Optimal observables for new-physics search at LEP2

P. Osland and A.A. Pankov\footnote{Permanent address: Gomel Polytechnical Institute, Gomel, 246746 Belarus.}

Department of Physics\footnote{Electronic mail addresses: P.e.Osland@fi.uib.no; pankov@gpi.gomel.by}
University of Bergen
Allégt. 55, N-5007 Bergen, Norway

Abstract

New observables $\sigma_\pm$ for the process $e^+e^- \rightarrow \mu^+\mu^-$ allow one to get more direct information on additional $Z'$ boson effects than what is obtained from the canonical ones, $\sigma$ and $A_{FB}$. Their deviations from the Standard Model predictions have very specific energy dependences, which are precisely determined by SM parameters. At energies varying from TRISTAN to LEP2, one can uniquely predict the signs of $\Delta\sigma_\pm$ induced by a $Z'$ as well as the locations of their extrema and zeros. This unambiguous energy correlation could be quite useful in distinguishing effects due to $Z'$ exchange from those caused by other new physics sources. Furthermore, there are two energy points, $\sqrt{s_+} \simeq 78$ GeV and $\sqrt{s_-} \simeq 113$ GeV, where the SM quantities $\sigma_{\pm}^{SM}$ as well as the deviations $\Delta\sigma_\pm$ attain their minimum values or vanish. These points could be very favourable for a search for new physics beyond the SM and beyond $Z'$ effects.
1 Introduction

After several years of successful LEP1 operation with high statistics, there is excellent agreement between the data and the Standard Model (SM) predictions at energies around the $Z$ resonance. In addition, these experiments at LEP1 have tested the SM expectations away from the $Z$ pole. In particular, experimental results from studies of events collected in the channel $e^+e^- \rightarrow \mu^+\mu^- \gamma_{\text{isr}}$, with $\gamma_{\text{isr}}$ being an initial-state radiation photon, are at LEP1 used to probe the cross section and forward-backward asymmetry in the energy region between LEP1 and TRISTAN and down to PETRA energies.

The investigation of the process $e^+e^- \rightarrow \mu^+\mu^-$ at energies below the $Z$ peak is attractive because preliminary results from TRISTAN could indicate a downward deviation of the cross section (by two standard deviations) from the prediction of the SM at 58 GeV. Measurable deviations in the $e^+e^- \rightarrow \mu^+\mu^-$ cross section are in this energy range predicted by several models beyond the SM, for instance those which introduce an additional $Z'$ boson. Alternatively, it could be induced by anomalous triple gauge boson couplings. Thus, $Z'$ effects at $\sqrt{s} < M_Z$, as well as at LEP2, would be of a similar type as those arising from anomalous triple gauge couplings, although the responsible mechanism would be of a totally different origin.

It is very important to optimize the strategy when searching for new physics beyond the SM, since any signal would most likely be very small. It is also important to exploit the available data at “low”-energy machines, namely TRISTAN and LEP1, as well as those at LEP1.5. We concentrate here on the strategy for new-physics search at these machines, in particular at LEP2, where data will be obtained in the next couple of years.

In general, one is not able to predict the magnitude of the $Z'$ effects because they depend on a priori unknown parameters: the couplings to the $Z'$ and its mass, $M_{Z'}$. Therefore, definite predictions of $Z'$ effects would be quite desirable and important in such searches, also in order to discriminate them from other possible new physics effects.

In a previous paper we studied the interference effects induced by an extra neutral gauge boson $Z'$ in the production of lepton pairs

$$e^+e^- \rightarrow l^+l^-, \quad (l = \mu, \tau). \quad (1)$$

We have shown that assuming lepton universality, the lepton channel has the advantage over the $q\bar{q}$ channel that the signs of the interference terms are given very simply by the propagators of $Z$ and $Z'$. This is caused by the fact that the observables $\sigma$ and $A_{\text{FB}}$ depend only on squares of coupling constants. Due to this dependence, the canonical observables have for the process certain properties which are useful for the identification of effects of $Z'$ origin. Namely, at LEP2 energies, the effect of a $Z'$ (with arbitrary vector and axial vector couplings) is to reduce both the cross section and the forward-backward cross-section difference, as compared with the SM expectation. These unique properties of $\sigma$ and $A_{\text{FB}}$ are due to the fact that the $\gamma-Z'$, as well as the $Z-Z'$ interference terms are both negative. However, predictions for another energy region, $\sqrt{s} < M_Z$, are less definite. For example, at energies below the $M_Z$ the modifications of the cross section and the forward-backward asymmetry depend crucially on whether the coupling is dominantly vector or dominantly...
axial vector. In such a situation, it may be quite difficult to uniquely identify and extract
effects due to extra gauge bosons from those caused by other new physics effects.

