Branes, Orientifolds and Chiral Gauge Theories

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ABSTRACT

We discuss some aspects of the physics of branes in the presence of orientifolds and the corresponding worldvolume gauge dynamics. We show that at strong coupling orientifolds sometimes turn into bound states of orientifolds and branes, and give a worldsheet argument for the flip of the sign of an orientifold plane split into two disconnected parts by an NS fivebrane. We also describe the moduli space of vacua of $\mathcal{N} = 2$ supersymmetric gauge theories with symplectic and orthogonal gauge groups, and analyze a set of four dimensional $\mathcal{N} = 1$ supersymmetric gauge theories with chiral matter content using branes.
1 Introduction and Discussion

In the last year important new insights were obtained into the classical and quantum vacuum structure of supersymmetric gauge theories in different dimensions with various numbers of supersymmetries. This was achieved by realizing the gauge theories in question as low energy worldvolume theories on branes in string theory (see [1] for a review and references), and studying the resulting brane configurations. Some of the results that were found are:

1. Nahm’s construction of the moduli space of magnetic monopoles has been obtained by using the description of monopoles as $D$-strings stretched between parallel $D3$-branes in type IIB string theory [2].

2. Montonen and Olive’s electric-magnetic duality in four dimensional $N = 4$ SUSY gauge theory as well as Intriligator and Seiberg’s mirror symmetry in three dimensional $N = 4$ SUSY gauge theory were shown to be consequences of the non-perturbative S-duality symmetry of type IIB string theory [3, 4, 5].

3. The auxiliary Riemann surface whose complex structure was proven by Seiberg and Witten to determine the low energy coupling matrix of four dimensional $N = 2$ SUSY gauge theory was shown in [6, 7] to be part of the worldvolume of a fivebrane. Hence it is physical in string theory.

4. Seiberg’s infrared equivalence between different four dimensional $N = 1$ supersymmetric gauge theories was shown [8, 9] to be manifest in string theory. The electric and magnetic theories provide different parametrizations of the same quantum moduli space of vacua. They are related by smoothly exchanging fivebranes in an appropriate brane configuration. Many additional features of the vacuum structure of $N = 1$ SUSY gauge theories were reproduced by studying the fivebrane configuration [10, 11, 12].

String theory has vastly more degrees of freedom than field theory but most of these are irrelevant at low energies. The conventional view in the past was that to study low energy gauge dynamics, it is sufficient to retain the light (i.e. gauge theory) degrees of freedom and, therefore, string theory can not shed light on issues having to do with strong coupling at long distances.

The recent results mentioned above suggest that while most of the degrees of freedom of string theory are indeed irrelevant for understanding low energy dynamics, there is a sector of the theory that is significantly larger than the gauge theory in question that should be kept to understand the low energy structure. This sector involves degrees of freedom living on branes and describing their internal fluctuations and embedding in spacetime.

Brane dynamics seems to provide a new perspective on gauge theory; the description in terms of gauge bosons and quarks appears as an effective low energy picture that is useful in some region of the moduli space of vacua. Different descriptions are useful in different regions of moduli space, and in some regions the infrared behavior cannot be given a field theory interpretation at all. The success of the brane description in reproducing strong coupling phenomena such as hidden relations between different gauge theories leads one to hope that it
can serve as a basis for an alternative formulation of gauge theory. Such a formulation might be useful for computing non-vacuum low energy properties, e.g. the masses and interactions of low lying non-BPS states. At present, its development awaits a better understanding of the dynamics of fivebranes in string theory.

The set of gauge theories that have been constructed to date using branes is rather limited. In four dimensional theories with $N \leq 2$ SUSY only the simplest kinds of matter have been studied (fundamentals and various two index tensors). It would be very interesting to study generic chiral $N = 1$ SUSY gauge theories, which in addition to their phenomenological appeal have in general rather rich dynamics. Some recent work on brane constructions of chiral gauge theories appears in [13, 14].

One of the most interesting phenomena discovered in four dimensional $N = 1$ SUSY gauge theory is Seiberg’s duality [15]. Seiberg’s original work on SQCD has been generalized in gauge theory in two directions. One class of examples [16, 17] involves introducing two index tensors with polynomial superpotentials. The duality properties of this class of theories have been understood using branes (and we will discuss a chiral example belonging to this class later). The other class [18] has the property that one of the members of a dual pair has an orthogonal gauge group with matter in the spinor representation. It has still not been constructed in brane theory. It would be very interesting to construct the theories of [18] using branes.

The purpose of this paper is to take a modest step towards describing non-trivial chiral gauge theories in four dimensions by studying the physics of branes near orientifolds. Orientifolds play an essential role in constructing orthogonal and symplectic gauge theories on branes [19, 4], and allow one to describe theories with interesting matter content (see for example [20, 14]). Therefore, it is likely that understanding brane dynamics near orientifolds is important for the program outlined above.

The plan of the paper is as follows. In section 2 we discuss configurations of parallel orientifolds and D-branes preserving sixteen supercharges. By using the relation between Montonen-Olive duality and S-duality of type IIB string theory we show that an orientifold threeplane (O3-plane) with positive Ramond-Ramond (RR) charge is exchanged under S-duality with an O3-plane with negative charge with a $D3$-brane embedded in it. Considering this system on a transverse torus we briefly discuss the disconnected components of the moduli space of $Dp$-branes near an $Op$-plane wrapped around $T^{p-3}$, in which $2^{p-3}$ D-branes are stuck at the orientifold.

In section 3 we discuss four dimensional $N = 2$ SYM with orthogonal or symplectic gauge group obtained by suspending $D4$-branes between $NS5$-branes near an orientifold plane. We describe the moduli space of vacua and show that it agrees with gauge theory. We also give a worldsheet argument for the fact that an NS fivebrane which intersects an orientifold plane, splitting it into two disconnected parts, acts as a domain wall for RR charge. This was previously observed from the worldvolume gauge theory point of view in [19].

In section 4 we study a configuration describing a chiral $N = 1$ SUSY four dimensional gauge theory. We discuss some of its deformations and show that the brane analysis reproduces the predictions of [17] regarding duality in this system.
2 Systems With Sixteen Supercharges

Consider a configuration of $N_c$ D3-branes and an O3-plane all stretched in $(x^0, x^1, x^2, x^3)$. The threebrane worldvolume gauge theory has $N = 4$ SUSY. The orientifold projection breaks the usual unitary gauge group on $N_c$ parallel $Dp$-branes to a symplectic or orthogonal one, depending on the RR charge of the orientifold. For an orientifold of charge $Q_{O3} = +Q_{D3}/2$ ($Q_{O3} = -Q_{D3}/2$) the gauge group is $G = Sp(N_c/2)$ ($G = SO(N_c)$). In the former case $N_c$ must be even while in the latter it can be odd. For odd $N_c$ there is a single D3-brane without a mirror image stuck at the orientifold plane. It is easy to see that such a brane survives the orientifold projection only for $Q_{O3} = -Q_{D3}/2$.

