Left-right mixing on leptonic and semileptonic $b \to u$ decays

Chuan-Hung Chen* and Soo-hyeon Nam†
Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan
National Center for Theoretical Sciences, Hsinchu 300, Taiwan
(Dated: July 6, 2008)

Abstract

It has been known that there exists a disagreement emerged between the determination of $|V_{ub}|$ from inclusive $B \to X_u \ell \nu$ decays and exclusive $B \to \pi \ell \nu$ decays. In order to solve the mismatch, we investigate the left-right (LR) mixing effects, denoted by $\xi_u$, in leptonic and semileptonic $b \to u$ decays. We find that the new interactions $(V + A) \times (V - A)$ induced via the LR mixing can explain the mismatch between the values of $|V_{ub}|$ if $\text{Re}(\xi_u) = -(0.14 \pm 0.12)$. Furthermore, we also find that the LR mixing effects can enhance the branching fractions for $B \to \tau \nu$ and $B \to \rho \ell \nu$ decays by 30% and 17%, respectively, while reducing the branching fraction for $B \to \gamma \ell \nu$ decays by 18%.

* Email: physchen@mail.ncku.edu.tw
† Email: shnam@mail.ncku.edu.tw
The dominant weak interaction in $b \to u$ decays in the standard model (SM) is strongly suppressed by the quark mixing matrix element $|V_{ub}| \sim \lambda^4$ where $\lambda \simeq 0.22$ in Wolfenstein parametrization \[1\]. Although the relevant decays occur at the tree level, such decays are often sensitive to a non-standard physics beyond the SM if the new physics effects are not directly proportional to the small weak mixing. One of the simplest extensions of the SM corresponding to such a scenario is the general left-right model (LRM) with gauge group $SU(2)_L \times SU(2)_R \times U(1)$ \[3\]. Although the new physics effects in the LRM are followed by suppression factors such as the $W_L - W_R$ mixing angle $\xi$, such suppression could be compensated by the right-handed quark mixing matrix $V_R$ if $V_R \neq V_L^\dagger$ (nonmanifest LRM) where $V_L$ is the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix \[2\]. Especially, if $V_R$ takes one of the following forms, the $W_R$ mass limit can be lowered to approximately 300 GeV \[4, 5\], and $V_{ub}^R$ can be as large as $\lambda$ (for $M_{W_R} \geq 800$ GeV) \[6\]:

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 
\end{pmatrix},
\begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 
\end{pmatrix},
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 
\end{pmatrix}.
\]

(1)

In this case, the right-handed current contributions in $b \to u$ decays can be maximal. The right-handed gauge boson mass $M_{W_R}$ and the mixing angle $\xi$ are restricted by a number of low-energy phenomenological constraints under various assumptions \[3\]. From the global analysis of muon decay measurements \[7\], the lower bound on $\xi$ can be obtained without imposing discrete left-right symmetry as follows \[8\]:

\[
\xi \leq \frac{g_R M_{W_L}^2}{g_L M_{W_R}^2} \lesssim 0.034 \frac{g_L}{g_R},
\]

(2)

where $g_L(g_R)$ is the left(right)-handed gauge coupling constant. Although this mixing angle $\xi$ is small, the combined parameter $\xi V_{ub}^R V_{ub}^L$ could significantly contribute to the value of $|V_{ub}|$ extracted from the data in $b \to u$ decays.

The general four-fermion interaction for $b \to q\ell\bar{\nu}_\ell$ decays with $V \pm A$ currents can be written as

\[
\mathcal{H}_{\text{eff}} = 2\sqrt{2} G_F V_{qb}^L [(\bar{q}L \gamma_\mu b_L) + \xi_q (\bar{q}R \gamma_\mu b_R)] (\bar{\ell}L \gamma_\mu \nu_L),
\]

