SYK Lindbladian

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We study the Lindbladian dynamics of the Sachdev-Ye-Kitaev (SYK) model, where the SYK model is coupled to Markovian reservoirs with jump operators that are either linear or quadratic in the Majorana fermion operators. Here, the linear jump operators are non-random while the quadratic jump operators are sampled from a Gaussian distribution. In the limit of large \( N \), where \( N \) is the number of Majorana fermion operators, and also in the limit of large \( N \) and \( M \), where \( M \) is the number of jump operators, the SYK Lindbladians are analytically tractable, and we obtain their stationary Green’s functions, from which we can read off the decay rate. For finite \( N \), we also study the distribution of the eigenvalues of the SYK Lindbladians.

I. INTRODUCTION

While quantum dynamics is often modeled by an idealized unitary time evolution, non-unitary time evolutions are ubiquitous and relevant since experimental systems are never completely isolated. Non-unitarity may arise in many different forms, such as dissipation, gain/loss, decoherence, measurements, and so on. Understanding and controlling these effects are of both fundamental and practical importance. Furthermore, non-unitarity may give rise to rich behaviors that do not have counterparts in systems governed by unitary time evolution. Our understanding of possible universal behaviors in open quantum systems, however, is still limited, particularly in the context of many-body quantum systems and quantum field theory.

In this paper, we study tractable many-body quantum systems with Lindbladian dynamics, aiming to deepen our understanding of open many-body quantum systems. The models we study consist of the SYK Hamiltonian, i.e., fermionic quantum many-body Hamiltonian with all-to-all interactions [1, 2] and jump operators that we will describe momentarily. Specifically, we consider a set of Majorana fermion operators, \( \{ \psi_i, \psi_i^\dagger \} \), where \( i = 1, \ldots, N \), and the associated Fock space where \( i = 1, \ldots, N \). The Lindbladian \( \mathcal{L} \) of our interest, which generates the dynamics \( \frac{d\rho}{dt} = \mathcal{L}(\rho) \), is given by

\[
\mathcal{L}(\rho) = -i[H_{\text{SYK}}, \rho] + \sum_{\alpha} \left[ L_\alpha \rho L_\alpha^\dagger - \frac{1}{2} (L_\alpha^\dagger L_\alpha, \rho) \right].
\]  

(1)

Here, the Hamiltonian part is given by the SYK (SYK\(_q\)) Hamiltonian with \( q \)-body interaction,

\[
H_{\text{SYK}} = q^{q+1} \sum_{i_1 < \cdots < i_q} J_{i_1 \cdots i_q} \psi_{i_1} \cdots \psi_{i_q},
\]  

(2)

where \( J_{i_1 \cdots i_q} \) are random coupling drawn from the Gaussian distribution. As for the jump operators \( \{ L_\alpha \} \), we consider the following two cases. (i) First, we consider \( N \) jump operators that are linear in fermion operators,

\[
L^i = \sqrt{\mu} \psi^i, \quad i = 1, \cdots, N.
\]  

(3)

Here, \( \mu \) is a non-random real parameter. (ii) In the second example, we consider \( M \) quadratic jump operators,

\[
L^a = \sum_{1 \leq i < j \leq N} K_{ij}^a \psi_i \psi_j, \quad K_{ij}^a \in \mathbb{C}, \quad a = 1, \cdots, M
\]  

(4)

where all \( K_{ij}^a \) are independent complex Gaussian distributed random variables with mean and variance given by

\[
\langle K_{ij}^a \rangle = 0, \quad \langle |K_{ij}^a|^2 \rangle = \frac{K^2}{N^2} \quad \forall \ i, j, a \quad \text{(no sum)}.
\]  

(5)

It is also possible to consider more generic jump operators that consist of \( p \) Majorana operators. For more details of these models, see later sections. As we will show, these models can be analytically studied in the limit \( N \to \infty \) in the first example, and in the limit \( N, M \to \infty \) while keeping the ratio \( R = M/N \) fixed in the second example.

The SYK model and its variants provided various tractable toy models for many-body problems, e.g., the butterfly effect, quantum information scrambling, quantum entanglement, non-fermi liquid, etc., and have been extensively studied recently [3–9]. Some of these models admit holographic dual descriptions. We note that non-unitary time evolutions of various kinds in the SYK type models have been also studied recently. See, for example, [10–20]. Our study using the Lindbladian dynamics is different from and complementary to these previous works. The effects of dissipation in these SYK models have been studied within unitary dynamics by including the heat bath degrees of freedom explicitly. There are two-coupled variants of SYK models, where one of the copies can be considered as a bath. See, for example, [21–23]. At more technical levels, there are other hermitian SYK type models (supersymmetric SYK and Wishert SYK models) that have some resemblance with our SYK Lindbladian model(s) [24–26].

