Squeezed vacuum reservoir effect for entanglement decay in the nonlinear quantum scissor system

A Kowalewska-Kudłaszyk and W Leoński

1 Nonlinear Optics Division, Department of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
2 Quantum Optics and Engineering Division, Institute of Physics, University of Zielona Góra, Prof. Z Szafrana 4a, 65-516 Zielona Góra, Poland

E-mail: annakow@amu.edu.pl and wleonski@proton.if.uz.zgora.pl

Received 16 July 2010, in final form 17 August 2010
Published 4 October 2010
Online at stacks.iop.org/JPhysB/43/205503

Abstract
We discuss the coupler system of two nonlinear oscillators excited by an external coherent field prepared in a maximally entangled state (Bell-like state). We show that as a result of the coupler interaction of the system with external broadband squeezed vacuum bath, entanglement decay dynamics can be considerably affected. Besides the phenomena of sudden entanglement death and its rebirth, a shortening (or lengthening) of the total disentanglement time \( \tau_D \) can be observed, depending on the squeezing parameters. Moreover, in the example of one of the reborn entanglement cases it is shown that by changing the values of these parameters the maximal values of the negativity for the \( 3 \otimes 2 \) system discussed can be tailored.

1. Introduction

Modern quantum physics still takes a very lively interest in quantum entanglement. The entanglement problems are exciting not only from the cognitive point of view, but also as a source of future applications in the field of quantum information theory. In real situations the system which is able to generate entangled states always interacts with an external environment, and this interaction leads to the losses of entanglement in the systems considered (for instance, for discussion concerning decoherence processes for a system of two qubits in a lossy cavity see [1] and the references quoted therein). In particular, if we deal with the cases when the system interacts with a zero-temperature reservoir, one can observe asymptotic entanglement decay (if there is no interaction between the qubits which produce an entangled state)—meaning that entanglement decays to zero for time \( \rightarrow \infty \). In contradiction, interaction with a thermal reservoir is able to shorten significantly that time and entanglement is destroyed in finite time (such a behaviour is referred to as the sudden entanglement death [2–4]). It is also known that when the entangled system interacts with squeezed vacuum bath, sudden death of entanglement can be observed [5]. It should be mentioned that under certain conditions it is possible to rebuild entanglement in the system considered after its sudden death (this is called the sudden entanglement birth [6–8]). Moreover, such ‘sudden’ phenomena can be observed not only for entangling optical systems but also for other models, for instance dealing with spins. Thus, quite recently Wang et al [9] have discussed the sudden spin-squeezing death effect.

Moreover, to avoid decoherence processes several schemes were proposed. For instance, some of them use the decoherence free subspace (DFS) method. In particular, one can show that the systems interacting with squeezed environment states taken from the DFS will never show sudden death phenomena. In contrast, when the interaction involves states that are not taken from the DFS, the system loses entanglement in a sudden way (with possible birth events) [10].

In this paper we focus on the decay of an initially maximally entangled state generated via the coherently excited nonlinear coupler (two nonlinear, Kerr-like oscillators interacting nonlinearly with each other) system. Such couplers can exhibit numerous interesting physical phenomena not only in the domain of quantum information theory but also in other fields of quantum optics (for the thorough review of the problem concerning nonlinear couplers see for instance [11] and the references quoted therein, whereas for problems of...
nonlinear quantum-optical oscillators in general see [12]). We show that we can manipulate the character of the entanglement decay to some extent via the phase of a squeezed reservoir in which the system is immersed.

