A method to measure the integral vertical intensity and angular distribution of atmospheric muons with a stationary plastic scintillator bar detector

Bryan Olmos Yáñez *, Alexis A. Aguilar-Arevalo
Instituto de Ciencias Nucleares. Universidad Nacional Autónoma de México
Circuito Exterior, Ciudad Universitaria, Apartado postal 70-543, 04510 CDMX, México

Abstract
A stationary thick plastic scintillator bar detector with PMTs at both of its ends was used to measure the integral vertical intensity and angular distribution of atmospheric muons over exposures of a few hours in a laboratory setting. A muon generator algorithm was developed and used together with a simple simulation including the effects of the detector resolution, nonlinear saturation of the PMTs, and laboratory building coverage. Assuming the standard \( \cos^n \theta \) omnienergetic angular distribution with \( n = 2 \) and different models for the energy spectrum of the source, we extract a measurement of the integral vertical intensity \( I_0 = (108.2 \pm 0.51 \text{(stat)} \pm 4.8 \text{(syst)}) \text{ m}^{-2}\text{s}^{-1}\text{sr}^{-1} \) at the location with geographic coordinates 19.33°N 99.19°W, altitude 2,268 m above sea level, and geomagnetic cut-off of 8.2 GV. Considering the exponent of the angular distribution as a free parameter we measure \( n = 2.12 \pm 0.19 \text{(stat)} \pm 0.11 \text{(syst)} \).

Keywords: Atmospheric muons, muon vertical intensity, muon angular distribution.

1. Introduction
Atmospheric muons are produced in the decay of secondary particles (mostly pions and kaons) created in the interaction of primary cosmic rays, such as protons and heavier nuclei, with the Earth’s atmosphere at high altitudes, typically of the order of 15 km. At sea level muons (\( \mu^+ \), \( \mu^- \)) are the most abundant charged particles in the extended air showers created by cosmic rays, and reach the ground with a mean energy of approximately 4 GeV. At various latitudes and altitudes it has been observed that the muon angular distribution is well described by \( I(\theta) = I_0 \cos^n \theta \), where \( I_0 \) is the integrated vertical flux intensity (m\(^{-2}\)s\(^{-1}\)sr\(^{-1}\)), and \( \theta \) is the zenith angle of the incident muon. Both, \( I_0 \) and the exponent \( n \) depend on many factors, such as altitude, latitude, geomagnetic cut-off, and, over shorter timescales, solar activity and atmospheric conditions. A summary of measurements can be found in [1]. For muons with energies above 1 GeV at typical sea level altitude locations, the integral vertical intensity is about 70 m\(^{-2}\)s\(^{-1}\)sr\(^{-1}\) [2, 3], although some measurements suggest a value that is lower by 10-15% [4, 5, 6]. Significantly higher values have been reported [7] within the South Atlantic Anomaly [8] where the Earth’s magnetic field is strongly reduced. In the flat-atmosphere approximation, the exponent \( n \), can be shown to be one unit lower than that of the assumed primary cosmic ray spectrum \( (\sim E^{-(n+1)}) \) [9], and is expected to lie around the value of 2 for energies > 1 GeV.

Reliable data on the energy and angular distribution of atmospheric muons at a variety of geographic locations exist for \( E > 200 \text{ MeV} \) [10, 11]. These data, together with a large body of measurements of other components of the cosmic radiation reaching Earth, have been used to validate simulations of the production of atmospheric showers which can be used to predict the muon flux at virtually any location on the Earth's surface [12, 13, 14, 15, 16].

Cosmogenic muons have been either used or proposed for many applications, for example: to search for hidden chambers in the Egyptian pyramids [17], in the non-destructive assessment of radioactive materials stored in sealed containers [18], to determine the location of high density material in reactor meltdowns like Chernobyl and Fukushima [19, 20], to image the inte-
rior of volcanoes and study the evolution of their internal structure \[21\].Muon tomography (muography) offers great potential as a new technology for the evaluation and screening of materials and structures beyond the conventional x-ray, and gamma techniques \[24\].

In this work, we implement a simple method to extract the vertical muon intensity and angular distribution (exponent \(n\)) for positive plus negative muons \((\mu^+ + \mu^-)\) at a given location on the Earth’s surface using a stationary, thick scintillator bar detector coupled to photomultiplier tubes (PMTs) at both of its ends. The thickness of the scintillator provides access to sampling the angular distribution of the incoming muon flux, which constraints its absolute normalization, and generic assumptions about the detector response allow for a good description of the spectrum shape without the need of a detailed simulation of the scintillation processes, requiring relatively little computing power.

The method relies on a simple muon generator algorithm suitable for simulating the cosmogenic muon flux raining over a detector of finite size situated on the Earth surface, and a basic Geant4 simulation of the energy deposited by muons on the scintillator material, as well as the coverage provided by the building where the measurement is performed.