In this paper we extend the analysis started in [8]. As we shall show below it is possible
to provide more definite information on $Z'$ effects. We shall here consider certain new
observables, for which the deviations from the SM predictions have very specific energy
deependences. These energy dependences are precisely determined because they involve only
the SM parameters such as the lepton couplings of the standard Z boson and the mass $M_Z$. In particular, one can uniquely predict the sign of any deviations of the observables
due to a $Z'$ at energies from TRISTAN to LEP2, as well as the locations of their extrema
and zeros. In such a case, one can easily distinguish the effects induced by $Z'$ from those
caused by other new physics effects.

2 New observables

A new neutral gauge boson would induce additional neutral current interactions, the corre-
sponding Lagrangian can be written as

$$-\mathcal{L}_{NC} = e J_\mu^i A_\mu + g_Z J_\mu^i Z_\mu + g_{Z'} J_\mu^i Z'_\mu,$$

where $e = \sqrt{4\pi\alpha}$, $g_Z = e/s_W c_W$ ($s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$) and $g_{Z'}$ are the gauge coupling
constants. The neutral currents are

$$J_\mu^i = \sum_f \bar{\psi}_f \gamma^\mu \left( L_f P_L + R_f P_R \right) \psi_f = \sum_f \bar{\psi}_f \gamma^\mu \left( V_{if} - A_{if}^\gamma \gamma_5 \right) \psi_f,$$

where $i \equiv \gamma, Z, Z'$, and $P_{L,R} = (1 \mp \gamma_5)/2$ are the left- and right-handed chirality projection
operators. The SM vector and axial-vector couplings of the vector boson $i$ to the fermions are

$$V_{if} = Q_f, \quad A_{if}^\gamma = 0, \quad V_{if}^Z = \frac{I_{3L}^f}{2} - Q_f s_W^2, \quad A_{if}^Z = \frac{I_{3L}^f}{2}.$$  

Here, $Q_f$ is the electric charge of $f$ ($Q_e = -1$), and $I_{3L}^f$ denotes the third component of
the weak isospin.

The lowest-order unpolarized differential cross section for the process (1), assuming e-$l$
universality, mediated by $\gamma$, $Z$, and the extra $Z'$ boson exchanges, is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \left[ (1 + \cos^2\theta) F_1 + 2 \cos\theta F_2 \right],$$

$$F_1 = F_1^{SM} + \Delta F_1, \quad F_2 = F_2^{SM} + \Delta F_2,$$  

(5)
with \((v \equiv v_t, a \equiv a_t\) and similarly for the primed quantities):

\[
\begin{align*}
F_{SM}^1 &= 1 + 2v^2 \text{Re}\chi_Z + (v^2 + a^2)|\chi_Z|^2, \\
F_{SM}^2 &= 2a^2 \text{Re}\chi_Z + 4(va)^2|\chi_Z|^2, \\
\Delta F_1 &= 2v'^2 \text{Re}\chi_{Z'} + 2(vv' + aa')^2 \text{Re}(\chi_Z\chi_{Z'}^*) + (v'^2 + a'^2)|\chi_{Z'}|^2, \\
\Delta F_2 &= 2a'^2 \text{Re}\chi_{Z'} + 2(va' + v'a)^2 \text{Re}(\chi_Z\chi_{Z'}^*) + 4(v'a')^2|\chi_{Z'}|^2.
\end{align*}
\]

(6)

The coupling constants are normalized to the unit of charge \(e\), and are expressed in terms of the couplings in the current basis (3) as

\[
\begin{align*}
v &= g_Z e V_f, \\
v' &= g_{Z'} e V_{f'}, \\
a &= g_Z e A_f, \\
a' &= g_{Z'} e A_{f'},
\end{align*}
\]

(7)

and the gauge boson propagators are \(\chi_V = s/(s-M_V^2+iM_V\Gamma_V)\), \(V = Z, Z'\). The total cross section and the forward-backward asymmetry can be written as

\[
\sigma = \sigma_{pt} F_1, \quad A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3F_2}{4F_1},
\]

(8)

with \(\sigma_{pt} = (4\pi\alpha^2)/(3s)\).

