New Evidence for “Confined Coherence” in Weakly Coupled Luttinger Liquids

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Abstract

Based upon the calculation of the exact interliquid hopping rate and an approximate single particle Green’s function, we present new evidence for the existence of a phase of \textit{relevant but incoherent} inter-Luttinger liquid transport. This phase of “confined coherence” occurs when the Luttinger liquid exponent $\alpha$ satisfies $\alpha_c < \alpha < 1/2$. We argue that $\alpha_c$ is strictly bounded above by $1/4$, and is probably substantially smaller, especially in spin-charge separated Luttinger liquids. We also discuss connections with the work of others.

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I. INTRODUCTION

The physics of a strongly correlated, highly anisotropic electron system represents a subtle problem in many-body physics. In previous work [1,2] we have considered the problem of one-dimensional (1D) electron liquids coupled by weak interliquid hopping. Implicit in our approach is the recognition that if one begins with a collection of truly 1D metals and then turns on weak interliquid hopping, it is not \textit{a priori} appropriate to consider the electron-electron interaction as a perturbation on an anisotropic (2D) free Fermi gas. Rather, one should consider the \textit{interliquid hopping} as a perturbation on the (otherwise decoupled) 1D liquids. The problem is nontrivial but tractable to some degree because the low energy physics of a 1D metal is described by Luttinger liquid theory. Unlike in a Fermi liquid, where the electron spectral function, $\rho(k,\omega)$ is dominated by a quasiparticle part, which sharpens up to a $\delta$-function as $k \to k_F$, in a Luttinger liquid there are no Landau quasiparticles, rather, $\rho(k,\omega)$ exhibits only power law singularities. For this reason, and others we have previously discussed [1,2], the problem of weakly coupled Luttinger liquids is closely analogous to that of weak tunneling in a two level system (TLS) coupled to an ohmic dissipative bath [3]. Exploiting this analogy led us to propose [1,2] that interliquid hopping between non-Fermi liquids may have three qualitatively distinct regimes: it may be irrelevant, relevant and coherent, or relevant but entirely incoherent. The incoherent interliquid hopping phase would represent a new state of matter with intrinsically incoherent transport in at least one direction. There is substantial experimental support for this proposal based on its ability to explain certain anomalous properties of the low dimensional organic conductor (TMTSF)$_2$PF$_6$, as has been discussed elsewhere [4–6]. In this paper we briefly report new results which address this question based upon the use of exact Luttinger liquid spectral functions, careful consideration of the analytic properties of Luttinger liquid Green’s functions, and a reinterpretation of a calculation made by others [7].
II. INTERLIQUID HOPPING RATE FOR WEAKLY COUPLED LUTTINGER LIQUIDS

We are interested in the problem of \( N \) coupled Luttinger liquids, \( N \rightarrow \infty \). At \( O(t_\perp^2) \), however, our results are equivalent to those for \( N = 2 \), and we therefore consider the problem of two Luttinger liquids coupled by a spatially uniform, single particle hopping (as in [1]). Our calculation is dynamical and involves taking a \( t = 0 \) state with \( \Delta N \) more (right moving) particles in liquid 2 than liquid 1 and no Tomonaga bosons excited in either, then turning on \( t_\perp \) and examining the time dependence of \( \Delta N \) (for motivation see [1] and [3]). The particle number difference \( \Delta N \) entails a Fermi momentum difference \( \Delta k \) and a chemical potential difference \( \Delta \mu \equiv v \Delta k \). Unlike our earlier work [1] based upon space-time Green’s functions, we use spectral function methods here, which is both physically more illuminating and permits the calculation of key correlation functions exactly.

