Multiple EIT and Frequency Comb in a Mechanically Modulated Quantum Emitter

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We propose an electromechanical scheme where the electronic degree of freedom of an embedded color center is coupled to the motion of the hosting resonator via dispersive forces. The localized emitters on a free-standing two-dimensional hexagonal boron nitride membrane are considered and studied, specifically. We show that the coupling of membrane vibrations to the electronic degree of freedom of the emitter can reach the strong regime. By suitable driving of a three-level Λ-system composed of two spin degrees of freedom in the electronic ground state as well as an isolated excited state of the emitter a multiple electromagnetically induced transparency spectrum becomes available. The experimental feasibility of the efficient vibrational ground state cooling of the membrane via quantum interference effects in the two-color drive scheme is numerically confirmed. More interestingly, the emission spectrum of the defect exhibits a frequency comb with frequency spacings as small as the fundamental vibrational mode, which finds applications in high-precision spectroscopy.

Introduction.—Quantum metrology relies on the resources provided in quantum systems to beat the standard limits set by quantum physics [1]. High-precision spectroscopy that has become available thanks to the development of optical frequency combs [2, 3] is a powerful tool which may be improved further by employing quantum resources [4]. The optical quantum communication as another major quantum technology relies on efficient and broadband quantum memories where the photons can be faithfully stored and retrieved [5, 6].

The discovery of single-photon emitters (SPE) in two-dimensional (2D) systems such as WSe2 [7] and h-BN [8] represents a significant step forward for quantum technology where miniaturization of the devices without compromising their performance is of crucial importance. Such systems also hold considerable promise in the field of optomechanics and the related technologies [9, 10]. The extremely low mass and high quality mechanical resonators achievable in free-standing 2D membranes make them promising objects for sensing weak forces and small displacements [11–15]. These are capabilities that allow for the exploration of various problems in fundamental physics, e.g. the validation of the exotic decoherence models [16–18] and are advantageous in technological applications [19, 20].

Here, we propose an electromechanical system where an emitter embedded in a free-standing hexagonal boron nitride (h-BN) membrane couples to its vibrational modes via the vacuum dispersive forces. The setup even reaches a regime where the strength of the coupling between electronic states of the emitter and mechanical oscillations of the h-BN membrane exceeds their dissipation rates and characteristic frequencies, the so-called ultrastrong coupling regime. By realizing a Λ-system in the emitter and employing a two-color drive scheme we study the dynamics of the system. The scheme is tuned to the electromagnetically induced transparency (EIT) regime, where the quantum interference between two different paths opens-up a transparency window in the single-atom level [21–23]. Due to the ultrastrong coupling to the mechanical mode the EIT signal exhibits multiple absorption and transparency windows with distances matching the mechanical frequency. The emission spectrum of the emitter in this working regime—when the mechanical mode is cooled down to its ground state—is a tightly spaced frequency comb. The high emission rate of photons [8] allows for formation of a time-bin entangled string of photons that spans the comb, by employing a sequential entangling protocol [24, 25]. This highly entangled configuration has many application in quantum metrology such as quantum boosted optical spectroscopy [26–28].

We finally propose to achieve the ground state mechanical cooling via the same emitter and by applying the EIT cooling technique originally developed for trapped ions [29]. When the optical drives are both far off-resonance and blue detuned, the absorption spectrum assumes a Fano-like shape, hence, allowing a resolved-sideband transition via the emitter which then cools the mechanical vibrations down to the ground state.

Model.—A sketch of our proposed setup is given in Fig. 1(a). The h-BN monolayer membrane is suspended at a distance z from the substrate surface. We single out three states of the emitter that realize a Λ-system composed of two electronic ground states (corresponding to two different spin states of the emitter that realize a Λ-system composed of two electronic ground states (corresponding to two different spin states with energy splitting Δω) and a bright excited state [30]. The reflection of virtual photons from the underlying surface induces off-resonant transitions in the emitter. This will modify transition energy of the emitter, and consequently, induce a dispersive force on it [31]. When the emitter is hosted by an oscillating object the lowest emitter transition frequency ωeg will depend on its position. For emitters sufficiently close to the surface of the substrate (z ≪ λ), in the first order of approximation, the change in the transition frequency is given by (see supplemental information for a derivation)

\[
\delta \omega_{eg}(z) \approx \frac{3}{32} \frac{c^3}{\tau_{eg} \omega_{eg}^3 z^3} \left| \epsilon(\omega_{eg}) \right|^2 - 1 \left| \epsilon(\omega_{eg}) + 1 \right|^2, \tag{1}
\]

where ωeg and τeg = 1/γ are the free-space transition frequency and the excited state lifetime, respectively. Here, γ is the photon emission rate and ε(ω) is the electric permittivity of the substrate material. The dependence of frequency shift on the displacement results in the dispersion-force-induced frequency shift \( \delta \omega_{eg} = \frac{\partial}{\partial z} \delta \omega_{eg} \), which provides the coupling between membrane vibrations and electronic state of the emitter.
First we consider the electromagnetically induced transparency scheme. It turns out that the EIT signal in our setup assumes multiple dips and peaks [35, 36]. In fact, the Λ-system is driven by one optical control drive tuned at the $|\uparrow\rangle\langle e|$ transition, while another weak probe drives the $|\downarrow\rangle\langle e|$ transition [Fig. 1(a)]. Hamiltonian of this system in the rotating frame of the two transition frequencies [37] (with the notation $\hat{\sigma}_{ij} \equiv |i\rangle\langle j|$) is given by

$$\hat{H}_\Lambda = -\delta_p \hat{\sigma}_{ee} - (\delta_p - \delta_e) \hat{\sigma}_{\uparrow\uparrow} + \Omega \hat{b}^\dagger \hat{b} + G \hat{\sigma}_{ee}(\hat{b}^\dagger + \hat{b}) + \hat{H}_d,$$