In the search for effects induced by the exchange of a \(Z'\), it will be advantageous to consider new observables, free of certain shortcomings of the canonical ones, \(\sigma\) and \(A_{FB}\). The observables we want to propose, are differences of cross sections obtained by integrating over suitable ranges of polar angle,

\[
\begin{align*}
\sigma_+ &\equiv \left( \int_{-z^*}^{1} - \int_{-1}^{-z^*} \right) \frac{d\sigma}{d\cos \theta} \ d\cos \theta, \\
\sigma_- &\equiv \left( \int_{1}^{z^*} - \int_{z^*}^{-1} \right) \frac{d\sigma}{d\cos \theta} \ d\cos \theta,
\end{align*}
\]

(9)

(10)

where \(z^* > 0\) is determined from the condition that the coefficients multiplying \(F_1\) and \(F_2\) be the same (cf. Eq. (5)),

\[
\int_{-z^*}^{z^*} (1 + \cos^2 \theta) \ d\cos \theta = \left( \int_{z^*}^{1} - \int_{-1}^{-z^*} \right) 2 \cos \theta \ d\cos \theta,
\]

(11)

or

\[
\frac{1}{2} z^{*3} + z^{*2} + z^* - 1 = 0,
\]

(12)

whose solution is \(z^* = 2^{2/3} - 1 = 0.5874\), corresponding to \(\theta^* = 54^\circ\). Thus,

\[
\sigma_\pm = \sigma_{pt}^* (F_1 \pm F_2),
\]

(13)
where

\[
\sigma_{pt}^* = \frac{3}{4} \left(1 - z^{*2}\right) \sigma_{pt} = \frac{\pi \alpha^2}{s} \left(1 - z^{*2}\right).
\] (14)

It should be noted, that the introduction and exploitation of the new independent observables \(\sigma_{\pm}\) is quite analogous to dealing with the canonical ones, \(\sigma\) and \(A_{FB}\). In fact, from Eqs. (8) and (13) one can simply express \(\sigma_{\pm}\) in terms of \(\sigma\) and \(A_{FB}\):

\[
\sigma_{\pm} = \frac{3}{4} \left(1 - z^{*2}\right) \sigma \left(1 \pm \frac{4}{3} A_{FB}\right) = 0.49 \sigma \left(1 \pm \frac{4}{3} A_{FB}\right).
\] (15)

Thus, they can be measured either directly according to Eqs. (9) and (10), or indirectly by means of \(\sigma\) and \(A_{FB}\).

For the sake of a simplified presentation, the discussion presented in the next section is based on several assumptions, whereas our numerical results are based on the full formulas. In particular, we assume:

(i) since the typical upper bound for the \(Z'\) boson mass, \(M_{Z'} > 600\) GeV \[9\], lies quite a bit higher than the energy available at LEP2, it suffices to take into account \(Z'\) interference effects only, the pure \(Z'\) exchange contributions being negligible;

(ii) since in the SM \(|v| \ll |a| < 1\), in the following we shall ignore \(v\) against \(a\). In addition, one can neglect the imaginary part of the \(Z'\) boson propagator.

According to these simplifying assumptions, we can write

\[
\frac{\sigma_{\pm}}{\sigma_{pt}^*} = F_1 \pm F_2 = \left(F_1^{SM} \pm F_2^{SM}\right) + \left(\Delta F_1 \pm \Delta F_2\right)
\]

\[
\approx \frac{1}{\sqrt{s} \ll M_{Z'}} \left|1 \pm a^2 \chi_{Z}\right|^2 + 2\left|v^2 \pm a'^2\right| \chi_{Z'}(1 \pm a^2 \text{Re} \chi_{Z}),
\] (16)

where the first term represents the SM contribution, and the second one the \(Z'\) effects.