The gauge coupling of the theory on the threebrane $g$ is related to the IIB string coupling $g_s$ via $g^2 = g_s$. If we send the string length $l_s \to 0$ holding $g$ fixed, the theory on the threebranes decouples from gravity and massive string modes, and the four dimensional dynamics becomes that of $N = 4$ SYM with gauge group $G$ at all energy scales.

The Coulomb branch of the theory is parametrized by the locations of the D3-branes in the transverse space labeled by $(x^1, x^5, \ldots, x^9)$. Since the D3-branes can only leave the orientifold plane in pairs, the dimension of the Coulomb branch is $6 \times [N_c/2]$ as expected from gauge theory. Generically in the Coulomb branch the gauge symmetry is broken to $U(1)^{[N_c/2]}$, and all the charged gauge bosons and dyons, which correspond to $(p, q)$ strings stretched between different threebranes, are massive. Singularities in moduli space correspond to points with enhanced unbroken gauge symmetry, at which charged gauge bosons and dyons go to zero mass. The most singular point in the moduli space is the origin, which corresponds to a non-trivial CFT parametrized by the exactly marginal gauge coupling $g$.

One application of this construction is to the study of the moduli space of monopoles in broken $SO(N_c)$ or $Sp(N_c/2)$ gauge theories. Monopoles in broken $SO/Sp$ gauge theory are described by $D$-strings stretched between different D3-branes. Consider for example the rank one case $N_c = 2$. For positive orientifold charge the gauge group is $Sp(1) \simeq SU(2)$ and the moduli space of $k SU(2)$ monopoles can be studied by analyzing the worldvolume theory of $k$ D-strings connecting the single “physical” D3-brane to its mirror image. For negative orientifold charge the gauge group is $SO(2) \simeq U(1)$ and one does not expect non-singular monopoles to exist. This means that D-strings cannot connect the single physical D3-brane to its mirror image. This is related by S-duality to the fact that for negative orientifold charge the ground states of fundamental strings stretched between the D3-brane and its image are projected out by the orientifold projection.

The identification of Montonen-Olive duality with S-duality of type IIB string theory can be used to learn about strong coupling properties of orientifolds. Recall that in gauge theory, electric-magnetic duality acts on the gauge coupling as strong-weak coupling duality $g \to 1/g$. It also takes the gauge algebra to the dual algebra; in particular $G = SO(2r)$ is self-dual, while $SO(2r + 1)$ and $Sp(r)$ transform to each other under Montonen-Olive duality.

The D3-brane is self-dual under S-duality, and so is the four-form gauge field that it couples

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4In the limit $l_s \to 0$ one must hold the energy scale $\Phi^i = x^i/l_s^2$ ($i = 4, \ldots, 9$) fixed.
to. For agreement with $N = 4$ supersymmetric $SO(2r)$ gauge theory, the $O3$-plane with negative charge must be self-dual as well. To study the non-simply laced case consider $e.g.$ a weakly coupled $SO(2r+1)$ gauge theory. The orientifold charge is $-Q_{D3}/2$; the 6-dimensional Coulomb branch corresponds to removing $r$ pairs of threebranes from the orientifold plane. A single threebrane which does not have a mirror remains stuck at the orientifold. When the gauge coupling becomes large there are two ways of thinking about the system. We can either continue thinking about it as a strongly coupled $SO(2r+1)$ gauge theory, or relate it to a weakly coupled $Sp(r)$ gauge theory by performing a strong-weak coupling S-duality transformation. The latter is described by an orientifold with charge $+Q_{D3}/2$.

Thus, Montonen-Olive duality of gauge theory teaches us that a “bound state” of an $O3$-plane with negative RR charge and a single $D3$-brane embedded in it (a configuration with RR charge $(-1/2 + 1)Q_{D3}$) transforms under S-duality of type IIB string theory into an $O3$-plane with Ramond charge $+Q_{D3}/2$. This is consistent with the fact that RR four-form charge cannot change under S-duality, since the corresponding gauge field is self-dual.

For higher dimensional orientifolds, similar considerations lead to a strong-weak coupling relation between the following two apparently different configurations. One has an $Op$-plane with positive charge wrapped around a $(p - 3)$ dimensional torus $T^{p-3}$. The other is a certain disconnected component of the moduli space of $2^{p-3} Dp$-branes in the presence of a negatively charged orientifold plane, $Op_-$ (all objects wrapped around a dual torus $\hat{T}^{p-3}$), in which the $2^{p-3}$ branes are stuck at the orientifold.

The relation is obtained by considering an $Op$-plane with positive RR charge wrapped around $T^{p-3}$. Clearly, this configuration has no moduli and this must be the case for any other configuration related to it by U-duality. Performing a T-duality transformation which inverts the volume of the $(p-3)$ torus we find $2^{p-3}$ $O3$-planes with positive charge distributed at equal distances on the dual torus $\hat{T}^{p-3}$. A further S-duality turns them into $2^{p-3}$ negative charge orientifold planes, each of which has a $D3$-brane embedded in it. A final T-duality on the $(p - 3)$ torus gives rise to an $Op$-plane with negative charge wrapped around a torus $\hat{T}^{p-3}$ and $2^{p-3} Dp$-branes embedded in it. Since the original system we started with (the positive charge $Op$-plane) did not have any moduli, this must be the case for the final configuration as well. In particular, the $2^{p-3}$ D-branes are stuck at the orientifold, without the ability to move. Similar disconnected components of the moduli space of D-branes near orientifold planes have been recently discussed in [24].

### 3 Systems With Eight Supercharges

Four dimensional $N = 2$ SUSY gauge theory with gauge group $G = U(N_c)$ and $N_f$ hypermultiplets in the fundamental representation of the gauge group can be studied [3, 4] by suspending $N_c$ $D4$-branes with worldvolume $(x^0, x^1, x^2, x^3, x^6)$ between two $NS5$-branes which are extended in $(x^0, x^1, x^2, x^3, x^4, x^5)$ and located at different values of $x^6$. The fourbranes are finite in $x^6$, stretching between the two $NS5$-branes; thus they describe four dimensional physics on their worldvolume. The matter hypermultiplets arise from $N_f$ $D6$-branes extended
in \((x^0, x^1, x^2, x^3, x^7, x^8, x^9)\) and placed between the two NS5-branes in \(x^6\) (see \([4]\) for a more detailed discussion).

