Analysing e-Commerce A/B Tests with Dependent Data
Empirical Evidence on Measurement Uncertainty in Average Basket Value and Other e-Commerce KPIs

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Abstract
Digital technology organizations often use A/B tests to guide their product and business decisions. In e-commerce, it is a common pitfall to ignore dependent transaction/item value and size that arises when one measure changes to key performance indices such as Average Basket Value (ABV), Average Basket Size (ABS), and Average Selling Price (ASP). We present empirical evidence on dependent transaction value/size, its impact on measurement uncertainty, and practical implications on A/B test outcomes if left unmitigated. By making the evidence available, we hope to drive awareness of the pitfall among experimenters in e-commerce and hence the adoption of many established mitigation approaches.

1 Introduction
A/B tests have become popular among digital technology organizations in measuring the impact of their products/services and guiding business decisions [1]. In an A/B test, we randomly split incoming users into a treatment and a control group. We calculate some decision metric(s) based on responses from both groups and compare the metrics using a statistical test to draw causal statements about the treatment.

Here we focus on A/B tests in e-commerce. Similar to many digital technology organizations, e-commerce organizations aim to provide a consistent user experience, and hence randomization in A/B tests is generally done on a per-user basis. On the other hand, e-commerce organizations are unique as they feature physical inventories and thus track a set of Key Performance Indices (KPIs) that are transaction- or item-based. These KPIs include Average Basket Value (ABV), Average Basket Size (ABS), and Average Selling Price (ASP).

Experiments that measure changes to ABV, ABS, and ASP often feature dependent responses. In these experiments, the responses (or analysis units) are at a more granular transaction- or item-level than the randomization units, which are at a user level. Given a user can make many transactions and purchase many items during an experiment, the value of these transactions and items are likely to be correlated based on the user’s preference. This creates a local dependence structure between users and transactions/items, which violates the usual independent and identically distributed (i.i.d. hereafter) assumptions in common statistical tests that A/B tests employ (e.g., a Student’s t-test). If left unmitigated, it risks experimenters having wrong estimates of sampling uncertainty and hence making wrong conclusions from a statistical test.

Having dependent responses in experiments due to differences in granularity between randomization and analysis units is not a recent revelation [2]. In some sense, one may regard such a setup as a cluster-randomized controlled experiment, where an experimenter randomly assigns clusters of transactions or items belonging to the same user to the A/B test groups. In terms of obtaining an unbiased estimate for

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1 Average amount spent by a user per transaction/basket. Also known as Average Order Value (AOV).
2 Average number of items purchased by a user per transaction/basket. Also known as Average Order Size (AOS).
3 Average price per item sold.
4 In such a setup, the size of a cluster can be zero and would only be determined after the experiment ends. This is due to experimenters not able to control how much a user buys on the website.
the sampling uncertainty, many viable approaches already exist: experimenters in medicine, economics, and social sciences often employ a crossed random effects model [3] or cluster robust standard errors [4]. Meanwhile, those in digital technology prefer using bootstrap [5] or the delta method [6] to estimate the sample variance and the standard error due to their ability to scale to large datasets and relatively lack of model assumptions.

Despite the number of work dedicated to dependent responses in digital experiments, there is little published evidence specifically in the context of A/B tests in e-commerce. Most of the work available is based on the context of A/B tests in digital advertising and content, where experimenters randomize by users and analyze by sessions or page views [5, 6]; or social networks, where experimenters randomize and analyze both by users whose responses may correlate via their social connection(s) [7]. We believe the lack of evidence contributes to insufficient awareness of the issue from experimenters in this area.

This work aims to address this gap in the literature by presenting some empirical evidence from e-commerce A/B tests. We show, via a real dataset from ASOS.com, a global fashion e-tail company, that the value of transactions and items made by the same user are often positively correlated (Section 2). Such a local dependence structure can lead to the standard error being up to 3.70 times the vanilla standard error calculated under i.i.d. assumptions. Moreover, we observe the increase in response variance may outpace the increase in the number of samples as an experiment progresses, which suggests designing long A/B tests solely to collect more responses may be unwise (Section 3). We finally clarify how the increased standard error impact decisions from null hypothesis statistical tests in terms of test power and confidence interval coverage (Section 4).