(2)

where $\delta_p \equiv \omega_p - \omega_0$ and $\delta_e \equiv \omega_e - \omega_0 + \Delta_0$ are detuning of the probe and control fields from their respective transitions. Here, the drive Hamiltonian is given by $\hat{H}_d = \mathcal{E}_p \hat{\sigma}_{ee} + \mathcal{E}_c \hat{\sigma}_{c\uparrow} + \text{H.c.}$ with the probe and control Rabi frequencies $\mathcal{E}_c$ and $\mathcal{E}_p$ such that $\mathcal{E}_c \gg \mathcal{E}_p$. The standard EIT signal which is proportional to the steady state absorption spectrum of $\hat{\sigma}_{ee}$ manifests itself when the control field is driving on-resonance ($\delta_e = 0$). To study the longtime behavior of the system we numerically solve for the steady-state solutions of the Lindblad master equation

$$\dot{\rho} = -i[\hat{H}_\Lambda, \rho] + \frac{\gamma}{2} \sum_{k} \{ (\tilde{N}_\Omega + 1) D_k \rho + \tilde{N}_\Omega D_k^\dagger \rho \},$$

(3)

where $\Gamma \equiv \Omega/Q$ is the damping rate of the mechanical mode with quality factor $Q$ and $\tilde{N}_\Omega = \langle \exp\{\frac{\hbar \omega}{k_B T} \} - 1 \rangle^{-1}$ is the thermal occupation number of a bosonic mode with frequency $\omega$ at temperature $T$. ($k_B$ is the Boltzmann constant). The results are presented in Fig. 2 where the modification of the absorption spectrum of $|\downarrow\rangle\langle e|$ with the coupling strength is presented in a density plot. The plot indicates that alongside the normal transparency window occurring at $\delta_p = 0$ the system exhibits multiple valleys and peaks at mechanical sidebands $\delta_p = \pm k \Omega$ ($k = \pm 1, \pm 2, \cdots$) when the emitter is strongly interacting with the mechanical mode. The stronger the coupling, the more sideband appear mostly towards the positive detuning values. The latter is indeed because of our choice of $\tilde{N}_\Omega = 0$, i.e. the assumption of a ground state cooled mechanical mode. We will justify this assumption, subsequently.

To better understand the behavior of the EIT signal, we derive an approximate analytical expression for $\rho_{\downarrow e}$ which is proportional to the probe absorption spectrum. We first apply the unitary transformation $\hat{U} = \exp\{\frac{i}{\hbar} \sum \hat{\sigma}_{ee}(\hat{b}^\dagger - \hat{b})\}$ to the system Hamiltonian in (2). Then by assuming no back-action from the emitter on the mechanical mode we calculate the steady-state mechanical correlation functions. This is justified for small drives $\mathcal{E}_p, \mathcal{E}_c < \gamma$ for which the excited state population remains negligible ($\rho_{ee} \approx 0$). With this we arrive at the following approximate relation [32]:

$$\rho_{\downarrow e} \approx \frac{i \mathcal{E}_p \alpha}{\gamma/2 - i \delta_p + \mathcal{E}_c^2 \alpha^2 \sum_{n=0}^\infty \sum_{l=0}^\infty \langle n | \frac{L(n,k)}{2n} | l \rangle^2 \Delta(n,k)},$$

(4)

where $\alpha \equiv \exp\{-\frac{1}{2} \langle \mathcal{G} \rangle^2 (2\tilde{N}_\Omega + 1)\}$ is the signal attenuation coefficient as a consequence of the mechanical vibrations and

In Fig. 1(b) the normalized value of $\mathcal{G}$ versus distance from the surface is given considering the inhomogeneous broadening of the observed emission frequencies $\omega_{eg}$.

**Parameters.**—Apart from distance to the substrate, the absolute value of the electromechanical coupling rate $G \equiv \mathcal{G} z_{zp}$ depends on the emission frequency $\omega_{eg}$, emission rate $\gamma$, and amplitude of the zero-point fluctuations $z_{zp} = \sqrt{\hbar/2m\Omega}$. The latter, in turn, depends on the geometry of the membrane via the vibrational frequency and the effective mass of its normal modes. We consider a long rectangular strip of dimensions $L \cdot w \cdot h$ clamped at both ends with tensile strain $\epsilon$ [Fig. 1]. The normal modes of this geometry are found by solving the elastic wave equation with proper boundary conditions [32, 33]. For a monolayer hBN nano-ribbon with determined dimensions the coupling rate can be tuned by varying clamping strain. The coupling strength for the fundamental vibrational mode of a $1 \mu$m ribbon to the electronic degree of freedom of an emitter in the middle of the strip is plotted in Fig. 1(c) as a function of strain. We notice that coupling rates as high as $\approx 50$ MHz are realistic in this setup. In our calculations we have assumed a strip of width $w = 10$ nm and a free-space emission lifetime of $\tau_{eg} \approx 3.2$ ns for the emitter [34]. With these parameters the system achieves the strong coupling regime, i.e. $|G| \geq 1/\tau_{eg}$. In the remainder of the paper we discuss the manifestations of such strong coupling in the optical properties of the emitter.

