A fractional-order mathematical model for analyzing the pandemic trend of COVID-19

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Communicated by: M. Efendiev

Many countries worldwide have been affected by the outbreak of the novel coronavirus (COVID-19) that was first reported in China. To understand and forecast the transmission dynamics of this disease, fractional-order derivative-based modeling can be beneficial. We propose in this paper a fractional-order mathematical model to examine the COVID-19 disease outbreak. This model outlines the multiple mechanisms of transmission within the dynamics of infection. The basic reproduction number and the equilibrium points are calculated from the model to assess the transmissibility of the COVID-19. Sensitivity analysis is discussed to explain the significance of the epidemic parameters. The existence and uniqueness of the solution to the proposed model have been proven using the fixed-point theorem and by helping the Arzela–Ascoli theorem.

KEYWORDS
basic reproduction number, COVID-19, equilibrium point, fractional-order derivative, predictor–corrector algorithm

MSC CLASSIFICATION
26A33; 65R10

1 | INTRODUCTION

The WHO announced COVID-19 to be a pandemic epidemic on January 22, 2020. Nowadays, the pandemic is spreading across the world and affects almost every aspect of life. In addition to health problems, it undermines the global economic system and restricting people’s contact. Researchers from diverse scientific fields have therefore committed modeling and studied the pandemic trend of COVID-19 has been presented with fractional-order derivative. Using the generation matrix method, the basic reproduction number is calculated that located whether the disease would persist or disappears from the population. The equilibrium points for this system are calculated, where the graphical solutions show that the results of the model converge to their equilibrium points. Based on the derivatives of the basic reproduction number, the sensitivity of the parameters was analyzed. The existence and uniqueness of the solution to the proposed model have been proven using the fixed-point theorem and by helping the Arzela–Ascoli theorem.
themselves to the study of COVID-19. The objectives were to contribute to the enhancement of the comprehension, forecast, and interpretation from different points of view for this disease. Any new suggestions are a step forward in solving this health crisis. Mathematical models are a significant and efficient way to comprehend epidemic transmission dynamics.1–9 These models can be helping predict disease transmission and thus help decision makers in planning and make necessary decisions.

The fractional-order models provide a more accurate fit than the integer-order models for the actual data for various diseases and other experiential studies around modeling and simulation. New mathematical models of fractional order are developed that can be used for simulation to forecast the disease outbreaks and flatten the infection and deaths curve.10–14 The mathematicians have also put their efforts forward in the analysis of various nonlinear dynamics of problems related to the infection like epidemics.15–18 Khan and Atangana19 described the mathematical modeling and dynamics of a novel coronavirus (2019-nCoV), then formulated a new mathematical model for simulation of the dynamics COVID-19 with quarantine and isolation.20 Atangana21 proposed a model for the spread of COVID-19, with new fractal-fractional operators and taking into account the potential of transmission of COVID-19 from dead bodies to humans and the effect of lockdown. Yadav and Verma22 investigated a fractional model based on Caputo–Fabrizio fractional derivative and developed for simulation the transmission coronavirus (COVID-19) in Wuhan Chinese.

Baleanu et al23 used the homotopy analysis transformation method to solve the COVID-19 transmission model with Caputo–Fabrizio derivative. Erturk and Kumar24 used the corrector–predictor algorithm with a new generalized Caputo type derivative to solve the novel coronavirus (COVID-19) epidemic fractional-order model. Abdulwasaa et al25 generalized a mathematical model based on a fractal-fractional operator to forecast future trends in the behavior of the COVID-19 pandemics in India for October 2020. Rezapour et al26 provided the SEIR model of the spreading epidemic of the COVID-19 globally and Iran for data reported from December 31, 2019, to January 28, 2020. Shaikh et al27,28 analyzed the fractional-order COVID-19 model for simulating the potential transmission of epidemic and suggested some possible control strategies. Zhang et al29 proposed and applied the time-dependent SEIR model to fit the time series of coronavirus evolution observed recently in China and then predict transmission. Matouk30 analyzed complex dynamics in a model for COVID-19 and its fractional-order counterpart with multidrug resistance. Tang et al31 adopted a deterministic model that was devised based on the clinical improvement to illuminate the dynamics for transmission of the novel coronavirus and determine the effect of the public health measures on the infection.

The paper is arranged as follows: In Section 2, we recalled some basic concepts and results used in the article. In Section 3, we developed the model pandemic trend Covid-19 with fractional order then calculated the basic reproduction number and equilibrium points along with sensitivity analysis for parameters. Using the fixed-point theory, the existence and uniqueness analysis of the model was performed in Section 4. In Section 5, we presented a numerical algorithm for solving the model based on the predictor-corrector method. In Section 6, we discussed the stability of the numerical method. In Section 7, we have displayed the numerical simulation results for the real data. A conclusion completes the paper.

2 | BASIC CONCEPT

In this section, we present some of the fundamental definitions and rand results of fractional order derivatives which, can be used throughout this manuscript.

**Definition 1** (Samko et al. and Miller and Ross32,33). For an integrable function \( f(t) \), the Caputo derivative of fractional order \( \alpha \in (0, 1) \) is given by

\[
_{t}^{c}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad n = [\alpha] + 1. \tag{1}
\]

The corresponding fractional integer of order \( \alpha \) with \( Re(\alpha) > 0 \) is given by

\[
_{t}^{c}I_{t}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau. \tag{2}
\]
Lemma 1 (Li and Zeng). If $0 < \alpha < 1$ and $n \geq 0$ is an integer, then there exists the positive constants $C_{\alpha,1}$ and $C_{\alpha,2}$ only dependent on $\alpha$, such that
\[ (n + 1)^{\alpha} - n^{\alpha} \leq C_{\alpha,1} (n + 1)^{\alpha - 1}, \]
and
\[ (n + 2)^{\alpha+1} - 2(n + 1)^{\alpha+1} + n^{\alpha+1} \leq C_{\alpha,2} (n + 1)^{\alpha - 1}. \]

Lemma 2 (Li and Zeng). Assume that $a_{k,n} = (n - k)^{\alpha - 1}$, $k = 1, 2, \ldots, n - 1$ and $a_{k,n} = 0$ for $k \geq n$. Let $\alpha, h, L, T > 0$, $\tau h \leq T$ where $\tau$ is a positive integer. Let $\sum_{k=m}^{n} a_{k,n} |e_k| = 0$ for $m > n \geq 1$. If
\[ |e_n| \leq L h^\alpha \sum_{k=1}^{n-1} a_{k,n} |e_k| + |\zeta_0|, \quad n = 1, 2, \ldots, \tau, \]
then,
\[ |e_n| \leq C |\zeta_0|, \quad \tau = 1, 2, \ldots, \]
where $C$ is a positive constant independent of $h$ and $\tau$.

