Analysis and Simulation Research on Radial Electromagnetic Force Wave of Permanent Magnet Synchronous Motor

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Abstract. Aiming at the problems of vibration and noise of permanent magnet synchronous motors, based on the principle of radial air gap magnetic density generation, a model of radial electromagnetic force waves is constructed, and the radial air gap magnetic density and diameter of permanent magnet synchronous motors are derived by analytical analysis. Analytical expression of the electromagnetic wave, to obtain the main order, frequency and amplitude of the radial electromagnetic wave. Using Maxwell 2D simulation software to establish a simulation model, verify it, prove the correctness of the proposed method, and provide a theoretical basis for further research on the vibration and noise of permanent magnet synchronous motors.

1. Introduction
Noise is ubiquitous in people's lives, which disturbs people's lives. According to research and investigation, long-term living in a noisy environment poses a serious threat to human health. Vibration and noise have increasingly become an important indicator of the quality of motors and have received much attention[1]. In the permanent magnet synchronous motor, the noise is mainly divided into three types: electromagnetic noise, aerodynamic noise and mechanical noise. Among them, electromagnetic noise is the main component of permanent magnet synchronous motor noise. It is generated by electromagnetic vibration[2]. The electromagnetic vibration is generated by the radial force wave generated by each harmonic magnetic field acting on the stator core in the air gap magnetic field of the motor. Therefore, the study of vibration and noise of permanent magnet synchronous motors should be based on the analysis of radial electromagnetic force waves.

In this paper, the permanent magnet synchronous motor is studied through analytical analysis, the expressions of radial air gap flux density and radial electromagnetic force wave are derived, and the main order, frequency doubling, and amplitude are obtained by studying the characteristics of the force wave. Through Maxwell 2D simulation software, the simulation images of the air gap magnetic density and radial force waves are mainly obtained, and the obtained data is imported into the compiled MATLAB program for further analysis in space, thereby verifying the correctness of the proposed analytical analysis method.
2. Deduction and analysis of radial electromagnetic force waves

The stator, permanent magnet magnetic potential and air gap permeability determine the magnitude of the radial electromagnetic force of the permanent magnet synchronous motor. The harmonic magnetic field generated by the magnetic potential harmonics will subject the permanent magnet motor stator core to alternating electromagnetic forces, thereby generating vibration and noise. Therefore, the electromagnetic vibration noise of the permanent magnet synchronous motor can be studied by analyzing the stator and rotor magnetic fields[3].

From Maxwell's law, the instantaneous value of the radial electromagnetic force per unit area of the air gap of the permanent magnet synchronous motor[4] can be obtained as:

$$p_r(\theta, t) = \frac{1}{2\mu_0} [b_r^2(\theta, t) - b_t^2(\theta, t)]$$  \hspace{1cm} (1)

In the formula:
$\mu_0$ -- vacuum relative permeability;
$b_r$ -- radial air gap magnetic density;
$b_t$ -- tangential air gap magnetic density.

The research shows that the air gap magnetic close component $b_t$ is much smaller than the radial component $b_r$. The radial force density is mainly determined by the radial component $b_r$ of the air gap magnetic density, and the tangential component $b_t$ can be ignored. Therefore, the radial electromagnetic force can be expressed as:

$$p_r(\theta, t) \approx \frac{1}{2\mu_0} b_r^2(\theta, t)$$  \hspace{1cm} (2)

If the core magnetic resistance is neglected, the radial air gap magnetic density can be expressed as:

$$b_r(\theta, t) \approx f(\theta, t) \lambda(\theta, t)$$  \hspace{1cm} (3)

In the formula:
$f(\theta, t)$ - air gap magnetomotive force
$\lambda(\theta, t)$ - air gap ratio magnetic permeability.

In the permanent magnet synchronous motor, due to the slotting of the stator, the air gap at the slot of the motor is relatively large, resulting in the air gap becoming uneven, and the air gap flux conductance \[5\] can be expressed as:

$$\lambda(\theta, t) = \Lambda_0 + \sum_j \Lambda_j \cos(j\theta)$$  \hspace{1cm} (4)

In the formula:
$\Lambda_0$ -- Constant component per unit air gap permeable area
$\Lambda_j$ -- The amplitude of the air gap permeable harmonic component per unit area.