2. The model and its solutions in a squeezed reservoir

The system we deal with is composed of two nonlinear oscillators labelled as \( a \) and \( b \), and characterized by the nonlinearity constants \( \chi_a \) and \( \chi_b \), respectively. The oscillators are described by the Hamiltonian expressed by the bosonic creation and annihilation operators in the following form:

\[
H_{nl} = \frac{1}{2}(\hat{a}^\dagger)^2 \hat{a}^2 + \frac{1}{2}(\hat{b}^\dagger)^2 \hat{b}^2,
\]

where the parameters \( \chi_a \) and \( \chi_b \) can be identified as third-order susceptibilities of the optical nonlinear media—we can refer to our oscillators as ‘Kerr-like’ ones. Such nonlinear systems can be a source of not only entangled states but also other relevant states commonly discussed in the papers devoted to the quantum optics problems. For instance, Miranowicz et al. [13] have shown that the Kerr-like oscillator can be a source of discrete superpositions of an arbitrary number of coherent states (so-called Schrödinger cat-like or kitten states). The oscillators interact with each other by a nonlinear coupling of the strength \( \epsilon \)—the Hamiltonian corresponding to this interaction can be written as

\[
H_\epsilon = \epsilon(\hat{a}^\dagger \hat{a}^2 + \hat{b}^\dagger \hat{b}^2) + \text{h.c.}
\]

Additionally, one of the oscillators (\( a \)) is externally driven by a coherent field. The intensity of this linear interaction with the field is equal to \( \alpha \) and the Hamiltonian corresponding to this process is

\[
H_\alpha = \alpha \hat{a}^\dagger + \text{h.c.}
\]

While the system is not subjected to the interactions with any kind of external environment it can be treated as a quantum scissors device under some assumptions. The number of two-mode states engaged in the dynamics can be substantially truncated as both internal (characterized via \( \epsilon \)) and external (described by \( \alpha \)) interactions are small if compared with nonlinearity parameters \( \chi \). Therefore, as explained in [14], the number of two-mode states effectively engaged in the system’s dynamics is drastically reduced. The fidelity between such a truncated state \( |\Psi\rangle = c_{02}(0)_a|2\rangle_b + c_{12}|1\rangle_a|2\rangle_b + c_{20}|2\rangle_a|0\rangle_b \)

\[
|\Psi\rangle = \sum_n \sum_m c_{n,m}(t)|n\rangle_a|m\rangle_b,
\]

and the couplings \( \alpha \) and \( \chi \) are almost equal to unity with the accuracy \( \sim 10^{-4} \). We show this fact in figure 1 where the fidelity is plotted for \( \epsilon = 0.1, \alpha = \epsilon \) and \( \chi = 25 \), and the numerical calculations were performed in the \( n \)-photon Fock basis where 100 two-mode states were involved. This result enables us to treat our system as a nonlinear quantum scissors device. In fact, such behaviour originates from the resonances between the energies of the levels generated by the Hamiltonian \( H_{nl} \) and the couplings discussed in the model. For further information concerning linear and nonlinear quantum scissors devices see for example [15–19] and the references quoted therein or the review papers [20, 21].

As mentioned above, the system can behave as a quantum scissors device described in detail in [14]. This system is able to form Bell-like states that are examples of maximally entangled states (MES). It should be mentioned that for the cases when interactions with the external field are absent, we deal practically with the qubit–qubit system, whereas when this interaction is present our system should be treated as a qutrit–qubit one. In consequence, thanks to the two interactions included in our model, we can generate the Bell-like states that are examples of maximally entangled states generated by the Hamiltonian

\[
|\Psi\rangle = \sum_n \sum_m c_{n,m}(t)|n\rangle_a|m\rangle_b,
\]

and a two-mode state

\[
|\Psi\rangle = \sqrt{n}|n\rangle_a|\text{cut}\rangle_b + \text{h.c.}
\]

\[
|\Psi\rangle = \sum_n \sum_m c_{n,m}^{(t)}(t)|n\rangle_a|m\rangle_b
\]

is almost equal to unity with the accuracy \( \sim 10^{-4} \). We show this fact in figure 1 where the fidelity is plotted for \( \epsilon = 0.1, \alpha = \epsilon \) and \( \chi = 25 \), and the numerical calculations were performed in the \( n \)-photon Fock basis where 100 two-mode states were involved. This result enables us to treat our system as a nonlinear quantum scissors device. In fact, such behaviour originates from the resonances between the energies of the levels generated by the Hamiltonian \( H_{nl} \) and the couplings discussed in the model. For further information concerning linear and nonlinear quantum scissors devices see for example [15–19] and the references quoted therein or the review papers [20, 21].

