A realistic non-local heat engine based on Coulomb coupled systems

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Optimal non-local heat-engines, based on Coulomb-coupled systems, demand a sharp step-like change in energy resolved system-to-reservoir coupling around the ground state of quantum-dots [1–5]. Such sharp step-like transition in the system-to-reservoir coupling cannot be achieved in a realistic scenario. Here, I propose realistic design for non-local heat engine based on Coulomb-coupled systems. The performance of the proposed heat engine is then theoretically investigated using quantum-master-equation (QME) approach. It is demonstrated that the theoretical maximum power output for the proposed set-up is limited to about 50% of the optimal design. Despite a lower performance compared to the optimal set-up, the novelty of the proposed design is the conjunction of fabrication simplicity along with reasonable power output. At the end, the sequential transport processes leading to a performance deterioration of the proposed design strategy are analyzed and a method to alleviate such transport processes is proposed. The design proposed in this paper can be used to fabricate high-performance non-local cryogenic heat engines.

FIG. 1. Schematic diagram of the proposed non-local heat-engine based on Coulomb-coupled quantum dots. The entire system consists of three dots $S_1$, $S_2$ and $G_1$ which are electrically coupled to the reservoirs $L$, $R$ and $G$ respectively. The dots $S_1$ and $S_2$ are tunnel coupled, while $S_1$ and $G_1$ are electrostatically coupled. The ground states of $S_1$ and $S_2$ form a staircase configuration with $\varepsilon_{2s} \approx \varepsilon_{1s} + \Delta \varepsilon$. In the proposed arrangement, current can be driven between the cold reservoirs $L$ and $R$ by absorbing thermal energy from the hot reservoir $G$.

I. INTRODUCTION

With the progress in fabrication and scaling technology, efficient heat harvesting in lower dimensional systems has gained a lot of attention [6–16]. One of the major issues affecting heat harvesting performance in nano-systems is the drastic lattice heat flux resulting from a small spatial separation between the heat reservoir and the heat sink. The large lattice heat flux severely limits the overall efficiency of the heat engine and poses a major performance issue in cases where supply of heat energy is limited. Tailoring the lattice thermal conductance, in an attempt to gain enhanced harvesting efficiency generally affects the current path, thereby deteriorating the peak harvested power. As such, one of the crucial focus of the modern thermoelectric community is to facilitate an independent optimization of the electron transport path and lattice heat conduction path, by introducing a spatial separation between the current path and the heat reservoir [17–19]. This phenomenon of harvesting heat from a reservoir, which is spatially separated from the current conduction path, is known as non-local heat harvesting [1–5, 17–20]. In this case, tailoring the lattice heat transport path, in an attempt to gain enhanced efficiency, can be accomplished without altering the current conduction path. Recently designs and concepts of non-local heat engines and refrigerators using Coulomb coupled quantum dots have been proposed and explored in literature [1–5, 20]. However, the operation of such non-local heat engines demand a sharp step-like change in the system-to-reservoir coupling around the ground state energy [1–5, 20], which is impossible to achieve in a practical scenario. In this paper, I propose a realistic design strategy to accomplish non-local heat harvesting using capacitively coupled quantum dots. Unlike the optimal non-local heat-engine based on Coulomb coupled systems [1–5, 20], the proposed design doesn’t demand an change in the system-to-reservoir coupling near the ground state. The performance proposed heat-engine is then evaluated and compared with the optimal set-up. It is demonstrated that the performance of the proposed heat engine hovers around 50% of the optimal set-up. However, the novelty of the proposed set-up is the conjunction of fabrication simplicity along with a reasonable power output. At the end, the
processes leading to a performance deterioration of the proposed set-up is discussed and analyzed. This paper is organized as follows. In Sec. II, I illustrate the proposed design strategy and elaborate the transport formulation employed to analyze the thermoelectric performance of the same. Next, Sec III elaborates a detailed analysis on the heat harvesting performance and regime of operation of the proposed heat-engine. A performance comparison between the proposed heat engine and the optimal set-up is also conducted along with a brief discussion on the transport processes leading to a performance deterioration of the proposed heat engine. I conclude this paper briefly in Sec. IV