The paper is organized as follows. In Section 2, we briefly describe the phenomenological model of the muon flux reaching the ground to be used as a benchmark in our study. In Section 3, we describe an algorithm to generate atmospheric muons reaching a plastic scintillator bar on the surface of the Earth. In Section 4, details of the simulation of the detector are given, and simple validation tests are performed. In Section 5, we go over the experimental setup used to acquire the energy spectra of atmospheric muons reaching the ground with a plastic scintillator bar. In Section 6, we fit the model of the simulation to the experimental spectrum including energy resolution and non-linearity effects. Finally, in Section 7, we extract the muon vertical intensity and angular distribution, and explore variations due to systematic uncertainties. The conclusions are discussed in Section 8.

2. Muon flux model

We will use the phenomenological model of Smith-Duller \[23\] to describe the differential intensity of atmospheric muons as a benchmark. This simple model captures the main characteristics of experimentally measured energy and angular distributions of cosmogenic muons on the ground, and has previously been used for similar purposes \[24\]. The model calculates the number of muons \((N)\) per unit time \((t)\), area \((A)\), energy \((E_p)\), and solid angle \((\Omega)\), reaching the ground at atmospheric depth \(y_0\) and air density \(\rho_0\), originating from the decay of pions (rest mass and lifetime \(m_p\) and \(\tau_p\), respectively) produced at high altitudes in the Earth’s atmosphere as follows

\[
\frac{dN}{d\Omega d\Omega dt dE_p}(E_p, \theta) = \frac{A}{E_p} \frac{E_p^2 P_\mu \Lambda_\pi b j_\pi}{E_\pi \cos \theta + b j_\pi},
\]

(1)

where \(A\) is a normalization parameter; \(k = 8/3\) is the spectral index inherited from pion production; \(E_\pi\) is the parent pion energy prior to its decay, \(\Lambda_\pi = 120 \text{ g/cm}^2\) is the pion absorption mean free path at high energies, \(j_\pi = y_0 m_p c / \tau_p \rho_0\), \(b = 0.771\) is a numerical factor introduced to correct the isothermal atmosphere approximation at high altitudes, and \(P_\mu\) is the probability that a muon coming down at zenith angle \(\theta\) reaches the Earth’s surface without decaying, approximated by

\[
P_\mu = \left[0.100 \cos \theta \left(1 - \frac{a(y_0 \sec \theta - 100)}{r E_\pi}\right)\right]^{x_\pi},
\]

(2)

where \(r = 0.76\) is the fraction of the parent pion energy taken by the muon (assumed constant), \(B_\mu = b_\mu y_0 m_p c / \tau_p \rho_0\), with \(m_p\) and \(\tau_p\) the muon mass and lifetime at rest, respectively, \(b_\mu = 0.8\) a constant introduced to correct the isothermal atmosphere model for muons, and \(a = \cos \theta (dE_p / dy) = 2.5 \text{ MeV/g cm}^2\) is the minimum ionizing particle energy loss of the muon along its path through the atmosphere, also assumed constant. In Eq. (2), Smith and Duller have assumed that the dependence of \(P_\mu\) on \(\gamma / \cos \theta\) can be substituted for an appropriate average \(\langle \gamma / \cos \theta\rangle_{\text{avg}} = 100 \text{ g/cm}^2\). Within the model assumptions, the parent pion energy is given by

\[
E_\pi = \frac{1}{r} \left[E_\mu + a y_0 (\sec \theta - 0.100)\right].
\]

(3)

In writing equations (1), (2), and (3), a number of assumptions and approximations have been made \[23\]: pions are produced with a constant average multiplicity, taking a constant fraction of the primary nucleon energy, and are exponentially attenuated with the primary nucleon absorption mean free path \(\lambda_p\), which is taken to be approximately equal to \(\lambda_\pi\); the forward direction of the primary nucleon is maintained by the produced particles; the curvature of the atmosphere is neglected and an isothermal atmosphere approximation is used including ad-hoc correction factors on the ratio \(y_0 / \rho_0\) for each particle species; only muons produced by pions are considered although it has been shown that kaon decay
contributes in the order of 20% to the total atmospheric muon flux \cite{25}. The model has built into it an omnienergetic angular distribution that to an excellent approximation is \( \propto \cos^n \theta \) with \( n = 2 \).

As noted by the authors of \cite{24}, the Smith-Duller model captures correctly the different features of the muon spectrum at various zenith angles, and the approximations and assumptions summarized above have little effect on its accuracy to describe the cosmogenic muon differential intensities at high atmospheric depths, over a wide range of energies.