(3)

where $\xi_q \equiv \xi (g_R V_{qb}^R)/(g_L V_{qb}^L)$ and $q = u, c$. As well as the above terms, one can include other terms with right-handed leptons. However, the interference of such terms with the dominant one is suppressed by the small lepton masses $m_\ell m_\nu$, and the second dominant term is suppressed by $\xi^2$ or $1/M_{W_R}^4$, so we can drop them. From the above expression, it is clear that $\xi_u \neq \xi_c$ in general. The bound on $\xi_c$, $\xi_c \approx 0.14 \pm 0.18$, was obtained by Voloshin.
from the difference $\Delta V_{cb} = |V_{cb}|_{incl} - |V_{cb}|_{excl}$ where $|V_{cb}|_{incl}$ and $|V_{cb}|_{excl}$ were extracted from the inclusive rate of the decays $B \rightarrow \ell \nu X_c$ and the exclusive decay $B \rightarrow D^* \ell \nu$ at zero recoil, respectively \cite{9}. One can see from Ref. \cite{9} that $|V_{cb}|_{incl}$ is related to $|V_{L_{cb}}|$ of Eq. (3) as $|V_{cb}|_{incl} \approx |V_{L_{cb}}| |1 - \xi_c f(x_c)|$ where $f(x_q)$ is a kinematic phase space function proportional to the ratio $x_q = m_q/m_b$. For $b \rightarrow u$ decays, neglecting the $u$-quark mass, one can safely use the approximation $|V_{ub}|_{incl} \approx |V_{L_{ub}}|$ assuming that $\xi_u$ is small.

Experimentally, unlike the case of $|V_{cb}|_{incl}$, the determination of $|V_{ub}|_{incl}$ is very difficult due to the large background from $b \rightarrow c$ decays since $|V_{ub}^L| \ll |V_{cb}^L|$. One may remove this large background by applying specific kinematic selection criteria such as the lepton-energy requirement, but in that restricted kinematic region the inclusive amplitude is governed by a non-perturbative shape function which is unknown theoretically from the first principle. In order to overcome this problem, various different theoretical techniques have been developed. In this letter, we adopt the following values obtained by the techniques called Dressed Gluon Exponentiation (DGE) \cite{10} and the Analytic Coupling Model (AC) \cite{11}:

$$|V_{ub}|_{incl} \times 10^3 = \begin{cases} 4.48 \pm 0.16 & (\text{DGE}) \\ 3.78 \pm 0.13 & (\text{AC}) \end{cases}$$

where each value is an average of independent measurements \cite{12}. Other than these two, there are several other methods to determine $|V_{ub}|_{incl}$ \cite{13}. However, we do not consider them here because they use inputs obtained from other measurements such as $b \rightarrow c\ell\nu$ moments which could also be affected by possible new physics contributions. So, the values of $|V_{ub}|_{incl}$ obtained from such methods are not suitable for our analysis. For numerical analysis, we use the weighted average of the two determinations of Eq. (4):

$$|V_{ub}|_{incl} = (4.09 \pm 0.20) \times 10^{-3}.$$  

Due to the large error, this average can only be provisional.

The determination of $|V_{cb}|_{excl}$ from exclusive semileptonic $B \rightarrow \pi$ decays requires a theoretical calculation of the hadronic matrix element parametrized in terms of form factors. The most recent values of the $B \rightarrow \pi$ form factors were calculated by the QCD light-cone sum rule (LCSR), and the extracted value of $|V_{cb}|_{excl}$ is \cite{14}:

$$|V_{cb}|_{excl} = (3.5 \pm 0.4 \pm 0.2 \pm 0.1) \times 10^{-3}.$$  

This updated result is in very good agreement with the earlier results from other groups, which can also be found in Ref. \cite{14} with detailed discussion, so we do not repeat them here. The amplitude of semileptonic $B \rightarrow \pi$ decays is determined only by the vector current.
From the mismatch between the values of $\xi$, given in Eqs. (5, 6), we roughly estimate the mixing parameter $\xi$ as

$$\xi^{\prime} = -(0.14 \pm 0.12),$$

where $\xi^{\prime} \equiv \text{Re}(\xi)$, and we assumed $\xi^{\prime} \gg |\xi|$. Of course, more accurate analysis of $|V_{ub}|$ extracted from the experimental data could further improve the bounds on $\xi$. As discussed above, the obtained $\xi^{\prime} \approx 0$ is negative while $\xi^{\prime} \approx 0$ is positive, which implies that the mixing parameter $\xi$ is not universal and, in this case, the manifest $(V^{R} = V^{L})$ LRM is disfavored. This negative value of the left-right mixing contribution commonly reduces the branching fractions for semileptonic $B \rightarrow P$ decays in $b \rightarrow u$ transitions where $P$ indicates a pseudo-scalar meson. Using the obtained value of $\xi$, we will also estimate the branching fractions for other types of $b \rightarrow u$ transitions such as $B \rightarrow \tau \nu$, $B \rightarrow \rho \ell \nu$, and $B \rightarrow \gamma \ell \nu$ decays.

