THE NATURE OF DARK MATTER AND THE DENSITY PROFILE AND CENTRAL BEHAVIOR OF RELAXED HALOS

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ABSTRACT

We show that the two basic assumptions of the model proposed by Manrique et al. (2003) for the universal density profile of cold dark matter (CDM) halos, namely that these objects grow inside-out in periods of smooth accretion and that their mass profile and its radial derivatives are all continuous functions are both well-understood in terms of the very nature of CDM. Those two assumptions allow one to derive the typical density profile of halos of a given mass from the accretion rate characteristic of the particular cosmology. This profile was shown by Manrique et al. to recover the results of numerical simulations. In the present paper, we investigate its behavior beyond the ranges covered by present-day N-body simulations. We find that the central asymptotic logarithmic slope depends crucially on the shape of the power spectrum of density perturbations: it is equal to a constant negative value for power-law spectra, and has central cores for the standard CDM power spectrum. The predicted density profile in the CDM case is well fit by the 3D Sérsic profile over at least ten decades in halo mass. The values of the Sérsic parameters depend on the mass of the structure considered. A practical procedure is provided that allows one to infer the typical values of the best NFW or Sérsic fitting law parameters for halos of any mass and redshift in any given standard CDM cosmology.

Subject headings: cosmology: theory – dark matter – galaxies: halos

1. INTRODUCTION

The universal shape of the spherically averaged density profile of relaxed dark halos in high-resolution N-body simulations is considered one of the major predictions of standard Cold Dark Matter (CDM) cosmologies. Down to one percent of the virial radius $R$, it is well fit by the so-called NFW profile (Navarro, Frenk, & White 1996, 1997),

$$\rho(r) = \frac{\rho_s r_s^3}{r(r_s + r)^2},$$

specified by only one mass-dependent parameter, the scale radius $r_s$, or equivalently the concentration $c \equiv r_s/R$.

At radii smaller than one percent of the virial radius, however, the behavior of the density profile is unknown. Based on recent numerical simulations, some authors advocate a central asymptotic slope significantly steeper (Moore et al. 1998; Jing & Suto 2000) or shallower (Taylor & Navarro 2001; Ricotti 2003; Hansen & Stadel 2006) than that of the NFW law. Others suggest an ever decreasing absolute value of the logarithmic slope (Power et al. 2003; Navarro 2004; Reed et al. 2005), which might tend to zero as a power of the radius like in the 3D Sérsic (1968) or Einasto (Einasto & Haud 1969) law (Merritt et al. 2005, 2006).

This uncertainty is the consequence of the fact that the origin of such a universal profile is poorly understood. Two extreme points of view have been envisaged. In one of these, it would be caused by repeated significant mergers (Syer & White 1998; Raig, González-Casado G., & Salvador-Solé 1998; Subramanian, Cen, & Ostriker 2000, 2004; Dekel, Devor, & Hetzroni 2003), while in the other it would be essentially the result of smooth accretion or secondary infall (Avila-Reese, Firmani, & Hernández 1998; Nusser & Sheth 1999; Del Popolo et al. 2000; Kull 2000; Manrique et al. 2003; Williams, Babul, & Dalcanton 2004; Ascasibar et al. 2004).

The fact that the $M-\sigma$ relation at $z = 0$ is consistent (Salvador-Solé & Babul 1998, hereafter SSM; Wechsler et al. 2002; Zhao et al. 2003a) with the idea that all halos emerge from major mergers with similar values of $\sigma$, which then decreases according to the inside-out growth of halos during the subsequent accretion phase seems to favor an important role of mergers. But, the pure accretion-driven scenario is at least as attractive, as the inside-out growth during accretion leads to a typical density profile that appears roughly to have the NFW shape with the correct $M-\sigma$ relation in any epoch and cosmology analyzed (Manrique et al. 2003, hereafter MRSSS).

