Tachyon Kinks on Unstable Dp-branes

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Abstract

In the context of tachyon effective theory coupled to Born-Infeld electromagnetic fields, we obtain all possible singularity-free static flat configurations of codimension one on unstable Dp-branes. Computed tension and string charge density suggest that the obtained kinks are D(p − 1) or D(p − 1)F1-branes.

Keywords: Kink, Tachyon effective action
1 Introduction

Instability of non-BPS Dp-brane in string theory is realized by the presence of a tachyonic mode in its spectrum. It is expected that condensation of the tachyon takes place and its energetic minimum is the closed string vacuum \[1\].

Study of tachyon dynamics on a non-BPS Dp-brane is accomplished in the scheme of either boundary conformal field theory (BCFT) \[2, 3\] or effective theory with the action \[4–6\]

\[
S = -T_p \int d^{p+1}x \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu})},
\]

where \(T\) is tachyon and \(F_{\mu\nu}\) is the field strength tensor of Abelian gauge field \(A_\mu\) on the Dp-brane. Since the tachyon potential \(V(T)\) measures varying tension, it should satisfy two boundary values such that \(V(T = 0) = 1\) and \(V(T = \infty) = 0\). Specific computation based on (boundary) string field theory \[7\] gives \(V(T) \sim e^{-T^2}\) and Ref. \[9\] suggests \(V(T) \sim e^{-T}\) for large \(T\).\(^1\) Here we adopt a runaway tachyon potential which is convenient for analysis \[10–14\] and is obtained from open string theory \[15, 16\]

\[
V(T) = \frac{1}{\cosh\left(\frac{T}{T_0}\right)},
\]

where \(T_0\) is \(\sqrt{2}\) for the non-BPS D-brane in the superstring and 2 for the bosonic string.

Since the theory of our interest is the effective theory of tachyon without physical states around its vacuum, an adequate proposal to understand the tachyon effective action \[11\] is made through comparison of the classical solutions from both the effective theory and the open string theory which is describable in terms of BCFT \[15, 14, 17\]. The most intriguing solution is so-called rolling tachyon which provides a real time process of homogeneous tachyon configuration and, at late time, becomes a pressureless gas \[2, 3, 9, 18\]. This is understood in terms of string (field) theory \[19\] and is also a representative candidate of S-brane \[20–22\]. In relation with generation of fundamental string (F1) from unstable D-brane \[23, 17\], coupling of Born-Infeld type electromagnetism leads to fluid state of electric flux tube \[24, 25\] or that of electromagnetic flux tube \[26, 11\].

Spatial inhomogeneity is another important issue \[27, 28\], particularly in the form of tachyon solitons. In the effective theory of pure tachyon, tachyon kinks of codimension one have been studied extensively, however the obtained configurations are either static singular solutions \[6, 29, 30, 31, 32\] or array of regular kink-antikink \[14, 33, 34\]. Time-dependent kink or periodic sinusoidal array is also suffered by encountering of a singularity after a finite time interval \[35\], which appears in a form of blowing-up energy-momentum tensor for Dp-branes in bosonic string theory \[36\]. Static topological tachyon kink is shown

\[^1\]Comparison of the S-matrix elements from string theory and those from effective theory predicts \(V(T) = e^{-T^2}\) \[8\].
to be BPS D($p-1$)-brane with keeping supersymmetry from the study of its worldvolume action of fluctuations $[32, 37, 38]$. Computed tension of the tachyon kink coincides with that of D($p-1$)-brane $[14, 32]$. In the context of string theory, corresponding BCFT description has recently been worked out $[17]$.

Remarkably, introduction of electromagnetic fields regularizes static topological kink $[33]$. For the various constant electric and magnetic fields, rich spectra of extended tachyon objects of codimension one are obtained, i.e., they include array of kink-antikink, topological kink, half-kink, hybrid of two half-kinks, and bounce. The corresponding tension and F1 charge density confined on the codimension-one kink imply that the single unit kink is naturally a candidate of D1 or D1F1. When pure electric field is less than or equal to 1, corresponding BCFT solutions are also obtained in $[17]$. In the work of Ref. $[33]$, the obtained kinks are codimension one objects of unstable D2-brane In this paper, we consider unstable D$p$-brane for arbitrary $p$ and find all possible static regular tachyon solutions of codimension one, which will be interpreted as general flat D($p-1$)- or D($p-1$)F1-branes. Probably static kinks will play an important role in resuming fundamental dynamical questions, e.g., strings from (rolling) tachyons $[39, 40]$ and emission of gravitational fields or closed strings $[11, 42, 14, 17]$. The rest of the paper is organized as follows. In section 2, we analyze in detail the case of $p = 3$. We obtain all possible configurations, including array, topological kink, half-kink, hybrid of two half-kinks, and bounce. In section 3, we consider the general case of arbitrary $p$ and show that structure of static regular kinks is the same independent of $p \geq 2$. We conclude in section 4.

2 Tachyon Kinks on Unstable D3-brane

In this section we analyze the effective tachyon action in (1+3)-dimensions in detail and find all possible static regular tachyon kink solutions. We will show that they consist of array of kink-antikink, topological kink, half-kink, hybrid of two half-kinks, and bounce in addition to homogeneous symmetric and broken vacua. In the context of string theory, the obtained configurations correspond to D2- and D2F1-branes from unstable D3-brane or to their hybrids. The solution spectra turn out to be identical to $D_2$ case analyzed in $[33]$.  

