Sivers function in light-cone quark model and azimuthal spin asymmetries in pion electroproduction

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Abstract

We perform a calculation of Sivers function in a light-cone SU(6) quark-diquark model with both scalar diquark and vector diquark spectators. We derive the transverse momentum dependent light-cone wave function of the proton by taking into account the Melosh-Wigner rotation. By adopting one-gluon exchange, we obtain a non-vanishing Sivers function of \textit{down} quark from interference of proton spin amplitudes. We analyze the \(|P_{h\perp}|\) weighted Sivers asymmetries in \(\pi^+\), \(\pi^-\) and \(\pi^0\) electroproduction off transverse polarized proton target, averaged and not averaged by the kinematics of HERMES experiment.

Key words: Sivers function, azimuthal spin asymmetries, light-cone quark model, Melosh-Wigner rotation, semi-inclusive DIS

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1 Introduction

The investigation of single-spin asymmetries in hadronic reaction has attracted great attention because it can uncover some novel structure of the nucleon. Various theoretical explanations have been put forward. One of them is the so-called Sivers effect \cite{1}, which indicates that the transverse motion of quarks in transverse polarized proton can lead to an azimuthal production asymmetry.

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The effect can be factorized as a convolution of a T-odd transverse momentum dependent parton distribution function (Sivers function) $f_{1T}^{\perp}(x, k_\perp)$, the number density of unpolarized quarks in transversely polarized proton, with the unpolarized fragmentation function. However, for several years the Sivers function was believed to vanish due to the time reversal invariance property of QCD. An alternative mechanism hereafter named the Collins effect was proposed in Ref. [2]: the chiral-odd fragmentation function convoluted with the transversity distributions of the proton can generate the single-spin production asymmetries. Collins’ idea motivated the experimental and theoretical efforts to reveal the transversity properties of the nucleon [3]. Semi-inclusive DIS (SIDIS) experiments on longitudinally polarized target by HERMES collaboration [4] and on transversely polarized target by SMC collaboration [5] showed significant azimuthal asymmetries. During the same time, a number of theoretical analyses on these observed asymmetries have been undergone in terms of the Collins effect [6,7,8,9]. Recently, Brodsky, Hwang and Schmidt [10,11] proposed a new mechanism of producing single spin asymmetries, and they demonstrated that the final state interactions from gluon exchange between the outgoing quark and target spectator system can lead to single-spin asymmetries in SIDIS or Dell-Yan process at leading-twist level in perturbative QCD. It was shown soon by Collins [12] that this mechanism can be recognized as Sivers effect, and the Sivers function is not vanish due to the presence of Wilson line in the operator definitions of parton density. This implies that the gauge link term in the gauge-invariant definition of the parton distribution function can generate the final or initial state interactions [13], which are necessary to produce a phase difference for a nonzero Sivers function.

The allowance of T-odd parton distribution opens up a wide range of phenomenological applications. In this work we attempt to give a full analysis of Sivers asymmetries in SIDIS in the scope of final state interactions. Based on the quark-scalar-diquark model employed in [10], we incorporate vector diquark structure into the proton wave function through a simple relativistic quark spectator-diquark model which is formulated in the light-cone frame. This model was originally proposed in order to study deep inelastic lepton nucleon scattering [14,15], based on the quark-parton model picture that deep inelastic scattering is well described by the impulse approximation, in which the incident lepton scatters incoherently off a quark in the nucleon, with the remaining nucleon constituents treated as a quasiparticle spectator. After taking into account Melosh rotation effects, this model is in good agreement with the experimental data of polarized deep inelastic scattering, and the mass difference between the scalar and vector spectators reproduces the up and down valence quark asymmetry [16]. By advantage of the proton wave function we find that the interference of the proton spin amplitudes can provide necessary phase for Sivers function of down quark. The inclusion of vector diquark has also been adopted in Ref. [17], where a version of the axial-vector spectator model [18] was employed. Although there are vector diquarks in both mod-
els, we will explain later that, our model is different from that in Ref. [17]. Both models can produce qualitatively similar Sivers functions and asymme-
tries, but our model has fewer parameters. Relying on Sivers functions of up and down quarks we give some prediction for the Sivers asymmetries in $\pi^+$ production in SIDIS, and also rough estimation of those in $\pi^-$ and $\pi^0$ production which is currently analyzed in HERMES Collaboration. One feature of our model which will be shown is that if we take away the vector diquark spectator, the result of our model returns to the result appearing in Ref. [10].

This paper is organized as follows. In Sec. 2 we derive the light-cone wave function for the two-body Fock state of the proton including vector diquark spectator utilizing the Melosh-Wigner rotation. Then we estimate the Sivers functions of up and down quarks in the presence of final state interactions in Sec. 3. In Sec. 4, we analyze the single-spin Sivers asymmetries of $\pi^+$, $\pi^-$ and $\pi^0$ off proton target respectively. Finally we give a brief summary in Sec. 5.