Certainly, the effects of major mergers cannot be neglected in hierarchical cosmologies, so both major mergers and accretion should contribute in shaping relaxed halos. However, as noted by MRSSS, if the density profile arising from a major merger were set by the boundary conditions imposed by current accretion, then the density profile of halos would appear to be independent of their past aggregation history, so halos could be assumed to grow by pure accretion without any loss of generality. All the correlations shown by relaxed halos in numerical simulations can be recovered under this point of view (Salvador-Solé, Manrique, & Solanes 2003). Simultaneously, this would explain why halos with very different initial conditions and aggregation histories have similar density profiles (Romano-Diaz et al. 2006).
In the present paper, we show that the MRSSS model relies on two basic assumptions, namely 1) that halos grow inside-out in periods of smooth accretion, and 2) that the mass profile and all its derivatives are continuous functions. The former assumption is supported by the results of numerical simulations (Salvador-Solé, Manrique, & Solanes 2005; Lu et al. 2006; Romano-Diaz et al. 2006, 2007), while the second one is at least not in contradiction with them. In the present paper we show that both assumptions are in fact sound from a theoretical point of view as they can be related to the very nature of CDM. This renders the predictions of the model beyond the range of current simulations worth to examine in detail as done below.

For simplicity, we are considering spherical structures, which at best is an approximation to the triaxial structures observed in numerical simulations. Secondary infall is known to be influenced by deviations from spherical symmetry (Bond & Myers 1996; Zaroubi, Naimi, & Hoffman 1996). Yet, in the accompanying paper (González-Casado et al. 2007), we show that this does not seem to affect the fundamental role of the two assumptions given above. Another simplifying assumption used here is the neglect of substructure. It has recently been shown in high resolution simulations that about 57% of the halo mass is collected in the previous major merger (Faltenbacher et al. 2005). Yet, substructures (and sub-substructures) contribute only about 5% of the total mass in halos (Diemand, Kuhlen, & Madau 2007) whose density profile is well described by equation 1. It seems therefore a good first approximation to ignore substructure.

The paper is organized as follows. In Section 2, we show how the MRSSS model would emerge from the properties of standard CDM. The behavior of the predicted density profile at extremely small radii and for a wide range of halo masses is investigated in Section 3. Our results are summarized in Section 4.

2. CDM PROPERTIES AND HALO DENSITY PROFILE

2.1. Inside-out growth during accretion

We now argue that halos grow inside-out during accretion, as found indeed in numerical simulations, because of some properties of CDM, in particular, its characteristic power-spectrum leading to a slow halo accretion rate. As shown in Section 2.2 this has important consequences for the inner structure of these objects.

Schematically, one may distinguish between minor and major mergers. In minor mergers, the relative mass increase produced, $\Delta \equiv \Delta M/M$, is so small that the system is left essentially unaltered, whereas in major mergers $\Delta$ is large enough to cause rearrangements.

The smaller $\Delta$, the most frequent are mergers (Lacey & Cole 1993). For this reason, although individual minor mergers do not affect the structure of halos, their added contribution yields a smooth secular mass increase, the so-called accretion, with apparent effects on the aggregation track $M(t)$ of the halo. The accretion scaled rate, $\dot{M}/M(t)$, is given by (Raig, González-Casado G., & Salvador-Solé 2001, hereafter RGS)

$$R_a(M, t) = \int_0^{\Delta_m} d\Delta \Delta R_m(M, t, \Delta),$$

where $R_m(M, t, \Delta)$ is the usual Lacey-Cole (1993) instantaneous merger rate (see eq. [A1]) and $\Delta_m$ is the maximum value of $\Delta$ for mergers contributing to accretion. In contrast, less frequent major mergers yield notable sudden mass increases or discontinuities in $M(t)$.