**Multiple EIT.**—First we consider the electromagnetically induced transparency scheme. It turns out that the EIT signal in our setup assumes multiple dips and peaks [35, 36]. In fact, the Λ-system is driven by one optical control drive tuned at the $|\uparrow\rangle\langle e|$ transition, while another weak probe drives the $|\downarrow\rangle\langle e|$ transition [Fig. 1(a)]. Hamiltonian of this system in the rotating frame of the two transition frequencies [37] (with the notation $\hat{\sigma}_{ij} \equiv |i\rangle\langle j|$) is given by

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To better understand the behavior of the EIT signal, we derive an approximate analytical expression for $\rho_{\downarrow e}$ which is proportional to the probe absorption spectrum. We first apply the unitary transformation $\hat{U} = \exp\{\frac{i}{\hbar} \sum \hat{\sigma}_{ee}(\hat{b}^\dagger - \hat{b})\}$ to the system Hamiltonian in (2). Then by assuming no back-action from the emitter on the mechanical mode we calculate the steady-state mechanical correlation functions. This is justified for small drives $\mathcal{E}_p, \mathcal{E}_c < \gamma$ for which the excited state population remains negligible ($\rho_{ee} \approx 0$). With this we arrive at the following approximate relation [32]:

$$\rho_{\downarrow e} \approx \frac{i \mathcal{E}_p \alpha}{\gamma/2 - i \delta_p + \mathcal{E}_c^2 \alpha^2 \sum_{n=0}^\infty \sum_{l=0}^\infty \langle n | \frac{L(n,k)}{2n} | l \rangle^2 \Delta(n,k)},$$

(4)

where $\alpha \equiv \exp\{-\frac{1}{2} \langle \mathcal{G} \rangle^2 (2\tilde{N}_\Omega + 1)\}$ is the signal attenuation coefficient as a consequence of the mechanical vibrations and
shows that for $\bar{N}_G \gg 1$ the signal will be completely demolished. Also we have introduced the Lorentzian function
\[ L(n, k) \equiv (\frac{\bar{G}}{\Omega})^{2n} \frac{\sqrt{n!}}{\pi^{2n/2} [1 - \ell(n/2)]^{1/2}} \] illuminating the mechanical sidebands. We emphasize that Eq. (4) is only valid for small coupling rates $G \ll \Omega$ where the dynamics of the mechanical mode is sufficiently independent of the emitter’s excited state.

For comparison, the analytical expression is also presented in the absorption spectra in Fig. 2. Although the analytical expression (4) captures most features of the spectrum, one obviously finds the build-up of discrepancy between the numerical and analytical results as the coupling rate increases. This includes the small blue sideband ($\delta_p = -\Omega$) spectral feature that is an indication of the mechanical back-action. Furthermore, the setup exhibits multiple sharp dispersive features in the imaginary part of the $\delta_{pe}$ transition located at integer multiples of the mechanical frequency, see the supplemental material [32]. In practice, one needs to enhance the signal by either placing the setup in a cavity [21] or focusing a light beam on the emitter [23]. The former can alternatively be realized in an h-BN phononic crystal cavity [38].

**Frequency Comb.**—We now turn to the study of the resonance fluorescence spectrum of the emitter. The spectrum about emitter transition frequencies is given by
\[ S(\omega) = \lim_{t \to \infty} \int_{-\infty}^{\infty} d\tau e^{-i\omega t} \left[ e^{i(\omega_{eg} - \Delta_0)\tau} \langle \delta_{ee}(t + \tau) \delta_{ee}(t) \rangle + e^{i(\omega_{eg} + \Delta_0)\tau} \langle \delta_{ee}(t + \tau) \delta_{ee}(t) \rangle \right], \]
where the steady state two-time correlation functions are calculated by the quantum regression theorem [39, 40]. We numerically evaluate this function for the strong $G \simeq \Omega$ and ultrastrong $G \gg \Omega$ coupling regimes. We find that in the strong coupling regime, the mechanical mode manifests itself as small peaks at the sidebands in the emission spectrum. However, in the ultrastrong coupling case a frequency comb with frequency distances determined by the mechanical mode frequency $\Omega$ emerges, see Fig. 3. Such equally spaced and tightly arranged frequency combs are useful in precision measurements and metrology as well as light storage quantum memory [41].

The comb that arises from the strongly mixed electronic–mechanical system provides an efficient quantum memory based on photon echo [41–43]. The comb finesse, defined as the ratio of peak separation and linewidth, is a limiting factor in such schemes, equals the mechanical quality factor in our setup and can assume very large values $F = \Omega / \Gamma \sim 10^4$. Hence, the efficiency of such quantum memory is anticipated to be close to unity $\eta \approx e^{-\gamma/2^2} \sim 1$ and only limited by the cavity finesse [38, 41]. Another interesting feature of the comb is the possibility of creating a time-frequency chain of entangled photons. Since the single photons emitted by the color center assume any of the peaked frequencies depicted in Fig. 3 with a finite probability, application of a photon entangling protocol that employs the same $\Lambda$-system as an ancillary will create a comb of time-bin entangled photons [25, 44, 45]. These are very useful in super-precision quantum imaging and metrology [1, 46].

