Second-formulation adaptive wavelet optimization technique applicable in material sciences

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Abstract. We developed a second-formulation adaptive wavelet optimize finite element (SFAWOFE) method on a general manifold with the help of approximation theory. The SFAWOFE method used to solve Burger’s equation with periodic and Dirichlet boundary conditions. The beauty of the SFAWOFE process is the numerical solution is optimized for the finite element method and does not affect the computational algorithm. We construct an algorithm for numerical results that have been optimized finite element method on a diffusion wavelet, which formed employing the adaptive wavelet plot. The algorithms and trial problems used to calculate magnetostatic networking range dipoles in material science. For the trial problem, the processing time checked out the SFAWOFE method has calculated and analyzed the analogous processing time checked out the finite element method on a general manifold. Finally, we verify the SFAWOFE method convergence for each trial problem and high efficient.

Key words: Adaptive wavelet; approximation theory; finite element method; trial problem.

1. Introduction
Wavelet first interpolated by Grossmann and Morlet in 1980 [1] since then they maintain expressively ingrowing a great deal of intellect and metallurgy [2, 3]. Wavelets are construction wedges for familiar data sheets or challenges. In general, we shall wavelet forms an orthonormal basis for multiresolution analysis with the climbing function of a wavelet frame [4]. Wavelets are exploited, in the wavelet frame assignment of an imperious numerical rank on an adaptive plot to discover the network has to be clarified or coarcted to characterize the solution optimally are faster to calculate. Alternatively, of developing the results in form of wavelets, the adaptive transform is utilized to dictate the wavelet lattice [5, 6, 7]. The significant arrogance of second-formulation wavelets is the same as always wavelets are concocted straight real-time domain while one-hit-wonder to solve your problems; therefore, the invite formulated is boundary conditions disparate.
Excellent wavelet compaction features[8, 9] short fulcrum, departure moment [10], and the capacity to determine confined framework formed the wavelets a useful appliance future exploited meant for the numerical solution of the partial differential equation(PDEs) [11, 12]. Wavelet optimized finite element (WOFE) interpolated by Jameson [4]. In WOFE, wavelets exploited
to obtain an adaptive plot. Alternatively, we developing the climbing function, and climbing transformation of the wavelet transform is utilized for the solution, while the finite element grid has to be clarified or coarcted [13]. In this framework, we are going to solve Burger’s equation with periodic and Dirichlet boundary conditions with the help of an adaptive wavelet optimized finite element technique. The least-squares method exploited for the approach of the partial differential equation implicated in that manner Burger’s equation, and such as second-formulation wavelet exploited to dictate wheres limited dissimilarity plot needs to be clarified or coarcted to express the solution precisely.

2. Second-formulation wavelet transform

This explanation of the second-formulation multiresolution analysis convinces expeditious second-formulation wavelet transform. Accustomed procedure in the vicinity of the framework for wavelets with rare up heave is aware of ensure:

1. Delineation the diffusion wavelet transform with out an allusion to thaw climbing functions $\phi, \overline{\phi}$ and wavelet function $\psi, \overline{\psi}$.
2. The climbing function $\phi, \overline{\phi}$ and wavelet function $\psi, \overline{\psi}$ a retail compile adopting diffusion wavelet transform.

Consequently, the most significant gait is the framework of diffusion wavelet transform and inverse diffusion wavelet transform, which is deliberate follow.

Conclave function $h \in l_2(R)$, the climbing function co-active $\eta_{k,m}$ and wavelet co-active $\kappa_{k,m}$ are defined as ensue:

$$\eta_{k,m} = \langle h, \bar{\phi}_{k,m} \rangle, \quad \kappa_{k,m} = \langle h, \bar{\psi}_{k,m} \rangle,$$

where $\bar{\phi}_{k,m} = 2^{k/2} \phi(2^k r - m)$. Let us postulate $\eta_{s,m}$ is given for stationary $s$ and we want to calculate $\eta_{k,m}$ and $\kappa_{k,m}$ for $k < s$. This can be done concluded the iterative application of the elaboration relation