After undergoing some major merger (and virializing), halos evolve as relaxed systems until the next major merger. The fact that standard CDM is non-decaying and non-self-annihilating guarantees that the mass collected during such periods is conserved. This does not yet imply that halos grow inside-out during accretion, because their mass distribution might still vary owing to energy gains or losses or to the action of accretion itself. Even if each individual minor merger would leave the halo unchanged their collective action might alter these systems. However, standard CDM is also dissipationless and therefore halos cannot loose energy. Furthermore, under the assumption of spherical symmetry halos cannot suffer tidal torques from surrounding matter, and hence the surrounding matter is unable to alter the kinetic energy of the halo. Therefore, the only possibility for a time-varying inner mass distribution of accreting halos is that the accretion process causes it itself.

This possibility, that the process of accretion itself could alter the internal mass distribution would be realized only if the accretion time $1/R_a$, were smaller than the dynamical time $\tau_{\rm cr}$. On the contrary, if $1/R_a$ is substantially larger than $\tau_{\rm cr}$, the adiabatic invariance of the inner halo structure will be guaranteed, and the halo will evolve inside-out. Thus, by requiring $1/R_a$ to be $C$ times larger than $\tau_{\rm cr}$, we are led to the equation

$$C \left[ \frac{4\pi}{3} G \Delta_{\text{vir}}(t) \bar{\rho}(t) \right]^{1/2} = R_a(M, t),$$

which gives the upper mass $M_a$ for inside-out growth at $t$. In equation [3], $\bar{\rho}$ is the mean cosmic density and $\Delta_{\text{vir}}(t)$ is the virialization density contrast given e.g. by Bryan & Norman (1998). $M_a$ is indeed an upper limit because $R_a(M, t)$ is an increasing function of $t$; see RGS. In any CDM cosmology analyzed, equation [3] appears to have no solution for $C$ significantly larger than unity in the relevant redshift range. This means that accretion is always slow enough for halos to grow inside-out as required by the MRSSS model.

As mentioned, the inside-out growth of halos in accreting periods is unambiguously confirmed by the results of N-body simulations (Salvador-Solé, Manrique, & Solanes 2003; Lu et al. 2006; Romano-Diaz et al. 2006, 2007). It is also consistent with the fact that dark matter structures preserves the memory of initial conditions, in the sense that the most initially overdense regions end up being the central regions of the final structures (Diemand, Madau, & Moore 2005), implying that the spatial positions of particles are not significantly perturbed by merging/accretion during the assembly of the structures. Likewise, the energy of the individual particles in the final structure (at $z = 0$) is very strongly correlated with their energies at much earlier times ($z = 10$) (Dantas & Ramos 2006). This shows that particles even preserve memory of the initial energies.

Both decay and annihilation rates are extremely small for realistic dark matter particle candidates.
2.2. Smoothness of the mass profile

Contrarily to an ordinary fluid, CDM is collisionless and free-streaming and, hence, cannot support discontinuities (shock fronts) in the spatial distribution of any of its macroscopic properties. As a consequence, all radial profiles in relaxed halos are necessarily smooth. This holds in particular for the mass profile, \( M(r) \), and its radial derivatives\(^2\), which has the following consequence.

The inside-out growth of a halo during accretion (section 2.1) implies that the mass profile \( M(r) \) built at that interval is the simple conversion, through the definition of the instantaneous virial radius

\[
R(t) = \left[ \frac{3M(t)}{4\pi \Delta_{\text{vir}}(t) \rho(t)} \right]^{1/3}, \tag{4}
\]

of the associated mass aggregation track \( M(t) \).

The smoothness condition implies that the old \( M(r) \) profile must match perfectly the new part of the profile built during that time. Since minor mergers only cause tiny discontinuities, the new piece of \( M(t) \) track they produce is well approximated by a smooth function and, as the functions \( \bar{\rho}(t) \) and \( \Delta_{\text{vir}}(t) \) in equation (4) are also smooth functions, the corresponding piece of \( M(r) \) profile automatically fulfills the right smoothness condition. Thus, the system can grow, during accretion, without the need to essentially rearrange its structure.