2.1 Effective theory

The effective tachyon action for the unstable D3-brane system is

$$S = -T_3 \int d^4x \, V(T) \sqrt{- \det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu})}.$$  (2.1)
To proceed, we introduce a few notations. We first define

\[ X_{\mu \nu} \equiv \eta_{\mu \nu} + \partial_{\mu} T \partial_{\nu} T + F_{\mu \nu}, \quad (2.2) \]

\[ X \equiv \text{det}(X_{\mu \nu}). \quad (2.3) \]

In \( X_{\mu \nu} \), we separate barred metric \( \bar{\eta}_{\mu \nu} \) and barred field strength tensor \( \bar{F}_{\mu \nu} \)

\[ \bar{\eta}_{\mu \nu} = \eta_{\mu \nu} + \partial_{\mu} T \partial_{\nu} T, \quad (2.4) \]

\[ \bar{F}_{\mu \nu} = F_{\mu \nu}. \quad (2.5) \]

Then we have determinant of barred metric \( \bar{\eta} \) and inverse metric \( \bar{\eta}^{\mu \nu} \)

\[ \bar{\eta} = -(1 + \partial_{\mu} T \partial_{\nu} T), \quad \bar{\eta}^{\mu \nu} = \eta^{\mu \nu} - \frac{\partial^{\mu} T \partial^{\nu} T}{1 + \partial_{\mu} T \partial_{\nu} T}. \quad (2.6) \]

Contravariant barred field strength tensor \( \bar{F}^{\mu \nu} \) and its dual field strength \( \bar{F}^*_{\mu \nu} \) are

\[ \bar{F}^{\mu \nu} = \bar{\eta}^{\mu \alpha} \bar{\eta}^{\nu \beta} F_{\alpha \beta}, \quad \bar{F}^*_{\mu \nu} = \frac{\bar{\epsilon}^{\mu \nu \alpha \beta}}{2} \bar{F}^{\alpha \beta} = \frac{\bar{\epsilon}^{\mu \nu \alpha \beta}}{2} \bar{\eta}^{\alpha \gamma} \bar{\eta}^{\beta \delta} F_{\gamma \delta}, \quad (2.7) \]

where \( \bar{\epsilon}^{\mu \nu \alpha \beta} = \sqrt{-\bar{\eta}} \epsilon_{\mu \nu \alpha \beta} \) with \( \epsilon_{0123} = 1 \). In terms of barred quantities Eq. (2.3) is computed as

\[ X = \bar{\eta} \left[ 1 + \frac{1}{2} \bar{F}_{\mu \nu} \bar{F}^{\mu \nu} - \frac{1}{16} \left( \bar{F}^*_{\mu \nu} \bar{F}^{\mu \nu} \right)^2 \right]. \quad (2.8) \]

Then equations of motion for the tachyon \( T \) and the gauge field \( A_{\mu} \) are

\[ \partial_{\mu} \left( \frac{V}{\sqrt{-X}} C_{S}^{\mu \nu} \partial_{\nu} T \right) + \sqrt{-X} \frac{dV}{dT} = 0, \quad (2.9) \]

\[ \partial_{\mu} \left( \frac{V}{\sqrt{-X}} C_{A}^{\mu \nu} \right) = 0. \quad (2.10) \]

Here \( C_{S}^{\mu \nu} \) and \( C_{A}^{\mu \nu} \) are symmetric and asymmetric part of the cofactor,

\[ C^{\mu \nu} = \bar{\eta} \left( \bar{\eta}^{\mu \nu} + \bar{F}^{\mu \nu} + \bar{\eta}^{\mu \alpha} \bar{\eta}^{\nu \beta} F_{\alpha \beta} \bar{F}^*_{\gamma \delta} + \bar{\eta}^{\mu \alpha} \bar{\eta}^{\nu \beta} F_{\alpha \beta} \bar{F}^*_{\gamma \delta} \bar{F}^{\gamma \delta} \right), \quad (2.11) \]

namely,

\[ C_{S}^{\mu \nu} = \bar{\eta} \left( \bar{\eta}^{\mu \nu} + \bar{\eta}^{\mu \alpha} \bar{\eta}^{\nu \beta} F_{\alpha \beta} \bar{F}^*_{\gamma \delta} \right), \quad (2.12) \]

\[ C_{A}^{\mu \nu} = \bar{\eta} \left( \bar{F}^{\mu \nu} + \bar{\eta}^{\mu \alpha} \bar{\eta}^{\nu \beta} F_{\alpha \beta} \bar{F}^*_{\gamma \delta} \bar{F}^{\gamma \delta} \right). \quad (2.13) \]

Energy-momentum tensor \( T_{\mu \nu} \) in the symmetric form is given by

\[ T^{\mu \nu} = \frac{T_{3} V(T)}{\sqrt{-X}} C_{S}^{\mu \nu}, \quad (2.14) \]
where $C_{\mu\nu} \equiv \eta_{\mu\alpha}\eta_{\nu\beta}C^{\alpha\beta}$.