II. PROPOSED DESIGN AND TRANSPORT FORMULATION

The proposed heat engine, schematically demonstrated in Fig. 1, consists of three dots $S_1$, $S_2$ and $G_1$ which are electrically coupled to the reservoirs $L$, $R$ and $G$ respectively. $S_1$ and $S_2$ are tunnel coupled to each other, while $G_1$ is capacitively coupled to $S_1$. The ground states of $S_1$ and $S_2$ form a stair-case configuration with $\varepsilon_1^2 \approx \varepsilon_1^1 + \Delta \varepsilon$. Any electronic tunneling between the dots $S_1$ and $G_1$ is suppressed via suitable fabrication techniques. Energy exchange between the two dots is, however, possible via capacitive coupling [21–23]. Quantum dots that are far from each other in space, may be bridged to obtain strong capacitive coupling , in addition to excellent thermal isolation between the hot and cold reservoirs [21, 22]. In addition, the bridge may be fabricated between two specific quantum dots to drastically enhance their mutual capacitive coupling, without affecting the electrostatic energy of the other quantum dots [21–23]. Thus, the change in electron number $n_{S_1} (n_{G_1})$ of the dot $S_1 (G_1)$ influences the electrostatic energy of the dot $G_1 (S_1)$. In general, the total electrostatic energy $U$ of the system, demonstrated in Fig. 1 (a), consisting of three dots can be given by:

$$U(n_{S_1}, n_{G_1}, n_{S_2}) = \sum_{x} U_{x}^{self} \left( n_x - \frac{V_x C_x^{self}}{q} \right)^2 + \sum_{x \neq x_2} U_{x_1, x_2}^{m} \left( n_{x_1} - \frac{V_{x_1} C_{x_1}^{self}}{q} \right) \left( n_{x_2} - \frac{V_{x_2} C_{x_2}^{self}}{q} \right),$$

where $n_x$ is the electron number, $C_x^{self}$ is the self-capacitance and $U_{x}^{self} = \frac{q^2}{2 C_x^{self}}$ is the electrostatic energy due to self-capacitance of quantum dot ‘$x’$. $U_{x_1, x_2}^{m}$ is the electrostatic energy arising out of Coulomb coupling between two different quantum dots that are separated in space. Here, a minimal physics based model is used to investigate the heat engine performance under the assumption that the change in potential due self-capacitance is much greater than than the average thermal voltage $kT/q$ or the applied bias voltage $V$, that is $U_{x}^{self} = \frac{q^2}{C_x^{self}} \gg (kT, qV)$. Hence, electron occupation probability or transfer rate via the Coulomb blocked energy level, due to self-capacitance, is negligibly small. The analysis of the entire system of dots may hence be approximated by limiting the maximum number of electrons in each dot to one. Thus the analysis of the entire system may be limited to eight multi-electron levels, which I denote by the electron occupation number in the ground state of each quantum dot. Hence, a possible state of interest in the system may be denoted as $|n_{S_1}, n_{G_1}, n_{S_2}\rangle = |n_{S_1}\rangle \otimes |n_{G_1}\rangle \otimes |n_{S_2}\rangle$, where $n_{S_1}, n_{G_1}, n_{S_2} \in (0,1)$. I also assume that the electrostatic coupling between $S_1$, $S_2$ and between $S_2$, $G_1$ is negligible, such that, for all practical purposes under consideration, $U_{S_1 S_2} \approx 0$ and $U_{G_1 S_2} \approx 0$. Since, the electronic transport and ground states in $S_1$ and $G_1$ are mutually coupled, I treat the pair of dots $S_1$ and $G_1$ as a sub-system ($S_1 G_1$), $S_2$ being the complementary sub-system ($S_2$) of the entire system consisting of three dots [24]. The state probability of $S_1$ is denoted by $P_{i, S_1}^{S_1}$, $i$ and $j$ being the number of electrons in the dot $S_1$ and $G_1$ respectively. $P_{k, S_2}^{S_2}$, on the other hand, denotes the probability of occupancy of the dot $S_2$ in the sub-system $S_2$. It can be shown that if $\Delta \varepsilon$ is much greater than the ground state broadening due to system-to-reservoir coupling, then the interdot tunneling rate between $S_1$ and $S_2$ is optimized when $\varepsilon_1^1 + U_{S_1 G_1} = \varepsilon_2^2$, that is when $\Delta \varepsilon = U_{S_1 G_1}^{m}$ [24]. To evaluate the optimal performance of the proposed heat-engine, I hence assume $\Delta \varepsilon = U_{S_1 G_1}^{m}$ [24]. Henceforth, I would simply represent $U_{S_1 G_1}^{m}$ as $U_m$. Under the assumption stated above, the equations governing sub-system state probabilities in steady state can be derived as [24]:

