We calculate the triviality bound on the Higgs mass in scalar field theory models whose global symmetry group $SU(2)_L \times SU(2)_{\text{custodial}} \approx O(4)$ has been replaced by $O(N)$ and $N$ has been taken to infinity. Limits on observable cutoff effects at four percent in several regularized models with tunable couplings in the bare action yield triviality bounds displaying a large degree of universality. Extrapolating from $N = \infty$ to $N = 4$ we conservatively estimate that a Higgs particle with mass up to $0.750 \text{ TeV}$ and width up to $0.290 \text{ TeV}$ is realizable without large cutoff effects, indicating that strong scalar self interactions in the standard model are not ruled out. We also present preliminary numerical results of the physical $N = 4$ case for the $F_4$ lattice that are in agreement with the large $N$ expectations.

1. Large $N$ analysis

We examine the regularization scheme dependence of the triviality bound on the Higgs mass in an $O(N)$ symmetric scalar field theory leading order in $1/N$[1]. Our purpose in doing this is two-fold: we wish to investigate the issue by explicit, analytical calculations and we need the results both directly and indirectly to complement numerical work at $N = 4$. The results are needed directly for estimating cutoff effects on physical observables that are not accessible by Monte Carlo and indirectly for guiding our search in the space of lattice actions to the region where heavier Higgs particles are possible.

A preliminary account of some of our results has been presented in [2]. Our general reasons for suspecting that the present numbers for the bound are too low in the context of the minimal standard model have been explained before [2,3] and will not be repeated here.

The basic logic of our approach [2] is that all possible leading cutoff effects can be induced by adding only a few higher dimensional operators with adjustable coefficients to the standard $\lambda \Phi^4$ action. We find that the highest Higgs masses are obtained in the nonlinear limit (infinite $\lambda$ limit). This leads us to consider nonlinear actions with the leading cutoff effects induced by four derivative terms with tunable couplings.

We use one class of continuum regularization schemes, of Pauli Villars type (PV), and three kinds of lattice regularizations: $F_4$, Hypercubic (HC), and Symanzik Improved Hypercubic (SI). For these regularization schemes the action, when expanded for slowly varying fields to order momentum to the fourth power, is of the form

$$S_c = \int \left[ \frac{1}{2} (\partial^2 \phi^2 + 2b_0 \partial^4 \phi^2) - \frac{b_1}{2N} (\partial_\mu \phi \cdot \partial_\mu \phi)^2 - \frac{b_2}{2N} (\partial_\mu \phi \cdot \partial_\nu \phi - \frac{1}{4} \delta_{\mu,\nu} \partial_\sigma \phi \cdot \partial_\sigma \phi)^2 \right],$$

where $\phi^2 = N \beta$. There are four control parameters in this action but one is redundant since to this order it can be absorbed into the other parameters by a field redefinition. We choose to absorb the parameter $b_0$ into $b_1$ and $b_2$. However, eliminating the dependence in $b_0$ to order momentum to the fourth power in the bare action does not necessarily imply that the dependence in $b_0$ has been eliminated to leading order in the inverse cutoff in the physical observables. The vacuum fluctuations are “aware” of the full bare action and will carry that information to the non

*Speakers: P. Vranas on large $N$ analysis and M. Klomfass on numerical analysis.
universal part of the physical observables. Still, the remaining effect of $b_0$ is probably small and, as far as the value of the bound is concerned, one may be able to cover the whole range of leading cutoff effects by varying $b_1$ and $b_2$ only. As we will see, our results indicate that this is realized to a good extend, leading to an approximate universality of the Higgs mass triviality bound.

For PV we simply set $b_0 = 0$, and for the lattice regularizations $b_0$ is set to the naïve value obtained in the expansion of the lattice kinetic energy term. We calculate the phase diagram for each regularization and find a second order line that ends at a tricritical point where a first order line begins. We study the physically interesting region close to the second order line in the broken phase. The parameter $\beta$ corresponds to the relevant direction and is traded, as usual, for the pion decay constant $f_\pi$. The parameters $b_1$ and $b_2$ control the size of the leading order cutoff effects for a given value of $f_\pi$. Our analysis shows that, to this order, the quantities we consider do not depend on the parameter $b_2$. Therefore, a very simple situation emerges with the scale set by $f_\pi$ and the leading cutoff effects parametrized by only one parameter $b_1$. This simplification at infinite $N$ makes it easy to relate very different regularization schemes, and leads to a reasonably “universal” bound on the renormalized charge $g$ and therefore on the Higgs mass.

