Behavioural modelling and system-level simulation of micromechanical beam resonators

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Abstract. This paper presents a behavioural modelling technique for micromechanical beam resonators that enables the simulation of MEMS resonator model in Analog Hardware Description Language (AHDL) format within a system-level circuit simulation. A 1.13 MHz clamped-clamped beam and a 10.4 MHz free-free beam resonators have been modelled into Verilog-A code and successfully simulated with Spectre in Cadence. Analysis has shown that both models behave well and their electrical characteristics are in agreement with the theory.

1. Introduction
A large variety of MicroElectroMechanical Systems (MEMS) includes complex interacting mechanical and electrical components that must be carefully designed for proper operation. In typical MEMS application areas such as inertial sensors, microfluidics, bioMEMS, and communication systems – devices ranging from accelerometers, gyroscopes, micromirrors, DNA sensors, resonators, filters, and RF switches – the designer is often faced with a challenge of predicting the performance of the MEMS device when it is integrated with external electronic circuitry. In addition to the initial device modelling, the system-level behaviour of the device with other signal processing electronics needs to be performed for accurately predicting its overall performance.

Initial device modelling for MEMS resonators is usually done with available Finite Element Method (FEM) and Boundary Element Method (BEM) tools such as ANSYS and ABAQUS. Although numerical simulation of mechanical and electrostatic interactions can be performed with these tools, system-level simulation of the mechanical device along with the transistor-level IC electronic circuits, such as with CMOS, cannot be performed. System-level simulation is usually desired by the designer for accurate optimization and reduction in design cycle. Currently, there are several tools available for circuit-level behavioural representation of MEMS, which include MEMSPro, MEMSMaster, NODAS, and SUGAR. The general strategy with these tools is to break up the MEMS structure into smaller elements and represent them as behavioural models, realized in Analog Hardware Description Language (AHDL), which can then be simulated in a circuit simulator.

A modelling approach has already been presented in [1] where FEM model is transformed into AHDL model after which it is included in a system level simulation, and is demonstrated for low frequency (~ 36.5 kHz) comb resonators and filters. In another behavioural modelling strategy reported by MEMSCAP in [2], a superior method has been demonstrated for a behavioural model of a 10-MHz clamped-clamped beam resonator embedded within a Pierce oscillator circuit, which was presented as a test case of the work previously reported in [3].
This paper presents a behavioural modelling approach, utilizing the tools presented in [2], without requiring the tedious fragmentation of the device into components at the initial modelling stage as presented in [1], and it can automatically integrate a MEMS device within a system-level circuit simulation. This work examines the electrical characteristics of the generated model in detail, and the simulated results are verified against theory. The verification of the method is demonstrated with two examples, first with a 1.13 MHz clamped-clamped beam resonator and second with a more complex 10.4 MHz free-free beam of the work presented in [4]. The model for resonator of [4] displays a resonant frequency tuning behaviour comparable to the experimental results reported in [4].

A description of the overall method is presented in section 2, followed by the simulation results of the clamped-clamped beam in section 3, and the free-free beam resonator in section 4. Finally, conclusions are summarized in section 5.

2. Behavioural modelling methodology

The usual practice in order to simulate the MEMS device with other electronic circuits is to derive an RLC equivalent circuit to represent the mechanical components of the device. The RLC equivalent circuit approach is very useful if the geometry of the resonator is simple while the electrical behaviour is reasonably accurate, and since this is done with device components centred on some operating point, it is limited to small signal behaviour. Modelling with AHDL can better incorporate nonlinear physical and electrical behaviours, as well as that of other energy domains.

The method presented in this paper begins with creation of a 3D finite element (FE) model of the device in ANSYS. It does not require the decomposition step of dividing the device into simpler components or functional blocks as described in [1]. Another advantage for initial 3D model creation is that the FE model can be imported from different graphics file formats allowed in ANSYS. Once the FE model is done, electrical and structural PHYSICS files are created in ANSYS, through which nonlinear electrostatic-to-structural coupling mechanisms could be better integrated.

