Study on the Quantitative Relationship Between Harmonic Amplification and Cable Length

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This work was supported in part by the National Natural Science Foundation of China under Grant 51877141 and Grant 51842703.

ABSTRACT With the large increase of the wide-frequency and high-order harmonic in power electronic based systems as well as the wide use of cables with large distributed capacitance in modern power grids especially renewable energy generations, the harmonic amplification in cables becomes a serious problem. Some researchers have paid attention to this problem, but the quantitative relationship between harmonic amplification and cable length has not been determined, which is studied in this paper to cover the shortage. First, the influence factors of harmonic amplification are converted into two rotation phasors. These factors, including cable parameters, harmonic frequency and equivalent harmonic impedance of power system, are converted into the initial amplitude, phase and angular frequency of the rotation phasors. Then, the solution of harmonic amplification factor (HAF) is converted to solve the reciprocal of the amplitude of the sum of the two rotation phasors. With the cosine law, HAF and the cable length segments where harmonic exceeds the limit can be figured out quantitatively. The analysis method and conclusions proposed can help to guide cable length planning for avoiding harmonic exceeding the limit or to analyze the cause of harmonic exceeding the limit. Correctness and effectiveness are verified by the simulation and field cases.

INDEX TERMS Harmonic amplification, distribution parameters, cable length, quantitative relationship.

I. INTRODUCTION

Driven by the massive increase of renewable energy generations and energy-saving applications, the legacy power grids are evolving as power electronic based power systems, and a large number of harmonics with wide-frequency and high-order are emitted by power electronics [1]–[6]. Meanwhile, the cables with large distributed capacitance are used widely in the power electronic based power systems, especially renewable energy generations [7]–[9]. Therefore, the harmonic amplification (HA) in cables becomes a serious problem, and the HA only considered in long cables previously may occur in shorter ones. HA may result in harmonic exceeding the limit and cause a series of problems, such as serious voltage distortion, equipment mal-operation, interference problems of secondary measurement systems etc [10]–[12]. HA is bringing new challenges for power electronic-based power systems.

At present, HA in cables has attracted some attention [13]–[17]. References [13]–[15] study the harmonic resonance amplification in wind farm through sensitivity analysis and real-time digital simulator, and the influence of cable distribution parameters is included in the wind farm modeling. However, these research about HA mainly focus on the whole wind farm, rather than the certain transmission cable. Reference [16] studies the resonance amplification caused by the distribution parameter of the high-voltage submarine transmission cable in wind farm. Reference [17] studies the influence of portable trailing cable length and short-circuit capacity of mine substation on resonance frequency. But [16], [17] only focus on the resonance peak value and resonance frequency. In the studies of wideband harmonic resonance bands in long overhead lines [18], [19] and cable feeders [20], the relationship between HA and power line length is involved, while the results about HA are obtained by simulation with specific power line parameters, and the theoretical derivation reflecting the relationship between HA and cable length is not proposed.
Therefore, the quantitative relationship between HA and cable length is studied in this paper. First, the influence factors of harmonic amplification are converted into two rotation phasors. These factors, including cable parameters, harmonic frequency and equivalent harmonic impedance of power system, are converted into in the initial amplitude, phase and angular frequency of the rotation phasors. Then, the solving of harmonic amplification factor (HAF), that is the harmonic amplitude ratio between the terminal and the beginning of cable, is transformed into the solving of the reciprocal of the amplitude of the sum of the two rotation phasors. Last, with the cosines law, HAF is calculated and the cable length segments where harmonic exceeds limit are figured out quantitatively. The main features and benefits of this paper are listed as follows:

1) The paper determines the quantitative relationship between HA and the cable length with the rigorous derivation. With the quantitative relationship, we can determine the HAF, harmonic amplification period and the length segment of harmonic exceeding the limit, etc.

2) With the quantitative relationship, we can guide the cable length planning for avoiding the harmonic exceeding the National standard limit;

3) With the quantitative relationship, we can analyze the cause of harmonic exceeding the limit at the length segment of harmonic exceeding the limit, etc.

This paper is organized as follows. Section II derives the quantitative relationship between HA and cable length. Section III and IV verifies the correctness and effectiveness of the studies by the simulation and field cases respectively. Section V concludes the paper.

II. DERIVATION OF THE QUANTITIVE RELATIONSHIP BETWEEN HARMONIC AMPLIFICATION AND CABLE LENGTH

Figure 1 shows a typical equivalent circuit of the cable line connecting the harmonic source and the power system. The connection points are bus 1 and bus 2 respectively, and the bus 2 is the PCC between the harmonic source and power system.

