Gravitational microlensing of gamma-ray bursts at medium optical depth

J. S. B. Wyithe\textsuperscript{1,2} and E. L. Turner\textsuperscript{2}*

\textsuperscript{1}School of Physics, University of Melbourne, Parkville, Vic. 3052, Australia \textsuperscript{2}Princeton University Observatory, Peyton Hall, Princeton, NJ 08544, USA

Accepted 2000 July 27. Received 2000 July 24; in original form 2000 June 6

\section{ABSTRACT}
Gravitational lensing of a gamma-ray burst (GRB) by a single point mass will produce a second, delayed signal. Several authors have discussed using microlensed GRBs to probe a possible cosmological population of compact objects. We analyse a closely related phenomenon: the effect of microlensing by low to medium optical depth in compact objects on the averaged observed light curve of a sample of GRBs. We discuss the cumulative measured flux as a function of time resulting from delays caused by microlensing by cosmological compact objects. The time-scale and curvature of this function describe unique values for the compact object mass and optical depth. For GRBs with durations larger than the detector resolution, limits could be placed on the mass and optical depth of cosmological compact objects. The method does not rely on the separation of lensed bursts from those that are spatially coincident.

\textbf{Key words:} gravitational lensing – gamma-rays: bursts.

\section{INTRODUCTION}

Press & Gunn (1973) were the first to propose that a cosmological abundance of dark compact objects could be detected by gravitational lensing of more distant sources. If a point mass lies along the observer–source line of sight, the relative motion between the lens, source and observer produces a change in the magnifications of two lensed images. The presence of foreground compact objects is therefore detected through a change in the observed flux of the background source (termed microlensing). This effect has been used successfully in the search for compact objects in the halo of the Milky Way galaxy (e.g. Alcock et al. 2000). Also, microlensing from stars in a galaxy at moderate redshift has been observed in the gravitationally lensed quasar Q2237+0305 (Irwin et al. 1989; Corrigan et al. 1991). The short duration of gamma-ray bursts (GRBs) offers an alternative way to study microlensing and hence to search for a cosmological population of compact objects, through observation of repeating bursts.

In this paper we assume GRBs to be at cosmological distances (Paczynski 1995), and therefore probable sources for gravitational lensing (Turner, Ostriker & Gott 1984). Paczynski (1986, 1987) noted that while multiple images of a single GRB cannot be resolved angularly by present-day detectors, their relative delay may be longer than the burst duration so that the lensed images could be resolved temporally [a pair of lensed GRB images that are produced by a single mass (M\textsubscript{CO}) have a relative time delay that is \(\Delta t \sim 50 s \times (M\textsubscript{CO}/10^6 M_\odot)\) (Mao 1992)]. Microlensing of a GRB by a compact object is therefore observed as a GRB that repeats. The utility of GRBs to explore cosmological dark matter, and the possible inference of properties of the GRB population itself were discussed in detail by Blaes & Webster (1992). The short event duration and the transparency of the universe to gamma-rays make GRBs ideal probes of dark matter in the form of compact objects over a wide range of masses.

Microlensing of existing and potential catalogues of GRBs have been used to discuss the cosmological abundance of compact objects. Marani et al. (1999) use non-detections of lensed images from the BATSE and Ulysses catalogues to set conservative limits on dark compact objects with masses between \(10^{-16}\) and \(10^{-7} M_\odot\). Also a universe proposed by Gnedin & Ostriker (1992) with \(M\textsubscript{CO} \sim 10^{6.5} M_\odot\) and \(\Omega_M = \Omega_C = 0.15\) was ruled out at a confidence level of 90 per cent. This scenario had been previously investigated in detail, using a three-dimensional (3D) lensing code by Mao (1993). He found results that did not depart significantly from those obtained by a single-screen approximation. Mao (1992) and Grossman & Nowak (1994) estimate that the waiting time for one lensed pair due to an intervening galaxy to be observed in the BATSE catalogue is between 1 and 10 yr, and conclude that it is not certain that such a lensed pair will be found by BATSE.