3. Atmospheric muon generator

Our atmospheric muon generator is based on the geometrical construct shown in Figure \ref{fig:fig1} and in default mode uses the Smith-Duller model. A hemisphere of radius \( R = 4.5 \text{ m} \) is placed on the XY plane with the detector resting at the origin of the coordinate system along the Y direction. We begin by drawing the muon zenith and azimuthal angles \( \theta \) and \( \varphi \) from the omnienergetic angular distribution

\[
\frac{dP(\theta, \varphi)}{d\Omega} = \mathcal{A} \cos^2 \theta \, d\Omega,
\]

with \( \theta \in [0, \pi/2] \) and \( \varphi \in [0, 2\pi] \). The direction of the generated muon will be given by the vector with Cartesian coordinates in 3D space

\[
\mathbf{u} = -\hat{r} = -(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta).
\]

At the point \( R \hat{r} \), we then construct a square \( \ell \times \ell \) plane \((\ell = 1.5 \text{ m}), \) tangent to the sphere. A random point \( p \) on this plane is selected by drawing two numbers \( a \) and \( b \) from a uniform distribution in the interval \([-\ell/2, \ell/2]\) such that

\[
p = a \hat{\theta} + b \hat{\varphi},
\]

where

\[
\hat{\theta} = \hat{x} \cos \theta \cos \varphi + \hat{y} \cos \theta \sin \varphi - \hat{z} \sin \theta,
\]

\[
\hat{\varphi} = \hat{x} \sin \varphi + \hat{y} \cos \varphi.
\]

The point \( p \) on the tangent plane will be the starting position of the muon, discarding points lying below the \( z = 0 \) plane. The muon energy is drawn from the energy distribution that corresponds to the angle \( \theta \) according to Smith-Duller. In this algorithm muons generated at the same \((\theta, \varphi)\) form a shower of parallel rays that rain over the detector’s extent, hence the tangent plane dimensions are chosen to cover the detector completely from all possible points on the sphere. In order to explore systematic variations due to the muon flux model, we implemented the capacity to sample from other energy and angular distributions.

4. Simulation

The muon generator algorithm and the detector geometry were implemented in a GEANT4 simulation using the standard electromagnetic physics list, and the QGSP\_BIC\_HP physics list for hadronic interactions. All electromagnetic and particle decay processes, including muon decay, were turned on in the simulation. The detector geometry included the plastic scintillator volume with density of 1.032 g/cm\(^3\), a 0.08 cm thick layer of PVC around all faces except the small 10 cm \( \times \) 10 cm faces where the PMTs are. At each end of the bar a coarse model of the PMTs and mu-metal shields was added for completeness.

The universe containing the hemisphere where muons are generated was filled with air. The Birks energy correction was turned on with a Birks constant of \( K_\text{B} = 1.26 \times 10^{-2} \text{ gMeV}^{-1} \text{ cm}^2 \), corresponding to PVT material. Only the plastic scintillator volume was defined as a sensitive volume where the deposited energy by traversing particles was collected.

The effects of PMT response, such as non-linear effects, DAQ noise and saturation, were implemented by means of a convolution of the simulated deposited energy spectrum with a generic Gaussian response function and a non-linear conversion from energy (MeV) to digitizer units (ADC). The parameters of the response function and non-linearity were determined from a fit to experimental spectra, and were later added to the simulation in order to include them in an event by event basis. This was accomplished by adding to the deposited energy of a given event a random Gaussian fluctuation dependent on the value of the deposited energy, and applying the conversion from MeV to ADC according to the non linear model.
Figure 2: Checks of the GEANT4 simulation. The upper plots show the (a) energy, (b) cosine of the zenith angle, and (c) azimuthal angle distributions of the generated muons. The lower plots show the (d) energy, (e) cosine of the zenith angle, and (f) azimuthal angle of the simulated muons entering through any of the faces of the detector. The continuous lines show the expected analytical behavior in each case.

The simulation was checked by comparing the energy and angular distributions of the muons entering through the detector faces with theoretical expectations based on the model of the source and the detector geometry. The top plots in Figure 2 show the distribution of input variables from the muon generator algorithm: (a) energy, (b) zenith angle, and (c) azimuthal angle, compared with the shapes from the energy spectrum parameterization, and the $dP \propto \cos^2 \theta \, d\Omega$ angular distribution. By construction the distribution of the zenith angle is proportional to $\sin \theta \cos^2 \theta$, and the distribution of $\varphi$ is uniform. Plots (d), (e), and (f) in the figure show the corresponding distributions for muons that have deposited some energy in the bar. As expected, the energy distribution of muons is mostly unaffected by the requirement that they penetrate the bar, while the angular distributions reflect the fact that the flux of muons entering each face depends on its orientation (horizontal vs vertical). For the assumed input angular distribution the zenith and azimuthal angle distributions can be shown to have the exact form

$$\Theta(\theta) \propto A \sin \theta \cos^2 \theta + B \sin^2 \theta \cos^2 \theta$$  \hspace{1cm} (7)

$$\Phi(\varphi) \propto C + |D| \cos |\varphi| + E |\sin |\varphi||$$  \hspace{1cm} (8)

where $(A, B, C, D, E) = (10, 22/\pi, 20/\pi, 5, 1/2)$. This behavior is verified in plots (e) and (f) in Figure 2. For the $\theta$ distribution in Eq. (7) the first term corresponds to the horizontal face, while the second term corresponds to the contribution of all 4 vertical faces. For the $\varphi$ distribution in Eq. (8) the constant term corresponds to the horizontal face, the $|\cos |\varphi||$ term corresponds to the long vertical faces perpendicular to the X-axis, and the $|\sin |\varphi||$ term corresponds to the small vertical faces perpendicular to the Y-axis. Note that for our bar the area of the small square faces is one tenth of the long rectangular faces.