![FIGURE 1. The typical equivalent circuit of the transmission cable.](image)

In Figure 1, \( \dot{I}_s \) and \( Z_s \) are the equivalent background harmonic source and the harmonic impedance of power system, respectively. \( \dot{U}_{1h} \), \( \dot{I}_{1h} \) and \( \dot{U}_{2h}, \dot{I}_{2h} \) denote the harmonic voltage and current of the beginning and terminal of the cable respectively, and the subscript \( h \) denotes the order of harmonic. \( l \) is the total length of cable line. Obviously, the harmonic of the PCC is jointly determined by the background harmonic from the power system and the harmonic source. This paper mainly focuses on HA along cable from harmonic source to power system, so the background harmonic can be regarded as very small and set to zero. In the situation that the background harmonic is large, we can analyze the HA with the superposition theorem. Moreover, there are many methods to distinguish the harmonic emission level of the system side and customer side [21]–[25].

According to Figure 1, we have [26]

\[
\begin{align*}
\dot{U}_{1h} &= \frac{1}{2} \left( \dot{U}_{2h} + Z_c \dot{I}_{2h} e^{j\gamma l} + \frac{1}{2} \left( \dot{U}_{2h} - Z_c \dot{I}_{2h} e^{-j\gamma l} \right) \right) \\
\dot{I}_{1h} &= \frac{1}{2Z_e} \left( \dot{U}_{2h} + Z_c \dot{I}_{2h} e^{j\gamma l} - \frac{1}{2} \left( \dot{U}_{2h} - Z_c \dot{I}_{2h} e^{-j\gamma l} \right) \right)
\end{align*}
\]

(1)

For brevity, the subscript “\( h \)” are omitted in the latter of this paper. \( Z_c \) and \( \gamma \) are the characteristic impedance and the propagation constant of the cable, respectively, and can be calculated as [20]

\[
\begin{align*}
Z_c &= \sqrt{(R_0 + j\omega L_0)/j\omega C_0} \\
\gamma &= \alpha + j\beta = \left( R_0/2 \right) \sqrt{C_0/L_0} + j\omega \sqrt{C_0 L_0}
\end{align*}
\]

(2)

(3)

In Eq. (2) and (3), the conductance of the cable is regarded as 0 and has been omitted, \( \alpha \) and \( \beta \) are the attenuation constant and phase shifting constant. \( C_0 \), \( L_0 \) and \( R_0 \) are the values of capacitance, inductance and resistance per kilometer, respectively. \( \omega \) denotes the angular frequency.

Define the harmonic voltage amplitude ratio and the harmonic current amplitude ratio between the terminal and the beginning of cable as harmonic voltage amplification factor \( H_u \) and harmonic current amplification factor \( H_i \), respectively. And for simplicity, the uniform name for harmonic voltage and current amplification is named as harmonic amplification factor (HAF) in the latter of this paper. According to Figure 1, with \( \dot{I}_2 = \dot{U}_2/Z_s \), we have

\[
H_u = \frac{1}{0.5 (1 + Z_c/Z_s) e^{al} e^{j\beta l} + 0.5 (1 - Z_c/Z_s) e^{-al} e^{-j\beta l}}
\]

(4)

\[
H_i = \frac{1}{0.5 (1 + Z_s/Z_c) e^{al} e^{j\beta l} + 0.5 (1 - Z_s/Z_c) e^{-al} e^{-j\beta l}}
\]

(5)

In Eq.(4) and (5), \( H_u > 1 \) or \( H_i > 1 \) represent that the terminal harmonic is amplified relative to the beginning of cable.

A. HARMONIC VOLTAGE AMPLIFICATION IN LOSSLESS CABLE

In this section, the harmonic voltage amplification characteristics are studied with the lossless and lossy cable, respectively. For lossless cable, i.e., \( R_0 = 0 \), we have \( \alpha = 0 \)
rotating counterclockwise (denoted as the magnitude of a phasor, which is the summation with the supplementary angle of the phase difference between \(\dot{\alpha}\) and \(\dot{\beta}\)). Are constant with \(l\) increasing. And let \(\dot{A}\) and \(\dot{B}\) denote the magnitudes of \(\dot{A}\) and \(\dot{B}\), \(\theta_1\) and \(\theta_2\) denote the initial phase angles of \(\dot{A}\) and \(\dot{B}\), respectively, then we have

\[
H_u = \frac{1}{|\dot{T}_u|} = \frac{1}{|\dot{A}e^{\dot{\alpha}l} + Be^{-\dot{\beta}l}|} = |\dot{A}' + \dot{B}'| = |\dot{T}_u| \tag{6}
\]