Recently, the BELLE [15] and BABAR [16] collaborations have found evidence for the purely leptonic $B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}$ decays. Their measurements are

$$Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}) = \begin{cases} (1.79 \pm 0.56 \pm 0.46) \times 10^{-4} & \text{(BELLE)} \\ (1.2 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4} & \text{(BABAR)} \end{cases},$$

where the BABAR result is an average of two results, $(0.9 \pm 0.6 \pm 0.1) \times 10^{-4}$ and $(1.8^{+0.9}_{-0.8} \pm 0.4 \pm 0.2) \times 10^{-4}$, from separate analysis with semi-leptonic and hadronic tags, respectively, and the latter one is newer. On the theory side, there have been numerous discussions on the mode $B \rightarrow \tau \nu$ in physics beyond the SM such as the two Higgs Doublet Model (2HDM) [17] and the Minimal Supersymmetric SM (MSSM) [18, 19]. This process occurs via annihilation of $b$ and $\bar{u}$ quarks, and its amplitude is determined only by the axial current $(\bar{u} \gamma_{\mu} b)$. So the branching ratio is give by

$$Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}) = \frac{G_{F}^{2}m_{B}m_{\tau}^{2}}{8\pi} \left(1 - \frac{m_{\tau}^{2}}{m_{B}^{2}}\right)^{2} f_{B}^{2}|V_{ub}^{L}|^{2}|1 - \xi|^{2} \tau_{B^{-}},$$

where $\tau_{B^{-}}$ is the lifetime of $B^{-}$ and $f_{B}$ is the $B$ meson decay constant. Using $f_{B} = (216 \pm 22)$ MeV obtained from unquenched lattice QCD [20], we arrive at the SM prediction for the $\tau^{-} \bar{\nu}_{\tau}$ branching fraction of $(1.38 \pm 0.31) \times 10^{-4}$. In the presence of right-handed currents for small $\xi$, our estimate of the branching fraction according to Eq. (8) is

$$Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}) = (1.78 \pm 0.53) \times 10^{-4}.$$


Interestingly, this value agrees very well with the BELLE result and the new BABAR result, but not with the old BABAR result.

The decay mode $B \to \rho \ell \nu$ has been studied earlier in the SM by many authors \cite{21}. Meanwhile, the left-right mixing effect in $B \to \rho \ell \nu$ decays was also studied for selected regions of $q^2$ in Ref. \cite{22}, where $\xi_u$ was assumed to be a positive real parameter. In this letter, we reexamine the mode $B \to \rho \ell \nu$ for the whole range of $q^2$ with the value of $\xi_u$ in Eq. (8). Since $\rho$ is a vector particle, all virtual $W$ polarizations are allowed in semileptonic $B \to \rho$ decays, and the hadronic matrix elements for $B \to \rho$ transitions can be written in terms of the four Lorentz-invariant form factors $V$ and $A_{0,1,2}$ as

$$
\langle \rho(p_\rho, \epsilon) | \bar{u} \gamma_\mu b | B(p_B) \rangle = -2 \frac{V(q^2)}{m_B + m_\rho} \varepsilon^{\mu \nu \rho \sigma} \epsilon^* \rho^\nu_\rho, 
$$

$$
\langle \rho(p_\rho, \epsilon) | \bar{u} \gamma_\mu \gamma_5 b | B(p_B) \rangle = i \frac{A_0(q^2)}{q^2} 2m_\rho (\epsilon^* \cdot q) q_\mu 
+ i A_1(q^2)(m_B + m_\rho) \left( \epsilon^* - \frac{\epsilon^* \cdot q}{q^2} q \right) \mu 
- i \frac{A_2(q^2)}{m_B + m_\rho} \left( \frac{m_B^2 - m_\rho^2}{q^2} q \right) \mu (\epsilon^* \cdot q),
$$

