A Model for High Energy Scattering in Quantum Gravity

Tom Banks
Department of Physics and Astronomy
Rutgers University, Piscataway, NJ 08855-0849
E-mail: banks@physics.rutgers.edu

Willy Fischler
Department of Physics
University of Texas, Austin, TX 78712
E-mail: fischler@mail1.ph.utexas.edu

Abstract: We present a model for high energy two body scattering in the quantum theory of gravity. The model is applicable for center of mass energies higher than the relevant Planck scale. At impact parameters smaller than the Schwarzschild radius appropriate to the center of mass energy and total charge of the initial state, the cross section is dominated by an inelastic process in which a single large black hole is formed. The black hole then decays by Hawking radiation. The elastic cross section is highly suppressed at these impact parameters because of the small phase space for thermal decay into a high energy two body state. For very large impact parameter the amplitude is dominated by eikonalized single graviton exchange. At intermediate impact parameters the scattering is more complicated, but since the Schwarzschild radius grows with energy, we speculate that a more sophisticated eikonal calculation which uses the nonlinear classical solutions of the field equations may provide a good approximation at all larger impact parameters. We discuss the extent to which black hole production will be observable in theories with low scale quantum gravity and large dimensions.

Keywords: M Theory, Black Holes, .
1. Introduction

The problem of very high energy scattering is deeply intertwined with the history of string theory and the quantum theory of gravity. Indeed, string theory was originally invented as a model of the Regge behavior expected for high energy scattering in hadron physics. More recently, there has been a great deal of effort devoted to elucidating the behavior of high energy scattering in string theory [3]-[4]. In parallel to this activity, 't Hooft and followers [5], have initiated a study of high energy scattering in the "quantum theory of gravity", which is to say that their considerations are supposed to be valid in any theory which obeys the principles of quantum theory and reduces to General Relativity at long distances. The aim of this latter program was, at least in part, to address the question of information loss in black hole formation. Not coincidentally, the work cited in [3]-[4] restricts attention to impact parameters larger than those at which General Relativity would predict that black holes are formed.

The claim of the present note is that the gross features of high energy scattering far above the Planck scale can be extracted from semiclassical considerations in General Relativity. We will always consider situations with four or more asymptotically Minkowski dimensions. For simplicity, we will restrict our attention to initial scattering states consisting of two particles of mass far below the Planck scale, but it should be possible to extend it to more complicated initial states. The basis of this claim is the following simple observation. The classical picture of this initial state, insofar as gravitational interactions are concerned, consists of two Aichelburg-Sexl shock wave metrics. General Relativity predicts that when the impact parameter of the two shock waves is smaller than a critical value $R_S$, a black hole is formed. The No-Hair theorem tells us that the classical final state is then uniquely specified by its representation
under the asymptotic symmetry group. $R_S$ is of order the Schwarzschild radius of the corresponding black hole and we will, by abuse of language, call it the Schwarzschild radius. The mass of the black hole is of order the center of mass energy of the collision. Thus, $R_S$ grows with the center of mass energy.

On the other hand, for asymptotically large impact parameters, scattering is also described by classical General Relativity. Indeed, all existant calculations are consistent with the claim that high energy large impact parameter scattering is dominated by eikonalized single graviton exchange. Thus, since $R_S$ grows with energy, we may expect that all aspects of the scattering up to the point of formation of the black hole are well described by the classical theory. Unfortunately, except in the case of $2+1$ dimensions with Anti-deSitter boundary conditions [2] the exact classical solution for black hole formation in the collision of Aichelburg-Sexl shock waves is unknown. The state of the art calculations for shock wave initiated processes in four dimensional flat space may be found in [6]. This fact will make it impossible for us to exhibit a complete formula for scattering cross sections at all energies and impact parameters.

For impact parameters below $R_S$ an exact description of scattering amplitudes would require us to enter into all of the mysteries of the black hole information problem. However, since for large energy the mass of the black hole is large, one needs only the familiar Hawking formulae to extract the gross features of inclusive cross sections. Furthermore, the thermal nature of these cross sections suggests that any more precise description of the scattering will be hopelessly complicated. We want to emphasize that, although we believe recent progress in M Theory suggests that the scattering matrix is unitary even in the presence of black hole production, our results do not depend heavily on this assumption since we only describe inclusive cross sections.

