Efficiency of different methods to predict settlement of shallow foundation in cohesionless soil

Zainab R Mohammed¹, Husain A Abdul-Husain¹
¹Department of Civil Engineering, Faculty of Engineering, University of Kufa, Iraq
E-mail: husainali2002@gmail.com; alyaszainab@yahoo.com

Abstract. During the last decades, many methods have been developed to predict the settlement of shallow foundations on cohesionless soils based on the results of SPT. However, such prediction methods with the required degree of accuracy and consistency have not yet been developed. The current study is an attempt to evaluate the performance of some of these methods and obtain the most efficient one. This goal has been performed by evaluating and ranking eight common methods. These methods are Terzaghi and peck, Meyerhof 1, Bowles (1986), Parry, Schultze and Sherif, Meyerhof 2, Burland and Burbidge and Anagnostopoulos et al.. A total number of 189 cases published in the literature were adopted to carry out the ranking analysis of the selected methods. The concept of rank analysis depends in general on four approaches. These approaches are Nash-Sutcliffe efficiency, arithmetic average & standard deviation, accumulative probability analysis, and Log-Normal distribution of the ratio between the predicted and measured settlement. The results of the analysis revealed that the Anagnostopoulos et al. method have got the first rank with a rank index of 6 while Terzaghi and peck have achieved the last rank with rank index of 27. Therefore, Anagnostopoulos et al. method can be used to estimate the settlement of shallow foundations based on the results of SPT with high confidence.

Keywords: Cohesionless soil, Shallow foundation, Settlement, Rank index

1. Introduction

The design of any foundation should meet two main criteria. The first one is that the supporting soil should be safe against shear failure due to the imposed pressure and the second is that the resulting settlement within the permissible limits. For cohesionless soil, settlement usually controls the design process rather than bearing capacity, especially when the foundation's width exceeds 1.2 meters [1]. Determination of settlement of shallow foundations on cohesionless soil is highly complex; due to the various uncertainties associated with the stress-strain history, applied stress distribution, and soil's compressibility. Moreover, many difficulties involved in sampling and determining the in-situ properties of cohesionless soil deposits [2]. During the last decades, several methods have been developed to predict elastic settlement in cohesionless soil.

Yet consistent success remains elusive. Accurate settlement estimation of shallow foundation relies on the accurate assessment of the deformation modulus of in situ cohesionless soil, which is almost impossible due to the high cost. As a result, most of these methods depend on in situ tests, such as standard penetration test (SPT), cone penetration test (CPT), dilatometer test (DMT), etc. [3].

Many comparative studies have been conducted to check the empirical method's capability to estimate the foundation's settlement and select the best universalize method to use in practice. The current study aims to assign the most efficient method of estimating settlement for foundation supporting by cohesionless soil based on the SPT test results.
2. Data Base

The data analyzed during this study are obtained from various published literature. It includes information concerning soil and foundation and the corresponding field measurements of the settlement for shallow foundations. These data provide a wide range of variations in soil types, soil properties, and footing dimensions. With a good range of variable values, the accuracy of the results will increase. The database comprises a complete of (189) individual field measurement cases from the literature as represented by Shahin [4]. Table 1 explains the references of the used database and the basic statistical information about the database is presented in Table 2.

Table 1. The sources of the collected database after Shahin [4].

| Reference                        | No of cases |
|----------------------------------|-------------|
| Bazaraa (1967)                   | 5           |
| Burbidge (1982)                  | 22          |
| Burland and Burbidge (1985)     | 125         |
| Wahls (1997)                     | 30          |
| Maugeri et al. (1998)            | 2           |
| Picornell and Del Monte (1998)   | 1           |
| Briaud and Gibbens (1999)        | 4           |

Table 2. Basic statistical information of the database.

| Model variables                  | Mean  | Standard Deviation | Minimum | Maximum |
|----------------------------------|-------|--------------------|---------|---------|
| Footing width, \( B_{(m)} \)     | 8.8   | 10.1               | 0.8     | 60.0    |
| Footing net applied load, \( q_{net} \) (kPa) | 187.1 | 123.3              | 18.3    | 697.0   |
| Footing geometry, L/B            | 2.2   | 1.8                | 1       | 10.5    |
| Average SPT blow count, N        | 24.6  | 13.5               | 4.0     | 60.0    |
| Footing embedment ratio, \( D/B \) | 0.53  | 0.58               | 0.0     | 3.4     |
| The depth of water table, \( H_w \) (m) | 13.57 | 27.84              | 0       | 103     |
| The compressed layer thickness, \( H_s/B \) | 1.83  | 0.37               | 0.25    | 2       |
| Measured settlement, \( S_m \) (mm) | 20.4  | 26.6               | 0.6     | 121.0   |
3. Traditional methods for Settlement Prediction

