Analysis of Fracture Mechanics Theory of the First Fracture Mechanism of Main Roof and Support Resistance with Large Mining Height in a Shallow Coal Seam

Dengfeng Yang 1, 2

1 School of Science, Qingdao University of Technology, Qingdao 266033, China; yang_dengfeng@qut.edu.cn
2 Cooperative Innovation Center of Engineering Construction and Safety in Shandong Blue Economic Zone, Qingdao 266033, China

Abstract: Because the first-weighting of a main roof with a large mining height has obvious sudden characteristics and is more severe, which causes large-scale support crushing and has a great impact on the ecological environment of the mining area, it is necessary to conduct an in-depth analysis. This paper studies the mechanical mechanism and asymmetric fracture conditions of a main roof with a large mining height, with the first-weighting occurring in a shallow coal seam. In combination with an asymmetric three-hinged arch structural model, the main roof was regarded as a finite plate model with a crack, and a fracture-mechanics model was established. The conditions and main controlling factors of main roof fracture asymmetry were analyzed, and the determination methods of the first-weighting interval and support resistance were further analyzed. The results show that the stress concentration and the stress-intensity factor increase at the crack tip with the advancement of the face; when the stress-intensity factors increase beyond the critical value, the crack expands until the first-weighting. The sufficient condition for modeling the instability was the length $s$ of the branch crack reaching the protection thickness $H$ of the main roof, and the necessary condition was the activation of the crack. The calculation equations of the first-weighting interval and the support resistance were obtained. The influence weights of each parameter on the support resistance are ordered as follows: overburden load $q >$ rock fracture toughness $K_C >$ crack length $a >$ main roof thickness $h >$ weighting interval $l$. Finally, the theoretical analysis results were verified by an in situ monitoring case of the no. 33,206 working face in the Bulianta coal mine, China. On this basis, a reasonable value of the support resistance is further calculated. The results mentioned above can provide a new method for researching the first-weighting of the main roof and can improve the accuracy of the roof control analysis. The research on the mechanisms of first-weighting and the support resistance can effectively promote the safety production of mine, which is in line with the concept of green and sustainable development of the mine.

Keywords: asymmetrical breaking; crack; first-weighting; stress-intensity factor; main controlling factor

1. Introduction

Shallow coal seams with burial depths of less than 150 m are widely distributed in Gansu, Ningxia and Inner Mongolia in Northwest China and are characterized by shallow burial depths, thin bedrock, and overlying thick and loose sand layers [1]. The mining height of the coal seam increases with the continuous improvement of the fully mechanized mining degree, and the range of overburden failure increases, which results in extremely violent strata behavior. The strata behavior is especially violent and difficult to control during the first-weighting. In the process of weighting, the roof is prone to the full thickness cutting to the surface, and the dynamic load phenomenon is obvious, resulting in support crushing or water/sand burst, causing substantial losses and safety risks to mining enterprises [2,3]. Due to the particularity and complexity of the strata behavior in the working face with a large mining height in shallow coal seams, the analysis of the...
overburden fracture mechanism and the determination of the support resistance must be further enhanced. This has important practical significance for improving the accuracy of roof control analysis.

Many scholars have already used various methods to determine the mine pressure characteristics and the relationship between the support and the surrounding rock in a large mining height face. Wang et al. [4] proposed a two-factor control method to determine the support resistance and established the structure of a cantilever beam + voussoir beam analysis model of the working face. Xu et al. [5–8] found that the breaking of the upper sub-key strata of the large mining height face causes the cantilever beam structure of the lower sub-key strata to break, and the step interval and the weighting intensity in the working face were changed alternately. Yang et al. [9] pointed out that the hard main roof was broken not only near the coalface regions but also near the side of the longwall panel. Xu et al. [10] found that the asymmetric deformations and failures of the coal pillar roadway surrounding rocks were closely related to caving thickness in the former panel. Ghosh and Sivakumar [11] analyzed the main roof fracture characteristics through monitoring microseism incident of the coal mine at South Eastern Coalfields Limited, India. Singh [12] pointed out that en masse caving of strata was responsible for dynamic loading leading to the collapse of supports. Yavuz [13] found that with the mining height increases, the bulking factor of the caved rock pile increases and resulted in an increase in cover pressure distance. Wang et al. [14] proposed a dynamic-load method for calculating the support resistance in combination with the instability characteristics of the roof of a large mining height working face in a shallow seam. Szurgacz and Brodny [15,16] developed an effective method for determining the position (geometry) of the support section during the operation process and analyzed the influence of dynamic loads on the working parameters of a powered roof support’s hydraulic leg through tests. The research results were conducive to the safe and efficient mining of coal. Yu et al. [17] introduced a new concept and criterion of the immediate roof and main roof and obtained the analytic expression of the support resistance combined with the structure of a cantilever beam-articulated rock beam formed by a large mining height. Zhou and Huang [18] took the dynamic load when the key stratum lost stability as the additional load and deduced the equation for calculating the support resistance in a working face with a large mining height. Yang et al. [19,20] deduced the computational formula for determining the roof pressure based on the conservation of energy principle. Guo et al. [21] pointed out that the hanging length of the key strata in the immediate roof increases with the increasing thickness of the rock layer and uniaxial compressive strength of the rock block and decreases with increasing thickness of the immediate overlying roof. Yu et al. [22,23] pointed out that the structure model for the near field key stratum was a cantilever beam + voussoir beam broken in a vertical O-X shape, while that of the far-field key stratum was a voussoir beam broken in a horizontal O-X shape when fully mechanized sublevel caving mining was carried out. Yang et al. [24] found that the main reasons for the strong ground pressure in a working face with a shallow buried depth and super high mining height were as follows: the high mining strength of the working face, intense roof activity, single key layer structure, low occurrence horizon, easy slipping and instability, and integral breakage of the overlying roof. Chinese scholars have focused on the analysis of the roof fracture characteristics, pressure mechanism and adaptability of the support of the first-weighting. Huang et al. [25,26] pointed out that an asymmetric three-hinged arch structure was formed when the main roof fractured first during shallow coal seam mining and provided a method for calculating the support resistance. Feng et al. [27] found that the mining height had an exponential function relationship with the average first-weighting intervals and a binomial functional relationship with the average periodic weighting intervals. Yang et al. [28] established a mechanical system model consisting of the main roof, immediate roof and support and obtained the sufficient and necessary conditions for the catastrophic instability of the system to occur. Jiang et al. [29] divided the initial collapse of a shallow buried working face roof into the following two forms:
buckling collapse occurring along the X yield zones and shear failure occurring along the O yield zones in front of the face. Huang et al. [30] proposed an inversion method of working resistance optimization by considering the roof control effect and working resistance overrun percentage. Zhao et al. [31] pointed out that the main roof hinged structure was more prone to slip and instability in the early stage of rotation. The increase in scale between the height and the length of the first broken block of the main roof resulted in the increase in the rotation angle required for the hinged structure to reach a balanced state, and the main roof was prone to sliding and instability [32]. The cut down of the entire thickness of the overlying strata along the working face during the first-weighting of the main roof was a dynamic evolutionary process [33]. Yang [34] used the initial post-buckling theory and the cusp catastrophic model to deduce the formulae of the main roof fracture convergence and build the step convergence criterion following the main roof fracture during the first-weighting. Yi et al. [35] pointed out that the unloading coefficient was affected only by the bench convergence and internal friction angle of the soil during the first-weighting. Wang et al. [36–38] divided the vertical load-displacement–horizontal reaction force curves of the roof under concentrated loads into the following four stages: initial adjustment before loading, rock plate fracture and hinged arch formation pre-peak hardening and post-peak softening of the rock arch load bearing. Sun et al. [39] proposed a new model to describe the movement and subsidence of the inner burden during shallow coal seam mining. Qian et al. [40–42] proposed the O-X model of the fracture of the main roof according to the characteristics of the first-weighting of the main roof. Wang et al. [43] analyzed the O-X fracture mechanism of the main roof through the process of forming and expanding the plastic hinge line in the roof. Pu et al. [44] obtained the complete spatial X-O fracture morphology of a roof fracture through numerical simulation, in which the failure speed of the roof along the long side was substantially faster and more severe than that of the short side.

