Realistic shortcuts to adiabaticity in optical transfer

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Abstract
Shortcuts to adiabaticity are techniques allowing rapid variation of the system Hamiltonian without inducing excess heating. Fast optical transfer of atoms between different locations is an important application of shortcuts to adiabaticity. We show that the common boundary conditions on the atomic position, which are imposed to find the driving trajectory, lead to highly non-practical boundary conditions for the optical trap. Our experimental results demonstrate that, as a result, previously suggested trajectories are likely to fall short of the expectation. We develop two complementary methods that solve this boundary conditions problem by adding more degrees of freedom to the trajectory parameter space. In the first method, this is achieved by the addition of a spectral component at the trapping frequency, while in the second we use a polynomial trajectory of an order high enough to account for the new boundary conditions. We experimentally demonstrate that this approach allows us to construct highly non-adiabatic movements with no residual sloshing. Our techniques can also account for non-harmonic terms in the confining potential.

1. Introduction

Future quantum technologies will require deep understanding and exquisite control of the underlying system dynamics. The Hamiltonian would have to be changed as fast as possible while still reaching a specific target state. However, rapid variation of the Hamiltonian might introduce undesired excitations and heating. **Shortcuts to adiabaticity (STA)** refers to a class of solutions to this problem, all based on cleverly tailoring the fast driving protocol such that the system reaches a desired final state which is adiabatically connected to the initial one, but without adhering to the adiabatic condition [1]. It can be roughly divided into two subcategories: **counterdiabatic** and **invariant-based** drivings. Counterdiabatic methods continuously suppress excitations by utilizing a time-dependent auxiliary term in the Hamiltonian, and are therefore harder to implement experimentally. Invariant-based methods, on which we focus in this work, take the system back to the adiabatic path only at the end of the process by engineering the trajectory to satisfy the boundary conditions at initial and final times and, in general, the Ermakov equation in between [2, 3]. Among the many fields who can benefit from STA are quantum computation [4–7], quantum control [8–11], quantum thermodynamics [12, 13] and transport [14–20] or manipulation [21–25] of ions and ultracold neutral atoms.

The rich toolbox available in ultracold atomic experiments makes these systems particularly attractive as platforms for quantum simulation and quantum computation [26–28]. In particular, the interaction of atoms with a far-off-resonance laser light introduces negligible dissipation and can be effectively described as a conservative potential for the atoms [29–31], with potential depth that is proportional to the laser intensity. Ultracold atoms can be trapped in the vicinity of potential minimum, which for the lowest energy optical mode, namely a Gaussian beam, is at the focal point (‘waist’). Rapid changes may be desired in the trap shape [12, 21, 22, 24, 32] or position [15, 16, 18, 19, 32–34]. Optical transfer of ultracold atoms can be used to move atoms between different sites or implement quantum gates [35–37]. Shorter transfer duration is advantageous considering the finite coherence time of any experimental system, but at the same time, it might have an adverse effect on the fidelity of single operations [37]. STA techniques applied to optical transfer can alleviate this conflict.
In a practical implementation of invariant-based STA driving, a problem arises: the boundary conditions are given for the atomic trajectory while the experimental control is over the position of the trap, which in turn is derived from the positions of the atoms through the equation of motion. This leads to boundary conditions for the trap which may be very difficult to satisfy in reality. For example, it may require the trap and atoms to start at the same position, with the trap having an initial velocity while the atoms are at rest. Here we study experimentally non-adiabatic optical transfer with ultracold atoms and find that indeed this issue almost always harms the performance of known STA based trajectories. We show that in order to lift conflicting boundary conditions, it is necessary to increase the number of degrees of freedom in the trajectory. We describe and demonstrate two complementary approaches to achieve this: one, by addition of a ‘correction’ spectral component at the trapping frequency with a specific phase and amplitude. This method can be applied to any existing trajectory. Two, by using a polynomial trajectory with high enough order and incorporating the boundary conditions by a correct choice of the coefficients.

The structure of this paper is as follows: in section 2 we briefly introduce the STA formalism, the required boundary conditions and the use of the final sloshing mode as a measure of undesired excitations added by the movement. In section 3, we introduce the apparatus and describe the experimental sequence and probing technique. We then present in section 4 our experimental study of several known STA based trajectories for optical transfer. Our results show that almost always the movement results in considerable heating. This leads us in section 5 to develop and demonstrate two new methods to construct proper STA trajectories. We conclude and give our outlook in section 6.

