Soft $A_4 \to Z_3$ Symmetry Breaking and Cobimaximal Neutrino Mixing

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Abstract

I propose a model of radiative charged-lepton and neutrino masses with $A_4$ symmetry. The soft breaking of $A_4$ to $Z_3$ lepton triality is accomplished by dimension-three terms. The breaking of $Z_3$ by dimension-two terms allows cobimaximal neutrino mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm \pi/2$) to be realized with only very small finite calculable deviations from the residual $Z_3$ lepton triality. This construction solves a long-standing technical problem inherent in renormalizable $A_4$ models since their inception.
For the past several years, some new things have been learned regarding the theory of neutrino flavor mixing. (1) Whereas the choice of symmetry, for example $A_4$ [1] [2] [3], and its representations are obviously important, the breaking of this symmetry into specific residual symmetries, for example $A_4 \to Z_3$ lepton triality [4] [5], is actually more important. (2) A mixing pattern may be obtained [6] independent of the masses of the charged leptons and neutrinos. (3) The clashing of residual symmetries between the charged-lepton, for example $A_4 \to Z_3$, and neutrino, for example $A_4 \to Z_2$, sectors is technically very difficult to maintain [7]. (4) The essential incorporation of $CP$ transformations [8, 9] may be the new approach [10] [11] [12] [13] [14] [15] which will lead to an improved understanding of neutrino flavor mixing.

In this paper, a model of radiative charged-lepton and neutrino masses is proposed with the following properties. (1) The masses are generated in one loop through dark matter [16], i.e. particles distinguished from ordinary matter by an exactly conserved dark symmetry. This is the so-called scotogenic mechanism. (2) The symmetry $A_4 \times Z_2$ is imposed on all dimension-four terms of the renormalizable Lagrangian with particle content given in Table 1. (3) Dimension-three terms break $A_4 \times Z_2$, but all such terms respect the residual $Z_3$ lepton triality. (4) Dimension-two terms break $Z_3$, which is nevertheless retained in dimension-three (and dimension-four) terms with only finite calculable deviations. This solves the problem of clashing residual symmetries. (5) The proposed specific model results in cobimaximal [15] neutrino mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm \pi/2$), which is consistent with the present data [17] [18]. It is also theoretically sound, because the residual $Z_3$ is protected, unlike previous proposals. Cobimaximal mixing becomes thus a genuine prediction, robustly supported in the context of a complete renormalizable theory of neutrino mass and mixing.

The dark $U(1)_D$ and $Z_2$ symmetries are assumed to be unbroken. The other $Z_2$ symmetry is used to forbid the dimension-four Yukawa couplings $\bar{l}_L l_R \phi^0$ so that charged leptons only
Table 1: Particle content under $U(1)_D \times Z_2 \times A_4 \times Z_2$.

| particles      | dark $U(1)_D$ | dark $Z_2$ | flavor $A_4$ | $Z_2$ |
|----------------|---------------|-------------|--------------|-------|
| $(\nu, l)_L$  | 0             | +           | 3            | +     |
| $l_R$         | 0             | +           | 3            | −     |
| $(\phi^+, \phi^0)$ | 0     | +           | 1            | +     |
| $N_{L,R}$     | 1             | +           | 3            | +     |
| $(\eta^+, \eta^0)$ | 1     | +           | 1            | +     |
| $\chi^+$      | 1             | +           | 1            | −     |
| $(E^0, E^-)_{L,R}$ | 0   | −           | 1            | +     |
| $F^0_L$       | 0             | −           | 1            | +     |
| $s$           | 0             | −           | 3            | +     |

acquire masses in one loop as shown in Fig. 1. Whereas this $Z_2$ is respected by the dimension-

![Figure 1: One-loop generation of charged-lepton mass with $U(1)_D$ symmetry.](image)

Figure 1: One-loop generation of charged-lepton mass with $U(1)_D$ symmetry.

four $l_R N_L \chi^-$ terms, it is broken softly by the dimension-three trilinear $\eta^+ \chi^- \phi^0$ term to complete the loop. This guarantees the one-loop charged-lepton mass to be finite. Note that a dark $U(1)_D$ symmetry [19, 20] is supported here with $\chi^+, (\eta^+, \eta^0)$, and $N_{L,R}$ all transforming as 1 under $U(1)_D$. The dimension-three soft terms $\tilde{N}_{L} N_{R}$ are assumed to break $A_4$ to $Z_3$ through the well-known unitary matrix [1, 21, 22] $U_\omega$, i.e.

