QCD Calculation of $B$ Meson Form Factors and Exclusive Nonleptonic Decays

A. Khodjamirian$^{a,b,1}$ and R. Rückl$^{a,c,2}$

$^a$ Sektion Physik der Universität München, D-80333 München, Germany
$^b$ Yerevan Physics Institute, 375036 Yerevan, Armenia
$^c$ Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-80805 München, Germany

Abstract

The method of QCD sum rules has proven to be particularly useful in heavy quark physics, where a small distance scale is provided by the inverse heavy quark mass. We present two new examples for the application of this method to B-physics: a calculation of the heavy-to-light form factors $B \to \pi, K$ and an estimate of the weak decay amplitude for $B \to J/\psi K$ beyond the factorization approximation.

$^1$ Alexander von Humboldt Fellow
$^2$ supported in part by the German Federal Ministry for Research and Technology (BMFT) under contract No. 05 6MU93P

* presented by A. Khodjamirian at the Workshop "Continuous Advances in QCD", TPI, University of Minnesota, Minneapolis, February 18-20, 1994
1 Introduction

Introduced fifteen years ago, the method of QCD sum rules [1] has become one of the most useful and reliable calculational schemes to derive hadronic properties from first principles with a minimal number of additional phenomenological assumptions. Originally, focusing on the operator product expansion (OPE) of two-point correlation functions one aimed at the determination of masses and coupling constants of ground states. The framework has then been extended and also applied to other dynamical characteristics of hadrons such as form factors and decay amplitudes. These extensions are based on OPE of three- and four-point correlation functions. Nevertheless, no new input parameters appear which is one of the most attractive features of this approach. All essential parameters such as vacuum condensates, coupling constants and thresholds of higher states are fixed by corresponding two-point sum rules.

On the other hand, straightforward application of QCD sum rules in their original version to many-point correlation functions runs into a number of technical problems. First of all, the correlation functions are to be calculated in the deep Euclidean region. The result is then continued to physical momenta with the help of dispersion relations. In the presence of many channels, this continuation is not at all trivial. In some cases like for on mass-shell hadronic vertices and amplitudes one must rely on extrapolations which are plausible but not rigorous. Furthermore, the OPE for n-point correlation functions is usually represented by Feynman-type diagrams which are used to calculate the Wilson coefficients in perturbation theory. These diagrams grow rapidly in number with n and become rather complicated. Finally, in order to include higher resonances and continua of states into the sum rules, one often has to resort to models based on the concept of quark-hadron duality. These models usually work well for two-point correlation functions, but may be too crude already in the case of double dispersion integrals.

For these and other reasons, it is very important to develop alternative, more economical methods which avoid these problems. Clearly, starting from vacuum correlation functions, applying OPE and deriving sum rules in terms of local vacuum condensates is only one way to account for nonperturbative quark-gluon dynamics. Other approaches leading to different versions of QCD sum rules include the external field method [2]-[4] and induced vacuum condensates, the concept of nonlocal condensates [5], and the combination with light-cone wave functions [6]-[10]. Each of these versions has advantages and disadvantages. The comparison of the results may shed some light on the accuracy and reliability of the different approaches.

In this report we want to illustrate some of the current trends in the development of the QCD sum rule method by presenting two recent calculations relevant for B-physics. In section 2 we discuss the calculation of the $B \to \pi$ and $B \to K$ form factors using an expansion in terms of pion and kaon wave functions on the light-cone with increasing twist, and compare the results with the predictions of conventional QCD sum rules. Then, in section 3, we study the problem how to calculate the amplitude of the prominent weak decay $B \to J/\psi K$ beyond the usual factorization approximation. In other words, we undertake the difficult task to estimate the nonfactorizable contributions to the decay amplitude originating from quark-gluon interactions at scales between the heavy b-quark mass and the hadronization scale. Following an idea suggested in ref. [11] for $D$ meson nonleptonic decays, we try to extract this amplitude from a suitable four-point correlation function treated within the framework of conventional QCD sum rules.
2 QCD sum rules on the light-cone for the $B \to \pi, K$ form factors

The standard way to derive QCD sum rules for the transition amplitude $<A|j|B>$ between given ground state hadrons $A$ and $B$ starts from the vacuum correlation function of three currents, $<0|T\{j_A(x)j(0)j_B(y)\}|0>$, where $j_A$ ($j_B$) is chosen such as to create the hadron $A$ ($B$) from the vacuum and $T$ denotes the time-ordered product. A double dispersion relation in $p_A^2$ and $p_B^2$, $p_A$ ($p_B$) being the four-momentum in the $A$ ($B$)-channel, is then used to relate this correlation function calculated in the Euclidean region with its imaginary part containing the desired transition amplitude. The remainder of the calculation is essentially the same as in the case of two-point QCD sum rules: derivation of OPE in terms of local quark-gluon condensates, subtraction of excited state and continuum contributions and Borel transformation. Since the quark-gluon condensates are universal, they can be taken from any QCD sum rule analysis. Furthermore, the coupling constants $f_A \sim <0|j_A|A>$ and $f_B \sim <0|j_B|B>$ as well as the thresholds of higher states in the $A$ and $B$ channels are fixed by the two-point sum rules for the correlation functions $<0|j_Aj_A|0>$ and $<0|j_Bj_B|0>$, respectively. Hence, as already pointed out, one has essentially no new parameters. This procedure has been applied to a variety of problems such as the pion form factor [12, 13], radiative charmonium transition rates [14]-[16], and semileptonic form factors of heavy mesons [17]-[22].