At \( O(t_\perp^2) \) the interliquid hopping rate \( \Gamma(t) \) can be written in a spectral function form as

\[
\Gamma(t) = 2t_\perp^2 L \int \frac{d\omega}{2\pi} \sin \frac{\omega t}{\omega} \{ A_{12}(\omega) + A_{21}(\omega) \} \tag{1}
\]

where

\[
A_{ij}(\omega) = \int \frac{d\omega'}{2\pi} \int \frac{dk}{2\pi} J_1^{(i)}(k, \omega') J_2^{(j)}(k, \omega' - \omega) \tag{2}
\]

and the spectral functions \( J_{1,2}(k, \omega) \) are the Fourier transforms of \( J_1(k, t) \equiv \langle c_1(k, t)c_1^\dagger(k, 0) \rangle \) and \( J_2(k, t) \equiv \langle c_2^\dagger(k, 0)c_2(k, t) \rangle \). In this paper we consider only the zero temperature limit, in which case \( J_{1,2}(k, \omega') = \theta_\pm(\omega' - \mu)\rho_{1,2}(k, \omega' - \mu) \) where \( \rho(k, \omega) \) is the electron spectral function as conventionally defined. We remark that Eqs. (1) and (2) are not specific to coupled 1D liquids: they may be extended to the case of coupled 2D liquids by replacing \( k \) by \( \mathbf{k} \) in the \( k \)-integrals and in the definitions of \( J_1 \) and \( J_2 \).

Physically, \( A_{12}(\omega) \) is the effective interliquid hopping spectral function for an electron hopping to liquid 1, from liquid 2, and \( A_{21}(\omega) \) the opposite. As \( A_{21}(\omega) \) never has a coherent component, it suffices, for the purposes of studying the question of coherence, to consider only \( A_{12}(\omega) \).
Before presenting the calculation of $\Gamma(t)$ for coupled Luttinger liquids, we first show how the coherence of interliquid hopping manifests itself in the case of coupled (Landau) Fermi liquids.

**Free Fermi Gasses, and Fermi Liquids:** For free Fermi gasses, $A_{12}(\omega) \propto \Delta \mu \delta(\omega)$ and $A_{21}(\omega) = 0$. Thus $\Gamma(t) \propto \Delta \mu t$, a clear signal of coherent hopping and hence of a fundamental rearrangement of the ground state.

In a Fermi liquid the (retarded) Green’s function is $G_{R}^{-1}(k, \omega) = Z\frac{1}{\omega - E_{k} + i\gamma \omega^{2}}$ where $Z$ is the quasiparticle renormalization factor, and $\gamma$ is a (positive) parameter characterizing the strength of the electron-electron interactions. The spectral function is then given by $\rho(k, \omega) = -2 \text{Im}G_{R}(k, \omega)$ from which we obtain

$$A_{12}(\omega) \sim v_{F}^{-1}\{Z^{2}\Delta \mu \delta(\omega) + (3\pi)^{-1}Z^{3}\gamma \omega\} \theta_{+}(\omega + \Delta \mu)$$  

We find that $\Gamma_{12}(t)$ is a sum of a term $\propto Z^{2}\Delta \mu t$ representing fundamentally coherent processes, and a term $\propto \gamma Z^{3}t^{-1}$ which is marginal. By choosing a sufficiently small $t_{\perp}$ one can find a time $t$ such that, while remaining in the perturbative regime, $N^{-1}\int_{0}^{t} \Gamma(t')dt' \ll 1$, the ratio of the coherent contribution to the marginal contribution is arbitrarily large. This is true regardless of how small $Z$ is. Thus, a perturbative calculation in $t_{\perp}$ does not reveal any likelihood of a loss of coherence of interliquid tunneling, and there is no impediment to the formation of an interliquid band of width $\sim Zt_{\perp}$. This is consistent with what we would expect from a calculation based upon (Landau) quasiparticles. Formally, the coherence is reflected in the fact that the spectral function $A_{12}(\omega)$ is dominated by the $\delta$-function at $\omega = 0$, indicating that hopping is almost entirely energy degenerate.

**Luttinger Liquids:** We now turn to the problem of coupled Luttinger liquids, considering the case of spin-independent electronic interactions, characterized by the anomalous exponent, $2\alpha$, of the single particle Green’s function, and charge- and spin-velocities $v_{c}$ and $v_{s}$. The calculation of $A_{12}(\omega)$ and $A_{21}(\omega)$ is lengthy, and we present only the final results here. Complete details are given in [6]. The exact result is

