The detection rate of Inspiral and Quasi-normal modes of Pop III binary black holes which can confirm or refute the General Relativity in the strong gravity region

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ABSTRACT
Using our population synthesis code, we found that the typical chirp mass defined by \((m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}\) of Population III (Pop III) binary black holes (BH-BHs) is \(\sim 30 M_\odot\) with the total mass of \(\sim 60 M_\odot\) so that the inspiral chirp signal as well as quasi normal mode (QNM) of the merging black hole (BH) are interesting targets of KAGRA. The detection rate of the coalescing Pop III BH-BHs is \(\sim 180\) events yr\(^{-1}\)(SFR\(_p^p/(10^{-2.5} M_\odot \text{yr}^{-1} \text{Mpc}^{-3}) \cdot (f_b/(1 + f_b))/0.33 \cdot \text{Err}_{\text{sys}}\) in our standard model where SFR\(_p^p\), \(f_b\) and Err\(_{\text{sys}}\) are the peak value of the Pop III star formation rate, the binary fraction and the systematic error with Err\(_{\text{sys}} = 1\) for our standard model, respectively. To evaluate the robustness of chirp mass distribution and the range of Err\(_{\text{sys}}\), we examine the dependence of the results on the unknown parameters and the distribution functions in the population synthesis code. We found that the chirp mass has a peak at \(\sim 30 M_\odot\) in most of parameters and distribution functions as well as Err\(_{\text{sys}}\) ranges from 0.046 to 4. Therefore, the detection rate of the coalescing Pop III BH-BHs ranges about 8.3 – 720 events yr\(^{-1}\) (SFR\(_p^p/(10^{-2.5} M_\odot \text{yr}^{-1} \text{Mpc}^{-3}) \cdot (f_b/(1 + f_b))/0.33\). The minimum rate corresponds to the worst model which we think unlikely so that unless (SFR\(_p^p/(10^{-2.5} M_\odot \text{yr}^{-1} \text{Mpc}^{-3}) \cdot (f_b/(1 + f_b))/0.33 \ll 0.1, we expect the Pop III BH-BHs merger rate of at least one event per year by KAGRA. Nakano, Tanaka & Nakamura (2015) show that if S/N of QNM is larger than 35, we can confirm or refute the General Relativity (GR) more than 5 sigma level. In our standard model, the detection rate of Pop III BH-BHs whose S/N is larger than 35 is 3.2 events yr\(^{-1}\) (SFR\(_p^p/(10^{-2.5} M_\odot \text{yr}^{-1} \text{Mpc}^{-3}) \cdot (f_b/(1 + f_b))/0.33 \cdot \text{Err}_{\text{sys}}. Thus, there is a good chance to check whether GR is correct or not in the strong gravity region.

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1 INTRODUCTION
The second generation gravitational wave detectors such as KAGRA\(^1\), Advanced LIGO\(^2\), Advanced VIRGO\(^3\) and GEO\(^4\) are under construction and the first detection of gravitational wave is expected in near future. The most important sources of gravitational waves are compact binary mergers such as the binary neutron star (NS-NS), the neutron star black hole binary (NS-BH), and the BH-BH. As the compact binary radiates gravitational wave and loses the orbital energy and the angular momentum, the compact binary coalesces. The merger rate of NS-NS can be estimated using the binary pulsar observation (e.g., Kalogera, Kim, Lorimer et al. 2004a,b). However NS-BH and BH-BH merger rates cannot be estimated using the observation since no such binaries have been observed so that they can be estimated only by the theoretical approach called the population synthesis. For Population I (Pop I) and Population II (Pop II) stars, the merger rates of compact binaries are estimated by Belczynski, Kalogera & Bulik (2002); Belczynski et al. (2007); a (2012); Dominik et al. (2012, 2013).

In this paper, we focus on Pop III stars which were formed first in the universe with zero metal after the Big

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\(^1\) http://gwcenter.icrr.u-tokyo.ac.jp/en/
\(^2\) http://www.ligo.caltech.edu/
\(^3\) http://www.ego-gw.it/index.aspx/
\(^4\) http://www.geo600.org/
Bang. The formation process of Pop III stars has been argued by many authors such as Omukai & Nishi (1998); Bromm, Coppi & Larson (2002); Abel, Bryan & Norman (2002); Yoshida, Omukai & Hernquist (2008); Greif et al. (2012). The simulations of rotating minihalo in the early universe suggest the formation of binaries and multiple star systems (Machida et al. 2008; Stacy, Greif & Bromm 2010).

There are four reasons why we focus on Pop III binaries as gravitational wave sources. Firstly, Pop III stars are more massive than Pop I stars (McKee & Tan 2008; Hosokawa et al. 2011, 2012; Stacy, Greif & Bromm 2012) so that Pop III binaries tend to be neutron star (NS) and BH. Secondly, the merger timescale of compact binaries due to gravitational wave is in proportion to the fourth power of the separation so that even the compact binary formed in the early universe can merge at present. Therefore, the Pop III compact binary mergers formed in the early Universe may be detected by gravitational wave detectors such as KAGRA. Thirdly, BH-BH and BH-NS binary formed from Pop III stars tend to be more massive than those formed from Pop I stars while the detectable distance of the chirp signal is proportional to 5/6 power of the chirp mass defined by \((m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}\) of the compact binary (Kinugawa et al. 2014). This means that the detectable distance of Pop III compact binaries is longer than that of Pop I compact binaries so that even if the merger rate per comoving volume is smaller, the detection rate can be larger for Pop III BH-BH binaries. Fourthly, Pop III compact binaries as gravitational wave source were considered by Belczynski, Bulik & Rudak (2004), Kowalska, Bulik & Belczynski (2012), they focused on Pop III stars of mass over hundred solar masses or only on the background. However the typical mass of Pop III stars is now considered as 10-100 M⊙ (Hosokawa et al. 2011, 2012) due to the evaporation of the accretion disk by the strong ultra-violet photons from the central star. Therefore, Pop III binary population synthesis should be calculated with more realistic lower mass range.

In our previous study (Kinugawa et al. 2014), we performed Pop III binary population synthesis with initial mass range of 10-100 M⊙. The results showed that Pop III binaries tend to be massive BH ones with chirp mass \(\sim 30 \text{ M}_\odot\). In our standard model, the detection rate of gravitational wave by the second generation detectors such as KAGRA is \(1.4 \cdot (\text{SFR}_p/10^{-2.5} \frac{\text{M}_\odot}{\text{yr}^{-1} \text{Mpc}^{-3}}) \cdot ([f_0/(1 + f_0)]/0.33) \cdot \text{Err}_{\text{sys}} \) events yr\(^{-1}\) where \(\text{SFR}_p\), \(f_0\) and \(\text{Err}_{\text{sys}}\) are the peak value of the Pop III star formation rate, the binary fraction\(^5\) and the possible systematic error due to the assumptions in Pop III Monte Carlo population synthesis, respectively. \(\text{Err}_{\text{sys}} = 1\) corresponds to our assumption of binary parameters and initial distribution functions for our standard model. However, there are some uncertainties in binary evolution and initial distribution function of Pop III stars so that \(\text{Err}_{\text{sys}}\) might not be unity.

\(^5\) The definition of the binary fraction is \(N_{\text{binary}}/(N_{\text{single}} + N_{\text{binary}})\) where the \(N_{\text{binary}}\) and \(N_{\text{single}}\) are the number of single stars and the number of binary systems, respectively. Thus, \(\text{SFR}_p \cdot f_0/(1 + f_0)\) means the binary system formation rate.

Therefore, in this paper, we perform Pop III binary population synthesis with several binary parameters and initial distribution functions to estimate the variability of merger rates and properties of Pop III compact binaries. That is, we estimate the possible range of \(\text{Err}_{\text{sys}}\) to see if the event rate is larger than 1 yr\(^{-1}\). In order to perform Pop III binary evolutions, we renewed Hurley’s binary population synthesis code (Hurley, Tout & Pols 2002) in our previous paper (Kinugawa et al. 2014). In this paper, we adopt the cosmological parameters of \((\Omega_\Lambda, \Omega_m) = (0.6825, 0.3175)\), the Hubble parameter of \(H_0 = 67.11 \text{ km s}^{-1} \text{ Mpc}^{-1}\) and the Hubble time \(t_{\text{Hubble}} = 13.8\) Gyrs (Planck Collaboration 2013).

The readers of this journal might not be familiar with the quasi-normal mode of BH in the title of this paper so that we will briefly explain what is the quasi-normal mode of BH here. Let us consider the Schwarzschild BH of mass \(M\) and a small particle of mass \(m \ll M\) falling into this BH. If the specific angular momentum \((L)\) of the particle is smaller than \(3GM/c^2\), the particle spirals into the BH. At first the gravitational wave depending on \(L\) is emitted but in the final phase the damped gravitational wave is observed as is shown in Figs. 5-4 of (Nakamura, Oohara & Kojima 1987). The wave form of this damped gravitational wave is expressed by \(e^{i(\omega t + i\omega_i t)}\) where \(\omega\) and \(\omega_i\) depend only on the mass of BH \(M\) but not on \(L\). Since \(\omega_i > 0\) in general, this damped oscillation is called the ringing tail or quasi-normal mode. Chandrasekhar & Detweiler (1975); Leaver (1985, 1986) obtained the complex frequency of quasi-normal modes including Kerr BH case. For the case of Schwarzschild BH, using the WKB approximation, Schutz & Will (1985) showed that \(\omega_i\) and \(\omega\) are determined by the space-time inside \(3GM/c^2\), that is, the space-time near the event horizon so that the detection of the expected quasi-normal mode of the BH can confirm General Relativity in the strong gravity region. If it is different from the expected value, the true theory of gravity is different from the General Relativity.

This paper is organized as follows. We describe the models and the calculation method of Pop III binary population synthesis simulations in §2. Results of our calculation such as the properties and the merger rates of Pop III compact binaries are shown in §3. The discussion & summary are presented in §4.

\(^6\) http://astronomy.swin.edu.au/~jrhurley/
while the details of these parameters and distribution functions will be shown later. We perform Monte Carlo simulations using $10^6$ zero age main sequence (ZAMS) binaries for each model. Table 1 shows the parameters and distribution functions of each model. Each column represents the model name, the IMF, the IEF, the natal kick velocity of supernova, the CE parameter $\alpha_{\lambda}$ and the lose fraction $\beta$ of transfer of stellar matter at the Roche lobe over flow (RLOF) in each model. Model name "worst" means the worst combination of the parameter and distribution functions each model, we calculate 6 cases with different mass range and the merger criterion at the CE phase so that the total number of the models is 84.

Firstly, we use three mass range cases such as the under100, the over100 and the 140. In the under100 case, the initial mass range is $10 M_\odot \leq M \leq 100 M_\odot$. If the stellar mass becomes over 100 $M_\odot$ by the binary interaction, the binary evolution calculation is stopped because of no fitting formula of the Pop III star evolution for $M > 100 M_\odot$ (Kinugawa et al. 2014) due to the lack of numerical data of the evolution of Pop III star. This treatment is the same as that in Kinugawa et al. (2014). In the over100 case, the initial mass range is also $10 M_\odot \leq M \leq 140 M_\odot$. However, in this case, if the stellar mass becomes larger than 100$M_\odot$ by the binary interaction, the binary evolution calculation continues by extrapolating the fitting formula of the evolution beyond 100 $M_\odot$. In the 140 case, the initial mass range is $10 M_\odot \leq M \leq 140 M_\odot$. We use the extrapolated fitting formula up to $M = 140 M_\odot$. The reason for up to 140 is that Pop III star of mass larger than 140$M_\odot$ explodes with no remnant (Heger et al. 2003).

We randomly choose the initial stellar mass from these mass ranges with the initial distribution functions such as the flat IMF, the log flat IMF and the Salpeter IMF (See §2.1 and 2.2.1). If the stellar mass is over 140 $M_\odot$ at the supernova, the binary evolution is stopped because such a high mass star becomes pair instability supernova without compact remnant (Heger et al. 2003). Numerical simulations of Pop III star formation by (e.g., Hosokawa et al. 2011, 2012; Stacy et al. 2012) suggest that the Pop III protostar grows only up to $\sim$ several 10 $M_\odot$ and the typical mass of Pop III star can be about 40 $M_\odot$. In recent simulations (Hirano et al. 2014; Susa et al. 2014), the typical mass is almost the same as previous study, that is, 40 $M_\odot$, however, the some Pop III stars can be more massive than 100 $M_\odot$. Therefore, we use the initial mass range as $10 M_\odot \leq M \leq 140 M_\odot$ in the 140 case and study the influences of high mass Pop III binaries for the event rate of gravitational wave sources.