Here we demonstrate an alternative method which may be used in cases where one of the hadrons, $A$ or $B$, is a light meson. An important example is provided by the $B \to \pi$ form factors $f_\pi^\pm$ which determine the matrix element

$$<\pi|\bar{u}\gamma_\mu b|B> = 2f_\pi^+(p^2)q_\mu + [f_\pi^+(p^2) + f_\pi^-(p^2)]p_\mu$$

(1)

with $p + q$, $q$ and $p$ being the $B$ and $\pi$ momenta and the momentum transfer, respectively. In the following, we concentrate on the phenomenologically more interesting form factor $f_\pi^+$. The corresponding $B \to K$ form factor $f_K^+$ can be treated in parallel by obvious formal replacements. Numerical results will be shown for both $f_\pi^+$ and $f_K^+$.

Instead of investigating the vacuum averaged correlation of the $b \to u$ transition current with two other currents carrying the $B$ and $\pi$ quantum numbers, we consider the matrix element

$$F_\mu(p,q) = i \int d^4x e^{ipx} <\pi(q)|T\{\bar{u}(x)\gamma_\mu b(x),\bar{b}(0)i\gamma_5 d(0)\}|0>$$

(2)

$$= F((p + q)^2,p^2)q_\mu + \bar{F}((p + q)^2,p^2)p_\mu$$

between the vacuum and an on-shell pion state. This object is represented diagrammatically in Fig. 1. The pion momentum squared, $q^2 = m_\pi^2$, vanishes in the chiral limit adopted throughout this discussion. Moreover, the light $u$ and $d$ quarks forming the pion eventually propagate to large distances. In contrast, the $b$-quark still propagates far off-shell provided that $(p + q)^2$ is taken sufficiently large and negative, and the time-like momentum transfer squared $p^2$ is far from the kinematical limit, $p^2 = m_B^2$.

Formally, contracting the $b$-quark fields in (2) and keeping only the lowest order term, i.e. the free $b$-quark propagator, yields

$$F((p + q)^2,p^2) = i \int d^4x \int \frac{d^4P}{(2\pi)^4} e^{i(p-P)x} \sum_a \frac{\phi_a(x^2,q \cdot x)}{P^2 - m_b^2},$$

(3)
where
\[ \phi_a(x^2, q \cdot x) = \langle \pi(q) | \bar{u}(x) \Gamma_a d(0) | 0 \rangle, \]  
(4)
\( \Gamma_a \) denoting certain combinations of Dirac matrices. This approximation corresponds to Fig. 1a. The high virtuality of the \( b \)-quark propagating between the points \( x \) and 0 guarantees that the \( u \) and \( d \) quarks are emitted at almost light-like distances. In that case, it is justified to keep only the first few terms in the expansion of the matrix elements (4) around \( x^2 = 0 \), that is near the light-cone:
\[ \phi_a(x^2, q \cdot x) = \sum_n \phi^n_a(q \cdot x)(x^2)^n. \]  
(5)
Logarithms in \( x^2 \) which may also appear in (5) are disregarded for simplicity. These terms can be consistently treated by means of QCD perturbation theory. They give rise to normalization scale dependence. Because of translation invariance, the coefficients \( \phi^n_a \) must have the form
\[ \phi^n_a(q \cdot x) \sim \int_0^1 du \varphi^n_a(u) \exp(iuq \cdot x). \]  
(6)
Inserting (5) and (6) into (3) and integrating over \( x \) and \( P \), one obtains, schematically,
\[ F((p + q)^2, p^2) \sim \sum_a \sum_n \int_0^1 du \frac{\varphi^n_a(u)}{[m_b^2 - (p + qu)^2]^{2n}}. \]  
(7)
It is thus possible to calculate the invariant function \( F \) with reasonable accuracy in the kinematical region of highly virtual \( b \)-quarks provided one knows the distribution functions \( \varphi^n_a(u) \) at least for low values of \( n \). These distributions contain essential information about the dynamics at large distances and play a similar role as the vacuum condensates in the conventional OPE of vacuum correlation functions.

As it turns out, the distributions \( \varphi^n_a(u) \) are nothing but light-cone wave functions of the pion the twist of which is growing with \( n \). These wave functions were introduced long ago in the context of hard exclusive processes \[23\]-\[26\]. The leading twist 2 wave function is defined by
\[ \langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 P \exp\{i \int_0^1 d\alpha x_\mu A^\mu(\alpha x)\} d(0) | 0 \rangle = -if_\pi q_\mu \int_0^1 du e^{iuq \cdot x} \varphi_\pi(u), \]  
(8)
where the exponential factor involving the gluon field is necessary for gauge invariance. The asymptotic form of \( \phi_\pi(u) \) can be obtained from perturbative considerations and is well known:
\[ \varphi_\pi(u) = 6u(1 - u). \]  
(9)
Over the years a great deal has been learned about these wave functions \[27\]-\[32\]. All important twist two, three and four wave functions have been identified and their asymptotic form has been determined. Besides two-particle wave functions reflecting the quark-antiquark structure of mesons, also three-particle wave functions associated with the quark-antiquark-gluon component have been studied. Very important for practical applications, nonasymptotic corrections have been estimated considering expansions in orthogonal polynomials and renormalizing the coefficients of this expansions in QCD perturbation theory. Furthermore, conventional two-point QCD sum rules have been used \[28\] to fix the first few moments of the low twist wave functions. Based on these estimates and improvements, various models...
for the nonasymptotic form of these functions have been suggested (see, for example, refs. [10,28,32]).