$$A_{12}(\omega) = A_{12}^{\text{low}}(\omega) \theta_{+}[\omega - (v_{s} - v)\Delta k] \theta_{+}[(v_{c} - v)\Delta k - \omega] + A_{12}^{\text{high}}(\omega) \theta_{+}[\omega - (v_{c} - v)\Delta k]$$
\[
A_{12}^{\text{low}}(\omega) = \frac{1}{\Gamma(1 + 4\alpha)} \frac{1}{2\Delta v} \left( \frac{a^2}{\bar{v}\Delta v} \right)^{2\alpha} (\omega + (v_s - v_s)\Delta k)^{4\alpha}
\]
\[
A_{12}^{\text{high}}(\omega) = \frac{1}{(1 + 2\alpha) \Gamma(2\alpha)\Gamma(1 + 2\alpha)} \frac{1}{\bar{v}} \left( \frac{a}{2v_c} \right)^{4\alpha} (\omega + (v_c + v_c)\Delta k)^{2\alpha+1}(\omega - (v_c - v_c)\Delta k)^{2\alpha-1}
\]
\[
_2F_1 \left( 1, 1 - 2\alpha; 2 + 2\alpha; -\left( \frac{\Delta v}{\bar{v}} \right) \left[ \frac{\omega + (v_c + v)\Delta k}{\omega - (v_c - v)\Delta k} \right] \right)
\]

where \(_2F_1\) is the hypergeometric function, \(a\) a short distance cutoff, and \(\bar{v} \equiv v_c + v_s, \Delta v \equiv v_c - v_s\). The typical morphology of \(A_{12}(\omega)\) is shown in Fig. 1. We observe that \(A_{12}(\omega)\) is both nonsingular and of wide support, having non-zero weight from just below \(\omega = 0\) all the way up to the ultraviolet cutoff. As \(\alpha \to 0\), we have \(A_{12}(\omega) \to \delta(\omega)\), and one needs to use degenerate perturbation theory to treat the interliquid hopping. For \(\alpha > 1/2\), \(t_{\perp}\) is a formally irrelevant operator, which is reflected in the fact that the spectral weight in \(A_{12}(\omega)\) is pushed to high energies. For \(\alpha < 1/2\), but not too small, \(A_{12}(\omega)\) is generically “flat” suggesting that much, if not most, of the hopping occurs via non-degenerate (i.e. “inelastic”) processes.

This is reminiscent of situations in more elementary quantum mechanical problems where Fermi’s “Golden rule” is applied, and clearly raises doubts over any claim that the action of \(t_{\perp}\) is to drive the system to a fixed point in which interliquid hopping is coherent.

For simplicity, we shall restrict our discussion from here on to the spinless case, which can be obtained by formally taking \(\Delta v \to 0\). The general case shall be discussed elsewhere.

In calculating \(\Gamma_{12}(t)\) it is simplest to consider its time derivative. We find

\[
\frac{d\Gamma_{12}(t)}{dt} = \frac{t_{\perp}^2 L}{\pi} \frac{1}{\Gamma(2\alpha)\Gamma(2 + 2\alpha)} \left( \frac{a}{2v_c} \right)^{4\alpha} \frac{1}{2v_c} \frac{1}{\Gamma(1 - 2\alpha)} t^{-(1+4\alpha)} \Re \left\{ e^{i(v_c-v)\Delta k t} \left[ e^{i2\pi\alpha} \Gamma(1 - 2\alpha) \Gamma(1 + 4\alpha) _1F_1(-1 - 2\alpha, -4\alpha; -ix) + \frac{1}{2} \frac{1 + 2\alpha}{2\Gamma(2\alpha)\Gamma(1 - 4\alpha)} x^{1+4\alpha} _1F_1(2\alpha, 2 + 4\alpha; -ix) \right] \right\}
\]

where \(_1F_1\) is the confluent hypergeometric function and, for convenience, we have introduced the variable \(x = 2v_c\Delta k t\).