2 The quark-diquark decomposition of proton in Fock space

The light-cone formalism [19] has remarkable properties in describing composite state such as hadron. It is suitable to describe the relativistic many-body problem and there have been many successful applications of the light-cone quark model to various physical processes [20]. In light-cone formalism, the hadronic wave function is expressed in terms of a series of light-cone wave functions in the Fock state basis. For the proton a convenient basis is the quark-diquark two-body state, which has been adopted in Ref. [10], where the two-body state wave function was derived from the relativistic field theory treatment by calculating the interaction vertex in light-cone frame [10]:

$$\frac{\bar{u}(k^+, k^-, k_\perp)}{\sqrt{k^+}} \cdot \frac{u(P^+, P^-, P_\perp = 0_\perp)}{\sqrt{P^+}},$$  

(1)

$\bar{u}(k^+, k^-, k_\perp)$ and $u(P^+, P^-, P_\perp)$ are the light-cone spinors of the quark and the proton respectively. A similar treatment was employed in Refs. [21,22] where the electron-photon two-particle Fock state decomposition was obtained from the interaction vertex:

$$\frac{\bar{u}(k^+, k^-, k_\perp)}{\sqrt{k^+}} \gamma^\mu \epsilon_{\mu} u(P^+, P^-, P_\perp),$$  

(2)

in which the light-cone spinors represent the electrons and $\epsilon^\mu$ represents the polarization of the photon.
The same issue can be also analyzed in the light-cone quark model. In this model, the light-cone wave function of a composite system can be obtained by transforming the ordinary equal-time (instant-form) wave function in the rest frame into that in the light-front dynamics, by taking into account relativistic effects such as the Melosh-Wigner rotation. The equivalence of two approaches has been demonstrated in Ref. [23], where both approaches are used to derive the pion light-cone wave function and lead to the same result. The Melosh-Wigner rotation is one of the most important ingredients of the light-cone formalism, and it relates the light-cone spin state $|J, \lambda\rangle_F$ to the ordinary instant-form spin state wave functions $|J, s\rangle_T$ by the general relation [24,25,26]

$$|J, \lambda\rangle_F = \sum_s U_{s\lambda}^J |J, s\rangle_T. \quad (3)$$

The effects due to the Melosh-Wigner rotation have been calculated for the nucleon axial charges [26,27], the magnetic moments [27], the nucleon helicity [16] and transversity [28] distributions, the quark orbital angular moments of the nucleon [29], and recently for the form factors of nucleons [30].

We now derive the light-cone wave function for the quark-diquark Fock state of the proton in the light-cone quark model, with both scalar diquark and vector diquark. The proton wave function in the SU(6) quark-diquark model in the instant form is written as [16,31]

$$\Psi_{\uparrow, \downarrow}^{\uparrow, \downarrow}(qD) = \sin \theta \varphi_V |qV\rangle_{\uparrow, \downarrow} + \cos \theta \varphi_S |qS\rangle_{\uparrow, \downarrow}, \quad (4)$$

with

$$|qV\rangle_{\uparrow, \downarrow} = \pm \frac{1}{3} |V_0(ud)u^\uparrow d^\downarrow - \sqrt{2}V_{\pm 1}(ud)u^{\uparrow, \downarrow} - \sqrt{2}V_0(uu)d^\uparrow d^\downarrow + 2V_{\pm 1}(uu)d^{\uparrow, \downarrow}|;$$

$$|qS\rangle_{\uparrow, \downarrow} = S(ud)u^\uparrow d^\downarrow, \quad (5)$$

where $\uparrow, \downarrow$ and $\uparrow, \downarrow$ label the spin projection $J_z = \pm \frac{1}{2}$ of the proton and $J_z = \pm \frac{1}{2}$ of the quark, respectively, $V_{s_z}(q_1q_2)$ stands for the $q_1q_2$ vector diquark Fock state with third spin component $s_z$, $S(ud)$ stands for a ud scalar diquark Fock state, and $\varphi_D$ represents the momentum space wave function of the quark-diquark state with $D$ denoting the vector or scalar diquarks and $\theta$ being a mixing angle that breaks SU(6) symmetry at $\theta \neq \pi/4$. Here we choose $\theta = \pi/4$ following Ref. [16]. Applying the transformation Eq. (3) on both sides of Eq. (4), we will obtain the spin space wave function of the quark-diquark Fock state in light-front frame. The proton state with zero transverse
momentum $\textbf{P}_\perp = \textbf{0}_\perp$ in the l.h.s. of Eq. (4) keeps identical under the Melosh-Wigner rotation:

$$\Psi_F^{\uparrow, \downarrow}(P^+, P^-, 0_\perp) = \Psi_F^{\uparrow, \downarrow}(P^+, P^-, 0_\perp).$$  \hfill (6)

For the r.h.s of Eq. (4), we first consider the spin part of quark-scalar-diquark Fock state, which is determined by the spin of the quark $\chi_q$:

$$\chi_{(qS)}^{\uparrow, \downarrow} = \chi_q^{\uparrow, \downarrow}. \hfill (7)$$