The approach outlined above together with the accumulated knowledge of pion and kaon wave functions has been employed in ref. [6] to calculate the $B \to \pi$ form factor at zero momentum transfer, in ref. [7] to get the $D \to \pi$ form factor, and in ref. [8] to derive the $B \to \pi$ and $B \to K$ form factors including the momentum dependence.

In our calculation of $f^+_\pi$ and $f^+_K$ we have included quark-antiquark wave functions up to twist four. In addition, we have also estimated the first-order correction to the free $b$-quark propagation shown in Fig. 1b which involves quark-antiquark-gluon wave functions. On the other hand, the perturbative $O(\alpha_s)$ corrections indicated in Figs. 1c and 1d have not been evaluated directly, but have been taken into account indirectly as explained below. The direct calculation of these corrections is on its way.

In order to extract the desired form factor from the resulting invariant function $F((p+q)^2,p^2)$ sketched in (4) we employ a QCD sum rule with respect to the $B$-meson channel. Writing a dispersion relation in $(p+q)^2$, we approximate the hadronic spectral function in the $B$-channel by the pole contribution of the $B$ meson and a continuum contribution. In accordance with quark-hadron duality, the latter is identified with the spectral function derived from the QCD representation (4) above the threshold $(p+q)^2 = s_0$. Formally, subtraction of the continuum then amounts to simply changing the lower integration boundary in (7) from 0 to $\Delta = (m^2_b - p^2)/(s_0 - p^2)$. After Borel transformation one arrives at a sum rule for the product $f^+_\pi f_B$ where $f_B$ is the $B$ meson decay constant. It is important to note that the numerical values to be substituted for $m_b$, $f_B$ and the threshold $s_0$ are interrelated by the QCD sum rule for the correlation function $< 0 | T\{\bar{\psi}(x)\gamma_5\psi(x), \bar{u}(0)\gamma_5\bar{b}(0)\} | 0 >$. We stress that the latter sum rule should be used without $O(\alpha_s)$ corrections in order to be consistent with the neglect of these corrections in the sum rule for $f^+_\pi f_B$ (see also ref. [20]).

The final expression of the form factor $f^+_\pi$ is given by

$$f^+_\pi(p^2) = \frac{f^+_\pi m^2_b}{2f_B m^2_B} \int^1_0 du \exp\left[ -\frac{m^2_B}{M^2} - \frac{m^2_b - p^2(1-u)}{uM^2} \right]$$

$$\left[ \varphi_\pi(u) + \frac{\mu}{m_b} u \varphi_p(u) + \frac{\mu}{6m_b} \varphi_\sigma(u)(2 + \frac{m^2_b + p^2}{uM^2}) \right],$$

(10)

where $M^2$ is the Borel parameter and $\mu = m^2_s/(m_u + m_d)$. While in the concrete calculation of ref. [8] the twist 2 wave function $\varphi_\pi$ is corrected for nonasymptotic effects [28,32], the twist 3 wave functions $\varphi_p$ and $\varphi_\sigma$ are taken in their asymptotic form. Numerically, the higher twist contributions turn out to be more important than the nonasymptotic corrections to the leading twist wave function. For brevity, in (10) we have omitted the contributions from the twist four quark-antiquark wave functions and the term involving the twist three quark-antiquark-gluon wave function. These contributions are quantitatively negligible. Further details can be found in ref. [8].

Numerical results for $f^+_\pi(p^2)$ are plotted in Fig. 2a for two values of the Borel parameter $M$ characterizing the fiducial range of the method. As can be seen, the predictions are quite stable under variation of $M$ as long as $p^2$ is not getting too close to $m^2_B$. For comparison, we also show results obtained by other methods. Furthermore, it is interesting to note that $f^+_\pi(p^2)$ is not very sensitive to the precise shape of the leading twist wave function $\varphi_\pi(u)$ at least at $p^2 \leq 10 GeV^2$. We have checked this by replacing the nonasymptotic two-humped wave function [28] used for Fig. 2a by the simple asymptotic wave function given in (1).
The \( B \to K \) form factor \( f_K^+ \) was calculated analogous to \( f_\pi^+ \) using the leading twist wave function \( \varphi_K \) from ref. [28]. The higher twist wave functions are left unchanged. The numerical result is presented in Fig. 2b. Although \( f_K^+ \) is not accessible directly in semileptonic \( B \) decays, it determines the factorizable part of the nonleptonic decay amplitude for \( B \to J/\psi K \) and is therefore phenomenologically very important. Later, in section 3 we shall make use of the value

\[
f_K^+(m_{\psi}^2) = 0.55 \pm 0.05 .
\]  

(11)

In conclusion, we emphasize that light-cone sum rules such as the ones exemplified in this section represent a well defined alternative to the conventional QCD sum rule method. In this variant, the nonperturbative aspects are described by a set of wave functions on the light-cone with varying twist and quark-gluon multiplicity. These universal functions can be studied in a variety of processes involving the \( \pi \) and \( K \) meson, or other light mesons. The most important advantage of the light-cone sum rules is the possibility to take hadrons on mass-shell from the very beginning. One can thus avoid the notorious model-dependence of extrapolations from Euclidean to physical momenta in light channels. Furthermore, in many cases the light-cone approach is technically much easier than the conventional QCD sum rule technique. Finally, the light-cone method is rather versatile. It can also profitably be employed to calculate heavy-to-light form factors such as \( B \to \rho \) and \( B \to K^* \), amplitudes of rare decays such as \( B \to K^*\gamma \) [9], and hadronic couplings such as \( B^*B\pi \) [33]. Just in passing we mention that the \( B^*B\pi \) coupling can be extracted from the correlation function (2) by performing a second Borel transformation in \( p^2 \). Needless to say, light-cone sum rule are equally useful in calculating properties of \( D \) mesons and also of light hadrons [10], [32].

