Ground state $1/2^+$ octet baryon sum rules predicting a chain of inequalities for hadron photoproduction total cross-sections on corresponding baryons

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Sum rules are derived relating Dirac mean square radii and anomalous magnetic moments of various couples of the ground state $1/2^+$ octet baryons with the convergent integral of the difference of hadron photoproduction cross-sections on the corresponding baryons. Taking into account the present knowledge of static parameters of baryons a chain of inequalities for total hadronic photoproduction cross-sections on baryons is found from those sum rules.

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I. INTRODUCTION

Recently, the new sum rule has been derived [1]

$$
\frac{1}{3} \langle r^2_p \rangle - \frac{\kappa_p^2}{4m_p^2} + \frac{\kappa_n^2}{4m_n^2} = \frac{2}{\pi^2 \alpha} \int_{\omega_N}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^{\gamma n \to X} - \sigma_{\gamma n \to X}^{\gamma p \to X} \right]
$$

relating Dirac proton mean square radius $\langle r^2_p \rangle$ and anomalous magnetic moments of proton $\kappa_p$ and neutron $\kappa_n$ to the convergent integral over a difference of the total proton and neutron photoproduction cross-sections, in which a mutual cancelation of the rise of the corresponding cross-sections for $\omega \to \infty$ ($\omega$ is the photon energy in the laboratory frame), created by the Pomeron exchanges, was achieved. Using similar ideas the new Cabibbo-Radicati [2] like sum rules for various suitable couples of the members of the pseudoscalar meson nonet have been found in Ref. [3]. In this work, to be fascinated just by the very precise satisfaction of the sum rule for a difference of proton and neutron total photoproduction cross-sections evaluating both sides of (1) (for more detail see [1]) and getting $(1.93 \pm 0.18)$mb and $(1.92 \pm 0.32)$mb, respectively, we extend the method for a derivation of all possible sum rules for various suitable couples of the members of the ground state $1/2^+$ octet baryons.

The paper is organized as follows. The next section is devoted to a brief derivation of Weizsäcker-Williams like relations between differential baryon electroproduction and total hadron photoproduction cross-sections. In Section III baryon sum rules in a general form are derived on the basis of analytic properties of the retarded photon on baryon. Applications to various couples of baryons are carried out in Section IV. Conclusions are given in the last Section.

II. WEIZSÄCKER-WILLIAMS LIKE RELATIONS BETWEEN DIFFERENTIAL BARYON ELECTROPRODUCTION AND TOTAL HADRON PHOTOPRODUCTION CROSS-SECTIONS

In a derivation of a such relation one considers currently a very high energy peripheral electroproduction process on baryon $B$

$$e^-(p_1) + B(p) \to e^-(p'_1) + X,$$

with the produced pure hadronic state $X$ moving closely to the direction of the initial baryon, which is described by a matrix element

$$M = i\frac{\sqrt{4\pi\alpha}}{q^2} \bar{u}(p_1')\gamma_\mu u(p_1) < X | J_\mu^{EM} | B > g^{\mu\nu},$$

in the one photon exchange approximation, where $m_X^2 = (p + q)^2$. Then the Sudakov expansion [4] of the photon transferred four-vector $q$

$$q = \beta_\gamma p_1 + \alpha_\gamma p + q^+,$$

$q^+ = (0, 0, q)$, $q^2 = -q^2$

into the almost light-like vectors

$$\tilde{p}_1 = p_1 - m_b^2 p/(2p_1 p), \quad \tilde{p} = p - m_B^2 p_1/(2p_1 p)$$

is applied and the Gribov prescription [5] for the numerator of the photon Green function

$$g_{\mu\nu} = g_{\mu\nu}^{\pm} + \frac{2}{s}(\tilde{p}_\mu \tilde{p}_\nu + \tilde{p}_\nu \tilde{p}_\mu) \approx \frac{2}{s} \tilde{p}_\mu \tilde{p}_\nu$$

with $s = (p_1 + p)^2 \approx 2p_1 p \gg Q^2 = -q^2$ is used in [6] in order to write down for very high electron energy in [2] and small photon momentum transfer squared $t = q^2 = -Q^2 = -q^2$ the corresponding cross-section in the form

$$d\sigma^\pm \frac{-e^e X}{B} = \frac{4\pi\alpha}{s(q^2)^2} p_1^\mu p_1^\nu \times$$

$$\sum_{X \neq B} \sum_{r=-1/2}^{1/2} \langle B^{(r)} | J_\mu^{EM} | X \rangle \langle X | J_\nu^{EM} | B^{(r)} \rangle d\Gamma$$