An approximate geometry of the building was implemented to generate spectra that would be compared to experimental data. The room where the experiment was run has 4.4 m long walls, 4 m in height. Therefore, the building was modeled as a box with these dimensions, having vertical concrete walls 20 cm thick and a ceiling with 28 cm of concrete. An additional 40 cm layer of brick was added on top of the ceiling to simulate the additional material from the two stories above the actual laboratory. This model of the building fitted within the hemispherical dome used in the muon generator algo-
The distribution of the energy deposited by muons in the simulation with and without building structure are compared in Figure 3. The building has two major effects: i) reduce the number of muons in the Landau-shaped peak at 20 MeV, corresponding to the energy deposited by vertical minimum ionizing particles (MIPs) traversing \( \sim 10 \) cm of material and depositing \( \sim 2 \) MeV/cm, and ii) enhance the lower energy part of the spectrum with particles from electromagnetic showers and neutrons.

The simulated muon rate can be calculated by integrating the omnienergetic angular distribution of muons reaching the surface \( dN/(dA, d\Omega, dt) = \frac{dN}{dA} \cos^2 \theta \) over the solid angle of the upper hemisphere above the detector, and over the area of the tangent plane from where the muons are originated. The integral vertical muon intensity in the simulation was arbitrarily set to the sea level reference value for muons with energies \( > \) 1 GeV (see Section 1) of \( I_{0}^{\text{sim}} = 70 \) m\(^{-2}\)s\(^{-1}\)sr\(^{-1}\). Integration gives the rate

\[
\frac{dN}{dt} = \frac{2\pi}{3} I_{0}^{\text{sim}} \ell^2 = 329.87 \text{ s}^{-1} = 1 \times 10^7 \text{ h}^{-1}.
\]  

(9)

A total of 50,000,000 muons were simulated, corresponding to a detector exposure time of \( T_{\text{sim}} = 42.10 \) h. For angular distributions with \( n \neq 2 \) the calculation in Eq. (9) is done changing the 3 \( \rightarrow \) \( n + 1 \) in the denominator. According to the simulation and the fitted parameters of the detector response, only muons with \( E \geq 110 \) MeV produce pulses with integrated charge \( > 100 \) ADC \( (E_d > 2.6 \) MeV\).

5. Experiment

A schematic of the experiment is shown in Figure 4. It consists of a PVT-based plastic scintillator bar (Rexon RP-408) with dimensions 100 cm \( \times \) 10 cm \( \times \) 10 cm with a 3” photomultiplier (PMT) tube (Rexon-R1200P) encased in a mu-metal shield, and attached to each end. The bar is wrapped in a reflective aluminum paper cover, an intermediate layer of threaded tape, and an outer layer of black PVC tape. The analog sum of the two PMT signals is processed in a self-trigger mode. One copy of the summed pulse is sent to a discriminator module whose output triggers the opening a 160 ns wide pulse in a gate generator, which is then input as the “Gate” signal of a CAEN V965 QDC charge integrator module. A second copy of the analog sum of the PMT signals, produced simultaneously with the first one, is sent through a delay module adjusted to fit the signal in the integration gate, and then sent into one of the QDC input channels. A CAEN V1718 USB bridge is used to interface with the DAQ computer where the spectrum of the integrated charge of the summed pulse is accumulated. With the discriminator level set at 50 mV, the trigger rate averaged \( \sim 410 \) Hz. At this low rate the probability to have two events in the same trigger window is \( < 10^{-6} \), and the efficiency of the DAQ was measured to be \( \epsilon = 0.998 \pm 0.003 \% \).

The experiment was run in the interior of the Detectors Laboratory at ICN-UNAM in Mexico City (19.33° latitude, 99.19° longitude, 2,268 m sea level altitude, geomagnetic cutoff 8.2 GV [26,27]), under an effective coverage of \( \sim 28 \) cm of concrete and 40 cm of brick from the ceiling, and \( \sim 20 \) cm of concrete from
the walls. Spectra were acquired over 5 hr with the detector oriented ~10° from the N-S direction (clock wise looking from above).

