Electron density measurement in a pulsed-power plasma by FIR laser beam deflection and/or interferometry

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Abstract. A pulsed-power experiment has been designed to produce arc-shaped magnetic flux tubes similar to ascending solar flares. The tubes are filled with hydrogen plasma (electron temperature \( \leq 10 \) eV, electron density \( 2 \times 10^{20} \) m\(^{-3} \)) and expand with a velocity of \( \sim 2.5 \) cm/\( \mu \)s, while keeping their cross section constant at a radius of about 1.5 cm. For measuring the spatial electron density distribution within the moving flux tube, a single cw laser beam can be used. The information taken from the laser beam, which traverses the vacuum vessel perpendicular to the plane of the plasma arch, can be either the phase shift or the beam deflection due to the density gradient. Assuming a parabolic distribution with a central electron density of \( 2 \times 10^{20} \) m\(^{-3} \), the maximum deflection angle occurring at an impact parameter of 0.7 \( \) amounts to \( \gamma_{\text{max}}/\text{deg} \equiv 10^{-5} \times (\lambda/\mu \text{m})^2 \). Hence, a FIR laser operating at \( \lambda = 433 \) \( \mu \)m would be deflected by \( \gamma_{\text{max}} = 1.9^\circ \) only. Alternatively, a beam passing through the plasma centre would experience a plasma-induced phase shift of \( \Delta \phi_{\text{max}}/\text{rad} \equiv 10^{-2} \times (\lambda/\mu \text{m}) \), yielding 4.3 rad for a FIR laser (\( \lambda = 433 \) \( \mu \)m) and 0.1 rad for a CO\(_2\) laser (\( \lambda = 10.6 \) \( \mu \)m). While the former is readily detectable in a standard interferometer, the latter requires a more advanced technique of measurement to achieve the necessary resolution. On the other hand, the short wavelength compared to FIR radiation allows for a very narrow beam and hence for a high spatial resolution. For these reasons a so-called coupled-cavity scheme for a CO\(_2\) laser interferometer is presently under development.

1. The FlareLab-Experiment

FlareLab is a laboratory experiment that has been built to perform detailed studies into the behavior of plasma configurations with initially loop-shaped magnetic field lines and to analyze the observed dynamics of the magnetic flux tubes by means of extensive 3-dimensional MHD simulations [1]. Well-known examples of such configurations are arched Solar prominences, which often result in spectacular eruptions like Coronal mass ejections (CMEs).

The lay-out of the experiment follows a scheme which was proposed and successfully operated by Bellan and Hansen [2]. A sketch of our present setup is shown in figure 1. In order to produce a plasma, hydrogen gas is puffed into the space above the electrodes, and a discharge is ignited. The current causes the plasma column to pinch along the guiding B-field and to form an expanding loop structure (see figure 1 for a snapshot of a discharge taken 5 \( \) after ignition with an exposure time of 50 ns by means of a fast ICCD camera).

At present our set of diagnostics is still limited to standard electro-technical devices for measuring currents and voltages, and to an electrostatic triple probe (supported by elementary spectroscopy) for
estimating electron densities and temperatures. In addition, a movable pick-up coil has been installed to obtain space- and time-resolved information about the magnetic field distribution near the flux tube. It turns out that the loop expands with a fairly constant speed of about 2.5 cm/µs while keeping its cross section at a diameter of ≤ 3 cm. Triple probe and spectroscopy provide electron temperatures and densities of the order of a few eV and a few times $10^{20}$ m$^{-3}$, respectively, with little variation throughout the loop expansion. On the other hand, MHD simulations of the dynamical behavior [1] have shown that the underlying model of the density evolution plays an important role for matching the experimental observations. Hence there is an urgent need to improve our diagnostic capabilities for a more precise determination of the electron density distribution.

2. Laser-aided electron density measurements

In our case, already a single cw laser beam (directed perpendicular to the plane of the flux loop) will be sufficient to resolve the spatial density profile. After ignition the plasma arch ascends from its foot points until it reaches the beam and travels across. As a result, the beam is scanning the whole plasma cross-section and provides all the information needed to apply standard Abel inversion techniques in order to convert the measured data into a spatial density distribution (assuming cylindrical symmetry and neglecting possible profile changes during the passage). The changes imposed on the beam are time-varying phase shifts and beam deflections due to the density gradient. While the former are routinely utilized in various interferometric schemes, the latter are less frequently exploited in special schlieren and shadowgraph techniques. We will briefly consider both options for our purpose.