2. Shortcut to adiabaticity in transport problems

The optical dipole potential induced by a Gaussian beam can be written as [29]:

\[
U(z, r) = -\frac{U_0}{1 + \left(\frac{z - z_d}{z_R}\right)^2} \exp\left[\frac{-2r^2/\sigma^2}{1 + \left(\frac{z - z_d}{z_R}\right)^2}\right],
\]

where \(z\) denotes the atoms coordinate in the laboratory frame of reference and \(z_d\) the position of the potential minimum, both of which will become time-dependent, \(r\) is the radial distance from the beam path, \(U_0\) is the maximal potential depth, and \(z_R = \pi\sigma^2/\lambda\) is the Rayleigh range, with \(\sigma\) the waist radius and \(\lambda\) the wavelength. Furthermore, gravity acts in one of the radial directions, but we disregard its effect since it is negligible for our experimental parameters.

In our experiment we induce motion along the axial direction and in addition the radial trapping frequency, \(\omega_{rr}\), is much higher than the axial one, \(\omega_{zz}/\omega_{rr} \approx 90\). Hence, we average out the radial motion and retain only the axial dependence in the Hamiltonian. This averaging is done over a period of \(\omega_{rr}^{-1}\) for which the axial movement is negligible. At low temperatures the atoms lie very close to the potential minimum and \(\langle r^2 \rangle / \sigma^2 \ll 1\), where \(\langle \cdot \rangle\) denotes the ensemble average. Thus, we expand the potential in powers of \(z - z_d\), and \(r\), then average over \(r\) and keep terms up to sixth order, which leads to the following potential (up to a constant):

\[
U = \frac{1}{2} m \omega_0^2 (z - z_d)^2 \left\{ 1 - 4 \frac{\langle r^2 \rangle}{\sigma^2} + 6 \frac{\langle r^2 \rangle^2}{\sigma^4} - \frac{(z - z_d)^2}{z_R^2} \left[ 1 - 6 \frac{\langle r^2 \rangle}{\sigma^2} + \frac{(z - z_d)^4}{z_R^4} \right] \right\},
\]

where \(\omega_0 = (2U_0/mz_R^2)^{1/2}\) is the axial harmonic trapping frequency.

Since the underlying motion in \(z\) is still oscillatory, we substitute a sinusoidal solution in \(z - z_d\) with a frequency \(\omega_0^2\) into the equation of motion. After omitting high-frequency terms, we arrive at the following effective Hamiltonian:

\[
H(t) = \frac{m}{2} \dot{z}^2(t) + \frac{m}{2} \omega_0^2 (z(t) - z_d(t))^2,
\]

with the effective frequency \(\omega_0:\)

\[
\omega_0^2 = \omega_0^2 \left[ 1 - 4 \frac{\langle r^2 \rangle}{\sigma^2} + 6 \frac{\langle r^2 \rangle^2}{\sigma^4} - 3 \frac{(z_d)^2}{z_R^2} \left( 1 - 6 \frac{\langle r^2 \rangle}{\sigma^2} + \frac{15(z_d)^4}{2z_R^4} \right) \right].
\]

The axial and radial variances depend on temperature, number of atoms and quantum statistics. In section 5.1.2 we employ their measured values to compute the frequency shift relative to \(\omega_0\). Somewhat surprisingly, we have found experimentally that in the effective frequency expression the second order terms need to be retained in order to account for the data. In what follows, when we discuss the harmonic Hamiltonian, we will denote \(\omega_0 \mapsto \omega_0^2\) for clarity of the expressions.
The Hamiltonian (3) can be written in operational form as:

\[ \hat{H}(t) = \frac{\hat{p}^2}{2m} + \frac{m}{2}\omega_0^2(\hat{q} - z_\omega(t))^2. \]  

(5)

Here \( z(t) = \langle \hat{q} \rangle \) denotes the classical trajectory of a particle evolving under the trap driving trajectory \( z_\omega(t) \).

We impose the following boundary conditions on the atoms motion:

\[
\begin{align*}
    z(0) &= 0, & \dot{z}(0) &= 0, & \ddot{z}(0) &= 0, \\
    z(t_f) &= d, & \dot{z}(t_f) &= 0, & \ddot{z}(t_f) &= 0.
\end{align*}
\]

(6a) (6b)

That is, the atomic movement of \( d \) meters during \( t_f \) seconds begins and ends with zero velocity and acceleration.