The Melosh transformation on a quark spin state with four-momentum $(k^0, \textbf{k})$ is

$$\chi_q^{\uparrow}(T) = \omega_q[(k^+ + m)\chi_q^{\uparrow}(F) - k^R\chi_q^{\downarrow}(F)],$$
$$\chi_q^{\downarrow}(T) = \omega_q[(k^+ + m)\chi_q^{\downarrow}(F) + k^L\chi_q^{\uparrow}(F)],$$  \hfill (8)

where $\omega_q = [2k^+(k^0 + m)]^{-\frac{1}{2}}, k^R = k^1+ i k^2$, and $k^+ = k^0 + k^3 = x \mathcal{M}$, $x$ is the momentum fraction of the quark, $m$ is the mass of the constituent quark, and $\mathcal{M}$ is the invariance mass of the composite state, which is approximately taken as the proton mass $M$ in this work. Therefore we get the light-cone spin wave function of the quark-scalar-diquark part for the $J_z^p = +\frac{1}{2}$ proton:

$$\chi_{(qS)}(x, \textbf{k}_\perp) = \sum_{J_z^q} C_{+\frac{1}{2}}^F(x, \textbf{k}_\perp, J_z^q) \chi_{J_z^q}^{J_z^q},$$  \hfill (9)

where the component coefficients $C_{+\frac{1}{2}}^F(x, \textbf{k}_\perp, J_z^q)$ have the forms:

$$C_{+\frac{1}{2}}^F(x, \textbf{k}_\perp, \uparrow) = \omega_q(x M + m),$$
$$C_{+\frac{1}{2}}^F(x, \textbf{k}_\perp, \downarrow) = -\omega_q(k^1_\perp + i k^2_\perp).$$  \hfill (10)

With the momentum space wave function for the diquark, we can write down the light-cone wave function for quark-scalar-diquark Fock state as follows

$$|Sq(P^+, \textbf{P}_\perp = \textbf{0}_\perp))^{\uparrow, \downarrow} = \int \frac{d^3\textbf{k}_\perp dx}{\sqrt{x(1-x)16\pi^3}} [\psi_S^{\uparrow}(x, \textbf{k}_\perp, \uparrow)|xP^+, \textbf{k}_\perp, \uparrow)$$
$$+\psi_S^{\downarrow}(x, \textbf{k}_\perp, \downarrow)|xP^+, \textbf{k}_\perp, \downarrow)],$$  \hfill (11)
\[ \psi^0_S(x, k_\perp, \uparrow) = (M + \frac{m}{x}) \varphi_S, \quad (12) \]
\[ \psi^0_S(x, k_\perp, \downarrow) = -\frac{k_1^\perp + ik_2^\perp}{x} \varphi_S, \quad (13) \]

which is consistent with the light-cone wave function employed in Ref. [10]. For the momentum space wave function of the scalar diquark we adopt

\[ \varphi_S = \frac{e/\sqrt{(1-x)}}{M^2 - (k_1^2 + m^2)/x - (k_1^2 + \lambda^2)/2}, \quad (14) \]

where \( \lambda_S \) is the mass of the scalar diquark. The \( \omega_q \) appearing in Eq. (10) can be considered to be incorporated into the momentum space wave function \( \varphi_S \).

It is shown that there is a component \( \psi^0_S(x, k_\perp, \downarrow) \) in the proton wave function coming from the Wigner-Melosh rotation effect, which does not exist in the non-relativistic constituent quark model. Note that each spin configuration satisfies the spin sum rule \( J_q^z + L^z = +\frac{1}{2} \), where \( L^z \) is the orbital angular momentum projection.

Now consider the quark-vector-diquark two-body Fock state in the right hand side of Eq. (4). The spin part of the state composed by up quark and vector diquark with \( J_p^z = +\frac{1}{2} \) is written as

\[ \chi^0_{\lambda V} \chi_u^\uparrow = \sqrt{2} \chi^1_{\lambda V} \chi_u^\downarrow, \quad (15) \]

where \( \chi_V \) stands for the spin of the vector diquark. For the spin-1 vector diquark, the Melosh transformations read [32]

\[ \chi^0 V(T) = \omega^2_{\lambda V} [(k_T^V + \lambda_V)^2 \chi^0 V(F) - \sqrt{2}(k_T^V + \lambda_V)k_T^R \chi^0 F(V) + (k_T^V)^2 \chi^{-1} V(F)], \]
\[ \chi^0 V(T) = \omega^2_{\lambda V} \{\sqrt{2}(k_T^V + \lambda_V)k_T^L \chi^1 V(F) + 2[(k_T^V + \lambda_V)k_T^V - k_T^R k_T^L] \chi^0 F(V) \}
\]
\[ -\sqrt{2}(k_T^V + \lambda_V)k_T^R \chi^{-1} V(F) \}, \]
\[ \chi^{-1} V(T) = \omega^2_{\lambda V} [(k_T^V)^2 \chi^1 V(F) + \sqrt{2}(k_T^V + \lambda_V)k_T^L \chi^0 V(F) + (k_T^V + \lambda_V)^2 \chi^{-1} V(F)], \]

where \( \lambda_V \) is the mass of the vector diquark, \( k_T^V = k_T^0 + k_T^3 = (1-x)M, k_T^R,L = k_T^1 \pm ik_T^2 \), here \( k_T^1 \) is the transverse momentum of the vector diquark satisfied \( k_T^1 = -k_\perp \), and \( \omega_V = [2k_T^V(k_T^0 + \lambda_V)] \).