$$P_{0, 0}^{S_1} \left( f_L(\varepsilon_1^1) + f_C(\varepsilon_1^1) \right) + P_{0, 1}^{S_1} \left( 1 - f_C(\varepsilon_1^1) \right) + P_{1, 0}^{S_1} \left( 1 - f_L(\varepsilon_1^1) \right) = 0$$

$$P_{0, 1}^{S_1} \left( 1 - f_L(\varepsilon_1^1) + f_C(\varepsilon_1^1 + U_m) \right) + P_{1, 1}^{S_1} \left( 1 - f_C(\varepsilon_1^1 + U_m) \right) + P_{0, 0}^{S_1 f_C(\varepsilon_1^1)} = 0$$

$$P_{0, 1}^{S_1} \left( 1 - f_R(\varepsilon_1^2) + f_C(\varepsilon_2^2 + U_m) + \frac{\gamma}{\gamma_c} P_{0, 1}^{S_1} \right) + P_{1, 1}^{S_1 \gamma_c f_C(\varepsilon_1^1 + U_m)} + P_{0, 1}^{S_1} \left( f_C(\varepsilon_1^2 + U_m) + \frac{\gamma}{\gamma_c} P_{1, 1}^{S_1} \right) = 0 \tag{1}$$

$$P_{0, 2}^{S_1} \left( f_R(\varepsilon_1^2) + \frac{\gamma}{\gamma_c} P_{1, 1}^{S_1} \right) + P_{1, 2}^{S_1} \left( 1 - f_R(\varepsilon_1^2) + \frac{\gamma}{\gamma_c} P_{1, 1}^{S_1} \right) = 0$$

$$P_{1, 2}^{S_1} \left( 1 - f_R(\varepsilon_1^2) + \frac{\gamma}{\gamma_c} P_{1, 1}^{S_1} \right) + P_{2, 2}^{S_1} \left( f_R(\varepsilon_1^2) + \frac{\gamma}{\gamma_c} P_{1, 1}^{S_1} \right) = 0, \tag{2}$$

where $\gamma_c$ and $\gamma$ are related to the reservoir-to-system
tunnel coupling and the inter-dot tunnel coupling respectively [24, 25]. In the above set of equations, \( f_\lambda(\varepsilon) \) denote the probability of occupancy of the reservoir \( \lambda \) at energy \( \varepsilon \). For the purpose of calculations in this paper, I assume an equilibrium Fermi-Dirac statistics at the reservoirs. From the set of Eqns. (1) and (2), it is clear that an electron in \( S_1 \) can tunnel into \( S_2 \) only when the ground state in the dot \( G_1 \) is occupied with an electron. The set of Eqns. (1) and (2) are coupled to each other and may be solved using any iterative method. Here, I use Newton-Raphson iterative method to solve the steady-state values of sub-system probabilities. On calculation of the sub-system state probabilities \( P_{\varepsilon_1}^{\lambda_3} \) and \( P_{\varepsilon_2}^{\lambda_3} \), the electron current flow into (out of) the system from the reservoirs \( L(R) \) can be given as:

Next, I use the set of Eqns. (1), (2) and (3), to evaluate the thermoelectric generation performance of the the set-up demonstrated in Fig. 1. To analyze the performance of the heat engine, I use a voltage-controlled set-up demonstrated in literature [26–28], where an applied bias voltage \( V \) is used to emulate the voltage drop across an external load. Assuming the equilibrium electrochemical potential across the entire set-up is \( \mu_0 \), and a voltage drop \( V \) across the external load, the quasi-Fermi levels at the reservoirs \( L \) and \( R \) may be written as \( \mu_{L(R)} = \mu_0 \pm \frac{V}{2} \). The generated power \( (P) \) and efficiency \( (\eta) \) can be defined as:

\[
P = I_{L(R)} \times V,
\]

\[
\eta = \frac{P}{I_Q},
\]

where \( V \) is the applied potential bias in the voltage controlled model and \( I_Q \) is the sum of electronic and lattice heat flux from the heat source \( (G) \). In the non-local heat engine the lattice heat flux can be favourably engineered [29–33] without affecting the current conduction path. In addition the lattice heat flux is generally independent of the system configuration [34–41]. Hence, to simplify our calculation, we assume ideal condition by neglecting the lattice heat flux as done in recent literature [6, 7, 42, 43]. The generation efficiency for our case, can hence be defined as:

\[
\eta = \frac{P}{I_{Qe}},
\]

where the electronic heat flux at the heat source \( I_{Qe} \) can be calculated as [24]:

\[
I_{Qe} = U_m \gamma_c \{ P_{L(1)}^{\mu_0} f_G(\varepsilon_g + U_m) - P_{L(1)}^{\mu_0} \{ 1 - f_G(\varepsilon_g + U_m) \} \}
\]

