Fatigue Life Prediction of Plain and Reinforced Concrete – A Review

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ABSTRACT

Many infrastructures like Bridge decks, airfield and highway pavements, offshore structure and machinery foundation are subjected to fatigue loading. This cyclic loading induces gradual, permanent internal changes in a material and thereby affecting the remaining life of the infrastructure. The heterogeneities in concrete add complexities in analysing fatigue failure of reinforced concrete. This review paper discuss about fatigue life prediction models for both plain and reinforced concrete structural member. This review paper comprises various deterministic and probabilistic models used in predicting the fatigue life of plain and reinforced concrete. Deterministic approach is dependent on some initial parameters and conditions and is unreliable to accurately determine the fatigue life of concrete. This results in the development of a more generalized model based on a probabilistic approach that accounts for the stochasticity in fatigue failure of concrete. In recent years, artificial neural network emerged as a new promising computational tool which adopts a probabilistic approach for modelling complex relationships.

Keywords: Fatigue, Plain concrete, Reinforced concrete, Deterministic, Probabilistic, Artificial Neural Network.

1 Introduction

Concrete is one of the conventional material used for the construction purpose and it is most durable construction materials. Although it is generally used it has some weakness also like low tensile strength, poor ductility, crack under tension, brittle failure, which leads to cracking. Cracking of concrete will affect the durability of structures and crack propagation will be more when structures subjected to fatigue loading. Fatigue is the phenomenon of gradual, perpetual internal changes occurring in a material due to repeated or cyclic loading. This leads to crack formation from the existing flaws within the material and results in fatigue fracture of materials. The separation of a material or object into two or more pieces on the action of stress is called fracture. Many infrastructures like Bridge decks, airfield and highway pavements, offshore structure and machinery foundation etc have to withstand a lot of repeated cyclic loading of uneven amplitudes. Due to this repeated loading fatigue fracture may develop in these structures and reduce the service life. So, it is important to consider the crack formation in the structural components which are subjected to fatigue loading. Concrete is heterogeneous in nature though it is considered as homogeneous in design perspective of view. Mainly three components of concrete are: cement matrix, aggregate and the matrix-aggregate interface. Due to the heterogeneous nature of the concrete internal flaws will be formed which in turn, triggers the initiation of the crack even before the loading is applied. There are three different zones of fatigue crack growth in concrete: crack initiation zone, crack propagation zone and unstable crack growth. Through the weakest path in the concrete crack propagation will occur.
Many experimental numerical and analytical investigations were done to know the fatigue failure mechanism of plain and reinforced concrete. Fatigue crack growth models for plain and reinforced concrete were developed on the basis of deterministic approach and probabilistic approach.

2 Fatigue Life Prediction of Plain and Reinforced Concrete

Since concrete is a heterogeneous material it has some defects such as shrinkage cracks, air voids, pores and spalling. There are three stages of fatigue failure mechanism in concrete: Flaw initiation, Microcracking, Macrocracking [2]. Fracture in concrete is governed by the existence of fracture process zone (FPZ) as concrete is quasi-brittle in nature. The fracture process zone is a region ahead of a crack tip and acts as an intermediate zone between cracked region and uncracked region. It is assumed that different loading cycle produces different failure mechanisms in concrete. For fatigue of low-cycle, the main mechanism is the formation of mortar cracks which result in the formation of continuous cracked networks. However, for fatigue of high-cycle bond cracks are developed in a slow and gradual process [3]. For plain concrete applied to repeated uniaxial tensile stresses show no fatigue limit under \(2 \times 10^6\) cycle [4].

For taking static loads reinforced concrete members were constructed. Due to fatigue loading cyclic stresses causes the structure to fail before attaining the ultimate load carrying capacity [5]. To resist tensile stresses concrete structures are reinforced with steel bars. For the design it is assumed that the tensile strength of reinforced concrete is zero. Due to fracture process zone concrete exhibit tension softening behaviour and requires an extra energy from the loading system for crack propagation. For the analysis, this could be included by using the concepts of fracture mechanics together with the softening behaviour of concrete. Crack growth in reinforced concrete depends on concrete toughness and elasticity of reinforcing steel [6].

