A Model for Addition Spectra in Quantum Dots

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A simple model for addition spectra in quantum dots is proposed and studied. It is an extension of the standard charging model which assumes that the charge spreads uniformly over the entire dot. The proposed model attempts to account for a nonuniform distribution of the charge, by introducing an extra parameter $U$. When $U$ increases, the distribution of the conductance peak spacings changes from the Wigner-Dyson shape towards a broader, more symmetric distribution.

I. INTRODUCTION

The combined effect of disorder and electron-electron interactions is an old theme, to which Michael Pollak made important early contributions. The topic is still very much alive. In particular, with the advance in fabrication of mesoscopic electronic systems, the new feature of the confinement, in one or more directions, has been added to the problem.

This paper deals with disordered quantum dots, i.e., small islands of interacting electrons, in the presence of the random potential of impurities. Let us denote by $E_0(N)$ the ground state energy of a dot with $N$ electrons and define the addition spectrum as $E_0(N+1) - E_0(N) = \mu(N+1)$. This quantity is a subject of extensive theoretical and experimental study. It is determined experimentally by measuring the conductance of the dot, as a function of the gate voltage $V_G$. The latter determines the electrostatic potential on the dot. For a dot weakly coupled to the leads, and at low temperatures, the conductance is negligible (Coulomb blockade), unless $V_G$ is equal to $\mu(N)$, at which point the blockade is lifted and an additional electron can enter the dot. Thus, the conductance exhibits a sequence of peaks. The distance between two consecutive peaks is

$$\Delta_2(N) = \mu(N+1) - \mu(N) = E_0(N+1) - 2E_0(N) + E_0(N-1).$$

(1)

The simplest possible description of the Coulomb blockade is given in terms of the charging model, also known as the constant-interaction model (for a recent review see Alhassid, 2000). It is assumed in this model that the added, $(N+1)$-th electron occupies the $(N+1)$-th single particle state in the dot, with energy $\epsilon_{N+1}$. The interaction between the added electron and each of the $N$ electrons in the dot is described by a constant potential $V_C$. These assumptions result in the following expressions for the addition spectrum $\mu(N)$ and peak spacing $\Delta_2(N)$:

$$\mu(N+1) = V_C N + \epsilon_{N+1},$$

$$\Delta_2(N) = V_C + (\epsilon_{N+1} - \epsilon_N).$$

(2)

It follows from Eq. (2), that fluctuations in $\Delta_2(N)$ are determined by the fluctuations in the single-particle level spacing which, for a weakly disordered dot, are given by the Wigner-Dyson distribution. Furthermore, if $N$ is odd, then the $(N+1)$-th particle occupies a level which is already occupied by a particle with opposite spin, so that $\Delta_2 = V_C$. Thus, according to the charging model, the distribution $P(\Delta_2)$ should exhibit a bimodal structure, composed of a $\delta$-function at $V_C$ plus a Wigner-Dyson function (shifted by $V_C$). Experiments (Sivan et al., 1996, Patel et al., 1998, Simmel et al., 1999) however, show an approximately Gaussian distribution for $\Delta_2$, with a width considerably larger than the average single-particle level spacing $\Delta$ and with no sign of a bimodal structure. The disagreement between experiment and the charging model led to a number of theoretical studies of $P(\Delta_2)$, based on exact diagonalization of the microscopic Hamiltonian (Prus et al., 1996, Berkovits, 1998) as well as on various approximations such as RPA (Blanter et al., 1997, Berkovits and Altshuler, 1997), the extreme classical limit (Koulakov et al., 1997) and the self-consistent Hartree-Fock scheme (Walker et al., 1999, Cohen et al., 1999, Levit and Orgad, 1999), or the somewhat related configuration interaction method (Benenti et al., 2000). In this paper we propose, and study, a simple model for the addition spectrum and the peak spacing of a weakly disordered (or chaotic) quantum dot. The electron spin is ignored, except for a few comments at the end of the paper.
II. THE MODEL