To describe orthogonal and symplectic gauge groups one can add to the brane configuration an \(O4\)-plane parallel to the \(D4\)-branes, or an \(O6\)-plane parallel to the \(D6\)-branes \([19, 9]\). Both cases are going to be useful below when we describe the chiral \(N = 1\) configuration. We will next discuss the better understood case of an \(O6\)-plane, confining our discussion of the \(O4\)-plane to a few comments. A more detailed discussion appears in \([4]\).

In the presence of an \(O6\)-plane we would like to stretch \(N_c\) fourbranes from an NS5-brane to its mirror image. The first question that we have to address is whether it is possible to stretch fourbranes this way without breaking SUSY. For example, in the previous section\(^2\) we saw that it is impossible to stretch a BPS saturated \(Dp\)-brane between a \(D(p + 2)\)-brane and its mirror image with respect to an \(O(p + 2)\)-plane with negative charge. One might worry that the same is true for \(D4\)-branes stretched between \(NS\)-branes.

In fact, it turns out that the relevant \(D4\)-branes are indeed BPS saturated \([4]\). To show that, one maps (using U-duality) the fourbrane connecting an NS5-brane to its image, to a fundamental string connecting two \(D5\)-branes in the presence of an orientifold nineplane, and uses known properties of D-branes.

We will next describe the classical gauge theories arising for the two choices of the sign of the orientifold charge, starting with the case of positive \(O6\_+\) charge, which leads to an orthogonal projection on the \(D4\)-branes. The case of \(O6\_-\), which leads to a symplectic gauge group, will be considered later.

The gauge group on \(N_c\) \(D4\)-branes connecting an NS5-brane to its mirror image with respect to an \(O6\_+\) plane is \(SO(N_c)\). \(N_f\) \(D6\)-branes parallel to the \(O6\)-plane located between the NS5-brane and the orientifold give \(N_f\) hypermultiplets in the fundamental \((N_c)\) representation of \(SO(N_c)\), arising as usual from \(4 - 6\) strings. In \(N = 1\) SUSY notation there are \(2N_f\) chiral multiplets \(Q^i, i = 1, \ldots, 2N_f\) which are paired to make \(N_f\) hypermultiplets. The global flavor symmetry of this gauge theory is \(Sp(N_f)\), in agreement with the projection imposed by the positive charge \(O6\_+\)-plane on the \(D6\)-branes.

The Coulomb branch of the \(N = 2\) SUSY gauge theory is parametrized by the locations of the \(D4\)-branes along the fivebrane, in the \((x^4, x^5)\) plane. Entering the Coulomb branch involves removing the ends of the fourbranes from the orientifold plane (which is located at a particular point in the \((x^4, x^5)\) plane). Since the fourbranes can only leave the orientifold plane in pairs, the dimension of the Coulomb branch is \([N_c/2]\), in agreement with the gauge theory description.

The different Higgs branches of the gauge theory are parametrized by all possible breakings of fourbranes on sixbranes. As for the unitary case there are many different branches; as a check that we get the right structure, consider the fully Higgsed branch which exists when the number of flavors is sufficiently large. From gauge theory we expect its dimension to be \(2N_cN_f - N_c(N_c - 1)\).

\(^2\)Where we discussed explicitly the case \(p = 1\); clearly the result is \(p\) independent.
The brane analysis gives

\[ \dim \mathcal{M}_H = \sum_{i=1}^{N_c} 2(N_f + 1 - i) = 2N_f N_c - N_c(N_c - 1) = [2N_f N_c - N_c(N_c + 1)] + 2N_c \]  

The term in the square brackets is the number of moduli corresponding to segments that do not touch the orientifold, and the additional \(2N_c\) is the number of moduli coming from the segments of the fourbranes connecting the \(D6\)-brane closest to the orientifold to its mirror image. These segments transform to themselves under the orientifold projection and thus are dynamical for positive orientifold charge.

The \(2N_c\) moduli coming from fourbranes connecting a \(D6\)-brane to its image have a natural interpretation in the theory on the \(D6\)-branes. At low energies this is an \(Sp(1)\) gauge theory with sixteen supercharges, and the \(D4\)-branes stretched between the \(D6\) and its mirror can be thought of as \(Sp(1)\) monopoles, as in the previous section. From this point of view the above \(2N_c\) moduli parametrize the space of \(N_c\) \(Sp(1)\) monopoles.

Thus, the total dimension of moduli space agrees with the gauge theory result. It is easy to similarly check the agreement with gauge theory of the maximally Higgsed branch for small \(N_f\), as well as the structure of the mixed Higgs-Coulomb phases. The analysis of the vacuum structure of the corresponding gauge theories appears in [22].

For negative charge of the \(O6\)-plane, the configuration discussed above describes an \(Sp(N_c/2)\) gauge theory with \(N_f\) hypermultiplets in the fundamental \((N_c)\) representation. Qualitatively, most of the analysis is the same as above, but the results are clearly somewhat different. For example, the dimension of the fully Higgsed branch is in this case \(2N_f N_c - N_c(N_c + 1)\), smaller by \(2N_c\) than the \(SO\) case discussed above.

From the point of view of the brane construction the Higgs branch is different because it is no longer possible to connect a \(D6\)-brane to its mirror image by a \(D4\)-brane. Such fourbranes transform to themselves under the orientifold projection, and are projected out when the \(O6\)-plane has negative charge. This is also clear from the point of view of interpreting these fourbranes as magnetic monopoles in the sixbrane theory. In this case the theory on the \(D6\)-brane adjacent to the orientifold and its image has gauge group \(SO(2)\), and there are no non-singular monopoles.

Therefore, the breaking of the \(D4\)-branes on \(D6\)-branes near the orientifold is modified. We have to stop the pattern (1) when we get to the last two \(D6\)-branes before the orientifold, and there we must perform the breaking in such a way that no fourbrane connects a \(D6\)-brane to its mirror image. It is not difficult to see that compared to (1) we lose \(2N_c\) moduli. Overall, the brane Higgs branch12(395,132),(650,191) is \(2N_f N_c - N_c(N_c + 1)\) dimensional, in agreement with the gauge theory analysis. One can again check that the full classical phase structure of the \(Sp(N_c/2)\) gauge theory [22] is similarly reproduced.

