Extended hypoplastic model incorporating the coordination number for the simulation of granular flow

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In this contribution, a hypoplastic viscous soil model incorporating viscous and grain-inertia rate effects observed in granular flows is enhanced by means of the coordination number, which allows to characterize the evolution of the microstructure of the granular material. Near a critical solid volume fraction and critical coordination number there exists a transition from a solid-like (rate-independent) to a flow-like (rate-dependent) behavior of granular materials. This information is used to calibrate a so-linear concentration factor, which controls viscous and grain-inertial rate effects in terms of the rate dependent evolution of the coordination number. The influence of this proposed hypoplastic model considering the coordination number is investigated in this paper by means of a simple undrained shear test.

1 Hypoplastic viscous model

The viscous hypoplastic model proposed by Guo et al. [1] is used as the basis for the hypoplastic model proposed in this work, which is characterized by an additive structure,

$$\sigma = \sigma_H + \sigma_D,$$

where $\sigma_H$ and $\sigma_D$ are the static and dynamic contributions to the total Cauchy stress tensor $\sigma$, respectively. The dynamic part

$$\sigma_D = \sigma_D (I H_d, d_{dev}, K_1, K_2)$$

depends on the second invariant of the rate of deformation gradient $I H_d$, on the deviatoric part of the rate of the deformation gradient $d_{dev}$, and on the viscous and grain-inertia scaling factors $K_1 (\mu, C, \lambda_N)$, $K_2 (\rho_s, d, \lambda_N)$, where $\mu$ is the dynamic viscosity of the interstitial fluid surrounding the grains, $C$ is the mean solid volume fraction, $\rho_s$ the density of the solid grains, and $d$ is the mean diameter. $\lambda_N$ is the proposed linear concentration factor given by

$$\lambda_N = \left( \frac{N_{\text{max}}}{N^*} \right)^\alpha - 1, \quad \beta,$$

where $N_{\text{max}}$ is the maximum coordination number corresponding to the maximum possible volume fraction $C_{\text{max}}, N^*$ is the rate dependent coordination number $N$, which is computed using a Prandtl-type equation [2], and $\alpha, \beta$ are parameters that are calibrated according to experiments. It should be noted that in Bagnold’s theory [3], the linear concentration factor is written as

$$\lambda = \left( \frac{C_{\text{max}}}{C} \right)^\frac{1}{2} - 1.$$

Figure 1 a) displays $\lambda_N - C$ plots for different values of $\alpha (\beta = -1)$.

2 Coordination number and evolution of soil microstructure

Discrete Element Method (DEM) simulations of steady and unsteady shear flows performed by Vescovi et al. [4] show, that there exists a critical volume fraction $C_c$ and critical coordination number $N_c$ below which the granular material exhibits a rate dependent (flow-like) behavior, whereas a rate independent (solid-like) behavior is found above these critical values. A curve fitting (performed by the authors) of the $N-C$ plots obtained from DEM simulations for steady shear flows [4] at different shearing rates is shown in Figure 1 b).

Steady state values provided by DEM results are subsequently combined with the model proposed by Rothenburg et al. [5], describing the strain-driven evolution of the coordination number. This allows its determination for a given strain level. The proposed non-linear equation relating the shear strain $\gamma$ and $N$ is given by

$$c\gamma = (N - N_0) - (\bar{N} - N_{\infty}) \ln \left( \frac{N - N_{\infty}}{N_0 - N_{\infty}} \right),$$

where $c$ is a constant independent of the coordination number, $N_0$ is the initial value of the coordination number, $\bar{N}$ is the maximum potential value for the coordination number and $N_{\infty}$ is an asymptotic value to which the solution of $N$ approaches. Figure 1 c) shows the transient evolution of the coordination number vs. the shear strain for different shearing rates at a given volume fraction.
Fig. 1: a) Linear concentration factor $\lambda$ vs. volume fraction $C$ for different values of parameter $\alpha$ showing an inflection point at critical volume fraction $C_c \approx 0.60$ (solid blue corresponds to $\lambda$ according to Bagnold’s theory), b) curves calibrated from DEM steady state shear flow simulations [4] for different shear strain rates and c) strain-driven evolution of $N$ for different shear strain rates at constant volume fraction, using the asymptotic values $N_{\infty}$ provided by b).

Fig. 2: a) Element test set-up and boundary conditions. b) Shear stress vs. shear strain rate plots for increasing shear strain rate and different values of parameter $\alpha$.

3 Numerical example

A numerical test performed on an undrained simple shear presented in [1] is re-analyzed and the influence of the modified linear concentration factor is evaluated. The initial vertical stress is set to $\sigma_y^0 = 1.5$ kPa, while an increasing shear velocity $v_x(y)$ is imposed in the right most side of the model (Figure 2 a)). As the test is performed under undrained conditions, the incompressibility constraint ($\Delta V = 0$) is imposed (i.e. the top plate is fixed against vertical motion). The hypoplastic parameters employed for the simulation are: $c_1 = -50$, $c_2 = -629.6$, $c_3 = -629.6$, $c_4 = 1220.8$, $c_6 = 0.88$, $c_{min} = 0.57$, $p_1 = 0.53$, $p_2 = 0.45$, $p_3 = 1.8 \times 10^{-6}$.

As seen in Figure 2 b) shows the shear stress-shear strain rate plots for different values of parameter $\alpha$. A peak in the shear stress for all plots is observed at a shear strain rate of $\dot{\gamma} = 8 \frac{1}{s}$, followed by a softening branch, which is a typical behavior for loose granular materials. With increasing levels of the shear strain rate, the rate effects on the stress-strain response become more significant. Investigating the influence of the parameter $\alpha$, it is observed that decreasing values of $\alpha$ lead to increasing shear stresses, in particular, when approaching the dynamic loading regime (i.e. $\dot{\gamma} > 30 \frac{1}{s}$). Furthermore, it is noted that the solution using the linear concentration factor provided by Bagnold’s theory (line with square black marks) is, for the investigated shear strain rates, within the range of the solutions obtained from the proposed linear concentration factor for the three selected values for $\alpha$ parameter.

4 Conclusions

In this contribution, a modification of the linear concentration factor, originally proposed by Bagnolds, is presented to model granular materials during the transition from a solid-like (rate-independent) to a flow-like (rate-dependent) behavior. Rate effects are incorporated into the coordination number by means of rate dependent terms correlating coordination number and volume fraction, which has been calibrated from DEM results documented in the open literature. The calibrated curves are then combined with the analytical model proposed by Rothenburg, describing the shear strain-driven evolution of the coordination number. Numerical results from shear undrained tests show the sensitivity of the shear stress to the shear strain rate for different values of the model parameter $\alpha$. The reference solution provided by Bagnold’s theory can be (approximately) recovered by appropriately choosing the $\alpha$ parameter.

References

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