We denote conjugate momenta of the gauge fields as $\Pi_i$,

$$\Pi_1 = T_3 \frac{V}{\sqrt{-X}} \left[ E_1 + B_1 (\mathbf{E} \cdot \mathbf{B}) \right],$$

$$\Pi_2 = T_3 \frac{V}{\sqrt{-X}} \left[ E_2 (1 + T'^2) + B_2 (\mathbf{E} \cdot \mathbf{B}) \right],$$

$$\Pi_3 = T_3 \frac{V}{\sqrt{-X}} \left[ E_3 (1 + T'^2) + B_3 (\mathbf{E} \cdot \mathbf{B}) \right].$$

We only consider the cases of $T = T(x)$, $\mathbf{E} = \mathbf{E}(x)$, and $\mathbf{B} = \mathbf{B}(x)$ without dependence on $y$ and $z$ coordinates. Then the equations of motion (2.9)–(2.10) become

$$\partial_1 \left[ \frac{V}{\sqrt{-X}} (1 + B_1^2 - E_2^2 - E_3^2) T' \right] = \sqrt{-X} \frac{dV}{dT},$$

$$\partial_1 \Pi_1 = 0,$$

$$\partial_1 \left\{ \frac{V}{\sqrt{-X}} [-B_3 + E_3 (\mathbf{E} \cdot \mathbf{B})] \right\} = 0,$$

$$\partial_1 \left\{ \frac{V}{\sqrt{-X}} [-B_2 + E_2 (\mathbf{E} \cdot \mathbf{B})] \right\} = 0,$$

where

$$-X = \left[ 1 - \mathbf{E}^2 + \mathbf{B}^2 - (\mathbf{E} \cdot \mathbf{B})^2 \right] + (1 + B_1^2 - E_2^2 - E_3^2) T'^2.$$ (2.22)

From Eq. (2.14) we have energy density $\rho$

$$\rho \equiv T_{00} = T_3 \frac{V}{\sqrt{-X}} \left[ (1 + T'^2)(1 + B_1^2) + B_2^2 + B_3^2 \right].$$

The system in this reference frame carries nonvanishing linear momentum density

$$P_i \equiv T^{0i} = T_3 \frac{V}{\sqrt{-X}} \left( \epsilon_{ijk} B^j E^k - \epsilon_{ij1} B^1 E^j T'^2 \right).$$

Other nonvanishing components of the energy-momentum tensor (2.14) are

$$T_{11} = -T_3 \frac{V(T)}{\sqrt{-X}} \left( 1 + B_1^2 - E_2^2 - E_3^2 \right),$$

$$T_{22} = -T_3 \frac{V(T)}{\sqrt{-X}} \left[ (1 + T'^2)(1 - E_3^2) - E_2^2 + B_2^2 \right],$$

$$T_{33} = -T_3 \frac{V(T)}{\sqrt{-X}} \left[ (1 + T'^2)(1 - E_2^2) - E_3^2 + B_3^2 \right],$$

$$T_{12} = -T_3 \frac{V(T)}{\sqrt{-X}} (E_1 E_2 + B_1 B_2),$$

$$T_{13} = -T_3 \frac{V(T)}{\sqrt{-X}} (E_1 E_3 + B_1 B_3),$$

$$T_{23} = -T_3 \frac{V(T)}{\sqrt{-X}} [E_2 E_3 (1 + T'^2) + B_2 B_3].$$
Conservation of the energy-momentum tensor, $\partial^\mu T_{\mu\nu} = 0$, leads to four constants of motion

$$T_{10} = T_{01}, \quad T_{11}, \quad T_{21} = T_{12}, \quad T_{31} = T_{13}. \quad (2.31)$$

From Faraday’s law $\partial^\mu F_{\mu\nu} = 0$, we find that $E_2$, $E_3$ and $B_3$ are constants. By an appropriate choice of coordinates we may assume that, without loss of generality,

$$E_3 = 0. \quad (2.32)$$

Then from Eq. (2.20) we see that

$$\gamma \equiv T_3 V(T) \sqrt{-\lambda} = \frac{\Pi_1}{E_1 + B_1(E \cdot B)} = \text{constant}. \quad (2.33)$$

Constancy of $T_{11}$ and $T_{13}$, in turn, implies that $B_1$ is also a constant. Finally, the remaining equations (2.19), (2.21) and (2.28) lead to constancy of $B_2$ and $E_1$. Therefore $E$ and $B$ are actually constants.

Substitution of the expression of $X$ (2.22) into Eq. (2.33) summarizes the static system of our interest as a single first-order equation

$$\mathcal{E} = \frac{1}{2} T'^2 + U(T). \quad (2.34)$$

Here $\mathcal{E}$ and $U(T)$ are

$$\mathcal{E} = -\frac{\beta}{2\alpha}, \quad (2.35)$$

$$U(T) = -\frac{1}{2\alpha \gamma^2} [T_3 V(T)]^2, \quad (2.36)$$

where $\alpha$ and $\beta$ are defined by

$$\alpha = 1 + B_2^2 - E_2^2, \quad (2.37)$$

$$\beta = 1 - E^2 + B^2 - (E \cdot B)^2. \quad (2.38)$$

Note that every static solution of Eq. (2.34) with $x$-dependence alone satisfies the second-order tachyon equation (2.18). Though the original system is complicated, we now have a simplified equation (2.34) specified by three constants $\alpha$, $\beta$ and $\gamma$.