Another way to study four dimensional \(N = 2\) SUSY gauge theories with symplectic and orthogonal gauge groups is by using branes and orientifold fourplanes. It is interesting and in some ways more mysterious than the construction using \(O6\)-planes described above (see [2] for a more detailed discussion).
Consider, for example, a pair of $NS5$-branes with worldvolume $(x^0, x^1, \cdots, x^5)$ stuck on an orientifold fourplane stretched in $(x^0, x^1, x^2, x^3, x^6)$. The two $NS5$-branes are separated by a distance $L_6$ (in the $x^6$ direction) along the orientifold. We can now stretch $N_c$ $D4$-branes parallel to the $O4$-plane between the two $NS5$-branes, and place $2N_f$ $D6$-branes with worldvolume $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$ between the $NS5$-branes as before. Alternatively, we can replace the $D6$-branes by $2N_f$ semi-infinite $D4$-branes attached to the $NS5$-branes and extending (say) to $x^6 = \pm \infty$.

Each of the $NS5$-branes divides the orientifold into two disconnected parts and acts as a domain wall for orientifold charge. If the charge on one side of the fivebrane is +1, on the other it is −1 and vice versa [14]. We will next motivate this interesting effect by comparing the brane picture to the worldvolume gauge theory, as well as by a direct worldsheet analysis.

To see that the orientifold charge flip is required by gauge theory it is useful to represent fundamental hypermultiplets by semi-infinite fourbranes. If the charge of the segment of the $O4$-plane trapped between the $NS5$-branes is positive, the gauge group on $N_c$ mirror pairs of $D4$-branes stretched between the $NS5$-branes is $Sp(N_c)$. $2N_f$ semi-infinite $D4$-branes extending to $x^6 = \infty$ (say) give rise to $N_f$ hypermultiplets in the fundamental representation of $Sp(N_c)$. It is well known (see e.g. [22]) that the global symmetry in gauge theory in this situation is $SO(2N_f)$. In brane theory, the global symmetry arises from the gauge symmetry of the $2N_f$ semi-infinite fourbranes. To get the right global symmetry, the charge of the segment of the orientifold to the right of the $NS5$-branes must be negative.

An alternative gauge theory explanation of the charge flip involves a six dimensional version of the above brane construction that will be useful in the next section. The four dimensional $N = 2$ SUSY gauge theory in question can be thought of as a dimensional reduction of a six dimensional $N = 1$ SUSY theory. To obtain the six dimensional theory using branes, we can compactify $(x^4, x^5)$, T-dualize the configuration, and then decompactify the dual $(x^4, x^5)$. Using the standard action of T-duality on branes [1], one finds that the $D4$-branes and $O4$-plane become $D6$-branes and an $O6$-plane stretched in $(x^0, x^1, x^2, x^3, x^4, x^5, x^6)$; the $NS5$-branes are invariant. Such brane configurations realizing six dimensional theories were studied in [28].

The $5+1$ dimensional $Sp(N_c)$ gauge theory with $N_f$ hypermultiplets is anomalous for generic $N_f$. It is anomaly free for $N_f = 2N_c + 8$. Anomaly freedom in six dimensions manifests itself in the brane construction as RR charge conservation on the $NS5$-branes. The $D6$-branes ending on them carry net RR charge that has nowhere to escape and thus has to be cancelled. We can split the $2N_c + 8$ flavors symmetrically between the two $NS5$-branes, by attaching $2N_c + 8$ semi-infinite $D6$-branes to the right of the rightmost $NS5$-brane and the other $2N_c + 8$ to the left of the leftmost $NS5$-brane. Naively, there appears to be a deficit of eight units of charge on each $NS5$-brane. It is cancelled by the jump of the orientifold charge from +4 to −4 described above[1].

To understand the jump of the charge of an $Op$-plane cut in two by an $NS5$-brane directly in worldsheet terms, recall that the RR charge of the orientifold can be computed by evaluating the one point function of the vertex operator of the closed string RR $(p + 1)$-form gauge field.

\[\text{In general, the charge of an } Op\text{-plane cut into two parts by an } NS5\text{-brane jumps from } +2^{p-4} \text{ to } -2^{p-4}.\]
to which the \(Dp\)-brane couples, on the leading non-orientable Riemann surface, \(RP^2\). This diagram is non zero and is localized at the orientifold. The charge of the orientifold to the left of the \(NS5\)-brane is given by a worldsheet path integral over an \(RP^2\) surface embedded in spacetime with \(x^6\) large and negative. We may refer to this surface as \(RP^2_L\). The charge of the segment to the right of the \(NS5\)-brane is given by a path integral over a surface with \(x^6\) large and positive, which we will refer to as \(RP^2_R\).

The path integral contains the term \(\exp i(\int_{RP^2} B)\) where \(B\) is the NS-NS sector Kalb-Ramond two-form gauge field under which the \(NS5\)-brane is magnetically charged. The ratio of the path integrals corresponding to the one point function of the RR \((p + 1)\)-form gauge field far to the left and to the right of the \(NS5\)-brane (which gives the ratio of the corresponding RR charges \(Q_L/Q_R\)) can thus be written by using Stokes’ theorem as

\[
Q_L/Q_R = \exp i \int_{C_3} H
\]

where \(H = dB\), and \(C_3\) is a three dimensional surface whose boundaries are \(RP^2_L\) and \(RP^2_R\). It is important that \(C_3\) necessarily encloses the \(NS5\)-brane and thus the integral receives a contribution which is due to the magnetic coupling of the \(NS5\)-brane to \(B\). If we replaced \(RP^2\) by a two-sphere in the above analysis, the integral \[2\] would give \(2\pi\), the total flux of the \(NS5\)-brane (by the Dirac quantization condition). Since the boundary is \(RP^2\) the result is \(\pi\), half of the total flux\[1\]. Substituting this into \[2\] we see that the charges of the left and right portions of the orientifolds are opposite, \(Q_L = -Q_R\).

The construction of four dimensional \(N = 2\) SYM using branes near an \(O4\)-plane can be used to study the moduli space of vacua of the theory. Some of the details of this analysis appear in \[4\].

4 Systems With Four Supercharges

In the previous section we discussed configurations of \(NS5\), \(D4\) and \(D6\)-branes in the presence of orientifold planes, which preserve eight supercharges and are useful for describing four dimensional \(N = 2\) SUSY gauge theories. To describe \(N = 1\) SYM we would like to break four supercharges by changing the orientation of some of the branes in the configuration \[5\]. Defining the complex variables \(v, w\) by:

\[
\begin{align*}
v &= x^4 + ix^5 \\
w &= x^8 + ix^9
\end{align*}
\]

one can check \[24\] that relative complex rotations of the different branes and orientifolds in the \((v, w)\) plane,

\[
\begin{pmatrix} v \\ w \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix}
\]

\[4\]No inconsistency with the Dirac quantization condition is induced by this, since there are no surfaces \(C_3\) with boundaries \(RP^2_L\) and \(RP^2_R\) which do not enclose the \(NS5\)-brane.
do the job. For generic rotation angles $\{\theta_i\}$ the configuration preserves four supercharges. For particular values of $\{\theta_i\}$ the SUSY is enhanced to eight supercharges, and one recovers the configurations of section 3.