2 Dependent data inflates standard error estimates

We first motivate why we may obtain inaccurate estimates of the sampling uncertainty when measuring changes to e-commerce KPIs based on transactions or items in A/B tests. Let \( X_1, X_2, \ldots, X_n \) be our responses with \( \mathbb{E}(X_i) = \mu \) and \( \text{Var}(X_i) = \sigma^2 \). The test statistic for a (one-sample) Student’s \( t \)-test, the most commonly used statistical test, is \( (\bar{x} - \mu) / \sqrt{s^2/n} \), where \( \bar{x} \) is the sample mean and \( s^2 \) is the sample variance. The denominator of the test statistic is an unbiased estimate of the standard error of the mean (standard error, or SE hereafter):

\[
\sqrt{\text{Var}(\bar{X})} = \sqrt{\text{Var}(\frac{1}{n} \sum_{i=1}^{n} X_i)} = \sqrt{\frac{1}{n^2} \left( \sum_{i=1}^{n} \text{Var}(X_i) + 2 \sum_{i<j} \text{Cov}(X_i, X_j) \right)} .
\]
If we assume \( X_i \) are i.i.d., which is the prevailing assumption when experimenters randomize by users and analyze by user \([8]\), then \( \text{Cov}(X_i, X_j) = 0 \forall i \neq j \) and thus \( \text{Var}(\bar{X}) = \sum_i \text{Var}(X_i)/n^2 = \sigma^2/n \).

The i.i.d. assumption is unlikely to hold when experimenters randomize by users and analyze by transactions or items. Many users tend to make a transaction that is similarly sized and valued to their previous transaction(s). They also tend to purchase items that are at a similar price point. Such behavior can be seen in Figures 1 and 2 where we plot the value and size of a user’s transaction against that of their next transaction (if it exists) using a real dataset at ASOS.com. The dataset records transactions occurred within a specific two-month period in 2022 on a particular mobile platform. It involves around 4M users purchasing roughly 33M items in approximately 10M transactions. As responses from the same user becomes correlated, often positively as indicated in the plot, we have \( \text{Cov}(X_i, X_j) > 0 \) for some \( i \) and \( j \) and thus a higher SE than the vanilla sample SE (\( \sqrt{\sigma^2/n} \) or its unbiased estimate \( \sqrt{s^2/n} \)).

### 3 Actual standard error can be estimated using bootstrap

We then explore how much the SE can inflate. We apply the bootstrap procedure described in Section 2.2 of \([5]\) to the dataset described above. Bootstrapping generates a resample by sampling the original set of responses with replacement (or applying a random weight in this case). This yields a different sample mean. By repeating the process many times and taking many bootstrap means, we can obtain an estimate for the SE by calculating the standard deviation of the bootstrap means.

We use a one-way bootstrap (a.k.a. block bootstrap) to account for the dependency between transactions/items and users. Instead of generating the resample by sampling each transaction/item individually, we sample clusters of transactions/items belonging to the same user, mirroring our randomization process. We use random weights generated from \( \text{Poisson}(1) \) to reweight our samples before calculating a bootstrap mean and estimate the SE from 500 bootstrap means.

Figures 3–5 show how the bootstrap SE differs from the vanilla sample SE in successive expanding windows, which corresponds to different potential A/B test duration. We observe the bootstrap SE is significantly higher than the vanilla sample SE, with the difference being roughly 1.40x for ABV, 1.95x for ABS, and 3.70x for ASP during a 30-day window. The exact ratio is heavily dependent on the KPI and duration. Given the magnitude of the SE affects many other quantities in a statistical test (see Section 4), experimenters should strive to obtain an accurate estimate.

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5 The data described are not representative of ASOS.com’s overall business operations and no conclusion of such should be drawn.

