Distributed Model Predictive Control of Spatially Decoupled Systems Using Switched Cost Functions

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Abstract—This note investigates the stabilization problem of a class of spatially decoupled systems by applying distributed model predictive control (DMPC) with switched cost functions. The proposed DMPC scheme switches the optimization index on a switching surface generated by control invariant sets. By applying the index-switching strategy, stability of the closed-loop system is ensured by the feasibility of a series of constrained optimal control problems. The stability conditions established in this note do not require terminal equality constraints of the optimization problem, and preserves the quadratic program property that is desired in practical applications. It is also observed that the proposed DMPC scheme has benefits dealing with systems that need to take into account safety-related spatial constraints.

Index Terms—distributed MPC, stabilization, networked systems, switched cost function.

I. INTRODUCTION

Spatially decoupled systems consist of independently actuated systems that are spatially distributed, dynamically decoupled, and are interconnected via communication networks. Such systems have wide application areas ranging from automated highway systems, to unmanned aerial vehicle (UAV) formation, to distributed mobile sensing agents [1], [2]. In a spatially decoupled system, cooperation of distributed subsystems (agents) is usually required to achieve global objectives of the overall system such as behavioral consensus, task allocation, economic savings, and so forth. Such cooperation of subsystems could be either systemwide or local depending on the communication network and cooperation strategy. While centralized control can deal with systemwide interactions by controlling all subsystems via a single agent, the difficulty of systemwide organization and maintenance via the central agent deters application of centralized control schemes developed to regulate spatially decoupled systems [3], [4]. Research on spatially distributed control schemes that are flexible, scalable, and "plug-and-play" has been attracting increasing attention in cyber-physical system study.

Distributed model predictive control (DMPC) is one of the distributed control schemes developed to regulate spatially decoupled systems that subject to system and communication constraints. DMPC inherits properties from model predictive control (MPC) of taking explicit account of state and input constraints. The distributed controllers cooperatively solve a constrained optimal control problem (COCP) in a receding horizon fashion, and implement respectively first items of optimized input sequences to corresponding subsystems. Different from decentralized MPC, DMPC solves a COCP of which the cost function couples dynamics of multiple subsystems. Additionally, DMPC implements an information exchange process via communication connections to fulfill distributed optimization. Typically, DMPC schemes can be grouped into two types according to the distributed optimization procedure. The first category involves systemwide information exchanges and distributively optimizes a unique cost function [5]. At each control step, distributed controllers optimize the systemwide cost in an iterative manner using distributed optimization algorithms. Stability and optimality of the iterative DMPC strategy have been well-studied in previous literature (see, for example [3], [5], [6]). Systemwide optimality is equivalent to the centralized MPC scheme if the iteration at each control step converges. It has been shown that, with certain bound assumptions of inputs, the intermediate iterations stabilize the nominal closed-loop system [3]. Similar DMPC schemes employing a coordination layer to allocate distributed optimization tasks have also been investigated [7]. The second category typically implements partial information exchanges and relies on the structure of spatially decoupled systems [8]–[10]. In this DMPC group, each distributed controller solves a COCP of which the cost function couples the dynamics of that subsystem and its neighbors. Only single information exchanges between a subsystem and its neighbors are conducted at each control step.

Comparing to the iterative DMPC strategy, although the noniterative DMPC strategy cannot achieve systemwide optimality, it has the advantage of flexibly arranging coordination tasks of spatially decoupled systems that have subsystems dynamically joining and leaving. However, one barrier hinders the implementation of noniterative DMPC is the complexity of ensuring closed-loop system stability due to partial information exchange at each control step. Although it is known analytically that the optimal control laws are piecewise affine functions of initial conditions when the cost function couples dynamics of all subsystems [11], stability conditions are still not straightforward when cost functions only comprise local dynamics. Existing stabilization results typically require bounds on mismatches between estimated and actual states of neighbor subsystems, and terminal equality constraint of COCPs [8], [9]. The constraints ensure that either the individual cost function or the sum of cost functions is a control Lyapunov function. However, the constraints could be too stringent to implement in practical applications.