Secondly, we use two merger criteria at the CE phase such as the optimistic core-merger criterion and the conservative core-merger criterion (Hurley et al. 2002; Belczynski et al. 2002; Kinugawa et al. 2014). In the case of the optimistic core-merger criterion, if the condition $R_1^t > R_{L,1}$ or $R_2^t > R_{L,2}$ is fulfilled, the primary star merges with secondary star, where $R_1^t$, $R_{L,1}$, $R_2^t$ and $R_{L,2}$ are the primary stellar radius, the Roche lobe radius of the primary star, the secondary stellar radius and the Roche lobe radius of the secondary star after the CE phase, respectively (Hurley et al. 2002). On the other hand, the conservative core-merger criterion is $R_1^t + R_2^t > a_f$, where $a_f$ is the separation after the CE phase (Belczynski et al. 2002).

2.1 Brief review of Paper I; our standard model

In this paper, our standard model is the same as the model III.f in Kinugawa et al. (2014). In this section, we review the model III.f in Kinugawa et al. (2014) briefly. The details are shown in Kinugawa et al. (2014). In order to simulate the binary evolution, we need to choose initial binary parameters such as the primary mass, the mass ratio, the separation and the eccentricity. These parameters are decided randomly by the initial distribution function and the Monte Carlo method. In our standard model, we adopt the flat initial distribution function for the primary mass, the flat function for the mass ratio $f(q) \propto 1$ (Kobulnicky & Fryer 2007; Kobulnicky et al. 2012), the log flat function for the separation $f(a) \propto 1/a$ (Abt 1983) and the thermal equilibrium distribution function of the eccentricity $\propto e$ (Heggie 1975; Duquennoy, Mayor & Halbwachs 1991). We use these initial distribution functions except the IMF referenced by the Pop I stars observations because there are no observations suggestions of Pop III binaries initial distribution functions. As for the IMF, some simulations suggest the flat or the log flat IMF (Hirano et al. 2014; Susa, Hasegawa & Tominaga 2014). Using these initial distribution functions, we put the ZAMS binary and start the evolution of the binary. In order to calculate each stellar evolution, we use the formula fitted to the numerical calculations of Pop III stellar evolutions by Marigo et al. (2001). This fitting formula is described by the stellar radius and the core mass as a function of the stellar mass and the time from the birth of each star. The details of fitting equations for Pop III are shown in Kinugawa et al. (2014). We can calculate the evolution the binary adding the binary interactions to the Pop III star evolution using the fitting formula. We also need to consider the binary interactions such as the tidal friction, the Roche lobe over flow, the CE phase, the effect of the supernova explosion and the back reaction of the gravitational wave.

Firstly, we review the tidal friction. The tidal force from the companion star changes the stellar radius and the shape. In general, the stellar spin angular velocity is different from the orbital angular velocity. Therefore the vector of the tidal deformation is different from the vector of the tidal force so that the tidal torque is generated. The tidal torque transfers the angular momentum from the stellar spin to the orbital angular momentum. This interaction changes the binary separation, eccentricity and the spins of each star.

Secondly, we review the RLOF. When the star evolves and the stellar radius becomes large, the stellar matter is captured by the companion star and is transferred to the companion star. This phenomenon is called as the RLOF. We call the donor star as the primary star. The recipient star is called as the secondary star. The region within which the stellar material is gravitationally bound to the star is called as the Roche lobe radius of that star. If the primary stellar radius becomes larger than the Roche lobe radius, the primary stellar matter migrates to the secondary star.
is the accretion time scale defined as

\[ \dot{M}_2 = -(1 - \beta)\dot{M}_1. \] (1)

In this case, the accretion rate to the secondary star and \( \beta \) is determined by the method of Hurley et al. (2002). If the secondary star is in the main sequence phase or in the He-burning phase, we assume the accretion rate is expressed by

\[ \dot{M}_2 = \min \left( \frac{10 \tau_M}{\tau_{KH,2}} - 1 \right) \dot{M}_1, \] (2)

where \( \dot{M}_1 \) is the mass loss rate of the primary star and \( \tau_M \) is the accretion time scale defined as

\[ \tau_M = \frac{M_2}{\dot{M}_1}. \] (3)

The Kelvin-Helmholtz timescale \( \tau_{KH,2} \) is defined as

\[ \tau_{KH,2} = \frac{GM_2(M_2 - M_{c,2})}{L_2 R_2^2}, \] (4)

where \( M_2, M_{c,2}, L_2 \) and \( R_2 \) are the secondary stellar mass, the core mass, the luminosity and the radius of the star, respectively. If the secondary star is in the He-shell burning phase, we assume the secondary star can get all matter of the primary. Thus,

\[ \dot{M}_2 = -\dot{M}_1. \] (5)

In our standard model, we use \( \beta \) function defined by Eq.(2) which is computed by the fitting formulae by Hurley et al. (2002). However, the accretion rate of the secondary which is not a compact object, is not understood well so that we use also the accretion rate of the secondary described by the constant \( \beta \) parameter (For details see §2.2.5).

If the secondary star is a compact object such as NS and BH, we always use \( \beta = 0 \) and the maximum of the accretion rate is limited by the Eddington mass accretion rate defined by

\[ \dot{M}_{\text{Edd}} = \frac{4\pi c R_2}{\kappa T} \] (6)

\[ = 2.08 \times 10^{-3} (1 + X)^{-1} \left( \frac{R_2}{R_\odot} \right) M_\odot \text{ yr}^{-1}, \]

where \( \kappa_T = 0.2(1 + X) \text{ cm}^2 \text{ g}^{-1} \) is the Thomson scattering opacity and \( X(= 0.76) \) is the H-mass fraction for Pop III star.

When the primary star is a giant and the mass transfer is dynamically unstable, the secondary star plunges into the primary envelope and the binary enters the CE phase. In this phase, the friction between the primary star and the secondary star yields the loss of the angular momentum of the secondary to decrease the binary separation, while the envelope of the primary is evaporated by the energy liberated through the friction. Consequently, the binary either becomes the close binary or merges during the CE phase. Now we define \( a_i, a_t, M_t, M_{c,1}, M_{env,1} \) and \( R_1 \) as the separation before the CE phase, the separation after the CE phase, the primary mass, the primary core mass, the primary envelope mass and the primary separation, respectively. The separation after the CE phase is calculated by the energy formalism (Webbink 1984) defined by

\[ \alpha \left( \frac{GM_{c,1} M_2}{2 a_t} - \frac{GM_1 M_2}{2 a_t} \right) = \frac{GM_1 M_{env,1}}{\lambda R_1}. \] (7)

where \( \alpha \) and \( \lambda \) are the efficiency and the binding energy parameter, respectively. In our standard model, we adopt \( \alpha \lambda = 1 \).

In our calculation, we adopt two merger criteria during the CE phase which is shown already in the last paragraph of §2 before §2.1.

When the supernova explosion occurs, the sudden mass ejection and the natal kick make the binary orbit to change drastically. In our standard model, we adopt the natal kick velocity equal to zero. In this case, the binary orbit changes
only by the mass ejection effect in the supernova event. The separation and the eccentricity after the supernova explosion are described as
\[ a' = \left( \frac{v^2}{G M_{\text{total}}} - \frac{|v|^2}{G M'_{\text{total}}} + \frac{1}{a^2} \right)^{-1}, \]
\[ e' = \sqrt{1 - \frac{|r \times v|^2}{G M_{\text{total}} a}}, \]
where \( M_{\text{total}} \) is the total mass, the superscript ' means the value after the supernova while \( v, v \) and \( r \) are the relative speed, the relative velocity and the separation vector before the supernova, respectively (Blaauw 1961).

After the binary becomes the compact binary due to above binary interactions, the orbit of the binary shrinks by the emission of the gravitational waves. The separation and the eccentricity are given by
\[ \dot{a} = -\frac{64G^3 M_1 M_2 M_{\text{total}}}{5c^6 a^5} \left( 1 + \frac{21}{16} e^2 + \frac{17}{16} e^4 \right), \]
\[ \dot{e} = -\frac{32G^3 M_1 M_2 M_{\text{total}}}{15c^6 a^5} \frac{1 + \frac{121}{80} e^2}{(1 - e^2)^{5/2}}. \]
(Peters & Mathews 1963; Peters 1964). We calculate the binary evolutions taking account all these binary interactions and estimate how many binaries become the compact binaries which merge within the Hubble time.

2.2 Parameter Surveys

To estimate the range of \( \text{Err}_{\text{sys}} \), we calculate the binary population synthesis using other initial distribution functions and binary parameters since for Pop III stars we have no information on these functions and parameters. However in this paper we do not take into account the dependence on the initial separation function and the initial mass ratio function because unlike an initial eccentricity functions, there are no suggestions for other distribution functions for massive binaries although possible dependence of \( \text{Err}_{\text{sys}} \) on the change of these initial distribution functions is discussed in §4 (Discussion).

2.2.1 log flat IMF and Salpeter models

These models correspond to the log flat IMF, that is IMF\( \propto d \log(M) \), and the Salpeter IMF. In recent numerical simulations (Hirano et al. 2014; Susa et al. 2014), IMF of Pop III stars might be the log flat IMF. On the other hand, Salpeter IMF is acceptable as Pop I IMF. We calculate these IMF models in order to estimate the IMF dependence. The other initial distribution functions and binary parameters are the same as in our standard model.

2.2.2 IEF:const. and \( e^{-0.5} \) models

In these models, the IEF are changed from our standard model. In general the initial eccentricity distribution might be the thermal-equilibrium distribution (IEF:2e) (Heggie 1975). However in recent observation of massive binaries, the eccentricity distribution is not the thermal-equilibrium distribution. The observation of massive multiple-star systems in the Cygnus OB2 (Kobulnicky et al. 2014) implies that the observed IEF is consistent with uniform one. On the other hand, the observation of massive binaries (\( M > 15 M_\odot \)) (Sana et al. 2012) suggests that the power law for the distribution function of eccentricity as \( \propto e^{-0.5} \). Thus, we can calculate these two initial eccentricity distribution function models. The other initial distribution functions and binary parameters are the same as in our standard model.

2.2.3 kick 100 km s\(^{-1}\) and kick 300 km s\(^{-1}\) models

The pulsar observations suggest the existence of the NS kick. It is observed that the young NSs move with velocities in range of 200 – 500 km s\(^{-1}\) (e.g. Lyne & Lorimer 1994; Hansen & Phinney 1997). Since the NS kick velocity either disrupts the binary or increases the separation, the formation rate and the coalescing time of the NS-NS and the NS-BH depend on the NS kick velocity. On the other hand, the formation rate and the coalescing time of the BH-BH might have nothing to do with the natal kick velocity because the BH progenitor directly collapses to the BH. However, Repetto, Davies & Sigurdsson (2012) suggests that the stellar mass BHs have the natal kicks comparable to NSs from the distance distribution of the Galactic BH (low mass X-ray binaries) above the galactic plane. Pop III BH-BHs are massive, so that they may not have such natal kick as stellar mass BHs. However there is no observation of Pop III BHs. Thus we cannot definitely claim that Pop III BHs do not have natal kicks. Therefore, we take into account the natal kick for both NS and BH. In order to estimate the dependence on the natal kick, we calculate two models. In these models, when stars become compact objects such as NS and BH, we assume that the natal kick speed \( v_k \) obeys an isotropic Maxwellian distribution as
\[ P(v_k) = \frac{\sqrt{2} \sigma_k^2}{\pi \sigma_k^4} \exp \left( -\frac{v_k^2}{\sigma_k^2} \right), \]
where \( \sigma_k \) is the dispersion. In the kick 100 km s\(^{-1}\) model and kick 300 km s\(^{-1}\) models, we uses \( \sigma_k = 100 \text{ km s}\(^{-1}\) and \( \sigma_k = 300 \text{ km s}\(^{-1}\) respectively. The details of the method how to calculate the natal kick are shown in Hurley et al. (2002). The other initial distribution functions and binary parameters are the same as in our standard model.

2.2.4 \( \alpha \lambda = 0.01, \alpha \lambda = 0.1 \) and \( \alpha \lambda = 10 \) models

If the primary star becomes a giant and it begins dynamically unstable mass transfer so that the secondary star can be engulfed into the envelope of the primary star. In such case, the binary enters the CE phase. Once the secondary star is swallowed up by the envelope of the primary star, it spirals into the core of the primary star due to the orbital energy and angular momentum loss by the friction. It is assumed that this spiral-in continues until all the envelope of
the primary star is ejected from the binary system. The separation after the CE phase $a_1$ is calculated using CE energy balance prescription of Eq. 7.