**Cooling.**—For accessing the multiple EIT and consequently the frequency comb structure discussed in the two above sections one has to work at low temperatures where the thermal occupation of the mechanical becomes negligible. In our setup the thermal occupation number of the mechanical fundamental mode will be still considerable even at temperatures as low as $T = 0.1$ K. Hence, a mechanism must be invoked for cooling the mechanical mode. Sideband cooling scheme is the most convenient method, however, at the chosen working point the conditions for ground state cooling are not satisfied [47]. Most importantly the mechanical sidebands are not resolved $\ell_{eg} \Omega > 1$ in the spectrum of the qubit. Yet, the emitter is strongly coupled to the mechanical mode and the mechanical back-actions are not suppressed [48]. We instead choose to use EIT cooling technique, originally developed for trapped ions, which has been proven to be even more efficient than sideband cooling [29]. In this technique the control
transition of the Λ-system $\hat{\sigma}_1$ is driven blue detuned from resonance with a Rabi frequency such that the large decay rate of the excited state is significantly reduced by quantum interference effects in a Fano-like resonance spectrum for a probe field. The system is thus tuned such that the phonon annihilation processes are significantly more probable than the phonon absorption processes. One finds that when the probe field is driven at $E_p = 0.1\varepsilon_c$ and $\delta_0 = +10\Omega$. (b) Absorption spectrum of the probe field in the absence (black) and presence (blue) of coupling to the mechanical mode. (c) Variations of the steady state mechanical occupation number with probe drive detuning (solid green line). The absorption spectrum as well as the phonon assisted transitions are also shown for reference (see the text to a discussion). In (b) and (c) $\varepsilon_c = 14\Omega$ and $N_\Omega = 210$ corresponding to the temperature $T = 0.1$ K.

The feasibility of steady-state ground state cooling of an h-BN nano-ribbon via dispersive electromechanical coupling is examined numerically by finding the steady state solution of the quantum optical master equation (3), $\hat{\rho}$. In Fig. 4 the numerical results are summarized where we present the final occupation number of the mechanical mode $\bar{n} = \text{Tr} \{ \hat{b}\hat{b} \hat{\rho} \}$ and the cooling efficiency $\bar{N}_\Omega$. The complicated cooling pattern in Fig. 4(a) is understood by generalizing the EIT cooling mechanism which results from the narrowed absorption spectrum of the Λ-system at large detunings to the multiple absorption pattern resulting from the strong coupling to the mechanical mode [Fig. 4(b)]. This is clearly observable in Fig. 4(c) where we plot $\bar{n}$ alongside the system absorption. One finds that when the probe field is driven at the proper detuning values the phonon absorption processes are enhanced while the phonon preserving and creating transitions are largely suppressed. The minimum steady state phonon number ($\tau_{\text{min}} \approx 0.12$) is obtained when the probe is driven next to the first Fano peak which is the global maximum of the spectrum. The follow-up local minima in $\bar{n}$ are understood with the same reasoning. The effect of pure dephasing in the excited state of the emitter can be included by adding $\propto (\hat{\gamma}/2) \hat{b} \hat{\rho} \hat{b}^\dagger$ to the right-hand-side of the master equation in Eq. (3). Throughout this paper we have omitted pure dephasing of the emitter excited state, an assumption based on the recent observation of emission rate-limited linewidth [49]. Nonetheless, our simulations show that a moderate dephasing rate affects the multiple EIT and frequency comb only insignificantly (not shown here), while it can be detrimental for the ground state cooling. For instance, with a dephasing rate $\hat{\gamma} = \gamma$ the minimum achievable mechanical occupation number with the same parameters used in Fig. 4(c) rises to $\approx 1.21$.

Summary.—We have proposed an electro-mechanical set-up based on the emitters in a free-standing membrane of h-BN. We have shown that the dispersive forces induced coupling strength can reach the ultrastrong regime. The system, therefore, is anticipated to exhibit a rich variety of interesting phenomena. By solving the quantum optical master equation the appearance of multiple EIT and formation of tightly spaced frequency comb are studied in this paper as well as efficient ground-state cooling of the mechanical mode by EIT-cooling technique.

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\[ \frac{\bar{n}}{n_{\text{in}}} \approx 0.12 \]

\[ \bar{N}_\Omega \approx 210 \]

\[ \varepsilon_c = 14\Omega \]

\[ N_\Omega = 210 \]

\[ T = 0.1 \text{ K} \]

\[ E_p = 0.1\varepsilon_c \]

\[ \delta_0 = +10\Omega \]

\[ \hat{\gamma} = \gamma \]

\[ \bar{N}_\Omega \approx 1.21 \]

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Supplemental Information

Dispersion forces

The frequency shift in ground state of a non-magnetic two-level atom at zero temperature in the leading order of perturbation theory is

\[
\delta \omega_g(r) = -\frac{\mu_0}{\pi \hbar} \frac{\omega_e^2}{\omega + \omega_e} \mathbf{d}^\prime \cdot \mathbf{G}(r, r, \omega) \cdot \mathbf{d},
\]

