A Language for Describing Optimization Strategies

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Abstract

Optimizing programs to run efficiently on modern parallel hardware is hard but crucial for many applications. The predominantly used imperative languages - like C or OpenCL - force the programmer to intertwine the code describing functionality and optimizations. This results in a nightmare for portability which is particularly problematic given the accelerating trend towards specialized hardware devices to further increase efficiency.

Many emerging DSLs used in performance demanding domains such as deep learning, automatic differentiation, or image processing attempt to simplify or even fully automate the optimization process. Using a high-level - often functional - language, programmers focus on describing functionality in a declarative way. In some systems such as Halide or TVM, a separate schedule specifies how the program should be optimized. Unfortunately, these schedules are not written in well-defined programming languages. Instead, they are implemented as a set of ad-hoc predefined APIs that the compiler writers have exposed.

In this paper, we present ELEVATE: a functional language for describing optimization strategies. ELEVATE follows a tradition of prior systems used in different contexts that express optimization strategies as composition of rewrites. In contrast to systems with scheduling APIs, in ELEVATE programmers are not restricted to a set of built-in optimizations but define their own optimization strategies freely in a composable way. We show how user-defined optimization strategies in ELEVATE enable the effective optimization of programs expressed in a functional data-parallel language demonstrating competitive performance with Halide and TVM.

1 Introduction

The tremendous gains in performance and efficiency that computer hardware continues to make are a key driving force for innovation in computing. This enables entire new areas of computing such as deep learning to deliver applications unthinkable even just a few years ago. With the end of Moore’s law and Denard’s scaling [23], these gains no longer come for free for software writers. Programs have to be optimized for an increasing diverse set of hardware devices by exploiting many subtle details of the computer architecture. Performance portability has emerged as a crucial concern as software naturally outlives the faster cycle of hardware generations. In addition, specialized hardware has proven to offer extreme benefits for performance and energy efficiency - if the specially optimized software exploits it.

The predominant imperative and low-level programming approaches such as C, CUDA, or OpenCL force programmers to intertwine the code describing the functional behavior of the program with optimization decisions. This makes them – by design – non performance portable. As an alternative, higher level domain-specific approaches have emerged that allow programmers to declaratively describe the functional behavior without committing to a specific implementation. Popular examples of this approach are virtually all machine learning systems such as TensorFlow [1] or PyTorch [27]. For these approaches, the compilers and runtime systems are responsible to optimize the computations that are expressed as data-flow graphs. Programmers have limited control about the optimization process. Instead large teams of engineers at Google and Facebook provide fast implementations for the most common hardware platforms, for TensorFlow including Google’s specialized TPU hardware. This labour intensive support of new hardware devices is currently only sustainable for the biggest companies in the market – and even they struggle [5]. To overcome this innovation obstacle and to achieve automated performance portability we will need to rethink how we separate, describe, and apply optimizations in a more principled way.

Encoding program transformations as rewrite rules has been a long established idea. Bird and de Moor [6] studied an algebraic programming approach where functional programs are rewritten by exploiting algebraic properties. The Glasgow Haskell Compiler allows the specification of rewrite rules for program optimizations [28]. More recently, Lift [33] encodes optimization and implementation choices as rewrite rules for optimizing a high-level pattern-based data-parallel functional language using an automated stochastic search method applying the rewrites. Rewrite based approaches, such as Lift, have the advantage of being easily extensible towards new application domains (such as stencils [21]) as well as supporting new hardware features (such as specialized vector instructions which are encoded as new low-level patterns and introduced by a rewrite rule [34]). Unfortunately, these rewrite approaches are limited in their...
practicality to deliver the high performance required in many real-world applications. They lack control over the rewriting process and the automated rewriting using stochastic search processes takes a long time to find a high performance implementation. In this paper, we are going to address these practical limitations of rewrite-based approaches for optimizing high-performance real-world applications by defining a strategy language that allows the definition of optimization strategies that precisely controls the rewrite process.

Halide [30, 31] has introduced the concept of separating programs into functional descriptions and schedules in the area of high-performance domain-specific code generators. A schedule describes the optimizations to be applied to the Halide algorithm that defines the functional behavior of the computation. Halide’s schedules – as well similar schedules in TVM [12] – are implemented using a set of predefined APIs that expose a fixed set of optimization options. Halide’s authors describe these APIs as a scheduling language but it lacks many desirable properties of a programming language. Most crucially, programmers are not able to define their own abstractions. Even the composition of existing optimization primitives is in some cases unintuitive due to the lack of a clear semantics and Halide’s compiler has default and implicit behavior limiting experts’ control. All of these reasons make writing schedules in Halide significantly harder than writing algorithms. Furthermore, for some desirable optimizations it is not sufficient to change the schedule but the algorithm itself has to be redefined – violating the promise of separating algorithm and schedule. In this paper, we build upon Halide’s general idea but provide a proper functional strategy language, called Elevate, with clear semantics of individual primitives and how they compose. It enables programmers to define their own abstractions for building optimization strategies in a composable and reusable way.

The design of Elevate is heavily inspired by research on strategy languages for rewrite systems used in other contexts – and largely unknown to the high-performance code generation community – such as Stratego [37]. Kirchner [24] provides a recent overview of the research of the rewriting community. We claim no novelty in the design foundations of strategy languages but instead in the strategies we present and their usage to facilitate the generation of highly efficient code on modern hardware.

Our paper makes the following key contributions:

- Description of the design of Elevate, a functional language for describing optimization strategies for high-performance code generation (Section 3);
- Demonstration of Elevate using three case studies: automatic differentiation (section 4), image processing (section 5) and deep learning (section 6). They show the flexibility and extensibility of Elevate and experimentally evaluate the practicality of a rewrite based approach for achieving competitive high performance.

## 2 Motivation and Background

We motivate the need for a strategy language with a closer look at Halide. We then argue for a more principled language approach for describing optimizations strategies.

### 2.1 Halide: Decoupling Algorithm from Schedules

Halide [30] has originally been designed to generate high performance code for image processing pipelines [31], but has since inspired similar approaches in other contexts such as TVM in deep leaning [12]. A crucial idea is the separation of a program in two parts: the algorithm describing the functional behavior, and the schedule specifying how the program should be optimized by the underlying Halide compiler.