N interacting electrons in a quantum dot are described by the Hamiltonian

\[ H = \sum_{i=1}^{N} \left[ -\frac{\hbar^2}{2m} \nabla_i^2 + V(\vec{r}_i) \right] + \frac{1}{2} \sum_{i<j} U(\vec{r}_i - \vec{r}_j), \]

where \( m \) is the electron effective mass, \( V(\vec{r}_i) \) includes the confining potential as well as the random potential of impurities, and \( U(\vec{r}_i - \vec{r}_j) \) is the interaction potential. Let us discuss first the two limits, of weak and strong interactions. In the first limit \( \Delta_2 \) is equal to the single-particle level spacing and, thus, obey the Wigner-Dyson statistics. Small corrections, due to interactions, can be treated in perturbation theory (Prus et al., 1996).

In the opposite case of very strong interactions (the "extreme classical limit") the potential energy dominates, and the charges settle down in a certain configuration which minimizes the potential energy of the system. For some special values of \( N \), called "magic numbers", one can expect particularly stable, i.e. low energy, configurations (Koulakov and Shklovskii, 1998, Morris et al., 1996). Specific values of the magic numbers depend on the shape of the dot, the boundary conditions and the type of interactions. For instance, for a dot of an equilateral triangle shape these numbers are \( N = \ell(\ell+1)/2 \), where \( \ell \) is an integer. Indeed, for these values of \( N \) one can view the magic configuration inside the triangle as being cut out of an infinite Wigner crystal, which is known to crystallize into a triangular lattice. In what follows, we consider a square dot, with periodic boundary conditions, often used in numerical calculations. The triangular Wigner crystal, characteristic for an isotropic interaction, is clearly incompatible with the square shape of the dot, and finding out the magic configurations for this geometry becomes a formidable problem. However, for anisotropic interactions, or in the presence of an underlying periodic potential, the charges can crystallize into a square lattice. In this case the magic configurations should occur at \( N = \ell^2 \). In order to keep the picture as simple as possible, without seriously damaging the forthcoming reasoning, we shall stick to this example.

Consider, thus, a magic configuration of \( N = \ell^2 \) electrons, in a square dot of size \( L \). The electrons inside the dot form a square lattice, with a lattice constant \( b = n^{-1/2} \), where \( n = NL^{-2} \) is the electron concentration. The lattice is pinned by the weak disorder in the dot. The main property of a magic configuration is its stability, i.e., it can accommodate few extra particles without a major rearrangement. An extra particle can cause some polarization effects, by pushing apart the charges in its neighborhood, but, basically, it has to reside in the center of a cell, formed by 4 lattice charges. Furthermore, the extra particles can somewhat lower their energy by making quantum hops between neighboring cells. Thus, the added particles can be viewed as "defects" moving on the intersticals of a magic configuration. For an infinite perfect crystal this kind of defects have been discussed long ago by Andreev and Lifshitz (Andreev and Lifshitz, 1969) in their theory of quantum crystals (see also Tsiper and Efros, 1997). They proposed that defects (or vacancies) can move through the crystal coherently and can be treated as a gas of excitations. For a disordered mesoscopic quantum dot this picture of "defects" becomes more arguable. It is clear, in particular, that when too many particles are added to a given magic configuration it is not appropriate to speak in terms of many defects moving on top of that particular configuration: one should rather speak about vacancies (holes) in the next magic configuration. For instance, for \( N = 102 \) one can envisage two defects in the magic configuration \( N = 100 \). However, for \( N = 119 \) it is clearly more appropriate to envisage two vacancies in the next magic configuration, \( N = 121 \) (compare to shells in atomic or nuclear physics). Moreover, somewhere in between, say, at \( N = 110 \) a major rearrangement is likely to occur, and at this point it is hardly possible to speak in terms of noninteracting defects or vacancies. Inspite of its limitations, we will use this picture of defects, embedded in the corresponding magic configuration, in order to formulate the forthcoming phenomenological model.