(12)

where $q$ is the momentum of lepton pair and $p_M$ is the momentum of $M$ meson. In the limit of massless leptons, the terms proportional to $q_\mu$ in Eq. (12) vanish, and the three helicity amplitudes $H_{\pm,0}$ depend effectively on only three form factors $V$ and $A_{1,2}$ as:

$$
H_\pm = \frac{1}{m_B + m_\rho} \left[ (m_B + m_\rho)^2 (1 - \xi_u) A_1(q^2) \mp 2m_\rho |\mathbf{p}_\rho| (1 + \xi_u) V(q^2) \right],
$$

$$
H_0 = \frac{m_B(1 - \xi_u)}{2m_\rho(m_B + m_\rho)^2} \sqrt{y} \left[ \left( 1 - \frac{m_\rho^2}{m_B^2} - y \right) (m_B + m_\rho)^2 A_1(q^2) - 4|\mathbf{p}_\rho|^2 A_2(q^2) \right],
$$

(13)

where $y = q^2/m_B^2$ and $\mathbf{p}_\rho$ is the $\rho$ meson three-momentum in the $B$-meson rest frame. In terms of these three helicity amplitudes, the differential decay rate is then given by:\footnote{The general form of the differential decay rate for semileptonic $B \to \rho$ transitions with the non-zero lepton masses in the SM can be found in Ref. [19]. The right-handed current contribution can simply be obtained by replacing the form factors $V$ and $A_i$ in the SM with $(1 + \xi_u)V$ and $(1 - \xi_u)A_i$, respectively.}

$$
\frac{d^2 \Gamma(B^0 \to \rho^- \ell^+ \nu_\ell)}{dy \cos \theta_\ell} = \frac{G_F^2 m_B^2 |\mathbf{p}_\rho|^2 |V_{ub}^L|^2 \left[ (1 - \cos \theta_\ell)^2 |H_+|^2 
+ (1 + \cos \theta_\ell)^2 |H_-|^2 + 2 \sin \theta_\ell^2 |H_0|^2 \right]}{256 \pi^3},
$$

(14)

where $\theta_\ell$ is the azimuthal angle between the directions of the $\ell \nu$ system and the lepton in the $\ell \nu$ rest frame. At large $q^2$, the axial current represented by the $A_i$ terms is dominant and
the corresponding decay rate can be expressed as \( \Gamma \sim |1 - \xi_u|^2 \Gamma_{\text{SM}} \) while the vector current represented by the \( V \) term could be important at low \( q^2 \).

**TABLE I:** Set of parameters for the \( B \to \rho \) and \( B \to \gamma \) form factors obtained from the fits of the LCSR results in Ref. [23] and Ref. [24], respectively.

| \( F(q^2) \) | \( f_1 \) | \( f_2 \) | \( m_1^2 \) | \( m_2^2 \) | \( n \) |
|------------|-------|-------|-------|-------|-----|
| \( V \)    | 1.045 | -0.721 | 28.30 | 38.34 | 1   |
| \( A_1 \)  | 0     | -0.240 |   -   | 37.51 | 1   |
| \( A_2 \)  | 0.009 | -0.212 | 40.82 | 40.82 | 2   |
| \( F_V \)  | 0     | 0.190  |   -   | 31.36 | 2   |
| \( F_A \)  | 0     | 0.150  |   -   | 42.25 | 2   |

A theoretical prediction of the \( B \to \rho \) decay rate requires a specific choice of form factors. For numerical analysis, we use the recent LCSR result [23], where the \( B \to \rho \) form factors \( V \) and \( A_i \) are parametrized as

\[
F(q^2) = \frac{f_1}{1 - q^2/m_1^2} + \frac{f_2}{(1 - q^2/m_2^2)^n},
\]

and the corresponding parameters are collected in Table I. Using these values, we plot the differential branching fraction for \( B^0 \to \rho^- \ell^+ \nu_\ell \) decays by varying \( q^2 \) in Fig. 1. We also show the \( d\Gamma/dq^2 \) distribution in Fig. 2 for each term of \( H_{\pm,0} \) given in Eq. (14). As one can see from the figures, \( H_- \) contributes the largest fraction of the total rate in the SM, but the left-right mixing effects in \( H_- \) is small due to the cancellation between the vector and axial currents. However, \( H_0 \) is only determined by the axial current, and receives the significant contribution from the left-right mixing term. As well as the branching fraction, one can consider the forward-backward asymmetry \( (A_{FB}) \) of charged lepton defined by

