Testing Abelian dyon–fermion bound system

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Abstract

Characteristics of Abelian dyon–fermion bound system, parity-violating effects, a new series of energy spectra, effects related to the non-vanishing electric dipole moment, feature of spin orientation etc., are analyzed and compared with hydrogen-like atom. These analyses explore possibility of a new approach of searching for dyons under bound condition.

1. Introduction

Since Dirac studied the problem of quantum mechanics of a particle in presence of a magnetic monopole [1], monopoles have been one of the interesting topics concerned in physics [2–35]. Their existence has been involved in explanation of phenomenon of electric charge quantization. Production of super-heavy monopoles or dyons (i.e., both electric and magnetic charged) in the early Universe is predicted in unified theories of strong and electroweak interactions [11] and its detection is one of few experimental handles for these theories.

Because Montonen–Olive duality conjecture [10] which is manifestation of classical electromagnetic duality in some spontaneously broken gauge theories, and its extension to an SL(2, Z) duality conjecture [22] suggested by Witten effect [12], plays important roles in recent developments of superstring theories during the last few years, monopoles have been receiving renewed attention.

So far search for free monopoles (or dyons) and for ones trapped in bulk matter (meteorites, schist, ferromanganese nodules, iron ores and others) has turned up negative. A summary of experiments can be found in review papers [13,34]. In various experimental schemes monopoles were assumed to have different properties [13,24,34]. Some of assumptions involved are (i) electromagnetic induction, (ii) energy losses, (iii) scintillation signature, (iv) catalysis of proton decay, and (v) trapping and extraction. Monopoles could be trapped in ferromagnetic domains by an image force of order 10 eV/Å. Trapped monopoles are supposed to be wrecked out of material by large magnetic force.

There are several difficulties in searching for free monopoles. First, we do not know how small the monopole flux is (according to the Parkker limit the up bound is $\phi < 3 \times 10^{-9} \text{ cm}^{-2} \text{ yr}^{-1}$ [4]), so we do not know that in order to record a event how long we
have to wait. Second, estimation of monopole masses is model-dependent, for example, masses of classical monopoles are about order 10–10^{12} GeV, and masses of super-heavy monopoles in grand unified theories are about order 10^{16} GeV. But we are ignorant of their definite values. Specially we do not know whether masses of monopoles are within the energy region which can be reached by accelerator experiments in the near future.

In view of the fact whether monopoles (dyons) exist or not is important, we may as well try to explore other means to find their existence. If monopoles (dyons) were produced in the early Universe, they would like to form bound states with charged fermions and remain in the present Universe. In this letter we examine some detailed properties of dyon–fermion bound system, including their parity-violating effects, a new series of energy spectra, trapping phenomenon by an inhomogeneous electric field through a non-vanishing electric dipole moment, feature of the spin orientation, compare this system with hydrogen-like atom, and suggest a number of experiments to detect them.

2. Charge of dyon

The Dirac quantization condition only determines possible values \( n \) of charge of fermion. Charge \( z_d \) of dyon is a free parameter which is not determined by the Dirac quantization condition. In order to quantitatively analyze dyon–fermion bound system, values \( z_d \) should be correctly determined. Consider two dyons with electric and magnetic charges, respectively, \( (q_d = z_d e, g) \) and \( (q_d' = z'_d e, g') \). The Zwanziger–Schwinger quantization condition [2]

\[
q_d g' - q_d' g = 2\pi n, \quad (n = 0, \pm 1, \pm 2, \ldots)
\]

determines only difference between electric charges of dyons, \( z_d - z'_d = n \), but does not determine values of either \( z_d \) or \( z'_d \). If CP is not violated, under CP transformation one determines that there are only two mutually exclusive possibilities: either \( z_d = n \) or \( z_d = n + 1/2 \) [23]. In presence of a CP-violating term, electric charge of dyon explicitly depends on \( \theta \) vacuum angle [12], \( z_d = n + \frac{\theta}{2\pi} \). Possibility \( z_d = n + 1/2 \) is excluded, thus we have

\[
z_d = n. \tag{1}
\]

In the following we review unusual properties of dyon–fermion bound system in detail. In order to provide experimental test all the results are calculated according to new estimation of charge \( z_d \) of dyon given by Eq. (1).

