A Test of the Standard Model, Using DaΦne

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Abstract

Both the light hypernucleus \( \Lambda H^4 \) and the nucleus He\(^4 \) have spin 0. The ratio \( R \) of the electron to muon rates for the pure Fermi transitions

\[
R = \frac{\Gamma (\Lambda H^4 \rightarrow e^- + \bar{\nu} + He^4)}{\Gamma (\Lambda H^4 \rightarrow \mu^- + \bar{\nu} + He^4)}
\]

(1)

is a sensitive measure of the presence of a second-class weak current (to the extent that SU(3) is valid in strong interactions), and hence, is a test of the Standard Model. Rates and sensitivities, using DaΦne, the \( e^+e^- \) machine under construction at Frascati, are discussed.

1 Introduction

The ground states of both the light hypernucleus \( \Lambda H^4 \) and the nucleus He\(^4 \) are spin 0 states[1]. Therefore, the weak decays

\[
\Lambda H^4 \rightarrow e^- + \bar{\nu} + He^4
\]

(2)

and

\[
\Lambda H^4 \rightarrow \mu^- + \bar{\nu} + He^4
\]

(3)

are both Fermi transitions, and hence, are both pure vector transitions[2, 3, 4, 5]. Thus, the hadronic current is given by the vector current

\[
V^\rho = G_\Lambda \gamma^\rho + g_2 i\sigma^{\mu\nu} Q_\nu + f_3 Q^\rho,
\]

(4)

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*to be published in the Proceedings of the XXIV International Symposium on Multiparticle Dynamics, Eds. A. Giovannini, S. Lupia and R. Ugocionni, World Scientific, Singapore. Work partially supported by Department of Energy grant DOE 0680-300-N008 Task B.
where the 4-momentum transfer $Q^\rho$ is given by $\Lambda - p = \ell - \nu$. The first term of eq. (4), $\gamma^\rho$, is the conventional (and dominant) term, the second term, $i\sigma^{\mu \nu} Q_\nu$, is the “weak magnetism” term (which will later neglected as very tiny), both of which are first class currents, and the third term, $Q^\rho$, is the “induced scalar”, which is a second-class current, and hence (to the level that SU(3) is valid for the strong interactions) is forbidden in the Standard Model. Thus, a non-zero $f_3$ would be a violation of the Standard Model.

This communication will suggest a method of testing for a non-zero $f_3$, using the Frascati accelerator DaΦne.

2 Experimental Outline

DaΦne, the new $e^+e^-$ machine being constructed at Frascati, is a copious source of Φ’s, which decay dominantly into very low energy K$^+K^-$ pairs. If one surrounds the intersection region with a Helium target (either a liquid or a sufficiently large gaseous target), one can stop the K$^-$ and produce the at-rest reaction

$$K^- + \text{He}^4 \rightarrow \pi^0 + \Lambda \text{He}^4.$$  \hspace{1cm} (5)

This reaction occurs copiously, being about 1% of all reactions in which a negative kaon stops in Helium[1]. This hypernucleus production has a unique experimental signature, producing a monoenergetic and isotropic $\pi^0$ of $\approx 260$ MeV. Subsequently, after the detection of the $\pi^0$, one can measure the ratio of the electron to muon rates for the pure Fermi transition

$$R = \frac{\Gamma(\Lambda \text{He}^4 \rightarrow e^- + \bar{\nu} + \text{He}^4)}{\Gamma(\Lambda \text{He}^4 \rightarrow \mu^- + \bar{\nu} + \text{He}^4)},$$  \hspace{1cm} (6)

which is a sensitive measure of $f_3$ in eq. (4), i.e., the presence of an induced scalar term (a second-class term) in the weak hadronic current. It is easy to show that the contraction of vector term $f_3 Q^\rho$ with the leptonic current $\bar{u}_\ell \gamma_\rho (1 - i\gamma_5) u_\nu$, (where $\ell$ stands for lepton, either $e$ or $\mu$), results in an effective induced scalar term $m_\ell f_3 [\bar{u}_\ell (1 - i\gamma_5) u_\nu]$, where the lepton mass $m_\ell$ is given by either $m_\mu$ or $m_e$. Since the electron mass is so small, the ratio $R$ in eq. (6) is sensitive to the presence of $f_3 Q^\rho$, because the induced scalar contributes negligibly in the electron channel, whereas it has a strong influence in the muon channel. The ratio of electrons to muons is thus very sensitive to $f_3$, whereas, at the same time, it is very insensitive to the details of the calculation, since the uncertainties in the nuclear physics, wave functions, etc., is about the same for the muon and the electron and tends to cancel in the ratio.