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with a summation through the created hadronic states $X$ and the spin states of the initial baryon. If the relation $\int d^4q \delta^{(4)}(p_1 - q - p') = 1$ is used in order to utilize in (7) the expression a difference of corresponding differential cross-sections of $m$ negligible contribution) (7) over $d\Gamma$ of the final particles one gets

$$d\Gamma = \frac{ds_1}{2s(2\pi)^3} d^2q d\Gamma_X$$

with

$$d\Gamma_X = (2\pi)^4 \delta^{(4)}(p + q - \sum q_i) \prod_i \frac{d^3q_i}{2E_i(2\pi)^3}$$

$$s_1 = 2(qp) = m_X^2 + q^2 - m_B^2 = s_Bq.$$ (10)

Now the current conservation condition ($\alpha q\beta$ gives a negligible contribution)

$$q^\mu <X, J_{\mu}^{EM}> B^{(r)} \approx$$

$$\approx (\beta q \beta_1 + q_\perp)^\mu <X, J_{\mu}^{EM}> B^{(r)} = 0,$$ (11)

is used in order to utilize in Fig. 1 the expression

$$\int p''_1 p''_2 \sum_{X \neq B, r = -1/2}^{1/2} <B^{(r)} | J_{\mu}^{EM} | X>*$$

$$<X, J_{\mu}^{EM} | B^{(r)}> d\Gamma_X = 2i s_1^2 \alpha q^2 Im \bar{A}^{(B)}(s_1, q),$$

(12)

with the imaginary part of the retarded forward Compton scattering amplitude $\bar{A}^{B}(s_1, q)$ on a baryon. Then for a difference of corresponding differential cross-sections of the electroproduction on $B$ and $B'$ (after integration in Fig. 1 over $d\Gamma_X$, as well as over the invariant mass squared $m_X^2$, i.e. over the variable $s_1$ to be interested only for $q$ distribution) one finds

$$\frac{ds_1}{2s(2\pi)^3} d^2q \frac{d\sigma^{-B\rightarrow e^-X}(s, q)}{d^2q} - \frac{ds_1}{2s(2\pi)^3} d^2q \frac{d\sigma^{-B'\rightarrow e^-X'}(s, q)}{d^2q} =$$

$$= \alpha q^2 \int_{s_1}^{\infty} \frac{ds_1}{s_1^2 q^2 + (m_e s_1/s)^2} \times$$

$$\times [Im \bar{A}^B(s_1, q) - Im \bar{A}^{B'}(s_1, q)].$$

Finally, if one neglects the second term in square brackets of the denominator of the integral in (13) (owing to the small value of $m_e$ and high $s$ in comparison with $s_1$) and takes the limit $q^2 \rightarrow 0$ along with the expressions $d^2q = \pi q^2$ and $Im \bar{A}^{B}(s_1, q) = 4s_1^2 \sigma_{tot}^{-B\rightarrow X}(s_1, q)$, one comes to the Weizsacker-Williams like relation

$$q^2 \frac{d\sigma^{-B\rightarrow e^-X}(s, q)}{d^2q} - \frac{d\sigma^{-B'\rightarrow e^-X'}(s, q)}{d^2q} |_{q^2 \rightarrow 0} =$$

$$= \alpha \int_{s_1}^{\infty} \frac{ds_1}{s_1^2 [\sigma_{tot}^{B\rightarrow X}(s_1) - \sigma_{tot}^{B'\rightarrow X}(s_1)]]}$$

relating the difference of $q^2$-dependent differential cross-sections of the processes in Fig. 1 to the convergent integral over the difference of the total hadron photoproduction cross-sections on baryons.