The main problem to be solved if one wants to fully exploit the light-cone approach is a reliable determination of the nonasymptotic effects in the wave functions. In this respect, measurements of hadronic form factors, couplings etc. can provide important information. A second, mainly technical problem, concerns higher order perturbative corrections, such as those shown in Figs. 1c and 1d, and the possible occurrence of large logarithms.

3 Weak decay amplitude for \( B \to J/\psi K \) beyond factorization

Since the earliest estimates [34]–[36], nonleptonic two-body decays of heavy mesons are usually calculated by splitting the appropriate matrix element of the weak Hamiltonian into a product of a semileptonic transition form factor and a decay constant of one of the final mesons. On the theoretical side, one still lacks a strict proof of this recipe. Empirically, with the accumulating data on \( D \) decays, it soon became clear that naive factorization fails [37]. In order to achieve agreement with experiment it is necessary to let the coefficients of the OPE of the weak Hamiltonian deviate from their values predicted in short-distance QCD. Phenomenologically [38], the coefficients are reinterpreted and treated as free parameters to be determined from experiment. For \( D \) decays, it has been shown [39] that the fitted values can be reconciled with the short-distance expectations if the matrix elements of the local operators appearing in the OPE are expanded in \( 1/N_c \) and if only the leading terms are kept. This observation gave rise to the rule of discarding nonleading in \( 1/N_c \) terms. Factorization combined with this rule leads to a satisfactory description of the majority of two-body \( D \)-decays.

For \( B \) decays, an instructive example is provided by the illustrious decay mode \( B \to J/\psi K \).
The effective weak Hamiltonian responsible for this decay is given by

\[ H_W = \frac{G}{\sqrt{2}} V_{cb} V_{cs}^* \{ c_1 O_1 + c_2 O_2 \}, \]  

(12)

\[ O_1 = (\bar{s} \Gamma^\rho c)(\bar{c} \Gamma_\rho b), \quad O_2 = (\bar{c} \Gamma^\rho c)(\bar{s} \Gamma_\rho b) \]  

(13)

where \( G \) is the Fermi constant, \( V_{cb} \) and \( V_{cs} \) are the relevant CKM matrix elements and \( \Gamma_\rho = \gamma_\rho (1 - \gamma_5) \). The coefficients \( c_1(\mu) \) and \( c_2(\mu) \) incorporate the short-distance effects arising from the renormalization of \( H_W \) from \( \mu = m_W \) to \( \mu = O(m_b) \). These coefficients are known to next-to-leading order (NLO) [40], [41]. While \( O_2 \) already possesses the appropriate flavour structure, this is the case for \( O_1 \) only after Fierz transformation:

\[ O_1 = \frac{1}{3} O_2 + 2 \tilde{O}_2, \]  

(14)

\[ \tilde{O}_2 = (\bar{c} \Gamma^\rho c)(\bar{s} \Gamma_\rho b). \]  

(15)

Assuming factorization, the matrix element of \( H_W \) for \( B \to J/\psi K \) simplifies to

\[ <J/\psi K | (c_2 + \frac{c_1}{3}) O_2 + 2 c_1 \tilde{O}_2 | B> \]  

(16)

Note that in this approximation

\[ <J/\psi K | \tilde{O}_2 | B> = <J/\psi | \bar{c} \Gamma^\rho c | 0> <K | \bar{s} \Gamma_\rho b | B> = 0 \]  

(17)

because of color conservation. Moreover, using a parametrization of \( <K | \bar{s} \gamma_\rho b | B> \) analogous to (1) and \( <J/\psi | \bar{c} \gamma_\rho c | 0> = m_\psi f_\psi \epsilon_\rho \) one readily obtains the following expression for the decay amplitude:

\[ A(B \to J/\psi K) = \sqrt{2} G V_{cb} V_{cs}^* (c_2 + \frac{c_1}{3}) f_\psi f_K^+(p^2 = m_\psi^2) m_\psi \epsilon \cdot q. \]  

(18)

The corresponding branching ratio is given by

\[ BR(B \to J/\psi K) = \frac{G^2}{32\pi} |V_{cb} V_{cs}^*|^2 (c_2 + \frac{c_1}{3}) f_\psi f_K^+(p^2 = m_\psi^2) \]  

\[ \times m_B^3 (1 - \frac{m_\psi^2}{m_B^2})^3 \tau_B \]  

(19)

where \( \tau_B \) is the \( B \) meson lifetime.