The main point of the above discussion is that in both limits, of small and large \( U \), the motion of a particle added to an interacting system is described in a single particle language: a bare electron, in the first case, and a "defect" in a magic configuration, in the second. Next, we interpolate between the two limits and arrive to the following, single particle, model for the addition spectra in quantum dots. The model is defined on a lattice with \( 2N \) sites. This lattice originates from the strong interaction limit, as described in the previous paragraph. In this limit one sublattice \((A)\) is occupied by the \( N \) electrons of a magic configuration, while the other one \((B)\) represents sites available for extra particles (Fig. 1). When an extra particle is placed on site \( i \) of sublattice \( B \), the energy of the system increases by \( W_i + U_i \), where \( W_i \) is the disorder related site energy, and

\[ U_i = \sum_{j \neq i} U(\vec{r}_i - \vec{r}_j) \]

(4)

describes the interaction of the added electron with the electrons in the dot. Consider, as an example, a model potential (Walker et al. 1999) consisting of a short range part \( V_\text{0} \delta_{i,i+\eta} \) (\( \eta \) designates the nearest neighbors of site
i) plus a constant term $V_C$, which describes the charging energy due to the long range part of the interaction. For this example $U_i = NV_C + 4V_0$. More generally, one can replace $4V_0$ by a phenomenological parameter $U$, whose relation to the pair potential remains unspecified. Thus, in this ”extreme classical limit”, the $(N + 1)$-th particle, added to the $N$-particle magic configuration, will reside on the site with the lowest value of $W_i$, on sublattice $B$. The corresponding increase in energy is $\mu(N + 1) = V_CN + U + W_i$. The $(N + 2)$-th particle will go into the site with the next lowest value of $W_i$, etc. Similarly, one can remove particles from a magic configuration and compare energies $E_0(N)$, $E_0(N - 1)$, etc. For instance, when one adds the $N$-th particle to the $(N - 1)$-th particle system, one fills in a vacancy in sublattice $A$. The main point is that the energy $U_i$, corresponding to a site on sublattice $A$, differs from that on sublattice $B$. This is because a site on sublattice $A$ ($B$) is surrounded by empty (occupied) sites. We assign values $U$ and zero to sites on sublattices $B$ and $A$, respectively.

The full model for the fluctuations in addition spectra is obtained by ”switching on” the kinetic energy and, thus, interpolating between the limits of small and large $U$. It is given by the matrix

$$H_{ij} = W_i\delta_{ij} - t_{ij} + U_i\delta_{ij}$$

(5)

where $t_{ij}$ accounts for hopping between sites $i$ and $j$. Energies $U_i$ assume two values: $U$ for $i \in B$ and 0 for $i \in A$. The model concentrates on the fluctuations, i.e., the charging energy $V_C$, due to the long range part of the interaction, is not included (it can be added to $\Delta_2$ at the end). The matrix $H_{ij}$ is of size $2N \times 2N$. Its $(N + n)$-th eigenvalue describes the energy (more precisely, its fluctuating part) of the $n$-th particle added to the system of $N$ particles, so that $\Delta_2$ (with $V_C$ subtracted) is given by the difference between a pair of consecutive eigenvalues. Let us stress that, although the picture in the limit of very strong interactions, discussed above, was essential for formulating the model, this limit is quite remote from the experimentally relevant situation: the latter corresponds to the ”liquid phase”, rather than the solid one. In the liquid phase the ”magic numbers” do not play any essential role and the predictions of the model are expected to be quite robust, i.e., independent of the specific assumptions (e.g. a square lattice, rather than a triangular one) used in the ”extreme classical limit”. (If the number of electrons is varied in too wide a range, one should allow for a weak dependence of the parameters of the model on the number of electrons.)