**III. UNIVERSAL SUM RULE FOR OCTET BARYONS**

Universal sum rule is derived by exploiting the analytic properties of the retarded Compton scattering amplitude $\bar{A}^{B}(s_1, q)$ in $s_1-$ plane as presented in Fig. 1a, defining the integral $I$ over the path $C$ (for more detail see [7]) in
the $s_1$-plane

$$I = \int_{C} ds_1 \frac{p^\mu_1 p^\nu_1}{s_1^2} \left( \hat{A}_{\mu\nu}^B(s_1, q) - \hat{A}_{\mu\nu}^{B'}(s_1, q) \right)$$  \hspace{1cm} (15)$$

from the gauge invariant light-cone projection $p^\mu_1 p^\nu_1 \hat{A}_{\mu\nu}^B(s_1, q)$ of the amplitude $\hat{A}^B(s_1, q)$ and once closing the contour $C$ to upper half-plane, another one to lower half-plane (see Fig. 1b). As a result the following sum rule

$$\pi(\text{Res}^{B'} - \text{Res}^B) = q^2 \int_{r.h.} ds_1 \left[ \text{Im} \hat{A}^B(s_1, q) - \text{Im} \hat{A}^{B'}(s_1, q) \right]$$  \hspace{1cm} (16)$$

appears with (an averaging over the initial baryon and photon spins is performed)

$$\text{Res}^B = 2\pi\alpha(F_{1B}^2 + \frac{q^2}{4m_B^2} F_{1B}^{2'})$$  \hspace{1cm} (17)$$

to be the residuum of the baryon intermediate state pole (see Fig. 1) contribution expressed through the Dirac and Pauli baryon electromagnetic form factors and the left-hand cut contributions expressed by an integral over the difference $\text{Im} \hat{A}^B(s_1, q) - \text{Im} \hat{A}^{B'}(s_1, q)$ are assumed to be mutually annulled. Then, substituting (17) into (16) and taking into account (13) from the previous Section with $d^2q = \pi d\mathbf{q}^2$, one comes to the $q^2$ dependent baryon sum rule

$$[F_{1B'}^2(-q^2) - F_{1B'}^2(0)] - [F_{1B}^2(-q^2) - F_{1B}^2(0)] + q^2 \left[ \frac{F_{2B'}^2(-q^2) - F_{2B}^2(-q^2)}{4m_{B'}^2} - \frac{F_{2B'}^2(-q^2) - F_{2B}^2(-q^2)}{4m_B^2} \right] = \frac{2}{\pi\alpha^2} \langle q^2 \rangle \left( \frac{d\sigma_{e^- B' e^- X}}{dq^2} - \frac{d\sigma_{e^- B e^- X}}{dq^2} \right),$$  \hspace{1cm} (18)$$

where the left-hand side was renormalized in order to separate the pure strong interactions from electromagnetic ones. Besides, substituting here for small values of $q^2$ the relation (13) from the previous Section, using the laboratory reference frame by $s_1 = 2m_B\omega$ and finally taking a derivative according to $q^2$ of both sides for $q^2 = 0$, one gets the universal octet baryon sum rule

$$\frac{1}{3} \left[ F_{1B}(0) \langle r_{1B'}^2 \rangle - F_{1B'}(0) \langle r_{1B}^2 \rangle \right] - \left[ \frac{\kappa_B^2}{4m_B^2} - \frac{\kappa_{B'}^2}{4m_{B'}^2} \right] = \frac{2}{\pi^2\alpha} \int_{\omega_B}^{\infty} d\omega \left[ \sigma_{B' \rightarrow X}^{\gamma} - \sigma_{B \rightarrow X}^{\gamma} \right]$$  \hspace{1cm} (19)$$

relating Dirac baryon mean square radii $\langle r_{1B}^2 \rangle$ and baryon anomalous magnetic moments $\kappa_B$ to the convergent integral, in which a mutual cancelation of the rise of the corresponding total cross-sections for $\omega \rightarrow \infty$ is achieved.

\section{IV. APPLICATION TO VARIOUS COUPLES OF OCTET BARYONS}

According to the SU(3) classification of existing hadrons there are known the following members of the ground state 1/2$^+$ baryon octet ($p, n, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$). As a result, by using the universal expression (19) one can write down $8!/(2!(8-2)!)$ = 28 different sum rules for total cross-sections of hadron photoproduction on ground state 1/2$^+$ octet baryons. The most precise of them (already experimentally verified) proton-neutron sum rule (1) has been previously published in [1]. The rest of 27 sum rules are explicitly presented in this paper.