$$M_N = U^T_\omega \begin{pmatrix} m_{N_1} & 0 & 0 \\ 0 & m_{N_2} & 0 \\ 0 & 0 & m_{N_3} \end{pmatrix} U_\omega,$$  \hspace{1cm} (1)
where

\[
U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega
\end{pmatrix}.
\]  

(2)

In the \(A_4\) limit, \(M_N\) is proportional to the identity matrix. With three different mass eigenvalues, the residual symmetry is \(Z_3\) lepton triality. Let the \((\eta^+, \chi^+)\) mass eigenvalues be \(m_{1,2}\) with mixing angle \(\theta\), then each lepton mass is given by \[19\]

\[
m_l = \frac{f_L f_R \sin \theta \cos \theta m_N}{16\pi^2} [F(x_1) - F(x_2)],
\]

(3)

where \(F(x) = x \ln x / (x - 1)\), with \(x_{1,2} = m_{1,2}^2/m_N^2\).

The dark \(U(1)_D\) symmetry forbids the quartic scalar term \((\Phi^i \eta)^2\), so that a neutrino mass is not generated as in Ref. \[16\]. It comes instead from Fig. 2, where the scalars \(s_{1,2,3}\) are assumed real \[10, 23, 24\] to enable cobimaximal mixing, hence a separate dark \(Z_2\) symmetry is required. Let the \(\bar{F}_L E_R\) mass term be \(m_D\) and assumed to be much smaller than \(m_E, m_F\).

![Figure 2: One-loop generation of neutrino mass from \(s\).](image)

then each neutrino mass is given by

\[
m_\nu = \frac{h^2 m_D^2 m_F}{16\pi^2 (m_F^2 - m_s^2)} [G(x_F) - G(x_s)],
\]

(4)

where

\[
G(x) = \frac{x}{1-x} + \frac{x^2 \ln x}{(1-x)^2},
\]

(5)
with \( x_F = m_F^2/m_E^2 \), \( x_s = m_s^2/m_E^2 \). The dimension-two \( s_i s_j \) terms are allowed to break \( Z_3 \) arbitrarily. However, since this mass-squared matrix is real, it is diagonalized by an orthogonal matrix \( O \), hence the neutrino mixing matrix is given by \( U_{\nu} = U_\omega O \),

resulting in \( U_{\mu i} = U_{\tau i}^* \), thus guaranteeing cobimaximal mixing: \( \theta_{13} \neq 0, \theta_{23} = \pi/4, \delta_{CP} = \pm \pi/2 \).

In a previous proposal \([10]\), instead of Fig. 1, the radiative charged-lepton masses also come from scalars, i.e. \( x_i^+ \sim 3, y_i^+ \sim 1, 1', 1'' \) under \( A_4 \). The \( A_4 \to Z_3 \) breaking is accomplished by rotating \( x_i^+ \) through \( U_\omega \) so that \( x_{1,2,3}^+ \) now correspond to \( y_{1,2,3}^+ \) under \( Z_3 \), and allowing the \((x_1, y_1), (x_2, y_2), (x_3, y_3)\) sectors to have separate arbitrary masses. Now the quartic scalar coupling \((x_1^+ s_1 + x_2^+ s_2 + x_3^+ s_3)(x_1^- s_1 + x_2^- s_2 + x_3^- s_3)\) is allowed under \( A_4 \). If the \( s_i s_j \) mass-squared terms break \( Z_3 \) as in Fig. 2, then the \( s_1 s_2(x_1^+ x_2^- + x_3^+ x_3^-) \) term from the above will induce a quadratic \( x_1 x_2 \) term as shown in Fig. 3. Whereas this diagram is not quadratically divergent, it is still logarithmically divergent. This means a counterterm is required for \( x_1^+ x_2^- + x_2^+ x_1^- \), thereby invalidating the \( Z_3 \) residual symmetry necessary to derive \( U_\omega \) and thus Eq. (6).

In this proposal, the \( A_4 \to Z_3 \) breaking comes from \( \bar{N}_L N_R \), with the Dirac fermions \( N_{1,2,3} \) distinguished from one another by the residual \( Z_3 \) lepton triality through \( U_\omega \) as shown in Eq. (1). The soft breaking of \( Z_3 \) by \( s_1 s_2 \) induces only a finite two-loop correction to the
$N_1 - N_2$ wavefunction mixing as shown in Fig. 4. Therefore this construction solves a long-standing technical problem in renormalizable theories of $A_4$ flavor mixing. To summarize, (1) $A_4$ is respected by all dimension-four terms; (2) $Z_3$ is respected by all dimension-three terms; (3) $Z_3$ is broken arbitrarily by dimension-two terms to allow cobimaximal mixing according to Eq. (6); (4) the $s_i s_j$ terms generate very small finite radiative corrections to $Z_3$ breaking in the dimension-three terms, justifying the use of $U_\omega$ to obtain Eq. (6).