As was shown by Lewis and Riesenfeld [2, 16], with these boundary conditions, and for a general class of Hamiltonians that includes the one in (3), there exists a dynamical invariant \( \hat{I}(t) \) that satisfies:

\[ \frac{d}{dt} \hat{I}(t) = \frac{\partial}{\partial t} \hat{I}(t) - \frac{i}{\hbar} [\hat{I}(t), \hat{H}(t)] = 0. \]

(7)

For the rigid harmonic case of constant trapping frequency this invariant is given by:

\[ \hat{I}(t) = m \left( \frac{\hat{p}^2}{2m} - \dot{z}(t) \right) + \frac{m}{2}\omega_0^2(\hat{q} - z(t))^2. \]

(8)

The third equalities in (6a) and in (6b) guarantee that \( [\hat{I}(0), \hat{H}(0)] = [\hat{I}(t_f), \hat{H}(t_f)] = 0 \). The state which we would like to conserve can therefore be written at \( t = 0 \) as a superposition of \( \hat{I} \) eigenstates, and since \( \hat{I} \) is invariant under the motion, their state at \( t_f \) is the same as at \( t = 0 \). Hence, under the harmonic approximation, a finite trajectory \( z(t) \) that respects the boundary conditions (6a)-(6b) is a proper STA that will end in an adiabatically connected state. Devising an STA trajectory amounts to finding a non-adiabatic path that satisfies (6a)-(6b). Several such trajectories were suggested in the literature [16, 18, 19, 33, 38-42] and implemented experimentally [14, 15, 20].

However, the boundary conditions (6a)-(6b) are given for the atomic position, while experimentally the control is over the trap position. The latter can be derived from the former using the equation of motion:

\[ m\ddot{z}(t) = -m\omega_0^2[z(t) - z_\omega(t)] \quad \Rightarrow \quad z_\omega(t) = z(t) + \ddot{z}(t)/\omega_0^2. \]

(9)

The boundary conditions (6a) imply that \( z_\omega(0) = 0 \) and \( \dot{z}_\omega(0) = \ddot{z}(0)/\omega_0^2 \). Similarly, we get that \( z_\omega(t_f) = d \) and \( \dot{z}_\omega(t_f) = \ddot{z}(t_f)/\omega_0^2 \). Since \( \ddot{z}(0) \) and \( \ddot{z}(t_f) \) are generally not zero, they enforce initial and final non-zero trap velocities. The first condition is very hard to implement in most physical realizations, as it requires that the atoms are initially at rest and that their position coincides with the trap minimum but there is a finite initial velocity to the trap. In most cases, both the trap and the atoms are at rest before the transport commences. The second condition means that even if the atoms are brought to the adiabatically connected state at the end of the motion, since the trap is still moving it will soon induce atoms excitation. Evidently, we need to require that the trap will also be at rest at the beginning and end of the transport. This implies:

\[
\begin{align*}
    z_\omega(0) &= 0, & \dot{z}_\omega(0) &= d, & \ddot{z}_\omega(0) &= 0, \\
    \dot{z}_\omega(t_f) &= 0, & \ddot{z}_\omega(t_f) &= 0.
\end{align*}
\]

(10)

Most STA based trajectories do not generally satisfy this extra set of boundary conditions for \( \ddot{z}(t) \) and, as we later demonstrate, their experimental implementation might leave excess energy in the cloud.

The only motion excited mode in a harmonic potential is the center-of-mass oscillation (sloshing mode). Non-harmonic terms in the Hamiltonian and interactions between the atoms couple between the sloshing mode and higher order modes which eventually lead to an increase of temperature. Hence, the sloshing mode and temperature are the observables one is required to measure when characterizing realistic implementations of STA in optical transport. As an example, we present in figure 1 a series of absorption images of atomic clouds taken during and following a non-adiabatic transport. The upper panel presents the result of a trajectory originally suggested in [16]. Considerable sloshing is apparent after the end of the trap motion. This is because the trajectory does not respect the third boundary condition given in (10). In contrast, the lower panel shows the same trajectory with our spectral correction method applied. The final state shows no detectable sloshing and minimal increase in temperature.

