Fermion Triplet Dark Matter and Radiative Neutrino Mass

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Abstract

The neutral member of a Majorana fermion triplet ($\Sigma^+ , \Sigma^0 , \Sigma^-$) is proposed as a candidate for the dark matter of the Universe. It may also serve as the seesaw anchor for obtaining a radiative neutrino mass.
Introduction: The cosmological and astrophysical evidence [1] for dark matter (DM) is a powerful incentive for considering new particles and interactions beyond those of the standard model (SM) of quarks and leptons. Whereas most studies have concentrated on supersymmetric extensions of the SM, other excellent DM candidates abound. For example, if the SM is extended to include just one new scalar or fermion multiplet, then there are many possible DM candidates [2]. In particular, a scalar doublet \((\eta^+, \eta^0)\) odd under an exactly conserved \(Z_2\) symmetry [3] is a very good choice [4, 5, 6, 7].

Such a “dark” scalar doublet is amenable to discovery at the Large Hadron Collider (LHC) [8]. It is also very useful for generating small radiative Majorana neutrino masses [4] if there exist neutral singlet fermions \(N_i\) which are odd under \(Z_2\). For a brief review of the further developments of this idea of “scotogenic” neutrino mass, see Ref. [9]. More recently, it has been extended to include \(A_4\) tribimaximal mixing [10] as well.

Now the lightest \(N_i\) may also be considered a DM candidate [11, 12, 13]. However, processes such as \(\mu \rightarrow e\gamma\) impose severe constraints on the Yukawa couplings of \(N_i\), making it difficult to satisfy the cosmological relic abundance required. One way to avoid this problem is to introduce additional interactions for \(N_i\) [14, 15, 16]. Other SM singlets have also been considered [17, 18, 19, 20, 21, 22, 23, 24, 25].

Whereas the canonical seesaw mechanism uses the fermion singlet \(N\) so that the neutrino mass is given by \(m_\nu \simeq -m_D^2/m_N\) where \(m_D\) is the Dirac mass linking \(\nu\) to \(N\), it is not the only way to realize the generic dimension-five effective operator [26]

\[
L_5 = -\frac{f_{ij}}{2\Lambda} (\nu_i \phi^0 - l_i \phi^+)(\nu_j \phi^0 - l_j \phi^+) + H.c. \tag{1}
\]

for obtaining small Majorana neutrino masses in the SM. In fact, there are three tree-level (and three generic one-loop) realizations [27]. The second most often considered mechanism for neutrino mass is that of a scalar triplet \((\xi^{++}, \xi^+, \xi^0)\), whereas the third tree-level realization, i.e. that of a fermion triplet \((\Sigma^+, \Sigma^0, \Sigma^-)\) [28], has not received as much attention.
However, it has some rather intriguing properties. It supports a new U(1) gauge symmetry \[29, 30, 31\] and may be important for gauge-coupling unification \[32, 33, 34\] in the SM. It may be probed \[30, 35, 36, 37\] at the LHC, and is being discussed in a variety of other contexts \[38, 39, 40, 41, 42\]. Now suppose \(\Sigma^0\) is also odd under \(Z_2\), then it may become a DM candidate \[32, 40\] and replace \(N\) in the radiative generation of neutrino mass as shown in Fig. 1. The difference between \(N\) and \(\Sigma^0\) is that whereas the former has only Yukawa inter-

\[
\begin{align*}
\phi^0 &\quad \eta^0 \\
\phi^0 &\quad \eta^0 \\
\nu_i &\quad \Sigma^0_k &\quad \nu_j
\end{align*}
\]

Figure 1: One-loop generation of seesaw neutrino mass.

actions in the minimal scenario, the latter has electroweak gauge interactions, i.e. \(\Sigma^0\Sigma^\pm W^\mp\), which will allow \(\Sigma^0\) and \(\Sigma^\pm\) to annihilate and coannihilate in the early Universe to account for the correct DM relic abundance without relying on their Yukawa couplings \[12\]. Note that \(\Sigma^\pm\) is slightly heavier than \(\Sigma^0\) from electroweak radiative corrections \[2\]. It is also possible \[43\] that \(\Sigma^0\) exists as DM without having anything to do with neutrino mass.