(S1)

where \( \mathbf{d} \) is the vector of atomic dipole moments, \( \mathbf{G} \) is the first-rank Green tensor of the body, and \( P \) denotes principal value integration. There is a contribution to this frequency shift which does not depend on the position of the atom, which is referred to as the Lamb shift. It is intuitive to write the frequency shift in terms of atom ground state polarizability as

\[
\delta \omega_g(r) = \frac{\mu_0}{2\pi} \int_0^\infty du \frac{u^2}{\omega_e^2 + u^2} \mathbf{d}^\prime \cdot \mathbf{G}(r, r, iu) \cdot \mathbf{d},
\]

where the principal value integration is coded inside atom polarizability definition

\[
\alpha(\omega) = \lim_{\zeta \to 0} \frac{2\omega_e \mathbf{d}^\prime}{\omega_e^2 - \omega^2 - i\omega \zeta}.
\]

(S3)

This equation clarifies the origin of the dispersion force, where the frequency shift is induced by the dipole interaction of the emitter with its own emissions.

The absolute frequency shift of the excited state cannot be calculated in the same way it is for the ground state, i.e. application of time-independent perturbation theory is problematic because the method does not consider the limited lifetime of the excited state. Moreover, the perturbative treatment of the problem breaks down in the strong atom-field coupling regime. Therefore, a dynamical method is invoked based on the principles of the motion of a charged particle in the electromagnetic fields. This eventually leads us to the following relation for the frequency shift in the excited state

\[
\delta \omega_e = -\frac{\mu_0 \omega_e^2}{\hbar} \mathbf{d}^\prime \cdot \text{Re}\{\mathbf{G}(r, r, \omega_e)\} \cdot \mathbf{d} + \frac{\mu_0 \omega_e^2}{\pi \hbar} \int_0^\infty du \frac{u^2}{\omega_e^2 + u^2} \mathbf{d}^\prime \cdot \mathbf{G}(r, r, iu) \cdot \mathbf{d}.
\]

(S4)

The transition frequency and rate shifts of a two-level atom next to a semi-infinite surface with respect to the free-space values in the first order approximation are [31]

\[
\delta \omega_{eg} = -\frac{\mu_0 \omega_{eg}^2}{\hbar} \mathbf{d}^\prime \cdot \text{Re}\{\mathbf{G}(r, r, \omega_{eg})\} \cdot \mathbf{d} + 2\frac{\mu_0 \omega_{eg}^2}{\pi \hbar} \int_0^\infty du \frac{u^2}{\omega_{eg}^2 + u^2} \mathbf{d}^\prime \cdot \mathbf{G}(r, r, iu) \cdot \mathbf{d},
\]

(S5a)

\[
\delta \gamma_{eg} = 2\frac{\mu_0 \omega_{eg}^2}{\hbar} \mathbf{d}^\prime \cdot \text{Im}\{\mathbf{G}(r, r, \omega_{eg})\} \cdot \mathbf{d}.
\]

(S5b)

From here on, the problem boils down to the specification of the Green’s tensor. The scattering Green tensor for a semi-infinite half space which contains a homogenous, dispersing, and absorbing magneto-dielectric medium (in \( z < 0 \)) is isotropic in the \( xy \)-plane (the direction of our interest) for \( z > 0 \) and reads [31]

\[
\mathbf{G}(r, r, \omega) = \frac{i}{8\pi} \int_0^\infty d\kappa \frac{\kappa}{q} e^{i\kappa q} \text{diag}\{r_s - \frac{c^2 q^2}{\omega^2} r_p, r_s - \frac{c^2 q^2}{\omega^2} r_p, 2\frac{c^2 \kappa^2}{\omega^2} r_p\},
\]

(S6)

where \( \text{diag}\{\cdots\} \) is a \( 3 \times 3 \) diagonal matrix with the specified diagonal elements, while \( r_s \) and \( r_p \) are the reflection coefficients for \( s \)- and \( p \)-polarized waves, respectively. Also, \( \kappa = \sqrt{k_x^2 + k_y^2} \) and \( q = k_z \) are the parallel and perpendicular components of the wave vector with respect to the planar interface between the substrate and the free space [Fig. 1].

For a homogeneous and isotropic substrate these coefficients are given by

\[
r_s = \frac{\mu q - \tilde{q}}{\mu q + \tilde{q}}, \quad r_p = \frac{\epsilon q - \tilde{q}}{\epsilon q + \tilde{q}},
\]

