Settlement and bearing capacity of the foundation of a finite width due to extrusion of a loose ground layer from the compressed footing mass

Zaven Ter-Martirosyan and Valery Demyanenko
Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, Russia
E-mail: demianenkovi@gmail.com

Abstract. The paper provides a quantitative assessment of the deflected mode of foundation stratum of finite width foundation, in the compressible thickness of which there is a slack clay soil layer. A number of criteria for assessing the possibility or impossibility of extruding a slack layer depending on its strength and rheological properties, as well as the relative thickness of the layer to its length (h/l) and the relative depth of the layer (h/d) have been given. Closed analytical solutions are given to determine the rate of Foundation precipitation depending on the rate of extrusion of the weak layer, including taking into account the damped and undamped creep.

1. Introduction
If the foundation mass incorporates loose ground layers (silts, flowable silt-loam soils, peat, peaty soils, low consolidated soils), which have low bearing capacity and increased compressibility, there is a need to quantify the bearing capacity and settlement of such footings in order to determine the impossibility or possibility of using them as natural bases. If the loose ground layer is found not fit for use, it may be economically more efficient to replace the loose ground layer with another one provided that it is not deeply situated under the earth as well as with sand or gravel piles or beds, drain piles, including at a depth [5, 6].

The present paper covers quantitative assessment of the rate of foundation settlement caused by extrusion of a loose ground layer from the compressed footing mass based on analytical and numerical (Finite Element Method) methods [2, 8]. For the loose ground layer, the Maxwell rheological model is considered as a design case. The computational scheme of the task set is shown in Fig. 1. Numerical simulation of the loose layer extrusion over time using PLAXIS PC by SoftSoilCreep is given as well.
Figure 1. Calculation scheme of analytical solution. Designation: \(a\) – half the width of Foundation, \(b\) – width of Foundation, \(N\) – distributed load on the Foundation, \(P\) – concentrated load on the Foundation, \(d\) – the capacity of the strong layer, \(h\) – half the power of the weak layer, \(p_0\) – is the maximum value of normal stress on \(z\) axis, \(\tau_{xz}\) – shear stresses, \(U(x,z)\) – the velocity of extrusion of the weak layer

### 2. Material and methods

Rheological models including Maxwell’s model [1, 4, 5] are considered as a computational model for the analytical solution of the problem.

\[
\dot{\gamma} = \frac{\tau}{\eta(t)} + \frac{\dot{\tau}}{G}
\]

(1)

where \(\dot{\gamma}\) is the rate of angular distortion of loose ground layers (1/sec); \(\tau\) is the effective value of shear stress in the layer (kN/m2); \(\eta(t) = \eta_0 \cdot f(t)\) is the varying viscosity of the loose layer over time.

The stress state of the base is considered according to Flaman, i.e. the problem of the distributed load action \(p = \text{const}\) on the surface of the soil half space in a bandwidth (flat distortion) is calculated using the formula:

\[
\sigma_z = \frac{P}{\pi} \cdot \left( \arctg \frac{a-x}{z} + \arctg \frac{a+x}{z} \right) - \frac{2ap}{\pi} \cdot \frac{z(x^2 - z^2 - a^2)}{(x^2 + z^2 - a^2)^2 + 4a^2z^2}
\]

\[
\tau_{xz} = \frac{4ap}{\pi} \cdot \frac{xz^2}{(x^2 + z^2 - a^2)^2 + 4a^2z^2}
\]

(2)
where $\sigma_z$ – normal stress along the z-axis (kN/m²), $p$ – distributed load on the surface of the soil half space, $a, x, z$ – geometrical characteristics, $\tau_{xz}$ – shear stress.