As mentioned above, the system can behave as a quantum scissors device described in detail in [14]. This system is able to form Bell-like states that are examples of maximally entangled states (MES). It should be mentioned that for the cases when interactions with the external field are absent, we deal practically with the qubit–qubit system, whereas when this interaction is present our system should be treated as a qutrit–qubit one. In consequence, thanks to the two interactions included in our model, we can generate the Bell-like states during the system evolution. They can be expressed in the following form:

\[
|B_1\rangle = \frac{1}{\sqrt{2}}(|2\rangle_a|0\rangle_b + i|0\rangle_a|2\rangle_b),
\]

\[
|B_2\rangle = \frac{1}{\sqrt{2}}(|2\rangle_a|0\rangle_b - i|0\rangle_a|2\rangle_b),
\]

\[
|B_3\rangle = \frac{1}{\sqrt{2}}(|2\rangle_a|0\rangle_b + |1\rangle_a|2\rangle_b).
\]
The parameters \( \gamma_j \) of the spontaneous emission rates for both of the modes \( (j = a, b) \); \( M_j \) characterizes degree of squeezing of external bath, whereas \( \phi \) is the phase of squeezing for the bath field. If we assume that \( M_j = 0 \) we deal with usual thermal bath characterized by the mean number of photons equal to \( N_j \) in the modes \( j = [a, b] \). However, if we look at the form of equation (8) the strength of the squeezing can be characterized by the value of \( N_j \) as well (and we shall use it in further considerations).

As already pointed out, the system populated under the interactions with external environment is more dimensional than that considered at the beginning of the evolution (3 \( \otimes \) 2 system). Anyway, we are interested in the dynamics within the subsystem which was occupied before the interactions were included. Therefore, to discuss the entanglement decay we extract 3 \( \otimes \) 2 subspace occupied initially from the whole two-mode subspace which is involved in the system’s dynamics, using the truncation procedure. Of course, one should keep in mind that entanglement can be temporally transferred to other subspace, different from the subspace defined by the states (equations (1)–(3)). The density matrix for the truncated subspace can be obtained from the full density matrix calculated for \( 6 \times 6 \). The truncation procedure is given by the following relation:

\[
\rho_{\text{trunc}} = (\langle 0 | 0 \rangle + | 1 | 1 \rangle + | 2 | 2 \rangle) \rho (| 0 | 0 \rangle + | 2 | 2 \rangle).
\]

In this way we trace the evolution of entanglement (in the case discussed here—Bell-like states) and for separable ones it is equal to zero. Using such an entanglement measure we are able to determine the degree of entanglement in the whole system considered.

As already pointed out in [14, 25], the system under consideration can be a source of the MES \( | B_1 \rangle \), \( | B_2 \rangle \) and \( | B_3 \rangle \) (see equations (1)–(3)). It should be mentioned that in [25] the entanglement dynamics in thermal and zero-temperature reservoirs has already been presented, however for the entanglement within two-qubit subspaces (in our paper we deal with the 3 \( \otimes \) 2 system). As shown in [25], the entanglement sudden death and rebirth phenomena within the 2 \( \otimes \) 2 subsystems were caused by the interaction of this subsystem with the third entangled state \( | B_3 \rangle \) from which the entanglement can be periodically transferred to the two-qubit subsystem (formed by the \( | B_1 \rangle \) and \( | B_2 \rangle \) states). We should point out that in this paper we deal with entanglement generated in the qutrit–qubit (3 \( \otimes \) 2) system and we shall concentrate on the...
influence of the squeezed reservoir on the entanglement decay process.

