The mass formula for an exotic BTZ black hole

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Abstract

An exotic Bañados-Teitelboim-Zanelli (BTZ) black hole has an angular momentum larger than its mass in three dimension (3D), which suggests the possibility that cosmic censorship could be violated if angular momentum is extracted by the Penrose process. In this paper, we propose a mass formula for the exotic BTZ black hole and show no violation of weak cosmic censorship in the gedanken process above by understanding properly its mass formula. Unlike the other black holes, the total energy of the exotic BTZ black hole is represented by the angular momentum instead of the mass, which supports a basic point of view that the same geometry should be determined by the same energy in 3D general relativity whose equation of motion can be given either by normal 3D Einstein gravity or by exotic 3D Einstein gravity. However, only the mass of the exotic black hole is related to the thermodynamics and other forms of energy are “dumb”, which is consistent with the earlier thermodynamic analysis about exotic black holes.

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I. INTRODUCTION

The two-parameter Banados, Teitelboim, and Zanelli (BTZ) solution [1] represents a rotating black hole in three-dimensional (3D) spacetime with a negative cosmological constant. The expressions for the parameters in terms of conserved quantities of a black hole is theory-dependent. In the normal 3D Einstein gravity, the identification of the mass $M$ and the angular momentum $J$ of a black hole gives some similar behaviors to the Kerr black-hole solution of 4D Einstein equations. In particular, the mass formula can be gotten directly along the line of the Smarr’s method [2] for Kerr black holes; that is

$$M = \frac{1}{2} TS + \Omega J$$  

where, according to black hole thermodynamics [3], $T$ is the temperature, $S$ is the entropy, and $\Omega$ is the angular velocity. Like the Smarr formula for Kerr black holes, the mass $M$ represents the total energy (in units for which $c = \hbar = 1$) which includes the surface energy and the rotation energy.

In the exotic 3D Einstein gravity [4], however, the parameters have to be reinterpreted with the reversed roles for mass $M_E$ and angular momentum $J_E$, compared with the case of normal Einstein gravity; i.e. $M_E \sim J, J_E \sim M_E$. Usually, such black holes are thought to be “exotic” which were discovered first in Ref. [5], and their thermodynamics have been discussed in our recent works [6, 7]. Coincidentally, the mass formula of an exotic BTZ black hole has the same form with Eq. (1),

$$M_E = \frac{1}{2} TS_E + \Omega J_E$$

where $S_E$ is the entropy of the exotic BTZ black hole, which is proportional to the length of the inner horizon instead of the event horizon [6, 8, 9]. To ensure the existence of an event horizon, the parameters must satisfy a condition

$$J_E \geq \ell M_E$$

where $\ell$ is the AdS radius. From this condition, it is seen that if the mass $M_E$ is regarded as the total energy for the exotic BTZ black hole, it cannot include all the energy that is contributed by the angular moment. Thus, to investigate the partition of energy for the exotic BTZ black hole is a natural and significant task for understanding the thermodynamics.
better. In fact, the reversed role for the mass and angular momentum of the exotic BTZ black hole also caused some other questions that need to be interpreted further. One is related to the total energy of an exotic BTZ black hole, as mentioned above. If the total energy is \( M_E \), the exotic black hole will have the different energy from the normal one (that is \( M \)). But the two kinds of black holes correspond to the same geometry of spacetime with the same geometric variables, i.e. the positions of the inner and outer horizon, the temperature, the angular velocity. In particular, it is noted that the two kinds of black holes are the solution of 3D Einstein equation which can be obtained either by 3D normal Einstein gravity or by exotic Einstein gravity. Thus according to general relativity, only the same energy leads to the same geometry of spacetime. We conclude that the total energy must not be \( M_E \), but what is the total energy of an exotic BTZ black hole? The other one is related to the condition \( (3) \). According to Penrose process \( [10] \), the energy associated with the angular momentum \( J_E \) can be extracted by a physically feasible process. But such extraction of energy will lead to violation of the condition \( (3) \) or weak cosmic censorship (WCC) \( [11] \). So how to clarify the question is also a purpose of this paper.