Listing 1 shows a snippet of Halide code used for generating an efficient matrix-matrix multiplication for an Nvidia GPU. Halide is a DSL embedded in C++, so the syntax used here is C++. The lines 2–4 define the matrix-matrix multiplication computation: \( A \) and \( B \) are multiplied by performing

```cpp
// functional description of matrix multiplication
Var x,y; Func prod("prod"); #Dom r(0, size);
prod(x, y) = A(x, r) * B(r, y);
out(x, y) = prod(x, y);

// schedule for Nvidia GPUs
const int warp_size = 32; const int vec_size = 2;
const int x_tile = 3; const int y_tile = 4;
const int y_unroll = 8; const int r_unroll = 1;
Var xi,yi,xii,yii,xo,yo,x_pair,xio,yi,ty; RVar rxi,rxo;
out(x, y,0).bound(0, size);
.tile(x, y, xi, yi, x_tile * vec_size * warp_size, 
  y_tile * y_unroll).
.split(xi, y, yi, y_unroll).
.vectorize(xi, vec_size).
.splitt(xii, xi, xii, warp_size).
.split(yii, yo, y_unroll).
.reorder(xii, yi, yi, xii, xio, y, y).
.unroll(xi, yi, xii, warp_size).
.gpu_blocks(x, y).
gpu_lanes(xi).
prod.store_in(MemoryType::Register).compute_at(out, x).
.split(x, xo, xi, x, warp_size * vec_size, RoundUp).
.split(y, ty, y, y_unroll).
.gpu_threads(ty).
.unroll(x, vec_size).compute_at(gpu_lanes(xi)).
.unroll(xo).unroll(y).
.split(x, xo, xi, x, warp_size * vec_size, RoundUp).
.split(y, ty, y, y_unroll).
.gpu_threads(ty).
.unroll(x, vec_size).compute_at(gpu_lanes(xi)).
.unroll(xo).unroll(r).
.unroll(xo, r).unroll(r).
.reorder(x, xo, xi, y, r, ty, xo).
.unroll(xo).unroll(y);
Var Bx = B.in().args()[0], By = B.in().args()[1];
Var Ax = A.in().args()[0], Ay = A.in().args()[1];
B.in().compute_at(prod, ty).split(Bx, xo, x, warp_size).
gpu_lanes(xi).unroll(Bx).
A.in().compute_at(prod, r).vectorize(Ax, vec_size).
.split(Ax, xo, xi, warp_size).gpu_lanes(xi).unroll(xo).
.split(Ay, yo, yi, y_tile).gpu_threads(yi).unroll(y);
A.in().compute_at(prod, r).vectorize(Ax, vec_size).
.split(Ax, xo, xi, warp_size).gpu_lanes(xi).
.unroll(xo).unroll(Ay);
```

Listing 1. Matrix matrix multiplication in Halide. Lines 2–4 define the computation \( A \times B \), the other lines define the schedule specifying the optimizations to be applied by the compiler. From: https://github.com/halide/Halide/blob/master/apps/cuda_matmul/mat_mul_generator.cpp.
ELEVATE: A Language for Describing Optimization Strategies

The dot product for each coordinate pair \((x, y)\). The dot product is expressed as pairwise multiplications and reducing over the reduction domain \(r\) using the ++ operator (line 3).

The other lines in the listing define the schedule specifying the optimizations to be performed. The Halide compiler takes this C++ program and produces efficient GPU code coming close to highly optimized low-level library code.

By looking at the code it is immediately clear that writing a schedule is significantly more challenging than writing the algorithm describing matrix-matrix multiplication. Schedules are written using a sequence of API calls on the C++ objects that represent the input (A, B) and output (out) data. prod represents the reduction operation in Halide’s internal representation. While the algorithm and schedule are separated they still share the same C++ identifiers and must, therefore, be written in the same C++ scope limiting the reuse of schedules across algorithms.

This schedule uses 12 built-in optimization primitives (bound, tile, split, vectorize, reorder, unroll, update, compute_at, store_in, gpu_blocks, gpu_threads, gpu_lanes). Some of these optimizations are specific for the hardware (like vectorize or gpu_threads), others are generally useful algorithmic optimizations for many applications (like tiling to increase data locality), and others are low-level optimizations (like unroll and reorder that transform loop nests). Halide is not easily extensible. Adding a new optimization primitive to the schedule API requires extending the Halide compiler. Even a primitive like tile that can be implemented with split and reorder is represented as a composition but provided as a built-in abstraction. Halide’s schedules lack the ability for user-defined abstractions.

The behavior of some primitives is not intuitive and the documentation provides only informal descriptions, e.g., for update: “Get a handle on an update step for the purposes of scheduling it.” The lack of clear descriptions of the optimization primitives makes reasoning about the schedule difficult. For example, is it unclear to us why lines 21–23 are repeated at lines 25–27 with calls to unroll and update in between.

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1 See: https://halide-lang.org/tutorials/tutorial_lesson_05_scheduling_1.html

Figure 1. Two-dimensional binomial filter in Halide (top) and separated version (bottom).

Figure 2. 2D binomial filter in a Lift-like language (top) and the separating optimization strategy in ELEVATE (bottom).

In the remainder of the paper, we will explain the language for writing such strategies and defining custom abstractions.
3 ELEVATE: A Language for Describing Optimization Strategies

In this section, we describe our language for describing optimization strategies: ELEVATE. It is heavily inspired by earlier works on strategy languages for term rewriting systems, particularly Stratego [41]. Our key contribution is not the design of ELEVATE itself but rather its application to define optimization for high-performance code generators. In this section, we focus on introducing ELEVATE as a practical programming language. For a more formal treatment of strategy languages see [39].

3.1 Language Features and Types

ELEVATE is a functional language with a standard feature set including function recursion, algebraic data types and pattern matching. Besides the standard scalar data types such as int, types of interests are function types, tuple types and list types. Our current implementation is an embedded DSL in Scala and we use Scala-like notation in the paper.

3.2 Strategies

A strategy is the fundamental building block of ELEVATE. Strategies encode program transformations and are modeled as functions with the following type:

\[
\text{type } \text{Strategy}[P] = \text{P} \Rightarrow \text{Success}[P] \quad | \quad \text{Failure}[P](s: \text{Strategy}[P])
\]

Here \( P \) is the type of the program that is rewritten. \( P \) could for example be \( \text{Rise} \) for programs written in the Rise language from fig. 2. A \( \text{RewriteResult}[P] \) is an algebraic datatype encoding the success or failure of applying a strategy to a program:

\[
\text{RewriteResult}[P] = \text{Success}[P](p: P) \quad | \quad \text{Failure}[P](s: \text{Strategy}[P])
\]

In case of a successful application, \( \text{Success} \) contains the transformed program, in case of a failure, \( \text{Failure} \) contains the strategy that has been unsuccessful.

