Bit Plane Slicing, Wavelet and Polynomials Mixing for Image Compression

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Abstract
This paper introduced a hybrid technique for lossless image compression of natural and medical images; it is based on integrating the bit plane slicing and Wavelet transform along with a mixed polynomial of linear and non-linear base. The experiments showed high compression performance with fully grunted reconstruction.

Keywords: Bit Plane Slicing, Wavelet Transform, Lossless Image Compression, Polynomial Prediction.

ضغط الصور باستخدام طريقة تشريحة الحدود worksheets التجميع و خليط متعدد الحدود

رنا طالب التميمي
قسم العلوم المالية والمصرفية، كلية الإدارة والاقتصاد، الجامعة المستنصرية، العراق

الخلاصة
في هذا البحث تم تقديم طريقة هجينة لضغط الصور للأغراض العملية والتقنية بدون خسارة. حيث اعتمدت الطرق على الجمع بين تشريحة الحدود والتحويل الموعي، ومزيج من التوابع الخطي واللاخطي متعدد الحدود. أظهرت النتائج نسبة ضغط عالية مع استجابة مطاربة بالأداء والصور الأصلية.

Introduction
Lossless compression of images is characterized by maintaining the quality of an image and therefore, the reconstructed image is the same as the original, but the compression ratio has a small values recorded as less than 10 [1]. This is because of focusing on the coding redundancy and/or inter pixel redundancy.

Researchers in recent years exploit ways to increase the lossless compression system’s efficiency either by designing a technique that selects certain blocks and ignores the others or by merging several techniques such as the prediction and wavelet [2-6].

By applying the wavelet transform, the best compression ratio will be attained compared to the spatial domain [7] where the image is decomposed into four sub-bands (LL, HL, LH, and HH sub bands images). This process may be applied several times on the LL sub band image. The approximation sub band (LL) contains all image information; thus, it is considered as the most important part, while the other sub-bands are considered less significant as they contain very little amount of image information [8].

Bit Plane Slicing (BPS) was adopted as a technique combined with the well-known image compression methods by several researchers [9-11]; it is a separation technique used to generate eight binary images (eight layers: layer0 - layer7) according to bit position, where Low Order Layers (LOLs) represented by layers0-layer3 have small significant image details and therefore is ignored, while the High Order Layers (HOLs) (layer4-layer7) contain the most significant image details [12].

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This paper introduces a technique to compress gray images based on determining the wavelet transform, along with a mixed polynomial of first and second order representation, that effectively improved the compression ratio.

**Materials and Methods**

The main concerns taken in the suggested method are:

Keeping the high order layers and neglecting the low order layers of bit plane slicing effectively reduces the number of bits from 8 to 4 bits, which means an implicit reduction in image information.

Integrating the Wavelet Transform with the mixed polynomial coding of linear and non-linear approximation model was achieved to compress the image. Figure-1 clearly illustrates the layout of the proposed method.

The proposed hybrid compression system implementation is described in details below.

1. The input: gray scale image (I) of size n×n.

2- Bit-Plane Slicing (BPS) was used to produce eight binary bit plane images form image I, each plane was of size n×n. The higher top order bit planes (4-7) of significant major image were used and the lower order bit planes (0-3) were discarded.

3- The wavelet transform was exploited to produce four quadrants sub bands (LL and detail sub bands LH, HL and HH) each of size n/2×n/2; then we applied the polynomial approximation model on the LL sub-band, such that:

For bit slicing 7 non-linear polynomial prediction was applied on approximation of the sub band LL7

Otherwise for bit planes 4,5,6 linear polynomial prediction was used on approximation of the sub bands LL4,LL5 and LL6.

4- for bit slicing 7:

- the approximated Image $LL^7$ was determined using equation (15).
- the error (residual) was found using equation (16).

5- for bits slicing 4,5,6:

- the approximated Images $LL^4$, $LL^5$ and $LL^6$ were determined using equation (20).
- the errors (residual) $LL_4$ Re $sd$, $LL_5$ Re $sd$ and $LL_6$ Re $sd$ were found using equation (21).

**non-linear** polynomial prediction [13,14] was applied using the following equations:

\[ a_1 = \frac{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} j \times (j - x_c)^2}{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} (j - x_c)^2} \] (1)

\[ a_2 = \frac{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} LL_7(i, j) \times (i - y_c)^2}{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} (i - y_c)^2} \] (2)

\[ a_3 = \frac{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} LL_7(i, j) \times (j - x_c)(i - y_c)}{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} (j - x_c)^2 \times (i - y_c)^2} \] (3)

\[ x_c = \frac{r - 1}{2} \] (4)

\[ y_c = \frac{n/2 \times n/2}{j=0} \] (5)

\[ w_2 = \frac{\sum_{j=0}^{n-1/2} \times (x_c)^2 - \sum_{i=0}^{n-1/2} (i - y_c)^2}{(i - y_c)^2} \] (6)
\[ W_3 = \sum_{j=0}^{n-1/2} (i - yc)^4 - \sum_{i=0}^{n-1/2} (i - yc)^4 \]  

(7)

\[ W_4 = \sum_{i=u}^{n-1/2} \sum_{j=0}^{1/2} (j - xc)^2 (i - yc)^2 \]  

(8)

\[ V_1 = a_0 W_1 + a_3 W_2 + a_4 W_3 \]  

(9)

\[ V_2 = a_0 W_2 + a_3 W_3 + a_4 W_4 \]  

(10)

\[ V_3 = a_0 W_2 + a_3 W_4 + a_4 W_3 \]  

(11)

\[ \begin{array}{ccc}
V_1 & W_1 & W_2 \\
V_2 & W_3 & W_4 \\
V_3 & W_4 & W_3 \\
\end{array} \]  