Many traditional formulas for settlement prediction of shallow foundations on cohesionless soils are developed in literature. Among these, eight empirical formulas adopted in the present study as shown in Table.3. These formulas are depending on the recorded data from the standard penetration test (SPT). The reasons for selecting these formulas as they are commonly used and the collected database included all the parameters that allow for the prediction of settlement by these formulas. These methods are Terzaghi and peck [5], Meyerhof [6], Bowles [7], Parry [8], Schultze and Sherif [9], Meyerhof [10], Burland and Burbidge [11] and Anagnostopoulos et al. [12].

Table 3. Adopted settlement methods.

| Methods                  | Settlement Expression |
|--------------------------|-----------------------|
| Terzaghi and Peck [5]    | $Sp(mm) = \frac{3q}{N} \left[ \frac{B}{B + 0.3} \right]^2 Cw Cd$ |
|                          | $C_w = \begin{cases} 2 & H_w \leq 0 \\ 1 & H_w \geq 2B \\ 2 - \frac{H_w}{2B} & \text{otherwise} \end{cases}$ |
|                          | $Cd = 1 - 0.25 \frac{Df}{B}$ |
|                          | $N' = 15 + 0.5(N - 15)$ for saturated very dense, fine or silty sand. |
| Meyerhof [6]             | $Sp = 1.25 \frac{q}{N} C_d$ for $B \leq 1.25$ m |
|                          | $Sp = 2 \frac{q}{N} \left( \frac{B}{B + 0.3} \right)^2 C_d$ for $B > 1.25$ m |
| Bowles [7]               | $Sp = 1.25 \frac{q}{N} C_w$ for $B \leq 1.25$ m |
|                          | $Sp = 2 \frac{q}{N} \left( \frac{B}{B + 0.3} \right)^2 \frac{C_w}{C_d}$ for $B > 1.25$ m |
|                          | $C_d = 1 + 0.33 \frac{D}{B} \leq 1.33$ |
|                          | $C_w = 2 - \frac{H_w}{B + D} \leq 2$ |
Parry [8]

\[ Sp = a \left( \frac{q \cdot B}{N} \right) Cd \cdot Cw \cdot Ct \]

\[ C_w = \begin{cases} 
1 + \frac{H_n}{D + 0.75B} & 0 < H_n < D \\
1 + \frac{H_n(2B + D - H_n)}{2B(D + 0.75B)} & D < H_n < 2B
\end{cases} \]

\[ a: \text{constant} = 200 \text{ in SI units.} \]
\[ q: \text{net applied load (MPa).} \]
\[ Cd: \text{calculated from Figure.1} \]
\[ Ct: \text{calculated from Figure.2} \]

\[ Sp = \left( \frac{q}{N^{0.87}} \right) \frac{C}{C_d} \]
\[ C_d = 1 + 0.4 \frac{D}{B} \]

Schultze and Sherif [9]

\[ C: \text{calculated from Figure.3 and Table.4} \]

\[ Sp = 0.825 \left( \frac{q}{N} \right) \sqrt{B \cdot C_d} \]

Meyerhof [10]

\[ Sp = 1.65 \left( \frac{q}{N} \right) \sqrt{B \cdot C_d} \text{ for submerged fine or silty sand} \]

\[ Sp = \begin{cases} 
qB^{0.7}I_cC_tC_s & \text{for N.C} \\
qB^{0.7} \frac{L}{3}C_tC_s & \text{for O.C and } q \leq \sigma_c^e \\
q - \frac{2}{3}\sigma_c^e B^{0.7}I_cC_tC_s & \text{for O.C and } q > \sigma_c^e
\end{cases} \]