In this paper, we will investigate the mass formula for exotic BTZ black holes along the line of Smarr’s method, and compare them with the reversible and irreversible transformations of a black hole made by Christodoulou \( [12] \). The two potential questions will also be answered in the process of investigation. Actually, the different recognition of parameters in the exotic BTZ black hole does not change the total energy which determines the geometry of spacetime according to Einstein’s general relativity. Meanwhile, WCC will be not violated. In the next section we will first review the thermodynamics of BTZ black holes for the two cases, based on which we will study the mass formula for the exotic black holes.

II. BTZ BLACK HOLES

In this section we will give the related information about BTZ black holes both for the normal and exotic cases, as obtained in our previous works \( [6, 7] \). Start with the BTZ metric

\[
\begin{align*}
 ds^2 &= -N^2 dt^2 + N^{-2} dr^2 + r^2 (N^\phi dt + d\phi)^2, 
\end{align*}
\] (4)
where $\phi$ is an angle with the period $2\pi$ as the identification of the black hole spacetime. The functions $N^2$ and $N^\phi$ are

$$N^2 = -8Gm + \frac{r^2}{\ell^2} + \frac{16G^2j^2}{r^2}, \quad N^\phi = \frac{4Gj}{r^2},$$

where $G$ the 3D Newton constant. Its Killing horizons are found by setting $N^2 = 0$; this gives

$$r_\pm = \sqrt{2G\ell(\ell m + j)} \pm \sqrt{2G\ell(\ell m - j)},$$

where $G$ is the 3D Newton constant. Its Killing horizons are found by setting $N^2 = 0$; this gives

$$r_\pm = \sqrt{2G\ell(\ell m + j)} \pm \sqrt{2G\ell(\ell m - j)},$$

We may assume without loss of generality that $j \geq 0$ and assume that $\ell m \geq j$, to ensure the existence of an event horizon at $r = r_+$. The BTZ metric can be solved either in the normal Einstein gravity with the Lagrangian

$$L = \frac{1}{8\pi G} \left( e_a R^a - \frac{1}{6\ell^2} \epsilon^{abc} e_a e_b e_c \right),$$

or in the exotic Einstein gravity with the Lagrangian

$$L_E = \frac{\ell}{8\pi G} \left[ \omega_a \left( d\omega^a + \frac{2}{3} \epsilon^{abc} \omega_b \omega_c \right) - \frac{1}{\ell^2} \epsilon_a T^a \right],$$

where the Lagrangians are given with 3-forms in which $e^a (a = 0, 1, 2)$ is the dreibein 1-forms, $\omega^a$ is Lorentz connection 1-forms, and their torsion and curvature 2-form field strengths are $T^a = de^a + \epsilon^{abc} \omega_b e_c$, $R^a = d\omega^a + \frac{1}{2} \epsilon^{abc} \omega_b \omega_c$. It is easily checked that the two Lagrangians give the same equation of motion which is the 3D Einstein equation, but their parities are different, which cause some different thermodynamic interpretations for the two cases. With the BTZ metric (4), the thermodynamic parameters for the normal BTZ black holes are

$$M = m, \quad J = j, \quad S = \frac{\pi r_+}{2G},$$

and the parameters for the exotic ones are

$$M_E = j/\ell, \quad J_E = \ell m, \quad S_E = \frac{\pi r_-}{2G},$$

where the exotic forms had been interpreted and the corresponding thermodynamic laws had been given in our recent work [6] and here we do not elaborate further. In particular, the temperature $T$ and the angular velocity $\Omega$ take the same values

$$T = \frac{r_+^2 - r_-^2}{2\pi r_+ \ell^2}, \quad \Omega = \frac{r_-}{\ell r_+}$$

for the two cases, which present the geometric properties for the 3D BTZ spacetime background.
III. MASS FORMULA

A. Normal BTZ black holes

In this subsection we will revisit the mass formula for a normal BTZ black hole. It has to be pointed out that the BTZ black hole is asymptotic anti-de Sitter, different from the Kerr black hole that is asymptotic flat. Generally, the asymptotic properties will influence the definitions of conserved charges \[13–15\], and for the case of 3D gravity see the discussion of Ref. \[16\]. But we can assume in advance that the mass and angular momentum of the BTZ black hole has been defined well, i.e. in the normal 3D Einstein gravity we can identify the mass and angular momentum from the metric (4) as given in the last section. For exotic BTZ black holes discussed in the next subsection, we will conform to this assumption.

