Exchange flow numerical method of dual-porosity model for different matrix size in 2D oil-water imbibition

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Abstract. Spontaneous imbibition is a common natural phenomenon in porous media. For natural fractured reservoir with a large number of hydrophilic and oleo-phobic rock blocks, the imbibition displacement of non-wetting phase by wetting phase with the dominant effect of capillary force is an important mechanism in exchange flow between matrix block and fracture in reservoir recovery. Scientists have concluded a scaling model which includes the permeability, porosity, boundary conditions, fluid-solid interaction and other factors though lots of experiments. In this study, after proposing appropriate characteristic scale in different periods of imbibition, the approximate saturation solution in one-dimensional imbibition obtained on the basis of scaling model is extended to two-dimensional process. Then, the exchange flow numerical method of dual-porosity model for two-dimensional multiphase imbibition is obtained. The results of simulation example show that the numerical method of exchange flow in this paper is more accurate than that of traditional dual-porosity model under different matrix rock size.

1. Introduction
Naturally fractured reservoir is widely existed, abundant natural fractures form complex networks that separate the matrix blocks in this kind of reservoir. The matrix blocks have plentiful pores, strong oil storage capacity, but poor connectivity and weak flow capacity. The fractures have a small proportion of reservoir volume, but large permeability and strong flow capacity. For the existence of the two completely different media in the natural fractured reservoir, Barrenblatt et al. (1960) proposed to idealize the fracture matrix system as two dynamic interaction systems, and the exchange flow between the matrix and fracture was considered to be related to the pressure difference between the matrix and fracture [1]. In the reservoir recovery, oil in matrix blocks is precipitated into the fracture and then goes into the wellbore through the fracture. Therefore, the rate of exchange flow flux between matrix blocks and fracture largely determines the rate of exploitation.

Because the stromal blocks in reservoir are mostly hydrophilic and oil-phobic media, the capillary forces will make the matrix blocks spontaneously absorb water in the fractures and discharge oil at the same time. Many small scale experiments have been developed to investigate the influence factors of spontaneous imbibition. Aronofsky et al. (1958) proposed the scale model of production and dimensionless time [2]:

\[ R / R_e = 1 - e^{-at_0} \]  

(1)
Where, $R$ is the imbibition production, $R_{\infty}$ is the ultimate production, $\alpha$ is constant. Mattax, C.C., Kyte, J.R. (1962) verified the influence of block size, permeability and viscosity on the oil production through quasi-one-dimensional experimental results, and summarized the dimensionless time $t_{D, MK}$:

$$t_{D, MK} = \frac{t}{\sqrt{\frac{\alpha \mu L}{\sigma k}}}. \quad (2)$$

Torsæter and Silseth (1985) analyzed the effect of sample shape and boundary conditions on capillary force imbibition [4]. Hamon and Vidal (1986) studied imbibition in samples with heterogeneous permeability and porosity, indicating that spatial heterogeneity of rocks should be considered when predicting production [5]. Kazemi (1992) modified the MK model by introducing characteristic scale $L_s$ instead of $L$ [6]. Ma S et al. (1997) detected that $L_s$ could not be applied to imbibition with only one side open, and defined a new characteristic length $L_c$. Meanwhile, the effect of non-wetting phase viscosity was involved in the study of Ma S et al, the imbibition rate was shown to be proportional to the geometric mean of the water and oil viscosities [7].

In the reviewed work, a large number of experiments have concluded a scaling model considers the influence of permeability, porosity, boundary conditions and fluid-solid interaction of porous media on imbibition. In this study, after proposing appropriate characteristic scale in different periods of imbibition, the approximate saturation solution in one-dimensional imbibition obtained on the basis of scaling model is extended to two-dimensional process. The influence of permeability, porosity, boundary conditions and fluid-solid interaction of porous media on imbibition in different periods are involved in the the approximate saturation solution. In the process of matrix-fracture exchange dominated by spontaneous imbibition, a new method for exchange flow is obtained by using the approximate saturation solution of two-dimensional imbibition. In this paper, numerical examples of different matrix block scale are verified. Compared with the traditional dual-porosity model, the solutions obtained by the new method are in better agreement with the reference solutions obtained by the fine grid.