In the weak interaction limit, when $U \rightarrow 0$, the model reduces to the Anderson model for a particle propagating in a random potential. In this limit the lattice has no physical meaning and, simply, provides discretization of the continuous space. Eq. (5) defines a simple, single-particle model for addition spectra. The model captures the right physics in the two limits, of small and large $U$. The weakness of the model is that it is no more than a simple interpolation between the two limits. Such an interpolation is, clearly, a big leap — especially, since some new quantum phases might exist between the two limits (Chakravarty et al., 1999, Benenti et al., 1999). The model, however, is not intended to describe subtle correlations in the ground state of $N$ interacting particles. Its only purpose is to describe the energetics of added particles. In fact, a rather similar picture — with the same characteristic feature of two energy bands in the large-$U$ limit, merging into one band for smaller $U$ — is often used in qualitative discussions of the Hubbard model (Mott, 1990). Below we study the addition spectra within the model of Eq. (5).

### III. NUMERICAL RESULTS

We assume that site energies, $W_i$, are uniformly distributed in the interval $[-W/2, W/2]$ and take $t_{ij} = t$ for nearest neighbors (and zero otherwise). $t$ is related to the particle mass by $t = \hbar^2/(2ma^2)$ where $a = (2n)^{-i/2}$ is the lattice constant. This relation insures the correct value, $\Delta = 2\pi\hbar^2/maL^2$, for the single particle level spacing. The disorder strength is measured by the parameter $W/t$ and the interaction strength by the value of $U/t$.

Let us discuss in more detail the limit of strong interaction and weak disorder, i.e., $W < t << U$. The $U$-term in Eq. (5) produces two levels, separated by energy $U$. Each of the levels is $N$-fold degenerate. The disorder and the kinetic energy remove this degeneracy and broaden the levels into bands. The lower band is occupied by the $N$ electrons, so that extra electrons must go into the upper band. They reside in the low energy tail of that band, so that the conductance peak spacings, $\Delta_2$, are given by the level spacings in the tail. Since the density of states in the tail is much smaller than that in the middle of the band, one can expect large (in units of $\Delta$) fluctuations in $\Delta_2$. Moreover, since the tail states do not obey the Wigner-Dyson statistics, the same must be true for the statistics of $\Delta_2$. Similarly, removing electrons from the $N$-particle system amounts to creating holes in the high energy tail of the lower band. Note that the width of the two bands is determined not by the hopping amplitude $t$ in the absence of interactions but, rather, by an effective hopping amplitude $\tilde{t} \simeq t^2/U$. This is because hopping between sites on sublattice $B$, such as sites 1 and 2 in Fig. 1, requires visiting an intermediate site (e.g. site 3), which belongs to the (occupied) sublattice $A$. Such virtual visits are costly and contribute only in second order of the perturbation theory.
producing the effective hopping amplitude $\tilde{t}$ for particles (holes) in the upper (lower) band. The estimate $\tilde{t}$ for the band width holds only as long as $\tilde{t} > W$, i.e., $t > \sqrt{UW}$. In the opposite case when $W < t < \sqrt{UW}$, the $t$-term in Eq. (5) can be neglected. In this case the bands have width $W$ and all eigenstates are localized.