When the permanent magnet synchronous motor is supplied with three-phase symmetrical sine waves, the fixed rotor magnetomotive force includes: stator magnetic field fundamental wave magnetomotive force, permanent magnet magnetic field harmonic magnetomotive force, and stator magnetic field harmonic magnetomotive force. Among them:

$$f(\theta, t) = f_0(\theta, t) + \sum_\mu f_\mu(\theta, t) + \sum_\nu f_\nu(\theta, t)$$  \hspace{1cm} (5)

In the formula:
$f_0(\theta, t)$, $f_\mu(\theta, t)$ and $f_\nu(\theta, t)$ are respectively the fundamental wave synthetic magnetomotive force, rotor permanent magnet harmonic magnetomotive force and stator harmonic magnetomotive force.

If the fundamental wave circumference is $2\pi r$ (the number of motor pole pairs is $p$, $r$ is the motor pole distance), then $f_0(\theta, t)$, $f_\mu(\theta, t)$ and $f_\nu(\theta, t)$ can be expressed as:

$$f_\nu(\theta, t) = F_\nu(\theta, t) \cos(\theta - \alpha_\nu - \phi_\nu)$$  \hspace{1cm} (6)
In the formula:

- \( F_0 \) and \( \varphi_0 \) are respectively expressed as the amplitude and initial phase angle of the fundamental wave synthesized magnetomotive force.
- \( F_v \) and \( \varphi_v \) are respectively expressed as the amplitude and initial phase angle of the stator harmonic magnetomotive force.
- \( F_\mu \) and \( \varphi_\mu \) are respectively expressed as the amplitude and initial phase angle of the harmonic magnetomotive force of the permanent magnet.

Substituting equations (4) and (5) into (3) can get the analytical expression of radial air gap magnetic density.

Substituting equations (6)-(8) into (9), the expression of air gap magnetic density is obtained

\[
b_r(\theta, t) = f(\theta, t)\lambda(\theta, t) = \left[f_0(\theta, t) + \sum_{v} f_v(\theta, t) + \sum_{\mu} f_\mu(\theta, t) \right] \times \left[\Lambda_0 + \sum_{l} \Lambda_l \cos(l\omega t)\right]
\]

\[
= \sum_{l} \Lambda_0 f_0(\theta, t) + \sum_{l} \sum_{v} f_v(\theta, t) \Lambda_l \cos(l\omega t) + \sum_{l} \sum_{\mu} \Lambda_0 f_\mu(\theta, t) \Lambda_l \cos(l\omega t) + \sum_{l} \sum_{\mu} f_\mu(\theta, t) \Lambda_l \cos(l\omega t)
\]

Substituting equations (6)-(8) into (9), the expression of air gap magnetic density is obtained

\[
b_r(\theta, t) = (1) + (2) + (3) + (4) + (5) + (6)
\]

In the formula, the air gap magnetic density components are expressed as:

\[
(1) = \sum_{l} \Lambda_0 F_0 \cos(p\theta - \alpha_l t - \varphi_0)
\]

\[
(2) = \sum_{l} \Lambda_l F_0 \cos((p \pm \ell z)\theta - \alpha_l t - \varphi_0)
\]

\[
(3) = \sum_{\mu} \Lambda_\mu F_v \cos(\mu\theta - \mu\omega_l t + p - \varphi_\mu)
\]

\[
(4) = \sum_{\nu} \Lambda_\nu F_\mu \cos(v\theta - \alpha_\nu t - \varphi_\nu)
\]

\[
(5) = \sum_{l} \sum_{\mu} \frac{\Lambda_0 F_\mu}{2} \cos((\mu \pm \ell z)\theta - \mu\omega_l t + p - \varphi_\mu)
\]

\[
(6) = \sum_{l} \sum_{\nu} \frac{\Lambda_\nu F_\mu}{2} \cos((v \pm \ell z)\theta - \alpha_\nu t - \varphi_\nu)
\]

Summarize the order and frequency of the above radial air gap magnetic density characteristics to Table 1.

| Source | Amplitude | Order | Frequency |
|--------|-----------|-------|-----------|
| (1)    | \( \Lambda_0 F_0 \) | \( p \) | \( f_0 \) |
| (2)    | \( \Lambda_l F_0 \) | \( p \pm \ell z \) | \( f_\ell \) |
| (3)    | \( \Lambda_l F_v \) | \( \mu \) | \( \mu f_\ell / p \) |
| (4)    | \( \Lambda_\nu F_\mu / 2 \) | \( \mu \pm \ell z \) | \( \mu f_\ell / p \) |
| (5)    | \( \Lambda_\nu F_\nu \) | \( \nu \) | \( f_\nu \) |
Then write formula (10) into the following matrix form:

\[ b_i(\theta, t) = \begin{bmatrix} (1) & (2) & (3) & (4) & (5) & (6) \end{bmatrix} \]