### 3 Improved Born results

The previous formula for the differential cross section (5) as well as those for all other observables are still valid to a very good (improved Born) approximation after one-loop electro-weak radiative corrections, with the following replacements \[10\]:

\[
\alpha \Rightarrow \alpha(M_Z^2),
\]

\[
v \Rightarrow \frac{1}{\sqrt{\kappa}} \left(I^e_{3L} - 2 Q_e \sin^2 \theta_W^{eff}\right), \quad a \Rightarrow \frac{I^e_{3L}}{\sqrt{\kappa}}
\]

\[
\sin^2 \theta_W \Rightarrow \sin^2 \theta_W^{eff}, \quad \sin^2(2\theta_W^{eff}) \equiv \kappa = \frac{4\pi \alpha(M_Z^2)}{\sqrt{2} G_F M_Z^2 \rho},
\] (17)

\[1\]The proposed observables are related to helicity amplitudes as follows: \(\sigma_{\pm} \propto |A_{RR}|^2 + |A_{LL}|^2; \sigma_{\pm} \propto |A_{RL}|^2 + |A_{LR}|^2\).

\[2\]It means that the available experimental data for \(\sigma\) and \(A_{FB}\) of the process \(1\) at TRISTAN to LEP1.5 energies \[3, 4, 7\] can be directly converted to \(\sigma_{\pm}\).
with
\[ \rho \approx 1 + \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}, \quad (18) \]
where only the main contribution to \( \rho \), coming from the top mass, has been given. This parameterization uses the best known quantities \( G_F, M_Z, \) and \( \alpha(M_Z^2) \). A final step consists in introducing the energy dependence in the width term of the \( Z \) propagator,
\[ \chi_Z(s) \Rightarrow \frac{s}{s - M_Z^2 + i(s/M_Z^2)M_Z\Gamma_Z}. \quad (19) \]
All numerical results presented in this section are based on the improved Born approximation with \( m_t = 170 \text{ GeV} \) and \( m_H = 300 \text{ GeV} \).

Let us start our discussion with the observable \( \sigma^+ \), defined by Eqs. (13) and (16). It has an SM part, \(|1 + a^2\chi|^2\) which tends quadratically to its minimum value (given by the \( Z \) width) at
\[ \sqrt{s^+} = M_Z \sqrt{1 + a^2} \approx 78 \text{ GeV}, \quad (20) \]
as is displayed in Fig. 1a. While the \( Z' \) interference term also vanishes at the same point (see Fig. 1b), it does so only linearly. Therefore, one may expect an enhanced sensitivity of \( \sigma^+ \) to \( Z' \) effects around this point, \( \sqrt{s^+} \). (In Figs. 1 and 2 we consider, as illustrative cases, the effects of a \( Z' \) with \( M_{Z'} = 600 \text{ GeV} \) and various leptonic couplings.)

From Eqs. (13) and (16) one can directly read off the deviations of \( \sigma^+ \) from the SM prediction at “low” energies
\[ \Delta \sigma^+ \equiv \sigma^+ - \sigma^+_{\text{SM}} \approx 2\sigma^*_\mu (\nu'^2 + a'^2)\chi_{Z'}(1 + a^2 \text{Re} \chi_Z). \quad (21) \]
Before embarking on a more detailed analysis, two important remarks are in order. First, as one can see from Eq. (21), the dependence of \( \Delta \sigma^+ \) on the \( Z' \) parameters is characterized by the expression \((\nu'^2 + a'^2)\chi_{Z'}\), where the \( Z' \) couplings appear only as a sum of their squares, i.e., as a positive definite quantity. This means that in contrast to the canonical observables, \( \sigma \) and \( A_{FB} \), in \( \sigma^+ \) the \( \gamma - Z' \) and \( Z - Z' \) interferences contribute coherently. Thus, in \( \Delta \sigma^+ \), there is no cancellation between the \( \gamma - Z' \) and \( Z - Z' \) interference effects, instead they enhance each other.

Secondly, the energy dependence of \( \Delta \sigma^+ \) is given by the factor \((1 + a^2 \text{Re} \chi_Z)\), which is completely determined by Standard-Model parameters. Hence, one can precisely predict the energy dependence of the deviation \( \Delta \sigma^+ \). Its important feature is that it is independent of the \( Z' \) lepton couplings and the mass \( M_{Z'} \). In particular, \( \Delta \sigma^+ \) vanishes at \( \sqrt{s} = \sqrt{s^+} \) and \( \sqrt{s} \approx M_Z \), and it achieves extrema at
\[ |\sqrt{s^+} - \sqrt{s}| \approx \frac{\Gamma_Z}{2}. \quad (22) \]
\footnotetext{This special energy, as well as the one given by eq. (26), have also been noted by Frère et al. \[11\] from the study of helicity cross sections.}

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It should be stressed once again that in the approximation \( v = 0 \) the locations of these particular points (zeros and extrema) are not affected by a variation of the \( Z' \) parameters. The finite value of \( v \) leads to a small shift, by a factor \( 1 + \delta \), where \( \delta = \frac{va \cos \gamma}{(1 + a^2)} \). This amounts to a shift of at most (for \( |\cos \gamma| = 1 \)) 1.6 GeV. Also, the non-zero value of \( v \) is responsible for the splitting of the three curves in Fig. 1a, for different values of \( v'a' \).

