I. THE SMALLEST ENCLOSED CIRCLES ALGORITHM

Suppose there are $k$ points $P = \{p_i = [x_i, y_i]\}_{i=1}^k$ in the scene, and we need to find a circle with the smallest radius to cover these points, regarded as the smallest enclosing circle for these $k$ points.

**Proposition 1:** The smallest enclosing circle for $k$ points should pass through at least two points.

**Proof:** It is straightforward to prove it.

**Proposition 2:** The smallest enclosing circle for $k$ points is generated by at most three points.

**Proof:** The proof is given in [1].

**Proposition 3:** If point $k$ is outside of the smallest enclosing circle for points $\{p_1, p_2, \ldots, p_{k-1}\}$, then the point $k$ should be on the boundary of the smallest enclosing circle for points $\{p_1, p_2, \ldots, p_k\}$.

**Proof:** The proof is given in [1].

Then, we divide the searching problem of the smallest enclosing circle into two cases to discuss separately.

- **Case 1:** The smallest enclosing circle for $k$ points pass through only two points of $P$.

If the circle passes through two points (denoted as $p_l$ and $p_j$) of $P$, then the line connecting these two points is the diameter of the circle. In this case, the radius and center can be given by $L_c = \frac{p_l + p_j}{2}$ and $r_c = \frac{|p_l - p_j|}{2}$.

- **Case 2:** The smallest enclosing circle for $k$ points should pass through at least three points of $P$.

If the smallest enclosing circle passes through three points (denoted as $p_l$, $p_j$, and $p_h$) of $P$, then the center of the circle (i.e. $L_c = [x_0, y_0]$) can be given by

$$
\begin{align}
  x_0 &= \frac{de - bf}{bc - ad}, \\
  y_0 &= \frac{af - cd}{bc - ad},
\end{align}
$$

where

$$
\begin{align}
  a &= x_i - x_j, \\
  b &= y_i - y_j, \\
  c &= x_i - x_h, \\
  d &= y_i - y_h, \\
  e &= \frac{(x_j^2 - x_i^2) - (y_j^2 - y_i^2)}{2}, \\
  f &= \frac{(x_j^2 - x_h^2) - (y_j^2 - y_h^2)}{2},
\end{align}
$$

and the radius is given by $L_c = \sqrt{x_0^2 + y_0^2}$.

Assuming that the points in $P$ are with weights $\{W_i | W_1 \geq W_2 \geq \cdots \geq W_k\}_{i=1}^k$, respectively. If we need to find the smallest enclosing circle for at most $n$ points of $P$, which owns the maximum sum of the weight value, an efficient and effective method is given as follows.

**Algorithm 1** Greedy Search for the Constrained Smallest Enclosing Circle

1. $\Omega^*_t = \{p_1, p_2\}$, $L_c = \frac{p_1 + p_2}{2}$, $r_c = |\frac{p_1 - p_2}{2}|$, $k = 3$.
2. while $|\Omega^*_t| \leq n$ do
3.   **if** $|p_k - L_c| \leq r_c$
4.     $\Omega^*_t = \Omega^*_t \cup p_k$.
5.     Search the remaining points from $P/\Omega^*_t$, and add them into $\Omega^*_t$.
6.   **else**
7.     Construct a circle $c$ whose diameter is the line connecting from point $p_k$ to the point that is the furthest from $p_k$ from the set $P/\Omega^*_t$.
8.     **if** all points of $\Omega^*_t$ is within the circle $c$:
9.       Update $L_c$ and $r_c$.
10. **else**
11.       For each two points in $\Omega^*_t$:
12.         Construct a circle based on the two points and $p_k$.
13.         Pick out the circle with the smallest radius, and update $L_c$ and $r_c$.
14.         Search the remaining points from $P/\Omega^*_t$, and add them into $\Omega^*_t$.
15. **end while**
16. Retain the $n$ points with the largest weight value in $\Omega^*_t$, and output $L_c$ and $r_c$.

Due to the fact that the Algorithm 1 is to search the smallest enclosing circle for partial points in a greedy way, thus the results of the Algorithm 1 can be regard as a periodic achievement of the algorithm proposed in [1]. Therefore, as proven in [1], the expectation of the computational complexity of the algorithm is given by $O(k^3)$, in which $k$ denotes the number of points.

II. EXTENSION IN PROBABILITY LoS CHANNELS

Here we discuss how to further extend this letter to probability LoS channel scenarios. In this case, the waypoint of the UAV in time slot $t$ is denoted as $L_u(t) = [x(t), y(t), z]^{T} \in \mathbb{R}^{3 \times 1}$ in a given 3-D space. Then, the LoS probability at each time slot $t$ can be modeled as a function of the UAV-SN elevation angle, which can be expressed in the form of

$$
P(e_{k_t}^{L_t} = 1) = \frac{1}{1 + \alpha e^{-b(a_{k_t-t})}},$$

where $\alpha$, $b$, and $a$ are parameters.
where $\theta_{k,t}$ denotes the angel between the UAV and SN $k$ in time slot $t$, which is given by

$$\theta_{k,t} = \frac{180}{\pi} \arctan\left(\frac{z}{\sqrt{(x_i - x(t))^2 + (y_i - y(t))^2}}\right), \quad (4)$$

where $\alpha$ and $\beta$ are modeling parameters to be specified. Then, the corresponding NLoS probability can be obtained by

$$P(c_{k,t}^L = 0) = 1 - P(c_{k,t}^L = 1). \quad (5)$$

In this case, the large-scale channel power gain between the UAV and SN $k$ in the time slot $t$, including both the path loss and shadowing, can be modeled as

$$h_{k,t} = c_{k,t}^L h_{k,t}^L + (1 - c_{k,t}^L) h_{k,t}^N, \quad (6)$$

where

$$h_{k,t}^L = \beta_0 d_{k,t}^{-\alpha_L}, \quad h_{k,t}^N = \mu \beta_0 d_{k,t}^{-\alpha_N} \quad (7)$$

denote the channel power gains in LoS and NLoS cases, respectively. Meanwhile, $d_{k,t}$ can be calculated as

$$d_{k,t} = ||L_i - L_u(t)||. \quad (8)$$

As a result, it only needs to replace $g_i(t)$ (i.e. (1) of the manuscript) with (6) of this technical report, and the optimization problem (i.e. problem P0 of the manuscript) stays the same. The UAV-NOMA system with probability LoS channels can still be solved by the proposed schemes in the manuscript.

REFERENCES

[1] E. Welzl, “Smallest enclosing disks (balls and ellipsoids)”. New Results and New Trends in Computer Science, Lecture Notes in Computer Science, vol 555. Springer, Berlin, Heidelberg, 1991.