Lemma 3 (Diethelm). The function $y \in C[0, T]$ is a solution of the following fractional differential equation:
\[ {}^C D_t^\alpha y(t) = f(t, y(t)), \quad y(0) = y_0, \]
if and only if it is a solution of the nonlinear Volterra integral equation of the second kind
\[ y(t) = y(0) + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f(\tau, y(\tau)) d\tau. \]

3 | THEORETICAL APPROACH

In this section, we are considered to examine a classical model developed by Fatima et al. for the pandemic trend of 2019 coronavirus, which was first identified in the Chinese city of Wuhan in December 2019 and then spread quickly across the world. The population is divided into six subcategories: susceptible people denoted by $S$, exposed people $E$, infected people $I$, asymptomatic infectious people $A$, isolated or hospitalized people $H$, recovered or dead people $R$, and the reservoir for COVID-19 is denoted by $W$. The pandemic trend model of COVID-19 is given by the system of ordinary differential equations as follows:
\[
\begin{align*}
\dot{S} &= Y - \xi_1 (I + \sigma A + \psi H) - \xi_2 WS - \delta S, \\
E &= \xi_1 (I + \sigma A + \psi H) + \xi_2 WS - (\nu + \delta) E, \\
I &= \nu E - (\beta_1 + \beta_2 + \delta) I, \\
A &= \nu (1 - \eta) E - (\rho + \delta) A, \\
H &= \beta_1 I + \rho A - (\beta_3 + \delta) H, \\
R &= \beta_2 I + \beta_3 H - \delta R, \\
W &= \theta_1 I + \theta_2 A - \lambda W.
\end{align*}
\]

Subject to non-negative initial conditions.
\[ S(0) = S_0, E(0) = E_0, I(0) = I_0, A(0) = A_0, H(0) = H_0, R(0) = R_0, W(0) = W_0, \]
where $Y = b \times N$, $b$ is the birth rate, and $N$ is the total number of people; $\delta$ the death rate of the population; $\xi_1$ and $\xi_2$ denote the rate of transmission infected of the susceptible people through sufficient contact with $I$ and $W$, respectively. The parameters $\sigma$ and $\psi$ denote the approximate transmissibility from $A$ and $H$ to $I$, respectively. $(\nu)^{-1}$ represented the
Equilibrium points and basic reproduction number

To find the disease-free equilibrium of the suggested fractional-order model (11), we solve the following system:

\[
\begin{align*}
\kappa^{a-1} D_t^a S(t) &= Y - \xi_1 S(t)(I(t) + \sigma A(t) + \psi H(t)) - \xi_2 W(t) S(t) - \delta S(t), \\
\kappa^{a-1} D_t^a E(t) &= \xi_1 (I(t) + \sigma A(t) + \psi H(t)) + \xi_2 W(t) S(t) - (\nu + \delta) E(t), \\
\kappa^{a-1} D_t^a I(t) &= \nu \eta E(t) - (\beta_1 + \beta_2 + \delta) I(t), \\
\kappa^{a-1} D_t^a A(t) &= \nu (1 - \eta) E(t) - (\rho + \delta) A(t), \\
\kappa^{a-1} D_t^a H(t) &= \beta_1 I(t) + \rho A(t) - (\beta_1 + \delta) H(t), \\
\kappa^{a-1} D_t^a R(t) &= \beta_2 I(t) + \beta_3 H(t) - \delta R(t), \\
\kappa^{a-1} D_t^a W(t) &= \theta_1 I(t) + \theta_2 A(t) - \lambda W(t).
\end{align*}
\]

Subject to non-negative initial conditions

\[
S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \quad A(0) = A_0, \quad H(0) = H_0, \quad R(0) = R_0, \quad W(0) = W_0.
\]

3.1 Equilibrium points and basic reproduction number

To find the disease-free equilibrium of the suggested fractional-order model (11), we solve the following system.

\[
\begin{align*}
0 &= Y - \xi_1 S(t)(I(t) + \sigma A(t) + \psi H(t)) - \xi_2 W(t) S(t) - \delta S(t), \\
0 &= \xi_1 (I(t) + \sigma A(t) + \psi H(t)) + \xi_2 W(t) S(t) - (\nu + \delta) E(t), \\
0 &= \nu \eta E(t) - (\beta_1 + \beta_2 + \delta) I(t), \\
0 &= \nu (1 - \eta) E(t) - (\rho + \delta) A(t), \\
0 &= \beta_1 I(t) + \rho A(t) - (\beta_1 + \delta) H(t), \\
0 &= \beta_2 I(t) + \beta_3 H(t) - \delta R(t), \\
0 &= \theta_1 I(t) + \theta_2 A(t) - \lambda W(t).
\end{align*}
\]
We get \( P_0 = (Y/\delta, 0, 0, 0, 0, 0, 0) \), which is the point where there is no disease. If the basic reproduction number \( R_0 > 0 \), then the system (13) has a positive endemic equilibrium point, which is represented by

\[
\begin{align*}
S^* &= -\frac{uxyz\lambda}{v(\lambda(a + \delta + bc + \beta_2 + (b\delta - z\gamma)\beta_3) + \beta_1(b\delta + d\psi + b\delta\beta_3))\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}, \\
E^* &= \frac{xz\delta^3\lambda + xz\delta\lambda(eu + \delta\beta_3) + \lambda v Y(a + bc + b\delta - z\gamma)\beta_3}{xv(\lambda(a + \delta + bc + (b\delta - z\gamma)\beta_3) + \beta_1(b\delta + d\psi + b\delta\beta_3))\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}, \\
I^* &= \frac{\eta(zx\delta^3\lambda + xz\delta\lambda(eu + \delta\beta_3) + \lambda v Y(a + bc + b\delta - z\gamma)\beta_3}{xv(\lambda(\psi + g\beta_2 + (u\delta - z\gamma)\beta_3) + \beta_1(\psi - f\phi\beta_3))\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}, \\
A^* &= \frac{\lambda(\psi + g\beta_2 + (u\delta - z\gamma)\beta_3) + \beta_1(\psi - f\phi\beta_3)\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}{v(\psi - f\phi\beta_3)} \\
H^* &= \frac{\lambda(\psi + g\beta_2 + (u\delta - z\gamma)\beta_3) + \beta_1(\psi - f\phi\beta_3)\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}{v(\psi - f\phi\beta_3)} \\
R^* &= \frac{\lambda(\psi + g\beta_2 + (u\delta - z\gamma)\beta_3) + \beta_1(\psi - f\phi\beta_3)\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}{v(\psi - f\phi\beta_3)} \\
W^* &= \frac{\lambda(\psi + g\beta_2 + (u\delta - z\gamma)\beta_3) + \beta_1(\psi - f\phi\beta_3)\xi_1 - u(\eta\theta_1 - by\theta_2)\xi_2}{v(\psi - f\phi\beta_3)}
\end{align*}
\]

where \( x = \delta + \nu, y = \delta + \beta_1 + \beta_2, z = \delta + \rho, u = \delta + \beta_3, a = -\eta z + b\delta + b\psi, b = \eta - 1, c = \delta \sigma + \rho \psi + \sigma \beta_3, d = \delta \eta + \rho. e = \beta_1 + \beta_2. \)

A vital indicator of the spread of infectious disease in the population is the basic reproduction number \( R_0 \), defined as the number of new infections caused by a typical infectious person in a disease-free equilibrium population. An epidemic will spread if \( R_0 > 1 \). While that an outbreak will most likely not accrue if \( R_0 < 1 \). To obtain \( R_0 \) for the system (13), we use the generation method.\(^{36}\) The infection components in the model system are \( E, I, A, H, \) and \( W \), so we consider the following fractional system:

\[
k^{\alpha - 1} D_t^\alpha \Phi(t) = F(\Phi(t)) - V(\Phi(t)),
\]

where

\[
F(\Phi(t)) = \kappa^{-1-a} \left[
\begin{array}{c}
\xi_1 (I(t) + \sigma A(t) + \psi H(t)) S(t) + \xi_2 W(t) S(t)
\end{array}
\right],
\]

and

\[
V(\Phi(t)) = \kappa^{-1-a} \left[
\begin{array}{c}
(v + \delta) E(t) - \nu Z(t) + (b_1 + b_2 + \delta) I(t)
\end{array}
\right] .
\]