(S7)
with \( q^2 + \kappa^2 = \omega^2/c^2 \) and \( q^2 + \kappa^2 = \varepsilon\mu\omega^2/c^2 \). Here, \( \varepsilon \) and \( \mu \) are the relative permittivity and permeability of the medium. The dielectric permittivity can be expressed as \( \varepsilon = \varepsilon' + i\varepsilon'' \), where \( \varepsilon' \) and \( \varepsilon'' \) are proportional to the dispersion and dissipation, respectively. Without loose of generality—in the case of V_{N}N_{B} defects—we assume a dipole moment on the \( xz \)-plane and rewrite the integral in terms of the normalized wave vector \( \vec{k} = \kappa c/\omega \):

\[
G(\vec{r}, \omega) = \frac{i\omega}{8\pi c} \int_{0}^{\infty} d\vec{k} \frac{\vec{k}}{\sqrt{1 - \kappa^2}} e^{2izc} \sqrt{1 - \kappa^2} \left\{ \frac{1}{\sqrt{1 - \kappa^2 + \sqrt{\varepsilon - \kappa^2}}} + (\kappa^2 - 1) \frac{\varepsilon\sqrt{1 - \kappa^2 + \sqrt{\varepsilon - \kappa^2}}}{\varepsilon\sqrt{1 - \kappa^2 + \sqrt{\varepsilon - \kappa^2}}} \right\} \text{diag}\{1,1,0\}
+ \left( 2\kappa^2 e\sqrt{1 - \kappa^2 + \sqrt{\varepsilon - \kappa^2}} \right) \text{diag}\{0,0,1\},
\]

(S8)

where we have assumed a non-magnetic substrate \( \mu = 1 \). We split integration into two pieces: low and high wave vector: \( G(\vec{r}, \omega) = \int_{0}^{\kappa_0} d\kappa G(\kappa) + \int_{\kappa_0}^{\infty} d\kappa G(\kappa) \), where \( G \) is a shorthand for the integrand in Eq. (S8).

For \( \kappa \gg 1,\sqrt{\varepsilon} \) the free space dispersion relation ensures that \( e^{2izc} = e^{-2izc} \), thus the integrand decays exponentially. This introduces a cutoff in the high wave vector contribution of the Green tensor \( G \). This cutoff depends on the distance of emitter from the interface \( z \). So, the high wave vector contribution is split into \( \int_{\kappa_0}^{\infty} d\kappa G(\kappa) = \int_{\kappa_0}^{\kappa_0/\varepsilon\omega} d\kappa G(\kappa) + \int_{\kappa_0/\varepsilon\omega}^{\infty} d\kappa G(\kappa) \). As the dipole emitter gets closer to the surface, larger domain of wave vectors contribute in the integration. If \( z\omega/c \ll 1 \) one expects that the integration is dominant within \( 1 \leq \kappa \lesssim c/\omega \). This results in

\[
G(\vec{r}, \omega) \approx \frac{\varepsilon}{32\pi\omega^3 c^3} \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} \text{diag}\{1,1,2\}.
\]

(S9)

Here, the optical emission wavelength is around \( \lambda \approx 650 \) nm, which implies the \( z \lesssim 20 \) nm for the validity of the above approximation. With these, we arrived at the following equations:

\[
\delta\omega_\varepsilon \approx -\frac{d^2\omega}{32\pi\varepsilon_0^2 c^3} \left\{ \frac{\varepsilon(\omega_\varepsilon)|\varepsilon(\omega_\varepsilon)|^2 - 1}{\varepsilon(\omega_\varepsilon) + 1}^2 \right\} + \frac{d^2\omega_\varepsilon}{8\pi\varepsilon_0^2 c^3} \int_{\omega_\varepsilon}^{\omega} \frac{du}{u^2} \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1},
\]

\[
\delta\gamma_\varepsilon \approx \frac{d^2\omega}{8\pi\varepsilon_0^2 c^3} \frac{\varepsilon(\omega_\varepsilon)|\varepsilon(\omega_\varepsilon)|^2 - 1}{\varepsilon(\omega_\varepsilon) + 1}^2.
\]

(S10a, S10b)

In our analysis, only resonant frequency shift (the first term in the first line) is taken into account and the (small) effect of the non-resonant term is neglected.

### Elasticity of the strip

In this section we present details of study on the elasticity of the strip. We denote the vertical displacement of the membrane with respect to the equilibrium position by \( \zeta(x,t) \). Elastic wave equation for this deflection variable is given by [33]

\[
\rho h \partial_t^2 \zeta = -D\partial_x^4 \zeta + T \partial_x^2 \zeta,
\]

(S11)

where \( \rho \) is mass density of h-BN and \( h \) is thickness of the membrane. In the paper we assume a monolayer h-BN membrane, therefore, \( h \approx 3.3 \) Å. In Eq. (S11) \( D = Eh^3/12(1 - \alpha^2) \) with modulus of elasticity \( E \) and Poisson ratio \( \alpha \). Also, \( T = Eh\varepsilon \) is the uniform tensile force exerted on the boundary of the membrane with tensile (positive) strain \( \varepsilon \).

By separation of variables into position and time functions \( \zeta(x,t) = \Phi(t)\Psi(x) \) the elastic wave equation simplifies to an eigenvalue problem

\[
\frac{d^4}{dx^4} \Psi - \frac{T}{D} \frac{d^2}{dx^2} \Psi - \frac{\lambda^4}{D} \Psi = 0,
\]

(S12)

whose solutions determine eigen-frequencies \( \omega^2 = \lambda^4/\rho h \) and eigen-modes \( \Psi \) of the membrane. To solve Eq. (S12) we note that it can be rewritten as

\[
\left( \frac{d^2}{dx^2} + \frac{\alpha^2}{L^2} \right) \left( \frac{d^2}{dx^2} - \frac{\beta^2}{L^2} \right) \Psi = 0,
\]