$$\phi(r) = 2 \sum_n f_n \phi(2r - n)$$

The inverse transform iterative benediction the formula

$$\eta_{k+1,m} = \sqrt{2} \sum_n f_{n-2m} \eta_{k,m} + \sqrt{2} \sum_n g_{n-2m} \kappa_{k,m}. \quad (1)$$
The consequence beeline programmer is certified as the adaptive wavelet transform. Now that the fundamental scoop method, we hoop against \( \{f, f^0, g^0, \overline{g}^0\} \) to place \( \{f, f, g, \overline{g}\} \) and hence from the set \( \{\phi^0, \phi^\overline{0}, \psi^0, \psi^\overline{0}\} \) to place \( \{\phi, \phi, \psi, \psi\} \). Thither diminish the quantity of operators as an analogy to thaw garden-variety process. The matters emulate this

\[
\tilde{\phi}_{k,m} = 2^{k/2} \bar{\phi}(2^k r - m) = 2^{k/2} \left( 2 \sum_m f^0_n \bar{\phi}(2(2^k r - m) - n) + \sum_n p_{-n} \overline{\psi}(2^k r - m - n) \right)
\]

\[
= \sqrt{2} \sum_n f^0_{n-2m} \overline{\psi}_{k,n}.
\]

Therefore,

\[
\eta_{k,m} = \sqrt{2} \sum_n f^0_{n-2m} \eta_{k+1,n} + \sum_n p_{m-n} \psi_{k,n}.
\]

We know

\[
\overline{\psi}(r) = 2 \sum_n g^0_n \phi(2r - n),
\]

then we have

\[
\kappa_{k,m} = \sqrt{2} \sum_n g^0_{n-2m} \eta_{k+1,n}.
\]

They acquire the inverse transform with Eq-1 like

\[
\eta_{k+1,n} = \sqrt{2} \sum_m f_{n-2m}(\eta_{k,m} - \sum_i p_{m-t} \kappa_{k,i}) + \sqrt{2} \sum_m g^0_{n-2m} \eta_{k,m}.
\]

Consequently, we hoop the following procedure for adaptive scoop wavelet transform (Diffusion wavelet transform DWT) and more polar (polar diffusion wavelet transform PDWT).

**DWT**

Step-1: (Computational of unliganded co-active)

\[
\eta_{k,m} = \sqrt{2} \sum_n f^0_{n-2m} \eta_{k+1,n}
\]

\[
\kappa_{k,m} = \sqrt{2} \sum_n g^0_{n-2m} \eta_{k+1,n}.
\]

Step-2 (Computation of uprear co-active)

\[
\eta_{k,m} = \eta_{k,m} + \sum n p_{m-n} \kappa_{k,m}.
\]

**PDWT**

Step-1

\[
\eta_{k,m} = \eta_{k,m} - \sum n p_{m-n} \kappa_{k,m}.
\]

Step-2

\[
\eta_{k+1,m} = \sqrt{2} \sum_m f_{n-2m} \eta_{k,m} + \sqrt{2} \sum_m g^0_{n-2m} \kappa_{k,m}.
\]
3. Second-formulation adaptive wavelet optimized finite element method

The many rear applications of Burger’s equation in mathematics and material sciences. For example, it observed as an equation that models convulsion, influx flows, audio communication in a haze, etc. It can also see as an agitation often-used ugly Laplace equation, what type of crook investigated are discussed in this framework. They evaluate a bar built arise of temperature fetching material. Let $E$ be the elongation of the bar, and $B$ be the fixed sliced region of the bar. We suppose that the only endpoint of the bar can replace the temperature, and the edgewise rectus of the bar is desolated. Again, it is considered that the temperature of the bar is stable all over any layer given the sliced region.

Let illustrate the heat of location $r$ and time $t$ by $v(t,r)$ and subsequent arete bounds often quilted substance tufted bar: $n$, the thermal energy; $\rho(r)$, the solidity often-used content at point $r$; $c(r)$, the particular temperature oftentimes bar at that position $r$, $h_{1}(t,r)$, the ratio of heat-producing respectively time. With the significance bounds, the equation controlling the heat within the bar is inclined

$$
\rho(r)c(r)\frac{\partial v(t,r)}{\partial t} = \frac{\partial}{\partial r}(k(r)\frac{\partial v(t,r)}{\partial r}) + h_{1}(t,r).
$$

(2)

We farther suppose that the solidity, the explicit temperature, and the thermal energy are isolated of the location; with this taking up Eq-2 gets

$$
\frac{\partial v(t,r)}{\partial t} = \frac{m}{\rho c} \frac{\partial^{2}u(t,r)}{\partial r^{2}} + \frac{h_{1}(t,r)}{\rho c}.
$$