At point \( P_0 \), the Jacobian matrix for \( F \) and \( V \) is given by

\[
J_F(P_0) = \kappa^{-1-a} \left[
\begin{array}{ccccccc}
0 & \xi_1 \sigma & \xi_1 \psi & \xi_2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\right].
\]

and

\[
J_V(P_0) = \kappa^{-1-a} \left[
\begin{array}{ccccccc}
v + \delta & 0 & 0 & 0 & 0 \\
0 & \beta_1 + \beta_2 + \delta & 0 & 0 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & -\beta_1 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & -\beta_1 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & -\beta_1 \\
\end{array}
\right].
\]
The basic reproduction number is given by the spectral radius of $J_V(P_0)[J_V(P_0)]^{-1}$ as follows:

$$R_0 = \frac{\eta \nu \xi_1 Y}{\delta(\delta + \nu)(\delta + \beta_1 + \beta_2)} + \frac{Y \xi_2(\eta \nu \theta_1 + (1 - \eta) \nu(\delta + \beta_1 + \beta_2) \theta_z)}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)}$$

$$+ \frac{Y(1 - \eta) \nu \sigma \xi_1}{\delta(\delta + \nu)(\delta + \rho)} + \frac{\psi(\delta \eta + \rho) \beta_1 + (1 - \eta) \rho(\delta + \beta_2) \xi_1}{\delta(\delta + \nu)(\delta + \rho)(\delta + \beta_1 + \beta_2)(\delta + \beta_3)}.$$  

(15)

### 3.2 | Sensitivity analysis

The sensitivity analysis of a few parameters used in the proposed model (11) is discussed in this section. That will make it easier for us to identify the parameters that favorably impact the basic reproduction number. To do this, we apply the technic given in Tuan et al. and Rezapour et al. 11,26 Using $R_0$, we have

$$\frac{\partial}{\partial \beta_0} R_0 = \frac{\eta \nu \xi_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)}, \quad \frac{\partial}{\partial \gamma} R_0 = \frac{R_0}{Y}.$$

$$\frac{\partial}{\partial \sigma} R_0 = \frac{(1 - \eta) \nu Y \xi_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)},$$

$$\frac{\partial}{\partial \theta_1} R_0 = \frac{Y \xi_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)} + \frac{(1 - \eta) \nu Y \theta_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)},$$

$$\frac{\partial}{\partial \theta_2} R_0 = \frac{Y \xi_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)} + \frac{(1 - \eta) \nu Y \theta_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)},$$

$$\frac{\partial}{\partial \beta_3} R_0 = \frac{Y \xi_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)} + \frac{(1 - \eta) \nu Y \theta_2}{\delta \lambda(\delta + \nu)(\delta + \beta_1 + \beta_2)}.$$
4 | EXISTENCE AND UNIQUENESS ANALYSIS

In this section, we investigate the existence and uniqueness of the proposed model with the assistance of fixed-point theory. First, let us write Equation (11) as follows:

\[
\begin{align*}
\kappa^{\alpha-1} D_t^\alpha S(t) &= \chi_1(t, S(t)), \\
\kappa^{\alpha-1} D_t^\alpha E(t) &= \chi_2(t, E(t)), \\
\kappa^{\alpha-1} D_t^\alpha I(t) &= \chi_3(t, I(t)), \\
\kappa^{\alpha-1} D_t^\alpha A(t) &= \chi_4(t, A(t)), \\
\kappa^{\alpha-1} D_t^\alpha H(t) &= \chi_5(t, H(t)), \\
\kappa^{\alpha-1} D_t^\alpha R(t) &= \chi_6(t, R(t)), \\
\kappa^{\alpha-1} D_t^\alpha W(t) &= \chi_7(t, W(t)),
\end{align*}
\]

where

\[
\begin{align*}
\chi_1(t, S(t)) &= Y - \xi_1 S(t)(I(t) + \sigma A(t) + \psi H(t)) - \xi_2 W(t)S(t) - \delta S(t), \\
\chi_2(t, E(t)) &= \xi_1 I(t) + \sigma A(t) + \psi H(t) + \xi_2 W(t)S(t) - \nu E(t), \\
\chi_3(t, I(t)) &= \nu E(t) - (\beta_1 + \beta_2 + \delta I(t), \chi_4(t, A(t)) = \nu (1 - \eta) E(t) - (\rho + \delta) A(t), \\
\chi_5(t, H(t)) &= \beta_1 I(t) + \rho A(t) - (\beta_2 + \delta) H(t), \chi_6(t, R(t)) = \beta_2 I(t) + \beta_3 H(t) - \delta R(t), \\
\chi_7(t, W(t)) &= \theta_1 I(t) + \theta_2 A(t) - \lambda W(t).
\end{align*}
\]

By Lemma 3, the corresponding Volterra integral system of the second kind for Equation (16) is given by

\[
\begin{align*}
S(t) &= S(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_1(\tau, S(\tau)) \, d\tau, \\
E(t) &= E(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_2(\tau, E(\tau)) \, d\tau, \\
I(t) &= I(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_3(\tau, I(\tau)) \, d\tau, \\
A(t) &= A(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_4(\tau, A(\tau)) \, d\tau, \\
H(t) &= H(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_5(\tau, H(\tau)) \, d\tau, \\
R(t) &= R(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_6(\tau, R(\tau)) \, d\tau, \\
W(t) &= W(0) + \frac{\kappa^{1-\alpha}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \chi_7(\tau, W(\tau)) \, d\tau.
\end{align*}
\]

Theorem 1. The kernel of the proposed fractional model will be satisfying the Lipchitz condition if the following inequality hold:

\[0 \leq L_i < 1, i = 1, 2, \ldots, 7,\]

where \(L_1 = \xi_1 (Z_1 + \sigma Z_2 + \psi Z_3) + \xi_2 Z_4 + \delta, L_2 = \nu + \delta, L_3 = \beta_1 + \beta_2 + \delta, L_4 = \rho + \delta, L_5 = \beta_3 + \delta, L_6 = \delta, and L_7 = \lambda.\)

Proof. We will prove for the first kernel and similarly for the other. Consider function \(S(t)\) and \(S_1(t)\), then

\[
\begin{align*}
\| \chi_1(t, S) - \chi_1(t, S_1) \| &= \| (\xi_1 (I(t) + \sigma A(t) + \psi H(t)) + \xi_2 W(t) + \delta) (S(t) - S_1(t)) \| \\
&\leq (\xi_1 \| I(t) \| + \sigma \| A(t) \| + \psi \| H(t) \| + \xi_2 \| W(t) \| + \delta) \| S(t) - S_1(t) \| \\
&\leq (\xi_1 (Z_1 + \sigma Z_2 + \psi Z_3) + \xi_2 Z_4 + \delta) \| S(t) - S_1(t) \| = L_1 \| S(t) - S_1(t) \|. \\
\end{align*}
\]
where \( Z_1 \leq \|I(t)\|, Z_2 \leq \|A(t)\|, Z_3 \leq \|H(t)\|, \) and \( Z_4 \leq \|W(t)\|.

Similarly, we get
\[
\begin{align*}
\|\chi_2(t, E) - \chi_2(t, E_1)\| & \leq L_2 \|E(t) - E(t_1)\| \\
\|\chi_3(t, I) - \chi_3(t, I_1)\| & \leq L_3 \|I(t) - I(t_1)\| \\
\|\chi_4(t, A) - \chi_4(t, A_1)\| & \leq L_4 \|A(t) - A(t_1)\| \\
\|\chi_5(t, H) - \chi_5(t, H_1)\| & \leq L_5 \|H(t) - H(t_1)\| \\
\|\chi_6(t, R) - \chi_6(t, R_1)\| & \leq L_6 \|R(t) - R(t_1)\| \\
\|\chi_7(t, W) - \chi_7(t, W_1)\| & \leq L_7 \|W(t) - W(t_1)\|.
\end{align*}
\]

By helping Equation (16), the proposed model Equation (11) can be written as follows:
\[
\kappa^{\alpha-1}C D_t^\alpha \chi(t) = \Psi(t, \chi(t)), \quad t \in [0, T], \quad \alpha \in (0, 1], 
\tag{18}
\]
with initial condition \( \chi(0) = \chi_0 \), where \( \chi(t) = \left[ S(t) E(t) I(t) A(t) H(t) R(t) W(t) \right]^T, \chi_0 = \left[ S_0 E_0 I_0 A_0 H_0 R_0 W_0 \right]^T \), and \( \Psi(t, \chi) = \left[ \chi_1(t, S), \chi_2(t, E), \chi_3(t, I), \chi_4(t, A), \chi_5(t, H), \chi_6(t, R), \chi_7(t, W) \right]^T \).