As an example, the $NS5$-brane used in section 3 was located at a particular value of $w$ and stretched in $v$. Rotating it by an angle $\theta$ we find a fivebrane which we may call the “$NS_\theta$ fivebrane.” It is located at $w = v \tan \theta$ and stretched in the orthogonal direction, $v_\theta = v \cos \theta + w \sin \theta$. Two particularly useful special cases are $\theta = 0, \pi/2$. The first corresponds to the original $NS5$-brane of section 3. The second is the $NS5'$-brane of [8], which is stretched in $w$ and located at a particular value of $v$.

Similarly, one can rotate each of the $D6$-branes of section 3 separately as in (4). This leads to a large variety of $N = 1$ SUSY models which we will not discuss here (see [1] for more details). Instead we will analyze below a particular example – a chiral $N = 1$ model that illustrates some of the issues discussed in the previous sections.

4.1 A Chiral Brane Configuration

Below we will consider brane configurations in which an $NS5'$-brane without a mirror image is embedded in an $O6$-plane extending, as in section 3, in $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$. One can achieve this situation by embedding a pair of $NS5'$-branes in the $O6$-plane, separating them along the orientifold (in $x^7$) and taking one of them to infinity. The remaining $NS5'$-brane is free to move inside the $O6$-plane, along the $x^7$ axis. In the absence of other branes nothing depends on its location; after we add more features it will become an important parameter on which low energy physics depends sensitively.

The $NS5'$-brane, located (say) at $x^7 = 0$, divides the $O6$-plane into two disconnected regions, corresponding to positive and negative $x^7$. As we saw in section 3, in this situation the RR charge of the orientifold jumps, from $+4$ to $-4$, as we cross the $NS5'$-brane. The part of the orientifold with negative charge (which we will take to correspond to $x^7 < 0$) has furthermore eight semi-infinite $D6$-branes embedded in it. As discussed in section 3, the presence of these $D6$-branes is required for charge conservation or, equivalently, vanishing of the six dimensional anomaly.

In addition to the eight semi-infinite $D6$-branes, we can place on the orientifold any number of parallel infinite $D6$-branes extending all the way from $x^7 = -\infty$ to $x^7 = \infty$. We will denote the number of such $D6$-branes by $2N_f$.

Then, an $NS_\theta$ fivebrane located at a distance $L_6$ in the $x^6$ direction from the $NS5'$-brane, but at the same value of $x^7$, is connected to the $NS5'$-brane by $N_c$ $D4$-branes stretched in $x^6$, as in section 3. We will see later that $N_c$ must be even for consistency. The mirror image of the $NS_\theta$ fivebrane, which is an $NS_{-\theta}$ fivebrane, is necessarily also connected to the $NS5'$-brane.

We can also place any number of $D6$-branes oriented at arbitrary angles $\theta_i$ between the $NS_\theta$ fivebrane and the orientifold (in $x^6$). We will mainly discuss the case where such branes are absent, but it is easy to incorporate them, following [25, 4].
As is by now standard [1], at low energies and weak string coupling the brane configuration describes a four dimensional gauge theory. Our next task is to identify the particular gauge theory that arises. In the next subsections we address this problem. Before studying the general case we describe the structure for $\theta = 0$ (when the external $NS_{\pm \theta}$ fivebranes are $NS$-branes), and $\theta = \pi/2$ (when they are $NS'$-branes). We also show that the field theory duality of [17] for the resulting gauge theory is reproduced by the brane analysis [8].

4.2 The Case $\theta = 0$

The gauge theory corresponding to the brane configuration described above can be pieced together as follows. Note that the configuration depends on a real parameter $r$ corresponding to the relative position in $x^7$ of the $NS$-brane (and its mirror image) and the $NS'$-brane embedded in the $O6$-plane. In the original configuration $r$ vanishes, but it is useful to study the physics as a function of it.

For positive $r$, the $D4$-branes connecting the $NS$-brane to the $NS'$-brane must reconnect to stretch between the $NS$-brane and its mirror image. At low energies we can ignore the $NS'$-brane and the region $x^7 < r$, and find the brane configuration analyzed in section 3 – an $NS$-brane connected to its image with respect to an $O6_+$ plane by $N_c$ $D4$-branes, in the presence of $2N_f$ $D6$-branes parallel to the orientifold. The low energy theory has in this case an accidental $N = 2$ SUSY; the gauge group is $G = SO(N_c)$ and the matter corresponds to $N_f$ hypermultiplets ($2N_f$ chiral multiplets) in the fundamental representation. Denoting the adjoint chiral multiplet in the $SO(N_c)$ vectormultiplet by $\tilde{A}$ and the fundamental chiral multiplets by $\tilde{Q}_a$, the $N = 2$ superpotential is

$$W = \tilde{Q} A \tilde{Q}$$  \hspace{1cm} (5)

The case of negative $r$ is similar, except now the charge of the $O6$-plane felt by the $D4$-branes intersecting it is negative, and there are $2N_f + 8$ $D6$-branes embedded in it. The corresponding gauge theory is an $N = 2$ SUSY $Sp(N_c/2)$ gauge theory (this makes it clear that $N_c$ must be even) with $N_f + 4$ hypermultiplets ($2N_f + 8$ chiral multiplets $Q^i$) in the fundamental representation. The $N = 2$ superpotential couples the adjoint chiral multiplet $\tilde{S}$ to the fundamentals

$$W = Q \tilde{S} Q$$  \hspace{1cm} (6)

What can we say about the theory with $r = 0$? In the absence of the orientifold, the gauge group would be $U(N_c) \times U(N_c)$ [1]. The orientifold projection preserves the diagonal $U(N_c)$. This is consistent with the low energy gauge groups for $r \neq 0$, both of which are subgroups of $U(N_c)$. $r$ is interpreted, as in other similar situations in brane theory [1], as a FI D-term for the $U(1)$ factor in $U(N_c)$.