6. Fit to data

A convolution function combining the predicted deposited energy spectrum from the Geant4 simulation, and the effects of random fluctuations and non linear-ity in the detector energy response was constructed to fit it to the experimental spectrum. The function has nine free parameters: The muon spectrum normalization ($N_\mu$), the normalizations of the low energy exponentially decaying backgrounds ($N_b_1$ and $N_b_2$), the exponential decay constants of the low energy backgrounds ($\epsilon_1$ and $\epsilon_2$), the fractional energy resolution at 20 MeV of deposited energy ($f$), and the three parameters of the non linear model for the detector response ($a_0$, $a_1$ and $a_2$). The fit function is given in Eq. (10)

$$F(E_d) = \frac{1}{(dE_d/dE_v)} \int_0^{E_{\text{max}}} \left( N_\mu F_\mu(E_d) + N_{b_1} \left(1/\epsilon_1\right) e^{-E_d/\epsilon_1} + N_{b_2} \left(1/\epsilon_2\right) e^{-E_d/\epsilon_2} \right) \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(E_d-E_0)^2}{2\sigma^2}} dE_d \quad (10)$$

and

$$\frac{dE_d}{dE_v} = \frac{a_1}{(1+a_2E_v)^2}, \quad E_v = \frac{E_d - a_0}{a_1 + a_2E_d - a_2E_0} \quad (12)$$

$F_\mu(E_d) = \left(1/N_\mu^\text{sim}\right) dN_\mu/dE_d$, is the unit-normalized deposited energy spectrum obtained by dividing the Geant4 simulated spectrum in Figure 3 (with building) by the bin width (0.5 MeV/bin) and the total number of entries in the simulated histogram; its integral from 0 to $E_{\text{max}} = 100$ MeV is by definition set to one. The shape of the low energy part of the spectrum, due to neutrons and electromagnetic showers, is not included in the simulation and is modeled as two simple exponentials $(1/\epsilon_{1,2})e^{-E_d/\epsilon_{1,2}}$ with decaying constants $\epsilon_{1,2}$, in MeV. Both spectral shapes (muons and low-$E_d$ backgrounds) are treated on equal footing with respect to the modelled detector response. Eq. (11) states that the width of the Gaussian scales as the square root of the energy $E_d$.

The function $F(E_d) = dN/dE_d$, gives the number of events per visible energy interval (events per ADC), and was fitted to the experimental spectrum in the interval from 100 to 3500 ADC, as shown in Figure 5. In the fit the constant $a_0$ from the non-linearity model was kept fixed to a value consistent with the DAQ pedestal.

The given assumed dependence of the Gaussian width in Eq. (11), for pulses with $E_d > 100$ ADC the effect of truncating the Gaussian response function at $E_d = 0$ in the convolution integral is negligible. Table 1 shows the values of the fitted parameters and their uncertainties.

The position and shape of the peak in the spectrum are determined by the last four parameters in Table 1. The overall energy response is consistent with a ~5%
was representing the physical effects and made the flat part of the muon spectrum more clearly visible.

As a check that the convolution function, Eq. (10), was representing the physical effects of fluctuations and non-linear effects properly, we completed the Geant4 simulation by adding these effects event-by-event. Given the deposited energy $E_d$ in an event, a Gaussian fluctuation with $\sigma$ given by Eq. (11) was added to it, $E'_d = E_d + \delta E_d$, and the energy in digitizer units was assigned using Eq. (13). Figure 6 shows the comparison of the simulated spectrum incorporating the resolution and non-linearity in an event-by-event basis, demonstrating the equivalence with the convolution approach.

7. Integral vertical muon intensity and angular distribution

The muon normalization parameter ($N_\mu$) in Table 1 is proportional to the integral vertical muon intensity at the experiment’s location ($I_0^{\text{lab}}$), and to the running time ($T$), therefore

$$I_0^{\text{lab}} = I_0^{\text{sim}} \left( \frac{N_\mu}{N_\mu^{\text{sim}}} \right) \left( \frac{T_{\text{sim}}}{T} \right) \times \left( \frac{1}{\epsilon} \right),$$

where $I_0^{\text{sim}}$ is the integral vertical muon intensity used in the muon generator in Section 5. $T_{\text{sim}}$ is the simulation time, and $\epsilon$ is an effective detection efficiency, which we took as the DAQ efficiency of Section 5. Due to being a small correction, in what follows we approximated $\epsilon \approx 1$, but considered its effect in the systematic uncertainty for $I_0$. For 5 h long exposures, this yields a measurement of the integral vertical muon intensity of $I_0 = (107.2 \pm 0.51) \text{ m}^{-2} \text{s}^{-1} \text{sr}^{-1}$ (statistical error only).