The simplest example of a strategy is the \( \text{id} \) strategy which always succeeds and returns its input program:

\[
\text{def id}[P]: \text{Strategy}[P] = (p: P) \Rightarrow \text{Success}(p)
\]

The \( \text{fail} \) strategy does the opposite and always fails while recording that the \( \text{fail} \) strategy was the one failing:

\[
\text{def fail}[P]: \text{Strategy}[P] = (p: P) \Rightarrow \text{Failure}(\text{fail})
\]

3.3 Rewrite Rules as Strategies

In ELEVATE, rewrite rules are also strategies, i.e., functions satisfying the same type given above. Let’s look at a concrete type of programs, such as \( \text{Rise} \) which is a pattern-based functional programming language where we want to apply well-known rewrite rules such as the fusion of two \( \text{map} \) calls:

\[
\text{(map } f \text{) } \odot \text{ (map } g \text{) } \Rightarrow \text{map } (f \odot g)\].
\]

In \( \text{Rise} \), the left-hand side of the rule is expressed as:

\[
\text{val } p: \text{Rise} = \text{fun}(xs \Rightarrow \text{map}(f)(\text{map}(g)(xs)))
\]

The AST representation of the body of this is shown in fig. 3 on the left, with function applications explicit as \text{app} nodes. This is the implementation of the fusion rule in ELEVATE:

\[
\text{def mapFusion: Strategy[Rise]} = p \Rightarrow p \text{ match } \{
\text{success } \text{app(app(map, f), app(app(map, g), xs)) } \Rightarrow
\text{Success( map(fun(x => f(g(x))))(xs) )}
\text{failure } \text{mapFusion}\}
\]

Note that we are mixing expressions of the \( \text{Rise} \) language (i.e., \( \text{map}(f) \)) and ELEVATE. The expression nested inside \( \text{Success} \) is the rewritten expression shown in fig. 3 on the right.

3.4 Strategy Combinators

A key idea that ELEVATE inherits from Stratego [39] is to describe strategies as compositions of other strategies. Therefore, we introduce strategy combinators.

The \( \text{seq} \) combinator is given two strategies \( ss \) and \( ss \) and applies the first strategy to the input program \( p \). Afterwards, the second strategy is applied to the result.

\[
\text{def seq}[P]: \text{Strategy}[P] \Rightarrow \text{Strategy}[P] \Rightarrow \text{Strategy}[P] = \text{fs} \Rightarrow \text{ss} \Rightarrow p \Rightarrow \text{fs}(p).\text{flatMapSuccess}(\text{ss})
\]

The \( \text{seq} \) strategy is only successful when both strategies are successfully applied in succession, otherwise \( \text{seq} \) fails.

In the implementation of \( \text{seq} \), we make use of the monadic interface of strategies: the \( \text{RewriteResult} \) ADT provides two versions of \text{map/flatMap} to compose strategies – one in case of a successful strategy application and one in case of failure.

The \( \text{lChoice} \) combinator is given two strategies and applies the second strategy only if the first strategy failed.

\[
\text{def lChoice}[P]: \text{Strategy}[P] \Rightarrow \text{Strategy}[P] \Rightarrow \text{Strategy}[P] = \text{fs} \Rightarrow \text{ss} \Rightarrow p \Rightarrow \text{fs}(p).\text{flatMapFailure}(_) \Rightarrow \text{ss}(p))
\]

We use \( \Leftarrow \) as notation for \( \text{lChoice} \) and \( ; \) for \( \text{seq} \) and define two more combinators:

The \( \text{try} \) combinator applies a strategy and in case of failure applies the identity strategy. Therefore, \( \text{try} \) never fails.

\[
\text{def try}[P]: \text{Strategy}[P] \Rightarrow \text{Strategy}[P] = s \Rightarrow p \Rightarrow (s \Leftarrow \text{id})(p)
\]

\( \text{repeat} \) applies a strategy until it is no longer applicable.

\[
\text{def repeat}[P]: \text{Strategy}[P] \Rightarrow \text{Strategy}[P] = s \Rightarrow p \Rightarrow \text{try}(s ; \text{'; repeat}(s))(p)
\]
fun(xs => map(f)(map(g)(map(h)(xs))))

Figure 4. Two possible locations for applying the map-fusion rule within the same program.

3.5 Traversal Strategies

The mapFusion strategy we saw in the previous subsection is implemented as a function in ELEVATE. Therefore, its match statement will try to pattern match its argument – the entire program. This means that a strategy on its own is very hard to reuse in different circumstances.

In addition, a strategy is often applicable at multiple places within the same program or only applicable at a specific location. For example, the mapFusion strategy is applicable twice in the following Rise program:

```haskell
def some(P): Strategy[Rise] => Strategy[Rise] =
  case body(argument(mapFusion)) => p => body(argument(mapFusion))
  case argument(s: Strategy[Rise]) => Strategy[Rise] =
    p => p match
    case app(f,a) => s(f).mapSuccess(nf => app(nf, a))
    case _ => Failure(s)
```

We may fuse the first or last two maps as shown in fig. 4.

In ELEVATE, we use traversal strategies to describe at which exact location a strategy is applied. Luttik et al. [25] proposed three basic traversal strategies:

```haskell
def all(P): Strategy[P] => Strategy[P]
def one(P): Strategy[P] => Strategy[P]
def some(P): Strategy[P] => Strategy[P]
```

all applies a given strategy to all sub-expressions of the current expression and fails if the strategy is not applicable to all sub-expressions. one applies a given strategy to exactly one sub-expression and fails if the strategy is not applicable to any sub-expression. some applies a given strategy to at least one sub-expression but potentially more if possible. one and some are allowed to non-deterministically choose sub-expressions.

In ELEVATE, we see these three basic traversal strategies as a type class: an interface that has to be implemented for each program type p. The implementation for Rise is straightforward. Rise programs are represented by ASTs such as the one in fig. 4, therefore, all, one, and some correspond to the obvious implementations on the tree-based representation.

To fuse the first two maps in fig. 4 we use the one traversal strategy: one(mapFusion)(threemaps). This will apply the mapFusion strategy not at the root of the AST, but instead one level down first trying to apply the strategy (unsuccessfully) to the function parameter and then (successfully) to the function body highlighted in the upper-right blue box.