3. Parity violation

In this system spatial parity is violated by magnetic charge of dyon [9,20] because of wrong transformation property of magnetic field \( H_D = g\vec{r}/r^3 \) of dyon under space reflection \( P: H_D \rightarrow -H_D \). Invariance of Dirac equation in external magnetic field under \( P \) requires that vector potential \( A(\vec{x}, t) \) transforms as \( P \bar{A}(\vec{x}, t) P^{-1} = -\bar{A}(-\vec{x}, t) \) which obviously contradicts transformation of \( H_D \), unless one artificially changes sign of magnetic charge \( g \) under \( P \). This parity violation leads to two effects:

(i) A modification of selection rules of electromagnetic transition for this system [20]. In hydrogen-like atom, electric dipole transitions are subject to strict selection rules as regards total angular momentum \( j \) and parity \( P: |j' - j| \leq 1 \leq j' + j \). Selection rules of total angular momentum \( j \) are \( \Delta j = 0, \pm 1 \); but parities of initial and final states must be opposite, thus \( \Delta j = 0 \) transition is strictly forbidden. But for dyon–fermion system, parity is violated, thus \( \Delta j = 0 \) electric dipole transitions are allowed.

(ii) This system, different from hydrogen-like atom, can possess a non-vanishing electric dipole moment [9].

According to Ref. [8] for a fixed \( q \) there are three types of simultaneous eigensections of \( j^2, J_y, \) and \( H \) in dyon–fermion system, types \( A \) and \( B \) \((j \geq |q| + \frac{1}{2})\), and type \( C \) \((j = |q| - \frac{1}{2})\). Their eigensections are:

for type \( A \) \((j \geq |q| + \frac{1}{2})\)

\[
y_{qnm}^{(1)} = \frac{1}{r} \left( h_{21}^{qnm}(r)\xi_{qnm}^{(1)} - iH_{21}^{qnm}(r)\xi_{qnm}^{(2)} \right),
\]

for type \( B \) \((j \geq |q| + \frac{1}{2})\)

\[
y_{qnm}^{(2)} = \frac{1}{r} \left( h_{32}^{qnm}(r)\xi_{qnm}^{(2)} - iH_{32}^{qnm}(r)\xi_{qnm}^{(1)} \right),
\]
for type C \((j = |q| - \frac{1}{2})\)

\[\psi_{qjm}^{(3)} = \frac{1}{r} \left( f^{(0)}_{qjm}(r) R_{qjm}^{(2)} \right),\]

where

\[s_{qjm}^{(1)} = c_{qj} \phi_{qjm}^{(1)} - s_{qj} \phi_{qjm}^{(2)},\]
\[s_{qjm}^{(2)} = s_{qj} \phi_{qjm}^{(1)} + c_{qj} \phi_{qjm}^{(2)},\]
\[c_{qj} = q \left[ (2j + 1 + 2q)^{1/2} - (2j + 1 - 2q)^{1/2} \right] \]
\[\times \left[ 2|q|(2j + 1)^{1/2} \right]^{-1},\]
\[s_{qj} = q \left[ (2j + 1 + 2q)^{1/2} - (2j + 1 - 2q)^{1/2} \right] \]
\[\times \left[ 2|q|(2j + 1)^{1/2} \right]^{-1},\]
\[\phi_{qjm}^{(1)} = \left( \frac{j + m}{2j + 1} \right)^{1/2} Y_{q,j - \frac{1}{2},m - \frac{1}{2}},\]
\[\phi_{qjm}^{(2)} = \left( \frac{j - m + 1}{2j + 1} \right)^{1/2} Y_{q,j + \frac{1}{2},m + \frac{1}{2}}.\]