The above research involves in-depth analyses of the fracture structure, fracture characteristics, weighting interval, and supports the resistance of the main roof with a large mining height. However, mining practices show that the existing theory still cannot fully guide the mining of shallow coal seams with large mining heights, and occasional disasters and accidents occur. In particular, the mechanical mechanism and the main influencing factors of the main roof fracture, the calculation of the first-weighting interval, and the determination of the support resistance to ensure the safety of the working face during the first-weighting still require further study.

Currently, fracture mechanics research methods are increasingly being used in the study of mine disasters. Chen et al. [45] developed a simplified fracture-mechanics model of water inrush through a hidden fault in the floor, analyzed the stress-intensity factors on a finite plate with an oblique crack, and obtained the critical water pressure allowed for hydraulic splitting. Wang et al. [46] established a simplified fracture mechanical principle model of buried fault progressive ascent and obtained the necessary and sufficient conditions for water inrush from buried faults to occur. Yang et al. [47] established a fracture-mechanics model of a central inclined crack in the main roof, obtained the conditions of the roof fracture instability, and deduced a reasonable support resistance value through the use of the fracture mechanics theory. Zhao et al. [48] pointed out that a seepage crack in the concealed structural floor extended in the opposite direction of the fault extension line and stopped at a depth of 30 m. Zhang et al. [49] used the fracture mechanics theory to derive the criterion for crack propagation in the main roof and discovered the relationship between the fracture length in the roof and the support resistance. Gao et al. [50] obtained the evolution rule of interlayer separation–rock beam cracking–dynamic cracks from dynamic loading on composite roofs under dynamic pressure by means of a model for analyzing the elastic–plastic mechanics and fracture principles. Huang and Gao [51] introduced the damage deformation energy into the Griffith energy theory of fracture mechanics and obtained the ultimate stress value of rockburst. The above research results
show that the fracture mechanics analysis method can be applied to study the failure mechanism of mining rock masses.

In the mining process involving large mining heights in shallow coal seams, with the advance of the working face, the suspended area of the main roof increases continuously. With the increase of suspended area, the overlying strata bend and rotate under the action of overlying loads, and damage cracks continue to form in the tensile stress area. The bending moment of the fixed ends on both sides of the main roof is the largest, and the fracture occurs first, and then the crack in the lower side of the main roof expands and penetrates; the rock beam fractures, rotation, or sliding instability occurs, and the first-weighting is formed [32]. Professor Huang proposed a structural model of an asymmetric three-hinged arch (as shown in Figure 1) for the first fracture in the main roof in a shallow coal seam based on mining practices and physical tests on a shallow coal seam. Professor Huang revealed the mechanism of the first breaking of the main roof by using the damage mechanics method and obtained the formulae for calculating the modulus of the limit span for the main roof. A similar result has been obtained by physical experiments (as shown in Figure 2) [52]. However, the fracture process of the main roof, the main influencing factors, and the support resistance to ensure the stability of the working face still require further study.

![Figure 1. Asymmetrical breaking of the main roof during the first-weighting [25].](image1)

![Figure 2. Physical simulation of the asymmetrical breaking of the main roof during the first-weighting [52].](image2)

Many scholars have already performed much work on the stability and weighting of rock strata based on the theory and method of theoretical mechanics and elastic mechanics. At present, the main roof structure model generally assumes that the rock stratum is a homogeneous and continuous medium. However, after long-term tectonic activity, there are inevitably some joints and fissures of different sizes in the main roof rock mass, which controls the main roof fracture and weighting. In the process of working face advancement, the unloading effect causes these joints and fissures to expand. Therefore, it is possible to study the formation of the damage zone and the characteristics of crack propagation and coalescence of the main roof by means of fracture mechanics.
Therefore, from the perspective of fracture mechanics, combined with the research conclusion of the asymmetric three-hinged arch structural model, this paper studies the mechanical mechanism of formation and expansion of the damage zone in the main roof, predicts the fracturing of the main roof, and analyzes the determination method of the first-weighting interval and support resistance. In this article, first, the fracture characteristics of the main roof with a large mining height are analyzed. We established a fracture mechanics analysis model of a main roof with a crack and solved the stress-intensity factors of crack propagation, and analyzed the crack propagation mechanism. The necessary and sufficient conditions for fracture occurrence and weighting of the main roof are studied. Furthermore, the calculation equation of the first-weighting interval and the support resistance based on the fracture-mechanics model are obtained. The main influencing factors of the support resistance, such as overburden load \( q \), fracture toughness of the rock \( K_C \), crack length \( a \), main roof thickness \( h \), and weighting interval \( l \), are analyzed and summarized. Finally, the theoretical analysis results are verified with an engineering example, and a reasonable value of the support resistance of the working face is obtained.

This paper is organized as follows: In Section 2, the fracture characteristics of a main roof with a large mining height increase during first-weighting are analyzed, and the process of crack propagation and coalescence in the main roof are discussed. In Section 3, a fracture-mechanics model is constructed for the analysis of the main roof asymmetric fracture mechanism, and the sufficient and necessary conditions for the fracture and weighting of the main roof are derived; additionally, the formulae of the first-weighting interval and the support resistance are obtained. In Section 4, the main factors that affect the support resistance are analyzed, and the theoretical research results are verified with an engineering example. The conclusions are drawn in Section 5.

2. Fracture Characteristics of a Main Roof with a Large Mining Height during First-Weighting

2.1. Roof Weighting Characteristics of a Large Mining Height Working Face

To analyze the various characteristics of the roof weighting interval and support resistance under the condition of a large mining height, we analyzed the data of the weighting interval and support resistance of 20 working faces with mining heights of 3–7 m, including the Daliuta, Shigetai and Baode coal mines (as shown in Table 1). The data change characteristics show that with the increase in the mining height, the first-weighting interval increases substantially, from 35.4 m at a mining height of 3 m to 58 m at a mining height of 7 m. The range in which the periodic weighting interval increases is relatively small, i.e., from 11.1 m at a mining height of 4 m to 16 m at a mining height of 7 m. Similarly, the support resistance also increases from 3758 kN at a mining height of 3 m to 10,918 kN at a mining height of 7 m. With the increase in the mining height, the dynamic load coefficient increases from 1.16 to 1.74. The increase in the mining height causes the roof to fracture and rotate differently from the normal mining height working face; the dynamic load is obvious, and there are higher requirements for support resistance. This introduces many problems for the prevention and control of strata behavior disasters and large-scale surface subsidence of the working face.

2.2. Simulation Analysis of the First-Weighting of a Main Roof with Different Mining Heights

Figure 3 shows the similar material model test results of the first-weighting of a main roof with the mining heights of 3 m, 5 m and 7 m. With the increase in the mining height, the main roof gradually presents the structural characteristics of an asymmetric three-hinged arch. When the mining height is 3 m (Figure 3a,b), the breaking of the main roof is basically symmetrical. When the mining height is 5 m (Figure 3c,d), the length of the primary fracture of the main roof is larger than that of the other side near the advancing direction of the working face. When the mining height is 7 m (Figure 3e,f), the breaking of the main roof presents asymmetric characteristics similar to Professor Huang’s conclusion.
The asymmetric fracture phenomenon becomes more obvious with the increase in the mining height.