Interestingly Eqn. (6) is not directly dependent on \( \varepsilon_g \). This is due to the fact that the net electronic current into or out of the reservoir \( G \) is zero. To better understand this, let us consider the situation where an electron tunnels into \( G_1 \) from \( G \) with an energy \( \varepsilon_g + U_m \), when the ground state of \( S_1 \) is occupied. Next, the electron tunnels out of \( S_1 \) with an energy \( \varepsilon_1^* + U_m \), followed by the electron tunneling out of \( G_1 \) with an energy \( \varepsilon_g \). In this process a heat packet \( U_m \) is lost from \( G \), independent of \( \mu_0 \). Without loss of generality, I assume that \( \gamma_c = 10^{-6} \frac{q}{R} \) and \( \gamma = 10^{-5} \frac{q}{R} \). The temperature of the reservoirs \( L(R) \) and \( G \) are assumed to be \( T_{L(R)} = 5K \) and \( T_G = 10K \). The average temperature between the hot and the cold reservoirs is, hence, given by \( T = \frac{T_{L(R)} + T_G}{2} = 7.5K \).

III. RESULTS

In this section, I discuss the optimal operation regimes of the proposed heat engine. In addition, I conduct a performance comparison of the proposed heat engine

FIG. 2. Variation of the proposed heat engine performance with variation in the ground states \( \varepsilon_g \) and \( \varepsilon_1^* \) for \( U_m = 3.9meV \approx 6 \frac{q_T}{h} \) and \( V = 1.3meV \approx 2 \frac{q_T}{h} \). Colour plot demonstrating the variation in (a) generated power \( (P) \) and (b) efficiency \( (\eta) \). \( T_0 = \frac{T_{L(R)} + T_G}{2} = 7.5K \) is the average temperature between the heat source and the heat sink. The efficiency of generation is measured with respect to the Carnot efficiency \( \eta_c = 1 - T_{L(R)}/T_G \):

\[
I_L = q_c \times \{ P_{L(0)}^{\mu_0} f_L(\varepsilon_1^*) + P_{L(1)}^{\mu_0} f_L(\varepsilon_1^* + U_m) \} - q_c P_{L(1)}^{\mu_0} \{ 1 - f_L(\varepsilon_1^*) \} - q_c P_{L(1)}^{\mu_0} \{ 1 - f_L(\varepsilon_1^* + U_m) \}
\]

\[
I_R = - q_c \times \{ P_{R(0)}^{\mu_0} f_R(\varepsilon_1^*) - P_{R(1)}^{\mu_0} \{ 1 - f_R(\varepsilon_1^*) \} \}.
\]
with the optimal set-up discussed in literature and investigate the transport processes leading to a performance deterioration of the proposed set-up. Fig. 2 demonstrates the performance of the heat engine, in particular the generated power $P$ and efficiency of generated power ($\eta$) over a range of values of $V$ and $U_m$. To find out the maximum power $P_m$ for a given value of $V$ and $U_m$, the ground states of the dots are tuned to optimal position. $T = \frac{T_{L(R)} + T_G}{2} = 7.5K$ is the average temperature between the heat source and the heat sink. The efficiency of maximum power ($P_m$) generation is normalized with respect to the Carnot efficiency $\eta_c = 1 - \frac{T_{L(R)}}{T_G}$.

![Graph](image-url)  
**FIG. 3.** Variation in the peak performance of the heat engine with variation in Coulomb coupling energy $U_m$ and applied bias $V$. Colour plot depicting the (a) peak generated power $P_m$ and (b) efficiency at the peak generated power for a range of values of $V$ and $U_m$. To find out the maximum power $P_m$ for a given value of $V$ and $U_m$, the ground states of the dots are tuned to optimal position. $T = \frac{T_{L(R)} + T_G}{2} = 7.5K$ is the average temperature between the heat source and the heat sink. The efficiency of maximum power ($P_m$) generation is normalized with respect to the Carnot efficiency $\eta_c = 1 - \frac{T_{L(R)}}{T_G}$.