In this work, we will study the properties of the above SYK Lindbladians by computing Green’s functions in the large \( N \) and \( M \) limit. This allows us to extract, for example, the dominant decay rate (the spectral gap of the Lindbladians). We also study the spectral properties of the SYK Lindbladians by exact diagonalization at finite
of as an infinite temperature Gibbs state, is mapped to $|\rho\rangle$. Similarly, the Lindbladian can be mapped to an operator acting on $H_+ \otimes H_-$, and the Lindblad equation is now written as $d|\rho\rangle/dt = \mathcal{L}|\rho\rangle$, where we continue to use $\mathcal{L}$ to represent the mapped operator. The explicit form of $\mathcal{L}$ for our models is given in equations (10) and (25).

The state $|\rho\rangle$ is annihilated by $\mathcal{L}$, $\mathcal{L}|\rho\rangle = 0$, as the infinite temperature state is stationary with respect to any Lindbladian.

With the operator-state map, for example, the “partition function” can be expressed as $\text{Tr}(\rho(t)) = \langle |\rho(t)\rangle = \langle |\psi^{t}\rangle |\rho_0\rangle = 0$ where $|\rho_0\rangle$ is an initial condition and we noted $\langle |\psi^{t}\rangle \mathcal{L} = |\rho\rangle$. Similarly, the expectation value of an operator $A$ is given by $\text{Tr}(\rho(t)A) = \langle |\psi^{t}\rangle A_{+} \otimes \mathbb{1} \mathcal{L} |\rho_0\rangle$. These quantities can be readily expressed in terms of the coherent state path integral over two copies of real fermionic fields, $\psi_{\pm}(t)$, as

$$Z = \langle |\psi^{t}\rangle |\rho_0\rangle = \int \mathcal{D} \psi_{+} \mathcal{D} \psi_{-} e^{i S[\psi_{+}, \psi_{-}]} ,$$

i.e., the Schwinger-Keldysh formalism.

For the SYK type models discussed below, we will analyze the Schwinger-Keldysh path integral (8) in the large $N$ limit. Furthermore, in this work, we will be interested in stationary properties that may emerge in the late time limit. In particular, we will assume in this limit that the memory of the initial state is lost, and the system relaxes into a stationary state independent of the initial state.

III. NON-RANDOM LINEAR DISSIPATOR

In this section, we consider the SYK model in the presence of the jump operators

$$\mathcal{L} = \sqrt{\mu} \psi_{\pm}^{t}, \quad i = 1, \cdots, N. \quad (9)$$

Here, we assume $\mu$ is a real parameter. Following the procedure outlined in the previous section, the Lindbladian acting on the doubled Hilbert space $H_+ \otimes H_-$ is given by

$$\mathcal{L} = -i H_{\text{SYK}}^{+} \otimes \mathbb{1}_{-} + i(-1)^{\frac{q}{2}} \mathbb{1}_{+} \otimes H_{\text{SYK}}^{-} - i \mu \sum_{i} \psi_{+}^{t} \psi_{-}^{t} - \mu N \mathbb{1}_{+} \otimes \mathbb{1}_{-} . \quad (10)$$

At least superficially, this model looks similar to the two-coupled SYK model discussed in [21]. We however note various differences. First, the relative phase between the $H_{\text{SYK}}^{+}$ and $H_{\text{SYK}}^{-}$ terms. For example, when $q = 4$, we have the opposite signs for these terms. The relative sign between the terms is necessary so that their sum is an isometry of $|\rho\rangle$. On the other hand, for the regular two-coupled SYK model, these terms have the same sign, and induces
time evolution. Another difference is that the Hamiltonian terms (the first two terms) are anti-hermitian whereas $-i\mu \sum_i \psi^+_i \psi^+_i$ is hermitian. Overall, $\mathcal{L}$ is not anti-hermitian ($\mathcal{L}^\dagger \neq -\mathcal{L}$) and evolution is non-unitary.