Burland and Burbidge [11]

\[ I_c = \frac{1.71}{N_{1.4}} \quad C_s = \frac{1.25L/B}{\left(0.25 + L/B\right)^2} \]

\[ C_t = \begin{cases} 
2 \left( \frac{T}{Z_i} \right) \left( \frac{T}{Z_i} \right) & T < Z_i \\
1 & \text{otherwise}
\end{cases} \]

\[ Z_i = \text{B}^{0.75} \text{ in the case of SPT-values} \]
\[ \text{increased or constant with depth and } 2B \]
\[ \text{in other cases.} \]
Anagnostopoulos et al. [12]  

\[ Sp = 2.37 \left( \frac{q^{0.87}}{N^{1.2}} \right) B^{0.7} \]

\[ B \]: is the foundation width in (m).
\[ C \]: is the correction factor due to foundation shape (L/B) and the depth to the rigid layer (T/B) in (mm/kPa).
\[ C_D \]: is the correction factor for embedment depth.
\[ C_r \]: correction factor of the compressible layer thickness
\[ C_w \]: is the correction factor for the groundwater level.
\[ D_f \]: is the foundation's depth within the soil.
\[ H_w \]: is the groundwater level from the foundation level.
\[ L \]: is the foundation length.
\[ N.C \]: is normal consolidation ratio.
\[ N \]: is the measured SPT-value.
\[ O.C \]: is over consolidation ratio.
\[ q \]: is the applied pressure in (kN/m²).
\[ S_p \]: is the foundation settlement (mm)
\[ T \]: is the thickness of the compressible layer from the foundation level.
\[ \sigma \]: is total overburden pressure.
\[ \sigma' \]: is effective overburden pressure.
\[ \sigma_c \]: is preconsolidation stress.
\[ Z_I \]: is the influence zone

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**Figure 1.** Excavation correction factor, C_d (After Parry [16]).

**Figure 2.** Correction factor for the thickness of the compressible layer, C_t (After Parry [16]).
Figure 3. Reduction factor (C) for T/B ≥ 2 (after Blake [17]).

Table 4. Reduction factor (C) for T/B < 2 (after Blake [17]).

| L/B | 1   | 2   | 5   | 100 |
|-----|-----|-----|-----|-----|
| T/B |     |     |     |     |
| 1.5 | 0.91| 0.89| 0.87| 0.85|
| 1.0 | 0.76| 0.72| 0.69| 0.65|
| 0.5 | 0.52| 0.48| 0.43| 0.39|

4. Methods Evaluation

For compare the performance of settlement methods' prediction and evaluate their accuracy and efficiency, rank index (RI) will be utilized, as shown in Equation 1 [13].

\[
RI = R_1 + R_2 + R_3 + R_4
\]

Where:

- \( R_1 \): is the method’s rank based on statistical analysis using the Nash-Sutcliffe efficiency index (NSE).
- \( R_2 \): is the method’s rank based on statistical analysis based on the arithmetic average (\( \bar{x} \)) and standard deviation (SD).
- \( R_3 \): is the method’s rank based on accumulative probability analysis, and
- \( R_4 \): is the method’s rank based on the Log-Normal probability distribution.
4.1 First Approach

In this approach (R₁) Nash–Sutcliffe efficiency (NSE) is utilized. Nash–Sutcliffe efficiency index (NSE) is a widely used and potentially reliable statistic for assessing the goodness of models' fit. It indicates how well the observed versus predicted data plot fits the 1:1 line. It is ranging between (−∞ and 1.0). Generally, values between (zero and one) indicate an acceptable performance level; where one is the optimal value. Whereas values less than zero indicate that the mean value of the observed is a better predictor than the model (unacceptable performance) [14]. Nash–Sutcliffe efficiency index (NSE) is computed, as shown in Equation 2:

\[
NSE = 1 - \frac{\sum_{i=1}^{n} (S_P - S_o)^2}{\sum_{i=1}^{n}(S_o - \bar{S}_o)^2}
\]  

Where:

- \( S_o \): is the observed settlement value.
- \( S_P \): is the predicted settlement value.
- \( \bar{S}_o \): mean of observed settlement values.
- \( n \): number of dataset.

4.2 Second Approach

The second approach (R₂) is employed, in which the coefficient of variation (COV) is calculated and utilized. The coefficient of variation knows as a standardized measure of the dispersion of a frequency distribution or probability distribution. It is often expressed as a percentage of the standard deviation (SD) to the mean (\( \bar{x} \)).

The standard deviation (SD) is a measure of the dispersion of a set of values. It's telling how closely the values of the data set clustered around the mean. A low standard deviation value indicates that the values tend to be close to the mean. In contrast, a high standard deviation indicates that the values are spread over a broader range [15]. The coefficient of variation (COV) is calculated as shown in Equation 3:

\[
COV = \frac{SD}{\bar{x}}
\]  

In this approach to use (COV), the arithmetic average (\( \bar{x} \)) and standard deviation (SD) of (R) ratio values are calculated. However, the lowest value of (COV) indicates good performance of the method.