**Gauge-coupling unification**: It is well-known that gauge-coupling unification occurs for the minimal supersymmetric standard model (MSSM) but not the SM. On the other hand, the addition of \(\Sigma\) improves the situation and gauge-coupling unification in the SM is possible \[32, 33, 34\] with the inclusion of some other fields. Consider the one-loop renormalization-group equations governing the evolution of the three gauge couplings of the standard \(SU(3)_C \times \)}
$SU(2)_L \times U(1)_Y$ gauge group as functions of mass scale:

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1},$$

(2)

where $\alpha_i = g_i^2 / 4\pi$ and the numbers $b_i$ are determined by the particle content of the model between $M_1$ and $M_2$. In the SM with one Higgs scalar doublet, these are given by

- $SU(3)_C$, $b_C = -11 + (4/3)N_f = -7$,

(3)

- $SU(2)_L$, $b_L = -22/3 + (4/3)N_F + 1/6 = -19/6$,

(4)

- $U(1)_Y$, $b_Y = (4/3)N_f + 1/10 = 41/10$,

(5)

where $N_f = 3$ is the number of quark and lepton families and $b_Y$ has been normalized by the well-known factor of $3/5$. Using the input

$$\alpha_L(M_Z) = (\sqrt{2}/\pi)G_F M_W^2 = 0.0340,$$

(6)

$$\alpha_Y(M_Z) = \alpha_L(M_Z) \tan^2 \theta_W = 0.0102,$$

(7)

$$\alpha_C(M_Z) = 0.122,$$

(8)

it is easy to check that the SM particle content is incompatible with the unification condition

$$\alpha_C(M_U) = \alpha_L(M_U) = (5/3)\alpha_Y(M_U) = \alpha_U.$$

(9)

Suppose $(\Sigma^+, \Sigma^0, \Sigma^-) \sim (1, 3, 0)$ and $(\eta^+, \eta^0) \sim (1, 2, 1/2)$ are added at the scale $M_X$, together with two real scalar color octets $\zeta_{1,2} \sim (8, 1, 0)$, then $\Delta b_L = 2(2/3) + 1/6 = 3/2$, $\Delta b_Y = 1/10$, and $\Delta b_C = 3(2)(1/6) = 1$ between $M_X$ and $M_U$, so that Eq. (9) implies

$$\ln \frac{M_U}{M_Z} = \left( \frac{\pi}{45} \right) \left( \frac{3}{\alpha_Y(M_Z)} + \frac{9}{\alpha_L(M_Z)} - \frac{14}{\alpha_C(M_Z)} \right) = 31.0.$$

(10)

Hence $M_U \simeq 2.65 \times 10^{15}$ GeV, which is an acceptable value for suppressing the proton decay lifetime above the experimental lower bound of about $10^{32}$ years. The scale $M_X$ is determined to be about 730 GeV. Thus the new particles have a chance of being observed at
the LHC. In particular, the $\zeta$ scalars would be produced in abundance at the LHC because they are color octets \cite{46,47} and would decay in one loop to two gluons \cite{32}, i.e. $\zeta \to \zeta \zeta \to gg$.

**$\Sigma^0$ as dark matter**: Consider the minimal case where the SM is extended to include only one fermion triplet $\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-) \sim (1, 3, 0)$ which is odd under $Z_2$ with all other fields even. In that case, $m_{\Sigma^\pm} = m_{\Sigma^0}$ at tree level, but the former is heavier than the latter from one-loop electroweak radiative corrections, namely \cite{2}

$$\Delta = m_{\Sigma^\pm} - m_{\Sigma^0} = \frac{\alpha_L m_{\Sigma^0}}{4\pi} \left\{ f \left( \frac{M_W}{m_{\Sigma^0}} \right) - \cos^2 \theta_W f \left( \frac{M_Z}{m_{\Sigma^0}} \right) \right\},$$

where

$$f(r) = -r^2 + r^4 \ln r + r(r^2 - 4)^{1/2}(1 + r^2/2) \ln[-1 - (r^2 - 4)^{1/2}r/2 + r^2/2]$$

$$\simeq 2\pi r - 3r^2, \quad \text{for } r \ll 1. \quad (12)$$

This splitting is positive and approaches $(\alpha_L/2) \cos \theta_W (1 - \cos \theta_W) M_Z \simeq 167$ MeV for large $m_{\Sigma}$. This means that $\Sigma^\pm$ is allowed to decay into $\Sigma^0$ plus a virtual $W^\pm$ which then converts into $\pi^\pm$ or leptons.