2. Analysis

The matrix water saturation equation in one-dimensional oil-water imbibition is:

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left( K \frac{k_w k_{ro}}{\mu_w + k_{ro} \mu} \frac{\partial P}{\partial x} \frac{\partial S_w}{\partial x} \right) = 0 \quad (3)$$

and in two-dimensional process, the matrix water saturation equation is:

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left( K \frac{k_w k_{ro}}{\mu_w + k_{ro} \mu} \frac{\partial P}{\partial x} \frac{\partial S_w}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{k_w k_{ro}}{\mu_w + k_{ro} \mu} \frac{\partial P}{\partial y} \frac{\partial S_w}{\partial y} \right) = 0 \quad (4)$$

where $S_w$ is water saturation, $K$ is absolute permeability, $P_c$ is capillary force, $k_w$ and $\mu$ is relative permeability and viscosity. Suppose the relative permeability curve and capillary force curve are as follows:

$$k_{ro} = k_{ro}^{\text{max}} S_w, k_{rw} = k_{rw}^{\text{max}} (1 - S_w)^{\phi}, P_c = -B \ln S_w \quad (5)$$

Define the normalized saturation as $S = \frac{S_w - S_{wr}}{1 - S_{wi} - S_{wr}}$, $S_{wi}$ and $S_{wr}$ are the residual water and oil saturations; the fluidity ratio is $M = \frac{\mu_w k_{ro}^{\text{max}}}{\mu_k k_{ro}^{\text{max}}}$, then equation (3) and (4) can be converted to equation (6) and (7):

$$\phi \frac{1 - S_{wi} - S_{wr}}{1 - S_{wi} - S_{wr}} \frac{\partial S}{\partial t} - \frac{KB}{\mu_w} \frac{\partial}{\partial x} \left[ k_{ro}^{\text{max}} S^{\phi - 1} (1 - S)^{\phi} \frac{\partial S}{\partial x} \right] = 0 \quad (6)$$

2
\[
\phi(1 - S_{wi} - S_{ro}) \frac{\partial S}{\partial t} - \frac{KB}{\mu_S} \left( \frac{\partial}{\partial x} \left[ k_{\text{max}} S^{-1} (1 - S)^{\phi} \frac{\partial S}{\partial x} \right] + \frac{\partial}{\partial y} \left[ k_{\text{max}} S^{-1} (1 - S)^{\phi} \frac{\partial S}{\partial y} \right] \right) = 0 \tag{7}
\]

Define time variable \( t_D = t \frac{K}{\phi(1 - S_{wi} - S_{ro}) \mu_S} \), the equation (6) and (7) can further be derived to:

\[
\frac{\partial S}{\partial t_D} = \frac{\partial}{\partial x} \left[ k_{\text{max}} S^{-1} (1 - S)^{\phi} \frac{\partial S}{\partial x} \right] = 0 \tag{8}
\]

\[
\frac{\partial S}{\partial t_D} - \nabla \left[ k_{\text{max}} S^{-1} (1 - S)^{\phi} \nabla S \right] = 0 \tag{9}
\]

Define a saturation function \( F(S) = \frac{k_{\text{max}} S^{-1} (1 - S)^{\phi}}{S^* + M(1 - S)^{\phi}} \), integrate equation (9) in two-dimensions:

\[
\int_{V_{xy}} \frac{\partial S}{\partial t_D} \, dxdy = \int_{V_{xy}} \nabla \left[ F(S) \cdot \nabla S \right] \, dxdy = 0 \tag{10}
\]

\( V_{xy} \) is the area of the domain; \( C_y \) is the length of boundary open to imbibition, suppose the velocity is the same on these boundaries:

\[
\int_{C_y} [F(S) \cdot \nabla S] \, dl = C_y \cdot [F(S) \cdot \nabla S] \bigg|_{C_y} \tag{11}
\]

Then, equation (10) can be derived to:

\[
\frac{\partial S}{\partial t_D} = \frac{1}{L_c} \left[ F(S) \cdot \nabla S \right] \bigg|_{C_y} \tag{12}
\]

Where average saturation \( \bar{S} = \frac{1}{V_{xy}} \int_{V_{xy}} S \, dxdy \) and characteristics scale \( L_c = \frac{V_{xy}}{C_y} \), define dimensionless scales and dimensionless time \( x_D = \frac{x}{L_c}, \, y_D = \frac{y}{L_c}, \, t_{DL} = \frac{t_D}{L_c^2} \), then the average saturation equations in 1D and 2D are obtained respectively:

\[
\frac{\partial \bar{S}}{\partial t_{DL}} = -k_{\text{max}} (1 - S)^{\phi} \frac{\partial S}{\partial S} \bigg|_{L_D = 0} \tag{13}
\]

\[
\frac{\partial \bar{S}}{\partial t_{DL}} = -k_{\text{max}} (1 - S)^{\phi} \nabla S \bigg|_{C_y} \tag{14}
\]