To summarize then, our proposed model for high energy scattering is the following: At impact parameters greater than $R_S$ elastic and inelastic processes (gravitational radiation, photon bremsstrahlung for charged particles and the like) will in principle be described by solving the classical equations of the low energy theory with initial conditions described by a pair of shock waves with appropriate quantum numbers\(^1\). Note that since $R_S^2$ grows much faster with energy than strong interaction cross sections are allowed to (by the Froissart bound), this behavior will be completely determined by the classical physics of the degrees of freedom with energies below the Planck scale (see however the discussion of weakly coupled string theory below).

At smaller impact parameters, scattering will be dominated by “resonant” production of a single black hole with mass equal to the center of mass energy. We put the

\(^1\)Note that we only attempt to describe the leading high energy behavior. The full series of corrections to this behavior would require more knowledge of M Theory than we possess.
word “resonant” in quotes because, despite their long lifetimes, black holes do not fit the profile of a classic Breit-Wigner resonance. Indeed they are most peculiar from the Breit-Wigner point of view. The Breit-Wigner formula for the contribution of a particular resonance to the elastic cross section for two body scattering is proportional to the square of the partial width for the resonance to decay into this particular channel. This is because, for a single narrow resonance, unitarity implies that the amplitude to produce the resonant state is the same as that for the resonance to decay back into the initial state. For black hole production, we expect the initial amplitude to be of order one whenever the impact parameter is smaller than $R_S$. On the other hand, the decay of the black hole is thermal; there will be a very small probability for it to decay back into the initial high energy two particle state. Thus, we expect the elastic cross section to be linear rather than quadratic in the partial width of the two body final state. Thus, the elastic cross section is larger than might have been expected.

More striking is the inelastic resonant cross section, $\sigma(A+B \rightarrow BH \rightarrow \text{Anything})$. This will be large even though the partial width to decay into the initial state is small. The cross section resembles that for a high energy collision of two bodies with an already existing, highly degenerate, macroscopic object. In such a situation, the energy of the initial particles is thermalized among the large number of degrees of freedom of the macroscopic body, the decay is thermal, and the probability of recreating the initial state much smaller than that of the initial collision.

A. Rajaraman has suggested to us that a fact from classical GR may help to explain this behavior. When a black hole is formed by the collapse of a thin spherical shell of matter, the horizon forms long before the shell has fallen past the Schwarzschild radius. Similarly, in the collision of two shock waves, we might expect the horizon to form long before the waves reach the distance $R_S$. In this sense we can view the scattering as being caused by the impact of the colliding particles with an already existing horizon, an object which has a macroscopic number of degrees of freedom.

The other important difference between black hole and resonance production is that resonances occur at discrete energies. By contrast, for every center of mass energy $E$ above some threshold of order the Planck mass, and every impact parameter below $R_S(E)$ we expect the high energy cross section to be dominated by the production of a single object, almost stationary in the center of mass frame. The object will have a long lifetime and will decay thermally (and thus isotropically in the center of mass frame). The elastic cross section will be small. Note in addition that two body final states with large momentum transfer will be even more highly suppressed than the generic two body state. This is in marked contrast with the behavior of the system below the black hole production threshold, where the proliferation of hadronic jets in the final state increases with energy. Once black holes with large enough radii can be formed,
the colliding particles never get close enough to perform a hard QCD scattering.

The dramatic nature of these processes suggests that they will be easy to see if we ever build a Planck energy accelerator. This is particularly exciting in view of recent suggestions that the world may have large extra dimensions and a true Planck scale of order a TeV \[^7\] . However, we argue that most of the Hawking radiation of the higher dimensional black hole will, for phase space reasons, consist of Kaluza-Klein modes of gravitons, and thus be undetectable. In the absence of experimental information about the final state, it is hard to distinguish these missing energy signals from those which come from production of a few KK gravitons \[^10\] . At sufficiently high energies the Hawking temperature of the black hole is small compared to the KK energy scale and the Hawking radiation will be dominated by observable particles. We show this occurs at about the point where the Schwarzchild radius is equal to the KK radius, which is at energies above the four dimensional Planck mass.

We suggest that the suppression of hard QCD processes is a possible signal for identifying this sort of invisible black hole production. Complete suppression of QCD jet phenomena requires that the Schwarzchild radius be larger than an inverse GeV, and this only occurs at unreachably high energies. However, the suppression of jets with transverse momenta higher than the inverse Schwarzchild radius should become apparent before this. The detailed investigation of this phenomenon is beyond the scope of the present paper.

