Strict bounds for the distance between Normal($m/2, m/4$) and Binomial($m, 1/2$) distributions are provided by Tusnády's inequality which “lies at the heart of the modern approach to the proof of the Komlós-Major-Tusnády approximation of the empirical process by a sequence of Brownian bridges”. The bounds are in terms of a standard $N(0, 1)$ distributed random variable $Z$. The main result is a remarkable improvement of previous bounds if the values allowed for $Z$ are restricted to $|Z| \leq \sqrt{\log(m)}$ (the “bulk” of $N(0, 1)$), where $m$ is subject to usually large lower bound. The proof relies among other rather technical evaluations on a refined continuity correction for the Binomial distribution.

Reviewer: Hans Daduna (Hamburg)

MSC:

60E15 Inequalities; stochastic orderings
62E17 Approximations to statistical distributions (nonasymptotic)
62E20 Asymptotic distribution theory in statistics
60F99 Limit theorems in probability theory

Keywords:

Tusnády’s inequality; KMT approximation; quantile coupling; local limit theorem; continuity correction; binomial distribution; Gaussian approximation

Full Text: DOI arXiv

References:

[1] Berkes, I.; Liu, W.; Wu, W. B., Komlós-Major-Tusnády approximation under dependence, Ann. Probab., 42, 2, 794-817 (2014), MR3184747 · Zbl 1308.60037
[2] Bretagnolle, J.; Massart, P., Hungarian constructions from the nonasymptotic viewpoint, Ann. Probab., 17, 1, 239-256 (1989), MR972783 · Zbl 0667.60042
[3] Carter, A.; Pollard, D., Tusnády’s inequality revisited, Ann. Stat., 32, 6, 2731-2741 (2004), MR2154001 · Zbl 1076.62012
[4] Cressie, N., A finely tuned continuity correction, Ann. Inst. Stat. Math., 30, 3, 435-442 (1978), MR538319 · Zbl 0445.62033
[5] Csörgő, M.; Horváth, L., Weighted Approximations in Probability and Statistics (1993), John Wiley & Sons Ltd. Chichester, MR1215046 · Zbl 0770.60038
[6] Csörgő, M.; Révész, P., Strong Approximations in Probability and Statistics, Probability and Mathematical Statistics (1981), Academic Press, Inc., New York-London, MR665468 · Zbl 0539.60029
[7] Dudley, R. M., An exposition of Bretagnolle and Massart’s proof of the KMT theorem for the uniform empirical process, (Lectures Notes for a Course Given in Aarhus. Lectures Notes for a Course Given in Aarhus, August 1999 (2005)) (1999)
[8] Einmahl, U., Extensions of results of Komlós, Major, and Tusnády to the multivariate case, J. Multivar. Anal., 28, 1, 20-68 (1989), MR996984 · Zbl 0676.60038
[9] Govindarajulu, Z., Normal approximations to the classical discrete distributions, Sankhyā, Ser. A, 27, 143-172 (1965), MR1215046 · Zbl 1021.60027
[10] Mason, D. M., Notes on the KMT Brownian bridge approximation to the uniform empirical process, (Asymptotic Meth-
[16] Mason, D. M.; van Zwet, W. R., A refinement of the KMT inequality for the uniform empirical process, Ann. Probab., 15, 3, 871-884 (1987), MR893903 · Zbl 0638.60040

[17] Mason, D. M.; Zhou, H. H., Quantile coupling inequalities and their applications, Probab. Surv., 9, 439-479 (2012), MR3087210 · Zbl 1307.62036

[18] Massart, P., Tusnady’s lemma, 24 years later, Ann. Inst. Henri Poincaré Probab. Stat., 38, 6, 991-1007 (2002), MR1955348 · Zbl 1016.60037

[19] Remmert, R., Classical Topics in Complex Function Theory, Graduate Texts in Mathematics, vol. 172 (1998), Springer-Verlag: Springer-Verlag New York, MR1483074 · Zbl 0895.30001

[20] Tusnády, G., A study of statistical hypotheses (in Hungarian) (1977), Hungarian Academy of Sciences: Hungarian Academy of Sciences Budapest, PhD thesis

[21] Zaitsev, A. Yu., Multidimensional version of the results of Komlós, Major and Tusnády for vectors with finite exponential moments, ESAIM Probab. Stat., 2, 41-108 (1998), MR1616527 · Zbl 0897.60033

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.