However CE parameters are uncertain. In general, it is assumed that $\alpha \lambda = 1$ since only $\alpha \lambda$ is the meaningful parameter. Our standard model uses $\alpha \lambda = 1$. However, these parameters can be other values. Therefore, we calculate three cases of the CE parameters as $\alpha \lambda = 0.01, 0.1, 10$. The other initial distribution functions and binary parameters are the same as in our standard model.

2.2.5 $\beta = 0$, $\beta = 0.5$ and $\beta = 1$ models

If the primary star fulfills Roche lobe and it begins dynamically stable mass transfer, the secondary gets the mass from the primary star. $\beta$ is called as the lose fraction of transferred stellar mass defined as Eq. 1. It is considered that $\beta$ varies depending on the binary (e.g. Eggleton 2000). In binary population synthesis study, $\beta$ is treated as a function or the constant parameter. In our standard model, we use $\beta$ as a function (See section 2.1). On the other hand, in other studies $\beta$ is treated as the constant (e.g. Belczynski et al. 2002). Thus, we estimate the variabilities of result for three constant $\beta$ cases. The other initial distribution functions and binary parameters are the same as in our standard model. Furthermore, if the mass transfer is nonconservative ($\beta > 0$), the criterion of the stability of the mass transfer should be changed from the criterion of (Kinugawa et al. 2014; Hurley et al. 2002) because this criterion assumes that the mass transfer is conservative. We use the criterion of Eggleton (2011) as

$$\zeta_L = \frac{\text{dlog} R_{L,1}}{\text{dlog} M_1} = \frac{0.33 + 0.13 q_1 (1 + q_1 - \beta q_1)}{1 + q_1} + (1 - \beta) (q_1^2 - 1) - \beta q_1$$

(13)

where $R_{L,1}$ and $q_1 = M_1/M_2$ are the Roche lobe radius and the mass ratio. If $\zeta_{ad} = \text{dlog} R_{ad,1}/\text{dlog} M_1 < \zeta_L$ where $R_{ad}$ is the radius when the star reaches hydrostatic equilibrium, the binary starts the dynamically unstable mass transfer.

2.2.6 Worst model

In this model, we adopt the initial conditions and binary parameters which make the worst result in IMF, IEF, kick, $\alpha \lambda$ and $\beta$ (See Section 3). Namely we adopt IMF:Salpeter, IEF:$e^{-0.5}$, kick 300 km s$^{-1}$, $\alpha \lambda = 0.01$ and $\beta = 1$. We, however, think that this worst case is unlikely so that the worst case will teach us the minimum merging rate of Pop III BH-BHs. Note that other combination of parameters may yield even lower rates. However, we cannot calculate and check all $3 \times 3 \times 3 \times 4 \times 4 = 432$ models. Thus, we choose the worst parameters from each parameter region.

3 RESULTS

3.1 The properties of Pop III compact binaries

In order to study the property of Pop III compact binaries, we now show the number of the compact binary formations, the number of the compact binaries which merge within 15 Gyrs and the distribution of the BH-BH chirp mass. The details of the difference between each model will be shown in the following sub-sections (a) to (g).

Tables 2 to 15 show the numbers of NS-NS, NS-BH and BH-BH binaries for each model from the initial $10^6$ zero age main star binary. The meanings of under 100, over 100 and 140 are explained in the first paragraph of §2. The title of each table comes from the change of some parameter or that of IMF or IEF from our standard model. In each table we also tabulated the number of the compact binaries which merge within 15 Gyrs. The numbers in the parenthesis are for the case of the conservative core-merger criterion while those without the parenthesis are for the case of the optimistic core-merger criterion. The meanings of these two criteria were explained in the last paragraph of §2 before §2.1.

In most cases, the number of merging NS-NSs and merging NS-BH are very small or zero as one can notice easily from Tables 2 to 14. The reasons are as follows. The wind mass loss and the mass loss by the binary interactions are not so effective for the Pop III binaries because of the zero metallicity and smaller radius of Pop III stars so that the Pop III binaries tend to disrupt or to increase the separation by the supernova mass ejection (Kinugawa et al. 2014). Therefore we focus on the description of the BH-BHs. Figs. 1 to 14 show the chirp mass distribution of BH-BH binaries which merge within 15Gyr for each model. In each figure, the red, green, blue, pink, light blue and grey lines correspond to under100 case with optimistic core-merger criterion, over100 case with optimistic core-merger criterion, 140 case with optimistic core-merger criterion, under100 case with conservative core-merger criterion, over100 case with conservative core-merger criterion and 140 case with conservative core-merger criterion, respectively. One can see that in all models, the peak of the observable chirp mass distribution is about $3 M_\odot$. Pop III binaries with each mass $M < 50 M_\odot$ are unlikely to be the CE phase. They evolve via some mass transfer phases and their mass loss is smaller than the evolution passes via a CE phase. They tend to lose 1/10-1/3 of their mass so that they tend to be 20-30 $M_\odot$ BH-BHs. Pop III binaries with each mass $M > 50 M_\odot$ are likely to be the CE phase and they lose 1/2-2/3 of their mass so that they tend to be 25-30 $M_\odot$ BH-BHs too. Therefore, the peak of chirp mass become 25-30 $M_\odot$.

Figs. 15 to 20 show the dependence of merger time distribution of BH-BH binaries for each model. In each figure, we describe under100 cases with optimistic core-merger criterion because the merger time distributions do not change a lot in other cases and core-merger criteria. The most important characteristic is that the merger rate for $t > 10^{9.5}$ yr is almost constant for every model.
(a) our standard model

Under 100 case is the same as the result of the model III for
Kinugawa et al. (2014). Over 100 case is equal to the
result of under 100 case plus the binaries whose star becomes
more massive than 100 M⊙ by the mass transfer so that
the number of massive BH-BH mergers in over 100 case is
more than that of the under 100 case (See Fig. 1). Thus, the
number of Pop III BH-BHs which merge within 15 Gyrs in
over 100 case increases about ten percent compared to the
that of under 100 case (See Table 2). However, the peak of
the chirp mass is not changed (See Fig. 1) In 140 case the
peak of the chirp mass distribution (Fig. 1) is almost the
same as that of under 100 and over 100 cases, but the chirp
mass distribution of 140 case has tail in the region of 25-60
M⊙ since the high mass Pop III binaries (> 100 M⊙) tend
to be high mass BH-BHs (40-60 M⊙) via the CE phase. The
event rate of high chirp mass BH-BHs is large because the
detectable volume (V) is \( V \propto M^{-5/2} \). The tail of chirp mass
distribution \( dN/dM \) in 140 case is proportional to \( M^{-5/2} \) in
the range of 25-60 M⊙.

(b) log flat and Salpeter IMF models

In these models, the initial masses tend to be smaller than
that of our standard model. Thus, the numbers of the BH-
BH formations and merging BH-BHs decrease because the
BH-BH progenitors decrease due to IMFs (See Table 3 and
4). On the properties of the chirp mass distributions in the
log flat and Salpeter IMF models, the number of merging
BH-BHs where each mass is more massive than 30 M⊙ is
smaller than our standard model due to the steepness of
IMFs. However, the peak mass is independent on the IMF
(See Figs. 2 and 3). In 140 cases, the chirp mass distribution
\( dN/dM \) of IMF for log flat model is proportional to \( M^{-3} \)
and that of Salpeter model is proportional to \( M^{-3.9} \) in a
range of 25-60 M⊙. In Fig. 15, the shape of each merger
time distribution is almost the same independent of IMFs,
although the number of BH-BHs decreases dependent on the
steepness of IMF.

(c) IEF: const. and \( e^{-0.5} \) models

In these models, the initial eccentricities tend to be smaller
than that of our standard model. If the initial eccentricity is
small, the decrease of the separation by binary interactions
such as the tidal friction and the gravitational radiation is
suppressed. Thus, the merger rate by the binary interaction
decreases. Therefore, the number of BH-BH formation rate
increases while the number of merging BH-BHs decreases al-
though the influence to the merger rate is very small (See
Table 5 and 6). The properties of the chirp mass distributions
are almost the same as in our standard model (See Fig.
4, 5). The properties of the merger time distributions are
almost the same as in our standard model too (See Fig. 16).
We can say that the effect of the eccentricity distribution is
not so large.

(d) kick 100 km s^{-1} and kick 300 km s^{-1} models

In these models, the natal kick disrupts binaries or makes
their orbits wide and eccentric. Firstly, we argue the kick
100 km s^{-1} model. The number of the BH-BH formations
decreases by the natal kick compared to our standard model.
However, the number of merging BH-BHs becomes about 1.5
times larger than that of our standard model (See Table 7).
The merger timescale by the gravitational radiation is given by

\[
T_{\text{merge}}(e_0 = 0) = \frac{5}{256} \frac{a_0^4}{c^2} \left( \frac{G M_1}{c^2} \right)^{-1} \left( \frac{G M_2}{c^2} \right)^{-1} \left( \frac{G M_{\text{total}}}{c^2} \right)^{-1}
\]

\[
= 10^{10} \text{yr} \left( \frac{a_0}{43 \text{ R}_\odot} \right)^4 \left( \frac{M_1}{30 \text{ M}_\odot} \right)^{-1} \left( \frac{M_2}{30 \text{ M}_\odot} \right)^{-1} \left( \frac{M_{\text{total}}}{60 \text{ M}_\odot} \right)^{-1}
\]

where \( a_0 \) and \( c_0 \) are the separation and the eccentricity when
the BH-BH is formed. Thus, the BH-BHs whose separation
is larger than about 50 R⊙ cannot merge within Hubble time.
On the other hand, the escape velocity and the orbital
velocity of the binary system are given by

\[ v_{\text{esc}} = \sqrt{\frac{2GM}{a}} \]

\[ = 500 \text{ km s}^{-1} \left( \frac{M}{30 \text{ M}_\odot} \right)^{1/2} \left( \frac{a}{43 \text{ R}_\odot} \right)^{-1/2}, \]

\[ v_{\text{orb}} = \sqrt{\frac{GM}{a}} \]

\[ = 350 \text{ km s}^{-1} \left( \frac{M}{30 \text{ M}_\odot} \right)^{1/2} \left( \frac{a}{43 \text{ R}_\odot} \right)^{-1/2}, \]

where \( a \) is the separation. Equations 15-18 tells us that even
though the kick velocity which is 100 km s^{-1} is aligned with
the orbital velocity, the BH-BH progenitors whose separa-
tion is smaller than 43 R⊙ cannot be disrupted by the
natal kick. Therefore, the BH-BHs disrupted by the natal
kick of 100 km s^{-1} rarely contribute to the merger rate of
Pop III BH-BHs from the beginning. While the increase of
the eccentricity by the natal kick reduces the merger
timescale due to the gravitational waves since
\( T_{\text{merge}}(e) \sim (1 - e^2)^{7/2} T_{\text{merge}}(e_0 = 0) \) (Peters & Mathews 1963; Peters
1964), so the number of merging BH-BHs is larger than
that of our standard model. The chirp mass distribution is,
however, almost the same as that of our standard model
(See Fig. 6). In Fig. 17, the merger time distribution of the
kick 100 km s^{-1} model is almost the same as in our standard
model. However, the number of merging BH-BHs which
merges at the early universe is larger than that of our stan-
dard model due to the increase of the eccentricity by the
natal kick.

Secondly, we argue the kick 300 km s^{-1} model. The
number of the BH-BH formations decreases by the natal kick
compared to our standard model and the kick 100 km s^{-1}
model. The number of the merging BH-BHs also decreases
unlike the kick 100 km s^{-1} model. The reasons are that
the sum of the orbital velocity and the kick velocity some-
times can exceed 500 km s^{-1} and that if the binary was not
due to the small luminosity of GW. Thus, the number of the merging BH-BHs does not decrease. In 140 case, the number of merging BH-BHs which are low mass becomes larger than our standard model (See Fig. 9) because the low mass BH-BHs can merge more easily than our standard model due to moderately small $\alpha \lambda$. In Fig. 18, the shortest merger time is smaller than that of our standard model due to moderately small $\alpha \lambda$. $\alpha \lambda$ is small but the binaries which become the CE phase do not tend to merge during the CE phase and they have close orbit due to small $\alpha \lambda$.