Thus, the model in Eq. (5) brings out in a simple way the origin of large fluctuations in the peak spacings $\Delta_2$. The point is that, for large $U$, the model exhibits two bands and the statistics of $\Delta_2$ is controlled by the states deep in the tails of those bands. For small $U$ the two bands merge into one band, and the statistics of $\Delta_2$ is determined by the single-particle levels in the middle of that band. This qualitative picture is supported by numerics. As an example consider a dot with a number of electrons near $N=64$. The situation is described, within our model, by a lattice of 128 sites, i.e., by a matrix of size $128 \times 128$. Separation between eigenvalues number $(N+i)$ and $(N+i-1)$ gives the spacing between the corresponding conductance peaks (the eigenvalues are ordered in energy). Let us look, for example, at the separation $\Delta_2$ between eigenvalues number 67 and 66. We take $t = 1$ so that disorder and interactions are described by the numbers $W$ and $U$ respectively. The distribution of $\Delta_2$, for an ensemble of 3000 matrices, for $W = 4$ and various values of $U$ is shown in Fig. 2 ($\Delta_2$ is normalized to $\Delta$, so that the $x$-axes is labeled by $\delta=\Delta_2/\Delta$). For $U = 0$ the distribution is close to Wigner-Dyson. This fact demonstrates that $W = 4$, although not small compared to unity, corresponds to the diffusive regime (to reach diffusion behavior with small $W$ one would need larger matrices, i.e., larger number of electrons in the dot). For $U$ near $2$ significant deviations from Wigner-Dyson statistics start to show up, and between $U \approx 2$ and $U \approx 3$ the distribution rapidly changes from Wigner-Dyson towards a more symmetric, broader distribution. For still larger values of $U$ (roughly, larger than $10$) the distribution gradually approaches a limiting Poissonian shape. The origin of this limiting distribution is clear: as was explained above, for sufficiently large $U$ the $t$-term in Eq. (5) can be neglected, and one ends up with an ensemble of diagonal matrices $H_{ij} = (W_i + U_i)\delta_{ij}$, with statistically independent eigenvalues. Histograms similar to those in Fig. 2 can be obtained also for other pairs of consecutive eigenvalues, not too far from the 64-th eigenvalue, $\epsilon_{64}$ (otherwise one should change the size of the matrix, adjusting it to the nearest magic number). This eigenvalue is somewhat exceptional, in the sense that it is between $\epsilon_{64}$ and $\epsilon_{65}$ that a gap opens up, for a sufficiently large $U$ (see Fig. 3, for $U=5$). Therefore, for large $U$, the distribution of $\Delta_2$ for this case is shifted by $U$ (Fig. 4). The other special feature of this particular case is that, for very large values of $U$, the distribution assumes a semi-Poisson shape (a semi-Poisson distribution is defined as $p(x) = xe^{-x}$).

The average density of states, for the same ensemble of matrices, is plotted in Fig. 3. For $U = 0$ one has the single band of the Anderson model. When $U$ increases, a gap starts to form in the middle of the band and, eventually, two separate bands emerge. There is a clear correlation between formation of the gap and deviation of the peak spacing statistics from the Wigner-Dyson shape. Note that significant deviations from Wigner-Dyson statistics develop long before a genuine gap opens up in the spectrum, i.e., standard metallic conductivity can coexist with "non-metallic" thermodynamic properties.

**IV. A RANDOM MATRIX MODEL**

One can further simplify the model, replacing the Anderson matrix, i.e., the first two terms in Eq. (5), by a $2N \times 2N$ Gaussian orthogonal ensemble (GOE) (Mehta, 1991). This leads to a matrix model

$$ H = G + A $$

where $G$ is taken from the GOE and $A$ is a $2N \times 2N$ (non-random) diagonal matrix, with half of the eigenvalues equal to $U$ and the other half equal to zero. It is interesting that the same matrix model can be arrived at by entirely different, more abstract arguments. Let's start with a standard Gaussian matrix, $G$, which is known to give a good description of the single particle level statistics in a chaotic or disordered quantum dot. The matrix ensemble $G$ has a $O(M)$ symmetry, where $M$ is the number of states (orbitals) spanning the relevant Hilbert space. Consider now $N$ interacting particles in the dot and employ some self-consistent description, e.g., the Hartree-Fock scheme. The outcome will be $M$ self-consistent orbitals, $N$ of which are occupied and the rest, $M - N$, are empty. One can make orthogonal transformations separately among the $N$ occupied and the $M - N$ empty orbitals, but not between the two types of orbitals. Thus, the symmetry is broken from $O(M)$ to $O(N) \times O(M - N)$. The matrix $A$ in Eq. (6) implements such symmetry breaking and introduces the two types of orbitals. Thus, the matrix model of Eq. (6) seems to emerge from some very general arguments.