Again, by Lemma 3, we have
\[
\chi(t) = \chi(0) + \frac{\kappa^{\alpha-1}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \Psi(\tau, \chi(\tau)) \, d\tau.
\tag{19}
\]

**Theorem 2** (Existence). Let \( T^* > 0, \chi_0 \in \mathbb{R}_+^7 \), and \( b > 0 \). Define\n\[
\mathcal{D} := \left\{ (t, \chi) \in [0, T^*] \times \mathbb{R}_+^7 : t \in [0, T^*], \|\chi - \chi_0\| \leq b \right\},
\]
and assume that \( \Psi : \mathcal{D} \to \mathbb{R}_+^7 \) be a continuous function; furthermore, define \( \zeta := \sup_{(t, \chi) \in \mathcal{D}} \|\Psi(t, \chi(t))\| \) then, for \( \alpha \in (0, 1] \), there exists a function \( \chi \in \left[ [0, T], \mathbb{R}_+^7 \right] \), which is a solution of Equation (18), where\n\[
T = \min \left\{ T^*, \left( \frac{b \Gamma(\alpha + 1)}{\kappa^{1-\alpha} \zeta} \right)^{\frac{1}{\alpha}} \right\}.
\]

**Proof.** For \( \chi \in C \left([0, T], \mathbb{R}_+^7 \right), \) define norm of \( \|\chi\|_* = \sup_{t \in [0, T]} \|\chi(t)\| \), with \( \|\cdot\|_* \) a Banach space. Define the set \( V := \left\{ \chi \in C \left([0, T], \mathbb{R}_+^7 \right) \right|: \|\chi - \chi_0\|_* \leq b \} \); it is evident that \( V \) is bounded, closed, and convex subset of the Banach space of all continuous function on \( C \left([0, T], \mathbb{R}_+^7 \right) \). It is clear that \( V \) is a non-empty set, since \( \chi_0 \in V \). We now define an operator \( \mathfrak{F} \) on \( V \). For each element \( \chi \in V \),
\[
(\mathfrak{F} \chi)(t) := \chi(0) + \frac{\kappa^{\alpha-1}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \Psi(\tau, \chi(\tau)) \, d\tau.
\tag{20}
\]

Using that operator, Equation (19) can be written as \( \chi = \mathfrak{F} \chi \); now we must prove that \( \mathfrak{F} \) has a fixed-point. This will be done by using Schuder's second fixed-point theorem. First, we will show that \( \mathfrak{F} \in V \) for \( \chi \in V \). For any \( \chi \in V \), we have
\[
\|(\mathfrak{F} \chi)(t) - \chi(0)\| = \left\| \frac{\kappa^{\alpha-1}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \Psi(\tau, \chi(\tau)) \, d\tau \right\|
\leq \frac{\kappa^{\alpha-1} \zeta}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \, d\tau = \frac{\kappa^{\alpha-1} \zeta \Gamma(\alpha + 1)}{\Gamma(\alpha + 1)} T^* = \frac{\kappa^{\alpha-1} \zeta b \Gamma(\alpha + 1)}{\Gamma(\alpha + 1)} \kappa^{1-\alpha} \zeta = b.
\tag{21}
\]

Hence, \( \|\mathfrak{F} \chi - \chi_0\|_* \leq b \), so \( \mathfrak{F} \chi \in V \) for \( \chi \in V \).
Second, we show that \( \mathfrak{F} \) is continuous. For every \( \chi_n, \chi \in V, n = 1, 2, \ldots, \) with \( \lim_{n \to \infty} \| \chi_n - \chi \|_s = 0, \) we have \( \lim_{n \to \infty} \chi_n(t) = \chi(t), t \in [0, T], \) since all components of \( \Psi(t, \chi(t)) \) are continuous on \( \mathbb{R}, \) thus \( \Psi \) is continuous on \( \{ \chi \in C([0, T], \mathbb{R}^s_+): \| \chi - \chi_0 \| \leq b \} , \) consequently, \( \lim_{n \to \infty} \Psi(t, \chi_n(t)) = \Psi(t, \chi(t)) \), so

\[
\sup_{t \in [0, T]} \| \Psi(t, \chi_n(t)) - \Psi(t, \chi(t)) \| \to 0 \text{ as } n \to \infty. \tag{22}
\]

On the other hand,

\[
\| (\mathfrak{F} \chi_n)(t) - (\mathfrak{F} \chi)(t) \| \leq \frac{k^{1-a}}{\Gamma(\alpha)} \int_0^t (t - \tau)^{a-1} \| \Psi(\tau, \chi_n(\tau)) - \Psi(\tau, \chi(\tau)) \| d\tau.
\]

Hence,

\[
\| (\mathfrak{F} \chi_n - \mathfrak{F} \chi) \|_s \to 0 \text{ as } n \to \infty. \tag{23}
\]

Thus, \( \| (\mathfrak{F} \chi)(t) \|_s \leq \| \chi_0 \| + b, \) this means that \( \mathfrak{F}(V) \) is uniformly bounded. For any \( 0 \leq t_1 \leq t_2 \leq T, \) we have

\[
\| (\mathfrak{F} \chi)(t_1) - (\mathfrak{F} \chi)(t_2) \| \leq \frac{k^{1-a}}{\Gamma(\alpha)} \left[ \int_0^{t_1} (t_1 - \tau)^{a-1} \Psi(\tau, \chi(\tau)) d\tau - \int_0^{t_2} (t_2 - \tau)^{a-1} \Psi(\tau, \chi(\tau)) d\tau \right]
\]

\[
= \frac{k^{1-a}}{\Gamma(\alpha)} \int_0^{t_2} \left[ (t_1 - \tau)^{a-1} - (t_2 - \tau)^{a-1} \right] \Psi(\tau, \chi(\tau)) d\tau + \int_0^{t_1} (t_2 - \tau)^{a-1} \Psi(\tau, \chi(\tau)) d\tau.
\]

Since \( \alpha \leq 1 \) and \( t_1 \leq t_2, \) then \( (t_1 - \tau)^{a-1} \geq (t_2 - \tau)^{a-1}, \) therefore,

\[
\| (\mathfrak{F} \chi)(t_1) - (\mathfrak{F} \chi)(t_2) \| \leq \frac{k^{1-a} \zeta}{\Gamma(\alpha)} \left[ \int_0^{t_1} (t_1 - \tau)^{a-1} d\tau + \frac{(t_2 - t_1)^a}{a} \right]
\]

\[
= \frac{k^{1-a} \zeta}{\Gamma(\alpha + 1)} \left[ (t_2 - t_1)^a + t_1^a - t_2^a + (t_2 - t_1)^a \right] \leq \frac{2k^{1-a} \zeta}{\Gamma(\alpha + 1)} (t_2 - t_1)^a \to 0 \text{ as } t_2 \to t_1. \tag{24}
\]
Hence, $\|(\mathbf{f}\chi)(t_1) - (\mathbf{f}\chi)(t_2)\|_e \to 0$ as $t_2 \to t_1$, and $\mathbf{f}(V)$ is equicontinuous on $[0, T]$. Thus, $\mathbf{f}(V)$ is relatively compact. Moreover, $\mathbf{f}$ has a fixed-point, which is the required solution of Equation (18). This completes the proof.