Start with the mass differential $dM$ according to the thermodynamic first law,

$$dM = TdS + \Omega dJ.$$ \hspace{1cm} (12)

Like Smarr’s discussion, we choose a path of integration in the space $(S, J)$ to define for a BTZ black hole two energy components: the surface energy $E_s$ by

$$E_s = \int_0^S T \left( S', 0 \right) dS';$$ \hspace{1cm} (13)

and the rotation energy $E_r$ by

$$E_r = \int_0^J \Omega \left( S, J' \right) dJ', \; S \; f i x e d.$$ \hspace{1cm} (14)

These integrals can be calculated directly with the variables $M$, $T$, and $\Omega$ that expressed in terms of $S$ and $J$,

$$M = \frac{GS^2}{2\pi^2 \ell^2} + \frac{\pi^2 J^2}{2GS^2},$$

$$T = \frac{GS}{\pi^2 \ell^2} - \frac{\pi^2 J^2}{GS^3},$$

$$\Omega = \frac{\pi^2 J}{GS^2}.$$

Then we obtain the results of these integrals as

$$E_s = \frac{GS^2}{2\pi^2 \ell^2}, \; E_r = \frac{\pi^2 J^2}{2GS^2}.$$ \hspace{1cm} (15)
Thus the total energy is
\[ E = E_s + E_r = M. \] (16)

According to Christodoulou [12], the total energy or the mass \( M \) of a BTZ black hole can be divided into an irreducible mass \( M_{ir} \) and a rotational energy \( M - M_{ir} \). Now we turn to the calculation of irreducible mass and see if it is equal to the surface energy \( E_s \).

Consider a particle of energy \( E_0 \) \((E_0 \ll M)\) sent from infinity into a 3D BTZ black hole and its motion had been analyzed in some previous papers [17, 18]. From its radial geodesics, one knows that the most efficient Penrose process has to satisfy a condition at the horizon, that is \( \frac{E_0}{L} = \frac{4Gj}{r_+^2} \) where \( L \) is the angular momentum of the particle. Due to the conservation laws of energy and angular momentum, after the particle drops into the black hole we obtain,
\[ \frac{dM}{dJ} = \frac{J}{\ell \left( \ell M + \sqrt{\left( \ell M \right)^2 - J^2} \right)}. \] (17)

Integrate the relation by taking the initial values \( J_0 = 0, M_0 = M_{ir} \) which is also the starting point of the reversible transformation, and the irreducible mass is gotten as
\[ M_{ir} = \frac{r_+^2}{8G\ell^2}. \] (18)

Using the entropy \( S = \frac{\pi r_+}{2G} \), we have \( M_{ir} = E_s \). Moreover, the relation \( \delta M_{ir} \geq 0 \) also implies the second law of black hole thermodynamics.

**B. Exotic BTZ black holes**

Since the first law holds for the exotic black hole, we can use the similar definitions as Eqs. (13) and (14) to discuss the energy partition. At first, we have to work out the expressions of these variables \( M_E, T \) and \( \Omega \) in terms of \( S_E, J_E \); they are

\[ M_E = \frac{1}{\ell} \left( \frac{2GS_E^2J_E}{\pi^2\ell} - \frac{G^2S_E^4}{\pi^4\ell^2} \right)^{\frac{1}{2}}; \]
\[ T = \frac{2GS_E}{\ell \pi^2} \left( J_E - \frac{GS_E^2}{\pi^2\ell} \right) \left( \frac{2GS_E^2J_E}{\pi^2\ell} - \frac{G^2S_E^4}{\pi^4\ell^2} \right)^{-\frac{1}{2}}; \]
\[ \Omega = \frac{GS_E^2}{\pi^2\ell} \left( \frac{2GS_E^2J_E}{\pi^2\ell} - \frac{G^2S_E^4}{\pi^4\ell^2} \right)^{-\frac{1}{2}}. \] (19)

where an implicit assumption that \( J_E \geq \frac{GS_E^2}{2\pi^2\ell} \) has been enforced in order to ensure the real values of these physical variables.
Thus, one can choose $\frac{GS^2}{2\pi^2\ell}$ as the lower limit of the angular momentum. With Eqs. (13) and (14), we get the surface energy of the exotic BTZ black hole as $E_s = 0$ and the rotation energy $E_r = ME$, which is consistent with the relation $ME = E_s + E_r$. But this takes the ground state with $M_{E0} = 0$ and $J_{E0} = \frac{GS^2}{2\pi^2\ell}$, which is inconsistent with the former assumption that $ME$ represents the total energy in the mass formula. Thus, either the total energy is not given by $ME$ or the analysis here is unfeasible. Since the thermodynamics about exotic BTZ black holes had been confirmed [6, 7], we guess that the analysis is feasible but some changes have to be made.