Fig. 1b shows the energy dependence of the relative deviation

\[
\frac{\Delta \sigma_+}{\sigma_+^{\text{SM}}} = \frac{\sigma_+ - \sigma_+^{\text{SM}}}{\sigma_+^{\text{SM}}},
\]

for different couplings: \( v'^2 + a'^2 = 0.25, 0.5, \) and 1. One can see from Fig. 1b that the \( Z' \) interference pattern is quite stable under variations of the couplings, only its scale is changed.

Another very important property of the observable \( \sigma_+ \) is that the sign of the deviation \( \Delta \sigma_+ \) is uniquely determined, cf. Eq. (21). In fact, at \( \sqrt{s} < \sqrt{s_+} \) and \( \sqrt{s} > M_Z \) it is negative (\( \Delta \sigma_+ < 0 \)), while for \( \sqrt{s_+} < \sqrt{s} < M_Z \) the quantity \( \Delta \sigma_+ > 0 \) (see Fig. 1b). In other words, there is a correlation of the signs of \( \Delta \sigma_+ \) at different energies. This means that the determination of the signs of the deviation \( \Delta \sigma_+ \) at different energies should help in distinguishing \( Z' \) effects from those induced by other possible new physics origins. For example, if at \( \sqrt{s} < \sqrt{s_+} \) or at \( \sqrt{s} > M_Z \) one observes \( \Delta \sigma_+ > 0 \), then one can definitely conclude that it is not induced by a \( Z' \). However, if \( \Delta \sigma_+ \) is negative at \( \sqrt{s} < \sqrt{s_+} \) or/and at \( \sqrt{s} > M_Z \), and positive for \( \sqrt{s_+} < \sqrt{s} < M_Z \), then one has a stronger case that it could be due to a \( Z' \) (see Fig. 1b).

Finally, another interesting feature of the energy dependence of \( \Delta \sigma_+ \) is associated with the energy point \( \sqrt{s_+} \). As mentioned above, at this energy the SM background (\( \sigma_+^{\text{SM}} \)) tends to its minimum value, and also \( \Delta \sigma_+ \) vanishes. Thus, this energy is very favourable for a search for new physics beyond the SM and beyond \( Z' \) effects.

In Fig. 1c we show the statistical significance

\[
S_+ \equiv \frac{|\sigma_+ - \sigma_+^{\text{SM}}|}{\delta \sigma_+} = \frac{\Delta \sigma_+}{\sqrt{\sigma_+^{\text{SM}}}} \sqrt{L_{\text{int}}},
\]

defined as deviation from the SM prediction in units of the standard deviation, where \( \delta \sigma_+ \) is the statistical uncertainty, and \( L_{\text{int}} \) the integrated luminosity, \( L_{\text{int}} = \int dt L \). We here consider \( L_{\text{int}} = 300 \text{ pb}^{-1} \) (as a typical value at TRISTAN), and \( M_{Z'} = 600 \text{ GeV} \). As can be seen from the figure, with the exception of the two minima, this function increases with \( \sqrt{s} \). These features are quite independent of the \( Z' \) mass. We note that \( S_+ \) has a minimum around \( \sqrt{s_+} \). Hence, this energy is very favourable for a search for new physics other than \( Z' \) effects.

Let us now turn to the observable \( \sigma_- \) defined by Eqs. (13) and (16). It has several properties in common with \( \sigma_+ \). In particular, the deviation from the SM prediction,

\[
\Delta \sigma_- \equiv \sigma_- - \sigma_-^{\text{SM}} = 2 \sigma_\text{pt} (v'^2 - a'^2) \chi_{Z'} (1 - a^2 \text{Re} \chi_{Z}).
\]

(25)
has an energy dependence given by SM parameters. However, the sign and magnitude of such a deviation is determined by \( v'^2 - a'^2 \), or whether the \( Z' \) couplings are predominantly vector or axial vector. Thus, this deviation, at a given energy, can be either positive or negative.