If one considers couples of the isoscalar isoscalar and separately couples of isodoublet of $\Xi$-hyperons, one finds

$$\frac{1}{3} \left[ \langle r_{1\Sigma^+}^2 \rangle - \langle r_{1\Sigma^-}^2 \rangle \right] - \left[ \frac{\kappa_{\Sigma^+}^2}{4m_{\Sigma^+}^2} - \frac{\kappa_{\Sigma^-}^2}{4m_{\Sigma^-}^2} \right] = \frac{2}{\pi^2\alpha} \int_{\omega_{\Sigma^+}}^{\infty} d\omega \left[ \sigma_{\Sigma^+ \rightarrow X}^{\gamma} - \sigma_{\Sigma^- \rightarrow X}^{\gamma} \right],$$  \hspace{1cm} (20)$$

and

$$\frac{1}{3} \left[ \langle r_{1\Xi^0}^2 \rangle - \langle r_{1\Xi^-}^2 \rangle \right] - \left[ \frac{\kappa_{\Xi^0}^2}{4m_{\Xi^0}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2} \right] = \frac{2}{\pi^2\alpha} \int_{\omega_{\Xi^0}}^{\infty} d\omega \left[ \sigma_{\Xi^0 \rightarrow X}^{\gamma} - \sigma_{\Xi^- \rightarrow X}^{\gamma} \right],$$  \hspace{1cm} (21)$$

respectively, which represent the second class of the baryon sum rules what is concerned of their precision.

The third class of the 23 baryon sum rules is found by a consideration of a couple of baryons always taken from different isomultiplets of the ground state 1/2$^+$ baryon octet and take forms as follows

$$\frac{1}{3} \left[ \langle r_{1\Lambda^0}^2 \rangle - \langle r_{1\Xi^-}^2 \rangle \right] - \left[ \frac{\kappa_{\Lambda^0}^2}{4m_{\Lambda^0}^2} - \frac{\kappa_{\Xi^-}^2}{4m_{\Xi^-}^2} \right] = \frac{2}{\pi^2\alpha} \int_{\omega_{\Lambda^0}}^{\infty} d\omega \left[ \sigma_{\Lambda^0 \rightarrow X}^{\gamma} - \sigma_{\Xi^- \rightarrow X}^{\gamma} \right],$$  \hspace{1cm} (22)$$

respectively.
\[
\frac{1}{3} \left( \langle r^{2}_{1p} \rangle - \langle r^{2}_{1\Sigma^{+}} \rangle \right) - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{+}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\gamma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{+} \rightarrow X}(\omega)], \quad (25)
\]

\[
\frac{1}{3} \langle r^{2}_{1p} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{+}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{+} \rightarrow X}(\omega)], \quad (26)
\]

\[
\frac{1}{3} \left( \langle r^{2}_{1p} \rangle + \langle r^{2}_{1\Sigma^{-}} \rangle \right) - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\gamma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{-} \rightarrow X}(\omega)], \quad (27)
\]

\[
\frac{1}{3} \langle r^{2}_{1p} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{-} \rightarrow X}(\omega)], \quad (28)
\]

\[
\frac{1}{3} \left( \langle r^{2}_{1p} \rangle + \langle r^{2}_{1\Xi^{-}} \rangle \right) - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} [\gamma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (29)
\]

\[
-\left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (30)
\]

\[
-\left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (31)
\]

\[
-\left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (32)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Sigma^{-}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\gamma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{-} \rightarrow X}(\omega)], \quad (33)
\]

\[
-\left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{-} \rightarrow X}(\omega)], \quad (34)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Xi^{-}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\infty} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (35)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Sigma^{+}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{+}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Sigma^{+} \rightarrow X}(\omega)], \quad (36)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Xi^{+}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{+}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{+} \rightarrow X}(\omega)], \quad (37)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Xi^{-}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (38)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Xi^{-}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (39)
\]

\[
\frac{1}{3} \langle r^{2}_{1\Xi^{-}} \rangle - \left[ \frac{\kappa^2_n}{4m^2_p} - \frac{\kappa^2_{2\Sigma}}{4m^2_{2\Sigma^{-}}} \right] = \\
\frac{2}{\pi^2 \alpha} \int_{\omega_n}^{\pi} \frac{d\omega}{\omega} [\sigma_{tot}^{n \rightarrow X}(\omega) - \sigma_{tot}^{\Xi^{-} \rightarrow X}(\omega)], \quad (40)
\]
\[ \frac{1}{3} \langle r_1^2 \rangle + \frac{1}{3} \langle r_2^2 \rangle - \left[ \frac{\kappa_{20}^2}{4m_{20}^2} - \frac{\kappa_{20}^2}{4m_{20}^2} \right] = \]