Define a new reasonable ground state with $\ell ME_0 = J_{E0}$ which is also the extreme state of the exotic BTZ black hole. With such ground state, we have to take the constraint further as $J_E \geq J_C = \frac{GS^2}{\pi^2\ell}$ by the expression for mass $ME$. Thus $J_C$ should be taken as the lower limit, which will be explained further below when discussing the irreducible mass. Now we calculate the surface energy

$$E_{Es} = \int_0^{S_E} T \left(S'_E, J_C\right) dS'_E = \frac{GS^2_E}{\pi^2\ell^2};$$

which is just the energy associated with the part of the angular moment $J_C$, or the energy of ground state, and the rotation energy

$$E_{Er} = \int_{J_C}^{J_E} \Omega \left(S_E, J'_E\right) dJ'_E = \frac{1}{\ell} \left(\frac{2GS^2_EJ_E}{\pi^2\ell} - \frac{G^2S^4_E}{\pi^4\ell^2}\right)^{\frac{1}{2}} - \frac{GS^2}{\pi^2\ell^2}.\tag{21}$$

Instantly, one obtain $ME = E_{Es} + E_{Er}$. From the calculation of $E_{Er}$, one might think that the rotation energy $E_{Er}$ is just the energy associated with the part with the range of angular momentum from $J_C$ to $J_E$ (we will call it the part of $J_E - J_C$), but a further analysis finds that $E_{Er} \leq$ energy (associated with the part of $J_E - J_C$), i.e. $\ell E_{Er} = \sqrt{\frac{2GS^2_EJ_E}{\pi^2\ell} - \frac{G^2S^4_E}{\pi^4\ell^2} - \frac{GS^2_E}{\pi^2\ell^2} = \sqrt{J^2_E - \left(J_E - \frac{GS^2}{\pi^2\ell}\right)^2} - J_C \leq J_E - J_C}$. So the energy $E_{Er}$ does not include all the energy contributed by the angular momentum $J_E - J_C$. An energy contributed by the part of angular moment $J_E - J_C - \ell E_{Er}$ has to be introduced for the energy conservation, that is, $E_{ER} = \frac{1}{\ell} \left((J_E - J_C) - E_{Er}\right) = \frac{J_E}{\ell} - ME$. Then the total energy for an exotic BTZ black hole described by the metric [4] can be written as

$$E_E = E_{Es} + E_{Er} + E_{ER} = \frac{J_E}{\ell}.$$

Comparing Eq. (16) with Eq. (22), we find that $E = E_E$ which shows that the total energies for two different black holes are the same, which determine the same geometry of
spacetime. In order to confirm this analysis, we will discuss the reversible and irreversible transformations for an exotic BTZ black hole. Using the same process of a particle falling into the black hole, it is given an equation as

\[
\frac{dM_E}{dJ_E} = \frac{M_E}{\ell (J_E + \sqrt{J_E^2 - (\ell M_E)^2})},
\]

(23)

where the equation is also derived from the relation \( \frac{E_0}{\ell} = \frac{4Gj}{r_+^2} \) which is obtained from the geodesic motion of particles [17, 18] and independent on the concrete 3D gravity models, but the corresponding parameters have to be matched with the exotic black holes. Integrating the equation but taking the initial values as \( J_E^0/\ell = M_E^0 = M_{E-ir} \), we obtain the irreducible mass as

\[ M_{E-ir} = \frac{r_-^2}{4G\ell^2}, \]