We calculate the leading cutoff effects on two physical quantities, namely the width of the Higgs particle and the 90 degrees $\pi - \pi$ scattering cross section. We find that they are given by the product of a “universal” factor (identical for all regularizations considered) which only depends on $g$ and the dimensionless external momenta, and a non–universal factor that depends on the parameter $b_1$ but not on $g$ or the momenta. The universal factor associated with the width is different from the one associated with the cross section, but the non–universal factors are identical for a fixed regularization scheme. These properties suggest that, to leading order in $1/N$, all cutoff effects in on–shell dimensionless physical quantities (viewed as functions of dimensionless momenta) are given by an effective renormalized action,

$$S_{\text{eff}} = S_R + c \exp \left( \frac{-96\pi^2}{g} |\mathcal{O}| \right),$$

where $\mathcal{O}$ is a renormalized operator, $c$ is a $g$ independent free parameter containing all the non–universal information (i.e. a function of $b_1$ only), and $S_R$ is describing the usual universal part of physical observables with the unit of energy set by $f_\pi = 1$. In the general RG framework this representation of $S_{\text{eff}}$ is not unreasonable if one accepts that at $N = \infty$ the number of independent operators that contribute to observables at order $1/\Lambda^2$ decreases by one relatively to $N < \infty$. Because of the nonlinear relationship between $c$ and the parameters in the bare action, the range in which $c$ is allowed to vary depends on the type of action chosen. Different actions that realize the same $c$ are indistinguishable to order $1/\Lambda^2$; all actions have large regions of overlap but the triviality bounds are obtained from the area near the edges of the ranges in which $c$ varies and therefore are somewhat dependent on the particular regularization scheme. Still, this dependence turns out to be weaker than what one would have been inclined to believe on the basis of the few simulations that had no continuous tuning ability for $c$ built in. Thus, some of the stronger “universality” evident in eq. [4] also makes its way into the triviality bound.

There is an approximate physical picture associated with the models we studied. When the regularized model is nonlinear one has to think about the Higgs resonance as a loose bound state of two pions in an $I = 0$, $J = 0$ state. Pions in such a state attract because superposing the field configurations corresponding to individual pions makes the state look more like the vacuum and hence lowers the energy. The four derivative term in the action can add or subtract to this attraction. We found that the smallest cutoff effects are obtained when the coupling $b_1$ of the four derivative term is set so that the term induces the maximal possible repulsion between the pions, postponing the appearance of the Higgs resonance to higher energies.

Our findings concerning the Higgs mass triviality bound are best summarized in figure 1 for
Figure 1. Leading order cutoff effects in the invariant $\pi - \pi$ scattering amplitude at 90° for the naïve actions (lines on the left) and for the actions with a four derivative term turned on to maximal allowed strength (lines on the right). The dotted line represents center of mass energies $W = 2M_H$, the dashed line $W = 3M_H$ and the solid line $W = 4M_H$. The PV case here involves a $(\partial^2)^3$ term.

The four different regularization schemes considered. There the leading cutoff effects in the cross section are plotted versus the Higgs mass in $TeV$ with the large $N$ results presented for $N = 4$. For all regularization schemes we extract the bound by restricting the allowed cutoff effects in the cross section to 4%, at center of mass energies up to twice the Higgs mass. From this figure it can be seen that turning on the four derivative term to maximal allowed strength (maximum repulsion between the pions) lowers the leading cutoff effects and results to an approximately universal bound. Similar results are obtained for the cutoff effects in the width. However these effects are very small (less than $\approx 1\%$) and we therefore chose to extract the Higgs mass bound using the more stringent cutoff effects in the cross section.

Figure 2. Leading order cutoff effects in the invariant 90° $\pi - \pi$ scattering amplitude vs. $\beta_2$ ($\beta_2$ is proportional to $b_1$) for three values of $M_H$ on the $F_4$ lattice. The center of mass energy is set at $W = 2M_H$.