Furthermore, the energy where \( \sigma_{-\text{SM}} \) has its minimum and \( \Delta \sigma_- = 0 \) is located above \( M_Z \), rather than below, namely at

\[
\sqrt{s_-} = \frac{M_Z}{\sqrt{1 - a^2}} \approx 113 \text{ GeV.} \tag{26}
\]

The corresponding set of energy correlations of a \( Z' \) signal in \( \sigma_- \) is given in Table 1. Finally, we note that due to the different dependences on the \( Z' \) couplings \( (v'^2 + a'^2 \text{ vs. } v'^2 - a'^2) \), the observables \( \sigma_+ \) and \( \sigma_- \) are quite complementary in the \( Z' \) search.

The energy dependences of the observable \( \sigma_- \), its relative deviation from the SM prediction,

\[
\frac{\Delta \sigma_-}{\sigma_{-\text{SM}}} = \frac{\sigma_- - \sigma_{-\text{SM}}}{\sigma_{-\text{SM}}}, \tag{27}
\]

and the statistical significance, defined as

\[
S_- \equiv \frac{|\sigma_- - \sigma_{-\text{SM}}|}{\delta \sigma_-}, \tag{28}
\]

are shown in Figs. 2b and 2c, respectively. In particular, in Fig. 2b we show the relative deviation \( \Delta \sigma_- / \sigma_{-\text{SM}} \) for two different possibilities of the parameters, \( (v'^2 - a'^2) > 0 \) and \( (v'^2 - a'^2) < 0 \), henceforth denoted as the \( V \) and \( A \) case, respectively. As can be seen from Fig. 2c the \( A \) case provides somewhat higher sensitivity to a \( Z' \) than the \( V \) case at \( \sqrt{s} > M_Z \). The reason is that in the \( V \) case there is at high energies some cancellation between the interference terms and the pure \( Z' \) contribution.

Finally, the energy region around \( \sqrt{s_-} \approx 113 \text{ GeV} \) where \( \sigma_{-\text{SM}} \) has its minimum value, and where \( \Delta \sigma_- \) also vanishes, would presumably be a convenient place to probe for new physics effects beyond those due to a \( Z' \) (see Fig. 2c).

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\[^{4}\text{One should note that the } \sigma_- \text{ dependence on } (v'^2 - a'^2) \text{ is exact, there is no dependence on } v'a'.\]
4 Model-independent bounds on Z' couplings

In this section we assess the sensitivities of the observables $\sigma_\pm$ to a $Z'$ and compare them with those obtained for the canonical observables $\sigma$ and $A_{FB}$ [8]. This analysis will be performed in a model-independent manner, and also includes initial state radiation (ISR) effects. In writing down the neutral current interaction of the $Z'$ in a model-independent way we follow [8, 12]. The $Z'$ mediated amplitude for fermion pair production in the Born approximation can be written as

$$M(Z') \propto \frac{g_{Z'}^2}{s - M_{Z'}^2} [\bar{u}_e \gamma_\mu (V_{Z'}^e - A_{Z'}^e \gamma_5) u_e] [\bar{u}_l \gamma_\mu (V_{Z'}^l - A_{Z'}^l \gamma_5) u_l]$$

$$= - \frac{4\pi}{M_Z^2} [\bar{u}_e \gamma_\mu (V_e - A_e \gamma_5) u_e] [\bar{u}_l \gamma_\mu (V_l - A_l \gamma_5) u_l],$$

(29)

with

$$V_l = V_{Z'}^l \frac{g_{Z'}^2}{4\pi} \frac{M_{Z'}^2}{M_{Z'}^2 - s}, \quad A_l = A_{Z'}^l \frac{g_{Z'}^2}{4\pi} \frac{M_{Z'}^2}{M_{Z'}^2 - s}.$$  (30)

It should be noted that the imaginary part of the $Z'$ propagator is irrelevant up to LEP2 energies, hence it is set to zero in the present analysis.

The sensitivity of observables, $\sigma_\pm$, has been assessed numerically by defining a $\chi^2$ function as follows:

$$\chi^2 = \left( \frac{\Delta \sigma_\pm}{\delta \sigma_\pm} \right)^2,$$

(31)

where $\sigma_\pm$ are given by Eqs. (23) and (27), the uncertainty $\delta \sigma_\pm$ combines both statistical and systematic errors (we take $\delta \text{syst} = 0.5\%$ [6]). As a criterion to derive allowed regions for the coupling constants if no deviations from the SM were observed, and in this way to assess the sensitivity of the process (1) to $V_l$ and $A_l$, we impose that $\chi^2 < \chi^2_{\text{crit}}$, where $\chi^2_{\text{crit}}$ is a number that specifies the desired level of significance.