6 For ASP, in addition to the dependency between items and users, there may be additional dependence between items and stock keeping units (SKUs). A natural extension is to account for both via a two-way bootstrap, which is also described in \([5]\).
Figure 3: The standard error (SE) estimate for Average Basket Value (ABV) obtained via a one-way bootstrap as described in [5] (solid orange line with error bars) versus the vanilla sample SE estimate ($\sqrt{s^2/n}$ - dashed blue line). They are plotted against different expanding time windows (i.e., Day 1 only, Days 1–2, Days 1–3, ..., and Days 1–61). The error bars represent the 95% confidence interval of the SE estimate, with the sampling uncertainty arising from the bootstrap procedure itself. All values on the left plot are normalized by the vanilla sample SE during Day 1. Each value on the right plot is normalized by the vanilla sample SE of the same expanding window, expressing the bootstrap SE estimate as a multiple of the vanilla sample SE estimate.

Figure 4: The standard error (SE) estimate for Average Basket Size (ABS) obtained via a one-way bootstrap (solid orange line with error bars) versus the vanilla sample SE estimate (dashed blue line). They are plot against different expanding time windows (i.e., Day 1 only, Days 1–2, Days 1–3, ..., and Days 1–61). See the caption in Figure 3 for further details on expanding time windows, error bars, and normalization.

Figure 5: The standard error (SE) estimate for Average Selling Price (ASP) obtained via a one-way bootstrap (solid orange line with error bars) versus the vanilla sample SE estimate (dashed blue line). They are plot against different expanding time windows (i.e., Day 1 only, Days 1–2, Days 1–3, ..., and Days 1–61). See the caption in Figure 3 for further details on expanding time windows, error bars, and normalization.
We also note the bootstrap SE may no longer drop as as more transactions over time are involved, and in some cases, may go up again. This is likely due to returning users making further transactions and thus creating more and larger clusters of transaction/item values and sizes. It leads to the response variance increasing, sometimes more quickly than the increase in the number of samples. Such observation suggests that lengthening an experiment with the sole purpose of collecting more responses (and thus lowering the SE) may backfire. In addition to established practices on sample size estimation [9], experimenters should also consider how the variance of their decision metric evolves when designing A/B tests with dependent data.

4 Unmitigated standard error inflation risks bad test decisions

We finally discuss how an inflated SE due to dependent data can affect decisions made from a null hypothesis significance test. Firstly, it reduces the power of the test, making any potential treatment effect harder to detect. Consider a two-tailed Student’s $t$-test with significance level $\alpha$ and $\nu$ d.f. Its power is

$$1 - T_\nu(t_{\nu,1-\alpha/2} - \theta/SE) + T_\nu(-t_{\nu,1-\alpha/2} - \theta/SE),$$

where $\theta$ is the effect size, $T_\nu(\cdot)$ is the CDF, and $t_{\nu,1-\alpha/2}$ is the $(1 - \alpha/2)$ quantile of a $t$-distributed r.v. If the SE increases, then both the standardized effect size $\theta/SE$ and the test power in Expression (2) decrease.

Secondly, having an inflated SE without knowing so will lead to tests producing confidence intervals that are too narrow, risking more false positives. In the test above, the $(1 - \alpha)$ confidence interval (CI) is

$$[\bar{x} \pm t_{\nu,1-\alpha/2} \cdot SE].$$

Fixing the CI bounds, we observe that $t_{\nu,1-\alpha/2}$ must decrease (and cease to be a $(1 - \alpha/2)$ quantile) when the SE increases. This means the said CI will no longer have a $(1 - \alpha)$ but a lower coverage, i.e., there is now a lower chance the interval will contain the actual effect size.