Taking for the \( B \to K \) form factor the value [11] from ref. [8] which agrees with other calculations, and using \( f_\psi = 409 \text{ MeV} \), \( V_{cb} = 0.04 \), and \( \tau_B = (1.489 \pm 0.038) \text{ ps} \) we obtain

\[ 0.005\% \leq BR(B \to J/\psi K) \leq 0.013\%. \]  

(20)

The range of this prediction mainly reflects the theoretical uncertainties in the coefficients \( c_1 \) and \( c_2 \) at \( \mu = m_b \), given in ref. [42]:

\[ \{ c_1 = 1.115, \ c_2 = -0.255 \} \div \{ c_1 = 1.146, \ c_2 = -0.312 \}. \]  

(21)
As a matter of fact, the combination $c_2 + c_1/3$ turns out to be particularly sensitive to the precise values of $c_1$ and $c_2$, as pointed out in ref. [43]. Despite of this uncertainty, the branching ratio expected from naive factorization is undoubtedly much lower than the recent experimental results [44]:

$$BR(B^- \to J/\psi K^-) = (0.11 \pm 0.015 \pm 0.009)\%$$
$$BR(B^0 \to J/\psi K^0) = (0.075 \pm 0.024 \pm 0.008)\% .$$

On the other hand, applying the rule of discarding the nonleading in $1/N_c$ terms, that is dropping the term $c_1/3$ in (19), yields

$$0.065\% \leq BR(B \to J/\psi K) \leq 0.095\%$$

in good agreement with (22).

Yet, from this observation one cannot conclude that the $1/N_c$-rule generally works in $B$ decays. The CLEO analysis [44], for example, shows that factorization is consistent with the measurements provided the short distance coefficients $c_1 + c_2/3$ and $c_2 + c_1/3$ are replaced by two a priori unknown parameters $a_1$ and $a_2$, respectively. Similarly as in $D$ decays, these coefficients are found to be universal, at least on the basis of the two-body $B$ decays analysed so far. However, unlike in $D$ decays the data favour a value $a_2 > 0$ which is inconsistent with the $1/N_c$-rule implying $a_2 \simeq c_2 < 0$. (see also the discussion in ref. [45]).

The present situation is very unsatisfactory and rather confusing. Theoretically, there seems to be no trustworthy reason why factorization should work. Moreover, on dynamical grounds [48] the universality of $a_1$ and $a_2$ is rather surprising than expected. Also, the empirical facts do certainly not prove factorization. Just the contrary may be the case. For example, in the matrix element (16) the factorizable term proportional to $c_1/3$ may cancel with sizeable nonfactorizable contributions neglected in eqs. (16) to (20). Effectively, this would lead to the result (23) and explain the success of the $1/N_c$ rule. Such a cancellation was first advocated in ref. [38] and then shown in ref. [11] to actually take place in two-body $D$-decays by estimating the nonfactorizable contributions to the decay amplitudes using QCD sum rule techniques (for a more recent discussion see refs. [46,47]). This cancellation may be different in $B$ decays. It may occur in certain channels and not in others. The nonfactorizable contribution may also happen to overcompensate the factorizable amplitudes nonleading in $1/N_c$, and thus change even the sign of the effective coefficient $a_2$. Clearly, one has to explain the coefficients $a_1$ and $a_2$ before one can claim some theoretical understanding.

Recently, we have started an attempt in this direction [48]. We are investigating the problem of factorization in $B$ decays using $B \to J/\psi K$ as a study case. Here we briefly explain our approach and present some preliminary results. Following the general idea suggested in ref. [11], we consider the four-point correlation function:

$$\Pi_{\mu\nu}(q,p) = \int d^4x d^4y d^4z e^{iqx+ipy} < 0 | T\{j^K_{\mu5}(x)j^\psi_\nu(y)H_W(z)j^B_5(0)\} | 0 \rangle ,$$

where $j^K_{\mu5} = \bar{u}\gamma_\mu\gamma_5s$ , $j^\psi_\nu = \bar{c}\gamma_\nu c$ and $j^B_5 = \bar{b}\gamma_5u$ are the generating currents of the mesons involved and $H_W$ is the effective weak Hamiltonian given in (12). The four-momenta assigned to the $K$, $J/\psi$ and $B$ channel are $q$, $p$ and $P = q + p$, respectively. The momentum transfer in the weak interaction point $z$ is zero. The bare diagram associated with (24) is depicted in Fig. 3a.
The correlation function $\Pi_{\mu\nu}$ is then calculated by means of the QCD operator product expansion. The appropriate Euclidean region located far enough from the physical thresholds in all three channels is $q^2 \leq -1 GeV^2$, $p^2 \leq 0$ and $P^2 \leq 0$. For the $J/\psi$ and $B$ channels, the conditions are more relaxed than for the $K$-channel, since the former are protected by the large $c$ and $b$ quark masses. The light quarks are considered massless. In the OPE for (24) we include all local operators up to dimension six. The Wilson coefficients are calculated to lowest nonvanishing order in $\alpha_s$.

In this approximation the nonfactorizable contributions to (24) only arise from the operator $\tilde{O}_2$ given in (17). This is just the operator the matrix element of which vanishes by factorization as pointed out in (17). However, nonperturbative gluon exchange between the loop and the triangle indicated in Fig. 4 breaks factorization and gives rise to a finite contribution from $\tilde{O}_2$. Fig. 4a shows one of the diagrams associated with the $d = 4$ gluon operator $G^a_{\mu\nu}G_{\mu\nu}$. Typical diagrams for the $d = 5$ quark-gluon operator $\bar{q}qG^a_{\lambda\mu\nu}\lambda\sigma_{\mu\nu}q$ and the $d = 6$ four-quark operator are depicted in Figs. 4b and 4c, respectively. Additional $d = 6$ contributions corresponding to the diagram Fig. 4b emerge from the operators $\bar{q}\nabla_\sigma qG^a_{\tau\lambda}q$ and $\bar{q}\mathcal{D}_\sigma G^a_{\tau\lambda}q$, where $\nabla_\sigma$ and $\mathcal{D}_\sigma$ denote the proper covariant derivatives.