Now substituting Eq. (16) and Eq. (8) into Eq. (15), we obtain the spin space wave function of the up-quark-vector-diquark state for the \( J_p^z = +\frac{1}{2} \) proton in the light-front frame:

\[ \chi_{(V0)}(x, k_\perp) = \sum_{J_q^z} C^{F}_{\frac{1}{2}, \frac{1}{2}}(x, k_\perp, J_q^z) \chi^0_{V} \chi^q_u, \quad (17) \]
where the coefficients read

\begin{align*}
C_{+1}^F(x, k_\perp, +1, \uparrow) &= w_q w_V^2 [-\sqrt{2}(k^+_\perp + \lambda_V)(k^+_\perp + m) - \sqrt{2}(k^+_\perp + \lambda_V)^2]k^L, \\
C_{+1}^F(x, k_\perp, +1, \downarrow) &= w_q w_V^2 [\sqrt{2}(k^+_\perp + \lambda_V)k^2_\perp - \sqrt{2}(k^+_\perp + \lambda_V)^2(k^+_\perp + m)], \\
C_{+1}^F(x, k_\perp, 0, \uparrow) &= w_q w_V^2 \{2[(k^+_\perp + \lambda_V)k^+_\perp - k^2_\perp](k^+_\perp + m) - 2(k^+_\perp + \lambda_V)k^2_\perp\}, \\
C_{+1}^F(x, k_\perp, 0, \downarrow) &= w_q w_V^2 2[-(k^+_\perp + \lambda_V)k^+_\perp + k^2_\perp - 2(k^+_\perp + \lambda_V)(k^+_\perp + m)]k^R, \\
C_{+1}^F(x, k_\perp, -1, \uparrow) &= w_q w_V^2 [\sqrt{2}(k^+_\perp + \lambda_V)(k^+_\perp + m) - \sqrt{2}k^2_\perp]k^R, \\
C_{+1}^F(x, k_\perp, -1, \downarrow) &= w_q w_V^2 [\sqrt{2}k^2_\perp (k^+_\perp + \lambda_V + k^+_\perp + m)].
\end{align*}

(18)

Similar to the quark-scalar-diquark two-body Fock state wave function, we notice that there are also higher helicity components in the case of vector diquark. They also appear in the analysis of the form factor of the nucleon [30]. With $k^{R/L}$ in the wave function, those transverse momentum dependent higher helicity components $(+1, \downarrow), (0, \downarrow), (-1, \uparrow) \text{ and } (-1, \downarrow)$ come from the Melosh-Wigner rotation and reflect the intrinsic transverse motion of quarks in the proton. Each spin configuration satisfies the spin sum rule $J_q^z + J_V^z + L^z = +\frac{1}{2}$. The component $(-1, \downarrow)$ is the higher twist component which can be ignored in contrast with other components. From Eq. (18) we write down the expression for the light-cone wave function of $u$-quark-vector-diquark Fock state with $J_p^z = +\frac{1}{2}$.

\begin{align*}
|Vu(P^+, P_\perp = 0_\perp)\rangle^\dagger &= \int \frac{d^2k_\perp dx}{\sqrt{x(1-x)}16\pi^3} [\psi_V^\uparrow(x, k_\perp, +1, \uparrow)|xP^+, k_\perp, +1, \uparrow) \\
+\psi_V^\uparrow(x, k_\perp, +1, \downarrow)|xP^+, k_\perp, +1, \downarrow) + \psi_V^\uparrow(x, k_\perp, -1, \uparrow)|xP^+, k_\perp, -1, \uparrow) \\
+\psi_V^\uparrow(x, k_\perp, 0, \uparrow)|xP^+, k_\perp, 0, \uparrow) + \psi_V^\uparrow(x, k_\perp, 0, \downarrow)|xP^+, k_\perp, 0, \downarrow)],
\end{align*}

(19)

in which

\begin{align*}
\psi_V^\uparrow(x, k_\perp, +1, \uparrow) &= -\sqrt{2}\frac{k^1_\perp - ik^2_\perp}{x(1-x)}\varphi_V, \\
\psi_V^\uparrow(x, k_\perp, +1, \downarrow) &= \sqrt{2}(M + \frac{\lambda_V}{1-x})\varphi_V, \\
\psi_V^\uparrow(x, k_\perp, -1, \uparrow) &= \sqrt{2}\frac{k^1_\perp + ik^2_\perp}{1-x}\varphi_V, \\
\psi_V^\uparrow(x, k_\perp, 0, \uparrow) &= [2(M + \frac{\lambda_V}{1-x}) - (M + \frac{m}{x})]\varphi_V, \\
\psi_V^\uparrow(x, k_\perp, 0, \downarrow) &= -\frac{1 + x k^1_\perp + ik^2_\perp}{1-x}\varphi_V.
\end{align*}

(20)
where $\varphi_V$ stands for the momentum space wave function, which has the same form of Eq. (14), but $\lambda_S$ replaced by $\lambda_V$. The above components of the wave function are similar to the case of the light-cone wave function of $|e\gamma\rangle$ Fock state in Ref. [21,22], except that there are two components with spin projection $J^z = 0$, according to the mass of the vector diquark. We should point out that Eq. (20) is the parametrization of the spin coupling of the quark-vector-diquark state, in which the three spin projections of the vector diquark have been specified explicitly. This is different from the spectator model employed in Ref. [17], where the spin coupling is parameterized by a covariant nucleon-quark-diquark interaction vertex.

Similarly we can obtain the Fock state of down-quark-vector-diquark which is as the same as Eq. (20) only with an extra factor $-\sqrt{2}$, as shown in following schematic form of the proton light-cone wave function with $J^z_p = +\frac{1}{2}$:

$$
\Psi_F^\uparrow = \frac{1}{\sqrt{2}}|S_u^\downarrow_F + \frac{1}{3\sqrt{2}}|V_u^\downarrow_F - \frac{1}{3}|V_d^\uparrow_F.
$$

(21)

The $J^z_p = -\frac{1}{2}$ proton wave function can be evaluated in the same way.