Equation (5) is an exact result for the (time derivative of the) interliquid hopping rate, to lowest order in \(t_{\perp}\). We use the expansion \(_1F_1(a, b; z) = 1 + ab^{-1}z + ab^{-1}(a + 1)(b + 1)\)
1) \(-z^2/2 + \ldots\) and, noting that it makes little physical sense to suppose that terms of \(O(x^2)\) or higher (i.e. terms of \(O(\Delta k^2)\) or higher) are important in determining the coherence or incoherence of single particle hopping, we retain only the \(O(x^0)\), \(O(x)\) and \(O(x^{1+4\alpha})\) terms. This gives

\[
\frac{d\Gamma_{12}(t)}{dt} = \frac{t_1^2 L}{\pi} \frac{1}{\Gamma(2\alpha)\Gamma(2 + 2\alpha)} \left( \frac{a}{2v_c} \right)^{4\alpha} \frac{1}{2v_c} \cos[(v_c - v)\Delta kt] t^{-(1+4\alpha)}
\]

\[
\{(1 + 2\alpha) \left\{ -\sin(2\pi\alpha) \frac{\Gamma(1 + 4\alpha)}{(1 + 2\alpha)} + \cos(2\pi\alpha)\Gamma(4\alpha) x + \frac{\Gamma(2\alpha)\Gamma(1 - 4\alpha)}{2(1 + 4\alpha)\Gamma(1 - 2\alpha)} x^{1+4\alpha} \right\}
\]

\[-\tan[(v_c - v)\Delta kt] \cos(2\pi\alpha)\Gamma(1 + 4\alpha) \}
\]

The latter two terms continuously develop into the correct, coherent result for free fermions as \(\alpha \to 0\) and their modification from the Fermi liquid result is closely analogous to the behavior of the appropriate terms in the derivative of the TLS transition rate upon turning on coupling to an ohmic bath. In the other well understood limit, \(\alpha > 1/2\), the entire expression leads to a finite integrated transition probability, \(P(t) = \int_0^t dt'\Gamma(t')\), in agreement with the known irrelevance of \(t_\perp\) in that limit. In between, the \(\Delta k\)-independent term gives \(P(t) \propto t^{1-4\alpha}\) which is long-time convergent and therefore irrelevant if \(\alpha > 1/4\), but represents fundamentally incoherent interliquid hops if \(0 < \alpha < 1/4\). The \(O(\Delta k^{1+4\alpha})\) term requires care to interpret when \(\alpha \neq 0\), but we note that the oscillatory prefactor \(\cos[(v_c - v)\Delta kt]\) will force \(\Gamma_{12}(t)\) to be essentially time-independent for times \(t > [(v_c - v)\Delta k]^{-1}\). This effect is analogous to that of non-degeneracy in the TLS where it has been argued to dramatically decrease coherence. In order for lowest order hopping to be coherent, one must remain at times short enough to avoid the cutoff effect of this prefactor and the maximum possible \(\Delta k\) for a given time \(t\) is \(\Delta k_{\max} \sim [(v_c - v)t]^{-1}\). The \(O(\Delta k^{1+4\alpha})\) term in \(d\Gamma/dt\) is therefore bounded by \(\sim \Delta k t^{-4\alpha}/(v_c - v)^{4\alpha}\) which has the same form as the term linear in \(\Delta k\) and we therefore consider only the latter term.

If the term linear in \(\Delta k\) decays slower than \(t^{-1}\) it should be interpreted as a potentially coherent term. For \(\alpha > 1/4\) it falls off faster than \(t^{-1}\) and at \(O(t_\perp^2)\) the interliquid single particle hopping is completely incoherent, signalled by the convergence of \(\Gamma(t \to \infty)\). This is despite the relevance of \(t_\perp\) in the RG sense for \(\alpha < 1/2\). We therefore expect an inco-
herent interliquid hopping phase for $1/4 < \alpha < 1/2$. There are, however, additional factors enhancing incoherence over and above the time exponent of the $O(\Delta k)$ term.

First, there is the “dephasing” prefactor $\cos[(v_c - v)\Delta k t]$, analogous to a bias term in a TLS. Based on the results from that problem [3], this should enhance incoherence. Further, there are the incoherent processes contributing to the $\Delta k$-independent term. For $0 < \alpha < 1/4$ the interliquid hopping rate and the integrated transition probability, $P$, are essentially sums of incoherent and coherent parts, defined by their respective time behaviors. Due to the presence of the dephasing term, the coherent term remains so only for times $t \lesssim [(v_c - v)\Delta k]^{-1}$. As such, $P_{12}^{\text{coh}}(t)$ is bounded above in magnitude by $\sim t_\perp^2 v_c \Lambda^{4\alpha} t^{1-4\alpha}/(v_c - v)$ so that,

$$\frac{P_{12}^{\text{incoh}}(t)}{P_{12}^{\text{coh}}(t)} > \alpha \frac{(v_c - v)}{v_c}$$