Table 1. Statistical table of the strata behavior in a large mining height working face.

| Mine Name | Working Face Name | Mining Height/m | First-Weighting Interval/m | Periodic Weighting Interval/m | Maximum Support Resistance /k N | Dynamic Load Coefficient |
|-----------|-------------------|-----------------|-----------------------------|-------------------------------|---------------------------------|-------------------------|
| Daliuta   | 12301             | 3               | 26.9                        | 11.8                          | 3758                            | 1.26                    |
| Shigetai  | 20601             | 4               | 35.3                        | 11.2                          | 5283                            | 1.16                    |
| Daliuta   | 22301             | 4.3             | 21.5                        | 12.6                          | 6987                            | 1.21                    |
| Daliuta   | 24101             | 4.3             | 20.3                        | 13.7                          | 5902                            | 1.28                    |
| Shaqu     | 20604             | 4.2             | 53.5                        | 14.7                          | 5063                            | 1.58                    |
| Kangjiatan| 81300             | 4.3             | 46                          | 13.8                          | 8235                            | 1.55                    |
| Baode     | 88101             | 4.5             | 51                          | 14                            | 7930                            | 1.26                    |
| Daliuta   | 12403             | 4.5             | 52.3                        | 14.5                          | 8457                            | 1.3                     |
| Bulianta  | 22613             | 4.8             | 53.4                        | 14.8                          | 8638                            | 1.32                    |
| Halagou   | 32206             | 5.2             | 57.2                        | 15.7                          | 8675                            | 1.46                    |
| Bulianta  | 22205             | 5.3             | 65                          | 12.7                          | 8540                            | 1.28                    |
| Sihe      | 62101             | 5.4             | 54                          | 15.2                          | 11494                           | 1.54                    |
| Nalinmiao | 2303              | 5.4             | 67                          | 13                            | 7838                            | 1.36                    |
| Yangchangwan | 51101          | 5.5             | 70                          | 16                            | 11202                           | 1.27                    |
| Shangwan  | 110206            | 6               | 49.3                        | 12.4                          | 8120                            | 1.32                    |
| Bulianta  | 31302             | 6               | 45.8                        | 15.9                          | 12718                           | 1.5                     |
| Zhangjiamao | 32301            | 6               | 54.1                        | 15.4                          | 11171                           | 1.5                     |
| Bulianta  | 15201             | 6.3             | 55                          | 15                            | 9976                            | 1.52                    |
| Bulianta  | 22303             | 7               | 58                          | 16                            | 10918                           | 1.57                    |

To analyze the structural characteristics of overlying strata with large mining heights, we used the discrete-element numerical simulation software UDEC to construct a numerical model and analyzed the first-weighting characteristics of the main roof with mining heights ranging from 4–7 m. Because the bedrock and surface loose layer of a shallow seam working face are elastic–plastic geological materials, the material model is calculated using the Moore–Coulomb elastic–plastic theory model. The direction of the numerical model was 200 m, and the vertical height was 60 m, reaching the thick loose layer on the ground. In order to eliminate the influence of the boundary effect on the calculation results, 50 m coal pillars were left on both sides of the model. The upper boundary of the model was free, the bottom boundary and the left and right horizontal boundary displacement were fixed.

The simulation results are shown in Figure 4. Under the condition of a large mining height, the fracture and weighting of the main roof show obvious asymmetric fracture phenomena, forming asymmetric three-hinged arch structures (the breaking angle of the main roof is approximately 76–61°, as shown in Figure 4a–d). The length ratio of the advancing side rock block and the open-off cut rock block was approximately 1.5. The space of the goaf increases in terms of the increase in the mining height, and thus, the collapsed main roof has difficulty forming a stable articulated structure; this makes the rotation angle of the asymmetric three-hinged arch increase continuously (as shown in Figure 4c,d, resulting in different degrees of sliding instability of the main roof (as shown in Figure 4c,d). The larger the mining height of the working face is, the more unstable the asymmetric three-hinged arch structure and the more violent the strata behavior. The subsidence of the overlying strata also increases greatly with an increasing mining height.
Figure 3. Comparison of the first-weighting of a main roof with different mining heights: (a,b) have 3 m mining heights; (c,d) have 5 m mining heights; (e,f) have 7 m mining heights.

Figure 4. Overburden structure under first-weighting at different mining heights: (a) 4 m, advance 10 m each step; (b) 5 m, advance 10 m each step; (c) 6 m, advance 10 m each step; (d) 7 m, advance 10 m each step.
2.3. The Fracture Structural Model of the First-Weighting of the Main Roof

There are joints and fissures in the rock mass that control the fracturing of the main roof rock. With the advance of the working face, the crack in the mining unloading and the overlying strata load effect start to expand until the first-weighting (as shown in Figure 5a) [45–47,53–56]. Finally, asymmetric three-hinged arch structures are formed (as shown in Figure 5b). The results show that the first-weighting of the main roof under the condition of a large mining height also satisfies the asymmetric three-hinged-arch structural model. Therefore, the research conclusion of Professor Huang is still applicable to the fracture analysis of the main roof under the condition of a large mining height.

![Figure 5](image)

**Figure 5.** The simplified structural model of the main roof before and after the first-weighting failure: (a) main roof damage cracks and (b) ground pressure after crack coalescence.

Because there are joints and small faults in the rock beam, it can be assumed that the main roof is a rock beam with an edge crack; when the working face advances to a certain position (the limit span of the main roof), the crack propagates and penetrates under the action of an external load. The process of crack propagation and coalescence is the process of roof rock beam rotation instability. Therefore, the method of studying cracks in fracture mechanics can be used to study the fracture and supporting conditions of main roof rock beams.

3. Construction and Analysis of the Mechanical Model

3.1. Construction of the Fracture-Mechanics Model

The O-X model proposed by Academician Qian [40–42] simplifies the main roof into a two-dimensional rock beam with a crack and hinged support on both sides. During the fracture and weighting of the main roof, the load on the roof is mainly the weight of the rock beam, the uniformly distributed load of the overlying strata \( q_l \), the supporting force of the immediate roof \( Q \), the horizontal squeezing force \( T \) of the rock mass on both sides of the rock mass, and the shear stress \( F \) of the rock mass on both sides. It is assumed that the length of the contact surface between the immediate roof and the main roof is \( m \), and the main roof length (ultimate span) is \( l \). The distance between the crack and the left coal wall is \( c \). Considering the research results of the asymmetric three-hinged arch structure model, \( c = 0.6 \ l \), and the distance between the crack and the open-off cut is \( 0.4 \ l \) [25]. The crack is simplified as a crack with a certain friction effect, the length is \( a \), and the width of the crack plate is \( h \). The main roof rock beam is regarded as a finite plate model with edge cracks. The mechanical analysis model is established as shown in Figure 6.
The crack in the main roof is usually a compound crack under complex loads. The stress-intensity factor can be decomposed into several simple load models for comprehensive analysis. The crack is mainly affected by the tensile stress, shear stress and bending moment; therefore, the decomposed stress-intensity factor calculation models of the three simple loads are shown in Figure 7. The horizontal squeezing force $T$ is decomposed into the uniformly distributed tensile stress $\sigma$ acting on the cross-section of the roof, where $\sigma = T/h$. The overburden load is decomposed into the concentrated force $ql$ and bending moment $M$, in which the concentrated force $ql$, supporting force of the immediate roof $Q$, and shear stress $F$ on both sides of the rock beam constitute the shear stress on the rock beam, and the bending moment $M$ acts on both ends of the rock beam, resulting in the propagation and penetration of the crack in the main roof rock beam.

![Diagram](image)

**Figure 6.** Fracture-mechanics model of the main roof rock beam.

**Figure 7.** Stress-intensity factor calculation for three basic loads: (a) tensile stress, (b) shear stress, and (c) bending moment.

### 3.2. Fracture Mechanics Analysis

According to the equation of the finite plate model, the equation for calculating the stress-intensity factor under various simple loads is as follows [57]:

1. The calculation of the stress-intensity factor $K_{ic}$ of the crack under the action of the horizontal squeezing force (Figure 7a).

   \[
   K_{ic} = \sigma F_\sigma \sqrt{\pi a} \left(\frac{a}{h}\right)
   \]  

   (1)

   Because $T$ is the compressive stress, substituting $\sigma = -T/h$ into Equation (1) gives the following:

   \[
   K_{ic} = \frac{-aT}{h^2} F_\sigma \sqrt{\pi a}
   \]  

   (2)

   where $F_\sigma$ can be obtained from the stress-intensity factor handbook.

   \[
   F_\sigma = 1.12 - 0.231\left(\frac{a}{h}\right) + 10.55\left(\frac{a}{h}\right)^2 - 21.72\left(\frac{a}{h}\right)^3 + 30.39\left(\frac{a}{h}\right)^4
   \]
(2) The calculation of the stress-intensity factor of the crack under the action of concentrated force (Figure 7b).