The choice $E_3 = 0$ allows us to identify additional constants: $T_{02}$, $T_{23}$ and $\Pi_3$. Let us summarize the results. The solution space of our tachyonic system coupled to Born-Infeld electromagnetism is classified by six independent constant parameters; here we choose

$$(\Pi_1, E_1, E_2, B_1, B_2, B_3) \quad (2.39)$$
with $E_3 = 0$. Then other eight constants ($\Pi_3, T_{01}, T_{02}, T_{11}, T_{12}, T_{23}, \gamma$) are expressed by the above 6 parameters (2.39). Nontrivial $x$-dependence appears in the quantities $(\Pi_2(x), T_{00}(x), T_{03}(x), T_{22}(x), T_{33}(x))$, which can be written as

\begin{align*}
\Pi_2 &= \gamma [E_2 + B_2 (E \cdot B)] + \gamma E_2 T'^2, \quad (2.40) \\
T_{00} &= \gamma (1 + B^2) + \gamma (1 + B_1^2) T'^2, \quad (2.41) \\
T_{03} &= \gamma (E_1 B_2 - E_2 B_1) + \gamma E_2 B_1 T'^2, \quad (2.42) \\
T_{22} &= -\gamma (1 - E_1^2 + B_2^2) - \gamma T'^2, \quad (2.43) \\
T_{33} &= -\gamma (1 - E_1^2 - E_2^2 + B_3^2) - \gamma (1 - E_2^2) T'^2. \quad (2.44)
\end{align*}

Here the inhomogeneous part $\gamma T'^2$ can also be written as a constant term plus a piece proportional to square of the tachyon potential,

$$
\gamma T'^2 = -\frac{\beta \gamma}{\alpha} + \frac{1}{\alpha \gamma} [T_3 V(T)]^2
$$

by use of Eq. (2.34). Note that in the above equations the constant terms are proportional to $\Pi_1$ while $(T_2 V)^2$ terms are inversely proportional to $\Pi_1$. Another interesting point is that, for the string charge density $\Pi_2$ along $y$-direction, the coefficients of inhomogeneous part is proportional to $E_2$. Therefore, the existence of $E_2$ is necessary to achieve a confined $F1$ charge on the kink. In addition, the inhomogeneous part of $T_{33}$ vanishes when $E_2^2 = 1$.

If we turn off $B_1$ and $B_2$, the magnetic field has only $B_3$ orthogonal to the electric field $E$ so that $E \cdot B = 0$. Then the system reduces to the case of unstable D2-brane and subsequently the obtained kink configurations are D1- or D1F1-branes [33].

### 2.2 Tachyon kink solutions

In this section we examine the equation (2.34) and find all possible regular static kink configurations. As mentioned previously, each solution is characterized by three parameters $\alpha$, $\beta$, and $\gamma$ defined in Eqs. (2.37)–(2.33). First of all, from Eq. (2.36) we see that the solution space is divided into two classes depending on the sign of $\alpha$ since the potential $U(T)$ flips the sign (see Fig. 1). The singular point of $\alpha = 0$ will be dealt with separately.

Suppose we have a positive fixed $\alpha$ with a given nonzero $\gamma$. Then, there are five cases classified by the value of $\beta$ or equivalently by $\mathcal{E}$.

(i) When $\mathcal{E} < U(0)$ ($\beta > T_3^2 / \gamma^2$), there exists no real tachyon solution.

(ii) When $\mathcal{E} = U(0)$ ($\beta = T_3^2 / \gamma^2$); see the dotted-dashed line in Fig. 1(a)), the constant ontop solution $T(x) = 0$ is allowed (see the dotted-dashed line in Fig. 1(b)). Correspondingly all the quantities in Eqs. (2.40)–(2.44) become constant (see the dotted-dashed line in Fig. 3). In the limit of $\Pi_1 \to 0$, $\gamma \to 0$ and then they vanish.

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(iii) When \( U(0) < \mathcal{E} < 0 \) (\( 0 < \beta < T_3^2/\gamma^2 \)), the tachyon field oscillates between \( T_{\text{max}} \) and \(-T_{\text{max}}\) where \( T_{\text{max}} = T_0 \arccosh(T_3/\gamma\sqrt{\beta}) \) (see the solid line in Fig. 1(a)). Rewriting Eq. (2.34) as an integral equation

\[
x = \int_0^T dT \frac{\sqrt{\alpha}}{\sqrt{\beta} \sqrt{(T_3 V)^2/\beta \gamma^2 - 1}},
\]

we find an exact solution with the tachyon potential \( \alpha \)

\[
T(x) = \pm T_0 \arcsinh \left[ \sqrt{u^2 - 1} \sin \left( \frac{x}{\zeta} \right) \right],
\]
where period $\zeta$ is
\[
2\pi \zeta = 2\pi T_0 \sqrt{\frac{\alpha}{\beta}},
\]
and $u$ is
\[
u = \frac{T_3}{\gamma \sqrt{\beta}}.
\]

The obtained configuration is a kink (or antikink) which is not topological. Since the period $\zeta$ is finite, space-filling configuration is an array of kink-antikink (see the solid line in Fig. 2). The localized part of the quantities in Eqs. (2.40)–(2.44) is given by, e.g.,
\[
\Pi_{2l} \equiv \gamma E_2 T^2 = \frac{E_2 \beta \gamma}{\alpha} \frac{1}{-1 + \sec^2(x/\zeta)}.
\]

Note that for single kink (antikink) Eq. (2.50) is peaked at the origin and localized in the region $-\pi \zeta/2 < x < \pi \zeta/2$ (see the solid line in Fig. 3). The localized piece of the energy density over the half period provides the tension of this codimension one object
\[
T_2 = \frac{(1 + B_1^2)T_3^2}{\alpha \gamma} \int_{-\pi \zeta/2}^{\pi \zeta/2} dx V^2(T(x))
\]
\[
= \pi T_0 T_3 \frac{1 + B_1^2}{\sqrt{\alpha}}.
\]