The relic abundance of $\Sigma^0$ is determined by the annihilation and coannihilation of itself and $\Sigma^\pm$. These cross sections are dominated by their $s$-wave contributions. For $\Sigma^0\Sigma^0 \to W^+W^-$ through $\Sigma^\pm$ exchange,

$$\sigma(\Sigma^0\Sigma^0)|v| \simeq \frac{2\pi \alpha_L^2}{m_{\Sigma}^2},$$

where $v$ is the relative velocity of the incident particles in their center of mass and $m_{\Sigma} \gg \Delta$ is assumed. As for coannihilation, several processes have to be included: $\Sigma^0\Sigma^\pm \to W^0W^\pm$ through $\Sigma^\pm$ exchange and $\Sigma^0\Sigma^\pm \to W^+\to \bar{f}f', \ W^\pmW^0, \ W^\pmH$, as well as $\Sigma^+\Sigma^- \to W^0W^0$ through $\Sigma^\pm$ exchange, $\Sigma^+\Sigma^- \to W^+W^-$ through $\Sigma^0$ exchange, $\Sigma^+\Sigma^- \to W^0 \to \bar{f}f, \ W^+W^-, \ W^0H$, and $\Sigma^\pm\Sigma^\pm \to W^\pmW^\pm$ through $\Sigma^0$ exchange.
They are also easily calculated to be

\[
\sigma(\Sigma^0\Sigma^\pm)|v| \simeq \frac{29\pi\alpha_L^2}{8m^2_{\Sigma}}, \quad \sigma(\Sigma^+\Sigma^-)|v| \simeq \frac{37\pi\alpha_L^2}{8m^2_{\Sigma}}, \quad \sigma(\Sigma^\pm\Sigma^\pm)|v| \simeq \frac{\pi\alpha_L^2}{m^2_{\Sigma}}.
\] (14)

In the above, we have kept only the \( a_{ij} \) coefficients in the relative-velocity expansion of the cross section: \( \sigma_{ij}|v| = a_{ij} + b_{ij}v^2 \). Note that \( \sigma(\Sigma^0\Sigma^0)|v| \) is smaller than \( \sigma(\Sigma^0\Sigma^\pm)|v| \) and \( \sigma(\Sigma^+\Sigma^-)|v| \). This means that \( \Sigma^\pm \) contributes importantly to the relic abundance of \( \Sigma^0 \).

Using the method developed in Ref. [48] to take coannihilation into account, we calculate below the relic abundance of \( \Sigma^0 \) as a function of \( m_{\Sigma} \) and \( \Delta \). The decoupling temperature \( T_f \) of \( \Sigma^0 \) is estimated by using the effective cross section \( \sigma_{\text{eff}} \) and the effective degrees of freedom \( g_{\text{eff}} \) from the condition

\[
x = \ln \frac{0.038 \, g_{\text{eff}} \, M_{\text{Pl}} \, m_{\Sigma} \, \langle \sigma_{\text{eff}}|v| \rangle}{\sqrt{g_{*}x}},
\] (15)

where \( x = m_{\Sigma}/T \), \( g_* = 106.75 \) is the SM number of relativistic degrees of freedom at \( T_f \), \( M_{\text{Pl}} = 1.22 \times 10^{19} \) GeV is the Planck mass, and

\[
\langle \sigma_{\text{eff}}|v| \rangle = \frac{g_0^2}{g_{\text{eff}}^2}\sigma(\Sigma^0\Sigma^0) + 4\frac{g_0g_\pm}{g_{\text{eff}}^2}\sigma(\Sigma^0\Sigma^\pm)(1 + \epsilon)^{3/2} \exp(-\epsilon x)
\]
\[
+ \frac{g_\pm^2}{g_{\text{eff}}^2}[2\sigma(\Sigma^+\Sigma^-) + 2\sigma(\Sigma^\pm\Sigma^\pm)](1 + \epsilon)^2 \exp(-2\epsilon x),
\]
\[
g_{\text{eff}} = g_0 + 2g_\pm(1 + \epsilon)^{3/2} \exp(-\epsilon x),
\] (16)

with \( g_0 = g_\pm = 2 \) and \( \epsilon = \Delta/m_{\Sigma} \). The relic abundance is then given by

\[
\Omega h^2 = \frac{1.04 \times 10^9 x_f}{g^{1/2}_{*} M_{\text{Pl}}(\text{GeV}) I_a},
\] (17)

where \( I_a = x_f \int_{x_f}^{\infty} a_{\text{eff}} x^{-2} dx \), \( x_f = m_{\Sigma}/T_f \), and \( a_{\text{eff}} \) is extracted from \( \sigma_{\text{eff}}|v| = a_{\text{eff}} + b_{\text{eff}}v^2 \).