The observed cross section is significantly distorted in shape and magnitude by the emission of real photons by the incoming electron and positron. The model predictions are corrected for ISR effects according to [13]. The hard photon radiation is calculated up to order $\alpha^2$ and the leading soft and virtual corrections are summed to all orders by the exponentiation technique. Each of the coefficients $F_1$ and $F_2$ is convoluted with the radiator functions $R_{T}^p(k)$ and $R_{FB}^p(k)$, respectively [13], where $k$ is the fraction of energy lost by the radiation. The final expression for the differential cross section is

$$\frac{d\sigma}{d\cos \theta} = \frac{3}{8} \left[ (1 + \cos^2 \theta) \sigma_s + 2 \cos \theta \sigma_a \right],$$

(32)

where $\theta$ is the angle between the $\mu^-$ and the $e^-$ beam direction in the $\mu^+\mu^-$ centre-of-mass system [14]. The symmetric and antisymmetric parts of the cross section are given by
convolutions with the “radiators”,

\[
\sigma_s = \int_0^\Delta dk R^s_T(k) \sigma_{pt}(s') F_1(s'), \quad \sigma_a = \int_0^\Delta dk R^a_{FB}(k) \sigma_{pt}(s') F_2(s'),
\]  

(33)

with \(s' = s(1 - k)\). Due to the radiative return to the \(Z\) resonance at \(\sqrt{s} > M_Z\) the energy spectrum of the radiated photons is peaked around \(E_\gamma/E_{\text{beam}} \approx 1 - M_Z^2/s\) \(^{13}\). In order to increase the \(Z'\) signal, events with hard photons should be eliminated from a \(Z'\) search by a cut on the photon energy, \(\Delta = E_\gamma/E_{\text{beam}} < 1 - M_Z^2/s\).

Since the form of the corrected cross section, Eq. (32), is the same as that of Eq. (5), it follows that the radiatively-corrected \(\sigma_\pm\) can also be defined by Eqs. (9) and (10), with the same value for \(z^*\). However, a convolution of the coefficients \(F_1\) and \(F_2\) with the radiator functions results in some shifts of the positions of the zeros \(\sqrt{s_\pm}\) and the extrema in the energy dependences of \(\Delta \sigma_\pm\). These modifications can be kept under control by Eq. (32). Our numerical analysis shows that these shifts are of the order of 100 MeV. It means that the ISR does not affect substantially the interference patterns shown in Figs. 1b and 2b.

A numerical analysis has been performed by means of the program ZEFIT, which has to be used along with ZFITTER \(^{15}\). In this way, all the SM corrections, as well as those of QED associated with the \(Z'\) contributions were taken into account. In Fig. 3 we compare the allowed bounds on the leptonic couplings in the \((A_l, V_l)\) plane obtained in the improved Born approximation with those where we take into account also ISR effects. The contours are obtained from two observables \(\sigma_\pm\) and correspond to 95\% CL \((\chi^2_{\text{crit}} = 6)\). According to Eq. (21) one can conclude that the \(\sigma_+\) yields ranges of observability in the \((A_l, V_l)\) plane bounded by a circle around the origin, whereas \(\sigma_-\), as can be seen from Eq. (25), yields detectability regions that are bounded by hyperbolas. Fig. 3 shows the role of ISR in affecting the sensitivity to \(Z'\) parameters. It results in some relaxation of the allowed bounds on the parameters with respect to the improved Born predictions. Also, a comparison of allowed bounds on \((A_l, V_l)\) depicted in Fig. 3 with those presented in Fig. 4 of Ref. \(^{8}\) from an analysis of \(\sigma\) and \(A_{FB}\), shows that the sensitivities of the new observables and the canonical ones are almost the same.