Only when a halo undergoes a major merger and its \( M(t) \) track suffers a marked discontinuity, the mass profile prior to the major merger will no longer match the piece that begins to develop after it. Since the \( M(r) \) profile cannot have any discontinuity, the halo is then forced to rearrange its mass distribution (through violent relaxation) to fulfill the required smooth condition.

In other words, the fundamental assumption of the MRSSS model that the mass distribution of relaxed halos is determined by their current accretion rate (through dramatic rearrangements of the structure on the occasion of major mergers and very tiny and negligible ones during accretion periods) would simply be the natural consequence of the slowly accreting, non-decaying, non-self-annihilating, dissipationless and collisionless nature of standard CDM.

2.3. The MRSSS model

The mass profile of a specific halo with mass \( M_l \) at time \( t_i \) accreting at a given rate during any arbitrarily small time interval \( \Delta t \) around \( t_i \) is therefore simply the smooth extension inwards of the small piece of profile being built during that interval\(^3\). Unfortunately, the smooth extension of a small piece of function is hard to find in practice, so the mass profile of real individual halos can hardly be obtained in this way.

There is one case, however, in which such a smooth extension can readily be achieved: that of halos with \( M_l \) at \( t_i \) accreting at the typical cosmological rate \( R_0 \)[(\( M(t), t \)] during any arbitrarily small interval of time around \( t_i \). In this case, the (unique) smooth extension we are looking for necessarily coincides with the smooth function \( M(t) \), solution of the differential equation

\[
\frac{\dot{M}}{M(t)} = R_0[M(t), t] \tag{5}
\]

for the boundary condition \( M(t_i) = M_i \), properly converted from \( t \) to \( r \) by means of the equation (4). Once the typical \( M(r) \) profile is known, by differentiating it and taking into account equations (4) and (5), one is led to the typical density profile for halos with \( M_l \) at \( t_i \) proposed by MRSSS,

\[
\rho(r) = \frac{1}{4\pi r^2} \left( \frac{\dot{M}}{R} \right)_{t(r)} = \left[ \Delta_{\text{vir}}(t) \bar{\rho}(t) \delta(t) \right]_{t(r)} \tag{6}
\]

where

\[
\delta(t) = \left[ 1 - \frac{1}{R_0[M(t), t]} \frac{d}{dt} \ln(\Delta_{\text{vir}}) \right]^{-1}. \tag{7}
\]

From equations (6) and (7) we see that the shape of this profile is ultimately set by the CDM power spectrum of density perturbations in the cosmology considered through the merger rate \( R_m(M, t, \Delta) \) entering the accretion rate \( R_0(M, t) \) (see eqs. [A1] and [2]). This dependence is however so convoluted that the density profile (6) must be inferred numerically. Only its central asymptotic behavior can be derived analytically as will be shown in the next section.

3. SOME CONSEQUENCES OF THE MODEL

From equation (2) we see that the exact shape of the density profile in equation (6) depends on \( \Delta_m \). This parameter marks the effective transition between minor and major mergers, and it can be determined from the empirical \( M - c \) relation at some given redshift.

\[ \text{Fig. 1.} - M - c \text{ relation at } z = 0 \text{ predicted in the concordance model by the MRSSS model for } \Delta_m = 0.26 \text{ (solid line) compared to the empirical relation traced by those points with minimal error bars obtained by Zhao et al. (2003b) from high-resolution simulations (triangles with error bars).} \]
Fig. 2.— Predicted density profiles (solid lines) compared to their fits to a NFW profile (dashed lines) for halo masses at $z = 0$ ranging from $10^{14} M_\odot$ to $10^{16} M_\odot$ (or, equivalently, from $10^3$ to $10^{-2}$ times the current critical mass $M_\text{\text{\odot}}$ for collapse) in the concordance model. Upper sections: profiles. Lower sections: residuals of the logarithmic fits. The halo mass and the corresponding best-fitting value of the NFW concentration parameter are quoted in each panel.