The multiplicative factor $(1 + B_1^2)/\sqrt{\alpha}$ is expected since the energy density of a Born-Infeld theory increases precisely by this factor when constant electromagnetic fields are
Figure 3: Profiles of string charge density $\Pi_2(x)$ when $\alpha > 0$.

turned on on the worldvolume. It allows us to interpret this codimension one object as a D2-brane in the context of string theory. Similarly, we have string charge per unit transverse area

$$Q_{F1} = \frac{E_2 T_3^2}{\alpha \gamma} \int_{-\frac{\pi \zeta}{2}}^{\frac{\pi \zeta}{2}} dx V^2(T(x))$$

$$= \pi T_0 T_3 \frac{E_2}{\sqrt{\alpha}}. \quad (2.54)$$

It means that the form of F1’s on the D2-brane is a string fluid confined on the D2-brane, where string charge is proportional to $E_2$.

In the limit of $\Pi_1 \to 0$ or equivalently $\gamma \to 0$ with fixed $\alpha$ and $\beta$, constant piece of Eqs. (2.40)–(2.44) vanishes and localized part becomes sharply peaked so that they are given by sums of $\delta$-functions

$$\rho(x) \simeq T_2 \sum_{n=-\infty}^{\infty} \delta(x - 2\pi \zeta), \quad (2.55)$$

$$\Pi_2 \simeq Q_{F1} \sum_{n=-\infty}^{\infty} \delta(x - 2\pi \zeta). \quad (2.56)$$

Therefore, the array of kink-antikink is interpreted as that of infinitely thin D2-\overline{D}2 or D2F1-\overline{D}2F1. The period $2\pi \zeta$ is unchanged in the singular limit under the tachyon potential of our consideration (1.2) but can be changed under a different potential [34].

The obtained array with electromagnetic fields shares the same property with the array of pure tachyon kink-antikink except for scaling factor. This phenomenon can easily be
understood through a rescaling of \(x\)-coordinate in the effective action (2.1)

\[
S = -T_3 \int dt \, dx \, d^2x_\perp \, V(T) \sqrt{\beta + \alpha \left( \frac{dT}{dx} \right)^2} = -\sqrt{\alpha} T_3 \int dt \, d\left( T_0 \frac{x}{\zeta} \right) \, d^2x_\perp \, V(T) \sqrt{1 + \left[ \frac{dT}{d(T_0 x/\zeta)} \right]^2}. \tag{2.57}
\]

The resultant action (2.57) is the same as that of pure tachyonic theory except for the rescaling of \(x\)-coordinate \(x \rightarrow (\sqrt{\beta/\alpha} x)\) and an overall factor \(\sqrt{\alpha}\).

(iv) When \(E = 0 \quad (\beta = 0\); see the dashed line in Fig. 1-(a)), the period of the tachyon kink stretches to infinity, \(\lim_{\beta \to 0} 2\pi \zeta = \lim_{\beta \to 0} 2\pi T_0 \sqrt{\alpha/\beta} \rightarrow \infty\). In addition, \(u\) in Eq. (2.49) diverges with finite ratio \(\zeta/u = \gamma T_0 \sqrt{\alpha}/T_3\). The solution obtained in this limit is a regular static single topological kink configuration with \(T'(\pm \infty) = 0\) (see the dashed line in Fig. 2)

\[
T(x) = T_0 \arcsinh \left( \frac{ux}{\zeta} \right). \tag{2.58}
\]

The localized piece of various quantities (2.40)-(2.44) including the energy density and the string charge density takes Lorentzian shape (see the dashed line in Fig. 3) since

\[
\gamma T'^2 = \frac{1}{\alpha \gamma^2 [T_3 V(T)]^2} = \pi T_0 T_3 \frac{\zeta \pi u}{\sqrt{\alpha} x^2 + (\zeta/u)^2}, \tag{2.59}
\]

where \(\zeta/u = (\gamma T_0 \sqrt{\alpha})/T_3\) stands for width of the topological kink. When \(\Pi_1\) goes to zero, the localized piece (2.59) approaches a \(\delta\)-function. This sharpening is shown in Fig. 4.

From the coefficients in front of the Lorentzian shape, we read the tension and the string charge

\[
T_2 = \pi T_0 T_3 \frac{1 + B_1^2}{\sqrt{\alpha}}, \quad Q_{F1} = \pi T_0 T_3 \frac{E_2}{\sqrt{\alpha}}. \tag{2.60}
\]

An intriguing point is that the action (2.1) is rewritten in a localized form

\[
S = -T_p \int dt \, dx \, d^2x_\perp \, V(T) \sqrt{\beta + \alpha T'^2} = 0 = -(\pm) \sqrt{\alpha} T_p \int dt \, dx \, d^2x_\perp \, V(T) T' \tag{2.61}
\]

\[
= - \int dt \, d^2x_\perp \sqrt{\alpha} T_p \int_{-\infty}^{\infty} dT \, V(T), \tag{2.62}
\]

where \(+(-)\) in the second line (2.61) corresponds to the kink (the antikink). The exact integral formula for the tachyon field in the third line (2.62) coincides with that of the tension

\[
T_{p-1} = T_p \int_{-\infty}^{\infty} dT \, V(T), \tag{2.63}
\]

which can be obtained only for the singular limit of the kink in the array with or without electromagnetic fields [32].
Figure 4: Profiles of string charge density $\Pi_2(x)$ for various $\Pi_1$’s. Dashed line with $\Pi_1 = 0.3$, dotted line with $\Pi_1 = 0.9$, and solid line with $\Pi_1 = 1.6$.