Observing Eq. (6), \(e^{\dot{\alpha}l}\) and \(e^{-\dot{\beta}l}\) are the rotation factors of \(\dot{A}\) and \(\dot{B}\) varying with \(l\), and \(|\dot{T}_u|\) can be regarded as the magnitude of a phasor, which is the summation with the supplementary angle of the phase difference between \(\dot{\alpha}\) and \(\dot{\beta}\), and the amplitudes of \(\dot{A}\) and \(\dot{B}\) are constant with \(l\) increasing. In Figure 2, |\dot{T}_u| depends on \(\sin(\theta_1 - \theta_2)\). And with \(\dot{A}\) and \(\dot{Z}_c/\dot{Z}_s\) in the fourth quadrant and \(\dot{B}\) in the first or second quadrant, we can determine that \(\dot{A}\) is in the fourth quadrant, and \(\dot{B}\) is in the first or second quadrant. Synthesize the above description, when \(\dot{B}\) is in the first quadrant, we can obtain Figure 2.

For lossless cable, \(R_0 = 0\), \(Z_c\) is a pure resistor according to Eq.(2), therefore, the impedance angle is 0, \(Z_c\) is the equivalent impedance of terminal power system and can be estimated by many methods [21]–[25], which can be regarded as inductive and the real and imaginary parts of \(Z_c\) are usually positive [2]. Therefore the impedance phase angle of \(Z_c\) is in the interval \(0 \sim \pi/2\) and \(Z_c/\dot{Z}_s\) is in the fourth quadrant. Meanwhile, we can derive that \(\dot{T}_u\) can be regarded as half of the summation of the complex phasor (1, 0j) and \(Z_c/\dot{Z}_s\), and \(\dot{B}\) is half of the difference between the complex phasor (1, 0j) and \(Z_c/\dot{Z}_s\). Thus, we can determine that \(\dot{A}\) is in the fourth quadrant, and \(\dot{B}\) is in the first or second quadrant. Synthesize the above description, when \(\dot{B}\) is in the first quadrant, we can obtain Figure 2.

According to Figure 2 and cosine law, we have

\[
T_u = \sqrt{A'^2 + B'^2 - 2A'B'\cos(\theta)} \tag{8}
\]

where \(T_u\), \(A'\) and \(B'\) denote the magnitude of the phasors \(\dot{T}_u\), \(\dot{A}'\) and \(\dot{B}'\), respectively, and \(A' = A, B' = B\). \(\theta\) is the supplementary angle of the phase difference between \(\dot{A}'\) and \(\dot{B}'\), therefore \(\theta = \pi - (2\dot{\beta}l + \theta_1 - \theta_2)\), Eq.(8) can be rewritten as

\[
T_u = \sqrt{A^2 + B^2 + 2AB\cos(2\dot{\beta}l + \theta_1 - \theta_2)} \tag{9}
\]

Note that in the derivation of Eq.(9), there is no constraint for the position of \(\dot{B}\). Therefore we can draw the following four conclusions for harmonic voltage amplification.

1) PERIOD OF HARMONIC AMPLIFICATION

\(T_u\) and \(H_u\) maintain the same periodic fluctuation dominated by cosine function with \(l\) increasing (see Eq.(9)), which indicates that the harmonic voltage is periodically amplified in lossless cable lines, and the period is

\[
T = \pi/\sqrt{(\omega/\sqrt{C_0L_0})} \tag{10}
\]

2) TREND OF HARMONIC AMPLIFICATION AT THE BEGINNING OF CABLE

By calculating the value and the sign of the derivative of \(T_u\) with respect to \(l\), in Figure 3, whether \(\dot{B}\) is in the first or second quadrant, \(|\dot{\theta}_1|, |\dot{\theta}_2| < \pi\) always holds since the sum of the two angles of a triangle is less than \(\pi\), i.e., \(-\pi < \dot{\theta}_1 - \dot{\theta}_2 < 0\) and \(\sin(\dot{\theta}_1 - \dot{\theta}_2) < 0\). Thus, we have \(\partial T_u/\partial l > 0\), \(T_u\) increases and \(H_u\) decreases with \(l\) increasing, and \(H_u = T_u = 1\) at \(l = 0\) according to Eq.(6). Therefore, at the beginning of lossless cable, the harmonic voltage is always attenuated.

3) MAXIMUM OF HARMONIC AMPLIFICATION

With the periodicity of \(H_u\), \(H_u\) will be amplified during some segments of cable. In Eq.(9), \(H_u\) takes the maximum when the value of cosine function is \(-1\), and the maximum value \(H_u^{\max}\) is

\[
H_u^{\max} = 1/|A - B| \tag{12}
\]
and the value of \( l \) corresponding to \( H_u^{\text{max}} \) is
\[
I^{\text{max}}(n) = \frac{(2n + 1)\pi + \theta_2 - \theta_1}{2\omega\sqrt{C_0L_0}}
\]
where \( n \) is an integer, and \( n \geq 0 \).