Further, in our case, the only second-class current allowed by Lorentz invariance is the “induced scalar” term $f_3 Q^\rho$, since only the vector transition is allowed. The axial “induced pseudoscalar” term of the form $Q^\rho \gamma_5 (which is allowed in the free decay of the $\Lambda^0$, and thus ‘mimics’ the vector term $f_3 Q^\rho$) is not allowed in the pure vector transition of the hypernucleus. Thus, we have a unique interpretation of the experimental results.

The remainder of this paper will be devoted to a quantitative analysis of the ratio $R$, with regard to its sensitivity to $f_3$. 
3 Decay Rate for $\Lambda H^4 \rightarrow \ell^- + \bar{\nu} + He^4$

The nuclear matrix element $\mathcal{M}^\rho$ for the $0 \rightarrow 0$ transition $\Lambda H^4 \rightarrow \ell^- + \nu + He^4$ is

$$\mathcal{M}^\rho = \sqrt{2} \int e^{-i\vec{p}_\nu \cdot \vec{r}_\nu + i\vec{p}_\ell \cdot \vec{r}_\ell} e^{-i\phi(r_\nu + r_\ell + \vec{r}_4)/4} \Phi^*_\Lambda(r_\nu, r_\ell, \vec{r}_4) \chi^*_\ell(p, 2; 3, 4) \times$$

$$V^\rho \times \Phi_\Lambda(r_\Lambda; r_2, r_3, r_4) \chi_\ell(\Lambda; 2; 3, 4) \delta(r_\nu - r_\Lambda) \delta(r_\ell - r_\Lambda) \delta(r_\nu - r_\Lambda) \delta_{p_\Lambda} \times$$

$$d\tau_\nu d\tau_\ell d\tau_\nu d\tau_2 d\tau_3 d\tau_4,$$

where $q_\nu$ and $p_\ell$ are the momenta of the recoil $He^4$, the anti-neutrino and the electron (muon), and $\vec{q} + \vec{p}_\nu + \vec{p}_\ell = 0$. The $\chi$'s are the spin-wave functions for $\Lambda H^4$ and $He^4$. The factor $\sqrt{2}$ arises from having two identical protons in $He^4$.

For the alpha-particle wave function in eq. (7), we use the Gaussian wave function

$$\Phi^*_\alpha(r_1, r_2; r_3, r_4) = N_4 e^{-\frac{1}{2} \sum r_i^2}, \quad (7)$$

where $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$, the normalization factor is given by $N_4 = 2^{3/4} (\frac{2\alpha_4}{\pi})^{9/4}$, and $\alpha_4 = \frac{9}{32R_4^3}$. The radius $R_4$ is fixed from electron scattering data, assuming that the matter distribution is the same as the charge distribution and allowing for the finite size of the nucleon. We will use $R_4 = 1.44 f$. We will assume that the hypernucleus wave function factors into a $\Lambda_0$ moving about a physical triton core, i.e.,

$$\Phi_\Lambda(r_\Lambda; r_2; r_3, r_4) = u_\Lambda \left( \vec{r}_\Lambda - \frac{\vec{r}_2 + \vec{r}_3 + \vec{r}_4}{3} \right) \Phi_t(r_2; r_3, r_4)$$

$$= u_\Lambda(\rho) \Phi_t(r_2; r_3, r_4)$$

$$= u_\Lambda(\rho) N_3 e^{-\frac{\alpha_3}{2} \sum r_i^2}, \quad (9)$$

where in eq. (9), the triton wave function is assumed to be Gaussian and $\vec{r} = \vec{r}_\Lambda - \frac{\vec{r}_2 + \vec{r}_3 + \vec{r}_4}{3}$. The triton normalization factor is given by $N_3 = \frac{(3\alpha_3)^{3/4}}{\pi^{1/2}}$ and $\alpha_3 = \frac{1}{3R_3^2}$. Again, the radius $R_3$ is fixed from electron scattering data, assuming that the matter distribution is the same as the charge distribution and allowing for the finite size of the nucleon. We use $R_3 = 1.66 f$. The wave function $u_\Lambda(\rho)$ has been numerically evaluated by Dalitz and Downs, after solving the Schrödinger equation using a binding energy $\epsilon = 1.9$ MeV for the $\Lambda H^3$.