(v) When $\mathcal{E} > 0$ ($\beta < 0$; see the dotted line in Fig. 1(a)), the solution is given by (see the dotted line in Fig. 2)

$$T(x) = T_0 \text{arcsinh} \left[ \sqrt{1 + \bar{u}^2} \sinh \left( \frac{x}{\bar{\zeta}} \right) \right],$$

where

$$\bar{\zeta} = T_0 \sqrt{-\frac{\alpha}{\beta}}, \quad \bar{u}^2 = -\frac{T_3^2}{\beta \gamma^2}.\quad (2.65)$$

The obtained configuration is also a topological kink with a finite asymptotic slope $T'(\pm \infty) = \pm \sqrt{-(\beta/\alpha)} \neq 0$. Therefore, the localized piece is represented by $V(T)^2$ as

$$\frac{1}{\alpha \gamma} [T_3 V(T)]^2 = \frac{T_3^2}{\alpha \gamma} \frac{1}{1 + (1 + \bar{u}^2) \sinh^2(x/\bar{\zeta})},$$

and then all the quantities in Eqs. (2.40) - (2.44) also have such localized piece in addition to a relatively large constant piece. As an example, the string charge density $\Pi_2$ is plotted by the dotted line in Fig. 3. Note that the pressure along z-direction flips its sign when $E^2_2 > 1$. Similar to the previous case, the tension and the string charge density are given by

$$T_2 = \frac{(1 + B^2_2) T_3^2}{\alpha \gamma} \int_{-\infty}^{\infty} dx \ V^2(T(x))$$

$$= \frac{2 T_0 T_3 (1 + B^2_2)}{\sqrt{\alpha}} \arctan (\bar{u}).$$

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and

\[
Q_{F1} = \frac{E_2 T_3^2}{\gamma\alpha} \int_{-\infty}^{\infty} dx \, V^2(T(x))
\]

\[
= \frac{2 T_0 T_3 E_2}{\sqrt{\alpha}} \arctan(\bar{u}),
\]

(2.68)

which are less than the quantities in Eq. (2.52), Eq. (2.54), and Eq. (2.60). In the limit of divergent \( \bar{u} \) with a fixed \( \alpha \), the previous values are reproduced. When \( \sqrt{-\beta} \) diverges with finite \( \bar{u} \) and \( \alpha \), the tachyon kink becomes sharply peaked.

When \( \alpha \) is negative \( (E_2^2 > 1 + B_1^2) \), the potential \( U(T) \) is flipped as shown in Fig. 1-(b) and then character of regular static solutions changes drastically. For fixed \( \alpha \) and \( \gamma \), the system of our interest is again specified by the value of \( \beta \) in Eq. (2.35). When \( \beta \) is positive, the action of our system (2.1) is rewritten as

\[
S = -T_3 \int dt \, dx \, d^2 x_\perp \, V(T) \sqrt{\beta - \left(-\alpha\right) \left(\frac{dT}{dx}\right)^2}
\]

\[
= -\sqrt{-\alpha} T_3 \int dt \, d\left(\frac{T_0 x}{\zeta}\right) \, d^2 x_\perp \, V(T) \sqrt{1 - \left[\frac{dT}{d(T_0 x/\zeta)}\right]^2}.
\]

(2.69)

But this action (2.69) is exactly the same as that of rolling tachyon which is given by

\[
S = -T_3 \int dt \, dx \, d^2 x_\perp \, V(T) \sqrt{\beta - (1 + B^2) \left(\frac{dT}{dt}\right)^2}
\]

\[
= -\sqrt{(1 + B^2)} T_3 \int dx \, d\left(\frac{T_0 t}{\zeta_B}\right) \, d^2 x_\perp \, V(T) \sqrt{1 - \left[\frac{dT}{d(T_0 t/\zeta_B)}\right]^2}
\]

(2.70)

where \( \zeta_B = \sqrt{(1 + B^2)/\beta} \). Thus, there exists a one-to-one correspondence between a regular configuration with spatial \( x \)-dependence and the time evolution of a homogeneous rolling tachyon solution. With this identification, the pressure \( -T_{11} \) of this system plays the same role as the Hamiltonian density \( \mathcal{H} \) in the rolling tachyon system. Since we will find static configurations in a closed form in what follows, it means that we obtain the most general rolling tachyon solutions in an arbitrary flat unstable \( D_p \)-brane [2, 3, 4, 8, 24, 25, 26, 11].

(i) When \( \mathcal{E} \to 0^+ \) \((\beta \to 0^+); \) see the dotted-dashed line in Fig. 1-(b)), constant vacuum solutions, \( T(x) = \pm \infty \), are the only possible configurations (see the dotted-dashed line in Figs. 5 and 6).

