Distribution of rain height over subtropical region: Durban, South Africa for satellite communication systems

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Abstract. Rain height is one of the significant parameters for prediction of rain attenuation for Earth-space telecommunication links, especially those operating at frequencies above 10 GHz. This study examines Three-parameter Dagum distribution of the rain height over Durban, South Africa. 5-year data were used to study the monthly, seasonal, and annual variations using the parameters estimated by the maximum likelihood of the distribution. The performance estimation of the distribution was determined using the statistical goodness of fit. Three-parameter Dagum distribution shows an appropriate distribution for the modeling of rain height over Durban with the Root Mean Square Error of 0.26. Also, the shape and scale parameters for the distribution show a wide variation. The probability exceedance of time for 0.01% indicates the high probability of rain attenuation at higher frequencies.

1. Introduction

Statistical information of rain height is an important factor in rain attenuation prediction when designing satellite communication links, especially at higher frequency bands. Hydrometeors such as rainfall, hail, cloud, and rain height can frequently cause degradation of satellite signals, especially those operating at the frequency above 10 GHz. The result could be detrimental to the Quality of Service (QoS) at the receiving end [1-4]. Rain height is associated with the attenuation of the satellite signal and the interference from co-channel as a result of scattering at any location [5, 6]. In this work, an indirect procedure was employed due to the inadequate information of rain height in South Africa. Zero-degree isotherm height (ZDIH) obtained from the Tropical Rainfall Measuring Mission-Precipitation Radar (TRMM-PR) as provided in the product 2A23 algorithm have been used. ZDIH can be defined as the height where the temperature is at 0°C in the free atmosphere. ZDIH also determines the effective path length, which rain is affected. Though ITU-R Recommendation [7] provides the method of predicting rain attenuation which requires rain height, the recommendation also stated that around the world where there is no availability of specific information, the mean ZDIH should be considered as the proper quantity of ZDIH [8].

Several types of research have been carried out using numerous statistical distribution models over other atmospheric data such as rain rate, raindrop size distribution among others. However, very limited work has been done on ZDIH over South Africa. Hence, this work observes probability density function using one of the statistical distributions ‘Three-parameter Dagum distribution’ to model ZDIH based on the 5-year (2011–2015) data obtained from TRMM-PR over Durban, one of the largest cities in KwaZulu-Natal province of South Africa, 29° 97', 30° 95', altitude 8 m above sea level. Information on the classification of the season and data sources obtained from TRMM-PR is well presented in the work of [5, 9, 10]. The model was chosen among others because of its ability to define real-positive numbers over a wide range of empirical data [11]. Also, Root Mean Square Error was used to estimate the quality of statistical fitness.
2. Methods

2.1. Dagum Distribution

Dagum distribution was introduced in [11] for modeling data as an alternative to the Pareto and Log-normal models. This type of distribution has been extensively used for modeling in various data such as meteorological data, income, and financial analysis among others. Also, this distribution is among the continuous probability distributions that can be used to define real positive numbers over a wide range of empirical data [11]. In this work, three-parameter Dagum distribution is used and denoted as Dagum. The probability density function (PDF) and cumulative distribution function (CDF) of Dagum are presented as shown in equation (1) and (2) respectively:

\[ f(x) = k\alpha x^{-\alpha-1}(1 + \beta x^{-\alpha})^{k-1} \]  

(1)

\[ F(x) = (1 + \beta x^{-\alpha})^{-k} \]  

(2)

where \( k \) and \( \alpha \) are the shape parameters (\( \alpha \) and \( k > 0 \)), \( \beta \) is the scale parameter (\( \beta > 0 \)) and \( x \) is the value of ZDIH (\( x > 0 \)).