Granting $\nu = \frac{m}{\rho c}$ and $h(t,r) = \frac{h_{1}(t,m)}{\rho c}$, the above equation grows

$$
\frac{\partial v(t,r)}{\partial t} = \nu \frac{\partial^{2}v(t,r)}{\partial r^{2}} + h(t,r).
$$

(3)

Now we examine a confusion Eq-3 in such a way

$$
\frac{\partial v(t,r)}{\partial t} + v(t,r)\frac{\partial v(t,r)}{\partial r} = \nu \frac{\partial^{2}v(t,r)}{\partial r^{2}} + h(t,r).
$$

(4)

Equation-4 viscosity even constant $\nu$. Because the partial equation involves in time and area respectively, the necessity of this type equation contains initial condition $v(0,r) = v_{0}(r)$ to solve the problem. The real thing two standard categories of two issues:

1. Periodic boundary condition

$$
v(t,r) = v(t,r + L)
$$

2. Dirichlet boundary condition

$$
v(t,0) = v_{L}(t), v(t,L) = v_{R}(t).
$$

3.1. Finite element method for two boundary conditions

Introduce the region isthmian interruption $[a,b]$ with point discretion the region be $a = r_{1} < r_{2} < r_{3} < \cdots r_{I} = b$. Applying the Riesz representation theory, the first and second derivatives for good approximation behavior of the finite-dimensional domain is inclined

$$
\frac{dv(r_{1})}{dr} = \frac{v(r_{2}) - v(r_{0})}{r_{2} - r_{0}}, \frac{dv(r_{2})}{dr} = \frac{v(r_{3}) - v(r_{1})}{r_{3} - r_{1}},
$$

$$
\ldots \ldots \ldots
$$

$$
\frac{dv(r_{I-1})}{dr} = \frac{v(r_{I}) - v(r_{I-2})}{r_{I} - r_{I-2}}, \frac{dv(r_{I})}{dr} = \frac{v(r_{I+1}) - v(r_{I-1})}{r_{I+1} - r_{I-1}}.
$$

(5)
Not even $r_0$ and $r_{I+1}$ are not inclined to in our discretion region. Conditional on the different kinds of the properties elaborate we-group disparate finite element method as clarified following.

### 3.2. Periodic boundary conditions

For fear that of periodic boundary conditions, every indecision is expected to be cogent modulo $I - 1$ (as $v(a) = v(b)$). Consequently, we handle $r_0$ are $r_{I+1}$ as $r_2$. Supplementary, we suppose that $r_1 - r_0 = r_2 - r_1$ and $r_{I+1} - r_I = r_I - r_{I-1}$. Hence, in this compact, $E^{(1)}$ and $E^{(2)}$ are first and second finite element matrix in such a manner

$$[v'(r_1), v'(r_2), \cdots v'(r_I)]' = E^{(1)}[v(r_1), v(r_2), v(r_3), \cdots v(r_I)]'$$

and

$$[v''(r_1), v''(r_2), \cdots v''(r_I)]' = E^{(2)}[v(r_1), v(r_2), v(r_3), \cdots v(r_I)]'$$

are specified

$$E^{(1)}(1,2) = -E^{(1)}(1,1 - 1) = \frac{1}{2(r_2 - r_1)},$$

$$E^{(1)}(I,2) = -E^{(1)}(I,1 - 1) = \frac{1}{2(r_I - r_{I-1})},$$

$$E^{(1)}(j,j - 1) = E^{(1)}(j, j + 1) = -\frac{1}{r_{j+1} - r_{j-1}},$$

$$E'(j,k) = 0, \forall \text{other jandk}$$

$$-\frac{1}{2}E^{(2)}(1,1) = E^{(2)}(1,2) = E^{(2)}(1,I - 1) = \frac{1}{(r_2 - r_1)^2},$$

$$E^{(2)}(I,2) = E^{(2)}(I,1 - 1) = -\frac{1}{2}E^{(2)}(I,1) = \frac{1}{(r_I - r_{I-1})^2},$$

$$E^{(2)}(j,j - 1) = \frac{2}{(r_{j-1} - r_j)(r_{j-1} - r_{j+1})}, 2 \leq j \leq I - 1,$$

$$E^{(2)}(j,j) = \frac{2}{(r_j - r_{j-1})(r_{j} - r_{j+1})}, 2 \leq j \leq I - 1,$$

$$E^{(2)}(j,j + 1) = \frac{2}{(r_{j+1} - r_{j-1})(r_{j+1} - r_j)}, 2 \leq j \leq I - 1,$$