3 Estimates of Sivers function for up and down quark

The final state interactions are generated by the gauge link term in the gauge-invariance definition of the parton distribution function. In model calculations [17,33,34,35] of non-vanishing Sivers function, one gluon-exchange approximation for the gauge link exponential was adopted. In the language of final state interactions, the nonzero phase comes from the interference of proton spin amplitudes, in which different proton spin states $J^z_p = \pm\frac{1}{2}$ couple to the same final state $|F\rangle$, requiring the orbital angular momentum of the two proton wave functions differing by $\Delta L^z = 1$. In previous section, we have derived the light-cone wave function including quark-vector-diquark structure. Like the quark-scalar-diquark state in Ref. [10], the quark-vector-diquark structure also contributes to the proton spin amplitudes, and further the final state interactions. Following the calculation in Ref. [10], we calculate all possible proton spin amplitudes, by adopting one gluon-exchange approximation:

$$
A(\uparrow \rightarrow -1 \uparrow) = \sqrt{2} \frac{r_1^1 + ir_2^2}{1 - \Delta} C(h_v + i \frac{e_1 e_2}{8\pi} g_{v1}),
$$

$$
A(\uparrow \rightarrow +1 \downarrow) = -\sqrt{2}(M + \frac{\lambda_V}{1 - \Delta})C(h_v + i \frac{e_1 e_2}{8\pi} g_{v1}),
$$

$$
A(\uparrow \rightarrow +1 \uparrow) = -\sqrt{2} \frac{r_1^1 - ir_2^2}{\Delta(1 - \Delta)} C(h_v + i \frac{e_1 e_2}{8\pi} g_{v1}),
$$
\( \mathcal{A}(\uparrow \to 0 \uparrow) = (M + 2 \frac{\lambda_V}{1 - \Delta} - \frac{m}{\Delta})C(h_\nu + i\frac{e_1 e_2}{8\pi}g_{v1}), \)

\( \mathcal{A}(\uparrow \to 0 \downarrow) = -\frac{1 + \Delta}{1 - \Delta} C \frac{r_{1\perp} + i r_{2\perp}^2}{\Delta} (h_\nu + i\frac{e_1 e_2}{8\pi}g_{v1}), \)

\( \mathcal{A}(\uparrow \to S \uparrow) = (M + \frac{m}{\Delta})C(h_s + i\frac{e_1 e_2}{8\pi}g_{s1}), \)

\( \mathcal{A}(\uparrow \to S \downarrow) = -\frac{r_{1\perp} + i r_{2\perp}^2}{\Delta} C(h_s + i\frac{e_1 e_2}{8\pi}g_{s2}), \)

(22)

for \( J^z_p = +\frac{1}{2} \) two-body states and

\( \mathcal{A}(\downarrow \to -1 \uparrow) = -\sqrt{2}(M + \frac{\lambda_v}{1 - \Delta})C(h_\nu + i\frac{e_1 e_2}{8\pi}g_{v1}), \)

\( \mathcal{A}(\downarrow \to +1 \downarrow) = -\sqrt{2}\frac{r_{1\perp} - i r_{2\perp}^2}{1 - \Delta} C(h_\nu + i\frac{e_1 e_2}{8\pi}g_{v2}), \)

\( \mathcal{A}(\downarrow \to -1 \downarrow) = \sqrt{2}\frac{r_{1\perp} + i r_{2\perp}^2}{\Delta(1 - \Delta)} C(h_\nu + i\frac{e_1 e_2}{8\pi}g_{v2}), \)

\( \mathcal{A}(\downarrow \to 0 \uparrow) = \frac{1 + \Delta}{1 - \Delta} (r_{1\perp} - i r_{2\perp}^2) C(h_\nu + i\frac{e_1 e_2}{8\pi}g_{v2}), \)

\( \mathcal{A}(\downarrow \to 0 \downarrow) = (M + 2 \frac{\lambda_V}{1 - \Delta} - \frac{m}{\Delta})C(h_\nu + i\frac{e_1 e_2}{8\pi}g_{v2}), \)

\( \mathcal{A}(\downarrow \to S \uparrow) = \frac{r_{1\perp} - i r_{2\perp}^2}{\Delta} C(h_s + i\frac{e_1 e_2}{8\pi}g_{s2}), \)

\( \mathcal{A}(\downarrow \to S \downarrow) = (M + \frac{m}{\Delta})C(h_s + i\frac{e_1 e_2}{8\pi}g_{s1}), \)