(ii) When \( 0 < \mathcal{E} < U(0) \) \((0 < \beta < T_2^2/\gamma^2); \) see the solid line in Fig. 1-(b)), there is a turning point \( T_{\min} (-T_{\min}) \) such that

\[
T(x) \geq T_{\min} = T_0 \text{arccosh}(u), \quad (T(x) \leq -T_{\min}),
\]

(2.71)
Figure 5: Profiles of tachyon kink and bounce $T(x)$ when $\alpha < 0$.

where $u$ is given in Eq. (2.49). The corresponding configuration is a bounce which is convex up (convex down) as shown by the two solid curves in Fig. 5

\[
T(x) = T_0 \arcsinh \left[ \sqrt{u^2 - 1} \cosh \left( \frac{x}{\zeta} \right) \right],
\]

(2.72)

where $\zeta$ is given in Eq. (2.48). Its asymptotic slopes are

\[
T'(\pm \infty) = \pm \frac{T_0}{\zeta},
\]

(2.73)

which are shown by the two solid straight lines in Fig. 5. Since $\alpha$ is negative, the localized pieces of Eqs. (2.40)–(2.44) change the sign,

\[
\frac{1}{\alpha \gamma} [T_0 V(T)]^2 = -\frac{T_3^2}{1 + (u^2 - 1) \cosh^2(x/\zeta)},
\]

(2.74)

which implies negative contribution of the localized energy density $\rho$ and the localized string charge density $\Pi_2$ to the constant background quantities as shown by the solid line in Fig. 5.

(iii) When $\mathcal{E} = U(0)$ ($\beta = T_3^2/\gamma^2$; see the dashed line in Fig. 11(b)), we have the trivial ontop solution $T(x) = 0$. In addition, we find nontrivial tachyon half-kink solutions connecting the unstable symmetric vacuum $T(-\infty) = 0$ and one of two stable broken vacua $T(\infty) = \pm \infty$ (see the dashed curve in Fig. 5),

\[
T(x) = \pm T_0 \arcsinh \left[ \exp \left( \frac{x}{\zeta} \right) \right].
\]

(2.75)
Since the half-kink connects smoothly two vacua with different vacuum energy as $V(T = 0) > V(T = \pm \infty)$, the localized piece of Eqs. (2.40)–(2.44) is monotonically increasing or decreasing as shown by the dashed line in Fig. 6.

\[
\frac{1}{\alpha \gamma} |T_3 V(T)|^2 = -\frac{T_3^2}{-\alpha \gamma} \frac{1}{1 + \exp \left( \frac{2x}{\bar{\zeta}} \right)}. \tag{2.76}
\]

In the limit of infinite $\Pi_1$ with finite $-\alpha$, \( \frac{1}{\alpha \gamma} |T_3 V(T)|^2 \) becomes a step function with infinite gap.

(iv) When $\mathcal{E} > U(0) \ (\beta > T_3^2/\gamma^2$; see the dotted line in Fig. 1(b)), we have

\[
T(x) = T_0 \arcsinh \left[ \sqrt{1 - u^2} \sinh \left( \frac{x}{\bar{\zeta}} \right) \right]. \tag{2.77}
\]

Configuration is monotonically increasing (or decreasing) from $T(-\infty) = \mp \infty$ to $T(\infty) = \pm \infty$ (see the dotted curve in Fig. 3) so that this solution can be regarded as hybrid of two half-kink solutions joined at the origin. Opposite to the similar kink solutions for positive $\alpha$, slope of the solutions has minimum value at the origin and maximum value at infinity. Thus the localized piece of Eqs. (2.40)–(2.44) has minimum at the origin due to the flip of its signature

\[
\frac{1}{\alpha \gamma} |T_3 V(T)|^2 = -\frac{T_3^2}{-\alpha \gamma} \frac{1}{1 + (1 - u^2) \sinh^2 \left( \frac{x}{\bar{\zeta}} \right)}. \tag{2.78}
\]

The string charge density $\Pi_2$ is plotted by the dotted line in Fig. 6.
Finally we consider the case $\alpha = 0$. If we multiply $\alpha$ to Eq. (2.34) and take the limit of $\alpha \to 0$, then $T'^2$ term disappears. The original tachyon equation (2.18) reduces to $dV/dT = 0$ so that we only have homogeneous vacuum solutions, $T(x) = 0$ or $T(x) = \pm \infty$.

3 Tachyon Kinks on Unstable D$p$-brane

The analysis in the previous section can be applied to general unstable D$p$-branes without much difficulty. In this section we will show that, even for general D$p$-branes, the equations of motion reduce to a single first-order equation of the form in Eq. (2.34) for static case with inhomogeneity in the $x$-direction. Thus what we have obtained in the previous section and also in Ref. [33] are actually the most general type of regular static kink solutions of codimension one for D$p$-branes.

Assuming only the $x(= x^1)$-dependences in the fields $T$ and $A_\mu$, the determinant $X$ in the action (1.1) can be written as

\[
-X = -\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu}) = \beta_p + T'^2 \alpha_p, \tag{3.1}
\]

where

\[
\beta_p = -\det(\eta_{\mu\nu} + F_{\mu\nu}), \tag{3.2}
\]

and $\alpha_p = C^{11}$ is the 11-component of the cofactor of $X_{\mu\nu}$. The equations of motion are

\[
\partial_1 \left( T_p \frac{V}{\sqrt{-X}} C^{11} \right) = -\sqrt{-X} T_p \frac{dV}{dT},
\]

\[
\partial_1 \Pi_1 = 0,
\]

\[
\partial_1 \left( T_p \frac{V}{\sqrt{-X}} C^{1i} \right) = 0, \quad (i = 2, 3, \ldots, p-1), \tag{3.3}
\]

where

\[
\Pi_1 = T_p \frac{V}{\sqrt{-X}} C^{01}_A. \tag{3.4}
\]