### A. Path integral and large $N$ effective action

Using the formalism in the previous section, we can study this model using the Schwinger-Keldysh path integral. The action is given by

$$iS[\psi_+, \psi_-] = \int_{t_i}^{t_f} dt \left[ -\frac{1}{2} \sum_i \psi^+_i \partial_t \psi^+_i - \frac{1}{2} \sum_i \psi^+_i \partial_t \psi^+_i - i^{q+1} \sum_{t_1 < \cdots < t_q} J_{1\cdots q} \psi^+_1 \cdots \psi^+_q + i^{q+1} \sum_{t_1 < \cdots < t_q} J_{1\cdots q} \psi^+_1 \cdots \psi^+_q - i\mu \sum_i \psi^+_i(t)\psi^-_i(t) - \mu N \int dt \right].$$

(11)

The action has to be supplemented with the proper boundary conditions at $t = t_i, t_f$, set by the initial ($|\rho_0\rangle$) and final ($|\rho\rangle$) states. When analyzing the stationary state, however, the boundary conditions are immaterial. This path integral can be studied in the large $N$ limit as in the regular SYK model. We introduce two kinds of matrix collective fields, $G_{\alpha\beta}(t_1, t_2)$ and $\Sigma_{\alpha\beta}(t_1, t_2)$, where $\alpha, \beta = \pm$. The effective action for the collective fields is

$$S[G, \Sigma] = -\frac{iN}{2} \log \text{Tr} [-i(G_0^{-1} - \Sigma)] + \frac{i^{q+1}J^2N}{2q} \int_{t_i}^{t_f} dt_1 dt_2 \sum_{\alpha\beta} s_{\alpha\beta} G_{\alpha\beta}(t_1, t_2)^q$$

$$+ \frac{iN}{2} \int_{t_i}^{t_f} dt_1 dt_2 \sum_{\alpha\beta} \Sigma_{\alpha\beta}(t_1, t_2) G_{\alpha\beta}(t_1, t_2) - i\mu \int_{t_i}^{t_f} dt [G_{++}(t, t) - G_{--}(t, t)] + i\mu N \int dt,$$

where $s_{\alpha\beta}$ is given by

$$s_{++} = s_{--} = 1, \quad s_{+-} = s_{-+} = -1/2.$$

(13)

In the saddle point approximation, the collective field $G_{\alpha\beta}$ is nothing but the Green’s functions of the fermion fields,

$$G_{\alpha\beta}(t_1, t_2) = -i \langle T(\psi_\alpha(t_1)\psi_\beta(t_2)) \rangle.$$

(14)

The correlation functions satisfy the symmetry relation $G_{\alpha\beta}(t_1, t_2) = -G_{\beta\alpha}(t_2, t_1)$. The partition function of the system in terms of the collective fields is $Z = \int \mathcal{D}G_{\alpha\beta} \mathcal{D}\Sigma_{\alpha\beta} \exp \{iS[G, \Sigma] \}$. The large $N$ saddle point equation is

$$i\partial_{t_1} G_{\alpha\beta}(t_1, t_2) - \int dt_3 \sum_{\gamma=+,-} \Sigma_{\alpha\gamma}(t_1, t_3) G_{\gamma\beta}(t_3, t_2) = \delta_{\alpha\beta} \delta(t_1 - t_2),$$

(15)

$$\Sigma_{\alpha\beta}(t_1, t_2) = -i^q J^2 s_{\alpha\beta} G_{\alpha\beta}(t_1, t_2)^q - \mu g_{\alpha\beta} \delta(t_1 - t_2).$$

(16)

### B. Stationary Green’s functions

#### a. Large $N$ limit with $q = 4$

The saddle point equation can be analyzed numerically, or by taking the large $q$ limit. We first take $q = 4$ and solve the Kadanoff-Baym equations (15) and (16) numerically. We note that assuming the memory of the initial state is lost in the long time limit, the time translation invariance is recovered and the collective fields depend only $t_1 - t_2 \equiv t$. In Fig. 1, we show an example of the numerical stationary solution for $J = 1$ and $\mu = 0.250$. For large enough $\mu \gg J$, the system crosses over to the case of dissipation only model, where the correlation function decays exponentially with the decay rate given by $\Gamma = \mu$. In Fig. 1, we plot the numerically determined decay rate as a function of $\mu$.

