The $\rho$ Meson in a Nuclear Medium

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Abstract

In this work, propagation properties of the $\rho$ meson in symmetric nuclear matter are studied. We make use of a coupled channel unitary approach to meson-meson scattering, calculated from the lowest order Chiral Perturbation Theory ($\chi$PT) lagrangian including explicit resonance fields. Low energy chiral constraints are considered by matching our expressions to those of one loop $\chi$PT. To account for the medium corrections, the $\rho$ couples to $\pi\pi$ and $K\bar{K}$ pairs which are properly renormalized in the nuclear medium.

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1 Description of the model

We study the $\rho$ propagation properties by obtaining the $\pi\pi$ and $K\bar{K}$ scattering amplitudes in the $I = 1$ channel. Our states in the isospin basis are

$$ |\pi\pi> = \frac{1}{2} |\pi^+\pi^- - \pi^-\pi^+ > $$

$$ |K\bar{K}> = \frac{1}{\sqrt{2}} |K^+\bar{K}^- - K^-\bar{K}^+ > . $$ (1)

Tree level amplitudes are obtained from the lowest order $\chi$PT and explicit resonance field lagrangians of refs. [1], [2]. We collect this amplitudes in a $2 \times 2$ $K$ matrix whose elements are

$$ K_{11}(s) = \frac{1}{3} \frac{p_1^2}{f^2} \left[ 1 + 2 \frac{G_V^2}{f^2} \frac{s}{M_{\rho}^2 - s} \right] $$

$$ K_{12}(s) = \sqrt{2} \frac{p_1 p_2}{3} \frac{1}{f^2} \left[ 1 + 2 \frac{G_V^2}{f^2} \frac{s}{M_{\rho}^2 - s} \right] $$

$$ K_{21}(s) = K_{12}(s) $$

$$ K_{22}(s) = \frac{2}{3} \frac{p_2^2}{f^2} \left[ 1 + 2 \frac{G_V^2}{f^2} \frac{s}{M_{\rho}^2 - s} \right] $$ (2)

with the labels 1 for $\pi\pi$ and 2 for $K\bar{K}$ states. In equation (2) $G_V$ is the strength of the pseudoscalar-vector resonance vertex, $f$ the pion decay constant in the chiral limit, $s$ the squared invariant mass, $M_{\rho}$ the bare mass of the $\rho$ meson and $p_i = \sqrt{s/4 - m_i^2}$. 

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The final expression of the $T$ matrix is obtained following the N/D method, which was adapted to the context of chiral theory in ref. [3]. It reads

$$T(s) = [I + K(s) \cdot g(s)]^{-1} \cdot K(s), \quad (3)$$

where $g(s)$ is a diagonal matrix given by the loop of two mesons. In dimensional regularization it reads

$$g_i(s) = \frac{1}{16\pi^2} \left[ -2 + d_i + \sigma_i(s) \log \frac{\sigma_i(s) + 1}{\sigma_i(s) - 1} \right], \quad (4)$$

where the subindex $i$ refers to the corresponding two meson state and $\sigma_i(s) = \sqrt{1 - 4m_i^2/s}$ with $m_i$ the mass of the particles in the state $i$. At this stage (vacuum case), the model has proved to be successful in describing $\pi\pi$ P-wave phase shifts and $\pi$, $K$ electromagnetic vector form factors [4] up to $\sqrt{s} \lesssim 1.2$ GeV. The $d_i$ constants contain the information of low energy chiral constraints. They are obtained by matching the expressions of the form factors calculated in this approach with those of one-loop $\chi PT$.

In our calculation, in which $g(s)$ is modified in the nuclear medium, we use cut-off regularization. The $g_i(s)$ function with a cut-off in the three-momentum of the particles in the loop can be found in Appendix A of ref. [5]. By comparing the expressions in both schemes we can get the equivalent $q_i^{\text{max}}$ in order to keep the information of the $d_i$ constants.

Medium corrections are incorporated in the selfenergies of the mesons in the loops. All the graphs in fig. 1 are considered. In addition to the single $p - h$ bubble we have to account for the medium modifications of the $\rho M M'$ vertex via the $\rho M N$ contact term requested by the gauge invariance of the theory.

Medium corrections graphs: double solid line represents the $\rho$ resonance, dashed lines are $\pi$, $K$ mesons in the loop and single solid lines are reserved for particle-hole excitations.
The pion selfenergy is written as usual in terms of the Lindhard functions. Both $N-h$ and $\Delta-h$ excitations are included. Short range correlations are also accounted for with the Landau-Migdal parameter $g'$, set to 0.7. The final expression is

$$\Pi_\pi(q, \rho) = q^2 \frac{\left(\frac{D+P}{2F}\right)^2 U(q, \rho)}{1 - \left(\frac{D+P}{2F}\right)^2 g' U(q, \rho)}$$

(5)

where $U = U_N + U_\Delta$, the ordinary Lindhard function for $p-h$, $\Delta-h$ excitations [6].

The $\bar{K}$ selfenergy has both S-wave and P-wave contributions. The S-wave piece is obtained from a self-consistent calculation with coupled channels ($\bar{K}N$, $\pi \Sigma$, $\pi \Lambda$, $\eta \Sigma$, $\eta \Lambda$, $K \Xi$) in which both meson and baryon selfenergies in the medium have been considered. The P-wave piece includes $\Lambda-h$, $\Sigma-h$ and $\Sigma^*(1385)-h$ excitations. The whole $\bar{K}$ selfenergy is borrowed from ref. [7].

At low energies the $K$ system interacts with nucleons only by S-wave elastic scattering. We use the expression for the selfenergy from ref. [8, 9],

$$\Pi_K(\rho) \simeq 0.13 \frac{m_K^2 \rho}{\rho_0} (MeV^2)$$

(6)

![Figure 2: Real (a) and imaginary (b) parts of the amplitude $T_{\pi \pi \rightarrow \pi \pi}$](image)

Figure 2: Real (a) and imaginary (b) parts of the amplitude $T_{\pi \pi \rightarrow \pi \pi}$. The curves are as follows: Solid line, zero density; long dashed line, $\rho = \rho_0/16$; short dashed line, $\rho = \rho_0/2$; dotted line, normal nuclear density. $\sqrt{s}$ is the invariant mass of the meson pair.
2 Results and discussion

We have plotted in fig. 2 the real and imaginary parts of $T_{22}$ for several densities. As can be seen, the resonance broadens significantly as density is increased, its width being around 350 MeV at $\rho = \rho_0$. The zero of the real part experiences a downward shift which amounts to 100 – 150 MeV at normal density.

Another interesting result comes from the comparison between coupled and decoupled cases. This is done by setting $K_{12}(s)$ to zero, what automatically makes the $T$ matrix diagonal. We have found very small differences in $T_{22}$ when calculating in these two cases. This tells us that in our model the $K\bar{K}$ system has almost no influence on the $\pi\pi \rightarrow \pi\pi$ channel even at normal nuclear density.

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