In the harmonic case, the delta-impulse response is given by a sinusoidal function [43]. The terminal position can be calculated by integrating over this response function times the instantaneous acceleration. Time differentiation of this position gives the ultimate velocity. The sloshing amplitude \( A \) encapsulates the energy stored in this mode, which can be expressed in terms of the position and velocity by:
The system is composed of three interconnected vacuum chambers. In the first chamber, a two-dimensional magneto-optical trap (MOT) [44] generates a stream of cold fermionic potassium atoms that fly through a narrow nozzle to the second chamber. There, the atoms are captured and cooled in a three-dimensional dark SPOT MOT [45] on the D$_2$ line and get farther cooled using gray molasses on the D$_1$ line [46]. Then, we optically pump the atoms and load them into a QUIC magnetic trap [47], where we perform forced microwave evaporation. Next, around $25 \times 10^6$ atoms at $T/T_F \approx 4.5$, with $T_F$ the Fermi temperature, are loaded into a far-off-resonance optical dipole trap of $\lambda = 1064$ nm. This trap is made of a single Gaussian beam, with a waist of 39 $\mu$m and a power of about 2.5 W. The atoms are then transported [48] in approximately a second to the third chamber. This is done by moving a single lens which is a part of an optical relay system that creates the optical trap. The actual movement is performed with an air-bearing translation stage, to reduce vibrations and heating of the ensemble, as depicted in figure 2. Upon arrival at the third chamber, the waist of the optical trap is 19.45 $\mu$m. Forced optical evaporation concludes the preparation stage. In this evaporation we decrease the laser power to some minimal value and then ramp it up back to its final value. By this, we increase the ratio between the trap depth and temperature which effectively narrows the atomic cloud extent with respect to the waist and Rayleigh range. At this final point, the laser power is typically 39 mW resulting in a heating rate of about 7 nK s$^{-1}$. The conditions for the experiments reported here are $N \approx 400,000$ atoms at a temperature of $T \approx 300$ nK. The atoms are in a balanced mixture of the $m = -9/2, -7/2$ Zeeman states in the $F = 9/2$ hyperfine level, which in the applied uniform magnetic field of $\sim 185$ G are weakly interacting with an $s$-wave scattering length of $\sim 236 \ a_0$, with $a_0$ being the Bohr radius. Typical trap oscillation frequencies are $\omega_z = 2\pi \times 646(7)$ Hz and $\omega_0 = 2\pi \times 7.16(15)$ Hz in the radial and axial directions, respectively. The typical trap Fermi energy is $E_F \approx \hbar \times 15$ kHz.

\[ A(t_f) \equiv \left\{ |z(t_f) - d|^2 + \frac{z(t_f)^2}{\omega_0^2} \right\}^{1/2} = \left| \int_0^{t_f} \exp(-i\omega_0 t) z(t) \, dt \right| . \]  

(11)

Where here we also used (10) to replace the integration along the trap acceleration to a one over its velocity. This is merely the Fourier component at the trap frequency of the trap velocity trajectory. For realistic trap trajectories that maintain (10) and (6a), the conditions in (6b) translate into zero ultimate sloshing amplitude. This can be used as a guideline for constructing STA driving trajectories: they should result in zero final sloshing.

3. The experimental apparatus

Figure 1. Sequences of absorption images capturing a non-adiabatic transport originally suggested in [16](a) and a corrected version of the same transport (b). The significant sloshing mode exhibited in (a) is suppressed in (b) and by this the associated excess energy is avoided. Each panel is composed of 44 atomic density distributions captured by absorption imaging at different times along and after the translation sequence. Darker shade stands for higher density. The blue curves denote the trap position and the red dashed vertical lines mark the motion end. The trap in (a) was driven along a polynomial trajectory (12) of 1.29 mm transport during $t_f = 186$ ms in a trap of axial frequency $\omega_0 = 2\pi \cdot 7.16(15)$ Hz. In (b) the suggested harmonic spectral correction (13) was implemented with $\omega_0 = 2\pi \cdot 7.11$ Hz, $\delta_0 = 279^\circ$ and $A_0 = 105$ $\mu$m, resulting in 2(2) $\mu$m sloshing amplitude compared to 206(4) $\mu$m in (a).
After the cloud reaches equilibrium conditions with a negligible sloshing of a sub-micron amplitude, we execute the driving protocol by moving again the lens mounted on the air-bearing stage. The stage specifications guarantee that the position of the trap minimum is accurate to within $1 \mu m$, and the trapping frequencies are constant to within $1.5\%$ during the movement. At each point along the translation we can stop and record the sloshing mode. This is done by waiting for some duration, then abruptly shutting off the trap, letting the atoms expand ballistically and then recording the atomic density distribution using absorption imaging. From these images we can extract the number of atoms, the center-of-mass position and $\theta$. The sloshing mode can be reconstructed by fitting a series of such images taken at different waiting times (see figure 1) with a decaying sine.

$A \sqrt{(1 - \frac{2}{n^2})} + (\omega_0 t)^2 \sin \left[ \frac{\omega_0 t}{1 - \frac{2}{n^2}} + \arctan \left( \frac{\omega_0}{1 - \frac{2}{n^2}} \right) \right] e^{-t/\tau},$ with a sloshing amplitude $A = 206(4) \mu m$, decay time $\tau = 175(6) \text{ ms}$, $\omega_0 = 2\pi \cdot 7.08(3) \text{ Hz}$ and $\phi = 133.0(8)^\circ$. The width of the blue curve designates the extracted sloshing amplitude 68% certainty ($1\sigma$) that will be presented as the amplitude error in following graphs, and the width of its green background represents the error in the other fitting parameters.