The formula 1 and 2 analysis implies that their use is inconvenient for the solution of the problem which is caused by the necessity of their differentiation and integration in the course of the problem solution. At the same time, the curves $\sigma_z(x, z = \text{const})$ and $\tau_{xz}(x, z = \text{const})$ represent a relatively simple form, similar to Fig. 1. Comparative analysis with the Gauss curve in the form of $\sigma_z = p_0 \cdot e^{-a_0 x^2}$ with $\sigma_z(x, z = \text{const})$ curve according to Flamant showed their good convergence in satisfying the condition of equality of the diagram area $\sigma_z(x, z = \text{const})$ and distributed load $p \cdot b$, therefore the equivalent Gauss formula is used in this paper to describe the $\sigma_z(x, z = \text{const})$ stress:

$$\sigma_{zp}(x) = p_0 \cdot e^{-a_0 x^2} \quad (3)$$

where $a_0$ – exponential parameter, $p_0$ – maximum value $\sigma_z$ along the z-axis $(x = 0)$ at the depth $z = d$ (see Fig. 1.) and is calculated according to Tables in the Set of rules SP 22.13330.2016 “Bases of buildings and structures” or scientific publications [7] on soil mechanics through the coefficient, i. e. $p_0 = K \left( \frac{x}{\alpha p} \right) \cdot P$

Equivalence of such replacement is provided by the equilibrium equation:

$$p \cdot b = \int_{-l}^{l} e^{-a_0 x^2} \, dx \approx p_0 \frac{\pi}{\alpha_0} \quad \text{при } 1 \gg b \quad (4)$$

The equilibrium condition of the layer elementary length $dx$ implies that:

$$\tau_{xz}(x) = \frac{dp}{dx} \cdot z \quad (5)$$

where $x$ and $z$ are determined according to the origin of coordinates in the center of the layer (Fig. 1). By differentiating (3) by $x$ and substituting in (7) the equation as follows is obtained (Fig. 2):

$$\tau_{xz}(x) = -2p_0 \cdot \alpha_0 \cdot x \cdot z \cdot e^{-a_0 x^2} \quad (6)$$

3. Results and discussion

Solving the problem and determining the initial critical load on foundation

The rate of layer extrusion on the base is determined under the assumption that there is a simple shift in the layer
\[ \dot{u}(x, z) = \int \hat{\tau}(x, z)dz + C \]  

where C is a constant value of integration, determined from a boundary condition \( z = \pm h \).

**Figure 2.** Diagrams of shear stress changes in the layer along x at different z = const by (8) at \( a_0 = 0,02; z = \pm 1,5; \pm 1; \pm 0,5 \). Designation: red line – value of tangential stresses at z = ±1,5, blue line-value of tangential stresses at z = ±1,0, green line - value of tangential stresses at z = ±0,5

According to the condition \( \dot{\tau}_{xz}(x,0)=0 \), the value \( x^* \) is determined, where \( \tau_{xz}(x,0) \) assumes a maximum value. Having differentiated \( \tau_{xz} \) from (6) and equated it to zero, the equation as follows is obtained:

\[ x^* = \frac{1}{\sqrt{2a}} \]  

(8)

By substituting \( x^* \) from (8) to (6), the maximum \( \tau_{xz}^{\text{max}} \) is obtained, which in the limiting state will be equal to \( \sigma_{xz}^{\text{max}}(x) \cdot \tan \varphi + c \), where \( \varphi \) is angle of friction, \( c \) – adhesion. By converting the obtained expression \( \tau_{xz}^{\text{max}} \), the starting critical load on the foundation will be as follows:

\[ P_{\text{max}}^* = \frac{c}{K(\frac{z}{b}) \left[ e^{-\sigma_{xz}^{\text{max}}(x) \cdot \frac{1}{\sqrt{2a_0}} - \tan \varphi} \right]} \]  

(9)
Rate of foundation settlement during the loose layer extrusion by Maxwell viscoelastic model

The rate of angular distortion in the loose layer is determined by (1), which, at a constant $\tau_{xz} = const$, $\dot{\tau}_{xz} = 0$ is as follows:

$$\dot{\gamma}_{xz} = \frac{\tau_{xz}}{\eta(t)}$$

(10)

By substituting this value $\dot{\gamma}_{xz}$ in (7) after integration, while considering that $C = 0$, the rate of the loose layer extrusion by $x$ and $z$ (Figs. 3, 4, 5) is obtained as follows:

$$\dot{u}(x,z) = \frac{P_0 \cdot \alpha_0 \cdot x \cdot e^{-\alpha x}}{\eta(t)} \cdot \left(h^2 - z^2\right)$$

(11)

**Figure 3.** Contours of the horizontal velocity of displacement in the weak layer (right side of Fig. 1) calculated by (15). Designation: the horizontal velocity isolines are shown by colors conditionally, from the maximum speed-the blue color isoline, to the minimum-the black color isoline.
Figure 4. Velocity plots of horizontal displacements in the middle of the weak layer. Designation: plots of speeds of horizontal displacement are shown by colors conditionally, from the maximum speed - a line of red color, to the minimum - a line of green color.