For each given $\Delta m$ value, the density profiles, down to $R/100$, predicted at $z = 0$ in the concordance model characterized by $(\Omega_m, \Omega_\Lambda, h, \sigma_8) = (0.3, 0.7, 0.7, 0.9)$ for halos with different masses have been fitted to the NFW profile to find the best fitting values of $c$. Then we searched the value of $\Delta m$ that minimizes the departure of the theoretical $M - c$ relations from the empirical one drawn from high-resolution simulations by Zhao et al. (2003b). As shown in Figure 1, $\Delta m = 0.26$ gives an excellent fit over almost 4 decades in mass ($8 \times 10^{10} h^{-1} M_\odot < M < 4 \times 10^{14} h^{-1} M_\odot$).

In Figure 2 we plot, down to a radius equal to the current resolution radius of most numerical simulations, the density profiles predicted in the concordance model for halo masses at $z = 0$ ranging from $10^{11}$ to $10^{16} M_\odot$. They are all well fit to a NFW profile although there is the tendency for the theoretical profiles for $M \gtrsim 10^{14} M_\odot$ to deviate from that shape and approach a power-law with logarithmic slope intermediate between the NFW asymptotic values of $-1$ and $-3$. This tendency also makes $c$ increase very rapidly at large masses where the fit by the NFW law is no longer acceptable. This causes the $M - c$ relation to deviate from its regular trend at smaller $M$ (see Fig. 2). Both effects, already reported in MRSSS, were later observed in simulated halos (Zhao et al. 2003b; Tatsiomi et al. 2004). This is a clear indication that the NFW profile is not providing an optimal fit for very massive structures. Of course, above $10^{14} M_\odot$ halos are hardly in virial equilibrium, so such a deviation has essentially no practical effects.

As explained in Salvador-Solé, Manrique, & Solanes (2005), another interesting consequence of the MRSSS model is that the $M(t)$ tracks traced by accreting halos (hence growing inside-out) coincide with curves of constant $r_s$ and $M_s$ values, with $M_s$ defined as the mass interior to $r_s$. Thus, the intersection of those accretion tracks at any arbitrary redshift sets the relation $M_s(r_s)$...
between such a couple of parameters, implying that the $M_s(r_s)$ relation satisfied by halos is time-invariant. In Figure 3 we show how the different $M_s(r_s)$ curves obtained by fitting to a NFW law the density profiles predicted at different redshifts overlap. There is only some deviation at large masses where the density profiles are not correctly described by the NFW profile. Such a time-deviation at large masses where the density profiles are predicted at different redshifts overlap. There is only some deviation at large masses where the density profiles are not correctly described by the NFW profile. Such a time-deviation at large masses where the density profiles are predicted at different redshifts overlap. There is only some deviation at large masses where the density profiles are not correctly described by the NFW profile.

Substituting equation (8) into the relation $M$ between such a couple of parameters, implying that the invariant not correctly described by the NFW profile. Such a time-deviation at large masses where the density profiles are predicted at different redshifts overlap. There is only some deviation at large masses where the density profiles are not correctly described by the NFW profile.

What about the central behavior of the predicted density profile? According to equation (4), small radii correspond to small cosmic times. In this asymptotic regime all Friedman cosmologies approach the Einstein-de Sitter model in which $\Delta_{\text{vir}}(t)$ is constant and $\dot{\rho}(t)$ is (in the matter dominated era when halos form) proportional to $t^{-2}$. If the power spectrum of density perturbations were of the power-law form, $P(k) \propto k^j$ with index $j$ satisfying $1 > j > -3$ to guarantee hierarchical clustering, then the universe would be self-similar. The mass accretion in equation (3) would take the asymptotic form: $M(t) \propto t^{3/4}$ (see the Appendix). The fact that both $\Delta_{\text{vir}}(t)$ and $M(t)$ would then be power-laws has two consequences. First, the time dependence of their respective logarithmic derivatives on the right side of equation (7) cancel, which implies that $\rho(t)$ is proportional to $\Delta_{\text{vir}}(t)\dot{\rho}(t)$, and hence to $\dot{\rho}(t)$. Second, the virial radius given by equation (3) is also a power-law, $R(t) \propto [M(t)/\bar{\rho}(t)]^{1/3} \propto t^{2(j+4)/(3(j+3))}$. From this we get $t(r)$, and thereby one finds