(12)

\[ \begin{array}{ccc}
W_1 & V_1 & W_2 \\
W_2 & V_1 & W_3 \\
W_3 & V_2 & W_4 \\
\end{array} \]  

(13)

\[ \begin{array}{ccc}
W_1 & W_2 & V_2 \\
W_2 & W_3 & V_3 \\
W_3 & W_4 & V_4 \\
\end{array} \]  

(14)

\[ L_7 = a_0 W_1 + a_1 (j - xc) + a_2 (i - yc) + a_3 (j - xc)^2 + a_4 (i - yc)^2 + a_5 (j - xc)(i - yc) \]  

(15)
Figure 1-Compression and decompression System Structure

$$\text{LL7 Resd} = \text{L7} - \text{LL7}$$  \hfill (16)
The linear prediction coefficients were adopted [14, 15] using the following equations:

\[ a_0 = \frac{1}{n/2 \times n/2} \sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} LL(i, j) \]  

(17)

\[ a_1 = \frac{n^{-1/2} \sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} LL(i, j) \times (j - x_c)}{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} (j - x_c)^2} \]  

(18)

\[ a_2 = \frac{n^{-1/2} \sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} LL(i, j) \times (i - y_c)}{\sum_{i=0}^{n-1/2} \sum_{j=0}^{n-1/2} (i - y_c)^2} \]  

(19)

\[ LL = a_0 + a_1(j - x_c) + a_2(i - y_c) \]  

(20)

6-The residual [15, 16, 17] was calculated as follows:

\[ LL_{Re sd} = L - LL \]  

(21)

7-To reconstruct the original image:

A- LL quadrant 4, 5, 6 & 7 were reconstructed by adding the residual to the predicted data:

\[ LL = LL_{Re sd} + LL \]  

(22)

B-the bit plane was reconstructed by using the inverse wavelet transforms.

**Results and Discussion**

As a lossless compression, the performance measures were based on determining the Compression Ratio (CR) as in equation (23).

\[ \text{Compression Ratio (CR)} = \frac{\text{Size of Original Image}}{\text{Size of Compressed Information}} \]  

(23)

Two sets of images (natural and medical) were tested to report the performance of the proposed hybrid system (Figure-2). All tested images were of 256 gray levels (8bits/pixel) and of the size 256x256. Figure-3 illustrates the output images after the decompression process, while Figure 4 shows the eight layers, bit planes slicing for the tested input images.

![Figure 2](image1)

**Figure 2**: The Tested Grayscale Images, (a) Pepper (b) Rose (c) Brain and (d) Knee

![Figure 3](image2)

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Figure 3 - The Output Images after the Decompression (a) Pepper (b) Rose (c) Brain and (d) Knee

Figure 4 - Bit plane slicing of Tested Images from Layer 0 to Layer 7.

Table 1 illustrates the compression performance of the suggested hybrid compression technique using two blocks sizes (4*4 & 8*8).

Tables (2 and 3) show the compression performance of two compression techniques, the traditional (nonlinear and linear, respectively).

The suggested hybrid lossless system was fast and simple. It was also obvious from the test results reported in Table 1 that the compression ratio was improved as compared with the results of the other methods (listed in Tables (2 and 3)).

| Test image | original image size (in bytes) | Block size 4*4 | Block size 8*8 |
|------------|-------------------------------|----------------|----------------|
| Pepper     | 65536                         | 5878           | 11.1494        |
|            |                               |                |                |
| Rose       | 65536                         | 5624           | 11.6529        |
|            |                               |                |                |
| Brain      | 65536                         | 5610           | 11.6820        |
|            |                               |                |                |
| knee       | 65536                         | 6824           | 9.6038         |
|            |                               |                |                |
Table 2-Performance of non linear Traditional Compression System

| Test image | original image size (in bytes) | compressed information size (in bytes) | Compression Ratio | compressed information size (in bytes) | Compression Ratio |
|------------|-------------------------------|----------------------------------------|------------------|----------------------------------------|------------------|
| Pepper     | 65536                         | 7738                                   | 8.4694           | 7110                                   | 9.2174           |
| Rose       | 65536                         | 7100                                   | 9.2304           | 5968                                   | 10.9812          |
| Brain      | 65536                         | 7324                                   | 8.9481           | 7310                                   | 8.9653           |
| knee       | 65536                         | 7234                                   | 9.0594           | 6320                                   | 10.3696          |

Table 3-Performance of Linear Traditional Compression System

| Test image | image size (in bytes) | compressed information size (in bytes) | Compression Ratio | compressed information size (in bytes) | Compression Ratio |
|------------|-----------------------|----------------------------------------|------------------|----------------------------------------|------------------|
| Pepper     | 65536                 | 8018                                   | 8.1736           | 77730                                  | 8.4781           |
| Rose       | 65536                 | 7312                                   | 8.9628           | 6434                                   | 10.1859          |
| Brain      | 65536                 | 8528                                   | 7.6848           | 8024                                   | 8.1675           |
| knee       | 65536                 | 6952                                   | 9.4269           | 6436                                   | 10.1827          |

Conclusions
The suggested hybrid technique is characterized by simplicity and efficiency. Combining the bit plane slicing with wavelet transforms leads to saving more storage space and hence affects the compression ratio due to preserving layers (4-7) and ignoring layers (0-3) from the bit plane, ultimately reducing the number of bits from 8 to 4 which implicitly means a reduction in image information. Finally, the compression ratio depended on the approximation of sub band LL block size, such that when the size gets bigger, fewer coefficients are needed, and this will implicitly improves the compression ratio.

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