At the next step, MemsModeler takes in files from ANSYS directly and performs the model reduction algorithm based on selected degrees of freedom (DOF), such as displacement of nodes. Proper selection of the retained nodes in reduction, optimum for a particular mode, will lead to higher accuracy of the model. Furthermore, tedious efforts of RLC circuit model generation to represent complex structures can be avoided. The output reduced-order behavioural model can be transferred into a suitable model format that can be simulated with circuit simulators of ADVance MS, SMASH, T-Spice, or Spectre in Cadence.

3. Simulation of clamped-clamped beam behavioural model

In order to illustrate the validity of the methodology, a clamped-clamped beam of $L = 200\mu m$, $w = 5\mu m$ and $h = 10\mu m$, as shown in figure 1, is initially modelled in ANSYS. Next, electrical and structural descriptions of the beam are extracted from ANSYS as PHYSICS files. Proper selection and description of conducting elements is critical at this point, along with the modelling of electrode-to-resonator gap, which is taken as a material with the relative permittivity of 1.0 to model ambient environment. The gap is set at 1\mu m and the electrode length $L_e$ is 20\mu m for simulation.

Then, the Verilog-A code for the beam provided by MemsModeler is simulated with Spectre in Cadence. In order to properly model the fundamental resonant frequency, the model reduction is done around the node at the middle of the resonator beam, with three DOF for $x$-, $y$-, and $z$-directions. The FEM simulation from ANSYS for the fundamental resonant frequency $f_0$ is 1.128 MHz, while the Verilog-A behavioural model frequency is 1.148 MHz, with a difference of 1.8 percent. The Verilog-A model has six pins: three mechanical pins to describe the displacement of the three DOF of the middle node; and three electrical pins to represent the drive and sense electrodes, and the resonator beam itself.

In order to evaluate the electrical behaviour, a DC bias voltage is applied to the pin corresponding to the resonator, named as COND2 as shown in figure 1, and an ac voltage to COND1 and the amplitude of the current output at COND0 is recorded. The simulated values can be found in table 1.
Figure 1. Schematic of clamped-clamped beam resonator and its mode shape in ANSYS.

where the ac wave with 0.2V amplitude at resonant frequency is fixed, and the sensing COND0 is biased with a DC voltage of 0V, while DC level at COND2 is varied. The transient vibration amplitude of the middle node of the resonating beam is included in table 1 as well.

Table 1. Simulated values for 1.13 MHz clamped-clamped beam model.

| \( V_P \) (V) | Vib. amplitude mid. node (nm) | Current at COND0 (pA) | Theoretical current (pA) | Difference in currents (%) | Frequency, \( f_0 \) (MHz) |
|--------------|-----------------------------|-----------------------|--------------------------|----------------------------|-----------------------------|
| 10           | 0.118                       | 15.3                  | 15.1                     | 1.3                        | 1.1476                      |
| 20           | 0.236                       | 61.3                  | 60.2                     | 1.8                        | 1.1463                      |
| 30           | 0.355                       | 138.1                 | 135.5                    | 1.9                        | 1.1439                      |
| 40           | 0.477                       | 246.5                 | 242.1                    | 1.8                        | 1.1405                      |
| 50           | 0.600                       | 385.7                 | 379.3                    | 1.7                        | 1.1363                      |
| 80           | 0.995                       | 1008.3                | 989.6                    | 1.9                        | 1.1174                      |
| 100          | 1.280                       | 1596.3                | 1566.3                   | 1.9                        | 1.0998                      |

The theoretical current output, \( i_o \), at COND0, assuming COND1 and resonator right at the gap are moving as two parallel plates, can be estimated as follows

\[
i_o = V_P \frac{\partial C}{\partial x} = V_P \frac{\partial C}{\partial \delta x} \frac{\partial \delta x}{\partial t}
\]  