In the above \(Y_{q,l,m}\) is monopole harmonic [6,16]. Radial wave functions \(R_{qjm}^{(2)}(\rho) = 2p_{i}^{(2)}(\rho)/\rho (i = 1, 2, 3, 4)\) are [19]:

for type A

\[R_{12}^{qjm}(\rho) = 4p^{2}(M \pm E_{qjm}^{D})^{1/2} A_{1}^{qjm} e^{-\rho/2} \rho^{v-1} \]
\[\times \left[ F(-n, 2v + 1, \rho) \mp \frac{n}{\mu + (\mu^{2} + n^{2} + 2nv)^{1/2}} \right] \times F(-n + 1, 2v + 1, \rho);\]

for type B

\[R_{34}^{qjm}(\rho) = 4p^{2}(M \pm E_{qjm}^{D})^{1/2} A_{3}^{qjm} e^{-\rho/2} \rho^{v-1} \]
\[\times \left[ F(-n, 2v + 1, \rho) \pm \frac{n}{\mu - (\mu^{2} + n^{2} + 2nv)^{1/2}} \right] \times F(-n + 1, 2v + 1, \rho).\]

In the above \(M\) is mass of fermion, \(E_{qjm}^{D}\) is energy of dyon–fermion bound system which is given by Eq. (2) below; \(\rho = 2pr, p = [M^{2} - (E_{qjm}^{D})^{2}]^{1/2},n = 0, 1, 2, \ldots\) is radial quantum number; \(v = (\mu^{2} - \lambda^{2})^{1/2} > 0; \mu = [(j + \frac{1}{2})^{2} - q^{2}]^{1/2} > 0; \lambda = zz_{d}e^{2}, z\) is electric charge of fermion, which is an integer; \(j \gg |q| + \frac{1}{2}; q = z\epsilon g \neq 0; \) Dirac quantization sets \(eg = \frac{N}{2}. (N = \pm 1, \pm 2, \pm 3, \ldots)\) (For the dyon case possibility \(eg = 0\) is excluded.) \(F(a, b, \rho)\) is confluent hypergeometric function. \(A_{1,3}^{qjm}\) are the normalization constants. It is not necessary to show detailed structures of radial wave functions \(f^{qjm}(\rho)\) and \(g^{qjm}(\rho)\) of type C for our purpose.

4. Energy spectrum

Energy spectrum of dyon–fermion bound system is [17–20]

\[E_{qjm}^{D} = M \left[ 1 + \frac{\lambda^{2}}{(m + n)^{2}} \right]^{-1/2}.\]

Spectrum (2) is hydrogen-like, but there is delicate difference between spectra of a dyon–fermion bound system and spectrum of a ordinary hydrogen-like atom. For hydrogen-like atom \(j\) takes only half-integer. Total angular momentum of dyon–fermion bound system includes a term \(−q\sqrt{r}\) contributed by monopole field, so \(j\) takes half-integer as well as integer.

(i) When \(q\) takes half-integer, total angular momentum \(j\) takes integer, leading to a new series of energy spectra that do not exist in ordinary hydrogen-like atom.

(ii) When \(q\) takes integer, \(j\) takes half-integer which is similar to the case of ordinary hydrogen-like atom. But compared with energy level of the latter

\[E_{nj}^{H} = M \left[ 1 + \frac{(ze^{2})^{2}}{(n + |j + \frac{1}{2}|^{2} - (ze^{2})^{2})^{1/2}} \right]^{-1/2}.\]

\(E_{qjm}^{D}\) shifts down. Consider the case \(z = -1, z_{d} = 1, |q| = 1, j = 3/2,\) the shifted amount for \(n = 1\) energy level is \((E_{D} - E_{H})/M \sim 10^{-2} \alpha^{2}\), where \(\alpha\) is the fine-structure constant. We also compare energy interval \(\Delta E = E(n' = 1) - E(n = 0).\) The shifted \((\Delta E_{D} - \Delta E_{H})/M\) is also at the order \(10^{-2} \alpha^{2}\). Notice that these differences can be measured by present experiments.

For a Dirac monopole-fermion bound system, there is LWP difficulty [3] in angular momentum state \(j = |q| - \frac{1}{2}\). For a Daric dyon–fermion bound system a new
singularity occurs even in angular momentum states $j > |q| + \frac{1}{2}$ when charge $z_d$ of dyon exceeds a critical value $z_d^c$ [15]. In order to avoid this difficulty, one way is to introduce terms [8,15] $-\kappa q/(2Mr^3)\vec{\Sigma} \cdot \vec{r}$ and $-\kappa q/(2Mr^3)\vec{\gamma} \cdot \vec{r}$. Using the above wavefunctions $\psi_{qnjm}^{(1,2)}$ of dyon–fermion bound system [19] we find that energy shifts from term $\vec{\Sigma} \cdot \vec{r}$ vanishes