(11)

Substitute formula (11) into formula (2), The expression of the radial electromagnetic force wave magnetic density matrix is:

\[ \begin{bmatrix} p_r(\theta, t) \end{bmatrix} = \frac{1}{2\mu_0} \begin{bmatrix} b_i(\theta, t) \end{bmatrix} \begin{bmatrix} b_i(\theta, t) \end{bmatrix}^T \]

(12)

In a symmetric matrix, the main radial electromagnetic force wave components are expressed as:

Radial electromagnetic force waves generated by the interaction of the permanent magnet magnetomotive force harmonics and the constant component of the magnetic permeability

\[ (3)(3)=\sum_{n=1}^{2\pi} F_{n1} F_{n2} A_n^2 \cos\left(\mu \pm \nu \theta - (\mu \pm \nu) \phi / p - 2\phi \right) \]

Radial electromagnetic force waves generated by the interaction of the permanent magnet magnetomotive force with the constant component of the permeability and the harmonic of the permeability

\[ (3)(5)=\sum_{n=1}^{2\pi} \sum_{l=1}^{2\pi} F_{n1} F_{n2} A_n A_l \cos\left(\mu \pm \nu \pm L \theta - (\mu \pm \nu \pm L) \phi / p - 2\phi \right) \]

Radial electromagnetic force waves generated by the interaction of stator and rotor magnetomotive force and invariant components of permeability

\[ (3)(4)=\sum_{n=1}^{2\pi} F_{n1} F_{n2} A_n^2 \cos\left(\mu \pm \nu \theta - (\mu / p \pm 1) \phi - \phi_\nu \pm \phi_\nu \right) \]

Radial electromagnetic force waves generated by the interaction of stator and rotor magnetomotive force and harmonic components

\[ (3)(6)=\sum_{n=1}^{2\pi} \sum_{l=1}^{2\pi} F_{n1} F_{n2} A_n A_l \cos\left(\mu \pm \nu \pm L \theta - (\mu / p \pm 1) \phi - \phi_\nu \pm \phi_\nu \right) \]

Radial electromagnetic force waves generated by the interaction of stator magnetomotive force and invariant component of permeability

\[ (5)(5)=\sum_{n=1}^{2\pi} \sum_{l=1}^{2\pi} F_{n1} F_{n2} A_n A_l \cos\left(\mu \pm \nu \pm L \theta - (\mu / p \pm 1) \phi - \phi_\nu \pm \phi_\nu \right) \]

Radial electromagnetic force waves generated by the interaction of stator magnetomotive force and harmonic components

\[ (4)(4)=\sum_{n=1}^{2\pi} \sum_{l=1}^{2\pi} F_{n1} F_{n2} A_n A_l \cos\left(\nu_1 + \nu_2 \theta - 2\phi - 2\phi \right) \]

Radial electromagnetic force waves generated by the interaction of stator magnetomotive force, invariant components of magnetic conductance and harmonic components
\( (4)(6)=\sum_{v_t} \sum_{v_z} \frac{F_{v_t} F_{v_z} A_{v_t} A_{v_z}}{4\mu_0} \cos((v_1 + v_z \pm l_z)\theta - 2\omega_r t) \) Radial electromagnetic force waves generated by the interaction of stator magnetomotive force and permeable harmonics.

\( (6)(6)=\sum_{v_t} \sum_{v_z} \sum_{l_z} \frac{F_{v_t} F_{v_z} A_{v_t} A_{v_z}}{16\mu_0} \cos((v_1 + v_z \pm l_z \pm l_z)\theta - 2\omega_r t) \)

The other items will be expanded by analogy and will not be repeated here. From the above radial electromagnetic force wave components, according to the values of and, the main components of each radial electromagnetic force wave can be further summarized and summarized in Table 2.