The plan of the rest of this paper is as follows. In the following section we outline the regime of parameters within M Theory in which we might expect our discussion to be valid. We point out in particular that physics in the regimes corresponding to weakly coupled string theories is considerably more complex than what we have discussed. There can be a plethora of scales and a variety of different high energy regimes. Readers of a more phenomenological bent are advised to skip this section, which will be of interest mostly to string theorists. In Section 3, we present some remarks relevant to the regime discussed in this introduction, and assess the likelihood of observing black hole production experimentally if theories with large dimensions are correct.

### 2. High Energy Scattering in Weakly Coupled String Regimes

Consider the moduli space of M Theory compactifications with four Minkowski dimensions. Much of this moduli space can be well approximated by compactifications of

\[^2\] We note that the authors of [1] have investigated the properties of and astrophysical constraints on, black holes in theories with large extra dimensions.
11 dimensional supergravity (11D SUGRA) on manifolds with dimensions large compared to the Planck scale. The discussion in the introduction applies primarily to such regions of moduli space. An example of regions not covered by this description are weakly coupled string compactifications. These can be viewed as the proper limits of compactifications of 11D SUGRA on manifolds with some dimensions much smaller than the Planck scale. The same is true for F theory compactifications. We will begin by discussing the simple case in which all dimensions are of order or much larger than the eleven dimensional Planck length. Then by examining the case of weakly coupled string theory, we will establish that other regimes have a much more complicated set of high energy behaviors.

Imagine first that some dimensions are of order the Planck scale, while \( n \) others are much larger. Let \( M_{4+n} \) denote the Planck mass in the effective theory below the eleven dimensional Planck scale. Given our assumptions, it is of order the eleven dimensional Planck scale and we will not bother to distinguish between them. The four dimensional Planck scale is given by

\[
M_P^2 \sim V_n M_{4+n}^2,
\]

where \( V_n \) is the volume of the large dimensions. Note that \( M_P > M_{4+n} \).

As the energy is raised, we will approach two thresholds, the first, the Kaluza Klein (KK) scale of the large dimensions, and the second, \( M_{4+n} \). The observation of [7] is that if \( V_n \) is large in eleven dimensional Planck units, if the standard model lives on a brane embedded in the large dimensions, and if gravitons and other fields with only nonrenormalizable couplings to the fields on the brane are the only bulk fields, then the first threshold may show up only in very high precision experiments. The couplings of ordinary matter to the new states will be suppressed by powers of the energy divided by \( M_P \) until we reach the threshold \( M_{4+n} \). Most analyses of what happens above this threshold have concentrated on the production of KK modes. We will argue that somewhere around this energy regime, the (in principle) much more dramatic phenomenon of black hole production sets in. We will reserve a more detailed description of these processes for the next section. Here we merely observe that the appropriate form of GR to use in these estimates is \( 4+n \) dimensional gravity.

We turn now to regimes described by weakly coupled string theory. Here there is generically a hierarchy of scales, starting with the string scale and proceeding to energy scales which are larger than the string scale by inverse powers of the coupling. Among these is the Planck scale associated with the \( 4+n \) dimensional space.\(^3\)

\(^3\)We now assume that \( 6-n \) dimensions are compactified at about the string scale. In string theory, T dualities and Mirror symmetries usually make this a lower bound on compactification dimensions in the weak coupling regime.
Since we are interested in scattering above the string energy the effective theory in the regime of interest will always be ten dimensional (see the previous footnote). There have been many attempts to study high energy scattering in string perturbation theory. We will argue that these are reliable only in a certain energy regime\(^4\).

Let us turn first to the fixed angle regime studied in \[3\]. A cartoon version of the analysis of this papers follows: All Lorentz invariants of the scattering process have the same order of magnitude, call it \(s\), in this regime. \(k\) loop amplitudes have the form

\[
A_k \sim \int dme^{-s\alpha' f_k(m)/k}
\]

where the integral is over the moduli space of genus \(k\) Riemann surfaces with 4 punctures and \(f_k\) depends only weakly on \(k\). For large \(s\) one does the integral by steepest descents, obtaining an amplitude which falls off like \(e^{-s\alpha' f_k(m_0)/k}\). The Gaussian fluctuations around the stationary point in moduli space give a coefficient of order \(6k\).

The facts that the exponential becomes flatter for large \(k\) and that the coefficient far exceeds the \(2k\) growth expected for the large order behavior of string perturbation theory \[8\] (facts which are mathematically related) lead us to be suspicious of this result at energies which scale like inverse powers of \(g_S\)\(^5\). Indeed, the large \(k\) behavior of the amplitude at fixed \(s\) is constant in \(s\) and is of order \(2k\) (the estimate of the coefficient comes from the volume of moduli space). This suggests an \(s\) independent, nonperturbative contribution to the amplitude of magnitude \(e^{-gs}\) such as that predicted by D-instantons in Type IIB string theory. One may expect similar pointlike contributions in other weakly coupled string theories (with the notable exception of the heterotic theory where these contributions may have something to do with the throats of NS 5 branes) from components of the wave functions of scattering states which contain D-object anti D-object pairs separated by distances of order the string length. At small impact parameter we should see a contribution from the pointlike scattering of individual D-branes \[9\].