For the system considered here we assume that the MES (Bell-like state $|B_1\rangle$) is generated initially and next, the external squeezed bath starts to influence the system. Since we are interested in the character of the negativity decay when the system interacts with the squeezed reservoir, we want to check whether the squeezed reservoir can influence this decay in any way. We examine the time evolution of the negativity for various values of the parameters describing the reservoir squeezed field. As shown in our previous paper [25], if the Bell state interacts with the zero-temperature reservoir and there is no interaction between the system’s parts forming the entangled state, the entanglement decays asymptotically, i.e. for $t \to \infty \ N(\rho) \to 0$. However, if we deal with the thermal reservoir, the sudden death of the entanglement phenomenon occurs in the system (entanglement decays to zero in a finite time). For the system interacting with the broadband squeezed vacuum bath (the case discussed here) we see that the moment of time of the last sudden death of entanglement phenomenon occurrence can depend on the squeezing parameters. In particular, for the system discussed here we assume that the bath is squeezed in the same mode as that in which the coupler is externally pumped. At first, looking at figure 2 we can see the influence of the squeezing parameter $N_a$ on the final disentanglement time $\tau_d$ for a constant squeezed vacuum phase $\phi$. Similarly as for the case when we deal with non-squeezed and zero-temperature bath ($N_a = 0$) the time $\tau_d$ reaches high values—we can conclude that we obtain almost an asymptotic decay of entanglement. However, higher values of $N_a$ make the sudden death of entanglement visible as finite values of time $\tau_d$ become considerably smaller. Shortening of $\tau_d$ can be observed for both cases considered here ($\phi = \{0, \pi\}$). Moreover, for these two cases the value of $\tau_d$ decreases continuously and tends asymptotically to its final value (see figure 2). However, comparing the two dependences of $\tau_d$ on $N_a$, we can easily identify some values of $N_a$ for which $\tau_d$ is considerably different for $\phi = 0$ and $\phi = \pi$.

It means that by changing the value of $\phi$ we may prevent the system from rebuilding the entanglement despite the fact that there is still an interaction between both oscillators and $\alpha$ (strength of the interaction with external pumping field) is large enough to produce another entanglement birth.

For further considerations we will choose non-zero values for $N_a$ and shall concentrate on the influence of the phase $\phi$ on the entanglement decay dynamics. In particular, we shall find such a value of $N_a$ for which we can observe the noticeable changes in the values of $\tau_d$ as $\phi$ varies from 0 to $\pi$, concentrating in further considerations on the influence of the squeezed reservoir properties on rebuilding of the entanglement in the system. For instance, if we neglect mutual interaction between two oscillators ($\epsilon = 0$), assume weak external pumping ($\alpha = 0.01$) and $N_a = 2$, $N_b = 0$ (squeezed bath in one mode) sudden death of entanglement appears. We see (figure 3) that when the phase of squeezing is equal to zero (or $\pi$) the entanglement death appears slightly later than for the non-squeezed, thermal field case ($M_a = 0$, $N_a = 2$). However, if we additionally assume that apart from the absence of coupling between the oscillators the external coherent field is also cut off, the changes are practically unnoticeable and the result is practically identical to that of the thermal reservoir case. Moreover, it is worth mentioning that any larger changes of the moment of time when the entanglement death appears are noticeable when the squeezing is present in the same mode of the reservoir modes that is externally pumped.

At this point we assume that two oscillators interact with each other (i.e. $\epsilon \neq 0$) and we will focus on the differences in the entanglement decay due to the interactions with standard thermal reservoir. As shown in [25], the oscillations of entanglement appear in the system and for the thermal reservoir case besides the regular oscillations (characteristic
for zero-temperature environment) it is possible to observe sequences of the sudden deaths and entanglement revivals. But what is more interesting, for the case of the squeezed external bath the phase of the squeezed reservoir mode can influence the time after which the final disentanglement is obtained and also influences the amount of the entanglement rebuilt in the system. In particular, we have noted that by changing the phase of the squeezed reservoir mode we can qualitatively change the decay of negativity. It is seen for the whole range of $\epsilon/\alpha$ (for all cases: $\epsilon/\alpha > 1$, $\epsilon/\alpha < 1$ and $\epsilon/\alpha = 1$). In particular, in figures 4(a)–(c) we plotted the time after which the final entanglement death occurs ($\tau_d$) as a function of the squeezed bath phase $\phi$ showing that one additional rebuilt maximum in negativity versus time appears for some range of squeezed reservoir phases ($\sim 0$; $\sim 12\pi/20$) and ($\sim 31\pi/20$; $\sim 2\pi$) (a similar feature can be observed for the thermal reservoir case, although there [25] the mean number of photons and external coupling strength dependences were discussed instead of phase dependence). For the system discussed here we can observe a significant shortening (by $\sim 20\%$) of total disentanglement time $\tau_d$ for $\epsilon/\alpha = 1$ (figure 4(b)) and shortening by $\sim 17\%$ for both $\epsilon/\alpha > 1$ and $\epsilon/\alpha < 1$ (see figures 4(a) and (c)). In figure 2 we can see that this shortening (when we compare the cases for $\phi = \pi$ and $\phi = 0$) varies from $4\%$ to $24\%$ for various values of the squeezing parameter $N_s$. Our considerations concern one of these $N_s$ values that cause the maximal $\tau_d$ shortening.