Since the calculation of eq. (7) involves (non-relativistic) wave functions for helium and the hypernucleus, we must expand the relativistic hadronic current $V^\rho$ non-relativistically to get

$$V^\rho = \left( G_\Lambda + f_3 Q^0, G_\Lambda \left[ \vec{\sigma} \cdot \nabla_{r_\Lambda} \frac{\vec{r}_\Lambda}{2m_\Lambda} - \left( \vec{\sigma} \cdot \nabla_{r_p} \frac{\vec{r}_p}{2m_\rho} \right) \vec{\sigma} \right] + f_3 \vec{Q} \right), \quad (10)$$

where the form factors $G_\Lambda$ and $f_3$ are assumed to be functions of the squared 4-momentum $Q^2$, and are taken to be

$$G_\Lambda(Q^2) = G_\Lambda(0) \frac{M_{K^*}}{M_{K^*} + Q^2},$$

$$f_3(Q^2) = f_3(0) \frac{M_{K^*}^2}{M_{K^*}^2 + Q^2}, \quad (11)$$
where $M_{K^*} = 894.1$ MeV, the mass of the $K^*$ resonance. The effect of the form factor variation will turn out to be very small ($< 5\%$). In the non-relativistic expression for $V^o$ in eq. (10), we have neglected completely the "weak magnetism" term $g_2 i \sigma^\mu \nu Q_\nu$ of eq. (4), completely, which is exceedingly small. As emphasized earlier, the term in $f_3$ is only important for muon decay, and is negligible for electron decay. After integration over triton coordinates, using the factorizable wave functions, we find the nuclear matrix element

$$M^0 = \sqrt{2} \chi f \chi_i F(q),$$

(12)

$$\vec{M} = \sqrt{2} \vec{f} \chi_i F(q) \left\{ G_{\Lambda}(q) \cdot \vec{\sigma} \frac{1 + 1.366}{2m_4} + f_3 Q \right\},$$

where $F(q)$, the nuclear overlap integral is

$$F(q) = \left[ \frac{48 \alpha_3 \alpha_4}{(3\alpha_3 + \alpha_4)^2} \right]^{3/2} G(q),$$

(13)

with

$$G(q) = (3\alpha_4/\pi)^{3/4} \int \exp(-\frac{3}{2} \alpha_4 \rho^2) \times \exp\left[ \frac{3}{4} \vec{q} \cdot \vec{\rho} \right] u_{\Lambda}(\rho) d\rho,$$

(14)

and $m_4 \equiv 4m$. Dalitz and Downs have evaluated $G(q)$ numerically, as a function of $q$, using the numerical wave function $u_{\Lambda}(\rho)$, for the binding energy $\epsilon(\Lambda^4) = 1.9$ MeV. In the correction term proportional to $\frac{1}{2m_4}$, the term $\frac{1.366}{2m_4}$ is due to the nuclear corrections arising from the nuclear wave functions employed. We have also substituted $m_4 = 4m_p$ in the above correction terms. The quantity $F^2(q)$ is recognized as the "sticking probability" that the decay proton from the $\Lambda_0$ inside the hypernucleus and the triton core of the hypernucleus overlap to form He$^4$.

The total matrix element $M$ is given by

$$M = \frac{1}{\sqrt{2}} \left( M^0 j_0 - \vec{M} \cdot \vec{j} \right),$$

(15)

where the lepton current is given by

$$j_\rho = (j_0, \vec{j}) = \bar{u}_\ell \gamma_\rho (1 - i\gamma_5) u_\nu.$$  

(16)

The squared matrix element, summed and averaged over spins, for the reaction $\Lambda^4 H \rightarrow e^- + \bar{\nu} + \text{He}^4$, is given by

$$\frac{1}{2J + 1} \left( m_\ell m_\nu \sum_{\text{spins}} |M|^2 \right) = 2F^2(q^2) \times \left\{ |G_A|^2 \left[ (E_\nu(E_\ell + p_\ell x) \left( 1 + 2.37 \frac{\Delta m}{m_4} \right) - 2.37 \frac{m_\ell^2 E_\nu}{m_4} \right] + 2 \Re(G_A f_3) \left[ m_\ell^2 E_\nu \left( 1 + 2.37 \frac{E_\nu + p_\ell x}{2m_4} \right) \right] + |f_3|^2 \left[ m_\ell^2 E_\nu (E_\ell - p_\ell x) \right] \right\}$$

(17)
where \( x = \vec{p}_\ell \cdot \vec{p}_\nu, \vec{p}_\ell + \vec{p}_\nu + \vec{q} = 0, \) \( \Delta m \) is the mass difference \( m_{\text{He}^4} - m_{\Lambda^4} \), and \( m_4 = 4m_p \), where \( m_p \) is the nucleon (proton) mass.