(23)

for \( J^z_p = -\frac{1}{2} \) two-body states. Where \( \Delta \) and \( r_{\perp} \) are the Bjorken variable and the transverse momentum of the struck quark respectively, \( e_1 \) and \( e_2 \) are the charges of the struck quark and spectator diquark, and

\[
C = -ge_1 P^+ \sqrt{2\Delta(1 - \Delta)}, \tag{24}
\]

\[
h_{s/v} = \frac{1}{r_{\perp} + \Delta(1 - \Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda_s v}{1 - \Delta})}, \tag{25}
\]

in which \( g \) is the proton-quark-diquark coupling constant. In literature there are two choice for \( g \), one is to take \( g \) as a constant [10,33], another as a form factor [17,35] relying on the momentum of the outgoing quark, to converge the transverse momentum integration of the distribution functions. As argued in Ref. [33], the inclusion of the form factor would add another complication on the evolution of the asymmetry, therefore we treat \( g \) as a constant in our model, which is equal for the scalar and the vector diquark. The last two equations in Eq. (22) and Eq. (23) are as the same as Eq. (5)-Eq. (8) in Ref. [10], while the other ten equations are contributed from vector-diquark
structure. With above amplitudes we calculate the asymmetries in quark level for \textit{up} and \textit{down} quarks through \cite{11}

$$P_{u/d} = \text{Im}(T_{int}^{u/d}) / M_{u/d}. \quad (26)$$

Here $T_{int}^{u}$ and $T_{int}^{d}$ are the interference terms coming from the final state interactions with the forms

$$T_{int}^{u} = A(\uparrow \rightarrow S \uparrow) \ast A(\downarrow \rightarrow S \uparrow) + \frac{1}{9} A(\uparrow \rightarrow -1 \uparrow) \ast A(\downarrow \rightarrow -1 \uparrow)$$

$$+ \frac{1}{9} A(\uparrow \rightarrow 0 \uparrow) \ast A(\downarrow \rightarrow 0 \uparrow),$$

$$T_{int}^{d} = \frac{2}{9} A(\uparrow \rightarrow -1 \uparrow) \ast A(\downarrow \rightarrow -1 \uparrow) + \frac{2}{9} A(\uparrow \rightarrow 0 \uparrow) \ast A(\downarrow \rightarrow 0 \uparrow), \quad (27)$$

respectively, and $M_{u/d}$ are denoted as:

$$M^{u} = \frac{1}{2} |A(\uparrow \rightarrow S \uparrow)|^2 + \frac{1}{18} |A(\uparrow \rightarrow S \downarrow)|^2 + \frac{1}{18} |A(\uparrow \rightarrow -1 \uparrow)|^2$$

$$+ \frac{1}{18} |A(\uparrow \rightarrow +1 \downarrow)|^2 + \frac{1}{18} |A(\uparrow \rightarrow -1 \downarrow)|^2 + \frac{1}{18} |A(\uparrow \rightarrow 0 \uparrow)|^2$$

$$+ \frac{1}{18} |A(\uparrow \rightarrow 0 \downarrow)|^2,$$

$$M^{d} = \frac{1}{9} |A(\uparrow \rightarrow +1 \uparrow)|^2 + \frac{1}{9} |A(\uparrow \rightarrow +1 \downarrow)|^2 + \frac{1}{9} |A(\uparrow \rightarrow 0 \uparrow)|^2$$

$$+ \frac{1}{9} |A(\uparrow \rightarrow 0 \downarrow)|^2. \quad (28)$$

Substituting Eq. (22) and Eq. (23) into the equations above we obtain

$$T_{int}^{u} = -\frac{1}{2} \left( M + \frac{m}{\Delta} \right) \frac{r_{\perp}}{\Delta} \frac{r_{\perp} e_{1} e_{2}}{4 \pi} \frac{1}{\Lambda_{s}(r_{\perp}^2)} \ln \frac{\Lambda_{s}(r_{\perp}^2)}{\Lambda_{s}(0)}$$

$$- \left[ \frac{2}{9} \left( M + \frac{\lambda_{V}}{1 - \Delta} \right) \right] \frac{r_{\perp}^4}{1 - \Delta} + \frac{1}{9} \left( M + 2 \frac{\lambda_{V}}{1 - \Delta} - \frac{m}{\Delta} \right)$$

$$\frac{1 - \Delta}{(1 + \Delta) \Delta} \frac{r_{\perp}^4}{8 \pi} \frac{e_{1} e_{2}}{\Lambda_{v}(r_{\perp}^2)} \ln \frac{\Lambda_{v}(r_{\perp}^2)}{\Lambda_{v}(0)}, \quad (29)$$

$$T_{int}^{d} = -\frac{e_{1} e_{2}}{4 \pi} \frac{1}{\Lambda_{v}(r_{\perp}^2)} \ln \frac{\Lambda_{v}(r_{\perp}^2)}{\Lambda_{v}(0)} \left[ \frac{2}{9} \left( M + \frac{\lambda_{V}}{1 - \Delta} \right) \right] \frac{r_{\perp}^4}{1 - \Delta}$$

$$+ \frac{1}{9} \left( M + 2 \frac{\lambda_{V}}{1 - \Delta} - \frac{m}{\Delta} \right) \frac{1 - \Delta}{(1 + \Delta) \Delta}, \quad (30)$$