Thirdly, we consider the $\alpha \lambda = 10$ model. In this case, the number of the BH-BH formation increases compared with our standard model. While the number of the merging BH-BHs decreases compared with our standard model. $\alpha \lambda$ is so large that the separation after the CE phase tends to become large and the binary does not merge during the CE phase. Thus, the number of the BH-BH formations increases. But, due to the large separation the BH-BH binary does not merge within 15 Gyrs. Therefore, the number of the merging BH-BHs decreases. On the other hand, the number of merging high chirp mass BH-BHs is more than our standard model (See Fig. 10), because the BH-BHs which are formed after the CE phase tend to have wide orbit due to high $\alpha \lambda$ and the wide massive BH-BHs can merge more easily than the wide low mass BH-BHs. Especially, in 140 case this effect is remarkably clear. In Fig. 18, the shortest merger time becomes larger than that of our standard model and the number of merging BH-BHs which merge at the early universe is smaller than that of our standard model due to the same reason.

$\varepsilon (\alpha \lambda = 0.01, \alpha \lambda = 0.1$ and $\alpha \lambda = 10$ models

In these models, the CE phase results in the merger of binary or the change of the separation. The separation after the CE phase is determined by the CE parameter $\alpha \lambda$. If $\alpha \lambda$ is small, the separation after the CE phase tends to be small and to merge during the CE phase due to the increase in the loss of the orbital energy. Easily the binary does not survive during the CE phase. But if binary survives during the CE phase, the binary easily merges by the large luminosity of GW emission due to the tight orbit. On the other hand, if $\alpha \lambda$ is large, the separation after the CE phase tends to be large due to the decrease in the loss of the orbital energy. Thus, the binary tends to survive during the CE phase. When the binary survives in the CE phase, however, it is hard to merge due to the small luminosity of GW.

Firstly, we consider $\alpha \lambda = 0.01$ model. In this case, the number of the BH-BH formations decreases and the number of the merging BH-BHs becomes about $1/3$ as large as our standard model. The parameter $\alpha \lambda$ is so small that the almost all binaries merge during the CE phase. The Pop III giant with initial mass larger than $50 M_\odot$ can enter the CE phase and these binaries tend to merge. Thus, the survived binaries evolved via the RLOF. The binaries which evolved via the RLOF tend to be smaller than $50 M_\odot$ and the massive binaries merge more easily. Thus, in the chirp mass distribution (Fig. 8), the number of the merging high mass BH-BHs becomes smaller than our standard model. Especially, 140 case is easily affected by these effects. In Fig. 18, the shortest merger time is larger than that of our standard model, because almost all progenitors of BH-BHs merge during the CE phase.

Secondly, we consider the $\alpha \lambda = 0.1$ model. In this case, the number of the BH-BH formation decreases compared with our standard model. While, the number of the merging BH-BHs is almost the same as that of our standard model. Owing to the small $\alpha \lambda$, the binary does not tend to survive during the CE phase. But if binary survives in the CE phase, the binary becomes close binary and easy to merge due to the emission of GW. Thus, the number of the merging BH-BHs does not decrease. In 140 case, the number of merging BH-BHs which are low mass becomes larger than our standard model (See Fig. 9) because the low mass BH-BHs can merge more easily than our standard model due to moderately small $\alpha \lambda$. In Fig. 18, the shortest merger time is smaller than that of our standard model due to moderately small $\alpha \lambda$. $\alpha \lambda$ is small but the binaries which become the CE phase do not tend to merge during the CE phase and they have close orbit due to small $\alpha \lambda$.

Thirdly, we consider the $\alpha \lambda = 10$ model. In this case, the number of the BH-BH formation increases compared with our standard model. While the number of the merging BH-BHs decreases compared with our standard model. $\alpha \lambda$ is so large that the separation after the CE phase tends to become large and the binary does not merge during the CE phase. Thus, the number of the BH-BH formations increases. But, due to the large separation the BH-BH binary does not merge within 15 Gyrs. Therefore, the number of the merging BH-BHs decreases. On the other hand, the number of merging high chirp mass BH-BHs is more than our standard model (See Fig. 10), because the BH-BHs which are formed after the CE phase tend to have wide orbit due to high $\alpha \lambda$ and the wide massive BH-BHs can merge more easily than the wide low mass BH-BHs. Especially, in 140 case this effect is remarkably clear. In Fig. 18, the shortest merger time becomes larger than that of our standard model and the number of merging BH-BHs which merge at the early universe is smaller than that of our standard model due to the same reason.

$\beta = 0, \beta = 0.5$ and $\beta = 1$ models

In these cases, the accretion rate during the RLOF is changed by the loss fraction of transferred stellar mass $\beta$. Firstly, we consider the $\beta = 0$ model.

The result of our standard model and the result of $\beta = 0$ model are the same (Table 12.2 and Figs. 1, 11, 19). We use the Hurley’s fitting $\beta$ which is fitted by Pop I binaries for the Pop III case. We found the Hurley’s fitting $\beta$ is same as $\beta = 0$ in Pop III case. Fig.11 and Fig.1 are the same. This means that the RLOF of our standard model (equation 2) is the conservative mass transfer prescription.

Secondly, we consider $\beta = 0.5$ model. In this case, the mass loss during the mass transfer makes the separation wide so that the mass transfer tends to be dynamically stable. The number of the binaries which are merged during the CE phase decreases. Therefore, the number of BH-BH formation increases compared to our standard model. However, the number of merging BH-BHs decreases because the binary does not tend to be a close binary by the CE phase. The peaks of the chirp mass distributions are almost the same as in our standard model (See Fig. 12). While the highest mass of merging BH-BHs decreases compared with our standard model because of the mass loss during the RLOF.

Thirdly, we consider $\beta = 1$ model. In this model, the number of BH-BH formation is larger than our standard model, but it is smaller than that of $\beta = 0.5$ model. Like $\beta = 0.5$ model, the mass transfer tends to be dynamically
stable. Especially, in this case, there are no paths to the CE phase via dynamically unstable mass transfer. There are only paths to the CE phase in which the secondary plunges into the primary due to the eccentric orbit or in which the each star is a giant and the binary become the contact binary due to the expanding. Thus, the number of the binaries which are merged during the CE phase decreases, so that the number of BH-BH formations increases compared with our standard model. However, the mass loss during the mass transfer is so large that some stars which can be BH by the mass accretion during the RLOF cannot be a BH but a NS. Therefore, the the number of BH-BH formations decreases compared with $\beta = 0.5$ model, but the number of NS-BH formations increases accordingly. Furthermore, since the progenitors of merging BH-BHs are hard to enter the CE phase and tend to become wide orbits due to the mass loss during the mass transfer. Thus, the number of merging BH-BHs decrease. The major progenitors of merging BH-BHs do not enter the CE phase and lose their mass during the mass transfer. But, since the Pop III star radius is small (See Fig.2 in Marigo et al. (2001) and Fig.1 in Kinugawa et al. (2014)), the mass loss during the RLOF tends to stop right away and the separation tends to be close enough that Pop III BH-BHs can merge within 15 Gyrs. The highest mass peak region of the chirp mass distributions becomes smaller than our standard model and the highest mass of merging BH-BHs decreases due to the mass loss during the RLOF (See Fig. 13).

(g) Worst model

In this model, we choose the initial conditions and binary parameters which will make the worst result in (b)IMF, (c)IEF, (d)kick, (e)$\alpha\lambda$ and (f)$\beta$. Thus, we adopt (b)IMF: Salpeter, (c)IEF:$e^{-0.5}$, (d)kick 300 km s$^{-1}$, (e)$\alpha\lambda = 0.01$ and (f)$\beta = 1$. Especially, we already know that (b)IMF, (d)kick, (e)$\alpha\lambda$ and (f)$\beta$ influence the result very much so that the result of BH-BH formation and the number of merging BH-BHs are determined by these parameters and IMF. Each effect of (b)IMF:Salpeter, (d)kick 300 km s$^{-1}$, (e)$\alpha\lambda = 0.01$ and (f)$\beta = 1$ makes the number of merging BH-BHs decrease (See Table 4, 8 and 14). Thus, the number of merging BH-BHs extremely decreases compared with our standard model (See Tabel 2 and 15). On the properties of chirp mass distribution, the number of merging BH-BHs more massive than 30 $M_\odot$ decreases and the gradient of the chirp mass distribution of merging BH-BHs is much steeper than that of our standard model (See Fig. 14 (a) (b)). The Salpeter IMF makes the number of high mass stars to decrease and non-conservative mass transfer ($\beta = 1$) prevents to merge BH-BHs while in our standard mode they can be merging BH-BHs after the CE phase. Furthermore, even though the massive binaries become the CE phase, they usually merge during the CE phase due to the very small $\alpha\lambda$. Therefore, the peak region of the chirp mass is 25-30 $M_\odot$ even in 140 case.
Table 2. our standard model

This table shows the numbers of NS-NS, NS-BH and BH-BH binaries and the numbers of each compact binary which merges within 15 Gyrs for our standard model. 15Gyrs is used in order to compare our results with previous works. To estimate the present merger rates, we use 13.8Gyrs as the present age of the universe. The meanings of under 100, over 100 and 140 are explained in the first paragraph of §2. The numbers in the parenthesis are for the case of the conservative core-merger criterion while those without the parenthesis are for the case of the optimistic core-merger criterion. The meanings of these two criteria were explained in the last paragraph of §2 before §2.1

|            | under100 | over100 | 140        |
|------------|----------|---------|------------|
| NS-NS      | 0 (279)  | 0 (279) | 0 (195)    |
| NS-BH      | 185335   | 185335  | 153435     |
| BH-BH      | 517067   | 534693  | 595894     |
| merging NS-NS | 0 (279) | 0 (279) | 0 (195)    |
| merging NS-BH | 50 (149)| 50 (149)| 825 (1255) |
| merging BH-BH | 115056 | 131060  | 128894     |

Table 3. IMF:logflat

Same as Table 2 but for IMF:logflat model.

|            | under100 | over100 | 140        |
|------------|----------|---------|------------|
| NS-NS      | 2 (789)  | 2 (789) | 1 (693)    |
| NS-BH      | 168100   | 168100  | 157106     |
| BH-BH      | 350169   | 357989  | 405922     |
| merging NS-NS | 2 (789)| 2 (789) | 1 (693)    |
| merging NS-BH | 68 (183)| 68 (183)| 374 (579) |
| merging BH-BH | 74745  | 81786   | 87590      |

Table 4. IMF:Salpeter

Same as Table 2 but for IMF:Salpeter model.

|            | under100 | over100 | 140        |
|------------|----------|---------|------------|
| NS-NS      | 5 (1994)| 5 (1994)| 3 (1957)   |
| NS-BH      | 93085   | 93085   | 92861      |
| BH-BH      | 132534  | 133880  | 144096     |
| merging NS-NS | 5 (1994)| 5 (1994)| 3 (1957)   |
| merging NS-BH | 64 (164)| 64 (164)| 97 (216)  |
| merging BH-BH | 25536 | 26720   | 28378      |

Table 5. IEF:const.

Same as Table 2 but for IMF:logflat model.

|            | under100 | over100 | 140        |
|------------|----------|---------|------------|
| NS-NS      | 0 (358)  | 0 (358) | 0 (255)    |
| NS-BH      | 183460   | 183460  | 152099     |
| BH-BH      | 522808   | 541264  | 602071     |
| merging NS-NS | 0 (358)| 0 (358) | 0 (255)    |
| merging NS-BH | 43 (130)| 43 (130)| 843 (1087)|
| merging BH-BH | 111106 | 127904  | 124714     |
Table 6. IEF: $e^{-0.5}$

|         | under100   | over100    | 140        |
|---------|------------|------------|------------|
| NS-NS   | 0 (365)    | 0 (365)    | 0 (258)    |
| NS-BH   | 181650 (182388) | 181650 (182388) | 150779 (151805) |
| BH-BH   | 523285 (526534) | 542015 (545389) | 602575 (607054) |
| merging NS-NS | 38 (100)    | 38 (100)   | 774 (964)  |
| merging NS-BH | 107594 (110832) | 124620 (127983) | 121494 (125955) |

Table 7. kick 100 km s$^{-1}$

|         | under100   | over100    | 140        |
|---------|------------|------------|------------|
| NS-NS   | 283 (794)  | 283 (794)  | 180 (516)  |
| NS-BH   | 32701 (34778) | 32701 (34778) | 32014 (34144) |
| BH-BH   | 191755 (197327) | 208268 (213962) | 234117 (243348) |
| merging NS-NS | 17 (526)    | 17 (526)   | 6 (342)    |
| merging NS-BH | 2527 (3016)  | 2527 (3016) | 3218 (3762) |
| merging BH-BH | 117415 (122830) | 132066 (137603) | 135758 (144554) |