Moreover, it is seen that for all cases shown in figures 4(a)–(c) for some values of $\phi$ we observe one entanglement rebirth event less as a result of the influence of the squeezing phase on the entanglement dynamics. This change is of sudden character and indicates that we have some sort of ‘phase transition’ within the system discussed. For $\epsilon/\alpha > 1$ and $\epsilon/\alpha < 1$ (figures 4(a) and (c)) we can see that the interaction of the system with the squeezed vacuum (for $\phi \in (\sim 12\pi/20; \sim 31\pi/20)$) can shorten the time $\tau_d$ as we compare it with the case when ordinary thermal bath is considered. However this shortening is less pronounced—see the inset. Beyond this range even though $\alpha$ is not strong enough to produce another entanglement rebirth, the interactions with the squeezed bath lead to the appearance of additional negativity re-occurrence and hence, to a significant ($\sim 17\%$) lengthening of $\tau_d$. When $\epsilon/\alpha = 1$ (figure 4(b)) the changes of $\phi$ are of the ‘phase transition’ character again (as for the previous cases), but for this case one can observe vanishing of the last negativity maximum and consequently, a rapid shortening of time $\tau_d$.

The character of this ‘transition’ is related to the fact that if the squeezing phase $\phi$ changes its value the terms in the master equation proportional to $\exp(\pm i\phi)$ add or subtract. In consequence, the populations and coherences in the system considered (and corresponding to them matrix elements of $\rho$) will be affected as well. Due to the fact that the definition of the negativity involves the max function changes in the matrix elements of $\rho$ lead to the rapid changes (zeroing) in the negativity. A similar behaviour was discussed in [26] where the squeezing phase dependence of the decoherence rate was considered for the atomic system located inside the cavity.

Thus, we can see that the phase of the squeezed reservoir can not only influence the time of entanglement death but also change the system’s ability to rebuild the entanglement. Especially, we can see that even though the value of $\alpha$ is enough to cause the entanglement rebuild, the appropriately chosen phase $\phi$ may prevent the last entanglement reconstruction.

One more aspect that can be influenced by the squeezed reservoir state phase $\phi$ is the amount of entanglement rebuild in the system during the last entanglement rebirth phenomenon observed. Figure 5 shows the dependence of the maximal value of the last $N_{\text{max,reb}}$ and last but one negativity maximum $N_{\text{max}}$ on the phase $\phi$. Similar to the case discussed earlier (figure 4) we can observe some kind of phase transition-like behaviour. We see sudden changes of the maximal value of

---

**Figure 3.** Negativity $N(\rho)$ versus time scaled in $1/\chi$ units; $\chi_a = \chi_b = 25$, $\alpha = 0.01$, $\epsilon = 0$, $\gamma_a = \gamma_b = 0.0025$ and $N_a = 2$, $N_b = 0$, $|\Psi\rangle_\text{in} = |B_3\rangle$. 