| Parameter | Smith & Duller | EXPACS ($\cos^2 \theta$) | EXPACS ($f(\theta)$) | Reyna | Units |
|-----------|----------------|--------------------------|-----------------------|-------|-------|
| $f$       | $4.51 \pm 0.18$ | $4.39 \pm 0.18$          | $5.20 \pm 0.15$       | $4.55 \pm 0.17$ | $(\times 10^{-5})$ |
| $N_\mu$   | $5.36 \pm 0.03$ | $5.29 \pm 0.03$          | $5.14 \pm 0.03$       | $5.34 \pm 0.03$ | $(\times 10^5)$   |
| $N_{b1}$  | $0.88 \pm 0.30$ | $0.91 \pm 0.26$          | $0.97 \pm 0.06$       | $1.00 \pm 0.10$ | $(\times 10^5)$   |
| $\sigma$  | $4.10 \pm 3.15$ | $4.22 \pm 2.75$          | $5.27 \pm 0.59$       | $3.06 \pm 1.02$ | $(\times 10^4)$   |
| $a_0$     | $1.24 \pm 0.004$| $1.24 \pm 0.004$         | $1.26 \pm 0.006$      | $1.24 \pm 0.005$| $(\times 10^5)$   |
| $a_1$     | $1.07 \pm 0.02$ | $1.07 \pm 0.02$          | $1.12 \pm 0.03$       | $1.03 \pm 0.02$ | $(\times 10^{-2})$|
using our benchmark Smith-Duller model of the flux.

As an attempt to assess the systematic uncertainty in $I_0$ arising from the modeling of the atmospheric muon flux we considered three alternative models. The first one used the energy distribution calculated with the EXPACS [13, 14] tool at the geographic coordinates of Mexico City, but forcing a $\cos^2 \theta$ angular distribution. A second one used both, the energy and angular distributions as predicted by EXPACS, forcing the angular distribution to have the same normalization as the $\cos^2 \theta$ case. The third model considered was that of Reyna [28], which also gives an angular distribution proportional to $\cos^2 \theta$.

Table 1 shows the result of the fits of the convolution function Eq.(10) with each of the models. The extracted parameters are in general very consistent, and are in reasonably good agreement with expectations derived from older measurements at varying altitudes, for locations away from strong geomagnetic anomalies [29]. Table 2 summarizes the systematic uncertainties that were considered for the measurement of $I_0$. The effect of the flux model on $I_0$ with respect to the error-weighted average is 3.5%. In addition, conservative variations on the effective building thickness in the simulation ($\pm50\%$) were studied and seen to produce a 2.6% variation on $I_0$. Including the DAQ inefficiency uncertainty of 0.3%, we estimate a total systematic error of 4.4%. Considering these systematic variations we report as our measurement of the integral vertical muon intensity the error-weighted average $I_0 = (108.2 \pm 0.51 \text{stat} \pm 4.7 \text{syst}) \text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}$.

Although an angular distribution with $n = 2$ provides a very good description of the observed spectrum, the data showed a preference for a slightly larger value. The preferred exponent in the muon angular distribution was measured by constructing a set of simulations with varying values of $n$ and performing, for each one, a fit for all 8 free parameters in Eq.(10). The simulations were constructed by applying a reweighing factor $\cos^n \theta/\cos^2 \theta$ to each event initially generated with the Smith-Duller model. Adding this weight to the events modifies the relative height of the Landau-shape peak and the flat portion to its left (the plateau) in the deposited energy distribution. The effect of varying $n$ is shown in Figure 7 in the observed energy distribution (in ADC units). A larger $n$ produces a spectrum with more vertical muons, hence adding to the population under the peak, relative to those entering at wider zenith angles, which contribute to the plateau (corner clipping muons). The inset in the figure shows the ratio of the peak height to the plateau height at 600 ADC (ratio peak-to-plateau, rpp).

**Table 2: Summary of systematic errors.** The percent error is relative to our average value of $I_0 = 108.2 \text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}$.

| Source                  | Error (m$^{-2}$s$^{-1}$sr$^{-1}$) | (%) |
|-------------------------|----------------------------------|-----|
| Flux model              | 3.8                              | 3.5 |
| Building coverage (±50%) | 2.8                              | 2.6 |
| DAQ efficiency          | 0.3                              | 0.3 |
| Total                   | 4.7                              | 4.4 |

**Figure 7:** Simulated muon energy spectra for different values of the exponent of the angular distribution. The spectra have been arbitrarily normalized to match in the region of the plateau ($< 1500 \text{ADC}$). The insert shows that the peak-to-plateau ratio ($r_{pp}$) increases linearly with $n$.

**Figure 8:** The $\chi^2$ minimum obtained from fits of the convolution function, Eq.(10), the experimental data, using different values of $n$ in the simulated deposited energy spectrum $F_\mu(E_d)$. The minimum and 1-$\sigma$ interval ($\Delta \chi^2 = 1$) represent the marginalization over the 8 free parameters in the fit.
as a function of \( n \), demonstrating that the sensitivity of the spectral shape to the angular distribution exponent.