As $r \to 0$ we expect all the light fields seen for either sign of $r$ to become massless. Thus, the $U(N_c)$ gauge group at the origin couples to an antisymmetric tensor $A$, a symmetric tensor $\tilde{S}$, $2N_f + 8$ quarks $Q$ in the fundamental representation, and $2N_f$ quarks $\tilde{Q}$ in the antifundamental representation. The superpotential is a combination of (5, 6):

$$W = Q \tilde{S} Q + \tilde{Q} A \tilde{Q}$$  \hspace{1cm} (7)

More precisely, in the limit $l_s, g_s, L_6 \to 0$, with the gauge coupling $g^2 = g_s l_s / L_6$ held fixed.
The transformation properties of the quarks can be understood following [26, 25, 27] who argued that fundamental chiral multiplets of the gauge group come from 4−6 strings connecting the D4-branes to D6-branes ending on an NS5′-brane from below (in $x^7$), while antifundamentals arise from D6-branes ending on the NS5′-brane from above. In our case there are therefore $2N_f$ fundamentals $Q^i$, and $2N_f$ antifundamentals $\tilde{Q}_a$. The global symmetry of the system is determined by the gauge symmetry on the D6-branes, $Sp(N_f) \times SO(2N_f + 8)$. The superpotential (7) is the unique one consistent with this symmetry.

Note that the theory is chiral and potentially anomalous as there are eight more fundamental than antifundamental chiral multiplets. The superpotential (7) implies that the symmetric tensor $\tilde{S}$ is in fact a symmetric bar (i.e. a symmetric tensor with two antifundamental indices). Thus the total anomaly $(2N_f + 8) - 2N_f + (N_c - 4) - (N_c + 4)$ vanishes, as one would expect for a consistent vacuum of string theory.

An interesting feature of the spectrum of light fields is the fact that there are $2N_f$ fundamental superfields $Q^i$ and $2N_f$ antifundamentals $\tilde{Q}_a$ that can be given a mass. Recall [1] that masses correspond to displacements of the D6-branes relative to the D4-branes (and thus to the orientifold as well) in the $(x^4, x^5)$ directions. Since one can only remove D6-branes from the orientifold in pairs, only $N_f$ independent mass parameters seem to be visible, which would appear to indicate that there are only $N_f$ flavors.

To see why there are in fact $2N_f$ flavors it is convenient to remove the D6-branes from the orientifold in the $x^6$ direction. Positions of D6-branes along the $x^6$ axis are often irrelevant parameters in the low energy limit of brane theory. However, it is well known that precisely in the case relevant here, the relative position of an NS5′-brane and a D6-brane is important for the infrared dynamics. When a D6-brane crosses an NS5′-brane (more generally, when parallel D and NS branes cross) the theory loses or gains a massless flavor [1].

In our case when a D6-brane and its image are displaced in $x^6$ from the orientifold (but remain at the origin in $x^4$, $x^5$) two of the four degrees of freedom arising from the corresponding 4−6 strings gain a mass, while two remain massless. The massive states correspond to strings stretched between the D6-branes and D4-branes on the opposite side of the orientifold. These strings have a finite length and hence describe massive states. The massless degrees of freedom correspond to short 4−6 strings connecting the sixbranes to fourbranes on the same side of the orientifold plane. Of course, as the D6-brane approaches the orientifold and the NS5′-brane embedded in it (in $x^6$), the previously massive matter becomes light. When the sixbrane and orientifold coincide the number of massless states jumps by a factor of two. This is why the theory has $2N_f$ fundamental − antifundamental pairs.

As a further check on the identification of the brane configuration and the chiral gauge theory one can analyze the moduli space of vacua as a function of various parameters one can add to the Lagrangian. An example is the FI D-term, which we have already discussed from the brane point of view. In the gauge theory, adding to the Lagrangian a FI D-term for the $U(1)$ vectormultiplet $\text{Tr}V$,

$$\mathcal{L}_D = r \int d^4 \theta \text{Tr}V$$

(8)
modifies the D flatness vacuum conditions:

\[ AA^\dagger - \tilde{S}\tilde{S}^\dagger + QQ^\dagger - \tilde{Q}\tilde{Q}^\dagger = -r \]  

(9)

Setting the quarks \( Q, \tilde{Q} \) to zero we see that when \( r \) is positive, \( S \) gets an expectation value which breaks \( U(N_c) \to SO(N_c) \). Due to the superpotential (7) the \( 2N_f + 8 \) chiral multiplets \( Q^i \) as well as \( \tilde{S} \) become massive and one if left with the \( N = 2 \) spectrum and interactions for gauge group \( SO(N_c) \), (8), with the antisymmetric tensor \( A \) playing the role of the adjoint of \( SO(N_c) \). All of this is easily read off the brane configuration. In particular, the fact that the \( 2N_f + 8 \) quarks \( Q^i \) are massive is due to the fact that the corresponding \( 4 - 6 \) strings have finite length (proportional to \( r \)).

Similarly, for negative \( r \) (9) implies that \( A \) gets an expectation value, breaking \( U(N_c) \) to \( Sp(N_c/2) \). The quarks \( \tilde{Q} \) get a mass and we end up with an \( N = 2 \) gauge theory with \( G = Sp(N_c/2) \) and \( 2N_f + 8 \) light quarks.

The Coulomb, Higgs and mixed branches of moduli space are realized in a similar way to that described in section 3 and in [1]. The Coulomb branch is obtained by removing the \( D4 \)-branes from the orientifold plane in pairs along the \( NS5 \)-brane and its mirror image (i.e. in the \( v \)-plane). This gives rise to an \( N_c/2 \) dimensional moduli space which in gauge theory corresponds to giving expectation values to the fields \( A, \tilde{S} \). Since when we enter the Coulomb branch the fourbranes are already reconnected and stretch between the \( NS5 \)-brane and its image, it doesn’t matter whether the FI D-term \( r \) is turned on or not. Thus the structure of the Coulomb branch is very similar to that of the corresponding \( N = 2 \) SUSY theory, and we will not discuss it in any detail here\(^6\). The Higgs branch of the \( N = 2 \) SUSY theories was described using branes in section 3. There is a jump in the dimension of the Higgs branch as we pass through \( r = 0 \) – the Higgs branches of the symplectic and orthogonal theories are different. This too is clearly seen in the brane construction.

### 4.3 The Case \( \theta = \frac{\pi}{2} \)

In this case the external fivebrane and its mirror image are \( NS5'-\)branes. If the orientifold was absent, the configuration would correspond to a \( U(N_c) \times U(N_c) \) gauge theory with two chiral multiplets \( \Phi_1, \Phi_2 \) transforming as \( (N_c^2, 1) \) and \( (1, N_c^2) \), respectively. \( \Phi_1 \) and \( \Phi_2 \) parametrize separate motions of the two groups of \( N_c \) fourbranes along the fivebranes (i.e. in the \( w \) direction). The orientifold again preserves one of the two \( U(N_c) \) factors, and the corresponding adjoint field \( \Phi \).