$$\Delta E_{qnj}^{(1)} = 0. \quad (3)$$

This result is unlike to be accidental, behind it there should be a simple symmetry which needs to be explored. For energy shift $\Delta E_{qnj}^{(2)}$ from term $\vec{\gamma} \cdot \vec{r}$ we consider $n = 0$ and the case of dyon carrying one Dirac unit of pole strength, $|q| = \frac{1}{2}$, thus $j = |q| + \frac{1}{2} = 1$; take $z = -1$, $z_d = 1$, we obtain

$$\Delta E_{qnj}^{(2)} = \begin{cases} -\frac{\kappa q}{2Mr^3} \vec{\gamma} \cdot \vec{r}, & qn0m, \\ \frac{2\kappa q}{\mu^2} \vec{\gamma} \cdot (2\vec{v}-1), & \end{cases} = C_1 \kappa a^4 M, \quad (4)$$

where $C_1$ is a number of order 1. $\Delta E_{qnj}^{(2)}$ can be neglected. The above results show that energy spectrum (2) is quite accurate for a Daric dyon–fermion bound system with terms $\vec{\Sigma} \cdot \vec{r}$ and $\vec{\gamma} \cdot \vec{r}$.

For a Dirac dyon–fermion bound system coupled to general gravitational and electromagnetic fields their energy levels $\tilde{E}_{qnj}^D$, in the closed or open Robertson–Walker metric are [21]

$$\tilde{E}_{qnj}^D = E_{qnj}^D \left[1 \pm \mu^2 \lambda^2 (R_0/a_0)^2 \times \left\{6(n^2 + \lambda^2 + 2n(\mu^2 - \lambda^2)^{1/2}) \right\}^{-1} \right], \quad (5)$$

where $R_0$ is the average radius of region of dyon–fermion system, $a_0$ is the cosmological radius; the plus and minus sign corresponds, respectively, to the closed and open space–time. After epoch of recombination, the cosmological radius $a_0(r)$ is about $10^{23}$ cm, $(R_0/a_0)^2 \sim 10^{-62}$ (if $R_0$ is about $10^{-8}$ cm). Thus correction of the curved space to energy levels (2) is

$$\frac{\tilde{E}_{qnj}^D - E_{qnj}^D}{E_{qnj}^D} \sim \frac{(R_0/a_0)^2}{(\mu^2 - \lambda^2)^{1/2}}, \quad (6)$$

which can be neglected. Only in the case of a large $z_d$ satisfied $\lambda \sim \mu$ correction of the curved space would become important.

5. Parity-violating transition

Matrix elements of the $\Delta j = 0$ parity violation electric dipole transition can be precisely estimated [36]. In electric dipole approximation, taking transverse Coulomb gauge, Hamiltonian of this system is $H_I = -ze\vec{a} \cdot \vec{e} A_0$, where $\vec{a}$ and $A_0$ are, respectively, polarization vector and amplitude of external electromagnetic field. For type $A$ we consider the case of $q = \frac{1}{2}$, $j = |q| + \frac{1}{2} = 1$, $n = 1$, $n' = 0$, $z = -1$, $z_d = 1$. Up to order $\alpha^2$, we have

$$(H_I)_{qp^j jm,jn jm}^{(A)} = iC_2A_0\epsilon_3m, \quad (7)$$

$$(H_I)_{qp^j jm,jn jm}^{(A)} = iC_3A_0(\epsilon_1 + i\epsilon_2)(2 \pm m)^{1/2}(1 \mp m)^{1/2}, \quad (8)$$

where $C_2$ and $C_3$ are numbers of order $10^{-2}$.

For electric dipole transitions within type $B$ states or between type $A$ and type $B$ states, results are similar to Eqs. (7) and (8).