As in quasar lensing, the detection of a lensed pair is separated from spatially coincident events by the comparison of light curves and spectra. However, the presence of noise and the faintness of images will alter the light curves so that macro images appear dissimilar (Wambsganss 1993; Nowak & Grossman 1994). In addition, if a foreground galaxy is responsible for the lensed pair then Williams & Wijers (1997) find that microlensing by individual stars in that galaxy can smear out image light curves.

\textsuperscript{*}E-mail: swyithe@astro.Princeton.edu (JSBW); elt@astro.Princeton.edu (ELT)

\copyright\ 2000 RAS
Further increasing the chance of their being misclassified as two spatially coincident events. Light-curve similarity of a lensed pair requires the source to be much smaller than the Einstein ring radius of the lens (projected on to the source plane), a condition likely to be filled in the case of GRBs owing to the short-event duration for all but the smallest lenses. However, an anisotropic source (e.g. a beamed source) is effectively viewed at two different angles in the two images. This raises the possibility that the time variation of the two images may be different (e.g. Babul, Paczynski & Spergel 1987), although Blaes & Webster (1992) find that a source should be isotropic to lensing provided that the beaming is not too strong. Paczynski (1987) noted that the potential difficulties in identifying lensed pairs will make confirmation of the cosmological origin of the GRBs difficult through a lensing argument. On the other hand, if we assume that GRBs have a cosmological origin, the misclassification of events will lead to an underestimate of the compact object population.

The microlensing effect of a cluster of objects on a GRB was first discussed by Paczynski (1987). He found that a single instantaneous burst lensed by a screen of objects such as that produced by a galactic halo or a cluster of galaxies results in the observation of many repeats of the original burst separated by different delays. Examples of the light curves produced can be found in Paczynski (1987) and Williams & Wijers (1997). At moderate optical depths the first images have magnifications that are not correlated with their delay, however, the faint images arriving later have observed fluxes that decrease monotonically with time. Williams & Wijers (1997) investigated the flux weighted rms of the delay of clusters of microimages and found this to be a sensitive function of optical depth and shear. The temporal spread of the cluster of microimages was shown to be related to the longest side of the area of the image plane containing microimages that account for most of the macroimage flux. This scalelength is considerably larger than that of the separation of a lensed pair produced by a single microlens, and so the spread in the flux-weighted arrival time is also considerably larger than the relative delay between images produced by a single mass. This phenomenon allows smaller masses to be probed modulo the detector resolution and intrinsic GRB duration.

A non-negligible fraction of the baryon content of the universe may exist in stellar remnant form (e.g. Kerins & Carr 1994). In addition, Galactic microlensing searches (e.g. Alcock et al. 2000) provide evidence for a significant fraction of the Galactic halo mass being in the form of stellar mass compact objects, an interpretation which is supported by observations of high proper motion, cold white dwarfs (Hodgkin et al. 2000; Ibata et al. 2000). While intervening galaxies produce relatively large delays, GRBs with microsecond durations or variability would resolve microlensing by cosmological stellar mass objects (Nemiroff et al. 1998). Nemiroff et al. (1998) note that there is no known fundamental reason for the non-existence of microsecond GRBs, and find that a significant rate of these might exist and go undetected by current telescopes. They find that at a given flux level there may be more GRBs with durations between 1 and 2 ms than between 8.192 and 16.384 s (BATSE duration bin), and an abundance of microsecond spikes that is an order of magnitude greater. However, Nemiroff et al. (1998) also note that the converse possibility of short-duration spikes not existing at all is equally consistent with the analysis of current data.

We propose that if short-duration bursts were to exist then they could be used to probe the cosmological stellar mass compact object population without resolving and identifying individual microimages. Also, in the absence of short spikes, the light curves of GRBs can place upper limits on typical compact object mass and optical depth. Rather than considering the probability of lensing by a single mass on individual GRBs, we consider the microlensing effect on an ensemble of GRB light curves of a collection of point masses at low to moderate optical depth. For a sample of microlensed spike GRBs we look at the average value of the cumulative flux as a function of time.

This paper presents calculations corresponding to the microlensing effect of populations of stellar mass objects on the observed light curves of hypothetical, very-short-duration GRBs. However, the results are also applicable to the presently known population of GRBs, with durations of 10s of seconds as probes of compact objects having masses of \( \sim 10^6 M_\odot \). The relevant scaling is pointed out where appropriate.

