrictions on the allowed parameter space of an effective dark-matter theory than mono-jet searches.

**Applicability to other effective models.** — In the preceding analysis we considered the specific effective model given in eq. (1) as an example. We now discuss how a different model might change the statements given so far.

**Different spins for χ.** If the introduced new particle χ is not fermionic but a scalar or vector particle, the Lorentz structure of its coupling to the mediator changes (see, e.g., [27]). However, this affects neither the effective Standard Model contact interaction nor the definitions of the effective coupling constants G. Therefore, both limits in eq. (6) and eq. (8) remain.

For the mono-jet/mono-photon searches, a different spin for χ might change the total orbital angular momentum of the final state and therefore affect the kinematics of the events. This would lead to a change in the expected signal and background in the kinematic region of interest such that the derived limits on Gχ could change. However, the cross-section for effective theories has its maximum for small angles and energies of the radiated object. In that kinematic regime, radiated objects are mainly described by spin-independent splitting functions, i.e., the Weizsäcker-Williams distribution in case of soft photons [41,42] or the solution of the DGLAP equations for soft gluons [43–45]. Thus, we do not expect a significant impact of the spin structure on the interaction and therefore no significant effect on the experimental limits.

**Different spins for the mediator.** A different spin for the mediator changes both effective operators and therefore affects the signal for both experimental searches. For the mono-jet/mono-photon searches we still do not expect large differences for the same reason we gave in the case of a different spin for χ. The di-jet analyses usually examine the angular distribution of the two final state objects, which changes with the spin of the mediator. However, the expected signal distributions should still be distinguishable from the QCD background, such that we expect the derived limits to not change drastically.

**Changing the interaction vertices.** Changing any vertex from a chiral interaction to, e.g., vector- or axial-vector–like couplings usually affects the dependence of any related cross-section on the initial-particle spin polarisation. However, since the LHC only measures unpolarised cross-sections, these changes are not expected to change the respective limits on the effective couplings and thus keep the aforementioned statements valid.

If either the mediator or the dark-matter particle is a fermion, a t-channel interaction of the form $V^μ(qγμχ + ̄χγμq)$ is also possible. In that case, the effective approximation does not give any q4 interaction to leading order in $s/M_V^2$ such that CI measurements are not sensitive anymore. Limits from mono-jet searches then give the strongest restrictions in parameter space in the regions allowed by perturbation theory.

We note that if the mediator V and the DM candidate χ are either both scalars or vectors, their respective coupling $χV$ has mass dimension 3 such that the corresponding coupling constant g is not dimensionless anymore. In that case neither the perturbative upper limit of $\sqrt{4\pi}$ can be applied, nor do we expect the experimental limits to stay constant since the cross-sections formulae most likely change non-trivially due to the different mass dimension of G.

**Coupling to Standard Model leptons.** The new mediator may also have an additional coupling to the leptonic sector of the Standard Model, from which one would expect an effective contact interaction of the form $qqll$. If the mediator couples universally to all Standard Model particles, this vertex would be described by the same effective coupling $G_q$ as the CI quark interaction we analysed before. This new coupling could then be probed by di-lepton searches [33,35]. The ATLAS Collaboration quotes a combined limit given by both di-electron and di-muon searches of

$$G_q \leq 4\pi(9.8\text{ TeV})^{-2}$$

at 95% CL, determined with an integrated luminosity of 5.9 fb–1 at 7 TeV. The resulting limits on $G_χ$ are shown in fig. 3. They are more restrictive than the di-jet limits given in eq. (4), which is not only due to the larger integrated luminosity but also because of the cleaner final state. However, the di-jet limits are valid for a larger set of models, i.e., for all that introduce quartic quark self-interaction, whereas the dilepton limits need to assume quark-lepton universality.

**Coupling to gluons.** Since we are discussing the experimental results of a hadron collider, not only quark couplings but also the interaction with gluons can be probed. There are different possibilities to construct effective interactions of gluons with the particle χ (see, e.g., [11]), that may lead to anomalous 4-gluon couplings. However, there are some drawbacks which make the direct analogy difficult:

- In our approach, we start with the underlying renormalizable ultraviolet complete theory. Writing down
an interaction for gluons with any mediator $V$ should both keep the dark-matter candidate a gauge singlet and be in agreement with $SU(3)$ color symmetry. This demands the existence of a $\chi\chiVV$ term, which leads to more involved effective theories due to the necessity of exchanging mediators in pairs. It can therefore be expected that the relation between the two effective coupling constants $G_g$ and $G_\chi$ becomes non-trivial and does not allow for an easy comparison as above.

Possible measurements of different gauge-invariant anomalous gluon self-couplings have been analysed in [46,47]. It is stated that di-jet analyses are practically impossible in that framework, since new operators contribute to the order $1/M_V^2$, whereas for quark operators as in eq. (3) they already arose at order $1/M_V^2$. This leads to a very small expected signal which is difficult to probe at LHC energies. Other proposed methods might form better alternatives but their sensitivity is still expected to be small.

We therefore do not expect that our statement about the relative strength of mono-jet and CI searches holds in the case of gluonic operators.

**Non-universal couplings.** In order to translate the measured LHC limits into bounds on the parton coupling $g_g$, it is necessary to know how the new mediator couples to different quark flavours. The experimental limits given in eqs. (4), (5) assume a universal coupling to all types of quarks. Changing this assumption will affect the size of these limits. As an example, in models with a new Higgs-like particle that couples in the Yukawa-like way to quarks, $g_g$ grows proportionally to the quarks’ masses. In that case, the interaction with virtual heavy-quark pairs would be the leading contribution to the scattering cross-section, in contrast to the universal scenario for which these are negligible. The kinematics of these new channels in addition to the new dependence on the heavy sea-quarks for the production cross-section can lead to sizable effects on both limits. Thus, we cannot draw any general conclusions about their relative strengths for these scenarios.

**Conclusion.** In this study we have compared the bounds placed on effective theories of dark matter from mono-jet searches and contact interactions. We have shown that for models which dominantly interact at the LHC via a $q\bar{q}$ initial state and an $s$-channel mediator, the bounds from contact interactions are the most stringent in the regions of parameter space where the effective theory is valid and the couplings are perturbative. For these models, the contact interaction searches can probe mediator masses up to 7 TeV. For lower mediator masses, the limit from contact interactions is the most constraining as long as the mediator mass is above ~ 1 TeV. Due to the nature of the contact interaction being probed there is no dependence of the dark-matter mass on the analysis.

Additionally we have also commented on the applicability of these limits to other effective models. We believe that the conclusions presented will only depend weakly on the spin of the dark matter and/or the mediator. We again note that the limit is only valid for $s$-channel mediators but any other change to the interaction vertex will leave our conclusions unchanged. In the case that the mediator couples to leptons in the final state, the bounds from contact interactions may even be increased. However, we unfortunately find that for gluonic couplings no definitive statements can be made from a contact interaction analysis.

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