Fatigue life prediction models for reinforced concrete are rare when compared to that of plain concrete. Based on deterministic and probabilistic approach many fatigue life propagation models have been developed. In deterministic approach the output is determined by the parameter values and initial values and also deterministic models does not include elements of randomness. Deterministic approach is one of the earliest and easiest method and the accuracy of failure probability are usually less and the variability of properties are also not considered. Whereas the probabilistic (stochastic) approach adopt distribution rather than fixed values which has variation and uncertainty. The probabilistic approach shows the stochastic behaviours involved in the process and is more dependable with the experimental data and it gives more realistic results. Therefore, the influence of heterogeneity in concrete like materials can be accounted by adopting a probabilistic approach.
Deterministic approach includes Fatigue life models, fracture mechanics method and fatigue damage method. Probabilistic approach includes Weibull distribution and Artificial Neural Network (ANN).

2.1 Fatigue Life Model

Fatigue property of a material can be defined by fatigue limit or S-N curve (Wohler curve) which gives the relationship between cyclic stress amplitude and number of cycles to failure. From fatigue test S-N curves are obtained. This test is usually done until failure of the specimen by applying a cyclic stress with constant amplitude on specimens. In several instances, the test is stopped after so many numbers of cycles (N>10⁶) and these materials are therefore considered to have an infinite life.

Miner [7] developed a typical empirical formula using S-N curve from many fatigue tests by analysing fatigue stress life data of concrete at varying stress levels. The developed fatigue stress-life relationship (S-N curve) is known as Palmgren-Miner (PM) hypothesis and it states that damage accumulates linearly with number of cycles applied at a particular stress level and is given by

$$\sum \frac{n_i}{N_i} = C$$  \hspace{1cm} (1)

where \(N_i\) is the number of cycles to failure at stress level i and \(n_i\) is the number of cycles at stress level i.

McCall [8] had done fatigue test on concrete beams and develops a relationship between applied stress level (S) and cycles to failure (N) by analysing the data from the test. This mathematical relationship fits for data at high stress level but fails at low stress level.

$$\log_{10}(S) = \log_{10}(C_1) - C_2 \log_{10}(N)$$  \hspace{1cm} (2)

\(N\) is the number of cycles to failure, \(S\) is the stress ratio and \(C_1\) and \(C_2\) are specific to each concrete.

Tamer et. al [9] developed a fatigue stress-life prediction model based on accelerated fatigue approach for RC beams. The model thus developed is on the basis of hypothesis of linear accumulative damage of the Palmgren–Miner rule; in this approach a linear increase of applied cyclic load range is there with respect to the number of cycles until the specimen fails. Using ANSYS software a three-dimensional RC beam was modelled and was validated. With different initial condition a numerical simulation was performed for the RC beam under linearly increased cyclic loading. Based on these analysed data fatigue stress-life model was developed. Comparing to standard testing approach, accelerated fatigue approach has rapid rate of damage accumulations.

$$\int_{\sigma_o}^{\sigma_u} \frac{\partial \bar{n}}{\partial \sigma} = \int_{\sigma_o}^{\sigma_u} \frac{\partial \bar{n}}{\partial N} \partial \sigma = 1$$  \hspace{1cm} (3)

Where, \(N(\sigma)\) is the lifetime from the S-N curve, \(\sigma_o\) is the initial cyclic stress range, and \(\sigma_u\) is the stress range at failure. S-N curve considers only overall fatigue life and failure mechanism is not considered. Thus, S-N curve method is not an appropriate method to determine fatigue life in concrete, as it does not account for the failure mechanism. A major drawback of this approach is that it needs large experimental data along with a statistical analysis. Also time consumption for this approach is more.

2.2 Fracture Mechanics Model

Fracture mechanics concept is introduced which are on the basis of crack growth due to fatigue. Paris law is the fundamental law for fracture mechanics concept, it was proposed by Paris and Erdogan [10]. Paris law gives the relationship between crack length increment per cycle and the stress intensity factor amplitude. The above-mentioned law is on the basis of fatigue crack propagation in plain concrete and is given by