To fuse the last two maps we use the one traversal strategy twice to apply mapFusion two levels down in the AST: one(one(mapFusion))(threemaps). This successfully applies the fusion strategy to the expression highlighted in the lower-left purple box in fig. 4.

3.6 Language-Specific Traversal Strategies

The traversals we have discussed so far are not specific to a particular language, such as Rise. These traversals are flexible, but offer only limited control as for one and some the selection of sub-expressions is either non-deterministic, or implementation-dependent (as for Rise) and in our context it makes rarely sense to apply a strategy to all sub-expressions.

In ELEVATE, one can easily specify program language specific traversal primitives. Rise is a functional language using $\lambda$-calculus as its representation. Therefore, it makes sense to introduce traversals that navigate the two core concepts of $\lambda$-calculus: function abstraction and application.

To apply a strategy to the body of a function abstraction we define the following traversal strategy:

```haskell
def body(s: Strategy[Rise]): Strategy[Rise] =
  p => p match
  case fun(x,b) => s(b).mapSuccess(nb => fun(x,nb))
  case _ => Failure(s)
```

A strategy s is applied to the function body and if successful a function is built around the transformed body.

Similarly we define traversals function and argument to traverse function applications:

```haskell
def function(s: Strategy[Rise]): Strategy[Rise] =
  p => p match
  case app(f,a) => s(f).mapSuccess(nf => app(nf, a))
  case _ => Failure(s)
```

```haskell
def argument(s: Strategy[Rise]): Strategy[Rise] =
  p => p match
  case app(f,a) => s(a).mapSuccess(na => app(f, na))
  case _ => Failure(s)
```

For the Rise program shown in fig. 4, we are now able to precisely describe a traversal path in the AST. To fuse the first two maps we may write body(mapFusion)(threemaps), and to fuse the others body(argument(mapFusion))(threemaps).

The traversals defined here are specific to Rise but similar traversals are obviously possible for any functional language. If the program is not a functional language, say e.g., a computational graph as used by Tensorflow, different language-specific traversals (e.g., leftOperand and rightOperand) could be defined to describe language-specific traversals.
3.7 Complete Expression Traversal Strategies

All of the traversal primitives introduced so far apply their given strategies only to immediate sub-expressions.

Using strategy combinators and traversals, we are able to define recursive strategies which traverse entire expressions:

```python
def topDown(p: Strategy): Strategy =
    p => (s => (topDown(s))(p))
def bottomUp(p: Strategy): Strategy =
    p => (bottomUp(s)) => s(p)
def allTopDown(p: Strategy): Strategy =
    p => (topDown(s) => s)(p)
def allBottomUp(p: Strategy): Strategy =
    p => (bottomUp(s) => s)p)
def tryAll(p: Strategy): Strategy =
    p => (tryAll(s))(p)
```

topDown and bottomUp are useful strategies traversing an expression either from the top or from the bottom, trying to apply a given strategy at every sub-expression and stopping at the first successful application. If the strategy is not applicable at any sub-expression, topDown and bottomUp fail.

allTopDown and allBottomUp do not use lChoice insisting on applying the given strategy to every sub-expression.

The tryAll strategy is often more useful as it wraps its given strategy in a try and thus never fails but applies the strategy wherever possible. Also note that the tryAll strategy traverses the AST bottom-up instead of top-down.

These traversals have also been proposed by Visser [39] and we use them here with slightly different names more fitting for our use cases.

3.8 Normalization, Confluence and Termination

When implementing rewrite rules, such as the mapFusion rule, as strategies, the match statement expects the program expression to be in a particular syntactic form. For a functional language like Risc, we might for example expect that expressions are fully \( \beta \)-reduced. To ensure that expressions satisfy a normal form we define:

```python
def normalize(p: Strategy): Strategy =
    p => repeat(topDown(s))(p)
```

The normalize strategy applies a given strategy repeatedly at every possible sub-expression until it can not be applied any more. Therefore, after normalize successfully finishes it is not possible to apply the given strategy to any sub-expression any more. By defining a strategy for \( \beta \)-reduction and using it together with normalize we ensure that expressions are in \( \beta \)-normal-form.

Confluence (multiple non-deterministic rewrite paths eventually produce the same result) and termination are desirable properties for normal forms in term rewriting systems [39]. In ELEVATE, confluence only becomes a factor when the implementation of one and some are non-deterministic. This can often be avoided such as for the use cases we consider with Risc and \( \bar{F} \) that we will discuss in section 4.

Termination of normal forms critically depends on the chosen set of strategies. Therefore, reasoning about terminating normal forms must be done on a case by case basis. For example, it is trivial to build a non-terminating normal form using the id strategy that is always applicable. We currently, do not prevent the creation of non-terminating strategies similar as almost all general purpose computational languages do not prevent writing non-terminating programs. In the future, we are interested to introduce a richer type system for ELEVATE to better assist the user in writing well behaved strategies.

3.9 Summary

We have introduced ELEVATE, a language for describing optimization strategies. In the next three sections we discuss three case studies of using ELEVATE in the domains of automatic differentiation, image processing, and deep learning.

4 Case Study 1: Automatic Differentiation

So far we have seen Risc as the only example of a language that we transform with ELEVATE, but ELEVATE is flexible and not restricted to a single language. In this first case study, we will look at the \( F \) language that has been introduced in [32].

\( F \) is a small functional language capable of automatically computing the derivative of arbitrary \( F \) functions. Implementing automatic differentiation is not too difficult but making it efficient is non-trivial. \( F \) achieves efficiency by rewriting the differentiated code. In the paper, rewrite rules are specified alongside examples. Example 5 in the paper shows that a \( F \) program transposing a matrix twice can be rewritten into a program without transposition, see fig. 5.

The paper does not provide an explanation how the rewriting between these programs happens or is specified. The authors only state “by applying the loop fusion rules and performing further partial evaluation the expression is derived”.