4.3 Third Approach

In accumulative probability approach (R₃) the predicted to measured value ratio (R) will be drawn versus accumulative probability. A series of settlement ratio (R) will be set in ascending order from with (1 to n). Subsequently, the cumulative probability factor calculated for each ratio as expressed by Equation 4:

\[
P\% = \frac{i}{n+1} * 100
\]  

Where:

- \( P \): is the accumulative probability factor.
- \( i \): is the index of the considered case, and
- \( n \): is the number of total cases.

To indicate the tendency of prediction’s output to be convergent or deviation; one of the following criteria will be referred:
1. The 50% accumulative probability value is considered a measurement of the tendency to overestimate or underestimate settlement. The closer to a ratio of unity, the better the agreement. The average error is determined by Equation 5.

\[ E_{ave} = \frac{(S_P)}{S_0}_{50\%} - 1 \]  

2. The slope of the line through the data points is a measurement of the dispersion or standard deviation. The flatter the line, the better the general agreement.

3. The difference between \( P_{50} \) and \( P_{90} \) also could be used as a scattering parameter. It should be stated that for the current study, the current point will be adopted.

4.4 Fourth Approach

The settlement ratio (R) is theoretically ranging from zero to an unlimited upper value. The non-systematic distribution of (R) around the mean will be derived; therefore, the under and over-prediction weight cannot be reached. To assessing the settlement’s prediction performance, a Log-Normal distribution of (R) is employed in this approach (\( R_4 \)). The Log-Normal distribution is defined as the distribution with the following density, as shown in Equation 6.

\[
\left[ f(x) = \frac{1}{\sqrt{2\pi} \cdot SD_{ln}} \cdot e^{-0.5 \left( \frac{lnx - \bar{ln}x}{SD_{ln}} \right)^2} \right]
\]  

Where: \( x \): is the settlement ratio R.

\( SD_{ln} \): is the standard deviation of \( ln \) R.

\( \bar{ln}x \): is the mean of \( ln \) R.

Depending on the Log-Normal distribution curve, it is possible to obtain another ranking criterion. This criterion is represented by calculating the probability that falls at the desired accuracy level. For this study, an accuracy level of 20 % was adopted, which is in line with other researchers’ proposals. So, prediction’s probability of the settlement at 20 % accuracy is an area under the probability density function within the limits of \( 0.8 \leq R \leq 1.2 \), as shown in Equation 7 and Figure 4.

\[ P\% = 100 \cdot \int_{0.8}^{1.2} f(x) \, dx \]  

\[ (7) \]
5. Results and Discussion

For the first approach concerning Nash–Sutcliffe efficiency, the analysis results are demonstrated in Table 5. According to the results, Anagnostopoulos et al. [12] method yields the highest (NSE) value that put the method in the first rank among the other methods. Burland and Burbidge [11] and Bowles [7] achieved a second and third rank, respectively. In contrast, the Scultze and Shierf [9] method yields a value pretty close to 0.21, which put the method in the last of method’s list.

As for the second approach concerning the coefficient of variation, the results of the analysis demonstrated in Table 5 are revealed that Anagnostopoulos et al. [12], Burland and Burbidge [11], and Meyerhof [10] with (0.55, 0.57, and 0.66) coefficient value; that the methods are achieved the first three ranks, respectively.

| Prediction Methods | NSE  | COV  |
|--------------------|------|------|
| TERZAGHI and PECK [5] | 0.28 | 0.69 |
| MEYERHOF [6] | 0.22 | 0.90 |
| BOWLES [7] | 0.54 | 0.87 |
| PARRY [8] | 0.52 | 0.67 |
| SCHULTZE and SHERIF [9] | 0.21 | 0.73 |
| MEYERHOF [10] | 0.39 | 0.66 |
| BURLAND and BURBIDGE [11] | 0.66 | 0.57 |
| ANAGNOSTOPOUL ET AL. [12] | 0.72 | 0.55 |

For the third approach, Figure 5 illustrates the relationship between the settlement ratio (R) ratio and the accumulative probability plot of all methods. The slope of the line through the data points is a measurement of the dispersion or standard deviation. The flatter the line, the better general agreement.
It can be noticed that all methods yield the same trend, approximately a straight line. Also, there are different bias and precision for each method, which means that the methods yield a different predicted probability for a specified settlement ratio. Also, it can be required the same settlement ratio for different probabilities.