State $\varepsilon_g$ is always occupied with an electron and so the asymmetry of the system with respect to the reservoir $L$ and $R$ disappears. Hence a directional thermoelectric current flow is not possible [5]. On the other hand, when $\varepsilon_g - \mu_0 > \text{few } kT_G$, the probability of an electron tunneling into the reservoir $G_1$ with an energy $\varepsilon_g + U_m$ (provided that the ground state of $S_1$ is occupied) is negligibly small, resulting in the deterioration of the unidirectional current flow and hence, power generation. Fig. 2(b) demonstrates the heat engine efficiency as a function of the ground state energy levels. The efficiency of heat harvesting increases monotonically with increase in $\varepsilon_g - \mu_0$. An equivalent behaviour can be noted in bulk and lower dimensional thermoelectric engines as the equilibrium Fermi-energy moves outside the band-edge [6, 7, 12, 42, 43]. The variation in generation efficiency with $\varepsilon_g$ is non-monotonic. Initially as $\varepsilon_g$ increases and approaches the Fermi energy, the generated power increases leading to an increase in efficiency. As $\varepsilon_g$ gradually increases the probability of reverse electronic flow from the system to reservoir $L$ at energy $\varepsilon_g + U_m$ increases. Such processes (discussed later) lead to a deterioration in the generation efficiency. The variation of the optimal performance of the heat engine with variation in the Coulomb coupling energy $U_m$ and applied bias $V$ is demonstrated in Fig. 3. In particular, Fig. 3 (a) demonstrates the maximum generated power ($P_m$), while Fig. 3 (b) demonstrates the efficiency at the maximum generated power for a range of values of the applied bias $V$ and the Coulomb coupling energy $U_m$. To calculate the the maximum generated power $P_m$ for a given value of $V$ and $U_m$, the ground states of the dots are tuned to the optimal energy position. It should be noted that the maximum generated power is low for low values of $U_m$. This is due to the fact that the ground state of the dots approach symmetrical arrangement with respect to the reservoir $L$ and $R$ as $U_m$ approaches towards zero. Hence, the directional flow of electrons decreases. As $U_m$ increases, the asymmetry of the system increases resulting in an increase in directional electron flow, and hence, the maximum generated power [5]. With further increase in $U_m$, the maximum generated power reaches its peak and then decreases due to lower probability of an electron tunneling into $G_1$ with an energy $\varepsilon_g + U_m$, when the ground state of $S_1$ is already occupied. For a fixed value of $U_m$, the maximum generated power first increases and then decreases with an increase in the bias voltage $V$. Such a behaviour is indeed expected from heat engines as the regime of operation approaches from short-circuited condition to the open-circuited condition [28]. Fig. 3(b) demonstrates the efficiency at the maximum generated power with variation in applied bias $V$ and Coulomb coupling energy $U_m$. The efficiency varies non-monotonically with the applied bias $V$, that is, the efficiency increases with an increase in $V$ as the regime of operation approaches the point of maximum power and then gradually decreases as the generated power.
FIG. 4. Performance comparison of the proposed non-local heat engine (solid lines) with the optimal set-up [1–4] (dashed lines) for different values of the Coulomb coupling energy. Plot of (a) maximum generated power $P_M$ vs bias voltage $V$, (b) efficiency (with respect to Carnot efficiency) at the maximum generated power vs bias voltage, (c) maximum power $P_M$ vs efficiency at the maximum power for the optimal set-up (d) maximum power $P_M$ vs efficiency at the maximum power for the proposed design.

decrees with the regime of operation approaching the open circuited condition. Such a trend can also be noted in bulk and lower dimensional heat engines [6, 7]. On the other hand with an increase in $U_m$, the efficiency at the maximum power increases monotonically as $\varepsilon_g + U_m$ gradually surpasses the Fermi energy. An equivalent trend, again, can be noted in bulk and lower dimensional thermoelectric engines as the band-edge gradually surpasses the Fermi-energy [6, 26–28, 44]. Fig. 4 demonstrates a performance comparison of the proposed heat engine with the optimal non-local heat engine put forward in literature [1–4]. In particular, Fig. 4(a) and (b) demonstrates the variation in the maximum power $P_M$ and efficiency at the maximum power respectively with applied load bias $V$ for different values of $U_m$. We note that the overall maximum generated power for the proposed design $P^{\text{prop}}_{\text{MAX}}$ is approximately 2.03fW, which is about 50% of the overall maximum power output of 3.8fW for the optimal design $P^{\text{opt}}_{\text{MAX}}$. In both these cases the maximum power is generated around $U_m \approx 4\text{meV}$. As already discussed in literature [5], the efficiency at the overall maximum generated power for the optimal set-up increases linearly with the applied bias for a given value of $U_m$. The efficiency at the overall maximum power for our proposed design and the optimal set-up are 24.5% and 60% of the Carnot efficiency respectively. In addition, it can also be noted that the open circuit voltage for the optimal set-up is slightly higher compared to the proposed design. Fig. 4 (c) and (d) demonstrates the the maximum power vs efficiency loops [6] for the optimal design and the proposed design respectively for various values of $U_m$. We note that the power-efficiency trade-off for the optimal set-up is somewhat mild compared to the proposed design.