\[ = \frac{2}{\pi^2 \alpha} \int_{\omega_{e+}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{20}^{\gamma \Sigma^+} - X(\omega) - \sigma_{20}^{\gamma \Xi^0} - X(\omega) \right], \quad (41) \]

\[ \frac{1}{3} \left[ \langle r_1^2 \rangle - \langle r_2^2 \rangle \right] - \left[ \frac{\kappa_{20}^2}{4m_{20}^2} - \frac{\kappa_{20}^2}{4m_{20}^2} \right] = \]

\[ = \frac{2}{\pi^2 \alpha} \int_{\omega_{e+}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{20}^{\gamma \Sigma^+} - X(\omega) - \sigma_{20}^{\gamma \Xi^0} - X(\omega) \right], \quad (42) \]

The latter follows from the careful analysis carried out in [6], where in analogy with QED the left-hand cut contribution can be associated with contribution to the cross-section of the process of baryon-antibaryon pair electro-production on baryon \( e^-B \to e^-2BB \), arising from taking into account the identity of final state baryons. Keeping in mind the minimal value of three baryon invariant mass squared \( s_{2\text{min}} = 8m_B^2 \), the left-hand cut contribution to the derivative according to \( q^2 \) for \( q^2 = 0 \) entering the derived sum rules have an order of magnitude

\[ I_B = \frac{g^4 m_B^2}{(2\pi)^3 s_{2\text{min}}} = \frac{g^4}{64(2\pi)^3 m_B^2}, \quad \frac{g^2}{4\pi} = 14.4 \quad (47) \]

which depends on the baryon mass squared and so, the error in derived sum rules increases with the increased difference in the masses of joining pairs of baryons (see Table I). Now it is clear that the proton-neutron sum rule is the most precise and then follow the sum rules \( \Sigma^+ - \Sigma^0, \Sigma^+ - \Sigma^-, \Sigma^0 - \Sigma^- \) and \( \Sigma^0 - \Xi^- \).

In order to evaluate the left hand sides of the derived sum rules (41-46) and to draw out some phenomenological consequences, one needs the reliable values of Dirac baryon mean square radii \( \langle r_1^2 \rangle \) and baryon anomalous magnetic moments \( \kappa_B \). The latter are known (besides \( \Sigma^0 \), which is found from the well known relation \( \kappa_{\Sigma^+} + \kappa_{\Sigma^-} = 2\kappa_{\Sigma^0} \)) experimentally (see the third column in Table I), however, to calculate \( \langle r_1^2 \rangle \) by means of the difference of the baryon electric mean square radius \( \langle r_1^2 \rangle \) and Foldy term (well known for all ground state octet baryons from the experimental information on the magnetic moments [8])

\[ \langle r_1^2 \rangle = \langle r_1^2 \rangle - \frac{3\kappa_B}{2m_B^2}. \quad (48) \]

we are in need of the reliable values of \( \langle r_1^2 \rangle \). They are known experimentally only for the proton, neutron and \( \Sigma^- \)-hyperon [8]. Fortunately there are recent results of Kubis and Meissner [8] to fourth order in relativistic baryon chiral perturbation theory (giving predictions for the \( \Sigma^- \) charge radius and the \( \Lambda\Sigma^0 \) transition moment in excellent agreement with the available experimental information), which solve our problem completely. All necessary information from [8] and [9] is collected in Table I, where also numerical values of corresponding \( \langle r_1^2 \rangle \) are presented.

Calculating the left-hand side of all sum rules one finds
Table I:

| B  | $I_B[mb]$ | $m_B[Gev]$ | $\kappa_B[\mu_N]$ | $\langle r_{pB}^s \rangle [fm^2]$ | $3\kappa_B/2m_B^2 [fm^2]$ | $\langle r_{pB}^2 \rangle [fm^2]$ |
|----|-----------|------------|-------------------|------------------------------|----------------------------|-------------------|
| $p$ | 0.9125    | 0.93827    | 1.7928            | 0.717                        | 0.119                     | 0.598             |
| $n$ | 0.9100    | 0.93957    | -1.9130           | -0.113                       | -0.127                    | -0.240            |
| $\Lambda^0$ | 0.6454    | 1.11568    | -0.6130           | 0.110                        | -0.029                    | 0.081             |
| $\Sigma^+$ | 0.5679    | 1.18937    | 1.4580            | 0.600                        | 0.060                     | 0.660             |
| $\Sigma^0$ | 0.5648    | 1.19264    | 0.6490            | -0.030                       | 0.027                     | -0.003            |
| $\Sigma^-$ | 0.5602    | 1.19745    | -0.1600           | 0.670                        | -0.007                    | 0.663             |
| $\Xi^0$  | 0.4647    | 1.31483    | -1.2500           | 0.130                        | -0.042                    | 0.088             |
| $\Xi^-$  | 0.4601    | 1.32131    | 0.3493            | 0.490                        | 0.012                     | 0.502             |