Figure 3. Center-of-mass position of the atomic cloud after a ballistic expansion time of $t_e = 12 \text{ ms}$ versus the waiting time after the trap has stopped. Red errorbars are the standard deviation between three independent measurements. The blue curve is a fit to a decaying sine: $A \sqrt{(1 - \frac{2}{n^2})} + (\omega_0 t)^2 \sin \left[ \frac{\omega_0 t}{1 - \frac{2}{n^2}} + \arctan \left( \frac{\omega_0}{1 - \frac{2}{n^2}} \right) \right] e^{-t/\tau},$ with a sloshing amplitude $A = 206(4) \mu m$, decay time $\tau = 175(6) \text{ ms}$, $\omega_0 = 2\pi \cdot 7.08(3) \text{ Hz}$ and $\phi = 133.0(8)^\circ$. The width of the blue curve designates the extracted sloshing amplitude 68% certainty ($1\sigma$) that will be presented as the amplitude error in following graphs, and the width of its green background represents the error in the other fitting parameters.

Figure 2. Schematic of the experimental system. The Gaussian focal point created by the translatable lens is relayed by the optical relaying system, to transport the atoms held in the focal point inside the ultra-high vacuum chamber. The lens is moved by Aerotech’s ABL1500 air bearing stage, specified to sub-micron accuracy and repeatability.

4. Experimental test of three non-adiabatic trajectories

We have implemented three non-adiabatic trajectories that maintain (6a) and (10) but not necessarily (6b). Their velocity profiles are depicted in the insets of figure 4. The first trajectory is a Sine with a velocity profile given by $z(t) = \frac{z_n}{2\pi} \sin \left( \frac{\pi t}{T} \right)$. It satisfies (6b) only for $T \cdot f_0 = \frac{\pi}{2} + n$ where $f_0 = \omega_0 / 2\pi$ and $n$ is an integer $n \geq 1$. The second trajectory has a Triangular velocity profile with a constant acceleration and deceleration given by $4d / T_2^2$ [15]. It satisfies (6b) only for $T \cdot f_0 = 2 - n$ with integer $n \geq 1$. The third trajectory is a Polynomial, which generally can be written for the atomic position as $z(t) = d \sum_{n=1}^{\infty} a_n \left( \frac{t}{T} \right)^n$. The lowest order polynomial to respect the boundary conditions in (6a)-(6b) is given by [16]:

\[ z(t) = d \left( \frac{t}{T} \right)^3. \]
For a trajectory to be realistic and flexible non-adiabatic trajectories.

5.1. Method I: spectral correction

5.1.1. Harmonic potential

The idea in this method is to introduce a new spectral component to an existing trajectory which maintains all required boundary conditions and the sloshing amplitude at this frequency such that their sum is zero. This method is very general as it does not assume anything regarding the original trajectory other than it satisfies all eight boundary conditions. In principle, for an ideal harmonic potential these trajectories can be calculated numerically, and the resulting sloshing amplitudes as given by (11). There is a satisfactory agreement between the experiment and theoretical calculations. As expected, zero sloshing is obtained only at specific $t_f \cdot f_0$ values. This places a strict constraint on potential applications of STA. Moreover, it puts a lower limit on the duration of the trajectory which is on the order of $f_0^{-1}$. In what follows we develop two methods to construct non-adiabatic trajectories that satisfy all eight boundary conditions. In principle, for an ideal harmonic potential these trajectories can be constructed for any desired duration above the fundamental limits [49]. A more practical limit on the shortest possible trajectory stems from the finite trap depth: a faster movement coherently drives the population during the motion via higher energy levels.

5.2. Method II: transport duration

For a trajectory to be realistic we require that both the trap and atoms will start and end at rest. As we explained earlier, this implies boundary conditions (6a),(6b) and (10). For a trajectory to be flexible, we require that it could be constructed for a wide range of distances $d$ and durations $t_f$.