\[
A_{FB} = \frac{\int_0^1 d\cos \theta \frac{d^2\Gamma}{dy d\cos \theta} - \int_{-1}^0 d\cos \theta \frac{d^2\Gamma}{dy d\cos \theta}}{\int_0^1 d\cos \theta \frac{d^2\Gamma}{dy d\cos \theta} + \int_{-1}^0 d\cos \theta \frac{d^2\Gamma}{dy d\cos \theta}}.
\]

The variation of \( A_{FB} \) as a function of \( q^2 \) is shown in Fig. 3. After an integration over whole phase space, we show the left-right mixing effects to the branching fraction and \( A_{FB} \) as

\[
Br(B^0 \to \rho^- \ell^+ \nu_\ell) \simeq (1 - 1.21 \xi_u) Br_{\text{SM}}(B^0 \to \rho^- \ell^+ \nu_\ell),
\]

\[
\int dy A_{FB}(B^0 \to \rho^- \ell^+ \nu_\ell) \simeq (1 + 1.21 \xi_u) \int dy A_{FB}^{\text{SM}}(B^0 \to \rho^- \ell^+ \nu_\ell).
\]
Note that the branching fraction can be enhanced by about 17% and the integrated $A_{FB}$ can be reduced by about 17% for $\xi_u^r = -0.14$. Of course, using the form factors from different theoretical methods would lead us to somewhat different results, and it is beyond the scope of this letter to discuss the detailed analysis of those results.

FIG. 1: $d\Gamma(B^0 \to \rho^-\ell^+\nu_\ell)/dq^2$ distribution for the SM (solid line), $\xi_u^r = -0.14$ (dashed line), and its error (dotted line).

FIG. 2: $d\Gamma(B^0 \to \rho^-\ell^+\nu_\ell)/dq^2$ distributions for each of three terms in Eq. (14) for the SM (solid line), $\xi_u^r = -0.14$ (dashed line), and its error (dotted line).

The radiative leptonic $B \to \gamma\ell\nu_\ell$ decays are governed by internal bremsstrahlung (IB) and structure-dependence (SD) [25], where the former corresponds to the photon emitted via the lepton and associates with the helicity suppressed factor $m_\ell/m_B$ while the latter photon couples to the quarks inside $B$ meson and is free of the suppressed factor. Therefore, for simplicity, we neglect the contributions of IB and only consider the contributions of SD.
FIG. 3: $A_{FB}(B^0 \to \rho^- \ell^+ \nu_\ell)$ as a function of $q^2$ for the SM (solid line), $\xi_u = -0.14$ (dashed line), and its error (dotted line).

In order to consider the hadronic effects for leptonic $B \to \gamma$ decays, we parametrize the transition matrix elements in terms of the form factors $F_V$ and $F_A$ as [26]:

$$
\langle \gamma(k, \epsilon) | \bar{u} \gamma_\mu b | B^-(p_B) \rangle = e \frac{F_V(q^2)}{m_B} \epsilon_{\mu \nu \rho \sigma} \gamma_\nu p_\rho^\nu k_\sigma,
$$

$$
\langle \gamma(k, \epsilon) | \bar{u} \gamma_\mu \gamma_5 b | B^-(p_B) \rangle = i \epsilon \frac{F_A(q^2)}{m_B} [(p_B \cdot k) \epsilon_\mu^* - (\epsilon^* \cdot p_B) k_\mu],
$$

where $\epsilon$ and $k$ are the polarization vector and the momentum of the photon, respectively.