The Coulomb branch is now \( N_c \) dimensional. It is labeled by locations in the \( w \) plane of the \( N_c \) \( D4 \)-branes stretched between the \( NS5' \)-brane outside the orientifold and the one inside it. Of course, the \( N_c \) mirror \( D4 \)-branes on the other side of the orientifold (in \( x^6 \)) follow the same motion. Generically along the Coulomb branch the gauge group is broken to \( U(1)^{N_c} \).

\(^6\)Except to note that the brane configuration in question can be used to show that the Coulomb branches of \( SO(N_c) \) gauge theory with \( N_f \) hypermultiplets, and \( Sp(N_c/2) \) gauge theory with \( N_f + 4 \) hypermultiplets four of which are massless, are identical. This can be checked directly by comparing the appropriate Seiberg-Witten curves.
The corresponding gauge theory has in addition to the matter content discussed in the previous subsection the adjoint field $\Phi$. The classical superpotential generalizing (7) is:

$$W = Q \tilde{S} Q + \tilde{Q} A \tilde{Q} + \Phi A \tilde{S} \quad (10)$$

Because of the last term in $W$, generically in the Coulomb branch $A$, $\tilde{S}$ are massive (except for the diagonal components of $\tilde{S}$). In the brane construction we saw that off-diagonal components of $A$ and $\tilde{S}$ are described by strings connecting different fourbranes on opposite sides of the orientifold (while $\Phi$ is described by strings connecting different fourbranes on the same side of the orientifold). When the $D4$-branes are separated in $w$, such strings become long and thus the corresponding components of $A$, $\tilde{S}$ become massive.

The moduli space associated with giving expectation values to $A$, $\tilde{S}$ is absent here. In the brane language this is because the fourbranes are not free to move in $v$; in gauge theory it is due to the superpotential (10), one of whose equations of motion is

$$\tilde{S} A = 0 \quad (11)$$

which together with the D-flatness condition (9) sets $A$, $\tilde{S}$ to zero.

As a check on the gauge theory we can again study the D-term perturbation (5) corresponding to relative displacement of the $NS5'$-branes. For positive $r$ we now find an $SO(N_c)$ gauge theory with $2N_f$ fundamental chiral multiplets, a symmetric tensor and vanishing superpotential. This can be understood by analyzing the D-flatness conditions (9) in the presence of the superpotential (10). As before, the symmetric tensor $\tilde{S}$ gets an expectation value, which for unbroken $SO(N_c)$ must be proportional to the identity matrix. The last term in the superpotential (10) then gives rise to the mass term $W \sim \Phi A$. Since $A$ is antisymmetric, this term gives a mass to the antisymmetric part of $\Phi$ (as well as to $A$). The symmetric part of $\Phi$ becomes the symmetric tensor mentioned above. Clearly, it does not couple to the $2N_f$ fundamental chiral multiplets. In the brane description the fact that fluctuations of the fourbranes in $w$ are described by a symmetric tensor is a direct consequence of the action of the orientifold projection [28].

The resulting $SO(N_c)$ gauge theory has an $N_c$ dimensional moduli space corresponding to giving an expectation value to the symmetric tensor. Generically in this moduli space the gauge group is completely broken\footnote{As a check, the symmetric tensor has $N_c(N_c + 1)/2$ components out of which $N_c(N_c - 1)/2$ are eaten up by the Higgs mechanism. The remaining $N_c$ massless fields parametrize the moduli space.}. In the brane description this moduli space corresponds to separating the $N_c$ $D4$-branes along the orientifold plane (in $w$). Generically in the moduli space no two fourbranes are at the same $w$ and the gauge symmetry is completely broken. The fact that the symmetric tensor does not couple to the fundamentals is reflected in the brane construction by the property that for generic locations of the $D4$-branes in $w$ they still intersect the $D6$-branes, and hence the $4 - 6$ strings giving rise to fundamental chiral multiplets can be short. The theory also has a fully Higgsed branch (for sufficiently large $N_f$) whose dimension is $2N_fN_c + N_c$ and many mixed Higgs-Coulomb branches. These branches can be analyzed using the brane description, as in section 3.
For negative $r$ one similarly finds an $Sp(N_c/2)$ gauge theory with $2N_f + 8$ fundamental chiral multiplets and an antisymmetric tensor. The moduli space corresponding to giving an expectation value to the antisymmetric tensor is $N_c/2$ dimensional. In the brane construction it corresponds to separating pairs of fourbranes in $w$; the fact that in this situation pairs of $D4$-branes cannot be separated is well known [28]. Generically in this moduli space $Sp(N_c/2)$ is broken to $Sp(1)$. There is again no Yukawa coupling between the antisymmetric tensor and the fundamentals. The brane and gauge theory moduli spaces can be checked to agree as before.

### 4.4 The General Case

For generic rotation angle $\theta$ (4) the adjoint field $\Phi$ discussed in the previous subsection is massive [24]. Its mass $\mu(\theta)$ varies smoothly between zero at $\theta = \pi/2$ and $\infty$ for $\theta = 0$. The superpotential describing this system is

$$W = Q\tilde{S}Q + \tilde{Q}AQ + \Phi A\tilde{S} + \mu(\theta)\Phi^2$$

(12)

For non-zero $\mu$ we can integrate $\Phi$ out and find the superpotential

$$W = Q\tilde{S}Q + \tilde{Q}AQ + \frac{1}{\mu(\theta)}(A\tilde{S})^2$$

(13)

for the remaining degrees of freedom. When $\theta \to 0$, $\mu \to \infty$, and (13) approaches (7). When $\theta = \frac{\pi}{2}$, the mass $\mu$ vanishes and it is inconsistent to integrate $\Phi$ out.

The analysis of the previous two subsections can be repeated in this case with very few qualitative differences. Consider e.g. turning on a positive FI D-term $r > 0$ (8). Its effect is still to induce an expectation value for the field $\tilde{S}$ and break $U(N_c) \to SO(N_c)$. The only difference with the previous analysis is that now for non zero $\langle \tilde{S} \rangle$ the superpotential (13) includes a mass term for $A$; hence we can integrate it out and find a quartic superpotential $W \sim (\tilde{Q}\tilde{Q})^2$. This is rather standard in brane theory [1] and the analysis of the moduli space of vacua can be repeated for this case.