We are interested in exploring the flexibility of ELEVATE and if we can specify the rewrite rule applications programmatically. We implemented the \( F \) language representing expressions using an algebraic data type FSmooth. We implemented the rewrite rules from the paper, such as fusion rules:

```plaintext
(build e_0 e_1)[e_2] <- e_1 e_2
length (build e_0 e_1) <- e_0
```

**Figure 5.** Transposing a matrix twice in \( \bar{F} \) (top) and the rewritten program without transposition (bottom). From [32]
These fusion rules are implemented as ELEVATE strategies:

```python
def buildGet(p: FSmooth): Strategy[FSmooth] = p match 
  case app(get,(app(build,(e0,e1)),e2)) => Success(app(e1,e2))
  case _ => Failure(buildGet) 

def lenBuild(p: FSmooth): Strategy[FSmooth] = p match 
  case app(length, (app(build, (e0, e1)))) => Success(e0)
  case _ => Failure(lenBuild) 
```

Using these rewrite rules encoded as ELEVATE strategies, we use `normalize` and `lChoice` to specify multiple strategies that should be applied repeatedly at every sub-expression:

```python
normalize(buildGet <= lenBuild <= letPartialEval <=
  letApp <= funToLet)(mt)
```

This ELEVATE program successfully rewrites the doubly-transposed $\tilde{F}$ program into the non-transposed form. By tracing the execution of the ELEVATE program we get a sequence of 12 basic rewrite rule applications explaining the program transformation step-by-step. A full version of the trace is shown in the supplementary material.

We also implemented the other examples in the paper which contain rewriting in ELEVATE – and identifying a minor bug in the description of example 6. This case study shows ELEVATE’s flexibility to implement an existing rewrite system. It also demonstrates ELEVATE’s ease of use: we did not have to think about where to apply the individual rewrite rules thanks to the abstraction provided by `normalize`. It is important to stress, that this is not a built-in abstraction but defined itself in terms of the smaller building blocks `repeat` and `topDown`. In the next case-study, we discuss how the ability to leverage these abstractions and build custom ones enables the definition of optimization strategies for image processing.

5 Case Study 2: Image Processing

In this case study, we are interested to see how our extensible strategy language ELEVATE compares directly to Halide. We will look at the binomial filter example which we already briefly showed in fig. 1 and 2. We will use Rise as our computational language. Rise is a Lift [33]-like language that is compiled by rewriting high-level-pattern-based programs into a low-level representation encoding implementation and optimization choices. It then uses a compilation process similar to [3] to compile the low-level pattern-based code into parallel imperative code.

5.1 Halide Schedules for the Binomial Filter

For Halide, we saw two algorithms describing the computation of the binomial filter in fig. 1. The naive version on the top is a straightforward implementation as a two-dimensional stencil. The separated version on the bottom specifies the computation as a composition of vertical and horizontal one-dimensional stencils. In Halide, it is not possible to take the naive algorithm and use a schedule to specify the separability optimization. When we do not specify a schedule in Halide, a default schedule is chosen that inlines computations as much as possible. Therefore, the naive and the separated version with the default schedule result in code executing two nested loops and accessing 9 elements of the input image. We can instruct Halide not to fully inline the vertical filter by writing: $b_y.y$ compute at(out, y). This schedule uses the variable names from the Halide algorithm implementing the separated version of binomial filter. The $compute$ at Schedule instruction tells the compiler to perform the computation of the vertical filter ($b_y$) inside the for loop iterating over the $y$ dimension of the out image. This computation will be stored in a temporary that is consumed in the nested loop iterating over the $x$ dimension of out. We call this version `scanline` as the temporary stores an entire line of the image.

For all versions, it is possible to parallelize the outermost loop with: `out.parallel(y)`;

5.2 ELEVATE Strategies Optimizing Binomial Filter

To compare with Halide, we express the binomial filter with Rise as shown at the top of fig. 2. This formulation is the naive way to describe a two-dimensional filter in Rise. The filter is expressed using two-dimensional variations of the `pad`, `slide`, and `map` high-level patterns in the style of [21]. `pad` models the boundary handling, the sliding window pattern `slide` describes a neighborhood of values each of which is then processed by the `map` pattern. The 20 versions of these patterns are just macros defined as compositions of the one-dimensional versions and a few additional basic patterns. The `dot` product computation is also defined as a composition:

```lambda
dot(x)(y) = reduce(add)(a)(map(mult)(zip(x)(y)))
```

To express the separability as an ELEVATE strategy we introduce an image-processing specific rewrite rule that describes how the dot product of the weights and the neighborhood inside the binomial filter is separated:

```python
def separateDot(w2d:Rise, wh:Rise, wv:Rise): Strategy[Rise] = p match 
  case app(app(app(reduce, add), g), app(app(map, mult),
    app(app(zip, app(join, w)), app(join, nbh)))) if w==w2d =>
    Success(nbh |> map(dot(wh)) |> dot(wv))
  case _ => Failure(sepDot) 
```

The underlined values in the pattern matching indicate that these must match for the pattern matching to succeed.

The strategy takes three parameters that are all expressions in the computational language Rise. $w2d$ represents the two-dimensional weights and $wh$ and $wv$ are the separate horizontal and vertical weights. ELEVATE does not automatically attempt to separate the weights nor does it attempt to prove that the horizontal and vertical weights are a valid separation of the two-dimensional weights. These considerations are left to the user. We aim to empower users to extend ELEVATE which such domain-specific strategies.
To rewrite the naive binomial filter into the separated one, we apply `separateDot` with `topDown` as shown earlier as an ELEVATE strategy at the bottom of fig. 2 resulting in:

```plaintext
1 ing |> pad2D(4) |> slide2D(3)(4) |> map2D(fun(nbh =>
2 nbh |> map(dot(weightsH)) |> map(dot(weightsV))) ))
```

We express the lowering of high-level Rise expressions with ELEVATE strategies that encode low-level implementation choices such as whether we would like to parallelize or not. In Halide, the built-in `compute_at` primitive is used for the scanline version. In ELEVATE a user-defined strategy encodes the same transformation. We will investigate more complex optimization and low-level implementation strategies in more detail in the third case study in section 6.

### 5.3 Performance Evaluation

Even though this paper discusses ELEVATE and not Rise we want to evaluate whether a rewrite-based approach by combining them is capable of achieving competitive performance compared to the industry-strength Halide compiler that uses more traditional compiler techniques. We have seen that ELEVATE allows the definition of optimization strategies in an extensible way and that this allows to express optimizations as strategies that are not expressible as Halide schedules (such as separability). We are now interested to see if these strategies encoding the same optimization decisions lead to competitive performance when compiled with Rise and compared to Halide.

Figure 6 shows the performance of the Halide and Rise generated code measured on an ARM Cortex A7 quad-core. We can see – not surprisingly – that the non-parallel versions on the left are significantly slower than the parallel versions. The Halide generated code is 10-15% faster than the Rise generated code. Improvements to the Rise compiler might close this gap in the future. Crucially, we observe the same trend for performance improvements due to optimizations. This demonstrates that our extensible and rewrite based approach is capable of achieving competitive performance.