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (4)
where, $\frac{da}{dN}$ is the crack growth per cycle, $\Delta K$ is stress intensity factor, $C$ and $m$ are material constants. For macroscopic cracks only Paris law is applicable, which means that only for very large specimens. Due to heterogeneous nature, this model cannot predict the crack growth behaviour of concrete and it needs some modification [10]. Bazant and Kazemi [11] modified the Paris law and was modified to incorporate the crack length increment and the size-effect, depends on size adjusted stress intensity factors. The size adjustment is done on the basis of structures brittleness number. Brittleness number gives the ratio of size of structure $d$ to transitional size $d_o$ which distinguish the responses governed by stress intensity factor and nominal stress [11]. The modified Paris law is given by

$$\frac{da}{dN} = C (K_{IC})^m$$

$$K_{IC} = K_{IF} \left[ \frac{\beta}{1 + \beta} \right]^{1/2}$$

Where, $K_{IF}$ is the fracture toughness, $\beta$ is the brittleness number $\left(\frac{d}{d_o}\right)$. One of the advantageous of this size effect method is that by calculating the maximum load values of geometrically similar specimen of different sizes, size independent fracture parameters can be obtained. Various experimental results show that $d_o$ for cyclic loading is greater than $d_o$ for monotonic loading, which means that the brittleness number for monotonic loading is greater than the brittleness number for cyclic loading.

Ray and Kishen [12] develop an analytical model for estimation of fatigue crack growth rate in plain concrete. An empirical formula is developed based on change in crack growth rate and depends on crack length ($a$), loading ratio ($R$), loading frequency ($\omega$), depth of beam ($D$), tensile strength ($\sigma_t$), fracture toughness ($G_f$) and change in energy release rate ($\Delta G$). The equation is given by,

$$\frac{da}{dN} = G_f^{1-\gamma_1-\gamma_2} \Delta G \gamma_1 \sigma_t \gamma_2^{-1} a \Phi_2 \left(\Pi_3, \Pi_4\right)$$

The exponents $\gamma_1, \gamma_2$ and dimensionless parameter $\Phi_2$ is obtained by calibration process using experimental results available in literature. The developed fatigue model is validated with the experimental results and it is well agreement with those experimental results. Also, a sensitivity analysis is done to find out which of the parameters used in this model are more sensitive and have an important role in the fatigue crack propagation. It is found that structure size is the most sensitive parameter and size independent fracture toughness is least sensitive.

A fracture mechanics model was developed by Carpinteri [13] to predict the fatigue life of reinforced concrete beam subjected to unidirectional cyclic bending moment. In the beam element presence of reinforcement is considered by applying an eccentric axial force applied to the concrete beam element and statically undetermined axial force has been calculated by applying the rotation congruence condition. It is observed that with increase in applied bending moment the steel force increases linearly, until yielding or slippage of reinforcement occurs. After this a perfectly elastic plastic behaviour is considered and it is assumed that before yielding of reinforcement there is no crack propagation and crack propagation occurs only when slippage is allowed between concrete and steel.

For $M \leq M_p$

$$K_f = \frac{1}{b^2} Y_M \left(\xi\right) \left[M - F \left(\frac{b}{2} - h\right)\right] - \frac{1}{b^2} Y_f(\xi) F$$
For $M > M_P$

$$K_I = \frac{1}{b^2 t} Y_M \left(\xi\right) \left[M - F_P \left(\frac{b}{2} - h\right)\right] - \frac{1}{b^2 t} Y_F \left(\xi\right) F_P$$  \hspace{1cm} (8)

The model thus developed is based on stress intensity factor ($K_I$), where $M$ is the bending moment, $F$ is the statically undetermined reaction of reinforcement, $M_P$ is the bending moment at which reinforcement yielding occurs, $F_P$ is the force of plastic flow collapse for reinforcement, $b$ is the depth of beam, $t$ is the beam thickness, $h$ is the distance of reinforcement from external surface, $\xi$ is the relative crack depth, $Y_M$ and $Y_F$ are the geometric functions.

Bosco and Carpinteri [14] modified the above-mentioned model to predict the fatigue life as well as the minimum percentage of reinforcement required for the concrete structures. In this approach instead of applying rotational congruence condition over the cross-section of the beam, rotational congruence condition is applied on the crack opening displacement. Even when steel is in the elastic condition the developed model has the capability to reproduce crack propagation. Through theoretical and experimental studies, when the depth of the RC beam exceeds a certain value even when the percentage of steel is low, a transition from ductile to brittle behaviour has been observed. The stress intensity factor is given by

For $M \leq M_P$

$$K_I = \frac{M}{h^2 b} Y_M \left(\xi\right) - \frac{P}{h^2 b} Y_P \left(\frac{c}{h}, \xi\right)$$  \hspace{1cm} (9)

For $M > M_P$

$$K_I = \frac{M}{h^2 b} Y_M \left(\xi\right) - \frac{P}{h^2 b} Y_P \left(\frac{c}{h}, \xi\right) - \frac{1}{h^2 b} Y_F \left(\frac{c}{h}, \xi\right)$$  \hspace{1cm} (10)

Where $P$ is the steel force, $c$ is the distance from the level of reinforcement to lower edge of beam, $Y_M$ and $Y_P$ are the geometric functions.