The remainder of the paper is presented in three parts. Section 2 describes the microlensing models and Section 3 proposes a new method for analysing microlensing in GRBs. A brief summary is presented in Section 4.

2 MICROLENSING MODELS

As a crude approximation to the lensing effect of compact objects distributed along the line of sight, we assume a screen of point masses distributed randomly in a disc. The model does not include a continuous matter distribution. We use standard notation for gravitational lensing. The Einstein radius of a microlens in the image plane is denoted by \( \xi_0 \) and when projected into the source plane by \( \eta_0 \). The normalized convergence or optical depth is denoted by \( \kappa \). The normalized lens equation for a field of point masses is

\[
y = \begin{pmatrix} 1 - \gamma & 0 \\ 0 & 1 + \gamma \end{pmatrix} x + \sum_{j=0}^{N^*} \frac{(x' - x)}{|x' - x|} m_j,
\]

(1)

Here \( x \) and \( y \) are the normalized image and source positions, respectively. \( \gamma \) is the applied shear, and the \( x_j \) and \( m_j \) are the normalized positions and masses of the individual microlenses. To construct a microlensed light curve for an instantaneous microlensed GRB, equation (1) is solved for the microimage positions at many points along a predefined source line through the inversion technique of Lewis et al. (1993) and Witt (1993). The time delay and magnification are then determined for each microimage \( i \), given by

\[
\Delta T_i = \frac{\xi_0^2}{c} \frac{D_s}{D_o D_L} (1 + z_o) \times \left( \frac{(x_i - y)^2}{2} - \sum_{j=0}^{N^*} m_j \ln(|x_i - x_j|) \right)
\]

(2)

and

\[
\mu_i = \frac{1}{|\det \mathbf{A}(x_i)|}
\]

where

\[
\det \mathbf{A}(x_i) = \frac{\partial y_1}{\partial x_{i1}} \frac{\partial y_2}{\partial x_{i2}} - \left( \frac{\partial y_1}{\partial x_{i1}} \right)^2.
\]

(3)

Here \( D_o D_s \) and \( D_L \) are the angular diameter distance between the observer and the source, the observer and the lens, and the lens and source. \( c \) is the speed of light and \( z_o \) is the lens redshift. We note that the delay is proportional to \( (1 + z_o) \), but is not explicitly
dependent on the source redshift. Also, as a result of the dependence on \( \tilde{\gamma} \), \( \Delta T \) is proportional to the mean mass.

At a sufficiently large angle from the point source, there is a low-magnification image located very close to each point mass. All solutions of the lens equation must therefore be found in a region that contains a sufficient percentage of the total macroimage flux. The region of the lens plane in which image solutions need to be found to ensure that 99.9 per cent of the total macroimage flux is recovered from all points on the source line is known as the shooting region. The number of stars in the region about any point which collects 99.9 per cent of the macroimage flux was calculated by Katz, Balbus & Paczynski (1986) and is given by

\[
N_s = 300 \frac{\langle m^2 \rangle}{\langle m \rangle^2} \frac{\kappa^2}{\left[ \langle 1 - \kappa \rangle^2 - \gamma^2 \right]^2}. \tag{4}
\]

In the absence of shear (\( \gamma = 0 \)) these stars are distributed in a disc with a radius \( R_d = \sqrt{\kappa / (N_s \times \langle m \rangle)} \). The shooting region is defined by the union of these discs centred on the points \( x_1 = y_1 / (1 - \kappa), x_2 = y_2 / (1 - \kappa) \) corresponding to all parts of the source line. The minimum number of stars \( N_{\text{min}} \) required for the model is contained in a disc having a radius \( R \), which covers the shooting region. We have used 500 stars in each of our models, which is larger than \( N_{\text{min}} \) in all cases. We assume that the source is stationary with respect to the microlenses for the duration of the microlensed light curve. We find the light curves for 10 bursts per microlens field distributed along a source track of length \( 10 \eta_0 \). At optical depths between \( \kappa = 0.025 \) and 0.25 (at intervals of 0.025) we compute the light curves for \( 10^4 \) bursts.