This note investigates the stabilization problem of spatially decoupled systems, and proposes a novel DMPC scheme using switched cost functions. The focus of this work is on spatially decoupled systems consisting of identical linear time-invariant (LTI) subsystems interconnected by state-dependant cost functions. It is assumed that each subsystem has spatial entries, e.g. position, in the system state, and can communicate with its neighbors. By switching the cost functions on a switching surface generated by control invariant sets, stability of the overall system is transformed into the feasibility of a series of COCPs that preserves quadratic program properties. Moreover, the distributed subsystems applying the proposed DMPC scheme are able to avoid stationary obstacles independently when a feasible avoidance state is selected inside control invariant sets.

The rest of this paper is outlined as follows. Section II gives the preliminaries to formulate noniterative DMPC of spatially decoupled systems. Section III presents the proposed DMPC scheme using switched cost functions. Stability conditions are established, and spatial properties are analyzed. Section IV gives a numerical example to illustrate the effectiveness of the proposed DMPC scheme. Section V concludes this work.
Throughout this work, $D J(x)$ denotes the first forward difference of $J$ along $x$. Let $||\cdot||$ denote the Euclidian norm, $||x||_p$ the quadratic form of $x$, i.e. $||x||_p = x^TPx$. $\Phi(k, x_0)$ denotes the state trajectory of $x$ at time $k$ with initial condition $x_0$. $B_i(0) \triangleq \{x \in \mathbb{R}^n : ||x|| \leq \rho\}$. $\mathbb{Z}^+$ denotes the set of nonnegative integers. We use $i : j$, $i, j \in \mathbb{Z}$, $i < j$ to denote the integers from $i$ to $j$. $A^T$ denotes the transpose of matrix $A$. The superscript $*$ denotes the optimizer or optimal cost based on the context.

II. Preliminaries

A. Spatially Decoupled Systems

Consider a set of $M$ spatially decoupled systems with the following discrete-time LTI dynamic of subsystem $i$

$$x^i(t+1) = Ax^i(t) + Bu^i(t)$$

where $x^i \in \mathbb{R}^n$, $u^i \in \mathbb{R}^m$ are the state and input of subsystem $i$, respectively, with initial condition $x^i(0) = x_0^i$. $(A, B)$ is a controllable pair. The origin of each subsystem is an equilibrium point. The system state and input subject to the following constraints

$$x^i(t) \in \mathcal{X}_i \subset \mathbb{R}^n, \quad u^i(t) \in \mathcal{U}_i \subset \mathbb{R}^m, \forall t \in \mathbb{Z}^+$$

The overall system is represented by an undirected graph structure by associating the $i$th subsystem to the $i$th vertex of the graph. Then, the interaction of system $i$ and $j$ is presented by an edge $(i, j)$ in the graph. To better describe information exchanges and interactions between subsystems, we now introduce the following definitions.

**Definition 1 (Undirected graph):** An undirected graph $\mathcal{G} = (V, E)$ consists of a vertex set $V = \{v_1, \ldots, v_M\}$ of $M$ vertices and an edge set $E \subset V \times V$ of unordered pairs $\{e_{ij} = e_{ij} \triangleq (v_i, v_j), \quad v_i, v_j \in V\}$.

For distributed systems employing asymmetric information exchange structures, e.g. leader-following mobile agents, directed graphs may be applied to formulate the overall system. Without loss of generality, information exchanges between distributed subsystems are assumed to be symmetric throughout this work.

**Definition 2 (Neighbor set):** The neighbor set of a vertex $v_i \in \mathcal{G}$ is $\mathcal{N}_i \triangleq \{v_j \in V : (v_i, v_j) \in E, j \neq i\}$.

The subsystems are interconnected by the means of coupled objective functions

$$J_i \triangleq J(x^i, u^i, \{x^j, u^j\}), \quad x^i \in \mathcal{X}_i$$

When $\mathcal{N}_i = \{\mathcal{X}^j : j = 1, \ldots, M\}$, the distributed control problem degrades to a centralized optimal control problem. In general, the graph edges representing subsystem interactions can be time-varying. For distributed control of spatially decoupled systems, the emphasis is on coordination of subsystems between high-level decision-making stages of the overall system. For the sake of simplicity, the graph structure is assumed to be time invariant throughout this note. To further analyze stability properties of DMPC, the following definitions are presented.