Ray and Kishen [6] developed a model to predict the fatigue life of reinforced concrete using fracture mechanics method by modifying the existing life prediction model developed for plain concrete as discussed earlier. In this work the reinforcement effect acting at the crack face is included as a pair of axial force. This model depends on parameters such as crack length, structural size, fracture toughness and loading ratio. Empirical equation is given by,

For $M < M_P$

$$K_I = \frac{M}{h^2 b} Y_M \left(\xi\right) - \frac{1}{h^2 b} Y_F \left(\frac{c}{h}, \xi\right) - \frac{1}{h^2 b} Y_F \left(\frac{c}{h}, \xi\right)$$  \hspace{1cm} (11)

For $M > M_P$

$$K_I = \frac{M}{h^2 b} Y_M \left(\xi\right) - \frac{F_P}{h^2 b} Y_F \left(\frac{c}{h}, \xi\right)$$  \hspace{1cm} (12)

$M_P$ is the moment at which reinforcement yielding occurs. The model thus developed is expressed in terms of energy release rate ($\Delta G$) and is given by,

$$\Delta G = \frac{K_I^2}{E}$$  \hspace{1cm} (13)

With the available experimental results, the model is validated and is found to be in good agreement. Further, a sensitivity analysis was done to identify the most sensitive parameter in the model and is found that structural size is the most sensitive parameter. The material parameters such as size independent fracture energy and tensile stress have only less significant in the computation of fatigue life.
Since the stress is very high at crack tip and the crack growth is based on stress intensity factor in fracture mechanics method. Like fatigue life model, fracture mechanics model is also extended from metals and comparing to S-N curve model, fracture mechanics gives a good prediction of crack growth rate and fatigue failure of concrete. Fracture mechanics model become more complex and is not dependable when varying parameters are there.

2.3 Fatigue Damage Model

By the presence of micro and macrocracks, fatigue damage in concrete can be identified. Propagation of macrocracks is handled by the theory of fracture mechanics, whereas the state of microcracking is handled by damage mechanics. Therefore, Fathima and Kishen [15] proposed a fatigue damage model by connecting fracture mechanics and damage mechanics theories through an energy equivalence within a thermodynamic framework by equating the energy dissipated according to each theory. By correlating these two theories, the damage corresponding to a given crack length is obtained and a discrete crack can be transformed into an equivalent damage zone. Thus the damage corresponds to given crack length is given by,

\[ D = \left( \frac{a - a_0}{a_c - a_0} \right)^{0.5} \]  

(14)

Where, \( a_0 \) is the notch length and \( a_c \) is the critical crack length. This model could predict the fatigue life with considerable accuracy as it incorporates the influence of micro and macrocrack.

2.4 Weibull Distribution

The two parameter Weibull distribution has been used for the probabilistic analysis of fatigue data at each stress level. Increasing hazard function of the two-parameter Weibull distribution makes it more suitable for fatigue studies. The fatigue life of concrete by Weibull distribution can be determined by a linear relationship between survivorship function \( L_N \) and number of cycles to failure. The survivorship function \( L_N(n) \) of the two-parameter Weibull distribution is given by,

\[ \ln[\ln(1/L_N)] = \alpha \ln(n) - \alpha \ln(u) \]  

(15)

Where, \( n \) is the specific value of number of cycles to failure, \( \alpha \) is the shape parameter at stress level S and \( u \) is the scale parameter at stress level S. This equation represents a linear relationship between \( \ln[\ln(1/L_N)] \) and \( \ln(n) \) and is used to verify the two-parameter Weibull distribution [16].

