Comparing the various slope stability methods to find the optimum method for calculating factor of slope safety

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Abstract There are various slope stability analysis methods normally used to find the factor of safety of a slope. All the methods have different analysis techniques and formulation, due to which the results also vary in some cases. In this research article, all those methods are compared and the results are provided in graphical form to show the variation. The results can be used in any slope stability project to find the optimum value for factor of slope safety and hence can minimize the risk while designing any slope in hilly areas or in case of designing an earthen dam.

1. Introduction

Land siding is a very common issue all over the world especially in hilly areas and in case of earthen dams. This issue causes human and economy loss time to time. Therefore researchers are always working how to minimize and overcome this issue. There are various methods to design slopes and know about the factor of safety. There are various methods to analyse the slope stability which are summarized by Duncan[1].The methods normally followed in analyzing slope stability issues are:

- Bishop simplified
- Bishop modified
- Janbu simplified and corrected
- Spencer
- GLE – Morgenstern – Price
2. Literature Survey

The influence of soil strength on the probability of failure of slopes using conventional limit equilibrium slip circle analysis has been explored by [2-5]. A brief introduction to some of these methods is explained below.

2.1. Bishop Simplified

In this method, the slipping mass above the failure plane is divided into slices. On each slice, force equilibrium conditions are applied and equation for factor of safety is generated. Considering the mechanical equilibrium, the forces acting on each slice are obtained[6]. All slices are considered individually and the interactions of slices is neglected because the resultant forces are parallel to the base of each slice. Figure 1 shows all the assumed forces acting on the slice and equation 1 is the generated equation for calculating the factor of safety in this method.

![Figure 1. Assumed slice](image)

Where; $F = \text{Factor of safety} = C + N' \tan \phi$

$S_a = \text{Available strength}$

$S_m = \text{Mobilized strength}$

$U_p = \text{Pore water force}$

$U_a = \text{Surface water force}$

$W = \text{Weight of slice}$

$Q = \text{External surcharge}$

$N' = \text{Effective normal force}$

$K_v = \text{Vertical seismic coefficient}$

$K_h = \text{Vertical seismic coefficient}$
\( Z_L = \text{Left interslice force} \quad Z_R = \text{Left interslice force} \)

\( \theta_L = \text{Left interslice force angle} \quad \theta_R = \text{Left interslice force angle} \)

\( h_L = Z_L, \text{ Force height, left} \quad h_R = Z_L, \text{ Force height, right} \)

\( \alpha = \text{Slice base inclination} \quad \beta = \text{Slice top inclination} \)

\( \sigma = \text{Surcharge inclination} \quad b = \text{width of slice} \)

\( h_c = \text{Height of centroid of slice} \)

\[
F = \frac{\sum_{i=1}^{n} (C + N' \tan \phi)}{\sum_{i=1}^{n} A_5 - \sum_{i=1}^{n} A_6 - \sum_{i=1}^{n} A_7} \quad (1)
\]

Where;

\[
A_5 = [W(1 - k_u) + U_\beta \cos \beta + Q \cos \sigma] R \sin \alpha \quad (2)
\]

\[
A_6 = [U_\beta \sin \beta + Q \sin \sigma] (\cos \alpha - \frac{h}{R}) \quad (3)
\]

\[
A_7 = k_h W (\cos \alpha - \frac{h}{R}) \quad (4)
\]

2.2. Bishop modified

The difference between ordinary and modified Bishop method is that in modified method, an assumption is made that the normal interaction forces between nearby and adjacent slices are collinear and the final interslice shear force is considered as zero.