The most probable scenario for lensing has the lens lying at a distance that is a reasonable fraction of that of the source (Turner et al. 1984). In addition, the distribution of optical depth with redshift is reasonably sharp. With this in mind we stress that our calculation uses a single screen to approximate the lensing effect of all masses along the line of sight. As an example we place the population of model GRBs at a redshift of \( z_\text{GRB} = 1 \) and a screen of compact objects at a redshift of \( z_\text{obj} = 2/3 \). For simplicity we assume that all compact objects have a unique mass \( m \).

### 3 Method of Analysis

In this section we consider the analysis of a hypothetical sample of spike GRBs that have an intrinsic duration smaller than 0.1 ms which we take as the detector bin size. From our sample of model microlensed light curves we find the cumulative flux (summed over all located microimages) that has arrived by the end of each bin. One minus the cumulative flux divided by the total flux gives the fraction of total flux that is yet to arrive \( F(\Delta T) \) for each burst \( i \) as a function of time. These curves are averaged over many bursts:

\[
F(\Delta T) = \frac{1}{N_{\text{burts}}} \sum_{i=0}^{N_{\text{burts}}} F_i(\Delta T)/N_{\text{burts}}. \tag{5}
\]

Equation (5) describes the average behaviour of the arrival time of lensed flux for a set of microlensing parameters \( \kappa \) and \( m \). Note the assumption that all lines of sight have the same average optical depth \( \kappa \). From the work on clusters of GRB microimages by Paczynski (1987) and Williams & Wijers (1997) we expect \( F(\Delta T) \) to have the following characteristics. Since the microlensing spread of the GRB is related to the size of the shooting region, which is proportional to the Einstein radius of the microlenses and therefore to \( \sqrt{m} \), \( \Delta T(m) \) such that \( F(\Delta T) = \text{constant} \) is proportional to \( m \). Secondly, at larger optical depths there are more microimages with large magnification. A given fraction of microlensed flux therefore arrives later at higher optical depths and so \( F(\Delta T) \) is larger for increased \( \kappa \). Note that when in a region where \( (1 - \kappa)^2 - \gamma^2 \approx 0 \), the magnification becomes very large, and flux from the burst will be observed at a comparable level for a time similar to the geometric delay for a trajectory at the edge of the region.

Fig. 1 shows the fraction of flux that is yet to arrive as a function of time. Cases are shown for mean masses of 1 and

![Figure 1](https://example.com/f1.png)

**Figure 1.** The fraction of flux that is yet to arrive as a function of time. The cases shown are for mean masses of 1 and \( 10 M_\odot \), for optical depths of \( \kappa = 0.025 \) (thin lines) and \( \kappa = 0.250 \) (thick lines).
Fig. 3 shows average values of the quantities was calculated using 100 model microlensed GRB light curves]. At each optical depth, the larger mass produces longer microimage delays, causing a given fraction of flux to arrive later on average. In addition, at a fixed mass a given fraction of flux arrives later at larger optical depths, producing a curve that has a smaller initial drop in the first bin followed by a decline that is more rapid than that at smaller optical depth. However, at higher \( k \), \( F(\Delta T) \) is larger for all \( \Delta T \).

We define \( T_{0.99} \) as the time \( \Delta T \) at which 99 per cent of the total microlensed flux from the GRB has arrived. For bin sizes that are small with respect to the microlensed spread of the event, \( T_{0.99} \) scales linearly with microlens mass. \( T_{0.99} \) is also a function of \( k \) and provides a natural scaling unit for time. The left-hand panel in Fig. 2 re-displays the curves from Fig. 1 with time normalized by \( T_{0.99} \) in each case \( F(\Delta T) \) where \( \Delta T = \Delta T / T_{0.99} \). At the end of each bin the normalized curves are independent of mass. The curvature therefore provides a probe of the optical depth. The right-hand panel of Fig. 2 displays \( F(\Delta T) \) for optical depths between \( k = 0.025 \) and \( 0.250 \) at a microlens mass of 10\( M_\odot \). We suggest the value of \( F(\Delta T) = 0.1 \) as an indicator of optical depth.

