Scalar mesons in QCD and tests of the gluon content of the $\sigma$ *

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We summarize the different features of the scalar mesons from QCD spectral sum rule analyses of the two- and three-point functions. The results do not favour the $\bar{u}u + \bar{d}d$ interpretation of the broad and low mass $\sigma(0.6)$, and the $\bar{s}s$ resonance nature of the eventually observed $\kappa(0.9)$ meson. We also discuss some OZI-violating and classic semileptonic and radiative decay processes which can reveal in a model-independent way the eventual gluon component $\sigma_B$ of the $\sigma$. In a meson-gluonium mixing scenario, one also expects an observation of the $K\bar{K}$ final states from the $\sigma_B$ which may compete (if phase space allowed) with the one from a low mass $\bar{s}s$ state assumed in the literature to be the SU(3) partner of the $\sigma(0.6)$ if this latter is a $\bar{u}u + \bar{d}d$ state.

1. Introduction

The nature of scalar mesons is an intriguing problem in QCD. Experimentally, there are well established scalar mesons with isospin $I = 1$, the $a_0(980)$, with isospin $I = 1/2$ $K^*_0(1410)$ meson, and with isospin $I = 0$, the $f_0$-mesons at 980, 1370 and 1500 MeV [1]. Besides these resonances there are different indications [2,3] for a low lying scalar isoscalar state, the famous $\sigma$. The isoscalar scalar states are especially interesting in the framework of QCD since, in this $U(1)_V$ channel, their interpolating operator is the trace of the energy-momentum tensor:

$$\theta_\mu^\nu = \frac{1}{4} \beta(\alpha_s)G^2 + \sum_i [1 + \gamma_m(\alpha_s)]m_i \bar{\psi}_i \psi_i , \quad (1)$$

where $G^\alpha_\mu$ is the gluon field strengths, $\psi_i$ is the quark field; $\beta(\alpha_s)$ and $\gamma_m(\alpha_s)$ are respectively the QCD $\beta$-function and quark mass-anomalous dimension. In the chiral limit $m_i = 0$, $\theta_\mu^\nu$ is dominated by its gluon component $\theta_g$, like is the case of the $\eta'$ for the $U(1)_A$ axial-anomaly, explaining why the $\eta'$-mass does not vanish like other Goldstone bosons for $m_i = 0$. In this sense, it is natural to expect that these $I = 0$ scalar states are glueballs/gluonia or have at least a strong glue component in their wave function. QCD spectral sum rules (QSSR) are an important analytical tool of nonperturbative QCD and especially well suited to address the question of the quark-gluon mixing since the principal nonperturbative ingredients are the quark condensate, the gluon condensate and the mixed quark-gluon condensate. In this talk, we summarize some essential features of QSSR analyses and discuss different production processes for obtaining information on the gluon content of the scalar mesons.

2. Unmixed scalar mesons from QSSR

- Masses and couplings of unmixed scalar $\bar{q}q$ mesons and gluonia have been extensively studied in the past using QSSR within the standard Operator Product Expansion (OPE) of the diagonal two-point correlator [4,5]:

$$\psi(q^2) = i \int d^4x e^{iqx} \langle 0|TJ(x)J(0)|0 \rangle , \quad (2)$$

associated to the quark or/and the gluonic currents. It has been emphasized that the mass of the isoscalar scalar $S_2 \equiv \bar{u}u + \bar{d}d$ meson is about 1 GeV, in agreement with the one of the observed $a_0(980)$, as expected from the good realization of the $SU(2)$ symmetry implying a degeneracy between the $a_0$ and $S_2$ states, while its width into $\pi\pi$ is about 100 MeV [4,5]. On the other, the mass...
of the mesons containing a strange quark is above 1 GeV due to $SU(3)$ breaking, where the same mechanism explains successfully the well-known $\phi$-$\rho$ and $K^*$$-\rho$ mass splittings. In recent analysis\textsuperscript{2}, it has been shown that instanton within the instanton liquid model\textsuperscript{3} as well as new $1/q^2$ induced by a tachyonic gluon mass\textsuperscript{4} from the linear term of the short distance part of the QCD potential\textsuperscript{5}, affect only slightly these mass predictions and do not allow to decrease these values in a stable way.

- In the gluonium channel, using a subtracted sum rule sensitive to $\psi_G(0) \simeq -16\beta_1/\pi(\alpha_s G^2)$, (where $\beta_1 = -1/2(11 - 2n/3)$ and $\langle \alpha_s G^2 \rangle \simeq 0.07$ GeV\textsuperscript{6}) and the unsubtracted sum rule, it was found\textsuperscript{7,8} that one needs two resonances for consistently saturating the two sum rules, where the lowest mass gluonium $\sigma_B$ should be below 1 GeV. A low energy theorem obeyed by the vertex $\langle \pi|\theta^\mu|\pi \rangle$ also shows that the $\sigma_B$ can be very wide with a $\pi^+\pi^-$ width of about $(0.2 - 0.8)$ GeV corresponding to a mass of $(0.7 - 1)$ GeV, while a $\sigma_B$ having a mass less than 0.6 GeV has a weaker coupling because $g_{\sigma\pi\pi} \sim \sigma B^2$ and thus cannot be broad. This result shows a huge violation of the OZI rule like in the case of the $\eta'$-channel\textsuperscript{9}.