The components of the shear stress of the crack are as follows: the main roof dead weight, the uniformly distributed load of the overlying strata $ql$, the supporting force of the immediate roof and the shearing force of the coal wall at the open-off cut. The resultant forces constitute the shear force $F_{(c)}$ and the bending moment force $M_{(c)}$ of the crack. According to the method for calculating the shear stress in material mechanics, the calculation result of the shear stress at the crack is obtained as follows:

$$F_{(c)} = \frac{2ql}{2} - \frac{ql}{2} + \frac{Qm^2}{\delta t}$$

$$M_{(c)} = -\frac{Qm^2}{\delta t^2} + \frac{8ql^2}{25} - \frac{ql^2}{5} + \frac{Qm^2}{\delta t}$$

$$K_{II} = \left( \frac{Qm^2}{3\delta t} - \frac{q^2}{5} \right) \frac{F_{\tau}}{\sqrt{\pi a}}$$

The stress-intensity factor under the shear stress at the crack can then be expressed as follows:

$$F_{\tau} = \frac{1.3 - 0.65\frac{a}{h} + 0.37\left(\frac{a}{h}\right)^2 + 0.28\left(\frac{a}{h}\right)^3}{\sqrt{1 - a/h}}$$

(3) The calculation of the stress-intensity factor of the crack under the action of the bending moment (Figure 7c).

The crack affected by the bending moment can be simplified into a purely curved single-side crack, in which the rock beam is mainly subjected to shear stress $F$ on the coal wall on both sides, the supporting force of the immediate roof, and the uniformly distributed load of the overlying strata $ql$. The resultant of these three forces constitutes the bending moment $M_{(c)}$ effect on the rock beam. Referring to the material mechanics calculation method, the calculation result for $M_{(c)}$ is shown in Equation (4), which causes the crack propagation. The stress-intensity factor under the action of the bending moment can then be expressed as follows:

$$K_I = F_M\sigma_M\sqrt{\pi a}$$

where $\sigma_M = \frac{6M}{\delta t^2}$ is the stress value of $\sigma_x$; when $x = a, x \in (-h/2, h/2)$. Substituting the stress value into Equation (4) and simplifying it to obtain the $\sigma_M$ expression at the crack gives the following:

$$\sigma_M = \frac{18ql^2}{25h^2}$$

Substituting Equation (7) into Equation (6) gives the following:

$$K_I = \frac{18ql^2F_M}{25h^2}\sqrt{\pi a}$$

where $F_M$ can be obtained from the stress-intensity factor handbook

$$F_M = 1.122 - 1.4\frac{a}{h} + 7.33\left(\frac{a}{h}\right)^2 - 13.08\left(\frac{a}{h}\right)^3 + 14\left(\frac{a}{h}\right)^4$$

From the above analysis, it can be found that the stress-intensity factor of the crack tip is directly related to the length of the crack and the thickness of the rock beam.
The stress-intensity factor of the crack tip of the rock beam is the superposition of the stress-intensity factor under the above three simple loads, namely,

\[
\begin{align*}
K_I &= \frac{18q l^2 F_M}{25h^2} \sqrt{\pi a} - \frac{aT}{h^2} F_c \sqrt{\pi a} \\
K_{II} &= \left(\frac{Q m^2}{3l^2} - \frac{q l}{5}\right) \frac{F_c}{\sqrt{\pi a}} 
\end{align*}
\]  
(9)

From Equation (19), we find that the stress-intensity factor is not only directly related to the length of the crack and the thickness of the rock beam, but is also related to the overlying strata load \( q \), the main roof weighting interval \( l \), the horizontal extrusion force \( T \) and the support resistance \( Q \). All of these factors determine the activation and propagation of the crack in the rock beam. With the increase in the crack length \( a \), \( K_I \) first increases and then decreases; it reaches the maximum value when \( a \) is near 5 m and then decreases continuously, and \( K_{II} \) decreases continuously, which indicates that the increase in crack length \( a \) is more likely to cause the opening mode propagation of damaged cracks. When the thickness of the main roof increases, \( K_I \) decreases in the form of a power function, while \( K_{II} \) is not affected. Because the crack propagation path is longer, the main roof is not easy to fracture. With the increase of the overlying strata load \( Q \) and weighting interval \( l \), \( K_I \) increases linearly, while \( K_{II} \) decreases linearly, which indicates that the increase in the overlying strata load \( Q \) and weighting interval \( l \) more easily achieves the fracture toughness of roof strata and causes the expansion of crack damage, which is similar to the analysis conclusion for crack length \( a \). With the increase in horizontal stress \( T \), \( K_I \) decreases linearly, which is not conducive to the propagation of crack damage. The above analysis results are consistent with engineering practice, which verifies the correctness of the theoretical model.

According to a large number of test studies [58], the criterion for rock and concrete compression and shear fracture can be expressed as follows:

\[
\lambda \sum K_I + \left| \sum K_{II} \right| = K_c
\]
(10)

where \( \lambda \) is the compression ratio of crack propagation, and \( K_c \) is the fracture toughness of the rock.

Substituting Equation (9) into Equation (10) gives the following:

\[
\lambda \left[ \frac{18q l^2 F_M}{25h^2} \sqrt{\pi a} - \frac{aT}{h^2} F_c \sqrt{\pi a} \right] + \frac{F_c}{\sqrt{\pi a}} \left(\frac{Q m^2}{6l} - \frac{q l}{10}\right) = K_c
\]
(11)

3.3. Analysis of the First-Weighting Interval

The shear stress has little effect on the crack growth of the main roof [45,53], so we can conveniently ignore the influence of shear stress when calculating the first-weighting interval \( l \); that is, \( F_{(c)} = 0 \), and \( M_{(c)} = 0 \). Then, Equation (11) can be simplified as follows:

\[
\lambda \left[ \frac{18q l^2 F_M}{25h^2} \sqrt{\pi a} - \frac{aT}{h^2} F_c \sqrt{\pi a} \right] = K_c
\]
(12)

The first-weighting interval of the main roof (or the limit span) based on fracture mechanics can be approximately expressed as follows:

\[
l = \frac{5}{3} \sqrt{\frac{h^2 K_c}{\lambda \sqrt{\pi a} + a T F_c}}/2q F_M
\]
(13)

Equation (14) is the ultimate span of the main roof at the first-weighting, which is derived from material mechanics.

\[
L_b = h \sqrt{\frac{2R_T}{q}}
\]
(14)
where \( R_T \) is the ultimate tensile strength of rock.

Comparison of Equations (13) and (14) reveals that the equation of the main roof first interval deduced from fracture mechanics is more complex than that deduced from material mechanics. Because the defects and the characteristics of the mining damage in the main roof are considered in the conclusion of fracture mechanics analysis, the factors such as crack length \( a \) and horizontal extrusion force \( T \) are comprehensively considered. However, only the bending and tensile fracture modes of the rock beam under uniform load are considered in the material mechanics derivation.

The influence of the crack length \( a \), the load of the overlying strata \( q \), the main roof thickness \( h \) and the horizontal squeezing force \( T \) on the first-weighting interval \( l \) are shown in Figure 8. It can be found from the curves that with the increase in the crack length, the weighting interval first increases and then decreases. The weighting interval varies greatly in the range of 1~3 m and varies slightly in the range of 4~11 m, which indicates that the influence of cracks on weighting interval is larger in a certain range, and the subsequent influence is relatively weak (Figure 8a). The overlying strata load has a great influence on the weighting interval, which indicates that the overlying strata load is the main factor controlling the weighting interval (Figure 8b). With the increase of the thickness of the main roof, the weighting interval increases approximately linearly, and the larger the main roof thickness, the more difficult it is to fracture (Figure 8c). With the increase of the horizontal extrusion stress \( T \), the weighting interval increases in the form of a parabola, but the change range is relatively small. (Figure 8d). In a shallow coal seam, the horizontal squeezing force of the rock beam is generally very small. With the decrease in horizontal squeezing force, the first-weighting interval of the main roof gradually decreases, which proves that the first-weighting interval of the main roof in a shallow coal seam is generally small. Comparing the four curves, we find that the influence weight of each parameter on the first-weighting interval is as follows: the load of the overlying strata \( q > \) main roof thickness \( h > \) horizontal squeezing force \( T > \) crack length \( a \).