| Amplitude | Order | Frequency |
|-----------|-------|-----------|
| Permanent magnet magnetomotive force harmonics and constant permeability component | \(2p_m\) | \(2nf_o\) |
| Permanent magnet magnetomotive force harmonics and permeable harmonics | \(2p_m \pm l_z\) | \(2nf_o\) |
| Stator and rotor magnetomotive force and permeable component | \(2p_m\) | \(2nf_o\) |
| Stator and rotor magnetomotive force harmonics and permeable harmonic components | \(2p_m \pm l_z\) | \(2nf_o\) |
| Stator magnetomotive force harmonics and invariant components of permeability | \(6n_{o,p} + 2\rho\) | \(2f_o\) |
| Stator magnetomotive force harmonic and permeable harmonic components | \(6n_{o,p} + 2\rho \pm l_z\) | \(2f_o\) |

3. Finite element verification

Taking a 550w permanent magnet synchronous motor as an example, the finite element simulation analysis is carried out, the radial air gap magnetic density and the radial electromagnetic force wave are analyzed to verify the correctness of the proposed analytical method. The basic parameters of the motor are shown in Table 3.

| Motor parameters | Numerical value | Motor parameters | Numerical value |
|------------------|----------------|-----------------|----------------|
| Rated power/kw   | 0.55           | Rated voltage/v | 127            |
| Stator inner diameter/mm | 75   | Stator outer diameter/mm | 120 |
| Rotor inner diameter/mm | 26   | Rotor outer diameter/mm | 74  |
| Number of pole pairs | 2     | Number of stator slots | 24  |
| Air gap length/mm | 1     | Permanent magnet material | NdFe35 |

3.1. Electromagnetic field modeling

Define Select the solver as the transient field, set the air gap to air and 1mm, set the stator winding material to copper, the stator iron core is silicon steel DW465-50, the permanent magnet material is NdFe35, use Ansoft maxwell to establish the finite element simulation model of the motor as As shown in Figure 3-1, the motor has a 4-pole internal rotor magnetic circuit structure, the number of stator slots is 24, and the winding is a three-phase single-layer structure. The maximum power speed is 1500rpm, each pole occupies 24/4=6 slots, each pole is divided into 3 phases, and each pole occupies 6/3=2 slots.
Set the three-phase sinusoidal current A phase input current to 
\[ I(t) = 1.414 \cos(2 \times \pi \times 50 \times \text{time} + \phi) \times (180 \times \pi), \]
Phase B and phase C lag behind phase A by 120° and 240°, respectively, so the input currents of phases B and C are 
\[ I_B(t) = 1.414 \cos(2 \times \pi \times 50 \times \text{time} + \phi) \times (\frac{2}{3}) \times (180 \times \pi - 2 \times \pi), \]
\[ I_C(t) = 1.414 \cos(2 \times \pi \times 50 \times \text{time} + \phi) \times (\frac{4}{3}) \times (180 \times \pi - 4 \times \pi), \]
respectively. Where \( I \) is rated current 5A, \( \phi \) is taken. Set the simulation time to an electrical cycle of 0.02s and the step size to 0.00004s to verify the validity of the model and parameter settings. The electromagnetic field model, magnetic field lines, Figures 1 and 2 are shown.

3.2. Electromagnetic field modeling
When the motor is running with no load, the radial air gap flux density is mainly composed of the following: the magnetomotive force generated by the permanent magnet and the radial air gap flux density generated by the constant permeability component; the magnetomotive force generated by the permanent magnet and the flux resonance Radial air gap magnetic density generated by wave; fundamental wave magnetomotive force, permanent magnet and permeable components produce radial air gap magnetic density. Using Matlab simulation, the spatial distribution waveform of radial air gap, the order and frequency distribution of no-load radial air gap magnetic density can be obtained as shown in Figures 3 and 4.
According to analytical analysis, the main harmonic component of the radial air gap flux density generated by the motor during no-load operation is \((\mu, \mu f_0 / p)\) and \((\mu \pm jZ, \mu f_0 / p)\). It can be seen from the figure that the 4 finite element simulation results show that the motor generates \((2k, k f_0)\) and \((2k \pm 24, k f_0)\) during no-load operation, of which \(k = 1, 3, 5 \cdots\). \((2k, k f_0)\) is the magnetic density component generated by the interaction between the permanent magnet magnetomotive force and the invariant component of the permeability, corresponding analysis \((\mu, \mu f_0 / p)\) in the formula. \((2k \pm 24, k f_0)\) is the analysis of the radial air gap flux density harmonics generated by the interaction between the permanent magnet magnetomotive force and the first-order tooth harmonics of the permeance, corresponding to \((\mu \pm jZ, \mu f_0 / p)\) in the analytical analysis. It can be explained from this that the magnetic density of the motor during no-load operation is mainly caused by the interaction of the permanent magnet magnetomotive force with the invariant component of the air gap permeability and the first-order tooth harmonic of the permeability. The correctness of the analytical analysis is verified by finite element simulation.