2.2. Dagum maximum likelihood method of estimation

The maximum likelihood method is one of the most regular methods for parameter estimation [12]. This method’s success stems from its many valuable properties including reliability, asymptotic proficiency, invariance and simply its intuitive appeal [13]. Let \( x_1, \ldots, x_n \) be a ZDIH data size \( n \) from the Dagum distribution with parameters \( k, \beta, \) and \( \alpha \). The likelihood can be given as from equation (1):

\[ L(k, \beta, \alpha) = (\beta k \alpha)^n \prod_{i=1}^{n} x^{-\alpha-1}(1 + \beta x^{-\alpha})^{k-1} \]  

(3)

and the log-likelihood function is also given as:

\[ l(k, \beta, \alpha) = n (\log b - \log (c + d)) \sum_{i=1}^{n} \log \left( \frac{1}{\alpha} \right) + \sum_{i=1}^{n} \log \left( \frac{x^{-\alpha}}{1 + \beta x^{-\alpha}} \right) \]  

(4)

where \( b = \alpha - 1, \ c = \log x - (k + 1), \ d = \log \left( 1 + \beta x^{-\alpha} \right) \) and \( e = \log k + \log \beta + \log \alpha \).

The maximum likelihood estimators of \( \hat{k}_{MLE}, \hat{\beta}_{MLE}, \) and \( \hat{\alpha}_{MLE} \) of the parameters \( k, \beta, \) and \( \alpha \) can then be obtained numerically by maximizing with respect to \( k, \beta, \) and \( \alpha \), in the log-likelihood function (4). In this case, the log-likelihood function is maximized by solving \( k, \beta, \) and \( \alpha, \) \( x, \) in the non-linear equations which are given as follows:

\[ \frac{\partial}{\partial k} l(k, \beta, \alpha) = \frac{n}{k} - \sum_{i=1}^{n} \ln \left( 1 + \beta x^{-\alpha} \right) = 0 \]  

(5)

\[ \frac{\partial}{\partial \beta} l(k, \beta, \alpha) = \frac{n}{\beta} - (k + 1) \sum_{i=1}^{n} \frac{x^{-\alpha}}{(1 + \beta x^{-\alpha})} = 0 \]  

(6)

\[ \frac{\partial}{\partial \alpha} l(k, \beta, \alpha) = \frac{n}{\alpha} - \sum_{i=1}^{n} \log x + (k + 1) \sum_{i=1}^{n} \frac{x^{-\alpha}}{(1 + \beta x^{-\alpha})} = 0 \]  

(7)

2.3. Performance Evaluation of the Distribution

Root Mean Square Error (RMSE) was used to estimate the quality of statistical fitness of distribution over the study location, as expressed in equation (8):

\[ RMSE = \left( \frac{1}{n} \sum_{i=1}^{n} \left( x_a(i) - x_p(i) \right)^2 \right)^{1/2} \]  

(8)

where \( n \) and \( i \) are the total numbers and cumulative rank of the data points, \( f(x) \) is the density function of the fitted data, \( x_a \) and \( x_p \) are the measured value and Dagum predicted value of ZDIH respectively.

3. Results

3.1 Monthly variations of ZDIH

Figure 1 shows the variations of the monthly and corresponding 5-year monthly error bars of the standard deviation of the ZDIH. For example, Figure 1(a) shows that ZDIH increased as the year of data sampling increases, hence, the year 2011 and 2015 records the lowest and highest ZDIH.
respectively. The peak value of ZDIH observed in the year 2015 might be due to the presence of El Nino in the Southern part of Africa as it transits over the South Africa region. Also, Figure 1(b) revealed that the month of April presents the least ZDIH, followed by the months of February and March with the values of about 4.22, 4.31 and 4.32 km respectively. The standard deviation observed over these months is 0.05, 0.08, and 0.03. Consequently, the highest ZDIH transpired in the month of September with the value of about 4.46, 4.44 and 4.42 km respectively. The corresponding standard deviation is 0.08, 0.10 and 0.03 respectively.