**Definition 3 (Asymptotic stability):** The equilibrium $x = 0$ of (1) is asymptotically stable if i) $\forall \epsilon > 0$ there exists a $\delta > 0$ such that $|\Phi(k, x_0) - B_i(0), \forall k \in \mathbb{Z}^+ \text{ whenever } x_0 \in B_i(0)$; ii) $\exists \eta > 0$ such that $\lim_{k \to \infty} |\Phi(k, x_0)| = 0$ if $x_0 \in B_i(0)$.

**Definition 4:** $\alpha : [0, a) \to [0, \infty)$ is said to be a class $K$ function, if it is continuous on $[0, a)$ and strictly increasing with $\alpha(0) = 0$.

**Definition 5 (Multi-parametric quadratic programs):** A multi-parametric quadratic program (mp-QP) is a multi-parametric program in the following form

$$J^*(x) = \min_{x} J(z, x) = \frac{1}{2} z^T H z$$

$$s.t. \quad Gz \leq w + Sx$$

where $z \in \mathbb{R}^n$ and $x \in \mathbb{R}^m$ are the optimization variable and the parameter, respectively. $J(z, x) : \mathbb{R}^n \to \mathbb{R}, H \succ 0, G \in \mathbb{R}^{p \times n}, w \in \mathbb{R}^p, S \in \mathbb{R}^{p \times n}$.

B. Distributed Model Predictive Control

A noniterative DMPC scheme of a subsystem employs actual states and predicted states of itself and its neighbors to solve the corresponding COCP at each control step. Throughout this work, let $x^i_{\tau}$ denote the actual state of subsystem $i$ at time $t$. We denote by $x^j_{\tau+k}$ the state of subsystem $j$ at time $t + k$, with initial condition $x^j_{\tau}$, predicted by subsystem $i$. In particular, $x^i_{\tau+k} = x^i_{\tau+k}$. For notation simplicity, the superscript $i$ denoting the subsystem index will be eliminated when only the dynamic of a single subsystem is discussed.

Each subsystem solves a local finite-horizon optimization problem to achieve local cooperation. The DMPC scheme is given as follows.

$$J^*_i(U_{0:N-1}^i; X_0^i) = \min_{U_{0:N-1}^i} \sum_{k=0}^{N-1} \ell^i_k(X_k^i, U_k^i) + \ell_N(X_N^i)$$

$$s.t. \quad x_{k+1}^i = Ax_k^i + Bu_k^i \quad x_{k}^i \in \mathcal{X}_i, \quad x_{N}^i \in \mathcal{X}^i$$

$$u_k^i \in \mathcal{U}_i \quad x_{k+1}^i = Ax_k^i + Bu_k^i \quad x_{k}^i \in \mathcal{X}_i$$

$$u_k^i \in \mathcal{U}_i$$

where $N$ is the prediction horizon, $\ell^i_k$ and $\ell_N$ are the process cost and the final penalty, respectively, $i \in \{1, \ldots, M\}$. $U^i_{0:N-1} = \{u_{k}^i \mid k = 0, \ldots, N-1\}$. $U^i_{0:N-1} = \{u_{k}^i \mid k = 0, \ldots, N-1\}$. $X^i_{0:N-1} = \{X_{k}^i \mid k = 0, \ldots, N-1\}$. $X^i_{0:N-1} = \{X_{k}^i \mid k = 0, \ldots, N-1\}$. $X_{0:N-1} = \{X_{k}^i \mid k = 0, \ldots, N-1\}$. $X_{0:N-1} = \{X_{k}^i \mid k = 0, \ldots, N-1\}$.
Existing approaches to distributed stabilization by applying DMPC of the form (4) and (5) typically require bounds on mismatches between estimated and actual states of neighbor subsystems to guarantee that the coupled cost function $J_i$ is a control Lyapunov function. Stability of the overall system is then ensured by the condition that each subsystem is asymptotically stable [8], [9], [12]. The stability conditions also require that the COCP solved at each control step has terminal equality constraints, i.e. $X_f = 0$, to simplify the bound on state mismatches. A drawback of such conditions is that the introduction of state inequality constraints makes the DMPC problem no longer a quadratic program, hence non-applicable to quadratic program solvers.