![Figure 8](image-url)  
**Figure 8.** Stress-intensity factor calculation for three basic loads: (a) crack length, (b) load of the overlying strata, (c) main roof thickness, and (d) horizontal squeezing force.
3.4. Analysis of the Support Resistance

Equation (11) is simplified, and the calculation expression for the supporting force of the main roof under static loading can be obtained as follows:

\[ Q = \frac{3lK_c\sqrt{\pi a}}{F_Tm^2} + \lambda \left[ \frac{3l_a^2TF_T\pi}{h^2F_Tm^2} - \frac{18qI^3F_M\pi a}{25h^2F_Tm^2} \right] + \frac{3qI^2F_T}{5F_Tm^2} \]  

(15)

According to the cantilever beam-masonry beam structural model of the roof with a large mining height, the cantilever beam and the main roof are supported by the support. The supporting force of the main roof in Equation (15) is the difference between the support resistance and the weight of the cantilever beam, so the weight of the cantilever beam must be included in calculating the support resistance. The load of the cantilever beam structure can be expressed as follows \[3\]:

\[ L_i = h_i \sqrt{R_{Ti}/3q_i} \]  

(16)
\[ G_i = L_i bh_i \]  

(17)
\[ x_i = \frac{L_i}{2} \]  

(18)
\[ F_X = k \frac{\sum_{i=1}^{n} G_i x_i}{\mu} \]  

(19)

Substituting Equations (16)–(18) into Equation (19), the expression for the force of the cantilever beam acting on the support can be obtained as follows:

\[ F_X = k \frac{\sum_{i=1}^{n} L_i bh_i^2}{2} \sqrt{R_{Ti}/3q_i} \]  

(20)

where \( F_X \) is the cantilever beam force, kN; \( h_i \) is the thickness of the \( i \)-th layer cantilever beam, m; \( L_i \) is the length of the \( i \)-th layer cantilever beam, m; \( R_{Ti} \) is the tensile strength of the rock, MPa; \( q_i \) is the load per unit length of the cantilever beam, MPa; \( G_i \) is the self-weight of the cantilever beam in the \( i \)-th layer, kN; \( x_i \) is the horizontal distance between the coal wall and the center of gravity of the cantilever beam in the \( i \)-th layer, m; \( l \) is the distance from the coal wall to the centerline of the support column, m; \( k \) is the design factor considering the advance of the adjacent support, which is usually 1.10–1.25; and \( b \) is the width of the hydraulic support.

Substituting Equation (20) into Equation (15) and considering the support efficiency of the support, the support resistance can be expressed as follows:

\[ R = \frac{Q+F_X}{\mu} \]  

\[ = \frac{3lK_c\sqrt{\pi a}}{\mu F_Tm^2} + \lambda \left[ \frac{3l_a^2TF_T\pi}{\mu h^2F_Tm^2} - \frac{18qI^3F_M\pi a}{25\mu h^2F_Tm^2} \right] + \frac{3qI^2F_T}{5\mu F_Tm^2} + \frac{k}{\mu} \frac{\sum_{i=1}^{n} L_i bh_i^2}{2} \sqrt{R_{Ti}/3q_i} \]  

(21)

where \( \mu \) is the support efficiency of the support.

We consider the activation of the crack as the necessary condition for roof fracture and weighting to occur. Equation (21) of the support resistance is deduced from the fracture mechanics analysis method, so the support resistance can be used as one of the necessary conditions for crack activation and propagation. When the actual value of the support resistance is less than the calculated value, the crack starts to activate. The function of the support is to provide enough supporting force to ensure that the main roof can rotate into the goaf after the crack is penetrated, instead of step sinking or cutting off above the roof while avoiding the impact of unstable weighting on the working face. The first-weighting of the working face is inevitably caused by the formation, propagation and penetration of the crack. When the support force is insufficient, the crack forms and propagates more easily, and the main roof is more prone to the rotation.
3.5. Analysis of the Crack Penetration Process

The model of compressive-shear cracks under the action of a complex load is shown in Figure 9. The crack length is \( u \), which is under the action of the maximum and minimum principal stresses \( \sigma_1 \) and \( \sigma_3 \). According to the compression-shear crack propagation theory, the crack will inevitably generate airfoil branch fissures along the direction of the maximum compressive stress. Because \( \sigma_3 \) on the crack surface is tensile stress, the crack propagation belongs to the growth of I and II compound cracks. Assuming that \( \sigma_1 \) is the maximum compressive stress, the branch crack generated during crack propagation will propagate along the direction of the maximum compressive stress, that is, vertically upward. When the crack extension length reaches \( H \) (as shown in Figure 6) (that is, the branch crack penetrates the main roof), it will cause the first-weighting. Therefore, it is a sufficient condition when the branch fracture penetrates the main roof, or the propagation length reaches \( H \).

![Figure 9. Compressive shear model of the crack.](image)

The stress state on the crack surface can be expressed as follows:

\[
\begin{align*}
\sigma_n &= \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha \\
\tau &= \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha
\end{align*}
\]  
\[
(22)
\]

where \( u \) is the angle between the long axis direction and the minimum principal stress direction of the crack.

The stress-intensity factor of the branch crack tip is composed of the stress-intensity factor \( (K_{I})_1 \) generated by the shear stress on the crack surface and the stress-intensity factor \( (K_{I})_2 \) generated by the far-field lateral stress \( \sigma_3 \) [45,46,59].

\[
K_I = (K_{I})_1 + (K_{I})_2 = \frac{2\tau \sin \alpha}{\sqrt{a}} - \sigma_3 \sqrt{\pi s}
\]

\[
(23)
\]

where \( s \) is the length of the branch crack.

According to the principle of fracture mechanics, the lateral stress of the crack under the mining stress is tensile stress. The stress-intensity factor and the length of the branch crack tip increase until it propagates to the main roof. Under the action of lateral compressive stress, the stress-intensity factor \( K_I \) of the branch crack tip decreases with increasing crack length, and the crack stops expanding when \( K_I = K_C \). Since the crack of the roof is
generally a vertical crack with an inclination angle of 90°, the length of the airfoil branch fissure propagation can be expressed as follows:

\[ s = \frac{K_C^2}{\pi \sigma_3^2} \]  

(24)

When the crack propagation length reaches \( H \), it indicates that the crack penetrates through the main roof, and the first-weighting occurs, which is a sufficient condition for the first-weighting of the working face. Equation (24) shows that the length of the branch crack is proportional to the square of the stress-intensity factor at the crack tip and is negatively related to the square of the resultant force on the crack surface. That is, the lower the rock mass strength is, the greater the crack propagation length and the easier it is to penetrate the roof. According to the theory of elasticity, the stress-intensity factor is much faster than the stress growth. The main roof protection thickness \( H \) decreases as the crack propagation length accumulates.

4. Analysis of the Influencing Factors and Engineering Examples

4.1. Analysis of the Influencing Factors of the Support Resistance

To thoroughly analyze the main influencing factors of the support resistance during the first-weighting, we analyzed the main parameters and the geological engineering conditions of the no. 33,206 fully mechanized coal mining face in the Bulianta mining area. Figures 10–14 shows the corresponding support resistance curve when the crack length is \( a \), the main roof thickness is \( h \), the main roof weighting interval is \( l \), the load of the overlying strata is \( q \), and the rock fracture toughness \( K_C \) changes.

![Figure 10. The relationship between support resistance and the main roof thickness.](image)

![Figure 11. The relationship between support resistance and the crack length.](image)
Figure 12. The relationship between support resistance and the weighting interval.