(1)

where the DC bias is \( V_P \), and the change in capacitance between COND1 and resonator with respect to vibration displacement is \( \frac{\partial C}{\partial x} \), which can be estimated as

\[
\frac{\partial C}{\partial x} = \varepsilon_o \varepsilon_r L_e \frac{h}{d_o^2}
\]

(2)

where \( \varepsilon_o \) is the permittivity, \( L_e \) is the length of the electrode, and \( d_o \) the electrode-to-resonator gap. Therefore, the output current can now be estimated further as

\[
i_o = V_P \cdot \varepsilon_o L_e \frac{h}{d_o^2} \cdot (2\pi f_0 \cdot X_o)
\]

(3)

where the displacement of the resonator \( x(t) \) is a sine wave, and the amplitude of its derivative \( \frac{\partial x}{\partial t} \) can be derived as \((2\pi f_0 \cdot X_o)\), and the term \( X_o \) is the amplitude of vibration [5]. Given the above derivation, the theoretical output current can now be calculated based on equation (3).

The difference between the theoretical and simulated values of the output current at COND0, as listed in table 1 of less than 2 percent for \( L_e = 20\mu\text{m} \), can be explained given the fact that the above derivations are based on the two parallel plates moving together with the gap along the edges moving in parallel. However, for the clamped-clamped beam that is resonating at the fundamental frequency, due to the nature of the vibrating mode, it has uneven parallel gap distance along the plates, as shown in figure 1. The simulated value of current deviates more from the theoretical value if the electrode...
length used for simulation is increased. Simulated output current shown in table 1 also shows that it varies proportionally with $V_P$ and the ac voltage applied to the drive electrode (COND1) verifying the equation (3).

In addition, the AC analysis of the Verilog-A code for the clamped-clamped beam has indicated a drop in $f_o$ of 4.2% as the DC bias voltage $V_P$ is increased from 10V to 100V. This phenomenon has been well documented in literature and according to [5], the resonant frequency can be estimated as

$$f_o = 1.03 \cdot K_1 \cdot \frac{w}{L^2} \sqrt{\frac{E}{\rho} \left[1 - \left(\frac{k_e}{k_m}\right)^2\right]}$$  \hspace{1cm} (4)

where the values $E$ and $\rho$ Young’s modulus and density, the simulated values of $E = 179$GPa and $\rho = 2330$ kg/m$^3$. The term $K_1$ is the scaling factor for surface topology effect [5], $k_m$ the mechanical stiffness, and $k_e$ is the electrical spring constant. With equation (4), if the scaling terms are ignored, the value of $f_o$ agrees with the ANSYS result of 1.128 MHz. The electrical spring constant can be related to DC voltage applied as described in the given differential [6]

$$dk_e(y) = V_P^2 \cdot \frac{\varepsilon_r h \cdot dy}{[d(y)]^2}$$  \hspace{1cm} (5)

with $k_e(y)$ being spring constant dependent on the specific $y$ locations along the length $L$ of the beam. From this relationship, it is evident that the electrical spring stiffness is strongly dependent on the electrode-to-resonator gap $d(y)$, as well as to the square of the DC bias $V_P$. To draw a relationship between $V_P$ and $f_o$, the expression for $k_e$ can be substituted into equation (4). When both sides of the equation are squared and rearranged, a linear relationship between $(f_o)^2$ and $(V_P)^2$ is obtained with a negative slope. To verify this with the simulated data, resonant frequency and DC bias from table 1 is plotted and shown in figure 2.

![Figure 2. Plot of data from table 1, (a) resonant frequency drop with DC bias increase and (b) linear relationship between $(f_o)^2$ and $(V_P)^2$.](image)

The linear relationship between $(f_o)^2$ and $(V_P)^2$ is clearly observed from figure 2(b) indicating an agreement between the simulated data and the theoretical analysis.