For values of \( k \) and \( m \) in the ranges \( 0.025 \leq k \leq 0.25 \) and \( 0.1 \leq m \leq 100 \), we made 1000 simulations of \( F(\Delta T) \) [each \( F(\Delta T) \) was calculated using 100 model microlensed GRB light curves]. Fig. 3 shows average values of the quantities \( T_{0.99} \) (dark contours) and \( F(\Delta T) = 0.1 \) (light contours) over a range of optical depths and microlens masses. In regions combining small values of optical depth with microlens masses \( m < 1 \, M_\odot \), the contours are nearly parallel, however, for most of the parameter space, values of \( T_{0.99} \) and \( F(\Delta T) = 0.1 \) describe a unique set of \( k \) and \( m \). For samples containing 100 GRBs the variance in values of \( T_{0.99} \) and \( F(\Delta T) = 0.1 \) are an order of magnitude lower than the corresponding mean.

We have demonstrated that a sample of GRBs could be used to measure the quantities \( T_{0.99} \) and \( F(\Delta T) = 0.1 \), and that this combination corresponds to measurements of \( k \) and \( m \). However, it will not be known whether the observed spread in the arrival time of GRB flux is due solely to microlensing effects or whether it is intrinsic to the source (unless the individual microimages are resolved). \( T_{0.99} \) and \( F(\Delta T) = 0.1 \) will therefore be upper bounds on the values for instantaneous bursts, and so will exclude a region of mass–optical depth parameter space rather than measure probable values. In addition, we note that Fig. 3 presents results covering only three orders of magnitude in mass. However, the bin size, mass and \( T_{0.99} \) are all linearly related, allowing scaling of the mass and \( T_{0.99} \) by the bin size divided by 0.1 ms. Estimates of upper limits for \( T_{0.99} \) and \( F(\Delta T) = 0.1 \) which will probe \( \sim 10^8 \, M_\odot \) compact objects are therefore possible using current data with GRB durations of \( \sim 10^8 \, s \).

The approach described requires that the source be integrated for long enough to be sure that the entire event flux (or some appropriately large fraction) has been accumulated. We now describe an alternative approach that does not require measurement of the total flux. Fig. 2 demonstrates that the shape of \( F(\Delta T) \) is unique for each combination of \( m \) and \( k \). For example, a curve produced by a sample of GRBs lensed by a low optical depth of high masses may have the same value \( F(\Delta T = T_{0.99}) \) as a sample lensed by smaller masses having a higher optical depth. However,
at all $\Delta T < T_{0.99}$ the former will have a smaller value of $F(\Delta T)$. We have produced the average function $F_{av}(\Delta T)$ for masses and optical depths covering the parameter space shown in Fig. 3. At each combination of mass and optical depth ($m_{\text{true}}$ and $\kappa_{\text{true}}$), we make 1000 mock observations of $F_i(\Delta T)$ each calculated from 100 GRBs. We look for the values of $m$ and $\kappa$ such that $F_{av}(\Delta T)$ best fits each mock observation $F_i(\Delta T)$. Our criteria for the best fit is to minimize the value

$$D = \max(|F_i - F_{av}|).$$  

This procedure provides a likelihood for measuring $m$ and $\kappa$ given the true values $m_{\text{true}}$ and $\kappa_{\text{true}}$:

$$p_{lh}(m, \kappa|m_{\text{true}}, \kappa_{\text{true}}).$$  

The construction of $F_{obs}(\Delta T)$ for an observed set of GRBs then provides estimates of $m$ and $\kappa$ through application of Bayes' theorem:

$$p(m, \kappa) = N \int p_{lh}(m, \kappa|m', \kappa') p_{\text{prior}}(m', \kappa') \, dm' \, d\kappa',$$

where $p_{\text{prior}}(m, \kappa)$ is the assumed prior probability for $m$ and $\kappa$, and $N$ is a normalizing constant.