Transitions from type $A$ ($B$) to type $C$ would presumably be crucial in identifying emissions from such a system. Electric dipole transition matrix elements from type $A$ to type $C$ are

$$(H_I)_{qp^j jm',jm'}^{(A,C)} = iA_0\delta_{j',j-1} \left[ \begin{array}{c} \delta_{m',m-1}(\epsilon_1 + i\epsilon_2) \frac{j + m}{2j} \\ -\delta_{m',m+1}(\epsilon_1 - i\epsilon_2) \frac{j - m}{2j} \\ +\delta_{m',m}(\epsilon_2) \frac{(j^2 - m^2)^{1/2}}{j} \end{array} \right] \times (I_{qp^j jm',jm'}^{(1)} R_{qj} + I_{qp^j jm',jm'}^{(2)} T_{qj}), \quad (9)$$

where $I_{qp^j jm',jm'}^{(1)}$ and $I_{qp^j jm',jm'}^{(2)}$ are radial integrals, $R_{qj}$ and $T_{qj}$ are numerical factors depending on $q$ and $j$. Eq. (9) shows that selection rule of transition from type $A$ to type $C$ is $\Delta j = -1$; the $\Delta j = 0$ parity-violating transition is absent. The result from type $B$ to type $C$ is similar to Eq. (9).
6. Lyman lines

Spectral series of transitions of this system can be accurately estimated by Eq. (2). In the general case, $zze^2 \ll 1$. By Eq. (2), the photon wavelength $\lambda(q; n'j', nj)$ of transition from $(n', j')$ state to $(n, j)$ state is

$$\lambda(q; n'j', nj) = \frac{4\pi}{M(zze^2)^2} \frac{(n' + \mu')^2(n + \mu)^2}{(n' + \mu')^2 - (n + \mu)^2}.$$  

(10)

We consider the case $|q| = \frac{1}{2}$. In this case $\mu = [j(j + 1)]^{1/2}$, $j = 1, 2, 3, \ldots$. For dyon–electron system, $z = -1$, $z_d = 1$. We calculate the first Lyman line. For transition from $(n' = 1, j' = 1)$ state to $(n = 0, j = 1)$ state is

$$\lambda_e(\frac{1}{2}; 11, 01) = 2.8 \times 10^3 \text{Å}$$

($\Delta j = 0$ parity violation transition).

For transition from $(n' = 1, j' = 2)$ state to $(n = 0, j = 1)$ state is

$$\lambda_e(\frac{1}{2}; 12, 01) = 2.2 \times 10^3 \text{Å}$$

($\Delta j = 1$ parity conservation transition).

For dyon–proton system, $z = 1$, $z_d = -1$:

$$\lambda_p(\frac{1}{2}; 11, 01) = 1.5 \text{Å}$$

($\Delta j = 0$ parity violation transition),

$$\lambda_p(\frac{1}{2}; 12, 01) = 1.3 \text{Å}$$

($\Delta j = 1$ parity conservation transition).

For dyon–electron system the first Lyman lines are in the infrared region. For dyon–proton system the first Lyman lines are in the x-ray region.

7. Dipole moment

Electric dipole moment $\vec{d} = e\vec{r}$ of this system can be represented by total angular moment

$$\vec{J} = \vec{r} \times (\vec{p} - ze\vec{A}) + \frac{1}{2} \vec{\Sigma} - q\vec{r}/r$$

as

$$\vec{d} = e(-q\vec{r} + \frac{1}{2} \vec{\Sigma} \cdot \vec{r}) \vec{J}/j(j + 1).$$

It is easy to show that only its $z$ component has non-vanishing expectation value $\langle d_z \rangle_{qnjm}$ in state $\psi_{(1,2)}^{(1,2)}$.

For the $n = 0$ case, we have

$$\langle d_z \rangle_{qnjm}^D = \frac{eqm}{2j(j + 1)} \frac{\mu}{\lambda M \Gamma(2v + 1)}.$$  

(11)

Taking $q = \frac{1}{2}$, $j = 1$, from Eq. (11) it follows that

$$\langle d_z \rangle_{01m}^D \sim -C_4(\text{em}/M),$$

(12)

where $C_4$ is a number of order 10.