4) CABLE LENGTH SEGMENTS OF HARMONIC AMPLIFICATION

When \( H_u > 1 \), the harmonic voltage will be amplified. Further, if \( H_u \) keeps increasing and exceeds a certain limit denoted as \( H_u^{\text{lim}} \), set according to some standards, the harmonic voltage will exceed the limit. By setting \( T_u = 1/H_u^{\text{lim}} \) in Eq.(9) and solving the \( l \), we can obtain the values of length corresponding to \( H_u = H_u^{\text{lim}} \),
\[
\begin{align*}
I_{\text{HEL}}^-(n) &= 2n\pi + \theta_2 - \theta_1 + ar\cos\left(\frac{1/H_u^{\text{lim}} - \theta_1 - \theta_2}{2AB}\right) \\
I_{\text{HEL}}^+(n) &= 2(n + 1)\pi + \theta_2 - \theta_1 - ar\cos\left(\frac{1/H_u^{\text{lim}} - \theta_1 - \theta_2}{2AB}\right)
\end{align*}
\]
where \( n \) is a integer, and \( n \geq 0 \). \( I_{\text{HEL}}^-(n) \) and \( I_{\text{HEL}}^+(n) \) represent the beginning and ending length of harmonic exceeding limit respectively, and the cable length segments between \( I_{\text{HEL}}^-(n) \) and \( I_{\text{HEL}}^+(n) \) is defined as the segments where harmonic exceeds limit (SHEL). Note that \( H_u = 1 \) corresponds to the critical length of HA, and the beginning and ending length of HA are denoted as \( l_{HA}^- \) and \( l_{HA}^+ \) respectively. For calculating \( l_{HA}^- \) and \( l_{HA}^+ \), we just need to replace \( H_u^{\text{lim}} \) in Eq.(14) with 1, and the cable length segments between \( l_{HA}^- \) and \( l_{HA}^+ \) is defined as the segments where harmonic is amplified (SHA).

Combined with the period, trend at the beginning of cable, maximum and cable length segments of harmonic amplification, we can determine the amplification situation of harmonic voltage in the cable easily and intuitively.

B. HARMONIC VOLTAGE AMPLIFICATION IN LOSSLY CABLE

When resistance is considered in the cable, \( \alpha \neq 0 \), \( e^{\alpha l} \) keeps increasing and \( e^{-\alpha l} \) keep decreasing with \( l \) increasing. Thus, the magnitude of \( \tilde{A} \) and \( \tilde{B} \) in Figure 2 are not constants anymore with \( l \) increasing, then Eq.(9) should be rewritten as
\[
T_u = \sqrt{(Ae^{\alpha l})^2 + (Be^{-\alpha l})^2 + 2AB\cos(2\beta l + \theta_1 - \theta_2)}
\]
In Eq. (15), the expression under the square root can be regarded as the sum of \( T_1 \) and \( T_2 \), which are respectively expressed as follows.
\[
\begin{align*}
T_1 &= (Ae^{\alpha l})^2 + (Be^{-\alpha l})^2 \\
T_2 &= 2AB\cos(2\beta l + \theta_1 - \theta_2)
\end{align*}
\]
\( T_2 \) is the same as the corresponding part in lossless cable, and the harmonic amplification characteristic difference between the lossy and lossless cable is determined by \( T_1 \). Take the derivative of \( T_1 \) with respect to \( l \), we have
\[
\frac{\partial T_1}{\partial l} = 2\alpha A^2 e^{2\alpha l} - 2\alpha B^2 e^{-2\alpha l}
\]
From Eq.(2), we can know \( R_0 \) has little effect on \( Z_c \) at harmonic frequency, therefore \( Z_c/Z_s \) can be considered the same as that in lossless cable and Figure 3 can be still applicable to harmonic amplification analysis in lossy cable. Let \( u = |Z_c/Z_s| \), combining Figure 3 and cosine law we have
\[
\begin{align*}
A &= 0.5\sqrt{1 + u^2 + 2u\cos\varphi} \\
B &= 0.5\sqrt{1 + u^2 - 2u\cos\varphi}
\end{align*}
\]
where \( \varphi \) is the angle between complex phasor \( (1, 0j) \) and \( Z_c/Z_s \). For \(-\pi/2 < \varphi < 0\), we have \( A > B \), meanwhile, with \( \alpha l > 0 \), then \( e^{2\alpha l} > e^{-2\alpha l} \) holds. Hence we have \( \partial T_1/\partial l > 0 \) and \( T_u \) can be determined combining the monotonically increasing function \( T_1 \) and the periodic function \( T_2 \). Therefore in lossy cable, we can draw the following conclusions.