To demonstrate the statistical uncertainty we have plotted in Fig. 4 the mean and variance of retrieved values of $m$ and $\kappa$ versus their assumed values ($\kappa_{\text{true}}, m_{\text{true}}$) at fixed mass and optical depth, respectively (i.e. sections through $p_{lh}$). As in the previous calculation, the figure demonstrates that a sample of 100 GRBs is sufficient to obtain a consistent result. The systematic bias (which is smaller than the statistical uncertainty) in the recovered values is caused by the finite grid of $m$ and $\kappa$ over which $F_{av}$ and $p_{lh}$ are computed. This calculation would obtain upper limits on the true values of $m$ and $\kappa$ owing to the possible intrinsic spread of the GRB. Also, the bin size and mass are linearly related allowing scaling of the case presented here.

4 DISCUSSION

If gamma-ray bursts have a cosmological origin then they provide a unique opportunity to study the cosmological abundance of compact objects through the identification of lensed pairs. However, there are several mechanisms that may lead to the misclassification of lensed pairs as spatially coincident but independent bursts. In addition, the identification of a lensed pair may be ambiguous if the relative delay is smaller than the event duration. Upper limits on the contribution to the mass density of compact objects determined from a lack of lensed pairs may therefore be underestimated.

We have demonstrated that simultaneous upper limits on the average optical depth and mass of compact objects can be estimated by measuring the average cumulative flux as a function of time for a collection of bursts and without resolving the individual lensed images. By selecting subsamples of bursts, specific populations of compact objects could be examined. For example, a collection of bursts that are spatially coincident with foreground galaxies [such as GRB 971214 (Kulkarni et al. 1998; Diercks et al. 1999)] will probe the compact object population in galactic haloes. In particular, if a population of short-duration (microsecond) cosmological gamma-ray bursts are identified, limits will be placed on the abundance of stellar mass compact objects.

ACKNOWLEDGMENTS

This work was supported by NSF grant AST98-02802. JSBW
acknowledges the support of an Australian Postgraduate award and a Melbourne University Overseas Research Experience Award. We would like to thank Rachel Webster for numerous helpful discussions.

REFERENCES

Alcock C. et al., 2000, ApJ, 542, 281
Babul A., Paczynski B., Spergel D., 1987, ApJ, 316, L49
Blaes O. M., Webster R. L., 1992, ApJ, 391, L63
Corrigan R. T. et al., 1991, AJ, 102, 34
Diercks A. H. et al., 1999, ApJ, 503, L105
Gnedin N., Ostriker J. P., 1992, ApJ, 400, 1
Grossman S. A., Nowak M. A., 1994, ApJ, 435, 548
Hodgkin S. T., Oppenheimer B. R., Hambly N. C., Jameson R. F., Smartt S. J., Steele I. A., 2000, Nat, 403, 57
Irbit R. A., Irwin M. J., Bienayme O., Scholz R., Guibert J., 2000, ApJ, 532, 41L
Irwin M. J., Webster R. L., Hewitt P. C., Corrigan R. T., Jedrzejewski R. I., 1989, AJ, 98, 1989
Katz N., Balbus S., Paczynski B., 1886, ApJ, 306, 2
Kerins E. J., Carr B. J., 1994, MNRAS, 266, 775
Kulkarni S. R. et al., 1998, Nat, 393, 35
Lewis G. F., Miralda-escude J., Richardson D. C., Wambsganss J., 1993, MNRAS, 261, 647
Mao S., 1992, ApJ, 389, L41
Mao S., 1993, ApJ, 402, 382
Marani G. F., Nemiroff R. J., Norris J. P., Hurley K., Bonnell J. T., 1999, ApJ, 512, L13
Nemiroff R. J., Norris J. P., Bonnell J. T., Marani G. F., 1998, ApJ, 494, L173
Nowak M. A., Grossman S. A., 1994, ApJ, 435, 557
Paczynski B., 1986, ApJ, 308, L51
Paczynski B., 1987, ApJ, 317, L51
Paczynski B., 1995, PASP, 107, 1167
Press W. H., Gunn J. E., 1973, ApJ, 185, 397
Turner E. L., Ostriker J. P., Gott J. R., III, 1984, ApJ, 284, 1
Wambsganss J., 1993, ApJ, 406, 29
Williams L. L. R., Wijers R. A. M. J., 1997, MNRAS, 286, L11
Witt H. J., 1993, ApJ, 403, 530

This paper has been typeset from a \TeX/L\TeX file prepared by the author.