- From the previous results, one can already conclude that the observed $\sigma(0.6)$ cannot be a pure $\bar{q}q$ state, but contains most probably a large gluon component in its wave function\textsuperscript{10}, while the $\kappa(0.9)$ meson cannot also be identified with an usual $\bar{q}q$ scalar resonance which mass is predicted to be about the $K^0(1.3 - 1.4)$.

- As a consequence, a quarkonium-gluonium mixing (decay mixing\textsuperscript{11}) scheme has been proposed in the $I = 0$ scalar sector\textsuperscript{12}, for explaining the observed spectrum and widths of the possibly wide $\sigma(< 1$ GeV) and the narrow $f_0(0.98)$. The data are well fitted when the mixing angle is maximal:

$$|\theta_S| \approx 40^0,$$

indicating that the $\sigma$ and $f_0$ have equal numbers of quark and gluon in each of their wave functions. This mixing scenario also implies a strong coupling of the $f_0$ to $\bar{K}K$ (without appealing to a four-quark state model) with a strength\textsuperscript{13}:

$$g_{f_0\bar{K}K} = 2g_{f_0\pi^+\pi^-}$$ \hspace{1cm} (4)

a property confirmed by the data. The physical on-shell $f_0$ is narrow ($< 134$ MeV) due to a destructive mixing, whilst the $\sigma(0.7 - 1)$ can be $(0.4 - 0.8)$ GeV wide (constructive mixing). Compared to the four-quark states and/or $\bar{K}K$ molecules models (see e.g.\textsuperscript{14}), this quarkonium-gluonium mixing scenario includes all QCD dynamics based on the properties of the scale anomaly $\theta^\mu$, which comes from QCD first principles. We propose some further tests of this scenario from semileptonic and radiative decay processes in the following\textsuperscript{15}.

### 3. Tests from $D_{(s)}$ semileptonic decays

#### $S_2(\bar{u}u + \bar{d}d)$ meson productions

- If the scalar mesons were simple $\bar{q}q$ states, the semileptonic decay width could be calculated quite reliably using QSSR, where the relevant diagram is a quark loop triangle. This analysis has been done with a good success for the semileptonic decays of the $D$ and $D_s$ into pseudoscalar and vector mesons\textsuperscript{16}. For the production of a pseudoscalar or scalar $\bar{q}q$ states several groups\textsuperscript{17} predict all form factors to be: $f_+(0) \approx 0.5$, yielding a decay rate:

$$\Gamma(D \rightarrow \bar{S}_2\ell) = (8 \pm 3) \times 10^{-16} \text{ GeV},$$

for $M_{S_2} \simeq 600$ MeV.

- However, because of the enigmatic nature of the $\sigma$, one has also considered, in\textsuperscript{18}, the case that the quark-antiquark current does not couple to a resonance but rather to an uncorrelated quark-antiquark pair. In that case the decay rate is reduced by a factor 2, but, in the spectral distribution, there is a broad bump visible with a maximum near the presumed $\sigma$ mass of 600 MeV. Unfortunately, even in high statistics experiments,
the estimated decay rates of the $D$-meson are at the edge of observation since the decays into an isoscalar are CKM-suppressed due to the $c-u$ transition at the weak vertex.

Scalar gluonium and/or $\bar{s}s$ productions
The evaluation of diagrams for a semileptonic decay into a gluonium state is, unfortunately, more involved than in the $\bar{q}q$ case. Therefore, we can give only semi-qualitative results which, however, are model independent.

- The only way to obtain a non-CKM suppressed isoscalar is to look at the semileptonic decay of the $D_s$-meson, where the light quark is a strange one and an isoscalar $s\bar{s}$ or/and gluonium state can be formed.
- If the $s\bar{s}$ is relatively light ($< 1$ GeV), which might be the natural partner of the $\bar{u}d + \bar{d}d$ often interpreted to be a $\sigma(0.6)$ in the literature, then, one should produce a $K\bar{K}$ pair through the isoscalar $s\bar{s}$ state. The QSSR prediction for this process is under quite good control $\frac{[8]}{[12]}$. The non-observation of this process will disfavour the $\bar{q}q$ interpretation of the $\sigma$ meson.
- If a gluonium state is formed it will decay with even strength into $\pi\pi$ and a $K\bar{K}$ pairs. A gluonium formation in semileptonic $D_s$ decays should result in the decay patterns:

$$D_s \rightarrow \sigma_B \ell\nu \rightarrow \pi\pi\ell\nu \quad D_s \rightarrow \sigma_c \ell\nu \rightarrow K\bar{K}\ell\nu$$

with about the same rate up to phase space factors. The observation of the semileptonic $\pi\pi$ decay of the $D_s$ would be a unique sign for glueball formation.