1) PERIOD AND AMPLITUDE OF HARMONIC AMPLIFICATION

Similarly to lossless cable, \( H_u \) fluctuates periodically and the period is same as that in lossy cable (see Eq.(10)). With \( l \) increasing, \( T_1 \) increases and the envelope of maximum value of \( T_u \) in each period increases. With \( H_u = 1/T_u \), the envelope of \( H_u^{\text{max}} \) in each period decreases, which represents that the resistance in cable can suppress harmonics amplification.

2) TREND OF HARMONIC AMPLIFICATION AT THE BEGINNING OF CABLE

Take the derivative of \( T_u \) with respect to \( l \) in Eq. (15) and taking \( l = 0 \), then
\[
\frac{\partial T_u}{\partial l} = \frac{1}{2T_u}\left(\frac{\partial T_1}{\partial l} - 4AB\beta\sin(\theta_1 - \theta_2)\right)
\]
In Eq.(19), \( T_u > 0 \) and \( \beta > 0 \), the sign of \( \partial T_u/\partial l \) depends on \( \sin(\theta_1 - \theta_2) \) and \( \partial T_1/\partial l \), but we can determine that the value of \( \partial T_1/\partial l \) is close to 0 in the supply distance of the cable, usually no more than 50km, for three reasons as follows.

a). The \( R_0 \) of cable is usually small and \( C_0 \ll L_0 \) at the same time. According to \( \alpha = 0.5R_0/\sqrt{C_0L_0} \), we determine the value of \( \alpha \) is small enough, usually in the order of magnitude of \( 10^{-3} \) or less.

b). Compared to fundamental frequency, \( Z_c \) has little change at harmonic frequency (see Eq.(2)), but \( Z_s \) increases significantly with the frequency. Therefore the amplitude of \( Z_c/Z_s \) is small, and the values of \( A \) and \( B \) are also small, usually no more than 1.

c). According to Eq. (17), \( \partial T_1/\partial l \) is the difference of two very small positive values, which is reduced further.
Therefore, similarly to lossless cable, when \( l = 0 \), \( \partial T_1/\partial l > 0 \), which means the harmonic voltage is always attenuated at the beginning of cable.

3) MAXIMUM OF HARMONIC AMPLIFICATION

The \( \max\)(n) occurs when Eq. (20) is satisfied.

\[
\frac{\partial T_1}{\partial l} + \frac{\partial T_2}{\partial l} = 0 \quad (20)
\]

For \( \frac{\partial T_1}{\partial l} \approx 0 \), the value of \( \max\)(n) in lossy cable is approximately equal to that in lossless cable, and we have

\[
H_u^{\max}(n) = \frac{1}{Ae^{\alpha\max(n)} - Re^{-\alpha\max(n)}} \quad (21)
\]

4) CABLE LENGTH SEGMENTS OF HARMONIC AMPLIFICATION

For \( l \), Eq.(15) is a transcendental equation, so we can get the analytical solutions of \( l_{\text{HEL}}^{-}(n) \), \( l_{\text{HEL}}^{+}(n) \) and \( l_{\text{HA}}^{-}(n) \), \( l_{\text{HA}}^{+}(n) \) in lossy cable, therefore the Newton iteration method can be employed to solve them. Because the value of \( \alpha \) is small enough, there is little difference between the \( l_{\text{HEL}}^{-}(n) \), \( l_{\text{HEL}}^{+}(n) \) and \( l_{\text{HA}}^{-}(n) \), \( l_{\text{HA}}^{+}(n) \) in lossless cable solved from Eq.(9) and in lossy cable solved from Eq.(15). Therefore, they can be used as the initial values of Newton iterative method to solve corresponding exact solutions in lossy cables. Newton iterative formula is (22), as shown at the bottom of the next page, where \( k \) denotes the number of iterations, \( l_k(n) \) corresponds to the \( l_{\text{HEL}}^{-}(n) \), \( l_{\text{HEL}}^{+}(n) \) and \( l_{\text{HA}}^{-}(n) \), \( l_{\text{HA}}^{+}(n) \) in lossy cable for different \( l_0(n) \), respectively. When solving the value of SHEL, the \( l_0(n) = l_{\text{HEL}}^{-}(n) \) or \( l_0(n) = l_{\text{HEL}}^{+}(n) \) and \( H_u = (1/H_u^{\text{lim}})^2 \) in Eq.(22). And when solving the value of SHA, the \( l_0(n) = l_{\text{HA}}^{-}(n) \) or \( l_0(n) = l_{\text{HA}}^{+}(n) \), and \( H_u = 1 \).