![Figure 6. Performance evaluation of Halide and Rise generated code for the binomial filter application. Optimization decisions for Rise are implemented as ELEVATE strategies.](image)

---

6 Case Study 3: Deep Learning

In our first two case studies, we looked at fairly simple strategies. In our final case study, we are interested to explore the practicability and scalability of ELEVATE. We are looking at the domain of deep learning where performance optimizations are particularly important. In this section, we explore the implementation of a scheduling language with ELEVATE using the ability to define custom abstractions. We use TVM [12] as an example for a state-of-the-art optimizing deep learning compiler with a scheduling API implemented in Python, similar to Halide. We use Rise again as the target language for the strategies we develop.

We start by looking at how TVM represents schedules and how we implement simple scheduling primitives such as `parallel` and `vectorize` in ELEVATE. Then we show how to implement more complex scheduling primitives like `tile` using composition in ELEVATE whereas it is built-in in TVM. We follow a tutorial from the TVM authors that discusses seven differently optimized versions of matrix multiplication. For each one, we show an equivalent strategy implemented with ELEVATE and evaluate the performance achieved.

### 6.1 TVM Schedules for Matrix Multiply

In TVM, computations are expressed similar to TensorFlow:

```python
1 C = tvm.compute((N, N), lambda x, y:
2 tvm.sum(A[x,k] * B[k,y], axis=k), name='C')
```

The "How to optimize GEMM on CPU" tutorial discusses seven versions applying different optimizations. The baseline version uses a default schedule with no instructions. The blocking version tiles the two outermost loops computing matrix C and then splits the reduction loop before reordering the loop nest:

```python
1 # blocking version
2 xo, yo, xi, yi = s[C].tile(C.op.axis[0],C.op.axis[1],32,32)
3 k, = s[C].op.reduce_axis
4 ko, ki = s[C].split(k, factor=4)
5 s[C].reorder(xo, yo, ko, ki, xi, yi)
```

The vectorized version vectorizes the innermost loop and, in addition, the loop permutation version changes to a cache friendly access of matrix A by reordering loops in a different way (switching `ki` and `xi`):

```python
1 # loop permutation version
2 xo, yo, xi, yi = s[C].tile(C.op.axis[0],C.op.axis[1],32,32)
3 k, = s[C].op.reduce_axis
4 ko, ki = s[C].split(k, factor=4)
5 s[C].reorder(xo, yo, ko, ki, xi, yi)
6 s[C].vectorize(yi) # added already in vectorized version
```

The array packing version enables better memory accesses to matrix B by introducing a temporary packedB but requires changing the description of the computation in TVM:

```python
1 # array packing version
2 xo, yo, xi, yi = s[C].tile(C.op.axis[0],C.op.axis[1],32,32)
3 k, = s[C].op.reduce_axis
4 ko, ki = s[C].split(k, factor=4)
5 s[C].reorder(xo, yo, ko, ki, xi, yi)
6 s[C].vectorize(yi) # added already in vectorized version
```

---

2The 4 LITTLE cores of a Samsung Exynos 5 Octa 5422

3https://docs.tvm.ai/tutorials/optimize/opt_gemm.html
With this rewritten computation we can now also describe the scheduling primitives that affect the computation of the temporary packedB matrix. Contrary to Halide, this computation is not inherently parallelized by default in TVM and results in a separate loop nest.

The write cache for blocks version allocates memory for the reduction accumulator and unrolls loops. Building up on this, the fully parallel version parallelizes the outermost loop:

```python
packedB = tvm.compute((W // 32, X, 32), lambda x, y, z:
   if y % 32 + z in packedB:
       return packedB[y // 32, k, tvm.indexmod(y, 32)],
   axis=x, name='C')
```

TVM’s `split` scheduling primitive implements loop-blocking (also known as strip-mining). In RSE, this is achieved by transforming the computation over an array expressed by `map(f)` first input is split into a 2D array using `split(n)`, then `f` is mapped twice to apply the function to all elements of the now nested array, and finally the resulting array is flattened into the original one-dimensional form using `join`. We write `f >> g` to indicate reverse function composition, i.e.: `f(x) = g(f(x))`. It is important to note, that RSE does not materialize the intermediate two-dimensional array in memory, but only uses this representation inside the compiler for code generation. There is a second case for splitting `reduce` resulting in two nested reductions.

```python
def vectorize(n: Int): Strategy[Rise] = p => p match {
    case map => Success(asVector(n) >> map(map(f))) >> asScalar)
    case _ => Failure( vectorize(n) )
}
```

The `vectorize` strategy is most efficient when applied to the innermost loop of a loop-nest. In RSE, this corresponds to applying the `vectorize` strategy to the innermost `map` of potentially nested loops. This is achieved in ELEVATE by `bottomUp(vectorize)`. The extra constraint isScalarFun(f) ensures that only functions operating on scalars are vectorized by inspecting f’s type. The restriction to scalar functions for vectorization is a current limitation of RSE.

```python
def unroll: Strategy[Rise] = p => p match {
    case map => Success( mapSeqUnroll )
    case reduce => Success( reduceSeqUnroll )
    case _ => Failure( unroll )
}
```

The `unroll` strategy rewrites the high-level `map` and `reduce` patterns into RSE low-level patterns that will be unrolled by the RSE compiler.

### Identifying Locations
In TVM, named identifiers describe the location at which the optimization should be applied. For example, TVM’s `parallel` is invoked with an argument specifying the loop to parallelize. Using named identifiers allows writing invalid schedules, e.g., trying to vectorize a reduction axis failing at runtime when TVM detects the error.

In RSE, low-level implementation choices such as performing a computation in parallel are encoded with low-level patterns. For example, the `map` pattern that applies a function to each element of an array might be performed in a data-parallel fashion as indicated by the `mapPar` variant of the pattern. This is precisely what the `mapPar` strategy encodes: rewriting a `map` pattern into its parallel variant. A rewrite into the sequential variant `mapSeq` is defined in the same style.

```python
def mapSeq(n: Int): Strategy[Rise] = p => p match {
    case app(map, f) =>
        case map => Success(mapPar)
        case _ => Failure(parallel)
}
```

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```python
def asScalarFun(f):
    return isScalarFun(f)
```

The `asScalarFun(f)` ensures that only functions operating on scalars are vectorized by inspecting `f`’s type. The restriction to scalar functions for vectorization is a current limitation of RSE.

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def asScalarFun(f):
    return isScalarFun(f)
```

The `asScalarFun(f)` ensures that only functions operating on scalars are vectorized by inspecting `f`’s type. The restriction to scalar functions for vectorization is a current limitation of RSE.
6.3 Tiling as an Elevate Strategy

Tiling is an important optimization improving the cache hit rate by exploiting locality within a small neighborhood of elements. TVM’s tile is a more complicated scheduling primitive to implement because it is essentially a combination of two traditional loop transformations: loop-blocking and loop-interchange. In fact, tile in TVM is a built-in combination of split for loop-blocking and reorder for loop-interchange. We already saw how to implement split using Elevate and we will now discuss how to implement tile using a combination of rules, normal-forms and domain-specific traversals.

