Winding Modes and Large Extra-Dimensions

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Abstract

We review briefly the main features of the Large Extra Dimensions scenario in the framework of weakly coupled Type I string theory. Kaluza-Klein (KK) excitations of the graviton are expected, whereas no KK modes for the gauge bosons arise if the gauge group is tied to a D3-brane. In this scenario, typical signatures such as direct production of KK modes of the graviton at high-energy colliders could test the size of the compactified dimensions. We point out that contrary to what considered in the literature on the subject, in the general case of anisotropic compactification Winding Modes of the Standard Model gauge bosons could also be directly observable, thus further constraining the model.
Large Extra Dimensions

The Newton law is tested at present from very large scale down to $R_{\text{exp}} \geq 1$ cm. However, the typical length scale where gravity is expected to become strongly interacting, thus requiring the inclusion of quantum effects, is $R_{\text{Planck}} \sim 10^{-33}$ cm. It is possible that deviations from the Einstein-Newton theory could be observed at the planned gravitational experiments [1] testing 4-dimensional gravity down to $R \sim 10 \mu$m.

In [2] a scenario was proposed where extra dimensions open at the sub-millimeter scale, thus modifying the 4-dimensional gravity. The relation between the fundamental $(4+n)$-dimensional Planck scale and the 4-dimensional one is

$$\frac{M_{\text{Planck}}^2}{8\pi} = R^n M_{(4+n)}^{2+n}$$

where $R$ is the size of the compactified dimensions. If the compactified volume ($V \sim R^n$) is large enough the fundamental Planck scale $M_{(4+n)}$ could be well below the 4-dimensional one, $M_{\text{Planck}} \sim 10^{19}$ GeV. If the fundamental Planck mass is at the TeV-scale, the long-standing hierarchy problem is solved in a natural way: in this framework, there is no hierarchy at all (or a very small one) between the electroweak scale and the fundamental Planck scale. New physics at the TeV-scale would be quantum gravity itself.

Since the only consistent theory of quantum gravity at present implies an underlying (10-dimensional) superstring theory and the string scale is related to the fundamental Planck scale, the scenario proposed in [2] naturally involves string theory at the TeV-scale, such as was soon noticed in [3]. The string scale is directly fixed to the 4-dimensional Planck scale in weakly coupled heterotic string theory, $M_s = \sqrt{\alpha} M_{\text{Planck}}$ (where $\alpha$ is the GUT gauge coupling), and thus it is not possible to lower it to the TeV-scale. In weakly coupled Type I string theory, however, the string scale is not uniquely fixed by the 4-dimensional Planck scale and the gauge coupling, the relation involving also the compactified volume $V$. It is then possible to achieve a low-energy string scale (see [4] for details on heterotic and Type I strings). The scenario proposed in [2, 3], then, implies the exciting possibility of direct experimental observation of stringy effects at the planned high energy colliders, such as LHC.

D-branes and String Modes

Due to the compactification of extra dimensions, gravitons and Standard Model particles acquire massive replicas, the Kaluza-Klein (KK) excitations, with mass $m_{KK} \propto 1/R$. If the radius is large enough to bring down the string scale to 1 TeV, these massive excitations are extremely light. If the compactification radius is $\sim 1$ mm, $m_{KK} \sim 10^{-12}$ GeV. Clearly, such a low mass for SM particles replicas is forbidden by the experiments. A possible solution to this
problem can be found by considering the SM particles confined on a D3-brane, whereas gravity lives in the full 10-dimensional space-time [3].