#### b. Large $q$ limit

In the large large $q$ limit, we expand the Green’s function as [4]

$$G_{\alpha\beta}(t_1, t_2) = G^0_{\alpha\beta}(t_1, t_2) \left( 1 + \frac{1}{q} g_{\alpha\beta}(t_1, t_2) + \cdots \right).$$

(17)

The Kadanoff-Baym equation then reduces to the Liouville equation

$$\partial_{t_1} \partial_{t_2} g_{++}(t_1, t_2) = -2J^2 e^{g++(t_1, t_2)},$$

$$\partial_{t_1} \partial_{t_2} g_{--}(t_1, t_2) = -2J^2 e^{g--(t_1, t_2)} - 2\mu \delta(t_1 - t_2).$$

(18)
By solving these conditions, we obtain

\[ \alpha = \hat{\alpha} = \frac{J}{2} \sqrt{\left(\frac{\mu}{2J}\right)^2 + 1}, \quad \gamma = \tilde{\gamma} = \text{arcsinh}(\frac{\mu}{2J}). \tag{22} \]

From these, we see that the correlation functions behave as \( G(t) \sim e^{\frac{\mu t}{q}} \) and decay exponentially. We can read off \( \frac{\mu}{q} \equiv \Gamma \) as the decay rate. This behavior also qualitatively agrees with the \( \mu \) dependence for the \( q = 4 \) case above analyzed numerically. Also, for large \( q \), we can confirm that as \( \mu \to 0 \), (after taking the long time limit) the Green’s function reduces to the infinite temperature thermal Green’s function.

C. Finite \( N \) spectrum

We now turn to the spectral properties of the SYK Lindbladian (10). The complex spectrum \( \{\lambda_i\} \) of the SYK Lindbladian (10) can be studied by numerical exact diagonalization for finite \( N \). We take \( N = 8 \) in our analysis below, which means, including both copies \( \psi_a^\dagger \) and \( \psi_a \), we have \( 2N = 16 \) flavors of Majorana fermion operators. Plotted in Fig. 2 are the numerical spectra \( \{\lambda_i\} \) for representative choices of \( \mu \) (we set \( J = 1 \)). We take 100 disorder realizations for each \( \mu \). For small \( \mu \), there are many eigenvalues centered around \( \text{Re} \lambda = -N\mu/2 \). As we increase \( \mu \), vertical bands of eigenvalues start forming along the real axis. Each band is located roughly along a line \( \text{Re} \lambda = -n\mu \) for \( n = 1, \ldots, N \). As we increase \( \mu \) even further, all the eigenvalues become close to real. This reminds us of a real-complex transition in some non-Hermitian systems [39]. Another effect of increasing \( \mu \) is the formation of clusters around \( \lambda = -n\mu \), with gaps in between. The cluster formation first occurs at the left and right edges of the spectrum, i.e. at small and large \( n \), and then subsequently at intermediate values of \( n \). Similar band and cluster formation and hierarchy of relaxation times were observed in Ref. [29, 34], although we should note that these works studied purely dissipative Lindbladians, while in our model the randomness enters only in the Hamiltonian part.

IV. RANDOM QUADRATIC JUMP OPERATORS

In this section we consider another open SYK system. Here we introduce \( M \) two-body jump operators \( L^a \) with random couplings:

\[
H_{\text{SYK}} = (-i)^{q/2} \sum_{1 \leq i_1 < \cdots < i_q \leq N} J_{i_1 \cdots i_q} \psi_{i_1} \cdots \psi_{i_q},
\]

\[
L^a = \sum_{1 \leq i < j \leq N} K^a_{ij} \psi_i \psi_j, \quad K^a_{ij} \in \mathbb{C}, \quad a = 1, \cdots, M. \tag{23}
\]
FIG. 2: Spectrum of the SYK Lindbladian operator $L$ (10) for $\mu = 0.1, 0.3, 0.5$ and $0.9$ with $J = 1$.

All $K^a_{ij}$ are iid complex Gaussian random variables with mean and variance given by

$$\langle K^a_{ij} \rangle = 0, \quad \langle |K^a_{ij}|^2 \rangle = \frac{K^2}{N^2} \quad \forall i, j, a \quad \text{(no sum).} \quad (24)$$

The Lindbladian acting on the doubled Hilbert space $\mathcal{H}_+ \otimes \mathcal{H}_-$ is given by

$$L = -iH^\pm_{SYK} \otimes 1_- + i(-1)^{\frac{q}{2}} \otimes H^\pm_{SYK} - \sum_a L^a_+ \otimes L^a_+ - \frac{1}{2} \sum_a L^a_+ \otimes L^a_- - \frac{1}{2} \sum_a L^a_- \otimes L^a_+ - \frac{1}{2} \sum_a L^a_+ \otimes L^a_-$$

$$= -iH^\pm_{SYK} \otimes 1_- + i(-1)^{\frac{q}{2}} \otimes H^\pm_{SYK} + \sum_a \sum_{i<j} \sum_{k<l} K^a_{ij} K^a_{kl} \left( \psi^i_+ \psi^j_- \psi^k_+ \psi^l_- + \frac{1}{2} \psi^i_+ \psi^j_+ \psi^k_- \psi^l_- + \frac{1}{2} \psi^k_+ \psi^l_+ \psi^i_- \psi^j_- \right). \quad (25)$$