(S13)

with \( \beta^2 - \alpha^2 = TL^2/D \) and \( \alpha^2\beta^2 = \lambda^4L^4/D \). Here, \( L \) is the length of strip. The above equation factorizes into

\[
\left( \frac{d^2}{dx^2} + \frac{\alpha^2}{L^2} \right) \Psi_1 = 0,
\]

(S14a)

\[
\left( \frac{d^2}{dx^2} - \frac{\beta^2}{L^2} \right) \Psi_2 = 0.
\]

(S14b)
Any superposition of \( \Psi_1 \) and \( \Psi_2 \) then satisfies Eq. (S12). The solutions thus are (\( \gamma^2 \equiv TL^2/D \))
\[
\Psi_\alpha(x) = A_\alpha \sin(\alpha \frac{x}{L}) + B_\alpha \cos(\alpha \frac{x}{L}) + C_\alpha \sinh \left[ \frac{x}{L} \sqrt{\alpha^2 + \gamma^2} \right] + D_\alpha \cosh \left[ \frac{x}{L} \sqrt{\alpha^2 + \gamma^2} \right]. \tag{S15}
\]
We apply clamped strip boundary conditions, which are \( \Psi_\alpha = \Psi_\alpha' = 0 \) at \( x = 0 \) and \( x = L \). These bring us to the following eigenvalue equation
\[
2 \cos \alpha \cosh[\sqrt{\alpha^2 + \gamma^2}] - \frac{\gamma^2}{\alpha \sqrt{\alpha^2 + \gamma^2}} \sin \alpha \sinh[\sqrt{\alpha^2 + \gamma^2}] = 2. \tag{S16}
\]
The mode profiles keeping in mind that \( \beta^2 = \alpha^2 + \gamma^2 \) are then
\[
\Psi_\alpha(x) = A_\alpha \left\{ \frac{\sin(\alpha \frac{x}{L}) - \frac{\alpha}{\beta} \sinh(\beta \frac{x}{L})}{\sin \alpha - \frac{\alpha}{\beta} \sinh \beta} - \frac{\cos(\alpha \frac{x}{L}) - \cosh(\beta \frac{x}{L})}{\cos \alpha - \cosh \beta} \right\}, \tag{S17}
\]
where \( A_\alpha \) is a normalization factor chosen such that \( \max(\Psi_\alpha(x)) = 1 \). The mode frequencies for a given \( \gamma \) which in turn is set by the strain in \( \gamma = 12(1 - \sigma^2)/(L/h)^2 \varepsilon \) are given by
\[
\omega_\alpha = \sqrt{\frac{D}{\rho}} \hbar (\alpha/L)^2 \left( \alpha^2 + \gamma^2 \right)^{\frac{1}{4}}. \tag{S18}
\]
In the main text we have only considered the fundamental mode which is specified by \( \Omega \equiv \min\{\omega_\alpha\} \).

Multiple EIT and slow-light

The \( \Lambda \) system allows for observation of the electromagnetically induced transparency (EIT) in the defect. However, the strong interaction of the excited state with the mechanical mode shall result-in its splitting into single and multiple phonon transitions and thus giving a multiple EIT pattern [35, 36]. In an EIT setup the legs of the \( \Lambda \)-system are driven by two coherent drives, where one of them is weak (probe) and the other is strong (pump). Let us assume that the \( |e\rangle \langle \uparrow| \) transition is driven by the control field with Rabi frequency \( \mathcal{E}_c \) and the \( |e\rangle \langle \downarrow| \) is driven with a weak probe \( \mathcal{E}_p \) where \( \mathcal{E}_p \ll \mathcal{E}_c \). Then the Hamiltonian written in the rotating frame of the drive frequencies reads [37]
\[
\hat{H}_\Lambda = -\delta_p \sigma_{ee} - (\delta_p - \delta_c) \sigma_{\uparrow\uparrow} + \Omega \hat{b}^\dagger \hat{b} + G \sigma_{ee} (\hat{b} + \hat{b}^\dagger) + \left( \mathcal{E}_p \sigma_{\uparrow\downarrow} + \mathcal{E}_c \sigma_{\uparrow\uparrow} + \text{H.c.} \right). \tag{S19}
\]
The set of emitter Langevin equations
\[
\dot{\sigma}_{ee} = -\left( \frac{\gamma}{2} - i \left[ \delta_p - G(\hat{b} + \hat{b}^\dagger) \right] \right) \sigma_{ee} + i \mathcal{E}_p (\sigma_{ee} - \sigma_{\uparrow\uparrow}) - i \mathcal{E}_c \sigma_{\uparrow\downarrow}, \tag{S20a}
\]
\[
\dot{\sigma}_{\uparrow\downarrow} = -i \left[ \delta_p - G(\hat{b} + \hat{b}^\dagger) \right] \sigma_{\uparrow\downarrow} + \mathcal{E}_c (\sigma_{ee} - \sigma_{\uparrow\uparrow}) - i \mathcal{E}_p \sigma_{\uparrow\downarrow}, \tag{S20b}
\]
\[
\dot{\sigma}_{\uparrow\uparrow} = -i \mathcal{E}_c (\sigma_{ee} - \sigma_{\uparrow\downarrow}) - i \mathcal{E}_p \sigma_{\uparrow\downarrow} + i \mathcal{E}_c \sigma_{\uparrow\downarrow}, \tag{S20c}
\]
along with the mechanical equation of motion \( \dot{\hat{b}} = -(\frac{\alpha}{2} + \Omega) \hat{b} - iG \sigma_{ee} + \sqrt{\Gamma} \hat{b}_{in} \) and their Hermitian conjugate equations describe the system dynamics. The non-vanishing mechanical noise correlation functions are \( \langle \hat{b}_{in}(t) \hat{b}_{in}^\dagger(t') \rangle = \tilde{N}_\Omega \delta(t - t') \) and \( \langle \hat{b}_{in}^\dagger(t) \hat{b}_{in}(t') \rangle = (\tilde{N}_\Omega + 1) \delta(t - t') \). In the steady state, for moderate control drives one expects to have \( \langle \dot{\sigma}_{ee} \rangle = \langle \dot{\sigma}_{\uparrow\downarrow} \rangle = \langle \dot{\sigma}_{\uparrow\uparrow} \rangle \approx 0 \) while \( \langle \dot{\sigma}_{\downarrow\uparrow} \rangle \approx 1 \). This simplifies the above equations to
\[
\dot{\sigma}_{ee} = -\left( \frac{\gamma}{2} - i \left[ \delta_p - G(\hat{b} + \hat{b}^\dagger) \right] \right) \sigma_{ee} - i \mathcal{E}_c \sigma_{\uparrow\uparrow} - i \mathcal{E}_p, \tag{S21a}
\]
\[
\dot{\sigma}_{\uparrow\downarrow} = -i \left[ \delta_p - G(\hat{b} + \hat{b}^\dagger) \right] \sigma_{\uparrow\downarrow} + i \mathcal{E}_c \sigma_{\uparrow\uparrow} - i \mathcal{E}_p, \tag{S21b}
\]
along with the mechanical equation of motion \( \dot{\hat{b}} = -(\frac{\alpha}{2} + i\tilde{\Omega}) \hat{b} + \sqrt{\Gamma} \hat{b}_{in} \), which omits the emitter back-action on the mechanical mode. To find an analytical expression for the EIT signal we apply the polaron transform \( \hat{U} = \exp \left\{ \frac{G}{\Omega} \sigma_{ee}(\hat{b}^\dagger - \hat{b}) \right\} \)
FIG. S1. Slow light in the EIT as function of probe detuning and the strength of coupling to the mechanical mode (left). In the right panel the cut through at $\tilde{G} = \Omega$ is illustrated. The parameters are the same as Fig. 2.