Dp-branes are extended objects with p spatial dimensions and appear naturally in many string theories as classical solutions of 10-dimensional gravity. In the classical limit, they can be naively viewed as static surfaces with open strings tied to, see [5]. Compactifying on a $T^2 \times T^2 \times T^2$ manifold the 10-dimensional effective action for weakly coupled Type I strings, we get

$$S_4 = - \int \frac{d^4x}{2\pi^2} \sqrt{-g} \left( \frac{R_i R_j R_k M_s^8}{\lambda^2} R + \frac{R_i R_j R_k M_s^6}{\lambda} \frac{1}{4} F_{(9)}^2 \right) + \sum_{i \neq j \neq k \neq i} \frac{R_i R_j M_s^4}{\lambda} \frac{1}{4} F_{(7_i)}^2 + \sum_{j=1}^{3} \frac{R_i M_s^2}{\lambda} \frac{1}{4} F_{(5_j)}^2 + \frac{1}{\lambda} \frac{1}{4} F_{(3)}^2 + ... \right) , \quad (2)$$

where $\lambda$ is the string coupling, $M_s$ the string scale, $R$ is the trace of the Ricci tensor (coming from the closed string sector) and $F_{(3)}, F_{(5)}, F_{(7)}$ and $F_{(9)}$ represent the gauge field strengths associated to the 3-, 5-, 7- and 9-brane sector respectively. These gauge fields can be imagined as the ending points of open strings tied to extended objects with 3, 5, 7 and 9 spatial dimensions. The compactified volume is $V = \Pi_i (2\pi R_i)^2$ and $R_i$ are the radii of the three complex tori [6].

The relation between the string scale, the 4-dimensional Planck scale and the gauge coupling is (for the 3-brane gauge group)

$$\frac{\alpha_3 M_{\text{Planck}}}{\sqrt{2}} = \left( M_s^4 R_1 R_2 R_3 \right) \quad (3)$$

and changing the size of the compactified manifold the string scale can be much lower than the Planck scale. If we assume that gauge SM particles are massless excitations (zero modes) of open strings starting and ending on, say, a 3-brane, at a first look it seems that gauge fields cannot see the extra dimensions, since these ending points cannot move freely in the dimensions transverse to the brane. This is not actually true, as will become clear by looking at the spectrum of string modes.

Consider first a closed string, whose zero-modes represent the graviton. The mass squared of a given excitation is:

$$M^2_{\text{closed}}(m_i, n_i) = M_0^2 + \sum_{i=1}^{3} \left( m_i^2 \frac{1}{R_i^2} + n_i^2 (M_s^2 R_i)^2 \right) , \quad m_i, n_i = 0, \pm 1, \pm 2, ... \quad (4)$$

where $M_0$ stands for $R_i$-independent contributions to the mass proportional to the string scale and $(m_i, n_i)$ are the Kaluza-Klein and winding numbers. These two kinds of excitations are related to the compactification from 10- to 4-dimensional space-time: KK modes represent the quantization of momenta in the compactified dimensions of the string; winding modes represent
the wrapping around the compact dimension of the string (and \( M_{\omega_i} = M_2^2 R_i \)). All the possible combinations of these two quantum numbers are possible for a closed string, since a closed string wrapped around the torus is topologically different from an unwrapped one.

For an open string the mass squared of a given excitation is:

\[
M_{\text{open}}^2(m_i) = M_0^2 + \sum_{i=1}^{3} \left( m_i^2 \frac{1}{R_i^2} \right). \tag{5}
\]

Only KK modes are possible for an open string, since an open string wrapped around the compact dimension is topologically equivalent to an unwrapped string.

Finally, for a D3-brane, we have

\[
M_{3\text{-brane}}^2(n_i) = M_0^2 + \sum_{i=1}^{3} \left( n_i^2 (M_2^2 R_i)^2 \right) \tag{6}
\]

and no KK modes are possible, since the endings of the open string are tied to the non-compact (3+1)-dimensional space-time and they cannot move into the compact dimensions. Notice, however, that such an open string wrapped around the compact dimension is topologically different from an unwrapped one, thus reintroducing winding modes in the spectrum.

If we consider a very large compact radius the KK modes are light whereas the winding modes are heavy and decouple from the low-energy spectrum. Moreover, if the string scale is still high enough, also terms proportional to \( M_0^2 \) decouple. In this framework, we can consider that SM particles have no KK replicas and only the graviton does. Winding modes of SM particles and of the graviton are heavy and are not relevant for low-energy physics.