When the motor load is running, the armature current is 5A, and the space-time distribution waveform of the air gap magnetic density is obtained by the finite element method for fast Fourier transform, as shown in Figure 5.

Relative to the no-load situation, analyze the magnetic density component of the radial air gap magnetic density increase when the load is running. The magnetic density harmonic component generated by the stator magnetomotive force and the constant permeability component is \((p, f_0)\), Harmonic component of flux density generated by stator magnetomotive force and first-order harmonic of permeance \((k, p \pm 48, f_0)\), where \(k = \pm 0 + 1, \pm 6 + 1, \cdots\). Therefore, the analytical analysis is consistent with the finite element simulation results.

3.3. Finite element verification of radial electromagnetic force waves
Using MATLAB program simulation software, the radial force wave with time and space distribution waveform in an electrical period is obtained as shown in Figure 6. Similarly, the fast electromagnetic
Fourier transform of the radial electromagnetic force wave can obtain the radial order and frequency of the motor during no-load operation Distribution, as shown in Figure 7.

It can be seen from Figure 11 that the main radial electromagnetic force wave components during motor no-load operation are mainly \((0, 0), (24, 0), (-24, 0), (4, 2f_0), (8, 4f_0), (12, 6f_0), (16, 8f_0), (20, 10f_0), (24, 12f_0), (28, 14f_0), (28, 2f_0), (32, 16f_0), (36, 18f_0), (32, 4f_0)\). Among them, \((0, 0), (4, 2f_0), (8, 4f_0), (12, 6f_0), (16, 8f_0), (20, 10f_0), (24, 12f_0), (28, 14f_0), (32, 16f_0), (36, 18f_0)\) correspond to the force wave component in analytical analysis \((2pn, 2nf_0)\) \((24, 0), (-24, 0), (32, 4f_0), (28, 2f_0)\) correspond to the radial force component of analytical analysis \((2pn \pm lZ, 2nf_0)\).

Fig. 6. Radial force wave with time and space distribution waveform

Fig. 7. No-load radial electromagnetic force wave order and frequency distribution

When the armature current is 5A, the space distribution waveform of the radial electromagnetic force wave generated by the steady state operation of the motor is subjected to Fourier decomposition using a two-dimensional fast Fourier transform to obtain the spatial order and frequency distribution map of the force wave, as shown in Figure 8.

Fig. 8. Load radial force wave order and frequency distribution

Excluding the force wave during no-load operation, the newly added force wave components generated during motor load operation are \((-8, 2f_0), (16, 2f_0), (28, 2f_0), (20, 2f_0), (36, 6f_0), (40, 8f_0)\). Among them, \((36, 6f_0)\) and \((40, 8f_0)\) correspond to the electromagnetic force wave components
generated by the interaction between the permanent magnet, the armature current magnetomotive force harmonics, and the permeable harmonics \((2pn \pm lZ, 2nf_0)\) \((28,2f_0)\). Corresponding to the electromagnetic force wave component generated by the interaction of the armature current magnetomotive force harmonic and the magnetic conduction harmonic \((6n,p + 2p \pm lZ, 2f_0)\), further proved the correctness of analytical analysis.

4. Conclusion
The analytic analysis method was used to analyze the radial air gap flux density and electromagnetic force wave characteristics of the permanent magnet synchronous motor in depth, and the analytic analysis conclusion was verified by the finite element method to verify the correctness of the analytic analysis. It is further obtained that when the motor is running without load, the radial electromagnetic force waves are mainly generated by the interaction of the permanent magnet magnetomotive force with the first-order harmonics and harmonic components of the magnetic permeability, and the generated radial electromagnetic force waves are \((2pn, 2nf_0)\) and \((2pn \pm lZ, 2nf_0)\), \(n = 0,1,2,3\ldots\). When the motor load is running, due to the influence of the armature current, the radial electromagnetic force wave component will be added, and the new radial electromagnetic force wave components are \((6n,p + 2p, 2f_0)\) \((6n,p + 2p \pm lZ, 2f_0)\), \(n = 0,1,2,3\ldots, n_1 = \pm 1,\pm 2,\pm 3\ldots\).

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