A. Large $N$: Schwinger-Dyson equations

Using the formalism of section II, we can obtain the Schwinger-Keldysh action for this model. Since we will be analyzing the stationary state, the initial state $|\rho_0\rangle$ is immaterial, and will be excluded from the path integral. We introduce complex Hubbard-Stratonovich (or auxiliary) fields $b^a_+ (t), b^-_+ (t)$, to make the dissipation term linear with respect to the jump operators. The resulting action is as follows:

$$iS[\psi_+, \psi_-, b^a_+, b^a_-, \bar{b}^a_+, \bar{b}^a_-]$$

$$= \int dt \left[ -\frac{1}{2} \sum_i \psi^+_i \partial_t \psi^+_i - \frac{1}{2} \sum_i \psi^-_i \partial_t \psi^-_i - i^{q+1} \sum_{i_1 < \cdots < i_q} J_{i_1 \cdots i_q} \psi^+_{i_1} \cdots \psi^+_{i_q} + i^{q+1} \sum_{i_1 < \cdots < i_q} J_{i_1 \cdots i_q} \psi^-_{i_1} \cdots \psi^-_{i_q} \right.$$  

$$- \frac{1}{2} \sum_a \left( t \bar{b}^a_+(t) \right) \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \left( \begin{array}{c} b^a_+(t) \\ b^a_- (t) \end{array} \right)$$

$$- \frac{1}{2} \sum_a \left( t \bar{b}^a_+(t) L^a_+ (t) + b^a_- (t) L^a_+ (t) + \bar{b}^a_+ (t) \bar{L}^a_+ (t) + \bar{b}^-_+ (t) \bar{L}^a_- (t) \right) \right]. \quad (26)$$

We then perform disorder averaging over the random couplings $J$ and $K$. Next, we introduce collective fields for both, the fermion fields and the auxiliary fields. We denote the fermion collective fields by $G_{\alpha \beta}$ and $\Sigma_{\alpha \beta}$, and the
auxiliary collective fields by \( G^b_{\alpha\beta} \) and \( \Sigma^b_{\alpha\beta} \), where \( \alpha, \beta = \pm \). Consider the limit \( N, M \to \infty \) with \( R = M/N \) constant. In this limit, the Green’s functions and self energies of the system are determined by the saddle point of the action. The saddle point equations are as follows:

\[
\Sigma^b_{\alpha\beta}(t_1, t_2) = \frac{K^2}{4} G^b_{\alpha\beta}(t_1, t_2)^2,
\]

\[
G^b(t_1, t_2) = \left[ \begin{array}{cc} 1 & 0 \\ -2 & 1 \end{array} \right] \delta(t_1 - t_2) - \Sigma^b(t_1, t_2) \right]^{-1},
\]

\[
\Sigma_{\alpha\beta}(t_1, t_2) = -i \gamma J^2 s_{\alpha\beta} G_{\alpha\beta}(t_1, t_2) \gamma^{-1} + \frac{K^2 R}{8} (G^b_{\alpha\beta}(t_1, t_2) + G^b_{\beta\alpha}(t_2, t_1)) G_{\alpha\beta}(t_1, t_2),
\]

\[
G(t_1, t_2) = [G^b_0(t_1, t_2) - \Sigma(t_1, t_2)]^{-1}.
\]

The boldface fields are \( 2 \times 2 \) matrices with \( \pm \) indices. The matrix inverses are with respect to this \( 2 \times 2 \) matrix multiplication as well as the time domain multiplication. Now let us apply the stationary state hypothesis to obtain the Schwinger-Dyson equations:

\[
\Sigma^b_{\alpha\beta}(t) = \frac{K^2}{4} G^b_{\alpha\beta}(t)^2,
\]

\[
G^b(\omega) = \left[ \begin{array}{cc} 1 & 0 \\ -2 & 1 \end{array} \right] - \Sigma^b(\omega) \right]^{-1},
\]

\[
\Sigma_{\alpha\beta}(t) = -i \gamma J^2 s_{\alpha\beta} G_{\alpha\beta}(t) \gamma^{-1} + \frac{K^2 R}{8} (G^b_{\alpha\beta}(t) + G^b_{\beta\alpha}(-t)) G_{\alpha\beta}(t),
\]

\[
G(\omega) = (G^b_0(\omega) - \Sigma(\omega))^{-1}.
\]