**4. Simulation of free-free beam behavioural model**

To further validate the modelling method, the FEM model of a 10.47 MHz free-free beam resonator, with the dimensions shown in [4] is first created in ANSYS, following technique described in section 2. The fundamental resonant mode of the middle “free-free” beam of the resonator is located at the fourth eigen frequency of the structure at 10.44 MHz in ANSYS, as shown in figure 3, whereas it is the first eigen frequency for the clamped-clamped beam presented in section 3, the difference due to
other lower modes introduced by inclusion of supporting beams. The fabricated resonator has been shown to have a high Q of 10,741 as stated in [4].

| Dimension          | Value   |
|--------------------|---------|
| Electrode gap      | 39.8 μm |
| Support beam Width | 2 μm    |
| Support beam Length| 14 μm   |
| Electrode width    | 2 μm    |
| FF-beam Length     | 10.44 MHz |
| FF-beam Width      | 1.2 μm  |
| Electrode gap      | 100 nm  |

**Figure 3.** Resonant mode of 10.44 MHz free-free beam simulated in ANSYS, along with the geometric dimensions from [4] used for the model.

For better matching of the FEM resonant frequency and the behavioural model frequency, several nodes along the middle beam are selected, with the overall 18 retained DOF for the reduction process, and the behavioural model frequency simulated is 11.59 MHz for the fourth eigen mode. The free-free beam Verilog-A model is set up with similar configuration shown in figure 1, one electrode to drive and the other to sense on either side of the middle beam, while the middle resonator itself is biased with DC voltage. The simulated data is similar to that from table 1, and the model is found to be properly modelling the resonator. The resonant frequency drop due to an increase in $V_p$ is also observed as well for this free-free beam resonator, and presented in figure 4 along with the data published in [4] for comparison, and the simulation follows the same trend as the results of [4].

![Resonant mode of 10.44 MHz free-free beam](image)

**Figure 4.** Plot of $f_o$ vs. $V_p$ for free-free beam.

![AC response of output current](image)

**Figure 5.** AC response of output current.

Figure 5 illustrates the AC response from Spectre simulation of output current at the sense electrode, while other is driven with AC voltage of 0.4V peak-to-peak at $f_o$, and the resonator is biased with $V_p = 20$V. The peak value of the AC plot at $f_o = 11.43$ MHz is -96.3 dB. Another peak at 31.6 MHz can also be observed for the seventh eigen frequency of the behavioural model simulation result, indicating that other neighboring modes around the desired mode are also included within the behavioural model for more realistic simulation of the resonator.
The behavioural model of a resonator can further be embedded within a closed-loop with a trans-impedance amplifier, as shown in figure 6, or within a Pierce oscillator circuit as demonstrated before in [2], transforming it into a free-running oscillator circuit that could be evaluated with several circuit simulators, valuable for communications applications.

![Figure 6. Schematic for a closed-loop simulation of the behavioural model.](image)

Given the AC response of the sense current output with its peak at the desired resonant frequency, as illustrated in figure 5, sense current can be connected to transresistance amplifier, whose voltage output, after passing through a voltage limiting stage, can be used to drive the resonator block, in a free running closed-loop oscillator simulation. The performance characteristics for the oscillator based on the designed resonator can then be evaluated. A free-free beam with different dimensions has already been fabricated and is currently under testing to further verify the validity of the modelling methodology presented in this paper.

5. Conclusion
A behavioural modelling method for MEMS devices has been presented in this report, beginning with FE model to generation of AHDL model, which is subsequently simulated in a circuit simulation where system-level evaluation with IC electronics such as with CMOS circuits is possible. The electrical behaviour of clamped-clamped beam and free-free beam Verilog-A models have been evaluated in detail with Spectre in Cadence, and found that the current output at the sense electrode is in agreement with the theory. The well-known effect of resonant frequency drop due to an increase in DC bias voltage applied to resonator has been observed from simulated results of both types of beam resonators.

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