$$\rho(r) \propto r^{-\frac{3(j+3)}{j+4}}.$$  

This central behavior, fully in agreement with the numerical profiles obtained from power-law power spectra, is particularly robust as it does not depend on $\Delta_m$. Note that it coincides with the solution derived in self-similar cosmologies by Hoffman and Shaham (1985) assuming spherical collapse. It is worth noting however that, in that derivation, such an asymptotic behavior is restricted to $j > -1$ so to warrant the required adiabatic invariance (Fillmore and Goldreich 1984), while, in the present derivation, there is no such a restriction as the central density profile is not assumed to be built by spherical infall, but results from smooth adaption to the boundary condition imposed by current accretion.

The CDM power spectrum is of course not a power-law. However, in the limit of small masses involved in that asymptotic regime, it tends to a power-law of index $j = -3$. Thus, according to the relation (11), we expect a vanishing central logarithmic slope of $\rho(r)$ for
the standard CDM case. This is confirmed by the numerical profiles obtained in this case: as one goes deeper and deeper into the halo center, they become increasingly shallower.

What is still more remarkable is that down to a radius as small as 1 pc, the density profiles appear to be well fit by the 3D Sérsic or Einasto law

\[ \rho(r) = \rho_0 \exp \left[ -\left( \frac{r}{r_n} \right)^{1/n} \right] , \quad (12) \]

over at least 10 decades in halo mass (see Fig. 4), from \(10^6 \, \text{M}_\odot\) to \(10^{16} \, \text{M}_\odot\). We remind that, for power-law spectra, equation (10) leads to density profiles with central cusps, so the Sérsic shape is not a general consequence of the MRSSS model, but it is specific of the standard CDM power spectrum. In fact, from the reasoning above we see that what causes the zero central logarithmic slope in the CDM case is the fact that the logarithmic accretion rate \(d \ln M(t)/dt = t \dot{\rho}(M(t), t)\) diverges in the limit of small \(t\). This is in contrast to the general power-law case, where the accretion rate remains finite.

Similar to the characteristic density \(\rho_c\) in the NFW profile (eq. (1)), the central density \(\rho_0\) entering the 3D Sérsic law (eq. (12)) can be written in terms of the mass \(M\) and the values of the two (instead of one) remaining parameters, \(n\) and \(r_n\) or \(c_n \equiv R/r_n\),

\[ \rho_0 = \frac{M}{4\pi n r_n^3 \Gamma(3n)} P^{-1}(3n, c_n^{1/n}) , \quad (13) \]

where \(\Gamma\) is the usual gamma function and

\[ P(a, x) = \frac{1}{\Gamma(a)} \int_0^x dt \, e^{-t} \, t^{a-1} \quad (14) \]

is the incomplete or regularized one. At \(z = 0\), the two free parameters \(n\) and \(c_n\) depend on \(M\) according to the relations plotted in Figure 5, which are well approximated by

\[ n(M) = 4.32 + 7.5 \times 10^{-7} \left[ \ln \left( \frac{M}{\text{M}_\odot} \right) \right]^{4.6} , \quad (15) \]

\[ \ln[c_n(M)] = 13.3 + 7.5 \times 10^{-8} \left[ \ln \left( \frac{M}{\text{M}_\odot} \right) \right]^{5.6} , \quad (16) \]

leading to the following combined relation

\[ c_n(M) = 5.84 \times 10^5 \left( \frac{M}{\text{M}_\odot} \right)^{\frac{n-4}{n-2}} . \quad (17) \]

These expressions are useful to infer the typical values of the Sérsc parameters for present-day halos with any mass. It is worth mentioning that the predicted values of \(n\) are of the same order of magnitude as the ones obtained by Merritt et al. (2003) from simulated halos with masses ranging from dwarfs galaxies to galaxy clusters (the values of \(c_n\) were not presented in that work).