Let us study the statistics of $\Delta_2 = \epsilon_{N+i} - \epsilon_{N+i-1}$, where $\epsilon_n$ is the $n$-th (ordered) eigenvalue of $H$ and $i$ is a small integer. Qualitatively, the picture is similar to that outlined above, i.e., under the increase of $U$, the distribution...
$P(\Delta_2)$ crosses over from the Wigner-Dyson shape to a broader, more symmetric distribution (Fig. 5). The crossover is related to the formation of a gap in the spectrum of $H$ (Fig. 6). Eventually, for sufficiently large $U$, the Wigner semicircle of the GOE splits into two well separated semicircles, and $P(\Delta_2)$ is controlled by the tail states, where the density of levels is extremely low (by a factor $N^{1/3}$ smaller than in the middle of the band (Mehta, 1991)). It is interesting to note that Brezin and Hikami (Brezin and Hikami, 1998) have studied the matrix model in Eq. (6), but with $G$ being the Gaussian unitary ensemble, instead of the GOE. They considered the "critical situation", when the two bands just touch one another, and showed that a new universal level statistics emerge in the large-$N$ limit.

V. CONCLUSIONS

A simple model for addition spectra is proposed and studied. It can be viewed as an extension of the charging model. The latter is based on the assumption that the added particles spread over the entire dot, charging it uniformly. This assumption is, clearly, inadequate in the limit of strong interactions, when the charge density in the dot exhibits strong spatial changes. The proposed model tries to account for this feature in the most crude phenomenological way, by introducing the parameter $U$. This turns out to be sufficient for obtaining a qualitatively correct behavior for the conductance peak spacings. The model, or rather its version in Eq.(6), is also of interest in the random matrix theory. The point is that the standard (orthogonal or unitary) Gaussian ensembles describe well the addition spectra in a disordered dot only in the absence of interactions. The interaction breaks the $O$- or $U$-invariance of a Gaussian ensemble. This is seen most clearly in the "extreme classical limit", when the electrons become localized on particular site-orbitals. Matrix $A$ in Eq.(6) implements such symmetry breaking. There are, of course, other ways to break this symmetry, some of which lead to a crossover between the Wigner-Dyson and the Poisson statistics for level spacings (for a review see Shapiro, 1996). It would be, perhaps, of some interest to investigate the connection between such generalized matrix ensembles and the addition spectra in disordered quantum dots.

The model in Eq. (5) was designed for spinless electrons. For spinfull electrons the extra particles can hop on occupied sites, so that there is no need for the second sublattice. The "minimal" model is defined on a $N$-site lattice, corresponding to the the $N$-electron magic configurations, which is the same as for the spinless case. However, each site of the lattice now contains 2 orbitals. They correspond to single and double occupation, respectively, and differ by energy $U$. For zero $U$ one has a single band, with doubly degenerate levels. The statistics of $\Delta_2$ exhibits in this case a bimodal structure, characteristic for non-interacting electrons with spin. For large $U$ the picture is essentially the same as for the spinless case: extra particles (holes) go into the tail of the upper (lower) Hubbard band (Mott, 1990). Since there is no spin degeneracy in these bands (they correspond to different site-occupation numbers), there is no trace of a bimodal structure. The absence of such structure, as well as other spin effects, have been studied earlier, both for disordered and clean quantum dots (Berkovits, 1998, Hirose and Wingreen, 1999, Baranger et al., 2000, Cha and Yang, 2000, Jacquod and Stone, 2000).

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REFERENCES

Alhassid, Y., 2000, Rev. Mod. Phys., 72, 845.

Andreev, A.F., and Lifshitz, A.M., 1969, Zh. Eksp. Teor. Fiz. 56, 2057 [Sov. Phys. JETP 29, 1107]

Baranger, H.U., Ullmo, D., and Glasman, L.I., 2000, Phys. Rev. B61, R2425.

Benenti, G., Waintal, X., Pichard, J.-L., and Shepelyansky, D., 2000, Europ. Phys. Jour. B17, 515.

Benenti, G., Waintal, X., and Pichard, J.-L., 1999, Phys. Rev. Lett. 83, 1826.

Berkovits R., 1998, Phys. Rev. Lett. 81, 2128.

Berkovits, R., and Altshuler, B.L., 1997, Phys. Rev. B55, 5297.

Blanter, Ya.M., Mirin, A.D., and Muzykantskii, B.A., 1997, Phys. Rev. Lett. 78, 2449.

Brezin, E., and Hikami, S., 1998, Phys. Rev. E58, 7176.