As mentioned in the beginning of this section, it is possible to construct brane configurations in which some of the $D6$-branes are placed outside the orientifold; they may also be rotated with respect to it as in (1). For example, we could remove the $2N_f$ infinite sixbranes placed on top of the orientifold before in $N_f$ mirror pairs, leaving behind only the eight semi-infinite sixbranes necessary for charge conservation and anomaly cancellation. We can then rotate each of the $N_f$ independent $D6$-branes by an arbitrary angle $\theta_i$, $i = 1, \cdots, N_f$. Such configurations contain $N_f$ fundamental + antifundamental chiral multiplets, with a superpotential that depends on the angles [23, 1].

A particularly symmetric configuration is obtained if we rotate the $D6$-brane by the angle $\theta$, such that the sixbranes near the $NS_\theta$ fivebrane are parallel to it (and their mirror images are parallel to its image). Since this configuration has a chiral $SU(N_f) \times SU(N_f)$ global symmetry the superpotential of the chiral multiplets arising from these sixbranes vanishes. Of course, the
eight quarks arising from the semi-infinite sixbranes stuck at the orientifold are still coupled to the symmetric tensor \( \tilde{S} \) by the superpotential (3). The analysis of this and other theories of this sort is very similar to that described above.

It is also possible to study using branes theories with higher polynomial superpotentials \( W = (\tilde{S}A)^{k+1} \) with \( k > 1 \). Such theories have been considered in the gauge theory context in \[17\]; the basic idea for their construction in brane theory appeared in \[8, 9\].

### 4.5 Duality

The gauge theories constructed using branes in the previous subsections are known to have a dual description, which was found in \[17\]. In the gauge theory analysis it is important to turn on the superpotential

\[ W_{el} = (\tilde{S}A)^2 \]  

(14)

to truncate the chiral ring. The dual theory has gauge group \( U(\tilde{N}_c), \tilde{N}_c = 6(N_f + 2) - N_c, 2N_f + 8 \) fundamental chiral multiplets \( q \), \( 2N_f \) antifundamentals, \( \tilde{q} \), a symmetric (bar) tensor, \( \tilde{s} \), an antisymmetric tensor, \( a \), and singlet meson fields \( P, \tilde{P} \), \( M_i \) \( (i = 1, 2) \), with the quantum numbers of \( M_1 = Q\tilde{Q}, M_2 = Q\tilde{S}A\tilde{Q}, P = Q\tilde{S}Q, \tilde{P} = \tilde{Q}A\tilde{Q} \). Thus, \( M_i \) are \( 2N_f \times (2N_f + 8) \) matrices, \( P \) is a \( (2N_f + 8) \times (2N_f + 8) \) matrix while \( \tilde{P} \) is a \( 2N_f \times 2N_f \) matrix. The magnetic superpotential is \[17\]:

\[ W_{mag} = (a\tilde{s})^2 + M_2q\tilde{q} + M_1qa\tilde{s}q + Pq\tilde{sq} + \tilde{P}qa\tilde{q} \]  

(15)

In brane theory \( N = 1 \) duality arises as an equivalence between the moduli spaces of vacua of the electric and magnetic theories, which can be seen by moving NS fivebranes past each other \[8, 1\]. It has been observed in many examples (although the general statement has not been proven) that when parallel NS fivebranes cross each other there is a phase transition in the low energy physics. On the other hand, when the fivebranes are not parallel, the transition in which they meet in space and exchange places is smooth \[1\], and gives rise to Seiberg’s duality.

In our case, \( N = 1 \) duality is obtained by displacing the \( N_{S\theta} \) fivebrane in \( x^6 \) until it meets the orientifold (with the \( N_{S\theta} \)-brane on top of it) and its mirror image, and then the \( N_{S\theta} \) fivebrane and its mirror image exchange places. For \( \theta = 0, \pi/2 \) two or more of the branes involved are parallel to each other and hence there is a phase transition. For generic \( \theta \) the branes are not parallel and one expects the transition to be smooth. Thus the brane construction “explains” the necessity to turn on the superpotential (14) in gauge theory to get a dual description.

Comparing (13) to (14) we see that the theory constructed from branes is obtained from that of \[17\] by adding to the electric superpotential the composite operators \( Q\tilde{S}Q \) and \( \tilde{Q}A\tilde{Q} \). The duality of \[17\] predicts that the corresponding magnetic theory has the superpotential

\[ W_{mag} = (a\tilde{s})^2 + M_2q\tilde{q} + M_1qa\tilde{s}q + Pq\tilde{sq} + \tilde{P}qa\tilde{q} + P + \tilde{P} \]  

(16)

The fields \( P, \tilde{P} \) are massive and can be integrated out. The F-term vacuum equations are:

\[ q^f_i \tilde{q}^{f_2} + \delta_i^{f_1}f_2 = 0 \]  

(17)
\[ \tilde{q}^{g_1} a \tilde{q}^{g_2} + J^{g_1 g_2} = 0 \]  

where \( J \) is the invariant tensor of the symplectic group, \( f_i = 1, \cdots, 2N_f + 8 \) and \( g_i = 1, \cdots, 2N_f \).

The first equation means that the rank of the matrix \( qq \) (and, therefore, of the matrix \( q \)) is \( 2N_f + 8 \), while the second equation means that the rank of \( \tilde{q} \tilde{q} \) and of \( \tilde{q} \) is \( 2N_f \). Therefore, the magnetic group is Higgsed, and the number of magnetic colors decreases by \( 4N_f + 8 \), to

\[ \tilde{N}_c = 2N_f + 4 - N_c \]  

The number of flavors does not decrease, since for each quark eaten up by the Higgs mechanism, a new one appears from decomposing \( s \) and \( a \) with respect to the lower rank gauge group. The two magnetic meson fields \( M_1, M_2 \) get a mass and decouple.

A quick way to derive the dual gauge group \( U(\tilde{N}_c) \) using branes is to turn on the FI D-term \( r \) and then exchange the \( NS_\theta \) fivebrane and its mirror image by moving them in \( x^6 \). Since this involves manipulations in \( SO(N_c) \) or \( Sp(N_c/2) \) \( N = 1 \) SQCD realized using an O6-plane, we can use the results of \[9\] for this system. For example, for \( r > 0 \) we have an \( SO(N_c) \) gauge theory with \( 2N_f \) fundamental chiral multiplets and exchanging the \( NS_\theta \) fivebrane and its mirror image leads to a magnetic theory with gauge group \( SO(\tilde{N}_c) \) given by (19). A similar analysis gives the same answer for \( r < 0 \) using the duality for \( G = Sp(N_c/2) \) with \( 2N_f + 8 \) fundamentals. Alternatively, one can use linking numbers \[5\] in the presence of an orientifold to determine the final configuration. Needless to say, this leads to the same results.

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Note added: After this work was completed we received \[29\] which discusses theories related to those studied in section 4.

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