As for dark matter, there are in principle two stable components: the lightest $N$ with $U(1)_D$ symmetry and the lightest $s$ with $Z_2$ symmetry. Whereas $N$ has only the allowed $\bar{N}_R (\nu_L \eta^0 - l_L \eta^+)$ interactions, $s$ has others, i.e. $s^2 \Phi^\dagger \Phi$, $s^2 \eta^\dagger \eta$, $s^2 \chi^+ \chi^-$, as well as $s (\bar{\nu}_L E_R^0 + \bar{l}_L E_R^-)$. Their interplay to make up the total correct dark-matter relic abundance of the Universe and how they may be detected in underground direct-search experiments require further study.

An immediate consequence of radiative charged-lepton mass is that the Higgs Yukawa coupling $h \bar{l} l$ is no longer exactly $m_l/(246 \text{ GeV})$ as predicted by the standard model, as studied in detail already [27, 28]. Because of the $Z_3$ lepton triality, large anomalous muon magnetic moment may be accommodated while $\mu \rightarrow e\gamma$ is suppressed [28].

In conclusion, cobimaximal neutrino mixing ($\theta_{13} \neq 0, \theta_{23} = \pi/4, \delta_{CP} = \pm \pi/2$) is achieved rigorously in a renormalizable model of radiative charged-lepton and neutrino masses. The key is the soft breaking of $A_4$ to $Z_3$ by dimension-three terms, so that the subsequent
breaking of $Z_3$ by dimension-two terms only introduces very small finite corrections to the $U_\omega$ transformation needed to obtain cobimaximal mixing as given by Eq. (6).

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References

[1] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001).
[2] E. Ma, Phys. Rev. D66, 117301 (2002).
[3] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B552, 207 (2003).
[4] E. Ma, Phys. Rev. D82, 037301 (2010).
[5] Q.-H. Cao, A. Damanik, E. Ma, and D. Wegman, Phys. Rev. D83, 093012 (2011).
[6] E. Ma, Phys. Rev. D70, 031901(R) (2004).
[7] G. Altarelli and F. Feruglio, Nucl. Phys. B720, 64 (2005).
[8] W. Grimus and L. Lavoura, Phys. Lett. B579, 113 (2004).
[9] R. N. Mohapatra and C. C. Nishi, Phys. Rev. D86, 073007 (2012).
[10] E. Ma, Phys. Rev. D92, 051301(R) (2015).
[11] P. Chen, C.-Y. Yao, and G.-J. Ding, Phys. Rev. D92, 073002 (2015).
[12] A. S. Joshipura and K. M. Patel, Phys. Lett. B749, 159 (2015).
[13] H.-J. He, W. Rodejohann, and X.-J. Xu, Phys. Lett. B751, 586 (2015).
[14] X.-G. He, Chin. J. Phys. 53, 100101 (2015).
[15] E. Ma, Phys. Lett. B752, 198 (2016).
[16] E. Ma, Phys. Rev. D73, 077301 (2006).
[17] Particle Data Group, K. A. Olive et al., Chin. Phys. C38, 090001 (2014).
[18] K. Abe et al., (T2K Collaboration), Phys. Rev. D91, 072010 (2015).
[19] E. Ma, Phys. Rev. Lett. 112, 091801 (2014).

[20] E. Ma, I. Picek, and B. Radovcic, Phys. Lett. B726, 744 (2013).

[21] N. Cabibbo, Phys. Lett. 72B, 333 (1978).

[22] L. Wolfenstein, Phys. Rev. D18, 958 (1978).

[23] S. Fraser, E. Ma, and O. Popov, Phys. Lett. B737, 280 (2014).

[24] E. Ma, A. Natale, and O. Popov, Phys. Lett. B746, 114 (2015).

[25] K. Fukuura, T. Miura, E. Takasugi, and M. Yoshimura, Phys. Rev. D61, 073002 (2000).

[26] T. Miura, E. Takasugi, and M. Yoshimura, Phys. Rev. D63, 013001 (2001).

[27] S. Fraser, and E. Ma, Europhys. Lett. 108, 11002 (2014).

[28] S. Fraser, E. Ma, and M. Zakeri, arXiv:1511.07458 [hep-ph].