Of course, there are also perturbative gluon corrections to the bare diagram, Fig. 3a, associated with the unity operator in the OPE. These are exemplified in Fig. 3b and 3c. The analogous corrections to two-point functions play an essential role in the corresponding sum rules. Therefore, they should also be included in a complete treatment of the four-point function (24). Some of the perturbative gluon corrections (Fig. 3b) are in fact included, summed up in the coefficients $c_1(m_W)$ and $c_2(m_b)$ of $H_W$, namely those originating at short-distances between $1/m_W$ and $1/m_b$. However, there are other contributions (Fig. 3c) coming from larger distances, say, between $1/m_b$ and $1/m_c$ or even $O(1 GeV^{-1})$, which are definitely not included in $H_W$. These may give rise to logarithms of the kind $ln(m_b/m_c)$ and induce important nonfactorizable contributions to the decay amplitudes. A systematic inclusion of all perturbative gluon effects requires three-loop calculations with massive quarks, a task which is postponed to later developments. Restricting in the first step of the analysis to the nonperturbative interactions sketched in Fig. 4, the correlation function (24) receives the following nonfactorizable contributions:

$$\tilde{\Pi}_{\mu\nu}^{QCD} = \bar{\Pi}_{\mu\nu}^{GG} + \bar{\Pi}_{\mu\nu}^{\bar{q}Gq} + \bar{\Pi}_{\mu\nu}^{\bar{q}\nabla qG} + \bar{\Pi}_{\mu\nu}^{q\bar{q}Dq} + \bar{\Pi}_{\mu\nu}^{q\bar{q}\bar{q}q},$$

where the tilde is a reminder to replace $H_W$ in (24) by $\tilde{O}_2$. The individual terms in (25) have been derived from the corresponding diagrams of Fig. 4 in the form of Feynman integrals. Their explicit expressions can be found in ref. [48].

With the result (25) at hand, one can now proceed in constructing sum rules which will allow to determine the matrix element (17) entering the correlation function through the resonance contribution

$$\Pi_{\mu\nu}^{B+J/\psi K} = i \frac{<0 | J^{K}_{\rho\delta} | K > <0 | J^{\psi}_{\nu} | J/\psi | K/J/\psi | \tilde{O}_2 | B > < B | j^B | 0 >}{(m^2_K - q^2)(m^2_{\psi} - p^2)(m^2_B - P^2)}.$$  

However, this problem is much more difficult than it sounds. One reason is that apart from the ground state contribution (26) and analogous contributions from excited states, one also has to take into account contributions to the spectral functions from continuum states with flavour quantum numbers which differ from those of the respective currents. Such intermediate
states emerge in every channel of the correlation function due to final state interaction. For example, in the $B$-meson channel the following contribution appears inevitably:

$$\tilde{\Pi}^{\pi_{DDs}^{*}}_{\mu\nu} = i \frac{<0 | j^{K}_{\mu5} | K > <0 | j^{\psi}_{\nu} | J/\psi > < KJ/\psi | DDs^{*} | DDs^{*} > | 0 >}{(m_{K}^{2} - q^{2})(m_{\psi}^{2} - p^{2})(m_{DDs}^{2} - P^{2})}$$ (27)

Here, the questionable intermediate state carries the quantum numbers of a virtual $DDs^{*}$ state. It is created by weak interaction and converted into the $J/\psi K$ final state by strong interaction. The problem is that this continuum of states has a mass threshold below the $B$-meson pole. It can therefore not be suppressed by Borel transformation and subtracted away similarly as outlined in section 2 for the normal continuum in the $B$-channel starting at a threshold $s_{0} > m_{b}^{2}$. On the contrary, this unwanted contributions will be enhanced by Borel transformation relative to the contribution from the ground state $B$ roughly by a factor $\exp[(m_{b}^{2} - 4m_{c}^{2})/M^{2}]$ which is quite large at characteristic values of the Borel parameter $M^{2} \simeq (m_{b}^{2} - m_{c}^{2})$.

Such dangerous contributions are also present in D decays. They have been estimated in ref. [11] using a simple model and found to be unimportant. Unfortunately, in the present case the solution of this problem seems to be less trivial. We have identified the analogous contributions in the QCD diagrams of Fig. 4 and find that they are numerically very important after Borel transformation. Indeed, examination of the analytical properties of the diagrams in Fig. 4 shows that in addition to a pole at $P^{2} = m_{b}^{2}$ the four quark condensate diagram, Fig. 4c, has a discontinuity at $P^{2} \geq 4m_{c}^{2}$ connected with the four-quark intermediate state with the flavour combination $u\bar{s}c\bar{c}$. This is just the flavour composition of the virtual $DDs^{*}$ continuum in (27). It seems to be reasonable to again use the quark-hadron duality principle in order to cancel this piece of (25) against the unwanted hadronic contribution (27). Practically, this results in neglecting the four-quark condensate diagram altogether.