On the boundary open to imbibition, \( S = 1 \); the approximate solutions of equation (13) has been found by Li. We can extend the solutions to two-dimensional case of imbibition after adjusting the characteristics scale. When the boundary is all open, in early time when the water front does not reach the boundary, the characteristics scale is:

\[
L_c(\text{early}) = \frac{L_c \cdot L_y}{2 \left( L_c + L_y \right)} \tag{15}
\]

In late time when the water front has reach the boundary, the characteristics scale takes the linear interpolation average of the value of early time and the value proposed by Ma S et al. (1997) with respect to saturation. When \( \bar{S} \leq S' \), \( L_c = L_c(\text{early}) \); when \( \bar{S} = 1.0 \), \( L_c,\text{late} = \frac{L_c \cdot L_y}{2 \sqrt{L_c^2 + L_y^2}} \). When \( S' \leq \bar{S} \leq 1.0 \), the characteristics scale is obtained:
Considering the conservation of mass \( q = \phi \frac{\partial S}{\partial t} \), the exchange flow related to the average saturation in matrix block in different period can be obtained:

When \( \overline{S} < S^* \),

\[
q = \frac{1}{2} \frac{\mu \kappa}{S} \left( \frac{B \cdot K}{1 - S_w - S_o} \right) \left( \frac{L_{x_{i}}} {L_c} + L_{y_{i}} \right)
\]

When \( \overline{S} > S^* \),

\[
q = \frac{b}{(1 - S^*)^n} \left( \frac{B \cdot K}{\mu \kappa} \right) \left( \frac{1}{1 - S_w - S_o} \right) \left( \frac{L_{x_{i}}} {L_c} + L_{y_{i}} \right)
\]

### 3. Examples

In this example, the numerical simulation results of the new numerical method, the traditional dual-porosity model and the fine grid numerical solution under different magnitude of capillary force are studied. A matrix rock is completely submerged in fractures filled with water, and the pressure boundary is distributed linearly along the boundary, the endpoint values are shown in Figure 1.

![Figure 1. Calculated area profile.](image)

Initially, the fracture is filled with water and the matrix is full of oil. The absolute permeability is \( 1.0 \times 10^{-3} \mu m^2 \) in fracture and \( 1.0 \times 10^{-9} \mu m^2 \) in matrix. The displacement effect of pressure field is very weak under the boundary condition shown in Figure 1. However, the oil from the matrix will flow out through the fracture rapidly because of the high absolute permeability in fracture. The porosity of the matrix block is 0.3. The residual water and oil saturations are both zero. The relative permeability and capillary is equation (5). The relevant parameters in example are shown is Table 1. Changing different matrix block sizes, we get the average oil saturation in matrix in these three model are shown in Figure 2.
Figure 2. The average oil saturation in matrix in these three model in example 2.

Table 1. The relevant parameters in example

|                | capillary force maximum value B | fluidity ratio M | the index a in relative permeability | the index b in relative permeability | matrix rock size, \(L_x \times L_y\) |
|----------------|--------------------------------|------------------|--------------------------------------|--------------------------------------|----------------------------------|
| (a)            | \(2.0 \times 10^5\) Pa         | 0.1              | 2                                    | 2                                    | 2m \times 2m                     |
| (b)            | \(2.0 \times 10^5\) Pa         | 0.1              | 2                                    | 2                                    | 2m \times 1m                     |
| (c)            | \(2.0 \times 10^6\) Pa         | 0.1              | 2                                    | 2                                    | 3m \times 1m                     |
| (d)            | \(2.0 \times 10^6\) Pa         | 0.1              | 2                                    | 2                                    | 3m \times 2m                     |

As can be seen from the results in Figure 2, compared with the traditional model, the results obtained by the new one are more consistent with the fine grid reference solutions under different matrix sizes.

4. Conclusion
In this paper, after proposing appropriate characteristic scale in different periods of imbibition, an approximate saturation solution in two-dimensional imbibition on the basis of scaling model is obtained. Further derivation of approximate saturation solution leads to a new exchange flow numerical method of dual-porosity model for 2D oil-water imbibition. The new dual-porosity model solutions with the new exchange flow numerical method are compared with fine grid reference solutions and traditional dual-porosity model solutions. The results show that the accuracy of this model is higher than that of the traditional one under different matrix rock size.

Acknowledgments
This work is supported by National Science and Technology Major Project (Grant Number No. 2017ZX05072-005)

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