I end the discussion with a brief description of the processes leading to a deterioration in generated power and efficiency for the proposed set-up. First, let us consider the cycles leading to electron transport from the reservoir $L$ to the reservoir $R$ against the applied bias while absorbing a heat packet $U_m$ from reservoir $G$. Let us consider the cycle $(n_{S_1}, n_{G_1}, n_{S_2}) \rightarrow (n_{S_1} + 1, n_{G_1}, n_{S_2}) \rightarrow (n_{S_1} + 1, n_{G_1} + 1, n_{S_2}) \rightarrow (n_{S_1}, n_{G_1} + 1, n_{S_2} + 1) \rightarrow (n_{S_1}, n_{G_1}, n_{S_2} + 1) \rightarrow (n_{S_1}, n_{G_1}, n_{S_2})$. In this cycle, the system starts with an initial state with the unoccupied ground state in all the three quantum dots. An electron tunnels from $L$ into $S_1$ at energy $\varepsilon_1$, followed by an electron tunneling into $G_1$ from $G$ at energy $\varepsilon_g + U_m$. At the next instant, the electron in $S_1$ tunnels into $S_2$, after which the electron in $G_1$ tunnels out into $G$ with energy $\varepsilon_g$. The cycle is completed and the system returns to the initial state when the electron in $S_2$ tunnels out into the reservoir $R$ with an energy $\varepsilon_2 = \varepsilon_1 + U_m$. It is clear that in this process an electron is transferred from $L$ to $R$ while absorbing a heat packet $U_m$ from $G$. Another cycle that again transfers electrons from $L$ to $R$, while absorbing heat packet from $G$ can be given by $(n_{S_1}, n_{G_1}, n_{S_2}) \rightarrow (n_{S_1} + 1, n_{G_1}, n_{S_2}) \rightarrow (n_{S_1} + 1, n_{G_1} + 1, n_{S_2}) \rightarrow (n_{S_1}, n_{G_1} + 1, n_{S_2} + 1) \rightarrow (n_{S_1}, n_{G_1}, n_{S_2} + 1) \rightarrow (n_{S_1}, n_{G_1}, n_{S_2})$. These transport processes contribute to thermoelectric power generation while absorbing heat energy from $G$. Next, consider the cycle $(n_{S_1}, n_{G_1}, n_{S_2}) \rightarrow (n_{S_1} + 1, n_{G_1}, n_{S_2}) \rightarrow (n_{S_1} + 1, n_{G_1} + 1, n_{S_2}) \rightarrow (n_{S_1}, n_{G_1} + 1, n_{S_2} + 1) \rightarrow (n_{S_1}, n_{G_1}, n_{S_2} + 1) \rightarrow (n_{S_1}, n_{G_1}, n_{S_2})$. This cycle consists of an electron tunneling into $S_1$ from $L$, with an energy $\varepsilon_3$, followed by an electron tunneling into $G_1$ with an energy $\varepsilon_g + U_m$. At the next step, the electron in $S_1$ exits into reservoir $L$ with an energy $\varepsilon_1 + U_m$. The cycle is completed with the electron in $G_1$ tunnels out into $G$ with energy $\varepsilon_g$. It is evident that in this process, a packet of heat energy $U_m$ is transmitted from reservoir $G$ to $L$ without any net flow of electrons between $L$ and $R$. So, effectively the heat packet $U_m$ is wasted without any power conversion.
FIG. 5. Colour plots demonstrating the electron flow into the system from the reservoir \( L \) with variation in the ground states \( \varepsilon_g \) and \( \varepsilon_1 \), for \( U_m = 3.9\text{meV} \approx 6\frac{kT}{q} \) and \( V = 1.3\text{meV} \approx 2\frac{kT}{q} \), when the (a) ground state of the dot \( G_1 \) is unoccupied (b) ground state of the dot is occupied (c) total average current between the system and the reservoir \( L \). Interestingly, when the ground state of the dot \( G_1 \) is occupied, the electron current flow into \( L \) is positive or against the voltage bias, generating a net value of thermoelectric power. We find that when \( \varepsilon_g \) is unoccupied, the electron current flow into \( L \) is to reduce the generated power and efficiency. Once the ground state of \( G_1 \) is occupied, an electron existing in \( S_1 \) can either tunnel into \( S_2 \) giving rise to directional electronic flow or tunnel out to \( L \) without any net electron flow. Hence, the power output as well as efficiency of the proposed set-up hovers around 50% of the optimal design. To further understand the situation, in Fig. 5, I separate out the current flow into the system from the reservoir \( L \) depending on the occupation of ground state of the dot \( G_1 \). In particular, Fig. 5(a) and (b) demonstrate the electron current flow into the system from reservoir \( L \) when the ground state of \( G_1 \) is unoccupied and occupied respectively. It should be noted that the electronic current demonstrated in Fig. 5, is opposite to the direction of conventional current flow. We find that when \( \varepsilon_g \) is unoccupied, the electron current flow into \( L \) is positive or against the voltage bias, generating a net value of thermoelectric power. Interestingly, we also find that when the ground state of \( G_1 \) is occupied, the electronic current from \( L \) to \( S_1 \) is negative, that is, the net electron current flows into the reservoir from the system in the direction of voltage bias. It is evident that this component of electron current flow from the system into the reservoir \( L \) transmits heat packets, but impacts negatively on the generated power. Thus, this component of electron current impacts both the generated power and efficiency. The deterioration in the heat engine performance, due to the current component discussed above, can be alleviated by adding an extra filter between \( L \) and \( S_1 \). However, doing so nullifies the novelty of the proposed set-up in terms of fabrication simplicity. In Fig. 5(c), I show the total electronic current flow from \( L \) to \( S_1 \). The negative values of total electronic current corresponds to current flow in the direction of the applied bias, resulting no net thermoelectric power generation.