For a trajectory to be realistic, we require that the trap position instead of the atomic trajectory then satisfies:

\[
\begin{align*}
\text{Sine} & : \quad \frac{z_{\text{es}}(t)}{d} = 10 \left( \frac{t}{t_f} \right)^3 - 15 \left( \frac{t}{t_f} \right)^4 + 6 \left( \frac{t}{t_f} \right)^5, \\
\text{Triangular} & : \quad \frac{z_{\text{es}}(t)}{d} = 10 \left( \frac{t}{t_f} \right)^3 - 15 \left( \frac{t}{t_f} \right)^4 + 6 \left( \frac{t}{t_f} \right)^5, \\
\text{Polynomial} & : \quad \frac{z_{\text{es}}(t)}{d} = 10 \left( \frac{t}{t_f} \right)^3 - 15 \left( \frac{t}{t_f} \right)^4 + 6 \left( \frac{t}{t_f} \right)^5.
\end{align*}
\]

In the experiment, we used the polynomial for the trap position instead of the atoms, thus satisfying (6b) and (10). However, similar to the other two trajectories, it satisfies (6b) only for a discrete set of points which can be calculated numerically, and the first four values are $t_f \cdot f_0 \approx 1.835, 2.895, 3.923, 4.938$.

Each of these trajectories was executed for a total movement of $d = 1.29$ mm. The resulting sloshing amplitudes $A(t_f)$ for three non-adiabatic durations $t_f$ are depicted in figure 4. For reference, we also plot the calculated sloshing amplitude as given by (11). There is a satisfactory agreement between the experiment and theoretical calculations. As expected, zero sloshing is obtained only at specific $t_f \cdot f_0$ values. This places a strict constraint on potential applications of STA. Moreover, it puts a lower limit on the duration of the trajectory which is on the order of $f_0^{-1}$. In what follows we develop two methods to construct non-adiabatic trajectories that satisfy all eight boundary conditions. In principle, for an ideal harmonic potential these trajectories can be constructed for any desired duration above the fundamental limits [49]. A more practical limit on the shortest possible trajectory stems from the finite trap depth: a faster movement coherently drives the population during the motion via higher energy levels.

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For a trajectory to be realistic, we require that the trap position instead of the atomic trajectory then satisfies:

\[
z(t) = d \left[ \frac{10}{(t/t_f)^3} - \frac{15}{(t/t_f)^4} + \frac{6}{(t/t_f)^5} \right],
\]
The boundary conditions (14a) are automatically fulfilled when there is no initial sloshing, as (6a). There are two more conditions that the atomic trajectory needs to fulfill given by (6b), namely, each of the additives in (14b) should vanish separately. The two degrees of freedom we have added, i.e. the correction amplitude (A₀) and phase (φ₀), are then used to suppress the amplitude in (11). This can be either done empirically by tuning the parameters and minimizing the resulting sloshing amplitude or numerically by solving the optimization problem of minimizing A(t_f) in (11). As an example, we plot in figure 5 the polynomial velocity trajectory of (12) with (dashed line) and without (solid line) our spectral correction. As can be clearly seen in the frequency domain, in the corrected trajectory the spectral component at ω₀ can be suppressed below any desired level.

We have experimentally tested our method with polynomial and sinusoidal trajectories. To obtain the minimal sloshing amplitude, we scanned the correction parameters around the calculated optimal values. The measured sloshing amplitudes (blue squares) and phases (gray circles) are plotted in figure 6 for the polynomial (upper panel) and sine (lower panel) trajectories. A clear minimum in the sloshing amplitude can be observed in both cases. At the optimal correction amplitude, the measured excess energy in the sloshing mode due to the non-adiabatic trajectory is consistent with zero. In contrast, the uncorrected trajectories (A₀ = 0) display a substantial sloshing. The atomic cloud dynamics in the non-corrected and corrected (A₀ = 105 μm) polynomial trajectories are presented in figures 1(a) and (b), respectively. When measuring the temperature after the center-of-mass motion has ceased, we do find an increase of about 200 nK for all non-adiabatic transports (corrected and uncorrected). This is probably due to high-frequency errors in the executed motion that couple through the anharmonic terms to higher vibrational modes of the cloud. We also measure an additional increase in temperature due to the excess energy in uncorrected trajectories compared to corrected ones.

Theoretical calculations shown as ribbons with matching colors in figure 6 agree reasonably well with the experiments at low sloshing amplitudes, but at higher values they deviate, probably due to contributions from non-harmonic terms in the potential. For both types of trajectories, the sloshing mode phase jumps sharply by π when crossing the optimal correction amplitude, as expected from an over-compensated driven harmonic oscillator. We find that the measured optimal correction phase deviates from the theoretical calculation by 23° and 41° for the polynomial and sinusoidal trajectories, respectively. This is most likely due to experimental imperfections in the execution of the trajectory, to which the phase is most sensitive. This exemplifies another advantage of our method: it can correct for experimental imperfections easily by parameters tuning.

