The London field in bulk layered superconductors

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Abstract

In a composite superconductor in uniform rotation, the London field strength at equilibrium is given by the usual expression \( B_L = 2m \omega / e \); here \( m \) corresponds to the bare electron mass, although the effective mass \( m^* \) can be different in different layers. In the presence of strong angular accelerations, however, transient phases and differences in \( m^* \) can become relevant. In particular, if the layers are orthogonal to the rotation axis and one of them is much thinner than the others, the superconductor may enter metastable states characterized by intense “slide” supercurrents in this layer. We investigate this phenomenon theoretically and with reference to current experiments.

Key words: Rotating superconductors, London field, high-Tc superconductors
1. Introduction

It is known that inside a rotating superconductor (SC) the magnetic field does not vanish, but takes a certain value \(B_L\) called “London field strength”; this is given, in terms of the electron charge \(e\), the electron mass \(m\) and the angular velocity \(\omega\) of the SC, by a very simple expression [1, 2], namely

\[
B_L = 2m\omega/e
\]  
(1)

This relation holds for any value of the external magnetic field, including the case \(B_{ext} = 0\). The strength of \(B_L\) is determined by the London equation (or better by a generalization of it—see [3]), written in the reference frame co-rotating with the SC.

The London field can be interpreted as the field needed to give the superelectrons the same rotation velocity of the crystal lattice. While the normal electrons keep pace with the positive ions due to ohmic friction, the Cooper pairs are mechanically decoupled from the lattice and need the field \(B_L\) to move on circular orbits with frequency \(\omega\). This interpretation also justifies a simple intuitive argument [4] to explain \textit{a posteriori} the presence of the London field inside a rotating SC, with strength given by (1): it is essential that over the sample as a whole \(v_{pairs} = v_{lattice}\), because otherwise very large bulk currents would flow.

Starting in the Sixties, the London field has been measured in several experiments. In all measurements it was found that the mass \(m\) defining the strength \(B_L\) corresponds to the bare electron mass, i.e., not the effective mass \(m^*\), renormalized by the interactions with the lattice, but the mass of the free electron (apart from small relativistic corrections [5] and some further minor corrections [6]). This made possible to use precise measurements of \(B_L\) to deduce precise values of the parameters \(e\) and \(m\) (and also \(\hbar\), since in a rotating ring of area \(S\) the flux is quantized and the quanta correspond to steps \(\Delta\nu = \hbar/4mS\) in the rotation frequency) [7].

The easiest way to check that \(m\) is the bare electron mass and not the effective mass is by starting from the full Hamiltonian of the solid including the electrons, the ions, and all their interactions. Transforming to the rotating frame, one finds that each electron “feels” the additional vector potential

\[
\mathbf{A}_\omega(\mathbf{r}) = m/e(\omega \times \mathbf{r})
\]

(2)
(with field \((m/e)\omega\)), regardless of its complicated interaction with the ions and the other electrons.
2. Acceleration-deceleration phases. Variations of $m^*$ in a layered disk

Within this well-established framework, two important variations can occur. Let us consider cases when

(i) The rotation velocity of the SC is suddenly changed

or

(ii) The SC has a composite structure, being made of two or more parts with different chemical and crystal properties and different values of the effective electron mass $m^*$,

or both (i) and (ii).

As we shall show in the following, the combined effect of accelerations-decelerations and inhomogeneities in the material can spoil the dynamical equilibrium between superelectrons and lattice ions usually associated with the London field.

When this happens, the relative velocity of the superelectrons with respect to the lattice ions can indeed lead to strong “slide” supercurrents—although we shall see that these are not bulk currents, but are always confined to thin layers.

It is quite straightforward to evaluate the order of magnitude of these currents. Let us consider a ceramic SC like YBCO. A typical value of the London length $\lambda$ in the conduction planes $ab$ is $\lambda = 0.2 \, \mu m$. From the expression for the London length $\lambda = \sqrt{m^*/(\mu_0 n_s e^2)}$ one finds for the density $n_s$ of superconducting charge carriers $n_s \sim 10^{27} \, m^{-3}$. In a material with critical current density of the order of $10^8 \, A/m^2$, this corresponds to an intrinsic velocity of the carriers $v_j = j/\rho = j/(en_s) \sim 0.6 \, m/s$. For comparison, in a SC rotating at 100 Hz (6000 rpm) the rotation velocity of the lattice 10 cm apart from the axis is $v_{rot} \simeq 63 \, m/s$. Therefore, in this case the slide current can be up to 100 times larger than the critical intrinsic supercurrent.