Summarizing, in this note we introduced the new observables \(\sigma_\pm\) and studied their role in getting more direct information on \(Z'\) effects compared with that obtained from the canonical ones, \(\sigma\) and \(A_{FB}\). The deviations from the SM predictions, \(\Delta \sigma_\pm\), have very specific energy dependences which are entirely determined by the SM parameters. In this case, one can uniquely predict the sign of \(\Delta \sigma_\pm\) induced by \(Z'\) exchange at energies from TRISTAN to LEP2, as well as the locations of their extrema and zeros. These features could be quite helpful in distinguishing the effects originated by a \(Z'\) from those caused by other new physics sources. In addition, we found that at the energy points \(\sqrt{s_\pm}\) (\(\approx 78\) GeV and 113 GeV) both the SM quantities \(\sigma_{\pm}^{\text{SM}}\) and their deviations \(\Delta \sigma_{\pm}\) induced by a \(Z'\) tend to their minimum values or vanish. These energies \(\sqrt{s_\pm}\) are very favourable for a search for new physics beyond the SM and beyond \(Z'\) effects.

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Figure captions

**Fig. 1** (a) The observable $\sigma_+$ for muon pair production in the improved Born approximation vs. c.m. energy in the SM and in the presence of a $Z'$ with mass $M_{Z'} = 600$ GeV and couplings $v'^2 + a'^2 = 1$. The labels 0, 1 and $-1$ correspond to the values of $v'a'$. (b) Relative deviation of $\sigma_+$, Eq. (23), at $M_{Z'} = 600$ GeV. The labels (1, 2, 3) attached to the curves correspond to $v'^2 + a'^2 = 1$, 0.5, 0.25, respectively. In all cases $v'a' = 0$. (c) Statistical significance $S_+$ of Eq. (24). Parameters are as in Fig. 1a, and the integrated luminosity is $L_{\text{int}} = 300$ pb$^{-1}$.

**Fig. 2** (a) The observable $\sigma_-$ for muon pair production in the improved Born approximation vs. c.m. energy in the SM and in the presence of a $Z'$ with mass $M_{Z'} = 600$ GeV. Labels $V$ and $A$ correspond to $v'^2 - a'^2 = \pm 1$, respectively. (b) Relative deviation of $\sigma_-$, Eq. (27). Parameters are as in Fig. 2a. (c) Statistical significance $S_-$ of Eq. (28). Parameters are as in Fig. 2a, and the integrated luminosity is $L_{\text{int}} = 300$ pb$^{-1}$. Labels $V$ and $A$ correspond to $v'^2 - a'^2 = \pm 1$, respectively.

**Fig. 3** Upper bounds on the model-independent couplings ($A_l$, $V_l$) at 95% CL, in the improved Born approximation, as well as those also corrected for ISR. The “circles” are derived from $\sigma_+$, whereas the hyperbolas are derived from $\sigma_-$. The energy corresponds to LEP2 with $\sqrt{s} = 190$ GeV and $L_{\text{int}} = 500$ pb$^{-1}$.
Figure 1: (a) The observable $\sigma_+$ for muon pair production in the improved Born approximation vs. c.m. energy in the SM and in the presence of a $Z'$ with mass $M_{Z'} = 600$ GeV and couplings $v'^2 + a'^2 = 1$. The labels 0, 1 and $-1$ correspond to the values of $v'a'$. 
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Figure 1: (c) Statistical significance $S_+ \text{ of Eq. (24)}$. Parameters are as in Fig. 1a, and the integrated luminosity is $L_{\text{int}} = 300 \text{ pb}^{-1}$. 
Figure 2: (a) The observable \( \sigma_\mu \) for muon pair production in the improved Born approximation vs. c.m. energy in the SM and in the presence of a \( Z' \) with mass \( M_{Z'} = 600 \text{ GeV} \). Labels \( V \) and \( A \) correspond to \( v'^2 - a'^2 = \pm 1 \), respectively.
Figure 2: (b) Relative deviation of $\sigma_-$, Eq. (27). Parameters are as in Fig. 2a.
Figure 2: (c) Statistical significance $S_\gamma$ of Eq. (28). Parameters are as in Fig. 2a, and the integrated luminosity is $L_{\text{int}} = 300 \text{ pb}^{-1}$. Labels $V$ and $A$ correspond to $v^2 - a^2 = \pm 1$, respectively.
Figure 3: Upper bounds on the model-independent couplings ($A_t, V_t$) at 95% CL, in the improved Born approximation, as well as those also corrected for ISR. The “circles” are derived from $\sigma_+$, whereas the hyperbolas are derived from $\sigma_-$. The energy corresponds to LEP2 with $E_{\text{cm}} = 190$ GeV and $\mathcal{L}_{\text{int}} = 500$ pb$^{-1}$. 