This is independent of $t_\perp$ and the purely incoherent channel cannot be eliminated in the $t_\perp \to 0$ limit, as it can in a Fermi liquid. As a result, we are forced to consider the influence of interliquid hops upon one another via correlations not automatically included in our $O(t_\perp^2)$ calculation. To begin with, interliquid hops through the coherent channel will be interrupted by the finite probability of a hop through the incoherent channel. Secondly, intraliquid interactions will lead to scattering of coherent hops by incoherent hops. In the limit $t_\perp \to 0$, the incoherent hops have an arbitrarily long time to scatter the coherent hops (although their density also vanishes in this limit) and we find that the effect of a given incoherent hop on the coherent hops grows at least linearly in time. If it grows faster than linearly, the scattering will be divergent and hopping should be incoherent as $t_\perp \to 0$ for any $\alpha$.

Combining all of these effects, we expect that as we decrease $\alpha$ from $1/4$, incoherence will be stabilized down to some critical value $\alpha_c < 1/4$ by a combination of the purely incoherent term, the dephasing prefactor which kills coherence if $\Delta k t$ is too large, and, in the case of fermions with spin, spin-charge separation, which further suppresses coherence for finite $\Delta k t$ [6]. We again emphasize the utility of the spectral function $A_{12}(\omega)$ in indicating the coherent
or incoherent nature of the interliquid hopping.

III. APPROXIMATE SINGLE PARTICLE GREEN’S FUNCTION:
CALCULATION AND INTERPRETATION

We now consider how these same effects might appear in the more conventional calculation of the Green’s function for $N \to \infty$ coupled Luttinger liquids of spinless fermions. We will neglect vertex corrections associated with $t_\perp$ and incorporate $t_\perp(k_\perp)$ as an energy independent self-energy. We are motivated by similar calculations by others [9], however we focus on analytic properties of the Green’s function not previously treated. Using $G^{-1} = G_0^{-1} - \Sigma$, $G_0(k, \omega) = (v^2k^2 - \omega^2)^\alpha(\omega - vk)^{-1}$, gives

$$G(k, k\perp, \omega) = \frac{(v^2k^2 - \omega^2)^\alpha}{(\omega - vk) - t_\perp(k_\perp)(v^2k^2 - \omega^2)^\alpha}$$

where we have set the dimensionful high energy cut-off to 1 and $k$ is momentum along the chains measured from the Fermi surface. Eq. 7 must be supplemented by a discussion of the analytic properties of $G$ and $G_0$, for whose discussion we consider only positive $k$ since negative $k$ is essentially identical for $t_\perp(k_\perp) \to -t_\perp(k_\perp)$.

First, recall that the singularities of the Green’s function, particularly poles, only have sensible physical interpretations in the second and fourth quadrants of the complex $\omega$ plane. For $k \neq 0$, $G_0$ has two branch cut singularities, one for each sign of $\omega$, which must originate in the second and fourth quadrants. Also, $G_0$ must be real for $-vk < \omega < vk$ since in that region no on-shell decay of an injected fermion is possible. This implies that the phase of $G_0$ for $\omega > vk$ should be given by $-\alpha\pi$, and by $-\pi - \alpha\pi$ for $\omega < -vk$. Now consider the pole equation, $G_0^{-1}(k, \omega) = t_\perp(k_\perp)$, for $k = 0$ and $t_\perp(k_\perp) > 0$. For $\alpha = 0$, the pole in the complex $\omega$ plane is at $t_\perp(k_\perp)$ and, as we turn on $\alpha$, it shifts into the fourth quadrant. Moving off the axis into the fourth quadrant an angle $\Theta$ changes the phase of $G_0^{-1}(0, \omega)$ to $\alpha\pi - (1 - 2\alpha)\Theta$ and it is again possible to have a pole if