---

[j.png] [figure3.png]
Figure 4. The dependence of \( \tau_d \) as a function of the squeezing phase \( \phi \). The system is initially prepared in a \( |B_3 \rangle \) state. The time is scaled in \( 1/\chi \) units, \( \chi_a = \chi_b = 25 \), \( \epsilon = 0.1 \), \( \gamma_a = \gamma_b = 0.0025 \) and \( N_a = 2 \), \( N_b = 0 \). At (a) \( \epsilon/\alpha = 10 \), (b) \( \rightarrow \epsilon/\alpha = 1 \) and at (c) \( \epsilon/\alpha = 0.5 \). The inset in (c) shows the central part of the whole plot. The stars correspond to a thermal non-squeezed reservoir characterized by the same mean number of photons.

the last negativity reconstruction \( N_{\text{maxlast}} \) for some values of \( \phi \) (the inset in figure 5) when the amplitude of the last negativity maximum is equal to zero. One should note that for the same range of \( \phi \) we observe significant shortening of the time \( \tau_d \) as well. Additionally, we can see from figure 5 that the interaction with the squeezed bath can reduce the amount of entanglement preserved in the system. This reduction depends on the value of \( \epsilon/\alpha \) for \( \epsilon/\alpha > 1 \) and \( \epsilon/\alpha < 1 \) it is about 30%, while for \( \epsilon/\alpha = 1 \) even 56%.

For the case of a smaller squeezing parameter (\( N_a = 1 \), mode b is not squeezed) we can only observe the regular changes in the time after which entanglement disappears and we do not see any ‘phase transitions’ in the system (figure 6). The shortening of the death time is only by about 4% and all the rebuild maxima of the negativity–time dependence are present for all squeezed reservoir phases—see figure 6. It confirms the earlier conclusion about the \( \tau_d \) dependence on the squeezing parameter \( N_a \) visible at figure 2. It should also be mentioned that for this case the changes in the value of the last rebuilt maximum are of the same character as for the case of stronger squeezing, but smaller in percentage.

As we look at the time evolution of both negativity and populations of individual states, the question arises if the reservoir effects and population transfer could influence considerably the negativity evolution. The answer can be obtained, for instance, from the analysis of the trace of the density matrix \( \rho_{\text{trunc}} \) corresponding to the 3 \( \otimes \) 2 subsystem considered. Thus, figure 7 shows the plots of the negativities and the trace of \( \rho_{\text{trunc}} \) for \( \phi = 0 \) and \( \pi \). We see that for both the values of \( \phi \) the whole system that was initially populated
Figure 5. The maximal value of the last but one negativity reconstruction $N_{\text{max}}$ for the entanglement reborn appearance as a function of the phase of the squeezing reservoir $\phi$. The system parameters are the same as in figure 4(b). Inset: the amplitude of the last negativity maximum $N_{\text{max last}}$.

Figure 6. The same as in figure 4(c) but for $N_a = 1$ and $N_b = 0$.

in this subspace ($\text{Tr}(\rho_{\text{trunc}}) = 1$ for $t = 0$) starts to populate other states—$\text{Tr}(\rho_{\text{trunc}})$ decreases and is practically identical for both the values of $\phi$. The character of this decay depends on the value of the negativity for a given moment of time. In particular, the decay rate increases when the negativity goes to zero, whereas it slows down as the negativity reaches its maximum (this effect is visible in particular for the first negativity reconstruction). However those effects do not play
a crucial role, especially for longer times. This means that the population leakage can effect the entanglement dynamics to some extent.