The correction term \([1 + 2.37(\Delta m/m_4)]\) has been evaluated using the approximate energy relation \( \Delta m \approx E_\ell + E_\nu \), and the form factor can be adequately numerically approximated by

\[
F^2(q^2) = a - bq^2 + cq^4 - dq^6, \quad \text{(18)}
\]

with \( q \) in units of 100 MeV/c, and \( a = 0.723, b = 0.170, c = 0.0166, d = 0.0025 \) (taken from Dalitz and Downs\([6]\)].

We consider the motion of the \( \text{He}^4 \) to be non-relativistic. Thus, the energy and momentum conservation conditions are

\[
\begin{align*}
\Delta m &= E_\ell + E_\nu + \frac{q^2}{2m_{\text{He}^4}}, \\
0 &= \vec{p}_\ell + \vec{p}_\nu + \vec{q},
\end{align*}
\]

(19) (20)

where \( m_{\text{He}^4} \), the mass of the \( \text{He}^4 \), will be approximated by \( m_4 \). We obtain \( q^2 = p_\ell^2 + E_\ell^2 + 2p_\ell E_\ell x \). In terms of the lepton energy \( E_\ell \), momentum \( p_\ell \) and the mass difference \( \Delta m \), we get

\[
E_\nu = \Delta m - E_\ell - \frac{p_\ell^2 + (\Delta m - E_\ell)^2 + 2p_\ell(\Delta m - E_\ell)x}{2m_4},
\]

(21)

using the approximate energy conservation relation \( E_\ell = \Delta m - E_\nu \) in the small correction term proportional to \( 1/m_4 \). Energy conservation limits the electron (muon) energy to

\[
m_\ell \leq E_\ell \leq E_{\text{max}},
\]

(22)

where

\[
E_{\text{max}} = \Delta m(1 - \frac{\Delta m}{2m_4}) + \frac{m_\ell^2}{2m_4}(1 - \frac{\Delta m}{m_4}).
\]

(23)

To obtain the phase space needed for the decay process, it is useful to find the quantity \( \frac{\partial E_\nu}{\partial \Delta m} \), which is given by

\[
\frac{\partial E_\nu}{\partial \Delta m} = 1 - \frac{\Delta m - E_\ell + p_\ell x}{m_4}.
\]

(24)

In order to find the lepton energy spectrum, \( d\Gamma/dE_\ell \), we must integrate over phase space the quantity

\[
d^6\Gamma = \frac{1}{(2\pi)^4} \left( \frac{1}{2J+1} \right) \left( m_\ell m_\nu \sum_{\text{spins}} |M|^2 \right) \frac{d^3\vec{p}_\ell}{(2\pi)^3 E_\ell} \frac{d^3\vec{p}_\nu}{(2\pi)^3 E_\nu} \frac{d^3\vec{q}}{(2\pi)^3} \times \delta^3(\vec{p}_\ell + \vec{p}_\nu + \vec{q}) \delta(\Delta m - (E_\ell + E_\nu + \frac{q^2}{2m_4})),
\]

(25)

where non-relativistic kinematics for the recoiling \( \text{He}^4 \) nucleus, including the substitution of \( m_4 \) for \( m_{\text{He}^4} \), have been used. Using \( J = 0 \), after integrating over \( p_\nu \), as well as over \( \vec{q} \), using energy and momentum conservation, we obtain

\[
d^5\Gamma = \frac{1}{(2\pi)^2} \left( m_\ell m_\nu \sum_{\text{spins}} |M|^2 \right) \frac{d^3\vec{p}_e}{(2\pi)^3 E_e} \frac{dE_\nu}{\partial \Delta m} \frac{\partial E_\nu}{\partial \Delta m} d\ell.
\]

(26)
We next substitute for $\partial E_\nu / \partial \Delta m$ and integrate over the azimuthal angle $\phi_{\ell \nu}$ ($f d\phi_{\ell \nu} = 2\pi$) to get, using the relation $p_\ell dp_\ell = E_\ell dE_\ell$,

$$d^4\Gamma = \frac{1}{(2\pi)^4} \left( m_\ell m_\nu \sum_{\text{spins}} |M|^2 \right) (1 - \frac{\Delta m - E_\ell + p_\ell x}{m_4}) p_\ell E_\nu dx dE_\ell d\Omega_\ell. \tag{27}$$

Using $q^2 = E_\ell^2 + E_\nu^2 + 2p_\ell E_\nu x$, and substituting for $E_\nu$ the relation (in terms of $E_\ell, p_\ell$ and $x$), and using $\int\int d\Omega_\ell = 4\pi$, we can rewrite the lepton (electron or muon) spectrum $\frac{d\Gamma}{dE_\ell}$ as

$$\frac{d\Gamma}{dE_\ell} = \frac{1}{4\pi^3} \int_{-1}^{+1} \left( m_\ell m_\nu \sum_{\text{spins}} |M|^2 \right) \left(1 - \frac{\Delta m - E_\ell + p_\ell x}{m_4}\right) p_\ell E_\nu dx. \tag{28}$$