2.3. Janbu simplified and corrected

The Janbu’s simplified method[7] is applicable to non-circular slip surfaces as shown in figure 2.6. In this method, the interslice forces are assumed to be horizontal and thus the shear forces are zero.
The factor of safety equation in this case is:

\[ F = f_0 F_0 \]  

(5)

Where \( F_0 \) is the factor of safety in simplified case and a correction factor \( f_0 \) is applied to the same equation to take account of the interslice shear forces.

\[ f_0 = 1 + b_1 \left[ \frac{d}{L} - 1.4 \left( \frac{d}{L} \right)^2 \right] \]  

(6)

\[ F_0 = \frac{\sum (c' \tan \phi + u \tan \phi) \sec \alpha}{\sum W \tan \alpha} \]  

(7)

Figure 3 shows the Janbu correction factor.
2.4. Spencer Method

Spencer’s method was developed for analyzing circular slip surfaces but later on it was extended to non-circular slip surfaces by assuming a frictional center of rotation. This method is based on the assumption that the inter-slice forces are parallel to each other and they have the same inclination. Figure 4 shows Spencer’s method diagram for the assumed slice.
The factor of safety equation in case of Spencer method is given below:

\[
F = \frac{\sum (c_i + (P - u_l) \tan \phi)\sec \alpha}{\sum (W - (X_R - X_L)) \tan \alpha}
\]  
\( (8) \)

2.5. Morgenstern – Price Method

This method was introduced by Morgenstern and Price[10], which considers both normal and tangential equilibrium along with the moment equilibrium for each slice in circular as well as non-circular slip surfaces. The factor of safety equation in this case is:

\[
F = \frac{\sum (c_i + P - u_l) \tan \phi \cos \alpha}{\sum P \sin \alpha}
\]  
\( (9) \)

Similarly, there are other methods of analysis such as:

- Ordinary / Fellenius
- Lowe – Karafiath
- Corps of Engineers #1
- Corps of Engineers #2

Summary of assumption, limitations and usage of some of these analysis methods is given in Table 1.
Table 1. Brief description of commonly used slope stability analysis methods[11]

| Method                      | Equilibrium condition                        | Slip Surface | Assumptions                                                                 | Unknowns                          |
|-----------------------------|----------------------------------------------|--------------|------------------------------------------------------------------------------|-----------------------------------|
| Logarithmic spiral method   | Moment equilibrium about centre of spiral    | Log spiral   | The slip surface is logarithmic spiral                                         | One, that is factor of safety      |
| Friction circle method      | Moment and force equilibrium                 | Circular     | Resultant of the normal and frictional component of shear strength tangent to friction circle. | One, that is factor of safety      |
| Ordinary method of slices   | Moment equilibrium about centre of circle    | Circular     | Side forces of the slices are neglected and normal force equals W\cos\alpha and the shear force W\sin\alpha. | One, that is factor of safety      |
| Simplified Bishop method    | Vertical equilibrium and overall moment equilibrium | Circular     | Zero interslice shear forces                                                 | n + 1                             |
| Janbu’s Simplified Method   | Force equilibrium (vertical and horizontal)  | Any shape    | The side forces are horizontal.                                              | 2n                                |
| Swedish circle (\(\phi = 0\)) Method | Moment equilibrium about centre of circle | Circular     | Circular slip surface and Zero Friction angle                               | One, that is factor of safety      |

In Corps of Engineer method, resultant interslice force direction is;

(i) The average slope from the beginning to the end of the slip surface or
(ii) Parallel to the ground surface.
Figure 5 shows the classification chart for few slope stability methods [11].

![Classification chart for slope stability methods](image)

**Figure 5.** Slope stability methods, field conditions and assumptions

3. Methodology

Three different material types are investigated in this analysis. All these three material types have different properties such as cohesion, friction, unit weight and angle of repose. Results for factor of safety were achieved using all the below methods:

- Bishop simplified
- Bishop modified
- Janbu simplified and corrected
- Spencer
- GLE – Morgenstern – Price
- Ordinary / Fellenius
- Lowe – Karafiath
- Corps of Engineers #1
- Corps of Engineers #2
A limit equilibrium software is used in this analysis namely slide. The variation of factor of safety is provided in graphical form out of which designer can select the optimum value.