Table 8. kick 300 km s$^{-1}$

|         | under100   | over100    | 140        |
|---------|------------|------------|------------|
| NS-NS   | 8 (112)    | 8 (112)    | 4 (78)     |
| NS-BH   | 11922 (13133) | 11941 (13152) | 12115 (13330) |
| BH-BH   | 70728 (75011) | 78058 (82496) | 86876 (93481) |
| merging NS-NS | 1 (85)      | 1 (85)     | 1 (60)     |
| merging NS-BH | 3893 (4483)  | 3900 (4490) | 4406 (5002) |
| merging BH-BH | 51928 (56021) | 58793 (63041) | 64084 (70252) |

Table 9. $\alpha \lambda = 0.01$

|         | under100   | over100    | 140        |
|---------|------------|------------|------------|
| NS-NS   | 0 (0)      | 0 (0)      | 0 (0)      |
| NS-BH   | 148290 (148770) | 148290 (148770) | 116548 (117117) |
| BH-BH   | 340893 (352047) | 345140 (363191) | 365526 (382686) |
| merging NS-NS | 0 (0)       | 0 (0)      | 0 (0)      |
| merging NS-BH | 32283 (43437)  | 36530 (54581) | 27790 (44950) |
Table 10. $\alpha \lambda = 0.1$

|       | under100 | over100 | 140       |
|-------|----------|---------|-----------|
| NS-NS | 0 (0)    | 0 (0)   | 0 (0)     |
| NS-BH | 162814 (173016) | 162814 (173016) | 130556 (138835) |
| BH-BH | 434590 (464369) | 448847 (480217) | 480520 (520031) |
| merging NS-NS | 0 (0) | 0 (0) | 0 (0) |
| merging NS-BH | 45 (181) | 45 (181) | 1065 (1877) |
| merging BH-BH | 111696 (141356) | 125953 (157204) | 124830 (164240) |

Table 11. $\alpha \lambda = 10$

|       | under100 | over100 | 140       |
|-------|----------|---------|-----------|
| NS-NS | 1116 (2215) | 1116 (2215) | 840 (1616) |
| NS-BH | 198408 (198758) | 198408 (198758) | 166173 (166408) |
| BH-BH | 542399 (542603) | 560156 (560360) | 624631 (624958) |
| merging NS-NS | 890 (1949) | 890 (1949) | 634 (1381) |
| merging NS-BH | 767 (975) | 767 (975) | 506 (645) |
| merging BH-BH | 91787 (91989) | 104656 (104858) | 93729 (94055) |

Table 12. $\beta = 0$

|       | under100 | over100 | 140       |
|-------|----------|---------|-----------|
| NS-NS | 5 (380) | 5 (380) | 6 (272)   |
| NS-BH | 193921 (196094) | 193921 (196094) | 158518 (160442) |
| BH-BH | 549893 (554150) | 554966 (559228) | 628253 (635698) |
| merging NS-NS | 5 (380) | 5 (380) | 6 (272) |
| merging NS-BH | 199 (286) | 199 (286) | 766 (1082) |
| merging BH-BH | 117094 (121310) | 119758 (123979) | 126090 (133512) |

Table 13. $\beta = 0.5$

|       | under100 | over100 | 140       |
|-------|----------|---------|-----------|
| NS-NS | 5 (380) | 5 (380) | 6 (272)   |
| NS-BH | 193921 (196094) | 193921 (196094) | 158518 (160442) |
| BH-BH | 549893 (554150) | 554966 (559228) | 628253 (635698) |
| merging NS-NS | 5 (380) | 5 (380) | 6 (272) |
| merging NS-BH | 199 (286) | 199 (286) | 766 (1082) |
| merging BH-BH | 117094 (121310) | 119758 (123979) | 126090 (133512) |
Table 14. $\beta = 1$

Same as Table 2 but for $\beta = 1$ model.

|        | under100 | over100 | 140     |
|--------|----------|---------|---------|
| NS-NS  | 1359 (2006) | 1359 (2006) | 898 (1344) |
| NS-BH  | 218311 (220521) | 218311 (220522) | 178444 (180375) |
| BH-BH  | 531452 (536579) | 531484 (536611) | 610732 (619230) |
| merging NS-NS | 1358 (2005) | 1358 (2005) | 898 (1344) |
| merging NS-BH | 119 (255) | 119 (255) | 578 (917) |
| merging BH-BH | 50119 (55214) | 50119 (55214) | 57025 (65121) |

Table 15. Worst

Same as Table 2 but for Worst model.

|        | under100 | over100 | 140     |
|--------|----------|---------|---------|
| NS-NS  | 1637 (1637) | 1637 (1637) | 1604 (1604) |
| NS-BH  | 4345 (4345) | 4345 (4345) | 4283 (4285) |
| BH-BH  | 5227 (5235) | 5227 (5235) | 5560 (5586) |
| merging NS-NS | 1562 (1562) | 1562 (1562) | 1532 (1532) |
| merging NS-BH | 1645 (1645) | 1645 (1645) | 1604 (1606) |
| merging BH-BH | 3195 (3203) | 3195 (3203) | 3376 (3399) |
Each line is the normalized distribution of the BH-BH chirp mass. The red, green, blue, pink, light blue and grey lines are the under100 case with optimistic core-merger criterion, the over100 case with optimistic core-merger criterion, the 140 case with optimistic core-merger criterion, the under100 case with conservative core-merger criterion, the over100 case with conservative core-merger criterion and the 140 case with conservative core-merger criterion, respectively. $N_{\text{total}} = 10^6$ binaries.
Pop III binary black holes and the quasi normal mode

Figure 6. kick 100 km s$^{-1}$
Same as Fig.1 but for kick 100 km s$^{-1}$ model.

Figure 7. kick 300 km s$^{-1}$
Same as Fig.1 but for kick 300 km s$^{-1}$ model.

Figure 8. $\alpha \lambda = 0.01$
Same as Fig.1 but for $\alpha \lambda = 0.01$ model.

Figure 9. $\alpha \lambda = 0.1$
Same as Fig.1 but for $\alpha \lambda = 0.1$ model.

Figure 10. $\alpha \lambda = 10$
Same as Fig.1 but for $\alpha \lambda = 10$ model.

Figure 11. $\beta = 0$
Same as Fig.1 but for $\beta = 0$ model.
Figure 12. $\beta = 0.5$

Same as Fig.1 but for $\beta = 0.5$ model.

Figure 13. $\beta = 1$

Same as Fig.1 but for $\beta = 1$ model.

Figure 14. Worst

Same as Fig.1 but for Worst model.

Figure 15. merger time: IMF

This figure describes the merger time distributions of Pop III BH-BHs. The red line, the green line and the blue line are our standard model, the logflat model and the Salpeter model. $N_{\text{total}} = 10^6$ binaries.

Figure 16. merger time: e

This figure describes the merger time distributions of Pop III BH-BHs. The red line, the pink line and the light blue line are our standard model, IEF:const. model and IEF:$e^{-0.5}$ model. $N_{\text{total}} = 10^6$ binaries.
This figure describes the merger time distributions of Pop III BH-BHs. The red line, the orange line and the black line are our standard model, the kick 100 km s$^{-1}$ model and the kick 300 km s$^{-1}$ model. $N_{\text{total}} = 10^6$ binaries.

This figure describes the merger time distributions of Pop III BH-BHs. The red line, the green line, the blue line and the pink line are our standard model, the $\beta = 0$ model, the $\beta = 0.5$ and the $\beta = 1$ model. $N_{\text{total}} = 10^6$ binaries.

This figure describes the merger time distributions of Pop III BH-BHs. The red line, the orange line, the grey line and the black line are our standard model, the $\alpha \lambda = 0.1$ model, the $\alpha \lambda = 0.1$ and the $\alpha \lambda = 10$ model. $N_{\text{total}} = 10^6$ binaries.

This figure describes the merger time distributions of Pop III BH-BHs as a function of cosmic time $[t', t' + dt]$ and merge at time $t$. $N_{\text{total}}$ is the total number of the simulated binaries.

Fig. 22 and not show the merger rate densities [Myr$^{-1}$ Mpc$^{-3}$] of BH-BHs as a function of cosmic time (lower abscissa) and redshift $z$ (upper abscissa) in our standard model and the worst model. It is seen that in each model the merger rate densities for the same redshift depend on neither the initial mass range ($[10, 100]$ or $[10, 140]$) nor the CE merger criterion. The other models have the same dependencies so that we do not show their figures. As a function of the redshift, the merger rate densities are nearly constant from $z = 0$ to $z \sim 1$ in each model. Ta-
SFR \([\text{Msun yr}^{-1} \text{Mpc}^{-3}]\)

Figure 21. The star formation rate density (comoving) calculated by de Souza et al. (2011). The unit of the rate is \(\text{M}_\odot\) per comoving volume per proper time. The red line is the total SFR density of Pop III stars.

Table 16 shows the merger rate density \([\text{Myr}^{-1} \text{Mpc}^{-3}]\) at \(z = 0\) \((t_{\text{Hubble}} = 13.8\ \text{Gyrs})\) for each model. The lowest rate is as expected in worst model while the highest rate is in \(\beta = 0.5\) model.

Fig. 24 shows the difference of the merger rate density of each model for under100 case. Table 17 describes the peak redshift of the BH-BHs merger rate density of each model in under100 case. It is seen that the peak redshift of the BH-BHs merger rate density ranges from 8.8 to 7.15. These peak redshifts are near the peak of the star formation rate at \(z \sim 9\). In the following, we discuss the difference of each model.

The IMF dependence of the peak redshift of the merger rate density is clear seen. Namely for the steeper IMF the peak redshift is small although the difference is not so large \((\sim 0.45\ \text{in} \ z)\). Since BH-BH progenitors whose initial mass is lower than 50 \(\text{M}_\odot\) tend to evolve via the RLOF but not via the CE, the steeper IMF can make BH-BH progenitors to evolve via RLOFs. BH-BHs which evolved via RLOFs tend to have the wider orbit than BH-BHs which evolved via CE phases. Therefore, the typical merger time for the steeper IMF tends to be long so that the peak redshift is smaller.

As for the IEF dependence, no tendency is seen while as for the natal kick velocity dependence, the peak redshift for 100 \(\text{km s}^{-1}\) model is smaller than our standard model, but that of the 300 \(\text{km s}^{-1}\) model is higher than our standard model. In the 100 \(\text{km s}^{-1}\) model, the kick makes the BH-BHs to eccentric orbit so that the merger time becomes smaller than that of the circular orbit. Thus, the number of the merging BH-BHs which merge at the high redshift should increase. However, the BH-BHs which cannot merge in our standard case due to wide orbit can merge due to the natal kick. Consequently, the number of the merging BH-BHs which merge at lower redshift tends to increase. In the 300 \(\text{km s}^{-1}\) model, the natal kick velocity is too large so that the binary tends to disrupt. However, if the natal kick direction is against the orbital direction, the natal kick behaves like the brake of the car or if the separation before the natal kick is very close, the binary can survive. Thus, the survived binary tends to have very close and eccentric orbit so that they can merge early. This explains the apparent strange behavior of the dependence of the peak redshift on the natal kick velocity.
In the case of the CE parameter, it changes the number of survived binaries during the CE phase and the merger time of the BH-BHs. If $\alpha = 0.1$ model, the parameter $\alpha$ is so small that the almost all binaries which enter the CE phase merge during the CE phase. Thus, merging BH-BH progenitors evolved via RLOF so that they have wide orbit. Therefore, their merger time tend to be long and the peak redshift is low. In $\alpha = 0.1$ model, the mass transfer is dynamically stable so that the number of the binaries which evolve via RLOF but not via the CE phase. Thus, the typical merger time is long and the peak redshift is low. In $\beta = 0$ model, the mass transfer is always dynamically stable. Furthermore, the mass accretion to the secondary during RLOF does not occur so that the orbit becomes wide. On the other hand, binaries which have the eccentric orbit have only the CE phase. Thus, the merging BH-BHs are separated into two groups. In one group the binaries evolve via RLOF while in the other group they evolve via the CE phase due to the eccentric orbit. The former group merges at low redshift and the latter group does at high redshift. Therefore, in the $\beta = 1$ model, the merger rate density is bimodal as shown in Fig. 24.