10
\[ M^u = \frac{1}{2} \left[ (M + \frac{m}{\Delta})^2 + \frac{r_\perp^2}{\Delta^2} \right] h_x^2 + \frac{1}{9} \left[ (M + \frac{\lambda_v}{1 - \Delta})^2 \frac{r_\perp^2}{(1 - \Delta)^2} \right. \\
\left. + \frac{r_\perp^2}{\Delta^2 (1 - \Delta)^2} + \frac{(1 - \Delta)^2 r_\perp^2}{2 \Delta^2} + \left( \frac{1}{2} M + \frac{\lambda_v}{1 - \Delta} - \frac{m}{2 \Delta} \right)^2 \right] h_v^2, \quad (31) \]

\[ M^d = \frac{2}{9} \left[ (M + \frac{\lambda_v}{1 - \Delta})^2 + \frac{r_\perp^2}{(1 - \Delta)^2} + \frac{r_\perp^2}{\Delta^2 (1 - \Delta)^2} + \frac{(1 - \Delta)^2 r_\perp^2}{(1 + \Delta)^2 2 \Delta^2} \right. \\
\left. + \left( \frac{1}{2} M + \frac{\lambda_v}{1 - \Delta} - \frac{m}{2 \Delta} \right)^2 \right] h_v^2, \quad (32) \]

in which

\[ \Lambda_{s/v}(r_\perp^2) = r_\perp^2 + \Delta (1 - \Delta) (-M^2 + \frac{m}{\Delta} + \frac{\lambda_{s/v}}{1 - \Delta}). \quad (33) \]

From Eq. (26) and the relation between the asymmetries \( P_y, f_{1T}^a(\Delta, r_\perp^2) \) and \( f_1(\Delta, r_\perp^2) \) [11,33]

\[ P_y^a = -\frac{r_\perp^2}{M} f_{1T}^{a}(\Delta, r_\perp^2)/f_1^a(\Delta, r_\perp^2), \quad (34) \]

we obtain the Sivers functions of up and down quarks by

\[ f_{1T}^a(\Delta, r_\perp^2) = -\frac{P_y^a}{r_\perp^2} f_1^a(\Delta, r_\perp^2). \quad (35) \]

Here \( a \) is the flavor index including \( u \) and \( d \). The difference between Eq. (34) and Eq. (32) in Ref. [33] is that we assume that the equation stands for both up and down quarks. From Eq. (34) we can see that if we exclude vector diquark, the result of our model will return to the results in Ref. [10] and Ref. [33]; when vector diquark is included, the Sivers function for down quark is evaluated. There is another advantage of Eq. (26): the coupling constant \( g \) will appear in both the numerator and denominator of Eq. (34) and cancel, thus the Sivers functions in our model is free from \( g \) and normalization factors. To avoid ambiguity, we use below the same notation used in Ref. [10] with \( \Delta \) denoting the Bjorken variable of parton and \( r_\perp \) for the transverse momentum of quark. In realistic application the moment of the Sivers function is more useful. The \( r_\perp^2 \)-moment of the Sivers function is defined as [36]

\[ f_{1T}^{(1)}(\Delta) = \int d^2 r_\perp \left( \frac{r_\perp^2}{2 M^2} \right) f_{1T}^{a}(\Delta, r_\perp^2). \quad (36) \]
Fig. 1. The \( r_\perp^2 \)-moments of Sivers functions of \( u \) and \( d \) quarks in SU(6) light-cone quark-diquark model.

In numerical estimates of this moment we adopt Gaussian distribution of the transverse momentum for the unpolarized distribution function

\[
f_1(\Delta, r_\perp^2) = \frac{f_1(\Delta)}{\pi \langle r_\perp \rangle^2} \exp\left(-\frac{r_\perp^2}{\langle r_\perp \rangle^2}\right),
\]

in order to guarantee convergence of the \( r_\perp \) integral in Eq. (36). Here \( \langle r_\perp \rangle \) is the average transverse momentum of the quark. For the transverse momentum integrated unpolarized distribution \( f_1(\Delta) \) we adopt CTEQ6M parametrization [38] for the valence quarks. Also we choose the parameters in the expression as: \( \lambda_S = 0.6 \) GeV, \( \lambda_V = 0.9 \) GeV as estimated to explain the \( N-\Lambda \) mass difference, \( m = 0.36 \) GeV, \( M = 0.94 \) GeV, \( \langle r_\perp \rangle = 0.4 \) GeV. To generalize the analysis to the corresponding calculation in QCD, one needs to extrapolate \( |a_1^{had}| \rightarrow C_F \alpha_S \) with \( C_F = \frac{4}{3} \). We choose \( \alpha_s = 0.2 \) to correspond the kinematics of HERMES experiment. With above parameters we obtain the Sivers functions of \( u \) and \( d \) quarks explicitly. The \( r_\perp^2 \)-moments of the Sivers functions of \( u \) and \( d \) quarks are figured by Fig. 1 which shows that in our model the Sivers function of \( u \) quark is positive, while that of \( d \) quark tends to be negative, and the former is \( 6-7 \) times larger than the later in scale. This agrees with the \( r_\perp \)-moments of the Sivers functions that appeared in Ref. [17]. And the size of the moments is similar to that of the bag-model result presented in Ref. [34].