- A semi-qualitative estimate of the above rates can be obtained by working in the large heavy quark mass limit $M_c$. Using, e.g., the result in $\frac{6}{12}$, the one for light $\bar{q}q$ quarkonium production behaves as:

$$\Gamma[D_s \rightarrow S_q(\bar{q}q) \ell\nu] \sim |V_{cq}|^2 G^2 F^4 M^5 \frac{f_c(0)^2}{M_c^2}.$$  \hspace{1cm} (7)

- For the $\sigma_B(gg)$ production, we study the $1/M_c$ behaviour of the $WWgg$ box diagram, where it is easy to find that the dominant (in $1/M_c$) contribution comes from the one involving one charmed propagator. Therefore the production amplitude can be described by the Euler-Heisenberg effective interaction:

$$\mathcal{L}_{eff} \sim \frac{g_W \alpha_s}{p^2 M_c^2} F_{\mu\nu} F^{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} + \text{perm.} + \cdots$$  \hspace{1cm} (8)

where $\cdots$ are h.o in $1/M_c$, $g_W$ is the electroweak coupling and $p^2 \simeq M_c^2$ is the typical virtual low scale entering into the box diagram. Using dispersion techniques similar to the one used for $J/\psi \rightarrow \sigma_B \gamma$ processes $\frac{[8]}{[12]}$, one obtains, assuming a $D_s$ and $\sigma_B$-dominances:

$$\Gamma[\bar{D}_s \rightarrow \sigma_B(gg) \ell\nu] \sim |V_{cs}|^2 G^2 \left| \frac{\langle 0|\alpha_s G^2|\sigma_B\rangle}{M_c^2} \right|^2.$$  \hspace{1cm} (9)

The matrix element $\langle 0|\alpha_s G^2|\sigma_B\rangle$ is by definition proportional to $f_s M_c^2$, where $f_s$ is hopefully known from two-point function QSSR analysis $\frac{[12]}{[13]}$. Using $f_s \approx 0.8$ GeV, one then deduces:

$$\Gamma[D_s \rightarrow \sigma_B(gg) \ell\nu] \sim \frac{1}{|f_s(0)|^2} \frac{f_s^2}{M_c^2},$$  \hspace{1cm} (10)

which is $O(1)$. This qualitative result indicates that the gluonium production rate can be of the same order as the $\bar{q}q$ one contrary to the naive perturbative expectation ($\alpha_s^2$ suppression), which is a consequence of the OZI-rule violation of the $\sigma_B$ decay.

- However, it also indicates that, due to the (almost) universal coupling of the $\sigma_B$ to Goldstone boson pairs, one also expects a production of the $K\bar{K}$ pairs, which can compete with the one from $s\bar{s}$ quarkonium state, and again renders more difficult the identification of the such $s\bar{s}$ state if allowed by phase space.

4. Tests from $J/\psi$ and $\phi$ radiative decays
Radiative decays of vector mesons have been often proposed to be the classical gluonia production processes. In $\frac{12}{[1]}$, one expects the rate:

$$B[J/\psi \rightarrow \gamma\sigma_B]|B[\sigma_B \rightarrow all]| \approx (4 - 6) \times 10^{-4},$$  \hspace{1cm} (11)

which is far below the upper rate of production for $B(J/\psi \rightarrow \gamma\pi\pi)$ of about $10^{-2}$ allowed by BES $\frac{[3]}{[3]}$. Analogous rate is expected for the $\sigma'(1.3)$ (radial excitation of the $\sigma$) and $G(1.5)$ productions.

- Extending the analysis of the $J/\psi$ into the one of the $\phi$ by replacing the charm quark constituent loop by the strange quark, it is easy to find $\frac{[1]}{[1]}$:

$$B \Gamma[\phi \rightarrow \gamma f_0(980)] \approx 1.3 \times 10^{-4},$$  \hspace{1cm} (12)
which despite the crude approximation (use of the $1/m_s$ constituent mass expansion) leads to a surprising satisfactory result compared with the averaged data $(1.08 \pm 0.07 \pm 0.06) \times 10^{-4}$ from [20].

- For the $a_0$, one also expects that the LrM + ChPT ($SU(3)$ symmetry) can give a reliable prediction of the production rate (see e.g. [21]) due to its isovector nature contrary to the case of the isoscalar mesons affected by their gluon content ($U(1)_V$ symmetry). In this case, one expects the decay chain process through kaon loops:

$$\phi \rightarrow \gamma \bar{K}K \rightarrow \gamma a_0 \rightarrow \gamma \eta \pi ,$$ (13)

where the coupling of the $a_0$ to $\bar{K}K$ can be obtained from e.g. the $SU(3)$ relation in [10].

5. Conclusions

We have reviewed the different features of scalar mesons from QCD spectral sum rules and the different OZI-violating and classic gluonia semileptonic and radiative decay processes for testing the eventual gluon component of the $\sigma$ meson. We wish that some progresses on this field will be accomplished in the near future, as after about a 1/4 century study, we still remain with more questions than answers on the true nature of scalar mesons.

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