We use matrix-matrix multiplication as the illustrative example to explain the optimization steps. Since we use Rise as the target language for the tiling strategy, we need to consider how matrix multiplication is expressed there:

```plaintext
def tileND(n: List[Int]) => Strategy[Rise] = 
{ 
val dot = fun((a,b) => zip(a,b) |> map(*)) |> reduce(*, +))
val mm = fun(x::M.K.float => fun(b::K.N.float =>
map(fun(bcol => // iterating over N
map(fun(arrow => // iterating over M
(dot(arrow, bcol) // iterating over K
(*)))(transpose))
for this 2D case we also write tile(x,y)(mm).
}
```

Essentially, the dot product is computed for each combination of rows and columns of matrix A and B.

In the following, we show how to use Elevate to systematically construct a strategy out of simple building blocks that has the same effect as TVM’s tile scheduling primitive. Specifically, we construct a generalized strategy that is able to tile an arbitrary number of dimensions whereas TVM’s tile only tiles in two-dimensions.

```plaintext
def tileND(n: List[Int]): Strategy[Rise] = 
{ 
DFNF 
'λ'; (n.size match {
    case 1 => function(split(n.head)) // loop-blocking
    case i => 
        fmap(tileND(d-1)(n.tail)) 'λ'; // recurse
        function(split(n.head)) 'λ'; // loop-blocking
        interchange(i) }) // loop-reorder
} 
```

In the following, we first introduce the required normalforms (e.g., DFNF), then explain how we recursively traverse (fmap) to apply loop-blocking and finally briefly explain how we interchange dimensions in Rise (interchange).

Normal forms In Elevate, we introduced normalize to ensure that expressions are in a particular form expected by the implementation of rewrite rules. λ-calculus (and Rise) allows for semantically equivalent but syntactically different expressions. For example, \( \text{fun}(x \mapsto f(x)) \) is equivalent to \( f \) if \( f(x) \) does not appear free in \( f \). Transforming between these representations is called η-reduction and η-abstraction.

The simplest normal-form we use is the \( \beta\eta \)-normal-form (BENF) which exhaustively applies \( \beta \) and η-reduction:

```plaintext
def BENF = normalize(betaReduction <= etaReduction)
```

As not every function abstraction is η-reducible, the function arguments of Rise’s higher-order patterns map and reduce might have different forms. Therefore, we introduce a normal form making the data flow explicit by ensuring a function abstraction is present in every higher order pattern:

```plaintext
def DFNF = BENF
```

The definition shows the normalization for map. A similar case exists for reduce. Using the not and isFun predicates, that are themselves Elevate strategies, we describe the desired form in a natural and elegant way.

Recursively Applying Loop-Blocking In order to recursively apply the loop blocking strategy to nested maps, we make use of the Rise-specific traversal fmap:

```plaintext
def fmap(s: Strategy[Rise]) = function(argOf(map, body(s)))
```

fmap essentially traverses to the function argument of a map primitive and applies the given strategy \( s \). For example,

```plaintext
fmap(fmap(split(n))(DFNF(map(map(map(f))))))
```

skips two maps applying loop-blocking to the innermost map.

Interchange in tile After recursively blocking all map \( s \), we use interchange to rearrange the dimensions in the correct order. For simplicity, we describe the two dimensional case: after loop-blocking the data is four-dimensional and we must swap the two inner dimensions. To achieve this we introduce two transpose patterns and then move one of the transpose into the right position. Doing it this way every strategy is a semantics-preserving transformation ensuring the correctness of the overall optimization.

6.4 Reordering as an Elevate Strategy

Due to the loopless nature of Rise, implementing TVM’s reorder primitive as a strategy is more complicated. Instead of simply interchanging perfectly nested loops, the same is achieved in Rise by interchanging the nesting of map and reduce patterns. Therefore, there are multiple possible combinations to consider and the implementation of each rewrite rule requires reasoning about why exchanging the specific patterns is possible in the first place.
We implemented a `reorder` strategy in ELEVATE but its implementation is non-trivial and, therefore, is not discussed here. While it is possible to implement TVM's `reorder` primitive, this particular loop transformation is just not a good fit for the pattern-based abstractions in the Rist language.

### 6.5 Matrix Multiply Schedules as ELEVATE Strategies

Using the TVM-like scheduling abstractions implemented as ELEVATE strategies, we are now able to discuss how we combine them together to describe entire schedules in ELEVATE.

For baseline, TVM does not provide a schedule, but we describe the implicit behaviour of the compiler explicitly:

```plaintext
(DFNF ';;' topDown(fuseReduceMap) ';;' lowerToC)(nm)
```

The TVM algorithm computes the dot product in a single

The `DFNF` strategy lowers every high-level pattern into

The `topDown` strategy lowers every high-level pattern into

The `split` strategy applies the following topDown traversal and the `isReduce` strategy predicate that applies the following `split` only if the current expression is a reduction. We use the `;;` combination to normalize using `DFNF` between each step.

The `loop permutation` version incorporates the changes of the `vectorized` version by adding `vectorize` and a different reordering of dimensions. In contrast to TVM we identify dimensions by index rather than by name.

```plaintext
val blocking = (topDown(tile(32,32)) ';;'
topDown(isReduce ';;' split(4)) ';;'
topDown(reorder(Seq(1,2,5,6,3,4))) )
)
(blocking ';;' lowerToC)(nm)
```

First we `tile`, then we `split` and then we `reorder`, just as specified in the TVM schedule. We describe locations using the `topDown` traversal and the `isReduce` strategy predicate that applies the following `split` only if the current expression is a reduction. We use the `;;` combination to normalize using `DFNF` between each step.