The nonperturbative amplitude competes with the perturbative one when \(s\alpha' \sim \frac{1}{g_S}\).

Note that the ten dimensional Planck energy squared is \(M_{10}^2 \sim \left(g_S^{1/2}\alpha'\right)^{-1}\), which is much smaller than this crossover energy. The semiclassical analysis of the introduction and the following section are valid only when the energy is much larger than \(M_P\) and the Schwarzchild radius larger than the impact parameter as well as the string length.

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\(^4\)The analysis of the next few paragraphs summarizes unpublished work done several years ago by one of the authors (TB) and S. Shenker. We thank S. Shenker for permission to include it here.

\(^5\)Physically, the reason for this flattening was explained in \[3\]: the lowest order amplitude gives an exponentially falling amplitude for large momentum transfer. At higher orders, the most efficient way to distribute the momentum transfer is to form a \(k\) string intermediate state, with each subprocess transferring momentum squared of order \(s/k\).
In ten dimensions the Schwarzschild radius is of order $s^{1/4}g_s^{2}$ in string units, so the semiclassical regime would apparently only set in for $s \sim g_s^{-4}$. This would seem to be a valid estimate in IIB string theory, but in IIA this energy is above the inverse compactification radius of the M Theory circle, so we should really make an eleven dimensional estimate. The eleven dimensional Schwarzschild radius only exceeds the string scale when $s \sim g_s^{-6}$ in string units. Thus in all cases it seems that the crossover between perturbative string and D-instanton behavior sets in in a regime in which gravitational corrections are negligible.

If the interpretation of the pointlike nonperturbative cross section in terms of D-brane “sea partons” in the string wave function is correct, we may expect that the description we have given of the crossover is not complete. Indeed, the authors of [9] showed that D0 brane scattering in weakly coupled type IIA string theory became soft at scales of order the eleven dimensional Planck mass, or $g_S^{-1/3}$ in string units. This is lower than the crossover scale.

To conclude, in the weakly coupled string regime, the semiclassical analysis of the introduction is expected to be valid only at energies which are parametrically (in $g_S$) higher than any relevant Planck scale. In the IIA theory it is only 11 dimensional SUGRA which eventually becomes relevant, and only at an energy scale parametrically larger than the inverse compactification radius of the M Theory circle. At energies below this true asymptopia we expect to see a rich structure of high energy amplitudes, dominated successively by perturbative strings, and nonrelativistic followed by relativistic scattering of “Dirichlet sea” constituents of the incoming states.

3. Black hole cross sections

We write the elastic amplitude for $2 \rightarrow 2$ scattering in eikonal form

$$A(s,q^2) \propto \int db e^{i\chi(b,s)}$$ (3.1)

where $b$ is the impact parameter and $s$ the square of the center of mass energy. For $n$ relatively large compact dimensions, the Schwarzschild radius of a $4+n$ dimensional black hole of mass $\sqrt{s}$ is approximately, $R_S \sim M_{4+n}^{-1}(s/M_{4+n}^2)^{1/(n+1)}$. In order to use flat space black hole formulae, we must have $R_S \ll L$, the radius of the compact dimensions. In terms of the energy, this bound is $\sqrt{s} \ll M_P(M_P/M_{4+n})^{2/(n+1)}$. For applications to theories with low scale quantum gravity, this bound is never exceeded, so we will not discuss larger values of $s$. We note however that when $R_S$ exceeds the compactification radius, the most likely outcome is that the system is described as a four dimensional black hole (a black brane wrapped on the compact dimensions).
For impact parameters smaller than $R_S$ the cross section will be completely dominated by black hole production. As outlined in the introduction, this will have the following consequences:

- The elastic cross section will be suppressed by a Boltzmann factor $e^{-\sqrt{s}/T_H}$, where the Hawking temperature $T_H \sim M_{4+n}(M_{4+n}/\sqrt{s})^{\frac{1}{n+1}}$.