**Winding Modes Phenomenology**

It has been mentioned that planned gravitational experiments could test the Newton law down to \( \sim 10 \, \mu \text{m} \), thus putting an upper bound on the size of the compactified manifold and therefore on the string scale. It is also possible that new high-energy experiments at the TeV-scale such as LHC or NLC could directly observe the production of KK modes of the graviton \[7\]. Although graviton emission in 4-dimensional quantum gravity is suppressed by \( 1/M_{\text{Planck}}^2 \), the integration over all the KK modes of the graviton trades this suppression factor with a much smaller one,

\[
\frac{1}{M_{\text{Planck}}^2} \rightarrow \frac{E^n}{M_{(4+n)}^{2+n}}, \tag{7}
\]

where \( n \) is the number of dimensions with large radius, \( M_{(4+n)} \) is the fundamental Planck scale and \( E \) is the c.m. energy. If the \((4+n)\)-dimensional Planck mass is at the TeV-scale, graviton production becomes sizeable at the planned accelerators. The typical signature will be the production of a SM particle and missing energy \[7\].
If the compactification radius is $R \sim 1$ mm and $M_{(4+n)} \sim 1$ TeV, this scenario can be tested at the planned gravitational and high-energy experiments. Slightly changing these scales dramatically changes its experimental appeal for the near future. In particular, the number of extra dimensions felt by gravity at the TeV-scale and the precise values of $M_{(4+n)}$ and of the compactification radii, $R_i$, are fundamental when making quantitative predictions on the decay rate into gravitons or on a specific cross-section.

In the isotropic case $n = 6$ ($R_1 = R_2 = R_3$) we face two possibilities:

- We observe deviations from the Newton law at the planned gravitational experiments, $R \sim 1$ mm. In this case, we immediately get $M_s \sim 10^{-5}$ GeV (for $\alpha_3 = 1/24$). Clearly, this situation is excluded by experiments, since the string scale is too low.

- $M_s \sim 1$ TeV. In this case we get $M_c \sim 10^{-2}$ GeV (i.e. $R \sim 10^{-11}$ mm), and thus no extra dimensions could be observed at the planned gravity experiments.

In the literature, it has been shown that the case with $n = 2$ gives the favoured signatures at the planned experiments. For $n = 2$ large dimensions ($R_1 > R_2 \sim R_3$), it is possible to have simultaneously $M_s \sim 1$ TeV and $R_1 \sim 0.1$ mm, thus observing deviations from the Newton law and graviton emission into the bulk at the same time.

This simple analysis shows that, from the experimental point of view, the most interesting scenario is represented by an anisotropically compactified manifold, with some large dimensions and some small ones. Although the isotropic $n = 6$ case is not excluded, its lower phenomenological interest with respect to the $n = 2$ case is manifest.

If we consider a large anisotropy, $R_1 >> R_2 \sim R_3$, the spectrum of the model changes with respect to the case of three equally large radii considered above. From eq. (6) we can see that the winding modes of SM particles along the large compact dimension actually decouple, whereas the winding modes along the small compact dimensions do not. This was first noticed in [8], pointing out that in this case the light winding modes can play a phenomenological role. Using eq. (6) with $R_1 = 1$ mm and a gauge coupling $\alpha_3 = 0.1$, we get for the light winding modes mass $M_\omega \sim 1$ TeV (the precise value depending on the gauge coupling $\alpha_3$ and hence on all the details of the specific model). Combining eq. (1) and eq. (3) we get ($R_2 = R_3 = R_c$):