3. Summary

We have analysed the influence of the squeezed reservoir on the entanglement decay for a nonlinear system initially prepared in a maximally entangled state (one of the Bell-like state). Such a state determines some subspace of the states that can be treated as a qutrit–qubit system and we have concentrated on it. Obviously, when the interaction with an external reservoir is included, other states are populated and in that way the whole system is no more a simple qutrit–qubit one, but more dimensional qudits should be considered. Anyway, our aim was to trace the evolution of the entanglement within the initially occupied subspace. Therefore, we look for the entanglement evolution inside the same qutrit–qubit system, keeping in mind that the other states (corresponding to the lower energies of the system) are involved in the dynamics and some losses of the entanglement in the subspace considered are related to the populating of these states. For the case when the oscillators could not interact at the squeezing phase. Moreover, this value does not differ considerably from that for the model involving a standard thermal reservoir. However, the more pronounced changes become visible when both the interactions (external characterized by $\alpha$ and internal characterized by $\epsilon$) are present. From the discussion of a similar model [25] (where the thermal bath was considered) we expect oscillations of negativity, but their character depends on the squeezed reservoir properties. In particular, we have shown that shortening of the death time for the parameters used, by about 17–20% of its original value can be obtained while $\epsilon/\alpha$ changes for some range of squeezed reservoir phases. The changes in this shortening can be of the step character resembling in some sense a 'phase transition' in the system. In fact, we observe one entanglement revival less or more, as a result of the interaction with the squeezed bath depending on the phase of squeezing. Moreover, some changes in the amplitude of the last negativity maximum due to the squeezing phase variations could also be observed. In consequence, we can reduce the value of the entanglement revival at the end of its life by about 30–56% of its original value. Concluding, we can say that the squeezing parameters of external bath can influence considerably the entanglement decay dynamics leading to various interesting features.

References

[1] Miranowicz A 2004 J. Phys. A: Math. Gen. 37 7909
[2] Życzkowski K, Horodecki P, Horodecki M and Horodecki R 2001 Phys. Rev. A 65 012101
[3] Diósi L 2003 Lect. Notes Phys. 622 157
[4] Yu T and Eberly J H 2004 Phys. Rev. Lett. 93 140404
[5] Ikram M, Li F and Zubairy M S 2007 Phys. Rev. A 75 062336
[6] Ficek Z and Tanaś R 2006 Phys. Rev. A 74 024304
[7] Ficek Z and Tanaś R 2008 Phys. Rev. A 74 054301
[8] López C E, Romero G, Lastra F, Solano E and Retamal J C 2008 Phys. Rev. Lett. 101 080503
[9] Wang X, Miranowicz A, Liu Y-X, Sun C P and Nori F 2010 Phys. Rev. A 81 022106
[10] Hernandez M and Orszag M 2008 Phys. Rev. A 78 042114
[11] Peřina J Jr and Peřina J 2000 Progress in Optics vol 41 ed E Wolf (Amsterdam: Elsevier) pp 361–419
[12] Pećinová V and Lukš A 1994 Progress in Optics vol 33 ed E Wolf (Amsterdam: Elsevier) pp 129–202
[13] Miranowicz A, Tanaś R and Kielich S 1990 Quantum Opt. 2 253
[14] Kowalewska-Kudłaszyk A and Leoński W 2006 Phys. Rev. A 73 042318
[15] Pegg D T, Phillips L S and Barnett S M 1998 Phys. Rev. Lett. 81 1604
[16] Leoński W and Tanaś R 1994 Phys. Rev. A 49 R20
[17] Koniorczyk M, Kurucz Z, Gábris A and Janszky J 2000 Phys. Rev. A 62 012802
[18] Miranowicz A 2005 J. Opt. B: Quantum Semiclass. Opt. 7 142
[19] Leoński W and Miranowicz A 2004 J. Opt. B: Quantum Semiclass. Opt. 6 S37
[20] Miranowicz A, Leoński W and Imoto N 2001 Quantum-Optical States in Finite-Dimensional Hilbert Space: I. General Formalism (Advances in Chemical Physics vol 119 part I) ed M W Evans (New York: Wiley) p 155
[21] Leoński W and Miranowicz A 2001 Quantum-Optical States in Finite-Dimensional Hilbert Space: II. States Generation (Advances in Chemical Physics vol 119 part I) ed M W Evans (New York: Wiley) p 195
[22] Peres A 1996 Phys. Rev. Lett. 77 1413
[23] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A 223 1
[24] Vidal G and Werner R F 2002 Phys. Rev. A 65 032314
[25] Kowalewska-Kudłaszyk A and Leoński W 2009 J. Opt. Soc. Am. B 26 1289
[26] Jakob M, Abranyos Y and Bergou J A 2001 Phys. Rev. A 64 062102