IV. CONCLUSION

To conclude, in this paper I have proposed a realistic design strategy for non-local heat engine based on Coulomb coupled systems. The performance of the proposed design was then theoretically analyzed and compared with the optimal set-up [5] using the QME approach. It was demonstrated that the proposed set-up outputs a maximum power of around 50% of the optimal set-up. However, the crucial advantage of the proposed design strategy is that along with a reasonable output power, it also circumvents the demand for a sharp step-like change in reservoir-to-system coupling, which is required for proper operation of the optimal set-up proposed in literature[5]. Although not shown here, the proposed system can also work as an efficient non-local heat engine when the reservoir \( G \) acts as a heat sink (cold) with respect to the reservoirs \( L \) and \( R \) (hot). In such a case, the direction of thermoelectric current flow is reversed. The various different possible design strategies
for non-local heat engines and their performance is left for future exploration. In addition, an investigation of the impact of electron-phonon interaction on the proposed design also constitutes an interesting research direction. Nevertheless, the set-up proposed in this paper can be employed to fabricate high performance non-local heat engines using Coulomb coupled systems.

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[1] N. Walldorf, A.-P. Jauho, and K. Kaasbjerg, Phys. Rev. B 96, 115415 (2017).
[2] A.-M. Daré, Phys. Rev. B 100, 195427 (2019).
[3] Y. Zhang and J. Chen, Physica E: Low-dimensional Systems and Nanostructures 114, 113635 (2019).
[4] A.-M. Daré and P. Lombardo, Phys. Rev. B 96, 115414 (2017).
[5] R. Sánchez and M. Böttiker, Phys. Rev. B 83, 085428 (2011).
[6] A. Singha, S. D. Mahanti, and B. Muralidharan, AIP Advances 5, 107210 (2015).
[7] A. Singha and B. Muralidharan, Scientific Reports 7, 7870 (2017).
[8] J. L. Mi, X. B. Zhao, T. J. Zhu, and J. P. Tu, Journal of Physics D: Applied Physics 41, 205403 (2008).
[9] J.-F. Li, W.-S. Liu, L.-D. Zhao, and M. Zhou, NPG Asia Mater 2, 152 (2010).
[10] S. Sumithra, N. J. Takas, W. M. Nolting, S. Sapkota, P. F. Poudeu, and K. L. Stokes, Journal of Electronic Materials 41, 1401 (2012).
[11] J.-H. Bahk, Z. Bian, and A. Shakouri, Phys. Rev. B 87, 075204 (2013).
[12] A. Agarwal and B. Muralidharan, Applied Physics Letters 105, 013104 (2014).
[13] R. Kim and M. S. Lundstrom, Journal of Applied Physics 111, 024508 (2012).
[14] R. Kim and M. S. Lundstrom, Journal of Applied Physics 110, 034511 (2011).
[15] S. V. Faleev and F. m. c. Léonard, Phys. Rev. B 77, 214304 (2008).
[16] N. Neophytou and H. Kosina, Journal of Applied Physics 114, 044315 (2013).
[17] K. Uchida, H. Adachi, T. Kikkaawa, A. Kirihiara, M. Ishida, S. Yorozu, S. Maekawa, and E. Saitoh, Proceedings of the IEEE 104, 1946 (2016).