Note that the maximum merger rate density of $3.7 \times 10^{-8}$ events yr$^{-1}$ Mpc$^{-3}$ from Table 16 with $\beta = 0.5$ and over100 case is consistent with the upper limit of $\sim 10^{-7}$ events yr$^{-1}$ Mpc$^{-3}$ by LIGO-Virgo(S6/VSR2/VSR3) (Aasi et al. 2013).
3.3 The detection rate of Pop III BH-BHs by the second generation detectors

In this section, we show how to calculate the detection rate of BH-BHs. Our Pop III population synthesis simulations produced a set of merging BH-BHs with component masses and merger time. To estimate the number of the event and the parameter decision accuracy according to the generated binary mass distribution, distances, various incident angles and orientations of the orbit plane, we employ the simple Monte-Carlo simulation. As the typical 2nd generation detectors that has a little advantage in a low frequency band in underground site, we employ KAGRA detector in our simulation. We use the detection range of Kanda & the LCGT collaboration (2011). The official sensitivity limit of the KAGRA7 is suitable for the detection of both inspiral and ring-down gravitational waves from the 10-30 M⊙ binaries. In the Monte-Carlo simulation, we placed each event at random position in the hemisphere, random direction of the binary orbit plane. The direction in the cosmological redshift and the mass are given by the Pop III binary simulation. We iterate many events for 1000 years, then, we estimate the expected detection rate for one year observation. The error of the rate is given as the square root of the number of detection in Monte-Carlo trials.

Tables 18 to 23 describe the detection rate of BH-BHs in each model. This table shows the detection rates of Pop III BH-BHs under100 cases with the optimistic core-merger criterion. The first column shows the name of the model. The second column shows the detection rate only by the inspiral chirp signal. The third, the fourth and the fifth columns show the detection rate only by the quasi normal mode (QNM) with Kerr parameter a/M = 0.70, the detection rate by the quadrature sum of the inspiral chirp signal and the QNM with a/M = 0.70 and the detection rate by the linear sum of the inspiral chirp signal and the QNM with a/M = 0.98, respectively. When signal-to-noise ratio of event that is calculated by matched filtering equation, over threshold S/N = 8, the event is detected. The QNM S/N is calculated by Eq. B14 in Flanagan & Hughes (1998). ε, in this equation is the fraction of binary total mass energy radiated in the QNM. We assumed the value ε = 0.03. Since their equation is averaged over the GW polarization and the sky location, a factor 1/5 is multiplied by the equation. However we have to take account of angular values of binary, we replace the factor with √((1 + cos²θ)/4 × F_x² + cos²θ × F_y²), where the θ is the inclination angle, and F_x, F_y are KAGRA antenna pattern functions. For the fourth and sixth columns, their S/N are calculated by the linear summation of S/N of the inspiral and the QNM with a/M = 0.70 and 0.98, respectively. All the rates are based on 1000 years Monte Carlo simulations.

In Zlochower & Lousto (2015), the reasonable value of Kerr parameter a/M is about 0.7. Thus, we focus on the detection rate by the quadrature sum of the inspiral chirp signal and the QNM with a/M = 0.70. The rates of the quadrature sum of the inspiral and the QNM is about 1/2 of the rates of the linear sum of the inspiral and the QNM. In our standard model with under100 case and optimistic core-merger criterion, the detection rate by the quadrature sum of the inspiral chirp signal and the QNM is ∼ 1.8 × 10^2 events yr⁻¹ (SFR_p/(10⁻²⁻⁵ M⊙ yr⁻¹ Mpc⁻³)) · (f_b/(1 + f_b))/0.33 where SFR_p and f_b are the peak value of the Pop III star formation rate and the binary fraction. Err_sys = 1 corresponds to the rate for our standard model with under100 case and the optimistic core-merger criterion. The definition of Err_sys is slightly different from that in our previous paper (Kinugawa et al. 2014). That is, the new definition is based on Monte Carlo simulations of the detection of the inspiral chirp signal and QNM with a/M = 0.7. The basic numerical data of population synthesis of our standard model is the same. Then Tables 18-23 show that Err_sys ranges from 4.6 × 10⁻² to 4 for a/M = 0.7. This means that the detection rate of the coalescing Pop III BH-BHs ranges 8.3 − 7.2 × 10² events yr⁻¹ (SFR_p/(10⁻²⁻⁵ M⊙ yr⁻¹ Mpc⁻³)) · (f_b/(1 + f_b))/0.33. The minimum detection rate of the coalescing Pop III BH-BHs corresponds to the worst model which we think unlikely so that unless (SFR_p/(10⁻²⁻⁵ M⊙ yr⁻¹ Mpc⁻³)) · (f_b/(1 + f_b))/0.33 < 0.1, we can expect the Pop III BH binary merger at least one event per year by the second generation gravitational wave detector.

Fig. 3 of Nakano et al. (2015) shows that if the S/N of the Pop III BH-BH QNM is 50, we can check the general relativity with the significance of much more than 5 sigma level. The criterion of S/N whether the significance has more than 5 sigma is 35. Therefore, we expect the enough accuracy to discuss about the GR test with at least one event of S/N = 35. In our standard model, the detection rate of Pop III BH-BHs whose S/N is more than 35 is 3.2 events yr⁻¹ (SFR_p/(10⁻²⁻⁵ M⊙ yr⁻¹ Mpc⁻³)) · (f_b/(1 + f_b))/0.33 < 0.1.
4 DISCUSSION & SUMMARY

In this paper, we performed the Pop III binary population synthesis and examined the parameter dependence of Pop III binary evolutions. We examined the dependence of the results on IMF, IEF, the natal kick velocity, the CE parameters and the lose fraction of stellar mass. As for the chirp mass distribution, each model has the peak at around 30 M\(_{\odot}\). In several models, the chirp mass distribution has a tail from 30 M\(_{\odot}\) to more massive region. However the robust property is that the chirp mass distribution has the peak at 30 M\(_{\odot}\).

In order to compare the variability of Err\(_{\text{sys}}\), we refer previous researches such as Belczynski et al. (2012); Dominik et al. (2015). In Belczynski et al. (2012), they calculated the solar metal (Z = 0.02) binaries and 10\% solar metal (Z = 0.002) binaries to estimate the detection rates assuming as half of the stars formed with solar metal and the other with 10\% solar metal. In Belczynski et al. (2012), they calculated 20 models by varying the maximum NS mass, the natal kick velocity, rapid or delayed supernova models which change the mass spectrum of supernova remnants, wind mass loss and \(\beta\). They also considered whether in the Hertzsprung gap donors always merge companion during the CE phase or not. The detection rate of their realistic Standard model in Belczynski et al. (2012) is 517.3 events yr\(^{-1}\). The detection rates of Belczynski et al. (2012) are from 14 events yr\(^{-1}\) to 12434.4 events yr\(^{-1}\). Thus, Err\(_{\text{sys}}\) of Belczynski et al. (2012) is from 2.7 \times 10^{-2} to 24.

On the other hand, Dominik et al. (2015) calculated binaries whose metallicity range is from Z = 10^{-4} to Z = 0.03 and estimated the detection rate using the metallicity and SFR evolution models. There are 16 models varying high-end or low-end metallicity models, whether in the Hertzsprung gap donors always merge companion during the CE phase or not, rapid or delayed supernova models, the natal kick and waveform models. The detection rate of their Standard model of high-end metallicity scenario in Dominik et al. (2015) is 306 events yr\(^{-1}\). The detection rate of Dominik et al. (2015) is from 8.2 events yr\(^{-1}\) to 3087 events yr\(^{-1}\) by the 3-detector network using inspiral and PhC waveform (S/N=10). Thus, Err\(_{\text{sys}}\) of Dominik et al. (2015) is from 2.7 \times 10^{-2} to 10. Therefore, the variability of Err\(_{\text{sys}}\) of Pop III is less than that of Pop I and Pop II, although the models are different. There are two reasons for this difference. Firstly, Pop III star binaries do not enter the CE phase so that the result does not depend on the treatment of the CE phase so much. Secondly, the Pop III compact binaries are more massive than the Pop I compact binary so that the Pop III binaries are hard to be disrupted by the natal kick. Therefore, the property of the chirp mass distribution and the detection rate are likely robust result. However, note that in the case of the detection rate of Pop III there are the dependence on the SFR and f\(_{\text{b}}\) yet.

There are some uncertainties yet such as the separation distribution function and the mass ratio distribution function for which we did not alter. The former will change the number of close binaries which can have binary interactions. Therefore this effect may change the event rate, but the property of chirp mass distribution is not likely changed a lot because the binary interaction is not changed. From our Monte Carlo simulations, the chirp mass distribution of Pop III BH-BHs is upward to the high mass and has a peak at \(\sim 30 M_{\odot}\) in each model. The compact objects in IC10 X-1 and NGC300 X-1 may be around 30 M\(_{\odot}\) and they might become coalescing massive BH-BHs whose chirp masses are 11-26 M\(_{\odot}\) (See Bulik et al. (2011)). Thus, Pop I stars or Pop II stars might become coalescing massive BH-BHs. However, the observed typical mass of Pop I BH-BHs is around 10 M\(_{\odot}\) and massive BH like IC10 X-1 and NGC300 X-1 would be rare (See also Fig. 1 in Belczynski et al. (2012)) so that the chirp mass distribution of Pop I BH-BHs might be flat or decreasing as a function of mass. The result of the binary population synthesis simulation for Pop I and Pop II stars by Dominik et al. (2015) also suggests that the chirp mass distribution of Pop I BH-BHs might be flat or downward to high mass (See the Fig. 7 in Dominik et al. (2015)). Furthermore, the Pop I and Pop II BH detection rate of the standard model in Dominik et al. (2015) is 306 yr\(^{-1}\). The fraction of the Pop I and Pop II BH-BH whose mass is larger than 20 M\(_{\odot}\) is about 25\%. Thus, the detection rate of the Pop I and Pop II high mass BH-BHs is expected as about 80 yr\(^{-1}\). Note that this value depends on the Err\(_{\text{sys}}\) of Dominik et al. (2015) which is from 2.7 \times 10^{-2} to 10. Therefore, if the detection rate of the coalescing Pop I and Pop II high mass BH-BHs is lower than that of Pop III, we may be able to confirm the existence of Pop III star by the detection of the chirp signal and QNM to determine the chirp mass and the total mass distribution since the typical mass of Pop III BH-BH binary is much larger than those of observed Pop I BH. On the other hand, if the detection rate of the coalescing Pop I and Pop II high mass BH-BHs is higher, Pop III BH-BHs contribute only some parts of the gravitational wave events of BH-BHs. In this case, the existence of Pop III binaries will be confirmed by the investigation of the merger rate history as function of redshift by DECIGO (DECi hertz Interferometer Gravitational wave Observatory) (Seto, Kawamura & Nakamura 2001).

As for the mass ratio distribution function, if the number of the high mass ratio (i.e. near 1) increases, the number of BH-BH probably increase. On the other hand, if the number of the low mass ratio increase, the number of BH-BH will decrease while the number of NS-BH will increase. We will check the dependence of these two initial distribution functions in future work. Development in the simulation

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This table shows the detection rates of Pop III BH-BHs for under100 cases with the optimistic core-merger criterion. The first column shows the name of the model. The second column shows the detection rate only by the inspiral chirp signal. The third, fourth and the fifth columns show the detection rate only by the quasi normal mode (QNM) with Kerr parameter $a/M = 0.70$, the detection rate by the quadrature sum of the inspiral chirp signal and the QNM with $a/M = 0.70$ and the detection rate by the linear sum of the inspiral chirp signal and the QNM with $a/M = 0.70$, respectively. The sixth, seventh and the eighth columns show the detection rates only by the QNM with $a/M = 0.98$, the detection rate by the quadrature sum of the inspiral chirp signal and the QNM with $a/M = 0.98$ and the detection rate by the linear sum of the inspiral chirp signal and the QNM with $a/M = 0.98$, respectively. When signal-to-noise ratio of event that is calculated by matche d filtering equation, over threshold $S/N = 8$, the event is detected. All the rates are based on 1000 years Monte Carlo simulations.

| 14models | Inspiral $S/N \geq 8$ | QNM(0.70) $S/N \geq 8$ | Linear sum of Inspiral and QNM(0.70) $S/N \geq 8$ | Quadrature sum of Inspiral and QNM(0.70) $S/N \geq 8$ | Linear sum of Inspiral and QNM(0.98) $S/N \geq 8$ |
|-----------|----------------------|-------------------------|---------------------------------|---------------------------------|-------------------------|
| our standard | 85747 | 67337 | 180806 | 392542 | 10680 |
| IMF: logflat | 74764 | 43130 | 139705 | 305428 | 6524 |
| IMF: Salpeter | 47055 | 16153 | 74899 | 156804 | 53960 |
| IEF: const. | 80947 | 65225 | 173059 | 380515 | 10680 |
| IEF: $e^{-0.5}$ | 77922 | 63050 | 167289 | 367922 | 10680 |
| kick 100 km s$^{-1}$ | 66901 | 57590 | 148370 | 330875 | 9243 |
| kick 300 km s$^{-1}$ | 10696 | 14166 | 36655 | 80711 | 2303 |
| $\alpha = 0.01$ | 88090 | 13327 | 37755 | 82585 | 23802 |
| $\alpha = 0.1$ | 67189 | 73207 | 165215 | 366096 | 10342 |
| $\alpha = 10$ | 74276 | 59901 | 156419 | 336621 | 10342 |
| $\beta = 0$ | 85578 | 67608 | 180362 | 393241 | 10990 |
| $\beta = 0.5$ | 117738 | 74535 | 229389 | 498512 | 11015 |
| $\beta = 1$ | 58823 | 45926 | 123468 | 268790 | 7353 |
| Worst | 4922 | 2322 | 8449 | 17433 | 316 |

