Fair Shares: Feasibility, Domination and Incentives

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We consider fair allocation of a set $M$ of indivisible goods to $n$ equally-entitled agents, with no monetary transfers. Every agent $i$ has a valuation function $v_i$ from some given class of valuation functions. A share $s$ is a function that maps a pair $(v_i, n)$ to a non-negative value, with the interpretation that if an allocation of $M$ to $n$ agents fails to give agent $i$ a bundle of value at least equal to $s(v_i, n)$, this serves as evidence that the allocation is not fair towards $i$. For such an interpretation to make sense, we would like the share to be feasible, meaning that for any valuations in the class, there is an allocation that gives every agent at least her share. The maximin share (MMS) was a natural candidate for a feasible share for additive valuations. However, Kurokawa, Procaccia and Wang [2018] show that it is not feasible.

We initiate a systematic study of the family of feasible shares. We say that a share is self-maximizing if truth-telling maximizes the implied guarantee (the worse true value of any bundle that gives the share with respect to the report). We show that every feasible share is dominated by some self-maximizing and feasible share. We seek to identify those self-maximizing feasible shares that are polynomial time computable, and offer the highest share values. We show that a SM-dominating feasible share – one that dominates every self-maximizing (SM) feasible share – does not exist for additive valuations (and beyond). Consequently, we relax the domination property to that of domination up to a multiplicative factor of $\rho$ (called $\rho$-dominating).

For additive valuations we present shares that are feasible, self-maximizing and polynomial-time computable. For $n$ agents we present such a share that is $\frac{3n}{5n-1}$-dominating, and is $\frac{3}{5}$-dominating when $n \leq 4$. For two agents we present such a share that is $(1 - \epsilon)$-dominating. Moreover, for each of these shares we present a polynomial time algorithm that computes allocations that give every agent at least her share.

CCS Concepts:
• Theory of computation → Algorithmic mechanism design;
• Applied computing → Economics.

Additional Key Words and Phrases: Fair Division, Indivisible Goods, Maximin Share, Self-Maximizing

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