Using Eq.(3), the decay amplitude for $B \to \gamma \ell \nu_\ell$ can be written as

$$
A(B^- \to \gamma \ell^- \bar{\nu}_\ell) = \frac{e G_F}{\sqrt{2}} V_{ub}^L \epsilon^{*\alpha} (\lambda) H_{\alpha \beta} [\bar{\ell}(p_\ell) \gamma^\beta (1 - \gamma_5) \nu(p_\nu)]
$$

with

$$
H_{\alpha \beta} = \frac{F_A'}{m_B} [-(p_B \cdot k) g_{\alpha \beta} + p_{Bo} k_\beta] + i \epsilon_{\alpha \beta \rho \sigma} F'_{V} k_\rho p_\sigma,
$$

where $F'_V = F_V(1 + \xi_u)$ and $F'_A = F_A(1 - \xi_u)$. With unpolarized photon, the double differential decay rate is then given by

$$
\frac{d^2\Gamma(B^- \to \gamma \ell^- \bar{\nu}_\ell)}{dy d\cos\theta} = \frac{\alpha_{em} G^2_F m^5_B}{512\pi^2} y (1 - y)^3 |V_{ub}^L|^2 \left(1 - \hat{m}_\ell^2\right)^2 I(q^2, \cos\theta)
$$

with

$$
I(q^2, \cos\theta) = |F_V' + F_A'|^2 \left[1 + \hat{m}_\ell^2 + (1 - \hat{m}_\ell^2) \cos\theta\right] (1 + \cos\theta)
$$

$$
+ |F_A' - F_V'|^2 \left[1 + \hat{m}_\ell^2 - (1 - \hat{m}_\ell^2) \cos\theta\right] (1 - \cos\theta),
$$

where $\hat{m}_\ell = m_\ell / \sqrt{q^2}$ and $\theta$ is the relative angle between photon and lepton.
Since both $\rho$ and $\gamma$ are vector particles, the numerical analysis of the $B \to \gamma \ell \nu_\ell$ transition can be done similarly to the $B \to \rho \ell \nu_\ell$ case. In order to clearly see the right-handed current contribution in $B \to \gamma \ell \nu_\ell$ decays and compare it with that in $B \to \rho \ell \nu_\ell$ decays, we use the LCSR result for the form factors parametrized as Eq. (15) obtained in Ref. [24], and plot the differential branching fraction for $B \to \gamma \ell \nu_\ell$ decays for zero lepton masses by varying $q^2$ in Fig. 4. One can see from the figure that the deviation from the SM is very small at low $q^2$. This is because $F_V \sim F_A$ at low $q^2$, and in this region the left-right mixing effect is suppressed by $\xi_u(F_V - F_A)$, which is clear from Eq. (22). However the deviation from the SM becomes larger as $q^2$ is increased. Beside the branching fraction, we also obtain the
angular asymmetry of lepton defined in Eq. (16) in $B \rightarrow \gamma \ell \nu_\ell$ decays as

$$A_{FB}(B \rightarrow \gamma \ell \nu_\ell) = \frac{6 \text{Re}(F'_V F_{A}^\ast)}{|F_A^\ast + F'_V|^2 + |F'_A - F'_V|^2} \left(2 + \frac{m_{\ell}^2}{q^2}\right)$$

(23)

The variation of $A_{FB}$ as a function of $q^2$ for zero lepton masses is shown in Fig. 5. After an integration over whole phase space, we show the left-right mixing effects to the branching fraction and $A_{FB}$ as

$$Br(B^− \rightarrow \gamma \ell^{-} \bar{\nu}_\ell) \simeq (1 + 1.25 \xi_u^r) Br^\text{SM}(B^− \rightarrow \gamma \ell^{-} \bar{\nu}_\ell),$$

$$\int dy A_{FB}(B^− \rightarrow \gamma \ell^{-} \bar{\nu}_\ell) \simeq (1 - 1.25 \xi_u^r) \int dy A^\text{SM}_{FB}(B^− \rightarrow \gamma \ell^{-} \bar{\nu}_\ell).$$

(24)

Note that the branching fraction can be reduced by about 18% and the integrated $A_{FB}$ can be enhanced by about 18% for $\xi_u^r = -0.14$. This result can be compared with those in semileptonic $B \rightarrow V$ transitions as shown in the previous example of $B \rightarrow \rho \ell \nu$ decays where $V$ indicates a vector meson.