To obtain more general values of these parameters for halos of any mass and redshift, we can proceed as in the NFW case above. For reasons identical to the ones leading to the time-invariant relation \(M_t(r_n)\), the relations \(\rho_0(r_n)\) and \(n(r_n)\) must be time-invariant. In Figure 6 we see how the corresponding curves obtained from the fit to the Sérsc profile of the same density profiles as used in Figure 4 overlap, indeed, even better than the \(M_t(r_n)\) curves do, since there is no large deviations at large masses. These invariant relations are well fit, for \(x \equiv \log(r_n/\text{Mpc})\) in the range \(-25 < x < -9\), by

\[ \rho_0(r_n) = \tilde{\rho}_0 \exp(A) , \quad (18) \]

\[ A = -2.48 - 4.73 x - 0.270 x^2 - 6.24 \times 10^{-3} x^3 , \]
\[ n(r_n) = -4.43 - 1.43x - 0.0313x^2 - 2.98 \times 10^{-4}x^3 . \]

Replacing these expressions into equation (13), we can solve for \( r_n \) and then use the relation (19) to find the value of \( n \).

This provides a very concrete prediction which can be tested with numerical simulations. One can take the very strong correlation shown in Figure 6, which allows one to fit the density profile of any dark matter structure with only two free parameters, e.g., \( n \) and \( \rho_0 \). With this value of \( n \) (purely from the shape of the density profile), one now gets a value for the mass (from Fig. 5). This mass can trivially be compared to the true virial mass (which is naturally known in the simulation), and hence one can confirm or reject the prediction of the accretion-driven model.

4. SUMMARY

We have shown how the basic properties of standard CDM can justify the MRSSS model previously shown to be in overall agreement with the results of numerical simulations. In this model, the density profile of relaxed halos permanently adapts to the profile currently building up through accretion and does not depend on their past aggregation history. As a consequence, the typical density profile of halos of a given mass at a given epoch is set by their time-evolving cosmology-dependent typical accretion rate.

Although halos have been assumed to be spherically symmetric throughout the present paper, this is not crucial for the MRSSS model. As shown in a following paper (González-Casado et al. 2007), the results presented here also hold for more realistic triaxial rotating halos. Furthermore, an approach similar to the one followed here allows one to explain not only their mass distribution, but also other of their structural and kinematic properties, such as the radial dependence of angular momentum.

According to the MRSSS model, the central asymptotic behavior of the halo density profile depends, through the typical accretion rate, on the power spectrum of density perturbations. The prediction made in the case of power-law spectra should be possible to check by means of numerical simulations provided one concentrates on massive halos as these reach the asymptotic regime at larger radii. In the case of the standard CDM power spectrum, the model predicts a vanishing central logarithmic slope. The way this asymptotic behavior is reached is surprisingly simple: down to a radius as small as 1 pc, the density profile is well fit by the 3D Sérsic or Einasto profile over at least 10 decades in halo mass.

Another consequence of the MRSSS model with useful practical applications is the existence of time-invariant relations among the NFW or 3D Sérsic power-law parameters (\( M_s \) and \( r_s \) in the former case and \( \rho_0 \), \( r_n \) and \( n \) in the latter) fitting the halo density profiles. A code is publicly available at http://www.am.ub.es/cosmo/gravitation.html that computes such invariant relations for any desired standard CDM cosmology.

Some of these consequences can be readily tested by numerical simulations or by (X-ray or strong lensing) observations, which should allow one to confirm or reject the prediction on the central behavior of the density profile of halos made by the MRSSS model.

