Application of free surface synthetic schlieren method in determining surface tension from a light floating object

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Abstract. Free surface synthetic schlieren (FS-SS) is a liquid surface topography visualization technique relying on apparent dot pattern displacement due to liquid surface deformation. It has gained attention in the fluid visualization field due to its relatively simple and inexpensive setup. One of the most powerful features of the FS-SS technique is the 3D topography of deformed liquid surface, in which submillimeter scale changes can be readily observed in great detail. The visualization of liquid meniscus at the solid interface is proposed in this study to calculate surface tension by equating the surface tension force with weight of light, floating object with circular geometry. It was found that surface tensions of liquids are much greater than anticipated, suggesting that the effect of buoyant force of displaced liquid due to deformed interface cannot be neglected. When the buoyant force is included, the calculated surface tension of liquids are within 10% of the reference values.

1. Introduction
Surface tension plays an important role in everyday phenomena ranging from water striders walking on water, droplet and bubble formations, capillary action in plant xylems, and floating of objects denser than liquid. Oftentimes, floating needles or double-sided razor blades are used in demonstrating the powerful effect of surface tension which is determined by balancing the surface tension force and the floating objects weight. In order to calculate the surface tension for a liquid, the inclination angle of the liquid surface has to be accurately measured. Until recently measuring the angle was not an easy task since liquid surface deformation created by small floating objects was only in the submillimeter scale. The recent development of the Free-Surface Synthetic Schlieren (FS-SS) technique for liquid surface topography visualization \cite{1} could be used to ease the surface curvature related measurement. The technique compares apparent images of random dot patterns between the flat and deformed liquid surfaces, creating a displacement field. The height of the deformed surface is reconstructed using computer programming. This technique provides extremely high surface height resolution when the deformed surface is not too steep. Owing to the FS-SS method’s relatively simple experimental setup and high accuracy surface topography, this work proposes applying the FS-SS technique to determine the surface inclination angle and thus surface tension via balancing surface tension and gravitational forces.
2. Theory

When placing a thin circular plate of radius $r$ on top of a liquid surface, the deformed liquid surface exerts a surface tension force of magnitude

$$F_\gamma = 2\pi r \gamma \sin \theta$$

on the plate, where $\gamma$ is a liquid surface tension and $\theta$ is the liquid surface inclination angle measured at a three-phase interface, as shown in figure 1. If the object is assumed to float without submerging into the liquid, the liquid surface tension can be determined by balancing the object’s weight and surface tension force.

$$mg = 2\pi r \gamma \sin \theta.$$  

![Figure 1](image)

Figure 1. The floating object on liquid surface. (a) A close-up image of circular floating object. (b) The surface tension force acting on a floating circular plate. The diagram is not drawn to scale.

3. Materials and methods

The FS-SS experiment on deformed liquid surface was conducted in a 40.0 cm wide and 60.0 cm length glass tray. The experiment was performed in a room with controlled temperature of 25 °C. The liquids used in the experiment were water and 0.1% (w/w) Tween-20 mixed with water. The properties of the two liquids are listed in table 1. For each experiment, the tray was filled to a depth of 1.00 cm. A random 0.4 mm diameter dot pattern with 50% dot density was placed underneath the tray. A digital camera (SONY RX10M3) with 3,840 x 2,160 pixels resolution placed above the tray was used in taking images of the apparent dot patterns. 0.16 mm thick circular cover slip of 18.00 mm diameter with mass 0.0905 g was used as floating object in creating liquid menisci along its perimeter. The random dot displacement field gradients were then generated using the digital image correlation (DIC) algorithm from MATLAB PIVlab toolbox [2] and the liquid surface topographies of the displacement field gradients were reconstructed using the least-squares integration algorithm from MATLAB PIVmat toolbox [1]. The surface inclination angles were later determined from the menisci cross-section profiles using MATLAB programming.

| Liquid            | Density (kg/m$^3$) | Refractive index | Surface tension (N/m) |
|-------------------|--------------------|------------------|-----------------------|
| Water             | 997                | 1.33             | 0.072                 |
| 0.10% Tween-20    | 1,042              | 1.33             | 0.035                 |
4. Result and discussion
The FS-SS process flow in reconstructing a deformed liquid surface is summarized in figure 2.

![Figure 2](image1.png)

**Figure 2.** The FS-SS process. (a) Flat and (b) deformed liquid surface images are used in generating a random dot pattern displacement gradient field indicated as green arrows in (b). The liquid surface height is reconstructed by integrating the gradient field, resulting in (c) a 3D generated deformed liquid surface topography. (d) The meniscus profile is generated by dissecting the surface topography in (c) through a plane perpendicular to the floating objects diameter.

![Figure 3](image2.png)

**Figure 3.** Magnified meniscus profile when the 18.00 mm circular cover slip was placed on the surface of (a) water and (b) 0.1% (w/w) Tween-20 mixed with water.

When the circular cover slip is placed on the surface of the liquid with different surface tensions, variations of surface deformation is observed, as shown in figure 3. Since the surface tension is reduced, the inclination angle has to be increased to compensate. The measured inclination angles and surface tensions of the two liquids are listed in table 2. By balancing the floating cover slip’s weight with the surface tension force, the surface tension obtained is 400% and 557% grater than the standard values for water and 0.10% Tween-20 mixed with water solution respectively. Oversimplifying the floating phenomenon by neglecting the buoyant force generated by displaced liquid volume due to deformed liquid surface results in vast difference in calculated and real surface tension values. The buoyant force is composed of two components. A force due to hydrostatic pressure acts on surface of floating object and is equilavent to a weight of liquid occupied a cylindrical volume of floating object surface area and height of meniscus depth. A surface tension force acts on curved meniscus interface and is equivalent to a weight of liquid occupying the volume outside of the
cylinder. By balancing weight of liquid in displaced volume outside the cylinder with the surface tension force, the liquid surface tension can be determined, as shown in table 2. The resultant surface tensions when include buoyant force into consideration indicates the extreme importance of the buoyant force as another force balancing the floating object’s weight, as the calculated surface tensions now are 10% percent off of the reference values.

![Diagram](image)

**Figure 4.** A 2D representation of displaced liquid volume composed of volume of column above the floating object and the volume outside the column.

| Liquid       | Surface inclination angle (°) | Depth of meniscus (mm) | Surface tension (N/m) | Weight | Weight+buoyancy |
|--------------|-------------------------------|------------------------|-----------------------|--------|-----------------|
| Water        | 5.36                          | 0.229                  | 0.367                 | 0.10%  | 0.069           |
| 0.10% Tween-20 | 7.56                          | 0.252                  | 0.233                 | 0.031  |                 |

5. Conclusion
The FS-SS technique can be used in visualizing deformed liquid surface topographies in order to find liquid surface inclination angles. However, determining liquid surface tension by simply equating the floating object’s weight with surface tension force is insufficient in achieving the accurate surface tension. The findings suggest oversimplification of physics exercise in surface tension determination could lead to large deviations between actual and derived surface tension values. In order to increase accuracy of surface tension calculations, buoyancy of displaced liquid volume due to deformed surface by floating objects must be included.

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