$$E^{(2)}(j,k) = 0, \forall \text{other jandk}$$

### 3.3. Dirichlet boundary conditions.

In anticipation of Dirichlet boundary condition the behavior is identified at the bound points, i.e. $v(a) = v_L$ and $v(b) = v_R$ that indicates $v(r_1) = v_L$ and $v(r_I) = v_R$. While solution is familiar at that bound points, we have lone one possible the $I - 2$ strangers especially $v(r_2), v(r_3), \cdots v(r_{I-1})$. Consequently, we examine the each one but first and last equations of 5 and 6 and write $v(r_1) = v_L, v(r_I) = v_R$. We acquire the succession
Coming to have found all the necessary materials for SFAWOFE, we describe the algorithm for solving PDEs.

4. Finite element algorithm for solving PDEs

where $E^{(1)}$ and $E^{(2)}$ are the matrices of order $(I - 2) \times (I - 2)$ with subsequent field

$$E^{(1)}(1, 1) = \frac{1}{(r_3 - r_1)}, E^{(1)}(I - 2, I - 3) = -\frac{1}{Z_I - r_{I-2}},$$

$$E^{(1)}(j, j - 1) = -\frac{1}{Z_{j+2} - r_j}, E^{(1)}(j, j + 1) = -\frac{1}{Z_{j+2} - r_j} \quad 2 \leq j \leq I - 3$$

$$E^{(1)}(j, k) = 0, \forall \text{other j and k}$$

$$E^{(2)}(1, 1) = \frac{2}{(r_2 - r_1)(r_2 - r_3)}, E^{(2)}(1, 2) = \frac{2}{(r_3 - r_1)(r_3 - r_2)},$$

$$E^{(2)}(I - 2, I - 3) = \frac{2}{(r_{I-2} - r_{I-1})(r_{I-2} - r_I)};$$

$$E^{(2)}(I - 2, I - 2) = \frac{2}{(r_{I-1} - r_{I-2})(r_{I-1} - r_I)};$$

$$E^{(2)}(j, j - 1) = \frac{2}{(r_j - r_{j+1})(r_j - r_{j+2})}, 2 \leq j \leq I - 3$$

$$E^{(2)}(j, j) = \frac{2}{(r_{j+1} - r_j)(r_{j+1} - r_{j+2})}, 2 \leq j \leq I - 3$$

$$E^{(2)}(j, j + 1) = \frac{2}{(r_{j+2} - r_j)(r_{j+2} - r_{j+1})}, 2 \leq j \leq I - 3$$

$$E^{(2)}(j, k) = 0, \forall \text{other j and k}$$

4. Finite element algorithm for solving PDEs

Coming to have found all the necessary materials for SFAWOFE, we describe the algorithm for SFAWOFE

1. Discretionary the domain of the PDEs.

2. Application Riesz representation theory for finite difference matrices disposed of in sec() to discretionary the function intricate in the PDE approaching.

3. Assume the time $t_1, t_2, \cdots$ are determined towards mesh transformation (note-taking the option of those times is dispute conditional, providing the problem is such a manner for a solution is quickly fluctuating then $t_j$’s must be favored nearer). Inception with the initial
condition, which takes an application of a time combination strategy (Least-Squares method in that matter), we acquire results at the time \( t_1 \), i.e., \( v(t_1) \).

4. We apply \( v(t_1) \) and the correlated mesh points \( T^{t_1} \) to acquire \( T^{t_1+\Delta t} \) applying the method explained in the behind section.

5. Computing the least-square approximation elaborate in the PDE on \( T^{t_1+\Delta t} \) which is convert PDEs to ordinary differential equations (ODEs).

6. Integrate the concluding mesh of ODEs in time using the Least-Squares method to acquire the results at \( t = t_1 + \Delta t \).

7. Repetition the step-6 as late as \( v(t_2) \) is achieved. Coming by found \( v(t_2) \) proceed to step-4.

4.1. Numerical outcomes

Investigation of the admitted SFAWOFE method weighs against deliberate Burger’s equation with two boundary conditions as periodic and Dirichlet s. The equation deliberate is specified as ensues

\[
\frac{\partial v(t,r)}{\partial t} + v(t,r) \frac{\partial v(t,r)}{\partial r} = \nu \frac{\partial^2 v(t,r)}{\partial r^2}
\]

with initial condition given by \( v(r,0) = f(r) \) and appropriate boundary conditions. Using the above algorithm on SFAWOFE, as clarified in Section-3. Calculating in operation boundary conditions exploited coming trial problems are deliberated.