C. HARMONIC CURRENT AMPLIFICATION IN LOSSLESS CABLE

In this section, the harmonic current amplification characteristics are studied in lossless and lossy cable, respectively. For lossless cable, Eq. (5) can be rewritten as

\[
H_i = \frac{1}{0.5(1 + Z_s/Z_c)e^{j\beta} + 0.5(1 - Z_s/Z_c)e^{-j\beta}}
\]

\[
= \frac{1}{|M|e^{j\beta} + |N|e^{-j\beta}} = \frac{1}{|M + N|} = \frac{1}{|T_1|} \quad (23)
\]

Let \( M \) and \( N \) denote the magnitudes of \( \hat{M} \) and \( \hat{N} \), \( \gamma_1 \) and \( \gamma_2 \) denote the initial phases of \( \hat{M} \) and \( \hat{N} \), respectively, then Eq.(23) can be rewritten as

\[
H_i = \frac{1}{|T_1|} = \frac{1}{|Me^{j\gamma_1}e^{j\beta} + Ne^{j\gamma_2}e^{-j\beta}|} \quad (24)
\]

Based on the analysis of the harmonic voltage amplification in the section A, the phase range of \( Z_s/Z_c \) is in the first quadrant of the coordinate axis. Meanwhile, \( \hat{M} \) can be regarded as half of the summation of the complex phasor \((1, 0j)\) and \( Z_s/Z_c \), and \( \hat{N} \) is half of the difference between the complex phasor \((1, 0j)\) and \( Z_s/Z_c \). Thus, we can determine that the phasor \( \hat{M} \) is in the first quadrant, and that \( \hat{N} \) is in the third or fourth quadrant. When \( \hat{N} \) is in the fourth quadrant, Figure 4 can be obtained.

According to Figure 4 and law of cosines, we have

\[
T_i = \sqrt{M^2 + N^2 + 2MN \cos(2\beta + \gamma_1 - \gamma_2)} \quad (25)
\]

Similar to Eq.(9), Eq.(25) also hold while \( \hat{N} \) is in the third quadrant and we can draw the following conclusions.

1) PERIOD OF HARMONIC AMPLIFICATION

Harmonic current and voltage have the same period of harmonic amplification.

2) TREND OF HARMONIC AMPLIFICATION AT THE BEGINNING OF CABLE

Take the derivative of \( T_i \) with respect to \( l \) in Eq.(25) and taking \( l = 0 \), then

\[
\frac{\partial T_i}{\partial l} = \frac{-2MN\beta}{T_i \sin(\gamma_1 - \gamma_2)} \quad (26)
\]

In Eq.(26), \( \gamma_1 > 0 \) and \( \gamma_2 < 0 \), through the analysis similar to Figure 3, it can be determined that \( 0 < \gamma_1 - \gamma_2 \leq \pi \). According to Eq.(26), \( \sin(\gamma_1 - \gamma_2) > 0, T_i > 0 \) and \( \beta > 0 \), thus \( \partial T_i/\partial l < 0 \), which means \( H_i \) increases with \( l \) increasing and \( H_i \approx 1 \) at \( l = 0 \), i.e., the harmonic current is always amplified at the beginning of lossless cable.

3) MAXIMUM AND CABLE LENGTH SEGMENTS OF HARMONIC AMPLIFICATION

Eq.(25) has the completely same structure as Eq.(9), so the expressions of \( H_i^{\max}(n), l^{\max}(n), l_{\text{HEL}}^{-}(n) \) and \( l_{\text{HEL}}^{+}(n) \) of harmonic current also have the same structure as Eq.(12)-(14) respectively, but \( A, B, \theta_1 \) and \( \theta_2 \) in these equations should be replaced by corresponding \( M, N, \gamma_1 \) and \( \gamma_2 \), respectively.

D. HARMONIC CURRENT AMPLIFICATION IN LOSSLY CABLE

When resistance is considered in cable, it can be concluded that the influence of resistance on harmonic current amplification characteristics in lossy cable is the same as that of harmonic voltage. The analysis can refer to the derivation of harmonic voltage. For simplicity, the similar content is omitted here.
III. SIMULATION CASE

In this section, to verify the validity of the proposed quantitative relationship between HA and cable length, we compare the results of calculated by formulas derived in section II with that obtained from simulation circuit in Matlab/Simulink based on Figure 1. The parameters of cable and $Z_s$ at fundamental frequency are shown in Table 1. In the simulation case, the harmonic orders include typical 5, 7, 11, 13, 17, 19, 23 and 25 in renewable energy generations.