which is equal to \( E_{Es} \) by using the entropy \( S_E = \frac{\pi r_-}{2G} \). Actually by the equality \( M_{E-ir} = E_{Es} \), one can also obtain the expression of the entropy as \( S_E = \frac{\pi r_-}{2G} \) for the exotic BTZ black hole. Again the relation \( \delta M_{E-ir} \geq 0 \) supports the second law of black hole thermodynamics for an exotic BTZ black hole, as expected. Moreover, it is noted that \( M_{E-ir} = J_C/\ell \), which gives the angular momentum of ground state as \( J_{E0}/\ell = M_{E-ir} \). When we put \( J_E/\ell = J_{E0}/\ell = M_{E-ir} \) into the expression of \( M_E (S_E, J_E) \), we get \( M_E = M_{E-ir} = M_{E0} \), which indicates that the ground state with \( \ell M_{E0} = J_{E0} \) are physically reasonable and self-consistent. Then can we take the values with such relation \( J_{E0}/\ell < M_{E0} \)? The answer is negative because such initial values will lead to an imaginary irreducible mass.

It is noted that \( M_E - M_{E-ir} \) cannot be considered as all the rotation energy, since \( M_E - M_{E-ir} \leq \frac{J_C}{\ell} - M_{E-ir} \). So we define the excess \( \left( \frac{J_C}{\ell} - M_{E-ir} \right) - (M_E - M_{E-ir}) = \frac{J_C}{\ell} - M_E \) as a part of the total energy, which is just \( E_{ER} \) mentioned before. Again we show that the total energy of an exotic BTZ black hole is \( J_E \) which consists of three parts: the surface energy, the rotation energy, and the redundant rotation energy. Here we make some discussions for these energies. Firstly, different from the general understanding for the surface energy, it consists of the rotation energy associated with the part of angular momentum \( J_C \). In particular, the rotation energy included in the surface energy cannot be extracted through the Penrose process, which ensure the validity of WCC. Secondly, the rotation energy is the same as usual, and can be extracted through the Penrose process. Finally, the redundant rotation energy is interesting, since it can be extracted through the
Penrose process but cannot be obtained by the standard calculation for the rotation energy, i.e. by Eq. (21). And its existence looks more like a kind of indication that WCC is preserved (i.e. only if $E_{ER} \geq 0$, WCC is not violated). Then through a brief process, these energies can be understood further. Given an exotic BTZ black hole with $M_E \leq \frac{\ell E}{\ell}$, one can extract the rotation energy $M_E - M_{E-ir}$ through Penrose process. Then the energy of the black hole is left as $\frac{\ell E}{\ell} - (M_E - M_{E-ir}) = \left(\frac{\ell E}{\ell} - M_E\right) + M_{E-ir} = E_{ER} + M_{E-ir}$. If the energy $E_{ER}$ is taken out, the extreme black hole appears, but according to the third law of black hole thermodynamics, the energy $E_{ER}$ cannot extracted completely during a finite time. So it is not necessary to worry about violation of WCC due to the excessive extraction of rotation energy.

Then, we want to investigate the extreme situation. For a normal BTZ black hole, its extreme condition is $\ell M = J$, for which $E_s = E_r = \frac{M}{2}$. The non-zero value of the rotation energy indicates that even in the extreme situation, the rotation energy can still be extracted through Penrose process. This will not cause any problems for the normal BTZ black hole, but at the extreme situation, the extraction of the rotation energy will lead to violation of WCC for an exotic BTZ black hole. Fortunately, under the extreme condition $\ell M_E = J_E$ of the exotic BTZ black hole, the energy that can be extracted through the Penrose process has been exhausted, i.e. $E_{Er} = E_{ER} = 0$, and the extra rotation energy that appears with the form of the surface energy or the irreducible mass cannot be reduced further and thus protects the WCC from violation by the extraction of rotation energy. In particular, due to the exotic components of the surface energy, the scenario of particle with the angular momentum infalling into a black hole will not lead to any violation of WCC, that can refer to the discussion in Ref. [19].