$$\Theta = \alpha\pi/(1 - 2\alpha)$$  (8)
For small $\alpha$, this pole could be sensibly interpreted as a weakly damped quasiparticle pole, as in a usual Fermi liquid. However, for $\alpha > 1/4$, $\Theta > \pi/2$, the pole enters the fourth quadrant and the solution has no sensible interpretation as a quasiparticle pole (it would imply an unoccupied, negative energy quasiparticle state). The last physical solution, which occurred for $\alpha = 1/4$, corresponds to a purely imaginary frequency, entirely in keeping with the idea that $t_{\perp}$ is acting incoherently at this value of $\alpha$. For a negative $t_{\perp}$, an exactly parallel scenario involving the second, instead of the fourth quadrant, results. In both cases, for $\alpha > 1/4$, there is no physically sensible pole resulting from incorporation of $t_{\perp}$ as a self-energy, and the results are extremely suggestive of incoherence.

The effect is very closely analogous to the behavior of the Laplace transform of $P(t)$ found in [3] at the onset of incoherence. A similar analogy between the locations of the poles of the single particle Green’s function approximated in this way and the Laplace transform of $P(t)$ in the TLS problem was noted in [10].

We now consider $k \neq 0$. Consider first the case $t_{\perp} > 0$. As we move some distance away from the Fermi surface, the singularity at $-vk$ becomes more distant and its effect on the phase less important. For $k^{1-2\alpha} \gg t_{\perp}$, it becomes possible to circle the singularity at $vk$ without moving appreciably with respect to the singularity at $-vk$, and the phase of $G_{0}^{-1}(k, \omega)$ close to $\omega = vk$ varies as $\alpha \pi - (1 - \alpha)\Theta$ where $\Theta$ is measured downward from the real $\omega > vk$ half-line. As before, at small $\alpha$ the pole has a small imaginary part to its frequency and it can be sensibly interpreted as a weakly damped quasiparticle pole. However, now $\alpha$ can be as large as $1/2$ before the pole is forced into an unphysical region. Note, however, that for $\alpha > 1/3$, $\Theta > \pi/2$, so that the addition of a positive, real self energy ($t_{\perp}$) shifts the singularity at $vk$ to a complex energy with a real part less than $vk$. Including spin-charge separation is more complicated and we state only two of our results for $v_{c} > v_{s}$: (1) for $\alpha > 1/6$, the pole lies at an energy whose real part is shifted lower than $v_{c}k$, and (2) for $\alpha > 1/4$, and large $k$ the pole equation is again not satisfiable for a physically sensible $\omega$.

Returning to the spinless case, let us follow the pole for $t_{\perp} < 0$ as we increase $k$. For
\[ \alpha < 1/4 \] this pole lies in the second quadrant and increasing \( k \) eventually pushes it to the imaginary axis. When \( k = k_c = v^{-1}t_{\perp}^{1/(1-2\alpha)} \cos(2\pi \alpha) \), the pole reaches \( \omega = \omega_c = ivk_c \tan(2\pi \alpha) \). Again, the last frequency with a possible physical interpretation is purely imaginary, paralleling what occurred for \( k = 0 \) and \( \alpha = 1/4 \).

In addition to this pole, a new pole appears, for negative \( t_{\perp}(k_{\perp}) \) and \( k > 0 \), at \( \omega \in (-vk, vk) \) given by the real solution of \( (vk - \omega)^{1-\alpha}(vk + \omega)^{-\alpha} = |t_{\perp}(k_{\perp})| \). This undamped pole is unphysical, however, in a number of ways. Firstly, \( G \) is purely real at the position of the pole only because there is nothing for a fermion at this momentum and energy to decay into in the unperturbed model. It is easy to see, however, that if the pole existed for \( k \) close to 0, then there would be accessible decay channels. These are neglected by the omission of vertex corrections. Secondly, for \( |t_{\perp}(k_{\perp})| > 2vk \), this pole approaches not \( vk - t_{\perp} \) but \(-vk \) (with rapidly vanishing weight) as \( \alpha \to 0 \). Finally, in a spin-charge separated Luttinger liquid, and at sufficiently large \( k \), this pole ceases to exist if \( \alpha > 1/4 \), while for \( \alpha < 1/4 \) the pole lies just below \( v_s k \). In a model with \( \alpha < 1/4 \) and vanishing spin velocity, e.g. the large \( U \) Hubbard model, the pole is completely dispersionless along the chains. The “quasiparticles” defined by it have the strange property of a vanishing bandwidth in the direction of large hopping, but a finite bandwidth in the direction of small hopping!