From here on, we follow the usual procedure. We perform a Borel transformation in $P^{2}$ and take moments in $p^{2}$ as is usually done in the charmonium channel. The momentum squared in the $K$-meson channel is kept fixed at $Q^{2} = -q^{2} \simeq 1 GeV^{2}$. In order to take into account the higher states in the resulting sum rules we adopt for simplicity a two-resonance description in each of three channels, i.e. above the effective thresholds $s_{0i}$ we describe the hadronic spectral functions by one effective resonance with a mass equal to $\sqrt{s_{0i}}$. Normalizing the matrix element (17) so that the amplitude (16) is proportional to combination

$$a_{2}f_{K}^{*} = (c_{2} + \frac{c_{1}}{3})f_{K}^{*} + 2c_{1}\tilde{f},$$ (28)

we obtain for the nonfactorizable contribution

$$\tilde{f} = -(0.045 - 0.075).$$ (29)

The range reflects our estimate of the uncertainty in the QCD calculation and the continuum subtraction.

Our analysis shows that the factorizable, nonleading in $1/N_{c}$ term and the nonfactorizable term in (28) are opposite in sign. Quantitatively, the nonfactorizable matrix element is considerably smaller than the factorizable one, $|\tilde{f}/f_{K}^{*}(m_{c}^{2})| \simeq O(10\%)$. Nevertheless, the nonfactorizable contribution to the decay amplitude is very important because of the large coefficient, $2c_{1}/(c_{2} + c_{1}/3) \simeq 20 \div 30$. In fact, if $|\tilde{f}|$ is close to the upper end of the range given in (29), the nonfactorizable contribution almost cancels the factorizable one proportional to
\(c_1/3\), thus leading to the prediction (23) which agrees with experiment. This is exactly the scenario anticipated by employing the \(1/N_c\)-rule.

At the same time, in our approach the nonfactorizable contributions are expected to be channel-dependent on quite general grounds. This expectation is corroborated by our preliminary analysis of other nonleptonic \(B\)-decays. In other words, as is obvious from eq. (28) there is no theoretical reason to expect a single universal value for the effective coefficient \(a_2\). Even the sign of \(a_2\), depending on the sign of \(\tilde{f}\) in a given channel, may vary. Universality can at most be anticipated for certain classes of decay modes, such as \(B \to D\pi\) or \(B \to D\bar{D}\), etc. Moreover, there is no simple relation between \(B\) and \(D\) decays in our approach since the OPE for the corresponding correlation functions significantly differ in the relevant diagrams and in the hierarchy of mass scales. We hope to be able to clarify this issue further. Theory seems to predict a much richer pattern in two-body weak decays than what is revealed by the present phenomenological approach to the data.

### 4 Conclusion

We have discussed applications of QCD sum rules in different variants to form factors and exclusive decay amplitudes of \(B\) mesons, both being important issues in \(B\)-physics. There are many other interesting aspects which can be studied in the framework of QCD sum rules and which have not been touched upon here. We just mention the dependence of form factors and couplings on the heavy quark mass. Here, the sum rule technique can be employed complementarily to the heavy-quark effective theory. From a theoretical point of view, heavy-light bound states such as the \(B\) meson are very suitable systems for studying the dynamics of the light quark and gluon degrees of freedom and for probing nonperturbative methods in QCD. It will be interesting to see whether or not the QCD sum rule technique passes these tests as successfully as the various tests in the past. For the time being, it is the only viable approach to some problems such as the calculation of weak decay amplitudes beyond factorization. Even the capabilities of current lattice calculations do not allow to solve this problem (as remarked in ref. [49]).

### 5 Acknowledgements

We are grateful to V. Belyaev and B. Lampe for collaboration and discussions. Without their contributions this report would not have been written. We also acknowledge helpful discussions with M. Shifman. A. Khodjamirian thanks the TPI, University of Minnesota for supporting his participation in the Workshop.
References

[1] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys.* B147 (1979) 385, 448.

[2] B.L. Ioffe, A.V. Smilga, *Nucl. Phys.* B232 (1984) 109.

[3] I.I. Balitsky, A.V. Yung, *Phys. Lett.* 129B (1983) 328.

[4] V.M. Belyaev, Ya.I. Kogan, *JETP Lett.* 37 (1983) 611.

[5] S.V. Mikhailov, A.V. Radyushkin, *JETP Lett.* 43 (1986) 712; *Phys. Rev.* D45 (1992) 1754.

[6] V.L. Chernyak, A.R. Zhitnisky, *Nucl. Phys.* B345 (1990) 137.

[7] P. Ball, V.M. Braun, H.G. Dosch, *Phys. Rev.* D44 (1991) 3567.

[8] V. Belyaev, A. Khodjamirian, R. Rückl, *Z. Phys.* C60 (1993) 349.

[9] A. Ali, V.M. Braun, H. Simma, Preprint CERN-TH.7118/93.

[10] V. Braun, I. Galperin, Preprint MPI-PhT/94-8 (1994).

[11] B.Yu. Blok, M.A. Shifman, *Sov. J. Nucl. Phys.* 45 (1987) 135, 301, 522.

[12] B.L. Ioffe, A.V. Smilga, *Nucl. Phys.* B216 (1983) 373.

[13] V.A. Nesterenko, A.V. Radyushkin, *Phys. Lett.* 115B (1982) 410.

[14] A.Yu. Khodjamirian, *Phys. Lett.* 90B (1980) 460; *Sov. J. Nucl. Phys.* 39 (1984) 614.

[15] A.V. Beilin, A.V. Radyushkin, *Sov. J. Nucl. Phys.* 39 (1984) 800.