Figure 5. Velocity plots of horizontal displacements $U(z)$ in the weak layer at different distances from the $x = 0$ axis (right side). Designation: the red line is the maximum value of the speed of horizontal movements depending on $x$.

Average rate of the loose layer extrusion both along its cross section $z = \pm h$ and along the length of the layer $x = \pm l$, that is, the rate at which the loose layer volume is shrinking $(2h \times 2l)$ by soil extrusion will be equal:

$$
\dot{Q}(t) = (2l \times 2h) \cdot \overline{u}(x, z)
$$

(12)

where $\overline{u}(x, z)$ is an average rate of the loose layer extrusion.
Rate of subsidence trough of the base surface:

\[
V = \frac{(1 - \nu) \cdot p}{\pi \cdot \eta} \cdot \left( (l + a)^2 \cdot \ln((l + a)^2) - (l + a)^2 \cdot \ln(l + a) - a^2 \cdot \ln(a)^2 + a^2 \ln(a) \right)
\] (13)

By equating (12) and (13), the rate of change in the surface subsidence trough volume \( z = 0 \) is obtained:

\[
\dot{u}(x, z) = \frac{(1 - \nu) \cdot p}{\pi \cdot \eta} \cdot \left( (l + a)^2 \cdot \ln((l + a)^2) - (a + l)^2 \cdot \ln(l + a) - a^2 \cdot \ln(a)^2 + a^2 \ln(a) \right) \cdot \frac{1}{l \cdot h}
\] (14)

Numerical simulation of the rate of loose layer extrusion by PLAXIS PC using SoftSoilCreep model (Fig. 6)

![Deformed mesh](image.png)

**Figure 6.** Calculation scheme for determining the rate of Foundation precipitation in PC PLAXIS 2d. Designation: black color-the Foundation, purple color-a strong layer of soil, yellow color-a weak layer of soil.

The parameters of the computational model are determined based on odometer testing results. Results of numerical simulation of the loose layer strain-stress state (Fig. 7), which are within the compressed mass at the foundation base of a finite width, are given.

Foundation width is \( b = 5 \) m, height \( 1 \) m. Loose foundation layer with a thickness \( 2h = 3 \) m is located between two layers with the same characteristics, with higher strength characteristics; thickness of each layer is \( 5 \) m.

Comparison of calculation results with the calculation results based on the analytical solution showed a qualitative convergence, since different rheological models of the loose layer were used in these calculations.

The color scale in the figures shows movements relative to the x axis from minimum – blue to maximum – red.
Figure 7. Isofield horizontal movements for 300 days (a) and 600 days (b). Designation: bright green-movements in this area have values from 0 to $20 \times 10^{-3}$ m, green-movements in this area have values from 0 to $-20 \times 10^{-3}$ m
b) Figure 8. Horizontal displacement contours in 300 days (a) and 600 days (b)

4. Conclusions

- The problem of the soft layer extrusion from the compressed foundation base mass is important today, as construction is carried out under tough engineering and geological conditions.
- The analytical solution of this problem was obtained using Maxwell rheological calculation model describing the rate of angular distortion of loose soils from shear stress.
- The isocurves of the loose layer extrusion rates are given, which are symmetrical with respect to the center (x = 0) and have an extreme point at a certain distance from it and in the center of the layer. As the distance from the center grows, the layer extrusion rate decreases to zero at $x \to \infty$.
- The comparative assessment of the loose layer extrusion rate determined by the results of the analytical and numerical (Finite Element Method) solution showed their qualitative convergence, which indicates the possibility of the analytical solution.

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