Figure 13. The relationship between support resistance and the load of the overlying strata.

Figure 14. The relationship between support resistance and the fracture toughness of the rock.

It can be found from the curves in Figures 10–14 that the support resistance has an exponential function correlation with the main roof thickness $h$, a positive parabolic correlation with the crack length $a$, a parabolic negative correlation with the weighting interval $l$, a negative linear correlation with the overlying strata load $q$, and a positive linear correlation with the rock fracture toughness $K_C$. As the main roof thickness increases, the path of the crack through the roof is extended, the rotation angle required for its fracture is increased, and the pressure on the support is reduced upon the rotation instability of the main roof. As the first-weighting interval $l$ increases, the load value acting on the support increases, and the support resistance increases accordingly. The first-weighting interval $l$ is inversely proportional to the support resistance (Figure 12). As the support resistance...
increases, the shear capacity of the support to the roof increases, which accelerates the collapse of the main roof and reduces the weighting interval. When the load $q$ of the overlying strata increases, the force transmitted to the support through the main roof fracture increases. The force on the support will increase with the increasing of the first-weighting interval and the load of the overlying strata, which is consistent with the actual situation of the working face. The support resistance is positively parabolic in relation to the crack length $a$. When the crack length $a$ is short, the larger the overhanging area of the main roof is when the crack penetrates, the greater the roof weighting when the rotation instability of the main roof is evident. As the length of the crack increases, the propagation path of the crack through the main roof decreases, the roof breaks and causes faster weighting, and the support resistance increases. Therefore, the effect of the crack length on the support is achieved through the fracture and instability of the main roof. By comparing the five parameters, we find that the influence weight on the support resistance is as follows: overburden load $q >$ rock fracture toughness $K_C >$ crack length $a >$ main roof thickness $h >$ weighting interval $l$.

4.2. Engineering Example

The Bulianta no. 33206 fully mechanized mining face of the Shendong coalfield was a Quaternary loose sand layer with a thickness of 60 m. The bedrock thickness was 64 m. The total length of the working face was 301 m. The strike length was 2474 m. The dip angle of the coal seam was 1–3$^\circ$, the unit weight was $1.28 \times 10^3$ kg/m$^3$, the average thickness was 5.96 m, and the design mining height was 5.5 m. The immediate roof was mainly composed of siltstone and mudstone, with a thickness of 6.5–12.5 m and an average thickness of 8.4 m. The main roof was mainly fine sandstone and siltstone. The floor was muddy sandstone and fine sandstone. The comprehensive histogram is shown in Table 2. The working face adopted 155 sets of domestic ZY11000 hydraulic supports, the set load was 7800 kN, and the rated support resistance was 11,000 kN.

**Table 2. Lithology of the no. 33,206 working face.**

| Geotechnical Name | Thickness/m | Burial Depth/m |
|-------------------|-------------|----------------|
| Sand layer        | 60          | 60             |
| Siltstone         | 7.22        | 67.22          |
| Fine sandstone    | 14.13       | 81.35          |
| Siltstone         | 15.15       | 90.01          |
| Fine sandstone    | 8.66        | 98.67          |
| Siltstone         | 7.38        | 106.05         |
| Fine sandstone    | 6.52        | 112.57         |
| Mudstone          | 5.77        | 118.34         |
| Siltstone         | 5.74        | 124.08         |
| 2−2 coal seam     | 5.5         | 129.58         |

The no. 33,206 large mining height working face was commissioned on 5 June 2019, and the open-off cut forced caving began on 8 June. When the working face advanced to 55 m, weighting was applied for the first time, which lasted for two days. During the applied weighting, the working face had obvious step sinking, and there were obvious vertical and horizontal movement cracks on the surface (as shown in Figure 15). The subsidence was between 180 mm and 220 mm, with an average of 205 mm. There was the frequent opening of the safety valve, rapid roof sinking, and great danger of support crushing. The rib fall of the coal wall was seriously cracked, and the average rib fall depth was 1200–1400 mm (as shown in Figure 16). The dynamic loading phenomenon was obvious in the weighting process. The dynamic load factor reached 1.86 on average, and the roof activity was intense. Therefore, it is necessary to analyze the applicability of the hydraulic support with a yield load of 11,000 kN.
Figure 15. Step sinking of the ground.

Figure 16. Rib fall of the coal wall.

The monitoring curves of the support resistance during the first-weighting are shown in Figure 17. The working resistance change curve of each support during the first-weighting is shown in Figure 18. It can be found from the analysis of the change characteristics of the monitoring curves that the first-weighting distance of the working face was approximately 55 m, and the support resistance of the working face before the first-weighting was small, i.e., generally between 5020 kN and 7530 kN. When the working face advanced by approximately 55 m, the support resistance of no. 50–120 in the middle of the working face increased rapidly, and the maximum value reached 11,848 kN (blue area in Figure 18). The rib fall of the coal wall was serious during the first-weighting, and the maximum depth reached 1400 mm. Frequent opening of the safety valve, rapid roof sinking, and great danger of support crushing occurred. This seriously affected the mining progress of the working face and introduced safety hazards to personnel and equipment.

Figure 17. The monitoring curve of the support resistance during the first-weighting.
Based on the above research, we analyzed the instability condition of the no. 33,206 working face in the Bulianta coal mine, calculated its reasonable support resistance and verified the theoretical analysis results. According to the actual mining conditions of the working face, the following parameters were determined: the support width is \( b = 1.75 \) m, the top distance of the support control is \( l_b = 2.2 \) m, the length of the top support beam is \( 5.5 \) m, the distance from the coal wall to the centerline of the support column is \( 3.8 \) m, the distance from the coal wall to the centerline of the support column is \( 3.8 \) m, \( \mu = 0.9 \), \( q = 0.38 \) MPa, \( R_f = 3.7 \) MPa, \( T = 2.3 \) MPa, \( \sigma_3 = 0.21 \) MPa, \( a = 4.2 \) m, \( \lambda = 1 \), \( K_c = 1.04 \text{ MN/m}^{3/2} \), \( h = 11.2 \) m, \( l = 55 \) m, and \( m = 13.3 \) m.

(1) **Judgment of crack propagation and penetration**

\[
S = \frac{K_c^2}{\pi \sigma_3^2} = \frac{(1.04 \times 10^3)^2}{3.14 \times (0.21 \times 10^3)^2} = 7.8 \text{ m}
\]

The minimum safe length \( s \) to ensure the stability of the main roof is 7.8 m, which exceeds the 7 m length of the main roof thickness \( H \). The no. 33,206 working face meets the sufficient conditions for instability to occur. Under the current mining conditions, the fracture and weighting of the main roof are prone to occur. It can be found from the in situ monitoring that the support resistance exceeds the rated value and that the rib fall of the coal wall is serious, which indicates that the theoretical calculation is consistent with the actual situation. Therefore, it is necessary to adjust the external conditions, such as increasing the support resistance or improving the advanced speed, to prevent roof accidents.

(2) **Calculation of the first-weighting interval**

The first-weighting interval can be calculated by Equation (13).

\[
l = \frac{5}{3} \sqrt{\frac{b^2 K_c}{2 a \sqrt{\lambda / q} \sigma_3} + a T F_c / 2 q F_M} = 54.3 \text{ m}
\]

The first-weighting interval can be calculated by Equation (14).

\[
L_b = h \sqrt{\frac{2 K_c}{q}} = 11.2 \times \sqrt{\frac{2 \times 3.7 \times 10^3}{0.38 \times 10^3}} = 49.42 \text{ m}
\]

Equation (22) is the limit span of the main roof considering damage accumulation [52], where \( n \) represents the number of excavation (advance) cycles when the limit span is...
reached, \( k \) is the parametric variable related to the material and loading condition, which is simplified to a constant for analysis, and \( L_0 \) is the advance distance of each excavation cycle.

\[
L_{iT}^* = h \sqrt{\frac{2R_T}{q} - \frac{kNR_TL_0^2}{6h} (2N^2 + 3N + 1)}
\]  

(25)

According to Equation (25), the first-weighting interval is 53.2 m.