To see more clearly how the bound changes with the action we show an example in figure 2 for the $F_4$ lattice (the parameter $\beta_2$ in the figure is proportional to $b_1$). Identical cutoff effects, of 4%, will be found at $N = \infty$ for a Higgs mass of 0.720 $TeV$ with the simplest action ($\beta_2 = 0$), and for a Higgs mass of 0.815 $TeV$ with an action that has the maximal amount of four field interaction allowed ($\beta_2 = -\beta_{2,t.c.}$). It is important to understand that this difference is substantial when the width is considered. In figure 3 we show the regularization independent part of the width as a function of the Higgs mass (note that the leading order weak coupling approximation severely underestimates the width when the Higgs mass becomes large). In our example the width went from 0.320 $TeV$ to about 0.500 $TeV$ and the heav-
ier Higgs is definitely strongly interacting. There is no doubt therefore that, at least at infinite $N$, stopping the search for the Higgs mass bound at the study of the simplest possible actions would have been misleading.

To make predictions for the physically relevant case $N = 4$ we use the known differences between the $N = \infty$ and the $N = 4$ numerical results, for the simplest lattice actions, to extrapolate to the actions with four derivative terms that have not yet been studied numerically. This extrapolation is sensible because the expansion in $1/N$ is expected to be “well behaved” in the region where the triviality bound is obtained. We find that the beta function to one loop is proportional to $1 + 8/N$ and thus the $N = 4$ answer is larger by a factor of 3 than the $N = \infty$ answer. However, this would make the $N = \infty$ predictions unreliable only for quantities calculated at energies much lower than the cutoff. The calculation of the Higgs mass bound involves processes at energies close to the cutoff and therefore there are not two very different scales to be connected and the beta function is not needed.

Based on our large $N$ results and using these extrapolations, it seems that a more realistic and not overly conservative estimate for the Higgs mass triviality bound is it is sometimes stated. At 0.750 TeV the Higgs particle is expected to have a width of about 0.290 TeV (after subtracting a possible overestimation of about 25%) and is therefore quite strongly interacting. Such a heavy and broad Higgs may be hard to detect experimentally.

2. Numerical analysis of the $N = 4$ case

The physical $N = 4$ case has to be studied numerically. We choose to perform the simulation on the $F_4$ lattice because it does not introduce lattice artifacts to the order we are interested in (order $1/\Lambda^2$ where $\Lambda$ is the cutoff) [7–9]. We use a single cluster Wolff type algorithm, and we measure the Higgs mass from the decay of the time slice correlation function of the zero spatial momentum Higgs field. In figure 4 we present our large $N$ and numerical results for the action with the four derivative term turned on to maximum allowed strength. For comparison we have included in the same figure the large $N$ and numerical [9] results for the naive action. It can be seen from this figure that, for a given value of the Higgs mass in lattice units ($m_H$), the Higgs mass in units of $f_\pi = 0.246$ TeV increases approximately in agreement with the large $N$ prediction when the four derivative term is turned on. This supports the conclusions of the previous section.

For the $16^4$ lattice the largest Higgs mass in lattice units is $\approx 0.70$ and it is therefore slightly larger than the energy of the lowest free two pion $I = 0, J = 0$ state which is $\approx 0.68$. It is therefore possible that the lowest lying two pion $I = 0, J = 0$ state may “mix” with the Higgs particle state. In that event the extraction of the Higgs mass from the time slice correlation function may...
Figure 4. The Higgs mass in units of $f_\pi$ versus the Higgs mass in lattice units. The solid line is the large $N$ result and the diamonds the numerical result for the naive action. The dotted line is the large $N$ result and the squares the numerical result for the action with the four derivative term turned on to maximum allowed strength.

To explore this problem we look at the low lying spectrum in that channel using the technique of reference [10]. The results, with the couplings set so that the largest Higgs mass with the smallest possible cutoff effects is obtained, are shown in figure 5. The x-axis is the lattice size and the y-axis is the energy in lattice units of the various states. The dotted lines describe the low lying free two pion $I = 0, J = 0$ states. The diamonds are the numerical values for the two pion $I = 0, J = 0$ states and the squares are the numerical values for the Higgs particle state. The stars are the numerical values for the Higgs particle state obtained by simply using the decay of the the time slice correlation function of the zero spatial momentum Higgs field. As can be seen, the stars and the boxes are the same, within errors, indicating that for our choice of operators the “mixing” is very weak. Therefore, the Higgs mass extracted from the decay of the the time slice correlation function of the zero spatial momentum Higgs field is reliable and the results in figure 4 can be trusted.

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