The integration over $x$ was performed analytically, using Mathematica. The total rate for both muons and electrons was found by integrating Eq. (27) numerically, from $m_\ell \leq E_\ell \leq E_{\text{max}}$. The energy spectra were then converted into momentum spectra, using the relation $\frac{d\Gamma}{dp_\ell} = \frac{d\Gamma}{dE_\ell} \frac{p_\ell E_\ell}{E_\ell}$. The momentum spectra, for the case of $f_3 = 0$, i.e., for the case of no second-class currents, are plotted in Fig. 1, with the electron spectrum being the full line and the muon spectrum being the dashed line. Both of these spectra have been normalized to unit area.

![Figure 1: The normalized lepton momentum spectrum $\frac{1}{p} \frac{d\Gamma}{dp}$ vs. the lepton momentum $p$, in units of 100 MeV. The solid curve is for the electron and the dashed curve is for the muon.](image)

In absolute units, the spectra have been integrated, using $G_A(0) = \sqrt{\frac{2}{3}} G_F \sin\theta_C$, where the Fermi coupling constant $G_F$ is given by $1.01 \times 10^{-5}$ and $\theta_C$, the Cabbibo angle, was taken as 0.26. The results for $f_3 = 0$ (no second-class current) are

$$\Gamma_e = 2.18 \times 10^6 \text{sec}^{-1},$$

$$\Gamma_\mu = 0.742 \times 10^6 \text{sec}^{-1}. \tag{29}$$
4 Experimental Predictions

Using a lifetime\cite{1,8} for the $\Lambda$H$^4$ of $2 \times 10^{-10}$ sec., and with the rates of eq. (29), we find branching ratios for the electron and muon decays of $R_e = \frac{\Gamma_e}{\Gamma_{\text{all}}} = 4.36 \times 10^{-4}$ and $R_\mu = \frac{\Gamma_\mu}{\Gamma_{\text{all}}} = 1.48 \times 10^{-4}$, respectively. Using a production rate\cite{1} of $10^{-2}$ $\Lambda$H$^4$ per stopped K$^-$ in He$^4$, we find, for $10^9$ stopped K$^-$ in a helium target, that 4600 decays of $\Lambda$H$^4 \rightarrow e^- + \bar{\nu} + \text{He}^4$ and 1570 decays of $\Lambda$H$^4 \rightarrow \mu^- + \bar{\nu} + \text{He}^4$ are expected. These results are summarized in Table I, where we have used the production rates and branching ratios of $\Lambda$H$^4 \rightarrow$ all $e^-$, $\Lambda$He$^4 \rightarrow$ all $e^-$ and $\Lambda$H$^3 \rightarrow$ all $e^-$ taken from ref. 5 to calculate these event rates.

In Table II, we summarize the sensitivity of the experiment to assumed values of $f_3$, the amplitude that violates the Standard Model. There, the amplitude $f_3$ has units of inverse mass $m^{-1}$. If instead, we reexpress the amplitude as $f_3 = \frac{1}{M}$, we see from Table II that we can reach a limit of $M \approx 10$ GeV for an experimental sensitivity of 2%. This is also the level where we might expect SU(3) violations to play a role.

References

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Table I

Summary—Experimental Rates for Hyperfragment $\beta$-Decay

For $10^9$ stopped $K^-$ in a He$^4$ target, we expect:

- 7600 decays of $\Lambda H^4 \rightarrow$ all e$^-$,
- 10,500 decays of $\Lambda He^4 \rightarrow$ all e$^-$,
- 1200 cases of $\Lambda H^3 \rightarrow$ all e$^-$,
  of which 400 are $\Lambda H^3 \rightarrow e^- + \bar{\nu} +$ He$^3$.

Pure Fermi Transitions

- 4600 $\Lambda H^4 \rightarrow e^- + \bar{\nu} +$ He$^4$,
- 1570 $\Lambda H^4 \rightarrow \mu^- + \bar{\nu} +$ He$^4$,

Table II

| $f_3 \, (m_\mu)$ | $R$ |
|------------------|-----|
| 1.0              | 0.72 |
| 0.1              | 2.46 |
| 0.01             | 2.88 |
| 0                | 2.94 |
| -.01             | 2.99 |
| -.1              | 3.52 |
| -1.0             | 5.50 |

Summary: At the level of 2%, we are sensitive to a mass scale of $\approx 10$ GeV.