Using the observational data \( \Omega h^2 = 0.11 \pm 0.006 \) [49], we find \( m_{\Sigma^0} \) to be in the range 2.28 to 2.42 TeV. Here the electroweak radiative contribution to \( \Delta \) is already at its asymptotic value of about 167 MeV and its effect on \( m_{\Sigma^0} \) is negligible. There is no tree-level contribution to \( \Delta \) unless a Higgs triplet \((s^+, s^0, s^-)\) is added [32] with \( \langle s^0 \rangle \neq 0 \). However, this value should
be less than about 1 GeV to conform to precision electroweak measurements; hence \( \Delta \) would still be negligible and our result is unchanged.

**Neutrino masses**: To have scotogenic neutrino masses, consider now the addition of the dark scalar doublet \( \eta \) and the specific choice of one \( \Sigma \) and two \( N \)'s, then under the assumption \( m_{\Sigma}^2, m_N^2 \ll m_\eta^2 \), the resulting radiative masses are given by \[ (M_\nu)_{\alpha\beta} = \frac{\lambda_5 v^2}{8\pi^2} \sum_{j=0,1,2} \frac{h_{\alpha j} h_{\beta j} M_j}{m_\eta^2}, \] (18)

where \( M_0 = m_\Sigma, M_{1,2} = m_{N_{1,2}}, h_{\alpha j} \) are their Yukawa couplings, \( v = \langle \phi^0 \rangle \), and \( \lambda_5 \) is the scalar coupling in the quartic term \( (\lambda_5/2)(\Phi^4\eta)^2 + H.c. \) which splits \( \text{Re}(\eta^0) \) and \( \text{Im}(\eta^0) \).

Since \( \lambda_5 \) and \( m_\eta \) are adjustable, it is clear that realistic neutrino masses may be obtained for \( h \sim 10^{-2} \), in which case processes such as \( \mu \to e\gamma \) are well below their experimental upper bounds. The problem with \( N \) as dark matter is the requirement of \( h > 1 \) for it to have a large enough annihilation cross section \[12\].

Looking at the form of Eq. (18), it is clear that it is possible to have a one-to-one correspondence between the neutrino mass eigenvalues \( m_{1,2,3} \) and the seesaw anchor masses \( M_{0,1,2} \). As an *anstaz*, let the \( 3 \times 3 \) Yukawa coupling matrix linking \( e, \mu, \tau \) to \( M_{0,1,2} \) be given by

\[
h_{\alpha j} = \begin{pmatrix}
\sqrt{2/3} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
h_0 & 0 & 0 \\
0 & h_1 & 0 \\
0 & 0 & h_2
\end{pmatrix},
\] (19)

then the tribimaximal mixing of neutrinos is obtained, and their mass eigenvalues are

\[
m_{i+1} = \frac{\lambda_5 v^2 h_i^2 M_i}{8\pi^2 m_\eta^2}, \quad i = 0, 1, 2.
\] (20)

**Relaxation of \( \mu \to e\gamma \) constraints**: Since \( \Sigma^0 \) has gauge interactions, its relic abundance is adequately accounted for. There is no need for it to have large Yukawa couplings, as is in the case \[12\] of choosing the singlet fermion \( N \) as dark matter, where \( m_\eta \) must also be close to \( m_N \). This means radiative flavor-changing decays are easily suppressed. In the above
example, using the experimental upper bound of $1.2 \times 10^{-11}$ on the branching fraction of $\mu \to e\gamma$, this corresponds to the condition

$$||h_0|^2 - |h_1|^2| < 0.77(m_\eta/2.35 \text{ TeV})^2.$$ \hfill(21)

Since $h$ is not required to be large and $\eta$ should be heavier than $\Sigma$, the tension between the constraints of dark-matter relic abundance and flavor-changing radiative decays is removed.

**Conclusion**: In this paper we have proposed the addition of a fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ to the standard model of quarks and leptons. We consider $\Sigma^0$ as a dark-matter candidate, being odd under an exactly conserved $Z_2$ symmetry. We show that with $\Sigma^\pm$ slightly heavier than $\Sigma^0$ from electroweak radiative corrections, $m^0_\Sigma \sim 2.35 \text{ TeV}$ yields the correct dark-matter relic abundance from the annihilation and coannihilation of $\Sigma$ through gauge interactions.

We also consider $\Sigma$ as the seesaw anchor in the radiative generation of neutrino mass with a second scalar doublet $\eta$. The constraints due to flavor-changing radiative decays such as $\mu \to e\gamma$ are then easily satisfied because the $\Sigma$ Yukawa couplings need not be large. (If $\Sigma$ is replaced by $N$, then $N$ must have large Yukawa couplings to be a dark-matter candidate.)

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