8. Spin orientation

For this system the expectation value $\mathbf{S} = \frac{1}{2} \vec{\Sigma}$ of spin of fermion is

$$\langle S_z \rangle_{qnjm}^D = \frac{m}{4j(j + 1)} \left( 1 + \frac{2\mu}{2j + 1} \frac{E_{\text{em}}^D}{M} \right).$$  

(13)

Here $j$ and $n$ dependence in Eq. (13) is different from that in hydrogen-like atom. For hydrogen-like atom $\langle S_z \rangle_{njjm}$ are $\langle S_z \rangle_{njj(j + \frac{1}{2})} = -m/[2(j + 1)]$, $\langle S_z \rangle_{nj(j - \frac{1}{2})} = m/(2j)$. In particular, Eq. (13) depends on the radial quantum number $n$, but the latter does not. The basic reason leading to the above difference is that in hydrogen-like atom spherical harmonic spinors $\Omega_{jlm}$ are eigenfunctions of $\vec{L}^2$, but in dyon–fermion bound state monopole spherical harmonic spinors $\hat{\psi}_{qjm}^{(1,2)}$ are not.

Based on the above examination of detailed properties of Dirac dyon–Fermion bound state, which are different from hydrogen-like atom, now we suggest the following experiments to detect them.

\footnote{Kazama in Ref. [9] estimated dominant term of $\vec{d}$ and $\vec{p}$ in a zero-energy bound state in the limit of very loosely bound approximation. In our case besides a kinematic factor $m/[j(j + 1)]$ dynamical behaviors of $\langle d_z \rangle_{qnjm}$ and $\langle p_z \rangle_{qnjm}$ are also obtained in detail.}
9. Analysis of astronomical spectrum

Approach of searching for dyon–fermion bound systems, compared with approach of searching for free monopoles, shows advantage. (i) Superheavy dyon is treated as an external potential so that its mass does not appear in energy spectrum (2). Spectrum (2) is quite accurate, corrections from the term $\vec{\gamma} \cdot \vec{r}$ and effect of curved space are completely negligible. (ii) If dyons were produced in plenty in the early Universe and formed into bound states with charged fermions, radial electromagnetic spectra of these bound systems should be recorded on astronomical observations during a long period. There are some astronomical spectra recorded at the Kitt Peak Observatory which cannot be explained by atomic or molecular spectrum [37]. We suggest to compare spectrum (2) and related Lyman lines with such unexplained astronomical spectrum.

10. Analysis of trapped dyon–fermion bound system

Because dyon–fermion (electron, proton, etc.) bound system possesses a non-vanishing electric dipole moment $\langle d_z \rangle_{qnm}$, it can be trapped by a well of inhomogeneous electric field through $-\langle d \cdot \vec{E} \rangle_{qnm}$.

(I) Residues in ferromagnetic. One way to obtain electric dipole trap of dusting material of elementary ferromagnetic in a trapping chamber is to use a strongly focused laser beam with Gaussian intensity profile, providing a field with an absolute maximum of laser intensity at the center of focus. A laser trap relies on the force of an inhomogeneous electric field of a laser acting on the dipole moment of dyon–fermion bound system. In order to violate the condition of optical Earnshaw theorem one needs to properly switch the laser field on and off. Because events are rare, we need high trapping density.

(II) Trapped events from cosmic rays. In order to trap rare events from cosmic rays, we need to use long duration static electric well with inhomogeneous distribution. In order to obtain a stable trap, it is necessary to violate the condition of static electric Earnshow theorem.

In such trapping devices it is possible to check whether trapped objects are dyon–fermion bound system:

(i) One can observe their absorption spectrum and compare it with hydrogen-like atomic one according to Eq. (2).

(ii) Using non-vanishing $\langle S_z \rangle_{qnm}$, adding a strong magnetic field $\vec{H}$ to orient the trapped system, one can examine $n$ and $j$ dependence of $\langle S \cdot \vec{H} \rangle_{qnm}$ according to Eq. (13).

Investigation of potential technical sensitivity of testing dyon–fermion bound system in the above suggested experiments is out of this Letter.

Discovery of monopoles would have far-reaching consequences. Of course, any attempt to detect monopoles or dyons is a challenging enterprise. The reason is that if they remain in the present Universe they are surely rare.

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3 Because of large mass and the permanent electric dipole moment of dyon–fermion bound system, laser cooling and trapping of events are easier than those of the neutral atom case. For this system maybe it is not so difficult to break sub-Doppler cooling limit and subrecoil cooling limit to reach very low temperature. Recent report of the low temperature in three-dimensional laser cooling beyond the single-photon recoil limit, see Ref. [38].
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