[18] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Rev. Mod. Phys. 87, 1213 (2015).
[19] E. Saitoh, M. Ueda, H. Miyajima, and G. Tatara, Applied Physics Letters 88, 182509 (2006).
[20] Y. Zhang, Y. Wang, C. Huang, G. Lin, and J. Chen, Energy 95, 599 (2016).
[21] A. Hbel, J. Weis, W. Dietsche, and K. v. Klitzing, Applied Physics Letters 91, 102101 (2007).
[22] I. H. Chan, R. M. Westervelt, K. D. Maranowski, and A. C. Gossard, Applied Physics Letters 80, 1818 (2002).
[23] L. W. Molenkamp, K. Flensberg, and M. Kemerink, Phys. Rev. Lett. 75, 4282 (1995).
[24] A. Singha, “Density matrix to quantum master equation (qme) model for arrays of coulomb coupled quantum dots in the sequential tunneling regime,” (2020), arXiv:2003.00522 [physics.app-ph].
[25] S. Datta, Quantum Transport:Atom to Transistor (Cambridge Press, 2005).
[26] M. Leijnse, M. R. Wegewijs, and K. Flensberg, Phys. Rev. B 82, 045412 (2010).
[27] B. Sothmann, R. Sánchez, and A. N. Jordan, Nanotechnology 26, 032001 (2014).
[28] N. Nakpathomkun, H. Q. Xu, and H. Linke, Phys. Rev. B 82, 235428 (2010).
[29] J. Androulakis, K. Hsu, R. Pcionek, H. Kong, C. Uher, J. D’Angelo, A. Downey, T. Hogan, and M. Kanatzidis, Advanced Materials 18, 1170 (2006).
[30] K. F. Hsu, S. Loo, F. Guo, W. Chen, J. S. Dyck, C. Uher, T. Hogan, E. K. Polychroniadis, and M. G. Kanatzidis, Science 303, 818 (2004).
[31] Y. Pan, G. Hong, S. N. Raja, S. Zimmermann, M. K. Tiwari, and D. Poulikakos, Applied Physics Letters 106, 093102 (2015).
[32] J. P. Feser, J. S. Sadhu, B. P. Azeredo, K. H. Hsu, J. Ma, J. Kim, M. Scong, N. X. Fang, X. Li, P. M. Ferreira, S. Sinha, and D. G. Cahill, Journal of Applied Physics 112, 114306 (2012).
[33] B. L. Davis and M. I. Hussein, Phys. Rev. Lett. 112, 055505 (2014).
[34] N. Mingo and D. A. Broido, Phys. Rev. Lett. 93, 246106 (2004).
[35] N. Mingo, Applied Physics Letters 84, 2652 (2004).
[36] F. Zhou, J. Szczzech, M. T. Pettes, A. L. Moore, S. Jin, and L. Shi, Nano Letters 7, 1649 (2007), pMID: 17508772.
[37] F. Zhou, A. L. Moore, M. T. Pettes, Y. Lee, J. H. Seol, Q. L. Ye, L. Rabenberg, and L. Shi, Journal of Physics D: Applied Physics 43, 025406 (2010).
[38] A. I. Boukai, Y. Bunimovich, J. Tahir-Kheli, J.-K. Yu, W. A. Goddard, and J. R. Heath, Nature 451, 168 (2008).
[39] A. I. Hochbaum, R. Chen, R. D. Delgado, W. Liang, E. C. Garnett, M. Najarian, A. Majumdar, and P. Yang, Nature Publishing Group 451, 163 (2008).
[40] A. Balandin, A. Khitun, J. Liu, C. Wang, T. Borca-Tasciuc, and G. Chen, in Eighteenth International Conference on Thermoelectrics (1999) pp. 189–192.
[41] G. Chen, Phys. Rev. B 57, 14958 (1998).
[42] R. S. Whitney, Physical Review Letters 112, 130601 (2014).
[43] R. S. Whitney, Phys. Rev. B 91, 115425 (2015).
[44] Y. Choi and A. N. Jordan, Physica E: Low-dimensional Systems and Nanostructures 74, 465 (2015).