Comparing the calculation results of Equations (13), (14) and (25), it can be found that the calculation result of Equation (14) is closer to the actual first-weighting interval of 55 m. The results show that it is scientifically feasible to analyze the first-weighting interval by fracture mechanics.

(3) Calculation of the support resistance

The support resistance can be determined by Equation (21).

\[
R = \frac{Q+F_{S}}{\mu} = 3lK_c\sqrt{\pi a} + \lambda \left[ \frac{3sl^2TF_{S} \pi}{h^2F_{S}m^2} - \frac{18ql^3F_{M} \pi a}{25F_{M}l^2m^2} \right] + \frac{3ql^2T_{F}}{5\mu l^2m^2} + \frac{k}{\mu l} \sum_{i=1}^{N} \frac{l_i^2h_i^2}{2} \sqrt{R_{Ti}/3l_i} 
\]

The calculated support resistance must be at least 12,075 kN to effectively control the main roof without the occurrence of coal wall sliding. The theoretical calculation result is close to the actual monitoring result of 11,850 kN but exceeds the rated support resistance of 11,000 kN. Under the current support conditions, roof fracture and weighting are inevitable. Therefore, it is necessary to improve the support resistance to ensure that the working face can advance safely during the first-weighting, and there will be no support crushing accidents.

5. Conclusions

In this paper, the asymmetric fracture mechanical mechanism of the first-weighting of the main roof was analyzed using the fracture mechanics method. A mechanical model was developed to study the main roof fracture mechanism, the expression of stress-intensity factors at the crack tip was obtained, and both the necessary and sufficient conditions for the fracture and weighting of the main roof were obtained. The calculation equations of the first-weighting interval and the support resistance were deduced, and the variation characteristics under the action of the main influencing factors were analyzed. The validity of the theoretical analysis was verified with an engineering example. The following conclusions can be drawn through the analysis of the theoretical results:

(1) The comprehensive field monitoring, physical simulation and numerical analysis show that the fracture of the main roof with a large mining height has an obvious asymmetric fracture phenomenon and still conforms to the asymmetric three-hinged arch structural model;

(2) The fracture mechanics analysis model for the first-weighting is established. The expressions of the stress-intensity factors at the crack tip are obtained by decomposing the stress-intensity factor into several simple load models for comprehensive analysis. The stress-intensity factor increases at the crack tip with the advancement of the face; when the stress-intensity factor increases beyond the critical value, the crack will expand until the first-weighting;

(3) When the airfoil branch fissure propagation length \( S \) reaches the critical value \( H \), the crack penetrates through the main roof, which is the sufficient condition for the fracture of the main roof to occur;

(4) The calculation equations of the first-weighting interval and the support resistance are deduced by using the fracture mechanics analysis method; the influence weights of each parameter on the first-weighting interval are in the following order: over-
burden load $q >$ rock fracture toughness $K_C >$ crack length $a >$ main roof thickness $h >$ weighting interval $l$;

(5) The stability of the no. 33,206 working face is calculated and analyzed by using the theoretical analysis results. The results show that the working face meets the instability condition, which is consistent with the monitoring results. The support resistance should be greater than 12,075 kN to ensure the stability of the main roof.

It should be pointed out that the criterion of the first-weighting of the main roof is derived by using fracture mechanics. However, the methods for researching the failure of rock materials are complex and diverse, and the differences in the main roof fracture characteristics under different failure criteria need to be further studied.

At present, there is still a lack of reasonable and effective research on large-scale strata behavior in the process of first-weighting. In this paper, the first fracture mechanism of large mining height main roof, first-weighting interval, support resistance and other contents are helpful to the analysis and control of roof disaster in the process of first-weighting, which has a certain guiding significance for the safety and stability of working face and is also conducive to reducing the occurrence of mining geohazards and eco-environmental issues such as land subsidence, collapse, water and soil loss. The research conclusions can have a positive impact on the safety and sustainable mining of coal resources in the Northwestern territory of China and are conducive to environmental safety and stability.

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**Abbreviations**

- $h$: The thickness of the main roof;
- $H$: The protection thickness of the main roof;
- $\Sigma H$: The total thickness of the immediate roof;
- $M$: The mining height;
- $q$: The weight of the overlying loose layer and main roof;
- $l$: The first weighting interval;
- $T$: The horizontal squeezing force;
- $Q$: The supporting force of the immediate roof;
- $F$: The horizontal squeezing force $T$ of the rock mass on both sides;
- $F$: The shear stress $F$ of the rock mass on both sides;
- $m$: The length of the contact surface between the immediate roof and the main roof;
- $c$: The distance between the crack and the left coal wall;
- $a$: The edge crack length;
- $\sigma_M$: The stress value;
- $\lambda$: The compression ratio of crack propagation;
- $K_C$: The fracture toughness of the rock;
- $F_X$: The cantilever beam force, kN;
- $h_i$: The thickness of the i-th layer cantilever beam, m;
- $L_i$: The length of the i-th layer cantilever beam, m;
- $R_{Ti}$: The tensile strength of the rock, MPa;
\( q_i \) The load per unit length of the cantilever beam, MPa;
\( G_i \) The self-weight of the cantilever beam in the i-th layer, kN;
\( x_i \) The horizontal distance between the coal wall and the center of gravity of the cantilever beam in the i-th layer, m;
\( l_r \) The distance from the coal wall to the centerline of the support column, m;
\( k_C \) The design factor considering the advance of the adjacent support, which is usually 1.10–1.25;
\( b \) The width of the hydraulic support;
\( a \) The angle between the long axis direction and the direction of the minimum principal stress of the crack;
\( s \) The length of the branch crack.

References
1. Huang, Q.X. Study on Roof Structure and Ground Control in Shallow Seam Longwall Mining; China University of Mining and Technology Press: Xuzhou, China, 2010.
2. Yan, S.H.; Yin, X.W.; Xu, H.J.; Xu, G.; Liu, Q.M.; Yu, L. Roof structure of short cantilever-articulated rock beam and calculation of support resistance in full-mechanized face with large mining height. J. China Coal Soc. 2011, 36, 1816–1820.
3. Li, H.M.; Jiang, D.J.; Li, D.Y. Analysis of ground pressure and roof movement in fully-mechanized top coal caving with large mining height in ultra-thick seam. J. China Coal Soc. 2014, 39, 1956–1960.
4. Wang, G.F.; Pang, Y.H.; Li, M.Z.; Ma, Y.; Liu, X.H. Hydraulic support and coal wall coupling relationship in ultra large height mining face. J. China Coal Soc. 2017, 42, 518–526.
5. Xu, J.F.; Xu, J.L. The structural morphology of key stratum and its influence on strata behaviors in fully-mechanized face with super-large mining height. Chin. J. Rock Mech. and Eng. 2011, 30, 1547–1556.
6. JU, J.F.; Xu, J.L.; Wang, Q.X. Cantilever structure moving type of key strata and its influence on ground pressure in large mining height face. J. China Coal Soc. 2011, 36, 2115–2120.
7. Yu, B.; Zhao, J.; Kuang, T.J.; Meng, X.B. In situ investigations into overburden failures of a super-thick coal seam for longwall top coal caving mining. J. China Coal Soc. 2015, 40, 277–288. [CrossRef]
8. Yang, S.L.; Wang, Z.H.; Kong, D.Z.; Cheng, Z.B.; Song, G.F. Overlying strata failure process and support resistance determination in large mining height face. J. China Coal Soc. 2017, 42, 590–596. [CrossRef]
9. Bai, J.Z.; Liu, Q.J. Analysis and measured of strata behavior law and mechanism of 8.8 m ultra—High mining. Coal Sci. Technol. 2013, 37, 737–742.

Sustainability 2021, 13, 1678
25. Huang, Q.X.; Qi, W.T.; Yang, C.L. Analysis of mechanism and form of main roof breaking during first weighting in longwall face. *J. Xi’an Univ. Sci. Technol.* 1999, 19, 193–197.