4.2. Trial problem

In the paper we contemplate Burger’s equation on the region \([0, 1]\)

\[
\frac{\partial v(t,r)}{\partial t} + v(t,r) \frac{\partial v(t,r)}{\partial r} = \nu \frac{\partial^2 v(t,r)}{\partial r^2}
\]

with initial condition \( v(r,0) = v_0(r) = \sin 2\pi r \) and boundary condition is periodic and Dirichlet i.e. \( v(r,t) = v(r+1,t) \) and \( v(t,0) = v_1(t) \), \( v(t,1) = v_1(t) \). Analytic solution for the Burger’s equation is

\[
v(r,t) = \frac{\int_{-\infty}^{\infty} \frac{r-r^*}{4\nu} \exp \left( -\frac{(r-r^*)^2}{4\nu t} \right) \exp \left( \frac{\cos(2\pi t)}{4\pi^2} \right) d\tau}{\int_{-\infty}^{\infty} \exp \left( -\frac{(r-r^*)^2}{4\nu t} \right) \exp \left( \frac{\cos(2\pi t)}{4\pi^2} \right) d\tau}.
\]

The analytic solution of an equation is a changeless wave, which is modified a sharp Grenadian at \( r = 0.5 \). For \( \epsilon = 10^{-5} \) and \( \nu = 0.004 \). Fig-1(a) exhibitions the comparison between a numerical and analytic solution of Burger’s equation. Fig-1(b) shows the relation between compaction error and the number of expressive coefficients. Fig-1(c) compares the PT time checked out the AWOFE method with the PT time checked out the finite element method for calculating the solution at the time \( t = 2^n \Delta t \). We can observe that from Fig-1(e) at time \( t = 2^5 \Delta t = 0.3601 \), the PT time is checked out the AWOFE method is 5% less than the time checked out finite element method. Fig-1(d) and Fig-1(f) exhibitions the convergence of the process corresponding to different boundary conditions and adaptive grid with time. Fig-1(g) exhibitions the point to point error at various times, at it is detected that error is the highest value for the diversion as anticipated. Therefore a large number of mesh grid will be taken in the taken interval, and titular lead to generating a diffusion plot. The error can be minimized while the decreases values of \( \epsilon \), an error is directly pro personal to \( \epsilon \). Therefore, \( \epsilon \) is to be taken a tiny number; the relation between error and mathematical cost stability properly handles. Now, weddings
Figure 1: Results for trial problem.
the time compaction diversion as \( \Theta = \frac{PT(\epsilon = 0)}{PT(\epsilon)} \). Fig-1(h) shows the diversion of \( \Theta \) being a purpose of \( \nu \); it can be checked that decrease isthmian value of \( \nu \), the increased cost of \( \Theta \), i.e., the technique executes superior for minimum costs of \( \nu \). Table-1 provides diversion of \( PT(\epsilon) \) with \( \epsilon \) for \( \nu = 0.0002 \). We hold checked that \( \epsilon \) lower \( \Theta \) also lower. The cost of \( \Theta \) increases the adaptive wavelet technique is more efficient as compare finite element method.

Table 1: The execution of SFAWOFE for trial problem.

| \( \epsilon \) | \( 10^{-3} \) | \( 10^{-4} \) | \( 10^{-5} \) | \( 10^{-6} \) |
|----------------|-------------|-------------|-------------|-------------|
| \( PT(\epsilon) \) | 0.0892      | 0.2361      | 0.2386      | 0.3034      |
| \( \Theta \)      | 3.7116      | 2.7353      | 2.6133      | 1.9840      |

5. Discussion and Conclusion

In this framework, the second-formulation adaptive wavelet optimized finite element method established, individually considered as SFAWOFE for the solution of Burger’s equation with periodic and Dirichlet boundary conditions. We have developed a finite element algorithm for the resolution of PDEs and calculate numerical results of Burger’s equation for the time integration trial problem. The trial problem used for computing magnetostatic networking rang dipole in material science, which used for fast computation of process time checked out the SFAWOFE method are analyzed with process time checked out the finite element method. We are getting verified the performance of the purposed way for Burger’s equation. We have comprehensive this method for disturbance designing and problem-solving for fluid analysis in our future work.

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