\[
\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) - \sigma_{\gamma p \to X}^{\gamma n \to X} (\omega) \right] = 2.0415 mb, \text{ then in averaged } \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) > \sigma_{\gamma n \to X}^{\gamma n \to X} (\omega) \quad (49)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_{\gamma \Sigma^+}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma \Sigma^+ \to X}^{\gamma \Sigma^+ \to X} (\omega) - \sigma_{\gamma \Sigma^0 \to X}^{\gamma \Sigma^0 \to X} (\omega) \right] = 2.0825 mb, \text{ then in averaged } \sigma_{\gamma \Sigma^+ \to X}^{\gamma \Sigma^+ \to X} (\omega) > \sigma_{\gamma \Sigma^0 \to X}^{\gamma \Sigma^0 \to X} (\omega) \quad (50)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_{\gamma \Sigma^0}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma \Sigma^0 \to X}^{\gamma \Sigma^+ \to X} (\omega) - \sigma_{\gamma \Sigma^0 \to X}^{\gamma \Sigma^{-} \to X} (\omega) \right] = 2.1829 mb, \text{ then in averaged } \sigma_{\gamma \Sigma^0 \to X}^{\gamma \Sigma^+ \to X} (\omega) > \sigma_{\gamma \Sigma^0 \to X}^{\gamma \Sigma^{-} \to X} (\omega) \quad (51)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_{\gamma \Xi^0}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma \Xi^0 \to X}^{\gamma \Xi^0 \to X} (\omega) - \sigma_{\gamma \Xi^0 \to X}^{\gamma \Xi^{-} \to X} (\omega) \right] = 1.5921 mb, \text{ then in averaged } \sigma_{\gamma \Xi^0 \to X}^{\gamma \Xi^0 \to X} (\omega) > \sigma_{\gamma \Xi^0 \to X}^{\gamma \Xi^{-} \to X} (\omega) \quad (52)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_{\gamma \Lambda^0}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma \Lambda^0 \to X}^{\gamma \Lambda^0 \to X} (\omega) - \sigma_{\gamma \Lambda^0 \to X}^{\gamma \Lambda^+ \to X} (\omega) \right] = 1.6673 mb, \text{ then in averaged } \sigma_{\gamma \Lambda^0 \to X}^{\gamma \Lambda^0 \to X} (\omega) > \sigma_{\gamma \Lambda^0 \to X}^{\gamma \Lambda^+ \to X} (\omega) \quad (53)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) - \sigma_{\gamma p \to X}^{\gamma \Lambda^0 \to X} (\omega) \right] = -0.4158 mb, \text{ then in averaged } \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) < \sigma_{\gamma p \to X}^{\gamma \Lambda^0 \to X} (\omega) \quad (54)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) - \sigma_{\gamma p \to X}^{\gamma \Sigma^+ \to X} (\omega) \right] = 1.6667 mb, \text{ then in averaged } \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) > \sigma_{\gamma p \to X}^{\gamma \Sigma^+ \to X} (\omega) \quad (55)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) - \sigma_{\gamma p \to X}^{\gamma \Sigma^0 \to X} (\omega) \right] = 3.8496 mb, \text{ then in averaged } \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) > \sigma_{\gamma p \to X}^{\gamma \Sigma^0 \to X} (\omega) \quad (56)
\]