26. Huang, Q.X.; Huang, K.J.; Zhao, M.Y. Research on roof structure and support resistance during first periodic weighting in shallow group coal seams. *J. Min. Saf. Eng.* 2018, 35, 940–944.

27. Feng, J.F.; Zhou, Y.; Zhang, K.Z.; Jiang, D.J.; Liu, C. The influence of mining height increase on weighting intervals in the fully-mechanized panels of Shendong coal field. *J. Min. Saf. Eng.* 2017, 34, 632.

28. Yang, D.F.; Zhang, Y.J.; Chen, Z.H. Analysis on catastrophe theory during first weighting sliding instability and support crushing of main roof with large mining height in shallow coal seam. *Appl. Sci.* 2020, 10, 5408. [CrossRef]

29. Jiang, H.J.; Cao, S.G.; Zhang, Y.; Wang, C. Study on the first failure and caving mechanism of key strata of shallow coal seam. *J. Min. Saf. Eng.* 2016, 33, 860–866.

30. Huang, P.; Ju, F.; Jessu, K.; Xiao, M.; Guo, S. Optimization and Practice of Support Working Resistance in Fully-Mechanized Top Coal Caving in Shallow Thick Seam. *Energies* 2017, 10, 1406. [CrossRef]

31. Zhao, Y.X.; Wang, X.Z.; Zhou, J.L.; Li, Q.S.; Zhang, C. Influence of main roof thickness-span ratio on the initial cracking induced instability in fully mechanized longwall face. *J. China Coal Soc.* 2019, 44, 94–104.

32. Wang, J.C.; Wang, Z.H. Stability of main roof structure during the first weighting in shallow high-intensity mining face with thin bedrock. *J. Min. Saf. Eng.* 2015, 32, 175–181.

33. Yang, D.F.; Zhang, Y.J.; Chen, Z.H. Analysis on catastrophe theory during first weighting sliding instability and support crushing of main roof in shallow coal seam. *Energies* 2016, 33, 860–866.

34. Yang, Z.L. Stability of nearly horizontal roof strata in shallow seam longwall mining. *Int. J. Rock Mech. Min. Sci.* 2010, 47, 672–677. [CrossRef]

35. Yi, K.; Gong, P.L.; Liu, C. Overlying strata structures and roof control of working face under thin topsoil and thin bedrock in shallow seam. *J. China Coal Soc.* 2018, 43, 1230–1237.

36. Wang, S.R.; Hagan, P.; Cheng, Y.; Wang, H. Experimental research on fracture hinged arching process and instability characteristics for rock plates. *Chin. J. Rock Mech. Eng.* 2012, 31, 1674–1679.

37. Wang, S.R.; Wu, X.G.; Zhao, Y.H.; Hagan, P.; Cao, C. Evolution characteristics of composite pressure-arch in thin bedrock of overlying strata during shallow coal mining. *Int. J. Appl. Mech.* 2019, 11, 3. [CrossRef]

38. Zhao, Y.H.; Wang, S.R.; Zou, Z.Z.; Ge, L.L.; Cui, F. Instability characteristics of the cracked roof rock beam under shallow mining conditions. *Int. J. Min. Sci. Technol.* 2018, 28, 437–444. [CrossRef]

39. Sun, Y.J.; Zuo, J.P.; Karakus, M.; Wang, J.T. Investigation of movement and damage of integral overburden during shallow coal seam mining. *Int. J. Rock Mech. Min. Sci.* 2019, 117, 63–75. [CrossRef]

40. Qian, M.G.; Shi, P.W.; Xu, J.L. *Mining Induced Pressure and Strata Control*; China University of Mining and Technology Press: Xuzhou, China, 2010; pp. 73–84.

41. Qian, M.G.; Xu, J.L. Behaviors of strata movement in coal mining. *J. China Coal Soc.* 2019, 44, 973–984.

42. Qian, M.G.; Zhu, D.R.; Wang, Z.T. The fracture types of main roof and their effects on roof pressure in coal face. *J. China Univ. Min. Technol.* 1986, 15, 9–18.

43. Wang, J.A.; Shang, X.C.; Liu, H.; Hou, Z.Y. Study on fracture mechanism and catastrophic collapse of strong roof strata above the mined area. *J. China Coal Soc.* 2008, 33, 850–855.

44. PU, H.; Huang, Y.G.; Chen, R.H. Mechanical analysis for X-O type fracture morphology of stope roof. *J. China Univ. Min. Technol.* 2011, 40, 835–840.

45. Chen, Z.H.; Hu, Z.P.; Li, H.; Chen, Q.F. Fracture mechanical model and criteria of insidious fault water inrush in coal mines. *J. China Univ. Min. Technol.* 2011, 40, 673–677.

46. Wang, J.S.; Yao, D.X.; Huang, H. Critical criterion and physical simulation research on progressive ascending water inrush in hidden faults of coal mines. *J. China Coal Soc.* 2018, 43, 2014–2020.

47. Yang, D.F.; Zhang, L.F.; Chai, M.; Li, B.; Bai, Y.F. Study of roof breaking law of fully mechanized top coal caving mining in ultra-thick coal seam based on fracture mechanics. *Rock Soil Mech.* 2016, 37, 2034–2039.

48. Zhao, J.W.; Zhou, H.W.; Xue, D.J.; Su, T.; Deng, H.L.; Yang, H.Z. Expansion law of seepage path in the concealed structural floor of coal seam in deep confined water. *J. China Coal Soc.* 2019, 44, 1836–1845.

49. Zhang, J.G.; Miao, X.X.; Huang, Y.L.; Li, M. Fracture mechanics model of fully mechanized top coal caving of shallow coal seams and its application. *Int. J. Min. Sci. Technol.* 2014, 24, 349–352. [CrossRef]

50. Gao, M.S.; Liu, Y.M.; Zhao, Y.C.; Gao, X.J.; Wen, Y.Y. Roof burst instability mechanism and dynamic characteristic of deep coal roadway subjected to rock burst. *J. China Coal Soc.* 2017, 42, 1650–1655.

51. Huang, Q.X.; Gao, Z.N. Mechanical model of fracture and damage of coal bump in the entry. *J. China Coal Soc.* 2001, 26, 156–159.

52. Yang, S.L.; Wang, J.C.; Yang, J.H. Physical analog simulation analysis and its mechanical explanation on dynamic load impact. *J. China Coal Soc.* 2017, 42, 335–343.

53. Chen, Z.H.; Feng, J.J.; Xiao, C.C.; Li, R.H. Fracture mechanical model of key roof for fully-mechanized top-coal caving in shallow thick coal seam. *J. China Coal Soc.* 2007, 32, 449–452.

54. Pang, Y.H.; Wang, G.F.; Ding, Z.W. Mechanical model of water inrush from coal seam floor based on triaxial seepage experiments. *Int. J. Coal Sci. Technol.* 2014, 1, 428–433. [CrossRef]
55. Wang, Y.T.; Zhou, X.P.; Xu, X. Numerical simulation of propagation and coalescence of flaws in rock materials under compressive loads using the extended non-ordinary state-based peridynamics. *Eng. Fract. Mech.* **2016**, *163*, 248–273. [CrossRef]

56. Kou, M.M.; Lian, Y.J.; Wang, Y.T. Numerical investigations on crack propagation and crack branching in brittle solids under dynamic loading using bond-particle model. *Eng. Fract. Mech.* **2019**, *212*, 41–56. [CrossRef]

57. Chinese Aeronautical Establishment. *Handbook of Stress Intensity Factors*; Science Press: Beijing, China, 1993; pp. 320–321.

58. Yu, X.Z.; Qiao, C.X.; Zhou, L.Q. *Rock and Concrete Fracture Mechanics*; Central South University of Technology Press: Changsha, China, 1991; pp. 230–278.

59. Kemeny, J.M. A model for nonlinear rock deformation under compression due to subcritical crack growth. *Int. J. Rock Mech. Min. Sci.* **1991**, *28*, 459–467. [CrossRef]