Table 19. over100 cases with the optimistic core-merger criterion, 1000 years, $S/N \geq 8$

The same as Table 18 but for over100 cases with the optimistic core-merger criterion.

| 14models | Inspiral $S/N \geq 8$ | QNM(0.70) $S/N \geq 8$ | Linear sum of Inspiral and QNM(0.70) $S/N \geq 8$ | Quadrature sum of Inspiral and QNM(0.70) $S/N \geq 8$ | Linear sum of Inspiral and QNM(0.98) $S/N \geq 8$ |
|-----------|----------------------|-------------------------|---------------------------------|---------------------------------|-------------------------|
| our standard | 84174 | 218283 | 332477 | 61583 | 41503 |
| IMF: logflat | 73805 | 130893 | 229061 | 435444 | 24041 |
| IMF: Salpeter | 47216 | 39218 | 98543 | 191702 | 6890 |
| IEF: const. | 79623 | 222826 | 332472 | 609477 | 42784 |
| IEF: $e^{-0.5}$ | 77427 | 221620 | 327610 | 598925 | 43430 |
| kick 100 km s$^{-1}$ | 66563 | 151555 | 244415 | 473699 | 27246 |
| kick 300 km s$^{-1}$ | 16843 | 40830 | 63663 | 123135 | 7334 |
| $\alpha = 0.01$ | 16864 | 31946 | 55038 | 111547 | 5170 |
| $\alpha = 0.1$ | 62295 | 169260 | 257276 | 499826 | 21764 |
| $\alpha = 10$ | 75876 | 181513 | 284367 | 531845 | 32400 |
| $\beta = 0$ | 84596 | 218993 | 33565 | 613502 | 41842 |
| $\beta = 0.5$ | 118459 | 98205 | 255012 | 540234 | 15457 |
| $\beta = 1$ | 58887 | 46219 | 123729 | 268745 | 7482 |
| Worst | 4873 | 2361 | 8388 | 17640 | 330 |
may make it possible to clarify initial conditions of Pop III binary.

The Pop III star formation rate will determine the merger rate. Our using Pop III SFR (de Souza et al. 2011) is Fig. 21. There are some arguments on Pop III SFR besides de Souza et al. (2011). For example, Johnson et al. (2013) simulated the Pop III SFR by the smooth particle hydrodynamics (SPH) simulations. In their simulations, the peak value of Pop III SFR is from $\sim 10^{-3.7} \text{M}_\odot \text{yr}^{-1} \text{Mpc}^{-3}$ to $\sim 10^{-3} \text{M}_\odot \text{yr}^{-1} \text{Mpc}^{-3}$ at $z \sim 10$. The difference between the high value and the low value comes from without Lyman-Werner (LW) feedback or with LW feedback. Note that the result of these simulations might change if the metal pollution model changes. On the other hand, Kulkarni et al. (2014) and Yajima & Kochchar (2015) studied the Pop III SFR by considering the contribution of Pop III stars to cosmic reionization. Kulkarni et al. (2014) suggest that the peak value of Pop III SFR is from $\sim 10^{-4.2} \text{M}_\odot \text{yr}^{-1} \text{Mpc}^{-3}$ to
Table 22. over100 cases with the conservative core-merger criterion, 1000 years, S/N ≥ 8

The same as Table 18 but for over100 cases with the conservative core-merger criterion

| 14models | Inspiral S/N > 8 | QNM(0.70) S/N > 8 | Linear sum of Inspiral and QNM(0.70) S/N > 8 | Quadrature sum of Inspiral and QNM(0.70) S/N > 8 | QNM(0.98) S/N > 8 | Linear sum of Inspiral and QNM(0.98) S/N > 8 | Quadrature sum of Inspiral and QNM(0.98) S/N > 8 |
|----------|-----------------|------------------|----------------------------------|----------------------------------|-----------------|----------------------------------|----------------------------------|
| our standard | 84864 | 214329 | 329356 | 608170 | 41205 | 142681 | 275555 |
| IMF:logflat | 76062 | 128962 | 227785 | 434489 | 23497 | 110212 | 265488 |
| IMF:Salpeter | 47104 | 38994 | 98198 | 191541 | 6847 | 59142 | 26943 |
| IEF:const. | 80521 | 219273 | 330324 | 606041 | 42209 | 139338 | 271312 |
| IEF:e^{-0.5} | 77318 | 218702 | 325574 | 595975 | 42196 | 135838 | 26943 |
| kick 100 km s^{-1} | 67736 | 150146 | 243859 | 474159 | 26286 | 108305 | 216333 |
| kick 300 km s^{-1} | 17139 | 40582 | 64028 | 122219 | 7137 | 27864 | 55602 |
| αλ=0.01 | 14194 | 57924 | 78827 | 153302 | 9617 | 28113 | 59044 |
| αλ=0.1 | 66817 | 166466 | 259438 | 497156 | 27926 | 109303 | 219794 |
| αλ=10 | 76476 | 182640 | 285088 | 532370 | 15356 | 152569 | 286062 |
| β=0 | 84415 | 213727 | 328158 | 607593 | 40698 | 141795 | 274286 |
| β=0.5 | 119110 | 97920 | 255288 | 540990 | 15356 | 152569 | 286062 |
| β=1 | 59051 | 45600 | 123150 | 267169 | 7314 | 75779 | 145191 |
| Worst | 4819 | 2337 | 8335 | 17592 | 333 | 5702 | 10007 |

~ 10^{-1.3} M⊙ yr^{-1} Mpc^{-3} at z ~ 10. The difference between the high value and the low value comes from that of the metal pollution timescale. While Yajima & Khochfar (2015) suggests that the peak value of Pop III SFR is ~ 10^{-3} M⊙ yr^{-1} Mpc^{-3} at z ~ 15 in order to recover the observed Thomson scattering optical depth of the cosmic microwave background. The SFR of Pop III is controversial now. However, these estimated value of the SFR tell us that except for the worst model, we might expect the detection of GW from massive Pop III BH-BH near future.

The present merger rate density which is calculated only from SFR between z=7 to z=11 is about 50% of the whole merger rate density. Therefore SFR_{peak} is a good parameter in our adopted model of SFR. However, there is Pop III SFR whose peak region is higher redshift than z ~ 10 such as the models in Yajima et al. (2015). In such a case, we have to

Table 23. 140 cases with the conservative core-merger criterion, 1000 years, S/N ≥ 8

The same as Table 18 but for 140 cases with the conservative core-merger criterion

| 14models | Inspiral S/N > 8 | QNM(0.70) S/N > 8 | Linear sum of Inspiral and QNM(0.70) S/N > 8 | Quadrature sum of Inspiral and QNM(0.70) S/N > 8 | QNM(0.98) S/N > 8 | Linear sum of Inspiral and QNM(0.98) S/N > 8 | Quadrature sum of Inspiral and QNM(0.98) S/N > 8 |
|----------|-----------------|------------------|----------------------------------|----------------------------------|-----------------|----------------------------------|----------------------------------|
| our standard | 55974 | 538382 | 616793 | 893170 | 249666 | 321038 | 489786 |
| IMF:logflat | 62224 | 337179 | 420100 | 644264 | 146827 | 221150 | 346650 |
| IMF:Salpeter | 45352 | 97882 | 155975 | 265370 | 37519 | 88623 | 142980 |
| IEF:const. | 53720 | 528461 | 603514 | 869645 | 239380 | 307730 | 470677 |
| IEF:e^{-0.5} | 51664 | 515375 | 587375 | 845467 | 295251 | 452615 |
| kick 100 km s^{-1} | 45815 | 506994 | 573221 | 830258 | 184386 | 244263 | 399027 |
| kick 300 km s^{-1} | 11241 | 135797 | 152230 | 267169 | 7314 | 75779 | 145191 |
| αλ=0.01 | 7431 | 62666 | 74189 | 123411 | 14129 | 24092 | 47107 |
| αλ=0.1 | 47406 | 366456 | 437298 | 710775 | 131247 | 176176 | 316298 |
| αλ=10 | 46402 | 395992 | 458184 | 710775 | 131247 | 176176 | 316298 |
| β=0 | 55773 | 532378 | 617239 | 892483 | 295251 | 452615 |
| β=0.5 | 74932 | 619682 | 722387 | 101940 | 277705 | 370639 | 570146 |
| β=1 | 34014 | 269616 | 317328 | 480619 | 79988 | 123652 | 216096 |
| Worst | 4276 | 2337 | 8335 | 17592 | 333 | 5702 | 10007 |
Table 24. under100 cases with optimistic core-merger criterion, 1000 years, $S/N \geq 35$

This table shows the detection rates of Pop III BH-BHs for under100 cases with the optimistic core-merger criterion. The first column shows the name of the model. The second column shows the detection rate only by the inspiral chirp signal. The third, fourth and the fifth columns show the detection rate only by the quasi normal mode (QNM) with Kerr parameter $a/M = 0.70$, the detection rate by the quadrature sum of the inspiral chirp signal and the QNM with $a/M = 0.70$ and the detection rate by the linear sum of the inspiral chirp signal and the QNM with $a/M = 0.70$, respectively. The sixth, seventh and the eighth columns show the detection rates only by the QNM with $a/M = 0.98$, the detection rate by the quadrature sum of the inspiral chirp signal and the QNM with $a/M = 0.98$ and the detection rate by the linear sum of the inspiral chirp signal and the QNM with $a/M = 0.98$, respectively. When signal-to-noise ratio of event that is calculated by matched filtering equation, over threshold $S/N = 35$, the event is detected. All the rates are based on 1000 years Monte Carlo simulations.

| Models       | Inspiral S/N≥35 (1000 yrs) | QNM(0.70) S/N≥35 (1000 yrs) | Quadrature sum of Inspiral and QNM(0.70) S/N≥35 (1000 yrs) | Linear sum of Inspiral and QNM(0.70) S/N≥35 (1000 yrs) | Quadrature sum of Inspiral and QNM(0.98) S/N≥35 (1000 yrs) | Linear sum of Inspiral and QNM(0.98) S/N≥35 (1000 yrs) |
|--------------|-----------------------------|-----------------------------|-------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
| our standard | 2166                        | 433                         | 4334                                                       | 594                                                       | 2156                                                       | 3590                                                       |
| IMF:logflat  | 1918                        | 326                         | 2591                                                       | 5544                                                      | 2062                                                       | 3292                                                       |
| IMF:Salpeter | 1087                        | 102                         | 1353                                                       | 2719                                                      | 1150                                                       | 1711                                                       |
| IEF:const.   | 2016                        | 476                         | 2945                                                       | 6679                                                      | 2238                                                       | 3732                                                       |
| IEF: e^{-0.5}| 1937                        | 433                         | 2855                                                       | 6362                                                      | 2156                                                       | 3590                                                       |
| kick 100 km/s | 1636                        | 404                         | 2456                                                       | 5623                                                      | 1850                                                       | 3123                                                       |
| kick 300 km/s | 387                         | 91                          | 589                                                        | 1342                                                      | 434                                                        | 732                                                        |
| $\alpha=0.01$ | 440                        | 96                          | 659                                                        | 1425                                                      | 485                                                        | 832                                                        |
| $\alpha=0.1$ | 1795                        | 520                         | 2882                                                       | 6717                                                      | 2050                                                       | 3569                                                       |
| $\alpha=10$  | 1829                        | 436                         | 2733                                                       | 5968                                                      | 2027                                                       | 3407                                                       |
| $\beta=0$    | 2118                        | 468                         | 3102                                                       | 7220                                                      | 2345                                                       | 4011                                                       |
| $\beta=0.5$  | 3176                        | 557                         | 4379                                                       | 9340                                                      | 3447                                                       | 5512                                                       |
| $\beta=1$    | 1574                        | 352                         | 2315                                                       | 5057                                                      | 1757                                                       | 2917                                                       |
| Worst        | 111                         | 21                          | 146                                                        | 297                                                       | 4                                           | 118                                                        |

Table 25. over100 cases with the optimistic core-merger criterion, 1000 years, $S/N \geq 35$