Remark 1. Assume hypotheses Theorem 1, the kernels $\chi_i, i = 1, 2, \ldots, 7$ satisfy Lipchitz condition; thus,

$$\left\|\Psi(t, \chi(t)) - \Psi(t, \bar{\chi}(t))\right\| = \max_{t \in [0,T]} \left\{ \left\| \chi(t) - \bar{\chi}(t) \right\| \right\} \leq \max_{t \in [0,T]} \left\{ L_1 \| S(t) - \bar{S}(t) \|, L_2 \| E(t) - \bar{E}(t) \|, L_3 \| I(t) - \bar{I}(t) \|, L_4 \| A(t) - \bar{A}(t) \|, L_5 \| H(t) - \bar{H}(t) \|, L_6 \| R(t) - \bar{R}(t) \|, L_7 \| W(t) - \bar{W}(t) \| \right\} \leq \max_{t \in [0,T]} \left\{ \| S(t) - \bar{S}(t) \|, \| E(t) - \bar{E}(t) \|, \| I(t) - \bar{I}(t) \|, \| A(t) - \bar{A}(t) \|, \| H(t) - \bar{H}(t) \|, \| R(t) - \bar{R}(t) \|, \| W(t) - \bar{W}(t) \| \right\} \leq L_\Psi \| \chi(t) - \bar{\chi}(t) \|,$$

where $L_\Psi = \max\{L_1, L_2, L_3, L_4, L_5, L_6, L_7\}$ and $L_i, i = 1, 2, \ldots, 7$ as defined in Theorem 1. Thus, $\Psi$ satisfies Lipchitz's condition.

Theorem 3 (Uniqueness). Assume all hypotheses in Theorems 1 and 2, the solution $\chi \in C([0,T], \mathbb{R}^7_+)$ is unique, where

$$T < \min \left\{ T^*, \left( \frac{b \Gamma(a + 1)}{\kappa^{1-a} \zeta L_\Psi} \right)^\frac{1}{2} \right\}.$$

Proof. Suppose that $\chi(t)$ and $\bar{\chi}(t)$ are solutions of Equation (18) on $C([0,T], \mathbb{R}^7_+)$, then

$$\left\|\chi(t) - \bar{\chi}(t)\right\| = \frac{\kappa^{1-a}}{\Gamma(a)} \left\| \int_0^t (t - \tau)^{a-1} \Psi(\tau, \chi(\tau)) d\tau - \int_0^t (t - \tau)^{a-1} \Psi(\tau, \bar{\chi}(\tau)) d\tau \right\| \leq \frac{\kappa^{1-a}}{\Gamma(a)} \int_0^t (t - \tau)^{a-1} \left\| \Psi(\tau, \chi(\tau)) - \Psi(\tau, \bar{\chi}(\tau)) \right\| d\tau.

By Lipchitz condition, we get

$$\left\|\chi(t) - \bar{\chi}(t)\right\| \leq \frac{\kappa^{1-a} L_\Psi}{\Gamma(a)} \int_0^t (t - \tau)^{a-1} \left\| \chi(\tau) - \bar{\chi}(\tau) \right\| d\tau \leq \frac{\kappa^{1-a} L_\Psi}{\Gamma(a)} \max_{t \in [0,T]} \left\| \chi(t) - \bar{\chi}(t) \right\| \int_0^t (t - \tau)^{a-1} d\tau \leq \frac{\kappa^{1-a} L_\Psi T^a}{\Gamma(a + 1)} \max_{t \in [0,T]} \left\| \chi(t) - \bar{\chi}(t) \right\| = C \max_{t \in [0,T]} \left\| \chi(t) - \bar{\chi}(t) \right\|,$$

where $C = \frac{\kappa^{1-a} L_\Psi T^a}{\Gamma(a + 1)} \in (0, 1)$.

On the other hand,

$$\left\|\chi(t) - \bar{\chi}(t)\right\| = \max_{t \in [0,T]} \left\| \chi(t) - \bar{\chi}(t) \right\| .$$

this means that

$$\max_{t \in [0,T]} \left\| \chi(t) - \bar{\chi}(t) \right\| \leq C \max_{t \in [0,T]} \left\| \chi(t) - \bar{\chi}(t) \right\|.$$

Thus, $\left\| \chi(t) - \bar{\chi}(t) \right\| = 0 \Rightarrow \chi(t) = \bar{\chi}(t)$. The solution is unique. □
5 | NUMERICAL ALGORITHM

This section presents the numerical algorithm based on the predictor–corrector method. Under the hypotheses of Theorem 2, there is a unique solution on \([0, T]\). Let \(t_0 = 0 < t_1 < \ldots < t_N = T\) be a uniformly divide of the interval \([0, T]\) where \(t_n = nh, n = 1, 2, \ldots, N\) and \(h = T/N\). By Lemma 3, the solution of

\[
\kappa^{a-1} C D^a_t S(t) = \xi_1(t, S(t)).
\]

is equivalent to

\[
S(t) = S(0) + \frac{\kappa^{1-a}}{\Gamma(a)} \int_0^t (t - \tau)^{a-1} \xi_1(\tau, S(\tau)) d\tau.
\]

By Equation (30), we have

\[
S(t_{n+1}) = S(0) + \frac{\kappa^{1-a}}{\Gamma(a)} \int_{t_n}^{t_{n+1}} (t_{n+1} - \tau)^{a-1} \xi_1(\tau, S(\tau)) d\tau.
\]

Let \(S_{n+1} \approx S(t_{n+1}), n = 0, 1, 2, \ldots, N-1\), by Lagrange interpolation, we approximate the kernel \(\xi_1(t, S(t))\) over \([t_k, t_{k+1}]\) as follows:

\[
\xi_1(\tau, S(\tau)) \approx \frac{\tau - t_k}{t_{k+1} - t_k} \xi_1(t_{k+1}, S_{k+1}) + \frac{\tau - t_{k+1}}{t_k - t_{k+1}} \xi_1(t_k, S_k).
\]

Substituting Equation (32) into Equation (31), we get

\[
S_{n+1} = S_0 + \frac{\kappa^{1-a}}{\Gamma(a)} \sum_{k=0}^{n} \left[ \frac{\xi_1(t_{k+1}, S_{k+1})}{h} \int_{t_k}^{t_{k+1}} (\tau - t_k)(t_{n+1} - \tau)^{a-1} d\tau - \frac{\xi_1(t_k, S_k)}{h} \int_{t_k}^{t_{k+1}} (\tau - t_{k+1})(t_{n+1} - \tau)^{a-1} d\tau \right]
\]

\[
= S_0 + \frac{\kappa^{1-a}}{\Gamma(a)} \sum_{k=0}^{n} \left[ \frac{\xi_1(t_{k+1}, S_{k+1})}{h} \left( (n+1-k)^{a+1} - (n-k)^a (n+1-k+a) \right) \right.
\]

\[
- \frac{\xi_1(t_k, S_k)}{a(a+1)} \left( (n-k)^a - (n+1-k)^a (n-k-a) \right) \left].
\]

After the rearrangement of the summation on the right-hand side of Equation (33), we get

\[
S_{n+1} = S_0 + \frac{\kappa^{1-a} h^a}{\Gamma(a+1)} \left[ \xi_1(t_{n+1}, S_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \xi_1(t_k, S_k) \right],
\]

where

\[
C_{k,n+1} = \begin{cases} 
\frac{n^{a+1} - (n+1)^a (n-a)}{a^n}, & k = 0, \\
(n-k+1)^a + (n-k)^a - 2(n-k+1)^{a+1}, & 1 \leq k \leq n.
\end{cases}
\]