We therefore see that, for \( \alpha \geq 1/4 \) and when analytic properties are treated carefully, this approximate calculation of \( G \) gives no indication of the existence of a transversely dispersing quasiparticle. This is in contrast to what has been suggested elsewhere [3]. In fact, if the poles found off the real axis are interpreted as quasiparticle poles and the Fermi surface is identified with the momenta at which the real part of their frequencies cross zero energy, then the conclusion within this approximation is that the Fermi surface warping vanishes completely for \( \alpha = 1/4 \).
IV. ANOTHER DIAGRAMMATIC CALCULATION: FERMI SURFACE WARPING

Finally, we briefly discuss another diagrammatic calculation addressing coupled Luttinger liquids. For the case of infinitely many coupled chains the behavior of $n(k_x, k_y)$ has been studied in lowest order perturbation theory in $t_\perp$ by Castellani et al. [7]. They find a shift of $n(k_x, k_y)$ proportional to $\cos(k_y)|k^F_x - k_x|^{-1+4\alpha}$ and interpret this as signaling the instability of the Luttinger liquid. The $k$ behavior arises from an integral given in our language by:

$$\langle \delta n(k_x, k_y) \rangle \propto \cos(k_y) \int \frac{d\omega}{2\pi} \frac{A^{\Delta N=0}(k_x, \omega)}{\omega}$$

which is infrared convergent everywhere for $\alpha > 1/4$.

We have previously argued [1] that the magnitude of the warping of the Fermi surface should be identified with the oscillation frequency of our dynamical calculation and provides the order parameter for the transition between the phase with “confined coherence” and the usual phase with coherent transport in all directions. When interpreted in this context, the finding of Castellani et al. [7], that a perturbation theory for the shifts in the Fermi occupation function has a qualitative change to convergent behavior when the hopping is still relevant, supports the notion of incoherence directly.

V. CONCLUSION

We have calculated exactly the inter-Luttinger liquid hopping rate to $O(t_\perp^2)$. Of great physical relevance is the effective spectral function for interliquid hopping, $A_{12}(\omega)$. We have shown that in a large region of Luttinger liquid parameter space below $\alpha = 1/2$ (the point where $t_\perp$ becomes a marginal operator), $A_{12}(\omega)$ is too broad a function to sustain coherent interliquid transport. Single particle coherence is confined to the one-dimensional chains, in the sense that it is impossible to observe any interference effects (beyond those observable for completely decoupled chains) between histories which involve interliquid hopping. Again,
we emphasize that this is not the result of an irrelevant $t_\perp$: the coherence is confined, but the electrons are not.

Our proposal is supported by a careful consideration of the analytic properties of approximate single particle Green’s functions. Even though such approximations are uncontrolled, we find no evidence in these calculations to suggest anything other than that there can exist a phase of relevant, but incoherent, interliquid transport. In fact, all of the results presented here indicate that motion of fermions transverse to the chains can be very different for different $\alpha$ (while still in the region of relevant $t_\perp$) and support the idea that the nature of the renormalization group instability of the $t_\perp = 0$ fixed point can also change. This gives further evidence for the existence of a novel fixed point (one of “confined coherence”) in which transport in one or more (but not all) directions is intrinsically (i.e., in a pure system at zero temperature) incoherent.
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FIGURES

FIG. 1. The interliquid hopping spectral function for various values of $\alpha$. Here $\omega_l = (v_s - v)\Delta k$, $\omega_i = (v_c - v)\Delta k$ and $\omega_u$ is the ultraviolet cutoff of order $v/a$. The plots do not include the weak power law cutoff dependent prefactors. The vertical arrow is the $\alpha = 0$ spectral function, $A_{12}(\omega) \propto \delta(\omega)$. 
Interliquid Hopping Spectral Function

$A_{12}(\omega)$

$\omega_i$ $\omega_f$ $\omega_v$

$\alpha = 1/12$
$1/8$
$1/6$
$1/4$
$1/2$