### TABLE 1. Parameters of the 220kV cable and equivalent harmonic impedance.

| $U_n$ (kV) | $R_0$ (Ω) | $L_0$ (mH) | $C_0$ (μF) | $l$ (km) | $Z_s$ (Ω) |
|------------|------------|------------|------------|----------|------------|
| 220        | 0.031      | 0.406      | 0.179      | 40       | 1.6+j16    |

A. THE HARMONIC VOLTAGE AMPLIFICATION

Suppose harmonic voltage of each order at the beginning of the cable is 0.8%, and the limit value of the terminal of cable is 1.6%, therefore, if the length of cable is in the length segments where $H_u$ is more than 2, the harmonic voltage will exceed the limit. We can obtain the $H_u^{\text{max}}$, $I_u^{\text{max}}$, SHEL and SHA of the harmonic voltage of each order in cable referring to Eq.(13)-(14). Eq.(21)-(22), which are shown in Table 2. In Table 2, the unit of length is km, the results for the length beyond the cable total length (i.e. 40km) are not shown, and if $l^{\text{max}} > 40$, the corresponding $H_u^{\text{max}}$ will be omitted.

### TABLE 2. Results of harmonic voltage amplification.

| $h$ | SHA | SHEL | $I_u^{\text{max}}$ (1) | $H_u^{\text{max}}$ (1) |
|-----|-----|------|------------------------|------------------------|
| 5   | >40 | >40  | >40                    | >40                    |
| 7   | >40 | >40  | >40                    | >40                    |
| 11  | >40 | >40  | >40                    | >40                    |
| 13  | >40 | >40  | >40                    | >40                    |
| 17  | >40 | >40  | >40                    | >40                    |
| 19  | >40 | >40  | >40                    | >40                    |
| 23  | >40 | >40  | >40                    | >40                    |
| 25  | >40 | >40  | >40                    | >40                    |

Meanwhile, the simulation results are shown in Figure 5. Figure 5(a) shows the amplification of each order harmonic voltage with different cable length. Figure 5(b) and (c) are the partial enlarged drawing of Figure 5(a). Figure 5(b) and (c) respectively show the regions where $0 < H_u < 3$ and cable length less than 2km.

According to Figure 5 and the results in Table 2, we can see that the $H_u^{\text{max}}$, $I_u^{\text{max}}$, SHEL and SHA in Figure 5 are consistent with what are obtained by the proposed method, which proves that the values of $H_u^{\text{max}}$, $I_u^{\text{max}}$, SHEL and SHA can be calculated accurately and quickly by the proposed analysis method and conclusions. In addition, from the Figure 5(c), we can see that each order harmonic voltage is always attenuated at the beginning of cable. Finally, it can be found that the higher the order is, the more obvious the harmonic voltage amplifies.

B. THE HARMONIC CURRENT AMPLIFICATION

Suppose harmonic current of each order at the beginning of the cable is 1A, and the limit value of the terminal of cable is 2A, therefore, if the length of cable is in some length segments where $H_i$ is more than 2, the harmonic current will exceed the limit. We can obtain the $H_i^{\text{max}}$, $I_i^{\text{max}}$, SHEL and SHA of the harmonic current of each order in cable referring to

$$
\begin{align*}
  l_0(n) &= l_{\text{HEL}}(n), l_{\text{HEL}}(n), l_{\text{HA}}(n), l_{\text{HA}}(n) \\
  l_{k+1}(n) &= l_k(n) - \frac{(Ae^{a_k(n)})^2 + (Be^{-a_k(n)})^2 + 2AB \cos(2\beta l_k(n) + \theta_1 - \theta_2) - H_u}{2\alpha (Ae^{a_k(n)})^2 - 2\alpha (Be^{-a_k(n)})^2 + 4AB\beta \sin(2\beta l_k(n) + \theta_1 - \theta_2)}
\end{align*}
$$

(22)
Eq. (13)-(14) and Eq. (21)-(22), which are showed in Table 3. The presentation of results in Table 3 is the same as that in Table 2.

The simulation results are shown in Figure 6. Figure 6(a) shows the amplification of each harmonic current with different cable length, Figure 6(b) and (c) are the partial enlarged drawing of Figure 6(a). Figure 6(b) and (c) respectively show the regions where \(0 < H_i < 3\) and cable length less than 2km.

IV. FIELD CASE

The schematic diagram of an offshore wind farm with power rating of 252 MVA and 63 wind turbines is shown in Figure 7.