Our results establish that the spectral correction technique is able to correct two different non-adiabatic driving trajectories. This exemplifies a major advantage of our method, namely that it can be applied to any given trajectory imposing relatively small changes. By this, it leaves a wide optimization playground for the motion parameters. For example, the trajectory can be first optimized for a specific goal (e.g., minimization of duration or displacement [39]), and only afterward corrected to suppress residual sloshing.
5.1.2. Anharmonic potential

Even in the anharmonic case, it is still true that the sloshing mode is the first to be excited from a rapid shift of the trap. Hence, nullifying the sloshing amplitude will provide us with a trajectory very close to optimum. In the case of a Gaussian beam, the frequency decreases with increasing temperature; an effect referred to as ‘softening’. A perturbative expansion of this shift in orders of the atomic spatial variances is given in (4). We therefore use the measured variances to compute the effect of anharmonicity on the oscillation frequency. We repeated the experiments in figure 6(a) with a similar number of atoms and with temperature increased by \( \times 1.6 \) using parametric excitation [29]. The axial in situ variance of the atomic cloud density \( \langle (z - z) \rangle^2 \) before the transport is about \( (229 \, \mu m)^2 \) and \( (272 \, \mu m)^2 \) for the cold and hot clouds, respectively. A good estimate for the radial variance \( \langle r^2 \rangle \) can be obtained using the known aspect ratio of the trapping frequencies in the axial and radial directions. Using this data together with (4), we calculate the ratio between the effective harmonic frequencies in the two experimental conditions and obtain \( \frac{\omega_{cold}}{\omega_{hot}} = 1.033 \). For this, we can numerically find a new optimum value for correction amplitude \( A_0 \). The correction phase \( \phi_0 \), however, is unaffected by this variation of the effective frequency.

**Figure 6.** Sloshing mode amplitude (blue squares) and phase (gray circles) versus the harmonic correction amplitude, after nonadiabatic movement with polynomial (a) and sinusoidal (b) trajectories. Theoretical harmonic calculations appear as ribbons with matching colors, where the trapping frequency uncertainty determines their width. These measurements were performed following transport of \( d = 1.29 \, \text{mm} \) lasting \( t_f = 186 \, \text{ms} \) in a trap of axial frequency \( \omega_0 = 2\pi \cdot 7.16(15) \, \text{Hz} \) (\( f_0 = t_f = 1.33(3) \)). Correction parameters for these measurements are \( \omega_0 = 2\pi \cdot 7.11 \, \text{Hz} \) frequency and optimal correction phase of \( \phi_0 = 279^\circ, 261^\circ \) for the polynomial and sinusoidal profiles, respectively. The expected correction phase, according to minimization of (11), is \( \phi_0 = 302^\circ \) for both cases.
While trying to implement previously suggested non-adiabatic trajectories in optical transfer, we have found that in most cases they leave excess energy in the cloud, which we measure as sloshing of the center-of-mass after the trap has stopped. We have identified the source for this behavior in a gap between the required boundary conditions and the effective harmonic trapping frequency decreases. The blue data is the same as in figure 6(a). The temperature and number of atoms are $T = 320(30)$ nK and $N = 390(20) \cdot 10^4$ for the blue squares, and $T = 510(70)$ nK and $N = 380(20) \cdot 10^4$ for the red triangles. Theoretical harmonic calculations appear as ribbons with matching colors, where the trapping frequency uncertainty determines their width. The sloshing measurements were performed following a 1.29 mm transport lasting $t_f = 186$ ms in a trap of axial frequency $\omega_0 = 2\pi \cdot 7.16(15)$ Hz and $\phi_h = 279^\circ$. The frequency shift for the red ribbon was calculated according to the measured extent of both atomic clouds in the trap before the motion with no fitting parameter, according to (4).

Figure 7. The effect of non-harmonicity in a non-adiabatic transfer. Shown are the sloshing mode amplitudes at $t_f$ versus the harmonic correction amplitude, $A_0$, for cooler (blue squares) and warmer (red triangles) atomic ensembles. As the temperature increases, the atoms experience more the non-harmonic terms and the effective harmonic trapping frequency decreases. The blue data is the same as in figure 6(a). The temperature and number of atoms are $T = 320(30)$ nK and $N = 390(20) \cdot 10^4$ for the blue squares, and $T = 510(70)$ nK and $N = 380(20) \cdot 10^4$ for the red triangles. Theoretical harmonic calculations appear as ribbons with matching colors, where the trapping frequency uncertainty determines their width. The sloshing measurements were performed following a 1.29 mm transport lasting $t_f = 186$ ms in a trap of axial frequency $\omega_0 = 2\pi \cdot 7.16(15)$ Hz and $\phi_h = 279^\circ$. The frequency shift for the red ribbon was calculated according to the measured extent of both atomic clouds in the trap before the motion with no fitting parameter, according to (4).