4. Results and Discussions

Figure 6 shows the slope model used in this analysis:

![Figure 6. Slope model](image)

Three types of material is used in this analysis. Table 2 shows the material properties details and factor of safety for all the analysis methods. Figure 7, 8 and 9 shows the variation of factor of safety for all the three material types.

| Material Number (M) | Cohesion (kN/m²) | Angle of Repose (AOR) | Unit Weight (kN/m³) | Friction angle (ϕ) | Material Type | Method                        | Factor of Safety (FS) |
|---------------------|------------------|-----------------------|---------------------|-------------------|---------------|-------------------------------|-----------------------|
| 1                   | 11               | 30                    | 14                  | 31                | Clay          | Bishop simplified            | 1.702                 |
| 1                   | 11               | 30                    | 14                  | 31                | Clay          | Ordinary / Fellenius          | 1.631                 |
| 1                   | 11               | 30                    | 14                  | 31                | Clay          | Janbu simplified             | 1.616                 |
| 1                   | 11               | 30                    | 14                  | 31                | Clay          | Janbu corrected              | 1.7                   |
| 1                   | 11               | 30                    | 14                  | 31                | Clay          | Spencer                      | 1.697                 |
| 1                   | 11               | 30                    | 14                  | 31                | Clay          | Corps of Engineers # 1        | 1.704                 |
|   |   |   |   |   |                      |          |
|---|---|---|---|---|----------------------|----------|
| 1 | 11| 30| 14| 31| Clay Corps of Engineers # 2 | 1.705    |
| 1 | 11| 30| 14| 31| Clay Lowe-Karafiath   | 1.701    |
| 1 | 11| 30| 14| 31| Clay GLE / Morgenstern-Price | 1.698    |
| 2 | 12| 35| 16| 33| Clay Bishop simplified  | 1.655    |
| 2 | 12| 35| 16| 33| Clay Ordinary / Fellenius | 1.601    |
| 2 | 12| 35| 16| 33| Clay Janbu simplified  | 1.59     |
| 2 | 12| 35| 16| 33| Clay Janbu corrected   | 1.633    |
| 2 | 12| 35| 16| 33| Clay Spencer           | 1.654    |
| 2 | 12| 35| 16| 33| Clay Corps of Engineers # 1 | 1.658    |
| 2 | 12| 35| 16| 33| Clay Corps of Engineers # 2 | 1.669    |
| 2 | 12| 35| 16| 33| Clay Lowe-Karafiath   | 1.66     |
| 2 | 12| 35| 16| 33| Clay GLE / Morgenstern-Price | 1.654    |
| 3 | 5 | 40| 17| 38| Clayey Sand Bishop simplified | 1.318    |
| 3 | 5 | 40| 17| 38| Clayey Sand Ordinary / Fellenius | 1.278    |
| 3 | 5 | 40| 17| 38| Clayey Sand Janbu simplified | 1.273    |
| 3 | 5 | 40| 17| 38| Clayey Sand Janbu corrected | 1.321    |
| 3 | 5 | 40| 17| 38| Clayey Sand Spencer | 1.313    |
| 3 | 5 | 40| 17| 38| Clayey Sand Corps of Engineers # 1 | 1.316    |
| 3 | 5 | 40| 17| 38| Clayey Sand Corps of Engineers # 2 | 1.32     |
| 3 | 5 | 40| 17| 38| Clayey Sand Lowe-Karafiath | 1.315    |
| 3 | 5 | 40| 17| 38| Clayey Sand GLE / Morgenstern-Price | 1.313    |
Figure 7. Bar graph showing factor of safety for Material 1

Figure 8. Bar graph showing factor of safety for Material 2
5. Conclusions

In case of material 1, the maximum value for factor of safety is 1.705 given by Corps of Engineer # 2 method while the minimum value for factor of safety is 1.616 given by Janbu simplified method. The overall average value for factor of safety by all the methods came out to be 1.684 which is very near to the value given by Spencer method that is 1.697.

In case of material 2, the maximum value for factor of safety is 1.669 given by Corps of Engineer # 2 method while the minimum value for factor of safety is 1.59 given by Janbu simplified method. The overall average value for factor of safety by all the methods came out to be 1.642 which is very near to the value given by Janbu corrected that is 1.633.

In case of material 3, the maximum value for factor of safety is 1.321 given by Janbu corrected method while the minimum value for factor of safety is 1.273 given by Janbu simplified method. The overall average value for factor of safety by all the methods came out to be 1.307 which is very near to the value given by Spencer that is 1.313.

This analysis shows that Spencer method gives optimum value for factor of safety in homogenous slopes.
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