In Figure 8 we plot the \( \chi^2 \) minimum from each fit of the convolution function as a function of \( n \). The minimum \( \chi^2 \) over all the fits, as well as the 1\( \sigma \) interval (\( \Delta \chi^2 = 1 \)) are shown. The measured value of the exponent of the angular distribution exponent obtained in this way is \( n = 2.12 \pm 0.19 \) (statistical error only). The same exercise was repeated using the EXPACS and Reyna flux models, and varying the building thickness in \( \pm 50\% \), resulting on an RMS variation of 5\% on the value of \( n \), comparable to the statistical error. Combining both effects we report a measured value of \( n = (2.12 \pm 0.19(\text{stat}) \pm 0.11(\text{syst})) \).

At the preferred value of \( n \), the simulation time \( T_{\text{sim}} \) must be recalculated, as noted near the end of Section 4. Taking this into account, the measured integral vertical intensity is changed to \( I_0 = (114.9 \pm 0.63(\text{stat}) \pm 12.9(\text{syst})) \text{ m}^{-2}\text{s}^{-1}\text{sr}^{-1} \), where the large systematic uncertainty (11\%\%) was estimated by varying \( n \) within the 1\( \sigma \) interval derived from Figure 8 and calculating \( I_0 \) with the corrected \( T_{\text{sim}} \) at each value. Within errors, this is consistent with the measurement reported assuming \( n = 2 \). Both measurements are in reasonable agreement with expectations for the \( \mu^+ + \mu^- \) flux at this altitude (see for example Figure 29.4 in [29]), absent strong geomagnetic anomalies.

8. Conclusions

A method to extract the integral vertical intensity and the angular distribution of atmospheric muons at a given location on the Earth’s surface using a stationary plastic scintillator bar detector was presented. The method relies on the accurate simulation of the observed deposited energy distribution of atmospheric muons in the detector, for which a muon generator algorithm was developed and coupled with a GEANT4 simulation of the detector response which included the effects of the energy resolution and PMT saturation, as well as a rough description of the laboratory building coverage. The simulated energy spectrum of muons, was fit to an experimentally measured spectrum considering also two exponentially decreasing background components representing the neutrons and electromagnetic showers entering the detector to match the lowest energy region.

Assuming that the angular distribution follows the usual \( \cos^2 \theta \) law, we extract a measurement of \( I_0 = (108.2 \pm 0.51(\text{stat}) \pm 4.8(\text{syst})) \text{ m}^{-2}\text{s}^{-1}\text{sr}^{-1} \) for this quantity in Mexico City, at the geographical coordinates 19.33°N 99.19°W, altitude of 2,268 m above sea level, and geomagnetic cut-off of 8.2 GV. This value fits reasonably well with expectations for a location at this altitude, and away from strong geomagnetic anomalies. The small statistical error indicates that this method can be used to track variations in this quantity of the order of or larger than 0.5% over periods of 5 h, and is a good option for monitoring applications. Allowing the exponent of the angular distribution to vary, we extract a measured value of \( n = (2.12 \pm 0.19(\text{stat}) \pm 0.11(\text{syst})) \), where the dominant error is from fit statistics. A 6\% higher muon vertical integral intensity is extracted when the exponent \( n \) in the angular distribution is set to the value preferred by the data.

Acknowledgements

The authors acknowledge the support of DGAPA-UNAM grant number PAPIIT-IN108917, and CONACYT through grants CB-2009/131598 and CB-2014/240666. We also thank ICN-UNAM personnel, Mr. José Rangel from the Advanced Manufacturing Laboratory for the machining of a piece to couple the PMTs to the plastic scintillator, and to Ing. Mauricio Martínez Montero for his technical assistance with the instrumentation of the Detectors Laboratory.