Notice that the terminal constraint set together with a nontrivial final penalty is used in generic MPC schemes to achieve closed-loop stability, we adopt a novel switched cost function together with terminal constraint sets $X_f^i$ of DMPC (4) to stabilize the nominal closed-loop system. The rationale is that the cost function of (4) switches to the cost function of generic decoupled MPC if the state is inside $X_f^i$. Thus, asymptotic stability of each subsystem in $X_f^i$ is ensured by conditions of a generic MPC scheme. The coupled cost function only accounts for collaboration and attractiveness of states out of $X_f^i$.

The decoupled MPC problem that only takes into account the state and input of subsystem $i \in \{1, \ldots, M\}$ is given as follows

$$J_i^*(u_{0, N-1}^i, x_{1}^i) = \min_{u_{0, N-1}^i} \left\{ \sum_{k=0}^{N-1} \bar{J}_k(x_{k,i}^i, u_{k,i}^i) + \bar{J}_N(x_{N,i}^i) \right\} \quad (6a)$$

subject to

$$x_{k+1,i}^i = Ax_{k,i}^i + Bu_{k,i}^i, \quad x_{0,i}^i \in X_i^i \quad (6b)$$

$$u_{k,i}^i \in U_i \quad \forall k \in \{0, \ldots, N-1\} \quad (6d)$$

where $N'$ is the prediction horizon. The process cost and final penalty are given as follows

$$\bar{J}_k(x_{k,i}^i, u_{k,i}^i) = \|x_{k,i}^i\|_Q + \|u_{k,i}^i\|_R \quad (7)$$

$$\bar{J}_N(x_{N,i}^i) = \|x_{N,i}^i\|_P \quad (8)$$

**Assumption 1:** For every subsystem $i \in \{1, \ldots, M\}$:

(a) $X_i^i, X_i^i, \bar{X}_i^i, \bar{X}_i^i$, and $U_i^i$ are convex polyhedra containing the origin as an interior point;

(b) $X_i^i \subseteq \bar{X}_i^i \subseteq \bar{X}_i^i$ is control invariant;

(c) $X_i^i$ (respectively, $U_i^i$) is decoupled with $X_j^j$ (respectively, $U_j^j$), $\forall j \in \{1, \ldots, M\} \setminus \{i\}$.

**Remark 1:** The terminal equality constraint $X_f^i = 0$ is a trivial condition that satisfies the invariance property.

**Assumption 2:** $Q = Q^T \succ 0$, $Q_k = Q^k \succeq 0$, $R = R^T \succ 0$, $P \succ 0$ is such that

$$\min_{x \in \mathbb{C}^n} \|x\|_P + \|x\|_Q + \|u\|_R + \|(Ax + Bu)\|_P \leq 0 \quad (9)$$

subject to

$$Ax + Bu \in \bar{X}_f^i, \forall x \in \bar{X}_f^i$$

**Remark 2:** $\bar{X}_f^i$ is convex and compact. Denote by $\bar{X}_0^i$ the $N'$-step backward reachable set of $\bar{X}_f^i$. Then $\bar{X}_0^i$ is convex, compact, and control invariant.

After combining the DMPC (4) with decoupled MPC (6), the DMPC scheme with switched cost functions is proposed by setting $X_{f}^i$.
The largest invariant set and forward invariant with respect to the nominal closed-loop system. 

\[ \Omega_e = \{ x \in \mathbb{R}^n : J^*_e(x) \leq c' \} \subset B_{\rho}(0) \]  

(13)

For every \( \epsilon > 0 \), let \( \sigma = \min\{\epsilon, \rho\} \). There exists \( 0 < c < c' \) such that \( \forall x \in \Omega_e, \forall \epsilon \subset B_{\rho}(0) \subset B_{\epsilon}(0) \). Moreover, \( D J^*_e(x) \leq 0 \), \( \forall x \in \Omega_e \). \( \Omega_e \) is forward invariant, and contains the origin as an interior point. Therefore, \( \exists \delta > 0 \) such that \( B_{\delta}(0) \subset \Omega_e \). Then \( \forall x_0 \in B_{\delta}(0), x_0 \in \Omega_e \). This means that \( \Phi(k, x_0) \in \Omega_e, \forall k \in \mathbb{Z}^+ \), hence, \( \Phi(k, x_0) \in B_{\delta}(0), \forall k \in \mathbb{Z}^+ \).