## 4 Single-spin Sivers asymmetries of pion electroproduction in SIDIS

Sivers effect can be observed in meson electroproduction in semi-inclusive DIS with unpolarized electron beam off transversely polarized nucleon target which is measured in HERMES. With the Sivers functions of \( u \) and \( d \) quarks obtained above, we calculate single-spin asymmetries in SIDIS contributed
from Sivers effect by selecting proper weighting factor \(|\frac{P_{h\perp}}{M}\sin(\phi - \phi_S)|\). \(\phi\) is the azimuthal angle between the lepton scattering plane and the transverse momentum of outgoing hadron, \(\phi_S\) is the azimuthal angle of the spin of proton target. The asymmetries are expressed by the convolution of the \(r_1^T\)-moment of Sivers function and the usual unpolarized fragmentation function \(D_1^q(z)\), divided by the unpolarized cross section [37]

\[
\langle \frac{P_{h\perp}}{M}\sin(\phi - \phi_S) \rangle_{UT} (\Delta, z) = \frac{\sum a e_j^2 f_j^{T}(\Delta) z D_1^q(z)}{\sum a e_j^2 f_j^{T}(\Delta) D_1^q(z)},
\]

(38)

where the subscript \(UT\) represents unpolarized beam on transversely polarized target, \(P_{h\perp}\) is the transverse momentum of the pion. For \(D_1(z)\), we consider both the favored fragmentation function

\[
D(z) = D^\pi^+(z) = D^\pi^-(z)
\]

(39)

and unfavored fragmentation function [7]

\[
\hat{D}(z) = D^\pi^+(z) = D^\pi^-(z)
\]

(40)

for \(\pi^\pm\) fragmentation. Also we assume

\[
D^\pi^0(z) = \frac{1}{2}[D^\pi^+(z) + D^\pi^-(z)] = \frac{1}{2}[D(z) + \hat{D}(z)]
\]

(41)

for \(\pi^0\) fragmentation. For the explicit analytically forms of \(D(z)\) and \(\hat{D}(z)\) we adopt [39]

\[
D(z) = 0.689 z^{-1.039} (1 - z)^{1.241},
\]

\[
\hat{D}(z) = 0.217 z^{-1.805} (1 - z)^{2.037}.
\]

(42)

For \(\pi^-\) production, the favored fragmentation process is \(d \rightarrow \pi^-\). Since we have obtained the Sivers function of down quark, we can give some prediction of the azimuthal asymmetries for the \(\pi^-\) production. We have ignored the sea quark contribution, and assume that the contribution comes mainly from the valence part, since we can not get information about sea quark from the SU(6) quark-diquark model. For comparison with future HERMES experimental data we also calculate the asymmetries averaged by the kinematics of HERMES: \(0.023 < x < 0.4\) and \(0.2 < z < 0.7\). The single-spin Sivers asymmetries of \(\pi^+, \pi^-\) and \(\pi^0\) plotted vs \(x\) and \(z\) are presented in Fig. 2. The figure shows that the \(x\)-dependent asymmetries of \(\pi^+\) production are larger than that of \(\pi^-\) production. The \(z\)-dependent asymmetries of \(\pi^+\) and \(\pi^0\) are similar, and different from that of \(\pi^-\) in shape and size.
Fig. 2. Single-spin Sivers asymmetries measured in SIDIS off proton target. The asymmetries have been weighted by $|\frac{P_{h\perp}}{M}|$. The first, second and third rows correspond to the asymmetries of $\pi^+$, $\pi^-$ and $\pi^0$ production respectively. The left column corresponds to the $x$-dependent asymmetries, the solid and dotted curves correspond to the asymmetries averaged and not averaged by the HERMES kinematics $0.2 < z < 0.7$. The right column corresponds to the $z$-dependent asymmetries, the solid and dotted curves correspond to the asymmetries averaged and not averaged by the HERMES kinematics $0.023 < x < 0.4$.

5 Conclusion

Final state interactions provide a reliable explanation for Sivers single-spin asymmetries in SIDIS. To get phenomenological estimation we derive the light-cone wave function of the proton based on the SU(6) quark-spectator-diquark model. We obtain the light-cone wave function containing both scalar diquark
and vector diquark. The transverse momentum dependence, which is responsible for the transverse momentum distribution function and single-spin asymmetries, is introduced to the wave function by employing the Wigner-Melosh rotation. The Wigner-Melosh rotation also brings the higher helicity components which contribute to the final state interactions. Utilizing the wave function we calculate the Sivers functions of $up$ and $down$ quarks in one-gluon exchange. We find that in our model the Sivers function of $up$ quark is positive, while that of $down$ quark tends to be negative, and that the former is much larger than the later in scale. We give analysis of $\frac{|P_{h\perp}|}{M}$ weighted Sivers asymmetries $A_{UT}^{\sin(\phi-\phi_s)}$ of $\pi^+$, $\pi^-$ and $\pi^0$ production off transverse proton targets plotted vs $x$ and $z$ from our model. The numerical result shows that the $x$-dependent asymmetries of $\pi^+$ production are a little larger than that of $\pi^-$ production. The $z$-dependent asymmetries of $\pi^+$ and $\pi^0$ are similar, and different from that of $\pi^-$ in shape and size.

In contrast to the result in Ref. [17], our Sivers functions and asymmetries are qualitatively similar but quantitatively different. We conclude that the differences come from two aspects. On the one hand, we use different quark-vector-diquark structure from that of Ref. [17]; on the other hand, we treat proton-quark-diquark coupling $g$ as a constant, different from that of Ref. [17]. As a result, the Sivers functions in our model are free from $g$ and normalization parameters, but rely on $\langle r_{\perp} \rangle$.

Similar to other model calculations, we use some assumptions and approximations, such as one Abelian gluon approximation and valence quark dominance, and do not consider the evolution of the Sivers function, thus at current level, our Sivers functions and asymmetries are rough estimates.

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