We solve these equations numerically for \( q = 4 \) and various values of the parameters \( J, K, \) and \( R \). For all these solutions, the Green’s functions of the Hubbard-Stratanovich fields are numerically consistent with the following trivial solution:

\[
G^b(t) = \left[ \begin{array}{cc} 1 & 0 \\ 2 & 1 \end{array} \right] \delta(t).
\]

In all cases, the fermion Green’s functions decay exponentially at late times. For small dissipation strength, the Green’s functions oscillate as they decay. To characterize these oscillation we try to fit the retarded Green’s function \( G^R(t) = -i \Theta(t)(G_{++}(t) - G_{--}(t)) \) with the following ansatz at late times.

\[
G^R(t) \approx A e^{-\Gamma t} \sin(\omega_0 t + \phi).
\]

Figure 3 shows the late time decay rate \( \Gamma \) and frequency \( \omega_0 \) of the retarded Green’s function, obtained by fitting this ansatz to the numerical solutions. Here we have fixed \( J = 1 \) and \( R = 1 \).

In the frequency domain, a useful quantity to analyze is the spectral function defined by

\[
A(\omega) = -2 \text{ Im} \left( G^R(\omega) \right).
\]

The spectral function can be interpreted as a probability distribution. Indeed, our numerical solutions satisfy the normalization condition \( \int_{-\infty}^{\infty} d\omega \frac{d\omega}{2\pi} A(\omega) = 1 \). We compare the spectral function to a Lorentzian distribution. Figure 4 demonstrates that for large dissipation strength \( K \),

\[
\Gamma \chi
\]

\[
\omega_0
\]

\[
K
\]

\[
A(\omega) \text{ is well approximated by a Lorentzian. We see the same effect when we vary the number of jump operators: at large } R, \text{ the spectral function is well described by a Lorentzian.}
\]

### B. Finite N: Spectrum of Lindbladian

The steady state Green’s functions do not contain information about the dynamics of the system. The time evolution of open quantum systems, in general, is completely determined by the eigenvalues and eigenvectors of the Lindbladian [40]. Therefore, here we analyze the spectrum to help us characterize the dynamics of the system. We set \( N = 10 \), which gives a total of 20 Majorana fields after the doubling described in section II. For each set of parameters, we collect 50 realizations of the random Lindbladian to plot the spectrum.
Recall that for large $K$ or large $R$, the steady state spectral functions are both close to Lorentzian, but evidently, the spectra are very different in the two cases.

**V. SUMMARY AND OUTLOOK**

In this work, we introduced SYK type Lindbladian models and studied their Green’s functions in the long time limit, and their spectral properties. The models admit exact analysis in various limits (large $N$, large $q$, simultaneous large $N$ and $M$ limits). Another merit of the models is that they exhibit very rich behaviors. In particular, the second model realizes many different behaviors by simply controlling the parameters $J$, $K$, and $R$, some of which can be well compared with different random Lindbladian models studied previously. There are many remaining questions. We close by listing a few of them. First, vast generalizations of the current models are possible, for example, by introducing $p$-body jump operators. Studying wider classes of models would allow us to explore different universal behaviors in open quantum many-body systems. Second, while we studied the distribution of the eigenvalues of the SYK Lindbladians, a more thorough characterization of the spectral properties is necessary. For example, it is of great interest to study the level statistics [20, 33, 41, 42]. It may show an interesting crossover as the distribution crosses over from the lemon shape to the one with many clusters [43]. Third, in this work, we mostly focused on stationary properties. However, it would be interesting to follow the time evolution by the SYK Lindbladians starting from some initial state. Technically, the Kadanoff-Baym equation can be solved numerically.

*Note added:* While finalizing the manuscript, [44] appeared on arXiv, which has a substantial overlap with our section IV.

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FIG. 5: Spectrum of the Lindbladian (25) for $J = R = 1$ and varying $K$. (a) The spectrum is elliptical, consistent with literature. (b,c) The spectrum is scaled and shifted according to equation (31) before plotting. As $K$ increases, the boundary of this (scaled) spectrum resembles the lemon-shaped contour derived in [27].

FIG. 6: Spectrum of the Lindbladian (25) for $J = K = 1$ and varying $R$.

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