In summary, we show that the difference between the values of $|V_{ub}|$ extracted from the total inclusive semileptonic decay rate of $b \rightarrow u$ transitions and from the exclusive decay rate of $B \rightarrow \pi \ell \nu$ transitions is sensitive to the admixture of right-handed $b \rightarrow u$ current characterized by the mixing parameter $\xi_u$. From the current mismatch between $|V_{ub}|^\text{incl}$ and $|V_{ub}|^\text{excl}$ obtained from the independent experiments, we estimate the size of the left-right mixing parameter $\xi_u$ to be $\text{Re}(\xi_u) = -(0.14 \pm 0.12)$. For $\text{Re}(\xi_u) = -0.14$, we show that the branching fraction for leptonic $B \rightarrow \tau \nu$ and semileptonic $B \rightarrow \rho \ell \nu$ decays can be enhanced by 30% and 17%, respectively, while the branching fraction for radiative leptonic $B \rightarrow \gamma \ell \nu$ decays can be reduced by 18%. The left-right mixing contributions obtained in this letter in leptonic and semileptonic $b \rightarrow u$ decays are not simply negligible. Therefore, our estimate could be a reasonable guide to search for the existence of the right-handed current, and future experimental progress can further improve the bound of the new physics parameter.

**Acknowledgments**

This work is supported in part by the National Science Council of R.O.C. under Grant #s:NSC-95-2112-M-006-013-MY2.

[1] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[2] For a review, see R. N. Mohapatra, *Unification and Supersymmetry* (Springer, New York, 1992).

[3] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975).

[4] F. I. Olness and M. E. Ebel, Phys. Rev. D **30**, 1034 (1984).

[5] P. Langacker and S. U. Sankar, Phys. Rev. D **40**, 1569 (1989).

[6] T. G. Rizzo, Phys. Rev. D **58**, 114014 (98).

[7] C. A. Gagliardi, R. E. Tribble, and N. J. Williams, Phys. Rev. D **72**, 073002 (2005).

[8] S.-h. Nam, Phys. Rev. D **66**, 055008 (2002).

[9] M. B. Voloshin, Mod. Phys. Lett. A **12**, 1823 (1997).

[10] J. R. Andersen and E. Gardi, J. High Energy Phys. **0601**, 097 (2006).

[11] U. Aglietti et al., [arXiv:0711.0860 [hep-ph]].

[12] Heavy Flavor Averaging Group, [http://www.slac.stanford.edu/xorg/hfag/].

[13] P. Gambino et al., J. High Energy Phys. **0710**, 058 (2007); also see Ref. [12].

[14] G. Duplancic et al., J. High Energy Phys. **0804**, 014 (2008), and references there in.

[15] K. Ikado et al., (BELLE Collaboration), Phys. Rev. Lett. **97**, 251802 (2006).

[16] B. Aubert et al., (BABAR Collaboration), Phys. Rev. D **76**, 052002 (2007); *ibid.* **77**, 011107 (2008).

[17] W. S. Hou, Phys. Rev. D **48**, 2342 (1993); A. G. Akeroyd and C. H. Chen, Phys. Rev. D **75**, 075004 (2007).

[18] A. G. Akeroyd and S. Recksiegel, J. Phys. G **29**, 2311 (2003); G. Isidori and P. Paradisi, Phys. Lett. B **639**, 499 (2006); A. G. Akeroyd, C. H. Chen, and S. Recksiegel, [arXiv:0803.3517 [hep-ph]].

[19] C. H. Chen and C. Q. Geng, J. High Energy Phys. **0610**, 053 (2006).

[20] A. Gray et al., (HPQCD), Phys. Rev. Lett. **95**, 212001 (2005).

[21] For instance, see L. K. Gibbons, Ann. Rev. Nucl. Part. Sci. **48**, 121 (1998).

[22] Y. Yang, H. Hao, and F. Su, Phys. Lett. B **618**, 97 (2005).

[23] P. Ball and R. Zwicky, Phys. Rev. D , 7 (12005014029).

[24] G. Eilam, I. E. Halperin, and R. R. Mendel, Phys. Lett. B **361**, 137 (1995).

[25] T. Goldmann and W. J. Wilson, Phys. Rev. D **15**, 709 (1977).

[26] C. Q. Geng, C. C. Lih, and W. M. Zhang, Phys. Rev. D **57**, 5697 (1998).