To show the full extent of the issues above, we plot the test power and CI coverage of a two-tailed $z$-test, essentially a Student’s $t$-test with a large degrees of freedom, against different SEs in Figure 6. We observe that the power of a $z$-test with a 5% significance level (dashed line) tumbles from 80% to around 29% when we merely double the vanilla sample SE. Moreover, the centered 95% CI calculated using the vanilla sample SE would only have roughly 67% coverage.
5 Conclusion

A/B tests with dependent data are common in e-commerce, especially when experimenters are randomizing by users and analyzing by transactions/items as they measure changes to KPIs such as Average Basket Value (ABV), Average Basket Size (ABS), and Average Selling Price (ASP). However, the risk they pose, namely experimenters obtaining an incorrect (usually a lower) standard error estimate from the vanilla sample standard error formula, is often not adequately mitigated. We attribute the phenomenon to the lack of related evidence, which leads to insufficient awareness from practitioners in e-commerce.

In this paper, we discussed why the vanilla standard error estimate could be incorrect, demonstrated the magnitude of the problem using a real dataset from an e-commerce company, and outlined its impact on A/B test outcomes. We did not offer any new solution to the problem as many viable mitigating approaches already exist (see discussion in Section 1). Instead, we intend for this paper to be a short but crucial piece of evidence that practitioners can utilize to justify introducing said mitigating approaches into the A/B test design and analysis processes within their organization.

References

[1] C. H. B. Liu, B. P. Chamberlain, and E. J. McCoy, “What is the value of experimentation and measurement?: Quantifying the value and risk of reducing uncertainty to make better decisions,” Data Science and Engineering, vol. 5, pp. 152–167, 2020. [Online]. Available: https://doi.org/10.1007/s41019-020-00121-5

[2] R. Kohavi, D. Tang, and Y. Xu, Trustworthy Online Controlled Experiments: A Practical Guide to A/B Testing, 1st ed. Cambridge University Press, 2020.

[3] R. H. Baayen, D. J. Davidson, and D. M. Bates, “Mixed-effects modeling with crossed random effects for subjects and items,” Journal of Memory and Language, vol. 59, no. 4, pp. 390–412, 2008, special Issue: Emerging Data Analysis. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0749596X07001398

[4] A. C. Cameron, J. B. Gelbach, and D. L. Miller, “Robust inference with multiway clustering,” Journal of Business & Economic Statistics, vol. 29, no. 2, pp. 238–249, 2011. [Online]. Available: http://www.jstor.org/stable/25800796

[5] E. Bakshy and D. Eckles, “Uncertainty in online experiments with dependent data: An evaluation of bootstrap methods,” in Proceedings of the 19th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ser. KDD ’13. New York, NY, USA: Association for Computing Machinery, 2013, p. 1303–1311. [Online]. Available: https://doi.org/10.1145/2487575.2488218

[6] A. Deng, U. Knoblich, and J. Lu, “Applying the delta method in metric analytics: A practical guide with novel ideas,” in Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, ser. KDD ’18. New York, NY, USA: Association for Computing Machinery, 2018, p. 233–242. [Online]. Available: https://doi.org/10.1145/3219819.3219919

[7] D. Eckles, B. Karrer, and J. Ugander, “Design and analysis of experiments in networks: Reducing bias from interference,” Journal of Causal Inference, vol. 5, no. 1, p. 20150021, 2017. [Online]. Available: https://doi.org/10.1515/jci-2015-0021

[8] A. Deng, J. Lu, and J. Litz, “Trustworthy analysis of online A/B tests: Pitfalls, challenges and solutions,” in Proceedings of the Tenth ACM International Conference on Web Search and Data Mining, ser. WSDM ’17. New York, NY, USA: Association for Computing Machinery, 2017, p. 641–649. [Online]. Available: https://doi.org/10.1145/3018661.3018677

[9] T. S. Richardson, Y. Liu, J. Mcqueen, and D. Hains, “A Bayesian model for online activity sample sizes,” in Proceedings of the 25th International Conference on Artificial Intelligence and Statistics, ser. Proceedings of Machine Learning Research, G. Camps-Valls, F. J. R. Ruiz, and I. Valera, Eds., vol. 151. PMLR, 28–30 Mar 2022, pp. 1775–1785. [Online]. Available: https://proceedings.mlr.press/v151/richardson22a.html