The quantity \(S_{n+1}\) on the right-hand side of Equation (34) is predicted by \(S_{n+1}^p\), applying the one-step Adams–Bashforth method to Equation (31) by replacing the function \(\xi_1(\tau, S(\tau))\) with the quantity \(\xi_1(t_k, S_k)\) as follows:

\[
S_{k+1}^p = S_0 + \frac{\kappa^{1-a}}{\Gamma(a)} \sum_{k=0}^{n} \xi_1(t_k, S_k) \int_{t_k}^{t_{k+1}} (t_{n+1} - \tau)^{a-1} d\tau = S_0 + \frac{\kappa^{1-a}}{\Gamma(a)} \sum_{k=0}^{n} \xi_1(t_k, S_k) \left( (n+1-k)^a - (n-k)^a \right).\]
Similarly, we can obtain the numerical algorithm of the other equation of system Equation (16). Thus, the approximate solution is given by

\[
\begin{align*}
S_{n+1} &= S_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_1 (t_{n+1}, S_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_1 (t_k, S_k) \right], \\
E_{n+1} &= E_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_2 (t_{n+1}, E_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_2 (t_k, E_k) \right], \\
I_{n+1} &= I_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_3 (t_{n+1}, I_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_3 (t_k, I_k) \right], \\
A_{n+1} &= A_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_4 (t_{n+1}, A_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_4 (t_k, A_k) \right], \\
H_{n+1} &= H_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_5 (t_{n+1}, H_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_5 (t_k, H_k) \right], \\
R_{n+1} &= R_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_6 (t_{n+1}, R_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_6 (t_k, R_k) \right], \\
W_{n+1} &= W_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a) + 1} \left[ \chi_7 (t_{n+1}, W_{n+1}) + \sum_{k=0}^{n} C_{k,n+1} \chi_7 (t_k, W_k) \right],
\end{align*}
\]

and

\[
\begin{align*}
S_{n+1}^p &= S_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_1 (t_k, S_k), \\
E_{n+1}^p &= E_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_2 (t_k, E_k), \\
I_{n+1}^p &= I_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_3 (t_k, I_k), \\
A_{n+1}^p &= A_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_4 (t_k, A_k), \\
H_{n+1}^p &= H_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_5 (t_k, H_k), \\
R_{n+1}^p &= R_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_6 (t_k, R_k), \\
W_{n+1}^p &= W_0 + \frac{k^{1-a} \Gamma(a)}{\Gamma(a)} \sum_{k=0}^{n} d_{k,n+1} \chi_7 (t_k, W_k),
\end{align*}
\]

where \( d_{k,n+1} = (n - k + 1)^a - (n - k)^a \).

### 6 STABILITY ANALYSIS OF ITERATION METHOD

**Theorem 4.** Assume the hypotheses of Theorem 1 and \( S_k, E_k, I_k, A_k, H_k, R_k, W_k, K = 1, 2, \ldots, n+1 \) are the solutions of systems Equations (36) and (37). Then, the fractional predictor-corrector method Equations (36), and (37) is conditional stable.
Proof. Assume that $S_0, S_{n+1}, S^p_{n+1}, n = 0, 1, 2, \ldots, N - 1$ have perturbations $\tilde{S}_0, \tilde{S}_{n+1}, \tilde{S}^p_{n+1}$, respectively. Let $\mu_0 = \max_{0 \leq \alpha \leq N} \left\{ \left| \tilde{S}_0 \right| + \frac{L_1 h^a k^{1-a}}{\Gamma(\alpha + 1)} \left| \tilde{S}_0 \right| \right\}$ and $\zeta_0 = \max_{0 \leq \alpha \leq N} \left\{ \left| \tilde{S}_0 \right| + \frac{L_1 h^a k^{1-a} C_{\alpha,2}}{\Gamma(\alpha + 2)} \left| \tilde{S}_0 \right| \right\}$. Then,

$$S_{n+1} + \tilde{S}_{n+1} = S_0 + \tilde{S}_0 + \frac{h^a k^{1-a}}{\Gamma(\alpha + 2)} \left( \chi_1 \left( t_{n+1}, S^p_{n+1} + \tilde{S}^p_{n+1} \right) + \sum_{k=0}^{n} C_{k,n+1} \chi_1 \left( t_k, S_k + \tilde{S}_k \right) \right),$$

(38)

$$S^p_{n+1} + \tilde{S}^p_{n+1} = S_0 + \tilde{S}_0 + \frac{h^a k^{1-a}}{\Gamma(\alpha + 1)} \sum_{k=0}^{n} d_{k,n+1} \chi_1 \left( t_k, S_k + \tilde{S}_k \right).$$

(39)

Subtracting Equations (34) and (35) from Equations (38) and (39), respectively, then

$$\left| \tilde{S}_{n+1} \right| = \left| \tilde{S}_0 + \frac{h^a k^{1-a}}{\Gamma(\alpha + 2)} \left( \chi_1 \left( t_{n+1}, S^p_{n+1} + \tilde{S}^p_{n+1} \right) - \chi_1 \left( t_{n+1}, S^p_{n+1} \right) + \sum_{k=0}^{n} C_{k,n+1} \left( \chi_1 \left( t_k, S_k + \tilde{S}_k \right) - \chi_1 \left( t_k, S_k \right) \right) \right) \right|$$

(40)

$$\leq \left| \tilde{S}_0 \right| + \frac{h^a k^{1-a}}{\Gamma(\alpha + 2)} \left( \left| \chi_1 \left( t_{n+1}, S^p_{n+1} + \tilde{S}^p_{n+1} \right) - \chi_1 \left( t_{n+1}, S^p_{n+1} \right) \right| + \sum_{k=0}^{n} C_{k,n+1} \left| \chi_1 \left( t_k, S_k + \tilde{S}_k \right) - \chi_1 \left( t_k, S_k \right) \right| \right)$$

By Lipchitz condition, we obtain

$$\left| \tilde{S}_{n+1} \right| \leq \zeta_0 + \frac{L_1 h^a k^{1-a}}{\Gamma(\alpha + 2)} \left( \left| \tilde{S}^p_{n+1} \right| + \sum_{k=0}^{n} C_{k,n+1} \left| \tilde{S}_k \right| \right).$$

(42)

$$\left| \tilde{S}^p_{n+1} \right| \leq \mu_0 + \frac{h^a k^{1-a}}{\Gamma(\alpha + 1)} \sum_{k=0}^{n} d_{k,n+1} \left| \tilde{S}_k \right|. $$

(43)

Substituting Equation (43) into Equation (42), we get

$$\left| \tilde{S}_{n+1} \right| \leq \zeta_0 + \frac{L_1 h^a k^{1-a}}{\Gamma(\alpha + 2)} \mu_0 + \frac{L_1 h^a k^{1-a}}{\Gamma(\alpha + 2)} \left( \frac{h^a k^{1-a}}{\Gamma(\alpha + 1)} \sum_{k=0}^{n} d_{k,n+1} \left| \tilde{S}_k \right| + \sum_{k=0}^{n} C_{k,n+1} \left| \tilde{S}_k \right| \right)$$

(44)

$$= \eta_0 + \frac{L_1 h^a k^{1-a}}{\Gamma(\alpha + 2)} \sum_{k=0}^{n} \left( \frac{h^a k^{1-a}}{\Gamma(\alpha + 1)} d_{k,n+1} + C_{k,n+1} \right) \left| \tilde{S}_k \right|,$$

where $\eta_0 = \left\{ \zeta_0 + \frac{L_1 h^a k^{1-a}}{\Gamma(\alpha + 2)} \mu_0 \right\}$. Using Lemma 1, we have

$$\left| \tilde{S}_{n+1} \right| \leq \eta_0 + \frac{L_1 h^a k^{1-a} C_{\alpha}}{\Gamma(\alpha + 2)} \sum_{k=0}^{n} (n - k + 1) a^{-1} \left| \tilde{S}_k \right|. $$

(45)
where, $C_{\alpha} = \max \{1, \alpha (\alpha + 1) 2^{1-\alpha}\}$, and by using Lemma 2, we get

$$\left| \bar{S}_{n+1} \right| \leq C \eta_0.$$  \hspace{1cm} (46)

where $C$ is a positive constant.