References

[1] S. Pethuraj, V. Datar, G. Majumder, N. Mondal, K. Ravindran, B. Satyanarayana, Measurement of cosmic muon angular distribution and vertical integrated flux by 2 m x 2 m rpc stack at ihep-madurai, Journal of Cosmology and Astroparticle Physics 2017 (021) (2017). doi:https://doi.org/10.1088/0305-4470/2017/09/021
[2] M. De Pascale, et al., Absolute spectrum and charge ratio of cosmic ray muons in the energy region from 0.2 gev to 100 gev at 600 m above sea level, J. Geophys. Res. 98 (3501) (1993).
[3] P. Grieder, et al., Cosmic rays at earth, Elsevier Science (2001).
[4] J. Kremer, et al., Measurements of ground-level muons at two geomagnetic locations, Phys. Rev. Lett. 83 (21) (1999) 4241.
[5] S. Haino, et al., Measurements of primary and atmospheric cosmic-ray spectra with the bess-tev spectrometer, Physics Letters B 594 (1) (2004) 35–46. doi:https://doi.org/10.1016/j.physletb.2004.05.019
[6] P. Archard, et al., Measurement of the atmospheric muon spectrum from 20 to 3000 gev, Physics Letters B 598 (2004) 15–32. doi:https://doi.org/10.1016/j.physletb.2004.08.003
[7] A. Fauth, J. Penereiro, E. K. I. W. Grizolli, D. Consalter, L. Gonzalez, Experimental demonstration of time dilatation and space contraction of cosmic-ray muons, Rev.Bras. Ensino Fis. 29 (4) (2007) 585–591. doi:https://dx.doi.org/10.1590/S1806-11172007000400017
[8] C. Augusto, C. Navia, K. Tsui, H. Shigueoka, P. Miranda, R. Ticona, A. Velarde, O. Saavedra, Simultaneous observation at sea level and at 5200 m.a.s.l. of high energy particles in the south atlantic anomaly, Astroparticle Physics 34 (1) (2010) 40 –
[9] P. Shukla, S. Sankrith, Energy and angular distributions of atmospheric muons at the earth, Int. Jour. Mod. Phys. A 33 (30) (2018) 1–12. doi:10.1142/S0217751X18501750

[10] M. Boezio, et al., Energy spectra of atmospheric muons measured with the caprice98 balloon experiment, Physical Review D 67 (072003) (2003). doi:https://doi.org/10.1103 PhysRevD.67.072003

[11] Y. Liu, L. Defome, B. M., Atmospheric muon and neutrino flux from 3-dimensional simulation, Physical Review D 67 (073022) (2003). doi:https://doi.org/10.1103 PhysRevD.67.073022

[12] D. Heck, J. Knapp, J. N. Capdevielle, G. Schatz, T. Thouw, CORSIKA: a Monte Carlo code to simulate extensive air showers. TIB Hannover, 1998.

[13] T. Sato, Analytical model for estimating the zenith angle dependence of terrestrial cosmic ray fluxes, PLOS ONE 11(8): e0160390 (2016).

[14] T. Sato, Analytical model for estimating terrestrial cosmic ray fluxes nearly anytime and anywhere in the world. Extension of parma/expacs, PLOS ONE 10(12): e0144679 (2016).

[15] F. Lei, S. Clucas, C. Dyer, P. Truscott, An atmospheric radiation model based on response matrices generated by detailed monte carlo simulations of cosmic ray interactions, IEEE Trans. Nucl. Sci. 51 (6) (2004) 3442 – 3451. doi:10.1109/TNS.2004.839193

[16] F. Lei, A. Hands, S. Clucas, C. Dyer, P. Truscott, Improvement to and validations of the qinetiq atmospheric radiation model (qarm), IEEE Trans. Nucl. Sci. 53 (4) (2006) 1851 – 1858. doi:10.1109/TNS.2006.880567

[17] L. W. Alvarez, J. A. Anderson, F. E. Bedwei, J. Burkhard, A. Fakhry, A. Girgis, A. Goneid, F. Hassan, D. Iverson, G. Lynch, Z. Miligy, A. H. Moussa, M. Sharkawi, L. Yazolino, Search for hidden chambers in the pyramids, Science 167 (3919) (1970) 832–839. doi:10.1126/science.167.3919.832

[18] S. Chatzidakis, P. Hausladen, S. Croft, J. Chapman, J. Jarrell, J. Scaglione, C. Choi, L. Tsoukalas, Exploring the use of muon momentum for detection of nuclear material within shielded spent nuclear fuel dry casks, Transactions of the American Nuclear Society 116 (131185) (2017) 190–193.

[19] P.J.O., Advanced applications of cosmic-ray muon radiography (thesis dissertation), University of New Mexico 16 (2003) 37–53.

[20] H. Miyadera, et al., Imaging fukushima daiichi reactors with muons, AIP Advances 3 (052133) (2013). doi:https://doi.org/10.1063/1.4808210

[21] V. Tsoukov, et al., First muography of stromboli volcano, Scientific Reports 9 (6695) (2019).

[22] C. Morris, et al., Tomographic imaging with cosmic ray muons, Science & Global Security 16 (2008) 37–53. doi:https://doi.org/10.1080/0892988080233578

[23] J. A. Smith, N. M. Duller, Effects of p meson decay-absorption phenomena on the high-energy mu meson zenithal variation near sea level, Journal of Geophysical Research 64 (12) (1959) 2297–2305. doi:10.1029/J28641012p023578

[24] S. Chatzidakis, S. Chrysokepoulou, L. Tsoukalas, Developing a cosmic ray muon sampling capability for muon tomography and monitoring applications, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 804 (2015) 33 – 42. doi:https://doi.org/10.1016/j.nima.2015.09.033

[25] B. C. Rastin, An accurate measurement of the sea-level muon spectrum within the range 4 to 3000 gev/c, Journal of Physics G: Nuclear Physics 10 (11) (1984) 1609.