to the equations (S21) to arrive at

$$\dot{\hat{\sigma}}_\downarrow \equiv -\left( \frac{\gamma}{2} - i\tilde{\delta}_p \right) \hat{\sigma}_\downarrow + i\hat{E}_c e^{-i\hat{P}} \hat{\sigma}_\downarrow - i\hat{E}_p e^{-i\hat{P}},$$  \hspace{1cm} \text{(S22a)}

$$\dot{\hat{\sigma}}_\uparrow \equiv +i(\tilde{\delta}_p - \tilde{\delta}_c) \hat{\sigma}_\uparrow - i\hat{E}_c \hat{\sigma}_\downarrow e^{i\hat{P}},$$  \hspace{1cm} \text{(S22b)}

where we have introduced the effective detuning $\tilde{\delta}_p = \delta_p - G^2/\Omega$ and the momentum operator $\hat{P} \equiv i\frac{G}{\Omega} (\hat{b}^\dagger - \hat{b})$. The rigorous steady-state solution for the mechanical equation gives

$$\hat{P}(t) = i\sqrt{\Gamma} \int_{-\infty}^{t} d\tau e^{-\Gamma(t - \tau)/2} \left[ \hat{b}_m e^{-i\Omega(t - \tau)} - \text{H.c.} \right].$$  \hspace{1cm} \text{(S23)}

One now solves for the steady-state solutions of Eqs. (S22) using Gaussian nature of operator $\hat{P}(t)$ in Eq. (S23). We first integrate (S22b), plugging the solution into (S22a) and integrating it brings us to

$$\hat{\sigma}_\downarrow(t) = \int_{-\infty}^{t} ds e^{-\Gamma(t - s)/2} \left\{ -i\hat{E}_p \langle e^{-i\hat{P}(s)} \rangle - E_c^2 \int_{-\infty}^{s} d\tau e^{i\delta_p(s - \tau)} \hat{\sigma}_\downarrow(\tau) \langle e^{-i\hat{P}(\tau)} e^{i\hat{P}(\tau)} \rangle \right\},$$  \hspace{1cm} \text{(S24)}

where for the sake of simplicity we have taken $\tilde{\delta}_c = 0$ and the expectation values are computed for the mechanical steady state. Note that in writing the above solution we have imposed a mean-field approximation disentangling the emitter and mechanical degrees of freedom. The two above mechanical expectation values are easily calculated thanks to the Gaussian nature of (S23) which in turn is inherited from the noise operators. For Gaussian operators one has $\langle \exp\{i\hat{\phi}\} \rangle = \exp\{-\frac{1}{2} \langle \hat{\phi}^2 \rangle \}$.

The effect associated with EIT is the slow light where the photon pulses experience significant slow-down in their group velocity due to a sharp slope in the real part susceptibility, i.e. the dispersion, which is equivalent to a high refractive index. In Fig. S1 we depict the multiple slow-light associated with the EIT in the setup presented in Fig. 2.