The `loop permutation` version incorporates the changes of the `vectorized` version by adding `vectorize` and a different reordering of dimensions. In contrast to TVM we identify dimensions by index rather than by name.

```plaintext
val loopPerm = (topDown(tile(32,32)) ';;'
topDown(isReduce ';;' split(4)) ';;'
topDown(reorder(Seq(1,2,5,6,3,4))) ';;'
topDown(vectorize(32)))
(loopPerm ';;' lowerToC)(nm)
```

For the `array packing` version, we are not required to change the Rist program manually, but can apply the array packing of matrix `B` as a rewrite step. Afterwards, we can reuse the `loopPerm` strategy before the packed representation of `B` is vectorized and then copied in parallel.

```plaintext
val arrayPacking = {packB ';;' loopPerm ';;'
topDown(vectorize(32)) ';;'
parallelizeCopy}
(arrayPacking ';;' lowerToC)(nm)
```

For the `parallel` version, we reuse the prior `array packing` strategy only changing the way we lower the high-level code. We parallelize the outermost loop with `topDown(parallel)` and unroll the innermost reduction using `bottomUp(isReduce ';;' unroll)` before lowering the remaining high-level patterns to sequential code as before.

```plaintext
1 (arrayPacking ';;' topDown(parallel) ';;'
2 bottomUp(isReduce ';;' unroll) ';;' lowerToC)(mm)
```

We have demonstrated that it is feasible to implement a TVM-like scheduling language in ELEVATE by expressing schedules as compositions of reusable strategies.

### 6.6 Experimental Evaluation

In order to evaluate the practicability and the scalability of ELEVATE, we performed two experiments.

#### Number of Rewrite Steps

In the first experiment, we are interested in the scalability of our approach by counting the number of successfully applied rewrite steps performed when applying a strategy to the Rist matrix multiplication expression. Figure 7 shows this number for every strategy shown in the previous subsection. Since no major optimization strategies are applied to the `baseline` version, only 211 rewrite steps are performed. However, as soon as interesting optimizations are applied, we easily reach about 100,000 steps for the next three versions and about 150,000 for the most complicated optimizations. The loop-permutation case slightly drops in numbers of applied rewrite rules because the specific nesting prescribed in TVM’s schedule required fewer loop-interchanges.

These high numbers clearly show that abstractions are required to control this many rewrite steps. It also shows the scalability of our compositional approach in which complex optimizations are composed out of a small set of fundamental building blocks. The high-level strategies encode practical optimizations and hide massive numbers of individual rewrite steps that are actually performed. Overall, applying the strategies to the Rist expression took less than 50 seconds per version on a commodity notebook.

![Figure 7. Total number of successful rewrite steps when applying different optimization strategies.](image-url)
Performance Comparison. In the second experiment, we are interested in the performance achieved when optimizing Rise programs with ELEVATE compared to TVM. Ideally, the code optimized with ELEVATE should be similar to the TVM-generated code and achieve competitive performance. We generated LLVM code with TVM (version 0.6.dev) and C code for Rise annotated with OpenMP pragmas for the version which include parallelization or vectorization. The Rise generated C code was compiled with clang (v:9.0.0) using -Ofast -ffast-math -fopenmp which echoes the settings used by TVM and Halide. The measurements were performed on an Intel core i5-4670K CPU (frequency locked to 3.4GHz) running Arch Linux (kernel 5.3.11-arch1-1). We measured wall-time for Rise-generated code and used TVM’s built-in measurement API. We measured 100 iterations per version reporting the median runtimes in milliseconds.

Figure 8 shows the performance of Rise and TVM generated code using a logarithmic scale. The code generated by Rise controlled by the ELEVATE optimization strategies performs competitive to TVM. Similar to the results of the Halide case study, our experiment shows a matching trend when comparing to TVM’s versions with equivalent optimizations. The most optimized parallel Rise generated version improves the performance over the baseline by a factor of about 110x. This means that the strategies we developed using ELEVATE, that are defined in an extensible style by composing individual rewrite steps, scale to a level where they actually encode practically useable and relevant optimizations.

7 Related Work

Term Rewriting and Strategy Languages. ELEVATE is inspired by existing strategy languages, especially ELAN [7, 8] and Stratego [39, 41], which introduce combinators to support user-defined strategies in the context of term rewriting systems. Similar rewriting systems include [2, 9, 14, 17, 20, 29, 36]. Program transformations using rewrite rules and strategy languages have since been used in many different domains including reverse engineering [13], refactoring [18], and obfuscation [15]. Visser [38, 40] and Kirchner [24] provide surveys covering term rewriting, strategy languages and their application domains.

To the best of our knowledge, ELEVATE is the first strategy language that has been used to specify state-of-the-art compiler optimizations such as tiling focusing on high performance code generation.

Rewriting in Compilers. Rewrite rules and rewriting strategies have also been used when building compilers. The Glasgow Haskell Compiler [28] uses rewrite rules as a practical way to optimize Haskell programs and Visser et. al. [41] describe how to build program optimizers using rewriting strategies. Other areas include building interpreters [16], instruction selection [10] or constant propagation [26]. More recently, Lift [21, 33, 35] showed how to use rewrite rules to generate high-performance code targeting accelerators.

Controlling the application of rewrite rules in compilers still largely is built-in in a fixed way based on heuristics. In this work, we showed how to use ELEVATE instead, allowing a more flexible and practical approach towards using rewrite rules for describing optimizations in compilers.

Schedule-based Compilers. Halide [30] introduced the concept of schedules describing program optimizations separate from the algorithm describing the computation. This concept has been adopted by many other frameworks in domains including machine learning (TVM [12]), graph applications (GraphIt [43]) or polyhedral compilation (Tiramisu [4], CHiLL [11, 22], AlphaZ [42], URUK [19]).

These existing scheduling APIs are not designed as principled programming languages. Instead, a fixed set of ad-hoc built-in primitives is exposed allowing users to specify which optimizations to apply. In this work, we showed how to use ELEVATE to implement scheduling languages from first principles as composition of rewrite rules.

8 Conclusion

In this paper, we presented ELEVATE: a language for describing optimization strategies. ELEVATE follows a tradition of prior systems used in different contexts that express optimization strategies as composition of rewrites. We showed that, in contrast to existing systems with scheduling APIs such as Halide and TVM, programmers are not restricted to a set of built-in optimizations but define their own optimization strategies. Using three case studies, we demonstrated ELEVATE’s flexibility to rewrite different languages (F and R5s), its extensibility to add custom abstractions including domain-specific optimizations, and its practicality to scale to complex optimization strategies requiring 150k rewrite steps used in deep learning. We showed that ELEVATE successfully optimizes programs in the R5s language achieving competitive performance compared to Halide and TVM.
Acknowledgments

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