2.5 Artificial Neural Network (ANN) Model

The complexities and time consumption involved in the aforementioned models resulted in the use of a new technology called the Artificial Neural Network (ANN) to predict the fatigue life of concrete-like materials. ANN emerged as a new promising computational tool which adopts a probabilistic approach for modelling complex relationships. ANN is a highly powerful computational tool inspired from biological nervous system which is used to perform special task like Statistical modelling, Pattern recognition, Classification of problems etc. ANN is developed as a part of artificial intelligence and is programmed to act and think like humans. ANN consists of several interconnected artificial neurons. Neuron is a nonlinear unit which receives input signals yielding an output. ANN has three layers of neurons, Input layer which receives information, Hidden layer which perceives and evaluate the information from input layer, and Output layer which provides the final output. Based on problem to be solved ANN architecture is divided into Single layer Neural Network and Multilayer Neural Network [17].
Renju and Simon [18] have developed an artificial neural network predictive model to determine the fatigue life of plain concrete. The material property and fracture mechanics property responsible for the softening behaviour of concrete is considered and the developed model can predict the critical crack length of concrete at which failure occurs with considerable accuracy. The predictive artificial neural network model is developed by collecting data’s from fatigue tests conducted on three different geometrically similar specimens of plain concrete beams. The input parameters considered are number of cycles corresponding to crack increment, size dependent fracture energy, nominal strength, brittleness number, tensile stress and stress ratio. The developed model can predict the fatigue life of concrete in a faster approach with reasonable accuracy.

Fathalla et. al [19] developed a fatigue life model to predict the life of a reinforced concrete deck using artificial neural network. For this work they collected data’s from 75 samples of reinforced concrete deck with wide range of dimensions, reinforcement ratio, material properties, load levels. The developed ANN model could predict the fatigue life in terms of number of cycles by considering span-width ratio, thickness, compressive strength, reinforcement ratio and wheel loads as the input parameters. The proposed ANN model can able to predict the fatigue life at the faster rate.

Avilla et. al [20] used ANN for the prediction of the maximum surface crack width of precast reinforced concrete beams joined by steel coupler connectors and anchor bars. The training algorithms used are Back propagation (BPANN) and Genetic Algorithms (GANN). The performance of both the algorithms was compared. The Input and output parameters were decided based on data from existing literatures. From the study it was concluded that the Genetic algorithm gives a better prediction performance than back propagation network.

### 3 Conclusions

This review paper deals with the fatigue life prediction models for plain and reinforced concrete. This paper has models based on both deterministic and probabilistic approach. Deterministic approach includes Fatigue life models, Fracture mechanics and Fatigue damage models. Probabilistic approach includes Weibull distribution, Artificial Neural Network.
Fatigue life model gives the relationship between cyclic stress amplitude and number of cycles leads to failure. This is not a proper approach for determining fatigue life in concrete as this does not accounts for the mechanisms involved. This is more applicable to brittle and ductile material.

Fracture mechanics models are based on crack growth due to fatigue. Fundamental law for fracture mechanics model is Paris law this law gives the relationship between crack length increment per cycle and the stress intensity factor amplitude. Many authors have modified this Paris law and various modes were proposed. In this model crack growth is based on stress intensity factor. This model gives a better prediction comparing to Fatigue life models

Fatigue damage in concrete can be identified by the presence of micro and macrocracks. The fatigue damage model developed by correlating the fracture mechanics and damage mechanics theories and an empirical equation is derived between damage and crack length.

In case of Weibull distribution, the fatigue life of concrete can be determined by a linear relationship between survivorship function $L_N$ and number of cycles to failure. Increasing hazard function makes Weibull distribution more suitable for fatigue studies.

Complexities and time consumption using the above model has resulted in utilizing ANN in predicting the fatigue life of cementitious members. ANN is inspired from biological nervous system and it can able to perform special task like pattern recognition, statistical modelling etc. Fatigue life prediction using this approach gives accurate results and is time-efficient.

After considering all the models it can be concluded that deterministic approach cannot produce a reliable result due to varying parameters and discreteness property of concrete. Probabilistic approach gives a better prediction in fatigue life assessment of plain and reinforced concrete. It can also be summarised that, fatigue life prediction model available for reinforced concrete is less compared to that of plain concrete and fatigue life prediction models for reinforced concrete is yet to be explored and ANN will emerge as a good life prediction method with the most reliable results in a faster approach.

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