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expansion of \([1 + \nu]\) of fluctuations on scale through both accretion and major mergers, accretion tracks \(M\) function approaches unity, we are led to

\[
\frac{d\delta_c(t)}{dt} = \frac{\delta_0}{t_0} \left( \frac{t}{t_0} \right)^{-2/3}, \quad \sigma(M) = \sigma(M_\infty) \left( \frac{M}{M_\infty} \right)^{-\frac{4}{3}} \tag{A2}
\]

where \(\delta_0\) and \(M_\infty\) are the current values of \(\delta_c(t)\) and \(M_\infty(t)\), respectively, \(M_\infty(t)\) being the critical mass for collapse at \(t\), solution of the implicit equation \(\sigma(M_\infty, t) = \delta_0\). Therefore, equation (A1) takes the form

\[
R_m(M, t, \Delta) = \sqrt{\frac{2}{\pi}} \frac{j + 3}{9t} \nu(M(t)) \left( 1 + \frac{\Delta}{M_\infty} \right)^{-\frac{j+3}{3}} \exp \left\{ -\frac{\nu^2(M(t))}{2} \left[ (1 + \Delta)^{\frac{j+3}{3}} - 1 \right] \right\}, \tag{A3}
\]

where

\[
\nu(M, t) = \left( \frac{t}{t_0} \right)^{-2/3} \left( \frac{M}{M_\infty} \right)^{\frac{j+3}{6}} = \left[ \frac{M}{M_\infty(t)} \right]^{\frac{j+3}{6}}. \tag{A4}
\]

Substituting expression (A3) in equation (2) and changing \(\Delta\) by \(x = \nu^2[(1 + \Delta)^{(j+3)/3} - 1]\), we obtain the following expression for the accretion rate

\[
R_a(M, t) = \frac{3A\nu^2}{(j + 3)t} \int_0^{x_m(M, t, \Delta_m)} dx \left[ (1 + \nu^{-2}x)^{\frac{j+3}{2}} - 1 \right] \left( 1 - x^{-2} \right) \frac{\exp(-x/2)}{x^\frac{j+3}{2}}, \tag{A5}
\]

where, for simplicity, we have dropped the explicit dependence of \(\nu\) and used the notation \(A = \sqrt{2/\pi}(j + 3)/9\) and \(x_m(M, t, \Delta_m) = \nu^2[(1 + \Delta_m)^{(j+3)/3} - 1]\).

In the differential equation (3), the variable \(M\) in \(R_a\) is replaced by the mass accretion track \(M(t)\). As halos grow through both accretion and major mergers, accretion tracks \(M(t)\) increase with increasing time less rapidly than does \(M_\infty(t)\) tracing the typical mass evolution of halos in any self-similar universe. Thus, \(\nu(t) = \nu(M(t), t)\) is a decreasing function of \(t\) (see eq. (A4)) and, in the small \(t\) asymptotic regime, \(\nu(t)^{-2}\) tends to zero. Taking the Taylor series expansion of \([1 + \nu^{-2}(t)x]^{(j+3)/3}\) inside the integral on the right of equation (A5), at leading order in \(\nu^{-1}(t)\), we obtain

\[
R_a[M(t), t] = \frac{9At^{-1}}{(j + 3)^2} \int_0^{x_m(t)} dx \frac{\exp(-x/2)}{\sqrt{x}} = \frac{9\sqrt{2\pi}At^{-1}}{(j + 3)^2} \text{erf} \left[ \nu(t) \sqrt{(1 + \Delta_m)^{\frac{j+3}{3}} - 1} \right], \tag{A6}
\]

where \(\text{erf}(x)\) is the error function. Given the value of constant \(A\) and given that, for \(\nu(t)\) tending to infinity, the error function approaches unity, we are led to

\[
R_a[M(t), t] \approx \frac{2}{j + 3} t^{-1}. \tag{A6}
\]

Finally, integrating equation (5) for such an accretion rate, we are led to the following asymptotic behavior for accretion tracks

\[
M(t) \propto t^{\frac{2}{j+3}}. \tag{A7}
\]

Note how it compares with the time dependence of the critical mass in a self-similar universe: \(M_\infty(t) \propto t^{\frac{1}{2+j}}\).