7 | NUMERICAL RESULTS

7.1 | Numerical simulation of the pandemic of COVID-19 in the world

This subsection presents a computational simulation of the pandemic trend model of COVID-19 in the world. To achieve this, we consider some of the literature’s parameter values, and estimated the other parameter values, as in Table 1. According to the WHO, the birth rate in 2020 for the world was 18.077 per 1000 people, the death rate was 7.612 per 1000 people, and the total population on February 4, 2020, was $N = 7610105452$. Thus, we have $\gamma = 0.018077 \times N = 391,347.066$ and $\delta = \frac{0.007612 \times N}{365} = 20.8547 \times 10^{-6}$. The initial values of infected people, death or recovery people, and hospitalized people as stated in the report of the WHO on February 4, 2020, are $I_0 = 24545$, $R_0 = 907$, and $H_0 = 12627$. The initial values of $A_0$, $E_0$, and $W_0$ assumed as $A_0 = 20000$, $E_0 = 80000$, and $W_0 = 50000$. Since $N = S_0 + E_0 + I_0 + A_0 + H_0 + R_0$, then $S_0 = 7609967373$.

Concerning the parameter values in Table 1, the basic reproduction number $R_0 > 1$. That means that the pandemic will spread, and the equilibrium point of the model Equation (11) is positive. $P_0 = (2.02 \times 10^7, 4.957 \times 10^6, 7.43 \times 10^5, 2.66 \times 10^6, 2.7 \times 10^6, 1.37 \times 10^6, 3.4 \times 10^5)$. All the approximate solutions are computed by using Wolfram Mathematica software with $\alpha = 0.99$. The graphical solutions of fractional system Equation (11) in the interval $[0, 250]$ have been described in Figures 1–3, where the unit of time is days. Figures 1–3 show that the results of the model converge to their equilibrium point for different fractional-order derivatives and stable at that points. These figures indicate that the obtained plots have the same behavior pattern for different values of $\alpha = 1, 0.9, 0.8, 0.7, 0.6$.

7.2 | Effect of parameters on the epidemic spread

The parameter values of the model play a significant role influencing the spread of the epidemic. So, the single most effective way to restrict the spread of a disease is to create quarantine to decrease the mobility of individuals. If we choose the

| Parameter | Value | Source | Parameter | Value | Source |
|-----------|-------|--------|-----------|-------|--------|
| $\xi_1$   | $2.6 \times 10^{-3}$ | Tuan et al. | $\xi_2$ | $1 \times 10^{-9}$ | Tuan et al. |
| $\xi_3$   | $0.0001$ | Estimated | $\beta_1$ | $0.02$ | Tuan et al. |
| $\sigma$  | $0.0023$ | Estimated | $\beta_2$ | $0.009$ | Tuan et al. |
| $\psi$    | $0.00023$ | Estimated | $\beta_3$ | $0.0074$ | Estimated |
| $\sigma$  | $0.000058$ | Tuan et al. | $\theta_1$ | $1 \times 10^{-6}$ | Tuan et al. |
| $\psi$    | $0.075$ | Tuan et al. | $\theta_2$ | $1 \times 10^{-6}$ | Tuan et al. |
| $\eta$    | $0.01$ | Tuan et al. |

FIGURE 1 Graphical approximate solutions of $S(t)$ and $E(t)$ for different values of $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ [Colour figure can be viewed at wileyonlinelibrary.com]
| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $\xi_1$   | 0.0026| $\beta_1$ | 0.014 |
| $\xi_2$   | 0.001 | $\beta_2$ | 0.004 |
| $\sigma$  | 0.04  | $\beta_3$ | 0.05  |
| $\psi$    | 0.023 | $\rho$    | 0.045 |
| $\delta$  | 0.09  | $\theta_1$| 0.001 |
| $\nu$     | 0.022 | $\theta_2$| 0.008 |
| $\eta$    | 0.065 | $\lambda$ | 0.033 |

TABLE 2  The numerical values of the parameters

FIGURE 2  Graphical approximate solutions of $I(t)$ and $A(t)$ for different values of $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 3  Graphical approximate solutions of $H(t)$, $R(t)$, and $W(t)$ for different values of $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ [Colour figure can be viewed at wileyonlinelibrary.com]

same parameter values and the same initial values of the classical model (see Fatima et al.\textsuperscript{1}) as in Table 2, then the basic reproduction number $R_0 < 1$ where $\gamma=0.00181$. Thus, we get the results shown in Figures 4–6. Comparing the results with Fatima et al.\textsuperscript{1} we note that the behavior of graphic solutions is the same. Figure 4–6 show that all variables in the pandemic model will be decreased and hit zero, indicating the system’s stability.
FIGURE 4  Approximate solutions of \( S \) and \( E \) with parameters of Table 2 for different values of \( \alpha = 1, 0.9, 0.8, 0.7, 0.6 \) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 5  Approximate solutions of \( I \) and \( A \) with parameters of Table 2 for different values of \( \alpha = 1, 0.9, 0.8, 0.7, 0.6 \) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 6  Approximate solutions of \( H, R, W \) with parameters of Table 2 for different values of \( \alpha = 1, 0.9, 0.8, 0.7, 0.6 \) [Colour figure can be viewed at wileyonlinelibrary.com]
8 | CONCLUSION

In this paper, modeling and studied the pandemic trend of COVID-19 has been presented with fractional-order derivative. Using the generation matrix method, the basic reproduction number is calculated that located whether the disease would persist or disappears from the population. The equilibrium points for this system are calculated, where the graphical solutions show that the results of the model converge to their equilibrium points. Based on the derivatives of the basic reproduction number, the sensitivity of the parameters was analyzed. The existence and uniqueness of the solution to the proposed model have been proven using the fixed-point theorem and by helping of the Arzela–Ascoli theorem. The fractional proposed model is solved using the predictor–corrector method in the sense of Caputo derivative. Using some essential lemmas, we proved that this method is conditionally stable. The results indicate that the disease will continue where that the basic reproduction number \( R_0 > 1 \). We selected the same parameter values and same initial conditions in the classical model\(^1\) for comparing the results of the fractional model with the classical model. We noted that both models have the same behavior. Moreover, parameters played a very significant role in limiting disease outbreaks.

ACKNOWLEDGEMENT

The authors would like to thank the anonymous referees for their valuable comments and suggestions. Praveen Agarwal thanks the SERB (project TAR/2018/000001), DST (projects DST/INT/DAAD/P-21/2019 and INT/RUS/RFBR/308), and NBHM (DAE) (project 02011/12/2020 NBHM (R.P)/RD II/7867) for their necessary support.