2) Attractiveness: for every \( x_0 \in \mathcal{X}_0 \), \( D J^*_i(x_0) \leq -\alpha_i(||x||) \). Denote by \( \partial \mathcal{X}_i \) the boundary of \( \mathcal{X}_i \). Since \( \mathcal{X}_i \) is compact, for \( x \in \partial \mathcal{X}_i \), there exists \( d = \min ||x|| \). Thus, for every \( x_0 \in \mathcal{X}_i \),

\[ D J^*_i(x_0) \leq -\alpha_i(||x||) \leq -\alpha_i(d) \]  

(14)

Moreover, \( J_i(x) \) is radially unbounded, the level set of \( J_i(x) \) is compact. For every \( x \in \partial \mathcal{X}_i \), there exists \( x^* = \arg\min J_i(x) \). Let \( T \) denote the time steps needed by the control law to drive an initial condition \( x_0 \) to \( \mathcal{X}_i \), \( T \in [0, \infty) \)

\[ T \leq 1 + \frac{J^*_i(x_0) - J^*_i(x^*)}{\alpha_i(d)} \]  

(15)

\( T \) is finite, \( x_0 \) can be driven into \( \mathcal{X}_i \) after \( T \) steps.

\( (9) \) implies that \( J^*_i(A x_0 + B u_0) \leq -||x|| + ||u|| \). i.e. \( D J^*_i(x) < 0, \forall x \in \mathcal{X}_i \) and \( D J^*_i(0) = 0 \). \( \mathcal{X}_i \) is compact and forward invariant with respect to the nominal closed-loop system. By LaSalle’s invariance principle, \( \forall x_0 \in \mathcal{X}_i \), \( \Phi(k, x_0) \) converges to the largest invariant set \( M \) in \( \mathcal{X}_i \), and

\[ M \subset W \triangleq \{ x \in \mathcal{X}_i : D J^*_i(x) = 0 \} = \{ 0 \} \]  

(16)

Therefore, \( \forall x_0 \in \mathcal{X}_i \), \( \Phi(k, x_0) \to 0 \), which indicates that \( \forall x_0 \in \mathcal{X}_i \), \( \Phi(k, x_0) \to 0 \).

The constraints in (4) may not guarantee the inequality (12) in Theorem 1. Adding extra constraints of state mismatches between actual states and estimated states could increase the complexity of solving (4). To address this issue, we propose the following algorithm.

**Algorithm 2:**

**Step 1** Each subsystem \( i, i \in 1, \ldots, M \), inquires the state of subsystem \( j \) at time \( t \), and updates the state estimation of system \( j \) by setting

\[ x^{i,j}_{0,t} = x^j_t \]
\[ x^{i,j}_{0,t} = x^j_t, \forall j \in N^i \]  

(17)

**Step 2** If \( \exists i \in \{1, \ldots, M\} \) such that \( x^i_t \notin \mathcal{X}_i \), then each subsystem solves the following COCP using updated estimations of the states \( \{x^j_t\} \)

\[ \mathcal{P}_{\text{copp}} : \begin{cases} 
\text{while } N \geq 1 \text{ do} \\
\text{solve (4) with updated } N \\
N \leftarrow N - 1 
\end{cases} \]  

(18)

else, each subsystem solves the decoupled MPC problem (6).

**Step 3** Each subsystem \( i \) implements the first term of the input sequence \( \{u^{i,i*}_{0,N-1}\} \) computed in Step 2.

\[ u^i_t = u^{i,i*}_{0,t} \]  

(19)

**Step 4** Each subsystem repeats steps 1-4 at time \( t + 1 \) based on the updated states \( \{x^j_{t+1}\} \).

**Lemma 3** ([14, page 133]): Consider the mp-QP (3) and let \( H > 0 \). Then the optimizer \( z^*(x) : \mathcal{K} \to \mathbb{R}^* \) is continuous and piecewise affine on polyhedra, where \( \mathcal{K} \) is the feasible set. In particular it is affine in each critical region.