![Offshore wind farm diagram](image)

The collector system comprises eight feeders with voltage level of 35 kV, every four the feeders converge at a collector bus, where the plant is integrated into the utility grid through 35/220kV main transformer in offshore station. Each wind turbines unit with a power rating of 4 MVA is connected to one feeder through a 0.69/35kV distributed transformer, and each feeder connects with 6–8 sets of wind turbines units. The length of the submarine cable connecting the offshore station and the onshore station is 10km. The parameters of the 220kV submarine cable at fundamental frequency are shown in Table 4. Defined the bus of the onshore station which connects the submarine cable as the point of common coupling (PCC, i.e. bus B), and the short-circuit capacity of PCC is 2644MVA.

![Submarine cable parameters](image)

TABLE 4. Cable line parameters of 220kV submarine cable.

| Voltage(kV) | \(R_d\) (\(\Omega\)) | \(L_d\) (mH) | \(C_d\) (uF) |
|------------|-----------------|-------------|-------------|
| 220        | 0.0283          | 0.306       | 0.106       |

Because the wind turbines are equipped with inverters, harmonics will be generated. Based on the harmonic measurement data of bus A, ICA method is used to divide the harmonic contributions [21]–[25], therefore, the harmonic voltage and harmonic current generated by the wind farm at bus A are obtained and shown in Figure 8. From Figure 8,
the main harmonics of the offshore wind farm are 5th, 7th, 11th and 13th. According to [27], the limits of harmonic current injected into PCC by the offshore wind farm and the allowable harmonic voltage are calculated and shown in table 5.

**TABLE 5. Limits of harmonic current and voltage at the PCC.**

| Harmonic order (h) | 5    | 7    | 11   | 13   |
|--------------------|------|------|------|------|
| Harmonic current (A) | 4.02 | 3.36 | 2.64 | 2.37 |
| Harmonic voltage (%) | 1.60 | 1.60 | 1.60 | 1.60 |

According to the limits in Table 5 and Figure 8, the limits of HAF of harmonic voltage and current ($H_{\text{lim} u}$ and $H_{\text{lim} i}$) can be calculated respectively, then the SHEL can be calculated and are shown in table 6.

From Table 6, the amplification length segment of the 13th harmonic current is 6.99~12.18 kilometers, and the 10km-length submarine cable is just contained in the length segment. Therefore, it can be predicted that the 13th harmonic current will exceed the limit at the PCC, but other harmonic current and all the harmonic voltage will not.

**TABLE 6. The SHEL of submarine cable.**

| h  | SHEL of harmonic current | SHEL of harmonic voltage |
|----|--------------------------|--------------------------|
|    | $l_{\text{HEL}} (1)$    | $l_{\text{HEL}} (1)$    |
| 5  | 35.36                    | 83.55                    |
| 7  | 14.87                    | 48.86                    |
| 11 | 10.14                    | 16.49                    |
| 13 | 6.99                     | 12.18                    |
|    | 35.36                    | 83.55                    |
|    | 205.10                   | 264.65                   |
|    | 140.90                   | 173.30                   |
|    | 85.47                    | 100.73                   |
|    | 69.16                    | 84.97                    |

Similar to bus A, based on the harmonic measurement data of bus B, divide the harmonic contributions, then the harmonic voltage and current generated by the wind farm and transmission cable are obtained and shown in Figure 9. From Figure 9 (b) we can know that the value of 13th harmonic current is 2.75A and does exceed the limit value of 2.37A, while other harmonic current and harmonic voltage don’t exceed. Meanwhile, because the $l_{\text{HEL}} (1)$ of the 11th harmonic current is 10.14km (nearly 10 km), the value of the 11th harmonic current is close to the limit value.

**FIGURE 9. The harmonic voltage and current at the PCC.**

Therefore, the results at the PCC are consistent with the calculated results. Further, comparing Figure 8 and 9, it is observed that all the harmonic voltage at the PCC reduce relative to bus A and that all harmonic current have the different degrees of amplification, which is consistent with the conclusions drawn in section II.

**V. CONCLUSION**

This paper proposes the quantitative relationship between harmonic amplification and cable length, and the main conclusions are drawn as follows:

1). The period, maximum value and cable length segments of harmonic amplification are determined by theoretical analysis, which is of great significance in cable length planning for avoiding harmonics exceeding the limit or in analyzing the cause of harmonic exceeding the limit.
2). It is proved that the harmonic voltage is always attenuated while the harmonic current is always amplified at the beginning of cable. And the higher the order of harmonic is, the more obvious the harmonic amplification is.

Although the distributed capacitance of overhead transmission line is small and the risk of harmonic amplification is lower than cable, the analysis method proposed is also applicable to the case that harmonic source is connected to the power system through overhead transmission line.

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