The expression of the energy-momentum tensor is again given by the symmetric part of the cofactor as in the D3-brane, namely,

\[
T^{\mu\nu} = T_p \frac{V}{\sqrt{-X}} C^{\mu\nu}_S. \tag{3.5}
\]

Then the conservation equation, $\partial_\mu T^{\mu\nu} = 0$, becomes

\[
\partial_1 T^{1\mu} = \partial_1 \left( T_p \frac{V}{\sqrt{-X}} C^{1\mu} \right) = 0. \tag{3.6}
\]
Addition of Eq. (3.3) and Eq. (3.6) leads to

$$\partial_1 \left( T_p \frac{V}{\sqrt{-X}} C^{\mu 1} \right) = 0.$$  \hspace{1cm} (3.7)

As in the case of unstable D3-brane, the Bianchi identity \( \partial_{(\mu} F_{\nu\lambda)} = 0 \) imposes \((p - 1)(p - 2)/2\) constraints on the \( p(p - 1)/2 \) components of field strength tensor with our ansatz: \( E_k = \text{constant} \) and \( F_{kl} = \text{constant} \), where \( k \neq 1 \) and \( l \neq 1 \). Now we argue that, in fact, all the field strength components are constant including \( E_1 \) and \( F_{1\mu} \). Since the cofactor \( C^{11} \) in Eq. (3.1) does not contain \( F_{\mu\nu} \) with \( \mu = 1 \) or \( \nu = 1 \), Eq. (3.7) implies that

$$\gamma_p \equiv T_p \frac{V}{\sqrt{-X}} = \text{constant},$$  \hspace{1cm} (3.8)

as in Eq. (2.33). Now the only remaining nontrivial equations are, from Eq. (3.7),

$$\partial_1 C^{01} = 0,$$
$$\partial_1 C^{k1} = 0 \quad (k = 2, 3, \ldots, p - 1).$$  \hspace{1cm} (3.9)

However, note that these equations are actually homogeneous coupled linear equations of \( \partial_1 F_{1\mu} \) since each \( X_{1\nu} \) (\( \nu \neq 1 \)) appears precisely once in every term of \( C^{\mu 1} \). Therefore, as long as the determinant made of the coefficients of \( \partial_1 F_{1\mu} \) does not vanish, we have \( F_{\mu\nu} = \text{constant} \) for all \( \mu, \nu \). (When the determinant does vanish, we can treat the case in a similar fashion as done in section 2.)

Combining Eqs. (3.11) and (3.8), we again obtain the single first-order equation (2.34) with

$$\mathcal{E} = -\frac{\beta_p}{2\alpha_p},$$
$$U(T) = -\frac{1}{2\alpha_p \gamma_p^2} [T_p V(T)]^2.$$  \hspace{1cm} (3.10)

Then, the rest of the analysis is the same as in D3 case, e.g., the solution space for regular static kinks of codimension one is classified by three constants \( \alpha_p, \beta_p, \) and \( \gamma_p \) and so on.

When \( p = 1 \), the field strength tensor has only one component of electric field and the solutions involve only those of \( \alpha_p > 0 \) case.

4 Conclusion

In this paper we have analyzed regular static solutions of codimension-one extended objects in the effective theory of a real tachyon, described by Born-Infeld type action with a runaway tachyon potential coupled to an Abelian gauge field. On arbitrary flat unstable
Dp-brane, the types of codimension-one extended objects are the same irrespective of \( p \), \((p \geq 2)\). The static regular kink-type solutions on unstable Dp-brane are shown to be classified by three parameters: \( \beta_p = -\text{det}(\eta_{\mu\nu} + F_{\mu\nu}) \), \( \alpha_p = C_{\mu\nu}^{11} \) (11-component of the cofactor of \( V_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu} \)) and \( \gamma_p = T_p V/\sqrt{-X} \). Detailed analysis has been carried out for D3 case. Species of the obtained solutions are summarized in Table 1 for various \( \alpha_p \) and \( \beta_p \) with fixed nonzero \( \gamma_p \).

| \( \beta_p \) | \( \alpha_p > 0 \) | \( \alpha_p < 0 \) |
|--------------|----------------|----------------|
| \( \beta_p < 0 \) | topological kink with \( T'(\pm \infty) \neq 0 \) | constant vacuum, \( T = \pm \infty \) |
| \( \beta_p = 0 \) | topological kink | bounce |
| \( 0 < \beta_p < 1/\gamma_p \) | array of kink-antikink | constant ontop, \( T = 0 \) |
| \( \beta_p = \frac{T_3^2}{\gamma_p^2} \) | constant ontop, \( T = 0 \) & half-kink | hybrid of two half-kinks |
| \( \beta_p > \frac{T_3^2}{\gamma_p^2} \) | | |

Table 1: List of regular static configurations.

For the single unit object listed in the left column of Table 1 (\( \alpha_p > 0 \)), the tension of lower dimensional branes is correctly reproduced. When the electric field along the kink direction is nonzero, the fundamental string charge per unit \((p - 1)\)-dimensional transverse volume has a confined piece. This suggests that it may be interpreted as D\((p - 1)\)- or D\((p - 1)\)F1-brane on the unstable Dp-brane. If \( \alpha_p < 0 \), due to the correspondence between the static case and time-dependent rolling tachyon case, the solutions found here may also be interpreted as the most general homogeneous rolling tachyon solutions of an arbitrary flat Dp-brane.

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