3. London-Maxwell equations for accelerating SCs

In order to achieve a better understanding of the problem, it is useful to recall first in short the theoretical ingredients employed for the description of accelerated SCs at thermodynamic

It is known for instance that the crystal structure of YBCO depends on the oxygen doping of the material. It happens quite frequently that the doping process of large ceramic samples results in portions of the samples having different oxygen content. The effective electron mass $m^*$ (typically 4-5 times larger than the bare mass) depends on this content. Compare also Section 4.
More generally, let us consider the interplay between a moving SC and the e.m. field. This includes the case of rotating SC samples, or samples which oscillate or accelerate along a line. Some of these cases were studied very early [1, 8]. A general formalism, suitable for the description of all these situations, has been given by Peng et al. [9, 10]. They proposed a unified phenomenological approach to study the electrodynamics of both an arbitrary moving SC and a SC under the influence of non-electromagnetic external forces including the Newtonian gravitation and gravitational waves. This theoretical work has provided a basis for the analysis of several experiments which exploit the London field for precise determinations of the Cooper-pair mass and the ratio $\hbar/m$ [7]; a similar analysis was applied to the readout systems of the Stanford gyroscope experiment [11].

Usually one starts from the covariant generalizations of the London or Ginzburg-Landau equations. The Ginzburg-Landau theory is needed if one wants to include non-linear effects and spatial variations in the order parameter $|\psi|^2$. If we focus on situations in which the perturbing fields and currents are so weak that $|\psi|^2 = n_s = \text{const.}$, then the Ginzburg-Landau equation reduces to the London equations which describe the motion of superelectrons. (In dealing with Type II SCs, we must further assume that the external magnetic field is zero, otherwise partial flux penetration will occur.)

In order to account for the motion of the SC, London introduced the concept that the net current should be the sum of the supercurrent and the current due to the motion of the ions. He then combined the London equations, the equations of motion of ions, and Maxwell equations to study the electrodynamics of a rotating SC in the presence of e.m. fields. More generally, denoting by $u$ and $U$ the 4-velocities of superelectrons and ions, respectively, the net electric 4-current will be

$$J^\mu = 2n_se(U^\mu - u^\mu)$$

This equation must be added to the Maxwell equations and the covariant equations of motion for the superelectrons. In the non-relativistic limit of low velocities ($v$ for the electrons, $V$ for the ions) these take the usual form

$$\frac{dv}{dt} \simeq -\frac{e}{m^*}E + \frac{1}{m^*}f$$
$$\partial \times v \simeq \frac{e}{m^*}B + \frac{1}{m^*}\int dt \partial \times f$$

implying

$$v = \frac{e}{m^*}A + \frac{1}{m^*}\int dt f$$
where $\mathbf{f}$ is the external force acting on superelectrons. In conclusion, after taking the curl and time derivative, we find

$$\partial^2 \mathbf{E} - \frac{d^2 \mathbf{E}}{dt^2} = \frac{2\mu_0 n_s e^2}{m^*} \left( \mathbf{E} - \frac{\mathbf{f}}{e} + \frac{m^*}{e} \frac{d\mathbf{V}}{dt} \right)$$  \hspace{1cm} (7)$$

$$\partial^2 \mathbf{B} - \frac{d^2 \mathbf{B}}{dt^2} = \frac{2\mu_0 n_s e^2}{m^*} \left( \mathbf{B} + \frac{1}{e} \int dt \partial \times \mathbf{f} - \frac{m^*}{e} \partial \times \mathbf{V} \right)$$  \hspace{1cm} (8)$$

By solving these equations with suitable boundary conditions and with the equations for the motion of ions and for the external forces acting on the superelectrons, one can describe the electrodynamics of an arbitrary moving SC in the presence of e.m. fields to lowest order. The forces acting on ions, which do not appear explicitly in these equations, are involved in the different expressions of $\mathbf{V}$ for various cases. We note, as stressed by Peng et al. [10], that:

- an external force $\mathbf{f}$, the electric field and the acceleration of ions are coupled;
- the curl of an external force, the magnetic field and the curl of $\mathbf{V}$ are coupled, too;
- the motion of ions, $\mathbf{V}$, and the vector magnetic potential, play the same role.