\[
\frac{2}{\pi^2\alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) - \sigma_{\gamma p \to X}^{\gamma \Sigma^{-} \to X} (\omega) \right] = 3.8496 mb, \text{ then in averaged } \sigma_{\gamma p \to X}^{\gamma p \to X} (\omega) > \sigma_{\gamma p \to X}^{\gamma \Sigma^{-} \to X} (\omega) \quad (57)
\]
\[
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma p \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma Z \rightarrow X} (\omega) \right] = 1.7259 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma p \rightarrow X} (\omega) > \sigma_{\text{tot}}^{\gamma Z \rightarrow X} (\omega) \quad (58)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega_p}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma p \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma Z \rightarrow X} (\omega) \right] = 3.3180 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma p \rightarrow X} (\omega) > \sigma_{\text{tot}}^{\gamma Z \rightarrow X} (\omega) \quad (59)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma A^0 \rightarrow X} (\omega) \right] = -0.3260 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) < \sigma_{\text{tot}}^{\gamma A^0 \rightarrow X} (\omega) \quad (60)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^\pm \rightarrow X} (\omega) \right] = -2.4573 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) < \sigma_{\text{tot}}^{\gamma \Sigma^\pm \rightarrow X} (\omega) \quad (61)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^\pm \rightarrow X} (\omega) \right] = -0.3747 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) < \sigma_{\text{tot}}^{\gamma \Sigma^\pm \rightarrow X} (\omega) \quad (62)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^- \rightarrow X} (\omega) \right] = 1.8082 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) > \sigma_{\text{tot}}^{\gamma \Sigma^- \rightarrow X} (\omega) \quad (63)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^0 \rightarrow X} (\omega) \right] = -0.3156 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma n \rightarrow X} (\omega) < \sigma_{\text{tot}}^{\gamma \Sigma^0 \rightarrow X} (\omega) \quad (64)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^- \rightarrow X} (\omega) \right] = 1.2766 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) > \sigma_{\text{tot}}^{\gamma \Sigma^- \rightarrow X} (\omega) \quad (65)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^+ \rightarrow X} (\omega) \right] = -2.0831 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) < \sigma_{\text{tot}}^{\gamma \Sigma^+ \rightarrow X} (\omega) \quad (66)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^- \rightarrow X} (\omega) \right] = -0.0026 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) \approx \sigma_{\text{tot}}^{\gamma \Sigma^- \rightarrow X} (\omega) \quad (67)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Sigma^0 \rightarrow X} (\omega) \right] = 2.1823 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) > \sigma_{\text{tot}}^{\gamma \Sigma^0 \rightarrow X} (\omega) \quad (68)
\]

\[
\frac{2}{\pi^2 \alpha} \int_{\omega}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) - \sigma_{\text{tot}}^{\gamma \Xi^0 \rightarrow X} (\omega) \right] = 0.0586 \text{mb}, \quad \text{then in averaged} \quad \sigma_{\text{tot}}^{\gamma \Lambda^0 \rightarrow X} (\omega) > \sigma_{\text{tot}}^{\gamma \Xi^0 \rightarrow X} (\omega) \quad (69)
\]
from where one gets the following chain of inequalities

\[
\sigma_{\text{tot}}^{\gamma \Sigma^+ \rightarrow X} > \sigma_{\text{tot}}^{\gamma \Lambda \rightarrow X} > \sigma_{\text{tot}}^{\gamma \Xi^0 \rightarrow X} \approx \sigma_{\text{tot}}^{\gamma \Xi^- \rightarrow X} > \sigma_{\text{tot}}^{\gamma n \rightarrow X} > \sigma_{\text{tot}}^{\gamma 
rightarrow X} > \sigma_{\text{tot}}^{\gamma 
 \rightarrow X} > \sigma_{\text{tot}}^{\n \rightarrow X} (77)
\]

for total cross-sections of hadron photoproduction on ground state 1/2+ octet baryons.

Experimental tests of the derived sum rules could be practically carried out provided there exist total hadron photoproduction cross-sections on hyperons as a function of energy, which are, however, missing till now. Nevertheless, the idea of intensive photon beams generated by the electron beams of linear \(e^+ e^-\) colliders by using the process of the backward Compton scattering of laser light off the energy electrons [10] is encouraging and one expects in near future that the measurements of the total hadron photoproduction cross-sections on hyperons can be practically achievable.

V. CONCLUSIONS

Considering the very high energy peripheral electron-baryon scattering with a production of a hadronic state \(X\) moving closely to the direction of initial baryon, then
exploiting analytic properties of the forward Compton scattering amplitude on the same baryon, for the case of small transferred momenta new sum rules, relating Dirac baryon mean square radii and the baryon anomalous magnetic moments with the convergent integrals over a difference of total hadron photoproduction cross-sections on baryons are derived. Evaluating the left-hand sides of the sum rules the chain of inequalities for total cross-sections of hadron photoproduction on all ground state $1/2^+$ octet baryons has been found. Possible practical tests of the derived sum rules are discussed as well.

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