The same as Table 24 but for over100 cases with the optimistic core-merger criterion.

| Models       | Inspiral S/N≥35 (1000 yrs) | QNM(0.70) S/N≥35 (1000 yrs) | Quadrature sum of Inspiral and QNM(0.70) S/N≥35 (1000 yrs) | Linear sum of Inspiral and QNM(0.70) S/N≥35 (1000 yrs) | Quadrature sum of Inspiral and QNM(0.98) S/N≥35 (1000 yrs) | Linear sum of Inspiral and QNM(0.98) S/N≥35 (1000 yrs) |
|--------------|-----------------------------|-----------------------------|-------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
| our standard | 2122                        | 595                         | 3707                                                       | 8269                                                      | 2287                                                       | 4693                                                       |
| IMF:logflat  | 1866                        | 543                         | 2867                                                       | 6241                                                      | 117                                                        | 3667                                                       |
| IMF:Salpeter | 1098                        | 174                         | 1485                                                       | 2976                                                      | 136                                                        | 1881                                                       |
| IEF:const.   | 1939                        | 953                         | 3475                                                       | 7756                                                      | 244                                                        | 4413                                                       |
| IEF: e^{-0.5}| 1914                        | 928                         | 3464                                                       | 7738                                                      | 216                                                        | 4408                                                       |
| kick 100 km/s | 1677                        | 660                         | 2850                                                       | 6442                                                      | 175                                                        | 3642                                                       |
| kick 300 km/s | 382                         | 172                         | 668                                                        | 1512                                                      | 40                                                         | 863                                                        |
| $\alpha=0.01$ | 410                        | 158                         | 689                                                        | 1454                                                      | 35                                                         | 839                                                        |
| $\alpha=0.1$ | 1692                        | 780                         | 3054                                                       | 6900                                                      | 159                                                        | 3923                                                       |
| $\alpha=10$  | 1762                        | 805                         | 3092                                                       | 6885                                                      | 197                                                        | 3900                                                       |
| $\beta=0$    | 2247                        | 1002                        | 3808                                                       | 8397                                                      | 266                                                        | 4799                                                       |
| $\beta=0.5$  | 2981                        | 637                         | 4349                                                       | 9394                                                      | 95                                                         | 5569                                                       |
| $\beta=1$    | 1554                        | 341                         | 2294                                                       | 5071                                                      | 53                                                         | 2909                                                       |
| Worst        | 104                         | 13                          | 148                                                        | 331                                                       | 2                                           | 116                                                        |
Table 26. 140 cases with the optimistic core-merger criterion, 1000 years, S/N $\geq 35$

The same as Table 24 but for 140 cases with the optimistic core-merger criterion

| 14models | Inspiral $S/N\geq 35$ | QNM(0.70) $S/N\geq 35$ | Linear sum of Inspiral and QNM(0.70) $S/N\geq 35$ | Quadrature sum of Inspiral and QNM(0.70) $S/N\geq 35$ | Linear sum of Inspiral and QNM(0.98) $S/N\geq 35$ | Quadrature sum of Inspiral and QNM(0.98) $S/N\geq 35$ |
|-----------|----------------------|------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
|           | [1/1000 yrs]         | [1/1000 yrs]           | [1/1000 yrs]                                  | [1/1000 yrs]                                  | [1/1000 yrs]                                  | [1/1000 yrs]                                  |
| our standard | 1410 4487 6553 | 12149 1017 2757 5342 | 2757 5342 |
| IMF:logflat | 1580 2585 4655 | 8876 572 2467 4371 | 2467 4371 |
| IMF:Salpeter | 955 624 | 1823 3652 | 3652 2018 |
| IMF:const. | 1387 4425 6146 | 11722 954 2677 5252 | 954 2677 5252 |
| IMF:$e^{-0.5}$ | 1318 4220 | 11353 966 2566 5002 | 966 2566 5002 |
| kick $100$ km s$^{-1}$ | 1040 2921 4415 8824 735 2028 4072 | 2028 4072 |
| kick $300$ km s$^{-1}$ | 288 968 1370 2489 206 538 1064 | 538 1064 |
| $\alpha\lambda=0.01$ | 236 139 436 933 44 318 556 | 933 44 318 556 |
| $\alpha\lambda=0.1$ | 1187 1606 3331 7358 397 1823 3789 | 7358 397 1823 3789 |
| $\alpha\lambda=10$ | 1210 4366 6072 10415 949 2418 4404 | 10415 949 2418 4404 |
| $\beta=0$ | 1381 4545 6617 12117 962 2718 5292 | 12117 962 2718 5292 |
| $\beta=0.5$ | 1904 4858 7457 14128 956 3223 6230 | 14128 956 3223 6230 |
| $\beta=1$ | 880 1377 2633 5571 393 1470 2877 | 5571 393 1470 2877 |
| Worst | 86 19 115 248 3 99 147 | 248 3 99 147 |

Table 27. under100 cases with the conservative core-merger criterion, 1000 years, S/N $\geq 35$

The same as Table 24 but for under100 cases with the conservative core-merger criterion

| 14models | Inspiral $S/N\geq 35$ | QNM(0.70) $S/N\geq 35$ | Linear sum of Inspiral and QNM(0.70) $S/N\geq 35$ | Quadrature sum of Inspiral and QNM(0.70) $S/N\geq 35$ | Linear sum of Inspiral and QNM(0.98) $S/N\geq 35$ | Quadrature sum of Inspiral and QNM(0.98) $S/N\geq 35$ |
|-----------|----------------------|------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
|           | [1/1000 yrs]         | [1/1000 yrs]           | [1/1000 yrs]                                  | [1/1000 yrs]                                  | [1/1000 yrs]                                  | [1/1000 yrs]                                  |
| our standard | 2109 498 3163 | 7092 54 2350 3986 | 2350 3986 |
| IMF:logflat | 1863 302 2646 | 5530 40 2034 3307 | 2034 3307 |
| IMF:Salpeter | 1132 132 3331 | 7358 397 1823 3789 | 7358 397 1823 3789 |
| IMF:const. | 2012 439 3016 | 6707 67 2241 3816 | 6707 67 2241 3816 |
| IMF:$e^{-0.5}$ | 1905 438 2792 | 6365 53 2122 3519 | 6365 53 2122 3519 |
| kick $100$ km s$^{-1}$ | 1641 384 2492 | 5613 61 1825 3131 | 5613 61 1825 3131 |
| kick $300$ km s$^{-1}$ | 376 80 565 | 1278 11 417 720 | 1278 11 417 720 |
| $\alpha\lambda=0.01$ | 396 164 670 | 1584 16 467 837 | 1584 16 467 837 |
| $\alpha\lambda=0.1$ | 1962 525 2939 | 6604 69 2191 3714 | 6604 69 2191 3714 |
| $\alpha\lambda=10$ | 1757 375 2551 | 5695 52 1913 3193 | 5695 52 1913 3193 |
| $\beta=0$ | 2221 521 3213 | 7150 64 2458 4108 | 7150 64 2458 4108 |
| $\beta=0.5$ | 3168 555 4401 | 9282 73 3454 5532 | 9282 73 3454 5532 |
| $\beta=1$ | 1506 330 2223 | 4925 48 1675 2843 | 4925 48 1675 2843 |
| Worst | 80 13 123 | 301 1 89 170 | 301 1 89 170 |

consider $\int SFR(z)dz$ in order to compare the dependence on the SFR.

We discuss also the binary fraction of Pop III. Recently, the resolution of the multi-dimension simulation becomes so high that the fragmentation of disk at the Pop III stellar formation can be studied like in (Clark et al. 2011). The recent cosmological hydrodynamics simulation (Susa et al. 2014) suggests that the binary fraction is about 50%. Thus, we use $f_b = 1/2$ since the total number of the stars in the binary is half of the total number of the stars in the binary. However, the binary fraction is controversial. Thus, we express the uncertainty of the binary fraction of Pop III as $f_b$. From Fig. 24, the peak of the rate of the merger of Pop III BH binary is around $z = 9$ so that the observed frequency of the chirp signal and the quasi normal mode are $\sim 10$ times small. To detect such a low frequency gravitational wave, DECIGO (Seto, Kawamura & Nakamura 2001) will be most appropriate. When DECIGO starts an observation around
Table 28. over100 cases with the conservative core-merger criterion, 1000 years, S/N ≥ 35

The same as Table 24 but for over100 cases with the conservative core-merger criterion.

| 14models | Inspiral S/N ≥ 35 [1/1000 yrs] | QNM(0.70) | QNM(0.70) | Linear sum of Inspiral and QNM(0.70) S/N ≥ 35 [1/1000 yrs] | Linear sum of Inspiral and QNM(0.70) S/N ≥ 35 [1/1000 yrs] |
|-----------|---------------------------------|-----------|-----------|-------------------------------------------------|-------------------------------------------------|
| our standard | 2103                            | 936       | 3682      | 8228                                           | 243                                             |
| IMF:logflat | 1918                            | 598       | 2945      | 6221                                           | 155                                             |
| IMF:Salpeter | 1107                           | 210       | 1483      | 2977                                           | 42                                              |
| IEF:const. | 1970                            | 884       | 3410      | 7700                                           | 224                                             |
| IEF:e^{-0.5} | 1881                           | 922       | 3364      | 7568                                           | 229                                             |
| kick 100 km s^{-1} | 1592                      | 690       | 2786      | 6430                                           | 147                                             |
| kick 300 km s^{-1} | 393                        | 172       | 666       | 1524                                           | 38                                              |
| αλ=0.01 | 385                             | 287       | 783       | 1715                                           | 59                                              |
| αλ=0.1 | 1758                            | 752       | 3030      | 6862                                           | 183                                             |
| αλ=10 | 1810                            | 834       | 3184      | 7080                                           | 216                                             |
| β=0 | 2157                            | 984       | 3707      | 8247                                           | 248                                             |
| β=0.5 | 3067                            | 677       | 4413      | 9573                                           | 71                                              |
| β=1 | 1554                            | 336       | 2249      | 4984                                           | 43                                              |
| Worst | 111                             | 18        | 160       | 358                                            | 4                                               |

Table 29. 140 cases with the conservative core-merger criterion, 1000 years, S/N ≥ 35

The same as Table 24 but for 140 cases with the conservative core-merger criterion.

| 14models | Inspiral S/N ≥ 35 [1/1000 yrs] | QNM(0.70) | QNM(0.70) | Linear sum of Inspiral and QNM(0.70) S/N ≥ 35 [1/1000 yrs] | Linear sum of Inspiral and QNM(0.70) S/N ≥ 35 [1/1000 yrs] |
|-----------|---------------------------------|-----------|-----------|-------------------------------------------------|-------------------------------------------------|
| our standard | 1373                            | 4983      | 6979      | 12584                                          | 1140                                           |
| IMF:logflat | 1548                            | 2862      | 4879      | 9012                                           | 614                                            |
| IMF:Salpeter | 1030                           | 714       | 1977      | 3769                                           | 153                                            |
| IEF:const. | 1336                            | 4820      | 6796      | 12216                                          | 1095                                          |
| IEF:e^{-0.5} | 1354                           | 4614      | 6574      | 11850                                          | 1029                                          |
| kick 100 km s^{-1} | 1144                      | 2951      | 4591      | 9106                                           | 760                                            |
| kick 300 km s^{-1} | 313                        | 1192      | 1616      | 2749                                           | 284                                            |
| αλ=0.01 | 170                             | 244       | 480       | 1053                                           | 71                                             |
| αλ=0.1 | 1167                            | 2007      | 3695      | 7679                                           | 522                                            |
| αλ=10 | 1144                            | 4244      | 5915      | 10287                                          | 854                                            |
| β=0 | 1432                            | 5100      | 7136      | 12715                                          | 1189                                          |
| β=0.5 | 1786                            | 5209      | 7792      | 14520                                          | 1085                                          |
| β=1 | 811                             | 1424      | 2596      | 5658                                           | 430                                            |
| Worst | 99                              | 35        | 151       | 313                                            | 2                                              |

2030, we can detect gravitational waves from Pop III BH-BHs which merged at z ~ 9. Thus, we might identify the peak of Fig. 24. The peak of Fig. 24 depends not only on binary parameters, but also on the Pop III SFR. Therefore, we might get the information of Pop III SFR.

In this paper, the merging NS-NS and NS-BH are not considered because they are negligibly small in number in almost all models. However, in some models they are not so. Since they are the candidates of the short gamma ray burst (GRB), the high redshift observation of GRB by Hi-z GUNDAM (Yonetoku et al. 2014) might be possible. Thus, the merging NS-NS and NS-BH might also be useful for Pop III binary parameter studying.
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