The effective electron mass $m^*$ coincides with the bare mass if the electrons are in relative equilibrium with the lattice ($\mathbf{v} = \mathbf{V}$), but in general it is different (the cyclotron frequency of charge carriers in a crystal depends on $m^*$—see for instance [12]).

Note that the London equations (4), (5)—a crucial component of the final eq.s (7), (8)—are equivalent to the minimization of the free energy $F$ of the SC; thus they hold at thermodynamical equilibrium. One might wonder if transient phases can be important in the presence of sudden accelerations. For the cases to which these equations have been previously applied [10], they seem to work well, and this means that the system is always close to equilibrium. Layered SCs in fast non-uniform rotation may represent a notable exception. In the next section we describe a first heuristic approach to this problem.

4. Heuristic description of non-equilibrium states

We have seen that a SC accelerating under the action of an external force can be described in a first approximation by the London-Maxwell equations (7), (8). All the quantities involved in these expressions are mutually coupled in a complex way, so the problem is hard to solve in general form. Moreover, in certain situations the system could be far from thermodynamic equilibrium. Let us then give here an heuristic description of the case of a rotating layered SC, taking advantage of the causal connections which are already known, namely:
(i) a bulk “slide current” due to a sudden acceleration generates, by virtue of the London equations (i.e., by minimization of the free energy of the SC), an increase in the surface supercurrent;

(ii) by virtue of the Maxwell equations, the surface supercurrent produces a London field inside the SC;

(iii) this field brings the Cooper pairs in relative equilibrium with the rotating lattice.

Let us proceed by steps and illustrate three different possible situations: the case of an homogeneous rotating bar; the case of a rotating bar with two parts of comparable thickness, made of different materials; the case of a rotating bar with a thin layer of different material.

**Case of an homogeneous rotating bar**

Suppose an homogeneous cylindrical SC bar is rotating at angular velocity $\omega$ and equilibrium has been reached, with a London field $B_L = e\omega/m$ inside the bar. Then the bar accelerates, reaching an angular velocity $\omega'$. At first the superelectrons are left behind; therefore they recover at once their effective mass and a large bulk slide current arises. After that, however, the supercurrent at the surface of the bar grows, in such a way to produce a London field $B_L^* = 2\omega'm^*/e$. This sets the superelectrons again in equilibrium with the lattice, and finally (possibly after a few further “oscillations” with respect to relative equilibrium) the field attains a new equilibrium value $B_L' = 2\omega'm/e$. The whole process is fast and usually not observed in the classical experiments involving slow and steady rotors (compare our conclusions in Section 5).

More precisely, note that in this situation the steps (i) and (ii) above can be best visualized through an analogy between the SC and an ideal solenoid, as follows:

(i') A driving electromotive force (EMF) is applied to the solenoid. Like many EMFs, it is of “thermodynamic” origin (minimization of the free energy of the SC) and takes a characteristic time $\Delta t_{EMF}$ to reach its maximum.

(ii') The current in the solenoid grows in response to the external EMF, but due to the self-inductance of the system, this growth takes a characteristic time $\Delta t_{induction}$.

The total time required to reach the new equilibrium configuration is thus of the order of $\Delta t_{equilibrium} = \Delta t_{EMF} + \Delta t_{induction}$. If this is much smaller than the characteristic acceleration time, then the system will just pass through a sequence of equilibrium states.
In the following, however, we shall consider “sudden accelerations”, with characteristic times smaller than $\Delta t_{equilibrium}$.

**Case of a rotating bar with two parts of comparable thickness**

Let us next consider a cylindrical bar made of two parts, 1 and 2, of comparable thickness (see figure, A). The material is different in the two parts, and so is the effective mass of the electrons. Suppose the system is initially in equilibrium: the bar rotates with constant angular velocity around its vertical axis and the London field is the same in both parts, corresponding to the bare electron mass ($B_{L1} = B_{L2} = e\omega/m$).