2.1. Plasma-induced beam deflection

If the gradients of an axisymmetric refractive index distribution $\mu(r)$ are large enough to cause detectable deflections of a test beam, it is possible to calculate $\mu(r)$ from the knowledge of the deflection angle $\gamma$ as a function of $x$ (see figure 2). Using the approximate expression for the refractive index of a fully ionized plasma without magnetic field (i.e. $\mu = (1 - \omega_p^2/\omega^2)^{1/2} \approx 1 - \omega_p^2/(2\omega^2)$ where $\omega_p$ denotes the electron plasma frequency and $\omega = 2\pi c/\lambda$ is the probing wave frequency), it can be shown that the electron density distribution $n_e(r)$ derives from the deflection profile $\gamma(x)$ according to [3,4]:

$$n_e(r) \approx \frac{2e_n m_e}{\pi e^2} \frac{\omega^2}{\omega_p^2} \int \gamma(x) \frac{dx}{(x^2 - r^2)^{1/2}}.$$  \hspace{1cm} (1)

Conversely, for a parabolic density distribution $n_e(r) = n_e0 (1 - r^2/a^2)$ the deflection profile is given by
\[ \gamma(\xi) \approx 2 \kappa \xi (1 - \xi^2) \quad \text{where} \quad \xi = \frac{x}{a} \quad \text{and} \quad \kappa = \frac{\omega_p^2}{\omega^2} = \frac{e^2}{4 \pi^2 \varepsilon_0 m_e} n_{e0} \lambda^2. \] (2)

It has a maximum value of \( \gamma_{\text{max}} = \gamma(\xi_1) = \kappa \) at a beam position of \( \xi_1 = 2^{-1/2} = 0.707 \) (cf. figure 2).

Taking a characteristic central density of \( n_{e0} = 2 \times 10^{20} \text{ m}^{-3} \) and asking for a maximum deflection angle of \( \gamma_{\text{max}} \geq 3^\circ \) to achieve reasonable detectability, the probing wavelength must be \( \lambda \geq 540 \mu \text{m} \). At such long wavelengths it would be difficult to restrict the laser beam to a ray which is much narrower than the plasma radius of about 1.5 cm. Moreover, suitable detector arrays providing high enough spatial and temporal resolution in the FIR regime would hardly be available. Therefore we have to conclude that beam deflection measurements are not a viable option for our experiment.

### 2.2. Plasma-induced phase shift

Interferometric phase-shift measurements are well established for determining line-averaged values of the electron density in a plasma. The relation between phase shift \( \Delta \phi \) and density \( n_e \) is given by [5]:

\[ \Delta \phi = r_e \lambda \int n_e \, dl \quad \text{where} \quad r_e = e^2 / (4 \pi^2 \varepsilon_0 m_e) = 2.82 \times 10^{-15} \text{ m} \] (3)

(the integration extends over the path length \( L \)). For an axisymmetric distribution \( n_e(r) \) as shown in figure 2 this expression can be transformed into an Abel integral equation according to

\[ \Delta \phi(x) = 2r_e \lambda \int_1^{a} n_e(r) \frac{r \, dr}{(r^2 - x^2)^{1/2}} \quad \Leftrightarrow \quad n_e(r) = \frac{-1}{\pi r_e \lambda} \int dx \frac{d \Delta \phi}{dx} \left( \frac{x^2}{(x^2 - r^2)^{1/2}} \right). \] (4)

Taking again a parabolic profile with a central density of \( n_{e0} = 2 \times 10^{20} \text{ m}^{-3} \) and a radius of \( a = 1.5 \text{ cm} \), a beam passing through the plasma centre at \( x = 0 \) would experience a wavelength-dependent phase shift of \( (\Delta \phi_{\text{max}} / \text{rad}) = 1.13 \times 10^2 \times (\lambda / \mu \text{m}) \), resulting in 0.12 rad for a CO\(_2\) laser (\( \lambda = 10.6 \mu \text{m} \)). In terms of interference fringes this number corresponds to 1/50 of a fringe, which is close to the resolution limit of common interferometers (as, e.g. Mach-Zehnder or Michelson configurations). On the other hand, the comparatively short CO\(_2\) wavelength allows for a fairly narrow probing beam, and the technical equipment (including detectors) is well developed and easy to operate.

### 3. Interferometer setup and first results

A technique, which is well suited for measuring small phase shifts, makes use of a coupled-cavity laser interferometer [6,7,8] as shown in figure 3. Here, the radiation of a grating-tuned CO\(_2\) laser is reflected back into the cavity by means of an external mirror on the opposite side of the discharge chamber. The mirror is placed at a distance of two times its focal length and mounted on a 3-axes micrometer table. The mounting contains a piezo actuator for shifting the mirror within a range of...
Figure 3. Coupled-cavity CO\textsubscript{2} laser interferometer.

appr. 20 \mu m. Depending on the position of the external mirror, the radiation reflected back into the laser is in phase or out of phase relative to the standing wave inside the laser resonator. This results in a change of the laser power as a function of the position of the external mirror (or the optical path length), which is monitored by collecting the residual light from the resonator at the grating by means of a photodiode detector.

In order to test the sensitivity of the device, the external mirror was moved by the piezo actuator, and the detector signal was recorded as a function of the mirror shift (figure 4). At the steepest slope a sensitivity of 3.7 V/\mu m is achieved over a range of roughly 1 \mu m. Since the detector noise is well below 100 mV, changes of the optical path length of less than 0.03 \mu m (corresponding to 0.3\% of a wavelength) can be detected, which will be sufficient to determine the electron density distribution in our FlareLab experiment.

Figure 4. Variation of the laser output power as a function of the position of the external feedback mirror. The laser is operated at the 9R20 line (~9.3 \mu m).

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