In figure 7 we present the measured sloshing amplitudes for both hot (red triangles) and cold (blue squares) clouds as a function of $A_0$, together with theoretical calculations based on $\omega_0$ and (11) (ribbons in matching colors). We find that using the same correction frequency and phase we get a clear minimum with respect to the correction amplitude also for the warmer case, correlative to the calculated amplitudes for $\omega_0 = 2\pi \cdot 7.16(15)$ Hz and $\omega_0 = 2\pi \cdot 6.94(16)$ Hz for the cooler and warmer cases, respectively, as calculated from the extents of the clouds before the experiment. In addition, the sloshing frequencies as obtained directly from the measured data following the transport agree with this shifted value of $\omega_0$. This demonstrates that the spectral correction technique can account for non-harmonic terms by using an effective trapping frequency.

5.2. Method II: septic polynomial trajectory

In the second method, in order to comply with all of the eight boundary conditions, we use a polynomial trajectory of the seventh order, written in terms of the normalized time as:

$$z_7(t) = d \left[ 35 \left( \frac{t}{t_f} \right)^4 - 84 \left( \frac{t}{t_f} \right)^5 + 70 \left( \frac{t}{t_f} \right)^6 - 20 \left( \frac{t}{t_f} \right)^7 \right]$$

This path for the atoms respects the invariant necessitated boundary conditions and its associated trap trajectory results with zero velocities at motion ends, so it is feasible to be implemented experimentally. The septic polynomial was also discussed by Tobalina et al in the context of launching atoms to a final velocity [34]. The trap trajectory is then given by (9), $z_7(t) = z_7(t) + \tilde{z}_7(t)/\omega^2_0$. Due to this dependency of the desirable path on the trapping frequency $\omega_0$, one is required to provide the later with great accuracy in order to respect the boundary conditions and accomplish the transport with zero sloshing amplitude. In figure 8 we present the sloshing amplitudes following such a trajectory where we scan the value of the frequency parameter. Indeed, for the correct value of the frequency parameter, we observe a sloshing amplitude consistent with zero. The right-hand side data points are in fact the case where we used the trajectory $z_7(t)$ directly for the trap itself, which results in considerable excess energy.

6. Discussion and outlook

While trying to implement previously suggested non-adiabatic trajectories in optical transfer, we have found that in most cases they leave excess energy in the cloud, which we measure as sloshing of the center-of-mass after the trap has stopped. We have identified the source for this behavior in a gap between the required boundary
conditions for the trap and realistic constraints. In reality, the trap holding the atoms is at rest before and after the movement. Formerly proposed STA trajectories, however, require the trap to have some initial and final velocity while the atoms are at rest. In principle, it is physically possible, for example by working with two precisely synchronized traps, one holding the atoms at rest and the other moving, then switching between them abruptly when they exactly spatially overlap as the motion commences. Clearly, it is not a practical solution. Hence, we introduced two new boundary conditions requiring the trap to be at rest before and after the motion. We also presented two methods to construct STA trajectories that comply with the new boundary conditions by either correcting an existing trajectory with an added spectral component at the trap frequency or by constructing a new trajectory with a higher order polynomial. Both techniques have demonstrated experimentally in the non-adiabatic regime where the transfer duration is on the order of the inverse trapping frequency. We have also shown that our technique can account for non-harmonic terms in the trapping potential. Thus, our approach is both realistic and flexible, and we anticipate it will be useful in the wide range of applications that can benefit from STA.

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Figure 8. Non-adiabatic transfer with a septic polynomial trajectory. Plotted are the sloshing amplitude (blue squares) and phase (gray circles) following a septic trajectory: \( z(t) = z(t) + z(t) / \omega_0^2 \) versus the frequency parameter \( \omega_1 \). Theoretical harmonic calculations appear as ribbons with matching colors, where the trapping frequency uncertainty determines their width. Minimum sloshing consistent with zero appears in the vicinity of the real oscillation frequency \( \omega_1 = \omega_0 \). The sloshing measurements performed following a 1.29 mm transport during \( t = 273 \text{ ms} \) in a trap of axial frequency \( \omega_p = 2\pi \cdot 7.53(8) \text{ Hz} \) (\( \omega_p - \omega_f \approx 2 \)).
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