[16] L.J. Reinders, H. Rubinstein, S. Yazaki, *Phys. Rep.* 127 (1985) 1.

[17] T.M. Aliev, V.L. Eletsky, Ya.I. Kogan, *Sov. J. Nucl. Phys.* 40 (1984) 527.

[18] C.A. Dominguez, N. Paver, *Z. Phys.* C41 (1988) 217.

[19] A.A. Ovchinnikov, *Sov. J. Nucl. Phys.* 50 (1989) 519.

[20] P. Ball, V.M. Braun, H.G. Dosch, *Phys. Lett.* 273B (1991) 316.

[21] P. Colangelo, G. Nardulli, A.A. Ovchinnikov, N. Paver, *Phys. Lett.* 269B (1991) 201.

[22] S. Narison, *Phys. Lett.* 283B (1992) 384.

[23] V.L. Chernyak, A.R. Zhitnitsky, *JETP Lett.* 25 (1977) 510.

[24] S.J. Brodsky, G.P. Lepage, *Phys. Lett.* 87B (1979) 359.

[25] A.V. Efremov, A.V. Radyushkin, *Phys. Lett.* 94B (1980) 245.

[26] G.R. Farrar, D.R. Jackson, *Phys. Rev. Lett.* 43 (1979) 246.

[27] N.S. Craigie, J. Stern, *Nucl. Phys.* B216 (1983) 209.
[28] V.L. Chernyak, A.R. Zhitnitsky, *Phys. Rep.* **112** (1984) 173.

[29] A.S. Gorsky, *Sov. J. Nucl. Phys.* **41** (1985) 1008; *ibid.* **45** (1987) 512.

[30] I.I. Balitsky, V.M. Braun, *Nucl. Phys.* **B311** (1988) 541.

[31] I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, *Nucl. Phys.* **B312** (1989) 509.

[32] V.M. Braun, I. Filyanov, *Z. Phys.* **C44** (1989) 157; *ibid.* **C48** (1990) 239.

[33] V.M. Belyaev, V.M. Braun, A. Khodjamirian, R. Rückl, *in preparation.*

[34] J. Ellis, M.K. Gaillard, D.V. Nanopoulos, *Nucl. Phys.* **B100** (1975) 313.

[35] D. Fakirov, B. Stech, *Nucl. Phys.* **B133** (1978) 315.

[36] N. Cabibbo, L. Maiani, *Phys. Lett.* **73B** (1978) 418.

[37] R. Rückl, *Weak decays of heavy flavours*, Habilitationsschrift, Munich University, 1983.

[38] M. Bauer, B. Stech, M. Wirbel, *Z. Phys.* **C34** (1987) 103.

[39] A.J. Buras, J.-M. Gerard, R. Rückl, *Nucl. Phys.* **B268** (1986) 16.

[40] G. Altarelli, G. Gurci, G. Martinelli, S. Petrarca, *Nucl. Phys.* **B187** (1981) 461.

[41] A. Buras, P.H. Weisz, *Nucl. Phys.* **B333** (1990) 66.

[42] A. Buras, M. Jamin, M.E. Lautenbacher, *Nucl. Phys.* **B408** (1993) 209.

[43] J.H. Kühn, R. Rückl, *Phys. Lett.* **135B** (1984) 477.

[44] M.S. Alam, et al., CLEO preprint CLNS 94-1270 (1994).

[45] I. Bigi, B. Blok, M. Shifman, N. Uraltsev, A. Vainshtein, Preprint CERN-TH.7132/94 (1994)

[46] M.A. Shifman, *Nucl. Phys.* **B388** (1992) 346.

[47] B. Blok, M. Shifman, *Nucl. Phys.* **B389** (1993) 534.

[48] A. Khodjamirian, B. Lampe, R. Rückl, *in preparation.*

[49] M.A. Shifman, in *QCD - 20 Years Later*, ed. P.M. Zerwas, H.A. Kastrup (World Scientific, Singapore, 1993), Vol.2, p.775.
Figure Captions

Fig. 1: QCD diagrams contributing to the matrix element (2) involving (a) quark-antiquark light-cone wave functions; (b) three-particle quark-antiquark-gluon wave functions; (c) and (d) perturbative \( O(\alpha_s) \) corrections. Solid lines represent quarks, dashed lines gluons, wavy lines are external currents.

Fig. 2: \( B \)-meson form factors calculated from light-cone sum rules: (a) the form factor \( f_\pi^+(p^2) \) of \( B \to \pi \) transitions and (b) the form factor \( f_K^+(p^2) \) of \( B \to K \) transitions at \( M^2 = 10 GeV^2 \) (upper solid curves) and \( M^2 = 15 GeV^2 \) (lower solid curves). The quark model predictions from ref. [38] (dash-dotted curves) and the QCD sum rule results for \( f_\pi^+(p^2) \) from ref. [20] (dashed curve) and for \( f_\pi^+(0) \) from ref. [6] (arrow) are shown for comparison.

Fig. 3: Diagrams associated with the correlation function (24): (a) bare diagram with \( H_W \) replaced by \( O_2 \); (b) and (c) diagrams corresponding to perturbative gluon corrections.

Fig. 4: Diagrams associated with (a) the gluon condensate, (b) the quark-gluon condensate and (c) the four-quark condensate contributions to the correlation function (24) with \( H_W \) replaced by \( \tilde{O}_2 \).
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405310v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405310v1
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405310v1
This figure "fig1-5.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405310v1