Cha, M.C., and Yang, S.R.E., 2000, Phys. Rev. B61, 1720.

Chakravarty, S., Kivelson, S., Nayak, C. and Voelker, K., 1999, Phil. Mag. B79, 859.

Cohen, A., Richter, K. and Berkovits, R., 1999, Phys. Rev. B60, 2536.

Hirose, K. and Wingreen, N.S., 1999, Phys. Rev. B59, 4604.

Jacquod, P., and Stone, A.D., 2000, Phys. Rev. Lett. 84, 3938.

Koulakov, A.A., Pikus, F.G. and Shklovskii, B.I., 1997, Phys. Rev. B55, 9223.

Koulakov, A.A. and Shklovskii, B.I., 1998, Phys. Rev. B57, 2352.

Levit, S. and Orgad, D., 1999, Phys. Rev. B60, 5549.

Mehta, M.L., 1991, Random Matrices (Academic Press).

Morris, J.R., Deaven, D.M., and Ho, K.M., 1996, Phys. Rev. B53, R1740.

Mott, N.F., 1990, Metal-Insulator Transitions (Taylor& Francis).

Patel, S.R., Cronenwett, S.M., Stewart, D.R., Huibers, A.G., Markus, C.M., Doroz, C.I., Harris, J.S., Campman, K., and Gossard, A.C., 1998, Phys. Rev. Lett. 80, 4522.

Prus, O., Auerbach, A., Aloni, Y., Sivan, U., and Berkovits, R., 1996, Phys. Rev. B54, R14289.

Shapiro, B., 1996, Int. Jour. Mod. Phys. B10, 3539.

Simmel, F., Abusch-Magder, D., Wharam, D.A., Kastner, M.A., and Kotthaus, J.R., 1999, Phys. Rev. B59, R10441.

Sivan, U., Berkovits, R., Aloni, Y., Prus, O., Auerbach, A., and Ben-Yoseph, G., 1996, Phys. Rev. Lett. 77, 1123.

Tsiper, E.U., and Efros, E.L., 1997, J. Phys. Cond. Matter 9, L561.

Walker, P.N., Montambaux, G., and Gefen, Y., 1999, Phys. Rev. B60, 2541.
FIG. 1. The large-$U$ limit of the model. In a magic configuration of $N$ electrons sublattice $A$ is occupied (full circles). Extra electrons can propagate on sublattice $B$ (empty circles). Vacancies can be created by removing particles from sublattice $A$. Hopping between sites 1 and 2 on sublattice $B$ occurs via occupied sites, like site 3.

FIG. 2. The distribution of conductance peak spacings, $P(\delta)$, is plotted for various values of $U$, between 0 and 5. The separation, $\Delta_2$, between eigenvalues number 67 and 66 is measured in units of $\Delta$, i.e., $\delta = \Delta_2 / \Delta$. The constant charging energy, $V_C$, is not shown in this plot. It can be accounted for by shifting all the curves by $V_C$. The dotted curve is the Wigner-Dyson distribution.

FIG. 3. The average density of states, $\nu$, for the same values of $U$ as in Fig. 2. Energy (the horizontal axes) is measured in units of $t$.

FIG. 4. The same as Fig. 2 but for the separation between eigenvalues 65 and 64. It is between these eigenvalues that a gap opens up, in the large-$U$ limit.

FIG. 5. Distribution for the separation between eigenvalues 67 and 66, for the matrix ensemble defined in Eq. (6), for values of $U$ between 0 and 2.5. The separation is measured in units of the average level spacing at $U = 0$, i.e., in the absence of matrix $A$. The matrix size is $128 \times 128$. The size of the ensemble is 3000.

FIG. 6. The average density of states, $\nu$, of the matrix ensemble, for the same values of $U$ as in Fig. 5.
\[ P(\delta) \] for different values of \( U \):

- \( U = 0 \)
- \( U = 1 \)
- \( U = 2 \)
- \( U = 2.5 \)
- \( U = 3 \)
- \( U = 5 \)