**Theorem 2:** Suppose conditions in Assumption 1 and Assumption 2 hold. Then,

(i) The spatially decoupled system comprising subsystems (1) by applying the DMPC law in Algorithm 2 is asymptotically stable with domain of attraction \( \prod_i \mathcal{X}_i, i = 1, \ldots, M \).

(ii) The DMPC scheme applying the strategy in Algorithm 2 is a series of quadratic programs that can be transformed into the following form

\[ J^*_i(X^0_i, \hat{U}_0) = \min_{U_0} \mathbb{U}_0 \quad x^0_i \quad \mathbb{H} \quad F \quad T \]
\[ \left[ \begin{array}{c}
\mathbb{U}_0^T \\
\mathbb{Y}
\end{array} \right] \left[ \begin{array}{c}
x^0_i \\
x^i \end{array} \right]^T \]  

s.t. \( \mathbb{G}_0 \leq w + \mathbb{E}_X \)

(20)

where \( \mathbb{U}_0 \) is the optimization variable, \( \mathbb{H}, F, \mathbb{Y} \) are matrix coefficients uniquely determined by the initial state \( X^0_i \).

The optimal control laws are piecewise affine functions of initial conditions on \( \prod_i \mathcal{X}_i \).

**Proof:** (i) The stability property of subsystem \( i \) follows the proof of Theorem 1. Asymptotic stability of the overall system will be obtained by showing that all subsystems are asymptotically stable.

By definition, \( \forall x_0^i \in \mathcal{X}_i \), there exists a control sequence \( \{u^i\} \) that drives the state of subsystem \( i \) into \( \mathcal{X}_i \) in \( N \) steps, i.e. \( x^i_N = \Phi(N, x_0^i) \in \mathcal{X}_i \).

Denote by \( u_{0,N-1}^i \) the optimal control sequence. Since the system is time invariant, without lose of generality, choose initial time \( t = 0 \) and assume \( x^0_{0,N-1} \notin \mathcal{X}_i \). By implementing the control law (19), at time \( t = 1 \)

\[ x^i_{1} = A x^i_0 + B u^i_{0,1} \]  

(21)

\( x^i_{1} \) is in the \( (N - 1) \)-step backward reachable set of \( \mathcal{X}_i \). According to Bellman’s optimality theorem [18], the control sequence \( u^i_{0,N-1} \) calculated at \( t = 0 \) is the optimal control sequence of (18) at \( t = 1 \) with the prediction horizon equals \( N - 1 \). This procedure repeats until \( t = N - 1 \). Therefore, the convergence of \( x^i_{1} \) is ensured by the feasibility of a series of COCPs. When \( x^i_{1} \) is in \( \mathcal{X}_i \), the convergence of \( x^i_{1} \) follows the proof of Theorem 1.

(ii) For the first stage when \( x^i_0 \notin \mathcal{X}_i \), \( X^0_i \) is affine of \( \hat{U}_0 \), and both the state and input constraints are convex polyhedra. Therefore, COCP (4) with (5) is a quadratic program as described by (20). By defining \( z = \hat{U}_0 + H^{-1} F X^0_i z \in \mathbb{R}^* \), and removing \( X^0_i Y X^0_i \) that is independent of \( \hat{U}_0 \), (20) is transformed into the following form

\[ \hat{J}^*(X^0_i) = \min_{z} z^T H z \]  

s.t. \( G z \leq w + S X^0_i \)  

(22)

where \( S = E + G H^{-1} F \).

From Lemma 3, the optimizer \( z^*(X^0_i) \) is continuous and piecewise affine on polyhedra.

For the second stage, the DMPC scheme changes into \( M \) decoupled MPC problems. The problem is quadratic program following the analysis of generic MPC schemes (see, for example [14, page 224]).

**Remark 3:** Different from a \( N \)-step COCP, the control law in Algorithm 2 preserves closed-loop feedback properties with a time-varying prediction horizon. For situations with modeling mismatches and disturbances, Theorem 2 still holds if the system state at time \( t + k \) applying the control law in Algorithm 2 is in the \( (N - k) \)-step backward reachable set.
B. Independent Obstacle-Avoidance Capability

The implementation of cost-switching strategy in the DMPC scheme introduces individually decoupled regulation capability of subsystems. Therefore, it is natural to investigate independent obstacle-avoidance capability of the decoupled subsystems. In particular, we have the following theorem.