A further comment contributes to such a question: since the total energy is $E_E$ that is not equal to $M_E$, it seems better to work out the differential form of energy $E_E$ to make the black hole behave like a thermal system satisfying the first law of thermodynamics. From the discussion above, we have

$$dE_E = dE_{Es} + dE_{Er} + dE_{ER};$$

where $dE_{Es} = 0$ since the energy associated with $E_{Es}$ represents its irreducible mass. Using the expression for $E_{ER}$, the first law becomes $dM_E = dE_{Er}$. Then using Eqs. [19] and [21]...
and noticing that $dJ_C = 0$, a straight calculation gives $dE_{Er} = TdS_E + \Omega dJ_E$. Thus we get

$$dM_E = TdS_E + \Omega dJ_E,$$

which is consistent with our earlier results for the first law of thermodynamics for an exotic BTZ black hole [6]. In particular, this interpretation also applies to the exotic BTZ black-hole solution obtained in “BCEA” theory [5, 20] which minimally couples topological matter to 3D Einstein gravity without the cosmological constant if one associates this term $dE_{ER}$ with the exotic topological matter. Moreover, such interpretation is very like that in the case of charged rotating BTZ black holes [21], in which the total energy is not represented by the mass, but the expression of the first law there includes an extra term $P_r dA$ that is from the contribution of electromagnetic matter.

Finally, the general Hamiltonian analysis gives the energy of the exotic BTZ black hole as $M_E$ which is the on-shell values of the asymptotic generators for time translations [22], but it has to be pointed out that such energy is only related to the dynamic or thermodynamic process. Whether there are other forms of energy that are not included in the thermodynamic process is unclear for the Hamiltonian form. For our case, a simple relation [3] tells us that the mass $M_E$ cannot represent the whole energy and it must require some other forms of energy. Based on our analysis, these forms of energy are the surface energy $E_{Es}$ which ensures that the initial state is consistent with WCC and the redundant rotation energy $E_{ER}$ which ensures that the final state will not violate WCC. Since the extra forms of energy are “dumb” which do not contribute to the thermodynamic process, the analysis here is consistent with the earlier understanding for the thermodynamics about exotic BTZ black holes.

IV. CONCLUSION

In this paper, we have investigated the mass formulas for the normal and exotic BTZ black holes and have presented their differences. For normal BTZ black holes, the understanding of the mass formula is nearly the same with that for Kerr black holes. For exotic BTZ black holes, however, the understanding of the mass formula is far from the usual situation. Firstly, the total energy is represented by the angular momentum, instead of the mass. Then the surface energy that can be interpreted as irreducible mass includes one part of the rotation
energy that cannot be extracted by the Penross process. This ensures the validity of WCC. We have also showed that the total energy will not change, although the parameters \((m, j)\) are read by the different conserved quantities of a black hole in the normal and exotic 3D gravity models. This ensures that the same geometry of spacetime is described by the metric \(4\) in the two different models of 3D general relativity. Thus the discussion of the mass formula in this paper improves further the understanding about thermodynamics of exotic BTZ black holes.

Further, we pointed out that the form presented in Eq. \(1\) can be applicable to all BTZ black holes in different 3D gravity models such as topological massive gravity \(23\), new massive gravity \(24\), general massive gravity \(24, 25\) and so on. The difference between these modified theories and the normal Einstein gravity is different from the difference between the exotic Einstein gravity and the normal Einstein gravity. For the former, they have different equations of motion, but for the latter, the equation of motion is the same one which is 3D Einstein equation. So in this paper, we discuss only the case for 3D general relativity which should inherit the basic point of view in 4D general relativity that says the geometry is determined by the total energy. But whether the modified gravity theories could also inherit such point of view should deserve further investigation. Moreover, it has to be stressed that the universal form \(1\) of mass formula is not the 3D extension of Smarr formula which requires an extended thermodynamic phase space in a new method called as “black hole chemistry” \(26\), as pointed out in a recent Ref. \(27\) that the scaling properties of the various thermodynamic parameters for BTZ black holes in Eq. \(1\) are inconsistent with the requirement of Smarr formula. But whether the extended Smarr formula in 3D can be consistently applied to all cases as done for the universal form of Eq. \(1\) is unclear now and deserve a further investigation.

V. ACKNOWLEDGEMENTS

The author would like to thank Prof. Townsend for bringing the problem to him and for reading and revising this paper, and thanks the anonymous referee for his/her critical comments and helpful advice. This work is supported by Grant No. 11374330 of the National Natural Science Foundation of China, the Open Research Fund Program of the State Key Laboratory of Low-Dimensional Quantum Physics in Tsinghua University, and the
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