If the angular velocity is suddenly increased, a transient phase will follow. The skin supercurrents $j_1$ and $j_2$ must increase, too, in order to produce a stronger London field. In the meanwhile, the superelectrons are unable to follow the rotation frequency of the lattice, and are in relative motion with respect to it.

The magnetic field needed to bring the electrons again to rest with respect to the lattice is different in the two parts, because the effective masses are different. The same is true for the skin supercurrents $j_1$ and $j_2$. Like in the previous case, there will be some oscillations around the relative equilibrium positions, but soon a new state is reached (if there are no further accelerations), with $B'_{L1} = B'_{L2} = e\omega'/m$. This is clearly the state with minimum total free energy $F = F_1 + F_2$.

**Case of a rotating bar with a thin layer of different material**

Finally, we consider the previous case in the limit when the Part 2 of the cylindrical bar is much thinner than Part 1 (see figure). Let us also suppose that $m_2^* > m_1^*$, i.e., the effective electron mass is larger in 1 than in 2. It is easy to see that after a sudden acceleration, the Part 2 cannot reach a new equilibrium situation, but remains in a sort of metastable state.

In fact, following the acceleration the skin supercurrent $j_1$ increases until the London field $B_{L1}$ brings again the superelectrons in 1 in relative equilibrium with the lattice (possibly after some oscillations); this field, however, is not strong enough to establish relative equilibrium in 2, where the effective electron mass is larger.

Note that being the Part 2 very thin (and much thinner than the radius of the bar), the magnetic field in it cannot be substantially different from $B_{L1}$. For the same reason, the free energy $F_2$ gives a negligible contribution to the total free energy, so while $F_1$ must be at a minimum and no bulk slide current can exist in 1, such a current can indeed be present in 2 in the circumstances we are considering.
Also note that the “feedback” magnetic field generated by this superficial current lies in a plane orthogonal to the bar axis; therefore it does not tend to compensate for the insufficient London field and does not oppose to the surface current.

In practice, a thin layer like the Part 2 considered above can be present in a SC not only because of intentional differences in the oxygen doping of the material, but also for other reasons. For instance, the bulk of the material might have been subjected to a melting treatment, while the base was less affected because in contact with a coolant.

The essential point, for the anomalous behavior described above to occur, is that the crystal structure of Part 2 must be different from that of Part 1, and the effective electron mass larger. This behavior could also depend on the temperature, because the different crystal structures of parts 1 and 2 could imply different critical temperatures, and typically we expect $T_{c2} < T_{c1}$. In this case, the metastable states will be most relevant at temperatures $T$ such that $T_{c2} < T < T_{c1}$.

5. Conclusions

In this work, after recalling in Section 3 the Maxwell-London equations for the general case of a SC in accelerated motion, we have set out in Section 4 an heuristic approach to the case of a layered SC in fast non-uniform rotation.

We have seen that in this situation the system can enter non-equilibrium states. In particular, if one of the layers is much thinner than the others, and the effective electron mass in it larger, “slide” surface currents can arise, with density higher than the critical density $j_c$ of the material. (Compare the estimate given in the Introduction, which yields $j \sim 100j_c$ for a 100% difference between the rotation velocity of the lattice and that of the superconducting carriers; for smaller differences, $j/j_c$ varies in direct ratio).

This phenomenon is interesting in itself, but also because in some experiments involving rotating SCs [13] the real operating conditions are not far from those considered here in principle: namely one has large ceramic disks (10-30 cm in diameter), rotating at frequencies of thousands of rpm, with acceleration and braking phases during which the rotation frequency varies by some % in a few seconds; moreover, these large superconducting samples are often made by several layers, having different crystal structure and oxygen doping. (For comparison, the classical experiments [7] involve rotors with a maximum size of $\sim 5$ cm and maximum rotation frequencies of $\sim 5$ Hz, driven by steady gas flows).

The possible presence, in these systems, of large surface currents like those predicted by
our analysis, could be checked directly through magneto-optical techniques \cite{14}. Alternatively, one could look for indirect evidence, for instance investigating the effect of these anomalous currents on the material that supports them.

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FIGURE CAPTION

Fig. 1 - A: rotating bar with two parts of comparable thickness; B: rotating bar with a thin layer of different material. The rotation axis is vertical in the figures.