**Theorem 3:** There exists a configuration of $\bar{X}$ and (6) for each subsystem, such that the subsystems applying control law in Algorithm 2 with initial condition $x_0 \in \bar{X}_0$ have no intersections in the spatial subspace, i.e. collision-free.

**Proof:** Since the systems are spatially decoupled, there exists a polyhedron set $\bar{X}^i$ of each subsystem such that $\forall i \in \{1, \ldots, M\}, \forall j \in \mathcal{N}^j, \bar{X}^i \cap \bar{X}^j = \emptyset$. Moreover, if $\bar{X}^j \subseteq \bar{X}^i$, and $\bar{X}^i_0 \subseteq X^i$, $\forall i \in \{1, \ldots, M\}$. Then,

$$\bar{X}^i_0 \cap \bar{X}^j_0 = \emptyset \rightarrow \bar{X}^i_0 \cap \bar{X}^j_0 = \emptyset$$  \hspace{1cm} (23)

Therefore, the terminal sets of (4) (i.e. the initial sets of (6)) have no intersections. With the control law in Algorithm 2, given initial condition $x_0$, the closed-loop trajectory of each subsystem will stay in $\bar{X}_f$, $\forall t \in \mathbb{Z}^+$. Therefore, subsystems applying Algorithm 2 will have no spatial intersections.

**Remark 4:** Theorem 3 shows that the terminal constraint sets $\{X^i_f\}$ have no intersections with each other by properly selecting state constraints $\{\bar{X}^i\}$. The distributed subsystems can independently regulate spatial disturbances if feasible solutions of (6) exist. Moreover, such independent regulation laws subject to collision-free constraints since the state constraint sets $\{\bar{X}^i\}$ have no intersections by design.

IV. **Numerical Results**

Consider an unmanned ground vehicle (UGV) formation of three vehicles with the following planar kinematic model

$$\begin{align*}
\dot{s} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega
\end{align*}$$  \hspace{1cm} (24)

where $s$, $y$ and $\theta$ are vehicle longitudinal position, lateral position, and heading angle, respectively. $v$ and $\omega$ are vehicle speed and steering rate, respectively. The DMPC scheme implements the following linearized discrete-time model

$$x(t + 1) = Ax(t) + Bu(t)$$  \hspace{1cm} (25)

where

$$A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0.5 \\
0 & 0 & 1
\end{bmatrix}, B = \begin{bmatrix}
0.1 & 0 \\
0 & 0 \\
0 & 0.1
\end{bmatrix}$$

with $x = [s, y, \theta]^T$, $u = [v, \omega]^T$. All three vehicles are set with initial state deviations. In the DMPC scheme, cost functions switch to decoupled indexes at $t = 0.7s$. The control problem switches from (4) to (6). Fig. 2 shows norm of state deviations of three vehicles. State deviations converge to zero after 2 seconds. Fig. 3 shows the results regarding independent obstacle-avoidance capability. In this case, the lead vehicle encounters a stationary obstacle that can be avoided by a state transition in the terminal constraint set. Since the control laws are decoupled for states inside $\bar{X}_f$, the lead vehicle individually generates an obstacle-avoiding path without disturbing the followers. Additionally, states of all vehicles are inside $\bar{X}_f$, the obstacle avoidance maneuver is collision-free for the formation.

V. **Conclusions**

In this note, stabilization by DMPC of a class of spatially decoupled systems has been investigated. By selecting the terminal constraint sets as control invariant sets, a DMPC scheme with switched cost functions has been proposed. With the cost-switching strategy, stability of the overall system is guaranteed by the feasibility of a series of COCPs; and the DMPC scheme preserves quadratic program properties. Independent obstacle-avoidance capability is also obtained by taking advantage of control invariance of terminal constraint sets. While the cost-switching strategy ensures attractiveness, it is imperative to investigate robustness properties of such strategies. For future work, it is of interest to extend the results by adopting results from robust MPC studies. DMPC with dynamic graph connections can be another possible extension.

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