As an example, Figure 6 demonstrates the accumulative probability of Anagnostopoulos et al. [12]. It can be noticed that accumulative probability of 50% corresponds to \((Sp/Sm)\) of (1.43) this means that 50\% of the time, the predictive method under predicts settlement by a factor of (1.43) or less. On the other hand, 90\% of the time, the predictive method over predicts settlement by a factor of (2.97) value.

Based on this approach's results, the lowest average error is attributed to the Schultz & Sherif (1973) method with a 78\% error. For Burland and Burbidge [11] and Parry [8] method, these values are 143\% and 160\%, respectively. Terzaghi and peck's [5] method yields the highest error value, about 295\%.

![Figure 5. Accumulative probability for settlement ratio of all adopted methods.](image1)

![Figure 6. Accumulative probability for settlement ratio based on Anagnostopoulos et al. [12] method.](image2)
In order to evaluate the selected methods based on the fourth approach, the Log-Normal distribution for all methods were drawn with aid of MINITAB 17 statistical software as shown in Figure 7. Also, with a 20% accuracy level was chosen to find the probability (area) bounded by R=0.8 and R=1.2.

It was found that Anagnostopoulos et al. [12] and the Burland and Burbidge [11] methods gave the highest probability with the above-mentioned limits about level of 23% and 22.6% respectively. This means that these methods (Anagnostopoulos et al. [12] and the Burland and Burbidge [11]) reveal absolute error of 20% in predicting settlement for about 43 cases out of 189 total cases. On the other hand, Terzaghi and peck [5] method shows the worst behavior with a probability of 12%.

Figure 8 demonstrates the probabilities of all methods for different accuracy levels (up to 50%). It is clearly noticed that Terzaghi and peck [5] reveals about 70% of the total cases with absolute error more than 50%.

![Figure 7. Log-Normal distribution of Sp/Sm ratio for all adopted methods.](image)

![Figure 8. Probabilities vs. different accuracy level for all adopted methods.](image)
It should be noted that the summary of the results of the ranking approaches (R₁, R₂, R₃, &R₄) is illustrated in Table 6 in addition to the total rank index for all methods. It can be pointed out that Anagnostopoulos et al. [12] method have got the first rank with rank index (RI) of 7 value followed by Burland and Burbidge [11] method with RI of 8. While the last ranking was for Terzaghi and peck [5] method with RI of 27.

Table 6. Results of rank index for all methods.

| Prediction methods                  | NSE  | R₁  | COV | R₂  | P₃₀ | P₉₀ | R₅ | P₉₀ | Prediction accuracy ±20 % | R₄  | RI  |
|------------------------------------|------|-----|-----|-----|-----|-----|----|-----|----------------------------|-----|-----|
| TERZAGHI and PECK [5]              | 0.28 | 6   | 0.69| 5   | 2.38| 5.33| 8  | 12.3| 8                          |     | 27  |
| MEYERHOF [6]                       | 0.22 | 7   | 0.90| 8   | 1   | 2.62| 6  | 19.3| 6                          | 27  |
| BOWLES [7]                         | 0.54 | 3   | 0.87| 7   | 1.2 | 2.78| 4  | 21  | 5                          | 19  |
| PARRY [8]                          | 0.52 | 4   | 0.67| 4   | 1.2 | 2.8 | 5  | 22  | 3                          | 16  |
| SCHULTZE and SHERIF [9]            | 0.21 | 8   | 0.73| 6   | 0.84| 1.62| 1  | 21.7| 4                          | 19  |
| MEYERHOF [10]                      | 0.39 | 5   | 0.66| 3   | 1.5 | 3.53| 7  | 19  | 7                          | 22  |
| BURLAND and BURBIDGE [11]          | 0.66 | 2   | 0.57| 2   | 1.42| 2.85| 2  | 22.6| 2                          | 8   |
| ANAGNOSTOPOUL ET AL. [12]          | 0.72 | 1   | 0.55| 1   | 1.43| 2.97| 3  | 23  | 1                          | 6   |

6. Conclusion

A ranking analysis of the selected formulas was conducted, which depends on several statistical requirements. The analysis results showed that the Anagnostopoulos et al. [12] formula was the best, as it scored a ranking index of 6. In comparison, the formulas proposed by Burland and Burbidge [11] and Parry [8] were in the second and third ranks. The analysis also showed that the last rank was for Terzaghi and peck [5] and Meyerhof [6] formulas, with rank indices of 27.

7. References

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