$$M_2^2 = \frac{1}{\sqrt{4\pi\alpha_3}} M_{\omega c}^2 (R_1 > R_c)$$

$$M_3^3 = \frac{1}{\sqrt{4\pi\alpha_3}} M_s^2 M_{\omega_1} (R_1 < R_c)$$

$$M_4^4 = \frac{1}{\sqrt{4\pi\alpha_3}} M_s^4 (R_1 = R_c)$$

respectively for $n = 2, 4, 6$ Large Extra Dimensions. Therefore, in the case $n = 2$ for which the most promising experimental signatures are foreseen, the relevant scale in the game is the light
Figure 1: Bounds on $M_{(6)}$ as a function of $M_{\omega c}$ for different values of $\alpha_3 = \alpha_{em}, \alpha_{GUT}, \alpha_s, 1$. The horizontal lines are the excluded region from present and planned gravitational experiments testing the Newton law down to $R \sim 1$ cm, 1 mm, 10 $\mu$m. The vertical shaded area is the excluded region from non-observation of massive replicas of SM bosons.

The winding mode mass $M_{\omega c}$. This scale is directly related to the 6-dimensional Planck mass, for which bounds can be extracted from the experiments (see [7]). Notice that in this particular case the string scale is completely irrelevant from the phenomenological point of view and could take any value.

Existing experimental limits on massive replicas of SM gauge bosons, such as $Z'_{SM}$ or $W'_{SM}$, can be directly translated into $M_{\omega c} \geq 700$ GeV [4]. This bound can be used to put limits on $R_1, M_{(6)}$ and $M_s$. For $n = 2$, with $\alpha_3 = \alpha_{GUT} = 1/24$, we obtain the limit $M_{(6)} \geq 1.8$ TeV. Contextually, using eq. (1) we get $R \leq 0.15$ mm. Non-observation of massive replicas of SM particles at the planned accelerators would imply even stronger bounds on $M_{(6)}$. For example, non-observation at NLC500 (i.e. $M_{Z'_{SM}} \geq 5$ TeV), translates into $M_{(6)} \geq 13$ TeV and $R \leq 3 \times 10^{-3}$ mm. The dependence of the previous limit on $\alpha_3$ is shown in Fig. 1 and Fig. 2.

Search for direct production of winding modes at LHC or NLC represents a useful tool to explore the parameter space of Large Extra Dimension models derived from Type I string theory.

We thank A. de Rujula, M. B. Gavela, L. Ibáñez and C. Muñoz for useful discussions.

References
Figure 2: Bounds on $R_1$ as a function of $M_{\omega_c}$ for different values of $\alpha_3 = \alpha_{em}, \alpha_{GUT}, \alpha_s, 1$. The upper shaded area is the excluded region from present gravitational experiments. The vertical shaded area is the excluded region from non observation of massive replicas of SM bosons.

[1] J. C. Price et al., NSF proposal, 1996; A. Kapitulnik and T. Kenny, NSF proposal, 1997; J. C. Long et al., Nucl. Phys. B 539 (1999) 23.

[2] N. Arkani-Hamed et al., Phys. Lett. B 429 (1998) 263; Phys. Rev. D59 (1999) 086004.

[3] I. Antoniadis et al., Phys. Lett. B 436 (1998) 257; G. Shiu and S. -H. H. Tye, Phys. Rev. D58 (1998) 106007.

[4] K. Dienes, Phys. Rept. 287 (1997) 447; J. Polchinski and E. Witten, Nucl. Phys. B460 (1996) 525.

[5] For a review see J. Polchinski, hep-th/9611050 and C.P. Bachas hep-th/9806199.

[6] L. Ibáñez et al., hep-th/9812397.

[7] G. Giudice et al., Nucl. Phys. B544 (1999) 3; E. A. Mirabelli et al., Phys. Rev. Lett. 82 (1999) 2236; T. Han et al., Phys. Rev. D 59 (1999) 105006.

[8] A. Donini and S. Rigolin, hep-ph/9901443.

[9] S. Abachi et al., D0 Collaboration, Phys. Rev. Lett. 76 (1996) 3271; F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 79 (1997) 2192. For NLC500 see, for example, A. Leike and S. Riemann, Z. Phys. C75 (1997) 341.