![Figure 1](image1.png)

**Figure 1.** (a) Monthly mean, and (b) 5-year monthly mean with standard deviation for ZDIH.

### 3.2 Fitting of PDF and CDF to 0°C Isotherm height data

Figure 2 presents PDF and CDF fitting to the ZDIH over the average observation period based on equation (8) to estimate the quality of fitness. The results from Figure 2 show that Dagum present good fitting to the measured values of ZDIH with RMSE of about 0.26. Also, the parameter values estimated from MLE are 0.45, 131.65 and 4.42 for k, α, and β respectively.

![Figure 2](image2.png)

**Figure 2.** (a) PDF, and (b) CDF of Dagum on the ZDIH data.

### 3.3 Mean Variations of the Dagum Distribution

Mean variations of monthly, seasonal, and annual Dagum distribution for the year of observation with the shape and scale parameters were presented in Table 1 and Figure 3. It could be seen from the Table 1 that both shape and scale parameters vary over a wide range of ZDIH values in the study location. For instance, in Table 1(a), the parameter k varies from 0.14 in April to 103.8 in August; α varies from 52.99 in the month of August to 135.2 in the month of October; while β varies from 4.02 in the month of August to 4.53 in the month of September. Also, Figure 3(a) has its lowest peak of 0.27 distribution in the month of December and the highest in the month of May with 0.56 peak distribution. Consequently, Figure 3(b) shows that the spring months have the highest distribution followed by winter; autumn and summer have the least distribution. Also, spring has the widest distribution followed by autumn, while summer has the narrowest distribution. In Figure 3(c), the year 2014 has the highest and widest distribution followed by the year 2011 while the year 2013 has the lowest peak and narrowest distribution. This indicates the wide variations of ZDIH distribution based on the studied location.

![Figure 3](image3.png)

**Figure 3.** (a) Monthly Mean, (b) seasonal, and (c) annual variation of Dagum distribution on ZDIH.
Table 1. (a) Monthly, (b) Seasonal, and (c) Annual Dagum distribution of the parameters.

| Months | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $k$    | 1.94| 0.43| 93.59| 0.14| 0.63| 0.75| 1.95| 103.8| 0.34| 0.40| 3.52| 1.22|
| $\alpha$ | 71.35| 99.94| 80.89| 129.82| 98.91| 103.8| 56.0| 52.99| 106.3| 135.2| 57.4| 76.38|
| $\beta$ | 4.26| 4.38| 4.06| 4.44| 4.38| 4.44| 4.30| 4.02| 4.53| 4.47| 4.28| 4.38|

| Season | Summer | Autumn | Winter | Spring |
|--------|--------|--------|--------|--------|
| $k$    | 0.87   | 0.26   | 0.99   | 0.71   |
| $\alpha$ | 78.81| 126.92| 73.24| 74.93|
| $\beta$ | 4.40  | 4.40  | 4.38  | 4.47  |

| Year | 2011 | 2012 | 2013 | 2014 | 2015 |
|------|------|------|------|------|------|
| $k$  | 0.22 | 0.27 | 1.01 | 1.04 | 96.47|
| $\alpha$ | 128 | 144.1 | 90.86 | 65.09 | 56.66|
| $\beta$ | 4.42 | 4.43 | 4.36 | 4.39 | 4.055|

3.4. Percentage exceedance of ZDIH
Figure 4 presents the percentage exceedance of the annual mean of ZDIH over the studied location. It could be noticed that ZDIH distribution varies from 3.58 to 4.59 km over probability range of all availabilities of signals. Also for 0.01% of time occurrence probability, about 4.56 km was recorded for ZDIH. This value corresponds to the high probability of rain attenuation at higher frequencies, especially at frequencies above 10 GHz.

Figure 4. Exceedance probability of ZDIH.

4. Conclusion
This paper examined the statistical distribution of Dagum distribution to model zero-degree isotherm height over Durban in South Africa. Maximum likelihood method was used to derive the parameters of the distribution. Dagum distribution gives a good fit to ZDIH data with the RMSE of about 0.26. This technique had shown a good performance and may be useful for satellite system engineers, for link planning and systems design. The results further suggested the use of locally generated data rain height for the prediction of rain-induced attenuation.

5. References
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