- Due to initial state bremsstrahlung the black hole will not be exactly at rest in the center of mass frame. The average energy emitted in bremsstrahlung should be calculable by the methods of [6]. The final state will be a black hole at rest in the frame determined by this bremsstrahlung calculation. It will decay thermally, and therefore isotropically in this frame. This prescription only allows one to calculate inclusive cross sections, but the thermal nature of these indicates that any more precise calculation of the amplitudes for various final states is beyond the range of our abilities.

- In the standard model, we expect high energy collisions to be characterized by a larger and larger multiplicity of QCD jets with higher and higher transverse momenta. One of the most striking features of black hole production is that processes with transverse momenta larger than $R_S^{-1}$ should be completely absent. The incoming particles never get close enough together to perform a hard QCD scattering. This characteristic shutoff of hadronic jets may be one of the most striking signals of black hole production processes.

- Although a long lived black hole will be produced at every sufficiently high energy and small impact parameter, the signature of these events does not look like a conventional Breit-Wigner resonance.

When the impact parameter is larger than $R_S$, we do not expect black hole formation to occur. When the impact parameter is very large, the elastic scattering is given by the eikonal formula coming from single graviton exchange. Note that here it is four dimensional gravitational physics which is relevant since we are talking about asymptotically large impact parameter. At energies relevant for discussing theories of low scale quantum gravity, these amplitudes are completely negligible.

Since, at sufficiently high energy, the Schwarzschild radius is larger than all microscopic scales beside the radius of the compact dimensions, we conjecture that the behavior of the elastic amplitude and at least gross features of multiparticle production cross sections, can be extracted from the solution of the equations of classical general relativity. It is possible that there is a small region in impact parameter near to
but larger than the Schwarzschild radius where a more detailed quantum mechanical treatment is necessary.

Thus, in summary we conjecture that most gross features of scattering at energies much higher than the Planck scale can in fact be determined by solving classical equations. This is still a very involved task. Even the problem of colliding Aichelburg-Sexl waves in four flat dimensions is not solved. For scenarios of low scale quantum gravity one would have to solve an analogous problem in a partially compactified space. Also, one would have to learn how to extract information about multiparticle amplitudes from the classical solutions. Despite the complication, we would imagine that these problems are amenable at least to numerical solution.

An important issue which might be clarified by this analysis is a more precise estimate of the threshold above which our description of high energy scattering would be expected to hold. At the moment we can only say that it should hold sufficiently far above the Planck scale. A better estimate of the threshold is crucial to any attempt to use the properties of black hole production to constrain theories of low scale gravity. We would guess that it is about an order of magnitude higher than the $4+n$ dimensional Planck mass.

However, even when we reach this threshold, it is not clear that black hole production will have striking experimental signatures. The most striking feature of black hole production is of course the Hawking decay of the final state, which will be nearly at rest in the center of mass frame. Unfortunately, for phase space reasons, this decay will be primarily into KK graviton modes, which are invisible to all detectors. Thus, although the final state of black hole decay is very different from that produced in the perturbative processes discussed in [10], it may not be different in a way that can be easily measured. One might hope that at sufficiently high energies, the Hawking temperature would be so low that KK modes could not be produced, and we would get a thermal distribution of standard model particles. This happens at temperatures where $R_{KK}T < 1$, since the Hawking temperature is just the inverse of the Schwarzschild radius, this is the point at which the Schwarzschild and KK radii cross. As noted above, this occurs only at energies larger than the four dimensional Planck mass.

A more promising signal is the suppression of hard QCD processes. Complete suppression requires a Schwarzschild radius of order an inverse GeV. This occurs at energies of order $(E/M_{4+n}) \sim (M_{4+n}/1\text{GeV})^{(n+1)}$. Even for a six dimensional scenario with $M_{4+n} = 1$ TeV, this is $10^{12}$ GeV. However, suppression of jets with transverse momenta larger than $M_{4+n}$ will occur as soon as the threshold for production of black holes is passed. Furthermore, the suppression will become more marked with increasing

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6The following paragraphs were a response to questions raised by E.Witten.
energy, \textit{i.e.} the average transverse momenta of jets should go down with the energy, precisely the opposite of the QCD expectation. This question deserves more detailed study, but we feel confident that a relatively clean experimental signature will emerge from such a study. Note that the rate of increase of the Schwarzschild radius with energy may be measurable in this way, thus providing a direct measurement of the number of large compact dimensions.

Clearly, all of these studies require the ability to probe a range of energies up to a few orders of magnitude above $M_{4+n}$, and it is unclear if anything can be seen in presently planned accelerators. If however, evidence for large extra dimensions is found at LHC, then one would be highly motivated to build a larger machine, which could study black hole production.

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