CONFLICT OF INTEREST

This work does not have any conflicts of interest

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REFERENCES

1. Fatima B, Zaman G, Alqudah MA, Abdeljawad T. Modeling the pandemic trend of 2019 coronavirus with optimal control analysis. Results in Phys. 2021;20:103660. https://doi.org/10.1016/j.rinp.2020.103660
2. Ansarizadeh F, Singh M, Richards D. Modelling of tumor cells regression in response to chemotherapeutic treatment. Appl Math Model. 2017;48:96-112. https://doi.org/10.1016/j.apm.2017.03.045
3. Pinkerton SD, Galletly CL. Reducing hiv transmission risk by increasing serostatus disclosure: a mathematical modeling analysis. AIDS Behav. 2007;11(5):698-705. https://doi.org/10.1007/s10461-006-9187-2
4. Mondaini RP, Pardalos PM. Mathematical Modelling of Biosystems, Vol. 102: Springer Science & Business Media; 2008.
5. Lepik U. Numerical solution of differential equations using haar wavelets. Math Comput Simul. 2005;68(2):127-143. https://doi.org/10.1016/j.matcom.2004.10.005
6. Djaoue S, Kolaye GG, Abboubakar H, Ari AAA, Damakoa I. Mathematical modeling, analysis and numerical simulation of the covid-19 transmission with mitigation of control strategies used in cameroon. Chaos, Solitons Fractals. 2020;139:110281. https://doi.org/10.1016/j.chaos.2020.110281
7. Yang C, Wang J. A mathematical model for the novel coronavirus epidemic in Wuhan, China. Math Biosci Eng MBE. 2020;17(3):2708. https://doi.org/10.3934/mbe.20200148
8. Zhang Z, Gul R, Zeb A. Global sensitivity analysis of covid-19 mathematical model. Alex Eng J. 2021;60(1):565-572. https://doi.org/10.1016/j.aej.2020.09.035
9. Atangana A, Araz SI. A novel covid-19 model with fractional differential operators with singular and non-singular kernels: analysis and numerical scheme based on newton polynomial. Alex Eng J. 2021;60(4):3781-3806. https://doi.org/10.1016/j.aej.2021.02.016
10. Peirlinck M, Linka K, Costabal FS, Kuhl E. Outbreak dynamics of covid-19 in china and the united states. Biomech Model Mechanobiol. 2020;19(6):2179-2193. https://doi.org/10.1007/s10237-020-01332-5
11. Tuan NH, Mohammadi H, Rezapour S. A mathematical model for covid-19 transmission by using the caputo fractional derivative. Chaos, Solitons Fractals. 2020;140:110107. https://doi.org/10.1016/j.chaos.2020.110107
12. Rajagopal K, Hasanzadeh N, Parastesh F, Hamarashe H, Jafari S, Hussain I. A fractional-order model for the novel coronavirus (covid-19) outbreak. Nonlinear Dyn. 2020;101(1):711-718. https://doi.org/10.1007/s11071-020-05757-6
13. Naik PA, Yavuz M, Qureshi S, Zu J, Townley S. Modeling and analysis of covid-19 epidemics with treatment in fractional derivatives using real data from pakistan. *Eur Phys J Plus*. 2020;135(10):1-42. https://doi.org/10.1140/epjp/s13360-020-00819-5

14. Ahmad S, Ullah A, Al-Mdallal QM, Khan H, Shah K, Khan A. Fractional order mathematical modeling of covid-19 transmission. *Chaos, Solitons Fractals*. 2020;139:110256. https://doi.org/10.1016/j.chaos.2020.110256

15. Garba SM, Lubuma JM-S, Tsanou B. Modeling the transmission dynamics of the covid-19 pandemic in south africa. *Math Biosci*. 2020;328:108441. https://doi.org/10.1016/j.mbs.2020.108441

16. Sher M, Shah K, Khan ZA, Khan H, Khan A. Computational and theoretical modeling of the transmission dynamics of novel covid-19 under mittag-leffler power law. *Alex Eng J*. 2020;59(5):3133-3147. https://doi.org/10.1016/j.aje.2020.07.014

17. Atangana A, Araz SI. Mathematical model of covid-19 spread in turkey and south africa: theory, methods, and applications. *Adv Differ Equ*. 2020;2020(1):1-89. https://doi.org/10.1186/s13662-020-03095-w

18. Atangana A, Araz SI. Modeling and forecasting the spread of covid-19 with stochastic and deterministic approaches: Africa and europe. *Adv Differ Equ*. 2021;2021(1):1-107. https://doi.org/10.1186/s13662-021-03213-2

19. Khan MA, Atangana A. Modeling the dynamics of novel coronavirus (2019-ncov) with fractional derivative. *Alex Eng J*. 2020;59(4):2379-2389. https://doi.org/10.1016/j.aje.2020.02.033

20. Khan MA, Atangana A, Alzahrani E, et al. The dynamics of covid-19 with quarantined and isolation. *Adv Differ Equ*. 2020;2020(1):1-22. https://doi.org/10.1186/s13662-020-02882-9

21. Atangana A. Modelling the spread of covid-19 with new fractal-fractional operators: can the lockdown save mankind before vaccination?. *Chaos, Solitons Fractals*. 2020;136:109860. https://doi.org/10.1016/j.chaos.2020.109860

22. Yadav RP, Verma R. A numerical simulation of fractional order mathematical modeling of covid-19 disease in case of wuhan china. *Chaos, Solitons Fractals*. 2020;140:110124. https://doi.org/10.1016/j.chaos.2020.110124

23. Baleanu D, Mohammadi H, Rezapour S. A fractional differential equation model for the covid-19 transmission by using the caputo–fabrizio derivative. *Adv Differ Equ*. 2020;2020(1):1-27. https://doi.org/10.1186/s13662-020-02762-2

24. Erturk VS, Kumar P. Solution of a covid-19 model via new generalized caputo-type fractional derivatives. *Chaos, Solitons Fractals*. 2020;139:110280. https://doi.org/10.1016/j.chaos.2020.110280

25. Abdulwasaa MA, Abdo MS, Shah K, Nofal TA, Panchal SK, Kawale SV, Abdel-Aty A-H. Fractal-fractional mathematical modeling and forecasting of new cases and deaths of covid-19 epidemic outbreaks in india. *Results in Physics*. 2021;21:103702. https://doi.org/10.1016/j.rinp.2020.103702

26. Rezapour S, Mohammadi H, Samei ME. Seir epidemic model for analyzing the pandemic trend of COVID-19. *Math Meth Appl Sci*. 2021;44(1):1-42. https://doi.org/10.1002/mma.8057

27. Shaikh AS, Jadhav VS, Timol MG, Nisar KS, Khan I. Analysis of the covid-19 pandemic spreading in india by an epidemiological model and fractional differential operator. *Adv Differ Equ*. 2020;2020(1):1-19. https://doi.org/10.1186/s13662-020-02834-3

28. Shaikh AS, Shaikh IN, Nisar KS. A mathematical model of covid-19 using fractional derivative: outbreak in india with dynamics of transmission and control. *Adv Differ Equ*. 2020;2020(1):1-19. https://doi.org/10.1186/s13662-020-02834-3

29. Zhang Y, Yu X, Sun H, Tick GR, Wei W, Jin B. Applicability of time fractional derivative models for simulating the dynamics and mitigation scenarios of covid-19. *Chaos, Solitons Fractals*. 2020;138:109959. https://doi.org/10.1016/j.chaos.2020.109959

30. Matouk AE. Complex dynamics in susceptible-infected models for covid-19 with multi-drug resistance. *Chaos, Solitons Fractals*. 2020;140:110257. https://doi.org/10.1016/j.chaos.2020.110257

31. Tang B, Wang X, Li Q, Bragazzi NL, Tang S, Xiao Y, Wu J. Estimation of the transmission risk of the 2019-ncov and its implication for public health interventions. *J Clin Med*. 2020;9(2):462. https://doi.org/10.3390/jcm9020462

32. Samko S, Kilbas AA, Marichev OI. Fractional Integrals and Derivatives: Theory and applications. Switzerland: Gordon and Breach Science Publishers; 1993.

33. Miller KS, Ross B. *An Introduction to the Fractional Calculus and Fractional Differential Equations*. Wiley; 1993.

34. Li C, Zeng F. The finite difference methods for fractional ordinary differential equations. *Numer Funct Anal Optim*. 2013;34(2):149-179. https://doi.org/10.1080/01630563.2012.706673

35. Diethelm K. *The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type*. Springer Science & Business Media; 2010.

36. Van den Driessche P, Watmough J. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Math Biosci*. 2002;180(1-2):29-48. https://doi.org/10.1016/S0025-5564(02)00086-6

37. Diethelm K, Ford NJ, Freed AD. A predictor-corrector approach for the numerical solution of fractional differential equations. *Nonlinear Dyn*. 2002;29(1-3):22. https://doi.org/10.1023/A:1016592219341

38. G.N. Xiong Y. This chinese doctor tried to save lives, but was silenced. now he has coronavirus n.d. https://edition.cnn.com/asia/live-news/coronavirus-outbreak-02-04-20/index.html